Quasi–fixed point scenario in the modified NMSSM

R. B. Nevzorov†,‡ and M. A. Trusov†

†ITEP, Moscow, Russia
‡DESY Theory, Hamburg, Germany

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Abstract

The simplest extension of the MSSM that does not contradict LEP II experimental bound on the lightest Higgs boson mass at tan β ~ 1 is the modified Next–to–Minimal Supersymmetric Standard Model (MNSSM). We investigate the renormalization of Yukawa couplings and soft SUSY breaking terms in this model. The possibility of $b$–quark and $\tau$–lepton Yukawa coupling unification at the Grand Unification scale $M_X$ is studied. The particle spectrum is analysed in the vicinity of the quasi–fixed point where the solutions of renormalization group equations are concentrated at the electroweak scale.
1 Introduction

A rapid development of experimental high–energy physics over the last decades of the XX century gave impetus to intensive investigations of various extensions of the Standard Model. Its supersymmetric generalization known as the Minimal Supersymmetric Standard Model (MSSM) is one of the most popular extensions of the Standard Model. The Higgs sector of the MSSM includes two doublets of Higgs fields, $H_1$ and $H_2$. Upon a spontaneous breakdown of gauge symmetry, each of them develops a vacuum expectation value; we denote the corresponding vacuum expectation values by $v_1$ and $v_2$. The sum of the squares of the vacuum expectation values of the Higgs fields is $v^2 = (246 \text{ GeV})^2$, the ratio of the expectation values being determined by the angle $\beta$. By definition, $\beta = \arctan(v_2/v_1)$. The value of $\tan\beta$ is not fixed experimentally. It is varied within a wide interval, from $1.3 - 1.8$ to $50 - 60$. Within supersymmetric (SUSY) models, the upper and lower limits on $\tan\beta$ arise under the assumption that perturbation theory is applicable up to the scale at which gauge coupling constants are unified, $M_X = 3 \cdot 10^{16}$ GeV – that is, under the assumption that there is no Landau pole in solutions to relevant renormalization group equations.

The spectrum of the Higgs sector of the MSSM contains four massive states. Two of them are CP–even, one is CP–odd, and one is charged. The presence of a light Higgs boson in the CP–even sector is an important distinguishing feature of SUSY models. Its mass is constrained from above as

$$m_h \leq \sqrt{M_Z^2 \cos^2 2\beta + \Delta},$$  \hspace{1cm} (1)

where $M_Z$ is the $Z$–boson mass ($M_Z \approx 91.2$ GeV) and $\Delta$ stands for the contribution of loop corrections. The magnitude of these corrections is proportional to $m_t^4$ ($m_t$ is the running mass of the $t$–quark), depends logarithmically on the supersymmetry breakdown scale $M_S$, and is virtually independent of the choice of $\tan\beta$. An upper limit on the mass of the light CP–even Higgs boson within the MSSM grows with increasing $\tan\beta$ and, for $\tan\beta \gg 1$, reaches $125 - 128$ GeV in realistic SUSY models with $M_S \leq 1000$ GeV.

At the same time it is known from [1] that, for $\tan\beta \ll 50 - 60$, solutions to the renormalization group equations for the $t$–quark Yukawa coupling constant $h_t(t)$ are concentrated in the vicinity of the quasi–fixed point

$$Y_{QFP}(t_0) = \frac{E(t_0)}{6F(t_0)},$$  \hspace{1cm} (2)

where

$$E(t) = \left[ \frac{\tilde{\alpha}_3(t)}{\tilde{\alpha}_3(0)} \right]^{16/9} \left[ \frac{\tilde{\alpha}_2(t)}{\tilde{\alpha}_2(0)} \right]^{-3} \left[ \frac{\tilde{\alpha}_1(t)}{\tilde{\alpha}_1(0)} \right]^{-13/99}, \quad F(t) = \int_0^t E(t') dt', \quad \tilde{\alpha}_i(t) = \left( \frac{g_i(t)}{4\pi} \right)^2, \quad Y_i(t) = \left( \frac{h_i(t)}{4\pi} \right)^2,$$

with $g_i$ being the gauge constants of the Standard Model group. The variable $t$ is defined in the standard way: $t = \ln(M_X^2/q^2)$. Its value at the electroweak scale is $t_0 = 2 \ln(M_X/M_t^{\text{pole}})$, where $M_t^{\text{pole}} \approx 174.3 \pm 5.1$ GeV is the pole mass of the $t$–quark.
Along with the $t$–quark Yukawa coupling constant, solutions to the renormalization group equations for the corresponding trilinear coupling constant $A_t$ characterising the interaction of scalar fields and the combination $M^2_t = m_Q^2 + m_U^2 + m_U^2$ of the scalar particle masses also approach the infrared quasi–fixed point. The properties of solutions to the renormalization group equations within the MSSM and the spectrum of particles in the infrared quasi–fixed point regime at $\tan\beta \sim 1$ were investigated in [2],[3].

