Entanglement behavior of quantum states of fermionic system in accelerated frame

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In this paper, we investigate the behavior of bipartite entanglement of fermionic systems when one of parties is traveling with a uniform acceleration. For the ordering problem in fermionic systems, we apply the recent result in [Montero and Martín-Martínez, Phys. Rev. A 83 052306 (2011)]. Based on the approach, we consider both pure and mixed entangled states, and we show that the behavior in terms of the entanglement measure, negativity, allows one to obtain physical results, i.e. its convergence in the infinite acceleration. The behavior shows that the ordering employed is relevant to derive physical results for fermionic entanglement. This also corrects the previous analysis of [Martín-Martínez and Fuentez].

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I. INTRODUCTION

Entanglement is one of central characteristics in quantum information theory, as it contains correlations that do not have a classical counterpart. Entangled states can be prepared by correlations stronger than those of local classical systems assisted with classical communication, but they do not exhibit arbitrarily strong correlations. Therefore, quantum theory is consistent with relativistic theories.

Along the line, it is of fundamental question in relativistic quantum information theory how the behavior of entanglement can be described in a relativistic setting, particularly when one of parties is traveling in an acceleration [1] [2]. This can be seen from two parties who share entangled states, while one remains in an inertial frame and the other is described in accelerated frame. It is highly nontrivial that, even though entanglement in bosonic systems disappears in the limit of infinite acceleration, entanglement in fermionic systems can survive in the limit [3][4]. This is remarkable in that entanglement related to correlations can behave differently, depending on fields, and can also persist in such a limit. However the analysis for the fermionic system was only based on the single-mode approximation [3][4]. To obtain a precise understanding in the limit, it follows that one has to give a full consideration beyond single-mode approximation. This has been attempted, for instance, in Ref. [5], which is however, hard to interpret physically. The question in fact lies with the peculiarity of fermionic system, that gives rise to the ambiguity in the ordering of operators. Note that, as it is mentioned in Ref. [6], what matters is to find an ordering that has the physical relevance, i.e. such that entanglement is characterized by what is observed by detectors. In Ref. [6], such an ordering beyond single mode approximation is also proposed. In ref.[7] the ordering is then applied to explain entanglement in the infinite acceleration so the physicality of the fermionic structure is provided and the independence of the choice of Unruh mode in the infinite acceleration limit is discussed.

However the ordering was tested only in the example considered in Ref. [7]. Therefore in this work, beyond the single-mode approximation, we investigate the behavior of entanglement of fermionic systems using the recent construction proposed in Ref. [6], for both pure and mixed states. In this way, the construction is extensively tested for a number of entangled states. We show that in all of these cases, the ordering constructed leads to convergence of fermionic entanglement in the infinite acceleration, i.e. it yields physical results. This also corrects the previously known analysis on the entanglement behavior in Ref. [8]. Finally, our results provide strong evidence that the ordering suggested recently would characterize entanglement as it gives physical results.

The organization of this article is as follows. In Sec. II we will briefly review how fermionic systems are described in a non-inertial frame. In Sec. III the approach in Ref. [6] is applied and the entanglement behavior of the fermionic system in an accelerated frame is shown. In Sec. IV we will conclude and discuss our results.

II. ACCELERATED FRAME

Let us begin with quantum fields in relativistic frames. A party traveling with a uniform acceleration is described by the so-called Rindler coordinate (τ,ζ,y,z), which has the following relation with Minkowski coordinate (t,x,y,z),

\[ ct = \zeta \sinh(\frac{a\tau}{c}), \quad x = \zeta \cosh(\frac{a\tau}{c}), \quad (1) \]

where a is a fixed acceleration of the frame and c is the velocity of light. For fixed ζ, the coordinate can be found as hyperbolic trajectories in space-time. This is shown in Fig. I, Equation (1) only covers the region (I) in Fig. I. The region (II) is covered by \( ct = -\zeta \sinh(\frac{a\tau}{c}) \) and \( x = -\zeta \cosh(\frac{a\tau}{c}) \). The other two regions (F and P) can be described as \( ct = \pm \xi \cosh(\frac{2\zeta}{c}) \) and \( x = \pm \xi \sinh(\frac{2\zeta}{c}) \).

