F-term moduli stabilization and uplifting

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Received March 17, 2019; Revised July 9, 2019; Accepted July 20, 2019; Published September 16, 2019

We study Kähler modulus stabilization in type IIB superstring theory. We propose a new moduli stabilization mechanism by the supersymmetry breaking chiral superfield which is coupled to Kähler moduli in the Kähler potential. We also study its uplifting of the Large Volume Scenario (LVS). In both cases, the form of the superpotential is crucial for moduli stabilization. We confirm that our uplifting mechanism does not destabilize the vacuum of the LVS drastically.

Subject Index B11, B29, B41

1. Introduction

Superstring theory is a promising candidate for a quantum theory of gravity. Also, it is a good candidate for a unified theory of all the gauge interactions and matter particles such as quarks and leptons, as well as the Higgs particle. Superstring theory predicts six-dimensional (6D) compact space in addition to the four-dimensional (4D) space-time. From the theoretical and phenomenological viewpoints, moduli stabilization of the 6D compact space is one of the most serious problems. Without moduli stabilization, we cannot determine the parameters of the 4D low-energy effective field theory of superstring theory, including the Kaluza–Klein scale, the supersymmetry (SUSY) breaking scale, gauge couplings, Yukawa couplings, and so on. (For phenomenological aspects of superstring theory, see Refs. [1,2] and references therein.)

In the mid 2000s, several moduli stabilization mechanisms were proposed. Among them, the Kachru–Kallosh–Linde–Trivedi (KKLT) scenario [3] and the Large Volume Scenario (LVS) [4,5] are two well-known mechanisms in type IIB superstring theory. In such scenarios of type IIB superstring theory, moduli stabilization is carried out in three steps. First, background three-form fluxes are turned on, and they induce a superpotential for the dilaton and complex structure moduli stabilization [6]. Second, some corrections, such as \(\alpha'\) corrections, string one-loop corrections, and non-perturbative corrections, are introduced. They generate a potential including Kähler moduli and stabilize them. The potential minimum is an anti-de Sitter (AdS) vacuum. Finally, a source of SUSY breaking such as anti-D-branes is introduced and the vacuum energy is uplifted to the Minkowski (or de Sitter) vacuum. The KKLT scenario and the LVS have been actively investigated since they can realize de Sitter vacua in controllable schemes.

In the second step, both the KKLT scenario and the LVS make use of non-perturbative effects, such as gaugino condensations and D-brane instanton effects, to stabilize the Kähler moduli. However, there is no reason why the non-perturbative effects behave as the leading-order contribution.
In this paper we propose a new Kähler modulus stabilization mechanism. We study the modulus potential from the Kähler potential with \( \alpha' \) corrections and the superpotential with a chiral superfield \( X \) which spontaneously breaks SUSY. When \( X \) couples to the Kähler modulus in the Kähler potential, it effectively generates a modulus potential. We show that when the modulus dependence satisfies certain conditions, the Kähler modulus can be stabilized. However, the vacuum energy in our model is positive definite for a non-vanishing vacuum expectation value (VEV) of the superpotential, and it is quite large for a natural VEV of the superpotential compared with the cosmological constant. This is because the chiral superfield \( X \) uplifts the vacuum energy. In order to realize the Minkowski vacuum, we need some effects to depress the vacuum energy. Indeed, instead of anti-D-branes, F-term uplifting by \( X \) has already been studied in the KKL T scenario [7–11]. Here, we also study F-term uplifting for the L VS by the chiral superfield \( X \).

This paper is organized as follows. In Sect. 2 we study a new model for Kähler modulus stabilization by the chiral superfield. There, we consider the Kähler potential, where the chiral superfield couples to the Kähler modulus. In Sect. 3 we study another scenario for uplifting the AdS vacuum of the L VS by the chiral superfield. Section 4 is devoted to conclusion and discussion.

2. Kähler moduli stabilization

In this section we study the moduli stabilization mechanism by the SUSY breaking chiral superfield. We consider IIB flux compactification: type IIB superstring theory compactified on a Calabi–Yau three-fold with background three-form fluxes. The theory has several types of moduli fields. They are classified into three types: the dilaton \( S \), complex structure moduli \( U_\alpha \), and Kähler moduli \( T_i \), where \( \alpha \) and \( i \) represent indices of a \((1, 2)\)-cycle and a \((1, 1)\)-cycle respectively [12]. Their effective theory is described by supergravity. The scalar potential is given by

\[
V = e^{K/M_P^2} \left( \sum_{I,J} K^{IJ} D_I W D_J \bar{W} - 3 \frac{|W|^2}{M_P^2} \right),
\]

where \( K \) and \( W \) are the Kähler potential and the superpotential, respectively, and \( M_P \) denotes the 4D reduced Planck mass. \( K^{IJ} \) is the inverse of \( K_{IJ} = \partial^2 K / \partial \phi_I \partial \phi_J \), and \( D_I W \) is the covariant derivative:

\[
D_I W = \frac{1}{M_P^2} W \partial K / \partial \phi_I + \partial W / \partial \phi_I,
\]

where \( \phi_I \) represent scalar components of all the chiral superfields that include the moduli fields. The three-form flux \( G_3 \) background induces the superpotential terms of the dilaton and the complex structure moduli [6]. The dilaton and the complex structure moduli are stabilized at the point satisfying \( D_{S,U_\alpha} W = 0 \). On the other hand, a potential for the Kähler moduli is not generated at the tree level. After integrating the dilaton and the complex structure moduli out, the Kähler potential for the Kähler moduli is given by

\[
K = -2M_P^2 \log (\mathcal{V}),
\]

where \( \mathcal{V} \) denotes the dimensionless volume of the compact space, and it is measured in units of the string length \( \ell_s = 2\pi \sqrt{\alpha'} \). From now on, to simplify the calculations we use units such that \( M_P = 1 \), but note that \( \mathcal{V} \) is still measured in units of the string length [4,5]. The volume \( \mathcal{V} \) is a function of the real part of the Kähler moduli, i.e.

\[
\mathcal{V} = \mathcal{V}(\tau_1, \tau_2, \ldots), \quad T_i = \tau_i + i\theta_i,
\]

where \( \tau_i \) denotes the dimensionless volume of the corresponding four-cycle. Moreover, after integrating the dilaton and the complex structure moduli out, the superpotential is a constant:

\[
W \big|_{S=\langle S \rangle, U_\alpha=\langle U_\alpha \rangle} = \int \Omega \wedge G_3 \big|_{S=\langle S \rangle, U_\alpha=\langle U_\alpha \rangle} \equiv W_0. \quad \text{Here, } \Omega \text{ is the holomorphic three-form.}
\]
Suppose that there is a single Kähler modulus $T$ and the whole volume is given by

$$V = (T + \bar{T})^{3/2}.$$ 

In this setup the tree-level potential is calculated such that

$$V = \frac{1}{y^2} \left[ \frac{1}{3} V^{4/3} (-3) V^{-2/3} - 3 \right] |W_0|^2 = 0.$$  \hspace{1cm} (2.3)$$

Thus, the potential of $T$ vanishes as mentioned above. This is known as the no-scale structure of supergravity. It is also true for the model including several Kähler moduli. Thus, we need some effects for moduli stabilization.$^1$

Non-perturbative effects can stabilize $T$ successfully. Non-perturbative superpotential is typically written as

$$W = W_0 + A e^{-\beta T},$$  \hspace{1cm} (2.4)$$

where $A$ and $\beta$ are constants. Such a superpotential is effectively induced by gaugino condensations and D-brane instanton effects. When $W_0$ is sufficiently small that it balances the non-perturbative term, $D_T W = 0$ has a nontrivial solution, e.g., $\beta = 1$ and $\tau = 10$ implies $|W_0/A| \sim e^{-10}$. This solution is known as the KKLT vacuum [3].

