A variable step-size adaptive notch filter for frequency estimation using combined gradient algorithm

Huiyue Yang¹,a*, Yaqing Tu¹,b, Ming Li¹,c

¹Army Logistics University of PLA, Chongqing 401311, China
*a correspond author: ahuuiyue_yang@163.com
          b yqtcq@sina.com
          c limitonly@126.com

Abstract To improve the performance of adaptive notch filter (ANF), a variable step-size ANF using combined gradient algorithm is proposed for frequency estimation. In this method, combined gradient algorithm is designed for improving constringency with both FIR-ANF and IIR-ANF advantages considered. According to the noise influence, a bias correction strategy is established for depressing the bias and MSE of frequency estimation. Additionally, variable step-size is adopted for the balance of convergence and precision. The proposed method process is given. In simulations, we discussed the influence of the ANF parameters and SNR on accuracy. Algorithms of DPG an d MPG are carried out as comparisons. Convergences of these methods are also analyzed. Coriolis mass flow meter (CMF) is taken as an application to test the proposed method. Simulation results and CMF application both confirm the availability and superiority of the proposed method.

1. INTRODUCTION
Adaptive estimation of the frequency of a single-tone signal in noise is an important research topic that has varied applications, such as Radar, Sonar, control engineering, communication systems, testing instrument, and so on. Adaptive notch filter (ANF) is one of many choices to serve such applications.

The frequency is estimated in ANF by minimizing the errors of filter, according to the parameters which is adaptive adopted with signal character. It is found in the literature survey that there two types of the ANF, namely, infinite-impulse-response notch filter (IIR-ANF) and finite-impulse-response notch filter (FIR-ANF), which will be both considered in this work. IIR-ANF has some advantages, such as simple structure, little computation and easy realized. But because of its gradient algorithm, IIR-ANF needs a long time for convergence especially when initial frequency of ANF is away from signal frequency. In the last several decades, methods for improving the structure of ANF or adaptive algorithm have been intensively studied. A majority of these methods adopt gradient-based adaptive algorithms, such as the plain gradient(PG)¹, the direct plain gradient(DPG)², the modified plain gradient(MPG)³, the modified sign algorithm(MSA)⁴, the unbiased plain gradient(UPG)⁵ and the unbiased modified plain gradient(UMPG)⁶ and so on, are available and suitable for real time frequency estimation. However, extensive studies have shown that the precision of these methods still need to improve as there are inherent biased estimators. ⁸ In addition, there is a trade-off between a small steady state error and a fast convergence. A small step-size provides small steady state error.
but also gives reduction to convergence. On the other hand, large step-size gives the opposite performance.

To serve the problem mentioned above, a variable step-size ANF using combined gradient algorithm (CGA) for frequency estimation is proposed in this paper. In the proposed ANF, CGA is developed to improve the performance, and biases corrected formula is deduced based on the correlation of noise in the processed signal. Synchronously, variable step-size technique is adopted to balance the steady state error and the convergence.

The rest of the paper is organized as follows. In Sec. II, the principle of the proposed method is introduced in detail. Sec.III displays the process of the proposed method. The simulation and experimental results validating the proposed method are reported in Sec.IV. Sec.V concludes this paper.

2. PRINCIPLE

2.1 Frequency iterative calculation

Assume that the signal waiting for frequency estimation has the form of

\[ x(k) = A \cos(\omega_0 k + \theta) + v_n(k), k = 1, 2, ..., N \]  

(1)

where \( A, \omega_0, \theta \) is the signal amplitude, frequency and phase, respectively. \( v_n(k) \) is an additional white noise with zero mean and variance \( \sigma^2 \). \( N \) is the length of sample data. The system function of adaptive notch filter used in this work is given by

\[ H(z, \hat{\omega}_0) = \frac{N(z, \hat{\omega}_0)}{D(z, \hat{\omega}_0)} = \frac{1 - 2\cos(\hat{\omega}_0)z^{-1} + z^{-2}}{1 - 2\rho \cos(\hat{\omega}_0)z^{-1} + \rho^2 z^{-2}} \]  

