NONLOCAL COLOR INTERACTIONS IN A GAUGE-INVARİANT FORMULATION OF QCD

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We construct a set of states that implement the non-Abelian Gauss’s law for QCD. We also construct a set of gauge-invariant operator-valued quark and gluon fields by establishing an explicit unitary equivalence between the Gauss’s law operator and the ‘pure glue’ part of the Gaussian law operator. This unital equivalence enables us to use the ‘pure glue’ Gauss’s law operator to implement the entire Gaussian law operator in a new representation. Since the quark field commutes with the ‘pure glue’ Gauss’s law operator, it is a gauge-invariant field in this new representation. We use the unitary equivalence of the new and the conventional representations to construct gauge-invariant quark and gluon fields in both representations, and to transform the QCD Hamiltonian into a gauge-invariant form in the temporal gauge so that it is expressed entirely in terms of gauge-invariant quantities. In that form, all interactions between quark fields mediated by ‘pure gauge’ components of the gluon field have been transformed away, and replaced by a nonlocal interaction between gauge-invariant color-charge densities. This feature — that, in gauge-invariant formulations, interactions mediated by pure gauge components of gauge fields have been replaced by nonlocal interactions — is shared by many gauge theories. In QED, the resulting nonlocal interaction is the Coulomb interaction, which is the Abelian analog of the QCD interaction we have identified and are describing in this work. The leading term, in a multipole expansion, of this nonlocal QCD interaction vanishes for quarks in color-singlet configurations, suggesting a dynamical origin for color confinement; higher order terms of this multipole expansion suggest a QCD mechanism for color transparency. We also show how, in an SU(2) model, this nonlocal interaction can be evaluated nonperturbatively.

1 Introduction

The program of the work we are discussing is designed to take advantage of a common thread that appears to underlie all gauge theories — non-Abelian as well as Abelian: When only gauge-invariant fields are used in constructing the Hamiltonian for a gauge theory, interactions between charged fields (electrical, color, etc.) and pure-gauge components of gauge fields cannot arise, and a nonlocal interaction between charge densities — the Coulomb interaction in QED — appears in their stead. Moreover, in gauge-invariant QED, the only interaction besides the Coulomb interaction is between the transverse (gauge-invariant) part of the gauge field and the current density; this interaction takes the form $-\int j_i(r)A_{T_i}(r)dr$, where $A_{T_i}(r)$ is the transverse (gauge-invariant) part of the gauge field, and $\vec{j}(r) = e\bar{\psi}(r)\gamma^i\gamma^5\slashed{v}\psi(r)$. Since the current density has a $v/c$ dependence in the nonrelativistic limit, the Coulomb interaction is, by far, the most important electromagnetic force in the low-energy regime.

We will show that a very similar effect occurs in QCD. The main difference between the two cases is that the fact that constructing gauge-invariant gauge fields is much more difficult in non-Abelian theories than in Abelian ones. Also, the nonlocal interaction in QCD is more complicated, and involves not only quark color-charge densities, but also gauge-invariant gluon fields. Significantly, in QCD, just as in QED, the only interaction other than the nonlocal one involves the gauge-invariant gauge (gluon) field and a quark color-current density that is the non-Abelian analog of the current density in QED, and that can be expected to also have a $v/c$ dependence in the nonrelativistic limit.

In this paper we will show how to implement the non-Abelian Gauss’s law, how to construct gauge-invariant quark and gluon states, and how to express the QCD Hamiltonian in terms of these gauge-invariant fields. We will also discuss the physical implications of the formal results we obtain.

