1 Introduction

There has recently been considerable attention focused on the study of supersymmetry (SUSY) breaking in the effective Lagrangian obtained from superstring compactification [1]. If one is to avoid the inclusion of an explicit mass scale associated with this breaking then it must proceed through nonperturbative effects [2]. Perhaps the most promising origin for such breaking is via gaugino condensate [3, 4, 5] in the hidden sector for it can easily lead to a large hierarchy between the Planck and gravitino mass. This is so because the scale of gaugino condensation is expected to correspond to the scale at which the gauge coupling constant
becomes strong. If the coupling at the compactification scale is small then, using the renormalization group equation, the scale at which the coupling becomes large is exponentially suppressed relative to the initial scale.

In order to study the SUSY breaking which results from gaugino condensation it is necessary to determine the effective low-energy theory below the scale $\Lambda_{\text{gut}}$ of string compactification. Most analyses start by introducing a “truncated superpotential” $W = \langle S\lambda\lambda \rangle$, where $S$ is the modulus superfield determining the gauge coupling, $g^2 = \frac{1}{16\pi}$, at the compactification scale, $\lambda$ the gaugino field and the brackets denote the vacuum expectation value (v.e.v.). The form of $\langle S\lambda\lambda \rangle$ has been determined by dimensional analysis, by instanton calculations \[7\] and by imposing an R-symmetry to the superpotential \[3, 5, 7\]. These all require $\langle S\lambda\lambda \rangle$ to have the form $c \exp(-3S/2b_0)$ where $c$ is proportional to $\Lambda_{\text{gut}}^3$ and $b_0$ is the coefficient of the $g^2/4\pi$ term in the beta-function associated with the hidden sector gauge group. Another approach to parameterize the gaugino condensate is via the effective Lagrangian \[8\]. In this approach the gaugino bilinear is assigned to a chiral superfield and its superpotential is obtained by demanding that it gives the correct terms to cancel the trace, axial and superconformal anomalies. The functional dependence of the gaugino condensate, $Y$, on the modulus fields $S$ and $T$ is given by minimizing the scalar potential.

Although these approaches give a general parameterization of the gaugino condensate they do not address the dynamical question why a condensate is energetically favoured. In practice this is an important consideration for the contribution to the vacuum energy from gaugino binding effects can play a significant role in determining the structure of the potential and the SUSY breaking effects in the visible sector. In order to study such effects it is necessary to evaluate the nonperturbative effects giving rise to gaugino condensation. The complete solution is clearly beyond our present-day technology so we are forced to employ approximation methods. Here we will apply Nambu-Jona-Lasinio (N-J-L) techniques \[9\] to obtain non-perturbative information about the gaugino binding.

We start by constructing the effective Lagrangian describing the Goldstone mode that results when a gaugino condensate forms due to the spontaneous symmetry breaking of R-symmetry. The form of the effective Lagrangian describing the Goldstone mode is strongly constrained by the requirement of N=1 local SUSY \[10\] and we show that this leads to a prediction of the dependence of the gaugino condensate on the modulus $S$ and $T$ consistent with the previous approaches mentioned above. However our approach shows that the resultant tree-level form with an effective four-fermion coupling should be radiatively corrected. In non-susy models it is known that, for sufficiently strong coupling, such a 4-Fermi interaction drives a fermion condensate dynamically and breaks the chiral symmetry spontaneously. By contrast, in the global SUSY version \[11\] the formation of bound states is not dynamically preferred, due to SUSY. In the local SUSY model obtained from 4-D superstring of interest here, a (non-perturbative) calculation of the radiative corrections via the Schwinger-Dyson equation shows...
that dynamical symmetry breaking is energetically favoured. The effective potential from the S and T fields is shown to be bounded from below in a simple Orbifold model once the constraints of duality invariance \[12, 13, 14\] are satisfied. (The inclusion of this symmetry is relevant for fixing the vacuum expectation value (v.e.v.) of the modulus T that parameterizes the compactified dimensions). By minimizing the full effective potential for the moduli and gaugino we show that a large mass hierarchy may develop, with a reasonable prediction for the compactification scale and the value of the fine structure constant.

The outline of the paper is as follows. In Section 2 we introduce the structure of the effective Lagrangian in the hidden sector of a four dimensional superstring theory and the consider the expected form of the gaugino condensate. By way of motivation we discuss in Section 3 the N-J-L analysis of fermion condensation in the non-supersymmetric case. In Section 4 we derive an effective Lagrangian which describes the Goldstone mode of broken R-symmetry and show that it leads to a parameterisation of the gaugino condensate equivalent, at tree level, to previous approaches. We conclude this section with a calculation of the 1 loop corrections. In Section 5 we discuss the N-J-L analysis in the locally supersymmetric case and show that, taking the constraints of duality into account, there is a stable minimum at which the scale of SUSY breaking is determined. Section 6 presents our conclusions.

2 The effective Lagrangian in D=4 superstrings

The effective D=4 superstring inspired model \[1\] is a N=1 supergravity (sugra) \[10\] with at least two gauge-singlet moduli S and T as well as an unspecified number of gauge chiral matter superfields. The N=1 sugra is specified by two functions, the Kahler potential, G, and the gauge kinetic function, f. In the simplest case a single (1,1) modulus superfield T determines the radius of the compactified space. The Kahler potential is given by (here and henceforth we set \(M_{Planck} = 1\))

\[
G = K - \ln\left(\frac{1}{4} |W|^2\right)
\]

where \(K = \ln(S + \bar{S}) + 3\ln(T + \bar{T} - 2 | \varphi_i |^2)\), \(\varphi_i\) are the (untwisted) chiral superfields and W is the superpotential that depends on the chiral superfields. The gauge coupling constant is given in terms of the real part of the gauge kinetic function, \(g^2 = Re f\), and at the compactification scale it is just \(g^2 = Re S\).

