Relaxation versus collision times in the cosmological radiative era

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We consider the Lemaître-Tolman-Bondi metric with an inhomogeneous viscous fluid source satisfying the equation of state of an interactive mixture of radiation and matter. Assuming conditions prior to the decoupling era, we apply Extended Irreversible Thermodynamics (EIT) to this mixture. Using the full transport equation of EIT we show that the relaxation time of shear viscosity can be several orders of magnitude larger than the Thomson collision time between photons and electrons. A comparison with the “truncated” transport equation for these models reveals that the latter cannot describe properly the decoupling of matter and radiation.

I. INTRODUCTION

The relaxation time $\tau$ for the decay of any dissipative flux in a hydrodynamic system is often misunderstood and uncritically assumed to vanish (an unphysical assumption) or be of the order of the relevant collision time $t_{\text{col}}$ of the particles making up the fluid -for a recent review see [1]. The root of this confusion may be traced to the great success of the Fourier-Navier-Stokes theory of dissipative transport [2]. This venerable theory is very useful for engineering applications, but it ignores altogether the relaxation times taken by the system to return to thermodynamic equilibrium state once the gradients of temperature, fluid velocity and so on are suddenly turned off, and therefore its range of validity is limited (at most) to steady-state situations [3]. Therefore, this theory displays a serious drawback: the prediction of instantaneous propagation of the dissipative signals which, on physical grounds, should be limited by the speed of sound. This fact becomes important when dealing with relativistic self–gravitating fluid distributions such as a collapsing star, as well as for the early Universe (see [4], [5] and references therein). Obviously $\tau$ and $t_{\text{col}}$ ought to be related to one another, however there is no generic rule on how to find such a relationship since it depends in each case on the particular system under consideration.

In a previous paper we studied the matter-radiation decoupling for an expanding inhomogeneous universe described by new exact solutions [6] characterized by the Lemaître–Tolman–Bondi (LTB) metric with flat space sections [7,8] and a viscous fluid source. The dissipative quantities involved (the shear stresses) in these models are well behaved and comply with relativistic causality and stability in the sense of Israel and Stewart [9–11]. As stressed in [8], the study of inhomogeneous cosmologies is a well motivated and justified endeavor, especially in view of the results of Mustapha et al. who show that inhomogeneous but spherically symmetric models can be compatible with current CBR observations [12]. This paper aims at showing that the solution found in [6] implies that the relaxation time of the dissipative shear stress of the cosmic matter–radiation mixture prior decoupling is several orders of magnitude larger than the collision time between photons and electrons. In section II we present our model, recall the exact solution derived in [6], and compare the aforesaid relaxation time with the collision time between photons and electrons before the matter-radiation decoupling is achieved. Section III summarizes our conclusions.

II. THE LTB DISSIPATIVE MODEL

Cosmic matter sources in the period from the end of cosmic nucleosynthesis to the decoupling of matter and radiation (roughly the temperature range $10^3K \leq T \leq 10^6K$) can be characterized as a mixture of ultra-relativistic and non-relativistic particles (“radiation” and “matter”), subjected to a tight interaction via various radiative processes (hence the name “radiative” era). The usual approach for studying this era is to consider a FLRW space-time background where matter sources are described by: (a) equilibrium kinetic theory, (b) gauge invariant perturbations, or (c) by hydrodynamical models, which, in general, fail to incorporate a physically plausible description of the matter-radiation interaction. Examination of these matter sources in an inhomogeneous hydrodynamic context requires an imperfect fluid whose state variables are compatible with the equation of state of a mixture of a non-relativistic gas (matter) and a extreme relativistic one (radiation). In [6] we derived a class of exact solutions along these lines, so that: (a) matter-radiation radiative interaction was modeled by a dissipative shear-stress tensor, (b) only the evolution along the radiative era was considered, (c) the background space-time was described by the LTB metric with flat spacelike slices:
\[ ds^2 = -c^2 dt^2 + Y'^2 dr^2 + Y^2 [d\theta^2 + \sin^2(\theta)d\phi^2], \quad Y = Y(t, r). \]  

