Electrostatic control of phase slips in Ti Josephson nanotransistors

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The investigation of the switching current probability distribution of a Josephson junction is a conventional tool to gain information on the phase slips dynamics as a function of the temperature. Here we adopt this well-established technique to probe the impact of an external static electric field on the occurrence of phase slips in gated all-metallic titanium (Ti) Josephson weak links. We show, in a temperature range between 20 mK and 420 mK, that the evolution of the phase slips dynamics as a function of the electrostatic field starkly differs from that observed as a function of the temperature. This fact demonstrates, on the one hand, that the electric field suppression of the critical current is not simply related to a conventional thermal-like quasiparticle overheating in the weak-link region. On the other hand, our results may open the way to operate an electrostatic-driven manipulation of phase slips in metallic Josephson nanojunctions, which can be pivotal for the control of decoherence in superconducting nanostructures.

Although a static electric field is almost ineffective on the conduction properties of metals, recent experiments demonstrated the possibility to suppress via conventional gating the critical current ($I_C$) of metallic Bardeen-Cooper-Schrieffer superconducting wires [1] and Dayem bridges, [2–5] and of proximity superconductor-normal metal-superconductor (SNS) Josephson junctions (JJ) [6]. Yet, by means of a superconducting quantum interference device (SQUID) consisting of two gated Dayem bridge constrictions, it has been possible to directly measure the impact of a static electric field on the quantum phase difference ($\phi$) across a Josephson weak-link [7]. The electric field was found to influence the SQUID current-phase relation via direct suppression of the critical current of a gated weak-link. In addition, unexpectedly, phase shifts in the SQUID current vs flux relation were measured for gate voltage values low enough to have no influence on $I_C$. Phase fluctuations present inside the superconductor [7] were shown to be a plausible cause for such an effect to occur, suggesting that the influence of the electric field on the phase may also lead to the occurrence of phase slips, i.e., local random $2\pi$ jumps of $\phi$ [8], which are responsible for the superconducting-to-normal state transition [9]. Phase slip events, indeed, reflect into the value of the switching current ($I_S$), that is, when the bias current is swept from zero to above the critical current, the bias value at which the superconductor switches to the normal state. Due to the stochastic nature of phase slip events, $I_S$ statistically spreads around $I_C$, and its distribution [also known as the switching current probability distribution (SCPD)] naturally provides information on the phase slips dynamics of mesoscopic superconducting devices [10–26] under the influence of external parameters such as, for instance, the temperature or an externally applied electric field. The latter was recently investigated in hybrid graphene-based Josephson junctions [27, 28], but no relationship between the electric field and the SCPDs has been observed so far in genuine all-metallic superconducting systems. The relevance of such topic lies in its natural link with the study of decoherence mechanisms in JJ devices, a matter of strong interest mainly in view of the realization of advanced superconducting quantum information architectures [29].

Here we tackle this point, and report the investigation of SCPDs in electrostatically-controlled titanium Dayem bridge Josephson weak-links in a temperature range from 20 mK to 420 mK, a regime explored so far only for Josephson tunnel junctions [31]. Our analysis of SCPDs of gated Dayem bridges JJs demonstrates the dramatic action of the electrostatic field on the phase slips dynamics in metallic superconductors, and opens the way to operate an electric field-driven control of phase slips and, thereby, of decoherence in Josephson weak-links. Moreover, we will show that the evolution of SCPDs as a function of the electrostatic field starkly differs from that measured as a function of temperature. This fact indicates that the electric field-driven critical current suppression is not related to a mere thermal-like quasiparticle overheating occurring in the junction region [32].

Our Ti-based Dayem bridges weak-links consist of 30-nm-tick, 150-nm-long, 120-nm-wide planar gated junctions fabricated by a single-step electron beam lithography of a poly methyl methacrylate (PMMA) resist mask deposited onto a sapphire ($\text{Al}_2\text{O}_3$) single-crystal wafer with a nominal resistivity larger than $10^{10} \Omega\cdot\text{cm}$. Titanium was evaporated at a rate of 1.2 nm/s in an ultra-high vacuum electron-beam evaporator with a base pressure of about $10^{-11}$ Torr. The 140-nm-wide gate electrode was separated by a distance of about 80 nm from the Dayem bridge constriction. Figure 1a shows the false color scanning electron micrograph of a representative Josephson device.

