Inclusive spectra of hadrons created by color tube fission 2. Inclusive spectra of primary hadrons

E.V. Gedalin

Department of Physics
Ben-Gurion University of the Negev
Beer-Sheva, 84105, Israel

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*E-mail address: gedal@bgumail.bgu.ac.il
Abstract

The primary inclusive spectra and correlation functions of particles created by color tube fission are considered. Using the previously obtained expression for probability of the tube breaking in \( n \) points we have calculated the one and two particle inclusive spectra of tube pieces as well as pseudoscalar and vector mesons in plateau area. It is shown that the plateau height of the one particle inclusive spectrum is determined by the flavor quark composition and spin of hadron. Small oscillations of the tube surface give only small correction to the main term. The correlation functions of fixed particles have the form of a product of the universal function that depends only on the particle rapidity difference and the scale factor dependent on the spin and flavor quark composition of hadron.
1 Introduction

In the preceding article [1] we have calculated the probability of the tube fission in one and many points. Here we apply the obtained expressions to the calculation of the inclusive spectra (IS) of primary hadrons produced in soft processes. Our approach to the tube fragmentation into hadrons is close to Artru-Menessier model [2] (as well as other models [3]-[8] based on Artru-Menessier approach). In our model [9]-[11] at each stage of hadronization process the tube splits into pieces with arbitrary masses until the distance between the $K$ and $\bar{K}$ walls of piece becomes of the order of thickness of the wall. This causes growth of the kink mass and rapid decrease of the probability of the tube fission and therefore the hadronization process stops. The produced pieces are of the length $\sim 2M/\epsilon^2$, where $M$ and $\epsilon^2$ are the kink mass and tube tension respectively.

The piece IS are proportional to the probability of the tube breaking in two (and more) points that forms the piece with the light cone momentum $p_+$ and $p_-$, mass $m$ and transverse momentum $k$. The usual mass shell condition $p_+ p_- = k^2 + m^2$ eliminates $p_-$ and we must calculate probability of the tube breaking in two or/and more points with the conditions for the light cone.
coordinates of piece edges

\[ u_1 - u_2 = p_+ / \epsilon \]
\[ v_2 - v_1 = p_- / \epsilon = (m^2 + k^2) / p_+ \epsilon \]  \hspace{1cm} (1)

etc. The obtained probabilities depend on \( p_+ \) as well as on the piece transverse mass squared, i.e. the breaking mechanism fully describes the IS of pieces (see the ref. [3]-[4]).

Since far we are interested only in the \( p_+ \)-dependence of IS the \( k^2 \) dependence must be integrated and we obtain the condition

\[ v_2 - v_1 \geq m^2 / p_+ \epsilon \]  \hspace{1cm} (2)

that greatly simplifies the calculations. Usually the rhs of this condition is taken equal to zero, neglecting all mass corrections. However, the mass of the piece plays an important role having influence on the value of the corresponding probability. We shall return to the mass corrections later.

When the mass corrections are neglected, the piece mass is not well defined that the pieces can be attributed to stable hadrons (pions and kaons) as well as hadron resonances. In other words, the piece with the flavor quantum numbers \( i \) and \( j \) of a kink and antikink has the probability \( C(i, j; S) \)
to be in state $S$, which is the stable hadron or resonance. The advantage of this concept is that all corrections necessary for including an additional resonances and final state interactions, can be implemented quite naturally (see for example refs. [3]-[4]).

We assume that the primary mesons are pseudoscalar and vector particles, neglecting temporarily the diquark and other type of $K, \bar{K}$ production that can be introduced in the same way (for a more complete discussion we refer to Ref.[3]-[4])

The article is organized as follows. First, we calculate the IS of the pieces that have fixed quark quantum numbers on the edges. The primary hadron IS then are built from the IS of pieces, using the quark flavor composition of hadrons. We take into account only $u, d$ and $s$ quarks. The probabilities $w_u$ and $w_d$ of the $u$ and $d$ quark production are taken the same $w_u = w_d = w$ and the (rare) strange quark creation probability $w_s$ is assumed to be sufficiently small. We have $2w + w_s = 1$. In the next section we calculate the one particle inclusive spectra (OPIS) of pieces. The two particle inclusive spectra (TPIS) of pieces and corresponding correlation functions (CF) are considered in Sec.3. The pseudoscalar and vector meson primary IS are calculated in Sec.4.In Conclusion we summarize and discuss the obtained results.
2 The OPIS of pieces

Here we calculate inclusive spectra of primary hadrons created by fission of a tube with small surface oscillations. We limit ourselves with particles with $p_+$ small in comparison with the momentum of initial tube $p_{0+}$ (but $p_+ >> m$). We restrict ourselves with IS integrated over transverse momenta and neglect mass corrections.

