Modification of triangular member’s function (TMF) based on firefly-chen fuzzy time series (FTS) method

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Abstract. Usually the applications of fuzzy forecasting methods use triangular fuzzy number for calculating the value of member’s function. In many previous researches on forecasting, FTS (FTS) methods were implemented using symmetric triangles in the fuzzification step. In the present paper, we study on how hypotenuse of the triangle can affect forecasting accuracy rates. We modify the TMF to be non-simetric triangle with different line of orientation (left and right leaning triangles) to find out the effect of FTS forecasting results. This modification is applying on Firefly-Chen FTS method to forecast the Indonesian Composite Stock Price Index (IHSG). Its performance verified through simulation by using Matlab.

1. Introduction
Fuzzy Time Series (FTS) is a method based on Fuzzy Linguistic (FL) that considered as statistical time series analysis applying on fuzzy sets [1]. In general, FTS methods are composed by three steps, i.e., divide the universe of discourse into several clusters, fuzzification, and defuzzification or forecasting. At the last research, we modified the universe of discourse into dynamic length of intervals by using Firefly Algorithm (FA) [2]. The error value shows that the modification outperforms the original Chen method in the case forecasting IHSG. The combination of Chen method and FA named Firefly-Chen method. Further, in this study we modify the fuzzification process to be different shape of member’s functions.

Fuzzification is a process for converting an input from a crisp value to be FL that usually presented in the form of fuzzy sets with its respective member’s functions. In principle, member’s function can be of different shape [3], but in practice, Triangular Member’s Function (TMF) are most frequently used [2-11]. TMF is one of the major components and it is more natural than a precise value in simulating uncertainty [4]. To quantify the vagueness, the expert faithfully expresses the lowerpoint of the preference intensity, the upper point, and the most probable point respectively [5].

In this paper, we modify the symmetric TMF to be different shape of triangle, i.e., left and right leaning triangle. We use IHSG data for demonstration. The structure of the article is arranged as follow, Section 1 Introduction, Section 2 discussed basic concept of FTS, Section 3 Modification of TMF, Section 4 Triangular Member’s Function (TMF) modifications, and Section 5 Conclusion.
2. FTS
The definition of fuzzy sets is given at Definition 1.

**Definition 1** [2]
Let \( W \) be the universe of discourse, a fuzzy sets \( \tilde{A} \) of \( W \) with member’s function \( \mu_{\tilde{A}} : W \rightarrow [0,1] \) is a fuzzy set \( \tilde{A} \) which expressed by this following form:

\[
\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) \mid x \in W \}
\]  

where the member’s function attribute every \( x \in W \) to every real number \( \mu_{\tilde{A}}(x) \) in the interval \([0,1]\). The value of \( \mu_{\tilde{A}}(x) \) shows the degree of membership of \( x \) at fuzzy set \( \tilde{A} \). The notation \((x, \mu_{\tilde{A}}(x))\) represent element \( x \) which has a membership degree \( \mu_{\tilde{A}}(x) \).

**Definition 2** [8]
Let \( A_1, A_2, A_3, ..., A_k \) are the fuzzy sets of linguistic variable \( W \) where \( s_{ij} \in [0,1] \) is member’s function of cluster \( r_i \), \( 1 \leq i \leq a \) and \( 1 \leq j \leq b \).
Fuzzy set \( \tilde{A} \) is also can be expressed by this following equation:

\[
\tilde{A} = \{ (r_i, \mu_{\tilde{A}}(r_i)) \mid r_i \in W \}
\]

(2) with \((r_i, \mu_{\tilde{A}}(r_i))\) indicate interval \( r_i \) which has member’s function degree \( \mu_{\tilde{A}}(r_i) \) \([0,1] \) and \( r_i \) be subset of universe of discourse \( W \).
For example:
\[
A_1 = \{ (r_1, s_{11}), (r_2, s_{12}), ..., (r_b, s_{1n}) \}
\]
\[
A_2 = \{ (r_1, s_{21}), (r_2, s_{22}), ..., (r_b, s_{2n}) \}
\]
\[
A_3 = \{ (r_1, s_{31}), (r_2, s_{32}), ..., (r_b, s_{3n}) \}
\]
\[
\vdots
\]
\[
A_n = \{ (r_1, s_{n1}), (r_2, s_{n2}), ..., (r_b, s_{nn}) \}
\]

