The Field Nature of Spin for Electromagnetic Particle

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Abstract. The field nature of spin in the framework of the field electromagnetic particle concept is considered. A mathematical character of the fine structure constant is discussed. Three topologically different field models for charged particle with spin are investigated in the scope of the linear electrodynamics. A using of these field configurations as an initial approximation for an appropriate particle solution of nonlinear electrodynamics is discussed.

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INTRODUCTION

The field electromagnetic particle concept in the framework of an unified nonlinear electrodynamics was discussed in my articles (see, for example, [1, 2, 3, 4]). Here I continue this theme.

ELECTROMAGNETIC PARTICLE WITH SPIN

Let us consider the electromagnetic particle which is a space-localized solution for a nonlinear electrodynamics field model. A field configuration corresponding to the solution is a three-dimensional electromagnetic soliton. It is not unreasonable to consider the field configuration which is more complicated than the simplest spherically symmetrical one with point singularity. The purely Coulomb field is the known example for such simplest configuration. We can consider the field configuration with singly or multiply connected singular region. This singular region can be considered to be small, so that it do not manifest explicitly in experiment. But its implicit manifestation is the existence of the spin and the magnetic moment of the particle.

Mass, spin, charge, and magnetic moment of the particle appear naturally in the presented approach when the long-range interaction between the particles is considered with the help of a perturbation method [4]. The classical equations of motion for electromagnetic particle in external electromagnetic field are derived but not postulated. These equations are a manifestation of the nonlinearity of the field model. Charge and magnetic moment in this approach characterize the particle solution at infinity. But mass and spin characterize the particle solution in the localization region and appear as the integrated energy and angular momentum accordingly. Thus we have the following definition for
spin:

\[ s = \int \mathcal{M} \, dV, \tag{1} \]

where \( \mathcal{M} \equiv r \times \mathcal{P} \) is an angular momentum density (spin density), \( r \) is a position vector, \( \mathcal{P} \equiv (\mathbf{D} \times \mathbf{B})/4\pi \) is a momentum density (Poynting vector).

The angular momentum density can appear in axisymmetric static electromagnetic field configurations with crossing electric and magnetic fields. In this case the crossing electric and magnetic fields give birth to the momentum (Poynting vector) density which is tangent to a circle with center located at the axis. Because of the axial symmetry, the full angular momentum contains only an appropriate axial component of the angular momentum density. Thus we have the spin density directed on the axis \( z \):

\[ \mathcal{M}_z = \rho \times \mathcal{P}, \]

where \( \rho \) is a vector component of the position vector which is perpendicular to the axis \( z \). This configuration is shown on Fig. 1.

\[ \mathbf{D} \mathbf{B} \mathcal{P} = \frac{1}{4\pi} \mathbf{D} \times \mathbf{B} \]

FIGURE 1. The origin of the spin density

SPIN AND FINE STRUCTURE CONSTANT

A value of the electron spin which we have in experiment is

\[ s = \frac{\hbar}{2} = \frac{e^2}{2\alpha}. \tag{2} \]

A particle solution of nonlinear electrodynamics, which can be appropriate to electron, has a free parameter associated with electron charge. The existence of this free parameter is connected with a scale invariance of the field model. Thus we have eleven free parameters for electromagnetic particle in all, viz. ten parameters for Poincaré group and one scale parameter.

One further known continuous symmetry can be considered. This is so called dual symmetry between electric and magnetic fields. An appropriate transformed field will have a magnetic charge. But this continuous transformation for the electromagnetic field be accompanied by an appearance of a vector potential field with a singular infinite line. Because this it is reasonable to consider that an appropriate free parameter does not exist for a removed particle.

We can assume that one particle solution has only these eleven free parameters. But a separate particle included in a many-particle solution of a nonlinear field model has not these free parameters. In the case of linear electrodynamics this separate particle has the
free parameters because of the known superposition property for the solutions. But for
the case of nonlinear electrodynamics only the entire many-particle solution has the free
parameters, and an included particle has not the free parameters.

Thus we can assume that the specified value of the electron charge is connected with
the nonlinearity of the model which is the cause of the interaction between particles in
the world solution.

