On the Detectability of Perturbations Induced by de Sitter-Gödel-de Sitter Phase Transition

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A geometrical phase transition in the very early universe, from de Sitter to Gödel and back to de Sitter (dGd) spacetime, was recently proposed. This phase transition is shown to induce fluctuations on the matter and radiation fields with possibly observable traces. In this work we simulate the dGd-induced inhomogeneities and use them as possible seeds of perturbations, along with the standard inflationary fluctuations in the cosmic microwave background sky and the distribution of the large scale structure. We show that the power spectrum of perturbations can be characterized by a parameter pair, labeled here as \( (p_1, p_2) \). With Planck observations we find \( p_1 = 0.008^{+0.003}_{-0.008} \) and \( p_2 = 0.002^{+0.001}_{-0.002} \) consistent with pure inflationary power spectrum and no hint for the dGd transition. We also estimate that future large scale surveys such as Euclid and SKA can further tighten the constraints up to an order of magnitude and probe the physics of the early Universe with much higher precision.

I. INTRODUCTION

The early universe, with its very high temperature, provides a unique laboratory to test theories of high energy physics, inaccessible to earth-bound experiments. Among these theories are possible cosmological phase transitions taking place at different epochs depending on the energy scales involved (e.g. see [1–3]). If these transitions leave observable cosmological imprints, e.g., if they generate fluctuations on the Cosmic Microwave Background (CMB) radiation and matter fields, they would have the chance to be tested against data and their parameter \( \beta \) would be constrained (see e.g. [4] for constraints on the cosmic string tension from Planck CMB observations).

Among plausible phase transitions in the early universe is the recently suggested quantum phase transition between space-times, the so-called de Sitter-Gödel-de Sitter (dGd) phase transition [5]. The dGd scenario assumes a scalar field, living in a de Sitter background, experiences a phase transition to a rotating geometry (Gödel) and slowly rolls back to the de Sitter phase. Quantum field theory calculations at finite temperature show that this second order phase transition has a chance to occur at high temperatures, and the transition probability depends on the rotation parameter of the Gödel phase, \( \alpha \), increasing as \( \alpha \) decreases. This rotation would be induced on the trajectories of test particles. Simulations show that local congruence of particles have nonzero induced rotation while the average global rotation is almost zero. The dGd transition could therefore be a source of initial rotation for large structures. This is particularly of interest, since simple inflationary theories do not seed vector modes, and therefore no initial spin is expected for the largest scale structure in pure inflationary cosmologies. The later growth of structures in the nonlinear regime could cause structures to spin, even in the absence of any initial angular momentum. In the linear regime, however, a mechanism is required to initiate the rotation. For example the "tidal torque theory" was proposed to produce initial spin for proto-halos in the linear regime [6].

It was also shown that Casimir forces in the dGd transition would induce inhomogeneities in the matter and radiation fields, possibly observable in the Cosmic Microwave Background (CMB) radiation or large scale structure data [7]. The predictions of the dGd transition can therefore be directly tested against the existing data. Observational assessment of the viability of this theory and estimating the model parameters are the main goals of this paper.

This paper is organized as follows: Sections 2 and 3 review the dGd mechanism as a possible origin of the cosmic rotation and its local features due to the Casimir effect in the Gödel phase, respectively. These sections form the theoretical basis of the current work. In Section 4
we simulate the primordial seeds of inhomogeneities produced by the dGd transition and assess the detectability of the trace of these primordial seeds (alongside with inflationary perturbations) on CMB anisotropies. We also explore how future large scale surveys would improve the current bounds. We conclude in Section 5.

II. GLOBAL FEATURES OF DGD AND INDUCED ROTATION

The extremely high temperature of early times allows for a scenario where a quantum mechanical phase transition could change the geometry of spacetime. This transition could occur due to a phase transition in the potential of a scalar field. For a spatially constant scalar field $\phi$ with negligible potential minimum and $\nabla \phi = 0$, one has $p_{\phi} = -\rho_{\phi}$[8]. Therefore $V(\phi)$ acts as an effective cosmological constant. This gives tools of using the effective potential in curved spacetimes. The action of a scalar field in a curved spacetime with the metric $g$ is [9]

$$S[\phi, g_{\mu\nu}] = \int d^4x \sqrt{-g} \left( \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) \right)$$  (1)

which gives the one-loop effective potential as

$$V_{\text{eff}}^{(1)} = - \frac{\Gamma^{(1)}}{\mathcal{V}}$$  (2)

where $\mathcal{V}$ is the spatial volume, $\Gamma^{(1)} = -\frac{1}{2} \ln(\mu^{-2} \det G)$ is the one-loop effective action and $G = \Box + V''$. $\Box$ is the Laplace operator defined with the metric and $\mu$ is introduced for dimensional considerations.

