Can the $\Lambda\pi$ scattering phase shifts be large?

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Abstract

Motivated by the presence of nearby thresholds in other baryon-meson channels with $I = 1$ and $S = -1$, we investigate whether the $\Lambda\pi$ scattering phase shifts at a center-of-mass energy equal to the $\Xi$ mass could be larger than suggested by lowest-order chiral perturbation theory. Within a coupled-channel $K$-matrix approach, we find that the $S$-wave phase shift could be as large as $-7^\circ$.  

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1 Introduction

The $CP$-violating observable, $A$, in weak nonleptonic hyperon decays of the form $B \rightarrow B'\pi$ depends on the strong-rescattering phases of the final state [1]. At leading order, this asymmetry is given by

$$A = -\tan(\delta_S - \delta_P) \sin(\phi_S - \phi_P),$$

(1)

where $\delta_S$ and $\delta_P$ ($\phi_S$ and $\phi_P$) are the strong-rescattering (weak) phases in the $S$- and $P$-wave components, respectively, of the decay amplitude. Currently the HyperCP (E871) experiment at Fermilab is in the process of measuring this $CP$-violating observable through the asymmetry sum $A(\Lambda) + A(\Xi)$ in the chain of decays $\Xi \rightarrow \Lambda\pi \rightarrow p\pi\pi$ [2]. The calculation of this observable, therefore, requires the knowledge of both the phase shifts for $N\pi$ scattering at the $\Lambda$ mass and those for $\Lambda\pi$ scattering at the $\Xi$ mass. The former phase shifts have been extracted from experiment (albeit with large errors) [3], but there is no experimental data for the latter.

An early calculation [4] of the $\Lambda\pi$ scattering phase shifts at $m_\Xi$ indicated that the $S$-wave phase shift was large, the result being $\delta_S = -18.7^\circ$ and $\delta_P = -2.7^\circ$. If correct, this would suggest that $CP$ violation in both decays $\Xi \rightarrow \Lambda\pi$ and $\Lambda \rightarrow p\pi\pi$ could yield similar contributions to the measurement of E871, making a theoretical prediction harder. More recently, this calculation has been repeated in the context of heavy-baryon chiral perturbation theory, with very different results. At leading order, it was found in Ref. [5] that $\delta_S = 0$ and $\delta_P = -1.7^\circ$. The implication of this result is that the $CP$-violating observable in E871 is probably dominated by $CP$ violation in $\Lambda \rightarrow p\pi\pi$. The vanishing of $\delta_S$ in this calculation results from the heavy-baryon limit. An estimate of relativistic corrections to the heavy-baryon result has been performed in Ref. [6], where it was found (within a leading-order calculation in the relativistic theory) that $\delta_S = 1.2^\circ$ and $\delta_P = -1.7^\circ$. Leading-order (in heavy-baryon chiral perturbation theory) calculations of the $N\pi$ [8] and $\Xi\pi$ [9] scattering phase shifts have also been carried out. They suggest that the smallness of $\delta_S$ at lowest order in chiral perturbation theory ($\chi$PT) is mostly a kinematic effect associated with the small pion-momentum available in $\Xi \rightarrow \Lambda\pi$.

Given the two very different results for $\delta_S$ in $\Lambda\pi$ scattering, it is important to estimate the effect of physics not present in the leading-order $\chi$PT calculation. A first attempt to investigate this question was carried out in Ref. [10]. Their approach was to look for possible resonant enhancements. To this effect, they considered the nearest resonance with the correct quantum numbers, the $\Sigma(1750)$ with $I = 1, J^P = \frac{1}{2}^-$ . Although the parameters of this resonance are not well known, the authors of Ref. [10] allowed them to vary in a reasonable range to conclude that the contribution to $\delta_S$ from this source was not more than about $0.5^\circ$.

In this paper we explore the possibility of an enhancement in $\delta_S$ due to the presence of nearby thresholds in other baryon-meson channels with the same quantum numbers. In particular, we

\footnote{We have redone this estimate and obtained the same result, but with $\delta_S$ having the opposite sign [7].}
wish to check the role of the $S$-wave $\Sigma\pi$ channel, which has a threshold only 10 MeV above $m_\Xi$. It is known from the Weinberg-Tomozawa theorem that the scattering length in this channel is very attractive [11]. To investigate this issue, we present two separate estimates. For the first one, we will take the point of view that any such effects can be parameterized by next-to-leading-order terms in chiral perturbation theory. The authors of Ref. [12] have recently studied the coupled-channel problem for the $S = -1, I = 1$ baryon-meson system, within a certain model, and have parameterized their results in terms of specific values for the coupling constants in the heavy-baryon chiral Lagrangian up to the next-to-leading order ($\mathcal{O}(p^2)$). We employ these values for the coupling constants in our first estimate. For our second estimate, we simply use leading-order $\chi$PT to derive all the amplitudes in the $S = -1, I = 1$ baryon-meson system and employ a $K$-matrix formalism to incorporate the effects of unitarity in the coupled-channel problem.