The low-temperature electric characterization of the devices was obtained by standard dc four-wire current versus voltage ($I$ vs $V$) technique in a filtered cryogen-free $^3\text{He}$–$^4\text{He}$ dilution refrigerator, carried out with a low-noise current generator and room-temperature differential voltage pre-amplifier. Figure 1b shows the back and forth $I$ vs $V$ characteristics of a typical JJ device registered at several bath temperatures. The curves exhibit the conventional hysteretical behaviour, which stems from heating induced in the weak-link when switching from the dissipative to the dissipationless regime [33] (the device normal-state resistance is $R_N \approx 550 \Omega$). $I_C$ decreases with temperature according
to the behaviour expected from the Bardeen’s formula [30]

\[ I_C(T) = I_C^0 \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]^{\frac{1}{2}} \]

where \( I_C^0 \) is the zero-temperature critical current, and \( T_C \) is the critical temperature of the superconducting weak-link. The fit of the \( I_C(T) \) characteristic with Bardeen’s equation (shown as the black line in Fig. 1c) yields \( I_C^0(\mu) \approx 6.02\mu \text{A} \) and \( T_C^0(\mu) \approx 348 \text{ mK} \). The latter value is in reasonable agreement with the critical temperature \( T_C^{\exp} \approx 310 \text{ mK} \) extracted from the low-frequency lock-in resistance \( (R) \) versus \( T \) measurement (see the inset of Fig. 1c).

The stochastic behavior of the switching current of a Dayem bridge can be modeled with the resistively and capacitively shunted junction (RCSJ) theory [34, 35] which schematizes a Josephson junction as the parallel of a resistance \( R \), a capacitor and a phase-dependent current generator \( I(\phi) \). According to this model, we can interpret the transition to the normal state as a phase particle moving in a tilted washboard potential under the effect of a friction force (see Fig. 2a) [36]. In this framework, switching events are represented by the escape of the phase particle from a minimum of the potential, corresponding to a \( 2\pi \)-rotation in \( \phi \). The probability distribution \( P(I,T) \) for this event to occur, as a function of the bias current \( I \) and of the electronic temperature \( T \), is given by the inverse KurkijrviFultonDunkleberger (KFD) transform [37, 38]:

\[ P(I,T) = \frac{\Gamma(I,T)}{v_I} \exp \left[ -\frac{1}{v_I} \int_0^I \Gamma(I',T) dI' \right], \]

where

\[ \Gamma(I,T) = \frac{L}{2\pi \xi(T) \tau_{GL}(T)} \sqrt{\frac{\Delta U(I,T)}{k_B T}} \times \exp \left[ -\frac{\Delta U(I,T)}{k_B T} \right], \]

is the phase slip rate, \( v_I = dI/dT \) is the ramp speed of the bias current, \( \xi(T) \) is the Ginzburg-Landau (GL) coherence length, \( \tau_{GL}(T) \) is the so-called GL relaxation time [8], and

\[ \Delta U(I,T) = aE_J(T) \left( 1 - \frac{I}{I_C(T)} \right)^b \]

