Search for the Rare Decay $B \to \pi \ell^+\ell^-$

B. Aubert, M. Bona, D. Boutigny, Y. Karyotakis, J. P. Lees, V. Poireau, X. Prudent, V. Tisserand, A. Zghiche, J. Garra Tico, E. Grauges, L. Lopez, A. Palano, G. Eigen, I. Ofe, B. Stugu, L. Sun, G. S. Abrams, M. Battaglia, D. N. Brown, J. Button-Shafer, R. N. Cahn, Y. Grosyman, R. G. Jacobsen, J. A. Kadyk, L. T. Kerth, Yu. G. Kolomensky, G. Kukartsev, D. Lopes Pegna, G. Lynch, L. M. Mir, T. J. Orimoto, M. Pripstein, N. A. Roe, M. T. Ronan, K. Tackmann, W. A. Wenzel, P. del Amo Sanchez, C. M. Hawkes, A. T. Watson, T. Held, H. Koch, B. Lewandowski, M. Pelizaeus, T. Schroeder, M. Steinke, J. T. Boyd, J. P. Burke, W. N. Cottingham, D. Walker, D. J. Asgeirsson, T. Cuhadar-Donszelmann, B. G. Fulsom, C. Hearty, S. N. Knecht, T. S. Mattison, J. A. McKenna, A. Khan, M. Saleem, L. Teodorescu, V. E. Blinov, A. B. Dukin, V. P. Druzhinin, V. B. Golubev, A. P. Ouchoun, S. I. Serednyakov, Yu. I. Skovpen, E. P. Solodov, K. Yu. Todyshev, M. Bondioli, M.Bruinsma, S. Curry, I. Eschrich, D. Kirkby, A. J. Lankford, P. Lund, M. Mandelkern, E. C. Martin, P. D. Stoker, S. Abachi, C. Buchanan, S. D. Foulkes, J. W. Gary, F. Liu, O. Long, B. C. Shen, L. Zhang, H. P. Paar, S. Rahatlou, V. Sharma, J. W. Berryhill, C. Campagnari, A. Cunha, B. Dahmes, T. M. Hong, D. Kovalskyi, J. D. Richman, T. W. Beck, A. M. Eisner, J. C. Flacco, C. A. Heusch, J. Kroseberg, W. S. Lockman, T. Schalk, B. A. Schumm, A. Seiden, D. C. Williams, M. G. Wilson, L. O. Winstrom, E. Chen, C. H. Cheng, A. Dvoretskyi, F. Fang, D. G. Hitlin, I. Narsky, T. Piatenko, F. C. Porter, G. Mancinelli, B. T. Meadows, K. Mishra, M. D. Sokoloff, F. Blanc, P. C. Bloom, S. Chen, W. T. Ford, J. F. Hirschauer, A. Kreisel, M. Nagel, U. Naueberg, A. Olivas, J. G. Smith, K. A. Ulmer, S. R. Wagner, J. Zhang, A. Chen, E. A. Eckhart, A. Soffer, W. H. Teki, R. J. Wilson, F. Winklmeier, Z. Qeng, D. D. Altenburg, E. Feltresi, H. Auke, H. Jaspers, J. M. Kerkl, A. Petzold, B. Spaan, K. Wacker, T. Brandt, V. Klose, H. M. Lacker, W. F. Mader, R. Nogowski, J. Schubert, K. R. Schubert, R. Schwierz, J. E. Sundermann, A. Volk, D. Bernard, G. R. Bonneau, E. Latour, Ch. Thiebaux, M. Verderi, P. J. Clark, W. Gradl, F. Muheim, S. Player, A. I. Robertson, Y. Xie, M. Andreotti, B. Bettoni, C. Bozzi, R. Calabrese, A. Cecchi, G. Cibinetto, P. Franchini, E. Luppi, M. Negri, A. Petrella, L. Piemontese, E. Precipe, V. Santor, F. Amuli, R. Baldini-Ferroli, A. Calcagno, R. de Sangro, G. Finocchiaro, S. Pacetti, P. Patteri, I. M. Peruzzi, M. Piccolo, M. Rama, A. Zallo, A. Buzzo, R. Contru, M. Lo