Recent Developments in $\bar{B} \to X_s \gamma$†

Ulrich Haisch
Institut für Theoretische Physik, Universität Zürich, CH-8057 Zürich, Switzerland
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We present a concise review of the recent theoretical progress concerning the standard model calculation of the inclusive radiative $\bar{B} \to X_s \gamma$ decay. Particular attention is thereby devoted to the calculations of the next-to-next-to-leading order fixed-order $\mathcal{O}(\alpha_s^2)$ contributions, non-local $\mathcal{O}(\alpha_s \Lambda/m_b)$ power corrections, and logarithmic-enhanced $\mathcal{O}(\alpha_s^2 \ln n)$ cut-effects to the decay rate. The current status of new physics calculations of the inclusive $b \to s \gamma$ mode is also briefly summarized.

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I. INTRODUCTION

As a flavor-changing-neutral-current process the inclusive radiative $\bar{B}$-meson ($\bar{B} = \bar{B}^0$ or $B^-$) decay is Cabibbo-Kobayashi-Maskawa (CKM) and loop-suppressed within the standard model (SM) and thus very sensitive to new physics (NP) effects. In order to exploit the full potential of $\bar{B} \to X_s \gamma$ in constraining the parameter space of beyond the SM physics both the measurements and the SM prediction should be known as precisely as possible.

The present experimental world average (WA) which includes the latest measurements by CLEO, Belle, and BaBar [1] is performed by the Heavy Flavor Averaging Group [2] and reads for a photon energy cut of $E_\gamma > E_{\text{cut}}$ with $E_{\text{cut}} = 1.6\,\text{GeV}$ in the $B$-meson rest-frame

$$B(\bar{B} \to X_s \gamma)_{\text{exp}} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}.$$  \hspace{1cm} (1)

The total error of the WA is below 8% and consists of (i) a combined statistical and systematic error, (ii) a systematic uncertainty due to the extrapolation from $E_{\text{cut}} = [1.8, 2.0]\,\text{GeV}$ to the reference value, and (iii) a systematic error due to the subtraction of the $\bar{B} \to X_d \gamma$ event fraction. At the end of the $B$-factory era the final accuracy of the averaged experimental value is expected to be around 5%.

II. BASIC PROPERTIES OF $\bar{B} \to X_s \gamma$

The $b \to s \gamma$ transition is dominated by perturbative QCD effects which replace the power-like Glashow-Iliopoulos-Maiani (GIM) suppression present in the electroweak (EW) vertex by a logarithmic one. This mild suppression of the QCD corrected amplitude reduces the sensitivity of the process to high scale physics, but enhances the $\bar{B} \to X_s \gamma$ branching ratio (BR) with respect to the purely EW prediction by a factor of around three. The logarithmic GIM cancellation originates from the non-conservation of the tensor current which is generated at the EW scale by loop diagrams involving $W$-boson and top quark exchange. The associated large logarithms $L = \ln M_W/m_b$ have to be resummed at each order in $\alpha_s$, using techniques of the renormalization group (RG) improved perturbation theory. Factoring out the Fermi constant $G_F$, the $b \to s \gamma$ amplitude receives corrections of $\mathcal{O}(\alpha_s^2 L^n)$ at leading order (LO), of $\mathcal{O}(\alpha_s^3 L^{n-1})$ at next-to-leading order (NLO), and of $\mathcal{O}(\alpha_s^4 L^{n-2})$ at next-to-next-to-leading order (NNLO) in QCD.

A suitable framework to achieve the necessary resummation is the construction of an effective theory with five active quarks, photons and gluons by integrating out the top quark and the EW bosons. Including terms of dimension up to six in the local product expansion (OPE) the relevant effective Lagrangian at a scale $\mu$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD} \times \text{QED}} + \frac{4G_F}{\sqrt{2}} V_{ts}^\ast V_{tb} \sum_{k=1}^8 C_k(\mu) Q_k.$$  \hspace{1cm} (2)

Here the first term is the conventional QCD and QED Lagrangian for the light SM particles. In the second term $V_{ij}$ denotes the elements of the CKM matrix and $C_k(\mu)$ are the Wilson coefficients of the corresponding operators $Q_k$ built out of the light fields.

