Dirac Sea Contribution in Relativistic Random Phase Approximation

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Abstract

In the hadrodynamics (QHD) there are two methods to take account of the contribution of negative-energy states in the relativistic random phase approximation (RRPA). Dawson and Furnstahl made the ansatz that the Dirac sea were empty, while according to the Dirac hole theory the sea should be fully occupied. The two methods seem contradictory. Their close relationship and compatibility are explored and in particular the question of the ground-state (GS) instability resulting from Dawson-Furnstahl’s ansatz is discussed.

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1 Introduction

In the framework of the conventional quantum hadrodynamics (CQHD), the relativistic mean-field theory (RMFT) based on effective Lagrangians and the no-sea approximation has achieved a remarkable success for the calculation of nuclear ground states (referred to as MFGS i.e. mean-field GS) [1]. Since the parameters are adjusted to fit the experimental data, one expects that a large part of vacuum contributions and some other neglected effects are already taken into account phenomenologically [1]. A formal justification of the above conjecture has been given by Furnstahl, Pickarewicz and Serot [2] on the basis of the effective field theory (EFT), where a more elaborate effective Lagrangian with non-renormalizable interactions (modern QHD [3]) has been considered. For the calculation of excited states in the relativistic random phase approximation (RRPA), Dawson and Furnstahl (D-F) pointed out long ago [4] that in order to separate out the spurious $J^\pi = 1^-$ state and to preserve current conservation, it is of vital importance to take the negative-energy (NE) states into account, because only then will the Dirac single-particle basis become complete. They suggested that based on the above MFGS one may assume the NE sea is empty and thus besides the positive-energy (PE) particle-hole pairs one should further consider pairs formed from a particle at one of the NE states and a hole in the occupied PE states (referred to as $\alpha h$ pairs as in Ref. [5]). Ma et al. [6] found that the D-F
method can improve the calculation of nuclear giant resonances significantly. Recently, Ring at al. [5] have made a detailed study of the effects of the $\alpha h$ pairs and found that they are indeed important. However, as is well-known [7, 8], an empty NE sea cannot avoid the problem of spontaneous radiation, i.e. of keeping the valence nucleons from jumping into NE states through emitting mesons and photons, if the corresponding interaction terms are included in the Lagrangian. As numerous NE states are available, the MFGS, even though successful in all the other aspects, will not be stable and will disappear almost instantly. For this reason we would like to emphasize that it is still important to follow Dirac’s original proposal of a filled NE sea. Evidently, a filled NE sea can equally fulfill all the above requirements pointed out by D-F. Besides, if any effect due to the filled sea has been encoded into the effective Lagrangian by fitting the parameters to experimental data, in the calculation it should and can be neglected in order to avoid redundancy (see section 2). Clearly, for such a purpose there is no need to drive the sea particles out of the sea. Since the calculation on the basis of a filled sea will be as simple as one with an empty sea if the vacuum loops are neglected, it shows that Dirac’s postulate is a preferable choice for both CQHD and MQHD, because it cannot only prohibit the valence nucleons from tumbling into the NE sea, but also take account of the existence of antiparticles in a natural way. In addition, it also provides a greater possibility to treat the vacuum contribution effectively, namely if it is already included in the parameters of the effective Lagrangian (EL), we may simply neglect it in the calculation; however, if a part of it
has been left out owing to a somewhat excessive truncation, we may then either add additional terms and parameters to EL or calculate the part as a contribution from the sea. It is known that the results of the one-N (nucleon) -loop and virtual $NN$ pair calculations based on structureless nucleons are questionable and earlier calculations with CQHD met difficulties of large effects from loop integrals caused by the vacuum contribution. It seems [9] that the above drawbacks can be overcome by a careful study of vertex corrections and relevant higher-order effects.

The relation between the effective D-F method and Dirac’s hole theory will be studied in some detail in section 2. In section 3 numerical results calculated in RRPA are presented and discussed, where phenomenological vertex corrections are also considered to make a preliminary study of the effect due to the nucleon compositeness. Finally a summary is given in the last section.

