Future dark energy constraints from measurements of quasar parallax: Gaia, SIM and beyond.

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ABSTRACT

A consequence of the Earth’s well-measured motion with respect to the Cosmic Microwave Background is that over a 10 year period it will travel a distance of $\sim 800$ AU. As first pointed out by Kardashev in 1986, this distance can be used as a baseline to carry out astrometric measurements of quasar parallaxes, so that only microarcsecond precision is necessary to detect parallax shifts of objects at gigaparsec distances. Such precision will soon be approached with the launch of the astrometric satellites Gaia and SIM. We use a Fisher matrix formalism to investigate the constraints that these and future, even more ambitious missions may be able to place on the cosmological distance scale and the parameters describing dark energy. We find that by observing around a million quasars as planned, an extended 10 year Gaia mission should have the capability to detect quasar parallax shifts at the $2.8\sigma$ level and so measure the Hubble constant to within $25$ km s$^{-1}$. For the interferometer SIM, (in its currently proposed SIMLite configuration) a Key Project using $2.4\%$ of the total mission time to observe 750 quasars could detect the effect at the $2\sigma$ level and dedicated use of the instrument at the $3.3\sigma$ level. In a concordance cosmological model, Gaia and dedicated SIMLite only weakly constrain the presence of a cosmological constant at the $\sim 1\sigma$ levels. We also investigate a range of future mission concepts, such as an interferometer similar in scope and design to NASA’s Terrestrial Planet Finder. This could in principle measure the dark energy parameters $w_0$ and $w_a$ with precision $\sigma_{w_0} = 0.02$ and $\sigma_{w_a} = 0.05$ respectively, yielding a Figure of Merit larger than the stage IV experiments considered in the report of the Dark Energy Task Force. Unlike perhaps all other probes of dark energy there appear to be no obvious astrophysical sources of systematic error on these measurements. There is however uncertainty regarding the statistical errors. As well as measurement error, there will be small additional contributions from image centroiding of variable sources, quasar peculiar motions and weak microlensing by stars along the line of sight.

Key words: Cosmology: observations

1 INTRODUCTION

The quest to measure the cosmological distance scale has been continued over the decades since the discovery of the expansion of the Universe in many different contexts, from studies of the deceleration parameter, to more recently dark energy parameters and modified gravity (see e.g., Frieman et al. 2008, Jain & Zhang 2008). The success of supernova standard candles in revealing the acceleration of the Universe (Perlmutter et al. 1999, Riess et al. 1998) has shown the power of classical tests, while at the same time much effort has been and will be spent in dealing with the many possible systematic errors in the measurements. The simplest and most direct classical test, using pure geometry to measure distances of objects from their parallax shift over time is arguably the most free of systematic uncertainty and easiest to interpret. More importantly it seems as though carrying out parallax measurements on cosmological scales should be feasible, through the combination of astrometric satellites, statistical averaging over many objects, and the long baseline afforded by the Earth’s motion with respect to the Cosmic Microwave Background (CMB). In this paper we investigate how well this combination can be expected to lead to cosmic distance scale and dark energy constraints in the future.

The parallax distance to an object in an expanding Universe was first calculated theoretically and published by Mc-Crea (1935), although he noted that it was unlikely to be
measurable. We give the result in the context of dark energy cosmological models in §2 below. We note that the calculation was also performed by Kardashev, Parlikii and Umarbaeva (1973) who explored the possibility of measurements using radio interferometry. It appears also in the textbook by Weinberg (1972), and the case of inhomogeneous universes was treated by Novikov (1977) (and also Kasai 1988).

Kardashev (1986) was the first to propose that the Earth’s motion with respect to the CMB would provide a much longer usable baseline for parallax measurements and make measurements much less technically challenging than using the Earth’s annual parallax. This effect is a variant of the “secular” parallax (see e.g. Binney and Merrifield, 1998, Section 2.2.3), and is in principle easier to measure because the signal increases linearly with time, while the annual (also known as “trigonometric”) parallax repeats at a constant (small) value.

Rosquist (1987) pointed out that parallax distances of distant objects can be used to determine number densities of conserved classes of objects such as galaxies even if no dynamical model is assumed. Pierce and Cash (2004) explored the possibility that future X-ray interferometers may be able to measure the differential parallax between quasar pairs, and hence characterize dark energy. Most recently, Quercellini et al. (2008) showed how alternative anisotropic models such as Lemaitre-Tolman-Bondi cosmologies with off-center observers would produce a secular parallax effect in distant quasars even for a stationary observer and how upcoming astrometric satellites may be able to put competitive constraints on those models.

Our plan for this paper is as follows. In §2 we summarize prior results for the parallax of extragalactic objects, and generalize them to the case of time varying dark energy models. In §3 we give details of planned future surveys of quasars with astrometric satellites, as well as outlining some hypothetical, more futuristic surveys. In §4 we deal with how the quasar datasets should be analyzed, and what the systematic and statistical uncertainties are likely to be. We describe how well these future surveys can be expected to constrain dark energy parameters in §5, and in §6 we summarize and discuss our results.

