FRP-Strengthening of Stretched Parts of Structural Elements by Cascade Multilayer Method

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Abstract. The purpose of research is the analysis of applicability and effectiveness of cascade type multilayer models in design of structural elements strengthening by gluing on their surfaces fiber reinforced polymers (FRP). Materials and methods: variants of using the FRP-reinforcement for stretched and bent elements with application of FEM simulation and analytic approach. A number of diagrams and tables represent the results. Design examples include the analysis of the adhesive joints application to attach FRP elements. Results are: justification of an efficient and cost-effective method of strengthening the stretched and/or bent elements to increase their bearing capacity reserves, the features of the bonded joint behavior, the equations and formulae for analysis and design. Analytical expressions are obtained and presented for designing the cascade strengthening schemes. Conclusions: the expression “cascade” reflects the features of the proposed strengthening layout, the reinforced (base) element unloading progress gradually along its length with each successive reinforcement element (strengthener) attached. The results suggest the possibility of effective use of adhesive joints to strengthen rather strong, including steel, elements. The cascade method allows to diminish the strength level of applied materials, thereby reducing the cost of reinforcement structures and increases the safety level of the reinforcement structure.

1. Introduction

Along with traditional methods of reinforcing elements of steel structures using welded and bolted joints, the use of high-strength fibre-reinforced polymer materials (for example, CFRP) can be quite effective, providing, with an insignificant increase in the weight of the structure, that there is no weakening of cross sections, additional stress concentrators or additional potential areas of corrosion, as well as a less labour intensive reinforcement process than traditional ones.

In [1-3] the authors consider the application of carbon polymers in building structures and recently obtained results of experimental studies of the behaviour of adhesive joints in the strengthening of damaged steel elements working in tension. The [4] considered principal aspects of the structure safety level control by incorporating reserve (intercepting) not loaded elements becoming active in emergencies. Polymer materials, due to their unique properties, may be ideal for such applications in stretch zones. The [5-9] analyse the advantages and disadvantages of using steel reinforcement...
elements (proved reliability but problems with stress concentration, welding, decrease in fatigue strength, etc.) and polymer reinforcement systems. In [10-18], various aspects of use of polymers in repair and strengthening of building structures for improving fatigue strength, stability with the use of adhesive and mechanical connections are considered.

The analysis of various parameters and properties of the adhesive layer effect on the adhesive joint response and the structure reinforcement effect on structure resistance forecast is possible on the basis of comparison of numerical and physical experiments results with approximate calculations on the basis of analytic relations allowing at relatively little time and effort to consider any required number of different combinations of joint parameters, especially in planning physical experiments [19, 20].

FRP-reinforcement of steel elements (by fibre reinforced polymeric materials) using the adhesive and combined joints can radically simplify and accelerate the technology of reinforcement operation for steel structures in many cases without interrupting the process of their normal service. The implementation of FRP-reinforcement with adhesive joints involves the reinforced element surface preparation for gluing. Allowing for the real dimensions of building structures, an important parameter is the area of gluing contact surfaces value that may the dimensions of glue layer dictate. The rational length of the bonding area may be dominant parameter, since the shear stress distribution in the adhesive layer may be extremely uneven along its length. Surface preparation, as the world experience of strengthening steel elements of bridge structures shows, is a labour intensive taking a lot of time operation of the reinforcement process.

This paper presents a peculiar scheme of structural element strengthening – cascade. It consists in fastening not a single but a number of reinforcement elements (strengtheners) of varying length and attachment points sharing the whole unloading force nearly proportionally to their stiffness \( EF \). The cascade strengthening method under consideration was not managed to be found in any publication in any form though some information about step by step reinforcement by steel elements of some continuous beam stretched flange over support of a bridge in Siberia in the 60-th of last century. On the other hand, there are lots of cable bridges where all cables are fastened by one end to the pylon with other ends attachments distributed along the span homogeneously.

2. Stretched element reinforcement

In figure 1 the reinforcing elements fastened in points 1 to \( n \) support the left half length of the stretched base element divided into \((n + 1)\) intervals under load \( N \). The base element has the mechanical characteristic \( (EA)^f_1 \) on each interval \( i \), the interval length is \( a_i \). The reinforcing elements have the mechanical characteristics \( (EA)^f_l \) on the length \( A_l \). On the right ends all the reinforcing elements and the base element are fixed. The scheme has double symmetry.