A reduction of the number of independent parameters in the vicinity of the infrared quasi–fixed point considerably increased the predictive power of the theory. On the basis of the equation relating the Yukawa coupling constant for the $t$–quark with its mass at the electroweak scale 

$$m_t(M^\text{pole}_t) = \frac{h_t(M^\text{pole}_t)}{\sqrt{2}} v \sin\beta,$$

and the value calculated for the running mass of the $t$–quark within the $\overline{MS}$–scheme ($m_t(M^\text{pole}_t) = 165 \pm 5 \text{ GeV}$), it was shown in [3]–[5] that, for a broad class of solutions satisfying the renormalization group equations within the MSSM and corresponding to the infrared quasi–fixed point regime, $\tan\beta$ takes values in the interval between 1.3 and 1.8. These comparatively small values of $\tan\beta$ lead to much more stringent constraints on the mass of the lightest Higgs boson. A detailed theoretical analysis performed in [3], [4], revealed that, in the case being considered, its mass does not exceed $94 \pm 5 \text{ GeV}$, which is $25 - 30 \text{ GeV}$ below the absolute upper limit in the minimal SUSY model. It should be noted that the LEP II constraints on the mass of the lightest Higgs boson [6] are such that a considerable fraction of solutions approaching a quasi–fixed point at $\tan\beta \sim 1$ have already been ruled out by existing experimental data.

All the aforesaid furnishes a sufficient motivation for studying the Higgs sector in more complicated SUSY models, as well renormalization group equations and solutions to these equations therein. The present article is devoted to an analysis of coupling constant renormalization within a modified Next–to–Minimal SUSY Model (MNSSM), where the mass of the lightest Higgs boson can be as large as $120 - 130 \text{ GeV}$ even at comparatively small values of $\tan\beta \sim 2$. In addition, the spectrum of superpartners of observable particles and of Higgs bosons is studied in the vicinity of the quasi–fixed point of the renormalization group equations within the MNSSM.

## 2 Modified NMSSM

The Next–to–Minimal Supersymmetric standard Model (NMSSM) [7]–[9] is the simplest extension of the MSSM. Historically, the NMSSM arose as one of the possible solutions to the problem of the $\mu$–term in supergravity (SUGRA) models [7]. Along with observable superfields, SUGRA theories contain a hidden sector that includes the dilaton and moduli fields ($S$ and $T_m$, respectively), which are singlet in gauge interactions. The total superpotential in SUGRA models is usually represented as an expansion in the superfields of the observable sector; that is,

$$W = W_0(S,T_m) + \mu(S,T_m)(\hat{H}_1\hat{H}_2) + h_t(S,T_m)(\hat{Q}\hat{H}_2)\hat{U}_R + \ldots,$$

(4)

where $W_0(S,T_m)$ is the superpotential of the hidden sector. The expansion in [10] presumes that the parameter $\mu$ appearing in front of the bilinear term $(\hat{H}_1\hat{H}_2)$ must be about
the Planck mass, since this scale is the only dimensional parameter characterising the hidden sector of the theory. In this case, however, the Higgs bosons $H_1$ and $H_2$ acquire an enormous mass ($m_{H_1, H_2}^2 \approx \mu^2 \approx M_{Pl}^2$) and $SU(2) \otimes U(1)$ symmetry remains unbroken.

In the NMSSM, an additional singlet superfield $\hat{Y}$ is introduced, while the term $\mu(\hat{H}_1 \hat{H}_2)$ is replaced by $\lambda \hat{Y}(\hat{H}_1 \hat{H}_2) + (\kappa/3)\hat{Y}^3$. A spontaneous breakdown of gauge symmetry leads to the emergence of the vacuum expectation value $\langle Y \rangle = y/\sqrt{2}$ of the field $Y$ and to generation an effective $\mu$-term ($\mu = \lambda \langle Y \rangle$). The resulting superpotential of the nonminimal SUSY model is invariant under discrete transformations of the $Z_3$ group [8]. The $Z_3$ symmetry of the superpotential of the observable sector naturally arises in string models, where all observable fields are massless in the limit of exact supersymmetry.

Upon the introduction of the neutral field $Y$ in the superpotential of the NMSSM, there arises the corresponding $F$-term in the potential of interaction of Higgs fields. As a result, an upper limit on the mass of the lightest Higgs boson becomes higher than that in the MSSM:

$$m_h \leq \sqrt{\frac{\lambda^2}{2} v^2 \sin^2 2\beta + M_Z^2 \cos^2 2\beta + \Delta}.$$  

In the tree approximation ($\Delta = 0$), relation (5) was obtained in [9]. For $\lambda \to 0$, the expressions for the upper limit in the MSSM and in the NMSSM coincide, after the substitution of $\lambda y/\sqrt{2}$ for $\mu$. The Higgs sector of the nonminimal SUSY model and one-loop corrections to it were studied in [10], [11]. In [12], the upper limit on the mass of the lightest Higgs boson within the NMSSM was contrasted against the analogous limits in the minimal standard and the minimal SUSY model.

From relation (5), it follows that the upper limit on $m_h$ grows with increasing $\lambda(t_0)$. It should be noted that only in the region of small values of $\tan \beta$ does it differ significantly from the analogous limit in the MSSM. As to the small $\tan \beta$ scenario, it is realised in the case of sufficiently large values of $h_t(t_0)$. The growth of the Yukawa coupling constants at the electroweak scale is accompanied by an increase in $h_t(0)$ and $\lambda(0)$ at the Grand Unification scale; therefore, the upper limit on the mass of the lightest Higgs boson in the nonminimal SUSY model attains a maximum value in the limit of strong Yukawa coupling, in which case both $h_t^2(0)$ and $\lambda^2(0)$ are much greater than $g_i^2(0)$.

Unfortunately, we were unable to obtain a self–consistent solution in the regime of strong Yukawa coupling within the NMSSM featuring the minimal set of fundamental parameters. Moreover, $Z_3$ symmetry, which makes it possible to avoid the problem of the $\mu$–term in the nonminimal SUSY model, leads to the emergence of three degenerate vacua in the theory. Immediately following the phase transition at the electroweak scale, the Universe is filled with three degenerate phases that must be separated by domain walls. However, the hypothesis of a domain structure of the vacuum is at odds with data from astrophysical observations. An attempt at destroying $Z_3$ symmetry and the domain structure of the vacuum by introducing nonrenormalizable operators in the NMSSM Lagrangian leads to the appearance of quadratic divergences – that is, to the hierarchy problem [13].

