Three Vertex and Parallelograms in the Affine Plane: Similarity and Addition Abelian Groups of Similarly \(n\)-Vertexes in the Desargues Affine Plane

Orgest Zaka

Department of Mathematics, Faculty of Technical Science, University “Ismail QEMALI” of Vlora, Vlora, Albania

Email address: gertizaka@yahoo.com

To cite this article:

Orgest Zaka. Three Vertex and Parallelograms in the Affine Plane: Similarity and Addition Abelian Groups of Similarly \(n\)-Vertexes in the Desargues Affine Plane. Mathematical Modelling and Applications. Vol. 3, No. 1, 2018, pp. 9-15. doi: 10.11648/j.mma.20180301.12

Abstract: In this article will do a’ concept generalization \(n\)-gon. By renouncing the metrics in much axiomatic geometry, the need arises for a new label to this concept. In this paper will use the meaning of \(n\)-vertexes. As you know in affine and projective plane simply set of points, blocks and incidence relation, which is argued in [1], [2], [3]. In this paper will focus on affine plane. Will describe the meaning of the similarity \(n\)-vertexes. Will determine the addition of similar three-vertexes in Desargues affine plane, which is argued in [1], [2], [3], and show that this set of three-vertexes forms a commutative group associated with additions of three-vertexes. At the end of this paperare making a generalization of the meeting of similarity \(n\)-vertexes in Desargues affine plane, also here it turns out to have a commutative group, associated with additions of similarity \(n\)-vertexes.

Keyword: \(n\)-vertexes, Desargues Affine Plane, Similarity of \(n\)-Vertexes, Abelian Group

1. Introduction

In Euclidean geometry use the term three-angle and non three-vertex, this because the fact that the Euclidean geometry think of associated with metrics, which are argued in [4], [6], [7]. In this paper will use the ”three-Vertex” term, by renouncing the metric. Will generalize so its own meaning in the Euclidean case. With the help of parallelism [1], [2], [3] will give meaning of similarity and will see that have a generalization of the similarity of the figures in the Euclidean plane. By following the logic of additions of points in a line of Desargues affine plane submitted to [3], herewill show that analogously this meaning may also extend to the addition of similarity three-vertex in Desargues affine plane, moreover extend this concept for the similarity \(n\)-vertexes to the Desargues affine plane.

The aim is to see if the move to three-vertexes as well as to \(n\)-vertexes has the group’s properties, which are arguing that the best in [5], [8], [9].

2. \(n\)-Vertexes in Affine Plane and Their Similarity

2.1. 3-Vertexes and Their Similarity

Let’s have the affine plane \(A = (P, L, T)\).

Definition 2.1.1 Three-Vertex will called an ordered trio of non-collinear points \((A, B, C)\) in an affine plane.

Definition 2.1.2 Two three-vertexes \((A_1B_1C_1)\) and \((A_2B_2C_2)\) will call similar if they meet conditions: \(A_1B_1//A_2B_2; A_1C_1//A_2C_2; B_1C_1//B_2C_2\)

Example 2.1.1 In affine plane of the second order have the similar three-vertexes (Figure 1): \((A, D, C) = (B, C, D): \text{because,}\)

\[ AD \parallel BC; DC \parallel CD; AC \parallel BD \]

\((A, B, D) = (C, D, B): \text{because,}\)

\[ AB \parallel CD; BD \parallel DB; AD \parallel CB \]
(A, B, D) = (D, C, A): because, 
AB \parallel DC; BD \parallel CA; AD \parallel DA

\textbf{Example 2.1.2:} In the third order affine plane.

\((2,3,9) \approx (7,9,3), \) because:
\(\ell_{(2,3)} \parallel \ell_{(7,9)}; \ \ell_{(2,9)} \parallel \ell_{(7,3)}; \ \ell_{(3,9)} \parallel \ell_{(9,3)};\)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{example2.1.2.png}
\caption{Two similar three-vertexes in affine plane of order 3.}
\end{figure}

\textbf{Proposition 2.1.1:} The similarity of the three-vertexes is equivalence relation.

\textbf{Proof:} 1) It is clear that every three-vertexes (A, B, C) is similar to yourself.

