SUPERSYMMETRIC CP VIOLATING PHASES AND THE LSP RELIC DENSITY AND DETECTION RATES

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ABSTRACT

For varying values of $\tan \beta$, we study the effect of CP violating phases from the soft supersymmetry breaking terms in string-inspired models on the relic abundance and detection rates of the lightest neutralino (LSP). We find that the phases have no significant effect on the LSP relic abundance but can have a substantial impact on the detection rates.

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1. Introduction

CP violation is an important test for physics beyond the standard model. In the standard model, only one CP violating phase exists in the Kobayashi-Maskawa matrix. However, in the supersymmetric standard model there are many complex parameters, in addition to Yukawa couplings, which lead to new sources of CP violation. These include the mass coefficient $\mu$ of the bilinear term involving the two Higgs doublets, the $SU(3)$, $SU(2)$ and $U(1)$ gaugino masses $M_3$, $M_2$ and $M_1$, and the parameters $A_f$ and $B$ which respectively are the coefficients of the supersymmetric breaking trilinear and bilinear couplings. (the subscript $f$ denotes the flavor index.)

In minimal supersymmetric standard model (MSSM), only two of these phases are physical. Through appropriate field redefinitions we end up with the phase of $\mu (\phi_\mu)$ and the phase of $A (\phi_A)$ as the physical phases which cannot be rotated away. The phase of $B$ is fixed by the condition that $B\mu$ is real. It is known that, unless these phases are sufficiently small, their contributions to the neutron electric dipole moment (EDM) are larger than the experimental limit $1.1 \times 10^{-25}$ e.cm. Recently, the effect of these phases on the EDM of the neutron was examined in a model with dilaton-dominated supersymmetry (SUSY) breaking, taking into account the cancellation mechanism between the different contributions. It was shown that for a wide region of the parameter space the phase of $\mu$ is constrained to be of order $10^{-1}$, while the phase of $A$ is strongly correlated with that of $\mu$ in order not to violate the bound on the neutron EDM.

The effect of SUSY CP violating phases on the relic density of the LSP has been considered for the MSSM case in Ref. [12], and for the supersymmetric standard model coupled to N=1 supergravity in Ref. [13]. It was shown in Ref. [12] that the upper bound on the LSP (from $\Omega h^2 \leq 0.25$) is relaxed from 250 GeV to 650 GeV. The effect of CP violation on the direct detection rates of the LSP in MSSM is also considered in Ref. [15, 16]. We argue that such a large upper bound on the LSP is not possible in the model we consider here. We show that in case of the bino-like LSP the chance of the CP phases to have a significant effect is very small. The impact of the phases on the direct and indirect detection rates is an important issue and we present some details here.

The paper is organized as follows. In section 2 we discuss the effect of the CP phases on the LSP mass and purity within the string inspired model considered in Ref. [3]. In section 3 we compute the relic abundance of the LSP for low and intermediate values of $\tan \beta$. We find that the CP phases have almost no effect on the LSP relic density, so that
the upper bound on the LSP mass obtained in Ref. [11,9] remains unchanged. In section 4 we discuss the large $\tan \beta (\simeq m_t/m_b)$ case and again find that there is no significant effect of CP phases on the LSP relic density. In section 5 we show that CP phases can have a substantial effect on the LSP detection rates. Our conclusions are given in section 6.

2. STRING INSPIRED MODEL

We will consider the string inspired model which has been recently studied in Ref. [3]. In this model, the dilaton $S$ and overall modulus field $T$ both contribute to SUSY breaking. The soft scalar masses $m_i$ and the gaugino masses $M_a$ are given as [4]

$$m_i^2 = m_{3/2}^2(1 + n_i \cos^2 \theta), \quad (1)$$

$$M_a = \sqrt{3} m_{3/2} \sin \theta e^{-i\alpha_S}, \quad (2)$$

where $m_{3/2}$ is the gravitino mass, $n_i$ is the modular weight of the chiral multiplet, and $\sin \theta$ defines the ratio between the $F$-terms of $S$ and $T$. (For example, the limit $\sin \theta \to 1$ corresponds to a dilaton-dominant SUSY breaking). The phase $\alpha_S$ originates from the $F$-term of $S$.

