Barrier function-based adaptive integral sliding mode finite-time attitude control for rigid spacecraft

Jie Wang · Yushang Hu · Wenqiang Ji

Received: 5 June 2021 / Accepted: 23 March 2022 / Published online: 24 August 2022
© The Author(s), under exclusive licence to Springer Nature B.V. 2022

Abstract This paper investigates the problem of attitude tracking control with predefined-time convergence for rigid spacecraft under external disturbances and inertia uncertainties. Firstly, the proposed nominal controller is designed to achieve attitude tracking control of the rigid spacecraft in the absence of disturbances and inertia uncertainties, and the convergence of the spacecraft attitude errors can be selected in advance. Then, the integral sliding mode combined with barrier function-based adaptive laws is proposed to reject the disturbances and inertia uncertainties, and at the same time, a barrier function-based adaptive method can also ensure the solutions of the rigid spacecraft system belonging to a stipulated vicinity of the intended variables starting from the initial moment and the uncertainties’ upper bound is not overestimated. Finally, a numerical simulation is provided to illustrate the efficiency of the proposed control protocol.

Keywords Barrier functions · Integral sliding mode control · Rigid spacecraft · Attitude control · Predefined-time convergence

1 Introduction

Attitude control of the rigid spacecraft has become an essential and practical issue in recent years, because of its critical responsibilities such as space surveillance, space support, space operations and spacecraft formation flying. During past decades, numerous control strategies have been developed to cope with this issue, such as adaptive control [1,2], sliding mode control [3,4], backstepping control [5,6] and fuzzy control [7,8]. Among these approaches, sliding mode control (SMC) is one of the most effective control strategies for rigid spacecraft attitude control in the presence of disturbances and uncertainties.

Recently, some significant results for attitude tracking have been obtained. The application of SMC for controlling spacecraft attitude was originally reported in [9], and since then, a number of SMC controllers have been developed by researchers. Traditional sliding mode [10,11], terminal sliding mode (TSM) [12–14] and fast terminal sliding mode (FTSM) [15,16] and integral terminal sliding mode (ITSM) [17,18] are gradually used for attitude tracking of the rigid spacecraft. In [10], traditional sliding mode control has been utilized to track attitude; however, its convergence rate and robustness are inferior to TSM. In [13], the attitude stabilization is considered for rigid spacecrafts and the use of TSM allows countries to converge with external disturbances into small regions of origin. In order to boost convergence speed even more, a new modi-
fied fast terminal sliding mode is being developed to overcome the problem of multiple spacecrafts attitude tracking [15]. Compared with TSM, FTSM effectively enhances the convergence rate and eliminates the singularity problem. Although the convergence speed is guaranteed and a process exists to reach the sliding surface, the global robustness cannot be guaranteed. To overcome this drawback, the global sliding mode is developed, such as integral sliding mode and integral terminal sliding mode. In [18], a recursive-structure integral terminal sliding mode is presented to eliminate the reaching phase and guarantee the finite-time stability of the system. The spacecraft fixed-time attitude tracking based on integral sliding mode control described in [19] eliminated the process of reaching the sliding mode surface while ensuring the global robustness. In the preceding literature, the upper bound of the external disturbance and the inertial matrix are assumed to be known.

For sliding mode control, the chattering problem is a paramount issue. The conventional SMC employs a control law with significant control gains, resulting in undesirable chattering while the control system is in sliding mode. Zhang [20] employs the concept of adding power integral to develop an attitude tracking controller, but it concentrated on dominating the rigid spacecraft’s nonlinearities rather than canceling them in the feedback design. Nevertheless, in order to avoid chattering and ensure that the states converge in fixed time, the control gains and the applied control torques must be considerable. In recent years, a more effective technique to reduce control chatter is also the usage of higher order control sliding mode (HOSM) [21–23]. Because the discontinuous control input is applied on a higher time derivative of the sliding variable in HOSM, the chattering is reduced. However, the theoretical analysis for HOSM control is difficult, and the computational overhead is high. The boundary is commonly used to reduce chattering, and numerous adaptation mechanisms have been devised to modify the control gains. To eliminate chattering, researchers developed an adaptive method [24] and a neural-network-based adaptive controller [25]. However, none of the aforementioned literatures addressed the uncertain bound of disturbances, making it arduous to apply the assumption of the uncertainties.

On the other hand, it is noted that convergence is the most important control objective for the rigid spacecraft and its corresponding index is the settling time that means the time when the tracking errors converge to zero. To the best of our knowledge, it can be divided into finite-time convergence [26], fixed-time convergence [27] and predefined-time convergence [28]. An output feedback attitude tracking control is designed for the rigid spacecraft, which ensures that the attitude tracks the time-varying reference attitude within finite time [29]. A finite-time neural adaptive observer for rigid spacecraft attitude tracking with disturbances and uncertainties was presented in [30]. Disturbance observer was used for attitude control and by using TSM control strategy, the rigid spacecraft attitude is stable in finite-time and reduced the chattering [31,32]. Nevertheless, the finite-time convergence depends on the initial condition of the system. To overcome this drawback, a fixed-time convergence protocol is proposed which depends on the system parameters. A novel fixed-time nonsingular terminal sliding mode surface is developed to achieve fixed-time trajectory tracking [27]. But the calculation of the fixed-time convergence is complex. Recently, a predefined-time control for distributed-order systems is proposed which achieve predefined-time convergence [33]. However, it is worth mentioning that there is still a lack of research on the optimal control with the predefined convergence time for the rigid spacecraft system, which motivates this study.

In fact, it is difficult to introduce an optimal adaptive sliding mode control method with a predefined convergence time to the attitude control problem for rigid spacecraft. Three of the main difficulties are shown as follows: (1) In practice, the external disturbances being often unknown, how can estimate the disturbance upper bound without disturbance upper bound information and, at the same time, weaken the jitter without affecting the convergence time. (2) The output of the actuator is limited, how to solve the problem of actuator saturation that may be caused by the adaptive law designed based on the barrier function. (3) How to design the optimal feedback control to achieve the predefined spacecraft attitude errors convergence time.

In this paper, inspired by the ISMC method [34] and barrier function-based adaptive techniques [35], the proposed design of attitude tracking for a rigid spacecraft system controller is provided satisfactory answers to the aforementioned questions. The main contributions of this paper are given as follows:

Firstly, an optimal feedback controller on a finite time interval is first applied to the rigid spacecraft atti-
tude control problem, which depends on the initial attitude position of the rigid spacecraft and allows the attitude errors to converge to a fixed final state in predefined time, and the convergence time is well known in advance, which is easily put into practice.

Secondly, a barrier-function-based adaptive law based on the integral sliding mode surface is firstly applied for the rigid spacecraft. In contrast to the barrier function-based adaptive techniques [35], the initial condition of the integral sliding mode surface is located in the domain of barrier function initially, which simplifies the proof of system stability.

The rest of this paper is organized as follows. In Sect. 2, the nonlinear spacecraft model and the barrier functions (BFs) are given. Section 3 presents the proposed barrier algorithms whose convergence time is known in advance. In Sect. 4, simulation results are provided to verify the effectiveness of the proposed control approach. Finally, the finding of the research and future work is presented in Sect. 5.

2 Model description and problem statement

2.1 Notations

Define $R$ as the real number, and $R^{m \times n}$ represents the $m \times n$ real vector. Denote a vector $s = [s_1, ..., s_n] \in R^{n \times 1}$, define $\text{Sign}(s) = [\text{sign}(s_1), ..., \text{sign}(s_n)]^T$ and $\text{sign}(\cdot)$ is the standard sign function, $\|x\| = \sum_{i=1}^n |x_i|$; $I_n$ is the identity matrix of order $n$. $\times$ is a mathematical operation on arbitrary vector $x = [x_1, x_2, x_3]^T$.