In order to avoid the domain structure of the vacuum and to obtain a self–consistent solution in the regime of strong Yukawa coupling, it is necessary to modify the nonminimal SUSY model. The simplest way to modify the NMSSM is to introduce additional terms in the superpotential of the Higgs sector, $\mu(\hat{H}_1 \hat{H}_2)$ and $\mu'\hat{Y}^2$ [14], that are not forbidden by gauge symmetry. The additional bilinear terms in the NMSSM superpotential destroy
Z\textsubscript{3} symmetry, and domain walls are not formed in such a theory. Upon the introduction of the parameter \(\mu\), it becomes possible to obtain the spectrum of SUSY particles in the modified model; for a specific choice of \(\mu'\), the mass of the lightest Higgs boson reaches its upper limit, taking the largest value at \(\kappa = 0\). In analysing the modified NMSSM, it is therefore reasonable to set the coupling constant for the self–interaction of neutral superfields \(Y\) to zero.

The MNSSM superpotential involves a large number of Yukawa coupling constants. At \(\tan\beta \sim 1\), they are all negligibly small, however, with the exception of the \(t\)–quark Yukawa coupling constant \(h_t\) and the coupling constant \(\lambda\), which is responsible for the interaction of the superfield \(Y\) with the doublets \(\hat{H}_1\) and \(\hat{H}_2\). Thus, the total superpotential of the modified NMSSM can be represented in the form

\[
W_{\text{MNSSM}} = \mu(\hat{H}_1\hat{H}_2) + \mu'\hat{Y}^2 + \lambda\hat{Y}(\hat{H}_1\hat{H}_2) + h_t(\hat{Q}\hat{H}_2\hat{U}^c) \tag{6}
\]

Within SUGRA models, the terms in the superpotential (6) that are bilinear in the superfields can be generated owing to the term \((Z(H_1H_2) + Z'Y^2 + h.c)\) in the Kähler potential [15],[16] or owing to the nonrenormalized interaction of fields from the observable and the hidden sector (this interaction may be due to nonperturbative effects) [16],[17].

Along with the parameters \(\mu\) and \(\mu'\), the masses of scalar fields \(m_i^2\), and the gaugino masses \(M_i\) are also generated upon a soft breakdown of supersymmetry. Moreover, a trilinear coupling constant \(A_i\) for the interaction of scalar fields is associated in the total Lagrangian of the theory with each Yukawa coupling constant, while a bilinear coupling constant \(B(B')\) is associated there with the parameter \(\mu(\mu')\). The hypothesis of universality of these constants at the scale \(M_X\) makes it possible to reduce their number to four: the scalar particle mass \(m_0\), the trilinear coupling constant \(A\) and the bilinear coupling constant \(B_0\) for the interaction of scalar fields, and the gaugino mass \(M_{1/2}\).

### 3 Analysis of the evolution of Yukawa couplings and determination of the quasi–fixed point

The MNSSM parameters

\[
\lambda, \ h_t, \ \mu, \ \mu', \ m_0, \ A, \ B_0, \ M_{1/2}
\]

specified at the Grand Unification scale evolve down to the electroweak scale or the scale of supersymmetry breakdown. Their renormalization is determined by the set of renormalization group equations, these equations for the coupling constants \(\lambda, h_t, A_i, m_i^2\), and \(M_i\) being coincident with the corresponding renormalization group equations within the NMSSM (see, for example, [11]) if one sets \(\varpi = 0\) in them. The equations describing the evolution of \(\mu, \ \mu', \ B, \) and \(B'\) within the modified NMSSM were obtained in [14].

Even in the one–loop approximation, the full system of renormalization group equations is nonlinear, so that it is hardly possible to solve it analytically. This set of equations can be broken down into two subsets. The first subset includes equations that describe the evolution of gauge and Yukawa coupling constants and of parameters \(\mu\) and \(\mu'\). The second subset comprises equations for the parameters of a soft breakdown of supersymmetry.
In studying the evolution of the Yukawa coupling constants, it is convenient to introduce, instead of the constants \( h_t, \lambda, \) and \( g_i, \) the ratios

\[
\rho_i(t) = \frac{Y_i(t)}{\lambda_3(t)}, \quad \rho_\lambda(t) = \frac{Y_\lambda(t)}{\lambda_3(t)}, \quad \rho_1(t) = \frac{\alpha_1(t)}{\lambda_3(t)}, \quad \rho_2(t) = \frac{\alpha_2(t)}{\lambda_3(t)},
\]

where \( Y_\lambda(t) = \lambda^2(t)/(4\pi)^2. \) The region of admissible values of the Yukawa coupling constants at the electroweak scale is bounded by the quasi–fixed (or Hill) line. Beyond this region, solutions to the renormalization group equations for \( Y_i(t) \) develop a Landau pole below the scale \( M_X, \) so that perturbation theory becomes inapplicable for \( q^2 \sim M_X^2. \) The results of our numerical calculations are presented in Fig. 1, whence one can see that, in the regime of strong Yukawa coupling, all solutions for \( Y_i(t) \) are attracted to the quasi–fixed line. The main reason behind this is that, in the regime of strong Yukawa coupling, all solutions for \( Y_i(t) \) are attracted not only to the quasi–fixed but also to the infrared fixed (or invariant) line. The latter connects two fixed points. One of them is the stable infrared fixed point for the set of renormalization group equations within the MNSSM \( (\rho_t = 7/18, \rho_\lambda = 0, \rho_1 = 0, \rho_2 = 0) \) [18]. As the invariant line approaches this point, \( \rho_\lambda \sim (\rho_t - 7/18)^{25/14} \). The other fixed point \( (\rho_\lambda / \rho_t) = 1 \) corresponds to large values of the Yukawa coupling constants, \( Y_\lambda \), \( Y_1 \), \( Y_2 \) develop a Landau pole. Integrating

\[
(7)
\]

The infrared fixed lines and their properties in the minimal standard and the minimal supersymmetric model were studied in detail elsewhere [20].

