Recombinant Charmonium in strongly coupled Quark-Gluon Plasma

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We update our previous work of a Langevin-with-interaction model for charmonium in heavy-ion collisions, by considering the effect due to recombination. We determine the contribution to \( J/\psi \) yields from \( \bar{c}c \) pairs whose constituent quarks originate from two different hard processes. Like the surviving \( J/\psi \) states, the recombinant \( J/\psi \) also undergo both a stochastic interaction, determined by a hydrodynamical simulation of the heavy-ion collision, and an interaction determined by the \( QQ \) potentials measured on the lattice for appropriate temperatures. From the results of these simulations, we determine both the direct and the recombinant contribution to the \( J/\psi \) yields for RHIC conditions, and find that for central collisions, between 30\% and 50\% of the \( J/\psi \) yield is due to recombinant production. We compare our results with other models and look for how the recombinant contribution differs from the surviving contribution in the differential \( p_T \) yields. Including the recombinant contribution improves the agreement with the latest analysis of charmonium at RHIC, which shows an absence of anomalous suppression except in the most central collisions.

I. INTRODUCTION

In a previous paper \cite{1}, we argued that the microscopic dynamics of charmonium in a heavy-ion collision should be modeled as a stochastic interaction with strongly-coupled quark-gluon plasma (sQGP). In such a plasma the diffusion coefficient is very small, leading to rapid thermalization in momentum space and slow spatial diffusion, which is further slowed by the attraction of the constituent charm and anti-charm quarks in the pair. We concluded that the amount of equilibration in space (and thus the \( J/\psi \) yield) depends strongly on the timescales of the collision. In realistic simulations of \( Au+Au \) collisions at RHIC, where the sQGP phase exists for \( \tau \sim 5 \text{ fm}/c \), our model predicted a survival probability for \( J/\psi \sim 1/2 \) even in the most central collisions, much larger than previously expected. Those results were able to explain qualitatively the data from PHENIX.

In \cite{1}, we did not consider another possibly important source of charmonium at RHIC, the “recombinant” contribution of \( J/\psi \) particles whose constituent quarks originate from different hard processes. At very high collision energies, as the number of charm pairs per event grows, recombinant charmonia could potentially lead to an enhancement of the final \( J/\psi \) yields in a heavy-ion collision, reversing the current suppression trend. Using the grand canonical ensemble approach, Braun-Munzinger and collaborators \cite{2} determine the fugacity of charm by the number of \( \bar{c}c \) pairs produced initially. The “statistical hadronization” approach to charmonium assumes complete thermal equilibration of charmonium. Another approach has been taken by Grandchamp and Rapp \cite{3}, who treat the \( J/\psi \) yields from heavy-ion collisions as coming from two sources: the direct component, which decays exponentially with some lifetime \( \tau_0 \), and the coalescent component, which is determined by the same mechanism in \cite{2}, with the additional consideration that spatial equilibration of charm does not happen. To account for enhanced local charm density, by small spatial diffusion, they had introduced another factor - the “correlation volume” \( V_{corr} \) - which was estimated. The present work can be viewed as a quantitative dynamical calculation of this parameter.

To gain insight, we should compare these models with our model in \cite{1}. The Langevin-with-interaction model for \( \bar{c}c \) pairs in medium makes no assumptions about complete thermalization, and shows how even in central \( Au+Au \) collisions at the RHIC, the majority of the \( J/\psi \) yields may survive the QGP phase. However, the model predicts rapid thermalization in the momentum distributions of charmonium, as well as equilibration in the relative yields of the various charmonium states due to the formation of “quasi-equilibrium” in phase space. This requires no fine-tuning of the rates for charmonium in plasma; it is just a natural consequence of the strongly coupled nature of the media, detailed by the Langevin dynamics. However, recombinant production of charmonium may still be an important effect in our model, due to the fact that in central collisions, the densities of unbound charm quarks can be quite high in some regions of the transverse plane.

