A Monte Carlo estimate for the fraction of thermal Comptonized photons that impinge back on the soft source in neutron star LMXBs.

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ABSTRACT
In earlier works, it was shown that the energy dependent soft time lags observed in kHz QPOs of neutron star low mass X-ray binaries (LMXBs) can be explained as being due to Comptonization lags provided a significant fraction ($\eta \sim 0.2 - 0.8$) of the Comptonized photons impinge back into the soft photon source. Here we use a Monte Carlo scheme to verify if such a fraction is viable or not. In particular we consider three different Comptonizing medium geometries: (i) a spherical shell, (ii) a boundary layer like torus and (iii) a corona on top of an accretion disk. Two set of spectral parameters corresponding to the 'hot' and 'cold' seed photon models were explored. The general result of the study is that for a wide range of sizes, the fraction lies within $\eta \sim 0.3 - 0.7$, and hence compatible with the range required to explain the soft time lags. Since there is a large uncertainty in the range, we cannot concretely rule out any of the geometries or spectral models, but the analysis suggests that a boundary layer type geometry with a 'cold' seed spectral model is favoured over an accretion corona model. Better quality data will allow one to constrain the geometry more rigorously.

Our results emphasise that there is significant heating of the soft photon source by the Comptonized photons and hence this effect needs to be taken into account for any detailed study of these sources.

Key words: stars: neutron – X-rays: binaries – X-rays: radiation mechanisms: thermal

1 INTRODUCTION
X-ray binaries are close binary systems where the compact object accretes matter from a companion star via an accretion disk. The compact object can be either a neutron star or a black hole, while the companion star is a main sequence one. The X-ray luminosity is usually generated in the inner accretion disk near to the compact object. There are two type of X-ray binaries, high mass X-ray binaries (HMXBs) where the companion star is a O or B star, and low mass X-ray binaries (LMXBs) where the companion star is a K or M star. The X-ray binaries are categorised into mainly two classes, transient and persistent, based on their long term X-ray variability. They have in general two distinct spectral states, a high luminous soft state, which is dominated by a black body like emission, and a low luminous hard state, which is dominated by power-law emission. They also typically show a third intermediate or transitional state where the X-ray flux is highly variable on time scale of milliseconds to seconds. The variability is sometimes of a quasi-periodic nature, and are termed as Quasi-periodic Oscillations, QPOs. In particular, Neutron star LMXBs have millisecond variability and their kHz QPOs are positioned in definite regions on their colour-colour plots (Altamirano et al. 2008, Straaten, van der Klis, Mendez 2003). These QPOs occur during the soft to hard state transition (see, eg. Belloni, Mendez & Homan 2007); and they seem to have no long term correlation with X-ray luminosity (Mendez et al. 1999, Misra & Shanthi 2004).

Important insights into the nature of these oscillations can be obtained by studying the fractional root mean square r.m.s. amplitude, and phase delay or time-lag as a function of energy which depend on the type of QPO and typically show complex behaviour (see for review, van der Klis 2000, 2006, Remillard & McClintock 2006, Tanaka & Shibazaki 1996). The energy dependence of the r.m.s and time-lag contain clues regarding the radiative processes that are involved in the QPO phenomena.