With increasing initial values of the Yukawa coupling constants \( Y_i(0) \) and \( Y_\lambda(0) \) at the Grand Unification scale, the region where solutions are concentrated at the electroweak scale shrinks abruptly and all solutions to the renormalization group equations within the MNSSM are focused near the point of intersection of the invariant and the quasi–fixed line:

\[
\rho_t^{\text{QFP}}(t_0) = 0.803, \quad \rho_\lambda^{\text{QFP}}(t_0) = 0.224.
\]

This point can be considered as the quasi–fixed point for the set of renormalization group equations for the modified NMSSM [21].

Among subsidiary constraints that are frequently imposed in studying supersymmetric models, we would like to mention the unification of the Yukawa coupling constants for the \( b \)–quark and the \( \tau \)–lepton at the scale \( M_X; \) this usually occurs in minimal schemes for unifying gauge interactions – for example, in those that are based on the \( SU(5) \), the \( E_6 \), or the \( SO(10) \) group. The unification of \( h_b \) and \( h_\tau \) within the MNSSM is realised only in the case where the constants satisfy a specific relation between \( Y_i \) and \( Y_\lambda. \) Integrating the renormalization group equations and substituting \( R_{br}(t_0) = m_b(t_0)/m_\tau(t_0) = 1.61, \)
which corresponds to $m_r(t_0) = 1.78$ GeV and $m_b(t_0) = 2.86$ GeV, we obtain

$$
\frac{Y_t(0)}{Y_t(t_0)} = \left[ \frac{R_{br}(0)}{R_{br}(t_0)} \right]^{\frac{21}{2}} \left[ \frac{\tilde{a}_2(0)}{\tilde{a}_3(0)} \right]^{48} \left[ \frac{\tilde{a}_1(0)}{\tilde{a}_1(0)} \right]^{\frac{463}{396}} \left[ \frac{Y_\lambda(0)}{Y_\lambda(t_0)} \right]^{\frac{1}{4}} \approx 3.67 \left[ \frac{Y_\lambda(0)}{Y_\lambda(t_0)} \right]^{\frac{1}{4}}.
$$

(9)

The results obtained here indicate that $b - \tau$ unification is possible under the condition that $Y_t(t_0) \gg Y_t(0)$, which is realised only in the regime of strong Yukawa coupling. By varying the running mass $m_b(m_b)$ of the $b$–quark from 4.1 to 4.4 GeV, we found that the equality of the Yukawa coupling constants at the Grand Unification scale can be achieved only at $\tan \beta \leq 2$.

The possibility of unifying the Yukawa coupling constants within the NMSSM was comprehensively studied in [21,22]. The condition $Y_b(0) = Y_\tau(0)$ imposes stringent constraints on the parameter space of the model being studied. Since $h_b$ and $h_\tau$ are small in magnitude at $\tan \beta \sim 1$, they can be generated, however, owing to nonrenormalizable operators upon a spontaneous breakdown of symmetry at the Grand Unification scale. In this case, $h_b$ and $h_\tau$ may be different. In studying the spectrum of superpartners below, we will not therefore assume that $R_{br}(0) = 1$.

### 4 Renormalization of the soft SUSY breaking parameters

If the evolution of gauge and Yukawa coupling constants is known, the remaining subset of renormalization group equations within the MNSSM can be treated as a set of linear differential equations for the parameters of a soft breakdown of supersymmetry. For universal boundary conditions, a general solution for the trilinear coupling constants $A_i(t)$ and for the masses of scalar fields $m_i^2(t)$ has the form

$$
A_i(t) = e_i(t)A + f_i(t)M_{1/2},
$$

$$
M_i^2(t) = a_i(t)m_0^2 + b_i(t)M_{1/2}^2 + c_i(t)AM_{1/2} + d_i(t)A^2.
$$

(10)

The functions $e_i(t)$, $f_i(t)$, $a_i(t)$, $b_i(t)$, $c_i(t)$, and $d_i(t)$, which determine the evolution of $A_i(t)$ and $m_i^2(t)$, remain unknown. The results of our numerical calculations reveal that these functions greatly depend on the choice of Yukawa coupling constants at the scale $M_X$.

In analysing the behaviour of solutions to the renormalization group equations in the regime of strong Yukawa coupling, it is more convenient to consider, instead of the squares of the scalar particle masses, their linear combinations

$$
M_i^2(t) = m_2^2(t) + m_3^2(t) + m_1^2(t),
$$

$$
M_\lambda^2(t) = m_1^2(t) + m_2^2(t) + m_3^2(t).
$$

(11)

For the universal boundary conditions, solutions to the differential equations for $M_i^2(t)$ can be represented in the same form as the solutions for $m_i^2(t)$ (see (10)); that is

$$
M_i^2(t) = 3\tilde{a}_i(t)m_0^2 + \tilde{b}_i(t)M_{1/2}^2 + \tilde{c}_i(t)AM_{1/2} + \tilde{d}_i(t)A^2.
$$

(12)
Since the homogeneous equations for $A_i(t)$ and $M_i^2(t)$ have the same form, the functions $\tilde{a}_i(t)$ and $\tilde{e}_i(t)$ coincide.

