Observation of the scissors mode in the quasicontinuum

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The experimental resonance parameters of the pygmy resonance in rare earth nuclei are compared to global observables of the scissors-mode states. It is argued that the pygmy resonance in rare earth nuclei can be described in terms of orbital M1 strength observed in (γ, γ′) experiments. The pygmy resonance is therefore interpreted as the scissors mode in the quasicontinuum.

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I. INTRODUCTION

The phenomena 'pygmy resonance' (PY) and 'scissors mode' (SC) have been discussed separately in the literature. In this work, we investigate for the first time quantitatively, if those two experimentally observed phenomena have the same physical origin.

The PY was introduced as a name for non-statistical features in quasicontinuous γ-ray spectra at energies below the giant dipole resonance. It is e.g. broadly discussed in a review of radiative strength functions as early as in 1973 [3]. These non-statistical features observed in different nuclear mass regions and at different energies have different physical origins. A PY in rare earth nuclei at an energy around 3.5 MeV was first reported in the γ-ray spectrum of $^{170}$Tm following a neutron capture reaction [4]. A systematic investigation of the PY parameters in deformed rare earth nuclei by radiative neutron capture has been carried out in [5], and very recently new investigations on the PY have appeared [6–8]. An important application of the PY is the calculation of astrophysical (n, γ) reaction rates in e.g. the astrophysical r-process [7].

In NRF experiments, the energy-integrated cross-section $I_f$ of resonant scattered γ-rays on the ground state, populating a state at excitation energy $E$ and then decaying down to a low-lying final state $f$, is given by

$$I_f = \frac{\pi^2\hbar^2c^2}{\Gamma} \frac{\Gamma_f}{\Gamma} \frac{\Gamma_0}{E^2}.$$

Here $\Gamma_0$ and $\Gamma_f$ are the partial radiative decay widths for transitions from the excited state to the ground state and the final state, respectively, $\Gamma$ is the total radiative width of the excited state and $g$ is a spin-factor. In NRF experiments on even-even nuclei, the excited states are usually observed to decay into two states, the ground state and the first excited state, denoted by '1' (in deformed nuclei the first rotational 2$^+$ state). The experimental information from NRF experiments on even-even nuclei consists therefore generally of the energy and the values $I_0$ and $I_1$ for every observed state. Often, instead of listing $I_1$ values, the branching ratio $R$ with

$$\frac{I_1}{I_0} = \frac{\Gamma_1}{\Gamma_0} = R \left( \frac{E_f}{E} \right)^3$$

is given [9,10], where $E_1$ is the energy of the γ-ray transition, populating the first excited state. Summing over all final states in Eq. (1), the total energy-integrated photon-absorption cross-section $I_{M1/E1,t}$ is $I_0 + I_1 + \ldots$ is obtained. Experimentally, one observes the partial energy-integrated photon-absorption cross-sections of excited states $I_{M1/E1,p}^{\text{NRF}} = I_0 + I_1$ where the word 'partial' refers to the fact, that only the two decay branches to the ground state and to the first excited state are taken into account in the sum. As can be seen from Eq. (1),

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The decay pattern in odd-nuclei is much more complex, since one can populate by dipole radiation excited levels with, in general, three different spins. In NRF experiments on odd gadolinium and dysprosium isotopes [13], branching ratios have been reported to several different low-lying states. However, very few individual excited states show $\gamma$ decay to more than one of these states at the same time. In addition, the observed branching ratios scatter strongly. For many excited states, only the ground state transition has been detected. A probable explanation for this observation is that the dipole strength in odd nuclei is highly fragmented, thus ground state transitions in odd nuclei are in general much weaker than in even nuclei. The even weaker branches to low-lying states might therefore easily fall below the experimental detection limit. On the other hand, due to the high fragmentation, the chances will increase that accidentally two excited states are separated in energy by the excitation energy of some low-lying state, and that the two respective peaks in the $\gamma$-ray spectrum are therefore interpreted as two branches from one excited state. This problem has e.g. already been observed in the even nucleus $^{172}\text{Yb}$ [10].

