Numerical analysis of an information propagation model

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Abstract. In recent years, information science theory has increasingly highlighted its important influence in the academia, and the numerical analysis method of information dissemination model has achieved rapid development. In this scenario, firstly, this study establishes an information dissemination model based on the classical information theory. Secondly, based on the system stability criterion in the control theory, the basic regeneration number of the model is derived. Thirdly, combined with the classical numerical analysis theory such as Runge-Kutta method, the numerical analysis of the established information propagation model is carried out. The proposed numerical analysis method shed important light on expanding the information science theory.

1. Introduction
In the era of "We Media", information propagates rapidly through BBS, social network, WeChat and other new media, with greater influence and faster propagation speed (Sahafizadeh and Ladani, 2018; Sattari and Zamanifar, 2018). Unverified information propagation on social networks has attracted close attention from public opinion management departments of governments at all levels. On the other hand, exploring the dynamic of unverified information propagation has become a hot topic in academic research (Bao, 2015; Liu et al., 2019).

The study of classic unverified information propagation model began in the 1960s, among which the most mature models are represented by DK (Daley and Kendall, 1965) and MT (Maki and Thompson, 1973). With the popularization and development of Internet and social network, complex network theory has been widely applied to the field of the dissemination of unverified information (Zhu et al., 2016). The main achievements are represented by the models proposed by Moreno (2004), Nekovee (2008) and Zanette (2001).

The propagation of unverified information is affected by network structure, psychological factors, identification capability and other factors. The propagation rules of unverified information are different in uniform network, BA scale-free network (Yu and Zhuang, 2015; Vegaoliveros et al., 2016) and weighted network (Nekovee, 2008; Singh and Nath, 2015). Counter-Rumor mechanism (Goh et al., 2017), forgetting mechanism (Zhao et al., 2011; 2013) and hesitating mechanism (Xia et al., 2015) significantly affect the controlling effect of the propagation of the unverified information.

Although these studies have made great contributions to enriching and expanding the dissemination theory of unverified information, the achievements in the influence of media reports on the
dissemination of unverified information are not mature enough. Media reports can increase science popularization knowledge, improve people's ability to identify unverified information and reduce the probability of spreading unverified information. The different degree of science popularization education makes people behave differently when they are exposed to unverified information.

This study focuses on the impact of media reports on the propagation of unverified information. In Section 2, a propagation model of unverified information considering the media reports is constructed. Simulation is performed in Section 3 to analyze the transmission behavior of different populations. Section 4 summarizes the conclusions.

2. Model description and analysis

In this section, a propagation model of unverified information (UIP) considering media reports is constructed and the free equilibrium point of the model is calculated. Thereafter, the basic reproduction number is derived for judging whether the unverified information can be spread on a large scale. At last, the stability of the model is analyzed by Routh-Hurwitz criteria.

Similar to the classic SIR model, the population is divided into the ignorants (not having heard of rumors), the spreaders (spreading rumors) and the stiflers (heard of but not spreading rumors). According to the degree of science popularization education, the ignorants and spreaders are divided into two categories, respectively, namely ignorants with strong identification ability, ignorants with weak identification ability, spreaders with strong identification ability and spreaders with weak identification ability. The stiflers no longer disseminate unverified information and therefore no longer considers the influence of popular science education. The density of ignorants with strong identification ability, ignorants with weak identification ability, spreaders with strong identification ability and spreaders with weak identification ability, and the density of stiflers at time $t$ are denoted by $I_0(t), I_2(t), S_0(t), S_2(t), R(t)$, respectively, which meet the normalization condition $I_0(t) + I_2(t) + S_0(t) + S_2(t) + R(t) = 1$. Let $k$ represents the average degree of the uniform social networks that spread unverified information. The rules for the propagation of unverified information are as follows: when an ignorant meets a spreader, the ignorant person becomes a spreader with the spreading rate $\beta$, when the spreader encounters the person receiving the unverified information (the spreader or stifler), the former spreader becomes a stifler with the stifling rate $\alpha$.

