Polya's Four Phases Exploration in Solving Linear Program Story Questions Based on Student Beliefs

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Abstract. This study aims to explore the four phases of Polya in solving story problems in linear programming material in terms of student beliefs. Polya's ideas about the four phases of solving math problems are of particular concern to mathematics educators. This study reveals the four phases of Polya by looking at students' beliefs in each process. The type of grounded theory research method design that emerges is used to reveal Polya's beliefs in class XI IPS in terms of high, medium, and low initial mathematical ability (IMA) students. Data collection techniques in Emerging Design use interviews and documentation in six ways, namely discussion as recitation, teacher-directed conversation, open conversation, asking challenging questions, as knowledge transfer with guidance, and as a verbal interaction practice. The results showed that 1) the students' beliefs in understanding the questions greatly influenced the understanding of the questions, 2) The students' beliefs in writing what was known in the questions affected the planning stage, 3) the students' beliefs in implementing the plan not strongly influenced the implementation of the planning stage, 4) the students' beliefs in the overall answer does not affect the hindsight stage.

Keywords: Polya's Four Phases, Belief, Linear Programming.

1. INTRODUCTION

The learning process of mathematics trains students in general problem solving or mathematical problem solving both inside and outside the classroom. The process of solving mathematical problems has at least two aspects, namely mathematics as a "process of determining from general to particular" and mathematics as a "process of discovering" science [1]. In the second aspect, Polya states four phases in solving mathematical problems based on her experience as a mathematician [1].

The fact about a person's success in solving problems depends on his mathematical abilities, which are specifically referred to as mathematical proficiency [2],[3]. The analysis of mathematical ability in previous studies shows that the indicators intersect with the four phases of Polya, but there are indicators that are not found in the four phases of Polya, namely the disposition of productive beliefs [2].

Meanwhile, there is an interwoven relationship between student beliefs and problem-solving approaches, but a causal relationship can be established between certain beliefs and problem-solving activities [4]. However, solving mathematical problems is generally influenced by student beliefs, both positive and negative, where the positive influence of students persists in finding solutions and the negative influence of students giving up in answering questions [5].

In this study, we explored Polya's four stages in solving linear program story problems based on students' beliefs where the material being studied is a...
method or application of mathematical techniques used to solve problems that occur in everyday life. Mathematically, the aim of linear programming is to determine the optimization of the linear function under one linear function condition [6], [7].

2. THEORETICAL BASIS

Polya's four phases are ideas raised by Polya when someone wants to solve a mathematical problem. First, understanding the problem, which means we have to understand the problem by asking questions such as: "What is not known yet?", "What data is there?" and "What is the condition of the problem?" [1].

Second, devising a plan which means action to find the relationship between data and unknown variables in the problem through self-reflection such as "Have we ever seen this problem before?", "Have we ever seen the same problem in another form" and "Do we know any possible mathematical theorems?". Third, carrying out the plan, which means implementing the plan that will be used in the second phase by checking every step that has been planned. Fourth, looking back, which means testing the results obtained [1].

The four phases of Polya have indicators that intersect with mathematical proficiency, but there is a loose indicator, namely belief which means the belief that someone can learn mathematics with the right effort [2]. In the view of social psychology, belief is defined as one of the cognitive units [8] which influences students in solving mathematical problems and motivation in making decisions [9].

Belief is sometimes not defined by researchers because of its research interest and variations in the notion of belief itself [10]. However, in this study, we use the definition of belief in the problem solving process as a belief that is believed to be a belief that influences a person to make decisions and get answers [5].

This study also uses the definition of belief about the problem of storytelling which is defined as belief in manipulating a situation [11]. While story problems are defined as text that describes situations that are assumed to be familiar to the reader, asking quantitative questions and answering them requires mathematical operations using the data contained in the text [11].

Regarding the four phases of Polya and the emerging design used in this study, there are other studies which state that there has been an increase in the application of strategies in foreign exchange plans to solve mathematical problems [12]. Therefore, we believe that this study is able to reveal a small part of the four phases of Polya in terms of student belief [13].

