SHADOWING OF ULTRAHIGH ENERGY NEUTRINOS

A. NICOLAIDIS and A. TARAMOPOULOS

Theoretical Physics Department
University of Thessaloniki,
Thessaloniki 54006, Greece

Abstract
The rise with energy of the neutrino–nucleon cross section implies that at energies above few TeV the Earth is becoming opaque to cosmic neutrinos. The neutrinos interact with the nucleons through the weak charged current, resulting into absorption, and the weak neutral current, which provides a redistribution of the neutrino energy. We Mellin transform the neutrino transport equation and find its exact solution in the moment space. A simple analytical formula is provided, which describes accurately the neutrino spectrum, after the neutrinos have traversed the Earth. The effect of the weak neutral current is most prominent for an initial flat neutrino spectrum and we find that at low energies (around 1 TeV) the neutrino intensity is even enhanced.
Neutrino telescopes will provide a new observational window on our cosmos and will help us probe the deepest reaches of distant astrophysical objects [1]. The enormous energies involved (cosmic neutrinos carry energy from few TeV to $10^8$ TeV) might also allow us to enlarge our knowledge of particle physics. Since neutrinos interact only weakly, they travel unhindered by intervening matter and can give us information about regions of the universe, which are not accessible to traditional photon astronomy. By the same token, it is extremely difficult to detect neutrinos and large effective volumes for detection are required.

Four neutrino observatories are under construction: DUMAND II in the Pacific Ocean off the coast of Hawaii [2], NESTOR in the bay of Pylos, Greece [3], AMANDA in the ice of Antarctica [4] and the Baikal NT-200 in the Siberian lake Baikal [5]. Expected neutrino sources include atmospheric neutrinos (neutrinos generated in the atmosphere by the cosmic rays), which dominate other sources at energies below few TeV, neutrinos generated in active galactic nuclei (AGN) with energies up to $10^6$ TeV and cosmological neutrinos, important at energies above $10^6$ TeV [1].

The neutrino–nucleon cross section rises linearly with energy up to 10 TeV. At higher energies, due to the propagator effect of the intermediate gauge bosons, the rise is slower. At around 40 TeV the cross section is large enough so that the Earth starts becoming opaque to neutrinos. To reduce the background from cosmic rays, at the detection site we are looking for upward moving muons induced by neutrinos coming from the other side of the Earth. Therefore, in all relevant calculations we should include attenuation factors describing how the neutrino fluxes change as the neutrinos travel through the Earth. It is the purpose of the present work to provide an analytic and accurate representation of the shadowing of highly energetic neutrinos.

Neutrinos interact with the nucleons through charged and neutral weak currents. In the first case a neutrino is transformed into a charged lepton and we have a neutrino loss, while in the second case the neutrino continues along its path with reduced energy. The inclusive neutrino–nucleon cross section looks generically like
\[ \frac{d^2 \sigma}{dx dy} = s_B \frac{A(x, Q^2) + (1 - y)^2 B(x, Q^2)}{(1 + s_B x y)^2}. \]  

(1)

In the above expression \( x \) is the Bjorken scaling variable, \( 1 - y \) is the momentum fraction of the produced lepton, \( A \) and \( B \) are structure functions expressed in terms of the appropriate parton densities and \( s_B = s/M_B^2 \) with \( M_B \) the mass of the intermediate gauge boson (W or Z). At high energies \( (s_B \gg 1) \) the propagator term (the term in the denominator) reduces the rise with energy of the total cross section. Also, at high neutrino energies the nucleon is probed at exceedingly small \( x \) values, where no experimental information is available. The neutrino–nucleon cross section at very high energies is a subject of current research [6–8]. For our purposes it is sufficient to parametrize the total cross sections in the following form

\[ \sigma_{cc}(E) = a_c \left( \frac{E}{E_0} \right)^{\beta_c}, \]  

(2)

\[ \sigma_{nc}(E) = a_n \left( \frac{E}{E_0} \right)^{\beta_n}. \]  

(3)

Recent calculations [8] provide for neutrino energies above \( 10^3 \) TeV, \( \beta_c \simeq \beta_n \simeq \beta \simeq 0.4 \), \( a_c/a_n \simeq 2.53 \). An important ingredient in the shadowing process is the \( y \) distribution of the neutral current cross section. We assume a factorized expression

\[ \frac{d\sigma_{nc}(E, y)}{dy} = a_n \left( \frac{E}{E_0} \right)^{\beta_n} f(y). \]  