In general, data on odd nuclei are very scarce (see e.g. [12] and references therein), and the results are not conclusive. The summed observed M1 strength in odd nuclei is by far weaker than in even-even nuclei. This has been explained in terms of unresolved strength hiding in the experimental background. A fluctuation analysis on the background has been performed [13], and indeed the resulting summed M1 strength in $^{157}\text{Gd}$ has been found to be approximately the same value as in the neighboring even nuclei. However, it has not yet been shown that similar results can be achieved for the much less fragmented M1 strength in $^{164}\text{Dy}$, which has been investigated with a quite similar detection threshold as $^{157}\text{Gd}$ [13], still, a new analysis is in progress and might prove the opposite [14]. In addition, the observed irregular branching properties of excited states in odd nuclei show that the NRF technique is taken to its very limit in the investigation of odd nuclei, and that global SC observables, like summed M1 strengths and average excitation energies of SC states from such experiments, have to be taken with great care. As a last and very important point one should mention that the M1 character of the observed radiation in NRF experiments on odd nuclei has never been measured experimentally. In fact, E1 and even E2 radiation can not be excluded [13]. In the further discussion, we will therefore mainly concentrate on even nuclei, where the deduced data are based on a more transparent experimental situation.

\[ \frac{I_{\text{NRF}}^{\text{M1/E1,p}}}{I_{\text{M1/E1,t}}} = \frac{\Gamma_0 + \Gamma_1}{\Gamma}. \] (3)

### III. PYGMY RESONANCE

The PY photon-absorption cross-section is usually parameterized by a Lorentzian function $\sigma_{\text{py}}(E)$

\[ \sigma_{\text{py}}(E) = \sigma_{\text{py}} \left( 1 + \frac{(E^2 - E_{\text{py}}^2)^2}{E^2\Gamma_{\text{py}}^2} \right)^{-1}, \] (4)

where $\sigma_{\text{py}}$, $E_{\text{py}}$ and $\Gamma_{\text{py}}$ are the strength, centroid and width of the PY, respectively. These parameters have been obtained by fits to experimental data [3], which are shown in Fig. [1]. Previously investigated M1 strength arises due to the tail of the spin-flip resonance (GMDR) [14]. This resonance can be described by a Lorentzian function $\sigma_{\text{sf}}(E)$ as well

\[ \sigma_{\text{sf}}(E) = \sigma_{\text{sf}} \left( 1 + \frac{(E^2 - E_{\text{sf}}^2)^2}{E^2\Gamma_{\text{sf}}^2} \right)^{-1}, \] (5)

where $\sigma_{\text{sf}}$, $E_{\text{sf}}$ and $\Gamma_{\text{sf}}$ are the strength, centroid and width of the GMDR, respectively.

Assuming that the PY is due to magnetic dipole strength, one can add the two cross-sections incoherently and obtain the total M1 photon-absorption cross-section $\sigma_{\text{M1,t}}(E)$

\[ \sigma_{\text{M1,t}}(E) = \sigma_{\text{py}}(E) + \sigma_{\text{sf}}(E). \] (6)

One can, at this point, discuss if deviations from the Lorentzian description of the PY and interference between the PY (presumably orbital M1 strength) and the spin-flip resonance might occur. Neither of the two effects is, however, manifested in the experimental data in Refs. [2–6] (see e.g. Fig. [1]), and therefore they must be very limited in size and are neglected in the further calculation. In the upper panels of Fig. [2] the two contributions to the total M1 photon-absorption cross-section and their sums for the nuclei $^{162}\text{Dy}$ and $^{172}\text{Yb}$ are shown.

