Macrodeterminism without non-invasive measurability

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We propose a definition of determinism for a physical system that includes, besides the measured system, the measurement device and the surrounding environment. This enlarged system is assumed to follow a predefined trajectory starting from some (unknown) initial conditions that play the role of hidden variables for the experiment. These assumptions, which are different from realism, allow us to derive some Leggett-Garg inequalities, which are violated by Quantum Mechanics in the particular case of consecutive measurements on individual photon polarizations.

I. INTRODUCTION

Using the terminology of [1], macroscopic realism or macrorealism for a system entails the following three postulates:

1. **Macrorealism per se**: A macroscopic object which has available to it two or more macroscopically distinct states is at any given time in a definite one of those states.

2. **Non-invasive measurability (NIM)**: Performing a measurement on the system has no effect on the capacity of predicting ulterior measurement outcomes.

3. **Induction**: The properties of the system are determined exclusively by the initial conditions and, in particular, not by final conditions.

It is well known that these assumptions allow us to prove the Leggett-Garg inequalities for successive measurements [2]. A violation of these inequalities leads to the macrorealism failure, through the invalidation of at least one of the above assumptions. Furthermore, in [3] it is pointed out that macrorealism also leads to a new property to add to the previous ones, the so-called no-signaling in time, which means that ‘a measurement does not change the outcome statistics of a later measurement’. The authors of [3] stress the independence of this third assumption from the other ones.

A system made of a photon, whose polarization is consecutively measured along different directions, randomly chosen among three selected and fixed directions, can be seen, in accordance with Quantum Mechanics (QM), to violate the no-signaling in time condition and also (see for instance Section III) the Leggett-Garg inequalities. This second violation had to be expected since the above NIM condition (2) is by no means satisfied in the present case.

But what about assumption (1), macrorealism per se, has it also to be invalidated in this case? Instead of trying to answer this question, we will recast it in the following one: Do the successive measurement outcomes obey determinism in the specific way this term is used in classical physics, say Laplacian determinism, where the evolution of the system is dictated by some initial conditions? At first sight, the question seems nonsense since the measured photon is not an isolated system: The measurement device acts on the measured system, after randomly choosing different measurement directions, leading to the corresponding collapse of the wave function. But our initial question is not whether each individual photon is a deterministic system, but whether we can define an isolated enlarged system, $E$, consisting of this measured particle, the entire measurement device, and perhaps some environment interacting with both of them, that behaves in a Laplacian deterministic way, at least during a significant period of time. In the next Section, we consider this question in detail.

II. ASSUMING LAPLACIAN DETERMINISM FOR THE ENLARGED SYSTEM

Before entering into the main subject of the paper, let us make some terminology precisions: Lately, some general consensus seems to have been reached on the specific meaning of the two current terms "local realism" and "de-
termism" (see [3], for example): The first term "demands that the probability for obtaining outcomes A and B under settings a and b can be written as a convex combination of products of probabilities which depend only on the local setting and a shared (hidden) variable"; the second term is a particular case of the first one "where the outcome probabilities are either 0 or 1". In the present paper, "determinism" will mean what is meant by this term in Newtonian mechanics, and by well known historical reasons we will call it here "Laplacian determinism". Obviously, its meaning does not coincide with the previous definition of "determinism". Then, hereafter, in order to avoid any confusion between both concepts, we call the last one "determinism as such". Let us be more specific:

Consider the time evolution of an isolated physical system, either classical or quantum. Does it always exist, as a matter of principle, a trajectory which, from some initial conditions, gives the values of the different quantities of the system during some finite duration of time, as it is, for example, the case in Newtonian mechanics, modulo some regularity conditions? This constitutes the concept of Laplacian determinism in nature. As we have already commented, this concept is sensibly different from the concepts of local realism, and determinism as such. More precisely, local realism and determinism as such, postulate the existence of some hidden variables values behind the outcome of a performed measurement, without requiring that these hidden values remain the same after performing the measurement. On the contrary, Laplacian determinism (hereafter LD) for the isolated physical system postulates the existence of the same standing hidden variable values (the unknown initial conditions) behind all the successive self-responses of the system along a certain finite time. In the present paper, this isolated system will be the above enlarged system $E$.