2 Implementing Gauss’s law for QCD

The Gauss’s law operator in QCD is given by

$$\tag{1} \hat{G}^{a}_{\alpha}(r) = \partial_{\alpha}P_{\beta}(r) + g\epsilon^{abc}\hat{A}(r)\hat{P}_{\beta}(r) + j_{a}^{\alpha}(r),$$

where $j_{a}^{\alpha}(r) = g\bar{\psi}(r)\gamma^{a}\gamma^{5}\slashed{v}\psi(r)$ is the quark color-charge density, and $P_{\alpha}(r)$ is the gluon color-charge density. To implement the non-Abelian Gauss’s law, we must construct states that are annihilated by the Gauss’s law operator, $i.e.$ we must solve

$$\tag{2} \hat{G}^{a}_{\alpha}(r)|\Psi\rangle = 0.$$
We will initially implement the ‘pure glue’ form of Gauss’s law for QCD,
\[ D_i \Pi_i^j(r) \Psi = 0, \]  
by constructing a state \( |\Psi\rangle = \Psi |\phi\rangle \), for which
\[ D_i \Pi_i^j(r) |\Psi\rangle = \{ \partial_i \Pi_i^j(r) + J_0^i(r) \} |\Psi\rangle = 0; \]  
\( |\phi\rangle \) represents a state that is annihilated by \( \partial_i \Pi_i^j(r) \) — the so-called ‘Fermi’ state. We then seek to construct an operator \( \Psi \), for which
\[ [ \partial_i \Pi_i^j(r), |\Psi\rangle ] = -J_0^i(r) |\Psi\rangle + B_Q^j(r), \]  
where \( B_Q^j(r) \) is an operator that has \( \partial_i \Pi_i^j(r) \) on its extreme right. Eq.\( (3) \) is essentially an operator differential equation, in which the commutator, \( [ \partial_i \Pi_i^j(r), |\Psi\rangle ] \), is a derivative.

We have found a solution of this equation,\( (4) \) in which \( |\Psi\rangle \) is expressed as
\[ |\Psi\rangle = || \exp(A) || \]  
with
\[ A = i \int dr A_i^j(r) \Pi_i^j(r), \]  
and with \( A_i^j(r) \) represented as the series
\[ A_i^j(r) = \sum_{n=1}^{\infty} g^n A_i^j_{(n)}(r). \]  
The \( A_i^j_{(n)}(r) \) are nonlinear functionals of transverse and longitudinal parts of gauge fields, but are independent of the canonical momentum \( \Pi_i^j(r) \). The ordered product \( || \exp(A) || \) is defined so that, in the \( n^{th} \) order term, \( ||(A)^n || \), all functionals of the gauge field \( A_i^j \) are to the left of all functionals of the canonical momenta \( \Pi_i^j \). We refer to \( A_i^j(r) \) as the resonant gauge field.

The requirement that \( |\Psi\rangle \) implement Gauss’s law can be translated into a condition on the resonant gauge field. Before we formulate this condition, we first define the following quantities:
\[ \chi_i^j(r) = \frac{\partial}{\partial x_i} A_i^j(r) \]  
and \( \mathcal{R}_{(n)}^\gamma(r) = \prod_{n=1}^{\gamma} \chi^{[m]}(r) \), which are functionals of gauge fields; \( \mathcal{Y}_{(n)}^\gamma(r) = \frac{\partial}{\partial x_i} A_i^j(r) \) and \( \mathcal{M}_{(n)}^\gamma(r) = \prod_{n=1}^{\gamma} \mathcal{Y}^{[m]}(r) \), which have structures similar to \( \chi_i^j(r) \) and \( \mathcal{R}_{(n)}^\gamma(r) \) respectively, but which are functionals of the resonant gauge fields. We also need to define:
\[ f(\gamma; (\eta_i)) = f^{\alpha\beta\gamma}(\eta) f^{\beta\gamma \delta}(\eta) f^{\eta \gamma \delta}(\eta) f^{\delta \gamma \delta}(\eta) \times \cdots \]  
\[ \times f^{\eta \gamma \delta}(\eta) f^{\delta \gamma \delta}(\eta) f^{\delta \gamma \delta}(\eta) f^{\delta \gamma \delta}(\eta) \]  
these are chains of structure constants whose ‘links’ are summed over repeated indices.