When comparing to the results of the NRF experiments, one should, however, in analogy with Eq. (8) define the partial M1 photon-absorption cross-section $\sigma_{\text{M1,p}}^{\text{QC}}(E)$, where branching only to the ground state and the first excited state is taken into account, i.e.

\[ \sigma_{\text{M1,p}}^{\text{QC}}(E) = \sigma_{\text{M1,t}}(E) \frac{\Gamma_0 + \Gamma_1}{\Gamma}. \] (7)

Here, the index ‘QC’ refers to the fact that the semi-experimental quantity $\sigma_{\text{M1,p}}^{\text{QC}}(E)$ is calculated by means of strength functions obtained in the quasicontinuum. The total radiative width in Eq. (8) is given by

\[ \Gamma = D_1 \sum_{X L} \sum_{I_f, \Pi_f} \int_0^E \text{d}E_\gamma \int_0^{E_\gamma} \text{d}E_\gamma \int_0^{E_\gamma} \text{d}E_\gamma \rho(E - E_\gamma, I_f, \Pi_f) \rho(E - 2E_\gamma, I_f, \Pi_f), \] (8)

where the second sum is going over all final levels with spin and parity $I_f, \Pi_f$ which are accessible by multipole
radiation of the type $XL$ from a $1^+$ state, $f_{XL}$ are the respective radiative strength functions, $\rho$ is the level density and $D_{1^+}$ is the average spacing of $1^+$ states. The sum of the partial radiative widths $\Gamma_0 + \Gamma_1$ can be calculated by taking into account in Eq. (8) only the two lowest states in the level density, i.e.

$$\rho(E - E_\gamma, I_f, \Pi_f) = \delta(E - E_\gamma) \delta_{I_f,0^+} + \delta(E_1 - E_\gamma) \delta_{I_f,1^+},$$

and performing the integral and the two sums, yielding finally

$$\langle \Gamma_0 + \Gamma_1 \rangle = D_{1^+} \left[ f_{M1}(E)E^3 + f_{M1}(E_1)E_1^3 \right].$$

In Eq. (9), electric quadrupole and higher-order multipole contributions can usually be neglected as has been done in Eq. (10).

In order to be able to evaluate Eqs. (7), (8) and (10), the level density and the radiative strength functions $f_{E1}$ and $f_{M1}$ have to be known. Recently, these data have been obtained experimentally [5] (see Fig. 1) for the four nuclei $^{161,162}\text{Dy}$ and $^{171,172}\text{Yb}$ from the $(^3\text{He},\alpha)$ reaction by using an iteration technique. The parameters of the PY have also been extracted from these experiments. The quality of the data allows us to calculate the partial photon-absorption cross-section and make comparison to NRF data.

A problem encountered here is that radiative strength functions, obtained for $\gamma$ transitions in the quasicontinuum, have to be used at low excitation energies where nuclear structure effects can be predominant. For instance, the observed branching ratios in NRF experiments [10,11] for even nuclei with $\Gamma_1/\Gamma_0 \approx 0.5$ and 2 for $K = 1$ and 0 states, respectively, cannot be reproduced accurately within this approach, where one obtains branching ratios of approximately 1 for both cases. The branching ratios are, on the other hand, well estimated by the Alaga rules, describing the nucleus in the rotational limit and thereby assuming a good $K$ quantum number. Unfortunately, a nuclear structure effect represented by e.g. a good $K$-quantum number can not be easily incorporated in the common definition of the radiative strength function. Therefore, we use here the statistical approach of Eq. (10). Since the sum in Eq. (10) is over two low-lying states only, we can not expect that all possible nuclear structure effects are averaged out by our treatment.

In Fig. 2 the total and partial M1-photon-absorption cross-sections, calculated according to Eqs. (2) and (3), and their ratio are shown for the nuclei $^{162}\text{Dy}$ and $^{172}\text{Yb}$. Branching ratios further further discussed in Sect. 3. The partial energy-integrated M1-photon-absorption cross-section $I_{M1,p}^{QC}$, which should be compared to the experimental SC value $I_{M1,p}^{NRF}$, can now be calculated by

$$I_{M1,p}^{QC} = \int dE \sigma_{M1,p}^{QC}(E),$$

where the integral covers the appropriate energy interval of interest.