\((A, B, C) = (A, B, C)\)

2) If three-vertexes \((A_1, B_1, C_1) \approx (A_2, B_2, C_2),\) are similar then also three-vertexes \((A_3, B_3, C_3) \approx (A_4, B_4, C_4),\) are similarity since from:

\(A_1 B_1 \parallel A_2 B_2; A_1 C_1 \parallel A_2 C_2; B_1 C_1 \parallel B_2 C_2\)

\(\implies A_3 B_3 \parallel A_4 B_4; A_3 C_3 \parallel A_4 C_4; B_3 C_3 \parallel B_4 C_4\)

3) If \((A_2, B_2, C_2) \approx (A_3, B_3, C_3)\), and three-vertexes \((A_3, B_3, C_3) \approx (A_4, B_4, C_4),\) then have to \((A_1, B_1, C_1) \approx (A_4, B_4, C_4),\) because parallelism in the affine plane is equivalence relation, which is described in [2], [3], [4].

So would have to:

\(A_1 B_1 \parallel A_2 B_2; A_1 C_1 \parallel A_2 C_2; B_1 C_1 \parallel B_2 C_2\)

and

\(A_2 B_2 \parallel A_3 B_3; A_2 C_2 \parallel A_3 C_3; B_2 C_2 \parallel B_3 C_3\)

since the parallelism in the affine plane is equivalence relation then will have to:

\(A_1 B_1 \parallel A_3 B_3\) and \(A_1 B_1 \parallel A_3 B_3\)

\(A_1 C_1 \parallel A_2 C_2\) and \(A_1 C_1 \parallel A_3 C_3\)

\(B_1 C_1 \parallel B_2 C_2\) and \(B_1 C_1 \parallel B_3 C_3\)

Well,

\((A_1, B_1, C_1) \approx (A_3, B_3, C_3).\)

\textbf{2.2. 4-Vertexes}

\textbf{Definition 2.2.1:} In affine plane \(A\), a set of four-point three out of three not-collineary will call 4-vertexes.

\textbf{Definition 2.2.2:} Two 4-vertexes \(ABCD\) and \(A'B'C'D'\) will call similar only if have the following parallels:

\(AB \parallel A'B', BC \parallel B'C', CD \parallel D'A'\) and \(DA \parallel D'A'.\)

\textbf{2.3. Parallelograms}

\textbf{Definition 2.3.1:} Parallelogram will call the ordered quartet of points \((A, B, C, D)\) from \(P\), that meets the conditions: \(AB \parallel CD\) and \(BC \parallel AD\) the lines \(AC\) and \(BD\) are called the diagonal of parallelogram.

\textbf{Example 2.2.1:} In affine plane of the second order (Figure 3. a.) have the following parallelogram:

\((A, B, C)\) with the diagonal \(AB\) and \(DC\) (Figure 3. b); \((A, B, D, C)\) with the diagonal \(AD\) and \(BC\) (Figure 3. c); \((A, B, C, D)\) with the diagonal \(AC\) and \(BD\) (Figure 3. d).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{example2.2.1.png}
\caption{4-parallelograms in the affine plane of order 2.}
\end{figure}

From the definition of parallelogram and the fact that parallelism is the equivalence relation is evident this Proposition:

\textbf{Proposition 2.2.1:} If you have two similar 4-vertexes, where each is parallelogram then another 4-vertexes will be parallelogram.

\textbf{2.4. n-Vertexes}

\textbf{Definition 2.4.1:} In affine plane \(A\), a set of \(n\)-points non-collinear will call similar if have the following parallelogram:

\((A_1, B_1, C_1, D_1) \approx (A_2, B_2, C_2, D_2)\)

where each is parallelogram then another \(n\)-vertexes will be called parallelogram.
collinearly three out of three will call n-vertex.

**Definition 2.4.2:** Two n-vertexes \((A_1A_2...A_n)\) and \((B_1B_2...B_n)\) will call similar just if have the following parallelisms:

\[ A_1A_j \parallel B_iB_j, \]

\[ \forall (i,j) \in \{(1,1),...,(1,n);(2,1),...,(2,n);...(n,1),...,(n,n)\}. \]

3. The Addition of Similarity Three-Vertexes in the Desargues Affine Plane

Let's have two similarity three-vertexes \((A_1,A_2,A_3)\) and \((B_1,B_2,B_3)\) in the Desargues affine plane \(A_D = \{P,L,I\}\).