**Definition 3** [8]
A fuzzy member’s function is called a TMF if it has 3 parameters, i.e., \( a, b, c \in R \) where \( a < b < c \), denoted by Triangular \( (x; a, b, c) \) with the following rules:

\[
\text{Triangular}(x; a, b, c) = \begin{cases} 
\frac{x-a}{b-a}, & a \leq x < b \\
\frac{c-x}{c-b}, & b \leq x \leq c \\
0, & \text{else}
\end{cases}
\]

(4)
The following Figure 1 is an example of \( \text{Triangular}(x; a, b, c) \).

![Figure 1](image)

Figure 1. Representation of symmetric TMF

The TMF can also be expressed by the following formula :

\[
\text{Triangular}(x; a, b, c) = \max \left( \min \left( \frac{x-a}{b-a}, \frac{c-x}{c-b} \right), 0 \right)
\]

(5)

If the symmetric TMF is used, parameter \( b \) is the midpoint of the interval \([a, b]\), so that the definition of symmetric TMF can use 2 parameters i.e., parameters \( a \) and \( c \) or can be expressed by the triangular formula \( (x; a, c) \). It can certainly be profitable in computing process.
### 3. Modification of Triangular Fuzzy Member’s Function (TMF)

In this paper, we modify the value of linguistic variable at fuzzification step. Example of a fuzzy set is given as follows.

\[ A_1 = \{ (r_1, 1), (r_2, 0.5), (r_3, 0), (r_4, 0), (r_5, 0), (r_6, 0), (r_7, 0), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

\[ A_2 = \{ (r_1, 0.5), (r_2, 1), (r_3, 0.5), (r_4, 0), (r_5, 0), (r_6, 0), (r_7, 0), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

\[ A_3 = \{ (r_1, 0), (r_2, 0.5), (r_3, 1), (r_4, 0.5), (r_5, 0), (r_6, 0), (r_7, 0), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

\[ A_4 = \{ (r_1, 0), (r_2, 0), (r_3, 0.5), (r_4, 1), (r_5, 0.5), (r_6, 0), (r_7, 0), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

\[ A_5 = \{ (r_1, 0), (r_2, 0), (r_3, 0), (r_4, 0.5), (r_5, 0.5), (r_6, 1), (r_7, 0.5), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

\[ A_6 = \{ (r_1, 0), (r_2, 0), (r_3, 0), (r_4, 0), (r_5, 0), (r_6, 0), (r_7, 0), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

\[ A_7 = \{ (r_1, 0), (r_2, 0), (r_3, 0), (r_4, 0), (r_5, 0), (r_6, 0), (r_7, 0), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

\[ A_8 = \{ (r_1, 0), (r_2, 0), (r_3, 0), (r_4, 0), (r_5, 0), (r_6, 0), (r_7, 0), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

\[ A_9 = \{ (r_1, 0), (r_2, 0), (r_3, 0), (r_4, 0), (r_5, 0), (r_6, 0), (r_7, 0), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

\[ A_{10} = \{ (r_1, 0), (r_2, 0), (r_3, 0), (r_4, 0), (r_5, 0), (r_6, 0), (r_7, 0), (r_8, 0), (r_9, 0), (r_{10}, 0) \} \]

The fuzzy set can be written in the fuzzification matrix \( [s_{ij}] \), with \( s_{ij} \) is membership degree values denoted as follows:

\[
\begin{bmatrix}
1 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

From matrix (6), we conclude that the fuzzification used symmetrical TMF. Then, we define the maximum value of member’s function degree for every \( A_i \) with \( i = 1, 2, ..., m \). If the maximum membership degree of \( A_i \) is located at \( U_i \), then the fuzzification is \( A_i \).