First we consider the one-particle IS (OPIS). Let us begin with the case of one type of $(K, \bar{K})$, that is, where we detect the primary hadrons of any quantum numbers. The procedure of one-particle IS calculation is the following. We must integrate over the probability $dP(1, 2)$ of two adjacent fission at $(u_1, v_1)$ and $(u_2, v_2)$, that form the piece with $p_+$, over all positions of fission the space-time points. From eq. (70) of ref. [1] we have

$$dP(1, 2; p_+) = dw(1)dw(2) \exp[-W(\tilde{S})] \vartheta(u_1 - u_2)\vartheta(v_2 - v_1)\delta(u_1 - u_2 - \tilde{p})$$

(3)

where the integration region $\tilde{S}$ is the rectangle area defined by lines $u = 0, u = u_1, v = 0, v = v_2$. The $\vartheta$ - functions in expression (3) reflect the ordering of fission points and $\delta$ - function accounts for the difference condition (4).
Now the OPIS is given by

\[
f_1(\tilde{p}) = \int dP(1, 2; p_+),
\]

(4)

where the integration is over the range \((0, \tilde{p}_0)\) for each variable.

As we have seen above the small-mass oscillations produce corrections to the fission probability of long tube that are proportional to the oscillation amplitude squared (see Ref.\[1\]), namely

\[
dw(\tau, \eta) = dw_0 \exp(a^2 D(\kappa s)).
\]

(5)

where

\[
dw_0 = (\epsilon^2/2\pi) \exp(-\pi M^2/\epsilon^2) d\eta d\tau = w_0 d\eta d\tau
\]

(6)

is the fission probability without oscillations, \(D = (1/\epsilon^2)D(\kappa_0)(s)\) depends on the initial shape of the color tube, \(\kappa\) is the (dimensionless) mass of the small mass surface oscillations, \(s_2 = \tau^2 - \eta^2 = uv\). It is important that \(D\) depends only on the product \(uv\) and not on \(u\) and \(v\) separately.

\(^1\)Only a small \(\sim 1/\tilde{p}_0\) contribution arise from one breaking point diagram when one wall of produced piece is the wall of initial tube and only second wall is produced by additional \(K\bar{K}\) pair. We will systematically neglect such contributions to IS in the plateau area.
Since we consider small oscillations with $a^2 \ll 1$ we expand all expressions in a power series in $a^2$ and keep terms up to $a^2$ order. After some calculations we have

$$f_1 = (1/\tilde{p})(1 - a^2 F(q)),$$

(7)

where $q = 2\kappa^2/w_0$ and

$$F(q) = \sum_{n=1}^{\infty} q^n Z_n \frac{n!}{n + 1},$$

(8)

$Z_n$ are the coefficients of power expansion of function $D = \sum q^n Z_n$.

We see that small oscillations do not affect the spectrum but change the height of the plateau \[12\]. Obviously this change depends on the probability of tube fission.

To proceed to the case of many type of kinks we note that the probability of the tube breaking at the point $(u, v)$ due to the creation of the $(K, \bar{K})$ pair of type $l$, can be written in the form

$$dw(u, v; l) = w_l e^{-a^2 Z_l(k_s)} du dv$$

(9)

and

$$W(\tilde{S}) = \frac{1}{2} \int_{\tilde{S}} du' dv' w_{tot}$$

$$= \frac{1}{2} \int_{\tilde{S}} du' dv' \sum_l w_l (1 + a^2 \sum_n Z_n(l)(ku'v')^n)$$

(10)
where \( w_l \) is the partial fission probability without oscillations and \( Z_n(l) \) are the coefficients of the power expansion of function \( Z_l(ks) \).

Repeating the procedure of OPIS calculation we obtain for particle with \( K \) of type \( j \) and \( \bar{K} \) of type \( l \) on the edges the expression

\[
f_1(j, l; \tilde{p}) = r_j r_l \frac{p}{\tilde{p}} (1 - a^2 F(j, l; q)) \tag{11}
\]

where \( r_l = w_l / w_{\text{tot}} \) is the relative weight of the kink of flavor \( i \) and \( w_l \) and \( w_{\text{tot}} = \sum_l w_l \) are the partial and total breaking probabilities without oscillations respectively and

\[
F(j, l; q) = \sum_n \Gamma(n + 1)q^n [Z_n(j) + Z_n(l) - \sum_k r_k \frac{n + 2}{n + 1} Z_n(l)] \tag{12}
\]