is the height of the potential barrier, \( E_J(T) = \hbar C(T)/2e \) is the Josephson energy, \( e \) is the electron charge, and \( (a,b) \) are parameters accounting for the typology of the Josephson weak-link [36]. Conventional, the critical current \( I_C \) of the superconducting junction is assumed...
FIG. 2. (a) Schematic diagram of the tilted washboard potential showing the quantum phase slip (QPS, blue arrow) and thermally-activated phase slip (TAPS, red arrow) processes. $\Delta U$ is the height of the barrier defined as the energy difference between a minimum and the following maximum of the washboard potential (black line). (b) Switching current probability distributions (SCPDs) vs current $I$ obtained at different temperatures from 20 mK to 300 mK. Dotted lines represent the best-fit curves obtained with KFD model. The gray vertical line indicates the crossover temperature from TAPS to MPS regime. (c) Experimental rate $\Gamma$ vs $I$ obtained with the direct KFD transform for the same temperature values as in panel (b). (d) Standard deviation $\sigma$ of the SCPDs vs bath temperature $T$. The crossover temperatures, $T_Q \approx 110$ mK and $T_T \approx 160$ mK, separate QPS/TAPS and TAPS/MPS regimes, respectively. For each SCPD the total sampling number of $I$ is $10^4$.

to be either the maximum current for which $P(I, T) \neq 0$ or the mode of the SCPD; in the following we will adopt the latter definition.

Figure 2(b) shows the SCPDs built through $10^4$ acquisitions of the switching current measured at several temperatures ranging from 20 mK to 300 mK. To perform SCPD measurements we used a 750-KHz bandwidth input/output analog-to-digital/digital-to-analog converter (ADC/DAC) board for the acquisition of the voltage drop signal and the generation of the bias current, respectively. The input signal consisted of an 8.7 Hz saw-tooth current wave obtained by applying a voltage signal generated by the digital board to an 1M$\Omega$ load resistor. The current wave was composed by a positive linear ramp with amplitude 10 $\mu$A, and slope $v_I = 133$ $\mu$A/s followed by a 100 ms zero-current plateau which turned out to be essential for the weak-link to cool down between two consecutive transitions to the normal state. In particular, the mode of the distributions decreases by raising the temperature, as a consequence of the reduction of $I_C$. The width and the shape of the distributions follow the conventional behaviour\[36, 39\] as a function of the $T$, quantitatively described by the evolution of the standard deviation $\sigma$. Firstly, the quantum phase slips (QPS) regime occurs when the transition from one minimum of the tilted washboard potential to the next one is due to quantum tunneling. Since the tunneling process does not require an activation energy, the standard deviation of the SCPDs in this regime is expected to be temperature independent. The temperature range where tunneling is the main source of phase slips defines the so-called crossover temperature $T_Q$, above which ($T > T_Q$) the thermal energy of the system allows the phase particle to hop over the potential barrier. In such thermally-activated phase slip (TAPS) regime, the thermal energy supplied to the system grows with the temperature, resulting into a widening of $\sigma$ as a function of $T$. Finally, when thermal energy is large enough to allow more than one phase slip event to occur simultaneously ($T > T_T$), the system falls in the so-called thermally-activated multiple phase slips (MPS) regime, and the standard deviation is known to decrease as a
The plot of $\sigma$ vs $T$ for the same Josephson nanotransistor, shown in Fig. 2d, demonstrates that our Ti Dayem bridge follows the RCSJ model and the conventional phase slip theory [36, 40–42]. Indeed, from 20 mK up to 110 mK it shows an almost constant value of $\sigma$ of $\sim 15$ nA (QPS regime), from 110 mK to $\sim 150$ mK the standard deviation is proportional to the temperature (TAPS regime), and for $T \gtrsim 150$ mK $\sigma$ decreases down to $\sim 10$ nA at 300 mK (MPS regime). We wish to stress that the independence of $\sigma$ in the QPS regime (i.e., $T \lesssim 110$ mK) cannot be ascribed to a saturation of the electronic temperature in the Josephson junction since the critical current turns out to increase in this range by decreasing $T$ (see Fig. 1c). Both in the QPS and TAPS regimes it is possible to fit the SCPDs curves through the KFD transform [36, 38, 43] in order to extract the characteristic parameters of our system. Dotted lines in Fig. 2b represent the inverse KFD transform fits of our data which show good agreement with the theory. Fit parameters, and a more detailed description about the fitting procedure are provided in the SI.

