Level-Triggered Harvest-then-Consume Protocol with Two Bits or Less Energy State Information
Sudarshan Guruacharya, Vandana Mittal, and Ekram Hossain

Abstract—We propose a variation of harvest-then-consume protocol with low complexity where the harvest and consume phases change when the battery energy level reaches certain thresholds. The proposed protocol allows us to control the possible energy outage during consumption phase. Assuming that the battery is perfect and that the energy arrival is a renewal process, we analyze the duty cycle and the operating cycle speed of the protocol. The proposed protocol also allows for limited battery energy state information. The cases when the system has two-bits, one-bit, and zero-bit of battery energy state information are studied in detail. Numerical simulations verify the obtained formulas.

Index Terms—Energy-harvesting wireless communication, harvest-then-consume, limited energy state information (ESI)

I. INTRODUCTION

Harvest-then-consume protocol is an instance of time-switching architecture of energy harvesting communication, where harvest and consume phases alternate between each other [1]. An obvious drawback of harvest-then-consume protocol is the delay induced by the harvest phase. Despite the drawback, the harvest-then-consume protocol has been studied in the context of relay communication [2], [3], cognitive networks [4], sensor networks [5]–[7], and wireless information and power transfer (WIPT) [8]–[12] where the alternation between the harvest and consume phase lends itself to the half-duplex nature of these applications. In these work, the size of the time frame of a harvest-consume cycle is assumed to be fixed, and the duty cycle is assumed as the control variable, with the goal of maximizing the system throughput. However, this can result in energy outage when the harvested energy is insufficient to power the consumer’s application.

In this letter, we propose a variation of harvest-then-consume protocol with low complexity where the phase change happens when the battery energy level crosses certain threshold; hence the qualifier level-triggered. The protocol we describe has a built-in guarantee over energy outage. A similar protocol was used for empirical work in [13] for a time slotted process. The current work is based on our prior work [14] where we investigated the recharge time distribution of an energy harvesting system. Being level-triggered also means that only finite bits are needed to monitor the changes in the battery level. This greatly simplifies the system design, since the circuit needed to monitor the battery’s energy state can be reduced or done away with completely. Also, since the goal of the proposed protocol is to ensure the energy sufficiency for a given time frame, complicated optimization problem over finite or infinite time horizon is avoided.

We will analyze the operating speed and the duty cycle of the proposed protocol, and establish upper bounds on performance. While these metrics are independent of the purpose of the energy consumption, nevertheless, we will assume that the energy is used to transmit messages in a wireless communications system. We will analyze the cases when two bits, one bit, and zero bit of energy state information (ESI) are available. The case of zero bit ESI is interesting in itself, since for this case the harvest and consume durations are deterministic. Note that such a simple harvest-then-consume protocol will be particularly suitable for low-complexity energy-harvesting wireless sensor nodes. To the best of our knowledge, we are not aware of any prior work that deals with limited ESI.

II. SYSTEM MODEL

Let $U(t)$ be the energy level of the battery at any given time $t$. At any given time, the battery can be in one of the three possible states: (i) Battery is empty, $U(t) = 0$, (ii) Battery is not empty but the energy level is below some required threshold $u$, such that $0 < U(t) < u$, (iii) Battery energy level is above the required threshold, $U(t) > u$. These three possible energy states can be represented by just two bits of information. Thus, we have a system with limited ESI.

In this letter, we will consider the following simple version of the harvest-then-consume protocol:

- The system switches off and goes into harvest phase when the battery is empty, i.e. after $U(t) = 0$.
- The system switches on and goes into consumption phase after the battery has acquired $u$ amount of energy, i.e. after $U(t) > u$.

Here the harvest and consume phases are triggered when the battery attains certain fixed levels, hence the name level triggered.

