Non-locality: what are the odds?

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Abstract

As part of a challenge to critics of Bell’s analysis of the EPR argument, framed in the form of a bet, R. D. Gill formulated criteria to assure that all non-locality is precluded from simulation-algorithms used to test Bell’s theorem. This is achieved in part by parceling out the subroutines for the source and both detectors to three separate computers. He argues that, in light of Bell’s theorem, following these criteria absolutely precludes mimicking EPR-B correlations as computed with Quantum Mechanics and observed in experiments. Herein, nevertheless, we describe just such a local algorithm, fully faithful to his criteria, that yields results mimicking exactly quantum correlations. We observe that our simulation-algorithm succeeds by altering an implicit assumption made by Bell to the equivalent effect that the source of EPR pairs is a single Poisson process followed by deterministic detection. Instead we assume the converse, namely that the source is deterministic but detection involves multiple, independent Poisson processes, one at each detector with an intensity given by Malus’ Law. Finally, we speculate on some consequences this might have for quantum computing algorithms.

1 State of Play

Richard D. Gill, a statistician, on the basis that probability and statistics have much to do with quantum mechanics (QM), and by his own declaration fascinated by the exotica of foundations disputes, has called for increased attention to these matters by his profession. To this purpose, he himself has published studies that translate some of the current notions oft seen in the foundations of QM into the jargon and style of his discipline. So far Gill’s results have tended to support the conventional wisdom, the majority viewpoint held by, for example, several who have done experiments that they credit even with extending the mystical facets of QM right into the realm of practical applications; e.g., quantum computing, teleportation and the like.

In addition, Gill has criticized the views of those who are distressed by the implications of modern foundations’ orthodoxy of QM, specifically and particularly that the non-locality promised by John Bell violates the order of cause and effect. 1 Some of

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1 A renowned proponent of Bell’s view told this writer, on the grounds that humankind probably is simply incapable of grasping all truth, that he was not distressed by this inversion. If, however, Science is that human enterprise, the point of which is to use the tools of logic (Mathematics) and observation as available to understand the world, then such an orientation is in a distinctly different category.
their efforts are based on a point first seen, to this writer’s best information, by Jaynes: Bell misapplied Bayes’s formula.\footnote{3}

An independent study in this vein initiated by Accardi, as an “anti-Bellist,” led to a local computer simulation of EPR-B experiments that he claimed exhibit the so-called non-local statistics of QM.\footnote{4} Accardi’s simulation fascinated Gill, who found the tactic meritorious but the results so far unconvincing, and so challenged Accardi to find a protocol that is beyond dispute. Convinced that it can’t be done, and to enliven the matter, Gill offered a 3000 Euro reward, essentially a bet, to Accardi should he succeed.\footnote{1, 5}

The present writer is also one long sceptical of arguments that non-locality is logically well founded.\footnote{6} He holds the view that some essential feature is being overlooked, much as von Neumann overlooked crucial contrary physics details in formulating his “theorem” to the same final effect as Bell’s. This writer’s work led to a series of arguments indicating that Bell’s argumentation must contain a flaw.\footnote{7, 8} Eventually this culminated in a study in which all the results from generic experiments credited with verifying Bell’s results were calculated using only principles of classical Physics, without reference to anything intimating that non-locality was in play.\footnote{9} In spite of a vigorous e-mail defense, these calculations failed to impress Gill, in part, apparently, for lack of clarity on how non-locality was precluded. Gill continued to insist that no protocol satisfying certain constraints that he formulated to enforce locality, could be envisioned.

It is the purpose of this note to present an extention of these calculations in the form of a simulation algorithm or protocol conforming with Gill’s criteria. In the following section the conditions set out by Gill will be presented; thereafter, the simulation we propose will be described. Finally, we delineate the technical details that make our simulation possible.

## 2 Gill’s desiderata

The purpose of Gill’s protocol is to preclude absolutely any structure that could covertly exploit non-locality. This is achieved by parceling out subroutines simulating the source and the two detector stations to three separate computers connected by wires conceptually equipped with diodes that allow only one way signaling. In this way, one can be sure that there is no feedback in the logic that mimics non-local interaction. This writer holds these specifications as a useful contribution to the discussion of this matter because, for lack of a unique definition of “non-locality,” alternates that are, e.g., actually only indirect statistical congruences, have clouded the matter.