As in the previous case the momentum spectrum is not affected. However the height of the plateau is defined by now the product \( r_j r_l \) (with small corrections \( \sim a^2 \)) and is \((j, l)\) dependent. In the case when on both edges of the piece are dominated kinks (i.e. the kinks of small mass with large creation probability \( w_l \approx w_{\text{tot}} \)) we obtain the plateau height close to unity as in the case of one type kink. The (strange) kinks with \( r_l \ll 1 \) contribute only into \( \sim a^2 \) corrections. When one (or both) kink is strange the height of the plateau is small. However the \( a^2 \) - order corrections contain contributions of dominant as well as strange kinks.
3 The TPIS and CF of the pieces

Now we pass to the two particle inclusive spectra (TPIS) and particle correlations. For TPIS we have two distinct contributions. The first one \( f_2^{(1)} \) is from the two pieces produced in three adjacent fission points \((u_1, v_1), (u_2, v_2)\) and \((u_3, v_3)\) ((1), (2) and (3)), having one of the edges created in the same fission at point (2) while there are no fission between points (1) and (2) as well as between (2) and (3). These pieces are the primary particles of adjacent ranges in Field-Feynman terminology [13].

The second contribution \( f_2^{(2)} \) is from two by two adjacent fission \([(u_1, v_1), (u_2, v_2)]\) and \([(u_3, v_3), (u_4, v_4)]\) and there is no fission between the points (1) and (2) as well as between (3) and (4).

Again first consider the one flavor case [12]. The first contribution has the form [12].

\[
f_2^{(1)}(\tilde{p}_1, \tilde{p}_2) = \int [\prod_{i=1}^3 dw(i)] \exp[-W(u_1, v_2) - W(u_2, v_3) + W(u_2, v_2)] \vartheta(u_1 - u_2) \vartheta(u_2 - u_3) \vartheta(v_2 - v_1) \vartheta(v_3 - v_2) \vartheta(v_1) \delta(u_1 - u_2 - \tilde{p}_1) \delta(u_2 - u_3 - \tilde{p}_2) + (\tilde{p}_1 \leftrightarrow \tilde{p}_2)
\]
where we have denoted by \( W(x, y) \) the \( W(S) \) defined by expression (10) for the rectangle area limited by lines \( u = 0, u = x, v = 0, v = y \) and \( (\tilde{p}_1 \leftrightarrow \tilde{p}_2) \) is the \( \tilde{p}_1 \) and \( \tilde{p}_2 \) interchanged term. The first term describes the case when tube \( (1) - (2) \) has \( u_1 - u_2 = \tilde{p}_1 \) and tube \( (2) - (3) \) has \( u_2 - u_3 = \tilde{p}_2 \) and the second term describes the situation when \( u_1 - u_2 = \tilde{p}_2 \) and \( u_2 - u_3 = \tilde{p}_1 \).

The calculation is straightforward but somewhat cumbersome and we obtain

\[
f_2^{(1)} = \frac{1}{\tilde{p}_1 \tilde{p}_2} [1 + \varphi(\tilde{p}_1, \tilde{p}_2)] [1 - a^2 F(q)] + \frac{a^2}{\tilde{p}_1 \tilde{p}_2} \varphi_1(\tilde{p}_1/\tilde{p}_2, q), \quad (14)
\]

where \( \frac{1}{\tilde{p}_1 \tilde{p}_2} [1 + \varphi(\tilde{p}_1, \tilde{p}_2)] \) is the spectrum of the two pieces produced in the adjacent fission without oscillations and

\[
\varphi(\tilde{p}_1, \tilde{p}_2) = \varphi(\tilde{p}_1/\tilde{p}_2) \quad (15)
\]

\[
\varphi(x) = x \ln(1 + \frac{1}{x}) + \frac{1}{x} \ln(1 + x) - 1 \quad (16)
\]

The function \( \varphi_1 \) depends only on rapidity difference \( y = \ln(\tilde{p}_1/\tilde{p}_2) \) and its explicit form is closely related to the form of \( Z(x) \).