Also, perturbative $\alpha'$ corrections can play an important role for the moduli stabilization. The $\alpha'$ corrections to the Kähler potential are calculated in Ref. [15], and the leading-order approximation is given as

$$K = -2 \log \left[ V + \frac{\xi}{2} \right], \quad \xi = -\zeta(3) \chi(M) / 2(2\pi)^3,$$  \hspace{1cm} (2.5)$$

where $\zeta(z)$ is the Riemann zeta function, i.e. $\zeta(3) \sim 1.2$, and $\chi(M)$ is the Euler number of the Calabi–Yau manifold $M$.

For instance, in the LVS [4,5], moduli fields are stabilized at a point where the non-perturbative effects and the $\alpha'$ corrections are balanced. This model has a SUSY breaking vacuum, which means that $\partial V / \partial \tau_i = 0$, but $D_T W \neq 0$.

In the above two models, the no-scale structure is broken by the (non-)perturbative corrections. Here, we propose a new mechanism for Kähler moduli stabilization.

### 2.1. Potential by chiral superfield

Suppose that there is a chiral superfield $X$ in addition to the Kähler modulus $T$. We assume that the Kähler potential is represented as

$$K = -2 \log \left[ (T + \bar{T})^{3/2} + \frac{\xi}{2} \right] + (T + \bar{T})^{-n} |X|^2.$$  \hspace{1cm} (2.6)$$

This form of the Kähler potential is given by a dimensional reduction of the effective action of superstring theory [15–18]. The modular weight $n$, which would be a fractional number, depends on the origin of $X$.$^2$ In this paper we do not specify a concrete origin of $X$. We treat $n$ as a free parameter.

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$^1$ Radiative corrections would violate the no-scale structure [13,14].

$^2$ The chiral superfield $X$ may be a position moduli of D-branes, a chiral matter field localized at an intersection of D-branes, or a localized mode at a singular point.
We assume the following superpotential:

\[ W = W_0 - f(T)X. \]  

(2.7)

The linear term \( X \) would be generated, e.g., from the Yukawa term \( W^{(Y)} = YX\bar{Q}Q \) after condensation \( \langle \bar{Q}Q \rangle \neq 0 \) by strong dynamics.\(^3\) When the Yukawa coupling \( Y \) depends only on the dilaton and complex structure moduli, i.e. the perturbative Yukawa coupling term, \( f(T) \) is just a constant, \( f \). When this Yukawa coupling term is induced by non-perturbative effects, the function \( f(T) \) would be written by \( f(T) = Ae^{-bT} \).

We postulate that \( X \) is coupled with other massive chiral fields \( \phi \) in the superpotential such as \( X\phi^2 \), and then radiative corrections generate the mass of \( X \) like the O’Raifeartaigh model [20] as we study explicitly in Sect. 2.2. Thus, the potential in our model is written as

\[ V = e^K \left[ \sum_{i,j} K^\bar{j}D_i WD_j W - 3|W|^2 \right] + \tilde{m}_X^2 |X|^2, \]

(2.8)

where \( \tilde{m}_X \) is the SUSY breaking mass of \( X \) generated by the quantum corrections. The tilde indicates that the chiral superfield \( X \) is not canonically normalized yet. We assume that the mass of \( X \) is much larger than that of \( T \). We justify this assumption later.

We expand the scalar potential as

\[ V = V_0 + V_1 + V_2 + \cdots, \]

where \( V_i \) is the \( i \)-th-order term of \( X \). When \( f \) is a real constant, each \( V_i \) is given as follows:

\[ V_0 = \frac{1}{(\mathcal{V} + \frac{\xi}{2})^2} \left[ (T + \bar{T}) f^2 + \frac{3\xi}{4\mathcal{V} - \xi} W_0^2 \right], \]

(2.9)

\[ V_1 = \frac{fW_0}{\mathcal{V} + \frac{\xi}{2}} \frac{n - 1}{\mathcal{V} - \frac{\xi}{4}} (X + \bar{X}), \]

(2.10)

\[ V_2 = \tilde{m}_X^2 |X|^2 + \cdots, \]

(2.11)

where the ellipsis represents mass terms of \( \mathcal{O}(\mathcal{V}^{-2}) \). The \( f^2 \) term comes from \( K^{X\bar{X}} \partial_X W \partial_{\bar{X}} \bar{W} \). After integrating \( X \) out, the modulus potential is given by

\[ V = \frac{1}{(\mathcal{V} + \frac{\xi}{2})^2} \left[ \frac{3\xi}{4\mathcal{V} - \xi} + \frac{f^2}{|W_0|^2} \mathcal{V}^{2n/3} \right] |W_0|^2 + \mathcal{O}(|X|^2). \]

(2.12)

We also assume that \( \langle X \rangle \) is small enough compared to the Planck mass, and higher-order terms of \( \langle X \rangle \) are negligible. We will justify this assumption later, too. This potential has a local minimum \( \mathcal{V} = \mathcal{V}_0 \), satisfying the equations

\[ V_T/|W_0|^2 = \frac{3\mathcal{V}_0^{4/3}}{(\mathcal{V}_0 + \frac{\xi}{2})^3} h(\mathcal{V}_0) = 0, \]

(2.13)

\[ V_{TT}/|W_0|^2 = \frac{9}{2} \frac{\mathcal{V}_0^{2}}{(\mathcal{V}_0 + \frac{\xi}{2})^3} h'(\mathcal{V}_0) > 0, \]

(2.14)

\(^3\) See, e.g., Ref. [19].
where \( h(V) \) is given by

\[
h(V) = \left[ -\frac{9\xi}{8(V - \xi/4)^2} + \frac{(n/3 - 1)}{6} \left(\frac{f}{W_0}\right)^2 V^{3n-1} + \frac{1}{6} \frac{n\xi}{f} \left(\frac{f}{W_0}\right)^2 V^{3n-2} \right],
\]

(2.15)

and \( V_T, V_{TT} \) and \( h'(V) \) are the derivatives of \( V \) and \( h \): \( V_T = dV/dT \), \( V_{TT} = d^2V/dT^2 \), and \( h'(V) = dh/dV \).

When \( n \) is smaller than or equal to 1, \( h(V) \) is always negative, and the above conditions cannot be satisfied.

