STABILITY OF SCALE-ININVARIANT COSMOLOGICAL CORRELATION FUNCTIONS IN THE STRONGLY NONLINEAR CLUSTERING REGIME

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Received 1997 January 21; accepted 1997 October 16

ABSTRACT

We have investigated the stability of the scale-invariant solutions of the BBGKY equations for two-point spatial correlation functions of density fluctuations in the strongly nonlinear regime. In the case in which the background skewness of the velocity field equals 0, we found that there is no local instability; i.e., the perturbations do not grow nor decay, the solutions are marginally stable. This result means that no special values of the power index of the two-point spatial correlation function are favored in terms of the stability of the solutions. In other words, the argument about the stability does not determine the power index of the two-point spatial correlation functions.

Subject headings: cosmology: theory — large-scale structure of universe

1. INTRODUCTION

Large-scale structure formation is one of the most important problems in cosmology. It is generally believed that these structures have been formed as a result of gravitational instabilities. Hence, it is very important to clarify the evolution of density fluctuations by a gravitational instability. Here we consider density fluctuations of collisionless particles, for example, dark matter, our interest being mainly concentrated on the effect of the self-gravity.

When the density fluctuations are much smaller than unity, their time evolutions can be analyzed by making use of the linear theory. In this linear regime, we can understand analytically how the small fluctuations grow (Peebles 1980, 1993). On the contrary, when the density fluctuations are much larger than unity, that is, in the strongly nonlinear regime, the analytical approach is very difficult. We believe, however, that it is necessary to understand clearly the nonlinear behavior of the density fluctuations on these strongly nonlinear scales. Galaxy formations are indeed much related to the density fluctuation on these small scales, and it is also a very interesting academic problem in the nonlinear dynamics of the self-gravity.

We generally use two-point spatial correlation functions of density fluctuations to quantify the clustering pattern. In the strongly nonlinear regime, it is found from N-body simulations that correlation functions obey the power law. This result is reasonable because the self-gravity is scale-free. Two-point spatial correlation functions have been investigated by using various methods, the correlation function power index being a good indicator of the dynamics of the self-gravity in this regime.

The power index of the two-point spatial correlation functions has been usually analyzed by using N-body simulations (Frenk, White, & Davis 1983; Davis et al. 1985; Suto 1993 and references therein). However, the physical process that determines the value of the power index cannot be clarified only by N-body simulations. There are some other methods besides those numerical simulations. One of them is the analysis by using the BBGKY equations. The work by Davis & Peebles (1977, hereafter DP) is a pioneer in that field. They showed first the existence of self-similar solutions for spatial correlation functions under some assumptions and that the power index γ of the correlation function in the strongly nonlinear regime is related to the index n of the initial power spectrum P(k) as follows:

\[
\xi(r) \propto r^{-\gamma} \quad \xi \gg 1: \gamma = \frac{3(3 + n)}{5 + n}. \tag{1}
\]

One of the assumptions that DP adopted is the stability condition that says that the mean relative physical velocity in the strongly nonlinear regime is equal to 0. It was tested by N-body simulations (Efstathiou et al. 1988; Jain 1995, 1997), but this condition is not completely verified. Furthermore, the scale-invariant solutions in the strongly nonlinear regime, which DP derived, were confirmed to be marginally stable by using the linear perturbation theory (Ramaswayer \\& Fy 1992, hereafter RF).

Other analyses besides the one proposed by DP were conducted in the past. One of them is given by Saslaw (1980). By using the cosmic energy equation under some assumptions, he concluded that the power index γ approaches 2. Some numerical simulations, however, do not support this result (Frenk et al. 1983; Davis et al. 1985; Fry \\& Melott 1985).

Another method can be followed. When the initial power spectrum has a sharp cutoff or when it is scale-free with negative and small initial power index, then caustics appear all over the density field. In these cases, after the first appearance of caustics, the power index becomes independent of the initial conditions on scales on the order of typical caustic thickness (in three-dimensional systems, they correspond to the pancake structures of highly clustered matter). The power index is determined by the type of these caustics classified in accord with the catastrophe theory. This idea is veriﬁed in one-dimensional systems (Kotok & Shandarin 1988; Gouda \\& Nakamura 1988, 1989), spherically symmetric systems (Gouda 1989), two-dimensional systems (Gouda 1996a), and also three-dimensional systems (Gouda 1996b). In these cases, it is suggested that γ ≈ 0 on small scales.

