Study on wind-induced response of a simplified model of transmission tower based on finite particle method

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Abstract. A reduced model of a transmission tower was established based on the principle of equivalent stiffness. The finite particle method was used to analyse the wind-induced response of the reduced model. The analysis of the dynamic characteristics of the reduced model shows that the accuracy of the equivalent formula proposed in this paper was very high. Compared with the first five order natural frequencies of the complete model, the maximum calculation error of the reduced model was only 0.87%. Since the first order shear deformation beam theory was used in the reduced model, the interpolation-independent element shape functions were developed to avoid the shear locking phenomenon. Moreover, the formula for calculating the internal force of Timoshenko beam for finite particle method was derived. By using two successive Euler angles, a new method for separating the rigid motion and pure deformation of the element was proposed. The computational efficiency of this method is significantly improved, and the computational time is less than half of that of the traditional finite particle method. The results of wind-induced response analysis of the transmission tower show that the simplified model proposed in this paper can obtain highly accurate displacement response of the structure, and the maximum error of the RMS of the displacements at the top of the tower is less than 3.2%. This model is suitable for studying the dynamic response of transmission tower line system and evaluating the effect of the wind-induced vibration control of damping device.

1. Introduction

As an important part of the electrical transmission systems, the safety of the transmission tower-line system is of great importance to the normal operation of the power grid. With the increase of the span and height of transmission tower-line system, the influence of strong wind load on the safety of the system becomes more and more obvious. Wind-induced dynamic response of transmission tower-line system has become an important subject in structural wind resistance field.

The transmission tower, consisting of large number of nodes and complicated structure connections, is the main load-bearing structure of the transmission tower-line system. Related researches are mainly developing in two directions, one is the accurate 3D model [1-3], the other is the simplified model [4-8]. The multi-particle model [4-6] is a widely used simplified model, which simplifies the transmission tower to a series of multi-degrees-of-freedom systems with multiple concentrated masses connected by rods, and the stiffness of the tower is usually obtained by inverse of the flexibility matrix using finite element method. Zhang et al [7] carried out wind tunnel test based on high frequency force balance and 3D finite element analysis, which proved that the multi-particle
model has enough accuracy in calculating the overall response of the tower. Limongelli et al [8] obtained a reduced model of the lattice transmission tower without stiffness matrix. In their method, the transmission tower is divided into several sections, and assumed that each section has the same material and stiffness properties. Compared with the multi-particle model, this model established the equivalent model of the cross arms, which can better consider the torsion effect and more accord with the actual situation. Based on this model, Dua et al [9] studied the dynamic response of transmission tower-line system under the action of random simulated wind fields, and proved that if the coupling effect of tower line is not taken into account, the displacement of the conductors and insulators will be wrongly estimated. Literature [8] gives a potential way to build the simplified model of transmission tower, but there are still some to be improved. For example, the torsion stiffness formula overestimates the first-order torsional frequency. In addition, the mass of the steel plate, bolt and nut is not considered in the calculation of equivalent density, which will have a great influence on the dynamic characteristics of the transmission tower, especially for a tower with a heavy tower top.

Finite particle method (FPM), developed from vector form finite element method [10,11], is a new structural analysis method which is more suitable for space structure analysis. This method does not need to assemble structural element stiffness matrix and to carry out iterative calculation. Therefore, it has advantages in calculation of the geometric and material nonlinearity, discontinuity, and has been successfully used in the analysis of various types of complex structural behaviors [12-16]. In FPM, Euler-Bernoulli beam model [12], rod model [14] and fine beam model [15,16] have been proposed to compute the internal force of spatial beam. In literature [14], the computational code of FPM was developed with C++ programming language, and it was found that the computational efficient of this method was significantly higher than that of ANSYS.

In this paper, the equivalent simplified model of a lattice transmission tower is established by the method of literature [8], and the formula of torsional stiffness is modified to improve the calculation of torsion natural frequency. In this model, the shear deformation is not negligible, so the formulas of internal forces based on Timoshenko beam model are derived. By using the method based on two successive Euler angles instead of the virtual inverse motion analysis, the calculation formula of rotation matrix is derived. The central difference method is adopted to solve the equations of motion.

2. Reduced model of transmission tower
A transmission tower can be subdivide in four substructures: the tower foot, the tower body, the tower top and the cross arms. The reduced model consists of straight beam elements along the center line of the substructures. The transverse cross arms are rigidly coupled to the vertical tower top [8].