When \( n \) is equal to 2, the local minimum conditions \( V_T = 0 \) and \( V_{TT} > 0 \) are rewritten as

\[
\frac{27\xi}{8} V_0^{2/3} + \left(\frac{f}{W_0}\right)^2 \left(V_0 - \frac{\xi}{4}\right)^2 (V_0 - \xi) = 0,
\]

(2.16)

\[
\frac{1}{(V_0 + \frac{\xi}{3})^3} \left(V_0 - \frac{\xi}{8} (17 + 3\sqrt{57})\right) \left(V_0 - \frac{\xi}{8} (17 - 3\sqrt{57})\right) < 0.
\]

(2.17)

If \( \xi \) is negative, these equations have no solutions. If \( \xi \) is positive, the second inequality means \( \frac{\xi}{4} < V_0 < \frac{\xi}{8} (17 + 3\sqrt{57}) \approx 4.96\xi \). The necessary condition for a solution of Eq. (2.16) to exist is \( \left|\frac{f}{W_0}\right|^2 (\frac{\xi}{4})^{4/3} > 6.83 \), and the range of solutions is

\[
\frac{\xi}{4} < V_0 < \xi.
\]

(2.18)

As a result, the volume of the compact space is positive definite and can be stabilized.\(^4\)

When \( n \) is equal to 3, the potential always has a non-trivial local minimum. If \( \xi \) is negative, the minimum \( V_0 \) is negative and it is not a valid stationary point, since the volume of the compact space must be positive. If \( \xi \) is positive, the solutions are given by \( V_0 = \frac{\xi}{4} \pm \frac{3|W_0|}{2f^2} \), and the potential is minimized by

\[
V_0 = \frac{\xi}{4} - \frac{3|W_0|}{2f^2}.
\]

(2.19)

If either \( \left|\frac{W_0}{f}\right| \) or \( \xi \) is large enough, the Kähler modulus is stabilized at \( V_0 \gg 1; V_0 \gg \xi \) for \( \left|\frac{W_0}{f}\right| \gg \xi \).

When \( n \) is larger than 3, the second term of Eq. (2.12) overcomes the prefactor \( \frac{1}{(V^2 + \xi/4)^2} \), and it diverges to positive infinity as \( V \to \infty \). If \( \xi \) is positive, Eq. (2.12) diverges to positive infinity as \( V \to \frac{\xi}{4} \), too, and we must have a global minimum in the range \( \frac{\xi}{4} < V \). If \( \xi \) is negative and \( \left|\frac{f}{W_0}\right|^2 > \left(\frac{\xi}{4}\right)^{2/3n} \), Eq. (2.12) diverges to positive infinity as \( V \to \frac{\xi}{4} \) and we have a non-trivial solution, too. Otherwise, we have no solutions.

Figure 1 shows typical shapes of the potentials with \( n = 1, 2, 3 \). It shows that large-\( n \) potentials stabilize the Kähler modulus.

\(^4\) We should comment on the \( \alpha' \) corrections. The \( \alpha' \) corrections to the Kähler potential in Eq. (2.5) are evaluated in the region \( V \gg \xi \) [15]. When \( V \) and \( \xi \) become of the same order, the leading-order approximation in Eq. (2.5) in terms of \( \alpha' \) is no longer reliable. We should take higher-order corrections into account. If these effects remain subdominant compared to Eq. (2.5), our scenario is still useful even for the model of \( n = 2 \).
On the other hand, when the $f$ in the superpotential is generated non-perturbatively, that is, $f$ is a function of $T$, the result of moduli stabilization is completely different. The superpotential is rewritten as

$$W = W_0 - AXe^{-bT}, \quad (2.20)$$

where the coefficients $A$ and $b$ are constant parameters. In this model the F-term potential is expanded as

$$V_0 = \frac{1}{(\mathcal{V} + \frac{\xi}{4})^2} \left[ \frac{3\xi}{4\mathcal{V} - \xi} |W_0|^2 + A^2 e^{-b(T + \bar{T})} \mathcal{V}^{2n/3} \right], \quad (2.21)$$

$$V_1 = \frac{1}{(\mathcal{V} + \frac{\xi}{4})^2} \left[ \frac{2\mathcal{V} + \xi}{4\mathcal{V} - \xi} \left\{ n - 1 - b(T + \bar{T}) \right\} A\bar{W}_0 e^{-bT}X + \text{h.c.} \right], \quad (2.22)$$

$$V_2 = \tilde{m}_X^2 |X|^2 + \cdots. \quad (2.23)$$

When $\langle X \rangle$ is sufficiently small, we can approximate the modulus potential by the above $V_0$. $T$ is stabilized at the point satisfying the equations

$$V_T/|W_0|^2 = \frac{3\mathcal{V}_0^{4/3}}{(\mathcal{V}_0 + \frac{\xi}{4})^3} g(\mathcal{V}_0) = 0, \quad (2.24)$$

$$V_{TT}/|W_0|^2 = \frac{9}{2} \frac{\mathcal{V}_0^5}{(\mathcal{V}_0 + \frac{\xi}{4})^3} g'(\mathcal{V}_0) > 0, \quad (2.25)$$

where $g(\mathcal{V})$ is given by

$$g(\mathcal{V}) = -\frac{9}{8} \frac{\xi}{(\mathcal{V} - \frac{\xi}{4})^2} + \frac{f^2}{3|W_0|^2} e^{-b\mathcal{V}^{2/3}} \mathcal{V}^{2n/3-1} \left[ (n - (3 + b\mathcal{V}^{2/3})) + \frac{1}{2} \frac{\xi}{\mathcal{V}} (n - b\mathcal{V}^{2/3}) \right]. \quad (2.26)$$

When $\xi$ is positive, to realize $g(\mathcal{V}) = 0$, the first and the second terms of Eq. (2.26) must be balanced, which implies that

$$(n - (3 + b\mathcal{V}^{2/3})) + \frac{\xi}{2\mathcal{V}} (n - b\mathcal{V}^{2/3}) > 0. \quad (2.27)$$

Thus, we need $n > b\mathcal{V}^{2/3}$. Here, $b\mathcal{V}^{2/3}$ is considered as an instanton action and it should be much larger than 1 for the single instanton approximation. $n$ is given by dimensional reduction, and it is typically of $\mathcal{O}(1)$ [16–18]. Therefore, we cannot satisfy the stationary condition.
When $\xi$ is negative there may be a stationary solution, but since there is the single Kähler modulus $T$ in our model, it is natural to assume that $(h_{1,2} - h_{1,1}) > 0$, hence $\xi$ is positive. Thus, this solution is invalid. We conclude that this form of superpotential cannot stabilize the whole volume.

### 2.2. Consistency

Here, we examine the consistency of our model. Hereafter, we consider the case that the prefactor $f(T)$ is a constant.

For a consistent moduli stabilization, the compact space should be large enough to justify the supergravity approximation, i.e. $V_0 \gg 1$ in units of the string length. In our scenario, the size of the compact space is characterized by $\xi$ and $|W_0|^2$. For $n = 2$, the stationary point is approximated by $\xi$. Three-dimensional Calabi–Yau manifolds admit a variety of topologies and Euler numbers [21,22]. It is possible to find a Calabi–Yau manifold whose Euler number is of $O(10^3)$, i.e. $\xi \sim 10$. Our moduli stabilization mechanism works well in such compactifications.\(^5\) For $n = 3$, the stationary point is given by Eq. (2.19). The volume of the compact space is stabilized at a large volume when either of $|W_0|$ and $\xi$ is much larger than 1. We have a range of parameters to realize the consistency of the supergravity approximation.

