Higgs boson decay into heavy quarks and heavy leptons: higher order corrections

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Theoretical predictions for the decay width of Standard Model Higgs boson into bottom quarks and τ-leptons, in the case when \( M_H \leq 2M_W \), are briefly reviewed. The effects of higher order perturbative QCD (up to \( \alpha_s^4 \)-level) and QED corrections are considered. The uncertainties of the decay width of Higgs boson into \( \bar{b}b \) and \( \tau^+\tau^- \) are discussed.

1. Introduction

Production cross-sections and decay widths of the Standard Electroweak Model Higgs boson are nowadays among the most extensively analyzed theoretical quantities (for a recent review, see, e.g., [1], [2]). Indeed, the main hope of scientific community is that this essential ingredient of the Standard Model may be discovered, if not at Fermilab Tevatron, then at the forthcoming LHC experiments at CERN. There is great interest in the “low-mass” region \( 114.5 \text{ GeV} \leq M_H \leq 2M_W \), because a “low-mass” Higgs boson is heavily favored by Standard Model analysis of the available precision data. The lower bound, 114.5 GeV, was obtained from the direct searches of Higgs boson at the LEP2 \( e^+e^- \)-collider primarily through Higgs boson decay into a \( \bar{b}b \)-pair.

It should be stressed, that the uncertainties in \( \Gamma(H \rightarrow \bar{b}b) \), analytically calculated in QCD using the \( \overline{\text{MS}} \)-scheme at the \( \alpha_s^4 \)-level [3], dominate the theoretical uncertainty for the branching ratio of \( H \rightarrow \gamma\gamma \) decay, which is considered to be the most important process in searches for a “low mass” Higgs boson by CMS and ATLAS collaborations at the LHC.

Here we briefly discuss the uncertainties of the QCD predictions for \( \Gamma_{H\bar{b}b} \), including those which come from the on-shell mass parameterizations of this quantity (previous discussions see in [4]-[9] and [10]) and from the resummations of the \( \pi^2 \)-terms, typical of the Minkowskian region (see [11]-[16]). We discuss also perturbative QED and QCD uncertainties for Higgs boson decay into heavy leptons, \( \Gamma_{H\tau\tau} = \Gamma(H \rightarrow \tau^+\tau^-) \).

2. QCD corrections for \( \Gamma_{H\bar{b}b} \) in terms of pole and running \( b \)-quark mass

There are several approaches for Higgs boson decay \( \Gamma_{H\bar{b}b} \) in perturbative QCD. One of them is based on pole (on-shell) mass consideration [4]-[4):

\[
\Gamma_{H\bar{b}b} = \Gamma_0^b \left[ 1 + \sum_{i \geq 1} \Delta \Gamma_i^b a_i(M_H) \right], \tag{1}
\]

where \( \Gamma_0^b = (3\sqrt{2}/8\pi)G_Fm_b^2, \ a_s(M_H) \equiv \alpha_s(M_H)/\pi, \ m_b \) and \( \ M_H \) are the pole \( b \)-quark and Higgs boson masses, and \( \Gamma_i^b \)-coefficients are \( i \)-th-order polynomials of large logarithms \( \ln(M_H^2/m_b^2) \). An another approach is based on \( \overline{\text{MS}} \)-scheme framework:

\[
\Gamma_{H\bar{b}b} = \Gamma_0^b \frac{m_b^2(M_H)}{m_b^2} \left[ 1 + \sum_{i \geq 1} \Delta \Gamma_i^b a_i(M_H) \right], \tag{2}
\]

where \( a_s(M_H) \equiv \alpha_s(M_H)/\pi \) and \( m_b(M_H) \) are the QCD running parameters, defined in the \( \overline{\text{MS}} \)-scheme. The coefficients \( \Delta \Gamma_i^b \) can be expressed through the sum of the following contributions:

\[
\Delta \Gamma_i^b = d_i^E + d_i^M. \tag{3}
\]
Here the positive contributions \( d_i^E \), calculated directly in the Euclidean region, and \( d_i^M \) are proportional to \( \pi^2 \)-factors, which are typical for the Minkowski time-like region.

