A doping-dependent switch from one- to two-component superfluidity at temperature above 100K in coupled electron-hole Van der Waals heterostructures

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The hunt for high temperature superfluidity has received new impetus from the discovery of atomically thin stable materials. Electron-hole superfluidity in coupled MoSe₂-WSe₂ monolayers is investigated using a mean-field multiband model that includes the band splitting caused by the strong spin-orbit coupling. The splitting leads to a large energy misalignment of the electron and hole bands which can be markedly changed by interchanging the doping of the monolayers. The choice of doping determines if the superfluidity is tuneable from one- to two-components. The electron-hole pairing is strong, with high transition temperatures in excess of $T_c \sim 100$ K.

Recently electron-hole superfluidity was reported in double bilayer graphene (DBG)¹, in which an n-doped bilayer graphene was placed in close proximity with a p-doped bilayer graphene, separated by a very thin insulating barrier to block recombination. The reported transition temperature was very low, $T_c \sim 1$ K, which can be traced back to the very strong interband screening² due to bilayer graphene’s tiny band gap³.

Monolayers of the transition metal dichalcogenides (TMDC) MoS₂, MoSe₂, WSe₂, and WS₂ are direct bandgap semiconductors with large bandgaps, $E_g \gtrsim 1$ eV⁴,⁵ that make interband processes and screening negligible. The effective masses in their low-lying nearly parabolic bands, larger than in bilayer graphene, result in much stronger coupling of the electron-hole pairs⁶.

Because of strong spin-orbit coupling, the MoSe₂-hBN-WSe₂ heterostructure, consisting of two TMDC monolayers, one n-doped and the other p-doped, separated by a thin hexagonal Boron Nitride (hBN) insulator⁷,⁸, is an interesting platform for investigating novel multicomponent effects for electron-hole superfluidity. Table I gives the parameters for these materials. Figure 1 shows their low-lying band structures.

The splitting of the conduction and valence bands by spin-orbit coupling into multibands consisting of two concentric parabolic spin-polarised subbands makes superfluidity in double TMDC monolayers resemble high-$T_c$ multiband superconductivity. This is emerging as a complex quantum coherent phenomenon with physical outcomes radically different, or even completely absent, from its single-band counterparts⁹. There are also close relations with multiband superconductivity in confined ultracold Fermi gases¹⁰ and with electric-field induced superconductivity at oxide surfaces¹¹,¹². Table I shows that the spin splitting of the valence bands $\lambda_v$ is an order of magnitude larger than the spin splitting in the conduction bands $\lambda_c$. This results in a misalignment between the electron and hole bands, as shown in Fig. 2 which we find strongly affects the electron-hole pairing processes. From Fig. 2 it is expected, due to the very different misalignments of the bands in systems A and B, that system A, an n-doped MoSe₂ with a p-doped WSe₂ will have markedly different properties from its system B counterpart, a p-doped MoSe₂ with an n-doped WSe₂.

![Figure 1.](image-url) (Color online) The low-lying band structures of monolayer MoSe₂ and WSe₂ centred in the $K$ valley. Red and blue lines are for the opposite spins. The spin configuration is opposite in the two valleys¹³.

| TMDC   | $a$ (nm) | $t$ (eV) | $E_g$ (eV) | $\lambda_c$ (eV) | $\lambda_v$ (eV) |
|--------|---------|---------|-----------|----------------|----------------|
| MoSe₂  | 0.33    | 0.94    | 1.47      | -0.021         | 0.18           |
| WSe₂   | 0.33    | 1.19    | 1.60      | 0.038          | 0.46           |

Table I. TMDC monolayer lattice constant ($a$), hopping parameter ($t$), band gap ($E_g$), and splitting of conduction band ($\lambda_c$) and valence band ($\lambda_v$) by spin-orbit coupling¹³-¹⁵.

