Observation of $D^0 - \bar{D}^0$ Mixing in $e^+e^-$ Collisions

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A weakly decaying flavored neutral meson is a two-state quantum system with an allowed transition between the two states. This transition is referred to as neutral meson mixing and originates from the difference between the flavor and mass eigenstates of the meson-antimeson system with a well-known rate depending on elements of the Cabibbo-Kobayashi-Maskawa matrix [1, 2]. Mixing phenomena are well established for $K^0$, $B^0$, and $B_s^0$ mesons and their mixing rates are consistent with predictions based on the standard model (SM) [3]. $D^0$ mixing has also recently been observed in hadron collider experiments [4, 5], confirming a previous $D^0 - D^{∗0}$ mixing signal [6] based mainly on combined evidence from three different experiments [7–9].

The phenomenology of meson mixing is described by two parameters, $x = \Delta m / \Gamma$ and $y = \Delta \Gamma / 2 \Gamma$, where $\Delta m$ and $\Delta \Gamma$ are the mass and width differences between the two mass eigenstates and $\Gamma$ is the average decay width of the mass eigenstates. While the finite mixing parameters where the experimental conditions are difficult to calculate [10, 11], which complicates the interpretation of experimental measurements against the SM. Nevertheless, it is still of great interest to improve the measurement of the $D^0$ mixing parameters to search for possible beyond-SM physics contributions [12]. It is also very valuable to confirm $D^0$ mixing in $e^+e^-$ collisions and provide further independent determinations of the $D^0$ mixing parameters where the experimental conditions are quite different from those in hadron collider experiments.

We observe $D^0 - \bar{D}^0$ mixing in the decay $D^0 \rightarrow K^+ \pi^−$ using a data sample of integrated luminosity 976 fb$^{-1}$ collected with the Belle detector at the KEKB $e^+e^−$ asymmetric-energy collider. We measure the mixing parameters $x^2 = (0.09 ± 0.22) \times 10^{-3}$ and $y^2 = (4.6 ± 3.4) \times 10^{-3}$ and the ratio of doubly Cabibbo-suppressed to Cabibbo-favored decay rates $R_D = (3.53 ± 0.13) \times 10^{-3}$, where the uncertainties are statistical and systematic combined. Our measurement excludes the no-mixing hypothesis at the 5.1 standard deviation level.

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experiments.

In this Letter, we report the first observation of $D^0$–$\bar{D}^0$ mixing from an $e^+e^-$ collision experiment by measuring the time-dependent ratio of the $D^0 \to K^+\pi^-$ to $D^0 \to K^-\pi^+$ decay rates. The consideration of charge-conjugated decays is implied throughout this Letter. We refer to $D^0 \to K^+\pi^-$ as wrong-sign (WS) and $D^0 \to K^-\pi^+$ as right-sign (RS) decays. We tag the RS and WS decays through the decay chain $D^{++}\to D^0(\to K^+\pi^\pm)\pi^\mp$ by comparing the charge of the $\pi$ from the $D^0$ decay and the charge of the low-momentum $\pi$, from the $D^{*+}$ decay. The RS decay amplitude is the sum of the amplitudes for Cabibbo-favored (CF) decay $D^0 \to K^-\pi^+$ and $D^0 \to \bar{D}^0$ mixing followed by the doubly-Cabibbo-suppressed (DCS) decay $D^0 \to K^+\pi^-$. Assuming charge-conjugation and parity (CP) conservation and that the mixing parameters are small ($|x| \ll 1$ and $|y| \ll 1$), the time-dependent RS and WS decay rates are

$$\Gamma_{\text{RS}}(i/\tau) \approx |A_{\text{CF}}|^2 e^{-\frac{t}{\tau}},$$

$$\Gamma_{\text{WS}}(i/\tau) \approx |A_{\text{CF}}|^2 e^{-\frac{t}{\tau}} \times \left( R_D + \sqrt{R_D y} \right)^2 \left( \frac{i}{\tau} \right)^2,$$

where $t$ is the reconstructed proper decay time and $R(t/\tau - \bar{i}/\tau)$ is the resolution function of the real decay time, $\bar{i}$.

