Strong Degrees in Single Valued Neutrosophic Graphs

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Abstract—The concept of single valued neutrosophic graphs (SVNGs) generalizes the concept of fuzzy graphs and intuitionistic fuzzy graphs. The purpose of this research paper is to define different types of strong degrees in SVNGs and introduce novel concepts, such as the vertex truth-membership, vertex indeterminacy-membership and falsity-membership sequence in SVNG with proof and numerical illustrations.

Keywords—Single valued neutrosophic graph (SVNG); neutrosophic set; sequence; strong degree

I. INTRODUCTION

In [1], [3] Smarandache explored the notion of neutrosophic sets (NS in short) as a powerful tool which extends the concepts of crisp set, fuzzy sets and intuitionistic fuzzy sets [2]-[6]. This concept deals with uncertain, incomplete and indeterminate information that exist in real world. The concept of NS sets associate to each element of the set a degree of membership $T_A(x)$, a degree of indeterminacy $I_A(x)$ and a degree of falsity $F_A(x)$, in which each membership degree is a real standard or non-standard subset of the nonstandard unit $[0, 1]^*$. Smarandache [1], [2] and Wang [7] defined the concept of single valued neutrosophic sets (SVNS), an instance of NS, to deal with real application. In [8], the readers can found a rich literature on SVNS.

In more recent times, combining the concepts of NSs, interval valued neutrosophic sets (IVNSs) and bipolar neutrosophic sets with graph theory, Broumi et al. introduced various types of neutrosophic graphs including single valued neutrosophic graphs (SVNGs for short) [9], [11], [14], interval valued neutrosophic graphs [13], [18], [20], bipolar neutrosophic graphs [10], [12], all these graphs are studied deeply. Later on, the same authors presented some papers for solving the shortest path problem on a network having single valued neutrosophic edges length [17], interval valued neutrosophic edge length [28], bipolar neutrosophic edge length [21], trapezoidal neutrosophic numbers [15], SV-trapezoidal neutrosophic numbers [16], triangular fuzzy neutrosophic [19]. Our approach of neutrosophic graphs are different from that of Akram et al. [26]-[28] since while Akram considers, for the neutrosophic environment (<=, <=, >=) we do (<=, >=, >=) which is better, since while T is a positive quality, I, F are considered negative qualities. Akram et al. include “I” as a positive quality together with “T”. So our papers improve Akram et al.’s papers. After that, several authors are focused on the study of SVNGs and many extensions of SVNGs have been developed. Hamidi and Borumand Saeid [25] defined the notion of accessible-SVNGs and apply it social networks. In [24], Mehra and Manjeet defined the notion of single valued neutrosophic signed graphs. Hassan et al. [30] proposed some kinds of bipolar neutrosophic graphs. Naz et al. [23] studied some basic operations on SVNGs and introduced vertex degree of these operations for SVNGs and provided an application of single valued neutrosophic digraph (SVNDG) in travel time. Ashraf et al. [22] defined new classes of SVNGs and studied some of its important properties. They solved a multi-attribute decision making problem using a SVNDG. Mullai [31] solved the spanning tree problem in bipolar neutrosophic environment and gave a numerical example.

Motivated by the Karunambigai work’s [29]. The concept of strong degree of intuitionistic fuzzy graphs is extended to strong degree of SVNGs.

This paper has been organized in five sections. In Section 2, we firstly review some basic concepts related to neutrosophic set, single valued neutrosophic sets and SVNGs. In Section 3, different strong degree of SVNGs are proposed and studied with proof and example. In Section 4, the concepts of vertex truth-membership, vertex indeterminacy-
membership and vertex falsity membership is discussed. Lastly, Section 5 concludes the paper.

II. PRELIMINARIES AND DEFINITIONS

In the following, we briefly describe some basic concepts related to neutrosophic sets, single valued neutrosophic sets and SVNGs.

**Definition 2.1** [1] Given the universal set \( \zeta \). A neutrosophic set \( A \) on \( \zeta \) is characterized by a truth membership function \( T_A \), an indeterminacy membership function \( I_A \) and falsity membership function \( F_A \), where \( T_A, I_A, F_A : \zeta \to [0,1] \). For all \( x \in \zeta \), \( x=(x,T_A(x),I_A(x),F_A(x)) \in A \) is neutrosophic element of \( A \).

The neutrosophic set can be written in the following form:

\[
A = \{ <x : T_A(x), I_A(x), F_A(x)> , x \in \zeta \}
\]  

(1)

with the condition

\[
-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+
\]

(2)

**Definition 2.2** [7] Given the universal set \( \zeta \). A single valued neutrosophic set \( A \) on \( \zeta \) is characterized by a truth membership function \( T_A \), an indeterminacy membership function \( I_A \) and falsity membership function \( F_A \), where \( T_A, I_A, F_A : \zeta \to [0,1] \). For all \( x \in \zeta \), \( x=(x, T_A(x), I_A(x), F_A(x)) \in A \) is a single valued neutrosophic element of \( A \).

