Exploiting Uncertainty in Popularity Prediction of Information Diffusion Cascades Using Self-exciting Point Processes

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Abstract

Hawkes processes have been successfully applied to understand online information diffusion and popularity of online items. Most prior work concentrate on individually modeling successful diffusion cascades, while discarding smaller cascades which, however, account for a majority proportion of the available data. In this work, we propose a set of tools to leverage information in the small cascades: a joint fitting procedure that accounts for cascade size bias in the sample, a Borel mixture model and a clustering algorithm to uncover latent groups within these cascades, and the posterior final size distribution of Hawkes processes. On a dataset of Twitter cascades, we show that, compared to the state-of-art models, the proposed method improves the generalization performance on unseen data, delivers better prediction for final popularity and provides means to characterize online content from the way Twitter users discuss about it.

1 Introduction

User-generated online information in the forms of posts, videos and images today stimulate widespread discussion within or across online social media platforms such as Twitter and Youtube. Among the broad classes of models successfully applied to understand the popularity of such online information [Jin et al., 2013; Zhang et al., 2013], a class of point process based models — dubbed the Hawkes processes — has seen increasing attention [Zhao et al., 2015; Kong et al., 2020]. Such processes learn from the temporal patterns of sharing events and from fine-grained features of online diffusions to produce explainable parameters to quantify and predict popularity. Most modeling efforts are generally aimed towards “popular” diffusions, whereas unpopular ones are usually discarded — Zhao et al. [2015] only study cascades with at least 50 retweets, and Mishra et al. [2016] threshold at 20 retweets —, the goal being to learn what makes a popular diffusion. Given that Cheng et al. [2014] and later Rizoiu et al. [2018] have shown that luck is an important factor in online popularity, and that the discarded diffusions make up for a large portion of the available data (more than 40\% events as seen in Section 6), this paper aims to establish a procedure to leverage the temporal information in “unpopular” diffusions by jointly modeling these with Hawkes processes.

Specifically, in this work, we address three open questions concerning online information diffusion modeling. Cheng et al. [2014] have shown that the final popularity, i.e., total number of events, of retweet diffusions is unpredictable due to uncertainties contributed by various factors. Recent work from [Rizoiu et al., 2018] seconds this conclusion with distributions of final popularity where the same information has high probabilities of both remaining unknown or getting extremely popular. The popular diffusions are the “lucky” ones, and the first open question is: \textbf{can we gain knowledge about popularity of online information from short diffusions?} Here, we first jointly model multiple diffusion sequences using a single Hawkes process by summing up their log-likelihood functions, and we show that this introduces bias in parameter estimation due to the non-representativity of the sample as only small cascades are observed. Next, we show how the parameter bias can be adjusted by accounting for the distribution of final event counts of Hawkes processes [Daw and Pender, 2018]. We also show that fitting short diffusions enjoys a better time efficiency.

The aforementioned joint modeling assumes that the observed diffusions are generated by the same process, however in real-life situations multiple latent processes are simultaneously generating cascades. The second challenge emerges — \textbf{how do we both heuristically and systematically uncover diffusions coming from same models, and how to simultaneously learn the models parameters?} The heuristic method is with regard to the information content, e.g., grouping retweet diffusions about the same video in Twitter. We design a Borel mixture model and a clustering algorithm to systematically regroup similar cascades, while simultaneously learning the model parameters.

After learning the models from short diffusions, the next question is: \textbf{what are the tools we can develop to explore the uncertainty of predicting popularity.} While fitted parameters or derived quantities are commonly used for analyzing content popularity [Rizoiu et al., 2017], we show that we can characterize the virality of online content based solely on how people discuss about it. Moreover, we present tools for predicting final popularity along with its uncertainty.

The main contributions of this work are:

- We refine the likelihood function of Hawkes processes
to jointly and correctly model size-biased diffusions.

- We design procedures to uncover latent clusters through heuristics and algorithms. Specifically, we apply a mixture model and the k-means on fitting branching factors and kernel functions, respectively. We also study the quantification of popularity with parameters fitted via the procedures on real data.

- On a real-world Twitter diffusion dataset, we show that better generalization performances are achieved on holdout proportions by learning from short diffusions compared to benchmarks trained on individual popular diffusions. We then show an improved early prediction of final popularities.

- We construct the ActiveRT2017 Twitter cascade dataset.

