Magnetotransport in Aharonov Bohm interferometers: Exact numerical simulations

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The linear conductance of a two-terminal Aharonov-Bohm interferometer is an even function of the applied magnetic flux, as dictated by the Onsager-Casimir symmetry. Away from linear response this symmetry may be broken when many-body interactions are in effect. Using a numerically-exact simulation tool, we study the dynamics and the steady-state behavior of the out-of-equilibrium double-dot Aharonov Bohm interferometer, while considering different types of interactions: Model I includes a closed interferometer with an inter-dot electron-electron repulsion energy. In model II the interferometer is interacting with a dissipative environment, possibly driven away from equilibrium. In both cases we show that depending on the (horizontal, vertical) mirror symmetries of the setup, nonlinear transport coefficients obey certain magnetosymmetries. We compare numerically exact simulations to phenomenological approaches: The behavior of model I is compared to self-consistent mean-field calculations. Model II, allowing heat dissipation to a thermal bath, is mimicked by an Aharonov Bohm junction with a voltage probe. In both cases we find that phenomenological treatments capture the relevant transport symmetries, yet significant deviations in magnitude may show up.

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I. INTRODUCTION

Microreversibility dictates linear response properties such as the Onsager-Casimir symmetry relations. Particularly, in a two-terminal conductor the linear conductance should be an even function of the magnetic field $B$. In Aharonov-Bohm (AB) interferometers, this symmetry is displayed by the “phase rigidity” of the conductance oscillations with $B^2$. Microreversibility is broken beyond linear response, thus magnetoasymmetries should develop at finite bias, as demonstrated in several experiments. What is then remarkable is not the failure of the Onsager symmetry away from equilibrium, rather the development of more general symmetries between nonlinear transport coefficients and high order cumulants.

Several studies explored magnetotransport in AB interferometers beyond the noninteracting limit, coupling electrons to either internal or external degrees of freedom. For example, the problem has been explored by implementing mean-field arguments within scattering theories, focusing on effective quantities such as the screening potential developing in the interferometer in response to an external bias. At this level, one can show that magnetoasymmetries develop since internal potentials (the result of many-body interactions) are asymmetric in the magnetic field away from equilibrium. In the complementary (phenomenological) Büttiker’s probes approach, elastic and inelastic scattering effects are introduced via probes whose parameters reflect the response of the conductor to the applied magnetic field and the voltage bias. Beyond phenomenological treatments, magnetotransport characteristics were investigated using microscopic models in the coulomb blockade limit and in the Kondo regime.

In this work we study characteristics of nonlinear transport in AB interferometers by means of an exact numerical technique. Our setup includes an AB interferometer with two quantum dots, one at each arm, and we introduce different types of many-body effects within the system: Model I includes an inter-dot Coulombic repulsion term, see Fig. 1. In model II a secondary fermionic environment interacts capacitively with one of the quantum dots. This environment can serve as a “charge sensor” or a “quantum point contact”, see Fig. 2. We simulate the dynamics and the steady-state properties of these (many-body out-of-equilibrium) setups by adapting an iterative influence functional path integral technique (INFPI), developed in Ref. to treat the dynamics of the single impurity Anderson dot model.

Our work includes the following contributions: (i) We study symmetries of magnetotransport far-from-equilibrium in the transient domain and in the steady-state limit including genuine many-body interactions, rather than using phenomenological (screening, probe) models. (ii) Magnetotransport characteristics were explored in the literature within different models, e.g., considering an interferometer made with one or two quantum dots, with or without thermal dissipation effects. Here, we study transport symmetries in different models using the same computational tool, allowing for a direct comparison. (iii) We compare exact simulations to phenomenological treatments, for clarifying the validity and accuracy of approximate techniques in magnetotransport calculations. Particularly, in model II our simulations reveal functionalities beyond the mean field level: diode (dc-rectification) effect at zero flux under spatial asymmetry and a finite coulomb drag current, driven by the fermionic environment.

The paper is organized as follows. In Sec. II we introduce two models of an interacting double-dot interferometer, and the principles of the numerical techniques...
adopted in this work. Results are presented in Sec. III. Sec. IV concludes. For simplicity, we set $e=1$, $\hbar=1$, and $k_B=1$.

![Figure 1](image1.png)

**FIG. 1:** Scheme of model I, a two-terminal double-dot Aharonov-Bohm interferometer with spinless electrons in two quantum dots, 1 and 2, with an inter-dot repulsion of strength $U$.

