The Observed Cosmic Star Formation Rate Density Has an Evolution that Resembles a $\Gamma(a, bt)$ Distribution and Can Be Described Successfully by Only Two Parameters

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Abstract

A debate is emerging regarding the recent inconsistent results of different studies for the cosmic star formation rate density (CSFRD) at high-$z$. We employ UV and IR data sets to investigate the SFR function (SFRF) at $z \sim 0$–9. We find that the SFRFs derived from the dust-corrected UV (UV$_{\text{corr}}$) data contradict those from IR on some key issues because they are described by different distributions (Schechter versus double power law), imply different physics for galaxy formation (UV$_{\text{corr}}$ data suggest an SFR limit/strong mechanism that diminish the number density of highly star-forming systems with respect to IR) and compare differently with the stellar mass density evolution obtained from spectral energy distribution fitting (UV$_{\text{corr}}$ is in agreement, while IR differs up to 0.5 dex). However, both tracers agree on a constant CSFRD evolution at $z \sim 1$–4 and point to a plateau instead of a peak. In addition, using both indicators, we demonstrate that the evolution of the observed CSFRD can be described by only two parameters and a function that has the form of a Gamma distribution ($\Gamma(a, bt)$). In contrast to previous parameterizations used in the literature, our framework connects the parameters to physical properties such as the SFR depletion time and cosmic baryonic gas density. The build-up of stellar mass occurs in $\Gamma(a, bt)$ distributed steps and is the result of gas consumption up to the limit at which no eligible gas for SF at $t = \infty$ remains, resulting in a final cosmic stellar mass density of $\sim0.5 \times 10^9 M_\odot$ Mpc$^{-3}$.

Unified Astronomy Thesaurus concepts: Galaxy evolution (594)

1. Introduction

In the past three decades, galaxy surveys and cosmological simulations have been used in our quest to understand how galaxies evolve. A particular focus has been given to the observed star formation rates (SFRs) and stellar masses of galaxies, which enable us to establish numerous constraints such as the galaxy stellar mass function, the SFR function (SFRF), the SFR-stellar mass relation ($\text{SFR} - M_*$), the cosmic SFR density (CSFRD), and the cosmic stellar mass density (Lapi et al. 2017; Driver et al. 2018; López Fernández et al. 2018; Blanc et al. 2019; Caplar & Tacchella 2019; Davies et al. 2019; Katsianis et al. 2019; Cheng et al. 2020; Hodge & da Cunha 2020; Tacchella et al. 2020; Thorne et al. 2021; Trčka et al. 2020; Lovell et al. 2021b; Vijayan et al. 2021).

However, different observational studies have been employing different methodologies/wavelengths in order to derive galaxy SFRs, such as IR luminosities (Guo et al. 2015; Qin et al. 2019), H$\alpha$ luminosities (Sánchez et al. 2018; Cano-Díaz et al. 2019), the spectral energy distribution (SED) fitting technique (Kurczynski et al. 2016; Zhao et al. 2020; Yang et al. 2021), or UV luminosities (Blanc et al. 2019; Moustard et al. 2020). All the above methods have been commonly used in the literature, and they have provided a great opportunity to study galaxy evolution, but they suffer from various shortcomings. For example, UV is a direct measurement of the SFR, but it is limited as follows:

1. Dust corrections for UV luminosities are uncertain for highly star-forming systems and are possibly underestimated (Dunlop et al. 2017),
2. UV-luminosity functions (LFs), and consequently, the UV-SFRFs, are probably incomplete at the bright end of the distribution because bright objects/highly star-forming galaxies have high dust contents and thus may be invisible to UV surveys (Katsianis et al. 2017a),
3. It is currently challenging to confirm how many stars a galaxy forms because the individual stellar populations cannot be resolved (Kuncarayakti et al. 2016). We do not have a cosmic timer to mark the time for the birth of stars, nor do we have a cosmic scale to weight their masses, so we cannot know if the actual SFRs we derived from their UV luminosities are actually correct. Any calibration for the UV-SFR conversion derived from stellar population synthesis (SPS) modeling like those from Kennicutt (1998) or Kennicutt & Evans (2012) relies on modeling and assumptions (Chen et al. 2010; Stanway 2020).  

On the other hand, IR studies are traditionally considered to be able to probe the SFRs of highly star-forming systems. We have to note the following shortcomings of deriving SFR, and consequently, CSFRDs, from IR data (e.g., Fumagalli et al. 2014; Hayward et al. 2014; Utomo et al. 2014; Katsianis et al. 2016, 2020), which can be summarized as follows:

1. Overestimation due to buried active galactic nuclei (AGNs) that boosts the IR luminosities (Brand et al. 2006; Ichikawa et al. 2012; Roebuck et al. 2016; Brown et al. 2019; Symeonidis & Page 2021),
2. overestimation due to the fact that dust can be heated by old populations that are not relevant to current star

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4 We note that individual stellar populations can be resolved for nearby objects. Olsen et al. (2021) used a sample of 36 nearby dwarf galaxies and demonstrated a broad agreement between the star formation histories derived from the SEDs and the color–magnitude diagrams.
formation (Viaene et al. 2017; Brown et al. 2019; Leja et al. 2019a; Nersesian et al. 2019). The IR luminosity can overestimate the instantaneous SFR during the post-starburst phase because stars that were formed in the starburst phase can remain dust obscured and thus produce a significant IR luminosity that is unrelated to newborn stars (Hayward et al. 2014),

3. overestimation due to higher polycyclic aromatic hydrocarbon emission of distant galaxies (Huang et al. 2009; Murata et al. 2014),

4. great uncertainties for the number density of galaxies with low SFRs (defined in our work as \( \text{SFR} = 0.01-0.5 \, M_\odot \, \text{yr}^{-1} \)) and intermediate SFRs (defined as \( \text{SFR} = 0.5-10 \, M_\odot \, \text{yr}^{-1} \)), which IR data cannot usually probe. This limitation is redshift dependent, and deriving any parameterization of the total SFRF or total LF using only IR data (i.e., the high star-forming end defined as objects with \( \text{SFR} > 10 \, M_\odot \, \text{yr}^{-1} \)) can be problematic (Katsianis et al. 2017a, 2017b),

5. highly star-forming/IR bright systems might be contaminated by the effect of gravitational lensing at high redshifts (Zavala et al. 2021),

6. insufficient wavelength coverage, especially at far-IR (FIR) wavelengths (Pearson et al. 2018),

7. ultraluminous IR galaxies are offset from the typically used SFR calibrations (De Looze et al. 2014),

8. IR estimates represent less instantaneous measurements than UV or \( \text{H}_\alpha \) tracers. Both UV and \( \text{H}_\alpha \) luminosities are sensitive only to the most massive stars, which have short lifetimes (Shivaei et al. 2015; Katsianis et al. 2016),

9. Like for the case of the UV SFRs, it is currently challenging to confirm how many stars a high-redshift galaxy forms per year. We stress that IR data are always important in order to build better SED models because they span an interesting part of the spectrum related to dust physics.

A natural question arises: (1) Because different indicators and methods rely on different principles (e.g., UV calibrations directly trace photons relevant to star formation, SF, while IR focus on the reprocessed light from dust) and have different shortcomings (e.g., UV light is affected by dust, while IR is contaminated by older population of stars) are they consistent with each other?

In the past 5 yr, an increasing number of authors have reported a severe discrepancy between the SFRs inferred by different methodologies (Hayward et al. 2014; Katsianis et al. 2015; Davies et al. 2016; Martis et al. 2019; Katsianis et al. 2020) in contrast with others, who insist on a consistent picture of galaxy SFRs among different studies and indicators (Domínguez Sánchez et al. 2012; Madau & Dickinson 2014; Rodighiero et al. 2014; Santini et al. 2017). A conclusive study is Required to establish if there is indeed a tension (and if so, estimate its magnitude). We have to keep in mind that numerous efforts have been made in the past 2 yr that indicate shortcomings of previous published work/methodologies that relied on the traditionally used UV, IR, or SED SFR indicators. There is evidence today that previous stellar masses reported in the literature may have been underestimated by 0.1–0.3 dex, while previously calculated SFRs may have been overestimated by \( \sim 0.1-1.0 \, \text{dex} \) (Leja et al. 2019b; Katsianis et al. 2020).