(1)

For a comoving 4-velocity \( u^a = c\delta_i^a \) in \([\text{[}][\text{]}]\), the expansion scalar and shear tensor are

\[ u^a_a \equiv \Theta = \frac{\dot{Y}'}{Y} + 2\frac{\dot{Y}}{Y}, \quad \sigma^a_b = \text{diag} \left[ 0, -2\sigma, \sigma, \sigma \right], \quad \sigma \equiv \frac{1}{3} \left( \frac{\dot{Y}'}{Y} - \frac{\dot{Y}}{Y^2} \right) \]  

(2)

where a prime denotes derivative with respect to \( r \) and \( \dot{Y} = u^a Y_a = Y, t. \)

The stress-energy tensor of the matter–radiation mixture is

\[ T^{ab} = \rho u^a u^b + \rho \dot{\Pi}^{ab} + \Pi^{ab} \quad (\dot{\Pi}^{ab} = c^{-2} u^a u^b + \eta^{ab}, \quad \Pi^{a}_a = 0), \]  

(3)

where the most general form for the shear-stress for (1) is given by: \( \text{diag} [0, -2P, P, P] \), with \( P = P(t, r) \) to be determined by the field equations. The relation between \( \rho \) and \( p \) follows by assuming that the matter component is “dust” (the low temperature limit of a classical ideal gas, hence its pressure can be neglected) and therefore the hydrostatic pressure is furnished by the radiation only, i.e.

\[ \rho \approx mc^2 n^{(m)} + 3n^{(r)} k_B T, \quad p \approx p^{(r)} = n^{(r)} k_B T \]

where the superscripts \( (m) \) and \( (r) \) denote the particle number densities of “matter” and “radiation”. The shear dissipative pressure tensor is governed by the transport equation of EIT

\[ \tau \dot{\Pi}_{cd} h^c_d h^d_b + \Pi_{ab} \left[ 1 + \frac{1}{2} T \eta \left( \frac{\tau}{Y \eta} u^c \right) \right] + 2\eta \sigma_{ab} = 0, \quad \text{with} \quad \eta = \eta^{(c)} = 4 \frac{5p^{(r)}}{3} \tau, \]  

(4)

where \( \tau \) is the relaxation time for the shear–stress and \( \eta \) is the coefficient of shear viscosity. The subscript \( r_g \) in \( \eta \) indicates that we use the form of this coefficient provided by kinetic theory for the electron-photon interaction (the “radiative gas” model). This transport equation is compatible with relativistic causality and stability, and is supported by kinetic theory, statistical fluctuation theory, and information theory \([\text{[}][\text{]}][\text{[}][\text{]}][\text{[}][\text{]}]\). In its turn the expression for \( \eta \) is well–known in the literature and it can be derived from different standpoints \([\text{[}][\text{]}][\text{[}][\text{]}][\text{[}][\text{]}]\). The entropy per particle takes the form

\[ s = s^{(c)} + \frac{\alpha}{nT} \Pi_{ab} \Pi^{ab}, \quad \Rightarrow \quad (sn^a)_a = \dot{s} n^a \geq 0, \]

where \( \alpha = -\tau/(2\eta^{(c)}) = -5/(8p^{(r)}) \), \( s^{(c)} \) is the equilibrium photon entropy per baryon, and we have used the fact that the number of particles of each fluid entering the mixture is independently conserved: \( (n^{(m)} u^a)_a = (n^{(r)} u^a)_a = 0 \), as well as \( n^{(m)}/n^{(r)} \ll 1 \). The positive–definite nature of \( \dot{s} \) and \( \tau \) above is crucial for the theory to comply with relativistic causality, notwithstanding the unjustified and widespread belief that \( \tau \) vanishes altogether or is at most of the order of the collision time.

