Inflation driven by scalar field and solid matter

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Abstract

Solid inflation is a cosmological model where inflation is driven by fields which enter the Lagrangian in the same way as body coordinates of a solid matter enter the equation of state, spontaneously breaking spatial translational and rotational symmetry. We construct a simple generalization of this model by adding a scalar field with standard kinetic term to the action. In our model the scalar power spectrum and the tensor-to-scalar ratio do not differ from the ones predicted by the solid inflation qualitatively, if the scalar field does not dominate the solid matter. The same applies also for the size of the scalar bispectrum measured by the non-linearity parameter, although our model allows it to have different shapes. The tensor bispectra predicted by the two models do not differ from each other in the leading order of the slow-roll approximation. In the case when contribution of the solid matter to the stress-energy tensor is much smaller than the contribution from the scalar field, the tensor-to-scalar ratio and the non-linearity parameter are amplified by factors $\epsilon^{-1}$ and $\epsilon^{-2}$ respectively.

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1 Introduction

Amongst numerous inflationary models there is a significant subgroup of single-field ones. In the simplest of them the primordial perturbations generated during inflation have a nearly flat spectrum and a small level of non-Gaussianity which arises only from non-linearities of the Einstein–Hilbert action and higher powers appearing in the potential of the scalar field [1]. On the other hand, in more complicated single-field models a significant level of non-Gaussianity is generated. For instance, models with non-canonical kinetic term [2] produce "orthogonal" non-Gaussianity described by bispectrum with positive peak at "equilateral" configuration of momenta ($k_1 \approx k_2 \approx k_3$) and negative peak at "folded" configuration ($k_1 \approx k_2 + k_3$). Examples with particular choices of non-canonical kinetic term are $k$-inflation [3] and Dirac–Born-Infeld (DBI) inflation [4, 5] with "equilateral" shape of bispectrum. Models with higher derivative interactions may generate either "folded" [2, 6] or "equilateral" type of non-Gaussianity. The latter case includes ghost inflation [7] and models arising from effective field theories [8]. "Folded" shape of bispectrum appears in models with non-Bunch-Davies vacuum [9, 10] as well.

Multi-field models of inflation lead to "local" non-Gaussianity peaking at "squeezed" configuration of momenta ($k_1 \approx k_2 \gg k_3$). One of less standard examples of such models is solid
inflation \cite{11,12}, driven by three-component scalar field $\phi^I$ which enter the Lagrangian in the same way as body coordinates of solid matter enter the equation of state, so that the matter action has to be invariant under internal translations and rotations,

$$\phi^I \rightarrow M^I_J \phi^J, \quad \phi^I \rightarrow \phi^I + C^I,$$

$$M^I_J \in SO(3), \quad C^I \in \mathbb{R}^3, \quad I,J = 1,2,3,$$  

where the capital indices are raised and lowered by the Euclidean metric. The simplest possible background configuration,

$$\phi^I = \delta^I_i x^i,$$  

$x^i$ being spatial coordinates, breaks the spatial translational and rotational symmetry, but in a flat universe it is invariant under the combined spatial-internal transformations. As shown by Endlich et al. \cite{12}, in this model there appears anisotropic dependence of the scalar bispectrum on how the squeezed limit is approached. Further development of the theory includes \cite{13,14,15,16}.

Apart from the inflationary models the idea of solid matter as one of the matter components present in the universe was studied in an attempt to give an alternative explanation of the accelerated expansion of the universe, see \cite{17,18,19,20,21}. This can be obtained by replacing the dark energy with a solid with negative pressure to energy density ratio and an important example of how such solid can be materialized are cosmic strings and domain walls \cite{22,23,24}.

In this paper we study a combined inflationary model including scalar field $\phi$ with standard kinetic term and three-component scalar field $\phi^I$ with symmetries defined above. Similar approach can be found in \cite{25} where the authors study a model with special form of equation of state of the solid but non-trivial coupling of scalar fields to gravity. In our Lagrangian

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + F(\phi, X, Y, Z),$$  

there is no non-trivial coupling of scalar fields to gravity, but we keep the form of the equation of state as general as possible, omitting derivative couplings only. Variables $X$, $Y$ and $Z$ are three independent quantities invariant under transformations \eqref{1.1}, for which we adopt definitions from \cite{12}:

$$X = B^{IJ}, \quad Y = \frac{B^{IJ} B^{IJ}}{X^2}, \quad Z = \frac{B^{IJ} B^{IK} B^{JK}}{X^3}, \quad B^{IJ} = -g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J,$$  

where $B^{IJ}$ is the body metric. (We have changed its sign in order to reconcile it with the signature of the metric tensor $(+ - - -)$, which we use throughout the paper.) Our model represents a straightforward combination of the solid inflation and the basic single-field models.

In section 2 we study evolution of the universe in which the cosmological perturbations are absent, section 3 contains summary of the perturbation theory including the quadratic actions for the scalar and tensor perturbations, and the detailed analysis of the scalar perturbations can be found in section 4, where also the scalar spectrum is derived. Section 5 is dedicated to the scalar bispectrum and the analysis of the tensor perturbations including the tensor spectrum and bispectrum is presented in section 6. In the last section we discuss our main results.

2 The unperturbed universe

In this section we provide an analysis of our inflationary model for the unperturbed case with flat Friedmann–Robertson–Walker–Lemaître (FRWL) metric

$$ds^2 = dt^2 - a^2 \delta_{ij} dx^i dx^j,$$  

(2.1)
where \( a = a(t) \) is the scale factor. The invariants \( X, Y \) and \( Z \) are then

\[
X = 3a^{-2}, \quad Y = 1/3, \quad Z = 1/9.
\] (2.2)

For two variables describing the universe, scale factor \( a(t) \) and scalar field \( \varphi(t) \), we have equations of motion serving as background equations in the perturbed theory,

\[
\dot{\varphi}^2 - 6M^2_{Pl}H^2 = 2F, \quad (2.3)
\]

\[
\varphi^2 + 2M^2_{Pl}\dot{H} = 2a^{-2}F_X, \quad (2.4)
\]

\[
\ddot{\varphi} + 3H\dot{\varphi} = F_\varphi, \quad (2.5)
\]

where the subscripts stand for partial derivatives, \( H = \dot{a}/a \) is the Hubble parameter and the dot denotes the time derivative. Of course, due to the Bianchi identity only two of these equations are independent. We can also see that in the unperturbed case the model is described by only two fields \( X \) and \( \varphi \), the former given in terms of the scale factor by the first relation in (2.2).

This is a manifestation of convenience of the definition (1.4). Note that the single-field model with potential dependent on the scale factor, which uses the same variables, differs from our model even in the absence of perturbations.

For different functions \( F \) there are different solutions of the background equations and a useful quantity measuring the deviation from the de Sitter solution is the slow-roll parameter

\[
\epsilon = -\frac{\dot{H}}{H^2}. \quad (2.6)
\]

Using equations (2.3) and (2.4) we find

\[
\epsilon = p + q - \frac{1}{3}pq, \quad p = \frac{\dot{\varphi}^2}{2M^2_{Pl}H^2}, \quad q = X\frac{F_X}{F}, \quad (2.7)
\]

where \( p \) and \( q \) are the slow-roll parameters of the single-field inflation and the solid inflation respectively. In our combined model we have an additional degree of freedom, so that the slow-roll parameter can be small also for finite values of the parameters \( p \) and \( q \). The region of the parameter space in which the slow-roll parameter is small and positive (so that superinflation is excluded) is depicted in fig. 1. Relation (2.7) can be rewritten in terms of pressure to energy

![Fig. 1: The slow-roll parameter is small near the hyperbolic contour given by relation (2.7). The hyperbola has asymptotics at \( p = 3 \) and \( q = 3 \).](image-url)
density ratios as
\[ 2w - 1 = w_\varphi + w_s - w_\varphi w_s, \quad (2.8) \]
where \( w_\varphi = 2p/3 - 1 \) and \( w_s = 2q/3 - 1 \) denote pressure to energy density ratio of the scalar field and the solid respectively, while \( w = 2\epsilon/3 - 1 \) is the overall pressure to energy density ratio of the system consisting of these two components. Similarly as with the dependence of \( \epsilon \) on \( p \) and \( q \), we can see that \( w \) can be close to \(-1\), in order to allow nearly de Sitter background solution, not only if \( w_\varphi \approx w_s \approx -1 \) but also for a wide range of parameters \( w_\varphi \) and \( w_s \), as long as the relation \( w_\varphi + w_s - w_\varphi w_s \approx -3 \) is satisfied.

The inflationary expansion, either exponential or power-law \([26, 27]\), requires that the slow-roll parameter \( \eta \) is not only small, but also has small enough time derivative. Thus, we need another slow-roll condition, \( |\eta| \ll 1 \), where
\[ \eta = \frac{\dot{\epsilon}}{\epsilon H}. \quad (2.9) \]

With the use of the background equations and definitions of slow-roll parameters we find
\[ 1 + \frac{1}{2} \eta - \epsilon = \frac{1}{3\varphi^2 - 2XF_X} \left( \sqrt{\frac{6M_P^2}{\varphi^2 - 2F}(3F_\varphi - XF_{X,\varphi})\dot{\varphi} + 2(2X^2F_{XX} - 3\varphi^2)} \right). \quad (2.10) \]

The background solution is given by two functions \( a(t) \) or \( X(t) \) and \( \varphi(t) \), and attempting to make the analysis of the inflationary solutions more transparent, we have replaced them by \( p \) and \( q \), which are functions of time as well. On the other hand, the theory under consideration is effectively described by a function of two variables \( F(X, \varphi) \). Therefore, it is possible that different functions \( F(X, \varphi) \) lead to the same solution \( \{X(t), \varphi(t)\} \), while it is also possible in principle that for some functions \( X \) and \( \varphi \) the corresponding function \( F \) does not exist, so that such pair of functions cannot be the background solution. The reason why the latter case cannot happen is summarized in the following paragraphs.

