Vibration Mitigation of Tunnel Structures via a Novel Passive Tuned Mass Damper

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Abstract. In this work, a novel passive tuned mass damper is introduced to mitigate the harmful vibrations in tunnel structures under moving trains/vehicles. First, a mechanism with inertial amplification effect is introduced. The transfer function of an SDOF system with the proposed inertial amplification system (SDOF-IAM) is presented. The adjustability of the proposed IAM is discussed. Second, replacing the SDOF by using the SDOF-IAM system, a novel passive tuned mass damper is proposed. Superior to the traditional TMD, the geometrical parameters of the present IAM-TMD can be changed to adjust to the vibration mitigation property of the TMD. A parametric study is also conducted to evaluate the influences of the geometrical parameters on the critical parameters of the TMD (the anti-resonant frequency, the effective mass parameter, and so on). Superior to the TMD, the vibration mitigation effect of the IAM-TMD can be adjusted effectively, which can be used to suppress vibrations of tunnel structures under moving trains.

1. Introduction

Vibrations induced by the dynamic interaction between the moving train/vehicle and the rail/road brings lots of harmful effect to the underground tunnel structures. To deal with this problem, various vibration mitigation methods, including both passive and active vibration mitigation methods, have been developed. Among those passive vibration methods, tuned mass damper (TMD) is widely accepted and used. When using this method, a mass-spring-damper system will be attached to the main structure. With proper design, the vibration energy of the main system may be absorbed and further dissipated by the attached mass-spring-damper system. As a result, responses of the main structure are mitigated.

In 1909, Frahm first proposed the basic concept of the undamped TMD system to reduce the dynamic response of ships. Further works pointed out that the performance of the undamped structure undamped TMD system deteriorates shapefly when the excitation frequency deviates away from the absorber's natural frequency. To eliminate this drawback, Ormondroyd and Den Hartog [1,2] introduced the damping into the absorber. Later, Bishop and Welbourn [3] considered the damping of the main mass. After that, more and more TMD configurations were proposed.

Recently, lots of works are reported to extend the possible applications of TMDs in civil engineering. For example, it is known that the effectiveness of tuned mass dampers decreases as the input duration shortens. And, their use is commonly discouraged against short-duration, pulse-like ground motions, such as those occurring in near-field(NF) zones in the presence of forward directivity or fling-step effects (add citations) [4]. As reported in several studies, in the case of mistuning, the beneficial effects of the TMD can be significantly reduced. Therefore, accounting for the uncertainties
in the design is very important [5]. Other tuned mass dampers with additional passive and active mechanical systems were also proposed.

In this work, a novel passive tuned mass damper is introduced to mitigate the harmful vibrations in tunnel structures under moving trains/vehicles. Section 2 introduces the concept of the proposed inertial amplification mechanism (IAM). The transfer function of an SDOF system with the proposed inertial amplification system (SDOF-IAM) is presented. Meanwhile, the adjustability of the proposed IAM is discussed. In Section 3, replacing the SDOF by using the SDOF-IAM system, a novel passive tuned mass damper is proposed. Theoretical derivations about the TMD-IAM are given. Superior to the traditional TMD, the geometrical parameters of the present IAM-TMD can be changed to adjust to the vibration mitigation property of the TMD. A parametric study is also conducted in Section 4 to evaluate the influences of the geometrical parameters on the critical parameters of the TMD (the anti-resonant frequency, the effective mass parameter, and so on). Conclusions are summarized in Section 5.

2. Conception of the Inertial Amplification Mechanism

2.1. Model and Equations
As shown in Figure 1, the proposed inertial amplification mechanism is a composite of two rigid bars and a small additional lumped mass \( m_a \). Rigid bars are connected to the masses through hinge connections. The angle between the direction of the rigid bar and the \( x \)-coordinate is \( \theta \). \( m_0 \), \( k_d \) and \( c_d \) are the mass, spring, and damping parameters, respectively. Removing the two additional masses and adding them to the lumped mass \( m_0 \), the corresponding SDOF system is also considered for comparison. Therefore, the mass parameter of the SDOF is \( m_d = 2m_a + m_0 \).

![Figure 1. Theoretical model of the proposed IAM system.](image)

Under the excitation of ground motion \( x_g(t) \), the two additional masses will move both in horizontal and vertical directions. And, the displacement components of the additional mass can be presented as

\[
x_a = \frac{x_d}{2} + x_d', \quad y_a = \frac{x_d}{2 \tan \theta}
\]

By using the Lagrange method, the governing equation of the IAM system can be given as

\[
m_{eff} \ddot{x}_d + c_d \dot{x}_d + k_d x_d = -(m_0 + m_a) \ddot{x}_d
\]

in which \( m_{eff} = m_0 + m_a b_1 \); \( b_1 = 0.5(1 + \tan^2 \theta) \). Further, Eq. (2) can be written as

\[
\ddot{x}_d + 2 \xi_{eff} \omega_{eff} \dot{x}_d + \omega_{eff}^2 x_d = -\frac{1-m_r}{m_r} \ddot{x}_s (b_1 - 1) + 1 \ddot{x}_s
\]

in which \( \xi_{eff} \) and \( \omega_{eff} \) are the damping ratio and frequency; \( m_r = m_s / m_d \).

