Electroweak Phase Transition in Minimal Extension of Scale Invariant Standard Model

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In an extension to the scale invariant standard model by two real singlet scalars $s$ and $s'$ in addition to the Higgs field, we investigate the strong first-order electroweak phase transition as a requirement for the baryogenesis. This is the minimal extension to the scale invariant standard model with two extra degrees of freedom that possesses the physical Higgs mass of 125 GeV. The scalar $s'$ being stable because of the $Z_2$ discrete symmetry is taken as the dark matter candidate. We then show that the electroweak phase transition is strongly first-order, the dark matter relic density takes the desired value $\Omega_{DM}h^2 \sim 0.11$ and the constraints from direct detection experiments are respected only if $m_{s'} \equiv m_{DM} \gtrsim 4.5$ TeV. The model puts also a lower bound on the scalon mass, $m_s \gtrsim 200$ GeV.

\section{Introduction}

After the discovery of the Higgs particle in July 2012 at the LHC \cite{1, 2}, the last missing piece of the standard model (SM) prediction, made almost a half-century ago \cite{3, 4}, was completed. The SM has been tested by the most stringent scrutinies over many different experiments and it has passed them successfully. However there are a number of issues, either theoretical or experimental/observational, that are not compatible with the SM predictions. The gauge hierarchy problem, the strong first-order electroweak phase transition (EWPT) and other conditions needed in the baryogenesis mechanism, and the problem of dark matter are some examples of unanswered puzzles in the SM. These inconsistencies led people to think of theories beyond the SM such as the GUT, SUSY, etc. Our goal in this paper is to address the above mentioned SM shortcomings rather in a minimal extension of the scale invariant standard model (MSISM).

The negative Higgs mass term, $-m_H^2 H^\dagger H$ in the SM potential causes a quadratical divergent term proportional to the energy scale cut-off $\Lambda^2$ after including the quantum corrections. In fact, the Higgs mass term is the only term that breaks the classical scale invariance in the SM. Therefore by omitting the Higgs mass term from the SM potential we have practically removed the problem of gauge hierarchy \cite{5}. In the seminal paper of Coleman and E. Weinberg \cite{6} it is shown that in a scale invariant gauge theory the radiative corrections breaks the scale invariance and that triggers the spontaneous symmetry breaking. Following their work Gildener and S. Weinberg \cite{7} argued that in the SISM the radiative corrections breaks the electroweak symmetry and thereby the Higgs mechanism is restored for the SISM. The SISM with only one classically massless scalar (Higgs) can not be realistic because as computed by Gildener and S. Weinberg from the quantum corrections the Higgs mass can be just as heavy as around 5 GeV, which is far lighter than the observed Higgs mass, $m_H \sim 125$ GeV. As discussed in \cite{7} in general among the $n$ scalars in addition to the Higgs scalar in the extended SISM, there is at least one heavy scalar that may be interpreted as the Higgs particle and there is one classically massless scalar that is dubbed scalon. In this paper we add only two extra scalars to the SISM, as the most minimal scale invariant extension of SISM that contains the correct Higgs mass. There are papers in the literature (see for instance \cite{8, 10}) that have sought similar scopes but none of them have presented an analytical investigation of the electroweak phase transition and furthermore they have involved extra fermionic degrees of freedom in the hidden sector. In this work, we are interested only in studying the pure scalar and the minimal extension of the SISM.

This paper is organized as the following. In the next section we build up the model extending the SISM with two real scalars. Then in section \ref{Minimal Extension of Scale Invariant Standard Model} we derive the critical temperature and the washout criterion for the electroweak phase transition. In section \ref{Stability Conditions} the stability conditions are given and the next section will be on dark matter computations. Finally in section \ref{Conclusion} we examine the model with the experimental bounds on dark matter elastic scattering off the nucleus. We conclude in section \ref{Conclusion}.