As we can see from the above different values of the power index γ, there are still uncertainties about the physical processes that determine the value of the power index.

By analyzing the scale-invariant solutions of the BBGKY equations, Yano \\& Gouda (1997, hereafter YG) investigated in the strongly nonlinear regime the conditions that deter-
mine the power index of the two-point spatial correlation function. We do not adopt the following DP assumptions, which are (1) the skewness is equal to 0, (2) the three-point spatial correlation function is represented by a product of two-point spatial correlation functions, and (3) the stability condition is satisfied. As a result, YG obtained a relation between a mean relative peculiar velocity, the skewness, the three-body correlation function, and the power index of the two-point spatial correlation function. YG found that the stability condition is satisfied only when both assumptions 1 and 2 are satisfied. As they are not satisfied in all cases, there is no absolute guarantee that the stability condition is assured. Furthermore, YG give, from the physical point of view, the probable range of the mean peculiar velocity that includes the value given by the stability condition. This fact results in the possibility that the power index of the two-point spatial correlation function takes various values according to the mean peculiar velocity that can indeed have a value between 0 (stable clustering) and the Hubble expansion velocity (comoving clustering). When the stable clustering picture is adopted and the self-similarity satisfied, the power index of the two-point spatial correlation function has the value derived by DP. On the other hand, when the comoving clustering picture is adopted and the self-similarity satisfied, the power index becomes 0, which is consistent with the expectation from the catastrophe theory (Gouda & Nakamura 1988, 1989; Gouda 1989, 1996a, 1996b). Although we found that there exist various scale-invariant solutions with different power indices, whether these solutions are stable or not is another interesting problem. As a matter of fact, the values of the power index leading to unstable solutions cannot represent processes taking place in the real world. RF investigated the stability of the DP solutions with the help of the linear perturbation. They showed in this way that the perturbations of the solutions were only marginally stable. Yet they did not investigate other solutions as obtained by YG. Furthermore, as we will discuss later, RF misunderstood the way to perturb the skewness. (Their result is correct, though, because of reasons we will explain later [§ 3]). Besides, they did not comment about the "strange" growing mode that exists in their solutions of the perturbation. We will show that this "strange" mode results from the fact that RF consider inapplicable perturbations that diverge on small or large scales. So, in this paper, we will investigate the stability of the general solutions that YG obtained by applying an acceptable form of perturbation. We will derive the perturbation equation by perturbed BBGKY equations from the background scale-invariant solutions in the strongly nonlinear regime. And we will show the solution to the linear perturbation equations when we assume a well-defined appropriate form of perturbation.

In § 2, we will briefly show the BBGKY equations that we use in this paper and also the scale-invariant solutions, in the strongly nonlinear regime, obtained by YG. In § 3, we will consider the stability of these solutions of the BBGKY equations by the analysis of the linear perturbation. Finally, we will devote § 4 to conclusions and discussions.

2. BASIC EQUATIONS AND THE SCALE-ININVARIANT SOLUTIONS

At first, we will show the BBGKY equations that we use in this paper. These equations are time evolution equations of statistical functions such as the two-body correlation function, the three-body correlation function, and so on. These equations can be derived from the ensemble mean of the Vlasov equation (DP; RF). The $N$th BBGKY equation represents the time evolution of the $N$-body correlation function. Because we are interested in the two-body correlation function, we analyze the second BBGKY equation by extracting its momentum moments. The zeroth moment of the second BBGKY equation is the time evolution equation of the two-point spatial correlation function. This evolution equation involves the first moment term, that is, the term of the mean relative peculiar velocity. The first moment of the second BBGKY equation is the time evolution equation of the mean relative peculiar velocity. This equation again involves the second moment term of the relative peculiar velocity dispersion, respectively (DP; RF; YG). The second moment of the second BBGKY equation is the time evolution equation of the relative peculiar velocity dispersion. These equations involve the skewness of the velocity field in the same way. In deriving the BBGKY equations, DP assumed that the skewness is equal to 0 and also the stability condition in which the mean relative peculiar physical velocity is equal to 0 ($\langle v \rangle = -d \xi$). Furthermore, DP assumed that a three-point spatial correlation function can be represented by a product of the two-point spatial correlation function as follows:

$$\zeta_{123} = Q(\xi_{12} \xi_{23} + \xi_{23} \xi_{31} + \xi_{31} \xi_{12}).$$

On the other hand, RF incorporated the skewness in the equations. RF expressed the skewness by using the values $A$ and $B$. These are related to our formulation by the next relations:

$$A(3\pi_{RF} + v^3) = 3\langle v \rangle + s_1, \quad Bv = \langle v \rangle + s_1, \quad (3)$$

where $v$ and $\Sigma$ in RF are the same as $\langle v \rangle$ and $\Sigma$ in our paper. Their definition of $\Pi_{RF}$ is different from ours ($\Pi_{RF} + v^3 = \Pi$). Here $s_1$ and $s_2$ are, as we will define later, the parallel component and the transverse component of the skewness, respectively. When we investigate the scale-invariant solutions of the BBGKY equations, the difference of the definition of the variables in RF's paper and ours is not important. But when we perturb each variable, the skewness also must be perturbed independently. However, it must be noted that RF treated $A$ and $B$ as constant values. Thus, RF did not deal correctly with the perturbation of the skewness. RF employed the same assumption, used by DP, concerning the three-point spatial correlation function. Furthermore, they also used the stability condition. We do not know whether the assumption of the three-point spatial correlation function is correct or not. Therefore, we assume that the three-point spatial correlation function has the following form:

$$\zeta_{123} = Q(\xi_{12}^{1+ \delta} \xi_{23} + \xi_{23}^{1+ \delta} \xi_{31} + \xi_{31}^{1+ \delta} \xi_{12}), \quad (4)$$

where $\delta$ is a constant value. This is not a general form, but an extension of the form adopted by DP. We use this form as a preliminary step in our analysis. Furthermore, because the stability condition is a special case of the mean relative peculiar velocity, we do not assume it, and we consider instead the general condition shown in YG. Because we are interested in the strongly nonlinear regime, we take the nonlinear approximation. In this regime ($x \ll 1$), the two-
point spatial correlation function is much larger than unity, $\xi > 1$, and we obtain the following four equations (DP; RF; YG):

Zeroth moment:

$$\frac{\partial \xi}{\partial t} + \frac{1}{a} \frac{\partial}{\partial x} \left( x^2 \xi \langle v \rangle \right) = 0 . \tag{5}$$

First moment:

$$\frac{1}{ax^2} \frac{\partial}{\partial x} \left( x^2 \xi \Pi \right) = \frac{2\xi \Sigma}{ax} + 2GmnaQ \frac{\chi^d}{x} \times \int \frac{x^d}{x_3^5} [\xi(x)(1+\delta) + \xi(z)(1+\delta)] \xi(z-x)(1+\delta) d^3x_3 = 0 . \tag{6}$$

Second moment (contraction 1):

$$\frac{1}{ax^2} \frac{\partial}{\partial x} \left( x^2 \xi \Pi \right) + \frac{1}{ax^2} \frac{\partial}{\partial x} \left[ x^2 \xi \langle v \rangle \Pi + s_j \right] = \frac{4\xi}{ax} \left( \langle v \rangle \Sigma + s_j \right) + 4GmnaQ^* \frac{\chi^d}{x^2} \int \frac{x^d}{x_3^5} [\xi(x)(1+\delta) + \xi(z)(1+\delta)] \xi(z-x)(1+\delta) \langle v \rangle d^3x_3 = 0 . \tag{7}$$

Second moment (contraction 2):

$$\frac{1}{ax^2} \frac{\partial}{\partial x} \left( x^2 \xi \Sigma \right) + \frac{1}{ax^2} \frac{\partial}{\partial x} \left[ x^2 \xi \langle v \rangle \Sigma + s_j \right] = 0 . \tag{8}$$

In the preceding equations, $G$ is the gravitational constant, $m$ is the mass of a particle, $\bar{n}$ is the mean number density of the particles, and $Q^*$ is the coefficient of the first momentum of the three-body correlation function (RF; YG). As YG commented, the fourth term of equation (8) is not always satisfied, although it is correct in the strongly nonlinear regime. However, DP showed the existence of a three-body correlation function that gives this term. As we will see later, the condition that $Q^* \neq Q$ is consistent with the zero skewness case (see eq. [43]). Since we consider the case of the zero skewness, we assume that the fourth term of equation (8) is satisfied in our analysis.

