The Dynamical Properties of Stellar Systems in the Galactic Disc

K2

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Abstract. We postulate that stars in the Galactic field are born in aggregates of binary stars with half mass radii $R_{0.5}$ and number of binaries $N_{\text{bin}}$ which are dynamically equivalent to the dominant mode cluster $(N_{\text{bin}}, R_{0.5}) \approx (200, 0.8 \text{ pc})$. Binary orbits are distributed according to an initial period distribution which is consistent with pre-main sequence data. Stellar masses are paired at random from the KTG(1.3) mass function. We develop a simple model for the redistribution of orbital angular momentum and energy in short-period proto-binary systems (pre-main sequence eigenevolution), which establishes the observed correlations between eccentricity, mass ratio and period. The evolution of orbital parameters owing to perturbations by neighbouring systems (stimulated evolution) within the dominant mode cluster places 1-2% of all orbits into the eigenevolution region ($P < 100 \text{ days}, e > 0.1$ approximately) of the eccentricity-period diagram. The number of such forbidden orbits at any time is a function of the stellar number density, the dynamical and the nuclear age of the cluster. Observations of binaries in clusters should reveal the odd binary with forbidden orbital parameters. Examples of such systems may be the pre-main sequence binaries P2486 and EK Cep and binaries in stellar clusters with eccentric orbits at periods smaller than the circularisation cutoff period. Eigenevolution is expected to depopulate the eigenevolution region within $10^5 \text{ yrs}$ for pre-main sequence binaries, but main sequence binaries with forbidden orbits should remain in the eigenevolution region for times of order $10^9 \text{ yrs}$. We show that the binary star population must have a birth eccentricity distribution which is approximately dynamically relaxed because stimulated evolution in the dominant mode cluster cannot sufficiently thermalise a significantly different distribution. After disintegration of the dominant mode cluster we have a population of Galactic field systems with orbital parameters as observed, with a surviving proportion of binaries of $f_{\text{tot}} = 0.48 \pm 0.03$ which compares favourably with the observed proportion $f_{\text{obs}} = 0.47 \pm 0.05$. The rise of the period distribution to a maximum at $\log_{10} P \approx 4.8$ reflects approximately the initial distribution, whereas the decay for $\log_{10} P > 4.8$ results from stimulated evolution. The mass-ratio distribution of G dwarf binaries is depleted at small mass ratios and has the shape of the main sequence distribution despite initially being the KTG(1.3) mass function. We predict and tabulate the mass ratio distribution for main sequence binaries with a primary star less massive than $1.1 \text{ M}_\odot$. Our model Galactic field stellar population has a binary proportion among G-, K- and M-dwarfs in good agreement with the observational data. Too few triple and quadruple systems form by capture to account for the number of observed systems. An example of a triple system which may have formed by capture in the birth aggregate may be Proxima Cen–$\alpha$ Cen A/B. We compare the specific angular momentum distribution of our initial binary star population with the observed distribution of specific angular momenta of molecular cloud cores. According to our model about 40% of all late-type stars are single after cluster dissolution but had companions with $\log_{10} P \geq 6$ at birth. These stars are expected to have circumstellar disks (and possible planetary systems) extending to at least about 40 AU.

Keywords: stars: low mass, formation – binary stars: orbital parameters, evolution, eccentricity–period diagram, individual: P2486, EK Cep, vB 164, vb 121, KW 181, S1284, Proxima Cen–$\alpha$ Cen A/B
1 INTRODUCTION

The dynamical properties of stellar systems are the stellar mass function, the proportion of binaries (and more generally of multiple systems) and their orbital parameters. In depth analysis of star count data provides the Galactic field stellar mass function (Kroupa, Tout & Gilmore 1993). The distribution of the remaining dynamical properties of stellar systems in the Galactic disk can be derived under simple assumptions about the initial distribution of the dynamical properties if the majority of low-mass stars form in aggregates which are dynamically equivalent to the dominant mode cluster (Kroupa 1995a, hereafter paper K1). The simple assumptions are consistent with data on pre-main sequence stars and are: (i) all stars are born in binary systems; (ii) which have uncorrelated component masses. Quantifying stimulated evolution (i.e. the evolution of orbital parameters owing to near-neighbour perturbations) allows K1 to derive an initial distribution of periods of the binary stars (equation 11 in K1):

\[ f_{P, \text{in}} = \frac{(\log_{10} P - \log_{10} P_{\text{min}})}{\delta + (\log_{10} P - \log_{10} P_{\text{min}})^2}, \]

where \( P \geq P_{\text{min}} \) is the orbital period in days and \( \eta \) and \( \delta \) are parameters which determine the shape and the maximum period which must satisfy \( \log_{10} P_{\text{max}} > 7.5 \) in order to account for observed systems with such large periods. K1 adopts \( \eta = 3.5, \delta = 100 \) and \( P_{\text{min}} = 1 \) day so that \( \log_{10} P_{\text{max}} = 8.78 \) because by our initial assumption that all stars form in binary systems equation 1 must have an integral over period equal to unity (equations 9-11 in K1). The initial distribution of eccentricities is taken to be thermal but is not critical for the conclusions of K1. The dominant mode cluster is assumed to be in virial equilibrium initially and is approximated by \((N_{\text{bin}}, R_{0.5}) \approx (200, 0.8 \text{ pc})\), where \( N_{\text{bin}} \) is the number of binaries distributed according to a Plummer density law with half-mass radius \( R_{0.5} \). This is consistent with direct imaging surveys of star-forming regions which identify similar structures to probably be the dominant mode of star formation (see K1 and references therein).

This paper (hereafter K2) is the second in a series of three papers K1, K2 and K3. In K1 we show that inverse dynamical population synthesis implies that clustered star formation may be the dominant mode of star formation. In K3 (Kroupa 1995b) we study in detail the dynamical evolution of the stellar aggregates simulated in K1 and K2. In this paper (K2) we study in detail the dynamical properties of stellar systems that result if stars are born in the dominant mode cluster.

In order to study the effects of stimulated evolution on the short period (\( \log_{10} P < 3 \)) binary star population we need to develop a model of the eccentricity–period diagram which reproduces its observational features. These include circular orbits for \( P < P_c \), where the circularisation cutoff period is \( P_c \approx 12 \) days for main sequence stars in the Galactic field, and the absence of eccentric orbits at small periods. The basic assumption of our model is that star formation is not responsible for the difference between the eccentricity and mass ratio distributions at \( \log_{10} P < 3 \) and \( \log_{10} P > 3 \) (see figure 2 in K1), but that this difference is due to the evolution of the orbital parameters in short period systems because of the interaction between the two accreting proto stars. We refer to this evolution as pre-main sequence eigenevolution. By modelling the initial \( e - \log_{10} P \) diagram we can hope to understand better the nature of ‘anomalous’ pre-main sequence and main sequence orbits which may appear in this diagram. We need to modify \( \eta \) and \( \delta \) in equation 1 to account for the redistribution of angular momentum and energy in short period systems owing to pre-main sequence eigenevolution.

We assume stars form in the dominant mode cluster, \((N_{\text{bin}}, R_{0.5}) = (200, 0.85 \text{ pc})\), listed under p.K2.K3 in table 1 of K1 and perform \( N_{\text{run}} = 20 \) simulations to improve statistical significance of our results. This cluster has an initial central number density of 320 stars pc\(^{-3}\), a mass of 128 \( M_\odot \), a crossing time \( T_{\text{cr}} = 3.5 \) Myrs and a median relaxation time \( T_{\text{relax}} = 11 \) Myrs (table 1 in K1). For the numerical integration of stellar orbits we use the N-body program \textit{nbody5} Aarseth (1994) has developed. A standard Galactic tidal field is modelled. The average stellar number density in the central 2 pc sphere decays to less than 0.1 stars pc\(^{-3}\) after 740 ± 150 Myrs when the cluster has completely disintegrated. We retain all stars, irrespective of whether they are bound to the initial aggregate or not. All final distributions are evaluated at time \( t = 1 \) Gyr, i.e. well after cluster disintegration. Our method of finding all bound binary systems at any given time is described in K1. We use the same definition for the distribution of periods, \( f_{P,i}(\log_{10} P, t) \), as K1 (equation 7 in K1). It is the number of orbits in the \( i \)th \( \log_{10} P \) bin divided by the total number of stellar
systems. The proportion of binary systems, $f_{\text{tot}}$, is the sum of $f_{\text{P,i}}$ over all periods (equations 2 and 7 in K1). We follow K1 and only consider stars less massive than 1.1 $M_\odot$ and use the initial stellar mass function derived by Kroupa et al. (1993). It is conveniently approximated by $\xi(m) \propto m^{-\alpha}$, where $0.70 \leq \alpha_1 \leq 1.85$ for $0.1 M_\odot \leq m < 0.5 M_\odot$, $\alpha_2 = 2.2$ for $0.5 M_\odot \leq m < 1 M_\odot$, $\alpha_3 = 2.7$ for $1 M_\odot \leq m$, and $\xi(m) \, dm$ is the number of stars in the mass range $m$ to $m + dm$. From here on we refer to $\xi(m)$ as the KTG($\alpha_1$) mass function and use $\alpha_1 = 1.3$ (K1). We assume component masses in binary systems are not correlated at birth.