The condition on the resonant gauge field that is equivalent to implementing the ‘pure glue’ Gauss’s law, is
\[ ig f^{\alpha \beta \gamma} A_i^\alpha(r) \int d^3 r' \bar{\Pi}_{(n)}^{\beta}(r') V_j^\gamma(r') \]  
\[ + \int d^3 r' [\partial_i \Pi_i^j(r), \bar{A}_i^\beta(r')] V_j^\gamma(r') + g f^{\alpha \mu \nu} A_i^\alpha(r) V_i^\mu(r) \]  
\[ = \sum_{\eta=1}^{\infty} g^{\alpha \beta \gamma} f^{\beta \gamma \delta}(\eta) \bar{A}_i^\delta(r) \bar{\Pi}_{(\eta)}^\gamma(r) \]  
\[ - \sum_{\eta=1}^{\infty} g^{\alpha \beta \gamma} \sum_{\eta=1}^{\infty} g^{\beta \gamma \delta}(\eta) \bar{\Pi}_{(\eta)}^\gamma(r) \]  
\[ = \sum_{\eta=1}^{\infty} g^{\alpha \beta \gamma} \sum_{\eta=1}^{\infty} g^{\beta \gamma \delta}(\eta) \bar{\Pi}_{(\eta)}^\gamma(r) \]  
\[ \sum_{\eta=1}^{\infty} g^{\alpha \beta \gamma} \sum_{\eta=1}^{\infty} g^{\beta \gamma \delta}(\eta) \bar{\Pi}_{(\eta)}^\gamma(r) \]  
where \( B(\eta) \) is the \( \eta^{th} \) Bernoulli number, and \( V_j^\gamma(r) \) represents any arbitrary vector field in the adjoint representation of SU(3).

We have solved Eq.\( (9) \):\( (10) \) The solution is given by
\[ \int d^3 r \bar{A}_i^\alpha(r) V_j^\gamma(r) = \sum_{\eta=1}^{\infty} \frac{ig}{\eta} \int d^3 r \{ \psi_{(\eta)j}^\gamma(r) + \]  
\[ f^{\gamma \delta \delta}(\eta) \bar{M}_{(\eta)}^\gamma(r) \bar{E}_{(\eta)j}^\beta(r) \} V_j^\gamma(r), \]  
where
\[ \psi_{(\eta)j}^\gamma(r) = (-1)^{\eta-1} f^{\gamma \delta \delta}(\eta) \bar{R}_{(\eta)}^\gamma(r) \bar{Q}_{(\eta)}^\beta(r) \]  
with \( \bar{Q}_{(\eta)j}^\beta(r) = [\bar{a}_i^\beta(r) + \frac{\eta}{(\eta+1)} x_i^\beta(r)] \), \( (11) \) and
\[ \bar{E}_{(\eta)j}^\beta(r) = a_i^\beta(r) + \left[ \frac{\delta_{ij} - \frac{\eta}{(\eta+1)} \bar{a}_i^\beta(r)}{\eta+1} \right] \bar{A}_i^\beta(r); \]  
\[ (13) \]
\( a_i^\alpha \) and \( x_i^\alpha \) designate the transverse and longitudinal parts of the gauge field \( A_i^j(r) \) respectively. Eq.\( (10) \) calls for summations over the multiplicity index \( \eta \), which labels the multiplicity of \( \bar{Y}^\gamma(r) \) factors in \( \bar{M}_{(\eta)}^\gamma(r) \), and the multiplicity of \( \chi^\gamma(r) \) factors in \( \bar{R}_{(\eta)}^\gamma(r) \). This multiplicity of factors makes Eq.\( (10) \) a nonlinear integral equation that specifies the resonant gauge field \( \bar{A}_i^\beta(r) \) recursively. The possibility that this nonlinear equation has multiple solutions, and that these multiple solutions correspond to different physical regimes, is interesting, but as yet unexplored.

In our earlier work,\( (10) \) we proved that Eq.\( (10) \) is a solution of Eq.\( (9) \) — a proof that we have referred to as the ‘fundamental theorem’ — and that the resonant gauge field, therefore, is the operator-valued field required for
the implementation of the non-Abelian Gauss’s law. We will see, in the remainder of this paper, that the resolvent gauge field plays a pivotal role in the construction of gauge-invariant fields and in the transformation of the QCD Hamiltonian to a functional of gauge-invariant fields.