2.2 Mathematical Model of spacecraft attitude system

Consider the attitude stabilization control of a rigid spacecraft during maneuvering, a singularity-free global representation is achieved by using unitary quaternion to describe the attitude of a rigid spacecraft. The kinematic and dynamic models of the spacecraft’s attitude are established, which are described as the following form [36]:

$$\begin{align*}
\dot{\Psi} &= \frac{1}{2}(\Psi_4 I_3 + \Psi_v \times) \Omega \\
\dot{\Psi}_4 &= -\frac{1}{2} \Psi_v^T \Omega
\end{align*}$$

(1)

$$J \Omega = -\Omega \times J \Omega + T_u + T_d$$

(2)

$\Psi = [\Psi_1, \Psi_2, \Psi_3, \Psi_4]^T = [\Psi_v, \Psi_4]^T$ satisfying $\|\Psi\| = 1$ is used to define the unit quaternion, with $\Psi_v \in R^3$ representing the vector part and $\Psi_4 \in R$ representing the scalar component. The moment of inertia, $J \in R^{3 \times 3}$, can be written as follows:

$$J = \begin{pmatrix}
J_{11} & J_{12} & J_{13} \\
J_{21} & J_{22} & J_{23} \\
J_{31} & J_{32} & J_{33}
\end{pmatrix}$$

(3)

$I_3$ is the $R^{3 \times 3}$ identity matrix. $\Omega = [\Omega_1, \Omega_2, \Omega_3]^T \in R^{3 \times 1}$ is the attitude angular velocity of the spacecraft, which represents projection of the angular velocity of the body coordinate system with respect to the geocentric inertial coordinate system in the body coordinate system. $T_u \in R^{3 \times 1}$ denotes the control torque, while $T_d \in R^{3 \times 1}$ denotes the disturbances torque. $\times$ is a mathematical operation on arbitrary vector $\vartheta = [\vartheta_1, \vartheta_2, \vartheta_3]^T$ such as:

$$\vartheta \times = \begin{pmatrix} 0 & -\vartheta_3 & \vartheta_2 \\
\vartheta_3 & 0 & -\vartheta_1 \\
-\vartheta_2 & \vartheta_1 & 0 \end{pmatrix}$$

(4)

Assumption 1 The full states of the quaternion units $\Psi$ and angular velocity $\Omega$ are available in the feedback control design in rigid model (1) and (2).

The control objective is to devise a robust adaptive control law that enables the rigid spacecraft system (1) to track the following reference path in the determined manner, notwithstanding the external disturbances and inertia uncertainties. Assume:

$$\dot{\Psi}_{d_1} = \frac{1}{2} (\Psi_{d_4} I_3 + \Psi_{d_4} \times) \Omega_d \quad \dot{\Psi}_{d_4} = -\frac{1}{2} \Psi_{d_4}^T \Omega_d$$

(5)

where $\Psi_d = [\Psi_{d_1}, \Psi_{d_2}, \Psi_{d_3}, \Psi_{d_4}]^T \in R^{4 \times 1}$ satisfying $\|\Psi_d\| = 1$. It is assumed that $\Omega_d$ and $\dot{\Omega}_d$ are bounded.

The purpose of tracking the attitude of the rigid spacecraft can be transformed into a stabilization problem, defined the attitude tracking error of the rigid spacecraft $e = [e_1 \ e_2 \ e_3 \ e_4]^T = [e_v^T \ e_4]^T$ as mentioned in [13]:

$$e_v = \Psi_{d_4} q_v - \Psi_{d_4}^\times \Psi_v - \Psi_4 \Psi_{d_4}$$

(6)

$$e_4 = \Psi_{d_4}^T \Psi_v + \Psi_4 \Psi_{d_4}$$

(7)

$$\Omega_e = \Omega - L \Omega_d$$

(8)

where the corresponding rotation matrix $L$ can be expressed as:

$$L = (e_3^2 - e_1^2 e_4^2) I_3 + 2 e_1 e_2^T e_3 - 2 e_4^e_1^\times$$

(9)
We can obtain:
\[ \dot{e}_v = \frac{1}{2}(e_4 I_3 + e_v^x)\Omega_e \quad \dot{e}_d = -\frac{1}{2}e_v^T \Omega_e \]  
(10)
\[ J \ddot{\Omega}_e = -(\Omega_e + L \Omega_d)^x J(\Omega_e + L \Omega_d) + J(\Omega_e^x L \Omega_d - L \dot{\Omega}_d) + T_u + T_d \]  
(11)

In this article, we propose to control the control torque \( T_u \) in order to follow the intended attitude angle \( \Psi_d \) of the rigid spacecraft, which can be described as follows:
\[ \lim_{t \to t_F} v_1(t) = 0, \quad v_1(t) = \Psi(t) - \Psi_d(t) \]  
(12)
\[ \lim_{t \to t_F} v_2(t) = 0, \quad v_2(t) = \Omega(t) - \Omega_d(t) \]  
(13)
where \( t_F \) is given in advance. The objective (12–13) can be attainable where a control law for system (10–11) exists, such that \( \lim_{t \to t_F} v_1(t) = 0 \) and \( \lim_{t \to t_F} v_2(t) = 0 \).

2.3 Integral sliding mode surface

The integral sliding mode (ISM) control law is a type of sliding mode control that is well known. It ensures the robustness of the system right from the outset. In this paper, the ISM surface is designed as follows:
\[ S(z) = [z_{2,1}, z_{2,2}, z_{2,3}]^T + \bar{z} \]  
(14)
where \( 1 \leq i \leq 3, \ 1 \leq j \leq 2 \), \( z_{i,j} = e_{v(i-j)} \), \( z_i = [z_{1,i}, z_{2,i}] \), \( S(z) = [s_1(z_1), s_2(z_2), s_3(z_3)]^T \in R^3 \) and \( \bar{z} = -\omega_{nom} = [-\omega_{nom,1}(z_1), \omega_{nom,2}(z_2), \omega_{nom,3}(z_3)]^T \) is designed in Sect. 3.1.

2.4 Barrier functions (BFs)

In order to make the sliding mode surface in a specified area of nominal curve and estimate the uncertainties’ upper bound online, define barrier functions as follows:

**Definition 1** [35]. Assume that \( \alpha > 0 \), and the barrier function can be described as an even continuous functions \( G_b : x \in [-\alpha, \alpha] \to G_b(x) \in [b, +\infty) \) strictly ascending on \( 0, \alpha \).

(i) \( \lim_{|x| \to \infty} G_b(x) = +\infty \).

(ii) \( G_b(x) \) has the unique minimum at origin and \( G_b(0) = b \geq 0 \).

In this paper, the following two separate groups of barrier functions are discussed as:

(i) Positive definite BFs(PBFs): \( G_{PB}(x) = \frac{a^{\beta}}{a - |x|} \), i.e., \( G_{PB}(0) = \beta > 0 \).

(ii) Positive semi-definite BFs(PSBF): \( G_{PSBF}(x) = \frac{a^{\beta}}{a - |x|} \), i.e., \( G_{PSBF}(0) = 0 \).

3 Design of the spacecraft attitude controller

In this section, the ISM controller is designed to guarantee a long and accurate tracking. The following assumptions are necessary before giving the main results.

**Assumption 2** The uncertain inertia matrix in (2) is uncertain but bounded, and it can be separated into a nominal element and an unknown uncertain bounded inertia matrix.
\[ J = J_0 + J_1 \]  
(15)
where the unknown inertia matrix satisfies the inequality \( \|J_1\| \leq \rho J \), and \( \rho J \) exists and is unknown positive upper bound constant.

**Assumption 3** The disturbance in (2) shall be considered as bound and as follows:
\[ \|T_d\| \leq \zeta_d \]  
(16)
where \( \zeta_d \) is the unknown constant.

The derivative of \( \dot{e}_v \) can be expressed as follows:
\[ \dot{e}_v = \frac{1}{2}(e_4 I_3 + e_v^x)\Omega_e + \frac{1}{2}(e_4 I_3 + e_v^x)\dot{\Omega}_e \]
\[ = -\frac{1}{4}[e_1(\Omega_1^2 + \Omega_2^2 + \Omega_3^2), e_2(\Omega_1^2 + \Omega_2^2 + \Omega_3^2), e_3(\Omega_1^2 + \Omega_2^2 + \Omega_3^2)]^T \]
\[ + \frac{1}{2}(e_4 I_3 + e_v^x)\Omega_e \]  
(17)
where the total uncertainties are denoted as \( R(t) = \frac{1}{2}(e_4 I_3 + e_v^x)J_1^{-1}T_d + \frac{1}{2}(e_4 I_3 + e_v^x)J_1^{-1}(-\Omega^x J_1 \Omega - J_1 \dot{\Omega}) \), which includes inertia uncertainties and external disturbances.