As a matter of fact, LD can be present under some circumstances, but the problem we want to address is whether LD is always present regardless of our capacity of prediction in practice. Therefore, to discard LD it suffices to find at least one example where it is contradicted by experimental data. This is what we are going to discuss along the present paper. Our claim is that this kind of determinism fails because it enters in contradiction with QM and, maybe, with experiments (modulo some loopholes, also present in the well known case where QM enters in contradiction with local realism).

QM is described in standard textbooks as an extremely successful non-deterministic theory, since the result of measurements can only be predicted in an statistical way. This is so even if the equation of motion (the Schrödinger equation, for non relativistic QM) provides the future state of the system when the initial conditions are specified. In this sense, the time evolution of QM might be considered as deterministic. It is the measurement process that introduces the non deterministic nature of QM, since the result of a measurement can not, in general, be predicted with certainty from the knowledge of the corresponding quantum state. Moreover, this state is modified by the measurement, leading to the well known collapse of the wave function. In other words, the measurement process is invasive (in the specific sense described in the Introduction) with respect to the system to be measured.

Thus, even if we admit that the quantum time evolution can be considered as deterministic, the action of the measurement device on the system under consideration appears as an extra ingredient that modifies the conditions of that system. Once we accept this, the question arises: Is it possible to postulate a Laplacian deterministic evolution that describes, not only the quantum system (represented by $Q$) under study, but also the measurement apparatus and, if necessary, the surrounding environment? We have referred to this system as the enlarged system $E$. Our Laplacian deterministic hypothesis will be formulated for $E$, and not for the quantum system, $Q$, to be measured. This has to be clearly stated in order to avoid further misunderstandings: The state of $Q$ may be (and in general will be) altered upon interaction with the measurement device in a way that the theory seems unable to predict. However, system $E$ as a whole will follow a predetermined trajectory that evolves from some initial conditions, according to the Laplacian deterministic hypothesis we want to be tested by confrontation with the experiment.

The definition of determinism we introduced above, Laplacian determinism, differs from local realism or determinism as such, can be discarded by proposed experiments, and is based on testing some Leggett-Garg inequalities, which take the form (see next Section)

$$|P(a,b) - P(a,c)| \leq 1 - P(b,c), \quad (1)$$

where $P(a,b)$ is the expected value associated to consecutive measurements along polarization directions $a$ and $b$ (similarly for the rest of magnitudes on this equation), on the same photon.

As shown in the next Section, we can find a scenario in which LD for the enlarged system, $E$, enters in contradiction with QM. Similarly to what happens with Bell inequalities, where local realism is falsified, modulo two well known loopholes (see next), LD enters in contradiction with QM, modulo the same loopholes. As already mentioned, this enlarged system includes, at least, the measurement device besides the measured system $Q$.

Similar results to the ones reported above have been previously obtained by De Zea [3] by adding to the deterministic postulate a non contextuality condition (freedom of choice, in the parlance of some authors), which amounts to assuming that initial conditions and measurement directions are uncorrelated. Nevertheless, this postulate needs to be justified, specially when LD is assumed, since then these directions depend on the initial conditions. In the present
paper, we propose a kind of experiment for which LD by itself would not entail contextuality, modulo the mentioned loopholes.

At the end of the paper we sketch the main differences between our paper and De Zela’s one [4].

III. LAPLACIAN DETERMINISM AND THE VIOLATION OF THE LEGGETT-GARG INEQUALITIES

We start with a source of individual free photons that will provide a large ensemble of photons numbered by 

\[ n = 1, 2, 3, \ldots, N \]

all of them prepared on the same quantum state, and consider the following ideal experiment: We first fix a set of three space directions given by the unit 3-vectors \( \vec{a}, \vec{b}, \vec{c} \). Then, on each photon we perform two successive polarization measurements along two randomly selected directions out of this set: We prepare one photon and make two consecutive measurements, then prepare the next photon, reset time to zero and proceed in the same way, etc ...