Next, we also have to justify our assumptions that the mass of $X$ is much heavier than that of $T$ and the modulus potential terms proportional to $\langle |X| \rangle$ are negligible. The mass of $X$ is generated by quantum corrections [10,20]. For a concrete discussion, we consider the O’Raifeartaigh-like model. To begin with, we briefly review the mass of $X$ generated by this model. Suppose that there are extra chiral superfields $\phi_1$ and $\phi_2$, and the superpotential is given by

$$W = m\phi_1\phi_2 + \lambda\phi_1^2X - fX.$$  

(2.28)

For simplicity, we assume that Kähler metrics of these chiral superfields depend on heavy (stabilized) moduli other than $T$, and then we have canonically normalized their Kähler metrics: $K = |\phi_1|^2 + |\phi_2|^2 + |X|^2$.\(^6\) The masses $m$ of $\phi_{1,2}$ are relatively heavier than that of $X$, and we can integrate them out in order to study the dynamics of $X$. We consider the case of $\lambda f \ll m$, which means the VEVs of $\phi_1$ and $\phi_2$ are sufficiently small compared to the Planck mass. Integrating $\phi_1$ and $\phi_2$ out, we obtain the Coleman–Weinberg potential. This is interpreted as a correction to the Kähler potential and written as

$$W = -fX,$$

$$K = X\bar{X} \left(1 - \frac{c\lambda^2}{16\pi^2} \log \left(1 + \frac{\lambda^2X\bar{X}}{m^2}\right)\right),$$

(2.29)

where we have assumed many $\phi_1$ and $\phi_2$, and the constant parameter $c$ denotes their multiplicity. Expanding the Kähler potential, we obtain

$$K \sim X\bar{X} - \frac{(X\bar{X})^2}{\Lambda^2},$$

(2.30)

\(^5\) For $n = 2$, the leading-order approximation of the $\alpha'$ corrections may be unreliable as mentioned in the previous subsection. In this subsection we consider its consistency, assuming that the higher-order corrections in terms of $\alpha'$ are negligible and our moduli stabilization mechanism works for $n = 2$ instead of considering the higher-order corrections deeply.

\(^6\) Similarly, we can discuss the case that their Kähler metrics depend on $T$. (See, e.g., Ref. [7].)
where \( \Lambda^2 = \frac{16\pi^2 m^2}{\alpha_X} \). Then, the mass of \( X \) comes from the F-term potential: \( e^K (K^{XX} D_X W D_X W - 3|W|^2) \). It is calculated as

\[
V = f^2 + \frac{4f^2}{\Lambda^2} X \bar{X} + \cdots.
\]

(2.31)

and the mass of \( X \) is \( \frac{2f}{\Lambda} \). In our model, a similar mass term can be generated. The difference only comes from the Kähler potential. Since we postulate that the Kähler potential is the sum of the Kähler potential of \( T \) and that of \( \phi_1 \) and \( \phi_2 \), the mass of \( X \) is given by

\[
\bar{m}_X^2 = 4e^{-2\log(V + \xi/2)} \frac{f^2}{\Lambda^2} \sim 4 \frac{f^2}{\Lambda^2} V_0^{-2}.
\]

(2.32)

To guarantee that \( \phi_{1,2} \) are heavier than \( X \), \( \Lambda^2 \) is much larger than \( f \). The canonically normalized masses are calculated as

\[
m_T^2 = 4 \frac{f^2}{\Lambda^2} \frac{1}{2} \left( \frac{1}{2V_0} + \frac{1}{V_0 - \xi/4} \right) \frac{9}{4} \frac{\xi}{(V_0 - \xi/4)^2} + \left(n - \frac{3}{4} - 1\right) \frac{f}{V_0} \left| \frac{\xi}{2V_0} \right| \frac{3n - 2}{2} |W_0|^2.
\]

(2.33)

When \( n \) is less than 3, \( V_0 \) is characterized by \( \xi \). We can estimate \( m_T \) and \( m_X \) as

\[
m_X^2 \sim 4 \frac{f^2}{\Lambda^2} \left(\frac{\xi}{4}\right)^{2n/3-2},
\]

\[
m_T^2 \sim \left(\frac{\xi}{4}\right)^{-\frac{3}{2}} \left[C_1 + C_2 \left| \frac{f}{V_0} \right|^2 \left(\frac{\xi}{4}\right)^{3n} \right] |W_0|^2.
\]

(2.34)

where the parameters \( C_1 \) and \( C_2 \) are of \( \mathcal{O}(1) \).\(^7\) When \( \left| \frac{f}{V_0} \right|^2 \) is much less than 1, \( m_T^2 \) is approximated as \( m_T^2 \sim \left(\frac{\xi}{4}\right)^{-5} |W_0|^2 \). The mass ratio is given as

\[
\frac{m_T^2}{m_X^2} \sim 1 \frac{f^2}{\Lambda^2 |W_0|^2} \left(\frac{\xi}{4}\right)^{\frac{3n-1}{3}}.
\]

(2.35)

\[
\frac{m_T^2}{m_X^2} \gg 1 \text{ implies } \Lambda \ll 1. \text{ On the other hand, when } \left| \frac{f}{V_0} \right|^2 \text{ is much larger than } 1, m_T^2 \text{ is approximated as } m_T^2 \sim \left(\frac{\xi}{4}\right)^{-\frac{3}{2}} |W_0|^2. \text{ The mass ratio is given as }
\]

\[
\frac{m_T^2}{m_X^2} \sim 1 \frac{\xi}{4} \left(\frac{\xi}{4}\right)^{-\frac{1}{3}}.
\]

(2.36)

\[
\text{This must be much greater than 1 since } \Lambda \text{ is measured in units of the Planck scale and } \Lambda < 1. \text{ In both cases, for } n < 3 \text{ a small } \Lambda \text{ justifies our assumption. When } n \text{ is equal to 3, } V_0 \text{ is given by Eq. (2.19). } m_T^2 \text{ is given by}
\]

\[
m_T^2 = \frac{\xi V_0^4}{2 \left( V_0 + \frac{1}{4} \right) |W_0|^2}.
\]

(2.37)

\[
\text{\(^7\) Here, we assume } V_0 \sim \xi \text{ and } V_0 - \xi/4 \sim \xi. \text{ When } \left(V_0 - \frac{\xi}{4}\right)^{-1} \text{ diverges, our estimation may be invalid.}
\]
The ratio of the masses is written as

\[
\frac{m_X^2}{m_T^2} = \frac{4}{\xi \Lambda^2} \frac{|W_0| V_0^{4/3}}{f V_0 + \frac{\xi}{2}} \sim \frac{4}{\xi \Lambda^2} \frac{|W_0| V_0^{1/3}}{f}. \tag{2.39}
\]

We can realize \(m_X^2 \gg m_T^2\) for \(\frac{|W_0|^2}{\xi \Lambda^2} \gg 1\).

