AdS/CFT applications to relativistic heavy-ion collisions: a brief review

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Abstract

We review some of the recent progress in our understanding of the physics of ultrarelativistic heavy-ion collisions due to applications of AdS/CFT correspondence.

(Some figures may appear in colour only in the online journal)

This article was invited by Robert E Tribble.

1. Introduction

Ultrarelativistic heavy-ion collision experiments are being carried out with the goal of creating a thermalized medium made out of deconfined quarks and gluons, known as the quark–gluon plasma (QGP) [1–3]. The aim is to study the properties of QGP, from those characterizing a static thermal bath, such as the equation of state and the order of phase transition, to the dynamical properties, such as the transport coefficients, including shear and bulk viscosities of QCD matter. With heavy-ion experiments running at the RHIC and the LHC, qualitative and quantitative theoretical progress is required both to better understand the existing data and to suggest new observables to measure.

Heavy-ion collision is a multi-scale process, with the harder momenta dominating the early times right after the collision, and the softer momentum scales becoming relevant at later time scales. Due to the running of the QCD coupling constant the hard modes are weakly coupled, while the soft modes are strongly coupled. In recent years a body of evidence has accumulated suggesting that the non-perturbative large-coupling effects could be more important for high-energy heavy-ion collisions than was previously thought. Perhaps the most important piece of evidence is the success of ideal hydrodynamics in describing the evolution of the medium produced in heavy-ion collisions [4, 5]. Corrections to ideal hydrodynamics are inversely proportional to the coupling constant and are small when the coupling is large. An example of this is the shear viscosity, which (at weak coupling) scales as $\eta \sim T^3/g^4 \ln(1/g)$ in a thermal QCD medium with the temperature $T$ [6]: extrapolating this result one sees that the shear viscosity is small when the coupling $g$ is large, generating a small correction to the energy–momentum tensor (EMT) of ideal hydrodynamics. Another argument in favor of strong-coupling dynamics playing an important role in heavy-ion collisions is the large amount of jet quenching needed to describe the data coming from the RHIC and the LHC. The relevant jet-quenching parameter $\hat{q}$, which is proportional to the energy loss per unit path length of an energetic parton traversing a medium [7], was found to be rather large in some analyses, possibly indicating that non-perturbative strongly coupled effects are at work [8]. Note however that $\hat{q}$ has mass dimension of $M^3$, and the existence of a large momentum scale $\hat{q}^{1/3}$ likely indicates that at least...
a part of the process is in fact weakly coupled, and, hence, perturbative. The last piece of evidence suggesting the possible importance of strong coupling effects in the QCD plasma comes from lattice simulations, which indicate that the energy density and pressure of the thermal QCD medium is about 80–85% of that in an ideal gas of quarks and gluons for a broad range of temperatures above the temperature of the QCD phase transition \( T_c \). As the strong-coupling constant \( \alpha_s(T) \) should become small at large-\( T \), one expects the high-temperature QCD medium to approach Stefan–Boltzmann ideal-gas behavior as \( T \) increases. The approach does take place, but is much slower than predicted by the perturbative QCD calculations, possibly indicating that non-perturbative effects are still important even at reasonably high-\( T \). Note however that the resummed perturbation series may successfully describe the lattice data [11].

While none of the arguments presented above definitively demonstrates dominance of strong-coupling effects in heavy-ion collisions at the RHIC and the LHC, we will proceed under the assumption that the strong-coupling effects are important and try to analyze heavy-ion collisions assuming that the coupling is large. One reason for this is that exploring heavy-ion collisions in the strong-coupling regime is very interesting in itself, providing a new theoretical angle on the process. In addition, it may be that the typical strong-coupling constant in these collisions is neither very small nor very large, such that both small- and large-coupling approaches would have some chance of describing the data.

The strong-coupling QCD analysis of such an ultrarelativistic process as a heavy-ion collision is impossible with the present state of the QCD theory. Instead an opportunity to study the strongly coupled field theories came up due to advances in string theory, culminating in the formulation of the Anti-de Sitter space/conformal field theory (AdS/CFT) correspondence [12–16], which is the duality between the \( \mathcal{N}=4 \) SU(\( N_c \)) super Yang–Mills (SYM) theory in four flat space–time dimensions (CFT) and the type-IIB superstring theory on AdS\(_5\)\( \times \)S\(_5\). In the limit of a large number of colors \( N_c \) and large \( 't \) Hooft coupling \( \lambda = g^2 N_c \) such that \( N_c \gg \lambda \gg 1 \), AdS/CFT correspondence reduces to the gauge-gravity duality: \( \mathcal{N}=4 \) SU(\( N_c \)) SYM theory at \( N_c \gg \lambda \gg 1 \) is dual to the (weakly coupled) classical supergravity in AdS\(_5\)\( \times \)S\(_5\). The AdS/CFT correspondence gives us a powerful tool we can use to systematically study a strongly coupled gauge theory (specifically, \( \mathcal{N}=4 \) SYM theory). Using the AdS/CFT correspondence one can try to study heavy-ion collisions at strong coupling. Indeed one should be mindful about the many differences between QCD and the \( \mathcal{N}=4 \) SYM theory: the former is confining, while the latter is not; QCD coupling runs, while it is a constant in the conformal \( \mathcal{N}=4 \) SYM theory; QCD plasma has a chiral phase transition, while \( \mathcal{N}=4 \) SYM plasma does not. Until the errors introduced by using a theory other than QCD are understood, any conclusions we derive from applying the AdS/CFT correspondence to the study of QCD-mediated processes should be taken as qualitative at best. However, in some cases one may hope that AdS/CFT predictions are universally valid for all strongly coupled theories, including QCD. This hypothesis is supported by some numerical successes of AdS/CFT, an example of which will be shown below.

