Effects of nuclear symmetry energy and in-medium $NN$ cross section in heavy-ion collisions at beam energies below the pion production threshold

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Based on the isospin-dependent Boltzmann-Uehling-Uhlenbeck(IBUU04) transport model, we explored effects of in-medium $NN$ elastic scattering cross section and nuclear symmetry energy on the sub-threshold pion production in $^{132}$Sn+$^{124}$Sn reaction. We find that with decrease of incident beam energy, effects of in-medium $NN$ elastic scattering cross section on the $\pi^-/\pi^+$ ratio are larger than that of the symmetry energy although the latter may be also larger. While keeping the effect of symmetry energy, the double ratio of $\pi^-/\pi^+$ from neutron-rich and neutron-poor reaction systems (with the same mass number of system) $^{132}$Sn+$^{124}$Sn and $^{128}$Pm+$^{126}$Pm almost fully cancels out the effects of in-medium $NN$ elastic scattering cross section.

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I. INTRODUCTION

Knowledge on the density dependence of the symmetry energy is crucial to understand the structure of exotic nuclei, dynamics of heavy-ion collisions, and many important issues in nuclear astrophysics such as neutron star cooling and supernova explosive $^{1-3}$-$^{1}$. While great progress has been made to constrain the symmetry energy at low densities $^{2,3}$-$^{1}$, the high-density behavior of the symmetry energy is divergent widely from interpretations of FOPI data $^{10-15}$. The $\pi^-/\pi^+$ ratio was found to be a sensitive probe to the high-density behavior of the symmetry energy by several transport models $^{16-20}$. In fact for pion production, at lower beam energies effects of symmetry energy and in-medium effect may both become larger $^{21,22}$. It is thus necessary to do a comparative study of effects of the in-medium $NN$ cross section and the effects of the symmetry energy on pion production in heavy-ion collisions at lower beam energies. And these experimental studies will become possible at facilities that offer fast radioactive beams, such as NSCL and FRIB in the US, FAIR in Germany, or RIBF in Japan.

II. THE IBUU MODEL

In this study, we adopt the semi-classical transport model IBUU04, in which the isospin-dependent initial neutron and proton density distributions of the projectile and target are obtained by using the Skyrme-Hartree-Fock with Skyrme $M^*$ (SM) force parameters $^{23}$. And an isospin- and momentum-dependent mean-field single nucleon potential is also used, i.e.,

$$U(\rho, \delta, \mathbf{p}, \tau) = A_u(x) \frac{\rho^\tau}{\rho_0} + A_l(x) \frac{\rho^\tau}{\rho_0} + B(\frac{\rho}{\rho_0})^2 (1 - x \delta^2) - 8 x \delta x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^\sigma} \delta \rho_t,$$

$$+ 2C_{\tau,\tau} \int d^3 \mathbf{p'} \frac{f_\tau(\mathbf{r}, \mathbf{p})}{1 + (\mathbf{p} - \mathbf{p'})^2/\Lambda^2},$$

$$+ 2C_{\tau,\tau'} \int d^3 \mathbf{p'} \frac{f_{\tau'}(\mathbf{r}, \mathbf{p'})}{1 + (\mathbf{p} - \mathbf{p'})^2/\Lambda^2},$$

where $\tau = 1/2(-1/2)$ for neutrons(protons), $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the isospin asymmetry, and $\rho_n$, $\rho_p$ denote neutron and proton densities, respectively. The parameters $A_u(x), A_l(x), B, C_{\tau,\tau}, C_{\tau,\tau'}, \sigma$, and $\Lambda$ are all given in Ref. $^{24}$. $f_\tau(\mathbf{r}, \mathbf{p})$ is the phase-space distribution function at coordinate $\mathbf{r}$ and momentum $\mathbf{p}$. Different $x$ parameters can be used to mimic different forms of the symmetry energy. In this model, reaction channels on
pion production and absorption are

\[ \begin{align*}
NN & \rightarrow NN, \\
NR & \rightarrow NR, \\
NN & \leftrightarrow NR, \\
R & \leftrightarrow N\pi,
\end{align*} \]

where \( R \) represents \( \Delta \) or \( N^* \) resonances. The experimental free-space nucleon-nucleon (NN) scattering cross section and the in-medium \( NN \) cross section can be used optionally. For the later, we use the isospin-dependent in-medium \( NN \) elastic cross section, which is from the scaling model according to nucleon effective masses [25–28].

\[
R_{\text{medium}}(\rho, \delta, \bar{p}) = \frac{\sigma_{NN,\text{elastic}}^{\text{medium}}}{\sigma_{NN,\text{elastic}}^{\text{free}}} = \left( \frac{\mu_N^*}{\mu_N} \right)^2,
\]

where \( \mu_{NN} \) and \( \mu_{N^*} \) are the reduced masses of the colliding nucleon pair in free space and medium, respectively. And the effective mass of nucleon in isospin asymmetric nuclear matter is given by

\[
m_N^* = \left\{ 1 + \frac{m_N}{p} \frac{dU}{dp} \right\}^{-1} m_N.
\]

From the definition and Eq. (1), it is seen that the effective mass depends not only on density and asymmetry of nuclear matter, but also the momentum of nucleon [29].

For the inelastic cross section we use the experimental data from free space \( NN \) collision since the in-medium inelastic \( NN \) cross section is still very controversial. The total and differential cross sections for all other particles are taken either from experimental data or obtained by using the detailed balance formula. The isospin-dependent phase-space distribution functions of the particles involved are solved by using the test-particle method numerically. The isospin-dependence of Pauli blockings for fermions is also considered.

