Designing H-shaped micromechanical filters

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Abstract. This paper investigates the design constraints and possibilities given when designing a micromechanical band pass filter for intermediate frequencies (e.g. 10 MHz). The class of filters are based on coupled clamped-clamped beams constituting an H-shaped structure. A primary beam can electrostatically be activated in one of its different harmonic modes, setting the filter center frequency. The motion is transferred to an accompanying beam of equal dimensions by a mechanical coupling beam. The placement or coupling points of the quarter-wavelength coupling beam which connects the vertically resonating beams is critical with respect to the bandwidth of the filters. Of special concern has been to investigate realistic dimensions allowing the filters to be processed by an actual foundry process and to find out how the choice of materials and actual dimensions would affect the performance.

1. Introduction
Various MEMS (Micro Electro Mechanical System) technologies have shown their possibilities to bring forth components of quite different kinds, e.g. pressure sensors, gyroscopes or micro mirrors for projectors. A class of MEMS components can preferably be used in RF (Radio Frequency) systems, RF MEMS, enabling compact implementations and high performance [1]. Micromechanical parts can replace their electronic counterparts in effective ways. Most mature in the RF MEMS area are mechanical switches which have demonstrated very low insertion loss and high isolation [2]. Of special interest to our work is mechanically resonating (or vibrating) structures which can be used to implement oscillators, mixers or filters. By taking advantage of the mechanical properties high performance and miniaturization can be obtained by cost effective batch processing. Such MEMS resonators can replace off-chip crystals or can be used in implementing filter banks with unique selection properties [1].

In this paper H-shaped filter structures based on coupled clamped-clamped (c-c) beams are studied. Figure 1 shows the 3-D model of our filter where one of the beams is electrostatically activated and output is taken from a second beam. The performance depends on various parameters related to the actual MEMS process, geometrical dimensions, material selection for the structural layer, offset voltage and placement of the coupling beam. A systematic way of choosing an appropriate set within the design constraints is vital. A commercial foundry process, [3], has been chosen to give a realistic parameter choice. In section 2 the design method is described, and results from a concrete design are shown and discussed in section 3, preceding the conclusion.
2. Method
As a starting point of our design procedure analytical models have been used by performing extensive Matlab calculations. The design space can thereby be investigated in an effective, although approximated way. Based on the introductory investigation and concrete processing parameters and design rules from the Europractice Integram program, offered by QinetiQ [4], detailed FEM (Finite Element Method) simulations have been done. The tool used for the FEM-simulations is CoventorWare, [5], which allows both mechanical, electrical and co-simulation of both domains to be done. Promising parameter sets have thus been simulated in detail and compared to results from the analytical formulas. A complete design and layout for a given filter specifications have been done according to the Integram Polysilicon Sacrificial Surface Micromachining (SSM) process [3]. A typical filter has a center frequency of 10 MHz and a BW of 10-100 kHz.

![Model of the H-shaped filter](image)

Figure 1. Model of the H-shaped filter with the beam length \( L_r \), beam width \( W_r \), electrode width \( W_e \), structural layer thickness \( h \), and gap distance \( g_d \), shown on the figure.

2.1. Analytical modeling of a clamped-clamped beam.
The natural or eigen frequency of a single c-c beam has been derived [6] from the general expression of mechanical resonance, \( \omega = \left( \frac{k}{m} \right)^{0.5} \) where \( k \) is the stiffness and \( m \) is the mass,

\[
f_{\text{nom}} = 1.03 \kappa \sqrt{\frac{E}{\rho L_r^2}} h
\]

In (1) \( \kappa \) is a scaling factor describing the surface topology [6], \( E \) and \( \rho \) are Young’s modulus and the material density respectively, \( h \) and \( L_r \) are the thickness and the length of the beam. As can be seen, there are two main parts that determine the frequency: material properties and geometry. The resonance frequency can be increased by minimizing the mass and maximizing the stiffness. The ratio between Young’s modulus and density, \( E/\rho \), is crucial for the resonance frequency. The material available in the chosen process (Polysilicon SSM) is polysilicon. Other processes could offer other materials, e.g. diamond has an \( E/\rho \) ratio approximately twice the ratio of that for polysilicon. This will give a significant increase in the resonance frequency. Diamond as structural layer has been demonstrated in other MEMS resonators like the disk-resonator [7], reaching considerably higher frequencies. Doping of Polysilicon can also be used to modify the \( E/\rho \) ratio.

Because the c-c beam is electrostatically driven, a DC-bias on the beam would be necessary to avoid resonance at half the frequency, [8]. A DC-bias will statically pull the beam downwards, soften the mechanical stiffness, \( k_m \), and create an electrical stiffness, \( k_e \), that will decrease the resulting beam stiffness, \( k_r \), and the resonance frequency. The resulting center frequency of the filter is given by (2), where \( <k_e/k_m> \) designates the beam softening and can be found in [9].

\[
f_0 = f_{\text{nom}} \left( 1 - \frac{k_e}{k_m} \right)^{1/2}
\]

The DC-bias gives a possibility to adjust the center frequency after the chip is produced and will also influence the input impedance. The series motional resistance can be approximated by [10]
where \( k_r \) is the spring stiffness at the center of the beam. As seen in (3), in addition to the DC-bias, \( V_p \), the gap, beam width and electrode width will influence the series motional resistance.

