Neural adaptive compensation control for a class of MIMO affine uncertain nonlinear systems with actuator failures

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ABSTRACT
A new neural adaptive compensation control approach for a class of multi-input multi-output (MIMO) uncertain nonlinear systems with actuator failures is proposed in this paper. In order to enlarge the set of compensable actuator failures, an actuator grouping scheme based on multiple model switching and tuning (MMST) is proposed for the nonlinear MIMO minimum phase systems with multiple actuator failures, and RBF neural networks are used to approximate the error of plant model. Then an adaptive compensation scheme based on prescribed performance bound (PPB) which characterizes the convergence rate and maximum overshoot of the tracking error is designed for the system to ensure closed-loop signal boundedness and asymptotic output tracking despite unknown actuator failures. Simulation results are given to show the effectiveness of the control design.

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1. Introduction

Complex technological systems are vulnerable to unpredictable events that can cause undesired reactions and as a consequence damage to technical parts of the plant, to personnel, or to the environment. Reliability and stability are of paramount importance for practical processes. Fault tolerant control (FTC) (Zhang & Jiang, 2008) systems are designed to be able to handle emergency situations arising from actuator and/or sensor faults, and improve the reliability of the overall system. FTC theory has attracted considerable academic interest, and a variety of techniques for FTC have been developed over the past two decades.

The FTC problem can be tackled using either a passive approach or an active one. The passive approach deals with the problem of finding a general controller able to satisfy control specifications both in nominal operation and after the occurrence of a fault. Passive FTC uses robust control techniques to ensure that the closed loop system remains insensitive to certain failures so that the impaired system continues to operate with the same controller and system structure. Hence, a unique controller, designed offline, can be used and online fault information is not required. In contrast, active FTC aims at achieving the control objectives by adapting the control law to the faulty system behaviour. Most existing active fault tolerance control designs for system with actuator failures either are based on some knowledge of actuator failures or depend on some extra detection techniques. There are many fault diagnosis technology achievements (G & Davide, 2017; Gomathi, Srinivasan, Ramkumar, & Muralidharan, 2017; Khaoula, Nizar, Sylvain, & Teodor, 2016; Yu & Jiang, 2015). An effective compensation controller is expected to guarantee satisfactory performance even in the presence of unknown actuator failures, as well as handle a larger class of systems with a simpler structure and lower cost. Recently, adaptive control (Tao, 2014) has been widely used to deal with actuator failures in various systems. Adaptive control, which can accommodate uncertainties in system parameters, system structure, and environment, is a promising approach to the problem of actuator failure compensation.

Focus on the adaptive compensation control for systems with actuator failures, the systems under consideration are from linear systems (Tao, 2008; Yang & Ye, 2010), multiple-input single-output (MISO) nonlinear systems to MIMO nonlinear systems with further research. In Tao, Joshi, and Ma (2001), an adaptive design schemes, known as direct adaptive control, is proposed to solve tracking problems for linear systems with unknown system parameters in the presence of total loss of effectiveness of actuators. In Li and Yang (2005), a fuzzy adaptive compensation scheme is proposed for a class of nonlinear systems with full state linearizable, but the
scheme can only solve limited tracking control problems of systems with actuator failures for the description of the plant. Tang, Tao, and Joshi (2003) propose a backstepping adaptive compensation tracking law for MISO nonlinear minimum phase systems with actuator failures. By using PPB originally presented in Bechlioulis and Rovithakis (2009), Wang and Wen (2010) design a backstepping adaptive compensation law based on PPB for a class of uncertain MISO nonlinear systems with actuator failures to improve the transient performance of adaptive systems, the method can characterize the convergence rate and the maximum overshoot of the tracking error. In Tang, Tao, and Joshi (2007), an adaptive compensation scheme is developed for MIMO systems based on a fixed grouping approach that actuators are regrouped into several groups and a parallel-control design is applied for each group. With such a fixed grouping condition, once all the actuators in one group are failure, the control design will fail. However, there may still be enough active actuators in the system to compensate for failures and achieve a required control objective. In Tang, Tao, and Joshi (2005), an adaptive design based on virtual grouping as a further study in actuator failure compensation for MIMO systems. Instead of grouping actuators ahead of a control design, an adaptive controller will virtually group actuators whenever actuator failures happen. This technique eliminates a critical design condition needed in the fixed grouping design, but only lock in space failure of the plant is considered in the scheme, so the virtual grouping is not applicable in other actuator failure cases. And Tang et al. (2005, 2007) do not consider the transient performance of the adaptive scheme.

Designing a controller with high performance for systems is not easy, because designers are likely to encounter lots of modelling uncertainties (nonlinear friction, external disturbances, and/or unmodelled dynamics), these uncertainties could severely deteriorate the achievable control performance, leading to undesirable control accuracy, limit cycles, and even instability. There are many achievements of technology to deal with the uncertainties exist in system. In Tang et al. (2003), Tang et al. (2007) and Tang et al. (2005), adaptive way is used to solve the uncertain of system, but the uncertain is requested to be linear. Using the RBF neural network to approximate the external interference of system during the process of controller design, an adaptive robust control scheme is presented for a class of uncertain MIMO nonlinear systems in Zhang, Xia, and Yi (2017); Yuan, Tan, Fan, and Yi (2014), the weights of neural networks are updated online with a new adaptive algorithm.

This paper is written with the following objectives. The first is to deal with the problem of the guaranteed transient performance for a class of MIMO nonlinear minimum phase systems in presence of actuator failures combination of lock in space, total and partial lose effectiveness. By employing PPB based controller design scheme (Zhang et al., 2014), an adaptive controller is designed for the transformed system. It is established that the tracking error can be guaranteed within the prescribed error bound all the time as long as the stability of the error system is ensured. The second is in order to reduce the effect of system uncertainties of plant model, a robust adaptive control scheme based on RBF neural network is proposed. The third is to propose a new actuator grouping technology based on MMST referencing the method of multiple model design for linear and nonlinear systems proposed in Narendra, Balakrishnan, and Ciliz (1995) and Boskovic and Mehra (1999). The MMST based grouping can enlarge the set of compensable actuator failures, by switching to the model close to the dynamics of the failed plant. The designed FTC law can ensure that the closed-loop system is stable when system has actuator failures, and track the given reference signal with prescribed performance bound. Simulation results verify the effectiveness of the adaptive compensation scheme.