For simplicity, during the consumption phase we will assume that the rate of energy consumption (or the consumed power) $p$ is constant. Let $\tau_c = \inf\{t : U(t) < u, U(0) = 0\}$ be the recharge duration and $\tau_d = \inf\{t : U(\tau_c + t) = 0\}$ be the discharge duration. Then, the total harvest-consume cycle duration is $T = \tau_c + \tau_d$. The performance metric of the harvest-then-consume protocol is taken to be

$$
\omega = \frac{1}{E[T]} \quad \text{and} \quad \rho = \frac{E[\tau_d]}{E[T]}.
$$

(1)
Here, $\omega$ represents the cycle speed at which the protocol can operate and has the units of Hertz. The other metric, $\rho$, is the duty cycle, such that $\rho \in (0, 1)$. It represents the fraction of time that the system does some useful work and is a dimensionless number. In this paper, we will derive formulas for these two metrics of interest.

During the harvest phase, we model the recharge process for perfect battery (i.e. no self-discharge) as

$$U(t) = \sum_{i=1}^{NA(t)} X_i,$$

where $NA(t) = \min\{k : A_0 + A_1 + \cdots + A_k \leq t\}$ counts the number of energy arrivals, $A_{i \geq 1}$ is the inter-arrival time of energy packets, $A_0$ is the residual time, and $X_i$ is the energy packet size. The energy arrival is assumed to be a delayed renewal process. The $\{A_{i \geq 1}\}$ and $\{X_i\}$ are assumed to be independent and identically distributed with finite mean and variance. Also, we assume that $\{A_i\}$ and $\{X_i\}$ are independent of each other. Lastly, we assume that the joint distribution of the random vectors $\{(A_i, X_i)\}$ are identically distributed as $(A, X)$. For notational convenience, we will denote $\lambda = 1/E[A]$ and $X = E[X]$.

With less than two bits of ESI, the system may not be able to tell if the battery has the desired energy level. As such, we need to impose a statistical guarantee on the energy outage. Let the energy outage constraint at the switching time $t_c$ be

$$P(U(t_c) \leq u) = \theta_1,$$

where $\theta_1 \in (0, 1)$. Here $t_c$ is a fixed duration of recharge, after which the system is turned on for the consume phase.

Given the recharge process in (2), the mean and variance of $\tau_c$ for large $u$ are, respectively [14] Eqns. (8), (9),

$$E[\tau_c] \sim C_1 + \frac{u}{\lambda X}, \quad \text{and} \quad \text{Var}[\tau_c] \sim C_2 + \frac{\sigma^2 u}{X^2}.$$

Here the constants $C_1 = (1 + \lambda^2 \sigma_A^2)/2\lambda$ and $C_2 = \frac{\mu^3}{3\sigma_A^2} - \frac{(\sigma^2 + \sigma_A^2)^2}{2p\sigma_A}$, where $\mu$ is the third moment of $A$; $\sigma_A^2$ and $\sigma_X^2$ are the variances of $A$ and $X$, respectively; and $\gamma^2 = \lambda^{-2}\sigma_X^2 + \sigma_A^2 X^2$. For large $u$, we can neglect the constant term and simply write $E[\tau_c] \sim u/\lambda X$ and $\text{Var}[\tau_c] \sim \gamma^2 u/X^2$. Thus, invoking the central limit theorem, for large $u$ the distribution of $\tau_c$ is given by [14] Eqn. (11),

$$P(\tau_c(u) \leq t_c) \approx \Phi \left( \frac{t_c - E[\tau_c]}{\sqrt{\text{Var}[\tau_c]}}, \right)$$

where $\Phi(\cdot)$ is the standard normal distribution.

If the harvested energy is used to transmit information in a narrow band channel, then in the transmit phase, assuming a point-to-point wireless communications system with transmit power $p$, flat fading channel gain $g$, and additive white Gaussian noise with power $N$, we have the signal-to-noise ratio (SNR) given by $Z = gp/N$. We assume that the transmitter always has data to transmit. Let the SNR outage constraint be

$$P(Z \leq \zeta) = \theta_2,$$

where $\theta_2 \in (0, 1)$ while $\zeta$ is the threshold SNR required for correct decoding of the message signal. Substituting the expression for SNR in (4), we have $P \left( g \leq \frac{\zeta N}{p} \right) = \theta_2$. Since

$$P \left( g \leq \frac{\zeta N}{p} \right) = F_G \left( \frac{\zeta N}{p} \right),$$

where $F_G$ is the distribution of $g$, we can solve for $p = \frac{\zeta N}{F_G^{-1}(\theta_2)}$.