2.2. The H-filter

A single c-c beam provides a too small bandwidth (\( BW \)) to be used as a realistic band-pass filter. To widen the pass-band, two identical c-c beams are coupled with an interconnecting beam. The \( BW \) is given by

\[
BW = \left( \frac{f_0}{k_{12}} \right) \left( \frac{k_{12}}{k_r} \right)
\]  

In (4), \( k_{12} \) is the normalized coupling coefficient found in filter books [11], \( k_r \) is the spring stiffness of the c-c beam and \( k_{st} \) is the stiffness of the coupling beam. The ratio between \( k_{st} \) and \( k_r \) decides the \( BW \). The dimension of the coupling beam is given by the design rules of the process and a quarter wavelength constraint [9]. Therefore, adjusting the spring stiffness of the c-c beams is the only way to be able to fulfill the \( BW \) specification. This can be done by adjusting the position of the coupling beam. From the equation for c-c beam spring stiffness [12] and equation (4), we see that \( k_r \) is largest near the anchors and a coupling point there will give a small \( BW \). The Q-factor of the filter is given by the ratio between the beam stiffness \( k_r \) and couplings beam stiffness \( k_{st} \) times the normalized coupling coefficient [9]

\[
Q_{filt} = k_{12} \left( \frac{k_r}{k_{st}} \right)
\]

Figure 2 shows how the \( BW \) and \( Q_{filt} \) varies with the coupling location along the beam. Clearly a coupling near the anchors gives the highest \( Q_{filt} \) and smallest \( BW \). The required \( BW \) can now be chosen by placing the coupling beam at an appropriate location along the beam.

3. Results and discussion

The less controllable parameters, given by specific processing steps, have been left out for further investigations. For the FEM simulation we concentrate mainly on the geometric design. Still a few parameters are ‘locked’ by the process, like the thickness of the sacrificial layer that determines the gap distance between the electrode and beam. A small gap makes the electrostatic driving effective and a minimum gap is preferable due to the series motional resistance. Table 1(a) shows how the
Table 1. Different approaches to adjust the series resistance. The decreasing Q when increasing the beam width [10] is not taken into account for the results in this table. (a) shows how \( R_z \) vary with both \( W_r \) and \( gd_0 \), here \( V_P \) is set to 20 volt and \( W_e = 33 \mu m \). (b) shows the variation in \( R_z \) with different \( V_P \) in addiction to the center frequency \( f_0 \), here \( gd_0 = 0.2 \mu m \) and \( W_e = 33 \mu m \). In (c) \( W_r = 10 \mu m \) and \( V_P = 20 \) V.

(a) |  | \( R_z \) (k\(
\Omega\)) | (\( \mu m \)) | \( gd_0 \) = | \( gd_0 \) = | \( gd_0 \) =
|---|---|---|---|---|---|
| 8 | 1280/Q | 24900/Q | 127600/Q | 0.1 \mu m | 0.2 \mu m | 0.3 \mu m |
| 10 | 1902/Q | 2000/Q | 102100/Q | 680/Q | 13300/Q | 68100/Q |
| 15 | 2010/Q | 9900/Q | 51100/Q | 510/Q | 9900/Q | 51100/Q |

(b) | \( V_P \) | \( R_z \) (k\( \Omega \)) | \( f_0 \) (MHz) | \( W_e \) (\( \mu m \)) | \( R_z \) (k\( \Omega \))
|---|---|---|---|---|---|
| 10 | 6.805 | 9.7 | 20 | 12.615 | 9.7 |
| 20 | 4.351 | 9.6 | 25 | 3.120 | 9.6 |
| 30 | 3.020 | 9.4 | 30 | 1.358 | 9.4 |
| 40 | 2.496 | 9.1 | 33 | 0.737 | 9.1 |

(c) | \( W_e \) (\( \mu m \)) | \( R_z \) (k\( \Omega \))
|---|---|
| 20 | 6.805 |
| 25 | 4.351 |
| 30 | 3.020 |
| 33 | 2.496 |

Series motional resistance can be reduced by decreasing the gap spacing between the beam and electrode, and by increasing the beam width. The Integram Polysilicon SSM process offers a minimum gap of 0.2 \( \mu m \), which will then be the natural choice of spacing. Unfortunately, the increased beam width also leads to a reduction of the Q-factor [10]. The beam width then has to be a traded off between high Q and low impedance. As seen in the table, the series resistance is decreased by increasing electrostatic force. In table 1(c) the effect of varying the electrode width is shown. The maximum allowed \( W_e \) is given by the beam length and the process rules. Table 1(b) shows how \( R_z \) depends on the DC-bias, and the frequency drop due to spring-softening is shown. The effect of spring softening can be seen more clearly in figure 3(a) and (b), when applying 10 and 20 volts respectively. The solid line in the graphs is the resulting beam stiffness \( k_r = k_m - k_e \). The stippled line is the mechanical stiffness \( k_m \), and is unchanged in figure 3(a) and (b). The total stiffness decreases when the DC-bias is increased. The DC-bias can be applied to adjust the frequency, but a too high value could lead to pull-in which must be avoided. This can be compensated by a relative thick structural layer. Table 2(b) shows how the frequency varies with the thickness of the structural layer. However, the chosen process can only offer a thickness of 2 \( \mu m \).

