Smart physics with an oscillating beverage can

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Abstract
A digital learning-teaching environment is introduced in which undergraduate students are challenged to connect the basic physical concepts of oscillation, buoyancy and data analysis via an authentic experiment. The damped oscillation of a cylindrical body swimming upright in water is measured via the MEMS acceleration sensor of a wireless MCU SensorTag. The data are recorded with the app phyphox on a smartphone or tablet. The theoretical oscillation period and the experimentally determined periods obtained via different data analysis roots are found to agree showing an excellent theory-experiment interplay. The proposed experiment is suited for the physics home lab e.g. under the current pandemic situation or for open university courses as well as for physics lab courses.

Keywords: smartphone experiments, wireless MCU SensorTag, damped oscillation, buoyancy and floatage

1. Introduction
Conducting experiments with smartphones or tablets is, in a way, sensational and helps to increase the motivation and curiosity of students for physics learning. Today’s mobile devices offer the possibility to bring real physical experiments into the classroom and the home lab, since they are equipped with many internal sensors [1, 2]. E.g., several articles show how to use the internal MEMS accelerometer and MEMS gyroscope in different pendulum setups [3–6]. Using smartphones in such experiments expands the physics task repertoire to authentic problems and to physical phenomena from students every day life. This will help to develop the competencies of the students later necessary for advantaged physics courses [1, 2].

However, in some experimental setups it is not advisable or not possible to use smartphones as measurement devices. For this situation, cost efficient wireless sensor boxes were proposed to extend the use of these mobile devices. For instance, measurements of pressure, temperature, voltage and current were realised with such sensor boxes for thermodynamic and electromagnetism experiments, respectively [7, 8].

This article presents a sensor box based experiment and its data analysis suitable for basic
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Figure 1. Forces acting on a cylindrical body swimming upright in a stable equilibrium position in water. The gravity force points downwards from the centre of mass $R_b$ of the body with additional weights on its bottom. It is compensated by the buoyancy force acting on the centre of mass $R_{fl}$ of the displaced water.

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In mechanics courses, the oscillation of a cylindrical body swimming in water is studied. The measurements are performed via a wireless MCU SensorTag (Texas Instruments, USA) coupled via Bluetooth to the app phyphox (RWTH Aachen, Germany) running on a smartphone to record the data [9, 10].

2. Oscillation of a swimming cylindrical body

For a swimming cylindrical body like a beverage can (mass $m$, volume $V = A \cdot h$) immersed into a liquid of density $\rho$, the gravity force $\vec{F}_g$ is equally counteracted by the upward directed buoyancy force $\vec{F}_b$ caused by the displaced liquid, see figure 1 [11, 12]:

$$\vec{F}_g + \vec{F}_b = 0 \quad \text{with} \quad \vec{F}_g = -m \cdot g \cdot \vec{e}_z$$

Therefore, in a stable equilibrium position, the cylinder sinks by a length $h_0$ into the liquid

$$h_0 = \frac{m}{\rho \cdot A}.$$  \hspace{1cm} (2)

In order to displace the cylinder in $z$-direction from this equilibrium position, an external force

$$\vec{F}(z) = \rho \cdot g \cdot A \cdot z \cdot \vec{e}_z$$

is required to compensate for the respective change in buoyancy. This force is obviously linearly dependent on the displacement of the cylinder. If the cylinder is now released, the excess buoyancy force $-\vec{F}(z)$ acts as restoring force. It accelerates the cylinder in a direction opposite to its current displacement. According to Newton’s second law, the equation of motion is

$$m \cdot \ddot{z} = -\rho \cdot g \cdot A \cdot z.$$  \hspace{1cm} (4)

This equation is equivalent to the equation of a harmonic oscillator, where the product of $\rho \cdot g \cdot A$ takes the role of the constant of proportionality for a Hooke’s law-like restoring force. Therefore, the expected angular velocity $\omega_0$ (eigenfrequency) and the oscillation period $T_0$ for the oscillation of the swimming cylinder are [11, 12]

$$\omega_0^2 = \frac{\rho \cdot g \cdot A}{m} = \frac{g}{h_0} \quad \text{and} \quad T = \frac{2\pi}{\omega_0}.$$  \hspace{1cm} (5)

A detailed solution of this problem may be found in references [11, 13], where similar physics exercise tasks are treated.

3. The experiment

Figure 2 illustrates the experimental setup. An emptied commercial beverage can with the mass $m_c$ and the radius $r$ (see table 1) swims upright in water. Additional weight $m_A$ is added at the bottom of the can to prevent the body from tipping over (compare figure 1). The phyphox application running on a smartphone is chosen to measure the acceleration during the oscillation via a wireless MCU SensorTag (Type: CC1350, Texas Instruments, USA, mass $m_T$) which is fixed at
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Figure 2. Setup of the experiment to record the damped oscillation of the swimming beverage can.

