Multiplicative graphs and their application to the equation

\[ n - \varphi(n) = c \]

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Abstract

We will study the solutions to the equation \( f(n) - g(n) = c \), where \( f \) and \( g \) are multiplicative functions and \( c \) is a constant. More precisely, we prove that the number of solutions does not exceed \( c^{1-\epsilon} \) when \( f, g \) and solutions \( n \) satisfy some certain constraints, such as \( f(n) > g(n) \) for \( n > 1 \). In particular, we will prove the following estimate: the number of solutions of the equation \( n - \varphi(n) = c \) (here \( G(k) \) is the number of ways to represent \( k \) as a sum of two primes) is:

\[ G(c + 1) + O(c^{\frac{3}{4} + o(1)}) \]

To obtain this we use so-called multiplicative graphs.

1 Introduction

Let \( \varphi \) be the Euler function, which value at \( n \) is defined as

\[ \varphi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right). \]

For a given \( c \) the equation \( \varphi(n) = c \) has been studied in works of Erdős [1] and Pomerance [2]. In particular, such bounds for \( T(c) = |\{n : \varphi(n) = c\}| \) were given:

\[ T(c) \leq ce^{-(1+o(1))} \frac{\log c \log \log \log c}{\log \log c}, \]

and also

\[ T(c) \geq c^\alpha, \]

for infinitely many \( c \) and \( \alpha = 0.55655\ldots \)

One can consider the cototient ([3]) Euler function:

\[ \psi(n) = n - \varphi(n). \]

For a given \( c \) the equation \( \psi(n) = c \) has been studied in the work of Banks and Luca [3]. In particular it was demonstrated that for almost all (i.e. of density 1) primes \( p \) equation \( \psi(n) = 2p \) can not be solved in \( n \). We show the following. Let \( G(n) \) be the amount of ways to represent \( n \) as a sum of 2 prime numbers. Then we have:

**Theorem 1.1.** The quantity of solutions of \( \psi(n) = c \) is equal to

\[ G(c + 1) + O(c^{\frac{3}{4} + o(1)}). \]

It is easy to see, that if \( c \) is an even number, then the term \( G(c + 1) \) in the formula above is \( O(1) \).

2 Multiplicative graphs

In this paragraph we introduce so-called multiplicative graphs, which turn out to be useful when one studies differences \( f(n) - g(n) \), where \( f \) and \( g \) are multiplicative functions.

Let \( \mathbb{A} = \{(A_i, a_i)\} \) and \( \mathbb{B} = \{(B_i, b_i)\} \) be two sets of points. We do not consider points \( x \in \mathbb{A} \) and
We consider the sequence of numbers. In this case the power 3 values, it makes sense to explore the behaviour of multiplicative graphs, built for sets of natural numbers. Since we are studying functions defined on natural numbers and attaining natural numbers as values, it makes sense to explore the behaviour of multiplicative graphs, built for sets of natural numbers. In this case the power \( \frac{3}{2} \) from remark 2 might be vastly decreased.

\( y \in \mathbb{B} \) equal even if their coordinates do coincide.

Let some \( c \in \mathbb{R} \) be fixed. We build the bipartite graph by the following rule. We connect point \((A, a) \in \mathbb{A}\) with \((B, b) \in \mathbb{B}\) by an edge if and only if \( AB - ab = c \) holds.

**Definition 1.** The obtained graph we call the multiplicative graph \( G(\mathbb{A}, \mathbb{B}, c) \) for sets \( \mathbb{A}, \mathbb{B} \) and given \( c \).

**Lemma 2.1 (on the multiplicative graph).** Let \( c \neq 0 \). For the given sets of points \( \mathbb{A} \subset \mathbb{R}^2, \mathbb{B} \subset \mathbb{R}^2 \), all of whose coordinates are nonzeros, the multiplicative graph \( G(\mathbb{A}, \mathbb{B}, c) \) does not contain cycles of length 4.

