There we used eighteen diquark-antidiquark previous QCD sum rule predictions [2], as partly shown in Table 1.

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Article

Strong decays of fully-charm tetraquarks into di-charmonia

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1. Introduction

Very recently, the LHCb Collaboration reported their results on possible fully-charm tetraquarks [1]. They investigated the di-$J/\psi$ invariant mass spectrum, where they observed a broad structure ranging from 6.2 to 6.8 GeV and a narrow structure at around 6.9 GeV with a global significance of more than 5σ. Especially, they assumed the latter narrow structure to be a resonance with the Breit-Wigner lineshape, and measured its mass and width to be either

\[ M = 6905 \pm 11 \pm 7 \text{ MeV}, \]  
\[ \Gamma = 80 \pm 19 \pm 33 \text{ MeV}, \]  
based on no-interference fit, or

\[ M = 6886 \pm 11 \pm 11 \text{ MeV}, \]  
\[ \Gamma = 168 \pm 33 \pm 69 \text{ MeV}, \]  
based on the simple model with interference.

The above results are in a remarkable coincidence with our previous QCD sum rule predictions [2], as partly shown in Table 1. There we used eighteen diquark-antidiquark (QQ)(\overline{QQ}) currentss
structure is a possible fully-bottom tetraquark state, but it was not confirmed in the latter LHCb and CMS experiments [14,15].

Driven by the above CMS and LHCb experiments [1,12–15], intensive theoretical studies have been performed to investigate fully-bottom and fully-charm tetraquark states [2,16–34], while the conclusions are quite model dependent. For examples, the bbbb tetraquark states are possible to lie below the di-bottonium threshold according to Refs. [2,16–21], but they are not according to Refs. [22–27,29]. Due to these controversial issues, relevant experimental studies are crucial to understand them. From the theoretical side, studies on their decay properties are also useful and important [35,36].

In this paper we shall study strong decay properties of fully-charm tetraquark states, based on our previous QCD sum rule study [2]. We shall investigate both S- and P-wave cccc tetraquark states. Our previous results in Ref. [2] suggest that their masses are above the di-J/Pψ and di-ηc thresholds, so their strong decays into these two channels can happen, together with several other di-charmonia channels. Assuming them to be compact diquark-antidiquark [cc][cc] states, in the present study we shall apply the Fierz rearrangement of the Dirac and color indices to calculate their relative branching ratios. This method has been used in Refs. [37–39] to study the Zc(3900), X(3872), and Pc states.

2. Currents and Fierz identities

All the diquark-antidiquark [QQ][QQ] currents without derivatives have been systematically constructed in Ref. [2], with only one current of \( J^{PC} = 2^+ \) missing:

\[
J^{2+}_{xy} = \Gamma^{2+}_{xy} Q_x C_{a\mu} Q_{a\mu} C_{b\mu} Q_y .
\]  

(5)

Here \( Q_a \) is the heavy quark field with the color index \( a \), and \( \Gamma^{a\mu}_{xy} \) is the projection operator,

\[
\Gamma^{a\mu\nu}_{xy} = g^{xy} g^{a\mu} + g^{y\nu} g^{a\nu} - \frac{1}{2} g^{x\nu} g^{a\mu} .
\]  

(6)

There are altogether twelve currents of the positive parity. Four of them well correspond to the S-wave [QQ][QQ] tetraquark states, and we shall only investigate these four currents in the present study. There are altogether seven currents of the negative parity, and we shall study all of them in the present study. Detailed expressions are given in the following subsections, together with the Fierz identities to transform them into meson-meson \( (|QQ\rangle |QQ\rangle) \) currents.

2.1. Currents of the positive parity

There are two S-wave diquarks, the "good" diquark of \( J^P = 0^- \) and the "bad" one of \( J^P = 1^- \) (other are "worse") [40]. We can combine them to construct S-wave tetraquark states. To do this we follow the diquark-antidiquark model proposed in Refs. [41,42]. In this model the S-wave tetraquark states can be written in the spin basis as \( S_j S_j \), where \( S = S_{Q\bar{Q}} \) and \( S = S_{Q\bar{Q}} \) are the diquark and antidiquark spins, respectively.