These "free" photons could evolve interacting, to some extent, with their environment in an uncontrollable way, and will certainly interact with the measurement apparatus. The latter includes the device that randomly selects the two measurement directions out of the three initially fixed directions. Consider now the physical system that includes the photons to be measured, the experimental facility and the interacting environment. In accordance with the two precedent sections, we refer to this system as the *enlarged system*, represented by \( E \), and assume that such system can be considered as an isolated system during the whole experiment.

We now define our notion of determinism following what we have already established, that is, following what happens, for example, with Newtonian determinism (where, aside from some “pathological” cases, the initial position and velocity, i.e., the initial conditions, allow us to know some piece of the trajectory of a particle). We will make the corresponding deterministic hypothesis, that we have called Laplace determinism (LD), for the enlarged system \( E \). Then the successive polarization measurement outcomes, ±\( \hbar \), are fixed from the initial conditions on \( E \). Notice that we do not impose any restrictions on the assumed initial conditions: In particular, they could be non local, i.e. they could range over non causally connected space-time regions.

Let us be more precise about our enlarged system and the measurement process, which is sketched in Fig. 1. We denote by \( \lambda \) the initial conditions that we postulate to exist, complementary to the prepared state of the quantum description, but leading to the same statistical predictions than this quantum description. These initial conditions will not belong exclusively to the photon, but to the whole system \( E \). Imagine that, each time we prepare a photon, system \( E \) starts from different initial conditions, i.e., different \( \lambda \) values. We will perform two consecutive polarization measurements on each prepared photon. These two consecutive polarization measurements will be performed at two randomly selected times, out of three fixed values \( t_1, t_2, \) and \( t_3 \), relative to the preparation time (which is always reset to zero). To each selected time, \( t_1, t_2 \) or \( t_3 \), we associate a constant measurement direction, \( \vec{a}, \vec{b}, \vec{c} \), respectively. In other words, we originally establish a given correspondence \( \{ t_1 \rightarrow \vec{a}, t_2 \rightarrow \vec{b}, t_3 \rightarrow \vec{c} \} \) and keep it unchanged during the measurement process for the entire set of photons. Thus, the randomness in the direction selection arises only as a consequence of the randomness in the selected pair of times out of the set \( \{ t_1, t_2, t_3 \} \). The device (hereafter referred to as \( P \)) that selects the pair of times, \( (t_1, t_2) \), \( (t_1, t_3) \) or \( (t_2, t_3) \) (and so the choice of the pairs of measurement directions) consists on a pseudo-random generator.

Then, let us denote by \( S \) the values of the photon polarization measurement outcomes, which are conveniently normalized to ±1. According to the LD postulate for our enlarged system, there exists a function (unknown to us) which provides those outcomes for each value of time \( t_i \), starting from the initial conditions, i.e., the parameter values \( \lambda \). Let us represent this function by \( S = S(\lambda, t_i, \vec{x}(t_i)) \), \( i = 1, 2, 3 \), with \( \vec{x}(t_i) \in \{ \vec{a}, \vec{b}, \vec{c} \} \). Notice that this notation for the function \( S \) is actually redundant: According to the above discussion, once we have fixed \( \lambda \) and \( t_i \), the value of \( S \) becomes determined, therefore we could drop the argument \( \vec{x} \) in \( S \), although we will keep it for convenience in the following discussion.

We now formally follow the original proof of usual Bell inequalities [5] in order to arrive to the Leggett-Garg inequalities [1] for our consecutive measurement outcomes. Let us consider the following three expectation values:

\[
P(a, b) = \int d\lambda \rho(\lambda) S(\lambda, t_1, \vec{a}) S(\lambda, t_2, \vec{b}),
\]

\[
P(a, c) = \int d\lambda \rho(\lambda) S(\lambda, t_1, \vec{a}) S(\lambda, t_3, \vec{c}),
\]

\[
P(b, c) = \int d\lambda \rho(\lambda) S(\lambda, t_2, \vec{b}) S(\lambda, t_3, \vec{c}),
\]
Figure 1: A sketch of the measurement apparatus, as described in the text. $M_a$, $M_b$, and $M_c$ perform polarization measurements on the system $Q$ (an individual photon) according to the selection made by the pseudorandom generator device $P$. Only two consecutive measurements are performed. The experimental setup is contained on some enlarged system $E$, which is assumed to be isolated.