The data used in this analysis are recorded at the $\Upsilon$(nS) resonances $(n = 1, 2, 3, 4, 5)$ or near the $\Upsilon$(4S) resonance with the Belle detector at the $e^+e^-$ asymmetric-energy collider KEKB [13]. The data sample corresponds to an integrated luminosity of 976 fb$^{-1}$. The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprising CsI(Tl) crystals (ECL) located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux return located outside the coil is instrumented to detect $K^0_S$ mesons and identify muons. A detailed description of the Belle detector can be found in Ref. [14].

We require that charged tracks originate from the $e^+e^-$ interaction point (IP) with an impact parameter less than 4 cm in the beam direction (the $z$ axis) and 2 cm in the transverse plane and have a transverse momentum greater than 0.1 GeV/$c$. All charged tracks are required to have at least two associated hits each in the $z$ and azimuthal strips of the SVD to assure good spatial resolution of the decay vertices of $D^0$ mesons. Charged tracks are identified as $K$ or $\pi$ candidates using the ratio of particle identification likelihoods, $P_{K/\pi} \equiv L_K/(L_K + L_\pi)$, reconstituted from the track-associated data in the CDC, TOF, and ACC. We require $P_{K/\pi} > 0.4$ for $K$, $P_{K/\pi} < 0.7$ for $\pi$, and $P_{K/\pi} < 0.9$ for $\pi_s$ candidates. The efficiency and $K/\pi$ misidentification rate of the $K$ selection are 91% and 12% and that of the $\pi$ selection are 94% and 18%. We also apply a loose electron veto criterion using the ECL information for all charged tracks. Oppositely-charged $K$ and $\pi$ candidates are combined to form a $D^0$ candidate by fitting them to a common vertex; the resulting $D^0$ candidate is fit to the IP to give the $D^{*+}$ vertex. A $D^{*+}$ candidate is reconstructed by combining a $D^0$ candidate—a $K\pi$ combination with invariant mass within $\pm 20$ MeV/$c^2$ (i.e., $\sim \pm 3\sigma$) of the nominal $D^0$ mass [3]—with a $\pi_s$. The $\pi_s$ is further constrained to pass through the $D^{*+}$ vertex. The sum of the reduced $\chi^2$ of the $D^{*+}$ vertex fit and $\pi_s$ fit to the $D^{*+}$ vertex is required to be less than 16.

There is a significant contribution to the WS sample from RS decays where both $K$ and $\pi$ candidates are misidentified as $\pi$ and $K$, respectively. We remove these with tighter particle identification requirements, $P_{K/\pi} > 0.99$ for $K$ and $P_{K/\pi} < 0.01$ for $\pi$, if $M(K\pi)$ is within $\pm 25$ MeV/$c^2$ of the nominal $D^0$ mass. To remove combinatorial background due to random unassociated charged track combinations that meet all the other requirements,
we require the $D^*$ meson momentum calculated in the center-of-mass system to be greater than 2.5, 2.6, and 3.0 GeV/c for the data taken below the $\Upsilon(4S)$, at the $\Upsilon(4S)$, and above the $\Upsilon(4S)$ resonance, respectively. This momentum requirement also removes $D^{*+} \to D^0\pi^+_s$ decays from $B$ meson decays, which do not give the proper decay time of the $D^0$ meson due to the finite $B$-meson lifetime.

The selection criteria described above are chosen by maximizing $R_{WS}N_{RS}^S / \sqrt{R_{WS}N_{RS}^S + N_{WS}^S}$, where $R_{WS}$ is the nominal ratio of WS to RS decay rates [3], $N_{RS}^S$ is the number of events in the RS signal region of the $D^{*+}-D^0$ mass difference, $\Delta M = M(D^{*+} \to D^0(\to K\pi)\pi^+_s) - M(D^0 \to K\pi)$, and $N_{WS}^S$ is that in the WS sideband regions of $\Delta M$. We define the signal region as $\Delta M \in [0.144, 0.147]$ GeV/c$^2$ and the background sidebands as $\Delta M \in [0.141, 0.142]$ or [0.149, 0.151] GeV/c$^2$. When counting $N_{RS}^S$, we subtract background candidates in the signal region using candidates in the RS sideband regions.

