Kinematical solution of the UHE-cosmic-ray puzzle without a preferred class of inertial observers

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Among the possible explanations for the puzzling observations of cosmic rays above the GZK cutoff there is growing interest in the ones that represent kinematical solutions, based either on general formulations of particle physics with small violations of Lorentz symmetry or on a quantum-gravity-motivated scheme for the breakup of Lorentz symmetry. An unappealing aspect of these cosmic-ray-puzzle solutions is that they require the existence of a preferred class of inertial observers. Here I propose a new kinematical solution of the cosmic-ray puzzle, which does not require the existence of a preferred class of inertial observers. My proposal is a new example of a type of relativistic theories, the so-called “doubly-special-relativity” theories, which have already been studied extensively over the last two years. The core ingredient of the proposal is a deformation of Lorentz transformations in which also the Planck scale $E_p$ (in addition to the speed-of-light scale $c$) is described as an invariant. Just like the introduction of the invariant $c$ requires a deformation of the Galileian transformations into the Lorentz transformations, the introduction of the invariant $E_p$ requires a deformation of the Lorentz transformations, but there is no special class of inertial observers. The Pierre Auger Observatory and the GLAST space telescope should play a key role in future developments of these investigations. I also emphasize that the doubly-special-relativity theory here proposed, besides being the first one to provide a solution for the cosmic-ray puzzle, is also the first one in which a natural description of macroscopic bodies is achieved, and may find applications in the context of a recently-proposed dark-energy scenario.

I. INTRODUCTION

Over the last few years there has been a large number of studies concerning the emerging “cosmic-ray puzzle.” UHECRs (ultra-high-energy cosmic rays) should interact with the Cosmic Microwave Background Radiation (CMBR) and produce pions. Based on the typical energies of CMBR photons, and on a straightforward special-relativistic analysis of the kinematics of the process $p + \gamma \rightarrow p + \pi$ (assuming the UHECR is a proton), one is led to the conclusion that photopion production should render observations of UHECRs with $E > 5 \times 10^{19}$ eV (the GZK cutoff) extremely unlikely. Above $10^{20}$ eV the cosmic-ray mean free path should be only of a few Mpc, and there are no astrophysical sources capable of accelerating particles to such energies within a few tens of Mpc from us. Still, more than a dozen UHECRs have been reported by AGASA with nominal energies at or above $10^{20}$ eV. It is plausible that these UHECRs, the highest-energy particles we have access to, may be providing us a window on a new realm of fundamental physics. Proposed solutions extend from the idea that previously unknown particles play one or another role in evading the GZK cutoff to the idea, which is the main focus of the study here reported, that the description of these UHECRs as protons coming from cosmological distances is correct but the GZK analysis requires modifications due to new (“quantum”) properties of spacetime (and the associated laws of kinematics). Of course, the simplest (and perhaps most likely) explanation is that these preliminary experimental results may be misleading: that for example AGASA might have overestimated the energies of the observed UHECRs.

For what concerns the experimental side the situation will become much clearer in the not-so-distant future: especially the Pierre Auger Observatory should definitely clarify whether violations of the GZK limit are a reality. In the meantime it is legitimate to take as working assumption that we are (or may be) confronted with a “cosmic-ray puzzle”. In this paper I propose a new solution for the puzzle. My solution can be seen as a third example of “kinematical solution” of the cosmic-ray puzzle. I describe as “kinematical solutions” the solutions that are based on the assumption that the description of these UHECRs as protons coming from cosmological distances is correct and that the GZK analysis requires modifications due to a failure of Lorentz symmetry in the realm of the relevant photopion-production processes. At first this possibility was examined at a purely phenomenological level: in the studies a general parametrization of possible deviations from Lorentz symmetry was introduced, and choices of parameters that would shift upward the GZK limit were found. Then, motivated by the proposal put forward in
Ref. [3], there has also been work on a specific idea and scheme for the breakup of Lorentz symmetry, in which the GZK limit is shifted upward by Planck-scale effects, reflecting some new quantum properties of spacetime.