2 QUASAR PARALLAX IN DARK ENERGY MODELS

The solar system is moving with respect to the CMB frame at a velocity of 369.5 ± 3.0 km/h towards an apex with galactic latitude and longitude $l = 264.4^\circ ± 0.3^\circ$, $b = 48.4^\circ ± 0.5^\circ$ (Kogut et al. 1993). As a result, all extragalactic objects will experience a parallax shift, increasing linearly with time, towards the antapex with amplitude proportional to $\sin \beta$, where $\beta$ is the angle between the object and the direction of the apex. Over a 10 year period a baseline of $l = 3800\mu$pc is therefore available for measures of parallax. We summarize below the expressions for the parallax shift of a distant extragalactic source (first computed by McCrea, 1935), using the notation due to Kardashev (1986) and Hogg (1999), and making them relevant for a Universe dominated by dark energy.

We write the equation of state for substances in cosmology as:

$$p = w \rho$$

(1)

where $p$ is the pressure, $\rho$ is a dimensionless number, and $\rho$ is the energy density. As usual, dark energy (e.g., Frieman et al. 2008) is defined as a substance that has $w < -\frac{1}{3}$, and thus negative pressure. We follow the usual parametrization for the equation of state for dark energy that varies with time as follows (Chevallier & Polarski 2001, Linder 2003):

$$w = w_0 + (1 - a)w_a,$$

(2)

where $w_0$ is the value of $w$ at redshift $z = 0$, $a$ is the cosmological scale factor given by $a = 1/(1 + z)$ and $w_a$ governs how $w$ changes with time.

The Hubble parameter in terms of redshift is given by (e.g., Hogg 1999):

$$H(z) = H_0 \cdot E(z),$$

(3)

where $H_0$ is the Hubble parameter at redshift $z = 0$. $E(z)$ for dark energy models is given by (Seo and Eisenstein 2003):

$$E(z) = \sqrt{\Omega_M (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_D}\frac{d(z)}{dz}$$

(4)

where $d(z)$ is the equation of state for dark energy (Equation 2) and

$$c(z) = 3 \int_0^z \frac{1 + w(z)}{1 + z} dz.$$  

(5)

The equation for the parallax angle is given by:

$$\pi = \frac{l}{r} + \frac{H_0}{c}$$

(6)

where $l$ is the baseline, and $r$ is the parallax distance defined below (for a flat Universe, it is equal to the comoving distance). In the case of a measurement made from a fixed baseline (such as the usual annual parallax), both terms are relevant. In the case of a measurement where the baseline is expanding with the Universe (which is the case for the present paper), only the first term should be used.

The expression for the parallax distance $r$ in Equation 6, depends on the curvature of the Universe:

$$r = \begin{cases} \frac{1}{H_0} \frac{1}{\sqrt{\Omega_k}} \tanh\left(\sqrt{\Omega_k} \frac{h_0}{c} D\right) & \Omega_k > 0 \\ D & \Omega_k = 0 \\ \frac{1}{H_0} \frac{1}{\sqrt{\Omega_k}} \tanh\left(\sqrt{\Omega_k} \frac{h_0}{c} D\right) & \Omega_k < 0 \end{cases}$$

(7)

where

$$D = \int_0^\pi \frac{d\zeta}{H(\zeta)},$$

(8)

and $\Omega_k$ is the curvature parameter expressed in terms of a fraction of the critical energy density.

We note that the expression for the parallax distance is therefore identical to the angular diameter distance for a flat Universe. For other cosmologies it differs by the substitution of tangent for sine and tangent for sinh (see e.g., Hogg 2000 for the angular diameter distance). For non-flat Universes, therefore, there is not a direct relationship between the other distance measures as there is between angular diameter and luminosity distance, (related by a factor of $(1 + z)$). Measurement of parallax distance therefore promises to reveal new information (see e.g., Rosquist, 1987 for more discussion) and different parameter degeneracies.
3 FUTURE SURVEYS

A measurement of quasar secular parallax is not part of the science requirements for any of the currently planned astrometry missions, Gaia (Lindegren et al. 2008), SIM (Unwin et al. 2008) and Jasmine (Yano et al. 2008), presumably due to the technical challenges. In the present paper, we will make predictions for what might be observable with Gaia and SIM, in the best possible scenario, but we recognize that for cosmological constraints to be competitive with other methods (at least in terms of statistical errors), significantly more futuristic instruments will need to be employed.

Table 1. Predicted number of quasars (with SDSS i band magnitude η 20) observable over the whole sky, (see §3)

| Redshift | Number of quasars |
|----------|-------------------|
| 0.25     | 40,000            |
| 0.75     | 33,824            |
| 1.25     | 49,404            |
| 1.75     | 40,7160           |
| 2.25     | 26,3650           |
| 2.75     | 14,5620           |
| 3.25     | 57290             |
| 3.75     | 19,020            |
| 4.25     | 5700              |
| 4.75     | 1880              |
| 5.25     | 480               |
| 5.75     | 90                |
| 6.25     | 20                |

Figure 1. The expected secular parallax shift of quasars over 10 years, as a function of redshift. We show curves for three different models. Error bars are given on the Ω_M = 0.3, Ω_Λ = 0.7 curve for 5 of the different experiments from Table 3 (all except “SIM-LiteKP”). The horizontal extent of each error box is arbitrary.

