Basics of automation in the formation of matrices of the numerical-analytical method of the boundary elements in calculation of spatial frame structures

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Abstract. The basic principles of the methodology for automating the formation of matrices of the numerical-analytical method of boundary elements (BEM) for the calculation of spatial frame structures are presented.

1. Research relevance

Extensive use of spatial frame structures necessitates calculations of their strength and stability in order to make a reasonable choice of parameters on which the magnitudes of stresses and deformations depend. Existing calculation methods are associated with a complex mathematical apparatus, as well as with a number of difficulties and conventions for modeling the individual elements of the bar structures.

The practice of engineering design requires reliable and alternative methods that allow determining the values of calculation forces and displacements that reflect the real work of structures.

To formulate the basic principles of the automation of the formation of matrices of the NA MBE in the calculation of spatial elastic frame structures.

2. Main part

Let us review the principle of the automation of the formation of matrices of the NA MBE using the example of a spatial frame structure shown in Figure 1.

![Figure 1. Numbering of elements of a spatial frame structure.](image)

The number of elements in spatial frame structures can be determined by the formula:
\[ K = (N + 1)(M + 1)P + (M + 1)NP + (N + 1)MP, \]  
(1)

where

\( K \) – number of elements;
\( N \) – number of steps;
\( M \) – number of stairs flights;
\( P \) – number of floors;
\((N+1)(M+1)P\) – number of racks;
\((M+1)NP\) – number of longitudinal crossbars;
\((N+1)MP\) – number of transverse crossbars.

The number of elements of the spatial frame structure shown in Fig. 1, which has two floors, two flights and two steps, according to formula (1) was 42.

As a result of the research carried out for spatial frame structures, general patterns of the formation of the vector for unknown boundary parameters \( X^* \) and the matrix of coefficients \( A^* \) were determined.

The system of equations describing the stress-strain state of an individual element of the spatial frame structure shown in Fig. 1, in matrix form (2) looks as follows:

\[ Y(l_i) = A(l_i)X(0) + B(l_i). \]  
(2)

The vectors \( Y(l) \), \( X(0) \), \( B(l) \) included in it, before the transformations look as follows:

\[
\begin{bmatrix}
Y_{10} \\
Y_{20} \\
Y_{30} \\
Y_{40} \\
Y_{50} \\
Y_{60} \\
Y_{70} \\
Y_{80}
\end{bmatrix}
= 
\begin{bmatrix}
X_{01} \\
X_{02} \\
X_{03} \\
X_{04} \\
X_{05} \\
X_{06} \\
X_{07} \\
X_{08}
\end{bmatrix}
+ 
\begin{bmatrix}
B_{00} \\
B_{01} \\
B_{02} \\
B_{03} \\
B_{04} \\
B_{05} \\
B_{06} \\
B_{07}
\end{bmatrix},
\]  
(3)

where

\[ Y_n = \begin{bmatrix}
EI_{r_s}(l_i) \\
EI_{s_p}(l_i) \\
M_{r}(l_i) \\
Q_{r}(l_i) \\
EI_{r_x}(l_i) \\
EI_{s_p}(l_i) \\
M_{s}(l_i) \\
Q_{s}(l_i)
\end{bmatrix}, \quad X_n = \begin{bmatrix}
EI_{r_s}(0) \\
EI_{s_p}(0) \\
M_{r}(0) \\
Q_{r}(0) \\
EI_{r_x}(0) \\
EI_{s_p}(0) \\
M_{s}(0) \\
Q_{s}(0)
\end{bmatrix}, \quad B_n = \begin{bmatrix}
B_{00} \\
B_{01} \\
B_{02} \\
B_{03} \\
B_{04} \\
B_{05} \\
B_{06} \\
B_{07}
\end{bmatrix}. \]  
(4)

The system of equations describing the stress-strain state of an individual element of the spatial frame structure shown in Fig. 1, in matrix form (2) as a result of a chain of equivalent transformations looks as follows:

\[ A^* X^* = -B. \]  
(5)

As a result of studying the topology of the vector of unknown boundary parameters of the resolving equation of the NA MBE, for spatial frame structures having \( P \) floors, \( M \) flights, \( N \) steps and consisting of \( K \) elements, all patterns of the location of the unknown parameters were identified.
As a result of studying the topology of the coefficient matrix \( A^* \) of the resolving equation of the NA MBE, for spatial frame structures having \( P \) floors, \( M \) flights, \( N \) steps and consisting of \( K \) elements, below the methodology for its formation is suggested.

Transformed matrix is proposed to be created by summing the matrix \( A^*_0 \) with the matrix \( Z^* \).

\[
A^* = A^*_0 + Z^*,
\]

(6)

where \( A^*_0 \) - a matrix of coefficients, consisting of compensating elements that appear in connection with the participation of dependent final parameters in the equations of equilibrium and compatibility of displacements;

\( Z^* \) - a matrix of coefficients, consisting of compensating elements that appear in connection with the participation of independent finite parameters in the equations of equilibrium and compatibility of displacements.

Decomposition of the coefficient matrix \( A^* \) into two components is adopted in connection with the possibility of the matrix blocks \( Z^* \) coinciding with the matrix blocks \( A^*_0 \), lying on the main diagonal.

The general order of the matrix for spatial frame structures can be determined by the formula:

\[
P_m = \left( (N + 1)(M + 1)P + (N + 1)NP + (N + 1)MP \right) \times 12.
\]

(7)

According to formula (7), the order of the matrix for a spatial frame structure with two floors, two flights and two steps was 504x504.

The coefficients matrix \( A^*_0 \) for a spatial frame structure, formed as a result of a chain of equivalent transformations and automated in the MatLab computer mathematics system, is shown in Figure 2.

![Figure 2](image2.png)

**Figure 2.** The coefficients matrix \( A^*_0 \) for a spatial frame structure, formed in the computer mathematics system MatLab.

The coefficients matrix \( Z^* \) for a spatial frame structure, formed as a result of a chain of equivalent transformations and automated in the MatLab computer mathematics system, is shown in Figure 3.
Figure 3. The coefficients matrix $Z'$ for a spatial frame structure, formed in the computer mathematics system MatLab computer mathematics system.

The structure of the $A'$ matrix formed in the MatLab computer mathematics system is shown in Figure 4.

Figure 4. The structure of the $A'$ matrix formed in the system of computer mathematics MatLab.
3. Conclusions
As a result of the research and analysis of its results, the general regularities of the formation of the vector of unknown parameters $\mathbf{X^*}$ and the coefficients matrix $\mathbf{A^*}$ for the NA MBE were revealed when calculating spatial frame structures.

The order of formation of matrices included in the resolving equation of the NA MBE for the simplest spatial frame structure is formulated. A technique is proposed for automating the formation of matrices of the NA MBE.

An algorithm has been compiled and a program has been implemented that allows calculating spatial frame structures in the elastic stage of work, in the MatLab computer mathematics system.

References
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