Water Distribution System Design Using Multi-Objective Genetic Algorithm with External Archive and Local Search

Mahesh B. Patil, M. Naveen Naidu, A. Vasan, and Murari R. R. Varma

1Department of Electrical Engineering, Indian Institute of Technology Bombay
2Department of Civil Engineering, BITS Pilani Hyderabad Campus

May 21, 2019

Abstract

Hybridisation of the multi-objective optimisation algorithm NSGA-II and local search is proposed for water distribution system design. Results obtained with the proposed algorithm are presented for four medium-size water networks taken from the literature. Local search is found to be beneficial for one of the networks in terms of finding new solutions not reported earlier. It is also shown that simply using an external archive to save all non-dominated solutions visited by the population, even without local search, leads to substantial improvement in the non-dominated set produced by the algorithm.

1 Introduction

Optimisation of water distribution systems (WDS) for the dual objectives of minimising cost and maximising network resilience is a challenging problem because of the large solution spaces involved (see [1], [2] and references therein). In this context, the benchmark water network problems made available by Wang et al. [1] have served as an excellent resource for researchers trying out new optimisation algorithms. Recently, hybridisation of local search and the multi-objective particle swarm optimisation algorithm (MOPSO) [2] was shown to be very effective [3] for the two-objective WDS design problem.

Table 1 presents a summary of the performance of this new “MOPSO+” algorithm [4] for the four medium-size water networks given in [1]. The table compares the sets of non-dominated (ND) solutions (loosely called “Pareto fronts” or PFs) by two algorithms. Algorithm 1 (called “UExeter”) is a combination of five multi-objective evolutionary algorithms (MOEAs) presented in [1], whereas Algorithm 2 is the MOPSO+ algorithm of [4]. $N^u_1$ is the total number of ND solutions obtained by algorithm 1 of which $N^a_1$ are accepted and $N^r_1$ are rejected (since they got dominated by some of the ND solutions given by algorithm 2). The number of unique solutions given by algorithm 1, i.e., solutions which could not be obtained by algorithm 2, is denoted by $N^u_2$, and the number of common solutions between the two algorithms by $N^c$. The total number of function evaluations over all independent runs of the concerned algorithm is denoted by $N_{FE}$. As seen from the table, $N^u_2$ is nearly zero in all cases which means that the MOPSO+ algorithm has covered all solutions given by algorithm 1. Furthermore, $N^c$ is significantly large, which means that algorithm 2 has produced many solutions which were not present in the ND set obtained by algorithm 1. Comparing the $N_{FE}$ values, we see that the computational efforts for the two algorithms are similar. In summary, the MOPSO+ algorithm has performed better without requiring a significantly larger computational effort.

The above beneficial hybridisation of local search with the MOPSO algorithm opens up the interesting possibility of improving the performance of other MOEAs using local search. It is the purpose of this paper to explore the effectiveness of local search when hybridised with another commonly used MOEA, viz., the NSGA-II algorithm [5], for the WDS design problem described in [1]. The paper is organised as follows. In Sec. 2 we describe the modifications of the basic NSGA-II algorithm to combine it with local search. In Sec. 3 we present results obtained with the different schemes of Sec. 2 for the four medium-size water networks described in [1]. Finally, we present the conclusions of this study in Sec. 4.

2 NSGA-II with local search

In the MOPSO+ scheme mentioned earlier, the current ND solutions are stored in an archive (usually referred to as “external archive” in the literature); local search (LS) is performed at regular intervals, and new ND solutions resulting from LS are added to the archive. One of the solutions in the archive is designated as the global leader using Roulette-wheel selection, favouring solutions in the least crowded regions of the archive. The position of the global leader affects the velocity of particles in the
In this work, we explore the effectiveness of local search when combined with one of the industry-standard MOEAs, viz., the NSGA-II algorithm [5], for WDS optimisation. In the following, we describe how various features can be added in a step-by-step manner to the NSGA-II algorithm to finally incorporate local search into the algorithm. The intermediate algorithms introduced in this process can also be used as stand-alone algorithms for WDS optimisation.

