Majorana phases in neutrino-antineutrino oscillations

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Abstract. If the massive neutrinos are Majorana particles, neutrinoless double beta (0νββ) decay experiments are not enough to determine the Majorana phases. We carry out a systematic study of CP violation in neutrino-antineutrino oscillations. In these processes, CP-conserving parts involve six independent 0νββ-like mass terms ⟨m⟩αβ and CP-violating parts are associated with nine independent Jarlskog-like parameters Vijαβ (for α, β = e, µ, τ and i, j = 1, 2, 3). With the help of current neutrino oscillation data, we illustrate the salient features of six independent CP-violating asymmetries between να → νβ and να → νβ oscillations.

1. Introduction

The lepton flavor mixing is described by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U[1]. Given neutrinos the Majorana nature, the PMNS matrix can be parametrized in terms of three flavor mixing angles (θ12, θ13, θ23) and three CP-violating phases (δ, ρ, σ) as follows:

\[
U = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23}
\end{pmatrix} \begin{pmatrix}
e^{i\rho} & 0 & 0 \\
0 & e^{i\sigma} & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

where cij ≡ cos θij and sij ≡ sin θij (for ij = 12, 13, 23). So far all the three neutrino mixing angles have been measured to precisely in neutrino oscillation experiments [2]. A determination of the phase parameter δ will be one of the major goals in the next-generation long-baseline neutrino oscillation experiments. The most challenging task is to detect the Majorana phases ρ and σ, which can only emerge in the lepton flavor violation (LNV) processes. Ref. [3] shows that it is in principle possible to determine all the three phases from the CP-violating asymmetries Aαβ between να → νβ and νμ → ντ oscillations. Here we report our work in Ref. [4], where we give a systematic analysis of the effective parameters and CP-violating asymmetries in the neutrino-antineutrino oscillations.

In section 2 we briefly review some issues of three-flavor neutrino-antineutrino oscillations. Six independent 0νββ-like mass terms ⟨m⟩αβ and nine independent Jarlskog-like parameters Vijαβ (for α, β = e, µ, τ and i, j = 1, 2, 3) are introduced and their main features are discussed. In section 3 we study the sensitivities of six CP-violating asymmetries Aαβ to the three phases, neutrino mass ordering, and the ratio of the neutrino beam energy E to the baseline length L. Section 4 is devoted to a summary of this report.
2. Neutrino-antineutrino oscillations and effective parameters

The neutrino-antineutrino oscillation probabilities for $\nu_\alpha \to \bar{\nu}_\beta$ and $\bar{\nu}_\alpha \to \nu_\beta$ are given by

$$P(\nu_\alpha \to \bar{\nu}_\beta) = \frac{|K|}{E^2} \left[ |\langle m \rangle|_{\alpha\beta}^2 - 4 \sum_{i<j} m_i m_j C_{\alpha\beta}^{ij} \sin^2 \phi_{ji} + 2 \sum_{i<j} m_i m_j V_{\alpha\beta}^{ij} \sin 2\phi_{ji} \right],$$

$$P(\bar{\nu}_\alpha \to \nu_\beta) = \frac{\bar{K}}{E^2} \left[ |\langle m \rangle|_{\alpha\beta}^2 - 4 \sum_{i<j} m_i m_j C_{\alpha\beta}^{ij} \sin^2 \phi_{ji} - 2 \sum_{i<j} m_i m_j V_{\alpha\beta}^{ij} \sin 2\phi_{ji} \right],$$

in which $\phi_{ji} = \Delta m_{ji}^2 L/(4E)$ with $\Delta m_{ji}^2 = m_j^2 - m_i^2$ being the mass-squared difference and $E$ the neutrino (or antineutrino) beam energy, $L$ the baseline length. $K$ and $\bar{K}$ are the kinematical factors, which satisfy $|K| = |\bar{K}|$. $m_i/E$ stands for the helicity suppression in the transition between $\nu_i$ and $\bar{\nu}_i$. The effective mass term $|\langle m \rangle|_{\alpha\beta}$ is defined as $|\langle m \rangle|_{\alpha\beta} = \sum_i m_i U_{\alpha i} U_{\beta i}$. We call them $0\nu\beta\beta$-like mass terms since $|\langle m \rangle|_{ee}$ is the effective mass term of the $0\nu\beta\beta$ decay. $C_{\alpha\beta}^{ij} = \text{Re}(U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta j}^*)$ and $V_{\alpha\beta}^{ij} = \text{Im}(U_{\alpha i} U_{\beta j} U_{\alpha j}^* U_{\beta j}^*)$ describe the CP-conserving and CP-violating contributions of the PMNS mixing, where the Greek and Latin subscripts running over ($e, \mu, \tau$) and (1, 2, 3), respectively. The CP-violating parts are referred to as Jarlskog-like parameters since they are similar to the definition of the Jarlskog parameter $J = \text{Im}(U_{\alpha e} U_{\beta \mu}^* U_{\alpha \mu} U_{\beta e}^*)$.

