Leptogenesis and gravity: baryon asymmetry without decays

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Abstract

A popular class of theories attributes the matter-antimatter asymmetry of the Universe to CP-violating decays of super-heavy BSM particles in the Early Universe. Recently, we discovered a new source of leptogenesis in these models, namely that the same Yukawa phases which provide the CP violation for decays, combined with curved-spacetime loop effects, lead to an entirely new gravitational mechanism for generating an asymmetry, driven by the expansion of the Universe and independent of the departure of the heavy particles from equilibrium. In this Letter, we build on previous work by analysing the full Boltzmann equation, exploring the full parameter space of the theory and studying the time-evolution of the asymmetry. Remarkably, we find regions of parameter space where decays play no part at all, and where the baryon asymmetry of the Universe is determined solely by gravitational effects.

1. Introduction

In a series of recent papers \cite{1,2} we described a new phenomenon whereby gravity drives the Universe towards a matter-antimatter asymmetry. Our main realisation was that matter and antimatter propagate differently in the presence of gravity when CP symmetry is violated. Specifically, we proved \cite{1,2} that in translation invariant environments, CPT symmetry necessarily forces matter and antimatter to propagate identically. Conversely, when this symmetry is broken by the background geometry, e.g., an expanding Universe, and when there is a source of CP violation, matter/antimatter propagators become distinct. This causes a spectral splitting for matter/antimatter and an energy cost difference which drives the system towards an asymmetric state, facilitated by particle number-violating reactions.

As in our previous papers, we shall illustrate this effect within the context of leptogenesis \cite{3}, though as will become apparent, it applies equally well in any theory with a source of CP violation and B or L violation. In this case, the Lagrangian is given by

\[ \mathcal{L} = \sqrt{-g} \left[ \overline{N} i D N + \overline{N} M N + h_\nu \overline{\nu} \phi N j + \text{h.c.} \right]. \]

where \( \ell_i \) are the left-handed lepton doublets, \( \phi \) is the charge-conjugate Higgs doublet, and \( N_j \) are sterile neutrinos, written here in the Majorana basis \cite{4} so that \( N^c = N \). As described above, at two-loops (figure 1) in a time-dependent gravitational background, lepton and antilepton self-energies are distinct \( \Sigma_\ell(x, x') \neq \Sigma_{\overline{\ell}}(x, x') \).

Minimal coupling ensures that at tree-level, the strong equivalence principle holds and leptons are insensitive to curvature, but when loop effects are taken into account, two things happen. Firstly, the propagators become sensitive to CP violation contained in the Yukawa couplings, a symmetry which obviously must be broken for distinct propagation. Moreover, as described in \cite{4,5} the screening cloud surrounding the propagating leptons causes them to acquire an effective “size” and experience gravitational tidal forces, violating the strong equivalence principle and causing the leptons to couple directly to curvature.

When the sterile neutrinos are integrated out from the diagrams in figure 1, the resulting effective action contains the following CP- and strong equivalence principle-violating operator for each lepton generation:

\[ \mathcal{L}_i = \partial_\mu R \overline{\ell_i} \gamma^\mu \ell_i \sum_{k,j} \frac{\text{Im} \left[ h_{ik}^* h_{ij} h_{jl}^* h_{jk} \right]}{3M_k M_l} \ell_j. \]

\[ \text{(2)} \]

\[ ^1 \text{In previous papers [1,2], as in [3], we used } N \text{ to label the basis of RH neutrinos, which are now more usually denoted } (\nu)_c. \]
Figure 1: Loop diagrams which give distinct matter/antimatter propagators and which generate the operator [2].

where $R$ is the Ricci scalar and $I_{ij} = I(M_i, M_j)$ is a loop-factor depending on the sterile masses $M_i$ and $M_j$ in the corresponding diagram and which was computed in full detail in [2]. As described in refs. [2, 6], this modifies the dispersion relations of leptons and antileptons to

$$p_{\mu} \pm \delta_{\mu R} \sum_{i,j} \frac{\text{Im} [h_i^* h_j]}{3 M_i M_j} I_{[ij]} = 0,$$  \hspace{1cm} (3)

This energy splitting together with $\Delta L = 2$ and $\Delta L = 1$ processes drives the system towards a non-zero B-L asymmetry, independently of the departure of sterile neutrinos from equilibrium. For cosmological space-times, isotropy and homogeneity mean that spatial derivatives of $R$ vanish and eq. (3) leads to an equilibrium B-L to photon ratio of the form

$$N_{B-L}^{eq} \approx \frac{\pi^2 R}{2 \zeta(3) T^6} \sum_{i,j} \frac{\text{Im} [K_{ij}^2]}{18 M_i M_j} I_{[ij]},$$  \hspace{1cm} (4)

where $K_{ij} = (h^i h_j)_{ij}$. In this sense, we have a mechanism satisfying all three Sakharov conditions [2], the first two of which (particle number and CP violation) are inherited from the usual seesaw mechanism. The third - usually stated as a departure from equilibrium - is provided by the time-dependence of the background itself, whose dynamical nature is probed by the lepton screening cloud.

