Extended methodology for deriving formal concepts

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Abstract. Two methodologies for formal concepts derivation are considered: the classical one, which focuses on the posterior analysis of the object’s properties of the studied knowledge domain, and non-classical, the cornerstone of which is the a priori formation of the set of measured object’s properties and the determination of existential relations on this set. In the article, firstly, a position is fixed in the technological chain of the target transformation of the source data, where the difference between considered methodologies shows itself. Secondly, the commonality of these two approaches is established in the aspect of the unity of their hypothetico-deductive basis. In this case, the cognitive activity of the subject is expressed first in a priori and then in a posteriori conceptual scaling of the measured properties. The work demonstrates the need for the joint use of the considered methodologies at processing incomplete and inconsistent empirical data about studied knowledge domain. The intermediate consolidation of these methodologies is possible only on the basis of multi-valued logic.

1. Introduction

As known, the standard protocol for the presentation of empirical information in various problems of data analysis is the table “objects-properties” [1-3]:

\begin{equation}
(G^*, M, V, I),
\end{equation}

where \(G^* = \{g_i\}_{i=1,...,r},\ r = |G^*| \geq 1\) is the set of observed objects; \(G^* \subseteq G,\ G\) – all hypothetically conceivable set of objects of the studied knowledge domain (KD), \(M = \{m_j\}_{j=1,...,s},\ s = |M| \geq 1\) – set of measured properties (attributes) of the objects; \(V\) – cumulative set of values of different properties, \(V = \bigcup_{j=1,...,s} V_j,\ V_j\) – area of value existence (domain) of property \(m_j\); \(I\) – ternary relation between \(G^*\) and \(V\) defined for all pairs from \(G^* \times M\).

The derivation of formal concepts from (1) is a variation of the clustering problem (more precisely, biclusterization [4, 5], when clusters – formal concepts – retain an object-attribute description of the data). This task involves a preliminary transformation (1) into a single-valued formal context

\begin{equation}
(G^*, M, I),
\end{equation}

where \(I \subseteq G^* \times M\) - the binary relation described by the incidence matrix “object-properties”, each element of which is the truth estimate of basic semantic proposition (BSP) about the studied KD: \(b_{ij} = \langle \text{object } g_i \in G^*\ has\ the\ property\ m_j \in M\rangle\). Therefore, the formal concepts derived from (2) – biclusters – will retain information about objects with a similar composition of properties and will be in relation of a partial order by nesting of compositions. The fundamental role of such data analysis is
that its results correspond to the definition of a concept and the generalization relation on the set of concepts in philosophy and classical logic [6, 7].

In [8], the difference in the methodologies for deriving formal concepts is determined by the orientation either to objects or to properties. This affects the “activation patterns” and “cognitive resource” of the subjects of the study. However, an analysis of [8] and the study that inspired this paper [9] leads to the conclusion that the existing difference in these methodologies appears exclusively at the stage of transformation (1) to (2).

The purpose of this article is to establish a fundamental commonality of the two existing methodologies for deriving formal concepts and to reason the need for their joint use at processing incomplete and inconsistent empirical data about studied KD.

2. Key points of competing approaches
From the standpoint of measurement theory [10, 11], transformation of (1) to (2) should not cause problems.

Indeed, each real measurement procedure (measuring tool, instrument) has a sensitivity threshold, a limited dynamic range. Therefore, for each value domain of the measured properties, None \( \in V_j \) is valid, where the constant None is the result of the measurement, which can be described as “no information” (but not in the sense of “measurement was not performed”), “outside the dynamic range”, or following [12], “the object and the measurement procedure are not semantically combined”. Then naturally we have

\[
I(g_i, m_j) = \begin{cases} 
\text{False}, & \text{if } I(g_i, m_j, \text{None}) = \text{True}; \\
\text{True}, & \text{otherwise}. 
\end{cases} \tag{3}
\]

Nevertheless, each of the considered methodologies in its own way "furnishes and extends" the transition from (1) to (2).

