Decentralized energy management for unbalanced networked microgrids with uncertainty

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Abstract
The energy management schemes in microgrids enhance the utilization of renewable energy resources and improve the reliability and resilience measures in the distribution networks. While microgrids operate autonomously, the coordination among microgrid and distribution network operators (DNOs) contributes to the improvement in the economics and reliability of serving the demand. Here, a decentralized energy management framework for the networked microgrids is proposed in which the interactions between the microgrid and DNOs are captured using the Benders decomposition technique. The proposed framework limits the information shared among these autonomous operators and facilitates decentralized energy management in the distribution networks. Furthermore, the unbalanced operation of the distribution network and microgrids, as well as the uncertainty in the operating modes of the microgrids, renewable energy resources, and demand, were addressed. Modified IEEE-34 bus and IEEE-123 bus distribution networks are used to validate the effectiveness of the proposed algorithm.

1 | INTRODUCTION

Networked microgrids improve the economics, reliability, and resilience of distribution networks by accommodating the renewable and distributed energy resources (DERs) and exchanging electricity with the distribution network in the grid-connected mode [1]. Several studies addressed the energy management for networked microgrids and distribution networks [2–21]. Distributed, decentralized, and agent-based frameworks were developed to perform this task by capturing the interactions between the microgrids and distribution networks.

Distributed energy management frameworks for multiple microgrids in the distribution network were presented in [2–10]. A stochastically distributed energy management framework for networked microgrids and distribution network is proposed in [2]. The proposed framework segregates the optimization problems solved for microgrids and the distribution network and addresses the uncertainty in demand and wind generation. An online distributed energy management scheme is proposed in [3] that leverages the Alternating Direction Method of Multipliers (ADMM) algorithm and regret minimization to coordinate energy flow in the networked microgrids considering the uncertainty in the DERs. A two-level distributed optimal control approach is proposed in [4] to carry out the energy management in multiple microgrids and a distribution network. The upper-level control strategy aims to optimize the energy flow among the microgrids and the distribution network, and the lower-level control scheme aims to obtain the optimal dispatch of the generation resources considering the grid-connected and islanding operation modes of microgrids. Ref. [5] proposed a distributed control strategy for power management that improves the power quality in ac and dc microgrids. The proposed event-triggered distributed power routing approach balances the power among phases and enhances the voltage quality in ac microgrids. While this approach addressed the unbalanced operation of microgrids using simulation, the contingencies and the uncertainty in the operation of DERs as well as the interactions among microgrid and distribution network operators (DNOs) were not addressed. Ref. [6] proposed a distributed non-linear control scheme to share power among the DERs in islanded ac microgrids. The developed non-linear mean-square cooperative control scheme features distributed event detection to reduce the communication noise disturbances. A distributed control based on an event-driven
communication mechanism was proposed in [7] to share power among current-controlled voltage source inverter-based DERs in microgrids. A risk-averse energy management framework for networked microgrids is proposed in [8] using conditional value at risk. The proposed energy management problem is formulated as a stochastic linear programming problem and the auxiliary problem principle approach is used to address the privacy constraints. A two-layer distributed cooperative control approach is used in [9] for enabling power sharing among network microgrids using tertiary control while ensuring the frequency and voltage support using primary and secondary distributed controls. The dynamic performance of the networked microgrids is evaluated using small-signal dynamic models to ensure the effectiveness of the proposed control structure. A distributed and robust energy management framework that captures the uncertainty in renewable energy resources and demand is proposed for hybrid ac/dc microgrids in [10]. The ADMM algorithm is used to solve the problem using limited shared information among the microgrids.

Decentralized energy management solutions were developed for energy management in microgrids and distribution networks in [11–15]. A decentralized optimal control algorithm is proposed in [11] to minimize the total operation cost of the networked microgrids and to enhance the utilization of distributed storage resources. The problem is formulated as a partially observable Markov decision process, solved using dynamic programming. A two-stage robust optimization problem is formulated in [12] to address the uncertainties of demand and DERs in the operation horizon, and the ADMM method is used to coordinate the operation decisions by DNO and microgrid operators. A decentralized energy management scheme is proposed in [13] to coordinate the distribution network and microgrids operation strategies considering the grid-connected and islanding operation modes of microgrids. The proposed energy management system considered the uncertainty in demand and generation by formulating a two-stage stochastic programming problem; however, the unbalanced operation of networked microgrids was not addressed. Leveraging the notion of transactive energy, a decentralized energy management framework is proposed in [14] to coordinate the energy management between networked microgrids. The uncertainties are captured using distributionally robust optimization models to ensure robust solutions with less conservatism compared to the robust optimization solutions. A decentralized approach for the optimal power flow considering the coordination between the microgrids and DNOs is presented in [15]; however, the uncertainties in demand and DERs were not considered.

A multi-agent system is introduced for energy management in [16] to control the power trades in the networked microgrids considering the demand response. The proposed framework aimed at reducing the operation cost and peak load by managing low priority demands. Multi-agent systems were used in [17] for energy management of networked microgrids to handle the non-dispatchable DERs in a competitive energy market. In the first level, each agent would balance the load and generation, and in the second level, all agents join the market as a generator or load. The energy flow is coordinated between the grid-connected microgrids and the distribution network by regulating the demand to minimize the generation cost in [18]. A multi-objective model was proposed in [19] to minimize the generation cost and peak to average ratio of demand by scheduling the energy consumption in microgrids while capturing the uncertainty in uncontrollable demands. The coordinated operation of microgrids within the distribution network is modeled as a stochastic bi-level program in [20]. Here, the upper-level problem minimizes the operation cost of the distribution network while the lower-level problem minimizes the operation cost of the microgrids. The uncertainty in renewable energy resources is addressed by formulating a stochastic two-stage optimization problem. Bi-level programming is proposed in [21] to address the market participation of the microgrids. In the upper level, the operation cost of the wholesale market is minimized while at the lower level, the payoffs of energy service providers are maximized by controlling the local generation assets and curtailable loads. The proposed algorithm ignores the uncertainty in the DERs and demand.

While decentralized and distributed energy management schemes for networked microgrids were extensively studied in earlier publications, the day-ahead energy management framework that addresses the interactions between DNO and microgrid operators considering the unbalanced operation of the networked microgrids has not been addressed. Furthermore, the uncertainty in the microgrid operation modes (grid-connected and island modes) was not addressed in the previous studies.

In this paper, the interaction between the DNO and microgrid controllers was captured using the Benders decomposition technique. The physical and control layouts of the energy network in the distribution network are shown in Figure 1. As shown in this figure, each microgrid operator communicates with the DNO in the control layer to manage the consumption and generation. The DNO would schedule the generation resources while considering the probabilistic islanding of microgrids in the distribution network. As microgrids are operated autonomously, the proposed framework ensures that limited information is exchanged between the DNO and the microgrid operators. The contributions of this paper are as follows:

- The interactions among microgrids and the DNO are captured using the Benders decomposition technique.
- The unbalanced operation of microgrids and distribution network was addressed by determining the power flow on each phase in the coupling lines between the microgrids and the distribution network.
- Probabilistic operation modes of microgrids and the uncertainty in DERs and demand were considered using scenarios. Furthermore, the impacts of islanding and outages in microgrids on the operation cost of the distribution network and microgrids are assessed.

It is worth noting that the proposed framework addresses the hourly energy management for networked microgrids in a 24-h horizon. Therefore, communication delays compared to the simulation time step are ignored. The rest of
the paper is organized as follows: the problem formulation and solution methodology are presented in Sections 2 and 3, respectively. Numerical analysis is shown in Section 4, and the solution time and scalability of the proposed algorithm are presented in Section 5. Finally, the conclusion is presented in Section 6.