Constructed the lines \(A_1B_1, A_2B_2, A_3B_3\) since are in Desargues affine plane and the similarity of three-vertexes have to: \(A_1A_2||B_1B_2; A_2A_3||B_2B_3; A_1A_3||B_1B_3\) \(\Rightarrow\) the lines \(A_1B_1, A_2B_2\) and \(A_3B_3\), or will be parallel or will cross the on a single point. Receive now a point \(O_1 \in A_1B_1\), and find points \(O_2\) and \(O_3\) how:

\[ O_2 = A_2B_2 \cap \ell_{A_1A_2} \quad \text{and} \quad O_3 = A_3B_3 \cap \ell_{A_1A_3}. \]

So have obtained thus three-vertexes \((O_1,O_2,O_3)\), (points \(O_1, O_2\) and \(O_3\) are non-collinearly, because from construction this three-vertexes will be similar with three-vertexes \((A_1,A_2,A_3)\) where \(O_1 \in A_1B_1, O_2 \in A_2B_2\) and \(O_3 \in A_3B_3\). This three-vertex called ‘zero’ three-vertex. So have three lines, to which each have its zero point. Now just as to [3], additions of the points of each line based on the algorithm of additions of points in a line in Desargues affine plans, and take:

\[ C_1 = A_1 + B_1, C_2 = A_2 + B_2, C_3 = A_3 + B_3, \]

**Definition 3.1:** The addition of two similarity three-vertexes \((A_1,A_2,A_3)\) and \((B_1,B_2,B_3)\), called three-vertexes \((C_1,C_2,C_3)\), where the points \(A_1B_1\), \(A_2B_2\) and \(A_3B_3\) found according to equation (1) (Figure 4).

From construction of three-vertexes as the addition of two similar three-vertexes have evident this Proposition:

**Proposition 3.1** Three-vertexes that obtained as the sum of two similar three-vertexes \((A_1,A_2,A_3)\) and \((B_1,B_2,B_3)\), it is similar to the first two.

Well

\[ (A_1 + B_1, A_2 + B_2, A_3 + B_3) = (A_1, A_2, A_3) \]

and

\[ (A_1 + B_1, A_2 + B_2, A_3 + B_3) = (B_1, B_2, B_3) \]

Proof: If renounce above from addition algorithm of the similar three-vertexes. In the same logic, are additions together two of whatever three-vertexes. Let's have two whatever three-vertexes \((A_1,A_2,A_3)\) and \((B_1,B_2,B_3)\) in the affine plane \(A = \{P,L,I\}\). Construct the line \(A_1B_1, A_2B_2\) and \(A_3B_3\). Get a whatever three-vertexes \((O_1,O_2,O_3)\) (the points \(O_1, O_2\) and \(O_3\) are non-collinearly) where \(O_1 \in A_1B_1, O_2 \in A_2B_2,\) and \(O_3 \in A_3B_3\). This three-vertex called the ‘zero’ three-vertex. So have three lines, where, in each line have hers zero point. Now just as to [3], the addition points of every line based on the addition algorithm given to [3], and take: \(C_1 = A_1 + B_1, C_2 = A_2 + B_2, \) and \(C_3 = A_3 + B_3, \)

**Figure 4. The Addition of two similarity three-Vertexes in the Desargues Affine Plane.**
Similarly \( n \)-Vertexes in the Desargues Affine Plane

Figure 5. The Addition of two non-similarity three-Vertexes in the Desargues Affine Plane is a three-Vertex.

Defined in this way, it seems as if does not have a contradiction. But the veracity of this Proposition are presenting with the help of a simple anti-example, shown in the following figure.

Remark 3.1: By following the addition algorithms for points in a line of Desargues affine plane, is sufficient to get only an auxiliary point \( P_1 \), for this obedient from [3], for three sums can either take one three-vertexes \((P_1,P_2,P_3)\), wherein each point of three-vertexes be auxiliary point for the relevant sum.

Remark 3.2: Marked the set of three-vertexes in the Desargues affine plans with symbol \( \mathcal{T}_a^{D,\text{Aff}} \).

Remark 3.3: Marked the set of similarity three-vertexes in the Desargues affine plans with symbol \( \mathcal{T}_a^{D,\text{Aff}} \).

It is clear that: \( \mathcal{T}_a^{D,\text{Aff}} \subseteq \mathcal{T}_a^{D,\text{Aff}} \).

