Photon helicity in $\Lambda_b \rightarrow pK\gamma$ decays

Federica Legger, Thomas Schietinger

Laboratory for High-Energy Physics, Ecole Polytechnique Fédérale, CH-1015 Lausanne, Switzerland

Abstract

Radiative decays of polarized $\Lambda_b$ baryons represent an attractive possibility to measure the helicity of the photon emitted in the $b \rightarrow s\gamma$ quark transition and thus to subject the Standard Model to a stringent test at existing and future hadron colliders. The most abundant mode, $\Lambda(1116)\gamma$, is experimentally very challenging because of the long decay length of the $\Lambda(1116)$. We show that the experimentally more accessible $\Lambda_b \rightarrow pK\gamma$ decays proceeding via $\Lambda$ resonances may be used to extract the photon helicity for sufficient $\Lambda_b$ polarization, if the resonance spin does not exceed 3/2. A direct comparison of the potential of such resonance decays to assess the photon polarization at a hadron collider with respect to the decay to $\Lambda(1116)$ is given.

Key words: Quark Masses and SM Parameters, B-Physics, Baryon decay

PACS: 11.30.Er, 13.30.-a, 13.88.+e, 14.20.Mr

1 Introduction

The helicity of the photon emitted in the $b \rightarrow s\gamma$ quark transition remains one of the last untested predictions of the Standard Model (SM) in the realm of $B$ physics [1]. Given the experimental difficulty of directly measuring the helicity of the emitted photon, several indirect methods for its determination in $B$ meson decays have been proposed, implying $B\overline{B}$ interference [1], photon conversion to $e^+e^-$ [2], resonant states in the $K\pi\pi^0$ final state [3], and interference with radiative charmonium decays [4]. A particularly attractive possibility arises from the decay of $b$-baryons, as first pointed out by Gremm, Krüger and Sehgal [5], and further elaborated on by Mannel and Recksiegel [6]. A rather complete study of polarized $\Lambda_b \rightarrow \Lambda\gamma$ decays in the context of a high-luminosity $Z$ factory (“Giga-Z”) has been given by Hiller and Kagan [7].

1 Now at Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland
Despite considerable efforts, the results obtained at the $e^+e^- B$ factories do not yet put significant constraints on the photon polarization in $b \to s\gamma$ [8]. With the Giga-$Z$ factory relegated to a distant future, but dedicated $B$ experiments at hadron colliders imminent, it is worth considering the potential of polarized $\Lambda_b$ decays at hadron colliders. At the Large Hadron Collider (LHC), for instance, the $\Lambda_b$ baryons produced in $pp$ collisions are expected to be polarized transversally with respect to the production plane, with a polarization possibly as large as 0.2 [9]. This polarization can be measured with an estimated statistical precision of 0.01 by an angular analysis of the decay $\Lambda_b \to \Lambda J/\psi$ [10].

An experimental issue arising at hadron colliders is the macroscopic decay length of the $\Lambda$ baryon ($c\tau = 7.89$ cm [11]). It typically escapes the innermost parts of a large detector system without leaving any trace, before weakly decaying, predominantly into a nucleon and a pion. This poses a severe problem to experiments relying on the observation of a decay vertex detached from the primary vertex to identify events containing $b$-hadrons [12,13], since neither the photon nor the $\Lambda$ baryon from the $\Lambda_b \to \Lambda\gamma$ decay produce a suitable signature. A possible way around this problem is afforded by considering radiative $\Lambda_b$ decays to $\Lambda$ resonances above the nucleon-kaon ($NK$) threshold, such as $\Lambda(1520)$ or $\Lambda(1670)$. With their prompt decay into $pK^-$ these resonances trace back the decay of the $\Lambda_b$, thus rendering it more accessible to the online and offline event selection.

The purpose of this Letter is to investigate the potential of $\Lambda_b \to \Lambda(X)\gamma$ decays ($X = 1520, 1670, 1690, \ldots$) for assessing the photon polarization in the $b \to s\gamma$ transition at hadron colliders, in particular in comparison to $\Lambda_b \to \Lambda(1116)\gamma$.

### 2 Photon polarization parameters

Decays of the type $\Lambda_b \to \Lambda\gamma$ are mediated by the quark transition $b \to s\gamma$. Long distance contributions, arising from $W$ or intermediate meson exchange, have been found to be negligible [6]. In the usual framework of an effective Hamiltonian, the relevant operators contributing at leading order (LO) in $\alpha_s$ are the electromagnetic dipole operators $O_{7}^{(i)} = (em_b)/(16\pi^2)\vec{\sigma}\sigma_{\mu\nu}R(L)bF^{\mu\nu}$, responsible for the emission of a left- or right-handed photon, respectively:

$$\mathcal{H}_{\text{eff}} = -4G_F^2 V_{tb}^* V_{ts}(C_7 O_7 + C_7^{'} O_7^{'}),$$

with $G_F$ the Fermi constant and $C_7^{(i)}$ the Wilson coefficient of the local operator $O_7^{(i)}$; $V_{tb}$ and $V_{ts}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix
elements. In the operator definition $e$ is the electric charge, $m_b$ the mass of the $b$-quark, $F_{\mu\nu}$ the electromagnetic field tensor and $\sigma_{\mu\nu} = \frac{i}{2} (\gamma_{\mu} \gamma_{\nu} - \gamma_{\nu} \gamma_{\mu})$. $R = (1 + \gamma_5)/2$ and $L = (1 - \gamma_5)/2$ are the right- and left-handed projectors, respectively. Thus in leading order, the occurrence of right-handed photons is given by the ratio of Wilson coefficients, $r = C_7'/C_7$. In the SM, $r = m_s/m_b$ by virtue of the chirality of the $W$ exchanged in the decay loop [1]. Various scenarios beyond the SM such as left-right symmetric models predict new contributions to $C_7'$ and therefore larger values for $r$. Hence the strong interest in constraining $r$ experimentally.