The corresponding expressions for \( \Delta \Gamma_b \) were derived at the \( \alpha_s^3 \)-level in Ref. [3, 19]. Detailed analysis and results for Higgs decay width \( \Gamma_{Hb} \) at \( \alpha_s^3 \)-level are presented in [9, 10], where the \( \beta \)-function of QCD renormalization group (RG) and mass anomalous dimension function \( \gamma_m \) [20-25] were considered at the 5-loop level:

\[
\frac{d\alpha_s}{d\ln \mu^2} = \beta(\alpha_s) = -\beta_0 a_s^2 \ldots - \beta_4 a_s^5 + \mathcal{O}(a_s^6), \tag{4}
\]

\[
\frac{d\ln \mu}{d\ln \mu^2} = \gamma_m(a_s) = -\gamma_0 a_s \ldots - \gamma_4 a_s^5 + \mathcal{O}(a_s^6). \tag{5}
\]

The 5-loop coefficients \( \beta_4 \) and \( \gamma_4 \) are still unknown, but it can be estimated by Padé approximation procedure, developed in [26] (see discussion in Refs. [3, 10]).

It should be stressed, however, that the uncertainties of the estimated 5-loop contributions to the QCD \( \beta \)-function and mass anomalous dimension function \( \gamma_m \) are not so important in the definition of the running of the \( b \)-quark mass from the pole mass \( m_b \) to the pole mass of Higgs boson \( M_H \). This effect of running is described by the solution of the following RG equation:

\[
\mathcal{m}_b^2(M_H) = \mathcal{m}_b^2(m_b) \exp \left[ -2 \int_{a_s(m_b)}^{a_s(M_H)} \frac{\gamma_m(x)}{\beta(x)} \, dx \right], \tag{6}
\]

\[
= \mathcal{m}_b^2(m_b) \left( \frac{a_s(M_H)}{a_s(m_b)} \right)^{2 \gamma_0/\beta_0} \left( \frac{AD(a_s(M_H))}{AD(a_s(m_b))} \right)^2,
\]

where \( AD(a_s) \) is a polynomial of 4-th order in the QCD expansion parameter \( a_s \) [9, 10].

In the Higgs boson mass region of interest, Eq. (2) may be expressed in numerical form as

\[
\frac{\Gamma_{Hb}}{\Gamma_0} = \frac{\mathcal{m}_b^2(M_H)}{m_b^2} \left[ 1 + 5.667 a_s(M_H) + 29.15 a_s(M_H)^2 \right.
\]

\[
+ 41.76 a_s(M_H)^3 - 825.7 a_s(M_H)^4 \left. \right] (7)
\]

Substituting the value \( a_s(M_H) \approx 0.0366 \) (which corresponds to \( a_s(M_H = 120 \text{ GeV}) \approx 0.115 \)) into Eq. (7), and decomposing the coefficients in the Minkowskian series into Euclidean contributions and Minkowskian-type \( \pi^2 \)-effects, one can get from Ref. [8] the following numbers

\[
\frac{\Gamma_{Hb}}{\Gamma_0} = \frac{\mathcal{m}_b^2(M_H)}{m_b^2} \left[ 1 + 20.27 + 0.039 \right.
\]

\[
+ 0.0020 - 0.0015 \left. \right] = \frac{\mathcal{m}_b^2(M_H)}{m_b^2} \left[ 1 + 20.27 + (0.556 - 0.017) \right.
\]

\[
+ (0.017 - 0.015) + (0.0063 - 0.0078) \right],
\]

where the negative numbers in the round brackets come from the effects of analytical continuation. Having a look at Eq. (8) we may conclude that in the Euclidean region the perturbative series is well-behaved and the \( \pi^2 \)-contributions typical of the Minkowskian region are also decreasing from order to order. However, due to the strong interplay between these two effects in the third and fourth terms, the latter ones are becoming numerically comparable. This feature spoils the convergence of the perturbation series in the Euclidean region. Therefore, to improve the accuracy of the perturbative prediction in the Minkowskian region it seems natural to sum up these \( \pi^2 \)-terms using the ideas, developed in the 80s (see, e.g., [27, 30]). These ideas now have a more solid theoretical background, see, e.g., Ref. [31].

