Research on Mathematical Methods of Improving Fama and French Three-factor Model Based on ETF Factors

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Abstract. Will ETF factors help to improve Fama and French 3 factor Model? Research has examined the Fama French model in different countries and its risk premium in different mathematical ways. This paper reconstructs the Fama-French model by replacing market capitalization size index (SMB) and book-to-market ratio index (HML) to other Exchange Traded Fund (ETF) factors to investigate whether ETF factors will improve the Fama French model. After comparing 15 ETFs, Factors EMD (GDP growth index) and SMG (factor IYC minus factor IYK) are chosen for our adapted risk model because of a relatively low rate of correlation and covariance. 20 sample stocks are selected from various industries and market capitalization size, and used to test to the performance of FFM and the new FFM. The result has been analyzed from 3 different ways: correlation and covariance, R squared, and p value. Although the new model fails to perform better than a traditional FFM, it may be improved by adding more factors or choosing factors which are more representative of the market.

Keywords: Fama French 3 factors model, ETFs, linear regression.

1. Introduction

Fama-French 3 Factor Model has been a relatively controversial tool in finance. Though it outplays CAPM greatly in maximizing R2 and minimizing P, Fama-French 3 Factor Model is not supported by traditional theory. The magic of SMB and HML cannot be explained; instead, Fama and French simply ran regressions for tens of thousands of times and obtained the optimal factors. In 2014, the latest Fama-French 5 Factor Model defeated the Fama-French 3 factor Model, astounding the market and academia. With more factors involved, it was believed that accuracy and specific risk will be controlled on a favorable scale. As a result, academics are now prone to finding some indexes to approximate the real market return, though no significant outcome has been yielded yet.

This paper aims to test whether it is possible to outplay the Fama-French 3 Factor Model with the same number of factors. The new model introduces two factors, EMD and SMG. EMD refers to the difference between VWO and VEA, representing the different behaviors of developed markets and
developing ones. SMG refers to the difference between IYC and IYK, representing the reverse relationship of consumer goods and services. These two new factors are expected to explain the market better than SMB and HML.

2. Portfolio selection

The factors that determine a good portfolio worth investing in is not just its ability to generate a considerable amount of returns. Moreover, the risk behind such a portfolio is also an essential factor that we need to consider.

Mathematically speculations that yield maximum returns could be achieved by using linear regression, optimization, and back testing. The challenge is to minimize the risk of a portfolio. In other words, one needs to lower the variance in a portfolio as much as possible.

When choosing the 20 stocks for our portfolio, we wanted to make those 20 stocks as diverse as possible, because the diversification of stocks can always efficiently reduce variance. Through a mathematical approach, variance is found to be positively related to the covariance of stocks in that portfolio. It is essential to lower the covariance of stocks, in order to minimize variance. In other words, portfolio diversification can be controlled by measuring the covariance between stocks.

Different stocks also belong to different sectors or fields, thus diversifying and lowering the variance as well as risk. We deliberately chose different stocks from different sectors. For instance, stocks are included from fields such as the internet (GOOG, FB, UIS), technology (AAPL, TSLA, JBL), national defense (LMT, KTOS), express delivery (FDX), banking and investment (JPM, WY), energy and resources (XOM, RRC), the medical industry (MDT, JNJ, XBIT), publishing (PCH), etc.

Although it may seem contradictory to include several stocks from the same sector, as this does not lower the covariance, other stock characteristics besides sectors are considered during the selection process. The other effect is the market capital size of each stock. Stocks could be generally divided into two categories: stocks of big-cap companies and stocks of small-cap companies. Twenty stocks can be divided into two groups following market rules: ten stocks of big-cap companies and another ten from small-cap companies. By doing so, the risk of stock portfolio is limited from the very beginning.

