Scour Downstream of Grade Control Structures under the Influence of Upward Seepage

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Abstract

The installation of free falling jet grade control structures has become a popular choice for river bed stabilization. However, the formation and development of scour downstream of the structure may lead to failure of the structure itself. The current approaches to scour depth prediction are generally based on studies conducted with the absence of upward seepage. In the present study, the effects of upward seepage on the scour depth were investigated. A total of 78 tests without and with the application of upward seepage were carried out using three different sediment sizes, three different tailwater depths, four different flow discharges, and four different upward seepage flow discharge rates. In some tests, the three-dimensional components of the flow velocity within the scour hole were measured for both the cases with and without upward seepage. The scour depth measured for the no-seepage results compared well with the most accurate relationship found in the literature. It was found that generally the upward seepage reduced the downward velocity components near the bed, which led to a decrease in the maximum scour depth. A maximum scour depth reduction of 49% was found for a minimum tailwater depth, small sediment size, and high flow discharge. A
decay of the downward velocity vector within the jet impingement was found due to the upward seepage flow velocity. The well-known equation of D’Agostino and Ferro was modified to account for the effect of upward seepage, which satisfactorily predicted the experimental scour depth, with a reasonable average error of 10.7%.

**Key words:** fluvial hydraulics, grade control structure, open channel flow, scour, sediment transport, seepage.

### 1. INTRODUCTION

Riverbed incision is associated with the formation of a knickpoint, which is developed as a result of waterfalls within the river course migrating upstream. As the flow plunges over a knickpoint face, downstream scour creates a plunge pool. As a result of this, the bank height is raised excessively, leading to a bank failure, the widening of the stream, damage to the surrounding infrastructures, and the feeding of a vast amount of sediment to the river (Papanicolaou et al. 2012). Therefore, natural river restoration is an important issue for environmental management to improve human life. Grade control structures are among the most widely used structures for river restoration and specifically for stabilizing a knickpoint. The spill-water over its crest can create a downflow jet, which impinges the downstream bed. Sediment particles are then picked up and transported downstream, creating a scour hole, which may result in the failure of the structure itself. The formulation of scour depth predictors downstream of a grade control structure has so far received considerable attention because of its practical importance. A literature review that included a data analysis was done by Mason and Arumugam (1985). Semi-theoretical relations based on the concept of the incipient motion of sediment particles and the jet diffusion analogy were proposed by Bormann and Julien (1991) and Shafai-Bejestan and Albertson (1992). The maximum scour depth for the scour geometry downstream of a grade control structure in a natural steep stream (> 0.02) was found to be 0.6-1.4 times the virtual jet energy per unit width (Lenzi et al. 2003).

D’Agostino and Ferro (2004) collected all the available previous data, conducted a series of experimental tests, and applied the incomplete-self-similarity theory to present predictor relations for the scour downstream of a grade control structure. Guven and Gunal (2008) applied explicit neural networks formulation combined with a genetic algorithm to present a new approach for determining the scour depth downstream of a grade control structure. A hybrid numerical-mathematical model was developed by Guven and Gunal (2010) to predict the temporal evolution of local scour and simulate the flow patterns in and around the scoured zone. Scurlock et al. (2012) proposed a predictor relation for the scour geometries downstream of A-, U-, and W-shaped weirs using a set of 27 laboratory experimental tests. An extensive
series of studies have been reported by Pagliara et al. (2011) on the effect of the sediment load on the morphology of a pool downstream of a block ramp, Pagliara and Palermo (2013) on rock grade control structures, Pagliara and Kurdistani (2013) on cross-vanes structures, Pagliara et al. (2013) on the behavior of J-hook vanes, and Pagliara et al. (2015) on log-vane structures.

The brief literature review revealed that no attempt has so far been made to study the effects of upward seepage (hereafter US), which usually exists due to a piezometric difference across the structure, on the scour downstream of a grade control structure.

