Chirally symmetric effective field theory for nuclei

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Abstract

The Lorentz-invariant nuclear lagrangian of Furnstahl, Serot and Tang (FST) is discussed. The FST lagrangian is derived in terms of an effective field theory and exhibits a nonlinear realization of chiral symmetry $SU(2)_L \times SU(2)_R$. The relevant degrees of freedom are nucleons, pions and the low-lying non-Goldstone bosons: isoscalar scalar ($\sigma$) and vector ($\omega$) mesons, and isovector vector ($\rho$) mesons. The terms in the lagrangian are organized by applying Georgi’s naive dimensional analysis and naturalness condition. As a consequence all coupling constants in theory are dimensionless and of order unity.
I. INTRODUCTION

The effective field theory (EFT) technique allows to construct in a controlled manner, below a characteristic energy scale, the most general lagrangian consistent with relevant degrees of freedom and symmetries of an underlying theory. In nuclear physics, the EFT method relies on the symmetries of QCD to construct the effective lagrangian. The main component of this construction is the chiral $SU(2)_L \times SU(2)_R$ symmetry. This almost perfect symmetry is spontaneously broken to its vectorial subgroup $SU(2)_V=L+R$ with the appearance of pseudo-Goldstone bosons (pions).

One of the recent attempts at formulating EFT for finite nuclei and nuclear matter is the generalization of Walecka quantum hadrodynamics (QHD) \cite{1,2} proposed by Furnstahl, Serot and Tang (FST) \cite{3,4}. The FST lagrangian is derived by expansion in powers of the lowest lying hadronic fields and their derivatives. The relevant degrees of freedom are nucleons, pions, isoscalar-vector fields ($\omega$-mesons), isoscalar-scalar fields ($\sigma$-mesons), and isovector-vector fields ($\rho$-mesons). In this lagrangian the chiral symmetry is realized nonlinearly using a standard procedure of Weinberg \cite{5}, and Callen, Coleman, Wess, Zumino \cite{6}, (WCCWZ). The terms of the lagrangian are organized by applying Georgi’s \cite{7,8} naive dimensional analysis (NDA) and a “naturalness” condition.

The framework of EFTs and Georgi’s naive dimensional analysis are detailed in Sect. II. In Sect. III the nonlinear realization of chiral symmetry and the FST chirally symmetric effective lagrangian are discussed. The Dirac-Hartree approximation of the FST lagrangian, i.e., a treatment of the lagrangian at the level of classical meson fields and valence nucleons is shown in Sect. IV. A short summary is given in the last section.

II. EFFECTIVE FIELD THEORIES

A. Principles of the EFTs

An essential idea underlying the effective field theories (EFTs), see e.g. Ref. \cite{9,10,11,12,13,14,15}, is relevant to the appearance of disparate characteristic energy scales, $E \ll E_0$, in quantum field theories. Suppose that we are interested in physics at lower scale $E$, then we can choose a cut-off scale $\Lambda$ at or slightly below $E_0$ and divide the generic fields $\phi$ into two parts: a low- $\phi_L$ and a high-energy $\phi_H$ ($\phi = \phi_L + \phi_H$), according their momenta
are smaller or greater than $\Lambda$

$$\phi_L(k): |k| < \Lambda, \quad \phi_H(k): |k| \geq \Lambda. \quad (1)$$

The effective lagrangian is obtained by path integrating over the high-energy part $\phi_H$ in the generating functional $Z$

$$Z = \int [d\phi_L][d\phi_H] e^{i\int d^4x \mathcal{L}(\phi_L, \phi_H)} = \int [d\phi_L] e^{i\int d^4x \mathcal{L}_{\text{eff}}(\phi_L)}, \quad (2)$$

where

$$\int d^4x \mathcal{L}_{\text{eff}}(\phi_L) = -i \ln \int [d\phi_H] e^{i\int d^4x \mathcal{L}(\phi_L, \phi_H)}. \quad (3)$$

This defines the procedure of eliminating the high-energy degrees of freedom $\phi_H$, referred to as “decimation”. The next step is to write $\mathcal{L}_{\text{eff}}$ in terms of local operators $O_i(\phi_L)$,

$$\mathcal{L}_{\text{eff}}(\phi_L) = \sum_{i}^\infty g_i(\Lambda) O_i(\phi_L), \quad (4)$$

where $g_i(\Lambda)$ are the coupling constants absorbing the contribution of the integrated out high-energy degrees of freedom $\phi_H$.