2 Relation between D-F’s effective method and Dirac’s hole theory

In order to expose the relation between D-F’s effective method and Dirac’s hole theory more clearly and in simpler terms, we shall restrict our discussion to symmetric nuclear matter and Walecka’s $\sigma - \omega$ model. The Lagrangian is written as

\[
L = -\bar{\psi}(\gamma_\mu \partial_\mu + M - g_s \sigma - ig_v \gamma_\mu \omega_\mu)\psi \\
- \frac{1}{2} \partial_\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{4} W_{\mu\nu} W_{\mu\nu} - \frac{1}{2} m_v^2 \omega_\mu \omega_\mu, \quad (1)
\]
where \( W_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \) and \( \psi, \sigma, \omega \) denote the nucleon, \( \sigma \)-meson and \( \omega \)-meson fields, respectively. As is wellknown [1], through fitting the parameters to nuclear properties, the \( \sigma - \omega \) model, though simple, has achieved considerable success in the description of many bulk and single-particle properties of nuclei. Thus we may assume that a part of vacuum contributions and some higher-order effects have been included in the Lagrangian \( L \), though the inclusion cannot be so complete as what the more elaborate effective Lagrangian may provide. We would like to show that if the vacuum contribution from Dirac’s filled NE sea is neglected, the Dirac results reduce almost to what one obtains according to the D-F ansatz. In order to explore the relationship between the two methods we may apply them to calculate the same set of physical quantities and compare their results. For this purpose we shall consider the correlation function

\[
C(A, B; x_1, x_2) = \langle T[A(x_1)B(x_2)] \rangle - \langle A(x_1) \rangle \langle B(x_2) \rangle, \tag{2}
\]

as a typical example. In Eq. (2) \( \langle O \rangle = \langle \Psi_0 | O | \Psi_0 \rangle \), \( K(x) = \overline{\psi}(x) \Gamma_\kappa \psi(x) \) \((K = A \text{ or } B)\), \( \psi(x) \) is the nucleon field operator, whereas operator \( \Gamma_\kappa \) is field- and time-independent. For instance, for the isovector multipole operator we have \( \Gamma_\kappa = \gamma_4 \tau_3 r^3 Y_{\lambda_\mu}(\theta, \varphi) \) and for the bilinear Dirac current and scalar density \( \Gamma_\kappa = \gamma_\mu \) \((\kappa = \mu = 1, 2, 3, 4)\) and \( \Gamma_\kappa = 1 \) \((\kappa = 5)\), respectively, etc. For simplicity we shall further assume that \( \Gamma_\kappa \) contains no differential operators. In the lowest order approximation Eq. (2) has the form

\[
C^0(A, B; x_1, x_2) = -\text{Tr}[\Gamma_A(x_1)C^0(x_1 - x_2)\Gamma_B(x_2)C^0(x_2 - x_1)], \tag{3}
\]
where \( G_0(x) \) is the relativistic Hartree approximation to the nucleon propagator

\[
G_{\alpha\beta}(x = x_1 - x_2) = \left\langle T[\psi_{\alpha}(x_1)\bar{\psi}_{\beta}(x_2)] \right\rangle,
\]

and \( x \equiv x_\mu = (x, ix_0) \) with \( x_0 = t \). The Fourier transform of Eq. (3) is given by

\[
C^0(A, B; kk') = \int d^4x_1 d^4x_2 e^{-ikx_1+ik'x_2} C^0(A, B; x_1, x_2)
\]

\[
= -2\pi \delta(k_0 - k'_0) \int \frac{d^4q}{(2\pi)^4} \frac{d^3p}{(2\pi)^3} \text{Tr}[\Gamma_A(k-k'-p) \times G^0(k' + q + p, k'_0 + q_0) \Gamma_B(p) G^0(q, q_0)].
\]

Eq. (5a)

If \( \Gamma_\kappa \) is further independent of \( x \) (indicated by \( \kappa \) taking a small letter), we have \( \Gamma_a(p) = (2\pi)^3 \delta^3(p) \Gamma_a \) and Eq. (5a) reduces to

\[
C^0(a, b; kk') = - (2\pi)^4 \delta^4(k-k') \int \frac{d^4q}{(2\pi)^4} \text{Tr}[\Gamma_a G^0(q + k) \Gamma_b G^0(q)]
\]

\[
\equiv (2\pi)^4 \delta^4(k-k') C^0(a, b; k).
\]