True information reported by media can improve the ability to identify unverified information. The influence of media is represented by $m$, which is closely related to the type, coverage rate and credibility of media. Strengthen the education of popular science knowledge to ignorants and spreaders can improve the ability to identify unverified information. It is assumed that the ignorant ($I_{u0}$) and spreader ($S_{u0}$) with weak identification ability are transformed into the ignorant ($I_s$) and spreader ($S_s$) with strong identification ability at the popular science education coverage rate of ignorant and spreader $\xi_1$ and $\xi_2$, respectively. Due to the influence of forgetting mechanism, spreaders with weak (strong) identification ability are transformed into stifler ones at the ratio $\gamma_1$ and $\gamma_2$, and will no longer spread unverified information. Figure 1 shows the propagation diagram of the UIP model.
The UIP model constructed is as follows:

\[ \frac{dI_w(t)}{dt} = -k\beta I_w(t)S_w(t) + \bar{S}_w(t) + \xi_1 I_w(t) \]  
\[ \frac{dS_w(t)}{dt} = -k\beta I_w(t)S_w(t) + \bar{S}_w(t) + \xi_2 S_w(t) - \xi_1 I_w(t) \]  
\[ \frac{dS_s(t)}{dt} = k\beta I_s(t)S_s(t) + \bar{S}_s(t) + \xi_2 S_s(t) - R(t) - \xi_2 S_s(t) \]  
\[ \frac{dR(t)}{dt} = k\alpha S_s(t)S_w(t) + \bar{S}_s(t) + \xi_2 S_s(t) + R(t) + \gamma_1 S_s(t) + \gamma_2 S_s(t) \]

Fig. 1 The framework of the UIP model

The UIP model constructed is as follows:

\[ \frac{dI_w(t)}{dt} = -k\beta I_w(t)S_w(t) + \bar{S}_w(t) + \xi_1 I_w(t) \]  
\[ \frac{dS_w(t)}{dt} = -k\beta I_w(t)S_w(t) + \bar{S}_w(t) + \xi_2 S_w(t) - \xi_1 I_w(t) \]  
\[ \frac{dS_s(t)}{dt} = k\beta I_s(t)S_s(t) + \bar{S}_s(t) + \xi_2 S_s(t) - R(t) + \gamma_1 S_s(t) + \gamma_2 S_s(t) \]  
\[ \frac{dR(t)}{dt} = k\alpha S_s(t)S_w(t) + \bar{S}_s(t) + \xi_2 S_s(t) + R(t) + \gamma_1 S_s(t) + \gamma_2 S_s(t) \]

Obviously, the free equilibrium point of the model is \( E_0 = (1,0,0,0,0) \). The newly added spreader with strong identification ability and the newly added spreader with weak identification ability are denoted by \( \mathcal{W} \). The removal rate of new spreaders is represented by symbol \( \mathcal{V} \). Thus

\[ \mathcal{W} = \begin{pmatrix} -k\beta I_w(t)S_w(t) + \bar{S}_w(t) + \xi_1 I_w(t) \\ -k\beta I_w(t)S_w(t) + \bar{S}_w(t) + \xi_2 S_w(t) - \xi_1 I_w(t) \\ k\beta I_s(t)S_s(t) + \bar{S}_s(t) + \xi_2 S_s(t) - R(t) + \gamma_1 S_s(t) + \gamma_2 S_s(t) \end{pmatrix} \]  
\[ \mathcal{V} = \begin{pmatrix} k\alpha S_s(t)S_w(t) + \bar{S}_s(t) + \xi_2 S_s(t) + R(t) \end{pmatrix} \]

Take the first partial derivatives of \( \mathcal{W} \) and \( \mathcal{V} \) with respect to \( S_s \) and \( S_w \), respectively

\[ \mathcal{W} = \begin{pmatrix} -k\beta I_w(t)S_w(t) + \bar{S}_w(t) + \xi_1 I_w(t) \\ -k\beta I_w(t)S_w(t) + \bar{S}_w(t) + \xi_2 S_w(t) - \xi_1 I_w(t) \\ k\beta I_s(t)S_s(t) + \bar{S}_s(t) + \xi_2 S_s(t) - R(t) + \gamma_1 S_s(t) + \gamma_2 S_s(t) \end{pmatrix} \]  
\[ \mathcal{V} = \begin{pmatrix} k\alpha S_s(t)S_w(t) + \bar{S}_s(t) + \xi_2 S_s(t) + R(t) \end{pmatrix} \]