In Eq. (4) the effective lagrangian is represented by a infinity series of interactions that involve the relevant degrees of freedom and satisfy the assumed symmetries of the underlying high-energy theory. In order to make this procedure useful we need some dimensional analysis. In units $\hbar = c = 1$ the action $S_{\text{eff}} = \int d^4x \mathcal{L}_{\text{eff}}(\phi_L)$ is dimensionless. If an operator $O_i$ has dimension $\delta_i$, $[O_i] = [m]^{\delta_i} \equiv \delta_i$, then $g_i$ has dimension $[g_i] = 4 - \delta_i$ and we can define dimensionless coefficients $c_i = \Lambda^{\delta_i - 4} g_i$, which additionally are assumed to be “natural”, i.e. of order $O(1)$. For a process at scale $E$, we can estimate dimensionally the magnitude of the $i$’th operator in the action as

$$\int d^4x O_i \sim E^{\delta_i - 4}, \quad (5)$$

so that the $i$’th term is of order

$$\int d^4x \frac{c_i}{\Lambda^{\delta_i - 4}} O_i \sim c_i \left( \frac{E}{\Lambda} \right)^{\delta_i - 4}. \quad (6)$$

Now we can see that at energies below $\Lambda$, the behaviour of the different operators is determined by their dimension. If $\delta_i < 4$ the operator is more and more important when $E \to 0$, and is termed relevant. Similarly, if $\delta_i > 4$ the operator is less and less important, and is
termed irrelevant. An operator with $\delta_i = 4$ is equally important at all scales and is called marginal.

At energies much below $\Lambda$, corrections due to the irrelevant (non-renormalizable) parts are suppressed by powers of $E/\Lambda$ and the effective lagrangian is able to describe the low-energy physics. The accurate procedure which connects the order of the expansion in powers $E/\Lambda$ with the terms in the effective lagrangian that need to be included at that order is called “power counting”.

**B. Naive dimensional analysis**

There are at least two relevant energy scales in nuclear physics: the pion-decay constant $f_\pi \approx 93$ MeV and the larger scale $\Lambda \sim 4\pi f_\pi \sim 1$ GeV, which characterizes the mass scale of physics beyond Goldstone bosons. Using a naive dimensional analysis (NDA) proposed by Georgi and Manohar [7, 8], for assigning the LECs of appropriate sizes, the effective lagrangian describing interactions of nucleons $N(x)$, pions $\vec{\pi}(x)$, and non-Goldstone bosons (scalar $\phi(x)$ and/or vector $V(x)$ mesons) takes a general form

$$L_{\text{eff}} = \sum_{\{\text{ndpb}\}} c_{\text{ndpb}} \left( \frac{\Gamma N N}{f_\pi^2 \Lambda} \right)^\frac{n}{2} \left( \frac{D}{m_\pi} \right)^d \left( \frac{\vec{\pi}}{f_\pi} \right)^p \frac{1}{b!} \left( \frac{\phi, V}{f_\pi} \right)^b f_\pi^2 \Lambda^2$$

$$= \sum_{\Delta=0}^{\infty} L^{(\Delta)},$$

where $\Gamma$ is a product of Dirac matrices, $D$ a covariant derivative, $m_\pi$ a pion mass (treated as derivative) and $c_{\text{ndpb}}$ the dimensionless LECs which are assumed to be natural, of $O(1)$.

In Eq. (8) the interactions are grouped in sets $L^{(\Delta)}$ of common index $\Delta \equiv \frac{n}{2} + d + b - 2$, according to (7), each of them carries a factor of the form $(1/\Lambda)^\Delta$. This formula has profound implications if we invoke chiral symmetry, Ref. [16]. For strong interactions (in absence of external gauge fields, e.g. photons) chiral constraint guarantees that $\Delta \geq 0$ and the large scale $\Lambda \sim 4\pi f_\pi \sim 1$ GeV does not occur with positive powers in Eq. (7).
III. EFFECTIVE CHIRAL LAGRANGIAN FOR NUCLEI

A. Nonlinear realization of chiral symmetry

Let us briefly collect the basic ingredients for considering the chiral effective lagrangian. In the chiral limit, where $N_f = 2$ or $3$ of quarks are massless ($u$, $d$ and possibly $s$), the underlying QCD lagrangian is invariant under a global group

$$U(N_f)_L \times U(N_f)_R \simeq SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A.$$  \hspace{1cm} (9)

However, at the quantum level, due to the axial anomaly, the $U(1)_A$ symmetry is broken that, e.g., leads to nonzero mass of $\eta'(958)$ meson even in the chiral limit. In the hadronic world, the chiral group $G \equiv SU(N_f)_L \times SU(N_f)_R$ is spontaneously broken in the vacuum