2. Problems statement

Consider the actuator failure compensation problem for

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i + \Delta f(x)$$

$$y = h(x)$$

where $x \in \mathbb{R}^n$ is the plant state; $y \in \mathbb{R}^q$, $q \geq 2$ is the output; $u_i$, $i = 1, \ldots, m > q$ is the $i$th input of the system; $\Delta f(x) \in \mathbb{R}^n$ is the error of plant model; $f(x) \in \mathbb{R}^n$, $g_i(x) \in \mathbb{R}^n$ and $h(x) \in \mathbb{R}^q$ are known smooth nonlinear functions, and we note $g = [g_1, \ldots, g_m] \in \mathbb{R}^{n \times m}$.

We aim at designing an adaptive compensation control scheme for plant (1) with actuator failures. The $i$th, $i = 1, \ldots, m$ actuator failure to be considered is modelled as

$$u_i = \lambda_i \bar{u}_i(x, t) + \bar{u}_i$$

$$\text{rank}(\text{diag}(\lambda_i, \bar{u}_i)) \leq 1$$

$$t \geq t_i, i = 1, \ldots, m$$

where $u_i$ and $\bar{u}_i$ are the input and output of the $i$th actuator; $0 \leq \lambda_i \leq 1$, $\bar{u}_i$, $t_i$ are unknown constant. (2) shows that the $i$th actuator fails suddenly at time $t_i$ and can implies the following four cases, in which actuator no-failure case, lock-in-space failure, loss of effectiveness and float type of failure are included.

(1) $\lambda_i = 1$ and $\bar{u}_i = 0$

In this case, the $i$th actuator with its input equal to its output, is regarded as a failure-free actuator.
(2) \( 0 < \lambda_i < 1 \) and \( \bar{u}_i = 0 \)

This case indicates partial loss of effectiveness of the \( i \)th actuator. For example, \( \lambda_i = 0.7 \) means that the \( i \)th actuator loss 30\% of its effectiveness.

(3) \( \bar{u}_i \neq 0 \) and \( \lambda_i = 0 \)

This case indicates that the \( i \)th actuator lock in space \( \bar{u}_i \).

(4) \( \lambda_i = 0 \) and \( \bar{u}_i = 0 \)

This case indicates that the \( i \)th actuator loss its effectiveness totally.

For plant (1) with actuator failures (2), the input \( u(t) = [u_1(t), \ldots, u_m(t)]^T \) can be expressed as

\[
\begin{align*}
u(t) &= \lambda u_c(t) + \bar{u} \\
\end{align*}
\]

where \( \lambda = \text{diag} \{ \lambda_1, \ldots, \lambda_m \} \), \( u(t) = [u_1(t), \ldots, u_m(t)]^T \), \( u_c(t) = [u_{c1}(t), \ldots, u_{cm}(t)]^T \) is the applied control to be designed.

With expression (3), we rewrite plant (1) as

\[
\begin{align*}
\dot{x} &= f(x) + g(x)(\lambda u_c(t) + \bar{u}) + \Delta f(x) \\
y &= h(x)
\end{align*}
\]

The control objective for actuator failures compensation is to design backstepping controller for plant (4) with up to \( m-q \) failures to ensure that the closed-loop signals are bounded and the plant output \( y = [y_1, \ldots, y_q]^T \) tracks the reference signal \( y_m = [y_{1m}, \ldots, y_{qm}]^T \) asymptotically. To accomplish this task, the following basic assumption for the actuator failure compensation problem is needed.

**Assumption 2.1:** With any types of actuator failures up to \( m-q \), the plant (4) can still achieves a desired control objective using the available actuators.

With the smooth assumption of \( f(\cdot), g(\cdot) \) and \( h(\cdot) \), there exist a diffeomorphism \( T(x) = [\xi, \eta]^T \in \mathbb{R}^n \), where \( \xi = [\xi_{11}, \xi_{12}, \ldots, \xi_{1p_1}, \xi_{21}, \ldots, \xi_{q_{pq}}]^T \in \mathbb{R}^{p_1+\cdots+p_q} \), \( \xi_j = L_{h_j}^{-1}h_i(x), i = 1, \ldots, q, j = 1, \ldots, p_j, \eta \in \mathbb{R}^{n-(p_1+\cdots+p_q)}, \rho = [\rho_1, \ldots, \rho_q]^T \in \mathbb{R}^q \) is relative degree, to transform the nominal system (4) into the canonical parametric-strict-feedback form as follow

\[
\begin{align*}
\dot{\xi}_{i1} &= \xi_{i2} + F_{i1}(x) \\
\dot{\xi}_{ij} &= \xi_{i(j+1)} + F_{ij}(x) \\
\dot{\xi}_{jm} &= \psi_i(\xi, \eta) + \beta_j u_c(t) + \bar{u} + F_{jm}(x) \\
\eta &= \psi(\xi, \eta) + \Phi_\sigma(\xi, \eta) u_c(t) + \Phi_\sigma(\xi, \eta) u_c(t) \\
y_i &= \xi_{i1}, i = 1, \ldots, q, j = 1, \ldots, \rho_i - 1
\end{align*}
\]

where \( \psi_i = L_{h_i}^{-1}h_i(x), \beta_j = \{L_{g_1}L_{f_i(x)}^{-1}h_i(x), \ldots, L_{g_q}L_{f_i(x)}^{-1}h_i(x) \}, F_{ij} = L_{\Delta f(x)}L_{f_i(x)}^{-1}h_i(x), i = 1, \ldots, q, j = 1, \ldots, \rho_i \) is unknown.