The $A$-terms can be written as

$$A_{ijk} = -\sqrt{3} m_{3/2} \sin \theta e^{-i\alpha_S} - m_{3/2} \cos \theta (3 + n_i + n_j + n_k) e^{-i\alpha_T}, \quad (3)$$

where $n_i$, $n_j$ and $n_k$ are the modular weights of the fields that are coupled by this $A$-term. One needs a correction term in eq (3) when the corresponding Yukawa coupling depends on moduli fields. However, the $T$-dependent Yukawa coupling includes a suppression factor [5], and so we ignore it. Finally, the phase $\alpha_T$ originates from the $F$-term of $T$.

The magnitude of the soft SUSY breaking term $B\mu H_1 H_2$ depends on the way one generates a ‘natural’ $\mu$-term. Here we take $\mu$ and $B$ as free parameters and we will fix them by requiring successful electroweak (EW) symmetry breaking.

As stated earlier, the gaugino masses as well as $A$-terms and $B$-term are, in general, complex. We have the freedom to rotate $M_a$ and $A_{ijk}$ at the same time [6]. Here we use the basis in which $M_a$ is real. Similarly, we can rotate the phase of $B$ so that $B\mu$ itself is real. In other words, $\phi_B = -\phi_\mu = \text{arg}(BM^*)$. In this basis, $A$-terms contain a single phase, $(\alpha_A \equiv \alpha_T - \alpha_S)$.
As shown in eqs. (1-3), the values of the soft SUSY breaking parameters at string scale depend on the modular weights of the matter states. The modular weights of the matter fields $n_i$ are normally negative integers. Following the approach of Ref. [7] the ‘natural’ values of modular weights for matter fields (in case of $Z_N$ orbifolds) are $-1,-2,-3$ and $-4$. It was shown in Ref. [8] that the following modular weights for quark and lepton superfields is favorable for EW breaking

\[ n_Q = n_U = n_{H_1} = -1, \]

and

\[ n_D = n_L = n_E = n_{H_2} = -2. \]

Under this assumption we have

\[ A_t = A_b = -\sqrt{3} m_{3/2} \sin \theta + m_{3/2} \cos \theta e^{-i\alpha}, \]

and

\[ A_r = -\sqrt{3} m_{3/2} \sin \theta + 2 m_{3/2} \cos \theta e^{-i\alpha}, \]

(4)

(5)

Given the boundary conditions in eqs. (1-3) at the compactification scale, we determine the evolution of the couplings and the mass parameters according to their one loop renormalization group equation in order to estimate the mass spectrum of the SUSY particles at the weak scale. The radiative EW symmetry breaking imposes the following conditions on the renormalized quantities:

\[ m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 > 2B\mu, \]

(6)

\[ (m_{H_1}^2 + \mu^2)(m_{H_2}^2 + \mu^2) < (B\mu)^2, \]

(7)

\[ \mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{M_Z^2}{2}, \]

(8)

and

\[ \sin 2\beta = \frac{-2B\mu}{m_{H_1}^2 + m_{H_2}^2 + 2\mu^2}, \]

(9)

where $\tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle$ is the ratio of the two Higgs VEVs that gives masses to the up and down type quarks, and $m_{H_1}^2$, $m_{H_2}^2$ are the two soft Higgs square masses at the EW scale. Using the above equations we can determine $|\mu|$ and $B$ in terms of $m_{3/2}$, $\theta$ and $\alpha_A$. The phase of $\phi_\mu$ remains undetermined.
Since we are interested in investigating the effect of the supersymmetric phases $\alpha_A$ and $\phi_\mu$ on the relic density of the LSP and its direct and indirect detection rates, we first study the allowed regions of these phases and later impose the constraints (derived in Ref [3]) from the experimental bounds on the electric dipole moments.

