Identifying the contributions of Universal Extra Dimensions in the Higgs sector at linear $e^+e^-$ colliders

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ABSTRACT

We study the dilepton-dijet signal in the dominant Higgs production channel at a linear $e^+e^-$ collider. We estimate the effects of Universal Extra Dimension (UED) by a simple analysis of the cross-sections. The heavy Kaluza-Klein excitations of the Standard Model fields in UED can significantly alter the decay properties of the Higgs boson to loop-driven final states. We show that by taking a simple ratio between cross-sections of two different final states this difference can be very easily highlighted.

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1 Introduction

Theories of large extra spacetime dimensions \[1,2\] and warped extra dimensions \[3,4\] have received a great deal of attention during the past several years. The most popular amongst them have been the models which consider localization of Standard Model (SM) fields on a 3-brane. In these scenario the SM fields can only feel the effects of the extra dimensions through interactions with the Kaluza-Klein (KK) excitations of the gravitational field in the bulk, projected onto the brane by the usual KK mechanism. However, within some other models motivated from the framework of string theory \[5\], other non-gravitational fields can also propagate in the bulk provided they do not disturb the experimental constraints, which puts a limit on the maximum size of the extra dimension.

One such scenario, referred to as the Universal Extra Dimension (UED) model \[6\], allows all the Standard Model (SM) fields to propagate in extra dimensions. At tree level, the momentum along the extra dimensions is also conserved, requiring pair production of the Kaluza-Klein (KK) modes at colliders and preventing tree level mixing effects from altering precision electroweak measurements. Values of compactification scale, as low as 250-300 GeV are allowed for one extra dimension \[7\]. The phenomenological implications of UED have been extensively studied in the literature \[6,8–27\].

Direct detection of UED KK states at future colliders requires them to be pair produced due to the KK number conservation and hence already puts a limit on the minimum energy at which the collider should run to produce these particles. However, effects of UED excitations can be also be observed in loops at much lower energies \[8–14\] than that required to produce them on-shell. It is therefore interesting to determine whether there are other, indirect ways in which the effects of UED can be detected at future high energy colliders. One such possibility is through the modification of Higgs decay properties due to the KK states.

Higgs boson discovery will prove to be a crucial ingredient towards understanding the mechanism of electroweak symmetry breaking. Once discovered, a major goal would be to determine its other intrinsic properties, couplings and its total width with high accuracy in a model independent way. The partial width of the Higgs decaying to the massless gauge boson is of special interest, since there are no tree level couplings of the Higgs to them and any contribution is generated at the one-loop level. The di-photon partial width gets contribution through massive charged particles in the loops while the gluon-gluon partial width gets contributions from the heavy quarks running in the loops. The effective loop induced couplings of $H\gamma\gamma$ and $Hgg$ are sensitive to new contributions
from particles which appear in various extensions of the SM. Not only do these decay modes provide for a possible probe of new physics particles which are too heavy to be produced directly but they are also sensitive to scales far beyond the Higgs mass. The partial decay widths for $H \to gg$, $H \to \gamma\gamma$ and $H \to \gamma Z$ decay modes which are driven by loops can be substantially modified due to KK excited modes of SM particles running in the loops. As pointed out in Ref. [28] there is a remarkably significant enhancement in the partial decay width of the Higgs in $H \to gg$ due to the excited top quark loops. This can greatly enhance the Higgs production at the Large Hadron Collider (LHC) viz. the $gg \to H$ mode of production. However, there are large uncertainties associated with the PDF’s and the choice of scale (scale uncertainties) [29, 30] at the LHC which might just mask this enhancement and make it difficult to differentiate the contributions coming from UED. Also the very fact that the $H \to gg$ or decays into light quarks cannot be observed at the LHC due to the huge QCD backgrounds, makes it difficult to look for UED contributions in the dijet final state. It is quite evident that one requires a linear electron-positron collider to study the properties of the Higgs boson, viz. its coupling to SM particles and its decay properties. In view of whether any new physics beyond the SM affects its properties, will be a crucial detail one needs to understand. The future lepton colliders would be instrumental in identifying the Higgs properties more precisely due to the much cleaner environment. In this work we try to point out a very robust and simple technique of identifying the effect of the contributions of the heavy KK-modes to the loop induced decay modes of the Higgs boson. In fact the technique is useful in testing the sensitivity of the scale of new physics beyond SM. For our study we focus on the UED model and we look at the dominant mode of Higgs production at a future $e^+e^-$ collider and its subsequent decay into two jets and try to identify the contribution coming from UED. In Section 2 we give a very brief overview about the model in consideration and how the partial width for $H \to gg$ gets modified. In Section 3 we present our results and conclusions.