Figure 1 shows that the maximum degree of membership occurs at the midpoint of an interval. It will affect the value of forecasting calculations or the forecasting value is the middle value of the interval \([a, b]\).

In this section we will discuss the comparison of member’s function simulations, i.e., the member’s function of symmetrical triangles and non-symmetrical triangles. The non-symmetrical triangle is a triangle which height line is not an axis of symmetry, as represented in the following Figure 2 and Figure 3.

![Figure 2. Representation of left leaning TMF](image)

![Figure 3. Representation of right leaning TMF](image)
4. Triangular Member’s Function (TMF) Modifications

The aim of TMF modification is to find out the difference between the use of the member’s function of symmetrical triangles and non-symmetrical triangles. Some forecasting algorithms with TMFs usually use symmetrical triangular shapes, because it is easier in calculating forecasting. Forecasting based on FTS with TMF predicts a data based on the maximum value of membership degree, or in other words the forecasting value is the value with the degree of membership equal to one or $\mu(x) = 1$. If the triangle member’s function used is a symmetrical triangle, the forecasting value is the middle or midpoint value of the cluster. However, this is not entirely appropriate depending on the characteristics of the data. The use of TMFs is generally only to simplify the forecasting calculation process, because it only uses the midpoint value in the cluster specified in the fuzzification process. Therefore, this paper modifies the fuzzification process where the TMF used is a non-symmetrical triangle for the composite stock price index (IHSG) forecasting case.

Modification of member’s functions is done in step 3, namely step fuzzification. The value of linguistic variables with the left-leaning non-symmetrical triangle member’s function can be written in the fuzzification matrix $[s_{ij}]$, with $s_{ij}$ being the value of the membership degree as follows.

\[
\begin{bmatrix}
0.67 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0.67 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0.67 & 0.3 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0.67 & 0.3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0.67 & 0.3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0.67 & 0.3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0.67 & 0.3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.67 & 0.3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.67 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  
(7)

Second, the value of linguistic variables with right-leaning non-symmetrical triangle member’s functions is also can be written in the fuzzification matrix $[s_{ij}]$, with $s_{ij}$ being the value of the membership degree as follows.

\[
\begin{bmatrix}
0.67 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.3 & 0.67 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.3 & 0.67 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.3 & 0.67 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.3 & 0.67 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.3 & 0.67 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.3 & 0.67 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0.67 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0.67 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.3 \\
\end{bmatrix}
\]  
(8)

The simulation use Chen method and Firefly-Chen method [2,5]. The following graphic shows the comparison of RMSE value on member’s function modification.
Based on Figure 4 and Figure 5, we compare the RMSE value of the non-symmetrical triangle member’s function. The results show that the modification is not too influential to reduce the error value. It can be seen that the error value for the symmetrical triangle member’s function is still better if compared with the error value for the non-symmetrical triangle member’s function, both left and right leaning triangles. This is due to the lack of criteria to determine how large the non-symmetrical triangle is. Modifying the member’s function in this paper is only limited to seeing the effect on numerical forecasting results. The use of TMFs in most forecasting method based on FTS is reasonable because certainly be profitable in computing process, it turns out that the results of forecasting with non-symmetrical TMFs are not better compared to forecasting using symmetrical TMFs.

5. Conclusion
Based on Figure 4 and Figure 5, when compared to the RMSE value for modification of the non-symmetrical TMF, it is not too influential to reduce the error value. It can be seen that the error value for the symmetrical TMF is still better compared to the error value for the non-symmetrical triangle
member’s function, both left and right leaning triangles. This is due to the lack of criteria to determine how large the non-symmetrical triangle is. To modify the member’s function in this paper is only limited to seeing the effect on numerical forecasting results. The use of TMFs in most forecasting algorithms based on FTS is reasonable because it certainly be profitable in computing process. In the case of forecasting IHSG, symmetrical triangle member’s functions better than forecasting using non-symmetrical triangle member’s functions. Because of the maximum degree of membership occurs at the midpoint of an interval, we can reduce the parameter to be just 2 parameters. It is very profitable in computing process, so the time needed in the calculation process is not too long.

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