A superspace module for the FeynRules package

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Abstract

We describe an additional module for the MATHEMATICA package FEYNRULES that allows for an easy building of any $N = 1$ supersymmetric quantum field theory, directly in superspace. After the superfield content of a specific model has been implemented, the user can study the properties of the model, such as the supersymmetric transformation laws of the associated Lagrangian, directly in MATHEMATICA. While the model dependent parts of the latter, \textit{i.e.}, the soft supersymmetry-breaking Lagrangian and the superpotential, have to be provided by the user, the model independent pieces, such as the gauge interaction terms, are derived automatically. Using the strengths of the FEYNRULES program, it is then possible to derive all the Feynman rules associated to the model and implement them in all the Feynman diagram calculators interfaced to FEYNRULES in a straightforward way.

\textbf{Keywords:} Supersymmetry, model building, superspace calculations.
PROGRAM SUMMARY

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Programming language: MATHEMATICA.
Computer: Platforms on which Mathematica is available.
Operating system: Operating systems on which Mathematica is available.
Keywords: Supersymmetry, model building, superspace calculations.
Classification: 11.1 General, High Energy Physics and Computing.
11.6 Phenomenological and Empirical Models and Theories.
External routines/libraries: FeynRules.
Nature of problem: Study of the properties of $N = 1$ supersymmetric field theories using the superfield formalism, derivation of the associated Lagrangians.
Solution method: We use the FeynRules package and define internally the $N = 1$ superspace. Then, we implement a module allowing to:

1. Perform the Grassmann variable series expansion so that any superfield expression can be developed in terms of the component fields. The resulting expression is thus suitable to be treated by the FeynRules package directly.

2. Execute a set of operations associated to the superspace, such as the superderivatives of an expression or the calculation of its supersymmetric transformation laws.

Restrictions: Superfields related to spin 3/2 and 2 particles are not implemented.
Unusual features: All calculations in the internal routines are performed completely. The only hardcoded core is the Grassmann variable algebra.
Running time: It depends on the user’s purposes. The extraction of a Lagrangian in terms of the component fields may take a few minutes for a complete model with complex mixing between the fields.
1. Introduction

Supersymmetric (SUSY) theories are among the most popular extensions of the Standard Model (SM) of particle physics and widely studied both on the theoretical and experimental sides. Apart from relating bosons with fermions and unifying internal and space-time symmetries, they address a set of conceptual problems of the SM, such as the large hierarchy between the electroweak and the Planck scale, gauge coupling unification at high energy or the dark matter in the universe. However, as the supersymmetric partners of the SM particles have not yet been observed, supersymmetry must be broken at low energies, and in order to remain a viable solution to the hierarchy problem, this breaking must be soft, predicting massive superpartners around the TeV scale. Therefore, the quest for SUSY particles is one of the main goals of the present high-energy collider experiments such as the Tevatron at Fermilab or the Large Hadron Collider at CERN.

Studying the hadron-collider phenomenology of models that go beyond the Standard Model requires the use of Monte Carlo event generators, based, on the one hand, on a proper modeling of the strong interactions to describe correctly the parton showering, fragmentation and hadronization, and, on the other hand, on the calculation of matrix-elements underlying the hard-scattering process. This last step requires the implementation of the complete set of Feynman rules associated to a given model, often one vertex at the time. This task, tedious if undertaken manually, has been rendered much easier with the use of packages such as LanHEP \cite{1,2} or FeynRules \cite{3,4,5}, starting from the Lagrangian of the theory and exporting the corresponding Feynman rules to one or several Monte Carlo generators. Specific tools dedicated to SUSY theories, such as the SARAH package \cite{6,7}, also focusing on the same issue, in addition to the generation of the model mass spectrum at the one-loop level. However, all of these tools only address the generation of the Lagrangian (partially automatically or not) in the usual spacetime, and are not suitable for computations in superspace.

Supersymmetric model building and the underlying calculations are often technically long and painstaking. The latter can be rendered easier if working within the so-called superspace formalism \cite{8,9}, a natural framework for SUSY model building. In this paper, we present an extension to the FeynRules package, based on Mathematica\textsuperscript{1}, which provides an environment

\footnote{MATHEMATICA is a registered trademark of Wolfram Research Inc.}
to build supersymmetric theories directly in superspace. The core of our program is a set of basic functions allowing to expand superfields, \textit{i.e.}, functions defined on the superspace, in term of their component fields, the usual scalar, fermionic and vector fields of particle physics, in an automated way. As a consequence, the user has only to worry about superfield expressions, in general much simpler than their component fields counterparts. In addition, the module contains a set of predefined functions for the supercharge and the superderivative operators that allow the user to study the properties of the SUSY theory under consideration.

Lagrangians for phenomenologically relevant supersymmetric theories can very often be expressed as a sum of four terms. The first two describe the kinetic terms for the chiral and vector supermultiplets and are independent of the specific model under consideration, since they are fixed entirely by supersymmetry and gauge invariance. For this reason, we have included into our code a set of functions to generate these pieces of the Lagrangian, for any model, in an automated way. Hence, the implementation of a SUSY theory in the \textsc{FeynRules} package consists only in the setting of the superfield content, the model parameters and the gauge symmetries of the theory as an input, together with the model dependent parts of the Lagrangian, \textit{i.e.}, the SUSY-breaking Lagrangian and the superpotential. This opens thus the way to a serious phenomenological study of entire classes of models whose implementation into Monte Carlo event generators was considered too complicated or too tedious so far. The \textsc{FeynRules} package including our superspace extension can be found, together with an up-to-date manual, at \url{http://feynrules.phys.ucl.ac.be}.

The outline of the paper is as follows: In Section 2 we briefly review the main functionalities of the \textsc{FeynRules} package and the structure of the \textsc{FeynRules} model files. In Section 3 we introduce our implementation of the Grassmann and supersymmetry algebras into \textsc{Mathematica}, before turning to the more specific case of the implementation of chiral and vector superfields in Section 4. The main part of the package, the simplification of superspace expressions and the generation and expansion of supersymmetric Lagrangians with the help of the superspace module, is discussed in Sections 5, 6 and 7. Finally, in Section 8 we illustrate the use of the package on the example of the implementation of the Minimal Supersymmetric Standard Model (MSSM) into \textsc{FeynRules} in terms of superfields.
2. The FeynRules package

FeynRules is a Mathematica package that allows to derive Feynman rules directly from a Lagrangian [3]. The information that the user needs to provide consists in the particle content and the parameters of the model, together with the Lagrangian that describes the interactions among the different particles. The Feynman rules can then be obtained automatically and the interaction vertices can be exported to various matrix-element generators by means of a set of translation interfaces included in the package. Presently, interfaces to CalcHep/CompHep [10, 11, 12], FeynArts/FormCalc [13, 14], MadGraph/MadEvent [15, 16, 17, 18], Sherpa [19] and Whizard/Omega [20, 21] are available. In the following we briefly describe the basic features of the package and the model files, and we give a very brief introduction on how to run the code in order to derive the interaction vertices. For more details on both the FeynRules package as well as the interfaces, we refer the reader to Refs. [3, 4, 22, 23].

The FeynRules model definition is an extension of the FeynArts model file format and consists in the definitions of the particles, parameters and gauge groups that characterize the model and the Lagrangian. Following the original FeynArts convention, particles are grouped into classes describing “multiplets” having the same quantum numbers, but possibly different masses. Each particle class is defined in terms of a set of class properties, given as a Mathematica replacement list. For example, a Weyl fermion $\chi$ could be written as

$$W[1] = \{\text{ClassName} \to \text{chi}, \text{SelfConjugate} \to \text{False}, \text{Chirality} \to \text{Left}, \text{Indices} \to \{\text{Index[Colour]}\}\}.$$ 

This defines a left-handed Weyl fermion (ψ) represented by the symbol chi. Note that the antiparticle is automatically declared and represented by the symbol chibar. The field carries an additional index labelled Colour, representing its gauge charge under the QCD gauge group. Additional information, like the mass and width of the particles, as well as the $U(1)$ quantum numbers carried by the fields can also be included. A complete description of the particle classes and properties can be found in the FeynRules manual.

A Lagrangian is not only defined by its particle content, but also by the local and global symmetries defining the model. FeynRules allows to define
gauge group classes in a way similar to the particle classes. As an example, the definition of the QCD gauge group can be written

\[
SU3C == \{ \text{Abelian} \rightarrow \text{False}, \\
\text{GaugeBoson} \rightarrow G, \\
\text{CouplingConstant} \rightarrow gs, \\
\text{StructureConstant} \rightarrow f, \\
\text{Representations} \rightarrow \{T, \text{Colour}\} \}
\]

where the gluon field \( G \) is defined together with the other fields during the particle declaration. The declaration of abelian gauge groups is analogous. \textsc{FeynRules} uses this information to construct the covariant derivative and field strength tensor which the user can use in the Lagrangian.

The third main ingredient to define a model is the set of parameters which it depends upon. The declaration of the parameters follows the same lines as the declarations of the particle and gauge group classes. However, since the declaration of the parameters is not needed in order to understand how the superfield module works, we do not review it here but refer the reader to the \textsc{FeynRules} manual [3].

After having loaded the \textsc{FeynRules} package into \textsc{Mathematica}, the user can load the model and the model restrictions via the commands

\[
\text{LoadModel[ file1, file2, ... ]},
\]

where the model can be implemented in as many files as convenient or it can be implemented directly in the \textsc{Mathematica} notebook in which case the list of files would be empty. The Lagrangian can now be entered directly into the notebook\footnote{Alternatively, the Lagrangian can also be included in the model file, in which case it is directly loaded together with the model file.} using standard \textsc{Mathematica} commands, augmented by some special symbols representing specific objects like Dirac matrices. As an example, we show the Lagrangian,

\[
L = -1/4 \text{FS}[G, \mu, \nu, a] \text{FS}[G, \mu, \nu, a] + I \text{chibar} \cdot \text{sibar}[\mu] \cdot \text{DC}[\chi, \mu]
\]

where \( \text{FS}[G, \mu, \nu, a] \) and \( \text{DC}[\chi, \mu] \) denote the \( SU(3)_C \) field strength tensors and covariant derivatives automatically defined by \textsc{FeynRules}, respectively. At this stage, the user can perform a set of basic checks on the
Lagrangian (hermiticity, normalization of kinetic terms, ...), or directly proceed to the derivation of the Feynman rules via the command

\[ \text{verts} = \text{FeynmanRules}[L] . \]

FEYNRULES then computes all the interaction vertices associated with the Lagrangian \( L \) and stores them in the variable \( \text{verts} \). The vertices can be used for further computations within MATHEMATICA, or they can be exported to one of the various matrix-element generators for further phenomenological studies of the model. The translation interfaces can be directly called from within the notebook, e.g., for the FEYNArts interface,

\[ \text{WriteFeynArtsOutput}[L] , \]

This will produce a file formatted for use in FEYNArts. All other interfaces are called in a similar way.

The \text{FeynmanRules} function, as well as the interfaces that call it, require the Lagrangian to be written in four-dimensional space time. For supersymmetric theories, however, the most natural and most convenient way to write a Lagrangian is in terms of superfields, \( i.e., \) in terms of the explicit supermultiplets of the supersymmetry algebra. For this reason, we have extended the FEYNRULES package by a superspace module, which allows to write a Lagrangian in terms of superfields and to transform the latter into a four-dimensional Lagrangian involving component fields only, hence expressing the superfield Lagrangian in a way that can be processed immediately by the \text{FeynmanRules} function. This new module will be described in detail in the next sections.

3. Superspace and supersymmetry algebra

3.1. Grassmann variables and fermionic fields

Supersymmetric theories are naturally formulated in superspace \([8, 9]\), an extension of the ordinary spacetime, defined, for \( N = 1 \) supersymmetric theories by adjoining a Majorana spinor \( (\theta_\alpha, \bar{\theta}^\dot{\alpha}) \) to the usual spacetime coordinates \( x^\mu \), where \( \theta \) and \( \bar{\theta} \) are (Grassmannian) two-component Weyl fermions. Using the recent implementation of Weyl fermions into FEYNRULES \([22]\), the \( \theta \) variables are hardcoded in the superspace module following the example of Section \([2]\) as
This field and its right-handed counterpart \( \bar{\theta} \) can be accessed, after loading FeynRules in Mathematica, through the symbols \( \text{theta} \) and \( \text{thetabar} \). By convention, the spin indices are assumed to be lowered for both the \( \theta \) and \( \bar{\theta} \) variables,

\[
\text{theta}[\text{alpha}] \leftrightarrow \theta_\alpha \quad \text{and} \quad \text{thetabar}[\text{alphadot}] \leftrightarrow \bar{\theta}^{\dot{\alpha}},
\]

where we relate undotted and dotted indices to left and right-handed fermions. Spin indices can be raised and lowered using the rank-two antisymmetric tensors \( \varepsilon_{\alpha\beta} \) and \( \varepsilon^{\dot{\alpha}\dot{\beta}} \)

\[
\theta^\alpha = \varepsilon_{\alpha\beta} \theta_\beta, \quad \bar{\theta}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\theta}_{\dot{\beta}} \quad \text{and} \quad \bar{\theta}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\theta}_{\dot{\beta}}, \quad (1)
\]

with \( \varepsilon_{12} = 1, \varepsilon^{12} = -1, \varepsilon_{i\bar{j}} = 1 \) and \( \varepsilon^{i\bar{j}} = -1 \). This strictly follows the conventions of Ref. [24], and so does the complete superspace module of FeynRules. Regardless of the left or right-handed nature of the spin indices, the tensors with lower and upper indices are implemented into FeynRules as \( \text{Deps} \) and \( \text{Ueps} \), respectively. An example is in order,

\[
\text{Ueps}[\text{alphadot},\text{betadot}] \leftrightarrow \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\theta}^{\dot{\beta}},
\]

\[
\text{Deps}[\text{alpha},\text{beta}] \text{ Ueps}[\text{beta},\text{gam}] \text{ theta}[\text{gam}] \leftrightarrow \varepsilon_{\alpha\beta} \varepsilon^{\gamma\gamma} \theta^\gamma = \theta_\alpha.
\]

The Levi-Civita tensors allow us to fix our conventions with respect to the summation on the spin indices, and we define the dot product of two Weyl spinors as

\[
\lambda \cdot \lambda' = \lambda^\alpha \lambda'^\alpha = \varepsilon^{\alpha\beta} \lambda_\beta \lambda'^\alpha, \quad \text{and} \quad \bar{\chi} \cdot \bar{\chi}' = \bar{\chi}_{\dot{\alpha}} \bar{\chi}'^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\chi}_{\dot{\alpha}} \bar{\chi}'^{\dot{\beta}}, \quad (2)
\]

where we have introduced some generic left and right-handed spinors \( \lambda^{(l)} \) and \( \bar{\chi}^{(r)} \), respectively.

