Coherence-Powered Charge and Discharge of a Quantum Battery

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A. Key question in quantum thermodynamics is how coherence impacts energy exchanges between quantum systems. Here we study theoretically and experimentally the role of coherence in work transfer processes of charging and discharging a quantum battery. During charging, work is transferred from a qubit into a reservoir of modes of the electromagnetic field, acting as a quantum battery. We show that the amount of work is proportional to the initial quantum coherence of the qubit, and is always lower than the energy change of the qubit, and is reduced by temperature. Using homodyne-type measurements, we discharge the quantum battery into a coherent field, acting as a work receiver. The amount of work transferred to the receiver is controlled by the relative classical optical phase between the two fields, the quantum purity of the charged battery field, and long-term fluctuations in the qubit environment. Complete discharge is reached in the low-energy limit where the battery field mostly overlaps with a coherent field.

Introduction Quantum thermodynamics aims to extend the concepts and laws of thermodynamics into the quantum realm [1]. Since its early days, it focused on the work value of quantum coherence and its potential to enhance the performance of heat engines [2–7]. In the context of quantum batteries, quantum coherence is predicted to impact the charging speed, work extraction [8], and the charging power [9–12]. Such coherence-enhanced thermodynamic properties have been experimentally investigated in the context of heat engines and refrigerators [13–20], Maxwell’s Demon experiments [15, 21, 22], among others.

These questions have recently acquired a renewed interest in the fully quantum regime, where energetic flows are studied between two coupled quantum systems, A and B, that are otherwise isolated [23–27]. This captures, for example, the charging of a quantum battery by a work provider, and reciprocally, its discharging into a work receiver [28–30]. In these situations, the energy change of each system verifies $-\Delta E_A = \Delta E_B$. The work $W_{A,B}$ exerted by A on B can be unambiguously defined as the fraction of energy transfer stemming from an effective unitary interaction between the two systems, while quantum correlations between the two systems limit the amount of work exchanged. The work transfer is upper bounded by the energy change of each system $W_{A,B} \leq |\Delta E_A|$, allowing to define the efficiency $\eta_{A,B}$ of the process as the ratio $\eta_{A,B} = W_{A,B}/|\Delta E_A|$. A unity efficiency, $\eta_{A,B} = 1$, indicates an optimal process where one of the systems behaves as a perfect work source.

In this new paradigm, a qubit coupled to a reservoir of electromagnetic modes, or a “one-dimensional atom” [31, 32] is a convenient and versatile platform to explore the impact of coherence on energetic transfers [33]. In charge and discharge scenarios the qubit and the field can both play the roles of the work provider, the work receiver, or the quantum battery [27, 28]. When light and matter are resonantly coupled through a Jaynes-Cummings Hamiltonian, it has been shown that the work exchanged between the qubit and the field is equal to the energy change in the coherent component of the field, making work an observable [21, 27, 28]. When a coherent and intense field provides energy to the qubit, both systems remain non-entangled as they exchange photons. This defines the classical limit in which the two systems solely exchange work. Since the qubit purity is unaltered, this is the proper regime to implement single qubit gates [27, 34, 35]. Conversely, when the initially inverted qubit acts as the work provider on the initially empty field, the qubit releases its energy via spontaneous emission. It was recently predicted that the work is then proportional to the initial quantum coherence of the qubit, and is always lower than the energy transfer, since the qubit and the field get entangled during the spontaneous emission process [36]. Such a prediction makes a direct, and thus far unobserved, connection between quantum coherence, entanglement and energetic properties [28].

Here we explore theoretically and experimentally the role of quantum coherence in the work transfers between a work provider, a quantum battery and a work receiver along two temporally separated steps, i.e. the charge and discharge of the battery. Our protocol is sketched in Fig. 1(a). In the charging step, the work provider is a qubit coupled to a reservoir of empty electromag-
ngetic modes, that plays the role of an initially uncharged quantum battery. Experimentally, the qubit is a semiconductor quantum dot inserted in an optical microcavity - a system that years of technological developments have brought close to the ideal- “one-dimensional atom” at low temperature [37, 38], but can naturally allow exploring the effect of decoherence increasing the temperature. At low temperatures, where decoherence is negligible, we theoretically show that the efficiency of the work transfer from the qubit to the battery corresponds to the visibility of fringes recorded in a self-homodyne measurement of the quantum battery field. Experimentally, we observe an efficiency and a work transfer close to the ideal value. Conversely, we observe reduced energetic performances when the qubit temperature is increased and decoherence perturbs the spontaneous emission mechanism.

In the second step, we discharge the quantum battery into a coherent field. We extend our framework to describe the work transfer between two light fields coupled by a balanced beam splitter. We show that such discharge can be implemented through a homodyne-type measurement where both fields interfere. We experimentally explore this discharge process and observe that the work transfer is limited by the overall quantum coherence of the quantum battery, but also by long-term fluctuations of the qubit environment. Complete discharge is reached in the low energy limit where the battery field mostly overlaps with a coherent field.

Quantum Battery Charging We first consider the ideal situation where the spontaneous emission process is not subject to decoherence [28]. At the beginning of the process, the qubit is resonantly excited by a classical drive so that at time \( t = 0 \) it is brought to the pure quantum superposition state \( |\Psi_q(0)\rangle = \cos(\theta/2) |g\rangle + \sin(\theta/2)|e\rangle \) where \( |g\rangle \) and \( |e\rangle \) are the ground and excited state of the qubit separated by an energy \( \hbar \omega_0 \), and \( \theta, \phi \in [0, \pi] \) are the pulse area and the classical phase of the driving laser pulse, respectively. The total initial energy brought to the qubit by the classical drive is \( \mathcal{E}_q(0) = \hbar \omega_0 \sin^2(\theta/2) \). After some time, this energy is entirely transferred to the quantum battery field via the spontaneous emission process, such that the battery final energy satisfies \( \mathcal{E}_b = \Delta \mathcal{E}_b = -\Delta \mathcal{E}_q = \mathcal{E}_q(0) \). The emitted field state, i.e. the charged battery field, is pure and reads \( |\Psi_b\rangle = \cos(\theta/2) |0\rangle + \sin(\theta/2)e^{i\phi}|1\rangle \), where \( |0\rangle \) and \( |1\rangle \) are the photon number states in the optical mode defined by the spontaneous emission process. The work provided by the qubit to the battery corresponds to the coherent part of the emitted field energy [21, 27, 28]. It is given by \( W_{q,b} = \hbar \omega_0 s^2 \), where \( s \) is the qubit initial coherence \( s = \cos(\theta/2)\sin(\theta/2) \). Thus the amount of extracted work is maximal if the qubit is initially prepared in an equally weighted superposition of ground and excited states, i.e. \( \theta = \pi/2 \). The charging efficiency is given by:

\[
\eta = \frac{\Delta \mathcal{E}_b}{\Delta \mathcal{E}_q} = \frac{\sin(\theta/2)}{\sin(\theta/2)} = 1
\]

To perform a homodyne measurement of the battery field, we need to perform a self-homodyne measurement on the battery field. The battery field is split into two paths at a beam splitter (BS2) and one arm is delayed such that two subsequently emitted photonic fields interfere at a 50:50 beam splitter (BS1). Configuration (2) corresponds to discharging processes where we perform a homodyne interference between the charged quantum battery and a coherent field acting as a work receiver. The coherent work receiver is derived from the same drive laser, and is reflected off a mirror in the same cryostat as the quantum battery field. The charged quantum battery and work receiver field interfere at a BS1, and the output intensities are recorded using two superconducting nanowire single-photon detectors (SNSPDs), D1 and D2.