**IV. COMPARISON**

In the following, experimental data of the PY will be compared to the global observables of SC data. First, a comparison of the centroids and widths is performed. For the SC, one obtains an average excitation energy of 2.87 MeV for $^{162}\text{Dy}$ [3]. This value changes by less than 10 keV when taking into account possible additional strength where no parity assignment was possible. The NRF experiment on $^{172}\text{Yb}$ was performed without polarimeter, thus, no model-independent parity assignment could be performed. If one, however, assumes that all states with $K = 1$ are populated by M1 radiation the average excitation energy of the SC states is 3.09 MeV [10]. Including states with uncertain $K$ assignment yields a higher value of 3.26 MeV, mainly due to the strong transition at 3.863 MeV. These values compare well with the centroids of the PY being 2.73(5) MeV and 3.48(7) MeV for $^{162}\text{Dy}$ and $^{172}\text{Yb}$, respectively.

The experimentally observed spreading of SC states in $^{162}\text{Dy}$ and $^{172}\text{Yb}$, which is estimated by the energy difference between the highest and the lowest observed SC state in the NRF experiment, are 0.67 MeV and 1.0 MeV, respectively [11,12]. The latter value increases to 1.3 MeV when taking the transition at 3.863 MeV into account. These values are in good agreement with the widths of the PY, which were determined to be 1.35 MeV and 1.30 MeV for $^{162}\text{Dy}$ and $^{172}\text{Yb}$, respectively.

Figure 3 shows the distribution of SC states for both nuclei. In the same figure, also $\sigma_{M1,p}^{QC}(E)$ is displayed. The distribution of observed SC states coincides very well with the maximum and the width of the $\sigma_{M1,p}^{QC}(E)$ curves.

In the second step, the strength of the PY is compared to the summed cross-section of SC states. For $^{162}\text{Dy}$,
one obtains $I_{M1,p}^{NRF} = 0.41(4) \text{ MeV mb}$ for all transitions with M1 assignment and 0.44(5) MeV mb, when also taking into account transitions to states with undetermined parity. For $^{172}\text{Yb}$, these quantities read 0.25(10) MeV mb and 0.34(15) MeV mb, respectively. The difference of some 0.09 MeV mb between the latter values is mostly due to the state at 3.863 MeV with uncertain $K$ assignment and hence uncertain parity. Here, one should recall once more that the M1 character of transitions in $^{172}\text{Yb}$ is not determined model independently by a polarimeter. For the PY, one has to integrate $\sigma_{M1,p}^{Q(C)}(E)$ over an appropriate energy interval. We choose, for both nuclei, a 2 MeV wide energy region between 2 MeV and 4 MeV. The integration intervals are marked by vertical lines in Fig. 3. The choice of the intervals is motivated by the fact that in NRF experiments the experimental conditions allow to observe possible SC states only in this energy region. The chosen integration region coincides also quite well with $E_{pp} \pm \Gamma_{pp}$, and is therefore covering most of the area of the PY. We obtain for $^{162}\text{Dy}$ and $^{172}\text{Yb}$ values of $\sigma_{M1,p}^{Q(C)} = 0.46 \text{ MeV mb}$ and 0.42 MeV mb, respectively. For convenience, all data are also collected in Table I. It is now difficult to judge the errors of these values. Certainly, the Lorentzian shapes of the PY and the GMDR are somehow idealized with respect to nature. In addition, the strength functions used for evaluating Eq. (7) are all obtained experimentally in the quasicontinuum, but they have to be applied to transitions to low-lying states in e.g. Eq. (14). Some shortcomings of the statistical approach to the region of discrete levels have already been discussed in Sect. I. Probably, a considerable systematical error is therefore introduced in $I_{M1,p}^{Q(C)}$. Nevertheless, the difference of 20–40% between the calculated and the experimental values for the $^{172}\text{Yb}$ nucleus and the agreement within 10% for the $^{162}\text{Dy}$ nucleus allow to hope that the systematic error of the calculation is not too high and within an approximate error of 30%, we find good agreement with the SC data.

