Analysis of energy systems for poly-generation using optimization modelling: the optimization process strategies and the formalization of an innovative multi-objective optimization model

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Abstract. The purpose of this paper is to describe the numerical modelling processes and optimization strategies of energy systems and to formalize an innovative optimization model for poly-generation systems. The model uses a multi-objective optimization approach, by means of three optimization functions: one of an economic nature, one technical and one relating to polluting emissions. The constraint equations of the problem, the boundary conditions and the input values, as well as some dimensionless parameters are described. In addition to a complete approach, the article proposes innovative aspects based on the studies in the literature, both from a technological and economic-regulatory standpoint. For each proposed innovation, an analysis of how this can influence the optimization model is presented and various types of optimization models have been evaluated, depending on the complexity of the physical system and the consequent numeric modelling.

1 Introduction

The new challenge for the whole energy sector is represented by the need of improving energy saving and efficiency, in order to allow a truly sustainable development. [1] The concept of sustainability in the energy sector is very broad, including not only climatic-environmental aspects, but also socio-political and economic aspects. In the last decades the major economic and industrial powers of the world have tried to combat the increase in energy consumption and polluting emissions that have led to worrying effects on the climate and the environment.

Among the various technologies developed to increase energy efficiency and primary energy savings, cogeneration and trigeneration are widely studied in literature, representing one of the most valid alternatives, both on an industrial scale and for residential applications [2 - 6]. Although the structure of a cogenerative or trigenerative system is well known from a technological point of view, the optimal management of a poly-generative plant is relatively complex, due to the high number of variables and constraints to be taken into account. The study of optimization models in the literature is very broad, [7 - 12] but in many cases a wide-angle approach is lacking, considering all the phases of interest for a CHP (Combined Heat and Power) plant, from the choice of the components, through the dimensioning, up to the choice of optimal operational management.

The first part of the article presents the strategies for numerical modelling and for the optimization of energy systems in a cogeneration structure. The definition of the optimization problem from a physical and economic point of view is described, evaluating the possible innovations to be introduced in the proposed model. Given the complexity of the problem, both from a physical and computational point of view, in the second part of the paper the use of multi-objective optimization is proposed, which allows to take into consideration different aspects that influence the optimal solution.

Finally, an ad hoc formalized multi-objective model is described, containing three optimization function, which take into account economic, technical and environmental aspect.

2 Optimization Strategies

The problem of technical-economic optimization of a CCHP (Combined Cooling, Heating and Power) system is very delicate, given the high number of variables,
constraints and boundary conditions, and has been deeply investigated in the literature. One of the problematic aspects is represented by having a global and complete vision of the system, considering therefore both the phase of choosing the components and the sizing up to the definition of the operational strategy. In [13] three levels of poly-generation energy systems optimization are described: Optimum synthesis, optimum design and optimum operation.

2.1 Numerical modelling of a trigeneration plant

The modelling of an energy system is the starting point for the optimization procedure that will be described in the following paragraphs. Modelling consists in identifying all the components of the system, quantifying the exchanges of energy and mass between the components themselves.

The definition of the objective function is linked to some economical KPIs (Key Performance Indicators).

The definition of the variables refers to the plant configuration.

The constraints of the system are of various kinds. The physical constraints are linked to the mass and energy balance equations for each component of the system. The constraints on operational strategy depend on the type of load and the functioning of the system. The constraints on the efficiency of the components are linked to the size of the plant but also to the loading conditions.

Moreover, from a numeric point of view, the study of physical phenomena is fundamental, in order to evaluate their linearity or non-linearity.

2.2 Definition of Physical Problem

The optimization of a poly-generation plant must take into account all the physical phenomena in the system. Compared to the separate generation, the system is relatively more complex, due to energy exchanges that take place between the various components. The physical definition of the problem consists in identifying, for each type of application, the correct layout of the plant and identifying each component, to which constraints and variables will be associated.

In [14] the optimal layout for a distributed district-scale CHP system was studied, consisting of a centralized cogeneration unit (ICE) and a series of MTGs placed in public buildings, by means of a MILP model.

