Joint Beamforming for Intelligent Reflecting Surface-Assisted Millimeter Wave Communications

Peilan Wang†, Jun Fang†, and Hongbin Li §, Fellow, IEEE
†University of Electronic Science and Technology of China, Chengdu, China
§Stevens Institute of Technology, Hoboken, USA,
Email: peilan.wangle@gmail.com, JunFang@uestc.edu.cn, and Hongbin.Li@stevens.edu

Abstract—Millimeter wave (mmWave) communications are evolving as a promising technology to meet the ever increasing data rate requirements. However, high directivity and severe path loss make it vulnerable to blockages, which could be frequent in indoor or urban environments. To address this issue, intelligent reflecting surfaces (IRSs) are introduced to provide additional adjustable reflected paths. Most prior works assume that elements of IRSs have an infinite phase resolution, which is difficult to be realized in practical systems. In this paper, IRSs with low-resolution phase shifters are considered. We aim to maximize the receive signal power at the user by jointly optimizing discrete phase shifts of IRSs and the transmit beamforming vector at the base station for mmWave downlink systems. An analytical near-optimal solution is developed by exploiting some important characteristics of mmWave channels. Our theoretical analysis reveals that low-resolution phase shifters can still achieve a receive signal power that increases quadratically with the number of reflecting elements. Simulation results are provided to corroborate our analysis and show the effectiveness of the proposed solution.

Index Terms—Millimeter wave communication, intelligent reflecting surfaces (IRSs), beamforming.

I. INTRODUCTION

MmWave communications are considered as a potential and promising technology to support multi-gigabit wireless applications [1], [2]. However, mmWave signals cannot diffract as well as their sub-6GHz counterparts and easily get blocked by obstacles [3], which is especially the case for indoor or dense urban environments [4]. To address this issue, intelligent reflecting surface (IRS) has recently emerged as a promising technology to support mmWave communications [5], [6]. IRS is a planar array consisting of a large number of passive elements, each of which can reflect the incident signal with a reconfigurable phase shift and amplitude via a smart controller [6], [7]. By smartly tuning the phase shifts of passive elements, IRSs can help create effective virtual LOS links, resulting in a more reliable mmWave connection [4], [5].

IRS-aided wireless communications have attracted much attention recently [8]. Prior works on IRS-assisted transmissions can be found in [3], [9]–[13]. For the single-user scenario, it was shown [5], [10] that IRSs are able to achieve a squared power gain in terms of the number of reflecting elements, thus creating a “signal hotspot” in the vicinity of the IRS. In the multi-user scenario, [9], [12] claimed that, by carefully adjusting the phase shift parameters of the IRS, an “interference-free” zone can be formed near the IRS to suppress interference for each user. Also, it was shown [13] that the IRS-assisted system can achieve massive MIMO like gain with much fewer active antennas. However, most of the above studies were based on the assumption that elements of IRSs have an infinite phase resolution. Several works considered IRS-aided systems with discrete phase shifts, the proposed algorithms either involve an exhaustive search [4] or an iterative procedure to jointly search for the optimal beamforming vector and discrete phase shift parameters [14], [15].

In this paper, we consider an IRS-assisted mmWave downlink system, where multiple IRSs with discrete phase shifters are deployed to assist the downlink transmission from a multi-antenna base station to a single-antenna user. Our objective is to maximize the receive signal power at the user by joint optimizing the phase shift parameters of each IRS and the transmit beamforming vector at the base station. Although such an optimization problem is generally non-convex, by exploiting some inherent characteristics of mmWave channels, we show that a near-optimal analytical solution can be developed. Our theoretical analysis reveals that, even with low-resolution phase shifters, our proposed solution can still achieve a receive signal power that increases quadratically with the number of reflecting elements. In addition, when compared with the receive power achieved by IRSs with infinite-resolution phase shifters, the receive signal power attained by our proposed solution decreases by a constant factor that depends on the number of quantization levels.