(4)

There is a mild energy dependence in the \( y \) distribution, but for our practical work we use an \( f(y) \) independent of energy. In the domain \( s_B \gg 1 \) the gauge boson propagator creates a peaking of the cross section near \( y = 0 \) and we adopt the form [9]

\[ f(y) = \frac{1}{\ln(1/\varepsilon)} \frac{1}{\varepsilon + y}. \]  

(5)

The average \( y \) and the parameter \( \varepsilon \) are related through \( < y > = 1/\ln(1/\varepsilon) \).

The transport equation for neutrinos traversing the Earth is

\[ \frac{dI(E, \tau)}{d\tau} = -(\sigma_{cc} + \sigma_{nc})I(E, \tau) + \frac{d\sigma_{nc}}{dy} \otimes I, \]  

(6)
where \( d\tau = n(z)dz \) and \( n(z) \) is the number density of the nucleons encountered by the neutrino along its path through the Earth. \( I(E, \tau) \) is the intensity of the neutrino flux at “depth” \( \tau \), with the initial intensity (before entering the Earth) \( \bar{I}(E) = I(E, \tau = 0) \). The convolution is defined by

\[
\frac{d\sigma_{nc}}{dy} \otimes I = \int \frac{d\sigma_{nc}(E', y)}{dy} I(E', \tau) \delta(E - E'(1 - y)) dE'dy.
\]

(7)

For \( n(z) \) we use \( n = \rho/M_N \), \( \rho \) being the density of the Earth. The maximum value \( \tau \) can attain is approximately \( 6 \times 10^{33} \) cm\(^{-2}\). To proceed further, we Mellin transform equ. (6), using also equ. (2)–(4). Defining

\[
I_k(\tau) = \int E^k I(E, \tau) dE,
\]

(8)

we obtain

\[
\frac{dI_k}{d\tau} = -c_k I_{k+\beta},
\]

(9)

where

\[
c_k = \left( \frac{1}{E_0} \right)^\beta (a_c + a_n - a_nf_k),
\]

(10)

\[
f_k = \int_0^1 (1 - y)^k f(y) dy.
\]

(11)

For \( k = 0, f_0 = 1 \) and the total number of neutrinos is reduced because of the weak charged current, the weak neutral current providing only a redistribution of energy. \( f_k \) is a decreasing function of \( k \), and for sufficiently large values of \( k \), \( c_k \) corresponds to absorption created by \( \sigma_{tot} \ (\sigma_{tot} = \sigma_{cc} + \sigma_{nc}) \).

Let us define the column vector \( J_n \)

\[
J_n = \begin{pmatrix}
I_{0+n\beta} \\
I_{1+n\beta} \\
\vdots \\
I_{k+n\beta} \\
\vdots
\end{pmatrix}
\]

(12)
and the matrix
\[ P = \begin{pmatrix} 0 & R_0 & 0 & 0 & \cdots \\ 0 & 0 & R_1 & 0 & \cdots \\ 0 & 0 & 0 & R_2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \] (13)

where \( R_n \) is the diagonal matrix
\[ R_n = \begin{pmatrix} c_{n\beta} & 0 & 0 & \cdots \\ 0 & c_{1+n\beta} & 0 & \cdots \\ 0 & 0 & c_{2+n\beta} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \] (14)

We obtain then
\[ \frac{d}{d\tau} \begin{pmatrix} J_0 \\ J_1 \\ J_2 \\ \vdots \\ J_k \\ \vdots \end{pmatrix} = P \begin{pmatrix} J_0 \\ J_1 \\ J_2 \\ \vdots \\ J_k \\ \vdots \end{pmatrix}. \] (15)

The formal solution of the above equation is
\[ \vec{J}(\tau) = \exp(P\tau) \vec{J}(\tau = 0). \] (16)