Despite the above limitations of the traditionally used IR and UV indicators, recently, a range of IR/CO studies (Rowan-Robinson et al. 2016; Gruppioni et al. 2020; Khusanova et al. 2021; Loiacono et al. 2020) suggested that a considerable amount of star-forming activity occurs at high-\( z \) that is not recovered by UV data, while cosmological simulations such as EAGLE (Schaye et al. 2015; Lagos et al. 2020) or IllustrisTNG (Pillepich et al. 2018; Kopenhafer et al. 2020) underpredict the CSFRD with respect their observations. Thus, models and previously reported UV, \( \text{H}_\alpha \), and SED cosmic SFRs are challenged by the authors. However, we present in the previous paragraphs that all indicators and methods possibly suffer from limitations, and thus the above statement can be reversed by some authors to the statement that the IR/CO studies mentioned above involve CSFRDs that are overestimated, and cosmological models, simulations, UV, SED, and \( \text{H}_\alpha \) measurements represent the “true” SFRs at high-\( z \) better.

A powerful way to investigate the ability of an observational tool in deriving physical quantities is using dust radiative transfer models and simulations (Baes et al. 2020; Dickey et al. 2021; Lu et al. 2020; Lovell et al. 2021b; Narayanan et al. 2021). Recent studies investigated morphology measures (Cochrane et al. 2019), SFR indicators (Katsianis et al. 2020; Lower et al. 2021), or galaxy dust attenuation curves (Trayford et al. 2020). More specifically, Katsianis et al. (2020) suggested that for high redshifts, UV/SED/\( \text{H}_\alpha \) SFR measurements are relatively robust, while numerous calibrations that relied on a combination of UV and IR luminosities (Heinis et al. 2014; Whittaker et al. 2014) overproduce the derived SFRs even by 0.5 dex, resulting in an illusionary tension between observed and simulated SFR – \( M_\odot \) relations. However, we stress that we cannot claim conclusively that the one or the other technique is better for real galaxies because cosmological models also suffer from numerous shortcomings. For example, the SFRs and stellar masses obtained by state-of-the-art cosmological simulations such as EAGLE or IllustrisTNG and any post-processing with radiative transfer codes can be affected by the two items below.

1. Resolution effects. It has been demonstrated that the simulated SFRFs and stellar mass functions are not converging among runs of different resolution (Schaye et al. 2015; Pillepich et al. 2018; Zhao et al. 2020). In order for convergence to be achieved, a resolution-dependent re-tuning of the subgrid prescriptions (e.g., Feedback) is required, and this raises the question whether our models reproduce galaxy properties for the right reasons. In addition, resolution limits cause uncertainties for the post-processing of the simulated galaxies via radiative transfer (Trayford et al. 2017; Nelson et al. 2018). For example, the luminosity at mid-infrared wavelengths mainly originates from star-forming regions that are well below the resolution limits of the current state-of-the-art simulations, so that further uncertain subgrid modeling is required (Baes et al. 2020).

2. Any comparison in the observed flux space (mock observations that are built upon the cosmological simulations) is always affected by the assumptions employed by the radiative transfer post-processing e.g.,

\[ \text{SFR}/M_\odot = \frac{\text{SFR}}{M_\odot} \]

We note that there are great uncertainties for the simulated SFRs (the SFR/\( M_\odot \) ratio) of passive galaxies (\( \text{SFR} < 10^{-11} \, M_\odot/\text{yr} \). The number density of the specific SFRs, namely the specific SFRF predicted from the simulations, is in severe tension with observations (Zhao et al. 2020; Corcho-Caballero et al. 2021; Katsianis et al. 2021). This disagreement indicates that galaxies are not quenched correctly in the simulations via feedback, and the results from the models should be treated with caution.
the assumed dust model (Calzetti et al. 1994; Zubko et al. 2004; Nelson et al. 2018) or the adopted metal fraction (Brinchmann et al. 2013; Camps et al. 2018).

In addition to these limitations, however, it is important to note that simulations/radiative transfer have progressed significantly and can provide strong indications and an insight into the shortcomings of tools that are being used to derive physical quantities from observations. Keeping all this in mind, the question that arises this time is: (2) In addition to using simulations/radiative transfer, is there a way for us to determine which method describes the “true” SFRs more successfully?

Severe limitations exist in the field of theory of SF as well, and especially in the parameterization of the SF histories of both individual galaxies and of the CSFRD. In addition to the fact that the SFHs of realistic galaxies can be complex, they are often modeled with simple functional forms. These forms, in addition to their limitations (Smith & Hayward 2015; Carnall et al. 2019; Leja et al. 2019a), provide a computationally fast approach and are able to be applied to SED fitting procedures (Leja et al. 2017). For the case of individual galaxies, the most common scenario is an exponential form with an SFR $\propto e^t$. When $B = 0$, the law describes a simple exponentially declining SFH, and when $B = 1$, a delayed exponentially declining SFH. More complex forms have been adopted to better describe the range of observations. For example, rising (Papovich et al. 2011), log-normal (Abramson et al. 2016), and double power law (Behroozi et al. 2013) SFHs have been proposed. For the case of the CSFRD, the most commonly used parameterizations are given by Madau & Dickinson (2014), who suggested that the cosmic SFH follows a rising phase, scaling as $\text{SFR}(z) \propto (1 + z)^{-2.9}$ at $z = 3–8$, slowing down and peaking at $z \sim 2$, followed by a gradual decline to the present day, roughly as $\text{SFR}(z) \propto (1 + z)^{-2}$. Similar models have been suggested earlier by Cole et al. (2001) as CSFRD = $\frac{a_{b} \int \tau_{b}^{c}}{1 + (\frac{z}{c})}$. Two questions arise: (1) These parameterizations are empirically motivated and represent “physics-free” models. Can the parameters be connected to the physical properties of galaxies? (2) Is it possible to decrease the number of parameters necessary (4) to broadly fit the data?

In our work, we construct SFRFs to demonstrate how different qualitative results the two schools of thought (IR versus UV) produce, pointing out the importance of the tension (Section 2.2). We perform sanity checks by making comparisons with other observables (the evolution of the cosmic stellar mass density at $z \sim 0–9$) as we deem this process a complementary approach to radiative transfer/simulations to start uncovering which indicator is more consistent with the current paradigm of galaxy formation (Section 4). Finally, we establish a strong simple parameterization that describes the CSFRD derived from the UV dust-corrected (UV$_{\text{corr}}$) and IR data using only two parameters and a function that resembles a Gamma distribution. This parameterization connects its parameters to properties of galaxies/halos/universe, and thus it is an effort to extend the current physics-free empirical fits (Section 5). In our work we adopt the Planck Collaboration et al. (2020) cosmology with $\Omega_m = 0.315$, $\Omega_{\Lambda} = 0.685$, $n_s = 0.965$, $h = 0.674$, and a Chabrier (2003) initial mass function (IMF).

2. The “Observed” Star Formation Rates of Galaxies from UV and IR Light

2.1. The Data

Bellow we summarize the data sets used for this work and the methodology for deriving galaxy SFRs from UV and IR light. We start with the observations of Moutard et al. (2020), who obtained the rest-frame far-UV (FUV; 1546 Å) LFs at redshifts $z \sim 0.2$ to $z \sim 3$. Their work spans over 4.3 million galaxies, selected from the Canada–France–Hawaii Telescope (CFHT) Large Area U-band Deep Survey (CLAUDS, Sawicki et al. 2019) and the HyperSuprime–Cam Subaru Strategic Program (HSC-SSP, Aihara et al. 2018). This combination of area and depth enabled the authors to probe both the faint end of the UV LF regime with excellent statistical precision, while also taking into account very rare galaxies at the bright end, which are essentially free of cosmic variance. Thus, this work can provide an excellent starting point for determining the UV SFRFs and provide comparisons with the predictions from IR data that actually are assumed to probe mostly the most star-forming/dusty objects at high-$z$. The authors noted that the LFs are described by classic Schechter forms. In addition to Moutard et al. (2020), we employ additional UV LFs to extend our work to higher redshifts (Ono et al. 2018; Bhatawdekar et al. 2019). Ono et al. (2018) presented results for very luminous galaxies at $z \sim 4–7$ based on the wide and deep optical Hyper Suprime–Cam, while Bhatawdekar et al. (2019) gave insight into $z \sim 9–10$ objects. Last, we use the SFRFs presented in Katsianis et al. (2017a) and Katsianis et al. (2017b), which span a redshift range of $z \sim 0–8$. The authors employed IR (Magnelli et al. 2013; Patel et al. 2013), UV (Bouwens et al. 2015; Parsa et al. 2016), Hα (Sobral et al. 2013), and radio data (Mauch &Sadler 2007). We note that most of our knowledge for the CSFRD at high redshifts ($z > 2$) is based mostly on galaxy samples selected in the rest-frame UV (Oesch et al. 2018), while the total SFRs are not measured, but rather inferred through dust-correction techniques. Following the commonly used IRX–$\beta$ relation (Meurer et al. 1999) as in Smit et al. (2012) and Katsianis et al. (2017a, 2017b), we correct the UV luminosities as follows:

