Neutrino mass and magnetic moment from neutrino-electron scattering.

J. Bernabéu, S.M. Bilenky, F.J. Botella, J.A. Peñarrocha and J. Segura

Departament de Física Teòrica
Universitat de València
and
IFIC, Centre Mixt Univ. Valencia-CSIC
E-46100 Burjassot, Spain

Abstract

We study both the elastic ($\nu e \rightarrow \nu e$) and the radiative process ($\nu e \rightarrow \nu e\gamma$) and discuss how these processes can shed light on some current topics in neutrino physics such as a neutrino magnetic moment and neutrino oscillations. The radiative process allows to reach low values of $Q^2$ without the need to operate at very small energies of recoil electrons, a favourable scenario to search for a neutrino magnetic moment. The elastic cross section contains a dynamical zero at $E_\nu = m/(4\sin^2\theta_W)$ and forward electrons for the electron antineutrino channel, which is reachable at reactor facilities and accessible after the convolution with the antineutrino spectrum. The implication for lepton flavour changing transitions in that energy region searched for in neutrino oscillation experiments, which combine disappearance and appearance rates, is discussed.
1 Introduction.

The neutrino-electron process plays a crucial role in the study of the standard model of electroweak interactions, as well as in searching for effects beyond the standard model. It gives relevant information about possible deviations from S.M. as, for instance, the possible existence of a large neutrino magnetic moment: the laboratory bound on the neutrino magnetic moment ($\mu_\nu < 2.4 \times 10^{-10}$) has been set \[1\] with $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$ in reactor experiments and several new proposals \[2\] plan to study the $\bar{\nu}_e$ magnetic moment at the level of $2 \times 10^{-11}$ Bohr magnetons.

The differential cross section for $\bar{\nu}_i e^- \rightarrow \bar{\nu}_i e^-$ including the neutrino magnetic moment contribution \[3\] and neglecting neutrino mass is given by

$$\frac{d\sigma_{\bar{\nu}e}}{dT} = \frac{G^2 m^2}{2\pi} \left[ (g^i_R)^2 + (g^i_L)^2 \left( 1 - \frac{T}{E_\nu} \right)^2 - g^i_L g^i_R \frac{mT}{E_\nu^2} \right] + \frac{\pi \alpha^2}{m^2} \left( \frac{\mu_\nu}{\mu_B} \right)^2 \left( 1 - \frac{T}{E_\nu} \right) \right] \quad (1)$$

where $G$ is the Fermi coupling constant, $\alpha$ the fine structure constant, $\mu_\nu$ the neutrino magnetic moment, $\mu_B$ the Bohr Magneton, $m$ the electron mass, $T$ the recoil kinetic energy of the electron and $E_\nu$ the antineutrino incident energy. In terms of $\sin^2 \theta_W$ the chiral couplings $g^i_L$ and $g^i_R$ ($i = e, \mu, \tau$) can be written as

$$g^i_L = -1 + 2 \sin^2 \theta_W + 2\delta_i e; \quad g^i_R = 2 \sin^2 \theta_W \quad (2)$$

( for neutrinos one should exchange $g_L$ by $g_R$).

In the laboratory experiments on neutrino magnetic moment, the sensitivity to $\mu_\nu$ is connected with the fact that at low enough values of $Q^2 = 2mT$ the contribution of the electromagnetic amplitude to the cross section of the process becomes comparable to the contribution of the weak amplitude. This is the case for $Q^2 \sim MeV^2$ at values $\mu_\nu \simeq (10^{-10}, 10^{-11})\mu_B$. The penetration in the region of such small $Q^2$ requires, however, to measure small energies of recoil electrons ($T \leq MeV$).

In section 2 we discuss the use of the radiative process to extract a neutrino magnetic moment. Section 3 presents the dynamical zero in elastic scattering, whereas section 4 shows a novel approach to neutrino oscillations based on this zero.
2 Neutrino magnetic moment and the radiative process.

