I. INTRODUCTION

There are two contrasting types of motion in classical dynamics. The first type are regular orbits in integrable systems, where $N$ independent conserved quantities exist ($N$ is the degree of freedom), restricting motion to an $N$-dimensional torus $\Sigma$ in phase space [1]. The second type is irregular motion in chaotic systems, where most orbits explore almost all points in a $2N - 1$ dimensional energy surface, in the sense of ergodicity and mixing. According to the well-known Kolmogorov-Arnold-Moser (KAM) theorem [2–4], there is a smooth crossover from an integrable system to an chaotic system. Specifically, Kolmogorov, Arnold, and Moser considered a Hamiltonian of the form $H = H_0 + \epsilon H'$, where $H_0$ is integrable. They found that a subset of the torus solutions under $H_0$ are deformed and survive under a sufficiently small perturbation $\epsilon H'$, while motion near the unstable tori is chaotic and fills regions with dimensionality $2N - 1$. As a result, the phase space is divided into integrable and chaotic regions, with the measure of the latter growing with $\epsilon$.

As classical dynamics is the semi-classical limit ($\hbar \to 0$) of quantum dynamics, one expects similar KAM effects in quantum mechanics. There has been lots of work extending KAM to quantum systems [5–13], especially, KAM in quantum many-body systems has become a recent interest [12, 13]. However, in this paper we focus on cases with a classical limit, where our understanding is more solid. In these systems, previous studies have shown that quantum KAM effects are manifested in eigen-energies and eigenfunctions. For systems of KAM types, both their eigen-energies and eigenfunctions have two parts: regular part and irregular part [14–18]. In particular, to quantitatively understand regular and irregular eigenfunctions, there have been serious efforts to compare quantum eigenfunctions to classical orbits in phase space either using Wigner distribution [16, 17] or Husimi localization.

II. QKR MODEL AND THE WANNIER BASIS APPROACH

A. QKR model

The dimensionless Hamiltonian of the QKR can be written as [24]

$$\hat{H} = \frac{\hat{p}^2}{2} + K \cos \hat{x} \sum_{j=-\infty}^{+\infty} \delta(t - j),$$

(1)

where $\hat{p}$ is the dimensionless angular-momentum operator, $\hat{x}$ is the angular coordinate operator, $t$ is the di-
mensionless time, and $K$ is the kicking strength. In the coordinate representation, $\hat{p} = -i\hbar_e(\partial/\partial x)$, where $\hbar_e$ is the dimensionless effective Planck constant. The dimensionless Schrödinger equation is $\hbar_e(\partial/\partial t)|\Psi\rangle = \hat{H}|\Psi\rangle$.

The evolution operator over one period is

$$\hat{U} = \exp\left(-i\frac{p^2}{2\hbar_e}\right)\exp\left(-i\frac{\hbar_e K \cos \hat{x}}{2}\right).$$  \hfill (2)

For this system, the momentum basis $\langle x|n\rangle = e^{inx}$ ($n$ is an integer) is the most convenient. The matrix elements of $\hat{U}$ are given by

$$U_{n'n} = \langle n'|\hat{U}|n\rangle = (-i)^{n-n'}J_{n-n'}(\frac{K}{\hbar_e})\exp\left(-i\frac{n'^2\hbar_e}{2}\right),$$  \hfill (3)

where $J_{n-n'}(K/\hbar_e)$ is the first kind Bessel function. Since $\hat{H}$ is periodic in time, the eigenstates of the Floquet operator $\hat{U}$ play the role as energy eigenstates in an autonomous system.

If $\hbar_e/(2\pi)$ is rational, i.e., $\hbar_e = 2\pi M/N$, where $M, N$ are coprime positive integers, $U_{n'n}$ can be reduced to a finite matrix by introducing a Bloch wave vector $\theta$ along $t$ [26] (Here $\theta$ is alike the Bloch momentum in crystal). This is called quantum resonance. In this work we consider the case that $N$ is even for simplicity. For even $N$, we find $U_{n'n} = U_{n+n'}$. Therefore $U_{n'n}$ have a translational invariance in $(n+n')/2$. Suppose $|\phi\rangle$ is the eigenstate of one step unitary operator $\hat{U}$, then according to Bloch theorem, we have

$$\phi_{\theta}(s+N\ell) \equiv \langle s+N\ell|\phi\rangle = e^{-i\theta}\phi_{\theta}(s), \quad 0 \leq \theta < 2\pi,$$  \hfill (4)

where $s = 1, \cdots, N$, $\ell = 0, \pm 1, \cdots$. This illustrates that all eigenstates are extended in $p$ space. Moreover, $\phi_{\theta}$ is the eigenstate of a $N \times N$ matrix $V_{\theta}$:

$$\sum_{s'=1}^{N} V_{\theta}(s,s')\phi_{\theta}(s') = e^{-i\omega(\theta)}\phi_{\theta}(s),$$  \hfill (5)

$$V_{\theta}(s,s') = \sum_{\ell=-\infty}^{\infty} U_{s,s'+N\ell}e^{-i\ell\theta}.$$  \hfill (6)