(2)

where \( N(z, \hat{\omega}_0) \) and \( D(z, \hat{\omega}_0) \) are, respectively all zeros and poles systems. \( \rho \) is pole radius restricting the bandwidth of ANF, \( 0 < \rho < 1 \). \( \hat{\omega}_0 \) is the estimation of frequency \( \omega_0 \). Accordingly, the iterative formula of frequency calculation is described by the following equation:

\[ \hat{\omega}_0(k + 1) = \hat{\omega}_0(k) + \frac{\mu}{2} \frac{\partial J(\omega(k))}{\partial \hat{\omega}_0(k)} \]  

(3)

Based on (2), we can get the relationship that the input \( x(k) \) to the output \( e_1(k) \) and \( e_2(k) \) of \( N(z, \hat{\omega}_0) \) and \( H(z, \hat{\omega}_0) \), denoted by

\[ e_1(k) = x(k) - 2\cos(\hat{\omega}_0)x(k-1) + x(k-2) \]
\[ e_2(k) = e_1(k) + 2\rho \cos(\hat{\omega}_0)e_2(k-1) - \rho^2 e_2(k-2) \]  

(4)

The gradient function of IIR-ANF is constructed as \( J(\omega(k)) = e_1^2(k) \), while gradient function FIR-ANF is \( J(\omega(k)) = e_2^2(k) \).

2.2 Combined gradient

To integrate the advantages of IIR-ANF and FIR-ANF, a combined gradient function is defined by

\[ J(\omega(k)) = [e_1(k) + e_2(k)]^2 \]  

(5)

In computation, \( J(\omega(k)) \) is obtained according to the equation \( \hat{J}(\omega) = \frac{1}{N} \sum_{k=1}^{N} (e_1(k) + e_2(k))^2 \).

Take the noise into consideration, the average \( E[J(\omega)] \) can be derived as follows:

\[
E[J(\omega)] = 2A^2(\cos(\omega_0) - \cos(\omega))^2 + A^2H^2(\omega)/2
+ A^2H(\omega)N(\omega) \cos(\phi_x(\omega) - \phi_y(\omega))
+ E[v_n^2(k)] + 2E[v_n(k)v_{\mu}(k)] + E[v_{\mu}^2(k)]
\]

(6)

where
\[ H(\omega) = \frac{2|\cos \omega_b - \cos \omega|}{\sqrt{[(1 + \rho^2) \cos \omega_b - 2 \rho \cos \omega]^2 + [(1 - \rho^2) \sin \omega_b]^2}} \]

\[ N(\omega) = 2(\cos \omega_b - \cos \omega), \quad \phi_b(\omega) = \begin{cases} \arctan \frac{(1 - \rho^2) \sin \omega_b}{(1 + \rho^2) \cos \omega_b - 2 \rho \cos \omega} & \omega_b \leq \pi/2 \\ \pi + \arctan \frac{(1 - \rho^2) \sin \omega_b}{(1 + \rho^2) \cos \omega_b - 2 \rho \cos \omega} & \omega_b > \pi/2 \end{cases} \]

\[ E[v_1^2(k)] = 2\sigma^2 \left(1 + 2 \cos^2 \omega \right) \]

\[ E[v_h(k)v_h(k)] = \sigma^2 \left(1 - \frac{1}{\rho^2} + \frac{4 \cos^2 \omega \rho^2 - (1 + \rho^2)^2}{\rho^2} \right) \]

\[ E[v_h^2(k)] = \sigma^2 \left(1 - \frac{1 - \rho (1 + \rho^2) (1 + \rho^2)^2 - 8 \rho^2 \cos^2 \omega}{1 + \rho - \rho^2 (\rho^2 - 2 \rho \cos 2\omega + 1)} \right) \]

In order to evaluate the property of the combined gradient function, \( J_1(\omega) = E[v_1^2(k)] \) introduced in Ref. [2] and \( J_2(\omega) = E[v_2^2(k)\tilde{e}_2(k)] \) introduced in [3] are taken as compares. Gradient function curves are shown in Fig.1 when frequency \( \omega = \pi \). In simulations, assuming that \( A = 1, \theta = \pi/6, \sigma^2 = 0, \rho = 0.95 \) and \( N = 200 \). From the Fig.1, we can see that \( J(\omega) \) has a similar form with \( J_1(\omega) \) and \( J_2(\omega) \), but differs in amplitude and gradient. \( J_1(\omega) \) and \( J_2(\omega) \) are smoother than \( J(\omega) \), while \( J(\omega) \) has a faster convergence rate.

