Experimental Requirements to Determine the Neutrino Mass Hierarchy Using Reactor Neutrinos

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This paper presents experimental requirements to determine the neutrino mass hierarchy using reactor neutrinos. The detector shall be located at a baseline around 58 km from the reactor(s) to measure the energy spectrum of electron antineutrinos ($\bar{\nu}_e$) precisely. By applying Fourier cosine and sine transform to the L/E spectrum, features of the neutrino mass hierarchy can be extracted from the $|\Delta m^2_{31}|$ and $|\Delta m^2_{21}|$ oscillations. To determine the neutrino mass hierarchy above 90% probability, requirements to the baseline, the energy resolution, the energy scale uncertainty, the detector mass and the event statistics are studied at different values of $\sin^2(2\theta_{13})$.

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Neutrino physics has undergone a revolution over the last decade and reaches now to an era of precision measurement for the neutrino oscillation parameters. However, $\theta_{13}$, CP-violating phase and the sign of $\Delta m^2_{32}$ (mass hierarchy) are still undetermined. Usually normal hierarchy (NH) is defined as $\Delta m^2_{32} > 0$ and inverted hierarchy (IH) is defined as $\Delta m^2_{32} < 0$. Accelerator neutrino experiments such as Nova [1, 2, 3] and T2KK [4] have the potential to determine the mass hierarchy using the matter effect of neutrinos at long baselines. There are also discussions to precisely measure the distortions of the reactor neutrino energy spectrum at an intermediate baseline (40 - 65 km) [5, 6]. A Fourier transform method was recently proposed to enhance and visualize the features of mass hierarchy in the frequency ($\Delta m^2$) spectrum [7].

A new study [8] based on Fourier transformation utilizing both the amplitude and phase information is presented recently to enhance the features distinguishing the mass hierarchy at a very small $\sin^2(2\theta_{13})$ value. This paper is complimentary to Ref. [8] by taking into account experimental details, such as the baseline, detector response including energy resolution, energy scale uncertainty, and the event statistics, etc. The study is based on Monte Carlo simulation.

Taking into account the detector response, the reactor neutrino $\bar{\nu}_e$ L/E spectrum $F(L/E)$ becomes

$$F(L/E') = \int R(E, E')F(L/E)dE,$$

$$F(L/E) = \phi(E)\sigma(E)P_{ee}(L/E),$$

where $L$ is the baseline, $E$ is the actual $\bar{\nu}_e$ energy, $E'$ is the observed $\bar{\nu}_e$ energy taking into account the detector response, and $R(E, E')$ represents the detector response including effects such as the energy resolution and energy scale. The reactor neutrino flux, $\phi(E)$, the neutrino inverse beta reaction cross section with detector, $\sigma(E)$, and the neutrino oscillation probability, $P_{ee}(E)$, have all been described in Ref. [8]. Here for completeness, we rewrite the $\bar{\nu}_e$ survival probability $P_{ee}(E)[8]$ as:

$$P_{ee}(L/E) = 1 - P_{21} - P_{31} - P_{32},$$

where $P_{21} = \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2(\Delta_{21})$, $P_{31} = \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{31})$, $P_{32} = \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{32}),$ (2)

The analytical formulas for Fourier cosine and sine transform [8] are:

$$FCT(\omega) = \int_{t_{\text{min}}}^{t_{\text{max}}} F(t)\cos(\omega t)dt,$$

$$FST(\omega) = \int_{t_{\text{min}}}^{t_{\text{max}}} F(t)\sin(\omega t)dt,$$ (3)

where $\omega$ is the frequency defined as 2.54$\Delta m^2$; $t = L/E$ is the variable in L/E space, varying from $t_{\text{min}} = L/E_{\text{max}}$ to $t_{\text{max}} = L/E_{\text{min}}$. In real experiments with a set of discrete events, the integral can be changed to the summation over all events as in the following:

$$FST(\omega) = \sum_{i=1}^{N} \sin(\omega L/E'_i),$$

$$FCT(\omega) = \sum_{i=1}^{N} \cos(\omega L/E'_i),$$ (4)

where $E'_i$ is the measured energy of individual events, and $N$ is the total number of events collected.

