A numerical study of fiber reinforced polymer laminate plates

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Abstract. Layered composites are used in various engineering fields like the aircraft, marine, automotive, sports and health sectors. The widespread use of these materials can be attributed to its high stiffness, high strength-to-weight ratio, and ease of installation. Among the various layered composites, fiber-reinforced polymers (FRP) laminates have high strength and stiffness of its constituent fibers and low-density due to its matrix properties. FRP laminate consists of unidirectional plies that can be tailor-made to achieve desirable performance. The demand for layered composite materials has driven the need for development of efficient analytical tools for accurate prediction of its performance. The present paper provides an insight into the analysis of a laminate plate. The effect of the number of layers, aspect ratio and the stacking sequence on the transverse deformation of a laminate plate under bending is analyzed. It is observed that the transverse deflection of the plate decreases with an increase in the number of layers. Also, the increase in aspect ratio leads to an increase in the deformation of the plates.

1. Introduction
The rising demand for improvisation in the construction industry has led to the development of new materials and building technologies, of which fiber-reinforced polymers (FRP) have been of great importance. An FRP laminate consists of unidirectional thin sheets called laminae or plies stacked and bound together. In a unidirectional lamina, the fibers are embedded along a single direction in the matrix. Hollaway [1], has reviewed the applications of FRP laminates in civil engineering. These have been widely used in the strengthening and retrofitting of structural members. Carbon FRP has been used extensively in order to meet the demands of aircraft designers which include reduction of mass and slender structures without compromising on the strength and stiffness, abrasion and impact resistance, improved fatigue life and corrosion resistance[2].

The laminate properties such as the number of layers and stacking sequence, impact the performance of a laminate. Reddy [3] performed bending, buckling and vibration analysis of FRP laminates using various theoretical approaches. Results indicated that under bending, the transverse deformation decreased with the increase in the number of laminate layers. Jauharia et al.[4] performed a numerical analysis of CFRP composite laminates under compressive edge loading. It was found that the increase in the number of layers of antisymmetric cross-ply laminate without increase in the total thickness of the plate, led to the decreased transverse deformation of the structure. Soufeiania et al. [5] conducted a numerical study on an FRP composite laminate slab for its dynamic characteristics. The laminates were made up of a combination of 0°, -45°, +45°, and 90° ply orientations. Walking loads causing floor vibrations were modelled and applied as a central point load on the slab. It was observed that the combination of 0° and 45° plies performed satisfactorily considering human comfort.
When the stacking sequence was changed, vibration on the floors was avoided without alteration of its geometry or mass. Thereby, a designer may modify the material properties and stiffness of a laminate by changing its stacking sequence.

This paper aims to study the effect of the number of layers, stacking sequence and aspect ratio on the transverse deformation of simply supported FRP laminates under bending.

2. Analytical approach

2.1. Finite element analysis

Finite element (FE) method is a beneficial tool in providing numerical solutions for many engineering problems. In civil engineering, its applications include deformations, stress analysis, damage, and failure analysis. In this method of analysis, the region under study is considered a continuum and divided into a number of finite elements. The material properties and the governing relationships are applied to the elements and are defined with respect to its nodes. The loading and constraints are then considered appropriately resulting in a set of equations whose solution gives the approximate behavior of the continuum [6]. Computer-aided systems help in modeling complex problems with relative ease. Modeling of structures requires a pre-requisite knowledge of the related theory and computation techniques. Reddy [3], describes the mechanics and theories used in the analysis of composite laminates. Various problems for FRP laminates and their analysis is discussed in the following sections.

2.2. Laminate plate theory

As per Hooke’s Law, the stress-strain relation when applied to an elastic material exhibiting anisotropy is expressed as shown in Eq 1.

\[ \begin{align*}
\sigma &= [C]\varepsilon \\
\end{align*} \]  

(1)

where \( \{\sigma\} \) and \( \{\varepsilon\} \) are stress and strain vectors respectively. The \([C]\) matrix is called the material stiffness matrix of order six, having 21 independent material constants.

The coordinate system followed for a unidirectional lamina is as depicted in Figure 1, where \( \alpha \) is the fiber orientation with the \( x \)-axis. The material has two principal directions; along the fiber orientation represented by 1 and the matrix directions 2 and 3 transverse to it.