We take for the multiband electron-hole Hamiltonian,

$$H = \sum_{k,\beta} \left\{ \xi^{(c)}(k) c^{\dagger}_{\beta,k} c_{\beta,k} + \xi^{(h)}(k) d^{\dagger}_{\beta,k} d_{\beta,k} \right\} + \sum_{k,k',q} V_{k,k'}^{D} c^{\dagger}_{\beta,k+q/2} c_{\beta',k'+q/2} d^{\dagger}_{\beta',-k'+q/2} d_{\beta',-k'+q/2} .$$

(1)
For the \( p \)-doped monolayer, we have used the standard particle-hole mapping of the valence to a conduction band, with positively charged holes filling condensation band states up to the Fermi level. Thanks to the large band gaps, we need consider only conduction band processes[2, 16]. For each monolayer, we label bottom and top conduction subbands \( \beta = b \) and \( \beta = t \). For the \( n \)-doped monolayer, the creation and annihilation operators for electrons in subband \( \beta \) are \( c_{\beta,k}^\dagger \) and \( c_{\beta,k} \), while for the \( p \)-doped monolayer, \( d_{\beta,k}^\dagger \) and \( d_{\beta,k} \) are the corresponding operators for holes. The kinetic energy terms are \( \xi^{(i)}(k) = \varepsilon^{(i)}_\beta(k) - \mu^{(i)} \) where \( \varepsilon^{(i)}_\beta(k) \) is the energy dispersion for the \( i = e, h \) monolayer[17]. We take bands with the same curvature so, since we consider only equal carrier densities \( n^e = n^h = n \), we have for the chemical potentials \( \mu^{(e)} = \mu^{(h)} \equiv \mu \). \( V_{kk'}^E \) is the bare attractive Coulomb interaction between electrons and holes in opposite monolayers separated by a barrier of thickness \( d \),

\[
V_{kk'}^E = -V_{kk'}^S e^{-d|k-k'|}, \quad V_{kk'}^S = \frac{2\pi e^2}{\epsilon} \frac{1}{|k-k'|}, \quad \tag{2}
\]

where \( V_{kk'}^S \) is the bare repulsive Coulomb interaction between carriers in the same monolayer. In contrast with conventional BCS pairing, the Coulomb pairing interaction has no dependence on the electron and hole spins. Because of the spin polarisation in the valleys, the spin degeneracy is \( g_s = 1 \), and due to the large valley separation in momentum space, the two inequivalent valleys contribute only to the valley degeneracy, \( g_v = 2 \).

In principle there are four possible electron-hole pairings, corresponding to four superfluid condensates[18] \( \{ \beta \beta' \} \). The first index \( \beta \) refers to the electron subbands and the second \( \beta' \) to the hole subbands. We find that the \( \{ bt \} \) and \( \{ tb \} \) cross-pairing make negligible contributions to the condensates, so for simplicity, we confine our attention to the mean-field equations for the superfluid gaps \( \Delta_{bb}(k) \) and \( \Delta_{tt}(k) \). Since there are no spin-flip scattering processes, Josephson-like pair transfer is forbidden. At zero temperature these gap equations are (See Appendix),

\[
\Delta_{bb}(k) = -\frac{1}{L^2} \sum_{k'} F_{kk'}^{bb} V_{kk'}^{eh} \frac{\Delta_{bb}(k')}{2E_b(k')}, \quad \tag{3}
\]

\[
\Delta_{tt}(k) = -\frac{1}{L^2} \sum_{k'} F_{kk'}^{tt} V_{kk'}^{eh} \frac{\Delta_{tt}(k')}{2E_t(k')}, \quad \tag{4}
\]

\( E_b(k) = \sqrt{\xi_\beta(k)^2 + \Delta_{bb}^2(k)} \) is the quasi-particle excitation energy for subbands \( \beta \), with \( \xi_\beta(k) = (\xi^{(e)}_\beta + \xi^{(h)}_\beta) / 2 \). \( E_t(k) = E_t(k) \pm \Delta \lambda \) with \( \delta \lambda = (\lambda_h - \lambda_e) / 2 \). \( \lambda_e \) is the spin-splitting of the conduction band of the \( p \)-doped monolayer, and \( \lambda_e \) the corresponding spin-splitting for the \( n \)-doped monolayer, with values taken from Table I. \( \theta[E_t(k)] = 1 - f[E_t(k), 0] \) is a step function that comes from the Fermi-Dirac distribution \( f[E(k), T] \) at zero temperature. \( F_{kk'}^{bb} = |\langle \beta k | \beta' k' \rangle|^2 \) is the form factor that accounts for the overlap of single-particle states in \( k \) and \( k' \) for subbands \( \beta \) in opposite monolayers[19] (See Appendix).