Now let us consider the process $\nu(\bar{\nu}) + e \rightarrow \nu(\bar{\nu}) + e + \gamma$. Even if it has an additional power of $\alpha$ in the cross section relative to the elastic case, the restriction to low recoil energies in order to reach down low values of $Q^2$ is a priori not necessary. The limit $Q^2 = 0$ at fixed values of the recoil energies can be reached for the maximal opening angle between electron and photon in the final state $(\theta_{e\gamma})$. Whatever the experimental limit on the total recoil energies $\nu$ could be, this process is able to lead to lower values of $Q^2$ than the elastic one, as shown by the ratio $x = Q^2/(2m\nu)$ varying from 1 to 0 ($\nu = T + E_{\gamma}$, being $E_{\gamma}$ the photon energy).

With this motivation we have calculated [4] the triple differential cross section for the process

$$\nu(l) + e(p) \rightarrow \nu(l') + e(p') + \gamma(k)$$

in terms of the three dimensionless variables

$$x = Q^2/(2m\nu) \quad y = \frac{\nu}{E_{\nu}} \quad \omega = \frac{E_{\gamma}}{E_{\nu}}$$

both for the weak and the magnetic contribution, which add incoherently due to the opposite final neutrino helicities induced by each of these two interactions for massless neutrinos. For fixed $x$ and $y$, the $\omega$-integration in the cross section can also be performed in an analytic way.

We are interested in the behaviour of both the weak and the electromagnetic cross sections at low $Q^2$, with a view to enhance the second contribution with respect to the first one. First we consider, at $y, \omega$ fixed, the expansion around $x \rightarrow 0$. The weak cross section is

$$\frac{d\sigma_W}{dx dy d\omega} \lessapprox \frac{G^2 m^2}{\pi^2} \alpha \frac{1}{y'\omega} \left\{ W(y, \omega) g_A^2 + \frac{E_{\nu} x y}{2m} [V(y, \omega, \frac{m}{E_{\nu}}) g_V^2 + A(y, \omega, \frac{m}{E_{\nu}}) g_A^2 + I(y, \omega) g_V g_A] \right\}$$

where
\[ W(y, \omega) = (1 - y)(y - \omega)^2 \]  
\( V(y, \omega, \frac{m}{E_\nu}), A(y, \omega, \frac{m}{E_\nu}) \) and \( I(y, \omega) \) are well behaved functions and all the explicit \( E_\nu \) dependences come in powers of \( m/E_\nu \).

The couplings are

\[ g_V = \frac{g_L + g_R}{2}, \quad g_A = \frac{g_L - g_R}{2} \]  

in terms of the chiral couplings of Eq. (2) (going form \( \nu \) to \( \bar{\nu} \) one should change the \( g_A \) sign).

There are interesting features associated with this result. At \( x = 0 \) the only survival term in the cross section goes like \( g_A^2 \). It is well known that, due to CVC, the structure function associated with inelastic excitations mediated by the vector current goes like \( Q^2 \) at fixed \( \nu \). So only the (PCAC) \( g_A^2 \)-term \([5]\) can survive at \( x = 0 \).

Nevertheless, \( W(y, \omega) \) will be the dominant term only in a very restricted range around \( x = 0 \). So, for example, this term gives a good approximation provided \( \nu >> m \) (high incoming energies) but within the restricted range \( Q^2 << 4m^2 \). This is so because the linear term in \( x \), in fact, goes as \( Q^2/4m^2 \). Furthermore, the \( W(y, \omega) \) dependence goes like the square of the recoil energy of the electron. If \( \nu << m \) there are high cancellations in this term, seen for example when one integrates over \( \omega \) at fixed \( y \). There is no such cancellation for \( V(y, \omega, \frac{m}{E_\nu}), A(y, \omega, \frac{m}{E_\nu}) \) or \( I(y, \omega) \).

We conclude that the \( x = 0 \) term is only important at high incoming energies with \( \nu >> m \), but with \( Q^2 << 4m^2 \). Our strategy will be just the contrary, i.e., have \( \nu < m \) with low \( Q^2 \), in order to suppress the \( x = 0 \) \( g_A^2 \)-term in the weak cross section.

