VOXET LATTICE MELTING AND THE DAMPING OF THE DHVA OSCILLATIONS IN THE MIXED STATE

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Phase fluctuations in the superconducting order parameter, which are responsible for the melting of the Abrikosov vortex lattice below the mean field $H_{c2}$, are shown to dramatically enhance the scattering of quasi-particles by the fluctuating pair potential, thus leading to enhanced damping of the dHvA oscillations in the mixed state. This effect is shown to quantitatively account for the detailed field dependence of the dHvA amplitude observed recently in the mixed state of a Quasi 2D organic SC.

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Much effort has been recently invested in elucidating the fundamental mechanisms in which the Abrikosov vortex lattice is involved in the damping process of the de-Haas van-Alphen (dHvA) oscillations in the mixed state of pure, type-II superconductors. Several theoretical attempts have been made to account quantitatively for this effect (see discussions in Refs. [1]). It seems, however, that none of the proposed theories is capable of fully achieving this goal, in particular, since there has been growing evidence recently to the significant sensitivity of the dHvA amplitude to extrinsic factors such as vortex lattice disorder and flux lines pinning. Furthermore, it has been shown theoretically that any mechanism destroying the phase coherence in the vortex lattice, such as disorder or vortex lines fluctuations, should dramatically enhance the inhomogeneous Landau level broadening of quasi-particles.

The effect of vortex lattice melting is of special interest since it is associated with phase fluctuations of the superconducting order parameter, which can be implemented self-consistently into the Gorkov-Ginzburg-Landau (GGL) formalism used in [2].

In a type-II superconductor under a magnetic field a soft shear Goldstone mode, which can be described by long wavelength phase fluctuations, is connected to the Abrikosov lattice melting. Unfortunately rigorous analytical approaches to this problem have encountered fundamental difficulties: large order high temperature perturbation expansion with Borel-Pade approximants to the low temperature behavior has no indication of an ordered vortex lattice even at zero temperature. The existing non-perturbative approaches have not completely clarified the situation: Renormalization group studies [3] have predicted no crystal vortex state in a pure 2D superconductor (SC) at finite temperature, while the novel functional integral formalism suggested in Ref. [4] has led to some kind of a vortex liquid freezing transition without breaking the $U(1)$ symmetry. Several Monte Carlo simulations, have recently shown [5] that even in a 2D SC a true vortex lattice melting phase transition takes place at finite temperature and that the transition is of the first order.

In the present letter we make the first attempt to take into account the effect of vortex lattice melting on the magnetization (dHvA) oscillations in the mixed state. A simple, non-perturbative analytical model of melting is developed for this purpose within the GGL theory for a pure 2D extremely type II SC. Our model is based on the observation that in the vortex lattice state the main correction to the mean field free energy arises from fluctuating “Bragg chains”, that is, from fluctuations which preserve long range periodic order along some crystallographic direction in the vortex lattice. This simplification reduces our 2D fluctuation problem to a 1D one, which is then solved exactly.

Our starting point is the microscopic BCS Hamiltonian for electrons interacting via an effective two-body attractive potential. We then write down the formal functional integral expression for the partition function of this system, eliminating the electronic field by introducing bosonic Hubbard-Stratonovich complex field $\Delta (\vec{r})$ , which describes all possible configurations of Cooper-pairs. By expanding the resulting log determinant in terms of the fully nonlocal Gorkov free energy functional $F_G [\{ \Delta (\vec{r}), \Delta^* (\vec{r}) \}]$ near $H_{c2}$:

$$Z = \int D\Delta (\vec{r}) D\Delta^* (\vec{r}) \exp \left[ -F_G / k_B T \right]$$ (1)

The stationary phase approximation for this functional integral yields the mean field (GGL) equation for $\Delta (\vec{r})$. In the lowest Landau level approximation this equation can be solved by expanding in a set of $\sqrt{N}$ Landau functions ($N$ being the total number of vortices) $\phi_n (x, y) = \exp [i q_n x - (y + q_n / 2)^2]$, with $q_n = \frac{2\pi}{a}$ a wave number along the $x$-axis, which is also proportional to the $y$-axis position of a given $x$-axis chain. Thus in the symmetric gauge
\[ \Delta(x, y) = e^{i z y} \sum_{n=-\sqrt{N}/2}^{\sqrt{N}/2} a_n \phi_n(x, y) \]  

where for an arbitrary 2D periodic lattice the coefficients 

\[ a_n = (2n)^{i/4} \Delta_0 \exp \left( i \gamma n^2 \right) \].  

