Further thoughts on supersymmetric $E_8$
as a family and grand unification theory

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ABSTRACT

We continue the analysis of the possibility of supersymmetric $E_8$ as a family unification and grand unification theory, this time under the assumptions that there is a vacuum gluino condensate, but that this condensate is not accompanied by dynamical generation of a mass gap in the pure $E_8$ gauge theory. Arguments supporting these assumptions are given. When the $E_8$ theory is coupled to supergravity, assuming vanishing of the cosmological constant and a supersymmetry breaking scale of around a TeV, we show that the gluino mass induced by gravitational coupling to the condensate is of order $10^{-3}$ eV or smaller, compatible with the fermion (and particularly the neutrino) mass spectrum. We suggest that composite scalar Higgs superfields can arise from a chiral glueball in the attractive $3875$ channel (and possibly other channels), permitting the breaking of the original $E_8$ gauge group to a $SO(10)$ grand unification group, times a $SU(3)$ family symmetry group and an extra $U(1)$ factor. A general analysis of the Higgs superpotential shows that in the absence of gravitational couplings, there is always a supersymmetric vacuum in the (unphysical) limit of infinite Higgs superfields. However, when gravitational couplings are included, dimensional analysis of the superpotential shows that the vacuum can be stabilized for finite Higgs superfields, with the occurrence of dynamical $\mathcal{F}$-type supersymmetry breaking. We conclude that an $E_8$ unification may be theoretically viable, providing an alternative paradigm for low energy model building, in which the supersymmetric partners of the standard model fermions are vectors, and in which the only chiral superfields are symmetry breaking Higgs fields.
1. Introduction

In a recent publication [1] we gave a mini-review of the literature suggesting $E_8$ as a grand unification and family unification group, and discussed a scenario for the realization of these ideas based on the Kovner–Shifman [2] proposal for a condensate-free vacuum state in supersymmetric Yang–Mills theory. Subsequently, the study of the chiral ring in supersymmetric Yang–Mills theory has ruled out the Kovner–Shifman state for the cases of the classical non-Abelian groups [3,4] and the exceptional group $G_2$ [5], and it seems likely that this analysis will eventually extend to the other exceptional groups, including $E_8$. Hence the scenario sketched in Ref. [1] is unlikely to work, and we must examine whether a unification model based on $E_8$ is compatible with a gluino condensate structure [6] for the supersymmetric Yang–Mills vacuum. This is the issue addressed in the present paper.

We begin with a brief review of the advantages, and problems, associated with $E_8$ unification. We then argue that even when a vacuum condensate is present, the conventional assumption that there is also a dynamically generated mass gap may fail, partly for reasons that are special to the group $E_8$, thus motivating our assumption that in the absence of gravitational couplings, there is no dynamically generated mass gap. We then analyze the implications of the gluino condensate picture when the Yang–Mills action is supplemented by the supersymmetric effective action arising from the coupling to linearized supergravity. We next discuss the possibility of dynamical generation of a chiral composite superfield, and then give a scenario for the dynamical breaking of supersymmetry and gauge symmetries, based on an augmented superpotential structure for this composite permitted in the presence of gravitation. We conclude by summarizing the implications of our analysis for low energy supersymmetric model building based on $E_8$ unification.
In the conventional MSSM approach to supersymmetrization of the standard model, the quarks and leptons are placed in a chiral supermultiplet, while the gauge bosons are placed in a vector supermultiplet. Thus, this approach does not exploit the possibility afforded by supersymmetry of unifying the quarks and gauge bosons of the standard model into a single supermultiplet. As sketched in Ref. [1], to which the reader is referred for extensive references to the prior literature, a natural candidate theory in which the standard model fermions and gauge bosons are superpartners of one another is supersymmetric Yang–Mills based on the group $E_8$.

The attractive features of $E_8$ unification may be briefly summarized as follows:

1. The exceptional group $E_8$ is the unique simple Lie group in which the adjoint representation, of dimension 248, is also the fundamental representation. Hence applying the natural grand unification paradigm of placing left-handed Weyl spinors in the fundamental, and gauge bosons in the adjoint, leads automatically [7] to a supersymmetric Yang–Mills theory in which the “matter” fermions and the “gluons” which bind them are superpartners. This theory is automatically free of gauge anomalies.

