Provably Fair Federated Learning via Bounded Group Loss

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Abstract

In federated learning, fair prediction across various protected groups (e.g., gender, race) is an important constraint for many applications. Unfortunately, prior work studying group fair federated learning lacks formal convergence or fairness guarantees. Our work provides a new definition for group fairness in federated learning based on the notion of Bounded Group Loss (BGL), which can be easily applied to common federated learning objectives. Based on our definition, we propose a scalable algorithm that optimizes the empirical risk and global fairness constraints, which we evaluate across common fairness and federated learning benchmarks. Our resulting method and analysis are the first we are aware of to provide formal theoretical guarantees for training a fair federated learning model.

1 Introduction

Federated learning (FL) is a training paradigm that aims to fit a model to data generated by, and residing in, a set of disparate data silos, such as a network of remote devices or collection of organizations [16]. Mirroring concerns around fairness in non-federated settings, many FL applications similarly require performing fair prediction across protected groups. However, naively estimating algorithmic fairness locally for each silo in a federated network may be inaccurate due to heterogeneity across silos—failing to produce a fair model over the entire dataset [25]. To address this, several recent works have aimed to implement limited notions of group fairness in federated networks [5, 19, 25]. Unfortunately, despite their promising empirical performance, these prior works are heuristic in nature in that they lack guarantees surrounding the resulting fairness of the solutions.

In this work we provide the first method we are aware for group fairness in FL that comes with formal convergence and fairness guarantees. In developing and analyzing our approach, we also take care to ensure that the proposed method addresses practical constraints of realistic federated networks—building off common communication-efficient federated optimization methods which can scale to networks of millions of devices. We demonstrate the effectiveness of our theoretically-grounded approach on common benchmarks from fair machine learning and federated learning.

The remainder of the paper is organized as follows. We present a detailed discussion of related work to Section 2 In Section 3, we formalize the fairness definition via Bounded Group Loss and the fair federated learning objective. In Section 4, we present a scalable algorithm to solve the proposed objective, and provide formal convergence and fairness guarantees for our objective and algorithm. In Section 5, we evaluate our algorithm on common fairness benchmark and show our method is able to achieve both better utility and fairness performance compared to vanilla FedAvg.

2 Background and Related Work

Algorithmic Fairness in Machine Learning. Algorithmic fairness in the machine learning literature often refers to protection of a protected attribute during the process of learning a model. Common approaches
for obtaining fairness include pre-processing methods that modify the input data \[1\] \[9\] \[24\]; post-processing methods that revise the prediction score \[8\] \[11\] \[17\]; or methods that optimize an objective subject to some fairness constraints \[1\] \[2\] \[21\] \[22\] \[23\]. All of these methods are based on using a centralized dataset to train and evaluate a model. In our setting where data is privately distributed across different data silos, it is not possible to directly apply these methods in order to achieve global fairness across all silos.

**Fair Federated Learning.** In federated learning, definitions of fairness may take many forms. One commonly used notion of fairness is representation parity \[12\], whose application in FL requires the model’s performance across all devices to have small variance \[7\] \[13\] \[18\]. In this work we instead focus on notions of group fairness, in which every data point in the federated network belongs to some (possibly) protected group, and the purpose of learning is to find a model that doesn’t introduce bias towards any group. Recent works have proposed various objectives for group fairness in federated learning. Zeng et al. \[25\] proposed a bi-level optimization objective that minimizes the difference between each group’s loss while finding an optimal global model. Chu et al. \[5\] proposed a similar constrained optimization problem by finding the best model subject to an upper bound on the group loss difference. Different from either approach, our method focuses on a fairness constraint based on upperbounding the loss of each group rather than the loss difference between any two groups. More closely related to our work, Papadaki et al. \[19\] weighs the empirical loss for obtaining fairness include pre-processing methods that modify the input data \[1\] \[9\] \[24\]; post-processing methods that revise the prediction score \[8\] \[11\] \[17\]; or methods that optimize an objective subject to some fairness constraints \[1\] \[2\] \[21\] \[22\] \[23\]. All of these methods are based on using a centralized dataset to train and evaluate a model. In our setting where data is privately distributed across different data silos, it is not possible to directly apply these methods in order to achieve global fairness across all silos.

In practice, we could define empirical bounded group loss constraint at level \(\zeta\) for any model \(h\) as follows:

\[
\min_{h \in \mathcal{H}} F(h) = \min_{h \in \mathcal{H}} \frac{1}{K} \sum_{k=1}^{K} \frac{1}{m_k} \sum_{i=1}^{m_k} l(h(x_{k,i}), y_{k,i}).
\]

For simplicity, we define \(f_k(h) = \frac{1}{m_k} \sum_{i=1}^{m_k} l(h(x_{k,i}), y_{k,i})\) as the local objective for client \(k\). Further, we assume \(h\) is parameterized by a vector \(w \in \mathbb{R}^p\) where \(p\) is the number of parameters. We will use \(F(w)\) and \(f_k(w)\) to represent \(F(h)\) and \(f_k(h)\) intermittently in the remainder of the paper.

