Application of Pascoletti-Serafini scalarization modification method to solve multi-objective optimization problems for stock portfolio

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Abstract. One example of a multi-objective optimization problem is stock portfolio management. There are at least two objective functions to be achieved simultaneously, namely to maximize returns and minimize risk. The desire to maximize return and minimize risk are conflicting objectives. In this study, the problem of multi-objective optimization in the selection of Islamic stock portfolios will use the Pascoletti-Serafini scalarization modification method. Furthermore, the solution to the multi-objective optimization problem is known as the Pareto optimal solution or efficient solution. In the Pascoletti-Serafini scalarization modification method, a set of Pareto optimal solutions can be constructed so that not only one solution is offered to decision makers, but a set of Pareto optimal solutions. From this research, the results obtained in the form of a set of efficient solutions that can be used as investor preferences in choosing the optimal stock portfolio.

1. Introduction
In many economic problems, decision maker no longer only consider one objective, such as only want to maximize profit, or just want to minimize production cost without considering other aspects. However, decision maker want to optimize several objectives functions at the same time. Furthermore, this problem is called as multi-objective optimization problem. In multi objective optimization problems, one objective function with another objective will conflict with each other so that it is difficult to find a single optimal solution. It is mean difficult to find a single solution that is able to optimize two or more objective functions at the same time.

One example of multi-objective optimization problem is stock portfolio management. There are at least two objective functions to be achieved simultaneously, namely to maximize return and minimize risk. The desire to maximize return and minimize risk are conflicting objectives.

To get an optimal stock portfolio requires a good investment management. According to Reilly and Brown [1], an investment is the current commitment of dollars for a period of time in order to derive future payments that will compensate the investor for (1) the time the funds are committed, (2) the expected rate of inflation during this time period, and (3) the uncertainly of the future payments.

According to Duan [2], portfolio optimization plays a critical role in determining portfolio strategies for investors. What investors hope to achieve from portfolio optimization is to maximize portfolio returns and minimize portfolio risk. Since return is compensated based on risk, investors have to balance the risk-return trade-off for their investments. Therefore, there is no a single optimized
portfolio that can satisfy all investors. An optimal portfolio is determined by an investor’s risk-return preference.

Many researches on optimization of stock portfolios have been done. Amiri, Ekhtiari, and Yazdani [3], Saputro and Qudratullah [4] use Nadir Compromise Programming to solve multi-objective optimization problems in stock portfolio. Pouya, Solimanpur, and Rezaee [5] use the method of invasive weed optimization. Fitria [6] uses Model Predictive Control (MPC) to solve the portfolio optimization problem. Oh, et.al. [7], and Skolpadungket, Dahal, and Hampupornchai [8] use Genetic Algorithm in solving stock portfolio optimization problems.

Nadir Compromise Programming (NCP) and Multi-Objective Genetic Algorithm (MOGA) are several methods for solving multi-objective optimization problems. There is another method that can solve the multi-objective optimization problem for both linear and nonlinear functions, namely the Pascoletti-Serafini Scalarization method [9]. The Pascoletti-Serafini method change the multi-objective optimization problem which is a vector optimization problem into a scalar optimization problem. Khorram, Khaleedian, and Khaledyan [10] modified the Pascoletti-Serafini scalarization method by restriction certain parameters to generate solutions.

In this research, the problem of multi-objective optimization in the selection of islamic stock portfolios will use the Pascoletti-Serafini scalarization modification method. Furthermore, multi-objective optimization problem solutions are known as Pareto Optimal solution or efficient solution or non-dominated solution. In the Pascoletti-Serafini scalarization modification method, a set of Pareto optimal solution can be constructed so that not only one solution is offered to decision makers, but a set of Pareto optimal solutions.