2 $\mathcal{O}(p^2)$ heavy-baryon chiral Lagrangian

We write the chiral Lagrangian for the strong interaction of the lightest (octet) baryons up to order $p^2$ in heavy-baryon $\chi$PT as the sum of two terms,

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)},$$

where the superscript refers to the chiral order. The first term is given by [13]

$$\mathcal{L}^{(1)} = \langle \bar{B}_v w \cdot \mathcal{D} B_v \rangle + 2D \langle \bar{B}_v S^\mu_v \{A_\mu, B_v\} \rangle + 2F \langle \bar{B}_v S^\mu_v [A_\mu, B_v] \rangle,$$

with $\langle \cdots \rangle \equiv \text{Tr} (\cdots)$. The second term can be written in the most general form as [12 14]

$$\mathcal{L}^{(2)} = \mathcal{L}^{(2)}_{\text{rc}} + b_D \langle \bar{B}_v \left\{ \chi_+, B_v \right\} \rangle + b_F \langle \bar{B}_v \left[ \chi_+, B_v \right] \rangle + b_0 \langle \chi_+ \rangle \langle \bar{B}_v B_v \rangle + \langle \bar{B}_v B_v \rangle \langle (v \cdot \mathcal{A})^2 \rangle + 2d_F \langle \bar{B}_v [(v \cdot \mathcal{A})^2, B_v] \rangle + 2d_D \langle \bar{B}_v B_v \rangle \langle (v \cdot \mathcal{A})^2 \rangle + 2d_1 \langle \bar{B}_v v \cdot \mathcal{A} \rangle \langle v \cdot \mathcal{A} B_v \rangle + 2g_D \langle \bar{B}_v \left\{ \mathcal{A} \cdot \mathcal{A}, B_v \right\} \rangle + 2g_F \langle \bar{B}_v \left[ \mathcal{A} \cdot \mathcal{A}, B_v \right] \rangle + 2g_0 \langle \bar{B}_v B_v \rangle \langle \mathcal{A} \cdot \mathcal{A} \rangle + 2g_1 \langle \bar{B}_v \mathcal{A} \rangle \cdot \langle \mathcal{A} B_v \rangle + 2h_D \langle \bar{B}_v i\mathbf{\sigma} \cdot \left\{ \mathcal{A} \times \mathcal{A}, B_v \right\} \rangle + 2h_F \langle \bar{B}_v i\mathbf{\sigma} \cdot [\mathcal{A} \times \mathcal{A}, B_v] \rangle + 2h_1 \langle \bar{B}_v i\mathbf{\sigma} \times \mathcal{A} \rangle \cdot \langle \mathcal{A} B_v \rangle,$$

where

$$\mathcal{L}^{(2)}_{\text{rc}} = \frac{-1}{2m_0} \langle \bar{B}_v [(D^2 - (v \cdot \mathcal{D}))^2] B_v - \bar{B}_v [S^\mu_v, S^\mu_v] [[A_\mu, A_\mu], B_v] \rangle - \frac{iD}{m_0} \left( \langle \bar{B}_v S^\mu_v \cdot \mathcal{D} \{v \cdot \mathcal{A}, B_v\} \rangle + \langle \bar{B}_v S^\mu_v \{v \cdot \mathcal{A}, \mathcal{D}_\mu B_v\} \rangle \right).$$
\[ -\frac{iF_{\text{LL}}}{m_0} \left( \langle \bar{B}_v S_v \cdot D[v \cdot A, B_v] \rangle + \langle \bar{B}_v S^\mu_v [v \cdot A, D_\mu B_v] \rangle \right) - \frac{DF_{\text{LL}}}{m_0} \langle \bar{B}_v [(v \cdot A)^2, B_v] \rangle \\
- \frac{D^2}{2m_0} \langle \bar{B}_v \{v \cdot A, \{v \cdot A, B_v\}\} \rangle - \frac{F^2}{2m_0} \langle \bar{B}_v [v \cdot A, [v \cdot A, B_v]] \rangle \]  

is the \(1/m_0\) (leading relativistic) correction to the leading-order Lagrangian at order \(p^2\), with \(m_0\) being the octet-baryon mass in the chiral limit. The constants, \(b, d, g\), are free parameters (in addition to the familiar \(D\) and \(F\)) that occur at this order, and we will obtain their values from the model of Ref. \([12]\). In these formulae, \(B_v\) is the usual \(3\times3\) matrix containing the (velocity dependent) octet-baryon fields, \(v\) the baryon velocity, \(S_v\) the spin operator, and

\[ A_\mu = \frac{i}{2} \left( \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right) = \frac{\partial_\mu \varphi}{2f} + \mathcal{O}(\varphi^3), \]

\[ \chi_+ = \xi^\dagger M_\varphi^2 \xi^\dagger + \xi M_\varphi^2 \xi = 2M_\varphi^2 - \frac{1}{4f^2} \{ \varphi, \{ \varphi, M_\varphi^2 \} \} + \mathcal{O}(\varphi^4), \]

where \(\varphi\) is the \(3\times3\) matrix for the octet of pseudo-scalar bosons, \(f = f_\pi = 92.4\,\text{MeV}\) is the pion decay constant, and \(M_\varphi^2 = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)\) the pseudo-scalar mass matrix in the isospin limit.