The actual experimental measurements of the neutrino energy usually have two aspects of detector responses: energy resolution and energy scale. The response of the detector due to energy resolution can usually be described by a Gaussian function $\frac{1}{\sqrt{2\pi}\sigma_E}\exp(-\frac{(E' - E)^2}{2\sigma_E^2})$, where $\sigma_E$ is the energy resolution. Since the neutrino energy are usually measured by scintillators, the energy is typically proportional to the number of photoelectrons, and the error is dominated by the photoelectron statistics. Therefore the neutrino energy resolution is proportional to $1/\sqrt{E_{\text{vis}}}$, where $E_{\text{vis}} = E_\nu - 0.8$ MeV is the neutrino visible energy in the detector. Previous experiments typically have an energy resolution of about 10%/$\sqrt{E_{\text{vis}}}$. Different detectors may have different forms of the energy scale uncertainty. For simplicity, we take two possible
The observed neutrino event number is proportional to the detector volume, exposure time and reactor(s) power. A very powerful reactor complex can consist of 8 reactor cores, each with ~3 GW thermal power. With a baseline of 58 km from such a reactor complex and taking into account the oscillation probability which is around $\sin^2(2\theta_{13}) \sim 3\%$, corresponding to a detector exposure of ~700 kt\textperiodcentered year, is taken as the default for an experiment. 

![Fig. 1: FCT and FST spectra from simulation with parameters $(\sin^2(2\theta_{13}), \sigma_E)$ = $(0.02, 3\% / \sqrt{E_{vis}})$, together with the analytical spectra for $\sin^2(2\theta_{13}) = 0.02$. Solid and long-dashed lines are spectra based on simulation for NH and IH cases, while dashed and dotted lines are analytical spectra.](image)

The impacts of the energy resolution and statistical errors are obviously seen as that the amplitudes of noisy peaks and valleys appear to be higher in the frequency range of $2.0 \times 10^{-3} \text{eV}^2 < \Delta m^2 < 2.8 \times 10^{-3} \text{eV}^2$. However, the main peak and valley are distinctive and can still be used to determine the neutrino mass hierarchy.

We introduce parameters RL and PV [8] to quantify the features of FCT and FST spectra:

$$RL = \frac{RV - LV}{RV + LV}, \quad PV = \frac{P - V}{P + V} \quad (5)$$

where RV is the amplitude of the right valley and LV is that of the left valley in the FCT spectrum; P is the amplitude of the peak and V is that of the valley in the FST spectrum.

For each set of input parameters \{$\sin^2(2\theta_{13}), L, \sigma_E, a, b, N$\}, we simulate 500 experiments and calculate the probability to determine the mass hierarchy based on the distributions of RL and PV values. The procedure is concluded as the following:

1. Given $\sin^2 \theta_{13}$ and $L$, we sample $N$ neutrino events with energy $E_i (i = 1, 2, ..., N)$ from energy spectrum both for NH and IH cases.

2. $E_i$ is smeared and/or shifted to $E'_i$ based on the given energy resolution $(\sigma_E)$ and energy scale uncertainty $(a$ and $b)$ parameters.

3. FCT and FST spectra are calculated using Eq. (4).

4. RL and PV values are calculated based on FCT and FST spectra using Eq. (5).
5. Repeat the above steps 500 times and obtain the distributions of RL and PV values.

6. Calculate the probability to determine the mass hierarchy correctly based on the distributions of RL and PV values.

Fig. 2 shows the distribution of RL and PV values for 500 experiments with input parameters \( (\sin^2(2\theta_{13}), \sigma_E) = (0.02, 2\%/\sqrt{E_{vis}}) \). Two clusters of points in the (RL, PV) plane corresponding to NH and IH cases show the probability to determine the mass hierarchy. Various input parameters have been tried and the distribution of \( RL + PV \) is shown in Fig. 3. Two clusters of points turn into two Gaussian distributions and the probability to determine the mass hierarchy can be correctly calculated.