The operators and the numerical values of their Wilson coefficients at $\mu_b \sim m_b$ are given by

$$Q_{1,2} = (s\Gamma_c)(c\Gamma'_b), \quad C_{1,2}(m_b) \sim 1,$n \hspace{1cm} Q_{3,6} = (s\Gamma_b) \sum_q (q\Gamma'_q), \quad |C_{3,6}(m_b)| < 0.07,$n \hspace{1cm} Q_7 = \frac{G_F}{\Lambda^{\frac{1}{2}} s_L \sigma^\mu\nu b_b F_{\mu\nu}}, \quad C_7(m_b) \sim -0.3,$n \hspace{1cm} Q_8 = \frac{G_F}{\Lambda^{\frac{1}{2}} s_L \sigma^\mu\nu T^a b_b G^a_{\mu\nu}}, \quad C_8(m_b) \sim -0.15,$\hspace{1cm} (3)$n$$

where $\Gamma$ and $\Gamma'$ denote the elements of the CKM matrix and $C_k(\mu)$ are the Wilson coefficients of the corresponding operators $Q_k$ built out of the light fields.

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loops that amount to $\sim 158\%$ of the total $b \to s\gamma$ decay amplitude. The top contribution is compared to the one from the charm quark with $\sim -60\%$ less than half as big and has the opposite sign. Diagrams involving up quarks are suppressed by small CKM factors and lead at the amplitude level to an effect of a mere $\sim 2\%$.

All perturbative calculations of $b \to s\gamma$ involve three steps: i) evaluation of the initial conditions $C_b(\mu_w)$ of the Wilson coefficients at the matching scale $\mu_w \sim M_W$ by requiring equality of Green’s functions in the full and the effective theory up to leading order in (external momenta)/$M_W$, ii) calculation of the anomalous dimension matrix (ADM) that determines the mixing and RG evolution of $C_b(\mu)$ from $\mu_w$ down to the $B$-meson scale $\mu_b \sim m_b$, and iii) determination of the on-shell matrix elements of the various operators at $\mu_b \sim m_b$. Due to the inclusive character of the $\bar{B} \to X_s\gamma$ mode and the heaviness of the bottom quark, $m_b \gg \Lambda \sim \Lambda_{QCD}$, non-perturbative effects arise in the last step only as small corrections to the partonic decay rate.

III. THEORETICAL PROGRESS IN $\bar{B} \to X_s\gamma$

At the NNLO level, the dipole and the four-quark operator matching involves three and two loops, respectively. Renormalization constants up to four loops must be found for $b \to s\gamma$ and $b \to s\gamma$ diagrams with four-quark operator insertions, while three-loop mixing is sufficient in the remaining cases. Two-loop matrix elements of the dipole and three-loop matrix elements of the four-quark operators must be evaluated in the last step.

The necessary two- and three-loop matching was performed in [4] and [5]. The mixing at three loops was determined in [6] and at four loops in [7]. The two-loop matrix element of the photonic dipole operator together with the corresponding bremsstrahlung was found in [8] and subsequently confirmed in [9]. These calculations have been very recently extended to include the full charm quark mass dependence [10]. The three-loop matrix elements of the current-current operators were derived in [11] within the so-called large-$\beta_0$ approximation. A calculation that goes beyond this approximation employs an interpolation in the charm quark mass [12]. The effect of still unknown NNLO contributions is believed to be smaller than the uncertainty that has been estimated after incorporating the above corrections into the SM calculation [12, 13]. To dispel possible doubts about the correctness of this assumption, calculations of the missing pieces are being pursued.

The most impressive bit of the various NNLO calculations is the one of the four-loop ADM that describes the $\mathcal{O}(\alpha_s^4)$ mixing of the four-quark into the dipole operators [7]. It has involved the computation of more than 20000 four-loop diagrams and required a mere computing time of several months on around 100 CPU’s.

Another crucial part of the NNLO calculation is the interpolation in the charm quark mass performed in [12].