While very different on many levels, the two previous kinematical solutions of the cosmic-ray puzzle, the general-particle-physics phenomenology of Refs. [1][2] and the class of quantum-spacetime phenomenological models of Refs. [3][4][5], have a common feature: the assumed deviations from Lorentz symmetry require the emergence of a preferred class of inertial observers (often, without any conceivable justification, identified with the natural frame for the description of the CMBR). The primary objective of the study I am here reporting is to show that this feature, which may be perceived as unattractive by some colleagues, is not a necessary ingredient of kinematical solutions of the cosmic-ray puzzle: I will discuss a Planck-scale deformation of Lorentz symmetry which evades the GZK limit kinematically, but does not require a preferred class of inertial observers.

My scheme is a new example of a class of relativistic theories, the so-called “DSR” or “doubly-special-relativity” theories, which I proposed in Ref. [10], motivated by a certain perspective on the quantum-gravity problem, and have already been studied extensively over the last two years [11-20] DSR theories are relativistic theories that describe the laws of transformation between inertial observers in such a way that the Planck scale, $E_p$, acquires a role completely analogous to the one of the speed-of-light constant $c$: in DSR both $c$ and $E_p$ are observer-independent scales of the transformation laws. It is indeed plausible that we might discover the need for this type of relativistic theories in ultra-high-energy physics just in the same sense that we discovered the need for special relativity in the study of processes involving high velocities.

In the specific type of DSR theory that I propose as a solution of the cosmic-ray puzzle there are no deviations from special relativity for massless particles, while for particles with finite mass, $m$, the deviations from special relativity are negligible at low energies but become significant when the particle’s energy $E$ is large enough that $m/E < E/E_p$. For a proton the condition $m/E < E/E_p$ requires $E > 3 \times 10^{18}$ eV, and therefore the GZK cutoff happens to occur when the deviations from special relativity are predicted to be rather large. In general I observe that the ratio $E/m$ sets the magnitude of the boost, while $E/E_p$ will set the magnitude of Planck-scale effects, and that in a large class of DSR theories one should expect that the deviations from ordinary Lorentz symmetry become significant when $E/m$ is large enough to compensate for the smallness of $E/E_p$.

In the next section I review the previously-proposed mechanism, based on Lorentz-symmetry breaking, for a quantum-gravity solution of the cosmic-ray puzzle. In Section 3 I give a brief review of results in DSR theories, in which Lorentz symmetry is deformed rather than broken. In Section 4 I propose my DSR-based solution of the cosmic-ray puzzle. Section 5 offers some closing remarks.

II. PREVIOUS PLANCK-SCALE SOLUTIONS OF THE COSMIC-RAY PUZZLE

It is plausible that the unification of general relativity and quantum mechanics will involve some sort of spacetime quantization, e.g. noncommutative geometry or spacetime discreteness. If spacetime is fundamentally discrete or noncommutative then this should in particular apply to the spacetimes that low-energy probes perceive as Minkowski (flat classical and continuous). From the point of view of experimental tests a key point is that spacetime discreteness or noncommutativity are likely to require departures from Lorentz symmetry (see, e.g., Refs. [21][22]).

As a starting point for a phenomenology aimed at exploring this type of possible quantum-gravity effects, Refs. [2][3][23] proposed the investigation of a scenario predicting violations of Lorentz symmetry set by the Planck scale and characterized by the emergence of a modified dispersion relation of the type

$$m^2 = E^2 - \vec{p}^2 + f(E, \vec{p}^2; E_p) \approx E^2 - c^2 \vec{p}^2 + \eta \left( \frac{E}{E_p} \right)^n E^2,$$

where $\eta$ is a dimensionless coefficient of order 1. The function $f(E, \vec{p}^2; E_p)$ is only specified in leading order in $1/E_p$ since anyway experiments could at best find some evidence of the leading contribution due to $f$.

There appears to be a certain tendency in the literature to confuse the proposals based on particle-physics models with a general parametrization of possible deviations from Lorentz symmetry with the proposals based on quantum-spacetime models with deviations from Lorentz symmetry. For example, in Ref. [1] the Coleman-Glashow model of Ref. [5] is described as a model of Planck-scale deviations from Lorentz symmetry, whereas in Ref. [6] there is no mention of the Planck scale, quantum gravity, and quantum spacetime. A phenomenology of Planck-scale deviations from Lorentz symmetry was only proposed later in Ref. [10], and found its way in the cosmic-ray-puzzle literature only starting with the studies reported in Refs. [11][12].
This type of quantum-gravity scenarios is seen in analogy with the deformed dispersion relations that apply to the description of collective modes (such as phonons) in certain materials, whose propagation satisfies a relativistic dispersion relation only up to corrections governed by the scale of atomic structure of the material. Similar considerations apply to the description of the propagation light in water. Intuitively one can indeed attempt to introduce in quantum gravity the (however vague) concept of “spacetime foam” and spacetime foam may affect particle propagation in a way that to some extent can be viewed in analogy with the way that the presence of other media affects particle propagation.