| Stock Names | Industry            | Cap size(big/small) |
|-------------|---------------------|---------------------|
| FB          | Internet            | Big                 |
| GOOG        | Internet            | Big                 |
| UIS         | Internet            | Big                 |
| AAPL        | Technology          | Big                 |
| TSLA        | Technology          | Big                 |
| FLEX        | Technology          | Small               |
| JBL         | Technology          | Small               |
| KTOS        | National defense    | Small               |
| LMT         | National defense    | Big                 |
| WY          | Investment          | Big                 |
| JPM         | Investment          | Small               |
| XOM         | Energy              | Big                 |
| RRC         | Energy              | Small               |
| MDT         | Medical             | Small               |
| JNJ         | Medical             | Big                 |
| XBIT        | Medical             | Small               |
| FDX         | Express delivery    | Big                 |
| PCH         | Publishing          | Small               |
| AER         | Transportation      | Small               |
| CVTI        | Transportation      | Small               |

(Note: Big Cap size is defined by a market capitalization of $10 billion and greater, Small Caps will be $10 billion and below.) Data retrieved from Yahoo Finance
The decision of what stocks to include in the portfolio then allowed us to extract data from Yahoo Finance. Since it is required to have data across five years’ periods, we obtained historical data for each stock from July 2015 to August 2020. (the data starts from July because we needed the percent change starting August 2015). Furthermore, after downloading data for each of the twenty stocks, we used the formula to calculate each month’s adjusted closing price percentage change. Then, all the percentage change data was put into one CSV file called stocks, with corresponding stock names and dates.

3. What Are Factor ETFs?
As outlined by the U.S. Security and Exchange Commission (SEC), ETFs or exchange-traded funds are referred to as investment companies registered at the SEC to provide people with a type of investment as an alternative to stocks, bonds, or other assets. From a general perspective, an ETF can be regarded as a hamper of securities trading on an exchange rate. By definition, most ETFs are traded depending on a rate or index, which usually reflects a section of market performance. For example, XLF (Financial Select Sector SPDR Fund) is an ETF managed by State Street Global Advisor (SSGA), from which investors can see a big picture of the financial industries’ performance. In fact, ETF factors are utilized by analysts and investors as a tool for gauging the fluctuation of a specific section of the market. Today, we will use ETF factors to assist the modeling.

In the following project, ETFs (exchange-traded funds) are used to replace the Fama French risk model’s traditional three factors. An Exchange-traded fund (ETF) is a type of investment fund, a collection of securities listed on exchanges that often tracks an underlying index, just like stocks.[1] Introduced in 1993, ETFs have proliferated. By the end of 2001, almost 50% of total assets in equity mutual funds were ETFs.[2] Unlike a stock, there are multiple underlying assets within an ETF. These assets can spread across various industry sectors or can be isolated to one particular sector. ETFs span hundreds of indices across various fixed-income markets and trace nearly 126 commodities and 22 currency indices. [3] Therefore, ETFs offer diversification and serve as a cost-effective alternative compared to equity mutual funds and individual stocks. With low turnover, tax efficiency, and daily trading, ETFs are creative solutions to investing. [4] ETFs’ trading volume can reflect the volatility changes of corresponding indexes at both monthly and daily frequencies. [5]

4. The Fama and French Three Factors Model
The Fama and French three factors model was an asset pricing model invented by Eugene Fama and Kenneth French in 1992. Building upon the traditional CAPM (Capital Asset Pricing Model), which only relied on a single variable — the returns of the whole market — to depict the performance of a portfolio. CAPM builds the relationship between returns and risks of the portfolio, but it does not fit the model very well. In investment, the ideal portfolios are the ones which have higher returns with smaller risks. Based on CAPM, stocks with small caps are preferred because portfolios with a higher proportion of small cap companies could be lower than the CAPM result, while large-cap stocks are predicted to be higher. Within the Fama and French model, two additional factors are considered: size risk (SMB) and value risk (HML). The first factor (SMB) is defined by small market caps minus those with big market cap; the second factor (HML) stood for the difference between value stocks with high book-to-market ratios and growth stocks.