Function \( F \) corresponding to a given background solution \( \{X(t), \varphi(t)\} \) can be constructed geometrically. First we define curve \( \Gamma \) in the \( X, \varphi-F \)-space as
\[ \Gamma : t \mapsto [X(t), \varphi(t), F(t)], \quad (2.11) \]
where \( F(t) \) can be found with the use of equation \((2.3)\), and \( F, \varphi \) and \( t \) must be replaced by dimensionless quantities \( \tilde{F} = M_P^{-1}F \), \( \tilde{\varphi} = M_P^{-1} \varphi \) and \( \tilde{t} = M_P t \), but for simplicity we skip the tilde over them in this paragraph. The curve \( \Gamma \) is constructed to belong to a surface given by the constraint \( F = F(X, \varphi) \), so that we know the function \( F(X, \varphi) \) for \( X \) and \( \varphi \) being the given background solution. For a given curve \( \Gamma \) there are obviously infinitely many functions \( F \), but not all of them lead to the desired solution \( \{X(t), \varphi(t)\} \), because the gradient of this function \([F_X, F_\varphi]\) is restricted by equations \((2.4)\) and \((2.5)\). As a result, the surface given by function \( F \) must contain not only curve \( \Gamma \) but also an infinitesimally shifted curve
\[ \Gamma^{(\varepsilon)} : t \mapsto \left[ X + \varepsilon \frac{F_X}{|\partial F|}, \varphi + \varepsilon \frac{F_\varphi}{|\partial F|}, F + \varepsilon |\partial F| \right], \quad |\partial F| = \sqrt{F_X^2 + F_\varphi^2}, \quad (2.12) \]
where all quantities in the square brackets except for infinitesimally small \( \varepsilon \) are functions of time obtained with the use of the background equations \((2.3)-(2.5)\).

The construction of function \( F \) fails if projections of curves \( \Gamma \) and \( \Gamma^{(\varepsilon)} \) to the \( X, \varphi \)-plane intersect while curves themselves do not, since at points where this happens the value of \( F \) cannot be defined. However, the Bianchi identity prevents such case to happen and the construction of \( F \) never fails. In this way the function \( F(X, \varphi) \) is defined uniquely only for \( X \) and \( \varphi \) which are infinitesimally close to the background solution. Outside this region it can be defined in
any way, even being discontinuous. This freedom of choice arises because our construction of function \( F \) is suited to a background solution with specific initial conditions.

Letting the initial conditions be arbitrary, the question of existence of a function \( F \) generating a given system of solutions \( \{X(X,0,t), \varphi(\varphi_0,t)\} \) changes considerably. For example solutions with an exponential growth of the scale factor, \( a(t) \propto \exp(HT) \) (\( H \) being constant) and the scalar field as a linear function of time, \( \varphi(t) \propto t \), can be obtained if \( p(t) \) and \( q(t) \) are constant functions such that \( q = 3p/(p - 3) \). Function \( F \) generating such solutions has to satisfy conditions

\[
F = M^2 Pl H^2 (p - 3), \quad FX = 3M^2 Pl H^2 p/X, \quad F_\varphi = \pm 3\sqrt{2} M^2 Pl H^2 \sqrt{p},
\]

so that, if \( p \neq 0 \), function \( F \) must be constant while its gradient is non-zero. Although such function does not exist, one can find a function satisfying these conditions for \( X \) and \( \varphi \) belonging to an arbitrary curve in the \( X-\varphi \)-plane, and we can chose this curve to be the desired background solution. The conclusion is that there exists a function \( F \) which generates an exponential growth of the scale factor and a linear dependence of the scalar field on the time only for special initial conditions \( \{a_0, \varphi_0\} \) and for any other initial conditions the form of the background solution must be different.

A trivial example of function \( F \) generating solutions of the form \( a \propto \exp(HT) \) and \( \varphi = \text{const.} \) regardless of initial conditions is the constant function \( F = -3M^2 Pl H^2 \), corresponding to parameters \( p \) and \( q \) which are both zero. One could find a way how to construct \( F(X, \varphi) \) from arbitrary \( \{X(X,0,t), \varphi(\varphi_0,t)\} \), which in contrast to our construction may not always be possible, but the result which we have obtained here will suffice for the purpose of the rest of our work.

### 3 Quadratic actions

For the perturbation theory we adopt the technical approach from \[1\] and we use the spacially flat slicing gauge as in \[12\]. This section contains a summary of the technicalities of this approach and the results obtained for our model within it. The most important results to be used in the following sections are the quadratic actions for the tensor and scalar perturbations (3.14) and (3.21), and useful additional relations (3.23)-(3.25). The scalar and tensor cubic action can be found in the following sections where bispectra are computed.

Leaving the unperturbed case, the flat FRWL metric (2.1) must be replaced by a more general one. In the ADM parametrisation the general metric is

\[
ds^2 = N^2 dt^2 - h_{ij} (dx^i + N^i dt)(dx^j + N^j dt).
\]

The components of the inverse metric are

\[
g^{00} = N^{-2}, \quad g^{0i} = -N^{-2} N^i, \quad g^{ij} = -h^{ij} + N^{-2} N^i N^j,
\]

and the flat FRWL metric corresponds to \( N = 1, N^i = 0 \) and \( h_{ij} = \alpha^2 \delta_{ij} \). For analyzing perturbations we will use spacially flat slicing gauge in which the scalar and vector perturbations of the three-dimensional metric are set to zero. The perturbed metric is then given by

\[
N = 1 + \delta N, \quad N^i = \xi_i + N^i_T, \quad h_{ij} = \alpha^2 \exp(\gamma_{ij}),
\]

where \( N^i \) is decomposed into scalar and vector parts (longitudinal and transversal parts in Helmholtz decomposition), \( N^i_T \) satisfying \( N^i_{T,T} = 0 \), and \( \gamma_{ij} \) is traceless and transversal, i.e. \( \gamma_{ii} = 0 \) and \( \gamma_{ij,j} = 0 \). Similarly, for perturbations of the inflationary fields we have

\[
\varphi = \varphi_0 + \delta \varphi, \quad \phi^i = \delta^i x^i + \pi^i, \quad \pi^T = \rho_T + \pi^T_T,
\]

where \( \pi^T_{T,T} = 0 \).
The overall action of our inflationary model consisting of the Einstein–Hilbert and matter part is

\[ S = \int \sqrt{-g} d^4 x \left[ \frac{1}{2} M_{Pl}^2 R - \frac{1}{2} g^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi + F(\varphi, X, Y, Z) \right], \]

and can be rewritten as

\[ S = \int d^4 x N \sqrt{h} \left\{ \frac{1}{2} M_{Pl}^2 \left[ R^{(3)} + \frac{1}{N^2} \left( E^i_i E^j_j - (E^i_i)^2 \right) \right] - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + F \right\}, \]

where we have followed notations of [1]. \( R^{(3)} \) denotes the three-dimensional scalar curvature corresponding to the spatial metric \( h_{ij} \) and the extrinsic curvature of equal-time hypersurfaces is

\[ K_{ij} = N^{-1} E_{ij} = \frac{1}{2N} \left( \delta_{ij} - \nabla_i N_j - \nabla_j N_i \right), \]

with the covariant derivative with respect to the spatial metric denoted by \( \nabla \). By varying this action with respect to \( N^i \) and \( \delta N \), we obtain the momentun and hamiltonian constraints,

\[ \frac{1}{2} M_{Pl}^2 \nabla_j \left[ N^{-1} (E^i_i - \delta^i_i E^k_k) \right] + N \partial_i \left( \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + F \right) = 0, \]

\[ \frac{1}{2} M_{Pl}^2 \left[ R^{(3)} - \frac{1}{N^2} \left( E^i_i E^j_j - (E^i_i)^2 \right) \right] + \partial_i N \left( \frac{1}{2} N \partial_\mu \varphi \partial^\mu \varphi + NF \right) = 0. \]

Considering \( \delta N, \xi \) and \( N_T^i \) in the form of the plane waves with the wavenumber \( k \), these constraints are satisfied up to the first order of the perturbation theory for

\[ \delta N = \frac{(M_{Pl}^2 H k^2 \dot{\varphi}_0 - F_X F_\varphi) \delta \varphi + \dot{\varphi}_0 F_X \delta \dot{\varphi} + 2a^{-2} F_X^2 k^2 \rho - 2M_{Pl}^2 H F_X k^2 \dot{\rho}}{2 FF_X + 2M_{Pl}^2 H k^2}, \]

\[ \xi = \frac{-(F_\dot{\varphi} + M_{Pl}^2 H F_\varphi) \delta \varphi + M_{Pl}^2 \dot{H} \dot{\varphi}_0 \delta \dot{\varphi} + 2M_{Pl}^2 H a^{-2} F_X k^2 \rho + 2 F F_X \dot{\rho}}{2 FF_X + 2M_{Pl}^2 H k^2}, \]

\[ N_T^i = \frac{4 F_X \delta^i_i \dot{\varphi}_0}{4 F_X - M_{Pl}^2 k^2}. \]

Knowing the higher order corrections is not necessary unless the fourth order terms in the action are needed, since the second order terms of \( N \) and \( N^i \) multiply the zeroth order constraints [1].

Expanding the action (3.6) up to the second order and using (3.10)-(3.12) we can find the quadratic action which determines the evolution of \( \delta \varphi, \rho, \pi_T^i \) and \( \gamma_{ij} \) in first order of the perturbation theory. We can also use integration by parts together with relations \( \pi_T^i T, I = 0 \), \( N_T^i = 0 \), \( \gamma_{ii} = 0 \) and \( \gamma_{ij,j} = 0 \) appearing in definitions of these perturbations. In this way we obtain the quadratic action decomposed into three parts

\[ S^{(2)} = S^{(2)}_S + S^{(2)}_V + S^{(2)}_T, \]

where the scalar, vector and the tensor parts are denoted by \( S^{(2)}_S \), \( S^{(2)}_V \) and \( S^{(2)}_T \) respectively.