To learn a model that satisfies any fairness constraint, a standard approach would be to solve:

\[
\min_{h \in \mathcal{H}} F(h) \quad \text{subject to } R(h),
\]

where \(R(h)\) represents a constraint set on \(h\). Prior works \[5\] \[25\] studying group fairness in federated learning proposed choosing a bounded group-specific parity difference for \(R(h)\). In this work, we focus on a different fairness definition known as Bounded Group Loss (BGL) \[2\] (defined below). We discuss the motivation of using BGL in Section 3.1.

**Definition 1.** A classifier \(h\) satisfies Bounded Group Loss (BGL) at level \(\zeta\) under distribution \(\mathcal{D}\) if for all \(a \in \mathcal{A}\), we have

\[
\mathbb{E}_{(x,y,a) \sim \mathcal{D}} [l(h(x), y) | A = a] \leq \zeta.
\]

In practice, we could define empirical bounded group loss constraint at level \(\zeta\) under the empirical distribution \(\hat{\mathcal{D}} = \frac{1}{K} \sum_{k=1}^{K} \mathcal{D}_k\) to be
\[
\frac{1}{m_a} \sum_{k=1}^{K} \sum_{a_{k,i}=a} l(h(x_{k,i}), y_{k,i}) \leq \zeta.
\]  \hspace{1cm} (4)

In the rest of the paper, we will refer problem 2 to be the constrained optimization problem with \( R(h) \) replaced by Equation 4, which is the main problem we propose to solve.

### 3.1 Fairness-aware objective

A common method to solve the constrained optimization problem (2) is to use Lagrangian multipliers. This converts the objective into the following saddle point optimization problem:

\[
\max_{\lambda \in \mathbb{R}^{|A|}, \|\lambda\|_1 \leq B} \min_w G(w; \lambda) = F(w) + \lambda^T r(w),
\]  \hspace{1cm} (5)

where the \( a \)-th index of \( r \) is \( \frac{1}{m_a} \sum_{k=1}^{K} \sum_{a_{k,i}=a} l(h(x_{k,i}), y_{k,i}) - \zeta_a \).

**Why use BGL rather than another fairness constraint?** Many prior works choose the gap between every two group’s loss as the fairness constraint and optimize the Lagrangian. Under such settings, the objective becomes non-convex in terms of the model weight, making it likely that a solver will find a local minima that either does not satisfy the fairness constraint or achieves poor utility. Unlike these approaches, BGL requires that for each group \( a \in A \), the classifier \( h \)'s loss evaluated on all data with protected attribute \( a \) is below a certain threshold. Therefore, given that the empirical risk is convex, adding the BGL constraint preserves convexity. As we will see, a major benefit of using BGL relative to other alternatives is that it can satisfy meaningful fairness constraints while preserving convexity, enabling both strong empirical performance and formal theoretical guarantees.

### 4 Provably Fair Federated Learning via Bounded Group Loss

In this section, we first provide a scalable solver for Equation 5 in Algorithm 1. We provide both a formal convergence and a formal fairness guarantee for our approach.

#### 4.1 Algorithm

To find a saddle point for Objective 5, we follow the scheme from Freund and Schapire [10] and summarize our solver for the fair FL with bounded group loss in Algorithm 1. Our algorithm is based off of FedAvg [16], a common scalable approach in federated learning. Our method alternates between two steps: (1) given a fixed \( \lambda \), find \( w \) that minimizes \( F(w) + \lambda^T r(w) \); (2) given a fixed \( w \), find \( \lambda \) that maximizes \( \lambda^T r(w) \). In Algorithm 1, we provide an example in which the first step is achieved by using FedAvg to solve \( \min_w F(w) + \lambda^T r \) (line 4-10). Note that solving this objective does not require the FedAvg solver; any algorithm that learns a global model in federated learning could be used to find a certain \( w \) given \( \lambda \). Following Algorithm 2 in [2], we use exponentiated gradient descent to update \( \lambda \) after training a federated model for each round.

