Optical Design of Astrograph for “Interplanetary Solar Stereoscopic Observatory”

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Abstract. In the project of the "Interplanetary Solar Stereoscopic Observatory" (ISSO) the astrograph with the high resolution is one of the basic tools. Optical requirements to the tool are discussed. Creation the astrograph as the three-mirror system is offered. Design variants of the scheme with all third-order aberrations correction are presented. Obscuration is considered in the obtained variants.

1. Introduction
The project of creation "Interplanetary Solar Stereoscopic Observatory" (ISSO) was proposed in 1993 [1, 2]. Estimations of technical potentials "a space stereoscope" with the base equal to $\sqrt{3} a$, where $a$ - astronomical unit of length, with installation of two identically equipped space apparatus in a vicinity of the triangular centers librations in system "the Sun - barycenter Earth + Moon" are presented in works [3,4], etc. The preliminary design of the project is represented in article [5].

The task of a telescope-astrograph in project ISSO is to provide forming image of fields of objects with permeation up to brightness $V = 22''$ at the attitude signal/noise $\leq 2$. Using this astrograph it is possible to perform measurements of position of elements of the image and position of maxima of intensity of a point source with accuracy of few milliseconds of an arch.

Different variants of construction completely mirror optical system in combination with a directing flat mirror are possible. At all possible variants the optical system should satisfy the main requirement: to provide diffraction quality of the image of point objects through all field of view. Requirements to parameters are:
- focal length 5000 mm and more, f-number 10;
- pixel size of CCD is $16\times16 \mu m$;
- diffraction quality of the image through the field $2\omega' \geq 45'$

2. Choosing the optical system
Let's consider the problem of choosing the optical system for an astrograph. First of all, diffraction quality of the image and zero distortion are necessary to perform the basic program of supervision in a telescope.
The problem of creating a telescope with zero distortion was already posed during designing of space astrometric system “Struve” [5]. As initial point catadioptric Schmidt objective with diameter of the primary mirror 0.4 meters, focal length of 2.5 meters and an angular field about 2 degrees was considered. Mirror systems were considered also: mirror Schmidt and a three-mirror objective (Korsh system) [6]. In a three-mirror telescope Smith's lens was placed near the image plane for field curvature and distortion correction.

As required optical characteristics of a telescope for “ISSO” differ essentially from optical characteristics of a telescope for the project "Struve", the experience of such system designing can be used only partially. The most essential difference is that in the project "Struve" telescopes had f-number 4 - 6, and in “ISSO” 10 - 20. At such small apertures using the mirror Schmidt telescope becomes inexpedient because of great increase in dimensions. In the family of known mirror systems only mirror Schmidt has no distortion. Thus, it appears the question of developing of a mirror telescope in which following aberrations are corrected: a spherical aberration, coma, astigmatism, image curvature and distortion. In two-mirror telescopes it is impossible to correct all these aberrations, so the research of three-mirror systems was carried out. In the three-mirror systems with small apertures it is expedient to use systems with the intermediate image after reflection from two mirrors [6-11]. Systems of such type are shown in figure 1(a), 1(b). The ray which parallel to the optical axis reflects of the primary mirror.

![Figure 1](image.png)

**Figure 1.** Variants of three-mirror systems: 1 - the main mirror, 2 - the second mirror, 3 - the third mirror, 4 - the additional flat mirror, 5 - the image plane.

Theoretically it is possible to correct all five aberrations in any system. However, most of obtained systems turn out structurally unfeasible, on the one hand, because of the big obscuration, on the other hand, light in general can not get on image plane. Even after correction only four aberrations: spherical aberration, coma, astigmatism and image curvature the number of structurally feasible systems is not large and these realizable systems exist not with all optical characteristics. If the condition of distortion correction is added the variety of feasible systems decreases greatly.

Obscuration in systems of the given type arises, as well as in usual two-mirror systems because of the second mirror obstructs an entrance bunch of beams. Besides a flat mirror causes the biggest obscuration, and it obstructs beams twice: first in a bunch of the beams reflected from the first mirror, and then in a bunch of the beams reflected from the third mirror. Therefore at optical system designing following requirements also should be fulfilled:
the third mirror should not get in a parallel beam path,
the image plane also should not get in a beam path,
the beams reflected from the third mirror, should not "run" into the second, the
beams reflected from the third mirror, should not "run" into the first.

If to all these requirements we add a condition of distortion correction the variety of systems
becomes even less.