Vetere, M. M. Macri, M. R. Monge, S. Passaggio, C. Patrignani, E. Robutti, A. Santroni, S. Tosi, K. S. Chaisanguanthum, M. Morii, J. Wu, R. S. Dubitzky, J. Marks, S. Schenk, U. Uwer, D. J. Bard, P. D. Dauncey, R. L. Flack, J. A. Nash, M. B. Nikolaich, W. Panduro Vazquez, P. K. Behera, X. Chai, M. J. Charles, U. Mallik, N. T. Meyer, V. Ziegler, J. Cochran, H. B. Crawley, L. Dong, V. Eyges, W. T. Meyer, S. Prell, E. I. Rosenberg, A. E. Rubin, A. V. Gritsan, K. C. Lai, A. G. Denig, M. Fritsch, G. Schott, N. Arnaud, J. Béguelin, M. Davier, G. Grosdidier, A. Höcker, V. Lепetit, F. Le Diberder, A. M. Lutz, S. Pruvot, S. Rodier, P. Roudeau, M. H. Schune, J. Serrano, A. Stocchi, W. F. Wang, G. Wornsunrern, D. J. Lange, D. M. Wright, C. A. Chavez, J. I. Forster, J. R. Fry, E. Gabathuler, R. Gamet, D. E. Hutchcroft, D. J. Payne, K. C. Schofield, C. Touramanis, A. J. Bevan, K. A. George, F. Di Lodovico, W. Menges, R. Sacco, G. Cowan, H. U. Flaecher, D. A. Hopkins, K. S. Jackson, T. R. McMahon, F. Salvatore, A. C. Wren, D. N. Brown, C. L. Davis, J. Allison, N. R. Barlow, R. J. Barlow, Y. M. Chia, G. L. Lafferty, T. J. West, J. I. Yi, J. Anderson, C. Chen, J. A. Jawahery, D. A. Roberts, G. Simi, J. M. Tuggle, G. Blaylock, C. Dallapiccola, S. S. Hertzbach, X. Li, T. B. Moore, E. Salvati, S. Saremi, R. Cowan, P. H. Fisher, G. Sciolla, J. S. Sekula, M. Spitznagel, T. Taylor, R. K. Yamamoto, H. Kim, S. E. Meacham, P. M. Patel, S. H. Robertson, A. Lazzaro, V. Lombardo, F. Palombo, J. M. Bauer, L. Cernaldi, V. Eschenburg, R. Godang, R. Kroeger, D. A. Sanders, D. J. Summers, H. W. Zhao, S. Brunet, D. Côte, M. Simard, P. Taras, F. B. Vialaud, H. Nicholson, G. De Nardo, F. Fabozzi, L. Lista, D. Monorchio, C. Sciaccia, M. A. Baak, R. Raven, H. L. Snoek, C. P. Jessop, J. M. LoSecco, G. Benelli, L. A. Corwin.
We have performed a search for the flavor-changing neutral-current decays $B \to \pi \ell^+ \ell^-$, where $\ell^+ \ell^-$ is either $e^+e^-$ or $\mu^+\mu^-$, using a sample of $230 \times 10^6 \Upsilon(4S) \to B\bar{B}$ decays collected with the BABAR detector. We observe no evidence of a signal and measure the upper limit on the isospin-averaged branching fraction to be $\mathcal{B}(B \to \pi \ell^+ \ell^-) < 9.1 \times 10^{-8}$ at 90% confidence level. We also search for the lepton-flavor-violating decays $B \to \pi e^+\mu^-$ and measure an upper limit on the isospin-averaged branching fraction of $\mathcal{B}(B \to \pi e^+\mu^-) < 9.2 \times 10^{-8}$ at 90% confidence level.