Note that the ultimate goal to solve for Objective 5 is to find a \( w \) such that it minimizes the empirical risk subject to \( r(w) \leq 0 \). Therefore, at the end of training, our algorithm checks whether the resulting model \( \bar{w} \) violates the fairness guarantee by at most some constant error \( \frac{M+2\nu}{B} \) where \( M \) is the upper bound for the empirical risk and \( \nu \) is the upper bound provided in Equation 7 (line 16-20). We will show in the Lemma 6 that this is always true when there exists a solution \( w^* \) for Problem 2. However, it is also worth noting that the Problem 2 does not always have a solution \( w^* \). For example when we set \( \zeta = 0 \), requiring \( r(w) \leq 0 \) is equivalent to requiring the empirical risk given any group \( a \in A \) is non positive, which is only feasible when the loss is 0 for every data in the dataset. In this case, our algorithm will simply output null if the fairness guarantee is violated by an error larger than \( \frac{M+2\nu}{B} \).
Algorithm 1 FedAvg with BGL

1: **Input:** \( T, \theta^0 = 0, \eta_w, \eta_\theta, w_0, \bar{w} = 0, M, \nu, B \)
2: **for** \( i = 1, \ldots, E \) **do**
3: \[ \lambda_a = B \frac{\exp(\theta_a^i)}{1 + \sum_{a'} \exp(\theta_{a'}^i)} \]
4: **for** \( t = 0, \ldots, T - 1 \) **do**
5: Server broadcasts \( w^t \) to all the clients.
6: **for** all \( k \) in parallel **do**
7: Each task updates its weight \( w^t_k \) for some \( J \) iterations
8: \[ w^{t+1}_k = w^t_k - \eta_w \left( \nabla w^t \left( f_k(w^t) + \lambda^T r \right) \right) \]
9: **end for**
10: Server aggregates the weight
11: \[ w^{t+1} = w^t + \frac{1}{K} \sum_{k=1}^K g^{t+1}_k \]
12: **end for**
13: Server updates \( \theta^{(i+1)} = \theta^i + \eta_\theta r \)
14: **end for**
15: Server updates \( \bar{w} \leftarrow \frac{1}{ET} \bar{w} \)
16: **if** \( \max_a r_a \leq \frac{M+2\nu}{B} \) **then**
17: **return** \( \bar{w} \)
18: **else**
19: **return** null
20: **end if**

4.2 Convergence guarantee

We now provide a formal convergence guarantee for Algorithm 1 in solving the empirical risk objective \( G(\cdot; \cdot) \). Note that \( G \) is linear in \( \lambda \). Hence, given a fixed \( w_0 \), we can find a solution to the problem \( \max_\lambda G(w_0; \lambda) \), denoted as \( \lambda^* \), i.e. \( G(w_0; \lambda^*) \geq G(w_0; \lambda) \) for all \( \lambda \). When \( G \) is convex in \( w \), we can argue that given a fixed \( \lambda_0 \), there exists \( w^* \) that satisfies \( w^* = \arg \min_w G(w; \lambda_0) \), i.e. \( G(w^*; \lambda_0) \leq G(w; \lambda_0) \) for all \( w \). Therefore, \((w^*, \lambda^*)\) is a saddle point of \( G(\cdot; \cdot) \).

To show how the solution found by our algorithm compares to an actual saddle point of \( G \), we introduce the notion of a \( \nu \)-approximate saddle point.

**Definition 2.** \((\bar{w}, \hat{\lambda})\) is a \( \nu \)-approximate saddle point of \( G \) if

\[
\begin{align*}
G(\bar{w}, \hat{\lambda}) &\leq G(w, \hat{\lambda}) + \nu \quad \text{for all } w \\
G(\bar{w}, \hat{\lambda}) &\geq G(\bar{w}, \lambda) - \nu \quad \text{for all } \lambda
\end{align*}
\]

As an example, \((w^*, \lambda^*)\) is a 0-approximate saddle point of \( G \). Now we present our main theorem of convergence below.

**Theorem 1.** Let Assumption 1-3 hold. Define \( \kappa = \frac{L}{\mu} \), \( \gamma = \max\{8\kappa, J\} \) and the learning rate \( \eta_Q = \)
2 and Algorithm 1 returns null or Algorithm 1 returns $w$ where $C$ is a constant.

We provide detailed descriptions of assumptions in the appendix. The upper bound in Equation 7 consists of two parts: the error for the FedAvg process to obtain $\bar{w}$ and the error for the Exponentiated Gradient Ascent process to obtain $\bar{\lambda}$. Following Theorem 1, we also provide the following corollary expressing the solution of Algorithm 1 as a $\nu$-approximate saddle point of $G$:

**Corollary 2.** Let $\eta_0 = \frac{\nu}{2\nu^2B}$ and $T \geq \frac{1}{\nu(\gamma+1)-2\nuC}\left(\frac{4\nu^2B^2\log(|A|+1)(\gamma+1)}{\nu}\right) + 2\kappa C(\gamma - 1)$, then $(\bar{w}, \bar{\lambda})$ is a $\nu$-approximate saddle point of $G$.

