Analysis of the Influence of the Critical Speed of Thermoelastic Instability of Transfer Case on the Torque Transmission

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Received June xx, 202x; revised February xx, 202x; accepted March xx, 202x
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Abstract: Based on the thermoelastic instability theory, the variation trend of film thickness on the surface of transfer case friction pair is studied, and the basic process in which the thickness of lubricating oil film changes due to thermoelastic instability was obtained. According to this process, the critical speed of transfer case friction pair is used to reflect transfer torque stability. Based on above, firstly, the dynamic simulation model of the transfer case is established from which the dynamic characteristic equation of the transfer case under the combined action of asperity torque and viscous torque is obtained. Secondly, according to the result that the thickness of the transfer case lubricating oil film is affected by the joint action of lubricating oil pressure, contact pressure and thermal expansion, the condition of thermoelastic instability of the transfer case torque transmission is discussed. The result shows that the critical speed of the thermoelastic instability increases correspondingly with the increase of the thickness of the oil film. The critical curve divides the engagement process into the stable region at the upper part of the curve and the unstable region at the lower part of the curve, so as to calculate the thermoelastic instability region of the torque transfer of the transfer case. Finally, in order to further analyze the influence of thermoelastic instability on the characteristics of the transfer case, three factors, namely the surface roughness of the friction plate, the lubricating oil viscosity and the thermal conductivity of the dual steel disc, are mainly selected for analysis, and the thermoelastic instability is taken into account. The conclusion is expected to be used in the engineering practice of torque distribution control of the transfer case.

Keywords: Transfer case • Multi-plate clutch • Torque transmission • Thermoelastic instability • Critical speed

1 Introduction

Transfer case is the critical part of four-wheel drive vehicle. The way to quickly and accurately realize the torque distribution of front and rear axle and left and right wheels determines the ability of vehicle to get out of trouble under bad working conditions. The thermoelastic instability of the friction pair in the transfer case occurs due to the friction heat. The way to judge the critical speed of the thermoelastic instability plays a critical role in preventing torque transfer from overheating and failure. The dynamic transfer characteristics of this part have been studied by scholars at home and abroad.

For example, scholars such as JANG and KHONSARI [1], on the one hand, they took the influence of friction material thickness into account, and established a comprehensive analysis model of the thermoelastic instability in the wet clutch. On the other hand, they derived and solved the control equation of the thermoelastic instability [2]. Then study the influence of different groove forms on the wet clutch, and establish a three-dimensional control equation for numerical solution [3]. MARKLUND [4-5] studied the working conditions of wet clutches under boundary lubrication conditions. The team of Professor Li Heyan [6-9] from Beijing Institute of Technology has conducted theoretical modeling and analysis using thermoelastic instability theory to investigate the effect of oil film thickness and local high temperature region on thermoelastic instability under fluid lubrication. At the same time, LI [10] and others studied the transient thermoelastic of wet multi-plate clutches.
In China, Chen Hufang [15] deduced the thermoelastic instability characteristic equation of friction plates for the problem of thermoelastic instability leading to the failure of wet multi-plate friction clutch power transmission. Yun Ruide [16] based on the classic Hertz contact mechanics theory and fractal contact model, taking the multi-scale contact state into account and establishing a contact model. Professor Ma Biao [17] of Beijing Institute of Technology used thermoelastic instability theory to study the inhomogeneous temperature field during the engagement process.

In the field of transfer torque transmission, scholars such as OKAMOTO [18] studied the variation of friction coefficients of friction plates with different factors during the engagement process. Scholars such as ZHAO [19] took the influence of fluid dynamics on the asperity surface into consideration and established a dynamics model of spiral groove friction plate. According to the structural characteristics of wet multi-disc clutch, Professor Ma Biao [20] et al. studied the influence of relevant factors such as the surface asperity contact of the friction pair, the material permeability, and the centrifugal force of the lubricating fluid.

