Chaotic evolutionary algorithms for dynamic instability enhancement

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Abstract. The algorithm principle is presented based on a geometry optimization of aeroelastic energy harvester. Efficiency of this class of devices highly depends on oscillation amplitude caused by dynamic instability of the system, which is related to its geometry. Optimization was performed using a genetic algorithm (GA) that processes data from CFD calculations. This algorithm generated a random population of twenty-arm geometrical figures. Each geometry was subjected to a numerical experiment during which its movement in a fluid-filled channel were simulated and resultant force acting on body was calculated. The calculations were repeated for angular orientation of the object varying from 0 to 180 degrees, at 5 degrees step, in order to obtain a complete characteristic of aerodynamic forces acting on body related to its angular orientation. For each of the obtained functions, satisfaction of Den Hartog's criterion is examined, which is the basis for geometry evaluation. In order to accelerate the calculations, classical GA has been modified by substituting random crossover process by operation determined by chaotic process — in this case, a logistic map. The numerical calculations was carried by Method of Fundamental Solutions.

1. Introduction

The issue of system stability has been of interest to scientists in various fields for hundreds of years. In the most general terms, it can be stated that the system is stable if it is not very sensitive to disturbances in initial conditions (Lyapunov and Poincare definitions) or to external excitation disorders (Malkin definition). Therefore, we consider the system to be stable if, after the disturbance, the nature of its motion remains sufficiently similar to the motion of the non-disturbed system.

Dynamic instabilities in technical facilities are associated rather with undesirable phenomena, as a result of which numerous works on stabilization of systems can be indicated. Reference [1] presents the innovative method of stabilization of dynamical system. Another approach to oscillation reduction describe authors of [2]. An innovative way to control unstable states in laser systems is presented in [3]. Preventing destabilization is particularly important in civil facilities such as bridges and buildings.

The stability analysis of bridges exposed to wind was carried out in [4]. The work [5] is a precise case study showing the influence of Karaman vortices on the stability of bridges. Gravity can also be caused by instability - for example in the case of seismic loads [6]. Given the dynamic climate change, it seems extremely important to study the possibilities of increasing the stability of existing structures [7]. Iced high voltage lines may also be subject to loss of stability – galloping wires may lead to catastrophic failure, hence the need to study this phenomenon [8]. A serious problem is also the loss of stability of the aircraft wings during the flight - flutter. Numerous papers are devoted to the analysis...
of this phenomenon and methods of its control [9-11]. However, loss of stability may be a desirable phenomenon. An example of a construction in which this phenomenon can be used is aeroelastic energy harvester (AEH). Aeroelastic energy harvester is a device that allows energy recovery from vibrations induced by the flow. Man has used the flow energy since the dawn of time, but only the recent development of electronics and autonomous devices has contributed to the need for small-scale systems that are capable for continuous low power generation.

Devices of this type use the phenomenon of galloping caused by negative aerodynamic damping, which assuming the quasistationarity of the phenomenon – was first described by Den Hartog [12] and extended in [13]. In these works, a body with one degree of freedom (translation parallel to axis Z), mounted on a damped spring system, subjected to flow in a direction parallel to the X axis was analyzed (see Fig. 1.). The dynamics of the system is described by the equation:

\[ m \ddot{z}(t) + c \dot{z}(t) + k z(t) = F_z(\alpha) = \frac{1}{2} b \rho U^2 C_z(\alpha) \]  

(1)

where: \( m \) – mass of the body, \( c \) – foundation’s damping coefficient, \( k \) – foundation’s stiffness coefficient, \( z(t) \) – displacement in Z direction, \( \dot{z} \) and \( \ddot{z} \) – first and second differential with respect to time, \( C_z(\alpha) \) – coefficient of aerodynamic force acting in the Z direction at \( \alpha \) angular orientation of the body, \( F_z \) – aerodynamic force component acting in the Z direction, \( b \) – characteristic length of the body, \( \rho \) – fluid density, \( U \) – flow velocity. Galloping occurs for \( (c - C_z(\alpha_0)) < 0 \).

