The electroweak theory with a priori superluminal neutrinos and its physical consequences

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In recent experiments conducted by the OPERA collaboration, researchers claimed the observation of neutrinos propagating faster than the light speed in vacuum. If correct, their results raise several issues concerning the special theory of relativity and the standard model of fundamental particles. Here, the physical consequences of superluminal neutrinos described by a tachyonic Dirac lagrangian, are explored within the standard model of electroweak interactions. If neutrino tachyonic behavior is allowed, it could provide a simple explanation for the parity violation in weak interactions and why electroweak theory has a chiral aspect, leading to invariance under a $SU_L(2) \times U_Y(1)$ gauge group. Right-handed neutrino becomes sterile and decoupled from the other particles quite naturally.

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INTRODUCTION

The OPERA collaboration$^1$ recently announced the astonishing results obtained in a long baseline neutrino experiment designed to measure, among other things, the neutrino time-of-flight with the best currently known estimation of time and distance using atomic clocks and GPS system. They claim that the neutrinos with energies in the range $10 - 60 \text{GeV}$ traveled the approximate 730km from the CERN facilities to the Gran Sasso detector faster than light would do, with $(v-c)/c = 2.5 \times 10^{-5}$ within a six-sigma level of confidence. The result more or less corroborate an earlier experimental finding conducted by the MINOS collaboration$^2$, but clearly contradicts astronomical available data. Measurement of neutrino with energies in the range of a few MeV coming from supernova SN1987A put the stringent limit of $(v-c)/c < 5 \times 10^{-9}$ [3, 4]. As a matter of fact, the sign of the muon-neutrino squared mass $m_\nu^2$ is still open to debate [5, 6], and its implications were discussed in Ref. [7]. In favor of the neutrino superluminal behavior, in Ref. [8] J. Ciborowski and J. Rembielinski proposed that tachyonic neutrinos could explain certain anomalies in the decay of tritium [10]. On the other hand, serious constraints on the existence of superluminal neutrinos were established by Cohen and Glashow [11], arguing that if the standard model as we know still applies, such neutrinos should lose energy by producing photons and $e^+ e^-$ pairs through $Z_0$ mediated processes, analogous to a bremsstrahlung radiation effect, which would alter the energy spectrum of detected neutrinos. The search of pair production by the ICARUS Collaboration found no evidence of anomalous $e^+ e^-$ pair production [12].

Currently, the OPERA result still waits to be confirmed or refuted by independent experiments, but if correct we are facing a ground-breaking moment in Physics comparable to that of the Michelson-Morley experiment. At this point we can only speculate on a few number of hypotheses to explain the observed data: i) The OPERA experimental results are not correct due to the presence of a subtle systematic error which is not being taken into account yet. Attempts to detect possible sources of such errors are being done, such as in Ref. [13], which posed the question on the influence of data filtering, but the OPERA team already refuted those objections conducting a series of improved experiments; ii) The neutrinos travel faster than light but are not tachyonic in character, which would mean that the true value of $c$ appearing in the special theory of relativity is the neutrino speed. As a matter of fact, one can speculate that since the photon is far more interacting than the neutrino the light speed in a quantum-mechanical vacuum is a renormalized value; iii) The neutrinos are tachyons and travel faster than light. The idea of tachyonic neutrinos are not new [9, 14], but there are a number of arguments against it [15]. In spite of the above, if we accept that the special theory of relativity remains valid including anti-orthochronous Lorentz transformations describing particles with $v > c$, this fact has to be taken into account in the standard model of particle physics; iv) The special theory relativity in four-dimensional spacetime reached its limit of validity and a profound reformulation of the physical theories are in order. For instance we may cite the possibility of theories having extra spacetime dimensions [16].