Computations in superspace require to keep track not only of the position of the different spin indices, but also of the ordering of the fermions. The
FeynRules superspace module addresses this issue in the following way. First, any explicit spin index will be considered, by default, as lowered. Second, we have implemented the environment nc[chain], where chain stands for an ordered sequence of fermions with lower spin indices. As simple examples, scalar products such as in Eq. (2) are implemented as

\[
\lambda \cdot \lambda' = \varepsilon^{\beta\alpha} \lambda_\alpha \lambda'_\beta \leftrightarrow \text{nc}[\lambda[\text{a}], \lambda'[\text{b}]] \ U_{\text{eps}[\text{b}, \text{a}]},
\]

\[
\bar{\chi} \cdot \bar{\chi}' = \varepsilon^{\dot{\beta}\dot{\alpha}} \bar{\chi}_{\dot{\beta}} \bar{\chi}'_{\dot{\alpha}} \leftrightarrow \text{nc}[\chi[\text{bd}], \chi'[\text{ad}]] \ U_{\text{eps}[\text{bd}, \text{ad}]},
\]

(3)

### 3.2. Supercharges and superderivatives

In \( N = 1 \) supersymmetric theories, there is a single spinorial SUSY generator, transforming as a Majorana spinor \((Q_\alpha, \bar{Q}^{\dot{\alpha}})\) under Lorentz transformations. Any translation in superspace can then be parameterized as

\[
G(x, \theta, \bar{\theta}) = e^{i \left[ x^\mu P_\mu + \theta^\alpha \sigma_\mu^\alpha + \bar{\theta}^{\dot{\alpha}} \sigma_\mu^{\dot{\alpha}} \right]},
\]

(4)

where \( P_\mu \) denotes the generator of space-time translations. A generic supersymmetric transformation then corresponds to \( G(0, \epsilon, \bar{\epsilon}) \), where \((\epsilon^\alpha, \bar{\epsilon}^{\dot{\alpha}})\) denotes a (spinorial) transformation parameter. Applying this transformation to a generic superspace point, \( G(0, \epsilon, \bar{\epsilon})G(x, \theta, \bar{\theta}) \), we can calculate the variations of the spacetime coordinates \( x \) and of the Grassmann variables \( \theta \) and \( \bar{\theta} \) using the Baker-Campbell-Hausdorff formula and the (anti)commutation relations among the generators. Comparing with a direct application of the supersymmetric generator \( i(\epsilon \cdot Q + \bar{Q} \cdot \bar{\epsilon}) \) on the coordinates \( x, \theta \) and \( \bar{\theta} \), we obtain the action of the generators as differential operators acting on functions on superspace. Similarly, starting from an action from the right, \( G(x, \theta, \bar{\theta})G(0, \epsilon, \bar{\epsilon}) \), we obtain the expressions of the superderivatives \( D_\alpha \) and \( \bar{D}_{\dot{\alpha}} \). In our conventions \([24]\), these four quantities are given by

\[
\begin{align*}
Q_\alpha &= -i \left( \frac{\partial}{\partial \theta^\alpha} + i \sigma_\mu^{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu \right), & \bar{Q}^{\dot{\alpha}} &= i \left( \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i \theta^\alpha \sigma_\mu^{\alpha\dot{\alpha}} \partial_\mu \right), \\
D_\alpha &= \frac{\partial}{\partial \theta^\alpha} - i \sigma_\mu^{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu, & \bar{D}_{\dot{\alpha}} &= -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i \theta^\alpha \sigma_\mu^{\alpha\dot{\alpha}} \partial_\mu.
\end{align*}
\]

(5)

The matrices \( \sigma^\mu \), and for later reference \( \bar{\sigma}^\mu \), are the four-vectors corresponding to the usual Pauli matrices. Note that, since the actions from the left and from the right commute, the supercharges and the superderivatives must anticommute.
The generators $Q_\alpha$ and $\bar{Q}_{\dot{\alpha}}$ of the supersymmetric transformations, as well as the superderivatives $D_\alpha$ and $\bar{D}_{\dot{\alpha}}$, are called in *FeynRules* via the functions,

\begin{align*}
QSUSY[expression, \alpha] &\leftrightarrow Q_\alpha(\text{expression}) , \\
QSUSYBar[expression, \alphadot] &\leftrightarrow \bar{Q}_{\dot{\alpha}}(\text{expression}) , \\
DSUSY[expression, \alpha] &\leftrightarrow D_\alpha(\text{expression}) , \\
DSUSYBar[expression, \alphadot] &\leftrightarrow \bar{D}_{\dot{\alpha}}(\text{expression}) .
\end{align*}

It is assumed that all the indices appearing in *expression* must be written explicitly, using the environment *nc* presented above in order to keep track of the correct fermion ordering. The only exception corresponds to the case of a single fermion, the ordering of the fermions being trivially irrelevant, and the *nc* environment can be omitted. Hence, both

\begin{align*}
\text{DSUSY}[\chi[\alpha], \beta] \quad \text{and} \quad \text{DSUSY}[\text{nc}[\chi[\alpha]], \beta]
\end{align*}

give a correct answer, whilst for two fermions, the correct syntax is

\begin{align*}
\text{DSUSY}[\text{nc}[\chi[\alpha], \lambda[\beta], \gamma], \beta]
\end{align*}

and not

\begin{align*}
\text{DSUSY}[\chi[\alpha], \lambda[\beta], \gamma] .
\end{align*}

4. Implementing superfields into *FeynRules*

4.1. Generic superfields

Any function $\Phi(x, \theta, \bar{\theta})$ defined on \( N = 1 \) superspace is called a superfield and can be expanded into a Taylor series with respect to the anticommuting coordinates $\theta$ and $\bar{\theta}$. Since the square of an anticommuting object vanishes, the series has only a finite number of terms and the most general expression for a scalar superfield is\(^3\)

\begin{align*}
\Phi(x, \theta, \bar{\theta}) &= z(x) + \theta \cdot \xi(x) + \bar{\theta} \cdot \bar{\xi}(x) + \theta \cdot \theta f(x) + \bar{\theta} \cdot \bar{\theta} g(x) \\
&\quad + \theta \sigma^\mu \bar{\theta} V_\mu(x) + \bar{\theta} \cdot \theta \cdot \omega(x) + \theta \cdot \theta \bar{\theta} \cdot \bar{\theta} d(x) , \quad (6)
\end{align*}

\[^3\]The extension to non-scalar superfields is immediate.
| Function     | Description                                                                 |
|--------------|-----------------------------------------------------------------------------|
| `theta[al]`  | The Grassmann variable $\theta_\alpha$, where $al$ denotes the carried spin index. |
| `thetabar[aldot]` | The Grassmann variable $\bar{\theta}_\dot{\alpha}$, where $aldot$ denotes the carried spin index. |
| `Ueps[i,j]` | The rank-two antisymmetric tensor $\epsilon^{ij}$. |
| `Deps[i,j]` | The rank-two antisymmetric tensor $\epsilon_{ij}$. |
| `nc[seq]` | Ordered sequence of fermionic field(s), labelled by `seq`, where each field carries explicitly its indices. |
| `QSUSY[exp,al]` | Calculates the action of the supercharge $Q_\alpha$ on the expression `exp`, where all indices must be written explicitly, using the `nc` environment if relevant. The symbol $al$ is related to the spin index of the supercharge. |
| `QSUSYBar[exp,ad]` | Calculates the action of the supercharge $\bar{Q}_{\dot{\alpha}}$ on the expression `exp`, where all indices must be written explicitly, using the `nc` environment if relevant. The symbol $ad$ is related to the spin index of the supercharge. |
| `DSUSY[exp,al]` | Same as `QSUSY`, but for the superderivative $D_\alpha$. |
| `DSUSYBar[exp,ad]` | Same as `QSUSYBar`, but for the superderivative $\bar{D}_{\dot{\alpha}}$. |
where the various coefficients of the expansion form a so-called supermultiplet and are referred to as the component fields of the superfield. They correspond to the usual scalar, fermionic and vector degrees of freedom used in particle physics. \( z, f, g \) and \( d \) denote complex scalar fields, whilst \( \xi, \zeta, \omega \) and \( \rho \) denote complex Weyl fermions and \( V_\mu \) is a complex vector field, leaving a total of 16 bosonic and 16 fermionic degrees of freedom.

Using the nc environment, the generic scalar superfield of Eq. (6) can be implemented into \textsc{FeynRules} in a straightforward way as

\[
\Phi = 
\begin{align*}
&z + \leftrightarrow z \\
&\text{nc[theta[sp], xi[sp2]] Ueps[sp2,sp] +} \leftrightarrow \theta \cdot \xi \\
&\text{nc[thetabar[spd], zetabar[spd2]]} \\
&\quad \text{Ueps[spd,spd2] +} \leftrightarrow \bar{\theta} \cdot \bar{\xi} \\
&\text{nc[theta[sp], theta[sp2]] Ueps[sp2,sp] f +} \leftrightarrow \theta \cdot \theta f \\
&\text{nc[thetabar[spd], thetabar[spd2]]} \\
&\quad \text{Ueps[spd,spd2] g +} \leftrightarrow \bar{\theta} \cdot \bar{\theta} g \\
&\text{nc[theta[sp], thetabar[spd]] Ueps[sp2,sp]} \\
&\quad \text{Ueps[spd2,spd] si[mu,sp2,spd2] V[mu] +} \leftrightarrow \theta \sigma^\mu \bar{\theta} V_\mu \\
&\text{nc[thetabar[spd], thetabar[spd2]]} \\
&\quad \text{Ueps[spd,spd2] Ueps[sp2,sp]} \\
&\quad \text{nc[theta[sp], omega[sp2]] +} \leftrightarrow \theta \cdot \theta \cdot \omega \\
&\text{nc[theta[sp], theta[sp2]] Ueps[sp2,sp]} \\
&\quad \text{nc[thetabar[spd], rhobar[spd2]]} \\
&\quad \text{Ueps[spd,spd2] +} \leftrightarrow \theta \cdot \theta \cdot \bar{\theta} \cdot \bar{\rho} \\
&\text{nc[theta[sp], theta[sp2]] Ueps[sp2,sp]} \\
&\quad \text{nc[thetabar[spd], thetabar[spd2]]} \\
&\quad \text{Ueps[spd,spd2] d} \leftrightarrow \theta \cdot \theta \cdot \bar{\theta} \cdot \bar{\theta} d ,
\end{align*}
\]

where we assume that the component fields have been defined properly in the \textsc{FeynRules} model file (See Section 2). In Eq. (7) we have made use of the object \text{si[mu, alpha, alphadot]}, corresponding to the Pauli matrices \( \sigma^\mu_{a\dot{a}} \). Note that the appearance of the Levi-Civita tensor \text{Ueps} is due to our convention that all spin indices carried by fermionic fields are lowered. Similarly, we could have used the matrices \text{sibar[mu, alphadot, alpha]} defined with two upper spin indices, \( \bar{\sigma}^{\mu a\dot{a}}, \) since

\[
\theta_\beta \epsilon^{a\beta} \sigma^\mu_{a\dot{a}} \epsilon^{\dot{a}\dot{\beta}} \bar{\theta}_\beta = -\bar{\theta}_\alpha \bar{\sigma}^{\mu a\dot{a}} \theta_\alpha .
\]
The second option would have allowed to get an expression free from any $\epsilon$-tensor.

Even though Eq. (7) is the canonical form in which superfields are represented inside the code, the form of Eq. (7) can however be very painful to use in practice. For this reason, the package contains a function denoted \texttt{ncc}, which has the same effect as \texttt{nc}, but all spin indices and $\epsilon$-tensors are suppressed. The code then assumes that, within a given \texttt{ncc} environment, all the suppressed indices are contracted according to the convention of Eq. (3), and outputs the result into the canonical form. As an example, using the \texttt{ncc} environment, Eq. (7) can be written in the more compact form,

$$\Phi = z + \leftrightarrow z$$
$$\text{ncc}[\text{theta, xi}] + \leftrightarrow \theta \cdot \xi$$
$$\text{ncc}[\text{thetabar, zetabar}] + \leftrightarrow \bar{\theta} \cdot \bar{\xi}$$
$$\text{ncc}[\text{theta, theta}] f + \leftrightarrow \theta \cdot \theta f$$
$$\text{ncc}[\text{thetabar, thetabar}] g + \leftrightarrow \bar{\theta} \cdot \bar{\theta} g$$
$$\text{ncc}[\text{theta, si[mu], thetabar}] V[\mu] + \leftrightarrow \theta_{\sigma} \mu \bar{\theta} V_{\mu}$$
$$\text{ncc}[\text{thetabar, thetabar}] \text{ncc}[\text{theta, omega}] + \leftrightarrow \bar{\theta} \cdot \bar{\theta} \cdot \theta \cdot \omega$$
$$\text{ncc}[\text{theta, theta}] \text{ncc}[\text{thetabar, rhobar}] + \leftrightarrow \theta \cdot \theta \cdot \bar{\theta} \cdot \bar{\rho}$$
$$\text{ncc}[\text{theta, theta}] \text{ncc}[\text{thetabar, thetabar}] d \leftrightarrow \theta \cdot \theta \cdot \bar{\theta} \cdot \bar{\theta} \cdot d \cdot d$$

The number of degrees of freedom of a generic superfield is too large to match those of the supermultiplets representing the $\mathcal{N} = 1$ supersymmetric algebra. Two special cases, with fewer degrees of freedom, are in general enough to build most of the phenomenologically relevant supersymmetric theories. These so-called chiral and vector superfields will be discussed in the next sections.

