Noise color and asymmetry in stochastic resonance with silicon nanomechanical resonators

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Abstract. Stochastic resonance with white noise has been well established as a potential signal amplification mechanism in nanomechanical two-state systems. While white noise represents the archetypal stimulus for stochastic resonance, typical operating environments for nanomechanical devices often contain different classes of noise, particularly colored noise with a $1/f$ spectrum. As a result, improved understanding of the effects of noise color will be helpful in maximizing device performance. Here we report measurements of stochastic resonance in a silicon nanomechanical resonator using $1/f$ noise and Ornstein-Uhlenbeck noise types. Power spectral densities and residence time distributions provide insight into asymmetry of the bistable amplitude states, and the data sets suggest that $1/f^\alpha$ noise spectra with increasing noise color (i.e. $\alpha$) may lead to increasing asymmetry in the system, reducing the achievable amplification. Furthermore, we explore the effects of correlation time $\tau$ on stochastic resonance with the use of exponentially correlated noise. We find monotonic suppression of the spectral amplification as the correlation time increases.

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1 Introduction

Although it is not surprising given the wealth of mechanical, electronic, biological and other systems which display stochastic resonance (SR) [1], the observation of noise-enhanced switching in nanoelectromechanical systems (NEMS) holds promise for the possible exploitation of noise to improve device performance [2]. In addition, the discovery of SR in NEMS has made available yet another experimental platform for the study of basic questions in the field, such as the effect of noise color. Combining these motivations, we discuss measurements of SR in a silicon nanomechanical resonator using various colored noise spectra.

Improving the model for SR in this complex application warrants additional attention as NEMS continue to emerge as a viable technology. In particular, the bistable amplitude states of a nonlinear nanomechanical resonator may see use as two-state devices such as memory elements and switches [3,4]. A particle subject to white noise in a periodically-modulated double-well potential remains a powerfully intuitive model for stochastic resonance, capturing the basic dynamics in a diverse array of systems. Still, much work has been devoted to generalizing this picture. Asymmetric potentials [5–8], spatially extended systems [9], and “non-conventional” stochastic resonance between coexisting periodic attractors [10] have been among the many extensions to the original idea, with the latter two offshoots both applying to nanomechanical SR. Such derivations continue to be important in understanding stochastic resonance in actual practice, where the experimental conditions are rarely straightforward.

Yet another important and ongoing consideration is in understanding the effect of noise color on stochastic resonance [11–17]. In particular, the ubiquity of $1/f$ noise makes it a prominent factor in a wide range of settings [18], including in the active electronic elements used to drive nanomechanical resonators [19]. This noise class has been used to induce SR in electronic [13], neurological [15], and most recently, nanoelectromechanical systems [17]. Although the suppression of SR persists as a common theme in these studies, to the authors knowledge, an accepted physical picture for the effect of $1/f^\alpha$ noise exponent is lacking. Asymmetry has been postulated as a consequence of $1/f$ noise, and evidence in this study further reinforces that idea.

Extending the study to another class of colored noise, we also observe SR with the use of exponentially correlated (Ornstein-Uhlenbeck) noise. We find that the
amplification of the signal declines with increasing correlation strength of the noise, in agreement with theoretical predictions and previous experimental results \[11,12,14\].

Previous studies of SR in nanomechanical resonators have focused mainly on traditional measures, such as signal-to-noise ratio (SNR) and spectral amplification \[2,17,20\]. Given the technological potential of these systems, enhancements seen in SNR and signal amplification provide obvious implications for the utility of SR in an applied setting. However, the wealth of analyses that the field has grown to embrace, including of power spectral densities and residence time distributions, afford further insights. We use these approaches here to discern the role of asymmetry on system dynamics.

2 Experimental methods

2.1 Device fabrication and characterization

Using e-beam lithography, resonators are patterned on the single crystal silicon layer of a silicon-on-insulator (SOI) wafer. The length \(L\) and width \(w\), 20 \(\mu\)m and 300 nm, respectively, place the in-plane resonance frequency of the doubly-clamped beam in the MHz range. A gap \(d\) of 250 nm at equilibrium separates the resonator from adjacent parallel electrodes used for actuation and detection, and the device layer thickness is 500 nm. Metal contacts are deposited via thermal evaporation, giving the resonator a thickness of 550 nm. Finally, reactive ion etching (RIE) and hydrofluoric acid etching are used to define and suspend the silicon structure.

To actuate the beam, we utilize the standard electrostatic technique at room temperature \([4]\) and references therein). With a DC bias \(V_B\) charging the resonator, a high frequency excitation signal \(v_D(t)\) applied to the drive electrode creates a capacitive force. The system can be modeled as a Duffing oscillator \[21\]:

\[
\ddot{x} + \gamma \dot{x} + \omega_0^2 x + k_3 x^3 = \frac{1}{2} \frac{dC}{dx} V_B v_D(t) \tag{1}
\]

where \(x\) is the effective resonator displacement, \(\gamma\) is a dissipation coefficient, \(\omega_0\) is the resonant angular frequency of oscillation, and \(k_3\) is the non-linear coefficient. Here, the resonator and drive electrode may be accurately approximated as parallel plates, making the capacitance between them \(C = \epsilon_0 L t / (d - x)\). In-plane oscillation of the biased resonator induces a current at the opposite electrode, which we amplify and measure with a network analyzer. The frequency response exhibits the expected Lorentzian line shape for small displacements (Fig. 1a).

As drive power is increased, nonlinearity leads to spring hardening, and two stable amplitude states emerge with the onset of hysteresis. These bistable amplitude states provide a two-state system for the study of SR.

2.2 Modulation and noise

Whereas previous studies of SR in nanomechanical systems have utilized an additional periodic signal for switching \[2,20\], we modulate the potential by means of a phase deviation \(\varphi_0\) in the drive signal itself. Recent experiments have shown that an abrupt phase shift can be used to induce a switch between the bistable amplitude states of a nanomechanical resonator \[4\]. Here, we drive the system in the bistable region and modulate the phase of the drive with a square wave:

\[
v_D(t) = v_D \cos \left(\omega t + \frac{\varphi_0}{2}\Theta(\Omega t)\right) \tag{2}
\]

where \(\Theta(\Omega t)\) represents a square wave of period \(\frac{1}{\Omega}\), alternating between +1 and -1 each successive half period. The drive frequency \(\omega/2\pi\) is a frequency in the bistable region, and \(\Omega \ll \omega/2\pi\) (\(\Omega = 50\) Hz for all results reported here). Expanding this expression, we see that the drive consists of two parts:

\[
v_D(t) = v_D \left[ \cos \left(\frac{\varphi_0}{2}\right) \cos(\omega t) + \Theta(\Omega t) \sin \left(\frac{\varphi_0}{2}\right) \sin(\omega t) \right]. \tag{3}
\]

The first term in the brackets provides simple periodic forcing. The second term, which changes phase by 180 degrees each half period of the square wave, induces switching of the resonator amplitude in synchronization with \(\Theta(\Omega t)\). The magnitude of \(\varphi_0\) dictates the amplitudes of