We have not made detailed simulations of the performance of any of the instruments (planned or hypothetical). We have merely used estimates from the literature of the expected performance of upcoming missions, and also crudely extrapolated those to make estimates for futuristic mission concepts. In any case, the most important inputs at the level we are doing the calculations are the number of quasars and the expected astrometric measurement accuracy per quasar. Because a secular parallax is being measured, the relevant error bars are those on the proper motion of objects, which are different to the trigonometric parallax errors. The proper motion errors for Gaia and SIM (see below) are quoted in terms of µarcsec/yr, for a measurement made over a 5 year period. If the observing period is increased, the proper motion errors will decrease, both because the secular parallax increases linearly with time and because the number of pairs of observations separated by time ∆t increases. This yields a proper motion error that scales approximately as t^−3/2 (C. Bailer-Jones, private communication), a factor which we use in our calculations. In this paper, for all the missions, we have assumed a 10 year baseline for observations. In the case of Gaia and SIM, this would require extensions to the nominal 5 year missions We note that the current planned extended Gaia mission is 6 years, (Lindegren et al. 2008), so that 10 years may be too optimistic.

Table 2. Sky-averaged rms proper motion error as a function of visual apparent magnitude for the Gaia satellite (data from Lindegren et al. 2008, see §3)

| V mag | 6-13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|-------|------|----|----|----|----|----|----|----|
| σ_{PM} (µas yr⁻¹) | 5 | 7 | 11 | 18 | 30 | 50 | 80 | 145 |

3.1 Gaia

An estimate for the number of quasars that Gaia is expected to observe is available in the Concept and Technology Study (from the year 2000) carried out by the GAIA Science Advisory Group (D. Schneider, private communication, data also available online). We reproduce the expected number of quasars with Sloan i magnitude < 20 in bins of redshift width Δz = 0.5 in table 1. We emphasize that although more recent estimates of the luminosity function have been made since, at the level of our exploratory analysis the Gaia official predictions are more than adequate. Finding targets for the astrometry satellites to observe will rely on prior surveys, such as SDSS (e.g., Richards et al. 2009) For comparison, the number of confirmed quasars with spectroscopically measured redshifts in the catalog by Veron-Cetty & Veron (2006) is 85221.

Gaia will observe large numbers of quasars (around a million), but the sky survey will not spend greater periods observing fainter objects, so that the positional accuracies will be a function of magnitude. We have used the magnitudes of the quasars in the Concept and Technology Study data to compute for each quasar the 1σ expected error on the secular parallax, σπ. This was done using the proper motion errors from Lindegren et al. (2008), which we repeat

1 http://www.rssd.esa.int/SA-general/Projects/GAIA_files/LATEX2HTML/report
in Table 2. The quasars in the predicted dataset have Sloan i magnitudes between 12 and 20, so that the faintest have a proper motion error $\sigma_{\text{PM}} = 145\mu\text{arcsec yr}^{-1}$. To convert this proper motion error into an estimate of the quasar secular parallax measurement uncertainty $\sigma_\pi$ after 10 years, we use $\sigma_\pi = \sigma_{\text{PM}} \times 10\text{yrs} \times (5\text{yrs}/10\text{yrs})^{1.5}$. This yields $\sigma_\pi = 510\mu\text{arcsec}$ for the faintest quasars, for example (see Table 3).

We have computed the weighted mean error on the measured parallax for each redshift bin, assuming Poisson errors, and use these to compute our Fisher matrix estimates (below). The errors were also used to show $1\sigma$ error boxes for a $\Lambda$CDM model on Figure 1. We have multiplied the error bars by $1.26$ to account for the fact that the measurements in each redshift bin will be an average over quasars which each have a parallax proportional to $\sin \beta$, where $\beta$ is the angle between the quasar position and the Solar System’s direction of motion.

3.2 SIM

The Space Interferometry Mission, SIM, also known as SIM PlanetQuest, is planned to be a Michelson Interferometer, with 50cm mirrors (there are 4) up to 9m apart. In the current most likely configuration, known as SIMlite, the relevant distance will be 6m. It will have an overall absolute precision of $\sim 3\mu\text{arcsec}$ and in narrow angle mode $\sim 1\mu\text{arcsec}$ differential accuracy between objects $< 15$ deg. apart. SIM will not be carrying out a sky survey in the same manner as Gaia, but will be targeted towards various objectives including extrasolar planet detection. It would therefore not in principle be a suitable mission to carry out the current study, given that only of the order of 100 quasars are planned to be observed currently and that measuring quasar secular parallax using a larger sample would require dedicating the telescope to the project at the exclusion of all other science, with (as we shall show) limited increases in possible constraints on cosmology. A detailed study of the precision astrometry which will be possible with SIM has been published by Unwin et al. (2008). According to this paper, approximately 1.5% of the mission will be used to measure 50 quasar positions to define the absolute reference frame. We do not include those measurements in what follows, although we note here that they would definitely be useful as they will be bright objects, with consequently small positional error bars. Instead, we compute two different scenarios for the use of SIMlite:

The first is a SIMlite Key Project (SIMliteKP in Table 3), with time use guided by the fact that the 15 currently planned Key Projects take up 36% of the observing time. We therefore take a time fraction equal to the average Key Project fraction, 2.4% of the mission. For the Key Project, we also limit ourselves to nearby quasars (redshift $z < 0.5$), this is the only survey for which we do not simply sample the full $z$ distribution in Table 1. Using the SIM time estimator, we find that this would enable 750 16th magnitude quasars to be observed with proper motion accuracy of $6.0\mu\text{arcsec yr}^{-1}$ (we assume 20 visits per quasar). Although many of the quasars within $z = 0.5$ are brighter than 16th magnitude, we make the conservative choice not to use that information. Properly optimizing an observing strategy is left for future work.

The second scenario is where 100% of the SIMlite observing time is used to observe quasars, at the expense of all other science. While this is not a realistic strategy, it does serve to illustrate what the satellite’s capabilities are. In this case, we find that (with 50 visits per quasar) 9500 quasars can be observed with proper motion accuracy of $4.0\mu\text{arcsec yr}^{-1}$ (see SIMlite100% in Table 3). This scenario, as we sample the entire quasar $n(z)$ distribution, will not be optimal in terms of parameter constraints. This should be borne in mind when comparing constraints from SIMliteKP and SIMlite100%.

For both of these scenarios, we convert the proper motion error into an absolute value of $\sigma_\pi$ after 10 years in the same fashion as was done for Gaia (Sec 3.1, above). Because the SIM time estimator assumes two dimensional results (i.e. errors on $x$ and $y$) are required, the proper motion error was reduced by an additional factor of $\sqrt{2}$. The resulting $\sigma_\pi$ values are given in Table 3.

3.3 Mission concepts

We can see from Figure 1 that the expected error bars on even our optimistic constraints from the Gaia and SIMlite satellites are large. A detection of quasar secular parallax from bins with $z < 2$ appears possible but constraints on cosmological parameters (we explore these in §5) will be weak. As a result, for precision cosmology it is necessary to explore constraints from instruments that can provide a higher measurement accuracy. We carry out a simple treatment of this by specifying the number of quasars to be observed and the proper motion accuracy per quasar for 3 futuristic satellite experiments. These are listed as the last 3 entries in Table 5. We emphasize that none of these has potential missions has been examined in detail and that it is beyond the scope of this paper to investigate what instruments would be required for measurements of quasars to actually be carried out to the accuracy listed.

The first of these hypothetical experiments (“$\mu\text{as}1$”) is meant to represent a satellite similar to SIM but which can achieve higher accuracy, on the order of $1\mu\text{arcsec}$ using a mode like SIM’s “narrow angle mode”. We assume for $\mu\text{as}1$ that 0.8 $\mu\text{arcsec yr}^{-1}$ proper motion errors are possible and carry out measurements for 10,000 quasars (this would involve some 17th magnitude quasars). Apart from these, we assume the same mission parameters as for SIMlite100% (i.e. a 10 year mission, using a dedicated satellite). The effective measurement error on the parallax after 10 years would therefore be 2.0 $\mu\text{as}$.

The second, (“$\mu\text{as}1$ w/4m”) is meant to roughly approximate a version of $\mu\text{as}1$ with 4m diameter mirrors (but still with the same baseline (6m)). This would enable $\mu\text{as}1$ w/4m to observe $\sim 100$ times more objects. We note that some of the mission time will be allocated to the substantial overhead time associated with the astrometric data taking. For short integrations ($\sim 30$ secs) this can be of the same order as the integration time. We also note that the largest mirrors which have yet been flown in space are those of the Herschel Observatory (3.5m diameter).
Table 3. A summary of various surveys discussed in §3. We give the name of the experiment, the total number of quasars to be observed, and the proper motion measurement error \( \sigma_{PM} \) (for the non-extended 5 year missions; see e.g., Lindegren et al. 2008 for Gaia). We also list the resulting effective measurement uncertainty in the parallax angle \( \pi \), after a 10 year observing period (see text). All missions have all sky coverage, and the quasars are drawn to uniformly sample the redshift distribution given in table 1 except for SIMlite KP for which only quasars with redshift \( z < 0.5 \) are used. The first three rows have parameters close to currently proposed experiments, the last three, below the horizontal line are hypothetical future missions. The last (60m int.) could be a similar design to NASA’s proposed Terrestrial Planet Finder satellite.

| Expt. | \( N_Q \) | \( \sigma_{PM} \) | \( \sigma_\pi \) | notes |
|------|-----------|----------------|-------------|-------|
| Gaia | 1700000  | 5-145          | 18-510      |       |
| SIMlite KP | 750  | 6.0          | 15.0        | 2.4\% time |
| SIMlite100% | 9500| 4.0          | 10.0        | 100 \% time |
| \( \mu as1 \) | 10000 | 0.8 | 2.0 |       |
| \( \mu as1 \ w/4m \) | 1700000 | 0.8 | 2.0 | 4m mirror |
| 60m int. | 1700000 | 0.08 | 0.2 | 4m + 60m b.l. |