One of the phenomenologically significant consequences of deformed dispersion relations of the type (3) is the associated deformation of the thresholds for certain particle-production processes. Let me illustrate this effect in the specific example of photopion production, \( p + \gamma \rightarrow p + \pi \), which is relevant for the cosmic-ray puzzle. The kinematic condition for photopion production is obtained by imposing conservation of energy and momentum and by applying the dispersion relation (which in this case is deformed). Denoting with \( E \) the energy of the incoming proton and denoting with \( \epsilon \) the energy of the incoming photon, one finds that photopion production is only possible if

\[
E \epsilon - \eta \frac{E^2 + n}{4E_p} \left( \frac{m_{\text{prot}}^{1+n} + m_\pi^{1+n}}{(m_{\text{prot}} + m_\pi)^{1+n}} - 1 \right) \geq \frac{(m_{\text{prot}} + m_\pi)^2 - m_{\text{prot}}^2}{4}.
\]

The \( E_p \rightarrow \infty \) limit, in which the second term on the left-hand side is neglected, of course reproduces the standard special-relativistic result for the photopion-production threshold condition. In the processes we study in laboratory experiments the \( E_p \)-dependent term is completely negligible, because of the large value of the scale \( E_p \). But in the analysis of the cosmic-ray puzzle one must consider an incoming proton of very high energy, of order \( 10^{20} \text{eV} \), while the photon is a CMBR photon with typical energy roughly of order \( 10^{-4} \text{eV} \), and therefore one finds that \( E \epsilon \sim 10^{17} \text{eV}^2 \), \( E^3/E_\epsilon \sim 10^{32} \text{eV}^2 \), \( E^3/E_\epsilon^2 \sim 10^{24} \text{eV}^2 \). From these estimates one concludes \((3)\) that, for \( n = 1 \) and \( n = 2 \), the formula \((3)\) predicts a significant shift upward of the GZK cutoff, and provides a solution of the cosmic-ray puzzle.

This striking observation has motivated several follow-up quantum-gravity studies based on the proposal \((3)\). More recently, other forms of Planck-scale modifications of the dispersion are also being considered \((24)\) in relation with the cosmic-ray puzzle.

These quantum-gravity-motivated solutions of the cosmic-ray puzzle, just like the mentioned non-quantum-gravity solutions \((3)\) that are also based on a breakup of Lorentz symmetry, necessarily require the existence of a preferred class of inertial observers. In fact, assuming that the Lorentz transformations maintain their familiar form, the only invariant dispersion relation is \( m^2 = E^2 - p^2 \). All deviations from this special-relativistic form of the dispersion relation will necessarily not be invariant: at least the values of the coefficients that characterize the modified dispersion relation (such as \( \eta \) in \((3)\)) will be different for different inertial observers and can be used to identify a preferred class of inertial observers. This is after all consistent with the mentioned quantum-gravity intuition based on “spacetime foam”, which would indeed be suitable for identifying a preferred class of inertial observers.

III. PREVIOUS DOUBLY-SPECIAL-RELATIVITY THEORIES

The fact that the quantum-gravity scenarios mentioned in the preceding section require a preferred class of inertial observers is not necessarily a disappointment. As mentioned certain quantum-gravity ideas, notably some perspectives on the spacetime-foam picture, can provide motivation for exploring this possibility. Moreover, in the study of particle physics the concept of spontaneous breaking of a symmetry has proven very useful, and it is conceivable that quantum gravity would host a similar mechanism for the spontaneous breaking of Lorentz symmetry at the Planck scale. For example, in string theory, the most popular approach to the quantum-gravity problem, mechanisms for the spontaneous breaking of Lorentz symmetry have been extensively investigated (see, e.g., Ref. \((25)\)).

