Performance of Multiple Antennas Selection in SIMO Systems with BPSK/QPSK Modulations

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Abstract—This paper studies the performance of single-input multiple-output (SIMO) systems under receive antenna selection (RAS) and BPSK/QPSK modulations. At the receiver, a subset of branches are selected and combined using maximal-ratio combining (MRC) to maximize the instantaneous Signal to Noise Ratio (SNR). By assuming independent and identical distributed (i.i.d.) Rayleigh flat fading, a closed-form expression, with considerably high precision, is developed to approximate the average input-output mutual information, also termed as symmetric capacity, of the whole system. Later, this approximated expression is further utilized to investigate the efficient capacity and energy efficiency of the SIMO system under BPSK/QPSK modulations and RAS. Besides analytical derivations, simulations are provided to demonstrate the approximation precision, feasibility and validity of the derived results.

Index Terms—SIMO system, receive antenna selection, symmetric capacity, BPSK/QPSK

I. INTRODUCTION

Multi-antenna system can drastically improve the spectral efficiency by deploying multiple antennas at the transceivers and is treated as a significant technology in the current cellular networks. However, to promise communication, each antenna should be connected with an expensive radio-frequency (RF) chain, which results in high hardware cost, thus there is an urgent need to address this challenge. To this end, antenna selection (AS) technology [1] has gained significant attentions in recent years aiming for design of high-efficiency transmission schemes.

Antenna selection can alleviate the requirement on the RF transceivers by selecting a subset of antennas to transceive signals. To analytically measure the performance of multiple-input multiple-output (MIMO) systems with antenna selection (AS-MIMO), Molisch et al., defined the upper capacity bound of AS-MIMO in [2] which treated one MIMO system as several independent single-input multiple-output (SIMO) systems, and this defined upper bound serves as an important evaluation criterion for the performance of antenna selection technology. Furthermore, their work was later extended to massive AS-MIMO systems by Gao et al., in [3]. Through the works of Molisch and Gao, it is clear that the discussion on antenna selection in SIMO systems (AS-SIMO) are fundamental to the explorations of AS-MIMO systems.

In the past decades, many researches on the AS-SIMO systems were presented [4]–[7]. For example, Win et al., in [4] investigated the received Signal to Noise Ratio (SNR) of SIMO systems after receive antenna selection (RAS). Molisch et al., in [5] and [6] derived the analytical expressions of the channel capacity for the AS-SIMO system. Later, the basic model in [6] was further utilized to estimate the outage probability of the single-antenna-selection-aided AS-MIMO systems [7]. In contrast to the explicit analysis in [4]–[7], the works in [8] and [9] proposed asymptotically approximated results for the ergodic capacity after RAS in the limit of large-scale SIMO systems. Nevertheless, nearly all these aforementioned works assumed the input signals followed Gaussian distribution. In fact, an important scenario which is necessary to be investigated when moving towards a practical implementation is the case where the channel inputs are constrained by finite constellation size.

Motivated by this, Li et al., in [10] explored the input-output mutual information (MI) with discrete inputs, also referred as symmetric capacity, of AS-SIMO systems in correlated fading channels. Later, Conti et al., in [11] investigated the MI of M-QAM in adaptive modulation aided AS-SIMO systems. However, existing literature on the symmetric capacity of AS-SIMO systems, including [10] and [11], failed to figure out any closed-form solutions for the average input-output MI but directly calculated it based on the definition of MI introduced in [12], the computation of which is prohibitively complicated especially for large-scale systems and high-order modulation modes. Therefore, it is significant to formulate a simple approximated expression for the average MI of AS-SIMO systems in order to reduce the computational burden.

This paper detailedly analyzes the average input-output mutual information for SIMO channels with receive antenna selection and finite-alphabet inputs. For simplicity, assume that the modulation mode is BPSK/QPSK. A high-precision closed-form approximated expression of the ergodic symmetric capacity is formulated, which holds a compact form and requires little computation. To the best of our knowledge, this is the first time to propose a closed-form expression of the ergodic symmetric capacity for the AS-SIMO systems with finite-alphabet inputs. Then, the derived approximated formula is further used to explore the efficient capacity and energy efficiency of the AS-SIMO systems. Analytical derivations and numerical simulations both show that antenna selection is an effective tool, which is able to reduce the power consumption.
but keep a relatively high transmission rate.  