From the experimental point of view, the observable of interest is the photon asymmetry

$$\alpha_\gamma = \frac{P(\gamma_L) - P(\gamma_R)}{P(\gamma_L) + P(\gamma_R)}, \quad (2)$$

where $P(\gamma_{L(R)})$ represents the probability of producing a left-(right-)handed photon in the decay. In the leading-order limit, where only $O_7$ and $O'_7$ contribute, $\alpha_\gamma$ is related to $r$ by

$$\alpha_\gamma^{LO} = \frac{1 - |r|^2}{1 + |r|^2}. \quad (3)$$

Recently, however, it was shown that gluon bremsstrahlung contributions to the matrix elements of operators other than $O_7$ and $O'_7$ can give significant contributions to $\alpha_\gamma$, such that an experimental determination of that asymmetry would only yield an effective ratio, $r_{\text{eff}}$ [14]. In the following, we will set $r_{\text{eff}} \equiv r$ to simplify our notation, but keep in mind that the relation between $\alpha_\gamma$ and $r$ may be more complicated. Furthermore, we assume that $r$ is the same for $\Lambda_b$ and $\bar{\Lambda}_b$ decays, i.e., we do not consider CP violating effects.

### 3 Properties of $\Lambda$ resonances

In Table 1 we list the properties of the better known $\Lambda(X)$ resonances, many of which can only be crudely estimated at this time, based on data compiled by the Particle Data Group [11]. The Table also lists our estimates for the branching fractions for the decays $\Lambda_b \to \Lambda(X)\gamma$, which are based on the kinematic suppression due to the larger mass of the higher resonances given by the factor $(1 - m_\Lambda^2/m_{\Lambda_b}^2)^3$ [7]. They do not take into account differences in the form factors, nor a possible spin-dependence of the decay probability, both to be determined by experiment. Judging from recent data on $B \to K^*\gamma$ decays [15] and from dedicated form factor studies on semi-leptonic $B$ and $B_s$ meson decays [16], we may expect these estimates to be correct up to a factor of 2–3 only. To evaluate the $\Lambda(X) \to pK$ decay probabilities we use the rough $\mathcal{B}(\Lambda(X) \to N\bar{K})$ estimates given in the Table and assume equal probabilities.
for decays to $pK^-$ and $n\overline{K}^0$ from isospin coupling, thereby neglecting possible suppression effects from angular momentum barriers.

Table 1
Table of $\Lambda$ resonances decaying to $pK$ that are established with at least a fair degree of certainty. The listed widths $\Gamma$ and branching fractions $\mathcal{B}_{\text{tot}}$ are those used to produce Fig. 1. In the Table, $\mathcal{B}_{\Lambda(N\overline{K})} \equiv \mathcal{B}(\Lambda(X) \rightarrow N\overline{K})$, $\mathcal{B}_{\Lambda(X)\gamma} \equiv \mathcal{B}(\Lambda_b \rightarrow \Lambda(X)\gamma)$, and $\mathcal{B}_{\text{tot}} \equiv \mathcal{B}(\Lambda_b \rightarrow \Lambda(X)\gamma \rightarrow pK\gamma)$. The values for $\mathcal{B}_{N\overline{K}}$ are estimates based on data compiled in Ref. [11], whereas the $\mathcal{B}_{\Lambda(X)\gamma}$ are our estimates derived from simple kinematic suppression (see text).

| $\Lambda(X)$ | $L_{I,2J}$ | $\Gamma$ (MeV) | $\mathcal{B}_{N\overline{K}}$ (%) | $\mathcal{B}_{\Lambda(X)\gamma}$ ($10^{-5}$) | $\mathcal{B}_{\text{tot}}$ ($10^{-5}$) |
|-------------|-------------|----------------|-----------------|----------------|----------------|
| $\Lambda(1520)$ | $D_{03}$ | 15.6 | 45 | 5.84 | 1.31 |
| $\Lambda(1600)$ | $P_{01}$ | 150 | 22 | 5.69 | 0.65 |
| $\Lambda(1670)$ | $S_{01}$ | 35 | 25 | 5.56 | 0.69 |
| $\Lambda(1690)$ | $D_{03}$ | 60 | 25 | 5.52 | 0.69 |
| $\Lambda(1800)$ | $S_{01}$ | 300 | 32 | 5.30 | 0.84 |
| $\Lambda(1810)$ | $P_{01}$ | 150 | 35 | 5.28 | 0.92 |
| $\Lambda(1820)$ | $F_{05}$ | 80 | 60 | 5.26 | 1.57 |
| $\Lambda(1830)$ | $D_{05}$ | 95 | 6 | 5.24 | 0.15 |
| $\Lambda(1890)$ | $P_{03}$ | 100 | 22 | 5.12 | 0.56 |
| $\Lambda(2100)$ | $G_{07}$ | 200 | 30 | 4.67 | 0.70 |
| $\Lambda(2110)$ | $F_{05}$ | 200 | 15 | 4.65 | 0.34 |
| $\Lambda(2350)$ | $H_{09}$ | 150 | 12 | 4.12 | 0.28 |

Figure 1 illustrates the $pK$ effective mass spectrum resulting from our simplifying assumptions. While the true spectrum, to be measured experimentally, may look different in detail, it is still useful to have a general overview of the $\Lambda(X)$ resonance properties, which allows us to identify the most promising decay modes. The mass spectrum is likely to feature the three rather distinct peaks visible in Fig. 1. The first and most prominent of these peaks is due to the well-established $\Lambda(1520)$. Since this resonance has spin 3/2, the extraction of $\alpha_\gamma$, and thus $r$, via angular decay distributions is not straightforward. We will see in Sec. 4.2 that it is possible under certain conditions. The second peak is made up of the $\Lambda(1670)$ (spin 1/2) and $\Lambda(1690)$ (spin 3/2) resonances. It may be assumed that the different angular decay distributions allow for the disentanglement of the two resonances, so that $\alpha_\gamma$ can be extracted from a combined fit applied to events in that region. A possible third peak is probably dominated by the $\Lambda(1820)$. Since this resonance has spin 5/2, we do not consider it useful for the determination of the photon polarization.
4 Decay angular distributions for radiative $\Lambda_b$ decays