As we will discuss in Section 5, the form of the superpotential should be restricted by the modular invariance of the underlying string theory. In order to make contact with previous results we first discuss in Section 4 the non-modular invariant form (equivalent to dropping the contributions of the Kaluza Klein and winding modes) and subsequently determine the full modular invariant form in Section 5.
It is believed that a gaugino condensation will form at a scale where the gauge coupling constant becomes strong. This scale is determined by the renormalization group equation and at the one loop level it is

$$\Lambda_c^2 = \Lambda_{gut}^2 e^{-ReS/b_0}$$  \hspace{1cm} (2)

with

$$\Lambda_{gut}^2 = \frac{1}{<ReSReT>}$$  \hspace{1cm} (3)

the compactification scale. From dimensional analysis or instanton calculations we know that the gaugino condensate, if it occurs, should be proportional to $\Lambda_c^3$. Note that gaugino condensation, if it forms below the compactification scale, is largely a field theory phenomenon coming from the strong Yang-Mills gauge forces in the hidden sector far below the scale of string excitations. The difference from usual Yang-Mills theory lies in the fact that the gauge coupling is itself a dynamical variable ($g^{-2} \propto ReS$). The value of the vev for the modulus $S$ determines the value of the gauge coupling at the compactification scale and hence the magnitude of the hierarchy.

Here we wish to discuss whether non-perturbative effects favour a gaugino condensate and to determine the effective potential for the $S$ field. The approach we employ is first to construct the effective Lagrangian describing the light degrees of freedom below the scale at which the gauge coupling becomes strong. If the effect of this interaction is to generate a gaugino condensate, the R-symmetry under which the gauginos transform will be spontaneously broken. The effective Lagrangian below $\Lambda_c$ will then describe the Goldstone modes associated with this spontaneous symmetry breaking and the stability or otherwise of the system to such breaking can be discussed in terms of these fields. This approach closely parallels the procedure adopted in the N-J-L model and by way of motivation we first discuss how the analysis proceeds in the non-supersymmetric case.

3 The non-supersymmetric N-J-L model

The non-SUSY N-J-L model starts with a four-fermion interaction described by the Lagrangian given by \[9\]

$$L = i\bar{\psi}\gamma^\mu \partial_\mu \psi + \frac{1}{4}g^2((\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2)$$  \hspace{1cm} (4)

or in two component notation \[\text{1}\]

$$L = i(\bar{\psi}_L\sigma^\mu\partial_\mu\psi_L + \bar{\psi}_R\sigma^\mu\partial_\mu\psi_R) + g^2\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L.$$  \hspace{1cm} (5)

\[1\]We define $\bar{\psi}_R.L = \frac{1}{2}(1 \pm \gamma_5)\psi, \bar{\psi}_R = (\psi_R)^\dagger \gamma_0$
Here $g^2$ is a dimensional coupling, $g^2 = \hbar^2/\Lambda^2$, where $\hbar$ is a dimensionless and $\Lambda$ is the mass scale at which the new physics generating the four-fermion interaction appears. The theory has a $U(1)_L \otimes U(1)_R$ chiral symmetry of independent phase rotations of the left and right handed fermion components. Eq.(3) can be rewritten in terms of an auxiliary scalar field $\phi$,

$$L = i(\bar{\psi}_L \sigma^\mu \partial_\mu \psi_L + \bar{\psi}_R \sigma^\mu \partial_\mu \psi_R) - |\phi|^2 + g\phi^* \bar{\psi}_R \psi_L + \bar{g}\phi \bar{\psi}_L \psi_R$$

and by its classical equation of motion $\phi$ is identified with $g\bar{\psi}_R \psi_L$. Eliminating $\phi$ just gives eq.(3).

The tree level potential

$$V_0 = |\phi|^2$$

is semipositive definite and the minimum is at $\phi = 0$, i.e. no condensation state. The one-loop corrections are properly taken into account by the Coleman-Weinberg result [18]

$$V_1 = -\frac{1}{8\pi^2} \int d^2 p p^2 \ln(p^2 + m_F^2).$$

(8)

Integrating eq.(8) out using a momentum-space cutoff, because the interaction is non-renormalizable, one obtains

$$V_1 = -\frac{\Lambda^4}{16\pi^2} (x + x^2 \ln\left(\frac{x}{1 + x}\right) + \ln(1 + x)).$$

(9)

with

$$x = \frac{m_F^2}{\Lambda^2} = \frac{4g^2|\phi|^2}{\Lambda^2}.$$  

(10)

The scalar potential is then the sum of $V_0$ and $V_1$ and it is given by

$$V = \frac{\Lambda^4}{16\pi^2} \left( \frac{2}{\alpha} x - x - x^2 \ln\left(\frac{x}{1 + x}\right) - \ln(1 + x)) \right)$$

(11)

with

$$\alpha = \frac{g^2\Lambda^2}{8\pi^2}.$$  

(12)

From eq. (11) it is easy to see that the extremum condition is

$$\frac{\partial V}{\partial \phi^*} = \phi \alpha \left( \frac{1}{\alpha} - 1 - x \ln\left(\frac{x}{1 + x}\right) \right) = 0.$$  

