Inclusion of $f(R)$ term in the action of Horava-Lifshitz quantum gravity with projectability but without detailed balance condition is investigated, where $R$ denotes the 3-spatial dimensional Ricci scalar. Conditions for the spin-0 graviton to be free of ghosts and instability are studied. The requirement that the theory reduce to general relativity in the IR makes the scalar mode unstable in the Minkowski background but stable in the de Sitter. It is remarkable that the dark sector, dark matter and dark energy, of the universe has a naturally geometric origin in such a setup. Bouncing universes can also be constructed. Scalar perturbations in the FRW backgrounds with non-zero curvature are presented.

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I. INTRODUCTION

There has been considerable interest recently on a theory of quantum gravity proposed by Horava [1], motivated by Lifshitz scalar theory [2]. A free scalar field in $(d + 1)$-dimensional flat spacetime is usually given by

$$S_\phi = \int dt d^d x \left( \dot{\phi}^2 + \phi \nabla^2 \phi \right),$$

(1.1)

which is Lorentz invariant,

$$t \rightarrow \xi^0(t, x), \ x^i \rightarrow \xi^i(t, x), \ (i, j = 1, 2, 3).$$

(1.2)

The corresponding propagator is given by $G_\phi(\omega, k) = 1/k^2$, indicating that the theory is not renormalizable. To improve the ultra-violet (UV) behavior, Lifshitz introduced high-order spatial derivatives, $\phi(-\nabla^2)^z \phi$, into the action, and found that the resulted theory becomes renormalizable for $z \geq d/3$. An immediate consequence of these terms is that the theory is no longer Lorentz invariant, and $t$ and $x$ scale differently,

$$t \rightarrow \ell^z t, \ x^i \rightarrow \ell x^i.$$  

(1.3)

Based on the above observations, Horava took the point of view that Lorentz symmetry should appear as an emergent symmetry at long distances, but can be fundamentally absent at short ones [4, 5]. To realize such a perspective, Horava started with the Arnowitt-Deser-Misner (ADM) form of the metric,

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} \left( dx^i + N^i dt \right) \left( dx^j + N^j dt \right),$$

(1.4)

and imposed the foliation-preserving diffeomorphisms,

$$t \rightarrow f(t), \ x^i \rightarrow \xi^i(t, x),$$

(1.5)

to be denoted by $\text{Diff}_F(M)$. At low energies, the theory is expected to flow to the IR fixed point $z = 1$, whereby the Lorentz invariance is “accidentally restored.”

The effective speed of light in this theory diverges in the UV regime, which could potentially resolve the horizon problem without invoking inflation [6]. The spatial curvature is enhanced by higher-order curvature terms $R^{ij}$, and this opens a new approach to the flatness problem and to a bouncing universe [7, 8, 10, 11]. In addition, in the super-horizon region scale-invariant curvature perturbations can be produced without inflation [12, 17]. The perturbations become adiabatic during slow-roll inflation driven by a single scalar field, and the comoving curvature perturbation is constant [17]. Due to all these remarkable features, the HL theory has attracted lot of attention lately, see, for example, Ref. [18] and references therein.

To formulate his theory, Horava started with two conditions – detailed balance and projectability [1]. The detailed balance condition restricts the form of a general potential in a $(d + 1)$-dimensional Lorentz action to a specific form that can be expressed in terms of a $d$-dimensional action of a relativistic theory with Euclidean signature, whereby the number of independent couplings is considerably limited. The projectability condition, on the other hand, restricts the lapse function $N$ to be space-independent, while the shift vector $N^i$ and the 3-dimensional metric $g_{ij}$ still depend on both time and space,

$$N = N(t), \ N^i = N^i(t, x), \ g_{ij} = g_{ij}(t, x).$$

(1.6)

Clearly, this condition is preserved by $\text{Diff}_F(M)$.

It should be noted that, due to the restricted $\text{Diff}_F(M)$, one more degree of freedom appears in the gravitational sector - a spin-0 graviton. In particular, in the projectable case this mode is unstable in the Minkowski background [8, 20, 21]. As shown below, this is a generic feature of the theory with projectability condition. However, this instability does not necessarily show up in physical environments [22].

