Cavity quantum electrodynamics (QED) (for a review, see [1]) provides a critical resource for quantum information processing [2–12]. For coherent manipulation, a key prerequisite is to reach the strong coupling regime, where the emitter-field coupling strength exceeds the decay rates of the emitter and the cavity field. In the past two decades great efforts have been made to improve the quality (Q) factor and reduce the mode volume (V) of the resonators for stronger interactions, using Fabry–Pérot cavities [13, 14], Bragg cavities [15–17], whispering-gallery mode cavities [18–23], photonic crystal cavities [24–30], hybrid plasmonic-photonic cavities [31] and transmission-line microwave cavities [32], along with theoretical studies of coupled-cavity QED through a waveguide [33–36]. However, it remains difficult to achieve high Q and small V simultaneously for the same-type resonator. Fundamentally, this is related to the diffraction limit. A smaller V corresponds to a larger radiative decay rate and more significant roughness scattering, leading to a lower Q. Different-type resonators possess their own unique properties, but the trade-off between high Q and small V still exists. For example, whispering-gallery mode cavities possess ultrahigh Q factors, while the mode volumes are relatively large; for photonic crystal cavities, sub-wavelength light confinement can be realized whereas the Q factors are relatively low.

Unlike the efforts to improve the $Q/\sqrt{V}$ figure of merit of the cavities, here we propose to reach the strong coupling regime via dark state resonances, which removes the requirement for high Q and small V for the same cavity. By coupling the originally weak-coupled cavity QED system with high cavity dissipation to an auxiliary cavity mode with high-Q but large V, a strong dark state interaction takes place. We demonstrate that vacuum Rabi oscillations and anharmonicity in the polariton dressed states occur even when the cavity decay rate is two orders of magnitude larger than the interaction rate.

As shown in Fig. 1(a), a cavity QED system, consisting of a dipole quantum emitter and a cavity, is coupled to an auxiliary cavity through a short-length single-mode waveguide. Here we take Fabry–Pérot cavity QED system as an example, while it allows generalization to other physical implementations, including solid-state circuit QED systems. In the frame rotating at the emitter’s resonance frequency $\omega_e$, the system Hamiltonian reads $H = \Delta_1 a_1^{\dagger}a_1 + \Delta_2 a_2^{\dagger}a_2 + g(a_1^{\dagger}\sigma_- + a_1\sigma_+) + J(a_1^{\dagger}a_2 + a_2^{\dagger}a_1)$, where $a_1$, $a_2$ are the annihilation operators of the two cavity modes; $\sigma_- \equiv |g\rangle\langle e| = \sigma_+^\dagger$ stands for the descending operator of the emitter with $|g\rangle$ ($|e\rangle$) being the ground (excited) state; $\Delta_1 \equiv \omega_1 - \omega_e$ and $\Delta_2 \equiv \omega_2 - \omega_e$ represent the detunings with $\omega_1$ ($\omega_2$) being the resonance frequency of mode $a_1$ ($a_2$); $g$ denotes the emitter.
field coupling strength between the emitter and mode $a_1$; $J$ describes the inter-cavity coupling strength between mode $a_1$ and $a_2$ [37–39]. Without loss of generality, we have assumed $g$ and $J$ to be real numbers. Taking the dissipations into consideration, the system is described by the quantum master equation

$$\dot{\rho} = i[\rho, H] + \kappa_1 D[a_1] \rho + \kappa_2 D[a_2] \rho + \gamma D[\sigma_-] \rho,$$

where $D[\sigma] = \sigma - \rho \sigma \rho$ is the standard dissipator in Lindblad form; $\kappa_1$, $\kappa_2$ and $\gamma$ represent the decay rates of modes $a_1$, $a_2$ and the emitter.