3. System research
It is convenient to calculate and research systems using the theory of the third order aberrations.
Conditions of elimination of a spherical aberration, coma, astigmatism, field curvature and distortion
for three mirrors systems look as following:

\[
S_I = h_1(P_1 + B_1) + h_2(P_2 + B_2) + h_3(P_3 + B_3) = 0
\]

\[
S_{II} = y_2(P_2 + B_2) + y_3(P_3 + B_3) + \frac{1}{2} = 0
\]

\[
S_{III} = \frac{v_2^2}{h_2}(P_2 + B_2) + \frac{v_3^2}{h_3}(P_3 + B_3) + 2 \frac{y_2^2}{h_2} W_2 + 2 \frac{y_3^2}{h_3} W_3
\]

\[
+ \frac{1}{h_1} \left( \frac{\alpha_2}{n_2} - \alpha_1 \right) + \frac{1}{h_2} \left( \frac{\alpha_3}{n_3} - \frac{\alpha_2}{n_2} \right) + \frac{1}{h_3} \left( \frac{\alpha_4}{n_4} - \frac{\alpha_3}{n_3} \right) = 0
\]

\[
S_{IV} = \frac{\alpha_2 n_2 - \alpha_1 n_1}{h_1 n_1 n_2} + \frac{\alpha_3 n_3 - \alpha_2 n_2}{h_2 n_2 n_3} + \frac{\alpha_4 n_4 - \alpha_3 n_3}{h_3 n_3 n_4} = 0
\]

\[
S_V = \frac{\gamma_2^2}{h_2^2}(P_2 + B_2) + \frac{\gamma_3^2}{h_3^2}(P_3 + B_3) + 3 \frac{\gamma_2^2}{h_2^2} W_2 + 3 \frac{\gamma_3^2}{h_3^2} W_3 + 2 \frac{\gamma_2^2}{h_2^2} (\alpha_3 + \alpha_2) - 2 \frac{\gamma_3^2}{h_3^2} (\alpha_4 + \alpha_3) = 0
\]

Parameters Pi, Bi, Wi of the components are defined by following expressions:

\[
P_1 = -\frac{\alpha_2^3}{4},
\]

\[
B_1 = -\frac{h_1 \alpha_2^3}{4},
\]

\[
W_1 = \frac{\alpha_2^3}{2},
\]

\[
P_2 = \frac{1}{4} (\alpha_3 - \alpha_2)^2 (\alpha_3 + \alpha_2),
\]

\[
B_2 = \frac{h_2}{4} (\alpha_3 + \alpha_2)^3,
\]

\[
W_2 = \frac{1}{2} (\alpha_3 - \alpha_2)^3,
\]

\[
P_3 = -\frac{1}{4} (\alpha_4 - \alpha_3)^2 (\alpha_4 + \alpha_3),
\]

\[
B_3 = -\frac{h_3}{4} (\alpha_4 + \alpha_3)^3,
\]

\[
W_3 = \frac{1}{2} (\alpha_4 - \alpha_3)^3,
\]

Where, \( b_i \) - deformation coefficient of a surface of the mirror, related to eccentricity \( e \): \( b_i = -e_i^2 \);
\( \alpha \) - tangent of a null-ray angle with an optical axis (\( \alpha_1 = 0, \ldots, \alpha_4 = -1 \));
h - height of the first null-ray \((h_1=1)\);
y - height of the second null-ray \((y_1=0)\)

The shape of mirrors is used for elimination of a spherical aberration, coma and astigmatism. The parameter \(D_2\) defines a relative aperture of the primary mirror, to make it more, than 1:1 is inexpedient because of difficulties with manufacturing and the controlling. Parameters \(D_3\) and \(h_3\) could be varied to receive system with admissible obscuration, and also to correct the image curvature and distortion.

As the result of substitutions and transformations of system of the equations (1), we obtain:

Condition of spherical aberration correction \((S_1=0)\):
\[
\alpha_2^3\left(b_1 + 1\right) - h_2\left(\alpha_3 + \alpha_2\right)\left((\alpha_3 - \alpha_2)^2 + b_2\left(\alpha_3 + \alpha_2\right)^2\right) + h_3\left(1 + \alpha_3\right)\left((1 - \alpha_3)^2 + b_3\left(1 + \alpha_3\right)^2\right) = 0
\]

Condition of coma correction \((S_{II} = 0)\):
\[
\frac{(1 - h_2)\left(\alpha_3 + \alpha_2\right)}{4\alpha_2}\left((\alpha_3 - \alpha_2)^2 + b_2\left(\alpha_3 + \alpha_2\right)^2\right) - \frac{\alpha_2 - \left(\alpha_2^2 - \alpha_3^2 - \alpha_3\right)(1 - h_2)}{4\alpha_2\alpha_3\left(\alpha_3 + \alpha_2(1 - h_2)\right)}\left((1 + \alpha_3)^2 + b_3(1 + \alpha_3)^2\right) = 0
\]

Condition of astigmatism correction \((S_{III} = 0)\):
\[
\frac{(1 - h_2)^2}{4\alpha_2^2 h_2}\left((\alpha_3 + \alpha_2)^2 + b_2(\alpha_3 + \alpha_2)^2\right) + \frac{\alpha_2 - \left(\alpha_2^2 - \alpha_3^2 - \alpha_3\right)(1 - h_2)}{\alpha_2\alpha_3 h_2\left(\alpha_3 + \alpha_2(1 - h_2)\right)}\left((1 - \alpha_3)^2 + b_3(1 + \alpha_3)^2\right) + \frac{1 - h_2}{\alpha_2^2 h_2}\left(\alpha_2^2 - \alpha_3^2\right) = 0
\]