\section{Minimal Extension of Scale Invariant Standard Model}

In the SM, if we set the Higgs mass term to zero, the only term remaining in the Higgs potential will be \( \lambda (H^\dagger H)^2 \) or \( \lambda h^4 \) after gauging away three components of the Higgs doublet. In Gildener-Weinberg notation \cite{7}, the scalar potential with $n$ scalars, \( s_i \), is shown as $1/24\lambda_{ijkl} s_is_js_vs_k$ where $\lambda_{ijkl}$ denotes the coupling. As discussed in \cite{11}, in order to have a scale invariant version of the standard model possessing a Higgs doublet with the observed Higgs mass of 125 GeV as well as other SM particles with their physical masses, at least two more scalars (singlet) must be added to the theory. The reason comes from eq. (4.6) of \cite{7} where the scalon gaining mass through the radiative corrections, depends only on...
the masses of the Higgs particle, the gauge bosons and the mass of the top quark. In the absence of any additional scalar except the Higgs and the scalon, this expression will be negative. In this paper, therefore we stay in the most minimal potential with only two extra scalars which we call them here $s$ and $s'$. We also assume that these scalars appear with the $Z_2$ symmetry in the potential to attribute them latter to the dark matter candidate,

$$V_{tr}(h, s, s') = \frac{1}{4} \lambda_h h^4 + \frac{1}{2} \lambda_{hs} s^2 h^2$$
$$+ \frac{1}{4} \lambda_s s^4 + \frac{1}{2} \lambda_{ss'} s^2 s'^2 + \frac{1}{4} \lambda_{s'} s'^4,$$

(1)

where $h$ stands for the Higgs field, $H^\dagger = \frac{\lambda}{\sqrt{2}} (0 \ h)$. At high temperature, above the electroweak phase transition temperature, the theory lives in its symmetric phase and the vacuum expectation values (vev) of the fields are temperature dependent. Let us assign the vev of each field as

$$v_h (T) \equiv \langle h \rangle, \quad v_s (T) \equiv \langle s \rangle, \quad v_{s'} (T) \equiv \langle s' \rangle.$$

(2)

We require that the vacuum expectation values after the phase transition be $(v_h = 246 \text{ GeV}, v_s \neq 0, v_{s'} = 0)$ so that the scalar field $s'$ remains stable because of the $Z_2$ symmetry and thereby it can play the role of the dark matter candidate. We see latter that the value of $v_s$ is not fixed and it depends on the value of the couplings in the model.

In the scale invariant standard model the flat direction is defined as the direction along which the tree-level potential is vanishing. This condition is equivalent to imposing the Ward identity of the scale symmetry in a scalar theory [12]. The flat direction for the potential in eq. (1) is obtained via a rotation in the $(h, s)$ space by the angle $\alpha$,

$$\cos^2 \alpha = -\frac{\lambda_{hs}}{\lambda_h - \lambda_{hs}}, \quad \lambda_s^2 - \lambda_h \lambda_s = 0.$$

(3)

The mass matrix (being meaningful after the phase transition at low temperature) is off-diagonal only in $(h, s)$ block. This is because of our special choice; we have taken non-zero vev for $h$ and $s$ and vanishing vev for $s'$. Finally, the mass eigenvalues after the EWPT read,

$$m_h^2 = 2v_h^2 (\lambda_h - \lambda_{hs}), \quad m_s^2 = 0, \quad m_{s'}^2 = -v_h^2 \frac{\lambda_h \lambda_{ss'}}{\lambda_{hs}}.$$

(4)

The one-loop correction at zero temperature gives a small mass to the classically massless eigenstate $s$, the so called scalon field [2],

$$\delta m_i^2 = \frac{-\lambda_{hs}}{32\pi^2 m_h^2} (m_h^4 + m_s^4 + 6m_W^4 + 3m_Z^4 - 12m_t^4).$$

(5)

Without the introduction of the second singlet scalar $s'$ while having the observed Higgs mass of $125 \text{ GeV}$ and the correct masses for the top quark and gauge bosons, the mass correction to the scalon field $s$ could not be positive. Now by means of the radiative correction, the scalon can be interpreted as the mediator in DM models.