The purpose of the present paper is to explore the physical consequences of having a priori superluminal neutrinos in the standard model of elementary particles. We assume that the special theory of relativity is rigorously valid and consider that a complete physical theory must include the superluminal sector of the full Lorentz-
Poincaré group, the so-called anti-orthochronous Lorentz transformations, in the form of a tachyonic Dirac equation. A particular interpretation of the special relativity says that massive particles may exist in states with \( c > v \) (subluminal or bradyonic particles) or \( v > c \) (superluminal or tachyonic particles) \[18–21\], but the frontier dividing subluminal and superluminal motion cannot be crossed, i.e., a bradyon cannot be converted classically into a tachyon and vice-versa. The standard model extended to include the tachyonic sector starts from a lagrangian density describing two Dirac fields, but one of them, identified with the neutrinos, is tachyonic. As we will show in the following Section, the Dirac equation describing superluminal fermions slightly differs from the subluminal Dirac equation but leads to important physical consequences.

THE DIRAC EQUATION FOR BRADYONS AND TACHYONS

In the present Section we briefly review the fundamental aspects of the Dirac equation in the bradyonic and tachyonic forms. The special theory of relativity requires the invariance of the quantity \( p_\mu p^\mu = m^2 \), which is a Casimir invariant of the Lorentz-Poincaré group, where \( \mu = 0, 1, 2, 3 \) are the spacetime indices, \( p^\mu = (E, \mathbf{p}) \) is the four-momentum vector, \( E \) is the energy and \( \mathbf{p} \) is the linear momentum and \( m \) is the rest mass of the particle. The Einstein convention of summing up the repeated indices is implied throughout this paper.

For a particle with rest mass \( m^2 > 0 \) the dispersion relation can be written as \( E^2 = p^2 + m^2 \), and the motion is subluminal (\( v < c \)). The Dirac equation can be directly obtained from \( E = \sqrt{p^2 + m^2} \). This procedure is now standard and found in textbooks \[18–21\]. For spin 1/2 fermions we get:

\[
(i\gamma^\mu \partial_\mu - m)\psi = 0, \tag{1}
\]

being \( \gamma^\mu \) the Dirac gamma matrices and \( \psi \) a four component Dirac spinor. The above equation is invariant under the symmetries of charge conjugation \( C \), parity \( P \) (corresponds change the sign of the spatial coordinates, i.e., \( \mathbf{x} \rightarrow -\mathbf{x} \)) and time reversal \( T \) (corresponds to reverse the time flow, \( t \rightarrow -t \)), separately and also the product \( CPT \) \[18–21\]. The Dirac equation can be obtained via Euler-Lagrange equations from the following lagrangian density for a massive Dirac field:

\[
\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi . \tag{2}
\]

The tachyonic dispersion relation, for which \( v > c \), is obtained by making the replacement \( m \rightarrow \pm im \), meaning that a superluminal particle has imaginary mass and \( m^2 < 0 \). This way the relation \( E^2 = p^2 - m^2 \) leads to the corresponding tachyonic Dirac equation, which was obtained in Ref. \[22\]. The result is:

\[
(i\gamma^5\gamma^\mu \partial_\mu - m)\psi = 0, \tag{3}
\]

where \( \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \) is the well known chirality operator. The detailed analysis of \( CPT \) invariance was performed in Ref. \[23\], showing that the tachyonic Dirac equation is invariant under the product \( CP \) and \( T \) separately, but not under \( C \) or \( P \) alone. Such a property of tachyonic Dirac equation could explain why the electroweak interactions violate parity symmetry \( P \), if neutrinos are tachyonic particles. The lagrangian density for a tachyonic Dirac field is written below for completeness:

\[
\mathcal{L} = i\bar{\psi}\gamma^5\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi , \tag{4}
\]

where \( m \) is now a real parameter corresponding to the particle's rest mass.

Notice that both the subluminal and tachyonic Dirac fields describe a luminal particle with \( v = c \) in the limit \( m \rightarrow 0 \), but the convergence to the speed limit \( v = c \) is obtained from the left for the case of massive field with \( v < c \), and from the right for the tachyonic massive field with \( v > c \).