Eq. (5b)

One observes that except for the \( \delta \)-function and a constant factor, Eq. (5b) is just the expression for the polarization tensor in the \( \sigma - \omega \) model, if \( \Gamma_\kappa \) \((\kappa = 1 \text{ to } 5) = \{\gamma_\mu, 1\} \). Since we are considering a nuclear matter whose GS is given by RMFT, \( G^0(p) = G^0_F(p) + G^0_D(p) \) can be written as

\[
G^0_F(p) = -(\gamma_\mu p_\mu + iM^*) \left[ \frac{1}{p^2 + M^*2} + i\pi \delta(p^2 + M^*2) \right]
\]

\[
= (\gamma_\mu p_\mu + iM^*) \frac{1}{2|E_p|} \left\{ \frac{\theta(E_p)}{p_0 - E_p + i\epsilon} - \frac{\theta(-E_p)}{p_0 - E_p - i\epsilon} \right\}
\]

\[
G^0_D(p) = (\gamma_\mu p_\mu + iM^*) \frac{i\pi}{E_p} \theta(p_0) \theta(k_F - |p|) \delta(p_0 - E_p),
\]

\( \theta \) being the step function.

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where $E_p = \pm[p^2 + M^*]^1/2$, $M^*$ is the effective mass, $k_F$ denotes the Fermi momentum, $F$ the Feynman part, $D$ the density-dependent part and $P$ the principle value, we may rewrite Eq. (5) as

$$C_0(A,B; kk') = C_0^{FF}(A,B; kk') + C_0^m(A,B; kk') + C_0^{DD}(A,B; kk'), \quad (7a)$$

$$C_0^m(A,B; kk') = C_0^{FF}(A,B; kk') + C_0^{F}(A,B; kk'). \quad (7b)$$

On the right-hand side of Eq. (7) the subscript $D$ or $F$ indicates $G_0$ in Eq. (5) is $G_0^D$ or $G_0^F$ and the three parts will be referred to as $FF$-, $m$- and $DD$-part, respectively. It is wellknown that $\psi(x) = e^{iHt}\psi(x,0)e^{-iHt}$ and $\psi(x,0)$ can be expanded in the form

$$\psi(x,0) = \sum_{r=1}^{4} \int \frac{d^3p}{(2\pi)^3/2} c(pr) u(pr) e^{ip\cdot x}, \quad (8)$$

where we note $c(pr) = d^+(-p, r - 2)$, $u(pr) = v(-p, r - 2)$ if $r = 3, 4$ and $v(pr)$ ($r = 1, 2$) denotes a NE spinor with momentum $-p$. Substituting Eq. (8) into Eq. (4), we easily find [10] that the Fourier transform of $G_{\alpha\beta}(x)$ can be represented in the form

$$G_{\alpha\beta}(p) = \sum_{r,s=1}^{4} u_{\alpha}(pr)iG(pr, ps; p_0)\bar{u}_{\beta}(ps), \quad (9a)$$

$$G(pr, ps; p_0) = \frac{1}{i} \int_{-\infty}^{+\infty} dt e^{ipt} \langle T[c(pr, t_1)c^+(ps, t_2)] \rangle. \quad (9b)$$

Hereafter, we shall use a tilde to indicate the result obtained according to the D-F method. D-F's ansatz [4] implies

$$\tilde{G}^0(pr, ps; p_0) = \delta_{rs} \left[ \frac{\theta(E_p - E_F)}{p_0 - E_p + i\epsilon} + \frac{\theta(-E_p)}{p_0 - E_p + i\epsilon} + \frac{\theta(E_F - E_p)\theta(E_p)}{p_0 - E_F - i\epsilon} \right]$$

$$= \delta_{rs} G^0(p). \quad (10a)$$
Inserting Eq. (10a) in Eq. (9a), we obtain

\[ \tilde{G}_0^0(p) = \tilde{G}_F^0(p) + G_D^0(p), \]

and

\[ \tilde{G}_F^0(p) = \left( \gamma \mu \mu + iM^* \right) \frac{1}{2|E_p|} \left\{ \frac{\theta(E_p)}{p_0 - E_p + i\epsilon} - \frac{\theta(-E_p)}{p_0 - E_p + i\epsilon} \right\}. \] (10b)

It is seen that the only difference between \( G_0^0(p) \) and \( \tilde{G}_0^0(p) \) consists in their \( F \) part. In \( \tilde{G}_F^0(p) \), just as \( \tilde{G}_0^0(p) \) in Eq. (10a), its NE pole is now shifted to the lower-half plane and there are no other changes from \( G_F^0(p) \) to \( \tilde{G}_F^0(p) \).