(13)

Provided $V$ has a negative slope at the origin eq.(13) admits a non-trivial solution which is dynamically preferred. This is possible only for

$$\alpha = \frac{g^2\Lambda^2}{8\pi^2} > 1,$$

(14)
i.e. a strong coupling constant. In this case \( q^2 \Lambda^2 \)

\[ 1 = \frac{g^2 \Lambda^2}{8\pi^2} (1 + \frac{m^2_F}{\Lambda^2} \ln(\frac{m^2_F}{\Lambda^2} + 1)) \]  

(15)

which is the familiar mass gap equation that could have been derived from the interaction of eq.(14). Note that this solution is necessarily non-perturbative since it equates tree level and one loop contributions. It is straightforward to show that it amounts to a non-perturbative summation of fermion bubble graphs, which are dominant in the large \( N_c \) limit where \( N_c \) is the number of colours.

The solution corresponds to the case where \( \phi \) acquires a v.e.v. and the \( U(1)_L \otimes U(1)_R \) chiral symmetry of the Lagrangian eq.(4) is broken to \( U(1)_{L+R} \). In this case the associated Goldstone mode may be identified with the field \( \phi \) which, through quantum effects, acquires a kinetic term \( L_k \) and becomes a propagating field. It may be seen that \( L + L_k \) is the effective Lagrangian describing the light (Goldstone) degrees of freedom, appropriate below \( \Lambda \), together with the fermion field. The effective potential calculated using this Lagrangian just reproduces the results using the more familiar Schwinger-Dyson equation following from the original Lagrangian eq.(4). Thus the effective Lagrangian describing the would-be Goldstone mode provides a convenient way of implementing the N-J-L scheme for summing the leading terms in the large \( N_c \) limit.

4 An effective Lagrangian description of gaugino condensation

We turn now to the superstring inspired N=1 sugra model. As in the case of the N-J-L model we look for a formulation that parameterizes the gaugino bilinear by a classically non-propagating auxiliary field which at the quantum level becomes the Goldstone mode associated with the breaking of a continuous symmetry. In this case, however, the formation of the original theory in terms of an auxiliary field must be made consistent with local SUSY.

4.1 The broken R-symmetry Goldstone mode

For gauginos, in the absence of superpotential terms, there is an R symmetry which is spontaneously broken if a gaugino condensate forms leading to a Goldstone mode. In this case the auxiliary field \( \phi \) describing this would-be mode must be embedded in a chiral superfield \( \Phi \) which is coupled in a supersymmetric way.

For a superpotential \( W \) and gauge kinetic function \( f \) depending on an auxiliary chiral superfield \( \Phi = (\phi, \chi, h) \), where \( \phi, \chi, h \) are the scalar, fermion and auxiliary field components, the terms in the N=1 sugra lagrangian involving these fields
are (suppressing all gauge indices) [11]:

\[
L_{\text{aux}} = \frac{1}{2} \frac{\partial W}{\partial \phi} h - \frac{e^{K/2}}{4} \frac{\partial f}{\partial \phi} \bar{\lambda}_R \lambda_L h + \left( \frac{1}{2} \tilde{\psi}_L \cdot \gamma W^i \chi_{Li} - \frac{1}{2} e^{K/6} W^{ij} \bar{\chi}_{Ri} \chi_{Lj} \right)
\]

\[
+ \frac{1}{4} e^{2K/3} \bar{\lambda}_R \lambda_L f^{ij} \bar{\chi}_{Ri} \chi_{Lj} - \frac{1}{4} e^{K/2} f^i \bar{\chi}_{Li} \psi_L \cdot \gamma \bar{\lambda}_R \lambda_L + \text{h.c.}
\]  

(16)

where \( \lambda_L \) represents the gaugino field\(^2\) (with kinetic term \( L_k = \frac{i}{2} \text{Re} f \bar{\lambda} \gamma^\mu \partial_\mu \lambda \)), \( \psi \) the gravitino field, \( \chi_i \) the fermion component of the chiral matter superfield \( \varphi_i \) and \( W_i \equiv \frac{\partial W}{\partial z_i} \) with \( z^i \) the scalar component of \( \varphi_i \). The classical equation of motion for \( \Phi \) yield

i) \( \delta L \over \delta h \)

\[
\frac{1}{2} \frac{\partial W}{\partial \phi} = \frac{e^{K/2}}{4} \frac{\partial f}{\partial \phi} \bar{\lambda}_R \lambda_L
\]

ii) \( \delta L \over \delta \chi \)

\[
\bar{\psi}_L \cdot \gamma \left( \frac{1}{2} \frac{\partial W}{\partial \phi} - \frac{e^{K/2}}{4} \frac{\partial f}{\partial \phi} \bar{\lambda}_R \lambda_L \right) + e^{K/6} \left( \frac{e^{K/2}}{4} \frac{\partial f^i}{\partial \phi} \bar{\lambda}_R \lambda_L - \frac{1}{2} \frac{\partial W^i}{\partial \phi} \bar{\chi}_{Ri} = 0
\]

and

iii) \( \delta L \over \delta \phi \)

\[
\frac{\partial V}{\partial \phi} = 0
\]  

(17)

with \( V = L - L_k \) and \( L_k \) the kinetic Lagrangian.