Related work. Generative models are commonly employed for modeling temporal diffusions of online information. Such models are designed to predict final popularities [Zhao et al., 2015; Samanta et al., 2017], uncover hidden diffusion networks [Gomez-Rodriguez et al., 2011] and detect rumors [Ma et al., 2016]. The same tasks were also approached using feature-driven models, which train machine learning algorithms that use temporal features — statistical summaries of temporal patterns — together with user features and content features [Bakshy et al., 2011; Martin et al., 2016]. However, to our knowledge, most of the prior work concentrate on large (popular) cascades, and the complete temporal information of the unpopular diffusions is rarely considered.

Hawkes processes [Hawkes and Oakes, 1974] are a class of self-exciting point processes — past events excites future events happening — widely applied in analyzing social media [Kobayashi and Lambiotte, 2016; Lukasik et al., 2016; Farajtabar et al., 2015], earthquake aftershocks [Ogata, 1988], crime rate [Mohler and others, 2013], invasive species [Gupta et al., 2018], energy consumption [Li and Zha, 2016] and finance [Bacry et al., 2015]. Event-level and sequence-level clusterings of Hawkes processes have been discussed in [Du et al., 2015] and [Yang and Zha, 2013; Xu and Zha, 2017], and particular attention has been given to the inference of Hawkes processes [Yan et al., 2018; Guo et al., 2018; Liu et al., 2018]. The present work extends the prior literature in several ways. First, we propose a joint inference procedure which accounts for length-biased diffusions — i.e. observing short cascades only. Second, we propose a Borel Mixture Model and an efficient clustering procedure that regroups similar diffusions together.

2 Preliminaries

In this section, we first define our data objects: the diffusion cascades. Next, we introduce the Hawkes processes, together with essential concepts including its cluster representations, branching factor, size distribution and likelihood functions.

Diffusion cascades. In online social media platforms, such as Twitter, users read contents posted by others, and they can share/retweet, exposing the contents to broader audience. This diffusion usually continues until the content shifts away from the users’ attention. The initial posting event and the following share/retweet events together constitute a diffusion cascade. Mathematically, we denote a cascade $i$ as $\mathcal{H}_i = \{t_0, t_1, t_2, \ldots, t_{N_i-1}\}$ where $N_i \geq 1$ is a random event number count, $\forall t_j \in \mathcal{H}_i$ are random event times on $(0, \infty)$ relative to $t_0$ and $t_0 = 0$ is the initial event time. Let $\mathcal{H}_i(t), N_i(t)$ represent the event set and the event count before time $T$, respectively, i.e., $\mathcal{H}_i(t) = \{t_j \mid t_j \in \mathcal{H}_i, t_j < T\}$ and $N_i(T) = |\mathcal{H}_i(T)|$. The popularity of the online content associated with the cascade $i$ is defined by $N_i$ its event count.

Hawkes processes are particular classes of self-exciting processes — in which the occurrence of new events will increase the likelihood of future event happening [Hawkes, 1971]. In Hawkes processes, the event intensity is a function conditioned on the past occurred events:

$$\lambda(t \mid \mathcal{H}_i(t)) = \mu + \sum_{t_j \in \mathcal{H}_i(t)} n^* g(t - t_j)$$

where $\mu$ is the background event rate, $n^*$ is known as the branching factor and $g : R^+ \rightarrow R^+$ is a memory kernel encoding the time-decaying influence of past events on future events. Note that for information cascades (such as retweet cascades on Twitter) there is no background intensity, as all the retweets are considered to be spawning from the original tweet. Therefore, $\mu = 0$, and $\int_0^\infty g(\tau) d\tau = 1$. Common choices of the memory kernels include the exponential kernel function [Xu et al., 2016], $g_{\text{exp}}(\tau) = \theta e^{-\theta \tau}$, the power-law kernel [Mishra et al., 2016], $g_{\text{pl}}(\tau) = \theta (\tau + c)^{-1(\tau)}$, among others (see [Kong et al., 2020] for a review of kernels used with diffusion cascades).

Cluster representation and size distribution. An alternate representation of the Hawkes self-exciting process is that of a latent cluster of Poisson processes, introduced by Hawkes and Oakes [1974]. Fig. 1 depicts the cluster representation of an example Hawkes process, with highlighted parent-offspring relation, and the event counts at each generation form a branching process, i.e., $\{Z_0, Z_1, Z_2, \ldots\}$.

