Transmission resonances on metallic gratings with very narrow slits

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In this letter we show how transmission metallic gratings with very narrow and deep enough slits can exhibit transmission resonances for wavelengths larger than the period of the grating. By using a transfer matrix formalism and a quasi-analytical model based on a modal expansion, we show that there are two possible ways of transferring light from the upper surface to the lower one: by the excitation of coupled surface plasmon polaritons on both surfaces of the metallic grating or by the coupling of incident plane waves with waveguide resonances located in the slits. Both mechanisms can lead to almost perfect transmittance for those particular resonances.

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Absorption anomalies in metallic gratings have attracted much attention since their discovery by Wood [1] in 1902. One of these anomalies is only observed for p-polarized light (H parallel to the grating grooves) and appears as a minimum on the specular reflectance. Now it is well known [2] that this anomaly stems from the excitation of surface plasmon polaritons (SPPs) by the incident electromagnetic radiation. The dependence of these SPP modes on the grating shape and the possible existence of modes localized in the grooves have been studied for a number of years and are still of interest [2–8]. On the other hand, the activity of the last decade in the field of photonic crystals [9] has originated a renewed interest in the properties of SPPs, as they can be viewed as surface electromagnetic modes propagating in 1D periodic dielectric media [10].

Besides, experimental evidence of the excitation of optical waveguide modes inside the narrow grooves of zero-order reflection gratings has been recently given [1]. Both from fundamental and practical points of view it would be very interesting to analyze the transmission properties of these waveguide modes. Also very recently some experiments carried out in arrays of submicrometre cylindrical holes in metallic films have shown an extraordinary optical transmission at wavelengths up to ten times larger than the diameter of the holes [12]. The similarities between this last structure and metal gratings suggest the possibility of equivalent resonant effects in transmission metal gratings of very narrow slits.

In this letter we test theoretically this possibility by analyzing the response of transmission metallic gratings to p-polarized electromagnetic radiation. We will show how, for very narrow slits, the coupling of the incident light with surface electromagnetic modes of the grating can lead to almost perfect transmission resonances appearing at wavelengths larger than the period of the grating and hence much larger than the lateral dimensions of the slits.

Inset of Fig. 1 shows a schematic view of the structures under study with the definition of the different parameters: the period of the grating \( d \), the width \( a \) and height \( h \) of the slits. The substrate is characterized by a dielectric constant, \( \epsilon \). Advances in material technology have allowed the production of transmission gratings with well controlled profiles, which have already been used in different interesting applications, such as polarizers or x-ray spectrometers [13]. From the theoretical point of view, there have been some studies of these structures in the last few years [14,15]. However, up to our knowledge, transmission properties of very narrow slits that are periodically structured remain unstudied.

In this letter we consider metal gratings made of gold and we use fixed values for the grating period \( d = 3.5 \mu m \) and the width of the slits \( a = 0.5 \mu m \), although the dependence of our results on \( a \) is also addressed. The thickness of the metallic grating \( h \) will be varied between 0 and 4\( \mu m \). The choice of these geometrical values is motivated by the experimental findings of waveguide resonances reported in Ref. [1] for reflection metal gratings with the same set of parameters. Nevertheless, it should be pointed out that the effects discussed in this letter do appear for any other range provided \( a \) is very small in comparison to \( d \) and the frequency of the incident light is well below the plasma frequency of the metal. The dielectric function of gold is described using the tables reported in Ref. [16].

We have analyzed the electromagnetic properties of these gratings by means of a transfer matrix formalism [17]. Within this formalism it is possible to calculate transmission and reflection coefficients for an incoming plane wave. Subsequently, the transmittance and reflectance of the grating as well as real-space electromagnetic fields can be calculated. Fig. 1 shows zero-order transmittance for normal incident radiation on metallic gratings in vacuum as a function of the wavelength of the incoming plane wave. The grating height \( h \) is varied in these calculations from 0.2 to 4\( \mu m \). As can be seen in Fig. 1a, for deep enough gratings \( h \geq 0.6 \mu m \) a remarkable transmission peak appears for a wavelength slightly larger than the grating period (in this case 3.5\( \mu m \)).
transmission peak moves to larger wavelengths as the grating height increases whereas its linewidth is broadened. And, as illustrated in Fig. 1b, subsequent transmission peaks emerge for deeper gratings. The behavior of the transmittance spectrum as a function of the metal thickness seems to suggest that these peaks could be linked to the coupling of incident plane waves with waveguide resonances of the slits.

In order to analyze the physical origin of these transmission resonances, we have also developed an approximated modal method. We incorporate two main simplifications to the exact modal method reported in [5]. First, as the frequency regime we are interested in is below the plasma frequency of the metal, surface-impedance boundary conditions (SIBC) [4] are imposed on the boundary of the metal. Second, we only consider the fundamental eigenmode in the slits, which are treated as perfect metal surfaces. The validity of these two approximations is confirmed by the fact that the transmission is completely governed by the behavior of the denominator of the complex part of the transmission coefficient. And, as illustrated in Fig. 1b, subsequent transmission peaks emerge for deeper gratings. The behavior of the transmittance spectrum as a function of the depth of the slits, $h$, is associated with the excitation of a surface plasmon with SPP character in each surface of the grating.