A. Exact Solution and Density Contrasts

Integration of Einstein’s field equations (see details in \([\text{[}][\text{]}]\)) leads to the following exact solution

\[ \frac{3}{2} \sqrt{\mu}(t - t_i) = \sqrt{y + \epsilon(y - 2\epsilon)} - \sqrt{1 + \epsilon(1 - 2\epsilon)}, \]  

(5)

where the definitions

\[ \mu \equiv \frac{\kappa M}{Y_i^3}, \quad \epsilon \equiv \frac{W}{M}, \quad y \equiv \frac{Y}{Y_i}, \]

were used along with

\[ M = \int \rho^{(m)}_i Y_i^2 Y_i' dr, \quad \rho^{(m)}_i \equiv mc^2 n^{(m)}_i, \quad W = \int \rho^{(r)}_i Y_i^2 Y_i' dr, \quad \rho^{(r)}_i \equiv 3n^{(r)}_i k_B T_i, \]

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so that \( \rho_i^{(m)}, \rho_i^{(r)} \) define the initial densities of the non-relativistic and relativistic components of the mixture, respectively. The subscript “i” refers to some initial hypersurface to be specified below, and \( \kappa \) denotes Einstein’s gravitational constant.

For the inhomogeneous solutions derived above it is convenient to characterize initial conditions by introducing suitably averaged initial densities: \( \langle \rho_i^{(m)} \rangle, \langle \rho_i^{(r)} \rangle \) and density contrast parameters \( \Delta_i^{(m)}, \Delta_i^{(r)} \) defined by

\[
3 \langle \rho_i^{(m)} \rangle Y_i^3 = \int \rho_i^{(m)} d(Y_i^3), \quad 3 \langle \rho_i^{(r)} \rangle Y_i^3 = \int \rho_i^{(r)} d(Y_i^3),
\]

\[
\rho_i^{(m)} = \langle \rho_i^{(m)} \rangle \left[ 1 + \Delta_i^{(m)} \right], \quad \rho_i^{(r)} = \langle \rho_i^{(r)} \rangle \left[ 1 + \Delta_i^{(r)} \right],
\]

as well as the ancillary functions

\[
\Gamma \equiv \frac{Y'/Y}{Y_i'/Y_i}, \quad \Psi \equiv 1 + \frac{(1 - \Gamma)}{3(1 + \Delta_i^{(r)})}, \quad \Phi \equiv 1 + \frac{(1 - 4\Gamma)}{3(1 + \Delta_i^{(r)})},
\]

that appear in the functional forms of the state variables: \( n_i^{(m)} = n_i^{(m)}/(y^3\Gamma), \quad n_i^{(r)} = n_i^{(r)}/(y^3\Gamma), \quad T = T_i\Psi/y \) and \( P = (1/2)n_i^{(r)}k_B T_i\Phi/(y^4\Gamma) \). These functions are linked to the density contrasts by

\[
\Gamma = 1 + 3A\Delta_i^{(m)} + 3B\Delta_i^{(r)}, \quad (6)
\]

\[
\Psi = 1 - \frac{A\Delta_i^{(m)} + B\Delta_i^{(r)}}{1 + \Delta_i^{(r)}}, \quad (7)
\]

\[
\Phi = \frac{-4A\Delta_i^{(m)} + (1 - 4B)\Delta_i^{(r)}}{1 + \Delta_i^{(r)}}, \quad (8)
\]

where the quantities \( A \) and \( B \) are known functions of \( y \)

\[
A = \frac{1}{3y^2} \left[ \sqrt{y + \epsilon} (1 - 4\epsilon - 8\epsilon^2) - (y^2 - 4\epsilon y - 8\epsilon^2) \right], \quad B = \frac{\epsilon}{y} \left[ \frac{1}{\sqrt{y + \epsilon}} - (y + 2\epsilon) \right].
\]

It should be noted that the solutions must satisfy the restriction \( \Gamma > 0 \) so that the particle number densities do not become negative and that no shell-crossing singularities occur.