We express the skewness by the following expression:

$$s^{\xi} \equiv \langle (v - \langle v \rangle)^3 (v - \langle v \rangle)^3 (v - \langle v \rangle)^3 \rangle = s_{||} P_{ppp}^{\xi} + s_{\perp} P_{pt}^{\xi} , \tag{9}$$

$$P_{ppp}^{\xi} = \frac{x^d x^d x^d}{x^3} , \quad P_{pt}^{\xi} = \frac{x^d}{x} \delta^d + \frac{x^d}{x} \delta^d + \frac{x^d}{x} \delta^d - 3 \frac{x^d x^d x^d}{x^3} . \tag{10}$$

where the subscripts $p$ and $t$ represent the parallel and transverse component of each two particles, respectively. Here $P_{ppp}$ and $P_{pt}$ vanish because of the symmetry of the background universe. On the other hand, we consider an Einstein–de Sitter universe because we are interested only in scale-invariant correlations.

Here we consider the scale-invariant solutions of these equations. In the strongly nonlinear regime, it is naturally expected that the effect of the nonlinear gravitational clustering dominates and that the solutions in this regime have no characteristic scales; that is, they are expected to obey a power law because of the scale-free nature of the gravity. Then we investigate the power-law solutions of $\xi, \langle v \rangle, \Pi, \Sigma$ (YG). We assume that the two-point spatial correlation function $\xi$ is given by

$$\xi = \xi_0 a^b x^{-\gamma} . \tag{11}$$

Then we obtain from the dimensional analysis of equation (5)

$$\langle v \rangle = - \bar{h} \bar{x} \right) , \quad \beta = (3 - \gamma) \bar{h} . \tag{12}$$

In this case, the solutions of the other variables are given by

$$\Pi = \Pi_0 a^{(1 + \delta)} - 1 x^{2 - \gamma(1 + \delta)} , \quad \Sigma = \Sigma_0 a^{(1 + \delta)} - 1 x^{2 - \gamma(1 + \delta)} , \quad s_{||} = s_{||} a^{(1 + \delta)} - 1 x^{3 - \gamma(1 + \delta)} , \quad s_{\perp} = s_{\perp} a^{(1 + \delta)} - 1 x^{3 - \gamma(1 + \delta)} . \tag{13}$$

From equation (9), it is found that

$$2\beta(1 + \delta) + 1 - [7 - 2\gamma(1 + \delta)](h - \Delta) = 0 , \tag{14}$$

and

$$h = \frac{1 + [7 - 2\gamma(1 + \delta)]\Delta}{1 - 6\delta} , \tag{15}$$

$$\Delta \equiv \frac{s_{\perp} \Delta_0}{s_{||} \Sigma_0} . \tag{16}$$

As we can see from equation (16), the parameter $h$ can take various values according to the skewness $\Delta$, the power index $\gamma$, and $\delta$. Only when $\Delta = \delta = 0$ is satisfied is the stability condition ($h = 1$) is correct. If the similarity solutions exist, the power index of the two-point spatial correlation function can be represented by the following form (Padmanabhan 1995; YG):

$$\gamma = \frac{3(3 + n)h}{2 + (3 + n)h} . \tag{17}$$

Even when we assume that self-similarity solutions exist, the power index of the two-point spatial correlation function can take various values depending on the mean relative peculiar velocity, $h$ (if the stability condition is satisfied, i.e., $h = 1$, the result of DP [eq. (1)] can be reproduced).

3. LINEAR STABILITY OF SCALE-INvariant SOLUTION

As reproduced in the previous section, we showed that there are various scale-invariant solutions in addition to the DP’s solutions (YG). But there is no guarantee that all the solutions are stable. We investigate thus the stability of these solutions by making use of the linear perturbations theory. With this aim in view, the two-point spatial correlation function in the scale-invariant solution is perturbed as follows:

$$\xi' = \xi(1 + \Delta_\xi) , \tag{18}$$

where $\xi$ is the scale-invariant solution, $\xi'$ the perturbed one and $\Delta_\xi \ll 1$. We also perturb the other variables such as the mean relative peculiar velocity $\langle v \rangle$, the relative peculiar velocity dispersions $\Pi, \Sigma$ and the skewness $s_{||}, s_{\perp}$ in the same way.
The equations of the linear perturbations are the following:

Zeroth moment:

\[
\frac{\partial \xi}{\partial t} + \frac{1}{ax^2} \frac{\partial}{\partial x} \left[ x^{2} \frac{\partial \xi}{\partial x} + \Delta_{\xi} + \Delta_{\xi}(\varepsilon) \right] = 0 .
\]  

(19)