2. Under the breaking chain $E_8 \supset SU(3) \times E_6$, the 248 of $E_8$ branches [8] as

$$248 = (8, 1) + (1, 78) + (3, 27) + (3, 27)$$

while under $E_8 \supset SU(3) \times SO(10) \times U(1)$, the 248 branches as

$$248 = (1, 16)(3) + (1, 16)(-3) + (3, 16)(-1) + (3, 16)(1) + (3, 10)(2)$$

$$+ (\overline{3}, 10)(-2) + (\overline{3}, 1)(-4) + (\overline{3}, 1)(4) + (8, 1)(0) + (1, 45)(0) + (1, 1)(0)$$
with the $U(1)$ generator in parentheses. Hence, the 248 of $E_8$ naturally contains three 27’s of $E_6$ and three 16’s of $SO(10)$, and so can unify the three families [9] into a single representation encompassing a $SO(10)$ grand unification group and a $SU(3)$ family group.

3. The beta function for supersymmetric Yang–Mills theory with an additional multiplet of scalar fields is given by

$$\beta = \frac{-3g^3}{16\pi^2}(C_{\text{adjoint}} - \frac{1}{18}C_{\text{scalar}}) \quad ,$$

with $C$ one half of the index $\ell$ tabulated in Ref. [8]. Since for $E_8$

$$\frac{C_{3875}}{C_{248}} = 25 > 18 \quad ,$$

the theory is no longer asymptotically free if any Higgs scalars capable of breaking $E_8$ are present [10]. Hence an asymptotically free theory is obtained only if the gauge symmetry is broken dynamically. In the absence of gravity, the theory is thus a one parameter theory, governed by the value of the running coupling at some high mass scale below the Planck mass.

The reason that unification through supersymmetric $E_8$ has been largely ignored for the last decade is that there are also three potentially serious problems:

1. Because $248 \times 248$ of $E_8$ contains an $E_8$ singlet, a gluino condensate $\langle \text{Tr} \chi \bar{\chi} \rangle$ can form, and is in fact expected under the standard picture [6] of the condensate structure of supersymmetric Yang–Mills theories. If this condensate is accompanied by a mass gap of order the $E_8$ scale mass, as is generally argued to be the case, then light gluinos cannot appear and $E_8$ unification is ruled out.
2. Because of well-established “no-go” theorems [11] and structural restrictions on the induced effective action [12], supersymmetric Yang–Mills theory in isolation cannot be spontaneously broken.

3. Finally, there are well-known phenomenological problems with having mirror families, associated with the observed values of the electroweak \( S \) and \( T \) parameters [13].

In this article we will address the first two problems on this list, by arguing that there are scenarios that are capable of surmounting them. Hence in principle, supersymmetric \( E_8 \) unification is not ruled out. This provides a motivation for a future detailed analysis, not attempted here, to address the third problem listed above.

3. Does supersymmetric \( E_8 \) necessarily have a mass gap?

Whereas the presence of a gluino condensate in supersymmetric Yang–Mills is now reasonably well established, the belief that this condensate should be accompanied by a mass gap is less well founded. One of the main arguments for a mass gap is based on the analogy with QCD: Since QCD is a strongly coupled gauge theory with both a chiral condensate and a mass gap, one would expect that supersymmetric Yang–Mills theories, which are also strongly coupled gauge theories, should have analogous features. However, a number of criticisms can be leveled at this analogy, particularly when applied to the \( E_8 \) gauge group.

