Selective excitation of vortex fibre modes using a spatial light modulator

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Abstract. A controllable excitation of fibre modes is examined theoretically and experimentally. We verify that the modes of a few-mode fibre can be selectively excited by means of a spatial light modulation of a laser beam focused to the fibre. The proposed method is demonstrated on a dynamical switching, phase coupling and weighted mixing of $LP_{01}$, $LP_{11}$ and $LP_{21}$ modes, achieved by an exchange of computer-generated holograms sent to the spatial light modulator. The possibility to excite a vortex fibre mode with a well-defined azimuthal phase dependence is also verified. It is promising for an encoding and transfer of information by vortices in fibre communications.

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1. Introduction

Optical vortex beams can be comprehended as specific light fields whose equiphase surface has the form of a helical spiral with a pitch of $m\lambda$, where $m$ is an integer called the topological charge of the vortex and $\lambda$ denotes the vacuum wavelength. At the vortex centre, the phase is singular and the intensity vanishes. The helical wavefront is associated with a spiral flow of electromagnetic energy resulting in the orbital angular momentum carried by the vortex beam. A modern treatment of the orbital angular momentum of light beams was proposed by Allen et al [1] and it generated great interest. Original works devoted to the identification of an orbital angular momentum of Laguerre–Gaussian beams and to its description using an analogy between paraxial optics and quantum mechanics [2] resulted in a very wide spectrum of problems concerning theoretical and experimental aspects of the subject (see [3, 4] for a review). Applying a spatial light modulator (SLM), advanced experimental methods have been proposed to create variable vortex arrays and coaxial vortex superposition [5]. Besides vortex structures nested in the Gaussian host beam, the pseudo-nondiffracting vortex beams and their superposition have also been examined [6]–[8].

Recently, optical vortices have been examined as information carriers providing additional degrees of freedom for information encoding. In a recently proposed method [9], the transferred information was encoded into a spatial structure of a mixed vortex field propagating in free-space as a pseudo-nondiffracting beam [8]. In that method, the separate vortices creating the mixed vortex field are superposed with the weight coefficients whose values 1 or 0 create an actual information chain. The topological charges of the vortices are used as ‘markers’ enabling their spatial separation and the decoding of information by an intensity detection. Independently, a similar method of information encoding operating with Laguerre–Gaussian vortex beams has been proposed and verified [10]. Both methods are applicable to free-space communications. An extension of the method to fibre communications requires a selective excitation of higher-order fibre modes and generation of their weighted superposition.

For a solution of that problem, a method proposed by Volpe and Petrov is promising [12]. It enables an efficient generation of light beams with radial, azimuthal and hybrid polarization through a few-mode fibre excited by a Laguerre–Gaussian beam. A vortex propagation in a
stressed optical fibre illuminated by the Hermite–Gaussian beam has been demonstrated in [11].
In our paper, an attention is focused on a spatial modulation of a laser beam illuminating the fibre.
A principle of the method is based on an analogy between ideal free-space nondiffracting beams
and fibre modes, and on the possibility to achieve their approximate mutual conversion. In this
paper, the simplest scalar description of the method is presented. It shows that in a general case,
the laser beam illuminating the fibre can be decomposed into a base of free-space nondiffracting
beams with a continuous spectrum of propagation constants. The fibre then serves as a spatial
filter accepting modes with guided propagation constants. A change of the complex amplitude
of the illuminating beam causes a redistribution of the weights of nondiffracting beams used in
its representation. In this way, the weights of guided fibre modes can be controlled. It is possible
to excite wanted or to suppress unwanted modes, to perform their phase control or to generate
weighted superposition of the chosen fibre modes. In a realized experiment, the required changes
of the laser beam exciting the fibre are performed by means of computer-generated holograms
sent to a SLM. We demonstrate a generation of helical fibre modes with a phase dependence
exp\left(\text{i} m \varphi\right) on the azimuthal coordinate \( \varphi \). By the SLM, a selective excitation and switching of
modes with \( m = 0, 1 \) and 2 is realized. It is also verified that relative phases and amplitudes of
the modes used in superposition can be changed by a SLM. Experiments are performed with
100 m long commercial fibre.

