The resolution of a mathematically bad-defined problem in the approximation of a cellular automation by the example of the description of human states

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Abstract. A mathematical model describing the mutual influence of bad-defined various human characteristics is constructed. This model is described by a system of differential equations that reflect the "rate" of change in a characteristic as a function of the frequency of interaction with other characteristics. The transition from differential equations to equations in finite differences and the introduction of the von Neumann neighborhood on the resulting square space of the frequency of interaction of various human characteristics allows us to introduce a cellular automaton. The sequential execution of iterations in the cellular automaton allows to track how each of the entered characteristics depends on the behavior of other characteristics.

Introduction

The situation with mathematically bad-defined problems is quite common. Such problems arise either due to the lack of suitable data describing the object, or because it is impossible to specify the necessary amount of data to describe the state of the object under study. For example, in quantum physics, the description of an object will always be incomplete, from the point of view of classical physics. This incompleteness of the description of the state is resolved by introducing a wave function, the square of which takes into account the probability of detecting a property of the object in question. Another type of resolution of defined object is the thermodynamic description of a large collective of particles. In this case, the impossibility of taking into account all possible states of the particles that make up the macroscopic object is replaced by the introduction of macroscopic variables that now describe the state of a large collective of microparticles, but do not take into account the detailed behavior of each microparticle of a large collective. One more example of a mathematically bad-defined problem is the problem of describing the states and evolution of human states. It is possible, of course, to consider the state of a person at the household level. Such states involve an assessment of a person's well-being. A person will feel good when he knows what to do, that he will be fed, that he will always have a roof over his head, when he will not suffer. – The list of such "wishes" that contribute to the good state of a person can be continued for as long as we want and they will still be missed. Modeling the behavior and activity of a person (and a community of people) under different conditions is mainly focused on developments related to the study of the behavior of neural networks [1-4], since artificial neural networks (ANN) are built in the image and likeness of the human nervous system. But the training of the ANN and its subsequent independent work (self-study) are concentrated on well-defined objects. And the state of the ANN can be characterized by the
number of parameters contained in the libraries and the processing speed (comparison with the data available in the libraries) of incoming external signals. However, more often, when talking about the description of the human state, we have to deal with very poorly defined concepts and objects that are even difficult to describe. For example, it is difficult to determine what is a person's satiety, joy or suffering, his health, mood, etc. how they are related to each other. All these descriptions or properties, in one way or another, allow us to judge the state of a person. (If we talk about satiety, this concept implies taking into account the physiological state, which takes into account the processes of assimilation of food associated with the conversion of energy stored in food into energy consumed by the body and distributed in the body). That is, to determine the same satiety of a person, we need to consider a huge number of microcharacteristics and their relationships in the body. And this is unlikely to fully determine all the possible states of the body, which could tell how, for example, the phenomenon of satiety affects mood or health. Nevertheless, all these poorly (or bad) defined concepts somehow make it possible to judge the state of a person. To identify the nuances of the mutual connection of poorly defined human characteristics and to find out how they describe the human state, we will use the idea of cellular automata [5-9]. But before to build a cellular automaton we will be interested in how each of these characteristics changes over time. Moreover, we assume that each of the other characteristics somehow affects the selected one. For example, for $A_1$, we can write the following sequence

$$\frac{dA_1}{dt} = a_{11}A_1 + a_{12}A_1A_2 + a_{13}A_1A_3 + \cdots + a_{1n}A_1A_n$$

This implies that the characteristic $A_1$ - (health) depends on the health itself, on satiety, mood, self-esteem, style, communication, rest, love, work, and all other characteristics of $A_j$. The $a_{jk}$ coefficients initially play the following roles. First, these coefficients order the sequence of actions of the characteristics on each other. This means that if we consider $j$ -its state $A_j$, then $A_k$ states act on it. Second, the $a_{jk}$ coefficient determines the frequency of interaction between the $j$ -th and $k$ -th states - how often these states affect each other. About the first product $a_{11}A_1$, we can say that, for example, the rate of change in health will depend on the health itself. (The same can be said about other characteristics). In general, the behavior of all human characteristics can be described by a system of equations

