Algebraic Reasoning in Solving Mathematical Problem Based on Learning Style

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Abstract. This study aimed to describe algebraic reasoning of secondary school’s pupils with different learning styles in solving mathematical problem. This study begins by giving the questionnaire to find out the learning styles and followed by mathematical ability test to get three subjects of 8th-grade whereas the learning styles of each pupil is visual, auditory, kinesthetic and had similar mathematical abilities. Then it continued with given algebraic problems and interviews. The data is validated using triangulation of time. The result showed that in the pattern of seeking indicator, subjects identified the things that were known and asked based on them observations. The visual and kinesthetic learners represented the known information in a chart, whereas the auditory learner in a table. In addition, they found the elements which makes the pattern and made a relationship between two quantities. In the pattern recognition indicator, they created conjectures on the relationship between two quantities and proved it. In the generalization indicator, they were determining the general rule of pattern found on each element of pattern using algebraic symbols and created a mathematical model. Visual and kinesthetic learners determined the general rule of equations which was used to solve problems using algebraic symbols, but auditory learner in a sentence.

1. Introduction

Algebraic reasoning is important because it is the basic of all mathematical contexts, including arithmetic which allows someone to explore mathematical structures [1]. Moreover, algebra is one of the topics in mathematics that is closely related to the use of symbols and numbers to solve equations, analyze functional relations, and determine the structure of representational systems which consist of expressions and relationships [2]. But in reality, many pupils experience difficulty in learning algebra. Tall [3] and Toshiakaira [4] state that the concept of variables and functions becomes a serious obstacle for many pupils and even teachers in Japan. The difficulties in learning algebra can be minimized by identifying the pupils’ algebraic reasoning so that teachers can design a learning which develops pupils' algebraic reasoning. Algebraic reasoning is a type of reasoning used in solving algebra problems [6] and problem solving can also be used to develop pupils' algebraic reasoning [7]. The problem solving strategy among pupils is different, this is influenced by the learning style that each pupil has [8]. Learning styles are a complex way in
which pupils perceive and feel most effective and efficient in processing, storing, and recalling what they have learned [9]. Learning styles are classified into 3 types; visual, auditory, and kinesthetic [10]. Based on these descriptions, it was concluded that learning styles can influence pupils’ algebraic reasoning in solving algebraic problems.

Some studies related to algebraic reasoning include relation on cognitive style and problem solving with algebraic reasoning. Chrystotomu [11] which mentioned the cognitive style of visualizer and verbalizer affect the number sense and algebraic reasoning of pupils in solving the problem. Nathan and Koedinger [12] found pupils’ problem solving strategies were versatile, so it indicated their algebraic reasoning is different. However, this study differs from previous research which had been explained. This study was not only limited to identifying the influence of cognitive style on the number sense and pupils’ algebraic reasoning as well as the difference of pupils’ completion strategy in solving the problems related to algebraic reasoning. This research was more focused to describe algebraic reasoning of pupils with visual, auditory, and kinesthetic learning style in solving algebraic problems deeply. The indicator of algebraic reasoning pupils was adapted from three stages of algebraic reasoning in solving the problems proposed by Herbert and Brown [13] that consist of seeking patterns, recognizing patterns, and generalizations. Based on the above explanation, the purpose of this study was to describe the algebraic reasoning secondary school’s pupils with visual, auditory, and kinesthetic learning style in solving mathematics problems.

2. Method
This study was an explorative study with qualitative descriptive approach. Explorative research means extract the necessary information in a study deeply. The approach used in this study is qualitative descriptive because the setting of study is natural background and the main instrument of research is the researcher. First, the researcher gave a questionnaire of learning style, after that a mathematics ability test which consists of 10 mathematical problems in secondary school was given. After getting one visual, one auditory, and one kinesthetic learning style pupils with equivalent mathematics ability, then the researcher gave the written test which consist of mathematical problem related to algebra. After each category had one subject, they were interviewed based on their answer in solving algebra related mathematics problem. The pupils were given a written test for twice. Then the consistency of the data gotten from the pupils’ solution and interview were checked by time triangulation. When the data was said valid, it was ready to be analyzed. Data analysis phase in this research consists of 5 steps; data categorization, data reduction, data presentation, data interpretation, and conclusion. After analysis the data, this study was eliciting the algebraic reasoning of secondary school’s pupils with visual, auditory, and kinesthetic learning style in solving mathematical problems.