\( V_{kk'}^{eh} \) in Eqs. (3-4) is the screened electron-hole interaction. We use the linear-response random phase approximation for static screening in the superfluid state[2],

\[
V_{kk'}^{eh} = \frac{\lambda e}{2} \left\{ \frac{1}{1 - 2[V_{kk'}^S + \Pi_n(q)][(V_{kk'}^S)^2 - (V_{kk'}^D)^2]} - \frac{1}{2[V_{kk'}^S + \Pi_n(q)][(V_{kk'}^S)^2 - (V_{kk'}^D)^2] + [\Pi_n^2(q) - \Pi_n^D(q)][(V_{kk'}^S)^2 - (V_{kk'}^D)^2]} \right\}, \quad \tag{5}
\]

where \( q = |k-k'| \). \( \Pi_n(q) \) is the normal polarizability in the superfluid state and \( \Pi_n(q) \) is the anomalous polariz-
ability \cite{20, 21}, which is only non-zero in the superfluid state. $\Pi_n(q)$ depends on the population of free carriers (See Appendix). $\Pi_s(q)$, with opposite sign, depends on the population of electron-hole pairs. The combined effect of $\Pi_n(q)$ and $\Pi_s(q)$ is that a large superfluid condensate fraction of strong-coupled and approximately neutral pairs is associated with very weak screening\cite{22}. This is because of the small remaining population of charged free carriers available for screening.

Equation (3) has the same form as for a decoupled one-band system, because the two $b$ bands are aligned\cite{23}. In contrast, Eq. (4) shows explicitly the effect of misalignment of the $t$ bands (Fig. 2) through the term $\theta[E_t^+(k') - \Delta_t^2(k)]$. This can only drop below unity at higher densities where the pair coupling strength is weak compared with the misalignment.

For a given chemical potential $\mu$, the carrier density $n$ of one monolayer is determined as a sum of the subband carrier densities $n_b$ and $n_t$ by,

$$n = g_s g_v \sum_{\beta = b, t} n_{\beta \beta} \tag{6}$$

$$n_{bb} = \frac{1}{L^2} \sum_k v_{\beta}^2(k) \tag{7}$$

$$n_{tt} = \frac{1}{L^2} \sum_k v_{\gamma}^2(k) \theta[E_t^+(k)] + u_{\beta}^2(k)(1 - \theta[E_t^-(k)]) \tag{8}$$

where $v_{\beta}^2$ and $u_{\beta}^2$ are the Bogoliubov amplitudes for the subbands $\beta$ (See Appendix).

The regimes of the superfluid crossover are characterized by the superfluid condensate fraction $c$\cite{24, 25}. $c$ is defined as the fraction of carriers bound in pairs relative to the total number of carriers. For $c > 0.8$ the condensate is in the strong-coupled BEC regime, for $0.2 \leq c \leq 0.8$ in the crossover regime, and for $c < 0.2$ in the BCS regime. In our system, the two condensate fractions are given by,

$$C_{\beta \beta} = \frac{\sum_k v_{\beta}^2(k) v_{\gamma}^2(k)}{\sum_k v_{\gamma}^2(k)} \tag{9}$$

Figure 3(b) shows the dependence on density of the maximum of the superfluid gaps $\Delta_{bb} = \max_k \Delta_{bb}(k)$ for the $b$ and $t$ bands (Eqs. (3-4)) in systems A and B. We took equal effective masses $m^*_c = m^*_\hbar = 0.44 m_e$, a barrier thickness $d = 1 \text{ nm}$, and dielectric constant $\epsilon = 2$, for monolayers encapsulated in a few layers of hBN\cite{26}.

Figure 3(c) shows the evolution of the condensate fractions (Eq. (9)) as a function of density, and Fig. 3(a) the evolution of the chemical potential.

