Two-sphere low-Reynolds-number propeller

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A three-dimensional model of a low-Reynolds-number swimmer is introduced and analyzed in this Brief Report. This model consists of two large and small spheres connected by two perpendicular thin rods. The geometry of this system is motivated by the microorganisms that use a single tail to swim; the large sphere represents the head of microorganism and the small sphere resembles its tail. Each rod changes its length and orientation in a nonreciprocal manner that effectively propels the system. Translational and rotational velocities of the swimmer are studied for different values of parameters. Our findings show that by changing the parameters we can adjust both the velocity and the direction of motion of the swimmer.

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I. INTRODUCTION

The propulsive motion of artificial and biological micron-scale objects is an interesting problem at low-Reynolds-number hydrodynamics. In this condition the dynamics is dominated by viscous forces. Examples of these micron-scale objects include biological microorganisms such as bacteria and also man made microswimmers, useful to operate at microfluidic investigations [1].

Propulsive motion at low Reynolds number is subject to the scallop theorem [2]. At small scales, where the Reynolds number is very low, the governing hydrodynamic equations, i.e., the Stokes and continuity equations, are linear and invariant under time reversal [3]. Any reciprocal shape deformation retracts its trajectory and the system stays back at the point where it started. In order to achieve a net translational displacement, the system should perform the body deformations in a nonreciprocal manner. As mentioned by Purcell a low-Reynolds-number propeller must have at least two internal degrees of freedom and he proposed a three-link swimmer. The detailed motion of Purcell’s swimmer was examined by Becker et al. where it was shown that Purcell’s system could swim and its dynamical properties were calculated [4]. Inspired by Purcell’s system, a low-Reynolds-number swimmer constructed by three linked spheres was introduced and analyzed by Najafi et al. [5] and experimentally realized by Leoni et al. [6]. After Purcell’s proposal there have been considerable scientific efforts in designing artificial swimmers. Such swimmers would be useful in developing microfluidic experiments. Furthermore, progresses in assembling microswimmers show the possibility of using micromachines inside the biological cells for noninvasive therapeutic treatments [7]. On the other hand, there are many theoretical works devoted to the study of different aspects in the motion of biological microorganisms at low-Reynolds-number condition [8–12]. Such interests include sperm swimming, metachronal waves in cilia, E. Coli chemotaxis, and coupling mediated by hydrodynamic interaction between nearby microorganisms [13,14]. For a review of recent progress on low-Reynolds-number hydrodynamics of microorganisms, see, for example, the review paper by Lauga and Powers [15].

Our first aim in this Brief Report is to present a simplified model that captures the characteristics of a swimming biological organism like a bacterium. Dipolar far velocity field and asymmetric shape, corresponding to the head-tail geometry of the organisms, are two important features of microswimmers. We model these systems by considering two spheres with different radii that are changing their separation. We will study the translational and angular motions of this system.

II. TWO-SPHERE MODEL

Figure 1 shows the schematic geometry of a model swimmer composed of two spheres. As shown in this figure two small and large spheres with radii a and R are connected by two perpendicular and negligible diameter rods. Let denote the lengths of long and short rods by L and l, respectively. The connection is established in a way that the angle between two rods is fixed to π/2 while the relative angular position of small rod with respect to the large sphere can be varied. Additionally, we assume that the length of the long rod can be dynamically changed. In this case, the system will have two internal degrees of freedom: the length of the long rod L(t) and the rotational angle of the short rod ϕ(t).

The geometry which we are introducing here resembles the body shape of a bacterium with a single flagellum or cilium. Bacteria use beating patterns in their tails to move. The small sphere in our two-sphere model acts as a beating tail and the large sphere resembles the head of animal. The minimum condition for swimming at low Reynolds number can be achieved in our three-dimensional model. By changing the length of long rod and the angle of small one in a prescribed form, we are able to choose the motion which breaks the time-reversal symmetry, the necessary condition for translational motion, and consequently propel the system.

As an example for the internal motion of the system, we let the angle ϕ(t) increase with constant angular velocity and the length of long rod change periodically around an average length. The explicit form of this motion is given by L(t)

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As argued by Higdon the total force acting by a point force on fluid bounded by a no-slip sphere is equivalent to the total Stokeslet strength \[ f = \frac{3}{2} \frac{R^3}{x_0^2} \mathbf{f} + \frac{1}{2} \frac{R^3}{x_0^2} \mathbf{m} + G \cdot \mathbf{f}, \]

where the tensor \( M \) and vector \( \mathbf{m} \) give the flow field due to the translational and rotational motions of a moving sphere. These quantities are given by

\[
M = \frac{3}{4X^4} \left[ \mathbf{I} + \frac{\mathbf{XX}}{X^2} \right], \quad \mathbf{m} = \frac{R^3}{X^4} \mathbf{X}. \tag{4}
\]
The average orientation of the long rod, which is not shown here is along the average swimming direction.

The average orientation of the long rod, which is not shown in the figure, is in the direction of the longitudinal axis of the helix. This is achieved by numerically solving for the rotational velocity. Controlling and adjusting the dynamical behavior of the swimmer are of prime importance in designing artificial micromachines. Here, we see that by changing the parameters of the system we can do this favor. In Fig. 3, we have shown that the overall swimming direction is sensitive to the initial phase $\phi_0$. Additionally and as another example, in Fig. 4, we have shown that by changing $L_0$, the length of long rod, the average swimming velocity can be changed.

As the geometry of the two-sphere swimmer is not symmetric, the far-field distribution of fluid velocity at the leading order of approximation resembles a velocity field of a single force dipole. This is the main characteristic of most swimming microorganisms with a dipolelike velocity pattern.