(A) NSGA-II: This is the real-coded NSGA-II algorithm [5], modified suitably for the WDS problem. The variables take on integer values corresponding to the indices for pipe diameters, but they are treated as real (continuous) variables. In the function evaluation step, each of them is converted to the nearest integer, following [1]. The algorithm parameters $p_c$ and $\eta_c$ are related to crossover, and $p_m$ and $\eta_m$ to mutation [5]. We will denote the population size by $N$, number of generations for a specific run by $N_{\text{gen}}$, number of independent runs by $N_r$, and the number of real parameters (same as the number of pipes in the WDS problem) by $N_{\text{real}}$. Note that, in each independent run, up to $N$ non-dominated solutions are produced by NSGA-II, and the final ND set is obtained by combining the ND sets given by the $N_r$ independent runs.

(B) NSGA-II with external archive: In this scheme [6], an external archive is used to store ND solutions. The solutions stored in the archive do not participate in the evolution of the population in any way; the archive is used purely as a storage mechanism. A “fixed hypergrid” without boundaries [6], which provides a memory-efficient implementation, is used as the external archive. In each generation, for each individual of the population not dominated by the solutions stored in the archive, a corresponding new solution is added to the archive, and any existing solutions in the archive which are dominated by this new solution are removed. There is no other interaction between the evolving population and the external archive. The hypergrid parameters [6] are selected so that the number of solutions in any hypercell remains smaller than the maximum allowed occupancy. This means that a current ND solution can get discarded during the evolution process only if it gets dominated by an incoming new solution, and not because of constraints on the hypergrid. All solutions in the external drive are written to a file at the end of a specific run. Note that the number of ND solutions in this case – even for a single independent run – can be larger than the population size, as demonstrated in [6] for several examples.

(C) NSGA-II with external archive and local search: This scheme is similar to scheme B except that local search is performed periodically (every $N_{\text{LS generations}}$ generations) around each solution stored currently in the external archive [4]. The archive is updated after the LS step by adding new ND solutions arising from LS and removing solutions which got dominated by the incoming solutions. Further details about implementation of local search for the WDS problem can be found in [4].

(D) NSGA-II with external archive, local search, and coupling: In the previous scheme, the external archive is (possibly) improved periodically by the local search process; however, that improvement does not get coupled to the individuals in the evolving population. The purpose of scheme D is to provide a way to couple (link) the external archive with the population. To this end, we use a mechanism similar to that described in [7]: Every $N_{\text{link generations}}$ generations, the child population is taken from the external archive using Roulette-wheel selection (favouring the least crowded regions of the archive) instead of using selection, crossover, and mutation. Through this mechanism, ND solutions in the archive can influence the evolution of the population.

Although our main interest in this paper is to compare the performance of algorithms A and D above, it is instructive to also consider algorithms B and C for WDS optimisation.

Table 1: Comparison of UExeter [1] and MOPSO+ [4] non-dominated solution sets (“PFs”) for four medium-size water networks.

| Network | UExeter (PF-1) | MOPSO+ (PF-2) | $N^c$ |
|---------|---------------|---------------|-------|
|         | $N_1^u$ | $N_1^a$ | $N_1^r$ | $N_1^{\text{net}}_{FE}$ | $N_2^u$ | $N_2^a$ | $N_2^r$ | $N_2^{\text{net}}_{FE}$ | $N_2^c$ |
| HAN     | 575   | 534   | 41    | 1    | 90 M  | 750   | 748   | 2    | 215  | 74.6 M | 533 |
| BLA     | 901   | 849   | 52    | 0    | 90 M  | 1045  | 1045  | 0    | 196  | 44.1 M | 849 |
| NYT     | 627   | 595   | 32    | 4    | 90 M  | 661   | 656   | 5    | 65   | 130.3 M | 591 |
| GOY     | 489   | 444   | 45    | 3    | 90 M  | 571   | 570   | 1    | 129  | 37.9 M | 441 |
Table 2: Comparison of PFs obtained in [1] and algorithm D for different values of $N_{\text{link}}$.

