The Θ⁺ (1540) as a heptaquark with the overlap of a pion, a kaon and a nucleon

P. Bicudo and G. M. Marques
Dep. Física and CFIF, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

We study the very recently discovered Θ⁺ (1540) at SPring-8, at ITEP and at CLAS-Thomas Jefferson Lab. We apply the same RGM techniques that already explained with success the repulsive hard core of nucleon-nucleon, kaon-nucleon exotic scattering, and the attractive hard core present in pion-nucleon and pion-pion non-exotic scattering. We find that the K − N repulsion excludes the Θ⁺ as a K − N s-wave pentaquark. We explore the Θ⁺ as a heptaquark, equivalent to a N + π + K borromean bound-state, with positive parity and total isospin I = 0. We find that the Θ⁺ may be a pentaquark state. We conclude with predictions that can be tested experimentally.

In this paper we study the exotic hadron Θ⁺ [1] (narrow hadron resonance of 1540 MeV decaying into a nK⁺) very recently discovered at SPring-8 [2], and confirmed by ITEP [3] and by CLAS at the TJNL [4]. This is an extremely exciting state because it may be the first exotic hadron to be discovered, with quantum numbers that cannot be interpreted as a quark and an anti-quark meson or as a three quark baryon. Exotic multiquarks are expected since the early works of Jaffe [5, 6], and some years ago Diakonov, Petrov and Polyakov [7] applied skyrmions to a precise prediction of Θ⁺. Very recent studies suggest that Θ⁺ is a pentaquark state [8]. The nature of this particle, its isospin, parity [9] and angular momentum, are yet to be determined.

Here we assume a standard Quark Model (QM) Hamiltonian, with a confining potential and a hyperfine term. Moreover our Hamiltonian includes a quark-antiquark annihilation term which is the result of spontaneous chiral symmetry breaking. We start in this paper by reviewing the QM, and the Resonating Group Method (RGM) [10] which is adequate to study states where several quarks overlap. To illustrate the RGM, we first apply it to compute the masses of all the possible s-wave and p-wave uuddσ interactions. We verify that the multiquarks computed with the Hamiltonian [11] are too heavy to explain the Θ⁺ resonance, except for the I = 0, Jπ = 1/2⁺ state. We then apply the RGM to show that the exotic N − K hard core s-wave interaction is repulsive [11, 12]. The result is consistent with the experimental data [13, see Fig. 1]. We think that this excludes, in our approach, the Θ⁺ as a bare pentaquark uuddσ state or as a tightly bound s-wave N − K narrow resonance. However the observed mass of Θ⁺ is larger than the sum of the K⁺ and n masses by 1540 − 940 − 494 = 106 MeV, and this suggests that a π could also be present in this system, in which case the binding energy would be of the order of 30 MeV. Moreover this state of seven quarks would have a positive parity, and would have to decay to a p-wave N − K system, which is suppressed by angular momentum, thus explaining the Θ⁺ narrow width. With this natural description in mind we then apply the RGM to show that π − N and π − K hard core interactions are attractive. Finally we put together π − N, π − K and N − K interactions to show that Θ⁺ is possibly a borromean three body s-wave boundstate of a π, a N and a K, with positive parity and total isospin I = 0. To conclude, we estimate the masses of other possible resonances of the Θ⁺ family and compare the QM formalism with other methods that address the Θ⁺.

Our Hamiltonian is the standard QM Hamiltonian,

\[ H = \sum_i T_i + \sum_{i<j} V_{ij} + \sum_{ij} A_{ij} \]  

(1)

where each quark or antiquark has a kinetic energy \( T_i \) with a constituent quark mass, and the colour dependent two-body interaction \( V_{ij} \) includes the standard QM confining term and a hyperfine term,

\[ V_{ij} = -\frac{3}{16} \hat{X}_i \cdot \hat{X}_j \left[ V_{con}(r) + V_{hyp}(r) \hat{S}_i \cdot \hat{S}_j \right] . \]  

(2)

For the purpose of this paper the details of potential [2] are unimportant, we only need to estimate its matrix elements. The hadron spectrum is compatible with,

\[ \langle V_{hyp} \rangle \simeq \frac{4}{3} (M_\Delta - M_N) \]  

(3)
Moreover we include in the Hamiltonian (11) a quark-antiquark annihilation potential $A_{ij}$. The quark-antiquark annihilation is constrained when the quark model produces spontaneous chiral symmetry breaking (15, 16). The annihilation potential $A$ is also present in the $\pi$ Salpeter equation,

$$\begin{bmatrix} 2T + V & A \\ A & 2T + V \end{bmatrix} \begin{pmatrix} \phi^+ \\ \phi^- \end{pmatrix} = M_\pi \begin{pmatrix} \phi^+ \\ -\phi^- \end{pmatrix}$$