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Now let us consider the field in Minkowski and Rindler spacetime, which can be written as
\[
\phi = N_M \sum_i (a_{i,M}v_{i,M}^+ + b_{i,M}v_{i,M}^-)
\]
\[
= N_R \sum_j (a_{j,I}v_{j,I}^+ + b_{j,I}v_{j,I}^- + a_{j,II}v_{j,II}^+ + b_{j,II}v_{j,II}^-),
\]
where \(N_M\) and \(N_R\) are normalization constants. Also \(v_{i,M}^+\) and \(v_{i,H}^+\) are the positive and negative energy solutions of the Dirac equation in Minkowski spacetime, and \(v_{j,I}^+\) and \(v_{j,II}^+\) are the positive and negative energy solutions of the Dirac equation in Rindler spacetime, with respect to the Killing vector field in regions I and II. In addition \(a_{i,\Delta}(a_{i,\Delta})\) and \(b_{i,\Delta}(b_{i,\Delta})\) are the creation (annihilation) operators for the positive and negative energy solutions (particle and antiparticle), where \(\Delta\) denotes \(M, I, \) II. Then they satisfy the anticommutation relations \(\{a_{i,\Delta}, a_{j,\Delta'}^\dagger\} = \{b_{i,\Delta}, b_{j,\Delta'}^\dagger\} = \delta_{ij}\delta_{\Delta\Delta'}\). It is known that the combination of Minkowski mode, called Unruh mode, can be transformed into a monochromatic Rindler mode and can annihilate the same Minkowski vacuum. The following relation thus holds:
\[
A_{i, R/L} \equiv \cos \gamma_i a_{i, I/II} - \sin \gamma_i b_{i, I/II}^\dagger,
\]
where \(\cos \gamma_i = (e^{-2\pi i\Omega} + 1)^{-1/2}\). A more general relation can also be found,
\[
a_{i, U}^\dagger = q_L(A_{\Omega, L}^\dagger \otimes I_R) + q_R(I_L \otimes A_{\Omega, R}^\dagger),
\]
by which one can go beyond the single-mode approximation.

Using these relations, in case of Grassmann scalar, the Unruh vacuum can be given by
\[
|0\Omega_U\rangle = \cos^2 \gamma_\Omega |0000\rangle_{\Omega} - \sin \gamma_\Omega \cos \gamma_\Omega |0011\rangle_{\Omega} + \sin \gamma_\Omega \cos \gamma_\Omega |1100\rangle_{\Omega} - \sin^2 \gamma_\Omega |1111\rangle_{\Omega} \tag{3}
\]
Here we use the notation \(|pqmn\rangle_{\Omega} = |p\Omega\rangle_{\Omega}^\dagger |q\Omega\rangle_{\Omega}^\dagger |m\Omega\rangle_{\Omega}^\dagger |n\Omega\rangle_{\Omega}^\dagger\). Then, one-particle states can be obtained as
\[
|1\Omega_U\rangle^\dagger = q_R(\cos \gamma_\Omega |1000\rangle_{\Omega} - \sin \gamma_\Omega |1011\rangle_{\Omega}) + q_L(\sin \gamma_\Omega |1101\rangle_{\Omega} + \cos \gamma_\Omega |0001\rangle_{\Omega}), \tag{4}
\]
and,
\[
|1\Omega_U\rangle = q_L(\cos \gamma_\Omega |0100\rangle_{\Omega} - \sin \gamma_\Omega |0111\rangle_{\Omega}) + q_R(\sin \gamma_\Omega |1110\rangle_{\Omega} + \cos \gamma_\Omega |0010\rangle_{\Omega}). \tag{5}
\]

From now on, for both convenience and simplicity, we restrict our consideration to cases when \(q_R\) and \(q_L\) are real numbers, and we also omit the index \(\Omega\) throughout. In Eqs. (4) and (5), the single-mode approximation can be found by putting \(q_R = 1\), so that the vacuum in the Minkowski frame can be written as, \(|0\rangle_M = \cos \gamma |00\rangle_1|0\rangle_II + \sin \gamma |1\rangle_1|1\rangle_II\).