The neutralinos $\chi_0^i$, ($i = 1, 2, 3, 4$) are the physical (mass) superpositions of the Higgsinos $\tilde{H}_1^0, \tilde{H}_2^0$ and the two neutral gaugino $\tilde{B}^0$ (bino) and $\tilde{W}_3^0$ (wino). The neutralino mass matrix is given by

$$M_N = \begin{pmatrix}
M_1 & 0 & -M_Z \cos \beta \sin \theta_W & M_Z \sin \beta \sin \theta_W \\
0 & M_2 & M_Z \cos \beta \cos \theta_W & -M_Z \sin \beta \cos \theta_W \\
-M_Z \cos \beta \sin \theta_W & M_Z \cos \beta \cos \theta_W & 0 & \mu e^{\phi_\mu} \\
M_Z \sin \beta \sin \theta_W & -M_Z \sin \beta \cos \theta_W & \mu e^{\phi_\mu} & 0
\end{pmatrix}, \quad (10)$$

where $M_1$ and $M_2$ now refer to ‘low energy’ quantities whose asymptotic values are given in equation (2). The lightest eigenstates $\tilde{\chi}_1^0$ is a linear combination of the original fields:

$$\tilde{\chi}_1^0 = N_{11} \tilde{B} + N_{12} \tilde{W}_3^3 + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0,$$  

(11)

where the unitary matrix $N_{ij}$ relates the $\tilde{\chi}_i^0$ fields to the original ones. The entries of this matrix depend on $m_{3/2}, \theta$ and $\phi_\mu$. The dependence of the $\tilde{\chi}_1^0$ (LSP) mass on $\phi_\mu$ is shown in figure 1 for $m_{3/2} \simeq 100 GeV$, $\cos^2 \theta \simeq 1/2$ and $\alpha_A \simeq \pi/2$.

![Figure 1. LSP mass as a function of the phase $\phi_\mu$.](attachment:figure1.png)
A useful parameter for describing the neutralino composition is the gaugino "purity" function

\[ f_g = |N_{11}|^2 + |N_{12}|^2 \]  

(12)

We plot this function versus \( \phi_\mu \) in figure (2) which clearly shows that the LSP is essentially a pure bino. These two figures show that the neutralino mass and composition are only slightly dependent on the supersymmetric phase \( \phi_\mu \).

3. **Relic Abundance Calculation for low and intermediate values of \( \tan \beta \)**

In this section we compute the relic density of the LSP in the case of low \( \tan \beta \) (i.e. \( \tan \beta \simeq 3 \)), as well as for intermediate \( \tan \beta \) values (i.e. \( \tan \beta \simeq 15 \)). Using a standard method [9] in which we expand the thermally averaged cross section \( \langle \sigma_A v \rangle \) as

\[ \langle \sigma_A v \rangle = a + bv^2 + \ldots, \]  

(13)

where \( v \) is the relative velocity, \( a \) is the s-wave contribution at zero relative velocity and \( b \) contains contributions from both the s and p waves, the relic abundance is given by [9].

\[ \Omega_\chi h^2 = \frac{\rho_\chi}{\rho_c h^2} = 2.82 \times 10^8 Y_\infty (m_\chi / GeV), \]  

(14)
where

\[ Y_{\infty}^{-1} = 0.264 \ g_s^{1/2} \ M_P \ m_\chi \left( \frac{a}{x_F} + \frac{3b}{x_F^2} \right), \]  

(15)

\( h \) is the well known Hubble parameter, \( 0.4 \leq h \leq 0.8 \), and \( \rho_c \sim 2 \times 10^{-29} h^2 \) is the critical density of the universe. The freeze-out temperature is given by

\[ x_F = \ln \frac{0.0764 M_P (a + 6b/x_F) c(2 + c) m_\chi}{\sqrt{g_* x_F}} \]  

(16)

Here \( x_F = m_\chi/T_F \), \( M_P = 1.22 \times 10^{19} \) GeV is the Planck mass, and \( g_* \) (8 \( \leq \sqrt{g_*} \leq 10 \)) is the effective number of relativistic degrees of freedom at \( T_F \). Also \( c = 1/2 \) as explained in Ref \[10\].