In a radiation dominated Universe as considered in this Letter,

$$\dot{R} = \sqrt{3} \sigma^{3/2} (1 - 3w)(1 + w) \frac{T^6}{M_P^2},$$  \hspace{1cm} (5)

where $\sigma = \pi^2 / 30 g_*$ and $g_* \approx 106.75$ counts the number of relativistic degrees of freedom in the plasma. Classically, the equation of state parameter $w$ is equal to $1/3$ for radiation, and so the expression (5) vanishes. However, trace-anomalies in the gauge sector give $(1 - 3w) \approx 10^{-1} [8]$, allowing for $R \neq 0$. Combining eqs. (4) and (5) we arrive at

$$N_{B-L}^{eq} \approx \frac{\sqrt{3} \sigma^{3/2} (1 - 3w)(1 + w) T^6}{36 \zeta(3) M_P^2} \sum_{i,j} \frac{\text{Im} [K_{ij}^2]}{M_i M_j} I_{[ij]}.$$  \hspace{1cm} (6)

A full description of the general theory of this gravitational leptogenesis mechanism and the calculation of the equilibrium asymmetry $N_{B-L}^{eq}$ was given in [3]. In that work, we also made a preliminary estimate of the gravitationally induced baryon asymmetry $\eta_B$ based on the assumption that the lepton number violating interactions, which maintain the asymmetry at its equilibrium value, freeze out for temperatures $T_D$ for which $z_D = M_1 / T_D \sim 1$. In order to achieve the observed value for $\eta_B$, we were then led to consider very high sterile neutrino masses and decoupling temperatures at the limits of existing physical bounds. However, as we demonstrate here, a complete dynamical analysis using the full $\Delta L = 2$ reaction cross-section shows that decoupling in fact occurs for significantly smaller values of $z_D$. Inspection of (6) then makes it clear that the observed asymmetry is achieved for lower, conventional values of $M_1 \sim 10^{10} - 10^{11}$ GeV with correspondingly lower decoupling temperatures.

Since our interest in ref. [2] was in the gravitational leptogenesis mechanism itself, we did not discuss the original mechanism whereby the out-of-equilibrium asymmetric decay rates $\Gamma(N \rightarrow \ell \bar{\phi}) \neq \Gamma(N \rightarrow \ell \phi)$ of sterile neutrinos in the region $z \sim 1$ contribute directly to the B-L asymmetry. Here, we consider the coupled Boltzmann equations involving both mechanisms and discuss in some detail the parameter space of the high-energy Yukawa phases in which one or other mechanism dominates in determining the final cosmological baryon asymmetry.

2. The Boltzmann Equation

We now study the Boltzmann equation to take into account the effect both of sterile neutrino decays and gravitational effects. We shall work in the hierarchical limit where $M_1 \ll M_2 \ll M_3$, so that the dynamics is dominated by the lightest sterile neutrino $N_1$, in which case the relevant Boltzmann equation is (see, e.g., [9])

$$\frac{dN_{N_i}}{dz} = -D \left( N_{N_i} - N_{N_i}^{eq} \right),$$  \hspace{1cm} (7)

$$\frac{dN_{B-L}}{dz} = -D \bar{e}_1 \left( N_{N_1} - N_{N_1}^{eq} \right) - W \left( N_{B-L} - N_{B-L}^{eq} \right),$$  \hspace{1cm} (8)
where each of the number densities is normalised by the photon density and where \( z = M_1/T \). This is the standard set of coupled Boltzmann equations encountered in lepto/baryogenesis (see e.g., \([8, 10, 11]\)) except that now, due to the gravitational interactions, we have \( N^\text{eq}_{\nu-L} \neq 0 \) in the RHS of (8) in the washout term. Conventionally one has \( N^\text{eq}_{\nu-L} = 0 \) and so any lepton asymmetry generated whilst the sterile neutrinos are in equilibrium is washed out. However, if one takes into account gravitational effects, a lepton asymmetry can be maintained even when \( N_{N_1} = N^\text{eq}_{N_1} \).

The CP asymmetry in the decays and inverse decays of sterile neutrinos is characterised by

\[
\epsilon_1 = \frac{\Gamma(N_1 \rightarrow \ell \phi) - \Gamma(N_1 \rightarrow \ell \bar{\phi})}{\Gamma(N_1 \rightarrow \ell \phi) + \Gamma(N_1 \rightarrow \ell \bar{\phi})},
\]

(9)
given in terms of \( M_i \) and \( h_{ij} \) by \([3, 10]\)

\[
\epsilon_i = -\frac{1}{8\pi} \sum_{j \neq i} \frac{\text{Im}[K_{ij}^*]}{K_{ii}} \left[ f \left( \frac{M_i^2}{M_j^2} \right) + g \left( \frac{M_j^2}{M_i^2} \right) \right],
\]

(10)
where

\[
f(x) = \sqrt{x} \left( 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right), \quad g(x) = \sqrt{x} \left( 1 - x \right).
\]

(11)

For a large hierarchy, \( x \gg 1 \),

\[
f(x) \sim -\frac{1}{2 \sqrt{x}}, \quad g(x) \sim \frac{1}{\sqrt{x}}.
\]

(12)

We shall return to the form of \( \epsilon_1 \) in subsequent sections.