Now, we examine our assumption that \(\langle X \rangle \ll 1\). From Eqs. (2.10) and (2.11), \(\langle |X| \rangle\) is given by

\[
\langle |X| \rangle \sim \frac{(n-1)\xi |W_0|}{m_X^2}. \tag{2.40}
\]

Since \(\tilde{m}_X\) is given by Eq. (2.32), \(\langle |X| \rangle\) is calculated as

\[
\langle |X| \rangle \sim \frac{(n-1)\Lambda^2 |W_0|}{4f}. \tag{2.41}
\]

To realize a small \(\langle |X| \rangle\), we need \(\Lambda^2 |W_0| \ll f\) in units of the Planck mass.

To illustrate the consistency conditions of our model, we study them for the case of \(n = 3\). In this case, the aforementioned conditions are written as

\[
\begin{align*}
3 |W_0| & \gg f M_P, \\
6 |W_0|^2 & \gg \xi f^2 \Lambda^2, \\
2 f M_P^3 & \gg \Lambda^2 |W_0|, \\
\Lambda^4 & \gg 4 f^2. \tag{2.42}
\end{align*}
\]

We explicitly indicate the Planck mass that has been omitted. Roughly speaking, these conditions imply \(|W_0| \gg f M_P\) and \(\Lambda M_P \gg f\). There is a large range of parameters satisfying the above conditions. For example, \(\xi = 1, W_0 = 10^{-3}, \Lambda = 10^{-2}, f = 10^{-5}\) is a typical solution.

### 2.3. The cosmological constant and other comments

In the above scenario, we can realize the modulus stabilization where the potential minimum is given by Eq. (2.12) and the stationary point is given by Eq. (2.13). Then we can approximate the vacuum energy as

\[
V_0 = |W_0|^2 \left( \frac{1}{(V_0 + \frac{\xi}{2} )^2} \left( \frac{3 \xi}{4 (V_0 - \frac{\xi}{2} )^2} + \frac{9 \xi V_0^2}{8 (V_0 - \frac{\xi}{4} )^2 (\frac{\xi}{4} - 1) V_0 + \frac{1}{6} n \xi} \right) \right). \tag{2.43}
\]

We can estimate

\[
V_0 \sim \frac{|W_0|^2}{M_P^2} \frac{1}{\xi^{18} V_0^{-3}}. \tag{2.44}
\]

This model has a positive cosmological constant. The vacuum energy is uplifted by the auxiliary component of \(X\). It may be interesting that the vacuum energy is proportional to \(V_0^{-3}\). In order to realize the Minkowski vacuum, we need some effects to depress the vacuum energy.

We should also comment on the imaginary part of \(T\), i.e. the axion. The axion is not stabilized in this scenario. That can be understood from the forms of the Kähler potential and the superpotential. We have shown that, in the superpotential in Eq. (2.7), \(f(T)\) must be a constant for the stabilization
of \( \tau \). The Kähler potential does not include the imaginary part of \( T \) either. Thus, the axion does not appear in the F-term potential. To stabilize it we need to consider additional effects, for example non-perturbative effects.

Although we have supposed a model that has one Kähler modulus only, we would apply this scenario to more general models that have many Kähler moduli. In general, the mass of the Kähler moduli is suppressed by the volume of the cycle related to the Kähler moduli, and the overall volume modulus would be lighter than the other Kähler moduli. In such cases we can apply our moduli stabilization mechanism after the other moduli are stabilized by another stabilization mechanism, e.g. D-terms [23–25,27], non-perturbative effects [3–5], etc.

Therefore, it is important to consider our moduli stabilization mechanism in collaboration with another one, such as the KKLT scenario and the LVS. In the next section we consider the latter: we discuss the possibility of the F-term uplifting of the LVS by our model.

3. F-term uplifting and the Large Volume Scenario

In this section we study uplifting the AdS vacuum of the LVS to the Minkowski vacuum by adding one chiral superfield. First, we briefly review the LVS, and then we study F-term uplifting mechanism.

3.1. Large Volume Scenario

The LVS was proposed in Refs. [4,5] about ten years ago. Here, we give a brief review of the LVS. In this scenario, the Kähler moduli are stabilized at the point where the \( \alpha' \) corrections and the non-perturbative effects are balanced. In this paper we study the LVS based on swiss cheese compactifications, which means that the dimensionless volume of the Calabi–Yau space is given like

\[
V = (T_1 + \bar{T}_1)^{3/2} - \sum_{i>1} y_i(T_i + \bar{T}_i)^{3/2},
\]

where \( T_i \) represents a volume modulus corresponding to the \( i \)th four-cycle on the Calabi–Yau manifold, and \( y_i \) is a geometrical parameter. With perturbative \( \alpha' \) corrections and non-perturbative effects taken into account, the Kähler potential and the superpotential can be represented as

\[
K = -2 \log \left[ V + \frac{\xi}{2} \right], \quad W = W_0 + \sum_{i>1} A_i e^{-a_i T_i}.
\]

Calculating Eq. (2.1), the scalar potential is given as

\[
V_{\text{LVS}} = \frac{A}{V^3} - \sum_{i>1} \frac{B_i a_i \tau_i e^{-a_i \tau_i}}{V^2} + \sum_{i>1} \frac{C_i \sqrt{a_i \tau_i} e^{-2a_i \tau_i}}{V} + O(V^{-4}),
\]

where \( A, B_i, C_i \) are given as

\[
A = \frac{3\xi |W_0|^2}{4}, \quad B_i = 4A_i|W_0|, \quad C_i = \frac{2\sqrt{2}a_i^{3/2}A_i^2}{3y_i^2}.
\]

The minimum of the potential is given by the point satisfying the equations

\[
A = \sum_{i>1} \frac{B_i^2(a_i \tau_i)^{3/2} a_i \tau_i (a_i \tau_i - 1)}{4C_i (a_i \tau_i - 1/4)^2}, \quad V = \frac{1}{2} \frac{B_i}{C_i} \sqrt{a_i \tau_i} e^{a_i \tau_i} \frac{a_i \tau_i - 1}{a_i \tau_i - 1/4}.
\]
When \( a_i \tau_i \) is much larger than 1, the solution is approximated by,

\[
\tau_i \sim \frac{1}{a_i} \left( \frac{4A}{\sum_{i>1} B_i C_i} \right)^{2/3}, \quad \mathcal{V} \sim \frac{1}{2} \frac{B_i}{C_i} \sqrt{a_i \tau_i e^{a_i \tau_i}}.
\]

(3.6)

As a result, all the Kähler moduli are stabilized successfully. The volume of the compact space is stabilized at an exponentially large value compared to \( \tau_i \).

The vacuum of the LVS breaks SUSY. In fact, the auxiliary fields of the Kähler moduli are not zero. However, its vacuum is still an AdS vacuum. The minimum value of the potential is calculated as

\[
V_{\text{min}} \sim -\frac{A}{2a_i \langle \tau_i \rangle \langle \mathcal{V} \rangle^3} + O(\mathcal{V}^{-4}),
\]

(3.7)

and it is negative.

In the original paper, anti-D-branes are introduced for uplifting. Here, we study uplifting by the chiral superfield \( X \).