Given that the LSP is bino-like, the annihilation is predominantly into leptons, with the other channels either closed or suppressed. It is worth noting that the squark exchanges are suppressed due to their large masses, as figure (3) shows.

![Figure 3](image_url)

**Figure 3.** The squarks (dashed-dotted) and sleptons (solid line) masses versus the LSP mass, for \( \cos^2 \theta \simeq 1/2 \) and \( \alpha_A \simeq \pi/2 \).

The annihilation process is dominated by the exchange of the right slepton. In fact, the masses of the right slepton are essentially independent of \( \alpha_A \) and \( \phi_\mu \), unless there is a significant amount of slepton mixing. Here, the off-diagonal element of the matrices are much smaller than the diagonal elements \( M^2_{l_L} \) and \( M^2_{l_R} \). Furthermore, since the LSP is essentially a bino, it only slightly depends on the phase of \( \mu \) as figure (1) confirms.
Therefore, we find that the constraint on the relic density: $0.1 \leq \Omega_{\text{LSP}} \leq 0.9$, with $0.4 \leq h \leq 0.8$ leads, as figure (4) shows, to the previously known upper bound on the LSP mass found in the case of vanishing SUSY phases [11, 13], namely, $m_\chi \leq 250$ GeV.

![Figure 4. The relic abundance of the LSP versus its mass.](image)

This result is different from the one discussed in Ref. [12] where it was claimed that the CP violating phases have a significant effect in that the cosmological upper bound on the bino mass is increased from 250 GeV to 650 GeV. This enhancement can be traced to the assumptions proposed in that model, namely that all the scalar masses are equal and of order $M_W$ at the weak scale. Also, the sfermion mixings were assumed to be large (with $\mu \sim TeV$). It turns out that such assumptions may lead to unacceptable charge and color breaking, as explained in Ref. [13]. Moreover, they cannot be motivated from supergravity or superstring models.

We have also considered the relic density for intermediate $\tan \beta$ (i.e. $\tan \beta \simeq 15$). We find that there is no significant difference between this and the case of low $\tan \beta$. The upper bound on the LSP mass is still of order 250 GeV.

4. SUSY CP phases with LARGE $\tan \beta$

We now extend our study to the case when $\tan \beta$ is large ($\simeq 50$). In a large class of supersymmetric models with flavor $U(1)$ symmetry, $\tan \beta \sim (\frac{m_t}{m_b})^{\epsilon^n}$, where $\epsilon \simeq 0.2$ ($\simeq$ Cabibbo angle) is a ‘small’ expansion parameter, and $n = 0, 1, 2$. (See for instance [14] and reference therein).
For \( \tan \beta \simeq \frac{m_\mu}{m_t} \), it is known that the phenomenological aspects of these models are very different compared with the small \( \tan \beta \) case. In particular, radiative EW symmetry breaking is an important non-trivial issue. Non-universality, such as \( m_{H_1}^2 > m_{H_2}^2 \) at the Planck scale, is favored for a successful EW breaking with large \( \tan \beta \). Further, non-universality of the squark and slepton masses can affect symmetry breaking as well as other phenomenological aspects. We have adopted this non-universality in our choice for the modular weights in section 2.