$$A_{1600} = 4.43 + 1.99 \beta,$$

where $A_{1600}$ is the dust absorption at 1600 Å, and $\beta$ is the UV-continuum spectral slope. We assume a relation between $\beta$ and the luminosity (Bouwens et al. 2012; Tacchella et al. 2013),

$$\langle \beta \rangle = \frac{d\beta}{dM_{\text{UV}}} (M_{\text{UV,AB}} + 19.5) + \beta_{M_{\text{UV}}},$$

we assume the same $\langle \beta \rangle$ as Arnouts et al. (2005), Oesch et al. (2010), Smit et al. (2012), Tacchella et al. (2013), Katsianis et al. (2017a), and Katsianis et al. (2020). Then, following Hao et al. (2011) and Katsianis et al. (2017b), we employ

$$L_{\text{UV,corr}} = L_{\text{UV,unc}} e^{-\tau_{\text{UV}}},$$

where $\tau_{\text{UV}}$ is the effective optical depth ($\tau_{\text{UV}} = A_{1600}/1.086$), $L_{\text{UV,corr}}$ is the observed (affected by dust luminosity), and $L_{\text{UV,unc}}$ represents the intrinsic luminosity. We convert the dust-corrected UV luminosities into SFRs following Kennicutt & Evans (2012),

$$\log_{10}(\text{SFR}_{\text{UV,corr}}) = \log_{10}(L_{\text{UV,corr}}) - 43.35.$$

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We label the above determination $\text{SFR}_{\text{UV,corr}}$. We note that in addition to the limitations of the indicator, its validity has been explored in cosmological simulations/radiative transfer codes in Katsianis et al. (2020), where it is shown that the $\text{SFR}_{\text{UV,corr}}$ is successful at deriving any SFRs at $z > 2$, but starts underperforming at lower redshifts ($z \leq 1$) for highly star-forming systems ($>10 M_{\odot} \text{yr}^{-1}$).\(^6\) This confirmation via radiative transfer/simulations (except for their limitations) is encouraging for high-$z$/low SFR determinations of SFRs from UV light. We note that the above method typically produces results that are in good agreement with SFRs obtained from SED fitting and $H_\alpha$ data.

In order to have a more complete picture of the SF, it is good practice to combine the UV SFRs with IR data. In our work for high-redshift galaxies, we employ the state-of-the-art data of Gruppioni et al. (2020), who used 56 sources that were blindly detected within the ALPINE survey to investigate the evolution of the dusty high SFR galaxy population at $z \sim 0.5$–6. The authors computed the rest-frame LFs at 250 $\mu m$ and compared them with the Herschel and SCUBA-2 LFs, suggesting that the ALPINE results are mostly complementary to the previous data. The authors computed the total IR luminosity by integrating the SEDs over 8–1000 $\mu m$, constructed IR LFs, and employed the Kennicutt (1998) relation to obtain the evolution of the CSFRD. The results were found in agreement with those from previous far-IR data, CO LFs (Riechers et al. 2019, Decarli et al. 2020), and gamma-ray burst data (Kistler et al. 2009). Gruppioni et al. (2020) report that the CSFRD from their IR data are significantly higher than those found by optical/UV surveys at $z > 2$ even by a factor of about 10 at $z = 6$, claiming they are witnessing obscured SF. In addition to the small number of objects (56), the authors support that the area covered by the survey guarantees that the contribution due to cosmic variance to the derived LFs and CSFRD is negligible. However, according to Zavala et al. (2021), the CSFRD derived from Gruppioni et al. (2020) may be actually artificially overestimated due to numerous reasons, as follows: (1) the fact that Herschel observations overestimate the derived IR luminosities, which conclusively overestimate the derived SFRs. (2) Clustering effects, which have their roots in the fact that the original targets are a few massive galaxies at $z \sim 4$–6, which represent an overdense region of the universe. (3) Uncertain extrapolations up to the faint end.

In addition to Gruppioni et al. (2020), in our work we employ the IR LFs from other studies (Gruppioni et al. 2013; Magnelli et al. 2013; Marchetti et al. 2016; Kilerci Eser & Goto 2018) in order to have a larger redshift and SFR coverage. We convert the TIR luminosities into SFRs following Kennicutt & Evans (2012).

$$\log_{10}(\text{SFR}_{\text{IR}}) = \log_{10}(L_{\text{TIR}}) - 43.41.$$  \hspace{1cm} \text{(5)}

We note that Katsianis et al. (2020) used radiative transfer/cosmological simulations and demonstrated that calibrations that involve IR wavelengths can overestimate the derived SFRs by 0.5 dex, especially at $z > 1$.\(^7\) In addition, Martis et al. (2019) used observations of the UltraVISTA DR3 photometry and Herschel PACS-SPIRE data and demonstrated that commonly adopted relations to derive SFRs from the observed 24$\mu$m are found to overestimate the SFR by a factor of 3–5.

2.2. The “Observed” Star Formation Rate Function

In this subsection we present the evolution of the SFRF at $z \sim 0$–9. Alongside, we plot the results from Katsianis et al. (2017a) and Katsianis et al. (2017b), represented by the open blue circles $(H_\alpha)$, open magenta pentagons/triangles (IR), and open green stars/diamonds (UV+$\text{dust corrections}$). In Figure 1 and Table 1, we summarize the results of this work. The evolution of the SFRF obtained from the CLAUDS++HCS UV observations (Moutard et al. 2020) is represented by the open black diamonds. The open black circles and the open black stars describe the SFRFs obtained from Livermore et al. (2017) and Ono et al. (2018), respectively. The solid black lines represent a fit for the above UV-SFRFs that is described by a Schechter form. In Figure 2 we present the evolution of the SFRF at three different redshift ranges (top left panel $-z \sim 4$–9, middle left panel $-z \sim 1$–4, and bottom left panel $-z \sim 0$–1 describe the UV SFRF evolution). We note the following:

1. The UV SFRFs of this work are broadly described by a standard Schechter form (Figure 1) as follows:

$$\Phi_{\text{SFR}} \odot \text{logSFR} = \ln(10) \times \Phi_{*,\text{Sch}} \times \left(\frac{\text{SFR}}{\text{SFR}_{*}}\right)^{1+\alpha} \times e^{-\text{SFR}/\text{SFR}_{*}}.$$  \hspace{1cm} \text{(6)}

The parameters at different redshifts can be found in Table 1.

2. There are almost no objects with $\text{SFR}_{\text{UV,corr}}$ more than $\sim 100 M_{\odot} \text{yr}^{-1}$ (Figure 2). A clear maximum SFR limit is implied. We note that the UV$_{\text{corr}}$ SFRFs at the highly star-forming end have been found to be in good agreement with the predictions from the EAGLE (Katsianis et al. 2017b) and IllustrisTNG simulations (Zhao et al. 2020), but are lower with respect other models such as Simba (Davé et al. 2019; Lovell et al. 2021a), which are able to produce more massive and highly star-forming galaxies.

3. The SFRF increases in normalization and becomes less steep with time (top left panel of Figure 2) from redshift 9 to 4. The evolution of the SFRF is mostly driven by the emergence of highly star-forming systems. This results in an increasing CSFRD\(^8\) (shown in Figure 2 as the dotted black line). However, the SFRF is mostly unchanged from redshift 4 to 1 (shown in the middle left panel of Figure 2), and this results in a CSFRD that is constant at this era. There is no peak at $z \sim 2$, in contrast with Madau & Dickinson (2014). The indicator suggests that there is instead a plateau that remains constant at $z \sim 1$–4, in agreement with Moutard et al. (2020) and Gruppioni et al. (2020). At $z \sim 1$ to 0, there is a uniform decrement of the SFRF that occurs similarly in all SFR bins, which causes a decrement in the CSFRD (shown in the top left panel of Figure 2). We note that according to the above UV$_{\text{corr}}$ SFRFs, there is no more aggressive quenching for

\(^6\) We note that due to the shortcomings of both cosmological simulations and observational techniques, outlined in the introduction, it is uncertain if the $\text{SFR}_{\text{UV,corr}}$ indicator described above is not successful at $z \leq 1$ and $\text{SFR} > 10 M_{\odot} \text{yr}^{-1}$ due to the fact that the dust is not modeled correctly in EAGLE or that indeed the method underestimates the SFRs of dusty galaxies at low redshifts.