**Assumption 2.2:** The nominal system (5) is minimum-phase, that is, the zero dynamics are input-to-state stable (ISS).

3. MMST-based actuators grouping

To deal with the actuator redundancy, based on the control characteristic indices \( \rho \), the actuators are grouped into \( q \) groups corresponding to \( q \) outputs by a grouping scheme denoted as

\[
\sigma = \{\{j_{11}, \ldots, j_{1d_1}\}, \ldots, \{j_{1k}, \ldots, j_{kd_k}\}, \ldots, \{j_{q1}, \ldots, j_{qd_q}\}\}
\]

such that for the \( d_k \), \( k = 1, \ldots, q \) inputs \( u_{jh_1}, \ldots, u_{jh_{d_k}} \) in the same group \( k \), and \( d_1 + \cdots + d_q = m \), one actuator can and only can be grouped into one group. A proportional actuation scheme is employed to deal with the actuation redundancy, that is

\[
u_c(t) = B \cdot w(t)
\]

where

\[
w(t) = [w_1(t), \ldots, w_q(t)]^T, B = \begin{bmatrix} b_{11} & \cdots & b_{1q} \\ \vdots & \ddots & \vdots \\ b_{m1} & \cdots & b_{mq} \end{bmatrix},
\]

\[
b_{ij} = \begin{cases} b_j, & \text{if } u(t) \text{ is assigned to group } k \\ 0, & \text{otherwise} \end{cases}
\]

With expression (7), the tracking dynamic subsystem of plant (5) can be rewritten as

\[
\begin{align*}
\dot{\xi}_{i1} &= \xi_{i2} + F_{i1}(x) \\
\dot{\xi}_{ij} &= \xi_{i(j+1)} + F_{ij}(x) \\
\dot{\xi}_{jm} &= \psi_i(\xi, \eta) + \beta_j \lambda L_b w(t) + \beta_i \bar{u} + F_{jm}(x) \\
\eta &= \psi(\xi, \eta) + \Phi_\sigma(\xi, \eta) u_c(t) + \Phi_\sigma(\xi, \eta) u_c(t) \\
y_i &= \xi_{i1}, i = 1, \ldots, q, j = 2, \ldots, \rho_i - 1
\end{align*}
\]

The actuator failures under consideration include lock-in-place and loss of effectiveness. \( \sigma = \{\sigma_1, \ldots, \sigma_N\}, N = \sum_{i=1}^{\rho} C_{\rho_i} \), is the set of all failure patterns, where \( N \) is the number of possible failure patterns of the system. We note \( \sigma_0 \) as the failure-free case. Let \( \Pi_\mu = \{\mu_1, \ldots, \mu_{\rho_p}\}\mu = 0, \ldots, N \) be the set of \( p \) faulty actuators of the failure pattern \( \sigma_\mu \); Let \( E = \{e_1, \ldots, e_\rho\} \) be the set of compensable failure patterns of the controller design, where \( L \) is the number of compensable failure patterns of the system. We will propose an actuators grouping scheme for each failure group \( \sigma_\mu \) based on the failure pattern \( \sigma_\mu \).

The grouping objective is to design a suitable algorithm to assure that the scheme switches to the actuators
grouping scheme \(\sigma_\mu\) corresponding to the failure pattern. When the system is no-failure, the actuators grouping is noted as \(\sigma_0\). In the case of failure pattern \(\sigma_\mu, \mu = 1, \ldots, L\), using \(\gamma_\mu \in \mathbb{R}, \mu = 0, \ldots, L\) to denote the result of switching, if the grouping scheme \(\sigma_\mu\) is chosen, then \(\gamma_\mu = 1\), otherwise \(\gamma_\mu = 0\). The MMST actuator grouping scheme can be described by Figure 1.

3.1. Observers design

Assumption 3.1: For any smooth nonlinear function \(\Delta f(x)\), there exist RBF neural network with \(h\) hidden layer nodes, the optimal weight matrix \(W^* \in \mathbb{R}^{h \times 1}\) and Gaussian basis functions \(\theta(x) \in \mathbb{R}^{h \times 1}\) to satisfy

\[
L(x) = W^* \theta(x) + \varepsilon
\]

where \(x\) are neural network inputs, \(\varepsilon\) is the approximation error of neural network. During the learning process, we note

\[
\hat{L}(x) = \hat{W}^T \theta(x) + E
\]

where \(\hat{W}\) is estimate of \(W^*\), \(\hat{L}(x)\) is estimate of \(L(x)\).

With the Assumption 3.1, the estimate of \(\Delta \hat{f}_\mu(x) = [\Delta \hat{f}_{\mu,1}(x), \ldots, \Delta \hat{f}_{\mu,n}(x)]^T\) of the \(\mu\)th, \(\mu = 0, \ldots, L\) model can be rewritten as

\[
\Delta \hat{f}_\mu(x) = [\hat{W}^T_{\mu,1} \theta_{\mu,1} + E_1, \ldots, \hat{W}^T_{\mu,n} \theta_{\mu,n} + E_n]^T
\]

where \(\Gamma_{\mu}\) is a positive definite design matrix, \(H_i \in \mathbb{R}^n, i = 1, \ldots, n\) is a matrix which the \(i\)th element is 1 and other elements are zero.

We next focus on the case of failures pattern of the actuators. It can be seen that each case of failure can be modelled by a different model resulting in the total of equations of the form

\[
\begin{align*}
\dot{x}_0^* &= f(x) + g(x)\lambda_0^0u + \Delta f(x) \\
\dot{x}_\mu^* &= f(x) + g(x)(\lambda_\mu^\ast u + \tilde{u}_\mu^\ast) + \Delta f(x)
\end{align*}
\]

\(\mu = 1, \ldots, L\)

In the above equations subscript ‘0’ corresponds to the no-failure case; \(x_\mu^* \in \mathbb{R}^n, \mu = 0, 1, \ldots, L\) is the states of system with actuator failures; \(\lambda_\mu^\ast \in \mathbb{R}^{m \times m}\) is the failure rate matrix, \(\tilde{u}_\mu^\ast \in \mathbb{R}^m\) is the stuck position matrix of failure pattern \(\sigma_\mu\).

