Analysis and Augmented Spatial Processing for Uplink OFDMA MU-MIMO Receiver under Transceiver I/Q Imbalance and External Interference

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Abstract—This paper addresses receiver (RX) signal processing in multi-user multiple-input multiple-output (MU-MIMO) systems. We focus on uplink orthogonal frequency division multiple access (OFDMA)-based MU-MIMO communications under in-phase/quadrature-phase (I/Q) imbalance in the associated radio frequency electronics. It is shown in the existing literature that transceiver I/Q imbalances cause cross-talk of mirror-subcarriers in OFDM systems. Opposed to typically reported single-user studies, we extend the studies to OFDMA-based MU-MIMO communications and incorporate also external interference from multiple sources at RX input, for modeling challenging conditions in increasingly popular heterogeneous networks. In the signal processing developments, we exploit the augmented subcarrier processing, which processes each subcarrier jointly with its counterpart at the image subcarrier, and jointly across all RX antennas. Furthermore, we derive optimal augmented linear RX in terms of minimizing the mean-squared error. Extensive numerical results show the signal-to-interference-plus-noise ratio as a function of different system and impairment parameters. Based on the results, the performance of the conventional per-subcarrier processing is heavily limited under transceiver I/Q imbalances, and is particularly sensitive to external interferers, whereas the proposed augmented subcarrier processing provides a high-performance signal processing solution being able to detect the signals of different users as well as suppress external interference efficiently.

Index Terms—external interference, heterogeneous networks, in-phase/quadrature-phase (I/Q) imbalance, interference suppression, massive multiple-input multiple-output (MIMO), multiuser multiple-input multiple-output (MU-MIMO), orthogonal frequency-division multiple access (OFDMA)

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I. INTRODUCTION

Modern communication systems need to provide sufficient performance for the ever-increasing user needs of faster data connections and cheaper devices. This has resulted e.g. in larger and more complicated symbol alphabets which are, unfortunately, more vulnerable to various signal distortions than conventional solutions. In addition, the user equipment (UE), including also the analog radio frequency (RF) circuitry, should be implemented with very low costs and silicon area. These things, among other requirements of maximum performance, low-cost, small size etc., have resulted in a situation where the RF imperfections and their mitigation methods by cost-efficient digital signal processing have become very important aspects in system design. One of these RF imperfections is the so-called in-phase quadrature-phase (I/Q) imbalance which occurs in direct-conversion transceivers. When the baseband signal is up-converted in the transmitters (TXs) or when the RF signal is down-converted in the receivers (RXs), the signals in the I and Q branches have slight differences in their amplitude and phase responses e.g. due to manufacturing tolerances. This leads to imbalance between the I and Q signals and thus distorts the overall signal waveforms.

I/Q imbalance effects and mitigation are widely studied for orthogonal frequency division division multiplexing (OFDM) waveforms. In [4–6], I/Q imbalance of a single data link in OFDM systems is studied comprehensively. The single link approach is extended to cover multiple TX antennas in [7] while [8–11] consider multiple antennas on both TX and RX sides, resulting in full multiple-input multiple-output (MIMO) communications. The joint effects of I/Q imbalance and power amplifier nonlinearities are studied in [12] whereas [13, 14] focus on I/Q imbalance with carrier frequency offset. Based on these studies, the so-called augmented subcarrier processing for I/Q imbalance mitigation in OFDM systems has been proposed in [4, 8, 11]. Therein, each subcarrier signal is processed jointly with the corresponding signal at the image, or mirror, subcarrier. This approach is very close to widely-linear processing [15] where the signal and its complex conjugate are processed jointly. The widely-linear processing is proposed for processing non-circular signals, see e.g. [15–17], and also for I/Q imbalance mitigation, see e.g. [18, 19]. Since I/Q imbalance results in non-circular signals even with originally circular signals.

Although the OFDM studies listed above concentrate on the I/Q imbalance challenges and their mitigation methods, they address neither multi-user MIMO (MU-MIMO) communication nor the influence of possible external interferers. Additionally, some of the studies make somewhat limited assumptions of equal I/Q imbalance coefficients between different subcarriers and/or transceiver branches. Our earlier study in [1] focused on external interference suppression with antenna array processing in OFDM systems but the study was limited only to the single-user single-input multiple-output (SU-SIMO) scenario. Therefore, in this paper, we extend the existing results towards more generic system scenario and consequently the novelty and main contributions of this paper are the following:

- In the analysis and mitigation, we focus on uplink MU-MIMO orthogonal frequency division multiple access (OFDMA) systems under transceiver I/Q imbalances. This means that multiple UEs can be active simultaneously at each of the available subcarriers, such as in LTE system specification release 11 [21]. In the base station (BS), the UEs are then spatially separated with antenna array processing.

- We also include the effects of external interferers into the analysis and show how antenna array processing can be efficiently used to suppress external interference having a given spatial response. This kind of external interferers may exist e.g. in increasingly popular heterogeneous networks where the UEs at the cell-edge of a macro cell, and consequently with considerably high TX power levels, severally interfere with the reception in a neighboring femto-cell BS.

- We formulate our analysis in a generic and flexible way by allowing arbitrary system parameters. This approach allows us to use frequency-selective and transceiver branch-dependent I/Q imbalance parameters in the analysis and signal processing.

- We provide an extensive set of simulations which illustrate explicitly the influence of different system parameters under the inevitable RF imperfections.

With these considerations we can provide valuable insight for MU-MIMO OFDMA system designers as well as a fundamental starting point for future research. One of the central technical findings is that the performance of conventional per-subcarrier spatial processing is heavily limited under transceiver I/Q imbalances, and is particularly sensitive to external interferers, whereas the proposed augmented spatial subcarrier processing provides a robust and high-performance RX signal processing solution being able to detect the signals of different users as well as suppress the effects of external interference in a highly efficient manner, despite of transceiver I/Q imbalances.

The paper is organized as follows. Section II presents the fundamental MU-MIMO signal and system models under transceiver I/Q imbalances. Linear minimum mean-square error (LMMSE) and augmented LMMSE combiners are described in Section III along with output signal-to-interference-plus-noise ratio (SINR) analysis. Section IV gives extensive numerical evaluations and illustrations as a function of numerous system parameters. Finally, the conclusions are drawn in Section V.

**Notation:** Throughout this paper, vectors and matrices are written with bold characters. The superscripts ($\cdot^T$, $\cdot^H$, $\cdot^*$) and ($\cdot^{-T}$) represent transpose, Hermitian (conjugate) transpose, complex conjugate and matrix inverse, respectively. The tilde sign ($\tilde{\cdot}$) is used to present an augmented quantity and the results obtained by the augmented processing. We write $\text{diag}(x_{11}, x_{22}, \ldots, x_{ii})$ to denote the diagonal matrix $X$ that is composed of the entries $x_{ii}$ on the main diagonal. The natural basis vector, where the $m$th entry is equal to one and the rest are zeros, is denoted as $e_m$. The statistical expectation is denoted with $\mathbb{E}[\cdot]$. A complex random variable $x$ is called circular if $\mathbb{E}[x^2] = 0$.