The total amplitude for \(\Lambda \pi \rightarrow \Lambda \pi\), up to order \(p^3\), is derived from the diagrams in Fig. [4]. In the center-of-mass (CM) frame, it is given by

\[ \mathcal{M}_{\Lambda \pi} = \frac{2m_\Lambda}{f^2} \chi_i^\dagger \left[ \frac{D^2}{3m_0} \left( m_\pi^2 - k^2 + \frac{k^4}{3E_\pi^2} \right) + \left( \frac{4bD}{3} + 4b_0 \right) m_\pi^2 - \left( \frac{2dD}{3} + 2d_0 \right) E_\pi^2 \right] \]

\[ + \frac{D^2}{3} \frac{3(k' \cdot k)^2 - k^4}{3m_0 E_\pi^2} + \frac{k' \cdot k}{E_\pi^2} \left( \frac{2D^2}{3} \frac{m_\Sigma - m_\Lambda}{E_\pi^2} - \frac{2gD}{3} - 2g_0 \right) \]

\[ + \frac{2D^2}{3} \frac{\mathbf{i} \sigma \cdot k' \times k}{E_\pi} \left( 1 + \frac{E_\pi}{m_0} - \frac{k'^2 + k \cdot k'}{2m_0 E_\pi} \right) \chi_i, \]

where \(\chi_i\) (\(\chi_f\)) is the Pauli spinor of the initial (final) \(\Lambda\), and \(\mathbf{k} (\mathbf{k}')\) is the three-momentum of the initial (final) pion. The partial-wave amplitudes are then extracted using standard techniques\(^2\) and one finds in the \(J = \frac{1}{2}\) channel

\[ f^{(S)}_{\Lambda \pi} = \frac{-m_\Lambda}{4\pi f^2 \sqrt{s}} \left[ \frac{D^2}{3m_0} \left( E_\pi^2 - 2k^2 + \frac{k^4}{3E_\pi^2} \right) + \left( \frac{4bD}{3} + 4b_0 \right) m_\pi^2 - \left( \frac{2dD}{3} + 2d_0 \right) E_\pi^2 \right], \]

\[ f^{(P)}_{\Lambda \pi} = \frac{-k^2 m_\Lambda}{4\pi f^2 \sqrt{s}} \left[ \frac{4D^2}{9E_\pi} \left( 1 + \frac{2E_\pi^2 - k^2}{2m_0 E_\pi} + \frac{m_\Sigma - m_\Lambda}{2E_\pi} \right) - \frac{2}{9} (gD + 3g_0) \right]. \]

\(^2\)At this order there are no loop contributions. The latter begin at \(\mathcal{O}(p^4)\).

\(^3\)See, e.g., Ref. [15].
Figure 1: Diagrams for $\Lambda\pi \to \Lambda\pi$ up to order $p^2$. A dashed (solid) line denotes a pion (octet baryon) field. The baryon in the intermediate states is $\Sigma$. Solid and hollow vertices are generated by $\mathcal{L}^{(1)}$ in Eq. (3) and $\mathcal{L}^{(2)}$ in Eq. (4), respectively.

For our numerical calculation, we will adopt the parameter values provided by Ref. [12]. In that work, the chiral Lagrangian $\mathcal{L}$ in Eq. (2) was used as a starting point for constructing a coupled-channel potential model to study $N\bar{K} \to N\bar{K}, \Lambda\pi, \Sigma\pi$ and other measured processes. We employ in particular the parameter values extracted in Ref. [16], $D = 0.782$, $m_0 = 0.869$ GeV,

\begin{align}
D &= 0.782, & m_0 &= 0.869 \text{ GeV}, \\
b_0 &= -0.320 \text{ GeV}^{-1}, & b_D &= 0.066 \text{ GeV}^{-1}, \\
d_0 &= -0.996 \text{ GeV}^{-1}, & d_D &= 0.512 \text{ GeV}^{-1}, \\
g_0 &= -1.492 \text{ GeV}^{-1}, & g_D &= 0.320 \text{ GeV}^{-1}.
\end{align}