The various reaction rates can be parametrised in terms of the standard quantity \( K = \tilde{m}_1/m_\ell \) \([9, 12, 13]\) given by

\[
\tilde{m}_1 = \sqrt{2} K \frac{\tilde{\Gamma}}{\sqrt{2} v^2} \approx 10^{-3} \text{ eV},
\]

(13)
where \( \tilde{m}_1 \) characterises the strength of the Yukawa interactions and \( v = 174 \text{ GeV} \) is the electroweak scale. The quantity \( D \) can then be written as

\[
D = \frac{\Gamma_{\text{tree}}(N_1 \rightarrow \ell \phi)}{zH} = K^2 \frac{\Gamma_{K}(z)}{\Gamma_{K}(z)}.
\]

(14)
and corresponds to the \( N_1 \rightarrow \ell \phi \) tree-level thermal decay width. \( W \) is the "washout term", so-called because when gravitational effects are neglected, \( N^\text{eq}_{\nu-L} = 0 \) and any lepton asymmetry established before the decays of sterile neutrinos is destroyed. The washout term consists of two parts:

\[
W = W_{ID} + 2W_{\Delta L=2}.
\]

(15)
The first is given by the tree-level inverse decay rate \([9]\)

\[
W_{ID} = \frac{\Gamma(\ell \phi \rightarrow N_1)}{zH} = \frac{1}{4} K^2 \frac{\Gamma(\ell \phi)}{\Gamma_{K}(z)}. \quad (16)
\]

The second part corresponds to \( \Delta L = 2 \) binary scatterings \( \ell \phi \leftrightarrow \ell \bar{\phi} \) in the s- and u-channel, and \( \ell \ell \leftrightarrow \phi \bar{\phi} \) in the t-channel. The reaction rates for these processes are given by the quantity \( W = \Gamma/zH \), with

\[
W = \frac{1}{64(2\pi)^3} \frac{1}{T^2} \int_0^\infty dss^{1/2} K_1 \left( \frac{\sqrt{s}}{T} \right) \frac{1}{s} |M(s)|^2, \quad (17)
\]

where

\[
|M(s)|^2 = \int_{-\pi}^\pi du |M(s, u)|^2 \quad (18)
\]

is the u-averaged amplitude for the process in question. The amplitudes for \( s, u \) and \( t \) processes are denoted by the subscripts + and t respectively and take the form

\[
|M_{\Delta L=2}(s)|^2 = 2s^2 \left\{ \frac{K_1^2}{M_1^2} F_{s, t}(s) - 6 \sum_{j=1}^3 \text{Re} \left[ \frac{K_{ij}}{M_i M_j} \right] G_{s, t}(s) \right\} + 3 \sum_{j=1}^3 \text{Re} \left[ \frac{K_{ij}}{M_i M_j} \right]. \quad (19)
\]

Introducing the variables

\[
c = \frac{K_{11}}{8\pi}, \quad x = \frac{s}{M_1^2}, \quad \text{ and } \quad x = \frac{s}{M_1^2}.
\]

(20)
the functions \( F \) and \( G \) are given by \([4, 12]\)

\[
F_+ = \frac{1}{\left( 1 - x \right)^2 + c^2} = \frac{\pi}{c} \delta(1 - x)
\]

\[
+ \frac{2}{x} \frac{2}{x^2} \left[ 1 + \frac{x^2 - 1}{(x - 1)^2 + c^2} + \frac{2(x - 1)}{x((1 - x)^2 + c^2)} \right],
\]

\[
G_+ = \frac{1}{x} + \frac{1}{2} \frac{x - 1}{(x - 1)^2 + c^2} + \frac{x + 1}{x^2 \ln(x + 1)}, \quad (21)
\]

and

\[
F_\ell = \frac{2}{x + 1} + \frac{2}{x(x - 1)} \ln(x + 1),
\]

\[
G_\ell = -\frac{1}{x} \ln(x + 1). \quad (22)
\]

The delta function subtraction in the first line for \( F_+ \) represents the real intermediate state subtraction from the s-channel. This is to avoid the well-known double counting problem \([3, 11, 12]\), where one over-counts the number of \( N_1 \leftrightarrow \ell \phi \) processes by including them in the s-channel \( N_1 \) exchange. Only with this subtraction does
the Boltzmann equation take the correct form, whereby no asymmetry can be generated when \( N_{N_i} = N_{\nu_i}^{\text{eq}} \). Of course, the whole point of our new mechanism is that \( N_{\nu\nu}^{\text{eq}} \neq 0 \) and so it is possible to generate an asymmetry when the sterile neutrinos are in equilibrium, but in the limit where \( N_{\nu\nu}^{\text{eq}} \to 0 \) we should still recover the traditional form of the Boltzmann equation.

Our next task is to parametrise the amplitude \( |\mathcal{M}_{\nu\nu}|^2 \) in terms of neutrino parameters. Firstly we note that

\[
\sum_{i,j=1,2,3} \text{Re} \left( \frac{K_i}{M_j} \right) \equiv \frac{\overline{m}_i}{v^4},
\]

where \( \overline{m}^2 = m_1^2 + m_2^2 + m_3^2 \) is the sum of the neutrino mass-squares. After a little algebra we can also write

\[
\sum_{i=1}^{3} \text{Re} \left( \frac{K_i}{M_j} \right) = \frac{\bar{m}_i}{v^4} \left( \sum_{i} x_i m_i - \bar{m}_i \right),
\]