4.2. Chiral superfields

Left and right-handed chiral superfields are superfields $\Phi_L$ and $\Phi_R$ that satisfy the constraints

$$\bar{D}_{\alpha} \Phi_L(x, \theta, \bar{\theta}) = 0 \quad \text{and} \quad D_{\alpha} \Phi_R(x, \theta, \bar{\theta}) = 0 ,$$

where the superderivatives have been introduced in Eq. (5). Since the superderivatives anticommute with the supercharges, the constraints are preserved by supersymmetry transformations. The most general solutions to
the constraints can be written as

\[
\Phi_L(y, \theta) = \phi(y) + \sqrt{2} \theta \cdot \psi(y) - \theta \cdot \theta F(y), \\
\Phi_R(y^\dagger, \bar{\theta}) = \phi(y^\dagger) + \sqrt{2} \bar{\theta} \cdot \bar{\psi}(y^\dagger) - \bar{\theta} \cdot \bar{\theta} F(y^\dagger),
\]

(11)

where we have introduced the variable \( y^\mu = x^\mu - i \theta \sigma^\mu \bar{\theta} \) and its hermitian conjugate \( (y^\mu)^\dagger = x^\mu + i \theta \sigma^\mu \bar{\theta} \). Chiral superfields are appropriate to describe the \( N = 1 \) matter supermultiplets containing one fermion and one scalar field. Indeed, the physical degrees of freedom of \( \Phi_L \) and \( \Phi_R \) consist of a single complex scalar field \( \phi \) and a single two-component fermion \( \psi \), the chirality of the Weyl fermion providing the name for this kind of superfield. The additional scalar field \( F \) has a mass dimension \([F] = 2\) and does hence not correspond to a physical degree of freedom, but it is necessary to restore the equality between the numbers of fermionic and bosonic degrees of freedom off-shell. Furthermore, these so-called \( F \)-terms can be used to break supersymmetry spontaneously if they develop a vacuum expectation value. As we will discuss in Section 7.2 these fields are non-propagating and can be eliminated through their equations of motion.

Left and right-handed chiral superfields can be declared in FeynRules following in the FeynRules model file in a way similar to the ordinary fields, e.g.,

```plaintext
M$Superfields = { 
  CSF[1] == { 
    ClassName -> PHI, 
    Chirality -> Left, 
    Weyl -> psi, 
    Scalar -> z, 
    Auxiliary -> FF}, 
  CSF[2] == { 
    ClassName -> OMEGA, 
    Chirality -> Right, 
    Weyl -> xibar, 
    Scalar -> zz} 
} .
```

\(^4\)The \( \sqrt{2} \) factors and the minus signs are purely conventional.
These Mathematica commands declare two chiral superfields, the label of the particle class CSF referring to chiral superfields. We consider the example of both a left and right-handed chiral superfield called PHI and OMEGA. The fermionic degrees of freedom of the superfields are psi and xibar, while the scalar degrees of freedom are denoted by z and zz. In addition to all the options available to all particle classes and briefly reviewed in Section 2 such as QuantumNumbers and Indices, the user must set the Chirality option to Left or Right, related to the chirality of the superfield. Moreover, it is mandatory to link a chiral superfield to its fermionic and scalar components through the options Weyl and Scalar, respectively. Note that the component fields have to be declared independently in the FeynRules model file (see Section 2 for a brief review or Refs. 3, 22 for a detailed description). We only emphasize here that the options for the component field classes, like Indices, QuantumNumber, etc., must be identical to the corresponding options of the superfield. The only exception to this rule are the F-terms, where the user can either point to an already declared complex scalar field via the Auxiliary options (as for CSF[1] in the example above, where the auxiliary field is denoted FF), or he leaves this option unspecified and FeynRules creates internally a symbol for the auxiliary field (as for CSF[2] in the example above). All the allowed options for the declaration of a chiral superfield are summarized in Table 2.

4.3. Vector superfields in the Wess-Zumino gauge

Besides matter fields, gauge theories contain also real vector fields describing the gauge bosons. Since the chiral superfields defined in the previous section do not contain any vector degree of freedom, they cannot be sufficient to describe supersymmetric gauge theories. We therefore introduce in this section the so-called vector (or gauge) supermultiplets, i.e., the representations of the \( N = 1 \) SUSY algebra containing one massless gauge boson together with the corresponding fermionic degree of freedom. These multiplets can be described by vector superfields, defined by the reality condition,

\[ V = V^\dagger. \]  

(12)

The constraint (12) on its own is not enough to reduce the number of degrees of freedom of the generic superfield of Eq. 6 to the required number and leads to a proliferation of unphysical fields that can be eliminated by a suitable gauge choice. A convenient choice is the so-called Wess and Zumino
### Table 2: Chiral superfield class options

| ClassName    | Defines the symbol by which a class is represented. |
|--------------|------------------------------------------------------|
| Chirality    | Defines the chirality, **Left** or **Right**, of the chiral superfield. |
| Weyl         | Contains the symbol of the fermionic component of the chiral superfield. |
| Scalar       | Contains the symbol of the scalar component of the chiral superfield. |

**Optional attributes**

| Auxiliary    | Contains the symbol of the auxiliary component of the chiral superfield. If absent, **FeynRules** generates the $F$-term automatically. |
|--------------|--------------------------------------------------------------------------------------------------------------------------------|
| Indices      | The list of indices, different from Lorentz and spin indices, carried by the superfield and all its component fields.               |
| QuantumNumbers | A replacement rule list, containing the $U(1)$ quantum numbers carried by the class.                                      |
gauge, in which a vector superfield is expressed as

\[ V_{W.Z.} = \theta \sigma^\mu \bar{\theta} v_\mu + i \theta \cdot \bar{\theta} \cdot \bar{\lambda} - i \bar{\theta} \cdot \bar{\theta} \cdot \lambda + \frac{1}{2} \theta \cdot \bar{\theta} \cdot \bar{\theta} \cdot D. \]  

(13)

The component fields of a vector superfield are hence a real vector field \( v_\mu \) and a Majorana fermion \((\lambda_\alpha, \bar{\lambda}^\dot{\alpha})\). Similar to the case of the chiral superfield, the multiplet also contains a non-propagating auxiliary scalar field \( D \) with mass dimension \([D] = 2\) that is necessary to restore the equality between the numbers of fermionic and bosonic degrees of freedom off-shell. These so-called \( D \)-terms can again be eliminated through their equations of motion.

Vector superfields in the Wess and Zumino gauge can be implemented into FeynRules in the same way as chiral superfield. Let us illustrate this by an example,

```feynrules
M$Superfields = {
    VSF[1] == {
        ClassName -> PHIV,
        GaugeBoson -> X,
        Gaugino -> lambda
    }
}.
```

In this example, \( PHIV \) denotes a vector superfield, whose vector component is denoted by \( X \) and the fermionic component by \( \lambda \). The label of the particle class \( VSF \) refers to its vector superfield nature. In addition to the usual options for particle classes, the user must associate to a vector superfield its bosonic and fermionic components by setting the options \( \text{GaugeBoson} \) and \( \text{Gaugino} \). Note that the corresponding vector field and the Weyl fermion must be declared independently in the FeynRules model file. As in the case of chiral superfields, the \( D \)-terms can either be defined explicitly via the \( \text{Auxiliary} \) option, or this option can be omitted and FeynRules will declare the \( D \)-terms internally. All the allowed options for the declaration of a vector superfield are summarized in Table 3. In addition, each vector superfield can be linked to a gauge group through the option \( \text{Superfield} \) which has been added to the gauge group class in a similar way as the we assigned the gluon field \( G \) to the QCD gauge group \( SU3C \) in Section 2. As an example, we could associate the superfield \( PHIV \) above to an abelian gauge group called \( U1X \) by declaring the gauge group as

```feynrules
U1X == {
}
```

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| Table 3: Vector superfield class options |
|------------------------------------------|
| **ClassName**                           |
| Defines the symbol by which a class is represented. |
| **GaugeBoson**                           |
| Contains the symbol of the vector field associated to the vector superfield. |
| **Gaugino**                              |
| Contains the symbol of the gaugino component of the vector superfield. |

**Optional attributes**

| **Auxiliary** |
| Contains the symbol of the auxiliary component of the superfield. If absent, \textsc{FeynRules} generates the $D$-term automatically. |

| **Indices** |
| The list of indices, different from Lorentz and spin indices, carried by the superfield and all its component fields. |

**New gauge group options**

| **Superfield** |
| This option points to the \texttt{ClassName} of the vector superfield associated to the gauge group. If the \texttt{Superfield} option is present, the option \texttt{GaugeBoson} becomes optional. If both options are there, they must be consistent. |

Abelian $\rightarrow$ True, 
CouplingConstant $\rightarrow$ $gX$, 
Superfield $\rightarrow$ PHIV}.

5. Simplification of superspace expressions

5.1. Simplification of expressions involving Grassmann numbers

In the previous sections we have discussed the implementation of superfields into \textsc{FeynRules}. The user can either use the predefined classes \texttt{CSF}}
and VSF for chiral and vector superfields, or implement superfields directly in terms of the component fields, as in the example of Eq. 7. In that case, however, the required syntax for the implementation can be complex, and as a consequence the Mathematica output could be difficult to read. Similarly, many calculations such as those involving the supersymmetric generators or the superderivatives could lead to rather long expressions after introducing the second rank antisymmetric tensors and the nc environment, which are necessary in order to have the fermions ordered correctly and all the spin indices carried by fields lowered, as is the convention in the superspace module of FeynRules. To bypass this issue, it might be useful to form dot products of spinors, and more specifically those involving the Grassmann variables $\theta$ and $\bar{\theta}$. This can be achieved with the help of the ToGrassmannBasis function, which proceeds in two steps. First, dot products are formed and simplified using relations among the Grassmann variables, such as

\[
\theta^\alpha \theta^\beta = -\frac{1}{2} \theta \cdot \theta \varepsilon^{\alpha \beta}, \quad \bar{\theta}^\dot{\alpha} \bar{\theta}^\dot{\beta} = \frac{1}{2} \bar{\theta} \cdot \bar{\theta} \varepsilon^{\dot{\alpha} \dot{\beta}}, \quad \theta^\alpha \bar{\theta}^{\dot{\alpha}} = \frac{1}{2} \theta \sigma^\mu \bar{\theta} \bar{\sigma}^{\dot{\mu}} \dot{\alpha}, \ldots
\] (14)

After this step, any function of the component fields is written in terms of a restricted set of scalar products involving Grassmann variables and Pauli matrices, forming hence a basis in which any superfield expression can be expanded.

Even though this procedure in principle solves the problem of simplifying superfield expressions, we have to deal at this stage with a purely technical issue. Expressing everything in terms of the basic objects results in Mathematica expressions that are equal up to the names of contracted indices, e.g.,

\[
\text{Dot}[\text{theta}[\text{al}], \text{theta}[\text{al}]] - \text{Dot}[\text{theta}[\text{be}], \text{theta}[\text{be}]] .
\] (15)

Although this difference is manifestly zero, the two terms represent different patterns in Mathematica, and hence the cancellation does not take place. In general, such a situation can occur relatively often with the ToGrassmannBasis function, and therefore the internal index naming scheme is optimized by the ToGrassmannBasis function after all the dot products are formed. Similar terms are collected and summed, and hence the readability of the final results is highly improved. The simplification function presented above can be called in FeynRules through the command

\[
\text{ToGrassmannBasis}[\text{expression}] .
\]
where expression is any function of the component fields. The \texttt{ToGrassmannBasis} function then expresses expression in the basis defined by the objects in Eq. (14).

As an example, the function \texttt{ToGrassmannBasis} could significantly help to improve the readability of the most general series in the $\theta$ and $\bar{\theta}$ variables presented in Eq. (7). Using the MATHEMATICA variable \texttt{Phi} defined in Eq. (7), the command

\texttt{ToGrassmannBasis[Phi]}

allows the user to obtain an expression much closer to the original form of Eq. (6),

\[ z + \text{theta}[sp].\text{xi}[sp] + \text{zetabar}[spd].\text{thetabar}[spd] + f*\text{theta}[sp].\theta[sp]+g*\text{thetabar}[spd].\thetabar[spd] + \text{theta}[sp].\text{thetabar}[spd]*\text{si}[mu,sp,spd]*V[mu] + \text{rhobar}[spd].\text{thetabar}[spd]*\text{theta}[sp].\text{theta}[sp] + \text{theta}[sp].\omega[sp]*\text{thetabar}[spd].\thetabar[spd] + d*\text{theta}[sp].\text{theta}[sp]*\text{thetabar}[spd].\thetabar[spd] . \]

(16)

It is important to note that the \texttt{ToGrassmannBasis} function can also work on spinorial or tensorial expressions, \textit{i.e.}, expressions with uncontracted spin (or vectorial) indices. After an application of the simplification module, the (upper or lower) free index could be attached to a single fermion, or to a chain containing one fermion and a given number of Pauli matrices,

\[ \chi^\alpha , \quad \chi^\alpha \sigma^\mu \alpha \dot{\alpha} \leftrightarrow (\chi\sigma^\mu)_\dot{\alpha} , \quad \sigma^\mu \alpha \dot{\alpha} \bar{\sigma}^{\nu \alpha \beta} \chi_\beta \leftrightarrow (\sigma^\mu \bar{\sigma}^\nu \chi)_\alpha , \quad \ldots , \]

(17)

where, in the examples above, $\chi$ denotes a generic left-handed Weyl fermion. The \texttt{ToGrassmannBasis} function has been implemented so that those chains are formed and stored in the \texttt{TensDot2} environment following the pattern

\texttt{TensDot2[ chain ][pos, chir, name] ,}

where \texttt{chain} is a sequence of one Weyl fermion and possibly one or several Pauli matrices, \texttt{pos} is the \texttt{up} or \texttt{down} position of the free spin index, \texttt{chir} its chirality, \textit{i.e.}, its dotted or undotted nature, and \texttt{name} its name. The three
examples of Eq. (17) could be implemented as

\[ \chi^\alpha \leftrightarrow \text{ToGrassmannBasis}[ \text{nc}[\chi[b]] \ast U_{\epsilon[a,b]} ] \],
\[ (\chi_{\sigma^\mu})_\dot{\alpha} \leftrightarrow \text{ToGrassmannBasis}[ \text{nc}[\chi[b]] \ast si[\mu,a,ad] \ast U_{\epsilon[a,b]} ] \],
\[ (\sigma^\mu\bar{\sigma}^\nu\chi)_\alpha \leftrightarrow \text{ToGrassmannBasis}[ \text{nc}[\chi[b]] \ast si[\mu,a,ad] \ast sibar[\nu,ad,b] ] \].