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In the Standard Model (SM), the decays $B \to \pi \ell^+ \ell^-$, where $\ell^+ \ell^-$ is either $e^+e^-$ or $\mu^+\mu^-$, proceed through $B \to d\ell^+ \ell^-$ flavor-changing neutral-current processes (FCNC) that do not occur at tree level. Three amplitudes contribute at leading order: a photon penguin, a $Z$ penguin, and a $W^+W^-$ box diagram. With highly suppressed SM rates, predicted to be $(3.3 \pm 1.0) \times 10^{-8}$, these decays provide a promising means to search for effects of new flavor-changing interactions. Such effects are predicted in a wide variety of models, usually in the context of new physics. The identification of electrons (muons) is about 92% (80%) efficient on average with a kaon misidentification rate of less than 1% (4%). Charged pion identification is more than 85% efficient and has a kaon misidentification rate of less than 5%. Neutral pions are identified as pairs of photons, each having an energy of at least 50 MeV. The invariant mass of the pair is required to satisfy $115 < m_{\gamma\gamma} < 150$ MeV/$c^2$.

Correctly reconstructed $B$ decays produce narrow peaks in the distributions of two kinematic variables: the beam-energy substituted mass, $m_{ES} = \sqrt{E_B^2 - p_B^2}$, and $\Delta E = E_B - E'_B$. Here, $E_B$ is the beam energy and $p_B^2$ is the momentum (or the reconstructed $B$ meson, evaluated in the center-of-mass (c.m.) frame). For signal events the $m_{ES}$ distribution is centered at the $B$-meson mass and the $\Delta E$ distribution is centered at zero. The mean and width of these distributions are determined from smearing and shifting the values from simulated signal events according to studies of $B^+ \to J/\psi K^+$ and $B^0 \to J/\psi K^0_s$ data in control samples and simulations. We find the width of $m_{ES}$ to be 2.5 (1.8) MeV/$c^2$ for the $\pi^\pm$ ($\pi^0$) modes and widths of $\Delta E$ to be 23, 50, 20 and 39 MeV for the $\pi^\pm e^+e^-$, $\pi^0 e^+e^-$, $\pi^\pm \mu^+\mu^-$ and $\pi^0\mu^+\mu^-$ final states, respectively. For events reconstructed as $e^+e^-$, we assume the same mean and width as for the corresponding $e^+e^-$ modes.

The primary sources of background are random combinations of particles from $e^+e^- \to q\bar{q}$ ($q = u,d,s,c$) and from $\Upsilon(4S) \to B\bar{B}$ decays. These combinatorial backgrounds typically arise from pairs of semileptonic decays of $B$ and $D$ mesons. Additionally, there is background from events that are peaking in $m_{ES}$ and $\Delta E$ as they have the same topology as signal events. These events include $B \to J/\psi \pi$ ($J/\psi \to \ell^+ \ell^-$), $B^\pm \to J/\psi K^\pm$ or $B^{\pm\mp} \to K^{\mp}\ell^+\ell^-$ (with $K^\pm$ misidentified as $\pi^\pm$), and $B \to \pi hh$ (with two hadrons $h = K^\pm, \pi^\pm$ misidentified as muons).

Contributions from $e^+e^- \to q\bar{q}$ processes are reduced by exploiting the difference between the spherical track distribution in $B\bar{B}$ events and the jetlike structure of $e^+e^- \to q\bar{q}$ events. We consider events for which the ratio of second to zeroth Fox-Wolfram moments $R_2$ is less than 0.5. Further suppression by a factor of ~45 is obtained by constructing a Fisher discriminant from the following four quantities $\mathcal{F}^2$ defined in the center-of-mass frame: $R_2$, $|\cos\theta_{\ell\gamma}|$, $|\cos\theta_{B\gamma}|$ where $\theta_{\ell\gamma}$ is the angle between the thrust axis of the signal particles and that of the remaining particles in the event, and $\theta_B$ is the
angle of the $B$ candidate’s momentum vector with respect to the beam axis, and the ratio of second- to zeroth-order Legendre moments [10].