There are altogether four S-wave QQ\bar{Q}Q tetraquark states, denoted as \( |J^P\rangle \):

\[
|X_1; 0^+\rangle = |0, 0\rangle_0 , \\
|X_2; 0^+\rangle = |1, 1\rangle_0 , \\
|X_3; 1^-\rangle = |1, 1\rangle_1 , \\
|X_4; 2^+\rangle = |1, 1\rangle_2 .
\]  

(7)

Similar to the quark-model picture, there are two S-wave diquark fields:

\[
Q_1^a C_{a\mu} Q_b J^P = 0^-, \\
Q_1^a C_{a\mu} Q_b J^P = 1^- .
\]  

(8)

We can combine them to construct four tetraquark currents corresponding to Eq. (7):

\[
J^{0+}_{xy} = Q_1^a C_{a\mu} Q_b ! J^P \rangle \langle Q_a C_{b\mu} Q_y , \\
J^{0+}_{xy} = Q_2^a C_{a\mu} Q_b ! J^P \rangle \langle Q_a C_{b\mu} Q_y , \\
J^{1-}_{xy} = Q_1^a C_{a\mu} Q_b ! J^P \rangle \langle Q_a C_{b\mu} Q_y , \\
J^{1-}_{xy} = Q_2^a C_{a\mu} Q_b ! J^P \rangle \langle Q_a C_{b\mu} Q_y .
\]  

(9)

(10)

(11)

(12)

The tensor diquark field \( Q_1^a C_{a\mu} Q_b \) couples to both \( J^P = 1^- \) and \( 1^+ \) channels. However, its positive-parity component \( Q_2^a C_{a\mu} Q_b \) (\( i, j = 1, 2, 3 \)) gives the dominant contribution to \( J^{1-}_{xy} \). Hence, this current \( J^{1-}_{xy} \) corresponds to \( |X_3; 1^-\rangle = |1, 1\rangle_1 \). Besides it, there exists another current directly corresponding to \( |X_3; 1^+\rangle \):

\[
J^{1+}_{xy} = Q_1^a C_{a\mu} Q_b ! J^P \rangle \langle Q_a C_{b\mu} Q_y - x \rightarrow \beta ,
\]  

(13)

but this current contains both positive- and negative-parity components, so we do not investigate it in the present study. We refer to Ref. [43] for its detailed discussions.

After applying the Fierz transformation on Eqs. (9)–(12), we obtain:

\[
f_1^{0+} = - \frac{1}{4} \eta_1^{00} - \frac{1}{4} \eta_2^{00} - \frac{1}{4} \eta_3^{00} - \frac{1}{4} \eta_4^{00} + \frac{1}{2} \eta_5^{00} ,
\]  

(14)

\[
f_2^{0+} = \eta_1^{11} - \eta_2^{11} + \eta_3^{11} + \eta_4^{11} ,
\]  

(15)

\[
j^{1+}_{15} = 2 \eta_1^{00} - \eta_5^{00} ,
\]  

(16)

\[
j^{1+}_{23} = 2 \eta_2^{00} - \eta_5^{00} + 2 \eta_5^{00} ,
\]  

(17)

Some of these relations have been derived in Refs. [44,45]. Here \( \eta_1^{0,2,3,4,5} \) are the meson-meson currents of \( J^P = 0^- \), \( \eta_1^{0,2,3,4,5} \) are of \( J^P = 1^- \), and \( \eta_1^{0,2,3,4,5} \) are of \( J^P = 2^+ \):
\[ \eta_1^{0-} = Q_a Q_b \gamma_5 Q_b \gamma_5 Q_a, \]
\[ \eta_2^{0-} = Q_a \gamma_5 Q_b \gamma_5 Q_b \gamma_5 Q_a, \]
\[ \eta_3^{0-} = Q_a \gamma_5 Q_b \gamma_5 Q_b \gamma_5 Q_a, \]
\[ \eta_4^{0-} = Q_a \sigma_{\mu\nu} Q_b \sigma_{\mu\nu} Q_b \gamma_5 Q_a, \]
\[ \eta_5^{0-} = Q_a \gamma_5 Q_b \gamma_5 Q_b \gamma_5 Q_a. \]