2. Research method

In this study, the stock data used to determine the optimal portfolio is Islamic stock data which is incorporated in the Jakarta Islamic Index (JII). The Jakarta Islamic Index (JII), which is an index of Islamic stocks in Indonesia, consists of the 30 most liquid stocks listed on the Indonesia Stock Exchange (IDX). The data used in this study are daily stock data from August 27th, 2018 to August 25th, 2020.

2.1. Calculation of return and risk value

The calculation of the discrete return value is formulated as follows:

\[ R_t = \frac{S_t}{S_{t-1}} - 1 \]  

where \( R_t \) is the stock return at time \( t \), and \( S_t \) is the stock price at time \( t \).

Next, to calculate the arithmetic average value of the expected return using the following formula:

\[ E(R_i) = \frac{\sum_{j=1}^{N} R_{ij}}{N} \]

where \( E(R_i) \) represents the expected return of stock \( i \), \( R_{ij} \) is the return of stock \( i \) at time \( t = j \), and \( N \) is the number of periods of observation.

The risk in investing can be determined by looking at the risk coefficient value of a stock. To calculate the risk coefficient (stock beta) of a stock, you can use the following formula:

\[ \beta_i = \frac{Cov(R_iR_m)}{\sigma_m^2} \]

where \( \beta_i \) states the risk coefficient of stock \( i \), \( \sigma_m^2 \) is the variance value of market shares, in this case the Jakarta Islamic Index (JII), and \( Cov(R_iR_m) \) states the covariance between stock \( i \) and market shares \( m \).
The value of the stock variance $i$, $\sigma_i^2$ from a number of $N$ stock data is formulated in the following equation:

$$\sigma_i^2 = \frac{\sum_{j=1}^{N} \left( R_{ij} - E(R_i) \right)^2}{N}$$  \hspace{0.5cm} (4)

Covariance shows the linear attachment of two variables. The covariance of two variables (stock $i$, and market share $m$) is formulated as follows:

$$Cov(R_i, R_m) = \frac{\sum_{j=1}^{N} \left( R_{ij} - E(R_i) \right) \left( R_{mj} - E(R_m) \right)}{N}$$  \hspace{0.5cm} (5)

2.2. Pascoletti-Serafini scalarization modification

Given a multi-objective optimization problem (MOP) which is mathematically expressed as:

$$MOP : \min_{x \in X} f(x) = (f_1(x), f_2(x), ..., f_p(x)),$$

with $X \subseteq \mathbb{R}^n$ is a non-empty phonetic assembly and $f$ is a vector value function composed of $p$ ($p \geq 2$) real value functions. The image of $X$ in $f$ is denoted as $Y := f(X) \subseteq \mathbb{R}^p$ and points to the image space.

Furthermore, the solution for $\bar{x} \in X$ is called the Pareto efficient/optimal solution of MOP (1) if there is no $x \in X$ such that $f(x) \leq f(\bar{x})$. If $\bar{x} \in X$ is Pareto optimal then $f(\bar{x})$ is called the point that is not dominated [10].

The Pascoletti-Serafini scalarization modification which is hereinafter called the problem $\overline{P}(a, r)$ is given as follows:

$$\min_{x \in X} t$$

with constraints:

$$tr - f(x) \geq 0,$$

$$x \in X, \quad t \in \mathbb{R}.$$ 

In this study, to limit the selection of the $r \in \mathbb{R}^p$ parameter, it is assumed that MOP (1) has an ideal point. The point $f^* = (f_1^*, ..., f_p^*)^T$, where $f_i^* = \text{min}_{x \in X} f_i(x)$ untuk $i = 1, ..., p$, is called the ideal point of MOP (1). Furthermore, after obtaining the ideal point from MOP (1) the objective function is redefined as $f(x) \leftrightarrow f(x) - f^*$. In other words, the ideal point is shifted to the starting point. In the case of two objective functions are shown in Figure 2.1. With this redefinition it is assumed that the ideal point is equal to zero and the objective function is non-negative [10].

