Allowed Gamow-Teller Excitations from the Ground State of $^{14}N$

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Abstract

Motivated by the proposed experiment $^{14}N(d, ^2He)^{14}C$, we study the final states which can be reached via the allowed Gamow-Teller mechanism. Much emphasis has been given in the past to the fact that the transition matrix element from the $J^\pi = 1^+ \ T = 0$ ground state of $^{14}N$ to the $J^\pi = 0^+ \ T = 1$ ground state of $^{14}C$ is very close to zero, despite the fact that all the quantum numbers are right for an allowed transition. We discuss this problem, but, in particular, focus on the excitations to final states with angular momenta $1^+$ and $2^+$. We note that the summed strength to the $J^\pi = 2^+ \ T = 1$ states,
calculated with a wide variety of interactions, is significantly larger than that to the $J^\pi = 1^+ T = 1$ final states.

1. INTRODUCTION

Much attention has been given over the past several decades to the fact that the Gamow-Teller ($GT$) matrix element between the $J^\pi = 1^+ T = 0$ ground state of $^{14}N$ and the $J^\pi = 0^+ T = 1$ ground state of $^{14}C$ (or that of its mirror nucleus $^{14}O$) is very close to zero, despite the fact that all the quantum numbers are right for an allowed Gamow-Teller transition. Of particular interest is the early work of Inglis [1] who showed that in the simplest shell model space (2 holes in the $0p$ shell), it is not possible to get this $GT$ matrix element to vanish if the residual nucleon-nucleon ($NN$) interaction consists of only a central part and a spin-orbit part. Inglis then commented upon the possibility that $A(GT)$ might vanish with only these two interactions if higher shells were included. He himself did not carry out such a calculation, but an attempt to do so was made a few years later by Baranger and Meshkov [2]. They concluded that it was possible that configuration mixing was the sole agent to cause $A(GT)$ to vanish; however, they had to speculate on the signs of certain matrix elements.

Following Inglis’ work, Jancovici and Talmi [3] showed that if one also had a tensor component present in the interaction one could get the $GT$ matrix element to vanish. Thus, the $A = 14$ system affords us one of the few instances where one can study the elusive effects of the tensor interaction in nuclear structure [4,5]. Visscher and Ferrel [6] plotted the strength parameters of the spin-orbit and tensor interactions for which one could get the $GT$ matrix element to vanish. They noted that if the spin-orbit interaction is too weak then they cannot get the $GT$ matrix element to vanish for any value of the tensor interaction strength parameter.

Zamick showed, in Ref. [7], that the $GT$ matrix element comes out too large when one uses the non-relativistic $G$—matrix elements which Kuo [8] obtained from the Hamada-
Johnston interaction \cite{9}. However, if the spin-orbit interaction was increased by about 50%, he could get this matrix element to vanish.

Whereas most early calculations were carried out in the model space of two holes in the $0p$ shell, more recently, Fayache, Zamick and Müther have reconsidered this issue by performing no-core shell model calculations ($NCSM$) in the larger model space ($[(0p)^{-2}] + 2\hbar\omega$) \cite{10}. First they used an interaction previously constructed by Zamick and Zheng \cite{11}:

$$V_{zz} = V_c + xV_{so} + yV_t,$$

where $c=$central, $so=$two-body spin-orbit, and $t=$tensor. For $x=y=1$, the matrix elements of $V_{zz}$ are in approximate agreement with those of the non-relativistic OBE potential Bonn A of \cite{12}. They then studied the effects of the spin-orbit and tensor components of the $NN$ interaction on the $GT$ matrix element by varying the strength parameters $x$ and $y$. They found the interesting result that in the small model space (2 holes in the $0p$ shell) they could (for the standard value $x = 1$ of the spin-orbit interaction strength parameter) find a value of $y$ for which the $GT$ matrix element vanishes. However, in a larger model space which also included $2\hbar\omega$ excitations and still using the standard value $x = 1$, they could not get the $GT$ matrix element to vanish for any value of the tensor interaction strength parameter $y$. Thus they reached the opposite conclusion to that of Baranger and Meshkov. However, if the spin-orbit interaction was enhanced by 50% (to $x = 1.5$), then they could find a reasonable value of $y$ for which the $GT$ matrix element vanishes (but not for $y = 0$). Furthermore, using a relativistic Bonn A $G$–matrix with a Dirac effective mass $m_D = 0.6m$ ($m$ being the mass of the free nucleon), they found that they can make the $GT$ matrix element vanish in both the small and large model spaces. It is known that the spin-orbit interaction gets enhanced by a factor $m/m_D$ in relativistic calculations \cite{13}.