Defining \( \Delta G_0^0(p) = \tilde{G}_0^0(p) - G_0^0(p) \), we have

\[ \Delta G_0^0(p) = \Delta G_F^0(p) = \left( \gamma \mu \mu + iM^* \right) \frac{i\pi}{|E_p|} \delta(p_0 - E_p)\theta(-E_p). \] (11)

Since all the poles of \( \tilde{G}_F^0(p) \) are in the lower-half plane, substituting Eq. (10) in Eq. (5), as pointed out in [4], we get \( \tilde{C}_{FF}(A,B;kk') = 0 \) which also follows from the fact that here there are no antiparticles. Besides, we find

\[ \tilde{C}_{DD}(A,B;kk') = C_{DD}^0(A,B;kk') \]

and

\[ \tilde{C}_m^0(A,B;kk') = C_m^0(A,B;kk') + \Delta C_m^0, \] (12a)

\[ \Delta C_m^0 = C_{DD,\Delta F}(A,B;kk') + C_{\Delta F,D}(A,B;kk'). \] (12b)

In Eq. (12b) the subscript \( \Delta F \) means that we substitute \( \Delta G_F^0(p) \) for \( G_F^0(p) \) in Eq. (7b). Comparing Eq. (12) with Eq. (7), we have

\[ \Delta C^0 \equiv \tilde{C}^0 - C^0 = \Delta C_m^0 - C_{FF}^0, \] (12c)

i.e. the ansatz of empty NE sea is equivalent to the assertion that \( C_{FF}^0 \) in \( C^0 \) is replaced by \( \Delta C_m^0 \). Thus, if the vacuum contribution \( C_{FF}^0 \) can be neglected, the difference between the two results consists solely of \( \Delta C_m^0 \). Note Eqs. (7)
and (12) show why the D-F method can give a better result than the no-sea approximation, because the latter only takes account of $C_{DD}^{0}$ and a part of $C_{m}^{0}$ given by the first term in Eq. (6b) or (10b), while the former has further correctly considered the full $C_{m}^{0}$ with even a correction $\Delta C_{m}^{0}$ (see below). In order to expose the physical implication of Eq. (12a) more clearly, consider, for instance, Eq. (5b) with $\Gamma_{a} = \gamma_{\eta}$ and $\Gamma_{b} = \gamma_{\lambda}$. Substituting Eq. (6a) in Eq. (7b), and using a prefix $\delta$ to denote the contribution of the second term of Eq. (6a), we obtain

$$\delta C_{m}^{0}(\eta, \lambda; k) = \text{Re} C_{m}^{0}(\eta, \lambda; k) = M_{1}(\eta, \lambda; k) + M_{2}(\eta, \lambda; k), \quad (13)$$

$$M_{1}(\eta, \lambda; k) = -\frac{1}{8\pi^{2}} \int d^{3}q \frac{\theta(k_{F} - |q|)}{E_{q}} \left\{ \frac{t_{\eta\lambda}(q + k)}{E_{q+k}} \delta(k_{0} + E_{q} - E_{q+k}) + \frac{t_{\eta\lambda}(q - k)}{E_{q-k}} \delta(k_{0} - E_{q} + E_{q-k}) \right\}, \quad (14a)$$

$$M_{2}(\eta, \lambda; k) = -\frac{1}{8\pi^{2}} \int d^{3}q \frac{\theta(k_{F} - |q|)}{E_{q}} \left\{ \frac{t_{\eta\lambda}(q + k)}{E_{q+k}} \delta(k_{0} + E_{q} + E_{q+k}) + \frac{t_{\eta\lambda}(q - k)}{E_{q-k}} \delta(k_{0} - E_{q} - E_{q-k}) \right\}, \quad (14b)$$

where $\text{Re}$ means the real part and

$$t_{\eta\lambda}(q \pm k) = [2q_{\eta}q_{\lambda} \pm q_{\eta}k_{\lambda} \pm q_{\lambda}k_{\eta} \mp \delta_{\eta\lambda}(q \cdot k)]_{q_{0} = E_{q}}. \quad (15)$$

