Degradation of Dynamic Parameters of Reinforced Concrete Buildings during an Earthquake

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Abstract. The article presents the results of a numerical study of the effect of local damage to load-bearing structures on the dynamic characteristics during an earthquake. The studies were carried out in the LS-DYNA software package, which implemented the nonlinear concrete model, Continuous Surface Cap Model (CSCM). The computational model of the building uses solid finite elements for concrete and bar elements for reinforcing bars. As the object of modelling, a 1-storey building with a frame structural layout was chosen. Studies have shown that during an earthquake with an intensity of 7 points, specified by the corresponding accelerogram, there is no change in natural frequencies, that is, the damage resulting from this earthquake is insignificant. During an earthquake with an intensity of 8 points, a significant (up to 55 %) reduction in natural frequencies occurs. This suggests that the structure is damaged, and the overall stiffness of the building is reduced. Analysis of the results shows that the most intense decrease in natural frequencies occurs in the active phase of an earthquake. After the end of the active phase, the frequencies stabilize and stop falling. The LS-DYNA software package enables to perform studies of the effect of local damage of the load-bearing structures on the dynamic characteristics during an earthquake with the direct reinforcement of concrete with reinforcing bars and can be used in the calculations and the design of buildings and structures.

1. Introduction

Modal analysis is performed to determine the frequencies and forms of natural oscillations of structures. Modal analysis enables to obtain integral characteristics of the stiffness of buildings and structures. For a more detailed study of the building response to the dynamic load it is necessary to make a direct dynamic analysis.

In the past 15–20 years, dynamic methods of structural survey of buildings and structures have been developing quite intensively in various research organizations of the Russian Federation. The essence of the methods lies in the fact that according to the dynamic characteristics of building structures, their state is assessed (the presence of defects in the load-bearing structures, degree of suitability for further operation of the building or structure, etc.). The methods use different approaches and differ in the types of measuring instruments used, software for processing the results of experiments, and methods for analysing the results of the survey.

There are various natural sources of excitation of vibrations of structures in order to determine the dynamic characteristics: microseism; man-made background fluctuations; dynamic component of the wind load on the building; impulse and vibration actions.
To assess the technical condition of buildings and structures according to their dynamic characteristics, the specialists of the Center for Research in Extreme Situations (CRES) have developed and repeatedly upgraded the Struna and Strela complexes [1]. They represent a set of hardware and software for experimental determination of the main dynamic characteristics of building structures and subsequent analysis of their strength and stability. Thus, in recent years, good development has been achieved; seismic engineering methods for determining the dynamic characteristics of building structures and their use for evaluating the reliable behaviour of such structures, until the end of their service life.

One of the objectives of the research is to study the effect of local damage on the dynamic characteristics during an earthquake, through numerical studies.

2. Problem statement

During the research, a LS-DYNA finite element software package was used, in which nonlinear, static, and dynamic methods were implemented.

Differential equations of motion of a system with a finite number of degrees of freedom in the matrix form are written in the following form:

\[ \mathbf{M} \ddot{\mathbf{u}} + \mathbf{C} \dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = \mathbf{f}, \]

where \( \mathbf{u} \) is the unknown vector of nodal displacements;
\( \dot{\mathbf{u}} = \mathbf{v} \) – nodal velocity vector;
\( \ddot{\mathbf{u}} = \mathbf{a} \) – nodal acceleration vector;
\( \mathbf{M} \) – mass matrix;
\( \mathbf{C} \) – damping matrix;
\( \mathbf{K} \) – stiffness matrix;
\( \mathbf{f} \) – vector of applied loads.

It is known that when solving the problem in a nonlinear formulation (considering physical, geometric nonlinearities) with the help of implicit integration schemes, elements of matrix \( \mathbf{K} \) depend not only on the properties of the material of structures, but also on the stress-strain state of the system, i.e. on the displacement vector \( \mathbf{u} \). The problem becomes much more complicated and reduces to solving nonlinear algebraic equations at each step of time integration.