Combinatorial background from $B\bar{B}$ events is reduced by a factor of $\sim 3$ by using a likelihood ratio composed of [9]: the missing energy of the event (computed from all charged tracks and neutral energy clusters), the vertex fit probability of all tracks from the $B$ candidate, the vertex fit probability of the two leptons, and $\cos \theta_B$. Missing energy provides the strongest suppression of these events, which typically contain energetic neutrinos from at least two semileptonic $B$ or $D^{(*)}$ meson decays.

We veto events that have a dilepton invariant mass consistent with the $J/\psi$ resonance ($2.90 < m_{\ell^+\ell^-} < 3.20 \text{ GeV}/c^2$ and $3.00 < m_{\ell^+\ell^-} < 3.20 \text{ GeV}/c^2$) or with the $\psi(2S)$ resonance ($3.60 < m_{\ell^+\ell^-} < 3.75 \text{ GeV}/c^2$). For electron modes, the vetoes are applied to $m_{\ell^+\ell^-}$ computed both with and without bremsstrahlung recovery. When a lepton radiates or is mismeasured, $m_{\ell^+\ell^-}$ may shift to values below the charmonium mass, with $\Delta E$ shifting downward accordingly. Therefore, we veto events that lie in linearly dependent $\Delta E$-$m_{\ell^+\ell^-}$ bands, whose widths are determined from simulation, similar to the technique applied in [8]. For $e^{\pm}\mu^{\mp}$ modes, we use the same vetoes as for the $e^{\pm}e^{\mp}$ modes. In modes with muons, in order to veto events with tracks that are consistent with hadronic decays $D \to K\pi$ or $D \to \pi\pi$, we require $m_{\ell\ell}$ and $m_{\pi\pi}$ to lie outside the range $1.84 - 1.89 \text{ GeV}/c^2$ when the $\ell$ is assigned the mass of a $\pi$ or $K$. For the $\pi^0$ modes, the range for $m_{\pi\pi}$ is increased to $1.79 - 1.94 \text{ GeV}/c^2$.

The events removed by the charmonium vetoes are kinematically similar to signal events and serve as large control samples for studying signal shapes, selection efficiencies, and systematic errors. The branching fraction of $B \to J/\psi \pi$ is also extracted from the control sample and found to be in agreement with the current world average [11]. We also select a control sample of $B^+ \to J/\psi K^+$ events to measure the efficiencies and systematic uncertainties of lepton identification and the Fisher and likelihood selection.

We extract the signal yield by counting events within a signal region defined as $\pm 2\sigma$ around the mean values of the $m_{ES}$ and $\Delta E$ distributions expected for signal events, and comparing observed event counts with estimations of the remaining background in the same region, summarized in Table II.

To determine the peaking background from hadronic $B \to \pi\pi\pi$ or $B \to K\pi\pi$ events, we select a control sample where one track is required to pass hadron identification in place of muon identification. This selects hadronic $B$-decays where the remaining track that passes the muon selection is a mis-identified hadron. Each event is further weighted by the probability that one more hadron is mis-identified as a muon. The expected contribution to the $B \to \pi\ell^+\ell^-$ signal region is extracted from a one-dimensional fit to the distribution of $m_{ES}$ for events that pass the $\Delta E$ signal selection.

Backgrounds from $B$ decays to final states with real leptons are estimated from high-statistics samples of simulated events. $B \to K\ell^+\ell^-$ events are the largest peaking background component for the $\pi^+e^-e^-$ mode, but are shifted towards lower $\Delta E$ than signal and fall outside the signal region. $B \to \rho\ell^+\ell^-$ events contribute even less, since the reconstructed $B$ mesons are missing a pion. Background from charmonium resonances are found to be negligible.