There are two currents of \(J^P = 1^-\):
\[ J_{10}^{1-} = Q_a^T C_{\gamma_5}^T Q_b^T \gamma_5 C_{\gamma_5}^T C_{\gamma_5}^T, \]
\[ J_{11}^{1-} = -Q_a^T C_{\gamma_5}^T Q_b^T \gamma_5 C_{\gamma_5}^T C_{\gamma_5}^T. \]

Similar to Eq. (7), we can write \(P\)-wave tetraquark states in the spin-parity basis as \(|s, s'\rangle\), where \(P\) and \(P\) are the diquark and antidiquark parities, respectively. The above tetraquark currents correspond to the following \(P\)-wave \(QQQQ\) tetraquark states:
\[ |X_{5}; 0^+\rangle = \frac{1}{\sqrt{2}} (|0^-, 0^+\rangle + |0^+, 0^-\rangle), \]
\[ |X_{6}; 0^+\rangle = |1^+, 1^+\rangle, \]
\[ |X_{7}; 0^-\rangle = \frac{1}{\sqrt{2}} (|0^-, 0^-\rangle - |0^+, 0^+\rangle), \]
\[ |X_{8}; 1^-\rangle = \frac{1}{\sqrt{2}} (|1^-; 0^+\rangle + |0^+, 1^+\rangle), \]
\[ |X_{9}; 1^-\rangle = \frac{1}{\sqrt{2}} (|1^-; 1^+\rangle + |1^+, 1^-\rangle), \]
\[ |X_{10}; 1^-\rangle = \frac{1}{\sqrt{2}} (|1^-; 0^+\rangle - |0^+, 1^+\rangle), \]
\[ |X_{11}; 1^-\rangle = \frac{1}{\sqrt{2}} (|1^-; 1^+\rangle - |1^+, 1^-\rangle). \]

The realistic physical states can be different from these states. However, if there exists some tetraquark current well (better) coupling to the physical state, the results extracted from this current should also be (more) consistent with that state. Hence, we investigate all the negative-parity tetraquark currents without derivatives in the present study, given that the internal structure of \(P\)-wave \(QQQQ\) tetraquark states are still unknown.

After applying the Fierz transformation on Eqs. (21)–(27), we obtain:
\[ J_{5}^{1-} = \eta_1^{1-} + \frac{1}{4} \eta_2^{1-}, \]
\[ J_{6}^{1-} = 6 \eta_1^{1-} - \frac{1}{2} \eta_2^{1-}, \]
\[ J_{7}^{1-} = -\eta_3^{1-}, \]
\[ J_{8}^{1-} = -\eta_4^{1-} + i \eta_5^{1-}, \]
\[ J_{9}^{1-} = -3 \eta_4^{1-} + \eta_5^{1-}, \]
\[ J_{10}^{1-} = \eta_6^{1-} - i \eta_7^{1-}, \]
\[ J_{11}^{1-} = -3 \eta_6^{1-} + \eta_7^{1-}. \]