This suggests that the Hilbert space can be reduced naturally to finite-dimensions without truncation, which is one benefit of the resonance condition. Our results have little dependence on the Bloch wave vector $\theta$, so we will always choose $\theta = 0$, and note $V(0)$ simply by $V$. The second benefit at resonance is that the quantum phase space is naturally constructed, while there is some insignificant ambiguity when $\hbar_e$ is generic, which we will see in the next section.

B. Construction of quantum phase space

In order to compare quantum dynamics with its classical counterpart, we need to construct a quantum phase space. This is accomplished by introducing a set of Wannier basis which are localized in both the coordinate space and the momentum space [24]. Suppose $N = N_x \cdot N_p$, where $N_x, N_p$ are integers. We apply Fourier transformation to each $N_x$ adjacent momentum eigenstates and obtain a set of new basis:

$$|\chi, \mathcal{P}\rangle = \frac{1}{\sqrt{N_x}} \sum_{n=1}^{N_x} \exp\left(-i\frac{2\pi \chi n}{N_x}\right)|n + \mathcal{P}N_x\rangle,$$  \hfill (7)

whose norm is plotted in [24].

Thus we obtain a phase space representation $|\psi\rangle = \sum |\chi, \mathcal{P}\rangle|\chi, \mathcal{P}|\psi\rangle$ for any state $|\psi\rangle$, and $P_{\chi,\mathcal{P}} = |\langle \chi, \mathcal{P}|\psi\rangle|^2$ is the probability for $|\psi\rangle$ to be in Planck cell $(\chi, \mathcal{P})$. We emphasize that this basis can be constructed as long as one has the action-angle pairs $(p, x)$, where $x$ has periodic boundary condition. If the natural coordinate of the classical system is not the angle variable, one can also numerically obtain an orthonormal and complete basis of Wannier functions efficiently [23].

If we push the limit $N_x, N_p \rightarrow \infty$ keeping $M$ constant, we get an unlimited resolution in the phase space: $h_e \rightarrow 0$, and it can be proved that the quantum dynamics will be reduced to the standard map for the CKR[24], that is $(\chi, \mathcal{P})\tilde{V}|\chi_0, \mathcal{P}_0\rangle$ will vanish unless

$$\tilde{\mathcal{P}} = \mathcal{P}_0 + \frac{K}{2\pi M} \sin \left(2\pi \frac{\chi_0}{N_x}\right),$$  \hfill (9)

$$\tilde{\chi} = \chi_0 + M\tilde{\mathcal{P}},$$  \hfill (10)

where $\tilde{\chi} = \chi/N_x \in [0, 1], \tilde{\mathcal{P}} = \mathcal{P}/N_p \in [0, 1]$. Taking $\mathcal{P}' = M\mathcal{P}$, one can see that the map for the pair $(\mathcal{P}', \tilde{\chi})$ is exactly the standard map in CKR. Thus the effect of $M$ is to divide the phase space $0 < \tilde{\chi} < 1, 0 < \tilde{\mathcal{P}} < 1$ into $M$ phase spaces of the standard map lined along the $\mathcal{P}$ direction. Each of the $M$ phase spaces will be referred to as a sub phase space in this paper.

C. Area and effective dimension of eigenstates

In the CKR model, the Hamiltonian is nonintegrable as long as $K$ is turned nonzero, but even in the region $1 < K < 5$, there is still a finite portion of quasi-periodic trajectories surviving under the strong kicking strength. Under these $K$, the classical phase space is divided clearly into two kinds of region: some small integrable islands.
and a large chaotic sea [27]. If an initial state lies in the chaotic region, it will explore almost everywhere in the chaotic sea during its long-time dynamics. On the contrary, if an initial state lies in one integrable island, it will remain on one trajectory which forms a 1-dimensional line inside the integrable island. Thus we can tell whether a trajectory is integrable or chaotic by its area in the phase space. In practice, we divide the phase space to \(N_x \times N_y\) cells and define the coarse-grained area of a trajectory by the number of cells passed through by the trajectory. Then the area of a chaotic trajectory will be proportional to \(N_y^2\), while that of an integrable trajectory will be proportional to \(N_x\), which gives a rigorous division at the limit \(N_e \rightarrow \infty\).