3.2. \( F \)-term uplifting

We study moduli stabilization and uplifting simultaneously. Suppose that there are two Kähler moduli, \( T_1, T_2 \), and one chiral superfield, \( X \). Their Kähler potential and the volume of the compact space are given by

\[
K = -2 \log \left[ \mathcal{V} + \frac{\xi}{2} \right] + (T_2 + \bar{T}_2)^{-m}(T_1 + \bar{T}_1)^{-n}|X|^2,
\]

(3.8)

\[
\mathcal{V} = (T_1 + \bar{T}_1)^{3/2} - \gamma_2(T_2 + \bar{T}_2)^{3/2}.
\]

(3.9)

Similar to the previous section, we consider two forms of superpotential:

\[
W = W_0 - fX + A_2 e^{-a_2 T_2}
\]

(3.10)

and

\[
W = W_0 - A e^{-b T_2} X,
\]

(3.11)

where \( f, a_2, A_2, A, \) and \( b \) are real constants. The superpotentials of Eqs. (3.10) and (3.11) correspond to the cases where the \( f(T)X \) term is induced by perturbative and non-perturbative effects, respectively. We assume that \( W_0 \) is real for simplicity. In both cases, we expect that the mass of \( X \) is generated by radiative corrections and the scalar potential is given by

\[
V = e^K \left[ K^{\bar{j}l} D_l W D_j \mathcal{W} - 3|W|^2 \right] + \tilde{m}_X^2 |X|^2.
\]

(3.12)

We also assume that \( X \) is much heavier than the other moduli, and we can integrate \( X \) out before studying the Kähler moduli stabilization. We confirm the validity of this assumption later.

We expand \( V \) in terms of \( X \) as

\[
V(T_1, T_2, X) = V_0(T_1, T_2) + V_1(T_1, T_2, X) + V_2(T_1, T_2, X) + \cdots,
\]

(3.13)

---

8 A similar model was considered in Ref. [26], too.
where $V_i$ is the $i$th-order term of $X$. For the case of Eq. (3.10), we obtain

$$V_0 = V_{LVS}(T_1, T_2) + \frac{1}{(V + \frac{\xi}{2})^2} (T_1 + \hat{T}_1)^n (T_2 + \hat{T}_2)^m f^2,$$

(3.14)

$$V_1 = \frac{1}{V^2} \left[ - (1 + m) f (W_0 + A_2 e^{-a_2 \hat{T}_2}) \tilde{X} + m (2 \tau_2)^{-1/2} \frac{2V}{3 \gamma^2} f (-A_2 a_2 e^{-a_2 \hat{T}_2}) \tilde{X} + h.c. \right] + \cdots,$$

(3.15)

$$V_2 = \tilde{m}_X^2 |X|^2 + \cdots,$$

(3.16)

where $V_{LVS}$ is the moduli potential of the LVS and the ellipses represent the higher-order terms of $V^{-1}$. Using Eq. (3.6), we can approximate $V_1$ as

$$V_1 \sim \frac{1}{V^2} \left[ (-1 + (3 \sqrt{2} - 1)m) f W_0 \tilde{X} + h.c. \right] + \cdots.$$

(3.17)

Then, the approximated VEV of $X$ is given by

$$\langle X \rangle \sim \frac{f}{V^2} \frac{(-1 + (3 \sqrt{2} - 1)m) W_0}{\tilde{m}_X^2}.$$

(3.18)

Since $V_{LVS}$ is of $O(V^{-3})$, the $O(\langle X \rangle^2)$ term is negligible. After integrating $X$ out, the moduli potential $\tilde{V}$ is evaluated as

$$\tilde{V} = V_{LVS}(\tau_1, \tau_2) + f'^2 V^{-2} (\tau_1)^n (\tau_2)^m + O(V^{-4}),$$

$$\sim V_{LVS}(\tau_1, \tau_2) + f'^2 V^{3/2 - n/2} (\tau_2)^m,$$

(3.19)

$$V_{LVS} = \frac{A}{\bar{\gamma}^3} - \frac{B a_2 \tau_2 e^{-a_2 \tau_2}}{\gamma^2} + \frac{C \sqrt{a_2} \tau_2 e^{-a_2 \tau_2}}{\gamma} + O(V^{-4}),$$

(3.20)

$$A = \frac{3 \xi |W_0|^2}{4}, \quad B = 4A_2 |W_0|, \quad C = \frac{2 \sqrt{2} a_2^3/2A_2^2}{\gamma^2}, \quad f'^2 = 2^{n+m} f^2.$$

(3.21)

We see that the potential is uplifted by $f'^2 \gamma^{2n/3 - 2} (\tau_2)^m$. If $f'$ is sufficiently small that the stationary point of our model is approximated by that of the LVS, its vacuum energy is approximated as

$$\tilde{V}(\nu_0, \tau_{2,0}) = - \frac{A_2}{a_2 \nu_{2,0} \gamma^2} + |f'|^2 \gamma^{2n/3 - 2} (\tau_{2,0})^m,$$

(3.22)

where $(\nu_0, \tau_{2,0})$ is the minimum point of the LVS potential. The Minkowski vacuum can be realized by

$$|f'|^2 = \frac{A}{2a_2} \gamma^{-2n/3 - 1} (\tau_{2,0})^{-m - 1}.$$

(3.23)

More precisely, the stationary point of our model is perturbed from that of the LVS. The true minimum is represented by

$$\nu = \nu_0 + \delta \nu,$$

(3.24)

$$\tau_2 = \tau_{2,0} + \delta \tau.$$

(3.25)
We assume $\delta_V/V_0, \delta_\tau/\tau_{2,0} \ll 1$. Using Eq. (3.23), we can calculate the leading-order deviations from the vacuum of the LVS as

$$\frac{\partial \tilde{V}}{\partial \tau_2} = \frac{a_2^2}{V^2} \left[ B(a_2 \tau_2 - 1)e^{-a_2 \tau_2} - CV \left( 2a_2 \tau_2 - \frac{1}{2} \right) e^{-2a_2 \tau_2} \right] + \frac{A}{2a_2} \frac{\gamma^{2n/3-2}(\tau_2)^{m-1}}{\gamma^{2n/3+1}(\tau_{2,0})^{m+1}} = 0,$$  

(3.26)

$$\frac{\partial \tilde{V}}{\partial \nu} = -\frac{3A}{V^4} + \frac{2Ba_2^2 e^{-a_2 \tau_2}}{V^3} - \frac{C\sqrt{a_2 \tau_2} e^{-2a_2 \tau_2}}{V^2} + \frac{A}{2a_2} \frac{2n}{3} - 2 \frac{\gamma^{2n/3-3}(\tau_2)^m}{\gamma^{2n/3+1}(\tau_{2,0})^{m+1}} = 0.$$  

(3.27)

That is, the deviations of the VEVs are estimated as

$$\frac{\delta_\tau}{\tau_{2,0}} = \frac{1}{x_0} \frac{2(\frac{n}{3} - 1)}{5 + 2(\frac{n}{3} - 1)(\frac{2n}{3} - 3)} + \mathcal{O}(x_0^{-2}),$$  

(3.28)

$$\frac{\delta_V}{V_0} = \frac{2}{5 + 2(\frac{n}{3} - 1)(\frac{2n}{3} - 3)} \frac{m}{2} + \mathcal{O}(x_0^{-1}),$$  

(3.29)

where $x_0 = a_2 \tau_{2,0}$. The vacuum of the LVS implies that $a_2 \tau_{2,0}$ is of $\mathcal{O}(10)$. Thus, $\delta_\tau/\tau_{2,0}$ is suppressed. On the other hand, there is no suppression factor for the deviation of the whole volume $\delta_V/V_0$, and it seems to be of $\mathcal{O}(1)$. However, since the denominator of Eq. (3.29) is of $\mathcal{O}(10)$, $\delta_V/V_0$ is successfully suppressed and of $\mathcal{O}(10^{-1})$. Therefore, the deviations are indeed small. The typical values of $\delta_V/V_0$ are summarized in Table 1, and we can confirm that the range of $\delta_V/V_0$ is small. The uplifting term does not destabilize the vacuum of the LVS drastically. Our rough estimation is valid and we can successfully uplift the vacuum energy of the LVS.