1. **Building a mathematical model of the human states**

   We do not have clear data and their boundaries that determine the state of a real person. We can collect some properties that are characterized by words that are familiar to human hearing. Of course, the set of these characteristics will be incomplete, and we will not even define the meaning of these "characteristics", which, more correctly, are a combination of some actions. Because, for example, health is not only the physical form of the object, the elasticity of the skin, good breathing, the absence of any chronic diseases - This is a complex or combination of actions in which a person is healthy or unhealthy. Here are a few such objects, suggested offhand by students, that could characterize the human condition: satiety, health, mood, self-esteem, style, communication, rest, love and work. Let's introduce the notation:

   - Health $- A_1$
   - Satiety $- A_2$
   - Mood $- A_3$
   - Self-assessment $- A_4$
   - Style $- A_5$
   - Communication $- A_6$
   - Rest $- A_7$
   - Love $- A_8$
   - Labour $- A_9$

   We will be interested in how each of these characteristics changes over time. Moreover, we assume that each of the other characteristics somehow affects the selected one. For example, for $A_1$, we can write the following sequence

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\[
\frac{dA_1}{dt} = a_{11}A_1 + a_{12}A_1A_2 + a_{13}A_1A_3 + \ldots + a_{1n}A_1A_n \\
\frac{dA_2}{dt} = a_{21}A_2A_1 + a_{22}A_2 + a_{23}A_2A_3 + \ldots + a_{2n}A_2A_n \\
\frac{dA_3}{dt} = a_{31}A_3A_1 + a_{32}A_3A_2 + a_{33}A_3 + \ldots + a_{3n}A_3A_n \\
\vdots \\
\frac{dA_n}{dt} = a_{n1}A_nA_1 + a_{n2}A_nA_2 + a_{n3}A_nA_3 + \ldots + a_{nn}A_nA_n
\]

(1)

This system of meshing differential equations describes the effect of each state \( A_j \) on all other states \( A_k \) and vice versa—all other states on the selected one. For example, the effect of the state of the second \( A_2 \) on the state of \( A_3 \) is described by the summand \( a_{32}A_3A_2 \) in the third row and in the second column, etc. In other words, in the row on the right, the first position is the state defined by its left side. The second position contains the state of the object acting on the object in the first position. The indexes of the \( a_{jk} \) coefficient specify the order of states in the \( j \)-th row and \( k \)-th column. In addition, the \( a_{jk} \) coefficient characterizes the frequency of interaction between states. It follows from system (1) that the behavior of each \( A_j \) depends on the behavior of the other elements of this collective. The interaction features described by the \( a_{jk} \) coefficients are not known. The system (1) is similar to the Volterra-Lotka equations [10, 11] for describing the development of populations and describes a large number of objects over a foreseeable but long time. For further convenience of solving and analyzing the system of meshing differential equations (1) with respect to a finite number of states and a finite time, it is convenient to introduce new notation

\[
A_jA_k = A_{jk}
\]

(2)

Then system (1) will be rewritten as:

\[
\frac{dA_{11}}{dt} = 0 + a_{12}A_{12} + a_{13}A_{13} + \ldots + a_{1n}A_{1n} \\
\frac{dA_{22}}{dt} = a_{21}A_{21} + 0 + a_{23}A_{23} + \ldots + a_{2n}A_{2n} \\
\frac{dA_{33}}{dt} = a_{31}A_{31} + a_{32}A_{32} + 0 + \ldots + a_{3n}A_{3n} \\
\vdots \\
\frac{dA_{nn}}{dt} = a_{n1}S_{n1} + a_{n2}S_{n2} + a_{n3}S_{n3} + \ldots + 0
\]

(1a)

In general, to solve this system of equations, we need to know everything about the \( a_{jk} \) coefficients. However, the lack of knowledge about such coefficients makes this system, at first glance, useless. However, the structure of the system of equations (1a) suggests another method for studying such a complex system based on the idea of cellular automata [5-9].