As the quantum phase space is naturally “coarse-grained” by Planck cells, we can define the area of an eigenstate, which serves as a criteria to distinguish integrable and chaotic eigenstates. We define the area \(A\) of a given state \(|\psi\rangle\) as

\[
A(|\psi\rangle) = \left(\sum_{X,P} |\langle X,P|\psi\rangle|^4\right)^{-1}.
\]  

It is clear that each Wannier basis has area \(A(|X,P\rangle) = 1\); if \(|\psi\rangle\) is equally distributed in \(N_y\) Planck cells while it has no overlap with other cells, its \(A\) will be equal to \(N_y\). So, this definition can reflect the extent of expansion of the state in the quantum phase space. Note that this quantity is called the inverse participation ratio defined for integrable ones. Since \(\hbar_e \propto 1/N\), we define the effective dimension of each eigenstate \(\phi\):

\[
D_{\text{eff}}(\phi) = -2 \lim_{\hbar_e \rightarrow 0} \frac{\ln A(\phi)}{\ln \hbar_e}.
\]  

which will be close to 1 for integrable eigenstates and 2 for chaotic ones. We note that although \(A\) is dependent on the construction detail of phase space, \(D_{\text{eff}}\) is universal. Instead of looking at the Husimi distribution of each eigenstate to determine which type that state belongs, we can make the discrimination directly from the value of its area or effective dimension by means of the Wannier phase space, which enables us to make the classification of all eigenstates, just as in classical mechanics where a single Poincaré section can depict the behavior of all orbits.

In the definition of \(D_{\text{eff}}\), one needs to relate eigenstates at different \(\hbar_e\). This is not straightforward as the number of all eigenstates varies with \(\hbar_e\). To relate eigenstates, we sort all eigenstates by \(A\), then label each eigenstate by its normalized position \(\phi \in [0, 1]\) in the sequence. When two states at different \(\hbar_e\) have the same \(\phi\), they are regarded as the same eigenstate at different \(\hbar_e\).

### III. CLASSIFICATION OF EIGENSTATES

#### A. Quantum resonance: \(\hbar_e = 2\pi/N_x^2\)

We first consider the simplest case \(\hbar_e = 2\pi/N_x^2\). As expected, there are two typical eigenstates of each kind: one is regular and the other is chaotic. Two examples are shown in Fig. 1.

![Fig. 1: Phase space representation of (a) an integrable eigenstate and (b) a chaotic eigenstate at \(K = 2, N_e = N_p = 128\). The value of each cell is \(|\langle X, P|\phi\rangle|^2\), where \(|\phi\rangle\) is the eigenstate.](image)

#### B. Chlorine resonance: \(\hbar_e = 2\pi/N_y^2\)

We calculate the area \(A\) for each eigenstate. There is a sharp step of \(A\) when the areas \(A\) of all the eigenstates are plotted as a function of the eigenstate index \(\phi\) (see Fig. 2(a)). The step gets sharper when \(N_x\) is increased or equivalently \(\hbar_e\) is decreased. This sharp step defines a critical value \(\phi_d\). One can roughly say that the eigenstates below \(\phi_d\) are integrable and those above \(\phi_d\) are

![Fig. 2: (a) \(A\) of all eigenstates at different \(N_x\). (b) \(A\) at \(\phi = 0.2\) for different \(N_x\). (c) Logarithmic fitting for 5 typical \(\phi\). (d) The effective dimension \(D_{\text{eff}}\) of all eigenstates \(\phi\), where \(D_{\text{eff}}\) is calculated from the slope of the logarithmic fitting. The parameters are \(\hbar_e = 2\pi/N_x^2, K = 2\).](image)
chaotic. Moreover, one expects that the area at \( \phi < \phi_d \) is \( A/N \propto 1/N_x \) (see Fig. 2(b)) while \( A/N \) tends to constant at \( \phi > \phi_d \).

The effective dimension \( D_{\text{eff}} \) is also calculated and is plotted in Fig. 2(d). As expected, \( D_{\text{eff}} = 1 \) for eigenstates below \( \phi_d \) and \( D_{\text{eff}} = 2 \) for eigenstates above \( \phi_d \). However, near \( \phi_d \), \( D_{\text{eff}} \) deviates from both 1 and 2. It may indicate the existence of hierarchical states described in Ref.[33]. These states correspond to classical orbits which are trapped in the vicinity of the hierarchy of integrable islands for a long time, but will finally leak into the chaotic sea. These states will disappear when \( \hbar_e \to 0 \) [33].