Table 1. Radius, height and mass of the can and the additional masses.

| Parameter                  | Value                          |
|----------------------------|--------------------------------|
| Radius of the can \( r \)  | \((2.88 \pm 0.01) \text{ cm}\) |
| Height of the can \( h \)   | \((16.97 \pm 0.01) \text{ cm}\) |
| Equilibrium position \( h_0 \) | \((15.96 \pm 0.02) \text{ cm}\) |
| Mass of the can \( m_c \)   | \((37.35 \pm 0.01) \text{ g}\)  |
| Mass of the SensorTag \( m_T \) | \((28.12 \pm 0.01) \text{ g}\)  |
| Additional mass \( m_A \)   | \((350.00 \pm 0.01) \text{ g}\) |
| Total mass \( m \)         | \((415.47 \pm 0.03) \text{ g}\) |
| Density of water \( \rho \) | \(0.999 \text{ g cm}^{-3}\) |

the top of the can. The MEMS accelerometer of the SensorTag is connected to the smartphone via Bluetooth. The remote access to the phyphox app can be used to control the measurement and to export the measured data from the smartphone to the laptop [9, 10]. The swimming can should be long and narrow, so that the oscillation amplitude and period may be as large and as long as possible (see equation (5)).

4. Results and discussion

Figure 3 depicts the \( z \)-component of acceleration \( a_z(t) \) measured with the SensorTag and corrected for the constant offset due to the component of the earth gravity field in direction of the oscillation. It shows a damped oscillation. Obviously, in addition to the restoring force, friction forces influence the movement of the can. Assuming that the friction force is proportional to the magnitude of the velocity \( \dot{z} \), one yields the differential equation of a damped oscillator [11]

\[
\ddot{z} + 2 \cdot \gamma \cdot \dot{z} + \omega_0^2 \cdot z = 0,
\]

where \( \gamma \) is the damping coefficient. Therefore, the data were fitted with the following equation

\[
a_z(t) = a_{z0} \cdot \exp\left(-\gamma \cdot t\right) \cdot \sin\left(2 \cdot \pi \cdot \frac{t-t_0}{T_d}\right)
\]

(7)

to determine the oscillation period \( T_D \) and the damping coefficient \( \gamma \). Both values are used to calculate the period of the undamped oscillation \( T_0 \) [11]

\[
T_0 = \frac{2\pi}{\sqrt{\left(\frac{2\pi}{T_D}\right)^2 + \gamma^2}}.
\]

(8)
Figure 3. Acceleration $a_z(t)$ of the can oscillating around the $z$ axis. The black line and the black circles represent the measured data. The red line shows the fit according to equation (7).

Table 2. The period of oscillation of the damped motion $T_D$, the damping coefficient $\gamma$ and the period of oscillation of the undamped motion $T_0$.

| $\gamma$ (s$^{-1}$)     | (0.165 ± 0.001) |
|-------------------------|------------------|
| $T_D$ (s)               | (0.83 ± 0.01)    |
| $T_0$ experimental via equation (8) (s) | (0.82 ± 0.01) |
| $T_0$ theoretical via equation (5) (s) | (0.80 ± 0.02) |

The results for the oscillation periods $T_0$ determined via equations (5) and (7) are given in table 2.

The comparison of the data for $T_D$ and $T_0$ shows that the damping only slightly decreases the oscillation period, although the damping is clearly observable in the enveloping function of the measurement data, see figure 3. The agreement between the experimental and the theoretical values of $T_0$ is remarkable. Students and experimentalists might even be fascinated by its excellent theory-experiment interplay.

The minor influence of damping suggests a simplification to data analysis. The oscillation period could simply be determined by reading the time required for a certain number of completed oscillations directly from the recorded $a_z(t)$-data. This approach would be suitable for courses with physics as minor subject at universities or for physics courses at secondary schools.

Another modification for students with advanced mathematical skills could be to apply the Fourier transformation to obtain the oscillation period. Figure 4 shows the transformed data i.e. the acceleration spectrum of the oscillating can. Fitting this spectrum with a Lorentz peak function yields a peak frequency of $f_F = (1.215 ± 0.001)$ s$^{-1}$ and a damping coefficient of $\gamma_F = (0.133 ± 0.002)$ s$^{-1}$.
The frequency $f_F$ corresponds to an oscillation period of $T_{0F} = (0.823 \pm 0.001)$ s. Both values agree with the corresponding data given in table 2.

5. Conclusion

In this paper we presented a digital learning environment with a wireless sensor bases smartphone experiment which we demanded to physics teacher trainees in our undergraduate physics course. It connects the concepts of oscillation, buoyancy and data analysis via an authentic experiment. The students have to discuss the influence of the damping and try to apply different solution models to their recorded data. The proposed experiment offers the possibility to be applied in physics courses with different levels of difficulty. It is applicable as demonstration experiment and could be used as a practical exercise for the ‘home-lab’. The experiment has particular value for those teachers and lecturers that are required to teach physics under the conditions of distance learning in the current pandemic situation or at open university courses.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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