3 Gauge-invariant quark and gluon fields

The observation that underlies the construction of gauge-invariant quark and gluon fields is that the Gauss’s law operator, $\hat{G}^a(r)$, and the ‘pure glue’ Gauss’s law operator $G^a_\alpha(r) = D_i \Pi^a_\alpha(r)$ are unitarily equivalent. In earlier work\cite{et.al.} we have shown that

$$\hat{G}^a(r) = \mathcal{U}_C \hat{G}^a_\alpha(r) \mathcal{U}_C^{-1},$$

where $\mathcal{U}_C = \exp(C_0) \exp(\mathcal{C})$ with $C_0$ and $\mathcal{C}$ given by

$$C_0 = i \int dr \lambda^\alpha(r) j^\alpha_0(r), \quad \text{and}$$

$$\mathcal{C} = i \int dr \overline{\lambda}^\alpha(r) j^\alpha_0(r).$$

We are therefore free to interpret the ‘pure glue’ Gauss’s law operator $\hat{G}^a_\alpha(r)$ as the complete Gauss’s law operator, $\hat{G}^a(r)$, in a different, unitarily equivalent representation. We will refer to the conventional representation, in which $\hat{G}^a(r)$ is the complete Gauss’s law operator and $G^a_\alpha(r)$ the ‘pure glue’ Gauss’s law operator, as the $\mathcal{C}$ representation; and the new representation, in which $\hat{G}^a_\alpha(r)$ is the unitarily transformed complete Gauss’s law operator, as the $\mathcal{N}$ representation. It is manifest that the spinor (quark) field $\psi(r)$ commutes with $\hat{G}^a_\alpha(r)$. Since the Gauss’s law operator is the generator of gauge transformations, $\psi(r)$ is a gauge-invariant spinor (quark) field in the $\mathcal{N}$ representation. To find the corresponding gauge-invariant quark field in the $\mathcal{C}$ representation, we apply the transformation that appears in Eq.\cite{et.al.}, and obtain

$$\psi_{\mathcal{C}}(r) = \mathcal{U}_C \psi(r) \mathcal{U}_C^{-1} = \psi_{\mathcal{N}}(r),$$

where

$$\psi_{\mathcal{C}}(r) = \exp \left( -ig \overline{\lambda}^\alpha(r) \lambda^\alpha \right) \exp \left( -ig \lambda^\alpha(r) \overline{\lambda}^\alpha \right),$$

and the $\lambda^b$ represent the Gell-Mann SU(3) matrices. $V_C(r)$ could be written as $\exp(-igZ^\alpha(r) \overline{\lambda}^\alpha)$, where the Baker-Hausdorff-Campbell theorem can be used to express $Z^\alpha$ as a functional of $\lambda^\alpha$ and $\overline{\lambda}^\alpha$. $V_C(r)$ takes the form of a unitary operator that carries out a gauge transformation on the spinor field $\psi(r)$; but, in this case, $Z^\alpha$ is not a c-number function in the adjoint representation of SU(3), but is a complicated functional of the gauge field. Under a gauge transformation, precisely compensating transformations are made on $\psi(r)$ and $V_C(r)$, so that $\psi_{\mathcal{C}}(r)$ is strictly gauge-invariant. When they appear in the $\mathcal{N}$ representation, the color-charge and color-current densities, $j_\alpha^a_0(r) = g \psi(r) \lambda^\alpha \psi(r)$ and $j_\alpha^a(r) = g \psi(r) \alpha_\mu \partial_\mu \psi(r)$ respectively, are gauge-invariant, although, in the $\mathcal{C}$ representation, both of these quantities transform gauge-covariantly, as vectors in the adjoint representation of SU(3). In the $\mathcal{N}$ representation, the quark field $\psi(r)$ implicitly includes enough of the gluon field to achieve this gauge invariance.