From Eqs. (11-13), it is easily seen that

$$\Delta C_{m}^{0}(\eta, \lambda; k) = -2M_{2}(\eta, \lambda; k), \quad (16)$$

$$\text{Re} \tilde{C}_{m}^{0}(\eta, \lambda; k) = \text{Re} C_{m}^{0}(\eta, \lambda; k) + \Delta C_{m}^{0}(\eta, \lambda; k) = M_{1}(\eta, \lambda; k) - M_{2}(\eta, \lambda; k), \quad (17a)$$
\[ Im\tilde{C}_m^0(\eta, \lambda; k) = ImC_m^0(\eta, \lambda; k). \] (17b)

According to Eqs. (16) and (17) one concludes that \( \tilde{C}_m^0 \) and \( C_m^0 \) differ from each other only in the sign before \( M_2 \), because their imaginary parts are the same. If we require \( k_0 > 0 \), it is seen from Eq. (14b) that the first term in \( M_2 \) is zero and only the second term will contribute. The \( \delta \)-function in Eq. (14b) shows \( M_2 \) becomes effective only if \( k_0 > 2M^* \), which is about 1.37\,\text{GeV} \text{ for } M^* \simeq 0.73M, \text{ i.e. if the excitation energy is not too high, we have } \tilde{C}_m^0 \simeq C_m^0. \text{ Since } M_2 = 0 \text{ if } 0 < k_0 < 2M^*, \text{ in the energy region where for instance, giant multipole resonances are studied we even expect } \tilde{C}_m^0 = C_m^0. \text{ Clearly the above derivation also applies to the other components of } C^0(a, b; k) \text{ (see Eq. (5b)) as well as to Eq. (5a) and similar results obtain. So far we have only considered the lowest order approximation to Eq. (2). In order to gain some idea about the accumulative effect, we have further made a RRPA calculation for } C(a, b; x_1, x_2) \text{ with } C^0 \text{ given by Eq. (5b). The Dyson-Schwinger equation for the full scalar-vector meson propagator in the } \sigma - \omega \text{ model can be written in the form [11]}

\[ \Delta_{ab}(k) = \Delta_{ab}^0(k) + \Delta_{ac}^0(k)\Pi_{cd}^0(k)\Delta_{db}(k), \] (18)

where the polarization tensor \( \Pi_{ab}^0(k) \) is related to \( C^0(a, b; k) \) in Eq. (5b) with \( \Gamma_\kappa = \{\gamma_\mu, 1\} \text{ for } \kappa = a \text{ or } b \) as follows:

\[
\Pi_{ab}^0(k) = \begin{pmatrix}
\Pi_{\mu\nu}^0 \\
\Pi_{5\nu}^0
\end{pmatrix} = \begin{pmatrix}
g_v^2C^0(\mu, \nu; k) & -ig_vg_sC^0(\mu, 5; k) \\
-ig_vg_sC^0(5, \nu; k) & -g_s^2C^0(5, 5; k)
\end{pmatrix}.
\] (19)
If we substitute $\Gamma_\kappa = \{ig_v\gamma_\mu, g_s\}$ for $\Gamma_\kappa$ and use an overhead bar to indicate an expression obtained in this way, we have $\overline{C}^0(a, b; k) = -\Pi^0_{ab}(k)$. In Eq. (18) the free meson propagator is given by $\Delta^0_{ab} = \begin{pmatrix} \Delta^0_{\mu}\nu & 0 \\ 0 & \Delta^0_{\delta} \end{pmatrix}$, where $\Delta^0_\kappa(k) = -i[k^2 + m^2_\kappa - i\epsilon]^{-1}$ for $\kappa = v$ or $s$, and $m_\kappa$ is determined by $[k^2 + m^2_\kappa + i\Pi^0_\kappa(k^2; F)|_{M^* = M} = 0$ with $\hat{m}_\kappa$ denoting the physical mass, and for our present purpose no medium dependence of meson masses is considered. It will be studied in more detail elsewhere. If only the summation of ring diagrams is considered, as illustrated in Fig. 1, RRPA to $C(A, B; kk')$ (see Eqs. (2) and (5a)) may be written as