2 The Model

The UED model, in its simplest form [6], has all the SM particles propagating in a single extra dimension, which is compactified on an $S_1/Z_2$ orbifold with $R$ as the radius of compactification. Conservation of KK number which is a consequence of momentum conservation along the extra dimension forces the KK particles to be pair produced. Consequently, UED predicts a stable lightest Kaluza-Klein particle (LKP) which would be much like the lightest supersymmetric particle (LSP). The LKP is a viable candidate for dark matter [31, 32]. Bulk and brane radiative effects [33–36] however break KK number down to a discrete conserved quantity, the so called KK parity, $(-1)^n$. 
where \( n \) is the KK level. KK parity conservation in turn, implies that the contributions to various precisely measured low-energy observables only arise at loop level and are small \([8–14]\). As a result, the limits on the scale of the extra dimension, from precision electro-weak data, are rather weak, constraining \( R^{-1} \) to be larger than 300 GeV.

The KK tower resulting on the four dimensional space-time has a tree level mass given by

\[
m_n^2 = m^2 + \frac{n^2}{R^2} \tag{1}
\]

where \( n \) denotes the \( n^{th} \)-level of the KK tower and \( m \) corresponds to the mass of the SM particle in question. This implies a mass degeneracy in the \( n^{th} \)-level of the spectrum at least for the leptons and lighter quarks. This degeneracy is however removed due to radiative corrections to the masses \([33–36]\). In the following, we use the tree-level masses, but our result is still valid with the mass degeneracies lifted.

In this work, we are mainly interested in the modification of the partial decay width of the Higgs in \( H \to gg \) due to the KK tower of the top quark running in the loops. Since the KK number is not violated at any of the vertices inside a loop, the contributions come from all the KK-excitations, with a decoupling nature for the higher modes. So only the first few KK modes have relevant contribution. Below we give the contributions to the two-gluon decay width of the Higgs boson in the SM and UED.

Since the \( H \to gg \) proceeds through diagrams containing fermion triangle loops and the coupling is proportional to the zero-mode mass of the fermion even in the case of UED, we consider contributions of the KK tower of the top quark only. The partial decay width for \( H \to gg \) with UED contribution is \([28]\),

\[
\Gamma(H \to gg) = \frac{G_F m_H^3}{36\sqrt{2}\pi} \left( \frac{\alpha_s(m_H)}{\pi} \right)^2 |I_q + \sum_n \tilde{I}_{\tilde{q}^{(n)}}|^2 \tag{2}
\]

where \( G_F \) is the Fermi constant, \( \alpha_s(m_H) \) is the running QCD coupling evaluated at \( m_H \) and \( I_q, \tilde{I}_{\tilde{q}^{(n)}} \) are the contributions of the loop integrals for the SM and UED case respectively. These loop integral functions can be written down as:

\[
I_q = \frac{3}{4} \lambda_q \left[ 2 - (1 - \lambda_q)F(\lambda_q) \right] \\
\tilde{I}_{\tilde{q}^{(n)}} = \frac{6}{4} \lambda_{\tilde{q}^{(n)}} \left[ 2 - (1 - \lambda_{\tilde{q}^{(n)}})F(\lambda_{\tilde{q}^{(n)}}) \right] \tag{3}
\]