We emphasise that if the suggestion by K1 is true that most stars may form in stellar aggregates that are dynamically equivalent to our dominant mode cluster then these aggregates must dissolve on a timescale less than 10 Myrs (see K1) which is significantly shorter than the evolution timescale of our dominant mode cluster. The presumed dominant mode embedded cluster probably expels most binding mass within 10 Myrs. After dissolution of the dominant mode embedded cluster we expect to obtain a very similar distribution of dynamical properties of stellar systems as we obtain here (dynamical equivalence: K1, K3).

The assumptions about star formation made here are not complete in so far as the birth of massive stars, nor of higher order multiple systems, nor a changing background potential in the young stellar aggregate are modelled. These will be future extensions of the present work.

In Section 2 we introduce our model for eigenevolution and in Section 3 we compare the dynamical properties of our model Galactic field stellar population with the observational constraints. The initial orbital parameters of our model binary star population are compared with the specific angular momentum distribution of molecular cloud cores in Section 4, and Section 5 contains a discussion of circumstellar disks. Finally, Section 6 concludes this paper.

2 EIGENEVOLUTION

In this section we device a model for the correlation between eccentricity and mass ratio with period. According to this model angular momentum and energy are redistributed in only about 10–20 per cent of all pre-main sequence binary systems, namely those with $P < 10^3$ days and those with longer periods and eccentric orbits at birth.

2.1 Introductory remarks

Stimulated evolution cannot account for the lack of orbits in the large eccentricity–small period region seen in all observational eccentricity–period diagrams of pre-main sequence binaries. An $e - \log_{10} P$ diagram of our orbits at any time is a rectangular region given by $0 \leq e \leq 1$ and $\log_{10} P_0 \leq \log_{10} P \leq \log_{10} P_1$, where $P_0 \approx 1$ days and $P_1 \approx 10^9$ days are not equal to $P_{\text{min}}$ and $P_{\text{max}}$ (equation 1), respectively, because stimulated evolution forces a few orbits outside this range. However, the distribution of observational data in any currently available $e - \log_{10} P$ diagram shows that for $\log_{10} P < 3$ there is an absence of systems with large eccentricity which is established at a very early age (at the latest by about 1 Myr, Mathieu 1994). The thick long-dashed line in the topmost panel in Fig. 1 approximately represents this upper envelope of the main-sequence data (Duquennoy & Mayor 1991), and the birth distribution (in Section 2.5 we discuss the difference between birth and initial distributions) of our model systems clearly shows a discrepancy with the observational data. The model distribution of eccentricities is in acceptable agreement with the observations for $\log_{10} P > 3$, but in bad agreement for $\log_{10} P < 3$ (central panel in Fig. 1). The birth distribution of periods we adopt shows there are too few model systems with $P < 100$ days (bottom panel in Fig. 1).

Pre-main sequence stars have larger radii than main sequence stars of the same mass, and significant interaction between the two stars can be expected if the periastron distance is comparable to their radii. For example, a 1 $M_\odot$ star at the birthline has a radius of about 5 $R_\odot$ (see Zahn & Bouchet 1989 and references therein). In a binary consisting of two such stars with a period of 10 days eccentricities larger than about 0.6 would imply collision of both components at periastron. Orbits with smaller eccentricities must evolve at a rate that depends on the ratio of the pre-main sequence stellar radius and the periastron distance. Orbital energy and orbital angular momentum are redistributed within this system at a rate which depends on the tidal deformation of the stars and on their internal structure, composition and rotation. The rate of tidal dissipation of kinetic energy is different for stars with radiative and convective envelopes, being much enhanced in the latter (Zahn 1992). In the present context the application by Zahn & Bouchet (1989) of tidal circularisation theory to post-birthline pre-main sequence binaries descending along the Hayashi track
is interesting. They suggest that tidal circularisation completes by 0.1 Myrs, i.e. on a very short time scale, because of the strong dependence of tidal dissipation on the stellar radius and the deep convection zones of pre-main sequence stars on the Hayashi track. We refer the reader to the more thorough discussion of tidal circularisation by Mathieu et al. (1992).

We do not adopt any of the proposed theories to describe the early evolution of short period orbits because the theories do not easily account for evolution if the initial eccentricity is larger than about 0.3. Also, we expect that significant evolution of the orbital elements occurs before the proto-stars reach the birth line. Major complications owing to continued accretion onto the proto-stars and dynamical friction in the circum-protostellar material make pre-main sequence “tidal circularisation” inaccessible to rigorous theoretical investigation. Direct hydrodynamical simulations cannot help because they cannot presently be extended beyond a few orbital periods at which stage most of the circum-protostellar material remains to be accreted. The resulting orbital parameters after the stellar masses have grown to approximately the final size thus remain inaccessible (Bonnell 1994).

Given this state of affairs we shall from here on refer to the evolution of orbital elements owing to system-internal redistribution of orbital energy and angular momentum in the proto-stellar binary as pre-main sequence eigenevolution rather than tidal circularisation.

We cannot model eigenevolution self consistently because this task (i.e. the interaction of the proto-stars and the subsequent changes to the phase-space variables) lies beyond the scope of the present investigation. We also do not model the possible increase in eccentricity owing to resonances with gaseous disks because eccentricity driving (Lubow & Artymowicz 1992) is difficult to verify observationally (Mathieu 1994). Instead, we assume proto-binary systems are born with periods distributed according to equation 8 below, with eccentricities distributed according to the thermal distribution (equation 4 in K1), and uncorrelated component masses. Prior to starting integration with nbody5 we evolve this distribution of birth orbital elements to an initial distribution according to our model of eigenevolution below, which was inspired by Zahn (1977) and Duquennoy, Mayor & Mermilliod (1992). This process models the very short (probably < $10^5$ yrs) duration of pre-main sequence eigenevolution during the proto-stellar phase so that the orbital parameters of short period binary systems are established during the proto-stellar accretion phase. According to our model, pre-main sequence eigenevolution empties the $e - \log_{10}P$ diagram of eccentric orbits with short period. We refer to the region above the thick long dashed curve in the top panel of Fig. 1 as the eigenevolution region of the diagram. However, while our model correctly reproduces the observed correlations of orbital parameters, we caution that our eigenevolution model is rather crude, i.e. it is primarily phenomenological in nature.

This approach allows us to obtain an insight into the effects stimulated evolution has on the distribution of orbits in this diagram. Any orbits found in the eigenevolution region subsequent to starting the numerical integration we refer to as having forbidden orbital parameters or simply as forbidden orbits. If stimulated evolution places a pre-main sequence binary with components on the Hayashi track into the eigenevolution region then we expect eigenevolution to rapidly (within about $10^3$ yrs) circularise the forbidden orbit (Zahn & Bouchet 1989). However, we are interested to learn how many forbidden orbits have been created, and so we do not eigenevolve these until we stop the N-body integration. Thus our simulation is not self-consistent because we do not take account of tidal dissipation on the equations of motion of each star and have to leave a binary with a forbidden orbit where stimulated evolution placed it in the $e - \log_{10}P$ diagram. Subsequent stimulated evolution of such an orbit is very unlikely though, because these hard binaries have small interaction cross sections, and so our approximation (i.e. pre-main sequence eigenevolution then stimulated evolution and finally main sequence eigenevolution) is reasonable for our purpose. Aarseth (1994 and private communication) is incorporating tidal dissipation in his N-body code so that a consistent treatment of a forbidden orbit will become possible in the future.

