Parameter-independent predictions for shape variables of heavy deformed nuclei in the proxy-SU(3) model

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Abstract. Using a new approximate analytic parameter-free proxy-SU(3) scheme, we make predictions of shape observables for deformed nuclei, namely $\beta$ and $\gamma$ deformation variables, and compare these with empirical data and with predictions by relativistic and non-relativistic mean-field theories.

PACS. 21.60.Fw Models based on group theory – 21.60.Ev Collective models

1 Introduction

Proxy-SU(3) is a new approximate symmetry scheme applicable in medium-mass and heavy deformed nuclei \cite{1,2}. The basic features and the theoretical foundations of proxy-SU(3) have been described in Refs. \cite{3,4}, to which the reader is referred. In this contribution we are going to focus attention on the first applications of proxy-SU(3) in making predictions for the deformation variables of deformed rare earth nuclei.

2 Connection between deformation variables and SU(3) quantum numbers

A connection between the collective variables $\beta$ and $\gamma$ of the collective model \cite{5} and the quantum numbers $\lambda$ and $\mu$ characterizing the irreducible representation $(\lambda, \mu)$ of SU(3) \cite{6,7} has long been established \cite{8,9}, based on the fact that the invariant quantities of the two theories should possess the same values.

The relevant equation for $\beta$ reads \cite{8,9}

\[
\beta^2 = \frac{4\pi}{5} \frac{1}{(Ar^2)^2} (\lambda^2 + \lambda \mu + \mu^2 + 3\lambda + 3\mu + 3),
\]

where $A$ is the mass number of the nucleus and $r^2$ is related to the dimensionless mean square radius \cite{10}, $\sqrt{r^2} = r_0 A^{1/6}$. The constant $r_0$ is determined from a fit over a wide range of nuclei \cite{11,12}. We use the value in Ref. \cite{8}, $r_0 = 0.87$, in agreement to Ref. \cite{12}. The quantity in Eq. (1) is proportional to the second order Casimir operator of SU(3) \cite{13}.

\[
C_2(\lambda, \mu) = \frac{2}{3} (\lambda^2 + \lambda \mu + \mu^2 + 3\lambda + 3\mu).
\]

The relevant equation for $\gamma$ reads \cite{8,9}

\[
\gamma = \arctan \left( \frac{\sqrt{3} (\mu + 1)}{2\lambda + \mu + 3} \right).
\]

3 Predictions for the $\beta$ variable

The $\beta$ deformation variable for a given nucleus can be obtained from Eq. (1), using the $(\lambda, \mu)$ values corresponding to the ground state band of this nucleus, obtained from Table 2 of Ref. \cite{14}.

A rescaling in order to take into account the size of the shell will be needed, as in the case of the geometric limit \cite{15} of the Interacting Boson Model \cite{16} in which a rescaling factor $2N_B/A$ is used, where $N_B$ is the number of bosons (half of the number of the valence nucleons measured from the closest closed shell) in a nucleus with mass number $A$. In the present case one can see \cite{2} that the $\beta$ values obtained from Eq. (1) should be multiplied by a rescaling factor $A/(S_p + S_n)$, where $S_p$ ($S_n$) is the size of the proton (neutron) shell in which the valence protons (neutrons) of the nucleus live. In the case of the rare
through $S_n = 82 - 50 = 32$ and $S_n = 126 - 82 = 44$, thus the rescaling factor is $A/76$.

Results for the $\beta$ variable for several isotopic chains are shown in Fig. 1. These can be compared to Relativistic Mean Field predictions [16] shown in Fig. 2, as well as to empirical $\beta$ values obtained from $B(E2)$ transition rates [17] shown in Fig. 3. Indeed such detailed comparisons for various series of isotopes are shown in Figs. 4-7. We remark that the proxy-SU(3) predictions are in general in very good agreement with both the RMF predictions and the empirical values. The sudden minimum developed in Fig. 1 at $N = 116$ could be related to the prolate-to-oblate shape/phase transition to be discussed in Ref. [14].