where \( \lambda_i = 4m_i^2/M_H^2 \). The \( q \) stands for the different quark flavours in the SM while \( \tilde{q}^{(n)} \) represents the \( n^{th} \) KK excitation of the particular quark flavour. The function \( F(\lambda) \) in the above expression
is given by

\[ F(\lambda) = -2 \left( \sin^{-1} \frac{1}{\sqrt{\lambda}} \right)^2 \text{ for } \lambda \geq 1 \]

\[ = -\frac{\pi^2}{2} + \frac{1}{2} \log^2 \frac{1 + \sqrt{1 - \lambda}}{1 - \sqrt{1 - \lambda}} - i\pi \log \frac{1 + \sqrt{1 - \lambda}}{1 - \sqrt{1 - \lambda}} \text{ for } \lambda < 1 \]

We find that only the KK excitations of the top quark has a relevant contribution to the partial decay width. The UED contributions include the sum over the KK towers of the respective particle. As the more massive modes in the loop will hardly make significant contributions, we ensure that the sum is terminated as the higher modes decouple. We include the corrections to all the decay modes affected by UED contributions in the decay package HDECAY [37] to evaluate the relative sensitivities to the branching ratios to the different decay channels of the Higgs boson.

To illustrate the effect of the KK-modes on the branching ratio of \( H \rightarrow gg \), we plot the UED enhanced result along with the SM result as a function of Higgs mass in Figure 1 for different values of \( R^{-1} \). One can clearly see that large effects from UED result for lower values of \( R^{-1} \). However, very low values of \( R^{-1} \) are constrained by precision electroweak data.

![Figure 1: Illustrating the effects of UED contributions on the branching ratio of \( h \rightarrow gg \) as a function of Higgs mass \((M_H)\) for different values of the inverse of radius of compactification \( R^{-1} \).](image-url)
3 Results and Discussions

We have considered the case of a 500 GeV linear $e^+e^-$ collider and calculated the production of a Higgs in association with a $Z$ boson [38–40] through a process of the form

$$e^+e^- \rightarrow Z + H$$

$$\rightarrow \ell^+\ell^- + H$$

In the preceding paragraph we discuss the final states relevant for our study.

1. $e^+e^- \rightarrow \ell^+\ell^- + \text{two jets}$, which arises when the Higgs boson decays to a pair of quarks ($u\bar{u}, d\bar{d}, c\bar{c}, s\bar{s}$ and $b\bar{b}$) or gluons, which then undergo fragmentation to form a pair of hadronic jets. Clearly, for a Higgs boson in the SM, final state will receive contributions mainly from the decays $H \rightarrow b\bar{b}$ and $H \rightarrow c\bar{c}$, with a minuscule contribution due to $H \rightarrow gg$. However, due to the increase in the partial width for $H \rightarrow gg$ due to the extra contribution coming from the additional KK excitations of the top quark in the loops, there will be an enhancement in the overall branching ratio to jets.

2. $e^+e^- \rightarrow \ell^+\ell^- + b\bar{b}$, which simply means that the final state in the above contains two tagged $b$-jets. The decay width for $H \rightarrow b\bar{b}$ is roughly the same in SM as well as UED, although the change of the two-gluon decay mode will have a small effect on the branching ratio for the $b\bar{b}$ mode. This would be observable in the rates for $b\bar{b}$ final state only if the change in the two-gluon mode is quite large, considering the fact that the corresponding branching ratios differ by more than an order of magnitude in the intermediate mass range of the Higgs boson. However, this small change in the branching ratio of $H \rightarrow b\bar{b}$ can still play a pivotal role in identifying effects of UED, as we show in our analysis.