**II. MODELS AND TECHNIQUES**

**A. Double-dot AB interferometer**

We begin with the noninteracting Hamiltonian, common to the two models. It includes a two-terminal ($\nu=L,R$) AB interferometer with two dots, $n=1,2$, one at each arm. For simplicity, we ignore the spin degree of freedom (absorbing Zeeman splitting into the definition of the energies), and take into account only one electronic level in each dot. The Hamiltonian includes the following terms:

$$H_{AB} = \sum_{n=1,2} \epsilon_n a_n^\dagger a_n + \sum_{l\in L} \epsilon_l a_l^\dagger a_l + \sum_{r\in R} \epsilon_r a_r^\dagger a_r + \sum_{n=1,2} v_{n,l} a_n^\dagger a_l e^{i\phi_l^n} + \sum_{n=1,2} v_{n,r} a_n^\dagger a_r e^{i\phi_r^n} + \text{h.c.}$$

(1)

Here $a_k^\dagger$ ($a_k$) are fermionic creation (annihilation) operators of electrons with momentum $k$ and energy $\epsilon_k$ in the $k\in \nu$ metal, $a_l^\dagger$ and $a_l$ are the respective operators for electrons on the dots, $\epsilon_n$ denotes the energy of spin-degenerate levels. The parameter $v_{n,l}$ stands for the coupling strength of dot $n$ to the $l$ state of the $L$ metal. A similar definition holds for $v_{n,r}$. These coupling terms are absorbed into the definition of the hybridization energy

$$\gamma_{\nu,n}(\epsilon) = 2\pi \sum_{j\in \nu} |v_{n,j}|^2 \delta(\epsilon - \epsilon_j).$$

(2)

In our simulations we set $\gamma_{\nu,n}$, use a constant density of states for the metals, up to a sharp cutoff $\pm D$, then construct the real-valued tunneling elements $v_{n,j}$ by using Eq. (2). The AB phase factors $\phi_l^n$ and $\phi_r^n$ are acquired by electrons in a magnetic field perpendicular to the device plane. These phases are constrained to satisfy

$$\phi_1^L - \phi_2^L + \phi_1^R - \phi_2^R = \phi = 2\pi \Phi / \Phi_0,$$

(3)

Here $\Phi$ is the magnetic flux threading through the AB ring, $\phi = 2\pi \Phi / \Phi_0$ the magnetic phase, and $\Phi_0 = h/e$ the magnetic flux quantum. We voltage-bias the AB interferometer, $\Delta \mu \equiv \mu_L - \mu_R$, with $\mu_{L,R}$ as the chemical potential of the metals; we use the convention of a positive current flowing left-to-right. We bias the system in a symmetric manner, $\mu_L = -\mu_R$, but this choice does not limit the generality of our discussion since the dots may be gated away from the so called “symmetric point” at which $\mu_L - \epsilon_n = \epsilon_n - \mu_R$.

The Hamiltonian (1) does not include interactions and one can readily obtain the exact form of the (steady-state) charge current flowing between the terminals, an even function of the magnetic flux. Assuming for simplicity that the two quantum dots are evenly coupled to the $\nu$ terminal, $\gamma_{\nu} \equiv \gamma_{\nu,n}$, we find that

$$I(\phi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\epsilon \frac{4\gamma_{L} \gamma_{R}}{((\epsilon - \epsilon_d)^2 - \frac{\Delta^2}{4} - \gamma_{L} \gamma_{R} \sin^2 \frac{\Delta \phi}{2})^2 + (\gamma_{L} + \gamma_{R})^2(\epsilon - \epsilon_d)^2} \left[ f_L(\epsilon) - f_R(\epsilon) \right].$$

(5)
with $\epsilon_d = (\epsilon_1 + \epsilon_2)/2$ and $\Delta \epsilon = \epsilon_1 - \epsilon_2$. The current is an even function in the magnetic flux at finite bias, irrespective of spatial asymmetries ($\gamma_L \neq \gamma_R$ and $\Delta \epsilon \neq 0$). Using the probe technique, a phenomenological tool for implementing scattering effects, we had recently proved that this “phase locking” behavior is preserved under elastic dephasing. In contrast, inelastic scattering processes, taken into account with a voltage probe or a voltage-temperature probe, break the even symmetry in $\phi$ in the nonlinear transport regime. We now augment the Hamiltonian with genuine many-body interactions: In model I we add an inter-dot repulsion interaction between electrons, overall conserving energy and charge in the interferometer. In model II energy exchange with an additional environment is allowed.