In the helicity formalism [17], we may write down the decay amplitude for the general case of the decay $\Lambda_b \to \Lambda\gamma \to p\gamma h$ ($h = K, \pi$) as

$$ A = \sum_{\lambda} D_{\lambda,\alpha}^{I*}(\phi_p, \theta_p, -\phi_p) D_{M,\lambda}^{J*}(\phi_\Lambda, \theta_\Lambda, -\phi_\Lambda) C_{\lambda,\alpha,\lambda} E_{\lambda}, \tag{4} $$

where $\lambda_i (J_i)$ is the helicity (spin) of particle $i$, $J$ and $M$ refer to the $\Lambda_b$ spin and its projection along the (arbitrary) quantization axis, respectively; the polar and azimuthal angles $\theta_\Lambda$ and $\phi_\Lambda$, defined in the $\Lambda_b$ rest frame, give the direction of the $\Lambda$ momentum relative to the quantization axis; the angles $\theta_p$ and $\phi_p$, defined in the $\Lambda$ rest frame, give the direction of the proton momentum relative to the $\Lambda$ flight direction. The quantities $C$ and $E$ parameterize the intrinsic helicity amplitudes for the decays $\Lambda_b \to \Lambda\gamma$ and $\Lambda \to p\gamma h$, respectively. If parity is conserved (i.e., in strong decays to $pK$), $|E_{\lambda_p}| = |E_{-\lambda_p}|$.

The decay probability is obtained by squaring the amplitude and summing over the final state helicities, which are not measured by the experiment,

$$ w = \sum_{M,\lambda,\lambda_p} \rho_{MM}|A|^2, \tag{5} $$

where the polarization density matrix $\rho$ takes account of the $\Lambda_b$ polarization with respect to the quantization axis. Since we do not consider correlations between the production and decay mechanisms, the non-diagonal elements of $\rho$ average out to zero [18] whereas the diagonal elements are characterized.

![Figure 1](image.png)

Fig. 1. Approximate $pK$ effective ("invariant") mass spectrum from $\Lambda_b \to pK\gamma$ decays, as obtained with the values in Table 1 by adding up simple non-relativistic Breit-Wigner forms. A possible non-resonant contribution is neglected, as are interference effects between the various resonances.
by \( \rho_{\frac{1}{2}, \frac{1}{2}} + \rho_{\frac{1}{2}, -\frac{1}{2}} = \text{Tr} \rho = 1 \) and the \( \Lambda_b \) polarization \( P_{\Lambda_b} = \rho_{\frac{1}{2}, \frac{1}{2}} - \rho_{\frac{1}{2}, -\frac{1}{2}} \).

The explicit form of the decay probability \( w \) then depends on the spin of the intermediate \( \Lambda \). We separately treat the cases \( J_\Lambda = 1/2 \) and \( J_\Lambda = 3/2 \).

### 4.1 The case \( J_\Lambda = \frac{1}{2} \)

Angular distributions for the spin-1/2 case have been given in Ref. [7]. We re-derive them here as a warm-up, and to introduce our notation. For \( J_\Lambda = 1/2 \) we have only two allowed helicity combinations which we may identify by the total helicity \( \lambda \equiv \lambda_\Lambda - \lambda_\gamma = \pm 1/2 \), with corresponding amplitudes \( C_\lambda \). The decay amplitude (4) becomes

\[
A = \sum_\lambda D_{\lambda_\Lambda, \lambda_\rho}^{\frac{1}{2}}(\phi_p, \theta_p, -\phi_p) D_{M, \lambda}^{\frac{1}{2}}(\phi_\Lambda, \theta_\Lambda, -\phi_\Lambda) C_\lambda E_{p, p},
\]

therefore (dropping the argument angles for better readability)

\[
|A|^2 = \sum_\lambda |D_{\lambda_\Lambda, \lambda_\rho}^{\frac{1}{2}}|^2 |D_{M, \lambda}^{\frac{1}{2}}|^2 |C_\lambda|^2 |E_{p, p}|^2,
\]

and the decay probability (5) becomes

\[
w_{\frac{1}{2}} = \sum_{\lambda_\rho, \lambda} |C_\lambda|^2 |E_{p, p}|^2 |D_{\lambda_\Lambda, \lambda_\rho}^{\frac{1}{2}}|^2 \left[ \rho_{\frac{1}{2}, \frac{1}{2}} |D_{M, \lambda}^{\frac{1}{2}}|^2 + \rho_{-\frac{1}{2}, -\frac{1}{2}} |D_{M, \lambda}^{\frac{1}{2}}|^2 \right].
\]

Inserting \( D_{m, m'}^{\lambda}(\alpha, \beta, \gamma) = e^{i \alpha m} d_{m, m'}^{\lambda}(\beta) e^{-i \gamma m} \) and explicit expressions for the \( d \)-functions it is a matter of straight-forward algebra to obtain

\[
w_{\frac{1}{2}} \propto 1 - \alpha_{p, \frac{1}{2}} P_{\Lambda_b} \cos \theta_p \cos \theta_\Lambda - \alpha_{\gamma, \frac{1}{2}} (\alpha_{p, \frac{1}{2}} \cos \theta_p - P_{\Lambda_b} \cos \theta_\Lambda),
\]

where we have defined, in addition to the \( \Lambda_b \) polarization \( P_{\Lambda_b} \), the photon asymmetry

\[
\alpha_{\gamma, \frac{1}{2}} = \frac{|C_{\frac{1}{2}}|^2 - |C_{-\frac{1}{2}}|^2}{|C_{\frac{1}{2}}|^2 + |C_{-\frac{1}{2}}|^2}
\]

and the proton asymmetry

\[
\alpha_{p, \frac{1}{2}} = \frac{|E_{\frac{1}{2}}|^2 - |E_{-\frac{1}{2}}|^2}{|E_{\frac{1}{2}}|^2 + |E_{-\frac{1}{2}}|^2}.
\]

