Observation of the semi-muonic decay $D^+ \rightarrow \omega \mu^+ \nu_\mu$
(BESIII Collaboration)

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We report the first observation of the semi-muonic decay $D^{+} \rightarrow \omega \mu^{+} \nu_{\mu}$ using an $e^{+}e^{-}$ collision data sample corresponding to an integrated luminosity of $2.93 \text{ fb}^{-1}$ collected with the BESIII detector at a center-of-mass energy of 3.773 GeV. The absolute branching fraction of the $D^{+} \rightarrow \omega \mu^{+} \nu_{\mu}$ decay is measured to be $B_{D^{+} \rightarrow \omega \mu^{+} \nu_{\mu}} = (17.7^{+1.8}_{-1.1} \text{ stat} \pm 1.1 \text{ syst}) \times 10^{-4}$. Its ratio with
the world average value of the branching fraction of the $D^+ \rightarrow \omega e^+\nu_e$ decay probes lepton flavor universality and it is determined to be $\mathcal{B}_{D^+ \rightarrow \omega e^+\nu_e}/\mathcal{B}_{D^+ \rightarrow \omega e^+\nu_e}^{PDG} = 1.05 \pm 0.14$, in agreement with the standard model expectation within one standard deviation.

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Semi-leptonic (SL) decays of charmed hadrons such as the $D^+$ are theoretically simple to interpret because the effects of the weak and strong interactions can be well separated. The branching fractions (BFs) have been extensively studied using several nonperturbative methods, e.g. lattice QCD, QCD sum rules, and quark models. Experimental determination of these BFs is important to test the different theoretical approaches. In contrast to the well studied semi-electronic $D$ decays, information on Cabibbo-suppressed semi-muonic $D$ decays is very limited at present [1], mainly due to low BFs and high backgrounds. The BFs of the $D^+ \rightarrow \omega \ell^+\nu_{\ell}$ ($\ell = e$ or $\mu$) decays are predicted to be $(1.78-2.46) \times 10^{-3}$, based on the light-front quark model (LFQM) [2], re-chiralized quark model (χUA) [3], covariant confined quark model (CCQM) [4], light-cone QCD sum rules (LCSR) [5], and relativistic quark model (RQM) [6]. Previously, the BF of the semi-electronic decay $D^+ \rightarrow \omega e^+\nu_e$ was measured by the CLEO [7] and BESIII [8] collaborations, but the semi-muonic decay $D^+ \rightarrow \omega \mu^+\nu_{\mu}$ has not been experimentally studied yet. Measurement of the BF of the $D^+ \rightarrow \omega \ell^+\nu_{\ell}$ decay can distinguish between these theoretical predictions, and offer deeper insight into nonperturbative effects in heavy meson decays [9, 10].

This paper reports the first measurement of the BF of $D^+ \rightarrow \omega \mu^+\nu_{\mu}$ based on 2.93 fb$^{-1}$ of data accumulated with the BESIII detector at a center-of-mass energy $\sqrt{s} = 3.773$ GeV [11]. Throughout this paper, charge conjugated channels are implied. The measured BF also probes lepton flavor universality [2-5] by a comparison with the known BF of $D^+ \rightarrow \omega e^+\nu_e$.

The BESIII detector is a magnetic spectrometer [12] located at the Beijing Electron Positron Collider (BEPCII) [13]. The cylindrical core of the BESIII detector consists of a helium-based main drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identifier modules interleaved with steel. The acceptance of charged particles and photons is 93% over 4$\pi$ solid angle. At 1 GeV/$c$, the charged-particle momentum resolution is 0.5%, and the $dE/dx$ resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end-cap) region. The time resolution of the TOF barrel part is 68 ps, while that of the end-cap part is 110 ps. More details about the BESIII detector are described in Ref. [12].