Figure 2c shows the JJ escape rate $\Gamma(I, T)$ computed through the direct KFD transform [38, 43]

$$\Gamma(I_n, T) = \frac{P(I_n, T)\nu_I}{1 - w \sum_{k=0}^w P(I_k, T)},$$

where $w$ is the bin size of the $P(I, T)$ histograms, and $P(I_k, T)$ is the switching probability in the current interval $[kw, (k + 1)w]$ with $k \in \mathbb{N}$. $\Gamma(I_n, T)$ provides a measure of the phase lifetime of our Dayem bridges, which spans between 1 $\mu$s ($\Gamma \sim 10^6$ Hz) and 10 ms ($\Gamma \sim 10^2$ Hz). The above escape rate range is in agreement with conventional switching current experiments performed so far [11, 36].

Let us now focus on the characterization of the impact of the electrostatic field on the phase slips dynamics. To verify the customary [1, 3, 4, 6, 7] dependence of $I_C$ on the electric field, a detailed measurement of $I_C(V_G)$ as a function of bath temperature was preliminary performed on our Dayem bridge (see Fig. 3d). The curves turn out to be symmetric function of $T$ [36].

![Figure 3](image-url)

**FIG. 3.** (a) Switching current probability distributions vs current $I$ at different gate voltage values from 12V to 30V. The curves are vertically offset for clarity. (b) Comparison of $\sigma$ vs $I_C$ obtained for SCPDs as a function of temperature and $V_G = 0$ (light red), and of gate voltage at 20 mK (light blue). (c) Mode-matched SCPDs, red and orange distributions were obtained for $V_G = 0$ at selected temperatures whereas yellow and green distributions were measured at $T = 20mK$ for different gate voltage values. (d) Dependence of the switching current $I_C$ on $V_G$ for different values of bath temperature from 20mK to 300mK. Data were obtained from the average computed over 25 acquisitions, and the error bars represent the standard deviation. The crossover voltage $V_Q \simeq 8V$ separates the QPS from the EAPS regime. (e) Standard deviation $\sigma$ of the SCPDs vs $V_G$. Crossover voltages $V_Q \simeq 8V$ and $V_E \simeq 21V$ separate QPS/EAPS and EAPS/MPS regimes, respectively.
forms the shape of the SCPDs. In particular, as shown in Fig. 4, the application of an electric field to the weak-link dramatically decreases the conductance at zero gate voltage (i.e., $V_G = 0$). The current $I_C$ flowing between the constriction and the gate electrode was also measured, and it was found to be at most $I_C^{\text{MAX}} \approx 15$ pA for $V_G = 34$ V, corresponding to a gate-bridge resistance $R_L \approx 2\,\Omega$. For a more comprehensive discussion on the effect of the gate current see the SI.

In order to assess the impact of the electric field on the SCPDs, we acquired the distributions at 20 mK for several different values of $V_G$. First of all, we emphasize that the application of an electric field to the weak-link dramatically deforms the shape of the SCPDs. In particular, as shown in Fig. 3a, for $V_G < 8$ V the SCPDs cannot be distinguished from the zero-gate one whereas a tail at low current values appears for $8 < V_G < 14$ V. Moreover, the SCPDs strongly widens for $14 < V_G < 24$ V. Finally, for $V_G > 24$ V, the SCPDs turn out to narrow. The above behaviour is quantitatively described by the standard deviation of the distributions displayed in Fig. 3e. Here, we note that in the $\sigma$ vs $V_C$ curve it can be still identified a region of constant standard deviation, thereby indicating a negligible contribution of the electric field to the phase slips for low $V_G$ values. This behavior turns out to be equivalent to the QPS regime. Notably, such a regime occurs in the voltage range (i.e., $|V_G| < V_G$, see Figs. 3 d and e) where not even $I_C$ is affected by the electrostatic field. For $|V_G| > V_G$, $I_C$ starts to monotonically decrease, phase slips are activated by the application of the electrostatic field, and $\sigma$ grows with $V_G$ obtaining its maximum value of $\approx 200$ nA. We define this region as the “electrically-activated” phase slip (EAPS) regime. This evidence suggests that, whatever the microscopic origin of $I_C$ suppression, the latter is accompanied by a corresponding increase of phase slip events.