2.1. Equivalent formulas of reduced model
The equivalent properties of tensile, bending, shear, and torsional of the i-th portion of the tower are as follows [8]:

\[ k_{A,i} = \frac{EA_{eq,i}}{l_i}, k_{F,i} = \frac{EI_{eq,i}}{l_i}, k_{S,i} = \frac{GA_{eq,i}}{l_i}, k_{T,i} = \frac{GJ_{eq,i}}{l_i} \]  \tag{1}

respectively, and

\[ A_{eq,i} = \sum_{j=1}^{n_{eq}} A_{ch,i} \cos^3 \theta_j \]  \tag{2}

\[ I_{eq,i} = A_{eq,i} h_{i,min} h_{i,max}^2 / 4 \]  \tag{3}
\[ A_{d,i} = \sum_{j=1}^{n_{ch,i}} A_{d,j} \cos^2 \delta_j \sin \delta_j \]  \hspace{1cm} (4)

\[ \chi_{eq,i} = \frac{A_{eq,i}}{2(1+v)A_{d,i}} \]  \hspace{1cm} (5)

\[ J_{eq,i} = 2I_{eq,i} / \chi_{eq,i} \]  \hspace{1cm} (6)

where, \( n_{ch,i} \) is the number of main bearing components of the \( i \)-th portion, \( \theta \) is the angle between the axial direction of the main bearing component and the axial direction of the equivalent element; \( n_{d,i} \) is the number of diagonal components of the \( i \)-th portion, \( \delta \) is the angle between the axial direction of the diagonal component and the shear direction of the equivalent element; \( A_{ch,j} \) and \( A_{d,j} \) are the transversal area of the \( j \)-th main bearing and diagonal components of the \( i \)-th portion; \( A_{eq,i} \), \( I_{eq,i} \), \( J_{eq,i} \), \( \chi_{eq,i} \) are the equivalent transversal area, moment of inertia, torsional moment of inertia, and shear factor of the \( i \)-th portion, respectively; \( h_{\text{min}} \) and \( h_{\text{max}} \) are the widths of the two ends of the \( i \)-th portion; \( l_j \) is the length of the \( i \)-th portion; and \( E \), \( G \), \( v \) are Young’s modulus, shear modulus, and Poisson’s ratio, respectively. See literature [8] for more details. Moreover, the equivalent density of the \( i \)-th portion is as follows:

\[ \gamma_{eq,i} = \rho \left( \frac{n_{ch,i}}{\sum_{j=1}^{n_{ch,i}} A_{ch,j} l_j} + \frac{n_{d,i}}{\sum_{j=1}^{n_{d,i}} A_{d,j} l_j} \right) / \sum_{j=1}^{n_{ch,i}} A_{ch,j} l_j \]  \hspace{1cm} (7)

where, \( \rho \) is density of steel, \( l_j \) is the length of the components of the \( i \)-th portion. In order to consider the influence of the mass of the steel plate, bolt and nut, a density correction factor \( \eta \) is introduced. The corrected density is \( \eta \gamma_{eq,i} \), and then \( \eta \gamma_{eq,i} A_{eq,i} \) equals to the total mass of the \( i \)-th portion.

2.2. Modified formula of torsional stiffness

Considering a lattice transmission tower shown in figure 1(a), the total height of the tower is 77.5 m. A complete finite element model (refer as complete model) of the tower was established with one element for each component. There were 652 nodes and 1802 elements in total. The element type was chosen as BEAM188, and cubic interpolation shape function was adopted. The reduced model only has 58 nodes and 58 elements, as shown in figure 1(b).

It is found that if the cross-section of the equivalent beam element is assumed to be circular, the torsional stiffness formula is the same as equation (6). In this paper, the cross-section of the equivalent beam element is assumed to be a square ring, then the torsional stiffness can be written as

\[ J_{eq,i} = I_{eq,i} / \chi_{eq,i} \]  \hspace{1cm} (8)

In table 1, the results of the reduced model using two torsional stiffness formulas are compared with those of the complete model. It can be seen that the error of the first-order torsion natural frequency obtained from equations (1)-(7) is up to 27.27%; while using the modified formulas (equation (8)) proposed in this paper, the errors of the first five natural frequencies are all less than 0.87%. The first five modes of the reduced model are shown in figure 2.
Figure 1. Models of transmission tower. (a) complete model and (b) reduced model.