The mathematical material used in this research is linear programming material with the assumption that the material fulfills the six steps of solving conditions [14] into the four phases of Polya and belief in solving problems. In addition, linear programming allows us to see the interactive process of mathematical models in certain situations, analyze the model and test the model of the problem [15].

3. RESEARCH METHODOLOGY

In order to reveal how students believe in the four phases of Polya in their work in solving linear program story problems, qualitative research was used with a grounded theory type of design that appeared to 3 students in terms of the three levels of students' initial mathematical abilities. The data collection procedures in this study were interviews and documentation of test results on linear program material. The interview used contains open and flexible questions where the results of the interview are used as a reference in six ways, namely discussion as a recitation, as a teacher-directed conversation, as an open conversation, asking challenging questions, as a transformation of knowledge along with guidance, and as a practice of verbal interaction [16].

4. RESULTS AND DISCUSSION

The research data is in the form of students' work in solving linear program story problems. "A boutique has 4 m of satin fabric and 5 m of prada cloth. From these materials will be made two party clothes. Party dress I requires 2 m satin and 1 m prada, party dress II requires 1 m satin and 2 m prada. If the selling price of the first party dress is Rp. 500,000.00 and party clothes II of Rp. 400,000.00, the maximum sales result for the boutique is…". In addition, the research data were obtained using interview techniques related to student work. Furthermore, these data are analyzed in a text based on MaxQDA to see the beliefs of each Polya process.

Respondent 1 (R1) is a student with a high IMA category who is analyzed for the first time. The result of R1’s work as in FIGURE 1, shows a complete answer that looks imperfect where there is no visible strain or barrier for. However, making a mathematical model is in accordance with the conditions of the problem.
The results of the interview and text analysis show that at the understanding the problem stage, R1 already understands the substance of the problem, as in the following interview excerpt:

R : Can you explain what is known in question number one?

R1 : What is known is that the boutique has 4 m of satin and 5 m of prada and what is being asked here is not the fabric but the party dress. Party dress one needs 2 m of satin and 1 m of prada and party dress two requires 1 m of satin and 2 m of prada.

Based on the answers given, both interviews and documentation, belief in R1 shows that belief in understanding the questions is quite significant. This can be seen in the following interview excerpt:

R : R1, do you understand what you mean by question number one?

R1 : I understand.

In the devising a plan stage, R1’s ability related to this stage can be seen in the following interview excerpt:

R : How do you change the known statement in question number 1?

R1 : Party clothes 1 are replaced with X and party clothes two are changed to Y, for example, if party clothes 1 requires 2 m of satin fabric, it means 2x and the second party clothes are like that.

In the looking back stage, R1’s ability can be seen in the following interview excerpt:

R : You already understand, next time, for example, if there is a problem like this written where x is more than zero and y is more than zero, what if...
you don't write it down later, there will be no corner points later.  
Can you mention a conclusion at each step of the solution in this problem?  
R1 : What do you mean, sir?  
R : For example, what was number 1 doing?  
R1 : Look for the maximum sales results  
R : This is what it's called for  
R1 : Make a mathematical model, then draw a DHP image and then look for the corner points  
R : Continue to look for what  
R1 : Ough, this is the last one to find the maximum result, so put it in that objective.  
R : Can you mention the conclusion of question number one?  
R1 : In conclusion the maximum value is 1.3 million.  
R : This (1, 2) right, what kind of 1 is this?  
R1 : This is X, this is Y  
R : What kind of party dress?  
R1 : Party dress 1  
R : Party clothes 2? Are the questions and answers synchronized yet?  
R1 : Yes.  
R : Next time, we will check again later.

These results show similarities to the results of research [17] which states that beliefs related to one's mathematical abilities have a direct effect on the ability to solve problems. On the other hand, the results of this study are similar to the results of other studies which state that belief in R1 is obtained from indirect questions about how it solves problems and from direct questions about beliefs related to the problem [18]. In this situation, [19] Stankov, L., & Lee states that belief affects intelligence [19] but in the case of R1 with high intelligence does not affect the last stage of Polya phases. Probably, there are other aspects influential in that stages but not revealed yet.

