Theoretical study on limited slip principle of differential

Jihao Dong¹ and Ye Sun¹

¹Department of Vehicle Application, Army Academy of Armored Forces, Economic and Technological Development Zone, Changchun, Jilin Province, China

Corresponding author and e-mail: Jihao Dong, 201701064314hbl@ncist.edu.cn

Abstract. As one of the important assemblies of off-road vehicles, the anti-skid performance of differential is closely related to the passability of the whole vehicle. Based on the analysis of the power distribution characteristics of ordinary bevel gear differential, this paper studies the anti-slip principle of variable transmission ratio differential theoretically through mechanical analysis and working process analysis. The variation of adhesion coefficient between tire and pavement is analyzed, and the reasonable value of locking coefficient is obtained.

1. Introduction

After World War II, in a series of "experience summaries", the U.S. military listed the differential lock technology that can improve the driving condition of automobiles as one of the five development directions of military trucks in the future (air-cooled engine, automatic transmission, sealed brake, tire pressure regulation, differential lock), which is enough to show the importance of differential anti-slip technology [1].

Ordinary symmetrical conical gear differential is widely used by wheeled vehicles because of its compact structure and convenient manufacture. However, its power distribution feature is that the torque ratio K is generally 1.1-1.4 because of the small internal friction torque. In fact, it can be considered that the torque at both ends of the drive axle is basically evenly distributed regardless of whether the rotational speeds of the left and right driving wheels are equal. Because of this characteristic of equal torque distribution transmission, when the vehicle runs on wet or soft road, the driving force will be reduced and even the passing capacity will be lost when one wheel slips. Therefore, in order to improve the power performance and passability of wheeled vehicles on roads with low adhesion coefficient such as ice and snow or on soft ground, it is necessary to adopt limited slip differential technology [2].

Modern wheeled off-road vehicles, especially medium and light off-road vehicles with high mobility, have higher requirements for safety with the increasing speed. When a car equipped with a common differential drives at a high speed in a curve and the adhesion coefficient is separated from the road surface, dangerous working conditions such as tail flick and sidesslip will occur. Therefore, it is required that the limited slip differential device of the car should not only meet the requirements of mobility, but also meet the requirements of active safety, reliability and convenient installation of the whole car [3].
2. Research status

Most researches on limited slip differential devices at home and abroad focus on the structural design and reliability of the devices themselves. In recent years, the influence of limited slip differential device on vehicle performance, especially on active safety, has also become the focus of international research. Up to now, the research on the influence of limited slip differential device on the active safety of vehicles at home and abroad is mainly reflected in the research on the handling stability of vehicles. At present, most of the research on limited slip differential device in China focuses on the development of independent products with specific structure and the influence of single limited slip differential device on automobile performance, while the mature application research results of medium and light off-road vehicles with high mobility are rare.

The locking coefficient is defined as the maximum value of the ratio of torque distributed by the left and right axle wheels, which is one of the important indexes to measure the vehicle passing capacity. The larger the locking coefficient is, the stronger the vehicle passing capacity is. The theoretical value of locking coefficient of ordinary bevel gear differential is always one, which results in low passing capacity of vehicles. In order to improve vehicle passing capacity and lock coefficient, many research achievements have been made abroad, including bevel gear differential with differential lock, high friction differential, self-locking differential and variable transmission ratio bevel gear differential.

The above-mentioned existing limited slip differentials improve the locking coefficient and the vehicle passing capacity to varying degrees, and reduce the tire wear due to the reduction of tire slip. However, it has its own inevitable shortcomings.

3. Performance analysis

The main performance parameters of the limited slip differential device include locking coefficient \( k \), torque distribution coefficient \( \xi \), differential efficiency \( \eta_D \) and differential transmission efficiency \( \eta_{DT} \).

As shown in figure 1, the locking coefficient \( k \) generally refers to the maximum torque ratio between the slow rotation side and the fast rotation side (here, it is assumed that the right half shaft transmits a large torque), that is, \( k = M_3/M_1 \); Sometimes it is also defined as the ratio of torque difference and torque sum on both sides, i.e.

\[
k = \frac{M_3 - M_1}{M_3 + M_1}
\]

It reflects the torque distribution capacity of limited slip differential, which depends on the structure and type of limited slip differential, but has nothing to do with vehicle and ground.