\(^7\) The analysis involved galaxies with SFRs of $0.3$–$100 M_{\odot} \text{yr}^{-1}$ and $M_{\text{f}}$ of $10^{8.5}$–$10^{11} M_{\odot}$ at $z \sim 1$–4. Objects with higher SFRs (and consequently, IR bright objects) were not included because we relied on the EAGLE cosmological simulation, which has been found to suffer from a scarcity of SFR/IR bright objects (Cowley et al. 2019; Wang et al. 2019).

\(^8\) We note that the integration limits for the calculation of the CSFRD from the SFRF are $SFR = 0.01$–$1000 M_{\odot} \text{yr}^{-1}$, consistently for both indicators and all redshifts considered.
Figure 1. The evolution of the SFRF obtained from CLAUDS+HCS UV observations (filled black diamonds, Moutard et al. 2020) and a compilation of IR studies (filled black squares, Gruppioni et al. 2013; Magnelli et al. 2013; Marchetti et al. 2016; Kilerci Eser & Goto 2018; Gruppioni et al. 2020). Alongside, we plot the results from Katsianis et al. (2017a, 2017b; Moutard et al. 2020). The solid black lines represent the Schechter fits of the UVcorr-SFRFs, while the dotted back line is a double power law fit to the highly star-forming end obtained from IR studies (Gruppioni et al. 2013; Magnelli et al. 2013; Marchetti et al. 2016; Kilerci Eser & Goto 2018) and the low star formation end probed by the UV studies (Katsianis et al. 2017a, 2017b; Moutard et al. 2020).
the highly or little star-forming objects, and this can be interesting for studies that focus on galaxy feedback.

The SFRFs obtained from the compilation of IR studies (Gruppioni et al. 2013; Magnelli et al. 2013; Marchetti et al. 2016; Kilerci Eser & Goto 2018; Gruppioni et al. 2020) are represented by the filled magenta squares. We see that the above IR SFRFs are able to probe only the highly star-forming end (SFR $> 10 \, M_\odot \, yr^{-1}$) at high redshifts, so we combine them with the UV SFRF for the low star formation end in order to construct an analytical form described by the dotted black line in Figure 1. We suggest that ambiguously extending the IR SFRFs to lower SFRs is problematic, as discussed in the introduction, because no data from the indicator at the low star formation end are available. We note that by doing so, we essentially compare the highly star-forming ends of the distributions, and this describes a lower limit for the tension between our UVcorr and UV+IR SFRFs when the analytic forms are compared. In Figure 2 we present the evolution of the SFRF at two different redshift eras (middle right panel - $z \sim 1-4$, and bottom right panel - $z \sim 0-1$ describe the IR SFRF evolution). We note the following:

1. The UV+IR SFRFs of this work are described by a double power law (Figure 1) instead of a Schechter form, which is described by the following form:

$$
\Phi_{\text{SFR}} \, d\log\text{SFR} = \frac{\Phi_{\text{double}}}{\text{SFR}_{\text{break}}} \left( \frac{\text{SFR}}{\text{SFR}_{\text{break}}} \right)^{1-\alpha_1} \left[ \text{SFR} < \text{SFR}_{\text{break}} \right],
$$

$$
\Phi_{\text{SFR}} \, d\log\text{SFR} = \frac{\Phi_{\text{double}}}{\text{SFR}_{\text{break}}} \left( \frac{\text{SFR}}{\text{SFR}_{\text{break}}} \right)^{1-\alpha_2} \times \left[ \text{SFR} > \text{SFR}_{\text{break}} \right].
$$

The parameters at different redshifts can be found in Table 1.

2. In contrast to the UVcorr SFRFs, there are numerous objects with SFRIR with values higher than $100 \, M_\odot \, yr^{-1}$.

3. In contrast with the UV SFRF, the IR SFRF varies with redshift at $z = 4$ to $z = 1$. However, overall, the derived CSFRD_{UV+IR} (using the double power laws of Table 1) remains almost constant at this epoch, except for some variations at $z = 3$ and $z = 1.5$ ($z = 4-0.106 \, M_\odot \, yr^{-1} \, Mpc^{-3}$, $z = 3-0.123 \, M_\odot \, yr^{-1} \, Mpc^{-3}$, $z = 2-0.107 \, M_\odot \, yr^{-1} \, Mpc^{-3}$, $z = 1.5-0.133 \, M_\odot \, yr^{-1} \, Mpc^{-3}$, and $z = 1-0.104 \, M_\odot \, yr^{-1} \, Mpc^{-3}$). As in the case of the UVcorr CSFRD, we see that there is a plateau for the (CSFRD_{UV+IR}) and not a peak, in agreement with Gruppioni et al. (2020). At $z \sim 1$ to 0.25, according to IR data, there is a fast decrement for the highly star-forming end that causes a decrement to the CSFRD as well (top panel of Figure 2). From redshift 0.25 to 0, the decrement is much slower.

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**Table 1**

| UV-SFRF, redshift | $\Phi_{\text{S,C}}$ | $\Phi_{\text{unc,S,C}}$ | $\alpha$ | $\alpha_{\text{unc}}$ | SFR$_1$ | SFR$_{\text{unc}}$ | CSFRD | CSFRD$_{\text{unc}}$ |
|-------------------|-------------------|-------------------|--------|-----------------|--------|-----------------|-------|-------------------|
| 8.0               | 0.18              | 0.07              | −2.23  | 0.08            | 23.01  | 3.23            | 0.009 | 0.002            |
| 7.0               | 0.39              | 0.01              | −2.10  | 0.01            | 33.17  | 3.63            | 0.015 | 0.003            |
| 6.0               | 0.26              | 0.02              | −2.16  | 0.08            | 40.05  | 1.94            | 0.018 | 0.002            |
| 5.0               | 3.30              | 0.08              | −1.68  | 0.03            | 41.04  | 5.68            | 0.034 | 0.004            |
| 4.0               | 11.82             | 0.08              | −1.60  | 0.02            | 22.12  | 0.85            | 0.055 | 0.008            |
| 3.0               | 7.24              | 1.41              | −1.56  | 0.08            | 42.89  | 4.00            | 0.061 | 0.007            |
| 2.6               | 13.68             | 1.25              | −1.42  | 0.02            | 26.91  | 1.35            | 0.056 | 0.006            |
| 2.15              | 23.74             | 1.04              | −1.37  | 0.05            | 19.78  | 0.82            | 0.066 | 0.010            |
| 1.55              | 12.31             | 4.5               | −1.61  | 0.16            | 22.72  | 3.65            | 0.061 | 0.011            |
| 1.1               | 18.23             | 4.22              | −1.60  | 0.2             | 17.88  | 1.66            | 0.069 | 0.008            |
| 0.7               | 17.4              | 3.0               | −1.57  | 0.08            | 10.4   | 0.8             | 0.037 | 0.004            |
| 0.37              | 25.9              | 2.63              | −1.43  | 0.03            | 6.10   | 0.3             | 0.024 | 0.002            |
| 0.18              | 26.5              | 4.0               | −1.42  | 0.04            | 4.068  | 0.36            | 0.016 | 0.003            |

Note. The normalization $\Phi_{\text{S,C}}$ and its uncertainty $\Phi_{\text{unc,S,C}}$ are given in units of $10^{-4} \, Mpc^{-3}$, while the $\Phi_{\text{double}}$ and its uncertainty $\Phi_{\text{unc, double}}$ are given in units of $10^{-3} \, Mpc^{-3} \, dex^{-1}$. The characteristic SFR$_1$, and power-law break SFR$_{\text{break}}$ are given in $M_\odot \, yr^{-1}$, while the CSFRD is given in units of $M_\odot \, yr^{-1} \, Mpc^{-3}$. The integrations of the CSFRDs occur between 0.01 to 1000 $M_\odot \, yr^{-1}$ consistently for all redshifts because these limits do not require extending our SFRFs to regimes where there are no data.
4. The discrepancies at the highly star-forming end result in differences between the inferred CSFRD$_{\text{UV,IR}}$ and CSFRD$_{\text{UV,corr}}$ evolutions (top right panel of Figure 2) from 0.3 dex to 0.5 dex.