The effective theory describing the interaction of \( \Phi \) is specified once \( W \) and \( f \) are given. The effects of gauge boson-gaugino interaction will be to generate an effective four fermion vertex. The form factor for this vertex vanishes rapidly above \( \Lambda_c \) as the gauge coupling becomes weak and is essentially constant below \( \Lambda_c \) for the gauge sector is confined and all masses are of the order \( \Lambda_c \).

If we demand that the effective theory given in terms of the auxiliary field \( \Phi \) generates this 4-Fermi interaction then the form of the \( W \) and \( f \) are uniquely determined (up to a constant),

\[
W = m^2 \phi,
\]

\[
f = \xi \text{ln}(\phi/\mu) + S
\]  

(18)

(19)

where \( m \) and \( \mu \) are mass parameters and \( \xi \) is a dimensionless constant. From the classical equation of motion eqs.(17) the scalar component of the auxiliary field \( \Phi \) is given in terms of the gaugino bilinear by

\[
\phi = \frac{e^{K/2} \xi}{2m^2} \bar{\lambda}_R \lambda_L,
\]  

(20)

\(^2\)Again we define \( \lambda_{R,L} = \frac{1}{2} (1 \pm \gamma_5) \lambda, \bar{\lambda}_R \equiv (\lambda_R)\gamma_0 \)
while the fermion component vanishes at the classical level. The third equation of eqs. (17) is an extremum condition on the scalar potential and once the one-loop corrections are included it is just the mass gap equation. As we will now show, this choice of $W$ and $f$ leads to an effective four-fermion interaction of the desired form once the auxiliary field is eliminated by its classical equation of motion, eq. (20).

4.2 Tree level potential

For a pure gauge theory in the hidden sector the tree level scalar potential is \[ V_0 = -(3e^{-G} + h_S h^S (G^{-1})^S_S + h_T h^T (G^{-1})^T_T) \] (21)

where $h_S$ and $h_T$ are the F-terms of the chiral superfields $S$ and $T$. In general they are given by

\[ h_i = e^{-G/2} G_i + \frac{1}{4} f^i \bar{\lambda}_R \lambda_L - C^j_i \bar{\chi}_{Rj} \chi_{Lk} - \frac{1}{2} \bar{\chi}_{Ri} G^j g_{L} \] (22)

SUSY will be broken if either $h_S$ or $h_T$ develop a non-vanishing v.e.v.. From the structure of $V_0$, eq. (21) one can see that it is possible to have broken SUSY and zero cosmological constant ($\langle (G^{-1})^j_j \rangle$ are negative), unlike in global SUSY.

The fermion partner of the auxiliary field $h_i$ (with $i=S$ or $T$) with non-vanishing v.e.v. will give rise to the Goldstino field. It is easily recognized as the combination of fermions that couple to the gravitino \[ \bar{\psi}_L \cdot \gamma \eta_L \]

with

\[ \eta_L = \left( \frac{1}{8} f^i \bar{\lambda}_R \lambda_L - e^{-G/2} G^i \right) \chi_{Li} + \frac{i}{2} g G^a T^j_i \bar{\chi}_{Rj} \lambda_L. \]

In local SUSY the Goldstino will be eaten by the gravitino which will acquire a mass

\[ m^2_{3/2} = \frac{1}{4} e^{-K} |W|^2. \] (23)

We may now apply this formalism to the choice of $W$ and $f$ given in eq. (18) and eq. (19). In this case the F-terms for the $S$ and $T$ fields are

\[ h_S = e^{-G/2} G_S + \frac{1}{4} f_S \bar{\lambda}_L \lambda_R \]

\[ h_S = e^{-G/2} \left( \frac{1 + S_r / \xi}{S_r} \right), \] (24)

\[ h_T = e^{-G/2} \frac{3}{T_r} \] (25)
with $S_r = S + \bar{S}$ and $T_r = T + \bar{T}$.

The tree level scalar potential is then
\[ V_0 = m^2_{3/2} H \]  
with
\[ H = (1 + \frac{S_r}{\xi})^2 \]
and the gravitino mass given by
\[ m^2_{3/2} = \frac{m^4|\phi|^2}{4S_r T^2}. \]

In terms of the gaugino field the scalar potential is
\[ V_0 = \frac{\xi^2 H}{16(Ref)^2} |\lambda'_{R}\lambda'_{L}|^2 \]
where the factor of $(Ref)^2$ in the denominator in eq.(29) appears because we have rescaled the gaugino fields appearing in this equation to have canonical kinetic terms. Thus we have shown that a choice of $W$ and $f$ in eqs.(18) and (19) leads to a four-fermion interaction as desired.

The form we have derived depends on the parameters $\xi$, $m$, and $\mu$. Since (cf.eq.(21)) $m^2\phi$ is proportional to $\lambda'\lambda$ which on dimensional grounds we expect to have the v.e.v. $\propto \Lambda^3$ (if it develops one), we obtain using eq.(19)
\[ m^2\phi = m^2 \mu e^{-ReS/\xi} e^{Ref/\xi} \sim \Lambda^3. \]
It is then naturally to chose $\xi = 2b_0/3$ and $m = \mu = \Lambda_{gut}$ (different choices of $m, \mu$ are possible as long as the relation of eq.(30) is satisfied). As we will see this identification is consistent with previous parameterizations of the gaugino condensate and with the one-loop running of the gauge coupling constant.