Our model simulates an ensemble of \( \bar{c}c \) pairs, generated initially by PYTHIA event generation and then evolved according to the Langevin-with-interaction model. We evolve the pairs not assuming any form of equilibrium, and then average over possible pairings of the quarks to form recombinant charmonium.

The outline of this work is as follows: in Section II we will describe how we simulated charm in plasma and took into account the contribution due to recombinant \( J/\psi \), and in Section III we take the opportunity to describe the progress in this model so far and also to summarize where
future work is needed for a Langevin-with-interaction description of \(J/\psi\) suppression. In Appendix \(A\) we discuss the statistics necessary to calculate the recombinant contribution to \(J/\psi\) yields.

II. RECOMBINANT CHARMONIUM IN HEAVY-ION COLLISIONS

A. Langevin-with-interaction model for \(\bar{c}c\) pairs in a heavy-ion collision

As we have done in our previous paper, we simulate \(J/\psi\) in medium with a hydrodynamical simulation of the collision. As before, we start with a large ensemble of \(\bar{c}c\) pairs whose momenta are determined with PYTHIA event generation \(B\). The positions of the initial hard collisions in the transverse plane at mid-rapidity are determined by sampling the distribution in \(N_{\text{coll}}\) determined from the Glauber model. In this way, our local densities of \(\bar{c}c\) pairs vary as one would expect from the Glauber model, which gives an enhancement for recombination towards the center of the transverse plane. Each element of the ensemble now contains \(N \bar{c}c\) pairs. The number of pairs \(N\) depends on the impact parameter of the collision and needs to be determined.

The average number of \(\bar{c}c\) pairs for a \(\text{Au+Au}\) collision at RHIC varies with impact parameter and has been investigated by the PHENIX collaboration at mid-rapidity \(C\). The measured cross sections for charm production vary somewhat with the centrality of the collision and achieves a maximum of about 800 \(\mu\)b for semi-central collisions. The nuclear overlap function \(T_{\text{AA}}(b)\) can be calculated with the Glauber model. We used a convenient program by Dariusz Miskowiec \(D\) to evaluate this function. With a centrality dependent cross-section \(\sigma_{cc}\), we can easily calculate the average number of \(\bar{c}c\) pairs in a collision: \(N_{\bar{c}c} = T_{\text{AA}} \sigma_{cc}\). The number of \(\bar{c}c\) pairs reaches a maximum in central collisions, with an average of 19 pairs per collision.

In order to determine the probability for two charm quarks from different hard processes to form recombinant charmonium, we must average over the different possible pairings of all of the unbound quarks in each element of our ensemble. This is discussed in Appendix \(A\) in generality. Since the number of \(\bar{c}c\) pairs approaches 20 for central \text{Au+Au} collisions at RHIC, we are faced with another issue: there are 20! possible pairings and it has become impractical to calculate the probability of each individual pairing this way. In general, we would be forced to perform permutation sampling of this partition function, preferably with some Metropolis algorithm. How to sample over permutations with a Metropolis algorithm is discussed thoroughly in the literature, for an excellent review of this see Ceperley \(E\). However, for RHIC, the situation simplifies due to the relatively low densities of \(\bar{c}c\) pairs involved. We ran our simulation for the most central \text{Au+Au} collisions at RHIC and examined how many “neighbors” each charm quark had. A “neighbor” is defined as a charm anti-quark, originating from a different pQCD event yielding the given charm quark, which is close enough to the charm quark that it could potentially form a bound state, in other words \(r\) is such that \(V_{\text{cornell}}(r) < 0.88\) GeV. The number of charm quarks expected to have one and only one neighbor in the most central \text{Au+Au} collisions was found to be 5.5%, while only 0.2% of the charm quarks are expected to have more than one neighbor. Therefore, even at the most central collisions at RHIC, we can be spared possibly complicated permutation samplings. Of course, this situation is not true in general, and for the numbers of pairs produced in a typical heavy-ion collision at the LHC one should modify these combinatorics.