In conclusion, both methods agree on a constant CSFRD at $z \sim 1–4$ (plateau) and then a decrement at lower redshifts. We expect that the constraints given above can be used in comparisons with cosmological simulations, like previous measurements (Katsianis et al. 2017b; Zhao et al. 2020; Lovell et al. 2021b). However, due to the severe differences of the indicators for the highly star-forming end, we can safely state that they cannot both be describing the same galaxy formation and evolution scenario in terms of SFRs. We note that IR SFRs/CSFRDs are not probing necessarily obscured SFR, as is adopted by numerous studies (e.g., Rowan-Robinson et al. 2016; Gruppioni et al. 2020). They just have higher values than other measurements. We test the legitimacy of the derived UV$_{\text{corr}}$-SFRFs and UV$+\text{IR}$ SFRFs with respect to the independent measurement of the stellar mass density (SMD) from $z \sim 0–8$. However, the main goal for the next sections is to determine a parameterization for both the UV$+\text{IR}$ and UV$_{\text{corr}}$ CSFRDs.

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Figure 2. Top left panel, middle left panel, and bottom left panel: Evolution of the UV SFRF at $z \sim 4–9$, $z \sim 1–4$, and $z \sim 0–1$, respectively. Top right panel: Evolution of the CSFRD from the integration of the SFRFs present in Figure 1. Middle right panel and bottom right panel: Evolution of the IR SFRF at $z \sim 1–4$ and $z \sim 0–1$, respectively. We recall that we adopt a Chabrier IMF (Chabrier 2003) and Kennicutt & Evans (2012) relations.

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We have to keep in mind that state-of-the-art SED modeling and radiative transfer simulations actually indicate that indeed, previous measurements/calibrations, especially those of IR, overestimated the derived SFRs (Katsianis et al. 2020; Leja et al. 2020).
3. The Star Formation Histories of Galaxies and the Cosmic Star Formation Rate Density

Several studies have focused on parameterizing the SFH of individual galaxies or/and the cosmic SFH with physics-free models (Abramson et al. 2016; Ciesla et al. 2017). The above functions are typically empirically motivated and have been described by the following parameterizations (Tacchella et al. 2018; Chattopadhyay et al. 2020):

1. Exponentially declining SFHs (McLure et al. 2018), in which the SF jumps from zero to its maximum value \( SFR_0 \) at some time \( T_0 \). After \( T_0 \), the SF declines exponentially with a timescale \( \tau \):

\[
SFR(t) = SFR_0 \times e^{-\frac{t}{\tau}}.
\]

Assuming \( T_0 = 0 \) and \( \lambda = 1/\tau \) produces the standard form of a negative exponentially declining SFH. Low values of \( \tau \) correspond to galaxies in which most of the stars were formed early on and in a short time, followed by a smooth decrease in SFR, while high values indicate a roughly constant SFR (Ciesla et al. 2017). However, the above parameterization is not appropriate at high redshifts, where SFHs of galaxies are expected to rise (Reddy et al. 2012; Carnall et al. 2019).

2. Delayed exponentially declining SFHs (Chevallard et al. 2019) are considered to be more realistic. After the first generation of stars are formed, some time is required for them to evolve and ultimately end their lives (via supernovae explosions in case of very massive stars). The ejection of material from the first-generation supernovae enrich the medium for the second generation of SF, but with a delay. The above can be written as

\[
SFR(t) = SFR_0 \times (t - T_0) \times e^{-\frac{t - T_0}{\tau}}.
\]

3. The Yang et al. (2013) SFHs of galaxies. In general, the evolution of the SFHs of central galaxies (which mostly drive the CSFRD) is governed by three processes: (1) its in situ SF; (2) the accretion of stars from satellite galaxies; and (3) its passive evolution (mass loss). Because the SFRs of central galaxies depend on halo mass and \( z \) in the model, this can be combined with the halo mass function (sum the individual SFRs of galaxies to generate the total), and the CSFRD can be described as

\[
CSFRD = \int_0^\infty SFR(M_{\text{halo}}, z) n(M_{\text{halo}}, z) dM_{\text{halo}},
\]

where \( SFR(M_{\text{halo}}, z) = SFR_{pk} \times e^{-\frac{\log(M_{\text{halo}})}{\sigma_{\text{pk}}}} \) and \( \sigma_{\text{pk}} \) describes the decay of the SFR with respect to the peak SFR_{pk}. This model has been found to perform well at low redshifts \( z < 2.0 \) since it is in good agreement with the observed CSFRD. However, it has some limitations:

(a) The model relies on the adopted stellar mass—halo mass relation to calculate the stellar mass of the central galaxy \( M_{*,1.0} \) at redshift \( z = 0.1 \) (Yang et al. 2012) and other functional forms to calculate the peak value \( SFR_{pk} = \left( \frac{M_{*,1.0}}{10^4 M_\odot \text{ yr}^{-1}} \right) \) of the SFHs within a halo and the redshift at which this peak occurs \( z_{pk} = \max\{a \times \log(M_{*,b}, 0)\} \). This involves multiple parameters, and the functional forms may not represent real galaxies.

(b) At \( z > 2.0 \), the model underpredicts the CSFRD with respect to observations.

The above functional forms, despite their successes, do not typically manage to model the early SFH or/and the position of the peak of SFR is offset with respect recent observations (Ciesla et al. 2017). In addition, as mentioned above, they are mostly physics-free parameterizations that have their roots in empirical motivation (Abramson et al. 2016) and sometimes require multiple parameters.\(^{12}\)

The most widely used way to describe the evolution of the SFR of the universe as a whole is by providing estimates of the CSFRD at various redshifts. These can be obtained by integrating LFs (UV, IR, Hα, and radio) or SF s (Madau & Dickinson 2014; Katsianis et al. 2017a). It is common practice to fit the CSFRD(\( z \)) and thus the evolution of the SFR of the universe by fitting the data with a model that summarizes the results. For example, Madau & Dickinson (2014) suggested that the cosmic SFH follows a rising phase, scaling as \( SFR(z) \propto (1 + z)^{-2.9} \) at \( z = 3-8 \), slowing down and peaking at \( z \sim 2 \), followed by a gradual decline to the present day, roughly as \( SFR(z) \propto (1 + z)^{-2.7} \). The above requires four parameters and can be written as

\[
CSFRD(z) = 0.015 \frac{(1 + z)^{-2.7}}{1 + [(1 + z)/2.9]^{3.6}} \frac{M_\odot \text{ yr}^{-1}}{\text{Mpc}^3}. \tag{11}
\]

Equation (11) has been widely used in the literature (Maniyar et al. 2018; Wilkins et al. 2019; Walter et al. 2020) and is represented by the solid red line in the top panel of Figure 2. An updated version is presented by Madau & Fragos (2017), which once again requires four parameters. Similar models have been suggested earlier by Cole et al. (2001) as

\[
CSFRD = \frac{a + b z}{1 + (z)^c}, \quad \text{but two questions arise. (1) Is it possible to decrease the number of parameters necessary to broadly fit the data? (2) Equation (11) represents a physics-free parameterization that is mostly empirical. Is there a physically motivated analytical form/fit that can reproduce the observations and at the same time connect its parameters to the properties of halos/galaxies?}
\]

We recall that in our work, in order to obtain the CSFRD(\( z \)), we integrate the UVcorr SFRFs (Schechter forms) and UV+IR SFRFs (double power forms) of Figure 1, adopting integration limits of \( SFR = 0.01-1000 M_\odot \text{ yr}^{-1} \). We summarize our results for the CSFRD in the top right panel of Figure 2 and Table 1. We adopt a parametric form for CSFRD(\( z \)) = \( \frac{C}{(1+z)^\nu} \times e^{-\frac{M_\odot}{Mpc}} \) that relies on three parameters, and the results are for the two indicators as follows:

\[
CSFRD_{UV+IR}(z) = \frac{29.1}{(1 + z)^{3.56}} \times e^{\frac{z}{0.7}} \frac{M_\odot \text{ yr}^{-1}}{\text{Mpc}^3}, \tag{12}
\]

\[
CSFRD_{UV,corr}(z) = \frac{79.48}{(1 + z)^{3.53}} \times e^{\frac{z}{0.7}} \frac{M_\odot \text{ yr}^{-1}}{\text{Mpc}^3}. \tag{13}
\]

The evolution given by Madau & Dickinson (2014) is in excellent agreement with our UVcorr CSFRDs for high redshifts (the measurements of the authors also rely on UV SFs via UV LFs). However, at lower redshifts, Madau & Dickinson (2014) employ a compilation of both IR and UV data. This combination causes the evolution described by the authors to have a sharper peak at \( z \sim 2 \) with respect our CSFRD_{UV, corr}(z).