**Assumption 4** Similar to the assumption in [1, 2, 10, 15, 16, 19, 31, 38]. The lumped system uncertainties \( R(t) \) in (17) are supposed to be constrained by the function below:
\[ \|R(t)\| \leq m + n \|\Omega_e\| \]  
(18)
where \( \|\Omega_e\| = \|\Omega\|^\alpha + \|\Omega\|^2 - \alpha + \|\Omega\|^2 + \|\Omega\|^\beta m \) and \( m \) and \( n \) are non-negative constants that are unidentified.

**Remark 1** In actuality, the inertia matrix operating on the spacecraft system is bounded [15, 16]. In practice, external unknown disturbances such as environmental disturbances, sun radiation, and magnetic influences
are all limited \([10,19]\). In actuality, control torque is bounded \([1,2,31,38]\). It is noted that \(\|e_v\| \leq 1\) and \(\|\Psi_v\| \leq 1\), as is similar in certain works such as \([1,2,10,15,16,19,31,38]\). Assumption 4 is reasonable and satisfied.

For the system (17) without total uncertainties, the following feedback control law is designed as:

\[
T_u = \left[ \frac{1}{2}(e_4 I_3 + e_v^2) J_{0}\right]^{-1} \left[ \frac{1}{2} e_v \Omega_e^T \Omega_e + \frac{1}{2} e_v I_3 + e_v^2 \right] J_{0}^{-1} \Omega^x J_{0} \Omega 
- \frac{1}{2} (e_4 I_3 + e_v^2) (\Omega_e^x L \Omega_d - L \dot{\Omega}_d) + \omega \right]
\]  
(19)

where \(\omega = [\omega_1, \omega_2, \omega_3]^T\) represents the auxiliary control input signals which will be designed in the next section. Applying the control input \(T_u\) to the system (17), we obtain:

\[
[\dot{e}_{v(1)}, \dot{e}_{v(2)}, \dot{e}_{v(3)}]^T = \omega + R(t)
\]  
(20)

Equation (20) can be rewritten as follows:

\[
\begin{cases}
\dot{z}_{1,i} = z_{2,i} \\
\dot{z}_{2,i} = z_{3,i} \\
\dot{z}_{2,2,2} = \dot{z}_{2,3} \\
\end{cases}
\]  
(21)

where \(1 \leq i \leq 3, 1 \leq j \leq 2, z_{j,i} = e_{v(j,i)}, z_i = [z_{1,i}, z_{2,i}, z_{3,i}]^T\).

In order to stabilize the rigid spacecraft system in presence of lumped uncertainties in \(t_F\), the auxiliary control input signals are developed as follows:

\[
\omega = \omega_{nom} + \omega_{disc}(\bar{z}, \bar{z})
\]  
(22)

where \(\omega_{nom} = [\omega_{nom,1}, \omega_{nom,2}, \omega_{nom,3}]^T\), named the nominal control, is continuous and guarantees the attitude tracking errors converge to zero in predefined convergence time \(t_F\). \(\omega_{disc}(\bar{z}, \bar{z})\) represents adaptive integral sliding mode control, which rejects the effect of the lumped uncertainties.

### 3.1 Continuous control part design

In this subsection, considering the rigid spacecraft system (21) in the absence of the total uncertainties which can be written as:

\[
\dot{z}_i = Az_i + B \omega_{nom,i} \quad \forall i \in \{1, 2, 3\}
\]  
(23)

where \(A\) and \(B\) can be defined as:

\[
A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]  
(24)

The control goal is to achieve the state \(z_i = 0\) at a given final time \(t = t_F < \infty\), where \(z_i(0)\) is a constrained initial state. The objective can be achieved by the optimum control law. The following performance indicators are used to complete the control law:

\[
I = \frac{1}{2} \int_0^{t_F} z_i^T Q_i z_i + \omega_{nom,i}^2 dt
\]  
(25)

with \(t_F < +\infty, |z_i(0)| < +\infty\) and \(Q_i\) being a symmetric positive defined matrix, and the state \(|z_i(t_F)| = 0\) is guaranteed.

**Theorem 1** Consider the spacecraft systems (10)-(11) without the lumped system uncertainties \(R(t)\), i.e., \(R(t) = 0\). The tracking errors \(e_v\) and \(\Omega_e\) will converge to the origin in \(t_F\) under the minimum performance indicators (25). The control law is as follows:

\[
\omega_{nom,i} = -B^T \mu_i
\]  
(26)

with \(\mu_i(t)\) and \(P_i\) can be defined by:

\[
\dot{\mu}_i = -(A^T - P_i B B^T) \mu_i, \quad 0 = P_i A + A^T P_i - P_i B B^T P_i + Q_i
\]  
(27)

where in order to satisfy the control objective \(z_i(t_F) = 0\), the initial condition \(\mu(0)\) is selected.

**Proof** The \(\omega_{nom,i}\) constants of two parts, the first part is \(\omega_{nom,i}(z_i)\) which represents the time-invariant feedback. The second part is \(\omega_{nom,i}(t)\) which represents an input independent of the state of the system and lets the system converge to the origin in predefined convergence time \(t_F\), such as:

\[
\omega_{nom,i} = \omega_{nom,i}(z_i) + \omega_{nom,i}(t)
\]  
(28)

Let \(q(t)\) be a three-dimensional Lagrangian multiplier vector. Define a scalar function \(L_i\) by:

\[
L_i = q^T \dot{z}_i - \frac{1}{2} (z_i^T Q_i z_i + \omega_{nom,i}^2)
\]  
(29)

Substituting (23) into (29), one gets:

\[
L_i(z_i, q, \omega_{nom,i}) = q^T (A z_i + B \omega_{nom,i}) - \frac{1}{2} (z_i^T Q_i z_i + \omega_{nom,i}^2)
\]  
(30)

Differentiating (30) to \(\omega_{nom,i}\), one gets:

\[
\frac{\partial L_i}{\partial \omega_{nom,i}} = q^T B - \omega_{nom,i}
\]  
(31)

Setting the equation (31) to the zero yields the optimum control \(\omega_{nom,i}^*\):

\[
\omega_{nom,i}^* = q^T B
\]  
(32)
Define the Hamiltonian function as:
\[ H_i(z_i, q_i, \omega_{\text{nom}, i}) = q^T (A z_i + B \omega_{\text{nom}, i}) \]
\[ - \frac{1}{2} z_i^T Q_i z_i + (\omega_{\text{nom}, i})^2 \]
\[ = q^T A z_i + q^T B (q^T B) - \frac{1}{2} z_i^T Q_i z_i + \frac{1}{2} (q^T B)^2 \]
\[ = q^T A z_i + q^T B (q^T B) - \frac{1}{2} z_i^T Q_i z_i \]  
(33)

and the canonic equations for the optimum system are:
\[ \dot{z}_i = \frac{\partial H_i}{\partial q} = A z_i + B B^T q \]  
(34)
\[ \dot{q} = -\frac{\partial H_i}{\partial z_i} = Q z_i - A^T q \]  
(35)

It is assumed that:
\[ q = P_i z_i + \mu_i \]  
(36)

Differentiate Eq. (36) and substitute Eq. (34–35), hence it obtains:
\[ (A^T P_i + P_i A - P_i B B^T P_i + Q_i) z_i = \dot{\mu}_i \]
\[ + (A^T - P_i B B^T) \mu_i = 0 \]  
(37)

If (36) is a solution of the (32), then for arbitrary values of \( z_i(t) \) are in accordance with the Eq. (37), let \( z_i = 0 \) one gets:
\[ \dot{\mu}_i + (A^T - P_i B B^T) \mu_i = 0 \]  
(38)
\[ A^T P_i + P_i A - P_i B B^T P_i + Q_i = 0 \]  
(39)

The optimal cost function \( V_1(z_i, t) \) is defined as:
\[ V_1(z_i, t) = \frac{1}{2} z_i^T P_1(t) z_i \]  
(40)

which satisfies the Hamilton–Jacobi–Bellman (HJB) equation:
\[ 0 = \min \left( H_i(z_i, \omega_{\text{nom}, i}, V_1) \right) \]  
(41)

It is assumed that \( \min \left( H_i(z_i, \omega_{\text{nom}, i}, V_1) \right) \) exists and unique. Then solve the HJB differential equation to look for the optimal control law \( \omega_{\text{nom}, i} \).
\[ \frac{\partial V_1}{\partial t} - H_i(z_i, \omega_{\text{nom}, i}, V_1) = 0 \]  
(42)

This yields the Riccati differential equation in matrix form:
\[ P_i(t) + P_i(t) A + A^T P_i(t) - P_i(t) B B^T P_i(t) + Q_i = 0 \]  
(43)

Then, the optimal control \( \omega_{\text{nom}, i} \) is:
\[ \omega_{\text{nom}, i} = -B^T P_i z_i(t) + B^T \mu_i(t) \]  
(44)

where the initial conditions on \( \mu_i(t) \) are selected to satisfy the terminal condition \( z_i(t_F) = 0 \) on the optimum system. The design of the initial conditions on \( \mu_i(t) \) as follows:

First, solve the Riccati equation (27). The solution \( P_i(t) \) of the Riccati equation is a symmetric positive definite matrix.