Based on the research results analyzed by many scholars, the previous research on the transfer case mainly focused on the groove form and material properties of the transfer case friction pair. According to the previous research, the importance of the torque transmission stability of the transfer case friction pair is obtained [21-23]. Therefore, it is necessary to calculate the critical speed of the transfer case when the thermoelastic instability occurs to obtain the influence of the thermoelastic instability on the transfer function of the transfer case, so as to provide reference and basis for the optimal design of the transfer case.
2 Mechanical Model of Transfer Case

When the pressing mechanism gradually acts on the friction pair, oil film thickness that between friction pairs is decreasing, so the engagement process of the transfer case is divided into three stages:

The first stage is the fluid lubrication stage. A certain oil clearance exists between the friction pairs, so there is no asperity torque between the two objects. The second stage is the mixed friction stage, the lubricating oil film and the asperity surface share the friction torque. In this case, the total friction torque at this stage is comprised of the viscous torque and the asperity torque produced by the friction pairs. The third stage is the asperity torque stage, that is, the pressure is borne by the asperity contact, The working process is shown in figure 1.

\[
\frac{dh}{dt} = \frac{1}{\rho(h)12\eta} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( 1 + 12\Phi d \phi h^{-1} \right) \frac{\partial p}{\partial r} \right) \right] + \frac{1}{r^2} \left[ \frac{1}{\phi h^{-1}} \frac{\partial p}{\partial \theta} \right] - \frac{3}{5} \lambda \omega^2
\]  

(2)

2.2 Asperity Torque of Transfer Case

The applied pressure \( F_{app} \) acting on the thrust plate, is composed of the oil film bearing capacity \( F \) and the friction plate asperity contact force \( F_c \). \( F_{app} \) is expressed as:

\[
F_{app} = F + F_c = \int_a^b p \, dA + \int_a^b p_c \, dA = \rho \omega \, d \theta \, dr
\]  

(3)

According to GW Model:

\[
p_c(h) = \sqrt{\frac{\pi}{2} \rho \sigma} \left[ e^{-\frac{h}{\rho \omega}} - \frac{h}{\rho \omega} \frac{\pi}{\rho \sigma} \text{erfc} \left( \frac{h}{\rho \omega \sqrt{2}} \right) \right]  
\]  

(4)

Taking into account of the symmetry of the friction plate, Eqs. 3 can be further expressed as:

\[
N_f \int_a^b p(r, \theta) r \, dr \, d \theta = F_{app} - N_g \theta_0 \frac{p_c(h) A}{2\pi}
\]  

(5)

With: \( \theta_0 = \frac{2\pi}{N_g} \left( \frac{w_s}{a + b} \right)^{1/2} \), \( A = \pi \left( b^2 - a^2 \right) \)

2.3 Total Torque of Transfer Case

The viscous torque \( T_p \) and asperity torque \( T_c \) constitutes total torque, thus the formula can be expressed as:

\[
T = T_c + T_p = N_f N_g \text{sgn}(\omega) \int_a^b \mu \rho \right\}^2 \, dr \, d \theta + N_f N_g \int_a^b \eta \omega r^2 \left( \frac{\phi_i - \phi_f}{h} \right) \, r \, dr \, d \theta
\]  

(6)

In the type: \( \phi_c = 11.1H^{2.31}e^{-2.38H+0.11} \)  

\[
H = \frac{h}{\sigma}
\]

The dynamic equations of the transfer case is:

\[
\phi_c + \phi_f = \begin{cases} 
3 \sin \left( \frac{\pi h}{2.6} \frac{\sigma}{\sigma} \right) & \frac{h}{\sigma} \leq 1.3 \\
-rac{\left( \frac{h}{\sigma} + 1.3 \right)^2}{1 + 2 \cdot e^{-\frac{\sigma h}{2}}} & \frac{h}{\sigma} > 1.3
\end{cases}
\]
Taking into account the symmetry of the annular friction plate, Eq. (6) can be further expressed as:

\[
T = T_c + T_v = N_r N_\theta \theta_0 \left( \frac{b^4 - a^4}{3} \mu p_{\omega} \sgn(\omega) \right) + N_r N_\theta \theta_0 \left( \frac{b^4 - a^4}{4} \phi_0 - \phi_h \right) / h \tag{7}
\]

3 The Characteristics of Transfer Case Oil Film

3.1 Surface Deformation Caused by Hydrodynamic Pressure

There is a film of lubricating oil thickness \( h_0 \) between the friction pairs, so a form of surface wave is used to represent the thick film of lubricating oil film:

\[
h' = h_1 \sin \Omega(x - vt) + h_2 \cos \Omega(x - vt) \tag{8}
\]