The tensor quadratic action can be written in the form

\[ S^{(2)}_T = \frac{1}{4} M_{Pl}^2 \int d^4 x a^3 \left( \frac{1}{2} \gamma_{ij} \dot{\gamma}_{ij} - \frac{1}{2} a^{-2} \gamma_{ij,k} \gamma_{ij,k} + 2 \dot{\rho} \gamma^2_T \gamma_{ij} \gamma_{ij} \right), \]

where \( \tilde{h} = \dot{H} + \dot{\varphi}_0^2/(2M_{Pl}^2) = H^2 (p - \epsilon) \) and \( c_T \) denotes the transverse sound speed,

\[ c_T^2 = \frac{1}{3} \frac{F_Y + F_Z}{F_X}. \]
The vector quadratic action is

\[ S_{V}^{(2)} = M_{p}^{2} \int \frac{d^{3}k}{(2\pi)^{3}} a^{3} \left( \frac{k^{2}}{4} \frac{\tilde{e}k^{4}}{1 - \tilde{e}k^{2}/(3a^{2}h)} \left( \frac{\dot{\rho}}{H} - \frac{\dot{h}}{H} \right)^{2} + \frac{\tilde{h}c_{T}^{2}k^{2}\pi_{T}^{f}}{\pi_{T}^{f}} \right). \]  

(3.16)

The quadratic terms in actions are written in the simplified form. Terms such \( \text{Re} \left\{ \psi \chi \right\} \) we denote as \( \psi \chi \), and we use this notation also in the rest of this section.

The scalar quadratic action can be written as a sum of three parts as

\[ S_{S}^{(2)} = M_{p}^{2} \int \frac{d^{3}k}{(2\pi)^{3}} a^{3} \left[ \frac{1}{3} \frac{\tilde{e}k^{4}}{1 - \tilde{e}k^{2}/(3a^{2}h)} \left( \frac{\dot{\rho}}{H} - \frac{\dot{h}}{H} \right)^{2} + \frac{\tilde{h}c_{T}^{2}k^{4}\rho^{2}}{\rho^{2}} \right] + \]  

\[ + S_{\delta\varphi}^{(2)} + S_{\delta\varphi,\rho}^{(2)}, \]

(3.17)

where \( \tilde{e} = 1 - \varphi_{0}^{2}/(2F) = 3/(3 - p) \), \( c_{L} \) is the longitudinal sound speed,

\[ c_{L}^{2} = 1 + \frac{2}{3} \frac{FX_{X}}{F_{X}} + \frac{8}{9} \frac{F_{Y} + F_{Z}}{F_{X}}, \]

(3.18)

and \( S_{\delta\varphi}^{(2)} \) denotes action quadratic in \( \delta\varphi \), while \( S_{\delta\varphi,\rho}^{(2)} \) is action consisting of \( \delta\varphi, \rho \)-type terms. The first part of action (3.17) is written in the explicit form for the sake of being easily compared to (6.4) in [12] as the special form of our action with parameter \( p \) set to zero. The same applies to the tensor and vector parts.

The full scalar quadratic action written with coefficients expressed in terms of the slow-roll parameter \( \epsilon \) and the parameter \( p \) is

\[ S_{S}^{(2)} = \int \frac{d^{3}k}{(2\pi)^{3}} a^{3} \left\{ M_{p}^{2} \tilde{Q}k^{4} \left( \dot{\rho} + HQ\rho \right)^{2} - M_{p}^{2}H^{2}Qc_{L}^{2}k^{4}\rho^{2} + \right\} \]  

\[ + \frac{1}{2} \left( 1 + p\tilde{Q} \right) \delta\varphi^{2} - \sqrt{\tilde{p}}pS_{+}\delta\varphi \delta\varphi + \]  

\[ + \frac{1}{2} \left( F_{\varphi\varphi} - \frac{k^{2}}{a^{2}} \right) \left[ \left( \frac{1}{2} \eta_{p} - Q \right) k^{2} + SS_{+} \right] \tilde{H}\sqrt{\tilde{p}} \delta\varphi^{2} + \]  

\[ + \left( -2FX_{\varphi} \pm \sqrt{2M_{p}\tilde{Q}S_{+}} \right) \frac{k^{2}}{a^{2}} \delta\varphi \rho \pm \]  

\[ \pm \sqrt{2M_{p}\tilde{Q}}k^{2} \left\{ H \left( \frac{1}{2} \eta_{p} - Q \right) \delta\varphi \tilde{\rho} - HQ\delta\varphi \rho - \delta\varphi \right\}, \]

where

\[ Q = \epsilon - p, \quad \tilde{Q} = a^{2}H^{2}Q/\left( M + k^{2} \right), \quad \tilde{\rho} = H\sqrt{\tilde{p}}, \quad M = a^{2}H^{2}(3 - p)Q, \]

\[ \eta_{p} = \frac{\tilde{\rho}}{pH}, \quad S = 3 - \epsilon + \frac{1}{2} \eta_{p}, \quad S_{+} = a^{2}H^{2}QS + k^{2}. \]

Unfortunately, equations governing evolution of perturbations obtained by variation of this action are coupled. Due to the effect of gravity, this occurs even if \( F_{X,\varphi} \) is set to zero. In order to quantize scalar perturbations properly, \( \delta\varphi \) and \( \rho \) then must be replaced by their linear combinations \( \delta\varphi \) and \( \tilde{\rho} \) such that the part of the action \( S_{\delta\varphi,\tilde{\varphi}}^{(2)} \) describing coupling vanishes. Finding the transformation relation \( (\delta\varphi, \rho) \mapsto (\delta\varphi, \tilde{\rho}) \), or finding the solution for \( \delta\varphi \) and \( \rho \) directly, is a matter of solving a complicated system of differential equations. The problem can be simplified in the special case if \( p \) is small, at most of the same order as \( \epsilon \), and \( F_{X,\varphi} \) is of a higher order, when the action written up to the next-to-leading order in the slow-roll
of plane waves with wavenumber $k$.

By varying the quadratic action (3.21) we obtain equations for scalar perturbations in the form

$$S_{\phi}^{(2)} = \int \frac{d^3k dt}{(2\pi)^3} a^3 \left[ M_p^2 a^2 H^2 (k^2 - 3a^2 H^2 Q) Q \rho^2 + 2M_p^2 a^2 H^2 k^2 Q^2 \rho \dot{\rho} + \frac{1}{2} \dot{\varphi}^2 + \left( F_{\varphi \varphi} - \frac{k^2}{a^2} + 3H^2 \rho \right) \delta \varphi^2 - H \rho \delta \varphi \delta \varphi \pm \sqrt{2}M_p a^2 H^2 \sqrt{\rho} Q \left( \frac{k^2}{a^2} \delta \varphi \rho - \delta \varphi \dot{\rho} \right) - 2F_{\varphi \varphi} \frac{k^2}{a^2} \delta \varphi \rho \right].$$

(3.21)

We can see that $Q = \epsilon - p$ must be positive in order to have the proper sign of the action.

For the rest of our work we will restrict ourselves to this special case assuming $p$ being of the same order as $\epsilon$. As a consequence, the parameter $\eta_p$ defined by the fifth relation in (3.20) can be expressed as

$$\eta_p = \frac{\epsilon}{p} \eta + \frac{1}{3} \frac{F_{\varphi \varphi}}{H^2},$$

(3.22)

where only the leading order terms of the slow-roll approximation have been kept. Moreover, $F_{X \varphi}$ being much smaller than the slow-roll parameter yields $F_{\varphi \varphi} \sim \epsilon$, and therefore parameter $\eta_p$ is small as well.

For the inflationary expansion of the universe, the smallness of parameter $p$ requires also smallness of parameter $q$, because up to the first order in the slow-roll parameter, relation (2.7) is simplified to $\epsilon = p + q$, and considering smallness of $\eta_p = p/(pH)$, $q/(qH)$ must be small as well. For this reason not only $\epsilon$ and $\eta$, but also $p$ and $\eta_p$ may be called slow-roll parameters. Consequently, higher derivatives of the function $F$ with respect to $X$ and $\varphi$ cannot be arbitrary either. By differentiating parameters $p$ and $q$ with respect to time and using the background equations (2.3)- (2.5) we find that if $p$ is not much greater than $\epsilon$ the partial derivatives of $F$ are constrained by slow-roll parameters,

$$XF_X, X^2F_{XX}, X^3F_{XXX}, \sqrt{\rho} F_\varphi \sim \epsilon, \quad X \sqrt{\rho} F_{X \varphi}, X^2 \sqrt{\rho} F_{XX \varphi}, ... \sim \epsilon^2.$$

(3.23)

Up to the leading order in slow-roll parameters the sound speeds (3.15) and (3.18) can be rewritten in the form

$$c_T^2 = 1 + \frac{2}{3} \frac{F_Y + F_Z}{X F_X}, \quad c_L^2 = \frac{1}{3} \left( \frac{1}{9} \frac{F_Y + F_Z}{X F_X} + \frac{8}{9} \frac{F_Y + F_Z}{X F_X} \right),$$

(3.24)

and neglecting the first order terms of the slow-roll approximation we obtain constraints

$$\frac{4}{3} c_T^2 - c_L^2 = 1, \quad -\frac{3}{8} \leq \frac{F_Y + F_Z}{X F_X} \leq 0,$$

(3.25)

so that the transverse sound speed must be greater than $\sqrt{3}/2$ and the longitudinal one must be smaller than $\sqrt{1/3}$, unless $\epsilon - p \sim \epsilon^2$. The assumption of real sound speeds was also taken into account, since if they were imaginary the undesired exponential growth of the perturbations would occur. We can also see that $\epsilon - p$ cannot be of order $\epsilon^{5/2}$ or smaller.