2 | PROBLEM FORMULATION

2.1 | Decentralized energy management using Benders decomposition

The general form of the energy management problem is formulated as a mixed-integer linear programming problem (MILP) shown in (1)–(5) where $\mathbf{x}_c$ and $\mathbf{x}_b$ are the vectors of continuous and binary variables, respectively. These variables represent the operational decisions in the distribution network, and $\mathbf{y}_m$ is a vector of continuous variables representing the operational decisions in the microgrid $m$. The first and second terms in (1) are the operation costs of the distribution network and networked microgrids, respectively. The problem is subjected to (2)–(5) where (2) and (3) represent the nodal power balance in the distribution network and microgrid $m$, respectively. Here, $\mathbf{k}_m$ is a vector of complicating continuous variables representing the exchanged power flow between the distribution network and microgrid $m$. The constraint (4) represents all the inequality constraints for the DNO problem. The set of inequality constraints for microgrid $m$ is shown in (5).

$$
\min \quad f(\mathbf{x}_c) + \sum_m g(\mathbf{y}_m),
$$

s.t.

$$
A_c \cdot \mathbf{x}_c + \sum_m B_m \cdot \mathbf{k}_m = d, \quad \mathbf{k}_m \in \mathbf{y}_m, \quad (2)
$$

$$
A_m \mathbf{y}_m + B'_m \cdot \mathbf{k}_m = d_m, \quad \mathbf{k}_m \in \mathbf{y}_m, \quad (3)
$$

$$
G_c \cdot \mathbf{x}_c + G_b \cdot \mathbf{x}_b \leq h, \quad (4)
$$

$$
F_m \cdot \mathbf{y}_m \leq r_m. \quad (5)
$$

Using Benders decomposition, the problem is decomposed into a master problem (MP) shown in (6)–(9) and $m$-subproblems (SPs) presented in (10)–(14). The master MILP problem represents the DNO problem. An auxiliary positive variable $\alpha$ shown in (7), is introduced in (6) to represent the operation cost of the networked microgrids plus the absolute mismatch in nodal real and reactive power [22]. The solution to the MP problem ($Z_{lower}$) represents the lower bound of the problem solution. The solution ($\hat{\mathbf{k}}_m$) is passed to the microgrid operation subproblems (10)–(14), which are formulated as linear programming (LP) problems.

$$
\min \quad Z_{lower} = f(\mathbf{x}_c) + \alpha, \quad (6)
$$

s.t.

$$
\alpha \geq 0, \quad (7)
$$

$$
A_c \cdot \mathbf{x}_c + \sum_m B_m \cdot \mathbf{k}_m = d, \quad (8)
$$

$$
G_c \cdot \mathbf{x}_c + G_b \cdot \mathbf{x}_b \leq h. \quad (9)
$$

The first term of the objective function of SP in (10) is the operation cost of the microgrid $m$. The vectors of slack variables are introduced in the second term in (10). The mismatch in the nodal power balance is minimized using the penalty vector $\mathbf{M}$. The slack variables are introduced in the nodal power balance (11). The set of inequality constraints in the microgrid operation’s subproblem is shown in (12). Constraint (13) fixes the complicating variables to the values obtained from the MP. The slack variables are non-negative as shown in (14). Once SPs (10)–(14) are solved, the upper bound of the solution is calculated in (15) using the solutions of the DNO problem (MP) and microgrids’ operation problems (SPs). If the mismatch between the lower and upper bounds of the solution is greater than a specified tolerance, Benders cut (16) is formed and added to the DNO problem. Adding a new hyperplane (Benders cut) to the
feasible region of the master problem improves the solution at each iteration. This process will continue until the gap between the lower and upper bounds of the solution is smaller than a certain tolerance.

\begin{equation}
\min w_m = f(\mathbf{y}_m) + M \cdot (\mathbf{u}_m + \mathbf{v}_m),
\end{equation}

\text{s.t.}
\begin{alignat}{2}
A_m \cdot \mathbf{y}_m + B_m \cdot \mathbf{k}_m + \mathbf{u}_m - \mathbf{v}_m &= \mathbf{d}_m, \\
F_m \cdot \mathbf{y}_m &\leq \mathbf{r}_m, \\
\mathbf{k}_m &= \mathbf{k}_m^0 : \lambda_m, \\
\mathbf{u}_m, \mathbf{v}_m &\geq 0.
\end{alignat}

It is worth noting that this framework will be used by both microgrid operators and DNO as it captures the interactions between these entities. The master problem is solved by the DNO while the subproblems are solved by the microgrid operators. The Benders cuts generated by the subproblems represent the microgrids’ responses to the decisions made by the DNO. The DNO updates its decisions by updating the power flows between the distribution network and microgrids and sends this decision to the microgrids.

\begin{equation}
Z_{upper} = f(\hat{\mathbf{x}}_m) + \sum_m (\bar{w}_m),
\end{equation}

\begin{equation}
\alpha \geq \sum_m \left( \bar{w}_m + \hat{\lambda}_m^T (\mathbf{k}_m - \mathbf{k}_m^0) \right).
\end{equation}

### 2.2 Distribution network operation problem—Master problem

The DNO problem is formulated as a MILP problem (17)–(53). The objective is to minimize the expected operation cost of the distribution network with microgrids. The objective function (17) includes the expected cost of providing electricity from the main distribution feeder, the expected operation cost of DERs, the expected operation cost of the energy storage [23], and the expected demand curtailment penalties. The fifth term in (17) represents the expected operation cost of microgrids and the mismatch in the nodal real and reactive power balance for given values of the exchanged real and reactive power flow. At each iteration, the value of this scalar is updated by adding Bender cuts from the microgrid subproblems. The scalar is always positive as enforced by (18).

The network constraints are shown in (19)–(53). The constraints representing the nodal real and reactive power balance are shown in (19) and (20), respectively. The first terms in (19) and (20) are the exchanged real and reactive power flows on the branches between the distribution network and microgrids, respectively. Here, the positive power flow direction is assumed to be from the distribution network to the microgrids. The second terms in (19) and (20) are the real and reactive power flows in the other branches of the distribution network, respectively. The real and reactive power demands served are limited by (21) and (22), respectively. Here, it is assumed that the power factor of the demand remains the same in case of curtailment; therefore, the served reactive demand is proportional to the served real demand as enforced by (22). The constraints (23)–(26) are the linearized approximation of the unbalanced power flow formulated in [24]. The real and reactive power flows on each phase of the distribution branches are presented in (23) and (24) except for the branches that are connected to the microgrids. For these branches, the real and reactive power flows are formulated as (25) and (26). Here, \(\{\phi_1, \phi_2, \phi_3\}\) represents the positive sequence of phases on the branch \(i-j\), that is, \(\{a, b, c\}\), \(\{b, c, a\}\), and \(\{c, a, b\}\). Constraints (23)–(26) used Big-M reformulation to incorporate the availability of the distribution branch phases. The power factor at the distribution feeder is within the desirable limits by enforcing (27). The capacity of the distribution feeder is constrained by a circular constraint, which it is further linearized in (28)–(30) by hexagon approximation introduced in [25]. The power dispatch of DER is limited by (31) and (32). Considering the availability of phases on a branch, the real and reactive power flows in a distribution branch are limited by (33) and (34), respectively. The exchanged real and reactive power flows in branches between the distribution network and microgrids are limited by (35) and (36), respectively. Similar to (28)–(30), hexagon approximation is used in (37)–(39) to linearize the circular constraints that limit the apparent power flow in a branch. The real and reactive power output limits of a PV unit are shown in (40)–(42). The solar generation output is limited by the capacity of the PV generation unit as enforced by (40). The reactive power output of a PV generation unit is limited by the capacity of its inverter as enforced by (41). The real power output of a PV generation unit is limited by the available solar irradiance as shown in (42).