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\(^{12}\) Some other widely used parameterizations with the same limitations are the double power law (Behroozi et al. 2013) and the log-normal (Gladders et al. 2013; Abramson et al. 2016) SFHs.
evolution (dashed black line) and actually slowly converges to IR data (dashed magenta line) at $z \sim 0$. As demonstrated in the previous subsection, UV$_{corr}$ and IR SFRFs are different distributions at the highly star-forming end, so we decide in contrast with Madau & Dickinson (2014) to investigate them separately. Focusing on the CSFRD$_{UV+IR}$ of our work, we see that we are in good agreement with the results of Gruppioni et al. (2020), which are represented by the open red circles in the top right panel of Figure 2.

4. The “Observed” Cosmic Star Formation Rate Density is Described by Two Parameters and a Function that Resembles a Gamma Distribution

In order to investigate more physically the evolution of the CSFRD, we perform the parameterization/fit with time (Gyr) in the left panel of Figure 3, while the CSFRD$_{UV+IR}$ is described as

$$\text{CSFRD}_{UV+IR}(T) = 0.10 \times T^{1.34} \times e^{-0.43 T} \frac{M_{\odot} \text{ yr}^{-1}}{\text{Mpc}^3},$$

(14)

where $T$ is time in Gyr, while for the UV-corrected data (solid black line of Figure 3), we have

$$\text{CSFRD}_{UV,corr}(T) = 0.037 \times T^{1.83} \times e^{-0.48 T} \frac{M_{\odot} \text{ yr}^{-1}}{\text{Mpc}^3}.$$  

(15)

The above form for the UV+IR data represents a hybrid of a power-law rise ($\sim T^{1.34}$) and an exponential decline ($\sim e^{-0.43 T}$). The decreasing amplitude via the exponential decline ($\exp(-CT)$, with a characteristic timescale $1/C$ in Gyr) seems that it originates in the fact that the gas supply to the inner galaxy is depleted with time and reflects gas consumption timescales (Dekel & Mandelker 2014; Burkert 2017). Bellow we present how this hybrid form emerges for the CSFRD:

A system that is isolated (a closed-box model scenario) has an initial gas mass $P_0$ and no further accretion occurs within it. It is expected to have a gas density evolution of $P_{\text{gas}} = P_0 \times e^{-b_0 t}$, where $b_0 = \epsilon_{\text{SFR}} \frac{\text{time fall}}{\text{time}}$, $\tau_{\text{free fall}}$ the free-fall timescale and $\epsilon_{\text{SFR}}$ the SF efficiency per free-fall time. This depletion is the result of gas consumption due to SF, which occurs at a rate of $\text{SFR} = b_0 \times P_0 \times e^{-b_0 t}$. All the above reflects the empirical Schmidt law (Schmidt 1959; Schaye & Dalla Vecchia 2008) that relates the SFR and gas via $\text{SFR} = b_0 \times M_{\text{gas}}$. The parameters that govern the decline in Equations (14) and (15) imply timescales of $1/0.43 \text{ Gyr} \sim 2.5 \times 10^9 \text{ yr}$ for the case of CSFRD$_{UV+IR}$ and $1/0.48 \text{ Gyr} \sim 2 \times 10^9 \text{ yr}$ for the case of CSFRD$_{UV,corr}$ respectively. These values are typically found in observations of individual galaxies (Kennicutt 1998; Bigiel et al. 2011; Davis et al. 2015, $T = 1-3 \times 10^9$) and semi-analytic models (Shamshiri et al. 2015, $T = 5 \times 10^3 \text{ yr}$), so it is a good start that our observations of the CSFRD find a similar value. As noted above (Section 3), the exponential declining scheme alone is not successful at reproducing the rising SFHs of individual high-redshift galaxies. Similarly to the individual SFHs, the CSFRD is anticipated to increase with time at $z \geq 0$. This era has been labeled the “gas accretion” epoch, and although there have been different ways to parameterize this behavior, the most straightforward way is by a simple power law ($\text{SFH}(T) = A \times T^\beta$), following Papovich et al. (2011) and Dutton et al. (2010). The value of $B$ has been found to be $\sim 1.5$ by Shamshiri et al. (2015, semi-analytic models), $\sim 1.7$ by Papovich et al. (2011, high-redshift ($z \sim 6-3$) observations), and $\sim 3.5$ by Behroozi et al. (2013). We find $B = 1.34$ for the case of CSFRD$_{UV+IR}$ (Equation (14)) and $B = 1.83$ for the case of CSFRD$_{UV,corr}$ (Equation (15)). We have to note that the hybrid form of the above processes (power law rise + exponential declining), i.e., $\text{CSFRD}(T) = A \times T^\beta \times e^{cT}$, has not been commonly used to parameterize the observed CSFRD for the whole history of the universe ($z \sim 0$–9), despite its success in describing our data. We can see the parameterization/model as a more general case of the delayed exponential declining SFH ($\text{SFR}_0 \times t \times e^{cT}$), where this time, the rising/delaying part $\text{SFR}_0 \times t$ has an extra parameter $B$. This parameter sets how quickly the SFR rise at the accretion era, and because the gas is finite, it sets the limit for the exponential part to take over. For $B = 1$, we have the simple case of delayed exponential declining SFH. We will gain more insight into the role of these parameters in the following paragraphs.

Interestingly, if we fit the UV+IR data with two parameters, we see that Equation (14) is almost identical (see Figure 3) to...
the following form:

$$\text{CSFRD}_{\text{UV+IR}}(T) \approx 0.44^{2.44} \frac{\Gamma(2.44)}{\Gamma(a)} \times T^{2.44-1} \times e^{-0.44T} \frac{M_\odot \text{yr}^{-1}}{\text{Mpc}^3},$$

(16)

where $T$ is time in Gyr. The above is represented by the dotted magenta line in Figure 3 and deviates from the actual fit with three parameters by only 0 to 0.05 dex. This is a function that closely resembles the formula of a gamma distribution, which has the following behavior:

$$\text{Gamma}(a, b) = \frac{b^a}{\Gamma(a)} \times \frac{\Gamma(a, bT)}{\Gamma(a)} \times e^{-bT}$$

(17)

where $a$ is the shape parameter and $b$ is the scale parameter. We show below that the UV$_{\text{corr}}$ CSFRD follows a similar behavior. The scale $b$ parameter describes the rate $(1/b)$ at which the process occurs and has the effect of stretching or compressing the range of the distribution. The parameter $a$ determines its skewness. The gamma distribution is commonly used to model stochastic processes, and an example is the reliability analysis, where partial failures (which are gamma distributed) have to occur before the item completely fails. The parameter $a$ describes the number of events necessary to generate the complete failure. A natural question that arises is how the dependence of the CSFRD on time is dictated by this elegant mathematical formula. Why is the normalization of Equation (14) in the end just a product of $a$ and $b$? We reproduce the above by a simple analysis inspired by a bathtub/equilibrium model that follows the evolution of the gas/SF in Section 5 and produce a physical motivated fit for the CSFRD with the parameters $a$ and $b$ being determined by halo/galaxy properties.