Table 1. First five modes and frequencies of the tower.

| Mode No. | Mode type                      | Frequencies (Hz)                  | Reduced model a | Reduced model b | Complete FE model |
|----------|--------------------------------|-----------------------------------|-----------------|-----------------|-------------------|
| 1        | 1<sup>st</sup> (bending xz plane) | 1.0470                           | 1.0470          | 1.0445          |                   |
| 2        | 1<sup>st</sup> (bending yz plane) | 1.0492                           | 1.0492          | 1.0468          |                   |
| 3        | 2<sup>nd</sup> (bending xz plane)  | 3.3590                           | 3.3590          | 3.3531          |                   |
| 4        | 2<sup>nd</sup> (bending yz plane)  | 3.4222                           | 3.4231          | 3.4266          |                   |
| 5        | 1<sup>st</sup> (torsion about z-axis)   | 5.4545                           | 3.9616          | 3.9962          |                   |

<sup>a</sup> Equation (6) was used to calculate the torsional stiffness;

<sup>b</sup> Equation (8) was used to calculate the torsional stiffness.
Figure 2. First five modes of the reduced model. (a) mode 1, (b) mode 2, (c) mode 3, (d) mode 4 and (e) mode 5.

3. Finite particle method
The motion of a straight beam element without initial curvatures and torsion can be decomposed into rigid motion and pure deformation by using local coordinate system. Therefore, the motion of an element can be decomposed into two steps: the first step is the rigid body translation and rotation of the element; and the second step is the pure deformation of the element in the local coordinate system. Assuming the length of the element is properly chosen, the pure deformation of the element is always small relative to the local coordinate system. Since the motion of the rigid body does not generate internal force, the internal force of the element is only related to the pure deformation.

3.1. Equation of motion of a particle
According to the characteristics of spatial frames, finite particle method uses a space beam element and two particles connected with it to describe each component. In FPM, it is assumed that each particle is in a state of dynamic equilibrium during the process of rigid motion and deformation. So the motion of all particles follows Newton’s second law, which can be expressed as

$$m_p \frac{d^2 \mathbf{u}_p}{dt^2} = \mathbf{F}^{\text{ext}}_p - \mathbf{F}^{\text{int}}_p - \mathbf{F}^{\text{damp}}_p, \quad I_p \frac{d^2 \mathbf{\theta}_p}{dt^2} = \mathbf{M}^{\text{ext}}_p - \mathbf{M}^{\text{int}}_p - \mathbf{M}^{\text{damp}}_p$$

(9)

where, $m_p$ and $I_p$ are the equivalent mass and rotatory inertia of particle $p$, respectively; $\mathbf{u}_p$ and $\mathbf{\theta}_p$ are the translation and angular displacement vectors of particle $p$, respectively; $\mathbf{F}^{\text{ext}}_p$ and $\mathbf{M}^{\text{ext}}_p$ are the equivalent external forces and moments acting on particle $p$, respectively; $\mathbf{F}^{\text{int}}_p$ and $\mathbf{M}^{\text{int}}_p$ are...
the internal forces and moments of particle \( p \) caused by the deformation of connecting elements, respectively; \( \mathbf{F}_{\text{damp}}^p \) and \( \mathbf{M}_{\text{damp}}^p \) are the damping forces and moments of particle \( p \), respectively, and they are usually assumed to be expressed as \( \mathbf{F}_{\text{damp}}^p = \mu \mathbf{m}_p \dot{\mathbf{u}}_p^c \) and \( \mathbf{M}_{\text{damp}}^p = \mu \mathbf{I}_p \dot{\mathbf{\theta}}_p^c \), where \( \mu \) is the damping factor. Finally, \( \mathbf{m}_p \), \( \mathbf{I}_p \), \( \mathbf{F}_{\text{ext}}^p \), and \( \mathbf{M}_{\text{ext}}^p \) are detailed in literature \[4\].