Condition of field curvature correction \((S_{IV} = 0)\):
\[
\alpha_2 - \frac{1}{h_2}(\alpha_3 + \alpha_2) + \frac{1}{h_3}(1 + \alpha_3) = 0
\]

Condition of distortion correction \((S_{V} = 0)\):
\[
\frac{(1 - h_2)^3}{4\alpha_2^3 h_2^2}\left((\alpha_3 + \alpha_2)^2 + b_2(\alpha_3 + \alpha_2)^2\right) - \frac{\alpha_2 - \left(\alpha_2^2 - \alpha_3^2 - \alpha_3\right)(1 - h_2)}{4\alpha_2^2\alpha_3^2(1 + \alpha_3)^2 h_2^2\left(\alpha_3 + \alpha_2(1 - h_2)\right)}\left((1 + \alpha_3)^2 + b_3(\alpha_4 + \alpha_3)^2\right) + \frac{2(1 - h_2)(\alpha_4 + \alpha_3)}{\alpha_2^2 h_2^2} + \frac{2(1 - h_2)(\alpha_4 + \alpha_3)}{\alpha_2^2 h_2^2} = 0
\]

Setting value \(\alpha_2\) which defines a relative aperture of the primary mirror both technological practicability and labour input of manufacturing of the main mirror, and also one of three parameters \(h_3, h_2\) or \(\alpha_3\) and solving the obtained system of equations by a numerical method using software, for example, Mathcad, it is possible to get different schemes variants. Constructive feasible systems which have admissible obscuration are selected of the obtained decisions.

Using the obtained formulas a number of systems with the eliminated five aberrations of the third order was calculated. These systems are given in table 1. Calculation was carried out for objectives with relative apertures 1:10 and obscuration value not more than 0,3. The maximum angular fields \(2\omega\) for systems in which the main mirror has relative apertures 1:1, 1:1.25, 1:1.5 are defined.
Table 1. System parameters for different primary f-number $k_1$ and different $\alpha_2$.

| $k_1$ | $\alpha_2$ | $\alpha_3$ | $h_1$ | $h_2$ | $S'_2$ | $d_1$  | $d_2$  | $2\omega$ | Notes |
|------|------------|------------|-------|-------|--------|--------|--------|-----------|-------|
| 1    | -10        | 3.800419   | -0.1  | 0.106882 | 0.0281 | -0.089312 | 0.054436 | 28'      | 1     |
| 1    | -10        | 3.331771   | -0.2  | 0.210628 | 0.063218 | -0.078937 | 0.123246 | 47'      | 2     |
| 1    | -10        | 2.920300   | -0.3  | 0.306910 | 0.105095 | -0.069309 | 0.207824 | 60'      | 2     |
| 1,25 | -8         | 3.254998   | -0.1  | 0.093868 | 0.028838 | -0.113266 | 0.059560 | 23'      | 1     |
| 1,25 | -8         | 2.913953   | -0.2  | 0.184479 | 0.063309 | -0.101940 | 0.131944 | 38'      | 2     |
| 1,25 | -8         | 2.611160   | -0.3  | 0.268942 | 0.102997 | -0.091382 | 0.217888 | 48'      | 2     |
| 1,25 | -8         | 2.475158   | -0.35 | 0.308151 | 0.12450  | -0.086481 | 0.265902 | 52'      | 2     |
| 1,5  | -6,67      | 2.88339    | -0.1  | 0.083215 | 0.02886  | -0.137449 | 0.063541 | 19'      | 1     |
| 1,5  | -6,67      | 2.62228    | -0.2  | 0.163337 | 0.062288 | -0.125437 | 0.138558 | 32'      | 2     |
| 1,5  | -6,67      | 2.38825    | -0.3  | 0.238349 | 0.0998   | -0.114190 | 0.225416 | 43'      | 3     |
| 1,5  | -6,67      | 2.28195    | -0.35 | 0.27345  | 0.119832 | -0.108928 | 0.273209 | 46'      | 3     |
| 1,5  | -6,67      | 2.24144    | -0.37 | 0.286997 | 0.128042 | -0.106897 | 0.293114 | 48'      | 2,3   |

Notes: 1 - the third mirror gets in a beam path, 2 - the variant can be realized, 3 - if we permit obscuration $\eta=0.4$ the angular field can be increased up to 1 deg.

The research showed that with identical obscuration the angular field turns out more in systems with high apertures of the primary and other mirrors.

Basing on the executed research a number of the systems was calculated. One of them has following characteristics: focal length $f' = 5000$ mm, entrance pupil diameter 500 mm and angular field $2\omega = 1$ deg, its optical layout is shown in figure 2.

![Figure 2. Three-mirror ortoscopic system.](image)

System dimensions do not exceed 1300 mm length. The system has diffraction image quality and zero distortion. Obscuration square ratio is about 10 %. The main mirror in a telescope is elliptic, close to parabolic, a secondary mirror is hyperbolic, and the third is elliptic.
4. Conclusion
Design and construction studies showed that using in the astrograph the three-mirror system and new optical and constructional materials mass and dimensional characteristics of the astrograph will not exceed given values.

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