Other reasons for discrepancy between estimated and experimental SC strength may exist. Firstly, some strength of the PY could in fact be E1 radiation. Such a resonance has been suggested by P. Van Isacker et al. in a schematic calculation. Within their model, the valence neutrons oscillate with respect to the core of neutrons and protons. However, this excitation mode has not yet been observed in NRF experiments. Another possibility is that in NRF experiments SC states are populated from the ground state where pairing correlations are important. The PY strength functions are, on the other hand, obtained in the quasicontinuum, where depairing has been observed. It has been shown, that depairing yields in general a higher summed M1 strength and also higher excitation energies of the SC.

Unfortunately, the M1 character of the PY in rare earth nuclei could not yet be determined experimentally. In fact, the PY in iron and lead nuclei have recently been measured to be of E1 character. Since the PY in these nuclei, which have other mass numbers and deformations than rare earth nuclei, occur at higher excitation energies and consequently may have different physical origins, no strong conclusions to rare earth nuclei should be drawn from these results. An indirect proof for the M1 character of the PY in rare earth nuclei comes from ($n,2\gamma$) experiments on $^{162}\text{Dy}$ with thermal neutrons, using the sum-coincidence method. Here, P. Becvár et al. have found good agreement with experiment when including a resonant component around 3 MeV in their M1 strength-function model. Since they only use schematic level density functions in their analysis, this result can, however, only serve as a strong hint of the M1 character of the PY in rare earth nuclei. Still, one can conclude that different experimental observations commonly labeled as ’pygmy resonances’ might have different physical origins.

V. APPLICATIONS

A. Numerical values of $\Gamma_0$ and $B(M1)$ from NRF experiments

In NRF experiments, often ground-state decay widths $\Gamma_0$ or reduced transition probabilities $B(M1/E1)$ are given as results. These quantities can be calculated from the total energy-integrated photon-absorption cross-section $I_{M1/E1,t}$ by

$$I_{M1/E1,t} = \frac{\pi^2 \hbar^2 c^2 q}{E^2} \Gamma_0$$

and

$$\Gamma_0 = \frac{16\pi}{9q} \left( \frac{E}{\hbar c} \right)^3 B(M1)$$

where the latter Equation is only valid for M1 transitions. However, only the partial energy-integrated photon-absorption cross-section $I_{M1/E1,t}^{NRF}$ is measured in NRF experiments, as discussed in Sect. I. Usually, one therefore assumes that $I_{M1/E1,t} = I_{NRF}^{M1/E1,p}$, i.e.

$$\Gamma_0 + \Gamma_1 = \Gamma$$

over the whole energy region where excited states are observed. On the other hand, Fig. 3 shows that this assumption may become incorrect for energies above ~3 MeV. Of course, the estimates in the lower panels of Fig. 3 are quite rough, since experimental radiative strength functions obtained in the quasicontinuum are applied to the energy region of discrete levels. Still, the large deviation from the common assumption (14) can not be neglected because we have obtained good agreement between estimated and experimental partial energy-integrated photon-absorption cross-sections, see Table I. As a consequence, too low $B(M1)$ values may have been reported in NRF works. We will therefore in the following estimate the summed $B(M1)$ value of the
SC, based on the assumption that the PY is entirely due to orbital M1 strength.

Combining Eqs. (13) and (13) and summing over all individual SC states, one obtains

\[ I_{M1,t} = \frac{16\pi^2}{9h_c} \sum E B(M1) = \frac{16\pi^2}{9h_c} \sum B(M1), \]  

(15)

where \( \bar{E} \) is the average excitation energy of the SC states. Now, we estimate \( I_{M1,t} \) by integrating over the Lorentzian PY model of Eq. (13) from zero to infinity, yielding

\[ I_{M1,t} = \frac{\pi}{2} \sigma_{py} \Gamma_{py}. \]  

(16)