The cross section induced by a neutrino magnetic moment \( \mu_\nu \neq 0 \) gives, in the limit \( x \to 0 \).

\[
\frac{d\sigma_M}{dxdyd\omega_{x<1}} \simeq \frac{\alpha^3}{2m^2} \left( \frac{\mu_\nu}{\mu_B} \right)^2 \frac{1}{y^2 \omega} \{ M(y, \omega, \frac{m}{E_\nu}) + xN(y, \omega, \frac{m}{E_\nu}) \} \]  

where

\[
M(y, \omega, \frac{m}{E_\nu}) = (1 - y)[(y^2 + \omega^2) - 2\frac{m}{E_\nu}(y - \omega) + \frac{m^2}{E_\nu^2 y\omega}(y - \omega)^2] \]
and \( N(y, \omega; \frac{m}{E_\nu}) \) is a well behaved function with the same explicit \( E_\nu \) dependences as \( V(y, \omega; \frac{m}{E_\nu}) \) and \( A(y, \omega; \frac{m}{E_\nu}) \).

The first point to be noticed in Eq. (8) is the absence of the \( 1/x \) singularity associated with the photon propagator in the magnetic contribution present in the elastic scattering cross section. This is again due to the conservation of the electromagnetic current in the electron vertex, implying a linear \( Q^2 \)-behaviour of the structure function, at \( \nu \) fixed, for inelastic excitations. The leading \( M(y, \omega) \) term is related to the Compton scattering cross section (like \( V(y, \omega) \) is, and also \( A(y, \omega) \) for \( E_\nu >> m \)). In fact, one can write

\[
\frac{d\sigma_M}{dx dy d\omega} \bigg|_{x=0} = \frac{\alpha}{2\pi} \frac{E_\nu}{m} (\frac{\mu_\nu}{\mu_B})^2 (1-y) \frac{d\sigma^{\gamma\gamma}}{d\omega} \tag{10}
\]

with \( \sigma^{\gamma\gamma} \) given by the Klein-Nishina formula identifying \( y \) with the energy of the incoming photon and \( \omega \) with the energy of the outgoing photon. Contrary to the behaviour that we have discussed for \( W(y, \omega) \) in the weak cross section, the term \( M(y, \omega) \) is not here suppressed with respect to the linear term in \( x \), \( N(y, \omega) \), so Eq. (10) is a very good approximation to the magnetic cross section at low energies and low values of \( Q^2 \). Taking the ratio of cross sections at \( Q^2 = 0 \), we have

\[
\frac{d\sigma_M}{d\sigma_W} \bigg|_{x=0} = (\frac{\mu_\nu}{\mu_B})^2 \frac{\pi^2 \alpha^2}{G^2 m^2} \frac{1}{2 m g_A T} \left\{ \frac{2E_\gamma(E_\gamma + T) + T^2}{mT} + \frac{mT - 2E_\gamma(E_\gamma + T)}{E_\gamma(E_\gamma + T)} \right\} \tag{11}
\]

where the global factor in front of the bracket is a typical measure of this ratio for the elastic scattering process at the same value of \( T \). A glance at eq. (11) would say that the highest cross section ratios are obtained for the hardest photon limit \( E_\gamma >> T \), with values higher than the elastic ones at will. Even more, one would say that higher neutrino energies are favoured in order to have hard photons but the discussion after eq. (7) should have clarified that a little departure from \( x = 0 \) under these conditions is enough to enhance the next linear term in \( x \) so that the ratio (11) becomes diluted.

To conclude, the strategy to reach low enough \( Q^2 \)-values, approaching \( \theta_{max} \) at fixed \( (y, \omega) \), works only in a very limited angular range around \( \theta \approx \theta_{max} \). Whenever the
results are integrated over a wider region of $\theta$, the ratio $d\sigma_M/d\sigma_W$ will be diluted, as illustrated in Fig. 1.

We can consider the approach to $Q^2 \to 0$ for fixed $x$. The vector contribution is in this case not penalized due to CVC with respect to the axial contribution, as it was the case for $x \to 0$: the structure function goes like $Q^2/\nu$ and the limit $\nu \to 0$ is not physically forbidden for our process. It is thus of interest to study the inclusive cross sections $d\sigma/dxdy$ and explore their behaviour when $y \to 0$ at fixed $x$. We can use the analytic results of the triple differential cross sections for the integration in $\omega$, with the condition $\nu << m$, and obtain

$$
\frac{d^2 \sigma_W}{dxd\nu} \simeq \frac{4 G^2}{3 \pi^2} \frac{\alpha}{x-1} \nu \left\{ x[(g_V^2 + g_A^2) - \frac{\nu}{E_\nu}(g_V^2 + g_A^2 - 2xg_Vg_A)] - \frac{\mu}{2E_\nu}(g_V^2 - g_A^2) \right\} + \frac{\nu}{m}[(\frac{17}{10} - 2)xg_V^2 + \frac{1}{10}(37x^2 - 60x + 20)g_A^2] + O(\nu^2)
$$