In our subsequent analysis, we have imposed a few kinematic acceptance cuts on the final state particles, viz.,

1. The final-state leptons should have transverse momentum $p_T^{(\ell)} > 10 \text{ GeV}$.

2. The final-state leptons should have pseudo-rapidity $\eta^{(\ell)} < 3.0$.

3. The final-state jets should be clearly separated from each other. This can be implemented in a parton level analysis like ours by imposing a cut: $\Delta R_{JJ}(\equiv \sqrt{\Delta \eta_{JJ}^2 + \Delta \phi_{JJ}^2}) > 0.4$, which is the usual criterion adopted at, for example, the LEP and Tevatron colliders.

\footnote{We exclude $H \rightarrow \tau^\pm$ decays because these produce narrow jets which can be identified as $\tau^\pm$ with 80-90% efficiency.}
4. The final-state jets should have transverse momentum $p_T^{(J)} > 10$ GeV.

5. The final-state jets should have pseudo-rapidity $\eta^{(J)} < 2.5$.

The $b$-tagging efficiency [41] has been taken to be 50%, which is probably a conservative estimate.

In Figure 2(a), we illustrate our result for the process discussed above, namely,

$$e^+e^- \rightarrow \ell^+\ell^- + \text{two jets}$$

at a $\sqrt{s} = 500$ GeV, $e^+e^-$ collider. The solid (red) line denotes the UED-included cross-section where we have chosen the value of $R^{-1} = 350$ GeV which gives a greater enhancement compared to values of $R^{-1}$ greater than the above, while the dashed (black) line denotes the SM contribution only. It should be noted that the graph shows the excess cross-section after removing the non-Higgs part of the Standard Model contributions (such as $e^+e^- \rightarrow ZZ, ZZ^*$ etc.). These lead to a large SM four-fermion background, which is however, easily reducible by selecting only events corresponding to peaks in the $\ell^+\ell^-$ around $Z$-mass and dijet invariant masses, but rejecting those events with 2-jet invariant mass corresponds to $Z$-mass. The continuum background ($\gamma^*\gamma^*$, $Z^*Z^*$)
too can be easily neglected as it lies below $10^{-3} \, fb$ (in the bins of $b\bar{b}$ invariant mass) and would hardly affect the rates for the signal in consideration. The cross-section shown in Figure 2(a) makes it clear that it is very hard to see for differences by just looking at the rates. The cross-sections for $\ell^+\ell^- +$ two jets final state are almost identical in the two cases. As the cross-sections look very similar, it would require very precise measurements to form a distinction between the two cases.

Thus, it seems that the two jet decay mode of the Higgs alone will not be able to show observable difference due to the enhancement in the decay channel of the two-gluon mode due to UED contributions. However if we consider the ratio of the two processes mentioned above, viz. $\frac{\sigma(e^+e^-\rightarrow \ell^+\ell^- + \text{two jets})}{\sigma(e^+e^-\rightarrow \ell^+\ell^- + b\bar{b})}$ we can see that the difference between the two cases becomes more prominent. The number of $2\ell + 2 \text{ jets}$ events do not differ in SM and UED to a great extent as revealed from Figure 2(a). However, efficacy of the above mentioned ratio is also evident from the following figure (Fig 2(b)). Enhancement of the gluonic width of the Higgs boson in UED case has two fold effect on the ratio. It not only increases the $gg$ branching ratio (which affects the numerator of the ratio), but also suppresses the branching ratio in $b\bar{b}$-channel (thus suppressing the denominator and hence amplifies the ratio). In Figure 2(b) we plot this ratio for different values of the compactification scale $R^{-1}$. We find that the ratio differs from that of the SM throughout the mass range of $120 \, \text{GeV} \leq m_H \leq 200 \, \text{GeV}$ with the lines converging towards the SM value as $R^{-1}$ is increased. In fact this also highlights the decoupling nature of the higher levels of the KK tower and justifies our termination of the sum of KK towers in the loop, to values where the contributions become negligibly small. The ratios tend to diverge more as the Higgs mass increases. This is because the branching ratios of $H \rightarrow gg$ and $H \rightarrow b\bar{b}$ become comparable and the enhancement in the $H \rightarrow gg$ mode starts playing a more significant role in the 2-jet final state. However, there is a caveat. For comparatively higher Higgs masses, cross-sections for both the above processes are small. Hence, we need higher luminosity to differentiate between the SM and the UED cases in a statistically significant way. The robustness of this method is, nevertheless, highlighted in the fact that although the $H \rightarrow gg$ branching ratio is more than an order smaller than that of $H \rightarrow b\bar{b}$ in the intermediate mass range for the Higgs boson, we are still able to identify the difference due to the UED contribution which would have been otherwise very difficult to see, by just looking at the cross-sections.