FIG. 1. Principle of charging and discharging a quantum battery. (a) Schematic of the experimental protocol. A classical drive generates coherence in a two-level system (qubit). In turn, the qubit exerts work on an empty quantum battery (vacuum of electromagnetic field) via spontaneous emission, charging the quantum battery (step 1). We discharge the quantum battery through homodyne-type interference with a coherent field (work receiver) at a 50:50 beam splitter (step 2). (b) Experimental realization for scheme presented in (a). The classical drive is a 7 ps laser pulse and resonantly excites the two-level system of the quantum dot (QD). After spontaneous decay, we separate the emitted photonic field (charged quantum battery) from the classical drive via cross-polarization at a PBS. Two configurations allow us to perform charging and discharging measurements. Configuration (1) with an additional flip mirror (FM) allows us to perform a self-homodyne measurement on the battery field. The battery field is split into two paths at a beam splitter (BS2) and one arm is delayed such that two subsequently emitted photonic fields interfere at a 50:50 beam splitter (BS1). Configuration (2) corresponds to discharging measurements where we perform a homodyne interference between the charged quantum battery and a coherent field acting as a work receiver. The coherent work receiver is derived from the same drive laser, and is reflected off a mirror in the same cryostat as the quantum battery field. The charged quantum battery and work receiver field interfere at a BS1, and the output intensities are recorded using two superconducting nanowire single-photon detectors (SNSPDs), D1 and D2.
\[ \eta_{q,b} = \mathcal{W}_{q,b}/E_b = \cos^2(\theta/2) \]

In a real system, the qubit is subject to various sources of decoherence such as charge noise [39], and phonon coupling [40] that alters both the excitation and the spontaneous emission processes and induces an overall loss of purity of the emitted photons. The energetic of work transfer during a spontaneous emission process subject to a phonon bath is beyond currently accessible theoretical frameworks. We adopt a simple approach here where we attribute all the decoherence to the excitation process so that the provided work still corresponds to the coherent component energy of the emitted field. The imperfect field state is now described by the density matrix \( \hat{\rho}_b = \cos^2(\theta/2)\hat{\rho}_0 + \sin^2(\theta/2)\hat{\rho}_1 + \cos(\theta/2)\sin(\theta/2)(\hat{\rho}_{01} + \hat{\rho}_{10}) \) (see Suppl.). \( \hat{\rho}_0 \) (resp. \( \hat{\rho}_1 \)) stands for the vacuum (resp. single photon) quantum state. The parameter \( M_b = \text{Tr}[\hat{\rho}_b^2] \) is the single photon indistinguishability. This well-known quantity models the loss of purity of the single photon component in the temporal domain [41]. The quantum state degradation also induces a reduction of quantum coherence between the vacuum and the single photon state that is captured by \( C = \text{Tr}[\hat{\rho}_{00}\hat{\rho}_{11}] \), and we show that it satisfies \( C \leq \sqrt{M_b} \) (see Suppl.). The work provided by the qubit to the quantum battery now reads:

\[ \mathcal{W}_{q,b} = \hbar \omega_0 C \cos^2 (\theta/2) \sin^2 (\theta/2) \]

We implement the charging step with a neutral InGaAs quantum dot (QD) coupled to a micropillar cavity as our qubit. The experimental set-up is shown in Fig. 1(b). The qubit is placed in a cryostation, and is resonantly driven by a 7 ps pulse from a Ti:Sapphire laser at a 81 MHz repetition rate. The emitted photonic field, or charged quantum battery, is separated from the classically drive—rejection (see Suppl.) and extract \( C(5K) = 0.975 \pm 0.007 \) when correcting for imperfect laser rejection. Combining this visibility measurement with the measurement of the field energy \( E_b \) (see Fig. 2(a)), we can deduce a lower bound to the amount of work transferred into the quantum battery: \( \mathcal{W}_{q,b} \geq E_b \times v \). In Fig. 2(c) we plot the experimental values of \( E_b \times v \) as a function of \( \theta \), as well as the theoretical prediction from Eq. 2. Our observations are remarkably close to the ideal situation where no decoherence affects the initial qubit state: in particular as expected from [28], a maximal work is extracted for \( \theta = \pi/2 \), which corresponds to an efficiency of \( \eta_{q,b} = 1/2 \).

These observations illustrate how the qubit initial energy impacts work transfers. At low \( \theta \), most of the energy transferred from the qubit to the electromagnetic field corresponds to work, which corresponds to a maximal efficiency [28]. Such behavior can be understood considering that, at low excitation regime, the radiated field comes from the qubit dipole: no entanglement takes place between the two systems, and the emitted field is remarkably close to a coherent field. Conversely, light-matter entanglement emerges during the spontaneous emission process when a qubit population is created for increasingly \( \theta \). Quantum correlations then reduce the amount of work eventually transferred to the electromagnetic field, reaching the situation where all the energy is transferred incoherently for \( \theta = \pi \) — when the qubit act as a deterministic single-photon source.

We underline that these observations evidencing the link between quantum coherence and work extraction in the quantum regime are obtained on a solid-state qubit. They reflect a high suppression of all decoherence processes at 5K — mostly owing to the acceleration of spontaneous emission provided by the coupling to the cavity. Such observations are consistent with the generation of single photon with near-unity indistinguishability with similar devices [43]. To explore the effect of decoherence, we performed the same measurement for an increased sample temperature of 20 K. Application of a bias on the
diode structure allows us to maintain the qubit-cavity resonance and keep all other parameters of the system mostly unchanged [44]. The coupling of the quantum dot to acoustic phonons affect both the coherent control of the quantum dot with the classical drive [45] as well as the single photon indistinguishability [44] which here is measured to be only $M_s = (58.0 \pm 1.0\%)$ at $\theta = \pi$. Repeating the self-homodyne measurement on the battery field, we still observe single photon interference, evidencing a work transfer between the qubit and the quantum battery. However, the work transfer efficiency is strongly reduced to $C(20K) = 0.594 \pm 0.007$ for $\theta \to 0$. The lower bound to the work transferred is still maximum for $\theta = \pi/2$ but for a maximum value reduced to $v = (15.8 \pm 0.6\%)$. Note that the relation $C \leq \sqrt{M_s}$ is verified in our experimental observations where we independently measured $M_s = (92.6 \pm 0.1\%)$ at 5 K, and $M_s = (58.0 \pm 1.0\%)$ at 20 K. Also, a seemingly slightly higher work transfer than theoretically expected is observed at high $\theta$. This observation is an artefact that arises from a small fraction of the classical drive leaking into the quantum battery field as we increase $\theta$ as shown by an increased second-order intensity correlation of the battery field. This small fraction of laser signal in the measured field leads to classical interference that artificially increases the amount of work transferred (solid lines in Fig. 2(c)).

Quantum Battery Discharging In the second step of the protocol described in Fig. 1(b), we discharge the quantum battery into a propagating coherent field which acts as a work receiver. To do so, we interfere the fields at a 50:50 beam splitter. The quantum battery is described by quantum state $\hat{\rho}_b$, mode $a_b(t)$ and a complex amplitude $\zeta(t) = \text{Tr}[a_b(t)\hat{\rho}_b]$, and the coherence field has complex amplitude $\beta(t)$. We have analyzed the nature of energy exchanges between the two modes coupled by a balanced beam splitter. In this new situation, the battery (b) and work receiver (w) energy changes read $\Delta E_b = -\Delta E_w = Q_{b,w} + W_{b,w}$ (see Suppl.), where we introduce $Q_{b,w} = (\mathcal{E}_w^{(in)} - \mathcal{E}_b^{(in)})/2$ as the energy difference between the work receiver and the battery input fields, and $W_{b,w} = h\omega_0 \cos(\theta/2) \sin(\theta/2) \int dt \text{Re}[\zeta(t)\beta^*(t)]$ the work exchanged between the two fields. Our calculation thus reveals that the work transferred equals the energy of the interference term between the two fields complex amplitudes. A maximal efficiency $\eta_{b,w} = 1$ is reached when $\mathcal{E}_w^{(in)} = \mathcal{E}_b^{(in)}$, i.e. where the energy of the two input fields is balanced and we henceforth choose this condition, corresponding to $\int dt |\beta(t)|^2 = \sin^2(\theta/2)$.

Let us consider the case of a pure battery field, i.e. $C = M_s = 1$. For an initial classical drive $\theta$, maximal work transfer is obtained when the complex amplitudes of the battery field and of the work receiver have the same phase, and when the two fields have the same temporal envelope, i.e. $\sin^2(\theta/2)|\zeta(t)|^2 = |\beta(t)|^2$. This boils down to maximizing the classical coherence between the two fields. The work in this ideal scenario reads $W_{b,w} = \cos(\theta/2) \mathcal{E}_b^{(in)}$, which corresponds to a fringe visibility of $v = W_{b,w}/\mathcal{E}_b^{(in)} = \cos(\theta/2)$.

Note that this situation does not lead to a complete discharge of the battery, that would be characterized by the extinction of one the beam splitter output ports. Indeed, although the two input fields have the same energy, they do not have the same complex amplitude since one is coherent, and the other quantum. The discharge is only complete in the limit where $\theta \to 0$. Remarkably in this case, the state of the battery field is very close to that of a coherent field. It thus perfectly interferes with the
work receiver field, yielding $W_{b,w} \sim C_{b,w}^{\text{in}}$ and $C_{b,w}^{\text{out}} \to 0$. On the other hand, when $\theta = \pi$, the battery field does not exhibit any photon number coherence: its complex amplitude vanishes and no discharge takes place.