3. Results & Discussion
Algebra is one of the branches of mathematics focusing on problem solving using symbols instead of constants and variables. Part of algebra used in this study is system of two linear equations. The two variable linear equation system is a system/unity of several similar linear two variables equations. linear two variables equations is an equation that has two variables and the rank of each variable is one. The similar linear two variables equations here is two linear equations which contain the same variable. The algebraic reasoning data of each subject was valid, so that it could be analyzed based on the algebraic reasoning indicator which was adapted from three stages of algebraic reasoning in solving the problems proposed by Herbert and Brown [13]. Here is the problem given to each pupil.

Mr. Toni has a garden which is divided into some plots and each plot consists of apple trees and evergreen trees. Mr. Toni spends different maintenance cost for each plot in every month. Both kind of trees were planted in a kind of pattern as shown in Figure 1.
3.1 The Algebraic Reasoning of Visual Learning Style’s pupil in Solving Mathematical Problem

When solving the problem, pupil observed the problem in several times and focused to understand the patterns. Then she identified the known information and the asked information. The pupil identified the known and asked information based on her observations on the given problem. Furthermore, in order to find the number of apple and evergreen trees in the sixth plot, pupil represented the amount of both kinds of trees between plots with logical reason. Representing the given problems was like Chrysostomou’s opinion [11] which stated algebraic reasoning as the use of various representations related to quantitative situations in a relational way. Furthermore, pupil stated that the number of apples and evergreen trees affected the pattern of each plot since both kinds of trees arranged each plot in the garden. Subject claimed that there was no relationship between the apple trees and evergreen trees regardless the fact that there was a relationship between the number of apple trees and evergreen trees with the order of plots. This indicated that the pupil made a relationship between two quantities.

Pupil declared rules for determining the number of apple and evergreen trees in symbol form. The rule had actually been found by pupil when she worked on the problem (point a). At point a, she had actually stated that to find out the number of apple trees was by using squaring, while to find out the number of evergreen trees was by multiplying the order of the plot by eight. In other words, when deciding on both kinds of trees in the sixth plot he had already recognised the pattern. It meant that the pupil had already made a conjecture related to the rule for determining the number of both kinds of trees in the next plot. To prove the conjecture, pupil used drawing to justify the reason. Pupil drew by observing the pattern of both kinds of trees on each plot. After a relation is established between the illustrations drawn and the rules which had already been found, pupil got same result which meant that the truth of conjecture was proven.

Pupil proved the truth of conjecture, then it could be stated that pupil determined the general rules of the constituent elements of the pattern directly. Pupil said that by using SPLDV, the maintenance cost required in the kth-plot could be determined. Pupils use x and y variables which represented the maintenance costs of each apple and evergreen tree, respectively. Then, pupil created a mathematical model for the first and second plots and look for the values of x and y. In order to determine the general rule of maintenance cost equation required for the kth-plot, pupil directly expressed it in algebraic equation $k^2(3500) + 8k(2000)$. 
3.2 The Algebraic Reasoning of Auditory Learning Style’s pupil in Solving Mathematical Problem

When solving the problem, pupil read the problem in several times and it sounds like talked to himself about what he understood. Then she identified the known information and the asked information. Furthermore, in order to find the number of apple and evergreen trees in the sixth plot, pupil represented the amount of both kinds of trees between plots using a table, accompanied by logical reason. Representing the given problem was like Chrysostomou’s opinion [11] which stated the algebraic reasoning as the use of various representations related to quantitative situations in a relational way. Pupil stated that the amount of apples and evergreen trees affected the pattern of each plot, since both kinds of trees arranged each plot in the garden. Then, pupil identified the relationship between both kinds of trees with the order of plots. Pupil claimed there were no relationship between the number of apples and evergreen trees, whereas there was a relationship between the number of apple and evergreen trees with the order of plots.