However, the existence of a preferred class of inertial observers is anyway not simple conceptually. At present we can only make wild guesses about this preferred class of observers. Many studies attempt to identify this preferred class of inertial observers with a natural class of observers for the CMBR, but this conjecture (although it cannot be excluded a priori) appears to lack any justification, since there is no connection between the classical physics responsible for CMBR physics and the Planck-scale realm. Moreover, in any case, even if one is not troubled by the possibility of a preferred class of inertial observers, it is natural to wonder \((14)\) whether there are other alternatives in addition to the cases in which Lorentz symmetry is exactly preserved at the Planck scale and the case in which Lorentz symmetry is broken, with associated emergence of a preferred class of inertial observers. The so-called “doubly-special relativity” (“DSR”) theories that I proposed in Ref. \((10)\) are scenarios in which ordinary Lorentz symmetry is not exactly preserved at the Planck scale, but there is no special class of inertial observers. Using a terminology which is popular in the mathematics community, my proposal can be described as a “deformation” of Lorentz symmetry,
without any true loss of symmetry. This proposal can be motivated \[10\] from the observation that some quantum-gravity scenarios, notably a certain type of scenarios based on noncommutative geometry, appear to require it. It has also been conjectured \[13\] that DSR theories may be applicable to the popular “loop quantum gravity” theory.

Here, since I am focusing on the cosmic-ray puzzle, I can motivate DSR theories following an analogy with the developments which led to the replacement of Galileian relativity with special relativity. In Galileian relativity there is no observer-independent scale, and in fact the dispersion relation is written as \( E = p^2/(2m) \) (whose structure fulfills the requirements of dimensional analysis without the need for dimensionful coefficients). As experimental evidence in favour of Maxwell equations started to grow, the fact that those equations involve a special velocity scale appeared to require (assuming the Galilei symmetry group should remain unaffected) the introduction of a preferred class of inertial observers (the “ether”). However, in the end we discovered that the Maxwell theory does not require a preferred class of inertial observers. It was wrong to assume that Galileian relativity should apply in all regimes. In the high-velocity regime it must be replaced by special relativity. Special relativity introduces the first observer-independent scale, the velocity scale \( c \), and its dispersion relation takes the form \( E^2 = c^2 p^2 + c^4 m^2 \). As interest in dispersion relations of the type \( c^4 m^2 = E^2 - c^2 p^2 + f(E, p^2; E_p) \) is starting to grow within the quantum-gravity community, also because of the analysis of cosmic-ray observations, the fact that these dispersion relations involve a special energy scale, \( E_p \), is leading, as mentioned, to the assumption that a preferred class of inertial observers might have to be introduced. Could it be that this assumption is wrong? Could it be that once again we do not need a preferred class of inertial observers, but rather we need to deform once more the laws of transformation between inertial observers? DSR theories modify special relativity in the exact same sense in which special relativity modified Galileian relativity.

In Ref. \[10\], in proposing the idea of DSR theories, I also derived the key features of a first illustrative example (sometimes called “DSR1”) of these theories. This example, whose key characteristic is the dispersion relation

\[
2E_p^2 \left[ \cosh \left( \frac{E}{E_p} \right) - \cosh \left( \frac{m}{E_p} \right) \right] = p^2 e^{E/E_p},
\]

has been analyzed in detail in a series of papers \[11-16\] and is at this point reasonably well understood.

More recently, Magueijo and Smolin proposed \[13\] a second example of DSR theory, sometimes called “DSR2”, whose key characteristic is the dispersion relation

\[
m^2 = \frac{E^2 - p^2}{(1 - E_p)^2},
\]

For both of these theories we have now several results \[10-20\] The deformed Lorentz generators that implement (2) (respectively (3)) as observer-independent laws have been explicitly constructed. It has been established that the laws of composition of energy-momentum (needed, for example, when we implement energy-momentum conservation in a collision process) must be Planck-scale modified, just in the same sense that the law of composition of velocity (which is scale-independent in Galileian relativity) must be speed-of-light-scale modified in special relativity. A description of the deformed-boost action in terms of a dependence of energy-momentum on the rapidity parameter \( \xi \) (the coefficient of a boost generator \( N \) in the exponentiation \( e^{\xi N} \) that implements a finite boost transformation) has also been derived.