3. Result

3.1. Calculating of expected return and risk coefficient values

Of the 30 stocks listed in the Jakarta Islamic Index (JII) for the period August 27th, 2018 to August 25th, 2020, the return value and the expected return of each stock are calculated in the following Table 1.
Table 1. The expected return value of each stocks in JII.

| Stock name | Value of expected return ($E(R)$) | Stock name | Value of expected return ($E(R)$) | Stock name | Value of expected return ($E(R)$) |
|------------|----------------------------------|------------|----------------------------------|------------|----------------------------------|
| ADRO       | 0.013054266                      | EXCL       | -0.003942662                     | MNCN       | -0.791666667                    |
| AKRA       | 0.001205266                      | ICBP       | 0.010502358                      | PGAS       | -0.008345024                    |
| ANTM       | 0.006998945                      | INCO       | 0.009009481                      | PTBA       | -0.023513961                    |
| ASII       | -0.007633199                     | INDIF      | 0.012538815                      | PTTP       | -0.031071859                    |
| BRPT       | 0.066208497                      | INTP       | -0.012004707                     | SCMA       | -0.032366620                    |
| BSDE       | -0.010783542                     | ITMG       | -0.041682694                     | TLKM       | -0.034259053                    |
| BTPS       | 0.045716680                      | JPFA       | -0.016709630                     | TPIA       | -0.035736733                    |
| CPIN       | 0.014477447                      | JSMR       | 0.005301349                      | UNTR       | -0.037517386                    |
| CTRA       | 0.012336491                      | KLBF       | 0.010940747                      | UNVR       | -0.038762560                    |
| ERAA       | 0.009666473                      | LPPF       | -0.047399529                     | WIKA       | 0.011659107                     |

Furthermore, stocks with a positive expected return value are selected and then the share risk coefficient value is calculated in the following Table 2.

Table 2. Stocks with positive expected returns.

| Stock name | Value of expected return ($E(R)$) | Stock name | Value of expected return ($E(R)$) |
|------------|----------------------------------|------------|----------------------------------|
| ADRO       | 0.013054266                      | ERAA       | 0.009666473                      |
| AKRA       | 0.001205266                      | ICBF       | 0.010502358                      |
| ANTM       | 0.006998945                      | INCO       | 0.009009481                      |
| BRPT       | 0.066208497                      | INDIF      | 0.012538815                      |
| BTPS       | 0.045716680                      | JPFA       | -0.016709630                     |
| CPIN       | 0.014477447                      | JSMR       | 0.005301349                      |
| CTRA       | 0.012336491                      | KLBF       | 0.010940747                      |

Of the 14 stocks with a positive expected return, the risk coefficient ($\beta$) is calculated. Furthermore, after obtaining the value of the expected return and the risk coefficient, the ratio of each stock is calculated by comparing the value of the expected return and the risk coefficient. The coefficient and ratio values of each share are presented in the following Table 3.

Table 3. Risk coefficient and ratio values of Stock with positive expected return.

| Stock name | Risk coefficient ($\beta$) | Ratio | Stock name | Risk coefficient ($\beta$) | Ratio |
|------------|---------------------------|-------|------------|---------------------------|-------|
| ADRO       | -3.78840746               | -0.00375248 | ERAA       | 1.195516024               | 0.008085607 |
| AKRA       | 0.188001228               | 0.006410949 | ICBF       | 11.43846871               | 0.00918147 |
| ANTM       | 1.1092694912              | 0.006309535 | INCO       | 1.545543312               | 0.005829329 |
| BRPT       | 6.950584620               | 0.009525601 | INDIF      | 12.97479585               | 0.00966398 |
| BTPS       | 11.29147788               | 0.004087939 | JSMR       | 1.027451655               | 0.00519707 |
| CPIN       | 10.17521513               | 0.001422815 | KLBF       | 3.040220735               | 0.003598669 |
| CTRA       | 2.008097185               | 0.006143374 | WIKA       | 1.838901341               | 0.006340257 |