### III. With Two Bits of Energy State Information

With two bits of ESI, the system can know when the battery is empty and when it has sufficient energy. The duration that the recharge process takes to cross the desired energy level $u$ is $\tau_c$, where $\tau_c$ is a random variable. Once the required energy level has been crossed, the system is turned on. The time it takes to fully discharge the battery is $\tau_d = U(\tau_c)/p$; and the total charging and discharging time is $T = \tau_c + \frac{1}{p}$. Here again $T$ is a random variable. Also, at the level crossing time $\tau_c$, $U(\tau_c) = u + V$, where $V \geq 0$ is the value by which $U(\tau_c)$ overshoots the required energy level $u$. Since $U(t)$ is renewal process, the overshoot is given by the stationary residual density of $X$, assuming $u$ is large, as $f_V(v) = X^{-1}[1 - F_X(v)]$. Thus, we have $T = \tau_c + (u + V)/p$. Here, $\tau_c$ and $V$ are independent of each other, thus the distribution of their sum can be obtained by the convolution of their distributions. We can find the mean value of $U$ as $E[U(\tau_c)] = u + C_3$, where $C_3 = (\sigma_X^2 + X^2)/2X$ is the mean of $V$ which does not depend on $u$. Also, using [1] and the mean of $U(\tau_c)$, we have the mean of $T$ as

$$E[T] = \left( \frac{1}{\lambda X} + \frac{1}{p} \right) u + C_1 + C_3.$$

In general, the duty cycle $\rho = \frac{E[\tau_d]}{E[T]} = \frac{E[U(\tau_c)]}{pE[T]}$ and the system speed $\omega = 1/E[T]$ are

$$\rho = \frac{u + C_3}{1 + \frac{u}{\lambda X} + \frac{p}{pC_1 + C_3}},$$

$$\omega = \left( \frac{1}{p} + \frac{1}{\lambda X} \right)^{-1}.$$ 

As a special case, as $u \to 0$, we have the duty cycle as $\rho = C_3/(pC_1 + C_3)$, and the system’s cycle speed as $\omega = p/(pC_1 + C_3)$. This is also the fastest speed that the system can attain; thus $\omega \leq p/(pC_1 + C_3)$.

Likewise, as $u \to \infty$, we can ignore the constant terms.

Thus, $E[U(\tau_c)] \sim u$ and $E[T] \sim \left( \frac{1}{p} + \frac{1}{\lambda X} \right) u$. In other words, larger the required energy, more we need to wait. Also, the duty cycle is $\rho \sim \frac{\lambda X}{\lambda X + p}$, and the system’s cycle speed is $\omega \sim \frac{p}{\lambda X}$. Interestingly, this limiting value of $\rho$ is not equal to its value at $u = 0$. Setting $u = 0$ represents an opportunistic scheme where the harvested energy is immediately consumed. When $u = 0$, we have $U(\tau_c) = X$ and $\tau_d = U(\tau_c)/p = X/p$. Hence, $E[\tau_d] = X/p$. Similarly, $E[T] = E[\tau_c] + E[\tau_d] = C_1 + X/p$. Therefore, $\rho = (1 + pC_1/X)^{-1}$ and $\omega = (C_1 + X/p)^{-1}$. For small values of $u$, these formulas for $\rho$ and $\omega$ will not be accurate, since we assume stationary residual distribution for $A_0$ and $V$, which is valid only for large $u$. Note that when $u = 0$, only one bit is required to check the battery status; thus this analysis is valid for Section [IV] as well.

---

1Here, $f(x) \sim g(x)$ if and only if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 1$. 

---
IV. WITH ONE BIT ENERGY STATE INFORMATION

Here, we assume that the system can discern whether or not the battery is empty. As such, during the charging process, only statistical guarantee \((3)\) can be given for the energy outage. Using \((4)\) and \((5)\) in \((3)\), the switching time \(t_c\) is given by

\[
 t_c = C_1 + \Phi^{-1}(1 - \theta_1) \sqrt{C_2 + \gamma^2 u / \lambda^2} + \frac{u}{\lambda \lambda X}. \tag{9}
\]

The minima at \(u = 0\) is \(t_{c, \text{min}} = C_1 + \Phi^{-1}(1 - \theta_1) \sqrt{C_2}\), which gives the minimum waiting time for an energy packet to arrive.