The relations involving rapidity are particularly useful for a characterization of a DSR theory. It is most convenient to focus on the amount of rapidity needed to take a particle from its rest frame to a frame in which its energy is \( E \) (and its momentum is \( p(E) \), which is fixed, once \( E \) is known, using the dispersion relation and the direction of the boost). In the DSR1 theory one finds

\[
cosh(\xi) = \frac{e^{E/E_p} - \cosh (m/E_p)}{\sinh (m/E_p)}, \quad \sinh(\xi) = \frac{p e^{E/E_p}}{E_p \sinh (m/E_p)},
\]

while in DSR2 one finds

\[
cosh(\xi) = \frac{E(1 - m/E_p)}{m(1 - E/E_p)}, \quad \sinh(\xi) = \frac{p(1 - m/E_p)}{m(1 - E/E_p)}.
\]

Of course, in the \( E_p^{-1} \to 0 \) limit these relations reproduce the corresponding special relativistic relations

\[
cosh(\xi) = \frac{E}{m}, \quad \sinh(\xi) = \frac{p}{m}.
\]
These rapidity relations are extremely useful in the analysis of DSR theories. On the basis of these relations one can introduce some useful functions of the physical energy, momentum, mass of a DSR theory. In DSR1 one introduces

$$ \frac{\mathcal{E}(E,m)}{\mu(m)} = \frac{e^{\lambda E} - \cosh (\lambda m)}{\sinh (\lambda m)}, \quad \frac{\mathcal{P}(E,p,m)}{\mu(m)} = \frac{p e^{E/E_p}}{E_p \sinh (m/E_p)}, \quad (9) $$

while in DSR2 one introduces

$$ \frac{\mathcal{E}(E,m)}{\mu(m)} = \frac{E(1 - m/E_p)}{m(1 - E/E_p)}, \quad \frac{\mathcal{P}(E,p,m)}{\mu(m)} = \frac{p(1 - m/E_p)}{m(1 - E/E_p)}. \quad (10) $$

The condition $\mu^2 = \mathcal{E}^2 - \mathcal{P}^2$ is understood everywhere. It is easy to show that in DSR1 and DSR2 the functions $\mathcal{E}$ and $\mathcal{P}$ defined, as described, on the basis of the rapidity relations, transform under the deformed Lorentz boosts (which act on their arguments $E, p$) just like special-relativistic energy and momentum would transform under ordinary Lorentz boosts. This allows to obtain most results in DSR1 or DSR2 following a very simple strategy: one does the analysis using the auxiliary momenta $\mathcal{E}$ and $\mathcal{P}$, which are easily handled because they transform under rotations and boosts in the familiar way, and then the DSR result is obtained by substituting in the final formula $\mathcal{E}$ and $\mathcal{P}$ with their expressions as functions $\mathcal{P}(E,p,m), \mathcal{E}(E,m)$ of the $E$ and $p$ which describe the physical energy and momentum in the DSR theory.

These useful properties of the functions $\mathcal{P}(E,p,m), \mathcal{E}(E,m)$ are a consequence of the fact that the symmetries implemented in the schemes DSR1 and DSR2 are actually a nonlinear realization of the Lorentz symmetry group. This simplicity is actually only present in the Lorentz sector: studies of Poincaré-like symmetries that would extended the Lorentz sectors of DSR1 and DSR2 are showing that this simplifications do not apply to the full Poincaré sector of the theory. The simplifications only apply to the descriptions of Lorentz transformations of energy-momentum space. This is rather obvious from the perspective of applications of the DSR framework in certain types of noncommutative spacetimes: the fact that the spacetime coordinates do not commute introduces (unless the commutators are themselves coordinate-independent) complications for what concerns the description of the product of plane waves (and the associated composition of momenta) and the description of translations. Rotations and boosts are not in a priori conflict with noncommutativity of the coordinates, but they are affected by this noncommutativity through the fact that their action of energy-momentum variables must reflect the new properties of energy-momentum.

Looking further ahead in the research programme on DSR theories one can conceive ways to introduce the second observer-independent scale $E_p$ such that even the Lorentz sector is affected by severe complexity. However, this possibility goes beyond the scope of the present study. Here I intend to show that exactly a DSR theory of the type described in this section, which in particular relies on a nonlinear realization of the Lorentz group, can be used to avoid the GZK cutoff.