**Proof.** Suppose this graph has a cycle of length 4. Since this graph is bipartite by definition, this cycle can be represented as

\[
(A_1, a_1) \rightarrow (B_1, b_1) \rightarrow (A_2, a_2) \rightarrow (B_2, b_2) \rightarrow (A_1, a_1),
\]

where all the points are different. We write down such equations:

\[
A_1B_1 - a_1b_1 = c, \\
A_2B_1 - a_2b_1 = c, \\
A_2B_2 - a_2b_2 = c, \\
A_1B_2 - a_1b_2 = c.
\]

From where we have

\[
(a_1b_1 + c)(a_2b_2 + c) = A_1A_2B_1B_2 = (a_2b_1 + c)(a_1b_2 + c).
\]

Expanding the brackets and reducing equal components we have \( a_1b_1 + a_2b_2 = a_2b_1 + a_1b_2 \), so \( (a_1 - a_2)(b_1 - b_2) = 0 \). Without loss of generality let \( a_1 = a_2 \). Subtracting second equality from the first we obtain

\[
B_1(A_1 - A_2) = b_1(a_1 - a_2) = 0,
\]

So either \( A_1 = A_2 \) holds (which means \((A_1, a_1)\) coincides with \((A_2, a_2)\)), i.e. contradiction), or \( B_1 = 0 \) holds (but all coordinates are nonzero, contradiction). So, this graph can not contain any 4-cycles.

**Remark 1.** In general, the statement "The multiplicative graph does not contain 6-cycles" would not hold even under some restrictions of prime multiplicative graph lemma below.

**Remark 2.** Let \( V = |\mathbb{A}| + |\mathbb{B}| \) be the amount of vertices in the graph. Then, since 4-cycles are missing, it is known, that it has no more than \( O(V \sqrt{V}) = O(\max(|\mathbb{A}|, |\mathbb{B}|)^{\frac{3}{2}}) \) edges.

Since we are studying functions defined on natural numbers and attaining natural numbers as values, it makes sense to explore the behaviour of multiplicative graphs, built for sets of natural numbers. In this case the power \( \frac{3}{2} \) from remark 2 might be vastly decreased.

**Definition 2.** Let \( \mathbb{A} \subset \mathbb{N}^2, \mathbb{B} \subset \mathbb{N}^2 \), and \( c \in \mathbb{N} \) be such that

\[
(A_1, c) = (A_2, c) = \ldots = (B_1, c) = (B_2, c) = \ldots = 1, \\
(a_1, c) = (a_2, c) = \ldots = (b_1, c) = (b_2, c) = \ldots = 1
\]

hold. Then we say that the associated multiplicative graph \( G(\mathbb{A}, \mathbb{B}, c) \) is prime.

**Lemma 2.2 (on the prime multiplicative graph).** The prime multiplicative graphs does not contain cycles of any length.

**Proof.** Suppose this graph contains cycle of length \( 2k, k \geq 3 \) (because of biparticty, cycles of odd length are not possible). This cycle can be represented as

\[
(A_1, a_1) \rightarrow (B_1, b_1) \rightarrow (A_2, a_2) \rightarrow \ldots \rightarrow (B_k, b_k) \rightarrow (A_1, a_1).
\]

We consider the sequence of 4 edges following each other:

\[
(X, x) \rightarrow (P, p) \rightarrow (Y, y) \rightarrow (Q, q) \rightarrow (Z, z).
\]
One can deduce following equations then:

\[ XP - xp = c, \]
\[ PY - py = c, \]
\[ YQ - yq = c, \]
\[ QZ - qz = c. \]

It follows easily that

\[ py + c = PY | XYPQ = (xp + c)(yq + c), \]

so

\[ py + c | (xp + c)(yq + c) - (py + c)(qx + c) = c(x - y)(p - q). \]

Because the graph is prime we have \((py + c, c) = (py, c) = 1\), so the \(c\) might be reduced, that’s why

\[ py + c | |x - y||p - q|. \]

Similarly, we have

\[ yq + c | |y - z||p - q|. \]

At this moment one can declare, that we have chosen such part of the cycle, where the value of \(y\) is maximal among \(x, y, z\). One can always declare that because all the numbers are different — if, for example, \(x = y\), then \(XP = xp + c = yp + c = YP\), so \(X = Y\), i.e. points \((X, x), (Y, y)\) of the same part do coincide.