We show how highly dissipative cavity QED systems ($\kappa_1 \gg g$) can be turned into the effective strong coupling regime via dark state interaction. By eliminating mode $a_1$, we obtain the effective interaction between the emitter and the auxiliary cavity mode $a_2$, with the effective Hamiltonian

$$H_{\text{eff}} = (\Delta_2 - \beta^2 \Delta_1) a_2 \dagger a_2 + \frac{1}{2} \alpha^2 \Delta_1 \sigma_z + g_{\text{eff}} (a_2 \dagger \sigma_- + a_2 \sigma_+),$$

(1)

where $\sigma_z = |e\rangle \langle e| - |g\rangle \langle g|$, $\alpha$ and $\beta$ represent the scaled dimensionless interaction parameters given by $\alpha = g/\sqrt{\Delta_1^2 + \kappa_1^2 / 4}$ and $\beta = J/\sqrt{\Delta_1^2 + \kappa_1^2 / 4}$, respectively. The effective coupling strength, detuning, decay rates of the cavity field and the emitter are described by

$$g_{\text{eff}} = \beta g, \quad \Delta_{\text{eff}} = \Delta_2 + (\alpha^2 - \beta^2) \Delta_1,$$

$$\kappa_{\text{eff}} = \kappa_2 + \beta \sqrt{\kappa_1^2}, \quad \gamma_{\text{eff}} = \gamma + \alpha^2 \kappa_1.$$

(2)

In Fig. 1(b) we plot the energy level diagram, which displays the lowest four energy levels of the system. It reveals that the emitter-field interaction between state $|e\rangle$ (short for $|e\rangle|0\rangle_1|0\rangle_2$) and state $|1\rangle$ (short for $|g\rangle|1\rangle_1|0\rangle_2$), together with the inter-cavity interaction between state $|2\rangle$ (short for $|g\rangle|0\rangle_1|1\rangle_2$) and state $|1\rangle$, yields the effective dark state interaction between state $|e\rangle$ and state $|2\rangle$. As shown in Eqs. (1)-(2) and illustrated in Fig. 1(c), after the the elimination of state states $|1\rangle$, the states $|e\rangle$ and $|2\rangle$ acquire energy shifts of $-\alpha^2 \Delta_1$ and $-\beta^2 \Delta_1$, together with broadenings of $\alpha^2 \kappa_1$ and $\beta \kappa_1$.

Equations (2) show that the effective coupling strength $g_{\text{eff}}$ depends linearly on $\beta$ while the effective decay rates $\kappa_{\text{eff}}$ and $\gamma_{\text{eff}}$ are quadratic functions of $\beta$ and $\alpha$, respectively. As a result, for $(\alpha, \beta) \ll 1$, the effective coupling strength will be larger than the decay rates, driving the effective interaction into the strong coupling regime. In Figs. 2(a) and (b) the parameters given by Eq. (2) as functions of inter-cavity interaction strength $J$ and the first cavity mode’s detuning $\Delta_1$ are plotted, respectively. It reveals that with a suitable $J$ and $\Delta_1$, the effective coupling strength $g_{\text{eff}}$ exceeds both decay rates $\kappa_{\text{eff}}$ and $\gamma_{\text{eff}}$, even for large cavity decay rate $\kappa_1/g = 100$. As shown in the insets of Figs. 2(a) and (b), the ranges of $J$ and $\Delta_1$ for effective strong coupling have both lower and upper bounds. To gain more insights on the parameter ranges, in Figs. 2(c)-(e) we plot $g_{\text{eff}}/\kappa_{\text{eff}}$, $g_{\text{eff}}/\gamma_{\text{eff}}$ and the cooperativity parameter $C_{\text{eff}} = g_{\text{eff}}^2 / (\kappa_{\text{eff}} \gamma_{\text{eff}})$ as functions of $\Delta_1$ and $J$. It reveals that a large $\Delta_1$ and a corresponding large $J$ lead the system deeply into the effective strong coupling regime. Examining Eq. (2), for $J > g > \kappa_2 \sim \gamma$, it gives $\kappa_{\text{eff}} > \gamma_{\text{eff}}$ with negligible $\gamma_{\text{eff}}$. In this case, the maximum effective coupling-to-decay rate ratio reads $g_{\text{eff}}/\kappa_{\text{eff}} = g/\sqrt{2 \kappa_1 \kappa_2}$, obtained when $\beta = \sqrt{\kappa_2 / \kappa_1}$. Thus the strong coupling condition $g_{\text{eff}} > \kappa_{\text{eff}}$ can be fulfilled when $\kappa_2 < g^2/(4\kappa_1)$. This is verified by the contour plot in Fig. 2(f), which displays $g_{\text{eff}}/\kappa_{\text{eff}}$ as a function of $\kappa_1$ and $\kappa_2$. The bottom left region indicates the strong effective coupling parameter regime, with $g_{\text{eff}}$ in excess of $\kappa_{\text{eff}}$ by more than one order of magnitude.