At this point it is expedient to point out first that no expression exists that gives \( \tau \) solely in terms of the macroscopic variables \( (\rho, p, T, n, \text{etc.}) \), i.e., there is not such thing as an “equation of state” for \( \tau \). Secondly, using the kinetic theory of gases (in general) the collision time (not \( \tau \)) could in principle be written as a collision integral, but that expression would be mathematically untractable. This is why we resort to (4) to get \( \tau \) in terms of the quantities introduced above

\[
\tau = \frac{-\Psi \Phi}{\sigma} \frac{9}{4} \left( 1 + \Delta_i^{(r)} \right)^2 \frac{3 + 4\Delta_i^{(r)} + \frac{11\Gamma}{32}}{3 + 4\Delta_i^{(r)} + \frac{11\Gamma}{32}} + \frac{17\Gamma}{256} \Gamma^2. \quad (9)
\]

This expression is justified as long as it behaves as a relaxation parameter for the interactive matter-radiation mixture in the theoretical framework of EIT.

Given a set of initial conditions specified by \( \epsilon, \Delta_i^{(m)}, \Delta_i^{(r)}, \) on some initial hypersurface, the forms of the state and geometric variables as functions of \( \eta \) and the chosen initial conditions are fully determined. However, for the solutions to be physically meaningful they must comply with the following set of physical restrictions,

\[
\dot{s} = \frac{15k_B}{16\tau} \frac{\Phi}{\Psi} > 0, \quad (10)
\]

(consistent with the condition that the entropy production be positive),
\[ \Psi > 0, \quad \sigma \Phi < 0, \] (11)
to ensure that \( \tau, \rho \) and \( T \) are all positive. Likewise the concavity and stability of \( s \) demand that
\[ \dot{\tau} > 0, \quad \frac{\dot{s}}{s} = \frac{2 \sigma \Gamma}{3 \Phi \rho_i^{(r)}} \left[ 1 + \frac{\langle \rho_i^{(r)} \rangle}{3 \rho_i^{(r)}} \right] - \frac{\dot{\tau}}{\tau} < 0. \] (12)

The collision time \( t_\gamma \) for the Thomson scattering of photons by electrons (the dominant process of shear transport in the radiative era) is given from the Saha equation by
\[ t_\gamma = \frac{1}{2 c \sigma_\gamma n^{(m)}} \left[ 1 + \left( 1 + \frac{4 h^3 n^{(m)} \exp(B_0/k_B T)}{(2 \pi m_e k_B T)^{3/2}} \right)^{1/2} \right], \] (13)
where \( \sigma_\gamma, B_0, m_e \) and \( h \) are, respectively, the Thomson scattering cross section, the hydrogen atom binding energy, the electron mass and Planck’s constant. For details of the derivation of (13) see [6]. Before matter–radiation decoupling \( t_\gamma < t_H \) (where \( t_H \equiv 3/\Theta \) denotes the expansion time, i.e., the “Hubble time”). For \( t_\gamma > t_H \) matter and radiation no longer interact between each other.

