New Statistical Results on the Angular Distribution of Gamma-Ray Bursts

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Abstract. We presented the results of several statistical tests of the randomness in the angular sky-distribution of gamma-ray bursts in BATSE Catalog. Thirteen different tests were presented based on Voronoi tessellation, Minimal spanning tree and Multifractal spectrum for five classes (short1, short2, intermediate, long1, long2) of gamma-ray bursts, separately. The long1 and long2 classes are distributed randomly. The intermediate subclass, in accordance with the earlier results of the authors, is distributed non-randomly. Concerning the short subclass earlier statistical tests also suggested some departure from the random distribution, but not on a high enough confidence level. The new tests presented in this article suggest also non-randomness here.

Keywords: gamma ray burst, factor analysis

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INTRODUCTION

There are increasing evidences that all the GRBs do not represent a physically homogeneous group (Kouveliotou et al. 1993, horváth 1998, balázs et al. 2003, Hakkila et al. 2003, horváth et al. 2006). Hence, it is worth investigating that the physically different subgroups are also different in their angular distributions. In the last years the authors provided (balázs et al. 1998, balázs et al. 1999, Mészáros et al. 2000) several different tests probing the intrinsic isotropy in the angular sky-distribution of GRBs collected in BATSE Catalogs. One may conclude the results of these studies: A. The long subgroup seems to be distributed isotropically (see also Briggs 1993); B. The intermediate subgroup (horváth 2002, horváth et al. 2006) is distributed anisotropically on the 96-97% significance level; C. For the short subgroup the assumption of isotropy is rejected only on the 92% significance level; D. The long and the short subclasses, respectively, are distributed differently on the 99.3% significance level. Independently (Litvin et al. 2001), confirmed the results A., B. and C. with one essential difference: for the intermediate subclass a much higher - namely 99.89% - significance level of anisotropy is claimed. Again, the short subgroup is found to be "suspicious", but only on the 85-95% significance level. In this paper, similarly to the previous studies, the intrinsic randomness is tested; this means that the non-uniform sky-exposure function of BATSE instrument was considered.
MATHEMATICAL SUMMARY AND THE TEST-VARIABLES

The randomness of the point field on the sphere can be tested with respect to different criteria. In the following we defined several test-variables.

**Voronoi tesselation (VT).** The Voronoi diagram - also known as Dirichlet tesselation or Thiessen polygons - is a fundamental structure in computational geometry (Voronoi 1908, Stoyan & Stoyan 1994). Generally, this diagram provides a partition of a point pattern according to its spatial structure. Assume that there are N points (N > 1) scattered on a sphere surface with an unit radius. The Voronoi cell of a point is the region of the sphere surface consisting of points which are closer to this given point than to any other ones of the sphere. This cell forms a polygon on this sphere. Every such cell has its area (A) given in steradians, perimeter (P) given by the length of boundary (one great circle of the boundary curve is called also as "chord"), number of vertices (Nv) given by an integer positive number, and by the inner angles. This method is completely non-parametric, and therefore may be sensitive for various point pattern structures in the different subclasses of GRBs.

Any of the four quantities characterizing the Voronoi cell can be used as test-variables or even some of their combinations, too. We defined the following test-variables: 1, Cell area A; 2, Cell vertex (edge) Nv; 3, Cell chords C; 4, Inner angle αi; 5, Round factor (RF) average $RF_{av} = 4\pi A/P$; 6, Round factor (RF) homogeneity $1 - \frac{\sigma(RF_{av})}{RF_{av}}$; 7, Shape factor A/P^2; 8, Modal factor $\sigma(\alpha_i)/N_v$; 9, The so-called "AD factor" defined as $AD = 1 - (1 - \sigma(A)/\langle A \rangle)^{-1}$.

**Minimal spanning tree (MST).** Contrary to VT, this method considers the distances (edges) among the points (vertices). A spanning tree is a system of lines connecting all the points without any loops. The minimal spanning tree (MST) is a system of connecting lines, where the sum of the lengths is minimal among all the possible connections between the points (Prim 1957). The statistics of the lengths and the MST angles between the edges at the vertices can be used for testing the randomness of the point pattern. To characterize the stochastic properties of a point patters we use three quantities obtained from a MST: 1, Variance of the MST edge-length $\sigma(L_{MST})$; 2, Mean MST edge-length $L_{MST}$; 3, Mean angle between edges $\alpha_{MST}$.

**Multifractal spectrum** is the third method which was used. Here the only used variable is the $f(\alpha)$ multifractal spectrum, which is a sensitive tool for testing the non-randomness of a point pattern.

### TABLE 1. Tested samples of BATSE GRBs.

| Sample   | Duration [s] | Peak flux $[\text{photons cm}^{-2}\text{s}^{-1}]$ | Number of GRBs |
|----------|--------------|-----------------------------------------------|----------------|
| Short1   | $T_{90} < 2$ s | $0.65 < P_{256} < 2$                            | 261            |
| Short2   | $T_{90} < 2$ s | $0.65 < P_{256} < 406$                         | 406            |
| Intermediate | $2 \text{s} < T_{90} < 10 \text{s}$ | $0.65 < P_{256}$                            | 253            |
| Long1    | $T_{90} > 2$ s | $0.65 < P_{256} < 2$                            | 676            |
| Long2    | $T_{90} > 10$ s | $0.65 < P_{256}$                             | 966            |
RESULTS

Completing 200 simulations in all of the subsamples (for them see Table 1.) we get a 13D sample representing the joint probability distribution of the 13 test variables. Using a suitable chosen measure of distance of the points from the sample mean we can get a stochastic variable characterizing the deviation of the simulated points from the mean only by chance. An obvious choice would be the squared Euclidean distance.

In case of a Gaussian distribution with unit variances and without correlations this would resulted in a $\chi^2$ distribution of 13 degree of freedom. But the test variables in our case are correlated and have different scales. Factor analysis (FA) is a suitable way to represent the correlated observed variables with fewer non-correlated variables of less in number (Wallet & Dussert 1998). The number of non-correlated variables, $k$, can be constrained by $k < 8.377$ in our case for $n = 13$. Hence, we retained 8 non-correlated variables.

Out of the 13 test-variables only the multifractal spectrum gave significant (>95%) deviation from the simulated sample in more than one group. The BATSE samples, however, were different in the number of test-variables giving positive signal (>95%) and in the level of significance. Among the tested samples short1 experiences four (96.5%, 97.5%, 97.5%, 95.5%), short2 two (99.98%, 96.02%), intermediate one (98.0%), long1 one (95.5%) and long2 no variables with >95% significance (see Table 2.). Calculating the joint significance level we assumed that they can be represented as a linear combination of non-correlated hidden factors of less in number. We obtained $k=8$ as the number of hidden factors. Then we computed the distribution of the squared Euclidean distances from the mean of the simulated variables. Comparing the distribution of the squared distances of the simulated with the BATSE samples we concluded that the short1, short2 and intermediate groups deviate significantly (99.90%, 99.98% and 98.51%) from the
FIGURE 1. Distribution of the Euclidean distances of the simulated samples from the stochastic mean of the variables in the 13D parameter space. There are altogether 1000 simulated points. Full line marks a $\chi^2$ distribution of 8 degree of freedom, normalized to the sample size. The distances of the BATSE samples are also indicated. The departures of samples "short1" and "short2" exceed all those of the simulated points. The probabilities, that these deviations are non-random, equal 99.9% and 99.98%.

Fully randomness but it is not the case at the long samples (see Fig. 1.).

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