On Thermalization in de Sitter Space

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Abstract

We discuss thermalization in de Sitter space and argue, from two different points of view, that the typical time needed for thermalization is of order $R^3/l_{pl}^2$, where $R$ is the radius of the de Sitter space in question. This time scale gives plenty of room for non-thermal deviations to survive during long periods of inflation. We also speculate in more general terms on the meaning of the time scale for finite quantum systems inside isolated boxes, and comment on the relation to the Poincaré recurrence time.

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1 Introduction

One of very few places where one has realistic hopes of detecting fundamental physics at or near the Planck scale is in the Cosmological Microwave Background Radiation (CMBR). The reason for this is that inflation expands microscopic fluctuations into macroscopic seeds for structure formation, and thereby possibly imprinting \textit{trans-Planckian} physics on large scales. For some recent reviews with references see [1, 2]. Unfortunately our understanding of Planckian or stringy physics, and the way inflation is embedded into a fundamental theory, is not sufficiently well developed to allow for reliable predictions of what to be expected.

Various models of possible trans-Planckian physics have been proposed indicating the rough nature of possible effects. In [3] it was argued that quite independently of the exact mechanism, one can expect a modulated primordial spectrum with an amplitude linear in \( \frac{H}{\Lambda} \). The main idea was to mimic the unknown physics in terms of a non-standard vacuum choice at the fundamental scale. This is quite natural since the vacuum is not unique in an expanding universe. In [4] this was phrased in terms of a (complex) one parameter family of de Sitter invariant vacua, called the \( \alpha \)-vacua, [5, 6, 7, 8].

The introduction of the \( \alpha \)-vacua has made it possible to analyze the trans-Planckian proposal in a systematic way. There has been a lively debate [9, 10, 11, 12, 13, 14, 15, 16, 17] as to whether a departure from the Bunch-Davies (or \textit{thermal}) vacuum is at all possible and whether field theoretical arguments put severe constraints on physics that is expected to be inherently Planckian. In this paper we will not address this issue, but instead focus on the more general question of to what extent non-thermal departures from the Bunch-Davies vacuum are possible.

The question we want to discuss is whether, and how long, non-thermal traces of physics before a possible beginning of inflation can survive and affect the CMBR. A similar question was asked in [21] where non-thermal initial conditions were introduced at a specific time and their effects on the CMBR analyzed. Technically, as will be explained in the next section, the analysis is a direct parallel to what was done in [3] even though the physics is slightly different.

It is reasonable to expect that any non-thermal deviation from the Bunch-Davies vacuum will experience thermalization due to the Hawking radiation present during inflation. In this paper we are in particular interested in the actual thermalization time \( \tau \), that is, the maximal time non-thermal deviations can survive in de Sitter space. We will argue, in two distinct but possibly related ways, that this time is of order \( \tau \sim \frac{R^3}{l_p^2} \), where \( R \) is the horizon radius. This is substantially longer than the naive expectation of a time scale of the order \( 1/T \).

This, one could argue, is generic for quantum systems inside isolated boxes and does not have anything particularly to do with de Sitter space or Hawking radiation. With this in mind, we discuss in more general terms the meaning of the relaxation time mentioned above. In particular we try to relate it to another typical time scale relevant for finite isolated systems, namely the Poincaré recurrence time.
The plan of the paper is as follows. In section 2 we briefly review the trans-
Planckian problem. Particular attention is given to generic signatures resulting from
the assumption that something special happens at a definite scale, above which new
physics is supposed to take over. The “transverse” case, where instead something
special happens at a definite time during the rolling of the inflaton field, is also
discussed. In section 3 the issue of thermalization is addressed. As already mentioned,
the typical thermalization time for non-thermal excitations in de Sitter space is argued
to be of order $\tau \sim R^3/l_p^2$. The section ends by attempts to put this into a more general
context. We conclude in section 4.