Also, we stress that the truncated perturbative expansions of Eq. (7) have some additional uncertainties. These include \( M_H \) and \( t \)-quark mass dependent QCD [32, 33] and QED [34] contributions:

\[
\Delta \Gamma_{Hb} = \frac{3\sqrt{2}}{8\pi} G_F M_H \mathcal{m}_b^2(M_H) \left[ \Delta^t + \Delta^{QED} \right] \tag{9}
\]

where \( \Delta^t \) and \( \Delta^{QED} \) are defined following Refs. [33, 34] as

\[
\Delta^t = a_s^2 \left( 3.11 - 0.667 L_t \right) \tag{10}
\]

\[
+ \frac{\mathcal{m}_b^2}{M_H^4} \left( -10 + 4 L_t + \frac{4}{3} \ln(\mathcal{m}_b^2/M_H^2) \right) \]
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\[ L_t = \ln(M_H^2/m_t^2), \quad X_t = g_F m_t^2/(8\pi^2 \sqrt{2}), \quad m_t = 175 \text{ GeV}, \quad M_H = 120 \text{ GeV}, \quad m_b = 2.8 \text{ GeV}, \quad G_F = 1.1667 \times 10^{-5} \text{ GeV}^{-2} \]

Using \( a = \alpha(M_H)/\pi = 0.0027 \) (\( \alpha(M_H)^{-1} \approx 129 \)), \( m_t = 175 \text{ GeV} \), \( M_H = 120 \text{ GeV} \), \( m_b = 2.8 \text{ GeV} \), \( G_F = 1.1667 \times 10^{-5} \text{ GeV}^{-2} \) we get

\[
\Delta_{t} = \left[ 4.84 \cdot 10^{-3} - 1.7 \cdot 10^{-5} \right] + 2.27 \cdot 10^{-3} + 1.85 \cdot 10^{-4} + 3.2 \cdot 10^{-3} - 5.75 \cdot 10^{-4} - 2.42 \cdot 10^{-4} \right] \approx 1.1 \cdot 10^{-3} - 4.5 \cdot 10^{-6} \] \tag{12}

\[
\Delta_{t}^{QED} = \left[ 1.1 \cdot 10^{-3} - 4.5 \cdot 10^{-6} \right] - 9 \cdot 10^{-6} - 1.2 \cdot 10^{-4} \] \tag{13}

Comparing the numbers presented in Eq. (8) and Eq. [13]-Eq. [13], we conclude that \( \alpha^3 \)-terms can be neglected at the current level of the experimental precision of “Higgs-hunting” at Fermilab and LHC. Indeed, one can see, that even for the light Higgs boson the numerical values of the order \( \alpha^3 \)-contributions to Eq. (8) are comparable with the leading \( M_H \)- and \( m_t \)- dependent terms in Eqs. (12)-(13).

An another approach for \( \Gamma_{H\bar{b}b} \), where the RG-controllable terms are summed up, may be written down as

\[
\Gamma_{H\bar{b}b} = \Gamma^b_0 \left( \frac{a_s(M_H)}{a_s(m_b)} \right)^{(24/23)} \tag{14}
\]

\[
\times AD(a_s(M_H))^2 \left[ 1 + \sum_{i \geq 1} \Delta \Gamma^b_1 a_i^b(M_H) \right] \times (1 - \frac{8}{3} a_s(m_b) - 18.556 a_s(m_b)^2 - 175.76 a_s(m_b)^3 - 1892 a_s(m_b)^4),
\]

where

\[
AD(a_s)^2 = 1 + 2.351 a_s + 4.383 a_s^2 + 3.873 a_s^3 - 15.15 a_s^4.
\] \tag{15}

Here, an important relation between pole and running masses of Refs. [35],[36],[10] has been used. Detailed comparison of \( \Gamma_{H\bar{b}b} \) in RG-improved (Eq. (14)) and in pole mass truncated (Eq. (2)) approaches was presented in Refs. [9],[10].

