Angle position influences over external magnetic field of a solenoid in wireless power transfer

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Abstract: The general form of the Biot-Savart law is referring to a closed loop of wire, but it can be integrated over a finite length solenoid to calculate the external magnetic field in a specific point, at a rated distance from the centre. The law states that the point of measure must be on the longitudinal axis of the solenoid. In real cases the point of measure has angle orientation in relation to the axis which results in modifications of the field value.

1. Introduction

A finite length of conducting wire coiled around an axis results in a closed loop and a multiple number of closed loops give a solenoid, or a coil. This is the basic principle of calculating the exterior magnetic field generated by a solenoid using the Biot-Savart law.

The magnetic field has the strongest value inside any solenoid, especially if it has a core made of conductive material. If the coil does not have a conductive core, the maximum magnetic field generated is located on the half distance between the length edges [1]. External magnetic field is calculated integrating the general form of Biot-Savart law over the total length of the solenoid, but is important that the point of measure to be on the transversal axis of solenoid.

Jean-Baptiste Biot and Félix Savart formulated the law around 1820, which is the fundamental formula for calculating the magnetic field. It corresponds to Coulomb's law for calculating the electric field. The Biot-Savart law is an equation describing the magnetic field generated by an electric current as a vector that has magnitude direction and length [2].

The purpose of this paperwork is to show the calculation of the exterior magnetic field produced by a solenoid using Biot-Savart law and the location impact of the point of measure. In real cases, the point of measure is not on the centre axis, but it has an angle orientation, which brings to modifications in the value of the field.

2. Mathematical Analysis

The basic principle used in finding the value of magnetic field is to split the solenoid into small parts known as elementary components.

The general form of Biot-Savart law is referring to a finite length of conductive wire [3]:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I \cdot d\mathbf{S} \times \hat{r}}{r^2}$$  \hspace{1cm} (1)
Figure 1. Vectorial components of exterior magnetic field of a solenoid

A solenoid with finite length $l$ and the centre at $l/2$ will have the point of measure $P$ located at $z$ distance from the centre on the Oz axis.

First, the general formula is derived for a single spire (closed loop). This is possible by taking into consideration any loop $dz'$, located at a random distance $z'$ from the centre of solenoid and $(z-z')$ distance from the point of measure and calculating the locations of corresponding vectors [4] [5].

$$d\vec{B} = \frac{\mu_0 \cdot I \cdot R}{4\pi \cdot (R^2 + (z - z')^2)^{3/2}} \cdot \left( (z - z') \cdot \cos \theta \cdot \hat{i} + (z - z') \cdot \sin \theta \cdot \hat{j} + R \cdot \hat{k} \right) \cdot d\theta \quad (2)$$

Because the point of measure has a deviation from the central axis, the angular dependence, $\cos \beta$, has been introduced [4] [5]:

$$d\vec{B} = \frac{\mu_0 \cdot I \cdot R}{4\pi \cdot (R^2 + (z - z')^2)^{3/2}} \cdot \left( \cos \theta \cdot \frac{(z - z')}{\cos \beta} \cdot \hat{i} + \sin \theta \cdot \frac{(z - z')}{\cos \beta} \cdot \hat{j} + R \cdot \hat{k} \right) \cdot d\theta \quad (3)$$

where:

- $R$ is the radius of loop
- $I$ is the current passing through solenoid

The current $I$ that pass through solenoid will distribute uniformly on all the spires, so that leads to a new derived form:

$$dI = I \cdot (n \cdot dz') \quad (4)$$

and

$$n = \frac{N}{l} \quad (5)$$

where $N$ is the total spire number of solenoids.