There have been few studies on the effects of US on scour, whereas numerous attempts have been made to study the effects of seepage on open channel flow characteristics and sediment motion (Cheng and Chiew 1998a, b; 1999; Dey and Zanke 2004, Dey and Cheng 2005, Liu and Chiew 2012, Yang 2013). Here, a brief review of the former is given. Dey and Singh (2007) studied the scour depth downstream of an underwater pipeline and showed a graphical relationship between the non-dimensional scour depth and dimensionless seepage velocity. Their results revealed that the scour depth is smaller in the presence of US, with a maximum reduction of about 34% when the dimensionless velocity was equal to 0.0015. Dey and Sarkar (2007) experimentally studied the effects of US on the scour downstream of a horizontal apron due to submerged jets. They found that the characteristic scour dimensions such as the maximum equilibrium scour depth, horizontal distance of the maximum scour depth from the sluice gate, horizontal extension of the scour hole from the sluice gate, dune height, and horizontal distance of the dune crest from the sluice increase with an increase in the seepage velocity. Sarkar and Dey (2007) reported another study on the effects of US on the scour downstream of a horizontal submerged jet downstream of a sluice gate with no horizontal apron and showed that the scour dimensions decrease as the US velocity increases. They concluded that the difference between the results of these two studies was that the flow patterns were different. Their final conclusion was that the flow characteristics can significantly affect the scour dimensions.

The aim of the present study was therefore to explore the context of US on the scour depth downstream of a grade control structure using a laboratory experimental study.

2. EXPERIMENTAL SETUP AND METHODOLOGY
Experiments were performed in a laboratory flume that was 0.88 m wide, 0.80 m deep, and 8.5 m long. The plan and section view of the experimental arrangement is shown in Fig. 1. A broad crested weir, having \( z = 0.361 \) m, was installed upstream of the test section. The test section incorporated 4 m of bed sediment, a recess that was 0.26 m deep and 1.2 m long, tailwater
control, a tailbox, a sharp rectangular weir, and an outlet drain. An arrangement similar to the technique previously reported by Dey and Sarkar (2007), with some modifications, was made to apply US from the bottom of the sediment through the first 1.2 m of the test section (see Fig. 1). The results of our base line tests, without US, showed that the maximum scour length and its associated point bar was less than 1.2 m for the range of variables tested in the current study. Our arrangement for providing a uniform distributed US included the following: a larger box (1.2 × 0.7 × 0.20 m) attached to the flume bottom at the test section with a perforated plate on its top and filled with gravel and boulders. To dissipate the kinetic energy of the incoming jet and produce a uniform distribution of the US, a smaller size box (0.35 × 0.35 × 0.05 m) was attached to the center of the bottom of the larger box. The sides of this smaller box were made of perforated plates. The US flow entered the smaller box through a pipe and spread to the larger box from its four sides. The seepage flow rate was controlled by a valve connected to the pipe. The valve remained closed for tests without US. Trial
tests were performed to determine the maximum concrete reservoir (henceforth CR) water level, corresponding to the condition of no sand boil or liquefaction, which is caused by a high piezometric drop across structure. The head (CR water level above the sediment bed surface) to discharge relationship was found by measuring the US rate for different CR water surfaces. The US flow rate was measured by an installed sharp edge rectangular weir (henceforth RW) at the end of the flume. During these trial tests, no flow was allowed to enter from the grade control structure. During the trial tests, attention was given to ensuring a uniform distribution of the US throughout the test area.