Then, given only the initial bounded condition \( z_i(0) \), solve the forcing term \( \mu_i(0) \) such that \( z_i(t_F) = 0 \). From the Eq. (27):
\[ \mu_i(t) = e^{A_p t} \mu_i(0) \]  
(45)

with \( A_p = -[A^T - P_i B B^T] \). Then, supposed that \( t \leq t_F \) and from the Eqs. (23–27), one gets:
\[ z_i = [A - B B^T P_i] z_i + B B^T \mu_i(t) \]
\[ = [A - B B^T P_i] z_i + B B^T e^{A_p t} \mu_i(0) \]
\[ = -A_p t z_i + B B^T e^{A_p t} \mu_i(0) \]  
(46)

Multiply both sides of the equation by
\[ e^{A_p t} z_i = -e^{A_p t} A_p t z_i + e^{A_p t} B B^T e^{A_p t} \mu_i(0) \]
\[ \rightarrow [(e^{A_p t})^T z_i(t)] = e^{A_p t} B B^T e^{A_p t} \mu_i(0) \]  
(47)

By integrating the above equation from \( t = 0 \) to \( t = t_F \), with \( z_i(t_F) = 0 \), one can obtain:
\[ z_i(0) = -\int_0^{t_F} e^{A_p t} B B^T e^{A_p t} dt \mu_i(0) \]  
(48)

Then, the initial condition \( \mu_i(0) \) can ensure that \( z_i(t_F) = 0 \), and one can get:
\[ \mu_i(0) = -\int_0^{t_F} e^{A_p t} B B^T e^{A_p t} dt ]^{-1} z_i(0) \]  
(49)

The proof is completed. \( \square \)

**Remark 2** This control law \( \omega_{\text{nom}} = [\omega_{\text{nom}, 1}, \omega_{\text{nom}, 2}, \omega_{\text{nom}, 3}]^T \) mainly guarantees the state of the system (23) to converge near the origin at predefined convergence time \( t = t_F \). In order to retain the system status after \( t > t_F \), delete the forced term \( \mu_i(t) \) after \( t > t_F \), so the control law becomes:
\[ \omega_{\text{nom}, i} = \begin{cases} -B^T P_i z_i(t) + B^T \mu_i(t) & 0 \leq t \leq t_F \\ -B^T P_i z_i(t) & t > t_F \end{cases} \]  
(50)

### 3.2 ISMC With Positive Definite Barrier
**Function-Based Adaptive Method**

For treating the attitude tracking problem with unknown boundary and inertia uncertainty disturbance presented
in the spacecraft system, the adaptive approach is the best solution.

According to the (14), (21) and (22), the time derivative of $S$ can be expressed:

$$
\dot{S}(z) = [\dot{z}_{2,1}, \dot{z}_{2,2}, \dot{z}_{2,3}]^T + \tilde{z}
= \frac{1}{4} e_v \Omega_e^T \Omega_e - \frac{1}{2} (e_4 I_3 + e_3^\infty) J_0^{-1} \Omega^\times J_0
+ \frac{1}{2} (e_4 I_3 + e_3^\infty) (\Omega_e^\times L \dot{\Omega}_d - L \dot{\Omega}_d)
+ \frac{1}{2} (e_4 I_3 + e_3^\infty) J_0^{-1} T_u + R(t) - \omega_{nom}
$$

(51)

Combined with ISM(14), the robust control law $\omega_{disc}$ can be designed as:

$$
\omega_{disc} = -G_1(t, S(z)) \text{Sign}(S(z))
$$

(52)

where $\text{Sign}(S(z, t)) = [\text{sign}(s_1), \ldots, \text{sign}(s_n)]^T$ and the adaptive control gain $G_1(t, S(z))$ is defined as follows:

$$
G_1(t, S(z)) = G_{PBF}(S(z)) = \frac{\alpha \beta}{\alpha - \|S(z)\|}
$$

(53)

Then the control law is designed as:

$$
T_u = \left[ \frac{1}{4} e_v \Omega_e^T \Omega_e - \frac{1}{2} (e_4 I_3 + e_3^\infty) J_0^{-1} \Omega^\times J_0 \right]^{-1}
\left[ \frac{1}{4} e_v \Omega_e^T \Omega_e
+ \frac{1}{2} (e_4 I_3 + e_3^\infty) J_0^{-1} \Omega^\times J_0
- \frac{1}{2} (e_4 I_3 + e_3^\infty) (\Omega_e^\times L \dot{\Omega}_d - L \dot{\Omega}_d)
+ \omega_{nom} + \omega_{disc}(z, \tilde{z}) \right]
$$

(54)

**Theorem 2** Considering the rigid spacecraft system (10) and (11) with integral sliding mode surface (14), the attitude controller (54) with the adaptive update law (53) rejects the effect of the lumped uncertainties and guarantees the inequality $\| S(z) \| < \alpha$ hold for $\alpha > 0$ in finite time.

**Proof** Based on the property of the integral sliding mode, the initial condition $S(0) = 0$ which can make the sliding mode surface locate in the region $[-\frac{\pi}{2}, \frac{\pi}{2}]$ initially. If $\| S(z) \| > s_1$, where $s_1$ is a less than boundary auxiliary variable can be expressed as:

$$
s_1 = \begin{cases} 
\alpha (1 - \frac{\beta}{\| R(t) \|}), & \text{if } \beta < \| R(t) \| \\
0, & \text{if } \beta \geq \| R(t) \|
\end{cases}
$$

(55)

The PBF guarantees that $\| S(z) \| \leq s_1$ in a finite time period $\tau_1$. Moreover, the sliding mode surface is demonstrated to remain within $\| S(z) \| \leq s_1 < \alpha$ for arbitrary $t \geq t_1 + \tau_1$. It should be noted that $\tau_1 = 0$ if $\| S(z(t_1)) \| \leq s_1$.

Choose a Lyapunov function candidate, as:

$$
V_2 = \frac{1}{2} S^T S + \frac{1}{2} \left( \frac{\alpha \beta}{\alpha - \| S(z) \|} - \beta \right)^2
$$

(56)

The time derivative of $V_2$ in (56) is:

$$
\dot{V}_2 = S^T \dot{S} + \left( \frac{\alpha \beta}{\alpha - \| S(z) \|} - \beta \right) \cdot \left( - \frac{\alpha \beta}{(\alpha - \| S(z) \|)^2} \right)
$$

$$
= \langle R(t) \rangle \cdot \| S(z) \| - \frac{\alpha \beta}{(\alpha - \| S(z) \|)^2} \cdot \| S(z) \|
$$

(57)

Substituting (54) into (57), one gets:

$$
\dot{V}_2 = -\left( \frac{\alpha \beta}{(\alpha - \| S(z) \|)^2} \right) \cdot \| S(z) \|
$$

(58)

Simplifying formula (58), one gets:

$$
\dot{V}_2 = -\Gamma_s \| S \| - \xi \cdot \Gamma_s \left( \frac{\alpha \beta}{\alpha - \| S(z) \|} - \beta \right)
$$