After splitting the result into Cartesian axis $xOyOz$, represented by the corresponding versors $\hat{i}, \hat{j}, \hat{k}$ the only component that will have value is $B_z$ as the others are null because of $\sin \theta$ and $\cos \theta$:

$$B_z = \frac{\mu_0 \cdot I \cdot R^2}{4\pi \cdot \left( R^2 + \frac{(z - z')^2}{\cos^2 \beta} \right)^{3/2}} \quad (6)$$
The contribution of this element will be derived in function to $I$ to find the field value for all the loops \[2\] \[3\]:

$$
\begin{align*}
\frac{dB_z}{dz} &= \mu_0 \cdot \frac{R^2}{4\pi \left( R^2 + \frac{(z-z')^2}{\cos^2 \beta} \right)^{3/2}} \cdot dI = \frac{\mu_0 \cdot R^2}{4\pi \left( R^2 + \frac{(z-z')^2}{\cos^2 \beta} \right)^{3/2}} \cdot (n \cdot I \cdot dz') \\
B_z &= \frac{\mu_0 \cdot R^2}{2} \cdot \int_{\frac{l}{2}}^{l} \frac{1}{\left( R^2 + \frac{(z-z')^2}{\cos^2 \beta} \right)^{3/2}} \cdot dz'
\end{align*}
$$

(7)

The $dz'$ element is characterized by $R$ and the $(z-z')$ which is the relative distance of the spire to the point of measure.

The result is integrated over the entire length of the solenoid, which leads to a possible final form of the magnetic field value.

$$
B_z = \frac{\mu_0 \cdot R^2}{2} \cdot \int_{\frac{l}{2}}^{l} \frac{1}{\left( R^2 + \frac{(z-z')^2}{\cos^2 \beta} \right)^{3/2}} \cdot dz'
$$

(8)

3. Results

The basic principle used in finding the value of magnetic field is to split the solenoid into small parts known as elementary components.

$$
u = \frac{z-z'}{\cos \beta}
$$

(9)

and the derived form becomes:

$$
dz' = -\cos \beta \cdot du
$$

(10)

After replacement the integral has a simpler form, consisting of values known on left side and an integral with complex number on right side:

$$
B_z = -\frac{\mu_0 \cdot R^2}{2} \cdot \int_{\frac{l}{2}}^{l} \cos \beta \cdot \frac{du}{\left( R^2 + u^2 \right)^{3/2}}
$$

(11)

A trigonometric form is desired to simplify more the equation, so the complex function number derived becomes:

$$
u = R \cdot \tan \rho
$$

(12)

and his derived form:

$$
du = R \cdot \sec^2 \rho \cdot d\rho
$$

(13)

With this solution, the magnetic field form becomes:

$$
B_z = -\frac{\mu_0 \cdot R^2}{2} \cdot \int_{\frac{l}{2}}^{l} \frac{\cos \beta \cdot \sec^2 \rho \cdot R \cdot (\sec^2 \rho \cdot R) \cdot d\rho}{\left( R^2 + \tan^2 \rho \right)^{3/2}}
$$

(14)

Using the basic trigonometric formulas:

$$
\tan^2 \rho + 1 = \sec^2 \rho
$$

(15)
and

\[
\frac{1}{\sec \rho} = \cos \rho \tag{16}
\]

a new simple form is given:

\[
B_z = -\frac{\mu_0 \cdot R^2 \cdot n \cdot I}{2} \int_2^1 \cos \rho \cdot d\rho \tag{17}
\]

After computing the notes, the final form of the magnetic field, with angle position influences is given:

\[
B_z = \frac{\mu_0 \cdot R^2 \cdot n \cdot I}{2} \cos \beta \cdot \frac{z' - z}{R^2 + \left(\frac{z' - z}{\cos \beta}\right)^2} \tag{18}
\]

4. Conclusions

The magnetic field is null on the Ox and Oy axis because of the dependence to the angle \( \theta \) and the Oz component, also it decreases exponential in relation to Euclidian distance from the source to the point of measure.

If the point of measure is not on the central transversal axis of the solenoid, the magnitude of the field is measured with the help of the corresponding axial component. In this case, the \( \cos \beta \) that appears increases the Euclidian distance, which causes a substantial drop of the field.

Also, if the angle between and reaches \( \pi/2 \) radians (the point of measure is perpendicular to the solenoid) the field value drops to 0 [5]:

\[
d\vec{B} = \cos \alpha \cdot d\vec{B}_z \tag{19}
\]

All expressions presented will help develop a general law, which can calculate the exterior magnetic field at a given point and to understand why the axial components are canceled. The differences between ideal and real cases help having a better knowledge of the field value and what is it impact of the orientation angle.

References

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