![Figure 1. A lamina with material and problem coordinate systems](image)

For plane stress problems, the stress-strain relation may be reduced as shown in Eq 2.

\[ \begin{align*}
\sigma_1 &= Q_{11}\varepsilon_1 + Q_{12}\varepsilon_2 + \tau_{12} \\
\sigma_2 &= Q_{21}\varepsilon_1 + Q_{22}\varepsilon_2 + \tau_{12} \\
\end{align*} \]  

(2)

where, \( \sigma_1 \) and \( \sigma_2 \) are the normal stresses along and transverse to fiber direction respectively, and \( \tau_{12} \) is the in-plane stress. \( \varepsilon_1 \) and \( \varepsilon_2 \) are the normal strains and \( \gamma_{12} \) is the shear strain. \([Q]\) is the reduced material stiffness matrix of the order 3, having 4 independent elements and is expressed as
where, \( E_1 \) and \( E_2 \) are Young’s modulus along fiber direction and transverse to it respectively, \( v_{12} \) and \( v_{21} \) are the Poisson’s ratio, and \( G_{12} \) is the shear modulus.

Transformation of the axes from the global coordinates to the natural material coordinates depends on the fiber orientation, \( \alpha \). On stress and strain transformations, the resultant constitutive relation of the plate can be expressed as,

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = [\bar{Q}]
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
\]

(4)

where, \([\bar{Q}]\) is the lamina stiffness matrix, which may be expressed as,

\[
[\bar{Q}] = [T]^{-1}[Q][R][T][R]^{-1}
\]

\[
[T] = \begin{bmatrix}
c^2 & s^2 & 2sc \\
s^2 & c^2 & -2sc \\
-2sc & sc & c^2 - s^2
\end{bmatrix}
\]

\[
[R] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\]

(5)

where \([T]\) and \([R]\) are the transformation and Reuter’s matrix respectively with \( c = \cos \alpha \) and \( s = \sin \alpha \).

Classical Laminate Plate Theory (CLPT) assumes that all layers are perfectly bonded together in a plate with no change in thickness of plate during deformation and the in-plane deformations are continuous. On loading, the ply strains along global coordinates are expressed as,

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_{x0} \\
\varepsilon_{y0} \\
\gamma_{xy0}
\end{bmatrix} + z
\begin{bmatrix}
\kappa_x \\
\kappa_y \\
\kappa_{xy}
\end{bmatrix}
\]

(6)

where \( \{\varepsilon\} \) represents the mid-plane strains \( \{\kappa\} \) represents the curvature, and \( z \) is the vertical distance of ply from mid-plane. For a laminated composite plate as shown in Figure 2, the relation between applied forces and plate strains is given as

\[
\begin{bmatrix}
\{N\} \\
\{M\}
\end{bmatrix} = \begin{bmatrix}
[A] & [B] \\
[B] & [D]
\end{bmatrix}
\begin{bmatrix}
\{\varepsilon\} \\
\{\kappa\}
\end{bmatrix}
\]

(7)

where, \( \{N\} \) and \( \{M\} \) are the force resultant and moment resultant vectors respectively. Elements of the extensional stiffness matrix \([A]\), coupling stiffness matrix \([B]\), and bending stiffness matrix \([D]\) matrices are defined as,

\[
A_{ij} = \sum_{k=1}^{n} (\bar{Q}_{ij})_k (z_k - z_{k-1})
\]

\[
B_{ij} = \sum_{k=1}^{n} (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2)
\]

\[
D_{ij} = \sum_{k=1}^{n} (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3)
\]

(8)
where \( n \) is the number of layers. Mid-plane strains and curvature are obtained on solving Eq (7), which is subsequently used to find out the ply strains. The stresses developed are then computed.

\[ \text{Figure 2. Coordinate system and layer numbering for laminated plate [3]} \]

CLPT does not consider shear deformation of plates and hence the theory was modified to obtain the First-order shear deformation theory (FSDT), proposed by Mindlin-Reissner. Shear correction factors are required to improve the accuracy of stresses obtained using FSDT. Higher-order shear deformation theories that include additional terms in the expansion series produce results with better accuracy thus avoiding the need for shear correction factors.

