Theory of internal transitions of charged excitons in quantum wells in magnetic fields

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For charged semiconductor complexes in magnetic fields $B$, we discuss an exact classification of states, which is based on magnetic translations. In this scheme, in addition to the total orbital angular momentum projection $M_z$ and electron and hole spins $S_e$, $S_h$, a new exact quantum number appears. This oscillator quantum number, $k$, is related physically to the center of the cyclotron motion of the complex as a whole. In the dipole approximation $k$ is strictly conserved in magneto-optical transitions. We discuss implications of this new exact selection rule for internal intraband magneto-optical transitions of charged excitons $X^-$ in quantum wells in $B$.

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Recently, there has been considerable experimental and theoretical interest in charged excitons $X^-$ and $X^+$ in magnetic fields $B$ in 2D systems. Experimentally, magneto-optical interband transitions of charged excitons have been studied extensively. Theoretically, the binding of charged excitons $X^-$ has been considered in quantum dots [3], in a strictly-2D system in the high-magnetic field limit [3], and in realistic quantum wells at finite $B$ [4]. In all these theoretical works on charged excitons, the existing exact symmetry — magnetic translations — has not been identified. The aim of the present theoretical work is to describe in some detail this symmetry and its manifestations in intraband magneto-optical transitions (see also [4]). Experimental evidences for internal $X^-$ singlet and triplet transitions have been reported very recently in [5], where also comparison with quantitative calculations is presented.

We consider a system of interacting particles of charges $e_j$ in a magnetic field $B = (0, 0, B)$ described by the Hamiltonian

$$H = \sum_j \frac{\hat{\pi}_j^2}{2m_j} + \frac{1}{2} \sum_{ij \neq j} U_{ij}(\mathbf{r}_i - \mathbf{r}_j),$$

(1)

here $\hat{\pi}_j = -i \hbar \nabla_j - e_j A(\mathbf{r}_j)$ is the kinematic momentum operator of the $j$-th particle in $B$ and $U_{ij}$ are the potentials of interactions that can be rather arbitrary. Dynamical symmetries of (1) are the following. In the symmetric Hamiltonian $V$, the Hamiltonian (1) is also invariant under a group of magnetic translations whose generators are the components of the operator $\mathbf{K} = \sum_j \mathbf{K}_j$, where $\mathbf{K}_j = \hat{\pi}_j - \frac{e_j}{\hbar} \mathbf{r}_j \times \mathbf{B}$ (see, e.g., [4]). $\mathbf{K}$ is the exact integral of the motion: $[H, \mathbf{K}] = 0$. The components of $\mathbf{K}$ and $\hat{\pi} = \sum_j \hat{\pi}_j$ commute in $B$ as

$$[\hat{K}_x, \hat{K}_y] = -[\hat{\pi}_x, \hat{\pi}_y] = -i \frac{\hbar B}{c} Q,$$

(2)

while $[\hat{K}_x, \hat{K}_y] = 0$, $p, q = x, y$. For neutral complexes (atoms, excitons, biexcitons) $Q = 0$, and classification of states in $B$ are due to the continuous two-component vector — the 2D magnetic momentum $\mathbf{K} = (K_x, K_y)$. For charged systems the components of $\mathbf{K}$ cannot be observed simultaneously. This determines the macroscopic Landau degeneracy of exact eigenstates of (1). For a dimensionless operator $k = \sqrt{e/\hbar B} |Q|$, $\mathbf{K}$ we have $[\hat{k}_x, \hat{k}_y] = -i Q/|Q|$. Therefore, $\hat{k}_\pm = (\hat{k}_x \pm i \hat{k}_y)/\sqrt{2}$ are Bose raising and lowering ladder operators: $[\hat{k}_+, \hat{k}_-] = -Q/|Q|$. It follows then that $k^2 = \hat{k}_+ \hat{k}_- + \hat{k}_+ \hat{k}_+$ has discrete oscillator eigenvalues $2k + 1$, $k = 0, 1, \ldots$. Since $[k^2, H] = 0$ and $[\hat{k}^2, \hat{L}_z] = 0$, the exact charged eigenstates of (1), in addition to the electron $S_e$ and hole $S_h$ spin quantum numbers, can be simultaneously labeled by the discrete quantum numbers $k$ and $M_z$. The labelling therefore is $|k M_z S_e S_h \nu \rangle$. Here $\nu$ is the “principal” quantum number, which can be discrete (bound states) or continuous (unbound states forming a continuum) [4]. The $k = 0$ states are Parent States (PS’s) within a degenerate manifold. All other Daugther states in each $\nu$-th family are generated out of the PS iteratively: for $Q < 0$ $|k, M_z - k, S_e S_h \nu \rangle = (\hat{k}_-)^{|k|} |0, M_z, S_e S_h \nu \rangle/\sqrt{|k|}$. Let us discuss now magneto-optical transitions of charged complexes. In the dipole approximation the photon momentum is negligibly small. Therefore, the quantum number $k$ should be conserved in intra- and interband magneto-optical transitions. For interband transitions in a translationally-invariant system with a simple valence band this leads to a striking result [5] that the ground triplet $X^- (S_e = 1)$ state is dark in photolumines-
The states are labeled by the total angular momentum projection $M_z$ and oscillator quantum number $k$. Separation between the 2D $e$- and $h$-layers $d = 0$ (see inset to Fig. 2). The energy $E_0 = \sqrt{\pi/2} e^2/\epsilon l_B$ parametrizes the 2D system in the limit of high $B$. Large (small) dots correspond to the bound parent $k = 0$ (daughter $k = 1, 2, \ldots$) $X^-$ states; see text for further explanations.