II. SYSTEM MODEL AND PROBLEM FORMULATION

As shown in Fig. 1 we consider an IRS-assisted mmWave downlink system, where multiple IRSs are deployed to assist the communication from the base station (BS) to a single-antenna user. Suppose K IRSs are employed to assist the downlink transmission, and each IRS is equipped with M reflecting elements. The BS is equipped with N antennas. Denote $G_k \in \mathbb{C}^{M \times N}$ as the channel from the BS to the $k$th IRS, and $h_{rk} \in \mathbb{C}^M$ as the channel from the $k$th IRS to the user.

Each reflecting element of the IRS can reflect the incident signal with a reconfigurable phase shift and amplitude via a
smart controller [14]. Define

$$\Theta_k \triangleq \text{diag}(\beta_{k,1}e^{j\theta_{k,1}}, \ldots, \beta_{k,M}e^{j\theta_{k,M}})$$ (1)

as the phase-shift matrix of the $k$th IRS, where $\theta_{k,m} \in [0, 2\pi]$ and $\beta_{k,m} \in [0, 1]$ denote the phase shift and amplitude reflection coefficient associated with the $m$th passive element of the $k$th IRS, respectively. For simplicity, we assume $\beta_{k,m} = 1, \forall k, \forall m$ in the sequel of this paper. Also, due to the hardware limitation, the phase shift cannot take an arbitrary value, instead, it has to be chosen from a finite set of discrete values \([4], [14]\). Specifically, the set of discrete values for the phase shift is defined as

$$\theta_{k,m} \in \mathcal{F} \triangleq \left\{0, \frac{2\pi}{2^b}, \ldots, \frac{2\pi(2^b - 1)}{2^b}\right\}$$ (2)

where $b$ denotes the resolution of the phase shifter. In our model, the direct link from the BS to the user is neglected due to unfavorable propagation conditions. Denote $w \in \mathbb{C}^{N \times 1}$ as the beamforming/precoding vector adopt by the BS. The signal received at the user is given by

$$y = \left(\sum_{k=1}^{K} h_{r_k}^H \Theta_k G_k \right) w + \epsilon$$ (3)

where $s$ is the transmitted signal modeled as a random variable with zero mean and unit variance, and $\epsilon$ denotes the additive white Gaussian noise with zero mean and variance $\sigma^2$. It should be noted that in the above model, we ignore the signals reflected by two or more times due to the high path attenuation of mmWave transmissions. Thus, the received signal power at the user is given as

$$\gamma = \left| \left(\sum_{k=1}^{K} h_{r_k}^H \Theta_k G_k \right) w \right|^2$$ (4)

Assuming the knowledge of global channel state information, we aim to jointly design the transmit beamforming vector and the diagonal phase-shift matrices $\{\Theta_k\}$ to maximize the received signal power, i.e.,

$$\max_{w,\{\Theta_k\}} \left| \left(\sum_{k=1}^{K} h_{r_k}^H \Theta_k G_k \right) w \right|^2$$ s.t. $\|w\|^2 \leq p$

$$\Theta_k = \text{diag}(e^{j\theta_{k,1}}, \ldots, e^{j\theta_{k,M}}), \quad \forall k$$

$$\theta_{k,m} \in \mathcal{F}, \quad \forall k, \forall m$$ (5)

where $p$ denotes the maximum transmit signal power at the BS.

Note that such a joint beamforming problem has been studied in our previous work [5], where the rank-one structure of BS-IRS channels was exploited to help obtain a near-optimal analytical solution. Nevertheless, [5] assumes elements of IRSs have an infinite phase resolution. Also, the direct link between the BS and the user was considered in [5]. In this work, following [5], we assume that the channel from the BS to each IRS is a rank-one matrix, i.e.

$$G_k = \lambda_k a_k b_k^T \quad \forall k$$ (6)

where $\lambda_k$ is a scaling factor accounting for the complex path gain and the antenna gain, $a_k \in \mathbb{C}^{M \times 1}$ and $b_k \in \mathbb{C}^{N \times 1}$ represent the normalized array response vector associated with the IRS and the BS, respectively. Such a rank-one channel assumption is reasonable in mmWave communication systems because the power of the mmWave LOS path is much higher (about 13dB higher) than the sum of power of NLOS paths, as suggested by measurement campaigns conducted in [10]. In practice, with the knowledge of the location of the BS, IRSs can be installed within sight of the BS.