![Cluster representation of a Hawkes process](image-url)
functions of independent parameters: 

$$\mathbb{B}(n^* | \kappa^*) = \mathbb{P}[N = \kappa | n^*] = \frac{(\kappa n^*)^{\kappa-1} e^{-\kappa n^*}}{\kappa!}$$

(2)

This analytical form for the Hawkes process size distribution has not been discussed until recently [Daw and Pender, 2018]. Eq. (2) holds for branching factors bounded by $n^* \leq 1$.

**Parameter estimation.** The parameters of a Hawkes process can be estimated by maximizing the likelihood function of a general point process [Daley and Vere-Jones, 2008]:

$$L(\Theta | \mathcal{H}(T)) = e^{-\int_{\mathcal{H}(T)} \lambda(r) dr} \prod_{t_j \in \mathcal{H}(T)} \lambda(t_j)$$

(3)

in which $\lambda(\cdot)$ is the intensity function defined in Eq. (1).

## 3 Joint Fitting of Hawkes Cascades

In this section, we propose a method to jointly learn a single set of parameters from a collection of Hawkes realizations biased in terms of event count. We first discuss the estimation bias when modeling on size-biased realizations, and next we propose a modified likelihood function to eliminate such bias.

**Joint likelihood function.** Let $\mathcal{H}^r = \{\mathcal{H}_1, \mathcal{H}_2, \ldots\}$ be a representative set of independent Hawkes realizations, assumed to be generated from the same model $\Theta$ and without any post-generation filtering applied. It is then straightforward to estimate $\Theta$ by maximizing the joint likelihood $L^r(\Theta | \mathcal{H})$ defined as the sum of the individual log-likelihoods (i.e., the log of Eq. (3)):

$$L^r(\Theta | \mathcal{H}) = \sum_{\mathcal{H}_i \in \mathcal{H}^r} \log L(\Theta | \mathcal{H}_i)$$

(4)

This is asymptotically equivalent to minimizing the Kullback-Leibler (KL) divergence between the underlying Hawkes process where cascades are sampled from and the theoretical distribution parameterized by $\Theta$.

**Jointly fitting a biased set.** Any filtering applied on the realizations post-generation — such as selecting realizations based on their final size — renders Eq. (4) non-applicable, and introduces systematic bias in parameter estimation (as shown empirically in our experiments in Section 6.1). Let $N^*$ be a set of positive integers defining selected realization sizes, and let $\mathcal{H}^b$ be the biased set of realizations of sizes in $N^*$, i.e. $\mathcal{H}^b = \{\mathcal{H}_i \in \mathcal{H}^r | N_i \in N^*\}$. Using Bayes theorem and the Borel distribution of Hawkes sizes, we compute the joint likelihood for the set $\mathcal{H}^b$:

$$L^b(\Theta | \mathcal{H}^b) = \sum_{\mathcal{H}_i \in \mathcal{H}^b} \log \frac{f(\mathcal{H}_i | \Theta)}{\mathbb{P}[N_i \in N^* | \Theta]}$$

$$= \sum_{\mathcal{H}_i \in \mathcal{H}^b} \log \frac{L(\Theta | \mathcal{H}_i)}{\sum_{j \in N^*} \mathbb{B}(j | \Theta)}$$

(5)

where $f(\mathcal{H}_i | \Theta)$ is the probability density of realization $i$ under model parameters $\Theta$, therefore $f(\mathcal{H}_i | \Theta) = L(\Theta | \mathcal{H}_i)$. Finally, we plug Eq. (3) into Eq. (5) and we see that the joint likelihood function can be rearranged as a sum of two functions of independent parameters:

$$L^b(\Theta | \mathcal{H}^b) = L_g(\Theta_g | \mathcal{H}^b) + L_n(n^* | \mathcal{H}^b)$$

(6)

$L_g$ is a function of $\Theta_g$ — the parameter set of $g(\cdot)$ — and $\Theta$ is a function of $n^*$ the branching factor, and $\Theta = \Theta_g \cup \{n^*\}$.

$$L_g(\Theta_g | \mathcal{H}^b) = \sum_{\mathcal{H}_i \in \mathcal{H}^b} \sum_{t_j \in \mathcal{H}_i, j \geq 1} \log g(t_j - t_k | \Theta_g)$$

(7)

$$L_n(n^* | \mathcal{H}^b) = \sum_{\mathcal{H}_i \in \mathcal{H}^b} \log \left(\frac{n_i}{N_i - 1} e^{-N_i n^*}\right)$$

(8)

The above results indicate that $\Theta_g$ and $n^*$ can be learned independently in two separate phases, by maximizing $L_g$ and $L_n$. This amounts to fitting $n^*$ from observed final realization sizes only, and $\Theta_g$ from inter-arrival times between events.