The expected number of combinatorial background events is extracted from a two-dimensional, unbinned maximum-likelihood fit to $m_{ES}$ and $\Delta E$ in a sideband defined by $5.2 < m_{ES} < 5.2724 \text{ GeV}/c^2$ and $|\Delta E| < 0.25 \text{ GeV}$, i.e., below the $m_{ES}$ value expected for signal $B$ events. The signal-region yield is obtained from extrapolation of this fit into the signal region. This procedure has been validated by studies of simulated background events and data events in the $e\mu$ channel where no signal-like events are expected. The background probability distribution function (PDF) is modeled as the product of an ARGUS function [12] for $m_{ES}$ and an exponential function for $\Delta E$. The slopes and normalizations are floating in the fit. Average biases in the background central value and its uncertainty were corrected for, based on a study of a large ensemble of simulated experiments generated from the background PDF obtained from data. The corrections amount to 35% in the low-statistics $B^0 \to \pi^0\mu^+\mu^-$ channel and < 10% in all others.

Systematic uncertainties due to the background estimates are summarized in Table II. The uncertainty in the combinatorial-background estimate is determined by varying the fit parameters by $\pm 1\sigma$ of the best fit. We also consider the effect of using alternative PDF parameterizations on the background estimates, and use the computed differences to bound the systematic uncertainty. Alternatives considered include a PDF that is correlated in $m_{ES}$ and $\Delta E$ via a linear $\Delta E$ dependence in the $m_{ES}$ slope parameter, and PDFs for which the $\Delta E$ shape is a linear or quadratic polynomial. For peaking background with real leptons the uncertainty is dominated by limited knowledge [11] of the branching fractions for these processes, and for hadronic $B$ peaking background the uncertainty is dominated by the control sample statistics from which it is derived.

Systematic uncertainties due to the signal efficiency include: charged-particle tracking efficiency (0.8% per lepton, 1.4% per charged hadron) and identification (0.7% per electron pair, 1.9% per muon pair, 0.5% per pion), neutral pion efficiency (3%), the Fisher and likelihood selection (1.4% for all modes involving electrons, 1.7% for $B^+ \to \pi^+\mu^+\mu^-$ and 1.9% for $B^0 \to \pi^0\mu^+\mu^-$), and signal simulation statistics (0.1%). A systematic uncertainty in signal-region selection efficiency arises from the uncertainty in the mean and width of the $m_{ES}$ and
The table represents the number of background events with associated systematic uncertainties in the signal region.

| $m_{ES}$–$\Delta E$ fit | $\pi^+\pi^-$ | $\pi^+\pi^-$ | $\pi^+\mu^+$ | $\pi^+\mu^-$ | $\pi^+\mu^+$ | $\pi^+\mu^-$ |
|------------------------|------------|------------|-------------|-------------|-------------|-------------|
| $m_{ES}$–$\Delta E$ fit | 0.84 ±0.24 | 0.43 ±0.23 | 0.90 ±0.25 | 0.23 ±0.20 | 1.55±0.34 | 1.22±0.43 |
| $m_{ES}$–$\Delta E$ correlations | ±0.02 | ±0.03 | ±0.06 | ±0.03 | ±0.17 | ±0.05 |
| $\Delta E$ shape | ±0.02 | ±0.01 | ±0.12 | ±0.02 | ±0.31 | ±0.24 |
| Peaking ($e^+e^-$) | 0.057±0.016 | 0.009±0.003 | 0.032±0.008 | 0.005±0.001 | 0.0 ±0.001 | 0.0 ±0.001 |
| Peaking (hadronic) | < 0.001 | < 0.001 | 0.027±0.033 | 0.035±0.022 | 0.0 ±0.02 | 0.0 ±0.02 |
| Total | 0.90 ±0.24 | 0.44 ±0.23 | 0.96 ±0.29 | 0.27 ±0.20 | 1.55±0.49 | 1.22±0.50 |

