Electromagnetic wave scattering and resonances in a complex plasma

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\textbf{ABSTRACT}

The electromagnetic wave propagation in a complex plasma containing macroparticle inclusions (dust) is reviewed. We focus on two classes of phenomena due to the presence of dust: (1), collection of plasma electrons and ions by macroparticle inclusions and related modification of dielectric properties, and (2), surface plasmon resonances on the macroparticle surfaces and their effect on electromagnetic wave propagation. It is demonstrated that the presence of these phenomena can significantly modify plasma electromagnetic properties by introducing additional (charging) damping, modifying field distributions near macroparticles, and leading to resonances and cut-offs in the effective permittivity. The conditions to observe such phenomena in laboratory dusty plasma and/or space (cosmic) dusty plasmas are discussed.

\textbf{1. Introduction}

Physics of dusty ‘complex’ plasma, i.e. plasma containing macroscopic inclusions (dust) has been attracting significant research attention for more than three decades. This interest is based not only on the wide scope of research, from space physics and astrophysics [1], to technological issues related to plasma...
devices [2,3], but also related to the new physics of collective processes in such plasmas [4,5] since the presence of macroscopic particles (macroparticles, ‘dust’) significantly modifies plasma properties [4–6], even with visualization of the kinetics of collective plasma dynamics in laboratory [7,8].

The rich variety of phenomena in complex dusty plasmas is mostly related to the high complexity of such plasmas. In particular, this complexity is related to finite sizes of dust particle inclusions, as compared with sizes of electrons and ions. For example, the ‘charging’ damping appears due to plasma currents on dust particles (collection of plasma by finite size dust particles) thus affecting propagation and scattering of electromagnetic waves [9,10]. Furthermore, resonances with surface oscillations (surface plasmons) on macroparticle surfaces can have important consequences on electromagnetic wave propagation in complex dusty plasmas resulting in a band mode related to surface plasmons and the frequency gap with the electromagnetic wave band [11].

One of the most important features is the proper account of the influence of plasma host environment. In the case of charging phenomena, that is mostly related to plasma currents on the surfaces of dust inclusions leading to dust charging and specific charging damping. In the case of surface plasmons on the surfaces, the important phenomenon is the surface plasmon resonance on dielectric particles, due to host plasma environment; this effect is absent for dielectric particles in dielectric (or vacuum) host environment. The presence of such resonance makes it possible for electromagnetic waves to propagate at frequencies below the electron plasma frequency.

In this brief review, we focus on electromagnetic wave propagation in an unmagnetized plasma containing macroparticles (dust) and consider two classes of phenomena taking place due to the presence of dust: (1), collection of plasma electrons and ions by macroparticle inclusions and related modification of dielectric properties, and (2), surface plasmon resonances on the macroparticle surfaces and their effect on electromagnetic wave propagation. To account for the charging phenomena, we employ the kinetic approach taking into account self-consistent kinetics of plasma electrons and ions [9,10,12]; to consider the presence of surface plasmons, we simplify the problem by excluding losses and effects of space dispersion and presenting of the effective plasma permittivity [11] by employing the Maxwell Garnett (MG) approximation [13] widely used in the studies of two-component mixtures [14,15].

The paper is organized as follows: in Section 2, we discuss the phenomenology relevant for the considered phenomena; Section 3 is devoted to the wave propagation and scattering in the presence of dust charging; Section 4 considers the new features of the wave propagation and dispersion due to the presence of surface plasmons on macroparticle inclusions; and in Section 5 we conclude and discuss the most important phenomena for the wave propagation in dusty plasmas in the presence of the effects reviewed.
2. Phenomenology

Plasma electron oscillations, by virtue of the collective nature of plasma, are used to identify the plasma characteristics and give the cut-off frequency for the propagation of electromagnetic waves [16]. The electromagnetic wave dispersion in plasma plays a critical role in determining the nature of cosmic environment [1].

Dust particles of various sizes, from a few nanometers to tens of micrometers, are often observed in nature as well as in low-temperature plasma laboratory experiments [2]. Dust can be found in the interstellar space, planetary atmospheres, ring structures of large planets in the Solar system, in cometary tails, etc. Dust grains play an important role in the synthesis of molecular species in diffuse nebulae and directly participate in various astrophysical processes [17]. Planetary atmospheres, including the Earth atmosphere, and planetary rings are yet another example of the complex plasma system in space environments [18,19]. On the Earth, dust particles are important constituents of colloidal suspensions, charged aerosols, high-pressure ionized gas systems, nucleation catalysts, and many others [3,20]. Dust particles with sizes in the nano-/micrometer range have been detected in low-temperature plasmas used for materials synthesis and processing applications, where fine dust powders appear in the mixtures of chemically active (reactive) gases [3,4,21]. In specially designed laboratory experiments, the dust particles levitating in the sheath region of a radio frequency discharge provide an important diagnostic tool, and the sheath characteristics can be studied in considerable detail using the dust probes [2,4].

In various plasma environments, dust particles are usually charged [5]. Notably, Alfvén was one of the very first researchers to suggest that electromagnetic forces acting on small charged dust particles played an important role in the formation of the entire Solar System [22]. In particular, he recognized planetary rings as an important laboratory for studying processes that not only act today, but also shaped the evolution of the early Solar System billions of years ago. The basic charging mechanism of the dust grain was given by Spitzer [1]: the motion of electrons and ions to the surface of an individual solid grain in a plasma background is limited by the orbital trajectories, with conservation of the energy and angular momentum. The simplicity of the orbit-motion-limited approach lies in the use of the conservation laws in calculating the cross section of the plasma attachment to the grain surface and allows us to analytically treat the self-consistent dust charging and derive the plasma permittivity in the presence of charging. This permittivity, appearing in the expression for the scattering cross-section of electromagnetic waves, makes it possible to consider the most important features of electromagnetic wave scattering by static (as well as moving) dust particles in the plasma environment.

The interaction of macroparticles embedded in a plasma with plasma itself has opened a new field when macroparticles are replaced by cells, micro-organisms or
living tissues. Considerable progresses have been achieved in therapeutic applications, sterilization, and decontamination [23]. The fundamental understanding of mechanisms involved in the interaction of living organisms and plasmas still needs wisdoms in cross disciplinary fields [24].