Clearly, \( \alpha_{p, \frac{1}{2}} = 0 \) for \( \Lambda(X) \to pK \) due to parity conservation, but \( \alpha_{p, \frac{1}{2}} = 0.642 \pm 0.013 \) for \( \Lambda(1116) \to p\pi \) [11]. Integration over the solid angle elements \( d\Omega_p \) and \( d\Omega_\gamma \) finally yields the well-known angular distributions

\[
\frac{d\Gamma}{d\cos \theta_\gamma} \propto 1 - \alpha_{\gamma, \frac{1}{2}} P_{\Lambda_b} \cos \theta_\gamma,
\]
where $\cos \theta_\gamma = - \cos \theta_\Lambda$, and

$$
\frac{d\Gamma}{d \cos \theta_p} \propto 1 - \alpha_{\gamma,1} \alpha_{p,1/2} \cos \theta_p.
$$

These distributions show that, if the $\Lambda_b$ polarization is known, $\Lambda_b \rightarrow \Lambda(1116)\gamma$ decays afford two independent ways to assess the polarization of the emitted photon, as pointed out in Ref. [7], whereas for decays to spin-1/2 $\Lambda$ resonances only the photon distribution is useful in that respect. Indeed, comparing (8) with (2) we identify

$$
\alpha_\gamma = \alpha_{\gamma,1/2},
$$

i.e., $r$ can be extracted directly from the angular distributions. We note that in a real experiment, possible selection bias effects in the photon angular distribution can easily be corrected for by applying the same selection to the abundant channel $B \rightarrow K^*\gamma \rightarrow K\pi\gamma$, which has a very similar topology but no intrinsic photon asymmetry.

\textbf{4.2 The case $J_\Lambda = \frac{3}{2}$}

In the spin-3/2 case, the number of allowed helicity combinations increases to four, defined by $\lambda_\Lambda = \pm 3/2$ (with $\lambda_\gamma = \pm 1$, $\lambda = \pm 1/2$) and $\lambda_\Lambda = \pm 1/2$ ($\lambda_\gamma = \pm 1$, $\lambda = \mp 1/2$) and governed by corresponding helicity amplitudes $C_{\lambda_\Lambda,\lambda_\gamma}$. In this case the decay amplitude (4) becomes:

$$
A = \sum_{\lambda_\Lambda} D^3_{\lambda_\Lambda,\lambda_\gamma}(\phi_p, \theta_p, -\phi_p) D^1_{\lambda_\Lambda,\lambda_\gamma}(\phi_\Lambda, \theta_\Lambda, -\phi_\Lambda) C_{\lambda_\Lambda,\lambda_\gamma} E_{\lambda_p}
$$

7
Proceeding analogously to the spin-1/2 case we obtain for the squared amplitude

\[ |A|^2 \propto \sum_{\lambda_\alpha,\lambda'_\alpha} d^{\frac{3}{2}}_{\lambda_\alpha,\lambda'\alpha} (\theta_\alpha) d^{\frac{3}{2}}_{\lambda_\alpha,\lambda'\alpha} (\theta_\alpha) d^{\frac{1}{2}}_{M,\lambda_\alpha,\lambda'_\alpha} (\theta_\lambda) \times e^{i(\phi_\alpha + \phi_\beta)} C_{\lambda_\alpha,\lambda'_\alpha} C_{\lambda'_\alpha,\lambda_\alpha} \]

\[ = |C_{\lambda,\lambda'}|^2 |d^{\frac{3}{2}}_{\lambda,\lambda'} (\theta_\alpha) d^{\frac{1}{2}}_{M,\lambda_\alpha,\lambda'_\alpha} (\theta_\lambda)|^2 + |C_{-\lambda,-\lambda'}|^2 |d^{\frac{3}{2}}_{\lambda,\lambda'} (\theta_\alpha) d^{\frac{1}{2}}_{M,\lambda_\alpha,\lambda'_\alpha} (\theta_\lambda)|^2 \]

\[ + 2 \operatorname{Re}\{C^*_{\lambda,\lambda'} C_{\lambda',\lambda} e^{i(\phi_\alpha + \phi_\beta)} \} |d^{\frac{3}{2}}_{\lambda,\lambda'} (\theta_\alpha) d^{\frac{1}{2}}_{M,\lambda_\alpha,\lambda'_\alpha} (\theta_\lambda)|^2 \]

where we have dropped the overall factor \(|E_{\lambda_\beta}|^2\), which is constant due to parity conservation. For the decay probability (5) we then have

\[ w_4 = \sum_{i=1}^{6} \rho_{MM} |A_i|^2 = \sum_{i=1}^{6} w_i \]  