## II. FUNDAMENTAL SIGNAL AND SYSTEM MODELS

We analyze a general uplink OFDMA MU-MIMO scenario from an arbitrary subcarrier point of view. The very generic model comprises a single BS which serves multiple UEs simultaneously at each subcarrier. The subcarriers are indexed with $c \in \{-C/2, -1, \ldots, C/2\}$ where $C$ is the total amount of active subcarriers. Additionally, the image (or mirror) subcarrier is defined as $c' = -c$. The number of UEs spatially multiplexed at subcarrier $c$ is denoted with $U$ while the corresponding number at subcarrier $c'$ is $V$. Correspondingly, the users are indexed by $u \in \{1, \ldots, U\}$ and $v \in \{1, \ldots, V\}$. Note that depending on the frequency allocation for the UEs, $u$ and $v$ might sometimes refer to the same UE if it is transmitting at both subcarriers $c$ and $c'$.

The BS has $N$ RX antennas whereas UE $u$ is equipped with $M_u$ TX antennas. In addition, the effect of $L$ external interferers is included to the model and external interferer $l$ is assumed to have $J_l$ TX antennas. The scenario under consideration is illustrated in Fig. [1]

We denote the transmitted baseband equivalent spatial signal vector of user $u$ at the desired subcarrier $c$ by $s_{u,c} \in \mathbb{C}^{M_u \times 1}$ where the elements of the vector model the parallel antenna signal snapshots. Similarly, the transmitted baseband equivalent signal snapshot vector of user $u$ at the image subcarrier $c'$ is denoted by $s_{u,c'} \in \mathbb{C}^{M'_u \times 1}$. Finally, $s_{ml,c} \in \mathbb{C}^{J_l \times 1}$ denotes the signal snapshot vector originating from the $l$th external interferer. All the signal vectors refer to subcarrier-level (frequency-domain) quantities in the considered OFDMA radio system, i.e., before inverse fast Fourier transform (IFFT) in the TXs and after fast Fourier transform (FFT) in the RXs. The most essential variables used throughout the paper are listed in Table [I].

### A. TX and RX I/Q Imbalance Characteristics

The imperfections in the analog electronics of direct-conversion transceivers create I/Q imbalance. On the one hand, the gain imbalance $g$ is created by unequal gains or attenuations between the I and Q branches in amplifiers, filters, mixers and digital-to-analog and analog-to-digital converters. On the other hand, the phase imbalance $\phi$ occurs mainly due to the imperfections in mixers and phase shifters, as well as due
to phase response differences of the branch filters. In general, both the gain and phase imbalance are frequency-dependent already within a few MHz processing bandwidths [22], [23].

For notational convenience, we first define TX I/Q imbalance parameters for a single TX antenna branch \( m \) of user \( u \) at subcarrier \( c \). They are given by

\[
K_{Tx1,m,u,c} = \frac{1 + g_{Tx,m,u,c} e^{j \phi_{Tx,m,u,c}}}{2},
\]

\[
K_{Tx2,m,u,c} = \frac{1 - g_{Tx,m,u,c} e^{j \phi_{Tx,m,u,c}}}{2},
\]

where \( g_{Tx,m,u,c} \) and \( \phi_{Tx,m,u,c} \) are the gain and phase imbalance coefficients for TX antenna branch \( m \) of user \( u \) at subcarrier \( c \), respectively [3]. Since the UE has \( M_u \) antennas and associated TX branches, we stack the I/Q imbalance parameters of different TX branches into diagonal matrices. Consequently, the TX I/Q imbalance matrices \( K_{Tx1,u,c} \) and \( K_{Tx2,u,c} \), both \( \in \mathbb{C}^{M_u \times M_u} \), are given by

\[
K_{Tx1,u,c} = \text{diag}(K_{Tx1,1,u,c}, \ldots, K_{Tx1,M_u,u,c}),
\]

\[
K_{Tx2,u,c} = \text{diag}(K_{Tx2,1,u,c}, \ldots, K_{Tx2,M_u,u,c}).
\]

Similarly, the I/Q imbalance characteristics for a single RX antenna branch \( n \) at subcarrier \( c \) are given by

\[
K_{Rx1,n,c} = \frac{1 + g_{Rx,n,c} e^{-j \phi_{Rx,n,c}}}{2},
\]

\[
K_{Rx2,n,c} = \frac{1 - g_{Rx,n,c} e^{j \phi_{Rx,n,c}}}{2},
\]

where \( g_{Rx,n,c} \) and \( \phi_{Rx,n,c} \) denote the gain and phase imbalance coefficients of RX antenna branch \( n \) [3]. We stack also (5) and (6) into diagonal matrices, resulting in the RX I/Q imbalance

### Table 1: Most Important Variables Used Throughout the Paper

| Variable | Dimensions | Definition |
|----------|------------|------------|
| \( c \) | scalar | Subcarrier index |
| \( \tilde{c} \) | scalar | Image subcarrier index |
| \( u, v \) | scalars | UE indexes for subcarriers \( c \) and \( \tilde{c} \), respectively |
| \( U, V \) | scalars | Number of spatially multiplexed UEs at subcarriers \( c \) and \( \tilde{c} \), respectively |
| \( L \) | scalar | Number of external interferers |
| \( N \) | scalar | Number of BS RX antennas |
| \( M_u \) | scalar | Number of TX antennas of UE \( u \) |
| \( l \) | scalar | Number of TX antennas of external interferer \( l \) |
| \( S \) | scalar | Total number of all transmitted data streams from all UEs, i.e., \( S = \sum_u M_u \) |
| \( s_{u,c} \) | \( M_u \times 1 \) | Transmitted baseband equivalent spatial signal vector of UE \( u \) at subcarrier \( c \) |
| \( s_{v,c} \) | \( M_v \times 1 \) | Transmitted baseband equivalent spatial signal vector of UE \( v \) at subcarrier \( c \) |
| \( s_{u,v,c} \) | \( J_l \times 1 \) | Transmitted baseband equivalent spatial signal vector of interferer \( l \) at subcarrier \( c \) |
| \( n_c \) | \( N \times 1 \) | Additive noise in the RX electronics at subcarrier \( c \) |
| \( z_c \) | \( N \times 1 \) | Sum of external interference and additive noise vectors at subcarriers \( c \) and \( \tilde{c} \), respectively |
| \( K_{Tx1,u,c} \) | \( M_u \times M_u \) | Diagonal TX I/Q imbalance matrices of UE \( u \) at subcarrier \( c \) |
| \( K_{Rx1,n,c} \) | \( N \times N \) | Diagonal RX I/Q imbalance matrices of BS at subcarrier \( c \) |
| \( H_{u,c} \) | \( N \times M_u \) | Channel response matrices of UE \( u \) at subcarriers \( c \) and \( \tilde{c} \), respectively |
| \( R_{Tx1,u,c} \) | \( 2N \times M_u \) | Matrices including the joint effects of the wireless channel and TX I/Q imbalance for UE \( u \) at subcarrier \( c \) |
| \( W_{c} \) | \( N \times S \), \( 2N \times S \) | Signal power of UE \( u \) in each of its TX antenna branches at subcarrier \( c \) |
| \( R_{Rx1,n,c} \) | \( 2N \times 2N \) | Covariance matrices of the received signal vector under joint TX+RX I/Q imbalances at subcarrier \( c \) |
| \( W_{v,c} \) | \( N \times S \) | Signal power of UE \( v \) in each of its TX antenna branches at subcarrier \( c \) |
| \( V_{Rx1,n,c} \) | \( N \times 1 \), \( 2N \times 1 \) | Cross-correlation vectors between the received signal vector and data stream \( m \) of UE \( u \) at subcarrier \( c \) |

The tilde sign (\( \tilde{\cdot} \)) refers to augmented quantities and the results obtained by the augmented processing.
matrices $K_{Rx1,c}$ and $K_{Rx2,c}$, now both $\in \mathbb{C}^{N \times N}$, given by

$$\begin{align*}
K_{Rx1,c} &= \text{diag}(K_{Rx1,1,c}, \cdots, K_{Rx1,N,c}), \\
K_{Rx2,c} &= \text{diag}(K_{Rx2,1,c}, \cdots, K_{Rx2,N,c}).
\end{align*}$$

These matrices are used in the modeling and analysis of the total effects of TX and RX imbalances in the considered MU-MIMO system. The above characterization allows setting the I/Q imbalances freely and independently, not only between different UEs but also between different antenna branches of a single UE. In addition, we assume that I/Q imbalance is frequency selective, i.e., I/Q imbalance parameters at different subcarriers are different. However, all derived expressions are valid also for the case where the I/Q imbalance parameters are equal in all transceivers, transceiver branches and/or subcarriers.