The three-loop $\mathcal{O}(\alpha_s^3)$ matrix elements of the current-current operators contain the charm quark, and the NNLO calculation of these matrix elements is essential to reduce the overall theoretical uncertainty of the SM calculation. In fact, the largest part of the theoretical uncertainty in the NLO analysis of the BR is related to the definition of the mass of the charm quark [14] that enters the $\mathcal{O}(\alpha_s)$ matrix elements $\langle s\gamma|Q_{1,2}|b\rangle$. The latter matrix elements are non-vanishing at two loops only and the scale at which $m_c$ should be normalized is therefore undetermined at NLO. Since varying $m_c$ between $m_c(m_c) \sim 1.25$ GeV and $m_c(m_b) \sim 0.85$ GeV leads to a shift in the NLO BR of more than 10% this issue is not an academic one.

Finding the complete NNLO correction to $\langle s\gamma|Q_{1,2}|b\rangle$ is a formidable task, since it involves the evaluation of hundreds of three-loop on-shell vertex diagrams that are presently not even known in the case $m_c = 0$. The ap-
proximation made in [12] is based on the observation that at the physical point $m_c \sim 0.25 m_b$ the large $m_c \gg m_b$ asymptotic form of the exact $O(\alpha_s^4)$ [3] and large-\(\beta_0\) $O(\alpha_s^2\beta_0)$ [11] result matches the small $m_c \ll m_b$ expansion rather well. This feature prompted the analytic calculation of the leading term in the $m_c \gg m_b$ expansion of the three-loop diagrams, and to use the obtained information to perform a interpolation to smaller values of $m_c$, assuming the $O(\alpha_s^2\beta_0)$ part to be a good approximation of the full $O(\alpha_s^2)$ result for vanishing charm quark mass. The uncertainty related to this procedure has been assessed in [12] by employing three ansätze with different boundary conditions at $m_c = 0$. A complete calculation of the $O(\alpha_s^2\beta_0)$ corrections to $\langle s\gamma|Q_{1,2}|b\rangle$ in the latter limit or, if possible, for $m_c \sim 0.25 m_b$, would resolve this ambiguity and should therefore be attempted.

Combining the aforementioned results it was possible to obtain the first theoretical estimate of the total BR of $B \to X_s\gamma$ at NNLO. For the reference value $E_{\text{cut}} = 1.6$ GeV the result of the improved SM evaluation is given by [12, 13]

$$B(B \to X_s\gamma)_{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4},$$

(4)

where the uncertainties from hadronic power corrections (5\%), parametric dependences (3\%), higher-order perturbative effects (3\%), and the interpolation in the charm quark mass (3\%) have been added in quadrature to obtain the total error.

The reduction of the renormalization scale dependences at NNLO is clearly seen in Fig. 1. The most pronounced effect occurs in the case of the charm quark mass renormalization scale $\mu_c$ that was the main source of uncertainty at NLO. The current uncertainty of 3\% due to higher-order effects is estimated from the variation of the NNLO curves. The central value in Eq. (4) corresponds to the choice $\mu_{w,b,c} = (160, 2.5, 1.5)$ GeV. More details on the phenomenological analysis including the list of input parameters can be found in [12].

It is well-known that the OPE for $B \to X_s\gamma$ has certain limitations which stem from the fact that the photon has a partonic substructure. In particular, the local expansion does not apply to contributions from operators other than $Q_7$, in which the photon couples to light quarks [15, 16]. While the presence of non-local power corrections was thus foreseen such terms have been studied until recently only in the case of the $(Q_8, Q_9)$ interference [15]. In [17] the analysis of non-perturbative effects that go beyond the local OPE have been extended to the enhanced non-local terms emerging from $(Q_7, Q_8)$ insertions. The found correction scales like $O(\alpha_s^2L/m_b)$ and its effect on the BR was estimated using the vacuum insertion approximation to be $[-0.3, 3.0]$\%. A measurement of the flavor asymmetry between $B^0 \to X_s\gamma$ and $B^- \to X_s\gamma$ could help to sustain this numerical estimate [17]. Potentially as or maybe even more important than the latter correction are those arising from the $(Q_{1,2}, Q_7)$ interference. Naive dimensional analysis suggests that some non-perturbative corrections to them also scale like $O(\alpha_s^2L/m_b)$. Since at the moment there is not even an estimate of those corrections, a non-perturbative uncertainty of 5\% has been assigned to the result in Eq. (4). This error is the dominant theoretical uncertainty at present and thought to include all known [17] and unknown $O(\alpha_s^2L/m_b)$ terms. Calculating the precise impact of the enhanced non-local power corrections may remain notoriously difficult given the limited control over non-perturbative effects on the light cone.