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \longrightarrow SU(N_f)_{V=L+R} \times U(1)_V,$$  \hspace{1cm} (10)

to the vectorial subgroup $H \equiv SU(N_f)_{V}$, either of isospin when $N_f = 2$ or flavor $SU(3)$ when $N_f = 3$. The preserve vector group $U(1)_V$ is realized as baryon number conservation. According to Goldstone’s theorem the number of Goldstone fields is the dimension of coset space $G/H$, which is the number of generators of $G$ that are not also generators of $H$. In our case of chiral symmetry $\text{dim } G/H = N_f^2 - 1$ and we can identify $N_f^2 - 1$ pseudoscalar Goldstone bosons, $\varphi \equiv \pi, K, \eta$, with the pions for $N_f = 2$, plus the kaons and an eta meson for $N_f = 3$.

The nonlinear realization of spontaneously broken chiral symmetry, denoted by WCCWZ, was suggested by Weinberg [5] and developed further by Callan, Coleman, Wess, and Zumino [6]. In the WCCWZ formalism the Goldstone bosons $\varphi$, being coordinates of the coset space $G/H$, are naturally represented by elements $\xi(x) = \xi(\varphi(x))$ of this coset space. The chiral symmetry is defined by specifying the action of $G$ on the representative $\xi(\varphi)$, with a canonical choice of coset representative this transformation takes the form

$$\xi(\varphi) \overset{g}{\rightarrow} g_R \xi(\varphi) h^\dagger(g, \varphi) = h(g, \varphi) \xi(\varphi) g^\dagger_L,$$  \hspace{1cm} (11)

where $g \equiv (g_L, g_R) \in G$. The equality in (11) is due to parity and it defines the so-called compensator (field) $h(g, \varphi(x)) \in H$. Its dependence on the Goldstone boson fields $\varphi(x)$ is a characteristic feature of the nonlinear realization of chiral symmetry.
Let us restrict ourselves to the two flavor case $N_f = 2$ with the isotriplet of pions collected in a $2 \times 2$ special unitary matrix

$$
\pi(x) \equiv \vec{\pi}(x) \cdot \frac{1}{2} \bar{\tau} = \frac{1}{2} \begin{pmatrix} \pi^0 & \sqrt{2} \pi^+ \\ \sqrt{2} \pi^- & -\pi^0 \end{pmatrix},
$$

(12)

where $\bar{\tau}$ are Pauli matrices. Applying an exponential parametrization a coset representative $\xi(x) = \xi(\pi(x))$ can be written as

$$
U(x) \equiv \xi^2(x) = \exp \left( \frac{2i\pi(x)}{f_\pi} \right),
$$

(13)

where $f_\pi \approx 93$ MeV is the pion-decay constant. The isospinor nucleon field is represented by a column matrix

$$
N(x) = \begin{pmatrix} p(x) \\ n(x) \end{pmatrix},
$$

(14)

where $p(x)$ and $n(x)$ are the proton and neutron fields, respectively. The relevant low-lying non-Goldstone bosons are an isovector-vector $\rho(770)$ meson $\rho_\mu(x) \equiv \vec{\rho}_\mu(x) \cdot \frac{1}{2} \bar{\tau}$, an isoscalar-vector meson $\omega(782)$ represented by a vector field $V_\mu(x)$ and an effective isoscalar-scalar field $S(x)$ ($\sigma$ meson) is described by the shifted field $\phi(x) \equiv S_0 - S(x)$, where $S_0$ is the vacuum expectation value of the scalar field $S$, see Ref. [17]. The $\omega$ meson is needed to describe the short-range repulsion and $\phi$ effective field is included to incorporate the mid-range attraction of the $NN$ interaction.

Following WCCWZ, the nonlinear realization of the chiral symmetry for the mentioned above degrees of freedom are

$$
\xi(x) \rightarrow g_R \xi(x) h^\dagger(g, \pi(x)) = h(g, \pi(x)) \xi(x) g_L^\dagger,
$$

(15)

$$
N(x) \rightarrow h(g, \pi(x)) N(x), \quad \overline{N}(x) \rightarrow \overline{N}(x) h^\dagger(g, \pi(x)),
$$