Let us be \((A_1,A_2,A_3)\) and \((B_1,B_2,B_3)\) two whatsoever three-vertexes in the set \( \mathcal{T}_a^{D,\text{Aff}} \). I associate pairs

\[ [(A_1,A_2,A_3),(B_1,B_2,B_3)] \in \mathcal{T}_a^{D,\text{Aff}} \times \mathcal{T}_a^{D,\text{Aff}}, \]

three-vertex \((C_1,C_2,C_3)\) \( \in \mathcal{T}_a^{D,\text{Aff}} \), that the vertexes are determines with algorithm in [3]. According to the preceding Theorems, three-vertexes \((C_1,C_2,C_3)\) is determined in single mode by [3].

Thus obtain an application

\[ \mathcal{T}_a^{D,\text{Aff}} \times \mathcal{T}_a^{D,\text{Aff}} \to \mathcal{T}_a^{D,\text{Aff}}. \]

Definition 3.2: In the above conditions, application

\[ +: \mathcal{T}_a^{D,\text{Aff}} \times \mathcal{T}_a^{D,\text{Aff}} \to \mathcal{T}_a^{D,\text{Aff}}, \]

defined by
\[ [(A_1, A_2, A_3), (B_1, B_2, B_3)] \mapsto (C_1, C_2, C_3) \]

\[ \forall [(A_1, A_2, A_3), (B_1, B_2, B_3)] \in T^{D, Aff}_u \times T^{D, Aff}_u. \]

The addition in \( T^{D, Aff}_u \) according to this Definitions, can write

\[ \forall (A_1, A_2, A_3), (B_1, B_2, B_3) \in T^{D, Aff}_u, \]

1. \( P_1 \not\in A_1B_1, A_2B_2, A_3B_3, \)

2. \( t_{A_1B_1} \cap t_{O_1P_1} = P_2, \)

3. \( t_{A_2B_2} \cap A_1B_1 = C_1, \)

4. \( t_{A_3B_3} \cap A_1B_1 = P_3, \)

5. \( t_{P_1P_2} \cap A_1B_2 = C_2, \)

6. \( t_{P_1P_3} \cap A_2B_3 = C_3, \)

7. \( t_{P_2P_3} \cap A_3B_1 = C_4. \)

\[ \Rightarrow (A_1, A_2, A_3) + (B_1, B_2, B_3) = (C_1, C_2, C_3). \]

**Theorem 3.1:** For every two three-vertexes \((A_1, A_2, A_3), (B_1, B_2, B_3) \in T^{D, Aff}_u\), algorithm (2) determines the single three-vertexes \((C_1, C_2, C_3) \in T^{D, Aff}_u\), which does not depend on the choice of hers auxiliary point \(P_1\).

*Proof:* From Theorem 2.1, in [3], have that for every two points is a line in the Desargues affine plane the addition is commutative, and consequently have to:

\[ (A_1, A_2, A_3) + (B_1, B_2, B_3) = (A_1 + B_1, A_2 + B_2, A_3 + B_3) \]

From Theorem 2.1, in [3], have to addition of two points in a line of Desargues affine plane does not depend on the choice of hers auxiliary point. For this reason keep as auxiliary points for addition of pairs points, the auxiliary point \(P_1\).

From Theorem 3.1, appears immediately true this

**Proposition 3.3:** Additions of three-vertexes in \( T^{D, Aff}_u \) there are element zero the three-vertexes \((O_1, O_2, O_3)\):

\[ \forall (A_1, A_2, A_3) \in T^{D, Aff}_u, \]

\[ (A_1, A_2, A_3) + (O_1, O_2, O_3) = \]

\[ = (O_1, O_2, O_3) + (A_1, A_2, A_3) = \]

\[ = (A_1, A_2, A_3). \]

As well as worth and below Propositions.