where \( x_i \) are \( O(1) \) parameters discussed in sec. 3. We make the standard choice in the literature \([9]\) and set \( \text{Re}(\bar{K}_{13}) = \text{Re}(\bar{K}_{23}) = 0 \), or equivalently, \( x_2 = x_3 = 0 \). Equation \((30)\) then implies \( x_1 = m_1/\bar{m}_1 \) and the RHS of \((24)\) simplifies to \( (\bar{m}_1 - \bar{m}_2^2)/v^4 \). Admittedly, this choice is somewhat arbitrary and its main aim is really to reduce the number of free variables, allowing for a simpler parametrisation of the theory. We shall work in this regime for the remainder of this Letter. Putting this together, the amplitudes become

\[
|\mathcal{M}_{\nu\nu}|^2 = \frac{3}{2} \frac{m_1 M_1}{v^2} \int_0^\infty dx \, x^{3/2} \left( \bar{m}_1 F_{\nu\nu}(s) + 6(G_{\nu\nu}(s) + 1) \left( m_1^2 - \bar{m}_1^2 \right) \right)
+ 3\left( \bar{m}^2 - \bar{m}_1^2 \right),
\]

allowing us to write eq. \((17)\), after a little manipulation, as

\[
W_{\nu\nu} = \frac{3}{32\pi^2} \frac{m_1 M_1}{v^2} \int_0^\infty dx \, x^{3/2} / M_1 \left( \sqrt{x} \right)
\left[ K^2 F_{\nu\nu}(x) + 6(G_{\nu\nu}(x) + 1) \left( m_1^2 - \bar{m}_1^2 \right) + 3\left( \bar{m}^2 - \bar{m}_1^2 \right) \right] .
\]

For fixed SM neutrino masses, the amplitude becomes a function of essentially two variables, \( M_1 \) and \( K \), which ultimately depend on the details of the high-energy theory. A short calculation also shows that the delta function term in \( F_+ \) gives a contribution \(-W_{\nu\nu} \) to \( W_{\nu\nu} \).

Making the substitution \( y = x/v^2 \) in the integral, we arrive at

\[
W_{\nu\nu} = \frac{1}{32\pi^2} \frac{m_1 M_1}{v^3} \frac{1}{\sqrt{y}} \int_0^\infty dy \, y^{3/2} / K_1 \left( \sqrt{y} \right)
\left[ K^2 F_{\nu\nu}(y) + 6(G_{\nu\nu}(y) + 1) \left( K^2 - \frac{m_1^2}{m_1^2} \right)
+ 3\left( \frac{m^2}{m_1^2} - K^2 \right) \right] .
\]

Since \( F_{\nu\nu}(y), G_{\nu\nu}(y) \to 0 \) as \( x \to \infty \), we see that in the high temperature limit \( z \to 0 \), \( W_{\nu\nu} \) takes the form

\[
W_{\nu\nu}(z \ll 1) \simeq \frac{3}{4\pi^2} \frac{m_1 M_1}{v^2} \frac{1}{z^2} \bar{m}_1^2 \left( \frac{m_1^2}{m_1^2} + K^2 - \frac{2m_1^2}{m_1^2} \right) ,
\]

where we used the result \( \int dy y^{3/2} / K_1(y) = 32 \). Similarly, at low temperatures \( F_1(0) = 3 \), \( G_1(0) = -1 \) leading to

\[
W_1(1 \ll z) \simeq \frac{3}{4\pi^2} \frac{m_1 M_1}{v^2} \frac{1}{z^2} \bar{m}_1^2 .
\]

Since \( F_1(0) = (3 + c^2)/(1 + c^2) \) and \( G_1(0) = -(2 + c^2)/(2(1 + c^2)) \), we also have

\[
W_1(1 \ll z) \simeq \frac{3}{4\pi^2} \frac{m_1 M_1}{v^2} \frac{1}{z^2} \left[ \frac{1}{v^2 m_1^2} + \frac{1}{3m_1^2 v^2} \right] .
\]

Given that \( c \ll 1 \), the second term is sub-dominant, so that to leading order the asymptotic form of eq. \((30)\) is the same as \((29)\). The contributions to \( W \) in eq. \((15)\) are shown in figure \(2\) where we took \( \overline{m} = \Delta m^2_{13} + \Delta m^2_{21} + 3m_1^2 = \Delta m^2_{sol} + \Delta m^2_{atm} \), setting \( m_1 = 0 \).

3. Parametrisation of the CP violation

The fundamental source of CP violation is of course the Yukawa phases contained in \( h_{ij} \), or more specifically, the quantities \( \text{Im}(\bar{K}_{ij}) \) which control the strength of CP violation both in the lepton propagator and \( N_{\nu\nu}^{\text{eq}} \) and also in the decays of sterile neutrinos via \( \epsilon_1 \). One might ask to what extent the CP violation in these two sectors is linked, and also how much each is constrained by low-energy neutrino physics. For hierarchical sterile neutrinos, \( M_1 \ll M_2 \ll M_3 \) we find that

\[
\epsilon_1 \simeq \frac{1}{8\pi} \sum_{j=1}^{3} \frac{\text{Im}(\bar{K}_{1j})}{K_{11}} \left( \frac{M_j}{M_1} \right) .
\]