The remaining parts of this manuscript is structured as follows: Section II describes the system model. In Section III the approximated expression for the ergodic symmetric capacity is derived. The applications of the approximated results and corresponding analysis are shown in Section IV. Finally, Section V concludes the paper.  

Notations: Scalars, vectors and matrices are denoted by non-bold, bold lower case, and bold upper letters, respectively. C stands for the complex plain and R stands for the real plain. The Hermitian and transposition of matrix H are indicated with H† and HT, respectively. Besides, || · || denotes the Euclidean norm and E [·] is the expectation operator. Additionally, the mutual information between the random variable X and Y is represented by I (X,Y).  

II. SYSTEM MODEL  

Consider a SIMO system, where the transmitter is equipped with one antenna and the receiver is equipped with Nr antennas. The received signal vector at the receiver is given by
\[ y = hx + w, \]
where x is the transmitted signal constrained by finite constellation size, such as BPSK, and unit power; w ~ CN(0, I Nr) is the additive white Gaussian noise (AWGN). Suppose that the channel suffers from independent and identically distributed (i.i.d) Rayleigh flat fading, thus the magnitude of each element in the channel matrix h ∈ C Nr×1 follows i.i.d. Rayleigh distribution with the same probability density function (PDF) as follows:
\[ f(x) = \frac{2x}{\bar{\gamma}} e^{-\frac{x^2}{\bar{\gamma}}}, \]
where \( \bar{\gamma} \) is the average per-antenna or per-branch SNR at the receiver.

Now, suppose that L (L ≤ Nr) receive antennas corresponding to the strongest L branches are activated. Additionally, assume that the channel state information (CSI) is only available at the receiver and maximal-ratio combination (MRC) is applied. Let \( \mathbf{h}, \mathbf{\tilde{y}} \) and \( \mathbf{\tilde{w}} \) denote the channel matrix, the received signal and the additive noise after RAS, thus \( \mathbf{\tilde{y}} = \mathbf{x} + \mathbf{\tilde{w}} \) holds. Moreover, the result obtained from MRC can be written as
\[ \mathbf{\tilde{y}} = \frac{\mathbf{\tilde{h}}^\dagger}{||\mathbf{\tilde{h}}||} \mathbf{\tilde{y}} = ||\mathbf{\tilde{h}}|| x + \mathbf{\tilde{w}}, \]
in which \( \mathbf{\tilde{w}} = \frac{\mathbf{\tilde{h}}^*}{||\mathbf{\tilde{h}}||} \sim \mathcal{CN}(0, 1). \)

Ergodic Symmetric Capacity  

Since the input signals, modulated by BPSK or QPSK, follow non-Gaussian distribution, the Shannon formula \[ \log_2 (1 + \text{SNR}) \] cannot be directly utilized to measure the input-output mutual information of this AS-SIMO system. Furthermore, it is sufficient to consider BPSK for QPSK is the superposition of two orthogonal BPSK modulations. Consequently, we assume that the transmitted data stream are i.i.d. zero-mean binary symbols with equal probabilities, and the input-output MI in terms of the SNR \( \gamma \) under BPSK modulation over AWGN channels can be formulated as [12]
\[ I(\gamma) = 1 - \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \log_2 \left( 1 + e^{-2\sqrt{\gamma}u - 2\gamma} \right) du. \]  

Additionally, on the basis of [13], the equation \( I(x; \mathbf{\tilde{y}}) = I(x; \mathbf{\tilde{y}}) \) holds, for the MRC is a lossless operation i.e., \( \mathbf{\tilde{y}} \) can be generated from \( \mathbf{\tilde{y}} \). Define \( \Gamma = ||\mathbf{\tilde{h}}||^2 \) as the received SNR after AS, the ergodic symmetric capacity of BPSK in AS-SIMO system can be written as
\[ C = \mathbb{E} [I(x; \mathbf{\tilde{y}})] = \mathbb{E} \left[ I(||\mathbf{\tilde{h}}||^2) \right] = \int_{0}^{+\infty} I(\gamma) F_T(\gamma) d\gamma = 1 - \int_{0}^{+\infty} \int_{-\infty}^{+\infty} \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}} \log_2 \left( 1 + e^{-2\sqrt{\gamma}u - 2\gamma} \right) F_T(\gamma) du d\gamma, \]
where \( F_T(\cdot) \) denotes the PDF of \( \Gamma \).