We have used Eq.\cite{et.al.} to find the gauge-invariant gluon field

$$[A_{GI}^h(r) \frac{\partial}{\partial x} ] = V_C(r) [ A_C^b(r) \frac{\partial}{\partial x} ] V_C^{-1}(r) + i g V_C(r) \partial_i V_C^{-1}(r),$$

or, equivalently,\cite{et.al.}

$$A_{GI}^h(r) = A_C^b(r) + [\delta_{ij} - \frac{\partial_i \partial_j}{\partial x^2} ] A_C^c(r).$$

We observe that, as in QED, the gauge-invariant gauge field $A_{GI}^b(r)$ is transverse. But it is not merely the transverse part of the gauge field. In contrast to the gauge-invariant gauge field in QED, $A_{GI}^b(r)$ also involves the transverse part of the resolvent gauge field.

We have expanded $\psi_{\mathcal{C}}(r)$ and $A_{GI}^b(r)$, and have verified that our gauge-invariant quark and gluon fields agree with the perturbative calculations of Lavelle, McMullan, et.al. to the highest order to which their perturbative calculations were available.\cite{et.al.}

4 A gauge-invariant QCD Hamiltonian

We have been able to express the QCD Hamiltonian

$$H_{QCD} = \int dr \left\{ \frac{1}{2} \Pi^a_\alpha(r) \Pi^a_\alpha(r) + \overline{F}^a_{ij}(r) F^a_{ij}(r) + \psi^\dagger(r) \left[ \beta m - ig \alpha_i \left( \partial_i - i g A^a_\alpha(r) \frac{\overline{\lambda}^\alpha}{2} \right) \right] \psi(r) \right\} ,$$

entirely in terms of gauge-invariant quark and gluon fields\cite{et.al.} by systematically transforming it, term by term, from the $\mathcal{C}$ to the $\mathcal{N}$ representation. Under this transformation, $\psi(r) \rightarrow V_C^{-1}(r) \psi(r), \psi^\dagger(r) \rightarrow \psi^\dagger(r) V_C(r)$, but the gauge field remains untransformed. Shifting $V_C(r)$ to the right and $V_C^{-1}(r)$ to the left until they encounter the $A^a_\alpha(r) \frac{\overline{\lambda}^\alpha}{2}$, turns the second line of Eq.\cite{et.al.} into

$$\tilde{H}_{J-A} = - \int dr \ g \psi^\dagger(r) \alpha_\mu \frac{\overline{\lambda}^\alpha}{2} \psi(r) \ A^h_{GI}(r).$$

$F^{a}_{ij}(r) F^{a}_{ij}(r)$ remains untransformed in this process, but we found that this similarity transformation, when applied to $\Pi^a_\alpha(r) \Pi^a_\alpha(r)$, generates nonlocal interactions between color-charge densities which we will discuss below.
The QCD Hamiltonian that results from the transformation to the $N$ representation is

$$\hat{H}_{\text{QCD}} = \int \left\{ \frac{i}{2} \left[ \Pi_{i}^a(r) \Pi_{i}^a(r) + \frac{1}{4} F_{i j}^a(r) F_{i j}^a(r) + \psi^\dagger(r) \left[ i \beta m - i \alpha_i \partial_i \right] \psi(r) \right] + \tilde{H}^\prime \right\},$$

(23)

where $\tilde{H}^\prime$ describes interactions involving the gauge-invariant quark field. The parts of $\tilde{H}^\prime$ relevant to the dynamics of quarks and gluons can be expressed as

$$\tilde{H}^\prime = \tilde{H}_{j-A} + \tilde{H}_{LR},$$

(24)

where $\tilde{H}_{LR}$ is the nonlocal interaction

$$\tilde{H}_{LR} = H_{g-Q} + H_{Q-Q}.$$

(25)

We will initially focus our attention on $H_{Q-Q}$ in this report, since it illustrates the most important features of our work. A useful formulation of $H_{Q-Q}$ can be given as

$$H_{Q-Q} = \frac{1}{2} \int d^4 x \int d^4 y \int d^4 z \left\{ \frac{\delta_{ab}}{4 \pi |x - y|} + 2 \sqrt{\delta} f^{(1)ba} \int \frac{dy}{4 \pi |r - y|} A^{(1)}_{G i}(y) \partial_i \frac{1}{4 \pi |y - x|} + \cdots \right\},$$