2.1. A posteriori analysis of the objects properties of the training sample
The basis of both methodologies is the use of very popular formal concept analysis (FCA) proposed by R. Wille [13]. FCA “in the narrow sense” derives formal concepts from context (2) using the following notation and models:

- Galois operators \( \rho, \omega \) (general notation “\( \langle \rangle \)”) for the context \((G^*, M, I)\):
  - \( \rho(X) = X' = \{ m_j \mid m_j \in M, \forall g_i \in X: I(g_i, m_j) = \text{True} \} \) – common objects properties making up \( X \subseteq G^* \);
  - \( \omega(Y) = Y' = \{ g_i \mid g_i \in G^*, \forall m_j \in Y: I(g_i, m_j) = \text{True} \} \) – objects that have all the properties of \( Y \subseteq M \);
  - for a set of objects \( X \), the set of their common attributes \( X' \) describes the similarity of objects from the set \( X \), and the closed set \( X' \) is a cluster of similar objects;
  - \((X, Y)\) – formal concept in which \( X \subseteq G^* \) is the extent, \( Y \subseteq M \) is the intent, and \( X = Y' \), \( Y = X' \);
  - \( B(G^*, M, I) \) – set of formal concepts of context \((G^*, M, I)\);
  - \( (B(G^*, M, I), \leq) \) – closed concept lattice, where \((X_1, Y_1) \leq (X_2, Y_2)\), if \( Y_1 \supseteq Y_2 \) (or \( X_1 \subset X_2 \)).

At the same time, undoubtedly, FCA “in the narrow sense” – an integral and classical methodology for deriving formal concepts (see, for example, [14-17]) which oriented in the terminology of [8] on “objects”. Within the framework of the proposed construct of differences between the considered methodologies, this expresses in the following:

- \( G^* \) objects are independent of the subject in the sense that there are no assumptions regarding their appearance in the problem, specifically, in (1);
- a posteriori subjective (that is, determined by some goals of the knowing subject) granulation of information is allowed in (1) using conceptual scales [14, 18, 19] (its elementary
manifestation is the expansion of the measured properties set \( M \). Only after this the transition to (2) is made based on (3).

If we leave aside the issue of fuzzy conceptual scales [19, 20], then said peculiarities completely characterize FCA as one of the most powerful methods of data mining.

2.2. A priori formation of a system of measured properties

In a “property-oriented” methodology, the subject a priori forms a system of measured properties (SMP), determining not only the composition of the set \( M \), but also the existential relations (or the properties existence constraints [9, 21]) on this set:

- **conditionality** \( C: M \times M \rightarrow \{ \text{True, False} \} \), when it is predetermined that if object \( g_i \) have the property \( m_j \), then it have property \( m_k \) (although the reverse may not be true), i.e. \( C(m_j, m_k) \leftrightarrow \forall g_i \in G: m_j \in \{ g_i \} \rightarrow m_k \in \{ g_i \} \), where \( \{ g_i \} \) - set of object \( g_i \) properties (q.v. subsection 2.1).
  
  Relation \( C \) is reflective, asymmetric and transitive, i.e. \( \forall a, b, c \in M: C(a, b) \land C(b, c) \rightarrow C(a, c) \) - q.v. figure 1;

- **incompatibility** \( E: M \times M \rightarrow \{ \text{True, False} \} \), when it is established beforehand that if object \( g_i \) have the property \( m_j \) then it does not have the property \( m_k \), and vice versa, i.e. \( E(m_j, m_k) \leftrightarrow \forall g_i \in G: m_j \in \{ g_i \} \rightarrow m_k \notin \{ g_i \} \). Relation \( E \) is antireflexive, symmetrical, and non-transitive, but is characterized by the so-called transitivity concerning conditionality, i.e. the implication \( \forall d, e, f \in M: C(d, e) \land E(e, f) \rightarrow E(d, f) \) is true – q.v. figure 1.

**Figure 1.** Examples of existential relations on a set of properties: 
- ○ - measurable property, ↗ - conditionality, / - incompatibility.