B. New analysis of the data including improved \(d\text{Au}\) sample

The data with which we now compare our results is different from that which we used for comparison in our previous work. New data analysis of \text{Au+Au} and \(d+\text{Au}\) described in \(F\) account for the (anti-)shadowing and the breakup of charmonium due to the cold nuclear matter effects (parameterized by \(\sigma_{\text{abs}}\)) in the framework of a Glauber model for the collision. The calculations at forward and mid-rapidity are now done independently, since shadowing and breakup could be considerably different at different rapidities. This new analysis is a significant success, demonstrating the high suppression at forward rapidity (previously very puzzling) as being due to cold nuclear matter effects. New ratios of observed suppression due to cold nuclear matter \(R_{\text{AA}}/R_{\text{CNM}}\), plotted versus the energy density times time \(\epsilon \tau\), show common trends for RHIC and SPS data, which was not the case previously. We use this new analysis as a measure of survival probability in our calculation.

C. The results

Before we show the results, let us remind the reader that our calculation is intended to be a dynamical one, with no free parameters. We use a hydrodynamical simulation developed in \(G\) which is known to describe accurately the radial and elliptic collective flows observed in heavy-ion collisions. Our drag and random force terms for the Langevin dynamics have one input – the diffusion coefficient for charm – constrained by two independent measurements \((p_T\) distributions and \(v_2(p_T)\) measurements for single lepton – charm – performed in Ref. \(H\). The interaction of these charm quarks are determined by the correlators for two Polyakov lines in IQCD \(I\).

Having said that, we still are aware of certain uncertainties in all the input parameters, which affect the results. In order to show how much the results change if we vary some of them, we have used the un-
FIG. 1: (Color online.) $R_{AA}^{anomalous} = R_{AA} / R_{AA}^{CNM}$ for $J/\psi$ versus centrality of the AuAu collisions at RHIC. The data points with error bars show the PHENIX Au+Au measurements with cold nuclear matter effects factored out as in [10]. Other points, connected by lines, are our calculations for the two values of the QCD phase transition temperature $T_c = 165$ MeV (upper) and $T_c = 190$ MeV (lower). From bottom to top: the (green) filled squares show our new results, the recombinant $J/\psi$, the open (red) squares show the $R_{AA}$ for surviving diagonal $J/\psi$, the open (blue) circles show the total.

As can be seen, a higher $T_c$ value improves the agreement of our simulation with the latest analysis of the data, because in this case the QGP phase is shorter in duration and the survival probability is larger. However the recombinant contribution (shown by filled squares) is in this case relatively smaller, making less than 1/3 of the yield even in the most central collisions at RHIC.

Our results for the total, direct, and recombinant contributions resembles considerably the results of Zhao and Rapp obtained from their two-component model [12]. However it is important to point out two important differences. First of all, what is described by Zhao and Rapp as the second component due to statistical coalescence includes, with the recombinant $J/\psi$, surviving $J/\psi$, destroyed by the medium, which ultimately coalesce in the end. Second, the direct $J/\psi$ states' abundance, when compared with the abundances of excited charmonium states, does not necessarily need to be as expected from these particles’ Boltzmann factors. For our model, these relative abundances do make sense for direct charmonium states, due to the formation of a quasi-equilibrium distribution.

D. Recombinant $J/\psi$ and $p_T$-distributions

So far, we have only considered the effect of the recombinant production on the overall yields of $J/\psi$ particles at the RHIC. We should test our model by considering whether or not adding the recombinant contribution can change the shape of any differential $J/\psi$ yields.

One differential yield where we may expect the surviving and recombinant component to have different behaviors is in the $p_T$-distributions for central Au+Au collisions. The surviving $J/\psi$ states tend to originate in the periphery of the collision region, since the $J/\psi$ states produced here endure the sQGP phase for the shortest times. However, the recombinant contribution should form toward the center of the collision region, since this is where the density of initial $\bar{c}c$ pairs is highest, and as we have been showing for some time, spatial diffusion is incomplete in the sQGP. Therefore, since the effect due to flow on the $p_T$-distributions has Hubble-like behavior, with the radial velocity of the medium scaling with distance from the center of the transverse plane, we would expect the recombinant contribution to exist, on average, in regions of the medium with significantly smaller flow velocities.