Simulated samples produced with the GEANT4-based [14] Monte Carlo (MC) software, which includes the geometric description [15, 16] of the BESIII detector and the detector response, are used to determine the detection efficiency and to estimate the backgrounds. The simulation includes the beam-energy spread and initial-state radiation (ISR) in the $e^+e^-$ annihilations modeled with the generator KKMC [17]. The inclusive MC samples consist of the production of the $D\bar{D}$ pairs, the non-$D\bar{D}$ decays of the $\psi(3770)$, the ISR production of the $J/\psi$ and $\psi(3686)$ states, and the continuum processes ($e^+e^- \rightarrow q\bar{q}$, ($q = u, d, s$)) incorporated in KKMC [17]. The known decay modes are modeled with EVTGEN [18] using BFs taken from the Particle Data Group (PDG) [1], and the remaining unknown decays from the charmonium states with LUNDCHARM [19]. The final-state radiation from charged final-state particles is incorporated with the PHOTOS package [20]. The $D^+ \rightarrow \omega \mu^+\nu_{\mu}$ decay is simulated by a model with the form parameter quotes from Ref. [8].

At $\sqrt{s} = 3.773$ GeV, the $\psi(3770)$ resonance decays predominantly into $D^0\bar{D}^0$ or $D^+D^-$ meson pairs. The $D^-$ mesons are reconstructed by their hadronic decays to $K^+\pi^0\pi^0$, $K_S^0\pi^+\pi^-$, $K_S^0\pi^+\pi^-$, and $K^+K^0\pi^-$, and $K^+K^-\pi^0$, and referred to as single-tag (ST) $D^-$ mesons. In the sides recoiling against of the ST $D^-$ mesons, the candidate $D^+ \rightarrow \omega \mu^+\nu_{\mu}$ decays are selected to form double-tag (DT) events. The absolute BF of $D^+ \rightarrow \omega \mu^+\nu_{\mu}$ is determined by

$$\mathcal{B}_{SL} = N_{DT}/(N_{ST}^{tot} \cdot \varepsilon_{SL} \cdot \mathcal{B}_\omega \cdot \mathcal{B}_{\pi^0}),$$

where $N_{ST}^{tot}$ and $N_{DT}$ are the ST and DT yields in the data sample, $\mathcal{B}_\omega$ and $\mathcal{B}_{\pi^0}$ are the BFs of the $\omega \rightarrow \pi^+\pi^-\pi^0$ and $\pi^0 \rightarrow \gamma\gamma$ decays, respectively, and $\varepsilon_{SL} = \Sigma_i[(\varepsilon_{DT} \cdot N_{ST}^i)/(\varepsilon_{ST} \cdot N_{ST}^{tot})]$ is the efficiency of detecting the SL decay in the presence of the ST $D^-$ meson. Here $i$ denotes the tag mode, and $\varepsilon_{ST}$ and $\varepsilon_{DT}$ are the efficiencies of selecting the ST and DT candidates, respectively.

The same selection criteria as reported in Refs. [21-25] are used in this analysis. Charged tracks are required to have polar angle ($\theta$) within $|\cos\theta| < 0.93$, and except for those from $K_S^0$ decays, are required to originate from an interaction region defined by $|V_{xyz}| < 1$ cm and $|V_z| < 10$ cm, where $|V_{xyz}|$ and $|V_z|$ refer to the distances of closest approach of the reconstructed track to the interaction point in the $xy$ plane and the $z$ direction (along the beam), respectively.

Particle identification (PID) of charged kaons and pions is implemented with the $dE/dx$ and TOF information. For $\mu$ identification, the EMC information
is also included. For each charged track, the combined confidence levels for the electron, muon, pion, and kaon hypotheses \((CL_e, CL_\mu, CL_\pi, CL_K)\) are calculated. The charged tracks satisfying \(CL_K(\pi) > CL_e(K)\) are identified as kaon (pion) candidates. The muon candidates are required to satisfy \(CL_\mu > 0.001, CL_\mu > CL_e,\) and \(CL_\mu > CL_K,\) and their deposited energy in the EMC is required to be within \((0.15, 0.25)\) GeV to suppress backgrounds misidentified from charged hadrons.