Some of these relations have been derived in Refs. [46–48]. Here \(\eta_{11}^{1-}\) are the meson-meson currents of \(J^P = 0^+\), \(\eta_{15}^{1-}\) is of \(J^P = 0^-\), \(\eta_{14}^{1-}\) are of \(J^P = 1^-\), and \(\eta_{16}^{1-}\) are of \(J^P = 1^-\):
\[ \eta_{11}^{1-} = Q_a Q_b \gamma_5 Q_b \gamma_5 Q_a, \]
\[ \eta_{12}^{0-} = Q_a \sigma_{\mu\nu} Q_b \sigma_{\mu\nu} \gamma_5 Q_b, \]
\[ \eta_{13}^{1-} = Q_a \gamma_5 Q_b \gamma_5 Q_b \gamma_5 Q_a, \]
\[ \eta_{14}^{1-} = Q_a \gamma_5 Q_b \gamma_5 Q_b \gamma_5 Q_a, \]
\[ \eta_{15}^{1-} = Q_a \gamma_5 Q_b \gamma_5 Q_b \gamma_5 Q_a, \]
\[ \eta_{16}^{1-} = Q_a \gamma_5 Q_b \gamma_5 Q_b \gamma_5 Q_a. \]
3. Relative branching ratios

In this section we investigate possible decay channels of fully-charm tetraquark states, both qualitatively and quantitatively. According to the recent LHCb experiment [1], we assume the S- and P-wave $cccc$ tetraquark states to have the masses about 6.5 and 6.9 GeV, respectively.

As depicted in Fig. 1, when one heavy quark and one heavy anti-quark meet each other and the rest heavy quark and antiquark also meet each at the same time, a compact diquark-antidiquark $[\bar{c}c][cc]$ state can fall-apart decay into two charmonium mesons. This process can be described by the Fierz identities given in Eqs. (14)–(17) and Eqs. (29)–(35). To study it we need the couplings of charmonium operators to charmonium states, which have been well studied in the literature [49–52] and summarized here in Table 2.

Firstly, let us perform qualitative analyses. Take Eq. (14) as an example, because both $\eta_0^{cc}$ and $\eta_1^{cc}$ can couple to the $J/\psi$/$\psi$ channel, the current $f_{1}^{cc}$ can also couple to this channel, so that the state $|X_1;0^{++}\rangle$ can decay into the $J/\psi$/$\psi$ final state. Similarly, we can derive six other possible channels to be $\eta_c|\bar{c}c\rangle$, $Z_{1059}$, $Z_{1530}$, $Z_{1531}$, $h_c$, $\eta_c$, and $J/\psi h_c$. Among them, the $J/\psi$/$\psi$/, $\eta_c$, $\eta_c$, and $\eta_c$ channels are kinematically allowed.

In principle one needs the coupling of $f_{1}^{cc}$ to $|X_1;0^{++}\rangle$ as an input to quantitatively calculate partial decay widths of these channels. We define this coupling to be

$$\langle 0|f_{1}^{cc}|X_1;0^{++}\rangle = f_{X_1},$$

Fig. 1. (Color online) The fall-apart decay of a compact diquark-antidiquark $[\bar{c}c][cc]$ state into two charmonium mesons. Quarks are shown in red/green/blue color, and antiquarks are shown in cyan/magenta/yellow color.

while it is not necessary any more if one only calculates relative branching ratios. Moreover, because couplings of meson operators to meson states are well studied but couplings of tetraquark currents to tetraquark states are not, the decay constant $f_{X_1}$ is not so well determined compared to the meson decay constants listed in Table 2. Accordingly, relative branching ratios can be calculated more reliably than partial decay widths.

To calculate relative branching ratios, we just need to keep $f_{X_1}$ as an unfixed parameter, calculate partial decay widths, and finally remove $f_{X_1}$. Still take Eq. (14) as an example, the couplings of $|X_1;0^{++}\rangle$ to the $J/\psi/\psi$ and $\eta_c$ channels can be extracted from it to be

$$\langle X_1(p);0^{++}|J/\psi(p_1, \epsilon_1)|J/\psi(p_2, \epsilon_2)\rangle \propto f_{X_1} \times c_{i} \epsilon_{i} \left( -\frac{1}{2} m_{J/\psi} f_{J/\psi}^2 g_{\mu\nu} - \frac{1}{2} \left( f_{J/\psi}^2 \right)^2 p_1^\mu p_2 \right).$$

$$\langle X_1(p);0^{++}|\eta_c(p_1, \eta)|\eta(p_2)\rangle \propto f_{X_1} \times \left( \frac{1}{2} m_{\eta} f_{\eta}^2 + \frac{1}{2} f_{J/\psi}^2 p_1^\mu p_2 \right).$$