To study the impact of the baseline, a total of 500 experiments have been simulated for each set of input parameters \( (\sin^2(2\theta_{13}), \sigma_E) \). Fig. 4 shows the results. The error bars are due to statistics since only a limited number of experiments are simulated. The optimal baseline is clearly 58 km, which is chosen as the default baseline.

Fig. 5 shows the impact of event number to the determination probability. Obviously, fewer events will induce larger statistical fluctuations, more noisy peaks and valleys in the FCT and FST spectra and hence reduce determination probability. As shown in Fig. 6, a total of \( 5 \times 10^5 \) events will reach 90% determination probability for \( \sin^2(2\theta_{13}) = 0.02 \) with an energy resolution of \( 3\%/\sqrt{E_{vis}} \). This number of events, probably the largest can be imagined nowadays, is chosen as the default in this paper.

The requirement to the event statistics strongly depends on the value of \( \sin^2(2\theta_{13}) \). Fig. 7 shows the number of neutrino events needed to determine the mass hierarchy at the 90% confidence level as a function of \( \sin^2(2\theta_{13}) \). Two cases of the energy resolution, \( 2\%/\sqrt{E_{vis}} \) and \( 3\%/\sqrt{E_{vis}} \), are studied. If \( \sin^2(2\theta_{13}) \) happens to be more than 0.05, as some of the recent global fit indicated, the number of events can be a factor of 5 smaller than that in the case of \( \sin^2(2\theta_{13}) = 0.02 \).

Impact of the energy resolution to the mass hierarchy determination is studied for the cases of \( \sin^2(2\theta_{13}) = 0.02, 0.01 \) and 0.005 as shown in Fig. 7. To achieve the mass hierarchy determination probability better than 90% at \( \sin^2(2\theta_{13}) = 0.02 \), the energy resolution shall be better than \( 3\%/\sqrt{E_{vis}} \). This is actually a very stringent requirement, at least a factor of two better than that of the existing reactor neutrino experiments. For a typi-
which results in the frequency spectra (FCT spectra) left shifted. The energy scale uncertainty only introduces a bias to the oscillation frequency and hence $\Delta m^2_{31}$ (shown as the main peak in the FCT spectrum). Since our
cal liquid scintillator experiment, substantial more light shall be collected to reach such a level.

Fig. 5 shows the impact of $\sin^2(2\theta_{13})$ to the determination probability in four cases of the energy resolution \{0/$\sqrt{E_{\text{vis}}}$, 1/$\sqrt{E_{\text{vis}}}$, 2/$\sqrt{E_{\text{vis}}}$, 3/$\sqrt{E_{\text{vis}}}$\}. It shows that the energy resolution is very important to determine the mass hierarchy, while a larger $\sin^2(2\theta_{13})$ can relax substantially such a requirement.

The impact of the energy scale uncertainty is studied by transforming the sampled neutrino energy $E$ to $E' = (1 + a')E + b$. For two cases of $a = -1\%$ or $b = -0.01$ MeV, which correspond to the shrinking or left-shift of the neutrino energy spectrum, the FCT spectra are calculated and shown in Fig. 6. It shows that the FCT spectra, both for NH and IH cases, are left shifted. After shrinking the energy spectrum, the L/E spectrum expands and the oscillation frequency becomes smaller, method only depends on the relative position of peaks or valleys in FCT and FST spectra, the mass hierarchy determination is not affected by the energy scale uncertainty.

In summary, we have studied experimental requirements to determine the mass hierarchy using Fourier cosine and sine transform to the reactor neutrino L/E spectrum. The parameters RL and PV are defined to extract features of the Fourier sine and cosine spectra, and the mass hierarchy can be determined from events collected in experiments similar to that in the analytical case. The impacts of baseline, event statistics, energy resolution and energy scale uncertainty to the mass hierarchy determination are studied in detail. This paper provides a guidance to the design of the experiment to determine the mass hierarchy using reactor neutrinos.

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[2] D. S. Ayres et al. [NOvA Collaboration],
FIG. 9: FCT spectrum for an energy shift of 0.01 MeV and shrinking of 1%.