The \(K^0_S\) candidates are selected from pairs of opposite charged tracks with \(|Z_2| < 20\) cm, but without requirements on \(|Z_{xy}|\). The two tracks are designated as pions without PID requirements, constrained to a common vertex and required to have an invariant mass satisfying \(|M_{\pi^+\pi^-} - m_{K^0_S}| < 12\) MeV/c\(^2\), where \(m_{K^0_S}\) is the \(K^0_S\) nominal mass \([1]\). The selected \(K^0_S\) candidate must have a decay length greater than two times the vertex resolution.

Photon candidates are selected using EMC information. It is required that the shower time is within 700 ns of the event start time, the shower energy must be greater than \(25\) (50) MeV in the barrel (end-cap) region \([12]\), and the opening angle between the candidate shower and any charged tracks must be greater than \(10^\circ\).

The \(\pi^0\) candidates are selected from photon pairs with invariant mass within \((0.115, 0.150)\) GeV/c\(^2\). To improve the momentum resolution, a one constraint (1-C) kinematic fit is performed constraining the pair’s \(\gamma\gamma\) invariant mass to the \(\pi^0\) nominal mass \([1]\), and the \(\chi^2_{1-C}\) of the 1-C (mass-constraint) kinematic fit is required to be less than 200.

The energy difference (\(\Delta E\)) and beam-constrained mass (\(M_{BC}\)) are used to select ST \(D^-\) candidates, where

\[
\Delta E \equiv E_D^- - E_{beam}
\]

and

\[
M_{BC} \equiv \sqrt{E_{beam}^2 - |\vec{p}_{D^-}|^2}.
\]

\(E_{beam}\) is the beam energy, and \(\vec{p}_{D^-}\) and \(E_{D^-}\) are the total momentum and energy of the ST candidate calculated in the \(e^+e^-\) rest frame. The \(D^-\) candidates are expected to concentrate around zero in the \(\Delta E\) distribution and around the nominal \(D^-\) mass in the \(M_{BC}\) distribution. For each tag mode, the one with minimum \(|\Delta E|\) is retained. Combinatorial backgrounds in the \(M_{BC}\) distributions are suppressed with a requirement of \(\Delta E \in (-0.055, 0.045)\) GeV for tags containing \(\pi^0\) and \(\Delta E \in (-0.025, 0.025)\) GeV for other tags.

For each tag mode, the ST yield is determined by fitting the \(M_{BC}\) distribution of the candidates surviving all above requirements. In the fit, the \(D^-\) signal is modeled with a shape obtained from an MC simulation convolved with a double Gaussian, and the combinatorial background is described by an ARGUS function \([26]\). The resulting fits to the \(M_{BC}\) distributions for each mode are shown in Fig. 1. Candidates in the \(M_{BC}\) signal region, (1.863, 1.877) GeV/c\(^2\), are kept for further analysis. The ST yields in data and the ST efficiencies for individual tags are shown in Table 1. Summing over the ST yields for all tags gives a total yield of \(N_{ST}^{TOT} = 1522474 \pm 2215\), where the uncertainty is statistical.

![Fig. 1. Fits to the \(M_{BC}\) distributions of the ST candidate events. The dots with error bars are data, the blue solid curves are the fit results, the red dashed curves are the fitted backgrounds, and the pair of red arrows in each sub-figure denote the ST \(D^-\) signal region.](image)

Table 1. Summary of ST yields \((N_{ST}^i)\), ST efficiencies \((\epsilon_{ST}^i)\) and DT efficiencies \((\epsilon_{DT}^i)\) for different tag modes. Uncertainties are statistical only. Efficiencies do not include the BF's of \(K^0_S \rightarrow \pi^+\pi^-, \pi^+\rightarrow \gamma\gamma\), and \(\omega \rightarrow \pi^+\pi^-\pi^0\).