With large nonlinearities, methods that implement explicit schemes for integrating the system motion equations become more effective. In the LS-DYNA software complex, the central difference method is used for explicit integration. To determine the displacement, an expression with time lag is used [2]:

\[ \mathbf{M} \ddot{\mathbf{u}}_i + \mathbf{C} \dot{\mathbf{u}}_i + \mathbf{K} \mathbf{u}_i = \mathbf{f}^e_i, \]

The peculiarity of explicit methods is that nodal accelerations \( \mathbf{a} \) and velocities \( \mathbf{v} \) are included in the calculation as unknowns (in the number of nodal degrees of freedom) and calculated directly, not by numerical differentiation of displacement functions.

Explicit methods use recurrence relations that express the displacements, velocities, and accelerations at a given step through their values in the previous steps.

Acceleration vector:

\[ \mathbf{a}_i = \mathbf{M}^{-1} \left( \mathbf{f}^e_i - \mathbf{f}^i \right), \]

where \( \mathbf{f}^e_i \) is a vector of external forces; \( \mathbf{f}^i \) is a vector of internal forces.

In the particular case:
\[ f^\text{int}_i = \sum \left( \int_{\Omega} B^T \sigma d\Omega + f^\text{cont}_i \right), \]  

(4)

- \(B\) – strain-displacement matrix;
- \(\sigma\) – vector of voltages;
- \(f^\text{cont}_i\) – vector of contact forces.

The velocity and displacement vectors in the corresponding step are determined as follows:

\[ \dot{v}_{t+\Delta t/2} = \dot{v}_{t-\Delta t/2} + \dot{a}_t \Delta t, \]  

(5)

\[ u_{t+\Delta t} = u_t + \dot{v}_{t+\Delta t/2} \frac{\Delta t + \Delta t_{t+\Delta t}}{2}. \]  

(6)

When using a diagonal mass matrix, it is possible to simplify the calculation and reduce the time of one iteration by calculating the inverse matrix. This shows that explicit methods are not related to solving systems of algebraic equations. The most time-consuming operation is the calculation of the vector of internal forces \(f^\text{int}_i\), which considers all types of nonlinearities.

To study the extent of damage to a building during an earthquake, we will perform a non-linear dynamic calculation using an explicit scheme with the selection of the forms and frequencies of natural oscillations (modal analysis) at certain fixed time points [3]. This considers changes in geometry, stresses and forces, contact conditions arising in the process of an earthquake, which will affect their own shape and vibration frequency. The process can be modelled using implicit or explicit methods. As stated above, we will use explicit methods for modelling.

The design diagram of the building uses direct reinforcement of load-bearing elements. The LS-DYNA implements a non-linear concrete model—Continuous Surface Cap Model (CSCM) that enables to consider the direct reinforcement of concrete with bars using solid (for concrete) and bar (for reinforcing bars) finite elements [4].

Let us consider a 1-story building frame structural layout (Figure 1). Overall plan dimensions are 6 x 6 x 3.3 (h) m. Floors are beam monolithic reinforced concrete. The slab thickness is 20 cm, the beam cross-section is 40x40 cm. The column cross section is 40x40 cm.

The reinforcing bars are made of steel; in the reinforcement modelling, an ideally elastic-plastic Prandtl model was used with the initial modulus of elasticity \(E= 2.1 \cdot 10^5\) MPa, the yield strength was assumed equal to \(\sigma_y = 245\) MPa, and the ultimate plastic strains \(\varepsilon_{pl} = 0.1\). A diameter of longitudinal reinforcing bars of beams and columns was assumed to be 28 mm, a diameter of longitudinal reinforcement plate was assumed to be 10 mm. Concrete corresponds to class B45 with respect to cube compressive strength, compressive strength \(R = 43\) MPa [5].

Figure 1. A design scheme with the FE mesh (a), the frame longitudinal reinforcement (b).
Calculation of the building is carried out with a rigid fixing at the base for the earthquake action, given in the form of a 2-component accelerogram, normalized to 7 and 8 (Figure 2) points on the MSK-64 scale [6].