We see in Fig. 3(b) that the form of $\Delta_{bb}$ is similar for systems A and B. At low densities the system is in the strong coupled BEC regime, with condensate fraction $C_{bb} > 0.8$. At these densities the $\{bb\}$ pairing is to a deep bound state with binding energy $E_{bb} \sim 400 \text{ meV}$ below the bottom of the $b$ band\cite{27, 28}. The chemical potential is $\mu \sim -E_{bb}/2$ (Fig. 3(a)). With increasing density, $\Delta_{bb}$ increases and then passes through a maximum. $\mu$ also increases and approaches zero. Eventually, $\Delta_{bb}$ drops sharply to zero at a superfluid threshold density $n_0$. For
n > n_0, the screening of the pairing interaction is so strong that it kills superfluidity [21].

In contrast, \( \Delta_u \) is only non-zero in system B. At low density, \( \Delta_{tt} = 0 \) also in system B, since the pairing population is zero. This is because the chemical potential \( \mu \) at these densities lies below the isolated bound state associated with the \( t \) bands, located at energy \( E_B^t = E_B^s - (\lambda_e + \lambda_h) \). It is only when \( \mu \) passes above \(-E_B^t/2\) that this state can be populated, so \( \Delta_u \) can become non-zero. Further increasing the density increases the \{\( tt \}\} pair population, \( \Delta_{tt} \) increases and then passes through a maximum. When \( \mu \) becomes positive, the build up of free carriers, as evidenced by \( C_{bb} < 0.8 \) in Fig. 3(c), combined with the misalignment of the \( t \) bands, starts to significantly weaken the effective electron-hole screened interaction. Eventually screening kills the superfluidity in both \{\( bb \}\} and \{\( tt \}\} channels at the same threshold density.

We see in Fig. 3(b) that the behavior of \( \Delta_u \) in systems A and B is completely different. In system A the chemical potential remains below the isolated bound state \( E_B^t \) associated with the \( t \) bands over the full range of densities up to \( n_0 \). With \( \mu \) lying below \( E_B^t \), the population of pairs in the \{\( tt \}\} channel remains zero. The only difference between system A and B is the choice of doping which results in the markedly different misalignment of the \( tt \) bands, leading one-component or two-component superfluidity.

The strikingly different behavior of \( \Delta_u \) in the two systems is a new and remarkable effect that can be probed using angle-resolved photoemission spectroscopy (ARPES)[29]. ARPES measures the spectral function, which in a one-component superfluid state like system A will have a single peak centred at a negative frequency corresponding to \( \Delta_{bb} \). However in system B, when it switches from one-component to two-component superfluidity, two peaks associated with the gaps \( \Delta_{bb} \) and \( \Delta_{tt} \) will appear in the spectral function at negative frequencies[30]. Other experimental techniques that can be used to detect the presence or absence of the second gap \( \Delta_{tt} \) are Andreev reflection spectroscopy[31, 32] and scanning tunneling microscopy[33].

In Fig. 3(b), we note that the threshold densities \( n_0 \) for the superfluidity are much larger than the threshold densities \( n_0 \sim 8 \times 10^{11} \text{ cm}^{-2} \) in double bilayer graphene[1, 2], and the \( n_0 \sim 4 \times 10^{12} \text{ cm}^{-2} \) predicted for double layer phosphorene[34]. \( n_0 \) is large for the double TMDC monolayers for two main reasons: (i) the large effective masses of the electrons and holes means a large effective Rydberg energy scale, thus large superfluid gaps \( \Delta \) that strongly suppress the screening; (ii) the large TMDC monolayer bandgaps \( E_g \) eliminate valence band screening, making the electron-hole pairing interaction very strong[2].

These large threshold densities in the double TMDC monolayers lead to high Berezinskii-Kosterlitz-Thouless transition temperatures \( T_{KT} \)[35]. The monolayers have near parabolic bands, so we can approximate[36, 37],

\[
T_{KT} = \frac{\pi}{2} \rho_s(T_{KT}) \sim n \frac{\pi^2 \hbar^2}{8 g_s g_m n^2}.
\]  

(10)

\( \rho_s(T) \) is the superfluid stiffness. Equation (10) gives transition temperatures for systems A and B at their threshold densities of \( T_{KT}^A = 110 \text{ K} \) and \( T_{KT}^B = 120 \text{ K} \).