The second contribution \( f_2^{(2)} \) can be written in the form

\[
f_2^{(2)} = \int [\prod_{i=1}^{4} dw(i)] \exp[-W(u_3, v_4) - W(u_1, v_2) + W(u_3, v_2)] \quad (17)
\]
\[ \partial(u_1 - u_2)\partial(u_2 - u_3)\partial(u_3 - u_4)\partial(v_2 - v_1)\partial(v_3 - v_2) \]
\[ \partial(v_4 - v_3)\partial(v_1)\delta(u_1 - u_2 - \tilde{p}_1)\delta(u_3 - u_4 - \tilde{p}_2) \]
\[ + (\tilde{p}_1 \leftrightarrow \tilde{p}_2) \]

After some calculation we obtain [12].

\[ f_2^{(2)} = f_2^{(1)} + \frac{1}{\tilde{p}_1 \tilde{p}_2}(1 - a^2[2F(q) + R(q, y)]) \] (18)

where

\[ R(q, y) = \frac{e^y}{1 + e^y}F\left(\frac{qe^y}{1 + e^y}\right) + \frac{1}{1 + e^y}F\left(\frac{q}{1 + e^y}\right) - F(q). \] (19)

and \( y = \ln(\tilde{p}_1/\tilde{p}_2) \) is the particle rapidity difference.

Thus \( f_2 \) has the form

\[ f_2 = \frac{1}{\tilde{p}_1 \tilde{p}_2}(1 - a^2[2F(q) - R(q, y)]) \] (20)

and the correlation function (CF) is proportional to \( a^2 \) and depends only on \( y \)

\[ C_2(e^y) = f_2(\tilde{p}_1, \tilde{p}_2)/f_1(\tilde{p}_1)f_1(\tilde{p}_2) - 1 \]
\[ = a^2[e^y(1 + e^y)^{-1}F(qe^y(1 + e^y)^{-1}) \]
\[ + (1 + e^y)^{-1}F(q(1 + e^y)^{-1}) - F(q)] \] (21)

As we can see \( C_2 \) in one flavor case the tube surface oscillations produce weak (\( \sim a^2 \)) short range rapidity correlations with correlation length \( \approx 1 \).
The CF $C_2$ is expressed by the same function $F(x)$ that $f_1$ is. At $y = 0$
$C_2(1, q) = a^2[F(q/2) - F(q)]$ and at large $|y| \gg 1$ it vanishes exponentially

$$C_2(e^y) \sim -a^2e^{-|y|}[F(q) + qdF(q)/dq]$$  \hspace{1cm} (22)

Below we shall show that this correlation can be seen as small correlation between resonances.

Now we proceed to the multilavor TPIS of particles $(i, k)$ and $(l, j)$ which internal quantum numbers defined by flavors of $K$ and $\bar{K}$ on the piece edges.

There are two different kinds of TPIS. The first one $f_{2s}(i, k: l, j; \tilde{p}_1, \tilde{p}_2)$ is symmetric in $\tilde{p}_1$ and $\tilde{p}_2$ as well as in $(i, k)$ and $(l, j)$ pairs. The second type $f_{2n}(i, k; \tilde{p}_1; l, j, \tilde{p}_2)$ is symmetric only with respect to simultaneous transposition of $(i, k; \tilde{p}_1) \leftrightarrow (l, j; \tilde{p}_2)$.

Let us consider first the $f_{2s}$. We begin with the three breaking point contribution $f_{2s}^{(1)}$. It is clear that $f_{2s}^{(1)}$ vanishes unless one of the $i, j$ or $/\text{and}$ $k, l$ pairs is the $(K, \bar{K})$ of same flavor. Therefore we have

$$f_{2s}^{(1)} = r_ir_k(r_j\delta_{ik} + r_l\delta_{ij})\frac{1}{\tilde{p}_1\tilde{p}_2} [\varphi(\tilde{p}_1, \tilde{p}_2) - a^2\Phi_{2s}^{(1)}(i, k; lj; \tilde{p}_1, \tilde{p}_2)]$$  \hspace{1cm} (23)

where the terms proportional to $\sim r_ir_kr_j\delta_{kl}$ and $\sim r_ir_kr_l\delta_{ij}$ are the contributions when at a middle breaking point are created the $l = k$ or $i = j$
kink-antikink pairs respectively, and function $\Phi_{2s}^{(1)}$ describes the influence of tube surface oscillations.

The four breaking point contribution $f_{2s}^{(2)}(i, k : l, j; \tilde{p}_1, \tilde{p}_2)$ has a more simple form

$$f_{2s}^{(2)} = r_i r_k r_j r_l \frac{2}{\tilde{p}_1 \tilde{p}_2} \left[ 1 - \varphi(\tilde{p}_1, \tilde{p}_2) - a^2 \Phi_{2s}^{(2)}(i, k; l j; \tilde{p}_1, \tilde{p}_2) \right]$$

where again the function $\Phi_{2s}^{(2)}$ describes the influence of small oscillations of tube surface.

