Secondary Teachers' Mathematics-related Beliefs and Knowledge about Mathematical Problem-solving

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Abstract. This study investigates secondary teachers’ belief about the three mathematics-related beliefs, i.e. nature of mathematics, teaching mathematics, learning mathematics, and knowledge about mathematical problem solving. Data were gathered through a set of task-based semi-structured interviews of three selected teachers with different philosophical views of teaching mathematics, i.e. instrumental, platonist, and problem solving. Those teachers were selected from an interview using a belief-related task from purposively selected teachers in Surabaya and Sidoarjo. While the interviews about knowledge examine teachers’ problem solving content and pedagogical knowledge, the interviews about beliefs examine their views on several cases extracted from each of such mathematics-related beliefs. Analysis included the categorization and comparison on each of beliefs and knowledge as well as their interaction. Results indicate that all the teachers did not show a high consistency in responding views of their mathematics-related beliefs, while they showed weaknesses primarily on problem solving content knowledge. Findings also point out that teachers’ beliefs have a strong relationship with teachers’ knowledge about problem solving. In particular, the instrumental teacher’s beliefs were consistent with his insufficient knowledge about problem-solving, while both platonist and problem-solving teacher’s beliefs were consistent with their sufficient knowledge of either content or pedagogical problem solving.

1. Introduction

Research on discussing relationship between teacher’s belief and practice within mathematics instruction have been addressed by many scholars with a variety of purposes. Some tried to conceptualise models on such relationship [1,2,3,4], while others reported how such relationship was confirmed into practical interest [5,6]. Either the models or the reports were identified to have several associated key variables which emphasize on explaining to what extend one variable influence another variable. Such key variables such as past school experience and immediate classroom situation, for instance, respectively have strong influence on teachers’ mathematics beliefs and teacher’s teaching practice [1]. Another important variable, teacher’s knowledge, is scrutinized to have strong interaction with beliefs in shaping teachers’ teaching practice with varying degree being given to particular types of knowledge or beliefs in different situations [5]. Fennema et al [3] added that beside students’ behaviours, such interaction will directly affect teacher’s decisions in planning classroom instructional activity. Hence, more recent studies reported how such interaction influence teachers’ teaching practice. Bray’s study results [6] argued that teacher’s beliefs seemed most likely influence on how teacher structure class discussion whereas teacher’s knowledge appeared to drive quality of teachers’ responses within classroom discussion. On the other hand, Anderson et al [1] summarised the knowledge component includes mathematical content and plans, professional development,
teachers’ knowledge and decisions, important mathematics and assessment procedures, use of mathematics texts, and teacher education program.

While such scholars above discussed general knowledge needed by teachers regarding mathematics instruction, in particular on problem solving, what knowledge does teachers need to hold? Some describe in pedagogical context, while others revealed in the context of content. Pedagogically, Franke, Kazemi, & Battey [7] explained that teachers need to orchestrate class discussion so that students share multiple problem-solving strategies, analyze relations among strategies, and explore contradictions in students’ ideas to provide greater insight into the mathematical focus. In brief, Chapman [8] summarised three types of knowledge for teaching problem solving: problem solving content knowledge consisting of knowledge of mathematical problems, problem-solving proficiency, and problem posing, as well as pedagogical problem solving knowledge consisting of knowledge of students as problem-solvers, and instructional practices for problem-solving, and affective factors and beliefs.

Regarding types of mathematics-related beliefs which might be held by teachers in teaching problem solving, several authors proposed some categories, such as Ernest [9] with his prominent categories: instrumental, platonist, and problem solving. Table 1 summarizes several authors’ view on how such three beliefs are characterized [10].

| Beliefs about the nature of mathematics | Beliefs about mathematics teaching | Beliefs about mathematics learning |
|----------------------------------------|---------------------------------|---------------------------------|
| Instrumentalist                        | Content focussed with an emphasis on performance | Skill mastery, passive reception of knowledge |
| Platonist                              | Content focussed with an emphasis on understanding | Active construction of understanding |
| Problem solving                        | Learner focussed                 | Autonomous exploration of own interests |

Our study aims to describe teachers’ mathematics-related belief and knowledge about mathematical problem solving as well as search for possible interaction between such two variables emerging in our teacher participants.