Thus, with $\sqrt{s} = m_\Xi$ and \(|k| \simeq 0.137 \text{ GeV}\), we obtain for the $J = \frac{1}{2}$ channel the phase shifts

$$\delta_S \simeq -2.5^\circ, \quad \delta_P \simeq -3.3^\circ.$$  

Of course, the parameters in Eq. (10) are not known precisely. If we allow them to take the following ranges of values (in the same units as before):

\begin{align}
0.4 < D < 0.8, & \quad 0.7 < m_0 < 1.2, \\
-0.6 < b_0 < -0.3, & \quad 0.02 < b_D < 0.08, \\
-1.0 < d_0 < -0.7, & \quad 0.3 < d_D < 0.6, \\
-1.5 < g_0 < -1.0, & \quad 0.3 < g_D < 0.5,
\end{align}

which are suggested by various tree- and loop-level $\chi$PT calculations [13, 17], as well as the results of Ref. [12], we find the following ranges for the phase shifts:

$$-3.0^\circ < \delta_S < +0.4^\circ, \quad -3.5^\circ < \delta_P < -1.2^\circ.$$  

The contributions of the lowest-order terms to these numbers are $\delta_S^{(1)} = 0$ and $-1.7^\circ < \delta_P^{(1)} < -0.4^\circ$. The $1/m_0$ terms, especially in the $S$-wave, give small corrections, $-0.06^\circ < \delta_S^{(e)} < -0.01^\circ$.
and $-0.4^\circ < \delta_P^c < -0.1^\circ$. Therefore, the rest of the $p^2$ terms in the $S$-wave generate the bulk of $\delta_S$, and those in the $P$-wave are comparable to the lowest-order term in their contribution to $\delta_P$.

It is useful to compare the result above with the leading-order result of Ref. [5]. In that calculation, the spin-$\frac{3}{2}$ decuplet-baryon degrees of freedom are also included in the chiral Lagrangian, so that at leading order [13]

$$\mathcal{L}^{(1)} = \left\langle \bar{B}_v \, i \sigma \cdot \mathbf{k} \cdot \mathbf{k}' \left( \frac{1}{2} D^2 \frac{1}{\sqrt{s - m_{\Sigma}}} + \frac{1}{3} D^2 \frac{1}{E_\pi - E_\Lambda + m_{\Sigma}} - \frac{1}{12} C^2 \right) \chi_i \right\rangle + \frac{2m_\Lambda}{f^2} \mathcal{M}_{\Lambda \pi} \left( \chi_i \right)
$$

where $D^2 = \sqrt{s - m_{\Sigma}}$ and $C^2 = \sqrt{s - m_{\Sigma}}$.

The resulting amplitude for $\Lambda \pi \rightarrow \Lambda \pi$ in the $J = \frac{1}{2}$ channel receives nonzero contributions from the last three diagrams in Fig. [2] and is given by

$$\mathcal{M}_{\Lambda \pi} = \frac{2m_\Lambda}{f^2} \mathcal{M} \left( \right) \left( \chi_i \right)
$$

the $\Sigma$ and $\Sigma^*$ being the intermediate baryons. This result, at order $p^1$, actually contains some contributions from the chiral Lagrangian of order $p^2$ which are implicit in the denominators. The partial-wave amplitudes in the $J = \frac{1}{2}$ channel are then

$$f_{\Lambda \pi}^{(S)} = 0 , \quad f_{\Lambda \pi}^{(P)} = \frac{-k^2 m_\Lambda}{4 \pi f^2 \sqrt{s}} \left( \frac{1}{3} D^2 \frac{1}{\sqrt{s - m_{\Sigma}}} + \frac{1}{3} D^2 \frac{1}{E_\pi - E_\Lambda + m_{\Sigma}} - \frac{1}{3} C^2 \right).
$$

Consequently, using $\sqrt{s} = m_{\Sigma}$ and the tree-level values $D = 0.8$ and $C = 1.7$, we find for the $J = \frac{1}{2}$ channel the phase shifts

$$\delta_S = 0 , \quad \delta_P \simeq -1.5^\circ.
$$

If chiral-symmetric masses $m_{\Sigma} = m_0 \simeq 1.15$ GeV and $m_{\Sigma^*} = m_T \simeq 1.38$ GeV are used instead, we obtain $\delta_P \simeq -1.0^\circ$. In each of these $\delta_P$ values, roughly $-2^\circ$ arises from the two diagrams involving the $\Sigma$, and about $+0.8^\circ$ comes from the diagram containing the $\Sigma^*$. One can see that the value of the $\Sigma$ contribution is compatible with the $\delta_P^{(1)}$ range quoted above. However, the $\Sigma^*$ contribution is opposite in sign to all the $O(p^2)$ contributions to $\delta_P$ that we have estimated using the Lagrangian in Eq. (13). This suggests that there are additional contributions beyond that of the $\Sigma^*$ which can be expected to be significant.

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4For hadronic fields, we follow the notation of Ref. [13].