So the total lubricant film thickness is:

\[
h = h_0 + h' \tag{9}
\]

Taking surface roughness into consideration, it assumes the surface roughness of the friction plate meets Gaussian height distribution. The average gap \( h' \) becomes:

\[
h' = \frac{\sigma}{2} H[1 + \text{erf}(\frac{H}{\sqrt{2}})] + \frac{\sigma}{2 \sqrt{2} \pi} e^{-(1/2)H^2} \tag{10}
\]

The formula \( H = h / \sigma \) represents the dimensionless nominal film thickness. In order to identify the deformation caused by lubricating oil pressure, the pressure distribution on the surface of the friction should be obtained. According to the theory of Patir and Cheng and Reynolds equation:

\[
\frac{\partial}{\partial x} \left( \frac{\phi_h h}{12 \eta} \frac{\partial p_h}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\phi_h h}{12 \eta} \frac{\partial p_h}{\partial z} \right) = - \frac{1}{2} \omega \phi_h \frac{\partial h}{\partial x} - \frac{1}{2} \omega \sigma \frac{\partial \phi}{\partial x} \tag{11}
\]

Parameter \( p_h' \) represents the lubricating oil pressure caused by asperity surface disturbance.

\[
\phi_p = 1 - 0.9 e^{-0.56H} \quad H > 0.5
\]
\[
\phi_c = \begin{cases} 
1.126 e^{-0.25H} & H > 5 \\
1.899 H^{0.98} e^{-0.92H + 0.05H^2} & 0.5 < H \leq 5 
\end{cases} \tag{12}
\]

The three influence factors ignore the influence of higher-order terms as:

\[
\phi_p = \phi_{p_o} + \phi_p' \frac{h}{h_0} \quad \phi_c = \phi_{c_o} + \phi_c' \frac{h}{h_0} \quad \phi_e = \phi_{e_o} + \phi_e' \frac{h}{h_0} \tag{13}
\]

Insert Eqs. (9), (12) and (13) into (11), and solve to get hydrodynamic pressure:

\[
p_h' = \frac{6 \eta \omega \phi_h}{H_0^3} \frac{dh'}{dx} \tag{14}
\]

Where: \( \phi_h = \frac{H_0 \phi_{p_o} + \phi_p'}{H_0 \phi_{p_o}} \tag{15} \)

The elastic deformation of the friction pair caused by the hydrodynamic pressure is expressed as:

\[
\delta_h = \frac{2 p_h'}{E \Omega} = \frac{12 \eta \omega \phi_h}{E \Omega} \frac{dh'}{dx} \tag{16}
\]

3.2 Deformation of Friction Plate Caused by Asperity Contact Pressure

A proportionality factor \( E_c \) is introduced in the paper. Therefore, \( P_c = E_c (A_c / A_n) = E_c A_c \). \( A_c \) and \( A_n \) mean the real and nominal area of asperity contact, \( A_c \) is calculated as follows:

\[
A_c = \pi \rho \gamma \sigma \left[ -\frac{1}{2 \sqrt{2} \pi} e^{-(1/2)H^2} + \frac{1}{2} H (\text{erf}(H / \sqrt{2}) - 1) \right] \tag{17}
\]

Using Gaussian distribution statistics method, the asperity contact area including disturbance is obtained as:

\[
A_c' = -\pi \rho \gamma \sigma \frac{h'}{h_0} \tag{18}
\]

Where:
\[ \varphi = \frac{H_0}{\sqrt{2\pi}} e^{-\frac{(z+z'}{2H_0^2}}} - \frac{1}{2} H_0 \left[ \operatorname{erf} \left( \frac{H_0}{\sqrt{2}} \right) + \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \frac{H_0}{\sqrt{2}} \right)^{2n+1} \right] \tag{19} \]

The surface deformation of the friction plate caused by the contact pressure is expressed as:

\[ \delta_c' = \frac{2p_c'}{E\Omega} \left[ 2 E_c A' \right] = -\frac{2 E_c \pi \rho \gamma \sigma_e - \h'}{E\Omega h_o} \tag{20} \]

### 3.3 Thermal Surface Deformation

The two-dimensional displacement potential \( \psi \) for the friction pair is solved first:

\[ \nabla^2 \psi = (1 + V)\alpha T' \tag{21} \]