Assuming that the average excitation energy of the SC states \( \bar{E} \) is equal to the centroid \( E_{py} \) of the PY, we finally obtain an estimate for the summed \( B(M1) \) value of SC states

\[ \sum B(M1) = \frac{9h_c \sigma_{py} \Gamma_{py}}{32\pi^2 E_{py}}. \]  

(17)

Using experimental PY parameters from Ref. 5, summed \( B(M1) \) strengths of \( 7 \pm 2 \mu_N^2 \) and \( 6 \pm 2 \mu_N^2 \) for \(^{162}\)Dy and \(^{172}\)Yb, respectively, are found. Here, one should stress that these values follow rather directly from experimental data. Neither branching ratios are needed, nor the assumption of a Lorentzian shape of the PY does influence the physical nature of this possible effect. Integrating Eq. (18) over an appropriate energy interval should now yield a value approximately equal to the sum of all partial energy-integrated E1 photon-absorption cross-sections \( \sigma_{E1,p}^{\text{NRF}} \) observed in NRF experiments and defined in Eq. (16).

In Fig. 3 the distribution of partial energy-integrated E1 photon-absorption cross-sections is displayed. From the NRF experiments, we obtain \( \sigma_{E1,p}^{\text{NRF}} = 0.07(1)\text{MeV mb} \) and \( 0.6(3)\text{MeV mb} \) for \(^{162}\)Dy and \(^{172}\)Yb, respectively, when taking into account only levels with certain parity assignments in the case of \(^{162}\)Dy and certain \( K \) assignments in the case of \(^{172}\)Yb. These values read \( 0.10(2)\text{MeV mb} \) and \( 0.7(3)\text{MeV mb} \) for the two nuclei when also taking into account levels with uncertain assignments. Integrating Eq. (18) from 2 MeV to 4 MeV, as it is done in the case of M1 radiation in Sect. VI, we obtain from the quasicontinuum 0.5 MeV mb and 0.7 MeV mb for \(^{162}\)Dy and \(^{172}\)Yb, respectively. Here, we judge the errors of these quantities to be around 30% due to systematical errors, see Sect. VI. For easier comparison all values can also be found in Table I.

Clearly, the observed strength in NRF experiments on \(^{172}\)Yb is very well described by the strength functions from the quasicontinuum, whereas this is not at all true for \(^{162}\)Dy. We have no good explanation for the discrepancy in the latter nucleus. One can of course argue that the E1 strength function from the quasicontinuum might not describe well the observed E1 strength in the region of discrete levels. We find, however, that our approach works well for \(^{172}\)Yb. Certainly, the situation would be even worse when assuming a substantial part of the PY to be of E1 character. The good agreement in terms of the M1 strength in both nuclei and the E1 strength in \(^{172}\)Yb would be degraded, while the overestimation of observed E1 strength in NRF experiments on \(^{162}\)Dy by the strength function from the quasicontinuum would even increase. Another possibility is that a substantial part of the E1 strength in the NRF experiment is hidden in the experimental background. If we, however, accept a detection limit of some 0.005 MeV mb, also this seems quite unrealistic. At last, one can imagine some distinct nuclear structure effect responsible for hindering the population of \( 1^- \) states at around 3 MeV from the ground state. The physical nature of this possible effect is, however, not clear at all.
C. Missing strength in NRF experiments on odd nuclei

An important issue of the SC is the question of missing strength in odd nuclei observed in NRF experiments. The solution of this problem is that in the case of odd nuclei, indeed a large portion of M1 strength is hidden in the experimental background \cite{12,13}. As a consequence, the summed M1 strength in odd nuclei might be comparable to the observed strength in neighboring even nuclei. The method for recovering the full strength yields, however, large errors and gives at the present unsatisfactory results for \(^{161}\text{Dy}\) \cite{12}. The latter statement might be disproved be a new experimental analysis \cite{14}. The PY parameters, on the other hand, show no odd-even effect at all \cite{5} and the summed \(B(M1)\) strengths in odd nuclei, calculated according to Eq. (17) are \(9 \pm 2 \mu^2_N\) and \(7 \pm 2 \mu^2_N\) for \(^{161}\text{Dy}\) and \(^{171}\text{Yb}\), respectively, which are equal within the errors to those of neighboring even nuclei. In the following, a possible explanation of the observations in NRF experiments on odd nuclei is given by means of strength functions from the quasicontinuum.

