Destruction of granular media under pore pressure

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Abstract. The researchers have modernized and used the boundary element method to calculate all components of displacement vector on a sufficiently complex surface. It is shown that under compression, given the rigid contact between grains, pore pressure induces tensile loads on the contact areas. With the increase in the loading frequency up to efficient wavelength conformable with the size of the contact area, the amplitude of the tensile loads reduces and the resonant frequencies are absent.

It is known that velocities of P- and S-waves in a micro-heterogeneous medium containing fluids depend on the difference of the external pressure and the internal pressure of the fluid, or the so-called effective pressure $P_e = P_{ex} - P_{pore}$ [1]. The common interpretation of this relation says that the pore pressure ‘hinders’ the external pressure to close pores and cracks. This is an unsatisfactory explanation as the both pressures are scalars (impossible to direct anywhere) and the both are compressive. An indirect proof of deficiency of the above interpretation is that on order to interpret experimental data, the relation above is added with a dimensionless member $n$ of the unclear physical nature, so that: $P_e = P_{ex} - n P_{pore}$, $n \leq 1$ [2].

Actually, a grain is subjected to a complex stress state when the grain contacts and the fluid–matrix interfaces experience different loads both by value and sign. Such loads can break the contacts and make the matrix a granular medium. That is why it is impossible to consider a granular medium as a solid, while averaging of stresses over the volume of a medium disables structural analysis of the mentioned phenomenon.

It has been shown in [3] that depending on the structure of the pore space, the compressive pore pressure in a granular medium can induce both compression (strengthening) and tension at the contacts. The amplitudes of the tensile stresses named as wedging pressure differ by tens per cents in case of rigid and slide contacts but have the same sign. Furthermore, the numerical modeling shows that wedging pressures grow at smaller contacts; accordingly, failure starts at the contact of smaller radii as the tensile load increases when the radius of a contact reduces. However, the mentioned study focused on the sphere with only two axially symmetrical contacts. It is interesting to analyze many contacts in order to take into account their interactions and to study the effect of the frequency of stationary fluctuations in the pore pressure with a view to modeling hydrofracture or other dynamic loads.

A representative granular medium may be a ball with eight flat contact sites if the rest medium is replaced by an adequate system of forces in accord with the Saint Venant principle. It is also reasonable to solve the problem in the framework of linear elasticity, i.e. under assumption that all energy losses due to stress raisors at the edge of the contacts are small. The surface depicted in Figure
was set in a parametric representation, i.e. each of 2500 points was either on the sphere or at a contact site. Actually, that meant that the edge of a contact was rounded so that the normal changed rapidly but continuously.

![Figure 1. A unit ball with eight remote segments which define the contact sites. The contacts preserve their coordinates and make a rigid frame structure. The rest of the surface is the matrix–fluid interface.](image1)

For the comparison of the calculations with the geometry of a grain, the same surface is presented in parametric coordinates using the indicator function (I) in Figure 2. The indicator function equals one if the point is at the contact site and zero if the point is at the matrix-fluid interface.

For the convenience, the distribution of the loads is given on the indicator surface.

Mechanically, we need to solve an elastic boundary value problem of mixed type. Zero displacements are set at the contact sites, and a hydrostatic load with the unit pore pressure is set at the rest surface of the sphere.

![Figure 2. Auxiliary (indicator) surface in the coordinates \(\theta, \varphi\) (angles in the spherical coordinate system). \(I = 0\) if the point is on the sphere, \(I = 1\) if the point is at the flat contact site.](image2)

As has been noticed in [4], the boundary element method is advantageous in case of an infinite length of one of the boundaries and in the case of the boundary value problems on the surfaces with the rapidly varying vector of the normal. When the boundary conditions are set at the off-smooth surfaces, the integral method has the advantage in terms of reliability of the results, including better causality of the system of equations (its dimension 7500×7500). In this method, the displacement is the solution of the equation of elastic stationary vibrations:
\[ \Delta U + k^2 U + \frac{\lambda + \mu}{\mu} \text{grad} \left( \text{div} U \right) = 0, \quad k^2 = \frac{\rho \omega^2}{\mu}. \]  

(1)

where \( U \) is the displacement vector; \( \lambda \) and \( \mu \) are the Lamé elasticity moduli; \( \rho \) is the density. The modified BEM [5] ensuring optimal balance of accuracy and causality consists in that the equations (1) satisfying boundary conditions at each fixed point of the surface are not solved as integrals but as the finite sums at a moving point of the surface, and each summand satisfies the equation (1):

\[ U_j(x_0) = \sum M_{ij}(x_0, x) f_j(x). \]

(2)