Three uniformly graded sediments of \( d_{50} = 1.5, 2.4, \) and 3.15 mm and specific gravities of 2.57, 2.52, and 2.56, respectively, were used in this study. The tail water depth \( Y_t \) was controlled by an adjustable tailgate at the downstream end of the test section. In order to minimize the undesirable sediment particle movement, the test section was first filled with water using a flexible pipe from downstream, while the tail-gate was fully closed. Once the downstream water level was high enough, the test was initiated by adjusting the inflow to the desired flow rate. At the same time, the tailwater was lowered gradually to the desired level by gradually opening the tail gate. The tests were carried out for three tailwater depths \( (Y_t = 0.16, 0.21, \) and 0.26 m), four flow discharges \( (Q = 10, 15, 20, \) and 25 l/s), and five different US rates \( (Q_s \) ranged from 0 to 14 l/s). The piezometric drop across the structure, \( H \), varied from 0.139 to 0.251 m, and the head above the weir crest, \( h_0 \), ranged from 0.038 to 0.07 m. During each test, the water surface level in the CR and within the flume was measured by a point gauge. The scour profiles were recorded at regular intervals by drawing the profile on a transparent paper attached to the flume walls or taking photographs. The duration of each test for the equilibrium scour condition was 6 h. The equilibrium configuration time was the duration of the test, after which the change in the scour depth was negligible. This time was found to be 40 min in the study by Pagliara and Palermo (2013), 12 h in that by Scurlock et al. (2012), and 2-4 days in the study by Bhuiyan et al. (2007). At the end of each test, the flume was slowly drained to minimize the reshaping of the scour geometry, and the bed topography was measured using a digital laser tape. These data were used to obtain the geometry of the scour hole. A total of 78 tests were carried out, and some of the collected data are presented in Table 1. In Table 1, \( h_0 \) is the head above the weir crest, \( Q \) is the flow discharge, \( H \) is the piezometric drop across the structure, \( D_{50} \) is the median sediment size, \( Y_t \) is the tailwater depth, \( H_s \) is the US head or CR water level above the bed, \( Q_s \) is the US rate, and \( d_s \) is the maximum scour depth at the end of the test.

Four of the above tests (nos. 74, 75, 77, and 78) were repeated to measure the three velocity components for both cases with and without US.
Table 1

Measured experimental data

| Test no. | $h_0$ [m] | $Q$ [l/s] | $H$ [m] | $D_{50}$ [m] | $Y_t$ [m] | $Q_s$ [l/s] | $d_s$ [m] |
|----------|-----------|-----------|---------|-------------|----------|-----------|--------|
| 1        | 0.038     | 10        | 0.239   | 0.0015      | 0.16     | 0.0       | 0.157  |
| 2        | 0.038     | 10        | 0.239   | 0.0015      | 0.16     | 8.64      | 0.126  |
| 3        | 0.038     | 10        | 0.239   | 0.0015      | 0.16     | 6.26      | 0.113  |
| 4        | 0.038     | 10        | 0.239   | 0.0015      | 0.16     | 3.28      | 0.12   |
| 5        | 0.038     | 10        | 0.239   | 0.0015      | 0.16     | 0.9       | 0.121  |
| 6        | 0.0495    | 15        | 0.251   | 0.0015      | 0.16     | 0.0       | 0.228  |
| 7        | 0.0495    | 15        | 0.251   | 0.0015      | 0.16     | 12.22     | 0.107  |
| 8        | 0.0495    | 15        | 0.251   | 0.0015      | 0.16     | 9.83      | 0.124  |
| 9        | 0.0495    | 15        | 0.251   | 0.0015      | 0.16     | 7.45      | 0.15   |
| 10       | 0.0495    | 15        | 0.251   | 0.0015      | 0.16     | 5.66      | 0.165  |
| 11       | 0.0495    | 15        | 0.251   | 0.0015      | 0.16     | 3.28      | 0.202  |
| 12       | 0.0495    | 15        | 0.251   | 0.0015      | 0.16     | 0.9       | 0.208  |
| 13       | 0.038     | 10        | 0.189   | 0.0015      | 0.21     | 0.0       | 0.145  |
| 14       | 0.038     | 10        | 0.189   | 0.0015      | 0.21     | 8.64      | 0.112  |
| 15       | 0.038     | 10        | 0.189   | 0.0015      | 0.21     | 8.04      | 0.116  |
| 16       | 0.038     | 10        | 0.189   | 0.0015      | 0.21     | 5.06      | 0.124  |
| 17       | 0.038     | 10        | 0.189   | 0.0015      | 0.21     | 0.9       | 0.134  |
| 18       | 0.0495    | 15        | 0.201   | 0.0015      | 0.21     | 0.0       | 0.196  |
| 19       | 0.0495    | 15        | 0.201   | 0.0015      | 0.21     | 8.64      | 0.155  |
| 20       | 0.0495    | 15        | 0.201   | 0.0015      | 0.21     | 8.04      | 0.148  |
| 21       | 0.0495    | 15        | 0.201   | 0.0015      | 0.21     | 5.06      | 0.144  |
| 22       | 0.0495    | 15        | 0.201   | 0.0015      | 0.21     | 0.9       | 0.157  |
| 23       | 0.060     | 20        | 0.211   | 0.0015      | 0.21     | 0.0       | 0.237  |
| 24       | 0.060     | 20        | 0.211   | 0.0015      | 0.21     | 8.64      | 0.132  |
| 25       | 0.060     | 20        | 0.211   | 0.0015      | 0.21     | 6.85      | 0.158  |
| 26       | 0.060     | 20        | 0.211   | 0.0015      | 0.21     | 3.87      | 0.229  |
| 27       | 0.060     | 20        | 0.211   | 0.0015      | 0.21     | 2.08      | 0.228  |
| 28       | 0.038     | 10        | 0.139   | 0.0015      | 0.26     | 0.0       | 0.115  |
| 29       | 0.038     | 10        | 0.139   | 0.0015      | 0.26     | 14.00     | 0.072  |
| 30       | 0.038     | 10        | 0.139   | 0.0015      | 0.26     | 11.02     | 0.063  |
| 31       | 0.038     | 10        | 0.139   | 0.0015      | 0.26     | 8.04      | 0.116  |
| 32       | 0.038     | 10        | 0.139   | 0.0015      | 0.26     | 5.06      | 0.102  |
| 33       | 0.038     | 10        | 0.139   | 0.0015      | 0.26     | 0.9       | 0.115  |
| 34       | 0.0495    | 15        | 0.151   | 0.0015      | 0.26     | 0.0       | 0.144  |
| 35       | 0.0495    | 15        | 0.151   | 0.0015      | 0.26     | 12.22     | 0.137  |
| 36       | 0.0495    | 15        | 0.151   | 0.0015      | 0.26     | 10.43     | 0.138  |
| 37       | 0.0495    | 15        | 0.151   | 0.0015      | 0.26     | 7.45      | 0.131  |