The macroscopic (with respect to sizes of electrons and ions) size of dust particle inclusions and therefore the finite size of their surfaces leads to the following major phenomena: (1) the collection of plasma electrons and ions by the (macro)particle surfaces, and (2), the possibility of surface (on the surface of dust particles, in particular, the surface electric field) wave phenomena influencing the field distributions in the plasma environment.

Surface waves on interfaces of two media with opposite signs of permittivities have long history of research [25]. Surface plasma waves (surface plasmons), when one of the media is plasma, attracted special interests for many applications, for example, in plasmonics [26], in light sources as spasers [27], and in nano-antennas [28]. The surface wave fields include information on the properties of adjacent media and therefore can affect electromagnetic waves in one of them by properties of another one [29].

The presence of surface plasmons on dielectric spherical inclusions is due to the oscillations of plasma particles in the surrounding host plasma. The electric field near the surface of a charged dielectric particle has been studied extensively and the polarization effect is found to produce a rather strong field in the vicinity of the dust surface; this field is confined in the region very close to the surface on the order of \( \lambda_E \approx a/\sqrt{Z_d} \), where \( a \) is the radius of the spherical inclusion and \( Z_d = |Q_d/e| \) is the dimensionless dust charge with \( Q_d \) the (average) charge of a dust particle [30]. In our present analysis, we have neglected such an electric field since the surface plasma oscillations under consideration are characterized by the wavelength much larger than \( \lambda_E \). On the other hand, the electrostatic surface plasmon oscillation fields are confined near the surface in the region of the order of Debye length \( \lambda_{De} \). These fields are produced by electrons that can come and leave the surface, and cannot necessarily always be present directly at the surface. Such physics has been discussed extensively, e.g. for the idealized case of mirror reflection of electrons from the plasma boundary [29]. Our present analysis involves distances and wavelengths much larger than \( \lambda_{De} \) which was used explicitly as the absence of the space dispersion in our analysis.

The presence of surface wave fields leading to the surface plasmon resonances, makes complex plasma qualitatively different from a classical electron–ion plasma (with no external fields): for electromagnetic waves with wavelengths much larger than interparticle distances, the complex plasma appears as a metamaterial (with positive permeability due to the absence of gyrotropy in the considered case) [15,31,32], either with epsilon near zero or epsilon very large [33] (there are also analogies between magnetized plasma and metamaterial wire medium [34,35]).
In a complex dusty plasma, chaotic as well as ordered arrangement of dust particles is possible [36], with interesting phase transitions between ordered and disordered phases [37,38]. The particle ordering will affect the metamaterial properties of complex plasma (wave propagation and scattering) if space dispersion effects are included. Naturally, the most pronounced effect of ordering (and in general, spatial dispersion effects) will be for dusty plasma as a photonic material, when the electromagnetic wavelength is of the order of the interparticle distances; the influence of surface plasmon resonances in this case was discussed in [39].

3. Electromagnetic wave propagation and scattering in the presence of dust charging phenomena

Here, we consider a kinetic theory of dusty unmagnetized plasmas with variable charges on dust particles due to plasma currents onto dust particle surfaces. The exact expressions for the longitudinal and transverse dielectric functions are presented [9,10,12], the corresponding rates are given, and the scattering of electromagnetic waves in such plasmas is investigated.

We consider spherical dust grains (with the same radius $a$) embedded in an unmagnetized host plasma. One such inclusion dust particle is shown in Figure 1. Due to motion of electrons and ions in the host plasma and finite size of the dust particle, the electron and ion currents $I_{e,i}$ on the dust surface appear as functions of the electron/ion charge ($-e/e$), mass ($m_{e,i}$), number density ($n_{e,i}$), and the distribution function ($f_{e,i}$). The electromagnetic wave field $E$ incident on such a particle is scattered by its charge.

The current on a dust particle can be described by

$$I(q) = \sum_{\alpha} I_{\alpha}, \quad I_{\alpha} = \int dv e_{\alpha} \sigma_{\alpha} v_{\alpha}, \quad (1)$$
where the subscript $\alpha = e, i$ describes electrons or ions, $v \equiv |v|$ is the particle speed, $f_\alpha$ is the distribution function of the corresponding particle velocities, $e_\alpha$ is the electron (ion) charge, $q$ is the charge of the dust particle, and $\sigma_\alpha = \sigma_\alpha(q, v)$ is the charging cross-section,

$$\sigma_\alpha = \pi a^2 \left(1 - \frac{2e_\alpha q}{am_\alpha v^2}\right) \text{ if } \frac{2e_\alpha q}{am_\alpha v^2} < 1, \quad \sigma_\alpha = 0 \text{ if } \frac{2e_\alpha q}{am_\alpha v^2} \geq 1,$$

$m_\alpha$ is the electron or ion mass. For equilibrium distribution functions $f_\alpha^{\text{eq}}$, we have $I^{\text{eq}}(-Zde) = 0$, where $q = -Zde$ is the equilibrium charge of a dust particle.

Furthermore, it is convenient to introduce the dimensionless variables

$$\tau \equiv \frac{T_i}{T_e}, \quad z \equiv \frac{Zde^2}{aT_e},$$

where $T_e(i)$ is the electron (ion) temperature. Also, we introduce the dimensionless Havnes parameter which explicitly includes the dust size $a$:

$$P \equiv \frac{n_d aT_e}{n_e e^2},$$

where $n_d(e)$ is the dust (electron) density.

The charging dissipative process is described by the charging frequency $v_d^{\text{eq}}$ which for thermal particle distributions is given by

$$v_d^{\text{eq}} = -\frac{\partial I(q)}{\partial q} \bigg|_{q=-Zde} = \frac{1}{\sqrt{2\pi}} \frac{\omega_{pi}^2 a}{v_{Ti}} \left(1 + \tau + z\right).$$

Here, $\omega_{pi} = (4\pi n_i e^2/m_i)^{1/2}$ is the ion plasma frequency (for simplicity, we assume $e_i = e$) and $v_{Ti} = (T_i/m_i)^{1/2}$ is the ion thermal speed. Furthermore, the frequency $v_{cd}^{\text{eq}}$ characterizing the rate of capture of electrons by dust particles in the equilibrium state can be introduced by

$$v_{cd}^{\text{eq}} = \frac{n_d}{n_e} \int dv \sigma_e v_f = 2\sqrt{2\pi} a^2 v_{Te} n_d e^{-z} = v_d^{\text{eq}} P \frac{\tau + z}{1 + \tau + z},$$

where $v_{Te} = (T_e/m_e)^{1/2}$ is the electron thermal velocity.