On the other hand, due to the use of a large number of antennas at the BS in mmWave systems and the fact that different IRSs, as seen from the BS, are sufficiently separated in the angular domain, it is reasonable to assume that array response vectors $\{b_k\}$ are near orthogonal to each other, i.e. $\|b_i^T b_j\| \approx 0$ for $i \neq j$.

III. PROPOSED SOLUTION

In this section, we propose a near-optimal analytical solution for the nonconvex problem (5), by exploiting the rank-one structure of BS-IRS channels and the near-orthogonality between steering vectors $\{b_k\}$. To solve (5), we first relax the discrete constraint placed on variables $\{\theta_{k,m}\}$:

$$\max_{w,\{\Theta_k\}} \left| \left(\sum_{k=1}^{K} h_{r_k}^H \Theta_k G_k \right) w \right|^2$$ s.t. $\|w\|^2 \leq p$

$$\Theta_k = \text{diag}(e^{j\theta_{k,1}}, \ldots, e^{j\theta_{k,M}}), \quad \forall k$$ (7)

Substituting $G_k = \lambda_k a_k b_k^T$ into the objective function of (5), we arrive at

$$\max_{w,\{\Theta_k\}} \left| \left(\sum_{k=1}^{K} h_{r_k}^H \Theta_k a_k b_k^T \right) w \right|^2$$ s.t. $\|w\|^2 \leq p$

$$\Theta_k = \text{diag}(e^{j\theta_{k,1}}, \ldots, e^{j\theta_{k,M}}), \quad \forall k$$

$$\theta_{k,m} \in \mathcal{F}, \quad \forall k, \forall m$$ (5)

where in (a), we define $\eta_k \triangleq b_k^T w$, $g_k \triangleq \lambda_k (h_{r_k}^* \circ a_k)$, $\circ$ denotes the Hadamard (elementwise) product, and $\theta_k \triangleq [e^{j\theta_{k,1}} \ldots e^{j\theta_{k,M}}]^T$, in (b), we express $\theta_k = \bar{\theta}_k e^{j\alpha_k}$, and
the inequality (c) becomes equality when arguments (i.e., phases) of all complex numbers are identical. It should be noted that we can always find a set of \( \{\alpha_k\} \) such that the arguments of \( \eta_k \tilde{\theta}_k \) are identical, although at this point we do not know the values of \( \{\alpha_k\} \). Therefore (7) can be converted into the following optimization

\[
\max_{w, \{\alpha_k\}} \sum_{k=1}^{K} \left| \eta_k \tilde{\theta}_k g_k \right|^2 + \sum_{i=1}^{K} \sum_{j \neq i} \left| \eta_i \tilde{\theta}_i g_i \right| \cdot \left| \eta_j \tilde{\theta}_j g_j \right|
\]

s.t. \( \|w\|_2^2 \leq p \) \hspace{1cm} (9)

From (9), it is clear that the optimization of \( \{\tilde{\theta}_k\} \) can be decomposed into a number of independent sub-problems, with \( \tilde{\theta}_k \) solved by

\[
\max_{\tilde{\theta}_k} \left| \tilde{\theta}_k \right|^2
\]

s.t. \( \tilde{\theta}_k = [e^{j\theta_{k,1}} \ldots e^{j\theta_{k,M}}]^T \) \hspace{1cm} (10)

It can be easily verified that the objective function reaches its maximum \( \|g_k\|_1 \) when

\[
\tilde{\theta}_k^* = [e^{-j\arg(g_{k,1})} \ldots e^{-j\arg(g_{k,M})}]
\]

where \( g_{k,m} \) denotes the \( m \)-th entry of \( g_k \), and \( \arg(x) \) denotes the argument of the complex number \( x \).