III. APPROXIMATION FOR THE AVERAGE MI OF BPSK  

As can be seen from Equ. (5), it is necessary to formulate \( F_T(\cdot) \) in order to solve the ergodic symmetric capacity. After the RAS and MRC, the received SNR is the sum of a series of i.i.d. ordered exponential-distributed statistics, that is
\[ \Gamma = \sum_{i=1}^{L} \gamma(i), \]
where \( \{\gamma(i)\}_{i=1,2,\ldots,N_r} \) denotes the i.i.d. ordered statistics, i.e., \( \gamma(1) \geq \gamma(2) \geq \cdots \geq \gamma(N_r) > 0 \), with the PDF as follows:
\[ g(x) = \frac{1}{\bar{\gamma}} e^{-\frac{x}{\bar{\gamma}}}. \]

Win et al., in [13] proved that \( \{\gamma(i)\}_{i=1,2,\ldots,N_r} \) can be expressed as the linear combination of a series of independent identical exponential-distributed random variables \( \{\upsilon_i\}_{i=1,2,\ldots,N_r} \) with the PDF \( h(x) = e^{-x} \). According to the conclusion in [13], \( \gamma(i) \) and \( \upsilon_i \) satisfy the following expression
\[ \mathbf{\gamma} = \mathbf{\Delta} \mathbf{\upsilon} \]
where \( \mathbf{\gamma} = [\gamma(1), \gamma(2), \ldots, \gamma(N_r)]^T, \mathbf{\upsilon} = [\upsilon_1, \upsilon_2, \ldots, \upsilon_{N_r}]^T \) are both column vectors and the coefficient matrix reads
\[ \mathbf{\Delta} = \begin{bmatrix} \tilde{\gamma}/1 & \tilde{\gamma}/2 & \ldots & \tilde{\gamma}/N_r \\ \tilde{\gamma}/2 & \tilde{\gamma}/2 & \ldots & \tilde{\gamma}/N_r \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\gamma}/N_r & \tilde{\gamma}/N_r & \ldots & \tilde{\gamma}/N_r \end{bmatrix}. \]

Moreover, define \( \mathbf{\beta} = [1, 1, \ldots, 1, 0, \ldots, 0]^T \in \mathbb{R}^{N_r \times 1} \), then
\[ \Gamma = \mathbf{\beta}^T \mathbf{\gamma} = \mathbf{\beta}^T (\mathbf{\Delta} \mathbf{\upsilon}) = (\mathbf{\Delta}^T \mathbf{\beta})^T \mathbf{\upsilon} = \sum_{n=1}^{L} \hat{\upsilon}_n + \sum_{n=L+1}^{N_r} \frac{L}{n} \hat{\upsilon}_n. \]
It can be seen from Eq. (9) that \( \Gamma \) is the linear combination of \( \psi_i \). Furthermore, as explained earlier, the PDF of \( \psi_i \) is given by \( h(x) = e^{-x} \), thus the characteristic function of \( \psi_i \) is derived as

\[
\Psi(y) = \int_{0}^{+\infty} e^{-x}e^{-yx}dx = \frac{1}{1+y} \quad (10)
\]

As a result, the characteristic function for \( \Gamma \) can be formulated as the production of \( \Psi(ky) \) (\( k \) is a coefficient), which holds the following expression

\[
\Psi_{\Gamma}(y) = \left[\Psi(y\gamma)\right]^{L} \prod_{n=L+1}^{N_r} \Psi \left( y^{2} \frac{L}{n} \right)
= \left( \frac{1}{1+y} \right)^{L} \prod_{n=L+1}^{N_r} \left( \frac{1}{1+y^{2} \frac{L}{n}} \right). \quad (11)
\]