Very recently, in Refs. [6, 7] it has been suggested that the ordering in fermionic systems should be rearranged by the sequence of particles and antiparticles in the separated regions, so that the entanglement behavior of those states will yield physical results. In the following section, we apply the construction and derive the entanglement behavior of fermionic systems.

### III. ENTANGLEMENT BEHAVIOR

In this section, we consider the construction shown in Refs. [6, 7], and we derive the entanglement behavior accordingly. For bipartite pure entanglement, the entanglement property of quantum states can be simplified greatly due to the Schmidt decomposition, by which a given quantum state can be expressed using a single parameter. That is, the decomposition allows any bipartite pure state to be in the expression \(|\psi(\alpha)\rangle = \cos \alpha |00\rangle + \sin \alpha |11\rangle\) for some \(\alpha\), with a set of orthonormal basis \(|0\rangle\) and \(|1\rangle\).

To quantify entanglement, we apply the measure called negativity, which is based on the partial transpose of quantum states, i.e. taking transpose to the state of either system of two parties [6]. Then, for a given quantum state \(\rho\), negativity \(N\) can be computed as follows:
\[
N(\rho) = \sum_i |\lambda_i| \quad \text{where} \quad \lambda_i \text{ are negative eigenvalues of} \quad \rho^\Gamma, \quad \text{where} \quad \Gamma \text{ denotes the partial transpose} [10].
\]
Note that the measure-negativity-is useful as a computable measure in various contexts.

With these, the entanglement behavior is to be studied in the following scenario. Suppose that two parties, called
Alice and Bob, share entangled states in inertial frames in the beginning. Afterward, Bob moves with a uniform acceleration. We then show that entanglement in the infinite acceleration allows us to obtain physical results. In particular, results shown in Sec. III A, III B and III C correct previously known analysis in Ref. [8].

A. Bipartite pure states I - particle and antiparticle Unruh excitations

We first consider pure entanglement between Alice and Bob. As was mentioned, suppose that two parties share a pure state and Bob travels with a uniform acceleration. Then, the state shared is described as

\[
|\Phi_\pm(\alpha)\rangle = \cos \alpha |0\rangle_M |0\rangle_U + \sin \alpha |1\rangle_M |1\rangle_U. \tag{6}
\]

Suppose that Bob’s detector cannot distinguish between the particle or the antiparticle. As it is depicted in Fig. 1, two regions I and II are causally disconnected and thus Bob does not have to assess both. Hence, the state between Alice and Bob can be found by tracing either regions. First, the state of Alice and Bob when Bob is in region I is described as

\[
\rho_{AB1}^{\Phi_+} = \cos^2 \alpha \cos^4 \gamma |000\rangle \langle 000| + \frac{q_R}{2} \sin 2\alpha \cos^3 \gamma (|000\rangle \langle 110| + |110\rangle \langle 000|) + \frac{q_L^2}{2} \sin^2 \alpha \cos^2 \gamma |100\rangle \langle 100| + \frac{1}{2} (1 + (1 - 2q_L^2) \cos 2\gamma) \sin^2 \alpha |110\rangle \langle 110| - \frac{q_L}{2} \sin 2\alpha \cos^2 \gamma \sin \gamma (|001\rangle \langle 100| + |100\rangle \langle 001|) - \frac{q_R q_L}{2} \sin^2 \alpha \sin 2\gamma (|100\rangle \langle 111| + |111\rangle \langle 100|) + \frac{1}{4} \cos^2 \alpha \sin^2 2\gamma (|001\rangle \langle 010| + |010\rangle \langle 001|) + \frac{q_R}{2} \sin 2\alpha \cos \gamma \sin^2 \gamma (|101\rangle \langle 111| + |111\rangle \langle 101|) + \frac{q_L^2}{2} \sin^2 \alpha \sin^2 \gamma |011\rangle \langle 011| + \frac{q_L}{2} \sin 2\alpha \sin^3 \gamma (|011\rangle \langle 110| + |110\rangle \langle 011|) + \cos^2 \alpha \sin^4 \gamma |011\rangle \langle 011|. \]

