A Graph-Database Approach to Assess the Impact of Demand-Responsive Services on Public Transit Accessibility

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ABSTRACT

This paper proposes an approach to analyze the impact of multimodal Public Transit (PT), combining conventional fixed-route transit and Demand-Responsive Transit (DRT), on equality in transport accessibility distribution. We construct a graph model of multimodal PT in Neo4j, based on General Transit Feed Specification (GTFS) data, modeling DRT analytically (using continuous approximation). We quantify service quality on any location using an accessibility measure, indicating how easily other places or opportunities can be reached. We quantify inequality of accessibility distribution. We state the problem of allocating a DRT fleet to minimize inequality and show its NP-completeness. We showcase our approach on the transportation network of the French town of Royan.

CCS CONCEPTS

• Applied computing → Transportation; • Information systems → Graph-based database models; • Mathematics of computing → Combinatorial optimization.

KEYWORDS

transportation, optimization, graph databases

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1 INTRODUCTION

Effective mobility shapes the quality of life and enables economic development of urban conurbations. At the same time, transport is one of the most pollutant human activities and urgent actions are needed in order to achieve the ambitious zero-emission goals that several (supra)national authorities set for the following years. While Public Transit (PT) plays a pivotal role to guarantee sustainable and effective mobility in urban conurbations, the quality of service is unequally distributed [5], keeping a large part of the population with poor PT service, thus forcing them to rely on private cars. Complementing conventional PT, solely based on fixed lines, with other more flexible modes can relieve this issue [3, 4, 7]. In this paper, we consider a Demand-Responsive Transit (DRT) service, in which vehicles adapt their routes to user requests. Despite the fact that DRT has been shown to be an effective complement of PT in low-demand areas [7, 10], no studies exist, to the best of our knowledge, to quantify its impact in the reduction of inequality of PT accessibility. Ours is a first preliminary work toward an open data-driven computational approach to enable such quantification.

To efficiently and scalably analyze PT, we resort to graph databases systems, which have been developed to address the ever-growing need for processing highly interconnected data, in a wide variety of areas. We leverage these to implement our formal model, to visualize and explore transport graphs, as well as to extract patterns and compute relevant metrics, using custom graph queries and algorithms. We summarize our contributions as follows:

• We formalize an optimization for allocating DRT vehicles with the objective of reducing accessibility distribution inequality and show its NP-completeness.
• We model multi-modal transit, consisting of conventional fixed lines and DRT (represented analytically via continuous approximation). The model is a single graph, amenable to simple computation of accessibility computation, via well-known graph-algorithms.
• We implement the above model in the Neo4j graph database [2] as a property graph, built based on open data (GTFS) [1].

Note that our proposed approach is generic and can be used to analyze any city for which the corresponding GTFS data is available. For illustrative purposes, we have chosen the city of Royan as use-case. The size of its PT graph is large enough to be representative, while maintaining tractability (≈ 18.8K nodes and ≈ 7.3K arcs) and its structure has all the elements needed to observe the impact that deploying DRT services has on the accessibility distribution. While the paper presents preliminary results, we are working on extending it into a usable tool for PT planners (§4).

2 MODEL

Table 1 provides the key notations and values used henceforth.

We compute accessibility on a generic weighted directed graph $G = (V, A, \omega(\cdot))$, where $V$ is the set of nodes, $A$ - the set of arcs, and $\omega(\cdot)$ - the weight function. We partition $V$ into: a set $C$ of centroids, from which we assume all passengers start, a set $S$ of physical stops and, for each $s \in S$, a set $ST(s)$ corresponding to the service of the stop, from a specific transit line, at a specific time (called hereinafter a stoptime). Further details are given in §2.3.
**Table 1: Notation and numerical values**

Within the set \(\mathcal{A}\), we distinguish the arcs \(\mathcal{A}^w\), which represent a walk connection between any centroid \(c\) and a close stop \(v\). In addition, a subset of stops \(S_{\text{DRT}}\) can be served by a DRT service deployed in a limited region close to each stop. We call \(C(s)\) the set of centroids within that region, for any \(s \in S_{\text{DRT}}\). The set of arcs \(\mathcal{A}_{\text{DRT}} \subseteq \mathcal{A}\) represent DRT connection between each centroid in \(C(s)\) and stops \(v \in S'\) (for any DRT-arc \(a \in \mathcal{A}_{\text{DRT}}\), the weight \(\omega(a)\) depends on the number \(q\) of DRT vehicles deployed around the corresponding stop. The formulas for computing the graph’s weights are described in §2.3.