1. Model I: Inter-dot Coulomb repulsion

We complement the Hamiltonian with a Coulomb repulsion term, nonzero when both quantum dots are occupied. The resulting inter-dot coulomb (C) model reads

$$H_C = H_{AB} + U n_1 n_2. \quad (6)$$

For a schematic representation see Fig. 1. Here $n_1 = a_1^\dagger a_1$ and $n_2 = a_2^\dagger a_2$ are the number operators for the dots. The behavior of the current and the occupation of the dots in this “interacting two-level quantum dot model” were investigated in different works: The case without the threading magnetic field was studied e.g. in Refs. $26-28$. The role of an external magnetic field was examined in different limits, particularly in the coulomb blockade regime. Recent studies further investigated transient effects, either analytically, disregarding interactions or numerically, considering relatively weak interactions.

In our simulations below we consider three geometries for Model I, see Fig. 1: (i) A setup with a mirror symmetry with respect to the horizontal axis, (ii) the case with a mirror symmetry along the vertical axis, and (iii) the model missing (horizontal and vertical) symmetries.

2. Model II: Coupling to a Fermionic environment

Dissipation effects can be included by capacitively coupling the interferometer to a fermionic environment (FE), set in equilibrium or out of equilibrium. For simplicity, we do not consider electron-electron interactions within the interferometer or within the FE. This dissipative (D) Hamiltonian includes the interferometer [Eq. 1], an additional FE, and the interaction energy between the units,

$$H_D = H_{AB} + H_F + V_D. \quad (7)$$

The FE is realized here by a tunneling junction

$$H_F = \epsilon_p c_p^\dagger c_p + \sum_{\alpha} \epsilon_{\alpha} c_{\alpha}^\dagger c_{\alpha} + \sum_{\alpha} g_{\alpha} c_{\alpha}^\dagger c_p + h.c. \quad (8)$$

It includes a quantum dot of energy $\epsilon_p$ coupled to two reservoirs ($\alpha = \pm$). The FE may be set at equilibrium when $\mu_+ = \mu_-$ (with the Fermi energy set at zero), or biased away from equilibrium, $\Delta \mu_F = \mu_+ - \mu_- \neq 0$. We distinguish between the AB interferometer and the FE by adopting the operators $c^\dagger$ and $c$ to denote creation and annihilation operators of electrons in the FE. We define the dot-reservoir hybridization energies in the FE by

$$\gamma_\alpha(\epsilon) = 2\pi \sum_{s \in \alpha} |g_s|^2 \delta(\epsilon - \epsilon_s). \quad (9)$$

Electrons in the AB interferometer and the FE are interacting (strength $U$) according to the form

$$V_D = Un_p n_1. \quad (10)$$

Here $n_p = c^\dagger p c_p$, $n_1 = a_1^\dagger a_1$ are number operators. Note that there is no leakage of electrons from the AB junction into the FE. However, this additional environment provides a mechanism for inducing elastic and inelastic scattering events of electrons on dot 1.

Model II has been examined in the literature in the context of charge sensing, and as a “which-path” detector, see for example Refs. $32-35$. The dephasing in an AB interferometer with a capacitively coupled charge sensor has been analyzed in Ref. $36$, using a second-order perturbation theory in $U$, yet limited to the linear conductance case. Here, using a numerical tool, we analyze the system away from equilibrium with the vertical mirror symmetry either preserved or violated, see Fig. 3.

B. Observables

The principal observable of interest in our work is the charge current in the interferometer. It is calculated from the current operator,

$$\dot{I}_L = - \frac{d N_L}{dt} = -i[H, N_L] = \sum_{l \in L} \left[ -i v_1 e^{i \phi_L} a_l^\dagger a_l + i v_{\perp} e^{-i \phi_L} a_l^\dagger a_1 \right]. \quad (11)$$

with $N_L = \sum_{l \in L} a_l^\dagger a_l$ as the number operator of the $L$ metal. The current at the $R$ contact, $\dot{I}_R$, can be defined in an analogous way. We identify the averaged current by $\bar{I} = \frac{1}{2}(\dot{I}_L - \dot{I}_R)$; we could simulate separately the currents at the $L$ and $R$ terminals, but we chose to directly compute the expectation value

$$I(t, \phi) = \text{tr}[\rho(0)e^{i H t} \hat{I} e^{-i H t}], \quad (12)$$

from a dot-metal-FE factorized initial state $\rho(0)$. Here $H$ denotes the total Hamiltonian of interest. Formally, the two-terminal current $I(t, \phi)$ can be expanded in powers of the applied voltage bias $\Delta \mu$,

$$I(t, \phi) = \sum_{k=1,2,\ldots} G_k(t, \phi)(\Delta \mu)^k. \quad (13)$$
We refer to $G_{k>1}$ as nonlinear conductance coefficients. The current can be separated into its odd and even terms in powers of the bias. Even terms represent the dc-rectification contribution,

$$R(t, \phi) = \frac{1}{2}[I(t, \phi) + I(t, \phi)] = G_2(t, \phi)(\Delta \mu)^2 + G_4(t, \phi)(\Delta \mu)^4 + \ldots \quad (14)$$