$$C(A, B; k, k') = C^0(A, B; k, k') - \int \frac{d^4q}{(2\pi)^4} \tilde{C}^0(A, c; k, q)\Delta_{cd}(q)\tilde{C}^0(B, d; -k', -q),$$  

$$\tilde{C}^0(E, f; k, q) = 2\pi\delta(k_0 - q_0) \int \frac{d^4p}{(2\pi)^4} \text{Tr}[\Gamma_E(k - q)G(p + q)\Gamma_fG(p)],$$

(20a)  

(20b)

where $E = A$ or $B$. Eq. (18) shows the ring summation is achieved by $\Delta_{cd}$ in Eq. (20). If $\Gamma_E$ is independent of $x$, from Eq. (18) one finds easily by iteration that Eq. (20) can be simplified to $\overline{C}_{RPA}(a, b; k) = -\Pi_{ab}(k)$ with

$$\Pi_{ab}(k) = \Pi^0_{ab}(k) + \Pi^0_{ac}(k)\Delta^0_{cd}(k)\Pi_{db}(k).$$

(21)

3 Numerical results

We have solved Eq. (21) with $\Pi^0_{ab}$ given by Eq. (19), where the scalar and vector mesons as well as their mixing are considered. The parameters are $\overline{g_s}^2 = 0.5263, \overline{g_v}^2 = 0.6842, (\overline{g}^2 \equiv g^2/16\pi^2)$, whereas $M^* = 0.73M, M = 939, \hat{m}_s = 550, \hat{m}_v = 783$ (all in MeV), and $k_F = 1.3fm^{-1}$ [1].
To illustrate the effects of $C_{FF}^0$ and $\Delta C_m^0$ (see Eqs. (7) and (12)) as well as the difference in results between the Dirac and D-F theories, we shall, as an example, discuss $C(4) = Re[C(4, 4; k)] = g_v^{-2} Re[\Pi_{44}(k)]$, which is nonnegative and closely related to the longitudinal response function. Fig. 2 shows the dependence of $C^0(4)$ and $\tilde{C}^0(4)$ on the energy transfer $k_0$. Indeed, in the small $k_0$ region, we have $C^0(4) = \tilde{C}^0(4)$, because both $C_{FF}^0(4)$ and $\Delta C_m^0(4)$ will not be zero only if $k_0 > 2M^*$. Thus, to the lowest order, the two methods yield the same results in case $k_0 < 2M^*$. In the large $k_0$ region since $C_{DD}^0(4) = \tilde{C}_{DD}^0(4) = 0$ (see Eq. (A. 26) in Ref. [13]) and according to Eq. (14a) $M_1(4, 4; k) = 0$ for time-like $k_\mu$ [11], one gets $\tilde{C}^0(4) = \tilde{C}_m^0(4)$, $C_m^0(4) = -\tilde{C}_m^0(4)$ and $C^0(4) = C_m^0(4) + C_{FF}^0(4)$. From Fig. 2 it is seen that the sign of $\tilde{C}_m^0(4)$ is correct, while $C_m^0(4)$ has a wrong sign. However, in the Dirac theory the relevant effect is represented by $C^0(4)$ whose sign is again correct owing to the contribution of $C_{FF}^0(4)$. It is interesting to find that $\Delta C_m^0$ included in $\tilde{C}_m^0$ in Eq. (12a) actually means a correction rather than a defect, though its effect is not very important. For the RRPA calculation we shall consider three cases: (1) Both $\Gamma_a$ and $\Gamma_b$ in the Eq. (5) are bare vertices. (2) One of $\Gamma_\kappa$ ($\kappa = a$ or $b$) is a dressed vertex $\hat{\Gamma}_\kappa$, which is phenomenologically taken as

$$\hat{\Gamma}_\kappa = \Gamma_\kappa \frac{\Lambda^2}{\Lambda^2 + |k^2|} \equiv \Gamma_\kappa F(k^2, \Lambda).$$