And, the state of Alice and antiBob (i.e. in Bob’s region II) is then expressed after tracing the region I, as follows:

\[
\rho_{AB2}^{\Phi_+} = \cos^2 \alpha \cos^4 \gamma |000\rangle \langle 000| + \frac{q_R}{2} \sin 2\alpha \cos^3 \gamma (|000\rangle \langle 110| + |110\rangle \langle 000|) + \frac{q_L^2}{2} \sin^2 \alpha \cos^2 \gamma |100\rangle \langle 100| + \frac{1}{2} (1 + (1 - 2q_L^2) \cos 2\gamma) \sin^2 \alpha |110\rangle \langle 110| - \frac{q_L}{2} \sin 2\alpha \cos^2 \gamma \sin \gamma (|001\rangle \langle 100| + |100\rangle \langle 001|) - \frac{q_R q_L}{2} \sin^2 \alpha \sin 2\gamma (|100\rangle \langle 111| + |111\rangle \langle 100|) + \frac{1}{4} \cos^2 \alpha \sin^2 2\gamma (|001\rangle \langle 010| + |010\rangle \langle 001|) + \frac{q_R}{2} \sin 2\alpha \cos \gamma \sin^2 \gamma (|101\rangle \langle 111| + |111\rangle \langle 101|) + \frac{q_L^2}{2} \sin^2 \alpha \sin^2 \gamma |111\rangle \langle 111| + \frac{q_L}{2} \sin 2\alpha \sin^3 \gamma (|111\rangle \langle 110| + |110\rangle \langle 011|) + \cos^2 \alpha \sin^4 \gamma |111\rangle \langle 111|. \]

Note that these are exact expressions, not obtained from the single-mode approximation.

Entanglement of these states is estimated using negativity in Ref. [10], which is shown in Fig. 2. As \(\gamma\) increases, the entanglement of \(\rho_{AB1}^{\Phi_+}\) decreases but that of \(\rho_{AB2}^{\Phi_+}\) increases. They are eventually coincident at \(\gamma = \frac{\pi}{4}\). It is thus shown that for states \(\rho_{AB1}^{\Phi_+}\) and \(\rho_{AB2}^{\Phi_+}\), entanglement is determined independently of \(q_R\) at the infinite acceleration, \(\gamma = \frac{\pi}{4}\). As has been discussed in Ref. [7], these allow us to obtain physical results since at infinite acceleration, entanglement of the states \(\rho_{AB1}^{\Phi_+}\) and \(\rho_{AB2}^{\Phi_+}\) is shown to be independent on \(q_R\).

Next let us consider the case in which Bob and antiBob detector can distinguish between the particle and the antiparticle. Then for the state in Eq. (6), one can find density matrices corresponding to cases of the Alice-Bob particle in region I, the Alice-Bob antiparticle in

\[\text{FIG. 2: (Color online) Negativity is computed for states } \rho_{AB1}^{\Phi_+} \text{ and } \rho_{AB2}^{\Phi_+}. \text{ Parts (a) and (b) show the cases of } \alpha = \frac{\pi}{4} \text{ and } \alpha = \frac{\pi}{10} \text{ respectively. The blue solid(red dotted) lines from top to bottom(from bottom to top) denote the negativity of } \rho_{AB1}^{\Phi_+}(\rho_{AB2}^{\Phi_+}) \text{ at } q_R = 0.85 \text{ and } q_R = 0.73 \text{ respectively. } \gamma = \frac{\pi}{4} \text{ means the infinite acceleration. As it can be seen, the entanglement of } \rho_{AB1}^{\Phi_+} \text{ and } \rho_{AB2}^{\Phi_+} \text{ coincides at infinite acceleration.} \]

\[\text{And, the state of Alice and antiBob (i.e. in Bob’s region II) is then expressed after tracing the region I, as follows:}\]
region I, the Alice-anti-Bob particle in region II, and the Alice-anti-Bob antiparticle in region II. From the density matrices, entanglement can be computed using negativity for those cases, see Fig. 3.