Finally, we study the mass of $X$. The heaviest mode of the Kähler moduli is the small volume moduli $\tau_2$, and its canonically normalized mass $m_{\tau_2}$ is estimated in Ref. [5] as

$$m_{\tau_2}^2 \sim \left( \frac{W_0}{\sqrt{4\pi} \nu_0} \right)^2.$$  

(3.30)

The mass squared of $X$ is given by Eq. (2.33). Substituting $f$ by Eq. (3.23), the mass of $X$ is estimated as

$$m_X^2 \sim 4\frac{f^2}{\Lambda^2} \nu_0^{\frac{2n}{3} - 2} \sim 3\xi(\tau_{2,0})^{-m-1} \frac{W_0^2}{2a_2 \Lambda^2} \nu_0^{-3}.$$  

(3.31)

*Our form for the mass of the modulus is different from that in Ref. [5]. The difference comes from the definition of the metric and the Kähler potential. We use the normal string frame in this paper; it is not the same that used in Ref. [5]. In addition, we ignore the overall factor of $e^{k(\Omega, S)}$. Here, we only concentrate on the ratio of the moduli masses and are not concerned with their physical values.*
Roughly speaking, $X$ is heavier than $T$ for $\Lambda^{-2} > \mathcal{V}_0$. For instance, when $\mathcal{V}_0 \sim 10^6$, $\Lambda$ should be smaller than $10^{-3}M_P$. When the above condition is satisfied, we can safely integrate $X$ out before the Kähler moduli stabilization, and succeed in uplifting the vacuum energy.

On the other hand, if the superpotential of $X$ includes the dependence of Kähler moduli $T_i$, that is, the superpotential is given as in Eq. (3.11), the result is completely different. Expanding its scalar potential in terms of $X$ and integrating $X$ out, we obtain moduli potential

$$V_0 = \frac{1}{2(\mathcal{V} + \frac{\xi}{2})^2} \left[ -3 \frac{\xi}{4} (\mathcal{V} + \frac{\xi}{2}) [W_0]^2 + (T_1 + \tilde{T}_1)^n (T_2 + \tilde{T}_2)^m A^2 e^{-2b\tau_2} \right].$$

(3.32)

Its stationary point must satisfy the following conditions:

$$\frac{\partial V}{\partial \tau_2} \sim \frac{2(2\tau_1)(2\tau_2)^{m-1}A^2 e^{-2b\tau_2}}{(\mathcal{V} + \frac{\xi}{2})^2} [m - 2b\tau_2] = 0,$$

(3.33)

$$\frac{\partial^2 V}{\partial \tau_2^2} \sim \left( \frac{\partial}{\partial \tau_2} \frac{2(2\tau_1)(2\tau_2)^{m-1}A^2 e^{-2b\tau_2}}{(\mathcal{V} + \frac{\xi}{2})^2} \right) [m - 2b\tau_2] - \frac{4b(2\tau_1)(2\tau_2)^{m-1}A^2 e^{-2b\tau_2}}{(\mathcal{V} + \frac{\xi}{2})^2} > 0,$$

(3.34)

where we used that $\tau_1$ is much larger than 1. Since $\partial V/\partial \tau_2 = 0$ implies $m = 2b\tau_2$, there are no stable vacua. Such a superpotential destabilizes the LVS vacuum. Hereafter, we treat $f(T)$ as a constant parameter and the superpotential is written as in Eq. (3.10).

### 3.3. Numerical analysis

In the previous subsection we considered the LVS model uplifted by the chiral superfield. We confirmed that if $f(T)$ is a constant, the vacuum of the LVS can be uplifted to the Minkowski vacuum successfully by expanding the moduli potential in terms of the deviations from the vacuum of the LVS. Here, we examine the previous conclusion numerically. In Fig. 2 we show the shape of a potential of the LVS (left) and that of the uplifted LVS (right), setting $A = 1$, $B = 0.2$, $C = 1$, and $a_2 = 2\pi$. In this case the approximated minimum of the original LVS in Eq. (3.6) is calculated as

$$\mathcal{V}_0 = 1.055 \times 10^9, \quad \tau_{2,0} = 3.429,$$

(3.35)

and the numerical result is

$$\mathcal{V}_0^{\text{Num}} = 1.449 \times 10^9, \quad \tau_{2,0}^{\text{Num}} = 3.484.$$

(3.36)

The value of the potential minimum is approximated well. From Eq. (3.23), we get the value of $f'$ that uplifts the minimum to the Minkowski vacuum. When $n = 1$ and $m = 2$, the value of $f'$ is calculated using the approximated solution and numerical solution:

$$f'^{2} \sim 1.344 \times 10^{-9}, \quad f'^{2}_{\text{Num}} \sim 1.007 \times 10^{-9}.$$

(3.37)

Analytical calculation of the potential minimum of the uplifted LVS is difficult. We only illustrate the existence of the minimum of the uplifted LVS and its rough position in Fig. 2. The orange surface represents the potential of the LVS and the uplifted LVS. The blue surface is $V = 0$. In the left figure, there is a large region where the moduli potential is negative around the curve of $\mathcal{V} \sim \frac{1}{2} \frac{\xi}{2} \sqrt{\frac{\xi}{2}} e^{2b\tau_1}$. Thus its potential minimum is definitely negative. However, in the right figure, the orange surface is above the blue surface in almost all of the region. The potential minimum must be located in a small region at the upper-left corner of the right figure, where the blue surface overcomes the orange
Fig. 2. Moduli potential of the L VS (left) and that of the uplifted L VS (right). We set \( A = 1, B = 0.2, C = 1, a_2 = 2\pi, n = 1, m = 2, \) and \( f'^2 = 1.00742 \times 10^{-9} \). Here, in order to visualize the minimum easily, we use the value of \( f' \) in the numerical solution. The orange surfaces denote the moduli potentials and the blue surface is \( V = 0 \). You can see that the vacuum energy of the L VS is negative, but that of the uplifted L VS is almost zero.

surface. This small region includes \( (V_0, \tau_{2,0}) \). The deviations from the vacuum of the L VS are not so drastic. The potential is almost uplifted by \( X \). We conclude that our uplifting mechanism works well.