In the formation of a multi-objective optimization problem, it is limited by choosing six decision variables, namely the six stocks with the highest ratio. The six shares were obtained as following in Table 4.
Table 4. List of stocks in the formation of a stock portfolio.

| Stock name | Value of expected return \((E(R))\) | Risk coefficient \((\beta)\) |
|------------|---------------------------------|--------------------------|
| AKRA       | 0.001205266                     | 0.188001228              |
| ANTM       | 0.006998945                     | 1.109264912              |
| BRPT       | 0.066208497                     | 6.95058462               |
| CTRA       | 0.012336491                     | 2.008097185              |
| ERAA       | 0.009666473                     | 1.195516024              |
| WIKA       | 0.011659107                     | 1.838901341              |

3.2. Mathematical model formulation of multi-objective optimization problems in the stock portfolio

In the multi objective optimization problem for stock portfolios, there are two aspects to be considered, namely risk and expected return. The first step taken to model the optimization problem is to determine the decision variable. The decision variables (variables) for the multi objective optimization problem for this stock portfolio are as follows:

\[ x_i \text{ = the proportion of funds to be invested in stocks } i \text{ ; } i = 1, 2, 3, 4, 5, 6 \]

where \(1 = \text{AKRA} ; 2 = \text{ANTM} ; 3 = \text{BRPT} ; 4 = \text{CTRA} ; 5 = \text{ERA} ; 6 = \text{WIKA} \).

The next step is to determine the objective function. There are two objective functions that are considered, namely as follows:

1. The objective function to maximize the expected return (profit)

\[
\text{Max. } f_1 = 0.001205266x_1 + 0.006998945x_2 + 0.066208497x_3 + 0.012336491x_4 \\
+ 0.009666473x_5 + 0.011659107x_6
\]

The above objective function can also be expressed as a minimization problem as follows:

\[
\text{Min. } f_1' = -(0.001205266x_1 + 0.006998945x_2 + 0.066208497x_3 + 0.012336491x_4 \\
+ 0.009666473x_5 + 0.011659107x_6)
\]

2. The objective function is to minimize risk

\[
\text{Min. } f_2 = 0.188001228x_1 + 1.109264912x_2 + 6.95058462x_3 + 2.008097185x_4 \\
+ 1.195516024x_5 + 1.838901341x_6
\]

In fulfilling the objective function above, there are several obstacles to consider, namely as follows:

i. The constraint function is the amount of the proportion of funds.
\[
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1
\]

ii. The constraint function is the lower and upper limits of the proportion of funds invested for each share:
\[
0 \leq x_i \leq 0.8; \ i = 1, 2, 3, 4, 5, 6.
\]
3.3 Solving multi-objective optimization problems on stock portfolios with the Pascoletti-Serafini scalarization modification method

The first step to solve the multi-objective optimization problem with the Pascoletti-Serafini scalarization modification method is to find the ideal point. In other words, trouble minimized. Obtained:

\[
\hat{x}_1 = \arg \min_{x \in \mathcal{X}} f_1(x) = (0; 0; 0.8; 0.2; 0; 0).
\]

Then,

\[
\hat{x}_2 = \arg \min_{x \in \mathcal{X}} f_2(x) = (0.8; 0.2; 0; 0; 0; 0).
\]

Furthermore, after getting the ideal point of the multi-objective optimization problem above, the next step is to redefine the objective functions into

\[
\begin{align*}
\tilde{f}_1(x) &= f_1(x) - f^* \\
\tilde{f}_2(x) &= f_2(x) - f^* \\
\end{align*}
\]

with constraints:

\[
\begin{align*}
\sum_{i=1}^{9} x_i &= 1 \\
0 \leq x_i \leq 0.8; i = 1, 2, \ldots, 9.
\end{align*}
\]