2. Experimental Method

For the purposes of our research, we describe qualitatively three Indonesian lower secondary mathematics teachers' belief and knowledge regarding problem-solving with different philosophical views of teaching mathematics: instrumental, platonist, and problem-solving. Those teachers were selected from the result of interviewing 12 teachers purposively chosen from Surabaya dan Sidoarjo city using a belief-related task discussing what best instructional sequences should be implemented into a mathematical problem within classroom activity. Of all the teachers, we selected teachers with the most significant feature describing their view about a particular type of belief. We selected Toro (male, pseudo name) as one of our further interest of participants since he seemed to portray his beliefs in an instrumental view. Two other selected teachers characterizing platonist and problem solving were represented by Yati (female, pseudo name) and Ana (female, pseudo name), each of whom agrees that students should be engaged actively in class activities as well as agree that lesson should be started by the contextual problem as shown by the task. The difference is that Yati tends to provide general steps on how to solve the problem, whereas Ana prefers to give feedback regarding her students' idea on solving the problem rather than recommend specific idea in solving the problem.

All the three teachers selected were individually interviewed about their beliefs and knowledge about mathematical problem solving. The beliefs include three components, i.e. nature of mathematics, how to teach mathematics, and how students learn mathematics; while the knowledge includes problem-solving content knowledge and pedagogical problem-solving knowledge. To collect the data of teachers' belief we used a set of task-based interview. Such tasks were arranged in the form of incomplete statements. We provided three options for each incomplete statement illustrating the
Each of the teachers was asked to choose one of the options which mostly represent his/her view and explain why he/she chose that option. Furthermore, we also asked them to compare his/her argumentation contrasted with the other two remaining options. As illustration, when asked about the what people should learn from mathematics, the teachers were confronted with options: (1) to have skills in calculating and applying mathematical formulae or procedures when solving daily life problem, (2) to be proficient in understanding topics in mathematics, such as algebra, statistics, probability, geometry, and the interrelationships among those topics entirely, and (3) to have thinking skills such as understanding regularities of phenomena, being critical and creative in solving any problems. Meanwhile, to collect data about teachers’ knowledge, our questions were primarily inspired by Chapman’s category of problem-solving knowledge for teaching, such as when a mathematics question becomes problem [the nature of problem], what processes needed to solve mathematical problem, how to implement problem solving process and strategies within classroom activity, etc.

Analysis of data was carried out by firstly reducing data, displaying data, and finally drawing conclusion and verification[11]. The conclusion was sought to understand the characteristics of belief and knowledge espoused by teachers and how such two variables interact each other shaping their idea of mathematical problem solving.

3. Results and Discussion

3.1. Toro’s beliefs and knowledge about mathematical problem-solving

Toro’s view of nature of mathematics was mostly aligned with Instrumental, especially when he argued about what one should learn from mathematics and when he defined mathematics. He prefers to have proficiency in calculating as well as applying formula and procedures because he said,’I think one needs to master mathematical concepts to understand how to apply a mathematical formula,...most of the mathematics teach us to calculate. ’This is in line with his option emphasizing that mathematics is a discipline discussing calculation, numbers, and formula. He asserted his view by arguing that strategies of solving a problem will only be successful if one knows the similar problem he/she ever encountered. However, when he was asked to link between real life problem and mathematics, he prefers to choose the statement that there are parts of topics in mathematics used to solve real life problems and there are also parts of mathematics used to develop the topics of mathematics itself. In this case, however, it indicates his platonist view.

His instrumental view also appeared in most cases examined in the group of ‘teaching mathematics’ statements, i.e. viewing the role of teacher, the precise time to teach contextual problem, ways to minimize students’ misunderstanding, and strategy to motivate students on particular topic. In viewing the latter two cases, for example, he told, “I choose to clarify their misunderstanding by giving a more detail explanation particularly on the parts in which they find difficulties instead of letting them discuss because if I pose questions, they are often be confused, no feed back.”, and “to motivate students learn something difficult to understand its application, I used to explain more clearly on the prerequisite knowledge for learning, in this case, central and circular angle.” Ultimately, when he responded on the best way to teach mathematics, he viewed, “teaching math should start with giving concepts, exercise from easy to difficult”.