First of all, in QCD the mass gap does not take the form of Lagrangian mass terms (so called “current” mass terms) for the quarks of the form \( m_q \bar{q}q \), but rather appears through effective masses (so called “constituent” masses) when the quarks are bound in hadrons. To see that no “current” masses are generated, we note that the presence of a chiral condensate in QCD is an indication that there is an effective potential for \( \bar{q}q \) with a minimum at
the vacuum expectation $\langle \overline{q}q \rangle$. Since this potential is expected to be quadratic around the minimum, substituting $\overline{q}q = N[\overline{q}q] + \langle \overline{q}q \rangle$ into the effective potential, with $N[\ ]$ denoting the normal ordered product, one finds an operator correction near the minimum proportional to $N[\overline{q}q]^2$, which is not a mass term but rather an effective four fermion interaction. Thus expansion around the minimum associated with the chiral condensate does not produce a Lagrangian mass term. This argument carries over directly to the Veneziano–Yankielowicz [6] effective potential for supersymmetric Yang–Mills theory, and shows that the presence of a gluino condensate does not imply the generation of Lagrangian mass terms for the gluinos.

This still leaves the possibility that supersymmetric Yang–Mills theory develops gluino effective masses analogous to the “constituent” masses in QCD, which are generated as a result of the confinement of quarks in hadrons. However, supersymmetric Yang–Mills is more closely analogous to QCD with adjoint quarks, than it is to standard QCD with fundamental quarks. In QCD with adjoint quarks the string tension vanishes, because the adjoint quark charges can be screened by adjoint gluons [14]; in other words, QCD with adjoint quarks is in a Higgs phase rather than a confining phase. For gauge groups other than $E_8$, a QCD analog can be created from supersymmetric Yang–Mills by adding a chiral multiplet in the fundamental representation, giving a probe to study the pure Yang–Mills limit. This method does not give a QCD analog in the case of $E_8$, because the fundamental representation is identical to the adjoint representation, and so chiral fundamental “matter” is also screened rather than confined. The same is true for probes using chiral “matter” in higher $E_8$ representations, since these representations are contained in multiple tensor products of the fundamental, and so their charges can be screened by the accumulation of multiple $E_8$ gluons. (These remarks are in accord with the study by Acharya [15] of con-
finement in supersymmetric Yang–Mills using $M$–theory methods, since in the $E_8$ case his method agrees with the trivial, nonconfining center of the $E_8$ group.) To sum up, $E_8$ supersymmetric Yang–Mills is not a confining theory, and thus it is not at all evident that the QCD analogy can be used to infer the generation of gluino constituent masses.

Other arguments for the presence of a mass gap in supersymmetric Yang–Mills theory are based on extrapolations from theories of different types, such as $N = 2$ supersymmetric Yang–Mills theory in four dimensions, or string theories or $M$–theory in higher dimensions. An example of the latter is the paper of Atiyah and Witten [16], which presents arguments for a continuous $M$–theory curve connecting a region of parameter space with a mass gap to a limiting region related to four-dimensional supersymmetric Yang–Mills theory. Here the problem is that there is no guarantee that the mass gap does not vanish in the course of the limiting operation needed to recover $N = 1$ supersymmetric Yang–Mills in four dimensions from a larger, qualitatively different theory. This could happen if either the limit of Yang–Mills theory is on a phase boundary, or even if no change of phase is involved, if the mass gap vanishes while the phase-defining order parameter (such as the gluino condensate) does not. In terms of the superconductive analog for condensate formation, the latter is just what happens in gapless superconductors [17], where as the impurity concentration is increased, the energy gap vanishes over an open interval in which the order parameter (the condensate wave function) is non-zero.

To summarize, not only is there no proof of the existence of a mass gap in supersymmetric $E_8$ Yang–Mills (even in the heuristic sense in which there is a “proof” that there is a gluino condensate), but the arguments advanced for existence of a mass gap are on somewhat shaky ground. Hence we shall make the assumption, in the remainder of this paper, that
The supersymmetric Yang–Mills theory is an exception to the conventional lore, and has no intrinsic mass gap.