Once the system is switched on at \(t_c\), the duration it takes for the battery to completely discharge is \(\tau_d = U(t_c)/p\). We can find the distribution for the discharge duration as

\[
P(\tau_d \leq t_d) = P(U(t_c) \leq pt_d) = \Phi \left(\frac{pt_d - \lambda X t_c}{\gamma \lambda^3/2 \sqrt{t_c}}\right). \tag{10}
\]

Thus, the mean discharge time is \(E[\tau_d] = \lambda X t_c / p\).

Since the system can detect when the battery is empty, we can start the recharging processes when the battery is completely discharged. Thus, the cycle duration is \(T = t_c + \tau_d\). The distribution of \(T\) is

\[
P(T \leq t) = P(\tau_d \leq t - t_c) = \Phi \left(\frac{pt - (p + \lambda X) t_c}{\gamma \lambda^3/2 \sqrt{t_c}}\right). \tag{11}
\]

Thus, the mean of \(T\) is \(E[T] = (p + \lambda X) t_c / p\). Hence, the duty cycle and the cycle speed are

\[
\rho = \frac{\lambda X}{p + \lambda X}, \quad \omega = \frac{p}{(p + \lambda X) t_c}. \tag{12}
\]

Interestingly, since the maximum value of \(\omega\) is obtained when \(t_c\) is minimum at \(u = 0\), we have the upper bound

\[
\omega \leq \frac{p}{(p + \lambda X)(C_1 + \Phi^{-1}(1 - \theta_1) \sqrt{C_2})}. \tag{13}
\]

If we neglect the constant term and the term with square root for \(t_c\), then we have \(\omega = \frac{p}{p + \lambda X} \sim \frac{p}{u}\).

V. NO ENERGY STATE INFORMATION

In this case, since we do not have any information on the battery state, we need to rely on the statistical constraints \((3)\). Unlike other cases, here \(T\) is a control parameter. Let the harvest duration be \(t_c\), as given by \((9)\), and the consumption duration be \(T - t_c > 0\). Here, we do not concern ourselves with complete discharge of the battery. Rather, we focus on the consumption of fixed \(u\) amount of energy within the consumption phase. Once this amount of energy is consumed, the system reverts to the harvest phase. Thus, some excess energy may remain in the battery after the consume phase. For simplicity, we will neglect the excess energy in the analysis. This is equivalent to assuming that any excess energy after a complete harvest-consume cycle is wasted or dissipated unproductively.

Since the consumed power is maintained at fixed \(p\), the consumed energy is \(u = pt_d = p\rho T\). Substituting this value of \(u\) in \((9)\), we have

\[
1 - \rho = d + \sqrt{c + bp + ap}, \quad a = p/\lambda X, \quad b = p(\gamma \Phi^{-1}(1 - \theta_1))^2/\lambda X^3, \quad c = C_2(\Phi^{-1}(1 - \theta_1)/T)^2, \quad d = C_1/T. \tag{14}
\]

Here, all the constants \(a, b, c, d \geq 0\). Now, solving for \(\rho\), we obtain

\[
\rho = \frac{2(1 + a)(1 - d) + b \pm \sqrt{b^2 + 4(1 + a)((1 + a)c + b(1 - d))}}{2(1 + a)^2}. \tag{15}
\]

When \(T \to \infty\), \(b \to 0\), \(c \to 0\), and \(d \to 0\), thus \(\rho \to 1/(1 + a)\), regardless of the value of \(\theta_1\). Thus,

\[
\rho \approx \frac{\lambda X}{p + \lambda X}, \quad \text{for large } T. \tag{16}
\]

A. Feasibility Conditions

For the solution \(\rho\) to be feasible, \(\rho\) should be within the interval \((0, 1)\). Thus we need to check the conditions when \(\rho > 0\) and \(\rho < 1\).