The model was built on authorized historical data of the American online stock market. To construct the model, different data were collected and twenty stocks were selected to form the portfolio. The purpose of the Fama and French Three Factors Model was to further specify the capital investment model, which was not precise in the construction of CAPM. As we regressed each stock’s returns in the portfolio by constructing regression models of historical stocks (factors) prices, and calculating the covariance, correlation, and specific risk, our team successfully derived the Fama and French Three Factors Model.
5. The New Model
With the rapid development of emerging markets, EM countries are taking a larger share in contributing to the global market's prosperity. Based on statistics provided by IMF World Economic Outlook, most countries classified as EMs showed great economic vitality in terms of real GDP growth in 2020. (Chart A) In the long term, developing economies in the emerging market outperformed advanced economies with a faster rate of development as observed through the trend of GDP growth from 1980 to 2021. (Chart B. Blue: emerging market; Yellow: world average; Red: advanced economies) World Economic Outlook (April 2020) - Real GDP Growth, n.d.) According to the research above, VOW, an ETF factor reflecting the investment return of stocks in emerging countries, was determined to be included in our model. we subtracted it with VEA, which is the ETF factor in measuring stocks' returns in developed markets, and we named the new factor as 'EMD' (Emerging minus developed), which accounts for stocks traded in developing countries with higher expected returns than those in the developed market.

![Fig. 1 Map of real GDP growth at annual percent change](image1)

Sources: IMF World Economic Outlook.

![Fig. 2 Graph of annual percent change trend from 1980-2025](image2)

Sources: IMF World Economic Outlook.
Another factor in our model which we paid attention to was the business types that could be influential factors in measuring manager performance. It is commonly known that two common business types exist, offering services or providing goods. In ETF factors, the IYK refers to an index to measure companies' performance by measuring profits gained mainly from providing consumer services. Simultaneously, the IYC represented the overall state of operation of firms primarily involved in production-related work, like manufacture, marketing, and sales. Similarly, we subtracted the difference between IYC and IYK, naming the newly generated factor as 'SMG' (Service minus Goods), which accounts for stocks in the service industry with better performance than those in the product industry.

6. Process Analysis
One of the purposes of using a factor model is to separate portfolio risk from the risk of factors (EMD, SMG), as well as each stock's specific risks. Jupyter notebook is used to analyze the data and form the risk model. In the beginning, the classic Fama French 3 factor model was built. To get a reasonable cross-section of the market, stocks are selected from different sectors and different caps. The three factors of FFM are SMB (small minus big), HML (high minus low), and ITOT (the portfolio's return less the risk-free rate of return). SMB stands for the difference of stock returns between publicly traded companies and small market caps, while HML accounts for stocks with high book-to-market ratios that usually generate higher returns.

Table 2. Covariance of factors in traditional Fama French Model

|       | Mkt-Rf     | SMB         | HML          | Rf           |
|-------|------------|-------------|--------------|--------------|
| Mkt-Rf| 20.361248  | 4.724336    | 3.342159     | -0.042641    |
| SMB   | 4.724336   | 6.036739    | 1.699699     | -0.035398    |
| HML   | 3.342159   | 1.699699    | 10.772663    | -0.037102    |
| Rf    | -0.042641  | -0.035398   | -0.037102    | 0.004986     |

Table 3. Correlation of factors in traditional Fama French Model

|       | Mkt-Rf     | SMB         | HML          | Rf           |
|-------|------------|-------------|--------------|--------------|
| Mkt-Rf| 1.000000   | 0.426125    | 0.225665     | -0.133827    |
| SMB   | 0.426125   | 1.000000    | 0.210766     | -0.204031    |
| HML   | 0.225665   | 0.210766    | 1.000000     | -0.160084    |
| Rf    | -0.133827  | -0.204031   | -0.160084    | 1.000000     |

The above result shows the covariance and correlation between 3 factors in the model. It is evident that these three factors have small correlations, and some of them approach zero. In finding ways to improve this model’s reliability, our solution is to structure a new risk model with three factors extracted from ETFs.