In such conditions, the object of the training sample can have only a normal subset of the measured properties set \([8, 9]\). A subset of the measured properties of \( Y \subseteq M \) is normal if and only if it is closed and compatible:

- \( Y \) is closed, if it contains all the properties conditioned by any element of \( Y \), i.e. \( \forall m_j \in Y: (\exists m_k \in M: C(m_j, m_k)) \rightarrow m_k \in Y; \)

- \( Y \) is compatible, if any two elements of \( Y \) are not related by incompatibility relation, i.e. \( \forall m_j \in Y: (\exists m_k \in M: E(m_j, m_k)) \rightarrow m_k \notin Y. \)

Within the framework of the used construct of differences between methodologies, a priori formation of the system of measured properties during transformation of (1) in (2) supplements (3) by checking the set of inherent properties of the object for normality. If this check yields a negative result, either the results of measuring the object properties are qualified as unreliable, and therefore corresponding lines in (1) and (2) are discarded, or the need to reconsider the notions of KD, which were a priori expressed by the subject in the description of the SMP, is recognized.

It should be noted that a “property-oriented” methodology does not exclude a posteriori conceptual scaling of properties. But such scaling changes the system of measured properties both in terms of composition and in terms of existential relationships between properties [18]. These changes should be reflected in the modification of the SMP as a tuple \((M, E, C)\), which will be used to check the set of inherent properties of the object for normality.

3. The unity of the hypothetical-deductive basis of methodologies

About a unified deductive basis of the considered methodologies for the derivation of formal concepts – FCA “in the narrow sense” – was said above (q.v. subsection 2.1). The nature of hypotheses, which
are produced as a result of the cognitive efforts of the subject and are tested in solving problems of deriving formal concepts, is unified as well.

3.1. Cognitive activity of the subject
The decision to measure some property \( m_i \) in the objects of the studied KD is a rational consequence of the subjective assumption about the presence in the KD of objects possessing this property [22, 23]. In conceptual terms, this means putting forward a hypothesis that the considered KD is characterized by the formal concept (\( \{m_i\}', \{m_i\} \)) (the hypothesis about the description of KD by the formal concept (\( \{\emptyset\}', \{\emptyset\} \)) – “nothing characterizes everything” – always put forward by default). Adding the new property \( m_i, m_j \neq m_i \), to the number of measured ones, brings the number of such hypotheses to 4: (\( \{\emptyset\}', \{\emptyset\} \)), (\( \{m_i\}', \{m_i\} \)), (\( \{m_i\}', \{m_j\} \)), (\( \{m_i, m_j\}', \{m_i, m_j\} \)). With \( M \) measured properties, this amount is \( 2^M \).

It is these \( 2^M \) of various hypothetical formal concepts that are studied in the classical methodology for deriving formal concepts from the context (2).

A priori formation of SMP, which distinguishes the competing methodology, should also be considered as hypothesizing about the conceptual structure of the studied KD. First, the formation of hypothetical concepts occurs according to the above rule with the usual replenishment of SMP with measured properties. Second, the introduction of existential relations on the set of measured properties limits the permissible conceptual description of the KD.

For all that, in the aspect of a priori formation of hypothetical concepts by the subject, the existential relations of the measured properties are not a cause, but a consequence.

Indeed, in classical logic, there are two possibilities for the formation of new concepts on the basis of existing ones [6, 7]:

- the division of an existing concept with the formation of new concepts replacing it is associated with the enumeration of all disjoint parts of its extent on a single basis – the attribute (the measured property), which is changing within the scope of the divisible extent (instead of division, the opposite action can be considered – a generalization [7]);

- the restriction of the concept means the introduction of a new concept, the extent of which is part of the extent of the original concept, and the intent is distinguished by an additional attribute (a new measured property), inherent only in that part of the objects conceivable in the original concept that forms the extent of the new concept introduced;

Let property \( m_i \) be the basis of division of a hypothetical concept. Note that, in reality, the fact of measuring the property \( m_i \) corresponds to set of hypothetical formal concepts that include the property \( m_i \) in its intent. However, for only one of them, \( m_i \) will be a distinctive (and not inherited according to the generalization relation on many concepts) property [14, 22]. We are talking about the division of this particular concept. Therefore, it is assumed that \( m_i \) is the basis of division of this particular concept.