So, let \(y\) be the maximal number. Without loss of generality, let \(p > q\) (case \(q < p\) is equivalent). Then we have \(0 < (y - x)(p - q) < yp < yp + c\), but \((y - x)(p - q)\) is divisible on \(yp + c\). Contradiction.

**Remark 3.** Due to lack of cycles, any prime multiplicative graph is a forest, that is why it has number of edges less than number of vertices \(|\mathcal{A}| + |\mathcal{B}| = O(\max(|\mathcal{A}|, |\mathcal{B}|))\).

### 3 Differences between multiplicative functions

In this paragraph we prove the lemma on the differences between multiplicative functions, which is used later to estimate amount of the solutions to the equation \(n - \varphi(n) = c\).

We first prove a helpful lemma.

**Lemma 3.1 (on the number partitioning).** Let \(n = x_1x_2\ldots x_k\) be such a number that any \(x_i\) does not exceed \(t \in \mathbb{R}_+\). Then such a partition \(n = ab\) \((a = x_1, x_2, \ldots, b = x_3, x_4, \ldots)\) exists so that \(1 \leq a, b \leq \sqrt{n\sqrt{t}}\).

**Proof.** From \(n = x_1 \ldots x_k\) we have:

\[ \log n = \log x_1 + \ldots + \log x_k. \]

We split \(x_i\)'s into two groups with sums \(s\) and \(r\) so that the absolute value of their difference is minimal possible. Now we demonstrate that \(|s - r| \leq \log t\) — then \(a = e^s, b = e^r\) would be the partition we desired to obtain. Suppose it does not hold. Without loss of generality \(s \geq r\). We can try to move some \(\log x \leq \log t\) from bigger sum to smaller sum: \(\hat{s} = s - \log x, \hat{r} = r + \log x\). It is clear that \(\hat{s} \leq \hat{r}\) (otherwise we would just make them closer and therefore their absolute difference was not minimal possible). Because the absolute difference was smallest possible, we have inequality: \(\hat{r} - \hat{s} \geq s - r\), thus \(2\log x \geq 2(s - r)\). But we supposed \(s - r\) strictly exceeds \(\log t\). Contradiction.

Now the main result of this chapter:
Lemma 3.2 (on the differences between multiplicative functions). Let \( f : \mathbb{N} \rightarrow \mathbb{N}, g : \mathbb{N} \rightarrow \mathbb{N} \) be the two multiplicative functions, \( c \) is a natural number, \( t > 0 \) is a real number and \( N \subset \mathbb{N} \) is the set of natural numbers. Assume next conditions hold:

(i) \( f(n) > g(n) \) for all \( n > 1 \).

(ii) \( f(n), g(n) = (f(m), g(m)) \Leftrightarrow n = m. \)

(iii) For arbitrary \( x > 0 \) there exists no more than \( O(x) \) such that \( f(n) \leq x \).

(iv) \( f(n) - g(n) = c \) for all \( n \) in \( N \).

(v) For any \( n = \prod p_i^{\alpha_i} \in N \) it is true that \( f(p_i^{\alpha_i}) \) does not exceed \( t \).

Then it is true that \( |N| = O(t^{\sqrt{\varphi(c)}}(c)) = O(t^{\frac{1}{2} + o(1)}) \).

Example 3.3. One may check easily that pair of functions \( f(n) = n, g(n) = \varphi(n) \) satisfies first three conditions.

Proof. Let elements of \( N \) be \( \{n_1, n_2, \ldots \} \). It is clear that for any \( n \) in \( N \) holds inequality \( f(n) \leq ct \).

Indeed, let us consider some primal divisor of \( n \), say \( p^\alpha \). Then \( n = mp^\alpha \), where \((m, p^\alpha) = 1 \). From \( f(n) - g(n) = c \) it follows that \( f(m)f(p^\alpha) - g(m)g(p^\alpha) = c \), i.e.

\[
   c = f(m)f(p^\alpha) - g(m)g(p^\alpha) = (f(m) - g(m))g(p^\alpha) + (f(p^\alpha) - g(p^\alpha))f(m) \geq (f(p^\alpha) - g(p^\alpha))f(m) \geq f(m).
\]

Then \( f(n) = f(mp^\alpha) = f(m)f(p^\alpha) \leq ct \).