In addition, it is also possible that the legitimate background in the HL theory is not Minkowski [1]. In particular, recent observations show that our universe is currently de Sitter-like [23]. Therefore, instead of the
Minkowski, one may take the de Sitter space as the background. As a matter of fact, it was shown recently that the de Sitter space is indeed stable in the framework of the HL theory with projectability condition 18, 19.

On the other hand, in the non-projectable case, this mode is also unstable 24. However, if one includes terms made of the spatial gradients of the lapse function,

$$a_i = \partial_i \ln N,$$  

the mode can be rendered stable 25. However, an immediate price to pay for this is the inclusion of more than 60 sixth-order derivative terms in the potential 26. When matter, such as a scalar field, is included, the number of such terms increases dramatically. In addition, strong coupling may still exist 27, unless the scales appearing in higher order terms are much lower than the Planck scale 25.

It should be noted that strong couplings also happen in the version with projectability condition 28. However, Mukohyama recently showed that, when nonlocal effects are taken into account, the spin-0 graviton decouples for spherically symmetric, static, vacuum spacetimes 29. Together with Wu, we showed that this is also the case in the cosmological setting 19. As a result, the relativistic continuation $\lambda \to 1^+$ exists in the IR, whereby the strong coupling problem is circumvented.

The most general form of the HL theory with projectability but without detailed balance condition was first developed in 20, 21 by Sotiriou, Visser and Weinfurtner (SVW), in which the highest order of spatial derivatives is assumed to be six, the minimal requirement for the theory to be power-counting renormalizable in (3+1)-dimensional spacetimes 1–3. However, in principle there is nothing to prevent one to construct actions with spatial derivatives higher than six. In addition, in condensed matter physics, the critical exponent $z$ is also not necessarily an integer. In this paper, we explore all these possibilities. Clearly, promoting all terms to high orders is out of control. Instead, we consider high order derivatives only coming from the Ricci scalar $R$, by simply replying the third-order polynomial of $R$ in the SVW setup by an arbitrary function, $f(R)$, while keep the rest the same. As shown below, this simple generalization results in very rich physical phenomena, and in particular can give rise to dark matter and dark energy. Since in such a setup the origin of them is purely geometric, it automatically explains why these objects are “dark”. Bouncing universes can be also easily constructed by properly choosing the form of $f(R)$.

It is interesting to note that in 30 it was advocated that the HL theory with projectability condition has a built-in dark matter component, due to the non-locality of the Hamiltonian constraint. In addition, $f(R)$ models have been investigated in the framework of the HL theory 31, 32, but in all these studies $R$ is different from the 3D Ricci scalar $R$. As a result, those $f(R)$ models are fundamentally different from the ones studied here. For example, in 34 $R$ was token as

$$R = \sum_{ij} (K_{ij} K^{ij} - \lambda K^2) + 2\mu \nabla_\beta (n^\beta n^\nu - n^\nu n^\beta) - \beta E_{ijkl} E^{ij},$$  

(1.8)

where $n^\nu (\nu = 0, 1, 2, 3)$ is a unit vector perpendicular to the hypersurfaces of $t = \text{Constant}$, $\lambda$ and $\mu$ are two coupling constants, $\beta$ is the “generalized” De Witt metric, and $E_{ijkl}$ is given in terms of a super-potential. For detail, we refer readers to 31, 32. Clearly, for any choice of $\lambda$, $\mu$ and $E_{ijkl}$, due to the presence of the first term on the right-hand side of Eq. (1.8), $R$ cannot reduce to the three-spatial dimensional Ricci scalar $R$. In addition, due to its presence, the corresponding theory usually involves high order time derivatives 33–35, a situation that was avoided in the first place by Horava, in order to circumvent the ghost problem 1.