1.1 Representation of a system of differential equations in discrete form

The listed properties of \( A_j \) form a collective of properties that affect each other. At the same time, it is not clear how this mutual influence is carried out. No matter how large the collective of these
properties is, it is finite. The team is finite and the interaction time is also finite. Moreover, finite time exhibits the properties of discreteness. To take these features into account, we reduce the differential equations (1a) to their discrete form in the form of equations in finite differences. To do this, we take into account that in finite differences, the derivative on the interval $\Delta t = (t + 1) - t = 1$ changes, for example, to a finite difference

$$\frac{[A_{11}(t + 1) - A_{11}(t)]}{\Delta t} = A_{11}(t + 1) - A_{11}(t) \quad (4)$$

The transition to the interval $\Delta t = (t + 1) - t$ and subsequent iterations completely negates the significance of the coefficients $a_{jk}$. Their main role now is only to determine the sequence of the test state and the states acting on it. However, the replacement of (2) and (3) makes this last role of the coefficients unnecessary, since at the minimum iteration step $\Delta t = 1$, the interaction of neighboring states either exists or does not exist, and the state in this case, during the interaction, either changes or does not change. Therefore, when passing to finite differences in (1a), taking into account (4), we come to a system of discrete equations in the form:

$$A_{11}(t + 1) = A_{11}(t) + A_{12}(t) + A_{13}(t) + \ldots + A_{1n}(t)$$

$$A_{21}(t + 1) = A_{21}(t) + A_{22}(t) + A_{23}(t) + \ldots + A_{2n}(t)$$

$$\vdots$$

$$A_{mn}(t + 1) = A_{n1}(t) + A_{n2}(t) + A_{n3}(t) + \ldots + A_{nn}(t) \quad (5)$$

In this case, the variable $t$ becomes a discrete variable that can change each time only by one. And the state variable $A_{jk}(t)$ now, in one step equal to $(t + 1) - t = 1$ can change and this change can be mapped to 1 or unchanged, and then this change is mapped to 0. So now $A_{jk}(t)$ is a Boolean function. We agree to compare the improvement of the corresponding property to the unit. Thus, the ranges of values of the state variables are limited to only two values of $A \in \{0,1\}$. Then, for example, the first line of (5) can be rewritten in a formal form:

$$A_{11}(t + 1) = f(A_{11}(t), A_{12}(t), \ldots, A_{1n}(t))$$

And the whole system (5) will be formally rewritten as follows

$$A_{jk}(t + 1) = f(A_{jk}(t)|j, k = 1,2, \ldots, n) \quad (6)$$

### 2. Computer model

#### 2.1. Constructing a cellular automaton

The system of equations (5) and (6) is still very confusing, because everything interacts in it at once. Therefore, in accordance with system (5), we will arrange $A_{jk}(t)$ in the table (matrix) according to the indexes

| $A_{11}$ | $A_{12}$ | $A_{13}$ | $\ldots$ | $A_{1k}$ | $\ldots$ | $A_{1n}$ |
|----------|----------|----------|----------|----------|----------|----------|
| $A_{21}$ | $A_{22}$ | $A_{23}$ | $\ldots$ | $A_{2k}$ | $\ldots$ | $A_{2n}$ |
| $A_{31}$ | $A_{32}$ | $A_{33}$ | $\ldots$ | $A_{3k}$ | $\ldots$ | $A_{3n}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $A_{j1}$ | $A_{j2}$ | $A_{j3}$ | $\ldots$ | $A_{jk}$ | $\ldots$ | $A_{jn}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $A_{n1}$ | $A_{n2}$ | $A_{n3}$ | $\ldots$ | $A_{nk}$ | $\ldots$ | $A_{nn}$ |

This arrangement of states (in the table) allows you to look at the problem of changing states in a different way. Each row $(A_{j1}, A_{j2}, \ldots, A_{jN})$, where $j$ is the "number" of the state ($j = 1,2,3,\ldots, N$) in the table can be perceived as a set of possible states that can be visited by $A_j$. Then the total number of
possible collective states is $N \times N$. But given that now any state in Table 1 is described by a Boolean function that has only two values, we get that the total number of states described by Table 1 is $2^{N \times N}$.