### 4.3 Connection with parameterisations of the gaugino condensate

From eqs.(26-28) we see that a v.e.v. for $\phi$, corresponding to gaugino condensate, will break SUSY giving the gravitino a mass. It is worth remarking at this point that the choice $\xi = 2b_0/3$ means the form of the scalar potential given by eq.(26) is identical to that obtained by the “truncated” approach [5, 6, 7] in which the effects of the gaugino condensate are included “by hand” by including in the superpotential the term $W = \Lambda^3_{gut} exp(-3S/2b_0)$. The form of this superpotential was (uniquely) determined by the condition that it should transform under the R-symmetry in the appropriate way for a gaugino bilinear. Under the R-symmetry
\[ \lambda_L \rightarrow e^{-i\delta} \lambda_L \]

9
The $S$ transformation eq.(32) cancels the anomalous term ($\delta L = \frac{b_0}{3} \delta F \tilde{F}$, where $\tilde{F}$ is the dual tensor of $F$) coming from the gaugino bilinear. In the approach adopted here when a gaugino condensate forms $\phi$ will be the Goldstone mode associated with the spontaneous breaking of this R-symmetry. Since the construction leading to eq.(29) respects both the underlying R-symmetry and local SUSY it must duplicate these results obtained in the “truncated” approach which rely on the R-symmetry. Thus we may understand the origin of the highly constrained form of eqs.(18) and (19) leading to the potential of eq.(29) as following from consistency with the symmetries of the system.

In fact, it is easily seen that the effective theory described by eq.(18), (19) and $\xi = 2b_0/3$ transforms correctly under the R-symmetry and it is anomalous free. From the R-transformation of the gaugino eq.(31) we deduce that

$$ \phi \rightarrow e^{-i2\delta} \phi $$

(33)

and the gauge kinetic function transforms as

$$ f \rightarrow f - i2 \xi \delta $$

(34)

leaving the effective Lagrangian invariant.

It is then of no surprise that by a simple reparametrization of the auxiliary field $\phi$ we obtain the effective superpotential derived by imposing that the trace, axial and superconformal anomalies cancel at the one-loop approximation. In this approach an effective superpotential $P_{eff}$ is given by $[8, 13, 15, 16]$

$$ P_{eff} = \frac{1}{4} Y (S + \frac{2b_0}{3} \ln(Y/\mu^3)) $$

(35)

where the scalar component of $Y$ is identified with the gaugino condensate. Its functional dependence on $S$ is obtained by minimizing the scalar potential and it is proportional to $\Lambda^2$.

As we said before, we may simply cast a superpotential $P$ as in this form by defining

$$ P = -m^2 \phi + f_{\alpha\beta} w^\alpha w^\beta $$

(36)

with $f$ given in eq.(19) and $w$ the gauge covariant chiral superfield (which has $\lambda \lambda$ as its scalar component). Then by using the equation of motion for $\phi$ ($\phi = \xi w w/m^2$) and rescaling the auxiliary field $\phi$, one obtains

$$ P = \frac{m^2 e}{\xi} \phi'(S + \xi \ln(\phi'/\mu^3)), $$

(37)

which is proportional to the superpotential used in the “effective” superpotential approach, eq.(35). Clearly the effects of the gaugino bilinear on the gauge
coupling obtained from eq. (19) and (30) can be interpreted as the one-loop renormalization of the gauge coupling given by

\[ g_{c}^{-2}(\Lambda_{c}) = g_{gut}^{-2}(\Lambda_{gut}) + 2b_{0}ln(\Lambda_{c}/\Lambda_{gut}) \]  

(38)

and since the beta-function is related by supergravity to the axial, trace and superconformal anomalies [15], this enforces the “effective” superpotential eq. (35) to take precisely the same from as in eq. (37).

4.4 The full Effective potential

As we have seen, the effective Lagrangian expressed in terms of the would-be Goldstone boson correctly parameterises the form of the gaugino condensate derived by other methods. What this connection shows is that these analyses just give the “tree level” form of the effective potential describing the gaugino condensate and that (cf. Section 3) radiative corrections must be included. Indeed the purpose of developing this formalism was to allow us to study non-perturbative effects in the strong hidden sector coupling using the N-J-L method. We proceed by calculating the \( \phi \) dependence of the effective potential, \( V \). If, at the minimum of \( V \), \( \phi \) develops a vacuum expectation value it will signal that a gaugino condensate is dynamically preferred, corresponding to the breaking of supersymmetry. To the extent that expressing the theory in terms of the auxiliary field \( \phi \) is just a re-parameterisation of the theory our results will be exact.

As we have seen, eliminating \( \phi \) leads to a Lagrangian involving a four-fermion interaction. Such a four-fermion interaction may be expected to occur due to (non-perturbative) gauge interactions involving gauge boson and gaugino exchange with coefficient \( \propto \Lambda_{c}^{-1} \). Here we choose to parameterise the strong gauge interaction in terms of this four fermion interaction rather than the primary gauge and gaugino couplings. We are then able to perform the non-perturbative sum of these interactions corresponding to the sum of all fermion bubble graphs. In this way we can get, albeit incomplete, information about the dynamics of such non-perturbative effects. We find that they can have a dramatic effect on the structure of the effective potential allowing for a stable non-trivial minimum for \( \phi \) corresponding to a supersymmetry breaking solution with a large mass hierarchy. This demonstrates the importance of including the binding effects and, at the very least, should encourage efforts to perform a more complete summation of such effects.