4 Scalar perturbations

By varying the quadratic action (3.21) we obtain equations for scalar perturbations in the form of plane waves with wavenumber $k$,

$$\ddot{\varphi}_k + \left( \frac{5}{2} - 2\epsilon + \eta_q - 6 \frac{a^2 H^2}{k^2} Q \right) H \dot{\varphi}_k + \left[ (5 + 3\epsilon_L) Q + \frac{k^2 c_T^2}{a^2 H^2} \right] H^2 \varphi_k =$$

$$= \pm \frac{\sqrt{p}}{\sqrt{2}M_p k^2} \left[ \delta \varphi_k + 5H \delta \varphi_k + \frac{k^2}{a^2} \left( 1 \pm \frac{\sqrt{2}M_p F_{X \varphi}}{\sqrt{p} F_X} \right) \delta \varphi \right],$$

(4.1)
\[
\delta \varphi_k + 3H \delta \varphi_k - \left( F_{,\varphi} - \frac{k^2}{a^2} + 6H^2 \rho \right) \delta \varphi_k = \\
= \pm \sqrt{2M_P a^2} H^2 \sqrt{p} q \left[ \rho_k + 5H \rho_k + \frac{k^2}{a^2} \left( 1 \pm \frac{\sqrt{2M_P F_X}}{\sqrt{p} X} \right) \rho_k \right], \\
\]
where
\[
\eta_q = \frac{\dot{Q}}{QH} = \frac{\epsilon q - p \eta_p}{\epsilon - p}. \\
\]

One can find the Fourier mode functions of \( \delta \varphi \) and \( \rho \) either by solving these equations numerically or employing some approximative methods such as the uniform approximation, see [28, 29], but we restrict ourselves to the case when a simple form of analytical solutions can be found. This requires not only assumptions we have imposed so far, but also two additional ones. The first assumption is that parameter \( \eta_q \) must be at most of the same order as slow-roll parameters, and the second assumption concerns parameter \( \eta_L \) defined as
\[
\eta_L = \frac{\dot{i}_L}{c_L H}, \\
\]
which must be small as well.

The right-hand side of equation (4.2) can be neglected if
\[
F_{,\varphi} = \pm \frac{\sqrt{p} X}{\sqrt{2M_P}} (c_L^2 - 1), \\
\]
since in this special case the combination of terms in brackets represents the equation of motion for the scalar perturbation \( \rho \) \( (4.1) \) in the leading order of the slow-roll approximation. Therefore, the equation of motion for the scalar field perturbation \( \delta \varphi \) is decoupled and can be easily solved. Equation for \( \rho \) can be decoupled by replacing \( \rho \) by the new variable \( S \) defined by
\[
S_k = \rho_k + \frac{\sqrt{p}}{\sqrt{2M_P} k^2} \delta \varphi_k, \\
\]
however, as we will see, to solve the equation for \( S \) is a bit more tricky than to solve the equation for \( \delta \varphi \).

In order to solve equations of motion for perturbations it is useful to introduce the conformal time \( \tau \) defined in the standard way as \( \tau = \int a^{-1} dt \), \( \tau \in (-\infty, 0) \). By replacing the cosmological time by it and considering assumptions imposed above including special form of \( F_{,\varphi} \) given by (4.2), we find
\[
\frac{d^2 S_k}{d\tau^2} = \frac{1}{\tau} \frac{dS_k}{d\tau} + \left( \frac{5 + 3c_L^2 - 6\eta_q}{\tau^2} Q_c + c_L^2 (\tau k)^2 \right) S_k = 0, \quad (4.7) \\
\delta \varphi_k'' - \frac{1}{\tau} \frac{d\delta \varphi_k}{d\tau} + \left( k^2 = \frac{6\rho_c + H^{-2} F_{,\varphi} \epsilon_c}{\tau^2} \right) \delta \varphi_k = 0, \quad (4.8)
\]
where the prime denotes the differentiation with respect to the conformal time, \( H = a' / a \), and the subscript \( c \) stands for quantities evaluated at the reference time \( \tau_c \) when the longest mode of observational relevance today with the wavenumber \( k_{\text{min}} \sim H_{\text{today}} \) \( (a_{\text{today}} \equiv 1) \) exits the horizon, i.e.,
\[
\left| \frac{k_{\text{min}}}{H_{\text{today}} a_c} \right| \sim |H_{\text{today}} \tau_c| = 1. \quad (4.9)
\]

Using this convention we also obtain relations
\[
a = a_c \left( \frac{\tau}{\tau_c} \right)^{-1 - \epsilon_c}, \quad H = \frac{1 - \epsilon_c}{a_c \tau_c} \left( \frac{\tau}{\tau_c} \right) = H_c \left( \frac{\tau}{\tau_c} \right)^{\epsilon_c}, \quad (4.10) \\
\epsilon = \epsilon_c \left( \frac{\tau}{\tau_c} \right)^{-\eta_c}, \quad p = p_c \left( \frac{\tau}{\tau_c} \right)^{-\eta_p,c}, \quad Q = Q_c \left( \frac{\tau}{\tau_c} \right)^{-\eta_Q,c}, \quad c_L = c_L,c \left( \frac{\tau}{\tau_c} \right)^{-\eta_L,c}.
\]
The Fourier modes of $\delta \varphi$ and $\rho$ can be quantized in the standard way as

$$\rho_k^- = \rho_k^{(cl)} e^{-i\delta \varphi} + \rho_k^{(cl)} e^{i\delta \varphi}, \quad (4.11)$$

$$\delta \varphi_k^- = \delta \varphi_k^{(cl)} e^{-i\delta \varphi} + \delta \varphi_k^{(cl)} e^{i\delta \varphi}, \quad (4.12)$$

where the classical solutions obeying equations of motion are denoted by the superscript (cl), and the creation and annihilation operators obey commutation relations

$$[a_k, a_k^+] = [b_k, b_k^+] = (2\pi)^3 \delta^{(3)}(k_1 - k_2). \quad (4.13)$$

Normalization of the classical solutions is determined by the equal time commutation relations for $\delta \varphi$ and $\rho$ and their conjugate momenta

$$[\rho(\bar{x}_1, t), \pi(\bar{x}_2, t)] = [\delta \varphi(\bar{x}_1, t), \pi(\bar{x}_2, t)] = i\delta^{(3)}(\bar{x}_1 - \bar{x}_2), \quad (4.14)$$

and it can be obtained by matching the canonically normalized fields

$$\delta \varphi^{(can)} = a \delta \varphi^{(cl)}, \quad \rho^{(can)} = \sqrt{2} M_{pl} H a^2 \sqrt{Q} \rho^{(cl)}, \quad (4.15)$$

to the mode functions of the free wave function of the Minkowski space vacuum, $\frac{1}{\sqrt{2\pi}} e^{-ikr}$ or $\frac{1}{\sqrt{2\pi}} e^{-i\xi L_k \tau}$ in the limit of very early time, $\tau \rightarrow -\infty$, when the modes are deep inside the horizon, $k \ll H a$, and the curvature of spacetime does not affect their evolution.

The correctly normalized classical solutions of equations (4.17) and (4.18) are

$$\rho_k^{(cl)} = \frac{i}{2} \sqrt{\frac{7}{2}} M_{pl} \sqrt{Q} k \rho^{(cl)}, \quad (4.16)$$

$$\delta \varphi_k^{(cl)} = \frac{i}{2} M_{pl} \sqrt{Q} k \delta \varphi^{(cl)}, \quad (4.17)$$

where $H^{(1)}_\nu$ denote Hankel functions of the first kind, and all parameters in equations of motion which are of the same order as slow-roll parameters or smaller have been omitted. This result is valid even without the restriction on $F_{X\varphi}$ (4.7) taken into account and it is sufficient for calculation of the scalar bispectrum in the leading order of the slow-roll approximation discussed in the next section, but the omitted parameters are needed to determine the deviation of the scalar power spectrum from the flat one. Unfortunately, when these parameters are taken into account, equation (4.17) cannot be solved immediately because of the term proportional to $\tau^{-2}$ in the coefficient in front of $S_k$. The extra term can be removed by performing one more transformation of dependent variable, mimicking the transformation used in [12]. The appropriate variable appears to be a scalar quantity $U$ defined by the solid matter velocity $u^{(s)i}$ as

$$U = H \delta u^{(s)} = a^2 H (\dot{\rho} - \xi), \quad (4.18)$$

where the $\delta \varphi$ is the scalar part of the solid matter velocity, $u^{(s)i} = \delta u^{(s)i} + u^{(s)T}_i \delta u^{(s)} = 0$, and the term $-\xi$ in the brackets originates from lowering the index with use the perturbed metric. By inserting (4.11) into (4.13) and keeping only the relevant terms in the slow-roll approximation we obtain

$$U_k = a^2 H \left[ 1 - 3 \frac{a^2 H^2}{k^2 Q} \right] S_k + H Q S^2_k. \quad (4.19)$$

The quantity $U$ is related to the quantity $\mathcal{R}$ defined with the use of the notation from (5.20) (see equation (5.4.22) there) as

$$\mathcal{R}_k = \frac{A_k}{2} + H \delta u_k, \quad (4.20)$$
where the signature \((-+++\) is used and \(\delta u\) is the scalar part of velocity of the system consisting of solid matter and scalar field. In order to express the right-hand side of this definition in the terms of scalar perturbations present in our model we need the \((0-i)\) components of stress-energy tensor up to the first order of the perturbation theory. By inserting the resulting velocity potential into the definition of \(R\) and returning to the signature of the metric tensor which we use, we find

\[ R = \frac{Q}{\epsilon} \mathcal{U} \mp \frac{\sqrt{p}}{\sqrt{2}\mpl H\epsilon} \delta \varphi. \quad (4.21) \]

In the case when the scalar field \(\varphi\) is not present in the universe, we simply have \(R = -\mathcal{U}\).