We introduce the distribution function of dust particles [4]

$$f_d = f_d(q, r, t),$$

where the charge $q$ is the additional independent variable; the distribution function $f_d$ is normalized by the dust number density $n_d = \int f_d dq$. The corresponding kinetic equation is given by (we suppose that the dust particles have infinite masses and consequently are immobile, or $v_d = 0$)
The state of equilibrium corresponds to the equilibrium distribution function $f_d^{eq}$ of charges on dust particles.

The kinetics of electrons and ions are described by usual distribution functions $f_{\alpha}$ [10]:

\[
\frac{\partial f_{\alpha}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{\alpha}}{\partial \mathbf{r}} + \frac{e_{\alpha}}{m_{\alpha}} \mathbf{E} \cdot \frac{\partial f_{\alpha}}{\partial \mathbf{v}} = - \int \sigma_{\alpha} \nu (f_d f_{\alpha} - f_{eq}^{\alpha} f_{eq}^{\alpha}) \, dq. \tag{9}
\]

We assume that in the equilibrium state the electron and ion capture by the dust is compensated by external sources and the equilibrium electron (ion) distribution $f_{eq}^{\alpha}$ is isotropic. A term with the magnetic field has been neglected on the l.h.s. of (9).

In the case of thermal particle distributions and sufficiently high frequency [$\omega \sim \omega_{pe} \gg \max(k v_T^{eq}, v_d^{eq}, v_{ed}^{eq})$, where $\omega_{pe} = (4 \pi n_e e^2 / m_e)^{1/2}$ is the electron plasma frequency] of longitudinal waves, we have the following approximate expression [9,10]

\[
\varepsilon^{(l)}_{k\omega} \sim 1 + \frac{4 \pi}{\omega^2 k^2} \sum_{\alpha} \frac{-e_{\alpha}^2}{m_{\alpha} v_{T\alpha}^2} \int d\mathbf{v} \left\{ 1 - i \frac{v_{ad}^{eq}(\mathbf{v})}{\omega} - \frac{[v_{ad}^{eq}(\mathbf{v})]^2}{\omega^2} \right\} (\mathbf{k} \cdot \mathbf{v})^2 f_{eq}^\alpha \\
= 1 - \frac{\omega_{pe}^2}{\omega^2} \left[ 1 - i \frac{2}{3} (2 + z) v_{ed}^{eq} - \sqrt{\pi} z A e^z \left( \frac{v_{ed}^{eq}}{\omega} \right)^2 \right], \tag{10}
\]

where

\[
A \equiv \frac{5}{4} - \frac{z}{6} + \left( \frac{5}{4} - z + \frac{z^2}{3} \right) \int_1^{\infty} d\tau \exp[-(\tau^2 - 1)z]. \tag{11}
\]

Therefore, the charging process leads to appearing of additional imaginary and real parts in the high-frequency longitudinal dielectric permittivity. These additional terms depend on relation between the charging frequency and electron plasma frequency. For longitudinal electron plasma waves, the damping due to the charging process is given by

\[
\gamma_d^L \sim -\frac{1}{3} (2 + z) v_{ed}^{eq} = -\frac{1}{3} P(2 + z) \frac{\tau + z}{1 + \tau + z} v_d^{eq}. \tag{12}
\]

For the transverse part of the permittivity function we have [9,10]

\[
\varepsilon^{(t)}_{k\omega} \sim 1 + \frac{2 \pi}{\omega^2 k^2} \sum_{\alpha} \frac{-e_{\alpha}^2}{m_{\alpha} v_{T\alpha}^2} \int d\mathbf{v} \left\{ 1 - i \frac{v_{ad}^{eq}(\mathbf{v})}{\omega} - \frac{[v_{ad}^{eq}(\mathbf{v})]^2}{\omega^2} \right\} |\mathbf{k} \times \mathbf{v}|^2 f_{eq}^\alpha \\
= 1 - \frac{\omega_{pe}^2}{\omega^2} \left[ 1 - i \frac{2}{3} (2 + z) v_{ed}^{eq} - \sqrt{\pi} z A e^z \left( \frac{v_{ed}^{eq}}{\omega} \right)^2 \right]. \tag{13}
\]
which exactly coincides with expression (10) for the high-frequency longitudinal permittivity. Equation (13) gives the following damping rate for electromagnetic waves due to the charging effects on dust particles

$$\gamma_{EM}^d \simeq -\frac{1}{3} \frac{\omega^2_{pe}}{\omega^2} (2 + z) \nu_{eq}^d.$$

If the frequency of electromagnetic waves does not far exceed \(\omega_{pe}\), when the phase velocity is significantly influenced by the conduction current, the damping rate (14) is of the order of the electron capture rate \(\nu_{eq}^d\). Note also that the real corrections to the electron plasma frequency appear; these corrections are also effective for electromagnetic waves.

To obtain the scattering of electromagnetic waves on a dust particle, we write the scattering cross-section [9]

$$\sigma_{em} = \sigma^T \frac{3e^3}{16\pi} \int dk \left( 1 + \cos^2 \Theta \right) \frac{|Z_{k-k_0}^e|^2}{\omega_0^2} \delta \left( \omega_0 - \sqrt{\omega_{pe}^2 + k_0^2 c^2} \right),$$

where

$$\sigma^T = \frac{8\pi}{3} \frac{e^4}{m_e^2 c^3}$$

is the Thomson scattering cross section, \(c\) is the light speed, \(\Theta\) is the angle between incident and scattered waves, subscript 0 corresponds to the incident wave with frequency \(\omega_0 = (\omega_{pe}^2 + k_0^2 c^2)^{1/2}\), and \(-Z_k^e e\) is the ‘effective charge’ of the dust particle. In the linear approximation, the effective charge is given by

$$Z_k^{eff} = Z_d \frac{\varepsilon_k^{(e)} - 1}{\varepsilon_k},$$

where \(\varepsilon_k = \varepsilon_k^{(e)} + \varepsilon_k^{(i)} - 1\) is the full static dielectric permittivity of the plasma (i.e. \(\varepsilon_k^{(\alpha)}\) corresponds to the dielectric function of \(\alpha\) plasma component).