3. Numerical analysis

The analysis is done using numerical computation with the help of analysis software ANSYS Workbench v19.0. The plate is modelled in 2D, laminate layers are defined and plate is meshed followed by application of loads and constraints. The transverse deflection is observed and non-dimensionalised.

The materials used for analysis are given in Table 1. The plate dimensions are given as ‘\( a \)’ along x-axis, ‘\( b \)’ along y-axis and ‘\( h \)’ along z-axis with layer offset referred from midplane. ‘\( w \)’ is the maximum transverse deflection of the midplane in the direction of z-axis. The elastic modulus along the fiber direction and transverse to it are given by \( E_1 \) and \( E_2 \) respectively. The shear modulus is given by \( G_{12} \) and Poisson’s ratio defined by \( v_{12} \). Various cases considered in the study are listed in Table 2.

### Table 1. Material Properties.

| Material | \( E_1 \text{(MPa)} \) | \( E_2 \text{(MPa)} \) | \( G_{12} \text{(MPa)} \) | \( v_{12} \) |
|----------|-----------------------|------------------------|--------------------------|------------|
| M1       | 20000                 |                        |                          | 0.3        |
| M2       | 132379                | 10756                  | 5654                     | 0.24       |
| M3       | 40E_2                 | 10756                  | 0.5E_2                   | 0.25       |
Table 2. Properties of plate under consideration.

| Case | Ply configuration | Number of layers | h (mm) | a/h |
|------|-------------------|------------------|--------|-----|
| P1   | Isotropic         | 1                | 120    | 33.3|
| P2   | (0/90/0)          | 3                | 10     | 100 |
| P3   | (0/90)n           | 2n               | 10     | 100 |
| P4   | (0/90/0)n         | 3n               | 10     | 100 |
| P5   | (-45/+45)n        | 2n               | 10     | 100 |
| P6   | (-45/+45/-45)n    | 3n               | 10     | 100 |
| P7   | (-θ/+θ)           | 2                | 10     | 100 |

3.1. Boundary and loading conditions
In this study, the plates under consideration are simply supported on all four sides. The transverse load applied is uniformly distributed with an intensity $q_0$, as shown in Figure 3.

![Figure 3. Loading pattern of uniformly distributed load](image)

3.2. Meshing
All models have been meshed with a mesh of size 50mm. The element used for meshing is SHELL281 as depicted in Figure 4. It is quadratic in nature with 8 nodes and 6 degrees of freedom. The degrees of freedom are $u_x$, $u_y$, $u_z$, $\theta_x$, $\theta_y$ and $\theta_z$. Assumptions of the FSDT are adopted and shear correction factors are computed using equivalent energy method, based on the section lay-up.

![Figure 4. Representation of SHELL281- 8 node shell element](image)
3.3. Finite element model
The plates have been modelled in 2D and analysed. The equations are reduced and solved with respect to the mid-layer equations. The meshed FE model for adopted for validation is shown in Figure 5.

![Meshed Finite Element Model](image)

**Figure 5.** Meshed Finite Element Model

3.4. Validation
To ensure the credibility of the work carried out, previous works have been validated. An isotropic square plate using material M1, has been modelled as specified in case P1. A three-layered symmetric (0/90/0), square composite laminate plate using material M2 has been modelled as specified in case P2. The validated results are as shown in Table 3.

| Case | Material | From literature | Present results | % difference |
|------|----------|----------------|----------------|-------------|
| 1    | M1       | 2.1166[7]      | 2.126          | 0.44        |
| 2    | M2       | 1.206[3]       | 1.289          | 6.9         |

Table 3. Validation.

4. Results and discussion

4.1. Effect of Number of Layers
Symmetric and antisymmetric laminates with cross-ply and angle ply configuration were considered, to study the effect of the number of layers on the transverse deflection of plates under bending. The number of layers was increased, keeping the thickness of the plate constant. Four different cases of square composite plates as mentioned in case P3, P4, P5 and P6 respectively were analyzed. The non-dimensionalised transverse deflection, $\bar{w} = \frac{1000E_hh^3w}{q_0a^4}$.
The effect of the number of layers on the transverse deflection is shown in Figure 6. As the number of layers was increased, the transverse deformation of the plates decreased. The optimum number of layers for minimum deflection is to be limited to n=6, as further increase will not be economical. It may be inferred that cross-ply laminates exhibit more flexibility than angle ply laminates. Also, anti-symmetric laminates exhibit lesser deformation as compared to symmetric laminates.