We recall that the nc environment is mandatory as soon as we are dealing with fermions and that by convention all spin indices carried by fermionic fields are considered to be lowered. We then obtain

\[ \text{nc}[ \text{TensDot2}[\chi[a]][\text{up},Left,a] ] \],
\[ \text{nc}[ \text{TensDot2}[\chi[a], si[\mu,a,ad]][\text{down},Right,ad] ] \],
\[ \text{nc}[ \text{TensDot2}[si[\mu,a,ad], sibar[\nu,ad,b], \chi[b]][\text{down},Left,a] ] \].

To conclude this section, let us comment on the index optimization routine used by the ToGrassmannBasis function, because there might be times where the user wants to handle the optimization of the index naming scheme without forming scalar products involving Grassmann variables, i.e., without calling the ToGrassmannBasis function. The standalone version of the index optimization that consistently renames the indices of an expression is called as

\[ \text{OptimizeIndex}[\text{expression, list}] \],

where list is an optional list of variables carrying indices to be included into the index renaming and that are neither a field nor a parameter included in the global variable of FeynRules M$Parameters. As an example, the application of the optimization of the index naming scheme on the sum of scalar products of Eq. (15) can be performed with the command

\[ \text{OptimizeIndex}[ \text{Dot}[\theta[al],\theta[al]] - \text{Dot}[\theta[be],\theta[be]] ] \],

which simplifies to zero. Equal terms have now been consistently subtracted and are not repeated anymore.
5.2. *Simplified input format for superfield expressions*

In Section 3 we introduced a canonical form in which every superspace function can be expressed. This canonical form is defined by the two simple rules

1. all spin indices carried by fermion fields are considered lowered,
2. the ordering of the fermions is implemented via the \( nc \) environment.

As we already discussed, this canonical form usually requires the explicit use of the \( \epsilon \)-tensors, and for this reason, we have introduced in the previous section the \texttt{ToGrassmannBasis} function which reduces any superfield expression to the basis of Grassmann variable monomials defined by Eq. (14). Conversely, this basis can also be used to input superspace expressions in an easier way. The rules for this input format are discussed in the rest of this section.

First, the dot products of spinors, connected or not with the help of Pauli matrices, are always written as

\[
\text{ferm}_1[sp1].ferm_2[sp2] \text{ chain}[sp1,sp2] ,
\]

where the symbols \texttt{ferm1} and \texttt{ferm2} denote the two fermions and \texttt{chain} contains a series of Pauli matrices linking the two spin indices \texttt{sp1} and \texttt{sp2}. As a simple example, a possible connected product of two left-handed spinors \( \chi \) and \( \psi \) would be

\[
\chi \sigma^\mu \bar{\sigma}^\nu \psi \leftrightarrow \text{chi}[sp1].psi[sp2] \text{ si}[mu,sp1,spd] \text{ sibar}[nu,spd,sp2] ,
\]

the symbol \texttt{chain} being here equal to \texttt{si}[mu,sp1,spd] \texttt{sibar}[nu,spd,sp2].

In the cases where there is no chain of Pauli matrices present in the expression, the spin indices carried by the fermions must be equal, the dot product being hence a regular scalar product of fermions, as those introduced in Eq. (2). A few additional examples can be found among the different terms of Eq. (16). Second, any fermionic expression carrying a free spin index must use both the \( nc \) environment as well as the \texttt{TensDot2} structure, following the syntahx explained in Section 5.1.

Even though this format allows to input superspace expressions in an easier way, most of the functionalities of the superspace module require the input expressions to be given in the canonical form of Section 3. In the next section we therefore introduce a function that allows to convert an expression written in terms of the basis objects of Eq. (14) to its canonical form.
5.3. Reverting simplifications: back to the nc environment

One drawback of the optimization routines is that several of the functions included in the superspace module of FeynRules, such as the QSUSY or DSUSY routines presented in Section 3, require expressions including the nc environment, second rank antisymmetric tensors, lower spin indices for fermions and not any explicit scalar products. A simple solution is provided by the Tonc function. Indeed, if one needs to perform additional operations on simplified expression within the superspace, it is recommended to first use the Tonc function, which allows to transform simplified expressions back to their original form in terms of the nc environment and the epsilon tensors. Hence, Tonc[Dot[theta[a],theta[a]]] would lead to

\[ \text{nc[theta[a],theta[sp$1$]] Ueps[sp$1$,a]} \]

where the second spin index has been automatically generated by FeynRules. Sometimes, it might be useful to call the OptimizeIndex function right after the use of the Tonc module in order to get shorter and simpler expressions.

Note that the Tonc function allows us at the same time to enter expressions directly with the use of the simplified syntax presented in Sections 5.1 and 5.2 and to convert them to their canonical form.

6. Manipulating superfield expressions

6.1. From superfields to Lagrangians

The main advantage of writing down supersymmetric Lagrangians in terms of (chiral and vector) superfields rather than in terms of the component fields is the size of the corresponding expressions. This is illustrated in the following simple example. Let us consider a left-handed chiral superfield Φ whose scalar, fermionic and auxiliary components are denoted by \( \phi \), \( \psi \) and \( F \). Performing the series expansion of \( \Phi^\dagger \Phi \) in terms of the Grassmann variables, it can be shown that the coefficient with the highest power in \( \theta \) and \( \bar{\theta} \), i.e., the coefficient of the \( \theta \cdot \theta \bar{\theta} \cdot \bar{\theta} \) term, is SUSY invariant and hence a good candidate for a Lagrangian density describing the free component fields. Indeed, we get

\[
\begin{align*}
\mathcal{L} = \Phi^\dagger \Phi_{\theta \bar{\theta} \hat{a}} &= -\frac{1}{4} \left( \phi^\dagger \Box \phi + \Box \phi^\dagger \phi - 2 \partial_\mu \phi^\dagger \partial^\mu \phi \right) + F^\dagger F \\
&+ \frac{i}{2} \left( \psi \sigma^\mu \partial_\mu \bar{\psi} - \partial_\mu \psi \sigma^\mu \bar{\psi} \right).
\end{align*}
\]  

(18)
| Function | Description |
|----------|-------------|
| **ToGrassmannBasis[exp]** | This function allows to express any function exp of the component fields in the basis of Grassmann monomial defined by Eq. (14). |
| **OptimizeIndex[exp,list]** | This function allows for the optimization of the index naming scheme used in the expression exp. The optional argument list is a list of variables carrying indices, which are neither a field nor included in M$\$Parameters. |
| **Tonc[exp]** | This function transforms simplified expressions back to their original form, with lower spin indices, epsilon tensors, etc... |

**New environments**

| Environment | Description |
|-------------|-------------|
| **TensDot2[chain][pos,chir,name]** | This environment contains a sequence, labelled by chain, of one Weyl fermion and possibly one or several Pauli matrices. The symbols pos, chir and name are the up or down position, the chirality and the name of the free index. |
The expansion of a superfield polynomial into a series in the Grassmann variables can be automatically performed in FeynRules via the SF2-Components function. Since this function allows in the same way to transform superfield expressions into four-dimensional Lagrangians, it is one of the most important functions available and at the heart of the superspace module of FeynRules. For any given function of chiral and vector superfields denoted expression, the correct syntax to use is simply

\[
\text{SF2Components[ expression ]}.
\]

The SF2Components function expands all the superfields appearing in expression in terms of their component fields and the \(x^\mu\) spacetime coordinates (rather than the \(y^\mu\) variable related to chiral superfields). In a second step, scalar products of Grassmann variables are simplified and the expression is reduced to the basis defined by Eq. (14) using the ToGrassmannBasis function. During this procedure representation matrices of the Lie algebra of the gauge groups are introduced and simplified, using, e.g., the commutation relations between the generators. The output of the SF2Components function consists in a list of two elements,

\[
\{ \text{Full series}, \text{List of the nine coefficients} \}.
\]

The first element of this list, labelled here Full series, is the full series expansion in the Grassmann variables which could also have been directly obtained with the GrassmannExpand function,

\[
\text{GrassmannExpand[ expression ]}.
\]

The second element of the list of Eq. (19) is a list containing the nine coefficients of the series. The first part of this last list is the scalar piece of the series, i.e. the terms independent of the \(\theta\) and \(\bar{\theta}\) variables, while the other elements are the coefficients of the \(\theta_\alpha, \bar{\theta}_\dot{\alpha}, \theta \sigma^\mu \bar{\theta}, \theta \cdot \theta, \bar{\theta} \cdot \bar{\theta}, \theta \cdot \theta \bar{\theta}_\dot{\alpha}, \bar{\theta} \cdot \bar{\theta} \theta_\alpha\) and \(\theta \cdot \theta \bar{\theta} \cdot \bar{\theta}\) terms, following this ordering. Each of them can also be obtained
using the functions,

\[
\begin{align*}
\text{ScalarComponent} \ [ \text{expression} ] , \\
\text{ThetaComponent} \ [ \text{expression} ] , \\
\text{ThetabarComponent} \ [ \text{expression} ] , \\
\text{ThetaThetabarComponent} \ [ \text{expression} ] , \\
\text{Theta2Component} \ [ \text{expression} ] , \\
\text{Thetabar2Component} \ [ \text{expression} ] , \\
\text{Theta2Thetabar2Component} \ [ \text{expression} ] , \\
\text{Theta2ThetabarComponent} \ [ \text{expression} ].
\end{align*}
\]

The results obtained by each of these functions are thus all independent of the Grassmann variables. It is important to note that each of these functions calls the \textit{SF2Components} module, the complete series being thus recalculated at each function call. Hence, if several of the coefficients have to be computed, it is much faster to store and re-use the results of the \textit{SF2Components} function than to call all the \textit{XXXXComponent} functions individually. Furthermore, these functions can also be used on expressions in terms of the component fields. In this case, we recall that the use of the \textit{Tonc} environment is mandatory. All the available functions to reexpress superfield expressions in terms of their component fields are summarized in Table 5.

As an example, let us derive the Lagrangian of Eq. (18) with \textit{FeynRules}, starting from the superfield expression, recalling that the superfield PHI has been defined in Section 4.2. This can be done via the \textsc{Mathematica} command

\[
\text{Theta2Thetabar2Component}[\text{PHIbar PHI}],
\]

and one obtains indeed the correct free Lagrangian for the component fields,

\[
\begin{align*}
&\frac{1}{2} \left( \text{del} [z, \mu] \right) \left( \text{del} [\bar{z}, \mu] \right) - \\
&\frac{1}{4} \left( \text{del} [z, \mu] \right) \left( \text{del} [\bar{z}, \mu] \right) + FF \cdot \bar{FF} - \\
&(\frac{1}{2}) \left( \text{del} [\xi [sp], \mu] \right) \left( \text{del} [\bar{xibar}, \mu] \right) \cdot si [\mu, sp, spd] + \\
&(\frac{1}{2}) \left( \text{del} [\bar{xibar}, \mu] \right) \left( \text{del} [\xi [sp], \mu] \right) \cdot si [\mu, sp, spd],
\end{align*}
\]

where \text{del} stands for the spacetime derivative.
**Table 5: From superfields to particles**

| Function                        | Description                                                                 |
|---------------------------------|-----------------------------------------------------------------------------|
| SF2Components[exp]              | Expands the superfield expression `exp` in terms of its component fields and simplifies the products of Grassmann variables. The result is a list of two elements, the complete series and a new list with all the individual coefficients. |

**Shortcuts to the individual component fields**

| Term                | Function                                      |
|---------------------|-----------------------------------------------|
| The full series.    | GrassmannExpand[exp]                          |
| The scalar term.    | ScalarComponent[exp]                          |
| The $\theta$ term. | ThetaComponent[exp]                           |
| The $\bar{\theta}$ term. | ThetabarComponent[exp]                        |
| The $\theta\sigma\bar{\theta}$ term. | ThetaThetabarComponent[exp]                  |
| The $\theta^2$ term. | Theta2Component[exp]                          |
| The $\bar{\theta}^2$ term. | Thetabar2Component[exp]                       |
| The $\theta^2\bar{\theta}$ term. | Theta2ThetabarComponent[exp]                 |
| The $\bar{\theta}^2\theta$ term. | Thetabar2ThetaComponent[exp]                 |
| The $\theta^2\bar{\theta}^2$ term. | Theta2Thetabar2Component[exp]                |
6.2. Supersymmetric transformation laws

The supersymmetric transformation laws of a superfield can be obtained by using the explicit representation of the supercharges given in Eq. (5). Hence, considering an infinitesimal supersymmetric transformation of a (spinorial) parameter \((\epsilon_\alpha, \bar{\epsilon}^{\dot{\alpha}})\), a superfield transforms as

\[
\Phi \to \Phi + \delta_\epsilon \Phi = \Phi + i(\epsilon \cdot Q + \bar{Q} \cdot \bar{\epsilon}) \Phi.
\]  

(22)

After expanding the superfield in terms of its component fields and using Eq. (5), we can immediately read off the transformations of the component fields. The operator \(\delta_\epsilon\) is implemented into the superfield module via the DeltaSUSY function,

\[
\text{DeltaSUSY} \[ \text{expression} , \text{epsilon} \]
\]

where expression can be any function of superfields and/or component fields and epsilon refers to the supersymmetric transformation parameter, without any spin index. There are ten such parameters predefined in the superfield module, labelled by \(\text{epsx}\) with \(x\) being an integer between zero and nine, which the user can use at his convenience. This parameter being a Majorana fermion, it is enough to only provide the associated two-component left-handed spinor\(^5\). The output of the DeltaSUSY module is the full series expansion in the Grassmann variables. Finally, the functions given in Eq. (20) allow for the identification of the variations of the various component fields.