Gill’s specifications are as follows,\footnote{5} we quote:

1. Computer O, which we call the source, sends information to computers X and Y, the measurement stations. It can be anything. It can be random (previously stored outcomes of actual random experiments) or pseudo-random or deterministic. It can depend in an arbitrary way on the results of past trials (see item 5). Without loss of generality it can be considered to be the same—send to each computer, both its own message and
the message for the other.

2. Computers A and B, which we call randomizers, send each a measurement-setting-label, namely a 1 or a 2, to computers X and Y. Actually, I will generate the labels to simulate independent fair coin tosses (I might even use the outcomes of real fair coin tosses, done secretly in advance and saved on my computers’ hard disks.

3. Computers X and Y each output ±1, computed in whatever way [an opponent] likes from the available information at each measurement station. He has all the possibilities mentioned under item 1. What each of these two computers do not have, is the measurement-setting-label which was delivered to the other. Denote the outcomes \( x^{(n)} \) and \( y^{(n)} \).

4. Computers A and B each output the measurement-setting-label which they had previously sent to X and Y. Denote these labels \( a^{(n)} \) and \( b^{(n)} \). An independent referee will confirm that these are identical to the labels given to [an opponent] in item 2.

5. Computers X, O and Y may communicate with one another in any way they like. In particular, all past setting labels are available to all locations. As far as I am concerned, [an opponent] may even alter the computer programs or memories of his machines.

At this point Gill proceeds to delineate how the data is to be analyzed. What he calls for is the total of the number of times the outputs are equal, i.e., \( N_{ab}^= := \# \{ x^{(n)} = y^{(n)} \} \), as well as the number of times they are unequal, \( N_{ab}^\neq \), and the total number of trials, \( N_{ab} \). With these numbers, the correlation, \( \kappa \), for each setting pair, \( ab \), is then:

\[
\kappa_{ab} = \frac{N_{ab}^=} - \frac{N_{ab}^\neq}{N_{ab}}.
\]

These correlations, in turn, are used to compute the CHSH contrast:

\[
S = k_{12} + k_{11} + k_{21} - k_{22},
\]

which is to be tested for violation of Bell’s limit, \( |S| \leq 2 \), as is well known.

3 A counter-protocol

Computer O is implemented as an equal, flat, random selection of one of two possible signal pairs, one comprised of a vertically polarized pulse to the left, say, and a horizontally polarized pulse to the right; the second signal exchanges the polarizations.

After the source pulse pair is selected, the measurements to be simulated at computers X and Y are selected by independent computers A and B. This they do for each run or each pulse pair individually by randomly specifying which of the two orientation angles for each side and iteration is to be used; i.e., they select \( \theta_l \) and \( \theta_r \).

The measurement stations X and Y are simulated by models of polarizers for which the axis of the left (right) one is the angle \( \theta_l(\theta_r) \); each polarizer feeds a photodetector.
These photodetectors are taken to adhere to Malus’ Law, that is, they produce photoelectron streams for which the intensity is proportional to the incoming field intensity, and the arrival time of the photoelectrons is a random variable described by a Poisson process. In so far as these photodetectors are independent, each Poisson process is uncorrelated with respect to the other. In conformity with Bell’s assumptions and the experiments, we take it that the source power and pulse duration are so low that, within a time-window, one and only one photoelectron will be generated if the polarization of the signal and the axis of the polarizer preceding the detector are parallel. Thus, when they are not parallel, the count rate is reduced in proportion to the usual Malus’ factor. This assumption is structurally parallel to assuming single photon states in QM. It is unrealistic to the extent that, even for the parallel regime, a true Poisson distribution would lead to some trials with no hits, which don’t contribute to the photoelectron count statistics, and some trials with two or more hits, which are so seldom they effectively don’t contribute.

As a matter of detail, the photoelectron generation process is modeled by comparing a random number with the intensity of the field entering the polarizer filter. If it is less, it is taken that a photoelectron was generated; if greater, none was generated. In other words, \( N_{lv} \), say, (where for \( N_{lm} \), \( s \) indicates the setting of \( \theta_s \) as chosen by A or B; and \( m \) indicates pulse mode from the source: vertical or horizontal) is increased by 1, when the random number \( \leq \cos^2(\theta_a - \phi_l) \). Also, \( N_{rh} \) is increased by 1 when another, independent random number \( \leq \cos^2(\theta_b - \phi_r) \). In each case \( \phi_j = r, \ l = 0 \) (vertical), \( \pi/2 \) (horizontal) is the polarization axis of the pulse sent from the source. The random input at the detector simulates background signals and detector noise.

The final step of the simulation is simply to register the simulated ‘creation’ of photoelectrons and to compute the correlations.

