The relational version of the modal interpretation offers both a consistent quantum ontology and solution for quantum paradoxes within the framework of nonrelativistic quantum mechanics. In the present paper this approach is generalized for the case of relativistic quantum field theories. Physical systems are defined as Hilbert spaces. The concept of the reduced density matrix is also generalized so that its trace may become smaller than one, expressing the possibility of annihilation. Superselection rules are shown to follow if the whole Universe has a definite electric charge, barionic number and leptonic number.

I. INTRODUCTION

Modal interpretations aim at extracting a consistent physical picture out of the formalism of quantum mechanics rather than relying on assumptions about an a priori classical world \[1\], \[2\], \[3\], \[4\]. The Dieks-Vermaas version of the modal interpretations \[4\] utilizes the Schmidt states, i.e., the eigenstates of the reduced density matrix, and identifies them with the actually existing, physical states. The significance of the Schmidt bases has previously been emphasized - in connection with decoherence and Everett’s many worlds interpretation - by Zeh \[5\]. The no-go theorem of Vermaas \[6\] stating the impossibility of defining probabilities for the simultaneous existence of certain physical states can be understood within the framework of an interpretation which can be called the relational version of the modal interpretation \[7\]. Relational ideas have a long history. They appeared first in the original version of Everett’s interpretation \[8\], then, in different forms, in \[9\] and in \[10\]. The essential idea is that states do not exist in an absolute sense but can only be defined with respect to another system (or another state \[8\]). This idea has been implemented in \[6\] in a way mathematically different from the previous propositions. The quantum reference systems here contain the system to be described. The physical states defined in the Dieks-Vermaas interpretation can be identified by the states of a system with respect to itself (i.e., when the quantum reference system coincides with the system to be described). The no-go theorem of Vermaas means now that certain states of different systems that are defined with respect to different quantum reference systems cannot be compared, not even in principle. This circumstance has been shown to be consistent with the experimental possibilities which are available according to the theory, on the other hand, it clearly goes beyond the usual ontology. Indeed, one expects that existing things, even if they are defined with respect to different reference systems, must somehow be comparable. This expectation is actually based on classical experience, and its failure does not violate any well founded physical principle. The quantum ontology emerging from the relational modal interpretation states that even the existence of the states cannot be imagined independently of the quantum reference systems. One cannot think of reality as a big book where all the states of any systems with respect to any quantum reference systems are carefully registered. Such a registration would readily imply that the simultaneous existence of any states can always be checked, i.e., any states are comparable. Precisely this is impossible. This startling statement of quantum ontology is closely related to the fact that the state of the whole Universe (this would be the “big book”) does not determine uniquely the state of a subsystem with respect to itself, only a set of possible states and their probabilities. On the other hand, this fundamental change of the ontology, i.e., of the very concept of realism, is necessary in view of Bell’s theorem \[11\]. By now it is well known that Bell’s theorem and the corresponding experiments which convincingly support quantum mechanical predictions \[12\], \[13\] imply that at least one fundamental concept should be given up or modified, either locality, causality or realism \[11\]. Modal interpretations satisfy all the requirements of locality and causality, thus the remaining option is that the concept of reality should be modified. Indeed, it has been shown that accepting the above quantum ontology Bell’s inequality does not follow \[14\]. It is instructive to

\[\text{This has been independently developed and it turned out later that it involves both the essential ideas of the modal interpretations and those of the relational interpretations.}\]

\[\text{†Sometimes other concepts like scientific inference are also questioned.}\]
consider the famous Einstein-Podolsky-Rosen (EPR) reality criterion from our point of view. This criterion states that

“If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.”

Now the point is that the value of a physical quantity depends on the state of the system, and this state must be given with respect to some quantum reference system. Thus, the EPR criterion is valid only if neither the system itself, nor the quantum reference system is disturbed. But in case of the EPR paradox, the quantum reference system is disturbed, so the EPR reality criterion is not applicable and the conclusion about the incompleteness of quantum mechanics does not follow. Note that already Bohr has claimed (albeit using different arguments) that the concept of realism changes in quantum mechanics.

All the above considerations have been done within the framework of nonrelativistic quantum mechanics. In the present paper the relational modal interpretation is generalized to the case of relativistic field theories. Such generalizations of the Dieks-Vermaas version of the modal interpretation has already been proposed in Refs. 18, 19. In Section II, the concept of the physical systems is given and discussed. Section III, contains the main result, i.e., the generalized postulates of the interpretation. These replace von Neumann’s measurement postulates, thus making quantum theory self consistent, i.e., removing the necessity of an a priori classical background. In Section IV, the possible origin of the superselection rules is discussed. In the concluding Section V, a summary of the results is given.