Let us illustrate the use of this function on the explicit example of chiral superfields, and let us start with the superfield \(\Phi\) introduced in Section 4.2. We then calculate its variation under a supersymmetric transformation of parameter \(\epsilon_1\),

\[
\text{DeltaPHI} = \text{DeltaSUSY} \[ \text{PHI} , \text{eps1} \]
\]

where we have used the supersymmetric transformation parameter \(\text{eps1}\). The scalar, \(\theta_\alpha\) and \(\theta \cdot \theta\) coefficients of \(\text{DeltaPHI}\) are finally identified with the variations of the scalar component \(\delta_{\epsilon_1} z\), the fermionic component \(\delta_{\epsilon_1} \psi\) and of

\(^5\)The objects \(\text{epsx}\) are in fact implemented into \textsc{FeynRules} as left-handed Weyl fermions. The corresponding right-handed objects are also available under the name \(\text{epsxbar}\).
the auxiliary component $\delta_\epsilon F$ of the superfield. The individual components can be easily extracted using the shortcuts of Eq. (20), and one recovers the well-known textbook expressions,

$$\begin{align*}
\delta_\epsilon z &= \sqrt{2} \epsilon_1 \cdot \psi , \\
\delta_\epsilon \psi &= - \sqrt{2} \epsilon_1 F - i \sqrt{2} \sigma^\mu \bar{\epsilon}_1 \partial_\mu z , \\
\delta_\epsilon F &= - i \sqrt{2} \partial_\mu \psi \sigma^\mu \bar{\epsilon}_1 ,
\end{align*}$$

(23)

where $z$, $\psi$ and $F$ are the scalar, fermionic and auxiliary component of the considered superfield.

7. Implementing supersymmetric Lagrangians into FeynRules

7.1. Supersymmetric Lagrangians

In this section we describe how to generate in an automated way (parts of) the Lagrangian describing the interactions between the different component fields of chiral and vector supermultiplets. The first part of this section is devoted to a brief review on how to construct supersymmetric Lagrangians. The most general supersymmetry-conserving Lagrangian describing the interactions between the various multiplets can be written as a sum of three pieces,

$$\mathcal{L} = \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{Yang–Mills}} + \mathcal{L}_{\text{superW}} ,$$

(24)

where $\mathcal{L}_{\text{chiral}}$ and $\mathcal{L}_{\text{Yang–Mills}}$ contain the kinetic terms as well as the gauge interactions of the different particles, and $\mathcal{L}_{\text{superW}}$ is the superpotential describing the interactions between the different chiral supermultiplets. Note that, since supersymmetric particles have not yet been observed, supersymmetry must be broken at low energies, which renders the superpartners heavy in comparison to their Standard Model counterparts. The corresponding Lagrangian does not involve superfields, but only some of the component fields, and so we exclude it from the present discussion and only concentrate on Lagrangians where supersymmetry is unbroken.

The first Lagrangian in Eq. (24), $\mathcal{L}_{\text{chiral}}$, contains the kinetic terms as well as the gauge interactions of the chiral superfields. It is completely fixed by gauge invariance and reads

$$\mathcal{L}_{\text{chiral}} = \left[ \Phi_i^\dagger e^{-2g_j V^j} \Phi_i \right]_{\theta \bar{\theta} \bar{\theta} \bar{\theta}} ,$$

(25)
where a sum over all chiral superfields \( \Phi^i \) and vector superfields \( V^j \) is understood. Furthermore, \( T_a \) denote the representation matrices of the supermultiplet \( \Phi^i \) and \( g_j \) is the associated gauge coupling constant. It can be shown that the \( \theta \cdot \bar{\theta} \cdot \bar{\theta} \) coefficient of the expansion in the Grassmann variables of the expression inside the squared brackets is invariant under supersymmetry transformations and reproduces for gauge singlet superfields the Lagrangian \( \mathcal{L} \) for a free chiral supermultiplet. This expression is hence a good candidate for a Lagrangian density. In general, if the fields carry some gauge charges, the expansion of Eq. (25) yields

\[
\mathcal{L}_{\text{chiral}} = D_\mu \phi_i^I D^{\mu} \phi^i + F_i^I F^i - \frac{i}{2} \left( D_\mu \bar{\psi}_i^a \bar{\sigma}^\mu \psi^i - \bar{\psi}_i^a \bar{\sigma}^\mu D_\mu \psi^i \right) + i \sqrt{2} g_j \lambda_i^i \cdot \bar{\psi}_i^a T_a \phi^i - i \sqrt{2} g_j \phi_i^I T_a \lambda^i - g_j D_{j a} \phi_i^I T_a \phi^i ,
\]

(26)

where \( (\phi^i, \psi^i, F^i) \) and \( (V_j^I, \lambda^i, D^i) \) denote the component fields of the chiral and vector supermultiplets \( \Phi^i \) and \( V^j \), and \( D_\mu \) is the covariant derivative

\[
D_\mu = \partial_\mu - i g_j V_{j a} T_a .
\]

(27)

The second Lagrangian of Eq. (24), \( \mathcal{L}_{\text{Yang–Mills}} \), contains the kinetic terms and self-interactions of the vector superfields. It consists in a sum over all the vector supermultiplets of the theory, and can be written, for one specific vector supermultiplet \( V \),

\[
\mathcal{L}_{\text{Yang–Mills}, V} = \frac{1}{16 g^2} \left[ W_\alpha^a W_\alpha^a \right]_{\theta \theta} + \frac{1}{16 g^2} \left[ \bar{W}_\alpha^a \bar{W}_\alpha^a \right]_{\bar{\theta} \bar{\theta}} .
\]

(28)

Similarly to \( \mathcal{L}_{\text{chiral}} \), the squared brackets indicate that we only take the coefficient of the corresponding term in the expansion in the Grassmann variables. The spinorial superfields \( W_\alpha^a \) and \( \bar{W}_\alpha^a \) in Eq. (28), the supersymmetric equivalents to the field strength tensor, are related to the quantities \( W_\alpha = W_\alpha^a T_a \) and \( \bar{W}_\alpha = \bar{W}_\alpha^a T_a \), the latter being given by

\[
W_\alpha = - \frac{1}{4} \bar{D} \cdot D e^{2gV} D_\alpha e^{-2gV} \quad \text{and} \quad \bar{W}_\alpha = - \frac{1}{4} D \cdot D e^{-2gV} \bar{D}_\alpha e^{2gV} ,
\]

(29)

where \( D \) and \( \bar{D} \) denote the superderivatives of Eq. (5) and \( g \) the coupling constant associated to the gauge group. After expanding \( W_\alpha \) into component fields, one obtains the expression of \( W_\alpha^a \),

\[
W_\alpha^a = -2g \left[ -i \lambda_\alpha^a + \frac{i}{2} \sigma^\mu \sigma^\nu \theta_\alpha V_{\mu \nu}^a + \theta_\alpha D^a \theta - \theta (\sigma^\mu D_\mu \lambda_\alpha^a) \right] ,
\]

(30)
a similar expression existing for $\overline{W}_\dot{a}^a$. We recognize in Eq. the expressions for the field strength tensors and the covariant derivative in the adjoint representation

$$D_\mu \tilde{\Lambda}^a = \partial_\mu \tilde{\Lambda}^a + g f_{bc}^a V_{\mu}^b \tilde{\Lambda}^c,$$

$$V^a_{\mu \nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu + g f_{bc}^a V^b_\mu V^c_\nu,$$

where $f_{bc}^a$ are the structure constants of the gauge group. In the abelian case, the expressions for the spinorial superfields can be drastically simplified,

$$W_\alpha = -\frac{1}{4} \bar{D} \cdot \bar{D} D \alpha V \quad \text{and} \quad \overline{W}_\dot{a} = \frac{1}{4} D \cdot \bar{D} \dot{a} V ,$$

and the Lagrangian reduces to

$$L_{U(1),V} = \frac{1}{4} [W^a W_\alpha]_{\theta \bar{\theta}} + \frac{1}{4} [\overline{W}_\dot{a} \overline{W}^\dot{a}]_{\bar{\theta} \bar{\theta}} .$$

Finally, the superpotential Lagrangian $L_{\text{super}W}$ contains the interactions among the chiral superfields derived from the superpotential $W(\Phi)$, a holomorphic function of the chiral superfields. It is given by

$$L_{\text{super}W} = [W(\Phi)]_{\theta \bar{\theta}} + [W^*(\Phi^\dagger)]_{\bar{\theta} \theta} ,$$

where $W^*(\Phi^\dagger)$ is the anti-holomorphic function complex conjugate to $W(\Phi)$. It can be shown that in a renormalizable model the superpotential can at most be cubic in the fields, thus taking the form

$$W(\Phi) = a_i \Phi^i + b_{ij} \Phi^i \Phi^j + c_{ijk} \Phi^i \Phi^j \Phi^k ,$$

for some model-dependent parameters $a_i$, $b_{ij}$ and $c_{ijk}$.

As the superpotential is simply a polynomial in the chiral superfields, it can be trivially implemented into FeynRules, and the Lagrangian density can easily be obtained from the Theta2Component and Thetabar2Component functions. The non-trivial part of any implementation of a supersymmetric model into FeynRules hence consists in the implementation of $L_{\text{chiral}}$ and $L_{\text{Yang-Mills}}$. Since these two Lagrangians are however completely fixed by gauge symmetry and their form is independent of the actual model under consideration, the superspace module of FeynRules comes with some predefined functions that allow to generate $L_{\text{chiral}}$ and $L_{\text{Yang-Mills}}$ in an automated way. These functions will be described in the next section and are summarized in Table 6.
Table 6: Predefined functions related to SUSY Lagrangians.

| Function                                      | Description                                                                                     |
|-----------------------------------------------|------------------------------------------------------------------------------------------------|
| CSFKineticTerms[csf]                          | Derives all the kinetic and gauge interaction terms associated to the chiral superfield csf. If the function is called without any argument, it will sum over the whole chiral content of the theory. |
| VSFKineticTerms[vsf]                          | Derives all the kinetic and gauge interaction terms associated to the vector superfield vsf. If the function is called without any argument, it will sum over the whole gauge content of the theory. |
| SuperfieldStrengthL[vsf,alpha,gaugeindex]     | Calculates the left-handed superfield strength tensor associated to the vector superfield vsf. The symbol alpha denotes the free spin index, whilst the optional symbol gaugeindex denotes the adjoint gauge index relevant for non-abelian gauge groups. |
| SuperfieldStrengthR[vsf,alphadot,gaugeindex]  | Calculates the right-handed superfield strength tensor associated to the vector superfield vsf. The symbol alphadot denotes the free spin index, whilst the optional symbol gaugeindex denotes the adjoint gauge index relevant for non-abelian gauge groups. |
| SolveEqMotionD[lag]                           | Computes and solves the equations of motion associated to the auxiliary D-fields, and then inserts the solution in the Lagrangian lag. |
| SolveEqMotionF[lag]                           | Computes and solves the equations of motion associated to the auxiliary F-fields, and then inserts the solution in the Lagrangian lag. |
7.2. Automatic generation of supersymmetric Lagrangians

The kinetic part of the Lagrangian for a chiral superfield \( \Phi \) can be obtained automatically in FeynRules from the \texttt{CSFKineticTerms} function. As an example, for a chiral superfield implemented as \texttt{PHI}[^6], the Lagrangian of Eq. (25) is obtained by issuing the command

\[ \texttt{LChiralPhi = CSFKineticTerms[ PHI ] .} \]

The expression returned by \texttt{CSFKineticTerms} is not automatically expanded in terms of the component fields but still expressed in terms of superfields. The component-field expression of the Lagrangian can be recovered by apply the \texttt{Theta2Thetabar2Component} function to the result,

\[ \texttt{Theta2Thetabar2Component[ LChiralPhi ] .} \]

The full Lagrangian \( \mathcal{L}_{\text{chiral}} \) is obtained by summing over all chiral superfields of the theory. In addition to the function described above which returns the kinetic term for a single chiral superfield, FeynRules allows the user to obtain directly the complete chiral Lagrangian expressed in terms of superfields via the command \texttt{CSFKineticTerms}. The expression returned by this command is equivalent to a sum of terms consisting each in an application of the \texttt{CSFKineticTerms} function to a single superfield. The extraction of the Lagrangian density can again be achieved via the \texttt{Theta2Thetabar2Component} function. Hence, the full Lagrangian \( \mathcal{L}_{\text{chiral}} \) can be obtained by simply issuing

\[ \texttt{Lchiral = Theta2Thetabar2Component[ CSFKineticTerms[ ] ] .} \]

The supersymmetric equivalents of the field strength tensors can be obtained automatically in a similar way. The left-handed superfield strength tensors \( W_\alpha \) and \( W_\alpha^a \) associated to a vector superfield \( V \) can be called in the superspace module of FeynRules via the commands[^7]

\[
\text{SuperfieldStrengthL [ V, sp ] ,} \\
\text{SuperfieldStrengthL [ V, sp, ga] ,}
\]

[^6]: We assume that \texttt{PHI} has been correctly declared in the \texttt{FeynRules} model file.
[^7]: We assume that the vector superfield \( V \) has been declared associated to some gauge group in the \texttt{FeynRules} model file.
in the abelian and non-abelian cases. The symbol $sp$ denotes the undotted spin index attached to the spinorial superfield while $ga$ is the adjoint gauge index relevant for non-abelian gauge groups. Similarly, the abelian and non-abelian right-handed superfield strength tensors $W_{\dot{\alpha}}$ and $W_{\dot{\alpha}a}$ can be obtained through

\[
\text{SuperfieldStrengthR} \ [ V, sp ] , \\
\text{SuperfieldStrengthR} \ [ V, sp, ga ] ,
\]

respectively, the only difference with the left-handed case being the variable $sp$ which stands this time for a dotted spin index. Note that the spinorial superfields defined by Eq. (29) and Eq. (32) are not hardcoded in the superspace module of FEYNRULES, and will be recalculated each time. However, the \text{SuperfieldStrengthL} and \text{SuperfieldStrengthR} functions will be evaluated by FEYNRULES only at the time of the expansion in terms of the component fields.