This algorithm fully satisfies Gill’s stipulations as presented in Section 2 above to preclude non-locality. All process steps can be implemented on separate computers such that the information flow is from A, B and O to X and Y. There is no connection between X and Y. At this point we note, however, that Gill’s analysis of EPR experiments is not faithful to the relevant physics. It fails to take the quadratic relationship between the source intensity and the subsequent measured current density, i.e., Malus’ Law, into account.

Thus, in our simulation, data acquisition and analysis proceed a bit differently. We take it, that currents are measured, that the total relative intensity between channels is given purely by geometrical considerations according to Malus’ Law; i.e., according to \( \cos^2(\theta_r - \theta_l) \) for like events, and \( \sin^2(\theta_r - \theta_l) \) for unlike events. This is essentially equivalent to requiring internal self consistency with respect to detection physics. That is, if the axis of one polarizer is parallel to the axis of the source, \( \theta_s = 0 \), say, then it is quite obvious that the relative intensity as measured by photodetectors on the output of the other polarizer must follow Malus’ Law by virtue of the photocurrent generation mechanism. Further, since this geometric fact must be independent of the choice of coordinate system, it follows that a transformation of angular coordinates can be effected using

\[
\cos(\theta_r - \theta_l) = \cos(\theta_r) \cos(\theta_l) + \sin(\theta_r) \sin(\theta_l),
\]
\[
\sin(\theta_r - \theta_l) = \sin(\theta_r) \cos(\theta_l) + \cos(\theta_r) \sin(\theta_l);
\]

just trigonometric identities. Note that although equations (3) are not in general factorable (sometimes said to be a criteria for ‘non-locality’), all the information required is available on-the-spot at both sides independently, i.e., locally. Thus, in modeling the “coincidence circuitry” we use the fact that the individual terms on the right side of (3) are, by virtue of photodetector physics, proportional to the square root of the number of counts in the channel, for example:

\[
\cos(\theta_l - \phi_h) = \cos(\theta_l - 0) = \cos(\theta_l) = \lim_{N \to \infty} \sqrt{N_{hl}/N},
\]

\[
\cos(\theta_l - \phi_v) = \cos(\theta_l - \pi/2) = \sin(\theta_l) = \lim_{N \to \infty} \sqrt{N_{vl}/N};
\]

where \(N\) is the total number of signals intercepted by the considered photodetector per regime. (Since there are four detector- and two source-regimes, for even distributions \(N \to T/8\) where \(T\) is the total number of pairs used in the simulation. Unlike experiments, this number is trivially available in simulations.) In other words, the relative frequency of coincidences is determined after-the-fact as a function of the intensity of the photocurrents. No communication between right and left sides is involved in determining these currents; what correlation there is, is there on account of the equality of the amplitudes (and therefore intensities) at the source of the signals making up the singlet state. Of course, in doing a simulation, provision must be made to resolve sign ambiguities; doing so, however, also does not violate locality as the required information is all available on-the-spot. In sum, instead of Eq. (1), we use:

\[
\kappa^*(\theta_r, \theta_l) = \cos^2(\theta_r - \theta_l) - \sin^2(\theta_r - \theta_l),
\]

as required by Malus’ Law, but in which the terms are expressed using (3) and (4) to get the result for each individual ‘photoelectron’ locally and independently.

To see intuitively how this algorithm works, it is perhaps advantageous to consider the individual steps in reverse order. The final step is the calculation of the CHSH contrast. This requires calculating \(\kappa^*(\theta_r, \theta_l)\) for the four combinations of polarizer settings according to Eq. (5). Now, each factor \(\kappa^*\), a coincidence correlation, according to Eq. (5) is expressed as the intensity of coincidences in like channels minus the intensity in unlike channels, all according to conventional formulas. The individual terms in \(\kappa^*\), however, involve information from each side, so it would appear that each requires in effect instantaneous communication between the sides for evaluation. However, each term can be expanded using Eqs. (3) in which the individual factors, e.g., \(\cos(\theta_l)\), can be computed at the detection event without regard to information from other events. These individual factors, in turn, are provided by Eq. (4), which again is just Malus’ Law. As the simulation is repeated, and more and more pairs of pulses are generated, the running ratios \(N_{ms}/N\) converge to provide ever improving estimates of the individual factors needed for Eq. (3). As described above, the individual \(N_{ms}\) are completely determined by the delayed input from the source, the local polarizer setting, and a random input at the detector. Only the after-the-fact calculation of \(\kappa^*\) mixes information from both sides. Thus, there is no non-locality involved — contrary to Bell’s claim that a model without it does not exist.
An example of the results from the simulation are presented on Fig.1. The top curve is the CHSH contrast; the lower four curves are the individual correlation coefficients for the four combinations of polarizer settings. The statistics stabilize after circa 700 trials and exhibit clear violation of the Bell limit of 2 by virtually exactly the amount calculated for the singlet state and observing angles $\theta_l = 0$, $\pi/4$ and $\theta_r = \pm \pi/8$ using Bell inequalities: $2\sqrt{2}$. Experimental verification can be found in [10]. That these statistics can be found in a sequence of individual events, each of which is calculated without recourse to non-locality, constitutes a direct, unambiguous counterexample to Bell-type “theorems.”