4. The gluino condensate and gluino mass in the presence of supergravity

If nature is supersymmetric, one expects a supersymmetric unified matter dynamics to be coupled to supergravity. To leading order in the gravitational coupling $\kappa^2 = 8\pi G_{\text{Newton}} = M_{\text{Planck}}^{-2}$, the effects of supergravity on the matter sector can be summarized by an effective action $S_{\text{eff grav}}$, given [18] by

$$S_{\text{eff}} = \kappa^2 \int d^4x \left[ -\frac{3}{16} j_\mu^{(5)} j^{\mu(5)} + \frac{1}{48} (P^2 + Q^2) \right] + \kappa^2 \int d^4xd^4y \left[ \frac{1}{4} \theta^{\nu\tau}(x) (\eta_{\nu\alpha} \eta_{\tau\beta} + \eta_{\nu\beta} \eta_{\tau\alpha} - \eta_{\nu\tau} \eta_{\alpha\beta}) \Delta_F(x-y) \theta^{\alpha\beta}(y) \right. \left. - \frac{1}{8} j_\tau(x) \left( \eta^{\tau\nu} \gamma \cdot \partial_x + \frac{1}{2} \gamma^{\tau} \gamma \cdot \partial_x \gamma^\nu \right) \Delta_F(x-y) j_\nu(y) \right].$$

Here $\Delta_F$ is the massless Feynman propagator, while $P, Q, j_\mu, j_\nu, \theta_{\mu\nu}$ are the components of the matter supercurrent, and it is straightforward to verify that Eq. (3a) is supersymmetry invariant when the conservation relations

$$\partial_\mu \theta^{\mu\nu} = \partial_\nu \theta^{\mu\nu} = \partial_\mu j^{\mu} = 0$$

are used. The total action will then be given by

$$S = S_{\text{matter}} + S_{\text{eff grav}},$$

with $S_{\text{matter}}$ the supersymmetric matter action. For the moment we leave the form of $S_{\text{matter}}$ unspecified, until we make statements further on that specifically assume a supersymmetric Yang–Mills form.

Let us now consider the vacuum energy implied by Eqs. (3a) and (4). Lorentz invariance implies that the only nonvanishing vacuum expectations of components of the current
supermultiplet are $\langle P \rangle$, $\langle Q \rangle$ and $\langle \theta_{\mu\nu} \rangle = \langle \theta_0^0 \rangle \eta_{\mu\nu}$, with $\eta_{\mu\nu}$ the Minkowski metric. Since we expect supersymmetry to be broken in the matter sector, the positive semidefinite matter vacuum energy density $\langle \theta_0^0 \rangle$ will be nonzero. Adding $\langle \theta_0^0 \rangle$ to the gravitational contributions coming from Eq. (3a), the total vacuum energy density becomes

$$\rho_{\text{VAC}} = \langle \theta_0^0 \rangle - \frac{\kappa^2}{48} (\langle P \rangle^2 + \langle Q \rangle^2) .$$  \hspace{1cm} (5)

By a current algebra calculation using the supersymmetry algebra, one can show [19] that the second order gravitino self-energy induced by the expectations $\langle P \rangle, \langle Q \rangle$ takes the form

$$\Delta S_{\text{mass}} = \frac{1}{2} m \int d^4x \bar{\psi}_\mu(x) \sigma^{\mu\rho} \psi_\rho(x) + \frac{1}{2} m' \int d^4x \bar{\psi}_\mu(x) i \gamma_5 \sigma^{\mu\rho} \psi_\rho(x) ,$$

$$m = \frac{\kappa^2}{12} \langle P \rangle , \quad m' = \frac{\kappa^2}{12} \langle Q \rangle ,$$  \hspace{1cm} (6)

with $\psi_\mu$ the gravitino field. The condition for the vacuum energy density or cosmological constant $\rho_{\text{VAC}}$ of Eq. (6) to vanish by cancellation between the matter and supergravity contributions is then [19]

$$\kappa \left[ \frac{\langle \theta_0^0 \rangle}{3} \right]^{\frac{1}{2}} = \frac{\kappa^2}{12} (\langle P \rangle^2 + \langle Q \rangle^2)^{\frac{1}{2}} = (m^2 + m'^2)^{\frac{1}{2}} .$$  \hspace{1cm} (7a)

When the CP nonconserving expectation $\langle Q \rangle \propto m'$ is zero, this reduces to the Deser- Zumino formula [20] for the gravitino mass $m$,

$$m = \frac{\kappa^2}{12} \langle P \rangle = \kappa \left[ \frac{\langle \theta_0^0 \rangle}{3} \right]^{\frac{1}{2}} .$$  \hspace{1cm} (7b)