Using the simplified modal method, we can also study in detail the behavior of transmission resonances as a function of the width of the slits, $a$. For this purpose, we show in Fig. 3 zero-order transmittance curves for gratings in vacuum of thickness (a) $h = 0.6 \mu m$ and (b) $h = 3.0 \mu m$ in the wavelength region where transmission peaks appear. The width of the slits is varied between $0$ and $1.5 \mu m$. For the case of transmission resonances linked to coupled SPPs (Fig. 3a) a minimum value of $a$ is needed in order to couple SPPs of each surface of the grating. Above this threshold (whose wavelength depends on the depth of the slits and for $h = 0.6 \mu m$ is
around 0.2μm), the resonance is extremely narrow and hence these structures could be used as filters of electromagnetic radiation for wavelengths close to the period of the grating. On the other hand, for waveguide resonances (Fig. 3b), even for extremely narrow slits the transmission peak could be close to 1. In this limit, the wavelength of the resonance tends to 2( that corresponds to the first zero of sin k0h) and its linewidth goes to zero. As shown in Fig. 3, transmission resonances associated with coupled SPPs are much narrower than the ones linked to waveguide modes and in both cases their linewidths are rapidly broadened as the width of the slits is increased.

Finally, two questions remain to be answered: how is light transmitted from one side of the metallic grating to the other one by these electromagnetic modes? and what is the difference in the transmission process between the two mechanisms mentioned above? In order to answer these questions, we show in Fig. 4 detailed pictures of the E-field for two cases, both with a = 0.5μm: (a) h = 0.6μm and λ = 3.6μm (that corresponds to coupled SPPs) and (b) h = 3.0μm and λ = 7.5μm (example of waveguide resonance). As can be seen in Fig. 4a, the normal incident plane wave is exciting first a SPP in the upper metal surface. Although metal thickness is much larger than the skin depth of the metal, this SPP couples with the corresponding SPP mode of the lower metal surface through a waveguide mode located in the slits. Then the SPP mode of the lower surface can match to an outgoing propagating plane wave of the same frequency and momentum as the incident one, leading to a large transmittance. Due to the nature of this process, these transmission resonances are very sensitive to the presence of a substrate in the lower surface: when energies of the two SPPs involved do not coincide, the coupling between them is less effective and transmittance is severely reduced. The transmission process associated with waveguide resonances is completely different to the one described above for coupled SPPs. As shown in Fig. 4b, for these electromagnetic modes only the metal walls of the slits play an active role in the process. Incident light induces current densities flowing parallel to the slits’ walls, having different signs on the two opposite surfaces of the slits. Therefore, and different from coupled SPPs, the transmittance associated with these waveguide resonances is not very sensitive to the refraction index of the substrate.

We believe that electromagnetic modes of a nature similar to coupled SPPs in transmission gratings are responsible for the extraordinary optical transmission reported in hole arrays [14]. There are several facts that support this belief. First, the positions of the transmission peaks in both structures (hole arrays and transmission gratings) are mainly determined by the periodicity of the system and are almost independent of the diameter of the holes or the slits width and of the particular metal used. Besides, transmission resonances in both structures disperse significantly with the angle of incidence. However, hole arrays and gratings are two different geometries and the correspondence between both systems must be established with certain caveats [19].

In conclusion, transmission properties of metallic gratings with very narrow slits have been analyzed by means of a transfer matrix formalism and a quasi-analytical approach based on a modal expansion. We have shown how for deep enough gratings, resonances in the zero-order transmission spectra appear for wavelengths larger than the period of the grating. For these resonances, zero-order transmittance could be close to 1 besides the fact that the wavelength of the transmitted light is much larger than the lateral dimension of the slits. Two different transmission mechanisms have been described: excitation of SPPs on both surfaces of the metal grating and coupling of the incident light with waveguide resonances of the slits.

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FIG. 1. Inset: schematic view of the lamellar transmission metallic gratings studied in this paper (see text). Zero-order transmittance for a normal incident plane wave calculated by means of the transfer matrix formalism for lamellar metal gratings in vacuum ($d = 3.5\mu m$, $a = 0.5\mu m$) for different values of the grating height ($h$), ranging from 0.2 $\mu m$ to 4.0 $\mu m$.

FIG. 2. Photonic band structure (black dots) of the surface plasmons responsible for the transmission resonances appearing at (a) $h = 0.6\mu m$ and (b) $h = 3\mu m$. In the same figure we plot the energetic positions (gray dots) of SPPs in the limit $h \rightarrow 0$. These bands are calculated using the simplified modal method. In the inset of this figure (a) we show a closed-up picture of the opening of the first band gap for this case.

FIG. 3. Zero-order transmittance curves, calculated by an approximated modal method (see text), for metallic gratings of period $3.5\mu m$ in vacuum and thickness (a) $h = 0.6\mu m$ and (b) $h = 3.0\mu m$ as a function of the width of the slits, $a$, and wavelength of the normal incident light. Transmittance is shown in a gray scale (black: transmittance between 0.9-1.0 and white: transmittance between 0.0-0.1).

FIG. 4. Detailed pictures of the $E$-field over two periods of transmission metal gratings ($d = 3.5\mu m$, $a = 0.5\mu m$) of thickness, (a) $h = 0.6\mu m$ and (b) $h = 3.0\mu m$ in vacuum. The wavelengths of the normal incident radiation are for (a) $\lambda = 3.6\mu m$ and (b) $\lambda = 7.5\mu m$, that correspond to different transmission peaks shown in Fig. 1. These $E$-fields have been obtained with the transfer matrix formalism.
Parallel momentum (in units of $\pi/d$)

Energy (eV)

Parallel momentum (in units of $\pi/d$)
Parallel momentum (in units of $\pi/d$) vs. Energy (eV)
(a) 

(b) 

Wavelength (µm)

Wavelength (µm)

a (µm)