The rest of the paper is organized as follows: In Section II, we present the Hamiltonian and momentum constraints, the dynamical equations, and the conservation laws of energy and momentum, after including the $f(R)$ term in the SVW setup. To separate the effect of this term from others introduced in 20, 21, in this section we only consider the $f(R)$ term. In Section III, we study the stability of the spin-0 graviton in the Minkowski background, and obtain the stability condition. We show explicitly that it is the requirement that the theory reduce to its relativistic limit at the IR leads to the instability of the spin-0 graviton. This is also true when all the other terms are included. In Section IV, we investigate its applications to cosmology, by first writing the corresponding Friedmann equations of the FRW universe with arbitrary curvature, and then their linear scalar perturbations. From these expressions, it can be easily shown that the de Sitter spacetime is stable, similar to the case without the $f(R)$ term 18. In this section, we also generalize our studies to include all the other terms introduced in 20, 21. In Section V, we present our main results and give some concluding remarks.

II. INCLUSION OF THE $f(R)$ TERM

To see the role that the term $f(R)$ may play, we shall first neglect all the other terms constructed in 20, 21, and simply consider the action,

$$S = \frac{\pi}{16} \int dt d^3 x N \sqrt{g}(\mathcal{L}_K + f(R)) + \frac{1}{4} \mathcal{L}_M, \quad (2.1)$$

where $g = \det g_{ij}$,

$$\mathcal{L}_K = K_{ij} K^{ij} - \lambda K^2,$$  

(2.2)

$\zeta^2 = 1/16\pi G$, $\lambda$ is a dynamical coupling constant, and $f(R)$ is an arbitrary function of $R$. However, to have the theory power-counting renormalizable, it is necessary to include terms that are equal or higher than $R^3$. The
extrinsic curvature $K_{ij}$ is defined as
\[ K_{ij} = \frac{1}{2N} (-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i), \quad (2.3) \]
where $N_i = g_{ij} N^j$, and the covariant derivatives refer to the three-metric $g_{ij}$.

Variation with respect to the lapse function $N(t)$ yields the global Hamiltonian constraint,
\[ \int d^3x \sqrt{g} \left( \mathcal{L}_K - f(R) \right) = 8\pi G \int d^3x \sqrt{g} J^i, \quad (2.4) \]
where
\[ J^i \equiv \frac{\delta(N \mathcal{L}_M)}{\delta N}. \quad (2.5) \]

Variation with respect to $N^i$ yields the super-momentum constraint,
\[ \nabla_j \pi^{ij} = 8\pi G J^i, \quad (2.6) \]
where $\pi^{ij}$ and $J^i$ are defined as
\[ \pi^{ij} \equiv \frac{\delta(N \mathcal{L}_K)}{\delta g_{ij}} = -K^{ij} + \lambda K g^{ij}, \]
\[ J^i \equiv -N \frac{\delta \mathcal{L}_M}{\delta N_i}. \quad (2.7) \]

Varying with respect to $g_{ij}$, on the other hand, leads to the dynamical equations,
\[ \frac{1}{N \sqrt{g}} \left( \sqrt{g} \pi^{ij} \right)' = 2\lambda K K^{ij} - 2 \left( K^2 \right)^{ij} + \frac{1}{2} \mathcal{L}_K g^{ij} + F^{ij} \]
\[ + \frac{1}{N} \nabla_k \left( N^k \pi^{ij} - \pi^{ki} N^j - \pi^{kj} N^i \right) + 8\pi G \tau^{ij}, \quad (2.8) \]
where $\left( K^2 \right)^{ij} \equiv K^{ii} K^j_j$, and
\[ \tau^{ij} \equiv \frac{2}{\sqrt{g}} \frac{\delta \left( \sqrt{g} \mathcal{L}_M \right)}{\delta g_{ij}}, \]
\[ F^{ij} \equiv \nabla^i \nabla^j F - g^{ij} \nabla^2 F - F g^{ij} + \frac{1}{2} f g^{ij}, \quad (2.9) \]
with $F \equiv df(R)/dR$.