### 2.1 Accessibility and inequality

We evaluate the inequality in the distribution of PT accessibility \([8]\). The accessibility of a location indicates how easy it is to reach other locations. For ease of representation, we focus on its opposite, i.e., the *inaccessibility*. Let \(T(c, v)\) be the weight of the shortest path from a centroid \(c\) to a stop in \(S'(c) \subseteq S\) - a set of potential destinations (Table 1). The inaccessibility of the centroid \(c \in C\) is \(\text{ia}(c) = \frac{1}{|\mathcal{D}(c)|} \cdot \sum_{s \in \mathcal{D}(c)} T(c, s)\). To quantify inequality, we compute the Palma index \([9]\). We denote with \(c^\text{worst}_{10}\) the set of 10% of centroids suffering from the largest inaccessibility and with \(c^\text{best}_{40}\) the set of 40% of centroids enjoying the lowest inaccessibility, we compute inequity as \(I(G, \{\omega(a)\}_{a \in \mathcal{A}}) = \sum_{c \in C} \text{ia}(c) / \sum_{c \in c^\text{worst}_{10}} \text{ia}(c)\).

This value can be obtained in polynomial time, by adapting the Dijkstra algorithm to compute \(T(c, v)\), for each pair of centroid and stop. Note that the more the accessibility values are equal, the less \(I(G, \omega)\) is. Our objective is then to minimize the value of \(I(G, \omega)\), without diminishing the accessibility of any centroid. To do so, we have one lever: we are given \(Q\) routing vehicles (DRT buses, for instance) that can be used to decrease the time needed to reach stop \(s\) from a centroid.

### 2.2 Problem Statement

We now state the problem of assigning DRT vehicles to different regions, in order to minimize inequality (§2.1), and show its NP-completeness. We will explore resolution heuristics in future work.

Let \(f: \mathcal{A}_{\text{DRT}} \times N \rightarrow N\) be a function that can be computed in polynomial time. By allocating \(Q\) DRT vehicles to the stop \(s \in S_{\text{DRT}}\), the weight of every arc \((c, v) \in \mathcal{A}_{\text{DRT}}\) from any centroid \(c \in C(s)\) to any stop \(s'\) is reduced by \(f((c, v), q)\). This increases the accessibility of those centroids. We define the assignment vector \(\text{q} = (q_s, s \in S_{\text{DRT}}) \in \mathbb{N}^{|S_{\text{DRT}}|}\), which allocates \(q_s\) vehicles to each \(s \in S_{\text{DRT}}\). Let \(\{\omega_q(a)\}_{a \in \mathcal{A}}\) be the weights obtained from \(Q\). Our optimization problem can thus be formulated as follows.

### Definition 2.1 (Problem (EQ))

Given a graph \(G = (V, \mathcal{A})\), a weight function \(\omega\) and an integer \(Q\), find the assignment \(q = (q_s, s \in S_{\text{DRT}})\) satisfying \(\sum_{s \in S_{\text{DRT}}} q_s \leq Q\) and minimizing \(I(G, \{\omega_q(a)\}_{a \in \mathcal{A}})\).

### NP-Completeness

We now prove the NP-Completeness of this problem in the general case, using a polynomial reduction from the knapsack problem, in which, given two sets \(\{d_1, d_2, \ldots, d_n\}\) and \(\{u_1, u_2, \ldots, u_n\}\) of integers and two integers \(D\) and \(U\), we search for \(J \subseteq \{1; \ldots; n\}\) such that \(\sum_{i \in J} u_i \geq U\) and \(\sum_{i \in J} d_i \leq D\).

