Accurate characterization of high-degree modes using MDI data

S G Korzennik\textsuperscript{1}, M C Rabello-Soares\textsuperscript{2,3}, J Schou\textsuperscript{2} and T Larson\textsuperscript{2}

\textsuperscript{1} Harvard-Smithsonian Center for Astrophysics, Cambridge, MA, USA
\textsuperscript{2} W.W. Hansen Experimental Physics Laboratory, Stanford University, CA, USA

E-mail: skorzennik@cfa.harvard.edu

Abstract. We present the first accurate characterization of high-degree modes (i.e., $\ell$ up to 1000), using the best MDI full-disk full-resolution data set available (90-day long time series, acquired in 2001). The Dopplergrams were spatially decomposed using our best estimate of the image scale and the known components of MDI’s image distortion. Multi-tapered power spectra were fitted for all degrees and all azimuthal orders, between $\ell = 100$ and $\ell = 1000$, and for all orders with substantial amplitude. Fitting at high degrees generates ridge characteristics, characteristics that do not correspond to the underlying mode characteristics. We used a sophisticated forward modeling to recover the best possible estimate of the underlying mode characteristics. We have derived a final set of corrected mode characteristics (frequencies, line widths, asymmetries and amplitudes) and their uncertainties.

1. Introduction

It has been shown that the inclusion of high degree modes ($\ell \leq 1000$) has great potential to further constrain the outer 4\% of the solar interior (see [4] for the stratification and [1] for the internal rotation). Unfortunately, modes merge into ridges at high degrees (i.e., $\ell > 300$ for f-modes, $\ell > 200$ for low order p-modes) and while the ridge can be fitted, the resulting ridge characteristics ($\tilde{\nu}$, $\tilde{\Gamma}$, $\tilde{A}$, $\tilde{\alpha}$) are not equal to the underlying mode characteristics. It is thus not enough to fit ridges, but one needs to correct the ridge characteristics using a precise and sophisticated ridge modeling procedure to derive accurate mode characteristics.

2. Data Analysis and Ridge Fitting

We have analyzed MDI’s \textit{Dynamics 2001}: 90 days of full disk images, with a 97\% fill factor, after gap filling (95.8\% prior). The full-disk two arc-second per pixel resolution Dopplergrams were spatially decomposed using our best spatial decomposition, including both components of the image distortion [2]. We used a 61-term sine multi-taper spectral estimator, since the ridges are wide (no need for high spectral resolution) and the resulting widening causes modes to blend into ridges at lower degrees (hence some overlap between mode fitting and ridge fitting). The resulting power spectra, for the zonal modes, is shown in Fig.1. All the power spectra, between $\ell = 100$ and $\ell = 1000$ and for all the azimuthal orders, were fitted with a sum of modified (asymmetric) Lorentzians, using a multi-step least-squares minimization. The number of orders included in the fitting was contrast limited (1:500 power ratio followed by rejecting the highest

\textsuperscript{3} Present address: Physics Department, Universidade Federal de Minas Gerais, Minas Gerais, Brazil.
Figure 1. Gap-filled *Dynamics* 2001 power spectrum, displayed on a logarithmic scale, for the zonal modes \((m = 0)\). The dots represent the fitted ridges (black) or modes (green).

\(n\) to limit the dynamical range of the fitting. This produced some \(5.2 \times 10^6\) singlets \((n, \ell, m)\), or 5,780 multiplets \((n, \ell)\)\(^4\).

3. Ridge Modeling
Over the years, we have developed a precise and sophisticated model of the ridge [2, 5, 3]. This model produces a simulated ridge, based on a mode characteristics input set. The simulated ridge is fitted to derive the resulting ridge characteristics, and thus a correspondence between the mode and ridge characteristics. The input model is refined by adjusting iteratively most of the parameters to produce a model that matches the observations. The input set was also perturbed to derive the sensitivity and precision of the correction scheme (see [3] for additional details.)