After calculating the two partial decay widths $\Gamma_{|X_1;0^{++}\rangle \rightarrow J/\psi/\psi}$ and $\Gamma_{|X_1;0^{++}\rangle \rightarrow \eta_c}$, we can remove the parameter $f_{X_1}$ and obtain:

$$\frac{B(|X_1;0^{++}\rangle \rightarrow J/\psi/\psi)}{B(|X_1;0^{++}\rangle \rightarrow \eta_c)} = 1 : 0.45. \tag{40}$$

Similarly, we can add the $\eta_c$ channel and obtain:

$$\frac{B(|X_1;0^{++}\rangle \rightarrow J/\psi/\psi \rightarrow \eta_c|X_1;0^{++}\rangle \rightarrow \eta_cZ_{1531})}{B(|X_1;0^{++}\rangle \rightarrow \eta_cZ_{1531})} = 1 : 0.45 : 2.1 \times 10^{-3}. \tag{41}$$

Following the same procedures, we shall separately investigate the S- and P-wave $cccc$ tetraquark states in the following subsections.

3.1. S-wave $cccc$ states

Firstly, we use the Fierz identities given in Eqs. (15)–(17) to perform qualitative analyses:

- Eq. (15) suggests the possible decay channels of $|X_2;0^{++}\rangle$ to be $J/\psi/\psi$, $\eta_c$, $Z_{1059}$, $Z_{1531}$, and $\eta_c$. Among them, the $J/\psi/\psi$, $\eta_c$, and $\eta_c$, channels are kinematically allowed.

- Eq. (16) suggests the possible decay channels of $|X_3;1^{-}\rangle$ to be $J/\psi h_c$, $J/\psi Z_{1531}$, $h_c$, $Z_{1531}$, and $J/\psi h_c$. Among them, only the $J/\psi h_c$ channel is kinematically allowed.

- Eq. (17) suggests the possible decay channels of $|X_4;2^{++}\rangle$ to be $J/\psi/\psi$, $\eta_c$, $Z_{1530}$, $Z_{1531}$, $Z_{1531}$, and $J/\psi/\psi$. Among them, the $J/\psi/\psi$, $\eta_c$, and $\eta_c$ channels are kinematically allowed.

Table 2

Couplings of charmonium operators to charmonium states. Color indices are omitted for simplicity.