With increasing $Y_i(0)$, the functions $e_i(t_0)$, $c_i(t_0)$, and $d_i(t_0)$ decrease and tend to zero in the limit $Y_i(0) \to \infty$. Concurrently, $A_i(t)$, $A_\lambda(t)$, $M_i^2(t)$, and $M_i^2(t)$ become independent of $A$ and $m_3^2$, while relations (10) and (12) are significantly simplified. This behaviour of the solutions in question implies that, as the solutions to the renormalization group equations for the Yukawa coupling constants approach quasi–fixed points, the corresponding solutions for $A_i(t)$ and $M_i^2(t)$ also approach the quasi–fixed points whose coordinates are

\begin{align*}
\rho_{QFP}^{A_i}(t_0) & \approx 1.77, \\
\rho_{QFP}^{M_i^2}(t_0) & \approx 6.09, \\
\rho_{QFP}^{A_\lambda}(t_0) & \approx -0.42, \\
\rho_{QFP}^{M_\lambda^2}(t_0) & \approx -2.28,
\end{align*}

where $\rho_{A_i}(t) = A_i(t)/M_{1/2}$ and $\rho_{M_i^2}(t) = M_i^2/M_{1/2}^2$. At the same time, the functions $a_i(t)$ approach some constants independent of $t$ and $Y_i(0)$:

\begin{align*}
a_y(t) & \to 1/7, \\
a_1(t) & \to 4/7, \\
a_2(t) & \to -5/7, \\
a_u(t) & \to 1/7, \\
a_q(t) & \to 4/7.
\end{align*}

In the case of nonuniversal boundary conditions at $Y_i(0) \simeq Y_\lambda(0)$, the required solution to the renormalization group equations for $A_i(t)$ and $M_i^2(t)$ can be represented as

\begin{align*}
\begin{pmatrix} A_i(t) \\
A_\lambda(t) \end{pmatrix} &= \alpha_1 \begin{pmatrix} v_{11}(t) \\
v_{21}(t) \end{pmatrix} (\epsilon_i(t))^{\lambda_1} + \alpha_2 \begin{pmatrix} v_{12}(t) \\
-3v_{22}(t) \end{pmatrix} (\epsilon_i(t))^{\lambda_2} + \ldots, \\
\begin{pmatrix} M_i^2(t) \\
M_\lambda^2(t) \end{pmatrix} &= \beta_1 \begin{pmatrix} v_{11}(t) \\
v_{21}(t) \end{pmatrix} (\epsilon_i(t))^{\lambda_1} + \beta_2 \begin{pmatrix} v_{12}(t) \\
-3v_{22}(t) \end{pmatrix} (\epsilon_i(t))^{\lambda_2} + \ldots,
\end{align*}

where $\alpha_i$ and $\beta_i$ are constants of integration that can be expressed in terms of $A_i(0)$, $A_\lambda(0)$, $M_i^2(0)$, and $M_\lambda^2(0)$; $\epsilon_i(t) = Y_i(t)/Y_i(0)$; $\lambda_1 = 1$ and $\lambda_2 = 3/7$. The functions $v_{ij}(t)$ are weakly dependent on the Yukawa coupling constants at the scale $M_X$, and $v_{ij}(0) = 1$. In equations (15), we have omitted terms proportional to $M_{1/2}$, $M_{1/2}^2$, $A_i(0)M_{1/2}$, and $A_i(0)A_j(0)$.

With increasing $Y_i(0) \simeq Y_\lambda(0)$, the dependence of $A_i(t_0)$ and $M_i^2(t_0)$ on $\alpha_1$ and $\beta_1$ quickly becomes weaker. The results of our numerical analysis that are displayed in Fig. 2 indicate that, for $h_i^2(0) = \lambda^2(0) = 20$ and boundary conditions uniformly distributed in the $(A_i, A_\lambda)$ and the $(M_i^2, M_\lambda^2)$ plane, the solutions to the renormalization group equations for the parameters of the soft SUSY breaking in the vicinity of the quasi–fixed point are concentrated near some straight lines. The equations of these straight lines can be obtained by setting $A_\lambda(0) = -3A_i(0)$ and $M_\lambda^2(0) = -3M_i^2(0)$ (that is, $\alpha_1 = \beta_1 = 0$) at the Grand Unification scale. As a result, we find that, at the electroweak scale, the parameters of a soft breakdown of supersymmetry satisfy the relations

\begin{align*}
A_i + 0.137(0.147)A_\lambda = 1.70M_{1/2}, \\
M_i^2 + 0.137(0.147)M_\lambda^2 = 5.76M_{1/2}^2.
\end{align*}

The equation for $M_i^2$ has been obtained for all $A_i(0)$ set to zero. In relations (16), the coefficients obtained by fitting the results of our numerical calculations (see Fig. 2) are indicated parenthetically. As the Yukawa coupling constants approach quasi–fixed points, the two sets of coefficients in (16) approach fast each other and, at $Y_i(0) \sim 1$, become virtually coincident.
Let us now proceed to study of the spectrum of the superpartners of observable particles and Higgs bosons in the vicinity of the quasi–fixed point within the MNSSM. The Yukawa coupling constants $h_t$ and $\lambda$ are determined here by relations (8). The value of $\tan \beta$ can be calculated by formula (3). In the regime of the infrared quasi–fixed point at $m_t(M_t^{pole}) = 165$ GeV the result is $\tan \beta = 1.88$.