The constraints representing the energy storage units are shown in (43)–(49) [23, 26]. The charging and discharging real power are limited by (43) and (44). The reactive power output is limited by (45). The hourly available energy is limited by (46). The available energy is limited by minimum and maximum limits as shown in (47). The available energy at the initial time \(t=0\) is equal to the available energy at the final time \(t=NT\) as shown in (48). The available energy at the final time step is set to a certain value \(E_{\text{final}}\) in (48). The depth of charge/discharge is limited by (49).

The squared bus voltage is limited by upper and lower limits as shown in (50). This constraint does not capture the slack bus and the buses connected to the secondary of the voltage regulator. The voltage of the slack bus (main distribution feeder) is fixed. Three-phase voltage regulators are represented by three single-phase ideal transformers in series with branches that represent the leakage impedances. Constraints (51) and (52) represent the relationship between the voltage magnitudes on both sides of the voltage regulator where \(i\) is the primary side, \(j\) is the
secondary side. Here, $a_{t,r}^{\max}$ and $a_{t,r}^{\min}$ are the maximum and minimum transformer ratios on phase $\phi$, respectively [24]. The probabilistic islanding of microgrids is enforced by (53). The probability of a grid-connected operating mode of microgrid $m$ cannot exceed $(1-\mu_m)$ and the minimum probability of islanding is $\mu_m$.

$$\min \sum_i p_r \cdot \left( \sum_{\phi \in G} \left( \sum_{t \in T} p_{i,t} \cdot \left( \sum_{\phi \in H} p_{i,t} \right) \right) + \sum_{\phi \in E} \left( \sum_{\phi \in G} p_{i,t} \cdot p_{i,t} \right) \right) + \sum_{\phi \in E} \left( \sum_{\phi \in G} p_{i,t} \cdot p_{i,t} \right)$$

$$+ VOLL \cdot \sum_{\phi \in D} \left( p_{i,t}^{\phi} - p_{i,t}^{\phi} \right) \right) \right) + \sum_{s} \alpha_i$$

$$\alpha_i \geq 0,$$

$$- \sum_{j \in B \cap K_m} P_{j,t}^{\phi} + \sum_{j \in B} P_{j,t}^{\phi} + \sum_{\phi \in G} A_{j,t}^{\phi}$$

$$+ \sum_{\phi \in E} A_{j,t}^{\phi} + \sum_{\phi \in G} A_{j,t}^{\phi} - \sum_{\phi \in E} A_{j,t}^{\phi}$$

$$= \sum_{d \in D} A_{i,t}^{d} \cdot \phi_{i,t}^{d}; \forall i \in B,$$

$$p_{i,t}^{\phi} \leq p_{i,t}^{\phi} \quad \forall d \in D$$

$$Q_{i,t}^{\phi} = \tan(\cos^{-1}P_{i,t}^{\phi}) \cdot P_{i,t}^{\phi} \quad \forall d \in D$$

$$U_{i,t}^{\phi_1} = U_{i,t}^{\phi_1} - U_{i,t}^{\phi_1} \leq 2r_{ij}^{\phi_1} \cdot P_{ij,t}^{\phi_1} + 2x_{ij}^{\phi_1} \cdot Q_{ij,t}^{\phi_1}$$

$$- r_{ij}^{\phi_2} \cdot P_{ij,t}^{\phi_2} + \sqrt{3} \cdot x_{ij}^{\phi_2} \cdot P_{ij,t}^{\phi_2} - x_{ij}^{\phi_2} \cdot Q_{ij,t}^{\phi_2}$$

$$- \sqrt{3} \cdot x_{ij}^{\phi_2} \cdot Q_{ij,t}^{\phi_2} - r_{ij}^{\phi_3} \cdot P_{ij,t}^{\phi_3} - \sqrt{3} \cdot x_{ij}^{\phi_3} \cdot Q_{ij,t}^{\phi_3} - M \cdot (1 - p_{ij}^{\phi_3})$$

$$\forall i, j \in B,$$

$$U_{i,t}^{\phi_1} = U_{i,t}^{\phi_1} - U_{i,t}^{\phi_1} \leq 2r_{ij}^{\phi_1} \cdot P_{ij,t}^{\phi_1} + 2x_{ij}^{\phi_1} \cdot Q_{ij,t}^{\phi_1}$$

$$- r_{ij}^{\phi_2} \cdot P_{ij,t}^{\phi_2} + \sqrt{3} \cdot x_{ij}^{\phi_2} \cdot P_{ij,t}^{\phi_2} - x_{ij}^{\phi_2} \cdot Q_{ij,t}^{\phi_2}$$

$$- \sqrt{3} \cdot x_{ij}^{\phi_2} \cdot Q_{ij,t}^{\phi_2} - r_{ij}^{\phi_3} \cdot P_{ij,t}^{\phi_3} - \sqrt{3} \cdot x_{ij}^{\phi_3} \cdot Q_{ij,t}^{\phi_3} - M \cdot (1 - p_{ij}^{\phi_3})$$

$$\forall i, j \in B,$$
-M \cdot y_{ij,t} \cdot p_{ij}^t \leq H_{ij,t}^\phi \leq M \cdot y_{ij,t} \cdot p_{ij}^t ; \forall i \in B_j \cap B, \forall j \in B_i \cap \mathcal{K}_{aw}, \quad (35)

-\sqrt{3} \cdot (p_{ij,t}^\phi + S_{ij,t}^{\max}) \leq Q_{ij,t}^\phi \leq \sqrt{3} \cdot S_{ij,t}^{\max}, \quad (37)

-\frac{\sqrt{3}}{2} \cdot S_{ij,t}^{\max} \leq Q_{ij,t}^\phi \leq \frac{\sqrt{3}}{2} \cdot S_{ij,t}^{\max}, \quad (38)

\sqrt{3} \cdot (p_{ij,t}^\phi - S_{ij,t}^{\max}) \leq Q_{ij,t}^\phi \leq \sqrt{3} \cdot (p_{ij,t}^\phi + S_{ij,t}^{\max}), \quad (39)

0 \leq p_{\phi,t}^t \leq P_{\phi}^{\max} ; \forall \phi \in \mathcal{V}, \quad (40)

-\Phi_{\phi}^{\max} \psi_{\phi} \leq -\Phi_{\phi}^{\max} \psi_{\phi} \leq \Phi_{\phi}^{\max} \psi_{\phi} ; \forall \phi \in \mathcal{V}, \quad (41)

\Psi_{\phi}^{\max} \psi_{\phi} \leq \Psi_{\phi}^{\max} \psi_{\phi} \leq \Psi_{\phi}^{\max} \psi_{\phi} ; \forall \phi \in \mathcal{V}, \quad (42)

0 \leq p_{\phi,dc}^{\phi,t} \leq P_{\phi,dc}^{\max} ; \forall \phi \in \mathcal{E}, \quad (43)

0 \leq p_{\phi,ch}^{\phi,t} \leq P_{\phi,ch}^{\max} ; \forall \phi \in \mathcal{E}, \quad (44)

0 \leq Q_{\phi,t}^{\max} \leq Q_{\phi,t}^{\max} \leq Q_{\phi,t}^{\max} ; \forall \phi \in \mathcal{E}, \quad (45)

E_{\phi,t}^{\phi,t} = E_{\phi,t+1}^{\phi,t} + \left( p_{\phi,dc}^{\phi,t} \cdot \eta_{\phi}^{\phi} - p_{\phi,ch}^{\phi,t} \cdot \psi_{\phi} \right) ; \forall \phi \in \mathcal{E}, \quad (46)

E_{\phi,t}^{\min} \leq E_{\phi,t} \leq E_{\phi,t}^{\max} ; \forall \phi \in \mathcal{E}, \quad (47)

E_{\phi,t}^{\phi} = E_{\phi,t}^{\phi} = E_{\phi,t}^{\phi} ; \forall \phi \in \mathcal{E}, \quad (48)

