Study of deformed quasi-periodic Fibonacci two dimensional photonic crystals

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Abstract. Quasi-periodic photonic crystals are not periodic structures. These structures are generally obtained by the arrangement of layers according to a recursive rule. Properties of these structures make more attention the researchers especially in the case when applying defects. So, photonic crystals with defects present localized modes in the band gap leading to many potential applications such light localization.

The objective of this work is to study by simulation the effect of the global deformation introduced in 2D quasiperiodic photonic crystals. Deformation was introduced by applying a power law, so that the coordinates y of the deformed object were determined through the coordinates x of the non-deformed structure in accordance with the following rule: y = x^{1+k}. Here k is the coefficient defining the deformation. Therefore, the objective is to study the effect of this deformation on the optical properties of 2D quasiperiodic photonic crystals, constructed by Fibonacci generation. An omnidirectional mirror was obtained for optimization Fibonacci iteration in a part of visible spectra.

1. Introduction:

Photonic crystals can be periodic and/or quasi periodic structures which are characterized by photonic band gaps (PBG). These structures can be in 1D, 2D or 3D dimensions. One more attractive property of photonic crystals is 2D configuration due to the inherent simplicity of calculation, visualization, and understanding in 2D. [1]

For example, two-dimensional (2D) photonic crystals can be integrated with existing planar optical waveguide technology [2]. So, two classes of two-dimensional photonic crystal are considered. The first is a photonic crystal formed by a square lattice of dielectric columns of dielectric constant $\varepsilon$, in a different dielectric background of dielectric constant $\varepsilon_p$. The second is a photonic crystal made by arranging the dielectric columns in a triangular lattice. One particularly interesting aspect of these systems is the possibility of creating crystal defects that confine light in localized modes [1].

Quasi-periodic crystals named quasi crystals were discovered by D. Shechtman et al [3], and exhibit unique electronic properties. It has been noticed that one advantage of these aperiodic structures is to present many inequivalent sites, and consequently many possible different defects [4, 5]. A significant property of photonic quasi crystals is that total band gaps can be obtained in the presence of sufficiently high statistical symmetry [6, 7], even if the value of the minimal index contrast has been subject of controversy.[3]
The most important and well-known quasi periodic structure is the Fibonacci sequence (FS) [8, 9]. In this paper we study photonic band gap (PBG) characteristics of deformed 2D quasi-periodic crystal which is constructed according to the Fibonacci sequence so that the coordinate y of the deformed object were determined through the coordinate x of the Fibonacci stack in accordance with the following rule: \(y = x^{k+1} \). \(k\) is the deformation degree. We assume all the regions to be linear, homogenous, no absorbing and we will show that the PBG depends on the Fibonacci iteration, the degree of deformation and the reference wavelength.

2. Model Description:
HFSS (High Frequency Simulator Structure) is a simulator of high-performance which models and feigns in 3D the global fields shone by the structures. The program is based on Finite Element Method which consists to solving the Maxwell equations.

Global defect is done by a power law \(y = x^{k+1} \). \(k\) is the coefficient defining the deformation or the asymmetry degree into the structure. So that 3 states can be occurred: \(k>0\), \(k<0\) and \(k=0\).

Fibonacci structures are the paradigm of quasi-periodic order [9]. In two spatial dimensions, they can be generated by combining two different materials from a seed letter A or B by applying two elementary one-dimensional Fibonacci sequence maps:

\[f_A: A \rightarrow AB, B \rightarrow A\]
\[f_B: A \rightarrow B, B \rightarrow BA\]

![Figure 1](image)

**Figure 1.** First three generations of 2D Fibonacci sequence shown along with the inflation rules,

In our study, A and B are two elementary materials, with high (TiO\(_2\)) and low refractive index (air). The refractive index of the study structure are taken as \(n_H = 5.29\) and \(n_L = 1\).