All spatial variables are expressed in units of the magnetic length. The parameter \(|\Delta_0|^2\) stands for the spatial average of the order parameter square, and \(a_x\) is the period along the \(x\)-axis. The summation in Eq.3 is performed over \(\sqrt{N}\) wavenumbers \(q_n\) only. The reduction of the number of degrees of freedom results from the assumed periodicity along the \(x\)-axis. Obviously \(a_x\) is not unique and depends on the choice of the coordinate system; any 2D lattice can be described by two different values of \(a_x\), corresponding to the principal diagonals of the elementary cell (Fig.1).

In Ref.1 it was shown that the free energy functional near stationary solutions can be approximated by a local expression. This important property, resulting from gross cancellations of nonlocal terms in the free energy due to destructive interference between phase factors of the order parameter, simplifies the problem considerably, reducing it to the Ginsburg-Landau model with the well known local free energy functional

\[ F_{GL} = \int d^2r \left[ -\alpha |\Delta(\vec{r})|^2 + \frac{1}{2} \beta |\Delta(\vec{r})|^4 \right] \]  

Here \(\alpha\) and \(\beta\) are known function of temperature and magnetic field (see below). It should be stressed, however, that the locality of the free energy functional can be very sensitive to phase fluctuations, especially in the vortex liquid state, since these fluctuations tend to destroy the phase coherence of the order parameter. For the sake of simplicity we ignore in this paper possible nonlocality in the Boltzmann factor of Eq.4. These assumption, though inconsistent with the nonlocal form of the free energy in the vortex liquid state, can be shown by a more careful analysis to have no significant influence on the final result.

It is convenient to express the GL free energy through the Abrikosov parameter \(\beta_a\): 

\[ F_{GL} = -N \varepsilon_0 / \beta_a, \]  

where \(\varepsilon_0 = \pi a^2 / 2\beta\). In the lattice state \(\beta_a\) is a function of \(a_x\) and \(\gamma\). The dependence of \(\beta_a\) on \(z = \pi \gamma^2 / a_x^2\) at \(\gamma = \pi / 2\) is plotted in Fig.(2a). Both minima with \(z_1 = \pi / 2 \sqrt{3}\) and \(z_2 = \sqrt{3} \pi / 2\) correspond to the triangular lattice with \(\beta_a \approx 1, 1596\). The maximum at \(z_3 = \pi / 2\) is obtained for the square lattice.

In discussing the effect of fluctuations we first note that the order parameter in Eq.2 with arbitrary coefficients \(a_n\) includes only those fluctuations which preserve periodic order along the \(x\)-direction. The number of independent coefficients \(a_n\) is \(\sqrt{N}\) whereas the total number of orbital centers is \(N\). Therefore each \(a_n = |a_n| e^{i \varphi_n}\) describes a set of \(\sqrt{N}\) orbital centers periodically arranged within a certain chain along the \(x\)-axis ("vortex Bragg chain"). The phase \(\varphi_n\) determines the relative position \(x_n = -\varphi_n / q_n\) of the \(n\)-th chain.

The scale of the melting temperature is determined by the value of the phase dependent terms in Eq.3. They can be readily calculated if we note that in the GL Hamiltonian there is a small parameter \(\lambda = \exp(-z)\), with \(z \gtrsim 1\) in the important regions near the minima of \(\beta_a\) (see Fig.(2a)). Thus the quartic term can be written as an expansion in \(\lambda\), i.e. 

\[ \sum_{n, s, t} \lambda^2 \Delta_0 a_n^{*} a_{n+s} a_{n+t} \phi_{n+t}, \]  

while the leading phase dependent term is of the order \(\lambda^2\). Integrating over amplitude fluctuation \(|a_n|\) by using the stationary phase approximation we find that the free energy can be written in the mean field like form 

\[ F_{GL} \approx -N \varepsilon_0 / \beta_a, \]  

where \(\beta_a = \sqrt{2 \left( 1 + 4 \lambda - 4 \lambda^2 \right) \langle \cos(\chi_n) \rangle_{\text{phase}}}, \) and \(\langle \cdots \rangle_{\text{phase}}\) means average over phase fluctuations. Note that \(\mu = \langle \cos(\chi_n) \rangle_{\text{phase}}\) is a characteristic crossover temperature from the crystal to the liquid state, with \(\beta_a = \sqrt{2 \left( 1 + 4 \lambda - 4 \lambda^2 \right)}\). Well below \(T_m\), \(\mu \rightarrow 0\), so that thermal fluctuations do not distort significantly the triangular lattice and the Abrikosov parameter is very close to its minimal (mean field) value, \(\beta_a\), i.e. 