where $\rho(\lambda)$ stands for the probability distribution of the $\lambda$ values, which satisfies $\int d\lambda \rho(\lambda) = 1$. There are some subtleties related to the derivation of the above equations. To begin with, the preparation of each photon, that is, the determination of the corresponding initial conditions $\lambda$, might be correlated with the chosen pair of directions corresponding to the two consecutive measurements. In such a case, in (2)-(4), we should write $\rho_{ab}(\lambda)$, $\rho_{ac}(\lambda)$, or $\rho_{bc}(\lambda)$, respectively instead of a common $\rho(\lambda)$, which will impede us to complete the proof that follows below for the Leggett-Garg inequalities. To avoid this, we assume that the times (and so the corresponding measurement directions) selected by the device $P$ in Fig. 1, and the events corresponding to the photon production, are space-like separated (in a similar way to the one described in [6]). Thus, we will assume that there is no such a correlation, that is, we will assume freedom of choice.

However, there are still two loopholes, which are sometimes referred to as "superdeterminism" (or in a equivalent way "superrealism") and "supercorrelation" [6, 7], also present in the proof of all kinds of Bell inequalities. We will consider these loopholes in the next Section, as well as a subtlety on the question of statistical "reproducibility" which is specific to the present case, where we consider an enlarged system $E$ which might have an enormous number of degrees of freedom.

Thus, we start from our expressions (2)-(4) and go on with the derivation of our Leggett-Garg inequalities. We take the difference
\[ P(a, b) - P(a, c) = \int d\lambda \rho(\lambda)S(\lambda, t_1, \vec{a}) \times [S(\lambda, t_2, \vec{b}) - S(\lambda, t_3, \vec{c})]. \]

Henceforth, the proof of the inequalities goes formally along the same lines as the proof of the original Bell inequalities in Bell’s seminal paper [8]. First, since \( S^2(\lambda, t_2, \vec{b}) = 1 \), the above difference can be written as

\[ P(a, b) - P(a, c) = \int d\lambda \rho(\lambda)S(\lambda, t_1, \vec{a})S(\lambda, t_2, \vec{b}) [1 - S(\lambda, t_2, \vec{b})S(\lambda, t_3, \vec{c})]. \]

Then, taking absolute values, we are led to

\[ |P(a, b) - P(a, c)| \leq \int d\lambda \rho(\lambda)[1 - S(\lambda, t_2, \vec{b})S(\lambda, t_3, \vec{c})], \]

that is, to the well known Leggett-Garg inequality

\[ |P(a, b) - P(a, c)| \leq 1 - P(b, c), \]

referring to two consecutive measurements on the same photon.

In QM, leaving aside the experimental difficulties to perform the kind of experiment we are considering, the three mean values in (9) can be theoretically calculated as the corresponding expected values. These values become \( P(a, b) = \cos 2\theta_{ab} \), where \( \theta_{ab} \) is the angle between \( \vec{a} \) and \( \vec{b} \), irrespective of the photon state prior to the first measurement and similarly for \( P(b, c) \) and \( P(a, c) \). Thus, inequality (9) becomes

\[ |\cos 2\theta_{ab} - \cos 2\theta_{ac}| + \cos 2\theta_{bc} \leq 1, \]

which is violated for example for \( \theta_{ab} = \theta_{bc} = \pi/6 \) and \( \theta_{ac} = \pi/3 \), in which case the left hand side of inequality (10) reaches the value 3/2.