Here $I$ is the current obtained when interchanging the chemical potentials of the two terminals (assuming identical temperatures). The complementary odd terms are grouped into

$$D(t, \phi) = \frac{1}{2}[I(t, \phi) - I(t, \phi)] = G_1(t, \phi)\Delta \mu + G_3(t, \phi)(\Delta \mu)^3 + \ldots \quad (15)$$

Below we relax the time variable when addressing long-time quasi steady-state values. Other quantities of interest are the occupation of the dots in the interferometer and the behavior of the coherences, off-diagonal terms comprising the exactly solvable (noninteracting) terms. Many-body interactions are collected into $H_1$, written in the form

$$H_1 = U \left[ n_1 n_a - \frac{1}{2} (n_1 + n_a) \right]. \quad (17)$$

In model I, $n_a = n_2$; in model II it corresponds to the number operator of the impurity level $p$ within the FE, $n_a = n_p = c_p^\dagger c_p$, see Eq. (10). The two-body term $\frac{1}{2}U(n_1 + n_a)$ is absorbed into the definition of $H_0$. This structure allows for the elimination of $H_1$ via the Hubbard-Stratonovich (HS) transformation and the propagation of quadratic expectation values with an influence functional path integral technique. We now briefly review the principles of INFPI, to explain why this method is fitting for the study of magnetotransport in far-from-equilibrium situations. We discuss the numerical errors associated with INFPI simulations, and point out that these errors do not interfere with the resolution of transport symmetries.

C. Numerically Exact Treatment: INFPI

We simulate the dynamics of electrons in Model I and II using a numerically exact influence function path integral technique, referred to as INFPI. The principles of this method have been detailed in several recent publications, therefore we only highlight here the aspects of relevance to the present work.

Eqs. (6) and (7) can be each organized into the generic form $H = H_0 + H_1$, with $H_0$ comprising the exactly

\[RH\]
ter. We further assume that \( \rho_{AB}(0) = \sigma(0) \rho_L \otimes \rho_R \), with \( \sigma(0) \) as the reduced density matrix of the isolated dots in the interferometer. The FE is similarly prepared in a factorized state with \( \rho_f(0) = \rho_f(0) \otimes \rho_+ \otimes \rho_- \). The four reservoirs \( \xi = L, R, \pm \) are separately prepared in grand canonical states at a given chemical potential and temperature, \( \rho_k = e^{-\beta (H_k - \mu_k \mathcal{N}_k)}/\text{tr}[e^{-\beta (H_k - \mu_k \mathcal{N}_k)}] \); we prepare all reservoirs at the same temperature \( 1/\beta \).

Using this initial state in Eq. (12), we apply the Trotter decomposition and the HS transformation, the latter eliminates the many-body term \( \mathcal{H}_1 \) by introducing auxiliary Ising variables. The result is a formally exact path integral expression; the integrand is referred to as the “Influence Functional” (IF) involving nonlocal dynamical correlations, generally missing an analytical form.

The fundamental principle behind INFPI is the observation that at finite temperature and/or nonzero chemical potential difference bath correlations exponentially decay in time, thus the IF can be truncated beyond a memory time \( \tau_c \). This allows us to define an auxiliary operator on the time-window \( \tau_c \), which can be time-evolved iteratively from the initial condition to time \( t \). The truncated IF can be evaluated numerically by discretizing the fermionic reservoirs and tracing out the baths’ degrees of freedom using trace identities.

The INFPI method involves three numerical errors: (i) The finite discretization of the reservoirs, each comprising \( L_s \) single-electron states. (ii) The time step adopted in the Trotter breakup, \( \delta t \). In our simulations the trotter error grows as \((\delta t U)^2\). (iii) The error associated with the truncation of the IF, to include only a finite memory time \( \tau_c \). The exact limit is reached when \( L_s \rightarrow \infty \), \( U \delta t \rightarrow 0 \) and \( \tau_c \rightarrow t \). Convergence is tested by studying the sensitivity of simulations to the energy discretization of the reservoirs, the time step, and the memory time \( \tau_c = N_s/\delta t \), with \( N_s \) as an integer.