It must be considered that the physical definition cannot be complete without the integration of the reference economic-regulatory context. In [15], for example, the possibility of increasing the profit management of a CHP plant was investigated with participation in the day-ahead energy markets and the spot market, by means of a multi-stage stochastic mixed-integer linear programming (MILP) model.

The process of optimizing a CHP system consists of three macro phases:

- Preliminary study for the definition of the system configuration,
- System sizing, starting from the prime mover to all the other components, depending on the size of the users and economic-regulatory constraints, such as the possible simplifications in the authorization process.
- Choice of the optimal operational strategy, in order to minimize plant operating costs, or the other objective functions.

2.3 Definition of Economical Problem

The feasibility of these energy systems is linked, in addition to the physical and environmental aspects already described, also to economic and regulatory assessments.

So, for the definition of the problem from an economic point of view it will be necessary to consider the following aspect:

- Regulatory scenario

Since cogeneration is considered, at European level, one of the most valid alternatives for achieving the Community objectives in terms of climate and environmental sustainability, in recent years various directives have been issued to incentivize this technology. These incentive mechanisms have been implemented by the various member states and have a great influence on the feasibility of the CHP and CCHP systems.

The model will take into account the regulatory aspects not only in terms of constraints, but also as a constituent part of the optimization functions. In particular, the economic objective function will take into account the possibility of mitigating investment risks through EPCs, or of evaluating the obtainment of state incentives to partially or totally cover the investment.

The optimization process can take place taking into account different energy policy scenarios, giving the possibility to evaluate in advance the "regulatory risk" which is very often a limiting factor for investments in the sector.

- Tariff scenario

The study of the tariff framework relating to electricity and gas prices is essential to develop the optimization model for a CCHP plant. Indeed, gas and electricity prices influence not only the most adequate plant choice, but also the plant's optimal operating strategy.

- Economical KPIs

The analysis of an investment in the industrial sector is based on the calculation of some fundamental economic parameters, the Key Performance Indicators. Below is the definition of the KPIs most used in the literature for optimization problems:

\[
NPV = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t}
\]

Where:

- \( R_t \) represents the net cash inflows-outflows during a single period \( t \).
3 Definition of Optimization Problem

From a computational point of view, defining an optimization problem means describing the variables, the constraints and the objective function. The structure of the mathematical model for the definition of these quantities changes as a function of the complexity of the energy system and as a function of the optimization approach chosen.

3.1 Formalization of Multi-objective optimization model

The following is a mathematical model, relating to a general system, which takes into consideration all the aspects discussed in the previous paragraphs. The block diagram of the system is shown in Fig. 1.

The model aims to optimize the design of a trigeneration plant, through a multi-level structure:

• Before the execution of the model it is necessary to carry out a preliminary analysis concerning the coupling between technology and fuel used
• The central routine of the model optimizes the operational management of the system. The input data is given by the "time" vector which indicates the temporal discretization of the model, and by the vectors which indicate the user's energy needs as a function of time.
• The second level concerns the optimization of the size of the plant, through technical-financial parameters to be maximized.

As shown in Fig. 2, for each level, the model optimizes three objective functions, obtaining three "result vectors", one that optimizes the economic function, one that optimizes the technical one and one that optimizes the normative one.

Then a Pareto dominance analysis is performed: if there is a result vector that optimizes all three object functions, there is a unique solution to the problem, otherwise a decision maker will have to be introduced who will determine the optimal local solution within the interval of Pareto Optimal Solutions, the Pareto Front.

3.1.1 Model Input and dimensionless parameter

The first input of the optimization model is represented by the vector N, which is the result of a preliminary analysis on the possibility of coupling a cogenerator to a given fuel. Each element j of the vector N therefore represents a Cogenerator-Fuel couple.

At the plant management optimization level, the inputs are represented by the time vector and the energy demand vectors.

The time vector implicitly indicates the chosen time discretization interval. To perform a fairly detailed analysis of the system and define the management mode, the model is set on a time interval of 1 hour. This type of discretization can however be modified according to the availability of data relating to the user's energy needs.