4.2. Effect of aspect ratio
Antisymmetric cross ply square laminates as mentioned in P3 were considered to understand the effect of the aspect ratio (b/a), on the transverse deflection of the plates. The in-plane plate dimension ‘b’ was varied with respect to ‘a’. In addition, the number of layers were also varied, maintaining a constant thickness of the plates. The effect of the aspect ratio on the transverse deflection of the laminates is depicted in Figure 7.
It was observed that as the aspect ratio increased, the transverse deflection also increased. The maximum deflection was observed for the aspect ratio of value 2, beyond which there is no further increase and the plates undergo cylindrical bending. The plate can then be considered as a plate strip, wherein the behavior of the plate is similar to the one-way action of slab.

4.3. Effect of ply orientation
To analyse the influence of ply orientation on the transverse deflection of the laminates, simply supported rectangular plates were considered. Antisymmetric angle-ply (-θ/θ) laminates as mentioned in P7, with aspect ratio 7 were studied. The deflection pattern observed for the rectangular plate under transverse loading is shown in Figure 8. For differing values of θ, the transverse deformation was calculated.

Figure 8. Deflection pattern of rectangular plate (+45/-45)

The results showed an exponential increase in the transverse deformation of the plate for an increase in the values of θ from 0° to 90°. The variation of transverse deflection with respect to fiber orientation is depicted in Figure 9.

Figure 9. Variation of transverse deflection in (+θ/-θ) two-layered rectangular laminates
The results are in agreement with the load transfer mechanism of a one way slab. Plate strips exhibit the characteristics of a one way slab; i.e., the bending action is primarily along the direction of the shorter span. Adequate longitudinal reinforcement is required for safe moment carrying capacity of the plate strip. Thus, plates with reinforced fiber along the longitudinal direction of the plate show least deformation. Plates with fibers oriented in the transverse direction of the plate undergo large deformation for the same loading and boundary conditions.

5. Conclusions
Composite laminates are excessively used owing to its tailor-made properties. In this study, the effect of the number of layers, aspect ratio and stacking sequence on the transverse deformation of laminates under bending is analysed. The optimum value of the parameters under study have been obtained and the following conclusions are drawn:
1. As the number of layers increases, the transverse deflection of a composite laminate under bending decreases. The optimum number of layers for minimal deformation obtained is 6.
2. Angle ply laminates perform better than cross ply laminates under transverse loading.
3. Antisymmetric laminates exhibit lesser deformation than symmetric laminates
4. The increase in aspect ratio, leads to an increase in the transverse deflection of laminates. Beyond an aspect ratio of 2, the plate behaves like a plate strip under cylindrical bending.
5. For rectangular plates (-θ/+θ) under cylindrical bending, transverse deformation shows a gradual increase with θ varying from 0° to 90°, owing to the one-way slab action.

Reference
[1] Hollaway LC 2010A review of the present and future utilisation of FRP composites in the civil infrastructure with reference to their important in-service properties Construction and Building Materials 24 2419–45
[2] Soutis C 2005 Fibre reinforced composites in aircraft construction Progress in Aerospace Sciences 41 143–151
[3] Reddy JN 2003 Mechanics of Laminated Composite Plates and Shells: Theory and Analysis, Second Edition CRC Press
[4] Jauhari N, Mishrab R and Thakur H 2016 Stress analysis in FRP composites Perspectives in Science 8 50–52
[5] Soufeiania L, Ghadyanib G, Kuehc A B H and Nguyena K T Q 2013 The effect of laminate stacking sequence and fiber orientation on the dynamic response of FRP composite slabs Journal of Building Engineering 13 41–52
[6] Chandrupatla T R, and Belegendu A D 2002 Introduction to Finite Elements in Engineering Prentice Hall
[7] Timoshenko S and Woinowsky S 1959 Theory of Plates and Shells Mc-Graw-Hill