The final row is for a mission with similar light collecting area, but with an interferometer baseline of 60m, so that the measurement error on the parallax has been reduced to 0.2 \( \mu arcsec \) over 10 years. Whether at this level, the measurement error will be dominated by other effects is uncertain. We explore this further in §4. We note that such a telescope could be similar in scope and design to the interferometer design for NASA’s Terrestrial Planet Finder satellite, TPF-\( ^4 \). That mission concept has 4 free-flying 4m diameter mirrors in formation with a maximum 200m baseline. ESA’s Darwin mission\( ^5 \) is another free-flying optical interferometer of similar size.

4 http://planetquest.jpl.nasa.gov/TPF/
5 http://www.esa.int/science/darwin

4. ANALYSIS AND UNCERTAINTIES

The analysis of a quasar dataset to extract the parallax distance measurements promises to be highly involved and be at the limit of the technical capabilities of the initial experimental setups we have considered. One of the complications is that quasars themselves are planned to be used by Gaia and SIM to define an inertial reference frame. For quasar secular parallax measurements, quasars obviously cannot be considered to have fixed angular positions, and so a different approach will have to be used. Also, in order to constrain dynamical cosmological models, redshifts will need to be known for all the quasars. As we have only made use of information in coarse \( (\Delta z = 0.5) \) bins, these could be photometric redshifts, although in practice, large surveys such as BOSS (Schlegel, White and Eisenstein 2009) will soon generate datasets of hundreds of thousands of quasars with spectroscopic redshifts. If some sort of redshifts are known for the quasars, then the differential secular parallax can be measured between quasars at different redshifts, and if this is done for all pairs of quasars, the result can be converted into a redshift distance relation. Exactly how best to do this is beyond the scope of this paper, but we note that one related, simpler approach which could be explored would be to consider all quasars beyond \( z = 5 \) (for example) to define a fixed coordinate system and make measurements with respect to that. In general, we note that although \( N_{\pi, quasar} \) angular separation measurements will be available, the Poisson error on the mean angular parallax shift for a given \( N_{\pi, quasar} \) objects will be \( \propto 1/\sqrt{N_{\pi, quasar}} \) and not \( 1/\sqrt{N_{\pi, quasar}} \).

Although the technical difficulties involved in making a measurement will be formidable, astrophysical systematic errors are likely to be subdominant to the effects of various statistical errors. The reason for this is that it is difficult to imagine effects that would cause a systematic shift of quasar angular positions across the sky in a manner consistent with the secular parallax shift. Because the angular shift we are searching for is in a precisely well known direction, the various errors are all likely instead to add to the random measurement error. One effect that will however vary across the sky is the shift caused by aberration due to our motion with respect to the barycenter of the Milky Way. The motion of the solar system around the center of the Milky Way causes an aberration of the positions of quasars in the same way that the Earth’s orbit around the Sun causes stellar aberration. Over time, with enough precision, this galactocentric acceleration of the Solar system can be detected from quasar astrometry (see e.g., calculations by Kopeikin & Makarov (2006) who show how SIM observations of 110 quasars can be used to do this). This (much larger) signal would need to be subtracted from the positions of quasars. Its different directional and redshift dependence to the signal we are looking for should enable this to be done cleanly.

One source of random error will be quasar peculiar motion. This can be divided into two components, motion of the host galaxy, and the potentially large orbital motion of black hole binaries as they merge. The effect of the first of these can be considered by assuming that the magnitude of the rms motion of galaxies wrt to the CMB will be similar to our own motion. By elementary trigonometry, such velocities will cause galaxies to have angular proper motions from our point of view that are similar in amplitude to the secular parallax shift due to our motion. The crucial difference is that these proper motions will be in random directions, and therefore their effect will be negligible once we average over several thousand (or tens of thousands) of quasars. We note that the overall cosmic variance limit for quasar parallax measurements of the distance scale will be due to either the effects of these peculiar velocities (which will also affect the redshifts we use) or else due to a bound on the number of faint magnitude quasars. We note that quasar peculiar velocities will not be totally random, due to velocity correlations that arise during the process of large-scale structure formation. Because quasars form an extremely sparse sampling of the cosmic density field, this effect will not be as important as it would be for galaxies, for example. We leave determination of the effect of velocity correlations to future work.

Consideration of the second peculiar motion effect, of
binary quasars is motivated by merger models of quasar formation (e.g., Di Matteo et al. 2005), and observations which show an excess in the number of quasar pairs at small separations (e.g., Hennawi et al. 2006). For these velocities to disrupt the parallax distance measurements, however, they would have to be extremely large, for example in a bin where we average over 10000 quasars, the rms motion of all quasars would have to be greater than $\sqrt{10000}$ times our motion wrt the CMB, or $\gtrsim 30000$ km s$^{-1}$ to dominate the overall error budget. Relativistic jets could also contribute a large proper motion, although they will only be relevant for a small fraction of quasars. Plots of the quasar autocorrelation function (e.g., Hoyle et al. 2002) show no sign of extreme elongation in the line-of-sight direction, and so such large peculiar motions cannot be common for quasars in any case.