So far we have obtained the optimal solution of \( \{\tilde{\theta}_k\} \), which is independent of \( \{\alpha_k\} \) and \( w \). Based on this result, (7) is simplified as optimizing \( w \) and \( \{\alpha_k\} \):

\[
\max_{w, \{\alpha_k\}} \left| \sum_{k=1}^{K} \lambda_k e^{j\alpha_k} h_{k}^H \tilde{\theta}_k^* \hat{a}_k b_k^T \right|^2
\]

s.t. \( \|w\|_2^2 \leq p \) \hspace{1cm} (12)

where \( \tilde{\theta}_k^* \triangleq \text{diag}(\tilde{\theta}_k^*) \). Note that \( \lambda_k h_{k}^H \tilde{\theta}_k^* \hat{a}_k = g_k^T \tilde{\theta}_k^* = \|g_k\|_1 \triangleq z_k \) is a real-valued number. Thus the objective function of (12) can be written in a more compact form as

\[
\left| \sum_{k=1}^{K} z_k e^{j\alpha_k} b_k^T \right|^2 \overset{(a)}{=} \|v^H D_z B w\|^2 \overset{(b)}{=} \|v^H \Phi w\|^2 \hspace{1cm} (13)
\]

where in (a), we define \( v \triangleq [e^{j\alpha_1} \ldots e^{j\alpha_K}]^T, D_z \triangleq \text{diag}(z_1, \ldots, z_K) \) and \( B \triangleq [b_1 \ldots b_K]^T \), and in (b), we define \( \Phi \triangleq D_z B \). Hence (12) can be simplified as

\[
\max_{w, v} \|v^H \Phi w\|^2
\]

s.t. \( \|w\|_2^2 \leq p \)

\[
v = [e^{j\alpha_1} \ldots e^{j\alpha_K}]^H \hspace{1cm} (14)
\]

Note that for any given \( v \), the optimal preceding vector \( w \) is the maximum-ratio transmission (MRT) solution, i.e., \( w^* = \sqrt{\|v^H \Phi\|^2} v^H \Phi v^* \). Substituting the optimal preceding vector \( w^* \) into (14) yields

\[
\max_{v} \|v^H \Phi v\|^2
\]

s.t. \( |v_k| = 1, \forall k \hspace{1cm} (15) \)

The above optimization is a non-convex quadratically constrained quadratic program (QCQP). It was shown this QCQP can be relaxed as a standard convex semidefinite program (SDP) [17]. Nevertheless, such an approach is computationally expensive and does not admit a closed-form solution. On the other hand, note that \( \{b_k\} \) are near orthogonal to each other. Hence we have

\[
v^H \Phi v = v^H D_z B B^H D_z v \approx \|z\|_2^2 \hspace{1cm} (16)
\]

which is a constant independent of the vector \( v \). Hence any vector \( v \) which satisfies the constraint \( |v_k| = 1, \forall k \) is a near-optimal solution to (15). For simplicity, we choose \( \alpha_k = 0, \forall k \) as a solution to (15). In this case, we have

\[
\theta_k^* = \tilde{\theta}_k^* \hspace{1cm} \forall k
\]

(17)

Considering the finite resolution constraint imposed on the phase shifters, each phase shift, \( \theta_{k,m} \), can take on a discrete value that is closest to its optimal value \( \theta_{k,m}^* \):

\[
\theta_{k,m}^* = \arg \min_{\theta \in \mathcal{F}} |\theta - \theta_{k,m}^*| \hspace{1cm} (18)
\]

where \( \theta_{k,m}^* \) denotes the \( m \)-th entry of \( \theta_k^* \). After the phase shifts are determined, the beamforming vector \( w \) can be obtained according to the MRT solution.