(12)

for the weak cross section, whereas

$$
\frac{d^2 \sigma_M}{dxd\nu} \simeq \frac{4\alpha^3}{3m^2} \left( \frac{\mu}{\mu_B} \right)^2 \left\{ 1 - \frac{\nu}{E_\nu} + \left( \frac{17}{10}x - 2 \right) \frac{\nu}{m} + O(\nu^2) \right\}
$$

(13)

gives the magnetic moment cross section, which is much less sensitive to low $x$ values. Note that the $g_A^2$ term is the only one which survives at $x = 0$ with a $\nu^2$ suppression due to the cancellation in $W(y, \omega)$.

Fig. 2 gives the ratio of the inclusive cross section $d^2\sigma^M/d\nu dx$ over $d^2\sigma^W/d\nu dx$ for electron-antineutrino scattering at $E_\nu = 1\,\text{MeV}$. This results confirms, as can be guessed from eqs. (12) and (13), that the highest sensitivity is obtained for the lowest values of $\nu$ and, by going down to low values of $x$, the sensitivity is higher than for the elastic scattering case with $x = 1$. On the contrary, once $\nu$ is high enough, the sensitivity is not improved when lowering the value of $x$. Then we see that, although the absolute cross sections are small (for instance, $\sigma_M/\sigma_W = 4.4$, $\sigma_M = 2.7 \times 10^{-47}\text{cm}^2$ for $\mu_\nu = 10^{-10}\mu_B$ integrating over $\nu < 0.5\,\text{MeV}$, $x < 0.5$), the standard model contribution is suppressed in these circumstances more strongly than in the elastic scattering case, thus giving a favourable ratio. The general features are not highly sensitive to the incoming neutrino energy within the range of the reactor antineutrino spectrum.
For \( x \to 1 \) and \( \nu \to 2E^2\nu/(2E\nu + m) \) one can see at \( E\nu = 1MeV \) a remarkable feature: the ratio \( d\sigma^M/d\sigma^W \) increases rapidly. The limit \( x \to 1 \) leads to the infrared behaviour; then the peak-effect should also take place for the elastic process. Taking a glance at Eq. (1) one concludes that this effect must be a consequence of some cancellation in the elastic weak cross section for forward electrons \( (T = 2E^2\nu/(2E\nu + m)) \), as we are going to show now.

3 The dynamical zero in \( \bar{\nu}_e e^- \) elastic scattering.

We see from eq. (1) that cancellations may happen in the elastic cross section provided \( g_L g_R > 0 \), as is the case for \( \bar{\nu}_e e^- \) scattering. In fact, a complete cancellation is going to take place.

In the LAB frame, the backward neutrino (forward electron) cross section for \( \nu_i e^- \to \nu_i e^- \) can be written as

\[
\left( \frac{d\sigma_{\nu_i}}{dT} \right)_{\text{back}} = \frac{G^2 m}{2\pi} \left[ g^i_L - g^i_R \frac{m}{2E\nu + m} \right]^2
\]  

We see this backward cross section does not cancel with \( g^e_L \) and \( g^e_R \) satisfying \( g^e_L > g^e_R > 0 \), which is the case for \( \nu_e \) as seen from equation (2). On the other hand, for \( \bar{\nu}_e e^- \) backward elastic scattering we have

\[
\left( \frac{d\sigma_{\bar{\nu}_i}}{dT} \right)_{\text{back}} = \frac{G^2 m}{2\pi} \left[ g^i_R - g^i_L \frac{m}{2E\nu + m} \right]^2
\]  

which vanishes for \( \bar{\nu}_e \) at

\[ E_{\nu} = \frac{m}{4\sin^2 \theta_W}. \]  

Therefore we have found that for the antineutrino energy \( E_{\nu} \) given by equation (16) and forward electrons (electrons with maximum recoil) the differential cross section for \( \bar{\nu}_e e^- \to \bar{\nu}_e e^- \) vanishes exactly at leading order. This cancellation, depending on the values of \( g_L \) and \( g_R \), is a dynamical zero [3].
For $\nu_\mu$ and $\bar{\nu}_\mu$ elastic scattering (or $\nu_\tau$ and $\bar{\nu}_\tau$) the corresponding $g^\mu_L$, $g^\mu_R$ parameters are such that $g^\mu_L g^\mu_R < 0$, thus preventing the corresponding cross sections from dynamical zeros for backward neutrinos.