| Operators | $f^c$ | Mesons | $f^c$ | Couplings | Decay constants |
|-----------|-------|--------|-------|------------|-----------------|
| $f = \bar{c}c$ | $0^{++}$ | $Z_{1059}$ | $0^{++}$ | $\langle 0|f|Z_{1059}\rangle - m_{Z_{1059}} f_{Z_{1059}}$ | $f_{Z_{1059}} = 343$ MeV $[50]$ |
| $f' = \bar{c}c_3c$ | $0^{--}$ | $\eta_c$ | $0^{++}$ | $\langle 0|f'|\eta_c\rangle - i_{\eta_c}$ | $\dot{i}_{\eta_c} = -\frac{f_{\eta_c}^2}{m_{\eta_c}}$ |
| $f''_o = \bar{c}c_3c$ | $1^{--}$ | $J/\psi$ | $1^{--}$ | $\langle 0|f''_o|J/\psi\rangle - m_{J/\psi} f_{J/\psi}$ | $f_{J/\psi} = 418$ MeV $[52]$ |
| $f''_o = \bar{c}c_3c$ | $1^{++}$ | $\eta_c$ | $0^{--}$ | $\langle 0|f''_o|\eta_c\rangle - i_{\eta_c}$ | $f_{\eta_c} = 387$ MeV $[52]$ |
| $f''_o = \bar{c}c_3c$ | $1^{--}$ | $Z_{1530}$ | $1^{--}$ | $\langle 0|f''_o|Z_{1530}\rangle - m_{Z_{1530}} f_{Z_{1530}}$ | $f_{Z_{1530}} = 335$ MeV $[51]$ |
| $f''_o = \bar{c}c_3c$ | $1^{++}$ | $J/\psi$ | $1^{--}$ | $\langle 0|f''_o|J/\psi\rangle - i_{\eta_c}$ | $f_{J/\psi} = 410$ MeV $[52]$ |
| $f''_o = \bar{c}c_3c$ | $h_c(1P)$ | $h_c(1P)$ | $1^{--}$ | $\langle 0|f''_o|h_c\rangle - i_{\eta_c}$ | $f_{h_c} = -235$ MeV $[52]$ |
Quantitatively, we assume masses of the S-wave $cccc$ tetraquark states to be about 6.5 GeV, and obtain:
\[
B(\xi_5; 0^+) \to \overline{J}/\psi \overline{J}/\psi ; \eta_c; \eta_c; \chi_{c0} \chi_{c1}) = 1 : 4.1 \times 10^{-5},
\]
\[
B(\xi_6; 0^+) \to \overline{J}/\psi \overline{J}/\psi ; \eta_c; \eta_c; \chi_{c0} \chi_{c1}) = 1 : 0.036 \times 6.0 \times 10^{-4}.
\]
Relative branching ratios of $\xi_5; 1^-$ are not given, because we only derive one of its possible decay channels.

3.2. P-wave $cccc$ states

Firstly, we use the Fierz identities given in Eqs. [29]–(35) to perform qualitative analyses:

- Eqs. (29) and (30) suggest the possible decay channels of $\xi_6; 0^+$ and $\xi_6; 0^-$ to be $\overline{J}/\psi \overline{J}/\psi ; \eta_c; \eta_c; \chi_{c0}$ and $\overline{J}/\psi \chi_{c1}$. Among them, the $\overline{J}/\psi \overline{J}/\psi ; \eta_c; \eta_c; \chi_{c0}$ channels are kinematically allowed.
- Eqs. (31) suggest the possible decay channels of $\xi_7; 0^-$ to be $\overline{J}/\psi \eta_c$, and $\eta_c \chi_{c1}$. These two channels are both kinematically allowed.
- Eqs. (32) and (33) suggest the possible decay channels of $\xi_8; 1^-$ to be $\overline{J}/\psi \eta_c; \eta_c; \chi_{c1}$ and $\overline{J}/\psi \chi_{c1}$. All these channels are kinematically allowed.
- Eqs. (34) and (35) suggest the possible decay channels of $\xi_9; 1^-$ and $\xi_11; 1^-$ to be $\overline{J}/\psi \eta_c; \eta_c; \chi_{c1}$ and $\overline{J}/\psi \chi_{c1}$. Among them, the $\overline{J}/\psi \eta_c; \eta_c; \chi_{c1}$ and $\eta_c \chi_{c1}$ channels are kinematically allowed.

Quantitatively, we assume masses of the P-wave $cccc$ tetraquark states to be about 6.9 GeV, and obtain:
\[
B(\xi_7; 0^-) \to \overline{J}/\psi \eta_c = 1 : 0.069 \times 0.21,
\]
\[
B(\xi_8; 1^-) \to \overline{J}/\psi \eta_c = 1 : 0.036 \times 0.011 \times 0.36 = 1 : 0.011 \times 0.36 = 0.30,
\]
\[
B(\xi_9; 1^-) \to \overline{J}/\psi \eta_c = 1 : 1.0 \times 3.2 \times 0.30 = 1 : 3.2 \times 0.30.
\]

4. Summary and discussions

Very recently, the LHCb Collaboration reported their results on possible fully-charm tetraquarks [1]. They investigated the di-$\overline{J}/\psi$ invariant mass spectrum, where they observed a broad structure ranging from 6.2 to 6.8 GeV and a narrow structure at around 6.9 GeV with a global significance of more than 5σ. Their results are in a remarkable coincidence with our previous QCD sum rule predictions [2], as discussed already in Section 1.