The single valued neutrosophic set can be written in the following form:

\[
A = \{ <x : T_A(x), I_A(x), F_A(x)> , x \in \zeta \}
\]  

(3)

with the condition

\[
0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3
\]

(4)

**Definition 2.3** [14] ASNV-graph \( G \) is of the form \( G=(A,B) \) where \( A \)

1. \( A=[v_1, v_2, \ldots, v_n] \) such that the functions \( T_A : A \to [0,1], I_A : A \to [0,1], F_A : A \to [0,1] \) denote the truth-membership function, an indeterminacy-membership function and falsity-membership function of the element \( v_i \in A \) respectively and

\[
0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \forall v_i \in A
\]

(5)

2. \( B=\{(v_i,v_j) ; (v_i,v_j) \in A \times A \} \) and the function \( T_B : B \to [0,1], I_B : B \to [0,1], F_B : B \to [0,1] \) are defined by

\[
T_B(v_i,v_j) = \min (T_A(v_i), T_A(v_j))
\]

(6)

\[
I_B(v_i,v_j) = \max (I_A(v_i), I_A(v_j))
\]

(7)

\[
F_B(v_i,v_j) = \max (F_A(v_i), F_A(v_j))
\]

(8)

Where \( T_B, I_B, F_B \) denotes the truth-membership function, indeterminacy membership function and falsity membership function of the edge \( (v_i,v_j) \in B \) respectively where

\[
0 \leq T_B(v_i,v_j) + I_B(v_i,v_j) + F_B(v_i,v_j) \leq 3
\]

(9)

**Definition 3.1** Let \( G=(V,E) \) be a single valued neutrosophic graph. The minimum strong degree of a vertex \( v_i \in V \) is defined as

\[
d_s(T)(v_i) = \sum_{c_i \in E} T_{ij} , e_{ij} \text{ are strong edges incident at } v_i.
\]

**Definition 3.2** Let \( G=(V,E) \) be a single valued neutrosophic graph. The minimum strong degree of a vertex \( v_i \in V \) is defined as

\[
d_s(I)(v_i) = \sum_{c_i \in E} I_{ij} , e_{ij} \text{ are strong edges incident at } v_i.
\]

**Definition 3.3** Let \( G=(V,E) \) be a single valued neutrosophic graph. The minimum strong degree of a vertex \( v_i \in V \) is defined as

\[
d_s(F)(v_i) = \sum_{c_i \in E} F_{ij} , e_{ij} \text{ are strong edges incident at } v_i.
\]

**Definition 3.4** Let \( G=(V,E) \) be a single valued neutrosophic graph. The strong degree of a vertex \( v_i \in V \) is as follows

\[
d_s(v_i) = \left[ \sum_{c_i \in E} T_{ij} , \sum_{c_i \in E} I_{ij} , \sum_{c_i \in E} F_{ij} \right] , \text{ where } e_{ij} \text{ are strong edges incident at } v_i.
\]

**Definition 3.5** Let \( G=(V,E) \) be a single valued neutrosophic graph. The minimum strong degree of a single valued neutrosophic graph is defined as

\[
\delta_s(G) = \left( \delta_s(T)(G), \delta_s(I)(G), \delta_s(F)(G) \right) , \text{ where}
\]

\[
\delta_s(T)(G) = \wedge \left\{ d_s(T)(v_i) \big/ v_i \in V \right\} \text{ is the minimum T-strong degree of } G.
\]
\[ \delta_{s(T)}(G) = \land \{d_{s(T)}(v_i) / v_i \in V \} \] is the minimum I-
strong degree of G.

\[ \delta_{s(F)}(G) = \land \{d_{s(F)}(v_i) / v_i \in V' \} \] is the minimum F-
strong degree of G.

**Definition 3.6** Given the SVN-graph \( G=(V, E) \). The maximum strong degree of G is defined as

\[ \Delta_s(G) = (\Delta_{s(T)}(G), \Delta_{s(I)}(G), \Delta_{s(F)}(G)) \], where

\[ \Delta_{s(T)}(G) = \lor \{d_{s(T)}(v_i) / v_i \in V \} \] is the maximum T-
stong degree of G.

\[ \Delta_{s(I)}(G) = \lor \{d_{s(I)}(v_i) / v_i \in V' \} \] is the maximum I-
strong degree of G.

\[ \Delta_{s(F)}(G) = \lor \{d_{s(F)}(v_i) / v_i \in V' \} \] is the maximum F-
stong degree of G.

**Definition 3.7** Let G be a SVNG, the T-total strong degree of a vertex \( v_i \in V \) in G is defined as

\[ td_{s(T)}(v_i) = d_{s(T)}(v_i) + T_i \] .

**Definition 3.8** Let G be a SVNG, the I-total strong degree of a vertex in G is defined as \( v_i \in V \)

\[ td_{s(I)}(v_i) = d_{s(I)}(v_i) + I_i \] .

**Definition 3.9** Let G be a SVNG, the F-total strong degree of a vertex \( v_i \in V \) in G is defined as

\[ td_{s(F)}(v_i) = d_{s(F)}(v_i) + F_i \] .

**Definition 3.10** Let G be a SVNG, the total strong degree of a vertex \( v_i \in V \) in G is defined as

\[ td_s(v_i) = \left[ td_{s(T)}(v_i), td_{s(I)}(v_i), td_{s(F)}(v_i) \right] \]

**Definition 3.11** Given the SVN-graph \( G=(V, E) \). The minimum total strong degree of G is defined as

\[ \delta_s(G) = (\delta_{s(T)}(G), \delta_{s(I)}(G), \delta_{s(F)}(G)) \], where

\[ \delta_{s(T)}(G) = \land \{d_{s(T)}(v_i) / v_i \in V \} \] is the minimum T-total strong degree of G.

\[ \delta_{s(I)}(G) = \land \{d_{s(I)}(v_i) / v_i \in V' \} \] is the minimum I-total strong degree of G.

\[ \delta_{s(F)}(G) = \land \{d_{s(F)}(v_i) / v_i \in V' \} \] is the minimum F-total strong degree of G.