The task is to calculate the tension force distributions in all the reinforcing elements and the base element with given geometry and applied force. Consider a steel tape with polymer reinforcing tapes bonded on its top and bottom surfaces.

\[
N^s - \text{tape stretching force in the interval } i; \quad N^p_l - \text{total force in two «tapes» attached at the interval } i; \quad \text{node } i - \text{the left node of interval } i.
\]

![Figure 1. Cascade scheme](image)
The problem can be reduced to solving \((n + 1) \times (n + 1)\) system of linear equations (1)-(2)

\[
\sum_{k=1}^{n+1} b_{ik}^{ps} \cdot N_k^s + h_{ii}^{ps} \cdot N_i^s - N_{i+1}^s = 0, \quad i = 1, \ldots, n; \quad N_{n+1}^s = N, \quad \text{where}
\]

\[
b_{ik}^p \equiv \frac{(EA)^{ps}_{ik}}{A_i}; \quad b_i^s \equiv \frac{(EA)^{ps}_i}{a_i}; \quad b_{ik}^{ps} \equiv b_i^p \cdot b_{ik}^s; \quad b_{ii}^{ps} \equiv b_i^p; \quad h_{ii}^{ps} = 1 + b_{ii}^p
\]

Then the tension force distributions in all the reinforcing elements can be calculated from

\[
N_i^p = \sum_{k=1}^{i} b_{ik}^{ps} \cdot N_k^s \quad i = 1, \ldots, n
\]

For \(i = 1\), the first term (sum) in the first equation should be ignored.

The special case is if the base specimen has the damage in the middle [19]. It means that \(N_i^s = 0\). Then in the matrix as shown in example the first line and the first column should be excluded thus reducing the order of the problem by 1.

For instance, in case of \(n = 5\) the system of equations in the matrix form looks as

\[
\begin{array}{cccccc}
 h_{11} & -1 & 0 & 0 & 0 & N_1 \\
 b_{21} & h_{22} & -1 & 0 & 0 & N_2 \\
 b_{31} & b_{32} & h_{33} & -1 & 0 & N_3 \\
 b_{41} & b_{42} & b_{43} & h_{44} & -1 & N_4 \\
 b_{51} & b_{52} & b_{53} & b_{54} & h_{55} & N_5 \\
\end{array} = \begin{array}{cccccc}
 0 & 1 & 0 & 0 & 0 & N_1 \\
 0 & 0 & h_{22} & -1 & 0 & N_2 \\
 0 & 0 & b_{32} & h_{33} & -1 & N_3 \\
 0 & 0 & b_{42} & b_{43} & h_{44} & N_4 \\
 0 & 0 & b_{52} & b_{53} & b_{54} & h_{55} & N_5 \\
\end{array}
\]

\[
\begin{array}{cccccc}
 N_1 & | & N_2 & | & N_3 & | & N_4 & | & N_5 \\
\end{array}
\]

\(a\)

\[
(h)
\]

\[\text{Figure 2. Cascade scheme matrix realization for continuous (a) and discontinuous (b) base element with five strengtheners}\]

Formula (3) gives the tension force distributions in the reinforcing elements

\[
T_i \equiv N_i^p = \sum_{k=1}^{5} b_{ik} \cdot N_k \quad i = 1, \ldots, 5
\]

The system does not seem difficult to solve even in such a relatively general case. At the same time allowing for the specific structure of the system it can be easily enhanced to higher orders by introducing quite formally some additional lines and columns. Anyone interested can spare some time to experiment with this algorithm.

This model does not take into account the deformability of attachment joints of reinforcement elements and can be useful just for preliminary evaluation of strengthening scheme applicability and effectiveness.

To illustrate the approach to effective strengthening problem solution, consider a simple FEM application.

\[\text{Figure 3. Scheme, general view, left and right ends}\]

The steel element of total length \(2 \cdot (200 + 10 \cdot 300)\) mm under tension force 250 kN applied at the ends having 50 x 20 mm cross section (250000 N / (50x20) = 250 MPa) is modelled in LIRA-SAPR as shown in figure 2. Here the strengthening FRP tapes each of 50x1 mm cross section are fastened through “absolutely” rigid fastening elements symmetrically on both top and bottom sides of the steel element. The fasteners’ interval is 300 mm. This bottom left quarter of the whole structure is loaded at the left free end (200 mm) with tension force of 125 kN. Steel tape cross section here is -50 x 10. All its nodes are secured from vertical displacement and rotation.
The results in table 1 show the tension force values distribution strengtheners. The a, b and c variants relate to continuous steel element. The rest three variants d, e and f are for the steel element with a gap in the span middle (right end on the model).

The internal tensile force in the steel element decreases with each next interval from left to right and in cases of the gap it diminishes to zero.