The one-loop effective potential consists of the tree-level potential in eq. (1), the Coleman-Weinberg one-loop correction at zero temperature, and the one-loop correction at finite temperature,

$$V_{\text{eff}} = V_{tr} + V_0^{1-\text{loop}} + V_T^{1-\text{loop}}.$$

(6)

If $T \gg m_i$ with $m_i$ the tree-level mass of particle $i$, the one-loop thermal contribution is approximated as $V_T^{1-\text{loop}} \approx CT^2 \phi^2$ and the total one-loop effective potential becomes [13],

$$V_{\text{eff}} \approx -\frac{1}{4}B\phi^4 + \frac{1}{2}B\phi^4 \log \frac{\phi^2}{v_\phi^2} + CT^2 \phi^2,$$

(7)

where $\phi$ is the radial field in the polar coordinate system, i.e. $(h, s) \equiv (\phi \cos \alpha, \phi \sin \alpha)$, hence $v_\phi^2 = v_h^2 + v_s^2$. The coefficients $B$ and $C$ are,

$$B = \frac{1}{6\pi^2 v_\phi^4} (m_h^4 + m_s^4 + 6m_W^4 + 3m_Z^4 - 12m_t^4),$$

(8)

$$C = \frac{1}{12v_\phi^4} (m_h^4 + m_s^4 + 6m_W^4 + 3m_Z^4 + 6m_t^4).$$

(9)

### III. CRITICAL TEMPERATURE AND WASHOUT CRITERION

The strong first-order electroweak phase transition is one of the three Sakharov conditions [14] for the baryogenesis. For the CP violation in minimal scale invariant extensions of the SM see [12]. The phase transition takes place at the critical temperature, $T_c$ at which the free energy (effective potential) has two degenerate minima at $T = T_c$. In this section we follow [15] to calculate analytically the washout criterion i.e. $v(T_c)/V_c > 1$ which guarantees the strong first-order phase transition.

The minimization condition on the thermal effective potential in eq. (7) with the derivative being along the radial field,

$$\frac{\partial}{\partial \phi} V_{\text{eff}} \bigg|_{v_\phi(T)} = 0,$$

leads to a set of $T$-dependent equations for the vacuum expectation value,

$$v_{\text{sym}}(T) = 0,$$

(10)

$$v_{\text{brk}}^2(T) \log \frac{v_{\text{brk}}^2(T)}{v_\phi^2} = - \frac{C}{B} T^2,$$

(11)

where $v_{\text{sym}}$ is the vev of the radial field in the symmetric phase and $v_{\text{brk}}$ is the vev in the broken phase. Eq. (12)
has no analytic solution for \( v_{\text{brk}} \). Nevertheless, the solution can be expressed in terms of the Lambert W function which is defined as,

\[
z = \text{we}^w \Leftrightarrow w = W(z),
\]

where \( z \) and \( w \) in general are complex numbers. In terms of the Lambert W function eq. (12) is written as,

\[
v_{\text{brk}}^2 = \frac{-CT^2/B}{W\left(-\frac{C}{Br_0}T^2\right)}.
\]

At the critical temperature \( T_c \) the effective potential in eq. (7) must vanish at the minimum \( v_{\text{brk}} \) as it is vanishing also at the minimum \( v_{\text{sym}} = 0 \) of the symmetric phase. Multiplying eq. (12) by \( v_s^2 \) and substituting its right hand side in \( V_{\text{eff}}(v_{\text{brk}}) = 0 \) from eq. (7) we arrive at \( v_{\text{brk}}^2(T_c) = (2C/B)T_c^2 \). Therefore the condition for the electroweak phase transition to be strongly first-order (the washout criterion) becomes,

\[
\frac{v_{\text{brk}}(T_c)}{T_c} = \left(\frac{2C}{B}\right) > 1.
\]

Finally, substituting \( v_{\text{brk}} \) from eq. (15) into eq. (14), expanding the effective potential and setting that to zero we obtain,

\[
T_c^2 \simeq \left(\sqrt{\Pi - 3}\right)\frac{B}{C}v_s^2.
\]

Before going further with more constraints on the parameters, regarding the values of \( B \) and \( C \) in eqs. (15) and (16), it is clear that the ratio \( v_c/T_c \) can easily be large enough leading to a very strong first-order phase transition.

FIG. 1. The plot compares the dark matter mass against the coupling \( \lambda_{hs} \) with washout criterion satisfied and \( \Omega_{\text{DM}}h^2 \sim 0.11 \).

FIG. 2. A histogram for the ratio \( v_c/T_c \) with the correct relic density \( \Omega h^2 \sim 0.11 \). It is shown that \( v_c/T_c \gtrsim 4 \) which guarantees a very strong first-order phase transition.