INFPI excellently fits for the simulation of magneto-transport in far-from-equilibrium situations: First, analytic considerations and numerical simulations suggest that the memory time characterizing the bath decorrelation process scales as \( \tau_c \sim 1/\Delta \mu^{24,35,39} \). Thus, the method should quickly converge to the exact limit at a large bias. Since we are specifically interested here in beyond-linear-response situations, INFPI is perfectly suited for the problem. Second, this is a deterministic time propagation scheme. Thus, it is an advantageous tool for testing magnetic field symmetries in nonlinear transport: Even if convergence is incomplete, \( I(t, \phi) \) and \( I(t, -\phi) \) deviate from the exact limit in the same (deterministic) form, conserving transport symmetries. In contrast, methods that rely on stochastic sampling of diagrams may accumulate distinct errors in the evaluation of the current at opposite phases, \( I(t, \pm \phi) \), thus one may need to approach the exact limit for validating transport symmetries. As we show below, at finite interactions the evolution of the current with time strongly depends on the magnetic phase, showing distinct relaxation times. Thus, it is important to adopt here techniques which accumulate identical errors for \( \pm \phi \). Finally, INFPI is a flexible tool and it can be easily adapted for the study of several related models, as long as the interacting contribution \( \mathcal{H}_1 \) follows Eq. (17). This allows us to analyze and compare the behavior of different many-body situations, e.g., with or without a dissipative bath.

### D. Phenomenological Approaches

Nonlinear transport characteristics in Model I and II can be explored based on perturbation theory expansions in \( U \), see for example Ref. In this approach we hybridize the quantum dot ‘1’ to a metal terminal (probe), then impose the condition of zero net charge current in this connection. Electrons can thus dephase and exchange energy in the probe, but (net) charge current only flows between the \( L \) and \( R \) terminals.

### III. RESULTS

In our simulations below we adopt the following parameters: \( \Delta \mu = 0.6 \), inverse temperature of the electronic reservoirs \( \beta = 50 \), \( U = 0.1 \), \( \gamma_{\nu,n} = 0.05 - 0.2 \), flat bands extending between \( D = \pm 1 \). INFPI numerical parameters are \( \delta t = 0.5 - 1.2 \), \( N_s = 3 - 6 \), and \( L_s = 120 \). Convergence was reached for \( \tau_c \sim 2 \), in agreement with the rough estimate \( \tau_c \sim 1/\Delta \mu \). We consider different setups, obeying or violating the horizontal and vertical mirror symmetries, see Fig. 8.

### A. Model I

We analyze nonlinear transport behavior in model I using INFPI simulations, then compare our results to the
achieved. We also examine the steady-state behavior of it obeys the symmetry relation (16) before convergence is

\[ D = (I + \hat{I})/2 \text{ and } D = (I - \hat{I})/2. \]

Using INFPI, we find that Eq. (16) is obeyed even before steady-state is reached. We exemplify the convergence behavior of this model in Fig. 8. In panels (a)-(b) we display \( R(t, \pm \pi/2) \) and demonstrate that it obeys the symmetry relation (16) before convergence is achieved. We also examine the steady-state behavior of the system using different values for the simulation time step, see panels (c)-(d). A large time-step \( \delta t = 1.2 \) does not allow convergence, but with \( \delta t = 0.6 \), \( R(\pm \pi/2) \) converges around \( \tau_c = 3 \). Odd conductance terms (panel d) converge quite well, but maintain transport symmetries (overlapping data for \( D(\pm \pi/2) \)) under a short memory time. This observation is not trivial: the dynamics under the phases \( \pm \phi \) is quite different, see for example Fig. 8(a).

The symmetry relations (16) are invalidated when the horizontal and vertical mirror symmetries are broken, see Fig. 8. Note that in panel (c), \( D(\phi) \neq D(-\phi) \); deviations are of order of \( 10^{-4} \). We can use Fig. 7 and estimate the magnitude of high order conductances. For example, from the behavior of \( D \) (assuming \( G_3 \) provides the largest contribution after \( G_1 \)) we find that at \( \phi = \pi/2 \), \( G_1 \sim D/\Delta \mu = 0.06 \), \( G_3 \sim (D(\pi/2) - D(-\pi/2))/2\Delta \mu \sim 5 \times 10^{-4} \) and \( G_2(\pi/2) \sim R(\pi/2)/\Delta \mu \sim 3 \times 10^{-3} \). Thus, at this phase, \( G_3/G_1 = 10^{-2} \) and \( G_2/G_1 = 2 \times 10^{-2} \). These conductances are translated to physical units when multiplied by the factor \( \frac{\hbar}{m_0} \). Note that \( G_2 \) and \( G_3 \) are of the same order of magnitude.