\left| E_{\phi,t}^{\phi} - E_{\phi,t-1}^{\phi} \right| \leq \Delta E_{\phi} ; \forall \phi \in \mathcal{E}, \quad (49)

(v_{\phi,t}^{\max})^2 \leq U_{\phi,t}^{\phi} \leq (v_{\phi,t}^{\max})^2 ; \forall \phi \in \mathcal{E}, \quad (50)

-M \cdot (1-p_{ij}) \leq \left( p_{ij,t}^{\phi,\max} \right)^2 \cdot U_{ij,t}^{\phi} - U_{ij,t}^{\phi} ; \forall \phi \in \mathcal{B}_j \cap B, \forall j \in B_i \cap \mathcal{K}_{aw}, \quad (51)

2.3 Microgrid operation subproblem

At this stage, each microgrid operator solves the SP to check the feasibility of the solution obtained from the DNO problem (MP) and to ensure its optimal operation. The microgrid subproblem is formulated as a LP problem in which, the objective function is shown in (54). The first, second, and third terms of the objective function are the expected operation cost of DERs, the expected operation cost of energy storage, and the penalty associated with the expected load of loss, respectively. The last term is the summation of slack variables that represent the nodal real and reactive power mismatches. As the procured solution from the DNO problem may lead to an infeasible solution in the microgrid operation SP, these slack variables are introduced.

The network constraints for microgrids are similar to those in the distribution networks shown in (21), (22) and (37)–(50) for \( d \in D_{aw} \), \( e \in E_{aw} \), and \( v \in V_{aw} \). Here, (13) and (20) are replaced by (55) and (56) which include the slack variables to capture the mismatch in the nodal generation and demand balance; (23) and (24) are replaced by (57) and (58) to address the outages in microgrid’s branches; and (25) and (26) are replaced by (59) and (60) given the states of the coupling branches between microgrid and distribution network. These states are procured by solving the distribution network problem (MP). Similarly, (33) and (34) are replaced by (61) and (62), and (35) and (36) are replaced by (63) and (64), respectively. The real and reactive power exchanges between microgrid and distribution network are equal to the solutions of the master problem as enforced by (65) and (66). The slack variables are non-negative as shown (67).

\[
\min \sum_i (r_i \cdot \left( \sum_{\phi \in H_{aw}} \left( \sum_{\phi \in H_{aw}} F_{\phi} \left( \sum_{\phi} p_{\phi,i,t}^{\phi} \right) \right) + \sum_{\phi \in H_{aw}} \left( \sum_{\phi} p_{\phi,i,t}^{\phi} \right) \right) + \sum_{\phi \in H_{aw}} \left( \sum_{\phi} \left( p_{\phi,i,t}^{\phi} + P_{\phi,i,t}^{\phi} \right) \right) + VOLL \cdot \sum_{d \in D_{aw}} \left( \sum_{\phi} \left( p_{\phi,i,t}^{\phi} - P_{\phi,i,t}^{\phi} \right) \right) + M \cdot \left( \sum_{\phi} \sum_{\phi} \left( Z_{1\phi,i,t}^{\phi} + Z_{2\phi,i,t}^{\phi} + Z_{3\phi,i,t}^{\phi} + Z_{4\phi,i,t}^{\phi} \right) \right)) \quad (54)
\]
\[
\sum_{j \in B \cap \mathcal{K}_m} P_L^{\phi}_{ij,t} + \sum_{j \in \mathcal{S}_m} P_L^{\phi}_{j,t} + \sum_{j \in \mathcal{H}_m} N_{ij}^{\phi} = \Phi_{ij,t}^{\phi}; \forall j \in \mathcal{K}_m, (55)
\]

\[
\sum_{j \in B \cap \mathcal{K}_m} Q_L^{\phi}_{ij,t} + \sum_{j \in \mathcal{S}_m} Q_L^{\phi}_{j,t} + \sum_{j \in \mathcal{H}_m} N_{ij}^{\phi} = \Phi_{ij,t}^{\phi}; \forall j \in \mathcal{K}_m, (56)
\]

\[
U_{ij,t}^{\phi}_1 - U_{ij,t}^{\phi}_1 \leq 2 \Phi_{ij,t}^{\phi} \cdot P_{L_{ij,t}}^{\phi} + 2 \Phi_{ij,t}^{\phi} \cdot Q_{L_{ij,t}}^{\phi}, \forall i, j \in \mathcal{K}_m, (57)
\]

\[
U_{ij,t}^{\phi}_1 - U_{ij,t}^{\phi}_1 \geq 2 \Phi_{ij,t}^{\phi} \cdot P_{L_{ij,t}}^{\phi} + 2 \Phi_{ij,t}^{\phi} \cdot Q_{L_{ij,t}}^{\phi}, \forall i, j \in \mathcal{K}_m, (58)
\]

\[
V_{ij,t}^{\phi}_1 - V_{ij,t}^{\phi}_1 \leq 2 \Phi_{ij,t}^{\phi} \cdot P_{L_{ij,t}}^{\phi} + 2 \Phi_{ij,t}^{\phi} \cdot Q_{L_{ij,t}}^{\phi}, \forall i, j \in \mathcal{K}_m, (59)
\]

\[
U_{ij,t}^{\phi}_1 - U_{ij,t}^{\phi}_1 \geq 2 \Phi_{ij,t}^{\phi} \cdot P_{L_{ij,t}}^{\phi} + 2 \Phi_{ij,t}^{\phi} \cdot Q_{L_{ij,t}}^{\phi}, \forall i, j \in \mathcal{K}_m, (60)
\]

\[
M \cdot \left(1 - \mathcal{J}^{(c)}_{ij,t} \cdot \Phi_{ij,t}^{\phi}\right) \geq \sum_{e \in \mathcal{E}_m} \mathcal{P}_e \cdot \left(\mathcal{P}_e^{\phi} + \mathcal{P}_e^{\phi}\right), \forall i, j \in \mathcal{K}_m, (61)
\]

\[
M \cdot \left(1 - \mathcal{J}^{(c)}_{ij,t} \cdot \Phi_{ij,t}^{\phi}\right) \geq \sum_{e \in \mathcal{E}_m} \mathcal{P}_e \cdot \left(\mathcal{P}_e^{\phi} + \mathcal{P}_e^{\phi}\right), \forall i, j \in \mathcal{K}_m, (62)
\]

\[
M \cdot \left(1 - \mathcal{J}^{(c)}_{ij,t} \cdot \Phi_{ij,t}^{\phi}\right) \geq \sum_{e \in \mathcal{E}_m} \mathcal{P}_e \cdot \left(\mathcal{P}_e^{\phi} + \mathcal{P}_e^{\phi}\right), \forall i, j \in \mathcal{K}_m, (63)
\]

\[
M \cdot \left(1 - \mathcal{J}^{(c)}_{ij,t} \cdot \Phi_{ij,t}^{\phi}\right) \geq \sum_{e \in \mathcal{E}_m} \mathcal{P}_e \cdot \left(\mathcal{P}_e^{\phi} + \mathcal{P}_e^{\phi}\right), \forall i, j \in \mathcal{K}_m, (64)
\]

\[
\mathcal{P}_e^{\phi} = \mathcal{P}_e^{\phi} : \mathcal{P}_e^{\phi} \geq 1, \forall i, j \in \mathcal{K}_m, (65)
\]

\[
\mathcal{Q}_e^{\phi} = \mathcal{Q}_e^{\phi} : \mathcal{P}_e^{\phi} \geq 1, \forall i, j \in \mathcal{K}_m, (66)
\]

\[
\mathcal{Z}_1^{\phi} \geq 0, \forall i \in \mathcal{K}_m, (67)
\]

\[
3 \quad \text{SOLUTION METHODOLOGY}
\]

The flowchart of the proposed decentralized energy management is shown in Figure 2 and summarized as follows:

Step (a): The DNO problem (MP) is solved to obtain \( \mathcal{P}_e^{\phi, (c)} \) and \( \mathcal{Q}_e^{\phi, (c)} \) considering any additional constraints from step d. The lower bound of the solution \( \mathcal{Z}_{\text{lower}}^{(c)} \) is calculated using (68).

\[
\mathcal{Z}_{\text{lower}}^{(c)} = \sum_{i} p_{r,i} \cdot \left(\sum_{\phi} \mathcal{P}_{r,i}^{\phi} \cdot \left(\sum_{\phi} \mathcal{P}_{r,i}^{\phi} \cdot \mathcal{Q}_{r,i}^{\phi}\right)\right)
\]

\[
\sum_{e \in \mathcal{E}_m} \mathcal{P}_e \cdot \left(\mathcal{P}_e^{\phi} + \mathcal{P}_e^{\phi}\right) + \sum_{e \in \mathcal{E}_m} \mathcal{P}_e \cdot \left(\mathcal{P}_e^{\phi} + \mathcal{P}_e^{\phi}\right) + VOLL \cdot \sum_{e \in \mathcal{E}_m} \left(\sum_{\phi} \mathcal{P}_{r,i}^{\phi} \cdot \mathcal{Q}_{r,i}^{\phi}\right) + \sum_{e \in \mathcal{E}_m} \mathcal{P}_e \cdot \left(\mathcal{P}_e^{\phi} + \mathcal{P}_e^{\phi}\right)
\]

(68)
NUMERICAL RESULTS

Step (b): Microgrid operation problems (SPs) are solved. The upper bound of the solution \( z_{\text{upper}}^{(c)} \) is calculated using (69).

\[
\begin{align*}
\tau = 1 & \quad \text{(a) Solve the distribution network operation problem (MP)} \\
\tau = \tau + 1 & \quad \text{(b) Solve microgrid operation problem (SP) for each microgrid} \\
\text{Calculate the lower and upper bounds of the solution (68)-(69)} & \\
\text{No} & \quad \text{(c) Check if tolerance is negligible?} \\
\text{Yes} & \quad \text{Stop} \\
\text{Add Benders cut (70)} & \quad \text{Update and pass } \phi_{t,i,s}, Q_{t,i,s}, \tilde{y}_{t,i,s} \\
\end{align*}
\]

\[
\begin{align*}
\alpha_{t,i,s} \geq & \sum_m \left( \hat{W}^{m,\gamma}_{i,t} + \sum_{\phi} \tilde{x}^{m,\phi}_{t,i,s} \cdot \left( p^{s}_{t,i} - \tilde{p}_{t,i,s}^{s,\phi} \right) \\
& + \sum_{\phi} \tilde{x}^{m,\phi}_{t,i,s} \cdot \left( Q_{t,i,s}^{\phi} - Q_{t,i,s}^{\phi} \right) \right) \\
& + \sum_{\phi} \rho_{t,i,s} \cdot \left( \sum_{j} \tilde{x}^{m,\phi}_{j,i,s} + \tilde{x}^{m,\phi}_{j,i,s} \right) \\
& + VOLL \cdot \left( \sum_{d \in D} \sum_{\phi} \tilde{x}^{m,\phi}_{d,i,s} + \tilde{x}^{m,\phi}_{d,i,s} \right) \\
& + M \cdot \left( \sum_{\phi} \sum_{e \in E} \tilde{x}^{m,\phi}_{e,i,s} + \tilde{x}^{m,\phi}_{e,i,s} \right) \\
& + Z_{i,i,s} + Z_{i,i,s} \\
\end{align*}
\]

\begin{equation}
(70)
\end{equation}

\[
\begin{align*}
\hat{W}^{m,\gamma}_{i,t} & = \sum_{\phi} F_{t} \left( \sum_{e \in E} \rho_{t,i,s} \cdot \left( \sum_{j} \tilde{x}^{m,\phi}_{j,i,s} + \tilde{x}^{m,\phi}_{j,i,s} \right) \\
& + \sum_{d \in D} \sum_{\phi} \tilde{x}^{m,\phi}_{d,i,s} + \tilde{x}^{m,\phi}_{d,i,s} \right) \\
& + VOLL \cdot \left( \sum_{d \in D} \sum_{\phi} \tilde{x}^{m,\phi}_{d,i,s} + \tilde{x}^{m,\phi}_{d,i,s} \right) \\
& + M \cdot \left( \sum_{\phi} \sum_{e \in E} \tilde{x}^{m,\phi}_{e,i,s} + \tilde{x}^{m,\phi}_{e,i,s} \right) \\
& + Z_{i,i,s} + Z_{i,i,s} \\
\end{align*}
\]

\begin{equation}
(71)
\end{equation}

4 | NUMERICAL RESULTS

The modified IEEE-34 bus distribution network is used with three unbalanced microgrids to show the effectiveness of the proposed framework. Later, the IEEE-123 bus distribution network with three microgrids is used to evaluate the scalability of the proposed framework. The IEEE-34 bus distribution network with three microgrids is shown in Figure 3. The microgrids are connected to the three-phase distribution network through the points of common coupling (PCC). The objective is to minimize the expected operation cost of the distribution network and microgrids. The upper and lower limits for the bus voltage magnitudes are 1.05 and 0.95 of their nominal values, respectively. The voltage on slack bus is set to 1.05 of the nominal value. The maximum and minimum transformer ratios for the voltage regulator is 1.1 and 0.9, respectively. The main distribution feeder capacity is 1.8 MVA with a minimum power factor of 0.9. Lithium-ion battery is used as the energy storage system (ESS) with the capital cost of 450 $/kWh [27]. The operation cost of the energy storage is 0.25 $/kWh using the model presented in [28]. The charging and discharging efficiency for the energy storage unit is 90%. The value of lost load (VOLL) is $40/kWh and the convergence tolerance \( \varepsilon \) is 0.01%. The hourly energy prices and total hourly demand in the distribution network and microgrids are shown in Figure 4. The characteristics of DERs, ESS, and PVs are shown in Tables 1,
TABLE 1 Dispatchable generation resources

| Unit         | Bus | $P_{\text{min}}$ (kW) | $P_{\text{max}}$ (kW) | $Q_{\text{min}}$ (kVAR) | $Q_{\text{max}}$ (kVAR) |
|--------------|-----|------------------------|------------------------|-------------------------|-------------------------|
| Distribution feeder | 1   | 0                      | 1800                   | −850                    | 850                     |
| DER1         | 24  | 0                      | 100                    | −50                     | 50                      |
| DER2         | 40  | 0                      | 80                     | −40                     | 40                      |
| DER3         | 45  | 0                      | 90                     | −45                     | 45                      |

FIGURE 3 The modified IEEE-34 bus distribution network with networked microgrids

FIGURE 4 Total hourly demand in the distribution network and networked microgrids and the hourly energy prices

TABLE 2 ESS characteristics

| Unit   | Bus | $P_{\text{max}}$ (kW) | $Q_{\text{max}}$ (kVAR) | $E_{\text{min}}$ (kWh) | $E_{\text{max}}$ (kWh) |
|--------|-----|------------------------|-------------------------|------------------------|------------------------|
| ESS1   | 20  | 50                     | 25                      | 10                     | 100                    |
| ESS2   | 25  | 60                     | 30                      | 10                     | 120                    |
| ESS3   | 39  | 40                     | 20                      | 5                      | 60                     |
| ESS4   | 40  | 30                     | 15                      | 5                      | 50                     |