To show this we shall consider the system of strictly-$2D$ $e$- and $h$-layers separated by a distance $d$ (see inset). Shown are the cases of three different separations $d = 0, 0.3l_B, \text{and } 0.7l_B$ between the strictly-$2D$ $e$- and $h$-layers (see inset). A filled dot shows the position of the forbidden bound-to-bound $X_{100} \rightarrow X_{110}$ transition for $d = 0$.

FIG. 2. Energies and dipole matrix elements of the inter-LL transitions from the ground $X_{100}$ state in the high-field limit. Shown are the cases of three different separations $d = 0, 0.3l_B, \text{and } 0.7l_B$ between the strictly-$2D$ $e$- and $h$-layers (see inset). A filled dot shows the position of the forbidden bound-to-bound $X_{100} \rightarrow X_{110}$ transition for $d = 0$.

[Diagram of two-dimensional system showing energy levels and transitions.]

FIG. 1. Schematic drawing of bound and scattering electron triplet $2e-h$ states in the lowest LL’s $(N_eN_h)=(00), (10)$. The states are labeled by the total angular momentum projection $M_z$ and oscillator quantum number $k$. Separation between the 2D $e$- and $h$-layers $d = 0$ (see inset to Fig. 2). The energy $E_0 = \sqrt{\pi/2} e^2/\epsilon l_B$ parametrizes the 2D system in the limit of high $B$. Large (small) dots correspond to the bound parent $k = 0$ (daughter $k = 1, 2, \ldots$) $X^-$ states; see text for further explanations.

To show this we shall consider the system of strictly-$2D$ $e$- and $h$-layers separated by a distance $d$ (see inset to Fig. 2) in the limit of high magnetic fields. To understand internal $X^-$ transitions, it is necessary to consider the eigenstates associated with higher Landau levels (LL’s). To this end we use expansion in free LL’s, which has been described in some detail elsewhere. The calculated three-particle $2e-h$ eigenspectra (electrons in the triplet state) in the two lowest LL’s are shown for $d = 0$ in Fig. 1. Generally, the eigenspectra associated with each LL consist of bands of finite width $\sim E_0 = \sqrt{\pi/2} e^2/\epsilon l_B$, where $l_B = (\hbar c/eB)^{1/2}$. The states within each such band form a continuum corresponding to the extended motion of a neutral magnetoelectron $e$- ($X^-$) as a whole with the second electron in a scattering state. As an example, the continuum in the lowest $(N_eN_h)=(00)$ LL consists of the MX band of width $E_0$ extending down in energy from the free $(00)$ LL. This corresponds to the 1s MX $(N_e = N_h = 0)$ plus a scattered electron in the zero LL, labeled $X_{00} + e_0$. The structure of the continuum in the $(N_eN_h)=(10)$ LL is more complicated: in addition to the $X_{00} + e_1$ band of the width $E_0$, there is another MX band of width $0.574E_0$ also extending down in energy from the free $(N_eN_h)=(10)$ LL. This corresponds to the $2p^+ e$ exciton $(N_e = 1, N_h = 0)$ plus a scattered electron in the $N_e = 0$ LL, labeled $X_{110} + e_0$. There are also bands (not shown in Fig. 1) above each free LL originating from the bound internal motion of two electrons in the absence of a hole. Internal transitions to such bands have extremely small oscillator strengths and not discussed here. Bound $X^-$ states (finite internal motions of all three particles) lie outside the continua (Fig. 1). In the limit of high $B$ the only bound $X^-$ state in the zeroth LL $(N_eN_h)=(00)$ is the $X^-$-triplet. There are no bound $X^-$-singlet states in contrast to the $B = 0$ case. The $X^-$-triplet binding energy in zero LL’s $(N_eN_h)=(00)$ is $0.043E_0$. In the next electron LL $(N_eN_h)=(10)$ there are no bound $X^-$-singlets, and only one bound triplet state $X_{100}^-$, lying below the lower edge...
of the MX band \([1]\). The \(X_{10}^-\) binding energy is 0.086\(E_0\), twice that of the \(X_{00}\), and similar to the stronger binding of the \(D^-\)-triplet in the \(N_c = 1\) LL \([3]\).