![Figure 1. Physical model of aeroelastic energy harvester](image)

On Figure 1 it can be seen that for small enough \( \alpha \) it is true that:

\[ F_z(\alpha) = -F_D \sin(\alpha) - F_L \cos(\alpha) \approx -F_L - F_D \alpha \]  

(2)

and after expanding in Taylor’s series:

\[ -F_L - F_D \alpha = -\left( F_L + \left( \frac{dF_L}{d\alpha} + F_D \right) + \frac{1}{2} \left( \frac{d^2F_L}{d\alpha^2} + \frac{dF_D}{d\alpha} \right) \alpha^2 + \frac{1}{6} \left( \frac{d^3F_L}{d\alpha^3} + 3 \frac{d^2F_D}{d\alpha^2} \right) \alpha^3 + O^4 \right) \]  

(3)

Since the constant force component \( F_L \) does not affect the dynamics of the system, it will be neglected in further considerations. The only factors affecting the aerodynamic force depending on the \( \alpha \) angle are shape coefficients, hence:

\[ C_z(\alpha) = \left( \frac{dG_L}{d\alpha} + C_D \right) + \frac{1}{2} \left( \frac{d^2G_L}{d\alpha^2} + \frac{dC_D}{d\alpha} \right) \alpha^2 + \frac{1}{6} \left( \frac{d^3G_L}{d\alpha^3} + 3 \frac{d^2C_D}{d\alpha^2} \right) \alpha^3 + O^4 \]  

(4)

As reported in [14] and [15] the approximation of the \( C_z(\alpha) \) function with a third order polynomial is sufficient for energy harvesting purposes:
The mathematical model of the phenomenon thus takes the form:

\[
\begin{aligned}
  \ddot{z}(t) + c \dot{z}(t) + k z(t) &= -\frac{1}{2} \rho U b \left( a_1 \frac{\dot{z}(t)}{U} + a_2 \left( \frac{\dot{z}(t)}{U} \right)^2 + a_3 \left( \frac{z(t)}{U} \right)^3 \right) \\
  \end{aligned}
\]  

(6)

Conversion of mechanical energy into electricity can be carried out using a variety of transducers, however, the most commonly used are piezoelectric ones. The piezoelectric vibration energy harvester (PVEH) mathematical model rewritten in dimensionless form can be express as [16]:

\[
\begin{aligned}
  \ddot{\eta}(\tau) + \epsilon \dot{\eta}(\tau) + \eta(\tau) + \kappa \nu(\tau) &= a_1 \overline{\nu}(\tau) + a_2 \bar{\nu}(\tau)^2 + a_3 \frac{\dot{\eta}(\tau)}{\Omega} \\
  \end{aligned}
\]  

(7)

where: \( \eta(\tau) = \frac{z(t)}{h} \), \( \nu(\tau) = \frac{\dot{z}(t)}{\theta c_p b} \), \( \epsilon = \frac{c}{m \omega_n} \), \( \kappa = \frac{\theta^2}{c_p m \omega_n^2} \), \( \bar{\nu} = \frac{h^2 \nu \rho}{2m} \), \( r = C_p \omega_n R \), \( \Omega = \frac{\omega}{\omega_n} \), \( \tau = \omega_n t \).

The efficiency of PVEH is significantly affected by its mechanical structure. The classic 1DOF beam devices ([17-19]) seem to give way in this respect to more complex systems with many degrees of freedom ([20], [21]). It is worth noting that devices showing also torsional vibrations should not be modeled using the Den Hartog's hypothesis – for torsional vibrations the quasistationarity condition is never satisfied. The nonstationary flow model was used, among others in works [22]. Researchers also signal the possibility of achieving higher efficiency by constructing devices with nonlinear dynamic properties whose movement is regular ([23]) or chaotic ([24], [25]). From the point of view of device efficiency, of course, the shape of the flowing body is of key importance. In [26], elliptical cross-sections with different ratios between the length of the semi-minor axis and the semi-major axis were examined. A substantial set of aerodynamic coefficients of various typical sections is included in [27]. The work [28] is devoted to the analysis of the impact of trapezoid arm inclination on its aerodynamic coefficients.

To the knowledge of the authors, so far there is no research that would include analysis of any, unusual and irregular geometry. The purpose of this work is to fill this gap.

2. Measure of instability

The optimization goal function is maximization of instability of the system, which leads to maximization of the power generated by the system. The analytical form of the expression for the device power as a function of the coefficients \( a_1, a_2 \) and \( a_3 \) can be obtained by adopting that both the vibration amplitude and the voltage have harmonic solutions in form of:

\[
\begin{aligned}
  \eta(\tau) &= A_{\eta} \cos(\Omega \tau) \\
  \nu(\tau) &= A_{\nu} \cos(\Omega \tau + \varphi) \\
\end{aligned}
\]  