![Graphs showing negativity for different values of qr]

**FIG. 3:** (Color online) For the state $\Phi_+$ in Eq. (6), negativity is computed for Alice-Bob particle in region I, Alice-Bob antiparticle in region I, Alice-anti-Bob particle in region II, and Alice-anti-Bob antiparticle in region II. Parts (a), (b), (c) and (d) show the cases of $qr = 1, qr = 0.75, qr = 0.5,$ and $qr = 0.25$ respectively. The blue solid line, the red thick dashed line, the green dotted line and the orange dot-dashed line one denote the negativity of Alice-Bob particle in region I, Alice-Bob antiparticle in region I, Alice-anti-Bob particle in region II, and Alice-anti-Bob antiparticle in region II respectively.

In Fig. 3 (a), the behavior of entanglement at $qr = 1$ in terms of negativity is shown. As $\gamma$ increases, the entanglement of the Alice-Bob particle in region I decreases, however that of the Alice-anti-Bob antiparticle in region II increases. At $qr = \frac{1}{2}$, we can see nonzero valued negativity only in cases of the Alice-Bob particle in region I, the Alice-Bob antiparticle in region I and the Alice-anti-Bob particle in region II.

**B. Bipartite pure states II - particle and antiparticle Unruh excitations**

We next consider the entanglement between Alice and Bob, when they share the following state:

$$|\Phi_-(\alpha)\rangle = \cos \alpha |0\rangle_M |0\rangle_U + \sin \alpha |1\rangle_M |1\rangle_U , \quad (7)$$

As it is explained previously, Bob has inaccessible part due to his acceleration. The state of Alice and Bob after tracing the region II is found as follows:

$$\rho_{AB_1}^{\Phi^-} = \cos^2 \alpha \cos^4 \gamma |000\rangle \langle 000| + \frac{qr}{2} \sin 2\alpha \cos \gamma |000 \rangle \langle 101| + |101\rangle \langle 000|) + \frac{q_r^2}{2} \sin^2 \alpha \cos^2 \gamma |101\rangle \langle 101| + \frac{1}{2} (1 + (1 - 2q_L^2) \cos 2\gamma) \sin^2 \alpha |101\rangle \langle 101| + \frac{qr q_L}{2} \sin 2\alpha \cos \gamma \sin \gamma |010\rangle \langle 100| + |100\rangle \langle 010|) - \frac{q_r q_L}{2} \sin 2\alpha \cos \gamma \sin \gamma |111\rangle \langle 111| - \frac{1}{4} \cos^2 \alpha \sin^2 \gamma |001\rangle \langle 001| + |001\rangle \langle 001|) + \frac{q_r q_L}{2} \sin 2\alpha \cos \gamma \sin \gamma |011\rangle \langle 101| + |101\rangle \langle 011|) + \frac{1}{4} \cos^2 \alpha \sin^2 \gamma |011\rangle \langle 011|.$$ 

The state that Alice-anti-Bob (in Bob’s region II) share can be obtained after tracing the region I,

$$\rho_{AB_1}^{\Phi^-} = \cos^2 \alpha \cos^4 \gamma |000\rangle \langle 000| + \frac{qr}{2} \sin 2\alpha \cos \gamma |000 \rangle \langle 101| + |101\rangle \langle 000|) + \frac{q_r^2}{2} \sin^2 \alpha \cos^2 \gamma |101\rangle \langle 101| + \frac{1}{2} (1 + (1 - 2q_L^2) \cos 2\gamma) \sin^2 \alpha |101\rangle \langle 101| + \frac{q_r q_L}{2} \sin 2\alpha \cos \gamma \sin \gamma |010\rangle \langle 100| + |100\rangle \langle 010|) - \frac{q_r q_L}{2} \sin 2\alpha \cos \gamma \sin \gamma |111\rangle \langle 111| - \frac{1}{4} \cos^2 \alpha \sin^2 \gamma |001\rangle \langle 001| + |001\rangle \langle 001|) + \frac{q_r q_L}{2} \sin 2\alpha \cos \gamma \sin \gamma |011\rangle \langle 101| + |101\rangle \langle 011|) + \frac{1}{4} \cos^2 \alpha \sin^2 \gamma |011\rangle \langle 011|.$$ 