2. Conversion of a fibre mode into a nondiffracting beam

2.1. Scalar theoretical concept

An ideal nondiffracting beam can be comprehended as an interference field created by plane
waves whose wave vectors \( \mathbf{k} \) have the same inclination \( \theta \) with respect to the beam axis
(figure 1). In that case, the relative phase differences of the plane waves remain constant
during propagation so that a complex amplitude of the interference field has a mode-like form,
\( U(x, y, z) = U_T(x, y, \alpha) \exp(-\text{i} \beta z) \). The complex amplitude exhibits oscillations depending
on a longitudinal propagation constant \( \beta = k \cos \theta \) but a transverse intensity profile \( |U|^2 = |U_T|^2 \)
remains unchanged during propagation. The size of a transverse spot depends on the

\[ \rho = \frac{\lambda v_0}{2} \]

Figure 1. A 4-\( f \) optical system enabling conversion of a guided fibre mode to a
free-space nondiffracting beam.
parameter $\alpha = k \sin \theta$; it is reduced if $\alpha$ increases. If the amplitudes and relative phase differences are the same for all interfering plane waves, the transverse intensity profile is rotationally symmetrical and can be expressed by the zero-order Bessel function of the first kind, $U(r, z) = J_0(\alpha r) \exp(-i\beta z)$, where $r^2 = x^2 + y^2$. By an appropriate azimuthal phase change of the interfering plane waves, the higher-order Bessel beam representing an ideal nondiffracting vortex beam is obtained,

$$U(r, \varphi, z) = J_m(\alpha r) \exp(i m \varphi - i\beta z),$$

where $\varphi = \arctan(y/x)$. Formally identical amplitude distribution can be obtained by taking into account the $LP_{m1}$ modes of a weakly guiding circular fibre. For each $m$, there are four modes with the electric fields whose transverse profiles are given by [13]

$$e_1 = J_m(\alpha r) [x \cos(m\varphi) - y \sin(m\varphi)],$$
$$e_2 = J_m(\alpha r) [x \cos(m\varphi) + y \sin(m\varphi)],$$
$$e_3 = J_m(\alpha r) [x \sin(m\varphi) + y \cos(m\varphi)],$$
$$e_4 = J_m(\alpha r) [x \sin(m\varphi) - y \cos(m\varphi)],$$

where $x$ and $y$ are unitary vectors. In a scalar theory, these modes have the same propagation constant $\beta$. If the polarization effects are involved, a small correction to $\beta$ must be performed for each mode. Nevertheless, the correction is the same for the modes with amplitudes $e_1$ and $e_3$. If they are superposed with a phase shift $\pm \pi/2$, the total propagating electric field can be written as $E = (e_1 + i e_3) \exp(-i\beta z)$. It can be rewritten to a form describing the circularly polarized field,

$$E = (x \pm iy) J_m(\alpha r) \exp(\pm im \varphi - i\beta z).$$

In a scalar approximation, a formal analogy between ideal nondiffracting beams (1) and fibre modes (6) is demonstrated. In both cases, the Fourier spectrum is given by the Dirac delta function $\delta(v - v_0)/v_0$, where the radial angular frequency is related to $\alpha$ by $v_0 = \alpha/(2\pi)$. At the Fourier plane of a lens with the focal length $f$, the spatial spectrum is represented by a circle with the radius $\lambda f/v_0$, where $\lambda$ is a wavelength. A transverse spatial shape of the nondiffracting beam or the fibre mode depends on an azimuthal modulation of the circle. Though mathematical expressions describing the amplitude profile and the spatial spectrum are identical for both an ideal nondiffracting beam and a fibre mode, the methods of their experimental realization are different.

A $\delta$-like spatial spectrum approving a free-space nondiffracting propagation is not exactly realizable in real experimental conditions. In place of a circle representing a single radial frequency, an annular ring with a finite width is obtained at the focal plane of the Fourier lens in real cases. Due to the spread of the spatial spectrum, phase differences of plane wave components slightly change under propagation and an invariance of the beam intensity profile is lost. Nevertheless, an experimentally realizable beam represents a good approximation of the ideal nondiffracting beam. It is known as a pseudo-nondiffracting beam. The simplest experiment
providing such a beam is based on application of an annular Fourier filter placed at the front focal plane of a lens.