The size and discretization of energy demand vectors follow temporal discretization. In particular, the model allows you to enter the demand values for electrical, thermal and cooling power as time changes. The assessment of current needs is an operation that each user or operator of the system will be able to perform according to his experience in the energy sector, by making a forecast, or based on statistical data.

In the case of industrial users who can take advantage of the presence of consumption data from a monitoring infrastructure, the values can be entered through a vectorized function. Within the model some parameters are then defined, such as efficiency or emission coefficients that represent input values of the optimization model.

3.1.2 The decision variables

Since the model has two levels of optimization, the decision variables also belong to two different categories: the operational management variables are expressed as a function of time, while the sizing variables are stationary. The following table shows all the variables, distinguished by category.

The variables with the subscript "t" indicate those that vary as a function of time, while those with the subscript "n" are stationary in nature, such as the...
electrical size of the cogenerator, gas compression and absorption heat pumps, or electrical and thermal storage.

### 3.1.3 The objective functions

The optimization model is multi-objective therefore three objective functions are defined, that are contrasting with each other. The three functions are related to reference values, and therefore represent dimensionless factors. The choice to manage dimensionless functions guarantees the possibility of comparing the three functions with each other, to obtain, according to the action of a decision maker, the optimal solution to the multi-objective problem.

The three objective functions are:

1. **Cash flow maximization function.**
   
   \[ F_C = \frac{R_{k,t} - C_{k,t}}{C_{k,t}} \]  \hspace{1cm} (2)

   Where:
   - \( R_{k,t} \) represents the revenues in the time interval considered. These are given by:
     - electricity produced by the cogeneration plant, net of electricity purchased from the network.
     - thermal energy produced by the cogeneration plant, net of that generated by traditional auxiliary systems.
   - \( C_{k,t} \) represents the operating costs associated with the cogenerator and they are given by:
     \[ C_{k,t} = F_{CHP,t} / (\text{LHV}_{\text{fuel}} \times C_{\text{fuel}}) \]  \hspace{1cm} (3)

   Function to maximize the overall efficiency of the system

   \[ F_T = \frac{E_{CHP,t} + H_{CHP,t}}{E_{CHP,t}} \]  \hspace{1cm} (4)

2. **Function to minimize the overall efficiency of the system**

   \[ E_{EM} = \frac{EM_{CHP,t} + EM_{int,H,t} + EM_{Grid,t}}{EM_{REF,E,t} + EM_{REF,H,t} + EM_{REF,C}} \]  \hspace{1cm} (5)

   For the emission function the parameter \( \mu \) is defined, in order to evaluate the production of CO2 per fuel entity. In particular \( \mu \) is defined as \( g_{CO2}/g_{fuel} \) and is given by the stochiometry of the combustion reaction. The value of \( \mu \) is equal to zero if the supplied fuel is hydrogen.

### 3.1.4 The constraints of the problem and the boundary conditions

An optimization problem for a complex energy system presents various types of constraints. In particular, the model is subjected to linear and non-linear constraints, of equality and inequality.

In detail, the optimization problem is subject to the following mathematical constraints:

**Constraints on the performance of the cogenerative motor**

These constraints express the definition of the two main production variables of the cogeneration engine. The electricity produced for each interval is defined by the electrical efficiency of the machine. The electrical efficiency is in turn a variable of the problem therefore equation 11 expresses a nonlinear constraint.

\[ E_{CHP,t} = \eta_{E_{CHP,t}} \times F_{CHP,t} \]  \hspace{1cm} (6)

\[ H_{CHP,t} = \left( F_{CHP,t} \times \left( 1 - \eta_{E_{CHP,t}} \right) - T_{D,t} \right) \times \eta_{HR} \]  \hspace{1cm} (7)

Where:
\[ T_{D,t} = 0.13 \times F_{CHP,t} \]  \hspace{1cm} (8)

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**Fig. 1. Block Scheme of Energetic Flows in the Plant**
Table 1. List of Decision Variables.

| Decision Variables in the Model | Equation |
|---------------------------------|----------|
| C_{THP,E}, t | C_{THP,E} = EER_{C,THP} * E_{C,THP,t} |
| C_{THP,H}, t | C_{THP,H} = EER_{ABSHP} * H_{CHP,cool,t} |
| H_{CHP,E}, t | H_{CHP,E} = COP_{C,THP} * E_{C,THP,t} |