We provide detailed proofs for both Theorem 1 and Corollary 2 in the appendix.

### 4.3 Fairness guarantee

In the previous section, we demonstrated that our Algorithm 1 could convergence and find a $\nu$-approximate saddle point of the objective $G$. In this section, we further motivate why we care about finding a $\nu$-approximate saddle point. Eventually, the model learned will be evaluated on test data and data from silo not seen during training. Define the true data distribution to be $D = \frac{1}{T} \sum_{k=1}^{K} D_k$. We would like to formalize how well our model is evaluated on the true distribution $D$ as well as how well the fairness constraint is satisfied under $D$.

The result is presented below in Theorem 3.

**Theorem 3.** Let Assumption 1-4 holds. Let $F$ be the expected risk over the true distribution $D$, $(\bar{w}, \bar{\lambda})$ be a $\nu$-approximate saddle point of $G$. Then with probability $1 - \delta$, either there doesn’t exist solution for Equation 2 and Algorithm 1 returns null or Algorithm 1 returns $\bar{w}$ satisfies

\[
F(\bar{w}) \leq F(w^*) + 2\nu + 4\mathbb{R}_m(\mathcal{H}) + \frac{2M}{\kappa} \sqrt{\sum_{k=1}^{K} \frac{1}{m_k} \log(1/\delta)},
\]

\[
r_a(\bar{w}) \leq \frac{M + 2\nu}{B} + 2\kappa_0(\mathcal{H}) + \frac{M}{m_a} \sqrt{\frac{K}{2} \log(1/\delta)}
\]

where $w^*$ is a solution for Equation 2 and $r_a = \mathbb{E}_{(x,y,a) \sim D}[l(h(x), y)|A = a] - \zeta_a$.

Note that the fairness constraint for group $a$ under true distribution in Equation 8 is upper bounded by $O\left(\frac{\sqrt{K}}{m_a}\right)$. For any group $a_0$ with sufficient data, i.e. $m_{a_0}$ is large, the BGL constraint with respect to group $a_0$ under $D$ has stronger formal fairness guarantee compared to any group with less data. We could also see that as the number of silo increases, the upper bound becomes weaker. We provide details and proof for Theorem 3 in the appendix.

### 5 Experiments

We evaluate our Algorithm 1 empirically on the US-wide ACS PUMS data, a recent group fairness benchmark dataset, and on CelebA [3], a common federated learning dataset. We compare our method with training a vanilla FedAvg model in terms of both fairness and utility (Section 5.1). We further show the empirical difference between training with our global BGL constraint vs. local BGL constraints in Section 5.2.

**Setup.** For all experiments, we evaluate the accuracy and the empirical loss for each group on test data that belongs to all the silos of our fair federated learning solver. We consider the ACS Employment task [6] with race as protected attribute and CelebA [3] with gender as a protected attribute. A detailed description of datasets and models can be found in the appendix.
5.1 Fairness-Utility Relationship of Algorithm 1

We first explore how test accuracy differs as a function of maximum group loss using our Algorithm 1. To be consistent with our method and theoretical analysis, we exclude the protected attribute \(a_i\) for each data as a feature for learning the predictor. For each dataset, given fixed number of training iterations \(E\) and \(T\), we finetune \(B\) and \(\zeta\) and evaluate both test accuracy and test loss on each group. Given a certain test accuracy, we select the hyperparameter pair \((B, \zeta)\) that yields the lowest maximum group loss. We show the relation between test accuracy vs. max group loss in Figure 1 (Left). On both datasets, our method not only yields a model with significantly smaller maximum group loss than vanilla FedAvg, but also achieves higher test accuracy than the baseline FedAvg which is unaware of group fairness. Therefore, our method yields a model where utility can coexist with fairness constraints relying on Bounded Group Loss.

5.2 Local vs Global fairness constraint

In federated learning, previous work has shown it is not feasible to use local fairness metrics to approximate global fairness metrics. In other words, applying fair training locally at each data silo and aggregate the resulting model is not able to provide strong fairness guarantee at the global level with the same fairness definition [25]. In this section, we present and compare the relationship between test accuracy and max group loss under local BGL constraint and global BGL constraint. The results are shown in Figure 2 for both datasets. On the ACS Employment dataset, compared to the proposed method, FL via local BGL achieves higher maximum group loss given the same accuracy. Contrary to what is shown in Zeng et al. [25], on both datasets, even with local BGL constraint, fairness aware federated learning with proper hyper parameters yields a more fair and accurate model than FedAvg.