\(^3\)The narrow width approximation means that \( c = (h^2)_{11}/8\pi = \Gamma_{N_1}/M_1 \ll 1 \). This ensures consistency in treating the sterile neutrinos as quasi-stable particle states in the Boltzmann equation.
which after a little algebra can be re-written in terms of light neutrino parameters as

\[ \varepsilon_1 \simeq \frac{3}{16\pi^2} \frac{M_1}{v^2} \sum_{i, j} \frac{\Delta m_{ij}^2}{m_i} \ln \left( \frac{\tilde{h}_{ij}^2}{\tilde{h}_i^2} \right) \]  

We can parametrise the CP violation in this quantity by using the parameters \( z_i \) defined as

\[ \tilde{h}_{ij} = \sqrt{m_j} \Omega_{ij}, \]

where \( \sum_i |z_i|^2 = 1 \) and \( \tilde{h} \) is the mass-eigenstate Yukawa coupling given by \( \tilde{h} = U h \) where \( U \) is the PNMS matrix. This satisfies

\[ \tilde{h}_{ij} = \frac{1}{\sqrt{m_j \Omega_{ij}}}, \]

where the see saw formula \( \tilde{h}_{ij}^2 / M_j = m_i \) implies that \( \Omega \) is orthogonal and therefore satisfies \( (\Omega^T \Omega)_{ij} = 1 \). This implies that

\[ \frac{y_1}{m_1} + \frac{y_2}{m_2} + \frac{y_3}{m_3} = 0, \]

and

\[ \tilde{m}_1 x_1 + \tilde{m}_2 x_2 + \tilde{m}_3 x_3 = 0. \]

Hence the strength of CP violation in \( N_1 \) decays can be neatly parametrised as

\[ \varepsilon_1 = \frac{3}{16\pi^2} \frac{M_1}{v^2} \left( \frac{\Delta m_{23}^2}{m_2} y_2 + \frac{\Delta m_{31}^2}{m_3} y_3 \right). \]  

One might now ask whether the size of \( \varepsilon_1 \), or more specifically the quantities \( y_i \), uniquely constrain the CP violation appearing in

\[ N_{B-L}^{eq} = \frac{\pi^2 R}{2\zeta(3)\pi^2} \sum_{i, j} \frac{\text{Im} \left[ K_{ij}^2 \right]}{18M_i M_j} I_{(ij)}. \]  

The answer to this question is no, as we now explain. Firstly, one should note that “CP violation” only really makes sense in the context of a particular process, since a given scattering amplitude or decay channel is determined not only by the Yukawa phases in \( h_{ij} \), but also by the combinations of masses \( M_i \) involved in the relevant diagrams. In this sense, there will be certain regions of parameter space for which CP violation in one process is strong and simultaneously weak in another. For instance, \( \varepsilon_1 \) depends only on the Yukawa couplings via the quantity \( \sum_i \text{Im} (K_{ij}^2) / M_j \), but this is invariant under the transformation

\[ \text{Im} \left[ K_{ij}^2 \right] \rightarrow \text{Im} \left[ K_{ij}^2 \right] + M_i \frac{\varepsilon_i}{M_j}, \]

where \( M_i \) is an arbitrary energy scale. This leaves \( \varepsilon_1 \) fixed, but changes \( \text{Im} \left[ K_{ij}^2 \right] \) and therefore the size of CP violation in \( (38) \), in which \( I_{(ij)} \) depends on a completely different combination of masses from those appearing in \( \varepsilon_1 \). Instead, for \( M_j \gg M_i \) we find that \( I_{(ij)} \) has the asymptotic behaviour \[ I_{(ij)} \sim \frac{1}{(4\pi)^4} \frac{M_i^2}{M_j} \ln \left( \frac{M_i^2}{M_j^2} \right), \]

so that

\[ N_{B-L}^{eq} \simeq \frac{\pi^2 R}{2\zeta(3)\pi^2} \sum_{i, j} \frac{\text{Im} \left[ K_{ij}^2 \right]}{18M_i M_j} \frac{1}{(4\pi)^2}. \]