\[ \beta_a \approx \beta_a \approx \sqrt{2 \left( 1 + 4 \lambda - 4 \lambda^2 \right)} \]  

At \(T \gg T_m\), on the other hand, \(\mu \rightarrow 0\) and the vortex system transforms to a new ("liquid") state for which \(\beta_a \approx \beta_a\). The crossover temperature \(T_m\) is determined by the minimum of the lattice entropy. The position of the cross-over \(T_m\) is obtained by minimizing the free energy 

\[ \langle F_{GL} \rangle \text{as functions of the parameter} t = -\sqrt{4 \pi / \beta_a}, \]  

which has been used in Ref.4. It is clear that at low temperatures the first state (\(z_1\)) is more stable than the second one (\(z_2\)). Since \(T_m(z_1) > T_m(z_2)\), the free energy of the first state increases faster with increasing temperature than the second one, and so there is an intersection point \(T_m\) at which energies of both states are equal but the corresponding entropies are a little different. Therefore we conclude that at this point there is a weak first order transition characterized by a small jump of the lattice entropy. The position of the crossover point \(t_m \approx -16\) and the jump of entropy \(S = -T \partial F / \partial T\) at \(T = T_m\): 

\[ T \Delta S / F_{MF} \approx 7.5 \times 10^{-3} \]  

are in good agreement with the Monte-Carlo simulations.

At higher temperatures \(\beta_a\) does not describe correctly the free energy. A straightforward calculation in this case yields for the partition function per unit vor-
tex: $Z_v \approx \sqrt{2 \beta n x^2} \text{erf}(x) \exp \left[ -\lambda \left( \frac{\partial \ln \text{erf}(x)}{\partial x} \right)^2 \right]$, where $x = \text{sign}(\alpha) \sqrt{\frac{n}{\beta n x^2}}$. In contrast to mean field theory, in which there is a second order phase transition, in our case there is only a crossover to the normal metal state, with significant fluctuations of the superfluid density appearing at magnetic fields far above the mean field $H_c$.

All current theories of the dHvA effect in the mixed state are restricted to the mean field approximation. The situation in the vortex liquid state is in some sense similar to that described by Stephen in his derivation of Maki's damping formula. However, in this approach a random distribution of vortex lines was assumed, while the (nonzero) mean field value of the Abrikosov lattice order parameter was used in the calculation. The averaging over all realizations of the random vortex lattice in such a model yields for the quartic term of the free energy: $\langle \alpha^*_q a^*_q a_q a_{q+i} \rangle \sim (\delta x_0 + \delta x_i)$. A similar expression, which reflects the complete destruction of phase coherence in the vortex lattice, can be naturally and consistently derived in the model discussed here at $T \gg T_m$ by averaging over phase fluctuations, while at $T \sim T_m$ it is approximately valid.

Thus, similarly to the MS theory, our fluctuation theory predicts an exponential damping: $M_{sc} = M_n \exp \left[ \frac{1}{2} \langle \Delta_0^2 \rangle \right]$, where $M_{sc}, M_n$ are amplitudes of the magnetization oscillations in the superconducting and normal states respectively, $\Delta_0 = \Delta_0 / \hbar \omega_c$, and $n_F = E_F / \hbar \omega_c$. A simple analytic expression for the damping parameter can be derived in the liquid state well above the melting point by integrating over amplitude fluctuations (after neglecting small linear corrections in $\lambda$):

$$\langle \Delta_0^2 \rangle = \frac{1}{\pi} \frac{\partial \ln Z_v}{\partial \alpha} \sim \frac{\alpha}{\beta} \left( 1 + \frac{\partial \ln \text{erf}(x)}{2x \partial x} \right)$$

(4)