The explicit central difference method was used to solve Eq. (9), so the displacements of a particle can be written as \[12\]

\[
\mathbf{X}_{p,t+\Delta t} = 2c_1 \mathbf{X}_{p,t} - c_2 \mathbf{X}_{p,t-\Delta t} + c_1 \Delta t^2 \mathbf{H}_p^{-1}(\mathbf{F}_{\text{ext}}^p - \mathbf{F}_{\text{int}}^p) \tag{10}
\]

where, \( c_1 = 1/(1 + \mu \Delta t/2) \), \( c_2 = c_1(1 - \mu \Delta t/2) \), and \( \Delta t \) is the time step; \( \mathbf{X} \) is the translation or angular displacement vectors; \( \mathbf{H}_p \) is a matrix consisting of \( \mathbf{m}_p \) or \( \mathbf{I}_p \).

3.2. Description of rigid body rotation and pure deformation

As shown in figure 3, \( \mathbf{i}_m \) is a fixed global coordinate system, and the subscript \( m = 1,2,3 \). Points \( p_1 \) and \( p_2 \) are the two endpoints of the beam element. At moment \( t_a \), the positions of the two endpoints are \( \mathbf{x}_1^a \) and \( \mathbf{x}_2^a \), respectively; and the rotation angle vectors about \( \mathbf{i}_m \) at the two endpoints are \( \mathbf{a}_1^a \) and \( \mathbf{a}_2^a \), respectively. At moment \( t_a \), the positions of the two endpoints are \( \mathbf{x}_1^b \) and \( \mathbf{x}_2^b \), respectively; and the rotation angle vectors about \( \mathbf{i}_m \) at the two endpoints are \( \mathbf{a}_1^b \) and \( \mathbf{a}_2^b \), respectively. At moment \( t_a \), the element length denotes \( l_a \); the local coordinate system denotes \( \mathbf{a}_m \), and the transformation matrix between \( \mathbf{a}_m \) and \( \mathbf{i}_m \) is \( \mathbf{R}_a \). At moment \( t_b \), the element length denotes \( l_b \); the local coordinate system denotes \( \mathbf{b}_m \), and the transformation matrix between \( \mathbf{b}_m \) and \( \mathbf{i}_m \) is \( \mathbf{R}_b \).

![Figure 3. Particle displacement and local coordinate system of spatial beam element.](image)

From moment \( t_a \) to \( t_b \), the increments of the rotation angle vectors in \( \mathbf{a}_m \) are \( \Delta \mathbf{a}_1 \) and \( \Delta \mathbf{a}_2 \), respectively, i.e.

\[
\Delta \mathbf{a}_1 = \mathbf{R}_a(\mathbf{a}_1^b - \mathbf{a}_1^a), \Delta \mathbf{a}_2 = \mathbf{R}_a(\mathbf{a}_2^b - \mathbf{a}_2^a) \tag{11}
\]

The displacement of endpoint \( p_2 \) in \( \mathbf{b}_m \) denotes \( \mathbf{u}_2 \). According to geometric relations, we have

\[
l_a \mathbf{b}_1 + \mathbf{u}_2 = l_b \mathbf{b}_1 \tag{12}
\]
\[ \mathbf{b}_1 = (\mathbf{x}_2^b - \mathbf{x}_1^b)/l_b = \mathbf{R}_b (\mathbf{x}_2^b - \mathbf{x}_1^b)/l_b = d_m \mathbf{a}_m \] (13)

Apparently, \( \mathbf{u}_2 = (l_b - l_a) \mathbf{b}_1 \). Notice that the last two equations of equation (13) describe the vector \( \mathbf{b}_1 \) in local coordinate system \( \mathbf{a}_m \).

The finite rotation of the rigid body motion part of the beam element can be described by two Euler angles [17]: firstly, \( \mathbf{a}_m \) is translated by \( \Delta \mathbf{x}_1 \) and rotated by angle \( \alpha = \sin^{-1}(\sqrt{d_z^2 + d_i^2}) \), where the rotation axis is \( \mathbf{n} = (\mathbf{b}_1 \times \mathbf{a}_i)/\|\mathbf{b}_1 \times \mathbf{a}_i\| \); Secondly, the rotated coordinate system is further rotated by angle \( \beta \) about \( \mathbf{b}_1 \), where \( \beta \) is the first component of \( \Delta \mathbf{a}_i \). Then, the rotation angle vector of \( \mathbf{a}_m \) can be written as

\[ \mathbf{\beta} = \begin{bmatrix} \beta \\ -\alpha d_3/\sqrt{d_z^2 + d_i^2} \\ \alpha d_2/\sqrt{d_z^2 + d_i^2} \end{bmatrix}^T \] (14)