**Definition 3.12** Given the SVN-graph \( G=(V, E) \). The maximum total strong degree of G is defined as:

\[ \Delta_s(G) = (\Delta_{s(T)}(G), \Delta_{s(I)}(G), \Delta_{s(F)}(G)) \], where

\[ \Delta_{s(T)}(G) = \lor \{d_{s(T)}(v_i) / v_i \in V \} \] is the maximum T-total strong degree of G.

\[ \Delta_{s(I)}(G) = \lor \{d_{s(I)}(v_i) / v_i \in V \} \] is the maximum I-total strong degree of G.

\[ \Delta_{s(F)}(G) = \lor \{d_{s(F)}(v_i) / v_i \in V \} \] is the maximum F-total strong degree of G.
**Definition 3.21** Let G be a SVNG. If \( d_{s(T)}(v_i) = k_1 \),
\( d_{s(I)}(v_i) = k_2 \) and \( d_{s(F)}(v_i) = k_3 \) for all \( v_i \in V \), then the SVNG is called as \((k_1, k_2, k_3)\) - strong constant SVNG (or) Strong constant SVNG of degree \((k_1, k_2, k_3)\).

**Definition 3.22** Let G be a SVNG. If \( td_{s(T)}(v_i) = r_1 \),
\( td_{s(I)}(v_i) = r_2 \) and \( d_{s(F)}(v_i) = r_3 \) for all \( v_i \in V \), then the SVNG is called as \((r_1, r_2, r_3)\) - totally strong constant SVNG (or) totally strong constant SVNG of degree \((r_1, r_2, r_3)\).

**Proposition 3.23** In a SVNG, G
\[
2S_{s(T)}(G) = \sum_{i=1}^{n} d_{s(T)}(v_i),
2S_{s(I)}(G) = \sum_{i=1}^{n} d_{s(I)}(v_i)
\] and
\[
2S_{s(F)}(G) = \sum_{i=1}^{n} d_{s(F)}(v_i)
\]

**Proposition 3.24** In a connected SVNG,
1) \( d_{s(T)}(v_i) \leq d_{s(I)}(v_i) \leq d_{s(F)}(v_i) \)
2) \( td_{s(T)}(v_i) \leq td_{s(I)}(v_i) \leq td_{s(F)}(v_i) \)

**Proposition 3.25** Let G be a SVNG where crisp graph \( G^* \) is an odd cycle. Then G is strong constant if \( f < T_{ij}, I_{ij}, F_{ij} \) is constant function for every \( e_{ij} \in E \).

**Proposition 3.26** Let G be a SVNG where crisp graph \( G^* \) is an even cycle. Then G is strong constant if \( f < T_{ij}, I_{ij}, F_{ij} \) is constant function or alternate edges have same true membership, indeterminate membership and false membership for every \( e_{ij} \in E \).

**Remark 3.27** The above proposition 3.25 and proposition 3.26 hold for totally strong constant SVNG, if \( < T_{ij}, I_{ij}, F_{ij} \) is a constant function.

**Remark 3.28** A complete SVNG need not be a strong constant SVNG and totally strong constant SVNG.

**Remark 3.29** A strong SVNG need not be a strong constant SVNG and totally strong constant SVNG.

**Remark 3.30** For a strong vertex \( v_i \in V \),
1) \( d_{s(T)}(v_i) = d_{s(T)}(v_i) = d_{s(F)}(v_i) = d_{s(F)}(v_i) \) and
2) \( td_{s(T)}(v_i) = td_{s(T)}(v_i) \), \( td_{s(I)}(v_i) = td_{s(I)}(v_i) \) and
\( td_{s(F)}(v_i) = td_{s(F)}(v_i) \)

**Theorem 3.31** Let G be a complete SVNG with V = \( \{v_1, v_2, \ldots, v_n\} \) such that \( T_{ij} = T_{ij} \leq T_{ij} \leq \ldots \leq T_{ij} \), \( I_{ij} \geq I_{ij} \geq I_{ij} \geq \ldots \geq I_{ij} \) and \( F_{ij} \geq F_{ij} \geq F_{ij} \geq \ldots \geq F_{ij} \) Then
1) \( T_{ij} \) is minimum edge truth membership, \( I_{ij} \) is the maximum edge indeterminacy membership and \( F_{ij} \) is the maximum edge falsity membership of \( e_{ij} \) emits from \( v_i \) to \( v_j \) for all \( j = 2, 3, 4, \ldots, n \).
2) \( T_{in} \) is maximum edge truth membership, \( I_{in} \) is the minimum edge indeterminacy membership and \( F_{in} \) is the minimum edge falsity membership of among all edges from \( v_i \) to \( v_n \) for all \( i = 1, 2, 3, 4, \ldots, n-1 \).
3) \( td_{T}(v_i) = \Delta_{td}(G) = n.T_{i1}, td_{I}(v_i) = \Delta_{td}(G) = n.I_{i1} \) and \( td_{F}(v_i) = \Delta_{td}(G) = n.F_{i1} \).
4) \( td_{T}(v_i) = \Delta_{td}(G) = \sum_{i=1}^{n} I_{i1}, td_{I}(v_i) = \Delta_{td}(G) = \sum_{i=1}^{n} I_{i1} \) and \( td_{F}(v_i) = \Delta_{td}(G) = \sum_{i=1}^{n} I_{i1} \).