The observers can be noted as

\[
\begin{align*}
\hat{x}_0 &= \Lambda_0 \hat{e}_0 + f(x)\hat{x}_0^* + \Delta \hat{f}(x) \\
\hat{x}_\mu &= \Lambda_\mu \hat{e}_\mu + f(x)(\hat{\lambda}_\mu^* u + \hat{\tilde{u}}_\mu^*) + \Delta \hat{f}(x)
\end{align*}
\]

\(\mu = 1, \ldots, L\)

where \(\hat{e}_\mu = \hat{x}_\mu - x, \mu = 0, 1, \ldots, L; \hat{\lambda}_\mu^*\) and \(\hat{\tilde{u}}_\mu^*\) denote respectively the estimates of \(\lambda_\mu^\ast\) and \(\tilde{u}_\mu^\ast; \Lambda_\mu \in \mathbb{R}^{n \times n}\) is a asymptotically stable matrix. The error model is obtained by subtracting (12) from (13)

\[
\begin{align*}
\hat{e}_0 &= \Lambda_0 \hat{e}_0 + g(x)\hat{x}_0^* - \Delta \hat{f}(x) \\
\hat{e}_\mu &= \Lambda_\mu \hat{e}_\mu + g(x)(\lambda_\mu^* u + \tilde{u}_\mu^*) - \Delta \hat{f}(x)
\end{align*}
\]

3.2. Adaptive law and switching scheme of MMST actuators grouping

The adaptive laws of the observers are chosen in the form

\[
\begin{align*}
\hat{\lambda}_\mu^* &= \text{Proj} [\lambda_{\mu, \min}, \lambda_{\mu, \max}][\hat{\lambda}_\mu^* (0)] \\
\hat{\tilde{u}}_\mu^* &= \text{Proj} [\tilde{u}_{\mu, \min}, \tilde{u}_{\mu, \max}][\hat{\tilde{u}}_\mu^* (0)]
\end{align*}
\]

where \(\lambda_{\mu, \min} < 1\) and \(\tilde{u}_{\mu, \max} > 0\). The RBF neural networks weight matrixes update laws are

\[
\begin{align*}
\hat{\lambda}_\mu &= \hat{\lambda}_\mu - \mu [\hat{\lambda}_\mu (0)] \\
\hat{\tilde{u}}_\mu &= \hat{\tilde{u}}_\mu - \mu [\hat{\tilde{u}}_\mu (0)]
\end{align*}
\]

where \(\mu = 0, 1, \ldots, L; i = 1, \ldots, m\) and \(u_i \in \Pi_\mu\);

\[
\begin{align*}
\hat{\lambda}_\mu &= \lambda_{\mu, \min} \lambda_{\mu, \max} \\
\hat{\tilde{u}}_\mu &= \tilde{u}_{\mu, \min} \tilde{u}_{\mu, \max}
\end{align*}
\]

The RBF neural networks weight matrixes update laws and robust terms are determined at the 1, 2th step as

\[
\hat{u}_i = \hat{\tilde{u}}_i \Gamma^{-1}_i \theta_{i, \mu} g(x)H_i
\]

\[
E_{i, \mu} = -\text{sign} (\hat{\tilde{u}}_i g(x)\epsilon_{i, \mu})
\]

where \(\epsilon_{i, \mu}\) is the max of \(\epsilon_i\).
Switching among the actuators grouping schemes is based on the following performance indices

\[
I_\mu = c_1 ||\hat{e}_\mu(t)||^2 + c_2 \int_{t_0}^{t} \exp(-l(t - \tau)) ||\hat{e}_\mu(t)||^2 d\tau,
\]

\[
\mu = 0, \ldots, L
\]

where \(c_1 > 0; c_2 > 0; l > 0; t_0\) is the time of last switching. The scheme is started with actuators grouping scheme \(\sigma_0\), and is implemented by calculating and comparing the above indices and finding their minimum. Once the minimum is found, the scheme switches to (or stays at) the corresponding actuator grouping scheme. Using the result of switching, we can rewrite (8) in the form

\[
\begin{align*}
\dot{\xi}_{i1} &= \xi_{i2} + F_{i1}(x) \\
\dot{\xi}_{ij} &= \xi_{i(j+1)} + F_{ij}(y) \\
\dot{\xi}_{ij0} &= \psi_i(\xi, \eta) + \beta_i \lambda \sum_{\mu=0}^{L} \gamma_{ij} B_i w(t) + \beta_i \bar{u} + F_{ij0}(x) \\
y_i &= \xi_{i1}, i = 1, \ldots, q, j = 2, \ldots, \rho_i - 1
\end{align*}
\]

**Theorem 3.1:** The above switching scheme with the adaptive law of observers (15–16), RBF neural networks weight matrixes update laws (7), robust terms (18) and performance indices (19) can assure the stability of the error model (14), and guarantee \(\lim_{t \to \infty} \hat{e}_1 = 0\) for the system with actuator failure pattern \(\sigma_{\mu, \mu} = 0, \ldots, L\).

**Proof:** When \(t = 0\), the scheme is started with and \(\hat{\chi}_{\mu}(0) = x(0), \mu = 0, \ldots, L\) with actuator grouping scheme \(\sigma_0\). We first focus on the nominal model and the case there is only loss of effectiveness failure. In this case, \(\bar{u}_0 = 0\), we choose \(V_0 = \frac{1}{2}(\bar{\varepsilon}_0^T \bar{\varepsilon}_0 + \bar{\mu}_0^T \bar{\mu}_0 + \sum_{i=1}^{n} \bar{W}_{i}^T \bar{W}_{i})\). In the case of simultaneous loss of effectiveness and float, lock-in-place of the failures pattern \(\sigma_{i, i} = 1, \ldots, L\), we have

\[
V_\mu = \frac{1}{2}(\bar{\varepsilon}_\mu^T \bar{\varepsilon}_\mu + \bar{\mu}_\mu^T \bar{\mu}_\mu + \sum_{i=1}^{n} \bar{W}_{i}^T \bar{W}_{i})
\]

where \(H = \text{diag}(H_1, \ldots, H_m) > 0\) is positive definite. Its first derivative

\[
\begin{align*}
\dot{V}_\mu &= \bar{\varepsilon}_\mu^T \bar{\varepsilon}_\mu + \bar{\mu}_\mu^T \bar{\mu}_\mu + \sum_{i=1}^{n} \hat{W}_{i}^T \hat{W}_{i} \\
&= \bar{\varepsilon}_\mu^T \bar{\varepsilon}_\mu + \bar{\mu}_\mu^T \bar{\mu}_\mu + \sum_{i=1}^{n} \hat{W}_{i}^T \hat{W}_{i}
\end{align*}
\]

Substituting parameter update laws (12–13) into (17), we get

\[
\dot{V}_i = \bar{\varepsilon}_i^T \bar{\varepsilon}_i \leq 0
\]

It can be shown that the observers can asymptotic tracking the system states, \(\lim_{t \to \infty} \hat{e}_1 = 0\). Since all indices have the exponentially decaying term \(\exp(-l(t - \tau))\), it follows that \(\lim_{t \to \infty} \hat{e}_1 = 0\). Further, since all other models indices \(l_i\) have integral terms, the error will accumulate over a time interval and the indices \(l_i\) will be greater than zero. Using the above switching scheme we can switch to the actuators grouping scheme corresponding
to the current failure pattern, $\gamma_i = 1$, $\gamma_j = 0, j = 0, 1, \ldots, L, j \neq i$.