From the superfield strength tensors we can easily built the kinetic terms for vector superfields in an automated way. This is achieved in FEYNRULES by issuing the command

\[
LV = \text{VSFKineticTerms} \ [ V ] ,
\]

The Super-Yang-Mills Lagrangian of Eq. (28) can then easily be obtained by extracting the $\theta \cdot \theta$ and $\bar{\theta} \cdot \bar{\theta}$ components,

\[
LSYM = \text{Theta2Component}[ LV ] + \text{Thetabar2Component}[ LV ] .
\]

If a model contains several gauge groups, we have to sum over the corresponding kinetic terms. Similarly to the automatic generation of the kinetic terms of chiral superfields, issuing \text{VSFKineticTerms} without any argument is equivalent to a sum over all possible vector superfields defined in the model.

At this stage, generating a Lagrangian density for any supersymmetric model reduces to an almost trivial task with the help of the functions that we have just described. Assuming that a superpotential $SP$ has been defined, the full Lagrangian density can be easily implemented into FEYNRULES as

\[
\text{Lag} = \text{Lchiral} + LSYM + LW ,
\]
where the terms in the sum in the right-hand side are given by

\[
\begin{align*}
LC &= \text{CSFKineticTerms}[ ] , \\
L_{\text{chiral}} &= \text{Theta2Thetabar2Component}[ LC ] , \\
LV &= \text{VSFKineticTerms}[ ] , \\
LSYM &= \text{Theta2Component}[LV] + \text{Thetabar2Component}[LV] , \\
LW &= \text{Theta2Component}[SP]+\text{Thetabar2Component}[HC[SP]] .
\end{align*}
\]

The Lagrangian density obtained in this way however still depends on the auxiliary \( F \) and \( D \) fields, which can be eliminated by their equations of motion. This can be performed automatically with the help of two functions \text{SolveEqMotionD} and \text{SolveEqMotionF}. Each of them computes, for the \( D \)-fields and \( F \)-fields respectively, the equations of motion directly from the Lagrangian, solves them analytically and subsequently inserts the solution into the Lagrangian in order to eliminate the auxiliary fields. Using the Lagrangian defined previously, the auxiliary fields are eliminated via the \text{MATHEMATICA} commands

\[
\begin{align*}
\text{Lag} &= \text{SolveEqMotionD}[\text{Lag}] , \\
\text{Lag} &= \text{SolveEqMotionF}[\text{Lag}] .
\end{align*}
\]

8. Implementation of the Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is the simplest supersymmetric model, resulting from a direct supersymmetrization of the Standard Model (SM) \cite{25,26}. In this section, we describe its implementation in \text{FeynRules} in terms of superfields. The corresponding model file can be downloaded from the \text{FeynRules} website:

\text{http://feynrules.phys.ucl.ac.be/wiki/MSSM}

8.1. Gauge groups and representations

The MSSM is based on the same gauge group as the SM, \( SU(3)_c \times SU(2)_L \times U(1)_Y \). The implementation of these three gauge group classes slightly differs from the one included into the previous MSSM implementation, expressed entirely in terms of the component fields \cite{4}. First, we associate to each gauge group one vector superfield instead of one gauge boson. The abelian factor \( U(1)_Y \) is hence implemented as
where the vector superfield $\text{BSF}$ will be specified below. Secondly, the implementation of the non-abelian direct factors of the gauge group, $SU(2)_L$ and $SU(3)_c$, includes a consistent definition of the representation matrices, together with the associated index type, related to the representations in which one or several chiral superfields of the model live. This allows to extract the gauge interaction and kinetic terms of the Lagrangian automatically with the help of the two functions CSFKineticTerms and VSFKineticTerms. For $SU(2)_L$, we only need the fundamental representation, labelled by the quantity $\text{Ta}$, and defined together with the associated gauge index $\text{SU2D}$. This is implemented as,

\[
SU2L == \{
\text{Abelian} -> \text{False},
\text{CouplingConstant} -> gw,
\text{Superfield} -> \text{WSF},
\text{StructureConstant} -> \text{ep},
\text{Representations} -> \{\text{Ta}, \text{SU2D}\},
\text{Definitions} -> \{
  \text{Ta}[\text{a\_\_}] -> \text{PauliSigma}[\text{a}]/2,
  \text{ep} -> \text{Eps}
\}
\}.
\]

For the $SU(3)_c$ gauge group, both the fundamental and its complex conjugate representation are needed, the gauge group being hence defined as,

\[
SU3C ==
\{
\text{Abelian} -> \text{False},
\text{CouplingConstant} -> gs,
\}
\]

\footnote{We let \texttt{FeynRules} handle automatically the adjoint representations necessary for the vector superfields of the model. We refer to the \texttt{FeynRules} manual for more information.}
DTerm -> dSUN,
Superfield -> GSF,
StructureConstant -> f,
Representations -> \{ \{T,Colour\}, \{Tb,Colourb\} \}

where Colour and Colourb are the fundamental and antifundamental representation indices and T and Tb the corresponding representation matrices. It is important to note that all antifundamental indices in colour space must be replaced by fundamental ones before exporting the Feynman rules to the interfaces to Monte Carlo generators, following the conventions of Ref. [3].

8.2. Field content

The Standard Model quarks and leptons are embedded into chiral supermultiplets, together with their squark and slepton partners, which are grouped into three generations of six chiral superfields,

\[
Q^i_L = (\begin{pmatrix} 3 & 2 & 1 \\ \bar{6} \end{pmatrix}), \quad U^i_R = (\begin{pmatrix} 3 & 1 & 2 \\ -3 \end{pmatrix}), \quad D^i_R = (\begin{pmatrix} 3 & 1 & 1 \\ -3 \end{pmatrix}),
\]

\[
L^i_L = (\begin{pmatrix} 1 & 2 & -1 \\ \bar{2} \end{pmatrix}), \quad E^i_R = (\begin{pmatrix} 1 & 1 \\ \bar{1} \end{pmatrix}), \quad V^i_R = (\begin{pmatrix} 1 & 1 & 0 \\ \bar{0} \end{pmatrix}),
\]

(36)

where \(i\) stands for a generation index and where we have indicated the representations of the different superfields under the MSSM gauge group. The component fields included in each superfield can be found in Table 7. For completeness, the right-handed neutrino superfield has been introduced, but it will be kept sterile, \textit{i.e.}, non-interacting with any other superfield.

The six chiral superfields of Eq. (36) are implemented following the instructions given in Section 4.2, while the component fields are implemented following the syntax presented in Refs. [3, 22] and the constraints introduced in Section 4.2. Hence, for each chiral superfield and the associated component fields, the hypercharge quantum number and the attached gauge indices are specified. This allows the automatic function CSFKineticTerms to correctly derive the associated gauge interactions. As an example, the weak isospin doublet of quarks \(Q^i_L\) is implemented using the \textsc{Mathematica} instructions

\footnote{It can then be seen that the representation matrices defined in Section 8.1 are enough to describe the entire superfield content given in Eq. (36).}
Table 7: The MSSM \((s)\)fermion sector, with the representations under the gauge group 
\(SU(3)_c \times SU(2)_L \times U(1)_Y\). The superscript \(c\) denotes charge conjugation.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Superfield} & \text{Standard Model fermion} & \text{Superpartner} & \text{Representation} \\
\hline
Q^i_L & q^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix} & \tilde{q}^i_L = \begin{pmatrix} \tilde{u}^i_L \\ \tilde{d}^i_L \end{pmatrix} & (3, 2, \frac{1}{6}) \\
U^i_R & u^{ic}_R & \tilde{u}^{ic}_R & (3, 1, -\frac{2}{3}) \\
D^i_R & d^{ie}_R & \tilde{d}^{ie}_R & (3, 1, \frac{1}{3}) \\
L^i_L & \ell^i_L = \begin{pmatrix} \nu^i_L \\ e^i_L \end{pmatrix} & \tilde{\ell}^i_L = \begin{pmatrix} \tilde{\nu}^i_L \\ \tilde{e}^i_L \end{pmatrix} & (1, 2, -\frac{1}{2}) \\
E^i_R & e^{ic}_R & \tilde{e}^{ic}_R & (1, 1, 1) \\
V^i_R & \nu^{ic}_R & \tilde{\nu}^{ic}_R & (1, 1, 0) \\
\hline
\end{array}
\]

In our model implementation, we follow a simple naming scheme for the component fields where the names of the Weyl fermionic and scalar components are obtained by suffixing \(w\) and \(s\) to the superfield class name, respectively.

To preserve the electroweak symmetry from chiral anomalies and in order to give masses to both up-type and down-type fermions, the MSSM Higgs sector contains two chiral supermultiplets,

\[
H_D = (1, 2, -\frac{1}{2}) \quad , \quad H_U = (1, 2, \frac{1}{2}) ,
\] (37)
Table 8: The MSSM Higgs(ino) sector, with the representations under the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

| Superfield | Higgs boson | Higgsino | Representation |
|------------|-------------|----------|----------------|
| $H_D$      | $H_d = \begin{pmatrix} H^0_d \\ H^+_d \end{pmatrix}$ | $\tilde{H}_d = \begin{pmatrix} \tilde{H}^0_d \\ \tilde{H}^+_d \end{pmatrix}$ | $(1, 2, -\frac{1}{2})$ |
| $H_U$      | $H_u = \begin{pmatrix} H^+_u \\ H^0_u \end{pmatrix}$ | $\tilde{H}_u = \begin{pmatrix} \tilde{H}^+_u \\ \tilde{H}^0_u \end{pmatrix}$ | $(1, 2, \frac{1}{2})$ |

Table 9: The MSSM gauge sector, with the representations under the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

| Superfield | Gauge boson | Gaugino | Representation |
|------------|-------------|---------|----------------|
| $\Phi_B$   | $B_\mu$     | $B$     | $(1, 1, 0)$    |
| $\Phi_W$   | $W_\mu$     | $\tilde{W}$ | $(1, 3, 0)$   |
| $\Phi_G$   | $g_\mu$     | $\tilde{g}$ | $(8, 1, 0)$   |

Each of which consisting in one scalar Higgs $SU(2)_L$ doublet and its fermionic Higgsino partner, as shown in Table 8. The superfield $H_U$ couples to up-type particles whilst $H_D$ couples to down-type particles. The two chiral superfields $H_U$ and $H_D$ are implemented in a similar fashion as presented above.

The gauge sector is described by three vector superfields associated each to one specific direct factor of the gauge group. They lie in the corresponding adjoint representation and are singlets under all the other group factors,

$$SU(3)_c \rightarrow \Phi_G = (8, 1, 0)$$
$$SU(2)_L \rightarrow \Phi_W = (1, 3, 0)$$
$$U(1)_Y \rightarrow \Phi_B = (1, 1, 0)$$

These superfields include, in addition to the Standard Model gauge bosons, their fermionic partners, the gauginos, as shown in Table 9. Each vector
superfield is implemented following the same pattern, with the Indices option set to the adjoint index of the relevant gauge group, labelled by SU\(2_W\) and Gluon for SU(2)\(L\) and SU(3)\(c\), respectively. As an example, the gluon superfield implementation is given by

```math
class VSF[3] == 

    ClassName -> GSF,
    GaugeBoson -> G,
    Gaugino -> gow,
    Indices -> {Index[Gluon] }

)
```

8.3. Lagrangian

As stated in Section 7, the kinetic and gauge interaction terms of the chiral and vector superfields are entirely fixed by gauge invariance. For the MSSM, these terms read,

\[
\mathcal{L}_{\text{SYM}} = \left[ \frac{W_B^a W_B^a}{4} + \frac{W_W^a W_W^k}{16g_w^2} + \frac{W_G^a W_G^a}{16g_s^2} \right] + \text{h.c.} ,
\]

\[
\mathcal{L}_{\text{chiral}} = \left[ Q_L^\dagger \left( e^{-\frac{1}{4}g'\Phi_B e^{-2g_w V_W} e^{-2g_s V_G}} \right) Q_L +
U_R^\dagger \left( e^{\frac{1}{4}g'\Phi_B e^{-2g_s V_G}} \right) U_R + D_R^\dagger \left( e^{-\frac{1}{4}g'\Phi_B e^{-2g_s V_G}} \right) D_R +
L_L^\dagger \left( e^{g'\Phi_B e^{-2g_w V_W}} \right) L_L + E_R^\dagger \left( e^{-2g'\Phi_B} \right) E_R + V_R^\dagger V_R +
H_D^\dagger \left( e^{g'\Phi_B e^{-2g_w V_W}} \right) H_D + H_U^\dagger \left( e^{-g'\Phi_B e^{-2g_w V_W}} \right) H_U \right]_{\bar{\theta} \theta \bar{\theta} \theta} ,
\]

where

\[
W_B^a = -\frac{1}{4} \bar{D} \cdot \bar{D} D \Phi_B ,
\]

\[
W_W^a = -\frac{1}{4} \bar{D} \cdot \bar{D} e^{2g_w V_W} D \alpha e^{-2g_w V_W} \quad \text{and} \quad W_W^a = W_W^a \frac{1}{2} \sigma_k ,
\]

\[
W_G^a = -\frac{1}{4} \bar{D} \cdot \bar{D} e^{2g_s V_G} D \alpha e^{-2g_s V_G} \quad \text{and} \quad W_G^a = W_G^a T_a .
\]

In the expressions above, we have introduced the non-abelian vector superfields \(V_W = \Phi_W^{\frac{1}{2}} \sigma_k\), \(V_G = \Phi_G T_a\) and \(V'_G = \Phi_G T_a\) where \(\sigma^k/2\) denote the generators of the fundamental representation of SU(2)\(L\), \(\sigma_k\) being the Pauli
matrices, and where $T_a$ and $\bar{T}_a$ are the generators of the fundamental and antifundamental representation of $SU(3)_c$. The gauge coupling constants are defined as $g', g_w$ and $g_s$ and all generation indices are understood. The two Lagrangians appearing in Eq. (39) are implemented in the model file as described in Section 7.2, using the automatized functions CSFKineticTerms and VSFKineticTerms.