To demonstrate that the effective parameters in Eq. (2) exactly describe the physical interaction, we dia-
shows that the eigenenergies of the two states are split by $\kappa$. Note that the global energy shift of $0|E_0|$ for the dark state doublets $|\pm\rangle$ exceed $100$. As shown in Fig. 3(b) we plot the eigenenergies and linewidths for states $|0\rangle_{1,2}$ as functions of the detuning between mode $a_2$ and the emitter ($\Delta_2/g$). Prominent avoided crossing phenomenon occurs for the eigenenergies, which occurs for the effective resonant case $\Delta_{eff} = 0$ (gray vertical line). Near the avoided crossing point the linewidths of the two polariton states are averaged compared with the large $\Delta_{eff}$ case (inset), and are swapped for increasing detuning as indication of the quantum strong coupling. The avoided crossing is further examined in Fig. 3(c), which shows the emitter’s spectra $S(\omega)$ for various detunings $\Delta_2$ through the weak excitation of mode $a_2$. It shows close agreement with the effective spectra $S_{eff}(\omega)$ obtained from the effective interaction [Fig. 3(d)].

In the time domain, vacuum Rabi oscillation is a direct evidence of the coherent energy exchange between the emitter and the cavity photon field. Here we numerically solve the quantum master equation to obtain the exact results. We assume initially the emitter is in the excited state and the two cavity modes are in their vacuum states, then we obtain the exact numerical results for the time evolution of the mean photon numbers $N_1(t) = \langle a_1^\dagger a_1 \rangle$, $N_2(t) = \langle a_2^\dagger a_2 \rangle$ and the probability for the emitter being in the excited state $P_e^\text{eff}(t) = (\langle \sigma_z \rangle + 1)/2$. As shown in Fig. 4(a), even for $\kappa_1/g = 100$, vacuum Rabi oscillation phenomenon occurs for several periods, revealing that the decoherence time is much longer than the energy exchange period. This is in contrary to the case without the auxiliary cavity as shown in the left inset of Fig. 4(a), where the emitter exponentially decays from the excited state. Note that the occupancy in mode $a_1$ oscillates with the maximum photon number below $10^{-5}$ as shown in the right inset of Fig. 4(a)], while the occupancy in mode $a_2$ oscillates with the maximum photon number exceeding 0.5. This reveals that the interaction is mainly between the emitter and mode $a_2$, while mode $a_1$ is only virtually excited. The analytical results for the emitter’s occupancy in the excited state, obtained from the effective parameters [Eq. (2)], is described by

$$P_e^\text{eff}(t) = \exp\left(-\frac{\kappa_{eff} + \gamma_{eff}}{2}t\right) \cos^2(g_{eff}t).$$

With vacuum Rabi frequencies $\Omega_R = 2g_{eff}$ and the decay rates $(\kappa_{eff} + \gamma_{eff})/2$, the results in the effective dark state picture (red solid curve) are in good accordance with the exact numerical results (red closed circles).

The effective strong coupling offers great potential for single-photon manipulation and quantum logic gate op-

FIG. 3. (color online) (a) and (b): Eigenvalues $E_{\pm}$ for states $|0\rangle_{1,2}$ as functions of $\kappa_1/g$ and $\Delta_2/g$. The main figures and the insets show the real and imaginary parts of the eigenvalues, respectively. The circles correspond to the exact results and the curves denote the results obtained from the effective Hamiltonian and effective parameters [Eqs. (1)-(2)]. The gray vertical line in (b) denotes $\Delta_2 = (\beta^2 - \alpha^2)\Delta_1$, (c) and (d): Normalized spectra $S(\omega)$ and effective spectra $S_{eff}(\omega)$ of the emitter for various $\Delta_2$. From top to bottom, $\Delta_2$ decrease from $(\beta^2 - \alpha^2)\Delta_1 + 9\gamma_{eff}$ to $(\beta^2 - \alpha^2)\Delta_1 + 9\gamma_{eff}$ with step $g_{eff}$. The common parameters are the same as Fig. 2(a)-(e).