Torque distribution coefficient \( \xi \) refers to the ratio of the torque on the slow side of the limited slip differential to the transmission torque \( M \) (input torque of the limited slip differential), that is, \( \xi = M_3/M_1 \). Torque distribution coefficient \( \xi \) has such a relationship with locking coefficient \( k \),

\[
\xi = \frac{k}{1+k}
\]

Differential efficiency \( \eta_D \) refers to the ratio of output power to input power when the differential case does not rotate and one half shaft drives the other half shaft, i.e.

\[
\eta_D = \frac{M_1 w_1}{M_2 w_2} = \frac{M_1}{M_3}
\]

The relationship between differential transmission efficiency \( \eta_D \) and locking coefficient \( k \) is,

\[
\eta_D = \frac{1}{k}
\]
Differential transmission efficiency $\eta_{DT}$ refers to the efficiency of power transmission to left and right axle shafts (limited slip differential between wheels) through differential case, namely:

$$\eta_{DT} = \frac{M_t w_1 + M_3 w_2}{M w_0}$$

Among the four parameters of limited slip differential, the locking coefficient $K$ directly determines the passing capacity of the vehicle, which is the most important performance parameter ($s$ and $\eta_D$ are both its functions), but with the development of automobiles, it has gradually developed into comprehensive performance evaluation.

All kinds of limited slip differential devices are comprehensively evaluated from six aspects: ground passability (hereinafter referred to as passability), active safety, economy, reliability, installation convenience and performance durability. For medium and light off-road vehicles, the variable transmission ratio differential can meet the required requirements by reasonably selecting and designing the transmission ratio change function. Moreover, due to the unique slip-limiting principle and structural form of the variable transmission ratio differential, it has obvious advantages in active safety and is a slip-limiting differential device with ideal comprehensive performance.

### 4. Analysis of limited slip principle of variable transmission ratio differential

When the resistance torque of the left and right axle wheels is unequal, the planetary bevel gear of the variable transmission ratio differential will rotate a little, which will change the included angle between the instantaneous axis of engagement between the left and right axle wheels and the axis of the planet wheels. The side with large resistance (the side with large resistance torque) will become smaller and the side with small resistance (the side with large resistance torque) will become larger, so that the driving torque transmitted to the side with large resistance will increase and the driving torque transmitted to the side with small resistance will decrease.

Figure 2 is a schematic diagram of a variable transmission ratio bevel gear limited slip differential. When the total torque $M$ transmitted by the planetary gear to the two half-axle wheels is constant, the torques $M_1'$ and $M_3'$ obtained by the two half-axle wheels should be:

$$M_1' + M_3' = M$$  \(1\)

$M_1'$ and $M_3'$ are equal to the resistance torque (i.e., the driving reaction torque obtained by driving the tire) $M_1$ and $M_3$, respectively, and the small internal friction is ignored here. According to the principle of virtual displacement, if the planetary gear 2 has an imaginary slight angular displacement $d\phi_2$ along its axis, then the half-axle gear has corresponding slight angular displacements $d\phi_1$ and $d\phi_3$, then the virtual work done by the system under the movement of all the people is zero, namely:
\[ M_1 \cdot \phi_1 - M_3 \cdot \phi_3 = 0 \]  \hspace{1cm} (2)

and
\[ M_1 + M_3 = M \]  \hspace{1cm} (3)

According to formula (2):
\[ M_1 / M_3 = \phi_3 / \phi_1 \]

Transmission ratio again:
\[ i_{32} = \phi_3 / \phi_2 \] and \[ i_{12} = \phi_1 / \phi_2 \]

\[ i_{12} \] and \[ i_{32} \] respectively represent the instantaneous transmission ratio between the left and right half axle wheels and the planet wheels, which can be obtained as follows:
\[ M_1 / M_3 = i_{32} / i_{12} \]  \hspace{1cm} (4)

As for the ordinary bevel gear differential, because \[ i_{32} = i_{12} = \] the number of teeth of planetary gears/the number of teeth of half axle gears, the torque \[ M_1 = M_3 \] distributed to the left and right driving wheels is given by Formula (4) and the foregoing. This formula holds true for ordinary differential at any time and any equilibrium position, and the driving torque is actually distributed symmetrically.