First moment:

\[
\frac{1}{ax^2} \frac{\partial}{\partial x} \left[ x^{2} \xi \xi(\Delta_{\xi} + \Delta_{\xi}(\varepsilon)) \right] = \frac{2 \xi \xi}{ax} (\Delta_{\xi} + \Delta_{\xi}(\varepsilon)) + 2Gm\xi Qx \xi^{2}(\varepsilon + \Delta_{\xi}) (1 + \varepsilon) M_{\xi}(1 + \varepsilon, q) \Delta_{\xi} = 0 .
\]  

(20)

Second moment (contraction 1):

\[
\frac{1}{ax^2} \frac{\partial}{\partial x} \left[ a^{2} \xi \xi(\Delta_{\xi} + \Delta_{\xi}(\varepsilon)) + \frac{1}{a^{2}} \frac{\partial}{\partial x} \right] 
\times \left\{ x^{2} \xi \left[ \left\langle \frac{\partial}{\partial x} \right\rangle (\Delta_{\xi} + \Delta_{\xi}(\varepsilon)) + \Delta_{\xi}(\varepsilon) + S_{\xi} (\Delta_{\xi} + \Delta_{\xi}(\varepsilon)) \right] \right\} = 0 .
\]  

(21)

Second moment (contraction 2):

\[
\frac{1}{ax^2} \frac{\partial}{\partial x} \left[ a^{2} \xi \xi(\Delta_{\xi} + \Delta_{\xi}(\varepsilon)) + \frac{1}{a^{2}} \frac{\partial}{\partial x} \right] 
\times \left\{ x^{2} \xi \left[ \left\langle \frac{\partial}{\partial x} \right\rangle (\Delta_{\xi} + \Delta_{\xi}(\varepsilon)) + \Delta_{\xi}(\varepsilon) + S_{\xi} (\Delta_{\xi} + \Delta_{\xi}(\varepsilon)) \right] \right\} = 0 .
\]  

(22)

Here it should be noted that in deriving the above equations (20)–(23) we neglect some terms: higher-order terms in the strongly nonlinear limit and higher order modes in the limit of small perturbations (larger than the first-order perturbation). The ordering parameters in both limits are generally independent of each other. However, we consider that higher order modes in the strongly nonlinear limit are much smaller than the first-order modes of the perturbation. Then we can derive the above equations by neglecting higher order terms in the nonlinear approximation. This case means that we neglect the effects of the higher order terms in the strongly nonlinear approximation in considering the linear perturbation.

RF considered the following power-law perturbation:

\[
\Delta_{\xi} = \varepsilon_{\Delta} a^{\theta} x^{\theta} .
\]  

(23)

In this case, \( M_{\xi}(1 + \varepsilon) \) and \( M_{\xi}(1 + \varepsilon, q) \) are given by

\[
M_{\xi}(1 + \varepsilon) = \int \frac{\mu}{y^2} s^{-\gamma(1 + \varepsilon)} (1 + y^{-\gamma(1 + \varepsilon)}) d^3 y ,
\]  

(24)

\[
M_{\xi}(1 + \varepsilon, q) = \int \frac{\mu}{y^2} [s^{-\gamma(1 + \varepsilon)} (1 + y^{-\gamma(1 + \varepsilon)}) + \gamma(1 + \varepsilon) \Delta_{\xi}] d^3 y ,
\]  

(25)

\[
\mu = \frac{x^2}{x}, \quad \gamma = \frac{3}{\alpha}, \quad s = \frac{2x^2}{x} = (1 + y^2 - 2y\mu)^{1/2} .
\]  

(26)

The integrals should not diverge. Then, \( 2 - \gamma(1 + \varepsilon) > 0 \) must be satisfied for \( y \to 0 \) and \( \gamma(1 + \varepsilon) > 0 \) for \( y \to \infty \) in the \( M \). Furthermore, \( 2 + q - \gamma(1 + \varepsilon) > 0 \) must be satisfied for \( y \to 0 \) and \( q - \gamma(1 + \varepsilon) < 0 \) for \( y \to \infty \) in the \( M' \). As a result, the following relations must be satisfied:

\[
0 < \gamma(1 + \varepsilon) < 2 , \quad \gamma(1 + \varepsilon) - 2 < q < \gamma(1 + \varepsilon) .
\]  

(27)

The four perturbation equations (20)–(23) are a little different from RF’s equations. This is because in perturbing the three-body correlation term, RF divide artificially the \( \left< v_{23} \right> \) into \( \left< v_{23} \right> \) and \( \left< v_{31} \right> \) and \( \left< v_{31} \right> \) independently, which is an incorrect treatment. Furthermore, although they obtained the \( q \)-dependent of \( M' \), they used the value of \( M' \) at only \( q = 0 \).