IV. PERFORMANCE ANALYSIS

In this section, we analyze the power scaling law of the average received power as \( M \to \infty \). For simplicity, we set the maximum transmit signal power \( p = 1 \). From (4), the average received power attained by our solution with \( b \)-bit phase shifters is given by

\[
\gamma(b) = E \left[ \left( \sum_{k=1}^{K} h_{r_k}^H \Theta_k G_k \right)^2 \right] \hspace{1cm} (19)
\]

where \( \Theta_k = \text{diag}(\theta_{k,1}, \ldots, \theta_{k,M}) \) with \( \theta_{k,m}^* \) given by (18).

Our main results are summarized as follows.

**Proposition 1:** Assume \( h_{r_k} \sim CN(0, \sigma_r^2 I) \), and the BS-kth IRS channel is characterized by (6), with \( \lambda_k = \sqrt{NM \rho_k} \), where \( \rho_k \) denotes the complex path gain. As \( M \to \infty \), we have

\[
\eta(b) \overset{\gamma(b)}{\approx} \frac{\pi^b}{2} \sin \left( \frac{\pi}{2^b} \right)^2 \hspace{1cm} (20)
\]

**Proof:** Substituting (6) into (19), we arrive at

\[
\gamma(b) = E \left[ \left( \sum_{k=1}^{K} \sqrt{NM \rho_k} h_{r_k}^H \Theta_k^* \hat{a}_k b_k^T \right)^2 \right] \hspace{1cm} \overset{(a)}{=} E \left[ \left( \sum_{k=1}^{K} z_k b_k^T \right)^2 \right] \hspace{1cm} \overset{(b)}{=} \sum_{k=1}^{K} E[z_k^2] \hspace{1cm} (21)
\]
where \( (b) \) comes from the fact that \( \{b_k\} \) are near orthogonal to each other, and in \((a)\), we define
\[
\tilde{z}_k = \sqrt{N}M \rho_k h_{rk}^H a_k = \sqrt{N} |\rho_k| \sum_{m=1}^M |h_{r_k,m}|e^{j \Delta \theta_{k,m}}
\]
(22)
in which \( h_{r_k,m} \) is the \( m \)th entry of the channel vector \( h_{rk} \), and \( \Delta \theta_{k,m} \) is the discretization error
\[
\Delta \theta_{k,m} = \theta_{k,m}^* - \theta_{k,m}
\]
(23)
Since discrete phase shifts in \( F \) are uniformly spaced, discretization errors \( \{\Delta \theta_{k,m}\} \) can be considered as independent random variables uniformly distributed on the interval \([-\pi/2^b, \pi/2^b]\). Therefore we have
\[
E[\tilde{z}_k^2] = N E[|\rho_k|^2] E \left[ \sum_{m=1}^M |h_{r_k,m}|^2 \right] + N E[|\rho_k|^2] E \left[ \sum_{m=1}^M \sum_{i \neq m}^M |h_{r_k,m}||h_{r_k,i}| e^{j(\Delta \theta_{k,m} - \Delta \theta_{k,i})} \right]
\]
(24)
in which
\[
E \left[ \sum_{m=1}^M |h_{r_k,m}|^2 \right] = M \theta_k^2
\]
(25)
\[
E[e^{j \Delta \theta_{k,m}}] = E[-e^{j \Delta \theta_{k,m}}] = \frac{2^b}{\pi} \sin \left( \frac{\pi}{2^b} \right)
\]
(26)
Finally, the average received power can be given as
\[
\gamma(b) \approx N M \sum_{k=1}^K \theta_k^2 E[|\rho_k|^2]
\]
\[
+ N M (M - 1) \sum_{k=1}^K E[|\rho_k|^2] \frac{\pi \theta_k^2}{4} \left( \frac{2^b}{\pi} \sin \left( \frac{\pi}{2^b} \right) \right)^2
\]
(27)
It is not difficult to verify that \( E[e^{j \Delta \theta_{k,m}}] \) increases monotonically with \( b \) and approaches 1 as \( b \to \infty \). Hence, it can be easily obtained that the ratio of \( \gamma(b) \) to \( \gamma(\infty) \) is given by \( 2^b/\pi \) as \( M \to \infty \). This completes the proof.