(8) \quad (9)

where: \( A_{\eta} \) and \( A_{\nu} \) – dimensionless amplitudes of vibrations and voltage respectively, \( \Omega \) – dimensionless natural frequency, \( \varphi \) – phase shift. The sine and cosine of the phase shift \( \varphi \) was...
determined from the second equation of the system (7) by implementing the assumed solutions (8) – (9) and examining satisfaction of equation for dimensionless time \( \tau = 0 \):

\[
\begin{align*}
\sin(\varphi) &= \frac{1}{\sqrt{(r \Omega)^2 + 1}} \\
\cos(\varphi) &= \frac{r \Omega}{\sqrt{(r \Omega)^2 + 1}}
\end{align*}
\]  

(10)  

(11)

The relation between dimensionless voltage and dimensionless displacement amplitudes was determined by integrating the second equation of system (7) with respect to dimensionless time in half-period boundary and substituting (9) – (10):

\[
A_v = \frac{r A_t \Omega}{\sqrt{1 + r^2 \Omega^2}}
\]  

(12)

Then, the energy balance was carried out on the basis of system of equations (8), which after taking into account equations (9) – (13) allowed to obtain an expression for dimensionless vibration amplitude and frequency:

\[
\begin{align*}
A_t &= \frac{2\sqrt{r^2 \kappa - \mathcal{U}^2 \bar{\rho} a_1 - r^2 \mathcal{U}^2 \bar{\rho} \Omega^2 a_1}}{3\sqrt{\bar{\rho} \Omega^2 a_3 + r^2 \bar{\rho} \Omega^4 a_3}} \\
\Omega &= \frac{1}{\sqrt{2}} \left( 1 - \frac{1}{r^2} + \kappa - \frac{\sqrt{4r^2 + (1 - r^2 - \kappa)^2}}{r^2} \right)
\end{align*}
\]  

(13)  

(14)

The dimensionless power of the system is therefore given by the formula:

\[
P = \frac{\kappa A_v^2}{r} = \frac{2r \mathcal{U} \kappa (\mathcal{U} \bar{\rho} a_1 (1 + r^2 \Omega^2) - r \kappa)}{-3\bar{\rho} (1 + r^2 \Omega^2)^2 a_3}
\]  

(15)

3. Optimization

As can be seen in section 1, to determine the body's galloping capacity, it is necessary to have an information about \( C_L(a_0) \) and \( C_D(a_0) \) functions determined by the body geometry. Thus, they will store information about extent that the geometry affect the power generated by the device. It is therefore reasonable to develop – through optimization – geometry that will be the most instable. The optimization process was carried out using a genetic algorithm, while the aerodynamic characteristics were obtained by simulating flow utilizing the Method of Fundamental Solutions.

![Figure 2. Examples of body geometries](image)

At the first stage of the GA, 200 random, having a horizontal axis of symmetry geometries were generated, defined as a closed broken line described on points, each of which lay on one (and only one) of twenty uniformly distributed radial axes (see Figure 2). Then each of them was evaluated.
according to the procedure described later in the paper. In order to improve calculations, the classic
crossover process has been replaced here by the chaotic process proposed in [29], based on logistic
map, given by the expression:

\[ z_{n+1} = \lambda z_n (1 - z_n) \]  

Function (17) implements the iterative process of generating the value of \( z_{n+1} \) on the basis
of any of the values chosen from the range \(<0; 1>\). Then the generated value is adopted as the new
value of \( z_n \) and the process is repeated. Depending on the arbitrarily chosen parameter \( \lambda \), the process
can be convergent for \( \lambda \in (1; 3>\), periodic for \( \lambda \in (3; 3.57) \), or chaotic for \( \lambda > 3.57 \) (this range includes
stable manifolds). The implementation of the chaotic crossover model consists in extending the
standard genome containing only the solution by two additional sections: information about the
randomly assigned value of the parameter \( \lambda \) and the random crossover mask (encoded in the Gray
code).

The crossover mask stores the information about which bits the offspring will inherit
from which parent - if the value 1 appears in the first place of the mask, the offspring inherits the gene
first from the first parent, if 0 appears in the second place of the mask, the offspring inherits the
second gene from the second parent, etc. The mask of each parent is used to generate one
offspring. The value of \( \lambda \) is inherited without any change and is used to generate the offspring mask. This
process consists in decoding the parent mask, normalizing its value to the range \(<0; 1>\), and then
processing this value by a logistic map with the assigned parameter \( \lambda \). Processed and re-encoded in
Gray code value is assigned as the offspring mask. The example of the chaotic crossover procedure is
presented in Table 1.