II. THE CONCEPT OF THE PHYSICAL SYSTEMS

In nonrelativistic quantum mechanics physical systems might be specified by the particles they contain. This definition becomes unsatisfactory, however, when macroscopic systems are concerned. Indeed, a description of a macroscopic system should contain its structure as well, which is not included if only its constituent particles are given. Moreover, this structure is much more important than the precise number of the particles. A straightforward possibility is to specify a system by the collection of states which correspond to the structure and functionality of that system. These states may even contain a different number of particles. Certainly, the superposition principle must be respected (at least to an extent allowed by the superselection rules), thus arbitrary linear combinations of these states are also allowed. This makes the collection of the states a vector space. As the scalar product of these states are defined as usually, we have an Euclidean vector space. Finally, the completion of this space gives rise to a Hilbert space, which is much narrower than the total Hilbert space of all the constituent particles. E.g., when constructing the Hilbert space of a measuring device as described above, one does not include states which correspond to a destructed device. This construction can be equally well applied in case of relativistic field theories. In that case states are given in Fock space, thus typically contain superpositions of states with different occupation numbers. In the nonrelativistic case interacting systems usually can be chosen such that they preserve their identity during the interaction, while this is in general impossible in the relativistic case. Mathematically, this means that in the nonrelativistic case the interaction moves the state of the composite system within the direct product of the subsystems’ Hilbert spaces, while in the relativistic case the state may leave the direct product space during the interaction. Note that this situation can appear in the nonrelativistic case as well, e.g., if a measuring device is destroyed by a too hard interaction (say, a too low measuring range has been set), the final state of the composite system can be outside of the direct product space. In the relativistic case this situation is typical which means that a system can disappear. This means that the direct product of the subsystems’ Hilbert spaces is just a subspace of the composite system’s Hilbert space. Certainly, for the description of the interactions one has to choose such a Hilbert space (i.e., such a composite system) which is broad enough to accommodate the state during the whole time evolution. Such a system can be called isolated (as it does not interact with the rest of the world). Strictly speaking, there is only one such system: the whole Universe itself.

Sometimes we may assume that in the absence of interactions with other systems time evolution moves the state of the system within its original Hilbert space. Even this condition can be released, as it is reasonable in case of open systems like living beings. Indeed, a living being would die at once in the absence of interactions and thus would leave the Hilbert space which defines it on the basis of its normal functionality.

III. POSTULATEs

Once physical systems are defined mathematically as suitable Hilbert spaces, the next technical problem is how to give the state of a system with respect to another (broader) one. Here we follow Ref. 6 and make the necessary generalizations to get consistent rules.

Let us denote the physical system to be described by A and the reference system by R. The state of A with respect to R will be denoted by \( \hat{\rho}_A(R) \).

Postulate 1. The system A to be described is a subsystem of the reference system R.

As the systems are now defined as Hilbert spaces, the
The concept of the subsystem needs to be suitably defined as well.

**Definition 0.** A is a subsystem of R if there exists another Hilbert space B so that
\[ A \otimes B \subseteq R \]  

The broadest system B is called the complemen ter system of A (with respect to R) and is denoted by R/A.  

Note that the reference system may coincide with the system to be described (A = R). In such a case we speak about an internal state.

**Definition 1.** \( \hat{\rho}_A(A) \) is called the internal state of A.

**Postulate 2.** The state \( \hat{\rho}_A(R) \) is a positive definite, Hermitian operator with a trace not larger than one, acting on A.

Note that the fact that the trace can be smaller than one is related to the possibility of annihilation.

**Postulate 3.** The internal states \( \hat{\rho}_A(A) \) are always projectors, i.e., \( \hat{\rho}_A(A) = |\psi_A><\psi_A| \).

In what follows these projectors will be identified with the corresponding wave functions \( |\psi_A> \) (as they are uniquely related, apart from a phase factor). In accordance with Postulate 2 we assume that the internal states are normalized to unity.

**Postulate 4.** The state of a system A with respect to the reference system R (denoted by \( \hat{\rho}_A(R) \)) is the reduced density matrix of A calculated from the internal state of R, i.e.
\[ \hat{\rho}_A(R) = Tr_{R/A}(\hat{\rho}_R(R)) \] ,

where R/A stands for the subsystem of R complemen ter to A (cf. Definition 0).