2. Elements of AdS/CFT correspondence and the strongly coupled supersymmetric plasma

For reviews of the AdS/CFT correspondence we refer the reader to [16–19]. Since our goals in this mini-review are rather applied, we will simply state the tools we need to accomplish them, without presenting any proofs, and illustrate these statements by analyzing some of the properties of the supersymmetric plasma.

AdS\(_5\)\( \times \)S\(_5\) space is obtained by stacking \( N_c \) parallel D3 branes on top of each other. The resulting metric for (the Poincare wedge of) AdS\(_5\)\( \times \)S\(_5\) in the \( z \gg L \) limit is

\[
\text{ds}^2 = \frac{L^2}{z^2} [-dt^2 + d\vec{x}^2 + dz^2] + L^2 \text{d} \Omega_4^2,
\]

where \((t, \vec{x}) = (t, x^1, x^2, x^3)\) are the regular four-dimensional coordinates, \( z \in [0, +\infty) \) is the fifth dimension of AdS\(_5\), and the branes are located at \( z = \infty \). \( L \) is the radius of the S\(_5\) sphere. For our purposes it is important to note that the AdS\(_5\) metric in equation (1) (everything on the right except for the last term) satisfies Einstein equations

\[
R_{MN} - \frac{1}{2} g_{MN} R + \Lambda g_{MN} = 0
\]

with \( M, N = 0, \ldots, 4 \) and the cosmological constant \( \Lambda = -6/L^2 \).

The AdS/CFT correspondence is the statement of equivalence of the \( \mathcal{N}=4 \) \( SU(N_c) \) SYM theory in four space–time dimensions and the type-IIB superstring theory on AdS\(_5\)\( \times \)S\(_5\) [12]. For practical calculations the duality means the following. Each operator \( \mathcal{O} \) in the four-dimensional CFT is paired up with the dual field \( \varphi \) in the AdS\(_5\) bulk. Then the duality can be mathematically formulated as [13–15]:

\[
\langle \exp \left[ \int d^4 x \mathcal{O}(x) \varphi_0(x) \right] \rangle_{\text{CFT}} = Z_{\text{string}}[\varphi(x, z)]\bigg|_{z=0} = z^{4-\Delta} \varphi_0(x).
\]

On the left-hand side of equation (3) we have an expectation value taken in the full four-dimensional CFT, while on the right there is a partition function for the type-IIB superstring theory, calculated with a particular boundary condition on the bulk field \( \varphi(x, z) \). \( \Delta \) is the scaling dimension of the operator \( \mathcal{O} \). One can show that in the \( N_c \gg \lambda \gg 1 \) limit the string partition function becomes classical, \( Z_{\text{string}} \approx \exp^{L \Delta S_{\text{bulk}}} \) with \( S_{\text{bulk}} \) the classical action on AdS\(_5\)\( \times \)S\(_5\) calculated with the \( \varphi(x, z)|_{z=0} = z^{4-\Delta} \varphi_0(x) \) boundary condition. Thus the incredibly complicated quantum field theoretical problem of calculating an expectation value in the full CFT is reduced to a classical calculation on AdS\(_5\)\( \times \)S\(_5\). This is a great simplification due to the AdS/CFT correspondence.

As an important example of the use of AdS/CFT duality which is also useful for heavy-ion collisions, let us consider a thermal medium made out of strongly-coupled \( \mathcal{N}=4 \) SYM matter. Due to thermodynamic properties of black holes, the dual of the thermal medium in the gauge theory is the AdS
Schwarzschild black hole (AdSSBH) with the metric [12]

$$ds^2 = \frac{L^2}{z^2} \left[ 1 - \frac{z^4}{z_h^4} \right] dt^2 + dz^2 + \frac{dz^2}{1 - \frac{z^4}{z_h^4}}. \tag{4}$$

The horizon of the AdSSBH metric is located at $z = z_h$. Introducing the Euclidean time $\tau = i t$ the metric (4) can be extended to the Euclidean space–time. Expand the Euclidean version of the metric (4) near the horizon by redefining $z = z_h (1 - \rho^2/L^2)$: at the lowest non-trivial order in $\rho$ we get

$$ds^2 \approx 4\frac{\rho^2}{z_h} d\tau^2 + \frac{L^2}{z_h} dz^2 + d\rho^2. \tag{5}$$

We see that the $(\tau, \rho)$-part of the metric looks like the polar coordinates in 2d, with the metric $ds_{2d}^2 = \rho^2 d\phi^2 + d\rho^2$, if we define the angle $\phi = 2\tau / z_h$. The metric $ds_{2d}^2$ has no kink singularity at $\rho = 0$ only if it is periodic in $\phi$ with the period of $2\pi$. This means that the Euclidean time $\tau$ should be periodic with the period of $\Delta \tau = \pi z_h$. Remembering that a thermal field theory in Euclidean space is periodic with the period $\Delta \tau = \beta = 1/T$ with $T$ the temperature, (i) we find the Hawking temperature $T_H = 1/(\pi z_h)$ of the AdS Schwarzschild black hole (4) and (ii) identifying the Hawking temperature of AdSSBH with the temperature of the dual gauge theory we obtain

$$z_h = \frac{1}{\pi T}. \tag{6}$$

The metric of (4) is illustrated in figure 1. We see from equation (6) that at high temperature $T$ the horizon is close to the boundary of the AdS space at $z = 0$, while for low $T$ the horizon is deep in the AdS$_5$ bulk.