6 Conclusion and Future Work

In this work, we propose a fair learning objective for federated settings via Bounded Group Loss. We then propose a scalable algorithm to find an approximate saddle point for the objective. Theoretically, we provide convergence and fairness guarantees for our method. Empirically, we show that on the ACS Employment and CelebA tasks, our method satisfies high accuracy and strong fairness simultaneously. We are interested in further empirically evaluating our approach in future work, as well as characterizing the difference between using local BGL and global BGL from a theoretic perspective.
Figure 2: Comparison between using local fairness constraint and global fairness constraint on ACS Employment (Left) and CelebA (Right). For each method we show the utility-fairness performance of different hyperparameter combinations and plot the pareto frontier.

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Table 1: Dataset and model details used in our experiment

| Dataset        | Number of Silos | Model         | Protected Attribute | Partition Type                     | Task Type               |
|----------------|-----------------|---------------|---------------------|------------------------------------|------------------------|
| ACS Employment | 50              | Logistic Regression | Race               | Natural partition by States        | Binary classification  |
| CelebA         | 50              | 4-layer CNN   | Gender              | Manual partition                   | Binary classification  |

## A Datasets and Models

We summarize the details of the datasets and models we used in our empirical study in Table 1. Our experiments include both convex (Logistic Regression) and non-convex (CNN) loss objectives on both fairness (ACS Employment) and federated learning (CelebA) benchmarks.

## B Proof of Theorem 1

We first introduce a few assumptions needed for Theorem 1.

**Assumption 1.** Let $f_k$ be $\mu$-strongly convex and $L$-smooth for all $k = 1, \cdots, K$.

**Assumption 2.** Assume the stochastic gradient of $f_k$ has bounded variance: $\mathbb{E}[\|\nabla f_i(w^k_t; \xi^k_t) - \nabla f_k(w^k_t)\|^2] \leq \sigma^2_k$ for all $k = 1, \cdots, K$.

**Assumption 3.** Assume the stochastic gradient of $f_k$ is uniformly bounded: $\mathbb{E}[\|\nabla f_k(w^k_t; \xi^k_t)\|^2] \leq G^2$ for all $k = 1, \cdots, K$.

**Assumption 4.** Let $F$ and $F^*$ be upper bounded by constant $M$.

**Lemma 1** (Li et al. [14]). Let $\Gamma = F^* - \sum_i p_i F^*_i$, $\kappa = \frac{1}{\mu}$, $\gamma = \max\{8\kappa, J\}$ and the learning rate $\eta_t = \frac{2}{\mu(\gamma + t)}$. Then FedAvg with full device participation satisfies

$$\frac{1}{T} \sum_{t=1}^{T} F(w^t) - F^* \leq \frac{1}{T} \sum_{t=1}^{T} \frac{\kappa}{\gamma + t - 1} \left( \frac{2C}{\mu} + \frac{\mu^\gamma}{2} \mathbb{E}[\|w^1 - w^*\|^2] \right)$$

where

$$C = \sum_{i=1}^{N} p_i^2 \sigma_i^2 + 6L\Gamma + 8(J - 1)^2 G^2$$

**Proof for Theorem 1**. Let $m_{a,k}$ be the number of data with protected attribute $a$ for client $k$. By Assumption 1, we have $G_i$ be $(1 + \sum_a \lambda_a \frac{m_{a,k}}{m_a})\mu$-strongly convex and $(1 + \sum_a \lambda_a \frac{m_{a,k}}{m_a})L$-smooth. Since $\|\lambda\|_1 \leq B$, we
have $G_t$ be $(1+B)\mu$-strongly convex and $(1+B)L$-smooth. We first present the regret bound for $w^t$

$$\frac{1}{ET} \sum_{t=1}^{ET} G(w^t; \lambda^t) - \min_{w} \frac{1}{ET} \sum_{t=1}^{ET} G(w; \lambda^t) = \frac{1}{ET} \left( \sum_{t=1}^{ET} G(w^t; \lambda^t) - \min_{w} \sum_{t=1}^{ET} G(w; \lambda^t) \right)$$

(9)

$$= \frac{1}{ET} \left( \sum_{t=0}^{E-1} \sum_{i=1}^{T} G(w^{t+1}; \lambda^i) - \min_{w} \sum_{t=1}^{ET} G(w; \lambda^t) \right)$$

(10)

$$\leq \frac{1}{ET} \left( \sum_{t=0}^{E-1} \left( \sum_{i=1}^{T} G(w^{t+1}; \lambda^i) - \min_{w} \sum_{t=1}^{T} G(w; \lambda^t) \right) \right)$$

(11)

$$= \frac{1}{T} \sum_{t=0}^{E-1} \left( \frac{1}{T} \sum_{i=1}^{T} G(w^t; \lambda^i) - G^*(\lambda^i) \right)$$

(12)

$$\leq \frac{1}{ET} \sum_{t=0}^{E-1} \sum_{i=1}^{T} \frac{\kappa}{\gamma + t - 1} \left( \frac{2C_i}{\mu} + \frac{\mu \gamma}{2} \max_i \mathbb{E}[\|w^1;i - w^*;i\|^2] \right)$$