Studying the conditions that define the potential dynamical zeros for each helicity amplitude at lowest order in electroweak interactions one obtains all the information about dynamical zeros for polarized and unpolarized differential cross sections. Let us denote by $M^{\nu_i(\bar{\nu}_i)}_{\lambda'\lambda}$ the helicity amplitudes in the LAB frame for $\nu_i(\bar{\nu}_i)e^- \rightarrow \nu_i(\bar{\nu}_i)e^-$, $i = e, \mu, \tau$, being $\lambda$ and $\lambda'$ the initial and final electron helicities respectively (the helicity of the target electron, at rest, is referred to the backward direction).

The conclusions about the dynamical zeros in the helicity amplitudes are the following:

i) $M_{++}^{\bar{\nu}_e}$ shows dynamical zeros given in the energy range $0 \leq E_\nu \leq m/(4 \sin^2 \theta_W)$. The upper value corresponds to the phase space point $\cos \theta = 1$ (where $\theta$ is the angle in the scattering plane of the final electron with respect to the incoming neutrino direction). At this end point the other three helicity amplitudes have kinematical zeros. This is the reason why this dynamical zero shows up in the unpolarized cross section in the backward configuration.

ii) $M_{+\nu_\mu}^{\bar{\nu}_e,\bar{\nu}_\tau}$ show dynamical zeros in the whole range of energies $0 \leq E_\nu < \infty$. In this case the helicity amplitudes never vanish simultaneously. Then, the dynamical zeros will only show up in polarized cross sections.

iii) There are no more dynamical zeros for any helicity amplitude in the physical region.

These results are summarized in Figure 3, where the dynamical zeros are plotted in the plane $(E_\nu, \cos \theta)$, together with the kinematical zeros.

4 The dynamical zero and a new kind of neutrino oscillation experiment.

It seems difficult to design a $\bar{\nu}_e e^-$ experiment where electron polarizations are involved. So we shall concentrate in the dynamical zero for the unpolarized $\bar{\nu}_e - e^-$ elastic cross section. The fact that the weak backward cross section for $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$ vanishes at leading order for $E_\nu = m/(4 \sin^2 \theta_W)$ clearly points out that this
kinematical configuration must be a good place to study new physics. Let us stress that backward neutrinos mean forward electrons, with maximum recoil energy; the electron recoil energy corresponding to the dynamical zero \( T \simeq 2m/3 \) is in fact within the range of the proposed detectors to measure recoil electrons. The neutrino energy for which we find the dynamical zero is on the peak of any typical antineutrino reactor spectra [3, 7], being precisely \( \bar{\nu}_e \) the flavour which is produced copiously in nuclear reactors. We have checked that although the dynamical zero appears for a given \( E_\nu \), the convolution of the cross section with the antineutrino spectrum still keeps the effect for the planned detectors that select neutrino energies by measuring \( T \) and the recoil angle of the electron. The kinematical region where the dynamical zero lies is thus in principle reachable by experiment.

As a first illustration of the interest of this dynamical zero we shall concentrate in the possibility of searching for a neutrino magnetic moment. In Figure 4 we denote by \( (d\sigma_W/dT)_{\text{back}} \) the standard contribution in the r.h.s. of eq. (1) and by \( (d\sigma_M/dT)_{\text{back}} \) the magnetic moment contribution, both for \( T = T_{\text{max}} \). The solid line represents the boundary where \( (d\sigma_W/dT)_{\text{back}} = (d\sigma_M/dT)_{\text{back}} \) for \( \bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^- \). The regions below the other lines are those for which \( (d\sigma_M/dT)_{\text{back}} > (d\sigma_W/dT)_{\text{back}} \) for the rest of neutrino species. It is quite apparent from this figure that electron antineutrinos with energies around 0.5 MeV give the possibility of studying low values for neutrino magnetic moment. With other kind of neutrinos this is only possible by going to much lower values of neutrino energy.