As an aside, we note that what is described here is a simulation, not an experiment. As the algorithm runs on a digital computer, in fact it is a realization of an EPR-B setup using information, which accords with some variations of current analysis of the issue.

4 Reconciliation

How does this protocol work? How is it possible that now, after generations have examined and reexamined seemingly everything surrounding EPR-B correlations and von Neumann’s and Bell’s theorems, a simple protocol can be found that delivers the offending statistics without non-locality?

The answer to these questions is in part, that so far the implied challenge has been so defined, that the option we used in constructing the simulation was precluded. The basic assumption constraining the orthodox approach is that the source emits “ready-
made” pairs of photons. At an abstract statistical level, this is tantamount to assuming that there is just a single Poisson process at the source, for which the time of conception of each pair is a random variable with a Poisson distribution, followed by deterministic detection. That is, in the terminology of QM, the probability of the random creation of a photon pair is proportional to the modulus of a solution to Schrödinger’s equation, followed by detection with a “quantum” efficiency of \( \approx 100\% \) of converting photons to photoelectrons without further stochastic input (except at the polarizer, see below).

For our protocol we reject these notions. Instead, we take it that the source is emitting pulses of classical, continuous radiation in both directions, which are finally registered in photodetectors that are the scene of independent Poisson detection processes proportional to the square of the source intensity. Although uncorrelated in terms of arrival times of elicited photoelectrons, the two processes, by virtue of the symmetry of the source (i.e., the source pulses have equal power and duration), nevertheless have correlated intensities. For the simulation, consistent with classical physics, polarizers are taken to be variable, linear attenuators, which are, therefore, deterministic. In ‘quantum’ imagery, polarizers are considered biased stochastic absorbers, i.e., not deterministic. The latter notion is not relevant to the issue of Bell inequalities, however, because they are meant to be limits imposed by local realism and are non-quantum from the start. In the end, the patterns seen in the correlations are due simply to the various Malus’ factors attenuating the equal energy source pulses.

Independently, underneath all the confused aspects of the dispute initiated by EPR, there is an inviolable mathematical truth, which has many forms. It is that both correlated and uncorrelated equal length dichotomic sequences with values \( \pm 1 \), tautologically satisfy Bell Inequalities. Being an ineluctable mathematical truth, it is also often mistaken for a physical indispensability.

There is, however, an intervening complication. Consider four dichotomic sequences comprised of \( \pm 1 \)'s and length \( N \): \( a, a', b \) and \( b' \). Now compose the following two quantities \( a_ib_i + a_ib'_i = a_i(b_i + b'_i) \) and \( a'_i b_i - a'_i b'_i = a'_i(b_i - b'_i) \), sum them over \( i \), divide by \( N \), and take absolute values before adding together to get:

\[
| < ab > + < a'b > | + | < a'b > - < a' b' > | \leq | < a|b + b'| > + | < a|b - b'| > \tag{6}
\]

The right side equals 2, so this equation is in fact a Bell inequality. This derivation demonstrates that this Bell inequality is simply an arithmetic tautology. Thus, certain dichotomic sequences comprised of \( \pm 1 \)'s, identically satisfy Bell inequalities.\[\text{[11]}\]