The dGd requires a situation where the background geometry changes from de Sitter to Gödel and back to de Sitter through a thermal phase transition of the scalar field. A typical scenario to realize this situation is shown in Figure 1 with three phases [5]. First, $V_{\text{eff}}$ acts like a positive cosmological constant and is much larger than the dust density $V_{\text{eff}} \gg \rho_{\text{dust}}$. The static patch of this de Sitter spacetime is described in $(t, \chi, \theta, \xi)$ coordinates by [10]

$$ds^2 = \cos^2 \chi dt^2 + a^2(d\chi^2 + \sin^2 \chi d\theta^2 + \sin^2 \chi \sin^2 \theta d\xi^2).$$  (3)

As the temperature drops, the shape of the renormalized effective potential turns into a Mexican hat, allowing for the required phase transition. The resulting spacetime is an exact dust solution of Einstein field equations via a negative cosmological constant and is described by the Gödel metric in Cartesian coordinates $(t, x, y, z)$ by [11]

$$ds^2 = (dt + \exp \sqrt{2\alpha x^2} dy)^2 - dx^2 - \frac{1}{2} \exp \sqrt{2\alpha x^2} dy^2 - dz^2.$$  (4)

Here $\alpha$ is a nonzero real constant and shows the rotation rate of dust around the $y$-axis. Being a rotating spacetime, the Gödel phase can naturally initiate the rotation. In this phase $V_{\text{eff}} = -\rho_{\text{dust}}/2$ and has the role of a negative cosmological constant. Finally the shape of the effective potential in the Gödel background would allow for rolling of the scalar field such that the Universe goes back to the de Sitter phase. A classical potential for $\phi$ is assumed as

$$V = \frac{1}{2} \sigma^2 \phi^2 + \frac{1}{24} \lambda \phi^4$$  (5)

with $\sigma$ and $\lambda$ being an arbitrary mass scale and a dimensionless coupling constant, respectively. Using the zeta-function regularization method, the corresponding effective potential to (5) would be

$$V_{\text{eff}}(\phi, \beta) = V(\phi) - \frac{1}{2\beta \mathcal{V}} \left[ \zeta'(0, \beta) + \log(\mu^2 a^2) \zeta(0, \beta) + \log(V''(\phi)\mu^{-2}) \right]$$  (6)

where $\beta = \frac{1}{k_B T}$ and $\zeta$ is the Zeta function [12]. Extensive computations and renormalization of effective potentials both in de Sitter and Gödel spacetimes prove there exist regions in the parameter space that can make the scenario work (for details see the Appendix of [5]). The scenario could be summarized as follows: In the beginning the scalar field was living in a de Sitter spacetime ($V_0 \sim \Lambda$). After enough cooling due to expansion, the Universe reached the critical temperature $\beta_c$ where $\frac{dV_{\text{eff}}}{d\sigma^2}|_{\phi=0} = 0$. 

FIG. 1. Typical scenario of the initial de Sitter, Gödel and final de Sitter phases, taken from [5].
Below $\beta_c$, the scalar field experienced a phase transition to the Mexican hat potential with negative cosmological constant which describes a Gödel phase. Then the scalar field rolled down the potential until $\phi = 0$. One finds after $\dot{t} \simeq \frac{\dot{\phi}}{\phi} \simeq \frac{\dot{\sigma}}{\sqrt{1}}$ (which is the time duration of dGd phase transition) passed, the Universe could go back to a de Sitter phase again.

Having shown that the dGd phase transition could have potentially happened in the early Universe, its impact on the equations of motion of a test particle was explored. As might be intuitively expected, a particle that enters the first de Sitter phase with a nonrotating trajectory, exits to the final phase rotating. In other words, this phase transition induces rotation in the motion of test particles. The value of the induced rotation depends on the position of the particle in the dGd space, its distance from the symmetry axis of Gödel and the initial velocity and is of order $\sqrt{1}$. It should also be noted that the Israel junction conditions applied to the boundary of the Gödel and de Sitter regions require non-zero distribution of matter on that boundary,

$$\Delta K_{ab} = K_{ab}^2 - K_{ab}^1 \propto S_{ab}$$

where $K^1$ and $K^2$ are extrinsic curvatures of Gödel and de Sitter spacetimes on the boundary and $S$ is the surface stress-energy tensor. One could go further and compute the junction conditions for this phase transition which is out of the scope of this paper.[13]

The above mechanism also works for a congruence of particles. Simulations show that a local congruence of particles would obtain nonzero local induced rotation. On the other hand, if we divide the space into cells and simulate the dGd transition based on the quantum tunneling probability and the randomness of the symmetry axis of Gödel space direction, we find that the average global induced rotation is nearly zero, as expected. See Figures 5 and 7 of [5].