In the large \( \tan \beta \) case the Higgs potential has two characteristic features. It follows from the minimization conditions that

\[
m_2^2 \simeq -\frac{M_Z^2}{2},
\]

\[
m_3^2 \simeq \frac{M_A^2}{\tan^2 \beta} \sim 0,
\]

with

\[
M_A^2 \simeq m_1^2 + m_2^2 > 0.
\]

Here, \( m_i^2 = m_{H_i}^2 + \mu, i = 1, 2 \) and \( m_3^2 = B\mu \). A combination of eqs.(17) and (19) gives the following constraint on the low energy parameters

\[
m_1^2 - m_2^2 > M_Z^2,
\]

i.e \( m_{H_1}^2 - m_{H_2}^2 > M_Z^2 \). In order to have electroweak breaking in the large \( \tan \beta \) case, the difference between the masses of the two Higgs fields should satisfy the above inequality.

In our model we find that this inequality is indeed satisfied, and the EW symmetry is broken at the weak scale. Also, one of the stau leptons \( \tilde{\tau}_R \) has a ‘small’ mass of order \( \mathcal{O}(100) \) GeV, and happens to be the lightest slepton. It therefore dominates the LSP annihilation process. This relaxes the upper bound on the LSP mass from 250 to 300 GeV. Thus, even in the of large \( \tan \beta \) case the effect of the supersymmetric phases are relatively small as figure (5) shows. This essentially follows because the diagonal elements of the stau mass matrices, in this model are larger than the off-diagonal ones, i.e., there is no large mixing, as well as from the fact that the LSP is bino like.
Figure 5. The relic abundance with non-vanishing phases (solid line) and vanishing phases (dashed line) versus the LSP mass in case of $\tan \beta \simeq \frac{m_t}{m_b}$.

5. CP phases and detection rates of the LSP

We have seen that the effects of CP violating phases on the neutralino relic density are very small. In this section we examine the effect of these phases on the event rates of relic neutralinos scattering off nuclei in terrestrial detectors. The direct detection experiments provide the most natural way of searching for the neutralino dark matter. Any large CP violating phases can affect the detection rate, as will see below. It is interesting to note that the measured event rate may shed light on the value of the supersymmetric phases.

The differential detection rate is given by

$$\frac{dR}{dQ} = \frac{\sigma \rho_\chi}{2m_\chi m_r^2} F^2(Q) \int_{v_{\text{min}}}^{\infty} \frac{f_1(v)}{v} dv,$$

where $f_1(v)$ is the distribution of speeds relative to the detector. The reduced mass is $m_r = \frac{m_\chi m_N}{m_\chi^2 + m_r^2}$, where $m_N$ is the mass of the nucleus, $v_{\text{min}} = (\frac{Q m_N}{2m_r^2})^{1/2}$, $Q$ is the energy deposited in the detector, and $\rho_\chi$ is the density of the neutralino near the Earth. $\sigma$ is the elastic-scattering cross section of the LSP with a given nucleus. In general $\sigma$ has two contributions: a spin-dependent contribution arising from $Z^0$ and $\tilde{q}$ exchange diagrams, and a spin-independent (scalar) contribution due to the Higgs and squark exchange diagrams. For $^{76}Ge$ detector, where the total spin of $^{76}Ge$ is equal to zero, we have contributions
only from the scalar part.

\[ \sigma = \frac{4m_f^2}{\pi} [Zf_p + (A - Z)f_n]^2, \]  

(22)

where \( Z \) is the nuclear charge, and \( A - Z \) is the number of neutrons. The expressions for \( f_p \) and \( f_n \), and their dependence on the SUSY phases can be found in Ref. [15, 16]. The effect of the CP violating phases enter through the neutralino eigenvector components \( N_{ij} \), and also through the matrices that diagonalize the squark mass matrices. Finally, \( F(Q) \) in (21) is the form factor. We use the standard parameterization [17]

\[ F(Q) = \frac{3j_1(qR_1)}{qR_1} e^{-\frac{1}{2}q^2s^2}, \]  

(23)

where the momentum transfer \( q^2 = 2m_N Q \), \( R_1 = (R^2 - 5s^2)^{1/2} \) with \( R = 1.2 fm A^{1/2} \), and \( A \) is the mass number of \( ^{76}Ge \). \( j_1 \) is the spherical Bessel function and \( s \approx 1 fm \).