### 4. Prescribed performance bounds (PPB) based control design

In this section, based on PPB technology, we design a backstepping adaptive compensation control scheme for the system. The objective is to ensure the transient performance in the sense that the tracking error $e_i(t) = y_i(t) - y_{\tilde{y}_i}(t), i = 1, \ldots, q$ is preserved within a specified PPB all the time no matter when actuator failures occur.

#### 4.1. PPB based transformed system

Wish the tracking error satisfying the condition that

$$-\hat{\delta} < S(v) < \hat{\delta};$$

where $0 < \hat{\delta}, \delta \leq 1$, $\tau(t)$ is performance to be designed, $\tau(t)$ is a decreasing function with $\tau_0 > \lim_{t \to -\infty} \tau(t) = \tau_\infty > 0$, where $\tau_0$ is the initial of $\tau(t)$.

We design a smooth and strictly increasing function $S(v)$ with the following properties

$$-\hat{\delta} < S(v) < \hat{\delta};$$

$$\lim_{v \to -\infty} S(v) = \hat{\delta}, \lim_{v \to -\infty} S(v) = -\hat{\delta};$$

$$S(0) = 0$$

From properties (10), performance (9) can be expressed as

$$e_i(t) = \tau(t) S(v)$$

In this paper, we use the $S(v)$ design of Wang and Wen (2010).

$$S(v) = \frac{\hat{\delta} e^{(v+\tau)} - \hat{\delta} e^{-(v+\tau)}}{e^{(v+\tau)} + e^{-(v+\tau)}}$$

where $r = (\ln(\hat{\delta}/\delta))/2$. It can be easily shown that $S(v)$ has the properties (11). From (11–12), the transformed error $v_i(t)$ is solved as

$$v_i = \frac{1}{2} \ln(\hat{\delta} e_i(t) + \hat{\delta}) - \frac{1}{2} \ln(\hat{\delta} - \hat{\delta} e_i(t))$$

where $e_i(t) = e_i(t)/\tau(t)$. Compute the time derivative of $v_i$ as

$$\dot{v}_i = \hat{\delta} S^{-1} \dot{e}_i = \frac{1}{2} [\frac{1}{\hat{\delta} + \delta} - \frac{1}{\hat{\delta} - \delta}] \left( \frac{\dot{e}}{e} - \frac{e \dot{e}}{e^2} \right)$$

$$= \dot{e} \left( \frac{e - \tilde{e}}{\tau} \right) = \xi_i \left( \tilde{e} - \tilde{e}_i \right)$$

where $\xi_i = \frac{1}{2} [\frac{1}{\hat{\delta} + \delta} - \frac{1}{\hat{\delta} - \delta}]$. With expression (14), we rewrite plant (8) as

$$\dot{v}_i = \xi_i \left( \delta \tilde{e}_i + F_{\tilde{y}_i}(x) - \tilde{y}_i - \frac{e_i \tilde{e}_i}{\tau} \right)$$

$$\dot{\xi}_i = \dot{\xi}_i - \tilde{F}_i(x)$$

$$\dot{\xi}_i = \xi_i - \tilde{F}_i(x)$$

$$i = 1, \ldots, q, j = 2, \ldots, \rho_i$$

From properties (25) of $S(v)$, $\lim_{t \to -\infty} \xi_i(t) = 0$ can be achieved if controller designed for plant (30) can make $\lim_{t \to -\infty} v_i(t) = 0$.

#### 4.2. Backstepping adaptive compensation control law design

In this section, RBF neural networks are used to approximate the error of plant model. We will design an adaptive compensation controller for plant (30) with actuator failures. The details of Step 1 and Step 2 are elaborated as following. Define

$$Z_{i1} = v_i$$

$$Z_{i2} = \xi_i - \alpha_{i-1}, i = 1, \ldots, q, j = 1, \ldots, \rho_i$$

Step 1. Consider the first subsystem

$$\dot{Z}_{i1} = \xi_i \left( \delta \tilde{e}_i + F_{\tilde{y}_i}(x) - \tilde{y}_i - \frac{e_i \tilde{e}_i}{\tau} \right)$$

To stabilize (32), where $\alpha_{i1}$ is the virtual control determined at 1th step that

$$\alpha_{i1} = -c_{i1} Z_{i1} - \dot{\tilde{W}}_{i1} \tilde{\theta}_{i1} - E_{i1} + \tilde{y}_{i1} + \frac{e_i \tilde{e}_i}{\tau}$$

where $c_{i1} > 0$, as $F_{\tilde{y}_i}(x, t)$ is unknown, $\dot{\tilde{W}}_{i1}(x, t) = \tilde{W}_{i1} \tilde{\theta}_{i1}$ is estimate of $F_{\tilde{y}_i}(x, t)$ respectively, $E_{i1}$ is robust term to attenuate approximation error of the neural network. We define a positive definite Lyapunov function $V_{i1}$ as

$$V_{i1} = \frac{1}{2} \dot{Z}_{i1}^2 + \frac{1}{2} \dot{\tilde{W}}_{i1} \Gamma_{i1}^{-1} \tilde{W}_{i1}$$

where $\dot{\tilde{W}}_{i1} = \dot{\tilde{W}}_{i1} - \dot{\tilde{W}}_{i1} \Gamma_{i1} \tilde{W}_{i1}$, $\Gamma_{i1}$ is a positive definite design matrix. Then

$$\dot{V}_{i1} = -c_{i1} Z_{i1}^2 + \xi_i Z_{i2} - \xi_i Z_{i1} \tilde{\theta}_{i1} \tilde{W}_{i1} + \xi_i Z_{i1} (\tilde{y}_{i1} - E_{i1}) + \dot{\tilde{W}}_{i1} \Gamma_{i1}^{-1} \tilde{W}_{i1}$$