(14)

with

\[ w_1 = \frac{3}{8} |C_{\lambda,\lambda'}|^2 \sin^2 \theta_\alpha (1 + P_{\lambda_\beta} \cos \theta_\lambda); \]

\[ w_2 = \frac{3}{8} |C_{-\lambda,-\lambda'}|^2 \sin^2 \theta_\alpha (1 - P_{\lambda_\beta} \cos \theta_\lambda); \]

\[ w_3 = \frac{1}{8} |C_{\lambda,\lambda'}|^2 (3 \cos^2 \theta_\alpha + 1) (1 - P_{\lambda_\beta} \cos \theta_\lambda); \]

\[ w_4 = \frac{1}{8} |C_{-\lambda,-\lambda'}|^2 (3 \cos^2 \theta_\alpha + 1) (1 + P_{\lambda_\beta} \cos \theta_\lambda); \]

\[ w_5 = \frac{\sqrt{3}}{2} \operatorname{Re}\{C^*_{\lambda,\lambda'} C_{\lambda',\lambda} e^{i(\phi_\alpha + \phi_\beta)} \} \cos \theta_\alpha \sin \theta_\beta \sin \theta_\lambda; \]

\[ w_6 = \frac{\sqrt{3}}{2} \operatorname{Re}\{C^*_{-\lambda,-\lambda'} C_{-\lambda',\lambda} e^{-i(\phi_\alpha + \phi_\beta)} \} \cos \theta_\alpha \sin \theta_\beta \sin \theta_\lambda. \]

Again we integrate over the appropriate solid angle elements and get

\[ \frac{d\Gamma}{d\cos \theta_\gamma} \propto 1 - \alpha_{\gamma,\frac{1}{2}} P_{\lambda_\beta} \cos \theta_\gamma, \]  

(15)
where now the photon asymmetry parameter is defined as

$$\alpha_{\gamma, \frac{3}{2}} = \frac{\lambda\Lambda - \lambda_{\gamma} = 1/2 \lambda\Lambda - \lambda_{\gamma} = -1/2}{|C_{\frac{3}{2}, 1}|^2 + |C_{\frac{3}{2}, -1}|^2 - |C_{\frac{3}{2}, 1}|^2 - |C_{\frac{3}{2}, -1}|^2}{|C_{\frac{3}{2}, 1}|^2 + |C_{\frac{3}{2}, -1}|^2 + |C_{-\frac{1}{2}, 1}|^2 + |C_{-\frac{1}{2}, -1}|^2},$$

(16)
i.e., it describes the asymmetry of the $\Lambda_b$ spin projection with respect to the photon momentum. It is obvious from (16) that the extraction of $\alpha_{\gamma, \frac{3}{2}}$ and therefore the photon polarization in the fundamental $b \to s\gamma$ process, from $\alpha_{\gamma, \frac{3}{2}}$ is only possible if we know the relative strengths of the $m = 1/2$ and $m = 3/2$ amplitudes. The parameter $\eta$, defined as

$$\eta = \frac{|C_{\frac{3}{2}, 1}|^2}{|C_{\frac{3}{2}, 1}|^2} = \frac{|C_{\frac{3}{2}, -1}|^2}{|C_{\frac{3}{2}, -1}|^2}$$

(17)

(where the second equals sign is justified by parity conservation in the hadronization process), allows us to relate $\alpha_{\gamma, \frac{3}{2}}$ and $\alpha_{\gamma}$ in a simple way:

$$\alpha_{\gamma, \frac{3}{2}} = \frac{1 - \eta}{1 + \eta} \alpha_{\gamma}$$

(18)

We can determine $\eta$ experimentally from the proton angular distribution. Indeed, integration of Eq. (14) over $\theta_{\Lambda}$ yields

$$d\Gamma d\cos\theta_{p} \propto 1 - \alpha_{p, \frac{3}{2}} \cos^2 \theta_{p},$$

(19)

with the proton asymmetry parameter

$$\alpha_{p, \frac{3}{2}} = \frac{|\lambda\Lambda = 3/2|}{|\lambda\Lambda = 3/2|} \frac{|\lambda\Lambda = 1/2|}{|\lambda\Lambda = 1/2|} = \frac{\eta - 1}{\eta + \frac{1}{3}}.$$  

(20)
The proton polar angle distribution is symmetric around $\cos\theta_{p} = 0$, as expected for a strong decay, but it still allows us to extract a value for $\eta$. The determination of $\alpha_{\gamma}$ from a combined measurement of photon and proton angular distributions is then possible according to

$$\alpha_{\gamma} = \frac{1}{2} \alpha_{\gamma, \frac{3}{2}} \left(1 - \frac{3}{\alpha_{p, \frac{3}{2}}}ight),$$

(21)

if $\eta$ is sufficiently far away from 1 (equal probability for $m = 1/2$ and $m = 3/2$). For $\eta \ll 1$ the $m = 1/2$ amplitude dominates, $\alpha_{p, 3/2} \simeq -3$, and $\alpha_{\gamma, \frac{3}{2}} \simeq \alpha_{\gamma}$. In the case where the $m = 3/2$ amplitude dominates ($\eta \gg 1$) $\alpha_{p, \frac{3}{2}} \simeq 1$ and $\alpha_{\gamma, \frac{3}{2}} \simeq -\alpha_{\gamma}$. 

9
5 Experimental prospects for a photon polarization measurement in $\Lambda_b \to pK\gamma$

We now compare the experimental prospects for a measurement of the photon polarization (parameter $|r|$) in $\Lambda_b \to \Lambda(X)\gamma \to pK\gamma$ decays to those in $\Lambda_b \to \Lambda(1116)\gamma \to p\pi\gamma$ decays at a hadron collider. For the $\Lambda_b$ polarization we will assume a mean value $[9]$ and experimental error $[10]$ of $P_{\Lambda_b} = 0.20 \pm 0.01$. For the sake of a concrete estimate of the sensitivity in $|r|$ we fix the number of fully reconstructed $\Lambda_b \to \Lambda(1520) \to pK\gamma$ to $10^4$ and scale the event yields for the other resonance channels according to their branching fractions. We note that this number is arbitrary but realistic. Indeed, the LHCb collaboration for example expects an annual yield of 35 000 events containing the topologically very similar decay $B \to K^*\gamma \to K\pi\gamma$ $[19]$. Factoring in the relevant production rates and branching fractions, but assuming equal reconstruction efficiencies, we find that it would take LHCb a little more than three years to collect $10^4\Lambda_b \to \Lambda(1520) \to pK\gamma$ decays.