In the static approximation, we have [9]

$$\varepsilon_{k,\omega=0} = 1 + \frac{1}{k_2 \lambda_D^2} \frac{1 + \tau}{\tau} \left( 1 + P \frac{\tau + z}{1 + \tau + z} F_k \right),$$

where the form factor \(F_{k,\omega}\) is defined by

$$F_k = \frac{1 + \Gamma_{k,\omega=0}}{1 + G_{k,\omega=0}}.$$

The factor \(G_{k,\omega}\) is given by

$$G_{k,\omega} = -\frac{1}{\omega + i\nu_{eq}^d} \sum_{\alpha} \int dv \frac{\nu_{eq}^d(v) e_{\alpha} \sigma_{\alpha}^d \nu_{eq}^d}{\omega - k \cdot v + i\nu_{eq}^d(v)},$$
where
\[ \nu_{eqd}(\nu) = \int \sigma_{\alpha} v_{f_d}^{eq} dq, \quad \sigma''_{\alpha} = \frac{\partial \sigma_{\alpha}(q)}{\partial q} \bigg|_{q=-Z_d} . \tag{21} \]

Furthermore, in expression (19)
\[ \Gamma_{k\omega} = \sum_{\alpha} \int dv \frac{-ie_{\omega} \sigma''_{\alpha} v_{f_d}^{eq}}{\omega - k \cdot v + iv_{eqd}(\nu)}. \tag{22} \]

We see that the charging process leads to appearing of additional real part in the static longitudinal dielectric permittivity. In particular, this additional term depends on parameter \( P \). Note also that factor before the function \( F_k \) on the r.h.s. of (18) is exactly equal to \( v_{eqd}/v_d \), see (6).

For the electron part of the longitudinal permittivity, which is present in Equation (17), we therefore can obtain
\[ \varepsilon_{k\omega=0}^{(e)} = 1 + \frac{1}{k^2 \lambda_{De}^2} \left( 1 + P \frac{\tau + z}{1 + \tau + z} F_k \right). \tag{23} \]

Thus, for the effective charge (17) we finally find
\[ Z_{k}^{eff} = Z_d \left( 1 + \frac{1}{\tau} + \frac{k^2 \lambda_{De}^2}{1 + PF_k(\tau + z)/(1 + \tau + z)} \right)^{-1}. \tag{24} \]

Thus the charging dissipative process on a spherical inclusion in a plasma (specifically, the effect of capture of plasma electrons by dust particles) affects the electromagnetic wave propagation and scattering: in particular, we found that the effective cross section of scattering of electromagnetic waves on a dust particle depends not only on the scattering parameter \( 1/k\lambda_{De} \) but also on parameters characterizing the charging process, in particular, the Havnes parameter \( P \) which explicitly includes the dust size \( a \). We note that the scattering of electromagnetic waves in plasma is a powerful diagnostic tool which is widely used in laboratory as well as geophysical experiments (see, e.g. [40–42]). For complex dusty plasmas, this technique is useful for many applications, e.g. not only in laboratory, but also in technological dust-contaminated plasmas [3,20]. Thus the effects of finite dust sizes are important for numerous applications of complex plasmas [2].

4. Electromagnetic wave dispersion in the presence of surface plasmons on macroparticle inclusions

In this section, we investigate the electromagnetic wave propagation in a dusty plasma as an effective medium containing finite size inclusions and focus on the effect of surface plasmons on the spherical dust inclusions in a host plasma [11]. In general, surface plasmons are excited at the boundary of two media characterized by permittivities of the opposite sign.
Consider a single sphere of radius $a$ filled by medium with permittivity $\varepsilon_{in} = \varepsilon_{in,0} - \omega_{in}^2/\omega^2$, i.e. we assume that in addition to permittivity $\varepsilon_{in,0} = \text{const} \geq 1$ there is a number of free charges (‘electrons’) contributing to Drude-like dispersion with plasma frequency $\omega_{in}$ (and no losses) as shown in Figure 2. In this figure, we show a dust inclusion (in) particle characterized by permittivity $\varepsilon_{in}(\omega)$ and electric fields/displacement $E_{in}/D_{in}$ embedded in the host (h) medium characterized by permittivity $\varepsilon_{h}(\omega)$ and electric fields/displacement $E_{h}/D_{h}$. On the surface of the inclusion particle, the possibility of surface plasmons with the characteristic frequency $\Omega_{sp}$ is taken into account.

In two limiting cases we have: metal for $\varepsilon_{in,0} = 1$, $\omega_{in} \neq 0$ and dielectric for $\omega_{in} = 0$ and $\varepsilon_{in,0} = \varepsilon_{d} = \text{const} \geq 1$ (vacuum when $\varepsilon_{in,0} = 1$). The sphere is embedded in a host medium with similar type of permittivity $\varepsilon_{h} = \varepsilon_{h,0} - \omega_{h}^2/\omega^2$, $\varepsilon_{h,0} = \text{const} \geq 1$, with plasma frequency $\omega_{h}$ (no losses). Thus, we account for dielectric host with $\omega_{h} = 0$ and $\varepsilon_{h,0} = \varepsilon_{d} = \text{const} \geq 1$ (vacuum when $\varepsilon_{h} = 1$) and plasma with $\omega_{h} \neq 0$ and $\varepsilon_{h,0} = 1$. While taking into account the effects of temporal dispersion we ignore effects of space dispersion and other collective plasma phenomena (such as Debye screening, sheath formation, dust charging plasma currents, etc.).