Here we consider the following form of perturbations. We have to consider perturbations that diverge neither at the strongly nonlinear limit \((x \to 0)\) nor at the linear limit \((x \to \infty)\). The following perturbations of the two-point spatial correlation function are generated:

\[
\Delta_{\xi} = \varepsilon_{\Delta} a^{\theta} x^{\theta} ,
\]  

(28)

\[
q = |q| i ,
\]  

(29)

where \( |q| \) is a real number \((q\) is a pure imaginary number while RF adopted the real number). In this case, the perturbations do not diverge in the strongly nonlinear limit or in the linear limit. If \( q \) is a real number, as RF adopted, the perturbations diverge on some scales. When \( q \) is negative, the perturbation diverges in the nonlinear limit \((x \to 0)\). On the other hand, when \( q \) is positive, the perturbation diverges in the linear limit \((x \to \infty)\). The perturbations of the other variables can also be written in the same form. From the dimensional analysis, all perturbations must have the same power \((q \) and \( p \) than the two-point spatial correlation function. When \( q \) is a pure imaginary number, \(|s| = 1\) and \(|y| = 1\), and the following relations are satisfied:

\[
|M_{\xi}(1 + \varepsilon, q)| \leq M_{\xi}(1 + \varepsilon, q = 0) = 2M_{\xi}(1 + \varepsilon) .
\]  

(30)

When \( 0 < \gamma(1 + \varepsilon) < 2 \), both \( M_{\xi}(1 + \varepsilon) \) and \( M_{\xi}(1 + \varepsilon, q) \) are finite for any value of the pure imaginary number \( q \).

These perturbed BBGKY equations are not closed by themselves in general and higher moment equations are needed. But if the coefficients of the perturbation of the skewness \((s_{\Delta} \) and \( s_{\Delta} \) ) are equal to 0, the perturbation equations for the other variables have no relation with the perturbation of it. In this case we can solve these perturbation equations independently from the higher moment equations. As we can see from equations (22) and (23), when the skewness of the background velocity field is equal to 0, the coefficient of the perturbation of the skewness becomes 0. Here we consider only this particular case. Then we can neglect the higher moment equations. RF treated \( A \) and \( B \) as constant values. So, their treatment of the perturbation of the skewness itself was conceptually wrong. However, when the skewness is equal to 0, RF’s treatment happen to be correct. In this case, the above equations can be rewritten by using the power-law perturbations given by equations (29) and (30):

\[
(p - hq)\Delta_{\xi} - h(3 - \gamma + q)\Delta_{\xi}(\varepsilon) = 0 ,
\]  

(31)

\[
2[\gamma(1 + \varepsilon) - 2 + \sigma](1 + 2\delta) + q + 2(1 + \delta)f_{\varepsilon} \Delta_{\xi}
\]  

\[
+ [4 - 2\gamma(1 + \varepsilon) + q] \Delta_{\xi}(\varepsilon) - 2\sigma \Delta_{\xi} = 0 ,
\]  

(32)
\[
\begin{align*}
&\left\{ 1 - 15h + (6 + 4\gamma)h(1 + \delta) + 4h\sigma + p - 3hq \\
&\quad - 8h \frac{Q^*}{\Omega} [\gamma(1 + \delta) - 2 + \sigma](1 + \delta) \\
&\quad - 4h \frac{Q^*}{\Omega} (1 + \delta)Dkq \right\} \Delta_t \\
&\quad + \left\{ -3h[5 - 2\gamma(1 + \delta) + q] \\
&\quad + 4h\sigma - 4h \frac{Q^*}{\Omega} [\gamma(1 + \delta) - 2 + \sigma] \right\} \Delta_{(v)} \\
&\quad + [1 - 15h + (6 + 4\gamma)h(1 + \delta) \\
&\quad + p - 3hq] \Delta_\Pi + 4h\sigma \Delta_2 = 0, \\
&(1 - h + 6h\delta + p - hq)\Delta_t - h[7 - 2\gamma(1 + \delta) + q] \Delta_{(v)} \\
&\quad + (1 - h + 6h\delta + p - hq) \Delta_2 = 0,
\end{align*}
\]

where

\[ D \equiv \gamma(1 + \delta) - 2 + \sigma, \quad \sigma = \frac{\Sigma}{\Pi}. \]