In Fig. 8 we display the long-time quasi steady-state data for \( R(\phi) \) and \( D(\phi) \). Within the present simulation times, we have not reached the steady-state limit for \( R \) using \( \phi/\pi < 1/23 \). We compare exact simulations to a
mean-field approach as explained in Sec. II.D, see Fig. 9. Both exact and approximate treatments satisfy the relations \( R = \gamma \) when the vertical mirror symmetry is preserved. However, the Hartree approach is unreliable as it predicts incorrect magnitudes for \( R \). Similarly, in the absence of mirror symmetries, exact and approximate tools demonstrate the violation of Eq. (16), but the Hartree approach overestimates the magnitude of \( R \).

Prior studies of nonlinear transport in quantum dot systems had indicated that the Hartree MF approximation suffers from fundamental artifacts, e.g., it predicts an incorrect hysteresis behavior in the single-impurity Anderson model. Here we find that the method conserves the correct transport symmetries, but it produces incorrect values for the nonlinear terms. We have also implemented a Hartree-Fock (HF) approach as described in Ref. 24 by further correcting off-diagonal elements in the Green’s function with the expectation values of the coherences \( \sigma_{1,2} \). This had reduced the amplitude of the oscillatory pattern around \( \phi = \pm 0.2 \), but HF results still overestimate \( R \) by almost an order of magnitude, for \( \phi \approx \pi/2 \). It is interesting to adopt an equations-of-motion treatment\(^{45}\) and explore these deviations maintaining higher-order correlation effects.

We conclude: (i) The Hartree mean-field approach properly describes the development of transport symmetries when the device acquires vertical or horizontal mirror symmetries; its quantitative predictions are unreliable. (ii) The AB interferometer can act as a charge diode (\( R \neq 0 \)) in a spatially symmetric device (\( \gamma_{\nu,n} \) are all identical) if the following conditions are simultaneously met: the dots’ energies are nondegenerate, the magnetic flux is nonzero \( \phi \neq \pi m \), and man-body interactions are in play, see Fig. 8(a). This observation agrees with recent simulations based on wave-packets propagation\(^{46}\).

**FIG. 7:** Model I. (a) Total current and its breakup into its (b) \( R \) and (c) \( D \) components for noncentrosymmetric and non-degenerate double dot system, \( \phi = \pi/2 \). \( \gamma_{L,1} = \gamma_{L,2} = 0.2 \), \( \gamma_{R,1} = \gamma_{R,2} = 0.05 \), \( \epsilon_1 = 0.1 \), \( \epsilon_2 = 0.2 \), \( \Delta \mu = 0.6 \), \( U = 0.1 \), \( N_s = 6 \) and \( \delta t = 0.6 \).

**FIG. 8:** Steady-state behavior of Model I with INFPI simulations using nondegenerate dots, \( \epsilon_1 = 0.1 \), \( \epsilon_2 = 0.2 \). Setups with a vertical mirror symmetry, \( \gamma_{\nu,n} = 0.1 \) (\( \circ \)), and missing mirror symmetries, \( \gamma_{L,n} = 0.2 \) and \( \gamma_{R,n} = 0.05 \) (\( \square \)). Other parameters are \( \Delta \mu = 0.6 \), \( U = 0.1 \), \( \beta = 50 \), \( N_s = 6 \) and \( \delta t = 0.6 \).

**FIG. 9:** Steady-state behavior of Model I using a mean-field scheme, and the same parameters as in Fig. 8. Setups with a vertical mirror symmetry, \( \gamma_{\nu,n} = 0.1 \) (\( \circ \)), and missing mirror symmetries, \( \gamma_{L,n} = 0.2 \) and \( \gamma_{R,n} = 0.05 \) (\( \square \)).

### B. Model II

Here we simulate with INFPI the dynamics of model II, an AB interferometer coupled to a dissipative environment. The FE provides a mechanism for elastic and inelastic scattering of electrons on dot ’1’, and it dissipates energy from the AB unit. We focus on an equilibrium environment, \( \mu_+ = \mu_- \); in Fig. 14 we further address the role of a nonequilibrium FE on transport symmetries within the AB interferometer. It should be noted that with dot ’1’ coupled to the FE, the model lacks a horizontal mirror symmetry by construction.

