Scheduling in Wireless Networks with Spatial Reuse of Spectrum as Restless Bandits

Vivek S. Borkar, Shantanu Choudhary, Vaibhav Kumar Gupta, Gaurav S. Kasbekar

Abstract—We study the problem of scheduling packet transmissions with the aim of minimizing the energy consumption and data transmission delay of users in a wireless network in which spatial reuse of spectrum is employed. We approach this problem using the theory of Whittle index for cost minimizing restless bandits, which has been used to effectively solve problems in a variety of applications. We design two Whittle index based policies—the first by treating the graph representing the network as a clique and the second based on interference constraints derived from the original graph. We evaluate the performance of these two policies via extensive simulations, in terms of average cost and packets dropped, and show that they outperform the well-known Slotted ALOHA and maximum-weight scheduling algorithms.

I. INTRODUCTION

Several wireless networks employ spatial reuse of spectrum, i.e., in such networks, multiple mutually far-apart transmitters simultaneously send data to their respective receivers on the same channel without interfering with each other. Examples of such networks are wireless cellular networks, mesh networks, ad hoc and sensor networks, and they have a variety of applications, e.g., in military and emergency operations, Internet access for communities, intrusion detection, precision agriculture, environmental monitoring and industrial monitoring [1], [2], [3], [4]. Since nodes in such networks are often battery-powered, a key objective is to achieve energy efficiency [2], [3]. Another important objective is to minimize the data transmission delay, especially that of real-time traffic such as audio and video calls, emergency alerts from security systems etc. Also, a basic function in wireless networks that employ spatial reuse of spectrum is scheduling, i.e., selecting a mutually non-interfering set (independent set) of nodes that will transmit in each time slot [5]. In this paper, we address the fundamental problem of scheduling packet transmissions with the objective of minimizing the energy consumption and data transmission delay of users in wireless networks in which spatial reuse of spectrum is employed.

Scheduling in wireless networks that employ spatial reuse of spectrum has been extensively studied in the research literature—see [7], [8], [9] for surveys. A throughput-optimal scheduling policy was provided in the seminal work [10]. The complexity of throughput-optimal scheduling in multi-hop wireless networks subject to interference constraints was studied in [6]. In [11], it was shown that a distributed scheduling strategy, called maximal scheduling, attains a guaranteed fraction of the maximum throughput region in multi-hop wireless networks. In [12], distributed scheduling schemes were designed that achieve throughput close to that of maximal schedules, but whose complexity is low. A distributed scheduling scheme that guarantees maximum throughput in multi-hop wireless networks was presented in [13]. However, the schemes in [6], [10], [11], [12], [13] were designed so as to maximize the achievable throughput. In contrast, in this paper our objective is to design a scheduling scheme that minimizes the delay and energy consumption. A large number of medium access control (MAC) protocols, including the well-known Pure ALOHA, Slotted ALOHA, CSMA/CA and IEEE 802.11 Distributed Coordination Function protocols, have been designed for wireless networks—see [14] for a survey. These MAC protocols can be used for scheduling in a wireless network that employs spatial reuse of spectrum. These protocols, however, do not in general minimize the energy consumption or delay.

Scheduling in wireless networks with the objectives of minimizing the energy consumption and/ or delay has been extensively studied in prior work. A survey of schemes for delay-aware resource control in a multi-hop wireless network is provided in [15]. A scheduling scheme for minimizing the energy-expenditure in a time-varying wireless network with adaptive transmission rates has been provided in [16]. In [17], the problem of allocating power to links as a function of current channel states and queue backlogs to stabilize the system while minimizing energy expenditure and maintaining low delay in a multiuser network is studied. In [18], the problem of designing opportunistic scheduling policies that minimize the average delay in a wireless network with multiple users sharing a wireless channel is studied. In [19], energy-efficient scheduling with delay constraints in a multiuser wireless network is studied. The problem of delay minimization under power constraints for uplink transmission in a multiuser wireless network is studied in [20]. The problem of minimizing the transmission power subject to a delay constraint in a multiuser wireless network is studied in [21]. However, with the exception of our prior work [22], no work has addressed the problem of scheduling in a wireless network with the objective of minimizing the energy consumption and delay using the theory of Whittle index [23]. In [22], at most one user can successfully transmit at a time on the channel. In this paper, we study a wireless network that employs spatial reuse of spectrum, allowing multiple simultaneous transmissions.