The tree level potential for \( \phi \), given by eqs. (26-28),

\[ V_{0} = \frac{m^{4}|\phi|^{2}}{4S_{r}T_{p}^{2}} (1 + \frac{S_{r}}{\xi})^{2} \]  

(39)

is clearly semipositive definite, as in the N-J-L model, and the minimum is at \( \phi = 0 \) which implies that there is no gaugino condensate and no SUSY breaking.
at tree level. This is consistent with the observation of Casas et al.\[19\] that gaugino condensation as usually parameterised does not occur in models with a single hidden sector gauge group factor. However we have argued it is essential to go beyond tree level to include non-perturbative effects in the effective potential which may allow for a non-trivial minimum even in the simple case of a single hidden sector gauge group. This non-perturbative sum (equivalent to the NJL sum) is readily obtained simply by computing the one loop correction to $V$. If these destabilise the potential the resultant minimum will correspond to a cancellation of tree level and one-loop terms which, as noted above, is necessarily non-perturbative in character \[20\].

The one-loop radiative corrections may be calculated using the Coleman-Weinberg one-loop effective potential,

$$V_1 = \frac{1}{32\pi^2} \text{Str} \int d^2p \frac{p^2}{p^2 + M^2} \ln(p^2 + M^2)$$

(40)

where $M^2$ represents the square mass matrices and $\text{Str}$ the supertrace. $V_1$ can be integrated to give

$$V_1 = \frac{1}{64\pi^2} \Lambda^4 \text{Str} J(x)$$

(41)

with

$$J(x) = x + x^2 \ln\left(\frac{x}{1 + x}\right) + \ln(1 + x)$$

(42)

and

$$x = \frac{M^2}{\Lambda^2}. \quad (43)$$

Since the 4-Fermi interaction is non-renormalizable we regularize it by introducing a momentum space cutoff $\Lambda$ that should be identify with the condensation scale eq.(2). The supertrace $\text{str}$ of a function $Q(M^2)$ is defined by

$$\text{str} Q(x) = 3\text{tr}Q(M_A^2) + \text{tr}Q(M_S^2) - 2\text{tr}Q(M_F^2) + 2Q(4m_{3/2}^2) - 4Q(m_{3/2}^2)$$

(44)

where $M_A^2, S, F$ are the (mass)\(^2\) matrices for vectors, scalars and spin=1/2 fields. The $2Q(4m_{3/2}^2)$ term is the contribution of the spin=3/2 particle, the gravitino, and $-4Q(m_{3/2}^2)$ is due to the gauge condition $\psi_R \cdot \gamma = 0$.

The scalar masses for the S and T fields are

$$m_S^2 = 4m_{3/2}^2, \quad (45)$$

$$m_T^2 = 8m_{3/2}^2 H. \quad (46)$$

In calculating the scalar masses one has to take into account that the scalar kinetic terms are not (and can not be) in a canonical form. The fermion masses can be read off of the N=1 sugra Lagrangian. The relevant terms are

$$L_{FM} = \frac{1}{2} \bar{\chi}_{Ri} B^{ij} \chi_{Lj} + h.c.$$\[47\]

\[3\]Once duality symmetry is included the scalar potential is no longer semipositive definite at tree level but there is no solution for reasonable values of the gauge coupling constant
with
\[ B^{mn} = m_{3/2} \left( G_i^n G_j^m \right)^{-1/2} \left[ D^{ij} + \frac{1}{4\xi} \left( 4 f^{ij} - 4 G_k^{ij} (G^{-1})^k_l f^l - (Re f)^{-1} f^i f^j \right) \right] \] (48)
and
\[ D^{ij} = G^{ij} - G^i G^j - G_k^{ij} (G^{-1})^k G^l. \] (49)

Using \( W \) and \( f \) of eqs.(18) and (19) the fermion masses are then the eigenvalues of
\[ \left( B^{nm} \right) = m_{3/2} \left( \begin{array}{cc} \frac{2s_\xi}{\sqrt{3} / 2} & \sqrt{3} / 2 \\ \frac{s_\xi}{\sqrt{3} / 2} & 1 \end{array} \right). \] (50)

Finally the gaugino mass is given by
\[ m_g^2 = m_{3/2} \frac{\xi^2 H^2}{4(Re f)^2}. \] (51)

These are the supersymmetry breaking masses following from the gaugino condensate. In addition we should allow for a supersymmetric contribution to the mass of the hidden sector states generated by the strong hidden sector forces which (in analogy with QCD) may be expected to be confining. Of course we are unable to determine these masses and so we proceed by examining the various possibilities. The first possibility is that the gaugino condensate forms at a scale above confinement and there is a domain in which the states are correctly described by the gauge bosons and gauginos with the only mass coming from the gaugino condensate as calculated above (It is thought the equivalent situation may occur in QCD with chiral symmetry breaking occurring before confinement).

In this case we may now compute, using eqs.(39-51), the one loop potential. Alternatively confinement may occur at, or above, the condensate scale. In this case the radiative corrections should be computed using the confined spectrum of states. Lacking knowledge of this spectrum we may still try to estimate the result by using the average description of these states in terms of gluons and gluinos but allowing for the confinement effects by giving them a common (supersymmetric) mass. We will discuss both these cases in the next section.