Identifying the AdS Schwarzschild black hole as the gravity dual of the strongly coupled SYM plasma allows us to calculate the entropy of the SYM plasma by equating it with the Bekenstein–Hawking entropy of a black hole

$$S_{BH} = \frac{A}{4 G_N}, \tag{7}$$

where $A$ is the horizon area and $G_N$ is Newton’s constant. Since the area of a 5-sphere is $A_S = \pi^3 L^5$, the horizon area for AdSSBH can be calculated via

$$A = \int dx^1 dx^2 dx^3 \sqrt{\det g_{3D}|_{z = z_h}} A_S^3, \tag{8}$$

where $\det g_{3D} = L^6 / z_h^6$ is the determinant of the part of the metric (4) in three dimensions $(x^1, x^2, x^3)$. Using equation (6) we get $A = L^6 V_{3D} \pi^6 T^3$ with $V_{3D}$ the 3D volume occupied by the plasma. Using this area in equation (7) along with

$$G_N = \frac{\pi^4 L^8}{2 N_c} \tag{9}$$

we obtain the Bekenstein–Hawking entropy of AdSSBH, or, by AdS/CFT duality, the entropy of the strongly coupled $\mathcal{N} = 4$ SYM plasma [20]

$$S = \frac{\pi^2}{3} N_c^2 T^3 V_{3D}. \tag{10}$$

We have derived an expression for the entropy of a strongly coupled medium in a thermal field theory by doing a very simple gravity calculation! Indeed the strong-coupling expression (10) corresponds to the sum of an infinite number of Feynman diagrams: AdS/CFT duality allowed us to arrive at the answer in a much simpler way.

Equation (10) should be compared with the Stefan–Boltzmann entropy of an ideal gas of non-interacting particles in $\mathcal{N} = 4$ SYM theory

$$S_{SB} = \frac{2\pi^2}{3} N_c^2 T^3 V_{3D}. \tag{11}$$

We see that $S/S_{SB} = 3/4$: the entropy of a strongly coupled supersymmetric plasma is $3/4$ of that for the ideal gas. As was mentioned before, this result seems to be in semi-quantitative agreement with the lattice QCD calculations for entropy above $T_c$ giving $S/S_{SB} = 80–85\%$ [9, 10], as illustrated in figure 2 for the entropy density $s = S / V_{3D}$. Note, again, that in interpreting this semi-agreement one has to remember that $\mathcal{N} = 4$ SYM and QCD are different theories, and the $\mathcal{N} = 4$ SYM prediction (10) for $S/T^3$ would correspond to a horizontal straight line in the plot of figure 2 exhibiting no temperature dependence.

Now suppose we want to find the expectation value of the EMT ($T_{\mu\nu}$) in the $\mathcal{N} = 4$ SYM plasma ($\mu, \nu = 0, \ldots, 3$ label the 4D space–time). Since gravity couples to the EMT of the matter fields in 4D, by AdS/CFT rules (3) the EMT operator is dual to the metric tensor $g_{MN}$ in the bulk [21]. The AdS/CFT duality (3) leads to the following prescription for the calculation of $(T_{\mu\nu})$, known as the holographic renormalization [21]. First we cast the metric in the Fefferman–Graham form [22]

$$ds^2 = \frac{L^2}{z^2} \left[ g_{\mu\nu} dx^\mu dx^\nu + dz^2 \right]. \tag{12}$$

Then we expand the 4D metric $g_{\mu\nu}(x, z)$ near the boundary of AdS$_5$ at $z = 0$. One can show that if the expansion starts with the Minkowski metric $\eta_{\mu\nu}$ at $z = 0$ (as is the case for AdS$_5$) then the next term in the expansion is order-$z^4$, such that [21]

$$g_{\mu\nu}(x, z) = \eta_{\mu\nu} + z^4 g^{(4)}_{\mu\nu}(x) + \ldots. \tag{13}$$
With the expansion (13) in hand one can find the expectation value of the EMT by the following simple relation:

$$\langle T_{\mu\nu} \rangle = \frac{N_c^2}{2\pi^2} g^{(4)}_{\mu\nu}(x).$$  \hspace{1cm} (14)

Applying this prescription to the AdSSBH metric (4) we first perform a substitution

$$z = \tilde{z} \frac{z^4}{z_h^4}$$  \hspace{1cm} (15)

with $z_h = z_h \sqrt{2}$ recasting equation (4) into Fefferman–Graham form [23]

$$ds^2 = \frac{L^2}{\tilde{z}^2} \left[ -\left(1 - \frac{\tilde{z}^4}{z_h^4}\right) dz^2 + \left(1 + \frac{\tilde{z}^4}{z_h^4}\right) d\tilde{z}^2 + d\tilde{z}^2 \right].$$  \hspace{1cm} (16)

Reading off the $z^4$ coefficients of $g_{\mu\nu}(x, z)$ from equation (16) we obtain the EMT of a static supersymmetric plasma

$$\langle T_{\mu\nu} \rangle = \frac{\pi^2}{8} N_c^2 T^4 \text{ diag} \{3, 1, 1, 1\}. \hspace{1cm} (17)$$

As we will see below, holographic renormalization is a powerful tool that can be used to study heavy-ion collisions at strong coupling.