2.3 Bias correction

By substituting \( J(\omega(k)) = [e_1(k) + e_2(k)]^2 \) into Eq. (3), the estimation formula with the combined function can be derived as

\[ \alpha(k+1) = \alpha(k) - \mu[e_1(k) + e_2(k)][g_1(k) + g_2(k)] \]  

Where

\[ g_1(k) = \frac{\partial e_1(k)}{\partial \alpha(k)} = 2x(k-1)\sin \alpha(k), \quad g_2(k) = \frac{\partial e_2(k)}{\partial \alpha(k)} = 2x(k-1)\rho e_2(k-1)\sin \alpha(k) \]

Frequency estimation by Eq. (7) is biased as a result of the noise. Therefore, correction for estimation results is needed. We figure out the bias denoted by \( R(k) = (3p - 5)\sin 2\alpha(k)\sigma^2 \). As \( \sigma^2 = E[x(k)e_2(k)] \), \( \sigma^2 \) at the point of \( k \) approximates to \( \sigma^2(k) = x(k)e_1(k) \). Accordingly, Eq. (7) can be rewritten as:

\[ \alpha(k+1) = \alpha(k) - \mu G(k) \]
where \( C(k) = (3\rho - 5)\sin 2\omega(k) \), \( G(k) = [e_1(k) + e_2(k)]\{g_1(k) + g_2(k)\} - C(k)x(k)e_1(k) \).

### 2.4 Variable step-size

Fast convergence and stability variance are two key targets of ANF. In the steady state, the mean of frequency estimation variance is\[14\]

\[
E[\Delta^2(k)] = \frac{Ln(k) + Rn(k)}{L(k)/\mu - Ls(k) - Rs(k)}
\]

where \( \Delta = \dot{\omega}_b - \omega_b, Ln, Rn, L, Ls, Rs \) are parameters depended on noise. It’s obviously from Eq.(9) that variance deduces with the decrease of step \( \mu \), while convergence of the proposed method decelerates at the same time. Simulations shown in Fig.2 validate the conclusion.

Varying step-size according to variance is a current strategy for balance the conflict between convergence rate and precision. To reduce the influence of noise correlation, step size updates according Eq.(10).

\[
\mu(k + 1) = Q\mu(k) + \gamma[e_1(k) + e(k)e(k-D)]
\]

where \( e(k) = e_1(k) + e_2(k), \mu_{\text{min}} < \mu(n) < \mu_{\text{max}}, 0 < \alpha < 1, \gamma > 0 \). \( D \) is great than correlation radius of noise and less than correlation radius of input signal time. We regulate the step size according to self-correlation of errors \( e(k)e(k-D) \). For this reason, convergence and precision are balanced. Sensitivity index of the method on noise is weakened.

![Fig.2 the frequency estimation bias and MSE for ANF at different steps](image)

### 3. PROCESS

In summary, the process of the proposed method is displayed in Fig.3. In the first place, sample the input signal and initialize the parameters. Second, compute the outputs \( e_1(k) \) and \( e_2(k) \) according to \( N(z, \omega_b) \) and \( H(z, \omega_b) \). Then, calculate the differential coefficient \( g_1(k) \) and \( g_2(k) \) of \( e_1(k) \) and \( e_2(k) \), respectively. To compensate the error, we then need to calculate \( C(k) = (3\rho - 5)\sin 2\omega(k) \) and \( G(k) \). And then, we update \( \mu(k) \) according to Eq. (10). To guarantee the stability of algorithm, the length of step size \( \mu(k) \) should be controlled. The control equation is given by

\[
\begin{cases}
\mu(k) = \mu_{\text{max}}, & \mu(k) > \mu_{\text{max}} \\
\mu(k) = \mu_{\text{min}}, & \mu(k) \leq \mu_{\text{min}}
\end{cases}
\]

Finally, the frequency \( \omega(k + 1) \) at the point of \( k + 1 \) can be estimated according to Eq. (8).
4. RESULTS AND ANALYSIS

In this section, the performances of the proposed method are compared with those of the DPG and MPG algorithms in terms of the analytical and simulation results.