(22)

(3) Only one $\Gamma_\kappa$ in $C_{FF}^0$ is replaced by $\hat{\Gamma}_\kappa$. For simplicity, the case of dressed vertex at each end of the loop will not be considered. The detailed formula for $\Pi_{ab}^0$ and the relevant renormalization procedure are known for all the
above three cases [11-14], thus they will not be written down here. We shall use a subscript \( \eta = 1 \) to 3 to indicate the case which the calculation refers to. In Fig. 3 the calculated RRPA results for cases (1) and (2) are shown, where we have chosen \( \Lambda = 1.1 GeV \) for case (2). Since according to Eqs. (18, 19) and (21) the real as well as the imaginary part of \( \Pi_{cd}^0(k) \) contributes, it is seen that there is a significant difference between \( C_1(4) \) and \( \tilde{C}_1(4) \) even in the small \( k_0 \) region. Hereafter we shall restrict our discussion to this region, as the parameters in the effective Lagrangian are fitted to nuclear ground-state properties. Fig. 3 shows that the difference between \( \tilde{C}_1(4) \) and \( \tilde{C}_2(4) \) is noticeably larger than that between \( C_1(4) \) and \( C_2(4) \). It shows that the form factor given in Eq. (22) is rather ineffective in the Dirac case. However, if we assume that the effect of nucleon compositeness has already been encoded in the effective Lagrangian, we may simply set \( F(k^2, \Lambda) = 1 \) i.e. then there would be no need to consider \( C_2(4) \) and \( \tilde{C}_2(4) \) in order to avoid redundancy. From Eqs. (7) and (19), it is easily seen that the large difference between \( C_1(4) \) and \( \tilde{C}_1(4) \) is caused by \( C_{FF}^0 \) calculated with bare vertices. For this reason we have further considered case (3), which is represented graphically in Fig. 4, where the graphs of case (1) are drawn for comparison. We shall use \( C_3(4, \alpha) \) to denote the results calculated with \( \tilde{\Gamma}_\kappa(\alpha) = \alpha \Gamma_\kappa F(k^2, \Lambda_e) \), where \( \alpha (= 0 \) to 1) is a percentage factor and \( \Lambda_e = 1.1 GeV \). If \( \alpha = 1 \), we have \( C_3(4,1) \simeq C_2(4) \simeq C_1(4) \), i.e. the result of Dirac’s case. On the other hand, if \( \alpha = 0 \), we have \( \tilde{\Gamma}_\kappa(\alpha) = 0 \), thus \( C_3(4,0) = \tilde{C}_1(4) \) as shown in Fig. 4, i.e. if \( C_{FF}^0 \) or the vacuum contribution is neglected, the Dirac and D-F
results are the same, because the effect of $\Delta C_m^0$ is insignificant. Note Fig. 4(b) shows that in the case of $k_a = 330\, MeV$ $C_3(4, \alpha)$ varies with $\alpha$ unusually. It first rises and then falls down finally to $C_2(4) \sim C_1(4)$ at $\alpha = 1$. Since $C_1(4) \sim C_2(4)$, this suggests that $\tilde{C}_2(4, \alpha)$ calculated with $\tilde{\Gamma}_\kappa$ replaced by $\hat{\Gamma}_\kappa(\alpha)$ may display such a similar behavior. This is indeed so as shown in Fig. 3(b) where $\tilde{C}_2(4, 1) = \tilde{C}_2(4)$. Clearly, the postulate of filled sea also gives a greater possibility to realize the experimental data than the ansatz of empty sea. In fact, if for instance, $C_3(4, \alpha = 0.1)$ fits more closely to the experimental data, it means that the effective Lagrangian has not yet included the vacuum effect and nucleon compositeness completely and there is no need to add additional terms and parameters to the Lagrangian, as they can be taken into account by means of Dirac’s method in the simple way. In the above, we have argued and demonstrated if the effective Lagrangian has encoded the vacuum contribution and some other higher-order effects (for instance, nucleon compositeness) in its parameters, we may simply neglect these contributions in the calculation to avoid redundancy without assuming that the NE sea is empty and nucleon is structureless.