Figure 3 shows 2D quasi-periodic system for the 3\(^{rd}\), 4\(^{th}\), 5\(^{th}\) and 6\(^{th}\) generation according to Fibonacci model.
3. Results and discussions:
To study the effect of deformation degree $k$ on the optical response of our system we start by modeling the 2D structure and then we launch simulation on the computer.

3.1 Effect of generation number of Fibonacci sequence:
To see the effect of generation number of Fibonacci sequence on the Photonic Band Gap, we proceed by increasing the iteration order ($N$) of this structure for the asymmetry degree $k=0$. So that we determined the reflectance for $N=4, 5$ and 6. The figure 3 shows the evolution of the structure response according to the number of Fibonacci generation.

![Figure 3. Variation of the reflectance spectrum according to the number of iteration of Fibonacci sequence for asymmetry degree $k=0$.](image)

From the figure 3 we can see that by increasing the number of Fibonacci sequence ($N$) an improvement in optical properties was observed. In fact we note that when $N<5$ there is no BIP. But from $N=5$ a Photonic Band Gap appears and an increase in width was observed. For $N=5$ the photonic band Gap cover the domain $(0.93...1.05) \mu m$ for 99% of reflection. This domain becomes $(0.89...1.09) \mu m$ for the 6th Fibonacci generation. The PBG width passes from $0.12 \mu m$ to $0.29 \mu m$. This number can be improved if we increase the number of Fibonacci sequence but it requires high computer performance, the reason why we will stop our research at the 6th Fibonacci generation.
3.2 Effect of asymmetry degree:
As it is mentioned, we use a 2D quasi-periodic structure obtained by applying the Fibonacci sequence for the 6th generation. The effect of deformation degree \(k\) on the BIP is given by modifying the values of \(k\). We will take negative and positive values of deformation degree and we will note the effect on the forbidden gap behavior with reference wavelength \(\lambda_0=0.6\mu\text{m}\).

The Figure 4 shows reflection spectra according to wavelength for different values of \(k\).

**Figure 4.** Reflection spectra according to wavelength while varying deformation degree \(k\) for the 6th generation of Fibonacci sequence applied on 2D structure of \(\text{TiO}_2\) on air for reference wavelength \(\lambda_0=0.6\mu\text{m}\)

The figure above shows that the photonic Band Gap widens gradually according to the deformation degree. When \(k=0\) the total bandwidth \((\Delta\lambda=\lambda_{\text{long}}-\lambda_{\text{short}})\) is 0.209 \(\mu\text{m}\) this value is increasing according to \(k\) and it reach 0.386 \(\mu\text{m}\).

The table 1 confirms the results above.

| \(K\)  | -0.1 | -0.05 | 0    | 0.03 | 0.05 | 0.1 |
|--------|------|-------|------|------|------|-----|
| \(\Delta\lambda(\mu\text{m})\) | 0.289 | 0.232 | 0.209 | 0.204 | 0.264 | 0.386 |

**Table 1.** Improvement of the PBG width for the 2D quasi-periodic structure according to deformation degree \(k\)

3.3 Effect of reference wavelength:
For studying the impact of the reference wavelength \(\lambda_0\) on the forbidden band we choose different values and we observe the response. The results are reported in figure 5.
Figure 5. Reflection response according to reference wavelength $\lambda_0$ for a deformed 2D quasi-periodic structure $k=0.1$.

Figure 5 shows that there is amelioration of the PBG behavior when we raise the reference wavelength. Indeed for $\lambda_0=300\mu$m the PBG shows some transmission peaks whose decreases when $\lambda_0=500\mu$m.

4. Conclusion

We have shown that the use of quasi-periodic photonic structures in the 2D configuration, and in the case where these structures are deformed as described above shows an improvement in their optical responses. Indeed, photonic band gap has been demonstrated dependent on the rank of the iteration, the reference wavelength, and the degree of deformation $k$. We varied these parameters in order that the BIP cover a wide band in the optical domain. The model that is based on the method of finite elements seems effective for extracting optical properties of 2D photonic crystals.

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