The interactions among the chiral superfields introduced in Eq. (36) and Eq. (37) are included in the superpotential

$$W_{\text{MSSM}} = (y_u)_{ij}U_R^iQ_L^j \cdot H_U - (y_d)_{ij}D_R^iQ_L^j \cdot H_D - (y_e)_{ij}E_R^iL_L^j \cdot H_D + \mu H_U \cdot H_D,$$

where $y_u$, $y_d$ and $y_l$ denote the $3 \times 3$ Yukawa matrices in flavor space, $\mu$ the Higgs off-diagonal mass-mixing and the dot products stand for $SU(2)$ invariant products. This superpotential is the most general function satisfying renormalizability, the gauge symmetries of the model and $R$-parity conservation. Our conventions regarding the parameters of the model follow the SUSY Les Houches Accord (SLHA) \cite{27, 28} and the Yukawa matrices must then be given flavor-diagonal. However, the fermionic components of the superfields given in Eq. (36) are gauge-eigenstates and not mass-eigenstates. In the model file, we address this issue by implementing the modified superpotential

$$W_{\text{MSSM}} = (\hat{y}_u)_{ij}U_R^iQ_L^j \cdot H_U - (\hat{y}_d^V_{\text{CKM}})_{ij}D_R^iQ_L^j \cdot H_D - (\hat{y}_e)_{ij}E_R^iL_L^j \cdot H_D + \mu H_U \cdot H_D,$$

where the hatted Yukawa matrices are flavor-diagonal, following thus the SLHA conventions, and where the superfields are gauge-eigenstates. All the rotations diagonalizing the gauge-eigenstate basis will be absorbed at the component field level rather than at the superfield level, as described in Section 8.4, and the role of the CKM matrix appearing in the second term of the superpotential is to compensate the only remaining misalignment between mass and gauge-eigenstates after these rotations. The Lagrangian associated to this superpotential is given by Eq. (34).

Finally, the supersymmetry-breaking Lagrangian is obtained by adding
explicitly all soft supersymmetry-breaking terms at low-energy,

\[ \mathcal{L}_{\text{Soft}} = -\frac{1}{2} \left[ M_1 \tilde{B} \cdot \tilde{B} + M_2 \tilde{W} \cdot \tilde{W} + M_3 \tilde{g} \cdot \tilde{g} + \text{h.c.} \right] \\
- (m^2_Q)_{ij} \tilde{Q}^i_R \tilde{Q}^j_L - (m^2_U)_{ij} \tilde{U}^i_R \tilde{U}^j_L - (m^2_D)_{ij} \tilde{H}^i_u \tilde{H}^j_d - (m^2_{E_{\tilde{L}_{\tilde{L}}}})_{ij} \tilde{E}^i_R \tilde{E}^j_R \\
- (m^2_{E_{\tilde{L}_{\tilde{L}}}})_{ij} \tilde{E}^i_R \tilde{E}^j_R - (m^2_{\tilde{E}_{\tilde{L}_{\tilde{L}}}})_{ij} \tilde{E}^i_R \tilde{E}^j_R - m^2_{H_u} H_u^i H_u^j - m^2_{H_d} H_d^i H_d^j \right) \\
+ \left[ - (T^u)_{ij} \tilde{u}^i_R \tilde{Q}^j_L \cdot H_u + (T^d)_{ij} \tilde{d}^i_R \tilde{Q}^j_L \cdot H_d + (T^e)_{ij} \tilde{e}^i_R \tilde{Q}^j_L \cdot H_d \right. \\
\left. - b H_u \cdot H_d + \text{h.c.} \right]. \tag{43} \\
\]

The first line of this equation contains the gaugino mass terms, the second and third lines the scalar mass terms, \( m^2_Q, m^2_U, m^2_D, m^2_{E_{\tilde{L}_{\tilde{L}}}} \) being 3 × 3 hermitian matrices in generation space and the fourth and fifth line the bilinear and trilinear scalar interactions derived from the superpotential. \( T^u, T^d, \) and \( T^e \) are 3 × 3 matrices in generation space. Similarly to the superpotential of Eq. (42), we write the scalar trilinear interactions as,

\[ \left[ - (T^u)_{ij} \tilde{u}^i_R \tilde{Q}^j_L \cdot H_u + (T^d)_{ij} \tilde{d}^i_R \tilde{Q}^j_L \cdot H_d + (T^e)_{ij} \tilde{e}^i_R \tilde{Q}^j_L \cdot H_d + \text{h.c.} \right] \tag{44} \]

rather than in their original form, following thus the SLHA conventions. Finally, the equation of motions for the auxiliary fields are solved so that they are eliminated from the Lagrangian as described in Section 7.2.

8.4. Electroweak symmetry breaking, particle mixings and Dirac fermions

Due to the strong Yukawa coupling between the superfields \( H_U, Q^3_L \) and \( U^3_R \) in the superpotential, the electroweak symmetry is radiatively broken to electromagnetism and the classical Higgs potential has a non-trivial minimum. Shifting the neutral scalar Higgs bosons by their vacuum expectation values (vevs),

\[ H^0_u \rightarrow \frac{v_u + h^0_u}{\sqrt{2}} \quad \text{and} \quad H^0_d \rightarrow \frac{v_d + h^0_d}{\sqrt{2}} \], \tag{45} \\
\]

where \( v_u \) and \( v_d \) denote the two vevs of the neutral Higgs bosons and \( h^0_u \) and \( h^0_d \) complex scalar fields, we can extract the mass matrices of the electroweak gauge bosons \( B_{\mu} \) and \( W^{\pm}_{\mu} \), diagonalize them, and derive the physical mass-eigenstates, the photon \( A_{\mu} \) and the weak bosons \( W^{\pm}_{\mu} \) and \( Z_{\mu} \). As in the
Standard Model, the transformation rules relating the mass and interaction bases are

\[ W^\pm_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \pm i W^2_\mu), \quad \text{and} \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_w & -\sin \theta_w \\ \sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}, \quad (46) \]

where the weak mixing angle \( \theta_w \) and the physical masses \( M_Z \) and \( M_W \) are defined by

\[ \cos^2 \theta_w = \frac{g^2_w}{g^2_w + g'^2_w}, \quad M_Z = \frac{g_w v^2}{2 \cos \theta_w} \quad \text{and} \quad M_W = \frac{g_w v^2}{2}, \quad (47) \]

with \( v^2 = v_u^2 + v_d^2 \). We define consistently all the mixing parameters in the model file, taking the masses as input parameters. The mixing angles and the vevs are then dependent on the latter. The rotations are implemented using the \texttt{Definitions} option of the particle class. As an example, the \( SU(2)_L \) boson redefinitions are implemented as

\[
\text{Definitions} \rightarrow \{
\text{Wi[\mu,1]} \rightarrow (\text{Wbar[\mu]}+\text{W[\mu]})/\text{Sqrt}[2], \\
\text{Wi[\mu,2]} \rightarrow (\text{Wbar[\mu]}-\text{W[\mu]})/(\text{I*Sqrt}[2]), \\
\text{Wi[\mu,3]} \rightarrow \text{cw Z[\mu]} + \text{sw A[\mu]}
\}
\]

where \( A, Z \) and \( W \) correspond to the model file definitions of the physical gauge bosons, containing all the options required in order to have the interfaces to the Monte Carlo codes and to \textsc{FeynArts} working properly (PDG, \texttt{PropagatorType}, ...), as presented in Ref. \[3\].

In the Higgs sector, three out of the eight real degrees of freedom of the two doublets are the pseudo-Goldstone bosons \( G^\pm \) and \( G^0 \) becoming the longitudinal modes of the weak bosons, while the five others mix to the physical Higgses, \( h^0, H^0, A^0 \) and \( H^\pm \). The diagonalization of the scalar, pseudoscalar and charged Higgs mass matrices leads to the transformation rules

\[
\begin{align*}
h^0_u &= \cos \alpha \ h^0 + \sin \alpha \ H^0 + i \cos \beta \ A^0 + i \sin \beta \ G^0, \\
h^0_d &= - \sin \alpha \ h^0 + \cos \alpha \ H^0 + i \sin \beta \ A^0 - i \cos \beta \ G^0, \\
H^+_u &= \cos \beta \ H^+ + \sin \beta \ G^+, \\
H^-_d &= \sin \beta \ H^- - \cos \beta \ G^-, \\
\end{align*} \quad (48)
\]
where $\alpha$ is the neutral Higgs mixing angle and the $\beta$ angle is defined by $\tan \beta = v_u/v_d$. Following the SLHA conventions [27, 28], all the mixing angles are external parameters included in the SLHA block $\text{HMIX}$, and the rotations are implemented in the scalar Higgs field class definition, e.g., as

\begin{verbatim}
Definitions -> {
    hus[1] -> Cos[beta]*H + Sin[beta]*GP,
    hus[2] -> (v_u + Cos[alp]*h0 + Sin[alp]*H0 + I*Cos[beta]*A0 + I*Sin[beta]*G0)/Sqrt[2]
}
\end{verbatim}

for the $H_u$ doublet labelled by $\text{hus}$, where $H, A0, h0, H0, GP$ and $G0$ are the labels of the physical Higgs and Goldstone bosons. The latter are implemented following the syntax presented in Ref. [3] and contain again all the options required by the interfaces to work properly.

In the fermionic sector, the mass matrix of the neutral partners of the gauge and Higgs bosons is diagonalized through a unitary matrix $N$ which relates the four physical (two-component) neutralinos $\chi_i^0$ to the interaction-eigenstates,

\begin{equation}
\begin{pmatrix}
    \chi_1^0 \\
    \chi_2^0 \\
    \chi_3^0 \\
    \chi_4^0
\end{pmatrix} = N \begin{pmatrix}
    i\tilde{B} \\
    i\tilde{W}^3 \\
    \tilde{H}_d^0 \\
    \tilde{H}_u^0
\end{pmatrix}.
\end{equation}

Similarly, the mass matrix of the charged partners is diagonalized through two unitary matrices $U$ and $V$ relating the interaction-eigenstates to the physical (two-component) charginos eigenstates $\chi_i^\pm$ according to

\begin{equation}
\begin{pmatrix}
    \chi_1^+ \\
    \chi_2^-
\end{pmatrix} = V \begin{pmatrix}
    i\tilde{W}^+ \\
    \tilde{H}_u^0
\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}
    \chi_1^- \\
    \chi_2^+
\end{pmatrix} = U \begin{pmatrix}
    i\tilde{W}^- \\
    \tilde{H}_d^0
\end{pmatrix},
\end{equation}

where the charged winos have undergone the same rotations as the charged gauge bosons,

\begin{equation}
\tilde{W}_\mu^\pm = \frac{1}{\sqrt{2}}(\tilde{W}_\mu^1 \mp i\tilde{W}_\mu^2).
\end{equation}

The factors of $i$ absorbed in the gaugino definition are required in order to obtain real mass matrices. These conventions may seem a bit different from the ones specified by the SUSY Les Houches Accord, accounting for factors of $-i$. However, this sign is purely conventional and is related to the sign of
the exponentials in the Lagrangian of Eq. (39). The choices of Eq. (49) and Eq. (50) ensures that the mixing matrices $N$, $U$ and $V$ are the same as those specified in the accord. In the MSSM model file, the real and imaginary parts of these mixing matrices are considered as input parameters, following the SLHA, while the full matrices themselves are dependent parameters. The physical two-component neutralino fields are implemented following the synthax of Ref. [22] and the rotations are included in the Definitions option of the gauge-eigenstate particle classes. As an example, the wino rotations are given by

\[
\text{Definitions} \rightarrow \{
\text{wow}[s_,1] :> \text{Module}[\{i\}, (\text{Conjugate}[UU[i,1]]*\text{chmw}[s,i] + \text{Conjugate}[VV[i,1]]*\text{chpw}[s,i])/(I*Sqrt[2]) ],
\text{wow}[s_,2] :> \text{Module}[\{i\}, (\text{Conjugate}[UU[i,1]]*\text{chmw}[s,i] - \text{Conjugate}[VV[i,1]]*\text{chpw}[s,i])/(-Sqrt[2]) ],
\text{wow}[s_,3] :> \text{Module}[\{i\}, -I*\text{Conjugate}[NN[i,2]]*\text{neuw}[s,i] ]
\} ,
\]

where $\text{chmw}$, $\text{chpw}$ and $\text{neuw}$ denote the labels of the physical $\chi^-$, $\chi^+$ and $\chi^0$ fields, respectively, and $NN$, $UU$ and $VV$ the mixing matrices.