Clearly, the total reconstruction efficiency, including trigger, for $\Lambda_b \to \Lambda(1116)\gamma \to p\pi\gamma$ decays will be significantly lower than that for $\Lambda_b \to \Lambda(X)\gamma \to pK\gamma$ decays. Since it is hard to predict the experimental difficulties at this time, we not only consider a default scenario where the reconstruction efficiency is ten times worse with respect to $\Lambda_b \to \Lambda(1520)\gamma \to pK\gamma$, but also a best (very optimistic) and a worst (very pessimistic) scenario in which the reconstruction efficiency is assumed to be equal and a hundred times worse, respectively.

In Fig. 2 we show the expected experimental (statistical only) reach for the parameter $|r|$ at a hadron collider, as obtained under the above assumptions and with the error evaluation described in Appendix A. In the top plot the reach is shown separately for the three resonances $\Lambda(1520)$, $\Lambda(1670)$, and $\Lambda(1690)$. The sensitivity curves are compared with the reach obtained using the $\Lambda(1116)$, where both the photon and the proton asymmetry contribute to the measurement, in the three scenarios of different reconstruction efficiency. Note that for the spin-3/2 $\Lambda$ resonances, the expected reach is a function of the parameter $\alpha_{p,\perp}$, to be determined by the experiment. The bottom plot in Fig. 2 illustrates the experimental reach for various plausible combinations of measurements in the $\Lambda(1116)$ default scenario:

- the case where only $\Lambda(1520)$, $\Lambda(1670)$ and $\Lambda(1690)$ are available ($\Lambda(1116)$ cannot be reconstructed),
- the case where only $\Lambda(1116)$ and $\Lambda(1520)$ contribute ($\Lambda(1670)$ and $\Lambda(1690)$ cannot be disentangled), and
- the case where all four $\Lambda$ states enter the determination of $|r|$.

We see that in the case of the $\Lambda(1116)$ the availability of both the photon
(10) and the proton asymmetry (11) for determining $|r|$ largely compensates even for large losses in statistics due to reconstruction problems. Under our default assumptions, decays to $\Lambda(1116)$ will allow a typical hadron collider experiment to probe $|r|$ down to 0.21, whereas the decays to $\Lambda$ resonances can only give constraints to about 0.33. A combination of measurements of the three $\Lambda$ resonances $\Lambda(1520)$, $\Lambda(1670)$ and $\Lambda(1690)$ can probe down to 0.27 in the most favourable case.

Since our current estimate for the $\Lambda_b$ polarization at a hadron collider may well be off by a large factor, it is worth examining the dependence of the relative error of $|r|$ on $P_{\Lambda_b}$. To illustrate the effect of the $\Lambda_b$ polarization on the statistical reach in $|r|$, we show in Fig. 3 for $\Lambda_b \rightarrow \Lambda(1670)\gamma$ and $\Lambda_b \rightarrow \Lambda(1116)\gamma$ (default reconstruction scenario) the relative statistical error on $(1-\alpha_\gamma)$ (cf. Appendix A) as a function of $|r|$ for the three cases $P_{\Lambda_b} = 0, 0.2$ and 0.5. As expected, in the case of the decay to a $\Lambda$ resonance the measurement of $|r|$ is much less robust against small values of the $\Lambda_b$ polarization than in the case of the $\Lambda(1116)$, where the proton asymmetry allows for a measurement of $|r|$ even if the $\Lambda_b$ is not polarized at all.

Another concern arises from systematic errors in the measurement of the photon and proton asymmetries. Similar to the reconstruction efficiency, these uncertainties depend on the specific experimental setup and cannot be estimated in a general way. Nevertheless, given the cancellation of a large class of experimental effects in asymmetry measurements, we may assume that these errors will not exceed the few-percent level for the parameter $\alpha_\gamma$. For the sake of illustration, we plot in Fig. 4 the expected total relative error on $(1-\alpha_\gamma)$ as a function of $|r|$ in the presence of a systematic error on $\alpha_\gamma$ of 0%, 5%, and 10% for the $\Lambda(1670)$ example. To explore the ultimate sensitivity we show the same curves for infinite statistics. We see that even in the case of vanishing statistical and (internal) systematic errors, the sensitivity would still be limited to about $|r| > 0.25$. In the case of the $\Lambda(1116)$ we find a sensitivity limit of about 0.15. These limits are a consequence of our assumptions on the uncertainties of the $\Lambda_b$ production polarization and the $\Lambda$ weak decay parameter.

It is interesting to compare the experimental reach for $|r|$ at a hadron collider with prospects at the $B$ factories. The only method applied so far at the $B$ factories is the one relying on $B-\bar{B}$ interference [1], where the CP-violation parameter $S$ [20] has been measured for the decay $B^0 \rightarrow K^0_S\pi^0\gamma$. The amplitude ratio $|r|$ may be extracted from $S_{K_S^0\pi^0\gamma}$ according to $S_{K_S^0\pi^0\gamma} = 2|r|\sin 2\beta$ [1], where $\sin 2\beta$ is the by now well known CKM parameter describing the $B-\bar{B}$ mixing phase. (Note that this method has a linear sensitivity in $|r|$
Fig. 2. Experimental reach for $|r|$ (as a function of $\alpha_{\frac{3}{2}}$) for decays involving a spin-3/2 $\Lambda(X)$. The plots show the values of $|r|$ that can be probed at 3σ (standard deviation) significance in single (top) and combined (bottom) measurements, at a hadron collider experiment capable of collecting $10^4 \Lambda_b \to \Lambda(1520)\gamma \to pK\gamma$ decays. The ranges are to be read from left to right starting from the curves. The three ranges for the $\Lambda(1116)$ in the top plot correspond to the best, default and worst reconstruction scenarios (see text). The bottom plot is based on the default reconstruction scenario and on the assumption of equal $\alpha_{\frac{3}{2}}$ for $\Lambda(1520)$ and $\Lambda(1690)$. 
Fig. 3. Expected relative statistical error on $(1 - \alpha)\gamma$ as a function of $|r|$ at a hadron collider experiment for the decays $\Lambda_b \to \Lambda(1670)\gamma$ (left) and $\Lambda_b \to \Lambda(1116)\gamma$ (right). The three curves represent different assumptions for the $\Lambda_b$ polarization $P_{\Lambda_b}$: 0.1 (solid), 0.2 (dashed) and 0.5 (dotted). For $\Lambda_b \to \Lambda(1115)\gamma$ the default reconstruction scenario is assumed (see text). Event yields are as in Fig. 2.