(59)

where $\xi = \frac{\alpha \beta}{(\alpha - \| S(z) \|)^2} > 0$ and

$$
\Gamma_s = -\left( \frac{\alpha \beta}{\alpha - \| S(z) \|} \right) \cdot \| \dot{R}(t) \|
$$

(60)

**Case 1.** Suppose $\beta < \| R(t) \|$ and when $\| S(z) \| > s_1$, the $G(t, S(z))$ is an increasing function in $[0, \alpha]$, so $G_{PBF}(S(z)) > G_{PBF}(s_1) = \| R(t) \|$. Therefore, $\Gamma_s > 0$.

$$
\dot{V}_2 \leq -\Gamma_s \| S \| - \xi \cdot \Gamma_s \left( \frac{\alpha \beta}{\alpha - \| S(z) \|} - \beta \right)
$$

$$
= -\Gamma_s \cdot \sqrt{2} \left( \| S \| + \frac{\alpha \beta}{\beta - \| S(z) \|} \right)
$$

$$
\leq -\Gamma_s \cdot \sqrt{2} \cdot \min(1, \xi) \cdot \left( \frac{\| S \|}{\sqrt{2}} + \frac{\alpha \beta}{\beta - \| S(z) \|} \right)
$$

(61)

$$
\leq -\Gamma_1 \cdot V_s^{\frac{1}{2}}
$$
Case 2. Suppose $\beta \geq \| R(t) \|$ and $\| S(z) \| > s_1 = 0$. Thus $\Gamma_s > 0$. Therefore, $G_{\text{PBF}}(t, S(z)) > G_{\text{PBF}}(0) = \beta \geq \| R(t) \|$. It satisfies: $V_2 = \leq -\Gamma_1 \cdot V_2^2$ where $\Gamma_1 = \Gamma_s \cdot \sqrt{2} \cdot \min(1, \zeta)$.

It follows from inequality $\Gamma_s > 0$ that if $\beta < \| R(t) \|$ or $\beta \geq \| R(t) \|$, it yields $V_2 \leq -\Gamma_1 \cdot V_2^2$, where $\Gamma_1 = \Gamma_s \cdot \sqrt{2} \cdot \min(1, \zeta)$, therefore, the condition $\| S(z) \| \leq s_1$ will be satisfied in the finite time and the period time can be estimated as:

$$\tau_1 = \frac{V_2^{-1}(S(z(t_1))), G_{\text{PBF}}(S(z(t_1)))) - V_2^{-1}(s_1, G_{\text{PBF}}(s_1))}{0.5 \cdot \Gamma_1}$$

(62)

Now, when $t \geq t_1 + \tau_1$, and $\| S(z) \| < s_1 < \alpha$ is studied. It is noted that $\tau_1 = 0$ if $\| S(z(t_1)) \| \leq s_1$. According to the property of integral sliding mode, $\| S(z(t_1)) \| \leq \frac{\alpha}{2}$, it becomes evident that for all $t \geq t_1$ the inequality $\| S(z) \| \leq \alpha$ holds. The proof of Theorem 2 is completed.

Remark 3 In the case, from Definition 1 and the Eq. (55), it can be seen that the output of PBF (i.e., the adaptive gain) is not zero even when $\| S \| = 0$, which can lead to overestimation of the uncertainty and then the unessential control signal chattering. Meanwhile, when $\beta > \| R(t) \|$ the adaptive law based on PBF will provide an ideal sliding mode resulting in chattering whose amplitude is proportional to the value of $\beta$ chosen. In order to overcome this problem, in Sect. 3.3, we will propose an adaptive technique based on a positive semi-definite barrier function.

3.3 ISMC with positive semi-definite barrier function-based adaptive method

In this subsection, the positive semi-definite barrier functional adaptive approach will be considered. Combined with ISM (14), the robust control method $\omega_{\text{disc}}$ can be designed as:

$$\omega_{\text{disc}} = -G_2(t, S(z)) \text{Sign}(S(z))$$

(63)

where $\text{Sign}(S(z)) = \text{sign}(s_1), \ldots, \text{sign}(s_m)^T$ and the adaptive control gain $G_2(t, S(z))$ is defined as:

$$G_2(t, S(z)) = G_{\text{PSBF}}(S(z)) = \frac{\| S(z) \|}{\alpha - \| S(z) \|}$$

(64)

Then control law is designed as:

$$T_u = \left[ \frac{1}{2} (e_4 I_3 + e_v^\times)^{-1} \frac{1}{2} e_v \Omega_e^T \Omega e + \frac{1}{2} (e_4 I_3 + e_v^\times) J_0^{-1} \Omega^\times J_0 \Omega - \frac{1}{2} (e_4 I_3 + e_v^\times)(\Omega_e^T L \Omega_d - L \dot{\Omega}_d) + \omega_{\text{nom}} + \omega_{\text{disc}}(z, \dot{z}) \right]$$

$$+ \frac{1}{2} (e_4 I_3 + e_v^\times) J_0^{-1} \frac{1}{2} e_v \Omega_e^T \Omega e + \frac{1}{2} (e_4 I_3 + e_v^\times) J_0^{-1} \Omega^\times J_0 \Omega - \frac{1}{2} (e_4 I_3 + e_v^\times)(\Omega_e^T L \Omega_d - L \dot{\Omega}_d) + \omega_{\text{nom}} - G_2(t, S(z)) \text{Sign}(S(z))$$

(65)

Theorem 3 Considering the rigid spacecraft system (10) and (11) with integral sliding mode surface (14), the attitude controller (65) with the adaptive update law (66) rejects the effect of the lumped uncertainties and guarantees the inequality $\| S(z) \| < \alpha$ hold for arbitrary $\alpha > 0$ in finite time.

Proof Based on the property of the integral sliding mode, the initial condition $S(0) = 0$ which can make the sliding mode surface locate in the region $[\frac{-\alpha}{2}, \frac{\alpha}{2}]$ initially. If $\| S(z) \| > s_2$, where $s_2$ is an auxiliary variable less than the border can be represented as:

$$s_2 = \alpha \left( \frac{\| R(t) \|}{1 + \| R(t) \|} \right)$$

(66)

then the adaptive gain based on PSBF guarantees that $\| S(z) \| \leq s_2$ in a finite time period $t_2$. Furthermore, it is demonstrated that for arbitrary $t$ greater than $t_1 + \tau_2$, the sliding mode surface is demonstrated to remain within $\| S(z) \| \leq s_2 < \alpha$. It should be noted that $t_2 = 0$ if $\| S(z(t_1)) \| \leq s_2$.

Choose a Lyapunov function candidate, such as:

$$V_3 = \frac{1}{2} S^T S + \frac{1}{2} \left( \frac{\| S(z) \|}{\alpha - \| S(z) \|} \right)^2$$

(67)

The time derivative of $V_3$ in (67) is:

$$\dot{V}_3 = S^T \dot{S} + \left( \frac{\| S(z) \|}{\alpha - \| S(z) \|} \right) \cdot \left( \frac{\alpha}{(\alpha - \| S(z) \|)^2} \right)$$

$$= S^T \cdot \left( \frac{1}{2} e_v \Omega_e^T \Omega e - \frac{1}{2} (e_4 I_3 + e_v^\times) J_0^{-1} \Omega^\times J_0 \right)$$

$$+ \frac{1}{2} (e_4 I_3 + e_v^\times)(\Omega_e^T L \Omega_d - L \dot{\Omega}_d) + \frac{1}{2} (e_4 I_3 + e_v^\times) J_0^{-1} T_u + R(t) - \omega_{\text{nom}}$$

$$+ \left( \frac{\| S(z) \|}{\alpha - \| S(z) \|} \right) \cdot \left( \frac{\alpha}{(\alpha - \| S(z) \|)^2} \right)$$

(68)
where

\[ \zeta = \frac{\alpha}{(\alpha - \|S(z)\|^2)^2} > 0 \quad \text{and} \quad \Gamma_s = -\|R(t)\| + \frac{\|S(z)\|}{(\alpha - \|S(z)\|^2)^2}. \]

Considering when the \( \|S(z)\| > s_2 \), the \( G(t, S(z)) \) is an increasing function in \([0, \alpha]\), so \( G_{\text{PSB}}(t, S(z)) > G_{\text{PSB}}(s_2) = \|R(t)\| \). Therefore, \( \Gamma_s > 0 \).