The diagonalization of the quark sector requires four unitary matrices,

\[
d_L^d \rightarrow V_d d_L^d, \quad d_L^{ic} \rightarrow U_d d_R^{ic}, \quad u_L^i \rightarrow V_u u_L^i, \quad u_R^{ic} \rightarrow U_u u_R^{ic},
\]

so that the superpotential of Eq. (41) is rotated to a form where the Yukawa matrices are diagonal. As a consequence, the charged current weak interactions become proportional to the CKM matrix

\[
V_{\text{CKM}} = V_u^t V_d .
\]

We adopt the standard choice of absorbing these rotations in a redefinition of the down-type quark fields alone,

\[
d_L^i \rightarrow V_{\text{CKM}} d_L^i,
\]

keeping the up-type quark fields unchanged. This field redefinition is implemented in \textsc{FeynRules} through the Definitions option of the \textsc{QLw} class,

\[
\text{Definitions} \rightarrow \{
\text{QLw}[s_, 1, ff_, cc_] \rightarrow uLw[s,ff,cc], \quad \text{QLw}[s_, 2, ff_, cc_] :> \text{Module}[\{ff2\}, \text{CKM}[ff,ff2] dLw[s,ff2,cc] ]
\} ,
\]

45
where $u_{Lw}$ and $d_{Lw}$ denote the (two-component) quark mass eigenstates and the CKM matrix is included in the model file through the SLHA blocks $V_{CKM}$ and $IM_{VCKM}$ for its real and imaginary parts, respectively, the complete matrix being thus an internal parameter. Similarly, the lepton sector is diagonalized through the rotations,

$$e^i_L \rightarrow V_{e}e^i_L, \quad e^i_R \rightarrow U_{e}e^{i^c}_R, \quad \nu^i_L \rightarrow V_{\nu}\nu^i_L,$$  \hspace{1cm} (55)

rendering the charged current interactions proportional to the PMNS matrix

$$V_{PMNS} = V^{\dagger}_e V_{\nu}.$$ \hspace{1cm} (56)

We adopt here the choice of absorbing the rotations in a redefinition of the neutrino fields alone,

$$\nu^i_L \rightarrow V_{PMNS}\nu^i_L,$$ \hspace{1cm} (57)

leaving the charged lepton fields unchanged. The fermion rotations presented above legitimate the implementation of the superpotential under the form of Eq. (42). It can be checked that the extracted quark and lepton mass matrices are indeed diagonal and no further rotation is necessary. The PMNS matrix is implemented in the model file within the SLHA blocks $UP_{PMNS}$ and $IM_{UP_{PMNS}}$ for its real and imaginary parts, respectively, while again, the entire matrix is considered as an internal parameter. On a similar fashion as for the quark sector, the implementation of the field redefinitions is given by

```math
Definitions -> {
    LLw[s_, 1, ff_] := Module[{ff2},
        PMNS[ff, ff2]*vLw[s, ff2],
        LLw[s_, 2, ff_] -> eLw[s, ff]
    ],
}
```

where $LLw$ is the gauge-eigenstate field and $vLw$ and $eLw$ the mass-eigenstate ones, the right-handed components being unchanged.

In the scalar sector, we define the super-CKM and super-PMNS bases as the bases in which the scalar fields undergo the same rotations as their fermionic counterparts. However, the fermion and sfermion fields can be

\footnote{There are many different models accounting for neutrino masses. Since there is thus not any unique way to define neutrino mass terms, we are not including them in the implemented Lagrangian and we only account for neutrino mixings through field redefinitions.}
misaligned due to possible off-diagonal mass terms in the supersymmetry-breaking Lagrangian $\mathcal{L}_{\text{Soft}}$, and four additional rotations $R^u, R^d, R^e$ and $R^\nu$ are (in general) required,

$$
\begin{pmatrix}
\tilde{u}_1 \\
\tilde{u}_2 \\
\tilde{u}_3 \\
\tilde{u}_4 \\
\tilde{u}_5 \\
\tilde{u}_6 \\
\end{pmatrix} = R^u
\begin{pmatrix}
\tilde{u}_L \\
\tilde{c}_L \\
\tilde{t}_L \\
\tilde{u}_R \\
\tilde{c}_R \\
\tilde{t}_R \\
\end{pmatrix},
\begin{pmatrix}
\tilde{d}_1 \\
\tilde{d}_2 \\
\tilde{d}_3 \\
\tilde{d}_4 \\
\tilde{d}_5 \\
\tilde{d}_6 \\
\end{pmatrix} = R^d
\begin{pmatrix}
\tilde{d}_L \\
\tilde{s}_L \\
\tilde{b}_L \\
\tilde{d}_R \\
\tilde{s}_R \\
\tilde{b}_R \\
\end{pmatrix},
\begin{pmatrix}
\tilde{e}_1 \\
\tilde{e}_2 \\
\tilde{e}_3 \\
\tilde{e}_4 \\
\tilde{e}_5 \\
\tilde{e}_6 \\
\end{pmatrix} = R^e
\begin{pmatrix}
\tilde{e}_L \\
\tilde{\mu}_L \\
\tilde{\tau}_L \\
\tilde{e}_R \\
\tilde{\mu}_R \\
\tilde{\tau}_R \\
\end{pmatrix},
\begin{pmatrix}
\tilde{\nu}_1 \\
\tilde{\nu}_2 \\
\tilde{\nu}_3 \\
\end{pmatrix} = R^\nu
\begin{pmatrix}
\tilde{\nu}_e \\
\tilde{\nu}_\mu \\
\tilde{\nu}_\tau \\
\end{pmatrix},
$$

(58)

where the flavor-eigenstates are denoted explicitly and where the physical eigenstates are mass-ordered, from the lightest to the heaviest. Again, the four rotation matrices are implemented after splitting their real and imaginary parts following the SLHA-2 conventions. The physical scalar fields are implemented following the syntax of Ref. [3] and contains all the information needed by the various FEYNRULES interfaces, including the field redefinitions in the Definitions option. As an example, the redefinition of the scalar component of the superfield $U_R^i$ is given by

```
Definitions -> {
    URs[ff_, cc_] := Module[{ff2}, subar[ff2, cc] * RuR[ff2, ff]]
},
```

where $\text{RuR}$ refers to the three last columns of the mixing matrix $R^u$ and $su$ denote up-type squark mass-eigenstates.

Finally, we define Dirac representations $\psi$ for the fermions, because most generators work in terms of Dirac fermion rather than Weyl fermions, the former being required at the Monte Carlo generator level. In terms of the Weyl fermions introduced above, the Dirac fermions read

$$
\psi_{u^i} = \left( \frac{u^i_L}{\bar{u}^c_R} \right), \quad \psi_{d^i} = \left( \frac{u^c_L}{\bar{d}^c_R} \right), \quad \psi_{e^i} = \left( \frac{e^i_L}{\bar{e}^c_R} \right), \quad \psi_{\nu^i} = \left( \frac{\nu^c_L}{\bar{\nu}^c_R} \right),
\psi_{\chi^{0}_i} = \left( \frac{\chi^{0}_i}{\bar{\chi}^{+}_i} \right), \quad \psi_{\chi^{\pm}_i} = \left( \frac{\chi^{\pm}_i}{\bar{\chi}^{0}_i} \right), \quad \psi_{\tilde{g}} = \left( \frac{i \tilde{g}}{-i \tilde{g}} \right).
$$

(59)
The four-component Dirac and Majorana fermions are implemented following the syntax of Ref. [3], where we associate a four-component fermion to its Weyl components through the WeylComponents option of the particle class [22]. As an example, the charged lepton is implemented as

\[
F[1] == \{
  \text{ClassName} \rightarrow \text{l},
  \text{SelfConjugate} \rightarrow \text{False},
  \text{Indices} \rightarrow \{\text{Index}[\text{GEN}]\},
  \text{FlavorIndex} \rightarrow \text{GEN},
  \text{WeylComponents} \rightarrow \{\text{eLw}, \text{ERwbar}\},
  \ldots
\},
\]

where the dots stand for additional options such as those required by the Monte Carlo tools.

The FEYNRULES function WeylToDirac allows to perform the expansion of the Weyl fermions in terms of the Dirac fields. However, we first need to address the issue related to the antifundamental color representation which the right-handed quark fields lie in. This is mandatory in order to have one single color representation for the Dirac fields, i.e., the fundamental one, and for the interfaces to the various tools linked to FEYNRULES to work properly. Denoting \( T \) and \( \bar{T} \) the fundamental and antifundamental color representations, and using the property \( \bar{T} = -T^t \), the problem is solved by implementing the instructions,

\[
\text{Colourb} = \text{Colour} \quad ,
\text{Lag} = \text{Lag} \/. \{ \text{Tb}[a_,i_,j_] \rightarrow -T[a,j,i] \} \ ,
\]

where \( \text{Lag} \) denotes the MSSM Lagrangian. Then, since Dirac fermions are defined only for physical particles, we start with an expansion of the \( SU(2)_L \) multiplets in terms of their components,

\[
\text{Lag} = \text{ExpandIndices}[ \text{Lag} \ , \text{FlavorExpand} \rightarrow \{\text{SU2W}, \text{SU2D}\} \] ,
\]

before eliminating the Weyl fermions from the Lagrangian,

\[
\text{Lag} = \text{WeylToDirac}[ \text{Lag} \] .
\]
The obtained Lagrangian is now suitable for the calculation of the Feynman rules through the function \texttt{FeynmanRules} or to be exported to \textsc{FeynArts} with the function \texttt{WriteFeynArtsOutput} or to any Monte Carlo tools linked to \textsc{FeynRules} via the functions \texttt{WriteCHOutput}, \texttt{WriteMGOOutput}, \texttt{WriteSHOutput}, \texttt{WriteUFO} \cite{29} or \texttt{WriteWOOutput} \cite{3}.

8.5. Validation

Our implementation has been validated against the (public) MSSM implementation in the current \textsc{FeynRules} version 1.4.x. The latter is based on component fields and Dirac fermions and has been validated both against the literature and the built-in MSSM implementations in various Monte Carlo tools \cite{4}. We have checked analytically that the Feynman rules obtained with \textsc{FeynRules} using the superfield MSSM implementation correspond to those found in the literature and verified that

\begin{verbatim}
FeynmanRules[ LagSF - LagComponents ]
\end{verbatim}

provides an empty list of Feynman rules, ensuring that the two Lagrangians are equal. In the expression above, \texttt{LagSF} denotes the Lagrangian obtained using the superspace module of \textsc{FeynRules} and \texttt{LagComponents} is the one available in the public version of the model.

8.6. Gauge choice

In Eq. \eqref{18}, we have introduced the Goldstone bosons absorbed by the weak gauge bosons to get their longitudinal polarization. Although absent in calculations performed in unitarity gauge, they must be included if another gauge is used. In addition, a gauge-fixing Lagrangian must be included in the model, as well as ghost fields associated to the gauge bosons, together with the corresponding Lagrangian constructed from the BRST formalism. These two Lagrangians are included in our MSSM implementation in the specific choice of the Feynman gauge.

The gauge-fixing Lagrangian is most conveniently derived after rewriting the Higgs doublets in terms of eight real scalar fields $\phi_i^q$ with $i = 1, \ldots, 4$. The Higgs doublets become

\begin{equation}
\begin{aligned}
H_u &= \frac{1}{\sqrt{2}} \sqrt{2} \left( \phi_u^1 + i \phi_u^2 \right) \equiv \left( \phi_u^1, \phi_u^2, \phi_u^3, \phi_u^4, 0, 0, 0, 0 \right), \\
H_d &= \frac{1}{\sqrt{2}} \left( \phi_d^1 + i \phi_d^2 \right) \equiv \left( 0, 0, 0, \phi_d^1, \phi_d^2, \phi_d^3, \phi_d^4, \phi_u^1 \right),
\end{aligned}
\end{equation}

(60)
and the gauge-fixing Lagrangian is given by
\[ \mathcal{L}_{\text{GF}} = -\frac{1}{2} \left( G_B^\dagger G_B + G_W^k G_W^k + G_g^a G_g^a \right) \] (61)
with
\[ G_B = \partial_\mu B^\mu - g' \left( -\frac{i}{2} \langle H_u \rangle \right) \cdot H_u - g' \left( \frac{i}{2} \langle H_d \rangle \right) \cdot H_d , \]
\[ G_W^k = \partial_\mu W_k^\mu - g_w \left( -\frac{i}{2} \langle \sigma^k H_u \rangle \right) \cdot H_u - g_w \left( \frac{i}{2} \langle \sigma^k H_d \rangle \right) \cdot H_d , \]
\[ G_g^a = \partial_\mu g_g^a , \]
where the dot product stands for the scalar product of the field space introduced above. Comparing with Eq. (45), we derive the only non-zero vevs as \( \langle \phi^3_u \rangle = v_u \) and \( \langle \phi^1_d \rangle = v_d \).

The Fadeev-Popov ghost Lagrangian is derived from the gauge variation of the gauge-fixing functions and reads, for the \( B \)-ghost \( u_B \), the \( W \)-ghost \( u_W \) and the gluon ghost \( u_g \),
\[ \mathcal{L}_{\text{ghost}} = -\bar{u}_B \partial_\mu \partial^\mu u_B - \bar{u}_W \partial_\mu D^\mu u_W - \bar{u}_g \partial_\mu D^\mu u_g + \mathcal{L}_{\text{ghost},\phi} , \] (63)
where the adjoint representation indices are understood and the covariant derivatives are taken in the adjoint representation,
\[ D_\mu u_W = \partial_\mu u_W + g_w \epsilon_{jk} W_j^k u_W^k , \quad D_\mu u_g^a = \partial_\mu u_g^a + g_s f_{bc}^a g_g^b u_g^c . \] (64)
Finally, the scalar piece of the ghost Lagrangian is given by
\[ \mathcal{L}_{\text{ghost,}\phi} = \bar{u}_A \left[ \langle T^A \langle H_u \rangle \rangle \cdot \langle T^A' [\langle H_u \rangle + H_u] \rangle + \langle T^A \langle H_d \rangle \rangle \cdot \langle T^A' [\langle H_d \rangle + H_d] \rangle \right] u_{A'} , \] (65)
with \( u_A = u_B, u_W^1, u_W^2, u_W^3 \), the corresponding real representation matrices being \( T^A = -iY, -i\sigma^1/2, -i\sigma^2/2, -i\sigma^3/2 \) and the dot product stands for the scalar product defined in the basis \( \{ \phi^i_u, \phi^i_d \} \) introduced above. The ghosts related to the physical gauge bosons are obtained by rotating those related to the gauge-eigenstates parallel to the latter.

The MSSM implementation in \textsc{FeynRules} contains the two Lagrangians of Eq. (61) and Eq. (63) and is hence fully expressed in Feynman gauge, the unitarity gauge being recovered by removing all the ghosts and Goldstone bosons from the Lagrangian.
9. Conclusions

In this paper we presented a superspace module for the FeynRules package, and the whole module is distributed together with the the FeynRules package starting from version 1.6.x. The module allows to perform computations of superspace quantities involving Grassmann variables and is well suited for the implementation of Lagrangians in terms of superfields. The package hence allows to implement supersymmetric models into FeynRules directly in terms of superfields, the superfield expressions can then be expanded into component fields and the corresponding coefficients of the Grassmann variables extracted in an automated way. Furthermore, since the only piece of a supersymmetric Lagrangian that is not fixed by supersymmetry and gauge invariance is the superpotential, the package allows for an automatic generation of all the kinetic terms and gauge interaction terms for the superfields, thus reducing the implementation of a supersymmetric Lagrangian to the almost trivial task of writing down the superpotential.

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