The ratio \( R \) of the event rate with non-vanishing CP violating phases to the event rate in the absence of these phases is presented in figure (6). The solid curve corresponds to the case \( \alpha_A = 0 \), while the dashed one corresponds to \( \alpha_A = \pi/2 \).

![Graph](image)

**Figure 6.** The ratio of the direct detection rates as function of \( \phi_\mu \).

From this figure, it is clear that, in the model we are considering, the CP violating phases can significantly affect the event rates of the direct detection of the LSP. The phase \( \phi_\mu \) reduces the value of \( R \), while the phase \( \alpha_A \) in the trilinear coupling increases
it. However, as explained in Ref. [3], $\phi_\mu$ is constrained from the electric dipole moment experimental limit to be $\leq 10^{-1}$

For completeness, we also examine the effect of CP violating phases on the indirect detection rates of the LSP in the halo. The observation of energetic neutrinos from the annihilation of the LSP that accumulate in the sun or in the earth is a promising method for detecting them. The technique for detecting such energetic neutrinos is through observation of upward going muons produced by charged current interactions of the neutrinos in the rock below the detector. The flux of such muons from neutralino annihilation in the sun is given by

$$\Gamma = 2.9 \times 10^8 m^{-2} yr^{-1} \tanh^2(t/\tau) \rho_\chi^{0.3} f(m_\chi) \zeta(m_\chi) \left(\frac{m_\chi^2}{GeV}\right)^2 \left(\frac{f_P}{GeV^{-2}}\right)^2.$$  \hspace{0.5cm} (24)

The neutralino-mass dependence of the capture rates is described by [9]

$$f(m_\chi) = \sum_i f_i \phi_i S_i(m_\chi) F_i(m_\chi) \frac{m_\chi^2}{(m_\chi + m_i)^2},$$  \hspace{0.5cm} (25)

where the quantities $\phi_i$ and $f_i$ describe the distribution of element $i$ in the sun and they are listed in Ref. [9], the quantity $S_i(m_\chi) = S(\frac{m_\chi}{m_{N_i}})$ is the kinematics suppression factor for the capture of neutralino of mass $m_\chi$ from a nucleus of mass $m_{N_i}$ [9], and $F_i(m_\chi)$ is the form factor suppression for the capture of a neutralino of mass $m_\chi$ by a nucleus $i$. Finally, the function $\zeta(m_\chi)$ describes the energy spectrum from neutralino annihilation for a given mass.

In Figure (7) we present the ratio of the muon fluxes resulting from captured neutralinos in the sun in the case of non-vanishing CP violating phases to that of vanishing $\phi_A$, for $\rho_\chi = 0.3 GeV/cm^3$. We see that the predicted muon flux increases as the phase of $A$-term is increased.

We can understand this important effect of CP violating phases on the detection rate as follows. The phases affect the neutralino eigenvector components $N_{ij}$ and the squark mass matrices. Consequently, they have a significant effect on the neutralino coupling to quarks. The spin independent contribution, as also shown in Ref. [15] and [16], is decreased by increasing the phase of $\mu$, and goes in the other direction if the phase of $A$-term is increased. This leads to the same behaviour for the elastic scattering cross
Figure 7. The ratio of the indirect detection rates as function of $\phi_\mu$.

section, which translates this dependence on the phases of $\mu$ and $A$ to the detection rates as figures 6 and 7 confirm.

6. Conclusions

We have studied the impact of CP violating phases from soft SUSY breaking terms in string-inspired models on the LSP, its purity and its relic abundance density. For different values of $\tan \beta$ (of order unity, intermediate and of order $m_t/m_b$), we found that these phases have no significant effect on the LSP relic density, so that the upper bound on the LSP mass is essentially unchanged. We have examined the effect of these phases on the direct and indirect detection rates. We found that increasing the value of the phase $\phi_\mu$ leads to a decrease the event rates, while the phase $\phi_A$ of the trilinear coupling has the opposite effect.

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