The fact that the weak cross section for \( \bar{\nu}_e \) behaves in such a peculiar way in contrast to the other neutrino species suggests a second phenomenological implication: measuring neutrino oscillations [8].

Let us consider the measurement of the elastic cross section of electron antineutrinos, coming from a nuclear reactor, using a detector at some distance \( x \) from the source. We know that, due to the dynamical zero, it is not possible to find forward electrons with \( T \simeq 2m/3 \) due to the \( \bar{\nu}_e e^- \) interaction. If one of these events is found one would conclude that the electron antineutrino has oscillated in the way to the detector to another type of neutrino ( \( \bar{\nu}_\mu \) or \( \bar{\nu}_\tau \) ).

Then, suppose we have a source of electron-antineutrinos \( \bar{\nu}_e(0) \) ( a nuclear reactor for example ) and we measure the differential cross section for the process \( \bar{\nu}_e(x)e^- \rightarrow \bar{\nu}_e(x)e^- \) at a distance \( x \) from the source. If vacuum oscillations take place we will
have
\[
\frac{d\sigma^\nu(E_\nu, T)}{dT} = P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}(x)\frac{d\sigma^\nu_e(E_\nu, T)}{dT} + \sum_{i=\mu, \tau} P_{\bar{\nu}_e \rightarrow \bar{\nu}_i}(x)\frac{d\sigma^\nu_i(E_\nu, T)}{dT} \tag{17}
\]
where \( P_{\bar{\nu}_e \rightarrow \bar{\nu}_i}(x) \) is the probability of getting a \( \bar{\nu}_i \) at a distance \( x \) from the source.

Making use of the conservation of probability (we will not consider oscillation to sterile neutrinos) and the identity \( d\sigma^\nu /dT = d\sigma^\nu_e /dT \) Eq. (17) can be written as

\[
\frac{d\sigma^\nu(E_\nu, T)}{dT} = \frac{d\sigma^\nu_e(E_\nu, T)}{dT} + \left( \frac{d\sigma^\nu_\mu(E_\nu, T)}{dT} - \frac{d\sigma^\nu_e(E_\nu, T)}{dT} \right) \sum_{i=\mu, \tau} P_{\bar{\nu}_e \rightarrow \bar{\nu}_i}(x) \tag{18}
\]

In the particular case of considering only two flavour oscillation we have:

\[
\sum_{i=\mu, \tau} P_{\bar{\nu}_e \rightarrow \bar{\nu}_i}(x) \rightarrow P_{\bar{\nu}_e \rightarrow \bar{\nu}_\mu}(x) = \sin^2 2\phi \sin^2 \left( \frac{\Delta m^2 x}{4E_\nu} \right) \tag{19}
\]

where \( \phi \) is the vacuum mixing angle and \( \Delta m^2 \) is the difference of the square of masses of the mass eigenstates \( \nu_1 \) and \( \nu_2 \).

From Eq. (18) it is quite evident that by measuring \( d\sigma^\nu /dT \) at the kinematical configuration where \( d\sigma^\nu_e /dT \) vanishes, the signal will be proportional to the oscillation probability times the \( \bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^- \) cross section thus simulating an "appearance" experiment.

There are some features of this appearance-like experiment which distinguish it from the usual appearance experiments. First, by measuring events on the dynamical zero we are sensitive to oscillations \( \bar{\nu}_e \rightarrow \bar{\nu}_x \), where \( \bar{\nu}_x \) is any non-sterile neutrino. The detection is not via purely charged current processes; on the contrary any signal of oscillation would be detected via neutral currents \( (\bar{\nu}_\mu(\bar{\nu}_\tau)e^- \rightarrow \bar{\nu}_\mu(\bar{\nu}_\tau)e^-) \). Hence there is no energy threshold; this experiment would use neutrinos with energies around 0.5 MeV, thus being in principle more sensitive to low \( \Delta m^2 \) values than in the standard appearance experiments.