B. MU-MIMO Transmission under I/Q Imbalance

The transmitted baseband equivalent signal snapshot vector of user $u$ at subcarrier $c$ under TX I/Q imbalance can be now written with the help of the TX I/Q imbalance matrices directly as [13]

$$s_{Tx,u,c} = K_{Tx1,u,c} s_{u,c} + K_{Tx2,u,c} s_{u,c}^*.$$  

(9)

Clearly, the structure of the transmitted signal is distorted, resulting in general in cross-talk between image-subcarriers $c$ and $c^\perp$. This is already a well-established phenomenon in the existing literature, see e.g. [3], [6], [13], [23]. Notice, however, that if the image subcarrier $c^\perp$ is not allocated for UE $u$ there is no cross-talk between the subcarriers of an individual UE and the resulting transmitted signal at subcarrier $c$ consists only of the scaled version of $s_{u,c}$. However, when the subcarrier $c^\perp$ is allocated to another UE $v$, through OFDMA principle, the corresponding emitted signal snapshot vector at subcarrier $c$ is of the form $s_{Tx,v,c} = K_{Tx2,v,c} s_{v,c}$. Then, when interpreted from receiver perspective, this implies cross-talk or interference between UEs. The corresponding transmitted signals vectors at the image subcarrier $c^\perp$ are given by $s_{Tx,u,c^\perp} = K_{Tx1,u,c^\perp} s_{u,c} + K_{Tx2,u,c^\perp} s_{u,c}^*$ and $s_{Tx,v,c^\perp} = K_{Tx1,v,c^\perp} s_{v,c} + K_{Tx2,v,c^\perp} s_{v,c}^*$.

1) Received signal under TX I/Q imbalance: When the transmitted signals from multiple UEs are received by the BS, equipped with $N$ antennas and assuming first perfect I/Q matching, the received signal vector $r_{Tx,c} \in \mathbb{C}^{N \times 1}$ at subcarrier $c$ is equal to

$$r_{Tx,c} = \sum_{u=1}^{U} H_{u,c} K_{Tx1,u,c} s_{u,c} + \sum_{v=1}^{V} H_{v,c} K_{Tx2,v,c} s_{v,c}^* + z_c,$$

(10)

where also perfect timing and frequency synchronization between the UEs and BS is assumed for simplicity. Here, $H_{u,c} \in \mathbb{C}^{N \times M_u}$ is the channel response matrix of user $u$ at subcarrier $c$. Similarly, $H_{v,c} \in \mathbb{C}^{N \times M_v}$ is the channel response matrix of user $v$ at subcarrier $c$. Throughout the paper, the channel response elements are assumed to be constants within each narrow subcarriers. Additionally, the external interference plus noise vector $z_c \in \mathbb{C}^{N \times 1}$ is given by

$$z_c = \sum_{l=1}^{L} H_{int,l,c} s_{int,l,c} + n_c,$$

(11)

where $H_{int,l,c} \in \mathbb{C}^{N \times 1}$ represents the channel response matrix of the $l$th interferer at subcarrier $c$. Since the interferers are not synchronized with the BS and since we are not limiting the study to any specific interference waveform, $s_{int,l,c}$ is basically the result of the sampled interference signal at the desired subcarrier after the RX FFT processing. The noise vector $n_c \in \mathbb{C}^{N \times 1}$ models the additive noise in the RX electronics. Noise elements in different RX branches are assumed to be complex circular and mutually uncorrelated. A corresponding formulation for the external interference and noise at the image subcarrier is obtained from (11) by substituting the subcarrier index $c$ with $c^\perp$.

2) Received signal under joint TX+RX I/Q imbalances: Next we take into account the fact that I/Q imbalance occurs also in the RX electronics of the BS. The received signal snapshot vector $r_{TxRx,c} \in \mathbb{C}^{N \times 1}$ at subcarrier $c$, under joint TX+RX I/Q imbalances is then given by

$$r_{TxRx,c} = K_{Rx1,c} r_{Tx,c} + K_{Rx2,c} r_{Tx,c}^*,$$

$$= \sum_{u=1}^{U} K_{Rx,u} \tilde{\Psi}_{u,c} s_{u,c} + \sum_{v=1}^{V} K_{Rx,v} \tilde{\Omega}_{v,c} s_{v,c}^* + K_{Rx1,c} z_c + K_{Rx2,c} z_c^*,$$

(12)

where the total RX I/Q imbalance matrix $K_{Rx,c} \in \mathbb{C}^{N \times 2N}$ equals

$$K_{Rx,c} = \left[ K_{Rx1,c}, K_{Rx2,c} \right].$$

(13)

In addition, matrices $\tilde{\Psi}_{u,c} \in \mathbb{C}^{2N \times M_u}$ and $\tilde{\Omega}_{v,c} \in \mathbb{C}^{2N \times M_v}$ include the joint effects of TX I/Q imbalance and the wireless propagation channel. They are given by

$$\tilde{\Psi}_{u,c} = \begin{bmatrix} H_{u,c} K_{Tx1,u,c} \\ H_{u,c}^* K_{Tx2,u,c} \end{bmatrix}, \quad \tilde{\Omega}_{v,c} = \begin{bmatrix} H_{v,c} K_{Tx1,v,c} \\ H_{v,c}^* K_{Tx2,v,c} \end{bmatrix}.$$

(14)

Notice also that the total effective linear channel, experienced by the spatial snapshot of UE $u$, is given by $K_{Rx,c} \tilde{\Psi}_{u,c} \in \mathbb{C}^{N \times M_u}$. Finally, $z_c$ represents the external interference and noise at the image subcarrier. Clearly, (12) includes contribution not only from subcarrier $c$ but also from the image subcarrier $c^\perp$. The UE signals transmitted at the image subcarrier leak to the considered subcarrier due to both TX and RX I/Q imbalances and consequently we call it inter-user-interference from the image subcarrier. In contrast to UE signals, the external interference and noise alias to subcarrier $c$ only due to RX I/Q imbalance. The overall spectral structure of the received signal is illustrated in Fig. 2.

In practice, a subcarrier with very high interference levels could be left unused for data transmission if there are subcarriers with better conditions available. However, due to RX I/Q imbalance, the strong interference from the image subcarrier aliases to the top of the desired signal, even if the image subcarrier is not used for data transmission at all. This
obtained from the signal models by substituting paper, the special case with I/Q imbalance only in the TXs is signal under joint TX+RX I/Q imbalances. Throughout the into account.