Other sources of error include microlensing by stars in the Milky Way and in the quasar host galaxies. Belokurov and Evans (2002) have estimated for Gaia that the all-sky astrometric microlensing optical depth is $2.5 \times 10^{-5}$, i.e. less than one in 10000 sources will have a significant centroid shift. Smaller centroid shifts will however be caused by so-called weak microlensing, which can also cause negative parallaxes (Sazhin et al. 2001). Sazhin et al. show that due to weak microlensing, all quasars are expected to show negative parallaxes on the order of a few nanoarcseconds, with a small fraction, of the order of 1 % reaching 1 microarcsecond. Apart from lensing errors, there will be image centroiding problems due to the underlying quasar host galaxies (see Bastian and Hefele 2004 for a summary). In order to make a convincing detection of quasar secular parallax, all of these will need to be dealt with. As for the measurement averages will be made over a large number of quasars, the hope is that these statistical errors will all average out, with no residual bias. The technical measurement challenges will of course be considerable. We note that the Gaia Concept and Technology Study has drawn the pessimistic conclusion that these effects will average between 10$\mu$ arcsec and 100$\mu$ arcsec for a typical quasar, much larger than we assume here (where for example our 60 m interferometer predictions assume a total error per quasar of 0.2$\mu$ arcsec.) This is at variance with other predictions (e.g., Bastian and Hefele 2004), and the hope is that further research (and Gaia observations themselves of nearby quasars) will show such large effects to be nonexistent.

5 PREDICTED CONSTRAINTS FROM FUTURE OBSERVATIONS

We use the survey parameters described in §3 and summarized in Table 3 to compute predicted constraints on cosmological parameters from future observations of quasar secular parallax.

In each case, we use as the underlying model the $\Lambda$CDM model plotted as a solid line in Figure 1 (i.e. $\Omega_{\Lambda} = 0.7$, $\Omega_M = 0.3$, $h=0.72$, $w_0=-1$, $w_a=0$). We compute estimated 1$\sigma$ uncertainties for the bins of redshift width $\Delta z=0.5$ using Poisson statistics.

For the cosmological parameter constraints themselves, we use the standard Fisher matrix approach (see e.g., Tegmark et al. 1997). The Fisher Matrix is given by:

$$F_{jk} = \sum_b \left( \frac{\partial f_b}{\partial p_j} \right) \left( \frac{\partial f_b}{\partial p_k} \right) \sigma_b^2$$

where $f_b$ is the function in terms of the bin $b$, $p_j$ and $p_k$ are parameters of $f$, and $\sigma_b^2$ is the variance in $b$.

A Gaussian prior on any of the parameters can be added by adding (\sigma_{prior}^2) to the appropriate diagonal element of the $F$. Inversion of $F$ yields the covariance matrix, the diagonal elements of which are 1$\sigma$ errors on the parameters. To compute the error ellipses, we use the fact that the square roots of the eigenvalues of the covariance matrix correspond to the error ellipse axis lengths, and their associated eigenvectors define the error ellipse axis directions. The 95% probability ellipse has axis lengths $\sqrt{0.17}$ times longer than the 68% ellipse.

5.1 Constraints on the Hubble constant

After a detection of secular parallax, the first constraints that could be usefully placed are those on the Hubble constant. This measure of $H_0$ would be based on large scale expansion of the Universe, on gigaparsec scales, and so would not suffer from peculiar velocity scatter which forms part of the error budget in the HST cepheid-based measurements of the distance scale (Freedman et al. 2001). Because the Universe has evolved over the light travel time to these high redshift quasars, it is necessary to assume a prior on the cosmological model to constrain $H_0$. We assume that the underlying model is a $\Lambda$CDM model and that $\Omega_{\Lambda}$ and $\Omega_M$ are each known to 10%.

The corresponding 1$\sigma$ error bars on $H_0$ for the different experiments from Table 3 are shown in Table 4. We can see that Gaia measurements, for example could yield a $\sim 35\%$ error bar on $H_0$. While this is not competitive with for example the constraints from HST key project (Freedman et al. 2001) it is within a factor of 3. We note also that the Gaia result amounts to a 2.8$\sigma$ detection of quasar parallax. The
SIMLiteKP constraint is somewhat worse, but still amounts to a 2σ detection. We note that the SIMLiteKP constraint is only a factor of 1.6 weaker than the SIMLite100% constraint, despite using only 2.4% of the observing time. This is because we restricted the redshift range of quasar targets to z < 0.5, where the parallax shift is largest. This is partly because there are sufficient m_v ≤ 16 quasars in that redshift range to do this for SIMLiteKP. The other reason is that we decided in the SIMLiteKP to trade detection significance with constraints on dark energy parameters, which are sensitive to quasars at higher z.

In these calculations, a 10 year duration has been assumed (§3). If this is instead reduced to 5 years, the significance of detections for both Gaia and SIMLiteKP shrinks to below 1σ. Moving through the other, hypothetical experiments, we can see the error bar on H_0 rapidly becomes smaller, as would be expected.