Specifically, we consider a wireless network with multiple users deployed in a region and communicating using a single channel. We represent the network using an undirected graph $G$, in which there is a node representing each user, and there is an edge between two nodes iff the transmissions of the corresponding users interfere. In each time slot, an independent set of users which will transmit in the slot needs to be selected. The energy consumed when a user transmits on the channel is modeled by an "energy cost", which is...
an increasing function of the number of packets transmitted. Note that the delay experienced by a packet is an increasing function of the number of packets ahead of it in its queue. Since we seek to minimize packet delays, we also consider a cost proportional to the queue length, referred to as the "holding cost". The cost incurred in a slot at a user is the sum of the energy cost and the holding cost. Our objective is to minimize the time-averaged total cost incurred at all the users in the network. To solve this problem, we use the Whittle index policy, which was introduced in [23] and has been used to effectively solve problems in a variety of applications [24, 25, 26, 27, 28, 29, 30, 31]. Specifically, we design two Whittle index based policies—the first by treating the graph representing the network as a clique (i.e., a complete graph) and the second based on interference constraints derived from the original graph. We evaluate the performance of these two policies via extensive simulations, in terms of average cost and packets dropped, and show that they outperform the well-known Slotted ALOHA [33] and maximum-weight scheduling (MWS) [10] algorithms.

In Section II, we describe the model and problem formulation and briefly review the theory of Whittle index. We present two scheduling algorithms based on Whittle indices to solve the above problem in Section III. We present simulation results in Section IV and conclude in Section V.

II. MODEL, PROBLEM FORMULATION AND BACKGROUND

A. Model and Problem Formulation

We consider a wireless network consisting of \( L \) users deployed in a region and communicating using a single channel. Each user is a transmitter-receiver pair, with a queue, at the transmitter, of packets which need to be sent to the receiver. Recall that the wireless medium has the property that simultaneous transmissions by two users that are close to each other interfere with each other, whereas the channel can be simultaneously used at far-off locations without interference. To model these spatial reuse (interference) constraints, we represent the network using an undirected graph \( G = (\mathcal{V}, \mathcal{E}) \), in which \( \mathcal{V} \) is the set of users and there is an edge between two users \( i, j \in \mathcal{V} \) iff the transmissions of users \( i \) and \( j \) interfere with each other. Let \( \mathcal{N}(i) \) be the set of neighbors of user \( i \), i.e., the set \( \{ j \in \mathcal{V} : \exists (i, j) \in \mathcal{E} \} \).

Time is divided into slots of equal durations. The queue of user \( i \) evolves according to the dynamics:

\[
X_{n+1}^i = [X_n^i - \nu_n^i (X_n^i \land \Psi^i) + \xi_{n+1}^i] \land M^i, \tag{1}
\]

where \( X_n^i \) is the length of the queue of user \( i \) in time slot \( n \), \( \xi_{n+1}^i \) is the number of arrivals at the queue of user \( i \) in time slot \( n \), \( M^i \) is the capacity of the buffer of user \( i \), \( \Psi^i \) is the maximum number of packets that may be transmitted by user \( i \) in a slot and \( \nu_n^i \) is 1 if user \( i \) transmits in slot \( n \) and 0 otherwise. We say that a user is "active" in a slot if it transmits and "passive" if not. We assume that the number of packet arrivals, \( \xi_{n+1}^i \), \( n = 0, 1, 2, \ldots \), in different slots are independent and identically distributed (IID) random variables with distribution \( \mu(\cdot) \).