$\Delta E$ distributions determined from charmonium control samples. This contributes a total uncertainty of 0.7% for charged modes for which a high-statistics sample of $B^+ \to J/\psi K^+$ events is used, and a total of 7% uncertainty for neutral modes for which a small statistics sample of $B^0 \to J/\psi \pi^0$ events is used. For the electron modes, we vary the amount of the bremsstrahlung tail in the $\Delta E$ distribution, introducing a systematic uncertainty of 1.4%. The number of $B\bar{B}$ events in the data sample is known to a precision of 1.1%. Additional systematic uncertainties for the efficiency result from the choice of the form factor model and the relative magnitudes of the $b \to d\ell^+\ell^-$ amplitudes, which affect the distribution of four-momentum transfer $q^2 = m^2_{\ell^+\ell^-}$ of the signal. We evaluate these systematics from the spread in efficiencies when using alternative form-factor models \[\text{[12]}\], and when varying the Wilson coefficients in the amplitudes by a factor of ±2. The former uncertainty varies from 1.1% for $B^+ \to \pi^+\pi^-\pi^-$ to 7.3% for $B^+ \to \pi^0\mu^+\mu^-$; the latter uncertainty varies from 0.3% for $B^0 \to \pi^0\mu^+\mu^-$ to 1.2% for $B^+ \to \pi^+\pi^-\pi^0$. For the $e\mu$ modes we use the spread in efficiency when applying two alternative theoretical models for these decays, which amounts to 17% (19%) for the $\pi^{0(0)}$ mode. The total systematic uncertainty of the signal efficiencies are 4% (9%), 6% (11%) and 17% (21%) for $\pi^{0(0)}e\mu, \pi^{0(0)}\mu^+\mu^-$ and $\pi^{0(0)}e\mu$ modes, respectively.

Figure 1 shows the distribution of events from data in the $m_{ES}$–$\Delta E$ plane. The rectangles in the plots indicate the signal regions. Three $B \to \pi^+\pi^-\pi^0$ candidates and one $B \to \pi^+\pi^-\mu^+$ candidate are observed in the signal regions, which is consistent with the expected background. In Table II we calculate the branching fraction upper limits at 90% confidence level (C.L.) using a frequentist method that takes systematic uncertainties and their correlations into account. We follow the algorithm of \[\text{[14]}\], but differ from it in that we assume Gaussian distribution truncated at zero for the systematic uncertainties in signal sensitivity and background expectation. We combine modes and determine the $e\mu$–averaged branching fractions to be $B(B^+ \to \pi^+\pi^-\pi^0) < 1.2 \times 10^{-7}$ and $B(B^0 \to \pi^+\pi^-\pi^0) < 1.2 \times 10^{-7}$ at 90% C.L., where charged conjugate modes are implied. Defining the isospin averaged branching fraction $B(B \to \pi^+\pi^-\pi^0) = B(B^+ \to \pi^+\pi^-\pi^0) + B(B^0 \to \pi^+\pi^-\pi^0)$, we find the combined upper limit

$$B(B \to \pi^+\pi^-\pi^0) < 9.1 \times 10^{-8} \text{ at 90% C.L.}$$

This is about a factor three above the nominal SM prediction \[\text{[1]}\]. We similarly compute the combined limit for the $e\mu$ modes of

$$B(B \to e\mu\pi^0) < 9.2 \times 10^{-8} \text{ at 90% C.L.}$$

In conclusion, we have presented the result of a search for $B \to \pi^+\pi^-\pi^0$ using a sample of $(230.1 \pm 2.5) \times 10^6$ $B\bar{B}$ pairs produced at the $\Upsilon(4S)$ resonance. No excess of events is observed in the signal regions, and at 90% confidence limit we measure the upper limit of $B(B \to \pi^+\pi^-\pi^0) < 9.1 \times 10^{-8}$, which is within a factor three of SM expectations. We also measure the upper limit of the lepton-flavor–violating branching fractions to be $B(B \to e\mu\pi^0) < 9.2 \times 10^{-8}$.
TABLE II: The observed yields, number of expected background events, signal efficiency, and branching fraction (B) upper limit (U.L.) at 90% C.L. in units of 10⁻⁷. The upper limits for combined modes are also given.