(4)

where the $\pi$ is the only hadron with a large negative energy wave-function, $\phi^- \approx \phi^+$. In eq. (4) the annihilation potential $A$ cancels most of the kinetic energy and confining potential $2T + V$. This is the reason why the pion has a very small mass. From the hadron spectrum and using eq. (4) we determine the matrix elements of the annihilation potential,

$$\langle 2T + V \rangle_{s=0} \approx \frac{2}{3} (2M_N - M_\Delta)$$

$$\Rightarrow \langle A \rangle_{s=0} \approx \frac{2}{3} (2M_N - M_\Delta).$$

(5)

We stress that the QM of eq. (11) not only reproduces the meson and baryon spectrum as quark and antiquark bound-states (from the heavy quarkonium to the light pion mass), but it also complies with the PCAC theorems. This includes the Adler zero (17), and the Weinberg theorem (15, 16), for pion-pion scattering. Therefore our model is adequate to address the $\Theta^+$, which was predicted by Diakonov, Petrov and Polyakov in an effective chiral model.

With the RGM (10) we compute the matrix elements of the microscopic Hamiltonian described in eq. (10). This method produces both the energy of a multiquark state and the effective hadron-hadron interaction. We arrange the wave functions of quarks and antiquarks in antisymmetrized overlaps of simple colour singlet quark clusters, the baryons, and mesons. This is illustrated in Fig. 2 where we show how a tetraquark system can be arranged in a pair of mesons $A$ and $B$. Once the internal energies $E_A$ and $E_B$ of the two hadronic clusters are accounted, the remaining energy of the meson-baryon or meson-meson system is computed with the overlap of the inter-cluster microscopic potentials,

$$V_{\text{mes}} \frac{A}{\text{mes}} \frac{B}{\text{mes}} = \langle \phi_B \phi_A \rangle - (V_{14} + V_{15} + 2V_{24} + 2V_{25})3P_{14} + 3A_{15}\phi_A \phi_B / (\phi_B \phi_A | 1 - 3P_{14}| \phi_A \phi_B)$$

FIG. 2: Jacobi coordinates of the incoming four quark wavefunction $\phi_A(\rho_A) \phi_B(\rho_B) \psi(\lambda_{AB})$.

FIG. 3: Examples of RGM overlaps: (a) norm overlap for meson-baryon interaction; (b) kinetic overlap for meson-meson interaction; (c) interaction overlap for meson-meson interaction; (d) annihilation overlap for meson-baryon interaction.
1775 MeV. All the other multiquarks, with higher spin, isospin, or angular momentum quantum numbers have a mass at least 400 MeV heavier than $M_N + M_K$. This is expected with our standard quark model Hamiltonian defined in eq. (1). The positive mass shifts of more than 400 MeV are either produced by the hyperfine interaction estimated in eq. (3), or by a p-wave excitation which is also directly estimated from the baryon spectrum.

We now proceed to study the $\Theta^+$ in hadron-hadron scattering, in particular we address the most favorable $I = 0, J^P = 1/2^-$ channel which is coupled to $K - N$ s-wave scattering. The RGM was used by Ribeiro [24] to show that in exotic hadron-hadron scattering, the quark-quark potential together with Pauli repulsion of quarks produces a repulsive short range interaction. For instance this explains the $N - N$ hard core repulsion [24], preventing nuclear collapse. Deus and Ribeiro [27] used the same RGM to show that, in non-exotic channels, the quark-antiquark annihilation could produce a short core attraction. Recently the short core attraction was further understood [17] [18], and it was shown that the QM and the RGM reproduce the Weinberg Theorem for $\pi - \pi$ scattering, with attraction in the $I = 0$ channel and repulsion in the exotic $I = 1$ channel. A convenient basis for the wave-functions is the harmonic oscillator basis,

$$\phi^\alpha_{000}(p_\rho) = \mathcal{N}_\alpha^{-1} \exp \left(-\frac{p_\rho^2}{2\alpha^2}\right), \quad \mathcal{N}_\alpha = \left(\frac{\alpha}{2\sqrt{\pi}}\right)^{\frac{3}{2}},$$