In the case of a fibre mode, the required δ-like spectrum is obtained due to the total reflections of the rays. If the fibre mode leaves the fibre end, it is strongly influenced by diffraction effects. Its spatial spectrum is changed to an annular ring whose width depends on a mode bounding by the fibre end face. In real cases, the ring is very wide and a fibre mode propagates with a large divergence in a free space. The mode leaving the fibre can be converted into a free-space pseudo-nondiffracting beam by a 4-f optical system illustrated in figure 1. The spatial spectrum of a divergent fibre mode is localized at the back focal plane of the first Fourier lens and sharpened by Fourier filtering in such a way that it again approximates an initial δ-like form. After Fourier transform realized by the second lens, a pseudo-nondiffracting beam is obtained. A natural question is how its spatial shape approximates a transverse profile of the original mode guided by the fibre.

The similarity of both fields can be examined in a simplified model. As a scalar guided field, a coherent superposition of higher-order fibre modes is assumed,

\[ U(r) = \exp(-i\beta z) \sum_{m=-M}^{M} q_m J_m(\alpha r) \exp(i m \phi). \]  

It represents a coherent mixture of the fibre modes (6) with weights \( q_m \) and transverse profiles given by the Bessel function \( J_m \). That field is restricted by a Gaussian aperture simulating an influence of the fibre end face. The beam spatial spectrum appearing at the focal plane of the first Fourier lens is filtered so that only the components related to the single radial angular frequency \( \nu_0 \) are transmitted. A pseudo-nondiffracting beam approximating an initial fibre mode is obtained behind the second Fourier lens. Its complex amplitude can be written as

\[ U'(r) = U'_0 \exp(-i\beta z) \sum_{m=-M}^{M} q'_m J_m(\alpha r) \exp(i m \phi), \]  

where

\[ q'_m = Q_m q_m, \quad Q_m = (-1)^m J_m \left( \frac{i\pi \alpha^2 a^2}{2} \right), \]

and \( U'_0 \) and \( a \) denote a constant amplitude and a waist radius of the Gaussian aperture, respectively. As is obvious from a comparison of (7) and (8), the influence of diffraction effects is not completely eliminated by the spatial filtering, because the spatial structures of the original fibre mode and the created free space nondiffracting beam are not identical. A spatial distortion remaining in the nondiffracting beam is expressed by the coefficients \( Q_m \) which modify the original weights \( q_m \). The influence of \( Q_m \) depends on the complex argument of the Bessel function \( J_m \). For large values of the argument, the Bessel function \( J_m \) provides nearly the same values for all \( m \). If the initial beam is simultaneously composed of the modes related to the indices \( m = 2m' \) or \( m = 2m' + 1 \), the factor \( (-1)^m \) has the same value for all superposed modes and the coefficient \( Q_m \) can be placed in front of the sum. In that case, the influence of diffraction is removed and the original fibre mode is exactly reconstructed. For small values of the argument, an apparent distortion can be observed. The parameters \( \alpha \) and \( a \) appearing in the argument represent the spot size of the original beam and the waist radius of the Gaussian aperture. The beam distortion depends on their ratio and can be examined numerically.
2.2. Numerical example

In a numerical simulation, we examine two-mode fibre field (7) whose weight coefficients are given as \( q_m = 1/2 \) and \( q_{-m} = (-1)^m/2 \). Its complex amplitude then exhibits a cosine modulation, 
\[
U = J_m(\alpha r) \cos(m\varphi) \exp(-i\beta z)
\]
valid for \( r < a \), where \( a \) is the fibre radius. At the fibre end face, the guided mode diffracts and propagates with a significant spreading depending on the ratio of the mode spot size and the fibre radius. To transform it into the free space nondiffracting beam, the spatial filtering realized by the 4-\( f \) optical system shown in figure 1 must be applied. Unlike the simplified model used in analytical calculations, a realistic situation is taken into account in a numerical analysis. The influence of the fibre end face is simulated by a hard circular aperture and finite apertures of the Fourier lenses are also included. A \( \delta \)-like spatial filter is replaced by an annular ring of a finite width. In that case, the pseudo-nondiffracting beam appears behind the 4-\( f \) optical system. Unlike an ideal nondiffracting beam, its propagation range \( L \) is finite. If both lenses have the same focal length \( f \), it can be approximated by 
\[
L \approx \frac{\rho_L}{\alpha} \frac{k}{2} \quad \text{and} \quad \frac{2L}{3}.
\]