THE ELECTROWEAK THEORY WITH A PRIORI SUPERLUMINAL NEUTRINO

Our starting point is a lagrangian density describing two massless Dirac fields written below:

\[
\mathcal{L} = i\bar{\psi}_e\gamma^\mu \partial_\mu \psi_e + i\bar{\psi}_\nu\gamma^5\gamma^\mu \partial_\mu \psi_\nu , \tag{5}
\]

where \( \psi_e \) and \( \psi_\nu \) describe the electron and the neutrino-electron field respectively. Notice that the electron becomes subluminal if a mass \( m > 0 \) is attributed to the field \( \psi_e \). By contrast, the neutrino field becomes superluminal in the case \( m > 0 \). For the sake of convenience, the fields can be split up into chirality eigenstates, using the projection operators \( \Pi_\pm = (1 \pm \gamma^5)/2 \), yielding:

\[
\psi_{eL} = \frac{1 - \gamma^5}{2}\psi_e , \quad \psi_{eR} = \frac{1 + \gamma^5}{2}\psi_e , \quad \psi_{\nu L} = \frac{1 - \gamma^5}{2}\psi_\nu , \tag{6}
\]

\[
\psi_{\nu R} = \frac{1 + \gamma^5}{2}\psi_\nu ,
\]

where the subscript \( L(R) \) are customary known as left(right)-handed particles, but in fact they correspond to negative(positive) chirality eigenvalues. Notice that for idempotent operators \( \Pi_\pm^2 = \Pi_\pm \) and for any Dirac spinor \( \psi \) we have \( \psi = (\Pi_+ + \Pi_-)\psi \), allowing to straightforwardly rewrite expression \[5\] as follows:

\[
\mathcal{L} = i\bar{\psi}_{eL}\gamma^\mu \partial_\mu \psi_{eL} + i\bar{\psi}_{eR}\gamma^5\gamma^\mu \partial_\mu \psi_{eL} + i\bar{\psi}_{\nu L}\gamma^\mu \partial_\mu \psi_{\nu L} - i\bar{\psi}_{\nu R}\gamma^5\gamma^\mu \partial_\mu \psi_{\nu R} . \tag{6}
\]
The above equation shows that the superluminal neutrino field ends up with a sign difference between the left and right chirality components, which is not the case of electrons. In order to appreciate the significance of such a fact, we merge the electron and neutrino fields into left-handed and right-handed isodoublets:

\[
\Psi_L = \begin{pmatrix} \psi_{eL} \\ \psi_{\nu L} \end{pmatrix}, \quad (7)
\]

\[
\Psi_R = \begin{pmatrix} \psi_{eR} \\ \psi_{\nu R} \end{pmatrix}, \quad (8)
\]

enabling us to recast equation (6) into the following form:

\[
\mathcal{L} = i \bar{\Psi}_L \gamma^\mu \partial_\mu \Psi_L + i \bar{\Psi}_R \gamma^\mu \partial_\mu \tau_2 \Psi_R, \quad (9)
\]

where \( \tau_2 \) is the Pauli matrix

\[
\tau_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

which operates in the so-called weak isospin space, yielding

\[
\tau_2 \Psi_R = \begin{pmatrix} \psi_{eR} \\ -\psi_{\nu R} \end{pmatrix}.
\]

Now, we can straightforwardly construct a gauge theory of electroweak interactions, requiring gauge invariance of the above lagrangian density \([20, 21]\). For the lagrangian density describing the left-handed isodoublet, \( \mathcal{L}_L = i \bar{\Psi}_L \gamma^\mu \partial_\mu \Psi_L \), we require invariance under a general transformation of the gauge group \( SU_L(2) \times U_Y(1) \):

\[
\Psi_L' = \exp \left[ i \Lambda_0 + i \frac{\tau \cdot \Lambda}{2} \right] \Psi_L,
\]

where \( \Lambda_0 \) and \( \Lambda \) are well-behaved spacetime functions and \( \tau \) are the Pauli isospin matrices. This is achieved by replacing ordinary derivatives \( \partial_\mu \) by covariant ones:

\[
D_{\mu L} = \partial_\mu + ig' X_\mu - ig \tau \cdot W_\mu, \quad (10)
\]