More factors lead to more interaction between them, making it more difficult to summarize stock returns' cross-section. (David, 2018) After comparing 15 different ETF factors, we decided to utilize the model with the same number of factors as with FFM. Factors EMD (factor VWO minus factor VEA) and SMG (factor IYC minus factor IYK) are chosen for our adapted risk model because they have a relatively low correlation and covariance rate. (see data at appendix C)

7. Statistics Analysis
After reconstructing FFM by Factors EMD (factor VWO minus factor VEA) and SMG (factor IYC minus factor IYK), it is time to compare this new model to traditional FFM. Using linear regression, FFM and improved FFM were separately used to analyze 20 various stocks in the same portfolio. The correlation and covariance of each factor in these two models will be shown and analyzed.
7.1. Correlation and Covariance

In the calculation section, our team compared the FFM's covariance data and the portfolio in a new combination of stocks. (see data at Appendixes) It is defined that a portfolio with higher covariance would lead to higher risk; therefore, lowering the covariance is crucial in investment. In the program, spec_risks_fama.cov() was used to indicate the covariance of FFM's specific risks, and spec_risks_new.cov() was the covariance of specific risks of the new model. According to the results, two stocks in the FFM clearly perform better than those in the new portfolio, while others cannot be counted as an absolute advantage or disadvantage. In another way, FFM is slightly better than the new portfolio in terms of risk control.

Moreover, we also constructed the correlation between two different models in the program. As for the 15 factors we chose in order to compare with FFM, only two stocks (EMD, SMG) have lower correlation, which is mostly lower than 0.5, while others have a correlation value close to 1. Two stocks that are highly correlated would lead to higher overall risks in the portfolio. On the contrary, each combination of stocks in FFM has a correlation that is lower than 0.5, which means that it would be less risky to use this model.

7.2. R squared

The R-squared and p-value are used to compare the performance of these two models. Morck et al. (2000) were the earliest to use the R-squared to measure the market model. In their paper, they discovered that R-squared might become a measure of price inefficiency.[6] After this discovery, scholars began to study the question of how to interpret R-squared statistics. In 2003, Durnev et al. found that lower R-squared indicates higher stock price informativeness.[7] In contrast, Pagano and Schwartz (2003) take the opposite perspective and assume that higher R-squared is associated with higher price efficiency. [7] Riccardo also suggested that R-squared is a measure of price efficiency. [8]

7.3. P-value

Another measurement is the p-value. A p-value is an area under the curve of a probability distribution defined by a mathematical model. (John, 2018) For example, a p-value of 0.05 means there is a 5% risk to get a false-positive result. The smaller the P-value, the stronger the new model's evidence in providing more information than the original FFM.

### Table 4. Statistics of two model’s R squared value

| Stock Names | Fama French Model | New Model | Larger R squared |
|-------------|-------------------|-----------|------------------|
| AAPL        | 0.484474          | 0.404289  | Fama             |
| GOOG        | 0.585200          | 0.552898  | Fama             |
| JPM         | 0.769982          | 0.578536  | Fama             |
| XOM         | 0.678246          | 0.578949  | Fama             |
| MDT         | 0.348255          | 0.311822  | Fama             |
| LMT         | 0.534643          | 0.470931  | Fama             |
| FDX         | 0.552777          | 0.549907  | Fama             |
| FB          | 0.468898          | 0.468170  | Fama             |
| JNJ         | 0.471832          | 0.479860  | New              |
| TSLA        | 0.154110          | 0.162839  | New              |
| WY          | 0.721465          | 0.714707  | Fama             |
| UIS         | 0.129929          | 0.126519  | Fama             |
| XBIT        | 0.116886          | 0.153487  | New              |
| PCH         | 0.540578          | 0.538647  | Fama             |
| RRC         | 0.263231          | 0.197458  | Fama             |
| KTOA        | 0.136785          | 0.137331  | New              |
| JBL         | 0.336999          | 0.326232  | Fama             |
| FLEX        | 0.523253          | 0.545448  | New              |
| AER         | 0.691285          | 0.655968  | Fama             |
| CVTI        | 0.380970          | 0.259380  | Fama             |
Table 5. Statistics of two model’s p value