Eq.(2) can be expressed in other forms, too. If \( |\xi_{A,j}> \) is a complete, orthonormed basis in A and \( |\xi_{R/A,j}> \) is a complete, orthonormed basis in R/A, then
\[ \hat{\rho}_A(R) = \sum_{j,k} |\xi_{A,j}> <\xi_{A,j}| (Tr_R (|\xi_{A,k}> <\xi_{A,k}|) <\xi_{A,k} | \hat{\rho}_R(R)| <\xi_{A,k} ) \]

or
\[ \hat{\rho}_A(R) = \sum_{j,k,l} |\xi_{A,j}> <\xi_{A,k}| \\
\times ( <\xi_{A,j}| \otimes <\xi_{R/A,l}|)|\psi_R> \\
\times <\psi_R| (|\xi_{R/A,l}> \otimes |\xi_{A,k}> ) \]  

One can see that Postulate 4 is consistent with Postulate 2 and Postulate 3. Because A \( \otimes (R/A) \) is not necessarily equal to R, Eq.(2) readily implies that \( Tr_A \hat{\rho}_A(R) \) can be smaller than unity:
\[ Tr_A \hat{\rho}_A(R) = \sum_j Tr_{R/A} <\xi_j| \hat{\rho}_R(R)| \xi_j > \leq 1 \] .

As in the nonrelativistic case, we introduce the notion of isolated and closed systems.

**Definition 2.** An isolated system is such a system that has not been interacting with the outside world. A closed system is such a system that is not interacting with any other system at the given instant of time (but might have interacted in the past).

An isolated system can be described by a wave function, and, moreover, this wave function always occurs as a factor in the internal state of any broader system. Thus we set

**Postulate 5.** If I is an isolated system then its state is independent of the reference system R:
\[ \hat{\rho}_I(R) = \hat{\rho}_I(I) \] .

Note that if only the whole universe can be considered an isolated system, then Postulate 5 becomes superfluous.

**Postulate 6.** If the reference system R = I is an isolated one then the state \( \hat{\rho}_A(I) \) commutes with the internal state \( \hat{\rho}_A(A) \).

This means that the internal state of A coincides with one of the eigenstates of \( \hat{\rho}_A(I) \). Note that usually there is no one-to-one correspondence between states with respect to different reference systems, thus one cannot tell (knowing \( \hat{\rho}_A(I) \)) which eigenstate corresponds to \( \hat{\rho}_A(A) \). In what follows, these eigenstates \( |\phi_{A,j}> \) will play an important role. They will be identified with the corresponding projector \( \hat{\pi}_{A,j} = |\phi_{A,j}><\phi_{A,j}| \), and we shall call them the possible internal states, as they constitute the set of those internal states of A that are compatible with \( \hat{\rho}_A(I) \). (Of course, at a given time only one of these states exists with respect to A.) For further reference, we set
Definition 3. The possible internal states are the eigenstates of \( \hat{\rho}_A(I) \) provided that the reference system \( I \) is an isolated one.

An important property of the possible internal states is their connection with the Schmidt decomposition:

**Proposition 1.** If \( A \) and \( B \) are two disjointed physical systems (i.e., they have no common subsystems) with possible internal states \( |\phi_{A,j}\rangle \) and \( |\phi_{B,j}\rangle \), respectively, and the joint system \( A \otimes B \) is an isolated one, then the internal state of \( A \otimes B \) can be written as

\[
|\psi_{A\otimes B}\rangle = \sum_j c_j |\phi_{A,j}\rangle \otimes |\phi_{B,j}\rangle .
\]  

(7)

Note that in the relativistic case the composite system suitable for the description of the interaction of \( A \) and \( B \) can be broader than \( A \otimes B \). Then, certainly, the states of that composite system \( C \) contain an additional term, namely,

\[
|\psi_C\rangle = \sum_j c_j |\phi_{A,j}\rangle \otimes |\phi_{B,j}\rangle + |\tilde{\psi}\rangle .
\]  

(8)

where \( |\tilde{\psi}\rangle \) is orthogonal to all the states \( |\phi_{A,j}\rangle \otimes |\phi_{B,k}\rangle \).

**Postulate 7.** If \( I \) is an isolated system, then the probability \( P(A,j) \) that the eigenstate \( |\phi_{A,j}\rangle \) of \( \hat{\rho}_A(I) \) coincides with the internal state of \( A \) is given by the corresponding eigenvalue \( \lambda_j \).

Due to the normalization of the internal states the sum of these eigenvalues is not larger than unity.