\( T' \) is the disturbance temperature in the dual steel disc:

\[ T' = T_o e^{\beta n^2} \sin(\Omega x + n y - \Omega vt) \tag{22} \]

\( \beta \) is the time index of temperature disturbance. The spatial index of temperature disturbance in y-direction (\( \xi \)) and the wave number in y-direction (\( n \)) are expressed as:

\[ \xi = \sqrt{\frac{1}{2} \left( \frac{\beta + \Omega^2}{\kappa} \right) + \frac{1}{2} \left( \frac{\beta + \Omega^2}{\kappa} \right)^2} + \frac{\Omega^2 v^2}{\kappa^2} \]

\[ n = \pm \sqrt{-\frac{1}{2} \left( \frac{\beta + \Omega^2}{\kappa} \right) + \frac{1}{2} \left( \frac{\beta + \Omega^2}{\kappa} \right)^2} + \frac{\Omega^2 v^2}{\kappa^2} \tag{23} \]

Using the boundary conditions, the two-dimensional displacement potential \( \psi \) is further expressed as:

\[ \psi = e^{\beta n^2 - \Omega x/v} \left( A \sin X_s + B \cos X_s \right) + e^{\beta n^2 + \Omega x/v} \left( C \sin X_s + D \cos X_s \right) \tag{24} \]

Where: \( X_s = \Omega(x - vt) \)

\[ A = -\frac{1}{\Omega} \left( \xi C - n D \right) \quad \quad C = \frac{(1 + V)\alpha T_h x \beta}{\beta^2 + \Omega^2 v^2} \]

\[ B = \frac{1}{\Omega} \left( n C - \xi D \right) \quad \quad D = \frac{(1 + V)\alpha T_h x \Omega v}{\beta^2 + \Omega^2 v^2} \tag{25} \]

The thermal stress on the friction plate surface is:

\[ \tau_{xy} = \frac{E \alpha \kappa \Omega}{\beta^2 + \Omega^2 v^2} \left[ \left( (\Omega - \xi) \beta - n \Omega \right) \sin X_s + \left( n \beta + \nu \Omega (\Omega - \xi) \right) \cos X_s \right] T_e e^{\beta n^2} \tag{26} \]

The surface deformation of the friction pair caused by thermal stress is:

\[ \delta_t' = \frac{2\tau_{xy}}{E\Omega} = \frac{2 \alpha \kappa}{\beta^2 + \Omega^2 v^2} \left[ \left( (\Omega - \xi) \beta - n \Omega \right) \sin X_s + \left( n \beta + \nu \Omega (\Omega - \xi) \right) \cos X_s \right] T_e e^{\beta n^2} \tag{27} \]

### 3.4 Total Perturbation Deformation

The total perturbation deformation is generated by the combined action of hydrodynamic pressure deformation \( \delta_h' \), aasperity contact pressure deformation \( \delta_c' \), and thermal deformation \( \delta_t' \). The total disturbance deformation is:

\[ \delta = \delta_h' + \delta_c' + \delta_t' \tag{28} \]

Combine Eqs. (16),(20), and (27)

\[ \delta' = \left[ \frac{2 \chi \phi}{E\Omega h_0} \right] h_1 - \left[ \frac{12 \rho \nu \Omega}{E\Omega h_0} \right] h_2 + \left[ \frac{2 \alpha \kappa}{\beta^2 + \Omega^2 v^2} \right] \left[ (\Omega - \xi) \beta - n \Omega \right] T_e e^{\beta n^2} \sin X_s + \left[ \frac{12 \rho \nu \Omega}{E\Omega h_0} \right] h_1 - \left[ \frac{2 \chi \phi}{E\Omega h_0} \right] h_2 + \left[ \frac{2 \alpha \kappa}{\beta^2 + \Omega^2 v^2} \right] \left[ n \beta + \nu \Omega (\Omega - \xi) \right] T_e e^{\beta n^2} \cos X_s \tag{29} \]

Where: \( \chi = E_c \pi \rho \gamma \sigma \)

### 3.5 Thermelastic Instability Analysis

The surface wave disturbance \( \delta \) causes three deformation components, including hydrodynamic pressure \( \delta_h' \), asperity contact pressure \( \delta_c' \), and thermal expansion \( \delta_t' \). Hence:

\[ \delta' = \delta_h' + \delta_c' + \delta_t' \tag{30} \]

If the disturbance film thickness \( \delta' \) between the friction pairs is greater than the sum of the deformations, the
friction pairs will be thermoelastic stable. In the opposite case, the system will experience thermoelastic instability and thermal ablation of the friction pair. In this situation, the stability parameter $\beta > 0$. The threshold of instability critical speed is obtained by identically satisfying Eqs. (30). This condition corresponds to $\beta = 0$.