The cost of holding packets in the queue of user \( i \) is \( C^i \) per packet per slot. That is, if there are \( x \) packets in queue \( i \) in a given slot, then a cost of \( xC^i \) is incurred. This cost models the delay requirement of a queue. In particular, the higher the value of \( C^i \), the more stringent the delay requirements of the packets stored in queue \( i \). For example, the value of \( C^i \) may be set to a low (respectively, high) value if queue \( i \) stores real-time traffic such as file transfer packets (respectively, real-time traffic such as audio and video flow packets). Let \( f(z) \) be the "energy cost", i.e., the cost incurred by user \( i \) due to expenditure of energy when it transmits \( z \) packets.

Let \( N^*(i) := \mathcal{N}(i) \cup \{i\} \). If two or more users from the set \( N^*(i) \) transmit in time slot \( n \), their transmissions interfere with each other, leading to the constraints:

\[
\sum_{j \in N^*(i)} \nu_n^j \leq 1, \forall i. \tag{2}
\]

If a subset of the users in \( \mathcal{V} \) transmits in a time slot subject to (2), that subset constitutes an independent set of nodes in the graph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \). For \( Z_n^i := X_n^i \land \Psi^i \), we seek to minimize the time-averaged cost incurred by all users, i.e.:

\[
\min_{\mathcal{N}(1) \cdots \mathcal{N}(N)} \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} \sum_{i \in \mathcal{V}} [\nu_n^i f(Z_n^i) + C^i X_n^i], \tag{3}
\]

subject to the interference constraints (2). That is, our objective is to select an independent set of users to activate, subject to (2), in each time slot so as to minimize (3).

Remark 1: The constraint in (2) may prevent two mutually non-interfering users from simultaneously transmitting. For example, consider the graph with \( \mathcal{V} = \{1, 2, 3\} \) and \( \mathcal{E} = \{(1, 2), (1, 3)\} \). The constraint in (2) for user \( i = 1 \) is:

\[
\nu_n^1 + \nu_n^2 + \nu_n^3 \leq 1.
\]

This constraint is violated if users 2 and 3 simultaneously transmit in slot \( n \). However, note that users 2 and 3 are mutually non-interfering.

Nevertheless, to facilitate the following analysis, we impose the constraint (2). In Section II-A, we provide an algorithm for activating users in different time slots that ensures that each user from a maximal independent set of users transmits in every time slot.

B. Background on Whittle index

We briefly recall here the basics of Whittle index [23] for cost minimizing restless bandits. The latter refers to a collection of \( N \geq 2 \) controlled Markov chains \( Y_{n,t}^i, n \geq 0, i \in \{1, \ldots, N\} \), taking values in discrete state spaces \( \mathcal{S}_t \), with two modes of operation, active and passive, with corresponding transition probabilities given by \( p_{i,1}(t|s), p_{i,0}(t|s) \) and running costs \( c_1(s), c_0(s) \) resp., where \( s, t \in \mathcal{S}_t \). The control process associated with \( i \)th chain is \( u_i(n), n \geq 0 \), taking values in \( \{0, 1\} \) with the interpretation that value 1 (resp., 0) corresponds to active (resp., passive) mode. Thus the transition probability at time \( n \) for the \( i \)th process is \( p_{i,u_i(n)}(t|s \mid Y_{n,t}^i) \). The objective is to minimize the average cost

\[
\lim_{n \to \infty} \sup_n \frac{1}{n} \mathbb{E} \left[ \sum_{i=1}^{N-1} \sum_{n=0}^{n-1} c_{u_i(m)}(Y_{n,m}^i) \right]
\]

subject to the per stage constraint

\[
\sum_{i=1}^{N-1} u_i(n) \leq M \quad \forall n
\]
for some $1 < M < N$, which couples the problems. This constraint makes the problem provably hard [23]. The Whittle device [23] is to relax it to the average constraint

$$\lim_{n \to \infty} \frac{1}{n} E \left[ \sum_{m=0}^{n-1} \sum_{i} u_i(m) \right] \leq M$$