### 3. Heavy-ion collisions in AdS$_5$

As mentioned above, heavy-ion collision is a multi-scale process involving an interplay of small- and large-coupling phenomena. Unfortunately no single theoretical framework incorporates both perturbative and non-perturbative effects. Therefore, applying AdS/CFT duality to heavy-ion collisions one has to make a choice between either treating the whole collision as a strongly coupled process, or, perhaps more realistically, limiting the application of AdS/CFT duality to the description of the produced medium at the later times after the collision when the QCD coupling is more likely to be large. In this section we will discuss the progress in the former direction, while the advances in the latter are presented in the next section.

#### 3.1. Shock wave collisions

We begin by modeling the whole heavy-ion collision in the strongly coupled framework of the AdS/CFT correspondence. The goal here is rather ambitious, and includes understanding particle production in the collision, followed by the thermalization of the produced matter, along with the subsequent evolution of the resulting thermal medium: essentially we want to understand the whole collision in a single unified framework. While there are no nuclei (or other bound states) that we could imagine colliding in the conformal $N = 4$ SYM theory, one can follow Bjorken [24] and model the colliding nuclei as the Lorentz-contracted thin sheets of matter, which are uniform and extend to infinity in the transverse direction. The collision of two such nuclei is illustrated in figure 3 using light-cone coordinates $x^\pm = (x^0 \pm x^3)/\sqrt{2}$.

The nucleus moving along the $x^+$-axis (nucleus 1’ in figure 3) has a large $T^{++}$ component of its EMT, with all other EMT components negligibly small. Moreover, due to assumptions we made about the uniform distribution in the transverse plane, $T^{++}$ is a function of $x^-$ only with its support localized near $x^- = 0$ due to Lorentz contraction. The metric dual to such an ultrarelativistic nucleus which solved Einstein equations (2) is that of a gravity shock wave in the
bulk [23]
\[ ds^2 = \frac{L^2}{z^2} \left\{ -2 dx^+ dx^- + \frac{2 \pi^2}{N_c} (T^{++}(x^-)) z^4 dx^-^2 + dx_+^2 + dx_2^2 \right\} \tag{18} \]
with \( dx_+^2 = (dx^1)^2 + (dx^2)^2 \). The metric dual to ‘nucleus 2’ in figure 3 is obtained from equation (18) by the + ↔ − interchange.

The problem of understanding heavy-ion collisions in this version of the AdS/CFT approach can be formulated as follows: we know the metric for the two nuclei with atomic numbers \( A_1 \) and \( A_2 \), corresponding to dimensionless Lorentz-invariant expansion parameters \( \mu_1 (x^-)^2 x^+ \) and \( \mu_2 (x^-)^2 x^- \). The idea of solving Einstein equations by an expansion in powers of \( \mu_1 \) and \( \mu_2 \) was formulated in [25] for the delta-function profiles (19) and extended to other realistic longitudinal profiles in [26] and to non-trivial transverse profiles in [29]. The all-order resummation of the powers of \( \mu_2 \) keeping \( \mu_1 \) at the lowest non-trivial order was carried out in [27] resulting in the following expectation value of the EMT in the forward light-cone (only non-zero components are listed):
\[ \langle T^{++} \rangle = \frac{N_c^2}{2 \pi^2} \frac{4 \mu_1 \mu_2 (x^+)^2 \theta(x^+) \theta(x^-)}{1 + 8 \mu_2 (x^+)^2 x^-} \tag{21a} \]
\[ \langle T^{--} \rangle = \frac{N_c^2}{2 \pi^2} \frac{\mu_1}{\mu_2} \frac{\theta(x^+) \theta(x^-)}{4 \mu_2 (x^+)^4 \left[ 1 + 8 \mu_2 (x^+)^2 x^- \right]^{3/2}} \times \left[ 3 - 3 \sqrt{1 + 8 \mu_2 (x^+)^2 x^-} + 4 \mu_2 (x^+)^2 x^- \right] \times \left( 9 + 16 \mu_2 (x^+)^2 x^- - 6 \sqrt{1 + 8 \mu_2 (x^+)^2 x^-} \right), \tag{21b} \]
\[ \langle T^{+-} \rangle = \frac{N_c^2}{2 \pi^2} \frac{\mu_1}{a_1} \frac{\theta(x^+) \theta(x^-)}{\sqrt{1 + 8 \mu_2 (x^+)^2 x^-}}, \tag{21c} \]

which, by construction, is applicable in the region defined by \( \mu_1 (x^-)^2 x^+ \ll 1 \). Due to this limitation, the EMT in equations (21a), (21b) and (21c) is not valid at asymptotically late proper times \( \tau = \sqrt{2} x^+ x^- \) when the system is likely to thermalize and should be described by ideal hydrodynamics; it does not yield us a new solution of ideal hydrodynamics equations. (For instance, \( \langle T^{++} \rangle < 0 \) in equation (21a), which is impossible in ideal hydrodynamics.) Rather equations (21a), (21b) and (21c) describe a non-equilibrium medium on its way to equilibration.