2 Signatures of new physics in the CMBR

2.1 Modified initial conditions on a fixed scale

It has been argued in several works that physics at or beyond the Planck scale (or
string scale) might leave a non negligible imprint in the CMBR. The basic argument
behind the claim is that the rapid expansion of the universe during inflation magnifies
small scale physics and makes it accessible on large scales. Going back in time the
quantum fluctuations responsible for the structure in the CMBR eventually reaches a
linear scale of the order the Planck scale and one would expect that Planckian physics
determine their form. In fact, in an expanding universe the notion of a vacuum is
not unique. The standard procedure is to ignore the presence of the Planck scale
and trace the quantum state back to the infinite past. In an accelerated universe
(where the scale factor grows faster than the Hubble radius) the linear scale of the
fluctuations can be made arbitrarily smaller than the Hubble scale and the vacuum
becomes unique. In exact de Sitter space the vacuum is called the Bunch-Davies
vacuum and happens to be thermal.

There is clearly something unsatisfactory with this procedure. A more conserva-
tive approach was advocated in \[3\] where it was argued that one should stop when
the modes reached the Planck scale (or string scale) and impose initial conditions
there. It was shown that a particular choice of initial conditions (the instantaneous
Minkowski vacuum) leads to a corrected primordial spectrum of the form

$$P(k) = \left( \frac{H}{\dot{\phi}} \right)^2 \left( \frac{H}{2\pi} \right)^2 \left( 1 - \frac{H}{\Lambda} \sin \left( \frac{2\Lambda}{H} \right) \right).$$

(2.1)

It was also argued that the linear dependence on $H/\Lambda$ in the correction term is rather
generic. If, furthermore, we are not in exact de Sitter space but instead experience a
slow roll where $H$ is changing with time (which is translated into scale on the sky)
the primordial spectrum will have a characteristic modulation. See also \[18\] \[19\] \[20\].
The precise amplitude and phase of the modulation depends in a detailed way on the
initial conditions chosen, but the modulation is a generic effect and a typical sign of
trans-Planckian physics. It is amusing to compare the effect with the acoustic peaks which are due to oscillations after the fluctuations have reentered the horizon. The modifications we are studying have their origin in oscillations before the fluctuations exit.

### 2.2 Modified initial conditions at a fixed time

In [21] an analysis very similar to the one in [3] was performed. The idea was to impose initial conditions different from the Bunch-Davies vacuum not at a fixed scale at all times, but at a fixed time on all scales. The main difference is that the fixed scale \( \Lambda \) in eq. (2.1) should be replaced by the physical scale of the mode at the initial time. This way of imposing initial conditions breaks de Sitter invariance and gives rise to a different kind of modulation. There is also a numerical factor, as explained in [21], depending on the change of the slowroll parameters at the initial time. It is useful to draw a diagram which illustrates the two possibilities (see Fig. 1).

![Figure 1: The two “transverse” modifications of initial conditions during inflation. The initial conditions are imposed either at a certain scale \( \Lambda \), or at a certain time \( t_c \). The modes responsible for structure formation cross the horizon at \( R = 1/H \).](image)

In [21] it was argued that one should choose the Bunch-Davies vacuum at the fundamental scale. As a consequence, as argued in [21], any departure from the Bunch-Davies vacuum will be washed away on small scales. It is only on the very largest scales that traces of the initial conditions remain, and if inflation has proceeded longer than the minimal 60 or 70 e-foldings, the scales where the imprints are located has not yet reentered (and never will if the universe now starts to accelerate again).

The aim of the next section is to argue that traces of non-thermal physics much further back can remain hidden in the trans-Planckian regime and manifest themselves as modified initial conditions at the fixed scale.
3 Time scales of thermalization

3.1 Thermalization in de Sitter space

Having a non-thermal state in a thermal environment suggests that the construction has a limited lifetime. The question we will try to answer in this section is how long this lifetime really is. That is, for how long can inflation, with a non-thermal vacuum, continue before the vacuum relaxes and becomes thermal? A naive first estimate would suggest that the thermalization time in de Sitter space would be related to the temperature and be given by $1/T$, where $T$ is the Gibbons-Hawking temperature \[22\]. It is easy to see, however, that this crucially depends on the strength of the interactions that are causing the thermalization. In an expanding universe the interactions must be faster than the expansion of the universe for the thermalization to happen in this way. Since the time scale of the expansion is $H^{-1} \sim T^{-1}$ this depends in a detailed way on the actual interactions. In a slow-roll inflation, with its flat potential, the interactions are not fast enough and thermalization will not take place.\footnote{Note that this assumption is implicit also in \[21\] when it is argued that physics can remember initial conditions at a fixed time all the way back through many e-foldings.}