Finally, for higher values of the electric field (i.e., $|V_G| > V_E \approx 20$ V), $\sigma$ decreases and saturates to $\approx 75$ nA, a value which is around 7.5 times larger than the corresponding one in the high temperature case. Therefore, this behaviour, yet resembling the thermally-activated MPS regime, cannot be ascribed to a conventional thermal trigger of phase slips. To emphasize this point, we compare the evolution of $\sigma(V_G, T = 20$ mK) and $\sigma(V_G = 0, T)$ by plotting them vs their corresponding critical current values (see Fig. 3b). We speculate that the electric field effect is responsible for a deep modification of the weak-link phase dynamics by enhancing the switching probability (i.e., fluctuations) in a wider current bias range. In addition, although both $\sigma$ vs $T$ and $\sigma$ vs $V_C$ curves present a similar behaviour, and three qualitatively similar regimes are recognisable in either curves, the average value of $\sigma(V_G)$ is around one order of magnitude larger than that of $\sigma(T)$.

To allow a further comparison between thermal and electric field distributions, we plot selected SCPDs corresponding to roughly the same $I_C$. Figure 3c shows such mode-matched distributions for $I_C = 2.2$, 2.8, 4.0 $\mu$A. The $I_C$-matched distributions show markedly different shapes and widths, a behavior which might stem from electrostatically-driven strong nonequilibrium induced in the superconducting Dayem bridge. Yet, this provides an additional confirmation that electrostatic field effect cannot be explained by a trivial local quasiparticle overheating of the superconductor. Indeed, assuming that the widening of the distributions were due to a thermal effect, the required effective quasiparticle temperature would be so high to be incompatible with the existence of superconductivity [44].

This observation reflects into a mean-
ingless attempt to fit the electrically-activated SCPDs with a conventional KFD transform since the necessary parameters would be totally outside the range of validity.

It is finally noteworthy to examine the response of the Josephson bridge under the simultaneous action of an electric field and thermal excitations. Figure 4a,b,c show the SCPDs as a function of temperature in the range between 20 mK and 300 mK for $V_G = 5, 15, 25$ V, respectively. At high temperature, the distributions seem to recover the thermal behaviour for each value of the applied electric field. Such an effect demonstrates a weakening of the electric field impact on phase slips at high temperature, which is consistent with what already observed in previous experiments [1, 3, 4, 6], and in our preliminary electric field characterization of the critical current (see Fig. 3d). Nonetheless, the evolution of $\sigma$ vs $T$ drastically changes when the electric field is applied. Figure 4d,e,f show the evolution of $\sigma$ as a function of temperature for $V_G = 5$, $15$, $25$ V. At low values of the gate voltage (i.e., $V_G = 5$ V) we can identify QPS, TAPS, and MPS regimes [45]. By contrast, for $V_G \geq 15$ V, the electric field seems to drive permanently the JJ into a MPS regime for every temperature value. In such a configuration, QPSs and TAPSs regimes cannot be observed anymore.

In conclusion, we have shown the occurrence of different phase slips regimes in a Ti Dayem bridge Josephson weak-link at several temperatures down to 20 mK. Firstly, the SCPDs of the system show the typical behaviour as a function of temperature. Secondly, the distribution shape is largely affected by an externally-applied electrostatic field. In particular, the standard deviation of the SCPDs far increases when the electric field is present. The drastic difference observed between the effect of the electric field and the temperature on SCPDs is a clear evidence of the non-thermal origin of the field effect-driven critical current suppression, and could be ascribed to a strong nonequilibrium condition set in the weak-link. Finally, the investigation of electric field-dependent SCPDs as a function of temperature shows how the Josephson weak-link is driven into a permanent MPSs regime for sufficiently large values of the gate voltage. On this background, our results might open the avenue to an electric field-driven manipulation of phase slips in all-metallic superconducting nanostructures and Josephson weak-links.