Assume that surface plasmon fields are electrostatic and oscillate with the frequency $\omega$ (i.e. their electrostatic field potential $\varphi \propto \sin (\omega t)$). In spherical geometry with azimuthal symmetry, solutions of the (linear) Poisson’s equation $\nabla \cdot \varepsilon_{in(h)} \nabla \varphi = 0$ are (see [29], Section 4.2):

\begin{align*}
\varphi &= \sum_{j=0}^{\infty} C_j(\omega) \left( r/a \right)^j P_j(\cos \theta), \quad \text{for } r < a, \quad (25) \\
\varphi &= \sum_{j=0}^{\infty} C_j(\omega) \left( a/r \right)^{j+1} P_j(\cos \theta), \quad \text{for } r > a, \quad (26)
\end{align*}

**Figure 2.** Spherical dust inclusion particle characterized by the permittivity $\varepsilon_{in}(\omega)$ embedded in a host plasma with the permittivity $\varepsilon_{h}(\omega)$. The surface plasmons are excited at the boundary of the inclusion sphere and the host medium and are characterized by the surface plasmon frequency $\Omega_{sp}$. 

where $P_j$ are the Legendre polynomials and $C_j(\omega)$ are (arbitrary) functions of frequency $\omega$. The multipole expansion (25)–(26) is in spherical harmonics and corresponds for $j = 0$ (with $P_0 = 1$) to constant (on $r, \theta$) potential inside the sphere, while for higher harmonics $j \geq 1$ to dipole field (with $P_1 = \cos \theta$) for $j = 1$, etc.

The boundary conditions are the continuity of potential $\varphi$ and normal (to the boundary, i.e. radial in the considered spherical geometry) component of displacement $\varepsilon_{in(h)} \nabla \varphi$ across $r = a$. Thus, we obtain dispersion equation for surface plasmons:

$$j \varepsilon_{in} = -(j + 1) \varepsilon_h.$$ (27)

For $j = 0$ [const$(r, \theta)$ potential inside the sphere], we have dispersion equation $\varepsilon_h = 0$ corresponding to bulk electrostatic plasma-type oscillations ($\omega^2 = \omega_{h,0}^2 \equiv \omega_0^2 / \varepsilon_{h,0}$); the same for any $j$ and $\varepsilon_{in,0} = \varepsilon_{h,0}$, $\omega_h = \omega_{in}$ (no inclusion and therefore no boundary). For $j \geq 1$, solutions (surface plasmons) are possible only for opposite signs of $\varepsilon_{in}$ and $\varepsilon_h$ (i.e. there is no surface-type solutions when $\omega_{in} = \omega_h = 0$ and both $\varepsilon_{in(h),0} > 0$). The (squared) eigenfrequencies of surface oscillations are

$$\Omega_{sp,j}^2 = \frac{j \omega_{in}^2 + (j + 1) \omega_h^2}{\varepsilon_{in,0} + (j + 1) \varepsilon_{h,0}}.$$ (28)

In the limits of metal sphere in vacuum ($\varepsilon_{in,0} = \varepsilon_{h,0} = 1$, $\omega_h = 0$, $\omega_{in} \neq 0$) and vacuum spherical void in plasma ($\varepsilon_{in,0} = \varepsilon_{h,0} = 1$, $\omega_{in} = 0$, $\omega_h \neq 0$), respectively, we have known ([29], Section 4.2) expressions for the eigenfrequencies of surface plasmon dipole ($j = 1$) oscillations $\Omega_{sp,1} = \omega_{in} / \sqrt{3}$ and $\Omega_{sp,h} = \omega_h \sqrt{2/3}$. The factors $1 / \sqrt{3}$ or $\sqrt{2/3}$ are different from $1 / \sqrt{2}$ for the fundamental frequency of surface plasmon oscillations on the boundary of semi-infinite plasma ([29], p.5). To proceed correctly to the limit of infinitely large radius, we need to sum up all (the infinite number of) spherical harmonics contributing to the resulting factor $1 / \sqrt{2}$ for the semi-infinite plasma.

In the presence of external electromagnetic wave field, we proceed in the dipole approximation ($j = 1$ above) [15]. The electric field $E_{in}$ and the displacement $D_{in}$ inside a single sphere in the presence of field $E_0$ are [43]

$$D_{in} + 2 \varepsilon_h E_{in} = 3 \varepsilon_h E_0,$$ (29)

and for a number of identical non-interacting spheres the dilute limit $\tilde{\varepsilon}_h^{dil}$ of the effective permittivity $\tilde{\varepsilon}_h$ is given by [44]:

$$\tilde{\varepsilon}_h^{dil} = \varepsilon_h[1 + 3f(\varepsilon_{in} - \varepsilon_h)/(\varepsilon_{in} + 2 \varepsilon_h)],$$ (30)

where the volume fraction $f = N_{in} V_{in}/V$, with the full volume $V = N_{in} V_{in} + V_h$, corresponds to the volume occupied by $N_{in}$ spherical inclusions (dust in plasma), each of the volume $V_{in} = 4\pi a^3 / 3$, in the host volume $V_h$. The dipolar interactions between spheres can be accounted for by the excluded volume
approach [45] where the field acting on inclusion particles is the average host field \( E_h \), which differs from \( E_0 \) due to the correlations between positions of different (non-overlapping) spheres. The averaged field over the entire system is \( E_0 \). Thus

\[
fE_{in} + (1 - f)E_h = E_0, \tag{31}
\]

and for \( E_h \) we now have

\[
E_h = E_0 / (1 - f + 3f\varepsilon_h / (\varepsilon_{in} + 2\varepsilon_h)). \tag{32}
\]

Calculating the volume (average) polarization, we arrive at the effective permittivity \( \tilde{\varepsilon}_h \) given by

\[
\tilde{\varepsilon}_h = \varepsilon_h \frac{1 + 2\tilde{f}}{1 - \tilde{f}}, \quad \tilde{f} = f \frac{\varepsilon_{in} - \varepsilon_h}{\varepsilon_{in} + 2\varepsilon_h}. \tag{33}
\]

Here, we introduced the ‘impedance’ volume fraction \( \tilde{f} \).