and consider the unconstrained problem of minimizing

$$\lim_{n \to \infty} \frac{1}{n} E \left[ \sum_{m=0}^{n-1} \sum_{i} \left( c_{ui}(m)Y^i_m + \lambda u_i(m) \right) \right], \quad (4)$$

where $\lambda$ is the Lagrange multiplier. Given $\lambda$, this decouples into individual control problems of minimizing

$$\lim_{n \to \infty} \frac{1}{n} E \left[ \sum_{m=0}^{n-1} \sum_{i} \left( c_{ui}(m)Y^i_m + \lambda u_i(m) \right) \right]$$

for each $i$. Whittle uses this to motivate the so-called Whittle index as follows. The problem is said to be (Whittle) indexable if the set of passive states (i.e., the states $Y^i_m$ for which $u_i(m) = 0$ is the optimal action) for each individual problem $i$ monotonically decreases from the whole state space to the empty set as the ‘tax’ $\lambda$ decreases from $+\infty$ to $-\infty$. If so, the Whittle index for the $i$th problem is the function $\lambda_i : S_i \mapsto R$ such that $\lambda_i(s) :=$ the smallest value of $\lambda$ for which both active and passive modes are equally desirable when the state is $s$. The index rule is then to order, at each time step, the current state $s$ in increasing order of $\lambda_i(s)$ for which $\lambda_i(s) \leq \lambda$. This fact motivates the approach for computing Whittle indices that we present in Section III-B1. Section III-B2 presents an alternative scheme that has an additional tweak to circumvent the need to consider the clique model.

### III. Scheduling Algorithm Based on Whittle Indices

In Section III-A, we provide an algorithm for selecting an independent set of users to activate in a given time slot, assuming that the Whittle indices of all the users in the slot have been already computed. In Section III-B, we provide two different approaches for computing Whittle indices.

#### A. Whittle Index Based Algorithm for Activating Users

Suppose the Whittle indices, $\lambda^i(X^i_n)$, of all the users $i \in \mathcal{V}$ have been computed in a given time slot $n$. An independent set of users to activate in the slot is selected as follows.

First, all users with empty queues are declared passive. Then, those users $i \in \mathcal{V}$ for which $\lambda^i(X^i_n) \leq \lambda^j(X^j_n) \\forall j \in \mathcal{N}(i)$ are declared active (ties are broken according to some tie-breaking rule, e.g., the user with smaller identifier (ID) is declared active). Next, for every active user $i$, all users $j \in \mathcal{N}(i)$ are declared passive. In the next step, all users $i \in \mathcal{V}$ for which $\lambda^i(X^i_n) \leq \lambda^j(X^j_n) \\forall j \in \mathcal{N}(i)$ which are not yet declared passive, are declared active, and their neighbors are declared passive if already not so. This process is repeated till all users have been declared either active or passive.

Note that the set of users that are declared active constitute an independent set; these users transmit in the slot. Furthermore, the procedure requires only local neighborhood information and can be implemented in a distributed manner.

#### B. Computation of Whittle Index

1) Computation Based on Constraints Derived from Clique Model: Recall from Section II-A that we represent the wireless network using a graph $G = (\mathcal{V}, \mathcal{E})$, which, in general, is not a clique. However, we now present an approach for computing Whittle indices by treating the graph as a clique, i.e., by assuming that a given user in $\mathcal{V}$ interferes with every other user in $\mathcal{V}$. After Whittle indices have been computed using this approach, an independent set of users to activate in the slot is selected using the algorithm provided in Section III-A.

The motivation for treating the graph as a clique is provided in Section III-B.