A further complication in the calculation of $B \to X_s\gamma$ arises from the fact that all measurements impose stringent cuts on the photon energy to suppress the background from other $B$-meson decay processes. Restricting $E_{\gamma}$ to be close to the physical endpoint $E_{\text{max}} = m_B/2$, leads to a breakdown of the local OPE, which can be cured by summing up an infinite set of leading-twist terms into a non-perturbative shape function [18]. A detailed knowledge of the shape function and other subleading effects is required to extrapolate the measurements to a region where the conventional OPE can be trusted.

The transition from the shape function to the OPE region can be described by a multi-scale OPE (MSOPE) [19]. In addition to the hard scale $\mu_a \sim m_b \sim 5$ GeV, this expansion involves a hard-collinear scale $\mu_{hc} \sim \sqrt{m_b\Delta} \sim 2.5$ GeV corresponding to the typical hadronic invariant mass of the final state $X_s$, and a soft scale $\mu_s \sim \Delta \sim 1.5$ GeV related to the width $\Delta/2 = m_b/2 - E_{\text{cut}}$ of the energy window in which the photon spectrum is measured. In the MSOPE framework, the perturbative tail of the spectrum receives calculable corrections at all three scales, and may be subject to large perturbative corrections due to the presence of terms proportional to $\alpha_s(\sqrt{m_b\Delta}) \sim 0.27$ and $\alpha_s(\Delta) \sim 0.36$.

A systematic MSOPE analysis of the $(Q_7, Q_7)$ interference at NNLO has been performed in [20]. Besides the hard matching corrections, it involves the two-loop logarithmic and constant terms of the jet [19, 22] and soft function [23]. The three-loop ADM of the shape function remains unknown and is not included. The MSOPE result can be combined with the fixed-order prediction by computing the fraction of events $1 - T$ that lies in the range $E_{\text{cut}} = [1.0, 1.6]$ GeV. The analysis [20] yields

$$1 - T = 0.07^{+0.03}_{-0.02}\text{pert} \pm 0.02\text{hadr} \pm 0.02\text{pars},$$

(5)

where the individual errors are perturbative, hadronic, and parametric. The quoted value is almost twice as large as the NNLO estimate $1 - T = 0.04 \pm 0.01\text{pert}$ obtained in fixed-order perturbation theory [12, 13, 21] and plagued by a significant additional theoretical error related to low-scale perturbative corrections. These large residual scale uncertainties indicate a slow convergence of the MSOPE series expansion in the tail region of the photon energy spectrum. Given that $\Delta$ is always larger than 1.4 GeV and thus fully in the perturbative regime this feature is unexpected.

Additional theoretical information on the shape of the photon energy spectrum can be obtained from the universality of soft and collinear gluon radiation. Such an
approach can be used to predict large logarithms of the form \( \ln(E_{\text{max}} - E_{\text{cut}}) \). These computations have also achieved NNLO accuracy [24] and incorporate Sudakov and renormalon resummation via dressed gluon exponentiation (DGE) [24, 25]. The present NNLO estimate of 1−\( T = 0.016 \pm 0.003_{\text{pert}} \) [24, 26] indicates a much thinner tail of the photon energy spectrum and a considerable smaller perturbative uncertainty than reported in [20].

The DGE analysis thus supports the view that the integrated photon energy spectrum below \( E_{\text{cut}} = 1.6 \text{ GeV} \) is well approximated by a fixed-order perturbative calculation, complemented by local OPE power corrections. To understand how precisely the tail of the photon energy spectrum can be calculated requires nevertheless further theoretical investigations.

### IV. NEW PHYSICS IN \( \bar{B} \rightarrow X_s \gamma \)

Compared with the experimental WA of Eq. (1), the new SM prediction of Eq. (4) is lower by more than 1σ. Potential beyond SM contributions should now be preferably constructive, while models that lead to a suppression of the \( b \rightarrow s \gamma \) amplitude are more severely constrained than in the past, where the theoretical determination used to be above the experimental one [3].