Using equation (4.19) together with (4.11), (4.22) and (4.5), the scalar quadratic action (3.21) can be rewritten into a more convenient form

\[
S_{\mathcal{S}}^{(2)} = \int \frac{d^3k}{(2\pi)^3} \frac{\rho}{c_s^2} \left[ -\dot{\mathcal{U}}^2 - 2H(3 - \epsilon - Q + \eta_Q)\dot{u}\mathcal{U} + \left( \frac{k^2}{\omega^2c_s^2} - 9 - 6\epsilon - 3c_s^2Q + 6\eta_QH^2 \right) \dot{u}^2 \right] + \frac{1}{2} \delta \varphi^2 + \frac{1}{2} \left( \mathcal{F}_{\varphi\varphi} - \frac{k^2}{\omega^2} + 3H^2p \right) \delta \varphi^2 - Hp\delta \varphi \delta \varphi. \quad (4.22)
\]

The sign of the kinetic term of \(\mathcal{U}\) in the action is the opposite as for \(\rho\), because in the gauge which we use, \(\rho\) measures the position of the solid matter elements while \(\mathcal{U}\) measures their velocity. As a simple example, such a change of the sign appears also in the action of the one-dimensional harmonic oscillator \(S = \int dt (\dot{x}^2 - \omega^2x^2)/2\), which rewritten in terms of the velocity \(v = \dot{x}\) takes the form \(S = \int dt (-\omega^2\dot{v}^2 + v^2)/2\).

Equation of motion for \(\mathcal{U}\) obtained by varying the action (4.22) reads

\[
\mathcal{U}'' - \frac{2 + 2\epsilon_c + \eta_Qc_e - 2\eta_Lc_e}{\tau} \mathcal{U}' + \left( k^2c_s^2c_e^2 + 3\frac{(1 + c_L^2c_e^2)Q_c - 2\eta_Lc_e}{\tau^2} \right) \mathcal{U} = 0, \quad (4.23)
\]

where the longitudinal sound speed as a conformal time dependent function is given by the last relation in (4.10). By matching the general form of the solution of this equation for the canonically normalized field

\[
\mathcal{U}^{(\text{can})} = i\sqrt{2}\mpl c_s\sqrt{Q}\mathcal{U}, \quad (4.24)
\]

to free wave mode function of the Minkowski space vacuum, and applying the same procedure to scalar field perturbation \(\delta \varphi\) with all small parameters taken into account up to the first order of the slow-roll approximation, we find

\[
\mathcal{U}_k^{(\text{cl})} = i\sqrt{\frac{\tau}{2\sqrt{2}\mpl\sqrt{Q}}}H_c(\tau)\epsilon_c\tau_{-\epsilon_c}^{-\epsilon_c} \left( 1 + \frac{1}{\tau}\eta_{L,c} - \epsilon_c \right) e^{\frac{i\pi}{3}p^{(\text{cl})}}. \quad (4.25)
\]

\[
\delta \varphi_k^{(\text{cl})} = -\sqrt{\frac{\tau}{2}}H_c(\tau)\epsilon_c\tau_{-\epsilon_c}^{-\epsilon_c} \left( 1 - \epsilon_c \right) e^{\frac{i\pi}{3}p^{(\text{cl})}}(\tau)\tau_{\frac{1}{2}+\epsilon_c}^\frac{1}{2+\epsilon_c} H_{\frac{1}{2}+\epsilon_c}(\tau). \quad (4.26)
\]

where

\[
\epsilon^{(\mathcal{U})} = \epsilon + \frac{1}{2}\eta_Q - \eta_L, \quad (4.27)
\]

\[
p^{(\mathcal{U})} = p + c_s^2Q + \frac{1}{2}\eta_Q + \frac{5}{2}\eta_L, \quad (4.28)
\]

\[
\epsilon^{(\delta \varphi)} = \epsilon + 2p + \frac{1}{3}\frac{F_{\varphi\varphi}}{H^2}. \quad (4.29)
\]
Our goal is to compute the correlation functions of a scalar quantity $\zeta$ that parameterizes the curvature perturbations, defined as

$$\zeta_k = \frac{A_k}{2} - H \frac{\delta p_k}{\dot{\rho}},$$  

(4.30)

where the notation follows [30] again. Expressed in term of $\delta \phi$ and $\mathcal{U}$, the scalar perturbation $\zeta$ in the leading order in slow-roll parameters is

$$\zeta = \pm \sqrt{p} \sqrt{2 M_p^{3/2}} \left( \frac{\delta \phi}{3H} - \delta \phi \right) + \frac{Q}{c_L^{3/2}} \left( \frac{\dot{\mathcal{U}}}{3H} + \mathcal{U} \right),$$  

(4.31)

and the corresponding two-point function in the late time limit is

$$\langle 0 | \zeta_{k_1} \zeta_{k_2} | 0 \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2) - \frac{H^2}{4 M_p^2} k^{-3} \left( \frac{\tau}{\eta_c} \right)^{2\epsilon_c + 2\eta_c} \cdot$$  

(4.32)

The scalar power spectrum $P_c(k)$ defined by

$$\langle 0 | \zeta_{k_1} \zeta_{k_2} | 0 \rangle = P_c(k_1) \frac{2 \pi^3}{2 k_1^3},$$  

(4.33)

is usually approximated by power-law function, $P_c(k) \propto k^{n_S-1}$, where $n_S$ is the scalar spectral index, being close to one for a nearly flat spectrum. The spectral tilt up to the leading order of the slow-roll approximation can be computed as

$$n_S - 1 = \frac{d \ln P_c}{d \ln k} = -2 \frac{c_L^3 \sigma_p c_p (\epsilon_c - p_c) p_c}{\epsilon_c + (c_L^3 \sigma - 1) p_c},$$  

(4.34)

where the subscript $c$ stands for quantities evaluated in the time when the inflation ends, $\tau_c \approx 0^-$, and $\sigma$ denotes

$$\sigma = \left( \frac{\tau_c}{\eta_c} \right)^{2(p(\mathcal{U}) - \epsilon(\mathcal{U}))},$$  

(4.35)

where $N_{min}$ is the minimal number of e-folds ($N_{min} \sim 60$), and $(k_{max}/k_{min})^{2(p(\mathcal{U}) - \epsilon(\mathcal{U}))}$, $k_{max} \sim 3000 k_{min}$ being the maximal wavenumber corresponding to the highest observed multipole moment of the cosmic microwave background, and $c_L^2 \epsilon \phi^{(4)}$ were replaced by one. (For example $3000^{0.01 \pm 0.08} = 1.08$ and $0.1^{0.01 \pm 0.08} = 0.98$.) The fifth power of the longitudinal sound speed appearing in relation $\eta_{L,c}$ cannot be larger than $3^{-5/2} = 0.064$, since the maximal value for the longitudinal sound speed allowing inflationary expansion of the universe is $1/\sqrt{3}$, and therefore, if $\sigma$ is not greater than of order unity, the dominant contribution to the spectral tilt is

$$n_S - 1 \approx -2 p_c^{(4)} = -2 c_L^2 \epsilon_c^2 - 2(1 + c_L^2 \epsilon_c) p_c - \eta_{L,c} + 5 \eta_{L,c},$$  

(4.36)

where the second power of the longitudinal sound speed with maximal allowed value $1/3$ (unless $\epsilon - p \sim \epsilon^2$) has been kept, whereas for $\sigma \gg 1$ we have

$$n_S - 1 \approx -2 \epsilon_c^{(4)} = -2 \epsilon_c - 4 p_c - \frac{2 F \phi^{(4)} e_c}{H^2 e_c},$$  

(4.37)
Our inflationary model contains two special cases. The first one is the most simple single-field inflation which can be obtained by taking the limit such that \( p = \epsilon \), when the scalar spectral tilt (4.34) reduces to

\[
\frac{d}{d\eta} = -6\epsilon - \frac{2}{3} \frac{F_{\phi\phi\phi}}{H^2} \bigg|_{p_c=\epsilon_c} = -2\epsilon - \eta_c,
\]

see also relation (54) in [31]. The second special case is the solid inflation model in which the scalar field \( \delta\phi \) is not present and parameter \( p \) must be set to zero. The corresponding spectral tilt is

\[
\frac{d}{d\eta} = 2\epsilon - \eta_c - 5\eta L_c,
\]

the same which can be found in [12].

5 Scalar bispectrum

In the linear order of the perturbation theory Gaussianity is preserved. Therefore, in order to compute bispectrum which encodes the non-Gaussianity, cubic terms in the action are needed. These terms include higher partial derivatives of the function \( F \), and those appearing in (3.23) that are suppressed by the slow-roll parameter, may be neglected. Moreover, from (3.25) follows

\[
0 \leq F_Y + F_Z \leq \frac{3}{8} X F_X = \frac{9}{8} M^2_{Pl} H^2 (\epsilon - p),
\]

so that \( F_Y + F_Z \) can be neglected as well. However, it must be small, its time derivative may not be, because the restriction (5.1) is just an inequality, and therefore there are no restrictions on \( F_{XY} + F_{XZ} \) and \( F_{Y\phi} + F_{Z\phi} \). On the other hand, small functions with not small derivative usually do not occur in physical problems, so that it is reasonable to restrict ourselves to the special case in which

\[
F_Y + F_Z = \frac{9}{8} A M^2_{Pl} H^2 (\epsilon - p),
\]

where \( A \) is a constant, or a slowly varying function, of order unity, \( A \sim 1 \). By differentiating this equation we obtain the restriction

\[
X (F_{XY} + F_{XZ}) = \frac{M_{Pl}}{\sqrt{2}} \sqrt{p} (F_{Y\phi} + F_{Z\phi}) \sim \epsilon^2,
\]

which is satisfied if we put

\[
\tilde{F} = X (F_{XY} + F_{XZ}) = \pm \frac{M_{Pl}}{\sqrt{2}} \sqrt{p} (F_{Y\phi} + F_{Z\phi}),
\]

and allow \( \tilde{F} \) to be of arbitrary order in the slow-roll parameters. As a reminder, the properly normalized classical mode of the scalar perturbation \( \rho \) (4.10) is of the order \( \rho \sim (\epsilon - p)^{-1/2} \). Now we have everything needed to keep track of orders in the slow-roll approximation when collecting cubic terms of the action.