«Modification» of \( m_i \) within the confines of divisible extent \( \{\ldots, m_i\}' \) (here «...» replaces set of inherited properties from more general hypothetical formal concepts) cannot be interpreted otherwise than as a transition in the domain \( V \), from one subset of property values to another, provided that these subsets do not intersect. Easy to see that a subjectively constructed disjunctive coverage of a domain \( V_s \):

\[
V_s = \bigcup_{k=1,\ldots,q} V_{sk}, q \geq 2, V_{si} \cap V_{sj} = \emptyset, i \neq j,
\]

corresponds to the nominal conceptual scaling of the property \( m_i \) [18], when the new measured properties \( \{m_k\}_k=1,\ldots,q \) with the domains \( \{V_{sk}\}_k=1,\ldots,q \) are incompatible.

Let us now turn to the method of restricting the concept. By definition, the restriction of the simplest hypothetical concept (\( \{m_k\}', \{m_k\} \)), where \( m_k \in M \), is the formal concept (\( \{m_k, m_i\}', \{m_k, m_i\} \)), where \( m_k \) is the new measured property, \( m_i \notin M \), such that specifies a certain part of the extent of restricted concept, i.e. \( \{m_k, m_i\}' \subset \{m_i\}' \). Then the detection of the property \( m_k \) in
the object of the KD is a reliable indication that this object belongs to the objects’ set \( \{ m_k \}' \). But since all objects in \( \{ m_k \}' \) possess the property \( m_k \), therefore, the presence of the property \( m_k \) in the object conditiones the presence of the property \( m_k \).

In the considered example the “restricting” property \( m_k \) cannot be interpreted otherwise than as an indication of the selection principle of the extent \( \{ m_k \}' \) part, when the choice is based on different values of the property \( m_k \) [23]. Therefore, for the domain of the property \( m_k \)

\[
V_k = V_k \cup V_k(m_k),
\]

where \( V_k(m_k) \) is the domain part of the measured property \( m_k \), the values from which characterize the objects that make up the extent part of the original concept, that was enumerated at its restriction. This means that the restriction of the concept \(( \{ m_k \}' , \{ m_k \} )\) is realized by the simplest ordinal conceptual scaling [18] of property \( m_k \) when its domain \( V_k \) is subjectively covered by only two sets: the domain \( V_k \) itself and its strict part \( V_k(m_k) \subset V_k \).

In general, the property \( m_k \notin M \) is introduced in the SMP to restrict some existing hypothetical concept \(( M_q' , M_q )\), \( M_q \subseteq M \), \( M_q = \{ m_k \}_{1,...,q1} \), \( q \geq 1 \). As explained in [24] this is realized by the simplest ordinal scaling of the nonstrict part of the properties from the intent of the restricted concept: \( M_q \subseteq M_q \), and then the presence of the property \( m_k \) in the object will determine the presence of the \( M_q \) properties in it.

Thus, both considered methodologies imply a priori cognitive activity of the subject associated with the formation of hypothetical concepts describing the unknown KD. In the “object-oriented” methodology, the subject predetermines the greatest variability of the potential results of formal concepts derivation. The “property-oriented” methodology is associated with the subjective narrowing of this variability. In this case, the researcher carries out the detailed elaboration of primary a priori hypotheses by means of two (and only two) types of a priori conceptual scaling of the properties to be measured – nominal and ordinal.

### 3.2. The structure of the system of measured properties

We have made several attempts to construct an adequate SMP that provides an effective solution to the problem of transition from (1) to (2) (see, for example, [23, 25, 26]). Finally, in [27], it was proposed to transform the “natural” description of the SMP in the form of a set of measured properties with two binary relations specified on it (which can be easily represented as a graph in which vertices represent properties, edges represent incompatibility, and arcs represent conditionality of properties) into a set of intersecting substructures, homogeneous in the form of existential conjugation of member-properties. It was shown in [27] that this model allows one to directly correlate the internal arrangement of such substructures with the pragmatically important concept of the normal subset of measured properties.