We focus here on internal transitions in the \(\sigma^+\) polarization governed by the usual selection rules: spin conserved, \(\Delta M_z = 1\). In this case the \(e\)-CR–like inter-LL (\(\Delta N_c = 1\)) transitions are strong and gain strength with \(B\). Both bound-to-bound \(X_{00} \rightarrow X_{10}^-\) and photoionizing \(X^-\) transitions are possible. For the latter the final three-particle states in the (10) LL belong to the continuum (Fig.1), and calculations show that the FIR absorption spectra reflect its rich structure \([5\)\]. Transitions to the \(X_{00} + e_1\) continuum are dominated by a sharp onset at the edge (transition 1) at an energy \(\hbar \omega_{ce}\) plus the \(X_{00}^-\) binding energy. In addition, there is a broader and weaker peak corresponding to the transition to the \(X_{01} + e_0\) MX band, transition 2. The latter may be thought of as the \(1s \rightarrow 2p^+\) internal transition of the MX \([10\)\], which is shifted and broadened by the presence of the second electron. In accordance with this picture, it is visible from Fig. 2 that with increasing separation \(d\) between the \(e\)- and \(h\)-layers (when the exciton binding and, thus, transition energies are reduced and the \(X^-e\) interaction is effectively diminished), the second peak is redshifted and sharpened. Thus the \(X^-\) triplet behaves physically in the photoionizing bound-to-continuum transitions as an exciton that very loosely binds an electron, and the two “parts” of the complex can absorb the FIR photon, to some extent, independently. The double-peak structure of the bound-to-continuum transitions is a generic feature for transitions from both the singlet and triplet ground \(X^-\) states in quasi-2D systems in strong \(B\). Such transitions in translationally invariant systems are discussed theoretically in \([3\)\], where also experimental results for bound-to-continuum transitions are reported and comparison between theory and experiment is made.

The inter-LL bound-to-bound transition, \(X_{00}^- \rightarrow X_{10}^-\), has a very specific spectral position: since the final state is more stronger bound, it lies below the \(e\)-CR energy \(\hbar \omega_{ce} = \hbar B/m_e c\). However, it has exactly zero oscillator strength, a manifestation of the magnetic translational invariance: the two selection rules – conservation of \(k\) and \(\Delta M_z = 1\) cannot be satisfied simultaneously. Indeed, e.g., the \(X_{00}^-\) PS (with \(k = 0\)) has \(M_z = -1\), while the \(X_{10}^-\) PS has \(M_z = 1\), so that the usual selection rule \(\Delta M_z = 1\) cannot be satisfied. Localization of charged excitons breaks translational invariance and relaxes the \(k\)-conservation rule. As a result, the bound-to-bound \(X_{00} \rightarrow X_{10}^-\) transition, which is prohibited in translationally-invariant systems, develops below the \(e\)-CR \([3\)\]. Such a peak is a tell-tale mark of localization of charged triplet excitons.

In conclusion, we have studied the exact symmetry — magnetic translations — for charged excitons in \(B\) and established its consequences for intraband magneto-optical transitions. In particular, we have shown that in translationally invariant quasi-2D system with a simple valence band the bound-to-bound transition from the triplet ground state \(X^-\) to the next electron Landau level is prohibited. In the presence of translationally-breaking effects (disorder, impurities etc.) the intraband bound-to-bound triplet transition develops below the electron cyclotron resonance. This suggests a method of studying localization of charged excitons.

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