These behaviors are altered in the more general case where the battery field is not pure. The work transfer is then lowered by a factor $C_{b,w}$, which captures the reduction of the overall coherence between the two fields (see Suppl.). Similar to the self-homodyne experiment, this parameter is upper bounded by the mean wave packet overlap between the two fields $C_{b,w} \leq \sqrt{M_{b,w}}$ (see Suppl.).

We experimentally study the discharge of battery fields obtained both at 5 K and 20 K with configuration (2) shown in Fig. 1(b). To ensure classical coherence between the fields, the work receiver is a coherent field derived from the same laser that is used to drive the qubit. We temporally shape the work receiver with a Fabry-Pérot etalon (FP) to match the temporal wave packet of the quantum battery field. Fig. 3(a) shows the temporal profiles of the charged quantum battery fields at 5 K (blue) and 20 K (red) and the work receiver field (dashed black) impinging on a 50:50 beam splitter BS1 displayed in Fig. 1(b). We adjust the energy of the work receiver to match that of the battery field for each $\theta$. The two fields interfere constructively or destructively depending on their relative classical phase which we let evolve freely. When the phase is such that we have constructive interference for the targeted output, work is transferred from the quantum battery to the work receiver, thereby discharging the quantum battery.

The visibility of the fringes is plotted as a function of pulse area $\theta$ in Fig. 3(b). As expected, an increasing interference visibility is observed when reducing $\theta$, both at 5 K and 20 K, evidencing an more efficient discharge process of the battery into the work receiver. We compare this behavior to the prediction given by $v = W_{b,w}/C_{b,w}^{\text{in}} = \cos(\theta/2)C_{b,w}$ (see Suppl.). The theoretical fits yield $C_{b,w}(5\text{K}) = (36.3\pm0.4)\%$ and $C_{b,w}(20\text{K}) = (27.2\pm0.4)\%$. As for the battery charging case, a reduced work transfer from the battery to the work receiver is observed at 20 K with respect to 5 K. To understand the origin of this reduction of work transfer we measured the mean wave packet overlap $M_{b,w}$ between the battery field and the coherent field. We extract these quantities from coincidence correlation measurements (see Suppl.), yielding $M_{b,w} = (48.9\pm0.3)\%$ at 5 K and $M_{b,w} = (32.3\pm0.7)\%$ at 20 K showing a decrease at the higher temperature. This is consistent with the observation of a reduced single photon indistinguishability $M_s$ of the battery field at 20 K. However, the corresponding upper bound for the work transfer is not reached experimentally, a discrepancy that may be attributed to blinking of the quantum dot charge state that is not captured by the extracted mean wave packet overlap.

Finally, the total work transfer is directly extracted from the interference visibility at 5 K (blue) and 20 K (red) and shown in Fig. 3(c). Again, these results are well accounted for by our theoretical model $W_{b,w} = h\omega_0 C_{b,w} \cos(\theta/2) \sin^2(\theta/2)$ (solid lines). The dependence with pulse area of the work transferred from the quantum battery to the work receiver $W_{b,w}$ is strikingly different from the dependence for the transfer from qubit to quantum battery $W_{q,b}$ (Fig. 2(c)). Fig. 3(c) evidences a maximum work transfer for $\theta > \pi/2$. This behavior results from a trade-off between the maximization of the battery field amplitude, which is reached at $\theta = \pi/2$, and the normalization condition imposed on the receiver, whose energy continuously increases with $\theta$. 

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**FIG. 3. Quantum battery discharge through interference with classical work receiver.** (a) The temporal profile of the work receiver, a classical coherent field (dashed black), and charged quantum battery at 5 K (blue, upper panel) and 20 K (red, lower panel). (b) Measured visibility $v$ as a function of pulse area $\theta$ when interfering the quantum battery with the work receiver. We fit the experimental data to $v = \cos(\theta/2)C_{b,w}$ as presented in the main text. (c) Measured work discharged from the quantum battery into the work receiver, $W_{b,w}$ as a function of pulse area $\theta$. The lines correspond to the theoretical predictions in the main and Supplementary. The error bars and shaded areas in (b) and (c) all correspond to 2$\sigma$ error on the data and fits.
**Conclusion** The present work provides powerful theoretical and experimental tools to understand the work exchanged between a two-level system and the electromagnetic field during spontaneous emission, as well as the work exchanged during the interaction of two light fields coupled through a simple beam splitter. Both types of interactions constitute key building blocks for a multitude of quantum technologies that are currently developed: atom-based quantum memories, distant connections between quantum processors, linear optical gates for both quantum communication protocols and optical quantum computing. Our study reveals how both classical and quantum coherence impacts energetic transfers and can be accessed by homodyne measurements. The observation of a maximum work transfer efficiency during the charging step at the onset of the qubit population inversion validates the picture that work transfer is reduced by the light-matter entanglement that takes place during spontaneous emission.

Conversely during the discharge, our study reveals that no work is exchanged during the interference between a pure single-photon Fock state and a coherent field - an experiment that is used to implement useful quantum processes such as quantum teleportation [46]. We thus believe that the present work carries the seeds of an energetic investigation of key processes for optical quantum technologies. Finally, we note that the maturity of the quantum-dot cavity system allowed us to study these various phenomena in a variety of situations by controlling the degree of decoherence exerted on the qubit. We believe that this will further stimulate theoretical developments to fully grasp the influence of coupling a qubit to a phonon bath in the energetics of spontaneous emission.

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Supplementary Material

I. Description of the quantum battery field

I.1 General formalism

In this section we provide a description of the battery field state, in realistic conditions where decoherence perturbs the spontaneous emission mechanism. The photonic density operator in the pulse-mode formalism [47] of a single propagating mode $\hat{a}_b(t)$ containing at most one photon reads:

$$\hat{\rho}_b = p_0 \langle 0 | 0 \rangle + p_1 \int dt dt' \xi(t, t') \hat{a}_b^\dagger(t) | 0 \rangle \langle 0 | \hat{a}_b(t') + \sqrt{p_0 p_1} \int dt \zeta(t) \hat{a}_b^\dagger(t) | 0 \rangle \langle 0 | + \text{h.c.} \tag{S.1}$$

where $\xi(t, t') = \xi^*(t', t)$ is a Hermitian function describing the temporal shape and coherence of the single photon and $\zeta(t)$ is the complex amplitude describing the time dynamics of the photon-number coherence. The photon-number probabilities $p_0$ and $p_1$ satisfy $p_0 + p_1 = 1$ and so $\text{Tr}[\hat{\rho}_b] = 1$ implies that $\int \xi(t, t) dt = 1$. The total purity of this photonic state is

$$\mathcal{P} = \text{Tr} \left[ \hat{\rho}_b^2 \right] = p_0^2 + p_1^2 M_s + 2 p_0 p_1 C,$$  

where

$$M_s = \text{Tr} \left[ \hat{\rho}_b^2 \right] = \int dt dt' | \xi(t, t') |^2$$  

is the single-photon indistinguishability, or purity in the temporal domain, and

$$C = \text{Tr} [\hat{\rho}_0 \hat{\rho}_1] = \int dt | \zeta(t) |^2$$  

is the number purity of the coherence between the single photon and the vacuum.

I.2. Relationship between the purity in the number basis and in the temporal domain

In Ref. [42], we discussed the self-homodyne measurement of two fields emitted by a quantum dot and introduced a formalism separating the coherence of the field in the photon number basis and in the time/frequency domain (indistinguishability). In this section, we present a more rigorous approach allowing to discuss both coherences.