Entanglement of states $\rho_{AB_1}^{\Phi^-}$ and $\rho_{AB_1}^{\Phi^-}$ is shown in Fig. 4. Their behavior are very similar to cases shown in Sec. III A. It is also shown that entanglement of states $\rho_{AB_1}^{\Phi^-}$ and $\rho_{AB_1}^{\Phi^-}$ is independent of $q_L$ at $\gamma = \frac{\pi}{4}$. This demonstrates proper entanglement behavior as it yields physical results.

As we discussed in the previous section, if the Bob and anti-Bob detector can distinguish between the particle and the antiparticle, for the state $\Phi^-$ in Eq. (7) one may find the density matrices of the Alice-Bob particle in region I, the Alice-Bob antiparticle in region I, the Alice-anti-Bob particle in region II, and the Alice-anti-Bob antiparticle in region II. In this way, density matrices corresponding to cases of the Alice-Bob particle in region I, the Alice-Bob antiparticle in region I, the
Alice-anti-Bob particle in region II, and the Alice-anti-Bob antiparticle in region II can be obtained. In Fig. 5, the entanglement behavior is shown for different values of $q_R$.

We now consider pure entangled states that describe correlations between particle and anti-particle degrees of freedom, when Bob is traveling with a uniform acceleration, as follows:

$$ |\Phi_+ (\alpha)\rangle = \cos \alpha |1\rangle_M |1^+\rangle_U + \sin \alpha |1\rangle_M |1^-\rangle_U. \quad (8) $$

As was done before, the state that Alice and Bob share can be obtained beyond the single-mode approximation. The state when Bob is in region I is obtained by tracing the other region,

$$ \rho^*_A |\Phi_+ (\alpha)\rangle = \frac{q^2_R \cos^2 \alpha \cos^2 \gamma}{2} |00\rangle + \frac{\cos \alpha (\gamma + 10)}{\cos^2 \alpha (\gamma + 10)} |+00\rangle $$

$$ + \frac{\cos \alpha (\gamma + 10)}{\cos^2 \alpha (\gamma + 10)} |+01\rangle + \frac{\cos \alpha (\gamma + 10)}{\cos^2 \alpha (\gamma + 10)} |+10\rangle $$

$$ + \frac{\cos \alpha (\gamma + 10)}{\cos^2 \alpha (\gamma + 10)} |+11\rangle + \frac{\cos \alpha (\gamma + 10)}{\cos^2 \alpha (\gamma + 10)} |+12\rangle $$

$$ + \frac{\cos \alpha (\gamma + 10)}{\cos^2 \alpha (\gamma + 10)} |+20\rangle + \frac{\cos \alpha (\gamma + 10)}{\cos^2 \alpha (\gamma + 10)} |+21\rangle + \frac{\cos \alpha (\gamma + 10)}{\cos^2 \alpha (\gamma + 10)} |+22\rangle $$

Where $

We observe that the entanglement behavior shown in Fig. 5 is different from those in Fig. 3. At $q_R = 1$ the non-zero values in negativity can be found in cases of the Alice-Bob antiparticle in region I and the Alice-anti-Bob particle in region II, but not in the others. At $q_R = \frac{3}{4}$, non-vanishing entanglement is found in cases of the Alice-Bob particle in region I, the Alice-anti-Bob antiparticle in region I, and the Alice-anti-Bob antiparticle in region II.
Also, the state that Alice and anti-Bob (in Bob’s region II) share is,