To summarize, after the fluctuations have grown larger than the horizon, there is no causal process which could affect them. As a consequence the fluctuations freeze and any non-thermal signatures are kept intact.\footnote{One way to see that a thermalization time of order $1/T \sim R$ is far too short, is to consider our present stage of cosmic acceleration. If we consider this state as a perturbed de Sitter space, one finds that with a thermalization time of order $R$ the lifetime for the perturbations would be roughly $R \sim 10$ billion years. Now our sun for example (being a small perturbation of the present de Sitter phase) will even 1000 billion years into the future still exist as a small perturbation, by then in shape of a cooling white dwarf.}

This global perspective could, however, be misleading from the point of view of a local observer \[23\]. It is important to note that taking a global perspective of the process ignores possible subtleties having to do with horizons. Since the physics of horizons, especially in de Sitter space, is largely mysterious one needs to be careful when they come into play. After all, the counterpart of black hole complementarity, \[24, 25, 26, 27\], is not well understood in de Sitter space (see, e.g. \[28, 29\]). Let us therefore take a local perspective and try to figure out what would be observed.

We start with a mode that is created at a definite scale and then expands to larger size. A local observer would see how the mode approaches the horizon but never crosses it. The closer the horizon it gets the more redshifted it becomes. From the local observer’s point of view anything approaching the horizon effectively enters into a region with ever higher temperature and will at some point in time be completely “thermalized” and turn into radiation. It therefore seems like there is indeed a finite lifetime for any excitations above the thermal state.

It is important to note that our first argument, where thermalization was excluded, was made using quantum field theory augmented with Hawking radiation. It is only when we take a further step, invoking physics related to the horizon – eventually
described by quantum gravity, holography and complementarity – that we are able to find signs of thermalization.

Before presenting the conditions we believe need to be fulfilled in order to give a correct estimate of the thermalization time, let us comment on a proposal made in [11], where the same issue was discussed. There thermalization also was invoked based on effects appearing only once the physics of horizons was taken into account. The condition the authors of [11] claim is determining the thermalization time is the time it takes an object (initially starting close to the center of the static patch) to be of order a Planck length away from the horizon. Indeed, from the local observers point of view the temperature at that point is of order the Planck scale, naively implying that gravitational interactions quickly thermalize the object in question. This estimate of the thermalization time is easily shown to give $\tau \sim R \ln R$.

We believe this way of arguing suffers from a number of different problems. First of all, it only concerns objects approaching the observers horizon, it has nothing to say about the observer herself and things bound to her. In particular, the time it takes for her to be thermalized, using this criterion, would be infinite. This is clearly wrong. In order to overcome this problem one could argue that, somehow, the time it takes for local objects to thermalize is also of order $R \ln R$. This, however, must also be wrong, as can be concluded from footnote 2 (and noting that $\ln R$ only gives a factor of order 100 for our present universe).

A second, perhaps more serious, objection concerns the actual validity of the argument even for objects approaching the horizon. If we focus on what an observer actually would see as an object recedes towards the horizon, one needs to be careful. Since the rate of the photons (emerging from the horizon) received by our observer is of order $1/R$ (see [29]), the time it would take for her to see the object being thermalized must be a much larger time than $R \ln R$, during which time only of order $\ln R$ photons could have been detected. In fact, the time it will take is of order $R^3$, as will be shown below.

The reason we find it important to focus on what actually would be seen is that, just like in inflation, one can imagine that the de Sitter phase is abruptly turned off, being followed by a more standard cosmological evolution. When this happens, the object will, of course, return to the observers causal patch at some time in the future [29]. Since the situation between the object and the observer is symmetric, it is clear that the object will be in as good shape as the observer. If the observer has not been thermalized by then, then neither should the object be, implying that the estimated thermalization time in [11] is far too short.