For forthcoming sections where RX spatial processing is taken forth onto the application under consideration. Furthermore, the primary target of separating the multiplexed streams of different UEs at subcarrier \( c \), under external interference and transceiver I/Q imbalances is a key concern. This will be elaborated in the forthcoming sections where RX spatial processing is taken into account.

Note that (12) expresses the generic form of the received signal under joint TX+RX I/Q imbalances. Throughout the paper, the special case with I/Q imbalance only in the TXs is obtained from the signal models by substituting \( \mathbf{K}_{\text{Rx1},c} = \mathbf{I} \) and \( \mathbf{K}_{\text{Rx2},c} = \mathbf{0} \) for all \( c \). Similarly, the case with I/Q imbalance only in the RX is obtained by substituting \( \mathbf{K}_{\text{Tx1},u,c} = \mathbf{I} \) and \( \mathbf{K}_{\text{Tx2},u,c} = \mathbf{0} \) for all \( u \) and \( c \).

C. Spatial Post-Processing with Digital Combiners

1) Classical per-subcarrier spatial combiner: Multiple RX antennas enable flexible combining of the antenna signals for obtaining the desired system performance. Usually, the combining process is implemented by digital signal processing due to its high computational power, reconfigurability and small physical size. Generally speaking, a digital linear combiner processes the received signal snapshots with complex weights \( \mathbf{w} = [w_1, w_2, \ldots, w_M]^T \in \mathbb{C}^{N \times 1} \) yielding an output signal \( y = \mathbf{w}_y^H \mathbf{r} \). When applying this method to MU-MIMO systems utilizing OFDMA waveforms, each of \( M_u \) transmitted signals of each \( U \) users needs an individual weight vector \( \mathbf{w}_{m,u,c} \) for separating different signals from each others in the RX at subcarrier \( c \). When stacking all these weight vectors into a matrix, we get the complete weight matrix \( \mathbf{W}_y = [\mathbf{w}_{1,1,c}, \cdots, \mathbf{w}_{M_u,U,c}] \in \mathbb{C}^{N \times S} \), where \( S = \sum_{u=1}^{U} M_u \) is the total amount of the transmitted signals. Then, the combiner’s output signal vector \( \mathbf{y}_c \in \mathbb{C}^{S \times 1} \) at subcarrier \( c \) is given conveniently by

\[
\mathbf{y}_c = \mathbf{W}_y^H \mathbf{r}_c
\]

where \( \mathbf{r}_c \in \mathbb{C}^{N \times 1} \) denotes the received signal vector at subcarrier \( c \), here under perfect I/Q matching. The entries of the output signal vector represent data streams originating from different TX antenna branches of different UEs, i.e. \( \mathbf{y}_c = [y_{1,1,c}, \cdots, y_{M_u,U,c}]^T \). The combiner weights can, in general, be selected with blind or non-blind methods, depending on a priori information, under a given optimization criteria. The basic approach is to combine the received signals from different RX branches coherently while trying to minimize the effect of the non-desired interference and noise. Since this classical processing is done at the subcarrier level, we call it per-subcarrier combiner.

2) Classical per-subcarrier spatial combiner under joint TX+RX I/Q imbalances: Stemming from the previous modeling, the output signal vector \( (c \in \mathbb{C}^{S \times 1}) \) of a per-subcarrier combiner under joint TX+RX I/Q imbalances is now equal to

\[
\mathbf{y}_{\text{TxRxI}} = \mathbf{W}_y^H \mathbf{r}_{\text{TxRxI}} = \sum_{u=1}^{U} \mathbf{W}_y^H \mathbf{K}_{\text{Rx},u,c} \bar{\Psi}_{u,c} \mathbf{s}_{u,c} + \sum_{v=1}^{V} \mathbf{W}_y^H \mathbf{K}_{\text{Rx},v,c} \bar{\Omega}_{v,c} \mathbf{s}_{v,c} + \mathbf{W}_y^H \mathbf{K}_{\text{Rx2},c} \mathbf{z}_c + \mathbf{W}_y^H \mathbf{K}_{\text{Rx2},c} \mathbf{z}_c^*.
\]

We have seen this kind of signal structure already in (12). However in (16), all signal terms are multiplied with the weighting matrix. The first term contains the spatial streams of all \( U \) desired UEs, which are to be separated by the spatial processing, while at the same time suppressing the effects of the other terms as much as possible. The second term is due to inter-user interference from the mirror UEs in the OFDMA framework while the third and fourth terms are due to external interference and noise. Notice that external interference contributes to the combiner output through direct co-channel coexistence as well as due to image subcarrier leakage. The above is clearly a challenge when optimizing the combiner weights and it becomes even more difficult when the number of UEs and external interferers is increased.

3) Augmented spatial combiner: As elaborated above, transceiver I/Q imbalances cause both inter-user interference and external interference through image subcarrier leakage, and classical per-subcarrier spatial processing can easily
run out of degrees of freedom to suppress all of them sufficiently. To alleviate this and enhance the interference suppression capabilities, we next augment the spatial combiner operating principle to process each subcarrier along with its image subcarrier jointly. This means augmented combiner processing where the signals from both subcarriers $c$ and $c'$ are combined with two separate sets of weights $[4], [8], [11]$. We denote these weight sets for transmitted stream $m$ of user $u$ by $w_{A,m,u,c} \in \mathbb{C}^{N \times 1}$ and $w_{B,m,u,c'} \in \mathbb{C}^{N \times 1}$ and stack them into the augmented weight vector $w_{m,u,c} = [w_{A,m,u,c}, w_{B,m,u,c'}]^T \in \mathbb{C}^{2N \times 1}$. Then the complete augmented weight matrix can be given by $\tilde{W}_c = [\tilde{w}_{1,1,c}, \cdots, \tilde{w}_{M_l,u,c}]^T \in \mathbb{C}^{2N \times S}$. When additionally defining the augmented spatial signal vector as $\tilde{r}_c = [r_c^T, r_c'^T] \in \mathbb{C}^{2N \times 1}$, the output signal of the augmented digital combiner at subcarrier $c$ finally becomes

$$\tilde{y}_c = \tilde{W}_c^H \tilde{r}_c$$

where $\tilde{y}_c = [\tilde{y}_{1,1,c}, \cdots, \tilde{y}_{M_l,u,c}]^T \in \mathbb{C}^{S \times 1}$. Note that despite of the seeming structural similarity of (17) and (13), there is a fundamental difference since now also the received signal at the image subcarrier is included in the processing. A detailed illustration of the augmented combining method is given in Fig. 3. This approach has been showed to be efficient for I/Q imbalance mitigation in SU-MIMO communication with OFDM waveforms $[4], [8], [11]$. Consequently, this study can be seen as a natural extension to MU-MIMO OFDM processing incorporating also external interference.