**Constraints on the performance of the Heat Pumps**

Equations 14-15-16 define the efficiency of gas compression and absorption heat pumps. For the CGHP two equations are defined, referring to winter and summer operation, while for ABSSHHP only one equation is defined with reference to summer operation. The introduction of heat pumps allows the recovery and use of surplus thermal and electric energy.

| Constraints relative to Electrical Energy Balance |
|-----------------------------------------------|
| E_{CHP,E} = E_{C,THP,E} + E_{C,THP,H} + \varepsilon |
| E_{CHP,cool,t} | E_{CHP,cool,t} = \delta_{grid} * * E_{CHP,store,t} |

Where: 
\varepsilon and \delta_{grid} represent parametric coefficients that describe the exchange of electricity with the national electricity grid and with the storage system.

Equation 18 describes the constraint on the balance of thermal power in the system.

\[ H_{CHP,1} = H_{CHP,1} + \varepsilon_{E} * H_{CHP,store,1} + \delta_{CHP,cool} \]

Equation 19 describes the constraint on the balance of thermal power in the system.

\[ C_{CHP,1} = C_{CHP,1} + \delta_{CHP,cool} \]

The constraint equation states that the demand for cooling energy will be met through heat absorption and compression heat pumps.

Energy Storage Constraints

The following inequalities 20-21 describe the constraints relating to electrical storage systems. The same conditions are fixed for thermal energy storage.

\[ E_{CHP,store,1} = E_{CHP,store,1} - E_{CHP,store,1} \]

| Environmental Constraints |
|---------------------------|
| E_{CHP,store,1} | E_{CHP,store,1} = E_{CHP,store,1} |

Regulation Constraints

With reference to Italian incentive policies, a cogeneration or trigeneration plant can access the mechanism of energy efficiency certificates if the value of the PES (Primary Energy Saving) parameter is greater than 0,10. This condition is expressed by the following equation.
\[
PES = \left(1 - \frac{1}{\eta_{\text{ref}} \cdot \eta_{\text{ref}}} \right) \times 100 \% \geq 0.10 \quad (18)
\]

**Boundary Conditions**

The boundary conditions, in terms of maximum and minimum values that can be assumed by the variables, take into account the production limits of the cogenerator, of the heat pumps, of the maximum storage capacity. The lower limit of all variables is equal to 0, since they are all physical quantities. The following inequalities express the boundary conditions for some of the most significant variables. The exhaustive list of boundary conditions is not reported for brevity.

\[
0 \leq F_{\text{CHP}} \leq F_{\text{CHP, size}} \quad (19)
\]
\[
0 \leq E_{\text{CHP}} \leq E_{\text{CHP, size}} \quad (20)
\]
\[
0 \leq C_{\text{CHP, size}} \leq G_{\text{CHP, cooling size}} \quad (21)
\]

**4 Conclusions**

This paper highlighted the importance of carrying out a technical, economic, regulatory and environmental analysis for the optimization of polygeneration energy systems. Cogeneration represents a valid possibility to guarantee a substantial increase in energy efficiency, leading to a reduction in consumption and polluting emissions.

A modelling approach based on multi-objective optimization is proposed, in which the optimal solution is given by a trade-off between three objective function, subject to a series of linear and non-linear constraints. After an initial phase of analysis on the coupling between fuel and technology, the model moves on to the definition of the optimal strategy for running the system, and then to the sizing, through the optimization of the size of the cogenerator, based on the economic-financial KPIs described in the model. The concept of dominance according to Pareto is indicated and the role of the decision maker is defined to order the optimal solutions found, based on objective preference criteria. This method is promising for the formulation of an optimization model that takes into account not only technical aspects, but also economic, environmental and regulation ones.

Future developments of the model will instead have to investigate in more detail the transitory aspect of some processes, such as the storage of thermal and electrical energy. The potential of the proposed approach lies in the possibility that the optimization process can take place taking into account different energy policy scenarios, giving the possibility to evaluate in advance the "regulatory risk" which is very often a limiting factor for investments in the sector.

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