Indeed, considering the symmetric situation we have between the observer and the object (being for example another observer) and the fact that they can meet again some time after the de Sitter phase has turned off, seems to imply that the estimated thermalization time should be the same for local objects as for those who approach the horizon, even from the perspective of one single observer.

With this last observation in mind let us make two independent estimates of the thermalization time, one considering objects approaching the horizon, and one
considering local object, bound to the observer. We will find that they are indeed of the same order.

As mentioned above, in order to decide how fast an object (falling towards the horizon) is being thermalized, one should focus on what is actually seen by a local observer. From this point of view the thermalization process will be very slow considering the fact that the rate of photons received by the observer is of the order $1/R$, i.e. one photon every $\frac{1}{R^2}$. Let us now try to find out what actually happens to the object (according to the observer). To do that we think of the horizon as an area consisting of $R^2/l_{pl}^2$ Planck cells, and remember that the photon has a wavelength of order Planck scale when emitted and can indeed resolve specific Planck cells.

Now first assume the object in question is something really simple, corresponding to an information content much smaller than the $R^2$ number of degrees of freedom of the horizon. This would mean that only a few of the Planck cells are involved in encoding the object. In the extreme case of an object with entropy of $O(1)$, one would need to wait until of the order $R^2$ photons has been emitted to be reasonably sure to see a photon coming from the burning of the object. In the other extreme one can think of an object consisting of the order $R^2$ degrees of freedom. In this case it is clear that one has to wait until of the order $R^2$ photons has been emitted, in order for all parts of the object to have been burnt. And so regardless of the size of the object, one has to wait a time,

$$\tau \sim \frac{R^2}{l_{pl}^2} R = \frac{R^3}{l_{pl}^2},$$

(3.1)

in order to actually see the destruction.\footnote{We note that precisely this time scale was found in \cite{30}, where in that paper $R^3$ could, in the context of black holes, be interpreted as the time, after some object had fallen into the black hole, an observer would need to wait in order to recover the corresponding information.} At this point one could object and say that since the blueshift is so strong near the horizon, anything close to it would thermalize almost immediately, leading to a much shorter thermalization time. However, taking the blueshift into account also requires taking the time dilation into account. Indeed, this makes processes happening close to the horizon appear extremely slow. This is one way to see that the large blueshift is perfectly consistent with the relatively long time scale in eq. (3.1).

We therefore suggest that the maximal lifetime of non-thermal excitations in de Sitter space is given by $\tau \sim \frac{R^3}{l_{pl}^2}$. It is important to emphasize that this relatively long time scale follows only if other non-gravitational interactions are frozen. If this is not the case, and those interactions occur faster than the expansion of the universe, the characteristic thermalization time will scale like the naive $1/T$. In that case we do not need any reference to holography or complementarity. Thermalization occurs regardless of perspective.

Now let us try to estimate the time it takes to thermalize a local object, bound to the observer. Let us reconsider the possibility that local interactions do give rise to a thermalization, but only if we take physics near the Planck scale into account. Again
it is the Hawking radiation that we expect can do the job. The probability that the
Hawking radiation, with wavelength of order $R$, will interact (and thereby thermalize)
with physics at a fundamental length scale, say at the Planck length, is small, but
obviously nonzero. The question is, therefore, how long do we need to wait in order
for the probability of scattering of a given Planck cell with the de Sitter radiation to
be of $O(1)$? The claim is that this will give a rough estimate of the thermalization
time of the underlying microphysics. The interaction rate is $\Gamma = \sigma n v$, where for this
particular process we have for the cross section $\sigma \sim l_{pl}^2$, the number density of the
radiation $n \sim T^3 \sim 1/R^3$ and the relative velocity $v = c = 1$. Then the time $\tau$ it
takes for this process to be likely is given by the condition $\Gamma \tau \sim 1$ implying,

$$1 \sim \sigma n v \tau \sim l_{pl}^2 \cdot 1/R^3 \cdot 1 \cdot \tau \Rightarrow \tau \sim \frac{R^3}{l_{pl}^2},$$  \hspace{1cm} (3.2)

which is seen to coincide, up to orders of one, with the previous result. Therefore, regard-
less of whether local objects or objects falling towards the horizon are concerned,
the thermalization time will be the same. We argued above that this must be the
case based on the symmetry between the observer and the object and by noting that,
if the de Sitter phase is only temporary, they will meet again. We find it encouraging
that the above results are in agreement with this assessment.