**Theorem 2.2.** The decision version of (EQ) is NP-Complete.

**Proof.** We associate a rational \(IN\) to the problem (EQ) and search for the existence of a feasible assignment \(q\) with inequality \(I(G, \{\omega_q(a)\}_{a \in \mathcal{A}}) < IN\) lower than \(IN\). The inclusion in \(NP\) is trivial. Considering that \(f\) can be computed in polynomial time, one can compute \(I(G, \{\omega_q(a)\}_{a \in \mathcal{A}})\) in polynomial time, for any given assignment \(q\).

We now prove there exists a polynomial time reduction from the knapsack problem to (EQ). From a knapsack instance \(J\), we build an instance \(I = (G, \omega, f, Q)\) as follows:

- We set an integer \(M\) to maxim \(u_i + 2\).
- We construct the graph \(G\) containing \(n + 1\) stops \(s_1, s_2, \ldots, s_n, s_{n+1}\), in which \(n + 1\) stop times \(v_1, v_2, \ldots, v_{n+1}\) and \(10n\) centroids \(c_1, c_2, \ldots, c_{n}, c_{n+1}, \ldots, c_{40n}\).
- We set \(C(s_1) = \{c_1\}\), \(S'T(s_1) = \{v_1\}\), and \(S_{\text{DRT}} = \{s_1, s_2, \ldots, s_n\}\).
- For \(i \in \{1; \ldots; n\}\), each centroid \(c_i\) is linked to the stop time \(v_i\) with one arc \((c_i, v_i) \in \mathcal{A}^w\) and another arc \((c_i, v_{i+1}) \in \mathcal{A}_{\text{DRT}}\). For \(i \in \{n + 1; \ldots; 10n\}\), each centroid \(c_i\) is linked to the stop time \(v_{n+i}\) with only one arc \((c_i, v_{n+i}) \in \mathcal{A}^w\). In addition, \(\mathcal{A}\) contains the arcs \((s_{n+i}, v_i), (v_i, v_{n+i})\), and \((v_{n+i}, v_{n+i+1})\), for \(i \neq n + 1\).
- For \(i \in \{1; \ldots; n\}\), \(f((c_i, v_i), q) = 0\) if \(q < d_i\), and \(M - u_i\) otherwise.
- Finally, we set \(Q = D\) and \(IN = \frac{m \cdot u_{\text{worst}}}{M}\).

This transformation from \(J\) to \(I\) is polynomial.

Note that we can consider only the feasible assignments \(q\) of \(I\), where \(q_i\) is either \(0\) or \(d_i\), for every \(i \leq n\). We call such a solution a canonical assignment. For any non-canonical assignment, there exists a canonical one with same Palma index, since by construction \(f((c_i, v_i), q)\) is null on \([0; d_i - 1]\) and constant on \([d_i; Q]\). In addition, there exists a bijection from the canonical assignments to the subsets of \(\{1; \ldots; n\}\) satisfying \(\sum_{i \in J} d_i \leq D\). From \(q\), we can define \(J \subseteq \{1; \ldots; n\}\) as the set \(\{i | q_i \geq d_i\}\). Thus, the associated canonical assignment has the inaccessibility values: \(ia(c_i) = 1\) if \(i \geq n + 1\), \(ia(c_i) = M\), if \(i \notin J\), and \(ia(c_i) = M - u_i\), otherwise. Note that the two last values are greater than \(2: c^\text{worst}_{10} = \{c_1, c_2, \ldots, c_n\}\) and \(c^\text{best}_{40} = \{c_{6n+1}, c_{6n+2}, \ldots, c_{40n}\}\). Thus, \(I(G, \{\omega_q(a)\}_{a \in \mathcal{A}}) = nM - \sum_{\forall i \in J} u_i /4n\). We then immediately have that \(\sum_{\forall i \in J} u_i \geq U\) if and only if \(I(G, \{\omega_q(a)\}_{a \in \mathcal{A}}) \leq IN\).