The Whittle index, $\lambda^i(X^i_n)$, for state $X^i_n = x^i$ is calculated by the following iteration (explained in Section III-B3): At step
m, solve the linear system of equations in variables $V^i(\cdot), \beta^i$ given by:

\[
V^i(y^i) = \sum_k V^i([y^i - y^i \land \Psi^i + k] \land M^i)\mu^i(k) - \beta^i + C^i y^i + f^i(y^i \land \Psi^i), \quad y^i \geq x^i,
\]

\[
V^i(0) = 0,
\]

and then perform a single iterate of

\[
\lambda_{m+1}^i = \lambda_m^i + \gamma \left[ f^i(x^i \land \Psi^i) - \lambda_m^i \sum_{(j: i \in N^*(j))} \nu^j \right.
\]

\[
+ \sum_k \mu^i(k)(V^i([x^i - x^i \land \Psi^i + k] \land M^i)) - V^i([x^i + k] \land M^i)\].
\]

Here $\gamma > 0$ is a small learning parameter. Note that $y^i - y^i \land \Psi^i = \max(y^i - \Psi^i, 0)$.

2) Computation Based on Constraints Derived from Original Graph: We now present an approach for computing Whittle indices based on interference constraints derived from the original graph $G = (V, E)$ (i.e., not by treating it as a clique). From (2), we get $\sum_{y \in N^*(i)} \nu^i = 1 \forall i$. Relax it to:

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{j \in N^*(i)} E \left[ \sum_{m=0}^{N-1} \nu^i_m \right] = 1.
\]

By (3) and using the Lagrange relaxation, the unconstrained problem has running cost:

\[
\sum_{i \in V} \left[ C^i x^i + f^i(x^i) + \lambda^i \sum_{(j: i \in N^*(j))} \nu^j \right].
\]

At time slot $n$, for each $i \in V$, the Whittle index, $\lambda^i(X^n_i)$, for state $X^n_i = x^i$ is calculated by the following iteration (explained in Section III-B3). At step $m$, solve the following linear system of equations in variables $V^i(\cdot), \beta^i$ for each fixed choice of $\nu^i$ such that $i \in N^*(j)$.

\[
V^i(y^i) = \sum_k V^i([y^i - y^i \land \Psi^i + k] \land M^i)\mu^i(k) - \beta^i + C^i y^i + f^i(y^i \land \Psi^i), \quad y^i \geq x^i,
\]

\[
V^i(0) = 0,
\]

and then perform a single iterate of

\[
\lambda_{m+1}^i = \lambda_m^i + \gamma \left[ f^i(x^i \land \Psi^i) - \lambda_m^i \sum_{(j: i \in N^*(j))} \nu^j \right.
\]

\[
+ \sum_k \mu^i(k)(V^i([x^i - x^i \land \Psi^i + k] \land M^i))
\]

\[
- V^i([x^i + k] \land M^i)\].
\]

Here we take $\sum_{(j: i \in N^*(j))} \nu^j$ to be the size of the maximum independent set in the subgraph formed by $N(i)$.

In both cases (Sections III-B1 and III-B2), for computational simplicity, the above iteration is performed for sufficiently large number of $x^i$ and then interpolated.

3) Explanation for Above Computational Schemes: We comment briefly on the theoretical underpinnings of the foregoing, omitting details both because of paucity of space and the fact that they are analogous to (22). To justify a Whittle-like index, one needs to establish Whittle indexability, i.e., the fact that the set of passive states depends monotonically on $\lambda_i$ for each $i$. For the first case (Section III-B1), the situation is exactly as in (22) and in the second case (Section III-B2), there is an $i$-dependent scaling of $\lambda$ that does not affect the argument. The computational scheme in each case is similar to that of (22): Whittle index is defined in terms of the value of $\lambda_i$ that, for given $x$, renders two functions of $x$ and $\lambda_i$ equal. The iterative scheme iterates candidate index values incrementally in the direction that decreases the difference between the two quantities and can be analyzed exactly as in (22).

IV. Simulations

In this section, we evaluate the performances of the proposed Whittle index based algorithms and compare them with those of the well-known Slotted ALOHA [33] and Max-Weight Scheduling (MWS) [10] algorithms via simulations. The performance is evaluated in terms of two metrics—average cost and average total number of packets dropped at all the users in the network per time slot. We briefly review the Slotted ALOHA and Max-Weight Scheduling Strategies in Sections IV-B and IV-C respectively.