By Eq. (11), the PDF of \( \Gamma \) can be obtained, that is

\[
F_{\Gamma}(\gamma) = \int_{-\infty}^{+\infty} \Psi_{\Gamma}(jy)e^{jy\gamma}dy, \quad (12)
\]

However, due to the complicated expression of the MI shown in Eq. (8), the exact value for the ergodic symmetric capacity is still difficult to extract even though \( F_{\Gamma}(\gamma) \) can be calculated. Fortunately, there exists a closed-form approximated formula for the MI in Eq. (14), with a compact form, which is written as

\[
\mathcal{C} \approx \mathcal{MI} = 1 - \int_{-\infty}^{+\infty} e^{-\phi\gamma} F_{\Gamma}(\gamma) d\gamma = 1 - \Psi_{\Gamma}(\phi) \quad (13)
\]

Moreover, Eq. (13) can be rewritten as the canonical structure similar to that in [14]. On the basis of [16], the canonical expansion of

\[
t(x) = \sum_{n=1}^{N} \sum_{k=1}^{\mu_n} A_{n,k} \left( \frac{c_n}{c_n + x} \right)^{k}. \quad (15)
\]

The weighting coefficients of the canonical expansion are given by

\[
A_{n,k} = \frac{1}{c_n^{k} (\mu_n - k)!} t_{n}(\mu_n - k)(0), \quad n = 1, \ldots, N, \quad k = 1, \ldots, \mu_n \quad (16)
\]

where \( t_{n}(x) \) represents the \( k \)th derivation of \( t_{n}(x) = x^{\mu_n} t(x - c_n) \) evaluated at \( x = 0 \). With Eq. (15) and Eq. (16), the canonical structure of Eq. (14) can be written as

\[
\mathcal{C} \approx \mathcal{MI} = 1 - \sum_{k=1}^{L} \sum_{n=L+1}^{N_r} A_{1,k} \frac{1}{1+\phi\gamma} \quad (17)
\]

Under this circumstance, the coefficients \( \{c_n\} \) and \( \{\mu_n\} \) in \( A_{n,k} \) are given by

\[
\mu_n = \begin{cases}     L & n = 1 \\     1 & n = 2, \ldots, N_r - L + 1 \\     1/\phi & n = 2, \ldots, N_r - L + 1. \end{cases} \quad (18)
\]

To confirm the accuracy of the approximation, we plot the approximated ergodic capacity, evaluated by Eq. (18), as a function of \( \gamma \) for different antenna deployment in Fig. 2 and compare it against the simulated results obtained by Monte-Carlo experiments. As can be seen from this figure, the approximation tracks the numerical results accurately for all the antenna set-up, which indicates that it is precise enough to estimate the ergodic symmetric capacity with Eq. (18).
IV. APPLICATIONS OF THE RESULTS

In this section, we employ the closed-form approximated expression of the ergodic symmetric capacity, derived in Equ. (13) or Equ. (18), to address several applications. Notably, all the simulation and numerical results are based on BPSK.

A. Application 1: Approximating Performance Measures

In fast fading channels, the ergodic capacity is regarded as an appropriate performance metric. On the basis of Equ. (18), the approximated ergodic capacity for different $L$, $N_r$ and $\tilde{\gamma}$ can be efficiently calculated. Fig. 3 shows the achievable ergodic symmetric capacity as a function of $L$ for $N_r = 50$ and selected values of $\tilde{\gamma}$. It can be observed that the higher the $\tilde{\gamma}$, the more capacity gain the antenna selection systems can achieve. In addition, the ergodic capacity is limited by 1 bits/symbol due to the finite-alphabet inputs, which is totally different from that driven by Gaussian inputs. And this observation further emphasizes the significance of the discussion on the systems constrained by finite-alphabet inputs. Most importantly, we can see that the approximated results match well with the simulations for the curves and circles almost coincided with each other, which verifies the validity of the previous deduction. Another important observation is that a large fraction of the maximal symmetric capacity achieved by full-complexity selection can be reached with only a small portion of the total antennas. For example, with $\tilde{\gamma} = -10$ dB, the achievable symmetric capacity for $L = 50$ is about 0.95 bits/symbol, while the achievable capacity for $L = 10$ is 0.8 bits/symbol, nearly 90% of that for $L = 50$. This interesting result suggests that most of the capacity can be achieved by a small subset of the total antennas, indicating the high efficiency of the AS technology. Moreover, the aforementioned phenomenon seems to be even more pronounced when $\tilde{\gamma}$ is higher, which means that antenna selection will be more efficient for the wireless multi-path channels with better quality.