It is difficult to treat exactly \( M' \) as a function of \( q \). So, we approximate \( M' \) by a linear function of \( q \). When \( q = 0 \), \( M'/M \) is equal to 2. Then we use the following approximation of \( M'/M \) in the above equations (21) and (22):

\[ \frac{M'}{M} \equiv 2 + kq, \]

where

\[ k = \frac{\int (\mu/y^2)s^{-\gamma(1+\delta)}(1 + y^{-\gamma(1+\delta)}) \log s \\
\quad + s^{-\gamma(1+\delta)}y^{-\gamma(1+\delta)} \log y) d^3y}{\int (\mu/y^2)s^{-\gamma(1+\delta)}(1 + y^{-\gamma(1+\delta)}) d^3y} \]

The integrals \( \int (\mu/y^2)s^{-\gamma(1+\delta)}(1 + y^{-\gamma(1+\delta)}) d^3y \) is dominated around \( s \) and \( y \approx 1 \). Around \( s \) and \( y \approx 1 \), \( \log s \) and \( \log y \) have values of the order of 1. Therefore, \( k \approx 1 \).

Furthermore, we use the following relation that can be derived from the first moment equation (eq. [7]):

\[ 4 - 2\gamma(1 + \delta) - 2\sigma + 2Gm \eta^2 Qx^2 \frac{\sigma^{1+\delta}}{\Pi} M(\gamma + 1) = 0. \]

(38)

The four equations (32)-(35) can be rewritten by using the matrix notation:

\[ N_{ij} u_j = 0, \]

(39)

\[ u_j = (\Delta_t, \Delta_{(v)}, \Delta_\Pi, \Delta_2). \]

(40)

If there exists a nontrivial solution, the determinant of the matrix \( N_{ij} \) should be equal to 0. Now we consider the zero skewness case. The relation between the three-point spatial correlation function and the mean relative peculiar velocity equation (16) becomes

\[ \delta = \frac{h - 1}{6h}. \]

(41)

By using this relation, we can eliminate \( \delta \) in equations (33)-(35). Furthermore, from the first moment equation (7) and the second moment (contraction 1) equation (8), we obtain the following relation:

\[ (-h + 1 + 6h\delta)\Pi + [5 - 2\gamma(1 + \delta)] \frac{s}{\alpha x} - 4s \frac{s}{\alpha x} \]

\[ - 4Gm \eta^2 x^2 M(\gamma + 1) h(Q^* - Q) = 0. \]

(42)

As we can see, when the skewness is equal to 0, \( Q^* - Q \) must be 0; that is, \( Q^*/Q \) is equal to 1.

Then the determinant of \( N \) is given by

\[ \det N = \frac{1}{2}(p - hq)^2 f(q, \gamma, h, \sigma, kD), \]

(43)

where

\[ f(q, \gamma', h, \sigma, kD) = [9h + kD(7h - 1)]q^2 + (4 - 2\gamma' + 29h - 10\gamma' h - 2\sigma + 8h\sigma)q \\
+ kD(-3 + 21h - 6\gamma' h)q + 12 - 6\gamma' - 6\sigma \]

(44)

and

\[ \gamma' \equiv \gamma(1 + \delta). \]

(45)

Here \( f \) is a quadratic equation of \( q \). Now we investigate whether the equation \( f = 0 \) has real solutions or not. Since we do not know the value \( \gamma', h, \sigma, \) and \( kD \) in the strongly nonlinear regime, we treat these values as parameters. Here we consider the allowed range for these parameter. As we can see from equation (28), the parameter \( \gamma' \) must satisfy 0 < \( \gamma' < 2 \). Since we have considered \( \xi \gg \xi \) in the strongly nonlinear regime, \( \xi^{2(1+\delta)} \) should be of an order larger than \( \xi \), so that \( 2(1 + \delta) > 1 \) must be satisfied. In this case, \( h > \frac{1}{3} \) must be satisfied from equation (42). YG showed the probable range of the mean relative peculiar velocity and obtained that the mean relative physical peculiar velocity must lies between 0 and the Hubble expansion velocity. This means that the parameter \( h \) (relative velocity parameter) has a value between 0 and 1. So in this case, the parameter \( h \) should be in the range \( \frac{1}{3} < h < 1 \). We do not know the range of the parameters \( \sigma \) and \( kD \). But the parameter \( \sigma \) should have a value around 1. Hence, we investigate \( \sigma \) in the range \( \frac{1}{3} < \sigma < 2 \). The value of the parameters \( D \) and \( k \) must also be around 1 as seen from the equations (36) and (38), respectively. So we investigate \( kD \) in the range \( -1 < kD < 1 \).