The remaining fundamental parameters of the MNSSM must be chosen in such a way that a spontaneous breakdown of $SU(2) \otimes U(1)$ gauge symmetry occur at the electroweak scale. The position of the physical minimum of the potential representing the interaction of Higgs fields is determined by solving the set of nonlinear algebraic equations

\[ \frac{\partial V(v_1, v_2, y)}{\partial v_1} = 0, \quad \frac{\partial V(v_1, v_2, y)}{\partial v_2} = 0, \quad \frac{\partial V(v_1, v_2, y)}{\partial y} = 0, \]  

(17)

where $V(v_1, v_2, y)$ is the effective potential of interaction of Higgs fields within the MNSSM [14].

Since the vacuum expectation value $v$ and $\tan \beta$ are known, the set of equations (17) can be used to determine the parameters $\mu$ and $B_0$ and to compute the vacuum expectation value $\langle Y \rangle$. Instead of $\mu$, it is convenient to introduce here $\mu_{eff} = \mu + \lambda y/\sqrt{2}$. The sign of $\mu_{eff}$ is not fixed in solving the set of equations (17); it must be considered as a free parameter of the theory. The results obtained in this way for the vacuum expectation value $y$, the parameters $\mu_{eff}$ and $B_0$, and the spectrum of particles within the modified NMSSM depend on the choice of $A$, $m_0$, $M_{1/2}$, and $\mu'$.

It is of particular interest to analyse the spectrum of particles in that region of the parameter space of the MNSSM where the mass of the lightest Higgs boson is close to its theoretical upper limit, since the remaining part of the parameter space is virtually ruled out by the existing experimental data. For each individual set of the parameters $A$, $m_0$, and $M_{1/2}$, the mass of the lightest Higgs boson reaches the upper bound on itself at a specific choice of $\mu'$. It is precisely at these values of the parameter $\mu'$ that we have calculated the particle spectrum presented in Tables 1 and 2. On the basis of our numerical results given there, one can judge the effect of the fundamental constants $A$, $m_0$, and $M_{1/2}$ on the spectrum of the superpartners of the $t$–quark ($m_{\tilde{t}_{1,2}}$), of the gluino ($M_3$), of the neutralino ($m_{\tilde{\chi}_{1,2,3}}$), of the chargino ($m_{\tilde{\chi}_{1,2}^\pm}$), and of the Higgs bosons ($m_h$, $m_H$, $m_S$, $m_{A_1,2}$). For each set of the aforementioned parameters, we quote the mass of the lightest Higgs boson according to the calculations in the one– and the two–loop approximation, along with the corresponding values of $\mu_{eff}$, $B_0$, $y$, and $\mu'$. As can be seen from the data displayed in Tables 1 and 2, the qualitative pattern of the spectrum within the MNSSM undergoes no changes in response to variations of the parameters $A$ and $m_0$ within reasonable limits.

The CP–even Higgs boson ($m_S$), which corresponds to the neutral field $Y$ is the heaviest particle in the spectrum of the modified NMSSM, while the neutralino ($m_{\tilde{\chi}_\nu}$) is the heaviest fermion there, the main contribution to its wave function coming from the superpartner of the field $Y$. With increasing $m_0^2$ the masses of the squarks, the Higgs bosons, and the heavy chargino and neutralino grow, whereas the spectrum of extremely light particles remains unchanged. Since the dependence of the parameters of a soft breakdown of supersymmetry on $A$ disappears at the electroweak scale in the regime of strong
Yukawa coupling, the parameters $\mu_{\text{eff}}$, $B$, and $\mu'$, together with the spectrum of the superpartners of observable particles and the mass of the lightest Higgs boson, undergo only slight changes in response to a variation of the trilinear coupling constant for the interaction of scalar fields from $-\frac{M_1}{2}$ to $\frac{M_1}{2}$. Despite this, the $A$ dependence of masses of one of the CP–even ($m_S$) and two CP–odd ($m_{A_1,2}$) Higgs bosons survives. It is due primarily to the fact that the bilinear coupling constant $B'$ for the interaction of neutral scalar fields is proportional to $A$. It should be noted in addition that, for a specific choice of fundamental parameters (in particular, of the parameter $A$), the mass of the lightest CP–odd Higgs boson may prove to be about 100 GeV or less. However, this Higgs boson takes virtually no part in electroweak interactions, since the main contribution to its wave function comes from the CP–odd component of the field $Y$. Therefore, attempts at experimentally detecting it run into problems.

Loop corrections play an important role in calculating the mass of the lightest Higgs boson. Their inclusion results in that the mass of the lightest Higgs boson proves to be greater for $\mu_{\text{eff}} < 0$ than for $\mu_{\text{eff}} > 0$. This is because $m_h$ grows as the mixing in the sector of the superpartners of the $t$–quark ($\tilde{t}_R$ and $\tilde{t}_L$) becomes stronger. The point is that the mixing of $\tilde{t}_R$ and $\tilde{t}_L$ is determined by the quantity $X_t = A_t + \mu_{\text{eff}}/\tan\beta$ and is therefore greater in magnitude for $\mu_{\text{eff}} < 0$ since $A_t < 0$. It should also be noted that the inclusion of two–loop corrections leads to a reduction of $m_h$ by approximately 10 GeV. The mass of the lightest Higgs boson depends only slightly on $A$ and $m_0$, because the squark masses depend slightly on the corresponding fundamental parameters (see Tables 1 and 2). The value of $m_h$ is determined primarily by the supersymmetry breaking scale $M_S$ – that is, by the quantity $M_3$. From our numerical results quoted in Tables 1 and 2, one can see that, at $m_t(M_t^\text{pole}) = 165$ GeV and $M_3 \leq 2$ TeV, the mass of the lightest Higgs boson does not exceed 127 GeV.

