The Roper resonance as a meson-baryon molecular state

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Abstract. The recently proposed mechanism for the formation of the Roper resonance, in which a dynamically generated state as well as a genuine three-quark resonant state play an equally important role, is confronted with the model proposed almost twenty years ago in which the Roper is pictured as a molecular state of the nucleon and the $\sigma$ meson.

Our recent investigation on the nature of the Roper resonance [1] has been motivated by the results of lattice QCD simulation in the P11 partial wave by the Graz-Ljubljana and the Adelaide groups [2,3] that have included beside three-quark interpolating fields also operators for $\pi N$ in relative p-wave and $\sigma N$ in s-wave, and have found no evidence for a dominant three-quark configuration below 1.65 GeV. In our research we use a coupled channel approach which has been previously successfully applied to describe meson scattering and photo- and electro-production in several partial waves in the intermediate energy region [4,5,6,7,8,9]. In the present analysis of the Roper resonance we include the $\pi N$, $\pi \Delta$, and $\sigma N$ channels and solve the Lippmann-Schwinger equation for the meson amplitudes to all orders in the approximation of a separable kernel. We have concluded that while the mass of the resonance is determined by the dynamically generated state, an admixture of the $(1s)^2(2s)^1$ component at an energy around 2 GeV turns out to be crucial to reproduce the experimental width and the modulus of the resonance pole. The mass of the dynamically generated state appears typically 100 MeV below the (nominal) nucleon-sigma threshold. This result agrees well with the prediction of a completely different approach that we studied in the 2001 paper [10] in which we discussed the possibility that the Roper was a molecular state of the nucleon and the $\sigma$ meson. In the following we review the main features of this molecular state and its relation to the dynamically generated state emerging in the coupled channel approach.

In our early approaches to describe the nucleon and the $\Delta(1232)$ we used a chiral version of the linear $\sigma$-model with quarks and determined the quark and meson fields self-consistently. This model does not work for higher nucleon excitations since the energy of the excited quark turns out to be higher than the free quark mass. In order to ensure confining we used in [10] a chiral version of the Cromodielectric model which included, beside the $\sigma$ and the pion fields, the chromodielectric field $\chi$. The coupling of the $\chi$ field to the quark and meson fields is
taken in the form:

$$\mathcal{L}_{\text{int}} = \frac{g}{\chi} \bar{q}(\hat{\sigma} + i\vec{r} \cdot \vec{\tau} \gamma_5)q,$$

(1)

such that for \( r \to \infty, \chi(r) \to 0 \), while the quark mass in this limit behaves as

$$m_q = \frac{g\sigma(r)}{\chi(r)} = \frac{gf_{\pi}}{\chi(r)} \to \infty,$$

which means that the quarks are bound. A typical self-consistent solution for the fields is shown in Fig. 1a).

We next expanded the field operators of the bosons around their expectation values in the ground state \( |N\rangle \); the \( \sigma \) operator can be written as:

$$\hat{\sigma}(r) = \sum_n \frac{1}{\sqrt{2\varepsilon_n}} \varphi_n(r) \frac{1}{\sqrt{4\pi}} [\tilde{a}_n + \tilde{a}_n^\dagger] + \sigma(r), \quad \tilde{a}_n|N\rangle = 0.$$

The stability conditions implies a Klein-Gordon equation for the \( \sigma \)-meson modes:

$$(-\nabla^2 + m_\sigma^2 + U_\sigma(r)) \varphi_n(r) = \varepsilon_n^2 \varphi_n(r), \quad U_\sigma(r) = \frac{d^2V(\sigma(r))}{d\sigma(r)^2}.$$

Here \( V \) stands for the potential originating from (1) and the potential parts of the \( \sigma \)-model. The potential \( U_\sigma \) (see Fig. 1b)) is attractive and supports a bound state which can be interpreted as a molecular state of the nucleon and (one quantum of) the \( \sigma \). The corresponding potential for the \( \chi \) field turns out to be repulsive, which means that the model does not predict glueball states.