4) Augmented spatial combiner under joint TX+RX I/Q imbalances: By defining next the augmented signal vector under joint TX+RX I/Q imbalances as $\tilde{r}_{\text{TXRX}} = \left[\begin{array}{c} r_{\text{TXRX},c} \\
_{\text{TXRX},c} 
\end{array}\right]^T \in \mathbb{C}^{2N \times 1}$, the output signal of the augmented combiner under joint TX+RX I/Q imbalances becomes

$$\tilde{y}_{\text{TXRX},c} = \tilde{W}_c^H \tilde{r}_{\text{TXRX},c} = \left(\begin{array}{c}
\sum_{u=1}^V \tilde{W}_c K_{\text{RX}}^2 \tilde{z}_{u,c} + \sum_{v=1}^V \tilde{W}_c \tilde{K}_{\text{RX}} s_{u,c} + \sum_{v=1}^V \tilde{W}_c \tilde{K}_{\text{RX}} K_{\text{RX}}^2 \tilde{z}_{v,c} + \tilde{W}_c \tilde{K}_{\text{RX}} K_{\text{RX}}^2 \tilde{z}_{c}.
\end{array}\right)$$

Here the augmented RX I/Q imbalance matrices $K_{\text{RX}}$ and $K_{\text{RX}}^2$, both $\in \mathbb{C}^{2N \times N}$, are given by

$$\tilde{K}_{\text{RX},c} = \left[\begin{array}{cc}
K_{\text{RX},c} & K_{\text{RX},c}^2 \\
K_{\text{RX},c}^2 & K_{\text{RX},c}
\end{array}\right] \in \mathbb{C}^{2N \times 2N}.$$  

Again, the output signal structures of the conventional and augmented combiners are very similar. However, the underlying difference is that (13) uses twice as many weights as (16) for processing both subcarrier signals simultaneously. Naturally, this doubles the computational complexity of the combining process but also gives us more degrees of freedom for obtaining the desired signal separation and interference suppression, even under challenging I/Q imbalances. Note that this flexibility is obtained by changing the combiner block only whereas the costly RF chains and demanding FFT processing remain the same as for per-subcarrier processing.

III. LMMSE AND AUGMENTED LMMSE COMBINERS AND OUTPUT SINRs

In this section, we seek to characterize the output performance of linear and augmented (widely-linear) combiners in terms of combiner output SINRs. Furthermore, the MMSE optimal linear and augmented (widely-linear) combiners are derived.

A. Covariance Matrices

In this subsection we derive expressions for the covariance matrix of the received spatial signal vector, first under perfect I/Q matching and then under joint TX+RX I/Q imbalances. We assume that the signals of different UEs, the signals at subcarriers $c$ and $c'$ (with perfect I/Q matchings), the interfering signals and the additive noise are all mutually uncorrelated. In addition, we assume that the external interference as well as noise at receiver input are complex circular.

The covariance matrix $R_c \in \mathbb{C}^{N \times N}$ of the received signal under perfect I/Q matching is given directly by

$$R_c = \mathbb{E} \left[ r_c r_c^H \right] = \sum_{u=1}^U \sigma_{u,c}^2 H_{u,c} H_{u,c}^H + R_{z,c}$$

where $\sigma_{u,c}^2 \in \mathbb{E} \left[ |s_{u,c}|^2 \right]$ denotes the signal power of the individual TX antenna branch $m$ of user $u$ at subcarrier $c$. Note that for simplicity, we assume equal TX powers in all TX branches of an individual UE $u$. In addition, the covariance matrix of the external interference plus noise equals

$$R_{z,c} = \mathbb{E} \left[ z_c z_c^H \right] = \sum_{l=1}^L \sigma_{m,l,c}^2 H_{m,l,c} H_{m,l,c}^H + \sigma_{n,c}^2 I$$

where $\sigma_{m,l,c}^2$ denotes the power of the $l^{th}$ external interferer at subcarrier $c$, while $\sigma_{n,c}^2$ denotes the noise power at subcarrier
Based on (20), the covariance matrix has a very intuitive structure since it depends directly on the signal power, channel matrices, and the external interference and noise. This is of course a well-known result in the literature. However, I/Q imbalances change the signal structure as we have already shown and consequently also the covariance matrix under I/Q imbalances becomes different. When considering the effects of joint TX+RX I/Q imbalances of all associated devices, the covariance matrix can now be written as

$$
\mathbf{R}_{\text{TX+RX}} = \mathbb{E} \left[ \mathbf{f}_{\text{TX+RX}} \mathbf{f}_{\text{TX+RX}}^H \right] = \sum_{u=1}^{U} \sigma_u^2 \mathbf{K}_{\text{Rx},c} \bar{\Psi}_{u,c} \bar{\Psi}_{u,c}^H \mathbf{K}_{\text{Rx},c}^H
$$

$$
+ \sum_{v=1}^{V} \sigma_v^2 \mathbf{K}_{\text{Rx},c} \bar{\Omega}_{v,c} \bar{\Omega}_{v,c}^H \mathbf{K}_{\text{Rx},c}^H
$$

$$
+ \mathbf{K}_{\text{Rx+Rx}} \mathbf{R}_{z,c} \mathbf{K}_{\text{Rx+Rx}}^H + \mathbf{K}_{\text{Rx+Rx}} \mathbf{R}_{z,c} \mathbf{K}_{\text{Rx+Rx}}^H
$$

(22)

Here, the first term represents the contribution of the desired spatially multiplexed signals of $U$ users at subcarrier $c$ whereas the second term corresponds to the inter-user interference from the image subcarrier. The third and fourth terms represent the effects of the external interference and noise from the considered subcarrier and the image subcarrier, respectively. Compared to the covariance matrix with perfect I/Q matching in (20), the covariance matrix under I/Q imbalances has obviously a more complicated structure and is affected especially by the inter-user interference from mirror UEs and by external interference imaging or aliasing.

The covariance matrix of the augmented signal model under joint TX+RX I/Q imbalances can, in turn, be given as

$$
\mathbf{R}_{\text{TX+RX}} = \mathbb{E} \left[ \mathbf{f}_{\text{TX+RX}} \mathbf{f}_{\text{TX+RX}}^H \right] = \sum_{u=1}^{U} \sigma_u^2 \mathbf{K}_{\text{Rx},c} \bar{\Psi}_{u,c} \bar{\Psi}_{u,c}^H \mathbf{K}_{\text{Rx},c}^H
$$

$$
+ \sum_{v=1}^{V} \sigma_v^2 \mathbf{K}_{\text{Rx},c} \bar{\Omega}_{v,c} \bar{\Omega}_{v,c}^H \mathbf{K}_{\text{Rx},c}^H
$$

$$
+ \mathbf{K}_{\text{Rx+Rx}} \mathbf{R}_{z,c} \mathbf{K}_{\text{Rx+Rx}}^H + \mathbf{K}_{\text{Rx+Rx}} \mathbf{R}_{z,c} \mathbf{K}_{\text{Rx+Rx}}^H \mathbf{R}_{z,c} \mathbf{K}_{\text{Rx+Rx}}^H
$$

Again, there are many similarities with the covariance matrix of the conventional signal model but the joint observation of both subcarriers offers increased degrees of freedom and can thus substantially improve the performance. This will be illustrated in Section IV, after we have quantified the combiner output performance in terms of the SINR.

B. Signal to Interference plus Noise Ratios

Next, we quantify the performance of the combiner output signals under I/Q imbalances for an arbitrary data stream $m$ of UE $u$ in terms of its SINR. Towards that end, we first take a short look on the signals in different TX branches of the UE $u$. For simplicity, we assume an open loop transmission, i.e. the channel state information (CSI) is not available on the TX side. This results in a transmission where the transmitted baseband signal snapshots in different TX antenna branches are essentially the same as the independent data streams. This corresponds to classical open-loop spatial multiplexing while other transmission scenarios like closed-loop spatial multiplexing are left for future work. Notice, however, that all our formulations directly support also closed-loop and other precoding schemes, implying simply an additional mapping from the actual data snapshot vector, say $\mathbf{x}_{u,c}$, to TX antenna snapshot vector $\mathbf{s}_{u,c}$ as $\mathbf{s}_{u,c} = \mathbf{G}_{u,c} \mathbf{x}_{u,c}$ where $\mathbf{G}_{u,c}$ denotes the precoder of UE $u$ at subcarrier $c$.