Spectral fitting reveals that thermal Comptonization is the main radiative mechanism for hard X-ray generation in X-ray binaries. In this process, the seed photons are Comp-
tonized by an hot thermal electron cloud or corona. The thermal Comptonization process is generally characterised by three parameters, the seed photon source temperature \( T_s \), the corona temperature \( T_c \), and the optical depth of the medium \( \tau \) or the average number of scattering \( <N_{sc}> \). That a photon would undergo thermal Comptonization process is generally characterised by three parameters, the seed photon source temperature \( T_s \), the corona temperature \( T_c \), and the optical depth of the medium \( \tau \) or the average number of scattering \( <N_{sc}> \). \( <N_{sc}> \) depends on the geometry of the corona and for a given optical depth, it is generally difficult to compute analytically for arbitrary corona shapes. Since the dominant spectral component in X-ray binaries is due to thermal Comptonization, the energy dependent r.m.s and time-lag of the QPOs may be related to the process. Indeed, the energy dependence of the r.m.s and time-lag for the lower kHz QPO can be explained in terms of a thermal Comptonization model and moreover, such an analysis can provide estimates of the size and geometry of the corona (Lee & Miller 1998; Lee, Misra & Taam 2001). In a more detailed work Kumar & Misra (2014) studied the expected energy dependent time lags and r.m.s for different kinds of driving oscillations such as in the seed photon temperature or in the coronal heating rate, while self-consistently incorporating the heating and cooling processes of the medium and the soft photon source. They showed that the observed soft lag for the kHz QPO could be obtained only when the heating oscillation is in the heating rate of the corona and if a substantial fraction, \( \eta \) of the Comptonized photons impinge back into the soft photon source. However, the quantitative results obtained depend on the the specific time-averaged spectral model used for the analysis. Typically in the Rossi X-ray Timing Experiment (RXTE), Proportional Counter Array (PCA) energy band of 3-20 keV, there are two spectral models namely the “hot” and “cold” seed photon models which are degenerate i.e. they both equally fit the data of neutron star LMXBs (Mitsuda et al. 1984; White et al. 1986; Barret 2001; Di Salvo & Stella 2002; Lin, Remillard & Homan 2007; Cocchi et al. 2011). In Kumar & Misra (2016, hereafter Paper I), we employed both these spectral models to infer the size of the medium and fraction of photons impinging on the soft seed source. While in these earlier works it has been treated as a parameter, in principle, it should be computed for a given geometry. In this work, we endeavour to do so, by implementing a Monte Carlo method to trace the photons as they scatter, escape from the medium and impinge into the soft photon source. The motivation here is to compute \( \eta \) as a function of size and for different simple geometries. We will then compare the results with the constraints obtained in Paper I, to find if any of the geometries are more viable. We will neglect General relativistic effects and any bulk (including orbital) motion of the Comptonizing medium.

In the next section, we briefly discuss the scheme of the Monte Carlo method used for the thermal Comptonization process. In Section 3, \( \eta \) is computed for three different geometries of the Comptonizing system and in Section 5, the results are summarised and discuss.

2 MONTE CARLO METHOD

In a Monte Carlo method a photon is tracked as it enters the Comptonizing medium and scatters multiply till it leaves the medium. The process is repeated for a large number of photons to build up the statistics that would give the emergent spectrum as well as the direction of each outgoing photon. The technique has been in use for several decades now (for e.g., Sazonov & Sunyaev 2000; Zdziarski & Pjanka 2013). Pozdnyakov, Sobol & Sunyaev (1983) have extensively reviewed the Monte Carlo method for the thermal Comptonization process. The algorithm used in this work for the Monte Carlo method in the lab frame has been adopted from their paper and the specific scheme used is from the Appendix of (Hua & Titarchuk 1995).

Since our analysis is in the non-relativistic regime, i.e. the electron temperature, \( kT_e \), and the photon energies considered are \( \ll mc^2 \), and that the size of the region is much larger than the scattering length, the diffusion limit is still valid. Thus, we can test the code with the analytical results obtained in this limit including the resultant spectrum from the Kompaneets equation (Kompaneets 1957). The test code in three stages. First, we compute the average energy change for a monochromatic photon of frequency \( \nu \) scattering once in a thermal medium, \( kT_e \), which is expected to be \( \Delta E = (4kT_e\hbar\nu)\frac{1}{m_c^2} \). We computed this average change in energy of the photon for different temperatures and found it to match with the above expectation. Next, for a spherical geometry we consider the average number of scatterings that a photon will undergo \( <N_{sc}> \), and the scattering number distribution. For such a spherical shape geometry, one can estimate in the diffusion limit that \( <N_{sc}> \approx \frac{x^2}{\tau} \) and the peak of scattering distribution should be around \( \sim 0.3\pi x^2 \) (Sunyaev & Titarchuk 1980). We find these expected results for the Monte Carlo code, for e.g. for \( \tau = 9.2 \), \( <N_{sc}> \) was found to be 41.4, and the peak of the distribution was around 25. Finally, we compare the output spectra of the code with the analytical ones and find a good match as shown in Figure 1. Here, the medium temperature is fixed at \( kT_e = 3.0 \) keV. The points with error-bars are from the Monte Carlo results while the lines are the analytical solutions of the Kompaneets equation (as described in Kumar & Misra 2014) in which the \((x^2 + \tau)\) term is equated with \( <N_{sc}> \). The curve marked 1 is for the case when the soft photon temperature \( kT_b = 0.1 \) keV and \( \tau = 9.2 \). For these values the spectrum around 1 keV should be of a power-law form and that indeed is seen. The curve marked 3 is for the
ESTIMATING THE FRACTION $\eta_e$ FOR DIFFERENT GEOMETRIES