\[ \rho_{ABII}^\phi = q_R^2 \cos^2 \alpha \cos^2 \gamma |+00\rangle \langle +00| + \frac{1}{2} \left( 1 + (1 - 2q_R^2) \cos 2\gamma \right) \cos^2 \alpha (|+10\rangle \langle +10|) + \frac{1}{4} \left( 1 + (1 - 2q_R^2) \cos 2\gamma \right) \sin 2\alpha |+10\rangle \langle -01| + \frac{1}{4} \left( 1 + (1 - 2q_R^2) \cos 2\gamma \right) \sin 2\alpha |-01\rangle \langle +10| + \frac{1}{2} \left( 1 + (1 - 2q_R^2) \cos 2\gamma \right) \sin 2\alpha |-01\rangle \langle -01| + \frac{q_R^2}{2} \sin^2 \alpha \cos^2 \gamma |00\rangle \langle 00| - \frac{q_R}{2} \cos^2 \alpha \sin 2\gamma (|100\rangle \langle +11| + |+11\rangle \langle 000\rangle) - \frac{q_R}{2} \sin^2 \alpha \sin 2\gamma (|-00\rangle \langle -11| + |-11\rangle \langle -00|) + q_L^2 \cos^2 \alpha \sin^2 \gamma |+11\rangle \langle +11| + q_L^2 \sin^2 \alpha \sin^2 \gamma |-11\rangle \langle -11|.

For these two states, entanglement is computed in terms of negativity, and is shown in Fig. 6.

Interestingly, compared to other cases in Sec. III A and Sec. III B (see Figs. 2 and 4), the behavior depicted in Fig. 6 is shown to be nearly equidistant according to different values of \( q_R \). It is also shown that the entanglement behavior of two states \( \rho_{ABI}^\phi \) and \( \rho_{ABII}^\phi \) is independent of \( q_R \) at the infinite acceleration, \( \gamma = \frac{\pi}{4} \).

Let us also mention about when Bob and anti-Bob detector can distinguish between the particle and the antiparticle. In this case, it turns out that all density matrices corresponding to cases of Alice-Bob particle in region I, Alice-Bob antiparticle in region I, Alice-anti-Bob particle in region II, and Alice-anti-Bob antiparticle in region II are separable. Thus, negativity remains zero for all parameters.

### D. Bipartite mixed states - particle and antiparticle Unruh excitations

Up to now, we have considered entanglement of pure states when one of the parties is traveling with a uniform acceleration. We have observed that, at the infinite acceleration, entanglement has the same convergence, which allows us to have physical results. In this subsection, we consider a more complicated scenario when two parties share a mixed state. It is our aim to discover how the entanglement behavior depends on the mixedness property, and also if its convergence is related to the mixedness. In particular, the case in which a white noise is added to a maximally entangled states, so-called Werner state, is to be considered. For Werner states, the mixedness is parameterized by a single parameter, depending on which one can see how noisy a state is.

Suppose that two parties, Alice and Bob prepare Werner states in inertial frames, and then Bob moves in the uniformly accelerated frame. That is, the state can be expressed as follows:

\[ \rho_W = F|\Psi_+(\alpha = \pi/4)\rangle \langle \Psi_+(\alpha = \pi/4)| + \frac{1 - F}{4} I, \]

where the maximally entangled state is taken from Eq. (6) when \( \alpha = \pi/4 \).

Suppose also that Bob’s detector cannot distinguish between the particle or the antiparticle. Beyond the single-mode approximation, both states that Alice and Bob share in Bob’s region I and region II, respectively, are obtained by tracing the other region, as follows:

\[
\rho_{ABI}^W = \frac{1}{2} F q_R \cos^3 \gamma (|000\rangle \langle 110| + |110\rangle \langle 000|) + \frac{1}{8} \cos^2 \gamma (3 - 2q_R^2 + F(1 - 2q_R^2) + (1 - F) \cos 2\gamma)|100\rangle \langle 100| \\
+ \frac{1}{8} \cos^2 \gamma (3 - 2q_R^2 - F(1 - 2q_R^2) + (1 + F) \cos 2\gamma)|000\rangle \langle 000| - \frac{F q_L}{2} \cos^2 \gamma \sin \gamma (|001\rangle \langle 100| + |100\rangle \langle 001|) \\
+ \frac{F q_R}{2} \cos \gamma \sin^2 \gamma (|001\rangle \langle 111| + |111\rangle \langle 001|) + \frac{F q_L}{2} \sin^3 \gamma (|011\rangle \langle 110| + |110\rangle \langle 011|) \\
+ \frac{1}{4} \sin^2 \gamma ((1 + F) q_R^2 + (1 - F) \sin^2 \gamma)|111\rangle \langle 111| + \frac{1}{4} \sin^2 \gamma ((1 - F) q_R^2 + (1 + F) \sin^2 \gamma)|011\rangle \langle 011| \\
- \frac{1}{8} (1 - F) q_L q_R \sin 2\gamma (|000\rangle \langle 011| + |011\rangle \langle 000|) - \frac{1}{8} (1 + F) q_L q_R \sin 2\gamma (|100\rangle \langle 111| + |111\rangle \langle 100|) \\
+ \frac{1}{16} (1 - F) \sin^2 2\gamma|101\rangle \langle 101| + \frac{1}{16} (1 + F) \sin^2 2\gamma|001\rangle \langle 001| + \frac{1}{16} (2(1 + F) - 2(1 + F)(1 - 2q_R^2) \cos 2\gamma \\
+(1 - F) \sin^2 2\gamma)|110\rangle \langle 110| + \frac{1}{16} (2(1 - F) - 2(1 - F)(1 - 2q_R^2) \cos 2\gamma + (1 + F) \sin^2 2\gamma)|010\rangle \langle 010|,
\]

and
\[ p_{AB_{li}}^{W} = \frac{1}{2} F q_{L} \cos^{3} \gamma \langle 000 \rangle \langle 110 \rangle + |110 \rangle \langle 000 \rangle + \frac{1}{8} \sin^{2} \gamma (3 - 2q_{R}^{2}) F(1 - 2q_{R}^{2}) - (1 - F) \cos 2\gamma |111 \rangle \langle 111 \rangle + \frac{1}{8} \sin^{2} \gamma (3 - 2q_{R}^{2}) - F(1 - 2q_{R}^{2}) - (1 + F) \cos 2\gamma |011 \rangle \langle 011 \rangle + \frac{F q_{R}^{2}}{2} \cos \gamma \sin \gamma |001 \rangle \langle 001 \rangle + \frac{F q_{R}}{2} \sin^{3} \gamma |011 \rangle \langle 110 \rangle + |110 \rangle \langle 011 \rangle \]

Entanglement of these states \( p_{AB_{li}}^{W} \) and \( p_{AB_{li}}^{W} \) are shown in terms of negativity in Fig. 7. It is shown that the entanglement behavior of \( p_{AB_{li}}^{W} \) and \( p_{AB_{li}}^{W} \) coincides at, and is independent of \( q_{R} \), at the infinite acceleration \( \gamma \). Next, when Bob and anti-Bob detector can distinguish between the particle and the antiparticle, density matrices can be found, for the following cases, Alice-Bob particle in region I, Alice-Bob antiparticle in region I, Alice-anti-Bob particle in region II, and Alice-anti-Bob antiparticle in region II. From the matrices, it is straightforward to compute entanglement using negativity. In Fig. 8, the entanglement behavior is shown.

**IV. DISCUSSION AND CONCLUSION**

In this article, we have investigated the entanglement behavior of bipartite quantum states in fermionic systems when one of parties is traveling with a uniform acceleration. We have employed the recent proposal in Ref. 8.

for the ordering of operators. This is because the ordering is suggested such that field entanglement is relevant to what is observed in detectors. We believe that this is a natural and relevant constraint, as it is designed to yield physical results, which is contrary to other previous approaches that consider various possibilities depending on mathematical ordering. Before the present consideration, the construction in Ref. 6 was only tested for a particular pure state in Ref. 7.
that, in all of these cases, the entanglement behavior allows one to obtain physical results so that in the infinite acceleration, entanglement converges to a single and finite value. Our considerations consist of exemplary states shown in Ref. [8], where a different ordering has been applied and consequently the convergence property has not been achieved in the entanglement behavior. This contrasts to what is shown in the present work, and thus our result has provided the correct behavior of entanglement in fermionic systems.

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