The calculated partial M1 and E1 photon-absorption cross-sections using Eqs. (8) and (18) are displayed in Fig. 2. Also, the sum of all energy-integrated partial photon-absorption cross-sections from Ref. \cite{11} is calculated, thereby taking into account all observed branching. Integrating the E1 and M1 curves of Fig. 2 as usual from 2 MeV to 4 MeV yields 0.062 MeV mb and 0.083 MeV mb, respectively, and again a systematical error of 30% is assumed. The sum of both contributions is 0.145 MeV mb and agrees well with the value from the NRF experiment 0.11(2) MeV mb. Unfortunately, in NRF experiments on odd nuclei, neither parity nor \(K\) assignments can be performed reliably \cite{11}. Thus one can not distinguish between M1 and E1 strength. Again, the numerical values are collected in Table I for convenience.

Figure 2 also shows the branching ratio \(\langle \Gamma_0 + \Gamma_1 \rangle / \Gamma\) in \(^{161}\text{Dy}\). This branching ratio is generally much smaller in odd nuclei than in even nuclei \(^{162}\text{Dy}\) due to the higher level density in odd nuclei entering Eq. (8). Here, the physical reason for the missing strength in odd nuclei becomes obvious. We find that the missing strength in NRF experiments might not only be due to unobserved elastic photon scattering below the experimental threshold, as suggested in Refs. \cite{12,13}, but also due to unobserved branching to higher lying states than the first excited states; a possibility which was not mentioned in earlier works and which can contribute much more to the summed strength than in even nuclei. These branches could, in principle, be detected in a similar type of fluctuation analysis on the experimental background as in Ref. \cite{12}; still, one has to expect a general tendency of these branches to appear at lower \(\gamma\)-ray energies in the \(\gamma\)-ray spectrum.

Additional sources of uncertainty in the calculation of partial photon-absorption cross-sections in odd nuclei comes from the use of the branching ratio \(\langle \Gamma_0 + \Gamma_1 \rangle / \Gamma\) and the fact that only \(7/2\) states are taken into account as intermediate states\(^2\). As discussed in Sect. \cite{11}, observed branching for odd nuclei in NRF experiments is much more complex than for even nuclei since it is dependent of the spin of the intermediate state and it does not only involve decay to the first excited state. The use of \(\langle \Gamma_0 + \Gamma_1 \rangle / \Gamma\) calculated for intermediate \(7/2\) states should therefore be regarded as an approximation to the average branching properties of all observed states in odd nuclei.

VI. CONCLUSION

In conclusion, global parameters of the SC have been compared to PY parameters. A good agreement has been found for the centroids, the widths and the partial energy-integrated M1 photon-absorption cross-sections. Therefore, although the M1 character of the PY has not been verified experimentally, we conclude, that the PY in rare earth nuclei with high probability originates from orbital M1 strength. Still, direct measurements of the M1 character of the PY by e.g. the sum-coincidence method, using experimental level densities, is very desirable.

Further, we have shown that branching to highly excited states from SC states might be important in the estimation of B(M1) strengths. Our approach, which does not depend on the knowledge of branching ratios, gives for even and odd nuclei summed B(M1) strengths of \(6.9 \mu^2_N\) for the SC in the quasicontinuum, which follow rather directly from experimental data and are in good agreement with a sum-rule approach using free \(g\)-factors. The E1 strength observed in NRF experiments could also be reproduced by the strength functions from the quasicontinuum in the case of \(^{172}\text{Yb}\), whereas in the case of \(^{162}\text{Dy}\) a discrepancy has been observed.