Figure 2 demonstrates this behavior existing in our simulation.

We should now determine whether or not the flow difference of the yield versus $r$ can be observed in the $J/\psi$ yield versus $p_t$. As we have shown in our previous paper,
during the phase transition from QGP to the hadronic phase in heavy-ion collisions, our model predicts a small change in the total $J/\psi$ yield but relatively large changes in the $J/\psi$ $p_t$ distributions, with these changes strongly dependent on the drag coefficient for quarkonium during this time, and $T_c$. We can easily run our code with an LH8 equation of state and make several predictions for the two components' $p_t$ distributions. However, for reasons which will become apparent, we are only interested in the upper limit of the effect of flow on $p_t$ in Au+Au collisions at the RHIC. Therefore, we ran our simulation where we assumed a phase transition which lasts 5 fm/c, in a Hubble-like expansion.

The $p_t$ distributions after this expansion are shown in Figure 3. It is visible from this plot that the recombinant contribution will observably increase the total yield at low $p_t$ (where the total yield is significantly higher than the surviving component alone) and have little effect at higher $p_t$ (where the total and the surviving component alone are nearly the same). However, we have found that even in this extreme limit, there is no clear signal in the differential $p_t$ yields for there being two components for $J/\psi$ production at the RHIC.

This test, however, should not be abandoned for measurements of the differential yields at higher collision energies. Since the recombinant contribution grows substantially as charm densities are increased, it should be checked whether or not the recombinant contribution is more strongly peaked in the center of the transverse plane of LHC collisions, and whether or not two components to the differential yields should become observable there. We will follow up on this issue in a work we have in progress.

III. DISCUSSION

We have found that at central Au+Au collisions at RHIC the fraction of recombinant pairs should be considerable, up to 30-50%, with smaller fractions at more peripheral collisions. The exact number depends on details of the model, such as duration of the QGP phase and the magnitude of the critical temperature $T_c$. We have also gone a step further, and attempted to find different behaviors of these two components in differential yields, so that these two components might be disentangled. This test (examining the differential $p_t$ yields) fails to identify clearly two different components. We will pursue whether or not this test works for the yields at the LHC.

Our model for charmonium in sQGP is rather conservative: we merely assume that the constituent charm quarks experience dynamics similar to the Langevin dynamics of single charm quarks in SQGP, which has already shown good agreement with the $R_{AA}(p_t)$ and $v_2(p_t)$ measured at PHENIX for single charm.

One final, careful observation of our results is worth mentioning. As one can see from our results of Fig. 1, the model seems to be working well for central collisions, in which there is a QGP phase lasting for several fm/c and leading to a possibility for charm quarks to diffuse away from each other, far enough so that $J/\psi$ states would not survive. However it overpredicts suppression for peripheral collisions, which – if the cold matter analysis will hold against further scrutiny – is nearly completely absent. One possible reason for that can be survival of the flux tubes (QCD strings) between quarks well into the mixed phase or even in small region of temperatures above $T_c$, as was recently advocated by one of us [13] in connection with “ridge” phenomenon.

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non-trivial, and for RHIC collisions we will take a binary
This averaging is possible computationally but is can therefore describe recombination in heavy-ion collision,
\(N\) is, an ensemble of very many systems, where each system consists of a single \(\bar{c}c\) pair. The interaction of charm quarks from different hard events is negligible compared to the stochastic interaction and the interaction within the pair, partly because near \(T_c\), the dynamics of charm pairs seems best described with some generalization of the Lund string model, which allows no interaction between unpaired charm quarks \[3\]. Therefore, it is simple bookkeeping to think now of the systems as each consisting of \(N \bar{c}c\) pairs.

