Calculation of transverse vibrations of the pump set shaft

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Abstract. The paper presents calculations of the transverse vibrations of the shaft of the pumping unit, represented as a rod (beam), on the basis of the Bubnov-Galerkin method. An estimate is made of the convergence of the results obtained, from which the main conclusions are formulated.

1. Introduction
Reliability of pumps, like any machinery and mechanisms, is determined by several factors. First, it depends on the perfection of the design and, accordingly, the completeness of its adaptation to the operating conditions of the machines. Second, it is determined by the quality of factory manufacturing. And, finally, it depends on the operating conditions of the pumps.

Increasing the reliability of pumping units due to the consideration of the first two groups of factors finds its quite successful solution in engineering organizations - design bureaux and factories. However, the degree of success of these solutions largely depends on how well the pumps are able to adapt to the possible changes in the operating conditions occurring in the conditions of their operation quite often. Not only does the operational reliability of the pump units significantly depend on this, but also the economy of their operation.

Increased vibration is among the factors most adversely affecting the reliability of pumping units. In modern conditions of operation and management of transport, oil and gas storage systems, as well as elimination of accidents, significant importance is placed on the issues of management, organization and control of technological processes.

With the toughening of operating conditions, the intensity and frequency of vibrations of pumping units are increasing, and the pressure pulsations that lead to breakdown of pumping units and, consequently, breakdown in the technological process have a negative effect. In connection with which the requirements for the vibration protection of pumping units are sharply increasing. However, research in this area is increasingly reduced to determining the frequencies of free oscillations that do not depend on the characteristics of the shaft. Therefore, research in this area is relevant.

2. Methodology
When deriving the equation of transverse vibrations of a rod (or beam), we shall assume that in the undeformed state the so-called elastic axis of the rod is rectilinear and coincides with the line of centers of gravity of the cross sections of the rod. We take this rectilinear axis as the coordinate axis x; and from it we will count the deviations of the rod elements for transverse vibrations. We will assume, at least at first, that the deviations of the individual points of the axis of the rod occur perpendicular to the rectilinear, undeformed direction, neglecting the displacements of these points parallel to the axis.
Further, we assume that the deviations of the points of the axis of the rod in transverse vibrations occur in the same plane ("plane of oscillations") and are "small" deviations in the sense that the restoring forces that arise remain within the limits of proportionality.

Under such assumptions, the deviations of the points of the axis of the rod for transverse vibrations are uniquely determined by a single function of two variables, the coordinate x and the time t:

\[ y = y(x, t) \]  (1)

This function satisfies a linear partial differential equation of the fourth order, which can be constructed as follows.

The equation that allows us to find transverse oscillations with the constant rigidity \( EJ \)

\[ \rho \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left( EJ \frac{\partial^2 w}{\partial x^2} \right) = 0 \]  (2)

where \( \rho \) - density (kg/m\(^3\)), \( EJ \) - stiffness on the deflection \([E \text{ (Pa)}\) - modulus of elasticity, \( J \) (m\(^4\)) - moment of inertia of the cross section of the rod with respect to the central axis of the cross section perpendicular to the plane of oscillations], \( L \) - length of the shaft m).

The obtained differential equation of the beam oscillations contains the second time derivative. Therefore, for \( t = 0 \), it is necessary to put two initial conditions:

\[ w(x, 0) = w_0(x), \]  (3)
\[ \dot{w}(x, 0) = 0 \]  (4)

In the variable \( x \), in the equation, there is the fourth-order derivative, therefore, at each end of the beam, in accordance with the conditions for its fixing, it is necessary to put two boundary conditions.

\[ w(0, t) = 0, \quad \dot{w}(0, t) = 0 \]  (5)
\[ \ddot{w}(1, t) = 0, \quad \dddot{w}(1, t) = 0 \]  (6)

The Bubnov-Galerkin method is used for the solution. The required displacements are specified as:

\[ u = a_1(t)b_1(x) + a_2(t)b_2(x) + \ldots + a_n(t)b_n(x) = \sum_{n=1}^{n} a_n(t)b_n(x) \]  (7)

where \( a_1, a_2 \ldots a_n \) are unknown functions of the variable \( t \); \( b_1, b_2 \ldots b_n \) are given basis functions of the variable \( x \).

We substitute \( u \) into the equations of motion. Since the displacements (7) are not an exact solution of the equations of motion, on the right-hand side, instead of zero, we obtain the discrepancy \( R_1 \):

\[ \mu(x) \frac{\partial^2 (a_n(t)b_n(x))}{\partial t^2} + EJ \frac{\partial^4 (a_n(t)b_n(x))}{\partial x^4} = R_1 \]  (8)

In accordance with the Bubnov-Galerkin method, it is required that orthogonality conditions of the residual \( R_1 \) of the basis function \( b_n \) be satisfied.

A basis function satisfying the main boundary conditions

\[ b_n(x) = \sin \frac{m\pi x}{L}; \quad n = 1, \ldots, m \]  (9)

The orthogonality condition can be written in the following form:

\[ \mu_x \int_0^L (\sum b_n \ddot{a}_n) b_n dx + EJ \int_0^L \left( \sum a_n \frac{\partial^4 b_n}{\partial x^4} \right) b_n dx = 0 \]  (10)
\[
\begin{align*}
\mu_x \int_0^L \left( \sum b_n \dddot{a}_n \right) b_1 dx + EJ \int_0^L \left( \sum a_n \frac{\partial^4 b_n}{\partial x^4} \right) b_1 dx &= 0 \\
\mu_x \int_0^L \left( \sum b_n \dddot{a}_n \right) b_m dx + EJ \int_0^L \left( \sum a_n \frac{\partial^4 b_n}{\partial x^4} \right) b_m dx &= 0
\end{align*}
\]

\[
\begin{align*}
\dddot{a}_1 \mu_x \int_0^L b_1 b_1 dx + a_1 EJ \int_0^L \frac{\partial^4 b_1}{\partial x^4} b_1 dx + \cdots + \dddot{a}_n \mu_x \int_0^L b_n b_1 dx + \\
+ a_n EJ \int_0^L \frac{\partial^4 b_n}{\partial x^4} b_1 dx &= 0 \\
\dddot{a}_1 \mu_x \int_0^L b_1 b_n dx + a_1 EJ \int_0^L \frac{\partial^4 b_n}{\partial x^4} b_1 dx + \cdots + \dddot{a}_n \mu_x \int_0^L b_n b_m dx + \\
+ a_n EJ \int_0^L \frac{\partial^4 b_n}{\partial x^4} b_m dx &= 0
\end{align*}
\]

Equations are a system of \( m \) ordinary linear differential equations with constant second-order coefficients with respect to unknown functions \( b_1(t), \ldots, b_m(t) \).

The condition for the existence of a nontrivial solution is that the principal determinant of the system disappears.

To solve the proposed method, it is necessary to determine the minimum number of basis functions that will allow calculating parameters without loss of accuracy. Let us estimate how the values of the parameters change when the number of basis functions changes (from 1 to 10).

\[\text{Figure 1. Evaluation of the convergence of the oscillation frequency.}\]
3. Conclusion

Analyzing the obtained values, it can be concluded that when the number of basis functions increase, the values of oscillation parameters tend to a certain limit. Deviation between decisions in which seven and eight functions were considered is 0.02%. Deviation at four and five terms is 0.01%. To obtain satisfactory results, it is sufficient to retain two functions.

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