IV. A NEW DSR THEORY THAT SOLVES THE COSMIC-RAY PUZZLE

In Section II I have discussed how quantum-gravity scenarios with Lorentz-symmetry breaking can solve the cosmic-ray puzzle, at the “price” of introducing a preferred class of inertial observers. In the previous section, Section III, I have discussed the first two examples, DSR1 and DSR2, of DSR theories. Since the DSR framework does not introduce a preferred class of inertial observers, if DSR1 and/or DSR2 were to solve the cosmic-ray puzzle, the solution would indeed not require a preferred class of inertial observers. However, the analysis of the threshold conditions for photopion production in DSR1 and DSR2 does not lead to a solution of the cosmic-ray puzzle. This analysis is actually very simple, because of the properties described above of the functions $\mathcal{P}(E,p,m), \mathcal{E}(E,m)$. In DSR1 and DSR2 the special relativistic condition for photopion production

$$ E\epsilon \geq \frac{(m_{prot} + m_\pi)^2 - m_{prot}^2}{4}, \quad (11) $$

is simply replaced by the condition

$$ \mathcal{E}(E, m_{prot})\epsilon \geq \frac{[\mu(m_{prot}) + \mu(m_\pi)]^2 - [\mu(m_{prot})]^2}{4}. \quad (12) $$

This result is valid to sufficient accuracy for the analysis of the cosmic-ray puzzle; in particular, for the energy of the CMBR photon I have not even introduced the deformation function because CMBR photons have such low energies that (as one can verify explicitly) their description in DSR1 and DSR2 is basically identical to the familiar special relativistic description.
As shown by Eqs. (10), DSR1 and DSR2 do not give the same prediction for the form of the function $\mathcal{E}(E, m_{\text{prot}})$, but in both cases one can easily verify that the difference between (12) and (11) leads to a negligibly small modification of the GZK cutoff.

This observation about DSR1 and DSR2 has already been known for some time and it has led to the expectation that DSR theories would, in general not provide solutions for the cosmic-ray puzzle, and therefore a kinematical solution of the cosmic-ray puzzle would require the emergence of a preferred class of inertial observers.

I will show here that instead there is a rather rich class of DSR theories that does provide a solution to the cosmic-ray puzzle. My analysis takes off from two related observations:

- The data that presently are at the basis of the cosmic-ray puzzle concern protons with energies in the range from $\sim 10^{19}\text{eV}$ to $\sim 10^{20}\text{eV}$. Taking into account that the proton has mass of $m_{\text{prot}} \sim 10^3\text{eV}$ (and the Planck scale is of about $E_p \sim 10^{28}\text{eV}$) it is noteworthy that the paradox emerges just above the scale $\sqrt{m_{\text{prot}} E_p} \sim 3 \cdot 10^{18}\text{eV}$.

- The relativistic theories DSR1 and DSR2 are deformations of ordinary special relativity which are governed by a single dimensionless ratio, $E/E_p$, but this cannot be assumed as a generic feature of DSR theories. Already in ordinary special relativity the ratio $E/m$ plays a central role (see, e.g., Eq. (5)), and it is therefore plausible (even natural) that in DSR theories the scales $E/m$ and $E/E_p$ would combine in such a way that the deformation would be also strongly characterized by the dimensionless quantity $E^2/(m E_p)$.

From these two observations the careful reader will be already guessing which type of DSR theories should be able to provide a solution of the cosmic-ray puzzle: theories in which the special-relativistic relation (7) is replaced by $P\Delta P = \alpha E^2/(m E_p)$ and denote by DSR3a a DSR theory in which the special-relativistic photopion-production threshold $P\Delta P = \alpha E^2/(m E_p)$ and denote by DSR3b a DSR theory in which (7) is replaced by $P\Delta P = \beta E^2/(m E_p)$.

Even just within the simple type of DSR theory reviewed in the previous section (DSR theories in which the rotation/boost generators act on the energy-momentum sector in a way that can be codified through functions of the type $P\Delta P = \alpha E^2/(m E_p)$, one can easily find deformations of the action of rotation/boost generators on the energy-momentum space that satisfy these conditions. It is useful to focus on a couple of explicit examples. Let me denote by DSR3a a DSR theory in which the special-relativistic relation (10) is replaced by

$$\cosh(\xi) = \frac{E}{m} - \frac{E^3}{m^2 E_p} . \tag{13}$$

and denote by DSR3b a DSR theory in which (10) is replaced by

$$\cosh(\xi) = \frac{E}{m} (2\pi)^{-E^2 \tanh[\pi E^2/(m E_p + E^2)]/(m E_p + E^2)} . \tag{14}$$

These conditions define implicitly the functions $\mathcal{E}(E, m)$ for DSR3a and DSR3b. I don’t even specify here the functions $P\Delta P = \alpha E^2/(m E_p)$, since their implications for the GZK threshold are negligible. The functions $P\Delta P = \alpha E^2/(m E_p)$ can be chosen, for example, in such a way as to impose the condition that $c$ remains the maximum attainable velocity.