Thus $f_{2s}$ has the form

$$f_{2s} = \frac{2}{\tilde{p}_1 \tilde{p}_2} \left[ 2r_i r_k r_j r_l + r_i r_k (r_j \delta_{lk} + r_l \delta_{ij} - 2r_l r_j) \varphi(\tilde{p}_1, \tilde{p}_2) - a^2 \Phi_{2s}(i, k; l j; \tilde{p}_1, \tilde{p}_2) \right]$$

where

$$\Phi_{2s} = 2r_i r_k r_j r_l \Phi_{2s}^{(2)} + r_i r_k (r_j \delta_{lk} + r_l \delta_{ij}) \Phi_{2s}^{(1)}$$

From expression (25) it follows that in the multiflavor case the two particle correlations are dependent on the internal quantum numbers of particles.

Let us consider the (symmetric) CF

$$C_{2s} = \frac{f_{2s}(i, k; l, j; \tilde{p}_1, \tilde{p}_2)}{f_1(i, k; \tilde{p}_1) f_1(l, j; \tilde{p}_2) + f_1(i, k; \tilde{p}_2) f_1(l, j; \tilde{p}_1)} - 1. \quad (27)$$
When all four \( K(i = k = l = j) \) are the same and dominant the CF has the form

\[
C_{2s} = \frac{1 - r}{r} \varphi(\tilde{p}_1, \tilde{p}_2) - a^2 \Phi_{2s}(q, Y)
\]  

(28)

The first term evidently describes the correlations without oscillations, the last term \((\sim a^2)\) describes the influence of the surface oscillations.

It is clear that only at \( r = 1 \) all correlations are produced by the tube surface oscillations. Even if \( r \) is very close to 1 it must the great influence of of first term must be expected. The oscillation role can be checked if we examine the \( y \) dependence of \( C_{2s} \). Then the difference \( C_{2s} - (1 - r)\varphi/r \) gives the measure of the influence of the oscillation term.

Let us consider now the case when there are \( n \geq 2 \) types of dominant kinks with equal \( r_d \) and one type of kink with \( r_s < r_d \) (for instance the kinks that correspond to the \( u, d \) and \( s \) quarks respectively). Then \( nr_d + r_s = 1 \) and in order to obtain any information on the tube surface oscillations we must sum over all types of kinks (sum over full multiplet of quarks), i.e. to study correlations irrespective to the internal quantum numbers of pieces. From (25) and (27) it easy to see that in this case \( C_{2s} = a^2 R(q, y) \) just as in the one flavor case. Otherwise the \( C_{2s} \) is strongly dominated by the nonoscillation
term. For correlation between particles with fixed quantum numbers the influence of tube surface oscillations is very small. However, in contrast to the one flavor case now we have strong rapidity correlations. When only one pair of kinks is present (for definiteness $k$ and $l$) we have

$$C_{2s}(i, k, k, i; \tilde{p}_1, \tilde{p}_2) = \left( \frac{1}{2r_k} - 1 \right) \varphi(\tilde{p}_1, \tilde{p}_2)$$  \hspace{1cm} (29)

The correlation become much more stronger for the rare ($K, \bar{K}$) pair ($s, \bar{s}$ quarks) creation in the middle point of three breaking point term. In this case $r_k$ is small and TPIS is dominated by the three breaking point contribution because the four breaking point term has an additional small $r_k$ factor.

The same holds also for the case of two different pairs with flavors $i = j$ and $k = l$ flavors where one pair of kinks is rare. The only difference from previous case is that we must observe the correlations between $K^0$ and $\bar{K}^0$ or $K^+$ and $\bar{K}^-$ (or corresponding resonances).

Let us assume now that we detect only charged primary hadrons with same charges. Such particles cannot be created as primaries of adjacent ranges and none of the $i, j$ and $k, l$ pair may be of the same flavor. Then the contribution $f_{2s}^{(1)}$ vanishes. Therefore $f_{2s}$ has only a four breaking point
contribution and we have

\[ C_{2s} = -\varphi(\tilde{p}_1, \tilde{p}_2). \] (30)

It must be noted that here the sign of CF is opposite to the sign of the previous case.

