SU(3) lattice gauge autocorrelations with anisotropic action

Terrence Draper, Constantine Nenkov, and Mike Peardon

Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506, USA

We report results of autocorrelation measurements in pure SU(3) lattice gauge theory. The computations are performed on the CONVEX spp1200 parallel platform within the CANOPY programming environment. The focus of our analysis is on typical autocorrelation times and optimization of the mixing ratio between overrelaxation and pseudo-heatbath sweeps for generating gauge field configurations. We study second order tadpole-improved approximations of the Wilson action in the gluon sector, which offers the advantage of working on smaller lattices ($8^3 \times 16$ and $6^3 \times 12 – 30$). We also make use of anisotropic lattices, with temporal lattice spacing smaller than the spatial spacing, which prove useful for calculating noisy correlation functions with large spatial lattice discretization (of the order of 0.4 fm).

1. NOTATION

- $s$ – spatial
- $t$ – temporal
- $\tau_{\text{int}}$ – integrated autocorrelation time
- $\xi$ – correlation length
- $a_s$ – spatial lattice spacing
- $a_t$ – temporal lattice spacing
- $\rho = a_t/a_s$ – anisotropy ratio
- $u_s$ – tadpole-improved mean spatial link
- $u_t$ – tadpole-improved mean temporal link
- $S_{nd}$ – lattice action without doublers
- $P_{ss}$ – spatial plaquette
- $P_{st}$ – temporal plaquette
- $R_{ss}$ – spatial $2 \times 1$ rectangle
- $R_{st1}$ – temporal $2 \times 1$ rectangle of type-I
- $R_{st2}$ – temporal $2 \times 1$ rectangle of type-II
- Act – average gluon action
- H-B – heatbath
- Over. – overrelaxation
- MC – Monte Carlo

2. MOTIVATION

Recently it has become apparent that coarse anisotropic lattices with $\rho < 1$ can be very useful in performing accurate Monte Carlo simulations of QCD at low computational cost. This is especially true when modeling heavy quantum states of QCD – like glueballs, for example. Because of the exponential fall off of the signal, small $a_t$ gives better resolution of the correlators at an early time step. On the other hand, $a_s$ should be kept relatively large because of critical slowing down.

Since the anisotropy of the lattice breaks the Euclidean invariance of the continuum theory, it induces temporal $\xi_t^{\text{lat}}$ and spatial $\xi_s^{\text{lat}}$ correlation lengths which scale as

$$\frac{\xi_t^{\text{lat}}}{\xi_s^{\text{lat}}} = \frac{a_s}{a_t} = \rho^{-1}$$

so that $a_t \ll a_s \Rightarrow \xi_t^{\text{lat}} \gg \xi_s^{\text{lat}}$.

On the other hand, the autocorrelations in Monte Carlo updates are proportional to a power of the correlation length

$$\tau_{\text{op}} \propto (\xi_s^{\text{relevant, op.}})^n,$$
where theoretically $n = 2$ for local stochastic updates and $n = 0$ for cluster/overrelaxation updates.

In practice, different lattice operators will have very different autocorrelation times, and we expect operators that couple strongly to $\xi_{t}^{\text{lat}}$ to have larger $\tau_{\text{int}}$. Unfortunately “interesting operators” in lattice QCD, like those for glueballs, live in the spatial domain and scale with $\xi_{t}^{\text{lat}}$ which is large.

This work tries to address the issue of the scaling behavior of different gluon operators on anisotropic lattices, and their relevance to the problem of MC algorithm optimization.

### 3. THE GLUON ACTION

The doubler-free gluon action used in this study is given by

$$S_{nd} = -\beta \sum_{x,s,s'} \left( \frac{a_{t}}{a_{s}} \right) \left( \frac{5}{3} \frac{P_{ss'}}{u_{s}^{4}} \right) + \beta \sum_{x,s,s'} \left( \frac{a_{t}}{a_{s}} \right) \left( \frac{1}{12} \frac{R_{ss'}}{u_{s}^{5}} + \frac{1}{12} \frac{R_{s's}}{u_{s}^{5}} \right) - \beta \sum_{x,s} \left( \frac{a_{t}}{a_{s}} \right) \left( \frac{4}{3} \frac{P_{st}}{u_{s}^{2}u_{t}^{2}} - \frac{1}{12} \frac{R_{st}}{u_{s}^{4}u_{t}^{2}} \right) \tag{3}$$

This is a Symanzik-improved lattice action [1], which is accurate up to errors $O(a_{s}^{4}, a_{t}^{2})$ (classically).

The tadpole-improvement scheme is

$$u_{t} = 1$$
$$u_{s} = (P_{ss'})^{1/4} \tag{4}$$

because of the assumption $a_{t} \ll a_{s}$.