By expanding the action (3.6) up to the third order in scalar perturbations and keeping only the leading order terms in the slow-roll approximation we find

\[
S^{(3)} = \int d^4x a^3 \left[ - \frac{8}{81} \left( \frac{2}{3} F_Y + \tilde{F} \right)^3 \right. (\rho_{ii})^3 + \frac{8}{27} (F_Y + \tilde{F})^3 \rho_{ii}^3 + \rho_{ij} \rho_{jk}^2 \frac{8}{27} F_Y \rho_{ij} \rho_{jk} \rho_{ij} \rho_{jk} \pm \frac{4\sqrt{2}}{9 M_{Pl}} \sqrt{p} \left( \rho_{ij} \rho_{ij} - \frac{1}{3} (\rho_{ii})^2 \right) \delta \varphi \left. \right].
\]
This action determines the interaction Hamiltonian responsible for the non-Gaussianity of scalar perturbations. The scalar bispectrum is given by the three-point function of the scalar $\zeta$, which can be computed with the use of the in-in formalism \cite{32} as

$$\langle \zeta_{k_1}(\tau)\zeta_{k_2}(\tau)\zeta_{k_3}(\tau) \rangle = -i \int_{-\infty}^{\tau} a(\tau')d\tau' \left[ \zeta_{k_1}(\tau)\zeta_{k_2}(\tau)\zeta_{k_3}(\tau), H_{\text{int}}(\tau') \right] |0\rangle, \quad (5.6)$$

where only the first order term with a single integration and a simple commutator is considered. By inserting \eqref{5.1}, \eqref{5.5} and classical modes \eqref{1.10} and \eqref{1.11} into this formula, and using the commutation relations \eqref{4.3}, we find the late time three-point function in the leading order of the slow-roll approximation,

$$\langle \zeta_{k_1}(0)\zeta_{k_2}(0)\zeta_{k_3}(0) \rangle = \frac{H_z^2(2\pi)^3\delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)}{2M_p^2c^2_e e^2_k (k_1k_2k_3)^3} \left[ \bar{Q}(\vec{k}_1, \vec{k}_2, \vec{k}_3)A_{k_1,k_2,k_3} + \right. \quad \left. + \ c_{L,c}^2 \bar{F} \left( Q^{(2,3)}(\vec{k}_1, \vec{k}_2, \vec{k}_3)\Omega_{k_1,cL,k_2,cL,k_3} + 2 \text{ permutations} \right) \right], \quad (5.7)$$

where

$$\bar{Q}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \left( F_Y + \bar{F} \right) \frac{k_1^2 (\vec{k}_2 \cdot \vec{k}_3)^2}{(k_1k_2k_3)^2} + \frac{2}{3} F_Y - \bar{F}, \quad (5.8)$$

and $A, B = 1, 2, 3,$ are given by the integrals

$$A_{k_1,k_2,k_3} = \Re \left\{ i \int_0^\infty \left[ 1 - ik_1z - \frac{1}{3}k_1^2z^2 \right] \left[ 1 - ik_2z - \frac{1}{3}k_2^2z^2 \right] \left[ 1 - ik_3z - \frac{1}{3}k_3^2z^2 \right] e^{ik_1+k_2+k_3}z^{-4}dz \right\}, \quad (5.10)$$

$$\Omega_{A,b,c} = \Re \left\{ \int_0^\infty (i + Az) \left[ 1 - ibz - \frac{1}{3}b^2z^2 \right] \left[ 1 - icz - \frac{1}{3}c^2z^2 \right] e^{(A+b+c)}z^{-4}dz \right\}. \quad (5.11)$$

These integrals obviously do not converge. The divergence due to unbounded upper limit of integration interval at $z = \infty$ can be avoided by tilting the integration contour, $z \to (1 + i\varepsilon)z$, with $\varepsilon \to 0^+$. This also provides projection on the right vacuum. The divergence of the integral \eqref{5.10} due to the lower limit of integration interval at $z = 0$ is consumed by evaluating the real part of the integral, however in the integral \eqref{5.11} a logarithmic divergence remains. By calculating integrals \eqref{5.10} and \eqref{5.11} in this way, we obtain

$$A_{k_1,k_2,k_3} = - \frac{1}{27} \left( \sum_i k_i \right)^2 \left[ \sum_i k_i^6 + 9 \sum_{i \neq j} k_i^2k_j + 12 \sum_{i \neq j} k_i^4k_j^2 + 6 \sum_{i \neq j} k_i^2k_j^3 + 18 \left( \prod_i k_i \right) \sum_{i \neq j} k_i^2k_j + 18 \left( \prod_i k_i \right) \sum_i k_i^3 + 20 \left( \prod_i k_i \right)^2 \right], \quad (5.12)$$
\[ \Omega_{A,b,c} = \frac{1}{3} A^3 [\gamma_{\text{EM}} + \ln(-\tau_c (A + b + c)) + \mathcal{O}(\tau_c (A + b + c))] - \frac{1}{9 (A + b + c)^2} \left[ b^5 + 2 b^4 c + 2 b^3 c^2 + 2 b^2 c^3 + 2 b c^4 + c^5 + 2 A (b^2 + b^2 c + b^2 c^2 + b c^3 + c^4) + 2 A^2 (2 b^3 + 3 b^2 c + 3 b c^2 + 2 c^3) + A^3 (10 b^2 + 17 b c + 10 c^2) + 11 A^4 (b + c) + 4 A^5 \right], \]

where \( \gamma_{\text{EM}} \) is the Euler–Mascheroni constant. The second integral has been computed with integration limits \( (-\tau_c, \infty) \), \( \tau_c \) being the time when the inflation ends, which can be expressed as \( \tau_c = -H_{\text{today}}^{-1} e^{-N_{\text{min}}} \). The integral is then dominated by

\[ \Omega_{A,b,c} = -\frac{1}{3} N_c A^3, \]

where \( N_c \) is a number of the order of number of e-folds, and we use this relation instead of \( (5.13) \) in what follows. We also neglect the part of \( Q(\vec{k}_1, \vec{k}_2, \vec{k}_3) \) defined by \( (5.8) \) which is proportional to \( \bar{F} \), since its contribution is much smaller than the contribution from \( (5.14) \).

As a result of computations above, the scalar bispectrum \( B_\zeta(k_1, k_2, k_3) \), defined by relation

\[ \left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_\zeta(k_1, k_2, k_3), \]

consists of two parts,

\[ B_\zeta(k_1, k_2, k_3) = F_Y B_\zeta^Y(k_1, k_2, k_3) + N_c c_{L,c}^2 \bar{F} \tilde{B}_\zeta(k_1, k_2, k_3), \]

parametrized by three independent parameters of the theory, \( F_Y, \bar{F} \) and \( c_{L,c} \). Due to the delta-function on the right-hand side of \( (5.15) \), three wavenumbers \( k_1, k_2 \) and \( k_3 \) can be identified with the sides of a triangle, and all information about bispectrum is encoded in a function of two variables which characterize the shape of the triangle. Following conventions of \( [33] \), we define \( x = k_2/k_1 \) and \( y = k_3/k_1 \) and describe the bispectrum by the function \( x^2 y^2 B_\zeta(1, x, y) \) defined in region \( 1 - x \leq y \leq x, \ 1/2 \leq x \leq 1, \ 0 \leq y \leq 1 \). Shapes of the functions \( x^2 y^2 B_\zeta^Y(1, x, y) \) and \( x^2 y^2 \tilde{B}_\zeta(1, x, y) \) are depicted in the first two panels of fig. \( [4] \) All functions in the figure are normalized to have value 1 in the equilateral limit, \( x = y = 1 \).

Function \( B_\zeta^Y(k_1, k_2, k_3) \) has the same shape as bispectrum derived by Endlich et al. in the model where the inflation is driven by the solid only \( [12] \). It peaks in the squeezed limit, \( x = 1, \ y = 0 \), with anisotropic dependence on how the limit is approached. The second part of the bispectrum \( \tilde{B}_\zeta(k_1, k_2, k_3) \) follows from the presence of the scalar field in our combined model and it has similar shape as the first one.

It is not unexpected that our model with the additional degree of freedom allows for a wider range of different shapes of the bispectrum. The overall bispectrum peaks in the squeezed limit, unless \( \bar{F}/F_Y = (5/6) N_c^{-1} c_{L,c}^{-2} \), when it peaks in the equilateral limit instead. This case is depicted in the third panel of fig. \( [2] \) An example of the overall bispectrum for \( \bar{F}/F_Y > (5/6) N_c^{-1} c_{L,c}^{-2} \), when the relative sign of the bispectrum in the squeezed limit and the bispectrum in the equilateral limits flips, is depicted in the fourth panel.

Apart from the shape of the bispectrum, we are interested also in its size. It is given by the non-linearity parameter \( f_{\text{NL}} \), defined for the Newtonian potential \( \Phi \), which is proportional to the scalar \( \zeta \) in the long-wavelength limit, \( \Phi = 3\zeta/5 \). Following the definition (4) in \( [33] \), we can use the formula

\[ f_{\text{NL}} = \frac{5}{72\pi^4} \frac{k^6 B_\zeta(k,k,k)}{P_\zeta^2(k)}, \]
and by inserting (4.32) and (5.7) into it, we find

\[
f_{\text{NL}} = \frac{\epsilon_c}{\epsilon_c + \left(c_L^2 - 1\right)p_c^2} \left(\frac{19415}{13122} \frac{F_Y}{c_L^2} - \frac{5}{18} \frac{N_\zeta}{\tilde{F}} F\right). \tag{5.18}
\]

We can see that if \(\epsilon - p \sim \epsilon \sim p\), the non-linearity parameter is of the order \(f_{\text{NL}} \sim (F_Y/F)c_L^{-2}\epsilon^{-1}\), the same as for the solid inflation without the scalar field, or \(f_{\text{NL}} \sim N_\zeta(\tilde{F}/F)\epsilon^{-1}\). Supposing that \(c_L^2 \sim \epsilon\) we have \(f_{\text{NL}} \sim (F_Y/F)c_L^{-2}\epsilon^{-3}\) or \(f_{\text{NL}} \sim N_\zeta(\tilde{F}/F)\epsilon^{-3}\) if \(\epsilon - p\) is of the order \(\epsilon^2\). The overall form of the non-linearity parameter is more complicated than in the solid inflation, since our model features more parameters of the theory.