*to be continued*
|   |       |   |       |       |   |       |
|---|-------|---|-------|-------|---|-------|
| 38 | 0.0495 | 15 | 0.151 | 0.0015 | 0.26 | 5.06  |
| 39 | 0.0495 | 15 | 0.151 | 0.0015 | 0.26 | 0.90  |
| 40 | 0.060  | 20 | 0.161 | 0.0015 | 0.26 | 0.0   |
| 41 | 0.060  | 20 | 0.161 | 0.0015 | 0.26 | 12.22 |
| 42 | 0.060  | 20 | 0.161 | 0.0015 | 0.26 | 11.02 |
| 43 | 0.060  | 20 | 0.161 | 0.0015 | 0.26 | 7.45  |
| 44 | 0.060  | 20 | 0.161 | 0.0015 | 0.26 | 4.47  |
| 45 | 0.060  | 20 | 0.161 | 0.0015 | 0.26 | 0.90  |
| 46 | 0.038  | 10 | 0.189 | 0.0024 | 0.21 | 0.0   |
| 47 | 0.038  | 10 | 0.189 | 0.0024 | 0.21 | 11.62 |
| 48 | 0.038  | 10 | 0.189 | 0.0024 | 0.21 | 8.64  |
| 49 | 0.038  | 10 | 0.189 | 0.0024 | 0.21 | 8.04  |
| 50 | 0.038  | 10 | 0.189 | 0.0024 | 0.21 | 5.06  |
| 51 | 0.0495 | 15 | 0.201 | 0.0024 | 0.21 | 0.0   |
| 52 | 0.0495 | 15 | 0.201 | 0.0024 | 0.21 | 13.41 |
| 53 | 0.0495 | 15 | 0.201 | 0.0024 | 0.21 | 11.02 |
| 54 | 0.0495 | 15 | 0.201 | 0.0024 | 0.21 | 8.64  |
| 55 | 0.0495 | 15 | 0.201 | 0.0024 | 0.21 | 8.04  |
| 56 | 0.0495 | 15 | 0.201 | 0.0024 | 0.21 | 0.90  |
| 57 | 0.060  | 20 | 0.211 | 0.0024 | 0.21 | 0.0   |
| 58 | 0.060  | 20 | 0.211 | 0.0024 | 0.21 | 12.22 |
| 59 | 0.060  | 20 | 0.211 | 0.0024 | 0.21 | 8.64  |
| 60 | 0.060  | 20 | 0.211 | 0.0024 | 0.21 | 6.85  |
| 61 | 0.060  | 20 | 0.211 | 0.0024 | 0.21 | 3.87  |
| 62 | 0.060  | 20 | 0.211 | 0.0024 | 0.21 | 2.08  |
| 63 | 0.0495 | 15 | 0.201 | 0.0032 | 0.21 | 0.0   |
| 64 | 0.0495 | 15 | 0.201 | 0.0032 | 0.21 | 11.02 |
| 65 | 0.0495 | 15 | 0.201 | 0.0032 | 0.21 | 8.64  |
| 66 | 0.0495 | 15 | 0.201 | 0.0032 | 0.21 | 8.04  |
| 67 | 0.0495 | 15 | 0.201 | 0.0032 | 0.21 | 5.06  |
| 68 | 0.060  | 20 | 0.211 | 0.0032 | 0.21 | 0.0   |
| 69 | 0.060  | 20 | 0.211 | 0.0032 | 0.21 | 14.00 |
| 70 | 0.060  | 20 | 0.211 | 0.0032 | 0.21 | 12.22 |
| 71 | 0.060  | 20 | 0.211 | 0.0032 | 0.21 | 8.64  |
| 72 | 0.060  | 20 | 0.211 | 0.0032 | 0.21 | 3.87  |
| 73 | 0.070  | 25 | 0.211 | 0.0032 | 0.21 | 0.0   |
| 74 | 0.070  | 25 | 0.221 | 0.0032 | 0.21 | 12.22 |
| 75 | 0.070  | 25 | 0.221 | 0.0032 | 0.21 | 8.64  |
| 76 | 0.070  | 25 | 0.221 | 0.0032 | 0.21 | 6.85  |
| 77 | 0.070  | 25 | 0.221 | 0.0032 | 0.21 | 3.87  |
| 78 | 0.070  | 25 | 0.221 | 0.0032 | 0.21 | 0.0   |