In the above discussion, we have focused on the ground-state-to-ground-state transition $^{14}N$ ($J^\pi = 1^+\, T = 0$) $\rightarrow$ $^{14}C$ ($J^\pi = 0^+_1\, T = 1$). But in a proposed experiment ($^{14}N(d, 2He)^{14}C$) \cite{14}, one can reach excited $T = 1$ states as well with spins $J^\pi = 0^+, 1^+$ and $2^+$ via the allowed $GT$ mechanism. In section II, we shall present the results of theoretical
calculations of the $GT$ reduced transition probability

$$B(GT) = \left( \frac{g_A}{g_V} \right)^2 \frac{1}{2J_i + 1} |A(GT)|^2,$$

(2)
as well as the summed strengths $\sum B(GT)$ to these states. In Eq. 2, $\frac{g_A}{g_V} = 1.251$ is the ratio of the Gamow-Teller to Fermi coupling constants introduced here for convenience \[15\]. The $GT$ matrix element itself, denoted as $A(GT)$, is given by the expression

$$|A(GT)|^2 = \sum_{M_i, M_f, \mu} \langle \psi_{J_f, M_f, T_f}^{T_f} | \sum_{k=1}^{A} \sigma_{\mu}(k) t_+(k) | \psi_{J_i, M_i, T_i}^{T_i} \rangle^2.$$  

(3)

We perform our calculations with a variety of realistic interactions in both the small and large model spaces. In light of the fact that in Ref. \[10\] there were such drastic differences between the results obtained in the small and large model spaces for the $GT$ transition from $J^\pi = 1^+_1$ to $J^\pi = 0^+_1$, we should also investigate the effects of going from small to large model spaces for the other allowed transitions, namely from $J^\pi = 1^+_1$ to $J^\pi = 1^+$ and to $J^\pi = 2^+$. This is one of the main points of the present work. Furthermore, we will do these calculations using both a phenomenological approach as in \[10\] and a purely theoretical one in which a modern realistic $N - N$ effective interaction is used in larger and larger model spaces.

The results of our calculations are presented in section II. Section III deals with the interpretation of the results, followed by concluding remarks.

II. RESULTS OF THE CALCULATIONS

We first show in Table I results of calculations of the $GT$ ground-state-to-ground-state transition strength $B(GT):$ $^{14}N \ (J^\pi = 1^+_1, \ T = 0) \rightarrow ^{14}C \ (J^\pi = 0^+_{1}, \ T = 1)$, followed by the summed strengths of the $GT$ transition from the ground state of $^{14}N$ to the $J^\pi = 0^+, \ 1^+$ and $2^+ (T = 1)$ states of $^{14}C$, using the interaction $V_{zz}$ (Eq. 1) of Zamick and Zheng \[11\]. Note that we never introduce a single-particle spin-orbit term, since in our case the average one-body spin-orbit interaction is implicitly generated by our two-body spin-orbit interaction.
in our no-core shell-model (NCSM) calculations, which we performed using the nuclear shell model code OXBASH.

First, let us compare the small- and large-space results for $J^\pi = 0^+$ final states obtained with the standard two-body spin-orbit strength $x = 1$. As we vary the strength parameter $y$ of the tensor interaction in the small model space calculation, we see that for $y = 0.5$ $B(GT)$ becomes vanishingly small. Indeed, for $y = 1.0$, $x = 1$ $A(GT)$ has an opposite sign to that for $y = 0$, $x = 1$. This verifies the contention of Jancovici and Talmi that one can get $A(GT)$ to vanish with a suitable tensor interaction.

However, when we go to the large $(0+2)\hbar\omega$ model space, we see that $B(GT)$ for $J^\pi = 0_1^+$ does not go to zero for any value of $y$ when $x = 1$, and we no longer get the Jancovici-Talmi behaviour.

The situation is restored if a combination of a weaker strength of the tensor interaction and an enhanced strength of the spin-orbit interaction (i.e. $x = 1.5$ and $y = 0.75$) is applied. In that case we get $B(GT)$ to vanish in both the small and large model spaces.