The condition \(\epsilon - p \lesssim \epsilon^2\) leading to an amplification of the non-linearity parameter can be rewritten as \(q \ll p\), which means that the contribution of the solid matter to the overall stress-energy tensor is negligible in comparison to the contribution of the scalar field.

So far, we have two constraints on our model given by observations. The first one concerning the scalar spectrum is that the spectral index must have the value \(n_S = 0.968\) \cite{35}, and the second one is \(f_{\text{NL}}\) to be not much larger than \(10\) \cite{36}. In our model there are three independent parameters of the theory which are not necessarily suppressed by the slow-roll parameters, \(F_Y\), \(\tilde{F}\) and \(c_L\), and in principle, the observational constraints can be satisfied. The way how to obtain more restrictions on our model is to study tensor perturbations, although they are beyond the reach of current observations. It is only known that the tensor-to-scalar ratio cannot be larger than of the order of 0.1 \cite{35,37}. However, in order to make the analysis of the model in consideration complete, in the next section we compute the tensor spectrum and bispectrum.
6 Tensor perturbations

Because the same technicalities which have been used for the analysis of the scalar perturbations are applicable also for the tensor perturbations, in this section the results are summarized more succinctly than in the previous ones. The only results in this section which differentiate our model from the solid inflation model are the tensor spectral tilt (6.8) and the tensor-to-scalar ratio (6.9), which now contain the additional slow-roll parameter $p$. The tensor bispectrum is affected by the presence of the scalar field only for higher orders of the slow-roll approximation, which are not included here.

The tensor modes can be decomposed into two independent polarizations,

$$\gamma_{ij} = \sum_{P=+,-} e_{ij}^{P} \gamma_{ij}^{P},$$  

where the polarization tensor $e_{ij}^{P}$ must satisfy the traceless and transversal conditions $e_{ii}^{P} = 0$ and $k_{i} e_{ij}^{P} = 0$, and as the normalization condition we use $e_{ij}^{P} e_{ij}^{P*} = \delta_{PP'}$. The quantized tensor modes can be written in the form

$$\gamma_{ij} = \sum_{P=+,-} \left( e_{ij}^{P} \gamma_{ij}^{(cl)} \frac{\gamma_{ij}^{(cl)} - \gamma_{ij}^{P}}{a_{ij}} + e_{ij}^{P*} \gamma_{ij}^{(cl)*} \frac{\gamma_{ij}^{(cl)*} - a_{ij}}{a_{ij}} \right),$$  

where the creation and annihilation operators obey the standard commutation relations and $\gamma_{ij}^{(cl)}$ denotes the classical solution of equation of motion given by the tensor quadratic action (3.14).

The first equation in (3.25) implies that if the parameter $\eta_{L}$ defined in (4.4) is small, and the parameter $\eta_{T}$ defined in the same manner,

$$\eta_{T} = \frac{\dot{c}_{T}}{c_{T} H},$$  

must be small as well. Therefore, under the assumptions which have been imposed in the previous sections, the equation for tensor perturbations can be written in the simplified form as

$$\gamma_{ij}^{(cl)} - \frac{1}{\tau} \gamma_{ij}^{(cl)} + \left( k^2 + \frac{4 \epsilon_{c} - p_{c}}{\tau^2} \right) \tau_{ij}^{(cl)} = 0,$$  

where only terms up to the first order of the slow-roll approximation have been kept, and notation follows the previous sections. By solving this equation and matching the canonically normalized tensor mode, $\gamma_{ij}^{(can)} = \frac{1}{\sqrt{2}} M_{Pl} a_{ij} \gamma_{ij}^{(cl)}$, to the free wave function of the Minkowski space vacuum, we find

$$\gamma_{ij}^{(cl)} = -\frac{\tau}{2 M_{Pl}} \left( -\epsilon_{c} \right) (1 - \tau_{c})^{-\epsilon_{c}} \epsilon_{c} \frac{\tau_{c}}{2} \left( -\tau_{c} \right) \frac{\tau_{c}}{2} \frac{\tau_{c}}{2} \dot{c}_{T}^{(cl)} (-c_{T}),$$  

where $\epsilon_{c} = (1 + L_{c}^{2})p - c_{L}^{2} \epsilon_{c}$.

The tensor power spectrum $P_{\gamma}(k)$ is defined by

$$\langle \gamma_{kij} \gamma_{klij} \rangle = \frac{P_{\gamma}(k_{1})}{2 k_{1}^{3}} (2\pi)^{3} \delta^{(3)}(k_{1} + k_{2}),$$  

where the late time two-point tensor function is

$$\langle \gamma_{kij} \gamma_{klij} \rangle = (2\pi)^{3} \delta^{(3)}(k_{1} + k_{2}) \sum_{P=+,-} \left( \frac{H^{2}}{M_{Pl}^{2}} k_{1}^{2} \right)^{2 c_{c} - 2 \epsilon_{c} \gamma}.$$  

As a result, the tensor spectral tilt is small,

\[ n_T - 1 = -2e_c^2 = 2c_L^2 \epsilon_c - 2(1 + c_L^2) \rho_c. \]  

(6.8)

Furthermore, the tensor-to-scalar ratio is

\[ r = \frac{P_T}{P_s} = \frac{4c_L^2 \epsilon^2}{\epsilon + (c_L^2 - 1) \rho}, \]

(6.9)

being of the order \( r \sim c_L^2 \epsilon^2 \sim \epsilon^2 \) if \( \epsilon - p \sim \epsilon \sim p \) and \( c_L^2 \sim \epsilon \), and it is amplified to the order of \( \epsilon \) if \( \epsilon - p \) is of the order \( \epsilon^2 \). This does not contradict the observational restrictions [35, 37].

The tensor three-point function can be computed in the same way as the scalar one in the previous section. In order to do so, we need the tensor cubic action,

\[ S_T^{(3)} = \int d^4x a^3 \left[ \frac{1}{4} M_T^2 a^{-2} \gamma_{ij} \gamma_{kl} \left( \gamma_{ik,jl} - \frac{1}{2} \gamma_{kl,ij} \right) + \frac{F_T}{24} c_T \gamma_{ij} \gamma_{jk} \right]. \]

(6.10)

Keeping only the leading order terms in the slow-roll approximation, we find the three-point function in the form

\[ \left\langle \gamma_{0} \gamma_{0} \gamma_{0} \right\rangle = \frac{16 \eta^2 (2 \pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)}{M_T^6 (k_1 k_2 k_3)^3} \cdot \left\{ \begin{array}{l}
- \frac{1}{4} M_T^2 \eta^2 \Pi_{ij,ab} (\vec{k}_1) \Pi_{ij,cd} (\vec{k}_2) \Pi_{ij,bc} (\vec{k}_3)(k_3)^{d - 4} + 5 \text{ permutations} \Gamma_{k_1,k_2,k_3} +
+ \frac{2}{9} F_T \Pi_{ij,ab} (\vec{k}_1) \Pi_{ij,cd} (\vec{k}_2) \Pi_{ij,bc} (\vec{k}_3) \Xi_{k_1,k_2,k_3},
\end{array} \right. \]

(6.11)

where \( \Pi_{abcd}(\vec{k}) = \sum_s e^P_{abkh} e^P_{cdkh} \), and \( \Gamma_{k_1,k_2,k_3} \) and \( \Xi_{k_1,k_2,k_3} \) are given by the integrals

\[ \Gamma_{k_1,k_2,k_3} = \text{Re} \left\{ \int_0^\infty (i + k_1 z)(i + k_2 z)(i + k_3 z)e^{i(k_2 + k_2 + k_3)z} z^{-2} dz \right\}, \]

(6.12)

\[ \Xi_{k_1,k_2,k_3} = \text{Re} \left\{ \int_0^\infty (i + k_1 z)(i + k_2 z)(i + k_3 z)e^{i(k_2 + k_2 + k_3)z} z^{-4} dz \right\}, \]

(6.13)

The first integral can be computed with the tilted integration contour, as the integral (6.10), and it is of the form

\[ \Gamma_{k_1,k_2,k_3} = \sum_i k_i - \frac{1}{2} \frac{\sum_{i \neq j} k_i k_j}{\sum_i k_i} - \frac{\prod_i k_i}{(\sum_i k_i)^2}, \]

(6.14)

but in the second integral a logarithmic divergence occurs due to the lower limit of the integration interval, similarly as in the integral (6.11). If we replace the integration limits \((0, \infty)\) by \((-\tau_c, \infty)\), we find that the second integral in the limit of small \( \tau_c \) is

\[ \Xi_{k_1,k_2,k_3} = \frac{1}{6} \left( \sum_i k_i \right) \left( \sum_{i \neq j} k_i k_j \right) - \frac{4}{3} \prod_i k_i + \left[ \frac{4}{9} - \frac{1}{3} \gamma_{EM} \right] \ln \left( -\tau_c \sum_i k_i \right) \sum_i k_i^3. \]

(6.15)
It is dominated by

\[ \Xi_{k_1, k_2, k_3} = \frac{1}{3} N_\gamma \sum_i k_i^3, \]  

(6.16)

where \( N_\gamma \) is a number of the order of number of e-folds, and the results presented in what follows were computed by using this relation instead of (6.15).

\[ \theta^P = \gamma_{kij} e^{P*}. \]  

(6.17)