Both in DSR3a, (13), and in DSR3b, (14), one finds an upward shift of the GZK cutoff that is large enough to provide a solution of the cosmic-ray puzzle. In fact, in DSR3a the special-relativistic photopion-production threshold condition (11) is replaced by

$$\left( E - \frac{E^3}{m_{\text{prot}} E_p} \right) \epsilon \geq \frac{(m_{\text{prot}} + m_{\pi})^2 - m_{\text{prot}}^2}{4} , \tag{15}$$

in which I have neglected other correction terms which are subleading with respect to the term $E^3/(m E_p)$, and in DSR3b on finds that, for $10^{19}\text{eV} < E < 10^{21}\text{eV},$

$$\left( E - \frac{(2\pi - 1)E}{2\pi} \right) \epsilon \geq \frac{(m_{\text{prot}} + m_{\pi})^2 - m_{\text{prot}}^2}{4} , \tag{16}$$

in which I have omitted again some negligibly small $E_p$-dependent corrections. The data reported by AGASA provide support for an upward shift of the GZK cutoff which is at least of a factor 6. This shift is clearly realized in (16), which

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\(^2\)Some authors have given up so completely on the hope of finding a DSR solution for the cosmic-ray puzzle that they arrived at proposing that solutions of the cosmic-ray puzzle would necessarily require two length/energy scales, rather than the single one available in DSR theories. The results I report in this paper provide counterexamples to this conjecture.
predicts a shift by a factor $2\pi$, and in (13), which actually predicts that there cannot be any photopion production involving CMBR photons (for $E > 10^{19}\text{eV}$, the quantity $E^2/(m_{\text{prot}}E_p)$ is greater than 1).

My two examples, DSR3a and DSR3b, of DSR deformations with dependence on $E^2/(mE_p)$ only serve the purpose of illustrating some mechanisms by which the dependence of the DSR deformation on $E^2/(mE_p)$ may lead to a relativistic theory that solves the cosmic-ray puzzle (without a preferred class of inertial observers). If this type of scenario is actually adopted by Nature it may well make use of some other type of dependence on $E^2/(mE_p)$.

The example DSR3a was chosen in order to show that a deformation with very simple structure can do the required task. A natural modification of the DSR3a scheme could be obtained by adopting a deformation function such that

$$\cosh(\xi) = f(E, m; E_p) \simeq \frac{E}{m} - \frac{E^3}{m^2 E_p},$$  

(17)

where $f$ is a function such that the approximation described in (17) holds at least for $10^{19}\text{eV} < E < 10^{21}\text{eV}$.

The example DSR3b was chosen in order to show that a deformation that leads to a solution of the cosmic-ray puzzle could also be structured in such a way that the new effects are confined in a relatively narrow range of scales. The DSR3b scheme automatically leads to negligible effects for massless particles (at all energies) while for particles with finite mass $m$ the effects are nonnegligible only for $\sqrt{mE_p} < E < (mE_p^2)^{1/3}$.

The (however ad hoc) deformation adopted in DSR3b also has the property that, even if one assumed that these formulas are applicable also to macroscopic bodies, the structure of the deformation function is such that the new effects are negligible for macroscopic bodies (the effects vanish in the $m \to \infty$ limit). In the general research programme on DSR theories a key open problem [13] concerns the DSR description of macroscopic bodies. The available data on macroscopic bodies allow us to exclude that deformations characterized by the dimensionless quantity $E/E_p$ be applicable to these macroscopic bodies. Since all DSR theories considered before the present paper were based on $E/E_p$ deformations, a key assumption is that the deformation only applies to microscopic particles. One can find [10,19] arguments to justify this assumption within DSR theories, but a satisfactory description is still lacking. When the deformation is governed, as here proposed, by the dimensionless quantity $E^2/(mE_p)$ and if the deformation is such that it can be neglected in the $E^2/(mE_p) \to \infty$ limit, as in the DSR3b case, then the deformation is automatically confined to the realm of microscopic particles (for a macroscopic body inevitably $E^2/(mE_p) \gg 1$), even before taking into account the DSR arguments that may hint at some fundamental reasons for confining the deformation to microscopic physics.