4 Predictions for the $\gamma$ variable

The $\gamma$ deformation variable for a given nucleus can be obtained from Eq. (3), using the $(\lambda, \mu)$ values corresponding to the ground state band of this nucleus, obtained from Table 2 of Ref. [14].

Results for the $\gamma$ variable for several isotopic chains are shown in Figs. 8 and 9. In Fig. 9, predictions by Gogny D1S calculations [18] are also shown for comparison. The sharp jump of the $\gamma$ variable from values close to 0 to values close to 60 degrees, seen in Fig. 9 close to $N = 116$, for both the proxy-SU(3) and the Gogny D1S predictions, can be related to the prolate-to-oblate shape/phase transition to be discussed in the next talk [14]. In contrast, in the series of isotopes shown in Fig. 8, $\gamma$ is only raising at large neutron number $N$ up to 30 degrees, indicating possible regions with triaxial shapes.

Minima appear in the proxy-SU(3) predictions for the neutron numbers for which the relevant SU(3) irrep, seen in Table 2 of Ref. [14], happens to possess $\mu = 0$, as one can easily see from Eq. (3). These oscillations could probably be smoothed out through a procedure of taking the average of neighboring SU(3) representations, as in Ref. [19].

Empirical values for the $\gamma$ variable can be estimated from ratios of the $\gamma$ vibrational bandhead to the first $2^+$ state,

$$R = \frac{E(2^+_2)}{E(2^+_1)},$$

through [20,21,22]

$$\sin 3\gamma = \frac{3}{2\sqrt{2}} \sqrt{1 - \left(\frac{R - 1}{R + 1}\right)^2}.\quad (5)$$

The proxy-SU(3) predictions for several isotopic chains are compared to so-obtained empirical values, as well as to Gogny D1S predictions where available, in Figs. 10 and 11. Again in general good agreement is seen.

5 Conclusions

The proxy-SU(3) symmetry provides predictions for the $\beta$ collective variable which are in good agreement with RMF predictions, as well as with empirical values obtained from $B(E2)$ transition rates. Furthermore, the proxy-SU(3) symmetry provides predictions for the $\gamma$ collective variable which are in good agreement with Gogny D1S predictions, as well as with empirical values obtained from the $\gamma$ vibrational bandhead.

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Fig. 1. Proxy-SU(3) predictions for $\beta$, obtained from Eq. (1).

Fig. 2. RMF predictions for $\beta$, obtained from Ref. [10].

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Fig. 3. Empirical predictions for $\beta$, obtained from Ref. [17].

Fig. 4. Proxy-SU(3) predictions for $\beta$, obtained from Eq. (1), compared with tabulated $\beta$ values [17] and also with predictions from relativistic mean field theory [16].

Fig. 5. Proxy-SU(3) predictions for $\beta$, obtained from Eq. (1), compared with tabulated $\beta$ values [17] and also with predictions from relativistic mean field theory [16].
Fig. 6. Proxy-SU(3) predictions for $\beta$, obtained from Eq. (1), compared with tabulated $\beta$ values [17] and also with predictions from relativistic mean field theory [16].

Fig. 7. Proxy-SU(3) predictions for $\beta$, obtained from Eq. (1), compared with tabulated $\beta$ values [17] and also with predictions from relativistic mean field theory [16].

Fig. 8. Proxy-SU(3) predictions for $\gamma$, obtained from Eq. (3).
Fig. 9. Proxy-SU(3) predictions for $\gamma$, obtained from Eq. (3) and from Gogny D1S calculations [18].

Fig. 10. Proxy-SU(3) predictions for $\gamma$, obtained from Eq. (3), compared with experimental values obtained from Eq. (5) [21, 22], as well as with predictions of Gogny D1S calculations [18].

Fig. 11. Proxy-SU(3) predictions for $\gamma$, obtained from Eq. (3), compared with experimental values obtained from Eq. (5) [21, 22], as well as with predictions of Gogny D1S calculations [18].