3. Controllable excitation and switching of fibre modes

3.1. Basic principle

A few-mode fibre with known parameters is able to guide a well-defined spectrum of spatial modes. Which modes are indeed excited depend on the conditions of the fibre illumination. Applying a scalar approximation, we examine amplitudes of the guided fibre modes depending on the spatial structure of a beam coupled to the fibre. As the main result, an experimental method enabling selective excitation and switching of fibre modes is proposed and verified.

If we use the circular cylindrical coordinates, a complex amplitude \( U(r, \varphi) \) of a field focused to the fibre can be expressed by its spatial spectrum \( u(v, \psi) \) as

\[
U(r, \varphi) = \int_0^\infty \int_0^{2\pi} u(v, \psi) \exp(-i2\pi vr \cos(\psi - \varphi)) dv \, d\psi.
\]
Figure 2. Transformation of a fibre mode-like field $U = J_2 \cos(2\varphi) \exp(-i\beta z)$ in the 4-f optical system (the field is bounded by a circular aperture simulating a fibre end face): (a) intensity profile of the bounded fibre mode, (b) spatial spectrum at the Fourier plane, and (c), (d) intensity profiles at the distances $z = L/3$ and $2L/3$, respectively.

Taking into account an azimuthal periodicity of the spatial spectrum $u(v, \psi) = u(v, \psi + 2\pi)$ and substituting $\alpha = 2\pi v$, we can write

$$U(r, \varphi) = \frac{1}{2\pi} \int_0^\infty \sum_{m=-\infty}^{+\infty} u_m(\alpha) J_m(\alpha r) \exp(i m \varphi) d\alpha.$$  (10)

The fibre illumination is now expressed as an infinite superposition of the nondiffracting Bessel beams with the weights $u_m$. They can be comprehended as vortices with the topological charges $m$ changing from $-\infty$ to $+\infty$. Each nondiffracting beam is specified by transverse and longitudinal constants $\alpha$ and $\beta$ related as $\beta = (k^2 - \alpha^2)^{1/2}$. Excluding evanescent waves, the illuminating field is obtained as an integral over the all nondiffracting fields with propagation constants $\beta$ continuously distributed in the interval $\beta \in (0, k)$. The spatial modes of a weakly guiding
Figure 3. Conversion of the bounded fibre mode to a pseudo-nondiffracting beam realized by a filtration in the 4-\( f \) optical system: (a) intensity profile of the bounded fibre mode, (b) spatial spectrum filtered by an annular ring, and (c), (d) intensity profiles of the pseudo-nondiffracting beam at the distances \( z = L/3 \) and \( 2L/3 \), respectively.

Fibre are formally identical with the fields appearing under integral (9), but their transverse and longitudinal propagation constants are not arbitrary. They are obtained as a solution of the dispersion equation. Each azimuthal index \( m \) is related to a spectrum of the transverse and longitudinal propagation constants \( \alpha_{mp} \) and \( \beta_{mp} \). From that point of view, the optical fibre serves as a spatial filter accepting only nondiffracting components of the illuminating field with proper propagation constants. Equation (10) provides a simple relation between a complex amplitude \( U \) of a laser beam illuminating the fibre and amplitudes \( u_m \) of the excited fibre modes. The amplitudes \( u_m \) can be expressed in terms of the complex amplitude \( U \),

\[
    u_m(\omega) = \frac{1}{(2\pi)^2} \int_0^\infty \int_0^{2\pi} U(r, \varphi) J_m(\alpha r) \exp(-im\varphi) r \, dr \, d\varphi. \tag{11}
\]
This relation is useful for a controllable excitation of fibre modes. To demonstrate it, we assume that the fibre is illuminated by a focused laser beam whose complex amplitude \( U \) can be written as

\[
U(r, \varphi) = U(r) \sum_{n=-N}^{+N} q_n \exp(\text{i} n \varphi).
\] (12)

Substituting (12) into (11), the integral over \( \varphi \) results in

\[
\int_0^{2\pi} \exp \left[ \text{i}(n - m)\varphi \right] d\varphi = 2\pi \delta(n - m).
\] (13)