2.2 The model

We assume significant eigenevolution occurs when the accreting proto-stars are at periastron

$$R_{\text{peri}} = (1 - e) P_{\text{yr}} \left( m_1 + m_2 \right)^{\frac{3}{4}}, \quad (2)$$

where $P_{\text{yr}} = P/365.25$ is the period in years. We assume that if the binary system is born with the birth eccentricity $e_{\text{birth}}$ then the system evolves according to
\[
\frac{1}{e} \frac{de}{dt} = -\rho'
\]

to the final eccentricity given by
\[
\log e_{\text{fin}} = -\rho + \log e_{\text{birth}},
\]

where
\[
\rho = \int_0^{\Delta t} \rho' \, dt = \left( \frac{\Lambda R_\odot}{R_{\text{peri}}} \right)^x,
\]
and we assume \( R_{\text{peri}} \) is constant (compare with Goldman & Mazeh 1994); \( R_\odot = 4.6523 \times 10^{-3} \) AU is the radius of the sun; \( \Delta t \) is the time interval after which pre-main sequence eigenevolution becomes insignificant \( (\Delta t < 10^5 \) yrs approximately, Zahn & Bouchet 1989) and \( \rho^{-1} \) is the circularisation or pre-main sequence eigenevolution timescale (compare with Duquennoy, Mayor & Mermilliod 1992). The period evolves to
\[
P_{\text{fin}} = P_{\text{birth}} \left( \frac{m_{\text{tot,birth}}}{m_{\text{tot,fin}}} \right)^{\frac{1}{2}} \left( \frac{1 - e_{\text{birth}}}{1 - e_{\text{fin}}} \right)^{\frac{3}{2}},
\]

where \( m_{\text{tot,birth}} \) and \( m_{\text{tot,fin}} \) are, respectively, the birth and final sum of the masses of the two companions. We merge two close proto stars if their semimajor axis after pre-main sequence eigenevolution is \( a \leq 10 R_\odot \). This is a very simple criterion for merging, and a different scenario based on a criterion that depends on eccentricity as well, such as merging if the periastron distance is smaller than a few stellar raies, might be more realistic. However, as it is not our present aim to model merging during the proto-stellar accretion phase, but merely to acknowledge the presence of merged binaries at the birth line and to ease the computational burden of integrating the equations of motion, we have chosen the simple criterion above, and have a dopted the somewhat arbitrary factor of 10.

Choosing appropriate dimensionless parameters \( \lambda \) and \( \chi \) allows us to model the distribution of systems in the \( e - \log_{10} P \) diagram. Basically, \( \lambda \) measures the length scale over which significant evolution of the orbital elements during the proto stellar phase occurs, and \( \chi \) measures the ‘interaction strength’ between the two proto stars in the binary system.

From the discussion in section 2 of K1 we know that the mass ratio distribution for short period solar-type binaries increases with increasing mass ratio, or it might also be flat. Long period systems on the other hand show the converse behaviour. At periastron it appears possible that the secondary proto star accretes mass from the larger circumstellar disk of the primary as noted by Bonnell & Bastien (1992). We adopt the following simple model by assuming the mass ratio of the system, \( q = m_2/m_1 \leq 1 \), changes from a birth value to a final value given by
\[
q_{\text{fin}} = q_{\text{birth}} + (1 - q_{\text{birth}}) \rho^*,
\]

where
\[
\rho^* = \begin{cases} 
\rho, & \text{if } \rho \leq 1; \\
1, & \text{if } \rho > 1.
\end{cases}
\]

The final mass of the secondary is given by \( m_{2,\text{fin}} = q_{\text{fin}} m_{1,\text{birth}} \), where \( m_{1,\text{birth}} \) is the birth mass of the primary, which we assume does not change. This implies a net gain in mass of the binary stellar system which may be as large as \( m_{1,\text{birth}} - m_{2,\text{birth}} \). An alternative model based on equations 6 and 7 but constraining the total mass of the binary system to remain constant proved to be unsatisfactory when comparing with the short period observational data.

We stress that we consider the feeding hypothesis merely to propose that a different formation mechanism for short period binaries need not be necessary. We do not exclude this possibility but note that some sort of mass exchange or accretion sharing in proto-stellar systems with \( \rho \approx 1 \) must be expected. We must however always bear in mind that the evidence for a different distribution of mass ratios for long and short period systems is weak because the number of data in the short period sample is small.
A reduction of eccentricity implies a reduction of orbital period and we need to find \( \log_{10} P_{\min} > 0 \) in equation 1 in order to reproduce the observed main sequence period distribution. Depending on the amount of pre-main sequence eigenevolution we allow, we need a lower cutoff, \( \log_{10} P_{\min} \), in the log(period) distribution which increases with increasing proto-stellar orbital evolution. However, \( \log_{10} P_{\min} < 3 \), because pre-main sequence binaries with \( \log_{10} P_{\min} > 3 \) show no strong orbital evolution (figure 1 in K1). There are not enough initial systems with \( \log_{10} P > 3 \) and sufficiently large eccentricity to fill the \( e - \log_{10} P \) diagram at \( \log_{10} P < 3 \). Also, in paper K1 we showed that stimulated evolution does not suffice to harden enough orbits. We thus expect \( \log_{10} P_{\min} < 2 \).

We choose in equation 1 \( \log_{10} P_{\min} = 1 \), \( \eta = 2.5 \), \( \delta = 45 \) and obtain

\[
\chi = \frac{2.5}{45 + (\log_{10} P - 1)^2},
\]

so that \( \log_{10} P_{\max} = 8.43 \) from equation 11b in K1.

### 2.3 Simulations – finding \( \lambda \) and \( \chi \)

We need to establish which values for \( \lambda \) and \( \chi \) in equation 4 best approximate the observational data. We proceed as follows: 600 birth component masses are chosen from the KTG(1.3) mass function. Birth eccentricities are generated from equation 4 in K1 and birth periods from equation 8. We have 300 systems with uncorrelated component masses and distribute these five times giving in total 1500 systems.

Increasing \( \lambda \) leads to an increasing circularisation cutoff period, and decreasing \( \chi \) leads to a flattening of the upper envelope in the \( e - \log_{10} P \) diagram. By adjusting both parameters we are able to model the observed upper envelope and the circularisation period. We find \( \lambda = 28 \) and \( \chi = 0.75 \) are the best approximations to the \( e - \log_{10} P \) data of Duquennoy & Mayor (1991). This model is depicted in Fig. 2, and from the central and bottom panels we observe that the shape of the eccentricity and period distributions for \( 1 < \log_{10} P < 3 \) are also in good agreement with the data.

### 2.4 The initial eccentricity distribution

In this section we demonstrate the very different character of eigenevolution and stimulated evolution and discuss the efficiency of thermalising an initial eccentricity distribution in a dominant mode cluster.

In the top panel of Fig. 3 we visualise the change of eccentricity and period owing to our model for eigenevolution if \( \lambda = 28 \) and \( \chi = 0.75 \). We generate binary systems with birth eccentricities 0.9, 0.5 and 0.1 with periods from equation 8, the range at birth of which are shown by the dotted lines. From the figure we see that orbits with \( e_{\text{birth}} > 0.5 \) are circularised for \( P < 1 \) days approximately (for this illustrative example we do not merge binaries with semi-major axis less than \( 10 R_{\odot} \)).