In summary, we have investigated multicomponent effects for electron-hole multiband superfluidity in \( n-p \) and \( p-n \) doped MoSe\(_2\)-hBN-WSe\(_2\) heterostructures (systems A and B, respectively). Both systems are multiband and can stabilize electron-hole superfluidity at temperatures above 100 K. Surprisingly we find that only in system B can superfluidity have two components. For both systems we would have expected to be able to tune from one- to two-component superfluidity by increasing the density, as recently observed in multiband superconductors[12], and this is indeed the case for system B. However for system A, the very large misalignment of the electron and hole top bands, means that there are no carriers available for pairing in the topmost band before screening has become so strong that it completely suppresses superfluidity. Therefore only one-component superfluidity is possible in system A. This is a remarkable result: activation of the second-component of the superfluidity in this heterostructure depends crucially on the choice of which TMDC monolayer is \( n \)-doped and which \( p \)-doped.

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Appendix: Mean field equations

The temperature dependent normal and anomalous multiband Matsubara Green functions, with subband indices \( \alpha \) and \( \beta \), are

\[
\begin{align*}
G^{\alpha\beta}(k, \tau) &= -T \langle \hat{c}_k^\alpha(\tau) \hat{c}_k^\beta(0) \rangle \\
F^{\alpha\beta}(k, \tau) &= -T \langle \hat{c}_k^\alpha(\tau) \hat{d}_k^\beta(0) \rangle.
\end{align*}
\]  

(11)

The mean field gap equations and density equations are calculated as [18]:

\[
\Delta_{\alpha\beta}(k) = -\frac{T}{L^2} \sum_{\alpha',\beta',k',\omega_n} F^{\alpha\beta\alpha'\beta'}_{kk'} V_{kk'}^c F^{\alpha'\beta'}_{k'k}(k',i\omega_n)\]

(12)

\[
n_{\alpha\beta} = \frac{T}{L^2} \sum_{k,i\omega_n} G^{\alpha\beta}(k,i\omega_n)\]

(13)

where \( F^{\alpha\beta\alpha'\beta'}_{kk'} = \langle \alpha' k | \alpha k \rangle \langle \beta k | \beta' k' \rangle \) is the overlap form factor. We are neglecting the cross-pairing contribution,
so we retain the Green functions and the form factors only for $\alpha = \beta (\alpha' = \beta')$. Furthermore Josephson-like pair transfer\cite{18} is forbidden since there are no spin-flip scattering processes and so $F_{kk'}^{\beta \beta'} = 0$ for $\beta \neq \beta'$. We adopt the abbreviated notation $F_{kk'}^{\alpha \beta} \equiv F_{kk'}^{\beta \beta}$.

In terms of Bogoliubov amplitudes:

$$u_\beta^2(k) = \frac{1}{2} \left(1 - \frac{\xi_\beta(k)}{E_\beta(k)}\right); \quad u'_\beta^2(k) = \frac{1}{2} \left(1 + \frac{\xi_\beta(k)}{E_\beta(k)}\right)$$

$$G^{\beta \beta}(k, i\omega_n) = \frac{u_\beta^2}{i\omega_n - E_\beta} + \frac{u'_\beta^2}{i\omega_n + E_\beta}$$

$$F^{\beta \beta}(k, i\varepsilon_n) = \frac{u_\beta v_\beta}{i\omega_n - E_\beta} + \frac{u'_\beta v_\beta}{i\omega_n + E_\beta}.$$  

Performing the summation over the Matsubara frequencies $\omega_n = \pi T(2n + 1)$, in the limit of zero temperature, the gap Eqs. (3-4) and the density Eqs. (7-8) are obtained. $\theta(E_\beta(k)) = 1 - f[E_\beta(k)]$, 0, is a step function that comes from the Fermi-Dirac distribution $f[E(k), T]$ at zero temperature.

The polarizabilities as functions of $q = |k - k'|$ are [20]:

$$\Pi_n(q, \Omega_l) = T \frac{g_\beta g_\gamma}{L^2} \sum_{\beta, k, i\omega_n}
\left\{F_{kk}^{\beta \beta} G^{\beta \beta}(k, i\omega_n + i\Omega_l) G^{\beta \beta}(k, i\omega_n)\right\}$$

$$\Pi_\sigma(q, \Omega_l) = T \frac{g_\beta g_\gamma}{L^2} \sum_{\beta, k, i\omega_n}
\left\{F_{kk}^{\beta \beta} F^{\beta \beta}(k, i\omega_n + i\Omega_l) F^{\beta \beta}(k, i\omega_n)\right\}$$

The polarizabilities in the effective electron-hole interaction (Eq. (5)) are obtained by evaluating Eqs. (17) and (18) at zero temperature in the static limit, $\Omega_l \to 0$.