As shown in Paper 1, an important parameter that determines the nature of the energy dependent time lag is the fraction of photons impinging back into the soft photon source. Perhaps a more physical quantity is the fraction $\eta_e$ lead to lower values of $\eta_e$ since the probability that a photon gets absorbed by the surface decreases. This is indeed the case as shown in Figure 3 where the left panel shows the computed $\eta_e$ as a function of $L$ for the four spectral parameters tabulated in Table 1. For comparison, the plots also show the range of $\eta$ and $L$ for the “cold” and “hot” seed photon models inferred by the energy dependent r.m.s and time-lag of the kHz QPO (Paper I). Although the range of $\eta_e$ and $L$ are rather large due to the quality of the data, it is heartening to see that for this geometry the computed $\eta_e$ fall within this range. If better quality data indicate a smaller $\eta_e$, then perhaps such a geometry can be ruled out. Naturally, a hollow geometry would lead to lower values of $\eta_e$. The right panel of Figure 3 shows this decrease of $\eta_e$ versus the gap size $R_H$ for fixed values of $L = 0.5$ (solid line) and 1.0 (dashed line) kms.

3.1 Spherical/hollow shell

We start with the simplest geometry depicted in Figure 2 where the neutron star is covered by a spherical shell which Comptonizes photons from the surface of the neutron star. The radius of the neutron star is fixed at $R_s = 10$ kms while the size of the shell $L$ is taken as a parameter. Although perhaps not physical, we also consider for completeness, the possibility that the Comptonizing medium is a hollow shell having a vacant region of size $R_H$ between it and the neutron star (right panel of Figure).

In the Monte Carlo code, a photon is released from the surface of the neutron star and is tracked till it either escapes or impinges back to the surface. One expects that the fraction $\eta_e$ will decrease with increasing $L$. This is indeed the case as shown in Figure 3 where the left panel shows the computed $\eta_e$ as a function of $L$ for the four spectral parameters tabulated in Table 1. For comparison, the plots also show the range of $\eta$ and $L$ for the “cold” and “hot” seed photon models inferred by the energy dependent r.m.s and time-lag of the kHz QPO (Paper I). Although the range of $\eta_e$ and $L$ are rather large due to the quality of the data, it is heartening to see that for this geometry the computed $\eta_e$ fall within this range. If better quality data indicate a smaller $\eta_e$, then perhaps such a geometry can be ruled out. Naturally, a hollow geometry would lead to lower values of $\eta_e$. The right panel of Figure 3 shows this decrease of $\eta_e$ versus the gap size $R_H$ for fixed values of $L = 0.5$ (solid line) and 1.0 (dashed line) kms.