Respondent 2 (R2) is a student with a moderate IMA category who is analyzed after students with a high IMA category. The results of R2's work are as in FIGURE 2 shows a complete answer which looks also imperfect where there is no visible strain or barrier for and the creation of a mathematical model that is still not in accordance with the problems in the problem.

The results of the interview and text analysis show that at the understanding the problem stage, R2 has understood the substance of the problem, as in the following interview excerpt:

R : Have you, try to explain what is known in question number one?  
R2 : It is known that the 1st party dress requires 2 m of satin fabric and 1 m of prada, the second party dress requires 1 m of satin and 2 m of prada, continue to be asked, then the price of the 1st party dress is 500,000 and the second party dress is 500,000 400,000  

Based on the answers given, both interviews and documentation, belief in R2 shows that belief in understanding the questions is quite significant. This can be seen in the following interview excerpt:

R : R2, do you understand what you mean by question number one?  
R2 : Yes, sir. ”  

In the devising a plan stage, R2's ability related to this stage can be seen in the following interview excerpt:

R : Then how do you change the statement that is known in question number 1, using what method did you change the shape of the story problem, how do you do it?
R2: Use the described method one by one.
R: How do you explain it?
R2: So it was dubbed \(2X + Y\).
R: Who is \(X\)?
R2: \(X\) is satin
R: I see, then \(Y\)?
R2: \(Y\) is prada cloth
R: Is that how you describe it?
R2: yes"

The interview quote above, shows that R2 is able to do devising a plan. In addition, beliefs in R2 shows belief in doing a devising plan even though at this stage R2 still makes mistakes in determining the supply limit and making models of the objective function. This can be seen in the following interview excerpt:
R: Are you sure you write what you know is appropriate in the questions?
R2: Sure
R: Are you sure?
R2: Sure"

Meanwhile, R2's ability in carrying out a plan can be seen in the following interview excerpt:
R: What steps do you know to solve this number one question?
R2: Determine the \(x\) and then each \(x\) and \(y\) will be determined later both of them will be eliminated,
R: So?
R2: Then the result will be put into \(4\) m of satin plus \(5\) m of prada
R: Hmm, does it continue to run out?
R2: After that, we will determine the maximum sales result.
R: Already?
R2: Alright.

On the other hand, belief R2 appears at this stage to show less confidence in the completion that has been made. This can be seen in the following interview excerpt:
R: Are you sure your answer is correct?
R2: aa ,, eee ,, half sir.
R: Half, you mean?
R2: Yes, half-heartedly, I'm not too sure.
R: Half-half, aren't you sure?
R2: Yes”

At the looking back stage, R2's ability can be seen in the following interview excerpt:
R: The procedure also has to be understood again, can you continue to mention the conclusion at each step of solving the problem?
R2: Ha, the conclusion?
R: Yes. What do you think about the conclusion on number one, try to state the conclusion?
R2: In conclusion, what is the maximum amount of satin and prada?
R: Who was asked what?
R2: The one being asked for party clothes 1 is Rp. 500 thousand and party clothes 2 is Rp. 400 thousand.
R: Then what was asked?
R2: The one asked for the maximum sales.
R: So what conclusion here do you get from this work?
R2: Ouh, it means the conclusion is cloth... the maximum is 1,643,000, but it's wrong sir.
R: What's wrong with that?
R2: You see, because this was the limit for party 1 and party clothes 2.
R: How much should the objective function be?
R2: It should be \(500,000x + 400,000y\).
R: Then it was checked again, didn't your answer after you finished the work on that question?
R2: Not yet, I didn't read it first, I didn't read it again, immediately did it, sir, just remember oh my God. "

In the case of R2 with moderate IMA shows a strong belief in phases of Polya one and two but less belief in the others. These results similar to the finding of J Wang et al. that claims belief had the power to solve problems [20], on the one hand, but on the other hand in this situation, R2 belief had several levels. Some interview results show that R2 had problems in how to solve the story question problems of linear programming and had decreases belief according to his mathematical ability. In this study, we do not reveal the factors specifically that make R2 had belief go down in the two
last stages of Polya but it makes sense if belief treats as media of mathematical performance [21] in problem-solving.