Applying Rodrigues rotation formula twice, the finite rotation tensor from \( \mathbf{a}_m \) to \( \mathbf{b}_m \) is obtained as

\[ \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} d_1 \\ -d_2 + d_i/(1 + d_1) \\ -d_2 d_i/(1 + d_1) \end{bmatrix} \begin{bmatrix} d_1 \\ -d_2 + d_i/(1 + d_1) \\ -d_2 d_i/(1 + d_1) \end{bmatrix} \] (15)

Therefore, at moment \( t_b \), the rotation matrix of the local coordinate system with respect to the fixed global coordinate system is

\[ \mathbf{R}_b = \mathbf{R} \mathbf{R}_a \] (16)

Usually, it is assumed that the rotation matrix of the local coordinate system with respect to the fixed global coordinate system is known at the initial time. Then, according to equation (16), the rotation matrix at any time can be obtained recursively. In local coordinate system \( \mathbf{b}_m \), the pure deformation of the two endpoints of the beam element can be written as

\[ \mathbf{\theta}_1 = \Delta \mathbf{a}_1 - \mathbf{\beta}, \mathbf{\theta}_2 = \Delta \mathbf{a}_2 - \mathbf{\beta} \] (17)

3.3. Internal force of the beam element

Consider the two-node space Timoshenko beam element shown in figure 4, each node has six degree-of-freedoms (DOFs), namely three translational DOFs and three rotational DOFs. Assuming that there is no external load acting between the two nodes, and ignoring the torsional warpings, the kinematics relationship of Timoshenko beam can be expressed as

\[ U = u - y\theta + z\psi, V = v - z\phi, W = w + y\phi \] (18)

![Figure 4. Two node spatial beam element.](image-url)
where, $u$, $v$ and $w$ are the displacement along the $x$, $y$ and $z$ axis at a point on the bema reference line, respectively; $U$, $V$ and $W$ are the displacements in $x$, $y$ and $z$ directions at any point of the cross-section, respectively; $\varphi$, $\psi$ and $\theta$ are the small rotation angles of the cross-section with respect to $x$, $y$ and $z$, respectively. The non-zeros strains of the beam element can be expressed as

$$\varepsilon = \{\varepsilon_x, \varepsilon_y, \varepsilon_z\}^T = [S] \{\chi\}$$

(19)

$$\{\chi\} = \{\chi_x, \chi_y, \chi_z, \chi'_x, \chi'_y, \chi'_z\}^T$$

(20)

$$[S] = \begin{bmatrix} 1 & 0 & 0 & 0 & z & y \\ 0 & 1 & 0 & -z & 0 & 0 \\ 0 & 0 & 1 & y & 0 & 0 \end{bmatrix}$$

(21)

where, $(\cdot)'$ denotes the derivative with respect to $x$. $\gamma_y$ and $\gamma_z$ are the shear strains in $y$ and $z$ directions, and can be written as

$$\gamma_y = \psi' - \theta, \gamma_z = w' + \psi$$

(22)

If the material is isotropic and homogeneous, and the beam reference point coincides with the mass centroid, then the variation of strain energy can be written as

$$\delta U = \int \int_A \left( \sigma_{xx} \delta \varepsilon_{xx} + \sigma_{yy} \delta \varepsilon_{yy} + \sigma_{zz} \delta \varepsilon_{zz} \right) dA dx = \int \{\delta \chi\}^T \{\tilde{F}\} dx$$

(23)

The resultant cross-sectional stress conjugated with the generalized strain are

$$\{\tilde{F}\} = \{F_x, Q_y, Q_z, M_x, M_y, M_z\}^T = [D] \{\chi\}$$

(24)

$$D_{11} = EA, D_{22} = k_y GA, D_{33} = k_z GA, D_{44} = k_y GJ, D_{55} = EI_y, D_{66} = EI_z$$

(25)

where, $k_y$ is the influence factor of torsion stiffness by warping, $k_y$ and $k_z$ are the shear correction factors, $A$ is the cross-section area, $I_y$ and $I_z$ are moment of inertia, and $J$ is polar moment of inertia. Since it is assumed that there is no load acting on the element, the relationship

$$M'_x - Q_y = 0, M'_y - Q_z = 0$$

(26)