6 Conclusions

We have studied coupling constant renormalization and the spectrum of particles within the modified Next–to–Minimal Supersymmetric Standard Model (MNSSM). We have shown that, in the regime of strong Yukawa coupling, solutions to the renormalization group equations for $Y_i(t)$ are attracted to the Hill line and that, under specific conditions $b - \tau$ unification is realised at the scale $M_X$. In the limit $Y_i(0) \to \infty$, all solutions for the Yukawa coupling constants are concentrated in the vicinity of the quasi–fixed point that is formed in the space of the Yukawa coupling constants as the result of intersection of the invariant and the Hill line.

As the Yukawa coupling constants approach the quasi–fixed point, the corresponding trilinear coupling constants and combinations of the scalar particle masses cease to depend on the boundary conditions at the scale $M_X$. In the case of nonuniversal boundary conditions, $A_i(t)$ and $M_2^2(t)$ are attracted to straight lines in the space spanned by the parameters of a soft breakdown of supersymmetry and, with increasing $Y_i(0)$, approach the quasi–fixed points, moving along these straight lines.

We have analysed the spectrum of particles in the infrared quasi–fixed point regime of the MNSSM. The CP–even Higgs boson, which corresponds to the neutral field $Y$, is the heaviest particle in this spectrum. At reasonable values of the parameters of the model
being studied, the gluinos, the squarks, and the heavy Higgs bosons are much heavier than the lightest Higgs boson and than the lightest chargino and the lightest neutralino as well. This is not so only for one of the CP–odd Higgs bosons whose mass changes within a wide range in response to variations in the fundamental parameters of the MNSSM. In the vicinity of the quasi–fixed point at $m_t(M_t^{\text{pole}}) = 165$ GeV, the mass of the lightest Higgs boson does not exceed 127 GeV.

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Table 1. Particle spectrum in the vicinity of the quasi–fixed point in the MNSSM at $m_t(M^\text{pole}) = 165$ GeV, $\tan \beta \approx 1.883$, and $\mu_{\text{eff}} > 0$ depending upon the choice of fundamental parameters $A$, $m_0$, and $M_{1/2}$ (all parameters and masses are given in GeV).

| $m^\prime_0$ | 0 | $M^2_{1/2}$ | 0 | 0 | 0 | 0 |
|--------------|---|-------------|---|---|---|---|
| $A$          | 0 | 0           | $-M_{1/2}$ | 0.5$M_{1/2}$ | 0 | 0 |
| $M_{1/2}$    | -392.8 | -392.8 | -392.8 | -392.8 | -785.5 | -196.4 |
| $m_t(t_0)$   | 165 | 165 | 165 | 165 | 165 |
| $\tan \beta$| 1.883 | 1.883 | 1.883 | 1.883 | 1.883 | 1.883 |
| $\mu_{\text{eff}}$ | 728.6 | 841.7 | 726.8 | 730.1 | 1361.2 | 380.4 |
| $B_0$        | -1629.1 | -1935.4 | -1260.0 | -1813.2 | -3064.4 | -861.8 |
| $\mu'(t_0)$  | -1899.8 | -2176.7 | -1905.9 | -1898.3 | -3544.6 | -993.1 |
| $m_h(t_0)$   | 118.4 | 118.5 | 118.4 | 118.4 | 123.2 | 111.9 |
| ($1$–loop)   |     |     |     |     |     |     |
| $m_h(t_0)$   | 118.4 | 118.5 | 118.4 | 118.4 | 123.2 | 111.9 |
| ($2$–loop)   |     |     |     |     |     |     |
| $m_H(1 \text{ TeV})$ | 953.9 | 1113.8 | 1245.7 | 925.2 | 1722.6 | 538.2 |
| $m_A(1 \text{ TeV})$ | 704.3 | 762.7 | 872.0 | 318.2 | 1366.2 | 302.2 |
| $m_{\tilde{\chi}_1}(t_0)$ | 164.6 | 164.6 | 164.6 | 164.6 | 326.9 | 84.3 |
| $m_{\tilde{\chi}_2}(t_0)$ | 327.8 | 327.6 | 327.8 | 327.8 | 649.4 | 170.1 |
| $m_{\tilde{\chi}_3}(1 \text{ TeV})$ | 755.1 | 870.8 | 753.3 | 756.7 | 1404.2 | 400.9 |
| $|m_{\tilde{\chi}_4}(1 \text{ TeV})|$ | 755.9 | 872.6 | 755.1 | 758.4 | 1405.0 | 404.3 |
| $m_{\tilde{\chi}_5}(1 \text{ TeV})$ | 1931.8 | 2212.3 | 1938 | 1930.3 | 3599.0 | 1015.4 |
| $m_{\tilde{\chi}_t}(t_0)$ | 327.8 | 327.6 | 327.8 | 327.8 | 649.4 | 169.9 |
| $m_{\tilde{\chi}_t}(1 \text{ TeV})$ | 757.0 | 872.6 | 755.2 | 758.5 | 1405.2 | 404.5 |
Table 2. Particle spectrum in the vicinity of the quasi–fixed point in the MNSSM at $m_t(M_t^{\text{pole}}) = 165$ GeV, $\tan \beta \approx 1.883$, and $\mu_{\text{eff}} < 0$ depending upon the choice of fundamental parameters $A$, $m_0$, and $M_{1/2}$ (all parameters and masses are given in GeV).