In general, the mixed photonic state can be described by a classical probability distribution $\hat{\rho} = \sum_k q_k | \psi_k \rangle \langle \psi_k |$ of pure photonic states $| \psi_k \rangle = p_{0,k} | 0 \rangle + p_{1,k} \int dt f_k(t) \hat{a}^\dagger(t) | 0 \rangle$ each occurring with the probability $q_k$. The single photon composing each state is described by a complex amplitude $f_k(t)$ normalized such that $\int dt | f_k(t) |^2 = 1$. In this notation, the total photon number probabilities are given by the weighted summations $p_0 = \sum_k q_k p_{0,k}$ and $p_1 = \sum_k q_k p_{1,k}$. By expanding out $\hat{\rho}$ in terms of $| \psi_k \rangle$, we can also see that the single-photon trace purity $M_s$ and number purity $C$ can be written in terms of $f_k(t)$ as:

$$M_s = \frac{1}{p_1^2} \sum_{k,k'} q_k q_{k'} | \hat{c}_{k,k'} |^2$$

$$C = \frac{1}{p_0 p_1} \sum_{k,k'} q_k q_{k'} \sqrt{p_{0,k} p_{0,k'} p_{1,k} p_{1,k'}} | \hat{c}_{k,k'} |$$

where $\hat{c}_{k,k'} = \int dt f_k(t) f_{k'}^\dagger(t)$ is the complex amplitude overlap. Note that $\hat{c}_{k,k} = \hat{c}_{k,k}^* k$ and so $C$ is real. Using a Cauchy-Schwarz inequality, we can now see that

$$C^2 \leq \frac{1}{p_0 p_1} \left( \sum_{k,k'} q_k q_{k'} p_{0,k} p_{0,k'} \right) \left( \sum_{k,k'} q_k q_{k'} p_{1,k} p_{1,k'} | \hat{c}_{k,k'} |^2 \right) \tag{S.6}$$

$$= \frac{1}{p_1^2} \sum_{k,k'} q_k q_{k'} p_{1,k} p_{1,k'} | \hat{c}_{k,k'} |^2 = M_s.$$
Hence, a vanishing temporal purity will prohibit coherence in the number basis, but number coherence can vanish even if there is high temporal purity. This inequality defines a useful figure of merit $\lambda^2 = C/\sqrt{M_s}$, where $0 \leq \lambda \leq 1$ quantifies the amount of decoherence in the photon-number basis which cannot be attributed to temporal decoherence of the involved single photon. Note that if $M_s = 1$, $\lambda = |\langle 0|\hat{a}_b(0)|1\rangle|/\sqrt{p_0p_1}$, describes the reduction in number coherence of the initial qubit state, otherwise it accounts for both the degradation of the initial state preparation and the excess decoherence occurring during emission that is not directly associated with the reduction in single-photon indistinguishability.

II. Relating energetic and optical quantities

In this section, we theoretically propose a framework to experimentally access work transfer mechanisms. We demonstrate that homodyne measurements, where two optical fields interfere on a balanced beam splitter (see Fig. 1), give direct access to the work exchanged between two fields. For two input fields impinging a balanced beam splitter, the transformation is given by:

$$\begin{pmatrix} \hat{a}_3(t) \\ \hat{a}_4(t) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\phi} \\ -e^{i\phi} & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_1(t) \\ \hat{a}_2(t) \end{pmatrix}. \quad (S.7)$$

The visibility of interference is defined as:

$$v = \frac{\mu_3 - \mu_4}{\mu_3 + \mu_4}, \quad (S.8)$$

where $\mu_j = \int \langle \hat{a}_j^\dagger(t)\hat{a}_j(t) \rangle dt$ is the average photon number detected in mode $j$ at the output of the beam splitter.

In what follows, we successively study the case of a self-homodyne experiment, where the input fields are copies of the battery field of Eq. (S.1), i.e. $a_1(t) = a_{b1}(t)$, and $a_2(t) = a_{b2}(t)$, and the case of a standard homodyne experiment where the input fields are the quantum battery ($b$) and the work receiver ($w$), i.e. $a_1(t) = a_{w}^{(in)}(t)$, $a_2(t) = a_{b}^{(in)}(t)$, $a_3(t) = a_{w}^{(out)}(t)$, $a_4(t) = a_{b}^{(out)}(t)$.

II.1. Measuring the efficiency of the battery charging with self-homodyne interference

In Refs.\cite{27, 28}, it has been demonstrated that when the qubit-field system is isolated, the total work provided by the qubit to the field’s mode (quantum battery) can be extracted from the final field’s state:

$$W_{q,b} = \hbar \omega_0 \int dt |\langle \hat{a}_b(t) \rangle|^2 \quad (S.9)$$

We extended the definition above to the experimental situation where the joint qubit-field state may not be perfectly pure. By evaluating Eq. (S.9) on the general state in Eq. (S.1) with $p_0 = \cos^2 (\theta/2)$, and $p_1 = \sin^2 (\theta/2)$, we find $W_{q,b} = \hbar \omega_0 \cos^2 (\theta/2) \sin^2 (\theta/2) \int dt |\langle \xi(t) \rangle|^2$. Now using Eq. (S.4), we find the expression for the work presented in the main text, $W_{q,b} = \hbar \omega_0 \cos^2 (\theta/2) \sin^2 (\theta/2) C$.

We directly measure $\eta_{q,b} = W_{q,b}/\mathcal{E}_b$ by performing self-homodyne interference. Two copies of the photonic state are incident on a balanced beam splitter and we measure the visibility of the interference at the output of the beam splitter. The two input states are identical except for a relative phase $\phi$ on the number coherence between them which is due to the difference in path length. The numerator of Eq. (S.8) is given by:

$$2\text{Re} \left[ \int dt e^{i\phi} \langle \hat{a}_{b1}^\dagger(t) \hat{a}_{b2}(t) \rangle \right] = 2\text{Re} \left[ \int dt e^{i\phi} \langle \hat{a}_{b1}^\dagger(t) \rangle \langle \hat{a}_{b2}(t) \rangle \right] = 2 \cos(\phi) \int dt |\langle \hat{a}_b(t) \rangle|^2 \quad (S.10)$$

where the first equality comes from the fact that the states of modes $b1$ and $b2$ are uncorrelated, the second from the fact that the states are identical, i.e. $\langle \hat{a}_{b1} \rangle = \langle \hat{a}_{b2} \rangle = \langle \hat{a}_b \rangle$.

Since the two fields carry the same number of photons, the denominator of Eq. (S.8) is simply $2\mu_b = 2\mathcal{E}_b/(\hbar \omega_0)$. Putting all together we get:
\[ v = \cos(\phi) \frac{\int dt |\hat{a}_b(t)|^2}{\mu_b} = \cos(\phi) \frac{W_{a,b}}{\xi_b} \equiv \cos(\phi) \eta_{a,b}, \]  

(S.11)

The visibility of the interference varies with the relative phase between the two inputs, \( \phi \), and reaches maximum constructive (destructive) interference when \( \phi = 0 (\pi) \). Thus, the maximum of the visibility is equal to the charging efficiency \( \eta_{a,b} \).

II.2. Discharging the quantum battery through homodyne interference

We discharge the quantum battery via interference with a coherent field, which acts as a work receiver. The visibility of this classical-homodyne measurement allows us to quantify the work transfer in the discharge process. The quantum battery (b) field enters the balanced beam splitter in channel 2, with \( \langle \hat{a}_b^{(in)}(t) \rangle = \sin(\theta/2) \cos(\theta/2) \zeta(t) \). The work receiver (w) field, \( |\beta\rangle \), enters the beam splitter in channel 1, its intensity matches that of the single-photon field, then we have \( \langle \hat{a}_w^{(in)}(t) \rangle = \beta(t) = \sin(\theta/2) \sqrt{\xi(t,\bar{t})} e^{-i\phi(t)} \), and \( \mu_w^{(in)} = \mu_b^{(in)} = \mu_b = \sin^2(\theta/2) \). Let us notice that, in the case where the single-photon field is pure, i.e. \( C = 1, \xi(t,\bar{t}) = |\zeta(t)|^2 \), and then \( |\beta(t)|^2 = \sin^2(\theta/2)|\zeta(t)|^2 \).

The interference’s visibility reads:

\[ v = \frac{1}{\mu_b} \text{Re} \left[ \int dt \langle \hat{a}_w^{(in)}(t) \rangle \langle \hat{a}_b^{(in)}(t) \rangle^* \right] = \cos(\theta/2) \text{Re} \left[ \int dt \zeta(t) \sqrt{\xi(t,\bar{t})} e^{i\phi(t)} \right] \]  

(S.12)

From the expression above, it is clear that the maximum of the visibility is reached when the two fields have same phases (classical coherence), as stated in the main text. We can define the quantity \( C_{b,w} = \text{Re} \left[ \int dt \zeta(t) \sqrt{\xi(t,\bar{t})} e^{i\phi(t)} \right] \) accounting for both the classical and the quantum coherence of the process.

Now we are ready to demonstrate that we can access the work exchanged during the discharging process by measuring the visibility \( v \) since \( W_{b,w} = v \xi_b \).

We adopt the energetic analysis used in [27] and model the interference among the fields (b and w) as a closed two-systems Hamiltonian interaction [48]. Each field is regarded as an ensemble of discrete temporal modes. To define those modes, we pass to the discrete-time notation, \( t \to t_n = n\Delta t \), and define the creation operators:

\[ \hat{a}_{b(w),n}^\dagger = \sqrt{\frac{\Delta t}{\delta}} \sum_k e^{i\omega_k t_n} \hat{b}_{b(w),k}^\dagger \]  

(S.13)

where \( \delta \) is the density of modes, and \( \hat{b}_{b(w),k}^\dagger \) creates an excitation of frequency \( \omega_k = (2\pi/\delta)k \) on the \( x(y) \) axis.