The boundary conditions at endpoints $p_1$ and $p_2$ in the local coordinate system $b_m$ are

$$u(0) = v(0) = w(0) = \varphi(0) = 0, \psi(0) = \psi_1, \theta(0) = 0,$$

$$u(l_a) = u_x, v(l_a) = w(l_a) = 0, \phi(0) = \phi_2, \psi(l_a) = \psi_2, \theta(l_a) = \theta_2$$

(27)

Linear interpolation was used for axial displacement $u$ and torsion angle $\varphi$, and cubic polynomial interpolation was used for transverse displacements $v$ and $w$. Considering equation (26) and applying boundary conditions (27), we have

$$\{u, v, w, \varphi, \psi, \theta\}^T = [N] \{q\}$$

(28)

where, $\{q\} = \{u_2, \phi_2, \psi_1, \psi_2, \theta_1, \theta_2\}^T$ is the nodal displacement vector, and the non-zero elements of the shape function matrix are
\[ N_{11} = N_{42} = \xi \]
\[ N_{23} = l_s \beta_1 [(\xi^3 - 2\xi^2 + \xi) + \frac{1}{2} \alpha_2 (\xi - \xi^2)] \]
\[ N_{26} = l_s \beta_1 [(\xi^3 - \xi^2) + \frac{1}{2} \alpha_2 (\xi^2 - \xi)] \]
\[ N_{33} = -l_s \beta_1 [(\xi^3 - 2\xi^2 + \xi) + \frac{1}{2} \alpha_2 (\xi - \xi^2)] \]
\[ N_{34} = -l_s \beta_1 [(\xi^3 - \xi^2) + \frac{1}{2} \alpha_2 (\xi^2 - \xi)] \]
\[ N_{53} = -l_s \beta_1, [3\xi^2 - 4\xi + 1 + \alpha_2 (1 - \xi)] \]
\[ N_{54} = -l_s \beta_1 [3\xi^2 - 2\xi + \alpha_2 \xi] \]
\[ N_{55} = l_s \beta_1 [3\xi^2 - 4\xi + 1 + \alpha_2 (1 - \xi)] \]
\[ N_{65} = l_s \beta_1 [3\xi^2 - 2\xi + \alpha_2 \xi] \]
\[ \alpha_i = \frac{12EI_s}{k_s GAl_0}, \beta_j = \frac{1}{1 + \alpha_i}, \alpha_\zeta = \frac{12EI_s}{k_s GAl_0}, \beta_\zeta = \frac{1}{1 + \alpha_\zeta} \]

(30)

Here, \( \xi = x/l_a \) is dimensionless axial coordinate. The relationship between generalized strains \( \{ \chi \} \) and nodal displacement vector \( \{ q \} \) is

\[ \{ \chi \} = \{ \Phi \} \{ q \} \]

(31)

where the non-zero elements of \( \{ \Phi \} \) are

\[
\Phi_{11} = N'_{11}, \Phi_{25} = N'_{25} - N'_{65}, \Phi_{26} = N'_{26} - N'_{66}, \\
\Phi_{33} = N'_{33} + N'_{53}, \Phi_{34} = N'_{34} + N'_{54}, \Phi_{44} = N'_{11}, \\
\Phi_{53} = N'_{53}, \Phi_{54} = N'_{54}, \Phi_{65} = -N'_{65}, \Phi_{66} = -N'_{66}
\]

(32)

Then, the variation of strain energy can be also written as

\[ \delta U = \int_0^l (\delta \chi)^T (\hat{F}) \delta x = \int_0^l (\delta q)^T [\{ \Phi \}^T [D] [\{ \Phi \}] \{ q \} \delta x = (\delta q)^T (\hat{F}) \]

(33)

The nodal force vector \( \{ \hat{F} \} \) corresponding to the nodal displacement vector \( \{ q \} \) is

\[ \{ \hat{F} \} = \{ F_2, M_{2z}, M_{1z}, M_{2y}, M_{1y}, M_{2z}, M_{1z} \}^T = \int_0^l [\{ \Phi \}^T [D] [\{ \Phi \}] \{ q \} \delta x 
\]

(34)

According to equation (34), the nodal force vector \( \{ \hat{F} \} \) can be easily obtained by symbolic software Mathematica, and hence it was not given here. Then, according to the static equilibrium conditions, the axial force \( F_i \) and the torque \( M_{1z} \) of point \( p_1 \), as well as the shear forces at the two endpoints can be obtained.