Step 2. With the help of (20) and the definition that $Z_{i2} = \xi_{i2} - \alpha_{i2}$, we obtain the subsystem of 2th step as

$$Z_{i1} = -c_{i1} Z_{i1} + \xi_i Z_{i2} + \xi_i (-\tilde{W}_{i1} \theta_{i1}(x) + \tilde{y}_{i1} - E_{i1})$$

$$Z_{i2} = \xi_{i2} + F_{\tilde{y}_i}(x) - \alpha_{i2}$$

(36)
To stabilize (36), where $a_{i2}$ is the virtual control determined at 2th step that
\begin{align*}
  a_{i2} = -c_{i2}Z_i - \zeta_i Z_i - \hat{W}_{i2}^T \theta_{i2} - E_{i2} + \hat{a}_{i1}
\end{align*}
where $c_{i2} > 0$.

Step 3. $j$ where $j = 3, \ldots, \rho_i$.
\begin{align*}
  a_{ik} = -c_{ik}Z_{ik} - Z_{i(k-1)} - \hat{W}_{ik}^T \theta_{ik} - E_{ik}
  + \hat{a}_{i(k-1)}, k = 3, \ldots, \rho_i - 1
\end{align*}
\begin{align*}
  a_{i\rho_i} = -c_{i\rho_i}Z_{i\rho_i} - Z_{i(\rho_i-1)} - \hat{W}_{i\rho_i}^T \theta_{i\rho_i} - E_{i\rho_i}
  + \hat{a}_{i(\rho_i-1)}
\end{align*}

The RBF neural networks weight matrices update laws and robust terms are determined at the 1, \ldots, $\rho_i$th step as
\begin{align*}
  \dot{W}_{i1} &= \Gamma_{i1} \theta_{i1} Z_{i1} Z_i
\end{align*}
\begin{align*}
  \dot{W}_{ij} &= \Gamma_{ij} \theta_{ij} Z_{ij}, j = 2, \ldots, \rho_i
\end{align*}
\begin{align*}
  E_{i1} &= \begin{cases}
  \text{sign}(\xi_i) \frac{Z_{i1}}{\rho_i} & \text{if } Z_{i1} \neq 0 \\
  0, Z_{i1} = 0
  \end{cases}
\end{align*}
\begin{align*}
  E_{ij} &= \begin{cases}
  \frac{Z_{ij}}{\rho_i} & \text{if } Z_{ij} \neq 0 \\
  0, Z_{ij} = 0
  \end{cases}, j = 2, \ldots, \rho_i
\end{align*}
where $\Gamma_{ij}, j = 1, \ldots, \rho_i$ is a positive definite design matrix. We write
\begin{align*}
  N = \beta \sum_{i=1}^{m} K_{i1} M_{i1} \sum_{\mu=0}^{L} \gamma_{i\mu} B_{i\mu}
\end{align*}
where $\beta = [\beta_1^T, \ldots, \beta_m^T]^T \in R^{m \times m}$, $\lambda_i = \sum_{i=1}^{m} K_{i1} M_{i1}$, $K_{i1} = \lambda_i$ and $M_{i1} \in R^{m \times m}$, $i = 1, \ldots, m$ is a matrix which the $i$th element of main diagonal is 1 and other elements are zero, and we note $K_1 = [K_{11}, \ldots, K_{1m}]^T$.

**Assumption 4.1:** $N$ is nonsingular.

Assumption 4.1 is precondition of the existence of the control law, and we note $\hat{N} = \beta \sum_{i=1}^{m} \hat{K}_{i1} M_{i1} \sum_{\mu=0}^{L} \gamma_{i\mu} B_{i\mu}$, where $\hat{K}_{i1}$ is estimate of $K_{i1}, i = 1, \ldots, m$.

The control law is designed as follow
\begin{align*}
  w(t) = \hat{N}^{-1} \begin{bmatrix}
  \alpha_{1\rho_i} - \beta_{1\rho_i} \hat{K}_2 \\
  \vdots \\
  \alpha_{q_{\rho_3}} - \beta_{q_{\rho_3}} \hat{K}_2
\end{bmatrix}
\end{align*}
where $C_m = C_{m1}\alpha + C_{m2}\alpha^2 + C_{m3} + C_{m4}(d_1\delta_{e1} + d_2\delta_{e2}) + C_{m5}q_i$; $\hat{K}_2$ is estimate of $K_2$. The parameter update laws are designed as
\begin{align*}
  \dot{\hat{K}}_1 &= \begin{bmatrix}
  \frac{1}{\Gamma_{11}} \sum_{l=1}^{q} \tilde{Z}_{i\rho_i} \beta_i M_{l1} \sum_{\mu=0}^{L} \gamma_{l\mu} B_{l\mu} w(t), |\chi| > a \\
  \frac{1}{\Gamma_{11}} \sum_{l=1}^{q} \tilde{Z}_{i\rho_i} \beta_i M_{l1} \sum_{\mu=0}^{L} \gamma_{l\mu} B_{l\mu} w(t), \chi = a \\
  \end{bmatrix} \\
  \dot{\hat{K}}_2 &= \Gamma_{2} \left( \sum_{l=1}^{q} z_{l\beta} \beta_l \right)^T
\end{align*}
where $l = 1, \ldots, m$; $\Gamma_1 = \text{diag}(\Gamma_{11}, \ldots, \Gamma_{1m}) \in R^{m \times m} > 0$, $\Gamma_2 \in R^{m \times m} > 0$ are positive definite design matrices; $\chi$ is the smallest absolute value element of $N = \beta \sum_{i=1}^{m} K_{i1} M_{i1} \sum_{\mu=0}^{L} \gamma_{i\mu} B_{i\mu}$; $\chi$ is the element of $N = \beta \sum_{i=1}^{m} K_{i1} M_{i1} \sum_{\mu=0}^{L} \gamma_{i\mu} B_{i\mu}$ corresponding with $\chi$; $b$ is the smallest absolute value element of $N$, although $N$ is unknown, but always exists a small positive number $a < b$. The parameter update law (46) can guarantee that $\dot{N}$ is nonsingular.