## 4 Uncovering Clusters of Hawkes Models

In practice, it is often unknown which realizations were generated from the same model parameters. In this section, we examine several strategies to construct clusters of diffusion cascades. Finally, we introduce a Borel mixture model (BMM) and a modified k-means algorithm to automatically discover clusters based on cascade sizes and time intervals.

**Heuristic grouping.** For diffusion cascades relating to online content, one natural grouping is based on the explicit features of the content. For instance, in the ActiveRT dataset [Rizoiu et al., 2018], each cascade records a retweet event series relating to a Youtube video, so one could group together cascades about the same video. Another example would be grouping cascades that are initiated by the same users. On the up side, the heuristic grouping builds content-related models depending on the grouping criterion — i.e., models describing the online videos or Twitter users — in addition to describing generated cascades. On the flip side, not all cascades relating to a video or a user might be generated by the same process, and they might in reality not share the same parameters.

**Algorithmic grouping.** We are given a set of cascades $\mathcal{H}^b$ with a known cascade size filtering condition $N^*$, and $K$ latent generative models with an unknown relation to the cascades in $\mathcal{H}^b$. We seek to learn the $K$ values of $n^*$ and $\Theta_g$, and the membership of each cascade to the models (denoted as clusters). As indicated in Section 3, we cluster $n^*$ and $g(\cdot)$ separately.

We model the $n^*$ for each cascade using a mixture model of Borel distributions (BMM), and we present an efficient EM estimation algorithm. A BMM can be fitted on $\mathcal{H}^b$ by maximizing the likelihood

$$L_{BMM} = \sum_{\mathcal{H}_i \in \mathcal{H}^b} \log \sum_{k=1}^{K} \frac{p_k \mathbb{B}(N_i | n_k^*)}{\sum_{j \in N^*} \mathbb{B}(j | n_k^*)}$$

(9)

where $N_i = | \mathcal{H}_i |$, and $p_k$ denotes the mixture probability of the $k$th cluster parameterized by $n_k^*$. As maximizing Eq. (9) directly suffers from the identifiability [Bishop, 2006], we apply the Expectation-Maximization (EM) algorithm commonly used for learning mixture models. Next we give the update formulas for the E and M steps, the detailed derivations can be found in [supplement, 2020].
Update membership probabilities (E-step): The probability of $N_i$ being a member of $k$ is defined as

$$p(k \mid N_i) = \frac{q(k, N_i)}{\sum_{j=1}^{K} q(j, N_i)} \quad (10)$$

Update $n_k^*$ and $p_k$ (M-step): When $N^*$ is the natural number set, namely there is no filtering, $n_k^*$ is updated analytically, and the update formulas for $n_k^*$ and $p_k$ are

$$(n_k^*)_{new} = \sum_{N_i} p(k \mid N_i)(N_i - 1), (p_k)_{new} = \frac{\sum_{N_i} p(k \mid N_i)}{|N|}$$

When filtering is imposed, $n_k^*$ can be efficiently solved by numerically finding roots of the simplified first partial derivative of Eq. (9) w.r.t. $n_k^*$.

For finding $g_k(t)$ kernel functions, we employ a K-means like procedure in which $\Theta_g$ serves as the equivalent of centroids. We start by randomly assigning cascades to clusters, and we use maximize Eq. (7) for each cluster to recompute its centroid. Cluster reassignment is performed by selecting for each cascade the cluster whose centroid ($\Theta_g$) maximizes its likelihood function (Eq. (3)).

As the branching factors $n_k^*$ and the kernel functions parameters $\Theta_g$ are inferred separately, there is no exact matched pair of parameters between them — two cascades might have the same $n^*$ but different $\Theta_g$, or the other way around.

5 Prediction for Partial Observations

In this section, we first describe how fitted parameters are chosen for newly observed sequences, based on previously trained clusters of Hawkes models. Next, we then derive predictions for their popularities — i.e., the final cascade size.