4. Wind induced response

The wind load acting on the transmission tower includes mean wind and turbulent wind [18], where the logarithmic function is used to express the mean wind, and the Kaimal fluctuating wind power spectrum is used to express the turbulent wind [2]. Moreover, the harmony superposition method is used to simulate the turbulent wind. Then, the along wind load is given by

\[ F(t) = \mu A_n u^2 / 1.6 \]

(35)

where, \( u \) is the wind speed of simulation point, \( \mu \), and \( A_n \) are the shape coefficient and wind area of a portion of the complete transmission tower, respectively.
In the following calculation, the angle of between the wind direction and line direction (y-direction in figure 1(b)) in horizontal direction is defined as wind angle. The modal damping ratio is set to 0.02.

4.1. Validation

At the top of the transmission tower, a concentrated constant load of $10^4$N is applied along the line direction. The total simulated time is 50 s. The time history of the displacement responses of the nodes connecting the four cross arms and the tower top are given in figure 5. It can be seen that the results of FPM (reduced model) and FEM (Finite Element Method) (complete model) are very close. Furthermore, the vibration responses caused by the sudden concentrated constant load are approximately zero after 50 s.

**Figure 5.** Comparison of displacement responses under concentrated constant load. (a) $z = 77.5$ m, (b) $z = 72.0$ m, (c) $z = 66.0$ m and (d) $z = 60.0$ m.

Spectrum analysis is carried out on the displacement and acceleration responses of the top of the tower, and the power spectrum curves are shown in figure 6. The response of the reduced model calculated by FPM shows that the first two natural frequencies in the along line direction are 1.04 Hz and 3.38 Hz, which are very close to those of the complete model calculated by FEM. It can be seen that the displacement response is dominated by the first mode, while the acceleration response is dominated by the first two modes. This example shows that the FPM can be used to calculate the dynamic characteristics of the tower, especially if the vibrations are dominated by low-order modes. In addition, the calculation time required by the present method is about 650s, while the time consumed by the traditional FPM is about 1480s, indicating that the proposed method in this paper, which uses two continuous Euler angles to describe the rotation of the local coordinate system of the beam.
element, is much more efficient.

Figure 6. Comparison of the displacement and acceleration power spectrums at the top of the tower. (a) displacement power spectrum and (b) acceleration power spectrum.

4.2. Response various with mean wind speed
In the case of 0° wind angle, the mean wind speeds at height of 10 m are taken 8 values equally spaced between 5 and 40 m/s, and the corresponding fluctuating wind speeds are superimposed to calculate the responses of the top of the tower. Figure 7 shows the time history of the displacement and acceleration of the top of the tower with mean wind speed at 40 m/s. Figure 8 compares the RMS (Root Mean Square) of the along and cross line displacements of the top of the tower at different wind speeds, where only the data after 50 s are taken for calculation. It can be seen that the displacement response obtained by FPM is very consistent with that of FEM, indicating that the reduced model presented in this paper has good accuracy in solving the dynamic response of the transmission tower under random wind loads.

Figure 7. Comparison of the displacement and acceleration responses at the top of the tower. (a) displacement and (b) acceleration.

Taking the results of the complete model as reference values, the error of the RMS of the along line displacements of the reduced model are listed in table 2. The errors are almost constant and relative small with the change of wind speed, which indicates that the reduced model has high reliability for different wind field intensities.
Figure 8. Comparison of the RMS of the displacements at the top of the tower with the wind speed. (a) along line direction and (a) cross line direction.

Table 2. Error of the RMS of the along line displacements at the top of the tower.

| Mean wind (m/s) | 5   | 10  | 15  | 20  | 25  | 30  | 35  | 40  |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Error (%)      | 2.38| 2.43| 2.39| 2.42| 2.39| 2.45| 2.36| 2.47|

4.3. Response various with wind angle

In the case of 30 m/s mean wind speed, the wind angles are taken 10 values equally spaced between 0° and 90°, and the corresponding fluctuating wind speed is superimposed to calculate the response of the top of the tower. Figure 9 shows the curves of the RMS of the displacements of the top of the tower various with different wind angles. In figure 9(a), the RMS of the along (cross) line displacement decreases (increases) gradually. In figure 9(b), the RMS of the along wind displacements are gradually smaller, while the RMS of the cross wind displacements first increases and then decreases, and the maximum value is achieved when the wind angle is between 40° to 50°. It can be seen that the maximum RMS errors of the displacements of the top of the tower is 3.2%.