Table 1. List of Comptonization spectral parameters used for the Monte Carlo code to compute $\eta_e$. The hot and cold seed photon models are represented by two sets of spectral parameters.

| Model     | index | Comptonization parameters |
|-----------|-------|---------------------------|
|           |       | $kT_e$ (keV) | $kT_b$ (keV) | $\tau$ |
| hot-seed  | Ia    | 3             | 1             | 9  |
|           | Ib    | 5             | 1             | 5  |
| cold-seed | IIa   | 3             | 0.4           | 9  |
|           | IIb   | 5             | 0.4           | 5  |

named Ia and Ib while the spectra corresponding to “cold” seed photon model are named IIa and IIb. The Monte Carlo computations have been done for each of these four set of spectral parameters.

3.2 Boundary layer geometry

The boundary layer is a region that connects the accretion disk to the neutron star surface, i.e. the accreting material makes a transition from centrifugal to pressure support near the star (e.g. Popham & Narayan 1995; Popham & Sunyaev 2001). Here, we approximate the geometry as shown in the left panel of Figure 4. We consider a rectangular torus surrounding the spherical neutron star. The radius of the neutron star is kept fixed at $R_s = 10$ kms. The gap between the torus and the neutron star $R_g$ is also fixed at a small distance of 50 m following Babkovskiaia, Brandenburg & Poutanen 2008 who estimate that the maximum distance between the star surface and the layer is about 100 m.

Table 1. List of Comptonization spectral parameters used for the Monte Carlo code to compute $\eta_e$. The hot and cold seed photon models are represented by two sets of spectral parameters.

| Model     | index | Comptonization parameters |
|-----------|-------|---------------------------|
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| hot-seed  | Ia    | 3             | 1             | 9  |
|           | Ib    | 5             | 1             | 5  |
| cold-seed | IIa   | 3             | 0.4           | 9  |
|           | IIb   | 5             | 0.4           | 5  |
The width of the torus in the radial direction is taken to be a parameter \( L_R \) while its half-height in the vertical direction is \( L_H \). This geometry allows for two definitions of the optical depth \( \tau \), one along the vertical and other in the radial direction and we do the analysis for both definitions. For the same optical depth defined in either fashion, the average number of scatterings \( \langle N_{sc} \rangle \) is smaller by a factor of \( \sim 1.5 \) than for the spherical shell case studied above. Hence we use a slightly higher values of \( \tau \), 10.4 and 5.8 instead of 9 and 5 mentioned in Table 1. We emphasise that these changes have little effect on the overall results.

We first consider the case when the optical depth is defined along the vertical direction and we explore the variation of \( \eta_e \) with the vertical height \( L_H \) for fixed radial extent \( L_R \) and for different spectral parameters. This is shown in Figure 5 where the top panels are for \( L_R = 1 \) km while the bottom ones are for \( L_R = 20 \) kms. The left panels are for the 'hot' seed photon model while the right ones are for the 'cold' seed photon one. The contours mark the estimated ranges of \( \eta \) and \( L \) from Paper I. Figure 5 is same as Figure 4 except that now the optical depth is defined along the horizontal direction and the top and bottom panels are for fixed values of \( L_H = 1 \) and 20 kms respectively. It is clear from both these Figures that for a wide range of sizes and spectral models, the fraction \( \eta_e \) falls within the range required to explain the energy dependent r.m.s and time-lags of the kHz QPOs.

### 3.3 Disk-Corona geometry

We next consider a third possible geometry, i.e., of an optically thick accretion disk sandwiched by an hot corona as shown in the right panel of Figure 4. The height of the corona is taken to be \( L \) while the disk and the corona above is considered to span from an inner radius of \( R_{\min} \) to \( R_{\max} \). For computational purposes we have introduced a thickness of the disk of \( R_c = 0.2 \) kms but the results, as expected, are insensitive to this value. In fact the determining parameter here is the ratio of the height of the corona to the annular width of the disk \( R_{\max} - R_{\min} \). Thus we fix \( R_{\max} - R_{\min} = 10 \) kms and vary \( L \).