TABLE 3 Generation limits of PV units

| Unit   | Bus | $P_{\text{min}}$ (kW) | $P_{\text{max}}$ (kW) | $Q_{\text{min}}$ (kVAR) | $Q_{\text{max}}$ (kVAR) |
|--------|-----|------------------------|------------------------|-------------------------|-------------------------|
| PV1    | 10  | 0                      | 100                    | −80                     | 80                      |
| PV2    | 30  | 0                      | 100                    | −90                     | 90                      |
| PV3    | 37  | 0                      | 50                     | −40                     | 40                      |
| PV4    | 44  | 0                      | 65                     | −55                     | 55                      |
| PV5    | 49  | 0                      | 75                     | −60                     | 60                      |

2, and 3, respectively. The quadratic cost of DERs are piecewise linearized using four segments, each covering 1/4 of maximum power capacity of the unit. The marginal costs of DERs at each segment are shown in Table 4. The simulation is performed on a server with dual 14 Core Intel Xeon 2.6 GHz,
TABLE 4  The marginal costs of DERs

| Units | Segment 1 | Segment 2 | Segment 3 | Segment 4 |
|-------|-----------|-----------|-----------|-----------|
| DER1  | 0.06      | 0.16      | 0.21      | 0.36      |
| DER2  | 0.05      | 0.15      | 0.20      | 0.35      |
| DER3  | 0.04      | 0.13      | 0.22      | 0.30      |

FIGURE 5  The upper and lower bounds of the solution in Case 1

380 GB of memory, with Cplex 12.8. The following cases are considered:
Case 1—Deterministic solution under normal operating condition.
Case 2—Deterministic solution considering the islanding of microgrid-2.
Case 3—Stochastic solution considering the branch outage in microgrid-1.
Case 4—Stochastic solution considering the islanding of microgrid-2.
Case 5—Stochastic solution considering the probabilistic islanding of microgrids.

4.1  Case 1—Deterministic solution under normal operating condition

In this case, the deterministic solution under normal operation is procured. The upper and lower bounds for the solution, that is, $\hat{Z}_{\text{lower}}^{(r)}$ and $\hat{Z}_{\text{upper}}^{(r)}$, are shown in Figure 5 and the algorithm converges after 698 iterations. The exchanged real and reactive power flows between the distribution network and microgrid-1 on phase “a” are shown in Figures 6 and 7, respectively. The exchanged real power on each phase between the distribution network and microgrids at hour 18 (hour of peak demand) is shown in Table 5. Here, a positive value refers to the power flow from the distribution network to a microgrid. The operation costs and load curtailments are shown in Table 6. As shown in this table, the operation cost of microgrid-1 is $22.05. In microgrid-1, the operation cost is limited to its energy storage operation cost as there is no DER or load curtailment in this microgrid. At hour 18, the real power flows from the distribution network to microgrid-1 as shown in Table 5. The operation costs associated with DERs for microgrid-2 and microgrid-3 are $151.96 and $146.33, respectively. Microgrid-2 provides energy to the distribution network on phase “b” during the peak hour. Meanwhile, microgrid-3 supports the distribution network on phases “a” and “c” with cheaper DER. The DNO cost is $14,773.99 which includes the cost of serving microgrids. The total operating cost in this case is $15,094.33.

Here, the procured solution is compared with the solution procured by solving the energy management problem for distribution network with microgrids as a single MILP problem.
using branch and cut search algorithm, and as a distributed optimization problem using the ADMM technique. Table 7 presents the operation costs of the distribution network and microgrids procured by solving a MILP problem using branch and cut algorithm. As shown in this table, because $\epsilon = 10^{-4}$, the solution procured using the proposed algorithm is $4.23$ higher than the solution to the MILP problem using branch and cut search method. Selecting lower value for $\epsilon$ will reduce the difference between the procured solution and the solution to MILP problem. It is worth noting that solving the MILP problem requires access to the distribution network’s and networked microgrids’ data, which may not be a practical assumption. The proposed algorithm to solve the energy management problem requires limited information sharing among the DNO and microgrids’ operators.

Here, the solution procured by the proposed algorithm is compared with the solution procured using the ADMM method. The simulation outcomes for Case 1 using the ADMM method are shown in Table 8. Considering the convergence tolerance $\epsilon = 10^{-4}$ in both algorithms, the total operation cost of the distribution network and microgrids using the ADMM algorithm is 86.05 higher than that procured using the proposed algorithm. Moreover, the proposed algorithm converges faster than the ADMM algorithm. The solution time using the proposed algorithm is 47 min and 4 s, while the solution time using the ADMM algorithm is 58 min and 36 s. To solve this problem using the ADMM algorithm, the real and reactive power in the coupling lines between the distribution network and microgrids, and the bus voltages at the PCCs are shared among the microgrids’ and distribution network’s operators. It is worth noting that ADMM method suffers from divergence when applied to solve non-convex optimization problems [29].

### 4.2 Case 2—Deterministic solution considering the islanding of microgrid-2

In this case, all microgrids are operating in grid-connected mode, except microgrid-2 which is islanded from the distribution network. Table 9 shows the exchanged real power between the distribution network and microgrids at hour 18. The operation costs and load curtailments for the distribution network and microgrids are shown in Table 10. Compared to Case 1,
at hour 18, the power flows on phases “a” and “c” from distribution network to microgrid-1 are reduced because of ESS operation in microgrid-1. Here, the operation cost of ESS in microgrid-1 is increased to $32.55 and the total operation cost is increased to $15,213.45 because of the increase in the operation costs of microgrid-1 and microgrid-2. The operation cost of microgrid-2 is increased to $409.99 as it is operated in the island mode. Compared to Case 1, as distribution network and microgrid-3 do not support microgrid-2, their operation costs are reduced to $14,689.95 and $80.96, respectively. The algorithm converged after 276 iterations.

4.3 Case 3—Stochastic solution considering the branch outage in microgrid-1

In this case, the stochastic solutions considering the uncertainty in demand and renewable energy resources are presented. Here, 3000 scenarios are generated using Monte Carlo simulation to capture the uncertainty in the PV generation and demand. The forecast errors in solar irradiation and demand are represented by Gaussian distribution function with the mean value equal to the forecasted values in Case 1, and the standard deviation equal to 3% of the mean values. The backward scenario reduction technique is used to reduce the number of scenarios to 10. The probability of each scenario is shown in Table 11. In this case all microgrids are operating in grid-connected mode. The expected operation costs and expected load curtailments in microgrids and distribution network are shown in Table 13. Compared to normal operating condition, the total expected operation cost is increased to $51,433.51 because of the penalty associated with the demand curtailment in microgrid-1. The total expected demand curtailments in microgrid-1 on phases “a”, “b”, and “c” are 303.270, 366.738, and 247.948 kWh, respectively. The expected operation cost of the distribution network is reduced to $14,452.74 because of the reduction in the served demand of microgrid-1. Similarly, the expected operation costs of microgrid-2 and microgrid-3 are reduced to $141.94 and $120.61, respectively. The algorithm converged after 307 iterations.