The discrete temporal modes arrive one after the other at the beam splitter position, where they are coupled by an interaction Hamiltonian reading:

\[ \hat{V}(t_n) = \frac{i\hbar \Omega}{2} \left( \hat{a}_{w,n}^\dagger \hat{a}_{b,n} - \hat{a}_{b,n}^\dagger \hat{a}_{w,n} \right). \]  

(S.14)

So at time \( t = t_n \) the \( n \)-th modes of the two fields interact (collide) at the BS position [31].

We can define for each time \( t_n \) an input and an output operator:

\[ \hat{a}_{w(b),n}^{(in)}(t_n) = \hat{a}_{w(b),n}/\sqrt{\Delta t}; \]
\[ \hat{a}_{w(b),n}^{(out)}(t_n) = \hat{a}_{w(b),n-1}/\sqrt{\Delta t}. \]  

(S.15)

These definitions can be understood as follows: at every time \( t_n \), each field contains \( n \) temporal modes that have already interacted; the output mode is the last of them, while the input is the first of the modes that have not interacted yet. It is clear that the continuous-time limit of \( \hat{a}_{w(b),n}^{(in)}(t_n) \) is \( \hat{a}_{w(b),n}(t) \), and that of \( \hat{a}_{w(b),n}^{(out)}(t_n) \) is \( \hat{a}_{w(b),n}(t) \).

Let us notice that \( [\hat{V}(t_n),\hat{V}(t_m)] = 0 \), therefore the evolution at any time can be factorized as a series of unitary operators \( \hat{U}_n = e^{-\frac{i}{\hbar}\Delta t \hat{V}(t_n)} \). Hence we find:

\[ \langle \hat{a}_w^{(out)}(t_n) \rangle = \langle \psi(t_n)| \hat{a}_{w,n-1} \psi(t_n) \rangle /\sqrt{\Delta t} \]
\[ = \langle \psi(t_{n-1})| \hat{U}_{n-1}^\dagger \hat{a}_{w,n-1} \hat{U}_{n-1} \psi(t_{n-1}) \rangle /\sqrt{\Delta t} \]
\[ = \cos (\Omega \Delta t/2) \langle \hat{a}_w^{(in)}(t_{n-1}) \rangle + \sin (\Omega \Delta t/2) \langle \hat{a}_b^{(in)}(t_{n-1}) \rangle, \]  

(S.16)
and similarly for $y$:

$$\langle \hat{a}_b^{(out)}(t_n) \rangle = \cos (\Omega \Delta t/2) \langle \hat{a}_b^{(in)}(t_{n-1}) \rangle - \sin (\Omega \Delta t/2) \langle \hat{a}_w^{(in)}(t_{n-1}) \rangle.$$  \hspace{1cm} (S.17)

Taking $\Omega \Delta t = \pi/2$ and coming back to continuous-time notation, the expressions above give respectively:

$$\langle \hat{a}_w^{(out)}(t) \rangle = \frac{1}{\sqrt{2}} \langle \hat{a}_w^{(in)}(t) \rangle + \frac{1}{\sqrt{2}} \langle \hat{a}_b^{(in)}(t) \rangle,$$  \hspace{1cm} (S.18)

and

$$\langle \hat{a}_b^{(out)}(t) \rangle = \frac{1}{\sqrt{2}} \langle \hat{a}_b^{(in)}(t) \rangle - \frac{1}{\sqrt{2}} \langle \hat{a}_w^{(in)}(t) \rangle,$$  \hspace{1cm} (S.19)

as expected from Eq.(S.7) when $\phi$ is incorporated in the definition of the input modes.

Using the definition of work of Refs. [23–27], we find that the two systems exchange and an equal and opposite amount of work reading:

$$W_{b,w} = \hbar \omega_0 \text{Re} \left[ \int dt \langle \hat{a}_b^{(in)}(t) \rangle \langle \hat{a}_w^{(in)}(t) \rangle^* \right]$$  \hspace{1cm} (S.20)

The two systems exchange also an equal and opposite amount of energy:

$$\mathcal{E}_w^{(out)} = \frac{1}{2} \left[ \mathcal{E}_w^{(in)} + \mathcal{E}_b^{(in)} \right] + W_{b,w}$$

$$\mathcal{E}_b^{(out)} = \frac{1}{2} \left[ \mathcal{E}_w^{(in)} + \mathcal{E}_b^{(in)} \right] - W_{b,w}$$  \hspace{1cm} (S.21)

Since the fields are matched in intensity we have $\mathcal{E}_w^{(in)} = \mathcal{E}_b^{(in)} = \hbar \omega_0 \sin^2 (\theta/2)$, and $W_{b,w} = \hbar \omega_0 \cos (\theta/2) \text{Re} \left[ \int dt \beta(t)^* \beta(t) \right] = \hbar \omega_0 \cos (\theta/2) \text{Re} \left[ \int dt \beta(t)^* \beta(t) \right] = \hbar \omega_0 \cos (\theta/2) C_{b,w}$, hence the ratio $W_{b,w}/\mathcal{E}_b^{(in)}$ is equal to the visibility of Eq. (S.12).

\section*{II.3 Upper bound to $C_{b,w}$}

As for the self-homodyne interference, we derive a relationship between $C_{b,w}$ and $M_{b,w}$. In terms of the amplitudes $f_k$ composing $\zeta$, we have $C_{b,w} = \sum_k q_k \sqrt{p_{0,k}p_{1,k}c_k}/\sqrt{p_{0,w}}$, where $c_k = \int dt \text{Re} \left[ \beta^*(t)f_k(t) / \sqrt{\mu_w} \right]$ is the real-valued normalized amplitude overlap of the coherent field with the $k$th pure state composing $\hat{\rho}$ and $\beta(t) = \langle \hat{a}_w(t) \rangle$ is the complex amplitude of the coherent state. Similarly, we can write the mean wave packet overlap in terms of $f_k$ as $M_{b,w} = \langle 1/p_{1} \rangle \sum_k q_k p_{1,k} M_k$, where $M_k = \left[ \int dt \beta^*(t)f_k(t) / \sqrt{\mu_w} \right]^2$ is the mean wave packet overlap between the coherent state and the $k$th pure state composing $\hat{\rho}$. Using a Cauchy-Schwarz inequality, we can again see that

$$|C_{b,w}|^2 \leq \frac{1}{p_{0}p_{1}} \left( \sum_k q_k p_{0,k} \right) \left( \sum_k q_k p_{1,k} |c_k|^2 \right)$$

$$= \frac{1}{p_{1}} \sum_k q_k p_{1,k} |c_k|^2 \leq M_{b,w},$$  \hspace{1cm} (S.22)

where the last inequality arises from the fact that $|c_k|^2 = |\text{Re} \left[ \int dt \beta^*(t)f_k(t) / \sqrt{\mu_w} \right]|^2 \leq \left[ \int dt \beta^*(t)f_k(t) \right]^2 / \mu_w = M_k$.

\section*{III. Experimental methods}

\subsection*{III.1. Device and setups}

The qubit is a neutral InGaAs quantum dot (QD) embedded in an electrically contacted micropillar $\lambda$-cavity with 14 (28) GaAs/AlAs Bragg pairs in the top (bottom) mirror, which has previously been used as a bright single-photon...
source [43, 49]. The device is placed in a cryostation (Cryostation s200, Montana Instruments), operating at either 5 K or 20 K, see Fig. 1. The qubit is resonantly excited using a Ti:Sapphire laser which generates 3 ps pulses centred at 925 nm at a 81 MHz repetition rate. The pulse duration is increased to 7 ps with a 4f-shaping line to optimize the mode overlap with the cavity. The excitation laser is aligned along the polarization axis of the cavity using a quarter wave plate (QWP) and half wave plate (HWP). An objective lens (L1) inside the cryostation focuses the excitation laser onto the micropillar cavity. The emitted single-photon field (charged quantum battery) is separated from the wave plate (QWP) and half wave plate (HWP). The setup as shown in Fig. 1, allows us to switch between two measurement configurations. The first configuration (1) allows us to probe the work transfer between qubit and quantum battery $W_{q,b}$. We measure this work transfer by interfering the charged quantum battery with a copy of itself (a subsequently emitted single-photon field) by inserting a second beam splitter (BS2) and flip mirror (FM) in the charged battery path. The two fields interfere at a 50:50 beam splitter (BS1) and two SNSPDs (D1, D2; Single Quantum Eos) at the outputs of BS1 record the photon intensities with 100 ms resolution over 20 min for each prepared qubit state $\theta$.