A measurement of neutrino-antineutrino oscillations is far beyond nowadays experimental capability as $P(\bar{\nu}_\alpha \to \nu_\beta)$ highly suppressed by the factor $m_1^2/E^2$. The typical oscillation lengths by taking $E \sim O(10) \text{ keV}$ for example, (1) $L_{31}^{\text{osc}} \simeq 1 \cdot 10^{-2} \text{ km} \times 10^3 \text{ m}$, and (2) $L_{21}^{\text{osc}} \simeq E_1^{\text{osc}} \simeq 10^5 \text{ m} \times 330 \text{ m}$, corresponding to $\Delta m_{21}^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{32}^2 \simeq 2.4 \times 10^{-3} \text{ eV}^2$, respectively. To make the oscillation effect detectable, the sizes of the neutrino (or antineutrino) source and the detector should be much smaller than $L_{21}^{\text{osc}}$ and (or) $L_{31}^{\text{osc}}$.

The Jarlskog-like parameters $V_{\alpha\beta}^{ij}$ satisfy the relations $V_{\alpha\beta}^{ij} = V_{\alpha\beta}^{ji} = -V_{\beta\alpha}^{ij} = -V_{\beta\alpha}^{ji}$, and $V_{\alpha\beta}^{ij} = 0$; but $V_{\alpha\alpha}^{ij} \neq 0$ for $i \neq j$. With the sum rules $V_{\alpha\beta} = (V_{\gamma\gamma} - V_{\alpha\alpha} - V_{\beta\beta})/2 (\alpha \neq \beta \neq \gamma \neq \alpha)$, we have only nine independent $V_{\alpha\beta}^{ij}$. For a numerical illustration, we take $\theta_{12} \simeq 33.4^\circ$, $\theta_{13} \simeq 40.6^\circ$ and $\theta_{23} \simeq 40.6^\circ$ as inputs. In general, $V_{\alpha\alpha}^{12}$, $V_{\alpha\alpha}^{23}$, $V_{\alpha\alpha}^{\mu\mu}$, and $V_{\alpha\alpha}^{\tau\tau}$ can maximally reach about $20\%$ in magnitude; in comparison, the Jarlskog invariant $J \lesssim 9.6\%$, suppressed by $s_{13}$. One special case is the “pseudo-Dirac” case with $\rho = \sigma = 0$. This case is interesting because appreciable CP- and T-violating effects are expected to show up in neutrino-antineutrino oscillations even if the Majorana phases $\rho$ and $\sigma$ vanish.

The effective mass terms $|\langle m \rangle|_{\alpha\beta}$ are important to understand the origin of neutrino masses, since they are the $(\alpha, \beta)$ elements of the symmetric Majorana neutrino mass matrix $M_\nu$ in the charged lepton flavor basis. These parameters also play important roles in some other LNV processes, such as the doubly charged Higgs decay $H^{++} \to e^+_\alpha e^+_\beta$ in the type-II seesaw mechanism and some LNV decays of $K$, $D$ and $B$ mesons. A measurement of the three CP-violating phases is absolutely necessary in order to fully reconstruct the neutrino mass matrix $M_\nu$. Figure 1 illustrates the profiles of six $|\langle m \rangle|_{\alpha\beta}$, with inputs shown above. We allow $\delta$ to randomly vary in $(0, 360^\circ)$ and $\rho, \sigma$ vary in $(0, 180^\circ)$. For some values of the lightest neutrino mass, texture zero $|\langle m \rangle|_{\tau\tau} = 0$ is allowed, either in the normal hierarchy (e.g., $|\langle m \rangle|_{ee} = 0$) or in the inverted hierarchy (e.g., $|\langle m \rangle|_{\mu\mu} = 0$), or in both of them (e.g., $|\langle m \rangle|_{\mu\mu} = 0$).