The matter quantities $(J^i, J^j, \pi^{ij})$ satisfy the conservation laws,
\[ \int d^3x \sqrt{g} \left[ \dot{\psi}_k \tau^{kl} - \frac{1}{\sqrt{g}} \left( \sqrt{g} J^l \right)' \right] + \frac{2 N_k}{N \sqrt{g}} \left( \sqrt{g} J^k \right)' = 0, \quad (2.10) \]
\[ \nabla^k \tau_{lk} - \frac{1}{N \sqrt{g}} \left( \sqrt{g} J^l \right)' - \frac{N_l}{N} \nabla_k N^l - \frac{J^l}{N} \left( \nabla_k N^l - \nabla_l N^k \right) = 0. \quad (2.11) \]

### III. Spin-0 Graviton in Minkowski Background

It can be shown that Minkowski spacetime $ds^2 = -dt^2 + \delta_{ij} dx^i dx^j$ is a solution of the vacuum field equations, provided that $f(0) = 0$. Considering its linear scalar perturbations,
\[ \delta g_{ij} = -2\psi \delta_{ij} + 2E_{ij}, \quad \delta N_i = B_{ii}, \quad \delta N = \phi, \quad (2.1) \]
in the quasi-longitudinal gauge \([9], \phi = 0 = E, \) we find that to second order the action without matter takes the form,
\[ S^{(2)}_g = \frac{\zeta^2}{c^2} \int d\eta d^3x N \sqrt{g} \left( \mathcal{L}^{(2)}_K + f^{(2)}(R) \right) \]
\[ = \frac{\zeta^2}{c^2} \int dt d^3x \left[ (1 - 3\lambda) \left( 3\psi^2 + 2\psi \partial^2 B \right) \right. \]
\[ + (1 - \lambda) B \partial^4 B + 2\gamma \psi \partial^2 \psi - 8\omega \zeta^2 \psi \partial^4 \psi \right], \quad (2.2) \]
where
\[ \gamma \equiv -f'(0), \quad \omega \equiv -\zeta^2 f''(0). \quad (2.3) \]

Variations with respect to $B$ and $\psi$ yield, respectively,
\[ (1 - 3\lambda) \ddot{\psi}_k - (1 - \lambda) k^2 \ddot{B}_k = 0, \quad (2.4) \]
\[ (3\lambda - 1) \left( 3\ddot{\psi}_k - k^2 \ddot{B}_k \right) = 2k^2 \left( \gamma + 4\omega k^2 / \zeta^2 \right) \psi_k, \quad (2.5) \]
in the momentum space. When $\lambda \neq 1$, from Eq. \([2.4] \) we can express $B_k$ in terms of $\psi_k$, and then Eq. \([2.2] \) becomes
\[ S^{(2)}_g = \frac{2c^2}{c^2} \int d\eta d^3x \left( \ddot{\psi}_k^2 - \omega^2 \psi_k^2 \right), \quad (2.6) \]
with
\[ c^2 \psi \equiv \frac{\lambda - 1}{3\lambda - 1}, \]
\[ \omega^2 \equiv \frac{c^2 k^2 \left( \gamma + 4\omega k^2 / \zeta^2 \right)}{c^2}. \quad (2.7) \]

Hence, $\psi_k$ satisfies $\ddot{\psi}_k + \omega^2 \psi_k = 0$, which has stable solutions only when $\omega^2 > 0$. Clearly, this is true in the IR limit only when
\[ \lambda > 1, \quad \gamma > 0. \quad (2.8) \]

These conditions also assure the kinetic term in $S^{(2)}_g$ always positive, that is, free of ghosts. When $\lambda = 1$, from Eq. \([2.4] \) we find that $\psi_k$ and $B_k$ are independent of $t$, and so are the two gauge-invariant quantities \([9] \) $\Phi_k = \ddot{B}_k$ and $\Psi_k = \psi_k$. Therefore, the spin-0 graviton is stable when $\lambda = 1$.