For any \( n_i \) we build such partitioning \( f(n_i) = f(a_i)f(b_i) \) (if \( n = p_1^{\alpha_1}p_2^{\alpha_2}\cdots p_k^{\alpha_k} \), then we use number partitioning lemma for \( f(n) = f(p_1^{\alpha_1})f(p_2^{\alpha_2})\cdots f(p_k^{\alpha_k}) \) ) so that

\[
   1 \leq f(a_i), f(b_i) \leq \sqrt{f(n_i)} \leq t\sqrt{c}.
\]

It is clear from \( f(n) - g(n) = c \) that \((f(n), c) = (g(n), c) \) holds. Consider all the possible tuples of length 5 \((l_1, l_2, l_3, l_4)\) of divisors of \( c \) so that \( l_1l_2 = l_3l_4 \). Consider the classes

\[
   N_{l_1, l_2, l_3, l_4} = \{ n \in N : (f(n), c) = (g(n), c) = l, l_1 | f(a), l_2 | f(b), l_3 | g(a), l_4 | g(b) \}
\]

Clearly \( N \) can be partitioned into union of such sets — not necessary strict. From \( l_1l_2 = l_3l_4 \) it follows that the class parameters are automatically determined by the triple \((l_1, l_2, l_4)\) which gives bound \( r^2(c) \) on the number of classes.

For each class \( N_{l_1, l_2, l_3, l_4} \) we build sets:

\[
   \mathcal{A}_{l_1, l_2, l_3, l_4} = \left\{ \left( \frac{f(a)}{l_1}, \frac{g(a)}{l_3} \right) \right\}, \mathcal{B}_{l_1, l_2, l_3, l_4} = \left\{ \left( \frac{f(b)}{l_2}, \frac{g(b)}{l_4} \right) \right\},
\]

for partitionings \( ab \) of numbers \( n \) from the given class. Clearly, if \( f(a)f(b) - g(a)g(b) = c \), then

\[
   \frac{f(a)}{l_1} \frac{f(b)}{l_2} = \frac{g(a)}{l_3} \frac{g(b)}{l_4} = \frac{c}{l} = c'.
\]

Now we build prime multiplicative graph \( G(\mathcal{A}_{l_1, l_2, l_3, l_4}, \mathcal{B}_{l_1, l_2, l_3, l_4}, c') \). This graph is indeed prime since we get lost of all the possible common divisors of \( c' \) with other numbers. Clearly, the number of vertices does not exceed \( O(t\sqrt{c}) \) (since all the \( f(a) \) and \( f(b) \) are bounded as \( t\sqrt{c} \)).

For each \( n_i = a_ib_i \) in \( N_{l_1, l_2, l_3, l_4} \) there is an unique edge connecting \((f(a_i))/l_1, (g(a_i))/l_3\) and \((f(b_i))/l_2, (g(b_i))/l_4\). Note, that the correspondence between the solutions and the edges is an injection. So, \(|N_{l_1, l_2, l_3, l_4}| \) does not exceed the number of edges in \( G(\mathcal{A}_{l_1, l_2, l_3, l_4}, \mathcal{B}_{l_1, l_2, l_3, l_4}, c') \). Since this graph is prime, and therefore does not contain cycles, the number of edges is less then number of vertices, i.e. is \( O(t\sqrt{c}) \).

Summing up the edges in all the classes, one gets the required bound on the size of \( N \).

Remark 4. One can replace condition (v) with \((v')\) for any \( n = \prod p_i^{\alpha_i} \in N \) inequality \( g(p_i^{\alpha_i}) \leq t \) must hold. Condition (iii) might be replaced with \((iii')\) for arbitrary \( x \) there is only \( O(x) \) such \( n \) so that \( g(n) \leq x \).
4 On the number of solutions to the \( n - \varphi(n) = c \).

In this section we apply obtained results to find out the amount of solutions of \( n - \varphi(n) = c \). Starting from here we consider \( c \) to be fixed. Also we remind, that \( G(k) \) is the number of ways to represent \( k \) as a sum of two numbers.

**Theorem 4.1.** For given \( c > 1 \) the equation

\[
n - \varphi(n) = c \quad (\ast)
\]

has \( G(c+1) + O(c^{\frac{3}{4} + o(1)}) \) solutions.