This proposition provides a quantitative analysis of the average received signal power in multiple-IRS assisted system with discrete phases shifts. We see that, when compared with the receive power achieved by IRSs with infinite-resolution phase shifters, the receive signal power attained by our proposed solution decreases by a constant factor that depends on the number of quantization levels \( b \). Specifically, we have \( \eta(1) = 0.4053 \), \( \eta(2) = 0.8106 \) and \( \eta(3) = 0.9496 \). This result also implies that the squared improvement \( 5 \) brought by IRSs with continuous phase shifts can still be achieved with low resolution phase shifters. Note that in [13], the author obtained a similar insight for a single IRS assisted MISO system by assuming that the BS is equipped with only one antenna. We extend this conclusion to the multiple-IRS assisted MISO system based on the assumption of rank-one structure of the channel matrix between the BS and each IRS.

\[\text{V. SIMULATION RESULTS}\]

In this section, we present simulation results to illustrate the performance of our proposed IRS-assisted beamforming solution. We consider a scenario where the BS employs a uniform linear array (ULA) with \( N \) antennas, and each IRS consists of a uniform rectangular array (URA) with \( M = M_y M_z \) reflecting elements, in which \( M_y \) and \( M_z \) denote the number of elements along the horizontal and vertical axis, respectively. In our simulations, the IRS-user channel is generated according to the following geometric channel model [18]:
\[
h = \sqrt{\frac{M}{T}} \sum_{l=1}^L \alpha_l \lambda_r \alpha_l (\phi_{a_l}, \phi_{c_l})
\]
(28)
where \( L \) is the number of paths, \( \alpha_l \) is the complex gain associated with the \( l \)th path, \( \phi_{a_l}(\phi_{c_l}) \) is the associated azimuth (elevation) angle of departure, \( \alpha_t \in \mathbb{C}^M \) is the normalized transmit array response vector, \( \lambda_r \) and \( \lambda_t \) denote the receive and transmit antenna element gain. According to [9], [19], \( \lambda_r \) and \( \lambda_t \) are respectively set to 0dBi and 9.82dBi for the IRS-user link. The complex gain \( \alpha_l \) is generated according to a complex Gaussian distribution [20]:
\[
\alpha_l \sim \mathcal{CN}(0, 10^{-0.1\kappa})
\]
(29)
with \( \kappa \) given as
\[
\kappa = e + 10 f \log_{10}(\bar{d}) + \xi
\]
(30)
in which \( \bar{d} \) denotes the distance between the transmitter and the receiver, and \( \xi \sim \mathcal{N}(0, \sigma^2) \). The values of \( e, f, \sigma \) are set to be \( e = 72, f = 2.92, \) and \( \sigma = 8.7 \)dB, as suggested by real-world channel measurements in the NLOS scenario at 28GHz [20]. The BS-IRS channel is characterized by a rank-one geometric channel model given as
\[
G = \sqrt{N M} \alpha_r \lambda_r \alpha_c (\vartheta_r, \vartheta_e) \alpha_c^H (\phi)
\]
(31)
where \( \vartheta_r (\vartheta_e) \) denotes the azimuth (elevation) angle of arrival associated with the BS-IRS path, \( \phi \) is the associated angle of departure, \( \alpha_r \in \mathbb{C}^M \) and \( \alpha_c \in \mathbb{C}^N \) represent the normalized receive and transmit array response vectors, respectively. In our simulations, \( \lambda_r \) and \( \lambda_t \) are set to 0dBi and 9.82dBi, respectively. The complex gain \( \alpha \) is generated according to [20] and the values of \( e, f, \sigma \) in (30) are set to be \( e = 61.4, f = 2, \) and \( \sigma = 5.8 \)dB, as suggested by real-world channel measurements in the LOS scenario at 28GHz [20]. Also,
unless specified otherwise, we assume \( N = 32, M_y = 10, \) and \( M_z = 5 \) in our experiments. Other parameters are set as follows: \( p = 30\text{dBm}, \sigma^2 = -85\text{dBm}. \)