**Theorem 4.1:** Under Assumptions 1–4, consider the closed-loop adaptive system (1), the PPB based controller (45) with RBF neural networks weight matrices update laws (40–41), robust terms (42–43) and the parameter update laws (46–47) in the presence of possible actuator failures (2) ensures the boundedness of all the signals and tracking error $e_i(t) = y_i(t) - y_i(t), i = 1, \ldots, q$ asymptotically approaching zero, i.e., $-\tilde{\xi} \tau(t) < e_i(t) < \tilde{\xi} \tau(t), \forall t \geq 0$ is guaranteed.

**Proof:** From (45–47) and (30), it is obtained that
\begin{align*}
  \dot{Z}_{i1} &= -c_{i1} Z_{i1} + \zeta_i Z_{i2} + \zeta_i (-\hat{W}_{i1}^T \theta_{i1}(x) + e_{i1} - E_{i1}) \\
  \dot{Z}_{i2} &= -c_{i2} Z_{i2} + Z_{i3} - \zeta_i Z_{i1} + (-\hat{W}_{i2}^T \theta_{i2}(x) + e_{i2} - E_{i2}) \\
  \dot{Z}_{ij} &= -c_{ij} Z_{ij} + Z_{i(j+1)} - Z_{i(j-1)} + (-\hat{W}_{ij}^T \theta_{ij}(x) + e_{ij} - E_{ij}) \\
  \dot{Z}_{ij} &= -c_{ij} Z_{ij} - Z_{i(\rho_i-1)} - \beta \sum_{n=1}^{m} \hat{K}_{i1} M_{i1} \sum_{\mu=0}^{L} \gamma_{i\mu} B_{i\mu} w(t) \\
  \dot{\hat{K}}_2 &= (-\hat{W}_{ij}^T \theta_{ij}(x) + e_{ij} - E_{ij}) \\
  \dot{\hat{K}}_3 &= (-\hat{W}_{ij}^T \theta_{ij}(x) + e_{ij} - E_{ij}) \\
  \end{align*}
\begin{align*}
  i = 1, \ldots, q, j = 1, \ldots, \rho_3 - 1
\end{align*}
where $\hat{K}_1 = \hat{K}_1 - K_1, \hat{K}_2 = \hat{K}_2 - K_2$. Define a positive definite $V(t)$ as
\begin{align*}
  V(t) &= \frac{1}{2} \sum_{l=1}^{q} \sum_{j=1}^{L} \tilde{z}_{lj}^2 + \frac{1}{2} \sum_{l=1}^{q} \sum_{j=1}^{L} \hat{W}_{lj}^T \Gamma_{ij}^{-1} \hat{W}_{lj} + \frac{1}{2} \hat{K}_1^T \Gamma_1 \hat{K}_1 \\
  &\quad + \frac{1}{2} \hat{K}_2^T \Gamma_2 \hat{K}_2
\end{align*}
Differentiating $V(t)$, we obtain
\[
\dot{V}(t) = \sum_{i=1}^{q} \left[ -\sum_{j=1}^{\rho_i} Z_{ij}^2 + \xi \tilde{Z}_1 (\varphi_{i1} - E_{i1}) + \sum_{j=2}^{j=\rho_i} Z_{ij} (\varphi_{ij} - E_{ij}) \right] \\
- \sum_{i=1}^{q} Z_{ij} \beta_i \sum_{n=1}^{m} \tilde{K}_{1n} M_n B \dot{\tilde{w}}(t) - \sum_{i=1}^{q} Z_{ij} \beta_i \tilde{K}_2 \\
+ \sum_{n=1}^{m} \Gamma_{1n} K_{1n} \tilde{K}_{1n} + \tilde{K}_2^T \Gamma_2^{-1} \tilde{K}_2 \\
= \sum_{i=1}^{q} \left[ -\sum_{j=1}^{\rho_i} Z_{ij}^2 + \xi \tilde{Z}_1 (\varphi_{i1} - E_{i1}) + \sum_{j=2}^{j=\rho_i} Z_{ij} (\varphi_{ij} - E_{ij}) \right] \\
+ \tilde{K}_2^T \Gamma_2^{-1} \sum_{i=1}^{q} Z_{ij} \beta_i \sum_{n=1}^{m} \tilde{K}_{1n} M_n \sum_{\mu=0}^{\mu=L} \gamma_{\mu} B_{\mu} \dot{w}(t) + \Gamma_{1n} \tilde{K}_{1n} \right] \\
\tag{50}
\]
Substituting parameter update laws (46–47) and robust terms (42–43) into (45), we get
\[
\dot{V}(t) = -\sum_{i=1}^{q} \left[ \sum_{j=1}^{\rho_i} Z_{ij}^2 + \xi \tilde{Z}_1 (\varphi_{i1} - E_{i1}) + \sum_{j=2}^{j=\rho_i} Z_{ij} (\varphi_{ij} - E_{ij}) \right] \\
< 0 \\
\tag{51}
\]
It can be shown that $\varphi_{i1} = \tilde{Z}_1$ is bounded and $\lim_{t \to \infty} \tilde{Z}_1(t) = 0$. From section 3.1, we get $\lim_{t \to \infty} \varphi_{i1}(t) = 0$ is thus satisfied. The closed-loop system stability is established and asymptotic tracking can still retained.  