We therefore see that constraining the size of \( \varepsilon_1 \) still leaves the three quantities \( \text{Im} \left[ K_{13}^2 \right], \text{Im} \left[ K_{23}^2 \right] \) and \( \text{Im} \left[ K_{12}^2 \right] \) underdetermined, so that the size of \( N_{B-L}^{eq} \) is not fully constrained in terms of \( \gamma_i \) of eq. (33). In this sense, the gravitational effect is sensitive to different details of the high-energy see-saw physics compared to the usual delayed decay picture and is less constrained by SM neutrinos. Therefore, the only reasonable constraint which can be placed on the couplings \( \text{Im} \left[ K_{ij}^2 \right] \) appearing in \( N_{B-L}^{eq} \) is that they should be perturbatively small, in the sense that \( K_{ij}^2 / 4\pi \), which plays the role of a fine-structure constant, must be less than 1.
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4. Evolution of the lepton asymmetry
We now describe the solution of the Boltzmann equa-
tions (1) and (2), highlighting the different leptogenesis
scenarios that occur depending on the value of the CP
violating parameter \epsilon_1 which governs the sterile
nuclide decays. These scenarios are illustrated in figures
3 and 4.
In all cases, even if we start from a vanishing ini-
tial net lepton number at high temperatures, the system
very rapidly attains its gravitationally-induced equilib-
rium asymmetry \(N_0^{\text{eq}}(z) \neq 0\). The asymmetry then
tracks this equilibrium value as the Universe cools. As
the corresponding rate for the lepton number-violating
interactions falls (see figures 2 and 8), the system can
no longer follow the extremely rapid \(1/z^5\) decrease in
\(N_0^{\text{eq}}\) and the asymmetry freezes out. The region of \(z\) at
which this decoupling takes place depends on the ster-
ile neutrino mass \(M_1\) and \(K\), which control the washout
coefficient \(W\). In the scenarios illustrated here, decou-
pling takes place for small values of \(z\), significantly be-
low the scale \(z \sim 1 - 10\) at which the effects of the \(N_1\)
resonance in \(W\) and the \(N_1\) decays are felt. In the first
scenario (figure 3), we consider maximal \(\epsilon_1 \approx 10^{-6}\) (set-
ing \(y_2 = 0, y_3 = 1\) in (37)) as in the standard delayed-
deay picture. Then, with the parameters shown, since
the asymmetry generated by the out-of-equilibrium \(N_1\)
decays is larger than the gravitational effect and occurs later (for \(z \gtrsim 1\)), the gravitationally-induced asymmetry
is washed out and the system then evolves according to
the conventional decay scenario with no memory of the
early-time gravitational effects.
A scenario where \(\epsilon_1\) is smaller is shown in figure 4. In
this case, although the sterile neutrino decays do gen-
erate an asymmetry as usual, this effect is smaller than the
gravitationally-induced asymmetry after freeze-out. Re-
markably, therefore, in this scenario the final asym-
metry is completely determined by the gravitational me-
chanism, with the decays playing no significant role. This
alters our understanding of the parameter space of lep-
togenesis, showing that regions which were previously
believed to give an asymmetry in terms of decays are
actually dominated by the gravitational mechanism.
Since our main interest here is in illustrating the
mechanism of gravitational leptogenesis, we now study

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Plot for \(K = 1, M_1 = 10^{10}\)GeV with \(\epsilon_1 = 10^{-6}\)
and \(\text{Im}(K_1^2)/(4\pi)^2 = 10^{-5}, M_2 = 10^{16}\). In the full
solution (pink), we see that at early times, there is a gravita-
tionally induced asymmetry, but the \(\epsilon_1 D(N_1 - N_0^{\text{eq}})\) term
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nuclide decays. These scenarios are illustrated in figures
3 and 4.
In all cases, even if we start from a vanishing ini-
tial net lepton number at high temperatures, the system
very rapidly attains its gravitationally-induced equilib-
rium asymmetry \(N_0^{\text{eq}}(z) \neq 0\). The asymmetry then
tracks this equilibrium value as the Universe cools. As
the corresponding rate for the lepton number-violating
interactions falls (see figures 2 and 8), the system can
no longer follow the extremely rapid \(1/z^5\) decrease in
\(N_0^{\text{eq}}\) and the asymmetry freezes out. The region of \(z\) at
which this decoupling takes place depends on the ster-
ile neutrino mass \(M_1\) and \(K\), which control the washout
coefficient \(W\). In the scenarios illustrated here, decou-
pling takes place for small values of \(z\), significantly be-
low the scale \(z \sim 1 - 10\) at which the effects of the \(N_1\)
resonance in \(W\) and the \(N_1\) decays are felt. In the first
scenario (figure 3), we consider maximal \(\epsilon_1 \approx 10^{-6}\) (set-
ing \(y_2 = 0, y_3 = 1\) in (37)) as in the standard delayed-
deay picture. Then, with the parameters shown, since
the asymmetry generated by the out-of-equilibrium \(N_1\)
decays is larger than the gravitational effect and occurs later (for \(z \gtrsim 1\)), the gravitationally-induced asymmetry
is washed out and the system then evolves according to
the conventional decay scenario with no memory of the
early-time gravitational effects.
A scenario where \(\epsilon_1\) is smaller is shown in figure 4. In
this case, although the sterile neutrino decays do gen-
erate an asymmetry as usual, this effect is smaller than the
gravitationally-induced asymmetry after freeze-out. Re-
markably, therefore, in this scenario the final asym-
metry is completely determined by the gravitational me-
chanism, with the decays playing no significant role. This
alters our understanding of the parameter space of lep-
togenesis, showing that regions which were previously
believed to give an asymmetry in terms of decays are
actually dominated by the gravitational mechanism.
Since our main interest here is in illustrating the
mechanism of gravitational leptogenesis, we now study

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{The other parameters are the same as figure 3 but we now take \(\epsilon_1 = 10^{-8}\). For this value of \(\epsilon_1\), the
full solution is solely dominated by gravitational ef-
facts (pink curve), i.e. the decays have no effect on the
relic asymmetry. This can be clearly seen by compari-
sion with the dotted purple curve which neglects decays
entirely by setting \(\epsilon_1 = 0\), and shows that the full sol-
ution is essentially independent of decays. From the
black dashed curve, we see that taking into account de-
cays alone does not give an accurate representation of the
true solution.
in detail the extremal case where the CP-violating decay parameter $|\epsilon_1| = 0$ is minimal. In this case, only binary scatterings contribute and the Boltzmann equation for $N_{B-L}$ simplifies radically:

$$\frac{dN_{B-L}}{dz} = -W( N_{B-L} - N_{B-L}^{eq}) . \quad (42)$$

As we now see, this scenario is readily realised by choosing opposite signs for the Yukawa phases in (31), (37). This places a constraint on the high energy physics of the form

$$e_1 = 0 \implies M_3 \text{Im}[K_{31}^2] + M_2 \text{Im}[K_{13}^2] = 0, \quad (43)$$
or equivalently, from eq. (37),

$$\frac{\Delta m_{21}^{y1}}{m_2} + \frac{\Delta m_{31}^{y1}}{m_3} \approx 0. \quad (44)$$