The behavior of the RG-resummed expressions for \( \Gamma_{H\bar{b}b} \) and \( R_{H\bar{b}b} \) are more stable than in the case, when RG-summation of the mass-dependent terms is not used [4]-[9],[10] (Figs. 1,2). Difference of \( \Delta \Gamma_{H\bar{b}b} \) calculated the truncated pole-mass approach and the RG-improved parametrization of \( \Gamma_{H\bar{b}b} \) is becoming smaller in each successive order of perturbation theory.

Indeed, for the phenomenologically interesting value of Higgs boson mass \( M_H = 120 \text{ GeV} \) we find that at the \( \alpha^2 \)-level \( \Delta \Gamma_{H\bar{b}b} \approx 0.7 \text{ MeV} \), while for the \( \alpha^3 \)-level it becomes smaller, namely \( \Delta \Gamma_{H\bar{b}b} \approx 0.3 \text{ MeV} \). At the \( \alpha^3 \)-level of the RG-improved
MS-scheme series one has $\Gamma_{H_{\tau\tau}} \approx 1.85$ MeV for $M_H = 120$ GeV. For this scale the value of $\Gamma_{H_{\tau\tau}}$ with the explicit dependence from the pole-mass is 16% higher, than its RG-improved estimate.

There are different approaches to the treatment of the typical Minkowskian $\pi^2$-contributions in the perturbative expressions for physical quantities, which demonstrated remarkable convergence properties [13]-[16]. At the moment, these approaches are developing for different phenomenological applications, which will allow a comparison with the existing methods.

3. Higgs boson decay into $\tau^+\tau^-$

Width of Higgs boson decay into $\tau^+, \tau^-$ - leptons in the MS-scheme can be read as [37]:

$$\Gamma_{H_{\tau\tau}} = \frac{m_{\tau}^2 (M_h)}{m_{\tau}^2} 1 + a(M_H) \Delta \Gamma_1^\tau + a(M_H)^2 \Delta \Gamma_2^\tau + a(M_H)^3 \Delta \Gamma_3^\tau + a_s(M_H) \Delta QEDxQCD$$

where $\Gamma_0^\tau = (\sqrt{2}/8\pi)G_F M_H m_{\tau}^2$, $a(M_H) = a_{MS}(M_H)/\pi$, $m_{\tau}(M_H)$ are QED running parameters and $a_s(M_H) = a_{sMS}(M_H)/\pi$ is QCD parameter, and $\Delta QEDxQCD$ is a mixed QED-QCD correction to the coefficient function. Evolution of running $\tau$-lepton mass in QED is similar to Eq. (6), but with $\beta_{QED}, \gamma_{QED}, \Delta \Gamma_2^\tau$ and $\Delta QEDxQCD$, complicated by quark fractional electric charge dependence [37]. $\beta_3^{QED}$ is known since [38], and $\gamma_3^{QED}$ is consistent with QED-limit of Ref. [21]. At present for $\Gamma_{H_{\tau\tau}}$ to get accuracy of $\Gamma_{H_{bb}}$ at $\alpha_3$-level it is enough to keep 2-loop running $\tau$-lepton mass and 1-loop coefficient function $\Delta \Gamma_1^\tau$ [37].

4. Summary

Different approaches based on the running and pole $b$-quark masses for the decay width of the $H \rightarrow b\bar{b}$ process become consistent in higher orders of perturbative QCD. However, different convergence in different approaches demonstrates an existence of additional theoretical QCD uncertainties, which are not usually considered in phenomenological studies.

Currently, for width of Higgs boson decay into heavy leptons $\Gamma_{H_{\tau\tau}}$ to have accuracy of $\Gamma_{H_{bb}}$ at $\alpha_3$-level it is enough to take into account 2-loop running $\tau$-lepton mass and 1-loop coefficient function $\Delta \Gamma_1^\tau$.

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