Another way to derive (33) is to use the Clausius–Mossotti equation that relates the permittivity of the medium with induced dipole moments in it and takes into account the renormalization due to the dipole–dipole interactions. The Clausius–Mossotti equation is given by (see, e.g. [46])

\[
\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{4\pi}{3} n_p \alpha_p, \tag{34}
\]

where \( \alpha_p \) is the polarizability and \( n_p \) is the number density of the dipoles induced in the medium. Thus for an isolated sphere (34) may be rewritten as

\[
V_{in} \frac{\varepsilon_{in} - 1}{\varepsilon_{in} + 2} = \frac{4\pi \alpha_p}{3}, \tag{35}
\]

where \( V_{in} \) is the volume of the (inclusion) sphere, characterized by the permittivity \( \varepsilon_{in} \). Now, consider the host background medium (occupying the volume \( V_h \)) which for simplicity is supposed to be vacuum. For the case of \( N_{in} \) spherical inclusions (since the polarization of the spherical inclusion outside its surface can be attributed to that created by the corresponding dipole placed at the center of the sphere) placed in such host (and occupying the volume \( V - V_h = N_{in}V_{in} \)),

we relate (34) written now with the effective permittivity \( \tilde{\varepsilon}_h \), and (35):

\[
\frac{\tilde{\varepsilon}_h - 1}{\tilde{\varepsilon}_h + 2} = f \frac{\varepsilon_{in} - 1}{\varepsilon_{in} + 2}, \tag{36}
\]

with \( n_{in}V_{in} = N_{in}V_{in}/V = f \). Generalizing (36) for the host medium with \( \varepsilon_h \) instead of vacuum supposed in (35)–(36), we again arrive at (33).

The effective permittivity (33) contains possibilities for cut-offs and resonances related to excitation of surface plasmons. The resonance corresponds to
1 = \tilde{f} while the cut-off to 1 = −2\tilde{f}. Here, there is the difference with the dilute approximation giving for the resonance the exact dispersion of surface plasmons (also taking place in \tilde{f}): \varepsilon_{in} + 2\varepsilon_h = 0, leading to the eigenfrequencies (28). We see that the dilute approximation corresponds not only to \tilde{f} \ll 1, but also to small impedance fraction, \tilde{f} \ll 1. Obviously, sufficiently close to the surface plasmon resonance, \varepsilon_{in} + 2\varepsilon_h = 0, the dilute approximation fails even for small \tilde{f} due to increased contribution of dipole interactions; this is analogous to the Landau theory of phase transitions, when fluctuations in the system become correlated (act ‘in resonance’) near the transition [47]. As a result, the account of other dipoles shifts the wave resonance frequency from the surface plasmon frequency; for \tilde{f} \ll 1, this shift is proportional to \tilde{f}, see below Equation (40).

Far from the resonance, in the limit \tilde{f} \to 0 (no inclusions in the medium) we have from (33) \tilde{\varepsilon}_h \to \varepsilon_h; in the opposite limit \tilde{f} \to 1 (inclusions occupy all space), \tilde{\varepsilon}_h \to \varepsilon_{in}. While the limit \tilde{f} \to 1 gives formally correct result, we stress that expression (33) takes into account only dipole interactions and therefore is approximate; with increased \tilde{f}, other types of interactions (quadrupole, etc.) should be taken into account even far away from the resonance (not mentioning the fact that we in general cannot pack by spheres a finite volume of arbitrary, not spherical, shape with \tilde{f} = 1). On the other hand, directly at the resonance, a renormalization theory (such as the turbulent resonance broadening theory [48]) should be constructed.

Now, we apply the dispersion equation

\[ \tilde{\varepsilon}_h = k^2 c^2 / \omega^2 \equiv n^2, \]  

(37)

where \( n \) stands for the refractive index, and obtain [11]

\[ \left( 1 - \frac{\tilde{\omega}_h^2}{\omega^2} \right) \left( 1 - \frac{\Omega_c^2}{\omega^2} \right) = \tilde{n}^2 \left( 1 - \frac{\Omega_r^2}{\omega^2} \right). \]  

(38)

We remind that here \( \tilde{\omega}_h^2 = \omega_h^2 / \varepsilon_{h,0} \). Furthermore in (38), \( \tilde{n}^2 = k^2 c^2 / \omega^2 \), with decreased, due to the presence of dielectrics \( \varepsilon_{h,0} \) and \( \varepsilon_{in,0} \), speed of light: \( \tilde{c}^2 = c^2 / \varepsilon_{h,0} \), where [compare with expression (33)] \( \tilde{\varepsilon}_{h,0} = \varepsilon_{h,0}(1 + 2\tilde{f}_0)/(1 - \tilde{f}_0) \), and \( \tilde{f}_0 = f(\varepsilon_{in,0} - \varepsilon_{h,0})/(\varepsilon_{in,0} + 2\varepsilon_{h,0}) \). In Equation (38), we also have defined two new characteristic frequencies related to the presence of surface plasmon oscillations on inclusion particle surfaces: the cut-off frequency (squared) given by

\[ \Omega_c^2 = \Omega_{sp}^2 + \frac{6f\varepsilon_{h,0}(\Omega_{sp}^2 - \tilde{\omega}_h^2)}{\varepsilon_{in,0} + 2\varepsilon_{h,0} + 2f(\varepsilon_{in,0} - \varepsilon_{h,0})}, \]  

(39)

and the resonance frequency (squared) given by

\[ \Omega_r^2 = \Omega_{sp}^2 - \frac{3f\varepsilon_{h,0}(\Omega_{sp}^2 - \tilde{\omega}_h^2)}{\varepsilon_{in,0} + 2\varepsilon_{h,0} - f(\varepsilon_{in,0} - \varepsilon_{h,0})}, \]  

(40)
where the eigenfrequency of (dipolar) surface plasmons \( \Omega_{sp}^2 = \Omega_s^2 \), see (28). In general, for \( \tilde{\omega}_{in} \equiv \omega_{in}/\sqrt{\varepsilon_{in,0}} > \tilde{\omega}_h \equiv \omega_h/\sqrt{\varepsilon_{h,0}} \), the cut-off frequency is above the resonance frequency: \( \tilde{\omega}_{in} > \Omega_c > \Omega_{sp} > \Omega_r > \tilde{\omega}_h \). In the opposite case \( \tilde{\omega}_{in} < \tilde{\omega}_h \), we have \( \tilde{\omega}_{in} < \Omega_c < \Omega_{sp} < \Omega_r < \tilde{\omega}_h \).