The all-order resummation of \( \mu_2 (x^+)^2 x^- \) in [27] allowed for one more insight into shock-wave collisions: smearing the profile of the first shock wave to generate a finite width \( a_1 \)
\[ t_1(x^-) = \mu_1 \frac{x^-}{a_1} \theta(x^-) \theta(x^-) = \frac{x^-}{a_1} \theta(x^-) \theta(x^-) \tag{22} \]
once following the evolution of the shock wave after the collision, which yields [27]
\[ \langle T^{++} \rangle = \frac{N_c^2}{2 \pi^2} \frac{\mu_1}{a_1} \frac{1}{\sqrt{1 + 8 \mu_2 (x^+)^2 x^-}} \], for \( 0 < x^- < a_1, x^+ > 0 \). \tag{23} \]

We see that \( \langle T^{++} \rangle \to 0 \) with increasing \( x^+ \). This implies that on the time scales of the order of \( x^+_{\text{stop}} \sim \frac{1}{\sqrt{\mu_2}} \) the shock wave would lose much of its light-cone momentum and would stop, i.e. it would either deviate from its light-cone trajectory or would simply dissolve into the produced medium. This stopping effect may be due to the following mechanism: the partons in the colliding nuclei branch unchecked due to the large value of the coupling, which results in the shock waves consisting of a huge number of very-low-momentum partons. When two systems of low-momentum partons collide, the partons easily slow each other down, resulting in the stopping’ of equation (23). This stopping conclusion, unfortunately, is a
strong argument against using AdS/CFT to describe the whole heavy-ion collision, since no such ‘stopping’ behavior appears to have been observed experimentally at the RHIC or the LHC.

A numerical solution of the shock-wave collision problem was carried out in [30] for the shock waves with Gaussian profiles (see also [31]). The energy density obtained in [30] is plotted in figure 4. The results of [30] appear (at least qualitatively) to confirm the analytical conclusion above regarding the fast depletion of the EMT along the shock wave’s light cone (23). In addition, the numerical simulation suggests a rather fast onset of viscous hydrodynamics, indicating a fast equilibration of the produced medium.

3.2. Trapped surface analysis and thermalization

Interestingly enough, one does not have to solve Einstein equations explicitly to figure out the fate of the colliding shock-wave system. It is possible to determine whether the system reaches thermal equilibrium, that is, whether a black hole is created in the bulk, by performing a trapped surface analysis [32, 33]. According to the Hawking–Penrose theorem, existence of a trapped surface implies that a gravitational collapse (and hence, a black hole creation) is inevitable. A trapped surface analysis for colliding shock waves in AdS5 was carried out in [34–37], with the original work on the subject [34] dealing with shock waves with non-trivial transverse coordinate profiles, obtained by putting ultrarelativistic point sources in the bulk. We now know that trapped surfaces are formed in a variety of shock waves with and without sources in the bulk [34–37], indicating that thermalization does take place in heavy-ion collisions at strong coupling.

The area of the trapped surface gives one the lower bound on the entropy of the black hole, and hence, on the entropy of the produced matter. For the sourceless delta-function shock waves of equations (18), (19) the produced entropy per unit transverse area \( A_\perp \) was found to be [37]

\[
\frac{S}{A_\perp} = \frac{N_c^2}{2\pi^2} (2\mu_1\mu_2)^{1/3}. \tag{24}
\]

The thermalization (black-hole formation) proper time is given by the only dimensionful boost-invariant parameter in the problem (\( \mu_1\mu_2 \)) [25, 37] as

\[
\tau_{th} \sim \frac{1}{(\mu_1\mu_2)^{1/6}}, \tag{25}
\]

which, as can be seen from equation (20), is very short parametrically, being suppressed by a power of the center-of-mass energy \( s \) as \( \mu_1\mu_2 \sim p_1^+ p_2^- \sim s \). This conclusion was supported by the numerical simulations of [30] performed after [25, 37]. Unfortunately, substituting realistic values for \( \mu_1,\mu_2 \) into equation (25) leads to the thermalization time which is too short to agree with the hydrodynamic analyses of the RHIC and LHC data [25, 37].

It is worth noting here that the numerical simulations of [30] give 0.35 fm/c as the time interval between when the Gaussian shocks used in the simulations start to overlap significantly with the onset of viscous hydrodynamics in RHIC kinematics. This number is somewhat closer to the time scale on the order of 1–2 fm/c used in hydrodynamic simulations [38–41] than the time scale of about \( \tau_{th} \approx 0.1 \) fm/c resulting from equation (25) evaluated for RHIC kinematics. The difference between the two numbers may be due in part to the finite longitudinal width of colliding shock waves used in the numerical simulations [30] versus the zero effective width of the delta-function shock waves (19). However, note that to properly compare thermalization time with the hydrodynamic simulations [38–41] one has to start measuring the proper time from \( \tau = 0 \), i.e. from the moment of the complete overlap of the two shocks: such a procedure, when applied to the results of [30], yields the thermalization time of about \( \tau_{th} \approx 0.17 \) fm/c, much closer to equation (25) and further away from the phenomenological times.