**Figure 2.** Force analysis diagram of variable transmission ratio differential.

Figure 2 is a stress analysis diagram of bevel gear differential with variable transmission ratio [3]. OQ1 and OQ2 are the axes of rotation of the two half-axle wheels respectively; OJ1 and OJ2 are the meshing instantaneous axes of the left and right half axle wheels and planetary bevel gears, namely the instantaneous contact lines of gear teeth. Because the normal line of contact points (lines) of tooth surfaces, that is, the action line of positive pressure between tooth surfaces must pass through the instantaneous axis, the normal positive pressure on the contact line of bevel gear as a gear pair in line contact must pass through OJ1 and OJ2. In the figure, \[ F_i = F_{ix} + F_{iy} + F_{iz} \] (\(i=1,2\)) is the resultant force of the positive pressure on the planet wheel at the contact position when the two half-axle wheels mesh with the planet wheel. \( \phi_1 \) and \( \phi_2 \) are the included angles between OJ1 and OJ2 and the axis op of the planetary gear respectively. The transmission ratio relationship between the left and right axle wheels and the planetary wheels is as follows:
\[ i_1 = \tan \phi_1 \]
\[ i_2 = \tan \phi_2 \]

When the planet wheels are in a relatively balanced position, that is, they only revolve around the axis of the half axle wheel without rotating, then the equation of moment balance is as follows:
\[ F_{1y} \circ \angle OJ1 \circ \sin \phi_1 = F_{2y} \circ \angle OJ2 \circ \sin \phi_2 \]

namely,
\[ F_{1y} / F_{2y} = OJ2 / OJ1 \circ \sin \phi_2 / \sin \phi_1 \]  \hspace{1cm} (5)

From the acting force and the reaction force, it is known that the torque transmitted by the planetary wheels received by the half axle wheels should be:
M0Q1 = - F1y \cdot OJ01 \cdot \cos \phi 1
M0Q2 = - F2y \cdot OJ02 \cdot \cos \phi 2
Therefore:
M0Q1 /M0Q2 = (F1y /F2y) \cdot (OJ01 \cdot \cos \phi 1 /OJ02 \cdot \cos \phi 2)
All of the above, and finally:
M0Q1 /M0Q2 = i2 /i1

From the above analysis, we can get enlightenment: if the designed bevel gear differential with variable transmission ratio can make the instantaneous transmission ratio i2 and i1 change according to the change of axle shaft wheel resistance torque, then within the locking range, the above two conclusions will hold, and the driving torque on the left side of the formula can be distributed proportionally (transmission ratio).

(i1/i2) max is called the theoretical locking coefficient of differential. Because of symmetry, when i1 takes the maximum value, i2 happens to be the minimum value. In this way, the locking coefficient becomes the ratio of the maximum value to the minimum value on the transmission ratio curve. It determines the limit of differential torque distribution. When this limit is exceeded, theoretically speaking, there is no relative equilibrium position of bevel gear pair, and the planetary bevel gears will rotate continuously while revolving. This also shows that it can still meet the basic differential requirements while improving the passability.

5. Analysis of locking coefficient of variable transmission ratio differential

Locking coefficient is the main evaluation parameter of the performance of limited slip differential. It is generally defined as the torque ratio kb of driving wheels on both sides, i.e.:
K = K_b = T_2 / T_1
Another more accurate definition is:
K = (T_2 - T_1) / (T_2 + T_1) = (K_b - 1) / (K_b + 1)

Where T_1 is the output torque of the fast-rotating wheel; T_2 is the output torque of the slowly rotating wheels. In order to keep consistent with the general literature, the former is adopted at the end of the paper.

The transmission ratio function of planetary gear and side gear is expressed as:
i = d \phi 1 / d \phi 2 = \tan \delta 1
Where, \phi 1, \phi 2- self-rotation angles of planetary gears and side gears;
\delta 1- the vertex angle of planetary gear cone.

Nowadays, the common bevel gear differential is widely used in automobiles, in which the two half-shaft gears mesh with the planetary gears at a constant transmission ratio, i21=const. Theoretically, the torque is evenly distributed to the left and right driving wheels under any circumstances, so the theoretical locking coefficient K=1. Even considering its own internal friction, the actual locking coefficient is only k = kb \approx 1.1 \sim 1.4 [4] because of its small internal friction.