Using the properties of the polarization tensor, the tensor three-point function (6.11) can be rewritten for the polarization mode \( \theta^+ \) as

\[ \langle \theta^+_{k_1} \theta^+_{k_2} \theta^+_{k_3} \rangle = -\frac{16 H_2^2}{M_{Pl}^2} \left( \sum k_i \right)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \cdot \]  

(6.18)

\[ G_{k_1, k_2, k_3}^{+++} = \frac{(\sum k_i)^2}{2} \bar{X}_{k_1, k_2, k_3}^{+++}, \]  

(6.19)

\[ \bar{X}_{k_1, k_2, k_3}^{+++} = \frac{(\sum k_i)^3}{64 (\prod k_i)^3} \left[ \left( \sum k_i \right)^3 - 2 \left( \sum k_i \right) \left( \sum_{i \neq j} k_i k_j \right) + 8 \prod k_i \right]. \]
The tensor bispectrum $B_{TTY}(k_1, k_2, k_3)$ consists of two parts,

\[
\langle \delta_{k_1} \delta_{k_2} \delta_{k_3} \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{TTY}(k_1, k_2, k_3) = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \left[ B_{TTY}^{(grav)}(k_1, k_2, k_3) + N_c F Y B_{TTY}^{(s)}(k_1, k_2, k_3) \right].
\]

The behavior of both parts as well as of their sum for two values of $F Y$ is depicted in fig. 3. The first part of the bispectrum $B_{TTY}^{(grav)}(k_1, k_2, k_3)$, given by the non-linear structure of the scalar curvature in general relativity, can be found in most inflationary models, in particular single-field ones, see [38]. The second part $B_{TTY}^{(s)}(k_1, k_2, k_3)$ represents the effect of the solid, while the presence of the scalar field in our model affects the tensor bispectrum only in higher orders of slow-roll approximation, which are not included in our work.

Both parts of the tensor bispectrum peak in the squeezed limit. The overall bispectrum does not peak in this limit if $F Y = F Y^{(lim)} = (81/16)M^2_F^2 N_c^{-1}$, and it peaks in the equilateral limit instead. For $F Y < F Y^{(lim)}$ the peak in the squeezed limit has the same sign as the bispectrum in the equilateral limit and for $F Y > F Y^{(lim)}$ their relative sign is minus. This is demonstrated in the third and fourth panel of the fig. 3. The tensor bispectrum is always zero in the folded limit.

7 Conclusion

In this paper we have studied a model in which inflation is driven not only by a solid as in [12], but also by a scalar field $\varphi$ with standard kinetic term. The object defining the model is the potential $F(\varphi, X, Y, Z)$, where the quantities $X, Y$ and $Z$ defined in (1.4) describe the solid. The model represents the most straightforward combination of solid inflation and the basic single-field inflationary model. It can be considered as, for instance, a simple toy model of interactions of fields driving the solid inflation with fields of an effective field theory of the standard model.

Due to the additional degree of freedom, the slow-roll parameter $\epsilon$ is a function of two independent parameters $p$ and $q$ defined in [14], which, in principle, allows for a wide range of inflationary scenarios. However, we have restricted ourselves to the special case such that both $p$ and $q$ are small, being of the same order as the slow-roll parameter. As a consequence, the scalar field mass squared $-F_{\varphi\varphi}$ is of the first order in the slow-roll parameter, which leads to the relation (3.25) between the transversal sound speed $c_T$ and the longitudinal sound speed $c_L$. Moreover, in case that $F_{\varphi\varphi}$ has a special form given in (1.5) the analysis of the cosmological perturbations can be treated analytically, since equations of motion for two scalar perturbations present in our model become decoupled if $p$ is replaced by $U$.

Under assumptions adopted above the scalar spectrum is nearly flat and for the scalar bispectrum different shapes are allowed. The reason is that there are three independent parameters of the theory which are not necessarily suppressed by the slow-roll parameters, $F Y$, $\bar{F}$ defined in [5,14] and the longitudinal sound speed. We computed the scalar bispectrum only in the leading order of the slow-roll approximation which does not require $F_{\varphi\varphi}$ to be of the special form given by [1,5]. In solid inflation the bispectrum peaks in the squeezed limit with an anisotropic dependence on how the limit is approached. This applies also for our combined model, unless $\bar{F}/F Y = (5/6)N_c^{-1}c_L^{-2}$, when the bispectrum peaks in the equilateral limit instead. The non-linearity parameter is of the order $f_{NL} \sim (F Y/F)c_L^2\epsilon^{-1}$, the same as for the solid inflation without scalar field, or $f_{NL} \sim N_c (\bar{F}/F)\epsilon^{-1}$, and is amplified by a factor of the order $\epsilon^{-2}$ when $\epsilon - p$ is of the order $\epsilon^2$, i.e. when the contribution of the solid matter to the overall stress-energy tensor is much smaller than the contribution from the scalar field. In this case the relation (6.20) between the sound speeds $c_T$ and $c_L$ is not valid. The case when $\epsilon - p$ is of order $\epsilon^{5/2}$ or smaller is excluded, since the sound speeds would be superluminal or imaginary.
The tensor power spectrum is nearly flat with spectral tilt given by the slow-roll parameters $\epsilon$ and $p$ and the longitudinal sound speed. The tensor-to-scalar ratio is of the order $r \sim c_s^2 \epsilon \sim \epsilon^2$ if $\epsilon - p \sim \epsilon \sim p$, while for $\epsilon - p$ being of the order $\epsilon^2$ we have $r \sim \epsilon$, which is in agreement with the observational restrictions. In our model the tensor bispectrum computed in the leading order of the slow-roll approximation does not differ from the tensor bispectrum in solid inflation. It is affected by presence of the scalar field only in the higher orders of the slow-roll approximation, which are not included in our work.

Although the analysis of cosmological perturbations presented in our work is valid only in the limit $p \ll 1$, $q \ll 1$, the problem can be studied also for other ranges of the parameter space. Of course, this would demand more complicated or even nonanalytic treatment. It is to be expected that some choices may lead to results which would be refuted by observations, but we cannot exclude that some of them, in addition to the one studied in this paper, may be allowed by the observational restrictions. We leave this for future work.

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References

[1] J. M. Maldacena, JHEP 0305, 013 (2003).
[2] L. Senatore, K. M. Smith, M. Zaldarriaga, JCAP 1001, 028 (2010).
[3] C. Armendariz-Picon, T. Damour, V. Mukhanov, Phys. Lett. B458, 209-218 (1999).
[4] E. Silverstein, D. Tong, Phys. Rev. D70, 103505 (2004).
[5] M. Alishahiha, E. Silverstein, D. Tong, Phys. Rev. D70, 123505 (2004).
[6] N. Bartolo, M. Fasiello, Sabino Matarrese, A. Riotto, JCAP 1008, 008 (2010).
[7] N. Arkani-Hamed, P. Creminelli, S. Mukohyama, M. Zaldarriaga, JCAP 0404, 001 (2004).
[8] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, L. Senatore, JHEP 0803, 014 (2008).
[9] X. Chen, M. Huang, S. Kachru, G. Shiu, JCAP 0701, 002 (2007).
[10] R. Holman, A. J. Tolley, JCAP 0805, 001 (2008).
[11] A. Gruzinov, Phys. Rev. D70, 063518 (2004).
[12] S. Endlich, A. Nicolis, J. Wang, JCAP 1310, 011 (2013).
[13] S. Endlich, B. Horn, A. Nicolis, J. Wang, Phys. Rev. D90, 063506 (2014).
[14] M. Akhshik, JCAP 1505, 043 (2015).
[15] N. Bartolo, S. Matarrese, M. Peloso and A. Ricciardone, JCAP 1308, 022 (2013).
[16] M. Sitwell, K. Sigurdson, Phys. Rev. D89, 123509 (2014).
[17] N. Bucher, D. N. Spergel, Phys. Rev. D60, 043505 (1999).
[18] R. A. Battye, A. Moss, Phys. Rev. D74, 041301 (2006).
[19] R. A. Battye, A. Moss, Phys. Rev. D80, 023531 (2009).
[20] R. A. Battye, A. Moss, Phys. Rev. D76, 023005 (2007).
[21] R. A. Battye, J. A. Pearson, Phys. Rev. D88, 084004 (2013).
[22] R. A. Battye, N. Bucher, D. Spergel, (1999), [astro-ph/9908047]
[23] A. Leite, C. Martins, Phys. Rev. D84, 103523 (2011).
[24] S. Kumar, A. Nautiyal, A. A. Sen, Eur. Phys. J. C73, 2562 (2013).
[25] A. Ricciardone, G. Tasinato, (2016), [arXiv:1611.04516 [astro-ph.CO]].
[26] F. Lucchin, S. Matarrese, Phys. Rev. D32, 1316 (1985).
[27] A. R. Liddle, Phys. Lett. B220, 502-508 (1989).
[28] S. Habib, K. Heitmann, G. Jungman, and C. Molina-Paris, Phys. Rev. Lett. 89, 281301 (2002).
[29] S. Habib, A. Heinen, K. Heitmann, G. Jungman, and C. Molina-Paris, Phys. Rev. D70, 083507 (2004).
[30] S. Weinberg: Cosmology, Oxford University Press (2008).
[31] D. H. Lyth, A. Riotto, Phys. Rept. 314:1-146 (1999).
[32] S. Weinberg, Phys. Rev. D72, 043514 (2005).
[33] D. Babich, P. Creminelli, M. Zaldarriaga, JCAP 0408, 009 (2004).
[34] P. Creminelli, A. Nicolis, L. Senatore, M. Tegmark, M. Zaldarriaga, JCAP 0605, 004 (2006).
[35] Planck Collaboration, (2015), [arXiv:1502.02114 [astro-ph.CO]].
[36] Planck Collaboration, Astron. Astrophys. 571, A24 (2014).
[37] BICEP2, Keck Array Collaboration, Phys. Rev. Lett. 116, 031302 (2016).
[38] X. Gao, T. Kobayashi, M. Yamaguchi, J. Yokoyama, Phys. Rev. Lett. 107, 211301 (2011).