Finally, noting again that \( \mu_1\mu_2 \sim s \) with \( s \) the center-of-mass energy of the collision, we get [34]

\[
\frac{S}{A_\perp} \propto s^{1/3}. \tag{26}
\]

Identifying the produced entropy with the number of degrees of freedom, and hence with the number of hadrons produced in the collision, we conclude that the AdS/CFT prediction is that the hadron multiplicity should grow as \( dN/d\eta \sim s^{1/3} \), with \( \eta \) the pseudo-rapidity. Unfortunately the experimental data gives a much smaller power of energy, \( dN/d\eta \sim s^{0.15} \) [42], possibly signaling again that the early stages of heavy-ion collisions (in which the majority of the entropy is produced) are not strongly coupled. Note, however, that modifications of AdS5 geometry allow for better agreement with the data [43].

4. Evolution of the produced medium:

hydrodynamics and the shear viscosity bound

Let us now change the strategy and assume that the actual collision is either weakly coupled or is described by the physics outside the gauge-gravity duality for some other reasons. Strong-coupling physics in this scenario becomes dominant at some later (though not necessarily very late) proper time \( \tau = \sqrt{2}x^+x^- \) in the collision. We are thus confined to the forward light cone of figure 3 without specific information about the system’s past (that is, without the shock waves). The question now is whether we can say anything specific about the strong-coupling dynamics of such a medium.

Assuming again that the system is uniform in the transverse direction, we see that the dynamics in the forward light cone in 4D depends only on two variables: the proper time and the space–time rapidity \( \eta_a = (1/2) \ln(x^+/x^-) \). Furthermore, let us assume for simplicity that the matter distribution is \( \eta_a \)-independent [24]; the assumption is justified by the weak rapidity dependence of the particle multiplicity near mid-rapidity in actual heavy-ion experiments. The most general metric in AdS5 describing \( \eta_a \)-independent medium in the forward light cone in the Feffermann–Graham...
coordinates is \[23\]
\[
\frac{dz}{c} = \frac{L^2}{2} \left[ 1 - \frac{e_0 (z^4/3)^2}{1 + \frac{e_0}{3} (z^4/3)} \right] \, d\tau^2 + \left( 1 + \frac{e_0}{3} (z^4/3) \right) (\tau^2 \, d\eta^2 + d\chi^2) + d\chi^2, \tag{29}
\]
which is the gravity dual of the celebrated Bjorken hydrodynamics \[24\]. Note that now \(\epsilon(\tau) = 3 \, p(\tau) = 3 \, p_3(\tau) = (N_c^2/2\pi^2) e_0/\tau^{4/3}\), in agreement with \[24\] (\(e_0\) is an arbitrary dimensionful constant). Since \(\epsilon \sim T^4\) in this conformal plasma, we conclude that \(T \sim \tau^{-1/3}\), again in agreement with \[24\]; the temperature falls off with time as the system cools. The metric (29) has a horizon at
\[
\tau = \tau_{\text{ch}} = \left( \frac{3}{e_0} \right)^{1/4} \, \tau^{1/3}. \tag{30}
\]
Comparing with figure 1 we see that the horizon of the black hole (29) falls deeper into the bulk as the time goes on.

The important conclusion we draw from the result of \[23\] is that a strongly-coupled \(\eta_{s_4}\) and \(x_{1s}\)-independent, \(N = 4\) SYM medium would invariably end up in a Bjorken hydrodynamics state. Hence one can derive the late-time asymptotics of this medium without an explicit knowledge of the medium’s origin.