If we assume that the matter supersymmetry breaking scale is of order a TeV (and thus, we assume no hidden sectors with higher supersymmetry breaking scales), Eq. (7b) leads to the estimate

$$\langle P \rangle^{\frac{1}{3}} \sim 10^9 \text{GeV}$$  \hspace{1cm} (8a)
for the mass scale associated with \( \langle P \rangle \), and to the estimate

\[
m \sim 10^{-13} \text{GeV} = 10^{-4} \text{eV}
\]  

(8b)

for the gravitino mass. This latter is compatible with the current accelerator bound [21] of \( m \geq 3 \times 10^{-13} \text{GeV} \) for the gravitino mass. In other words, in great generality, if one assumes cancellation of the cosmological constant between the matter and supergravity sectors, and a matter supersymmetry breaking scale of a TeV, one concludes that the gravitino must be superlight.

Let us now specialize to the case in which \( S_{\text{matter}} \) is the supersymmetric Yang–Mills action, and estimate the corresponding gluino condensate and gluino mass. For supersymmetric Yang–Mills, the tree level operator \( P \) is zero, with a contribution first appearing from the anomaly supermultiplet given by

\[
P = \frac{\beta(g)}{g} \text{Tr} \chi \chi,
\]  

(9)

with \( \chi \) the gluino field, with \( \beta \) the beta function, and with \( \text{Tr} \chi \chi = \sum_A \chi^A \chi^A \). Thus the estimate of Eq. (8a) shows that the gluino condensate must be nonzero, and gives an estimate of its magnitude. To get a corresponding estimate of the gluino mass induced by gravitational couplings to this condensate, we substitute \( P = \langle P \rangle + \frac{\beta(g)}{g} \text{Tr} N[\chi \chi] \), with \( N[ \ ] \) denoting normal ordering, into Eq. (3a) for the supergravity induced effective action. Linearizing in the normal ordered terms, this gives a gluino mass Lagrangian density term of

\[
\frac{\kappa^2}{12} \langle P \rangle \frac{\beta(g)}{g} \frac{1}{2} \text{Tr} N[\chi \chi],
\]  

(10a)
corresponding to a gluino mass of

\[ m_{\text{gluino}} = \frac{\kappa^2}{12} \langle P \rangle \left| \frac{\beta(g)}{g} \right| = \left| \frac{\beta(g)}{g} \right| m, \quad (10b) \]

a formula familiar [22] from the theory of anomaly mediated supersymmetry breaking. If we assume that \( \left| \frac{\beta(g)}{g} \right| \leq 10 \), then Eqs. (8b) and (10b) give an estimate for the \( E_8 \) singlet mass term,

\[ m_{\text{gluino}} \sim 10^{-3} \text{eV}, \quad (10c) \]

which sets a lower bound for the observed fermion masses. (Other, non-singlet dynamically generated mass terms will of course be needed to give a realistic fermion mass spectrum.) This bound is compatible with our knowledge of the neutrino mass spectrum, and so we conclude that the \( E_8 \) supersymmetric Yang–Mills vacuum, with its gluino condensate but assuming no intrinsic mass gap, is compatible with the idea that the fermions of the standard model may be the gluinos of a supersymmetric Yang–Mills theory.
5. A scenario for dynamical generation of chiral Higgs superfields

If an $E_8$ theory is to describe observed standard model physics, three kinds of symmetries must be broken: (i) gauge symmetry, (ii) the discrete symmetries P, C, and CP, and (iii) supersymmetry. In order for these symmetries to be broken by a supersymmetric analog of the Higgs mechanism, we need to have both chiral Higgs superfields, and an effective potential for these superfields that breaks the symmetries (i)–(iii). In this section we address the issue of obtaining the needed superfields, and in the next section we shall analyze whether suitable effective potentials can be generated.