**Proof:** Throughout the proof, suppose that \( T_{i1} \leq T_{i2} \leq T_{i3} \leq \ldots \leq T_{in} \), \( I_{i1} \geq I_{i2} \geq I_{i3} \geq \ldots \geq I_{in} \) and \( F_{i1} \geq F_{i2} \geq F_{i3} \geq \ldots \geq F_{in} \).

1) To prove that \( T_{ij} \) is minimum edge truth membership, \( I_{ij} \) is the maximum edge indeterminacy membership and \( F_{ij} \) is the maximum edge falsity membership of \( e_{ij} \) emits from \( v_i \) to \( v_j \) for all \( j = 2, 3, 4, \ldots, n \).

Being a complete SVNG,
\[
T_{ii} = \min \{ T_{i1}, T_{i1} \}, I_{ii} = \max \{ I_{i1}, I_{i1} \} \text{ and } F_{ii} = \max \{ F_{i1}, F_{i1} \}
\]

Then \( T_{ki} = \min \{ T_{k1}, T_{i1} \}, I_{ki} = \max \{ I_{k1}, I_{i1} \} \text{ and } F_{ki} = \max \{ F_{k1}, F_{i1} \} \)

Since \( T_{ki} < T_{ii} \Rightarrow \min \{ T_{ki}, T_{i} \} < \min \{ T_{i1}, T_{i1} \} \)

Thus either \( T_{ki} < T_{i1} \text{ or } T_{i1} < T_{i1} \).

Also since \( I_{ki} > I_{ii} \Rightarrow \max \{ I_{ki}, I_{i} \} > \max \{ I_{i1}, I_{i1} \} \), so either \( k < i \text{ or } i < i \).

Since 1, \( k \neq 1 \), this is contradiction to our vertex assumption that \( T_{i1} \) is the unique minimum vertex true membership, \( I_{i1} \) is the maximum vertex indeterminate membership and \( F_{i1} \) is the maximum vertex false membership.

Hence \( T_{ij} \) is minimum edge true membership, \( I_{ij} \) is the maximum edge indeterminate membership and \( F_{ij} \) is the maximum edge false membership of \( e_{ij} \) emits from \( v_i \) to \( v_j \) for all \( j = 2, 3, 4, \ldots, n \).

2) On the contrary, assume let \( e_{kn} \) is not an edge with maximum true membership, minimum indeterminate membership and minimum false membership emits from \( v_k \) for 1 \( \leq k \leq n-1 \). On the other hand, let \( e_{kn} \) be an edge with maximum true membership, minimum indeterminate membership and minimum false membership emits from \( v_k \) for 1 \( \leq r \leq n-1 \), \( k \neq r \).

Then \( T_{kr} > T_{kn} \Rightarrow \min \{ T_{k1}, T_{r1} \} > \min \{ T_{k1}, T_{n1} \} = T_{k1} \), so \( T_{kr} > T_{k1} \).

\( k < r \Rightarrow k < r \Rightarrow \max \{ I_{k1}, I_{r1} \} < \max \{ I_{k1}, I_{n1} \} = I_{k1} \), so \( I_{r1} < I_{k1} \) and...
Similarly \( F_{kr} < F_{kn} \) implies that \( \{ F_{k}, F_{r} \} \) is a maximum for \( \{ F_{k}, F_{n} \} \).

So \( T_{kr} = T_{k} = T_{n} \) and \( F_{kr} = F_{k} = F_{kn} \), which is a contradiction. Hence \( \epsilon_{kn} \) is an edge with maximum true membership, minimum indeterminate membership and minimum false membership among all edges emits from \( v_{k} \) to \( v_{n} \).

3) Now

\[
\begin{align*}
& \left( \sum_{e_{ij} \in T} T_{1} + 1 \right) = (n-1)T_{1} + 1 = nT_{1} - T_{1} + 1 = nT_{1}, \\
& \sum_{e_{ij} \in T} T_{ij} + 1 = \sum_{i=1}^{n} T_{ij} + 1,
\end{align*}
\]

Since \( T_{ij} \) for \( i = 1, 2, 3, \ldots, n \) and for all other indices \( j \), it follows that

\[
(n-1)T_{ij} + 1 < \sum_{i=1}^{n} T_{ij} + 1 < (n-1)T_{1} + 1,
\]

Hence, \( T_{ij} + 1 = \left\{ \sum_{i=1}^{n} T_{ij} + 1 \right\} \), a contradiction.

Therefore, \( T_{ij} = 0 \).

Suppose that \( T_{ij} = 0 \) and \( v_{k} \), \( k \neq 1 \) be a vertex in \( G \) with maximum T-total degree.

Then,

\[
\begin{align*}
& T_{ij} = 0 \implies \sum_{i=1}^{n} T_{ij} = 0, \\
& \sum_{i=1}^{n} T_{ij} + 1 = \sum_{i=1}^{n} T_{ij} + 1,
\end{align*}
\]

Since \( T_{ij} \) for \( i = 1, 2, 3, \ldots, n \) and for all other indices \( j \), it follows that

\[
(n-1)T_{ij} + 1 < \sum_{i=1}^{n} T_{ij} + 1 < (n-1)T_{1} + 1,
\]

Hence, \( T_{ij} + 1 = \left\{ \sum_{i=1}^{n} T_{ij} + 1 \right\} \), a contradiction.

Therefore, \( T_{ij} = 0 \).