Use Eqs. (29), set $\beta = 0$, the following dimensionless equations emerge:

$$\bar{\nu} = \frac{\nu}{\Omega \kappa}, \quad \bar{\omega} = \frac{\omega}{\Omega \kappa}, \quad \bar{\xi} = \frac{1}{2\sqrt{2}} [1 + \sqrt{1 + \bar{v}^2}], \quad \bar{\eta} = \pm \frac{1}{2} \sqrt{1 + \bar{v}^2}$$

(31)

The relationship between critical speed and oil film thickness is obtained by solving the following equations:

$$s_1 \bar{\nu} \bar{\omega} \bar{\omega} + [s_2 \bar{\nu} + s_1 \bar{\omega}^2] \bar{\omega} \bar{\omega} - [1 + s_4 \bar{\nu}] \bar{\nu} = 0 \quad s_1 (1 - \bar{\xi}) \bar{\omega} + [s_2 (1 - \bar{\xi}) - s_3 \bar{\nu}] \bar{\omega} \bar{\omega} - [1 + s_4 \bar{\nu}] \bar{\nu} = 0$$

(32)

$$s_1 = \frac{2 \alpha \sigma \eta \kappa^2}{k h_0 h_0} \varphi_a, \quad s_2 = \frac{2 f a c \eta \gamma}{E h_0 k_s} \varphi_a$$

Where:

$$s_3 = \frac{12 \eta \kappa}{E h_0^3} \varphi_a, \quad s_4 = \frac{2 \gamma}{E h_0} \varphi_a$$

(33)

Parameters $s_1$ and $s_2$ are related to the thermal deformation caused by the viscous shear and the asperity contact pressure, respectively. Parameters $s_3$ and $s_4$ are linked to the mechanical deformation caused by the hydrodynamic pressure and the asperity contact pressure, respectively.

4 Results, Validation and Discussion of Transfer Case Characteristics

4.1 Critical speed analysis

The characteristics of a certain type of SUV vehicle transfer case are analyzed. The friction pairs of parameters are shown in table 1. The simulation results of critical speed and the thickness of the oil film in the friction pairs are shown in Figure 2 and Figure 3.

| Parameters | Numerical Value (Unit) |
|------------|------------------------|
| $a$        | 0.057 (m)              |
| $b$        | 0.066 (m)              |
| $\sigma$   | 6E-6                   |
| $d$        | 5E-4 (m)               |
| $\Phi$     | 4E-12                  |
| $w_f$      | 1.5E-4 (m)             |
| $h_0$      | 6E-5 (m)               |
| $N_e$      | 32                     |
| $N_f$      | 8                      |
| $E$        | 200 (GPa)              |
| $E_c$      | 3.1E7 (Pa)             |
| $\rho$     | 3E7 (kg/m$^3$)         |
| $\eta$     | 0.035 (N·s/m$^2$)      |
| $\gamma$   | 5E-4 (m)               |
| $k_s$      | 40 (W/(m·K)$^2$)       |
| $\alpha$   | 1.71E-5 (K$^{-1}$)     |
| $\kappa$   | 8.58E-5 (m$^2$/s)      |

Figure 2 shows the relationship between the thickness of the oil film $h_0$ and the critical speed $\varphi_a$ based on the change of the stationary wave. The solid line represents the predicted value of the critical velocity of the dual steel disc (Equation 46), and dashed line indicates the critical velocity of the friction plate (Equation 47). It can be observed that the curve divides the diagram into two different regions. All critical velocities below the shown curve feature thermoelastic stability, and engaging speeds greater than the critical value will accelerate the generation of thermoelastic instability.