**Postulate 8.** The result of a measurement is contained unambiguously in the internal state of the measuring device.

**Postulate 9.** If \( A \) and \( B \) are two disjointed physical systems with possible internal states \( |\phi_{A,j}\rangle \) and \( |\phi_{B,j}\rangle \), respectively, and both systems are contained in the isolated reference system \( I \), then the joint probability that \( |\phi_{A,j}\rangle \) coincides with the internal state of \( A \) at the same time \( |\phi_{B,k}\rangle \) coincides with the internal state of \( B \) \((j,k = 1,2,3,...)\) is given by

\[
P(A,j,B,k) = Tr_{A\otimes B}(\hat{\tau}_{A,j}\hat{\tau}_{B,k}\hat{\rho}_{A\otimes B}(I)) .
\]  

(9)

where

\[
\hat{\tau}_{A,j} = |\phi_{A,j}\rangle \langle \phi_{A,j}| , \quad \hat{\tau}_{B,k} = |\phi_{B,k}\rangle \langle \phi_{B,k}| .
\]

Due to the completeness of the possible internal states \( \sum_j \hat{\tau}_{B,j} = 1 \) holds, therefore \( \sum_k P(A,j,B,k) = P(A,j) \), like in the nonrelativistic case.

More generally, we can consider the joint probabilities for more than two disjointed physical systems. Then we set

**Postulate 10.** If there are \( n \) disjointed physical systems, denoted by \( A_1,A_2,...,A_n \), all contained in the isolated reference system \( I \) and having the possible internal states \( |\phi_{A_{i,j}}\rangle \) \((i = 1,...,n)\) respectively, then the joint probability that \( |\phi_{A_{i,j_i}}\rangle \) coincides with the internal state of \( A_i \) \((i = 1,...,n)\) is given by

\[
P(A_1,j_1,A_2,j_2,...,A_n,j_n) = Tr_{A_1\otimes A_2\otimes...\otimes A_n}[\hat{\tau}_{A_{1,j_1}}\hat{\tau}_{A_{2,j_2}}...\hat{\tau}_{A_{n,j_n}}\hat{\rho}_{A_1\otimes A_2\otimes...\otimes A_n}(I)] ,
\]  

(10)

where \( \hat{\tau}_{A_{i,j_i}} = |\phi_{A_{i,j_i}}\rangle \langle \phi_{A_{i,j_i}}| \).

Note that \( n = 1 \) and \( n = 2 \) correspond to Postulate 7 and Postulate 9, respectively.

Finally, we require that **Postulate 11.** The internal state \( |\psi_C\rangle \) of a closed system \( C \) satisfies the time dependent Schrödinger equation

\[
\frac{\partial}{\partial t}|\psi_C\rangle = i\hbar \hat{H}|\psi_C\rangle .
\]  

(11)

Here \( \hat{H} \) stands for the Hamiltonian. Here we have used the Schrödinger picture.

**IV. SUPERSELECTION RULES**

As a check of the consistency of the present approach I show here that superselection rules follow for any system if they are valid for the whole universe. According to Postulate 6., it is enough to show that the state of a system with respect to the whole universe is such a density matrix that is diagonal in the electric charge, barionic and leptonic number. Let us apply Postulate 4. and Eq. (4) to calculate this state. Here we can choose - without restricting the generality - the states \( |\xi_{A,j}\rangle \), \( |\xi_{R/A,k}\rangle \) to be charge eigenstates. Now it is clear that if the state \( |\psi_R\rangle \) is a charge eigenstate, all those terms in Eq. (4) vanish where the states \( |\xi_{A,j}\rangle \) and \( |\xi_{A,k}\rangle \) correspond to different charges. This is because charge is an additive conserved quantity. Thus, the state (4) is indeed diagonal in the charge. The statement can be proven in the same way for the case of barionic and leptonic number.

**V. SUMMARY AND CONCLUSION**

As we have seen the postulates of the relational modal interpretation can be generalized for the case of relativistic field theories. As a mathematical description of physi-
ical systems Hilbert spaces has been constructed starting from state vectors which express the structure and functionality of the system. The postulates of Ref. [7] have been generalized accordingly. Note that the present formalism offers a useful generalization of the previous approach even within the framework of the nonrelativistic case. In the relativistic case the trace of the states is usually smaller than unity when the quantum reference system is broader than the system to be described. The postulates are consistent and, as a result of using the trace and eigenvalue equations as basic operations, they are also independent of the representation. Finally, it has been demonstrated that the postulates accommodate the superselection rules in a consistent way.

VI. ACKNOWLEDGEMENTS

Several enlightening discussions with Andreas Bringer, Dennis Dieks, Gert Eilenberger, Michael Eisele, Géza Györgyi, Frigyes Károlyházy, Hans Lustfeld, Roland Omnès, Zoltán Perjés, László Szabó, Miklós Rédei and Pieter Vermaas are gratefully acknowledged.

This work has been partially supported by the Hungarian Academy of Sciences under Grant No. OTKA T 029752, T 031 724 and the János Bolyai Research Fellowship.

* Electronic address: bene@poe.elte.hu

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