Parameter selection. Given a partially observed event sequence $H_i(T)$ where $T$ is the observation time, we select for it a parameter set $\Theta_g$ and a branching factor $n^*$ from the candidates constructed using the clustering procedure. We first choose the kernel function parameters that maximize its $L_n$ likelihood function (Eq. (7)), denoted as $\hat{\Theta}_g$. We then use $\hat{\Theta}_g$ to select the best $n_k^*$ that maximizes the probability

$$P[K = k \mid H_i(T)] = \frac{p_k L(n_k^*; \hat{\Theta}_g) \mid H_i(T)}{\sum_{j=1}^{K} p_j L(n_j^*; \hat{\Theta}_g) \mid H_i(T)} \quad (11)$$

When $T = 0$, however, the kernel function cannot be identified and we only maximize the $L_n$ function (Eq. (8)).

Posterior size distribution. To be able to make prediction using the chosen parameters, it is desirable to derive the posterior size distribution given $H_i(T)$. The future events after time $T$ are of two kinds: direct offspring of observed events (their count denoted as $N_d^n$) and indirect offspring (children of children, total count denoted as $N_i^{ind}$). The process generating direct offspring is an inhomogeneous Poisson process of conditional intensity $\lambda(t) H_i(T), t > T$ — note that this is not a stochastic function as only the history up to time $T$ is accounted in the intensity function. Consequently, $N_d^n$ follows a Poisson distribution of parameter $\Lambda_i(T) = \int_T^\infty \lambda(t) H_i(T) dt$. Furthermore, each direct offspring initiated a Hawkes process and its total progeny number follows a Borel distribution. Given the number of direct offspring $N_d^n$, the total number of direct and indirect offspring follows a Borel-Tanner distribution (also known as the generalized Borel distribution):

$$B(k \mid n^*, N_d^n) = \frac{(kn^*)^{N_d^n} e^{kn^*}}{k^{N_d^n} N_d^n!} \quad (12)$$

The proof of this is straightforward and leverages the general hitting time theorem [Van der Hofstad and Keane, 2008], which can be found in [supplement, 2020].

Finally, the posterior cascade size distribution is therefore

$$P[N_i = n \mid H_i(T)] = N_i(T)$$

$$+ \sum_{z=0}^{n-N_i(T)} Poi(z \mid \Lambda_i(T)) B(n - z - N_i(T) \mid n^*, z)$$

where $Poi(\cdot)$ is the Poisson distribution. Eq. (13) leads to a quadratic complexity in computing the final size distribution, which is intractable in most real-life scenarios. We apply a numerical trick to reduce the complexity by introducing a threshold probability $\epsilon_p$ and summing until $Poi(z \mid \Lambda_i(T)) < \epsilon_p$.

Prediction. The size of real-life diffusion cascades is mechanically limited by the available population and the span of human attention. Therefore, it is logical impose an upper bound on the cascade size $n_{max}$, leading to $P[N_i = n \mid H_i(T), n \leq n_{max}]$. In particular, for reweet cascades, $n_{max}$ can be estimated by the cumulative number of users exposed by the online item (computed as the sum of the followers of the users that were observed reweeting). Given the distribution, one is able to compute the expected final event count, its variance and the probability of a cascade diffusing beyond certain sizes. It is worth noting that when $n_{max} = \infty$, the expected final event count has an analytical solution as applied in [Zhao et al., 2015; Mishra et al., 2016].

6 Experiments

This section provides evaluation results of the proposed modeling procedures on both synthetic data and real data. On synthetic data, we assess the effect of different cascade size filtering conditions. We then present a reweet cascade dataset together with some measurements on it. Finally, we conduct experiments evaluating model generalization on unseen data and final popularity prediction performance compared to the state-of-art models [Zhao et al., 2015; Mishra et al., 2016].
Table 1: Chi-square tests of BMM fitted on size-biased cascades with various filtering conditions. Each cell shows the percentage of tests passing 0.05 significance level.

| #cluster | $N^{max} = 20$ | $N^{max} = 30$ | $N^{max} = 40$ | $N^{max} = 50$ | All sizes |
|---------|----------------|----------------|----------------|----------------|-----------|
| 1       | 90.1%          | 92.4%          | 93.7%          | 95.3%          | 96.1%     |
| 2       | 82%            | 89.7%          | 93.7%          | 94.7%          | 98.2%     |
| 3       | 80.5%          | 88.5%          | 93.9%          | 94.7%          | 98.8%     |

Table 2: Purity tests of the power law kernel function clustering experiments with various filtering condition. Each cell shows the mean purity measures and the standard deviation.