In the lower panel of Fig. 3 we plot the distribution of birth eccentricities (dotted lines) and the distribution of eccentricities after cluster dissolution for \( N_{\text{run}} = 2 \) simulations of a cluster consisting initially of 200 binaries with \( R_{0.5} = 0.85 \) pc. We observe that hardening and softening of binaries is a rare occurrence and that stimulated evolution becomes significant (i.e. the distribution of orbits in the diagram is affected noticeably) only at \( \log_{10} P > 3 \) approximately in such a cluster. For \( e_{\text{birth}} = 0.9, 0.5, 0.1 \) we find that the final proportion of binaries after the cluster has disintegrated is \( f_{\text{tot}} = 0.47, 0.46, 0.49 \), respectively. There is no measurable correlation of the ionisation of binaries with initial eccentricity because the binding energy of the binary is a function of orbital period only. The resulting distributions of periods and mass ratios also do not change significantly if different initial eccentricities are used. We note however from Fig. 3 that after aggregate dissolution the maximum period, \( P_{\text{max}} \), below which orbits with original eccentricity remain decreases with increasing initial eccentricity: For \( e_{\text{birth}} = 0.1, 0.5, 0.9 \) we infer from the figure that \( \log_{10}(P_{\text{max}}) \approx 7.4, 6.3, 5.9 \), respectively. This suggests that circular orbits are more stable against stimulated evolution, which is to be expected because stellar components on an eccentric orbit spend a large time at large relative separation thus being more susceptible to near-neighbour perturbation (see Aarseth 1992 for a more in depth discussion). We expect to observe this in the distribution of orbital elements and the proportion of final binaries if a sufficiently large number of simulations of each initial eccentricity is performed to reduce the statistical uncertainties significantly, but clearly the effect is not very pronounced.
Excluding $e = e_{\text{birth}}$, we find that the resulting distributions after cluster disintegration approach the thermal distribution, as shown in Fig. 4.

These simulations show that an initially significantly different eccentricity distribution than the thermal distribution cannot dynamically relax to the thermal distribution (equation 4 in K1) in our dominant mode cluster. If most stars are born in aggregates that are dynamically equivalent to our dominant mode cluster then binary systems must be born with a birth eccentricity distribution which is approximately dynamically relaxed. This conclusion is also arrived at by Aarseth (1992).

### 2.5 ‘Birth’ vs ‘initial’ orbital parameter distributions

We differentiate between the birth distributions and initial distributions. The initial distributions result from the birth distributions after pre-main sequence eigenevolution has largely completed. The initial distributions are the distributions of orbital elements of pre-main sequence stars after most of the circum-proto stellar material has disappeared through accretion or expulsion. With ‘initial’ we refer to the time ($t = 0$) at which the N-body integration starts, which is equivalent to that ‘instant’ in time when the age of the system becomes sufficiently large that dynamical evolution of the stellar cluster becomes effective. Stimulated evolution then evolves the initial distribution to the main sequence distribution we observe in the Galactic field. Thus, strictly speaking, our birth distribution must be seen in context with our model for pre-main sequence eigenevolution only.

The model data in Fig. 1 represent our birth distributions, whereas the data in Fig. 2 are the initial distributions that are established at an age of about $10^5$ yrs.

### 3 THE DOMINANT MODE IN STAR FORMATION?

We show here that when our initial orbital parameter distributions are subject to stimulated evolution in the dominant mode cluster they evolve to distributions which reproduce the observed distributions for main sequence stars. We also discuss the formation of triple and quadruple systems by capture.

We assume $N_{\text{bin}} = 200$ binary systems are formed in the dominant mode cluster, as described in the Introduction. The half mass radius of 0.85 pc (table 1 in K1) is somewhat larger than our previous $R_{0.5} = 0.77$ pc (K1). We increase $R_{0.5}$ somewhat on the basis of additional numerical experiments with our new period distribution by monitoring depletion of the mass-ratio and period distribution, $f_q$ and $f_P$, respectively. Birth eccentricities and stellar masses are generated as in Section 3.2.1 of K1. Birth periods are given by equation 8. We perform 20 simulations to obtain useful statistics. In addition to the eigenevolution of the orbital elements introduced in Section 2.2 we merge components of a binary to a single star with combined mass of the components if the semi-major axis after eigenevolution is $a \leq 10R_\odot$.

In our simulations stars are approximated by point particles. The equations of motion of the stars in a binary system are not subject to dissipative forces that drive the eigenevolution simultaneously to being subject to perturbative forces from other systems in close proximity. We believe this approximation is permissible on grounds of our argumentation in Section 2.1.

#### 3.1 The eccentricity–period diagram

Evolution of the cluster begins with the binary systems having eccentricity and period distributions after pre-main sequence eigenevolution as shown in the top panel of Fig. 5a. We have initially 4000 binary systems of which 124 merge so that at $t = 0$, 3 per cent of all systems have merged to single stars. While we expect merging to occur during pre-main sequence eigenevolution we caution against overinterpreting this number given the simple nature of our eigenevolution model.

The initial distribution of binaries (top panel of Fig. 5a) evolves before and during cluster disintegration to the $e - \log_{10} P$ distribution shown in the bottom panel of Fig. 5a. We now find binaries in the eigenevolution region which have been placed there via stimulated evolution. The number of binaries to the left of the constant periastron curves (short dashed lines) with $e > 0.3$ is small amounting to between 1 and 2 per cent of all systems. We expect to observe forbidden orbital elements in stellar clusters. Examples of such orbits (see table 3 in Duquennoy et al. 1992) may be vB 164, vB 121 in the Hyades cluster, KW 181 in the Praesepe cluster and S1284 in M67, all of which have $e > 0.2$ although their periods are significantly less than the
cutoff period. However, these binaries have $e < 0.4$, whereas our forbidden orbits (Fig. 5a) have $e > 0.6$. This discrepancy is not serious because tidal circularisation will begin reducing the eccentricity of a forbidden orbit immediately it has been created so that our forbidden orbits will eigenevolve to smaller eccentricities. 

For example, a forbidden orbit with $e > 0.8$, $P = 10^2$ days and $m_{\text{tot}} = 0.64 M_\odot$ has $R_{\text{peri}} < 16 R_\odot$ so that eigenevolution may proceed rapidly, even for main sequence stars. Assessment of the time it takes for this to occur is, however, difficult, but may be less than 500 Myrs (see the examples presented in table 3 in Duquennoy et al. 1992) which is sufficient since the Hyades and Praesepe Clusters are older than this.

The short eigenevolution timescale for pre-main sequence binaries (Zahn & Bouchet 1989) may make chance detection of forbidden orbits unlikely in very young clusters. Examples of such systems may, however, be P2486 and EK Cep, the orbital elements for which are listed in table A2 of the review by Mathieu (1994). Both may have been pushed into the eigenevolution region with larger eccentricities than currently observed. P2486 is located in the Trapezium Cluster whereas EK Cep is isolated. It may have been ejected within the last $10^5$ yrs from an embedded aggregate or a small group of pre-main sequence stars. A proper motion and radial velocity measurement would be illuminating.

It is important to emphasise that any discussion of tidal circularisation theory (e.g. see Mathieu et al. 1992) must bear in mind that stimulated evolution rather than incomplete tidal circularisation or eccentricity driving from disks may be responsible for eccentric orbits with periods shorter than the circularisation cutoff period. The number of forbidden orbits observed in Galactic clusters is a function of the rate of production of such orbits through encounters, and the rate at which such orbits are removed through eigenevolution, as well as the rate at which systems are lost from the cluster.

After cluster dissolution we have 2555 binaries and 2766 single stars (in Section 3.4 we discuss how these are distributed in triple and quadruple systems) of which 124 are merged binaries. Thus, 4 per cent of all single main sequence stars are merged binaries according to the present model (but see caveat above).

Eigenevolution proceeds also for main sequence binaries with small period via tidal dissipation but on a much longer time scale (see Mathieu et al. 1992 for a discussion) and can in principle be simulated by a direct N-body integration program (Aarseth 1994). However, this is not possible yet and we model this with equations 2–5 but with different $\lambda$ and $\chi$. Feeding (equations 6 and 7) is not assumed to occur during main sequence eigenevolution, and we do not merge components. Since we compare our model distribution with the observed main sequence distribution (Duquennoy & Mayor 1991) for which all orbits with a period less than 12 days are circularised we require $\lambda_{\text{ms}} = 24.7$ assuming a mean mass of 1.4 $M_\odot$. Also, for small initial eccentricities Zahn (1977) obtains $\chi_{\text{ms}} = 8$ ($\chi_{\text{ms}} = 5$ on the other hand is suggested by Goldman & Mazeh 1991, but the difference is not important here). The mean stellar age of the Duquennoy & Mayor (1991) sample is about $5 \times 10^9$ yrs which sets the timescale for main sequence eigenevolution. We note that a circularisation cutoff period of 12.4 days is suggested by the Galactic cluster M67 which is 4 Gyrs old (Mathieu 1994).