Since we have seen that an asymptotically free $E_8$ supersymmetric theory cannot have fundamental Higgs bosons, the Higgs fields must occur as dynamically generated composites. Letting $\lambda^A$, $A = 1, 2, ..., 248$ be the generator matrices for $E_8$, the strength of the vector exchange force between two 248 supermultiplets 1 and 2 in a channel with group representation $T$ is proportional to

$$\sum_A \lambda^A_1 \lambda^A_2 = \lambda_1 \cdot \lambda_2$$

$$= \frac{1}{2}[(\lambda_1 + \lambda_2)^2 - \lambda_1^2 - \lambda_2^2]$$

$$= C_2(T) - 2C_2(248)$$

(11a)

with $C_2(T)$ and $C_2(248)$ respectively one half of the Casimirs for the representations $T$ and 248. The representations $T$ that are potentially of interest for the formation of dynamical scalar composites are the the $1$, $3875$, $27000$, and $30380$, since these are the representations that appear symmetrically in the decomposition

$$248 \times 248 = 1_s + 248_a + 3875_s + 27000_s + 30380_s$$

(11b)

(Only the symmetrical terms are of interest because forming a Lorentz scalar from two anti-
commuting spinors requires an antisymmetric $\epsilon$ factor in the spinor indices, as in Eq. (13b) below, requiring the group structure factor to be symmetric \cite{23}.

Using the formula \cite{8}

$$C_2(T) = \frac{N(\text{adjoint})}{N(T)} \frac{1}{2} \ell(T) = \frac{N(248)}{N(T)} C_T,$$

(11c)

with $N(T)$ the dimension of the representation $T$, and $C_T$ as before one half of the index $\ell(T)$, we find the values

$$C_2(1) = 0, \quad C_2(248) = 30, \quad C_2(3875) = 48$$

$$(12a)$$

$$C_2(27000) = 62, \quad C_2(30380) = 60,$$

from which we find

$$C_2(1) - 2C_2(248) = -60,$$

$$C_2(3875) - 2C_2(248) = -12,$$

$$C_2(27000) - 2C_2(248) = 2,$$

$$C_2(30380) - 2C_2(248) = 0.$$

(12b)

The most attractive channel is the singlet, which as we have already seen is expected to contain a glueball condensate. According to the often used most attractive channel (MAC) rule \cite{24}, only the most attractive channel is supposed to contain a dynamical composite, but we see no compelling justification for excluding the possibility that other attractive channels may have composites as well. Thus, the 3875 channel, which is also attractive, may contain a dynamical composite. We note furthermore that since the 30380 is on the borderline between attractive and repulsive, and the 27000 is only weakly repulsive, renormalization effects may make one or both of these effectively attractive, leading to the more speculative possibility of further composites beyond the 3875.

Chiral superfields corresponding to the various possible composite channels can be written in terms of the chiral gaugino/gluino superfield $W_\alpha = \frac{1}{2} \sum_A \lambda^A W^A_\alpha$. The usual
glueball, corresponding to the channel \( T = 1 \), is given (with the spinor indices \( \alpha, \beta \) summed over) by

\[
\Phi(1, 0) \equiv \text{Tr} W_\alpha W_\beta \epsilon^{\alpha\beta} = \sum_A W^A_\alpha W^A_\beta \epsilon^{\alpha\beta}, \tag{13a}
\]

and is an \( E_8 \) gauge invariant chiral superfield. Using the \( E_8 \) Clebsch \((Tm|248A, 248B)\) one can form further chiral superfields

\[
\Phi(T, m) \equiv \sum_{A,B} (Tm|248A, 248B) W^A_\alpha W^B_\beta \epsilon^{\alpha\beta}, \tag{13b}
\]

which are Lorentz scalar chiral superfields transforming as the \( m \)th basis element of the representation \( T \). Hence the formation of non-singlet composites can preserve supersymmetry, through the formation of dynamical chiral superfields in attractive channels. Thus an \( E_8 \) gauge theory can, in principle, generate chiral Higgs fields that can lead to dynamical symmetry breaking when these chiral superfields develop non-vanishing vacuum expectation values.