We have projected unitarily one set of basis (eigenstates) to another (Wannier basis), which gives information about how many Planck cells each individual eigenstate occupies. We can inverse the unitary transformation, and expand Wannier basis in terms of the eigenstates; the expansion coefficients tell us if the rotor starts at one Planck cell how many other Planck cells it will traverse dynamically in the long time limit. This unitarity of projection further shows the power of our Wannier approach. Similar to \( A \), we can define the length \( L \) of a Planck cell represented by \( |\mathcal{X}, \mathcal{P}\rangle \)

\[
L = \left( \sum_{\phi} |\langle \mathcal{X}, \mathcal{P} | \phi \rangle|^4 \right)^{-1} \quad \text{(13)}
\]

We have computed \( L \) for each Wannier function \( |\mathcal{X}, \mathcal{P}\rangle \), and the results are plotted in Fig. 3, which resembles the classical Poincaré section that is divided into integrable and chaotic regions. To be specific, it is those Wannier basis which are localized in the classical integrable region that have small \( L \), while the others which are localized in the classical chaotic region have large \( L \), and the proportion of Wannier basis which have small \( L \) is exactly the same as the proportion of integrable eigenstates (Fig. 3(b)). These results imply that the set of Wannier basis lie in one classical trajectory, whose amount is of order \( O(N_x^2) \), span an invariant subspace of \( V \) approximately. On the other hand, the Wannier basis that fall in the chaotic region, whose amount is of order \( O(N^2) \), are mixed all together to form an approximate invariant subspace of \( V \).

Since the QKR becomes more chaotic as the kicking strength \( K \) increases, one expects that the critical value \( \phi_d \) decreases and eventually becomes zero with increasing \( K \). This is indeed the case as shown in Fig. 4. We have also compared these results to their classical counterparts in Fig. 4. For the classical results, we divide the phase space into \( N = 100 \times 100 \) cells, choose \( 10^4 \) random initial points and evolve long enough time \( (T = 10^6 \text{ kicks}) \). Then each trajectory contains \( T \) points. For each trajectory, \( A \) is calculated similar to the definition in the quantum case: \( A = \left( \sum_j (n_j/T)^2 \right)^{-1} \), where \( n_j \) is the number of points in the \( j \)-th cell. There is great consistency between the quantum results and the classical results. There are also differences. First of all, the saturation value of the classical \( A \) is much larger and close to the area of chaotic sea in the phase space, which indicates that the chaotic sea is classically ergodic. The saturation value of the quantum \( A \) is smaller; this is due to the fact that the probability distribution of chaotic eigenstates on the phase space has large fluctuations[34]. Second, the classical demarcation point \( \phi_d \) differs from its quantum counterpart, which means there are more integrable eigenstates in QKR than integrable trajectories in CKR, especially when \( K \) is small. This is because there are hierarchical states which are supported by the chaotic region but behave like integrable states, as \( \hbar_e \) is finite. Moreover, in CKR the hierarchical regions of integrable islands are larger with smaller \( K \).

FIG. 3: (a) Length \( L/N \) of each Wannier basis. (b) Sorted area \( A/N \) of each eigenstate and sorted \( L/N \) of each Wannier basis. The parameters are both \( K = 2, N_x = N_p = 128 \).

FIG. 4: (a) Area of each eigenstate (solid line) and coarse-grained area of classical trajectories (dashed line). (b) Demarcation point \( \phi_d \) for classical and quantum cases. \( \phi_d \) is obtained by \( A(\phi_d) = 0.018N, \hbar_e = 2\pi/2^{14} \).

B. Generic \( \hbar_e \) and Anderson localization

In general, \( \hbar_e/(2\pi) \) is irrational and the matrix \( U \) cannot be reduced to a finite one. However, we can build a series of rational numbers \( M_1/N_1, M_2/N_2, \cdots \), which has irrational number \( \hbar_e/(2\pi) \) as its limit. For each \( j \), we have a resonant matrix \( U \) with effective Planck constant
\( h_{e,j} = 2\pi M_j / N_j \), and we can do the previous reduction and construct the Wannier phase space. The properties of the system with the original \( h_e \) are approximated by increasing \( j \).

Without loss of generality, we let \( h_e = 2\pi / (N_x + \Delta)^2 \), where \( N_x \) is an even integer and \( \Delta \in \{-1,1\} \) is irrational. In order to approximate, we construct a series \( \{ M_j / (N_x N_p j) \} \) such that the series \( \{ M_j / N_p j \} \) approaches \( N_x^2 / (N_x + \Delta)^2 \). For this series, the quantum phase space has \( N_j = N_x \cdot N_p j \) Wannier states in total, which is very close to the case \( h_e = 2\pi / N_x^2 \).