4.4 Case 4—Stochastic solution considering the islanding of microgrid-2

Here, microgrid-2 is islanded from the distribution network for 24 hours. The expected exchanged real power on three phases between the distribution network and microgrids at hour 18 are presented in Table 14. The expected operation costs and the expected demand curtailments are presented in Table 15. Compared to Case 2, the expected real power flow between the distribution network and microgrid-1 is increased because

| Scenario | 1     | 2     | 3    | 4    | 5    |
|----------|-------|-------|------|------|------|
| Probability | 0.1093 | 0.09  | 0.1077 | 0.1163 | 0.111 |

| Scenario | 6     | 7     | 8    | 9    | 10   |
|----------|-------|-------|------|------|------|
| Probability | 0.0823 | 0.0931 | 0.1023 | 0.1207 | 0.0673 |

| Operator | Expected operation cost ($) | Total expected load curtailment (kWh) |
|----------|-----------------------------|--------------------------------------|
| DNO      | 14,785.19                   | 0                                    |
| Microgrid-1 | 21.67                      | 0                                    |
| Microgrid-2 | 179.70                     | 0                                    |
| Microgrid-3 | 142.28                     | 0                                    |
| Total    | 15,128.84                   | 0                                    |

| Operator | Expected operation cost ($) | Total expected load curtailment (kWh) |
|----------|-----------------------------|--------------------------------------|
| DNO      | 14,452.74                   | 0                                    |
| Microgrid-1 | 36,718.22                  | 917.956                              |
| Microgrid-2 | 141.94                    | 0                                    |
| Microgrid-3 | 120.61                     | 0                                    |
| Total    | 51,433.51                   | 917.956                              |

| Power flow on phases (kW) | Line |
|--------------------------|------|
|                          | a    | b    | c    |
| DN-Microgrid-1           | 40.227 | 42.124 | 37.985 |
| DN-Microgrid-2           | 0     | 0    | 0    |
| DN-Microgrid-3           | -17.37 | 4.849 | 8.604 |

| Operator | Expected operation cost ($) | Total expected load curtailment (kWh) |
|----------|-----------------------------|--------------------------------------|
| DNO      | 14,672.52                   | 0                                    |
| Microgrid-1 | 22.37                     | 0                                    |
| Microgrid-2 | 409.36                    | 0                                    |
| Microgrid-3 | 132.88                    | 0                                    |
| Total    | 15,237.13                   | 0                                    |
of the increase in the total expected demand in this microgrid. Here, the total expected demand of microgrid-1 is increased by 1.366 kWh compared to Case 2. Furthermore, the total expected operation cost is increased by $23.68 as the expected power outputs of the DERs are increased to address the uncertainty in demand and PV generation.

Compared to the stochastic solution under normal condition, the total expected operation cost is increased to $15,237.13 because of the increase in the expected operation cost of microgrid-1 and the higher expected operation cost of microgrid-2 with more expensive DER. Here, microgrid-2 has enough generation capacity to supply its load, and the expected operation cost is increased to $409.36 as more expensive local generation assets are used to serve the load in the island mode. The expected operation cost of the distribution network is reduced to $14,672.52 as the demand is reduced by islanding microgrid-2. Similarly, the expected operation cost of microgrid-3 is reduced from $142.28 in normal condition to $132.88 in this case, as this microgrid will not supply microgrid-2. The algorithm converged after 224 iterations.

4.5 Case 5—Stochastic solution considering the probabilistic islanding of microgrids

In this case, the lower bound for the probability of islanding in each microgrid is 13%. The expected exchanged real powers between the microgrids and distribution network at hour 18 are shown in Table 16. Compared to the stochastic solution under normal operation, the expected power flow between the distribution network and microgrid-1 is reduced as the demand is curtailed in the islanded operation of microgrid-1 in some scenarios. Here, microgrid-1 is in island mode in scenarios 6 and 10 with the total probability of 14.96%. Microgrid-2 is in island mode in scenarios 2 and 4 with the total probability of 20.63% and microgrid-3 is in island mode in scenarios 1 and 2 with the total probability of 19.93%. The expected operation costs and the expected demand curtailments are presented in Table 17. Compared to the stochastic solution under normal condition, the expected operation cost of the distribution network is reduced to $14,659.33 as the microgrid demands are not served by the distribution network in some scenarios. The expected operation cost of microgrid-1 is increased to $13,538.49 because of the penalty associated with the loss of demand in some scenarios. In microgrid-2, as the power dispatch of the expensive DER is increased, the expected operation cost is increased to $242.31. Similarly, the expected operation cost of microgrid-3 is increased to $147.52. There is no demand curtailment in microgrid-2 and microgrid-3 and the algorithm converged after 517 iterations.

Table 18 summarizes the operation cost of the microgrids and distribution network in Cases 1–5. Furthermore, the sensitivity of the total expected operation cost to the probability of islanding in the microgrids is evaluated. Here, the probability of islanding in each microgrid is changing from 0% to 50%. As shown in Figure 8, the total expected operation cost is increased as the probability of islanding increases.

![Figure 8](image-url)  
**Figure 8** Sensitivity of the total expected operation cost to the probability of islanding in microgrids
5 | SCALABILITY AND COMPUTATION EFFICIENCY

In this section, the computation efficiency of the proposed approach is discussed. Table 19 shows the solution time of the proposed algorithm for five cases applied to the IEEE-34 bus system with the relative optimality gap $\varepsilon = 10^{-4}$. To validate the scalability of the proposed algorithm, the modified IEEE-123 bus distribution network with three unbalanced microgrids shown in Figure 9 is used. Five scenarios with the probabilities shown in Table 20 are considered. The characteristics of DER, ESS, and PV units are shown in Tables 21, 22, and 23, respectively. The marginal costs of DERs are similar to those in Table 4. Four voltage regulators are considered as shown in Figure 9. The relative optimality gap for this case is 0.01. If the probability of islanding of each microgrid is 5%, the total expected operation cost of the distribution network and microgrids is $30,422.83. The solution time for the day-ahead operation, in this case, is 12 h, 58 min, and 4 s. The solution time reduces as the optimality gap increases as shown in Table 19 and therefore, there is a trade-off between accuracy and solution time [22]. The solution time could be further decreased by solving the microgrid

![Figure 9](image)

**FIGURE 9** The modified IEEE-123 bus distribution network with networked microgrids

### TABLE 19 Solution time of the proposed algorithm for Cases 1–5

| Case                          | Tolerance (%) | Solution time       |
|-------------------------------|---------------|---------------------|
| IEEE-34 bus system—Case 1     | 0.01          | 47 min 04 s         |
| IEEE-34 bus system—Case 2     | 0.01          | 17 min 55 s         |
| IEEE-34 bus system—Case 3     | 0.01          | 2 h 41 min 42 s     |
| IEEE-34 bus system—Case 4     | 0.01          | 2 h 12 min 14 s     |
| IEEE-34 bus system—Case 5     | 0.01          | 6 h 46 min 55 s     |
| IEEE-123 bus system—Case 5    | 1             | 12 h 58 min 4 s     |
| IEEE-123 bus system—Case 5    | 2             | 9 h 38 min 19 s     |
| IEEE-123 bus system—Case 5    | 5             | 5 h 39 min 45 s     |

### TABLE 20 Probability of scenarios for the modified IEEE-123 bus system

| Scenario | 1      | 2      | 3      | 4      | 5      |
|----------|--------|--------|--------|--------|--------|
| Probability | 0.1707 | 0.1593 | 0.2217 | 0.1610 | 0.2873 |
The interaction between the distribution and microgrids' operators aims to reduce the total operation cost under multiple operating conditions. The proposed algorithm is applied to the IEEE-34 bus distribution network with three connected microgrids. It is shown that the islanding in microgrids with expensive generation resources will increase the operation cost of the microgrid while reducing the operation cost of the distribution network. Furthermore, the expected total operation cost will increase as the probability of the grid-connected operation mode of the microgrids decreases.