5These are extracted from hyperon semileptonic decays (which also give $F = 0.5$) and the strong decays $T \rightarrow B\phi$, respectively.
In Ref. [6], the baryons are not treated as heavy and the phase shifts are computed using the relativistic version of the lowest-order Lagrangian in Eq. (14). We have repeated the calculation (with \( D = 0.8 \) and \( \mathcal{C} = 1.7 \)) and found \( \delta_S = -1.2^\circ \) and \( \delta_P = -1.7^\circ \), which agrees with the result of Ref. [3], except that \( \delta_S \) has the opposite sign. \( \delta_S \) here, only \(-0.1^\circ\) is generated by the \( \Sigma\)-mediated diagrams, with the rest, \(-1.1^\circ\), coming from the \( \Sigma^*\)-mediated diagram. We can see that the \( \Sigma \) contribution is comparable to the \( \delta_S^{\Sigma} \) values in the \( \mathcal{O}(p^2) \) heavy-baryon estimate. In contrast, the dominant \( \Sigma^* \) contribution is roughly only half of the \( \delta_S \) value in the \( \mathcal{O}(p^2) \) heavy-baryon result, although the two have the same sign. This again suggests that other contributions in addition to that of the \( \Sigma^* \) may be important. In the \( P \)-wave, the \( \Sigma \) and \( \Sigma^* \) contributions to \( \delta_P \) are \(-2.7^\circ \) and \(+1.0^\circ\), respectively, and so these are similar to their \( \mathcal{O}(p^1) \) heavy-baryon counterparts.

3 K-matrix approach

The SU(3) picture that we have implies that the \( \Lambda\pi \) state is coupled to the states \( \Sigma\pi, N\bar{K}, \Sigma\eta, \) and \( \Xi K \) with \( S = -1 \) and \( I = 1 \). Thus the \( \Lambda\pi \) scattering can be treated as a problem with five coupled channels. Although at \( \sqrt{s} = m_\Xi \) all the inelastic channels are below threshold, they may significantly affect the elastic one through unitarity constraints. The inclusion of such kinematically closed channels has been recently shown to be important in the case of \( N\bar{K} \) interactions [19].

In order to estimate the impact on the \( \Lambda\pi \) channel of the others coupled to it, we employ a \( K \)-matrix approach. This method guarantees that the resulting partial-wave amplitudes satisfy unitarity exactly. We follow the formalism described in Ref. [15]. For the \( K \)-matrix elements, we will make the simplest approximation and use only the partial-wave amplitudes at leading order in \( \chiPT \), obtained from \( L^{(1)} \) in Eq. (14).

The relevant partial-waves can be extracted by choosing the five isospin states

\[
\begin{align*}
|\Lambda\pi, I = 1\rangle &= |\Lambda\pi^-\rangle, \quad |\Sigma\pi, I = 1\rangle = \frac{1}{\sqrt{2}}(|\Sigma^0\pi^-\rangle - |\Sigma^-\pi^0\rangle), \\
|N\bar{K}, I = 1\rangle &= -|nK^-\rangle, \quad |\Sigma\eta, I = 1\rangle = |\Sigma^-\eta\rangle, \quad |\Xi K, I = 1\rangle = -|\Xi^-K^0\rangle. 
\end{align*}
\]  

(18)

The phase convention here is consistent with the structure of the \( \varphi \) and \( B_v \) matrices.

\footnote{We have checked that expanding the part of the amplitude arising from the \( \Sigma \) diagrams up to order \( p^2 \) does lead to the \( D^2 \) terms in the heavy-baryon \( \mathcal{O}(p^2) \) amplitude in Eq. (16).}
The lowest-order $S$-wave amplitude for $B\phi \rightarrow B'\phi'$ with $S = -1$ and $I = 1$ is derived from the first diagram in Fig. 2 and, in the CM frame, has the form

$$f_{B\phi,B'\phi'}^{(S)} = -C_{B\phi,B'\phi'} \sqrt{m_B m_{B'}} \frac{E_\phi + E_{\phi'}}{16\pi f_\pi^2 \sqrt{s}},$$

where $C_{B'\phi',B\phi} = C_{B\phi,B'\phi'}$. Using the isospin states above, one obtains

$$C_{\Lambda\pi,\Lambda\pi} = C_{\Lambda\pi,\Sigma\pi} = 0, \quad C_{\Lambda\pi,N\bar{K}} = \sqrt{3}/2, \quad C_{\Lambda\pi,\Sigma\eta} = 0, \quad C_{\Lambda\pi,\Xi K} = \sqrt{3}/2,$$

$$C_{\Sigma\pi,\Sigma\pi} = -2, \quad C_{\Sigma\pi,N\bar{K}} = -1, \quad C_{\Sigma\pi,\Sigma\eta} = 0, \quad C_{\Sigma\pi,\Xi K} = +1,$$

$$C_{N\bar{K},N\bar{K}} = -1, \quad C_{N\bar{K},\Sigma\eta} = \sqrt{3}/2, \quad C_{N\bar{K},\Xi K} = 0,$$

$$C_{\Sigma\eta,\Sigma\eta} = 0, \quad C_{\Sigma\eta,\Xi K} = \sqrt{3}/2, \quad C_{\Xi K,\Xi K} = -1.$$  