nalize the system Hamiltonian in the subspace of the first excited states. Using the non-Hermitian Hamiltonian where the decays are taken into account, the eigenenergies and broadenings of each states are obtained as the real and imaginary parts of the eigenvalues, respectively. For the first excited states, after the diagonalization under large detuning $\Delta_1$, the eigenstates read $|1\rangle_{1,2} \approx |g\rangle |1\rangle_{1,2} |0\rangle_{1,2}$, $|0\rangle_{1,2} \approx (|e\rangle |0\rangle_{1,2} \pm |g\rangle |1\rangle_{1,2})/\sqrt{2}$. It reveals that the states $|0\rangle_{1,2}$ $|e\rangle_{1,2}$ are dark state doublets with respect to the decay of mode $a_1$. In Figs. 3(a) we plot the real and imaginary parts of the eigenvalues $E_{\pm}$ for the dark state doublets $|0\rangle_{1,2}$ as functions of $\kappa_1/g$, where the real (imaginary) parts represent the eigenenergies (linewidths) of the states. It shows that the eigenenergies of the two states are split by $2\gamma_{eff} = 0.01g$, and the linewidths are much smaller than the energy splitting (inset), even for $\kappa_1/g$ exceeding 100. Note that the global energy shift of 0.001g ($= -\alpha^2\Delta_1$) can be eliminated by applying a unitary transformation to the effective Hamiltonian [Eq. (1)]. The results obtained from the effective Hamiltonian and effective parameters [Eqs. (1) and (2)] are in good accordance with the exact results for both the real and imaginary parts of the eigenvalues. For $\kappa_1/g \gtrsim 800$, discrepancy occurs because $\Delta_1 \gg \kappa_1$ is not satisfied.
strong coupling regime, with an external field pumping the system, it is also promising for the generation of one-atom lasing [42–44].

It should be noted that, although the auxiliary cavity is required to be high-\(Q\) (\(\kappa_2 < g\)), it does not need to interact directly with the emitter, and its mode volume is not necessary to be small. Therefore, the scheme does not require a high figure of merit \(Q/\sqrt{V}\) for the auxiliary cavity. Together with the allowed low \(Q\) factor for the primary cavity, both the two cavities can be low in figure of merit \(Q/\sqrt{V}\). This approach is also generic and can be applied to any cavity QED systems with different physical implementations, including solid-state circuit QED systems. In the viewpoint of mode density shaping [45], at the second cavity mode’s resonance frequency the system’s mode density is enhanced, and it leads to the effective interaction between the second cavity mode and the emitter.

In summary, we have presented a protocol for realizing effective strong coupling in a highly-dissipative cavity QED system. By employing the coupled cavity configuration, we show that a highly dissipative cavity interacting simultaneously with a single emitter and an auxiliary cavity leads to the dark state resonance between the emitter and the auxiliary cavity. It is demonstrated that effective strong coupling can be achieved even with low \(Q/\sqrt{V}\) cavities, with prominent vacuum Rabi oscillation and ladder anharmonicity phenomena for photon blockade. The cavity coupled to the emitter can be highly dissipative even with the decay rate in excess of the interaction strength by two orders of magnitude. The system enables single photon manipulation like photon blockade and quantum logic gate operations. This approach offers opportunities to exploit both theoretical and experimental physics in the strong light-matter interaction regime without stringent cavity requirements.

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[1] G. Khitrova, H. M. Gibbs, M. Kira, S. W. Koch, and A. Scherer, Nature Phys. 2, 81 (2006).
[2] S.-B. Zheng and G.-C. Guo, Phys. Rev. Lett. 85, 2392 (2000).
[3] T. Pelizzari, S. A. Gardiner, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 75, 3788 (1995).
[4] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi,