(16)

$$
\rho_\mu(x) \rightarrow h(g, \pi(x)) \rho_\mu(x) h^\dagger(g, \pi(x)),
$$

(17)

where $g = (g_L, g_R) \in SU(2)_L \times SU(2)_R$. The matrix field $U(x)$ transforms linearly under chiral transformations

$$
U(x) \rightarrow g_L U(x) g_R^\dagger,
$$

(18)

and the other degrees of freedom, neutral-isoscalar fields $V_\mu(x)$ and $\phi(x)$, can be treated as the chiral singlets.
We can also define an axial vector field \( a_\mu \) and a polar vector field \( v_\mu \)

\[
a_\mu \equiv -\frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu (\xi^\dagger)) = a_\mu^\dagger, \tag{19}
\]

\[
v_\mu \equiv -\frac{i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu (\xi^\dagger)) = v_\mu^\dagger, \tag{20}
\]

where the hermiticity follows from \( \partial_\mu (\xi^\dagger \xi) = 0 = \partial_\mu (\xi \xi^\dagger) \). Under the chiral symmetry the transformation of \( a_\mu \) is homogeneous

\[
a_\mu \xrightarrow{g} h a_\mu h^\dagger, \tag{21}
\]

whereas that of \( v_\mu \) is inhomogeneous

\[
v_\mu \xrightarrow{g} h(v_\mu - i\partial_\mu)h^\dagger. \tag{22}
\]

In fact \( v_\mu \) is the connection on the coset space and with it we can construct the covariant derivatives on this space. For example, since transformations (16) and (17) are not only nonlinear but also local, via compensator field \( h(g, \pi(x)) \), that requires the introduction of the chirally covariant derivatives for nucleon and rho meson fields:

\[
D_\mu N = (\partial_\mu + iv_\mu)N, \quad D_\mu \rho_\nu = \partial_\mu \rho_\nu + iv_\mu [\rho_\nu, \rho_\nu], \tag{23}
\]

which transform covariantly under the chiral group. Also, the curvature (strength) tensor, \( v_{\mu\nu} \), associated with the connection can be expressed in terms of axial vector fields, \( a_\mu \), as

\[
v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + iv_\mu [v_\nu, v_\mu] = -i[a_\mu, a_\nu]. \tag{24}
\]

The covariant derivative of rho meson can be used to construct the covariant field tensor

\[
\rho_{\mu\nu} = D_\mu \rho_\nu - D_\nu \rho_\mu + ig_\rho [\rho_\mu, \rho_\nu], \tag{25}
\]

which is the last ingredient needed to build the FST effective chiral lagrangian.

\section*{B. FST effective chiral lagrangian}

The low-energy effective lagrangian of Furnstahl, Serot and Tang (FST) \cite{Furnstahl:1999kp, Furnstahl:1999gi}, see also Ref. \cite{Serot:1984ey, Serot:1997xq, Serot:1998yc}, incorporates the symmetries of QCD: Lorentz invariance, parity invariance, nonlinear realization of chiral \( SU(2)_L \times SU(2)_R \) symmetry, this lagrangian is also invariant
under the electromagnetic $U(1)_{em}$ and isospin $SU(2)$ groups. The FST lagrangian is expanded in powers of fields and their derivatives in the procedure of power counting with index $\tilde{\Delta} = \frac{n}{2} + d + b$, where $n$ is the number of nucleon fields, $d$ is the number of derivatives and $b$ is the number of non-Goldstone boson fields in each term. Taking as the large energy scale $\Lambda$ in Eq. (7) the nucleon mass $M = 939$ MeV, we may write the effective chiral lagrangian through quartic order ($\tilde{\Delta} \leq 4$) as the sum

$$\mathcal{L}_{\text{eff}}(x) = \mathcal{L}_N^{(4)}(x) + \mathcal{L}_M^{(4)}(x) + \mathcal{L}_{\text{EM}}^{(4)}(x).$$

(26)

The part of lagrangian involving nucleons takes the form

$$\mathcal{L}_N^{(4)}(x) = \mathcal{N}\left[\gamma^\mu (i\partial_\mu - v_\mu - g_\rho \rho_\mu - g_\omega V_\mu) + g_\lambda \gamma^\mu \gamma_5 a_\mu - (M - g_\phi \phi)\right] N$$

$$- \frac{f_\rho g_\rho}{4M} \rho_{\mu\nu} \sigma^{\mu\nu} N - \frac{f_\omega g_\omega}{4M} V_{\mu\nu} \sigma^{\mu\nu} N - \frac{\kappa_\pi}{M} N v_{\mu\nu} \sigma^{\mu\nu} N$$

$$+ \ldots,$$

(27)

where $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$, $V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$ is the covariant tensor of the $\omega$ meson, $g_\lambda \approx 1.26$ is the axial coupling constant, $g_\rho$, $f_\rho$, and $g_\omega$, $f_\omega$ are vector and so-called tensor couplings for $\rho$ and $\omega$ mesons, see Ref. [21], $g_\phi$ is a Yukawa coupling for the effective scalar field $\phi$, and $\kappa_\pi = \frac{f_\pi}{4}$ is the coupling for higher-order $\pi N$ interaction. The ellipsis represents redundant or tiny additional terms with $\pi N$ and $\pi \pi$ interactions, which have been omitted in the FST lagrangian.