Note that the addition of other terms presented in \([20, 21] \) does not change the IR behavior, and their contributions only change the expression $\omega^2$ to the form,
\[ \omega^2 \psi^2 = c^2 k^2 \left( \gamma + \frac{(4\omega - 3g_3) k^2}{\zeta^2} + \frac{(3g_8 - 8g_3) k^4}{\zeta^4} \right), \quad (2.9) \]
where $g_i$ are the coupling constants introduced in \[21, 21\]. In addition, the condition that the theory reduces to general relativity (GR) requires $\gamma = -1$, a condition that was assumed in \[20, 21\]. This explains why in the SVW setup the spin-0 mode is not stable \[9\]. Our analysis given above shows that the instability is a generic feature of the HL theory with projectability condition. However, this instability does not necessarily show up in a physical environment, as longer as some conditions are satisfied \[22\]. In particular, if $L/|C_v| > L_t$, the instability will not show up, where $L$ is the length scale of interest, and $t_J$ denotes the timescale of Jeans instability \[22\]. In addition, the linear instability is stabilized by higher derivative terms if $|C_v| < 1/(LM_s)$, where $M_s$ is the energy scale suppressing higher derivative terms. In the current setup, it is given by $M_s \simeq |g_{13}|^{-1/2}M_{pl}$, $|g_{23}|^{-1/4}M_{pl}$. The linear instability can be also tamed by Hubble friction, if $L/(|C_v| > 1/H)$, where $H$ is the Hubble expansion rate at the time of interest. Therefore, even a solution is not stable, one may properly choose the form of the function $f(R)$, so that at least one of these conditions is satisfied. Then, the corresponding instability will not show up.

It is also interesting to note that, although the SVW setup is not stable in the Minkowski background, it is stable in the de Sitter \[15\]. This is also true in the current setup. Then, one may consider the instability found in the Minkowski background is just an indication that the latter is not the ground state of the theory. Instead, the legitimate background in the HL theory might be the de Sitter spacetime. This point of view is further supported by current observations that our universe is currently de Sitter-like \[23\].

### IV. COSMOLOGICAL MODELS

#### A. FRW Background

The homogeneous and isotropic universe is described by the FRW metric, $ds^2 = -dt^2 + a^2(t) dx^i dx^j$ where $\gamma_{ij} = (1 + \frac{4\kappa}{3}a^2)^{-2} \delta_{ij}$, with $\kappa = 0, \pm 1$. For this metric, $K_{ij} = -a^2 H \gamma_{ij}$ and $\Phi_{ij} = 2\kappa \gamma_{ij}$, where $H = \dot{a}/a$ and an overbar denotes a background quantity. Setting \[9\]

$$J^i = -2\rho, \quad J^i = 0, \quad \tau_{ij} = \bar{p} \bar{g}_{ij},$$

where $\bar{\rho}$ and $\bar{p}$ are the total density and pressure, we find that the Hamiltonian constraint \[22\] reduces to

$$\tilde{\lambda} H^2 = \frac{8\pi G}{3} \bar{\rho} - \frac{1}{6} f(\bar{R}),$$

where $\bar{R} = 6\kappa/a^2$, and $\tilde{\lambda} \equiv (3\lambda - 1)/2$. It can be shown that the momentum constraint \[23\] is satisfied identically, while the dynamical equations \[2.8\] yield,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\bar{\rho} + 3\bar{p}) - \frac{1}{6} f(\bar{R}) + \frac{\kappa}{a^2} F(\bar{R}).$$

The conservation of energy \[2.11\], on the other hand, is also satisfied identically, while the conservation of energy \[2.10\] gives

$$\dot{\bar{\rho}} + 3H (\dot{\bar{p}} + \bar{p}) = 0,$$

which can be obtained from Eqs. \[4.2\] and \[4.3\]. When $\kappa = 0$ we have $\bar{R} = 0$, and these two equations reduce exactly to those given in GR with a cosmological constant $\Lambda \equiv -f(0)/2$. In addition, Eqs. \[4.2\] and \[4.3\] also admit a de Sitter point that corresponds to a vacuum solution $(\bar{\rho} = \bar{p} = 0)$, at which $f(\bar{R}) - Rf'(\bar{R}) = -2\Lambda$, which has the solution $f(\bar{R}) = -2\Lambda - \gamma \bar{R}$.