In Fig. 5b we show the $e - \log_{10} P$ diagram which results from the distribution shown in the lower panel of Fig. 5a after main sequence eigenevolution. The model main sequence $e - \log_{10} P$ distribution is now in good agreement with the observational data. In particular the upper envelope is reproduced, and orbits with approximately $P < 11$ days are circularised although the precise cutoff period is mass dependent. The proportion of all systems appearing in Fig. 5b with circularised orbits amounts to approximately 4 per cent, and depends on $\log_{10} P_{\text{min}}$ (equation 8), eigenevolution and merging during pre-main sequence eigenevolution. Again we caution against overinterpreting this number, and stress that the astrophysical details of the evolution of close binary systems (tidal circularisation, Roche lobe overflow, merging) are quite beyond our simple model and our present aim.

3.2 The main sequence orbital parameter distributions

In Fig. 6 we compare our initial model eccentricity distributions at $t = 0$ (but after pre-main sequence eigenevolution) and after cluster disintegration (and after main sequence eigenevolution as in Fig. 5b) with main sequence data for short-period and long period binaries. Agreement in both cases is good.

The period distributions initially (but after pre-main sequence eigenevolution) and after cluster dissolution (after main sequence eigenevolution as in Fig. 5b) are compared with the observational data in Fig. 7. Agreement is very good. The final binary proportion is $f_{\text{tot}} = 0.480 \pm 0.032$ which compares very well with the observed $f_{\text{tot}} = 0.47 \pm 0.05$ in the Galactic field (K1). We also note that the initial period distribution
(which is the pre-main sequence eigenevolved birth distribution) is well approximated by the distribution given by equation 1 (the 2nd iteration in K1).

The observed mass ratio distributions for G dwarf binaries (Duquennoy & Mayor 1991) are contrasted with our model distributions of secondary masses for primaries with a mass in the range 0.9 to 1.1 $M_\odot$ in Fig. 8. The top panel shows that the short period model and observational distributions are in acceptable agreement. The very different character of this distribution contrasted with the long period distribution shown in the bottom panel may be due to shared accretion (i.e. feeding, Section 2.2). Agreement with the observed main sequence distribution of secondary masses for long-period systems is excellent, and we care to remember that the initial distribution is the power-law KTG(1.3) mass function.

3.3 Variation of binary proportion with system mass

The variation of the proportion of binaries with the mass of the primary can be used to constrain binary formation scenarios. We define

$$f_m = \frac{N_{\text{bin},m}}{N_{\text{sing},m} + N_{\text{bin},m}},$$  \hspace{1cm} (9)$$

where $N_{\text{bin},m}$ is the number of binaries with a primary mass $m$, and $N_{\text{sing},m}$ is the number of single stars with mass $m$.

Fischer & Marcy (1992) argue that the proportion of M dwarfs that are binary ($f_M = 0.42 \pm 0.09$) is smaller than the proportion of G dwarfs that are binary ($f_G \approx 0.57$) and suggest this is consistent with capture into potential wells of two different depths. Kroupa et al. (1993), on the other hand, show that variation of $f_m$ can be understood by random pairing of stellar masses if $f_{\text{tot}} \approx 0.6$. Leinert et al. (1993) find that the proportion of pre-main sequence binaries does not vary with K-band magnitude and, on the basis of the results of Kroupa et al. (1993), interpret this finding as evidence for $f_{\text{tot}}(t = 0) = 1$. We emphasise, however, that the observational data shown in Fig. 9 do not necessarily imply a decreasing binary proportion with decreasing primary mass!

In Fig. 9 we show how $f_m$ varies with primary mass if stars form in binary systems with uncorrelated masses. At $t = 0$ (after pre-main sequence eigenevolution) we have $f_m \approx 1$. If the majority of stars in the Galactic field are born in aggregates that are dynamically equivalent to the dominant mode cluster, as suggested by K1, then $f_m \approx 0.5$ on the main sequence (Fig. 9). The small increase in $f_m$ from $m = 1 M_\odot$ to $m = 0.4 M_\odot$ comes about because the highest mass stars (the G dwarfs in our simulations) spend more time near the cluster centre than the less massive stars. We expect $f_m$ to flatten or decay somewhat if more realistic embedded clusters are modelled with no upper mass cutoff and a changing background potential. The decrease to $f_m = 0.42$ at $m = 0.2 M_\odot$ results because these low mass binaries are, on average, less bound than the higher mass binaries.

3.4 Triple and quadruple systems

On the main sequence Duquennoy & Mayor (1991) observe the following ratio of single : double : triple : quadruple G-dwarf systems: 1.50 : 1 : 0.105 : 0.026, although they emphasise that the number of triple and quadruple systems may be larger.

For the above ratios we have in our model 0 : 1 : 0 : 0 at birth. We expect that some triple and quadruple systems form by capture at later times. We search the output data of our 20 simulations for such systems by replacing all binary systems by centre of mass particles, and searching for all binary centre of mass systems using the same algorithm as described in K1. We identify all those systems which have a binding energy between the centre of masses $-E_{\text{bin}} < 0$ and which persist for at least 43 Myrs at $t = 1$ Gyr (i.e. the same system must appear in two subsequent output lists). This is necessary because transient binary centre of mass systems with very small binding energy do appear. We find at $t = 1$ Gyr 37 binary centre of mass systems.

Our analysis identifies 2743 single stars, 2504 binary, 23 triple and 14 quadruple systems with relative proportions 1.10 : 1 : 0.0092 : 0.0056. Comparison with the main sequence proportions demonstrates that formation by capture cannot account for the number of triple and quadruple systems in the Galactic field.
In Fig. 10 we show the eccentricity–period diagram of the outer orbit of the triple and quadruple systems formed by capture. We refer to 'outer' orbital parameters as the orbital parameters of the binary which consists of two centre of mass particles, each of which can either be a single star or an 'inner' binary. All orbits of triple systems lie in the range $10^7 < P_{\text{outer}} < 10^{10.2}$ d, whereas the quadruple systems have $10^{8.2} < P_{\text{outer}} < 10^{11}$ d. The upper limit is given approximately by the Galactic tidal field (K1) and increases with system mass. We observe from Fig. 10 that quadruple systems have outer eccentricities $e_{\text{outer}} < 0.8$ approximately, while eccentricities close to unity are quite common in triple systems. The outer mass ratio distribution is biased towards one in quadruple systems (by the nature of our stellar mass range) whereas triple systems show a peak at $q_{\text{outer}} \approx 0.3$ (Fig. 10). The distribution of the ratio of inner to outer semi-major axes is bell shaped for both triple and quadruple systems. As apparent from Fig. 10 both have a broad maximum at log$_{10}$ ($a_{\text{inner}}/a_{\text{outer}}$) $\approx -4$. However, there are no quadruple systems with log$_{10}$ ($a_{\text{inner}}/a_{\text{outer}}$) $< -2.5$ (see also Fig. 11). The distribution of orbits in the $e_{\text{outer}}$–log$_{10}$ ($a_{\text{inner}}/a_{\text{outer}}$) diagram is shown in Fig. 11, where we plot two log$_{10}$ ($a_{\text{inner}}/a_{\text{outer}}$) values for one quadruple system outer eccentricity. The distribution of eccentricities is approximately thermal for triple systems, but $e_{\text{outer}} > 0.8$ does not occur in quadruple systems owing to the larger interaction cross section of the two inner binaries.

The bound nature and origin of the Proxima Cen–α Cen A/B triple system remains unclear (Kamper & Wesselink 1978, Matthews & Gilmore 1993, Anasova, Orlov & Pavlova 1994). Proxima Cen has a mass of $m_\odot = 0.12 M_\odot$ (based on $M_V = 15.49$ and the mass–$M_V$ relation tabulated by Kroupa et al. 1993), the mass of α Cen A is $m_A = 1.10 M_\odot$ and of α Cen B it is $m_B = 0.91 M_\odot$. The inner binary α Cen has $a_{\text{inner}} = 23.4$ AU and $e_{\text{inner}} \approx 0.5$. The current separation of Proxima from the inner binary is approximately 13000 AU, i.e. 0.063 pc. Two systems in our model Galactic field data resemble this triple system: (i) $m_\odot = 0.39 M_\odot$, $m_{A+B} = 1.72 M_\odot$, $e_{\text{inner}} = 0.40$, $a_{\text{inner}} = 33$ AU with a separation at $t = 1$ Gyr of 7685 AU. The outer orbit has $a_{\text{outer}} = 6419$ AU, $e_{\text{outer}} = 0.92$ and log$_{10} P_{\text{outer}} = 8.1$; (ii) $m_\odot = 0.13 M_\odot$, $m_{A+B} = 1.53 M_\odot$, $e_{\text{inner}} = 0.44$, $a_{\text{inner}} = 61$ AU with a separation at $t = 1$ Gyr of 68858 AU. The outer orbit has $a_{\text{outer}} = 46985$ AU, $e_{\text{outer}} = 0.51$ and log$_{10} P_{\text{outer}} = 9.5$.