1) SINR with the classical per-subcarrier spatial combiner:

At RX combiner output, the total output power of data stream $m$ of UE $u$ at subcarrier $c$ under joint TX+RX I/Q imbalances is given by

$$
P_{m,u,c} = \mathbb{E} \left[ |y_{\text{TX+RX},m,u,c}|^2 \right] = \mathbf{w}_{m,u,c}^H \mathbf{R}_{\text{TX+RX}} \mathbf{w}_{m,u,c} \quad (24)
$$

where, $\mathbf{w}_{m,u,c}$ refers to the combiner weight vector belonging to data stream $m$ of UE $u$ at subcarrier $c$ and is easily obtained by selecting the corresponding column from the weight matrix $\mathbf{W}_c$. However, in order to express the SINR for this data stream, we need to divide the total power into signal and interference power terms. For that purpose, the total output power $P_{m,u,c}$ can be expressed also through

$$
P_{m,u,c} = P_{s,m,u,c} + P_{I\text{SL},m,u,c} + P_{I\text{UL},m,u,c} + P_{I\text{UL}',m,u,c} + P_{z,c} + P_{z,c}' \quad (25)
$$

where, $P_{s,m,u,c}$ denotes the output power of the desired data stream and $P_{I\text{SL},m,u,c}$ represents the effect of the inter-stream interference originating from the other streams of the same UE $u$. These terms are both originating from the same UE $u$, but they have to be separated because when examining the received signal from an individual but arbitrary stream $m$ of UE $u$ perspective, the other streams of the same UE are also treated as interference. In addition, $P_{I\text{UL},m,u,c}$ and $P_{I\text{UL}',m,u,c}$ represent inter-user interference from subcarriers $c$ and $c'$, respectively. Finally, $P_{z,c}$ and $P_{z,c}'$ denote the output powers of the external interference and noise originating from subcarriers $c$ and $c'$, respectively. All these power terms can be expressed easily, since the covariance matrix $\mathbf{R}_{\text{TX+RX}}$ in (22) is a sum of multiple independent terms. We only need to define the two stream selection matrices $\Gamma_{m,u,c} = \text{diag} (\mathbf{e}_m) \in \mathbb{R}^{M_u \times M_u}$ and $\Delta_{m,u,c} = \mathbf{I} - \Gamma_{m,u,c} \in \mathbb{R}^{M_u \times M_u}$ which refer to the data stream transmitted from TX antenna branch $m$ of user $u$ at subcarrier $c$, and to the interfering other streams of the same UE $u$, respectively. Then the individual power terms in (25) can be given with the help of (22) and the stream selection matrices by

$$
P_{s,m,u,c} = \sigma_u^2 \mathbf{w}_{m,u,c}^H \mathbf{R}_{\text{Rx},c} \mathbf{K}_{\text{Rx},c} \bar{\Psi}_{u,c} \bar{\Psi}_{u,c}^H \mathbf{K}_{\text{Rx},c} \mathbf{w}_{m,u,c} \quad (26)
$$

$$
P_{I\text{SL},m,u,c} = \sigma_u^2 \mathbf{w}_{m,u,c}^H \mathbf{R}_{\text{Rx},c} \mathbf{K}_{\text{Rx},c} \bar{\Psi}_{u,c} \bar{\Omega}_{u,c} \bar{\Omega}_{u,c}^H \mathbf{K}_{\text{Rx},c} \mathbf{w}_{m,u,c} \quad (27)
$$

$$
P_{I\text{UL},m,u,c} = \sum_{i=1,i \neq u}^{U} \sigma_i^2 \mathbf{w}_{m,u,c}^H \mathbf{R}_{\text{Rx},c} \bar{\Psi}_{i,c} \bar{\Psi}_{i,c}^H \mathbf{K}_{\text{Rx},c} \mathbf{w}_{m,u,c} \quad (28)
$$
Then the SINR for the augmented signal model under joint TX+RX I/Q imbalances is given straightforwardly by

$$\text{SINR}_{\text{TxRxi,m,u,c}} = \frac{\tilde{P}_{s,m,u,c}}{\tilde{P}_{\text{ISL,m,u,c}} + \tilde{P}_{\text{IUL,u,c}} + \tilde{P}_{\text{IUL,c}}' + \tilde{P}_{z,c} + \tilde{P}_{z,c}'.}$$

(40)

Although (40) looks very similar to (32), the fundamental differences of the underlying signal model and augmented processing still exist, due to doubled degree of freedom. We will illustrate and discuss the practical performance differences in more detail with numerical evaluations in Section IV but first we concentrate on the weight optimization problem below.

### C. Linear and Augmented Linear MMSE Combiners

The above SINR expressions are in principle valid for any possible combiner coefficients, while we have not addressed the optimization of the coefficients yet. A well-known statistical method for solving stationary estimation problems is the so-called Wiener filter which yields the optimal linear solution in the MMSE sense [25]. We have shown in [11] that the Wiener filter approach, when generalized to augmented or widely-linear processing, can be successfully used for the channel and hardware characteristic estimation problem under I/Q imbalance in SU-SIMO systems. Here this approach is extended to cover the weight selection problem in MU-MIMO OFDMA systems.

We first define the ordinary Wiener filter or LMMSE weights as

$$W_{\text{LMMSE,c}} = R_c^{-1}v_c$$

(41)

where $v_c = [v_{1,c}, \cdots, v_{M,c}] \in \mathbb{C}^{N \times S}$ is a matrix consisting of the cross-correlation vectors between the received signal snapshots and transmitted data streams, i.e. $v_{m,u,c} = E[r_c s_{m,u,c}^*] \in \mathbb{C}^{N \times L}$. Under joint TX+RX I/Q imbalances the cross-correlation vector, related to the data stream $m$ of user $u$, is easily shown to be

$$v_{\text{TxRxi,m,u,c}} = E[r_{\text{TxRxi,m,u,c}}^*] = \sigma_{s,m,u,c}^2 \Omega_{u,c} \Psi_{u,c} e_m. \quad (42)$$

When all individual cross-correlation vectors are then stacked into the matrix $V_{\text{TxRxi,c}}$, the complete LMMSE weight matrix under joint TX+RX I/Q imbalances consequently becomes

$$W_{\text{LMMSE,TxRxi,c}} = R_c^{-1}V_{\text{TxRxi,c}}. \quad (43)$$

where the column related to data stream $m$ of UE $u$ at subcarrier $c$ is equal to

$$w_{\text{LMMSE,TxRxi,m,u,c}} = R_c^{-1}r_{c\text{TxRxi,m,u,c}}. \quad (43)$$

Notice that if I/Q imbalances were set to zero, (43) would reduce to the classical Wiener filter as expected.