\[
\begin{align*}
V_3 & \leq -\Gamma_s \|S\| - \zeta \cdot \Gamma_s \left( \frac{\|S(z)\|}{(\alpha - \|S(z)\|^2)^2} \right) \\
& = -\Gamma_s \sqrt{2} \left( \frac{\|S\|}{\sqrt{2}} + \zeta \cdot \frac{\|S(z)\|}{(\alpha - \|S(z)\|^2)^2} \right) \\
& \leq -\Gamma_s \sqrt{2} \cdot \min(1, \zeta) \left( \sqrt{2} \frac{\|S\|}{\sqrt{2}} + \sqrt{2} \frac{\|S(z)\|}{(\alpha - \|S(z)\|^2)^2} \right) \\
& \leq -\Gamma_s \sqrt{2} \cdot \min(1, \zeta).
\end{align*}
\]

where \( \Gamma_2 = \sqrt{2} \cdot \min(1, \zeta) \). Therefore, the condition \( \|S(z)\| \leq s_2 \) will be satisfied in finite time and the period time can be estimated as:

\[
\tau_2 \leq \frac{V_3^\frac{1}{2}(S(t_2), G_{\text{PSB}}(S(t_2))) - V_3^\frac{1}{2}(s_2, G_{\text{PSB}}(s_2))}{0.5 \cdot \Gamma_2}
\]

The proof is completed.

Remark 4 Theorem 3 encourages the use of PSBF rather than PBF to alter the system’s control gain, which decreases chattering and allows the adaptive gain to be reduced below \( \beta \).

3.4 ISMC with modified positive semi-definite function-based adaptive method

In addition, considering the problem of the rigid spacecraft actuator saturation, while the barrier function-based adaptive method leads to unbounded value when \( \|S\| \) approaches \( \alpha \). Inspired by [37], the adaptive method is improved by limiting the control gain which avoids generating the actuator saturation problem.

The following assumptions are necessary before giving the main results.

Assumption 5 The maximum permissible control gain \( G(t, S(z)) \) is a constant \( \tilde{G} \), i.e., \( G(t, S(z)) < \tilde{G} \). Furthermore, \( \tilde{G} \) is sufficiently small such that \( \|o\| \leq \|o_{\text{hom}}\| + \|o_{\text{disc}}\| \leq \|o_{\text{hom}}\| + \tilde{G} \leq \omega_{\text{max}} \) holds for arbitrary \( t \geq 0 \).

Assumption 6 The amplitude of the system lumped uncertainties is bounded by \( \|R(t)\| \leq \tilde{G} \).

Lemma 1 [37] Considering the rigid spacecraft system (10) and (11) with integral sliding mode surface (14), and suppose Assumptions 5 and 6 fulfilled, the attitude controller (65) with the adaptive update law:

\[
G_3(t, S(z)) = \min(\tilde{G}, G_{\text{PSB}}(S(z)))
\]

rejects the effect of the lumped uncertainties and guarantees the inequality \( \|S(z)\| < \alpha \) hold for arbitrary \( \alpha > 0 \) in finite time.

Proof According to Definition 1, it is noted that when \( S(z) \) increases such that \( G_3(t, S(z)) \) reaches \( \tilde{G} \), the value of \( \|S(z)\| \) as denoted by \( \xi \) equals to:

\[
\xi = \frac{\tilde{G}}{\alpha} < \alpha
\]

Case 1: \( \|S(z)\| \leq \xi \).

In this case, when \( \|S(z)\| \leq \xi \), \( G_3(t, S(z)) = G_{\text{PSB}}(S(z)) \) guarantees the inequality \( \|S(z)\| < \alpha \) hold for arbitrary \( \alpha > 0 \) in finite time. The proof can be seen in Theorem 3.

Case 2: \( \|S(z)\| > \xi \).

In this case, when \( \|S(z)\| > \xi \). It is noted that \( G_3(t, S(z)) = \tilde{G} \). Choose the Lyapunov function \( V_4 = \frac{1}{2} S^T S \). By using (45), one gets:

\[
\begin{align*}
\dot{V}_4 = S^T \left( \frac{1}{4} e_1 \Omega_e^T \Omega_e - \frac{1}{2} (e_4 I_3 + e_5 \dot{x}) J^{-1} \Omega^x J \Omega \\
+ \frac{1}{2} (e_4 I_3 + e_5 \dot{x}) (\Omega_e L \dot{\Omega}_d - L \dot{\Omega}_d) \\
+ \frac{1}{2} (e_4 I_3 + e_5 \dot{x}) J^{-1} T_u + R(t) - o_{\text{hom}} \right) \\
\leq S^T (R(t) - \tilde{G} \text{Sign}(S)) \\
\leq -\Gamma_3 V_4^\frac{1}{2}
\end{align*}
\]

where \( \Gamma_3 = \tilde{G} - \|R(t)\| > 0 \). Hence, it will converge from the initial condition to \([-\xi, \xi]\) in a finite time.
Remark 5 Based on the above analysis, an improved Positive Semi-Definite Barrier Function-Based Adaptive controller is proposed to guarantee that the saturation problem not occurs. At the same time, satisfying the inequality \( ||S(z)|| < \alpha \) hold for arbitrary \( \alpha > 0 \) in finite time and the robust controller compensates the effect of the lumped uncertainties.

Under auxiliary control law (22) with the previous robust adaptive laws fulfilled, the rigid spacecraft tracking errors system evolves on the sliding manifold. The attitude tracking errors converge to zero within a predefined time and are maintained the state. Meanwhile, the lumped uncertainties are estimated in real time and compensated by the barrier function-based integral sliding mode control.

Theorem 4 Considering the rigid spacecraft system (10) and (11) with the auxiliary control law \( \omega \) which can be described as:

\[
\omega = \omega_{\text{nom}} + \omega_{\text{disc}}(z, \tilde{z})
\]

where \( \omega_{\text{nom}} \) defined by (44), and \( \omega_{\text{disc}}(z, \tilde{z}) \) defined as:

\[
\omega_{\text{disc}} = -G(t, S(z))\text{Sign}(S(z))
\]

where \( G(t, S(z)) \) is given by (53),(64) or (72). The auxiliary control law \( \omega \) allows the attitude errors converge to zero in predefined convergence time with \( t = t_F < \infty \).

Remark 6 With respect to the choice of \( t_F \), there is theoretically no restriction on the choice of \( t_F \), but in the actual operating environment, the output of the rigid spacecraft actuator is finite. \( t_F \) is chosen small, and the actuator needs a larger torque in order to satisfy its convergence to zero in \( t_F \). As a result, the choice of \( t_F \) cannot be small.

4 Simulation results

In this section, we present the simulation examples to verify the theoretical results in the previous. Considering the rigid spacecraft attitude model (1) with the inertia matrix [38]:

\[
J = J_0 + \Delta J = \begin{pmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{pmatrix} \text{kg} \cdot \text{m}^2
\]

where the nominal inertia matrix and the uncertainties of inertia matrix are:

\[
J_0 = \begin{pmatrix} 20 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 15 \end{pmatrix} \text{kg} \cdot \text{m}^2
\]

\[
\Delta J = \begin{pmatrix} 0 & 1.2 & 0.9 \\ 1.2 & 0 & 1.4 \\ 0.9 & 1.4 & 0 \end{pmatrix} \text{kg} \cdot \text{m}^2
\]

The disturbances torque can be described as:

\[
T_d = \begin{pmatrix} 0.1 \sin(0.1t) + 0.1 \\ 0.2 \sin(0.2t) + 0.2 \\ 0.3 \sin(0.2t) + 0.3 \end{pmatrix}
\]

The intended angular velocity of the rigid spacecraft is provided by:

\[
\Omega_d(t) = 0.05 \begin{pmatrix} \sin \left( \frac{\pi t}{100} \right) \\ \sin \left( \frac{3\pi t}{100} \right) \\ \sin \left( \frac{15\pi t}{100} \right) \end{pmatrix} \text{ rad}
\]