Finally we consider the \( f_{2n} \) TPIS. The \((i, k, \tilde{p}_1) \leftrightarrow (l, j, \tilde{p}_2)\) symmetry influences only on the three breaking point term and instead of \( f_{2s}^{(1)} \) we now have

\[ f_{2n}(i, k; \tilde{p}_1; l, j, \tilde{p}_2) = \]

\[ \frac{2}{\tilde{p}_1 \tilde{p}_2} r_i r_k [r_j \delta_{ik} \varphi_n(\tilde{p}_1, \tilde{p}_2) + r_l \delta_{ij} - 2 r_l r_j] \] (31)

\[ \varphi_n(\tilde{p}_2, \tilde{p}_1)]

\[ -O(a^2) \]

where

\[ \varphi_n(x, y) = \frac{y}{x} \ln[(x + y)/y] - \frac{y}{x + y} \] (32)

and \( O(a^2) \) denotes the term proportional to \( a^2 \). Respectively for \( C_{2n} \) we obtain the expression

\[ C_{2n} = f_{2n}(i, k; \tilde{p}_1; l, j, \tilde{p}_2)/(f_1(i, k; \tilde{p}_1) f_1(l, j; \tilde{p}_2) - 1) \] (33)
and all conclusions of preceding discussion remain unchanged also for $C_{2n}$ with obvious substitution $\varphi \to \varphi_n$ in formulas (25)-(30).

The above consideration of IS and CF of pieces is sufficiently general and can be used also for pieces with barion quantum numbers. The only change is the substitution ”rare quarks” $\to$ ”diquarks” with corresponding $r_{dq}$.

4 IS of primary hadrons

The primary hadron IS can be constructed from IS of pieces in accordance with hadron quark flavor composition and spin state. Here we restrict ourselves by two nonets of mesons that are the pseudoscalar ($\pi, K, \bar{K}, \eta, \eta'$) and the vector $\rho, K^*, \bar{K}^*, \omega, \varphi$ mesons. The other words we shell consider $P$ and $V$ meson nonets built from $u, d$ and $s$ quarks and antiquarks and have similar flavor structure. The oktet-neton mixing for physical mesons is taken into account for $\eta$ and $\eta'$ and $\omega$ and $\Phi$ mesons according to [14]-[16].

Let us denote by $C(i, j; L)$ the probability for piece $(i, j)$ to form hadron $L$. Then we have

$$C(i, j; L) = A(i, j; L)C(s)$$  \hspace{1cm} (34)$$

where $A(i, j; L)$ is the probability for piece $(i, j)$ to form hadron with quark
flavor structure of hadron $L$ and $C(s)$ is the probability for piece to form the $P$ ($s = 1$) or $V$ ($s = 2$) meson state. In our case $C(0) + C(1) = 1$. Then using the standard recipe of the quark model \[14\]-\[16\] we obtain for the OPIS of primary hadrons

$$f(L, \tilde{p}) = g(L)/\tilde{p}$$

where the plateau height $g(L)$ now depends on the flavor composition and spin of the meson

$$g(L) = \sum_{i,j} A(i, j; L)C(s)r_ir_j.$$

The plateau heights of primary mesons are given in Table 1.

From (36) the relation between plateau heights becomes

$$g(\pi) : g(\rho) : g(K) : g(K^*) : g(\eta) : g(\eta^') : g(\omega) : g(\Phi) =$$

$$1 : \frac{C(1)}{C(0)} : \frac{r_s}{r} : \frac{C(1)}{C(0)}r_s : \frac{1}{3}(1 + \frac{2r_s^2}{r^2}) : \frac{1}{3}(2 + \frac{r_s^2}{r^2}) : \frac{C(1)}{C(0)} : \frac{C(1)r_s^2}{C(0)r^2}.$$

Thus to fix all plateau heights we need only two adjustable parameters $r_s/r$ and $C(1)/C(0)$. The first one may be fixed from relation of heights of $\rho$ and $K^*$ plateau since the $P$ meson plateau heights are badly disguised by resonance decay.

If we choose one of the most frequently used set of parameters \[3\]-\[4\] $r_s/r = 1/3$ and $C(0) = C(1)$ ($r = 3/7, r_s = 1/7, C(0) = 1$) we obtain for rhs
of eq. (38)

\[ 1 : 1 : \frac{1}{3} : \frac{1}{3} : \frac{11}{27} : \frac{10}{27} : 1 : \frac{1}{9}. \tag{38} \]

Therefore the measurement of heights of the vector meson plateau is of considerable interest for checking of hadronization mechanism.