![Figure 2](image)

**Figure 2** The relationship between oil film thickness and critical speed

Figure 2 shows that as the thickness of the oil film increases, the critical speed of thermoelastic instability also increases accordingly. The main reason is that the increase in the thickness of the oil film leads to the reduction of friction heat which is generated between the
friction pairs, resulting in a relatively small thermal expansion of the dual steel disc, more heat is generated in the lubricating oil as the film becomes thinner. By contrast, as the film thickens, the viscous torque produces less heat.

At the same time, the critical speed of the friction plate surface is much lower than that of the dual steel disc, because the oil film on the asperity surface generates relatively more frictional heat. The principle explains that the rougher the surface of the friction plate, the more vulnerable it is to be affected by thermoelastic instability. With the increase of oil film thickness, the roughness effect gradually decreases. When the surface roughness reaches \( h_0 = 3\sigma \), the simulation curves coincide because the critical speed of the friction pairs reach the same value.

![Figure 3](image-url)  
**Figure 3** The influence of different friction surface characteristic parameters on the critical speed

Figure 3 shows the impact of friction surface characteristic parameters \( \chi \) on critical speed. The critical velocity decreases with increasing surface characteristic parameters, because more frictional heat is generated at larger surface characteristic parameters. The simulation results also show that when \( \chi = 1 \) the critical speed of friction plate is higher than that of dual steel disc with lubricant film thickness lower than \( h = 3\mu m \). The reason is that when the thickness is less than \( h = 3\mu m \), the amount of elastic deformation caused by asperity contact pressure is greater than the amount of thermal deformation caused by the asperity contact pressure. When \( h = 3\mu m \), the amount of deformation caused by asperity contact pressure is equal to the amount of thermal deformation, so the critical speed of the asperity surface is consistent with that of the smooth surface.

Based on previous research\(^\text{[21-23]}\), the simulation obtained the relationship between the oil film thickness and the critical speed, and the relationship between the oil film and the thermoelastic instability of hydrodynamic lubrication, as shown in Figure 4. It can be obtained from the figure that the thickness of the oil film in the original article decreases rapidly with the speed difference. The friction pairs quickly transmit from the fluid lubrication stage to the full asperity contact stage, while the intermediate mixed lubrication stage is difficult to reflect. Since the oil film of thermoelastic instability changes slowly, the whole engaging process is divided into the stable region on the upper part of the curve and the unstable region on the lower part of the curve. It can be seen from the figure that the variation trend of the lubricating oil film in the previous paper has been in the unstable region of the thermoelastic instability of hydrodynamic lubrication. So the maximum torque transmitted by the transfer case obtained in the original paper will produce a certain degree of thermoelastic instability, and friction plates are more prone to ablation and warping. Therefore, in the actual torque transfer of the friction pair, the region of thermoelastic instability should be avoided.

![Figure 4](image-url)  
**Figure 4** The boundary of stable area

![Figure 5](image-url)  
**Figure 5** Comparison and analysis results of transfer torque
According to the variation trend of the lubricating oil film with the speed difference in Figure 4, the simulation obtains the friction torque transmitted by the original model and the transfer case considering the thermoelastic instability model during the engagement process. It can be seen from the figure that the original model and the model with thermoelastic instability under consideration not only have a gap of 0.15s to reach the maximum torque, but also the difference of the torque value transmitted by them is about 30 N·m. The torque value of the original model is greater than that of the model considering thermoelastic instability, which indicates that thermoelastic instability has occurred in the original model. That is, in the actual torque transmission process, it is necessary to actively control the torque value transferred by the transfer case to be greater than the simulation results of the thermoelastic instability model, so as to ensure the stability of torque transmission.

4.2 Test Verification
In order to verify the correctness of the transfer case model, a test bench was built as shown in Figure 6. The results obtained after related tests and data processing are showed in Figure 7.

The test results in Figure 7 show that the trend of the test value and the simulation value is generally the same. When the other conditions are consistent, the data of the test is slightly higher than the simulated result of the modified model. If the friction pair is operated for a long time, hot spots and thermal failure are prone to occur. In the course of controlling transfer case, it is necessary to ensure that the value of torque transmission is not more than the revised model of the critical value.