| Tag mode | \(N_{ST}^i\) | \(\epsilon_{ST}^i\) (%) | \(\epsilon_{DT}^i\) (%) |
|----------|-------------|----------------|----------------|
| \(D^+ \rightarrow K^+\pi^-\pi^-\) | 782669 ± 990 | 50.61 ± 0.06 | 4.28 ± 0.05 |
| \(D^+ \rightarrow K^0_S\pi^-\pi^-\) | 91345 ± 320 | 50.41 ± 0.17 | 4.57 ± 0.06 |
| \(D^+ \rightarrow K^+\pi^-\pi^0\) | 251008 ± 1135 | 26.74 ± 0.09 | 1.89 ± 0.04 |
| \(D^+ \rightarrow K^0_S\pi^+\pi^0\) | 215364 ± 1238 | 27.29 ± 0.07 | 2.26 ± 0.06 |
| \(D^+ \rightarrow K^0_S\pi^+\pi^-\) | 113054 ± 889 | 29.29 ± 0.12 | 2.16 ± 0.09 |
| \(D^- \rightarrow K^-\pi^-\) | 69034 ± 460 | 40.87 ± 0.24 | 3.05 ± 0.05 |

The \(D^+ \rightarrow \omega\mu^+\nu_\mu\) candidates are selected from the remaining charged tracks and photons that have not been used for the ST reconstruction. Each candidate must have three good charged tracks and one \(\pi^0\) candidate. If there are multiple neutral pions, the one with the minimum \(\chi^2_{1-C}\) is chosen. One of the three charged tracks must be identified as a muon, and the other two as \(\pi^+\pi^-\). The total charge of the DT event is required to be zero. The \(\omega\) candidates are selected from \(\pi^+\pi^-\pi^0\) combinations, and we require \(|M_{\pi^+\pi^-\pi^0} - m_\omega| < 0.025\) GeV/c\(^2\), where \(m_\omega\) is the \(\omega\) nominal mass \([1]\) and \(M_{\pi^+\pi^-\pi^0}\) is the invariant mass of the \(\pi^+\pi^-\pi^0\) combination. If two \(\pi^+\pi^-\pi^0\) combinations can be formed due to mis-identification between \(\pi^\pm\) and \(\mu^\pm\), the one with \(M_{\pi^+\pi^-\pi^0} \approx m_\omega\) closer to \(m_\omega\) is kept as the \(\omega\) candidate. To suppress backgrounds from the SL decays \(D^+ \rightarrow K^+\bar{K}^0\mu^+\nu_\mu\) with \(K^+\bar{K}_S(\pi^+\pi^-)\pi^0\), we require \(|M_{\pi^+\pi^-} - m_{K^0_S}| > 0.015\) GeV/c\(^2\) and \(|M_{\pi^+\pi^-} - m_{K^0_S}| > 0.015\) GeV/c\(^2\),
where $M_{\pi^+\pi^-}$ and $M_{\pi^+\pi^-}$ are the invariant masses of the $\pi^+\pi^-$ and $\mu^+\pi^-$ combinations, respectively. These requirements correspond to approximately four times the fitted mass resolution of $K_S^0$ around its nominal mass. To suppress backgrounds from the hadronic decays $D^+ \rightarrow K_S^0 (\pi^0 \pi^0) \pi^+ \pi^-$, the invariant mass of the system recoiling against the $D^- \pi^- \pi^+ \pi^-$ combination ($M_{\text{recoil}}^{D^- \pi^- \pi^+ \pi^-}$) is required to be outside the range of (0.45, 0.55) GeV/$c^2$. Here, $\pi^+_{\text{miss}}\pi^-$ denotes that the mass of the muon track has been replaced by the $\pi^+$ mass when calculating $M_{\pi^+ \pi^-}$ and $M_{\text{recoil}}^{D^- \pi^- \pi^+ \pi^-}$. The peaking backgrounds from the hadronic decays $D^+ \rightarrow \omega \pi^+$ and $D^+ \rightarrow \omega \pi^0$ are suppressed by requiring $M_{\omega \pi^+} < 1.5$ GeV/$c^2$ and $E_{\text{extra}}^{\max} \gamma < 0.15$ GeV. Here, $M_{\omega \mu^+}$ is the invariant mass of the $\omega \mu^+$ combination and $E_{\text{extra}}^{\max}$ is the maximum energy of any photon that is not used in the DT selection.