III. RESULTS AND DISCUSSIONS

As shown in Fig. 1, the maximal baryon densities reached in \( ^{132}\text{Sn} + ^{124}\text{Sn} \) collisions are about 1.5~2 times saturation density at \( E_{\text{beam}}=100, 200 \) and 300 MeV/nucleon. We can also see that the maximal baryon density reached increase with incident beam energy. However, existing time of supradensity matter becomes shorter with increase of beam energy.

Figure 2 shows effects of in-medium \( NN \) elastic scattering cross section on the \( \pi^-/\pi^+ \) ratio in central collision \( ^{132}\text{Sn} + ^{124}\text{Sn} \) at \( E_{\text{beam}}=100, 150, 200, 250, \) and 300 MeV/nucleon, respectively. From Fig. 2 we can see that the value of \( \pi^-/\pi^+ \) ratio decreases with increase of beam energy, which is consistent with that in Ref. [21]. This is partially because the production of pions is from repeated nucleon-nucleon collisions at higher beam energies, i.e., a neutron converts a proton by producing \( \pi^- \) and subsequent collisions of that proton can convert again to neutron by producing \( \pi^+ \). More interestingly, one can see that the effects of in-medium \( NN \) elastic cross section on the value of \( \pi^-/\pi^+ \) ratio become larger and larger with decrease of beam energy. At lower beam energy 100 MeV/nucleon, effects of in-medium \( NN \) elastic cross section on the value of \( \pi^-/\pi^+ \) ratio can reach about 40%. When one changes \( NN \) elastic cross section, total \( NN \) collision number would also changes accordingly. As there is certain probability of inelastic process in total \( NN \) collision, \( NN \) inelastic collision is thus affected by \( NN \) elastic cross section. At high beam energy, \( NN \) inelastic processes may be more than elastic processes. So elastic \( NN \) cross section should have small effects on pion production. But at low beam energy, pion production is via many \( NN \) scatterings, large number of \( NN \) elastic scatterings increases the whole \( NN \) scatter-
To see more clearly how the in-medium effects on the production are affected by NN elastic scatterings, since there is certain probability of inelastic process in NN collision, pion production should be affected by NN elastic scatterings.

To see more clearly how the in-medium NN cross section on the value of \( \pi^-/\pi^+ \) ratio and effect of the symmetry energy on pion production, we plot evolution of pion meson production with different NN cross sections and symmetry energies. Shown in figure 3 and figure 4 are time evolutions of \( \pi^- \) and \( \pi^+ \) mesons with different symmetry energies (\( x = 0, x = 1 \)) and different NN elastic scattering cross sections (in-medium, free) in the central collision \( ^{122}_{124} \text{Sn} + ^{124}_{124} \text{Sn} \) at the beam energy of 100 MeV/nucleon.

In order to reduce the effects of in-medium and retain effects of the symmetry energy on \( \pi^-/\pi^+ \) ratio, we calculate double ratio of \( \pi^-/\pi^+ \) using two reaction systems of neutron rich and neutron poor of same isotopes \(^{122}_{124} \text{Sn} \). Shown in Fig. 5 is time evolution of the double ratio from \( ^{122}_{124} \text{Sn} + ^{124}_{124} \text{Sn} \) and \( ^{100}_{100} \text{Sn} + ^{100}_{100} \text{Sn} \). It is seen that effects of the in-medium NN cross section on the double ratio is about 30%, but effects of the symmetry energy is about 42%. However, from the top panel of Fig. 5 we see that effects of the in-medium NN cross section become smaller.

FIG. 3: (Color online) Time evolution of \( \pi^- \) and \( \pi^+ \) mesons with different symmetry energies (\( x = 0, x = 1 \)) and different NN elastic scattering cross sections (in-medium, free) in the central collision \( ^{122}_{124} \text{Sn} + ^{124}_{124} \text{Sn} \) at the beam energy of 100 MeV/nucleon.

FIG. 4: (Color online) Same as figure 3 but at the beam energies of 200 MeV/nucleon.
medium $NN$ cross section on the $\pi^-/\pi^+$ ratio is about 50%, but effects of the symmetry energy is about 25%. Thus the double ratio of $\pi^-/\pi^+$ from two reaction systems of neutron rich and neutron poor of same isotopes can indeed reduce uncertainties from in-medium properties of hadrons and keep the effects of symmetry energy. While from Fig. 6 we can see that even we use the double ratio of observable, effects of in-medium $NN$ cross section are still very obvious. Besides theoretical efforts [27] and advanced method such as photon emission in heavy-ion collisions [30], we also simulated the double ratio of $\pi^-/\pi^+$ from two reaction systems (with the same mass number of system but different isotopes) $^{132}Sn + ^{124}Sn$ and $^{128}Pm + ^{128}Pm$. Interestingly, from Fig. 7 one sees that effects of symmetry energy are kept but the effects of in-medium $NN$ elastic scattering cross section are almost fully cancelled out. This is because the latter two neutron-rich and neutron-poor reaction systems have the same baryon number, evolutions and distributions of baryon density in the two reactions are almost the same.

IV. CONCLUSIONS

In summary, we studied effects of in-medium $NN$ cross section and effects of symmetry energy on $\pi^-/\pi^+$ ratio at lower incident beam energies. We find that at lower incident beam energy, for $\pi^-/\pi^+$ ratio, effects of in-medium $NN$ cross section are larger than that of symmetry energy. The double ratio of $\pi^-/\pi^+$ from reaction systems of neutron rich and neutron poor of same isotopes can not fully cancel out the effects of in-medium $NN$ cross section. However, the double ratio of $\pi^-/\pi^+$ from reaction systems of neutron rich and neutron poor of different isotopes but same mass number of reaction system almost fully cancels out the effects of in-medium $NN$ cross section.

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