Then the set is defined \( R = \{ \beta \in \mathbb{R}^p \mid \beta \in \mathbb{R}^p, \sum_{i=1}^{9} \beta_i = 1, \beta_i \geq 0 \} \). To generate vector spread \( \beta = (\beta_1, \beta_2) \) selected \( \delta = \frac{1}{10} \) so that eleven unit vectors are obtained \( \frac{R}{\| \beta \|_2} \) who are members of the set \( R \), as shown in Table 5.
Table 5. Vector spread \( \beta = (\beta_1, \beta_2) \) for \( \delta = \frac{1}{10} \).

| \( \beta = (\beta_1, \beta_2)^T \) | \( \|\beta\|_2 \) | \( \begin{pmatrix} \beta \\ \|\beta\|_2 \end{pmatrix} \) |
|---|---|---|
| \((0.1)^T\) | 1 | \( r_1 = (0.1)^T \) |
| \((0.1, 0.9)^T\) | 0.906 | \( r_2 = (0.110, 0.994)^T \) |
| \((0.2, 0.8)^T\) | 0.825 | \( r_3 = (0.242, 0.970)^T \) |
| \((0.3, 0.7)^T\) | 0.762 | \( r_4 = (0.394, 0.919)^T \) |
| \((0.4, 0.6)^T\) | 0.721 | \( r_5 = (0.555, 0.832)^T \) |
| \((0.5, 0.5)^T\) | 0.707 | \( r_6 = (0.707, 0.707)^T \) |
| \((0.6, 0.4)^T\) | 0.721 | \( r_7 = (0.832, 0.555)^T \) |
| \((0.7, 0.3)^T\) | 0.762 | \( r_8 = (0.919, 0.394)^T \) |
| \((0.8, 0.2)^T\) | 0.825 | \( r_9 = (0.970, 0.242)^T \) |
| \((0.9, 0.1)^T\) | 0.906 | \( r_{10} = (0.994, 0.110)^T \) |
| \((1.0)^T\) | 1 | \( r_{11} = (1.0)^T \) |

I. Troubleshooting \( \mathbb{SP}(a, r) \) for \( r_2 = (0.110, 0.994)^T \)

By substituting \( r = (0.110, 0.994)^T \) and \( f(x) \) as in MOP (1) problem \( \mathbb{SP}(a, r) \) becomes:

\[
\min f(t, x_1, x_2, x_3, x_4, x_5, x_6) = \min t
\]

with constraints:

\[
\begin{pmatrix}
0.110t \\
0.994t
\end{pmatrix} - \begin{pmatrix}
f'_1 + 0.05543410 \\
f_2 - 0.3722540
\end{pmatrix} \geq 0
\]

\[
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1
\]

\[
x_1 \leq x_i \leq 0.8 \; ; i = 1, 2, 3, 4, 5, 6.
\]

\[
t \in \mathbb{R}
\]

where

\[
f'_1 = -(0.001205266x_1 + 0.006998945x_2 + 0.066208497x_3 + 0.012336491x_4 + 0.009666473x_5 + 0.011659107x_6)
\]

and

\[
f_2 = 0.188001228x_1 + 1.109264912x_2 + 6.950584623x_3 + 2.008097185x_4 + 1.95516024x_5 + 1.838901341x_6.
\]

By using the LINGO program, optimal values are obtained \((t, x_1, x_2, x_3, x_4, x_5, x_6) = (0.3429, 0.0, 0.101, 0.8, 0, 0.099, .)\).

