A Fractal Permeability Model for Shale Oil Reservoir

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Abstract. In this work, a fractal analytical model is proposed to predict the permeability of shale reservoir. The proposed model explicitly relates the permeability to the micro-structural parameters (tortuosity, pore area fractal dimensions, porosity and slip velocity coefficient) of shale.

1. Introduction

With the depletion of conventional resources, the efficient development of unconventional resources, such as shale oil reservoirs, has become a hot topic. Shale oil reserves are abundant worldwide, and the US has seen a shale oil boom in recent years, attracting the attention of the whole world. Rich organic matter and nanoporosity are the main characteristics that distinguish shale oil reservoirs from other resources. And, MDS (molecular dynamics simulations) was the powerful tool needed for preliminary studies [1]. Mattia and Calabro derived an expression for the slip velocity of water flow in CNT (carbon nanotube). Based on this expression, a theoretical formula describing flow enhancement was achieved. The formula can incorporate the effects of solid-liquid interactions via surface diffusion and the work of adhesion, as well as the geometric characteristics of the tubes. The effects of surface roughness and mixed wettability on the flow are included in the flow enhancement model, superior to the MDS method [2]. Cui et al. derived a flow enhancement model incorporating boundary slip and physical adsorption. Based on this model, sensitivity analysis are conducted, and the contributions of physical adsorption and boundary slip to flow are studied. Employing the concept of normalized velocity, the shape of the velocity profile in the organic nanopores is studied quantitatively as a function of the pore length and pore radius [3].

Although many researchers have studied the transport properties of organic capillary bundle through experimental investigation and numerical simulations, analytical studies on both organic matter and inorganic matter are needed to further elucidate the permeability of natural core. In this work, a fractal analytical model was proposed to predict the permeability of shale oil reservoir.

2. Fractal Characteristics of Capillary Bundle in Shale Oil Reservoir

2.1. Liquid permeability of organic nanopores in shale

Based on the result of relevant MDS, including the mechanisms of boundary slip physical adsorption and complicated structural properties, a mathematical model was developed to calculate the liquid permeability of organic nanopores in shale.
\[ q_o = \xi_c \left[ \frac{\rho_{ads}}{\rho_{bulk}} (1 - \xi_{ads}) + \xi_{ads} \right] \frac{\pi R^4 \Delta p}{8 \mu L} \]  

Where \( q_o \) is volume flux in organic capillary bundle, \( \rho_{ads} \) is the density of physically adsorbed oil, \( \rho_{bulk} \) is the density of free oil, \( R \) is pore radius, \( \Delta p \) is pressure difference, \( \mu \) is liquid viscosity, \( L \) is pore length. \( \xi_c \) is flow enhancement, and is expressed as 

\[ \xi_c = 1 + \frac{8 \mu C}{R^2} = 1 + \frac{8 \mu D_t L}{R^2 W_t} \]

Where \( D_t \) is surface diffusion, \( W_t \) is adhesion work. And \( \xi_{ads} \) is adsorption factor, is expressed as 

\[ \xi_{ads} = (1 - \frac{h}{R})^2 \]

Where \( h \) is the thickness of the adsorption region [3].

2.2. Fractal Characteristics of porous media

A porous medium that has various pore sizes can be considered as a bundle of tortuous capillary tubes with variable cross-sectional areas. Let the diameter of a capillary in the medium be \( \lambda \) and its tortuous length along the flow direction be \( L_t(\lambda) \). Due to the tortuous nature of the capillary, \( L_t(\lambda) \geq L_0 \), with \( L_0 \) being the representative length. For a straight capillary, Chen and Yu developed a fractal tortuosity relationship of fluid flow through porous media

\[ L_t(\lambda) = \lambda^{1-D_T} L_0^D_T \]  

Where \( D_T \) is the tortuosity fractal dimension, with 1<\( D_T <2 \), representing the extent of convolutedness of capillary pathways for fluid flow through a medium [4].

Besides, the number of capillary pathways whose pore size is \( \lambda \) are another important properties. For the fractal porous media, the cumulative size distribution of pores should follow the fractal scaling law

\[ -dN = D_p \lambda_{\text{max}}^{D_p} \lambda^{-(D_T+1)} d(\lambda) \]  

Where \( D_p \) is the fractal dimension of an object. \( N \) is the total number of capillary bundle, \( \lambda_{\text{max}} \) is the size of the maximum pore.

Fig. 1 Fractal models for shale oil through porous medium

2.3. Fractal permeability of capillary tubes

Consider a unit cell, which consists of a bundle of tortuous capillary tube in Fig. 1. The total flow rate \( Q \) can be obtained by integrating the individual flow rate \( q_o \) over the entire range of pore sizes from the minimum \( \lambda_{\text{min}} \) to the maximum \( \lambda_{\text{max}} \) in a unit cell. According to Eqs. (1), (2) and (3), we have
\[ Q = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} q(\lambda) dN(\lambda) = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} \left[ \frac{\rho_{\text{ref}}}{\rho_{\text{ref}}} (1 - \frac{\xi}{\rho_{\text{ref}}} + \frac{\xi}{\rho_{\text{ref}}}) \right] \pi \lambda^{3+D_T} \Delta p \frac{D_p \lambda_{\text{max}}^3}{128 \mu L_0^3} \left[ 1 - \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \left( \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \right)^{3+D_T - 2D_p} \right] d(\lambda) \]

\[ = \frac{\pi}{128} \frac{\Delta p L_0^{-D_p}}{\mu} \frac{D_p}{3 + D_T - D_p} \lambda_{\text{max}}^{3+D_T} \left[ 1 - \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \left( \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \right)^{3+D_T - 2D_p} \right] \]

\[ + \frac{2\pi}{128} \frac{\Delta p L_0^{-D_p}}{C} \frac{D_p}{1 + D_T - D_p} \lambda_{\text{max}}^{3+D_T} \left[ 1 - \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \left( \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \right)^{3+D_T - D_p} \right] \]