5 Dynamical breaking of SUSY

We are now in a position to determine whether it is energetically favourable for a gaugino condensate to form. From eqs.(41-46) it is clear that in order to have a SUSY breaking solution to the gap equation
\[ \frac{\partial}{\partial \phi} (V_0 + V_1) = 0 \] (52)
the negative contribution from the fermion loops must dominate (note that the contribution to the one-loop scalar potential from each individual massive state
is a monotonic function of the mass (for a fixed cutoff) being zero only for vanishing mass). In this limit since the (supersymmetry breaking) gaugino mass in eq.(51) is proportional to $g^2$ a strong gauge coupling constant will be dynamically preferred, i.e. $g^{-2} = Rf << 1$. As it stands the one-loop potential will go to $-\infty$ for $Rf = 0$. This is an unphysical singularity which will be removed when non-perturbative effects are included for it corresponds to infinite coupling. As we discussed in Section 4 the value of $Rf$ is not a free parameter for it defines the initial four fermion interaction used to define the strong binding interaction in the N-J-L approach we have adopted to study gaugino condensation (cf. eq.(29).

Below the scale of gaugino condensation the effective four fermion interaction must have the form $\frac{c^2}{\Lambda^2} (\bar{\lambda}\lambda)^2$, $c = O(1)$, where the condensation scale $\Lambda_c$ is also the confinement scale. This will be true provided $[Rf(\phi)]^{-1}$ in eq.(24) reaches a maximum “frozen” value $[Rf]^{-1} = \frac{c}{\Lambda_c}$. As may be seen from eqs.(19),(20) and (30) the residual ambiguity in $Rf$ parameterised by $c$ corresponds to an ambiguity in determining $m^2\phi$ (and hence $\bar{\lambda}\lambda$) in terms of $\Lambda_c^2$, relatively unimportant when considering whether a condensate will form. We impose this physically motivated condition as a reasonable parametrization of the strong coupling effects which must eliminate the unphysical divergence associated with the vanishing of $Rf$ and which we are presently unable to calculate.

Unfortunately this still does not cure the problem of an unbounded potential for the potential still goes to $-\infty$ along the direction in which a gaugino condensate forms with $T$, the modulus setting the radius of compactification, in the limit $T \rightarrow 0$. Clearly this is physically unacceptable. As we will see this problem may be traced to an inadequate treatment of the effective potential in the region $T \rightarrow 0$ (the large radius limit) which can be corrected by demanding the potential to be invariant under the space duality symmetry. The origin of this problem is that in the $T \rightarrow 0$ limit corresponding to the large radius ($R$) limit Kaluza Klein modes with masses $\alpha 1/R^2$ cannot be neglected in writing the low-energy effective Lagrangian. Recent work has shown [12, 21, 22, 13, 14] that the effect of these modes may be included by constructing a superpotential invariant under space duality symmetry which is known to hold for compactified string models, and so to study the $T \rightarrow 0$ limit we turn now to the inclusion of these Kaluza-Klein effects.

5.1 Space-Duality symmetry and the effective potential

Space duality is a symmetry of the string Lagrangian and it is found to all orders in perturbation theory [22, 13, 14]. Incorporating this symmetry in the effective D=4 theory, one has to impose duality invariance on the Kahler potential eq.(11) and the gauge kinetic function $f$.

Under this symmetry the S field remains invariant while the T field transforms
as an element of the $SL(2,\mathbb{Z})$ group,

$$T \rightarrow \frac{\alpha T - i\beta}{i\gamma T + \delta}$$

(53)

with $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$ and $\alpha \delta - \beta \gamma = 1$. The superpotential must transform as a modular function of weight -3,

$$W(T) \rightarrow \frac{W(T)}{(i\gamma T + \delta)^3}.$$  

(54)

The general form for the $T$ dependence in $W(T)$ is given in terms of $\eta(T)$ the Dedekind eta-function ($\eta(T) = q^{1/24} \prod_n (1 - q^n)$, $q(T) = \exp(-2\pi T)$), with modular weight 1/2, and $P(T)$, a polynomial of the absolute invariant function $j(T)$.

The compactification scale should also be redefined so that it is modular invariant

$$\Lambda^2_{\text{gut}} \rightarrow \frac{1}{<\text{Re}S\text{Re}T>|\eta(T)|^4}$$

(55)

With these modifications the effective $N=1$ sugra model and the gauge kinetic function are modular invariant. We turn now to a consideration of how these effects change of the locally supersymmetric N-J-L model. In place of eqs. (18) and (26) the superpotential and the tree level scalar potential are now given by

$$W = m^2 \eta^{-6}(T)\phi,$$

(56)

$$V_0 = m_{3/2}^2 H$$

(57)

with

$$H = \frac{3T^2}{4\pi^2} |\hat{G}_2(T)|^2 + (1 + \frac{S_r}{\xi})^2 - 3.$$  

(58)

$\hat{G}_2(T)$ is the Eisenstein modular form with modular weight 2. Since $V_0$ is modular invariant we only need to consider $T \geq 1$.

The minimum of $V$ is now well defined. A gaugino condensate is energetically favoured, with $\phi$ acquiring a value to minimize $\text{Re}f$. As discussed above this corresponds to $\text{Re}f = \Lambda_c/cm_p$ and

$$|\phi|^2 = \mu^2 e^{-S_r/\xi} e^{\Lambda_c/cm_p}$$

(59)

for $\Lambda_c << cm_p$ we have

$$\phi = \frac{\Lambda_c^3}{\Lambda^2_{\text{gut}}}.$$  

(60)

Using this identification the scalar potential given by

$$V_{\text{tot}} = V_0 + V_1$$

is

$$V_{\text{tot}} = \Lambda^4_c \left( \frac{H\Lambda^2_c}{4S_r T^3 |\eta|^2} - \frac{n_g}{32\pi^2 J(m^2_{\Lambda_c})} \right) + V'_1$$