**AUTHOR CONTRIBUTIONS**

C.P. fabricated the samples. C.P. and G.D.S. performed the experiment and analyzed the data with inputs from F.G. F.G. conceived the experiment. C.P. wrote the manuscript with input from G.D.S. and F.G. All of the authors discussed the results and their implications equally.

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**SUPPLEMENTARY INFORMATION**

A. Gate-Dayem bridge current

The current between the Dayem bridge constriction and the gate was measured by a standard two-probes technique with a low noise voltage generator and a room-temperature current pre-amplifier. Figure 5a shows the $I_G$ vs $V_G$ characteristic for $T = 20$ mK from -35 V to 35 V. The corresponding gate-Dayem bridge current $I_L$ (See Fig. 5b) as a function of $V_G$ displays, in agreement with the conventional theory for electron tunnel-injection at low biases [46], a linear behaviour for almost the entire explored range, with a maximum value of $I_L^{\text{MAX}} = 12.5$ pA at $V_G = 35$ V which is on par with previous similar experiments [1, 3, 4, 6, 7]. The green line in Fig. 5b represents the total power injected in the system. Its maximum $P_L = 400$ pW occurs for $|V_G| = 35$ V. We emphasize that such a power is unlikely to be directly injected into the weak link. Indeed, for typical gate voltages of our experiments, a single electron field-emitted into the vacuum, and ballistically reaching the Dayem bridge, would immediately quench superconductivity [32, 44]. This suggests that only a negligible fraction of the power generated by the gate-bridge current is effectively transferred to and dissipated.
into the weak-link.

B. Inverse KFD transform fit

The fit was performed with the inverse KFD transform [36, 38, 43]:

\[
P(I, T) = \frac{\Gamma(I, T)}{\nu_I} \exp \left[ -\frac{1}{\nu_I} \int_0^I \Gamma(I', T) dI' \right]
\]

where \( \nu_I = dI/dT \) is the ramp speed of the bias current, and \( \Gamma(I, T) \) is the phase slip rate which assumes the following expression for the TAPS and the QPS regime, respectively:

\[
\Gamma_{TAPS}(I, T) = \frac{L}{2\pi \xi(T) \tau_{GL}(T)} \times \sqrt{-\frac{aE_j(T)}{k_B T} \left( 1 - \frac{I}{I_c(T)} \right)^b} \exp \left[ -\frac{aE_j(T)}{k_B T} \left( 1 - \frac{I}{I_c(T)} \right)^b \right]
\]

\[
\Gamma_{QPS}(I, T_{QPS}) = \frac{L}{2\pi \xi(T) \tau_{GL}(T)} \times \sqrt{-\frac{aE_j(T)}{k_B T_{QPS}} \left( 1 - \frac{I}{I_c(T)} \right)^b} \exp \left[ -\frac{aE_j(T)}{k_B T_{QPS}} \left( 1 - \frac{I}{I_c(T)} \right)^b \right]
\]

where \( L \) is the geometric length of the weak link, \( \xi(T) \) is the Ginzburg-Landau (GL) coherence length, \( \tau_{GL}(T) \) is the GL time constant, \( E_j \) is the Josephson energy, and \( a \) and \( b \) are parameters accounting for the typology of the Josephson weak-link. Their values can be analytically derived for tunnel Josephson junctions \( a_{tunnel} = 4\sqrt{2}/3, \ b_{tunnel} = 3/2 \) [36] and for long metallic superconducting wires \( a_{LW} = \sqrt{5}/4 \) [10], but are not known for Dayem bridges, therefore they are left as free parameters of the fit. For all the fit of SCPD curves, \( a_{DB} \) and \( b_{DB} \) converged to the same values \( a_{DB} = 1.0 \pm 0.1 \) and \( b_{DB} = 1.40 \pm 0.01 \). Also, in the QPS regime \( (20 \text{ mK} \leq T \leq 90 \text{ mK}) \) we introduced the effective temperature \( T_{QPS} \) as a fitting parameter.