In Fig. 5 we plot the rational approximation of \( h_e \) and area of eigenstates for each \( j \). The area of integrable eigenstates remains a small constant when \( N \) increases, because each eigenstate is confined in one integrable island of one sub phase space, which contains a constant amount of Planck cells. On the other hand, the area of chaotic eigenstates increases with \( N \) when \( N \) is small, and saturates when \( N \) is large enough. The initial growth is consistent with the classical version, in which the chaotic regions of each sub phase space are connected and one point can transport freely in the chaotic sea of the whole phase space. However, the effect of Anderson localization comes in when \( N \) is sufficiently large [35], which is a pure quantum effect and sets an upper bound of \( A \).

To be specific, the localization length in \( p \) space of each eigenstate is approximately \( n_{loc} = \frac{1}{2} D_c / \hbar_e^2 \), where \( D_c \) is the classical diffusion coefficient [36]. If \( N > n_{loc} \), although the chaotic eigenstates are not confined in one integrable island, they are also localized in some part of the phase space, whose area is of the order of \( n_{loc} \) and independent of \( N \).

For a one step evolution matrix \( U \) with generic \( h_e \), we can also simply set a large cutoff (\( \gg n_{loc} \)) in \( p \) space and only consider those eigenstates which are localized in the center of the whole \( p \) space. The quantum phase space can also be constructed as follows. Choose \( N = N_x \cdot N_p \) and construct the Wannier phase space. The properties of the system with the original \( h_e \) are approximated by increasing \( j \).

![Fig. 5](image)

**Fig. 5**: (a) Area \( A \) of eigenstates for each \( j \). (b) Rational approximation of generic \( h_e \). For each \( j \), \( \delta h_e \equiv |h_e - h_{e,j}| \). (c) \( A \) at \( \phi = 0.5 \), which saturates when \( N \to \infty \). The parameters are \( K = 2, N_x = 26, \Delta = 1/\sqrt{2} \), and the index \( j \) is omitted.

The ambiguity here is that the phase space depends on \( n_0 \), which is insignificant because the change of \( n_0 \) only causes a slight displacement of Planck cells in the phase space. Then we diagonalize the finite size Floquet operator \( U \) to obtain the eigenstates. In a similar manner, we can project these eigenstates in the phase space we have constructed. In Fig. 6 we show that these eigenstates are also separated to integrable and chaotic ones, which justifies that this structure of eigenstates does not depend on the previous rational approximation and reduction process.

![Fig. 6](image)

**Fig. 6**: Eigenstates of \( U \) with generic \( h_e \) without reduction. After diagonalization, we only choose \( \sim 2 \times 10^3 \) eigenstates \( \phi \) whose average momentum \( \langle n \rangle \sim 1.5 \times 10^4 \) where the whole \( p \) space is \( 1 \leq n \leq 3 \times 10^4 \). The quantum phase space is constructed by \( 3N_x^2 \) adjacent momentum eigenstates near \( n \sim 1.5 \times 10^4 \), where \( N_x = 26, h_e = 2\pi / (N_x + \Delta)^2, \Delta = 1/\sqrt{2}, K = 2 \). (a) \( \ln |\langle n|\phi\rangle|^2 \). (b) Area of eigenstates. The value of \( A \) is normalized by the projection of each eigenstate to the phase space: \( A(\phi) = \frac{\sum_{X,N} |\langle X,N|\phi\rangle|^2}{\sum_{X,N} |\langle X,N|\phi\rangle|^4} \). (c) One typical integrable eigenstate in quantum phase space. (d) One typical chaotic eigenstate in quantum phase space.

**IV. CONCLUSION**

In sum, we have developed a method based on Wannier phase space to approach KAM effect in quantum systems. In this approach, each Planck cell in the quantum phase space is represented by a Wannier function; all the Wannier functions together form a complete and orthonormal basis. With the example of QKR, this approach has
been shown quite powerful. First, it has lead us to define the area and effective dimension of eigenstates, which then give us quantitative measures to divide all eigenstates into integrable and chaotic classes. Second, it has allowed us define the length of each Planck cell, which measures quantitatively how many Planck cells the system will travers if it starts at one Planck cell. Thirdly, this approach was also used to clarify the distinction between KAM and Anderson localization in QKR. We have used this approach in systems with a classical limit, and it is interesting to ask whether it can be generalized to other quantum systems like spin chains. This work complements our understanding of the quantum-classical correspondence, and may provide insight into short-wavelength physics such as microcavity photonics.

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