Also the missing strength in NRF experiments on odd nuclei compared to even nuclei has been interpreted in terms of additional branching to higher lying states than the first excited states. Since branching might play a crucial role in the interpretation of NRF data, we would like to confront our branching estimates to experimental measurements of branching ratios from the sum-coincidence method. Here, one should especially keep in mind that the radiative strength functions used for estimating branching ratios in this work are only based on experimental data down to energies of \(\sim 1.5\) MeV.

\(^2\)Since the ground state spin of \(^{161}\text{Dy}\) is \(5/2\), one can also populate 3/2 and 5/2 states by absorption of dipole radiation.
whereas below, extrapolations of the experimental data by theoretical models have to be used.

The investigation of the SC in the quasicontinuum is complementary to the conventional NRF method. It might yield valuable data on branching ratios, summed M1 strengths and possible odd-even effects, as well as the influence of depairing on the summed M1 strength.

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TABLE I. Partial energy-integrated photon-absorption cross-sections for M1 and E1 radiation from NRF experiments and estimated from strength functions (details see text). All values are in MeV mb. The errors of the calculated values are estimated to be around 30%.

|       | 162Dy | 152Yb | 161Dy |
|-------|-------|-------|-------|
|       | M1    | E1    | M1    | E1    | M1    | E1    |
| $f_{M1/E1,p}^{NRF}$ | only good assignment | 0.41(4) | 0.07(1) | 0.25(10) | 0.6(3) | 0.11(2) |
|       | also uncertain assignment | 0.44(5) | 0.10(2) | 0.34(15) | 0.7(3) | 0.083 |
| $f_{M1/E1,p}^{QC}$  | 0.46   | 0.5   | 0.42   | 0.7   | 0.083 |
|       | 0.062  |       |       |       |       |
FIG. 1. Experimental radiative strength function data for four different rare earth nuclei. The PY is manifested in the data at energies around 3 MeV. The solid line is a fit to the data. For details see Ref. [5].
FIG. 2. In the upper panels, the M1 model, obtained in the quasicontinuum, is displayed. The dashed lines indicate the Lorentzian descriptions of the PY and the GMDR [Eqs. (4) and (5), respectively], the solid line shows the incoherent sum of both models [Eq. (6)]. In the central panels, the total (dashed lines) and the partial (solid lines) M1 photon-absorption cross-sections according to Eqs. (6) and (7) are given. In the lower panels, the ratio of these two quantities, i.e. \( \left< \Gamma_0 + \Gamma_1 \right> / \Gamma \) is shown. The parameters needed for calculating the displayed curves are based on experimental data, see Ref. [5].
FIG. 3. The partial M1 photon-absorption cross-section $\sigma_{\text{QC M1}}^E (E)$ according to Eq. (7) is displayed by solid lines (scale on the left hand side). The maxima of these curves coincide well with the energy distributions of partial energy-integrated photon-absorption cross-sections of SC states $I_{\text{NRF M1 p}}^{\text{p}} [10,11]$ (scale on the right hand side). The full symbols represent states with certain parity or $K$ assignment for $^{162}\text{Dy}$ and $^{172}\text{Yb}$, respectively, the open symbols denote states with uncertain assignments. All data necessary for evaluating Eq. (7) are taken from [5]. The vertical lines show the energy region where the integration of $I_{\text{NRF M1 p}}^{\text{p}}$ for the calculation of $I_{\text{NRF M1 p}}^{\text{p}}$ is performed.
FIG. 4. The same as in Fig. 3 but for E1 radiation. The dashed lines denote the $\sigma_{\text{M1, b}}^{Q_c}(E)$ curves for comparison.
FIG. 5. Upper panels: The partial M1 and E1 photon-absorption cross-sections $\sigma_{QC}^{M1}(E)$ and $\sigma_{QC}^{E1}(E)$ according to Eqs. (7) and (18), respectively. Lower panels: The branching ratio $\langle \Gamma_0 + \Gamma_1 \rangle / \Gamma$. The solid lines denote $^{161}$Dy data and the dashed lines $^{162}$Dy data. The parameters needed for calculating the displayed curves are based on experimental data, see Ref. [5].