![Figure 3. Logistic Map](image-url)
Table 1. Chaotic crossover algorithm

| Parent 1 | Parent 1 |
|----------|----------|
| Solution | $\lambda$ | Mask | Solution | $\lambda$ | Mask |
| 0110     | 3.9      | 1101 | 1001     | 2.5      | 0101 |

| Offspring 1 | Offspring 2 |
|-------------|-------------|
| Solution    | $\lambda$ | Mask | Solution | $\lambda$ | Mask |
| 0010        | 3.9      | 1100 |         | 2.5      | 1110 |

| Binary representation | 11.111 | 1101 | 10.100 | 0110 |
|------------------------|--------|------|--------|------|
| Decimal representation | 3.88   | 13   | 2.5    | 6    |
| Normalized value ($z_n$) | 0.87   | 0.40 |
| $z_{n+1}$              | 0.45   | 0.60 |
| Expanded value         | 6.72   | 9.00 |
| Binary representation  | 0111   | 1001 |
| Gray code representation | 0101 | 1101 |

As reported in [20], $\lambda$ values that appear in the initial population have a significant impact on the algorithm performance. Individuals with this parameter within the convergent and chaotic range are desirable, and parameters causing periodic solutions should be avoided. The distribution of parameters in the population does not seem to matter. The crossover process was carried with the strategy of elitism, assuming the probability of its occurrence at the level of 0.8. In addition, the solution may have been subjected to a flip bit mutation with a probability of 0.05. The condition for the algorithm to end was the invariability of the best solution for a 100 iterations. The aerodynamic coefficients $C_l$ and $C_D$ necessary to determine the parameters $a_1$, $a_2$ and $a_3$ of the body were obtained by numerical simulation of the movement of the body in a channel filled with fluid (air). The physical model was prepared according to the benchmark [30], with the difference that the cylinder was replaced by the tested geometry. The simulation was repeated for angular orientations of the object from 0 to 180 degrees at 5 degrees step so as to obtain full characteristics describing the aerodynamics of the body $C_l(a_0)$ and $C_D(a_0)$. For each simulation constant Reynolds number $Re = 10^5$ was maintained. Based on the received characteristics and according to model (7), the functions $a_1(a_0)$, $a_2(a_0)$, $a_3(a_0)$ were calculated, these in turn, after substituting them into (15), were utilized to characterize the power $P(a_0)$ generated by the device equipped with the resonator of the studied geometry. The maximum value of $P(a_0)$ was the basis for the evaluation of geometry by the genetic algorithm.

4. Results and discussion

Figure 4 shows the optimized geometry, while Figure 5 shows its aerodynamic characteristics. Individual arm lengths are shown in Table 2. The blunt side orientation in the normal direction to the wind flow corresponds to the 90 degree orientation and for this orientation the body will exhibit the greatest aerodynamic instability.
Table 2. Length of individual arms of optimized geometry [mm]

|    | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|    | 30.20 | 35.50 | 24.40 | 20.95 | 21.55 | 25.40 | 34.00 | 48.70 | 85.10 | 75.60 | 71.00 |

Table 3 lists the coefficients \( a_1, a_2 \) and \( a_3 \) calculated for optimized geometry and for standard geometries that are commonly studied. Based on these coefficients and using the formula (17), the potential power \( P \) generated by a device equipped with a resonator of a given geometry was calculated and related to the power generated by a device with optimal geometry \( P_{opt} \).

Table 3. Summary of obtained results

|                | Optimized geometry | D-section | Equilateral triangle | Square |
|----------------|--------------------|-----------|----------------------|--------|
| \( a_1 \)      | 3.56               | 0.097     | 1.87                 | 2.69   |
| \( a_2 \)      | 0.28               | 4.25      | 5.11                 | 0      |
| \( a_3 \)      | 9.74               | −28.83    | −1418                | −0.0068|
| \( P/P_{opt} \) | 1                  | 0.77      | 0.87                 | 0.59   |

Figure 4. Optimized geometry

Figure 5. Aerodynamic characteristic of geometry

As can be seen from the table above, the efficiency of using optimal body geometry is closest to the efficiency of a triangular body (13% difference). Both figures are geometrically similar, however, the shape of optimized model seems to facilitate the detachment of the fluid stream from surface. More complex geometry promotes performance, although, can be problematic from a technological point of view, which in turn can be a barrier when using it in prototype devices.

5. Conclusions

The body geometry constituting the energy harvester resonator was optimized. The optimization goal function was to maximize the efficiency of the device, thus the body with a given geometry had to show maximum dynamic instability. In order to analytically formulate a goal function, a mathematical model of the device was derived and then through its solution, an analytical expression for the system was obtained. The optimization process was carried out using a chaotic genetic algorithm, while the necessary flow calculations were made using the Method of Fundamental Solutions. The efficiency of the generated geometry was compared with other typical, commonly used ones and it was shown to be significantly more effective.

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