### 4. Combination of methodologies

The combination of the considered methodologies is understood as the necessity and the sufficiency of their joint use for the derivation of formal concepts. The analysis allows us to argue that at transforming (1) into (2) this combination is reduced to taking into account the results of both a priori and a posteriori conceptual scaling of the measured properties.

#### 4.1. Genesis and processing of the “soft” formal context

The need to combine the methodological orientations “on objects” and “on properties” arises if the empirical information (1) is inconsistent, which results in inapplicability of formula (3). The source of contradictions and incompleteness of data in (1) can be a posteriori fuzzy granulation of empirical information [19, 20] and / or taking into account the realities of accumulation of empirical data when (1) takes the form of a generalized table “object-properties” (GTOP) [25, 26].

Then (3) is forcedly replaced by data (1) transformation on the basis of a suitable multi-valued logic, for example, fuzzy or vectorial [28], which makes the “object-property” correspondence in (2) accordingly fuzzy or non-strict. The methods developed by the FCA for the derivation of formal
The analysis clearly shows that the methodologies for deriving formal concepts are immersed in the common hypothetical-deductive theory of formal concept analysis, an essential element of which is the reflection of the cognitive activity of the subject at the formation stage of a single-valued formal context representing empirical information about the studied knowledge domain. This activity is
expressed in a \textit{priori} and a \textit{posteriori} granulation respectively of ideal and real informational entities, which is carried out using subjectively constructed conceptual scales. Figure 2 shows the matching steps of data analysis in the derivation of formal concepts:

**Figure 2.** Formation of a single-valued formal context in extended methodology for deriving formal concepts.

- 1, 4, 9 – the perception by a subject (at a certain stage of problem solving) of the result of any of subsequent steps of the analysis (for example, the 4th step can be the perception of the result of the 12th – the set of formal concepts (SFC); to exclude the pile-up of connections, these transitions are depicted only within the selected cognitive acts);
- 2, 5 – formation/editing of SMP and GTOP, respectively, including conceptual scaling of measured properties (inverse to the subject impacts are associated with informing about the impossibility of performing the selected action);
- 3 – measurement (in a wide sense) of the object properties of KD;
- 6 – formation of SMP and GTOP models: transformation of SMP into a set of substructures that are homogeneous in the type of existential conjugation of the measured properties and interpretation of measurement results in GTOP as truth estimates of the BSP;
- 7 – formation of the initial formal context (IFC): combining meaningfully equivalent BSP (accumulation of corresponding evidences), taking into account the confidence in the
measurement procedures and the reliability of the measurement series in GTOP and the
standard alpha-section of the resulting “soft” formal context with an extremely soft confidence
threshold;

- 8 – identification of the IFC with the formal context, which is used further for the derivation of
formal concepts – the operating formal context (OFC);

- 10 – setting a subjective confidence threshold in the BSP and updating the OFC, by converting
the IFC into the OFC with marking all BSP, the truth estimates of which does not reach the set
threshold, as false and the rest – as true;

- 11 – transformation of the OFC, during which the contradiction of empirical data with the a
priori hypotheses about the desired FC is eliminated, namely, the properties sets of the
measurement objects are normalized by setting local confidence thresholds that ensure the
assignment of the least reliable BSP, previously considered true, to false (inverse to the
subject impact is associated with informing about the impossibility of performing this action);

- 12 – extraction of a set of formal concepts SFC from the OFC.

Since the functionality of the most famous data analysis systems based on FCA (Fuzzy FCA-
Wizard, Python FCA Tool, Galicia, ToscanaJ, ConExp [29, 32]) is not complete (in light of the
above), it is necessary to develop the corresponding algorithmic base and computer resources. Our
efforts at that point are focused on the development of the tool of ontological data analysis
OntoWorker [33].

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