where, in the case of vanishing external momenta $p_A$ and $p_B$, the momentum integral in eq. (6) is simply $\mathcal{N}_\alpha^{-2}$. For instance in the matrix element of the exchange operator $P_{13}$ in a meson-meson system, the coordinates of the incoming $\phi_A\phi_B$ functions are illustrated in Fig. 2 while the coordinates of the outgoing $\phi'_A\phi'_B$ have the quark 1 and 3 exchanged. We summarize [11] [18] [24] the effective potentials computed for the different channels,

$$V_{K-N} = \frac{4}{3}(M_\Delta - M_N)\frac{1}{2} + \frac{8}{3}r_K \cdot r_N \mathcal{N}_\alpha^{-2},$$

$$V_{\pi-N} = \frac{2}{9}(2M_N - M_\Delta)\tau_\pi \cdot \tau_N \mathcal{N}_\alpha^{-2},$$

$$V_{\pi-K} = \frac{8}{27}(2M_N - M_\Delta)\tau_\pi \cdot \tau_K \mathcal{N}_\alpha^{-2},$$

where $\tau$ are the isospin matrices. The results of eqs. [6] and [8] are recognised in eq. [9]. The vanishing momentum case of eq. [5] is sufficient to compute the scattering lengths with the Born approximation, and to estimate that $\alpha \approx 3$ Fm$^{-1}$. The estimation of $\alpha$ is an important by-product of this method because the hadronic size can not be estimated directly by the hadronic charge radius which is masked by the vector meson dominance.

However we are interested in binding and therefore we have to proceed to the finite momentum case. Then the effective potentials in eq. [5] turn out to be multiplied by the gaussian separable factor, exp $[\frac{-p_\rho^2}{2\beta^2}] \int \frac{d^3p}{(2\pi)^3} \exp [\frac{-p_\rho^2}{2\beta^2}]$. In the exotic $K - N$ channels, we can prove this result and we also show that the new parameter $\beta = \alpha$. This occurs because the overlaps decrease when the relative momentum of hadrons $A$ and $B$ increases. In the non-exotic $\pi - N$ and $\pi - K$ channels the present state of the art of the RGM does not allow a precise determination of the finite momentum overlap. In this case $\beta$ is a new degree of freedom that will be fitted with the experiment. In particular, in pionic channels the Adler zero [13] disappears at finite momentum, thus contributing to the enhancement of the overlap when momentum increases. Nevertheless, we expect that eventually the overlap decreases due to the geometric wave-function overlap in momentum space. So we expect that $\beta >> \alpha$ in the pionic channels. This parameterization in a separable potential enables us to use standard techniques [11] to exactly compute the scattering $T$ matrix. The scattering length $d\delta_0/dk_{c.m.}$ is,

$$a = -\frac{\sqrt{\pi}}{\alpha} \frac{4\mu v}{\alpha^2 + \frac{\alpha^2}{4} 4\mu v}.$$

The parameters and results for the relevant channels are summarized and compared with experiment [13] [24] [27] in Table I. We have fitted $\alpha$ with the $I = 1$ $K - N$ scattering. We use $\beta = \alpha$ in the repulsive channels. We fit $\beta$ with the appropriate scattering lengths in the attractive $I = 1/2$ pionic channels.

Let us first apply our method to the $K - N$ exotic scattering. The only channel where we find a pentaquark with a mass close to 1540 MeV is the $I = 0, J^P = 1/2^+$ channel. This pentaquark is open to the $K - N$ s-wave channel, which is repulsive for $I = 0$, therefore its decay width is quite large. We think that this excludes $\Theta^+$ as a $uudd\bar{s}$ pentaquark, because its experimental decay width is quite narrow. With our method we reproduce the $K - N$ exotic s-wave phase shifts, see Fig. 1 where indeed there is no evidence for the $\Theta^+$ state. In what concerns the $\pi - N$ system and the $\pi - K$ systems, the corresponding parameters in Table I are almost identical. There we find repulsion for $I = 3/2$ and attraction for $I = 1/2$. The repulsion in the $I = 3/2$ channel prevents a bound state in this channel. In what concerns the $I = 3/2$ channel.