4. Results
Mode frequencies were estimated from ridge frequencies as follows:

\[ \nu_{n,\ell,m} = \tilde{\nu}_{n,\ell,m} - \Delta\nu_{n,\ell,m} \]  

(1)

where \(\tilde{\nu}\) represents the ridge frequency and \(\Delta\nu\) the ridge-to-mode offset:

\[ \Delta\nu_{n,\ell,m} = \tilde{\nu}^M_{n,\ell,m} - \nu^M_{n,\ell,m} \]  

(2)

where \(\nu^M\) is the input mode frequency and \(\tilde{\nu}^M\) the resulting ridge frequency of our model.

\(^4\) The tables are available at https://www.cfa.harvard.edu/~sylvain/research/tables/HiL/.
Figure 2. Comparison of estimate of mode singlet frequencies derived from ridge fitting to actual mode frequency computed by fitting resolved modes at intermediate degrees. The frequency differences (left panels), and the scaled differences (differences divided by their respective uncertainties, $\sigma$), right panels) are plotted as a function of degree, frequency, and $m/\ell$, and as a histogram (top to bottom). The color coding in the top three rows corresponds to, $n$. The red curves in the histograms are the Gaussians corresponding to the mean and standard deviation of the distributions (indicated by the vertical dotted lines), drawn to show the excess and skewness in the wings of the actual distribution.

The overlap between mode and ridge fitting, i.e., $100 \leq \ell \leq 300$ for f-modes and $100 \leq \ell \leq 200$ for p-modes, provides validation of the methodology. Figure 2 compares mode frequencies measured at low and intermediate degrees$^5$ to corrected mode frequencies derived from ridge fitting, for all the overlapping singlets. This figure demonstrates clearly that there is almost no bias in the corrected mode frequencies derived from ridge fitting, except for some excess and skewness in the wings of the distribution of the differences (bottom panels of Fig. 2.)

$^5$ Where the modes are resolved.
Figure 3. Comparison of ridge FWHM for a selection of multiplets, as a function of the ratio \( m/\ell \). The dots correspond to individual ridge width measurements, and the black curves the corresponding binned values. The red curves correspond to our best model of the ridge width, as evaluated at 51 equispaced \( m \) values. In most cases, our model reproduces rather well the variation of ridge width with the azimuthal order, \( m \), although the input mode width of the model is constant with \( m \).

Figure 3 shows that the ridge width is a function of \( m \), although the input mode width is assumed constant. That variation is well modeled and is thus taken out to estimate a ridge width for each multiplet, that is then corrected using the ridge modeling to estimate a mode width for that multiplet.

The correction scheme we developed to estimate mode widths uses a Gaussian widening model:

\[
\tilde{\Gamma}_{n,\ell}^2 = (\Gamma_{n,\ell}^E)^2 + \mathcal{W}_{n,\ell}^2
\]  

(3)
where \((\Gamma_{E_n}^{\ell})^2 = \Gamma_{n,\ell}^2 + W_{N,T}^2\) and \(W_{N,T}\) is the spectral resolution of the \(N\)th order sine multitaper \((N = 61)\).

Figure 4 illustrates how the ridge amplitude is also a function of \(m\). It shows also that the model ridge amplitude does not match the observations very closely. We suspect that this mismatch results from our poor knowledge of the point-spread function (PSF) of the MDI instrument.

5. Conclusions
We have derived the most accurate high-\(\ell\) mode characteristics to date, using *Dynamics 2002* MDI observations. The limiting factor remains our poor knowledge of MDI’s PSF. Since we have co-eval HMI & MDI data, we may be able to better characterize MDI’s PSF. We anticipate that the method will be extended to more MDI *Dynamics* segments and ported to HMI and GONG observations.

References
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Acknowledgments
The Solar Oscillations Investigation - Michelson Doppler Imager project on SOHO is supported by NASA grant NAG5–8878 and NAG5–10483 at Stanford University. SOHO is a project of international cooperation between ESA and NASA. SGK was supported by Stanford contract PR–6333 and NASA grants NAG5–9819 and NNG05GD58G.
Figure 4. Comparison of ridge amplitude, as a function of the ratio \( m/\ell \). The dots correspond to individual ridge power amplitude measurements, and the black curves the corresponding binned values. The colored curves correspond to different models of the ridge amplitude, resulting from different models computed for different leakage matrices.