The TPIS and CF of primary hadrons can be calculated the same way and for symmetric TPIS we obtain

\[
f_{2s}(L_1, L_2; \tilde{p}_1, \tilde{p}_2) = \sum_{i,j,k,l} A(i, k; L_1)C(s_1)A(l, j; L_2)C(s_2)f_{2s}(i, k; l, j; \tilde{p}_1, \tilde{p}_2) \tag{39} \]

\[
eq [f_1(L_1, \tilde{p}_1)f_1(L_2, \tilde{p}_2) + f_1(L_1, \tilde{p}_2)f_1(L_2, \tilde{p}_1)]
\]

\[
+ [G(L_1, L_2) - 2g(L_1)g(L_2)]\frac{\varphi(\tilde{p}_1, \tilde{p}_2)}{\tilde{p}_1\tilde{p}_2} - \frac{a^2}{\tilde{p}_1\tilde{p}_2}\Phi(L_1, L_2; \tilde{p}_1, \tilde{p}_2)
\]

where

\[
\Phi_{2s}(L_1, L_2; \tilde{p}_1, \tilde{p}_2) = 2g(L_1)g(L_2)\Phi_{2s}^{(2)} + G(L_1, L_2)\Phi_{2s}^{(1)} \tag{40} \]

and

\[
G(L_1, L_2) = C(s_1)C(s_2)\sum_{i,j,k,l} A(i, k; L_1)A(l, j; L_2)r_ir_j(r_k\delta_{lk} + r_l\delta_{lj}). \tag{41} \]

Thus from (39) we obtain for CF of primary particles the expression

\[
C_2(L_1, L_2; \tilde{p}_1, \tilde{p}_2) = S(L_1, L_2)\varphi(\tilde{p}_1, \tilde{p}_2) - a^2[\Phi_{2s}^{(2)} + (1 + S(L_1, L_2))\Phi_{2s}^{(1)}] \tag{42} \]
where

\[ S(L_1, L_2) = \frac{G(L_1, L_2)}{2g(L_1)g(L_2)} - 1 \]  \hspace{1cm} (43)

Apart from the oscillation contribution term all primary particle correlations are governed by the same function of particle rapidity difference, i.e. up to \( \sim a^2 \) corrections the all primary two particle CF have the same form and all dependence of CF on the type of particles is factorized in the scaling factor \( S(L_1, L_2) \).

It is quite obvious that the tube oscillation term is significant only when we detect whole nonet of particles. This is just the above considered case of detection of pieces irrespective to its flavor quantum numbers. This means that the role of tube surface oscillations may be studied in full nonet of vector particle correlations (since the hadron resonance decay badly disguise the pseudoscalar particle CF).

For individual particles the CF are strongly dominated by nonoscillation term. Next we consider this dominant part of CF for \( P \) and \( V \) particles. We have

\[ C_2(L_1, L_2; \tilde{p}_1, \tilde{p}_2) = S(L_1, L_2) \varphi(\tilde{p}_1, \tilde{p}_2) \]  \hspace{1cm} (44)

(The factors \( S(L_1, L_2) \) are given in Table 2).
Let us begin with CF that are independent of the particle spin. The first example is that both particles belong to the highest or lowest $I_3$ in isomultiplet ($\pi^+\pi^+, \pi^-K^0$ etc). This is just the same situation as we have seen above for correlation of two pieces with same charges. The $S(L_1, L_2) = -1$ and the function $\varphi(\tilde{p}_1, \tilde{p}_2)$ can be measured directly using $K^{++}K^{++}, K^{-*}K^{-*}$ CF. It is slightly more difficult to measure the function $\varphi(\tilde{p}_1, \tilde{p}_2)$ using the $\rho^+\rho^+$ or $\rho^-\rho^-$ CF because of distortion rising from resonance decay.

For nonstrange particles with opposite charges we have

$$S(\pi^+\pi^+) = S(\pi^+\rho^-) = S(\pi^-\rho^+) = S(\rho^+\rho^-) = \frac{1 - r}{r}$$  \hspace{1cm} (45)

i.e. in this case the CF coincide with CF of oppositely charged pieces. Then the measure of the relations $C_2(\rho^+\rho^-)/C_2(\rho^+\rho^+)$ and $C_2(\rho^-K^{*+})/C_2(\rho^+\rho^+)$ or $C_2(\rho^+K^{*-})/C_2(\rho^+\rho^+)$ defines the parameters $r$ and $r_s$ respectively. Thus the measurement of the CF of the vector hadrons especially the CF of $\rho$ and $K^*$ mesons is highly desirable despite of difficulties of data processing and necessity to take into account the corrections due the resonance decay.
5 Conclusions

We have seen above that the OPIS of primary hadrons for small rapidities have the universal plateau form $g(L)/\tilde{p}$ where $g(L)$ is the plateau height and $\tilde{p}$ is the light-cone momentum of hadron, where $g(L)$ depends on the flavor composition as well as on the spin of the hadron. The tube surface oscillations contribute only small correction term into $g$ proportional to oscillation amplitude squared.