| #cluster | $N^{max} = 20$ | $N^{max} = 30$ | $N^{max} = 40$ | $N^{max} = 50$ | All sizes |
|---------|----------------|----------------|----------------|----------------|-----------|
| 2       | 88.5 ± 11.4%   | 91.8 ± 9.3%    | 92.1 ± 9.5%    | 92.2 ± 8.6%    | 93.0 ± 8.0%|
| 3       | 76.5 ± 11.2%   | 80.5 ± 9.9%    | 79.3 ± 10.9%   | 80.9 ± 10.3%   | 83.6 ± 9.6%|

Figure 3: Profiling the ActiveRT2017 dataset: (a) the empirical complementary cumulative density (CCDF) of cascade sizes in the dataset; (b) the percentages of events filtered when imposing various mininum cascade sizes. The two points highlight the event loss as a minimum threshold of 20 and 50 events, introduced in [Mishra et al., 2016] and [Zhao et al., 2015], respectively.

6.1 Synthetic experiments

Bias from joint fitting. Given various $n^*$ values, we simulate 100 cascades for each. On simulated cascades, we fit Hawkes processes with three different settings: fit on all cascades with Eq. (4), and fit on cascades filtered at the event size 20 with Eq. (4) and Eq. (5). The absolute bias of fitted $n^*$ is reported along with the average times the fitting takes. Fig. 3 shows the bias in learning $n^*$ values when fitting on size-biased cascades with Eq. (4) and the correctness of using Eq. (5) to adjust this bias. It also highlights the efficiency when learning on small cascades. As the computational complexity of Eq. (4) is quadratic to cascade sizes, Eq. (5) allows one to bound the complexity to a maximum size thus scaling to more cascades.

BMM goodness-of-fit. We train BMMs on different size-biasing conditions imposed on simulated cascades and examine goodnesses of the fitted BMM on complete cascade data by conducting Chi-square tests between the empirical and learnt size distributions. For each experiment, branching factors are randomly sampled for clusters and used to simulate to the same number of cascades for each cluster summing to 1000 cascades in total. This is repeated for 1000 times.

Table 1 shows the proportion of repeated experiments that pass the tests at a 0.05 significance level. We tabulate the tests against two dimensions: the number of clusters — up to 3 clusters — and cascade size filters — cascades with sizes less than or equal to $N^{max}$ are kept where $N^{max} ∈ \{20, 30, 40, 50\}$ or $N^{max} → \infty$. The passing rates decrease as less cascades and more clusters are provided. However, we can see that, overall, BMMs fitted only on short diffusions can still generalize well to the whole dataset.

Kernel clustering. We measure the correctness of clustering on kernels and power law kernels are used. During each experiment, the sampled kernel parameters are enforced to have absolute difference 1 between cluster parameters to ensure the clusters are distinguishable. Other setup follows the BMM experiments and the clustering purity values are reported in Table 2. The same observation of performance increase during the growth of $N^{max}$ and cluster numbers is presented.

6.2 Jointly modeling diffusions on Twitter

Dataset. Retweet cascade datasets provided in prior works typically have filters on cascade sizes. In order to obtain a complete set of cascades, we produce the ActiveRT2017 dataset crawled via Twitter public APIs through the entire 2017. In this dataset, all tweets are related to Youtube videos whose video ids are found in tweet contents. Selected videos are published by the active Youtube channels where each video has the maximum cascade size larger than 50 and associated to at least 500 cascades. The definition of active videos can be found in [Rizoiu et al., 2017] and video meta data is collected using [Wu et al., 2018]. In total, there are about 110k videos and 45 million cascades. Fig. 3a presents a statistical summary of cascade sizes, in which the CCDF shows a long tail distribution of cascade sizes. To quantify the data loss due to cascade size filters, Fig. 3b presents the proportions of events (tweets) being lost when filtering out cascades smaller than certain sizes. We note that, in our experiments, we adopt the filter applied in [Zhao et al., 2015] at cascade size 50 to distinguish short diffusions, accounting for about 46% of the total events.