Then, there exists a polynomial time reduction from the knapsack problem to (EQ). □

Note that this reduction could be adapted to any fixed network. Changing the network would add a fixed value to the accessibility of all the centroids that can be balanced with the weights of the arcs and the function \(f\). However, this idea is not immediately achievable.
if we consider realistic functions \( f \) where the value \( f(v, q) \) should increase continuously with \( q \), possibly with a threshold representing the number of vehicles needed before the driving time takes an advantage on the waiting time. Further work is then needed to search for the complexity of this problem in realistic instances.

2.3 Graph model of multimodal public transit

2.3.1 Graph model of fixed public transit. We now render the generic graph model from §2 more specific. We detail the construction of the arcs and weights and how such concepts map to GTFS data. Let us denote with \( L \) and with \( R \) the set of runs, each corresponding to a vehicle traveling along the line and departing from the terminal at a specific time. GTFS data associate a stop-time \( v = (s, r) \) the event of a run \( r \), of a line \( l \), passing by physical stop \( s \). We denote with \( t(v) \) the time in which this even happens. In our graph model stop-times are represented as nodes. The stop times related to a physical stop and line \( l \) serving that stop are

\[
ST(s, l) = \{ v = (s, r) \mid r \in R^l \}.
\]

We add arcs \( a \) between stop-times \( s \) and \( v \)’ based on the following rules. An arc \( a \) is added between stop time \( s \) and \( v \)’ if there exists a run (of any line) that serves \( v \) and then \( v \)’ immediately after. Take a stop-time \( v \in ST(s, l) \). We add the following set of arcs, that allow to change from a line to another at stop \( s \):

\[
\{ a = (v, v') \mid l' \in L \text{ serving } s, v', \in ST(s, l'), \text{ } v' = \arg \min_{v' \in ST(s, l')} t(v') \}.
\]

To let paths end to physical stops, we add the following arcs:

\[
\{ a = (v, s) \mid v \in ST(s), s \in S \}
\]

To define the centroids \( C \), we tessellate the area into square tiles of side \( \xi \). The centroids are the centers of such tiles. We consider journeys start at time \( t_0 \). The time at which a user can reach stop \( s \) is \( t_{s,v} = t_0 + d(v, s)/v^w \), where \( v^w \) is the walking speed and \( d(v, s) \) is the distance between centroid \( c \) and stop \( s \). We connect each centroid \( c \) with walk arcs to stop-times within walk distance \( \tau \) that can be reached before the time of service:

\[
\{ a = (v, v') \mid l' \in L \text{ serving } s, v, v' \in ST(s, l'), \text{ } v' = \arg \min_{v' \in ST(s, l')} t(v') \}.
\]

We compute arc weights as follows. The arcs defined in (2) have weight 0. Arcs \( a = (c, v) \) defined in (2) have weight \( w(a) = t(v) - t_0 \). Arcs \( a = (v, v') \) defined in (1) have weight \( w(a) = t(v) - t(v') \).

2.3.2 Model of Demand-Responsive service. When deploying the DRT feeder bus for stop \( s \in S^{\text{DRT}} \), such service spans a rectangular area of size \( L \times W \) on the right of stop \( s \), as in [10]. \( C(s) \) is the set of centroids within such an area. The DRT is cyclic: during each cycle, a bus starts from \( s \), picks-up/drops-off passengers in/to the area and returns to \( s \). It thus stops for 2 minutes before starting the next cycle. A new bus departs every \( h(s) \) units of time (headway). We apply the continuous approximation model of [7], of which we give here only minimum information, for lack of space. The expected time needed to complete a cycle is \( T(s) \), which depends on the number of passengers departing from/arriving to the region. We assume demand density \( \rho \) (passengers willing to be picked-up) and the same density of passengers to be dropped-off is uniform across the entire urban area. The higher \( \rho \), the higher the detours imposed to the DRT, the higher \( T(s) \). On average, the time needed to go from a centroid to stop \( s \) via DRT is \( T(s)/2 \). Therefore, a passenger can arrive at stop \( s \) at time \( t_{c,s}^D = t_0 + T(s)/2 \). We thus add the following DRT arcs:

\[
\{ a = (c, v) \mid l \in L \text{ serves } s, v \in ST(s, l), \text{ } v = \arg \min_{v' \in ST(s, l')} t(v'), s \in S^{\text{DRT}}, c \in C(s) \}
\]

As usual, the weight on such arcs \( a = (c, v)^{\text{DRT}} \) are \( t(v) - t(c) \). Imposing a certain headway \( h \) requires to deploy a certain number of vehicles \( M_h \) (see [7] for the formula).