The fact is, however, that real data can not comply with this inequality; it just does not fit the conditions of the derivation of Eq. (6). This is a result of the fact that two of the sequences on the left side are counterfactual statements, i.e., what would have been measured if the setting had been otherwise. In real experiments, such data can never be obtained. Moreover, real data can not be rearranged to fit Eq. (6) either. Suppose to start, that the second term is rearranged so that the factor sequence \( a(2) \) where the argument 2 indicates that it is from the second term, is rearranged to match as closely as possible \( a(1) \). (For ever longer samples, this becomes ever more precise.) Let the rearranged version be denoted \( \tilde{a}(2) \). Then the second term becomes: \( a(2)b'(2) \Rightarrow \tilde{a}(2)\tilde{b}'(2) \equiv a(1)\tilde{b}(2) \). A similar rearrangement on the third and fourth terms converts
the right side of Eq. (6) to:

\[< |a(1)||b(1) + \tilde{b}'(2)| > + < |a(4)||\tilde{b}(3) - \tilde{b}'(2)| >, \tag{7} \]

from which it is obvious that unless \(b(1) \cong \tilde{b}(3)\) by virtue of the structure of the setup, that Eq. (6) is not germane; in other words, data from real experiments can not conform to the conditions of derivation of Bell inequalities. Variations of this argument hold for all derivations of Bell inequalities.\[12\]

5 Secondary arguments

Note also that the results of our simulation are in full accord with Jaynes’ criticism of Bell’s arguments, to wit: Bell, by ascribing the correlations to the hidden variables, effectively misused Bayes’ formula for conditional probabilities with respect to the overt variables. The result is that he encoded statistical independence instead of non-locality, so that the resulting inequalities are valid only for uncorrelated sequences. In ref [5], Gill endeavors to respond to this observation with the argument that Jaynes “refuses to admit” that:

\[P(a|b, \lambda) = P(a|\lambda), \tag{8} \]

where \(P(c|d)\) is the conditional probability for \(c\) occurring if \(d\) has occurred, \(a\) and \(b\) represent filar marks on a measurement device and \(\lambda\), represents a “hidden variable” specifying the quality whose magnitude influences the magnitude registered as \(a\) and \(b\).\[5\] Of course, if one is measuring an assortment of objects (nails, say), each engraved with its length (\(\lambda\)), then we can dispense with the comparison to the filar marks on a ruler (\(a\)) in favor of using the \(\lambda\). But when the \(\lambda\) are not knowable, not to mention uncontrollable, the measured objects must be characterized by the result of comparison with the ruler, i.e., the \(a\)’s. As this is exactly the situation with respect to EPR and Bell’s arguments, all statistical characteristics, including correlations, of an ensemble must be specified also in terms of results from measurements in the form of comparisons with the filar marks, the only variables actually available. EPR’s purpose was to discern the possible existence of such hidden variables exclusively in terms of their effects on quantities expressed in terms of overt variables; all evidence must appear, therefore, in the overt variables. That is, when Jaynes rejects \(5\), he is just faithful to reality if \(\lambda\) remains unaccessible.

The conventional understanding also overlooks the fact that the space of polarization is fundamentally not quantum mechanical in nature.\[5\] This is so in the first instance, because its variables, the two states of polarization, are not Hamiltonian conjugates, and therefore do not suffer Heisenberg uncertainty (HU), and do not involve Planck’s constant. Gill rejected this argument on the basis of the opinion that “quantum mechanics is as much about incompatible observables as Planck’s constant.” In essence this view implies that all commutivity somehow involves QM, which is manifestly false. Non commutivity can arise for different reasons, for example, non parallel Lorentz boosts do not commute, and this obviously has nothing to do with QM. In the case at hand, non commutivity of polarization vectors, or Stokes’ operators, arises only when the \(k\)-vector common to both polarization states rotates in space, which brings in
the geometric fact that the generators of rotations on the sphere do not commute. This has nothing specifically to do with QM, EPR or Bell inequalities, just geometry.

6 Conclusions

A consequence of the existence of a local-realistic simulation of EPR-B correlations is that it shows that such correlations do not arise because of the structure of QM. This could have significant practical consequences. In particular, to the extent that algorithms, usually considered to depend on exploiting quantum entanglement, for example, can be seen not to be dependant on intrinsically quantum phenomena. Thus, these algorithms should also not be restricted to realization at the atomic level, where QM reigns. In so far as it is much more practical to fabricate and operate devices at a larger scale, this may be a very promising development for the exploitation of what is (from this viewpoint, inaccurately) denoted as quantum computing.

In the end, does this simulation put Gill in debt? Whatever the call, it is clear that his considerations, like that of many others, was based on some mathematical facts regarding dichotomic sequences, comprised of both hits and non-hits, and not on the actual details of done experiments. These mathematical facts concerning dichotomic sequences can also be simulated by doing the data analysis as envisioned by Gill, and indeed this seems not to lead to a violation of Bell inequalities no matter how the parameters are adjusted. In short, what is shown here is, that EPR-B correlations are a direct manifestation of Malus’ Law, not mystical gibberish or QM.

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