We next proceed to the augmented combiner coefficient optimization in the MMSE sense, referred to as augmented LMMSE or augmented Wiener filter in the following. The weight selection or optimization problem corresponds to solving the weights as

$$\tilde{W}_{\text{LMMSE,c}} = R_c^{-1}v_c.$$ \quad (44)

where $R_c = E[r_c r_c^*] \in \mathbb{C}^{2N \times 2N}$ denotes the spatial covariance matrix of the augmented received signal under perfect I/Q
matchings and $\vec{V}_c = [\vec{v}_{1,c}, \cdots, \vec{v}_{M_c,U_c,c}] \in \mathbb{C}^{2N \times S_c}$. Under joint TX+RX I/Q imbalances the augmented cross-correlation vector of data stream $m$ of UE $u$ at subcarrier $c$ is equal to

$$
\vec{V}_{\text{TXRI},m,u,c} = \mathbb{E} \left[ \vec{f}_{\text{TXRI},c,s}^* \vec{c}_{m,u,c} \right] = \begin{bmatrix}
\sigma_{s,u,c}^2 \vec{K}_{\text{RXI},c} \vec{\Psi}_{u,c} \vec{e}_m \\
\sigma_{s,u,c}^2 \vec{K}_{\text{RXI},c}^* \vec{\Omega}_{u,c} \vec{e}_m
\end{bmatrix}.
$$

(45)

When the individual cross-correlation vectors are stacked into the matrix $\vec{V}_{\text{TXRI},c}$, we obtain the complete augmented LMMSE weight matrix under joint TX+RX I/Q imbalances by $\vec{W}_{\text{LMMSE},\text{TXRI},c} = \vec{R}_{\text{TXRI},c}^{-1} \vec{V}_{\text{TXRI},c}$. Here the weight vector related to data stream $m$ of UE $u$ at subcarrier $c$ is given by

$$
\vec{w}_{\text{LMMSE},\text{TXRI},m,u,c} = \vec{R}_{\text{TXRI},c}^{-1} \vec{V}_{\text{TXRI},m,u,c}.
$$

(46)

In general, the Wiener combiner as well as its augmented counterpart build on the exact statistical information for working reliably. These statistics are rarely directly available but fortunately the Wiener solutions can be approximated by various adaptive estimation methods based on known training or reference signals [26]. E.g. in [1] we showed that the least mean squares (LMS) method [27] provides a simple and accurate approximation for the Wiener filter and for the augmented Wiener filter under different I/Q imbalance scenarios in SU-SIMO systems with reasonable training data lengths. LMS as well as other adaptive estimation methods such as recursive least squares (RLS) [27] can be easily extended also for the MU-MIMO scenario. However, to keep the presentation length feasible, we exclude the exact adaptive filter formulations here.

IV. NUMERICAL EVALUATIONS AND ILLUSTRATIONS

A. Simulation Setup

In the numerical evaluation, we consider an uplink OFDMA MU-MIMO scenario with five UEs transmitting towards a single BS, all being active at subcarrier $c$ simultaneously. In addition, there are five other UEs which communicate with the BS at the image subcarrier $c'$. The BS is equipped with an antenna array consisting of 20 antenna elements. Each UE has two TX antennas, illustrating a typical UE level capability in LTE networks. The input signal to noise ratio (SNR) in the single BS has two TX antennas, illustrating a typical UE level capability in LTE networks. The input signal to noise ratio (SNR) in the single BS is equal to 20 dB. Here, we define the SNR as the ratio between the total averaged received signal power originating from all TX branches of a single user, and the total received power originating from the external interferers. Note that if the amount of the interferers is increased, the power of each individual interferer has to be decreased in order to obtain the same total SIR. The transmission channels between all TX-RX antenna pairs as well as between all interferer-RX antenna pairs are independent and Rayleigh distributed.

I/Q imbalance is defined in terms of the image rejection ratio (IRR) which is given in decibels for a single transceiver branch by $\text{IRR} = 10 \log_{10} \left( \frac{|K_1|^2}{|K_2|^2} \right)$ [19]. Firstly, the minimum allowable IRR ($\text{IRR}_{\text{min}}$) is set to 25 dB which is the minimum requirement for UE TX/RX IRR in the LTE specification [31]. Secondly, we select phase imbalance coefficients $\phi_{m,u,c,m',u,c}$ independently from $\mathcal{U}(-\alpha, \alpha)$ where $\alpha$ guarantees the selected IRR$_{\text{min}}$ when the gain imbalance is set to zero. Finally, the gain imbalance coefficients $g_{m,u,c,m',u,c}$ are sampled independently from the conditional distribution $\mathcal{U}(g_{\text{min}}, g_{\text{max}})$ where the range edges correspond to IRR$_{\text{min}}$ with the earlier selected $\phi$. The I/Q imbalance parameters at different subcarriers are assumed to be independent for modeling arbitrarily frequency selective I/Q imbalance. The basic simulation parameters are summarized in Table II while some parameters are also varied in the evaluations.

All results describe the performance from a single yet arbitrary subcarrier stream point of view. In order to illustrate the obtainable performance on average, the results are averaged over all data streams, UEs and 2000 realizations. For each realization, the channel matrices and I/Q imbalance parameters are randomly and independently generated according to the aforementioned criteria. All evaluations are carried out both the linear and the augmented linear MMSE combiner.

B. Simulation Results and Analysis

1) SINR as a function of the SIR: The SINR as a function of the SIR is depicted in Fig. [4]. Here the power of the external interference is swept at both the desired subcarrier and the image subcarrier while the signal powers are kept equal and constant at both subcarriers. First of all, we notice saturation of the performance with high and low SIRs, even with perfect I/Q matching. In the high SIR region, the combiners can suppress the inter-user interference effectively and the influence of the external interference is very small. Therefore, the ceiling effect

| Parameter          | Symbol | Value |
|--------------------|--------|-------|
| RX antennas        | $N$    | 20    |
| Number of UEs      | $U, V$ | 5     |
| TX antennas in UEs | $M_u, M_v$ | 2     |
| Number of interferers | $L_{c}, L_{c}'$ | 8     |
| TX antennas in ext. interferers | $d_{c}$ | 1     |
| Signal to noise ratio | $\text{SNR}$ | 20 dB |
| Signal to interference ratio | $\text{SIR}_s, \text{SIR}_c$ | -20 dB |
| Minimum image rejection ratio | $\text{IRR}_{\text{min}}$ | 25 dB |
is mainly caused by the additive noise. The resulting SINR is actually better than the input SNR since the effect of the noise can be decreased with the antenna array processing, as in the noise limited case, 20 receiver antennas provide extra degrees of freedom relative to separating $5 \times 2 = 10$ overall streams. Under I/Q imbalances, on the other hand, the performance is limited due to the signal leakage from the image subcarrier UEs even in the high SIR region. In that region TX and RX I/Q imbalances are equally deteriorating the overall performance. The worst SINR is seen with joint TX+RX I/Q imbalances which results in an approximately 2.7 dB worse SINR compared to the perfect I/Q matching case. When the SIR decreases, combiners have to put more effort into external interference suppression and consequently the SINRs decrease due to the finite degrees of freedom. With the augmented combiners on the other hand, the performance is the same for all I/Q imbalance scenarios as for perfect I/Q matching. This means that joint subcarrier processing results in the best possible performance. The per-subcarrier Wiener processing under TX I/Q imbalance only floors to 2.3 dB worse SINR, which is caused by the inter-user interference from the image subcarrier UEs. Clearly, the worst performance with per-subcarrier Wiener processing is obtained under RX or joint TX+RX I/Q imbalances. In this case, the significant drop in the SINR is caused not only by the aforementioned interferences but also due to the attenuated desired signal. This is a consequence of the case where the uncontrollable interference from the image subcarrier UEs and external interference sources gets so strong that the Wiener combiner has to start attenuating all incoming signals in order to minimize the mean square error. Note that the difference between TX and RX I/Q imbalances originates from the fact that RX I/Q imbalance causes external interference leakage from the image subcarrier whereas TX I/Q imbalance does not. In general, as is obvious from Fig. 4 the augmented combiner can provide substantially enhancement in output SINR, especially under high levels of external interference.