**Proposition 3.4:** Additions of three-vertexes is commutative in \( T^{D, Aff}_u \):

\[ \forall (A_1, A_2, A_3), (B_1, B_2, B_3) \in T^{D, Aff}_u \]

\[ (A_1, A_2, A_3) + (B_1, B_2, B_3) = \]

\[ = (B_1, B_2, B_3) + (A_1, A_2, A_3). \]

*Proof:* By definition of additions of three-vertexes that have:

\[ (A_1, A_2, A_3) + (B_1, B_2, B_3) = (A_1 + B_1, A_2 + B_2, A_3 + B_3) \]

From Theorem 2.1, in [3], have that for every two points is a line in the Desargues affine plane the addition is commutative, and consequently have to:

\[ (A_1, A_2, A_3) + (B_1, B_2, B_3) = (A_1 + B_1, A_2 + B_2, A_3 + B_3) \]

\[ = (B_1 + A_1, B_2 + A_2, B_3 + A_3) = (B_1, B_2, B_3) + (A_1, A_2, A_3). \]

**Proposition 3.5:** Addition of three-vertexes is associative in \( T^{D, Aff}_u \):

\[ \forall (A_1, A_2, A_3), (B_1, B_2, B_3), (C_1, C_2, C_3) \in T^{D, Aff}_u \]

\[ (A_1, A_2, A_3) + [(B_1, B_2, B_3) + (C_1, C_2, C_3)] = \]

\[ = [(A_1, A_2, A_3) + (B_1, B_2, B_3)] + (C_1, C_2, C_3). \]

*Proof:* Let's have three whatever three-vertexes

\[ (A_1, A_2, A_3), (B_1, B_2, B_3), (C_1, C_2, C_3) \in T^{D, Aff}_u \]

Appreciate now,

\[ (A_1, A_2, A_3) + [B_1 + B_2 + B_3 + C_1 + C_2 + C_3] = \]

\[ = (A_1, A_2, A_3) + (B_1 + C_1, B_2 + C_2, B_3 + C_3) = \]

\[ = [A_1 + (B_1 + C_1), A_2 + (B_2 + C_2), A_3 + (B_3 + C_3)] \]

\[ = [(A_1 + B_1), (A_2 + B_2), (A_3 + B_3)] + (C_1, C_2, C_3) \]

\[ = (A_1 + B_1, A_2 + B_2, A_3 + B_3) + (C_1, C_2, C_3). \]

**Proposition 3.6:** For every three-vertex in \( T^{D, Aff}_u \) exists her right symmetrical according to addition:

\[ \forall (A_1, A_2, A_3) \in T^{D, Aff}_u, \exists (A_1, A_2, A_3) \in T^{D, Aff}_u, \]

\[ (A_1, A_2, A_3) + (A_1, A_2, A_3) = (O_1, O_2, O_3) \]

*Proof:* Let us have whatever \((A_1, A_2, A_3) \in T^{D, Aff}_u\), fix the ‘zero’ three-vertexes \((O_1, O_2, O_3) \in T^{D, Aff}_u\) (which would be similar to three-vertexes \((A_1, A_2, A_3)\)) if apply the Proposition 3.4, in [3] pp34990, have that, for points \(A_a, A_2\) and \(A_b\) find points respectively \(A_1 = O_4A_1, A_2 = O_2A_2, A_3 = O_3A_3\) such that:

\[ A_1 + A_1 = O_1; A_2 + A_2 = O_2; A_3 + A_3 = O_3. \]

Well \( \exists (A_1, A_2, A_3) = (A_1, A_2, A_3) \in T^{D, Aff}_u \) such that it:
\[(A_1, A_2, A_3) + (A_4, A_5, A_6) = (O_1, O_2, O_3)\]

I summarize what was said earlier in this

**Theorem 3.2:** The Groupoid \( \mathcal{T}_n^{D AFF} , + \) is commutative (abelian) Group.

**4. The Addition of Similarity n-Vertexes in the Desargues Affine Plane**

Equally as addition of three-vertexes in Desargues affine plane, by the same logic, additions and n-vertexes in this plane.

**Remark 4.1:** The set of similarity n-vertexes in Desargues affine plane marked with symbol \( \mathcal{N}_n^{D AFF} \).

The addition algorithm of n-vertexes, by analogy with addition algorithm of the three-vertexes are presenting below:

Let's have two whatever similarity n-vertexes in Desargues affine plane:

\[(A_1, A_2, A_3, \ldots, A_n), (B_1, B_2, B_3, \ldots, B_n) \in \mathcal{N}_n^{D AFF} .\]

The definitions of the similarity n-vertexes have the following parallelisms:

\[A_1 A_2 \parallel B_1 B_2, A_2 A_3 \parallel B_2 B_3, \ldots, A_{n-1} A_n \parallel B_{n-1} B_n, A_n A_1 \parallel B_n B_1 .\]

constructed the lines \( A_1 B_1, A_2 B_2, A_3 B_3, \ldots, A_n B_n \) since are in Desargues affine plane, and from the parallels the above, are the conditions of the Desargues theorem, it results that the above lines or crossing from a fixed point \( V \) or they have a bunch of parallel lines.