Thus, for the enlarged system consisting on our photons and the measurement apparatus, including the device selecting the time pairs and the corresponding measurement directions, plus the affecting environment if any, the assumed LD enters in contradiction with quantum mechanics. In other words, the essence of the present paper is the following: could we consider any system \( E \) larger than the particle under study, so as to encompass the measurement device, perhaps the laboratory and even beyond, in order to attain an isolated enlarged system whose evolution fulfills the Laplacian deterministic assumption? Our claim is that, if the appropriate measurements were performed and inequality (10) was found to be violated, as we expect from QM, then (up to two loopholes), no matter how large \( E \) was assumed to be, the answer to this question would be negative.

At this point, it is interesting to compare our result with a similar one stated in [8]: There, under the following three postulates, macroscopic realism per se, noninvasive measurability and induction, Leggett derives some Clauser-Horn-Shimony-Holt (CHSH) inequalities [9, 10], relating the successive outcomes of a system with random dichotomic responses against four types of measurements. Our LD postulate entails macroscopic realism per se and the induction postulate. Thus, a difference between Leggett-Garg’s approach and ours is that we replace the non-invasive measurability for the measured system \( Q \) by a Laplacian deterministic postulate for the isolated enlarged system, which in our opinion is full of epistemological meaning. Since this determinism is assumed for the whole system \( E \), this implies that all quantities in this system, including the measuring device, are defined at all times once some starting initial conditions \( \lambda \) belonging to \( E \) are specified (even if unknown). Furthermore, since the measured system, \( Q \), is a part of \( E \) and \( E \) is Laplacian deterministic, so is the \( Q \) system, although its corresponding initial conditions, \( \lambda \), do not belong exclusively to \( Q \), but to \( E \). This reasoning formally converts \( Q \) into a non-invasive measured system, at the cost of allowing for initial conditions not entirely belonging to \( Q \).

**IV. TWO LOOPHOLES AND A SUBTLE QUESTION**

Because of the assumed space-like separation of the two events, particle preparation and choice of the measurement directions, we have discarded any kind of correlation between the initial conditions \( \lambda \), on the one hand, and the selected pair of measurement directions \((\vec{a}, \vec{b}), (\vec{a}, \vec{c}), \) or \((\vec{b}, \vec{c})\), on the other.
As pointed out above, there is, however, a first loophole against this assumed no correlation, the so called "superdeterminism" (also called "superrealism") (see, for example, [6,7]): Let us consider the past light cone associated to the particle preparation that fixes the corresponding initial conditions \( \lambda \), and also the two past light cones associated to selecting the first and second directions, say \( \vec{a} \) and \( \vec{b} \). The \( \lambda \) past cone and the \( \vec{a} \) cone will share a common region, and similarly for the \( \lambda \) and \( \vec{b} \) cones. Then, these common regions could be the source of some correlation between \( \lambda \) and \( (\vec{a}, \vec{b}) \).

The second loophole or "supercorrelation" [7] goes this way: without resorting to any shared region among the backward cones of the "superdeterminism", some correlations between initial conditions and measurements settings could be originally arranged between space-like separated places on the basis of what Bell [11] called a possible conspiratory arrangement in Nature.

As we have already commented in the precedent Section, both loopholes are also present in the proofs of the different versions of Bell’s inequalities. In [12,13], the question of these two loopholes has been addressed in the perspective of a procedure to reduce a given, small enough, degree of correlation to a virtually absence of correlation, assuming the non-signaling postulate, a procedure that we cannot apply here since this postulate is not satisfied in the case of our measured system \( Q \), i.e., in the case of the considered photons.