It is also interesting to study the approach to the Bjorken hydrodynamics/Janik–Peschanski metric (29). This was done in \[44,45\] by expanding the coefficients of the metric (27) around the scaling solution (29) for late times,
\[
a(\tau, z) = a(v) + a_1(v) \frac{1}{\tau^{7/3}} + a_2(v) \frac{1}{\tau^{4/3}} + \cdots \tag{31}
\]
with similar expansions for \(b(\tau, z)\) and \(c(\tau, z)\). The coefficients \(a_1, a_2, b_1, \ldots\) were found by solving Einstein equations perturbatively in \(1/\tau^{2/3}\) with the matching near the AdS boundary onto viscous hydrodynamics for the EMT of the gauge theory. The latter yields the following EMT \[46\]
\[
\langle T^{\mu \nu} \rangle = \begin{pmatrix}
\epsilon(\tau) & 0 & 0 & 0 \\
0 & p(\tau) + \frac{2}{\tau} & \frac{2}{\tau} & 0 \\
0 & \frac{2}{\tau} & p(\tau) + \frac{2}{\tau} & 0 \\
0 & 0 & 0 & p(\tau) - \frac{4}{3} \frac{\eta}{\tau}
\end{pmatrix} \tag{32}
\]
with \(\eta\) now denoting shear viscosity. In a conformal medium where \(\epsilon \sim T^4\), the shear viscosity scales as \(\eta \sim T^3 \sim 1/\tau\) in the Bjorken expansion. We hence write \(\eta = (N_c^2/2\pi^2) e_0^{3/4} \eta_0/\tau\), with \(\eta_0\) a dimensionless constant. Requiring that the resulting metric has no singularities in the bulk yields \[45\]
\[
\eta_0^2 = \frac{\sqrt{3}}{18}, \tag{33}
\]
such that the shear viscosity is \[45,47\]
\[
\eta = \frac{N_c^2}{2 \sqrt{2}/3 \pi^2 \pi^2} e_0^{3/4} \eta_0/\tau. \tag{34}
\]
Comparing the EMT of a SYM plasma (17) with \(\epsilon = (N_c^2/2\pi^2) e_0^{4/3}\) we read off the temperature
\[
T(\tau) = \frac{2^{1/2}}{3 \sqrt{3}} \frac{e_0^{1/4}}{\pi^{1/3} \tau.} \tag{35}
\]
which, when used in equation (34), gives
\[
\eta = \frac{\pi}{8} N_c^2 T^3. \tag{36}
\]
This important relation between the shear viscosity and temperature of the strongly coupled \(N = 4\) SYM plasma was derived originally by Kovtun, Policastro, Son, and Starinets \[48–52\] using the Kubo formula to relate \(\eta\) to the absorption cross section of a graviton by the AdSSBH.

Note that combining equations (36) and (10) one gets \[48–52\]
\[
\frac{\eta}{s} = \frac{1}{4 \pi}, \tag{37}
\]
where \(s = S/V_{3D}\) is the entropy density. Based on the consistency of this result for a variety of dual geometries it has been conjectured in \[51,52\] that the ratio (37) is a universal lower bound on \(\eta/s\) for any theory (including QCD):
\[
\frac{\eta}{s} \geq \frac{1}{4 \pi}. \tag{38}
\]
This is known as the Kovtun–Son–Starinets (KSS) bound. It was proven for theories with gravity duals in \[52\].

The possibility of a violation of this bound was discussed in \[53–55\]. The question of the value of \(\eta/s\) in QCD has been extensively studied in lattice QCD \[56\] and by viscous hydrodynamics simulations for heavy ion collisions \[38–41\]. The current estimates place \(\eta/s\) in QCD (extracted from heavy-ion collisions data) very close to the KSS bound \[41\], though
one has to remember that QCD is not a conformal theory, and \( \eta/s \) is \( T \)-dependent in QCD unlike \( N = 4 \) SYM [56].

Returning to the time evolution of the strongly coupled rapidity-independent medium, note that further higher-order \( 1/\tau^{2/3} \) corrections have been calculated in [47, 57]. Reversing the problem, one can investigate the possible early time asymptotics of this medium. It was shown in [58] that for \( \epsilon \sim \tau^\Delta \) only even integer \( \Delta \) are allowed as \( \tau \to 0 \). The non-negativity of the energy density demands that \( \Delta = 0 \) [58, 59]. In such a scenario the energy density \( \rho \) starts out as a constant in time and eventually begins to fall off like \( \epsilon \sim \tau^{-2/3} \) as dictated by Bjorken hydrodynamics. (Note however that if one expands equations (21a), (21b) and (21c) to the lowest order in \( \tau \) (as \( x^\pm = \tau e^{\pm \eta_0} / \sqrt{2} \)) one would obtain \( \epsilon \sim \tau^2 \) [25], such that \( \Delta = 2 \), though the medium produced in a shock-wave collision is not boost invariant.) For numerical simulations of rapidity-independent (boost-invariant) plasmas see [60, 61].

The weakness of the approach presented in this section is that at the early times: one has to specify the metric to be used as the initial condition for Einstein equations at some small value of \( \tau = \tau_0 \). The EMT of the particles formed in the early stages of a heavy-ion collision would only specify \( g^{(4)}_{\mu\nu} \) at \( \tau_0 \) in the expansion of the metric near \( z = 0 \). Higher-order metric coefficients \( g^{(i)}_{\mu\nu}, i \geq 6 \), have to be specified too at \( \tau_0 \): in the AdS/CFT dictionary they are likely to be dual to some higher-dimensional operators in \( N = 4 \) SYM theory. It appears that to insure the matching between the (presumably) weakly coupled early time dynamics and the later-time dynamics described by AdS/CFT one has to specify an infinite tower of expectation values of operators at \( \tau_0 \) resulting from the former to be used to initiate the latter. To date the problem of how to do this in a both rigorous and realistic way remains open.

5. Jet quenching in a strongly coupled medium

Many experimental signals of Quark–Gluon plasma (QGP) formation and evolution can be calculated in the framework of AdS/CFT duality. Above we have discussed the implications of the AdS/CFT correspondence for the hydrodynamic description of heavy-ion collisions. In the spirit of a brief review, we will only mention one more signal of QGP: jet quenching. The idea that jets can lose energy in QGP, which was first tackled in [8], with the goal of determining a strong-coupling value of the jet-quenching parameter \( \hat{\tau} \) originally defined in [7] in the perturbative QCD framework. The purely AdS approach to jet quenching was first advocated in [66–71] and involved calculating the drag force on a heavy quark moving through the SYM plasma. At zero temperature a heavy quark is dual in AdS to the endpoint of an open Nambu–Goto string terminating on a D7 (flavor) brane wrapped around \( S^3 \), with the other end of the string stretching to the stack of D3 branes at \( z = \infty \) [72]. For a finite temperature medium, if we work in the coordinate system employed in writing down the AdS/CFT one has to specify an infinite tower of expectation values of operators at \( \tau_0 \) resulting from the former to be used to initiate the latter. To date the problem of how to do this in a both rigorous and realistic way remains open.