A. Slotted ALOHA and Max-Weight Scheduling Strategies

1) Slotted ALOHA: Slotted ALOHA is a widely used randomized medium access control protocol [33]. Under this protocol, whenever a given user has packets to transmit in a given time slot, it transmits them with probability $p$ and does not transmit them with probability $1 - p$ in the time slot, where $p \in (0, 1)$ is a parameter. If two or more neighbouring users transmit in a given slot, then a collision takes place; in this case, each user involved in the collision repeats the above protocol, i.e., transmits (respectively, does not transmit) with probability $p$ (respectively, $1 - p$), in the next time slot.

2) Max-Weight Scheduling: The Max-Weight Scheduling algorithm has been used in the context of resource allocation in wireless networks [10], [14], [35] scheduling in input-queued switches [36] and several other contexts, and has been analytically shown to result in a high throughput and stability region in several prior works. Under the Max-Weight
Scheduling algorithm, in each time slot $n$, each user is assigned a weight equal to the length of the queue of packets waiting at the transmitter that need to be sent to the receiver of that user. The weight of an independent set of users is defined to be the sum of the weights of the users in the independent set. In each time slot $n$, the independent set with maximum weight is found and each user belonging to that independent set transmits. Note that computation of a maximum weight independent set is an NP-complete problem [37]. Hence, practical implementation of the Max-Weight Scheduling algorithm is computationally prohibitive. Nevertheless, we use the Max-Weight Scheduling algorithm as a benchmark for comparison with our scheme.

B. Simulation Model

In our simulations, we consider the model described in Section II with $L = 20$ users and buffer capacity $M_i = 100$ for each user $i \in V$. The location of each user is selected uniformly at random in a square of dimensions $1 \text{ unit} \times 1 \text{ unit}$. Two users are neighbors if the distance between them is less than $d_{\text{threshold}}$, where $d_{\text{threshold}}$ is a parameter. Throughout the simulations, we use $d_{\text{threshold}} = 0.6$ units. Let $\Psi^i$ be the maximum number of packets that may be transmitted by user $i$ in a given time slot. For the Whittle index based algorithms, we consider two cases: (i) $\Psi^i = \infty$, and (ii) $\Psi^i$ is uniformly chosen to be some value between $1$ and $M_i/5$ for user $i$ independent of other users. We refer to cases (i) and (ii) as the “unrestricted transmission” and “restricted transmission” cases respectively. Note that in case (i), a user that transmits in a time slot sends all the packets in its queue. Throughout our simulations, under both the Slotted ALOHA and Max-Weight Scheduling algorithms, the value $\Psi^i$ for user $i$ is the same as that in the restricted case of the Whittle index based algorithms. We assume that the number of packets, $\xi^i_m$, that arrive at user $i$ in time slot $n$ is a Poisson random variable with mean $l^i$; also, unless otherwise mentioned, $l^i$ is selected uniformly at random to be a value between $1$ and $M_i/10$ for each $i$ independent of other users.

C. Simulation Results

Henceforth, we refer to the algorithms based on computation of the Whittle indices as in Sections III-B1 and III-B2 as the “Clique Whittle Policy” and “Graphical Whittle Policy” respectively.

Fig. 1 shows the Whittle Index $\lambda(x)$ for the Clique Whittle Policy versus the number of packets, $x$, in the queue for each of the holding cost values $C = 20, 50$ and $100$. We see that for each value of $C$, $\lambda(x)$ decreases in $x$. Also, when $C$ is increased, for a given value of $x$, $\lambda(x)$ decreases. Since under the algorithm described in Section III-A, we preferably select users with low values of $\lambda(x)$ for transmission, the trends in Fig. 1 show that users with high holding costs $C$ and large queue lengths $x$ are preferred for transmission. Intuitively, this leads to a low average cost, since the average cost is an increasing function of the holding cost $C$ as well as the queue length $x$ (see Fig. 3).