In both cases equally found the zero n-vertex. Take one first point \( O \in A_1 B_1 \), and then find all the other vertexes of n-vertexes how:

\[O_2 = A_2 B_2 \cap \ell_{A_1 A_2} \cap O_{A_1}, O_3 = A_3 B_3 \cap \ell_{A_2 A_3} \cap O_{A_2}, \ldots, O_n = A_n B_n \cap \ell_{A_{n-1} A_n} \cap O_{A_{n-1}} .\]

**Definition 4.1:** In the above conditions, application

\[\oplus : \mathcal{N}_n^{D AFF} \times \mathcal{N}_n^{D AFF} \rightarrow \mathcal{N}_n^{D AFF} ,\]

defined by

\[\left[(A_1, A_2, \ldots, A_n), (B_1, B_2, \ldots, B_n)\right] \mapsto (C_1, C_2, \ldots, C_n)\]

\[\forall (A_1, A_2, \ldots, A_n), (B_1, B_2, \ldots, B_n) \in \mathcal{N}_n^{D AFF} \text{ call the addition in } \mathcal{N}_n^{D AFF} \text{ according to this Definitioni, can write the addition algorithm of the n-vertexes:}\]

\[\forall (A_1, A_2, A_3, \ldots, A_n), (B_1, B_2, B_3, \ldots, B_n) \in \mathcal{N}_n^{D AFF} \]

1. \( P_i \notin A_i B_i, A_i B_{i+1}, \ldots, A_i B_n, \)

2. \[
\begin{align*}
&\left[i\right] \left[\ell_{A_i B_i} \cap \ell_{A_i A_{i+1}} \cap O_{A_i} P_i = P_2, \\
&\left[ii\right] \ell_{P_i O_{A_i}} \cap A_i B_i = C_1 \right]
\end{align*}
\]

3. \[
\begin{align*}
&\left[i\right] \left[\ell_{A_i B_i} \cap \ell_{O_{A_i} P_i} = P_3, \\
&\left[ii\right] \ell_{P_i O_{A_i}} \cap A_i B_i = C_2 \right]
\end{align*}
\]

4. \[
\begin{align*}
&\left[i\right] \left[\ell_{A_i B_i} \cap \ell_{A_i P_i} = P_4, \\
&\left[ii\right] \ell_{P_i O_{A_i}} \cap A_i B_i = C_3 \right]
\end{align*}
\]

\[\vdots\]

\[n+1. \]

\[
\begin{align*}
&\left[i\right] \left[\ell_{A_i B_i} \cap \ell_{A_i O_{A_i} P_i} = P_{n+1}, \\
&\left[ii\right] \ell_{P_i O_{A_i}} \cap A_i B_i = C_n \right]
\end{align*}
\]

\[\Leftrightarrow (A_1, A_2, \ldots, A_n) + (B_1, B_2, \ldots, B_n) = (C_1, C_2, \ldots, C_n).\]

And for n-vertexes, have true analog the statements had to three-vertexes (everything proved equally).

Well have the verities of following statements

**Theorem 4.1:** For every two n-vertexes \((A_1, A_2, \ldots, A_n), (B_1, B_2, \ldots, B_n) \in \mathcal{N}_n^{D AFF} \), algorithm (7) determines the single three-vertexes \((C_1, C_2, \ldots, C_n) \in \mathcal{N}_n^{D AFF} \), which does not depend on the choice of hers auxiliary point \( P_i \).

From Theorem 4.1, appears immediately this

**Proposition 4.1:** Additions of n-vertexes in \( \mathcal{N}_n^{D AFF} \) there are element zero the three-vertexes \((O_1, O_2, \ldots, O_n)\):

\[\forall (A_1, A_2, \ldots, A_n) \in \mathcal{N}_n^{D AFF} \exists (O_1, O_2, \ldots, O_n) \in \mathcal{N}_n^{D AFF} \]

\[\left[(A_1, A_2, \ldots, A_n) + (O_1, O_2, \ldots, O_n) = (O_1, O_2, \ldots, O_n)\right]
\]

\[= (A_1, A_2, \ldots, A_n) \]

Also as worth and below Propositions.