(13)

$$\leq \frac{1}{T} \sum_{t=0}^{E-1} \frac{\kappa}{\gamma + t - 1} \left( \frac{2\max_i C_i}{\mu} + \frac{\mu \gamma}{2} \max_i \mathbb{E}[\|w^1;i - w^*;i\|^2] \right)$$

(14)

Now we present the regret bound for $\lambda^t \in \mathbb{R}^{|A|}_+$. For any $\lambda^t$, let’s define $\tilde{\lambda}^t \in \mathbb{R}^{|A|}_+ \cup \mathbb{R}$ such that $\tilde{\lambda}^t$ satisfies $\|\tilde{\lambda}^t\|_1 = B$ and the first $|A|$ entries of $\tilde{\lambda}^t$ is the same as $\lambda^t$. Let $\bar{\mathbf{r}}^t \in \mathbb{R}^{|A|}_+ \cup \mathbb{R}$ such that the first $|A|$ entries of $\bar{\mathbf{r}}^t$ is the same as $\mathbf{r}^t$ and the last entry of $\bar{\mathbf{r}}^t$ is 0. Therefore, we have

$$\mathbf{\lambda}^T \mathbf{r}^t = \tilde{\mathbf{\lambda}}^T \bar{\mathbf{r}}^t$$

(15)

for all $\lambda$.

By Shalev-Shwartz et al. [20], for any $\tilde{\mathbf{\lambda}}$, we have

$$\sum_{t=1}^{ET} \tilde{\mathbf{\lambda}}^T \bar{\mathbf{r}}^t \leq \sum_{t=1}^{ET} \tilde{\mathbf{\lambda}}^T \bar{\mathbf{r}}^t + B \log(|A| + 1) \frac{1}{\eta_0} + \eta_0 \rho^2 BET$$

(16)

$$= \sum_{t=1}^{ET} \tilde{\mathbf{\lambda}}^T \bar{\mathbf{r}}^t + B \log(|A| + 1) \frac{1}{\eta_0} + \eta_0 \rho^2 BET$$

(17)

Therefore, we have

$$\min_{\lambda} \frac{1}{ET} \sum_{t=1}^{ET} G(w^t; \lambda) - \frac{1}{ET} \sum_{t=1}^{ET} G(w^t; \lambda^t) = \min_{\lambda} \frac{1}{ET} \sum_{t=1}^{ET} \mathbf{\lambda}^T \mathbf{r}^t - \frac{1}{ET} \sum_{t=1}^{ET} (\tilde{\mathbf{\lambda}}^t)^T \bar{\mathbf{r}}^t$$

(18)

$$\leq \frac{B \log(|A| + 1)}{\eta_0 ET} + \eta_0 \rho^2 B$$

(19)

Hence, we conclude that

$$\min_{\lambda} \frac{1}{ET} \sum_{t=1}^{ET} G(w^t; \lambda) - \min_{w} \frac{1}{ET} \sum_{t=1}^{ET} G(w; \lambda^t) \leq \frac{1}{T} \sum_{t=1}^{T} \frac{\kappa}{\gamma + t - 1} \left( \frac{2\max_i C_i}{(1+B)\mu} + \frac{(1+B)\mu \gamma}{2} \max_i \mathbb{E}[\|w^1;i - w^*;i\|^2] \right)$$

(20)

$$+ \frac{B \log(|A| + 1)}{\eta_0 ET} + \eta_0 \rho^2 B$$

(21)
By Jensen’s Inequality, \( G(\frac{1}{E^T} \sum_{i=1}^{E^T} w^i; \lambda) \leq \frac{1}{E^T} \sum_{i=1}^{E^T} G(w^i; \lambda) \). Therefore, we have

\[
\begin{align*}
\min_{\lambda} G(\tilde{w}; \lambda) - \min_{w} G(w; \tilde{\lambda}) &\leq \frac{1}{T} \sum_{t=1}^{T} \frac{\kappa}{\gamma + t - 1} \left( \frac{2 \max_i C_i}{(1 + B)\mu} + \frac{(1 + B)\mu \gamma}{2} \max_i E[\|w^{1,i} - w^{*,i}\|^2] \right) \\
&\quad + \frac{B \log(|A| + 1)}{\eta_0 ET} + \eta_0 \rho^2 B
\end{align*}
\]

(22)

Let \( C_1 = \max_i C_i \) and \( C_2 = \max_i E[\|w^{1,i} - w^{*,i}\|^2] \), we get Theorem\( \square \)