(61)

with $J$ given in eq. (42), $V'_1$ the one-loop potential for the $S$ and $T$ chiral superfields and $n_g$ the dimension of the hidden sector gauge group.
5.2 Determination of the Supersymmetry Breaking scale

We have now constructed the modular invariant potential which determine the values for the S and T fields and hence the hierarchy relating $m_{3/2}$ to $m_P$ (the Planck mass). We start with the first case discussed above in which confinement masses are neglected. The scalar potential eq.(61) can be minimized numerically. At the minimum the modulus $S$ is

$$ S_r \simeq \frac{8\pi}{c} \sqrt{\frac{1}{n_g}} $$

and the gravitino $(mass)^2$ is

$$ m_{3/2}^2 \simeq \frac{4\sqrt{2}}{c^3} (\frac{16}{3})^3 \frac{b_0^{9/2}}{S_r^{17/2}} e^{-3S_r/4b_0} $$

where $c$ is the proportionality constant in $Ref = \frac{\Lambda_c}{cm_p}$ (we will take it to be one (i.e. $c = 1$)). As seen in eq.(12) and (13) the value of the gravitino mass is highly dependent on the one-loop beta function $b_0$ and $n_g$. Nevertheless it is very interesting that for certain values of $b_0$ and $n_g$ one can obtain a phenomenologically acceptable solution with the existence of only one gaugino condensate. Previous attempts to obtain realistic values for the gravitino mass [24, 25, 19] needed at least two gaugino condensates with different gauge groups and either an intermediate scale and/or the presence of matter fields with non-vanishing v.e.v. As we have emphasised the difference occurs because these analyses did not include the radiative corrections to the effective potential corresponding to the strong gaugino binding effects.

We may demonstrate that a large hierarchy may occur with reasonable choices for the hidden sector multiplet content by means of a simple example. We choose the gauge group SU(5) in the hidden sector together with 6 hidden sector fermion fields in the fundamental representation ($b_0 = (3N - \frac{n_f}{2})/16\pi^2$ for an SU(N) group with $n_f$ fermion fields in the fundamental representation). In this case the values of the different fields, condensation scale and gravitino mass are

$$ ReS = S_r/2 = 2.65, $$

$$ ReT = T_r/2 = 8.25, $$

$$ \Lambda_c^2 = 1.99 \times 10^{-13}, $$

$$ m_{3/2}^2 = 1.24 \times 10^{-32}. $$

This gives a value of $m_{3/2} = 249 GeV$ for the gravitino mass and $\Lambda_c = 8.93 \times 10^{11} GeV$ for the condensation scale both of which are phenomenologically realistic. The corresponding value for the gauge coupling at the compactification scale is $g^2 = \frac{1}{2.65}$. 

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As discussed above confinement effects may cause the spectrum determining one loop effects to differ from that used above so we turn to a consideration of how these effects may alter our conclusions. Since we do not know the details of this spectrum we will assume that, as an average of these effects, the states will get a common (supersymmetric) mass $m_{\text{con}}$ which we expect to be proportional to the condensation scale ($m_{\text{con}} = a \Lambda_c$). We can now minimize the scalar potential including these confinement effects. At the minimum the value of the modulus $S$ and the gravitino mass are:

$$S_r \simeq 8\pi \sqrt{\frac{1}{\eta g} \frac{1}{\sqrt{1 - a \ln\left(\frac{1+a}{a}\right)}}}$$

$$m_{3/2}^2 = m_{3/2}^2 (1 - a \ln\left(\frac{1+a}{a}\right))^{3/2}$$  \hspace{1cm} (65)

with $m_{3/2}^2$ given in eq. (63). We see from eq. (65) that the effects of the confinement mass is to shift (increase) the v.e.v. of $S$. Clearly it is still possible to obtain phenomenologically interesting results for reasonable values of $a$ (i.e. $a \sim O(1)$).

In all the solutions so far obtained the value of the potential at the minimum is negative corresponding to a non-vanishing cosmological constant of order $\Lambda_c^4$, many orders of magnitude larger than present bounds. This problem is shared by all attempts to generate supersymmetry breaking and we have nothing new to add to the discussion. It is possible to cancel the cosmological constant by modifying the superpotential and fine tuning but we postpone a discussion of this and of the implications of the supersymmetry breaking discussed here for the moduli and scalar fields to a subsequent paper [26].

6 Conclusions and Summary

If gaugino condensate forms in a N=1 supergravity theory supersymmetry will be broken. The condition that the Goldstone mode, associated with the spontaneous breaking of the $R$ symmetry by the gaugino condensate, be coupled in an N=1 locally supersymmetric way leads to a highly restricted form of the effective Lagrangian describing this mode. We have determined this Lagrangian and demonstrated that it leads to a description of the effective potential describing the gaugino condensate equivalent to the “truncated” and “effective” superpotential approaches.

However, the description in terms of the Goldstone mode shows that the effective potential should be corrected by radiative effects which may present approach allows for the study of the gap equation and it was found that a strong effective coupling between the gauginos is dynamically preferred and a gaugino condensate energetically favoured. After imposing a cut-off on the effective coupling consistent with dimensional analysis we determine the fine structure constant at
the compactification scale as well as the magnitude of the hierarchy between the gravitino and the Planck masses. They are largely determined by the $\beta$-function and dimension of the hidden sector gauge group. Different phenomenologically interesting solutions are possible and we showed, as an example, how a choice based on a SU(5) group gave very reasonable values both for the gauge coupling and the mass hierarchy.

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