Finally, by removing the flip mirror (FM), we obtain the second configuration (2) in Fig. 1 and measure the amount of work transfer between the charged battery and a work receiver (the coherent field derived from the classical drive). To minimize the effect of the vibrations in our closed cycle cryostation, the work receiver is sent into the same cryostation of work transfer between the charged battery and a work receiver (the coherent field derived from the classical drive).

III.2. Battery field indistinguishability

To measure $C$ and $M_b$ for the battery field, two sequentially generated copies of the state interfere at a 50:50 beam splitter. We monitor the single-photon intensity at the two outputs of the beam splitter to observe interference fringes, and also simultaneously monitor the two-photon coincidences where we observe bunching due to Hong-Ou-Mandel interference [36, 42, 50]. To obtain the single-photon indistinguishability $M_b$, we first measure the mean wave-packet overlap between two subsequently emitted battery fields, $M$ [50]. The mean wave packet overlap is defined by $M = \left(1/\mu_b^2\right) \int dt \int dt' |G_1(t, t')|^2$, where $\mu_b = \int dt I_b(t) = \sum_n n p_n$ is the average photon number, $I_b(t) = \langle \hat{a}_b(t) \hat{a}_b(t) \rangle$ is the wave packet temporal envelop, and $G_1(t, t') = \langle \hat{a}_b(t) \hat{a}_b(t') \rangle$ is the first-order (amplitude) correlation function. Experimentally, the integrated coincident counts after HOM interference binned with respect to the detection delay $\tau$ produce histograms proportional to $\int dt \int dt' G_2^{(2)}(t, t') = I_b(t)I_b(t') + G_2(t, t') - |G_1(t, t')|^2$ is the phase-averaged intensity correlation after HOM interference and $G_2(t, t') = \langle \hat{a}_b(t) \hat{a}_b(t') \rangle$ is the intensity autocorrelation measured using a Hanbury Brown–Twiss setup.

The required normalization $\mu_b^2$ for $M$ is obtained by comparing the coincident counts for when the inputs are co-polarized to when they are cross-polarized. In this latter case, we have $G_1(t) = 0$. Then, if the fully integrated and normalized intensity correlation $g_2^{(2)} = \left(1/\mu_b^2\right) \int dt \int dt' G_2^{(2)}(t, t')$ is very small, the HOM histograms allow us to quantify $M$ via the HOM visibility defined as:

$$V_{\text{HOM}} = \frac{g_2^{(2)} - g_{\text{HOM},||}}{g_2^{(2)} - g_{\text{HOM},\perp}},$$

(S.23)

where $g_{\text{HOM},||} = A_{\theta_0,\perp}/|\overline{T}_{\theta_0,\perp}|$ and $g_{\text{HOM},\perp} = A_{\theta_0,\perp}/|\overline{T}_{\theta_0,\perp}|$ are the central peak areas ($A_{\theta_0}$) in co- (||) and cross- (\perp) input polarization configurations normalized by the average peak area of the histogram peaks arising from uncorrelated states ($|\overline{T}_{\theta_0,\perp}|$ or $|\overline{T}_{\theta_0,\perp}|$). Note that, due to the interferometer delay needed to interfere subsequently generated states, the first side peak of the || case is partially suppressed due to antibunching and hence is excluded from the normalization. In addition, Eq. (S.23) is an accurate measurement of $M$ only when there is not much first-order coherence in the number basis so that $|\overline{T}_{\theta_0,\perp}|$ is not suppressed by possible interference fringes. Hence, this measurement approach is accurate for pulse areas $\theta \approx \pi$; however, $M$ is expected to take similar values for all $\theta$.

If the mode contains no more than one photon, then $G_1(t, t') = \mu_b^2 \xi(t, t')$ and so $V_{\text{HOM}} = M = M_b$. For small nonzero $g_2^{(2)}$, both $A_{\theta_0,\perp}$ and $A_{\theta_0,\perp}$ are increased equally, which increases the denominator in Eq. (S.23) and causes $V_{\text{HOM}}$ to underestimat $M$. This small underestimate of $M$ can be corrected by taking $M = V_{\text{HOM}}(1 + g_2^{(2)})$. Note
FIG. S.1. Performance of Single-Photon Source  

(a) Auto-correlation measurement of the single-photon wave packets generated by the single-photon source with $\theta = \pi$ at 5 K. The resulting coincidence histogram allows for extraction of the single-photon purity $g^{(2)}$. (b) The coincidence histogram obtained from Hong-Ou-Mandel interference measurements $g^{(2)}_{\text{HOM}}$ for $\theta = \pi$ at 5 K. From Eq. 24 and the single-photon purity extracted from (a) we can deduce the mean wave packet overlap, or single-photon indistinguishability, $M_s$. (c) and (d) the same as (a) and (b) but at 20 K. Note that the small peaks observed in the coincidence histograms at non-integer multiples of the pulse separation time of 12 ns are due to electronic reflections in our measurement setup and can be neglected.

that this measurement approach and subsequent $g^{(2)}$ correction are different than those used in Ref. [50], where the visibility was defined as $V_{\text{HOM}} = 1 - 2g^{(2)}_{\text{HOM},||}$, which implies $M = V_{\text{HOM}} + g^{(2)}$. Both approaches should predict identical values of $M$. Furthermore, if $g^{(2)}$ is small but nonzero and one of the two photons is approximately distinguishable from the emitted single photon state [50], then $M$ is related to $M_s$ via $M \simeq M_s(1 - g^{(2)})$. Hence, in our experiments, $M_s$ is accurately estimated by

$$M_s \simeq V_{\text{HOM}} \frac{1 + g^{(2)}}{1 - g^{(2)}}.$$  \hspace{1cm} (S.24)

Fig. S.1 shows the results of these second-order intensity correlation measurements with Fig. S.1(a) and Fig. S.1(b) (Fig. S.1(c) and Fig. S.1(d)) corresponding to the auto-correlation $g^{(2)}$, and the time integrated HOM measurement $g^{(2)}_{\text{HOM}}$ at 5 K (20 K), all taken at $\theta = \pi$. We extract similar auto-correlation values at 5 K and 20 K: $g^{(2)}$ of (2.84 ± 0.08)% and (2.28 ± 0.08)%, respectively. However, the indistinguishability between two subsequently emitted single-photon wave packets is reduced at the higher temperature. From the indistinguishability measurements we deduce a single-photon indistinguishability of $M_s = (92.6 \pm 0.1)\%$ at 5 K and $M_s = (58.0 \pm 1.0)\%$ at 20 K.

III.3. Battery and work receiver fields overlap

To access the mean wave packet overlap $M_{b,w}$, we now perform a HOM experiment with the quantum battery field and the work receiver field for $\theta = \pi$. Here it is important to account for the non-negligible classical intensity correlation $g^{(2)}_c = 1$ of the coherent state input. The coincidence counts can arise in three ways. (1) A photon from each input do not bunch when leaving the beam splitter, leading to a contribution of $2\mu_b \mu_w(1 - M_{b,w})$. The factor of two in this term arises from the two ways to obtain a coincidence count (both photons reflected or both photons
transmitted). (2) Two photons arrive from the coherent state when the battery field is vacuum (or lost), leading to an additional contribution $\mu_w^2 g^{(2)} = \mu_w^2$ to the coincidence counts. (3) Two photons arrive from the battery field input, leading to a small contribution $\mu_w^2 g^{(2)}$ to the coincidence counts. If we now assume that $\theta$ is near $\pi$-pulse, the average uncorrelated histogram peak area $A_{\tau>0}$ is given by $2\mu_w^2$. Thus, for the co-polarized case we have $g_{\text{HOM,||}}^{(2)} = 1 - M_{b,w} + \bar{g}^{(2)}$, where $\bar{g}^{(2)} = (\mu_w/\mu_b)g_{w}^{(2)}/2 + (\mu_b/\mu_w)g_{b}^{(2)}/2$ is the weighted average input intensity correlation. The intensity correlation after interference of inputs in cross-polarization is then simply obtained by $g_{\text{HOM,\perp}}^{(2)} = 1 + \bar{g}^{(2)}$. Hence, $M_{b,w}$ is measured by

$$M_{b,w} = \frac{g_{\text{HOM,\perp}}^{(2)} - g_{\text{HOM,||}}^{(2)}}{g_{\text{HOM,\perp}}^{(2)}} \left(1 + \bar{g}^{(2)}\right). \tag{S.25}$$

When the inputs are balanced ($\mu_w = \mu_b$), we have $1 + \bar{g}^{(2)} = 1 + (g^{(2)} + 1)/2$. If $g^{(2)} \ll 1$, the correction factor becomes $3/2$.