Other versions of the parallax can extend geometric distance measurement out as far as local group galaxies. For example, the “Rotational Parallax” technique which compares measurements of proper motions and galaxy rotation curves could be used to estimate bias free distances to M33, M31 and the LMC to the percent level (Olling 2007). Olling (2007) has also shown how important such local anchoring of the H_0 distance ladder is to measurements of other cosmological parameters.

We note that detection of quasar parallax at the level predicted would be incontrovertible evidence that quasar redshifts are cosmological (c.f. Burbidge et al. 2003).

5.2 Cosmological constant dominated models

The Type IA supernova measurements of the accelerating Universe were initially used to constrain the parameters Ω_Λ and Ω_M (e.g. Perlmutter et al. 1999). Measurements of quasar parallax could also be used to do this, it being interesting to compare the kind of constraints that the first generation of quasar parallax measurements could make with the first measurements from supernovae. We have first computed constraints on Ω_Λ for the different experiments, assuming a prior on Ω_M of 10% (i.e. σ_{Ω_M} = 0.03). The results are extremely insensitive to the size of the prior. The 1σ errors on Ω_Λ are given in Table 2, where we can see that Gaia and SIMLite100% can only in principle achieve very weak 0.7σ and 1.3σ constraints on Λ respectively. To constrain dark energy parameters therefore we will need to wait for future mission concepts to become reality. These other survey concepts could do significantly better, with extremely tight constraints possible from the more futuroistic ones (at the level of 0.07% for 60m int. experiment).

The reason for the relative tightness of the constraints becomes apparent once we plot the Fisher matrix contours for Ω_Λ and Ω_M (Figure 2), where we can see that the contours are quite close to horizontal, i.e. quasar parallax measurements constrain Ω_Λ well but are unable to say much about Ω_M. The difference between the expression for the parallax distance (Equation 7) and the luminosity distance (or angular diameter distance which is just related by (1+z) factors) causes a difference in the direction of least degeneracy. SN measurements (e.g., Riess et al. 1998) constrain Ω_Λ and Ω_M approximately equally. Measurements of galaxy cluster abundances (e.g., Allen et al. 2002) on the other hand constrain Ω_M much better.

5.3 Redshift-varying dark energy models

The nature of dark energy itself is not probed by a precise determination of Ω_Λ. To do that, the Dark Energy Task Force (DETF, Albrecht et al. 2006) has recommended that the variation of dark energy with redshift be determined, using the parametrization given in Equation 2. We have computed the constraints on w_0 and w_a, with our standard prior on Ω_M of 10%. We have also assumed a flat model in this calculation, although we describe the effect of relaxing this assumption below. The 1σ constraints on the parameters are listed in Table 3 and the confidence contours are plotted in Figure 3 for the various experiments. We can see that as is usual with other probes of dark energy, w_a is significantly more loosely bounded than w_0. For experiments smaller than mas1 w/4m there is little useful constraint on w_a. For the extremely ambitious experiment 60m int., measurement of w_0 at the 1% level would be possible and a 0.03 value of w_a would be detectable. Without the flat space prior, we
have calculated that these would increase to $\sigma_{w0} = 0.032$ and $\sigma_{wa} = 0.12$.

We have computed the inverse of the area enclosed by the 95% confidence contours, to be compared to the DETF Figure of Merit (FOM). These values are also shown in Table[I in the Table of results (page 77) in the DETF report, values are tabulated not for the FOM, but for a quantity proportional to it, $1/(\sigma_{w0}\sigma_{wa})$, where $\sigma_p$ is the value of $w$ at a pivot redshift chosen so that the errors in $w_0$ and $w_a$ are uncorrelated. They are related by $FOM/6.17\pi = 1/(\sigma_{w0}\sigma_{wa})$, so that $1/(\sigma_{w0}\sigma_{wa}) = 0.008, 0.016, 0.29, 10.7, 1070$ for Gaia through 60m int. Comparing these values to those in the DETF report, we can see that only the extreme experiments, $\muas1$ w/4m and 60m int, are competitive with so called stage III and stage IV surveys. The 60m int is more constraining than all the DETF survey models (the highest FOM considered by the DETF is weak lensing with the Square Kilometer Array ["optimistic case"], $1/(\sigma_{w0}\sigma_{wa}) = 645.76$, a factor of 1.7 times smaller than for 60m int). The $\muas1$ w/4m case is as good as several of the listed stage IV surveys.

In carrying out these comparisons, we note that the DETF report does not assume a flat space prior, and also that the important results for the DETF are how the constraints can be combined with other measurements (for example from stage II surveys) to increase the FOM. Without a flat space prior, the constraints from the parallax experiments become worse, but they are still competitive: $1/(\sigma_{w0}\sigma_{wa}) = 9.0$ and 605, respectively for $\muas1$ w/4m and 60m int., the latter still comparable to the best case studied by the DETF. As for the combination of constraints with those from other methods, although it is beyond the scope of the present paper to investigate this, it is clear from the equation for the parallax distance (Equation 7) that since they are not related to the angular diameter and luminosity distance by simple factors of $(1+z)$ there should be scope for using their different degeneracy directions to get good combined constraints. In general though, because in principle the parallax constraints have the potential for less systematic errors than other methods it would be beneficial to have small errors on parameters for parallax alone.