| channel | $\mu$ | $v_{th}$ | $\alpha$ | $\beta$ | $a_{th}$ | $a_{exp}$ |
|---------|-------|-----------|-----------|-----------|-----------|-----------|
| $K - N_{I=0}$ | 1.65 | 0.50 | 3.2 | 3.2 | -0.14 | -0.13 ± 0.04 |
| $K - N_{I=1}$ | 1.65 | 1.75 | 3.2 | 3.2 | -0.30 | -0.31 ± 0.01 |
| $\pi - N_{I=1}$ | 0.61 | -0.73 | 3.2 | 11.4 | 0.25 | 0.246 ± 0.007 |
| $\pi - N_{I=0}$ | 0.61 | 0.36 | 3.2 | 3.2 | -0.05 | -0.127 ± 0.006 |
| $\pi - K_{I=1}$ | 0.55 | -0.97 | 3.2 | 10.3 | 0.35 | 0.27 ± 0.08 |
| $\pi - K_{I=0}$ | 0.55 | 0.49 | 3.2 | 3.2 | -0.06 | -0.13 ± 0.06 |

TABLE I: This table summarizes the parameters $\mu, v, \alpha, \beta$ (in Fm$^{-1}$) and scattering lengths $a$ (in Fm).
1/2 channel, the attraction is not sufficient to provide for a bound state, because the \( \pi \) is quite light and the attractive potential is narrow. From eq. (10) we conclude that we have binding if the reduced mass is \( \mu \geq -\frac{\alpha^2}{2v^2} \). With the present parameters this limit is \( 0.86 \rightarrow 1.07 \) Fm\(^{-1} \). This is larger than the \( \pi \) mass, therefore it is not possible to bind the \( \pi \) to a \( K \) or to a \( N \). All that we can get is a very broad resonance. For instance in the \( \pi - K \) channel this is the kapa resonance [28], which has been recently confirmed by the scientific community. However, with a doubling of the interaction, produced by a \( K \) and a \( N \), we expect the \( \pi \) to bind.

We now investigate the borromean [14] binding of the exotic \( \Theta^+ \) constituted by a \( N, K \) and \( \pi \) triplet. In what concerns isospin, we need the \( \pi \) to couple to both \( N \) and \( K \) in \( I = 1/2 \) states for attraction. It turns out that the possible \( K-N \) states decaying in the observed \( K^+ - n \) are obtained with a linear combination of one total \( I = 0 \) and two total \( I = 1 \) states. Because the Hamiltonian does not mix the total \( I = 0 \) with the total \( I = 1 \), we can study them separately. The \( I = 0 \) state includes both the \( \pi - N \) and \( \pi - K \) in \( I = 1/2 \) states, however the \( N - K \) pair is in a \( I = 1 \) state, nevertheless we expect that this is not sufficiently repulsive to cancel the binding. In what concerns the \( I = 1 \) states, they both have a significant \( I = 3/2 \) either in the \( \pi - N \) pair or in the \( \pi - K \) pair. This excludes the \( I = 1 \) states because either the \( N \) or the \( K \) would not be bound. The only real candidate for binding is the total \( I = 0 \) state, see Fig. 4.

Since the \( \pi \) is much lighter that the \( N - K \) system, we study the borromean binding adiabatically. As a first step, we start by assuming that the \( K \) and \( N \) are essentially motionless and separated by \( r_N - r_K = 2b \). This will be improved later. We also take advantage of the similarities in Table 1 to assume that the two heavier partners of the \( \pi \) have a similar mass of 3.64 Fm\(^{-1} \) and interact with the \( \pi \) with the same separable potential. Then we solve the bound state equation for a \( \pi \) in the potential \( V_{\pi - N} + V_{\pi - K} \),

\[
v \frac{\phi_{\pi 00}(r_\pi - \vec{b})}{N_\alpha} \int d^3 r' \frac{\phi_{\pi 00}(r_\pi - \vec{b})}{N_\alpha} + (-\vec{b} \leftrightarrow \vec{b}) = 0
\]

where this potential is wider in the direction of the \( K-N \) axis. Nevertheless this potential remains separable and we can apply standard techniques to find the poles of the \( T \) matrix, which occur exactly at the \( \pi \) energy. The \( \pi \) energy is depicted if Fig. 5 and it is negative as expected.

Once the \( \pi \) binding energy is determined, we include it in the potential energy of the \( K-N \) system, which becomes the sum of the repulsive \( K-N \) potential and the \( \pi \) energy. We find that for short distances the total potential is indeed attractive. Finally, using this \( K-N \) potential energy we solve the Schrödinger equation for the system, thus including the previously neglected \( K \) and \( N \) kinetic energies. However here we are not able to bind the \( K-N \) system, because the total effective \( K-N \) potential is not sufficiently attractive to cancel the positive \( K-N \) kinetic energy. Nevertheless the \( K \) is heavier than the \( \pi \), thus a small enhancement of the attraction would suffice to bind the heptaquark. We remark that existing examples of narrow resonances with a trapped \( K \) are the \( f_0(980) \) [28] the \( D_s(2320) \) [29] very recently discovered at BABAR [30], and possibly the \( \Lambda(1405) \). Moreover we expect that meson exchange interactions and the irreducible three-body overlap of the three hadrons, that we did not include here, would further increase the attractive potential. Therefore it is plausible that a complete computation will eventually bind the \( K-\pi-N \) system.