The neutrino of the SL D decay is undetectable by the BESIII detector. The information of the $D^+ \rightarrow \omega \mu^+ \nu_\mu$ decay is inferred from the missing energy ($E_{\text{miss}}$) and the missing momentum ($|p_{\text{miss}}|$) of the observed particles of the DT event calculated in the $e^-e^-$ center-of-mass frame, $U_{\text{miss}} \equiv E_{\text{miss}} - |p_{\text{miss}}|$. Here, $E_{\text{miss}} \equiv E_{\text{beam}} - E_{\omega} - E_{\mu^+}$ and $p_{\text{miss}} \equiv p_{D^+} - p_{\omega} - p_{\mu^+}$, where $E_{\omega}(\mu^+)$ and $p_{\omega}(\mu^+)$ are the energy and momentum of the $\omega(\mu^+)$ candidates, respectively. The $U_{\text{miss}}$ resolution is improved by constraining the $D^+$ energy and momentum with the beam energy and $p_{D^+} = \sqrt{E_{\text{beam}}^2 - m_D^2}$, where $p_{D^+}$ is the unit vector in the momentum direction of the tagged $D^-$ and $m_D$ is the $D^-$ nominal mass [1].

The $U_{\text{miss}}$ distribution of the accepted DT events of data is shown in Fig. 2. An unbinned maximum likelihood fit to this distribution is used to determine the SL decay yield. The shapes of all the components in the fit are obtained from MC simulations, including the SL signal, the peaking background from the hadronic decays $D^+ \rightarrow \omega \pi^+ \pi^0$, and other backgrounds, while their yields are left free. The number of $D^+ \rightarrow \omega \mu^+ \nu_\mu$ decays obtained is $N_{\text{DT}} = 194 \pm 20$, where the uncertainty is statistical.

The fourth column of Table 1 lists the DT efficiencies for individual tag modes. The signal efficiency weighted by the ST yields in data is $\varepsilon_{\text{SL}} = (8.15 \pm 0.07)\%$. Detailed studies show that the momentum and $\cos \theta$ distributions of $\omega$ and $\mu^+$ of data are modeled well by MC simulations. The BF of the $D^+ \rightarrow \omega \mu^+ \nu_\mu$ decay is obtained by Eq. (1) to be

$$B_{D^+ \rightarrow \omega \mu^+ \nu_\mu} = (17.7 \pm 1.8 \pm 1.1) \times 10^{-4},$$

where the first uncertainty is statistical and the second systematic.