The TPIS and CF depend on the particle detection mode. When the whole multiplet of hadrons is detected irrespective of particle inner quantum numbers the CF contains only the tube oscillation contribution term. Otherwise the oscillations give only a small correction term. The main (nonoscillation) term of CF of fixed particles has the form $S(L_1, L_2)\varphi(\tilde{p}_1, \tilde{p}_2)$ where all dependence on the particle type factorizes in $S$ and $\varphi(\tilde{p}_1, \tilde{p}_2)$ is a function of the difference of particle rapidities only. Thus all primary CF have the same rapidity dependence that is scaled by $S(L_1, L_2)$ factor.

The parameters of the model can be unambiguously fixed by measuring the one and two particle IS of vector mesons. Such experimental investigations are highly desirable because of large disguise of IS of pseudoscalar
particles (pions and kaons) by decay of hadron resonances. We consider the influence of the resonance decay on IS of pions and kaons in the next paper.

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Table 1:
\[
g(\pi) = C(0)r^2, \quad g(\rho) = C(1)r^2, \\
g(K) = g(\bar{K}) = C(0)rr_s, \quad g(K^*) = g(\bar{K}^*) = C(1)rr_s, \\
g(\eta) = C(0)(r^2 + 2r_s^2)/3, \quad g(\omega) = C(1)r^2, \\
g(\eta') = C(0)(2r^2 + r_s^2)/3, \quad g(\Phi) = C(1)r_s^2.
\]

Table 2: The \(S(L_1, L_2)\) - factors for \(\rho\) and \(K^*\) mesons are equal to corresponding \(S\)-factors of \(\pi\) and \(K\). We have omitted the charge labels when \(S(L_1, L_2)\) are equal for entire isomultiplet.

\[
S(\pi^+\pi^+) = S(\pi^-\pi^-) = -1,
S(\pi^+K^+) = S(\pi^-K^-) = S(\pi^+\bar{K}^0) = S(\pi^-K^0) = -1
S(K^+K^+) = S(K^-K^-) = S(K^0\bar{K}^0) = S(\bar{K}^0K^0) = -1;
S(K^+K^0) = S(K^-\bar{K}^0) = -1;
S(\pi^+\pi^-) = \frac{1 - r_s}{r_s};
S(\pi^0\pi^0) = S(\pi^+\pi^-) = S(\pi^0\pi^-) = S(\pi\omega) = S(\omega\omega) = \frac{1 - 2r_s}{2r_s}
S(\pi^+K^0) = S(\pi^-\bar{K}^0) = S(\pi^+\bar{K}^0) = S(\pi^-K^0) = 1 - \frac{r_s}{r_s}
S(K^+\bar{K}^0) = S(K^0K^-) = S(K^0\bar{K}^0) = S(\Phi\Phi) = \frac{1 - r_s}{r_s}
S(\pi^0K^+)= S(\pi^0K^-) = S(\pi^0K^0) = S(\pi^0\bar{K}^0) = S(K\omega) = S(\bar{K}\omega) = \frac{1 - 4r_s}{4r_s}
\]
\[ S(K^+K^-) = S(K^0\bar{K}^0) = \frac{1}{2} \left( \frac{1-r}{r} + \frac{1-r_s}{r_s} \right) \]

\[ S(\pi, \eta) = -1 + \frac{r}{r^2 + 2r_s^2} \]

\[ S(\pi \Phi) = S(\omega \Phi) = -1 \]

\[ S(K \Phi) = S(\bar{K} \Phi) = \frac{1-2r_s}{2r_s} \]

\[ S(K \eta) = S(\bar{K} \eta) = -1 + \frac{3(r + 4r_s)}{2(r^2 + 2r_s^2)} \]

\[ S(K \eta') = S(\bar{K} \eta') = -1 + \frac{r + r_s}{2(2r^2 + r_s^2)} \]

\[ S(\eta \eta) = -1 + \frac{r^3 + 8r_s^3}{2(r^2 + 2r_s^2)^2} \]

\[ S(\eta' \eta') = -1 + \frac{2r^3 + r_s^3}{(2r^2 + r_s^2)^2} \]

\[ S(\eta \eta') = -1 + \frac{r^3 + 2r_s^3}{(r^2 + 2r_s^2)(2r^2 + r_s^2)} \]
FIGURE CAPTIONS 1. The integration area for OPIS.

2. The two contributions to the TPIS: a) the three breaking point contribution; b) the four breaking point contribution.

3. The function $\varphi$ versus particle rapidity difference $y = \ln(\tilde{p}_2/\tilde{p}_1)$
Fig. 3