5 Thermoelastic Instability Factors Analysis and Discussion
In order to further analyze the influence of thermoelastic instability on transfer case characteristics, three factors are selected for analysis, including the surface roughness of the friction plate, thermal conductivity of the dual steel disc and lubricating oil viscosity. The influence of thermoelastic instability on the critical speed and torque transmission of the transfer case is also considered. The simulation results are shown in Figure 9:
From the graph a in Figure 8, it is found that the critical speed of the friction plate with high surface roughness is much lower than that of the smooth surface, because the rougher the surface of the friction plate, the more heat is generated by the friction. Thus it can be predicted that the rougher the surface of the friction plate, the more prone to thermoelastic instability. As the thickness of the lubricating oil film increases, the surface roughness effect tends to decrease. And when the friction plate surface roughness reaches to $h_0 = 3\sigma$, the critical speed of the smooth surface is consistent with that of the asperity surface.

From the d diagram, surface roughness has a great influence on the torque transmission of the transfer case. The greater the surface roughness is, the smaller the transmission gap between the two models. In the first 0.5s of the joint, the torque transferred by the original model is above the thermoelastic instability model, which is in the state of thermoelastic instability. In the next process, as the surface roughness increases, the critical speed between the two models gradually approaches, which reveals the greater the surface roughness is, the more likely the frictional transfer torque will be in the stable region. If the surface roughness is a fixed value, the transfer torque which is greater than that of the thermoelastic instability model is the unstable torque, which is prone to overload.

It can be obtained from Figure b that the viscosity of different lubricants has no obvious effect on the critical speed. The graph e shows that the influence of the viscosity of the lubricating oil on the original model and the thermoelastic instability model is mainly reflected in the value of torque transmission. The maximum torque difference between the two models is close to 30 N·m, and it takes about 0.9s to enter the thermoelastic stable region. The whole process provides a great reference for the thermal load management of the transfer case.

Figure c reveals that thermal conductivity has a certain effect on the critical speed. The better the thermal conductivity is, the larger the critical speed value is. Figure f reveals that different thermal conductivities not only affect the torque transfer, but also influence the time they take to reach the area of thermoelastic instability varies. With the increase of thermal conductivity, the smaller torque is transferred, and the longer time it takes to reach the region of the thermoelastic instability.

### 6 Conclusions

1. Thermoelastic instability is one of the main causes of the instability of the transfer torque. The critical speed is an important criterion to judge the stability of the torque transmission of transfer case. Through the analysis of the joining process of the transfer case, the mathematical model is established. Then the lubricating film thickness of transfer case is analyzed by the joint action of lubricating oil pressure, contact pressure and thermal expansion. It is found that as the thickness of the oil film increases, the critical speed of thermoelastic instability also increases accordingly. At the same time, the critical speed of friction plate is much lower than that of the dual steel disc due to frictional heat generation.

2. Compared with the published torque transfer model, the film thickness in the original paper decreases rapidly with the speed difference, and the transfer case transmit rapidly from the fluid lubrication stage to the full asperity contact stage, while the intermediate stage of mixed lubrication is difficult to be reflected. Given that the variation trend of the oil film is relatively slow, the whole joining process is divided into the stable region of the upper part of the curve and the unstable region of the lower part of the curve, and finally the unstable region of the transfer torque is obtained. In order to further analyze the influence of thermoelastic instability on the features of the transfer case, the three factors of friction plate surface
roughness, lubricating oil viscosity and thermal conductivity of the dual steel plate are mainly analyzed. The greater the surface roughness is, the smaller the transmission gap is between the two models. The influence of the viscosity of the lubricating oil on the original model and the thermoelastic instability model mainly affects the torque transfer. The maximum torque difference between the two models is close to 30 N·m. These research results have theoretical and practical value on the design of transfer case.

7 Declaration

Acknowledgements
Not applicable.

Funding
This project is supported by the National Key Research and Development Project of China (Grant No. 2016YFD0700902) and Science and Technology Project of Nantong (Grant No. JC2019093).

Authors’ contributions
YW took most of the research work, including the literature research, modeling, results analysis and paper writing. LC assisted with the results analysis, paper revision and online submitting. XL provided technical supporting for modeling. HL assisted with structure and language of the manuscript. All authors read and approved the final manuscript.

Competing interests
The authors declare no competing financial interests.