Even with this restriction, there still remains much freedom in the choice of CP violation in the quantities $\text{Im}[K_{ij}]$ contained in (43) - for instance, eq. (44) places no constraints on the phases of $K_{31}^2$. For simplicity, we shall set $\text{Im}[K_{31}^2] = 0$ and from eqs. (41) and (43) we find

$$\sum_{i,j} \frac{\text{Im}[K_{ij}]}{M_i M_j} I_{i,j} = \frac{1}{M_1^2} \frac{\text{Im}[K_{13}^2]}{(4\pi)^4} \left(1 - \frac{M_2^2}{M_3^2}\right) \left(\frac{M_3}{M_1}\right) \quad (45)$$

so that if $M_1 \ll M_2 \ll M_3$ we have

$$\sum_{i,j} \frac{\text{Im}[K_{ij}]}{M_i M_j} I_{i,j} \approx \frac{\text{Im}[K_{13}^2]}{(4\pi)^4 M_1^2} \left(\frac{M_3}{M_1}\right) . \quad (46)$$

Notice that the size of the CP asymmetry is enhanced by the hierarchy between $M_3$ and $M_1$. In what follows, we shall treat $\text{Im}[K_{13}^2]$ as a free parameter controlling the strength of CP violation. Putting this together we find

$$N_{B-L}^{eq} \approx \frac{\sqrt{3\pi^2} \, \sigma(3/2)(1 - 3w)(1 + w)}{36 \zeta(3)} \left(\frac{M_1}{M_P}\right)^3 M_3 \ln \left(\frac{M_3}{M_1}\right) \frac{\text{Im}[K_{13}^2]}{(4\pi)^4} \left(\frac{M_3}{M_1}\right) \frac{\beta}{z^2} . \quad (47)$$

The corresponding solution of the Boltzmann equation (42) in this scenario is shown in figure 5. In this case, following the freeze-out of the asymmetry from its equilibrium value, the only further new feature is the late-time reduction of $N_{B-L}$ in the region $z \sim 1 - 10$ which is due to the contribution to the $W$ near the $N_1$ resonance. This raises the value of $W$ and pulls the asymmetry back, albeit only slightly with the parameter choice in figure 5 in the direction of the equilibrium value. This is also apparent from figure 8 where it is clear that $\Gamma_W / H$ once again becomes of order 1 in this region. The size of this late-time reduction in $N_{B-L}$ depends on the choice of parameters $M_1$ and $K$, in particular increasing sharply with $K$ as illustrated in figure 6. The key observation, however, is that even in this model

Figure 5: Solutions to the Boltzmann equation (42), for $K = 1$ for fixed $M_1 = 10^{10}$GeV and $\text{Im}(K_{13}^2)/(4\pi)^2 = 10^{-6}$, $M_3 = 10^{16}$GeV. The blue line shows the numerical solution, the red the analytic solution, valid at early times whilst the black dashed line gives the equilibrium curve. The vertical dashed line shows the value $z_d$ where $\Gamma / H \approx 1$.

Figure 6: Some of the solutions corresponding to figure 5 with $\text{Im}(K_{13}^2)/(4\pi)^2 = 10^{-6}$.

with the CP-violating parameters chosen such that the sterile neutrino decays produce a negligible asymmetry,
the gravitational leptogenesis mechanism on its own can produce the observed cosmological baryon asymmetry for an otherwise conventional choice of see-saw neutrino parameters. For example, in figure 5 the sterile neutrino masses were chosen to be $M_1 = 10^{10}$ GeV, $M_3 = 10^{16}$ GeV and $K = 1$, with $\Im(K_{13}^2)/(4\pi)^2 = 10^{-6}$. The corresponding value for the final relic baryon asymmetry is given by

$$\eta_B = \frac{1}{f} C_{\text{ph}} N_{\text{B-L}}^f,$$  

where $f = 2387/86$ is a photon production factor and $C_{\text{ph}} = 28/70$ is the sphaleron efficiency factor [9, 10]. Clearly, the observed asymmetry, $\eta_B \approx 10^{-10}$ can be obtained for a significant range of the parameters $M_1, M_3, \Im(K_{13}^2)$ and $K$. For example, in figure 7 we illustrate the dependence of $\eta_B$ on $\Im(K_{13}^2)/(4\pi)^2$ and $K$ for fixed $M_1, M_3$.