where \( X_\mu \) and \( W_\mu \) are the \( U_Y(1) \) and \( SU_L(2) \) gauge fields, respectively, and \( g' \) and \( g \) are the corresponding couplings. The \( SU_L(2) \) gauge group could be interpreted as a local ‘rotation’ mixing of the subluminal and superluminal spinor fields, i.e., connecting the orthochronous to the anti-orthochronous Lorentz groups in the limit \( v \to c \). By contrast, the lagrangian density representing the right-handed isodoublet, \( \mathcal{L}_R = i \bar{\Psi}_R \gamma^\mu \partial_\mu \tau_2 \Psi_R \), is not invariant under general \( SU_R(2) \) ‘rotations’ due to the presence of the \( \tau_2 \) matrix. Only rotations around the \( z \)-axis of the weak isospin would preserve the gauge invariance of the right-handed sector. This way, we can break the right-handed isodoublet into two isosinglets \( \psi_{eR} \) and \( \psi_{\nu R} \) by postulating a special case of \( U_Y(1) \) gauge transformation connected to the weak hypercharge \( Y \) as follows:

\[
\Psi_R' = \exp [i(1 + \tau_2) \Lambda_0] \Psi_R,
\]

where \( \Lambda_0 \) is a gauge change related to the \( U_Y(1) \) gauge group. The covariant derivative of the right-handed sector is written as:

\[
D_{\mu R} = \partial_\mu + ig' (1 + \tau_2) X_\mu, \quad (11)
\]

allowing us to obtain the usual Weinberg-Glashow-Salam electroweak theory, described in more details elsewhere\([20, 21]\). The Higgs sector of the theory, which provides mass to the physical fields via symmetry breaking, can be introduced in the next stage, making the neutrinos superluminal, \( m^2 < 0 \). Notice that the coupling constant of the right-handed sector of the theory to the \( SU_R(2) \) gauge field \( W_\mu \) vanishes in order to preserve the gauge invariance of the theory, i.e., the right-handed particles do not transform as an isodoublet. The right-handed neutrino is also decoupled from the gauge field \( X_\mu \), thus becoming ‘sterile’.

As a final remark, we mention that if the fields acquire mass the chirality eigenstates get mixed up \([21]\) \((-m \bar{\psi}_\nu \psi_\nu = -m[\bar{\psi}_{\nu L} \psi_{\nu R} + \bar{\psi}_{\nu R} \psi_{\nu L}])\), allowing the conversion of a left-handed neutrino into a right-handed one, despite the fact that only the left-handed eigenstate is physically observable through the electroweak interactions. This is a new kind of neutrino oscillation which mixes left and right-handed neutrinos of the same flavor (the other kind of neutrino oscillation being the well-known neutrino flavor oscillation \([24]\)). The mean velocity of the neutrino field is given by:

\[
\langle \frac{dx}{dt} \rangle = \int d^3x \bar{\psi}_\nu \gamma^5 \gamma \psi_\nu,
\]

where \( \gamma = (\gamma^1, \gamma^2, \gamma^3) \) is the vector containing the spatial components of the Dirac gamma matrices. Writing the above expression in terms of chirality eigenstates, we obtain:

\[
\langle \frac{dx}{dt} \rangle = \int d^3x [\bar{\psi}_{\nu L} \gamma \psi_{\nu L} - \bar{\psi}_{\nu R} \gamma \psi_{\nu R}].
\]

Looking at the above expression, the mean velocity of the neutrino field depends on both left and right-handed components. Even if the physically observable left-handed neutrino field \( \bar{\psi}_{\nu L} \) becomes subluminal when it acquires mass, the quantum interference effect due to the mixing of the left-handed and the sterile right-handed neutrinos provided by the mass term, would allow for a superluminal velocity, which could depend on the propagation distance of the neutrino beam.

**CONCLUSION**

In summary, we discussed the possibility of existence and its physical consequences of an \textit{a priori} tachyonic neutrino in the standard model of electroweak interactions. Despite the conceptual difficulties introduced by
the presence of superluminal particles in physical theories, as well as the stringent limits imposed on such behavior by experimental observations, the existence of a tachyonic neutrino, if allowed and not ruled out by future experiments, would explain in a ‘natural’ way the parity violation, the existence of chiral gauge transformations and the decoupling of the right-handed neutrino, which becomes sterile, from the electroweak field.

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