2.2. Transfer Function of the SDOF-IAM
The steady-state relative displacement responses of the SDOF with IAM under harmonic ground motion can be obtained easily

\[
H_{id}(\omega) = \frac{(1 - m_r)}{(b_1 - 2)m_r + 1} - \frac{\omega^2 / \omega_{eff}^2}{\left(\omega^2 / \omega_{eff}^2\right) + 2 \xi_{eff} \left(\omega / \omega_{eff}\right) + 1}
\]
Taking $\theta = 10^\circ, m_r = 0.1$, figure 2 presents the relative displacement response functions of the two systems. First, ignoring the damping effect ($\xi_d = 0$), transfer functions of the SDOF and the SDOF-IAM are compared. Responses of the two systems rise to infinity at their eigenfrequencies. The interesting point is that, because of the inertial amplification mechanism, the effective mass of the IAM is enlarged. Therefore, the resonant frequency of the IAM is smaller than that of the SDOF. In another word, the inertial amplification mechanism system can adjust the eigenfrequency of the system. Second, the damping effect is included. It is seen in figure 2 that, once the damping effect is considered, peak values of transfer functions are attenuated dramatically. Of particular interest is that even the damping of the two systems are the same, the peak responses of the IAM are much smaller than those of the SDOF. That means the inertial amplification system enhances the energy dissipation effect.

![Figure 2](image)

Figure 2. The transfer function of SDOF and IAM with damping

### 2.3. Adjustability of the IAM

To indicate the tuning ability of the inertial amplification system, the frequency ratio parameter ($R_\omega$) and the attenuation ratio parameter ($\psi$) are defined

$$R_\omega = \frac{\omega_{\text{eff}}}{\omega_d} = \sqrt{\frac{1}{(b - 2)m_d + 1}}, \quad \psi = \frac{H_{\text{d(max)}} - H_{\text{d(max)}}}{H_{\text{d(max)}}}$$

Figure 3(a) presents the relations between the frequency ratio and the governing parameters of the IAM. Here, the damping ratio is taken as $\xi_d = 0.1$. The proposed inertial amplification system can adjust the eigenfrequency of the system effectively, even the physical masses of the two systems are the same. When $\theta = 30^\circ$, the effective mass $m_{\text{eff}} = m_d$. Therefore, the frequency ratio is equal to 1. When $\theta < 30^\circ$, the effective mass $m_{\text{eff}}$ is larger than the $m_d$ and the frequency ratio is smaller than 1. For a given $\theta$, the frequency ratio decreases with the increase of the mass ratio $m_r$. Inversely, when $\theta > 30^\circ$, the effective mass $m_{\text{eff}}$ is smaller than the $m_d$ and the frequency ratio is larger than 1. For a given $\theta$, the frequency ratio increases with the increase of the mass ratio.

Figure 3(b) shows the relations between the attenuation ratio and the governing parameters of the IAM. The attenuation ratio is larger than 0 for all cases. That means the inertial amplification system can mitigate the vibration responses of the system effectively. For a given mass ratio $m_r$, the attenuation ratio decreases drastically with the increase of the $\theta$. For a given $\theta$, the attenuation ratio increases with the increase of the mass ratio $m_r$. In particular, when the mass ratio is small, the attenuation ratio tends to zero with the increase of the $\theta$. That means the benefit of the IAM can be ignored, even the frequency ratio may be larger than 1. However, when the mass ratio is large, the attenuation ratio tends to 100% with the decrease of the $\theta$. That means the main benefit of the IAM lies in the ability to attenuate the vibration responses of the structure in the lower frequency region.
Figure 3. (a) Frequency ratio and (b) attenuation ratio versus the governing parameters of the IAM.

3. A passive TMD with IAM

3.1. Theoretical Model

Figure 4. A theoretical model of the tunnel structure with the IAM-TMD.

Figure 4 shows the theoretical model of the tunnel structure with TMD. Without loss of generality, the main structure of the tunnel structure is simplified as a single-degree-of-freedom (SDOF) system, and $k_s$, $m_s$, $c_s$ are the stiffness, mass and damping coefficients, respectively. The aforementioned SDOF-IAM is considered as the IAM-TMD damper to mitigate the vibration of the main structure.

3.2. Governing Equations

Considering the external forcing ($p(t)$), governing equation of the SDOF system with the IAM-TMD is

$$MX + CX + KX = P$$

Here, $X = [x_s, x_d]^T$ is the displacement vector. $P = [p(t), 0]^T$ is the vector of the external excitation.