II. Problem solving \( \mathbb{SP}(a, r) \) for \( r_{11} = (1.0)^T \)

By substituting \( r = (1.0)^T \) and \( f(x) \) as in MOP (1) problem \( \mathbb{SP}(a, r) \) becomes:

\[
\min f(t, x_1, x_2, x_3, x_4, x_5, x_6) = \min t
\]

with constraints:

\[
\begin{pmatrix}
t \\
0
\end{pmatrix} - \begin{pmatrix}
f'_1 + 0.05543410 \\
f_2 - 0.3722540
\end{pmatrix} \geq 0
\]

\[
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1
\]

\[
0 \leq x_i \leq 0.8 \; ; i = 1, 2, 3, 4, 5, 6.
\]

\[
t \in \mathbb{R}
\]
where \( f_1^* = -(0.001205266x_1 + 0.006998945x_2 + 0.066208497x_3 + 0.012336491x_4 + \\
0.009666473x_5 + 0.011659107x_6) \)
and \( f_2 = 0.188001228x_1 + 1.109264912x_2 + 6.950584623x_3 + 2.008097185x_4 + \\
1.195516024x_5 + 1.838901341x_6. \)

By using the LINGO program, optimal values are obtained \((t, x_1, x_2, x_3, x_4, x_5, x_6) = \\
(0.0404, 0.0, 0.0519, 0.8, 0.0, 0.1481). \)

The same process is continued to obtain eight other solutions for \( r_3, r_4, ..., r_{10} \) so that the results are as shown in the following table 6.

| \( k \) | \( r_k \) | \( (t^{(k)}, \bar{x}^{(k)}) \) optimal | \( f(\bar{x}^{(k)}) \) |
|---|---|---|---|
| 1 | \((0,1)^T\) | \((0,0,0,0.8,0.2,0,0)\) | \((0.055434096, 5.962087133)\) |
| 2 | \((0.110,0.994)^T\) | \((0.3429,0.0,0.1010, 0.8, 0,0.099)\) | \((0.017710503, 2.490538027)\) |
| 3 | \((0.242,0.970)^T\) | \((0.1618,0.0,0.0745, 0.8, 0,0.1255)\) | \((0.016264944, 2.355078420)\) |
| 4 | \((0.394,0.919)^T\) | \((0.1007,0.0,0.0652, 0.8, 0,0.1348)\) | \((0.015757634, 2.307539766)\) |
| 5 | \((0.555,0.832)^T\) | \((0.0719,0.0,0.0605, 0.8, 0,0.1395)\) | \((0.015501252, 2.283514855)\) |
| 6 | \((0.707,0.707)^T\) | \((0.0567,0.0,0.0577, 0.8, 0,0.1423)\) | \((0.015348514, 2.269202141)\) |
| 7 | \((0.832,0.555)^T\) | \((0.0483,0.0,0.0557, 0.8, 0,0.1443)\) | \((0.015239415, 2.258978775)\) |
| 8 | \((0.919,0.394)^T\) | \((0.0438,0.0,0.0544, 0.8, 0,0.1456)\) | \((0.015168501, 2.252335387)\) |
| 9 | \((0.970,0.242)^T\) | \((0.0416,0.0,0.0533, 0.8, 0,0.1467)\) | \((0.015108497, 2.246710735)\) |
| 10 | \((0.994,0.110)^T\) | \((0.0406,0.0,0.0525, 0.8, 0,0.1475)\) | \((0.015064857, 2.242621388)\) |
| 11 | \((1,0)^T\) | \((0.0404,0.0,0.0519, 0.8, 0,0.1481)\) | \((0.015032128, 2.239554378)\) |

4. Conclusion

From the optimization results of six Islamic stocks using the Pascoletti-Serafini Scalarization Modification method, several efficient solutions with 11 different \( r \) parameters were obtained. The following results were obtained.

a. The proportion of invested funds is BRPT shares of 0.8 and CTRA shares of 0.2. From the proportion of funds, the expected return value is 0.055434096 and the risk is 5.962087133. This result is obtained in the selection of the parameter \( r = (0,1)^T \).

b. The other 10 efficient solutions have the same proportion value for CTRA shares, which is 0.8. Furthermore, the shares that get the proportion of funds to be invested are BTPR and WIKA with the sum of the proportions of both of them being 0.2.

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