The resulting $K$-matrix is written as

$$K = \begin{pmatrix} K_{oo} & K_{oc} \\ K_{co} & K_{cc} \end{pmatrix},$$

where, with $f_{B\phi,B'\phi'}^{(S)} \equiv f_{B\phi,B'\phi'}^{(S)}$,

$$K_{oo} = f_{\Lambda\pi,\Lambda\pi}, \quad K_{co} = K_{oc}^T = \begin{pmatrix} f_{\Lambda\pi,\Sigma\pi} \\ f_{\Lambda\pi,N\bar{K}} \\ f_{\Lambda\pi,\Sigma\eta} \\ f_{\Lambda\pi,\Xi K} \end{pmatrix},$$

$$K_{cc} = \begin{pmatrix} f_{\Sigma\pi,\Sigma\pi} & f_{\Sigma\pi,N\bar{K}} & f_{\Sigma\pi,\Sigma\eta} & f_{\Sigma\pi,\Xi K} \\ f_{N\bar{K},\Sigma\pi} & f_{N\bar{K},N\bar{K}} & f_{N\bar{K},\Sigma\eta} & f_{N\bar{K},\Xi K} \\ f_{\Sigma\eta,\Sigma\pi} & f_{\Sigma\eta,N\bar{K}} & f_{\Sigma\eta,\Sigma\eta} & f_{\Sigma\eta,\Xi K} \\ f_{\Xi K,\Sigma\pi} & f_{\Xi K,N\bar{K}} & f_{\Xi K,\Sigma\eta} & f_{\Xi K,\Xi K} \end{pmatrix} = K_{cc}^T,$$

the subscripts $o$ and $c$ referring, respectively, to open and closed channels at $\sqrt{s} = m_\Xi$. The unitarized $S$-wave amplitude for $\Lambda\pi \rightarrow \Lambda\pi$ is then given by

$$T_{oo} = \frac{K_{r}}{1 - i q_o K_{r}} = \frac{e^{2i\delta_S} - 1}{2i |K_{\Lambda\pi}|},$$

where

$$K_{r} = K_{oo} + i K_{oc} (1 - i q_c K_{cc})^{-1} q_c K_{co}, \quad q_o = q_{\Lambda\pi}, \quad q_c = \text{diag}(q_{\Sigma\pi}, q_{N\bar{K}}, q_{\Sigma\eta}, q_{\Xi K}),$$

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with \( q_{B\phi} = |k_{B\phi}| \), the magnitude of the particle three-momentum in \( B\phi \to B\phi \) scattering in the CM frame. We note that \( T_{oo} \) not only satisfies elastic unitarity exactly, but also reproduces the lowest-order \( \chi \)PT amplitude \( J_{\Lambda\pi,\Lambda\pi} \) (which happens to vanish in the \( S \)-wave case) as the chiral limit is approached. We further note that the diagonal elements of \( q_k \) are purely imaginary at \( \sqrt{s} = m_\Xi \), their corresponding channels being below threshold. It follows that the \( S \)-wave phase shift in \( \Lambda\pi \to \Lambda\pi \) scattering at \( \sqrt{s} = m_\Xi \) is calculated to be

\[
\delta_S = \tan^{-1}(q_{\Lambda\pi} K_t) \simeq -7.3^\circ .
\]  

If we drop the heavier \( \Sigma \eta \) and \( \Xi K \) channels, the phase shift is reduced in size to \( \delta_S \simeq -2.8^\circ \). If only the \( \Xi K \) channel is dropped, we find instead \( \delta_S \simeq -3.6^\circ \). These numbers are consistent with the fact that the \( \Lambda\pi \) state has nonzero \( S \)-wave couplings at leading order only to the \( N\bar{K} \) and \( \Xi K \) states. Interestingly, the last two numbers, \( \delta_S \sim -3^\circ \), are similar to the \( \delta_S \) in Eq. (11), calculated using \( \chi \)PT at order \( p^2 \) with the parameter values from Ref. [12], in which the heavier channels were not explicitly considered. In Fig. 3 we show the real part of \( \delta_S \) (which becomes complex above the \( \Sigma \pi \) threshold) as a function of the center-of-mass energy, with all the four inelastic channels contributing.

We remark here that we have not included contributions from the \( B\phi\phi' \) states in our \( K \)-matrix as they are not expected to be dominant, only entering at the two-loop level in \( \chi \)PT. Similarly, with the lowest-order vertices that we have, there are no \( S \)-wave couplings to states with a decuplet baryon and a pseudo-scalar meson.