Model II with an equilibrium FE may be mimicked by a noninteracting model with dot ’1’ connected to a voltage probe. The voltage probe is a metal terminal; its parameters are set so as the net charge current directed towards it, from the AB systems, vanishes. It provides a mechanism for implementing dissipative inelastic scattering of electrons in the interferometer, while allowing one to work in the Landauer formalism of noninteracting electrons. In Ref. 23 we used this machinery and proved that in systems with a vertical mirror symmetry even and odd conductance compo-
tem, obeys phase rigidity \( I(t, \phi) = I(t, -\phi) \). When the AB setup is coupled to the FE, the transient current and the steady-state value do not transparently expose any symmetry, but panels (b)-(c) demonstrate that in a geometrically symmetric setup, \( \gamma_L = \gamma_R \), the symmetries (16) are satisfied in the transient regime and in the steady-state limit.

It is important to recall that the occupations of the dots in the AB interferometer do not satisfy analogous symmetries, even in the isolated \( U = 0 \) limit. In Fig. 11 we display the occupation of dot ‘1’ and show that, in agreement with analytical results, \( \sigma_{1,1}(t, \phi) \) does not satisfy a phase symmetry away from the symmetric point. Note that at short time, \( \gamma_L t \lesssim 0.5 \), the current and the occupation of dot ‘1’ are insensitive to both interactions and the magnetic phase. When \( U \) is turned on, the case with \( \phi = \pi/2 \) approaches steady-state significantly faster than the opposite \( \phi = -\pi/2 \) situation. This is reflected in both the occupation dynamics and the current.

In Fig. 12 we present steady-state data for \( R \) and \( D \). We demonstrate the validity of Eq. (16) in junctions with a vertical mirror symmetry, and its violation in general situations. Note that \( D(\phi) \neq D(-\phi) \) under a spatial asymmetry, but deviations are small, for example \( D(\pi/2) - D(-\pi/2) \sim 1.5 \times 10^{-4} \). We compare these results to the probe technique as described in Ref. 23. The coupling of dot ‘1’ to the probe (hybridization strength \( \gamma_p \)) does not directly correspond to the capacitive coupling \( U \), thus we can only make a qualitative comparison here. Results are displayed in Fig. 13. Note that we used here a higher electronic temperature, \( 1/\beta = 25 \), to facilitate convergence. It was shown in Ref. 23 that an increase of the metals’ temperature only leads to a weaker visibility of the current with the magnetic flux, but it does not alter the oscillation of the current with flux.

Comparing Fig. 12 (INFP1) to Fig. 13 (probe), we observe that the probe technique provides qualitatively correct features. However, in asymmetric setups it brings \( R(\phi = 0) = 0 \). In contrast, INFPI yields a nonzero value for \( R(\phi = 0) \), see Fig. 12. This disagreement has an important implication: The phenomenological probe approach predicts that in the absence of a magnetic flux the AB system cannot act as a diode, though an asymmetry is introduced and (effective) many-body interactions are playing a role. In contrast, INFP1 simulations show that the system can act as a diode at zero flux if \( \gamma_L \neq \gamma_R \).

We now explore the role of nonequilibrium effects in the FE, \( \Delta \mu_F = \mu_+ - \mu_- \neq 0 \). As long as we keep the interferometer biased \( \Delta \mu \neq 0 \) we recover the symmetries as before, reaching the dynamics as in Fig. 11. Transport symmetries are thus unaffected by the nonequilibrium environment, and this could be justified by noting that in our model dot ‘1’ is coupled to a number operator in the FE, as in Ref. 22, rather than to scattering states 25.

Model II further allows us to explore the development of the “Coulomb drag current” in the interferometer, a result of its coupling to the FE. This effect has important implications in nanoscale electronic junctions:
FIG. 12: Steady-state values of $\mathcal{R}$ and $\mathcal{D}$ in Model II. (o) symmetric $\gamma_{\nu,n} = 0.2$, and (□) asymmetric $\gamma_{L,n} = 0.05 \neq \gamma_{R,n} = 0.2$ setups. Other parameters are the same as in Fig. 10.

FIG. 13: Model II, with the FE replaced by a voltage probe coupled to dot ‘1’. The probe equations are detailed in Ref. 25; (o) symmetric $\gamma_{\nu,n} = 0.2$, and (□) asymmetric $\gamma_{L,n} = 0.05 \neq \gamma_{R,n} = 0.2$ setups. Parameters are the same as in Figs. 10 and 12 besides the temperature which is set at $1/\beta = 25$ and $\gamma_p = 0.05$.