The measured $D^0$ proper decay time is calculated as $t = m_{D^0} \vec{L} \cdot \vec{\rho}/|\vec{\rho}|^2$ where $\vec{L}$ is the vector joining the decay and production vertices of the $D^0$, $\vec{\rho}$ is the $D^0$ momentum, and $m_{D^0}$ and $\tau$ are the nominal $D^0$ mass and lifetime [3]. We require the uncertainty on $t$ to satisfy $\sigma_t/\tau < 1.0$, and $t/\tau \in [-5, 10]$. These selections are determined from 5000 simplified simulated experiments by maximizing our sensitivity to the mixing parameters and minimizing the systematic biases in them.

Using these selections, we find no significant backgrounds in WS candidates that peak in the signal region from a large-statistics sample of fully simulated $e^+e^- \to$ hadrons events in our GEANT3-based [15] Monte Carlo (MC) simulation. Figure 1 shows the time-integrated distributions of $\Delta M$ from RS and WS candidate events after applying all the selections described above.

![FIG. 1: Time-integrated distributions for the mass difference of RS (left) and WS (right) candidates. Points with error bars are the data; full and dashed lines are, respectively, the signal and background fits described in the text.](image1)

The time-integrated RS signal shown in Fig. 1 is parametrized as a sum of Gaussian and Johnson $S_U$ [16] distributions with a common mean. The time-dependent RS signal in each bin of the proper decay time is fit with a Johnson $S_U$ only. The shapes of the WS signal are fixed using the corresponding RS signal shapes, and fit with only the signal normalization allowed to vary. The backgrounds in RS and WS decay events are fit independently and are parametrized with the form $(\Delta M - m_{\pi^+})^a e^{-b(\Delta M - m_{\pi^+})}$, where $a$ and $b$ are free fit parameters, and $m_{\pi^+}$ is the nominal mass of $\pi^+$ [3]. The fits give 2 980 710±1885 RS and 11 478±177 WS decays, giving an inclusive ratio of WS to RS decay rates of $(3.851 ± 0.059) \times 10^{-3}$. The uncertainty is statistical only.

We obtain the resolution function of Eq. (3) from the proper decay time distribution of RS decays after subtracting a small level of background events using the sideband regions defined above. This is shown in Fig. 2. We parametrize the proper decay time distribution of RS decays with the convolution of an exponential and a resolution function that is constructed as the sum of four Gaussians, $R(t/\tau) = \sum_{i=1}^{4} f_i G_i(t/\tau; \mu_i, \sigma_i)$, where $G_i$ is a Gaussian distribution with mean $\mu_i$ and width $\sigma_i$, and $f_i$ is its weight. The mean $\mu_i$ is further parametrized with $\mu_i = \mu_1 + a\sigma_i$, where $\mu_1$ is the mean of the core Gaussian $G_1$ ($i = 2, 3, 4$). The parameters $a$ and $\mu_1$ describe a possible asymmetry of the resolution function.

All parameters of the resolution function float freely and the fit is shown in Fig. 2. The $D^0$ lifetime is also a free fit parameter, for which we obtain $(408.5 ± 0.9)$ fs, where the uncertainty is statistical only. This $D^0$ lifetime is consistent with the world-average value [3] and the other Belle measurement [17], which gives further confidence in our parametrization of the resolution function.

To calculate the time-dependent WS to RS decay rate ratio, we divide the samples shown in Fig. 1 into ten bins of proper decay time. Our binning choice is made us-
ing 5000 simplified simulated experiments to maximize the sensitivity to the mixing parameters. Figure 3 shows the time-dependent ratios of WS to RS decay rates. The average value of the proper decay time in each bin is determined with the parametrization for the reconstructed RS proper decay time distribution shown in Fig. 2.