6. Structure of effective potentials for the chiral Higgs superfields

We must now check whether the theory is allowed to dynamically generate an effective superpotential function \( f \) of the superfields of Eqs. (13a,b) that obeys the following requirements: First, it should be of dimension 3, so that its \( F \) projection is of dimension 4 as required for an action density. Second, under the classical chiral \( U(1) \) transformation (the \( R \)-symmetry transformation) which scales \( W_\alpha \) as \( W_\alpha \rightarrow \exp(i\phi)W_\alpha \), the terms in the superpotential that do not solely involve \( \Phi(1, 0) \) should scale as \( \exp(2i\phi) \). In other words, these terms should have an \( R \) quantum number 2, so that their contribution to the effective action is independent of \( \phi \). (The part of the superpotential that is solely a function of the
singlet glueball $\Phi(1, 0)$ is affected by the chiral anomaly, leading to an extra logarithm in its effective superpotential, as shown by Veneziano and Yankielowicz [6], and so do not show simple $R = 2$ scaling.) Finally, as the gauge coupling $g$ approaches zero, or equivalently, as the $E_8$ subtraction-independent scale mass $M$ given by

$$M = \mu \exp \left( -\frac{8\pi^2}{3C_{\text{adjoint}}} \frac{1}{g^2(\mu)} \right)$$

(14)

approaches zero, the effective superpotential for fixed chiral superfield arguments should approach zero. We shall require this approach to zero to be uniform in the nonsinglet chiral superfield arguments $\Phi(T, m)$ of the effective potential, permitting interchange of taking the zero coupling limit with taking derivatives acting on these arguments.

Abbreviating the singlet glueball by $Z = \Phi(1, 0)$, with corresponding vacuum expectation $\langle Z \rangle$, and abbreviating the general non-singlet glueball by $Y_i = \Phi(T, m)$, in the absence of a coupling to gravitation the most general holomorphic superpotential satisfying the first two requirements, and incorporating the Veneziano–Yankielowicz superpotential, is

$$f = Z \log \left( \frac{Z}{e^{\langle Z \rangle}} \right) + Z F[\{ \frac{Y_i}{Z} \}] \hspace{1cm} (15a)$$

with $F$ a general function of its arguments. Since the scale of $Z$ is set by $\langle Z \rangle \propto M^3$, the third requirement will be satisfied if

$$ZF[\{ \frac{Y_i}{Z} \}] \to 0 \hspace{1cm} (15b)$$

uniformly in the nonsinglet chiral superfields $Y_i$ as $Z \to 0$. Differentiating with respect to $Y_i$, this implies that

$$\frac{\partial f}{\partial Y_i} = Z \frac{\partial F}{\partial Y_i} \hspace{1cm} (15c)$$
also approaches zero as $Z \to 0$. However, since the right hand side of Eq. (15c) is a function of the ratios $\kappa_i = Y_i/Z$, this also implies that $\partial f / Y_i \to 0$ as all of the Higgs superfields $Y_i$ are uniformly scaled to infinity, or equivalently, as all of the $\kappa_i$ are uniformly scaled to infinity.

Since

$$\frac{\partial f}{Z} = \log \left( \frac{Z}{\langle Z \rangle} \right) + F[\{\kappa_i\}] - \sum_j \kappa_j \frac{\partial F[\{\kappa_i\}]}{\partial \kappa_j}, \quad (16a)$$

the equation $\partial f / Z = 0$ always has a solution for any set $\{\kappa_i\}$. At this solution, the potential

$$V = \left| \frac{\partial f}{Z} \right|^2 + \sum_i \left| \frac{\partial f}{\partial Y_i} \right|^2 \quad (16b)$$

vanishes in the limit as all of the $Y_i$ or $\kappa_i$ are scaled to infinity, and so there is at least one supersymmetric (although not physically realistic) ground state. (If the potential $V$ vanishes monotonically as the $\kappa_i$ are scaled to infinity, then there cannot even be a metastable supersymmetry breaking vacuum for finite values of the Higgs superfields). Therefore, in the absence of coupling to supergravity, the $E_8$ gauge theory, even with the dynamical generation of Higgs superfields, does not break supersymmetry. This conclusion agrees with general expectations, based for example on the Witten index [11], that pure supersymmetric Yang–Mills theory cannot break supersymmetry.