| $m_0^2$ | 0 | $M_{1/2}^2$ | 0 | 0 | 0 | 0 |
|----------|---|------------|---|---|---|---|
| $A$      | 0 | 0          | $-M_{1/2}$ | $M_{1/2}$ | 0 | 0 |
| $M_{1/2}$ | -392.8 | -392.8 | -392.8 | -392.8 | -785.5 | -196.4 |
| $m_t(t_0)$ | 165 | 165 | 165 | 165 | 165 | 165 |
| $\tan \beta$ | 1.883 | 1.883 | 1.883 | 1.883 | 1.883 | 1.883 |
| $\mu_{\text{eff}}$ | -727.8 | -840.9 | -726.0 | -731.2 | -1360.7 | -378.9 |
| $B_0$ | 1008 | 1320.3 | 1366.7 | 647.9 | 2050.4 | 495.8 |
| $y$ | -0.00149 | -0.001 | -0.00128 | -0.00177 | -0.00020 | -0.0112 |
| $\mu'(t_0)$ | 1671.5 | 1950.6 | 1656.8 | 1690.3 | 3172.7 | 857.8 |
| $m_h(t_0)$ (1-loop) | 134.1 | 134.9 | 134.0 | 134.2 | 143.1 | 124.1 |
| $m_h(t_0)$ (2-loop) | 124.4 | 124.8 | 124.3 | 124.5 | 127.2 | 119.6 |
| $M_3(1$ TeV) | 1000 | 1000 | 1000 | 1000 | 2000 | 500 |
| $m_{\tilde{t}_1}(1$ TeV) | 890.2 | 935.6 | 890.5 | 889.8 | 1682.8 | 507.9 |
| $m_{\tilde{t}_2}(1$ TeV) | 630.3 | 652.2 | 632.2 | 628.0 | 1328.1 | 283.5 |
| $m_{H}(1$ TeV) | 896.2 | 1078.5 | 893.5 | 899.3 | 1689.9 | 464.4 |
| $m_{S}(1$ TeV) | 2147.4 | 2565.9 | 2309.2 | 1972.3 | 4126.5 | 1097.7 |
| $m_{A_1}(1$ TeV) | 1123.2 | 1219.3 | 931.0 | 1437.9 | 1984.8 | 623.1 |
| $m_{A_2}(1$ TeV) | 857.6 | 1017.8 | 545.0 | 886.9 | 1657.5 | 412.8 |
| $m_{\tilde{\chi}_1}(t_0)$ | 311.1 | 313.7 | 311.0 | 311.2 | 639.9 | 141.4 |
| $m_{\tilde{\chi}_2}(t_0)$ | 311.1 | 313.7 | 311.0 | 311.2 | 639.9 | 141.4 |
| $m_{\tilde{\chi}_3}(1$ TeV) | 753.7 | 896.6 | 751.9 | 757.2 | 1403.4 | 398.5 |
| $m_{\tilde{\chi}_4}(1$ TeV) | 764.7 | 878.1 | 763.0 | 768.1 | 1410.0 | 416.7 |
| $m_{\tilde{\chi}_5}(1$ TeV) | 1700.7 | 1983.2 | 1685.8 | 1719.6 | 3221.8 | 879.1 |
| $m_{\tilde{\chi}_1^+}(t_0)$ | 310.7 | 313.4 | 310.7 | 310.8 | 639.8 | 139.4 |
| $m_{\tilde{\chi}_2^+}(1$ TeV) | 763.3 | 877.0 | 761.6 | 766.7 | 1409.1 | 414.5 |
Figure captions

**Fig. 1.** Boundary conditions for the renormalization group equations of the MNSSM at the scale \( q = M_X \) uniformly distributed in the \((\rho_t, \rho_\lambda)\) plane in a square \(2 \leq h^2_t(0), \lambda^2(0) \leq 10\) – Fig. 1a, and the corresponding values of the Yukawa couplings at the electroweak scale – Fig. 1b. The thick and thin curves in Fig. 1b represent, respectively, the invariant and the Hill line. The dashed straight line in Fig. 1b is a fit of the values \((\rho_t(t_0), \rho_\lambda(t_0))\) for \(20 \leq h^2_t(0), \lambda^2(0) \leq 100\).

**Fig. 2.** Boundary conditions for the renormalization group equations of the MNSSM at the Grand Unification scale \((t = 0)\) at \(h^2_t(0) = \lambda^2(0) = 20\) uniformly distributed in the \((A_t/M_{1/2}, A_\lambda/M_{1/2})\) plane – Fig. 2a, and the corresponding values of the trilinear couplings at the electroweak scale \((t = t_0)\) – Fig. 2b. The straight line in Fig. 2b is a fit of the values \((A_t(t_0), A_\lambda(t_0))\).

**Fig. 3.** Boundary conditions for the renormalization group equations of the MNSSM at the Grand Unification scale \((t = 0)\) at \(h^2_t(0) = \lambda^2(0) = 20\) and \(A_t(0) = A_\lambda(0) = 0\) uniformly distributed in the \((\mathcal{M}_t^2/M_{1/2}^2, \mathcal{M}_\lambda^2/M_{1/2}^2)\) plane – Fig. 3a, and the corresponding values of the trilinear couplings at the electroweak scale \((t = t_0)\) – Fig. 3b. The straight line in Fig. 3b is a fit of the values \((\mathcal{M}_t^2(t_0), \mathcal{M}_\lambda^2(t_0))\).
Fig. 1a. Fig. 1b.
Fig. 2a

Fig. 2b.
Fig. 3a.

Fig. 3b.