4 Summary and conclusions

To compare the Dirac and D-F methods, we have considered the calculation of correlation function as a typical example. As shown in section 2, if the vacuum contribution can be neglected, the Dirac and D-F results are the same in the small $k_0$ region not only to the lowest order but also in RRPA,
because according to Eq. (16) the effect of $\Delta C_m^0$ is insignificant in the model studied. Thus, in such cases, the D-F ansatz will yield a good effective method of calculation, though it is unnecessary, because the Dirac theory gives the same result and the calculation will be no more complicated if the vacuum effect and some higher order effects can be neglected. Since the Dirac theory can identify all the higher order effects unambiguously, to neglect them we only need to neglect the relevant terms, as displayed in section 2. Actually, no-sea approximation does not imply that the sea must be empty. It only means that in the calculation the vacuum contribution can be neglected, because it has already been encoded in the parameters of the Lagrangian. As is well known, there is no way to avoid the spontaneous radiation which will cause MFGS to disappear instantly if the sea is empty. We have illustrated in sections (2) and (3) why we have emphasized in the introduction that it is not only important, but also advantageous to follow Dirac’s original postulate of filled sea. Clearly if the effect of $\Delta C_m^0$ is not small, the D-F and Dirac results may differ, even if the vacuum contribution can be neglected, thus the difference may then serve as a discrimination between the two methods. This is a problem under further study.

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Figure Captions

Fig. 1: Graphical representation of RRPA for the calculation of the correlation function \( C(A, B; x_1, x_2) \).

Fig. 2: Real part of the lowest order approximation to the correlation function \( C(4, 4; k) \): \( C^0(4) \equiv \text{Re}[C^0(4, 4; k)] \). In the small \( k_0 \) region the three curves coincide.

Fig. 3: Curves represent \( C(4) \equiv \text{Re}[C(4, 4; k)] \) in RRPA. The solid curve denotes \( C_1(4) \), the thin one \( \tilde{C}_1(4) \), whereas the heavily dashed one is \( C_2(4) \), and the dashed one \( \tilde{C}_2(4, \alpha) \) where \( \tilde{C}_2(4, 1) = \tilde{C}_2(4) \). (both with \( \Lambda = 1.1 \text{GeV} \)). (a) momentum transfer \( k_a \equiv |\vec{k}| = 560 \text{MeV} \), (b) \( k_a = 330 \text{MeV} \).

Fig. 4: Effects of vacuum contribution and nucleon compositeness in RRPA. The solid and thin curve again reprint \( C_1(4) \) and \( \tilde{C}_1(4) \), respectively. The dashed curves attached with the value of \( \alpha \) (\( \alpha = 0.3, 0.6 \text{ and } 1 \)) denote the respective \( C_3(4) \). (a) \( k_a = 560 \text{MeV} \), (b) \( k_a = 330 \text{MeV} \).
\[ x_1 \Gamma_A \rightarrow x_2 \Gamma_B + x_1 \Gamma_A \rightarrow y_1 \rightarrow \Delta_{ab} \rightarrow y_2 \Gamma_B \rightarrow x_2 \Gamma_B \]
$k_a = 560 \text{ MeV}$

- solid line: $C_0^0(4)$
- dash dot line: $C_0^0(4)$
- dash dash line: $C_0^0(4) - C_{\mathrm{FF}}^0(4)$
\[ \alpha = 1 \]

\[ C(4) \text{ (MeV)}^2 \]

(a) \[ k_a = 560 \text{ MeV} \]

(b) \[ k_a = 330 \text{ MeV} \]

\[ \alpha = 0.6 \]

\[ \alpha = 0.3 \]

\[ \alpha = 0.1 \]
\[ C(4) (\text{MeV}^2) \]

(a) \( k_a = 560 \text{ MeV} \)

\[ \alpha = 0.1 \]
\[ \alpha = 0.3 \]
\[ \alpha = 0.6 \]

(b) \( k_a = 330 \text{ MeV} \)

\[ \alpha = 0.3 \]
\[ \alpha = 0.6 \]
\[ \alpha = 0.1 \]