To summarize, we have made three attempts to estimate the thermalization time.
The one where the effects of horizons was ignored led to an apparent contradiction
with an estimate based on horizons and holography. A reconciliation of these two
estimates need an understanding of complementarity. A reasonable conclusion is,
though, that $\tau$ represents the longest time that a non-thermal deviation can survive
from a local perspective.

We also found that the longer time scale can be obtained from Hawking radiation
acting through Planck scale physics. It would be interesting to understand this con-
nection better. The Planck scale argument suggests that the longer time scale could
have global importance by affecting the initial conditions. That is, the vacuum, at
Planck scale, would be thermalized after a time $\tau$.

Now let us see what the implications of these results are for inflation. In inflation
the Hubble constant is constrained from observations to be no larger than $H \sim 10^{-4} m_{pl}$. With this input the thermalization time for non-thermal excitations ($\alpha$-
vacua included) is found to be of order $\tau \sim R^3 = 1/H^3 \sim 10^{12} t_{pl}$. Comparing this
with the time needed for the required number of e-foldings, which for 70 e-foldings
is $t_{infl} \sim 70/H \sim 7 \cdot 10^5 t_{pl}$, one can conclude that the thermalization time allows
for visible effects of non-thermal behavior in the CMBR, with room to spare. On the
other hand, if we imagine that inflation went on uninterrupted for a very long time,
thermalization effects become important.\footnote{More generally, the thermalization time, in units of e-foldings, relevant for initial conditions on a
scale $\Lambda$ can be expected to be $(\Lambda/H)^2$. If the desired amplitude for the corrections to the primordial
spectrum is $10^{-2}$ the number of allowed e-foldings become $10^5$.}
3.2 General considerations of thermalization in cosmological like boxes

Let us investigate the thermalization a bit further. The time scale is in general given by
\[ \tau \sim \Gamma^{-1} \sim E^2/T^3. \]  
(3.3)

In the expanding de Sitter space, with a constant Hawking temperature, the expanding modes redshift like \( E \sim e^{-Ht}\varepsilon \). Cutting of at the horizon scale, where everything freezes, we find \( E \sim T \) and as a consequence the naive \( \tau \sim 1/T \). A slightly more detailed calculation focuses on the integral \( \int dt \Gamma \). It is only if this integral diverges before horizon crossing – i.e. there is time for an infinite number of interactions – that we get exact thermalization. In our case we find
\[ \int_{t_{\text{cross}}}^\infty dt \Gamma \sim \frac{T^3}{H^2}(e^{2Ht_{\text{cross}}}-1) \sim T/H \sim 1 \]  
(3.4)

where we have used \( e^{2Ht_{\text{cross}}} \sim \varepsilon H \sim \varepsilon/T \gg 1 \). Note that we have taken a global perspective for our argument. The only way for the integral to become large is if the interactions are truly strong, unlike what is the case in inflation, giving rise to a large dimensionless prefactor in the above expression.

A main point of our paper has been to argue that there could be another way to get thermalization. We first argued for this from a holographic point of view. We then found a local derivation which gave the same result. Let us now proceed along the second line of approach to see whether we can learn something more. We assume that Planckian physics can be modelled by a fixed \( E \) of order \( \Lambda \) and that the Planckian physics not only affects what is going on at lower energies, but also suffers a possibly thermalizing back reaction from low energy physics. Under these assumptions the integral trivially diverges and the system, as we have argued in the previous subsection, eventually thermalizes. To be slightly more general, we can consider a temperature which is not constant (unlike the Gibbons-Hawking temperature in de Sitter space), but instead redshifts with the expansion of the universe. We then find
\[ \int dt \Gamma \sim \int dt \frac{T^3}{\Lambda^2} \sim \int dt \frac{1}{a^3\Lambda^2}, \]  
(3.5)

which diverges only if the scale factor grows slower than \( t^{1/3} \). For this to be the case, one needs an equation of state \( p = \sigma \rho \) where \( \sigma > 1 \) which is unphysical. In conclusion we find that we must put the system in a box that prevents the temperature from redshifting. This is precisely what happens in de Sitter space.\(^5\)