![Accelerograms](image)

**Figure 2.** Accelerograms (components X and Y), normalized by 8 magnitude.

The destruction of elements in the loading process, as well as the interaction of elements in their contact are considered, i.e. the calculation is made considering the physical, geometric, and structural nonlinearities.

3. Calculation results

Figure 3–5 shows the results of calculation in the LS-DYNA software package.

![Natural oscillations](image)

**Figure 3.** a – 1st, b – 2nd, c – 3rd forms of natural oscillations (corresponding frequencies: 6.59 Hz, 6.59 Hz, 10.18 Hz).
Tables 1 and 2 present the frequencies of the three main modes of natural oscillations at given time points, which were determined during and after the earthquake.

**Table 1.** The frequency change in the process of a 7-magnitude earthquake

| Time $t$, sec | 0.2  | 2.5  | 5.0  | 7.5  | 10   | 15   | 20   | 25   | 29   |
|---------------|------|------|------|------|------|------|------|------|------|
| Frequency 1 (Hz) | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 |
| Frequency 2 (Hz) | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 | 6.59 |
| Frequency 3 (Hz) | 10.18| 10.18| 10.18| 10.18| 10.18| 10.18| 10.18| 10.18| 10.18|

Analysis of the results shows that the accelerogram intensity of 7 points does not change the frequency (Table 1). From this it follows that the damage resulting from this earthquake is insignificant.

**Table 2.** The frequency change in the process of a 8-magnitude earthquake

| Time $t$, sec | 0.2  | 2.5  | 5.0  | 7.5  | 10   | 15   | 20   | 25   | 29   |
|---------------|------|------|------|------|------|------|------|------|------|
| Frequency 1 (Hz) | 6.59 | 6.59 | 6.57 | 6.04 | 4.67 | 4.23 | 4.24 | 4.24 | 4.24 |
| Frequency 2 (Hz) | 6.59 | 6.59 | 6.58 | 6.27 | 5.58 | 4.75 | 4.76 | 4.76 | 4.76 |
| Frequency 3 (Hz) | 10.18| 10.18| 10.17| 9.95 | 8.38 | 7.16 | 7.18 | 7.18 | 7.18 |

During an earthquake with an intensity of 8 points, a significant (up to 55 %) reduction in natural frequencies occurs (Table 2). This suggests that a decrease in the overall stiffness of the building occurs as a result of damage. Analysis of the calculation results shows that concrete fracture occurs at the base of the columns and at the junction points of the columns with the beams (Figure 4). At the base of the columns, an increase in plastic deformations and a failure of longitudinal reinforcement are observed.

**Figure 4.** The intensity of plastic deformation (1 on the scale corresponds to the complete exhaustion of the bearing capacity of the element).

Figure 5 shows a graph of the change of the first 3 natural frequencies of a building during an 8-point earthquake.
Figure 5. A graph of change of natural frequencies in the process 8-magnitude earthquake. 

Analysis of the results shows that during an intense earthquake in the bearing elements of the building significant damage occurs that can be assessed by numerical or field studies. The proposed approach can be used in assessing the effects of earthquakes, including for the informed decision-making on the possibility of further operation of damaged buildings and structures.

4. Conclusions
1. The structure under consideration in case of earthquake with an intensity of 7 points does not suffer significant damage and retains its operational suitability.
2. During an earthquake with an intensity of 8 points, a significant (up to 55 %) reduction in the natural oscillation frequencies (degradation of dynamic parameters) is observed, which is caused by significant damage to the load-bearing elements.
3. The greatest amount of damage occurs at the stage of the active phase of earthquake action.
4. The approach proposed in the article to the assessment of the extent of the damage enables to assess the condition of load-bearing elements of buildings and structures by analyzing dynamic characteristics in the process of nonlinear dynamic analysis.
5. The LS-DYNA software package makes it possible to study the effect of local damage to load-bearing structures on the dynamic characteristics during an earthquake, considering the direct reinforcement of concrete with reinforcing bars, and can be used in research, calculations, and design of load-bearing reinforced concrete elements of buildings and structures in a non-linear dynamic formulation.

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