4.1 Simulation results

Simulation signals are produced with these parameters: $A = \sqrt{2}$, fixed value $\theta$ located in $[0, 2\pi)$, signal to noise ratio SNR=5dB, and frequency $\omega_0$ respectively initialized as $0.01\pi$, $0.05\pi$ and $0.1\pi$. Supposing $\rho = 0.9$, we obtain the frequency estimation shown in Fig.4. It can be seen from Fig.4 that the convergence time of the proposed algorithm is obviously shorter than DPG and MPG. Additionally, convergence time of MPG increases rapidly when initialization frequency is far from the signal frequency, while DPG and the proposed algorithm sit by.

The movement of step size is shown in Fig.5. The ANF owns comparative great step size at the convergence moment, which is good for improving the rate of convergence. Comparatively, the step size localizes in a small range at stability, which guarantees the accuracy of frequency estimation.
To obtain the bias and MSE estimations of these algorithms from the simulations, 20 runs are calculated with the same parameters. Fig. 6 shows the bias and MSE estimations. From Fig. 6, we can see both the bias and the MSE of the proposed algorithm is lower than MPG.

The bandwidth of ANF depends on $\rho$. The bias and MSE of frequency estimation at different initializations of $\rho$ are shown in Fig. 7. The accuracy of frequency estimation increases as $\rho$ tends to 1. Although computational results have some distortion with theoretical, it still reveals the precision trend of frequency estimation. The precision of MPG decline at the condition of $\rho < 1$, while the proposed algorithm is not sensitive to the initialization of $\rho$. Therefore, ANF parameters initialization is simplified in the proposed algorithm.
Fig. 8 Bias and MSE of frequency estimation with different SNR

The bias and MSE of frequency estimation at different SNR are shown in Fig. 8. It can be seen from Fig. 8 that the precision improves with SNR increases. What’s more, the proposed method performs better than MPG at the low SNR condition, which illuminates the anti-jamming character of the proposed method.

4.2 Application in CMF

To experimentally validate the proposed method, we take the Coriolis mass flowmeter (CMF) as an application. CMF calculates the mass flowrate by measuring the frequency and phase difference between two signals detected by electromagnetic sensors. In this work, oscillation signals come from the F200S CMF (with a 1700R transmitter). Sampling frequency is set at 20 kHz. As we cannot get the real frequency of CMF, only DPG, MPG and the proposed method are compared in the application, as shown in Fig. 9 and Tab. 1. As we know, the CMF frequency is fixed at steady flow. Results of the proposed method maintain equable compared with DPG and MPG, which validates the serviceability in application of the proposed method.

Fig. 9 Estimation curves of CMF frequency

Table. 1 Frequency estimation under different flow rates

| Mass flow rate (kg/min) | DPG (Hz) | MPG (Hz) | Proposed method (Hz) |
|-------------------------|----------|----------|----------------------|
| 2.9                     | 198.5803 | 198.7230 | 198.3688             |
| 10                      | 198.4521 | 198.5008 | 198.3704             |
| 82.1                    | 198.5237 | 198.6224 | 198.3658             |
| 102.2                   | 198.5179 | 198.6217 | 198.3540             |

5. CONCLUSION

In this paper, a variable step-size ANF using combined gradient algorithm is proposed for frequency estimation. Combined gradient algorithm is designed for improving constringency, with both FIR-ANF and IIR-ANF advantages been considered. The bias and MSE of frequency estimation are depressed as bias correction strategy been used in the proposed method. What’s more, variable step-size
is adopted for the balance of convergence and precision. Simulation results and CMF application validate the availability and superiority of the proposed method compared with DPG and MPG.

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