2) **SINR as a function of the SIR**

   The leakage of the external interference at the the image subcarrier is further illustrated in Fig. 5. It shows the SINR when the SIR at the image subcarrier is swept and SIR at the desired subcarrier is fixed to -20 dB. Based on the results, the augmented combiner has a flat and robust response over all SIR\(c^\prime\) values and with all I/Q imbalance scenarios. This means that it can suppress the effect of the signal leakage completely and thus provides good performance in all conditions. Also the per-subcarrier processing under TX I/Q imbalance yields a flat response which has 2.3 dB lower SINR level than that with the augmented combiner. This difference is purely caused by the inter-user interference from the image subcarrier since the external interference and noise at the image subcarrier are not affected by TX I/Q imbalance. The response of the Wiener combiner is not flat when considering I/Q imbalance in the RX side. In that case, the SINR drops drastically as SIR\(c^\prime\) decreases. When comparing Figs. 4 and 5 with each other, we notice that actually the interference leakage is the main reason for performance degradation with high external interference levels since in those cases the resulting SINRs are almost the same in both figures. The results clearly indicate that the overall SINR performance of the per-subcarrier Wiener combiner is heavily deteriorated by the strong external interference at the image subcarrier, even if the contribution of external interference at the considered subcarrier can be efficiently suppressed.

3) **SINR as a function of the SNR**

   Fig. 6 visualizes the SINR as a function of the SNR. The performance saturates under I/Q imbalances and the worst performance with the Wiener combiner is obtained if I/Q imbalance occurs in the RX electronics. The ceiling effect, due to the unavoidable signal leakage from the image subcarrier, is very strong and the SINR saturates at around 20 dB SNR with RX and TX+RX imbalance scenarios and at around 35 dB SNR with TX I/Q
imbalance. Again, the augmented combiner outperforms the conventional one clearly and results in a linear growth of the SINR as the SNR increases. The results in Fig. 6 also extend the work with the SU-SIMO scenario in [1] and show somewhat similar behavior in both cases.

4) SINR as a function of L: The effect of increasing the number of external interferers is depicted in Fig. 7. With the default simulation parameters in Table III there are \( M_u U + J I L = 18 \) incoming signals at the desired subcarrier as well as at the image subcarrier. In theory, the combiners are able to separate \( N = 20 \) signals as long as all the signal sources have separable spatial characteristics, i.e. their channel responses are not fully correlated. Thus the number of single antenna interferers could be even increased to 10, resulting in 20 incoming signals in total, without losing the ability for signal separation in theory. However, based on the figure, the SINR decreases as the number of interferers increases, even for less than 10 external interferers. This is natural as the optimizing the MSE at the combiner output, corresponds to finding a proper compromise between coherent combining of desired signal as well as suppressing inter-stream interference, inter-user interference, external interference and noise, and all of their mirror images, and thus when the amount of signals is increasing, this becomes increasingly difficult. The augmented combiner under any I/Q imbalance scenario provides the best SINR. The per-subcarrier Wiener processing under RX and TX+RX I/Q imbalances turns out to have the worst SINRs. This is again caused by the interference leakage from the image subcarrier and is now emphasized since the number of the interferers is swept at both subcarriers. When the number of interferers exceeds 10, also the augmented combiner runs out of degrees of freedom in interference suppression and consequently the SINRs of all scenarios drop steeply towards lower levels.

5) SINR as a function of the IRR: Fig. 8 shows the SINR performance when the minimum allowable IRR is varied. The augmented combiner produces a flat response for all IRR\(_{\min}\) values, meaning that the effects of I/Q imbalances are mitigated completely even for low IRRs. The performance of the per-subcarrier processing under TX I/Q imbalance is deteriorated by the inter-user interference from the image subcarrier and therefore the SINR degrades fairly slowly as IRR\(_{\min}\) decreases. In contrast, the SINR under RX I/Q imbalance is heavily degraded, again due to the increasing external interference leakage from the image subcarrier. It is worth noting that under RX or TX+RX I/Q imbalances and even with very moderate values of IRR\(_{\min}\) the SINR is degraded by several decibels.

6) SINR as a function of N: Finally, Fig. 9 illustrates the SINR as a function of the number of RX antennas. Although the current LTE specification does not support more than 8 antennas in the BSs, this figure shows the capability of antenna array processing and thus gives an important insight also for the behavior of emerging massive MIMO systems under I/Q imbalances, see e.g. [32]. Based on the results with varying number of RX antennas, the performance is really poor when the number of RX antennas is around 12 or less due to too little degrees of freedom to spatially separate the signals. Beyond that point, the RX starts to be able to separate different signals and the SINR of the augmented combiner grows very steeply as \( N \) increases. Also the per-subcarrier Wiener processing under TX I/Q imbalance gets a similar performance boost. However, both curves start to saturate after the point where the number of antennas match the number of incoming signals which is in this case equal to 18. In contrast to these curves, RX and TX+RX I/Q imbalances cause slower increase in the resulting SINR but their saturation starts later, around the point \( N = 28 \). That point coincides with \( N = M_u U + 2J I L \) which means that at this point the per-
subcarrier Wiener processing is finally able to separate the signals from the desired subcarrier and strong interferers at both subcarriers from each others. Thus it is able to provide the same SINR as the augmented combiner has already with 20 antennas. As the number of antennas becomes very high, both combiners perform well under all I/Q imbalance scenarios. Additionally, the SINR increases only slightly when adding RX antennas to the BS side. This is a consequence of the situation where both combiners have more than enough spatial resources and they can use the extra degrees of freedom purely for noise optimization and interference suppression purposes.

V. CONCLUSION

Radio transceiver I/Q imbalances in MIMO communications with OFDM waveforms have been widely studied in the existing literature. This paper, however, extends the system approach to multiuser OFDMA-based MIMO uplink where multiple UEs are active simultaneously at each subcarrier and in addition to that also frequency division multiplexing is deployed. We also included the effects of possible external interference in the analysis and thus provide a valuable insight for the future heterogeneous network designs where system coexistence and interference suppression are key issues. It was explicitly shown that I/Q imbalances of UE transmitters and BS receiver distort the signal properties and cause inter-carrier and inter-user interference originating from the image subcarrier. This phenomenon turns out to be especially harmful when the external interference at the image subcarrier is strong. Furthermore, I/Q imbalance complicates separating the spatially multiplexed UEs at a given considered subcarrier.

The provided extensive SINR analysis, as a function of multiple system parameters, shows that the performance of the conventional per-subcarrier processing is heavily limited under I/Q imbalances and external interference. Stemming from that, an augmented spatial combiner was proposed, combining the signals jointly between mirror subcarriers and across all antennas. The proposed augmented subcarrier processing mitigates the effects of the transceiver I/Q imbalances efficiently and indeed provides combined output SINRs practically identical to a reference system with I/Q imbalance free transceivers. Note that the augmented processing is implemented completely by digital signal processing in the BS RX. Thus the number of costly RF chains and demanding FFT processing blocks are equal to those of the conventional per-subcarrier processing. Moreover, the augmented processing integrates the data stream separation, interference suppression, noise suppression and I/Q imbalance mitigation all into a single processing stage, thus avoiding separate transceiver calibration. Overall the results demonstrate that reliable and high-performance spatial processing characteristics can be obtained by the proposed augmented combiner principle, despite of challenging levels of external interference, transceiver I/Q imbalances and high number of spatially multiplexed users.

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