We consider a setup as depicted in Fig. 2 where \( K \) IRSs are equally spaced on a straight line which is in parallel with the line connecting the BS and the user. Specifically, the horizontal distance \( d_b \) between the BS and the first IRS is set to \( d_b = 11\text{m} \) and the vertical distance is set to \( d_v = 1.5\text{m} \). Also, the distance between the nearest IRS and the farthest IRS is set to be \( d = 50\text{m} \).

To verify the near-optimality of our proposed solution, an upper bound of the receive SNR can be obtained by solving a relaxed convex formulation of (15) [5]. Note that this upper bound is attained by assuming infinite-precision phase shifters. Also, to show the benefits brought by IRSs, a conventional system without IRSs is considered, where the channel between the BS and the user is generated according to [18]

\[
h_d = \sqrt{\frac{N}{L}} \sum_{l=1}^{L} \alpha_l a_l(\phi_l)
\]

where \( L \) is the number of paths, \( \alpha_l \) is the complex gain associated with the \( l \)-th path, \( \phi_l \) is the associated azimuth angle of departure, \( a_l \in \mathbb{C}^N \) is the normalized transmit array response vector. The complex gain \( \alpha_q \) is generated according to (29) and (30) in the NLOS scenario at 28GHz [20]. The optimal MRT solution is employed to achieve the maximum receive SNR.

Fig. 3 and Fig. 4 plot the receive SNRs of our proposed solution and the MRT solution vs. the distance between the BS and the user. It can be observed that the IRS-assisted system can help substantially improve the receive SNR, especially when the user is far away from the BS. Also, each IRS creates a “signal hotspot” in its vicinity: when the user moves closer to the IRS, its receive SNR becomes higher. In addition, we see that with 2-bit quantized phase shifts, our proposed solution can achieve a receive SNR close to the upper bound attained by assuming infinite-precision phase shifters. This result validates the near-optimality of our proposed solution.

To verify our power scaling law analysis, we plot the receive SNRs of different schemes versus the number of reflecting elements at each IRS in Fig. 5 where we set \( K = 3, d_u = 41\text{m} \), and we fix \( M_y = 10 \) and change \( M_z \). It can be observed that even with discrete phase shifters, the squared improvement still holds for our proposed solution. For example, when one-bit phase shifters are used, the receive SNR at the user is about 14dB when \( M = 50 \), and it gains 6dB increase as \( M \) doubles. Also, the receive SNR loss due to the use of low-resolution phase shifters is analyzed and given by [20]. Specifically, we have \( \eta(1) = -3.9224\text{dB} \) and \( \eta(2) = -0.9121\text{dB} \). It can be observed that simulation results are consistent with our theoretical results.

To show the robustness of IRS-assisted systems against blockages, we calculate the outage probability as follows

\[
P_{\text{out}}(\tau) = P\left( \mathbb{E} \left[ 10 \log_{10} \left( 1 + \frac{\gamma(b)}{\sigma^2} \right) \right] < \tau \right)
\]