It is necessary to mention here that the differences in the ratios as evident from the graphs is due to the difference in the corresponding branching ratios. The ratios are not susceptible to uncertainties like different efficiency factors associated with particle identifications as they would cancel out. The
efficiency factors will however give a more realistic estimate of the events that will be observed at the experiments and which gives us an estimate of the uncertainties in the statistics.

A similar exercise can be carried out for a 1 TeV machine too, but the rates for the above processes would rapidly fall due to the s-channel suppression. Infact, we find that the cross-section for the $\ell^+\ell^- + \text{ two jets}$ process falls by an order of magnitude compared to that of $\sqrt{s} = 500 \text{ GeV}$ collider. A better option at the higher $\sqrt{s}$ machine would be the production of Higgs through the $WW-$fusion channel with the final states being “two jets + $E_T$” and “$b\bar{b} + E_T$”. The ratio between the final states will again show differences, but it would be challenging to isolate the signal from the large continuum background for the above process without losing out much on the signal. Since the major background will be due to the $e^+e^- \rightarrow \gamma^*Z, \gamma^*Z^*$, etc. a large proportion of that can be suppressed by rejecting events for $M^{\text{missing}}_{\text{inv}} < 100 \text{ GeV}$. Due to the t-channel nature of the background processes, the momentum vector reconstructed from the two jets, would be mostly in the high rapidity region, in contrast to the same momentum vector coming from the Higgs boson produced by the $WW-$fusion process. The distribution of the latter, would be expected to peak in the central rapidity region. This too can provide for a substantial suppression of the background by putting an appropriate kinematic cut. However, we should accept the possibility that it might just be kinematically allowed, at a $\sqrt{s} = 1 \text{ TeV}$ machine, to pair produce the $n=1$ KK excitations for direct observation. In which case our results for the $\sqrt{s} = 500 \text{ GeV}$ machine becomes all the more interesting and worthy of attention.

To summarise, we have shown that the effects of KK excitations in the UED scenario can be evident at future lepton colliders, much before the direct observation of these particles (which are constrained to be produced in pairs), through the decay mode of Higgs, driven by loops. We find that, although these enhancements might not be distinguishable by just looking at a single cross-section viz. $\ell^+\ell^- + \text{ two jets}$, this difference can be highlighted by looking at the ratio of cross-sections for two different final states. Another advantage of taking ratios is that many systematic errors and efficiency factors cancel out, making it a more useful option. Infact, considering ratios to determine parameters have been a usual practice where large uncertainties and errors are involved. We also find that the differences are more pronounced for higher Higgs masses, but are constrained by the low cross-sections for the specific process in consideration, in that mass range. However, the high luminosity achievable at the future lepton colliders is expected to probe this mass range too.

In fact the above technique can be used to identify other new physics scenarios predicting massive

\footnote{\(M^{\text{missing}}_{\text{inv}}\) is defined to be the mass of the system recoiling against the jet-pairs}
particles, which play a similar role in modifying the partial width of the Higgs to massless gauge bosons. It can also be used to distinguish scalars of other theories which behave similar to the Higgs boson, like the Radion which has similar couplings like the Higgs boson [42]. A major difference is the enhanced coupling of Radion to gluons through the trace anomaly.

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