### 2.2. The short order (number of neighbors)

The placement of possible states indicates the maximum number of neighbors relative to the selected cell within the table. For the selected cell inside the table, we can determine the number of nearest neighbors. They can be defined in this table in two ways. The first method: if we move strictly along the rows and columns, then the selected cell has four neighbors. This neighborhood is called the von Neumann neighborhood, Figure 1.

![Figure 1. Von Neumann Neighborhood](image)

The second method: if the movement along the columns and rows is allowed to move along the diagonals of the selected cell, then the selected cell will have eight neighbors. This neighborhood is called the Moore neighborhood, Figure 2. If we now take into account the short-range order $\xi$ in one of the two variants in the resulting cell space in the iteration problems of expression (5) and (6), we get a Cellular automaton [5-9]. We introduce the near-order $\xi$ in the von Neumann form only for the neighborhood of Fig. 2.

$$\xi(j,k) = \{(m,n)| |j-m| + |k-n| \leq 1\}$$  \hspace{1cm} (7)

That is, in the vicinity of a cell with indices $j$ and $k$, when shifting one step (one cell) to the left or right, or up or down, only the nearest cell that meets the definition (7) is taken into account. Then equation (6) can be rewritten in the form

$$A_{jk}(t+1) = f(A_{jk}(t) \in A || A_{jk} \in \xi)$$  \hspace{1cm} (8)

Finally, the operator $f$ for equations in finite differences sets the transition state function for the variables $A_{jk}$. In our case, $f$ is a shift on the time scale by one minimum iteration step $\Delta t = 1$ of the Boolean state function $A_{jk}$. Thus, the cellular automaton (CA) is constructed:

$$CA = \langle G, A, \xi, f \rangle$$  \hspace{1cm} (9)

where $G$ is the number of cells ($= N \times N$), $A$ is a set of states (in our case $2^{N \times N}$), $\xi$ - number of nearest neighbors (8), $f: A(t) = A(t+1) \in \{A = \{0,1\}\}$ - transition function

The "cellular automaton" (CA) system evolves over a discrete time interval and updates the state variable in accordance with the transition function of states by taking into account its own state and the states of local (nearest) neighbors at the present time. And modeling a system using CA differs from modeling the same system in the language of differential equations in that on a discrete interval $(t+1) - t$, the state variables turn out to be discrete state variables of $CA$, and their values turn out to be bounded by finite values $\{0,1\}$. Thus, $CA$ describes a complex system not by complex equations, but by modeling a complex system using the interaction of simple components and by simple rules [5,7,8].
2.3. Choosing a neighborhood

This is an important point in the development and application of the constructed model. The resulting cell space has the property of inhomogeneity in terms of determining the neighborhood. In this space, you can select cells that are surrounded by a different number of neighbors. For certainty, we choose the von Neumann neighborhood. In the case of a von Neumann neighborhood, it is easiest to ensure uniformity. This is given by turning the "square" space CA into a torus by defining periodic boundary conditions

\[ A_{(j+N),k} = A_{jk} \quad A_{j,(k+N)} = A_{jk}. \] (10)

In this case, the CA becomes homogeneous. In the case of the Moore neighborhood (Fig. 2) and the transition to a torus-shaped space, we need to set periodic conditions "for the diagonal", and to stitch the boundaries, we need to set a shift on each side by one cell. This makes the calculations cumbersome and does not justify the goal).

3. Behavior of states on the CA

When building a program for CA, the following main tasks can be distinguished:
(a) building the initial configuration,
(b) determining the algorithmic solvability and complexity of problems in CA,
(c) determining the number of steps that solve the problem.

For a cellular automaton considered as a whole, the most important parameters are the distribution of the initial states of the cells (initial configuration) and the method (rule, \( f \)) displays the previous states in the current state of the cell, in accordance with (8) and (10).

3.1. The results of a study of the behavior of a cellular automaton built for bad-defined human states (a). the initial state is given by a random distribution of one of two states: "1" - positive reaction or "0" - negative reaction. This suggests that as a result of this distribution, cells with "1" responded positively to interaction with their neighbors, and cells with "0" were negatively affected by neighbors. Two variants were considered in the study of the behavior of CA.