Respondent 3 (R3) is the student with the last low IMA category analyzed. The results of R3 work as shown in FIGURE 3, show a complete answer that looks imperfect where there is no visible strain or barrier for and also the creation of a mathematical model that is not in accordance with the problem conditions.

FIGURE 3. R3 Work Results

The results of the interview and text analysis show that at the understanding the problem stage, R3 does not yet understand the substance of the problem, as in the following interview excerpt:

R : Not all? ok, try to explain what is known in question number one?
R3 : At first, 4 m of cloth was put in with 5 m of prada cloth, that is, what does it look like, the same size, then it continues to run out 2 and 1 like the usual formula.

Based on the answers given, both interviews and documentation, belief from R3 shows that belief is still lacking in understanding the problem or it can be said to be insignificant. This can be seen in the following interview excerpt:

R : R3, do you understand the meaning of question number one?
R3 : ee not all sir.

In the devising a plan stage, R3's capabilities related to this stage can be seen in the following interview excerpt:

R : How do you change this known number 1 statement?
R3 : I just need to enter the numbers in the usual way, just generate it and find the maximum value.

Meanwhile, R3's ability in carrying out a plan can be seen in the following interview excerpt:

R : Are there any steps that you know to solve the problem?
R3 : I forgot a little, I forgot the first steps, if the sequel has been found, you can do that.

On the other hand, belief R3 is not visible at this stage, but from the answers given both interviews and
documentation, implicitly shows that R3 has insignificant beliefs as in the following quote:

R : No, this is the maximum value, are you sure if you try to check the steps for the solution, why can you answer it all the points are entered?
R3 : Because as far as I know the formula is like that sir when looking for the maximum value "

[R3 INTERVIEW PROBLEM NUMBER 1; Position: 24-25; Author: nuramanah noviyanti; 12/17/2019 2:44 PM; Weight score: 0]

At the looking back stage, R3's ability can be seen in the following interview excerpt:

R : So, then, if you look for the maximum value, what is tested if you use the test point being tested, this is the corner point in the area of the settlement set, so tomorrow, be more careful, Can you mention a conclusion at each step of the solution in this problem?
R3 : eee, what do I do like this, sir, it's more or less specific, taken from the answers here, so it seems like the steps just need to be taken from here put the usual formulas like that.
R : Can you mention the conclusion of question number one?
R3 : the conclusion is that the answer I get 2,500,000, the conclusion is to use a formula like that was taught "

[R3 INTERVIEW PROBLEM NUMBER 1; Position: 26-29; Author: nuramanah noviyanti; 12/17/2019 3:03 PM; Weight score: 0]

In the case of R3 with low IMA shows low belief in all stages of Polya and affects problem solving abilities. These results seem reasonable if the IMA and low belief is in a person. Although, Laforce et al. claim that someone belief not influence the problem-based learning [22] but in the problem solving ability, belief had great effect to solving problem[20].

5. CONCLUSION

Based on the results of observations, the results of respondents' answers and interviews can be stated that the four phases of Polya in solving linear story questions, based on student beliefs, have a different level. Our findings show that students with high IMA and high belief are able to solve story problems on linear programming material using the four Polya phases. In this case, R1 does not show high belief in the hindsight stage but shows strong confidence in the previous three phases.

In the case of R2, students with moderate IMA, beliefs in phases one and two show a strong belief, but less so in phases three and four. This can be seen from the results of the answers and interviews which are both weak. In R3 case, students with low IMA, overall the Polya phase has low belief, where the results of the answers and interviews show mistakes and are sure of the answer itself.

In this study, it was not generally found that students' beliefs in understanding questions greatly influenced the stage of understanding the problem, students' beliefs in the analysis of what was known in the questions influenced the stage of designing a plan, students' beliefs in carrying out a plan not strongly influenced the stage of implementing a plan, while student beliefs in the answer do not affect the hindsight stage.