When placing two quantum wires (with independent contacts) into a close proximity, a “drive current” passing through one conductor can induce a “drag current” in the other wire, a result of Coulomb (capacitive) interactions between charges in the two wires, see e.g. Ref. 25 for an experimental demonstration. In Fig. 14 we explore this effect using INFP1: The interferometer is unbiased, $\Delta \mu = 0$, but we voltage bias the FE. We show that in a system with a vertical mirror symmetry the drag current is nonzero, an odd function of the magnetic flux. We can drive a positive or a negative current in the AB interferometer; the directionality is induced here through the magnetic flux, not the hybridization coefficients as in other works. Furthermore, by plotting in panel (b) the measure $\mathcal{R} = [I(t, \phi, \Delta \mu_F) + I(t, \phi, -\Delta \mu_F)]/2$ we confirm that the current includes only even powers in $\Delta \mu_F$, missing altogether a linear response term.

We emphasize that the drag current observed here does not emerge from the transfer of momentum between charges, rather, we harness here charge fluctuations in the FE. It is thus expected that an unbiased-thermal FE could induce a net current in a centrosymmetric AB interferometer, if the magnetic flux is nonzero. However, this situation cannot be explored at present by the INFP1 technique since its convergence requires a large voltage biasing or high temperatures, with the memory time approximately given by $\tau_e = \delta t N_s \sim \min\{1/\Delta \mu_F, \beta\}$.

The Coulomb drag effect has been examined so far by breaking the spatial symmetry using uneven contacts, adopting phenomenological rate equations or perturbative treatments, see e.g. Refs. 48, 50, 51. Our work here is a first step towards the exploration of this many-body phenomenon with a broken time reversal symmetry, by means of an exact numerical tool.

We summarize our main observations for model II: (i) The relations (10) are satisfied in the transient regime and in the steady-state limit when the vertical mirror symmetry is obeyed. The approach to steady-state depends on the magnetic flux. (ii) The probe technique, an effective mean, provides the correct features for $\mathcal{R}$ and $\mathcal{D}$, but it predicts no dc rectification current in the absence of a magnetic flux, for $\gamma_L \neq \gamma_R$. (iii) The FE may generate a positive or a negative drag current in an unbiased centrosymmetric AB interferometer, given a nonzero magnetic phase in the system.

IV. SUMMARY

We examined the double-dot AB interferometer with controlled many-body effects, either internal, between electrons on the dots (Model I), or between the AB electrons and a dissipative environment (Model II). Using a flexible numerically exact tool, we studied the transient and the steady-state characteristics of the charge current in the AB system. We validated magnetic field symmetries of nonlinear transport when the system preserves horizontal or vertical mirror symmetries. Transport asymmetries were displayed and quantified in general geometries. Applications beyond the mean field level
were exemplified, including a charge diode, charge sensing, and the Coulomb drag current.

Earlier studies of Magnetotransport properties were limited to steady-state situations, mostly analyzed at the mean-field level. Here, we studied a double-dot AB interferometer with genuine many-body interactions, and we simulated it with a numerically exact tool. The comparison to effective treatments, Hartree MF and the probe technique, reveals that these simplified methods capture correctly transport symmetries, though magnitudes of nonlinear terms may substantially deviate from the exact limit.

In future studies we plan to examine other many-body models, for example, a nanojunction coupled to internal vibrations. This will be done using INFPIT and other perturbative-analytical and numerical schemes such as the Green’s function technique and Quantum master equations. Such an analysis would not only resolve transport behavior, but further serve as a critical test for examining the consistency of analytical and numerical truncation schemes. Other ideas involve a detailed analysis of the Coulomb drag effect, harnessing (hot) thermal charge fluctuations to drive a dc current in the interferometer. Finally, we plan to study symmetries of the thermoelectric current using both the phenomenological probe technique and INFPIT, with the objective to suggest means for increasing heat to work conversion efficiency in nonlinear situations.

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48 The data for δt = 0.6 and δt = 1 reach a different plateau at large τc, see Fig. 6 (c). Given the accumulation of the Trotter error, we take below the values obtained with the smallest time step, δt = 0.6, to represent INFPI’s numerical solution. More extensive simulations, with δt ∼ 0.5 and Ns = 2−10, are necessary to achieve a more accurate value for R.
49 INFPI simulations allow us to explore the quasi steady-state regime, limited by the recurrence time which is dictated by the finite discretization of the fermionic baths. Numerical simulations in Fig. 3 are limited to certain phases since the relaxation time towards steady-state significantly grows when the magnetic phase is small, φ/\pi < 1/2. Indeed, in Ref. 32 we had shown analytically that the two-dots dynamics is controlled by rate constants of the form [1 ± cos(φ/2)]; the rate proportional to [1 − cos(φ/2)] apparently controls the dynamics of R(t, φ).
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