The mesonic part of the effective lagrangian is

$$\mathcal{L}_M^{(4)}(x) = \frac{1}{2} \left(1 + \alpha_1 \frac{g_\phi}{M}\right) \partial_\mu \phi \partial^\mu \phi + \frac{f_\pi^2}{4} \text{tr} \left(\partial_\mu U \partial^\mu U^\dagger\right)$$

$$- \frac{1}{2} \text{tr} \left(\rho_{\mu\nu} \rho^{\mu\nu}\right) - \frac{1}{4} \left(1 + \alpha_2 \frac{g_\omega}{M}\right) V_{\mu\nu} V^{\mu\nu} - g_{\rho\pi\pi} \frac{2f_\rho^2}{m_\rho^2} \text{tr} \left(\rho_{\mu\nu} \rho^{\mu\nu}\right)$$

$$+ \frac{1}{2} \left(1 + \eta_1 \frac{g_\phi}{M} + \frac{\eta_2 g_\omega^2}{2 M^2}\right) m_\psi^2 V_{\mu\nu} V^{\mu\nu} + \frac{1}{4!} \zeta_2 g_\omega^2 (V_{\mu\nu} V^{\mu\nu})^2$$

$$+ \left(1 + \eta_0 \frac{g_\omega}{M}\right) m_\rho^2 \text{tr} \left(\rho_{\mu\nu} \rho^{\mu\nu}\right) - m_\phi^2 \phi^2 \left(\frac{1}{2} + \frac{\kappa_3 g_\phi}{3! M} + \frac{\kappa_4 g_\omega^2}{4! M^2}\right),$$

(28)

where $m_\omega = 782$ MeV, $m_\rho = 770$ MeV, and $m_\sigma$ are $\omega$, $\rho$ and $\sigma$ mesons masses, $g_{\rho\pi\pi}$ is the coupling of $\rho\pi\pi$ interaction, which (assuming vector-meson dominance) is $g_{\rho\pi\pi} = g_\rho$. The trace “tr” is in the $2 \times 2$ isospin space. 

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The electromagnetic interactions are described by

\[ \mathcal{L}_{EM}^{(4)}(x) = -\frac{1}{4} F^\mu\nu F_{\mu\nu} - \frac{1}{2} e N \gamma^\mu (1 + \tau_3) N A^\mu - \frac{e}{4M} F_{\mu\nu} N \lambda \sigma^{\mu\nu} N \\
- \frac{e}{2M^2} N \gamma_\mu (\beta_s + \beta_v \tau_3) N \partial_\mu F^{\mu\nu} - 2 e f_\pi^2 A^\mu \text{tr}(v_\mu \tau_3) \\
- \frac{e}{2g_\gamma} F_{\mu\nu} \left[ \text{tr}(\tau_3 \rho^{\mu\nu}) + \frac{1}{3} V^{\mu\nu} \right] + \cdots, \]  

where \( A^\mu \) is the electromagnetic field, \( F_{\mu\nu} \) is the electromagnetic field-strength tensor. According to vector-meson dominance and phenomenology one can find that \( g_\gamma = 5.01 \). The lagrangian \( \mathcal{L}_{EM}^{(4)} \) is invariant under the \( U(1)_{em} \) group. The composite structure of the nucleon is included through an anomalous moment \( \lambda \equiv \frac{1}{2} \lambda_p (1 + \tau_3) + \frac{1}{2} \lambda_n (1 - \tau_3) \), with \( \lambda_p = 1.793 \) and \( \lambda_n = -1.913 \) the anomalous magnetic moments of the proton and the neutron, respectively. The ellipsis represents redundant terms of \( \mathcal{O}(e^2) \).

The effective chiral lagrangian Eq. (26) at a given order contains certain parameters that are not constrained by the symmetries, the so-called low-energy constants (LECs). Apart from the isoscalar (\( \beta_s \)), isovector (\( \beta_v \)) electromagnetic form factors and the tensor coupling for \( \rho \) meson (\( f_\rho \)), which are fixed from the free-space charge radii of the nucleon, the remaining thirteen LECs \{ \( g_s \frac{4}{4\pi}, g_v \frac{4}{4\pi}, \eta_1, \eta_2, \eta_\rho, \kappa_3, \kappa_4, \zeta_0, \frac{m_\rho}{M}, f_v, \alpha_1, \alpha_2 \} \) have to be determined from experimental data. The LECs are defined applying the naive dimensional analysis so that they are assumed to be of order unity, \( i.e. \) “natural”.