To end the present Section we refer to a subtle difficulty: perhaps the different initial conditions \( \lambda \) never repeat themselves completely. This should not be so strange in the present case of an enlarged macroscopic system whose number of degrees of freedom might be huge. Because of this hypothetical non repetition, we could have three hypothetical subsets, let us designate them, in an evident notation, \( \{ \lambda_{ab} \} \), \( \{ \lambda_{ac} \} \) and \( \{ \lambda_{bc} \} \), that might never achieve a standing equality \( \{ \lambda_{ab} \} = \{ \lambda_{ac} \} = \{ \lambda_{bc} \} \equiv \{ \lambda \} \). The lack of this equality would impede the proof of the Leggett-Garg inequalities that we have displayed in the previous Section. Nevertheless, this hypothetical inequality is not the problem, since what is physically relevant is not the, for example, \( \lambda_{ab} \) values by themselves, but the corresponding function values \( S(\lambda_{ab}, t_1) \), entering in the expected value \( P(a, b) \), in (9). But in the present case with photons, we can rely on the statistical "reproducibility" of the measurement outcomes, meaning by this that, in accordance to QM, those expectation values become stabilized for a sufficiently large number, \( N \), of measurement pairs. This implies that, beyond \( N \), we still could formally write \( P(a, b) \) in (2) using the same finite subset, \( \{ \lambda_{ab} \} \), that we have used just for \( N \). Thus, in practice, because this "reproducibility" property, (which, in the present case, because of the quantum predictions, can be taken as an experimental fact) we can use a finite number of \( \lambda_{ab} \) values, and the same for the \( \lambda_{ac} \) and the \( \lambda_{bc} \), to avoid our initial difficulty.

Conclusions

Let us summarize the hypothesis we introduced in order to derive our Leggett-Garg inequalities [9], based on LD postulate for the enlarged system, as compared to other hypothesis currently made in similar areas:

- We assume Laplace determinism defined as follows: One can define an enlarged isolated system \( E \) that includes the particles to be measured, the experimental setup and the surrounding environment, such that all variables in that system evolve predictively from some unknown initial conditions \( \lambda \) belonging entirely to this isolated system. This amounts to consider the measured system as another Laplacian deterministic system, one whose corresponding initial conditions do not belong entirely to it, but to the isolated enlarged system. LD implies induction: Initial conditions are not affected by measurements performed later in time.

- We assume that there is no correlation between the initial conditions \( \lambda \), on the one hand, and measurements directions, on the other hand, by arranging the experimental setting such that particle preparation and the corresponding two measurement directions choice are space-like separated events.

- We need to use statistical reproducibility, which in the case of photons is predicted by QM, and thus could be considered as an experimental fact.

Leaving the particular case of photons, and going to the general case of a measured system, macroscopic or not, whose measured outcomes are dichotomic (say, for instance, \( \pm 1 \)) and apparently random, it is straightforward to see that we still could derive some Leggett-Garg inequalities by postulating LD for the corresponding enlarged system, provided that we experimentally guarantee a space-like separation between system preparation and selection setting, and provided that the measurement outcomes obey statistical reproducibility.

Obviously, in the general case, the two loopholes, "superdeterminism" and "supercorrelation" remain open, although one could help to partially close them by resorting to the procedure of free randomness amplification explained in [12], provided that the non-signaling postulate be actually satisfied in practice.

In all, modulo these two loopholes, the meaning of the attained violation of Leggett-Garg inequalities [9] is the impossibility of finding any enlarged system, \( E \), whatever large it be, that allow us to describe the photon polarization
measurement outcomes as belonging to this system $E$ obeying LD. In other words, this means the violation of LD in Nature.

Then, before finishing, let us sketch which are the main differences between our paper and the preceding one from De Zela [4]:

First: We have made his non contextuality assumption (freedom of choice assumption) more consistent with the central postulate of LD by virtually adopting an experimental setting where preparation of the system and setting choice are space-like separated events.

Second: In a case like the present one, and also like the one by De Zela, where the considered enlarged system has a huge number of degrees of freedom, we have shown how unavoidable is to rely on the statistical "reproducibility" in order to prove Leggett-Garg inequalities from LD.

Third: We have clearly identified the two loopholes left, "superdeterminism" and "supercorrelation", and in the general case, that is, apart from the particular case considered in the present paper, we point out the possibility of approaching their closeness by using a protocol to reduce the scope of these two loopholes [12], provided the non-signaling postulate be actually satisfied.

Finally: contrarily to De Zela, we have not needed to treat separately the two cases of non-invasive or invasive measurability since our central postulate, LD, for the enlarged system, $E$, entails by itself LD (and so formal non-invasiveness) for the measured system $Q$, although the $Q$ initial conditions do not exclusively belong to $Q$ itself.

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