In Figure 6 we plot the fraction \( \eta_e \) versus height \( L \) for different spectral parameter values and as before compare with ranges obtained in Paper I. As expected, there is only a weak dependence of \( \eta_e \) on \( L \) and it has a rather large value of \( \sim 0.7 \). In fact for the "cold" seed photon case \( \eta_e \) is marginally larger than the maximum value obtained in Paper I. This seems to suggest that for this case at least, such a disk-corona geometry is unfavourable. However, given the
large uncertainties it is difficult to make concrete statements. Nevertheless, our results show that for such a geometry the value of $\eta_e$ is expected to be large, more or less independent of the thickness of the corona.

4 SUMMARY AND DISCUSSION

Using a Monte Carlo scheme, we estimate the fraction of Comptonized photons that impinge back into the seed photon source $\eta$ for different geometries and spectral parameters relevant to neutron star low mass X-ray binaries. The primary motivation was that to explain the observed soft lags in KHz QPOs, one needed to invoke a large value $\eta$ in the range of 0.2-0.6 and it was important to find out if this range can be achieved for any reasonable accretion geometry.

We consider three kinds of geometries for the Comptonizing medium which are (i) a spherical shell around the neutron star, (ii) a boundary layer system where the medium is taken to be a rectangular torus around the star and (iii) a corona sandwiching a thin accretion disk. We consider different sizes for the medium and a range of spectra parameters. In particular we consider two extreme cases of spectral parameters for the two degenerate spectral models which are called the hot and cold seed photon models.

Our basic result is that for a wide range of reasonable sizes and spectral parameters, the values of $\eta_e$ computed by the Monte Carlo method lie between 0.2 and 0.8 and hence are compatible with the values used by Kumar & Misra (2016) to explain the soft time lags of the KHz QPOs. Since the range of $\eta$ and size inferred from fitting the time-lags are rather broad, we cannot concretely rule out any of the three geometries considered. However, it seems that the boundary layer geometry can have $\eta$ values more in line with what is required and the disk-corona geometry produces $\eta$ values which are marginally larger. Our results show that it is possible to constrain the geometry of the system if high quality data for energy dependent time-lags are available. We look forward to data from the recently launched satellite ASTROSAT (Agrawal 2006; Singh et al. 2014), which might provide such high quality data. Perhaps it would then be warranted to consider other complexities such as the seed photon for the boundary layer case may be produced in the accretion disk rather than the neutron star surface or that the corona on top of the accretion disk maybe in the form of inhomogeneous clumps rather than being a uniform medium. Also, not all the photons that impinge back into the source, will be absorbed and one needs to solve the radiative transfer equations self consistently to find the fraction reflected. This reflected emission will have light travel time delays which may significantly effect the time-lags between different energy bands.

Finally, it is interesting to note that for the geometries considered here a significant fraction of the photons impinge

\[ \eta_e \text{ as a function of the coronal width } L \text{ for the accretion disk-corona geometry. Here the extent of the disk } R_{\text{max}} - R_{\text{min}} = 10 \text{ kms. The left and right panel are for the hot and cold seed photon models. The solid and dashed lines are for two corresponding spectral parameters. The closed curves show the allowed range of } \eta \text{ and size obtained by Kumar & Misra (2016).} \]

\[ \eta_e \text{ as a variation of size for the Boundary layer geometry when the optical depth is defined along the vertical direction. The top and bottom two panels are for the case when the horizontal width is taken to be } L_H = 1 \text{ and 20 kms respectively. The left and right panels are for the hot and cold seed photon models. The solid and dashed lines are for two corresponding spectral parameters. The closed curves show the allowed range of } \eta \text{ and size obtained by Kumar & Misra (2016).} \]

\[ \eta_e \text{ as a variation of size for the Boundary layer geometry when the optical depth is defined along the radial direction. The top and bottom two panels are for the case when the vertical height is taken to be } L_H = 1 \text{ and 20 kms respectively. The left and right panels are for the hot and cold seed photon models. The solid and dashed lines are for two corresponding spectral parameters. The closed curves show the allowed range of } \eta \text{ and size obtained by Kumar & Misra (2016).} \]
back into soft photon source. This effect needs to be taken into account in any detailed study of these X-ray binaries.

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