If the fibre is illuminated by a focused Gaussian beam possessing rotational symmetry, its complex amplitude is given by (12) used with \( N = 0 \). In that case, the only nonzero weight coefficient is \( u_0 \),

\[
u_0(\alpha) = \frac{q_0}{2\pi} \int_0^\infty U(r) J_0(\alpha r) r \, dr, \quad \text{for } N = 0.
\] (14)

In a real case, the coefficients \( u_m \) would also be nonzero but small in comparison with \( u_0 \) due to lateral and/or angular misalignment of the free-space beam axis and fibre core axis. A completely different situation appears if the field with the helical wavefront is focused to the fibre (\( N \neq 0 \)). In that case, the higher-order fibre modes are excited with significant weight coefficients \( u_m \) given by

\[
u_m(\alpha) = \frac{q_m}{2\pi} \int_0^\infty U(r) J_m(\alpha r) r \, dr, \quad \text{for } N \neq 0.
\] (15)

As is obvious, the higher-order fibre modes are excited if the laser beam focused to the fibre represents a mixed vortex field (12) composed of the helical phase components with the topological charges \( n \). Their weight coefficients \( q_n \) are then directly related to the amplitudes of the excited fibre modes \( u_m \) given by (15). By a convenient modulation of the laser beam focused to the fibre, the wanted modes can be excited and unwanted modes suppressed. An excitation of a weighted superposition of the fibre modes is also possible. The required field (12) illuminating the fibre can be realized experimentally by a transmission of a standard Gaussian beam through a complex mask or by its shaping by a SLM. A mathematical description of the laser beam transmitted through the complex mask and focused to the fibre is presented in the appendix. A method based on the spatial light modulation is discussed in the next section devoted to experimental results.

The method enabling creation of vortex beams by a stressed optical fibre [11] is also in agreement with a model of vortex fibre-coupling proposed in this paper. In the region where a stress is applied, an amplitude of the propagating fibre mode is locally redistributed and serves as a boundary condition for the following propagation. The redistributed field plays the same role as an illuminating field in the presented model. Its decomposition requires the higher azimuthal components with significant weights and continuous spatial frequencies. The fibre accepts only such components whose spatial frequencies are related to the propagation constants of the acceptable modes. If they are present in the boundary field distribution, the higher-order modes of the fibre are successfully excited.
3.2. Experimental realization

In the experiment, the commercially available fibre ‘Corning SMF-28’ was used to verify the proposed selective excitation of the fibre modes. It works as a single-mode fibre at the communication wavelengths $\lambda = 1310–1550$ nm, but in our experiment it was illuminated by a linearly polarized He–Ne laser beam with $\lambda = 632.8$ nm. In that case, the fibre can guide the linearly polarized modes $LP_{01}$, $LP_{11}$ and $LP_{21}$. As follows from the presented theory, the fibre must be illuminated by a focused helical beam (12) to excite its higher-order modes. In the experiment, the laser beam focused to the fibre is shaped by a SLM. The used set-up is illustrated in figure 4. The He–Ne laser beam is expanded and spatially filtered to achieve a homogeneous plane-wave-like illumination of the computer-generated hologram created in an amplitude SLM (CRL Opto, 1024 × 768 pixels). The diffracted beam is then transformed by the front Fourier lens FL I of the 4-f optical system ($f_i = 500$ mm). The Fourier spectrum composed of 0th, +1st and −1st diffraction orders can be observed at its back focal plane. By means of a spatial filter, the unwanted 0th and −1st diffraction orders are removed. The required beam modulation is encoded into the transmitted +1st diffraction order. The light localized in the +1st diffraction order is transformed by the back Fourier lens FL II ($f_{II} = 500$ mm) so that a collimated beam is obtained. It is strongly focused to the fibre by an aspherical lens AL ($f_{AL} = 11$ mm). The spatial modulation of the laser beam can be described mathematically. The hologram is created by an interference of a signal beam $U$ given by (12) with an inclined reference plane wave $U_R = u_R \exp [-i k(x \sin \gamma + z \cos \gamma)]$, where $\gamma$ denotes the inclination angle of the transverse component of the wave vector with respect to the $z$-axis. Its intensity profile is sent as a bitmap to an SLM. It is illuminated by a collimated laser beam which can be approximated by a plane
wave, \(U_r = u_r \exp(-ikz)\). The complex amplitude of the beam transmitted through a SLM can be written as \(U_T = U_r|U + UR|^2\). Its Fourier spectrum localized at the back focal plane of the lens FL I then can be expressed in the form \(\overline{U}_T = \overline{U}_0 + \overline{U}_{+1} + \overline{U}_{-1}\), where \(\overline{U}_0\), \(\overline{U}_{+1}\) and \(\overline{U}_{-1}\) represent the diffraction orders \(0\), \(+1\) and \(-1\), respectively. If we assume that the hologram is created at a distance \(z = 0\), we can write