3. CP violation in neutrino-antineutrino oscillations

To eliminate the $|K|^2/E^2$ and $|\bar{K}|^2/E^2$ factors, we define the CP-violating asymmetry between $\nu_\alpha \to \bar{\nu}_\beta$ and $\bar{\nu}_\alpha \to \nu_\beta$ oscillations

$$A_{\alpha\beta} = \frac{P(\nu_\alpha \to \bar{\nu}_\beta) - P(\bar{\nu}_\alpha \to \nu_\beta)}{P(\nu_\alpha \to \bar{\nu}_\beta) + P(\bar{\nu}_\alpha \to \nu_\beta)} = \frac{2 \sum_{i<j} m_i m_j V_{\alpha\beta}^{ij} \sin 2\phi_{ji}}{|\langle m \rangle|_{\alpha\beta}^2 - 4 \sum_{i<j} m_i m_j C_{\alpha\beta}^{ij} \sin^2 \phi_{ji}}. \quad (3)$$

Only six of the nine CP-violating asymmetries are independent and nontrivial because of
$A_{\alpha\beta} = A_{\beta\alpha}$. The $\nu_\alpha \rightarrow \bar{\nu}_\beta$ oscillation is actually a kind of “appearance” process and thus it can accommodate the CP-violating effects. Because of the fact $|\Delta m_{21}^2| \simeq |\Delta m_{32}^2| \simeq 32\Delta m_{31}^2$, there may exist two oscillating regions dominated respectively by $\Delta m_{21}^2$ and $\Delta m_{31}^2$. We classify our analysis into three cases: (1) the normal neutrino mass hierarchy with $m_1 = 0$, (2) the inverted neutrino mass hierarchy with $m_3 = 0$, and (3) the nearly degenerate mass hierarchy with $m_1 \simeq m_2 \simeq m_3$. By fixing the CP phases, we schematically show some results in Figs. (2), (3) and (4), where the inputs of mixing angles and mass-squared differences are the same as above. Note that some of $A_{\alpha\beta}$ are remarkably dependent on the CP phases. This has been considered in Ref. [4], and more detailed discussions can be found over there.

4. Summary
In principle, one may determine the Majorana phases of the PMNS matrix in neutrino-antineutrino oscillations. Such an experiment might be feasible in the very distant future, but a systematic study of CP violation in neutrino-antineutrino oscillations is still useful, so as to enrich the Majorana neutrino phenomenology. Six independent $0\nu\beta\beta$-like mass terms $\langle m \rangle_{\alpha\beta}$ and nine independent Jalskog-like parameters $V_{\alpha\beta}^\mu$ have been analyzed in detail, and allowed parameter spaces for $\langle m \rangle_{\alpha\beta}$ have been presented. We have also carried out an analysis of the sensitivities of six CP-violating asymmetries $A_{\alpha\beta}$ to the three phase parameters and the ratio $E/L$ in different mass hierarchies. Our analytical and numerical results provide a complete description of the distinct roles of Majorana CP-violating phases in neutrino-antineutrino oscillations.

Acknowledgement
The author is indebted to Z.Z. Xing for his collaboration. This work was supported in part by the National Natural Science Foundation of China under Grant No. 11135009.

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Figure 1. The profiles of $|\langle m \rangle_{\alpha\beta}|$ versus the lightest neutrino mass $m_1$ (normal hierarchy or NH: red region) or $m_3$ (inverted hierarchy or IH: green region).

Figure 2. The CP-violating asymmetries $A_{\alpha\beta}$ versus $L/E$ in the normal neutrino mass hierarchy with $m_1 = 0$, $\delta = 90^\circ$, $\sigma = 45^\circ$, and $\rho$ being arbitrary.

Figure 3. The CP-violating asymmetries $A_{\alpha\beta}$ versus $L/E$ in the inverted neutrino mass hierarchy with $m_3 = 0$: (a) $\delta = 0^\circ$ and $\rho - \sigma = 45^\circ$ (red dashed lines); (b) $\delta = 90^\circ$ and $\rho - \sigma = 0^\circ$ (blue solid lines). The absolute values of $\rho$ and $\sigma$ are set to be arbitrary.

Figure 4. The CP-violating asymmetries $A_{\alpha\beta}$ (blue solid lines) versus $L/E$ in the nearly degenerate neutrino mass hierarchy with $m_1 \simeq m_2 \simeq m_3$, $\rho = 0^\circ$, $\sigma = 45^\circ$ and $\delta = 90^\circ$, where the red dashed lines stand for the oscillations driven by $\Delta m_{31}^2$ and $\Delta m_{32}^2$ being averaged out.