For \( 0 < f < 1 \) (excluded degenerate cases \( f = 0,1 \)), dispersion equation (38) has cut-offs at \( \omega^2 = \Omega_c^2 \) and resonances at \( \omega^2 = \Omega_s^2 \). Its solutions correspond to two branches (real frequency, \( \omega^2 \geq 0 \), and refractive index, \( n^2 \geq 0 \)). When \( \tilde{\omega}_{in} > \tilde{\omega}_h \), the presence of surface plasmon oscillations changes the plasma branch by interrupting it at the surface plasmon resonance; as a result, two branches appear asymptotically approaching the electromagnetic plasma wave in the low-frequency and the high-frequency limits, respectively. For \( f \ll 1 \), the band gap \( \Delta \omega_g \) is small, \( \Delta \omega_g \sim f \Omega_{sp} \ll \Omega_{sp} \), while the band width of the first branch \( \Delta \omega_w \) is finite, \( \Delta \omega_w \sim \Omega_{sp} \). When \( \tilde{\omega}_{in} < \tilde{\omega}_h \), the presence of surface plasmon resonances does not affect the existence of host (plasma) branch but adds completely new branch fully related to surface plasmon oscillations. For \( f \ll 1 \), the band width of the surface plasmon branch is narrow \( \Delta \omega_w \sim f \Omega_{sp} \) while the band gap is finite \( \Delta \omega_g \sim \Omega_{sp} \).

Consider three limiting cases: (a), metal spherical inclusions in gaseous plasma, \( \varepsilon_{in,0} = \varepsilon_{h,0} = \tilde{\varepsilon}_{h,0} = 1, \tilde{\omega}_{in} = \omega_{pm}, \tilde{\omega}_h = \omega_{pe}, \Omega_{sp}^2 = (\omega_{pm}^2 + 2\omega_{pe}^2)/3 \), where \( \omega_{pm(e)} \) is the plasma frequency of metal (plasma) electrons, \( \Omega_c^2 = \Omega_{sp}^2 + 2f(\Omega_{sp}^2 - \omega_{pe}^2) \), and \( \Omega_r^2 = \Omega_{sp}^2 - f(\Omega_{sp}^2 - \omega_{pe}^2) \); (b), metal spherical inclusions in dielectric medium, \( \varepsilon_{in,0} = 1, \varepsilon_{h,0} = \varepsilon_d = \text{const}, \tilde{\omega}_{in} = \omega_{pm}, \tilde{\omega}_h = 0, \Omega_{sp}^2 = \omega_{pm}^2/(1 + 2\varepsilon_d), \Omega_c^2 = \Omega_{sp}^2[1 + 6f\varepsilon_d/(1 + 2\varepsilon_d + 2f(1 - \varepsilon_d))], \) and \( \Omega_r^2 = \Omega_{sp}^2[1 - 3f\varepsilon_d/(1 + 2\varepsilon_d - f(1 - \varepsilon_d))] \); and (c), dielectric spherical inclusions in plasma, \( \varepsilon_{in,0} = \varepsilon_d = \text{const}, \varepsilon_{h,0} = 1, \tilde{\omega}_{in} = 0, \tilde{\omega}_h = \omega_{pe}, \Omega_{sp}^2 = 2\omega_{pe}^2/(\varepsilon_d + 2), \Omega_c^2 = \Omega_{sp}^2[1 - 3f\varepsilon_d/(\varepsilon_d + 2 + 2f(\varepsilon_d - 1))], \) and \( \Omega_r^2 = \Omega_{sp}^2[1 + 3f\varepsilon_d/2(\varepsilon_d + 2 - f(\varepsilon_d - 1))] \). Note that for \( \varepsilon_d = 1 \), we have in (b) \( \Omega_c^2 = \Omega_{sp}^2(1 + 2f) \) and \( \Omega_r^2 = \Omega_{sp}^2(1 - f) \), coinciding with (a) when \( \omega_{pe} = 0 \); for \( \varepsilon_d = 1 \), we have in (c) \( \Omega_c^2 = \Omega_{sp}^2(1 - f) \) and \( \Omega_r^2 = \Omega_{sp}^2(1 + f/2) \).

For the case (a), the calculated (squared) refraction index follows the qualitative behavior shown in Figure 3 [11]. For the case (c), the calculated (squared) refraction index follows the qualitative behavior shown in Figure 4 [11].

Thus the presence of dust particle inclusions of the finite size and the possibility of related surface plasmon resonances makes complex plasma qualitatively different from a classical electron–ion plasma (with no external fields) by producing cut-offs and resonance in the propagation of electromagnetic plasma waves. A complex plasma with metallic dust is characterized by the resonance frequency of the order of the surface plasmon frequency in the metal; a complex plasma with dielectric dust is characterized by the resonant frequency of the order of the plasma frequency of electrons. The band gaps are controlled by the number density of inclusions. The surface plasmon resonance on dielectric particles in
Figure 3. Qualitative behavior, Ref. [11], of the squared index of refraction $\tilde{n}^2$ vs squared frequency $\omega^2$ for electromagnetic wave in the case $\tilde{\omega}_m > \tilde{\omega}_h$. The lower cut-off is at $\tilde{\omega}_h^2$.

Figure 4. Qualitative behavior, Ref. [11], of the squared index of refraction $\tilde{n}^2$ vs squared frequency $\omega^2$ for electromagnetic wave in the case $\tilde{\omega}_m < \tilde{\omega}_h$.

host plasma environment thus may lead to the possibility of electromagnetic wave propagation at frequencies below the electron plasma frequency.