### IV. DIRAC-HARTREE APPROXIMATION

The mean-field approximation (ignores) dismisses all quantum fluctuations of the meson fields and treats them by their expectation values. Assuming the time reversal invariance the spatial components of the vector field vanish and we can define scaled mean meson fields (potentials) by including couplings: \( W(\mathbf{r}) = g_v V_0(\mathbf{r}), \Phi(\mathbf{r}) = g_\Phi \phi_0(\mathbf{r}), R(\mathbf{r}) = g_\rho b_0(\mathbf{r}) \) and \( A(\mathbf{r}) = eA_0(\mathbf{r}) \). Since the nuclear ground state has a well-defined charge, only the neutral rho meson field (denoted by \( b_0 \)) has been used, also since the ground state is assumed to have well-defined parity the pseudo-scalar pion field does not contribute in this approximation.

If we restrict consideration to static nuclear systems the Dirac equation with eigenvalues \( E_\alpha \) and eigenfunctions \( \psi_\alpha(\mathbf{r}) \) is, see Ref. [2],

\[ h \psi_\alpha(\mathbf{r}) = E_\alpha \psi_\alpha(\mathbf{r}), \quad \int d^3x \psi_\alpha^\dagger(\mathbf{r}) \psi_\alpha(\mathbf{r}) = 1, \]  

(30)
with
\[ h(r) = -\mathbf{i} \alpha \cdot \nabla + W(r) + \frac{1}{2} \tau_3 R(r) + \beta \left[ M - \phi(r) \right] + \frac{1}{2} (1 + \tau_3) A(r) \]
\[-\frac{i}{2M} \beta \alpha \cdot \left( f_{\nu} \frac{1}{2} \tau_3 \nabla R(r) + f_{\nu} \nabla W(r) \right) + \frac{1}{2M} (\beta_{\nu} + \beta_{\nu} \tau_3) \Delta A(r) \]
\[-\frac{i}{2M} \lambda \beta \alpha \cdot \nabla A(r), \] (31)

where \( \beta = \gamma_0, \alpha = \gamma_0 \gamma. \)

The mean-field equations for \( \phi(r), W(r), R(r) \) and \( A(r) \) are

\[ (-\Delta + m^2_\phi) \phi(r) = g^2_s \rho_s(r) - \frac{m^2_s}{M} \phi^2(r) \left[ \frac{\kappa_3}{2} + \frac{\kappa_4 \phi(r)}{3!} \right] \]
\[ + \frac{g^2_s}{2M} \left[ \eta_1 + \eta_2 \frac{\phi(r)}{M} \right] m^2_s W^2(r) + \frac{g^2_s \eta_3 m^2_\rho}{2M g^2_\rho} R^2(r) \]
\[ + \frac{\alpha_1}{2M} \left[ (\nabla \phi(r))^2 + 2 \phi(r) \Delta \phi(r) \right] \]
\[ + \frac{\alpha_2 g^2_s}{2M g^2_\nu} (\nabla W(r))^2, \] (32)

\[ (-\Delta + m^2_W) W(r) = g^2_\nu \left[ \rho_B(r) + \frac{f_\nu}{2M} \nabla \cdot (\hat{r} \rho_B^\top(r)) \right] \]
\[ - \left[ \eta_1 + \eta_2 \frac{\phi(r)}{M} \right] \frac{\phi(r)}{M} m^2_s W(r) - \frac{1}{3!} \zeta_0 W^3(r) \]
\[ + \frac{\alpha_2}{M} \left[ \nabla \phi(r) \cdot \nabla W(r) + \phi(r) \Delta W(r) \right] \]
\[ - \frac{e^2 g_\nu}{3g_\nu} \rho_{\mathrm{chg}}(r), \] (33)

\[ (-\Delta + m^2_R) R(r) = \frac{1}{2} g_\rho^2 \left[ \rho_3(r) + \frac{f_\rho}{2M} \nabla \cdot (\hat{r} \rho_3^\top(r)) \right] \]
\[ - \eta_\rho \frac{\phi(r)}{M} m^2_\rho R(r) - \frac{e^2 g_\rho}{g_\nu} \rho_{\mathrm{chg}}(r), \] (34)
\[ - \Delta A(r) = e^2 \rho_{\mathrm{chg}}(r). \] (35)