| Network | $N_{\text{link}}$ | UExeter (PF-1) | Scheme D (PF-2) | $N^c$ |
|---------|-----------------|----------------|----------------|------|
|         | $N_1^f$ | $N_2^f$ | $N_1^r$ | $N_2^r$ | $N_1^u$ | $N_2^u$ | $N_1^c$ | $N_2^c$ |
| HAN     | 1     | 575  | 547  | 28   | 44   | 692  | 659  | 33   | 156  | 503  |
|         | 10    | 575  | 547  | 28   | 4    | 707  | 702  | 5    | 159  | 543  |
|         | 50    | 575  | 545  | 30   | 4    | 713  | 706  | 7    | 165  | 541  |
|         | 100   | 575  | 538  | 37   | 3    | 713  | 708  | 5    | 173  | 535  |
| BLA     | 1     | 901  | 851  | 50   | 33   | 1023 | 1000 | 23   | 156  | 503  |
|         | 10    | 901  | 849  | 52   | 0    | 1040 | 1040 | 0    | 191  | 849  |
|         | 50    | 901  | 849  | 52   | 0    | 1036 | 1036 | 0    | 187  | 849  |
|         | 100   | 901  | 849  | 52   | 0    | 1040 | 1040 | 0    | 191  | 849  |
| NYT     | 1     | 627  | 591  | 36   | 30   | 643  | 631  | 12   | 70   | 561  |
|         | 10    | 627  | 591  | 36   | 22   | 648  | 640  | 8    | 71   | 569  |
|         | 50    | 627  | 591  | 36   | 22   | 647  | 640  | 7    | 71   | 569  |
|         | 100   | 627  | 591  | 36   | 22   | 648  | 640  | 8    | 71   | 569  |
| GOY     | 1     | 489  | 448  | 41   | 89   | 521  | 465  | 56   | 106  | 359  |
|         | 10    | 489  | 444  | 45   | 55   | 535  | 510  | 25   | 122  | 388  |
|         | 50    | 489  | 444  | 45   | 55   | 526  | 510  | 16   | 121  | 389  |
|         | 100   | 489  | 444  | 45   | 56   | 544  | 509  | 35   | 121  | 388  |

3 Results and discussion

We consider four medium-size problems described in [1], viz., the HAN, BLA, NYT, and GOY networks. For each of these, we employ algorithms A-D of Sec. 2. To compute the network resilience for a given network, we use the EPANET program [8] as in [1]. The NSGA-II algorithm parameter values, taken from [1], are $\eta_c = 15$ (distribution index for crossover), $\eta_m = 7$ (distribution index for mutation), $p_c = 0.9$ (crossover rate), $p_m = 1/N_{\text{real}}$ (mutation rate). Following [4], local search – applicable in algorithms C and D – is carried out more frequently in the beginning with $N_{\text{LS}} = 100$ from generation 1,000 to 5,000, and with $N_{\text{LS}} = 1$,000 thereafter. Coupling between the archive and the population – applicable in algorithm D – is implemented only after the first 1,000 generations.

Coupling between the archive and the population – applicable in algorithm D – is implemented only after the first 1,000 generations.

The selection of the population size $N$, number of independent runs $N_r$, and number of generations $n_{\text{gen}}$ was made after studying their effect of the ND set obtained for each network. For example, for the BLA network, with $N = 200$ and $n_{\text{gen}} = 15,000$, it was observed that increasing $N_r$ beyond 20 did not produce any improvement in the ND set, and it was therefore fixed at 20. The following parameter values were selected: (a) $N = 200$ for all networks, (b) $n_{\text{gen}} = 10,000$ for the HAN network and 15,000 for the other three, (c) $N_r = 20$ for the BLA network and 30 for the others. It should be mentioned that, although a more systematic selection of the above parameters is desirable, it is not expected to alter the conclusions of the present study significantly.