However, even though the dynamics of the system is not changed when considering many \(\bar{c}c\) pairs per collision, the hadronization (“pairing”) of these \(2N\) charm quarks is now a non-trivial issue. For simplicity, assume that the quarks all reach the freezeout temperature \(T_c\) at the same proper time. There are \(N!\) different possible pairings of the quarks and anti-quarks into charmonium states (each pairing is an element of the permutation group \(S_N\)). Call a given pairing \(\sigma\) (which is an element of \(S_N\)). Near \(T_c\), the relative energetics of a pairing \(\sigma\) is given by

\[
E_i = \sum_i V(|\vec{r}_i - \vec{r}_{\sigma(i)}|),
\]

where \(V(r)\) is the zero-temperature Cornell potential (with some maximum value at large \(r\), corresponding to the string splitting), \(\vec{r}_i\) the position of the \(i\)-th charm quark, \(\vec{r}_{\sigma(i)}\) the position of the \(i\)-th charm antiquark, and \(\sigma(i)\) the integer in the \(i\)-th term of the permutation.

One way to proceed is to average over these pairings according to their Boltzmann factors. In this way, the probability of a given pairing would be given by

\[
P(i) = \frac{1}{Z} \exp(-E_i/T_c), \quad Z = \sum_{i=1}^{N!} \exp(-E_i/T_c).
\]

However, this averaging ignores the possibility of survival of bound \(\bar{c}c\) states from the start to the finish of the simulation, in that pairings which artificially “break up” bound states are included in the average. This goes against the main point of our last paper: that it is actually the incomplete thermalization of \(\bar{c}c\) pairs which explains the survival of charmonium states.

For this reason, the averaging we perform rejects permutations which break up pairs that would otherwise be bound: we average over a subgroup \(S'_N\) of \(S_N\), and determine the probability based on this modified partition function:

\[
P(i) = \frac{1}{Z} \exp(-E_i/T_c), \quad Z = \sum_{\sigma \in S'_N} \exp(-E_i/T_c),
\]

where \(E_i\) specifies the energy of a pairing we permit. We will average over the permutations in this way.

By doing this, we will use a fully canonical ensemble description for charm in plasma, which holds for any value for \(N\), large or small. Previous work in statistical hadronization used the grand canonical approach to explain relative abundances of open and hidden charm \[2\], which can only be applied where thermalization may be assumed to be complete.

**APPENDIX A: CANONICAL ENSEMBLES FOR \(N \bar{c}c\) -PAIR SYSTEMS**

In this section we will determine a partition function for a canonical ensemble of \(N\) charm pair systems (that is, an ensemble of very many systems, where each system contains \(N \bar{c}c\) pairs) which correctly averages over different possible pairings of charm and anticharm quarks and can therefore describe recombination in heavy-ion collisions. This averaging is possible computationally but is non-trivial, and for RHIC collisions we will take a binary approximation which makes this averaging much easier. We argue, however, that the unsimplified approach is necessary for describing collisions at the Large Hadron Collider, and for this reason we include this discussion here.

Our simulation could be thought of as a canonical ensemble description of charmonium in plasma: we can think of our large set of \(\bar{c}c\) pairs as a set of systems, each system containing \(N\) \(\bar{c}c\) pairs, with each system’s dynamics modeled as a stochastic interaction with the medium, with a deterministic interaction of each heavy quark with the other quark in the pair. Each system in this set samples the distribution of \(\bar{c}c\) pairs in the initial collision, the geometry of the collision, and also samples the stochastic forces on the heavy quarks. Up to this point, we have only thought of each system of this set as consisting of a single \(\bar{c}c\) pair. The interaction of charm quarks from different hard events is negligible compared with the stochastic interaction and the interaction within the pair, partly because near \(T_c\), the dynamics of charm pairs seems best described with some generalization of the Lund string model, which allows no interaction between unpaired charm quarks \[3\]. Therefore, it is simple bookkeeping to think now of the systems as each consisting of \(N \bar{c}c\) pairs.

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