Since \( 1 < D_T < 2 \) and \( 1 < D_p < 2 \), the exponent \( 3 + D_T - 2D_p > 0 \). Also, because \( \lambda_{\text{min}}/\lambda_{\text{max}} \sim 10^{-2} \) and \( (\lambda_{\text{min}}/\lambda_{\text{max}})^{D_p} \sim 0 \), therefore \( 0 < \left( \lambda_{\text{min}}/\lambda_{\text{max}} \right)^{3+D_T - 2D_p} < 1 \). It follows that Eq. (4) can be reduced to

\[ Q = \frac{\pi}{128} \frac{\Delta p L_0^{-D_p}}{\mu} \frac{D_p}{3 + D_T - D_p} \lambda_{\text{max}}^{3+D_T} \left[ 1 - \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \left( \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}} \right)^{3+D_T - 2D_p} \right] \]

For creeping flow through a porous medium, using Darcy law, we obtained the expression for calculating the permeability of a porous medium as follow

\[ k = \frac{\mu L_0 Q}{\Delta p A} = \frac{\pi}{128} \frac{D_p}{3 + D_T - D_p} \lambda_{\text{max}}^{3+D_T} + \frac{2\pi}{128} \frac{\mu L_0^{-D_p}}{C} \frac{D_p}{1 + D_T - D_p} \lambda_{\text{max}}^{3+D_T} \]

The pores in a cross section can be considered as circles with different diameters \( \lambda \). Consequently, the total pore area in the cross section \( A_p \) can be obtained with the aid of Eq. (3).

\[ A_p = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} \pi \left( \frac{\lambda}{2} \right)^2 dN(\lambda) = \frac{\pi}{4} \frac{D_p A_{\text{max}}^2}{2 - D_p} (1 - \phi) \lambda_{\text{max}}^2 \]

Whereas the cross sectional area \( A \) is [5]

\[ A = \frac{A_p}{\phi} \frac{D_p}{4} \frac{(1 - \phi) \lambda_{\text{max}}^2}{\phi} \]

Xiao and Yu considered that the maximum hydraulic diameter of capillary can be expressed as a function of average fiber diameter, porosity and the area fractal dimensions of pores [6]

\[ \lambda_{\text{max}} = \frac{\phi}{1 - \phi} \left( \frac{4 - D_p}{D_p} \right)^{\frac{1}{D_p}} \lambda_f \]

Where \( \lambda_f \) is the average pore size. By inserting Eqs. (8), (9) into Eq. (6), we can obtain an analytical model for calculating the fractal permeability

\[ K = \frac{2^{D_p} \pi}{64} \frac{1}{(3 + D_T - D_p) D_p^2} \left( \frac{1}{1 - \phi} \right)^{\frac{1+D_T}{D_T}} \lambda_f^2 + \frac{\pi}{64} \frac{\mu C}{1 + D_T - D_p} \frac{\phi}{\pi D_p} \left( \frac{1+D_T}{D_T} \right) \frac{1}{3 + D_T - D_p} \]

It indicates that permeability is a function of pore fractal dimension \( D_p \), tortuosity fractal dimension \( D_T \) and structural parameters, \( \phi \), \( \lambda_f \) and so on. Besides, the fractal permeability is very sensitive to
average pore size and positively correlated with the average pore size and porosity. It can be seen that the present fractal intrinsic permeability do not contain any empirical constant, every parameter of the proposed model has clear physical meaning, which is closely related to the microstructures in porous media.

If we ignore the influence of organic matter, Eq. (10) can be reduced to

\[
K = \frac{2^{D_f} \pi^{\frac{1}{2}}}{64} \frac{(4 - D_p)^{\frac{1}{2}}}{(3 + D_T - D_p)D_p} \left(\frac{\phi}{1 - \phi}\right)^{\frac{2 + D_T}{2}} d_f^2
\]

(11)

3. Results and discussions

We first compared our fractal permeability values, which based on the fractal tortuous capillary model, with Kozeny-Carman equation and the experimental data by Sun et al. [7] (at $\phi=0.04$–$0.23$). The results are presented in Fig. (2). It is obvious that our fractal results are in better agreement with experimental data than the classic Kozeny–Carman equation. Fig. 1 also shows that the permeability of the present model is higher than those ignore the influence of organic matter. As the porosity increases, the gap becomes smaller. Because when the porosity is relatively low, organic pores are more developed and the ratio of organic pores is higher, which enhances the permeability of the core.

![Fig. 2. A comparison between the permeability from the present fractal model, Kozeny-Carman Equation and the experimental data.](image)

Fig (3) plots the fractal permeability at three different levels of $D_T$ against varying $D_p$. It is seen there is a small decrease in the fractal permeability when pore fractal dimension $D_p$ and tortuosity fractal dimension $D_T$ increase.
4. Conclusion
In this work, a fractal analytical model is proposed to predict the permeability of shale reservoir. The proposed model explicitly relates the permeability to the micro-structural parameters (tortuosity, pore area fractal dimensions, porosity and slip velocity coefficient) of shale. There is no empirical constant and every parameter in the proposed model has specific physical significance. The present model based on the fractal geometry theory is more closely related to the microstructures in porous media than those obtained by conventional methods. It is found that the permeability strongly depend on porosity and average pore size. Besides, there is only a small increase in permeability when pore fractal dimension and tortuosity fractal dimension decreases.

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