Pupil declared rules for determining the number of apple and evergreen trees in symbol form. The rule had actually been found by pupil when she worked on the problem (point a). At point a, she had actually stated that to find out the number of apple trees was by using squaring, while to find out the number of evergreen trees was by multiplying the order of the plot by eight. In other words, when deciding on both kinds of trees in the sixth plot he had already recognised the pattern. It meant that the pupil had already made a conjecture related to the rule for determining the number of both kinds of trees in the next plot. To prove the truth of her conjecture, pupil proved it by drawing. Pupil drew by observing the pattern of both kinds of trees on each plot. After being matched between the illustrations drawn with the rules which had already found, pupil got same result which meant that the truth of conjecture was proven.

Pupil proved the truth of conjecture, then it could be stated that pupil determined the general rules of the constituent elements of the pattern directly. Pupil said that by using SPLDV, the maintenance cost required in the kth-plot could be determined. Pupils use x and y variables which represented the maintenance costs of each apple and evergreen tree, respectively. Then she made a mathematical model for the first and second plots, then found out the values of x and y. She added the maintenance cost required for each kind of tree to determine the general rule of the maintenance cost required equation in the kth-plot. However, pupil declared it in a word (verbally) but she had not expressed it directly by using algebraic symbols. This confirmed that the pupil has been determine the general rule of equation used to solve the problem even though the pupil still wrote it in a sentence rather than in algebraic symbols.

3.3 The Algebraic Reasoning of Kinesthetic Learning Style’s pupil in Solving Mathematical Problem

When solving the problem, pupil read the problem in several times and it sounds like talked to himself about what he understood. Then she identified the known information and the asked information. Furthermore, in order to find the number of apple and evergreen trees in the sixth plot, pupil represented the amount of both kinds of trees between plots using a table, accompanied by logical reason. Representing the given problem was like Chrysostomou’s opinion [11] which stated the algebraic reasoning as the use of various representations related to quantitative situations in a relational way. Pupil stated that the amount of apples and evergreen trees affected the pattern of each plot, since both kinds of trees arranged each plot in the garden. Then, pupil identified the relationship between both kinds of trees with the order of plots. Pupil claimed there were no relationship between the number of apples and evergreen trees, whereas there was a relationship between the number of apple and evergreen trees with the order of plots.

Pupil expressed rules for determining the number of apple and evergreen trees in algebraic symbol form. The rule was obtained from the difference between the number of apple trees and evergreen trees between plots, thus the pattern of both kinds of trees was gotten. This implied that the pupil made a rule-related to conjecture in determining the number of both kinds of trees in the next plot. Pupil drew the pattern of the 6th-plot which consist of 13 evergreen trees surrounding and apple trees inside for confirming the truth of the conjecture. After being matched between the illustrations drawn with the
rules which had already found, pupil got same result which meant that the truth of conjecture was proven. It meant the pupil had proven the truth of the conjecture she had made accompanied by logical reason.

Pupil proved the truth of conjecture, then it could be stated that pupil determined the general rules of the constituent elements of the pattern directly. Pupil said that by using SPLDV, the maintenance cost required in the $k^{th}$-plot could be determined. Pupils use $x$ and $y$ variables which represented the maintenance costs of each apple and evergreen tree, respectively. Then, pupil created a mathematical model for the first and second plots and look for the values of $x$ and $y$. She added the maintenance cost required for each kind of tree to determine the general rule of the maintenance cost required equation in the $k^{th}$-plot. However, the pupil declared it in separated algebraic symbol form, she had not yet written the equation in algebraic symbol form completely. She said that she forgot to write the equation in the complete form which was desired in the given problem. This confirmed that the pupil had been determine the general rule of equation used to solve the problem.

4. Conclusion
As kinesthetic pupils sought the pattern, visual and auditory pupils had already recognized the patterns formed based on the representations they had already made. In addition, when pupils proved the truth of conjecture, the three pupils proved it by drawing the patterns. But there was a difference in finding the number of both kinds of trees. Visual and auditory pupils looked for the number of evergreen trees by counting, whereas kinesthetic pupil used arithmetic series. The visual and kinesthetic pupil determined the general rule of equation used in solving problems using algebraic symbols, while the auditory pupil expressed the general rule that she made verbally (in a word). Based on this it could be said that the visual pupil, auditory pupil and kinesthetic pupil had already satisfied all three algebraic reasoning indicators which were adapted from Herbert and Brown [13].

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