To obtain some bounds on the likely outer orbital parameters of the Prox Cen–α Cen A/B system we note from the upper panel of Fig. 10 that log$_{10} P_{\text{outer}} < 10.2$. Kepler’s equation and the total mass of the system ($2.1 M_\odot$) gives $a_{\text{outer}} < 1.58 \times 10^6$ AU, i.e. log$_{10}$ ($a_{\text{inner}}/a_{\text{outer}}$) $\approx -3.8$. More generally, assume $R_{\text{peri,outer}} = 10^n a_{\text{inner}}$. Then $e_{\text{outer}} < 1 - 10^n + \kappa$. Since $\kappa = -1.5$ implies $e_{\text{outer}} = 0$, approximately (top panel of Fig. 11), we obtain $\theta = 1.5$. Thus our numerical experiment implies that triple systems formed by capture are stable if $R_{\text{peri,outer}} > 10^{1.5} a_{\text{inner}}$, $a_{\text{outer}} > 10^{1.5} a_{\text{inner}}$, and $e_{\text{outer}} < 1 - 10^{1.5 + \kappa}$. In the upper panel of Fig. 11 we indicate the approximate region region which is allowed for Proxima Cen–α Cen A/B.

A detailed study of the stability of triple systems initially on circular orbits is provided by Kiseleva, Eggleton & Orlov (1994). From their table 1 we find that the Prox α Cen A/B system can be stable only if $P_{\text{outer}}/P_{\text{inner}} > 4$ approximately, i.e. $\kappa < -0.4$. This represents a strict upper limit on $\kappa$, and we note that if $a_{\text{outer}} \approx 13000$ AU then $\kappa \approx -2.7$.

The capture scenario for the Proxima Cen–α Cen A/B system presented here does not alleviate the problem posed by the apparent youth of Proxima and the advanced age of α Cen A/B, which may be older than the Sun (Matthews & Gilmore 1993), unless either flare activity in late type M dwarfs persists for about $5 \times 10^9$ yrs (see also the discussion in section 10.3 in Kroupa et al. 1993) or Proxima was captured in the Galactic field by α Cen A/B which is highly improbable.

However, we cannot exclude the possibility that this triple system had initially a much larger $\kappa$ and that we are currently observing Proxima being ejected, or nearly ejected. Although this is very unlikely, this hypothesis may provide an explanation for the persisting significant flare activity of Proxima (Kroupa, Burnam & Blair 1989). Relatively recent close passages of Proxima past α Cen A/B may have temporarily induced flare activity through tidal deformation.

3.5 The mass ratio distribution

In the top panel of Fig. 12 we show as the dashed histogram the initially uncorrelated mass-ratio distribution of all binaries. The minor peak at $q \approx 0.9$ and the systems with $q = 1$ result from our feeding hypothesis (equations 6 and 7). We find that after the cluster has dissolved about 70 per cent binaries survive with $q \approx 0.9$, whereas only 40 per cent survive with $q \approx 0.2$. The initial and final mass-ratio distributions are tabulated in Table 1.
Table 1. The Mass-Ratio Distribution

| q   | $N_q$ | $\delta N_q$ | $N_q$ | $\delta N_q$ |
|-----|-------|--------------|-------|--------------|
|     | t=0   |              | final |              |
| 0.05| 0.0   | 0.0          | 0.2   | 0.1          |
| 0.15| 7.0   | 0.6          | 2.7   | 0.4          |
| 0.25| 17.3  | 1.0          | 8.2   | 0.7          |
| 0.35| 21.1  | 1.1          | 12.4  | 0.8          |
| 0.45| 23.0  | 1.1          | 15.1  | 0.9          |
| 0.55| 22.0  | 1.1          | 15.2  | 0.9          |
| 0.65| 22.4  | 1.1          | 14.3  | 0.9          |
| 0.75| 20.6  | 1.0          | 15.7  | 0.9          |
| 0.85| 26.2  | 1.2          | 17.5  | 1.0          |
| 0.95| 26.4  | 1.2          | 19.7  | 1.0          |
| 1.05| 8.1   | 0.7          | 7.7   | 0.6          |
| 1.15| 0.0   | 0.0          | 0.0   | 0.0          |

Notes to table:

$q = m_2/m_1 \leq 1$, $m_1 \leq 1.1 M_\odot$ and $m_2 \leq 1.1 M_\odot$ are the mass of the primary and secondary, respectively. $N_q$ is the average number of systems in mass ratio bin $q_i$, $N_q = \frac{1}{N_{\text{run}}} \sum_{i=1}^{N_{\text{run}}} N_{q_i}$. $\delta N_q$ is the standard deviation of the mean, $\delta N_q = \left( \frac{\sum_{i=1}^{N_{\text{run}}} N_{q_i}}{(N_{\text{run}}-1)} \right)^{\frac{1}{2}} N_{\text{run}}^{-\frac{1}{2}}$.

These model mass ratio distributions result from our uncorrelated birth distribution and the KTG(1.3) mass function after pre-main sequence eigenevolution (columns 2 and 3) and after stimulated evolution in the dominant mode cluster (columns 4 and 5). The data tabulated here are averages of $N_{\text{run}} = 20$ simulations.

In the bottom panel of Fig. 12 we show the stellar mass functions of the primaries and secondaries initially (after pre-main sequence eigenevolution) and after cluster disintegration. The mass function of the secondaries is steeper than that of the primaries. In Fig. 12 both the primary and secondary star mass functions show about the same depletion except at $m_2 > 0.5 M_\odot$, where the secondary star mass function is more depleted after the cluster has disintegrated. Mass segregation implies that the massive binaries spend more time in a dense environment so that the systems with massive components are ionised to a greater extent than systems with low mass components (see also Fig. 9). Depletion of orbits as a function of $q$ is not a process that can be modelled straightforwardly (see also the discussion of equation 6 in K1) and depends on the lifetime of the cluster and its initial configuration.

4 THE INITIAL ANGULAR MOMENTUM DISTRIBUTION

The initial distributions of orbital elements are equivalent to an initial distribution of angular momenta of the binary systems. The angular momentum is given by (masses in $M_\odot$, $P$ in days)

$$J = 1.24 \times 10^{52} (1 - e^2)^{\frac{1}{2}} P^{\frac{1}{2}} \frac{m_1 m_2}{(m_1 + m_2)^{\frac{3}{2}}} \left( \frac{g \text{ cm}^2}{\sec} \right).$$

The distributions of specific angular momenta, $J/M = J/(m_1 + m_2)$, are shown in Fig. 13 at $t = 0$ (pre-main sequence eigenevolution has a negligible effect on the specific angular momentum distribution) and after cluster dissolution. Ionisation of the least bound binaries leads to a depletion of the specific angular momentum distribution at high specific angular momenta. Our model initial binary star distribution adjoins the molecular cloud distribution without a gap (compare with Simon 1992). The molecular cloud core...
data are incomplete at small \( J/M \) owing to the observational difficulties in detecting slowly rotating cores (Goodman et al. 1993) and the distribution may extend to much lower values. An in depth discussion of angular momentum redistribution during star formation can be found in Bodenheimer et al. (1993).