The basic regeneration number is the spectral radius of \( J = \mathcal{W} - \mathcal{V}^{-1} \), which can be expressed by the formula \( R_0 = \frac{k\beta}{m\gamma_2} \). Let \( I \) denotes the identity matrix and \( \lambda \) is the characteristic root, the characteristic equation of the model can be obtained
\[ |\lambda - J| = \begin{vmatrix} \lambda - \xi_1 & -\frac{k\beta}{m^2} & \frac{k\beta}{m} & 0 \\ 0 & \lambda + \xi_1 & 0 & 0 \\ 0 & 0 & \lambda - \frac{k\beta}{m^2} + \frac{\gamma_2}{m} & -\frac{k\beta}{m} - \xi_2 \\ 0 & 0 & 0 & \lambda + \xi_2 + \gamma_1 \end{vmatrix} = \lambda^2 (\lambda + \xi_1)(\lambda + \xi_2 + \gamma_1) \left( \lambda - \frac{\gamma_2 (R_0 - 1)}{m} \right) \]  

(10)

Five characteristic roots can be found:

\[ \lambda_1 = \lambda_2 = 0, \lambda_3 = -\xi_1, \lambda_4 = -\xi_2 - \gamma_1, \lambda_5 = \frac{\gamma_2 (R_0 - 1)}{m} \]  

(11)

According to Routh-Hurwitz criteria, if \( R_0 > 1 \), the system is unstable and the unverified information can be spread on a large scale. If \( R_0 < 1 \), the system is asymptotically stable.

3. Numerical analysis

A social network formed by university community users on Facebook is selected for simulation analysis. The basic data of the social network is shown in Table 1, with a total number of 6,596 users and an average degree of 88.93 (Meng, 2013). The influence of media is divided into three levels. Let \( m=1 \) represents there is no real information reported by media, \( m=5 \) means the media influence of releasing information is general, and \( m=9 \) denotes the media influence of releasing rumor refuting information is tremendous. The values of parameters involved in the simulation are shown in Table 2. It is assumed that there is only one initial spreader.

| Table 1. Data of a university community network on Facebook |
|--------------|--------------|--------------|---------------|---------------|
| Number of note N | Average degree \( \bar{k} \) | Maximum K-shell | Average clustering coefficient | Maximum degree |
|----------------|----------------|----------------|-----------------------------|----------------|
| 6596           | 88.939         | 75             | 0.2369                      | 638            |

| Table 2. The values of parameters involved in the simulation |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| \( \beta \) | \( \bar{k} \) | \( \alpha \) | \( \gamma_1 \) | \( \gamma_2 \) | \( \xi_1 \) | \( \xi_2 \) |
| 0.1           | 88.939       | 0.1          | 0.1          | 0.2          | 0.1          | 0.2          |

The changes of population density of the five groups over time obtained by runge-kutta method are shown in Figure 2. The results show that as time goes on, the population density of the ignorants with weak identification ability decreased gradually, and then tended to be stable; the population density of the ignorants with strong identification ability first increased and then decreased; the two types of spreaders showed the same trend, which first increased, then decreased and finally stabilized. However, spreaders with weak identification ability were far higher than those with strong identification ability. The population density of the stiflers first increased rapidly and then stabilized.

The results indicate that the release of truthful information by official media can reduce the number of people who spread the unverified information. The peak value of the spreaders with weak identification ability is far greater than that of the spreaders with strong identification ability, but the time of reaching the peak value of the two spreaders is almost the same.
Figure 2 Population density of five groups of people over time $t$

Figure 3 shows that the stronger the media influence of releasing real information, the lower the peak value of the spreader will be, and the longer the transmission time to reach the peak value of the density of spreaders. Governments at all levels should enhance the credibility and coverage of official media, so as to quickly and effectively deal with the propagation of unverified information on social networks.

Figure 4 shows that the stronger the media influence, the lower the steady-state value of the density of stiflers becomes. Meanwhile, increasing the media influence increases the density of the stiflers. The result indicates that the release of true information by the media with stronger influence has a delayed and inhibitory effect on the propagation time and spreading influence of the unverified information.

Figure 3 The influence of media reports on the density of spreaders
4. Conclusions
The study has provided numerical analysis of an information dissemination model. The main contributions are as follows:

(1) An information dissemination model considering media report is constructed based on the classic information science theory.

(2) Based on the system stability criterion in the control theory, the basic regeneration number is derived for judging whether the unverified information can be spread on a large scale. The stability of the model is analyzed by Routh-Hurwitz criteria.

(3) Combined with the classical numerical analysis theory, the numerical analysis is carried out and the numerical solution is obtained by Runge-Kutta method. Results show that media has an inhibitory effect on the propagation of the information.

This study enriches the information science theory and provides effective suggestions for dealing with information dissemination. In the future, the numerical analysis of information dissemination in scale-free networks will be studied.

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