For the considered approach, the wavelengths should exceed the interparticle distance and/or the space dispersion length. In typical laboratory dusty plasma, Debye length (in experiments $\lambda_{De} \sim 10^{-2}$ cm) stands approximately for both distances; thus of most interest for experiments in plasma are relatively low frequencies (as compared to the optical range), with the corresponding wavelength $\lambda_w \gtrsim 10^{-1}$ cm. This range corresponds to the new (due to surface plasmon resonances) wave band in the case of dielectric particles in plasma, see case (c); these wavelengths should be less than the size of the dust cloud (for typical laboratory experiments the dust cloud size does not exceed 10 cm, but this size can be significantly larger for ionospheric or cosmic dust clouds).
5. Discussion and conclusion

In dusty plasmas, the dust particles are charged due to various processes; in laboratory plasmas, this is mostly by plasma currents [4,5]. The charging by plasma currents introduces new characteristic (charging) frequency $\nu_{ch}$, which introduces a new damping and affects the wave scattering at the ion time scale. For the electromagnetic wave band structure, the strongest effect of the dust charge fluctuations would have been for the very high frequency range, of the order of $\Omega_{sp}$. Even in the case of dielectric spherical inclusions in gaseous plasma with $\Omega_{sp} \sim \omega_{pe}$, this is much higher than the typical frequency of actual dust charge variations $\nu_{ch}$ which is affected by ion processes, and $\nu_{ch} \ll \omega_{pe}$. Thus when considering the electromagnetic wave band structure, the charges can be treated as constant affecting the interparticle separation and therefore the dust packing fraction. The packing fraction is directly related to the dust number density $n_d$ and size (radius $a$) of dust particles as $f = 4\pi n_d a^3 / 3$; e.g. $f \sim 10^{-3}$ corresponds to $n_d \sim 10^8 \sim 10^9 \text{ cm}^{-3}$ for micrometer-size dust particles. In experiments, the dust number density can be controlled by the interparticle distance; thus the packing fraction can be changed by choosing the spherical dust radius and controlling the plasma density. For larger dust spheres, of a few tens of micrometers in radius in a rather dense plasma (say, $10^{12} \text{ cm}^{-3}$), a larger packing fraction can be achieved; for smaller dust particles in the tens/hundreds nanometer range, the packing fraction becomes significantly less. However, even for smaller packing fraction, the surface plasmon resonance is still present and contributes to the wave dispersion although with narrower band width/band gap.

The control of the charge and number density of dust and therefore the packing fraction of dust particle inclusions is critical for many dust-related phenomena to be observed. The presence of dust particles is limited in the region of sheath in the ground laboratory experiment thus limiting their numbers; on the other hand, in gravity-free plasma environment, the number density can be controlled in a wider range [2]. Furthermore, the metallic dust particles are much heavier than dielectric ones, and for laboratory experiments there might be a problem with their levitation while there is no such restriction for microgravity experiments; we therefore can expect such experiments with, e.g. gold or silver particles onboard the ISS [2]. Note that the lower frequency band in the case of metallic dust is wide so the restriction for large wavelengths compared to the interparticle distance can be satisfied in, e.g. IR or THz range, if small enough (a few hundred nanometers) particles with small charges are separated by distances less than micrometer.

In considered effects, we have neglected all losses in the considered media; the losses are usually related to collision processes in host gaseous plasma as well as in metallic plasma of inclusion dust particles. In host gas-discharge weakly ionized plasmas, the most effective are usually collisions of charged plasma
particles with neutrals [2,4,5]. For the dust charging phenomena, the approach of Section 3 corresponds to the dust charging frequency \( \nu_{ch} \gg \nu_{coll} \) where the effective collision frequency is the largest of the respective rates for charged plasma particle collisions with neutrals. For Section 4, the band gap effects with inclusion (dust) particles, the strongest are effects due to losses in metals (i.e. metallic dusts) with \( n_{em} \approx (1 \sim 10) \times 10^{22} \text{ cm}^{-3} \) and \( \omega_{pm}/2\pi \approx 9000\sqrt{n_{em}} \approx 10^{16} \text{ Hz} \) (in dusty plasma, \( \omega_{pm} \gg \omega_{pe} \), with \( \omega_{pe}/2\pi \) of order few GHz in experiments). The relaxation times are \( \tau_m \approx (1 \sim 10) \times 10^{-15} \text{ s} \) [49]; this gives \( \omega_{pm} \tau_m \approx (1 \sim 10) \times 10^{2} \). When \( f \) is small, \( f \ll 10^{-3} \), we have \( \Delta \omega_{g,m} \tau_m \approx f \omega_{pm} \tau_m \lesssim 1 \), and the band gap as well as the surface plasmon resonance may completely disappear due to high damping on dust metal particle surfaces [11].

To conclude, the finite size of dust particles, as compared with sizes of electrons and ions, can lead to size-related effects; here, we presented two types of such phenomena affecting the electromagnetic wave propagation: (1), the collection of plasma electrons and ions by macroparticle inclusions and related modification of such plasma dielectric properties, and (2), the surface plasmon resonances on the macroparticle surfaces and their effect on electromagnetic wave propagation. To account for the charging phenomena, we employed the kinetic approach taking into account self-consistent kinetics of plasma electrons and ions; to consider the presence of surface plasmons, we used the effective plasma permittivity by employing the Maxwell Garnett approximation.

We demonstrated that dust particle inclusions of a finite size can significantly modify the electromagnetic wave propagation and scattering characteristics. In particular, the effective scattering cross section of electromagnetic waves on a dust particle depends not only on the scattering parameter containing such plasma characteristics as Debye length, but also depending on parameters characterizing the charging process, in particular, the Havnes parameter which explicitly depends on the dust size. Since the scattering of electromagnetic waves in plasma is a powerful diagnostic method widely used in laboratory and processing complex dusty plasmas, these effects of finite dust sizes are important for numerous applications. Also, the presence of dust will affect the dispersion nature to determine the propagation frequency and scattering characteristics in such dusty plasmas as the mesospheric/ionospheric plasma and cosmic plasma clouds. Moreover, the electromagnetic wave dispersion relation has been used to identify the distance to the interstellar object like pulsars [50]. Due to modification of the electromagnetic frequency cut-off, the proposed dispersion relation could affect the estimated distance of the known pulsars in the abundant presence of dust in the interstellar environment.

**Disclosure statement**

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