We next come to one of the main points of the paper: a comparison of the $GT$ summed strengths to the $J^\pi = 1^+$ and $J^\pi = 2^+$ ($T = 1$) final states in $^{14}C$. We see consistently that the excitation strengths to the $J^\pi = 2^+$ states are much larger than to the $J^\pi = 1^+$ states.

For example, in the large space with $x = 1.5$, $y = 0.75$, the values of the summed strength to the $1^+$ states is only 0.193, but to the $2^+$ states it is 3.113. We will discuss this further in the next section.

Table II presents results of calculations done with the relativistic Bonn A interaction of Mütter et. al. In this approach, one has a Dirac effective mass $m_D$ such that $m_D/m$ is typically less than one with $m_D/m = 1$ corresponding to the non-relativistic limit. In our case, the value of $m_D/m = 0.6$ seems to work best in as far as achieving a vanishing $GT$ transition between the ground states of the $A = 14$ system. This is true in both the small and the large model spaces.

In Table III, we present the results of calculations done with the Argonne V8’ effective interaction in four model spaces: $0\hbar\omega$, $(0+2)\hbar\omega$, $(0+2+4)\hbar\omega$ and $(0+2+4+6)\hbar\omega$, all
performed with the Many-Fermion Dynamics code of [17]. For this set of calculations, we followed the procedure described in Refs [18,19] in order to construct the two-body effective interaction. Note that in Tables I-III we give the ground-state-to-ground-state transition $B(\text{GT}) : ^{14}N \ (J^\pi = 1^+, \ T = 0) \to ^{14}C \ (J^\pi = 0^+, \ T = 1)$, as well as the summed strength $\sum B(\text{GT})$, i.e. summing the $B(\text{GT})$ values starting from the $^{14}N \ (J^\pi = 1^+, \ T = 0)$ ground state to all final $0^+$, $1^+$ and $2^+ \ T = 1$ states in $^{14}C$. In the smallest model space, the Argonne V8’ interaction gives a poor result for the ground-state-to-ground-state $B(\text{GT})$, a value of 2.518 which is far from the desired result of zero. When the model space is enlarged to $(0+2)h\omega$, the $B(\text{GT})$ to the $0^+_1$ state obtained with the Argonne V8’ interaction goes down to 1.403, then it goes further down to 0.430 in the larger model space $(0+2+4)h\omega$, and in the yet larger model space $(0+2+4+6)h\omega$ it goes way down to 0.164. This shows that the results are quite sensitive to the model space used, but overall they are rather encouraging in the sense that one seems to be converging to the desired result that the ground-state-to-ground-state $B(\text{GT})$ vanishes in the limit that the model space becomes sufficiently large. Indeed, it is clear that the many-body correlations in the large model spaces are causing the decrease in the transitions to the $0^+$ states and their increase for $2^+$ states.

III. INTERPRETATION OF THE RESULTS

A. The L – S picture

We can make sense of the results obtained in the small model space $0 \ h\omega$ by following the approach of Zheng and Zamick [11] and use an LS representation ($^{2S+1}L_J$) for the two-hole $A = 14$ system. For instance, the ground state $(J^\pi = 1^+, \ T = 0)$ wavefunction of $^{14}N$ (i.e. the initial state) is represented as follows:

$$\psi_i = C_i^S \ |^3S_1\rangle + C_i^P \ |^1P_1\rangle + C_i^D \ |^3D_1\rangle ,$$

(4)

whereas for final $J^\pi = 0^+, \ T = 1$ states the wavefunctions are of the form

$$\psi_f = C_f^S \ |^1S_0\rangle + C_f^P \ |^3P_0\rangle .$$

(5)
The expression for the transition amplitude $A(GT)$ (see Eq. 3) is then

$$A(GT) = \sqrt{6}[C_f^S C_i^S - C_f^P C_i^P / \sqrt{3}]. \quad (6)$$

It should be noted that if the $^{14}\text{N}$ ground-state wavefunction had a pure $^3D_1$ configuration then the Gamow-Teller transition amplitude $A(GT)$ to $J^\pi = 0^+$ and $1^+$ states would vanish. The reason for this, of course, is that the $GT$ operator $\sum_k \sigma_\mu(k)t_+(k)$ cannot change the orbital quantum number $L$. But from the above expression for $A(GT)$, it is not a necessary condition to have $C_i^D = 1$ and $C_i^S = C_i^P = 0$ in order for $A(GT)$ to vanish. Interference from the $L = 1$ contributions can and does make $A(GT)$ vanish before $C_i^D = 1$. Nevertheless, $C_i^D$ is very close to one at the point where $A(GT)$ vanishes.