**Table 1 (continuation)**
An electro-magnetic velocity meter from JFE ALEC (ACM3-RS model, with an accuracy of ± 1 mm/s) was used for this purpose. The WinLabEM data acquisition software provided a real-time display of the data. The advantage of this instrument was that it could be used closer to the bed (2 cm above the bed). To avoid any disturbance in the velocity measurements due to moving sediment particles within the scour hole, the flow was stopped after 6 h of test duration, and the water was drained out slowly from the scour hole. Then, the scour bed profile was stabilized by spraying a mixture of dilute slurry cement and albumen over the bed surface. After a few hours, the bed surface was hard enough to allow the test to be repeated. Prior to the test, the CR water level was adjusted according to the desired US flow rate. If the measured US flow rate was found to be affected by the solidified bed surface (the sediment pores could be clogged by the sprayed mixture), the pores were carefully opened by penetrating them with a very thin needle to the sediment until the proper seepage rate was reached. The three-dimensional flow velocity components were measured along the flume axis in different vertical sections at equal streamwise distances from the weir.

The uncertainties of the experimental data presented in this paper are as follows: (a) for bed levels, the maximum uncertainty would appear to arise from positioning the measuring laser tape at the scour bed, which may be considered to be half of the sediment size (± 1.5 mm or ± 1.3%); (b) for the water surface level measured by a point gage with an accuracy of ± 0.1 mm, where considering some small surface waves led to a maximum of ± 1%, and ± 2.5% for the flow discharge measured by a sharp edge rectangular weir.