**Proposition 4.2:** Additions of n-vertexes is commutative in \( \mathcal{N}_n^{D AFF} \):

\[\forall (A_1, A_2, \ldots, A_n), (B_1, B_2, \ldots, B_n) \in \mathcal{N}_n^{D AFF} \]

\[\left[(A_1, A_2, \ldots, A_n) + (B_1, B_2, \ldots, B_n) = (B_1, B_2, \ldots, B_n) + (A_1, A_2, \ldots, A_n)\right]
\]

\[= (B_1, B_2, \ldots, B_n) + (A_1, A_2, \ldots, A_n) \]

(9)

**Proposition 4.3:** Addition of n-vertexes is associative in \( \mathcal{N}_n^{D AFF} \):

\[\forall (A_1, A_2, \ldots, A_n), (B_1, B_2, \ldots, B_n), (C_1, C_2, \ldots, C_n) \in \mathcal{N}_n^{D AFF} \]

\[\left[(A_1, A_2, \ldots, A_n) + [(B_1, B_2, \ldots, B_n) + (C_1, C_2, \ldots, C_n)] = \right]

\[= [(A_1, A_2, \ldots, A_n) + (B_1, B_2, \ldots, B_n)] + (C_1, C_2, \ldots, C_n) .\]

(10)
**Proposition 4.4:** For every n-vertex in $\mathcal{N}_n^{D,\text{Aff}}$ exists her right symmetrical according to addition:

$$\forall (A_1, A_2, ..., A_n) \in \mathcal{N}_n^{D,\text{Aff}}, \exists(A_1, A_2, ..., A_n) \in \mathcal{N}_n^{D,\text{Aff}}$$

$$(A_1, A_2, ..., A_n) + (A_1, A_2, ..., A_n) = (O_1, O_2, ..., O_n) \quad (11)$$

(Here have that $(A_1, A_2, ..., A_n) = (\overline{A_1}, \overline{A_2}, ..., \overline{A_n})$)

By Theorem 4.1, Propositions 4.1, 4.2, 4.3 and 4.4 we have this true theorem:

**Theorem 4.2:** The Groupoid $\left(\mathcal{N}_n^{D,\text{Aff}}, +\right)$ is commutative (Abelian) Group.

---

**References**

[1] Dr. Orgest ZAKA, Prof. Dr. Kristaq FILIPI, (2017) *An Application of Finite Affine Plane of Order n, in an Experiment Planning*, International Journal of Science and Research (IJSR), http://www.ijsrpublications.com/ijsr.net/archive/v6i6/v6i6.php, Volume 6 Issue 6, June 2017, 1744 - 1747, DOI: 10.21275/ART20174592

[2] Zaka, O., Filipi, K. (2016). One construction of an affine plane over a corps. Journal of Advances in Mathematics, Council for Innovative Research. Volume 12 Number 5. 6200-6206.

[3] Zaka, O., Filipi, K. (2016), “The transform of a line of Desargues affine plane in an additive group of its points”, *International Journal of Current Research*, 8, (07), 34983-34990.

[4] FRANCIS BORCEUX (2014). *An Axiomatic Approach to Geometry&An Algebraic Approach to Geometry* (Geometric Trilogy I& Geometric Trilogy II). Springer International Publishing Switzerland.

[5] GRRILLET, P. A (2007). Abstract Algebra(Second Edition). Graduate Texts in Mathematics v242. ISBN-13: 978-0-387-71567-4 Springer Science + Business Media, LLC (Grrillet, P. A, et al, 2007).

[6] HILBERT, D, VOSSEN, S. C (1990). Geometry And The Imagination. Chelsea Publishing Company.

[7] Buenkhouf, F, (Editors)(1995). *HANDBOOK OF INCIDENCE GEOMETRY*. Elsevier Science B. V. ISBN: 0 444 88355 X.

[8] Hungerford, TH. W (1974). *Algebra (Graduate Text in Mathematics vol 73)*. Springer-Verlag New York, Inc. ISBN 0-387-90518-9.

[9] Lang, S (2002). *Abstract Algebra (Third Edition)*. Springer-verlag new york, inc. ISBN 0-387-95385-X.

[10] Ueberberg, Johannes (2011), *Foundations of Incidence Geometry*, Springer Monographs in Mathematics, Springer, doi:10.1007/978-3-642-20972-7, ISBN 978-3-642-26960-8.