4.1. A Comparison Between the UV$_{\text{corr}}$/UV+IR CSFRDs and Stellar Mass Density Evolution

Before studying the emergence of the above delicate form, we can start to see its physical meaning, while we have to note that a huge advantage in modeling processes through gamma functions is that any related mathematical calculations become straightforward and can be calculated analytically. This is the reason why both the gamma function and the gamma distribution find numerous applications from biology to engineering (Maghsoodloo 2014; McCombs & Kadelka 2020; Vazquez 2020). For example, integrating the above and multiplying it by a factor of $(1 - R)$, where $R = 0.41$ is the return fraction for a Chabrier (2003) and Madau & Dickinson (2014) IMF (i.e., the mass fraction of each generation of stars that is instantly returned to the interstellar/intergalactic medium), gives us the build-up of the stellar mass in the universe.$^{13}$

$$\text{SMD}_{\text{UV+IR}}(T) = (1 - R) \times \frac{b^a}{\Gamma(a)} \times \int_0^T T^{a-1} \times e^{-bT} dt = (1 - R) \times \frac{\Gamma(a,bT)}{\Gamma(a)} \times 10^9 \frac{M_\odot}{\text{Mpc}^3},$$

(18)

where $t$ is time in years, and $\gamma(a, bT)$ is the lower incomplete gamma function. The above is represented by the dashed magenta line in the right panel of Figure 3. So UV+IR SFRs indicate a build-up of cosmic stellar mass that occurs following an incomplete gamma function that is normalized by the total gamma function (i.e., the regularized gamma function). The ultimate fate of the stellar mass of the universe at $T = \infty$ is according to Equation (18), $\text{SMD}_{\text{UV+IR}}(\infty) \sim 0.59 \times 10^9 M_\odot \text{Mpc}^{-3}$ (because at $T = \infty$, $\gamma(a, bT) / \Gamma(a) = 1$), while the stellar mass density grows following gamma-distributed increments. Mathematically, it appears to be that partial stochastic events occur before the final event and total collapse of the system. The open black squares in the right panel of Figure 3 represent the observational studies of Bielby et al. (2012), Ilbert et al. (2013), Muzzin et al. (2013), Madau & Dickinson (2014), and Driver et al. (2018; filled blue circles) that are obtained through the SED fitting technique. The solid magenta line represents the evolution of the stellar mass density, integrating the three-parameter function (Equation (14)), which closely follows the result from the two-parameter function. We note that the UV+IR SFRs have a discrepancy with respect stellar masses by $\sim 0.3$ dex at $z \sim 0$, while this increases to $\sim 0.5$ dex at higher redshifts ($z \sim 2$). What we see is the emergence once again of the long-standing factor of 2–3 disagreement between CSFRD and stellar mass density (Leja et al. 2015; Davidson et al. 2018). However, as mentioned above, Katsianis et al. (2020) used EAGLE combined with the radiative transfer code SKIRT (Baes et al. 2020; Trčka et al. 2020) and demonstrated that numerous UV+IR calibrations that are used to determine galaxy SFRs (Heinis et al. 2014; Whitaker et al. 2014; Tomczak et al. 2016) typically overestimate the results at high redshifts. In addition, Leja et al. (2019b) inferred via prospector SFRs that are lower by 0.1–1 dex with respect to SFR$_{\text{UV+IR}}$ measurements (Whitaker et al. 2014). The authors suggest that this is due to the inclusion of additional physics (e.g., light from old stars). In this work, we show that the observed UV+IR SFRs and observed stellar masses are compared and are found to be in disagreement. Stellar mass is an accumulating property that is highly dependent on the past (i.e., high redshifts). Overestimating the SFRs at early epochs results in an overestimation of the stellar mass density for the entire history of the universe, including $z \sim 0$, which is a widely accepted constraint that has been commonly employed by the community.$^{14}$ We note that our result is independent of the parameterization of our model (which just facilitates the procedure of calculating the stellar mass density). But what about the UV$_{\text{corr}}$, SFRFs and CSFRD?

$^{13}$In addition, it is common practice in the literature to adopt a constant return fraction $R$ with respect to time (Muñoz-Mateos et al. 2007; Madau & Dickinson 2014; Grazian et al. 2015; Walter et al. 2020). The mass fraction is not returned instantly. Instead, the return fraction is sensitive to the age of the stellar population and is a function of time. For example, Yu & Wang (2016) employed the flexible SPS (FSPS) code (Conroy & Gunn 2010) to derive an evolving return fraction $R(t)$. The authors demonstrated that the SFRM derived from the CSFRD using the evolving $R(t)$ instead of the constant $R$ is higher with increasing redshift by up to a factor of 1.2 at $z = 8$.

$^{14}$The GSMF (integration of which produces the SMD) is usually the key calibration for most galaxy formation models (Leauthaud et al. 2011; Schaye et al. 2015; Wright et al. 2017; Pillepich et al. 2018) because it is considered robust.
Following the same steps with the CSFRD_{UV+IR}, this time we explore the evolution of the UV dust-corrected SFRs (black line in the right panel of Figure 2), which can be written in the form:

\[
\text{CSFRD}_{\text{UV,corr}} \approx 0.50 \times 0.50^{0.9} \frac{\Gamma(2.9)}{\Gamma(2.9)} \times \tau^{2.9-1} \\
\times e^{-0.50 \tau} \frac{M_\odot \text{yr}^{-1}}{\text{Mpc}^3},
\]

which is shown in the left panel of Figure 3, which once again requires only two parameters. Integrating it gives us the evolution of the cosmic stellar mass density according to the UV + dust-corrected SFRs as follows:

\[
\text{SMD}_{\text{UV,corr}}(T) = (1 - R) \times b \times \frac{b^a}{\Gamma(a)} \times \int_0^t \tau^{a-1} \\
\times e^{-b \tau T} dt = (1 - R) \times b \times \frac{\gamma(a, b \tau T)}{\Gamma(a)} \times 10^9 \frac{M_\odot}{\text{Mpc}^3}.
\]

The above is represented by the dotted black line in the right panel of Figure 2. The solid black line of the same form:

\[
\text{SMD}_{\text{UV}}(T) = (1 - R) \times b \times \frac{b^a}{\Gamma(a)} \times \int_0^t \tau^{a-1} \\
\times e^{-b \tau T} dt = (1 - R) \times b \times \frac{\gamma(a, b \tau T)}{\Gamma(a)} \times 10^9 \frac{M_\odot}{\text{Mpc}^3}.
\]

1. The observed UV+IR SFRs imply cosmic SFR densities that if integrated with time result in a stellar mass density evolution that is inconsistent with other measurements that are derived through SED fitting. These measurements involve the whole history of the universe, including the GSMF at \(z \sim 0\), which represents constraints for numerous models and a cornerstone for extragalactic astrophysics. The tension increases at high redshifts by even up to 0.5 dex, re-creating the long-standing factor of 2–3 disagreement. However, the case is different for the CSFRD derived from SFR_{UV}. We suggest that the success of the indicator at high redshifts, also confirmed by simulations/radiative transfer, where dust attenuation effects are less severe, leads to a good agreement between the observed CSFRD derived from the dust-corrected UV data and SMD. The above agreement represents a suggestion for the long-standing problem of the tension between observed CSFRDs and SMD, which seems to be actually the result of the uncertainties of SFR indicators and not a problem related to the theory of galaxy formation.

5. A Simple Physical Model that Reproduces the Observed CSFRD and Links its Parameters to the Physics of Halos and Galaxies

There has been considerable effort in cosmological simulations (Tescari et al. 2014; Crain et al. 2015; Lagos et al. 2018; Pillepich et al. 2018; Davé et al. 2019) and theory (Yang et al. 2013; Dekel & Mandelker 2014; Peng & Maiolino 2014; Sharma & Theuns 2020) to create galaxies that resemble those we observe. The CSFRD has always been one of the key observables that constrain the above. Below we demonstrate how the gamma CSFRD presented in this work is obtained using a simple model.