Suppose that \( T_{ij} = 0 \) and \( v_{k} \), \( k \neq 1 \) be a vertex in \( G \) with maximum I-total degree.

Then,

\[
\begin{align*}
& \sum_{i=1}^{n} I_{ij} + 1 = \sum_{i=1}^{n} I_{ij} + 1, \\
& \sum_{i=1}^{n} I_{ij} + 1 = \sum_{i=1}^{n} I_{ij} + 1,
\end{align*}
\]

Since \( I_{ij} \) for \( i = 1, 2, 3, \ldots, n \) and for all other indices \( j \), it follows that

\[
(n-1)I_{ij} + 1 < \sum_{i=1}^{n} I_{ij} + 1 < (n-1)I_{1} + 1,
\]

So that \( I_{ij} = 0 \), a contradiction.

Therefore, \( I_{ij} = 0 \).

Suppose that \( I_{ij} = 0 \) and \( v_{k} \), \( k \neq 1 \) be a vertex in \( G \) with maximum F-total degree.

Then,

\[
\begin{align*}
& F_{ij} + 1 = \sum_{i=1}^{n} F_{ij} + 1, \\
& \sum_{i=1}^{n} F_{ij} + 1 = \sum_{i=1}^{n} F_{ij} + 1,
\end{align*}
\]

Since \( F_{ij} \) for \( i = 1, 2, 3, \ldots, n \) and for all other indices \( j \), it follows that

\[
(n-1)F_{ij} + 1 < \sum_{i=1}^{n} F_{ij} + 1 < (n-1)F_{1} + 1,
\]

So that \( F_{ij} = 0 \), a contradiction.

Therefore, \( F_{ij} = 0 \).

Since \( I_{ij} + I_{k} = I_{i} \) for \( i = 1, 2, 3, \ldots, n \) and for all other indices \( j \), \( I_{k} + I_{j} < I_{i} \), it follows that

\[
(n-1)I_{i} + I_{k} < \sum_{j=1,k \neq j}^{n} I_{j} + I_{k} < (n-1)I_{i} + I_{k},
\]

So that \( T_{ij} = 0 \), a contradiction.

Therefore, \( T_{ij} = 0 \).

Also, Suppose that \( T_{ij} = 0 \) and \( v_{k} \), \( k \neq 1 \) be a vertex in \( G \) with maximum F-total degree.

Then

\[
\begin{align*}
& T_{ij} = 0 \implies \sum_{i=1}^{n} T_{ij} = 0, \\
& \sum_{i=1}^{n} T_{ij} + 1 = \sum_{i=1}^{n} T_{ij} + 1,
\end{align*}
\]

Since \( T_{ij} \) for \( i = 1, 2, 3, \ldots, n \) and for all other indices \( j \), it follows that

\[
(n-1)T_{ij} + 1 < \sum_{i=1}^{n} T_{ij} + 1 < (n-1)T_{1} + 1,
\]

Hence, \( T_{ij} + 1 = \left\{ \sum_{i=1}^{n} T_{ij} + 1 \right\} \), a contradiction.

Therefore, \( T_{ij} + 1 = 0 \).

Hence,

\[
\begin{align*}
& T_{ij} = 0 \implies \sum_{i=1}^{n} T_{ij} = 0, \\
& \sum_{i=1}^{n} T_{ij} + 1 = \sum_{i=1}^{n} T_{ij} + 1,
\end{align*}
\]

Since \( T_{ij} \) for \( i = 1, 2, 3, \ldots, n \) and for all other indices \( j \), it follows that

\[
(n-1)T_{ij} + 1 < \sum_{i=1}^{n} T_{ij} + 1 < (n-1)T_{1} + 1,
\]

Hence, \( T_{ij} + 1 = \left\{ \sum_{i=1}^{n} T_{ij} + 1 \right\} \), a contradiction.

Therefore, \( T_{ij} + 1 = 0 \).

Hence,

\[
\begin{align*}
& F_{ij} + 1 = \sum_{i=1}^{n} F_{ij} + 1, \\
& \sum_{i=1}^{n} F_{ij} + 1 = \sum_{i=1}^{n} F_{ij} + 1,
\end{align*}
\]

Since \( F_{ij} \) for \( i = 1, 2, 3, \ldots, n \) and for all other indices \( j \), it follows that

\[
(n-1)F_{ij} + 1 < \sum_{i=1}^{n} F_{ij} + 1 < (n-1)F_{1} + 1,
\]

So that \( F_{ij} = 0 \), a contradiction.

Therefore, \( F_{ij} = 0 \).

Hence,

\[
\begin{align*}
& F_{ij} = 0 \implies \sum_{i=1}^{n} F_{ij} = 0, \\
& \sum_{i=1}^{n} F_{ij} + 1 = \sum_{i=1}^{n} F_{ij} + 1,
\end{align*}
\]

Since \( F_{ij} \) for \( i = 1, 2, 3, \ldots, n \) and for all other indices \( j \), it follows that

\[
(n-1)F_{ij} + 1 < \sum_{i=1}^{n} F_{ij} + 1 < (n-1)F_{1} + 1,
\]

Hence, \( F_{ij} = 0 \).

Therefore, \( F_{ij} = 0 \).

And \( F_{ij} = 0 \).