We note that as pointed out by the DETF the FOM for the different experiments should scale as the number of quasars observed for equal precision in each measurement.

6 SUMMARY AND DISCUSSION

6.1 Summary

In this paper we have briefly explored the potential of astrometric measurements of quasar secular parallax as a probe of dark energy in the Universe. Our conclusions can be summarized as follows:

(1) Both the currently planned astrometric satellite Gaia (due for launch by ESA in early 2012) and NASA’s SIMlite mission (no launch date yet) could in principle detect (at the 2-3 $\sigma$ level) the quasar secular parallax shift due to the solar system motion with respect to the CMB over extended mission lifetimes of 10 years.

(2) Gaia may be able to constrain the Hubble constant at the $\sim 25$ km s$^{-1}$ level but only very weakly constrain the acceleration of the Universe (at the 0.$7\sigma$ level). SIMlite

would also be able to offer weak constraints at the 1.3$\sigma$ level on the acceleration but only in the unrealistic case of using 100% of the satellite’s observing time.

(3) Futuristic astrometric survey instruments, such as a large scale free flying interferometer similar in size to NASA’s Terrestrial Planet Finder could in principle carry out precision cosmology. Such a mission could achieve constraints on the dark energy parameters $w_0$ and $w_a$ significantly better than the best of the “Stage IV” surveys considered by the Dark Energy Task Force.

(4) Less ambitious mission concepts, such as an interferometer with the same baseline as SIM but with 4m diameter mirrors are approximately in the same class in terms of both mirror size and dark energy constraining power as some candidates proposed for the NASA/DOE Joint Dark Energy Mission.

(5) Although the astrophysical systematic errors in the method seem likely be non-existent, the measurement errors and other statistical errors are significantly uncertain and their investigation requires much future work.
6.2 Discussion

Although the ultimate aim of the kind of work set out in this paper is ambitious: precision measurement of dark energy parameters, the immediate aim of our work is merely to draw attention to the fact that upcoming interferometers may actually make direct geometric measurement of gigaparsec distances possible. Making use of this exciting possibility to do cosmology is probably as feasible as other suggested measurements in “real time” cosmology, such as the Sandage-Loeb test (Sandage 1962, Loeb 1998). In that probe, the Universe’s expansion rate is probed directly by looking at the variation of the redshifts of extragalactic objects with time (say a 10 year timeline, similar to the one we assume in this paper for our real time parallax shift measurements). Common to both approaches is the need for immense control of measurement error and the design of extremely futuristic instruments (e.g., Liske et al. 2008) to achieve the goals. Whether such a vast undertaking should be motivated only by the task of measuring dark energy parameters is debatable (e.g., White 2007), but of course other equally exciting problems require precision astrometry (e.g., Schneider et al. 2008).

The main advantage of a pure geometric method for constraining dark energy is the apparent lack of astrophysical systematic effects. This said, however, it is important to devise tests for unknown and unthought of errors. As the parallax shift is with respect to a well defined direction on the sky, this should be straightforward in principle. For example, perhaps the simplest null test one could imagine would involve recomputing the parallax shifts assuming that the solar system direction of motion is perpendicular to its actual motion, expecting to find a result of zero within the statistical errors. The statistical errors are perhaps the largest source of uncertainty in whether the cosmological measurements will be feasible. Precise modelling of image centroiding of variable sources, and actual Gaia observations will be needed to determine how much these will affect measurements. Quasar peculiar motions will need to be investigated, for example for nearby objects by Gaia and SIM, for which the peculiar motions should be readily detectable if they are large enough to be a problem for the cosmological constraints in this paper. As we have mentioned, there are some reasons to believe that statistical errors will be subdominant to the measurement errors, but without significant future work we cannot be sure.

One of the topics we have not explored here is the optimal redshift distribution of quasars needed for the measurement. Of course this depends on the type of dark energy being tested, but for the standard parametrization, it is possible that observing time would be wasted looking at very high redshift quasars, with little gain in parameter constraints. On the other hand, due to the high redshifts themselves, quasar distance measurements could perhaps be used to constrain exotic models of early dark energy better than lower $z$ supernova measurements.

We have also not explored how Gaia and SIM results could be combined to yield stronger constraints on cosmology. The general question of how to optimize use available resources to give the best possible constraints is one which should be considered. A related interesting question is whether longer timelines could be used to good effect, particularly in combining Gaia and SIM with missions which might come later. Also, if Gaia is launched first, SIM can be used to observe the brighter quasars later and so make use of a longer timeline.

What some have called the upcoming “age of astrometry” has the potential to yield the most direct constraints on cosmological models possible. Achieving precise measurements of dark energy from quasar parallax will require overcoming enormous technical challenges, but given the attention focused on many other probes of dark energy, many of which have astrophysical systematics which will be very difficult to control, it is worth exploring the matter further.

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