Prior to our fit to the time-dependent ratios of WS to RS decay rates, we estimate possible systematic effects. We validate the analysis procedure with the fully simulated MC events with several different input values of the mixing parameters and find results consistent with the input parameters. The dominant sources of systematic uncertainties are from fitting the $\Delta M$ distributions and uncertainties on the resolution function that do not cancel out in Eq. (3). However, these are estimated to be less than a tenth of the statistical uncertainty, which is estimated in simulated simplified experiments. Other sources of uncertainty are the binning of the proper decay time and the reconstruction efficiencies of WS and RS decays. These effects should cancel in the WS to RS ratio measurement. We estimate these with simulated simplified experiments and, indeed, find a negligible contribution of $<\mathcal{O}(10^{-4})$ on the mixing parameters and so ignore them. The systematic uncertainties due to fitting the $\Delta M$ distributions are estimated in the bins of the proper decay time and are added to the statistical uncertainties of the bin in quadrature, albeit with negligible effect.

Our fits to the time-dependent ratios of WS to RS decays using Eq. (3) are shown in Fig. 3. We test two hypotheses, with and without mixing, and the results are listed in Table I. The mixing parameters measured in this analysis agree with previous results from both hadron collider experiments [5, 18] using a similar method, as well as

![FIG. 3: The time-dependent ratios of WS to RS decay rates. Points with error bars reflect the data and their total uncertainties. The lines show the fit with (solid) and without (dashed) the mixing hypothesis.](image)

| Test hypothesis Parameters | Fit results (χ²/DOF) $\times 10^{-3}$ | Correlation coefficient $x^2$ | $y'$ |
|---------------------------|-----------------------------------|--------------------------|-----|
| Mixing (4.2/7)            | $R_D$ 3.53 ± 0.13 1                | $-0.865$                  | +0.737 |
| No Mixing (33.5/9)        | $R_D$ 3.864 ± 0.059                |                          |     |

with the results of alternate experimental methods from $e^+e^-$ collision experiments [7, 19] and are summarized in Table II.

As a check of our results in Table I, we repeat the analysis in two independent sub-samples. One corresponds to an integrated luminosity of 400 fb$^{-1}$ (the “old sample”) that is used in our previous publication [19] with a different method than used here. The other is the rest of our full data sample, corresponding to an integrated luminosity of 576 fb$^{-1}$ (the “new sample”). These two independent sub-samples are fed through this analysis separately. The results from the old and new samples (with statistical uncertainty only) are $(R_D, y', x^2) = (3.65 ± 0.22, -0.2 ± 5.4, 0.36 ± 0.32) \times 10^{-3}$ and $(3.45 ± 0.17, 7.6 ± 4.4, -0.09 ± 0.30) \times 10^{-3}$, respectively, which are compatible with the results from the full data sample. Furthermore, the results of this analysis using the old sample are consistent with our previous publication [19], which is superseded by the results of this analysis.

The $\chi^2$ difference between the “no-mixing” and “mixing” hypotheses, $\Delta \chi^2 = \chi^2_{\text{no-mixing}} - \chi^2_{\text{mixing}}$, is 29.3 for two degrees of freedom, corresponding to a probability of $4.3 \times 10^{-7}$; this implies the no-mixing hypothesis is excluded at the 5.1 standard deviation level. Thus, we observe $D^0 \rightarrow D^0$ mixing for the first time in an $e^+e^-$ collision experiment. We also show this in Fig. 4 with the $1\sigma, 3\sigma$, and $5\sigma$ contours around the best fit point in the $(x^2, y')$ plane.
In summary, we report the first observation of $D^0 - \bar{D}^0$ mixing in $e^+e^-$ collisions by measuring the time-dependent ratios of the WS to RS decay rates, providing $x^2 = (0.09 \pm 0.22) \times 10^{-3}$, $y' = (4.6 \pm 3.4) \times 10^{-3}$, and $R_D = (3.53 \pm 0.13) \times 10^{-3}$. Our results agree well with those from hadron collider experiments [5, 18] performed in very different experimental conditions.

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