In Table IV, we present the values of the coefficients $C_i^S$, $C_i^P$ and $C_i^D$ related to the $0\hbar\omega$ model space calculations done with various interactions considered earlier in Tables I, II and III, as well as the corresponding $A(GT)$. By comparing the values of the $LS$ coefficients shown in the upper half of this table to those in the lower one, it becomes clear that the argument presented above, about the crucial role of the $^3D_1$ component in the $J^\pi = 1^+ T = 0$ $^{14}\text{N}$ ground-state wavefunction in insuring the vanishing of $A(GT)$, holds for the other interactions as well.

We can also see why the $2^+$ final states are more strongly excited than the $J^\pi = 1^+$ final states. In the two-hole model space, and by virtue of the generalized Pauli exclusion principle, there is only one $J^\pi = 1^+$, $T = 1$ final state, corresponding to $L = 1$ $S = 1$ $T = 1$ and denoted by $^3P_1$. It can be excited by the $GT$ mechanism only via the $^1P_1$ component of the $^{14}\text{N}$ $J^\pi = 1^+ T = 0$ ground-state wavefunction. We see from Table IV that the $^1P_1$ component $C_i^P$ is rather small when $A(GT)$ vanishes. It is possible, however, to form two $J^\pi = 2^+ T = 1$ states in the two-hole model space (corresponding to $L = 2$ $S = 0$ and $L = 1$ $S = 1$), and the first one of these two configurations ($^1D_2$) will carry most of the strength of the $GT$ excitation emanating from the dominantly $^3D_1$ $^{14}\text{N}$ ground-state wavefunction ($C_i^D \geq 0.96$ when $A(GT)$ vanishes).
B. Renormalization of the spin-orbit interaction

As mentioned in the introduction, it has been noted in the past [6,7,10,11] that the vanishing of the ground-state-to-ground state $GT$ matrix element in the $A = 14$ system as calculated in the valence space (i.e. $0\hbar\omega$ model space) requires either an enhancement of the two-body spin-orbit interaction and/or a weakening of the tensor interaction -see the lower half of table IV. In a different but somewhat related context, Fayache et. al. had come to a similar conclusion in their study of $M1$ excitation rates in the $0p$ and $1d - 0d$ shells [20]. It is useful to note here, as pointed out by Wong [21], that the tensor interaction in an open-shell acts to some extent like a spin-orbit interaction of the opposite sign of the basic spin-orbit interaction, so that these two types of adjustments to the $N - N$ interaction are really equivalent for our purpose.

In the present work, we have shown in table III that, using a modern realistic effective $N - N$ interaction and performing no-core shell-model calculations with it in progressively larger and larger model spaces, we were able to obtain the desired vanishing of the ground-state-to-ground-state $GT$ matrix element in a natural way, i.e. without having to adjust any parameters. This suggests that some renormalization of the effective spin-orbit interaction coupling strength (in the sense of an enhancement of the latter relative to its strength in the $0\hbar\omega$ model space) must be taking place as one works in larger and larger model spaces.

In Table V, we present results of calculations that further corroborate the interpretation just given. Loosely speaking, the $J^\pi = 2_1^+$ state is mainly a $(p_{1/2}^{-1})(p_{3/2}^{-1})$ two-hole state, so that its excitation energy scales mainly as the spin-orbit splitting $E(3/2^-) - E(1/2^-)$ in the $A = 15$ system. Evidently, the latter can be thought of as a measure of the strength of the effective two-body spin-orbit interaction as calculated in a given model space. A striking systematics then emerges when one combines the results of tables III and V. Clearly, there is a one-to-one correlation between the re-distribution of the $GT$ strength in the $A = 14$ system (table III) and the effective spin-orbit strength as the size of the shell-model space is varied (table V). Indeed, there is a clear trend taking place in the sense that, as the size of
the shell-model space gets larger, the calculated excitation energy $E_x(2^+_1)$ in $^{14}C$ as well as the calculated energy splitting $E(3/2^-) - E(1/2^-)$ in $^{15}N$ are increasing, and all the while a re-distribution of the $A = 14$ GT strength is taking place, with all the combined results becoming in better agreement with experiment.