Proof for corollary 1. Note that \( \log(t + 1) \leq \sum_{n=1}^{t} \frac{1}{n} \leq \log(t) + 1 \) Let

\[
C = \frac{2 \max_i C_i}{(1 + B)\mu} + \frac{(1 + B)\mu \gamma}{2} \max_i E[\|w^{1,i} - w^{*,i}\|^2]
\]

(24)

we have

\[
\begin{align*}
\min_{\lambda} G(\tilde{w}; \lambda) - \min_{w} G(w; \tilde{\lambda}) &\leq \frac{\kappa C}{T} (\log(\gamma + T - 1) + 1 - \log(\gamma + 1)) + \frac{B \log(|A| + 1)}{\eta_0 ET} + \eta_0 \rho^2 B
\end{align*}
\]

(25)

Denote the right hand side as \( \nu_T \). Pick \( \eta_0 = \frac{\nu}{2\rho^2 B} \) and \( T \geq \frac{1}{\nu(\gamma + 1) - 2\nu C} \left( 4\rho^2 B \log(|A| + 1)(\gamma + 1) + 2\kappa C(\gamma - 1) \right) \).

\[
\nu_T \leq \frac{\kappa C \gamma + T - 1}{T} \frac{(\gamma + 1)}{\gamma + 1} + \frac{2\rho^2 B^2 \log(|A| + 1)}{\nu ET} + \frac{\nu}{2}
\]

(26)

\[
= \frac{\kappa C(\gamma - 1) \nu E + 2\rho^2 B^2 \log(|A| + 1)(\gamma + 1)}{\nu E(\gamma + 1)} + \frac{\kappa C}{\gamma + 1} + \frac{\nu}{2}
\]

(27)

\[
\leq \frac{\nu E(\nu(\gamma + 1) - 2\nu C)}{2\nu E(\gamma + 1)} + \frac{\kappa C}{\gamma + 1} + \frac{\nu}{2}
\]

(28)

\[
= \frac{\nu E(\nu(\gamma + 1) - 2\nu C)}{2\nu E(\gamma + 1)} + \frac{\kappa C}{\gamma + 1} + \frac{\nu}{2}
\]

(29)

\[
= \frac{\nu}{2} + \frac{\nu}{2}
\]

(30)

\[
= \nu
\]

(31)

\[\square\]

C Proof for Theorem 3

We first introduce a few lemmas necessary for proof for Theorem 3

Lemma 2. Let

\[
\mathcal{R}_m(\mathcal{H}) = \mathbb{E}_{\mathcal{S}_k \sim D_k^{m_k, \sigma}} \left[ \sup_{h \in \mathcal{H}} \frac{1}{K} \sum_{k=1}^{K} \frac{1}{m_k} \sum_{i=1}^{m_k} \sigma_{k,i} \mathbb{E}(h(x_{k,i}), y_{k,i}) \right]
\]

then for any \( h \in \mathcal{H} \), with probability \( 1 - \delta \), we have

\[
|F(h) - F(h)| \leq 2\mathcal{R}_m(\mathcal{H}) + \frac{M}{K} \sqrt{\sum_{k=1}^{K} \frac{1}{2m_k} \log(1/\delta)}
\]

(32)
Proof for lemma 2. Lemma 2 directly follows proof for Theorem 2 in Mohri et al. [18] with $\lambda_k = \frac{1}{K}$. \qed

Lemma 3. Let
\[
\mathfrak{R}_a(\mathcal{H}) = \mathbb{E}_{S_k \sim D_k^m, \sigma} \left[ \sup_{h \in \mathcal{H}} \sum_{k=1}^{K} \frac{1}{m_a} \sum_{a_i = a} \sigma_{k,i} l(h(x_{k,i}), y_{k,i}) \right]
\]
then for any $h \in \mathcal{H}$ and $a \in A$, with probability $1 - \delta$, we have
\[
|r_a(h) - r_a(h) - \mathfrak{R}_a(\mathcal{H})| \leq 2\mathfrak{R}_a(\mathcal{H}) + \frac{M}{m_a} K \sqrt{\frac{\log(1/\delta)}{2}} \tag{33}
\]

Lemma 4 (Lemma 1 in Agarwal et al. [1]). Let $(\bar{\omega}, \bar{\lambda})$ is a $\nu$-approximate saddle point, then
\[
\bar{\lambda}^T r(\bar{\omega}) \geq B \max_{a \in A} r_a(\bar{\omega}) + \nu \tag{34}
\]
where $x_+ = \max\{x, 0\}$.

Lemma 5 (Lemma 2 in Agarwal et al. [1]). For any $w$ such that $r(w) \leq 0_{|A|}$, $F(\bar{w}) \leq F(w) + 2\nu$.