To assess the performance of any of the algorithms (A-D) of Sec. 2, we compare the ND set produced by that algorithm with the benchmark UExeter ND set [1]. First, we present the effect of the parameter $N_{\text{link}}$ of algorithm D in Table 2. This parameter determines the frequency of interaction between the evolving population and the archive. In the extreme case of $N_{\text{link}} = 1$, the child population in every generation is taken from the archive. From the table, we see that $N_{\text{link}} = 1$ generally gives poor results. For example, consider the HAN network. With $N_{\text{link}} = 1$, 44 of the benchmark solutions (the $N_1^u$ column) have not been covered by algorithm D whereas With $N_{\text{link}} = 10$, that number drops to 4. We notice also that, for the HAN network, increasing $N_{\text{link}}$ results in a larger number of unique solutions ($N_2^u$). However, in general, we see that $N_{\text{link}} = 10, 50, and 100$ give similar results. In the following, we use a fixed value $N_{\text{link}} = 100$.

The results obtained with algorithms A-D of Sec. 2 are summarised in Table 3. We can make the following...
Table 3: Comparison of PFs obtained in [1] and algorithms A-D.

| Network | Algorithm | UExeter (PF-1) | Algorithm A/B/C/D (PF-2) | Nc |
|---------|-----------|----------------|--------------------------|----|
|         |           | N1^i | N1^u | N1^r | N1 NE | N2^i | N2^u | N2^r | N2 NE | N3^i | N3^u | N3^r | N3 NE |
| HAN     | A         | 575  | 568  | 7    | 132   | 90 M | 537  | 492  | 45   | 56    | 60 M | 436  |
|         | B         | 575  | 544  | 31   | 4     | 90 M | 706  | 701  | 5    | 161   | 60 M | 540  |
|         | C         | 575  | 543  | 32   | 6     | 90 M | 725  | 721  | 4    | 184   | 102.7 M | 537 |
|         | D         | 575  | 538  | 37   | 3     | 90 M | 713  | 708  | 5    | 173   | 102.2 M | 535 |
| BLA     | A         | 901  | 884  | 17   | 425   | 90 M | 678  | 497  | 181  | 38    | 60 M | 459  |
|         | B         | 901  | 850  | 51   | 4     | 90 M | 1034 | 1033 | 1    | 187   | 60 M | 846  |
|         | C         | 901  | 849  | 52   | 0     | 90 M | 1036 | 1036 | 0    | 187   | 101.4 M | 849 |
|         | D         | 901  | 849  | 52   | 0     | 90 M | 1040 | 1040 | 0    | 191   | 101.2 M | 849 |
| NYT     | A         | 627  | 604  | 23   | 97    | 90 M | 573  | 544  | 29   | 37    | 90 M | 507  |
|         | B         | 627  | 591  | 36   | 22    | 90 M | 647  | 640  | 7    | 71    | 90 M | 569  |
|         | C         | 627  | 591  | 36   | 28    | 90 M | 645  | 633  | 12   | 70    | 113.7 M | 563 |
|         | D         | 627  | 591  | 36   | 22    | 90 M | 648  | 640  | 8    | 71    | 113.1 M | 569 |
| GOY     | A         | 489  | 459  | 30   | 123   | 90 M | 458  | 401  | 57   | 65    | 90 M | 336  |
|         | B         | 489  | 443  | 46   | 56    | 90 M | 545  | 509  | 36   | 122   | 90 M | 387  |
|         | C         | 489  | 447  | 42   | 82    | 90 M | 519  | 473  | 46   | 108   | 122.1 M | 365 |
|         | D         | 489  | 443  | 46   | 56    | 90 M | 544  | 508  | 36   | 121   | 121.8 M | 387 |

observations from this table.