\(^5\)It is amusing to speculate whether there are astrophysical systems where the time scale eq. \( 3.3 \) could be important. In this context one may note that the hottest stars, with internal temperatures of the order \( 1 \) GK (or \( 10^5 \) eV) give rise to a time scale of \( 10^{17} \) years, which clearly is much too long to be of any interest. However, there are several systems emitting radiation in the TeV range which
3.3 Remarks on time scales in finite isolated systems

One should note that the time scale we have discussed, although long, is very much shorter than the Poincaré recurrence time (see Fig. 2). The Poincaré recurrence time is the time scale over which macroscopic thermal fluctuations are expected to happen (for recent qualitative discussions on this in the context of de Sitter space, see for example [29, 31, 32, 33, 34, 35]). The typical time scale is given by $t_{\text{rec}} \sim e^S$, where $S$ is the entropy of the system. The second law is only meaningful near a macroscopic fluctuation and states that time always points towards increasing entropy. Our time scale eq. (3.1) is the maximum decay time (or, by time reversal symmetry, rise time) of a macroscopic fluctuation.\(^6\)

We suggest that this is a general behavior for finite isolated quantum systems, not limited to de Sitter space. Another such system of particular interest is large black holes in Anti de Sitter space (for discussions on long, and very long, time scales in this context, see for example [36, 37, 38, 39]). Since these black holes are eternal they are particularly intriguing laboratories to test some of the ideas mentioned above. Interestingly, both time scales (maximal relaxation and recurrence) seems to tell us something about retrieval of information that has fallen into the black hole [36, 37, 38, 39]. The mechanism for this and the meaning of these time scales is, however, still obscure [39]. Even though tempting, we do not to attempt to address

translated into temperature becomes $10^{16}$ K. If this is inserted into eq. (3.1), one finds a relaxation time of about an hour. If our reasoning is correct, one could speculate that in a region where TeV temperatures are sustained for an hour or so, effects of quantum gravity could give rise to a complete thermalization of any physical system (including baryon number violation etc.).

\(^6\)In [29] it was noted that there is an interesting geometrical connection between these time scales in a space time where inflation suddenly stops.
4 Conclusions

In this paper we have investigated the problem of thermalization in de Sitter space. We have argued, based on holography, that the maximum thermalization time – relevant if all non-gravitational interactions are weak and slower than the expansion of the universe – is given by eq. (3.1). We have also argued that this allows trans-Planckian physics to retain a memory of physics preceding inflation for up to \( N = H \cdot \tau \sim 1/R \cdot R^3 = R^2/l_{\text{pl}}^2 \) e-foldings, which for \( R \sim 10^4 l_{\text{pl}} \) gives \( N \sim 10^8 \).

We have also speculated that the time scale could have a more universal importance and not only be relevant for de Sitter space. In particular there seems to be two typical time scales involved in box-like configurations. One is the maximal relaxation time (which we interpreted in terms of thermalization in de Sitter space), which is the typical decay (or rise) time for macroscopic fluctuations. The other is the recurrence time, which is the typical time scale for which these macroscopic fluctuations are expected to happen.

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These issues here.\(^7\)

It is important to point out though that in the discussion of thermalization in de Sitter space, we mean thermalization strictly in the thermodynamic sense. We do not mean actual loss of coherence at the microscopic level. However, these residual correlations are not likely to be detectable and so effectively the spectrum would be thermal.

This is provided that the trans-Planckian physics can be viewed as a closed system and there are no sources of negentropy.\(^8\)
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