5 IMPLICATIONS FOR CIRCUMSTELLAR DISKS

The orbital period of Jupiter is \( \log_{10} P = 3.64 \) and of Pluto it is \( \log_{10} P = 4.96 \). The Sun may have had a companion with \( \log_{10} P \geq 6 \) permitting stability of the solar system. From Fig. 7 we find that initially approximately 44 per cent of all G dwarfs were in such binary systems. These consist predominantly of one G dwarf and a late-type companion, and rarely two G dwarfs. After cluster disintegration about 12 per cent of all G dwarfs remain in such binary systems. Of all G dwarfs \( 44 - 12 = 32 \) per cent end up as single stars that came from binary systems with \( \log_{10} P \geq 6 \). The majority of the ex-companions of these are K- and M-dwarfs. Since the components from ionised binary systems initially with \( \log_{10} P \geq 6 \) are are not likely to be ejected from the aggregate immediately we need to estimate the proportion of systems which have encounters that are destructive for \( \log_{10} P \leq 5 \) to estimate the proportion of components that may retain a possible planetary system. From Fig. 7 we find that 9 per cent of all systems with \( \log_{10} P \leq 5 \) are ionised. Thus, \( 0.91 \times 0.32 = 0.29 \), i.e. 29 per cent of all G dwarfs are single and are likely to have escaped additional close encounters that are destructive at \( \log_{10} P \leq 5 \).

While reality will be much more complicated than discussed here, we have shown that in principle our Sun may be one such single G dwarf. Systems with \( \log_{10} P \geq 6 \) may also have a solar-type planetary system. We may thus expect in total \( 29 + 12 = 41 \) per cent of all G dwarfs may harbour solar-type planetary systems.

The total angular momentum vector of the solar planetary system is tilted by 7° with respect to the rotation axis of the Sun, and the orbit of Pluto is inclined by 17° to the ecliptic. This may indicate an early perturbation of the outer reaches of the solar system by either a young companion or a “visit” by a neighbour in the stellar group in which the sun was born. Heller (1993) models the perturbations of circumstellar disks in stellar groups and finds that this interpretation of the tilt of the total angular momentum vector of our planetary system is reasonable.

We can extend these considerations to circumstellar disks around all late-type stars. From fig. 1 in K1 or Fig. 7 we note that \( f_p \) does not depend on spectral type. Two stars result from each ionisation event. If the present model of star-formation is a valid approximation to reality then about 40 per cent of all late-type stars we see on the sky are single stars which at birth had a companion in an orbit with \( \log_{10} P \geq 6 \). These stars may have disks (when younger than a few Myrs) or planetary systems that extend to at least 40 AU.

6 CONCLUSIONS

In K1 we show that stimulated evolution in a stellar aggregate consisting of a few hundred binaries and with a half mass radius between 0.08 pc and 2.5 pc cannot evolve a period distribution which is initially limited to \( 3 \leq \log_{10} P \leq 7.5 \) into a distribution with a sufficient number of orbits at \( \log_{10} P < 3 \) to account for the observational data. The observed distribution of periods can only be accounted for if binary systems appear at the birth line with periods ranging from a few days to \( 10^9 \) days. We approximate the initial period distribution by equation 1. The resulting initial distribution of specific angular momenta for binary systems (Section 4) adjoins the specific angular momentum distribution of molecular cloud cores (Fig. 13).

We also assume that, rather than reflecting a different star-forming mode for short-period systems, the difference between the short and long period mass ratio and eccentricity distributions are manifestations of orbital element evolution which affects systems with small periastron (Section 2). We assume this mechanism is intrinsic to the young binary system and occurs to near-completion during the protostellar accretion phase. We develop a simple model of eigenevolution (Section 2.2) according to which the eccentricity–period diagram is depopulated at large eccentricities and small periods. We term this the eigenevolution region of the diagram and refer to any orbits within this region as forbidden orbital parameters. Thus the eigenevolution region is assumed to depopulate on a timescale of about 0.1 Myr (for pre-main sequence stars), establishing the upper envelope of the initial distribution of orbits in the \( e - \log_{10} P \) diagram by the time the pre-main sequence binaries appear near the birth-line in the HR diagram. Stimulated evolution depletes the distribution of orbits with approximately \( \log_{10} P > 4 \) on the dynamical evolution timescale of the stellar cluster.
Correcting for pre-main sequence eigenevolution and stimulated evolution we derive a birth distribution of periods (Section 2.5, equation 8).

To study in detail the evolution of the initial orbital parameters we perform 20 simulations of the dominant mode cluster (Section 3) and are left with a model Galactic field stellar population after its disintegration. We find that thermalisation of orbits in the dominant mode cluster is not complete enough to allow other than an initially approximately thermal eccentricity distribution (Section 2.4). The final binary star population has a period distribution in good agreement with the observed distribution of periods of G-, K- and M-dwarf binaries (Fig. 7) whereby the rise of the \( \log_{10} P \) distribution to \( \log_{10} P \approx 4.8 \) approximately reflects the initial distribution. The decay of the distribution at \( \log_{10} P > 4.8 \) results from stimulated evolution. The final short and long period mass ratio and eccentricity distributions are in good agreement with observational data for G dwarfs (Figs. 8 and 6, respectively). In particular, the maximum in the observed long-period mass-ratio distribution of G-dwarf binaries at a mass ratio of about 0.25 is reproduced even though the underlying KTG(1.3) mass function rises monotonically with decreasing stellar mass. We tabulate the overall model mass-ratio distribution for Galactic field systems with component masses less massive than \( 1.1 M_\odot \) in Table 1. The proportion of binaries in our model, \( f_{\text{tot}} = 0.48 \pm 0.03 \), compares very well with the observed proportion \( f_{\text{obs}} = 0.47 \pm 0.05 \).

The proportion of systems as a function of the mass of the primary star, \( f_m \), in our model Galactic field population is in good agreement with the observational constraints (Fig. 9). We emphasise that the latter are consistent with no decrease of \( f_m \) with decreasing primary mass.

When comparing our model with the observational data we keep in mind that in the observational period distribution triple systems are counted as two orbits and quadruple systems as three orbits adding data at the long-period end of the observational period distribution. Our model does not extend to higher multiplicities at birth. The number of triple and quadruple systems formed by capture in our model is too small to account for the observed numbers (Section 3.4). Proxima Cen–α Cen A/B has orbital parameters similar to triple systems formed in our simulations by capture in the dominant mode cluster.

Stimulated evolution pushes about 1–2 per cent of all final orbits into the eigenevolution region so that we expect to find in clusters binaries which have eccentric orbits although their period is shorter than the circularisation cutoff period. The pre-main sequence binaries P2486 (in the Trapezium Cluster) and EK Cep (isolated) may be examples of such systems with forbidden orbital elements. This interpretation of EK Cep requires it to have had a close encounter with another system approximately within the last \( 10^5 \) yrs and as a result of this it may have a relatively large velocity.

The existence of our planetary system does not contradict the hypothesis that all stars form in multiple systems (Section 5), nor that most stars originate in clustered star formation. We expect about 40 per cent of all young late-type stars to be single and have disks extending to about 40 AU. The majority of the remaining stars have companions.

We have demonstrated that reasonable assumptions about star formation that are consistent with observations of young stars, and the assumption that most stars form in aggregates that are dynamically equivalent to our dominant mode cluster (see K1), lead to a model population of stellar systems in the Galactic field which is in good agreement with the available observational constraints. Future observations (high-resolution observations of very young stars, large-scale near-infrared imaging surveys of star forming regions, radial velocity and proper motion surveys [see K3]) will verify if the assumptions made in this paper are true. These are: (i) most stars form as binaries, (ii) these have uncorrelated component masses, (iii) a distribution of orbits similar to equation 1 (equation 8 is our approximation to the birth distribution and is unlikely to be ever observable) and (iv) they form in aggregates dynamically equivalent to our dominant mode cluster.

Our assumptions and results would appear to be valid if most proto-stellar binary systems would form by dissipative capture in small groups of a few to ten accreting proto-stars that are clustered in aggregates of tens of such groups (K1).

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implying that forbidden orbital parameters have been merged. Stimulated evolution in the dominant mode cluster populates the eigenevolution region (Figure 4). The eccentricities (equation 8) are shown as the thick continuous line (equation 8) and the solid histogram. The dashed lines are the first and second iterations (fig. 8 in K1). Pre-main sequence eigenevolution is not implemented here. Observational data are as in fig. 1 of K1 (solid circles: G dwarfs; open squares and triangles: pre-main sequence binaries).