Choosing as initial time \( t = t_i \) roughly corresponding to \( T = 10^6 \) Kelvin, the evolution of the models crucially depends on initial conditions specified by the useful quantity \( \Delta_i^{(s)} = \frac{3}{2} \Delta_i^{(r)} - \Delta_i^{(m)} \) (formally analogous to the definition of adiabatic perturbations). As shown in [3] (see also Figure 3 of this paper), the collision time \( t_\gamma \) is quite insensitive to the values of \( \Delta_i^{(s)} \) given by initial conditions, therefore for some some \( y = y_D \) we find that \( t_\gamma \) overcomes \( t_H \), i.e., there is always a decoupling hypersurface. Taking the standard decoupling temperature \( T_D \approx 4 \times 10^3 \), numerical evaluation of \( T_D = (10^6/y_D) \Psi(y_D, \Delta_i^{(s)}, \Delta_i^{(r)}) \) where \( \Psi \) is given by [3] leads to \( y_D \approx 10^{2.4} \). Also, observational constraints on the decoupling hypersurface \( \delta T_{eq}/T_{eq} \approx 10^{-5} \) further restrict the maximum values of both \( \Delta_i^{(m)} \) and \( \Delta_i^{(r)} \) to about \( 10^{-4} \). However, \( \tau \) is very sensitive to \( \Delta_i^{(s)} \), for example, only if \( \Delta_i^{(s)} = 0 \) (i.e., for adiabatic initial conditions) \( \tau \) has an adequate asymptotic evolution, in the sense that as \( y \to \infty \) (and so, \( t \to \infty \)) it grows monotonically (as \( \tau \to y^{3/2} \) and \( s \to 0 \) in this limit, though \( \tau \) in this case remains smaller than \( t_H \) for all the evolution as \( y \to \infty \), a very different behavior from that of \( t_\gamma \) which overtakes \( t_H \) at the decoupling surface. The relaxation time for adiabatic initial conditions is then unphysical near the decoupling surface. For non-adiabatic initial conditions, \( \Delta_i^{(s)} \neq 0 \), if \( \Delta_i^{(r)} \Delta_i^{(s)} > 0 \) and \( \sigma \Phi < 0 \) hold, we can have \( \sigma \to 0 \) for non-adiabatic \( \Phi \), therefore \( \tau \) diverges and overtake \( t_H \), a behavior that is qualitatively analogous to that of \( t_\gamma \). In order to have \( \tau \) overtake \( t_H \) at \( y_D \approx 10^{2.4} \), we find that \( \Delta_i^{(s)} \) cannot be greater than approximately \( 10^{-8} \) (quasi-adiabatic initial conditions). Therefore, under the assumptions of EIT and using the full transport equation (3) and \( \tau \) given by (3), we find that initial conditions do exist so that \( \tau \) has a physically reasonable form near the decoupling surface. Figures 2, 3 and 5 illustrate this conclusion.

B. Comparing the different times

On the initial hypersurface \( \Gamma_i = \Psi_i = 1, \Phi_i \approx \Delta_i^{(r)} \) and \( \tau_i \) is evaluated to be \( \tau_i \approx 10^9 \) seconds. Likewise \( |t_\gamma| \approx 2227 \) seconds. As a consequence \( |\tau/t_H| \approx 5/16 \) while \( |t_\gamma/t_H| \approx 10^{-6} \). As the decoupling hypersurface is approached (i.e., \( y \to y_D \)) \( \Psi \) and \( \Gamma \) slightly depart from unity as depicted in Figure 1. On that hypersurface one has \( t_\gamma(y_D) \approx \tau(y_D) \approx 10^{13} \) seconds. Figure 2 shows how sensitive is the latter quantity to \( \Delta_i^{(s)} \). We see therefore that contrary to the usual belief, the model examined in [3] shows that \( \tau \) is much larger than \( t_\gamma \) for most of the radiative era and the timescales \( \tau, t_\gamma \) become comparable only as the decoupling hypersurface is approached. This behavior is shown in Figure 3 depicting the ratios \( \tau/t_H \) and \( t_\gamma/t_H \) as well their dependence on \( |\Delta_i^{(s)}| \).