Consent for publication
Not applicable

Ethics approval and consent to participate
Not applicable

References
[1] J Y Jang, M M Khonsari. On the formation of hot spots in wet clutch systems[J]. Transactions of the ASME, 2002, (124): 336-345.
[2] J Y Jang, M M Khonsari. Thermal elastic instability with consideration of surface roughness and hydrodynamic lubrication[J]. Journal of Tribology, 2000, (122): 725-732.
[3] J Y Jang, M M Khonsari, R Maki. Three-dimensional thermohydrodynamic analysis of a wet clutch with consideration of grooved friction surfaces[J]. Journal of Tribology, 2011, (133): 6-18.
[4] P Marklund, R Larsson. Wet clutch under limited slip conditions simplified testing and simulation[J]. Proc. Inst. Mech. Eng., Part J: J. Eng. Tribol., 2007, (221): 545-551.
[5] P Marklund, R Larsson. Modelling and simulation of thermal effects in wet clutches operating under boundary lubrication conditions[J]. Proc. Inst. Mech. Eng., Part J: J. Eng. Tribol., 2009, (223): 1129-1141.
[6] J X Zhao, B Ma, H Y Li. The effect of lubrication film thickness on thermal elastic instability under fluid lubricating condition[J]. Wear, 2009, (303): 146-153.
[7] J X Zhao, B Ma, H Y Li, et al. Thermal elastic stability of wet clutches during engaging process[J]. Journal of Jilin University (Engineering and Technology Edition), 2015, 45(1): 22-28.
[8] B Ma, J X Zhao, H Y Li, et al. Effect of clutches structural parameters on thermalelastic instability[J]. Journal of Jilin University (Engineering and Technology Edition), 2014, 44(4): 933-938.
[9] J X Zhao, B Ma, H Y Li, et al. Research on hot spots of the multi-disk clutch and stability of the system[J]. Journal of Jilin University (Engineering and Technology Edition), 2014, 44(4): 933-938.
[10] J Li, J R Barber. Solution of transient thermal elastic contact problems by the fast speed expansion method[J]. Wear, 2008, 265(3): 402-410.
[11] S W Lee, Y H Jang. Effect of functionally graded material on frictionally excited thermal elastic instability[J]. Wear, 2009, (267): 1715-1722.
[12] S W Lee, Y H Jang. Frictionally excited thermal elastic instability in a thin layer of functionally graded material sliding between two half-planes[J]. Wear, 2009, 267(9-10): 1715-1722.
[13] S W Lee, Y H Jang. Effect of functionally graded material on frictionally excited thermal elastic instability[J]. Wear, 2009, 266(1-2): 139-146.
[14] S Ahn, Y H Jang. Frictionally excited thermoelastoplastic instability[J]. Tribology International, 2010, 43(4): 779-784.
[15] H F Chen, W Yang, Q Mei. Research on thermal elastic instability of wet clutch[J]. Journal of Jilin University (automation and instrumentation), 2016, (5): 77-80,83.
[16] R D Yun, B Ding. New Fractal Contact Model Considered Multi-scale Levels[J]. Journal of Mechanical Engineering, 2019, 59(9): 80-89.
[17] B Ma, J X Zhao, M Chen, et al. Effect of friction material properties on thermal elastic instability of clutches[J]. Journal of Mechanical Engineering, 2014, 50(8): 111-118.
[18] D Okamoto. A Study on Friction Characteristics at Low Pressure Slip Condition of Wet-Clutch[J]. SAE Technical Paper, 2014, (01): 459-466.
[19] Y M Zhao, J B Hu, C Wei. Dynamic analysis of spiral-groove rotary seal ring for wet clutches[J]. Journal of Tribology, 2014, (136): 47-58.
[20] B Ma, G Q Li, H Y Li, et al. Simulation of wet clutch engagement characteristics based on advanced average flow model[J]. Journal of Jilin University (Engineering and Technology Edition), 2014, 44(6): 1557-1563.
[21] Y M Wang, Q D Wang, L Q Chen, et al. Mathematical model research in electromagnetic multi-plate clutch on torque distribution control[J]. Journal of Mechanical Engineering, 2017, 47(5): 472-483.
[22] Y M Wang, Q D Wang, L Q Chen, et al. Research on transfer case transmission power of heat load characteristics[J]. Journal of
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