3. RESULTS AND DISCUSSIONS

3.1 No upward seepage

D’Agostino and Ferro (2004) proposed an equation using a wide range of experimental data from previous researchers in their study, as listed in Table 2, to present a scour depth predictor formula, which can be written as follows:

\[
\frac{d_s}{z} = 0.54 \left( \frac{b}{z} \right)^{0.593} \left( \frac{Y_t}{H} \right)^{-0.126} A_{50}^{0.544} \left( \frac{d_{90}}{d_{50}} \right)^{-0.856} \left( \frac{b}{B} \right)^{-0.751},
\]

(1)

Table 2

Range of variables for developing Eq. 1 (D’Agostino and Ferro 2004)

| $B$ [cm] | $h_0$ [cm] | $Q$ [lit/s] | $D_{50}$ [mm] | $Y_t$ [cm] |
|-----------|------------|-------------|----------------|-------------|
| 15-108    | 0.02-0.46  | 1.4-2300    | 2-11           | 3-65        |
where $d_s$ is maximum scour depth, $b$ — width of the falling jet, $z$ — crest elevation from the downstream bed, $H$ — piezometric drop across the structure, $Y_t$ — tailwater depth, $d_{90}$ and $d_{50}$ are 90% and 50% finer sediment sizes, respectively, $b$ — channel width, and $A_{50}$ is a dimensionless parameter defined as follows:

$$A_{50} = \frac{Q}{bz\left[gd_{50}\left(\frac{\rho_s-\rho}{\rho}\right)\right]^{0.5}}, \quad (2)$$

where $Q$ is jet flow rate, $g$ — acceleration due to gravity, and $\rho_s$ and $\rho$ — sediment and fluid mass densities, respectively. Bhuiyan et al. (2007), by comparing their experimental data on the scour downstream of a W-weir with six empirical relations, found that Eq. 1 “predicts well” the maximum scour depth. The equation was found to over-predict by Scurlock et al. (2012) in their tests applying submerged types of grade control structures. As stated by Bhuiyan et al. (2007), Eq. 1 has the advantage of taking into account the effect of most of the variables responsible for the development of the scour depth, including the structure geometry (the contraction of the width, the sill height), the sediment characteristics (particle size and their gradation), and the hydraulic conditions (flow discharge, head above crest, and the tailwater depth). Equation 1 also fits well with our experimental setup of a free falling jet. Therefore, the results of the first series of tests, tests with $Q_s = 0.0$ in Table 1, were compared to those obtained from the expression of D’Agostino and Ferro (2004). The calculated scour depth was plotted against the measured results, which are shown in Fig. 2. The absolute relative error of each piece of predicted data ($R$) was calculated from the following:

![Fig. 2. Comparison of our experimental data without US with D’Agostino and Ferro (2004)’s relation.](image-url)
where $d_{sm}$ and $d_{sc}$ are measured and calculated scour depths, respectively. The average $R$ value was found to be 13%, which can be considered to be a satisfactory range. For this reason, the well known expression of D’Agostino and Ferro (2004) was modified to account for the effect of US.

### 3.2 Applying upward seepage

The experimental data from tests with different US flow rates revealed that the scour depth generally decreases as the US rate increases. To assess the quality of its variation, two non-dimensional parameters were calculated: (1) the non-dimensional scour depth (hereafter DDS), which is defined as the ratio of the maximum scour depth with US, $d_s(Q_s > 0)$, to the measured maximum scour depth with no seepage, $d_s(Q_s = 0)$; and (2) the non-dimensional flow discharge, $Q_s/Q$, which is the ratio of the US discharge rate to the weir flow discharge. Plots of the calculated non-dimensional variables are shown in Fig. 3. An approximate trend line was fitted, which generally shows that the scour depth is decreasing as the US flow rate increases. The slope of this line shows the rate of decay of the scour depth (henceforth RDSD). Figure 3a and b displays data from tests with minimum $d_{50}$ and $Y_t$. The difference is the flow discharge is larger for Fig. 3b, which shows an increase in RDSD as US increases. Such a conclusion can also be drawn from Fig. 3c, d, e, which displays data from tests with $d_{50} = 1.5$ mm, $Y_t = 0.26$ m, and $Q = 10, 15, \text{ and } 20$ l/s, respectively. Comparing these figures with previous figures shows that an increase in tailwater depth can decrease the RDSD. A closer look at Fig. 3 makes it clear that US has a maximum effect when $Y_t$ and $d_{50}$ are small, while $Q$ is high. This is because the downward jet flow velocity impinging the bed is high, and its destructive energy is more dissipated by the US flow velocity. In the present study, the maximum reduction of scour depth ($\approx 49\%$) due to US was obtained in test no. 7, which was carried out with the minimum $d_{50}$ and $Y_t$ and high $Q$.