Suppose that \( F_{ij} = 0 \) and \( v_{k} \), \( 1 \leq i \leq n \) be a vertex in \( G \) such that \( T_{ij} = 0 \) and \( F_{ij} = 0 \) and \( T_{ij} = 0 \). In addition,

\[
T_{ij} = \left[ \sum_{i=1}^{n} T_{ij} + \sum_{i=1}^{n} T_{ij} + T_{ij} \right] + T_{ij}
\]
\[
\begin{align*}
\leq & \left[ \sum_{i=1}^{n} T_i + (n-1)T_i + T_1 \right] + T_l \\
\leq & \sum_{i=1}^{n} T_i + T_l \\
\leq & \sum_{i=1}^{n} T_i = t d_T(v_n) . \text{ Thus } t d_T(v_n) \geq t d_T(v_l) , \text{ contradiction. So, } td_T(v_n) = \Delta_{d_T}(G) = \sum_{i=1}^{n} T_i. \\
\end{align*}
\]

Suppose that \( td_I(v_n) \neq \delta_{d_I}(G) \). Let \( v_l, 1 \leq l \leq n-1 \) be a vertex in G such that \( td_I(v_l) = \delta_{d_I}(G) \) and \( td_I(v_n) > td_I(v_l) \).

In addition,
\[
\begin{align*}
& td_I(v_l) = [ \sum_{i=1}^{l-1} I_i + \sum_{i=l+1}^{n} I_i + I_n ] + I_l \\
\geq & [ \sum_{i=1}^{l-1} I_i + (n-l)I_l + I_n ] + I_l \\
\geq & \sum_{i=1}^{l-1} I_i + I_l \\
\geq & \sum_{i=1}^{n} I_i = td_I(v_n) . \text{ Thus } td_I(v_n) \leq td_I(v_l) , \text{ contradiction. So, } td_I(v_n) = \delta_{d_I}(G) = \sum_{i=1}^{n} I_i. \\
\end{align*}
\]

Also, suppose that \( td_F(v_n) \neq \delta_{d_F}(G) \). Let \( v_l, 1 \leq l \leq n-1 \) be a vertex in G such that \( td_F(v_l) = \delta_{d_F}(G) \) and \( td_F(v_n) > td_F(v_l) \).

In addition,
\[
\begin{align*}
& td_F(v_l) = [ \sum_{i=1}^{l-1} F_i + \sum_{i=l+1}^{n} F_i + F_n ] + F_l \\
\geq & [ \sum_{i=1}^{l-1} F_i + (n-l)F_l + F_n ] + F_l \\
\geq & \sum_{i=1}^{l-1} F_i + F_l \\
\geq & \sum_{i=1}^{n} F_i = td_F(v_n) . \text{ Thus } td_F(v_n) \leq td_F(v_l) , \text{ contradiction. So, } td_F(v_n) = \delta_{d_F}(G) = \sum_{i=1}^{n} F_i. \\
\end{align*}
\]

Hence the lemma is proved.

**Remark 3.32** In a complete SVN G,

1) There exists at least one pair of vertices \( v_i \) and \( v_j \) such that \( d_T(v_i) = d_T(v_j) = \Delta_T(G) \), \( d_I(v_i) = d_I(v_j) = \delta_I(G) \) and \( d_F(v_i) = d_F(v_j) = \delta_F(G) \),

2) \( \Delta_T(G) = \delta_T(G) \), \( \Delta_I(v) = \delta_I(v) = \Delta_{d_I}(G) \) and \( \delta_F(G) = \delta_F(G) \) for a vertex \( v \in V \),

3) \( \sum_{i=1}^{n} td_T(v_i) = 2 \sum_{i=1}^{n} T_i + 2 \sum_{i=1}^{n} I_i \) and \( \sum_{i=1}^{n} td_I(v_i) = 2 \sum_{i=1}^{n} I_i + 2 \sum_{i=1}^{n} F_i \).

**IV. VERTEX TRUTH MEMBERSHIP, VERTEX INDETERMINACY MEMBERSHIP AND VERTEX FALSEITY MEMBERSHIP SEQUENCE IN SVN G**

In this section, vertex truth membership, vertex indeterminacy membership and vertex falsity membership sequences are defined in SVN Gs.

**Definition 4.1** Given a SVN-graph G with \( |V| = n \). The vertex truth membership sequence of G is defined to be \( \{ y_i \}_{i=1}^{n} \) with \( x_1 \leq x_2 \leq x_3 \leq \ldots \leq x_n \), where \( x_1 \), \( 0 < x_1 \leq 1 \), is the truth membership value of the vertex \( v_i \) when vertices are arranged so that their truth membership values are non-decreasing.

Particular, \( x_1 \) is smallest vertex truth membership value and \( x_n \) is largest vertex truth membership value in G.

**Note 4.2** If vertex truth membership sequence \( x_i \) is repeated more than once in G, say \( r \neq 1 \) times, then it is denoted by \( x_i^r \) in the sequence.

**Example 4.3** In Fig. 2 the vertex truth membership sequence of G is \( \{ 0.1, 0.1, 0.3, 0.3, 0.4, 0.8 \} \) or \( \{ 0.1^2, 0.3^2, 0.4, 0.8 \} \).

**Definition 4.4** Let G be a SVN-graph with \( |V| = n \). The vertex indeterminacy membership sequence of G is defined to be \( \{ y_i \}_{i=1}^{n} \) with \( y_1 \leq y_2 \leq y_3 \leq \ldots \leq y_n \) where \( y_1 \), \( 0 < y_1 \leq 1 \), is the indeterminacy membership value of the vertex \( v_i \) when vertices are arranged so that their indeterminacy membership values are non-increasing.