V. CLOSING REMARKS

I have shown here that there exist relativistic theories in which the GZK cutoff is shifted significantly upward (enough to explain the cosmic-ray puzzle) but there is no preferred class of inertial observers. My proposal is a new type of DSR theory, and it is therefore based on the idea that, just like a speed-of-light deformation of Galilean transformations turned out to be necessary when we acquired the capability of exploring contexts involving ultra-high velocities, we might need a Planck-scale deformation of Lorentz transformations in order to describe data obtained in ultra-high-energy contexts.

Of course, the fact that it is possible to find a solution of the cosmic-ray puzzle of the type here considered (kinematical, and without a preferred class of inertial observers) should not in any way reduce interest in the other kinematical solutions of the cosmic-ray puzzle. My analysis shows that a kinematical solution is also possible through a deformation of Lorentz symmetry, and therefore without a preferred class of inertial observers, but the ultimate answer will come from observations, and it may well be that Nature hosts a kinematical mechanism of violations of the GZK limit, based on a violation of Lorentz symmetry, and therefore requiring a preferred class of inertial observers. If indeed Planck-scale physics is responsible for the violations of the GZK limit, it is difficult to favour one or the other scenario: we still know very little about quantum gravity, and at present one appears to find equally plausible arguments that favour a Planck-scale violation of Lorentz symmetry [14,28,27] and arguments that favour a Planck-scale deformation of Lorentz symmetry [14,13].

In this paper, which is mainly intended for the astrophysics community, I have given a brief overview of DSR concepts and techniques and I have described my new DSR proposal only to the extent that is of interest for the cosmic-ray puzzle. In a paper now in preparation [23], which will be primarily intended for the quantum-gravity community, I will describe the proposal in detail, working out all aspects of a couple of DSR theories with deformation governed by the quantity $E^2/(mE_p)$.

The element of analysis provided here should be sufficient for DSR novices to get started in the task of attempting to figure out if there is a particularly compelling example of the type of DSR theories here proposed, with deformation governed by the dimensionless quantity $E^2/(mE_p)$. The deformations DSR3a and DSR3b that I discussed here should
be seen only as the starting point for these studies, since, of course, I cannot argue in any way in favour of a special compellingness of those proposals. I needed to formulate some specific proposals in order to illustrate the fact that $E^2/(mE_p)$ deformations of DSR type are possible and that they can provide solutions for the cosmic-ray puzzle.

While the search of a conceptually compelling proposal for a $E^2/(mE_p)$ deformation of DSR type may prove insightful, of course the ultimate task is to test the idea experimentally. As discussed in detail elsewhere [1][2][22][23], some planned observations can provide important input for this type of research endeavor. Clearly the key input will come from the Pierre Auger Observatory [3], which should establish whether or not there are indeed violations of the GZK cutoff and should provide important hints concerning the possibility that the violations be due to deformed kinematics. Concerning the specific structure of the DSR deformation, studies such as the ones planned by the GLAST space telescope [30] should provide important hints, since they are sensitive [6][10][23][29] to deformations of the $E(p)$ dispersion relation, which is a key feature of a DSR theory [10][19].

Among the possible avenues for extending the number of contexts in which the class of DSR theories here proposed may find applications it is worth mentioning, in closing, the scenario for dark energy considered in Refs. [31]. That scenario is based on a specific assumption concerning the structure of the dispersion relation. On the basis of the assumed deformation of the dispersion relation it is also assumed that Lorentz invariance should be broken at the Planck scale, with the associated emergence of a preferred class of inertial observers. The type of structure of the dispersion relation which is adopted in these dark-energy scenarios appears to be well suited for description within the type of DSR theories here proposed. If such a description is achieved one would then have a dark-energy scenario with phenomenological motivation analogous to the one advocated in the studies [31], but without the assumption of the existence of a preferred class of inertial observers.

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