| \( T \) (K) | \( T_{QPS} \) (mK) |
|----------|-----------------|
| 0.02     | 99 ± 4          |
| 0.03     | 110 ± 5         |
| 0.04     | 114 ± 7         |
| 0.05     | 115 ± 6         |
| 0.06     | 119 ± 7         |
| 0.07     | 125 ± 9         |
| 0.08     | 135 ± 7         |
| 0.09     | 153 ± 8         |

TABLE I. \( T_{QPS} \) values yielded by the fitting procedures of the SCPDs with the inverse KFD transform.

The value for \( T_{QPS} \) yielded by the fitting procedure are shown in the Table 1. We note from the point of view of the Josephson coupling, our weak links are in-between a tunnel junction and a long metallic superconducting wire just like the value found for parameter \( b_{DB} \) is in-between the other types of junctions \( b_{LW} < b_{DB} < b_{SIS} \).

C. Switching current cumulative probability distributions

The switching current cumulative probability distribution (SCCPD), or S-curve, is defined as the integral of the switching current probability distribution (SCPD) [22]. In this work, the S-curves are obtained upon summation of the frequency counts of the SCPD histograms shown in the main text. The
FIG. 7. a) Dependence of the switching current $I_C$ on $V_G$ for different values of bath temperature from 25 mK to 250 mK. Data were obtained from the average computed over 25 acquisitions, and the error bars represent the standard deviation. b) Switching current probability distributions vs current $I$ at different gate voltage values from 0 V to 24 V. The curves are vertically offset for clarity. c) Standard deviation $\sigma$ of the SCPDs vs $V_G$. Crossover voltages $V_Q \approx 8$ V and $V_E \approx 12$ V separate QPS/EAPS and EAPS/MPS regimes, respectively. d) Mode-matched SCPDs, red and orange distributions were obtained for $V_G = 0$ V at selected temperatures whereas yellow and green distributions were measured at $T = 20$ mK for different gate voltage values.

SCCPDs describe the probability to find the system in the normal state for a given value of the current bias.

Figure A shows the evolution of the SCCPD as a function of the temperature $T$ ($V_G = 0$ V) and the gate voltage $V_G$ ($T = 20$ mK). In particular, the application of the electric field results in a sizable widening of the S-curves. A comparison between S-curves with similar $I_C$ values obtained in the thermal excitation case and in the electrostatic case allows to appreciate how broader are the SCCPDs in the latter case. This fact is a further evidence that field effect cannot be ascribed to a conventional "thermal-like" quasiparticle overheating in the weak-link region.

D. Characterization of a second Josephson Dayem bridge

In this section, we show the characterization of a Ti Dayem bridge weak-link similar to the one described in the main text. Qualitatively, the results obtained on this device resembles those presented in the body of the manuscript. The thermal investigation of the system confirmed the typical behaviour of this kind of weak links [3, 34–36]. The preliminary characterization of $I_S$ response to the electric field (see Fig. 7a) shows the nearly-symmetric suppression [1, 4–7] for both positive and negative gate voltage values, with with full quenching at $|V_G| \approx 24$ V. More interesting is the evolution of the SCPDs as a function of the gate voltage $V_G$ displayed in Fig. 7b. As already similarly shown in the main text, the electric field modifies dramatically the shape of the SCPDs following the behaviour described in the main body of the paper. Here, we note that the evolution of $\sigma$ as a function of $V_G$ (see Fig. 7c) clearly displays as well the three regimes of quantum phase slips, electrical activated phase slips, and multiple phase slips. To compare the thermal and electric field distributions, we plot selected SCPDs with almost mode-matched $I_C$s. Figure 7d shows such distributions for $I_C = 0.9$, 1.2, 1.9 $\mu$A. The $I_C$-matched SCPDs show drastically different shapes and widths.