In the rest of this paper, we shall consider only the cases where $\kappa \neq 0$. Then, Eqs. \[4.2\] and \[4.3\] are quite different from the ones of 4-dimensional $f(R^{(4)})$ models \[36, 37\]. In particular, since $R^{(4)}$ contains second derivatives of $a$, the generalized Friedmann equations are usually high-order differential equations in the 4-dimensional $f(R^{(4)})$ models \[36, 37\]. But, in the HL theory $\bar{R} = 6\kappa/a^2$ is a function of $a$ only. As a result, $f(\bar{R})$ is a polynomial of $a$ only, too. Therefore, in the current setup these terms act like sources. For example, if $f(\bar{R}) \propto \bar{R}^{3/2}$, then from Eq. \[4.2\] we can see that this corresponds to dark matter. Note that to have $f(\bar{R})$ real, in this case we must assume $\kappa > 0$. In addition, for $f(\bar{R}) \propto \bar{R}^n$ Eq. \[4.2\] shows that $H^2 \propto a^{-2n}$. Then, with $n < 1$ this term can mimic dark energy. On the other hand, for $n \geq 2$ it can give rise to a bouncing universe \[7, 8, 10, 11\]. Since all of these terms are purely geometric, they do not subjected to the energy conditions \[38\] and matter instabilities. Note that the unification of the dark sector in $f^{(n)}(\bar{R})$ models was first considered in \[39\].

#### B. Linear Scalar Perturbations

Consider scalar perturbations in the quasi-longitudinal gauge \[9\],

$$\delta N = 0, \quad \delta N_i = a^2 B_i, \quad \delta g_{ij} = -2a^2 \psi \gamma_{ij},$$

it can be shown that the Hamiltonian constraint \[2.4\] to first order yields,

$$\int \sqrt{-g} d^3 x \left[ F(\bar{R}) \left( \nabla^2 \psi + 3\kappa \psi \right) - \lambda H \left( \nabla^2 B + 3\psi' \right) \right] = 4\pi G a^2 \int \sqrt{-g} d^3 x \delta \mu,$$

where $\delta \mu = -\delta J^i/2$, $\nabla^2 f \equiv \gamma^{ij} f_{ij}$, and $|i|$ denotes the covariant derivative with respect to $\gamma_{ij}$. The momentum constraint \[2.6\] takes the form,

$$(3\lambda - 1) \psi' - 2\kappa B + (\lambda - 1) \nabla^2 B = 8\pi G a \psi,$$

with $\delta J^i = a^{-2} q^i$. The dynamical equations \[2.8\], on the other hand, yield,

$$\psi'' + 2\frac{F(\bar{R})}{3(3\lambda - 1) - 1} \left( \nabla^2 \psi + 3\kappa \psi \right)$$
+ \frac{8F'(\bar{R})}{3(3\lambda - 1)a^2} \left( \bar{\nabla}^2 \psi - 3\kappa \bar{\nabla}^2 \psi + 9\kappa^2 \psi \right) \\
+ \frac{1}{3} \bar{\nabla}^2 (B' + 2\mathcal{H}B) = \frac{8\pi Ga^2}{3\lambda - 1} \delta \mathcal{P}, \quad (4.8) \\
B' + 2\mathcal{H}B = \mathcal{F}(\bar{R})\psi - \frac{4F'(\bar{R})}{a^2} \left( \bar{\nabla}^2 \psi + 3\kappa \psi \right) \\
- \frac{8\pi Ga^2}{3\lambda - 1} \Pi, \quad (4.9)

where

\delta \tau^{ij} = a^{-2} \left[ (\delta \mathcal{P} + 2\bar{\rho}\psi) \gamma^{ij} + \Pi^{(ij)} \right], \end{equation}

\[ \Pi^{(ij)} = \Pi^{ij} - \frac{1}{3} \gamma^{ij} \bar{\nabla}^2 \Pi. \quad (4.10) \]

The conservation laws give,

\[ \int \sqrt{\gamma} d^3 x \left[ \delta \mu' + 3\mathcal{H} (\delta \mathcal{P} + \delta \mu) - 3 (\bar{\rho} + \bar{\rho}) \psi' \right] = 0, \quad (4.11) \]

\[ q' + 3\mathcal{H} q - a\delta \mathcal{P} - \frac{2a}{3} \left( \bar{\nabla}^2 + 3k \right) \Pi = 0. \quad (4.12) \]

Following [18] it can be shown from Eq. (4.8) that the de Sitter background is indeed stable in the current setup.