According to formula (2) we have that the dimensionless constant $2\alpha$ is the aspect
ratio between the square of the electron charge $e^2$ and the value of electron spin $s$. We
can consider that the electron charge is the given constant. But the value of electron spin
is calculated in the presented approach by the formula (1). Thus we can consider the fine
structure constant as a mathematical one calculated by the formula

$$\alpha = \frac{e^2}{2s},$$

where $s$ is calculated by the formula (1) for the appropriate particle solution.

A value of the constant $\alpha$ calculated by the formula (3) for a particle solution in the
framework of a field model can be considered as a test for an appropriateness of these
field model and particle solution.

**THREE TOPOLOGICALLY DIFFERENT MODELS FOR CHARGED PARTICLE WITH SPIN**

Let us consider three topologically different models of the electron in the scope of the
linear electrodynamics. These solutions can be considered as an initial approximation to
the appropriate veritable solution for a nonlinear electrodynamics model.

**Two-dyon configuration** with equal electric and opposite magnetic charges was
considered in my article [1]. The spin calculated by formula (1) with the appropriate
solution of the linear electrodynamics does not depend on a distance between the dyons.
It is defined by the values of the electric and the magnetic charges. If an absolute value
of the ratio between the electric and the magnetic charges (which is an additional free
parameter) equals to the fine structure constant and the full charge equals to the electron
charge, then the value of spin equals to electron spin (2). For the details see my article
[1].

**Configuration with singular disk** is obtained from the Coulomb field by a space
shift with a complex number parameter (see, for example, [5]). We have the field in
the complex representation for the electromagnetic field by means of the following
formal complex shift: $D + iH = e r / r^3$, where $x_1 \rightarrow x_1$, $x_2 \rightarrow x_2$, $x_3 \rightarrow x_3 + i\rho$. The
spin calculated by formula (1) with this field does not depend on the shift parameter $\rho$
(radius of the disk). But the numerical value of the spin for this case is not equal to the
electron spin: $s \approx 0.027 e^2/(2\alpha)$. Note that here an additional parameter might be used
to obtain the value of spin for the electron. The magnetic part of the solution must be
multiplied on this parameter. But in this case the solution is not obtained by the complex
shift from the Coulomb field.

A consideration of the static linear electrodynamics equations in toroidal coordinates
$(\xi, \eta)$ gives the appropriate solution with toroidal symmetry. This solution can in-
clude an electric and a magnetic parts. They can be represented with the help of toroidal harmonics which are the spheroidal harmonics with half-integer index: \( P_{\frac{1}{2}}^n (\cosh \xi) \), where \( n \) and \( l \) are integer [6]. To obtain the right behaviour of the electromagnetic field at infinity for a charged particle with magnetic moment we must take the toroidal harmonics \( P_0^{\frac{1}{2}} (\cosh \xi) \), \( P_0^{\frac{3}{2}} (\cosh \xi) \) for the electric field and \( P_1^{\frac{1}{2}} (\cosh \xi) \), \( P_1^{\frac{3}{2}} (\cosh \xi) \) for the magnetic one. Because we intend to consider this solution as an initial approximation to a solution of a nonlinear electrodynamics model, it is reasonable to take the condition of vanishing of two electromagnetic invariants near the singular ring. This condition will be satisfied when the ratio between the electric and magnetic vector magnitudes tends to unit near the ring.

The spin calculated by formula (1) with this toroidal field configuration does not depend on a radius of the singular ring. But the numerical value of the spin for this case is not equal to the electron spin: \( s \approx 0.0006 e^2/(2 \alpha) \). An additional parameter might be used to obtain the value of spin for the electron. The magnetic part of the solution must be multiplied on this parameter. But in this case the solution does not satisfy the condition of vanishing of two electromagnetic invariants near the ring.

**CONCLUSIONS AND DISCUSSIONS**

Thus to obtain the right value of fine structure constant for the considered field configurations, we must introduce some additional free parameter. But, as mentioned above, for the solution of a nonlinear electrodynamics model we must have only one free parameter that is electron charge. We must obtain the right value of the fine structure constant in the case of an appropriate nonlinear electrodynamics model and an appropriate solution. The examined field configurations can be considered as possible initial approximations to the right electron particle solution.

It should be noted also that the considered here field configurations may describe the charged particles with spin but not neutral or massless one. However, another particle solutions may describe neutral and massless particles with spin according to the general approach based on the formula (1). These solutions may be more complicated than considered here.

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