To conclude, in this paper we address the \( \Theta^+ \) hadron with a standard quark model Hamiltonian, where the quark-antiquark annihilation is constrained by the spontaneous breaking of chiral symmetry. We first find that the \( \Theta^+ \) hadron very recently discovered cannot be an s-wave or p-wave pentaquark. To provide sufficient binding, it would be necessary to change the spin-independent interaction, for instance to a bag model potential [3], or to change the hyperfine interaction to an effective pion exchange interaction [20, 21].

We also find that the \( \Theta^+ \) may be essentially a heptaquark state, composed by the overlap of a \( \pi \), a \( K \) and a \( N \). This scenario has many interesting features. The \( \Theta^+ \) would be, so far, the only hadron with a trapped \( \pi \). Moreover the \( \pi \) would be trapped by a rare three body borromean effect. And the decay rate to a \( K \) and a \( N \) would be suppressed since the \( \pi \) needs to be absorbed with a derivative coupling, while the involved hadrons have a very low momentum in this \( \Theta^+ \) state. Because
the $\Theta^+$ would be composed by a $N$ and two pseudoscalar mesons, its parity would be positive, $J^P = \frac{1}{2}^+$. Our result with a positive parity agrees with the prediction of Diakonov, Petrov and Polyakov, although these authors suggested that parity was due to a $p$-wave excitation. The isospin of the $\Theta^+$ would be $J = 0$ in order to ensure the attraction of the $\pi$ both by the $N$ and the $K$.

We now detail other possible three body hadronic molecules that can be searched experimentally. Our mechanism is also expected to bind either a $K\pi K$ or a $N\pi N$ into a $J = 0$ narrow resonance with negative parity, slightly below threshold. Because the antiquark $\bar{s}$ is just a spectator in our scenario, similar states with flavour $uudd$ or $uudd\bar{b}$ are also expected. Moreover we estimate in a three meson scenario the masses of the positive parity anti-decuplet predicted by Diakonov, Petrov and Polyakov. A three hadron molecule $NNK$ is expected in the iso-doublet including the state $uud$, with a mass close to 1540 MeV. In the iso-triplet including the state $uus\bar{d}$, the binding is comparable with the $\Theta^+$ case and a $NK\pi$ is suggested, again at a mass close to 1540 MeV. The iso-quadruplet including the state $uuss\bar{d}$ is also expected to include a three body $NNK$ binding with a mass close to 1900 MeV. Therefore an anti-decuplet is possible in the three hadron scenario, but the masses differ from the Diakonov, Petrov and Polyakov prediction.

We plan to proceed with the challenging study of the heptaquark state, including the $p$-wave $K - N$ coupled channel, meson exchange interactions and three-body potentials. This will be done elsewhere.

We are very grateful to George Rupp for pointing our attention to the pentaquark state.

The work of G. Marques is supported by Fundação para a Ciência e a Tecnologia under the grant SFRH/BD/984/2000.

* Electronic address: bicudo@ist.utl.pt
† Electronic address: gmarques@cfif.ist.utl.pt

[1] For a recent online review see P. Schewe, J. Riordon, B. Stein; Physics News Update 644, 1, June 30 (2003).
[2] T. Nakano et al., arXiv:hep-ex/0301020, submitted to Phys. Rev. Lett.
[3] V. Barmin et al., arXiv:hep-ex/0304040.
[4] K. Hicks, to be published in the proceedings of the Conference on the Intersections of Particle and Nuclear Physics, New York may 2003, edited by Zohreh Parsa (2003).
[5] R. L. Jaffe, Phys. Rev. D 15 281 (1977).
[6] D. Strottman, Phys. Rev. D 20 748 (1979).

| flavour | $NK\pi_{I=0}$ | $NK\bar{K}_{I=1/2}$ | $N\bar{K}_{I=3/2}$ | $N\bar{K}\bar{K}_{I=2}$ |
|---------|---------------|----------------------|---------------------|----------------------|
| mass [MeV] | 1540 | 1900 | 1540 | 1900 |

TABLE II: Approximate masses of the anti-decuplet.