**Proof.** We say that the number is primal, if it can be represented as \( p^n \), where \( p \) is the prime number. Clearly any natural \( n > 1 \) is a product of some primal numbers. We would first consider cases where \( n \) is a product of no more than 2 primal numbers.

**Lemma 4.2 \( (n — \text{primal}). \)** *Primal* \( n \) result in just \( O(1) \) solutions.

**Proof.** Let \( n = p^a \), then since \( c > 1 \), we have \( a > 1 \), thus \( p^{a-1}(p-1) = c \) and \( p|c \), and it is clear that \( p \) is a greatest prime divisor of \( c \), so the \( a \) is defined explicitly. It gives us \( O(1) \) solutions.

**Lemma 4.3 \((n — \text{product of 2 primal numbers}).\)** Those \( n \) which are products of 2 primal numbers result in \( G(c+1) + O(\ln^2 c) \) solutions.

**Proof.** Consider \( n \) of the form \( n = p^aq^b \). If \( a = b = 1 \) holds, then \( pq - \varphi(pq) = p + q - 1 \), so \( c + 1 = p + q \), which results in \( G(c+1) \) solutions. If one of them (say \( a \)) is greater than 1, then we have \( p|c \), and iterating over \( a \) (which is obviously less than \( \omega(c) \)) gives us equations \( q^{b-1}(q+p-1) = \frac{c}{pq} \), each of which has just \( O(1) \) solutions, when \( b = 1 \), and \( O(\omega(c)) \) solutions when \( b > 1 \) and \( q|c \), so we have just \( O(\omega^2(c)) \) solutions in common.

As a result we have \( G(c+1) + O(\ln^2 c) \) solutions.

Because of these lemmas we can now consider cases when \( n \) is a product of at least 3 primal numbers. Our goal now is to show, that there exists no more than \( O(e^{c^{\frac{1}{4} + o(1)}}) \) solutions of equation (\( \ast \)) in this case.

Let \( N(c) \) be the number of such solutions \( n \), which are products of at least 3 primal numbers. Let \( N^*(c) \) be the number of such solutions \( n \) from \( N(c) \) which are square-free. Then the following holds:

\[
N(c) \leq \sum_{d|n} N^*(d) = \sum_{d|n} N^*(\frac{c}{d}).
\]

We will demonstrate that \( N^*(c) = O(c^{1-\epsilon + o(1)}) \), \( \epsilon > 0 \), and same holds for its sum over divisors of \( c \), so we will only consider the case when \( n \) is square-free in further text. Indeed, since \( c^{1-\epsilon} \) is monotonically increasing, one has

\[
\sum_{d|n} N^*(\frac{c}{d}) \leq \sum_{d=1}^\tau \left(\frac{c}{d}\right)^{1-\epsilon + o(1)} = c^{1-\epsilon + o(1)} \sum_{d=1}^\tau \frac{1}{d^{1-\epsilon + o(1)}} \ll c^{1-\epsilon + o(1)} \frac{\tau^\epsilon}{\epsilon} = c^{1-\epsilon + o(1)},
\]

where \( \tau = O(e^{\ln \ln c}) \) is a number of divisors of \( c \). Since bounding of \( N^*(c) \) leads to bounding of \( N(c) \), we are only estimating \( N^*(c) \) in further text.

Note that if \( n = Ap \) then \( n - \varphi(n) = Ap - \varphi(A)\varphi(p) \) holds, i.e.

\[
Ap - \varphi(A)(p-1) = c.
\]

The value of \( Ap - \varphi(A)(p-1) \) might also be expressed as:

\[
Ap - \varphi(A)(p-1) = (A - \varphi(A))p + \varphi(A) = (A - \varphi(A))(p-1) + A,
\]

from where it follows that \( A < c \).

Now we represent \( n \) as \( n = Bpq \), where \( p \) and \( q \) are some prime divisors of \( n \). Consider 2 cases:
4.1 \( n \) can be expressed as \( n = Bpq \), where \( B < c^{\frac{3}{4} + o(1)} \) holds.