In summary, inspired by bacterium swimming, we proposed and analyzed a swimmer, constructed by two joint spheres. We have shown that this simple three-dimensional swimmer is a model for a low-Reynolds-number propeller that captures a number of dynamical features in microorganisms. It will be interesting to use this model swimmer and study many interesting problems such as the hydrodynamic interaction between such swimmers, the effects due to the confinement in the bounded fluids, and also chemotaxis phenomena. Inspired by the colonies of microorganisms, we are extending our model to investigate the hydrodynamic interaction in an ensemble of two-sphere swimmers.

**APPENDIX: MATHEMATICAL DETAILS**

Here, we present the explicit form of the matrix elements $a_{ij}$, $b_{ij}$, and $c_{ij}$ which were introduced in the text,

$$a_{ii} = -\frac{1}{6\pi \eta R} \left[ (1 + c_i) + (c_i - c_i) \left( \frac{X_0^2}{X_i^2} \right) \right],$$

$$a_{ij} = a_{ji} = -\frac{1}{6\pi \eta R} (c_i - c_j) \left( \frac{X_0 X_{ij}}{X_0^2} \right) \quad \text{for } i \neq j,$$

where $X_0 = 0.1$, $h_0 = 0.1$, $l = 0.3$, and $\omega_0 = \omega_0 = 1$. Line shows the real space trajectory of the large sphere. As one can see, the overall swimming direction can be varied by changing the parameters of the system. Average orientation of the long rod which is not shown here is along the average swimming direction.

The fluid velocity field at the location of small sphere is subject to the boundary condition $u(X_0) = \dot{X}_0$. Together with this boundary condition, the above equations make a complete set of dynamical governing equations for the swimmer.

To solve the dynamical equations for the system, we can use the force and torque balance conditions and obtain a set of equations which relate the different components of the translational or angular velocities of the system to the component of the vector $\dot{X}_0$ in the following matrix form:

$$\mathbf{V} = \mathbf{A} \mathbf{C}^{-1} \dot{X}_0,$$

$$\mathbf{\Omega} = \mathbf{B} \mathbf{C}^{-1} \dot{X}_0,$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are given in the Appendix.

**V. RESULTS AND DISCUSSION**

In this section we will present the numerical solution for the governing equations and obtain the real-space trajectory of the swimmer. For this purpose we plot the trajectory of large sphere. For the prescribed internal motion given by Eq. (1) and a special choice of parameters, we have plotted in Fig. 3 the space trajectory of the large sphere. As one can distinguish, the trajectory is a helical-shaped path with an overall translational movement in each turn. The different characteristics of the trajectory, preferred direction, average swimming velocity, and the effective radius of the helix can be controlled by the geometrical as well as dynamical parameters of the swimmer.

FIG. 3. The trajectory of the swimmer in the $(x,y,z)$ space is plotted for two different values of $\phi_0$. Other parameters are $R=1$, $a=0.5$, $L_0=4$, $h_0=0.1$, $l=0.3$, and $\omega_0=2\omega_0=1$. Line shows the real path of the large sphere. The swimmer starts its motion from the initial state where the large sphere is located at the origin and the long rod is orientated along the $−z$ direction. As one can see, the overall swimming direction can be varied by changing the parameters of the system. Average orientation of the long rod which is not shown here is along the average swimming direction.

FIG. 4. Average swimming velocity is plotted in terms of the length of long rod. Other parameters are set to $R=1$, $a=0.5$, $\phi_0 = 0$, $h_0=0.1$, $l=0.3$, and $\omega_0=\omega_0=1$. 

As the geometry of the two-sphere swimmer is not symmetric, the far-field distribution of fluid velocity at the leading order of approximation resembles a velocity field of a single force dipole. This is the main characteristic of most swimming microorganisms with a dipolelike velocity pattern.
\[ b_{ij} = 0, \]
\[
 b_{ij} = -b_{ji} = \frac{1}{8\pi \eta R^3} \left( 1 - \frac{R^3}{X_0} \right) X_{ijkl} \quad \text{for } i \neq j \neq k, \]
\[
 c_{11} = (M_{xx} - 1)a_{11} + M_{xy}a_{21} + M_{xz}a_{31} + (m_z - z_0)b_{21} - (m_y - y_0)b_{31} + G_{xx}, \]
\[
 c_{12} = (M_{xx} - 1)a_{12} + M_{xy}a_{22} + M_{xz}a_{32} - (m_y - y_0)b_{32} + G_{xy}, \]
\[
 c_{13} = (M_{xx} - 1)a_{13} + M_{xy}a_{23} + M_{xz}a_{33} + (m_z - z_0)b_{23} + G_{xz}, \]
\[
 c_{21} = M_{yx}a_{11} + (M_{yy} - 1)a_{21} + M_{yz}a_{31} + (m_y - y_0)b_{31} + G_{yx}, \]
\[
 c_{22} = M_{yx}a_{12} + (M_{yy} - 1)a_{22} + M_{yz}a_{32} + (m_y - y_0)b_{32} - (m_z - z_0)b_{12} + G_{yy}, \]
\[
 c_{23} = M_{yx}a_{13} + (M_{yy} - 1)a_{23} + M_{yz}a_{33} - (m_z - z_0)b_{13} + G_{yz}, \]
\[
 c_{31} = M_{zx}a_{11} + M_{zy}a_{21} + (M_{zz} - 1)a_{31} - (m_x - x_0)b_{21} + G_{zx}, \]
\[
 c_{32} = M_{zx}a_{12} + M_{zy}a_{22} + (M_{zz} - 1)a_{32} + (m_y - y_0)b_{12} + G_{zy}, \]
\[
 c_{33} = M_{zx}a_{13} + M_{zy}a_{23} + (M_{zz} - 1)a_{33} + (m_y - y_0)b_{13} - (m_x - x_0)b_{23} + G_{zz}. \]

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