IV. CONCLUDING REMARKS

We have performed theoretical calculations of the allowed Gamow-Teller transitions from the ground state of $^{14}N$ to the lowest lying states in $^{14}C$ in anticipation of a proposed experiment involving the reaction $^{14}N(d,^2He)^{14}C$. We discussed the problem of the near vanishing of the GT transition to the $J^\pi = 0^+_1$, $T = 1$ ground state of $^{14}C$, but principally focused on the transitions to the final states with angular momenta $1^+$ and $2^+$.

In calculations limited to a $0\,\hbar\omega$ model space, it is necessary to effect a phenomenological enhancement of the $N-N$ two-body spin-orbit interaction in order to obtain a vanishing ground-state-to-ground-state GT matrix element in the $A = 14$ system. We have found that this in turn results in the GT strength going overwhelmingly to the lowest $2^+$ state. Such a result can be easily accounted for by the fact that the $^{14}N\,J^\pi = 1^+_1\,T = 0$ ground state wavefunction is predominantly composed of an $LS$ component $^3D_1$.

Using an effective interaction theoretically derived from the realistic $N-N$ Argonne V8’ interaction, and performing shell-model calculations in progressively larger and larger model spaces (with up to 6 $\hbar\omega$ excitations), we were able to achieve a similar degree of success in agreement with experiment as that obtained phenomenologically earlier in the $0\hbar\omega$ model space calculations, but this time without any adjustments of parameters. We have interpreted this as an indication of a natural renormalization of the effective two-body spin-orbit interaction affected by the many-body correlations taking place in the larger model spaces.

In concluding, we note that, when only the charge-symmetry-conserving strong interactions are taken into account (as we did in this paper), the matrix element for the transition
from $^{14}\text{N}$ to $^{14}\text{C}$ is the same as that from $^{14}\text{N}$ to $^{14}\text{O}$. However, since the transitions to the $J = 0^+$ states are strongly suppressed, large charge-symmetry effects can be induced by the Coulomb interaction. Indeed the $ft$ values in the decays of $^{14}\text{C}$ and $^{14}\text{O}$ to the ground state of $^{14}\text{N}$ are respectively $1.1 \times 10^9$ and $2 \times 10^7$, quite different values indeed. Talmi $^{[22]}$ was able to verify that indeed the Coulomb interaction could explain this difference to a large extent. It will be interesting in the near future to see the effects of other charge-symmetry-breaking interactions.

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TABLE I. Ground-state-to-ground-state Gamow-Teller strength $B(GT)$ (denoted by $0^+_1$ in columns three and seven) and summed strengths ($\sum B(GT)$) from the $J^\pi = 1^+_1 T = 0$ ground state of $^{14}N$ to the $J^\pi = 0^+, 1^+$ and $2^+$ states in $^{14}C$, using the $(x, y)$ interaction.

| $x$ | $y$ | $0^+_1$ | $0^+$ | $1^+$ | $2^+$ | $0^+_1$ | $0^+$ | $1^+$ | $2^+$ |
|-----|-----|--------|------|------|------|--------|------|------|------|
| 1   | 0   | 0.738  | 0.792| 0.280| 2.617| 1.235  | 1.345| 0.276| 2.030|
| 1   | 0.5 | 0.002  | 0.092| 0.148| 3.187| 1.682  | 2.158| 0.009| 0.933|
| 1   | 0.75| 0.668  | 1.047| 0.017| 2.100| 1.555  | 1.985| 0.005| 1.091|
| 1   | 1.0 | 0.991  | 1.427| 0.003| 1.707| 1.532  | 1.929| 0.004| 1.132|
| 1.5 | 0   | 0.359  | 0.364| 0.345| 3.111| 0.448  | 0.456| 0.351| 2.970|
| 1.5 | 0.5 | 0.161  | 0.169| 0.317| 3.278| 0.093  | 0.119| 0.287| 3.230|
| 1.5 | 0.75| 0.062  | 0.099| 0.281| 3.312| 0.003  | 0.126| 0.193| 3.113|
| 1.5 | 1.0 | 0.004  | 0.097| 0.230| 3.263| 0.165  | 0.422| 0.103| 2.709|
TABLE II. Same as Table I, but using M"uthers relativistic Bonn A interaction, characterized by the ratio of the nucleon’s Dirac mass $m_D$ to its free mass $m$.