C. The truncated transport equation

In many cases, just for the sake of mathematical simplicity, the transport equation (3) governing the evolution of the dissipative stress tensor has been replaced in by the more manageable Maxwell–Cattaneo–like expression, namely the so called “truncated” transport equation
\[ \tau \Pi_{cd} h^c_\alpha h^d_\beta + \Pi_{ab} + 2 \eta \sigma_{ab} = 0. \] (14)
an equation still complying with relativistic causality and stability in the sense mentioned earlier, and justified when the four–dimensional gradient of the combination \( \tau \sigma \epsilon / (T \eta) \) becomes negligibly small. The models presented here provide a theoretical context in which to test how suitable can this truncated transport equation be for studying the evolution of an interactive matter–radiation mixture. To do this consider the relaxation time that follows from (14)

\[
\tau_{mc} = \frac{P}{P + \frac{2}{5} \rho \sigma} = \frac{4 + 3 \Delta^{(r)}_i - 4 \Gamma}{4 \sigma \frac{[4 + 3 \Delta^{(r)}_i - 4 \Gamma]}{\sigma} - \frac{3}{2} \sigma [31(4 + 3 \Delta^{(r)}_i) - 15 \Gamma]},
\]

where \( \tau_{mc} \) is the relaxation time associated with the Maxwell–Cattaneo equation. Comparing (13) with the relaxation time (9) that emerges from the full theory (i.e. from (9))

\[
\tau = \frac{P}{P + \frac{2}{5} \rho \sigma - \frac{2}{5} \rho p} = \frac{1}{4 \sigma} \frac{(4 + 3 \Delta^{(r)}_i - 4 \Gamma)(4 + 3 \Delta^{(r)}_i - \Gamma)}{\frac{3}{2} \sigma [4 + 3 \Delta^{(r)}_i - 4 \Gamma] + \frac{121}{256} \Gamma^2}.
\]

we can see that \( \tau = \tau_{mc}(1 + \tau_{mc} (\dot{p}/p))^{-1} \), therefore since \( \dot{p}/p < 0 \) (as long as \( \Psi > 0 \), which is always the case) we have \( \tau > \tau_{mc} \). Now two possibilities arise:

1. **Adiabatic condition** \( \Delta^{(s)}_i = 0 \), in this case Figure 4 shows that \( \tau_{mc} < t_H \) for the whole evolution.

2. **Quasi–adiabatic condition** \( \Delta^{(s)}_i \neq 0 \), in this case as seen from Figure 5, \( \tau_{mc} \) does not diverge nor overtakes \( t_H \) at any time in the whole evolution, while \( \tau \) diverges and overtakes \( t_H \) for \( y > y_D \approx 10^{2.4} \).

This together with the fact that close to the decoupling surface \( t_s \) becomes larger than \( \tau_{mc} \) it follows that the Maxwell–Cattaneo equation (14) applied to the LTB models derived here and under the constraints imposed on the initial density contrasts \( (\Delta^{(r)}_i) \approx 10^{-4} \) and \( 10^{-9} < |\Delta^{(s)}_i| < 10^{-7} \) by the observed CMB anisotropies, does not provide a relaxation time that behaves reasonably in conditions near the matter–radiation decoupling.

It is widely assumed nowadays that the bulk of the matter of the Universe (about ninety percent) is not baryonic but belongs to some hitherto unknown form of “dark matter” (see e.g. [14])—though voices of dissent can be heard [15]. We have considered a baryon–photon mixture and so have not taken this into account in our calculations. However, we can roughly verify to what extent our results would change had we incorporated some form of this missing matter.

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It is worthwhile remarking that the introduction of a small cosmological constant or a quintessence fluid would not make sense for the models we are dealing with. Since these forms of “dark energy” contributions (as it is nowadays in vogue) are used to explain the alleged accelerated expansion of the present day Universe, they should be sub–dominants at the epoch contemplated in this paper (see e.g. [16]).