2.3.3 Shortest path. Note that after the construction explained above, graph \( G \) models multimodal PT, as it includes both fixed PT and DRT services. We can compute shortest path \( \hat{T}(c, s) \) from any centroid \( c \) to any stop \( s \), using Dijkstra algorithm or others. In case no path exist, we set \( \hat{T}(c, s) = \infty \). To compute accessibility (§2.1), we use \( T(c, s) = \min(\hat{T}(c, s), d(c, s)/v^w) \), i.e., we do not consider PT paths that would be longer than walking.

2.4 Graph Database Model

To store and process transportation data, we leverage the leading open-source Neo4j graph database. Its property graph model consists of a labeled, directed multigraph, in which lists of properties (key-value pairs) can be attached to both nodes and edges. For querying, its Cypher language allows to extract label-constrained reachability information encoding complex graph patterns [6]. The Neo4j Graph Data Science (GDS) library contains efficient implementations of common graph algorithms, useful for analytics.

We build our transportation graph model using GTFS data, a standard open data format detailing transit schedules. Specifically, we import the information corresponding to the Roayan urban region in Neo4j, as part of a custom graph instance we construct.

We model the urban transport network as a property graph instance (see Figure 1). Centroid nodes (tessellation centroids) are linked to Stoptime nodes, which represent scheduled passage times and correspond to specific Stop nodes (stations), as indicated by Locates_at arcs. The arcs connecting a given Centroid and Stoptime are labeled Walk and/or DRT, depending on the pedestrian and, respectively, demand-responsive accessibility. Stoptime nodes are connected with Precedes-labeled arcs (temporal ordering) or with Correspondance-labeled arcs (correspondance, using a different line, between a pair of stop-times of the same stop) The full graph instance amounts to 18892 nodes and 73714 relations (arcs), additionally annotated with 16 different node properties and 6 different relation properties. Note that the node values in Figure 1-right correspond to the identifiers of the corresponding Stop (also propagated to its stoptimes) or Centroid node.

For our analysis, we enrich the property graph instance with accessibility information, stored using custom edge properties. We illustrate an example of this with the Cypher query below, in which we compute the shortest path between a given centroid and a stop.
We showcase our approach on the French town of Royan, counting within a distance of 5km from 749 PT bus stops. We select 20 stops to be served by DRT (bold ones in Figure 2-left). The considered numerical values are in Table 1.

Figure 3-left, we plot the reduction of inaccessibility against $\Delta(c)$, the increase of DRT usage for each centroid $c$. $\Delta(c)$ is the amount of potential destinations for which the shortest path uses DRT, when available. As expected, the more $\Delta(c)$, the more inaccessibility is reduced.

Some features of stops $s \in S^{DRT}$ may suggest where benefit from introducing DRT can be expected to be higher. In Figure 3-right, we analyze the proximity, i.e., the number of other stops within a distance of 5km from $s$. We observe that the benefits of introducing DRT are, as expected, more pronounced when DRT serves PT stops that are close enough to other stops. In such cases, passengers can arrive more easily to $s$ and from $s$ they can reach more potential destinations. Such consideration may help construct efficient heuristics to solve the assignment problem.

3 NUMERICAL RESULTS

We showcase our approach on the French town of Royan, counting within a distance of 5km from 749 PT bus stops. We select 20 stops to be served by DRT (bold ones in Figure 2-left). The considered numerical values are in Table 1.

We measure the reduction in the inaccessibility of centroids when introducing the DRT service, with headway $h = 4$ min (resulting in 21.4 vehicles and 27.7 passengers per vehicle). We plot in Figure 2-right the reduction of inaccessibility observed by the centroids in $S^{DRT}$, only some centroids benefit from DRT, which suggests that it is of capital importance to solve the assignment problem (Def. 2.1) to not waste resources with no benefits.

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