III. LOCAL FEATURES OF DGD AND THE CASIMIR EFFECT

As stated in the previous section, the value for the globally induced rotation is found to be below the observational limit. We therefore investigate local properties of dGd phase transition. These effects were studied through computing the Casimir force in Gödel spacetime [7] and are briefly reviewed in the following.

The Casimir force is sensitive to the rotation angle of the Gödel spacetime. It is observed that for two parallel plates with a separation comparable to the rotation of Gödel spacetime ($\alpha$), the force becomes repulsive and then approaches zero. This effect, when considered collectively due to many layers, could induce inhomogeneities which are potentially observable. In fact, after the dGd phase transition, the distance between local layers of matter would shrink in a direction perpendicular to the random Gödel rotation axis in an inhomogeneous way. The Casimir energy of a scalar field in the Gödel background and at finite temperature is given by

$$\hat{E}_{\text{Casimir}}(\bar{d}, \bar{\beta}) = \hat{E}_0(\bar{d}) + \Delta_{F.T.}(\bar{d}, \bar{\beta}).$$

The zero temperature term $\hat{E}_0(\bar{d})$, and the finite temperature contribution $\Delta_{F.T.}(\bar{d}, \bar{\beta})$ are respectively given by

$$\hat{E}_0(\bar{d}) = -\frac{\pi}{24d} - \frac{1}{4\sqrt{2d}} \exp\left\{-2\left(\frac{1}{2} + \bar{m}^2\right)\frac{d^2}{\pi}\right\} ,$$

and

$$\Delta_{F.T.}(\bar{d}, \bar{\beta}) = \frac{1}{\lambda} \ln \left\{ \frac{1}{\exp(-\bar{\beta}\sqrt{\frac{1}{2} + \bar{m}^2 + \bar{n}_1^2 + \frac{\pi^2\bar{n}_2^2}{d^2}})} \right\} \times \left\{ \frac{1}{\exp(-\bar{\beta}\sqrt{\frac{1}{2} + \bar{m}^2 + \frac{\pi^2\bar{n}_2^2}{d^2}})} \right\} \times \left\{ \frac{1}{\exp(-2\bar{d}\sqrt{\frac{1}{2} + \bar{m}^2 + \bar{n}_4^2}} \right\} \right\}^{\frac{1}{2}}.$$ (10)

Here $\bar{\beta} = \bar{\beta}\alpha$, $\bar{d} = \bar{d}\alpha$, $\bar{m} = \bar{m}/\alpha$ and $\bar{E} = E/\alpha$ are dimensionless quantities in natural unites, $\alpha$ is the rotation rate of the Gödel metric (see Eq. 4), $d$ is the distance between plates and $n_1, n_2, n_3, n_4$ are introduced in [7].

Straightforward calculation would yield the expression for the Casimir force and the general behavior can be plotted numerically (see Figure 4 of [7]). Here we only keep the major contribution to the force given by $n_1 = n_2 = n_3 = n_4 = 1$. The $\bar{d}$-dependence of the Casimir force would then be

$$F_{\text{Casimir}} \simeq O\left(\frac{1}{\bar{\beta}^2 \bar{d}^{1/2}}\right) + O\left(\frac{\bar{d}^{1/2}}{\bar{\beta}^2}\right) + O\left(\frac{1}{\bar{\beta}}\right) + O\left(\frac{1}{\bar{\beta}\bar{d}^2}\right),$$

or, in terms of $\alpha$,

$$F_{\text{Casimir}} \simeq \alpha^2 \left\{ O\left(\alpha^{-5/2}\right) + O\left(\alpha^{-3/2}\right) + O\left(\alpha^{-1}\right) + O\left(\alpha^{-3}\right) \right\}.$$ (12)

Asymptotically, in the limit of small rotations we get

$$F_{\text{Casimir}} \simeq \frac{1}{\bar{\beta}\bar{d}^2}.$$ (13)

In the following, we investigate the observable consequences of dGd transition produced by local Casimir forces.