Assuming the two structures observed by LHCb [1] to be compact diquark-antidiquark $cc|\bar{c}\bar{c}$ states, in this paper we systematically study their fall-apart decay properties. To do this we use the Fierz rearrangement of the Dirac and color indices to transform diquark-antidiquark $(QQ|\bar{Q}\bar{Q})$ currents into meson-meson $(|\bar{Q}\bar{Q}||Q\bar{Q})$ currents. The obtained Fierz identities are given in Eqs. (14)–(17) and Eqs. (29)–(35), based on which we study their possible fall-apart decays within the naive factorization scheme.

In this paper we have calculated as many as possible relative branching ratios, and the results are given in Eqs. (41)–(50) and summarized in Table 3. Before drawing conclusions, let us generally discuss about their uncertainty. In the present study we have worked within the naive factorization scheme, so our uncertainty is larger than the well-developed QCD factorization method [53–55], that is at the 5% level when being applied to study weak and radiative decay properties of conventional (heavy) hadrons [56]. On the other hand, the tetraquark decay constants, such as $f_X$, are removed when calculating relative branching ratios. This significantly reduces our uncertainty. However, it is still not easy to explicitly evaluate our uncertainty, since the corrections to the naive factorization are still not calculable, as of now.

The LHCb experiment [1] observed a broad structure at around 6.2 – 6.8 GeV. It can be interpreted as an S-wave $cccc$ tetraquark state, whose mass was predicted to be about 6.5 GeV in our previous QCD sum rule study [2]. To study its fall-apart decays, we use the four $cccc$ currents, which well correspond to the four S-wave $cccc$ tetraquark states. The obtained results are summarized in the 3rd-6th rows of Table 3.

| \( f^P \) | Configuration | Decay channels |
|-------|----------------|----------------|
| 0++  | \( x_1 = |0^+_c| \bar{Q}^0_{c2,0} \) | \( J/\psi/\eta_c/\eta_c = 1:0.45 \) |
| 1++  | \( x_2 = |1^+_c| \bar{Q}^0_{c2,0} \) | \( J/\psi/\eta_c/\eta_c = 1:4.1 \times 10^{-5} \) |
| 2++  | \( x_3 = |2^+_c| \bar{Q}^0_{c2,0} \) | \( J/\psi/\eta_c/\eta_c = 1:0.036 \times 6.0 \times 10^{-4} \) |
| 0+  | \( x_4 = |0^+_c| \bar{Q}^0_{c2,0} \) | \( J/\psi/\eta_c/\eta_c = 1:0.011 \times 0.36 = 0.36 \) |
| 1+  | \( x_5 = |1^+_c| \bar{Q}^0_{c2,0} \) | \( J/\psi/\eta_c/\eta_c = 1:1.0 \times 0.30 = 3.2 \) |
| 0-  | \( x_6 = |0^+_c| \bar{Q}^0_{c2,0} \) | \( J/\psi/\eta_c/\eta_c = 1:0.79 \times 1.5 = 0.43 \) |
| 1-  | \( x_7 = |1^+_c| \bar{Q}^0_{c2,0} \) | \( J/\psi/\eta_c/\eta_c = 1:7.1 \times 1.5 = 0.43 \) |
Our results suggest that the broad structure observed by LHCb at around 6.2–6.8 GeV [1] has the quantum numbers $f_{PC} = 0^{++}$ or $2^{++}$. We propose to confirm it in the $d_{-} \eta_{c}$ channel. This channel is also helpful to determine its quantum numbers and understand its internal structure. Besides, we propose to search for another $f_{PC} = 1^{-+}$ state in the $J/\psi \eta_{c}$ channel also at around 6.5 GeV.