Next, by “large arrival rates” (respectively, “small arrival rates”), we mean that $l^i$ is chosen uniformly at random between $1$ and $M_i/3$ (respectively, between $1$ and $M_i/15$). Figs. 2 and 3 compare the performances of the Clique Whittle Policy, Graphical Whittle Policy, Slotted ALOHA algorithm and Max Weight Scheduling (MWS) algorithm in terms of average cost and average total number of packets dropped at all the users in the network per time slot for the cases with (i) small arrival rates and restricted transmissions and (ii) large arrival rates and restricted transmissions respectively. Fig. 3 shows that in the case with small arrival rates and restricted transmissions, the Graphical Whittle Policy performs the best in terms of average cost, whereas the Clique Whittle Policy performs the best in terms of packets dropped. Fig. 4 shows that in the case of large arrival rates with restricted transmissions, the Clique Whittle Policy outperforms the other three algorithms in terms of both average cost and packets dropped. Overall, Figs. 2 and 3 show that both the Whittle index based policies outperform the Slotted ALOHA algorithm in all the cases and the MWS algorithm in most of the cases considered. While transmissions in practice will always be restricted, we also tried the above comparison in the unrestricted transmissions case. The performance of our proposed policies (not reported here due to paucity of space) continued to be superior to the other two schemes for low arrival rates, but not so for high arrival rates. This is presumably because the errors due to ad hoc tweaks in both variants of our policies become more pronounced in the high traffic (i.e., high arrival and transmission rates) regime.

Graphical Whittle Policy, Slotted ALOHA algorithm and Max Weight Scheduling (MWS) algorithm in terms of average cost and average total number of packets dropped at all the users in the network per time slot for the cases with (i) small arrival rates and restricted transmissions and (ii) large arrival rates and restricted transmissions respectively.

Fig. 1. Variation of Whittle Index versus number of jobs in the queue with holding costs 20, 50 and 100

Fig. 2. Average cost and packets dropped under different algorithms for small arrival rates with restricted transmissions

Fig. 3. Average cost and packets dropped under different algorithms for large arrival rates with restricted transmissions
Finally, we consider the following common generalization of the Clique Whittle and Graphical Whittle policies: during the computation of the Whittle indices, instead of using the constraints derived from the original graph as in Section [II-B], constraints derived from the graph in which two users are neighbors iff the distance between them is less than a parameter \(d_{\text{threshold}}\) are used. However, when users actually transmit, as before, the transmissions of two users interfere iff the distance between them is less than \(d_{\text{threshold}}\). Note that since all users are located in a 1 unit \(\times\) 1 unit square, the special case \(d_{\text{threshold}} = \text{computation}\) of the above generalized policy corresponds to the Graphical Whittle policy (respectively, Clique Whittle policy). We investigate as to what values of \(d_{\text{threshold}}\) result in the best performance. For the parameter values \(\Psi = 20\) and \(C^i = 20\ \forall i \in \mathcal{V}\), Fig. 4 shows the average cost and average total number of packets dropped at all the users in the network per time slot under the policies with different values of \(d_{\text{threshold}}\) for the restricted transmissions case. The average cost as well as packets dropped are minimized when \(d_{\text{threshold}} = 0.4\). Thus, by using plots such as those in Fig. 4 we can find out as to what values of \(d_{\text{threshold}}\) result in the best performance for given parameter values.

V. CONCLUSIONS

We proposed two Whittle index based scheduling policies for scheduling packet transmissions with the objective of minimizing the energy consumption and data transmission delay of users in a wireless network in which spatial reuse of spectrum is employed. The first treats the graph as a clique and the second is based on interference constraints derived from the original graph. We evaluated the performance of these two policies via extensive simulations, in terms of average cost and packets dropped, and showed that they outperform the well-known Slotted ALOHA and maximum-weight scheduling algorithms.

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