In tests nos. 28 and 33, with a high tailwater depth, the ratio of $Q_s/Q$ was more than one. Such conditions can be observed in a natural stream during a low-surface-flow season, when more flow enters the stream from underground. The results show that the RDSD in Fig. 3f is higher than that in Fig. 3g and h, despite the fact that the flow discharge in Fig. 3g is larger. This of course is because of the high US flow velocity, which dissipates more of the incoming downward jet velocity near the bed.

A closer look at Fig. 3d, j, and l shows that, as the bed particle size increases, the rate of RDSD decreases. Generally speaking, the increases in
Fig. 3a-h. Variation of dimensionless scour depth versus dimensionless flow rate for sediment size of 1.5 mm.

RDSD due to US depend on the bed particle size, tailwater depth, and flow discharge.

Our result, namely the decrease in scour depth due to US, is in agreement with the results of Sarkar and Dey (2007) and Dey and Singh (2007). However, this result is in disagreement with the results of the studies of Cheng and Chiew (1998a, b; 1999), Yang (2013), and Dey and Sarkar (2007), who stated that the US can increase the scour depth. Such a paradox among the above mentioned studies about the effect of the US on the scour depth is most likely due to flow pattern differences in these studies.

The movement of sediment particles from the scour hole is basically due to the action of both the bed particle weight and the strength of the near bed jet flow velocity. Generally speaking, the sediment particles become lighter...
with an increase in the US velocity. On the other hand, a substantial decay of the downward jet flow velocity component occurs as the US flow velocity increases. In order to assess the extent of the US effect on the downward jet flow velocity component within the scour hole, the measured near-bed vertical flow velocity components along the scour axis are plotted, as shown in Fig. 4. Within the scour hole, the downward velocity jet is decayed as a greater US flow rate is applied. Consequently, less sediment is expected to be removed from the scour hole in the presence of US. Therefore, in those cases in which the entrained bed particles are mainly due to downward jet flow velocity components, such as in the present study, and the studies of Sarkar and Dey (2007) and Dey and Sing (2007), the US flow velocity can increase the decay of the downward velocity component within the flow field, which results in a smaller scour depth. However, the US does not affect the horizontal component of the jet flow velocity. Thus, in those cases in which the particle movements are more affected by the horizontal flow velocity component, as the US increases, the scour depth increases because of the weightless particles. In the study of Dey and Sarkar (2007), a horizontal submerged jet downstream of an apron was studied, in which the horizontal
component of the velocity was mainly responsible for the particle movement. The same argument can be applied to the studies of Cheng and Chiew (1998a,b; 1999) and Yang (2013), in which the particle movement was more affected by the horizontal flow velocity.

Beyond the area of the jet impingement, the direction of the downward jet flow velocity component was changed to an upward component, which is increased by the upward seepage flow velocity component because both velocity vectors are upward (as seen in Fig. 4).