REFERENCES

[1] G. Polya, How to Solve It: A New Aspect of Mathematical Method, Second Edi. New Jersey: Princeton University Press, 1973.
[2] J. Soebagyo, “Profil Pembelajaran dalam Mengakomodasi Mathematical Proficiency,” Euclid, vol. 3, no. 2, pp. 474–490, 2007.
[3] J. Kilpatrick, J. Swafford, and B. Findell, Adding It Up: Helping Children Learn Mathematics, Washington DC: National Academy Press, 2001.
[4] M. L. Callejo and A. Vila, “Approach to mathematical problem solving and students’ belief systems: Two case studies,” Educ. Stud. Math., vol. 72, no. 1, pp. 111–126, 2009.
[5] T. Ozturk and B. Guven, “Evaluating students’ beliefs in problem solving process: A case study,” Eurasia J. Math. Sci. Technol. Educ., vol. 12, no. 3, pp. 411–429, 2016.
[6] S. Danø, Linear Programming in Industry: Theory and Applications, Fourth Edi. New York: Springer-Verlag, 1974.
[7] E. Oki, Linear Programming and Algorithms for Communication Networks. 2012.
[8] Gilah C. Leder, E. Pehkonen, and G. Törner, “Setting the Scene,” in Beliefs: A Hidden Variable in Mathematics Education?, New York: Kluwer Academic Publishers, 2003, pp. 1–10.
[9] P. O. Eynde, E. De Corte, and L. Verschaffel, “Framing Students’ Mathematics-Related Beliefs.pdf,” in Beliefs: A Hidden Variable in Mathematics Education?, New York: Kluwer Academic Publishers, 2003, pp. 13–37.
[10] F. Furinghetti and E. Pehkonen, “Rethinking Characterizations of Beliefs,” in Beliefs: A Hidden Variable in Mathematics Education?, New Jersey: Kluwer Academic Publishers, 2003, pp. 39–57.
[11] B. Greer, L. Verschaffel, and E. De Corte, “‘The Answer Is Really 4 . 5 ‘: Beliefs about Word,” in Beliefs: A Hidden Variable in Mathematics Education?, New York: Kluwer Academic
K. K. R. Hensberry and T. Jacobbe, “The effects of Polya’s heuristic and diary writing on children’s problem solving,” Math. Educ. Res. J., vol. 24, no. 1, pp. 59–85, 2012.

P. Kloosterman, A. M. Raymond, and C. Emenaker, “Students’ Beliefs about Mathematics: A Three-Year Study,” Elem. Sch. J., vol. 97, no. 1, pp. 39–56, 1996.

B. Kolman and R. E. Beck, Elementary Linear Programming with Applications, no. June. 1980.

H. A. Eiselt and C.-L. Sandblom, Linear Programming and Its Applications. Springer-Verlag, 2007.

J. W. Creswell, Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research, 4th editio. Boston: Pearson Education, Inc., 2012.

F. Pajares and J. Kranzler, “Self-efficacy beliefs and general mental ability in mathematical problem-solving,” Contemp. Educ. Psychol., vol. 20, no. 4, pp. 426–443, 1995.

N. J. McNeill, E. P. Douglas, M. Koroljungberg, D. J. Therriault, and I. Krause, “Undergraduate Students’ Beliefs about Engineering Problem Solving,” J. Eng. Educ., vol. 105, no. 4, pp. 560–584, 2016.

L. Stankov and J. Lee, “Self-beliefs: Strong Correlates of Mathematics Achievement and Intelligence,” Intelligence, vol. 61, pp. 11–16, 2017.

J. Wang, H. Otgaar, M. L. Howe, T. Smeets, H. Merckelbach, and Z. Nahouli, “Undermining Belief in False Memories Leads to Less Efficient Problem-Solving Behaviour,” Memory, vol. 25, no. 7, pp. 910–921, 2017.

S. Henschel and T. Roick, “Relationships of Mathematics Performance, Control and Value Beliefs with Cognitive and Affective Math Anxiety,” Learn. Individ. Differ., vol. 55, pp. 97–107, 2017.

M. Laforce, E. Noble, and C. Blackwell, “Problem-Based Learning (PBL) and Student Interest in STEM Careers: The Roles of Motivation and Ability Beliefs,” Educ. Sci., vol. 7, no. 4, 2017.