For \(\rho > 0\), from \((13)\), after some simplification, we obtain the condition \((a + 1)^2(c - (d - 1)^2) > 0\). Since \(a + 1\) is always positive,

\[
c > (d - 1)^2. \tag{17}
\]

Substituting the definitions of \(c\) and \(d\), we find that this condition reduces to \(T > t_{c, \text{min}}\), where \(t_{c, \text{min}}\) is as given in Section IV. If the terms \(c\) and \(d\) were neglected, then the condition would have reduced to \(a + 1 > 0\), which is always true.

For \(\rho < 1\), from \((13)\), after some simplification, we obtain

\[
(a + d)^2 > b + c. \tag{18}
\]

Substituting the expressions for \(b, c,\) and \(d\), results in the condition \(f(T) > 0\), where \(f(T)\) is a quadratic equation in terms of \(T\), given as

\[
f(T) = KT^2 + LT + M,
\]

where \(K = a^2, L = 2aC_1 + p\Phi^{-1}(1 - \theta_1)^2/\lambda X^3,\) and \(M = C_1^2 - C_2(\Phi^{-1}(1 - \theta_1))^2\). The condition \(f(T) > 0\) is satisfied for any \(T\) if the discriminant of \(f(T)\) is negative. That is, if \(L^2 - 4KM < 0\). When this is not the case, we have \(T > T_+\), where \(T_+\) is the largest root of \(f(T) = 0\) given by \(T_+ = (-L + \sqrt{L^2 - 4KM})/2K\). Had we neglected \(c\) and \(d\), the condition \((16)\) would have simplify to \(a^2 > b\); and substituting the expression for \(a\) and \(b\), and solving for \(T\) would have given us \(T > \frac{4K^2}{pX}[\Phi^{-1}(1 - \theta_1)]^2\).

Hence, we have the lower bound on \(T\) as \(T > \max(t_{c, \text{min}}, T_+)\) and the upper bound on \(\omega\) as

\[
\omega < \frac{1}{\max(t_{c, \text{min}}, T_+)}. \tag{19}
\]

B. Possible Variation

If proper discharge is to be ensured for fixed cycle period \(T\), then allowed discharge time is \(t_d = T - t_c\). Thus, we have from \((10)\)

\[
P(\tau_d \leq t_d) = \Phi \left(\frac{pt - (p + \lambda X) t_c}{\gamma \lambda^3/2 \sqrt{t_c}}\right). \tag{20}
\]

Let the probability that battery is fully discharged by time \(t_d\) be constrained at \(P(\tau_d \leq t_d) = \theta_3\). Then, we have

\[
T = \left(1 + \frac{\lambda X}{p}\right) t_c + \frac{\gamma \lambda^3/2}{p} \sqrt{t_c \Phi^{-1}(\theta_3)}. \tag{21}
\]
If we ignore the square root terms, for large $u$, we have the approximation $T \sim \left( \frac{1}{\lambda X} + \frac{1}{p} \right) u$; and similarly, the duty cycle $\rho = 1 - t_c/T$ is $\rho \sim \frac{\lambda X}{\lambda X + 1/p}$. Likewise, the cycle speed of the system is $\frac{1}{T} \sim \frac{1}{u} \left( \frac{\lambda X}{\lambda X + 1/p} \right) = \frac{\rho}{u}$.

VI. Numerical Verification

In this section, we verify the obtained formulas with Monte Carlo simulations for the case of two-bit and one-bit ESI. In Fig. 1a and Fig. 1b, we plot the duty cycle $\rho$ and operating cycle speed $\omega$ with respect to the threshold energy level $u$. For the two-bit case, since the time $\tau_c$ and $\tau_d$ are known, it is easy to calculate the charging and discharging time and hence the duty cycle and operating frequency. However, for one bit case, charging time $\tau_c$ is calculated using (9) and discharge time $\tau_d$ is calculated using $U(t_c)/p$. Both the energy packet size and energy arrival are assumed to follow a uniform distribution $U(0, 2)$, with unit mean and variance $1/3$. We assume that the power consumption $\rho = 2$ and $\theta_1 = 0.1$. For a given $u$, 10,000 simulations are run to obtain a single value of $\rho$ and $\omega$.