NP affects the initial conditions of the Wilson coefficients of the operators in the low-energy effective theory and might also induce new operators besides those already present in the SM. Complete NLO matching calculations are available only in the case of the two-Higgs-doublet models (THDMs) [27, 28], the minimal supersymmetric SM (MSSM) with minimal-flavor-violation (MFV) for small and large tan \( \beta \) [28–33], and left-right (LR) symmetric models [28]. In the general MSSM [34], extra dimensional models like minimal universal extra dimensions (mUED) [36] or Randall-Sundrum (RS) scenarios [37], and littlest Higgs (LH) models without [38] and with \( T \)-parity (LHT) [39], the accuracy is in general strictly LO and hence far from the one achieved in the SM. The main features and results of recent analyses of beyond SM physics in \( \bar{B} \rightarrow X_s \gamma \) are listed in Tab. I. In the following we will briefly review the most important findings.

| Model       | Accuracy | Effect | Bound                              |
|-------------|----------|--------|------------------------------------|
| THDM type II| NLO [27, 28] | ⬆️ | \( M_H > 295 \text{ GeV (95\% CL)} \) [13] |
| MFV MSSM    | NLO [28–33] | ⬇️ | —                                  |
| LR          | NLO [28] | ⬇️ | —                                  |
| general MSSM| LO [34]  | ⬇️ | \( |\langle \delta^\gamma_{\text{LL}} \rangle| \leq 4 \times 10^{-1}, |\langle \delta^\gamma_{\text{LR}} \rangle| \leq 8 \times 10^{-1} \) [34] |
| mUED        | LO [36]  | ⬇️ | \( 1/R > 600 \text{ GeV (95\% CL)} \) [41] |
| RS          | LO [37]  | ⬆️ | \( M_KK \geq 2.4 \text{ TeV} \)      |
| LH          | LO [38]  | ⬆️ | —                                  |
| LHT         | LO [39]  | ⬆️ | —                                  |

**TABLE I:** Theoretical accuracy, effect on \( \bar{B} \rightarrow X_s \gamma \) relative to the SM prediction, and if applicable, constraint on the parameter space following from \( \bar{B} \rightarrow X_s \gamma \) in popular NP scenarios. Arrows pointing upward (downward) indicate that the NP effects interfere constructively (destructively) with the SM \( b \rightarrow s \gamma \) amplitude. Single (double) arrows specify whether the maximal possible shift is smaller (larger) than the theoretical uncertainty of the SM expectation. See text for details.

Even though the effect of charged Higgs boson contributions in the THDM type II model is necessarily constructive [27, 28], the lower bound on \( M_{H^\pm} \) following from \( \bar{B} \rightarrow X_s \gamma \) remains in general stronger than all other direct and indirect constraints. In particular, \( \bar{B} \rightarrow X_s \gamma \) still prevails over \( \bar{B} \rightarrow \tau \nu \) [40] for all values of tan \( \beta \) apart from those lying in the range tan \( \beta \sim [45, 65] \). This is illustrated in the upper panel of Fig. 2. The derived 95\% confidence level (CL) limit amounts to \( M_{H^\pm} > 295 \text{ GeV} \) independently of tan \( \beta \) [13]. In the THDM type I model, the strongest constraint on \( M_{H^\pm} \) stems from the ratio of the widths of the Z-boson decay into bottom quarks and hadrons, \( R_b \), and not from \( \bar{B} \rightarrow X_s \gamma \).

In the MFV MSSM the complete NLO corrections to \( \bar{B} \rightarrow X_s \gamma \) are also known. The needed two-loop diagrams containing gluinos and gluinos were evaluated in [28, 29] and [30, 31], respectively. Since EW interactions affect the quark and squark mass matrices in a different way, their alignment is not RG invariant and MFV can only be imposed at a certain scale \( \mu_{\text{MFV}} \) that is related to the mechanism of supersymmetry (SUSY) breaking [31]. For \( \mu_{\text{MFV}} \) much larger than the SUSY masses \( M_{\text{SUSY}} \), the ensuing large logarithms can lead to sizable effects in \( \bar{B} \rightarrow X_s \gamma \), and need to be resummed by solving the RG equation of the flavor-changing gluino-quark-squark couplings.