Pre-learn from short diffusions. In experiments, we conduct cascade joint fitting on the diffusions happened at the early part of 2017, and we then compare it to the state-of-art models on the cascades initiated later in 2017. While methods in Section 3 requires terminated cascades, we assume all cascades in our dataset have stopped diffusing given the data collection time. Specifically, within ActiveRT2017 dataset, cascades have sizes less than 50 and stopped earlier than 1st of May in 2017 are selected, resulting 12,690,817 cascades.
We evaluate the improvement of model generalization performances by leveraging pre-learnt parameters from short diffusions. Given the trained models for each video, the generalization performance is evaluated on all cascades related to the same video with more than 50 events (33,023 cascades). With a cascade $H_i$ and an observation time $T$, we first obtain the cluster parameters through its video id and parameter selections are conducted on the observed part $H_i(T)$ following Section 5. We apply this setting to Hawkes processes with an exponential kernel function (Joint EXP) and a power-law kernel function (Joint PL), comparing to their counterparts fitted only on $H_i(T)$ ([Hawkes, 1971] and [Mishra et al., 2016]). We report the negative log-likelihood values normalized by event counts on the remaining part, i.e., $L(\theta \mid H_i(T)) - L(\theta \mid H_i)$.

Fig. 5 gives the generalization performances as boxplots with mean values. At individual observation times, the direct comparison between models with pre-learnt parameters on short diffusions consistently outperform models trained on given observations. This highlights the improvements from the proposed fitting approach especially for the exponential kernel function where a large enhancement is shown. We also note that models with power law kernels are better than those with exponential kernels which reinforces the known conclusion from prior works [Mishra et al., 2016]. When observation times are concerned, we can see that the advantage of applying the pre-learnt parameters diminishes as time lengths increase. This provides a hint for our proposed pre-learning procedure to handle the early-start modeling.

7 Conclusion

Overall, this work is concerned with the way how short cascades are handled for modeling information diffusions with Hawkes processes. Instead of filtering, we propose to jointly pre-train Hawkes processes on cascades from same groups. We first adjust the Hawkes likelihood function to correctly fit on size-biased cascades by leveraging an analytical diffusion size distribution. To group cascades in real data, apart from applying simple heuristics, we future propose the procedures to automatically identify groups from a collection of cascades which we validate by conducting experiments on synthetic data. On a retweet cascade dataset, we analyze fitted models as indications of content virality. We also measure the improvement on models augmented with our pre-trained parameters and compare to the state-of-art generative model on predicting final popularities.

Limitations and future work. Due to the restriction of the size distribution of Hawkes processes, the current joint fitting on size-biased cascades is restricted to complete and un-marked processes. We plan to relax these constraints to allow for joint modeling with more flexibility.
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A Borel mixture model for branching factors

As the final cascade size distribution of Hawkes processes is only determined by the branching factor (Section 2), i.e. the Borel distribution, we are able to model sizes of a group of cascades as a Borel mixture distribution. Specifically, given a filtered cascade set $H^b$ and a cluster number $K$, we aim to find for each Borel distribution, a mixture probability $p_k = \mathbb{P}[K = k]$ and a branching factor $n^*_k$. We denote this parameter set as $\Theta_{BMM} = \{p_1, \ldots, p_k, n^*_1, \ldots, n^*_k\}$. The parameters are estimated via the EM algorithm following follows [Tomasi, 2004]. The log likelihood function is

$$L_{BMM} = \sum_{H_i \in H^b} \log \sum_{k=1}^K p_k \sum_{N_j \in N^*, H} \mathbb{B}(N_i \mid n^*_k)$$

(14)

For simplicity, let $q(k, N_i) = p_k \sum_{N_j \in N^*, H} \mathbb{B}(N_j \mid n^*_k)$. We first introduce the probability of $H_i$ being a member of $k$:

$$p(k \mid N_i) = \frac{q(k, N_i)}{\sum_{j=1}^K q(j, N_i)}$$

(15)

By employing Jensen’s inequality, we get

$$L_{BMM} = \sum_{H_i \in H^b} \log \sum_{k=1}^K q(k, N_i)$$

(16)

$$= \sum_{H_i \in H^b} \log \sum_{k=1}^K p(k \mid N_i) \frac{q(k, N_i)}{p(k \mid N_i)}$$

(17)

$$\geq \sum_{H_i \in H^b} \sum_{k=1}^K p(k \mid N_i) \log \frac{q(k, N_i)}{p(k \mid N_i)}$$

(18)

Optimizing Eq. (18) is equivalent to optimizing the following $Q_{BMM}$ function

$$Q_{BMM} = \sum_{H_i \in H^b} \sum_{k=1}^K p(k \mid N_i) \log q(k, N_i)$$

(19)

At the Maximization step, the parameters are updated by maximizing $Q_{BMM}$.