In the above probable value of the parameters, we can easily ascertain that \( f = 0 \) has real solutions. In other words, the solutions of \( f = 0 \) are not complex. Since we consider the case that \( q \) is an imaginary number, \( f = 0 \) can not be satisfied. \( p - hq \) should be equal to 0 to allow the determinant of the matrix \( N \) to be null. In this case, \( p \) has the following value:

\[ p = hq = h|q| i. \]

(46)

This means that the perturbations do not grow. And the solutions are stable. Furthermore we consider the strict condition that determines the stability of the two-point spatial correlation function. In the strongly nonlinear regime, the mean relative peculiar velocity takes the value \( \langle v \rangle = -h\dot{x} \) depending on the process of clustering. In this case, the scale of \( \dot{a}x \) for the two particles whose mean comoving distance is \( x \) does not change, as we can see from the following relation:

\[ \frac{d}{dt}(\dot{a}x) = \dot{a}^{2}(ax + \dot{h}ax) = \dot{a}^{2}(\langle v \rangle + \dot{h}ax) = 0. \]
Then we should determine the stability of the two-point spatial correlation function at the fixed scale $a^3x$. The solutions of the perturbations are rewritten as

$$\Delta_x = \epsilon_x (a^3 x)^q = \epsilon_x e^{q \log (a^3 x)}. \quad (48)$$

These perturbations do not grow, nor do they decay. At the fixed scale of $a^3 x$, the perturbation never even oscillates. This means that the perturbations are marginally stable. RF used the real number $q$ in investigating the behavior of the perturbations. As we can see from equation (49), the perturbation in the $p - hq = 0$ mode works well even when $q$ is a real number. That is, the perturbation is marginally stable in this mode. However, there also exist a "strange" growing mode for the real number of $q$ because $f = 0$ is satisfied in this case. RF did not comment sufficiently about this "strange" growing mode.

Besides, when $q$ is negative, the perturbations diverge in the nonlinear limit ($x \to 0$). On the other hand, when $q$ is positive, the perturbations diverge in the linear limit ($x \to \infty$). Those perturbations are thus not adequate in investigating the local stability of the two-point spatial correlation function in the strongly nonlinear regime.

4. RESULTS AND DISCUSSION

In this paper, we investigated the stability of the scale-invariant solutions of the cosmological BBGKY equations in the strongly nonlinear regime with the skewness of the velocity field being equal to zero. The reason we consider only the case is that perturbed BBGKY equations for $\xi, \langle v \rangle, \Pi$, and $\Sigma$ can be closed independently of the higher moment perturbations. When power-law perturbations are put in the solutions as investigated by RF, in other words, when $q$ is a real number, the perturbations in the linear limit or the nonlinear limit diverge. As a matter of fact, when $q$ is negative, the perturbations diverge and do not work in the nonlinear limit. When $q$ is positive, the perturbations behave well in the nonlinear regime. However, in the linear regime, those perturbations may diverge if the power-law form of the perturbations are retained, and so the form of the perturbations should be changed in order to avoid the divergence in the linear limit. In this case, it is insufficient to solve the nonlinear approximated equations, because we do not have information about the evolutions on large scales. Thus, we investigated only the local stability of the nonlinear regime. In investigating the local stability of the two-point spatial correlation function in the strongly nonlinear regime, we should put perturbations only on the scales in which we are going to investigate.

That is, we should put the wave packet-like perturbation. In order to put such a wave packet-like perturbation, the number $q$ must be imaginary. In this case, we found that there is no unstable mode. It seems stable for any value of the power index of the two-point spatial correlation function. However, we do not know whether a global instability exists because we consider only the local stability. It is certain that there is no local instability. So, in the strongly nonlinear regime, the solutions are marginally stable, and it does not seem that the power index of the two-point spatial correlation function approaches some stable point values. The power index of the two-point spatial correlation function that was derived by DP, $\gamma = 3(3 + n)/(5 + n)$, is not the special one also in terms of the stability of the solution. As a result, the argument of the stability does not determine the power index of the two-point spatial correlation function.

We are grateful to M. Nagashima for useful discussions. We would like to thank T. Tanaka for important comments and S. Ikeuchi and M. Sasaki for useful suggestions and continuous encouragement. We also thank E. Van Drom for reading carefully the manuscript of our paper. This work was supported in part by the grant-in-aid 06640352 for the Scientific Research Fund from the Ministry of Education, Science and Culture of Japan.

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