We examine anisotropic lattices with spatial lattice spacings $a_{s}$ from 0.25 to 0.4 fm and anisotropy $\rho$ ranging from 1/2 to 1/5. The mean spatial tadpole-improvement factor $u_{s}$ varies from 0.772 to 0.807. For every lattice considered we generate a corresponding Markov chain of 4000 configurations (i.e. MC sweeps), which are viewed as Monte Carlo time series of appropriate lattice operators.

On these time series we perform autocorrelation measurements on six lattice observables: $P_{ss}$, $P_{st}$, $R_{ss}$, $R_{st1}$, $R_{st2}$ and $A_{ct}$, which are shown in Fig. 1.

### 4. AUTOCORRELATION FUNCTIONS

In Fig. 2 and Fig. 3 we present comparison graphs of the autocorrelation functions for the simple plaquettes and the $2 \times 1$ rectangles. At small anisotropy of the lattice the autocorrelation times of the spatial and temporal gluon operators are small and almost indistinguishable.

With increasing (inverse) anisotropy, $\rho^{-1}$, the degeneracy lifts. The autocorrelation times increase for $P_{ss}$ and $R_{ss}$, and remain small and degenerate for $P_{st}$, $R_{st1}$ and $R_{st2}$ observables. This is to be expected, since $P_{ss}$ and $R_{ss}$ couple strongly to $\xi_{t}^{\text{lat}}$.

As can be observed from the graphs our autocorrelation functions distinguish well between the spatial and temporal modes on the lattice, and can be a good starting point for the optimization procedure.

### 5. AUTOCORRELATION TIMES

The integrated autocorrelation times for an observable $X$ are obtained using the method proposed by Sokal [2,3], namely

$$\tau_{\text{int}} = \frac{1}{2} \sum_{t=-n+m}^{n-m} \frac{R(t)}{R(0)} \tag{5}$$

![Figure 1. Types of planar loops used in this study.](image-url)
Figure 2. Effects of anisotropy on the autocorrelation functions for the plaquettes.

with

$$R(t) = \frac{1}{n - |t|} \sum_{i=1}^{n-|t|} (X_i - \bar{X})(X_{i+|t|} - \bar{X})$$

(6)

and \(m\) chosen so that \(\tau_{\text{int}} \ll m \ll n\). The smallest value of \(m\) for which \(m/\tau_{\text{int}} \geq 4\) has been chosen in a self-consistent manner. From here we derive an estimate for the error of \(\tau_{\text{int}}\) given by the formula

$$\sigma^2_{\tau_{\text{int}}} = \frac{2(2m + 1)}{n} \tau^2_{\text{int}}$$

(7)

We call this criterion – “criterion \(m/4\)”.

We also double check our results against the procedure employed by the QCD-TARO Collabor-}

oration \[3\], wherein the autocorrelation time is defined as:

$$\tau_{\text{int}} = \rho(0) + 2 \sum_{t=1}^{N} \rho(t) \frac{N - t}{N}$$

(8)

where \(N\) is determined so that \(\tau_{\text{int}}\) is maximized, but \(N < 10\%\) of the total sample and \(N < 3\tau_{\text{int}}\). Likewise, we call this criterion – “criterion \(\tau_{\text{max}}\)”.

We use hybrid overrelaxation updating scheme, which is considered to be the state-of-the-art algorithm for bosonic systems when no cluster algorithm is available \[8\]. It simply consists in the mixing of Cabibbo-Marinari pseudo-heatbath and the Brown-Woch microcanonical overrelaxation sweeps with a ratio 1 : \(n_o\) \[8\].
6. ANISOTROPY-DEPENDENCE OF $\tau_{\text{int}}$

In Fig. 4 we show our final results for the dependence of the integrated autocorrelation time on the anisotropy of the lattice for the six lattice observables. We find small variation of the amplitude of the integrated autocorrelation time $\tau_{\text{int}}$ for the temporal loops. The error bars for these observables are small. It is apparent that the modes are strongly suppressed because they effectively propagate in the spatial domain with characteristic correlation length $\xi_{\text{lat}}$, which is small. In contrast, the integrated autocorrelation times for the spatial loops increase as $\rho^{-1}$ increases.

7. CONCLUSIONS

This study is an initial attempt to perform accurate studies of QCD gluon update dynamics on lattices with large spatial lattice spacing and small temporal lattice spacing.

This work is at an early stage of development. We have found clear evidence that the spatial and temporal link variables have different autocorrelation behavior, and that this difference becomes more extreme as the anisotropy is increased. We hope to look for an optimal update scheme for anisotropic simulations. This will be accomplished by exploiting the freedom we have on such lattices to vary pseudo-heatbath and overrelaxation updates on the spatial and temporal links independently. Since the action has explicitly broken Euclidean symmetry, we can expect that an optimal update scheme will involve different update methods for spatial and temporal links. We anticipate the optimal update method will involve overrelaxation of the spatial links (which have been shown here to have the longest autocorrelation times) with heatbath updates of temporal links to ensure the overall update is ergodic.

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