In terms of the first component
\[ J_0 = \vec{J}_0 + R_0 \vec{J}_1 \tau + \frac{1}{2!} R_0 R_1 \vec{J}_2 \tau^2 + \cdots + \frac{1}{n!} R_0 R_1 \cdots R_{n-1} \vec{J}_n \tau^n + \cdots \] (17)

with \( \vec{J}_k \equiv J_k(\tau = 0) \). Finally in terms of the original moments we get
\[ I_k(\tau) = \vec{I}_k - c_k \vec{I}_{k+\beta} \tau + \frac{1}{2} c_k c_{k+\beta} \vec{I}_{k+2\beta} \tau^2 + \cdots + (-1)^n \frac{1}{n!} c_k c_{k+2\beta} \cdots c_{k+(n-1)\beta} \vec{I}_{k+n\beta} \tau^n + \cdots. \] (18)
The above expression is the exact solution to equ. (9).

The inversion of equ. (18) in order to obtain $I(E, \tau)$ is not an easy task. We prefer to study limiting behaviors of $I(E, \tau)$ and then establish a consistent expansion. It is already stressed that $f_k$ tends to zero as $k$ grows. At sufficiently large $k$, $c_k$ becomes a constant independent of $k$

$$c_k \simeq \left(\frac{1}{E_0}\right)^\beta (a_c + a_n) \equiv q \quad \text{(large } k\text{).}$$

(19)

Substituting the above expression into equ. (18) we get

$$I_k(\tau) \simeq \int \bar{I}(E)E^k \exp \left[-\sigma_{tot}(E)\tau\right] dE.$$  

(20)

Therefore, at large energies (large $k$) the neutrino intensity is well represented by the standard absorption formula

$$I_{abs}(E, \tau) = \bar{I}(E) \exp \left[-\sigma_{tot}(E)\tau\right].$$  

(21)

A systematic approach emerges, where the coefficients $c_k$ for $k$ larger than some value are replaced by $q$ (equ. (19)) and for the others we use the exact expression (equ. (10)). The first term in our expansion is obtained by keeping $c_k$ in equ. (18) and replacing all $c_{k+n}\beta(n \geq 1)$ by $q$. We find

$$I_k(\tau) \simeq \sum_{n=0}^{\infty} \bar{I}_{k+n}\beta \left(-q\tau\right)^n \frac{1}{n!} + \frac{a_n}{a_c + a_n} f_k \left[\bar{I}_k - \sum_{n=0}^{\infty} \bar{I}_{k+n}\beta \left(-q\tau\right)^n \frac{1}{n!}\right].$$  

(22)

Returning to the energy variable we obtain

$$I_1(E, \tau) = I_{abs}(E, \tau) + \frac{a_n}{a_c + a_n} f(y) \otimes \left[\bar{I} - \bar{I}_{abs}\right].$$  

(23)

Under the assumption of a power law initial neutrino spectrum

$$\bar{I}(E) = \bar{I}(E_0) \left(\frac{E}{E_0}\right)^{-\gamma}$$  

(24)

equ. (23) provides

$$I_1(E, \tau) = \bar{I}(E) \exp \left[-\sigma_{tot}(E)\tau\right] +$$

$$+ \bar{I}(E) \frac{\sigma_{abs}(E)}{\sigma_{tot}(E)} \int_0^1 dy f(y) (1-y)^{\gamma-1} \left[1 - \exp \left(-\left(\frac{y}{1-y}\right)^\beta \sigma_{tot}(E)\tau\right)\right].$$  

(25)
Successive terms in our expansion can be obtained by using the exact expression for more than one $c_k$ (two, three, \ldots).

One could treat the second term in equ. (6) as a source function $F$. Defining

$$F(E, \tau) = \int_0^1 \frac{d\sigma_{nc}}{dy}(E_{1-y}, y) I(E_{1-y}, \tau) \frac{dy}{1-y}$$

and multiplying equ. (6) by $\exp[\sigma_{tot}(E)\tau]$ results in

$$\frac{dH}{d\tau} = \exp(\sigma_{tot}) F,$$

where $H = \exp(\sigma_{tot}) I$. Direct integration yields

$$I_s(E, \tau) = \bar{I}(E) \exp(-\sigma_{tot} \tau) + \exp(-\sigma_{tot} \tau) \int_0^\tau F(E, t) \exp(\sigma_{tot} t) dt.$$