In responding how students should learn mathematics, Toro’s beliefs also indicates an instrumental view. He argued that students need to practice many times to master concepts or procedures inherent in the problem, need to only focus on particular strategy which can be applied to many types of problem by repeating such strategy many times rather than learning various strategy that may not applicable in the task students are solving, and must not use calculator whatever the reason is. Nevertheless, in viewing formula, he told, ”they [students] may use the formula given in the book or learning how such formula is constructed, but more importantly, they should develop their own ‘formula’ to solve any problem.”. He seemed to have the problem-solving view in this case.
Regarding knowledge, Toro seemed to have partial understanding, primarily on the meaning of mathematical problem, types of problem, problem-solving process, and strategies. When asked about whether particular mathematical questions are a problem or not for students, he knows that a problem should be challenging, interesting, and has no immediate solution, but he asserted that a question which demands insufficient prerequisite knowledge is a problem. He said, "question 'find the area bounded by curves y=x²+1, x=-4, x=5, and x-axis' is a problem for my students because they require certain formula which is not learned yet". Besides, Toro was seemingly unable to select an appropriate problem for her lesson as indicated when he incorrectly explained about for what purpose a particular problem is designed. He said, "this problem [a problem about finding the concept of circle area] is designed for mastering students' skill in applying the formula of circle area. I don't think it works on my students if I give it in early learning of circle area". Data also indicates that Toro incompletely mentioned problem-solving process. He noticed, "when solving problem, I need to firstly understand, then choose suitable strategies, and apply such strategies". No indication appeared that solving problem needs also process of looking back as suggested by scholars. When explaining about implementation of problem solving process within classroom, Toro was not able to explain an ideal process of guiding students to solve problem as indicated by his statement, "...students are asked to read the problem and listened to what I explained to understand the problem...let them choose methods...". Accordingly, his knowledge about guiding process is rather teacher-centered in the beginning of solving problem.

3.2. Yati's beliefs and knowledge about mathematical problem-solving

Yati expressed a deep belief that her nature of mathematics regards with platonist view as indicated by her responses to all the incomplete statements. She concluded that mathematics is a discipline of logic which is permanent over time. She argued, 'something that can change from it is only the methods which may be more various in the future, but something such as numbers, geometrical shape or other objects in mathematics will be remaining same.' Her idea is supported by her statement that the strategy of solving problem will find its success depends on whether one ever works on it or not, regardless of whether his concepts and procedures is correct or not. She also agrees that mathematics teach us to understand topics in mathematics, such as algebra, statistics, probability, geometry, and the interrelationships among those topics entirely. Such interrelationships, she said, are important as basic skill to solve a more complex problem. Thus, her view on defining mathematics influences her view on the scope of what to learn from mathematics and her successful experiences when solving a mathematical problem.

While Yati’s belief about nature of mathematics is likely significant, her belief in teaching mathematic is equally mixed between instrumental, platonist and problem solving. Her instrumental view is indicated by her responses on the role of teacher when real life problem should be taught, and what to place reward, while her problem-solving view is described by her responses on how to minimize students' misunderstanding during the lesson. Meanwhile, platonist view is pointed out by her statements on responding cases on motivating students to learn and the best way to teach the particular topic and how to help students when finding difficulties in solving the problem. Her statements such as 'I prefer to choose to provide a figural display to represent problem of this system of linear equation and let students explore such display to find further strategy rather than let them find their own form representation, it makes students not confident because of their insufficient knowledge of strategies' and supports such view.

Yati’s view on learning mathematics, however, is aligned with problem-solving view. All the cases examined, except the response on using calculator, are indicated to support an idea of autonomous exploration of students' interests. In viewing which strategies should be learned in solving a particular problem, for instance, she suggested students figure out all the strategies provided because she maintained, "although some strategies are not effective enough to solve a certain problem, but such strategies will be likely more effective for other problems."
Regarding Ana's beliefs and knowledge about mathematical problem-solving

The view of nature of mathematics espoused by Ana refers mostly to problem-solving, except on the way she defined mathematics. Her idea of defining mathematics includes his view that mathematics is dynamic and changeable on methods to invent such objects. She said, "you will surely find any changes in mathematics, even it is very easy to change, however, I'm not really sure whether the object which we learned will change." She also agrees that mathematics is about thinking and grasping real world phenomena. She argued, “by enhancing what really mathematics is, you will be more creative in using various types of representation presented in this world.” Regarding strategy of solving problems, she believes that she will not always be successful to find the solution of the problem even though the concepts and mathematical procedures that she used are correct. In this case, she asserted, "I often encounter this matter primarily when solving a problem which demands higher-order thinking. I am not sure whether this is caused by my lack knowledge or, indeed, there is no perfect solution to that problem."

Ana's beliefs about teaching mathematics also aligned with problem-solving. Except for her idea of choosing a suitable reward for motivating students to learn mathematical problem solving which is likely instrumental, Ana showed consistent view in putting learner as a focus of learning in instructional practice as indicated by the other cases. Her idea such as, “providing guidance on the form of representation the students develop is more fruitful than letting them think of strategy based on the form of representation, like figure shown in this option, will make students more freely express their idea”, “the way to motivate students learn central angle, even though they do not find any application in real life is by convincing them that the thinking skill used to find the relationship between central angle and circular angle is much more important in their future life”, and “I will provide students opportunity to discuss ideas emerging from their thinking and determine which one is the best to clarify their misunderstanding” support her problem-solving view.