Assuming spherical symmetry and parity conservation the eigenfunctions of Dirac equation (30) (the positive-energy spinors) can be written as

\[ \psi_a(r) \equiv \psi_{n\kappa m t}(r) = \begin{pmatrix} i \left[ G_a(r) / r \right] \phi_{n tm} \\ - \left[ F_a(r) / r \right] \phi_{-n tm} \end{pmatrix}, \quad a \equiv \{ n, \kappa, t \}, \] (36)
\[ \int_0^\infty \mathrm{d}r \left( |G_a(r)|^2 + |F_a(r)|^2 \right) = 1, \] (37)
where $\Phi_{\kappa m} = \sum_{m_s} (\ell m_s | \frac{1}{2} m_s | jm) Y_{\ell m}(\Omega) \chi_{m_s}$ are spin spherical harmonics, $n$ is the principal quantum number, $\kappa$ is a nonzero integer uniquely determining $j$ and $\ell$ through $\kappa = (2 j + 1)(\ell - j)$ and $\zeta$ is a two-component isospinor labeled by the isospin projection $t = \frac{1}{2}$ for protons and $t = -\frac{1}{2}$ for neutrons. The radial equations for upper ($G$) and lower ($F$) components become

$$
\left( \frac{d}{dr} + \frac{\kappa}{r} \right) G_a(r) - \left[ E_a - U_1(r) + U_2(r) \right] F_a(r) - U_3(r) G_a(r) = 0, \quad (38)
$$

$$
\left( \frac{d}{dr} - \frac{\kappa}{r} \right) F_a(r) + \left[ E_a - U_1(r) - U_2(r) \right] G_a(r) + U_3(r) F_a(r) = 0, \quad (39)
$$

where single-particle potentials are given by

$$
U_1(r) \equiv W(r) + t_a R(r) + \left( t_a + \frac{1}{2} \right) A(r) + \frac{1}{2M^2} (\beta_s + 2t_a \beta_v) \Delta A(r), \quad (40)
$$

$$
U_2(r) \equiv M - \Phi(r), \quad (41)
$$

$$
U_3(r) \equiv \frac{1}{2M} \left\{ f_s \frac{dW(r)}{dr} + t_a f_\rho \frac{dR(r)}{dr} \right. \right.
$$

$$
+ \left. \frac{dA(r)}{dr} \left[ \frac{1}{2} (\lambda_p + \lambda_n) + t_a (\lambda_p - \lambda_n) \right] \right\}. \quad (42)
$$

The various densities that appear on the r.h.s. of the meson equations for spherically symmetric systems are defined as follows:

$$
\rho_s(r) = \sum_\alpha \sum_\text{occ} \bar{\psi}_\alpha(r) \psi_\alpha(r) \equiv \sum_\alpha \sum_\text{occ} \frac{2j_a + 1}{4\pi r^2} \left( G^2_a(r) - F^2_a(r) \right), \quad (43)
$$

$$
\rho_B(r) = \sum_\alpha \sum_\text{occ} \bar{\psi}_\alpha^+(r) \psi_\alpha(r) \equiv \sum_\alpha \sum_\text{occ} \frac{2j_a + 1}{4\pi r^2} \left( G^2_a(r) + F^2_a(r) \right), \quad (44)
$$

$$
\rho_B^T(r) = \sum_\alpha \sum_\text{occ} \bar{\psi}_\alpha^+(r) i \beta \alpha \cdot \hat{\tau} \psi_\alpha(r) \equiv \sum_\alpha \sum_\text{occ} \frac{2j_a + 1}{4\pi r^2} 2G_a(r) F_a(r), \quad (45)
$$

$$
\rho_3(r) = \sum_\alpha \sum_\text{occ} \bar{\psi}_\alpha(r) \tau_3 \psi_\alpha(r) \equiv \sum_\alpha \sum_\text{occ} \frac{2j_a + 1}{4\pi r^2} (2t_a) \left( G^2_a(r) + F^2_a(r) \right), \quad (46)
$$

$$
\rho_3^T(r) = \sum_\alpha \sum_\text{occ} \bar{\psi}_\alpha^+(r) i \tau_3 \beta \alpha \cdot \hat{\tau} \psi_\alpha(r) \equiv \sum_\alpha \sum_\text{occ} \frac{2j_a + 1}{4\pi r^2} (2t_a) 2G_a(r) F_a(r), \quad (47)
$$

where the summation superscript “occ” means that the sum runs only over occupied (valence) orbitals up to some value of $n$ and $\kappa$. The quantum numbers are denoted by $\{\alpha\} = \{a; m\} \equiv \{n, \kappa, t; m\}$.