For the cars that mainly travel on the highway, the road surface is generally good, and the adhesion coefficient between each driving wheel and the road surface will not change greatly. Therefore, the ordinary bevel gear differential is widely used because of its simple structure, convenient manufacture, stable and reliable operation and no side effects. However, for many off-road vehicles, they often have to drive on muddy and soft roads, even in roadless areas, and the driving conditions are extremely bad, so it is obvious that ordinary bevel gear differential is far from enough [5].

One of the ways to solve this problem is to increase the locking coefficient of differential. At present, differential locks are generally used on heavy vehicles. When a wheel is on a road with less adhesion, the differential lock can be operated to lock the differential housing and the half shaft together, and the differential loses the differential function, so the locking coefficient K=\infty. This makes full use of the adhesion of the left and right driving wheels, so that the obtained traction force reaches the maximum possible value. On medium and light vehicles, the disadvantages of differential lock are particularly prominent, so it is not widely used. At this time, other anti-skid measures can be adopted to increase the locking coefficient of differential. As far as improving the passing capacity of
off-road vehicles is concerned, it seems that the greater the locking coefficient, the better. However, practical experience proves that excessive locking coefficient not only has adverse effects on the steering flexibility, running stability, load of transmission system, tire wear and fuel consumption, but also reduces the passability of automobile under certain conditions. Therefore, when designing and selecting high-pass differential, we must consider its comprehensive performance, and correctly select the appropriate locking coefficient value, not the bigger the better.

The test shows that the adhesion coefficient between the low-pressure tires of off-road vehicles and the ground is the maximum on dry asphalt pavement or concrete pavement, with a value of $\phi_{\text{Max}} = 0.7 \sim 0.8$, and the minimum value on melting ice, with a value of $\phi_{\text{Min}} = 0.1 \sim 0.2$. The minimum value is considered as the ratio when the adhesion coefficient between the driving wheels on both sides and the ground is the same, and its value is 1. If the special case that one side of the driving wheels on both sides is suspended is removed here (in this case, differential lock is a better solution, and it is also an important reason for heavy vehicles that require absolute reliable locking to adopt differential lock), the adhesion coefficient between the driving wheels and the ground can vary from 1 to 8.

The design index of locking coefficient of differential is directly related to the change of adhesion coefficient between automobile tire and ground. The former is mainly based on the actual problem of the latter, and puts forward a solution. The ratio of adhesion coefficient between tires on both sides of the same driving axle and the ground is also reflected in the ratio of output torque of two axle wheels, and the ratio of the two is equal. Therefore, in order to make full use of the traction force of automobiles, it is enough that the locking coefficient of differential reaches 8 for medium and small vehicles.

The extreme value of the ratio of adhesion coefficients is the test value under two limit conditions. In most cases, the driving tests of automobiles on bad roads or roadless areas show that the ratio of adhesion coefficient between each driving wheel and the ground is generally not more than 3 ~ 4.5. Therefore, it is appropriate to select the differential with locking coefficient $k = 3 \sim 4.5$ for medium and small vehicles. At this time, the passability of the automobile can be significantly improved, but its steering maneuverability and other service performance do not deteriorate, and its comprehensive performance is better.

6. Conclusions
By analyzing the advantages and disadvantages of ordinary conical differential, a variable transmission ratio limited slip differential is put forward, and the functions of limited slip and differential achieved by the variable transmission ratio differential are analyzed from the angles of mechanics and working conditions. By analyzing the adhesion coefficient between tire and road surface, the locking coefficient index of the designed variable transmission ratio of limited slip differential is obtained.

References
[1] Qiu xuanhuai et al. mechanical design (fourth edition). higher education press, 1998.6
[2] Liu Weixin. Drive axle. People's Communications Press, June 1987.
[3] Editorial Committee , Automobile Engineering Manual (Experimental Section). People's Communications Press, May 2001
[4] Gong weihan. modern automobile design and manufacture. people's communications press, 1995.8
[5] Si Lizen g. Automobile anti-skid control system ---ABS and ASR. People's Communications Publishing House, June 1996