3.3. Steady-State Response

Consider the excitation to be periodic of frequency $\omega$,

$$p(t) = P_0 e^{i\omega t}$$

and the steady-state displacement responses of the system can be given as

$$X_0 = \frac{P}{-\omega^2 M + i\omega C + K}$$

in which $P_0$ is the amplitude of the external force, $i = \sqrt{-1}$, $X_0 = [X_s, X_d]^T$ is the amplitude vector.
4. Discussion

4.1. Normalized Parameters
To obtain the general result, the following normalized parameters are introduced
\[ \mu = \frac{m_d}{m_s}, \ f = \frac{\omega_d}{\omega_s}, \ r = \frac{\omega}{\omega_s}, \ \xi = \frac{c}{(2\omega m_s)}, \ \bar{\xi} = \frac{c_d}{(2\omega m_d)} \] (9)
in which \( \omega_s = \sqrt{k/m_s} \) and \( \omega_d = \sqrt{k_d/m_d} \).

4.2. Transfer Function of the TMD-IAM
Ignoring the damping effect of the SDOF system and the absorber (\( \xi = 0, \bar{\xi} = 0 \)), transform functions of the displacement response of the main structure and the TMD can be obtained easily
\[
H_s(t) = \frac{X_s}{F / k_s} = \frac{r^2 (b \mu m_s + 1) - f^2}{r^4 (1 - 2 b) \mu m_s^2 - (1 + \mu) b m_s^2 - 1 + r^2 ((2 \mu f^2 + b) m_s + f^2 \mu + f^2 + 1) - f^2}.
\]
\[
H_d(t) = \frac{X_d}{F / k_s} = \frac{r^2 (m_s + 1)}{r^4 (1 - 2 b) \mu m_s^2 - (1 + \mu) b m_s^2 - 1 + r^2 ((2 \mu f^2 + b) m_s + f^2 \mu + f^2 + 1) - f^2}.
\] (10)

4.3. Effective Zone and Anti-Resonant Frequency
Taking \( f = 1, \mu = 0.05, \theta = 15^\circ, m_s = 0.05 \). Figure 5(a) shows the transfer function curves of the main structure. One can find that, when the TMD is used, the response of the main structure around the characteristic frequency is lowered. At a certain frequency, namely the anti-resonant frequency, the response of the main structure is 0, which means the inputted energy is absorbed by the TMD. Interestingly, it is found that the anti-resonant frequency of the system with IAM-TMD is much smaller than that of the TMD. That is to say, the attached inertial amplification mechanism system can adjust the tuning frequency of the TMD efficiently. Further, the transfer function means the inputted excitation is mitigated at the output end. It is found in Figure 5(a) that, once the TMD is used, the transfer function will be smaller than 1 in a smaller frequency region, named effective zone, around its anti-resonant frequency. As using the IAM, the width of the effective zone is enlarged and the effective zone moves to the low-frequency region.

![Figure 5](image-url)

**Figure 5.** Transfer functions for (a) the main structure and (b) the TMD for the un-damped SDOF system with un-damped absorbers.

In general, the relative displacement response between the main structure the TMD is very large, which is a very important constraint condition for TMD in actual design. Here, one can see from Figure 5(b) that, the minimum responses of the IAM-TMD is smaller than that of the TMD. That is to say, the attached inertial amplification mechanism system can enhance the performance of the TMD by suppressing the relative displacement response.
4.4. Parametric Study

Influences of the governing parameters of the IAM on the effective zone and the anti-resonant frequency are investigated. For comparison, two normalized parameters are introduced

\[ R_r = \frac{W_{IAM-TMD}}{W_{TMD}}, \quad R_\alpha = \frac{\Omega_\alpha}{\omega_\alpha} \]

in which \( W_{IAM-TMD} \) and \( W_{TMD} \) are the effective zone widths of the IAM-TMD and the TMD, respectively. \( \Omega_\alpha \) and \( \omega_\alpha \) are the anti-resonant frequency of the IAM-TMD and the TMD, respectively.

Figure 6. (a) Relative effective zones and (b) anti-resonant frequency ratio versus \( \theta \) and \( m_r \).

It is found in figure 6(a) that when the IAM-TMD is used, the effective attenuation zone is larger than 1 (the attenuation zone of the traditional TMD with the same physical mass). In particular, when the angle of the IAM is smaller than 10 degrees. With the increase of the degree, the effective attenuation zone of the IAM-TMD is lower exponentially and tends to 1. For a certain angle, the attenuation zone increases when a large mass is attached to the IAM. In Figure 6(b), it is observed that when the IAM-TMD is used, the anti-resonant frequency is always smaller than 1, indicating the proposed IAM enhances the low-frequency vibration mitigation effect of the TMD. Similarly, larger attached mass and smaller angle result in lower anti-resonant frequency.

5. Conclusion

This work presents the basic theory of a novel passive tuned mass damper for tunnel structures under moving trains/vehicles. Theoretical results and parametrical results validated that the proposed mechanical system can effectively adjust the eigenfrequency of the SDOF. Equipping it on the traditional tuned mass damper, the location and the width of the effective zone can be effectively modified. In particular, it can mitigate the low-frequency vibration in tunnel structures.

Acknowledgments

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