With regards to belief in how students should learn mathematics, Ana's view seemed to align with problem-solving view too. When asked to complete statement about learning strategy on being good in solving a mathematical problem, she said, "student should attempt to solve a problem by their own strategies based on their knowledge and experience, so they will get used of thinking first rather than practicing many times on similar problem. If they only attempt to apply what teachers teach, higher order thinking will not appear on their idea". Besides, Ana also suggests students to learn all strategies provided since she argued, “if students do not learn all the strategies, they will not likely find which strategy best for solving problem.” Surprisingly, Ana is the only one participant who agrees to permit students to use calculator. She said, "using calculator does not influence students' understanding of basic procedures or concepts, though and even it is recommended for facilitating students to solve problem which emphasizes on the process, not results, like in statistical problem." Despite all such indicate that her view seemed to support problem-solving view, her final conclusion regarding learning mathematics is rather platonist. She said, "students need to solve problem by their own way
using their experience, and of course, need aims from teacher when experiencing difficulties, and especially about mathematical formula, they need to understand how such formula is constructed.”

Ana’s knowledge about problem solving seemed better than Toro and Yati, particularly on problem-solving content knowledge. Unlike their idea, Ana claimed that not all mathematics questions are problem for students. She said, “it depends on students’ ability and whether the questions embody something challenging and interesting.” However, again, Ana, as well as Toro and Yati, regarded that question ‘find the area bounded by curves y=x^2+1, x=-4, x=5, and x-axis’ is a problem for her students. She also expressed her good understanding on the types of problem. When interviewed about the idea to exemplify open-ended problems, she said, "I will reverse the question, like if the area of a rectangle is 225 cm^2, what are its probable measure? Draw as many as possible irregular plane with the area of 500 cm^2.” Ana’s pedagogical knowledge seemed also sufficient to hold problem-solving instruction. She told, "teaching problem-solving, you need to begin with asking students understand the problem such as selecting relevant and irrelevant information, listing given and not given but needed, encouraging them to think creatively on any ideas in attempting to find precise mathematical model, giving feedback on their strategies and finally asking them to consider whether the solution they found make sense or not."

4. Conclusion

Our findings indicate although our participants espoused their view of the relationship between mathematics and real life as well as the success of problem-solving strategy in a problem solving or platonist view, no participant who defined mathematics in a problem-solving view. Yati and Ana only view mathematics as a static discipline with regards to platonist view, while toro defined it as a discipline studying numbers and formula. However, generally their view on teaching and learning mathematics is seemingly more constructivist which corresponds to at least platonist view particularly on Yati and Ana. this finding may indicate that belief about mathematics and about teaching and learning are two distinct belief sets, as suggested by Thompson [12]. Such result also is in line with the study of Lui & Bonner [13] suggesting that despite teachers may hold constructivist belief for mathematics teaching and learning, they may hold traditional beliefs about what mathematics is.

Regarding knowledge, our findings point out that generally teachers hold insufficient knowledge primarily on the nature of problem like deciding when a problem is suitable for building concept or applying concept This is aligned with findings in the study of Siswono et al [14] reporting that teachers found many difficulties when dealing with the nature of problem and identifying problem-solving strategies. In search of a possible interaction between teachers’ beliefs and knowledge about mathematical problem solving, we found that, seemingly, Toro shows less conceptual knowledge about problem solving, while Ana shows sufficiency. This finding is consistent with expectations that a problem-solving belief, would be associated with skills in conceptual knowledge as Lui & Bonner [13] explained that teachers with a constructivist beliefs had strong skills in conceptualknowledge.

To conclude, we highlight that all the teachers did not show a high consistency in responding views of their mathematics-related beliefs. Beside, they showed weaknesses primarily on problem-solving content knowledge. It is also noticed that teachers’ beliefs have a strong relationship with teachers’ knowledge about problem-solving. In particular, the instrumental teacher’s beliefs were consistent with his insufficient knowledge about problem solving, while both platonist and problem-solving teacher’s beliefs were consistent with their sufficient knowledge of either content or pedagogical problem solving. A possible further explanation of such relationships may be investigated from their observed teaching practice to examine whether teachers’ belief and knowledge consistent with their practice and how each belief and knowledge influence their practice.
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