Figure 2. As Fig. 1. The initial distribution of model orbital parameters obtained after applying a pre-main sequence eigenevolution model with $\chi = 0.75$ and $\lambda = 28$ to the data shown in Fig. 1 (Section 2.3). In the top panel the short dashed lines are constant periastron loci (equation 2 with $R_{\text{peri}} = \lambda R_{\odot}$) for system masses of $m_{\text{tot}} = 2.2, 0.64, 0.2 M_{\odot}$ in increasing thickness. Systems lying to the left of these lines have periastron distances less than $28 R_{\odot}$. Binaries with semi major axis less than $10 R_{\odot}$ are not merged here. Note that in the bottom panel the solid histogram represents the initial period distribution, whereas the solid curve is the birth distribution (equation 8). See also discussion in Section 2.5. The observational data are as in Fig. 1.

Figure 3. Eigenevolution and stimulated evolution of orbital parameters (Section 2.4). Upper panel: the effects of our model for pre-main sequence eigenevolution on birth eccentricities $e_{\text{birth}} = 0.1, 0.5, 0.9$, shown by the horizontal dotted lines, is presented for $\chi = 0.75$ and $\lambda = 28$. The thick long dashed line is as in Fig. 1. The short dashed lines are as in Fig. 2. Lower panel: Assuming no eigenevolution we show the distributions after cluster disintegration that result from stimulated evolution in the dominant mode cluster for birth eccentricities as in the top panel. Histograms are shown in Fig. 4. The results of two simulations ($N_{\text{run}} = 2$) are shown for each birth eccentricity. The birth period distribution in both panels is given by equation 8.

Figure 4. (Section 2.4) The eccentricity distributions after cluster dissolution are shown for the birth eccentricities ($e_{\text{birth}} = 0.1, 0.5, 0.9$) which are plotted in the lower panel of Fig. 3. The histograms, represented by the same symbols as in Fig. 3, have been normalised to unit area after discarding the remaining peaks at the original eccentricities. The mean of two simulations are shown per birth eccentricity. The solid line histogram is the observed distribution of eccentricities for main sequence G dwarfs with $\log_{10} P > 3$ and normalised to unit area (Duquennoy & Mayor 1991).

Figure 5. (Section 3.1) a: The $e - \log_{10} P$ diagram for our model of the pre-main sequence eigenevolution ($\lambda = 28, \chi = 0.75$) at $t = 0$ (top panel) and after cluster disintegration (bottom panel) when 1164 bound binaries appear in the plot. Systems with semimajor axis after pre-main sequence eigenevolution $a \leq 10 R_{\odot}$ have been merged. Stimulated evolution in the dominant mode cluster populates the eigenevolution region implying that forbidden orbital parameters should be observable in stellar clusters. b: Eigenevolution on the main sequence with $\lambda_{\text{ms}} = 24.7, \chi_{\text{ms}} = 8$ applied to the data in the bottom panel of Fig. 5a depopulates the eigenevolution region and circularises all orbits with period less than about 11 days on a timescale of about 5 Gyrs. The thick long-dashed and the short dashed curves in Fig. 5a and b are as in Fig. 2. We note the sparsity of data with $\log_{10} P > 2$ with small eccentricity does not imply eccentricity driving (Section 2.1). This figure contains data of 9 of the 20 simulations (table 1 in K1).

Figure 6. (Section 3.2) Top panel: The short period eccentricity distribution for all simulated data, a subset of which is shown in Fig. 4. The dashed line represents the initial ($t = 0$) distribution after pre-main sequence eigenevolution and the solid histogram represents the final distribution after application of main sequence eigenevolution (as in Fig. 5b). The model distributions are averaged from 20 simulations (table 1 in K1). Open circles are the data for main sequence G dwarf binaries (Duquennoy & Mayor 1991). Lower
panel: The same as the upper panel but for long period systems. The solid dots are main sequence G dwarf binary system data from Duquennoy & Mayor (1991). All distributions are normalised to unit area.

Figure 7. (Section 3.2) The main sequence period distribution for all simulated data, a subset of which is shown in Fig. 5b, is presented here as the solid histogram. This distribution of periods results after the birth distribution (solid curve, equation 8) is subjected to pre-main sequence eigenevolution, leading to the initial distribution shown as the long-dashed histogram, and then to stimulated evolution in the dominant mode cluster and main sequence eigenevolution (the latter having a negligible effect). The model distributions are averaged from 20 simulations (table 1 in K1). The short dashed curves are the first and second iteration (fig. 8 in K1). The data are as in fig. 1 in K1 (solid circles: G dwarfs; open circles: K dwarfs; stars: M dwarfs; open squares and triangles: pre-main sequence binaries).

Figure 8. The G dwarf mass-ratio distribution. Top panel: short period distribution of secondary star masses in systems with primaries with a mass between 0.9 and 1.1 $M_{\odot}$ (Section 3.2). The dashed histogram represents the initial distribution ($t = 0$) after pre-main sequence eigenevolution, and the solid histogram is the final distribution. The model distributions are averaged from 20 simulations (table 1 in K1) and have been normalised to the data (open circles) at $q > 0.4$. The open circles are G dwarf main sequence short period binary star data (Mazeh et al. 1992 and fig. 2 in K1). Bottom panel: the same as top panel but for long period systems. The solid dots are G dwarf main sequence long period binary star data (Duquennoy & Mayor 1991 and fig. 2 in K1).

Figure 9. Variation of the proportion of binaries with mass of the primary (Section 3.3 and equation 9). The upper ($t = 0$) and lower (after cluster disintegration) small open circles denote our model (average of 20 simulations), and the solid circles are measurements of G dwarfs ($f_G = 0.53 \pm 0.08$, following Leinert et al. 1993), K dwarfs ($f_K = 0.45 \pm 0.07$, following Leinert et al. 1993) and M dwarfs ($f_M = 0.42 \pm 0.09$, Fischer & Marcy 1992). The large open circle is derived from the 5.2 pc stellar sample (Kroupa et al. 1993). Horizontal ‘errorbars’ represent the primary star mass range.

Figure 10. The orbital parameters of triple and quadruple systems (Section 3.4). Upper panel: The eccentricity–period diagram for outer orbits. The thick dashed line and the scale of axis are as in Fig. 1. Middle panel: the distribution of mass ratios. Lower panel: The distribution of the ratios of inner to outer semi-major axis. Stars with three rays denote triple systems and stars with six rays denote quadruple systems.

Figure 11. (Section 3.4) The $e_{\text{outer}} - \log_{10}(a_{\text{inner}}/a_{\text{outer}})$ diagram for triple systems (upper panel) and quadruple systems (lower panel). The shaded region in the upper panel constrains the outer orbital parameters of Proxima Cen–α Cen A/B if it is a bound triple system. Each quadruple system has two inner orbits for one outer eccentricity.

Figure 12. (Section 4) Top panel: The overall mass ratio distribution $q = m_2/m_1 \leq 1$, where $m_1$ and $m_2$ are the mass of the primary and secondary, respectively. The initial distribution, with both component masses chosen at random from the KTG(1.3) mass function and after pre-main sequence eigenevolution, i.e. feeding (Section 2.2), is shown as the dashed histogram. After the dominant mode cluster has disintegrated we obtain the distribution shown as the thick solid histogram. A small number of systems appear with $q > 1$ because of our binning. These have $q = 1$ and result from our feeding hypothesis in Section 2.2. The data are tabulated in Table 1. Lower panel: The stellar mass function of primaries initially after pre-main sequence eigenevolution (short dashed histogram) and after cluster disintegration (solid histogram) is contrasted to the stellar mass function of secondaries initially after pre-main sequence eigenevolution (dotted histogram) and after cluster dissolution (thick long dashed histogram). On adding the mass function of the secondaries and primaries at any instant we obtain approximately the KTG(1.3) mass function. The model distributions are averaged from 20 simulations.
Figure 13. The initial distribution of specific angular momenta of our binary star population (solid histogram, normalised to unit area) is compared with the distribution of specific angular momenta in molecular cloud cores (dashed histogram, normalised to unit area) obtained from table 2 of Goodman et al. (1993). The binary star specific angular momentum distribution decays to the form shown as the dotted histogram after cluster dissolution which corresponds to that for main sequence binary stars in the Galactic field. The model distributions are averaged from 20 equivalent simulations (Section 4).