Reasonable approximations may be obtained by inserting “appropriate” forms for $I$ in equ. (24). Assuming the generic form

$$I_p(E, \tau) = \bar{I}(E) \exp[-\sigma_p(E) \tau]$$

with $\sigma_p(E)$ to be specified, equ. (28) provides

$$I_s(E, \tau) = \bar{I}(E) \exp[-\sigma_{tot} \tau] + \bar{I}(E) \frac{\sigma_{nc}(E)}{\sigma_{tot}(E)} \int_0^1 dy f(y)(1-y)^{\gamma-1} \frac{\exp[-\sigma_{tot} \tau] - \exp\left[-\left(\frac{\bar{I}(E)}{\bar{I}(E)}\right)^\beta \sigma_{tot}(E) \tau\right]}{\left[\frac{\sigma_p(E)}{\sigma_{tot}(E)} - (1-y)^\beta\right]^{\gamma-1}}.$$

Adopting $\sigma_p = 0$ (i.e. $I_p = \bar{I}(E)$) we obtain an expression similar to equ. (25). Both expressions are valid for short $\tau$ and they overestimate significantly the actual result at large $\tau$. The other obvious choice is $\sigma_p(E) = \sigma_{tot}(E)$, i.e. we insert in equ. (26) the zeroth-order answer for $I(E, \tau)$. We proceeded to a numerical solution of equ. (3) and in fig. 1 we show the shadowing factor defined by

$$S(E, \tau) = \frac{I(E, \tau)}{I(E)}$$

as a function of the energy for the maximum value of $\tau$. For the initial neutrino spectrum we used the form of equ. (24) with $\gamma = 2$. Existing calculations of the unresolved AGN
neutrino flux \[10^{12}\] give differential spectra with spectral index \(\gamma\) varying between 0 and 2 over the energy range. The absorption factor suggested by equ. (21) is shown also in fig. 1 (long dashed curve). It remains always below the numerical shadowing factor (solid line) and the two curves approach each other at large energies, as expected. The dash-dotted curve represents the shadowing factor implied by equ. (30) with \(\sigma_p(E) = \sigma_{\text{tot}}(E)\). It is a significant improvement over the simple absorption formula (equ. (21)), although it lags behind the numerical result.

Searching a more “appropriate” form for the neutrino intensity, we notice that for the specific form of \(f(y)\) (equ. (5)), we have
\[
f_k = 1 - \frac{1}{\ln \left(\frac{1}{\varepsilon}\right)} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}\right). \quad (32)
\]
Then for moderate values of \(k\) (\(k \geq 1\), but not very large) \(c_k\) remains relatively constant at
\[
c_k \simeq \left(\frac{1}{E_0}\right)^\beta (a_c + < y > a_n). \quad (33)
\]
With the above \(c_k\), the neutrino intensity is provided by an absorption formula, equ. (29), with
\[
\sigma_p(E) = \sigma_{cc}(E) + < y > \sigma_{nc}(E). \quad (34)
\]

The shadowing factor given by equus. (30) and (34) is shown in fig. 1 (short dashed curve) and it is in excellent agreement with the numerical simulation.

Another approximate solution to the neutrino transport equation has been suggested in ref \[13\]. Defining the effective cross section
\[
\sigma_{\text{eff}}(E) = \sigma_{cc}(E) + \int_0^1 dy(1 - y)^{\gamma-\beta-1} f(y), \quad (35)
\]
the neutrino “regeneration” effect is (incorrectly) exponentiated through the formula
\[
I_R(E, \tau) = \bar{I}(E) \exp [-\sigma_{\text{eff}}(E)\tau]. \quad (36)
\]
The ratio of the suggested shadowing factor by equ. (36) to the numerical one is shown in the inset of fig. 1 (dashed line). Clearly equ. (36) overestimates the neutral current contribution.
In the same inset we show the corresponding ratio for our formula, equs. (30) and (34) (solid line).