3.4. F-terms

In this subsection we study the auxiliary components of the Kähler moduli fields and \( X \). F-term SUSY breaking is characterized by the VEV of the auxiliary components of chiral superfields. They are given by

\[ F^{\bar{i}} = -e^{-K/2} K^{\bar{i}j} D_j W. \]  

(3.38)

\( D_{T_1} W, D_{T_2} W, \) and \( D_X W \) are estimated by

\[ D_{T_1} W = \left[ -3(2\tau_1)^{1/2} \left( W_0 + e^{-a_2\tau_2} \right) \right] \sim V_0^{-2/3}, \]  

(3.39)

\[ D_{T_2} W = \left[ A_2 a_2 e^{-a_2\tau_2} + \frac{3\gamma_2(2\tau_2)^{1/2}}{V + \xi} \left( W_0 + e^{-a_2\tau_2} \right) \right] \sim V_0^{-1}, \]  

(3.40)

\[ D_X W = [-f + (2\tau_1)^{-n}(2\tau_2)^{-m} (X) (W_0 + e^{-a_2\tau_2})] \sim V_0^{-n/3-1/2}. \]  

(3.41)

We used that \( \langle X \rangle \) is of \( \mathcal{O}(V_0^{-2}) \) and its linear term is a subleading term. \( K_{ij} \) and \( K^{ij} \) are calculated as

\[ K_{ij} \sim \begin{pmatrix} V_0^{-2n/3} & 0 & 0 \\ 0 & V_0^{-4/3} & V_0^{-5/3} \\ 0 & V_0^{-5/3} & V_0^{-1} \end{pmatrix}, \]  

(3.42)

\[ K^{ij} \sim \begin{pmatrix} V_0^{2n/3} & 0 & 0 \\ 0 & V_0^{4/3} & V_0^{2/3} \\ 0 & V_0^{4/3} & V_0 \end{pmatrix}. \]  

(3.43)

The VEVs of the F-terms are estimated as

\[ F^X = -\frac{1}{V + a} K^{X\bar{X}} D_X W \sim V_0^{n-1/2}, \]  

(3.44)
\[ F^{T_1} = -\frac{1}{V + a} (K^{T_1 \bar{T}_1} D_{T_1} W + K^{T_1 \bar{T}_2} D_{T_1} W) \sim V_0^{-1/3}, \]  
\[ F^{T_2} = -\frac{1}{V + a} (K^{T_2 \bar{T}_1} D_{T_1} W + K^{T_2 \bar{T}_2} D_{T_2} W) \sim V_0^{-1}. \]  
Thus, if \( n \) is larger than \( \frac{7}{2} \), we obtain \( V_0^{\frac{6}{3} - \frac{1}{3}} \sim F^X \gg F^{T_1} \gg F^{T_2} \sim V_0^{-1} \). If \( \frac{7}{2} > n > \frac{3}{2} \), we find \( V_0^{-1/3} \sim F^{T_1} \gg F^X \gg F^{T_2} \sim V_0^{-1} \), and if \( n \) is smaller than \( \frac{3}{2} \), we have \( V_0^{-1/3} \sim F^{T_1} \gg F^{T_2} \gg F^X \sim V_0^{\frac{3}{2} - \frac{1}{2}} \).

On the other hand, the gravitino mass \( m_{3/2} \) is independent of \( n \), and it is given by
\[ m_{3/2} = e^{K/2} W_0 \sim V_0^{-1}. \]  

Using the above equations we can calculate soft terms. Although \( F^X \) can overcome or be comparable to \( F^{T_1, T_2} \), the soft masses are not affected from those of the LVS.

### 4. Conclusion and discussion

We have studied the new type of Kähler moduli stabilization and the F-term uplifting of the Large Volume Scenario. For realistic string models, moduli stabilization is crucial. In addition, since our universe has a positive cosmological constant, there must be a source of uplifting in superstring theory.

First, we have investigated whether the chiral superfield \( X \) which is coupled with the whole volume in the Kähler potential can stabilize the Kähler modulus or not. The mass of \( X \) is much heavier than the Kähler modulus and the VEV \( \langle X \rangle \) is almost negligible, but its F-term potential can affect the moduli potential. We assumed that the superpotential is written as \( W = W_0 - f(T)X \) and the Kähler potential is given as \( K = -2 \ln(V + \frac{\xi}{2}) + (T + \bar{T})^{-n}|X|^2 \). We showed that if \( f \) is a constant and \( n \) is equal to 3 or larger, the whole volume of the extra dimension can be successfully stabilized. When \( n = 2 \), the modulus potential has a non-trivial stationary point, but its value is too small for the leading-order approximation in terms of \( \alpha' \) of the Kähler potential. If the higher-order corrections remain subdominant, our moduli stabilization mechanism works well even for \( n = 2 \).

If \( f \) is generated non-perturbatively, i.e. \( f = f(T) \), the modulus potential has no stationary points and the Kähler modulus cannot be stabilized. Thus, the form of the superpotential is very important for the modulus stabilization. We also studied the condition for heavy \( X \). The mass scale of \( X \) must be heavier than that of the Kähler modulus. We explicitly show the parameter set satisfying the consistency conditions for \( n = 3 \).

However, our moduli stabilization scenario has a few drawbacks. To stabilize the axions and other Kähler moduli, extra moduli stabilization mechanisms are required. The energy density of the stationary point is positive definite, and it must be depressed by another mechanism. Hence, it is important to consider this moduli stabilization mechanism in collaboration with another one. In this paper we consider it with the Large Volume Scenario. In this case, the chiral superfield \( X \) plays as a source of the F-term uplifting.

In the original LVS, the vacuum energy is uplifted by anti-D-branes. In this paper we studied F-term uplifting of the LVS vacuum by the chiral superfield. We found that F-term uplifting requires a certain form of the superpotential too. We need the \( T_i \)-independent \( fX \) term in the superpotential. If such a superpotential is induced, the vacuum energy of the LVS can be uplifted to the Minkowski vacuum (or de Sitter vacuum) by fine tuning the prefactor \( f \).
In both cases, the form of the superpotentials is crucial. The superpotential including $X$ is written as

$$ W = W_0 - f(T)X. \quad (4.1) $$

$f$ must be a constant for the moduli stabilization and the F-term uplifting, otherwise it destabilizes the moduli stabilization mechanism completely. For example, such a constant prefactor may be induced by non-perturbative effects on a D3-brane (or D(-1)-brane instanton), since it is suppressed by $e^{-S}$, where $S$ is the dilaton. $S$ is stabilized by a three-form flux at the tree level, and it can be substituted by its VEV. Another possibility is non-perturbative effects on D7-branes (D3-brane instantons) wrapping four-cycles whose sizes are already stabilized by other effects. Such a mechanism may be provided by D-terms in magnetized D-branes\(^{10}\) or other flux effects. Studying a concrete origin for such a superpotential in superstring theory would be interesting.

Acknowledgements

This work is supported by Ministry of Education, Culture, Sports, Science and Technology (MEXT) KAKENHI Grant Number JP17H05395 (TK), and Japan Society for the Promotion of Science (JSPS) Grant-in-Aid for Scientific Research 18J11233 (THT).

Funding

Open Access funding: SCOAP3.

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\(^{10}\) See, e.g., Ref. [27].
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