C. Inclusion of Other Terms Upto Six-Order

As mentioned above, one can add the terms [20, 21]

\[ \delta \mathcal{L}_V = \frac{\gamma_1}{\zeta^2} R_{ij} R^{ij} + \frac{1}{\zeta^3} \left( \gamma_2 R_{ij} R^{ij} + \gamma_3 R_{ij} R_{kj} R_{ik} \right) + \frac{1}{\zeta^4} \left[ \gamma_4 R \bar{\nabla}^2 R + \gamma_5 (\bar{\nabla}_i R_{jk}) (\bar{\nabla}^i R^{jk}) \right]. \quad (4.13) \]

This simple generalization leads to very rich physical phenomena. In particular, it naturally gives rise to dark matter and dark energy. We also studied scalar perturbations in the FRW backgrounds with non-zero curvature, and presented the general formulas, which will make the studies of perturbations in such a setup considerably easier. It would be very interesting to study their applications to large-scale structure formation, early universe, and investigate constraints of the models from solar system tests.

When high-order curvature terms, \( f^{(4)}(\bar{R}) \), are included, it was shown that spin-2 massive models also exist and are ghosts [40]. The Hamiltonian constraint can be obtained from Eq. (4.1) with the replacement of \( f(R) \) by \( -\mathcal{L}_V \), where

\[ \mathcal{L}_V = \delta \mathcal{L}_V - f(R), \quad (4.14) \]

while the super-momentum constraint (4.6) and the conservation laws (4.10) and (4.11) remain the same. The dynamical equations are also given by Eq. (2.8), but now with

\[ F^{ij} = F^{ij}_{f(R)} + \sum_{s=1}^{5} \gamma_s \zeta^{-2m_s} (F_s)^{ij}, \quad (4.15) \]

where \( F^{ij}_{f(R)} \) is defined by Eq. (2.9), and \( m_s = (1, 2, 2, 2, 2) \). and \( (F_s)^{ij} = (F_1, F_5, F_6, F_7, F_8)^{ij} \), where \( (F_s)^{ij} \) is defined in [9, 20, 21].

Due to the inclusion of the above terms in the action, Eqs. (4.2) and (4.3) become

\[ \lambda a^{-2} = \frac{4\pi G}{3} (\bar{\rho} + 3\bar{\rho}) - \frac{1}{6} f(\bar{R}) + \frac{2\beta_1 \kappa^2}{a^4} + \frac{4\beta_2 \kappa^3}{a^6}, \quad (4.16) \]

where

\[ \beta_1 \equiv \frac{\gamma_1}{\zeta^2}, \quad \beta_2 \equiv \frac{3\gamma_2 + \gamma_3}{\zeta^4}. \quad (4.18) \]

Similarly, one can obtain the corresponding scalar perturbations from the above and \[9\], which will not be presented here.

V. CONCLUSIONS

We studied the efforts of spatial derivatives higher than six, and the possibility of the critical exponents \( z \) being non-integer, in the HL gravity with projectability but without detailed balance condition, by replacing the third-order polynomial of the Ricci scalar in the SVW setup [20, 21] by an arbitrary function, \( f(R) \), while keeping the rest the same. The requirement that the theory reduce to GR in the IR makes scalar perturbations unstable in the Minkowski background, but stable in the de Sitter. This might imply that the legitimate background in the HL theory is not Minkowski spacetime, but the de Sitter.

In addition, dark matter and dark energy have been studied extensively [12, 14, and current observational data impose severe constraints on various models [22]. It is also very interesting to fit the data to our current models by using the MCMC code, recently developed by us [15], including the studies of the growth factor of perturbations [46].

Strong couplings [28] may exist here, too. One way is to provoke the Vainshtein mechanism [47], similar to what was done in [19, 26].

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