| Stock Names | Fama French Model | New Model | Larger P value |
|-------------|------------------|-----------|---------------|
| AAPL        | 3.760178e-08     | 1.980462e-06 | New          |
| GOOG        | 9.335670e-11     | 7.420367e-10 | New          |
| JPM         | 7.187503e-18     | 1.450768e-10 | New          |
| XOM         | 8.159683e-14     | 1.411948e-10 | New          |
| MDT         | 2.292451e-05     | 1.000107e-04 | New          |
| LMT         | 2.239816e-09     | 7.670111e-08 | New          |
| FDX         | 7.475955e-10     | 8.920874e-10 | New          |
| FB          | 8.522210e-08     | 8.849000e-08 | New          |
| JNJ         | 7.319104e-08     | 4.803991e-08 | Fama         |
| TSLA        | 2.382122e-02     | 1.822685e-02 | Fama         |
| WY          | 1.480464e-15     | 2.883757e-15 | New          |
| UIS         | 4.894284e-02     | 5.403303e-02 | New          |
| XBIT        | 7.120302e-02     | 2.427751e-02 | Fama         |
| PCH         | 1.571691e-09     | 1.764583e-09 | New          |
| RRC         | 6.264121e-04     | 6.058866e-03 | New          |
| KTOOS       | 4.003565e-02     | 3.939522e-02 | Fama         |
| JBL         | 3.648022e-05     | 5.643652e-05 | New          |
| FLEX        | 4.363894e-09     | 1.171138e-09 | Fama         |
| AER         | 2.585507e-14     | 5.234432e-13 | New          |
| CVTI        | 5.647421e-06     | 7.201705e-04 | New          |

The R-squared and p-value of each model were calculated. If the new model had a larger r-squared, the result would appear as “New”. Otherwise, FFM is kept as our optimal choice. In addition, the result will appear as New if the new model had a larger p-value. This means that the new model is not the better choice. According to the above table, the new model has shown a smaller r-squared in 5 stocks, consisting of only 25% of the total portfolio. This result is apparently not confident of proving that the new model is better than the classical FFM. Moreover, FFM had a better performance in p-value, as 75% of the adapted model had a larger p-value than FFM, which means traditional FFM is more reliable than the adapted model.

Using these three methods for analysis, we conclude that FFM explains more information about stocks than new models. Therefore, the Fama French model is a better choice.

8. Conclusion and discussion

In general, our model's outcomes (R-squared and p-value) are inferior to the outcomes of FFM. Our p-values, which indicate reliability, is higher than FFM's in 75% of outcomes, meaning that our model cannot explain the market efficiently. At the same time, the R-squared of our model, which refers to the goodness of fit, is lower than FFM's, indicating that the possible return of our model will be lower. However, though our model failed in competing against FFM, we observed some interesting facts during our experiment:

1. Indexes with negative correlations could sometimes explain the market better. (though the principle behind this phenomenon remains unknown)
2. Selected factors must be highly representative of the whole market to obtain high goodness fit.
3. FFM is almost impossible to be defeated using two factors selected from indexes on hand
4. Though it is inferior, our model outplays FFM in some circumstances, meaning that our model has the potential to be used in a specified field.

Instead of classic theory, FFM has dominated the market for almost 30 years based on empirical facts. For decades, economists strove to gain a more precise model by adding factors to the model. Numerous essays and articles have been published to address this problem. However, from the perspective of an
observant and student, it is believed that we will find the real impetus behind market fluctuations in a graceful mathematic form, instead of mere factors.

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