\[
\overline{U}_0 = F^i\{U_r(|U|^2 + |UR|^2)\},
\]

\[
\overline{U}_{+1} = u_R^\ast u_r \overline{U}(v_x + v_s, v_y),
\]

\[
\overline{U}_{-1} = u_Ru_r \overline{U}^\ast(v_x - v_s, v_y),
\]

where \(v_x = \sin(\gamma/\lambda)\) and \(\lambda\), \(F^i\) and \(\overline{U}\) denote the wavelength, the Fourier transform and the spatial spectrum of the signal field \(U\), respectively. A change of the inclination angle \(\gamma\) enables a sufficient separation of the diffraction orders at the focal plane. In this way, the undesirable diffraction orders \(0\) and \(-1\) can be removed. The spatial spectrum of the signal beam is represented by the diffraction order \(+1\). After an inverse Fourier transform realized by the lens FL II, the collimated beam with the complex amplitude \(U_T\) approximating the profile of the required beam \(U\) is obtained. The aspherical lens \(AL\) is used to focus the beam into the fibre. An exchange of holograms sent to the SLM enables a dynamical modulation of the beam resulting in a selective excitation and switching of the guided fibre modes.

A possibility to control the excited modes was verified in the following way. The SLM was addressed by a hologram creating a common Gaussian beam. It was focused to the fibre and precisely adjusted to remove any significant symmetry imperfections and to excite the basic \(LP_{01}\) mode. The mechanical set-up was kept fixed and the SLM was successively addressed by holograms created by the signal beams with variable helical wavefronts

\[
U = [\cos(n\varphi) + \exp(i\Delta\varphi_n) \sin(n\varphi)] \exp(-r^2/w_0^2), \quad n = 0, 1, 2.
\]

Their complex amplitude follows from (12) used with two nonzero coefficients given as

\[
q_n = \frac{1}{2}[1 - i \exp(i\Delta\varphi_n)],
\]

\[
q_{-n} = \frac{1}{2}[1 + i \exp(i\Delta\varphi_n)].
\]

As follows from the presented theory, the laser beam modulated by the hologram realized with \(n = 0\) excites the basic \(LP_{01}\) fibre mode, while the laser beams with the helical phases defined by the topological charges \(n = 1\) and \(2\) excite the higher-order fibre modes \(LP_{11}\) and \(LP_{21}\). From the performed experiment, the theoretical prediction was fully proved. An exchange of holograms sent to the SLM resulted in switching of basic and higher-order fibre modes without changes of the set-up. As the used fibre does not preserve polarization, four polarization modes \((2)\)–\((5)\) appeared for each index \(n\) of the \(LP_{n1}\) modes. Their relative phases were changed by the phase \(\Delta\varphi_n\) encoded into the hologram. The results obtained with 100 m long fibre Corning SMF-28 are illustrated in figures 5–7. In figure 5(a), the hologram used for an excitation of the \(LP_{01}\) fibre mode is shown. A spatial spectrum of a Gaussian beam transmitted through an SLM is shown in figure 5(b). After spatial filtering in the 4-\(f\) system, a collimated beam (figure 5(c))
is obtained. A collimated fibre output beam is shown in figure 5(d). A similar situation obtained for higher-order fibre modes $L_P_{11}$ and $L_P_{21}$ is shown in figures 6 and 7. Each $L_P_{n1}$ mode is composed of the modes (2)–(5). Their phase coupling can be changed by the phase $\Delta \varphi_n$ written into the hologram. A continuous change of the phase $\Delta \varphi_n$ in the range $\langle 0, 2\pi \rangle$ is demonstrated in movie 1 and movie 2. In the former case, we illustrate holograms, fibre inputs and fibre outputs for the $L_P_{11}$ mode; in the latter case, the same situation is demonstrated for the $L_P_{21}$ mode. By a convenient choice of the coefficients $q_n$ used in the preparation of the holograms, a predetermined superposition of different $L_P_{n1}$ modes can also be created by means of a SLM.

