Numerical solution of NLO $Q^2$ evolution equations for spin-dependent structure functions

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ABSTRACT

Numerical solution of DGLAP $Q^2$ evolution equations is studied for polarized parton distributions by using a “brute-force” method. NLO contributions to splitting functions are recently calculated, and they are included in our analysis. Numerical results in polarized parton distributions and in the structure function $g_1$ are shown. In particular, we discuss how numerical accuracy depends on number of steps in the variable $x$ and in $Q^2$.

Spin-dependent structure functions are measured in polarized lepton-nucleon scattering. They depend on two kinematical variables, $x$ and $Q^2$. The $Q^2$ dependence is calculated within perturbative QCD, and it is described by integrodifferential equations so called DGLAP equations. Because the $Q^2$ evolution equations are often used in theoretical and experimental studies, it is useful to have a computer code for solving the equations numerically. Our studies are important, for example, in getting optimal parton distributions by analyzing $g_1$ experimental data, particularly in obtaining gluon polarization by studying scaling violation of $g_1$.

In general, the DGLAP equations are coupled integrodifferential equations:

\[ \frac{\partial}{\partial t} \Delta q_i (x, t) = \int_x^1 \frac{dy}{y} \left[ \sum_j \Delta P_{q_j q_j} \left( \frac{x}{y} \right) \Delta q_j (y, t) + \Delta P_{q g} \left( \frac{x}{y} \right) \Delta g (y, t) \right] \], \quad (1a)

\[ \frac{\partial}{\partial t} \Delta g (x, t) = \int_x^1 \frac{dy}{y} \left[ \sum_j \Delta P_{g_j q_j} \left( \frac{x}{y} \right) \Delta q_j (y, t) + \Delta P_{g g} \left( \frac{x}{y} \right) \Delta g (y, t) \right] \]. \quad (1b)

where the variable $t$ is defined by $t = -(2/\beta_0) \ln[\alpha_s(Q^2)/\alpha_s(Q_0^2)]$, $\Delta q_j (x, t)$ and $\Delta g(x, t)$ are polarized $j$-flavor quark and gluon distributions, and $\Delta P_{ij} (x)$ are splitting functions. Each term in Eqs. (1a) and (1b) describes the process that a parton $p_j$ with the nucleon’s momentum fraction $y$ splits into a parton $p_i$ with the momentum fraction $x$ and another parton. The splitting function determines the probability of such a splitting process.

Next-to-leading-order (NLO) splitting functions for spin-dependent parton distributions are evaluated recently [1]. Therefore, we can study numerical solution of the NLO spin-dependent $Q^2$ evolution equations [2]. A brute-force method was studied for solving spin-independent evolution equations and for those with parton-recombination effects [3]. The same method is applied to the spin-dependent case. We divide variables into small steps ($N_x$ steps in the Bjorken variable $\ln x$ and $N_t$
steps in the variable $t$) and calculate differentiation and integration in the evolution equations. They are simply defined by \( df(x)/dx = [f(x_{m+1}) - f(x_m)]/\Delta x_m \) and \( df(x) = \sum_{m=1}^{N_x} \Delta x_m f(x_m) \). In this way, the integrodifferential equations are solved step by step if initial parton distributions are provided. We also studied a Laguerre polynomial method [4] as an alternative one in the unpolarized case. Although the computing time is very short in the Laguerre method, convergence is not good for “valence-like” distributions at small $x$. However, it should be mentioned that the Laguerre is still an excellent method in handling singlet-quark and gluon distributions. We decided to apply the brute-force method to the polarized case first by regarding the accuracy, instead of the computing time, as important.

There are various parametrizations for polarized parton distributions. Because only available data are $g_1$ for the proton and deuteron, it is impossible at this stage to have accurate information on each parton polarization. We use one of popular distributions, the Gehrmann-Stirling set A [5], which are given at $Q^2 = 4 \text{ GeV}^2$. We study evolution of nonsinglet, singlet, and gluon distributions to $Q^2 = 200 \text{ GeV}^2$ with $N_f=4$ and $\Lambda_{\overline{MS}}=231 \text{ MeV}$. For example, results of the singlet-quark evolution are shown in Figs. 1 and 2. Numerical results depend on the step numbers, $N_t$ and $N_x$. In Fig. 1, $N_t$ is varied from 20 to 1000 steps with fixed $N_x=1000$. As it is obvious from the figure, the evolved distributions are almost the same. It means that merely $N_t=50$ steps are enough for getting accurate evolution. This conclusion is expected because the scaling violation is a small logarithmic effect. Next, $N_t$ is fixed at 200 steps and $N_x$ is varied from 100 to 4000 steps in Fig. 2. From this figure, we find that several hundred $x$ steps are necessary for obtaining good accuracy. Therefore, if we choose the parameters $N_t=200$ and $N_x=1000$, the evolution results are good enough. We also analyzed evolution of nonsinglet and gluon distributions. The obtained results show similar accuracy. It should be mentioned that our analysis are still in progress, so that presented numerical results should be considered preliminary at this stage.

![Figure 1: $N_t$ dependence in $xq_s$ evolution.](image1)

![Figure 2: $N_x$ dependence in $xq_s$ evolution.](image2)

Experimental information on the polarized distributions is, for example, given by the structure function $g_1$. In the renormalization scheme $\overline{\text{MS}}$, the $g_1$ should be calculated by the convolution of polarized parton distributions with coefficient
functions:

\[ g_1(x, Q^2) = \sum_i e_i^2 g_{1,i}^+(x, Q^2) , \]  

\[ g_{1,i}^+(x, Q^2) = \int_x^1 \frac{dy}{y} \Delta C^q_1 \left( \frac{x}{y}, \alpha_s \right) \Delta q_i^+(y, Q^2) + \int_x^1 \frac{dy}{y} \Delta C^g_1 \left( \frac{x}{y}, \alpha_s \right) \Delta g(y, Q^2) , \]  

where \( \Delta q_i^+ = \Delta q_i + \Delta \bar{q}_i. \)

We show results of the \( g_1 \) evolution in Fig. 3. The initial distributions are given again by the GS set A, and \( g_1 \) at \( Q^2 = 4 \text{ GeV}^2 \) is calculated by Eqs. (2a) and (2b). The \( g_1 \) at \( Q^2 = 200 \text{ GeV}^2 \) is calculated first by evolving the initial distributions, then by taking into account the coefficient functions by Eqs. (2a) and (2b). We fix the parameters \( N_t = 200 \) and \( N_x = 1000 \) and show the evolved structure functions in the leading order (LO) and in the NLO. Note that the GS distributions, which are obtained in the NLO analysis, are also used in the LO input distributions at \( Q^2 = 4 \text{ GeV}^2 \). The NLO contributions are conspicuous at small \( x \). Because the gluon polarization should be extracted from the scaling violation at small \( x \), our NLO studies are important.

Our numerical analysis are still in progress, particularly in comparison with scaling violation data of \( g_1 \). However, the results indicate that accurate solution is obtained in the region \( 10^{-4} < x < 0.8 \) by taking more than two-hundred \( t \) steps and more than one-thousand \( x \) steps.

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