| Mode | Observed events | Expected background | Signal efficiency | 90% C.L. B U.L. |
|------|-----------------|---------------------|------------------|----------------|
| $B^+ \to \pi^- e^+ e^-$ | 1 | 0.90 ± 0.24 | (7.1 ± 0.3)% | 1.8 |
| $B^0 \to \pi^0 e^+ e^-$ | 0 | 0.44 ± 0.23 | (5.7 ± 0.5)% | 1.4 |
| $B^+ \to \pi^+ \mu^+ \mu^-$ | 1 | 0.96 ± 0.29 | (4.7 ± 0.3)% | 2.8 |
| $B^0 \to \pi^0 \mu^+ \mu^-$ | 1 | 0.27 ± 0.20 | (3.1 ± 0.3)% | 5.1 |
| $B^+ \to \pi^+ e^+ \mu^\mp$ | 1 | 1.55 ± 0.49 | (6.3 ± 1.1)% | 1.7 |
| $B^0 \to \pi^0 e^\mp \mu^\mp$ | 0 | 1.22 ± 0.50 | (3.7 ± 0.8)% | 1.4 |
| $B^+ \to \pi^+ \ell^+ \ell^-$ | 1.2 |
| $B^0 \to \pi^0 \ell^+ \ell^-$ | 1.2 |
| $B \to \pi \ell^+ \ell^-$ | 0.91 |
| $B \to \pi e^\pm \mu^\mp$ | 0.92 |

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* Deceased
† Also with Università della Basilicata, Potenza, Italy
‡ Also with IPPP, Physics Department, Durham University, Durham DH1 3LE, United Kingdom

[1] T. M. Aliev and M. Savci, Phys. Rev. D 60, 014005 (1999).
[2] G. Burdman, Phys. Rev. D 52, 6400 (1995); J. L. Hewett and J. D. Wells, Phys. Rev. D 55, 5549 (1997); G. Eilam, J. L. Hewett, and T. G. Rizzo, Phys. Rev. D 34, 2773 (1986); T. M. Aliev, A. Ozpineci, and M. Savci, Eur. Phys. J. C 29, 265 (2003).
[3] A. Ali, E. Lunghi, C. Greub, and G. Hiller, Phys. Rev. D 66, 034002 (2002).
[4] S. Davidson, D. C. Bailey, and B. A. Campbell, Z. Phys. C 61, 613 (1994).
[5] A. J. Weir et al. (Mark II Collaboration), Phys. Rev. D 41, 1384 (1990); K. W. Edwards et al. (CLEO Collaboration), Phys. Rev. D 65, 111102 (2002).
[6] B. Aubert et al. (BABAR Collaboration), Nucl. Instrum. Methods A 479, 1 (2002).
[7] P. Ball and R. Zwicky, Phys. Rev. D 71, 014015 (2005).
[8] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 73, 092001 (2006).
[9] Distributions can be seen in I. Ofte, Ph.D. thesis, University of Bergen, Bergen, Norway, 2006. ISBN 82-308-0272-6.
[10] The $i^{th}$ moment $L_i$ is defined by $L_i = \sum_j |p_j^x| \cos(\theta_j^x)$, where the $p_j^x$ are the c.m. momenta of all particles not used in reconstructing the signal $B$ candidate, and the angle $\theta_j^x$ is between the particle’s momentum and the thrust axis of the signal $B$.
[11] W.-M. Yao et al. (Particle Data Group), J. Phys. G 33, 1 (2006).
[12] The function is $f(x) \propto x^\zeta (1-x^2) \exp[-\zeta (1-x^2)]$, where the slope $\zeta$ is a fit parameter and $x = m_{BS}/E_{BS}$; H. Albrecht et al. (ARGUS Collaboration), Z. Phys. C 48, 543 (1990).
[13] D. Melikhov and N. Nikitin, hep-ph/9909503; D. Melikhov, Phys. Lett. B 380, 363 (1996); D. Melikhov, Phys. Rev. D 53, 2460 (1996); D. Melikhov and B. Stech, Phys. Rev. D 62, 014006 (2000).
[14] R. Barlow, Comput. Phys. Commun. 149, 97 (2002).