Figure 3. Spring softening by DC biasing. (a) shows the difference between \( k_m \) (stippled line) and \( k_r \) (solid line) for the center of the beam, when \( V_P = 10 \) V, and (b) \( V_P = 20 \) V.

Table 2. (a) shows the frequency dependency of the beam length \( L_r \), with \( V_P = 20 \) V, \( W_r = 8 \mu m \), \( W_e = 20 \mu m \) and \( h = 2 \mu m \). (b) shows the dependency of the thickness of the structural layer \( h \), with \( V_P = 20 \) V, \( W_r = 8 \mu m \), \( W_e = 20 \mu m \) and \( L_r = 40 \mu m \).

(a) | \( L_r \) (\( \mu m \)) | \( f_0 \) (MHz)
|---|---|
| 38 | 10.67 |
| 40 | 9.61 |
| 42 | 8.69 |

(b) | \( h \) (\( \mu m \)) | \( f_0 \) (MHz)
|---|---|
| 1 | 4.11 |
| 2 | 9.61 |
| 3 | 14.55 |
The remaining parameters of the c-c beams are the length and the width of the structures, which can be designed to fit the requirements. Table 2(a) showed a few selected lengths with the corresponding frequencies. A beam length of 40 \( \mu \text{m} \) was considered to be an appropriate length. As mentioned earlier the beam width is a trade-off between Q-factor, impedance and the frequency. The chosen \( W_c \) for the FEM simulations is therefore a width of 8 \( \mu \text{m} \), which should give a reasonable Q-value. From figure 2 and corresponding equations, a coupling location around 4 \( \mu \text{m} \) from the anchors should provide a \( BW \) around 10 kHz.

![Figure 4](image1.jpg)

**Figure 4.** In (a) the two distinct peaks from the two resonating modes. (b) shows the variations in \( BW \) when moving the coupling location.

**Table 3.** Mode Frequencies and bandwidth according to coupling beam location. From CoventorWare simulation.

| Coupling location (\( \mu \text{m} \)) | Mode 1 (MHz) | Mode 2 (MHz) | \( BW \) (kHz) |
|----------------------------------------|--------------|--------------|---------------|
| step 1                                 | 10.06671     | 10.07819     | 11.48         |
| step 2                                 | 10.05295     | 10.09083     | 37.88         |
| step 3                                 | 10.02548     | 10.11850     | 93.02         |

With the chosen parameter set, a FEM simulation is now done with CoventorWare. The two distinct peaks from the resonating modes are shown in figure 4(a). Figure 4(b) shows how the BW increases while moving the coupling beam away from the anchors. Corresponding results are also given in table 3. In figure 5 we see the total harmonic response with different damping coefficients. There is no easy way to do an analytic analysis of the total damping in the structure. A variety of different factors contribute to the damping, e.g. damping through the anchors. The y-axis in each figure describes the velocity at the center of the beams, while the x-axis shows the frequency. If the beams could vibrate freely with no damping, the two distinct peaks shown in figure 5(a) would be the result. However, the damping decreases the Q-factor, and thus broadens the peaks and binds them together. The result is a band-pass filter characteristic as shown in figure 5(b) and (c).

![Figure 5](image2.jpg)

**Figure 5.** FEM simulation with variation in damping coefficient. In (a) there is no damping. In (b) the damping is set to 1% of critical damping. (c) shows critical damping.
The result of the FEM simulation gives a somewhat higher resonance frequency and larger BW than the analytic models. Still the preliminary Matlab calculations for actual design parameters would give the designer a good overview for selecting an appropriate set and guidance for pursuing more detailed investigations. Not all equations have been shown, and several of the parameters depend on each other, and more complex equations can be used to get more accurate results. Still simple analytical models can give a good first estimate. How these analytical and FEM results would actually fit to a fabricated MEMS filter, remains for further investigations.

4. Conclusion
It has been shown that an appropriate first set of parameters can be selected using coarse grain analytical models. Difference between analytic and FEM modeling can vary, but with at pre-analyzed parameter set, the FEM simulation time can be reduced, and changes are easily understood and done. To find out how the choice of materials and actual dimensions would affect the performance of a micromechanical filter, FEM simulations can often be hard to capture to get an overview of design choices. An analytic model were you can play with parameters are much less time consuming, and you can much easier follow the changes that take place. This paper has shown the feasibility of first performing an analytical investigation in a systematic way. Then more thorough simulations should be done by FEM to increase the insight in the design before processing. Actual parameters of a concrete filter design have been found and discussed.

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