(a) Very significant improvement is obtained by algorithm B over algorithm A (NSGA-II) for the same computational effort N_F_E. This means that simply storing all ND positions visited by the population is greatly beneficial. For example, for the BLA network, NSGA-II could not cover 425 of the UExeter solutions whereas algorithm B missed only 4 of the UExeter solutions.

(b) For the HAN network, the use of local search (algorithm C) gave 184 unique solutions (not found in the UExeter set) whereas algorithm B gave 161, thus pointing to the effectiveness of local search for this problem. However, for other problems, local search either did not improve the ND set (over algorithm B) or made it worse.

(c) For the BLA, NYT, and GOY networks, local search together with coupling the population and archive (algorithm D) has produced a larger number of unique solutions as compared to only local search (algorithm C).

(d) The most significant improvement in the ND set comes from the use of external archive (compare the algorithm A and B results).

(e) For the GOY network, NSGA-II (algorithm A) as well as the proposed modifications of NSGA-II (algorithms B, C, D) are unable to cover a substantial number of UExeter solutions. Fig. [1] compares the UExeter ND set with that obtained with algorithm B. Note that a large number of UExeter solutions in the high resilience (or high cost) region are missed out by algorithm B (as also by algorithms C and D). As mentioned in [1], NSGA-II generally captured solutions in the low- and medium-cost regions but not in the high-cost regions. With algorithms B, C, D, this drawback could be eliminated for the HAN and BLA networks and to some extent for the NYT network. However, for the GOY network, none of the modifications is effective in obtaining the high-cost region of the PF.

4 Conclusions

In conclusion, three step-by-step modifications of the NSGA-II algorithm have been presented in this work.
The new algorithms have been used for the medium-size water network problem described in [1]. For three of the four problems, the proposed algorithms have given substantial improvement over the best-known Pareto fronts (ND sets) available in the literature. It was found that the most significant contribution in this improvement arises from the use of an external archive to store all ND positions visited by the population.

Compared to the recently proposed MOPSO+ algorithm [4], the algorithms presented in this work are found to be less effective for the two-objective WSD optimisation problem of [1] (see Tables 1 and 3). A mechanism other than that described in this paper for coupling the archive and the evolving population needs to be explored for improved performance.

References

[1] Q. Wang, M. Guidolin, D. Savic, and Z. Kapelan, “Two-objective design of benchmark problems of a water distribution system via MOEAs: Towards the best-known approximation of the true pareto front,” Journal of Water Resources Planning and Management, vol. 141, no. 3, p. 04014060, 2014.

[2] N. Moosavian and B. Lence, “Nondominated sorting differential evolution algorithms for multiobjective optimization of water distribution systems,” Journal of Water Resources Planning and Management, vol. 143, no. 4, p. 04016082, 2016.

[3] C. A. C. Coello, G. T. Pulido, and M. S. Lechuga, “Handling multiple objectives with particle swarm optimization,” IEEE Trans. Evol. Comput., vol. 8, no. 3, pp. 256–279, 2004.

[4] M. B. Patil, M. N. Naidu, A. Vasan, and M. R. R. Varma, “Water distribution system design using multi-objective particle swarm optimisation,” submitted to Sadhana.

[5] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: NSGA-II,” IEEE Trans. Evol. Comput., vol. 6, no. 2, pp. 182–197, 2002.

[6] M. B. Patil, “Using external archive for improved performance in multi-objective optimization,” submitted to Sadhana.

[7] E. Barlow and T. T. Tanyimboh, “Multiobjective memetic algorithm applied to the optimisation of water distribution systems,” Water resources management, vol. 28, no. 8, pp. 2229–2242, 2014.

[8] L. A. Rossman et al., “EPANET 2: Users Manual,” 2000.