as opposed to the $|r|^2$ dependence of $\alpha\gamma$, the principal observable in radiative $\Lambda_b$ decays). The most recent measurement of $S_{K_S^0 p \gamma}$ has been presented by the Belle Collaboration [21]. Based on $5.35 \times 10^8$ $B\overline{B}$ pairs, Belle finds $S_{K_S^0 p \gamma} = -0.10 \pm 0.31 \pm 0.07$, where the first error is statistical and the second is systematic. Using the latest world average $\sin 2\beta = 0.674 \pm 0.026$ [22], the error on $S$ translates to an error on $|r|$ of $\sigma_{|r|} = 0.23$; in other words: $|r|$ would have to be as large as 0.7 for the observation of a right-handed component at the $3\sigma$ level at today’s $B$ factories. Recent assessments of the physics potential of a next-generation $B$ factory find uncertainties on $S_{K_S^0 p \gamma}$ of 0.1 and 0.03 for integrated luminosities of 5 and 50 ab$^{-1}$, respectively [23]. This would correspond to $3\sigma$ reaches of $|r| > 0.22$ and $|r| > 0.07$, respectively, which may in principle be compared to the curves in Fig. 2. One should, however, keep in mind that the curves in Fig. 2 do not contain effects from detector-related systematic errors, which could substantially limit the sensitivity at hadron colliders (Fig. 4).

6 Conclusion

To summarize, radiative $\Lambda_b$ decays to $\Lambda$ resonances above the nucleon-kaon threshold provide an interesting alternative to the experimentally challenging decay $\Lambda_b \to \Lambda(1116)\gamma$ for assessing the photon polarization in the quark transition $b \to s\gamma$ at hadron colliders. The principal unknown for spin-3/2 res-
Fig. 4. Expected relative total error on $(1 - \alpha_\gamma)$ as a function of $|r|$ at a hadron collider experiment for the decay $\Lambda_b \to \Lambda(1670)\gamma$. The three curves represent different assumptions for the detector-related systematic relative error $\sigma_{\alpha_\gamma, \text{syst}}$ on the measurement of the photon asymmetry $\alpha_\gamma$: 0% (solid), 5% (dashed), and 10% (dotted). Event yields are as in Fig. 2 (left plot), and corresponding to infinite statistics (right plot).

onances such as $\Lambda(1520)$ and $\Lambda(1690)$, the repartition of $m = 1/2$ and $m = 3/2$ amplitudes, can be extracted directly from experiment.

We have studied the experimental prospects for constraining the presence of anomalous right-handed currents in the $b \to s\gamma$ transition, parameterized by the ratio of Wilson coefficients $r = C_7^\prime / C_7$, in the context of a generic hadron collider experiment. Our comparison between the decays $\Lambda_b \to \Lambda(1116)\gamma \to p\pi\gamma$ and $\Lambda_b \to \Lambda(X)\gamma \to pK\gamma$ shows that although the decay to $\Lambda(1116)$ offers by far the best sensitivity thanks to the simultaneous contributions from photon and proton asymmetries, a combined analysis of decays to $\Lambda$ resonances is capable of recovering a large fraction of the discovery range in $|r|$ in the case where the $\Lambda$ is not detectable due to its escape of the inner detector system.
A Statistical error estimation

In the linear approximation, the relative statistical error on \(|r|\) is given by

\[
\sigma_{|r|} = \frac{1}{|r|} \frac{\sigma_{\alpha_\gamma}}{(1 + \alpha_\gamma)^2}.
\]

(A.1)

We note, however, that the application of this formula can give misleading results when evaluating the sensitivity of an experiment to new physics. In our case new physics means \(|r| > 0\) or \(\alpha_\gamma < 1\). Since only \(\alpha_\gamma\) is the experimental observable, an experiment will have established new physics at the 3\(\sigma\) level if it finds \((1 - \alpha_\gamma)/\sigma_{\alpha_\gamma} > 3\). Evaluating the corresponding sensitivity for \(|r| > 0\) with Eq. (A.1) (and Eq. (3)) would result in

\[
\frac{|r|}{\sigma_{|r|}} = (1 + \alpha_\gamma) \frac{1 - \alpha_\gamma}{\sigma_{\alpha_\gamma}},
\]

a value that is too large by a factor of typically almost two\footnote{We thank Yuehong Xie for bringing this point to our attention.}. We therefore only use \((1 - \alpha_\gamma)/\sigma_{\alpha_\gamma}\) to estimate sensitivities\footnote{The reader should be advised that the sensitivity estimates presented in Ref. [7] are based on Eq. (A.1) and therefore suffer from the same problem.}

In the case of spin-1/2 \(\Lambda\) baryons (Sec. 4.1), \(\alpha_\gamma\) will be extracted from a fit to a distribution of the type \(1 - s_\gamma \cos \theta\) (Eqs. 10 and 11), i.e., \(\alpha_\gamma = s_\gamma/a\), where \(a\) is either the \(\Lambda_b\) polarization \(P_{\Lambda_b}\) or the weak decay parameter \(\alpha_{p_1^+}\). The statistical error on \(\alpha_\gamma\) is therefore

\[
\sigma_{\alpha_\gamma} = \frac{1}{a} \sqrt{\alpha_\gamma^2 \sigma_a^2 + \sigma_{s_\gamma}^2}.
\]

For the statistical error on the slope \(s_\gamma\) expected for a fit to \(N\) events we use the empirical formula

\[
\sigma_{s_\gamma} = 1.752 \sqrt{\frac{1 - 0.71 \cdot s_\gamma^2}{N}}
\]

obtained with a fast simulation tool [24].