The initial attitude is \( \Psi(0) = [0.3, -0.2, -0.3, 0.8832]^T \), and the initial target unit quaternion is \( \Psi_d = [0, 0, 0, 1]^T \). The angular velocity initial value is \( \Omega(0) = [0, 0, 0]^T \). The value of the parameter \( \alpha \) for all algorithms is selected as \( \alpha = 0.01 \). The PBF parameter \( \beta \) is set as \( \beta_1 = 0.1, \beta_2 = \beta_3 = 0.2 \). The initial attitude is \( \Psi_e(0) = [0.3, -0.2, -0.3, 0.8832]^T \), which implies that \( z_1(0) = [0.3, 0]^T, z_2(0) = [-0.2, 0]^T \) and \( z_3(0) = [-0.3, 0]^T \). As state in [39], the initial conditions of \( \mu_1 \) are computed such that the attitude tracking errors \( e_i (i = 1, 2, 3) \) equal 0 at exactly \( t_F = 5 \text{ s} \). For arbitrary \( i = 1, 2, 3 \), the matrix \( Q_i \) can be expressed as: \( Q_i = [1, 0; 0, 1], P_i = [1.7321, 1.0000; 1.0000, 1.7321] \), then the forcing term can be given by (27), it yields:

\[
\mu_1(0) = [0.0002683, 0.0004475 \text{ kg m}^2 T] \\
\mu_2(0) = [0.0001789, 0.0002983 \text{ kg m}^2 T] \\
\mu_3(0) = [0.0002683, 0.0004475 \text{ kg m}^2 T]
\]

4.1 PBF

In this simulation, the predefined-time controller of the rigid spacecraft is given by (22) with the nominal control is given by (50) and the adaptive method is given by (52) and (53).
Figure 1 shows the tracking errors of the attitude quaternion and angular velocity. The tracking responses of the PBF-based adaptive controllers in the presence of the external disturbances are shown in Fig. 2. Meanwhile, Figs. 1 and 2 can show that the control objection is achieved, which is stable at $t_F = 5s$. This is because the convergence time $t_F = 5s$ of the optimal control part is set in the nominal control law, and the robust control law based on the PBF adaptive method estimates and compensates the upper bound of the lumped uncertainties.

It is noted that due to the limited output of the actuator, the converge time of the tracking error cannot be chosen too small. In this paper, the maximum actuator output torque is $T_{u,\text{max}} = 10N \cdot m$. The external disturbance torque and the input control of the rigid spacecraft for the PBF are depicted in Fig. 3. For $32s < t < 50s$, the amplitude of the disturbance becomes less than $\beta_1 = 0.1$ in the controller $T_{u1}$; then, for $16s < t < 32s$, the amplitude of the disturbance becomes less than $\beta_2 = 0.2$ in the controller $T_{u2}$. For $18s < t < 30s$, the amplitude of the disturbance becomes less than $\beta_3 = 0.2$ in the controller $T_{u3}$. That is why starting for this moment, the chattering will appear with the amplitude of the disturbance equal to the $\beta$. Furthermore, as seen in Fig. 4, the adaptive gain can be increased and decreased, which estimates the upper bound of the lumped uncertainties. Nevertheless, the input control of the rigid spacecraft for the PSBF is continuous and will go to zero without chattering. The sliding mode surface $S(z)$ is maintained close
to the stipulated curve of $\alpha = 0.01$ from the original time moment, as illustrated in Fig. 5.

4.2 Comparison simulation

Considering the actuator saturation of the rigid spacecraft, to verify the integral sliding mode with modified positive Semi-Definite Barrier function-based controller, setting $T_{u, \text{max}} = 10N$. The simulation results of the rigid spacecraft attitude system (1) under the sliding mode control protocol (63) with the adaptive method (PSBF) (64) and the adaptive method (MPSBF) are shown in Figs. 6, 7, 8, 9, 10 and 11. The auxiliary control inputs $\omega_{\text{nom},i} (i = 1, 2, 3)$ are given by (44). $\bar{G} = [0.5; 0.9; 1.0]^T$ is satisfied the condition as stated in Assumption 6.

As shown in Fig. 6, because of the role of the attitude predefined convergence time controller with adaptive method of MPSBF, the attitude positions of the rigid spacecraft are tracking at $t_F = 5s$, and Fig. 7 shows that the attitude positions of the rigid spacecraft are tracking at $t_F = 5s$. Due to the action of the barrier function-based adaptive law, the sliding mode surface $S(z)$ belongs to the stipulated vicinity of zero $\alpha = 0.01$ from the original time moment can be shown in Fig. 8, which indicates that the sliding surface is bounded. The behavior of adaptive gains is seen in Fig. 9. It can be shown in Fig. 9 that the maximum adaptive control gain $\bar{G}$ under MPSBF is indeed constrained by $\bar{G} = [0.5; 0.9; 1.0]^T$ as desired. Figure 10 presents the control input amplitude as compared to PSBF while the control objective is guaranteed.

At the same time, compared with the finite-time controllers in the existing literature with:

$$\omega_i = -k_1 \cdot \text{sign}(S_i)|S_i|^\nu_1 - k_2 \cdot \text{sign}({\dot{S}_i})|{\dot{S}_i}|^\nu_2 - G_{\text{MPSBF}} \cdot \text{sign}(S_i)$$  \hspace{1cm} (82)

where the design parameters $k_1 = 10, k_2 = 20, \nu_1 = 3/5, \nu_2 = 3/4$, the adaptive parameters are not re-tuned. It can be seen from Figs. 11 and 12 that unlike the finite-time controller, the convergence time of the rigid spacecraft systems under the proposed controller is chosen in advance; meanwhile, the tracking performance is guaranteed.

According to theoretical analysis and numerical simulations, it is known that the convergence time is proposed in advance, and the smaller adaptive control gain can be guaranteed by applying the method of MPSBF. Meanwhile, the upper bound of lumped uncertainties which can be estimated in real time and prevented the actuator saturation by the modified positive semi-definite barrier function-based adaptive method.
Fig. 7 Tracking errors

(a) Attitude quaternion tracking errors with MPSBF and PSBF

(b) Angular velocity tracking errors with MPSBF and PSBF

Fig. 8 Sliding mode surface with MPSBF

Fig. 9 The gain parameter $G$ with MPSBF and PSBF

Fig. 10 Control input with MPSBF and PSBF

Fig. 11 Tracking errors under finite-time controllers
Compared to adaptive method (PBF), the chattering is reduced and the tracking performance is ensured. However, it is noted that the shorter convergence time requires the large actuator torque; therefore, the adjustment of the tracking performance should depend on the actual actuator.

5 Conclusion

In this paper, an adaptive integral sliding mode control schemes based on barrier function with predefined convergence time is proposed to solve the problems of disturbance and inertia parameters of rigid spacecraft. The convergence time of the attitude tracking is specified in order to achieve proper tracking accuracy and flexibility. Upon the integral sliding mode theory and barrier function, the upper bound of the lumped uncertainties including disturbances and uncertain inertia are estimated in real time to ensure that the sliding mode around the range of desired variables envisaged starting from the initial moment. At the same time, the actuator saturation is avoided by the modified adaptive method. Finally, numerical examples verify the effectiveness of the proposed methodologies.

Further research work includes two aspects. Firstly, only the external disturbance and inertia uncertainties are investigated in this paper; actuator faults such as the loss-of-effectiveness fault and actuator faults with unknown structure will be considered. Furthermore, the convergence time of the system should be considered, and a predefined-time scheme [40,41] will be investigated in our future study.

Acknowledgements The authors would like to express their gratitude to the Editor and all of the reviewers for their professional and constructive comments and suggestions, which have greatly improved the quality of this paper.

Funding This work is supposed in part by the Foundation under Grant 2019-JCJQ-ZD-049. The National Natural Science Foundation of China under Grants 61703134, 62022060, 62073234, and 61773278. The China Postdoctoral Science Foundation under Grant 2019M650874. The Science and Technology Research Project of Colleges and Universities in Hebei Province under Grant B12020017. The Undergraduate Education and Teaching Reform Research and Practice Project 202004023. The Key R&D Program of Hebei Province 20310802D, Natural Science Foundation of Hebei Province under Grants F2019202369, F2018202279 and F2019 202363.

Data availability Data sharing not applicable to this article as no data sets were generated or analyzed during the current study.

Declarations

Conflicts of interest The authors declare that they have no conflict of interest.