With the DT method, most systematic uncertainties arising from the ST side cancel. In the BF measurement, the systematic uncertainties arise from the following sources. The uncertainty in the total ST yield, which is mainly from the uncertainty due to the $M_{\text{BC}}$ fit of the ST candidates, has been studied in Refs. [21] and is assigned as 0.5%. The tracking and PID efficiencies of the pion and muon are studied by analyzing the DT hadronic $D\bar{D}$ events and $e^+e^- \rightarrow \gamma \mu^+\mu^-$ events, respectively. The systematic uncertainties associated with the pion tracking (PID), muon tracking (PID) are assigned to be 0.2% (0.3%) and 0.3% (0.3%), respectively. The $\pi^0$ efficiency, including effects of photon selection, the 1-C kinematic fit, and the mass window, is studied with DT hadronic $D\bar{D}$ decays [21, 22], and a systematic uncertainty of 0.7% is assigned to each $\pi^0$. The uncertainty of the $E_{\text{extra}}^{\max} \gamma$ requirement is estimated to be 4.4% by analyzing the DT $D\bar{D}$ events of $D^+ \rightarrow \omega (\pi^+\pi^-\pi^0) e^+\nu_e$, $D^+ \rightarrow K_S^0 (\pi^+\pi^-\pi^0) e^+\nu_e$, $D^+ \rightarrow K_S^0 (\pi^+\pi^-) e^+\nu_e$, and $D^+ \rightarrow K_S^0 (\pi^+\pi^-) \pi^+\pi^0$. The uncertainties due to the $M_{\omega \mu^+}$ requirement and the $K_S^0$ rejection ($M_{\pi^+\pi^-}, M_{\pi^+\pi^-}, and M_{\text{recoil}}^{D^- \pi^- \pi^+ \pi^-}$) are evaluated by repeating measurements varying the nominal requirements by $\pm 0.05$ GeV/$c^2$ and $\pm 0.005$ GeV/$c^2$, respectively, and they are found to be negligible. The uncertainty originating from the $U_{\text{miss}}$ fit is assigned to be 3.4%, which is estimated with alternative fit ranges and signal and background shapes. The uncertainty due to the limited MC size is 0.5%. The uncertainty in the MC model is assigned to be 2.5%, by comparing our nominal DT efficiency with one obtained using an ISGW model [9]. All these systematic uncertainties are assumed to be independent, and their quadratic sum gives a total systematic uncertainty of 6.3%.

To summarize, by analyzing the data sample with an integrated luminosity of 2.93 fb$^{-1}$ collected at $\sqrt{s} = \ldots$
3.773 GeV with the BESIII detector, we determine
the BF of the SL decay $D^+ \to \omega \mu^+ \nu_\mu$ for the first
time. Table 2 shows the comparison of our BF to
various theoretical calculations of $D^+ \to \omega \mu^+ \nu_\mu$ decay.
Our BF is consistent with the predicted values based
on the LFQM, CCQM, and LCSR methods [2, 4, 5],
but differs from those based on the $\chi$UA and RQM
methods [3] by 2.5$\sigma$ and 1.5$\sigma$, respectively. Combining
the $B_{D^+\to\omega\mu^+\nu_\mu}$ measured in this work with the world
average $B_{D^+\to\omega\mu^+\nu_\mu} = (16.9 \pm 1.1) \times 10^{-4}$ [1], we obtain the BF ratio to be
$B_{D^+\to\omega\mu^+\nu_\mu}/B_{D^+\to\omega e^+\nu_e} = 1.05\pm0.14$,
which agrees with the SM prediction (0.93-0.96) [2-6]
within uncertainties.

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Table 2. Comparison of the BF$s between $D^+ \to \omega e^+\nu_e$ and $D^+ \to \omega \mu^+\nu_\mu$.

|                      | CCQM [2] | $\chi$UA [3] | LFQM [4] | LCSR [5] | RQM [6] | Measurement |
|----------------------|----------|---------------|----------|----------|---------|-------------|
| $B_{D^+ \to \omega \mu^+\nu_\mu}$ ($\times 10^{-4}$) | 17.8     | 22.9          | 20 $\pm$ 2 | 18.5$^{+1.9}_{-1.3}$ | 20.8    | 17.7 $\pm$ 1.8 $\pm$ 1.1 |
| $B_{D^+ \to \omega e^+\nu_e}$ ($\times 10^{-4}$)  | 18.5     | 24.6          | 21 $\pm$ 2 | 19.3$^{+2.0}_{-1.3}$ | 21.7    | 16.9 $\pm$ 1.1 $\pm$ 1 |
| $\frac{B_{D^+ \to \omega \mu^+\nu_\mu}}{B_{D^+ \to \omega e^+\nu_e}}$ | 0.96     | 0.93          | 0.95      | 0.96     | 0.96    | 1.05 $\pm$ 0.14 |