### Nomenclature

- \( \alpha_{i,s} \): Auxiliary variable
- \( \Delta Q_e \): The allowable variation in the state of charge of battery storage \( e \)
- \( \epsilon \): Convergence tolerance
- \( \eta_e^{ch} \): Discharging efficiency for energy storage \( e \)
- \( \eta_e^{dc} \): Charging efficiency for energy storage \( e \)
- \( \eta_{p,v} \): The efficiency of solar panel \( v \)
- \( \lambda_{m,i,t,s} \): Dual variable
- \( \gamma_{m,i,t,s} \): Dual variable
- \( B \): Set of buses in the distribution network
- \( B_f \): Set of buses connected to the distribution feeder \( f \)
- \( B_i \): Set of buses connected to bus \( i \)
- \( B_r \): Set of buses connected to the secondary of voltage regulator \( r \)
- \( D \): Set of demands in the distribution network
- \( D_m \): Set of demands in microgrid \( m \)
- \( \mathcal{E} \): Set of energy storage units in the distribution network
- \( \mathcal{E}_m \): Set of energy storage units in microgrid \( m \)
- \( \mathcal{H} \): Set of DERs in the distribution network
- \( \mathcal{H}_m \): Set of DERs in microgrid \( m \)
- \( \mathcal{K}_m \): Set of buses in microgrid \( m \)
- \( R_{r,t,s} \): Solar irradiation at time \( t \) and scenario \( s \)
- \( \mathcal{V} \): Set of photovoltaic generation units in the distribution network
- \( \mathcal{V}_m \): Set of photovoltaic generation units in microgrid \( m \)
- \( \mu_{i,m} \): Islanding probability for microgrid \( m \)
- \( \phi \): Index of phase
- \( \psi_{p,v}^{e} \): Availability of phase \( \phi \) on PV panel \( v \)
- \( \rho_{e} \): Operation cost of battery storage \( e \)
- \( \rho_{f} \): Hourly price of electricity for feeder \( f \)
- \( \tau \): Index of iteration
- \( A_i^{(s)} \): Element of unit-bus incidence matrix in the distribution network
- \( d \): Index of demand
- \( e \): Index of energy storage unit
- \( E_{e,\text{max}} \): Maximum available energy in battery storage \( e \)
- \( E_{e,\text{min}} \): Minimum available energy in battery storage \( e \)
- \( E_{e,t,s}^{\phi} \): Available energy in battery energy storage \( e \) connected to phase \( \phi \) at time \( t \) and scenario \( s \)
- \( f \): Index of distribution feeder
- \( F_{g}(\cdot) \): Production cost of DER \( g \)
- \( i_{i,j} \): Index of distributed generation unit
- \( i_{i,j} \): Index of bus
- \( l_{i,j} \): Availability of line \( i-j \)
- \( m \): Index of microgrid

### Conclusion

In this paper, a decentralized operation framework for the distribution network and microgrids is proposed that leverages Bender's decomposition technique. The algorithm aims to coordinate the operation decisions between the distribution network and the microgrids considering the unbalanced demand and PV generation in the distribution network. The uncertainties in PV generation and demand in microgrids and distribution network are captured using Monte Carlo simulation and the risk associated with the islanded operation of microgrids was addressed. The interaction between the distribution and microgrids’ operators aims to reduce the total operation cost under multiple operating conditions. The proposed algorithm is applied to the IEEE-34 bus distribution network with three connected microgrids. The modified IEEE-123 bus distribution network with three unbalanced microgrids is also used to validate the performance of the proposed algorithm. The impact of the

### References

1. [Alobaidi et al. (2023)]
2. [Bender’s decomposition technique](https://example.com)
3. [Monte Carlo simulation](https://example.com)
4. [Microgrid islanding](https://example.com)

### Tables

**TABLE 21** Dispatchable resources' characteristics in the modified IEEE-123 bus system

| Unit          | Bus | P_{min} (kW) | P_{max} (kW) | Q_{min} (kVAR) | Q_{max} (kVAR) |
|---------------|-----|--------------|--------------|----------------|----------------|
| Distribution feeder | 1   | 0            | 3500         | -2500          | 2500           |
| DER1          | 24  | 0            | 100          | -50            | 50             |
| DER2          | 8  | 0            | 80           | -70            | 70             |
| DER3          | 44  | 0            | 90           | -70            | 70             |

**TABLE 22** ESS characteristics in the modified IEEE-123 bus system

| Unit | Bus | P_{max} (kW) | Q_{max} (kVAR) | E_{min} (kWh) | E_{max} (kWh) |
|------|-----|--------------|----------------|----------------|---------------|
| ESS1 | 48  | 50           | 25             | 10             | 100           |
| ESS2 | 44  | 60           | 30             | 10             | 120           |
| ESS3 | 125 | 40           | 20             | 5              | 60            |
| ESS4 | 127 | 30           | 15             | 5              | 50            |

**TABLE 23** PV units' characteristics in the modified IEEE-123 bus system

| Unit | Bus | P_{min} (kW) | P_{max} (kW) | Q_{min} (kVAR) | Q_{max} (kVAR) |
|------|-----|--------------|--------------|----------------|----------------|
| PV1  | 10  | 0            | 100          | -80            | 80             |
| PV2  | 31  | 0            | 100          | -90            | 90             |
| PV3  | 18  | 0            | 100          | -50            | 50             |
| PV4  | 25  | 0            | 200          | -100           | 100            |
| PV5  | 40  | 0            | 100          | -50            | 50             |
| PV6  | 125 | 0            | 50           | -40            | 40             |
| PV7  | 127 | 0            | 65           | -55            | 55             |
| PV8  | 129 | 0            | 75           | -60            | 60             |
$P^\phi_{(i,j,s)}$ Real power dispatch on phase $\phi$ at time $t$ in scenario $s$

$P_{\text{max}}^\phi$ Maximum real power dispatch of a unit on phase $\phi$

$P_{\text{min}}^\phi$ Minimum real power dispatch of a unit on phase $\phi$

$P_{i,j,s}^{\phi,+}$ Real power of energy storage $\epsilon$ in charging mode on phase $\phi$ at time $t$ in scenario $s$

$P_{i,j,s}^{\phi,-}$ Real power of energy storage $\epsilon$ in discharging mode on phase $\phi$ at time $t$ in scenario $s$

$P_{(i,j,s)}$ Availability of phase $\phi$ on line $i$-$j$.

$P_{i,j,s}^\phi$ Real power demand on phase $\phi$ at time $t$ in scenario $s$

$P_{\text{max}}^\phi$ Served real power demand on phase $\phi$ at time $t$ in scenario $s$

$P_{(i,j,s)}^\phi$ Real power flow of line $ij$ on phase $\phi$ at time $t$ in scenario $s$

$Q_{(i,j,s)}$ Probability of scenario $s$

$Q_{\text{max}}^\phi$ Reactive power dispatch on phase $\phi$ at time $t$ in scenario $s$

$Q_{\text{min}}^\phi$ Maximum reactive power dispatch of a unit on phase $\phi$

$Q_{i,j,s}^\phi$ Minimum reactive power dispatch of a unit on phase $\phi$.

$Q_{(i,j,s)}^\phi$ Reactive power demand on phase $\phi$ at time $t$ in scenario $s$.

$Q_{i,j,s}^\phi$ Served reactive power demand on phase $\phi$ at time $t$ in scenario $s$.

$Q_{(i,j,s)}^\phi$ Reactive power flow of line $i$-$j$ on phase $\phi$ at time $t$ in scenario $s$.

$r$ Index of voltage regulator.

$r_{ij}$ $X_{ij}$ Resistance and reactance of line $i$-$j$.

$s$ Index of scenario.

$S_{ij}^{\max}$ The maximum apparent power of the distribution feeder $f$.

$S_r$ The total area of solar PV panel $r$.

$T$ The total number of hours.

$t$ Index of time.

$U_{(i,j,s)}^\phi$ Squared voltage at bus $j$ on phase $\phi$ at time $t$ in scenario $s$.

$v$ Index of photovoltaic generation unit.

$V_{i,s}^{\max}$ Maximum allowable voltage at bus $j$.

$V_{i,s}^{\min}$ Minimum allowable voltage at bus $j$.

$V_{\text{OLL}}$ Value of lost load.

$Y_{(i,k,s)}$ The binary variable representing the microgrid operation mode connected to bus $i$ in the distribution network.

$Z_{(i,j,s)}^{\min}$ Slack variable.

$Z_{(i,j,s)}^{\text{lower}}$ The lower bound of the solution.

$Z_{(i,j,s)}^{\text{upper}}$ The upper bound of the solution.

$N_{m(c)}$ Elements of unit-bus incidence matrix in microgrid $m$.

$S_{i,j}^{\max}$ The maximum apparent power of line $i$-$j$.

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