4.1. Analytic solution for $z \ll 1$

To gain a little more insight into these numerical solutions, recall from sec. 2 that for small $z$ we have $W \sim 1/z^2$ so that the Boltzmann equation (42) takes the form

$$N_{\text{B-L}}^f(z) = \frac{\alpha}{z^2} \left( N_{\text{B-L}}^f(z_0) - \frac{\beta}{z^2} \right), \quad z \ll 1,$$  

where $\alpha$ is a constant depending on $K$ and $M_1$ which can be inferred from the small $z$ behaviour of $W$ given in equation eq. (28). For small $m_1 \ll \bar{m}$, we have

$$\alpha = \frac{6}{\pi^2} \frac{M_1 m_1}{v^2} \left( m_0^2 + K^2 \right).$$  

Rather surprisingly for a Boltzmann equation, eq. (49) has an analytic solution, which for zero initial lepton asymmetry (at $z_0$) is given by

$$N_{\text{B-L}}^f(z_0, \alpha) = \alpha \beta \left( f(z) - f(z_0) \epsilon^{-\alpha \beta \left( \frac{z}{z_0} \right)} \right),$$  

where

$$f(z) = \frac{120}{\alpha^2} + \frac{1}{\alpha^2 z^2} + \frac{5}{\alpha^2 z^4} + \frac{20}{\alpha^2 z^6} + \frac{60}{\alpha^2 z^8} + \frac{120}{\alpha^2 z^{10}},$$

providing a nice consistency check with our numerical solutions for $z \lesssim 1$. This analytic solution is shown along with the full numerical solution of the Boltzmann equation (42) in figure 5.

As noted above, the dip in the solution at late times is due to the departure from the 1/z^2 of W as it approaches a local maximum (see figure 8) shortly after $z = 1$, raising the reaction rate momentarily, and bringing the solution back slightly closer to equilibrium. Before this resonance effect, which is difficult to estimate analytically, the asymmetry after initial decoupling from $N_{\text{B-L}}^{eq}$ is given approximately from eqs. (51), (52) as

$$N_{\text{B-L}}^f(z_0) \approx 120 \beta \frac{z}{\alpha^2}.$$  

This gives a good approximation to the full numerical result and is a useful guide in scanning the parameter space of $M_1$ and $K$.

4.2. Decoupling

Finally, we wish to briefly emphasise a few subtleties concerning the nature of the decoupling temperature of the lepton number violating interactions. Traditionally one argues that the lepton asymmetry freezes out at $z = z_D$ when $H(z_D) \approx 1$, and estimates the freeze-out asymmetry by $N_{\text{B-L}}^{eq}(z_D)$. Of course, as is clear from figure 5, the decoupling of the asymmetry is not a sharp transition but takes place gradually over a range of values of $z$ in the vicinity of $z_D$. However, while this is not in itself a big difference in terms of $z \sim z_D$, because of the extremely steep 1/z^2 dependence characteristic of the gravitationally-induced $N_{\text{B-L}}^{eq}$, it can translate into a difference of several orders of magnitude in the corresponding estimate for $N_{\text{B-L}}^{eq}$ at freeze-out. For example, we see from figures 5 and 8 that $\Gamma(z)/H(z)$ reaches 1 for the first time at $z_D \approx 0.001$, where $N_{\text{B-L}}^{eq}(z_D) \approx 2 \times 10^{-10}$.
However, the actual value to which $N_{B-L}$ freezes between $0.01 \lesssim z \lesssim 1$ is in fact $N_{B-L} \approx 2 \times 10^{-9}$, meaning that the true value is actually two orders of magnitude different from the naive approximation. In general, $N_{B-L}$ is over-abundant compared to $N_{\nu}^{eq}$ unless $\Gamma/H$ is quite a bit larger than 1, requiring a high reaction rate keep up with the rapidly falling equilibrium value. This means that the estimates used in, for example, [8] for the relic asymmetry in general gravitational leptogenesis models based on an effective interaction of the form $\mathcal{L} \sim \partial_{\mu} R f_{B-L}^{\mu}/M^2$ may be increased by a few orders of magnitude when the full Boltzmann analysis is used. Of course, since as we have seen the subsequent local maximum in $\Gamma/H$ shown in figure 8 in the vicinity of $z = 1$ causes the asymmetry to drop again in a way which is difficult to estimate analytically, it is clear that the only way to determine the final asymmetry reliably is to solve the full Boltzmann equation numerically as in figure 7.

5. Conclusions

In this Letter, we have presented a detailed study of the dynamics of lepton number generation in the early Universe, taking into account both the conventional out-of-equilibrium decays of the sterile neutrinos in the see-saw model and our new mechanism of gravitational leptogenesis [1, 2]. This has demonstrated clearly for the first time that this gravitational mechanism indeed provides a viable scenario to explain the observed baryon asymmetry $\eta_B \approx 10^{-10}$.

This study, which sheds new light on traditional perspectives in leptogenesis, involved a full numerical analysis of the coupled Boltzmann equations, modified to include the non-vanishing equilibrium asymmetry generated at two-loop order by the gravitational interactions. The parameter space of high-energy Yukawa phases was explored fully, showing that the CP violation in the gravitational and sterile neutrino decay sectors can be dialled independently. Whether the final asymmetry is determined by the gravitational or decay effects is then controlled by the size of the CP-violating decay parameter $\epsilon_1$. In particular, even in the limit of minimal $\epsilon_1 \approx 0$, we showed that the observed value of $\eta_B$ may be obtained for otherwise standard choices of neutrino parameters in the see-saw model. This establishes radiatively-induced gravitational leptogenesis as a viable mechanism for explaining the matter-antimatter asymmetry of the Universe.

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