In a reactor the antineutrino spectrum is continuous so in Figure (5) we have plotted (dashed lines) the curves for constant neutrino energies in the plane \((T, \theta)\). The solid lines represent the curves for constant ratio \( \frac{d\sigma^\nu_\mu}{dT} / \frac{d\sigma^\nu_e}{dT} \); of course they have an absolute maximum in the dynamical zero. From this figure it is quite evident that far from the dynamical zero there still remain important effects associated to its presence.
As an illustration of the exclusion plots that one could obtain from the observable (18) we have integrated it over a typical reactor spectrum in the kinematical region where \( \frac{d\sigma_{\bar{\nu}e}}{dT}/d\sigma_{\bar{\nu}e} \geq 5 \) and imposed that the ratio \( \int \frac{d\sigma_{\bar{\nu}}}{dT}/\int d\sigma_{\bar{\nu}e} \) is less than 1.5. With the detector placed at 20 meters, the would-be exclusion plot we get is represented in Figure (6). Inside the excluded region we have inserted the by now allowed region of oscillations coming from atmospheric neutrino experiments [9]. Taking into account the original MUNU proposal [2], the numbers we have considered correspond roughly to detect a few (\( \sim 10 \)) events per year if no oscillations are present placing an upper bound (with oscillations) around 15 events.

We have drawn Figure (6) supposing a complete knowledge of the neutrino spectrum. From Figure 5 it is evident that by measuring the cross section at different kinematical points with the same neutrino energies we can avoid uncertainties from the neutrino flux. Note that for points with different \((\theta, T)\) but corresponding to the same energy \(E_\nu\) the dependence of \(\frac{d\sigma_{\bar{\nu}}}{dT}(\theta, T)\) on \(\Delta m^2\) and \(\phi\) is different, so that performing ratios the dependence on the flux can be cancelled out without cancelling the effect. If this ratio is performed by integrating over a reasonable kinematical region we have checked that errors coming from the flux uncertainty can be reduced to a few percent. This result makes this proposal very appealing.

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Figure Captions.

- **Fig. 1)** Regions in the plane $(\theta, E_\gamma)$ where the $\bar{\nu}_e$ radiative cross sections, when integrated from $\theta$ to $\theta_{max}$ ($Q^2 = 0$), satisfy that the ratio $d\sigma_M/d\sigma_W$ is 5, 4, 3 or 2 times larger than the elastic ratio at the same $T$-value ($T = 0.2$MeV). The solid line represents the $Q^2 = 0$ curve. In this figure $E_\nu = 1$MeV.

- **Fig. 2)** Ratio of the inclusive cross sections $d^2\sigma_M/d^2\sigma_W$. The physical region is bounded by $0 \leq x \leq 1$ and $0 \leq \nu \leq 2E_\nu^2/(2E_\nu + mx)$; the flat region on the right is unphysical.

- **Fig. 3)** Kinematical and dynamical zeros for the helicity amplitudes in the plane $(E_\nu, \cos\theta)$. The kinematical ones correspond to the line $\cos\theta = 1$. The curves have been done for $\sin^2\theta_w = 0.233$.

- **Fig. 4)** Regions of dominance of weak or magnetic backward differential cross sections in the plane $(\mu_\nu, E_\nu)$ for $\bar{\nu}_e$; there are three different zones divided by the solid line. For the rest of (anti-)neutrinos there are only two regions, being the magnetic backward cross section dominant below the corresponding line ( long-dashed for $\nu_e$, dashed-dotted for $\bar{\nu}_\mu$ and short-dashed for $\nu_\mu$ ) and the opposite above the line.

- **Fig. 5)** Curves for constant values of $\log\left(\frac{d\sigma_{\mu_\nu}}{d\sigma_{\mu_e}}/\frac{d\sigma_{\bar{\nu}_e}}{d\sigma_{\nu_e}}\right)$ (solid lines) and for constant $E_\nu$ values in MeV (dashed lines) in the plane $(T, \theta)$.

- **Fig. 6)** Exclusion plot obtained by imposing that the ratio $\int \frac{d\sigma_{\nu_e}}{d\ell} / \int \frac{d\sigma_{\bar{\nu}_e}}{d\ell}$ (ratio oscillation/non-oscillation) is less than 1.5, integrating the cross sections over a typical reactor spectrum in the kinematical region where $\frac{d\sigma_{\bar{\nu}_\mu}}{d\ell}/\frac{d\sigma_{\bar{\nu}_e}}{d\ell} \geq 5$ and considering the detector is 20 meters away from the reactor. The shaded zone corresponds to the allowed region for atmospheric $\nu_e \leftrightarrow \nu_\mu$ oscillations.
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