IV. SIMULATIONS AND RESULTS

It was argued in the previous section that an early dGd phase transition (happening around the end of inflation)
would generate Casimir forces. These forces would generate potentially observable inhomogeneities in the Universe. In this section, we first simulate the fluctuations in the inflaton filed by the dGd scenario and then consider them as possible seeds of inhomogeneities in the Universe. Our goal is to assess their detectability in the observations of CMB anisotropies and large scale structure.

Consider the Universe as a 3D lattice with $n^3$ cubic cells (with side $d$) as schematically illustrated in Figure 2. We find $n = 30$ to be a proper choice in this work, yielding converged results with reasonable computational cost. The location of each cell is represented by its center coordinate. In the Gödel phase each cell would experience some shrinkage, $\delta$, along a random direction. In general, one can calculate $\delta$ by using the geodesic equation of a test particle moving under the Casimir effect. Therefore, its density contrast against the background, is then calculated by taking into account both of these shrinkage and overlapping cell effects. It should be noted that since the rotation is due to a quantum phase transition, more precise simulations should take into account the quantum tunneling nature of the transition and therefore the rotation. In this work, however, we ignored this effect for simplicity. The result of each simulation would be an array of local density variations $\Delta(\vec{x}) = \frac{\delta \rho}{\rho}(\vec{x})$. One then gets the correlation function $\xi(\vec{r}) = \langle \Delta(\vec{x}+\vec{r})\Delta(\vec{x}) \rangle$ of the predicted primordial density field $\Delta(\vec{x})$ where $\langle \ldots \rangle$ represents averaging over the $N_{\text{sim}}$ simulations. The Fourier transform of the correlation function of the density field would give the power spectrum of the primordial field $\langle \Delta(\vec{k})\Delta(\vec{k}') \rangle = (2\pi)^3\delta_D(\vec{k}+\vec{k}')P_{\delta\phi}(k)$, where $\Delta(\vec{k})$ represents the Fourier transform of $\Delta(\vec{x})$. The required conversion from $P_{\delta\phi}$ (as directly calculated from simulations) to the primordial power spectrum $P(k)$ of curvature perturbations is the same as in standard inflationary scenarios, with the only difference being the shape of $P_{\delta\phi}$.

By repeating the simulations for different values of $\delta$ which can be considered the main physical free parameter of the scenario, we find that the dimensionless power spectrum for the induced curvature perturbations, $P(k)$, can be fitted by

$$P(k) = \left(p_0 + p_1(k/k_p) + \frac{p_2}{(k/k_p)^n}\right) \times 10^{-10},$$

where $k_p = 0.05\text{Mpc}^{-1}$ is the pivot scale for scalar perturbations and $p_{0,1,2}$ are dimensionless coefficients. It turns out that the functional form of the fitted curve is quite insensitive to the choice of $\delta$ and $\delta$ only affects the parameter values. We also find that $n \approx 1$. Figure 3 illustrates the dependence of the dGd parameters $p_1$ and $p_2$ on $\delta$ over a wide span.

Given the proposed shape for the power spectrum...
FIG. 4. The sensitivity of the CMB temperature power spectrum (top) and matter power spectrum at $z = 1$ (bottom) to variations in the two dGd parameters, $p_1$ and $p_2$.

(Eq. 15), we proceed by assessing the detectability of these fluctuations by CMB and large scale data.

A. Results

In this section we study the imprints of dGd parameters on the Planck measurement of the CMB power spectrum (Section IV A 1) and make forecast for the detectability of the dGd imprint with future large scale data. Figure 4 compares the expected impact of the dGd parameters on the CMB (top) and matter power spectrum (bottom) and illustrates where the maximum sensitivity of these observables to the parameters are. It should be noted that $p_0$ is hardly distinguishable from the amplitude of primordial inflationary scalar perturbations $A_s$ (assuming a nearly scale-independent power spectrum). Therefore we do not consider it as a new parameter in our analysis.

FIG. 5. The posterior probability of the dGd parameters $p_1$ and $p_2$ using Planck dataset.