The components of the tensor \( M_{ij} \) satisfy Eq. (1) analytically and are the appropriately oriented responses with respect to displacements in the direction \( i \) towards the finite analog of delta-load applied in the the direction \( j \). The use of the finite analog of delta-load instead of the classical delta-function allows obtaining smooth and finite kernels with the minimized loss of causality. The components of the vector of the potential \( f_i \) in (2) are calculated so that to meet the boundary conditions. Then, the dependence between the displacement vectors and the loads is calculated analytically, by differentiation and Hooke’s law (\( \sigma_{jk} = \lambda \delta_{jk} + 2 \mu \varepsilon_{jk} \)):

\[ p_j(x_0) = -\sum P_{ij}(x_0, x) f_j(x). \]

(3)

Here, \( p_i \) is the component of the load vector, \( P_{ij} \) is the tensor of loads calculated with the assumption that \( M_{ij} \) is three columns of the displacement vectors.

The calculations are illustrated by Figs. 3 and 4. Figure 3 shows the distribution of the normal loads at the contact under the unit pressure of fluid. The distribution of the loads is complex. On the other hand, the normal load has a negative sign, which conform with tension despite compression at the matrix–fluid interface. The peak tension is at the edge of the contact and reaches the value of the pore pressure. At the center of the contact, the tension is half as much.

**Figure 3.** Distribution of tensil loading (wedging pressure) at the grain contacts. This distribution is complex but loading has the negative sign, which means tension; the value of the tensile is approximately equal to the fluid pressure.

Figure 4 shows a shearing load induced by the pore pressure. The forces have different signs due to different signs of the normals at the opposite contact sites, but the stresses are the same. It is seen that the shearing loads reaches the value of the pore pressure at the edge of the contact and promotes destruction of the latter. At the same time, the stationary vibrations under similar boundary conditions weaken the traced phenomena even at the frequencies when the wavelength is comparable with the microstructure size. When a fluid is isolated by impermeable beds from the top and the bottom, and when the medium experiences the elementary stress state, i.e. the side and vertical stresses are related as \( \frac{V}{1 - V} \) (\( V \) is Poisson’s ratio), the pressure in the fluid is higher than in the matrix. In case that the wedging pressure ruptures the matrix and grains become isolated, the pressure in the bed jumps by 30–50% depending on Poisson’s ratio. Such situations govern the necessity of predicting anomalously
high formation pressures when toping oil reservoir. On the other hand, tensile loads should decrease P- and S-waves in a reservoir, which may a certain sign of pre-failure of the matrix and its transformation to a granular medium. The calculation results also show that the reduction in the contact area intensifies tension. This means that the process, if initiated, continues until total failure.

![Image](image_url)

**Figure 4.** Distribution of shearing load at the contacts. The loads change sign due to different signs of the normals at the opposite contacts (stresses are the same). At the edge of the contact, the loads reach the value of the pore pressure and contribute to failure of the contact periphery.

**Conclusion**

When fluid pressure affects a granular medium where contact sites make a rigid frame structure, two mechanisms of failure arise in the medium. One mechanism is tension at the contacts upon the increase in the pressure at the matrix–fluid interface. The tension (wedging pressure) is comparable with the pore pressure and depends on the geometry of the contacts. The second mechanism of failure is connected with the high shearing loads of the same order at the edges of the contact, which contribute to destruction of the contact periphery. The both mechanisms condition hazardous, anomalously high reservoir pressures in the course of striking oil.

Under stationary vibrations at a growing frequency, the both mechanisms of failure weaken up to very high frequencies conformable with the wavelengths comparable with the microstructure sizes.

The boundary element method results in a linear system of good causality, which allows avoiding regularizing procedure and provides reliable numerical results even at the contact edges where vector of the normal changes very rapidly.

The decrease in the formation velocities of P- and S-waves may be an indication of the pre-failure state in the granular matrixes.

**References**

[1] Serge A Shapiro & Axel Kaselow 2005 Stress and pore pressure depending anisotropy of elastic waves *Poromechanics—Biot Centennial (1905-2005)* Taylor & Francis Group London pp 167–172

[2] Yu G, Vozoff K and Durney DW 1991 Effect of pore pressure on compressional wave velocity in coals *Exploration Geophysics* 22(2) ASEG pp 475–480

[3] Sibiryakov EB and Sibiryakov BP 2010 Pore space structure and wedging pressure in granular medium *Fiz. Mezomekh.* Vol 13 No 1 pp 40–43

[4] George D Manolis and Petia S Dineva 2015 Elastic waves in continuous and discontinuous geological media by boundary integral equation methods: A review *Soil Dynamics and Earthquake Engineering* 70(2015) pp 11–29

[5] Sibiryakov EB 2016 Boundary element-based modeling of reflection from rough boundaries *Proc. 12 Int. Conf. InterExpo GEO-Sibir 2016* Novosibirsk: SGUGoT Vol 1 pp 262–267 (in Russian)