In the limit of \( M_{\text{SUSY}} \gg M_W \), SUSY effects can be ab-
In particular, for small and moderate values of \( \tan \beta \) all four mass insertions \((\delta_{ij}^{d})_{AB}\) with \( A, B = L, R \) except for \((\delta_{23}^{d})_{RR}\) are determined entirely by \( \bar{B} \to X_{s}\gamma \). The bounds on the mass insertions \((\delta_{ij}^{d})_{AB}\) corresponding to \( \tan \beta = 10 \) are given in Tab. I. For large values of \( \tan \beta \) neutral Higgs Penguin contributions become important and the constraints from both \( B_{s} \to \mu^{+}\mu^{-} \) and \( B_{s} \to \bar{B}_{s} \) mixing surpass the one from \( \bar{B} \to X_{s}\gamma \). The effect of the precision measurement of the mass difference \( \Delta M_{s} \) is especially strong in the case of \((\delta_{23}^{d})_{RL,RR}\). At large \( \tan \beta \) the limits on both mass insertions are now imposed by the \( B_{s} \to \bar{B}_{s} \) mixing constraint alone.

Since Kaluza-Klein (KK) modes in the mUED model interfere destructively with the SM \( b \to s\gamma \) amplitude [36], \( \bar{B} \to X_{s}\gamma \) leads to a very powerful bound on the inverse compactification radius of \( 1/R > 600 \text{ GeV} \) at 95\% CL [41]. This exclusion is independent from the Higgs mass and therefore stronger than any limit that can be derived from EW precision measurements. The 95\% CL bound on \( 1/R \) as a function of the SM central value and error is shown in the lower panel of Fig. 2. In RS models, KK modes enhance the BR relative to the SM [37], and the bound on the KK masses is in consequence with \( M_{KK} \gtrsim 2.4 \text{ TeV} \) significantly weaker than the constraint that derives from EW precision data.

The contributions to \( \bar{B} \to X_{s}\gamma \) from new heavy vector bosons, scalars, and quarks appearing in LH models, have been studied in [38] for the original model, and in [39] for an extension in which an additional \( Z_{2} \) symmetry called T-parity is introduced to preserve custodial SU(2) symmetry. While in the former case the new contributions always lead to an enhancement of \( \mathcal{B}(\bar{B} \to X_{s}\gamma) \) [38], in the latter case also a suppression with respect to the SM expectation is possible [39]. As the found LH effects in \( \bar{B} \to X_{s}\gamma \) are generally smaller than the theoretical uncertainties in the SM, they essentially do not lead to any restriction on the parameter space.

An alternative avenue to NP analyses of \( \bar{B} \to X_{s}\gamma \) consists in constraining the Wilson coefficients of the operators in the low-energy effective theory. This model-independent approach has been applied combining various \( B \)- and \( K \)-meson decay modes both neglecting [42, 43] and including [33, 44] operators that do not contribute in the SM. In particular, in the former case, merging the information on \( \bar{B} \to X_{s}\gamma \) with the one on \( \bar{B} \to X_{t}t^{-} \) [45], one can infer that the sign of the \( b \to s\gamma \) amplitude is in all probability SM-like [46]. In the case of the \( Z \)-penguin amplitude the same conclusion can be drawn on the basis of the precision measurements of \( R_{b} \) and the other \( Z \to bb \) pseudo observables [43].

V. CONCLUSIONS

The inclusion of NNLO QCD corrections has lead to a significant suppression of the renormalization scale dependences of the \( \bar{B} \to X_{s}\gamma \) branching ratio that have been the main source of theoretical uncertainty at NLO.
The central value of the SM prediction is shifted downward relative to all previously published NLO results. It is now more than 1σ below the experimental average. This revives the possibility for explorations of new physics contributions to rare flavor-changing B-decay processes. The dominant theoretical uncertainty in the SM is currently due to unknown non-perturbative effects. A reduction of this error, together with a calculation of the three-loop matrix elements of the current-current operators and a better understanding of the tail of the photon energy spectrum is essential to further increase the power of $\bar{B} \rightarrow X_s \gamma$ in the search for new physics.

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