In [10] this excitation of the \( \sigma \) field was confronted with the excitation of the quark core in which one quark was promoted to the \( 2s \) orbit. In the self-consistent solution the \( 2s - 1s \) energy splitting turned out to be smaller than the
corresponding vibrational energy $\epsilon_1$, and the conclusion of our work was that the Roper consisted of the dominant quark excitation and a $\sim 10\%$ admixture of the molecular state. However, in that work we used – in accordance with then accepted values – a relatively large $\sigma$ mass between $0.7$ GeV and $1.2$ GeV. With the present value $\sim 0.5$ GeV, the lowest eigenmode $\epsilon_1$ decreases (see Fig. 1b), while, assuming a somewhat smaller nucleon size, the $2s-1s$ splitting increases, such that the molecular state may eventually become the dominant component of the Roper resonance.

In our recent paper [1] we study the formation of the resonance in this partial wave in a coupled-channel approach including the $\pi N$, $\pi \Delta$ and $\sigma N$ channels. The Cloudy Bag Model is used to fix the quark-pion vertices while the $s$-wave $\sigma$-baryon vertex is introduced phenomenologically with the coupling strength $g_\sigma$ as a free parameter and two choices for the mass and the width of the $\sigma$ meson, $m_\sigma = \Gamma_\sigma = 0.6$ GeV and $m_\sigma = \Gamma_\sigma = 0.5$ GeV. Labeling the channels by $\alpha, \beta, \gamma$, the Lippmann-Schwinger equation for the meson amplitude $\chi_{\alpha \gamma}$ for the process $\gamma \to \alpha$ can be cast in the form:

$$\chi_{\alpha \gamma}(k_\alpha, k_\gamma) = K_{\alpha \gamma}(k_\alpha, k_\gamma) + \sum_\beta \int d k \frac{K_{\alpha \beta}(k_\alpha, k)\chi_{\beta \gamma}(k, k_\gamma)}{\omega(k) + E_\beta(k) - W}.$$ 

Approximating the kernel $K$ by a separable form, the integral equation reduces to a system of linear equations which can be solved exactly. For sufficiently strong coupling $g_\sigma$ the kernel $K$ may become singular and a (quasi) bound state arises.

![Fig. 2](image.png)

**Fig. 2.** The lowest eigenvalue $w_{\text{min}}$ for four different values of the $\sigma N$ coupling.

**Table 1.** Poles in the complex $W$-plane for four typical values of $g_\sigma$. The PDG values are from [11].

| $g_\sigma$ | Re$W_p$ [GeV] | −2Im$W_p$ [GeV] |
|-----------|---------------|-----------------|
| PDG       | 1.370         | 0.175           |
| 1.80      | 1.397         | 0.157           |
| 1.95      | 1.383         | 0.112           |
| 2.00      | 1.358         | 0.111           |
| 2.05      | 1.331         | 0.044           |
|           | 1.438         | 0.147           |

In order to study this process we follow the evolution of the lowest eigenvalue of the matrix pertinent to the system of linear equations, $w_{\text{min}}$, as a function of $W$ for different values of $g_\sigma$ (see Fig. 2). Along with this evolution we observe the evolution of the resonance $S$-matrix pole in the complex $W$-plane using the Laurent-Pietarinen expansion [12,13,14,15] (see Table 1). We see that the lowest eigenvalue indeed touches the zero line for $g_\sigma = 2.0$, the pole, however, emerges already for considerably weaker couplings and starts approaching the real axis.
Beyond the critical value, $w_{\text{min}}$ crosses zero twice, producing two poles in the complex energy plane. It is interesting to note that for the values below the critical value, the real part of the pole position almost coincides with $W$ at which $w_{\text{min}}$ reaches its minimum. This value of $W$ is of the order of 100 MeV below the nominal $\sigma N$ threshold. The result agrees well with the molecular picture of the Roper resonance discussed in the first part of this contribution. Let us note that because the $\sigma N$ channel is coupled to other channels, the molecular state has a finite width (i.e. finite $\text{Im} W_p$) even for $g_\sigma$ greater than the critical value.

In the present approach we have also studied the influence of including a genuine three quark state with one quark excited to the 2s orbit. Using $g_\sigma \approx 1.5$, the results for the position as well as the modulus and the phase come close to the PDG value [11], and are rather insensitive to the mass of the genuine three-quark state. This leads us to the conclusion that the mass of the S-matrix pole is determined by the energy of the molecular state while its detailed properties may still considerably depend on the three-quark excited state. The simple model discussed in the first paper provides a simplified picture which enables a deeper insight into the mechanism of the resonance formation, hindered by the complex formalism of the coupled-channel approach.

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