Fig. S.2 shows the coincidence histograms of HOM measurements with the quantum battery and the work receiver field at 5 K and 20 K. Fig. S.2(a) and Fig. S.2(b), respectively. We perform the measurements with the two fields cross-polarized ($g_{\text{HOM,\perp}}^{(2)}$, grey), and co-polarized ($g_{\text{HOM,||}}^{(2)}$, blue and red). From the coincidence histograms we extract a mean wave packet overlap of $M_{b,w} = (32.60 \pm 0.19)^%$ at 5 K and $M_{b,w} = (21.53 \pm 0.49)^%$ at 20 K.

III.4. Measuring $C$ and $C_{b,w}$

To determine both $C$ and $C_{b,w}$, homodyne measurements are performed where we measure the difference in count rate at each detector as the interferometer phase evolves. We detail here the protocol for the self-homodyne measurement (Step 1). The same protocol applies for the battery discharge with some small adjustments as indicated later on. The maximum visibility of these interference fringes $v$ give the integrated first-order coherence $c^{(1)}$ [36, 42] of the battery field, where $v \equiv c^{(1)} = (1/\mu_b) \int dt |\langle \hat{a}_b(t) \rangle|^2$. If the mode contains no more than one photon, then $\langle \hat{a}_b(t) \rangle = \sqrt{\rho_0 \rho_1} \zeta(t)$ and $\mu_b = \rho_1$. Thus, $C$ can be determined from $c^{(1)} = \rho_1 C$.

Experimentally, we interfere two fields which are matched in intensity, polarization, and time-of-arrival at a 50:50 beam splitter. We monitor the intensity at the two outputs of the beam splitter using two superconducting nanowire single photon detectors (SNSPDs, Single Quantum Eos), which register the single-photon counts as a function of time, with 100 ms resolution. When interfering two fields that show photon-number coherence, the count rates at the two outputs of the beam splitter are found to be anti-correlated. If the two fields are in (out of) phase then we see maximum constructive (destructive) interference in output 1, corresponding to a maximum (minimum) count rate in detector 1, respectively. In order to determine the maximum interference visibility we record the intensities measured by the two detectors for approximately 20 minutes whilst we let the phase between the two input fields, $\phi$, freely evolve, to ensure that the full phase space has been explored. An example of the intensities measured by two
the integrated first-order coherence $\theta$ becomes

$$C = \langle a^2(t) \rangle$$

where $\mu = \sqrt{p_1 / p_2}$ and $\eta = 1 - 2p_1(1 - p_2)$ quantifies the first-order coherence between the noise and the quantum state, and $c_n^1 = \langle a_n(t) \rangle / \langle a_n(t) \rangle^2$ quantifies the first-order coherence of the classical noise itself. In our experiments, the noise arises from reflected unfiltered laser from the fast qubit excitation pulse, which is temporally separate from the light emitted into the battery field by the qubit. Hence, the noise is not coherent with itself. In our experiments, the noise arises from reflected unfiltered laser from the fast qubit excitation pulse, which is temporally separate from the light emitted into the battery field by the qubit. Hence, the noise is not coherent with itself. In our experiments, the noise arises from reflected unfiltered laser from the fast qubit excitation pulse, which is temporally separate from the light emitted into the battery field by the qubit. Hence, the noise is not coherent with itself.

$$C = \langle a^2(t) \rangle$$

where $\mu = \mu_s + \mu_n = p_1 + p_n$, $c_{s,n}^1 = \langle a_{s,n}(t) \rangle / \langle a_{s,n}(t) \rangle^2$ quantifies the first-order coherence of the classical noise itself. In our experiments, the noise arises from reflected unfiltered laser from the fast qubit excitation pulse, which is temporally separate from the light emitted into the battery field by the qubit. Hence, the noise is not coherent with the quantum state $c_{s,n}^1 = 0$ but is itself coherent by definition $c_n^1 = 1$. In this notation, $g^{(2)} \ll 1$ can be similarly written as in the supplementary of Ref. [50]

$$\mu_b g^{(2)} = 2(1 + M_{s,n})p_1\mu_s^2 \cos^2(\theta) \sin^2(\theta) + \mu_n^2 \sin^4(\theta)$$

where $M_{s,n} = (1/\mu_s\mu_n) \int dt \int dt' \text{Re} \left[ G_{s}^{(1)}(t, t') G_{n}^{(1)}(t', t) \right] \approx 0$ is the mean wavepacket overlap between the emitted quantum state and the temporally-separate classical noise photons. Note that here we used $g_n^{(2)} = 0$ and $g_n^{(2)} = 1$. Defining $\cos^2(\eta) = p_1 \cos^2(\theta) / \mu_b$ and $\sin^2(\eta) = \mu_n \sin^2(\theta) / \mu_b$, we can re-write our expressions in terms of a single parameter:

$$c^{(1)}(\eta) = p_0 \cos^2(\eta) + \sin^2(\eta)$$

$$g^{(2)}(\eta) = 2 \cos^2(\eta) \sin^2(\eta) + \sin^4(\eta)$$

FIG. S.3. Extracting visibility of interference $\nu$. (a) Intensities measured by detector 1 (red) and 2 (blue) at the output of the interference beam splitter (BS1 in Figure 1b) as a function of time, where each point at $t_i$ is normalized by the sum of the two intensities at $t_i$. The inset shows a zoom of the signal, demonstrating anti-correlated intensities. Over the full measurement time we let the relative phase between the two signals freely evolve. (b) The visibility $V$ as a function of time obtained from Eq. 8 (see Suppl.) corresponding to the data shown in (a). (c) Histogram of visibility time trace presented in (b). Dashed vertical lines indicate the extracted maximum visibility $\nu$.

 detectors as a function of time for $\theta = \pi/2$ is given in Fig. S.3(a) with red (blue) being the intensity measured by detector 1 (2). The inset Fig. S.3(a) shows a zoom of the signal, clearly displaying anti-correlated intensity signals. The measured intensities vary over time due to the freely evolving phase.

Fig. S.3(b) shows the extracted visibility (Eq. 9) corresponding to the raw data shown in Fig. S.3(a). We build a histogram of the measured visibility values over time, as shown in Fig. S.3(c). We extract the maximum visibility of interference, $\nu$, by taking the average of the absolute value of the $N$ highest, and $N$ lowest visibility data points (with $N = 100$). The error is calculated through error propagation and standard counting error. For the example data set shown in Fig. S.3, we have indicated in Fig. S.3(c) the corresponding $\nu$ with two vertical dashed lines and label.

If the wave packet has a small probability of containing more than one photon, then $\nu \equiv c^{(1)}$ can overestimate $C$. To correct for this effect, we use the approach detailed in Ref. [50] and consider that a small amount of noise is added to the ideal state in Eq. (S.1) by a beam splitter interaction. We then decompose the battery field amplitude $\langle \hat{a}_b(t) \rangle$ into a contribution from the desired quantum state (subscript $s$) and the additional noise (subscript $n$) $\langle \hat{a}_b(t) \rangle = \cos(\vartheta) \langle \hat{a}_s(t) \rangle + \sin(\vartheta) \langle \hat{a}_n(t) \rangle$, where $\vartheta$ is a noise parameter governing the amount of added noise. Then, the integrated first-order coherence becomes

$$\mu_b c^{(1)} = p_1 p_0 C \cos^2(\vartheta) + 2 \sqrt{p_1\mu_s} c_{s,n}^1 \cos(\vartheta) \sin(\vartheta) + \mu_n c_{n}^1 \sin^2(\vartheta)$$

(S.26)

where $\mu_b = \mu_s + \mu_n = p_1 + p_n$, $c_{s,n}^1 = (1/\sqrt{\mu_s\mu_n}) \int dt \text{Re} \left[ \langle \hat{a}_s(t) \rangle \langle \hat{a}_n(t) \rangle \right]$ quantifies the first-order coherence between the noise and the quantum state, and $c_n^1 = (1/\mu_n) \int dt \langle \hat{a}_n(t) \rangle^2$ quantifies the first-order coherence of the classical noise itself. In our experiments, the noise arises from reflected unfiltered laser from the fast qubit excitation pulse, which is temporally separate from the light emitted into the battery field by the qubit. Hence, the noise is not coherent with the quantum state $c_{s,n}^1 = 0$ but is itself coherent by definition $c_n^1 = 1$. In this notation, $g^{(2)} \ll 1$ can be similarly written as in the supplementary of Ref. [50]

$$\mu_b g^{(2)} = 2(1 + M_{s,n})p_1\mu_s^2 \cos^2(\vartheta) \sin^2(\vartheta) + \mu_n^2 \sin^4(\vartheta)$$