TABLE I. Best-fit parameters describing the initial conditions of the Universe in the dGd model (both inflationary and dGd parameters) and their 1σ errors as measured by Planck.

| $p_1$  | $p_2$  | $n_s$  | Log($10^{10} A_s$) |
|-------|-------|-------|-------------------|
| $0.008^{+0.003}_{-0.008}$ | $0.002^{+0.001}_{-0.002}$ | $0.9623^{+0.0075}_{-0.0053}$ | $3.0410^{+0.0161}_{-0.0259}$ |

1. Cosmic Microwave Background

We modify the publicly available code CosmoMC [14] to take into account the contribution of the dGd induced inhomogeneities as a primordial source of inhomogeneities and leave the dGd parameters $p_1$ and $p_2$ as free parameters to be estimated by data. We assume uniform priors on these parameters and only require that the total dGd power spectrum, including contribution from both $p_1$ and $p_2$, be non-negative. Therefore, these two parameters are not separately restricted to non-negative values. As stated before, we use Planck measurement of CMB temperature and polarization anisotropies [4]. We work in the ΛCDM theoretical framework, with the only modification possibly coming from the dGd phase transition. Our parameter set therefore includes the standard cosmological base parameters ($\Omega_b h^2$, $\Omega_c h^2$, $\theta$, $\tau$, $A_s$ and $n_s$, with $k_{\text{pivot}} = 0.05\text{Mpc}^{-1}$ as the pivot for scalar perturbations) along with the dGd parameters. We have assumed uniform priors on the dGd parameters, in the range $[0, 10]$. We find this prior range to be safe in the sense that it covers all the dGd parameter space with non-negligible likelihood, and the posterior is not cut in the edges of the parameter space due to prior biases. We take the number of relativistic species to be $N_\nu = 3.046$ and assume the neutrinos to be massless. We have also assumed the primordial tensor perturbations have negligible contribution to CMB temperature and $E$-mode polarization anisotropies. The helium abundance is set from the BBN consistency relation. We use eight chains...
of parameters and use the Gelman and Rubin R-statistic to assess their convergence. We find that, with a total of about 32000 samples, the chains are converged with $R - 1 < 0.01$.

Table I summarizes the results of this dGd-parameter measurement and Figure 5 shows the posterior probabilities of $p_1$ and $p_2$. The two dGd parameters are almost uncorrelated with each other. That is expected since the two parameters affect different scales. Small scales, or large $k$’s, are most sensitive to $p_1$, while large scales, or small $k$’s, are mostly affected by $p_2$ (see equation 15 and Figure 4). The standard parameters also have little correlation with the dGd ones due to distinct imprints they leave on the power spectrum and are therefore almost unchanged. The results indicate no deviation from the inflationary power-law spectrum in the form predicted by the dGd formalism.

2. Large Scale Structure

Features in the primordial power spectrum also leave imprints on matter distribution. We investigate the detectability of the dGd-induced features characterized by the two parameters $p_1$ and $p_2$ in the future large scale surveys. In this work we make forecast using simulations for the European Space Agency’s Euclid mission, referred to as Euclid-like, and the Square Kilometer Array (SKA), with two different sets of proposed specifications, referred to as SKA1-like and SKA2-like. In particular we use the weak lensing (WL) and galaxy clustering (GC) probes, following specifications assumed in [15–17]. We do a Fisher matrix analysis in the linear regime of perturbations assuming a near-Gaussian distribution for the parameter. The formalism and the details of the analysis are similar to the analysis thoroughly described in [18].

Tables II and III present the forecasted errors of the two dGd parameters for the various experimental scenarios used in this section, for the WL and GC probes, and with the standard cosmological parameters assumed fixed and free respectively. The constraints from GC and WL are tightest from Euclid-like and SKA2-like and comparable to Planck measurements (Table I).

V. SUMMARY AND DISCUSSION

In this work we investigated the observational consequences of a possible phase transition of the spacetime at the end of inflation, the so-called dGd phase transition. We simulated fluctuations in the inflaton field induced by this transition and found the fit to the corresponding power spectrum. The amplitudes of the various terms in the dGd power spectrum were considered as free parameters and were constrained by Planck data. No significant deviations from the standard power-law inflationary power spectrum were found. The high precision observations of the large scale structures in the near future could improve these constraints. We made Fisher-based forecasts for Euclid and SKA-like surveys and found comparable bounds on the dGd parameters from the weak lensing and galaxy clustering probes.

If deviations from pure inflationary power law are observed, the consistency of these perturbations with the dGd scenario could be tested by extracting the $\delta$’s corresponding to each observed dGd parameter, $p_1$ and $p_2$, from Figure 3. The agreement of the deduced $\delta$’s (within the error bars) would imply the consistency of the observed deviation as seeded by an early dGd phase transition. The derived value for $\delta$ would also shed light on the physics of the phase transition through constraining its duration $\tilde{t}$ (as discussed in Section IV), which itself depends on the free parameters of the theory $\sigma$, $\lambda$ and $\Lambda$ through Equation 14.

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