The internal tensile force values in the FRP elements decrease with each next interval from left to right in cases of continuous steel element and increase in cases of gap.

Table 1. Tension force values (kN) in the strengtheners

| Intervals | a (300 mm) | b (600 mm) | c (900 mm) | d (300 mm) | e (600 mm) | f (900 mm) |
|-----------|------------|------------|------------|------------|------------|------------|
| 1         | 8.242      | 9.514      | 10.059     | 9.576      | 15.647     | 21.084     |
| 2         | 7.900      | 10.059     | 9.397      | 9.288      | 16.912     |            |
| 3         | 7.569      | 9.094      |            | 9.275      |            |            |
| 4         | 7.247      | 8.688      | 9.596      | 9.407      | 19.564     | 24.113     |
| 5         | 6.935      | 8.293      | 9.130      | 10.563     |            |            |
| 6         | 6.633      | 7.910      | 8.512      | 12.217     |            |            |
| 7         | 6.339      | 7.910      | 8.512      |            |            |            |
| 8         | 6.055      | 7.910      |            |            |            |            |
| 9         | 5.779      | 7.910      |            |            |            |            |
| 10        | 5.511      | 43.498     | 37.296     | 125.000    | 125.000    | 125.000    |

The value under each column of the table is the sum of tensile forces in strengtheners that equals to the amount of force reduction in the middle region of the steel tape. Thus the effect of strengthening in cases a, b, c is 68.210/125 = 55%, 35% and 30%. The maximum tensile stress level in strengtheners is 8242/(50 · 1) = 165 MPa, 190 MPa and 201 MPa respectively. In cases d, e and f it reaches the values of 588, 941 and 615 MPa respectively.

Theoretically, any group of strengtheners could be replaced, although maybe not without possible local damage for the reinforced base element, with one strong enough reinforcement element, especially with pretension. But not in case of glue connections.

In [19] some useful expressions were presented for a damaged specimen with discontinuity that should be quoted here. The ultimate value of tension force may be determined from strength conditions of adhesive (4)

\[
P \leq [P] = \frac{b_p}{\beta} \cdot R_{as}, \quad \beta = \sqrt[\gamma - 1]{\frac{1}{G_{st}} + \frac{1}{E_{ta} A_{ta}}} - 1
\]

\[R_{as} - \text{glue material shear design resistance, } R_{aY} - \text{glue material design resistance, } E_s - \text{elasticity modulus of steel, } A_s - \text{reinforced beam cross-section area, } E_p - \text{elasticity modulus of composite tape material, } A_p - \text{tapes cross-section area, } G_s - \text{shear modulus of glue material, } t_s - \text{shear material layer depth, } b_p - \text{tapes width.}

Minimal necessary glue layer length (5)

\[
d \geq d_{\text{min}} = \frac{1}{2\beta} \ln \left( \frac{1}{Y} + 1 \right), \quad Y = \frac{b_p}{\beta} \cdot \frac{R_s}{\tau_{\text{max}}}
\]

The effective (rational) for accepted value of \( P \leq [P] \) glue layer length (further lengthening does not result in decrease of shear stress in glue) may be calculated by a number of iterations using the expression (6)

\[
\tau_{\text{max}} = \frac{P \cdot R_s}{b_p \cdot cth(\beta d)} \leq \frac{R_s}{\beta}
\]

For the above sample the joint parameters are given in table 2.
Table 2. Dimensions and mechanical properties of reinforcement joint

| Steel | FRP |
|-------|-----|
| $b$, mm | $b$, mm |
| $h$, mm | $h$, mm |
| $A$, mm$^2$ | $A$, mm$^2$ |
| $E$, MPa | $E$, MPa |
| $(AE)$, N | $(AE)$, N |
| 50 | 50 |
| 10 | 0.5 |
| 500 | 25 |
| 206000 | 300000 |
| 1.03E+08 | 7500000 |

Glue: 

| $G_a$, MPa | $t_a$, mm | $G_a / t_a$ | $R_{as}$, MPa |
|---------|---------|-------------|-------------|
| 300 | 0.5 | 600 | 14 |

So $\beta = 0.065508$, $[P] = 10686$ N, $d_{min} = 26$ mm calculated for applied $P = 10$ kN. The glue length $d = 26$ mm results in $\tau_{max} = 14.00$ MPa. A further increase in the length of the gluing does not reduce the shear stress in the adhesive below 13 MPa.

Comparing this result with data in table 1 (case $d$) allows to suppose that applying more strengtheners bonded with shorter intervals will do.