where \( \tau \) denotes the required threshold level and set to \( \tau = 1.5\text{dB} \). For simplicity, we assume the link between BS and each IRS is always connected and the link between each IRS and the user is blocked with a pre-specified probability \( P \). We set \( d_u = 61\text{m} \) and \( b = 2 \). From Fig. 6 we observe that the outage probability can be substantially reduced by deploying IRSs. Also, the more the IRSs are deployed, the lower the outage probability can be achieved.
In this paper, we studied the problem of joint active and passive beamforming design for IRS-assisted mmWave systems, where multiple IRSs with discrete phase shifters are deployed to assist the downlink transmission from the BS to a single-antenna user. The objective is to maximize the received signal power by jointly optimizing the transmit beamforming vector and the discrete phase shift parameters at each IRS. By exploiting some inherent characteristics of mmWave channels, we derived a near-optimal analytical solution. Theoretical analysis reveals that low-resolution phase shifters can still achieve a receive signal power that increases quadratically with the number of reflecting elements at each IRS. Simulations were provided to corroborate our analysis and illustrate the near-optimality of our proposed solution.

ACKNOWLEDGMENT

This work was supported in part by the National Science Foundation of China under Grant 61829103 and Grant 61871091.

REFERENCES

[1] T. S. Rappaport, J. N. Murdock, and F. Gutierrez, “State of the art in 60-GHz integrated circuits and systems for wireless communications,” Proceedings of the IEEE, vol. 99, no. 8, pp. 1390–1436, August 2011.

[2] A. Ghosh, T. A. Thomas, M. C. Cudak, R. Ratasuk, P. Moorut, F. W. Vook, T. S. Rappaport, G. R. MacCartney, S. Sun, and S. Nie, “Millimeter-wave enhanced local area systems: a high-data-rate approach for future wireless networks,” IEEE Journal on Selected Areas in Communications, vol. 32, no. 6, pp. 1152–1163, June 2014.

[3] O. Abari, D. Bharadia, A. Duffield, and D. Katabi, “Enabling high-quality untethered virtual reality,” in 14th USENIX Symposium on Networked Systems Design and Implementation (NSDI 17). Boston, MA: USENIX Association, March 27-29, 2017, pp. 531–544.

[4] X. Tan, Z. Sun, D. Koutsonikolas, and J. M. Jornet, “Enabling indoor mobile millimeter-wave networks based on smart reflect-arrays,” in IEEE International Conference on Computer Communications (INFOCOM), Honolulu, Hawaii, April 15-19, 2018, pp. 270–278.

[5] P. Wang, J. Fang, X. Yuan, Z. Chen, H. Duan, and H. Li, “Intelligent reflecting surface-assisted millimeter wave communications: Joint active and passive precoding design,” submitted to IEEE Journal on Selected Areas in Communication, 2019, [Online]. Available: https://arxiv.org/abs/1908.10734.

[6] T. J. Cui, M. Q. Qi, X. Wan, J. Zhao, and Q. Cheng, “Coding metamaterials, digital metamaterials and programmable metamaterials,” Light: Science & Applications, vol. 3, no. 10, p. e218, 2014.

[7] C. Liaskos, S. Nie, A. Tsiofilarioud, A. Pitsillides, S. Ioannidis, and I. Akylidiz, “A new wireless communication paradigm through software-controlled metasurfaces,” IEEE Communications Magazine, vol. 56, no. 9, pp. 162–169, September 2018.

[8] E. Basar, M. Di Renzo, J. de Rosny, M. Debbah, M.-S. Alouini, and R. Zhang, “Wireless communications through reconfigurable intelligent surfaces,” 2019, [Online]. Available: https://arxiv.org/abs/1906.09490.

[9] Q. Wu and R. Zhang, “Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming,” IEEE Transactions on Wireless Communications, pp. 1–1, 2019.

[10] ——, “Intelligent reflecting surface enhanced wireless network: Joint active and passive beamforming design,” in 2018 IEEE Global Communications Conference (GLOBECOM), Abu Dhabi, UAE, December 10-12, 2018, pp. 1–6.

[11] S. Li, D. Bin, X. Yuan, Y.-C. Liang, and M. D. Renzo, “Reconfigurable intelligent surface assisted UAV communication: Joint trajectory design and passive beamforming,” 2019, [Online]. Available: https://arxiv.org/abs/1908.04082.