The charge density is given by

$$
\rho_{\text{chg}}(r) \equiv \rho_d(r) + \rho_m(r), \quad (48)
$$
where the “direct” nucleon charge density is

\[ \rho_d(r) = \rho_p(r) + \frac{1}{2M} \nabla \cdot \left[ \hat{r}\rho_a^T(r) \right] + \frac{1}{2M^2} \left[ \beta_s \Delta \rho_B(r) + \beta_v \Delta \rho_3(r) \right], \]  

(49)

and the vector mesons contribution is

\[ \rho_m(r) = \frac{1}{g_\gamma g_\rho} \Delta R(r) + \frac{1}{3g_\gamma g_v} \Delta W(r). \]  

(50)

Here the “point” proton density and nucleon tensor density are given by

\[ \rho_p(r) \equiv \frac{1}{2} \sum_\alpha \psi_\alpha^\dagger(r)(1 + \tau_3)\psi_\alpha(r) = \frac{1}{2} \left[ \rho_B(r) + \rho_3(r) \right], \]  

(51)

\[ \rho_a^T(r) \equiv \sum_\alpha \psi_\alpha^\dagger(r)i\lambda \beta_\alpha \cdot \hat{r}\psi_\alpha(r), \]  

(52)

respectively, where \( \lambda \) is the anomalous magnetic moment. Thus the spherical nuclear ground state with the presence of time reversal symmetry is described by coupled, one-dimensional differential equations that may be solved by an iterative procedure. Once the solution has been found, the total energy of the system is given by

\[ E = \sum_\alpha \frac{\sigma_c}{2} E_a(2j_a + 1) - \int d^3x U_m(r), \]  

(53)

where

\[ U_m \equiv -\frac{1}{2} \phi \rho_\sigma + \frac{1}{2} W \left[ \rho_B + \frac{f_v}{2M} \nabla \cdot \left( \hat{r}\rho_a^T \right) \right] + \frac{1}{2} R \left[ \rho_3 + \frac{f_\rho}{2M} \nabla \cdot \left( \hat{r}\rho_a^T \right) \right] + \frac{1}{2} A \rho_d + \frac{m^2}{g_\rho^2 M} \left[ \frac{c_3}{12} + \frac{c_4}{24} M \right] - \frac{\eta_\sigma}{4} \frac{\phi m^2}{g_\rho^2} R^2 \]

\[ -\frac{\phi}{4M} \left[ \eta_1 + \eta_2 \frac{\phi}{M} \right] \frac{m^2}{g_\rho^2} W^2 - \frac{1}{4!g_\rho^2} \zeta_0 W^4 + \frac{\alpha_1}{4g_\rho^2 M} (\nabla \psi)^2 - \frac{\alpha_2}{4g_\rho^2 M} (\nabla W)^2. \]  

(54)

One of the most prominent observables, the binding energy of a system of \( A = Z + N \) nucleons is defined by

\[ E_B = E - E_{CM} - AM, \]  

(55)

where \( E_{CM} \) is the center-of-mass (c.m.) correction which can be estimated nonrelativistically, e.g., its an empirical estimate given by Reinhard[22] is \( E_{CM} \approx 17.2A^{-0.2} \text{ MeV} \). An older estimate from the harmonic oscillator shell model is \( E_{CM} \approx \frac{2}{3}41A^{-1/3} \text{ MeV} \).
The mean-square radius of the charge distribution, with the (c.m.) motion correction, is given by

$$\langle r^2 \rangle_{\text{chg}} = \langle r^2 \rangle - \frac{3}{4 \langle \hat{P}_{\text{CM}}^2 \rangle},$$

(56)

where

$$\langle r^2 \rangle = \frac{1}{Z} \int d^3 r r^2 \rho_{\text{chg}}(r), \quad \langle \hat{P}_{\text{CM}}^2 \rangle = 2 AM E_{\text{CM}}.$$

(57)

Since the additional nonrenormalizable interaction between the nucleon and electromagnetic field were included in $\mathcal{L}_{\text{EM}}$, Eq. (29), the charge density $\rho_{\text{chg}}$ automatically contains the effects of nucleon structure, and it is unnecessary to introduce an ad hoc form factor in formula (56).

V. SUMMARY

One of the major challenges in nuclear physics is to establish a connection between nuclear dynamics and the fundamental QCD. The chiral effective field theories are considered to offer a natural and useful framework for this purpose. Thanks to the implementation of nonlinear realization of chiral symmetry, Georgi’s naive dimensional analysis and the “naturalness” condition, the FST approach is the extension of Walecka’s hadrodynamics and may be used in nuclear physics to cross the border from QCD to a nuclear theory.

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