- For updating $n^*_k$, we take the derivative of $Q_{BMM}$ w.r.t. $n^*_k$

$$\frac{\partial Q_{BMM}}{\partial n^*_k} = \sum_{H_i \in H^b} \frac{\partial}{\partial n^*_k} \sum_{k=1}^K p(k \mid N_i) \log q(k, N_i)$$

(20)

$$= \sum_{H_i \in H^b} p(k \mid N_i) \frac{\partial}{\partial n^*_k} \log q(k, N_i)$$

(21)

$$= \sum_{H_i \in H^b} p(k \mid N_i) \frac{\partial}{\partial n^*_k} \log \mathbb{B}(N_i \mid n^*_k) - \frac{\partial}{\partial n^*_k} \log \sum_{N_j \in N^*, H} \mathbb{B}(N_j \mid n^*_k)$$

(22)

$$= \sum_{H_i \in H^b} p(k \mid N_i) \left[ \frac{\partial}{\partial n^*_k} \mathbb{B}(N_i \mid n^*_k) - \frac{\partial}{\partial n^*_k} \mathbb{B}(N_j \mid n^*_k) \right]$$

(23)

$$= \sum_{H_i \in H^b} p(k \mid N_i) \left[ \frac{\partial}{\partial n^*_k} \mathbb{B}(N_i \mid n^*_k) - \frac{\sum_{N_j \in N^*, H} \partial}{\partial n^*_k} \mathbb{B}(N_j \mid n^*_k) \right]$$

(24)
we note that \( \frac{\partial \mathbb{B}(N_i | n_k^*)}{\partial n_k^*} \) has a special solution

\[
\frac{\partial \mathbb{B}(N_i | n_k^*)}{\partial n_k^*} = \frac{\partial}{\partial n_k^*} \left[ \frac{(N_in_k^*)^{N_i-1} e^{-N_i n_k^*}}{N_i!} \right]
= \frac{N_i(N_i - 1)(N_i n_k^*)^{N_i-2} e^{-N_i n_k^*} - N_i (N_i n_k^*)^{N_i-1} e^{-N_i n_k^*}}{N_i!}
= \frac{N_i^{-1} (N_i n_k^*)^{N_i-1} e^{-N_i n_k^*} - N_i (N_i n_k^*)^{N_i-1} e^{-N_i n_k^*}}{N_i!}
= \frac{N_i - N_i n_k^*-1}{n_k^*} \mathbb{B}(N_i | n_k^*)
\]

Plugging this result back to Eq. (24)

\[
\frac{\partial Q_{BMM}}{\partial n_k^*} = \sum_{H_i \in H^b} p(k | N_i) \left[ \frac{N_i - N_i n_k^* - 1}{n_k^*} - \frac{\sum_{N_j \in N^*} N_j - N_i n_k^* - 1 \mathbb{B}(N_j | n_k^*)}{\sum_{N_j \in N^*} \mathbb{B}(N_j | n_k^*)} \right]
\]

Let the derivative be 0 will lead to the equation

\[
\sum_{H_i \in H^b} p(k | N_i) \sum_{N_j \in N^*} (N_i - N_j)(1 - n_k^*) \mathbb{B}(N_j | n_k^*) = 0
\]

Although there is no clean analytical solution available to this equation, numerical roots can still be efficiently found within \( n_k^* \in [0, 1] \) which give the optimal \( n_k^* \), i.e., \( (n_k^*)^{\text{new}} \). Specifically, we note that when there is no filtering imposed on \( H \), an analytical solution exists,

\[
(n_k^*)^{\text{new}} = \frac{\sum_{H_i \in H^b} p(k | N_i)(N_i - 1)}{\sum_{H_i \in H^b} p(k | N_i)N_i}
\]

- Updating \( p_k \) shares same derivation steps from [Tomasi, 2004]

\[
p_k^{\text{new}} = \frac{\sum_{H_i \in H^b} p(k | N_i)}{|H^b|}
\]

Because final sizes of Hawkes processes are highly skewed towards small sizes, the estimation complexity can be reduced by counting the number of presences of various cascade sizes in \( H^b \), i.e., obtaining a set \( C' = \{ (c_i, N_i) \} \) where there are \( c_i \) cascades with size \( N_i \). The summation over \( H^b \) can be then replaced by this set for efficiency.

**Appendix References**

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