| $m_D/m$ | $0\hbar\omega$ Model Space | $(0+2)\hbar\omega$ Model Space |
|---------|-----------------------------|---------------------------------|
|         | $0_1^+$ | $0^+$ | $1^+$ | $2^+$ | $0_1^+$ | $0^+$ | $1^+$ | $2^+$ |
| 1.0     | 1.765   | 2.144 | 0.004 | 0.990 | 0.779   | 1.064 | 0.013 | 2.001 |
| 0.75    | 0.051   | 0.194 | 0.124 | 3.061 | 0.043   | 0.168 | 0.119 | 2.986 |
| 0.60    | 0.033   | 0.085 | 0.255 | 3.300 | 0.001   | 0.078 | 0.180 | 3.115 |

TABLE III. Same as Table I, but using two-body effective interactions derived from the Argonne V8' NN potential without Coulomb. The HO frequency of $\hbar\omega = 14$ MeV was employed. The calculated $6\hbar\omega$ binding energies of $^{14}$N is 110.52 MeV. The binding energy is expected to decrease with a further enlargement of the basis size.

| Model Space | $0_1^+$ | $0^+$ | $1^+$ | $2^+$ |
|-------------|---------|-------|-------|-------|
| $0\hbar\omega$ | 2.518   | 2.905 | 0.0262| 0.2511|
| $(0+2)\hbar\omega$ | 1.403   | 1.839 | 0.0004| 1.230 |
| $(0+2+4)\hbar\omega$ | 0.430   | 0.799 | 0.0291| 2.264 |
| $(0+2+4+6)\hbar\omega$ | 0.164   | 0.480 | 0.318 | 3.081 |
TABLE IV. The LS-representation coefficients of the $J^\pi = 1^+ T = 0$ ground state wavefunction of $^{14}N$ (Eq. 4), and the Gamow-Teller amplitude $A(GT)$ to the $J^\pi = 0^+ T = 1$ ground state of $^{14}C$ (Eq. 6) for various interactions considered in the previous tables. For the AV8' interaction, the $0h\omega$ basis results are shown.

| Interaction            | $C_i^S$ | $C_i^P$ | $C_i^D$ | $A(GT)$ |
|------------------------|---------|---------|---------|---------|
| $x=1.0, y=1.0$         | 0.675   | 0.032   | 0.737   | 1.378   |
| Bonn A ($m_D/m = 1$)   | 0.827   | -0.037  | 0.561   | 1.839   |
| Argonne V8'            | 0.963   | -0.093  | 0.255   | 2.197   |
| $x=1.0, y=0.49$        | 0.086   | 0.224   | 0.971   | 0.000   |
| $x=1.44, y=1.0$        | 0.116   | 0.256   | 0.960   | 0.000   |
| Bonn A ($m_D/m = 0.6$) | 0.057   | 0.364   | 0.930   | 0.250   |
TABLE V. Calculated excitation energy of the $2^+_1$ state in the $A = 14$ system and calculated spin-orbit splitting in the $A = 15$ system (in MeV) in various model spaces, using two-body effective interactions derived from the Argonne V8’ NN potential without Coulomb. The HO frequency of $\hbar\omega = 14$ MeV was used. The calculated $6\hbar\omega$ binding energies are 108.65 MeV and 126.73 MeV for $^{14}\text{C}$ and $^{15}\text{N}$, respectively. The binding energy is expected to decrease with a further enlargement of the basis size. The experimental values of $E_x(2^+_1)$ in $^{14}\text{O}$ and of $E(3/2^-) - E(1/2^-)$ in $^{15}\text{O}$ given under Expt are from Nuclear Data Retrieval (http://www.nndc.bnl.gov).

| Model Space       | $A = 14$ $E_x(2^+_1)$ | Expt | $A = 15$ $E(3/2^-) - E(1/2^-)$ | Expt |
|-------------------|------------------------|------|---------------------------------|------|
| $0\ h\omega$     | 3.152                  | 6.59 | 3.314                           | 6.176 |
| $(0+2)\hbar\omega$ | 4.854                  | "   | 5.366                           | "   |
| $(0+2+4)\hbar\omega$ | 5.564                  | "   | 6.326                           | "   |
| $(0+2+4+6)\hbar\omega$ | 5.874                  | "   | 6.731                           | "   |