As for continuous steel element the strengtheners’ tensile force values in all three cases $a$, $b$ and $c$ do not exceed the ultimate value $[P] = 10.686$ kN.

In a cascade system, with its efficient layout, the total additional resource of the carrying capacity can be quite rationally distributed over a number of individual strengtheners. So, the tension force in each of the strengtheners takes very modest values, that eliminates the necessity for the indispensable use of highly expensive high-strength materials and makes possible to use effectively the glue connections.

3. Bent element reinforcement

In this case the polymer tapes should be located over and/or under the stretched zones of structural element as it was shown above for stretched elements. For instance, in simply supported I-beam under transverse load the tapes may be bonded at the external and/or internal surfaces of the flange in tension.

Let the left half of the simply supported steel beam of total span $2 \cdot (3000 + 100) = 6200$ mm to be under vertical line load 15 N/mm (1.5 ton per m) applied at the top flange (-100x8 mm cross section. Web is -200x4 mm.). At the bottom and top surfaces of bottom flange (-100x8 mm) the polymer reinforcement elements (tapes) are layered with the ends glued to bottom flange surfaces. The total assumed cross-section of each strengthener tape is -200x1 mm.

As a final variant the layout with ten strengthening FRP tapes located one over the other is assumed. Each strengthener may include several polymer tapes. Say, one or two tapes at the bottom surface of bottom flange of beam and one tape at each side of the web at the top surface of bottom flange. The glue fasteners are located here in 300 mm increments. The modelled bonding areas are supposed to be limited by 50 -100 mm length. The first attachment (for the longest 6 m strengthener) starts at 100 mm from the left support.

Here the results for several layout of strengtheners cases are compared. Zero and basic one is a beam without strengthening, the first case is a single six-meter-long strengthenner attached at the ends of beam, second – two strengtheners with 300 mm attachments step, then four, six, eight and ten.

For each layout case the three variants of strengtheners’ material are compared: $E = 200$ GPa (typical in building for CFRP – carbon fiber), $E = 80$ GPa (typical for GFRP – glass fiber) and also $E = 20$ GPa (also possible for glass fiber or some other material). The results obtained in LIRA-SAPR for $E = 200$ GPa are in table 3.
The tensile stress in the bottom flange (table 1) of 298000 N / (100 x 8 mm) = 372 MPa is reduced here to 109 MPa. It makes about (298 – 87) / 2.98 = 71% reduction. In top flange it is about (297 - 261) / 2.97 = 12%. For the max vertical displacements reduction, it makes (71 – 45) / 0.71 = 36%. The tensile stress in the polymer strengtheners varies from 46000 / (200 x 1) = 230 MPa with single tape to maximum 28000 / 200 = 140 MPa in case of ten. Here the strength stops being a dominant property of strengtheners and the application of high strength polymer materials is not inevitably necessary. Therefore, in a cascade reinforcement layout, you can use economical materials of relatively low strength to obtain good result.

For the same layouts but with tapes of E = 80 GPa (say, for glass fiber) the results change (table 4).

For the same layouts but with tapes of E = 20 GPa (glass fiber) the results change (table 5).

Thus the elastic properties of tape including material factor and cross-section area (EA) together with attachments layout become the dominant factors of reinforcement effectiveness.

It should be noted that in a cascade system the failure of individual reinforcement elements or its attachment does not lead to an immediate collapse of the whole structure or its part as it may take
place with a single hardly loaded strengthener. The sequence of these failures has the character of a gradual relatively slow staged process with a series of dynamic effects of low intensity. So the total impact on the structure is rather quasi-static in nature without catastrophic dynamic effects. That is, the dynamic factor at each stage of destruction nearly does not exceed unity and this significantly increases the safety level of the structure.

4. Conclusions
1. An apparently new efficient and cost-effective method for structural elements strengthening is suggested. It must give the motivation to the application of polymeric materials with glue joints for metal structures.
2. The formulation "cascade" reflects the features of the strengthening layout. The tension force in each individual strengthener can be relatively small, thus eliminating the necessity for the indispensable use of expensive high-strength materials and allowing the glue connections application to become effective.
3. The relatively simple FE models let consider a number of design samples to analyze the features of cascade reinforcement system functioning and evaluate the effectiveness of its application for various materials and system parameters. Analytical expressions make it possible to evaluate the glue joints parameters for calculating the cascade strengthening scheme.
4. The cascade layout application increases the safety level of structures.
5. The results suggest the possibility of the effective use of adhesive joints in structural elements of rather high-modulus materials including steel strengthening. The cascade approach is only a foundation for further experimental and theoretical studies to optimize and increase the safety of design solutions in real practical activity.

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