A model of short-range correlations in the charge response

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Abstract
The validity of a model treating the short-range correlations up to the first order is studied by calculating the charge response of an infinite system and comparing the obtained results with those of a Fermi Hypernetted Chain calculation.

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The interest in the study of Short Range Correlations (SRC) in nuclear systems has increased in these last few years. In this field, the experimental activity has been concentrated in the search for observables allowing a clean identification of SRC effects [1, 2]. From the theoretical point of view there has been a development of calculations which explicitly consider SRC.

The theoretical situation is quite satisfactory for the few–body systems where Faddeev [3], Correlated Hyperspherical Harmonics Expansion [4] and Green Function Monte Carlo [5] theories solve exactly the Schrödinger equation. This last technique has been recently applied to investigate light nuclei up to \( A = 7 \) [6]. Unfortunately the straightforward application of these theories to the study of medium and heavy nuclei is not yet technically feasible, in spite of the rapid progress of the computer technology.

The other satisfactory situation, from the theoretical point of view, regards the opposite side of the isotope table: the infinite nuclear systems such as neutron and nuclear matter. For the study of these systems, perturbations techniques have been developed such as Brueckner Bethe Goldstone [7] or Correlated Basis Function theories (CBF) [8]. These approaches do not provide an exact solution of the Schrödinger equation, but they can reproduce rather well the empirical properties of nuclear matter because they sum complete sets of terms of their perturbation expansions.

The application of CBF to the description of the ground state of finite nuclear systems, has been recently carried out [9] using various levels of Fermi Hypernetted Chain (FHNC) approximations. The results are promising and it is conceivable that CBF theories may be applied to the description of excited states in the future. For the time being, these properties have to be studied by using simpler models.

The major part of the finite models developed up to now to describe finite nuclear systems treat SCR only at the lowest orders in the correlation. The main field of application has been the investigation of nuclear ground state properties [10]. Recently, nuclear models dealing with SRC have been implemented to study the electromagnetic two–nucleon emission [11, 12]. Given the wide use of these models and their relevance for predictions and comparisons with two-nucleon emission data, we believe that a test of their validity is necessary.

A model treating SRC up to the first order in the correlation, has been developed in Ref. [13] to study charge density and momentum distribution of some doubly closed shell nuclei. The model was able to reproduce rather well the finite nuclei FHNC results of Ref. [9] and it has been extended to describe the two-nucleon emission produced by the same electromagnetic operator, the charge operator, for inclusive electron scattering experiments [14]. It is however not obvious that the good agreement with the FHNC results obtained for the ground state can be conserved also for the excited states. Since complete FHNC calculations are available for one-particle one-hole (1p-1h) nuclear matter charge responses [15] we have applied our model to calculate these responses.

The basic idea of the model, already presented in [13] and [14], consists in truncating the CBF expansion in order to consider only those terms containing a single Jastrow-type correlation line, \( h(r) = f^2(r) - 1 \). In Fig. [1] we show the diagrams we have retained in the
present calculation. It is worth pointing out that by setting in each diagram the particle line equal to the hole line, we reproduce the set of diagrams used to calculate the ground state properties of the charge operator \[13\]. An important property of the expansion in powers of \( h(r) \) is that the normalizations of the wave function are exactly preserved at each order \[10\]. In Ref. \[13\], the nuclear charge was conserved as well as the proper normalization of the correlated many-body wave functions in the present calculation. On the contrary, the nuclear charge is not conserved in Refs. \[11, 12\], where the expansion adopted, and truncated at the second order, is based on the number of particles of the cluster and not on the powers of \( h(r) \).

The FHNC and the model calculations of the charge responses have been done using the same correlation, i.e. the scalar part of a complicated state dependent correlation fixed to minimize the nuclear binding energy in a FHNC calculation with the Urbana V14 nucleon–nucleon potential \[8\].