The leading-order \( P \)-wave amplitude for \( B\phi \to B'\phi' \) with \( S = -1 \) and \( I = 1 \) is derived from the last three diagrams in Fig. 2. In the CM frame, its \( J = \frac{1}{2} \) component can be written in the form

\[
f_{B\phi,B'\phi'}^{(P)} = -D_{B\phi,B'\phi'} \frac{|k_\phi||k_{\phi'}|\sqrt{m_B m_{B'}}}{8\pi f^2 \sqrt{s}},
\]  

where \( D_{B'\phi',B\phi} = D_{B\phi,B'\phi'} \), and \( k_\phi \) (\( k_{\phi'} \)) is the three-momentum of the initial (final) meson. We have collected in the Appendix the expressions for \( D_{B\phi,B'\phi'} \) corresponding to the five coupled channels. The \( P \)-wave \( J = \frac{1}{2} \) phase-shift in \( \Lambda\pi \to \Lambda\pi \) scattering at \( \sqrt{s} = m_\Xi \) is then calculated using the same method as in the \( S \)-wave case. The result for \( D = 0.8, \ F = 0.5, \) and \( C = 1.7 \) is

\[
\delta_P \simeq 0.2^\circ ,
\]  

where isospin-symmetric masses have been used for the intermediate baryons. If chiral-symmetric masses \( m_0 \simeq 1.15 \text{ GeV} \) and \( m_T \simeq 1.38 \text{ GeV} \) are used for the intermediate baryons, the result is instead

\[
\delta_P \simeq 0.5^\circ .
\]
We have found that dropping one or more of the inelastic channels would not change these numbers dramatically in size, yielding a phase shift within the range $-1.5^\circ < \delta_P < -0.4^\circ$.

As in the $S$-wave case, we have not included contributions from the $B\phi\phi'$ states in our $P$-wave $K$-matrix. Neither have we considered states with only one decuplet baryon and no meson in the $s$-channel as they have $J = \frac{3}{2}$, but they were included as intermediate states in the $J = \frac{1}{2}$ $u$-channel. Beyond the simplest approximation that we have made, one could add contributions from heavier states, such as those with one decuplet baryon and one pseudo-scalar meson, as well as those with heavier resonances. The neglect of heavier states is taken as part of the uncertainty of our estimate.

4 Conclusion

We have studied the $\Lambda\pi$ scattering phase shifts at $\sqrt{s} = m_\Xi$ beyond leading order in chiral perturbation theory. With next-to-leading-order $\chi$PT, we find results that are consistent within factors of two with the lowest-order phase shifts. Within a $K$-matrix approach, we find that unitarity effects from coupled channels enhance the $S$-wave phase shift, which could be as large as $-7^\circ$, but they do not change the $P$-wave phase shift significantly. The large $S$-wave value is driven mostly by couplings to the heavier channels. Since the two approaches do not incorporate exactly the same physics, their results may be combined to indicate that $-7.3^\circ < \delta_S < +0.4^\circ$ and $-3.5^\circ < \delta_P < +0.5^\circ$, passing.
leading to $-7.8^\circ < \delta_S - \delta_p < +3.9^\circ$. Our results also indicate that more-refined future calculations with $\chi$PT as a starting point should include the effects of coupled channels, as has been done in the $N\bar{K}$ case $[12, 13, 20]$. Finally, it is possible to extract these phase shifts from experiment. We expect E871 to present results in the near future from their analysis of polarization in $\Xi \rightarrow A\pi$ decays.

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**Appendix**

For the five coupled channels considered here, one obtains

\[
D_{\Lambda\pi,\Lambda\pi} = \frac{2D^2}{\sqrt{s} - m_{\Sigma}} - \frac{2}{9}D^2 \frac{E_{\Lambda} - E_{\pi} - m_{\Sigma}}{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}} + \frac{2}{9}C^2 \frac{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}}{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}},
\]

\[
D_{\Lambda\pi,\Sigma\pi} = \frac{4DF}{\sqrt{s} - m_{\Sigma}} + \frac{4}{3\sqrt{6}}DF \frac{E_{\Lambda} - E_{\pi} - m_{\Sigma}}{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}} + \frac{4}{9\sqrt{6}}C^2 \frac{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}}{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}},
\]

\[
D_{\Lambda\pi,N\bar{K}} = -\frac{\sqrt{2}D(D - F)}{\sqrt{s} - m_{\Sigma}} - \frac{1}{3\sqrt{6}}(D^2 + 4DF + 3F^2) \frac{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}}{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}},
\]

\[
D_{\Lambda\pi,\Sigma\eta} = \frac{2}{9}D^2 \frac{E_{\Lambda} - E_{\eta} - m_{\Lambda}}{E_{\Lambda} - E_{\eta} - m_{\Lambda}},
\]

\[
D_{\Lambda\pi,\Xi\kappa} = \frac{2}{9}D^2 \frac{E_{\Lambda} - E_{\pi} - m_{\Lambda}}{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}} - \frac{2}{3}F^2 \frac{E_{\Xi} - E_{\pi} - m_{\Xi}}{E_{\Xi} - E_{\pi} - m_{\Xi^*}} + \frac{2}{27}C^2 \frac{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}}{E_{\Lambda} - E_{\pi} - m_{\Sigma^*}},
\]