Similarly, the extraction of \(\alpha_\gamma\) from spin-3/2 \(\Lambda\) baryon decays (Sec. 4.2) proceeds via (cf. Eq. (21))

\[
\alpha_\gamma = \frac{s_\gamma}{2P_{\Lambda_b}} \left(1 - \frac{3}{\alpha_{p_1^+}}\right)
\]
with statistical error

\[ \sigma_{\alpha_\gamma} = \sqrt{\frac{\alpha^2_\gamma}{s^2_\gamma} \left( \frac{\sigma^2_{s_\gamma}}{s^2_\gamma} + \frac{\sigma^2_{P_{\Lambda_b}}}{P^2_{\Lambda_b}} \right) + \frac{9}{4} \frac{s^2_\gamma}{P^2_{\Lambda_b}} \frac{\sigma^4_{\alpha_\gamma_P}}{\alpha^4_{\alpha_\gamma_P}},} \]

and the statistical error on the proton asymmetry parameter from \( N \) events is approximated by another empirical formula obtained from simulation,

\[ \sigma_{\alpha_{P_{\frac{3}{2}}}} = \frac{3.48 - 3.16 \cdot \alpha_{P_{\frac{3}{2}}}}{\sqrt{N}}. \]

Acknowledgments

We are indebted to Gudrun Hiller for very fruitful discussions. We also thank Maurice Gailloud and Stefano Villa for carefully reading the manuscript. This work was supported by the Swiss National Science Foundation under grant Nr. 620-066162.

References

[1] D. Atwood, M. Gronau, A. Soni, Phys. Rev. Lett. 79 (1997) 185.

[2] D. Melikhov, N. Nikitin, S. Simula, Phys. Lett. B 442 (1998) 381; Y. Grossman, D. Pirjol, JHEP 0006 (2000) 029; L.M. Sehgal, J. van Leusen, Phys. Lett. B 591 (2004) 235.

[3] M. Gronau, Y. Grossman, D. Pirjol, A. Ryd, Phys. Rev. Lett. 88 (2002) 051802; M. Gronau, D. Pirjol, Phys. Rev. D 66 (2002) 054008.

[4] M. Knecht, T. Schietinger, Phys. Lett. B 634 (2006) 403.

[5] M. Gremm, F. Krüger, L.M. Sehgal, Phys. Lett. B 355 (1995) 579.

[6] T. Mannel, S. Recksiegel, Acta Phys. Pol. B 28 (1997) 2489; T. Mannel, S. Recksiegel, J. Phys. G: Nucl. Part. Phys. 24 (1998) 979.

[7] G. Hiller, A. Kagan, Phys. Rev. D 65 (2002) 074038.

[8] For a recent brief review, see T. Schietinger, Review of experimental results on rare radiative, semileptonic and leptonic \( B \) decays, in: Proceedings of the XLIIst Rencontres de Moriond (La Thuile, Italy, March 11–18, 2006), eds. J.M. Frère, J. Trần Thanh Vân, and G. Unal, (Thế Giới Publishers, Vietnam, 2006) p. 19, hep-ex/0605081.

[9] Z.J. Ajaltouni, E. Conte, O. Leitner, Phys. Lett. B 614 (2005) 165.
[10] J. Hřívňáč, R. Lednický, M. Smižanská, J. Phys. G: Nucl. Part. Phys. 21 (1995) 629.
[11] S. Eidelman, et al., Particle Data Group, Phys. Lett. B 592 (2004) 1.
[12] W. Ashmanskas, et al., Nucl. Instr. and Meth. in Phys. Res. A 447 (2000) 218.
[13] T. Schietinger (for the LHCb Collaboration), Nucl. Instr. and Meth. in Phys. Res. A 549 (2005) 137.
[14] B. Grinstein, Y. Grossman, Z. Ligeti, D. Pirjol, Phys. Rev. D 71 (2005) 011504.
[15] E. Barberio, et al., Heavy Flavor Averaging Group (HFAG), [hep-ex/0603003]
[16] S. Veseli, M.G. Olsson, Z. Phys. C 71 (1996) 287.
[17] M. Jacob, G.C. Wick, Ann. Phys. 7 (1959) 404; J. Richman, An Experimenter’s Guide to the Helicity Formalism, CALT-68-1148 (1984), unpublished.
[18] Y. Ueda, S. Okubo, Nucl. Phys. 49 (1963) 345.
[19] R. Antunes Nobrega, et al., LHCb Collaboration, LHCb Reoptimized Detector Design and Performance Technical Design Report, CERN/LHCC 2003-030.
[20] See, for instance, D. Kirkby and Y. Nir, CP Violation in Meson Decays, in [11].
[21] K. Abe, et al., Belle Collaboration, [hep-ex/0608017]
[22] M. Hazumi, plenary talk given at the XXXIII International Conference on High Energy Physics, Moscow, Russia, July 26–August 2, 2006.
[23] M. Hazumi, talk given at the 4th meeting on Flavour in the Era of the LHC, CERN, Geneva, Switzerland, October 9–11, 2006.
[24] W. Verkerke, D. Kirkby, The RooFit toolkit for data modeling, [physics/0306116]