(S.27)

where $M_{s,n} = (1/\mu_s\mu_n) \int dt \int dt' \text{Re} \left[ G_s^{(1)}(t, t') G_n^{(1)}(t', t) \right] \approx 0$ is the mean wavepacket overlap between the emitted quantum state and the temporally-separate classical noise photons. Note that here we used $g_n^{(2)} = 0$ and $g_n^{(2)} = 1$. Defining $\cos^2(\eta) = p_1 \cos^2(\vartheta) / \mu_b$ and $\sin^2(\eta) = \mu_n \sin^2(\vartheta) / \mu_b$, we can re-write our expressions in terms of a single parameter:

$$c^{(1)}(\eta) = p_0 C \cos^2(\eta) + \sin^2(\eta)$$

$$g^{(2)}(\eta) = 2 \cos^2(\eta) \sin^2(\eta) + \sin^4(\eta)$$

(S.28)
Clearly, we have $c^{(1)} = p_0 \mathcal{C}$ when $\eta \to 0$ as expected for the ideal case. If nonzero, the lowest-order correction is then given by $\lim_{\eta \to 0} \left( d c^{(1)}(\eta)/d g^{(2)}(\eta) \right) = (1 - p_0 \mathcal{C})/2$. Hence we have

$$p_0 \mathcal{C} \simeq \frac{c^{(1)} - g^{(2)}/2}{1 - g^{(2)}/2}.$$  

(S.29)

This formalism allows to account for the residual $g^2$ in the visibility measurement (solid lines in Fig. 2(a) and Fig. 2(b))
IV. Glossary

| Name                                      | Symbol | Definition or constraint |
|-------------------------------------------|--------|--------------------------|
| Qubit initial state                       | $|\Psi_q(0)\rangle$ | $\cos(\theta/2) |g\rangle + \sin(\theta/2)e^{i\phi} |e\rangle$ |
| Qubit initial energy                      | $E_q(0)$ | $\hbar \omega_0 \sin^2(\theta/2)$ |
| Qubit initial coherence                   | $s$ | $\cos(\theta/2) \sin(\theta/2)$ |
| Battery mode                              | $\hat{a}_b$ | - |
| Battery envelope (intensity profile)      | $I_b(t)$ | $\langle \hat{a}_b^\dagger(t)\hat{a}_b(t) \rangle$ |
| Battery initial energy (charged)          | $E_b$ | $\hbar \omega \mu_b = E_q(0)$ |
| Battery ideal state                       | $|\Psi_b\rangle$ | $\cos(\theta/2) |0\rangle + \sin(\theta/2)e^{i\phi} |1\rangle$ |
| Battery ideal charging work               | $W_{q,b}$ | $\hbar \omega s^2$ |
| Qubit-Battery charging efficiency         | $\eta_{q,b}$ | $W_{q,b}/E_b$ |
| Work receiver mode (coherent state)       | $\hat{a}_w$ | - |
| Work receiver amplitude                   | $\beta(t)$ | $\langle \hat{a}_w(t) \rangle$ |
| Work receiver envelope (intensity profile)| $I_w(t)$ | $\langle \hat{a}_w^\dagger(t)\hat{a}_w(t) \rangle = |\beta(t)|^2$ |
| Work receiver initial average photon number| $\mu_w$ | $\int dt I_w(t)$ |
| Work receiver initial energy              | $E_w$ | $\hbar \omega \mu_w$ |
| Work exchange during battery discharge    | $W_{b,w}$ | $\int dt \text{Re} \left[ \langle \hat{a}_b(t) \rangle \langle \hat{a}_w^\dagger(t) \rangle \right]$ |
| Battery discharge output energies         | $E_{\text{out} \pm}$ | $(E_b + E_w)/2 \pm \sqrt{W_{b,w}}$ |
| Name                                                      | Symbol                              | Definition or constraint                                                                 |
|-----------------------------------------------------------|-------------------------------------|----------------------------------------------------------------------------------------|
| Photon creation operator (time basis)                     | \( \hat{a}^\dagger(t) \)           | \([\hat{a}(t), \hat{a}^\dagger(t')] = \delta(t - t')\)                              |
| Photon density operator (number basis)                    | \( \hat{\rho} \)                   | \( p_0\hat{\rho}_0 + p_1\hat{\rho}_1 + \sqrt{p_0p_1}(\hat{\rho}_01 + \hat{\rho}_10) + \cdots \) |
| Vacuum state                                              | \( \hat{\rho}_0 \)                 | \( |0\rangle\langle 0| \)                                                          |
| Single-photon state                                       | \( \hat{\rho}_1 \)                 | \( \int dt\int dt'\xi(t, t')\hat{a}^\dagger(t)|0\rangle\langle 0|\hat{a}(t') \)       |
| Single-photon temporal density function                   | \( \xi(t, t') \)                   | \( \text{Tr}[\hat{a}^\dagger(t')\hat{a}(t)\hat{\rho}_1] \)                          |
| Number coherence (between \( |0\rangle \) and \( |1\rangle \)) | \( \hat{\rho}_{01} \)              | \( \int dt\zeta(t)\hat{a}(t)|0\rangle\langle 0|)                                    |
| Temporal number coherence amplitude                       | \( \zeta(t) \)                     | \( \text{Tr}[\hat{a}(t)\hat{ \rho}_{01}] \)                                         |
| Temporal wavepacket (first-order correlation)*            | \( G^{(1)}(t, t') \)               | \( \langle \hat{a}^\dagger(t')\hat{a}(t) \rangle \)                                |
| Temporal envelope (intensity profile)                     | \( I(t) \)                         | \( \langle \hat{a}^\dagger(t)\hat{a}(t) \rangle \)                                |
| Average photon number                                     | \( \mu \)                          | \( \int dtI(t) = \sum_n np_n \)                                                   |
| Mean wavepacket overlap                                   | \( M_{i,j} \)                       | \( \frac{1}{\mu_i\mu_j} \int dt\int dt' \text{Re}[G_i^{(1)}(t, t')G_j^{(1)}(t', t)] \) |
| Indistinguishability                                       | \( M \)                            | \( \frac{1}{\mu^2} \int dt\int dt' |G^{(1)}(t, t')|^2 \)                      |
| Single-photon indistinguishability (trace purity)         | \( M_s \)                          | \( \int dt\int dt' |\xi(t, t')|^2 = \text{Tr}[\hat{\rho}_1^2] \)                                    |
| Normalized\(^†\) first-order coherence amplitude overlap  | \( c_{i,j}^{(1)} \)                 | \( \frac{2}{\mu_i\mu_j} \int dt\text{Re}[\langle \hat{a}_i(t) \rangle \langle \hat{a}_j^\dagger(t) \rangle] \) |
| Number purity                                             | \( \mathcal{C} \)                   | \( \int dt |\zeta(t)|^2 = \text{Tr}[\hat{\rho}_0\hat{\rho}_{10}], 0 \leq \mathcal{C} \leq \sqrt{M_s} \) |
| Number purity parameter                                   | \( \lambda^2 \)                     | \( \mathcal{C}/\sqrt{M_s}, 0 \leq \lambda \leq 1 \)                                 |
| Second-order (intensity) correlation                      | \( G^{(2)}(t, t') \)               | \( \langle \hat{a}(t)\hat{a}^\dagger(t')\hat{a}(t')\hat{a}(t) \rangle \)           |
| Normalized time-integrated second-order correlation\(^†\)  | \( g^{(2)}(\tau) \)                | \( \frac{1}{\mu^2} \int dt G^{(2)}(t, t + \tau) \)                                |
| Normalized fully integrated second-order correlation\(^†\) | \( g^{(2)} \)                       | \( \frac{1}{\mu^2} \int dt\int dt' G^{(2)}(t, t') = \frac{2}{\mu^2} \sum_n \binom{n}{n} p_n \) |

* Upper-case \( G \) is used for un-normalized correlations, lower-case \( g \) is used for normalized correlations.

\(^†\) An alternative convenient normalization is \( 1/\sqrt{\mu_i\mu_j} \), which is equivalent when \( \mu_i = \mu_j \).

\(^\dagger\) Note that \( g^{(2)} \neq g^{(2)}(0) \) in this notation.