SCATTERING BY A NIHILITY SPHERE

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ABSTRACT: On interrogation by a plane wave, the back-scattering efficiency of a nihility sphere is identically zero, and its extinction and forward-scattering efficiencies are higher than those of a perfectly conducting sphere.

Keywords: back-scattering efficiency; extinction efficiency; forward-scattering efficiency; nihility

1 INTRODUCTION

The concept of nihility as an electromagnetic medium has emerged [1] from the rather extraordinary developments on negatively refracting materials during this decade [2, 3]. Whereas the relative permittivity and relative permeability of vacuum are \( \varepsilon_r = \mu_r = 1 \), and those of anti-vacuum are \( \varepsilon_r = \mu_r = -1 \), those of nihility are \( \varepsilon_r = \mu_r = 0 \). The so-called perfect lens of Pendry [4] is made of anti-vacuum [5], and any perfect lens in the present context is required to simulate nihility [5, 6].

Although quite some attention has been devoted to the electromagnetic response characteristics of anti-vacuum (or some approximation thereof), nihility has been neglected in comparison [7, 8]. Reflection and refraction of plane waves due to nihility half-spaces has recently been reported [9], and so has the plane-wave response of nihility slabs [5, 6]. Along the same lines,
this communication focuses on the plane–wave response of a nihility sphere. An \( \exp(-i\omega t) \) time–dependence is implicit in the following sections.

## 2 THEORY

Consider the spherical region \( r < a \) occupied by nihility, whereas the region \( r > a \) is vacuous. Without loss of generality, the incident plane wave is taken to be linearly polarized and traveling along the \(+z\) axis; thus,

\[
\mathbf{E}_{\text{inc}}(r) = E_0 \hat{u}_x \exp(ik_0 z),
\]

where \( E_0 \) is the amplitude, \( k_0 \) is the free–space wavenumber, and \( \hat{u}_x \) is the unit vector parallel to the \(+x\) axis.

As is commonplace, the incident plane wave is represented in terms of vector spherical harmonics \( M_{\sigma mn}^{(j)}(w) \) and \( N_{\sigma mn}^{(j)}(w) \) [10, 11] as follows:

\[
\mathbf{E}_{\text{inc}}(r) = E_0 \sum_{n=1}^{\infty} i^n \frac{2n + 1}{n(n + 1)} \left[ M_{\sigma 1n}^{(1)}(k_0 r) - i N_{\sigma 1n}^{(1)}(k_0 r) \right].
\]

The scattered field is also stated in terms of vector spherical harmonics as [11]

\[
\mathbf{E}_{\text{sc}}(r) = E_0 \sum_{n=1}^{\infty} i^n \frac{2n + 1}{n(n + 1)} \left[ ia_n N_{\sigma 1n}^{(3)}(k_0 r) - b_n M_{\sigma 1n}^{(3)}(k_0 r) \right], \quad r \geq a,
\]

where

\[
a_n = \frac{\epsilon_r j_n(N\xi) \psi_n^{(1)}(\xi) - j_n(\xi) \psi_n^{(1)}(N\xi)}{\epsilon_r j_n(N\xi) \psi_n^{(3)}(\xi) - h_n^{(1)}(\xi) \psi_n^{(1)}(N\xi)}
\]

and

\[
b_n = \frac{\mu_r j_n(N\xi) \psi_n^{(1)}(\xi) - j_n(\xi) \psi_n^{(1)}(N\xi)}{\mu_r j_n(N\xi) \psi_n^{(3)}(\xi) - h_n^{(1)}(\xi) \psi_n^{(1)}(N\xi)};
\]

\( j_n(\xi) \) and \( h_n^{(1)}(\xi) \) are the spherical Bessel function and the spherical Hankel function of the first kind;

\[
\psi_n^{(1)}(w) = \frac{d}{dw} \left[ w j_n(w) \right]
\]

and

\[
\psi_n^{(3)}(w) = \frac{d}{dw} \left[ w h_n^{(1)}(w) \right];
\]
\[ N = \sqrt{\varepsilon_r \mu_r} \text{ and } \xi = k_0 a; \text{ and } \varepsilon_r \text{ and } \mu_r \text{ are, respectively, the relative permittivity and the relative permeability of the matter occupying the region } r < a. \]

Taking the limits \( \varepsilon_r \to 0 \) and \( \mu_r \to 0 \) for the scattering medium (i.e., nihility), we obtain

\[ a_n = b_n = \frac{j_n(\xi)}{h_n^{(1)}(\xi)}. \]  

(8)

The equality of the coefficients \( a_n = b_n \forall n \) is remarkable, and possibly unique to nihility spheres.

3 DISCUSSION

Figure 1 contains a plot of the extinction efficiency

\[ Q_{ext} = \frac{2}{\xi^2} \sum_{n=1}^{\infty} (2n + 1) \Re(a_n + b_n) \]  

(9)

of the nihility sphere as a function of its normalized radius \( \xi \). The overall profile is similar to that for a perfectly conducting sphere [13], which is also shown in the same figure, but extinction by the nihility sphere is larger than that by a perfectly conducting sphere. Furthermore, the peak extinction by a nihility sphere occurs at a larger value of \( \xi \) (\( \approx 2.981 \)) than by a perfectly conducting sphere (\( \xi \approx 1.209 \)).

Calculations associated with (4) and (5) show that no electromagnetic field exists inside nihility spheres. Hence, there is no absorption, and the extinction efficiency equals the total scattering efficiency [11]

\[ Q_{sca} = \frac{2}{\xi^2} \sum_{n=1}^{\infty} (2n + 1) \left( |a_n|^2 + |b_n|^2 \right). \]  

(10)

The forward–scattering efficiency

\[ Q_{forw} = \frac{1}{\xi^2} \left| \sum_{n=1}^{\infty} (2n + 1) (a_n + b_n) \right|^2 \]  

(11)

of a nihility sphere is plotted in Figure 2 as a function of \( \xi \), and compared with that of a perfectly conducting sphere. That of the nihility sphere is higher.
The most interesting feature of the plane-wave response of a nihility sphere is its back-scattering efficiency [12]

\[
Q_{\text{back}} = \frac{1}{\xi^2} \sum_{n=1}^{\infty} (-)^n (2n + 1) (b_n - a_n)^2.
\] (12)

By virtue of (8),

\[
Q_{\text{back}} \equiv 0
\] (13)

for a nihility sphere; of course, \(Q_{\text{back}} \neq 0\) for perfectly conducting spheres [12, 13].

Equation (13) is a remarkable result, because it implies that the probability of detection of a nihility sphere by a monostatic radar system is very low. This result would not change even if the ambient (isotropic) medium were to have relative permittivity and relative permeability other than unity. This conclusion can be understood by realizing that nihility is impedance–matched to any isotropic, achiral, dielectric–magnetic medium [14].

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Figure 1: Extinction efficiency $Q_{\text{ext}}$ of a sphere as a function of its normalized radius $\xi = k_0 a$. 
Top: nihility sphere. Bottom: perfectly conducting sphere.
Figure 2: Forward-scattering efficiency $Q_{\text{forw}}$ of a sphere as a function of its normalized radius $\xi = k_0 a$. Top: nihility sphere. Bottom: perfectly conducting sphere.