[1] G. W. Burg, N. Prasad, K. Kim, T. Taniguchi, K. Watanabe, A. H. MacDonald, L. F. Register, and E. Tutuc, “Strongly enhanced tunneling at total charge neutrality in double-bilayer graphene-WSe$_2$ heterostructures,” Phys. Rev. Lett. 120, 177702 (2018).

[2] S. Conti, A. Perali, F. M. Peeters, and D. Neilson, “Multicomponent screening and superfluidity in gapped electron-hole double bilayer graphene with realistic bands,” Phys. Rev. B 99, 144517 (2019).

[3] Y. Zhang, T. T. Tang, C. Girit, Z. Hao, M. C. Martin, A. Zettl, M. F. Crommie, Y. R. Shen, and F. Wang, “Direct observation of a widely tunable bandgap in bilayer graphene,” Nature (London) 459, 820 (2009).

[4] K. F. Mak, C. Lee, J. Hone, J. Shan, and T. F. Heinz, “Atomically thin MoS$_2$: a new direct-gap semiconductor,” Phys. Rev. Lett. 105, 136805 (2010).

[5] H. Jiang, “Electronic band structures of molybdenum and tungsten dichalcogenides by the GW approach,” J. Phys. Chem. C 116, 7664 (2012).

[6] M. M. Fogler, L. V. Butov, and K. S. Novoselov, “High-temperature superfluidity with indirect excitons in van der Waals heterostructures,” Nat. Commun. 5, 4555 (2014).

[7] C. R. Dean, A. F. Young, I. Meric, C. Lee, L. Wang, S. Sorgenfrei, K. Watanabe, T. Taniguchi, P. Kim, K. L. Shepard, and J. Home, “Boron nitride substrates for high-quality graphene electronics,” Nat. Nanotechnol. 5, 722 (2010).

[8] K. K. Kim, A. Hsu, X. Jia, S. M. Kim, Y. Shi, M. Dresselhaus, T. Palacios, and J. Kong, “Synthesis and characterization of hexagonal boron nitride film as a dielectric layer for graphene devices,” ACS Nano 6, 5853 (2012).

[9] F. G. Kocherbe and M. E. Pustlari, “Superconductivity in graphene,” Phys. Rev. B 89, 060502 (2014).
tivity in a two-band system with low carrier density,” J. Exp. Theor. Phys. 77, 442 (1993).

[24] L. Salasnich, N. Manini, and A. Parola, “Condensate fraction of a Fermi gas in the BCS-BEC crossover,” Phys. Rev. A 72, 023621 (2005).

[25] P. López Ríos, A. Perali, R. J. Needs, and D. Neilson, “Evidence from quantum Monte Carlo simulations of large-gap superfluidity and BCS-BEC crossover in double electron-hole layers,” Phys. Rev. Lett. 120, 177701 (2018).

[26] P. Kumar, Y. S. Chauhan, A. Agarwal, and S. Bhownick, “Thickness and stacking dependent polarizability and dielectric constant of graphene-hexagonal boron nitride composite stacks,” J. Phys. Chem. C 120, 17620 (2016).

[27] M. Randeria, J.-M. Duan, and L.-Y. Shieh, “Superconductivity in a two-dimensional Fermi gas: Evolution from Cooper pairing to Bose condensation,” Phys. Rev. B 41, 327 (1990).

[28] F. Pistolesi and G. C. Strinati, “Evolution from BCS superconductivity to Bose condensation: Role of the parameter $k_F \xi$,” Phys. Rev. B 49, 6356 (1994).

[29] M. Saberi-Pouya, M. Zarenia, A. Perali, T. Vazifehshenas, and F. M. Peeters, “High-temperature electron-hole superfluidity with strong anisotropic gaps in double phosphorene monolayers,” Phys. Rev. B 97, 174503 (2018).

[30] J. M. Kosterlitz and D. J. Thouless, “Ordering, metastability and phase transitions in two-dimensional systems,” J. Phys. C: Solid State 6, 1181 (1973).