Let us find the drag force on a heavy quark moving with a constant speed \( v \) through a supersymmetric plasma (assuming that some external force is applied to keep the quark velocity constant). To do this, one first has to find the string configuration in the AdSSBH background (4) dual to the moving quark. This is done by extremizing the Nambu–Goto action of an open string

\[
S_{NG} = -\frac{\sqrt{\lambda}}{2\pi L^2} \int dt \, d\sigma \sqrt{-\det g_{ab}},
\]

\[
g_{ab} = g_{MN} \frac{\partial_a X^M}{\partial \sigma} \frac{\partial_b X^N}{\partial \sigma},
\]

with the boundary condition requiring that the string endpoint attached to the D7 brane moves with velocity \( v \) along the \( x \)-axis. Here \( g_{MN} \) is the AdSSBH metric (4) and \( X^M = X^M(\tau, \sigma) \) are string coordinates: they specify the mapping from the string world-sheet coordinates \( \sigma^a = (\tau, \sigma) \) with \( a = 0, 1 \) to space–time coordinates \( x^M \). (Note that \( \partial_a = \partial / \partial \sigma^a \).)

The resulting classical string configuration is

\[
X^M(\tau = t, \sigma = z) = (t, 0, 0, x(t, z), z)
\]

with [66–68]

\[
x(t, z) = v t - \frac{z_h}{2} \left[ \arctan \frac{z_h}{z} + \frac{1}{2} \ln \left( \frac{z_h + z}{z_h - z} \right) - \frac{\pi}{2} \right].
\]

This is illustrated in figure 5. The drag force (momentum change) is given by the momentum flow down the string from its endpoint attached to the D7 brane,

\[
\frac{dp}{dt} = -\pi^1_s
\]

where \( p \) is the 3-momentum of the quark and \( \pi^1_s \) is the canonical momentum of the string defined by

\[
\pi^a_s = -\frac{\sqrt{\lambda}}{2\pi L^2} \frac{\partial \sqrt{-\det g_{bc}}}{\partial (\partial_a X^b)}. 
\]

![Figure 5](image-url)
Using equations (41), (40), (43) in equation (42) gives us the drag force on a heavy quark in the strongly coupled $\mathcal{N} = 4$ SYM plasma \[ \frac{dp}{dt} = -\frac{\pi \sqrt{\mathcal{N}}}{2} T^{2} \frac{p}{M} \] (44)

where $M$ is the mass of the heavy quark. (The mass appears since $p/M = v/\sqrt{T - v^{2}}$.) Equation (44) was derived for a static medium: it was generalized to the case of Bjorken hydrodynamics corresponding to the Janik–Peschanski metric (29) in [73] with the conclusion that equation (44) still applies in this dynamic case if one replaces $T \rightarrow T(\tau)$ in it, with $T(\tau)$ given by equation (35).

The AdS/CFT result for the drag force (44) allowed for some interesting phenomenology of the heavy-quark energy loss at the RHIC [74]. However, one has to be careful applying this large-coupling result to QCD jet physics: due to asymptotic freedom, at least some part of the jet coupling to the medium should be perturbative. On top of that, the model considered here involves a quark which is being dragged through the medium by an external force which prevents it from slowing down, while in real life the hard partons simply plow through the medium losing energy and momentum. Modeling of jets in the medium along these more realistic lines was carried out in [75]. The conclusion was that, due to strong coupling, the partons are highly likely to branch into more partons, distributing the energy democratically between them. In the end one obtains an isotropic distribution of partons, very different from the jet cone of perturbative QCD and from the jet cones seen in the actual collider experiments (see also [76]). This difference between the ‘AdS jets’ and the real-life jet gives us another argument in favor of being cautious in applying AdS results for jet quenching to real-life heavy-ion collisions.

Applications of the AdS/CFT correspondence to jet quenching are by no means limited by the calculation we have just presented. Further research addressed issues of studying Mach cones generated by supersonic jets, energy loss of light quarks and gluons, momentum broadening of hard partons traversing the medium, quark and gluon stopping distance, dissociation and melting of mesons, along with other topics. A proper discussion of these important results lies outside of the scope of this brief review. We refer the interested reader to [77] for a detailed review of these subjects.

6. Conclusions

We hope that we have convinced the reader that practically any heavy-ion observable can be modeled using AdS/CFT correspondence, generating interesting qualitative and, sometimes, quantitative insight. In comparing the results of AdS/CFT calculations to the actual heavy-ion data or to lattice QCD simulations one has to take care to remember the many differences between QCD and $\mathcal{N} = 4$ SYM theory, though one hopes that certain universal features are common for both theories. One may also hope that future research would put these differences between QCD and $\mathcal{N} = 4$ SYM under quantitative theoretical control.

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