Particular, \( y_1 \) is largest vertex indeterminacy membership value and \( y_n \) is smallest vertex indeterminacy membership value in G.

**Note 4.5** If vertex indeterminacy membership sequence \( y_i \) is repeated more than once in G, say \( r \neq 1 \) times, then it is denoted by \( y_i^r \) in the sequence.

**Example 4.6** In Fig. 3 the vertex indeterminacy membership sequence of G is \( \{ 0.7, 0.6, 0.6, 0.5, 0.4, 0.4 \} \) or \( \{ 0.7, 0.6^2, 0.5, 0.4^2 \} \).

**Definition 4.7** Let G be a SVN-graph with \( |V| = n \). The vertex falsity membership sequence of G is defined to be \( \{ z_i \}_{i=1}^{n} \) with \( z_1 \leq z_2 \leq z_3 \leq \ldots \leq z_n \) where \( z_1 \), \( 0 < z_1 \leq 1 \), is the falsity membership value of the vertex \( v_i \) when vertices are arranged so that their falsity membership values are non-increasing.

Particular, \( z_1 \) is largest vertex falsity membership value and \( z_n \) is smallest vertex falsity membership value in G.
**Note 4.8** If vertex falsity membership sequence \( z_i \) is repeated more than once in \( G \), say \( r \neq 1 \) times, then it is denoted by \( z_i^r \) in the sequence.

**Example 4.9** In Fig. 4 the vertex falsity membership sequence of \( G \) is \( \{0.8, 0.8, 0.7, 0.6, 0.6, 0.5\} \) or \( \{0.8^2, 0.7, 0.6^2, 0.5\} \).

**Definition 4.10** If a SVNG with \(|V| = n\) has vertex truth membership sequence \( \{x_i\}_{i=1}^n \), vertex indeterminacy membership sequence \( \{y_i\}_{i=1}^n \) and vertex falsity membership sequence \( \{z_i\}_{i=1}^n \) in same order, then it said to have vertex single valued neutrosophic sequence and denoted by \( \langle x_1, y_1, z_1 \rangle_{i=1}^n \).

**Example 4.11** In Fig. 5 the vertex truth membership, vertex indeterminacy membership and vertex falsity membership sequence of \( G \) is \( \{0.4, 0.4, 0.5\}, \{0.2, 0.3, 0.5\}, \{0.1, 0.2, 0.6\}, \{0.5, 0.4, 0.8\}, \{0.4, 0.5, 0.4\}, \{0.3, 0.1, 0.7\} \).

**Theorem 4.12** Let \( G=(V,E) \) be a complete SVNG with \(|V| = n\). Then

1) If the vertex truth membership sequence of \( G \) is of the form \( \{x_1^n, x_2, \ldots, x_k\} \), vertex indeterminacy membership sequence of \( G \) is of the form \( \{y_1^n, y_2, \ldots, y_k\} \) and vertex falsity membership sequence of \( G \) is of the form \( \{z_1^n, z_2, \ldots, z_k\} \), then
   \[ a. \Delta_{tG}(G) = nT_1 \text{ and } \Delta_{dG}(G) = \sum_{i=1}^n T_i \]
   \[ b. \Delta_{lG}(G) = nI_1 \text{ and } \delta_{lG}(G) = \sum_{i=1}^n I_i \]
   \[ c. \Delta_{aG}(G) = nF_1 \text{ and } \delta_{aG}(G) = \sum_{i=1}^n F_i \]

2) If the vertex truth membership sequence of \( G \) is of the form \( \{x_1^n, x_2^n, \ldots, x_k^n\} \), vertex indeterminacy membership of \( G \) is of the form \( \{y_1^n, y_2^n, \ldots, y_k^n\} \) and vertex falsity membership sequence of \( G \) is of the form \( \{z_1^n, z_2^n, \ldots, z_k^n\} \) with \( 0 < r_1 \leq n-2 \), then there exist exactly \( r_1 \) vertices with minimum \( T \) total degree \( \delta_{aG}(G) \), maximum I-total degree \( \Delta_{lG}(G) \) and maximum F-total degree \( \Delta_{dG}(G) \) and exactly \( (n-r_1) \) vertices with maximum \( T \) total degree \( \Delta_{aG}(G) \), minimum I-total degree \( \delta_{lG}(G) \) and minimum F-total degree \( \delta_{dG}(G) \).

3) If the vertex truth membership sequence of \( G \) is of the form \( \{x_1^n, x_2^n, \ldots, x_k^n\} \), vertex indeterminacy membership sequence of \( G \) is of the form \( \{y_1^n, y_2^n, \ldots, y_k^n\} \) and vertex falsity membership sequence of \( G \) is of the form \( \{z_1^n, z_2^n, \ldots, z_k^n\} \) with \( r_k > 1 \) and \( k > 2 \), then there exist exactly \( r_k \) vertices with minimum \( T \) total degree \( \delta_{aG}(G) \), maximum I-total degree \( \Delta_{lG}(G) \) and maximum F-total degree \( \Delta_{dG}(G) \). Also, there exists exactly \( r_k \) vertices with maximum \( T \) total degree \( \Delta_{aG}(G) \), minimum I-total degree \( \delta_{lG}(G) \) and minimum F-total degree \( \delta_{dG}(G) \).