IV. STABILITY CONDITIONS

The stability conditions impose already strong constraints on the parameters of the model. The first derivative of the tree-level potential in eq. (1) must vanishes at the vevs,

\[
\frac{\partial V}{\partial h} \big|_{(h)} = \frac{\partial V}{\partial s} \big|_{(s)} = 0,
\]

which in turn leads to,

\[
\lambda_h v_h^2 = -\lambda_{hs} v_s^2, \quad \lambda_s v_s^2 = -\lambda_{hs} v_h^2.
\]

The positivity of the second derivatives of the potential in eq. (1) gives rise to,

\[
\lambda_{hs} < 0, \quad \lambda_{ss'} > 0.
\]

From eq. (19) and eq. (4) we get \( \lambda_h > 0 \). Now the radiative correction to scalon mass in eq. (5) is positive if \( m_{s'} > 316.5 \text{ GeV} \). Using eq. (1) for \( m_{s'} \) one arrives at \( \lambda_{s'} > -1.65\lambda_{hs}/\lambda_h \). Still we can make use of the Higgs mass relation in eq. (1) to constrain more the Higgs coupling: \( \lambda_h = \lambda_{hs} + 0.128 \). As \( \lambda_h > 0 \) and \( \lambda_{hs} < 0 \) then \(-0.128 < \lambda_{hs} < 0 \).

V. DARK MATTER

The scalar \( s' \) taking a zero expectation value is stable and can play the role of the thermal dark matter candidate within the freeze-out scenario. In this section we add the relic density condition to the washout criterion obtained in the previous section and probe the space of the parameters. The independent parameters in the model are not many; \( \lambda_{hs}, \lambda_{s'} \) and \( \lambda_{ss'} \), among which only
the parameter $\lambda_{hs}$ takes part in the relic density computation. The dark matter sector interacts with the visible sector via the scalar mediator $s$ which has become massive through the radiative correction and its mass is given by eq. (4). In fact the scalar mediator $s$ mixes with the Higgs field in the SM and the mixing angle is that of the flat direction in eq. (3). The Higgs vacuum expectation value is known experimentally; $v_h = 246$ GeV and the $vev$ of the scalar $s$ is determined by other known parameters of the theory as seen from eq. (15).

The thermal evolution of the dark matter number density, $n_s$, in the early universe is given by the Boltzmann equation,

$$\frac{dn_s}{dt} + 3H n_s = -\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \left[ n_s^2 - \left( n_s^{\text{EQ}} \right)^2 \right], \quad (20)$$

where $H$ is the Hubble expansion rate, $v_{\text{rel}}$ stands for the dark matter relative velocity and the $\sigma_{\text{ann}}$ is the dark matter annihilation cross section. We compute the relic abundance using the MicrOMEGAs4.3 package [10] that numerically solves the Boltzmann differential equation. We recall that the potential we use to compute the relic density is the potential in eq. (1) after the electroweak symmetry breaking which is given by,

$$V(h, s, s') = \frac{1}{2} m_h^2 h^2 + \frac{1}{2} m_s^2 s'^2 + (\lambda_h + \lambda_{hs}) \sqrt{1 - \frac{\lambda_{hs}}{\lambda_h}} v_h h^3 + \frac{1}{4} \frac{(\lambda_h + \lambda_{hs})^2}{\lambda_h} h^4$$

$$+ (\lambda_h + \lambda_{hs}) \sqrt{-\frac{\lambda_{hs}}{\lambda_h}} h^3 s$$

$$+ 2 \sqrt{-\lambda_{hs}} (\lambda_h - \lambda_{hs}) v_h h^2 s$$

$$- \lambda_{hs} h^2 s^2 + \frac{\lambda_h \lambda_{s's'} v_h}{\sqrt{-\lambda_{hs} (\lambda_h - \lambda_{hs})}} h s'^2$$

$$- \frac{\lambda_{s's'} \lambda_h v_h}{\sqrt{\lambda_h (\lambda_h - \lambda_{hs})}} h s'^2 + \frac{\lambda_{s's'} \lambda_h \lambda_{s's'}}{\lambda_h - \lambda_{hs}} h s'^2$$

$$+ \frac{1}{2} \frac{\lambda_h \lambda_{s's'}}{\lambda_h - \lambda_{hs}} h s'^2 - \frac{1}{2} \frac{\lambda_{s's'} \lambda_h}{\lambda_h - \lambda_{hs}} h^2 s'^2 + \frac{1}{4} \frac{\lambda_{s's'}}{\lambda_h} s'^4. \quad (21)$$