We start with the SFR within galaxies. We assume that the total baryonic mass is conserved and is separated between the available gas for SF within the galaxies (\(M_{\text{SF,T}}\)), stellar mass (\(M_{\ast}\)), outflows (\(M_{\text{out}}\)), and inflows (\(M_{\text{in}}\)). Star-forming galaxies in simulations are usually seen to lie near the equilibrium condition (Davé et al. 2012; Peng & Maiolino 2014), where SF is sustained by the inflowing gas \(M_{\text{in}}\), thus we assume the same for our simple model. We divide by volume (labeled Vol) in order to work with densities, and all the above can be written as

\[
\frac{dP_{\text{in,t}}}{dt} = \frac{P_{\text{out,t}}}{dt} + \frac{P_{\ast,t}}{dt},
\]

where \(P_{\text{in,t}} = \frac{M_{\text{in}}}{\text{Vol}}\), \(P_{\text{out,t}} = \frac{M_{\text{out}}}{\text{Vol}}\), and \(P_{\ast,t} = \frac{M_{\ast}}{\text{Vol}}\). Star-forming galaxies fluctuate around this relation, but are generally driven back to it on short timescales. We assume that the outflows are dependent on the SFR, thus the SFR density, labeled SFRD, is related to \(P_{\text{out,t}}\) as \(\frac{dP_{\ast,t}}{dt} = n \times \text{SFRD}\), where n is the mass-loading factor (Springel & Hernquist 2003; Barai et al. 2013; Puchwein & Springel 2013; Katsianis et al. 2017a; Lopez et al. 2020; Tejos et al. 2021) and \(P_{\ast,t} = (1 - R) \times \text{SFRD}\). Thus, the SFRD is related to the inflows \(P_{\text{in,t}}\) as

\[
\text{SFRD} = \frac{dP_{\text{in,t}}}{dt} \frac{1}{1 + n - R}.
\]

The inflowing baryonic gas is assumed to scale with the dark matter halo growth as \(\frac{dM_{\text{in}}}{dt} = f_{\text{gal}} \times f_b \times \frac{dM_{\text{Halo}}}{dt}\), where \(f_b\) is the cosmic baryon fraction and \(f_{\text{gal}}\) is the fraction of the incoming baryons that they are able to penetrate and reach the galaxy and be used for SF (Peng & Maiolino 2014). \(f_{\text{gal}}\) is equivalent to the accretion efficiency defined by Bouche et al. (2010), the preventive feedback parameter of Davé et al. (2012), and the penetration parameter \(p\) presented by Dekel & Mandelker (2014). Thus, Equation (22) can be written as

\[
\text{SFRD} = \frac{f_{\text{gal}} \times f_b}{1 + n - R} \times \frac{dP_{\text{Halo}}}{dt}.
\]
following:

\[ A \times \int_0^t \tau^{a-1} \times e^{-b \tau} \, d\tau = f_{\text{gal}} \times f_b \]

\[ \times P_{\text{C,Halo},t} \times \frac{1}{1 + n - R}. \quad (24) \]

Integrating up to \( t = \infty \), we have

\[ A = \frac{b^a}{\Gamma(a)} \frac{f_{\text{gal}} \times f_b \times P_{\text{C,Halo},\infty}}{1.0^9 \times (1 + n - R)}. \quad (25) \]

The above just reflects the endgame for the whole baryonic gas of the universe that is mostly accreted to halos and trapped in stars and stellar remnants (except for the \( 1 - f_{\text{gal}} \) fraction). We assume \( f_b = 0.157 \), all the eligible \( f_{\text{gal}} \) dark matter at \( \infty \) has been accreted in dark matter halos, so \( P_{\text{C,Halo},\infty} = \Omega_{\text{Matter}} \times P_{\text{crit}} \), where \( P_{\text{crit}} \) is the critical density of the universe \((3 H_0^2)/8\pi G)\) and \( \Omega_{\text{Matter}} \sim 0.315 \) is the matter density parameter. Thus, substituting \( P_{\text{C,Halo},\infty} = 0.315 \times \frac{3 H_0^2}{8 \pi G} \) \( = 0.315 \times 1.245 \times 10^{11} \frac{M_\odot}{Mpc^3} \), the mass-loading factor \( n = 2^{15} \), and the return fraction \( R = 0.41 \), the \( A \) parameter can be written as

\[ A = \frac{b^a}{\Gamma(a)} \frac{f_{\text{gal}} \times f_b \times \Omega_{\text{M}} \times P_{\text{crit}}}{(1 + n - R)} \]

\[ = \frac{b^a}{\Gamma(a)} \frac{0.0494 \times 1.245 \times 10^{11}}{10^9 \times (1 + 2 - 0.41)} \]

\[ = \frac{b^a}{\Gamma(a)} \times f_{\text{gal}} \times 2.37. \quad (26) \]

where \( f_{\text{gal}} \) is considered to have a value of \( \sim 0.5 \) (Dekel & Mandelker 2014; Peng & Maiolino 2014). Davé et al. (2012) suggest that the preventive parameter \( f_{\text{gal}} \) is halo dependent, while Katsianis et al. (2017b) has demonstrated using EAGLE that most of the CSFRD below redshift 5 occurs in halos between \( 10^{11} M_\odot \) and \( 10^{12} M_\odot \), so we can focus our analysis on this halo range. In this mass regime, values of 0.2 to 0.5 are expected for the \( f_{\text{gal}} \) parameter, according to Davé et al. (2012). A realistic value of \( f_{\text{gal}} = 0.45 \) or \( 0.5 \) that is mostly used in the literature would result in a value of \( A \sim 2.37 \) and be in perfect agreement with the relation (16), the UV+IR CSFRD, and its gamma distribution evolution. A lower value of \( f_{\text{gal}} \sim 0.25 \) would result in the relation implied by the CSFRD that relies on dust-corrected UV data. \( f_{\text{gal}} \) has a maximum of 1, so it is anticipated that the parameter \( A \) has a maximum of \( \frac{b^a}{\Gamma(a)} \times 2.37 \). We note that both values \( f_{\text{gal}} = 0.45 \) and \( f_{\text{gal}} = 0.25 \) are within the constraints from cosmological simulations. These values explain the emerging forms for the CSFRD of Equations (16) and (19) and result in an evolution for the SFR density that can mostly be described by two parameters, despite the fact that numerous physical processes are involved. We note that the three-parameter equation is physically motivated, while the two-parameter fit occurs because the parameters of
described as CSFRD(T) = \( \frac{f_{\text{gal}} \times f_b \times f_{\text{UV corr}} \times P_{\text{crit}}}{(1 + n - R)} \times b \times e^{-b T} \times \frac{M_\odot}{Mpc^{3}} \)

\[ \times T^{a-1} \times e^{-b \tau} \times \frac{M_\odot}{Gyr} \times \frac{Mpc^3}{M}. \]

For \( a = 1 \), we have an exponentially declining SFH with an outflow mass-loading factor \( n = 2 \). We note that for individual halos, a more appropriate assumption would be to associate outflows with the circular velocity, i.e., \( n = 2 \times \sqrt{\frac{500 \text{Km/s}}{2 \times V_c}} \) (Springel & Hernquist 2003; Barai et al. 2013; Puchwein &Springel 2013; Katsianis et al. 2017a).
2. he observed CSFRD can be described by three parameters \((C \times T^{-a-1} \times e^{-b \cdot T})\) and follows a hybrid of a power-law rise and an exponential decline. Individual galaxies have been seen to follow this parameterization broadly via what is commonly labeled the delayed SF history model. Exploring the CSFRD further, we see that it is mostly governed by two parameters and a function that resembles a gamma distribution, 
\[
\left( \frac{b^a}{\Gamma(a)} \times T^{-a-1} \times e^{-b \cdot T} \right) \times \left( \frac{b^a}{\Gamma(a)} \times T^{-a-1} \times e^{-b \cdot T} \right)
\]
3. We support that the parameterization of the CSFRD with two to three parameters is interesting because it reflects physics related to galaxy formation. In contrast to previous efforts, our framework connects the parameters to physical properties (SFFR depletion times, cosmic baryonic gas density, penetration parameter from the halo to the galaxy, etc.). The build-up of stellar mass in our model occurs in \((\Gamma(a, b))\) distributed steps and is the result of gas consumption up to the limit that there is no available gas at \(t = \infty\).
4. The final value of the cosmic stellar mass density is \(\sim 0.5 \times 10^9 \text{M}_{\odot}\) \(\text{Mpc}^{-3}\) at \(t = \infty\). We are approaching this value, so we can infer that most of the stars that were “supposed” to be born from the available baryonic gas have already been born.
5. The observed UV+IR SFRFs imply cosmic SFR densities that if integrated with time, result in a stellar mass density evolution from \(z \approx 0-9\) that is inconsistent with measurements that are obtained through SED fitting. The tension increases at high redshifts by even up to 0.5 dex. This could be seen as the long-standing problem of the tension between observed CSFRDs and stellar mass densities. Does it reflect the fact that high-redshift SFRs derived from UV+IR calibrations are overestimated? The above is supported by radiative transfer simulations and SED fitting techniques (Katsianis et al. 2020). On the other hand, the SFR_{UV,corr} indicator produces SFRFs and a CSFRD evolution that is actually consistent with the stellar mass density evolution, and the problem of the long-standing tension can be solved.

We are obligated to stress once again that any observational techniques (including those employed in this work) rely on assumptions and calibrations that can be incomplete, so we caution any other author to avoid treating our assumptions and calibrations that can be incomplete, so we work to be done both in terms of data and modeling until we have already been born. The long-standing tension can be solved.