### 3.3 Scour depth formula

From the aforementioned discussion, it is clear that US can decrease the scour depth downstream of a free falling grade control structure by as much as 49% in some cases. Therefore, it is logical that the extent of its influence on the scour depth can be predicted. To develop a functional relationship for predicting the maximum scour depth downstream of a free falling jet that is applicable for both cases (with and without US), the DDS is related to $Q_s/Q$ to account for the effects of the US rate. Considering Fig. 3, the following general equation is considered:

$$\frac{d_s}{d_s\text{ (Eq. 1)}} = 1 - \lambda Q_s^{n}$$

where $\lambda$ and $n$ are the coefficients to be determined from the experimental data. To do so, the $d_s$ (Eq. 1) values were determined from Eq. 1, and then $\lambda$ and $n$ were determined to be 0.28 and 0.75 by fitting the experimental data as the best fit method using Minitab software. The RMSE of Eq. 4 was 0.109. The same values were also found using genetic algorithm and particle swarm optimization techniques. Thus, the new expression, which is an ex-
tension of the equation of D’Agostino and Ferro (2004), is applicable for the scour downstream of a grade control structure where the US flow is presented.

\[
\frac{d_s}{z} = 0.54 \left[ 1 - 0.28 \left( \frac{Q_s}{Q} \right)^{0.75} \right] \left( \frac{b}{z} \right)^{0.593} \left( \frac{Y_l}{H} \right)^{-0.126} A_{50}^{0.544} \left( \frac{d_{90}}{d_{50}} \right)^{-0.856} \left( \frac{b}{B} \right)^{-0.751} . \tag{5}
\]

Equation 5 reduces to Eq. 1 when the US flow discharge is equal to zero \((Q_s = 0)\). Figure 5 displays the maximum scour depth calculated by Eq. 5 for all the measured data, showing that the \(d_s\) values fall out of the error band of ±20%. The absolute percentage error of Eq. 5 was determined using Eq. 3, and ranged from a minimum of 1% to a maximum of 31%, with an average absolute error of 10.7%. Equation 5 is similar to Eq. 1, which was developed for predicting the scour depth downstream of a grade control structure with the presence of US.

4. CONCLUSIONS

Predicting the maximum scour depth downstream of a grade control structure is necessary to prevent failure as a result of undermining the structure’s foundation. A total of 78 tests were carried out to obtain the equilibrium scour depth downstream of a free falling jet type structure. Most of the tests were conducted by applying different US flow discharges from a flume bed. The scour depth measured in those tests without seepage compared well with the scour depth predicted by the equation of D’Agostino and Ferro (2004). In general, it was found that the decay of the downward jet flow velocity due to the US flow velocity caused the scour depth to decrease. It was found that the US can have the greatest impact on reducing the scour depth where the
jet flow velocity near the bed is high. A reduction of 49% was found when the tailwater and sediment size were minimum, and the flow discharge was high. A general relationship for predicting the scour depth with respect to our finding was considered, in which as the US increases the scour depth decreases. The coefficients of the aforementioned expression were obtained by applying the experimental data using Minitab software. The new relation, which is a modified version of the equation by D’Agostino and Ferro, was found to satisfactorily predict the experimental scour depth with an average absolute error of 10.7%.

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Notation
The following symbols are used in this paper:

- $A_{50}$ – dimensionless parameter,
- $B$ – channel width,
- $b$ – width of jet,
- $DDS$ – ratio of $d$, with upward seepage to $d$, without upward seepage,
- $d_s$ – maximum scour depth,
- $d_{sc}$ – maximum scour depth calculated from Eq. 1,
- $d_{sm}$ – maximum scour depth measured in this study,
- $d_{50}$ – bed grain size for which 50% of sampled particles are finer,
- $d_{90}$ – bed grain size for which 90% of sampled particles are finer,
- $g$ – acceleration of gravity,
- $H$ – upstream and downstream water level difference,
- $H_s$ – upward seepage head,
- $h_0$ – head above the weir crest,
- $Q$ – main flow discharge,
- $QQS$ – ratio of upward seepage discharge to weir flow discharge,
- $Q_s$ – upward flow discharge,
- $v_s$ – seepage velocity,
- $Y_t$ – tailwater depth,
- $z$ – weir crest elevation above the bed,
- $\lambda$ – coefficient of proportional,
- $\rho$ – fluid density,
- $\rho_s$ – grain density.
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