5. Simulation studies

To test the effectiveness of the proposed adaptive actuator failure compensation, we use the twin otter aircraft longitudinal nonlinear dynamics model as in Zhang, Qiu, Jiang, and Liu (2015).

\[
\dot{V} = \frac{F_x \cos \alpha + F_z \sin \alpha}{m} + \Delta f_1 \\
\dot{\alpha} = q + \frac{-F_x \sin \alpha + F_x \cos \alpha}{mV} \\
\dot{\theta} = q, \quad \dot{q} = \frac{M}{I_y} \\
\tag{52}
\]
where $V$ is the velocity, $\alpha$ is the attack angle, $\theta$ is the pitch angle, $q$ is the pitch rate, $m$ is the mass, $I_y$ is the moment of inertia, $\rho$ is the air density, $S$ is the wing area, $c$ is the mean chord, and $F_x = qSC_C(\alpha, q, \delta_{e1}, \delta_{e2}) + T_x - mg \sin \theta; F_z = \tilde{q}SC_C(\alpha, q, \delta_{e1}, \delta_{e2}) + T_z + mg \cos \theta; M = \tilde{q}SC_C(\alpha, q, \delta_{e1}, \delta_{e2}); \tilde{q} = \frac{1}{2} \rho V^2; C_x = C_{x1} \alpha + C_{x2} \alpha^2 + C_{x3} + C_{x4} (d_1 \delta_{e1} + d_2 \delta_{e2}); C_z = C_{z1} \alpha + C_{z2} \alpha^2 + C_{z3} + C_{z4} (d_1 \delta_{e1} + d_2 \delta_{e2}) + C_{z5} \delta_{e1} + C_{z6} \delta_{e2}; T_x$ and $T_z$ are the components of the thrust along the body $x$ and $z$. $\delta_{e1}$ and $\delta_{e2}$ are the elevator angles of an augmented two-piece elevator used as two actuators.

Choosing $V, \alpha, \theta, q$ as the states $x_1, x_2, x_3, x_4$. Choosing $\delta_{e1}, \delta_{e2}, T_x, T_z$ as the inputs $u_1, u_2, u_3, u_4$, and outputs as $y = [x_1, x_3]^T$.

There exist diffeomorphism $\xi_1 = [x_1], \xi_2 = [x_3, x_4]^T$ that can be transformed into the parametric-strict-feedback form, relative degrees are $\rho_1 = 1, \rho_2 = 2$; the error of plant model is $\Delta f_1 = 10 \sin \alpha$; the reference model are $1/(s + 1) + 1/(s^2 + 6s + 18)$; the input of reference model are $60 + \sin(0.05t)$ and $\sin(0.05t)$; PPB performance function is $\tau(t) = 199e^{-0.2t} + 0.1$; the initial states are set as $x(0) = [55, 0, 0.02, 0]^T$; other design parameters are chosen as $\Gamma_1 = 100I_{4x4}, \Gamma_2 = \Gamma_3 = 0.01I_{4x4}$. The initial states are set as $x(0) = [55, 0, 0.02, 0]^T$.

We can conclude that the compensable failure pattern set is $E = \{ \alpha | i = 0, \ldots, 9 \}$, where $\alpha_0 = [0, 0, 0, 0], 9 = [1, 0, 0, 0], \alpha_2 = [0, 1, 0, 0], \alpha_3 = [0, 0, 1, 0], \alpha_4 = [0, 0, 0, 1], \alpha_5 = [1, 0, 1, 0], \alpha_6 = [1, 0, 0, 1], \alpha_7 = [0, 1, 1, 0], \alpha_8 = [0, 1, 0, 1]$ and $\alpha_9 = [0, 0, 1, 1]$. Actuator $u_1$ loses 40% of its effectiveness from $t = 100s$; actuator $u_3$ is stuck at $s_3(200)$ from $t = 200s$; actuator $u_4$ is stuck at $s_4(300)$ from $t = 300s$. We can get that the failure pattern in $t = 0s$–$200s$ is $\alpha_0$; the failure pattern in $t = 200s$–$300s$ is $\alpha_3$; the failure pattern in $t = 300s$–$400s$ is $\alpha_9$. The tracking errors are given in Figures 2 and 3.

The system outputs and reference signals $y_1, y_{r1}$ and $y_2, y_{r2}$ are given in Figures 2 and 3 to show that all signals are bounded and asymptotic tracking can be ensured, the approach without neural approximation (named normal method) are given to show the proposed neural adaptive compensation scheme. The system states $x_2$ and $x_4$.

![Figure 2. Output $y_1$ and reference signal $y_{r1}$.](image-url)
are given in Figures 4 and 5, it can be seen that the system is ISS, the 2-norm of observer approximation error $e_0, e_3$ and $e_9$ are shown in Figures 6–8. System with no failure in $t = 0 s–100 s, ||e_0|| = 0$, the actuators grouping scheme is $\mathcal{m}_0 = \{(u_3, u_4), (u_1, u_2)\}$; actuator $u_1$ loss effectiveness from $t = 100 s$, observer can track system states asymptotically, the actuators grouping scheme is $\mathcal{m}_0 = \{(u_3, u_4), (u_1, u_2)\}$; actuator $u_3$ lock in space from $t = 200 s, ||e_0|| > 0, ||e_3||$ tends to 0 asymptotically, the actuators grouping scheme is $\mathcal{m}_2 = \{(u_4), (u_1, u_2)\}$; actuator $u_4$ lock in space from $t = 300 s, ||e_0||$ tend to 0 asymptotically, the actuators grouping scheme is $\mathcal{m}_9 = \{(u_1), (u_2)\}$. It can be seen that system can switch to actuators grouping scheme corresponding to current failure pattern rapidly.

Figure 9 shows the response of $y_2$ and $y_{r2}$ by fixed grouping approach in Tang et al. (2007). The simulation shows that the overshoot of the compensation controller

Figure 3. Output $y_2$ and reference signal $y_{r2}$.

Figure 5. State $x_4$.

Figure 4. State $x_2$.

Figure 6. 2-norm of identification model residuals in 0–200 s.

Figure 7. 2-norm of identification model residuals in 200–300 s.
without PPB is big, and the fixed grouping way loses effectiveness after $u_4$ failure. The control law proposed in this paper can compensate a larger failure pattern set, and have a better dynamic performance.

6. Conclusion

A neural adaptive backstepping control scheme is presented in this paper for a class of MIMO affine nonlinear systems with unknown actuator failures and error of plant model. The actuator failures under consideration include lock in space, total and partial loss of effectiveness, and the error of plant model is nonlinear. The control law can ensure asymptotic output tracking and closed-loop signal boundedness. Simulation studies also verify the theoretical results.

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