Note that the Higgs scalar field has a mass term now in eq. (21). The phase transition (going from the symmetric phase with $v_h = 0$ to the broken phase with $v_h \neq 0$) is followed by the scale symmetry breaking through the radiative correction to the scalar mass. We constrain the model by the observed dark matter relic abundance from the WMAP/Planck [17, 18] to be $\Omega_{\text{DM}} h^2 \sim 0.11$. In section IV the mass of the dark matter had already a lower bound due to the positivity of the scalon mass; $m_s' \equiv m_{\text{DM}} > 316.5$ GeV. In Fig. 3 the dark matter mass is plotted against the only independent coupling i.e. the $\lambda_{hs}$. As seen from this figure the viable range of the coupling shrinks into $-0.007 \lesssim \lambda_{hs} \lesssim 0$. The DM mass however sits almost in the same limit we obtained in section III i.e. $m_{\text{DM}} > 318.3$ GeV. In Fig. 2 we have also demonstrated a histogram of the values $v_c/T_c$ which are bounded by the correct relic density. It is understood that interestingly $v_c/T_c$ is greater than 3.8 and much bigger that guarantees a very strong first-order electroweak phase transition.

VI. DIRECT DETECTION CONSTRAINT

There are experiments that have been set up with the goal to detect the elusive dark matter directly. Among which the XENON1t experiment located at Gran Sasso in Italy is the most recent and the more accurate one [19]. Although the XENON1t experiment and no other experiments such as LUX (see [20] for the recent results), have not detected the dark matter but they have put a very stringent constraint on the elastic scattering cross section of the dark matter off the nucleus. We examine the current model by data from the direct detection experiments.

The DM-nucleus cross section can be described simply by the following effective potential,

$$\mathcal{L}_{\text{eff}} = \alpha_q s' \bar{q} q, \quad (22)$$
where $q$ stands for the quark in the nucleon and $s'$ is the dark matter field. The coupling $\alpha_q$ is given by,

$$\alpha_q = m_q \frac{2\lambda_h \lambda_{ss'}}{\lambda_h - \lambda_{hs'}} \left( \frac{1}{m_h^2} - \frac{1}{m_s^2} \right).$$

(23)

The DM-nucleus scattering is obtained from a tree-level Feynman diagram leading to the following spin-independent elastic scattering cross section,

$$\sigma_{SI} = \frac{4\pi\alpha_N^2}{m_{DM}^2} \mu_N^2,$$

(24)

where $\mu_N$ is the reduced mass for the DM-nucleus system and $\alpha_N$ denotes a coefficient that depends on the nucleon form factors. For more details on $\alpha_N$ see [21] and the references therein.

For the viable parameter space obtained in section V we have computed the elastic scattering cross section in eq. (24) using the MicrOMEGAs4.3 package. The result is shown in Fig. 3. According to Fig. 3 only dark matter mass $m_{DM} \gtrsim 4.5$ TeV survives the XENON1t/LUX cross section limits [22] while respecting both the relic density constraint and the washout criterion. It is interesting also to determine the allowed masses of the scalon $s$ which is the mediator connecting the DM sector to the SM. As seen in Fig. 4 the scalon mass $m_s$, takes only values above 200 GeV.

VII. CONCLUSION

In this paper we have studied the minimal extension of the scale invariant standard model (MSISM) with two extra scalars, $s$ and $s'$ in addition to the Higgs particle. Two scalars are the minimum number of scalars we should add to the scale invariant standard model (SISM) to give a mass of 125 GeV to the Higgs and correct masses for other particles in the SM. The classically massive scalar, $s'$ is interpreted as a dark matter candidate and the classically massless scalar, $s$ called the scalon plays the role of the DM-SM mediator. We showed that this model supports a very strong first-order electroweak phase transition even if we constrain the model with the observed DM relic density by WMAP/Planck. Imposing the limits from the direct detection experiments such as XENON1t/LUX on the elastic scattering cross section of the DM-nucleus still allows the dark matter mass $m_{DM} \gtrsim 4.5$ TeV and the scalon mass $m_s \gtrsim 200$ GeV.

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