Figure 9. Comparison of the RMS of the displacements at the top of the tower with different wind angles. (a) along and cross line and (b) along and cross wind.

5. Conclusions

Based on the principle of stiffness equivalence, a reduced model of lattice transmission tower with high accuracy can be quickly established according to the geometric characteristics and material
properties of the tower. Compared with the multi-particle model, the stiffness matrix of the reduced model is obtained without the need of finite element analysis. Furthermore, since the equivalent model of the cross arms are also established, the wind load acting on the cross arms can be better considered. In order to verify the reduced model, the complete model is also established as reference and solved by finite element method. In the numerical examples, the RMS errors of the displacements of the top of the tower obtained by the reduced model are less than 3.2%, indicating that the calculation accuracy is very high, which also indicates that the correctness of the reduced model and the finite particle method presented in this paper. In this work, two continuous Euler angles are used to describe the rotation of the local coordinate system of the beam element, so that the computational efficiency is significantly improved compared with the traditional finite particle method. Compared with the complete model, the reduced model has fewer degrees of freedom, which provides a potential analysis method for studying the wind-induced response of transmission tower-line system and evaluating the vibration control effect of damping devices.

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References
[1] Shehata A Y, El Damatty A A and Savory E 2005 Finite element modeling of transmission line under downburst wind loading Finite Elem. Anal. Des. 42 71-89
[2] Tian L, Yu Q Q, Ma R S et al 2014 The collapse analysis of a transmission tower under wind excitation Open Civ. Eng. J. 8 136-42
[3] Edgar T-H and Sordo E 2017 Structural behaviour of lattice transmission tower subjected to wind load Struct. Infrastruct. E. 13 1462-75
[4] Ozono S, Maeda J and Makino M 1988 Characteristics of in-plane free vibration of transmission line system Eng. Struct. 10 272-80
[5] Ozono S and Maeda J 1992 In-plane dynamic interaction between a tower and conductors at lower frequencies Eng. Struct. 14 210-6
[6] Li H N and Wang Q X 1997 Dynamic characteristics of long-span transmission lines and their supporting towers China Civ. Eng. J. 30 28-35 (in Chinese)
[7] Zhang Q H, Gu M and Huang P 2012 Multi-degree-freedom of a lattice transmission tower J. Vib. Shock 31 148-52 (in Chinese)
[8] Limongelli M P, Marinelli L and Perotti F 2003 A reduced model for the dynamic analysis of power transmission lines with truss supporting towers Proceedings of the 5th International Symposium on Cable Dynamics (Santa Margherita, September) pp 125-32
[9] Dua A, Clobes M, Hobbel T and Matsagar V 2015 Dynamic analysis of overhead transmission lines under turbulent wind loading Open J. Civ. Eng. 5 359-71
[10] Ding E C, Shih C and Wang Y K 2004 Fundaments of a vector form intrinsic finite element: part 1. Basic procedure and a plane frame element J. Mech. 20 113-22
[11] Ding E C, Shih C and Wang Y K 2004 Fundaments of a vector form intrinsic finite element: part 2. Plane solid elements J. Mech. 20 123-32
[12] Yu Y, Glauocio H P and Luo Z Y 2011 Finite particle method for progressive failure simulation of truss structures J. Struct. Eng. 137 1168-81
[13] Luo Z Y, Zheng Y F, Yang C et al 2014 Review of the finite particle method for complex behaviors of structures Eng. Mech. 31 1-7+23 (in Chinese)
[14] Yao D, Shen G H, Pan F et al 2015 Wind-induced dynamic response of transmission tower using vector-form intrinsic finite element method Eng. Mech. 32 63-70 (in Chinese)
[15] Yuan X F, Chen C, Duan Y F et al 2018 Elastoplastic analysis with fine beam model of vector form intrinsic finite element Adv. Struct. Eng. 21 365-79
[16] Long X H, Wang W and Fan J 2018 Collapse analysis of transmission tower subjected to earthquake ground motion *Model Simul. Mater. Sc.* **2018** 2687561

[17] Pai P F 2014 Problems in geometrically exact modeling of highly flexible beams *Thin Wall Struct.* **76** 65-76

[18] Holmes J D 2015 *Wind Loading of Structures* 3rd ed. (Boca Raton, USA: CRC Press)