(26)

where the Green’s function $F^{(a)}(r, x)$ is represented as

$$F^{(a)}(r, x) = \frac{\delta_{ab}}{4 \pi |r - x|} + 2 \sqrt{\delta} f^{(1)ba} \int \frac{dy}{4 \pi |r - y|} A^{(1)}_{G i}(y) \partial_i \frac{1}{4 \pi |y - x|} + \cdots$$

(27)

We make the following observations about $F^{(a)}(r, x)$: The initial term resembles the Coulomb interaction. The infinite series of further terms consists of chains through which the interaction is transmitted from one color-charge density to the other. Each chain contains a succession of ‘links’, which have the characteristic form

$$\text{link} = g f^{s_{(1)}s_{(2)}} A^{(s)}_{GL}(x) \partial_i \frac{1}{4 \pi |y - x|}.$$

(28)

The ‘links’ are coupled through summations over the $s_{(n)}$ indices and integrations over the spatial variables. As noted before, all the quantities in $H_{Q-Q}$ are gauge-invariant — the color-charge density $\tilde{j}^{(b)}_0(r)$ as well as the gauge field $A^{(b)}_{G i}(x)$. $H_{g-Q}$ — the other nonlocal interaction in $\tilde{H}_{LR}$ — couples quark to gluon color charge density. In $H_{g-Q}$ the quark-color-charge density is coupled, through the same Green’s function $F^{(ba)}(r, x)$, to a gauge-invariant expression describing ‘glue’ color, which we have found to be

$$K^{(a)}_g(r) = g f^{d e c} \mathcal{Tr} \left[ V_C^{-1}(r) \frac{\lambda^a}{2} V_C(r) \right] A^{(b)}_{GL}(r).$$

The chains that constitute $F^{(ba)}(r, x)$ — coupled links, in which the $n^{th}$ order chain is a product of $n$ links — suggest features closely associated with QCD: flux tubes, ‘string’-like structures tying colored objects to each other, etc. But these rudimentary analogies only serve to direct our attention to potentially important features of this interaction — they do not, by themselves, demonstrate a physical effect.

With regard to $\tilde{H}_{j-A}$, we observe that the color-current density $\tilde{j}^{(b)}_0(r)$ has a configuration-space structure that is similar to that of the electric current density in QED. We can therefore reasonably expect that it, too, will manifest a $\nu/c$ dependence for nonrelativistic quarks interacting with the purely transverse $A^{(a)}_{GL}(x)$, and that $\tilde{H}_{LR}$ will be of predominant importance in the low-energy QCD regime.

5 Quark confinement and color transparency

The use of Eqs. (26) and (27) to explicitly evaluate an effective ‘potential’ between quark color-charge densities is precluded, at the present time, by the fact that we have not yet found an expression for the gauge-invariant gauge field $A^{(a)}_{GL}(x)$. However, we can draw some important conclusions from the general form of Eqs. (26) and (27).

If we assume that the Green’s function $F^{(ba)}(r, x)$ varies smoothly as a function of $x$, when evaluated in an appropriately chosen state, and that quarks, or configurations of quarks, are localized in wave packets whose size is small enough so that $F^{(ba)}(r, x)$ varies only moderately over the space occupied by these wave packets, then we are entitled to make a ‘color-multipole’ expansion of the expression $\int F^{(ba)}(r, x) \tilde{j}^{(b)}_0(x) dx$ — which appears in Eq. (26) — about the point $x = x_0$, in the form

$$\int dx \left\{ F^{(ba)}(r, x_0) + X_i \partial_i F^{(ba)}(r, x_0) + \frac{1}{2} X_i X_j \partial_i \partial_j F^{(ba)}(r, x_0) + \cdots \right\} \tilde{j}^{(b)}_0(x).$$