Lemma 6. Assume there exists $w^*$ satisfies $r(w^*) \leq 0_{|A|}$, we have
\[
B \max_{a \in A} r_a(\bar{\omega})_+ \leq M + 2\nu \tag{35}
\]

Proof for lemma 6. Note that
\[
F(\bar{w}) + B \max_{a \in A} r_a(\bar{\omega})_+ - \nu \leq F(\bar{w}) + \bar{\lambda}^T r(\bar{\omega}) \tag{36}
\]
\[
= G(\bar{\omega}, \bar{\lambda}) \tag{37}
\]
\[
\leq \min_w G(w, \bar{\lambda}) + \nu \tag{38}
\]
\[
\leq G(w^*, \bar{\lambda}) + \nu \tag{39}
\]
\[
= F(w^*) + \bar{\lambda}^T r(w^*) + \nu \tag{40}
\]
\[
\leq F(w^*) + \nu. \tag{41}
\]
Therefore, we have
\[
F(\bar{w}) \leq F(w^*) + 2\nu. \tag{42}
\]
Hence,
\[
B \max_{a \in A} r_a(\bar{\omega})_+ \leq F(w^*) - F(\bar{w}) + 2\nu \tag{43}
\]
\[
\leq M + 2\nu, \tag{44}
\]
\qed

Note that Lemma 6 tells us when there exists a solution for problem 2, the empirical fairness constraint violates by at most an error of $\frac{M + 2\nu}{B}$. In other words, this guarantees that our algorithm always output a model when problem 2 has a solution.

Now we provide a proof of Theorem 3.
Proof for Theorem 3. When there exists a solution to problem \( w^* \), by lemma 2, we have

\[
F(\bar{w}) \leq F(w^*) + 2\mathfrak{R}_m(\mathcal{H}) + \frac{M}{K} \sum_{k=1}^{K} \frac{1}{2m_k} \log(1/\delta)
\]

(45)

\[
\leq F(w^*) + 2\nu + 2\mathfrak{R}_m(\mathcal{H}) + \frac{M}{K} \sum_{k=1}^{K} \frac{1}{2m_k} \log(1/\delta)
\]

(46)

\[
\leq F(w^*) + 2\nu + 4\mathfrak{R}_m(\mathcal{H}) + \frac{2M}{K} \sum_{k=1}^{K} \frac{1}{2m_k} \log(1/\delta).
\]

(47)

Combined with lemma 3, we have

\[
r_a(\bar{w}) \leq r_a(\bar{w}) + 2\mathfrak{R}_m(\mathcal{H}) + \frac{M}{m_a} \sqrt{\frac{K}{2}} \log(1/\delta)
\]

(48)

\[
\leq \frac{M + 2\nu}{B} + 2\mathfrak{R}_m(\mathcal{H}) + \frac{M}{m_a} \sqrt{\frac{K}{2}} \log(1/\delta)
\]

(49)

Therefore, Theorem 3 holds in this case.

When there doesn’t exist a solution to problem \( w^* \), Algorithm 1 outputs \( \bar{w} \) only when \( \max_{a \in A} r_a(\bar{w}) \leq \frac{M + 2\nu}{B} \). In certain scenario, we are still able to obtain

\[
r_a(\bar{w}) \leq \frac{M + 2\nu}{B} + 2\mathfrak{R}_m(\mathcal{H}) + \frac{M}{m_a} \sqrt{\frac{K}{2}} \log(1/\delta)
\]

(50)

by applying lemma 3. Since \( w^* \) doesn’t exist,

\[
F(\bar{w}) \leq F(w^*) + 2\nu + 4\mathfrak{R}_m(\mathcal{H}) + \frac{2M}{K} \sum_{k=1}^{K} \frac{1}{2m_k} \log(1/\delta)
\]

(51)

holds vacuously.

Therefore, Theorem 3 holds for both cases.

\[ \square \]

D Comparison with Papadaki et al. [19]

Besides FedAvg, we also compare our method to other fair federated learning methods. In particular, we study FedMinMax [19], a method aims at learning a global model that provides fair prediction for different protected attributes across all silos in the network. Similar to our objective, FedMinMax scales each group’s loss using a dual variable and solves a minimax objective to find a fair model. However, it is unclear what formal group fairness guarantee is provided by the proposed algorithm. Empirically, we compare our method and FedMinMax on both ACS Employment and CelebA. The results are shown in Figure 3. Using Bounded Group Loss as the fairness criterion, our Algorithm achieves utility (test accuracy) and fairness (max group loss) performance that is either comparable to or stronger than FedMinMax on both datasets.
Figure 3: Comparison between our FL via BGL and FedMinMax in terms of test accuracy and max group loss on ACS Employment (Left) and CelebA (Right). For each dataset, we select the best model for both methods to show the relation between test accuracy and fairness guarantee.