References

1. Li, Z., Chen, X.: Adaptive actuator fault compensation and disturbance rejection scheme for spacecraft. Int. J. Control Autom. Syst. 19(2), 900–909 (2021)
2. Gui, H., Vukovich, G.: Adaptive fault-tolerant spacecraft attitude control using a novel integral terminal sliding mode. Int. J. Robust Nonlinear Control 27(16), 3174–3196 (2017)
3. Alatorre, A., Espinoza, E. S., Sánchez, B., Ordaz, P., Muñoz, F., García Carrillo, L. R.: Parameter estimation and control of an unmanned aircraft-based transportation system for variable-mass payloads. Asian. J. Control. (in press).https://doi.org/10.1002/ASJC.2565
4. Ordaz, P., Ordaz, M., Cuvas, C., Santos, O.: Reduction of matched and unmatched uncertainties for a class of nonlinear perturbed systems via robust control. Int. J. Robust Nonlinear Control 29(8), 2510–2524 (2019)
5. Wei, Y., Sheng, D., Chen, Z., Wang, Y.: Fractional order chattering-free robust adaptive backstepping control technique. Nonlinear Dyn. 95(3), 2383–2394 (2019)
6. Jiang, T., Zhang, F., Lin, D.: Finite-time backstepping for attitude tracking with disturbances and input constraints. Int. J. Control Autom. Syst. 18(6), 1487–1497 (2020)
7. Ji, W., Qiu, J., Wu, L., Lam, H.-K.: Fuzzy-Affine-Model-Based output feedback dynamic sliding mode controller design of nonlinear systems. IEEE Trans. Syst. 51(3), 1652–1661 (2021)
8. Ji, W., Qiu, J., Hamid, R.K., Fu, Y.: New results on fuzzy integral sliding mode control of nonlinear singularly perturbed systems. IEEE Trans. Fuzzy Syst. 27(9), 2062–2067 (2021)
9. Vadali, S.R.: Variable-structure control of spacecraft large-angle maneuvers. J. Guid. Control. Dyn. 9(2), 235–239 (1986)
10. Wu, B., Wang, D., Poh, E.K.: Decentralized sliding-mode control for attitude synchronization in spacecraft formation. Int. J. Robust Nonlinear Control 23(11), 1183–1197 (2013)
11. Zhu, Z., Xia, Y., Fu, M.: Adaptive sliding mode control for attitude stabilization with actuator saturation. IEEE Trans. Ind. Electron. 58(10), 4898–4907 (2011)
12. Tabatabaei-Asab, F.S., Naserifar, N.: Nanopositioning of an electrostatic MEMS actuator: adaptive terminal sliding mode control approach. Nonlinear Dyn. 105(1), 213–225 (2021)
13. Zong, Q., Shao, S.: Decentralized finite-time attitude synchronization for multiple rigid spacecraft via a novel disturbance observer. ISA Trans. 65, 150–163 (2016)
14. Hou, H., Yu, X., Xu, L., Rsetam, K.A., Cao, Z.: Finite-Time continuous terminal sliding mode control of servo motor systems. IEEE Trans. Ind. Electron. 67(7), 5647–5656 (2020)
15. Gan, C., Lu, P., Liu, X., Yang, D.: Distributed cooperative attitude tracking control for multiple rigid spacecraft using fast terminal sliding mode. In: Proceeding of the 11th world congress on intelligent control and automation, pp. 2687–2692 (2014)
16. Shao, S., Zong, Q., Tian, B., Wang, F.: Finite-time sliding mode attitude control for rigid spacecraft without angular velocity measurement. J. Frankl. Inst. 354(12), 4656–4674 (2017)
17. Wang, Z., Li, Q., Li, S.: Adaptive integral-type terminal sliding mode fault tolerant control for spacecraft attitude tracking. IEEE Access. 7, 35195–35207 (2019)
18. Shao, K.: Nested adaptive integral terminal sliding mode control for high-order uncertain nonlinear systems. Int. J. Robust Nonlinear Control 31(14), 6668–6680 (2021)
19. Sui, W., Duan, G., Hou, M., Zhang, M.: Distributed fixed-time coordinated tracking for multiple rigid spacecraft via a novel integral sliding mode approach. J. Frankl. Inst. 357(14), 9399–9422 (2020)
20. Zhang, J., Zhao, W., Shen, G., Xia, Y.: Disturbance observer-based adaptive finite-time attitude tracking control for rigid spacecraft. IEEE Trans. Syst. Man Cybern. Syst. 51(11), 6606–6613 (2021)
21. Pukdeboon, C., Zinober, A.S.I., Thein, M.-W.L.: Quasi-continuous higher order sliding-mode controllers for spacecraft-attitude-tracking maneuvers. IEEE Trans. Ind. Electron. 57(4), 1436–1444 (2010)
22. Tiwari, P.M., Janardhanan, S., Nabi, M.: (2016) Attitude control using higher order sliding mode. Aerosp. Sci. Technol. 54, 108–113 (2016)
23. Tian, B., Zong, Q., Wang, J., Wang, F.: Quasi-continuous high-order sliding mode controller design for reusable launch vehicles in reentry phase. Aerosp. Sci. Technol. 28(1), 198–207 (2013)
24. Lu, K., Xia, Y.: Finite-time fault-tolerant control for rigid spacecraft with actuator saturations. IET Control Theory Appl. 7(11), 1529–1539 (2013)
25. Chen, Q., Xie, S., He, X.: Neural-network-Based adaptive singularity-free fixed-time attitude tracking control for spacecrafts. IEEE Trans. Cybern. pp. 1–14 (2020)
26. Shi, X., Zhou, Z., Zhou, D.: Finite-time attitude trajectory tracking control of rigid spacecraft. IEEE Trans. Aerosp. Electron. Syst. 53(6), 2913–2923 (2017)
27. Chen, Q., Xia, Y., Zhang, J., Cui, B.: Adaptive fixed-time trajectory tracking control for Mars entry vehicle. Nonlinear Dyn. 102(4), 2687–2698 (2020)
28. Gao, J., Fu, Z., Zhang, S.: Adaptive fixed-time attitude tracking control using higher order sliding mode. Aerosp. Sci. Technol. 66, 2689–2700 (2021)
29. Zong, Q., Zhang, X., Shao, S., Tian, B., Liu, W.: Disturbance observer-based fault-tolerant attitude tracking control for rigid spacecraft with finite-time convergence. Proc. Inst. Mech. Eng. G. J. Aerosp Eng. 233(2), 616–628 (2017)
30. Muñoz-Vázquez, A.J., Fernández-Anaya, G., Sánchez-Torres, J.D., Meléndez-Vázquez, F.: Predefined-time control of distributed-order systems. Nonlinear Dyn. 103(3), 2689–2700 (2021)
31. Laghrouche, S., Plestan, F., Glumineau, A.: Higher order sliding mode control based on integral sliding mode. Automatica (Oxf). 43(3), 531–537 (2007)
32. Obeid, H., Fridman, L.M., Laghrouche, S., Harmouche, M.: Barrier function-based adaptive sliding mode control. Automatica (Oxf). 93, 540–544 (2018)
33. Gao, J., Fu, Z., Zhang, S.: Adaptive fixed-time attitude tracking control for rigid spacecraft with actuator faults. IEEE Trans. Ind. Electron. 66(9), 7141–7149 (2019)
34. Shao, K., Zheng, J., Wang, H., Wang, X., Lu, R., Man, Z.: Tracking control of a linear motor positioner based on barrier function adaptive sliding mode. IEEE Trans. Ind. Electron. 58(2), 647–659 (2011)
35. Rekasius, Z.: An alternate approach to the fixed terminal point regulator problem. IEEE Trans. Automat. Control 9(3), 290–292 (1964)
36. Sánchez-Torres, J.D., Muñoz-Vázquez, A.D., Defoort, M., Aldana-López, R., Gómez-Gutiérrez, D.: Predefined-time integral sliding mode control of second-order systems. Int. J. Syst. Sci. 51(16), 3425–3435 (2020)
41. Dong, Y., Zou, A.-M., Sun, Z.: Predefined-Time predefined-bounded attitude tracking control for rigid spacecraft. IEEE Trans. Aerosp. Electron. Syst. 58(1), 464–472 (2021)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.