(29)

where $X_i = (x - x_0)_i$ and $\partial_i = \partial / \partial x_i$. When we perform the integration in Eq. (29), the first term contributes $F^{(ba)}(r, x_0) Q^a$, where $Q^a = \int dx \tilde{j}^{(a)}_0(x)$ (the integrated “color charge”). Since the color charge is the generator
of rotations in SU(3) space, it will annihilate any multi-quark state vector in a singlet color configuration. Multi-quark packets in a singlet color configuration therefore are immune to the leading term of the nonlocal $H_{Q-Q}$. Color-singlet configurations of quarks are only subject to the color multipole terms, which act as color analogs to the Van der Waals interaction. The scenario that this model suggests is that the leading term in $H_{Q-Q}$, namely $Q^b \mathcal{F}^{ba}(r_0, x_0) Q^a$ for a quark color charge $Q^a$ at $r_0$ and another quark color charge $Q^b$ at $x_0$, may be responsible for the confinement of quarks and packets of quarks that are not in color-singlet configurations. Moreover, assuming that $\mathcal{F}^{ba}(r, x_0)$ varies only gradually within a volume occupied by quark packets, the effect of the higher order color multipole forces on a packet of quarks in a color-singlet configuration becomes more significant as the packet increases in size. As small quark packets move through gluonic matter, they will experience only insignificant effects from the multipole contributions to $H_{Q-Q}$, since, as can be seen from Eq. (29), the factors $X_i, X_i X_j, \cdots, X_i^{(1)} \cdots X_i^{(n)}$, keep the higher order multipole terms from making significant contributions to $\int d^x \mathcal{F}^{ba}(r, x) j_0^a(x)$ when they are integrated over small packets of quarks. As the size of the quark packets increases, the regions over which the multipoles are integrated also increases, and the effect of the multipole interactions on the color-singlet packets can become larger. This dependence on packet size of the final-state interactions experienced by color-singlet states — i.e. the increasing importance of final-state interactions as color-singlet packets grow in size — is in qualitative agreement with the characterizations of color transparency given by Miller and by Jain, Pire and Ralston.

In making this argument, the following features of this gauge-invariant formulation must be borne in mind: The state vectors that can appropriately be chosen for such a representation must implement the non-Abelian Gauss’s law applicable to QCD — i.e. the states would have to be annihilated by the Gauss’s law operator, which is $\mathcal{G}$ in the $\mathcal{N}$ representation. That still allows us to apply quark creation operators to such states to form multi-quark Fock states with orbitals whose configuration-space dependence is arbitrary as far as the gauge problem is concerned. These states are far from being Fock states in the gluon sector — they have to be complex enough to implement Gauss’s law. But that fact does not inhibit us in using Fock states for multi-quark configurations, because, in the $\mathcal{N}$ representation, the quark fields contain enough gluon contributions to be gauge-invariant by themselves. The multi-quark states therefore have a status similar to multi-electron states in a gauge-invariant formulation of QED (e.g. in the Coulomb gauge), which are surrounded by a Coulomb field and therefore are gauge-invariant.

In order to test these ideas quantitatively, we have made use of Yang-Mills theory — the SU(2) version of this model — for which the structure constants $f^{aba}$ are $\delta^{ba}$. We model $A_{GI}^b(r)$, which, in this case, are transverse fields in the adjoint representation of SU(2), as the manifestly transverse “hedgehog” configuration

$$\langle A_{GI}^b(r) \rangle = \epsilon^{ijk} r_j \phi(r). \quad (30)$$

Although there is no reason to believe that the ansatz given in Eq. (30) follows from the dynamical equations that determine $A_{GI}^b(r)$, it is a convenient choice for examining to what extent the structure of $\mathcal{F}^{ba}(r, x)$ — as distinct from the precise form of $A_{GI}^b(r, x)$ — enables us to nonperturbatively evaluate the infinite series given in Eq. (27). As we were able to show, the perturbative evaluation of $H_{Q-Q}$ for this SU(2) model could be bypassed, and the entire series in $\mathcal{F}^{ba}(r, x)$ obtained from the solution of a sixth-order differential equation.

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