The situation changes qualitatively when supergravity couplings are included, since then the superpotential $f$ can depend on the dimensionless quantity $\kappa M = M / M_{\text{Planck}}$, which vanishes as either the gauge coupling or the gravitational coupling approaches zero. One can then form O’Raifeartaigh [25] or $\mathcal{F}$-type superpotentials that satisfy all of the requirements, and break supersymmetry, with a vacuum stabilized at finite values of the Higgs superfields. For example, letting $X$, $Y_1$ and $Y_2$ be any of the nonsinglet Higgs superfields, and letting
ψ(u) be any dimensionless positive function that vanishes as $u \to \infty$, consider

$$f = Z \log \left( \frac{Z}{\psi(Z)} \right) + Y_1 \left[ \psi\left( \frac{X}{Z} \right) - \kappa^2 M^2 \right] + Y_2 \psi\left( \frac{X}{Z} \right), \quad (17a)$$

which satisfies all of our requirements. Then we see that $\partial f/\partial X$ vanishes on the surface $Y_1 + Y_2 = 0$, and $\partial f/\partial Z$ then vanishes for $Z = \langle Z \rangle$, but

$$\left| \frac{\partial f}{Y_1} \right|^2 + \left| \frac{\partial f}{Y_2} \right|^2 = \left| \psi\left( \frac{X}{Z} \right) - \kappa^2 M^2 \right|^2 + \left| \psi\left( \frac{X}{Z} \right) \right|^2 \geq \frac{1}{2} \kappa^4 M^4, \quad (17b)$$

and so supersymmetry is necessarily broken. Since the dimension one Higgs superfields are obtained by dividing the dimension three fields $Y_i$ by the scale mass squared $M^2$, the correctly normalized potential is obtained by multiplying Eq. (17b) by a factor of $M^4$, and so equating the supersymmetry breaking scale to a TeV gives the estimate $M^8 / M_{\text{Planck}}^4 \sim (1 \text{TeV})^4$, which gives $M \sim 10^{11} \text{GeV}$ for the scale mass.

The variant of the superpotential of Eq. (17a) in which the $Y_2$ term is omitted preserves supersymmetry, but for suitable choices of $X, Y_1$ and $\psi$ can spontaneously break the $E_8$ gauge symmetry or various discrete symmetries. Thus, supergravity couplings can generate effective superpotentials which stabilize the vacuum and break all of the requisite symmetries, with the natural appearance of a hierarchy when $\kappa^2 M^2$ is small. Finally, we remark that since the kinetic energy comes from $D$ terms in the effective action, which are not required to be holomorphic, the considerations of $R$ invariance used above place no restrictions, since one can always generate $R = 0$ terms as the squared modulus of terms with nonzero $R$. Hence even in the absence of gravitational couplings there will be effective kinetic terms for the composite Higgs superfields.
7. Implications for low energy supersymmetric model building

The proposals just sketched for a unification theory based on $E_8$ involve many non-perturbative phenomena, and so their verification or falsification will be challenging. However, they suggest an alternative paradigm for low energy supersymmetric model building in which the supersymmetric partners of the observed fermions are vectors, rather than scalars, and in which the only chiral superfields are the Higgs fields needed for gauge symmetry breaking. One test of the proposals we have made is to see whether phenomenologically acceptable supersymmetric extensions of the standard model can be constructed using this alternative paradigm. Such model building should be feasible within a conventional perturbative framework, and will be needed to see whether the third objection to $E_8$ unification, involving the presence of mirror fermions and their contributions to the electroweak precision parameters, can be satisfactorily dealt with. Within the decade, experiments may also make decisive statements about both the presence of additional families of mirror fermions, and the Lorentz structure (scalar versus vector) of squarks and sleptons.

Acknowledgments

This work was supported in part by the Department of Energy under Grant #DE-FG02-90ER40542. I wish to thank Bobby Acharya, Hooman Davoudiasl, Ryuichiro Kitano, Graham Kribs, Tianjun Li, Hitoshi Murayama, Marc Thormeier, and especially Nathan Seiberg and Edward Witten, for helpful conversations or email correspondence. This work was also stimulated by remarks made to me long ago by Freeman Dyson, Murray Gell-Mann, and Richard Slansky.
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