The first variant. Cells without memory. During the iterations, the state "1", as the state corresponding to a positive reaction, could be replaced by either the same or "0" as a result of the next step. The results are shown in Fig. 3. Here "0" and "1" are randomly distributed.

![Figure 3](image)

**Figure 3.** The cells of the space of the cellular automaton (Fig. 1) are filled randomly (the upper square). Lower square: the final result after the conditions are met. Right picture: the number of "units" versus number of iterations.

Now let the initial configuration of CA (Fig. 4), consists only of the values "1". This situation corresponds to the condition under which all interactions of different characteristics would lead to a
positive result. And the cellular automaton itself would correspond to the fact that all wishes are fulfilled:

![Image](image.jpg)

**Figure 4.** The population of the space of the cellular automaton and its transition to end state as a result of evolution. (a) In the initial state, all cells are occupied by units. (b) The evolution of CA led to a chess distribution of filling cells with zeros and ones. (c) Relaxation of CA to a chessboard distribution

Any other setting of the initial state of the CA leads to the fact that the CA relaxes to a chess distribution, which provides an alternation of positive and negative effects as a result of interaction with neighboring cells. And this indicates another one that characterizes a person as a self-consistent system. The cellular automaton, as a system corresponding to the human state, leads to a state in which the positive and negative effects of its various characteristics on each other are balanced.

**The second variant. Cells with memory.** In this case, each cell remembers the positive attitude "1" and assigns a weight equal to one to each successful interaction with neighboring cells. Thus, the cell gains weight, which is not destroyed at each subsequent iteration, but is saved and, if successful, increases by one each time. Now each cell is described by two positions (Fig. 5). The first position corresponds to the current interaction with neighbors and is filled with either "0" or "1". The second position of the cell, each time a positive outcome of the impact of neighboring cells adds one. At the same time, the behavior of the cellular automaton in the first position remains the same as for the first case (Fig. 3,4), i.e. the number of positive outcomes with "1" is approximately balanced by the number of negative outcomes with "0" when exposed to neighbors. However, taking into account the evolution of the cell weight shows, nevertheless, that the cells develop and the number of cells with a positive outcome increases (Fig. 5). As a result, the cells prefer a positive outcome. Let's consider two cases with this situation. Let's take a random automaton, and replace the second line in it, which is responsible for
satiety, with the values "1". In this case, we believe that the person suffers from overeating.

**Example:**

- Rows: 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 1, 0, 1, 1
- A11 = 4
- A21 = [4, 13, 22, 31, 40, 49, 58, 67, 76, 85, 94]

**Figure 5.** Evolution of CA for memory cells. The evolution of CA for cells with memory. The upper part shows the initial and final state of the CA. The lower part shows the change in the weight of the cells in the selected characteristics versus the iteration steps.

Looking at the final weight of the rows, it could be seen that the 1 and 3 rows do not gain weight above the weight of the 2 rows. The maximum weight is given to the row where there were originally more units. For a more detailed consideration of the overeating option, let's take an automaton, in which lines 1 and 3 are filled with the value "0", and the satiety line is filled with the values "1". All other values are random.
Figure 6. Evolution of CA for memory cells. The evolution of CA for cells with memory. The satiety line is filled with the values "1". The upper part shows the initial and final state of the CA. The lower part shows the change in the weight of the cells in the selected characteristics versus the iteration steps.

Here is a different situation: despite the fact that row 2 consisted of the values "1", and row 1 and 3 consisted of the values "0", row 4 showed the greatest weight due to the fact that the row above it consisted of the values "0", which allowed the mood row to gain weight.

Conclusion
A method of describing the human states using bad defined characteristics is proposed (the number of characteristics is not enough. And these characteristics are extremely conventional). For this purpose, a mathematical model was built that describes the various characteristics of a person and their mutual impact. A cellular automaton was built to analyze the effect of different characteristics (of a person) on each other. The operation of this cellular automaton is implemented in Python. It was demonstrated that it is possible to describe the human condition and the evolution of this condition using its
mathematically poorly defined characteristics. The conducted research allows us to track how the state of a person as a whole changes and how its different characteristics at different times affect each other and each of the initially introduced characteristics of a person.

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