B. Application 2: Efficient Symmetric Capacity

At the receiver, it is necessary to acquire the full CSI in order to select the optimal antenna subset. A time division duplex (TDD) AS-SIMO system is considered, in which $L$ ($L \leq N_r$) RF transceivers are recycled to estimate the uplink channels by receiving the training pilots from the transmitter. Let $t_{tr}$ and $t_{coh}$ denote the training duration and the coherence period, then the time used for communication is $(t_{coh} - \frac{N_r}{L} t_{tr})$. Similar to the spectral efficiency defined in [3], the efficient symmetric capacity, considering the CSI acquisition, can be defined as

$$SC = \frac{t_{coh} - \frac{N_r}{L} t_{tr}}{t_{coh}} C \approx \left( 1 - \frac{N_r \eta}{L} \right) \mathcal{M} \mathcal{I}$$

(20)

where $\eta = \frac{t_{tr}}{t_{coh}}$. From the definition of $\eta$, it is clear that smaller $\eta$ indicates higher efficiency of channel estimation. Besides, as $N_r$ grows, $(1 - \eta \frac{N_r}{L})$ decreases, but $\mathcal{M} \mathcal{I}$ increases, which implies that there is probably an optimal value, $N_{r, opt}$, to reach the largest $SC$. 

![Fig. 2. Simulated and approximated ergodic capacity versus $\gamma$.](image)

![Fig. 3. Simulated and approximated ergodic capacity versus $L$, $N_r = 50$.](image)

![Fig. 4. Simulated and approximated efficient ergodic capacity versus $(N_r/L)$ when $L = 4$ and $\tilde{\gamma} = -10$ dB.](image)
where $\zeta < 1$ is the efficiency of the power amplifier, and $P_{\text{cir}}$ is given by

$$P_{\text{cir}} = 2P_{\text{syn}} + N_rP_{\text{CT}} + N_rP_{\text{CR}},$$

where $P_{\text{syn}}$ is the frequency synthesizer’s power, $P_{\text{CT}}$ and $P_{\text{CR}}$ denote the circuit power consumption per RF chain at the transmitter and receiver, respectively. In the AS-SIMO systems, the energy efficiency is approximated as

$$\varepsilon \approx \frac{MI \times R}{\zeta^{-1}P_T + 2P_{\text{syn}} + P_{\text{CT}} + LP_{\text{CR}}}. \quad (23)$$

Fig. 5 plots the simulated and approximated energy efficiency in terms of $L$ for $R = 1$ symbol/s and selected values of $\gamma$. During the simulation, $P_T = 1$ W, $P_{\text{CT}} = 48.2$ mW, $P_{\text{syn}} = 50$ mW, and $P_{\text{CR}} = 62.5$ mW and $\zeta = 0.35$ which is typical for class-A RF power amplifiers [17], [18]. As can be seen from this figure, the simulated results meet accurately with the approximated results. Besides, there exists an optimal $L^*$, much smaller than $N_r$, which suggests that antenna selection can improve the energy efficiency. Furthermore, $L^*$ can be solved by numerical search in Eq. (23).

V. CONCLUSION

This paper studies the performance of multiple antennas selection in SIMO systems with BPSK/QPSK modulations. A closed-form approximated expression of the average input-output mutual information, with considerably high precision, is derived, which is further utilized to discuss the efficient capacity and energy efficiency. The derivations meet accurately the results given via numerical simulations, which suggests that the result of this paper can be employed for accurately approximating performance measures on SIMO fading channels.

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