### III. CONCLUDING REMARKS

It is worthwhile mentioning that the main results of this paper: (a) inadequacy of the truncated equation (13) to provide a relaxation time that behaves reasonably near matter–radiation decoupling, and (b) that \( \tau \) is much larger than \( t_s \) for most of the radiative era, strictly apply for the specific model examined in [6], based on the LTB metric. These results also rely on the expression of \( \tau \) obtained in terms of the transport equations (3) and (14). However, we argue that in spite of these limitations these results can complement and enrich the existing discussion in the literature concerning the adequacy of truncated transport equations of dissipative stresses in a cosmological context—see [17] and references therein. Our analysis suggests that, in any case, these equations should be used with due care. We believe it is important to stress that (as far as we are aware) these important features of the equations of irreversible thermodynamics have not been examined previously in the context of inhomogeneous metrics able to accommodate dissipative shear stresses.

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LIST OF CAPTION FOR FIGURES

Figure 1
The plot depicts the functions $\Psi$ and $\Gamma$, given by (6) and (7), in terms of $\log_{10}(y)$ and $\log_{10}(|\Delta^{(s)}_i|)$, from the initial hypersurface $y = 1$ corresponding to $T_i = 10^6$ Kelvin. Notice how these functions are almost unity for all the evolution range.

Figure 2
Logarithmic plot of $\tau$ vs $|\Delta^{(s)}_i|$ for $y = y^{2.5}$ corresponding to $T = T_D$. Note that $\tau$ diverges for $|\Delta^{(s)}_i| \approx 10^{-8}$. Since $t_\gamma$ also diverges as $T \to T_D$, the behavior of $\tau$ for this value of $|\Delta^{(s)}_i|$ is reasonable.

Figure 3
Logarithmic plot depicting the ratios $\tau/t_H$ (shaded surface) and $t_\gamma/t_H$ (non-shaded surface) vs the parameters $\log_{10}(y)$ and $|\Delta^{(s)}_i|$. The former ratio is far larger than the latter for most of the evolution range. For $|\Delta^{(s)}_i| \approx 10^{-8}$ the two ratios become comparable in magnitude near the decoupling surface ($t_\gamma/t_H = 1$).

Figure 4
Logarithmic plots of the ratios $\tau/t_H$ (top surface) and $\tau_{mc}/t_H$ (bottom surface) vs the parameters $y$ and $|\Delta^{(r)}_i| = (4/3)|\Delta^{(m)}_i|$ under adiabatic conditions ($|\Delta^{(s)}_i| = 0$). Neither one of these relaxation times diverges nor overtakes $t_H$, therefore they are not physically realistic near the decoupling surface $t_\gamma/t_H = 1$.

Figure 5
The same plot as figure 4 but under quasi–adiabatic conditions. The ratio $\tau/t_H$ diverges for $\log_{10}y \approx 2.4$ while the the ratio $\tau_{mc}/t_H$ remains finite for all $y$, therefore the full transport equation yields a reasonable relaxation time while the truncated equation does not.

Figure 6
This figure compares $\tau/t_H$ (A), $\tau_{mc}/t_H$ (B) and $t_\gamma/t_H$ (C) vs $\log_{10}(y)$. The thick curves were obtained assuming $T = 10^6$ Kelvin and an initial photon to baryon ratio $n^{(r)}_i/n^{(m)}_i = 10^3$, hence $\epsilon \approx \rho^{(r)}_i/\rho^{(m)}_i = 10^3$. If we assume that CDM does not interact with the photon gas and that it constitutes most of non–relativistic matter, say 20 times the abundance of baryons, then we would need to use $\epsilon = 50$, lower by a factor of 20. The thin curves were obtained using this value of $\epsilon$ with $|\Delta^{(s)}_i| = 10^{-8}$ (upper curves) and $|\Delta^{(s)}_i| = 10^{-9}$ (lower curves). Notice that these two curves are indistinguishable in (C), thus indicating that $t_\gamma$ is rather insensitive to changes in $|\Delta^{(s)}_i|$. The changes in $\epsilon$ due to the inclusion of the rest–mass of CDM are also minimal, shifting the best value of $|\Delta^{(s)}_i|$ to an order of magnitude less.
\[ \log(y) \]