In Fig. 2 we compare the results of our model with those obtained with a FHNC calculations. In this figure we show the proton structure functions, therefore no electromagnetic nucleon form factors have been included. The structure functions have been calculated for three values of the momentum transfer, and for Fermi momentum of 1.09 fm\(^{-1}\). We found this value of the Fermi momentum adequate to describe the quasi–elastic responses of \(^{12}\)C \[17\]. In the panel (a) of Fig. 2 the full lines represent the results of the model and the dashed lines those of the FHNC calculation. The difference between the two calculations are very small, and they are explicitly shown, multiplied by a factor \(10^5\), in the panel (b).

In Fig. 3 we show the response functions calculated using the electromagnetic nucleon form factors of Ref. \[18\]. These response functions have been obtained considering, in addition to the proton structure functions shown in Fig. 2, also the neutron contribution, which in any case, turns out to be negligible in the longitudinal response. The dashed lines show the Fermi gas responses, corresponding in our model to the first diagram of Fig. 1. The dashed–dotted lines have been obtained adding the contribution of the two–point diagrams, i.e. the diagrams multiplied by the factor \(1/2\) in Fig. 1. The full lines have been obtained by including all the diagrams of Fig. 1. The contribution of the two–point diagrams is partially cancelled by the inclusion of the three–point diagrams. This is an effect similar to that obtained in the calculation of the ground state charge and momentum distributions \[13\].

The results we have presented show that a model considering only those terms with a single correlation line can reproduce extremely well the FHNC charge response functions. This conclusion is consistent with the finding of Ref. \[13\] for the ground state charge distribution. We should remark that our calculations have been done for the charge operator only. An extension of the calculation to evaluate responses induced by other electromagnetic operators is necessary to test the validity and the range of applicability of the model.

We like to stress again that a good agreement with the FHNC results as been obtained, most probably, because, in our model, the proper normalization of the many–body wave
function has been conserved by evaluating both two- and three-point diagrams. Calculations of 1p–1h responses which include the two-point diagrams only, overestimate the effect of the correlations. One may expect that the same problem could affect also the two-nucleon emission calculations, like those of Refs. [1, 2], where only two-point diagrams are considered. On the other hand, there are indications that in the 2p-2h responses two- and three-point diagrams act differently than in the 1p-1h responses [4]. A similar analysis, as the one performed here, for the 2p-2h response is needed to further clarify the situation.

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Figure 1:
Diagrams considered in our model. The dotted lines represent the correlation function. The oriented lines represent particle and hole wave functions. The black circle indicates an integration point while the black square indicates the integration point where the charge operator is acting.

Figure 2:
In the panel (a) the 1p-1h nuclear matter proton structure functions calculated with the present model (full lines) are compared with those obtained from the FHNC calculation of Ref. [15]. The calculations have been done for $q = 300, 400$ and $550$ MeV/c and $k_F = 1.09$ fm$^{-1}$. In the panel (b) we show the differences, multiplied by $10^5$, between the structure functions obtained with the present model and those obtained using FHNC.

Figure 3:
Nuclear matter longitudinal responses for $q = 300$, 400 and 550 MeV/c and $k_F = 1.09$ fm$^{-1}$ calculated with the proton and neutron form factors of ref. [18]. The dashed lines represent the Fermi gas responses, the dashed–dotted lines have been obtained adding the two–point diagrams, while the full lines show the results of the complete calculations where all the diagrams of Fig. [1] have been considered.
\[ \begin{align*}
2 + \frac{1}{2} \left[ & 
\begin{array}{ccc}
\text{Diagram 1} & \text{Diagram 2} & \text{Diagram 3} \\
- & - & - \\
\end{array}
\right] \\
+ \frac{1}{3} \left[ & 
\begin{array}{ccc}
\text{Diagram 4} & \text{Diagram 5} & \text{Diagram 6} \\
- & - & - \\
\end{array}
\right] \\
+ & 
\begin{array}{ccc}
\text{Diagram 7} & \text{Diagram 8} & \text{Diagram 9} \\
- & - & - \\
\end{array}
\right] \\
\end{align*} \]