The theoretical expressions for $\rho$ and $\omega$ for two bit ESI are given by equations (6) and (7), respectively, and for one bit ESI, $\rho$ and $\omega$ are given by equations (10) and (11), respectively. From Fig. 1a and Fig. 1b, we see that both $\rho$ and $\omega$ do not vary much for higher values of $u$. The results from the simulations match closely with the theoretical predictions.

VII. Conclusion

A level-triggered harvest-then-consume protocol has been proposed. The duty cycle and operating cycle speed of the system have been derived for cases when the system has two-bits, one-bit, and zero-bit of battery energy state information. Upper bounds on the system’s speed have been obtained. Monte Carlo simulations have been performed to verify the obtained formulas.

REFERENCES

[1] M.-L. Ku, et al., “Advances in energy harvesting communications: Past, present, and future challenges,” IEEE Commun. Surveys & Tutorials, no. 2, vol. 18, pp. 1384–1412, 2016.
[2] I. Krikidis, S. Timotheou, and S. Sasaki, “RF energy transfer for cooperative networks: Data relaying or energy harvesting?,” IEEE Commun. Lett., vol. 16, no. 11, pp. 1772–1775, Nov. 2012.
[3] He Chen et al., “Harvest-then-cooperate: Wireless-powered cooperative communications,” IEEE Trans. Signal Process., vol. 63, no. 7, pp. 1700–1711, Apr. 2015.
[4] C. Wu et al., “Energy utilization efficient frame structure for energy harvesting cognitive radio networks,” IEEE Wireless Commun. Lett., vol. 5, no. 5, pp. 488–491, Oct. 2016.
[5] S. Park, et al., “Optimal mode selection for cognitive radio sensor networks with RF energy harvesting,” Proc. IEEE 23rd Int. Symp. Pers. Indoor Mobile Radio Commun. (PIMRC), pp. 2155–2159, Sep. 2012.
[6] N. Jain and V.A. Bohara, “Energy harvesting and spectrum sharing protocol for wireless sensor networks,” IEEE Wireless Commun. Lett., vol. 4, no. 6, pp. 697-700, Jun. 2015.
[7] W. Liu, et al., “Energy harvesting wireless sensor networks: Delay analysis considering energy costs of sensing and transmission,” IEEE Trans. Wireless Commun., vol. 15, no. 7, pp. 4635–4650, Jul. 2016.
[8] S. Luo, R. Zhang, and T.J. Lim, “Optimal save-then-transmit protocol for energy harvesting wireless transmitters,” IEEE Trans. Wireless Commun., vol. 12, no. 3, pp. 1196–1207, Feb. 2013.
[9] H. Ju and R. Zhang, “Throughput maximization in wireless powered communication networks,” IEEE Trans. Wireless Commun., vol. 13, no. 1, pp. 418–428, Jan. 2014.
[10] T.A. Zewde and M.C. Gursoy, “Wireless-powered communication under statistical quality of service constraints,” IEEE Int. Conf. Commun. (ICC), 22-27 May 2016.
[11] Z.H. Velkov, et al., “Wireless networks with energy harvesting and power transfer: Joint power and time allocation,” IEEE Signal Process. Lett., vol. 23, no. 1, pp. 50–54, Jan. 2016.
[12] F. Zhao, L. Wei, and H. Chen, “Optimal time allocation for wireless information and power transfer in wireless powered communication systems,” IEEE Trans. Veh. Tech., vol. 65, no. 3, pp. 1830–1835, Mar. 2016.
[13] P. Lee, et al., “Empirical modeling of a solar-powered energy harvesting wireless sensor node for time-slotted operation,” IEEE Wireless Commun. Network. Conf. (WCNC), 28-31 March 2011.
[14] S. Gurucharya, V. Mittal, and E. Hossain, “On the battery recharge time in a stochastic energy harvesting system,” IEEE Wireless Commun. Lett., submitted, 2017. Available [Online]: https://arxiv.org/abs/1706.03183
[15] F.E. Beichelt and L.P. Fatti, Stochastic Processes and Their Applications. CRC Press, 2002.