**Proof:** The proof of (1) and (2) are obvious. 3 Let \( v_i^{(j)} \) be the set of vertices in \( G \), for \( j = 1, 2, 3, \ldots, r_1 \), \( 1 \leq i \leq k \). Then by the **Theorem 3.31**

\[ td_T(v_i^{(j)}) = \delta_{td_T}(G) = nT_i = n, \]
\[ td_I(v_i^{(j)}) = \Delta_{td_I}(G) = nI_i = n, \]
\[ td_F(v_i^{(j)}) = \delta_{td_F}(G) = nF_i = n. \]

Since \( T(v_i^{(j)}) = T(v_i^{(j)}) > x_1 \) for \( 2 \leq i \leq k \), \( j = 1, 2, 3, \ldots, r_1 \), \( 1 \leq i \leq k \), \( j = 1, 2, 3, \ldots, r_1 \), no vertex with truth membership more than \( x_1 \) can have degree \( \delta_{td_T}(G) \),

\[ l(v_i^{(j)}) = l(v_i^{(j)}) < y_1 \] for \( 2 \leq i \leq k \), \( j = 1, 2, 3, \ldots, r_1 \), no vertex with indeterminacy membership less than \( y_1 \) can have degree \( \Delta_{td_I}(G) \)

And \( F(v_i^{(j)}) = F(v_i^{(j)}) > z_1 \) for \( 2 \leq i \leq k \), \( j = 1, 2, 3, \ldots, r_1 \), no vertex with falsity membership less than \( z_1 \) can have degree \( \delta_{td_F}(G) \).

Thus, there exist exactly \( r_1 \) vertices with degree \( \delta_{td_T}(G) \), \( \Delta_{td_I}(G) \), \( \Delta_{td_F}(G) \).

To prove \( td_T(v_k^{(1)}) = \Delta_{td_T}(G) \),

\[ td_I(v_k^{(1)}) = \delta_{td_I}(G) \text{ and } \]
\[ td_F(v_k^{(1)}) = \delta_{td_F}(G), t=1,2,3 \ldots, r_k. \]

Since, \( T(v_k^{(1)}) \) is maximum vertex truth membership,

\[ T_{(v_k^{(1)})} = x_k, t \neq j, t,j=1,2,3, \ldots, r_k \]
\[ T_{(v_k^{(1)})} = \min \{ T(v_k^{(1)}), T(v_i^{(j)}) = T(v_i^{(j)}) \} \] for \( t = 1,2,3 \ldots, r_k, j=1,2,3, \ldots, r_1, i=1,2,3, \ldots, k-1 \).
Thus for \( t = 1, 2, 3, \ldots, r_k \),
\[
td_t(v_k^{(t)}) = \sum_{i=1}^{n} \sum_{j=1}^{r_i} T(v_{i}^{(j)}) + (r_k-1)x_k
= \sum_{i=1}^{n} T_i
= \Delta_{tdr}(G) \text{ by Theorem 3.31}
\]

Now, if \( v_m \) is vertex such that \( T_m = x_{k-1} \), then
\[
td_t(v_m) = \sum_{i=1}^{n} \sum_{j=1}^{r_i} T(v_{m, i}^{(j)}) + (r_k-1)\sum_{i=1}^{k-1} x_{k-1} + T_m
= \sum_{i=1}^{n} \sum_{j=1}^{r_i} T(v_{i}^{(j)}) + (r_k-1)\sum_{i=1}^{k-1} x_{k-1} + T_m
= \sum_{i=1}^{n} T_i
= \Delta_{tdr}(G) \text{ by Theorem 3.31}
\]

Similarly, it can be proved that \( t = 1, 2, 3, \ldots, r_k \),
\[
I(v_k^{(t)}) = \max \{ I(v_{k}^{(1)}), I(v_{k}^{(2)}), \ldots, I(v_{k}^{(r_k)}) \} \text{ for } t = 1, 2, 3, \ldots, n
\]

\[\text{for } i = 1, 2, 3, \ldots, k-1.\]

Thus for \( t = 1, 2, 3, \ldots, r_k \),
\[
I(v_k^{(t)}) = \sum_{i=1}^{n} \sum_{j=1}^{r_i} I(v_{i}^{(j)}) + (r_k-1)\sum_{i=1}^{k-1} x_{k-1} + I_m
= \sum_{i=1}^{n} T_i
= \delta_{tdr}(G) \text{ by Theorem 3.31}
\]

Now, if \( v_m \) is vertex such that \( I_m = x_{k-1} \), then
\[
td_t(v_m) = \sum_{i=1}^{n} \sum_{j=1}^{r_i} I(v_{m, i}^{(j)}) + (r_k-1)\sum_{i=1}^{k-1} x_{k-1} + I_m
= \sum_{i=1}^{n} \sum_{j=1}^{r_i} I(v_{i}^{(j)}) + (r_k-1)\sum_{i=1}^{k-1} x_{k-1} + I_m
= \sum_{i=1}^{n} T_i
= \delta_{tdr}(G)
\]

So, there exist exactly \( r_k \) vertices with degree \( \delta_{tdr}(G) \).

\[\text{V. Conclusion}\]

In this paper, the idea of strong degree is imposed on the existing concepts of degrees in SVNGs. After that, we defined the vertex truth-membership, vertex indeterminacy-membership and vertex falsity membership sequence in SVNG with proofs and suitable examples. In the next research, the proposed concepts can be extended to labeling neutrosophic graph and also characterize the corresponding properties.

\[\text{Acknowledgement}\]

The authors are very grateful to the chief editor and reviewers for their comments and suggestions, which is helpful in improving the paper.

\[\text{Reference}\]

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