Note that if \( B \) is some fixed number so that \( 1 < B < c^{\frac{3}{4} + o(1)} \), then there exists no more than \( e^{o(1)} \) such \( n \), so that \( n \) can be expressed as \( n = Bpq \) (\( p, q \) — primes) and satisfying equation \((*)\).

Indeed, let \( B > 1 \) be fixed. Then \((*)\) with such \( n \) is equivalent to:

\[
Bpq - \varphi(B)(p-1)(q-1) = c,
\]

which, on it’s turn, is equivalent to

\[
\left((B - \varphi(B))p + \varphi(B)\right)\left((B - \varphi(B))q + \varphi(B)\right) = (B - \varphi(B))c + B\varphi(B).
\]

Since \( B \) is bounded, RHS does not exceed \( c^2 \). So it has no more than \( e^{O(\frac{\ln \ln n}{\ln n})} = e^{o(1)} \) solutions. Summing up by \( B \), we get estimate \( O(c^{\frac{3}{4} + o(1)}) \) on amount of solutions.

4.2 \( n \) can’t be expressed as \( n = Bpq \), where \( B < c^{\frac{3}{4} + o(1)} \) holds.

In this case we can assume that if \( n = Bpq \) (\( p, q \) — primes), then \( B > c^{\frac{3}{4} + o(1)} \).

From \( n = Bpq = (B)pq \) it follows that \( Bp \leq c \), and \( p \leq \frac{c}{B} \leq c^{\frac{3}{4} + o(1)} \). Since we could have taken \( p \) as any prime divisor of \( n \), this inequality holds for all prime divisors of \( n \).

We now apply lemma on differences between multiplicative functions to functions \( f(n) = n, g(n) = \varphi(n) \) and \( t = c^{\frac{3}{4} + o(1)} \). It gives us bound \( c^{\frac{3}{4} + o(1)} \) on number of solutions immediately

Summing up bounds from both cases we have bound on \( N^*(c) \) equal to \( O(c^{\frac{3}{4} + o(1)}) \), which we desired to prove.

\( \square \)

Remark 5. One may check, that the "obstacle" because of which we can not have better bound than \( c^{\frac{3}{4} + o(1)} \), is a case where \( n \) is a product of 5 prime divisors, each of which is roughly \( c^\frac{3}{4} \). As soon as bound in this case is improved, the overall bound will be improved immediately.

Remark 6. As we noticed, it is enough to consider only square-free \( n \). Let \( M_k \) be a set of such \( n \), which may be represented as \( n = p_1p_2\ldots p_k \) so that \((*)\) holds. Then we have a hypothesis that for \( k \geq 2 \) holds estimation \( |M_k| = O(c^{\frac{3}{4} + o(1)}) \). One may check (by Pigeonhole principle and lower bounds on amount of solutions in the interval) that those bounds are true in "average" case (meaning those bounds are lower-bounds in those cases).

This hypothesis is easily verifiable in cases \( k = 2 \) and \( k = 3 \): when \( k = 2 \) it straitghly follows from equality \( p + q = c + 1 \).

For other \( k \) one can use representation of \( n \) as \( n = Bpq \) (as we did above) and thus getting unconditional inequalities in the form

\[
|M_k| < c^{\frac{3}{4} + o(1)}.
\]

For \( k = 3 \) this bound coincides with the one stated by the hypothesis.

Using lemma on the differences between multiplicative functions one may obtain inequalities of such type: \( |M_k| < c^{\frac{3}{4} + \epsilon_k} \), where \( M_k \) are such members of \( M_k \), which do not have "too big" prime divisors, and \( \epsilon_k \) depends on what we mean by "too big" prime divisor. In other words, the main complexity of the problem is about non-smooth numbers \( n \).

Remark 7. If one considers only even \( c \), then we have a restriction that \( n \) from \((*)\) is also even, which leads to inequality \( c < n < 2c \). Obviously the "main term" of the sum, i.e. \( G(c+1) \) would be \( O(1) \), because \( c + 1 \) is odd. By analogous implications one may obtain bound \( O(c^{\frac{3}{4} + o(1)}) \) on a number of solutions. Despite that, one has a hypothesis that there exists just \( e^{o(1)} \) solutions of \((*)\) when \( n \) is even.
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