The LHCb experiment [1] observed a narrow structure at around 6.9 GeV. It can be interpreted as a $P$-wave $cccc$ tetraquark state, whose mass was predicted to be also about 6.9 GeV in our previous QCD sum rule study [2]. To study its fall-apart decays, we investigate all the negative-parity tetraquark currents without derivatives, and the results are summarized in the 7th-13th rows of Table 3. Their correspondences to physical states are not so clear. However, if some of them well (better) couples to the physical state, the results extracted from this current should also be (more) consistent with that state. Note that the internal structure of $P$-wave $cccc$ tetraquark states are still unknown and difficult to be known, since there can be as many as twenty states [30].

Our results suggest that the narrow structure observed by LHCb at around 6.9 GeV [1] has the quantum numbers $f_{PC} = 0^{++}$ or $1^{--}$. We propose to confirm it in the $d_{-} \eta_{c}$, $J/\psi \eta_{c}$, and $J/\psi \chi_{c}$ channels. These channels are helpful to determine its quantum numbers as well as understand its internal structure. We also propose to search for the $f_{PC} = 0^{--}$ and $1^{--}$ states in the $J/\psi \eta_{c}$, $J/\psi \chi_{c}$, and $J/\psi \chi_{c}$ channels at around 6.9 GeV.

To end this paper, we kindly note that there have been many other theoretical studies on the fully-charm tetraquark states, but the obtained conclusions are quite model dependent. Accordingly, there can be many other possible interpretations for the two structures observed by LHCb [1], e.g., their possible quantum numbers can be different from those extracted in the present study. Hence, further experimental and theoretical studies are crucial to understand them.

Conflict of interest

The authors declare that they have no conflict of interest.

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Author contributions

All the authors provided the theoretical calculations, discussed the results, and wrote the manuscript. All the authors have given approval to the final version of the manuscript.

References

[1] LHCb Collaboration, Aaij R, et al. Observation of structure in the $f_{PC}$-pair mass spectrum. Sci Bull 2020;65:1983–93.

[2] Chen W, Chen HX, Liu X, et al. Hunting for exotic doubly hidden-charm/bottom tetraquark states. Phys Lett B 2017;773:247–51.

[3] Chao KT. The $cc^{\ast}cc^{\ast}$ (diquark-antidiquark) states in $e^{+}e^{-}$ annihilation. Z Phys C 1981;1:317.

[4] Iwasaki Y. A possible model for new resonances-exotics and hidden charm. Prog Theor Phys 1975;54:492.

[5] Ader JP, Richard JM, Taxil P. Do narrow heavy multi-quark states exist? Phys Rev D 1982;25:2370.

[6] Keller L, Tjon JA. On bound states of heavy $Q\bar{Q}$ systems. Phys Rev D 1985;32:755.
[51] Novikov VA, Okun LB, Shifman MA, et al. Charmonium and gluons: basic experimental facts and theoretical introduction. Phys Rept 1978;41:1–133.

[52] Bećirević D, Duplancić G, Klajn B, et al. Lattice QCD and QCD sum rule determination of the decay constants of $\eta_c$, $J/\psi$ and $h_c$ states. Nucl Phys B 2014;883:306–27.

[53] Beneke M, Buchalla G, Neubert M, et al. QCD factorization for $B \to \pi\pi$ decays: strong phases and CP violation in the heavy quark limit. Phys Rev Lett 1999;83:1914–7.

[54] Beneke M, Buchalla G, Neubert M, et al. QCD factorization for exclusive, nonleptonic $B$-meson decays: general arguments and the case of heavy-light final states. Nucl Phys B 2000;591:313–418.

[55] Beneke M, Buchalla G, Neubert M, et al. QCD factorization in $B \to \rho K$; $\pi\pi$ decays and extraction of Wolfenstein parameters. Nucl Phys B 2001;606:245–321.

[56] Li HD, Lü CD, Wang C, et al. QCD calculations of radiative heavy meson decays with subleading power corrections. J High Energy Phys 2020;04:023.

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