\[
D_{\Sigma\pi,\Sigma\pi} = \frac{4F^2}{\sqrt{s} - m_{\Sigma}} + \frac{2}{9}D^2 \frac{E_{\Sigma} - E_{\pi} - m_{\Lambda}}{E_{\Sigma} - E_{\pi} - m_{\Sigma}} - \frac{2}{3}F^2 \frac{E_{\Sigma} - E_{\pi} - m_{\Sigma}}{E_{\Sigma} - E_{\pi} - m_{\Sigma^*}} + \frac{2}{27}C^2 \frac{E_{\Sigma} - E_{\pi} - m_{\Sigma^*}}{E_{\Sigma} - E_{\pi} - m_{\Sigma^*}},
\]

\[
D_{\Sigma\pi,N\bar{K}} = -\frac{2}{9}(D - F) \frac{E_{\Sigma} - E_{\pi} - m_{\Lambda}}{E_{\Sigma} - E_{\pi} - m_{\Sigma^*}} + \frac{1}{3\sqrt{6}} \frac{E_{\Sigma} - E_{\pi} - m_{\Sigma^*}}{E_{\Sigma} - E_{\pi} - m_{\Sigma^*}},
\]

\[
D_{\Sigma\pi,\Sigma\eta} = \frac{4DF}{\sqrt{s} - m_{\Sigma}} + \frac{4}{3\sqrt{6}}DF \frac{E_{\Sigma} - E_{\pi} - m_{\Sigma}}{E_{\Sigma} - E_{\pi} - m_{\Sigma^*}} + \frac{4}{9\sqrt{6}}C^2 \frac{E_{\Sigma} - E_{\pi} - m_{\Sigma^*}}{E_{\Sigma} - E_{\pi} - m_{\Sigma^*}},
\]

\[
D_{\Sigma\pi,\Xi\kappa} = -\frac{2}{9}(D + F) \frac{E_{\Sigma} - E_{\pi} - m_{\Lambda}}{E_{\Sigma} - E_{\pi} - m_{\Xi^*}} - \frac{1}{3\sqrt{6}} \frac{E_{\Sigma} - E_{\pi} - m_{\Xi^*}}{E_{\Sigma} - E_{\pi} - m_{\Xi^*}},
\]

\[D_{\Sigma\pi,\Xi\kappa} = -\frac{2}{9}(D + F) \frac{E_{\Sigma} - E_{\pi} - m_{\Lambda}}{E_{\Sigma} - E_{\pi} - m_{\Xi^*}} - \frac{1}{3\sqrt{6}} \frac{E_{\Sigma} - E_{\pi} - m_{\Xi^*}}{E_{\Sigma} - E_{\pi} - m_{\Xi^*}},\]
\[ D_{NK, NK} = \frac{(D - F)^2}{\sqrt{s - m_\Sigma}}, \quad D_{NK, \Sigma \eta} = -\frac{\sqrt{s}}{3} \frac{D(D - F)}{\sqrt{s - m_\Sigma}} - \frac{\sqrt{6}}{15} \frac{(D^2 - 4DF + 3F^2)}{E_N - E'_\eta - m_N}, \]

\[ D_{NK, \Xi K} = \frac{D^2 - F^2}{\sqrt{s - m_\Sigma}} - \frac{\sqrt{6}}{15} \frac{(D^2 - 9F^2)}{E_N - E'_K - m_\Lambda} + \frac{1}{6} \frac{(D^2 - F^2)}{E_N - E'_K - m_\Sigma} + \frac{2 \sigma C^2}{E_N - E'_K - m_\Sigma^*}, \]

\[ D_{\Sigma \eta, \Sigma \eta} = \frac{\sqrt{s}}{3} \frac{D^2}{\sqrt{s - m_\Sigma}} - \frac{\sqrt{6}}{15} \frac{D^2}{E_\Sigma - E'_\eta - m_\Sigma} + \frac{2 \sigma C^2}{E_\Sigma - E'_\eta - m_\Sigma^*}, \]

\[ D_{\Sigma \eta, \Xi K} = -\frac{\sqrt{6}}{3} \frac{D(D + F)}{\sqrt{s - m_\Sigma}} - \frac{\sqrt{6}}{15} \frac{(D^2 + 4DF + 3F^2)}{E_\Sigma - E'_K - m_\Xi} - \frac{2 \sigma C^2}{E_\Sigma - E'_K - m_\Xi^*}, \]

\[ D_{\Xi K, \Xi K} = \frac{D^2 + 2DF + F^2}{\sqrt{s - m_\Sigma}} + \frac{4 \sigma C^2}{E_\Xi - E'_K - m_\Omega}, \]

where \( \sqrt{s} = E_B + E_\phi \) and \( E'_\phi \) is the energy of \( \phi \) in the final state.

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