This expression has no singularity at the mean field transition, where both $\alpha \to 0$ and $x \to 0$, and smoothly interpolates between the high field ($x \to -\infty$) value, $T / \pi |\alpha|$, and the low field ($x \to \infty$) mean field value, $\alpha / \beta$. We have compared this damping with the experimental results on the organic quasi 2D SC: (ET)$_2$Cu(SCN)$_2$. In this computation the coefficients $\alpha$ and $\beta$ derived previously in Refs. for the GGL expansion have been used, namely $\alpha = 5.7 \hbar \omega_c \ln \sqrt{H_c / H}$, and $\beta = 1.38 \hbar \omega_c / n_F$, together with the well documented Fermi surface and SC parameters of the studied material. The only adjustable parameter in this scheme is the mean field $H_c$. We have found that a single value of about 4.77 $T$ (see Fig.3) fits well the two different sets of available experimental data, which were taken at quite different temperatures.

It should be stressed that in the fluctuation theory the magnetic oscillations are smoothly damped well above $H_c$, and disappear below the mean field $H_c$, in remarkable quantitative agreement with experiment. This contrasts the result of the mean field MS theory, where the additional damping begins abruptly at $H_c$ and then increases below $H_c$ with significantly stronger rate than what is observed experimentally.

The interesting point in our derivation of the four-particle correlation function concerns the fact that its factorization in the high temperature (or high field) limit into a product of two-particle correlation functions is solely due to integration over phase fluctuations, which leads to the limiting behavior: $\langle a^*_q a^*_q a^*_q a_q \rangle \sim (\delta \alpha_0 + \delta \alpha_s)$. This is a generalized form of the condition for the vanishing of the shear modulus $\mu$ in the liquid state. The apparent correlation between the vanishing of the shear modulus of the Abrikosov lattice and the factorization of the average phase factor of the quartic term in the SC free energy is a general result, independent of the nature of the melting transition. It is thus conceivable that at a first order melting transition, the vanishing of $\mu$ at some finite value of the parameter $T / T_m$ predicted by the Monte-Carlo simulation reported will be accompanied by a transition from 'coherent' (i.e. relatively weak) damping in the crystal state to 'incoherent' MS-like damping in the liquid state at a certain magnetic field below $H_c$.

Such a transition will be difficult to observe in the quasi 2D organic SC investigated due to the very small value of the melting temperature $T_m$ and to the strength of the 2D fluctuations. In 3D superconductors where the role of fluctuations is less important and the melting transition is shifted significantly closer to $H_c$, one could expect such a transition to be observable. It is thus interesting to note that in the torque dHvA measurement performed on the borocarbide SC $YNi_2B_2Cu_3$,[1] where vortex lines pinning seemed to be weak (no peak effect near $H_c$), the Dingle plot exhibited a rather sharp upward (turn of its (negative) slope at a certain field below $H_c$, which could indicate a freezing transition into an ordered vortex lattice state.[3]

In conclusion, we have developed a simple analytical model of the vortex lattice melting in a 2D type-II SC, which is in good quantitative agreement with the state of the art numerical simulations in this field. We have shown that fluctuations in the SC order parameter, which are responsible for this melting, destroy the phase coherence in the quasi particle scattering and thus lead to enhanced damping of the dHvA oscillations in the mixed state, in remarkable quantitative agreement with the experiment.

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Figure Captions
Fig. (1): Two different choices of the vortex chains in the triangular lattice: dash lines - $z_1 = \pi / 2\sqrt{3}$, solid lines $z_2 = \sqrt{3}\pi / 2$.

Fig. (2): (a) Mean field Abrikosov parameter $\beta_a$, as a function of $z = (\pi / a_x)^2$. (b) Dependence of the local minimal values of the free energy $-\langle F \rangle / F_{MF}$ on the parameter $t = -2\sqrt{\epsilon_0} / T$.

Fig. (3): The magnetization damping factor $R_s = M_{sc} / M_n$, as a function of magnetic field $H$ in the quasi 2D SC: $(ET)_2 Cu(SCN)_2$. Solid lines are our theoretical curves for $T = 20mK$, and for the dHvA frequency $F = 690T$ (1), and $T = 120mK, F = 600T$ (2); stars and circles are the corresponding experimental data taken from Refs. 16, 17, respectively. The other parameters used in the calculations are: $T_c = 10.4K$ (from which $\Delta_0$ is calculated using BCS weak coupling formula), and $m^* = 3.5m_e$.

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Abrikosov parameter

\[ \frac{\langle F \rangle}{F_{MF}} \]
$H_c2 = 4.7 \, T$