A robustness of the excited vortex fibre modes was examined by applying a three-loop polarization controller. An introduced fibre bending and squashing resulted in a change of relative phases of polarization modes. If a moderate bending and squashing allowing a stable propagation of modes was applied, no stress-induced switching of the vortex modes was observed.

4. Conclusions

In this paper, a mutual conversion of the free space nondiffracting vortex beams and the guided fibre modes was examined. In particular, a method enabling selective excitation of fibre modes was proposed, explained by a simplified scalar theory and verified experimentally. The method is based on the possibility of expressing a laser beam focused to the
Figure 6. The same as in figure 5, but for an excitation of the $LP_{11}$ fibre mode.

Figure 7. The same as in figure 5, but for an excitation of the $LP_{21}$ fibre mode.
fibre as a superposition of ideal nondiffracting beams possessing a formal analogy with the guided fibre modes. By a convenient modulation of the illuminating beam, the amplitudes of the excited fibre modes can be controlled. In the realized experiment, the beam modulation is performed by an amplitude SLM. In this way, it is possible to excite wanted or to suppress unwanted fibre modes and to perform their dynamical switching. In the experiment, a dynamical switching of $LP_{01}$, $LP_{11}$ and $LP_{21}$ modes was demonstrated. A commercial fibre with a length of 100 m was used in the experiment. It was also demonstrated that the phases of four polarization modes creating each $LP_{n1}$ mode can be changed by a phase term encoded into the hologram. In this way, the output fibre mode with a well-defined helical phase can be generated. The excited fibre modes are stable, and moderate fibre bending and introduced stress change only phases of the four constituents of the excited $LP_{n1}$ mode and do not cause switching of $LP$ modes with different $n$. The superposition of different $LP_{n1}$ driven by the SLM can also be created. The fibre output beam then represents a mixed vortex field which is perspective for information encoding [9, 10].

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Appendix

The mixed vortex field (12) is obtained if a collimated Gaussian beam $U_G$ is transmitted through a complex mask adjacent to a focusing lens with the transparency $t_L$. The transparency of the mask is given by

$$t_M(\varphi) = \sum_{n=-N}^{N} q_n \exp(i n \varphi), \quad (A.1)$$

where $q_n$ are the weight coefficients.

The complex amplitude of the beam transmitted through the complex mask and the lens can be written as

$$U_0(r, \varphi, 0) = \sum_{n=-N}^{N} q_n \exp(-Wr^2 + i n \varphi), \quad (A.2)$$

where

$$W = \left( \frac{1}{w_0^2} - i \frac{k}{2f} \right) \quad (A.3)$$

and $w_0$ denotes the waist radius of the beam impinging on the complex mask. The Fourier spectrum of the field at a distance $z = Z$ behind the lens can be expressed as

$$u(\nu, \psi, Z) = 2\pi H(\nu, Z) \sum_{n=-N}^{N} q_n \exp(i \nu \psi) \int_{0}^{\infty} \exp(-Wr^2) J_n(2\pi \nu r) r \, dr, \quad (A.4)$$
where $H(n, Z) = \exp(-ikZ + i2\pi\lambda Zv^2)$. The complex amplitude of the field illuminating the fibre placed at the focal plane of the lens is of the form (12)

$$U(r, \varphi) = U(r) \sum_{n=-N}^{N} q_n \exp(i n \varphi), \quad \text{ (A.5)}$$

where

$$U(r) = (2\pi)^2 \int_{0}^{\infty} H(n, f) J_n(2\pi vr) \int_{0}^{\infty} t_F(r') \exp(-W r'^2) J_n(2\pi vr') r' dr' \, dv \quad \text{ (A.6)}$$

and $t_F$ denotes a fibre aperture function.

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