The neutral current removes neutrinos from the large energy part of the spectrum and populates the lower energy part of the spectrum. It is evident that the shadowing factor depends upon the initial spectrum, since a steeply falling spectrum does not provide considerable regeneration, while a spectrum extending to high energies (i.e. flat spectrum) induces large regeneration. Calculations of the AGN neutrino flux [10] suggest a flat energy distribution ($\gamma = 0$) up to energies $E_{\text{max}} = 10^3$ TeV. Fig. 2 presents the shadowing factor for $\gamma = 0$ with the other assumptions unchanged as in fig. 1. The disagreement of the shadowing factor implied by equ. (36) to the numerical calculation (dashed line in the inset) increases. Fig. 2 shows that, contrary to simple absorption, the shadowing factor at low energies can become greater than 1.0. This is an effect we could anticipate. Neutral current shifts the neutrino energy to lower values. If the new energy is low enough, such that $\sigma_{\text{tot}}(E)\tau \ll 1$, then the neutrino traverses the Earth unabsorbed. Therefore, at such nominal energy, apart from the initial neutrinos, we will have the neutrinos displaced in energy by the neutral current. In our calculations (fig. 2) the shadowing factor at 1 TeV is 1.36.

The use of ultrahigh energy neutrinos in order to image the Earth’s internal structure [14–19] has been considered for some time. It is highly interesting to obtain further information on the density distribution of the Earth, independently of the seismological determinations. In these investigations the modified neutrino flux is given by simple absorption, ignoring the neutrino regeneration by the neutral current. We provided a simple, analytical and accurate description, equs. (30) and (34), for the neutrino propagation inside the Earth, for all the values of energy $E$ and depth $\tau$. Our formalism will be essential for all reliable evaluations of the event rates in the neutrino telescopes under construction [2–5]. In the present work we used simple parametrizations for the neutrino cross sections, which may not be very accurate. Our main concern here is to establish a correct procedure, rather than to provide detailed numbers. At very high energy, the cross sections sample the nucleon parton densities at small $x$, a kinematical region where new physical phenomena (BFKL physics
are operative. The shadowing factor is very sensitive to the actual magnitude of the cross section and therefore the neutrino passage through the Earth will provide information about the structure of the nucleon at small $x$. It seems that the detection of ultrahigh energy cosmic neutrinos will be important not only for astrophysics, but also for particle physics and geophysics.

Acknowledgements.

One of us (A.N.) would like to acknowledge useful discussions with P. Gondolo, V. Stenger, J. Learned, L. Resvanis, I. Sarcevic, F. Stecker, A. Tarantola, D. Kazanas and S. Ichtiaroglou. This research was supported in part by the Fulbright Foundation and the EU program “Human Capital and Mobility”.
FIG. 1. The shadowing factor for neutrinos traveling along the Earth’s diameter, as a function of the energy for spectral index $\gamma = 2.0$. The long-dashed curve is the absorption factor given by equ.(21), the dot-dashed curve is the shadowing factor implied by equ.(30) with $\sigma_p = \sigma_{tot}$, the short-dashed curve is the shadowing factor implied by equ.(30) and (34), while the solid line represents the numerical evaluation of the shadowing factor. The inset shows the ratio of the proposed shadowing factor to the numerical one. The dashed-curve is derived from equ.(36) and the solid curve is derived from equ.(30) and (34).
FIG. 2. The shadowing factor for neutrinos traveling along the Earth’s diameter, as a function of the energy, for spectral index $\gamma = 0$ and $E \leq 1100$ TeV. Different curves correspond to the same conditions as in fig. 1.
REFERENCES

[1] For a recent review see T. Gaisser, F. Halzen and T. Stanev, Phys. Reports 258, 173 (1995)

[2] R. Grieder, in NESTOR, 3rd NESTOR International Workshop, Pylos, 1993, edited by L. Resvanis, p. 168

[3] L. Resvanis, in NESTOR, 3rd NESTOR International Workshop, Pylos, 1993, edited by L. Resvanis, p. 1

[4] R. Morse, in Neutrino Telescopes, Fifth International Workshop, Venice, 1993, edited by M. Baldo Ceolin, p. 309

[5] R. Wischnewski, in NESTOR, 3rd NESTOR International Workshop, Pylos, 1993, edited by L. Resvanis, p. 213

[6] G. Frichter, D. McKay and J. Ralston, Phys. Rev. Lett. 74, 1508 (1995)

[7] G. Frichter, J. Ralston and D. McKay, Phys. Rev. D53, 1684 (1996)

[8] R. Gandhi, C. Quigg, M. Reno and I. Sarcevic, Ultrahigh-energy neutrino interactions, Fermilab–Pub-95/221 – T preprint (electronic archive hep-ph/9512364)

[9] V. Berezinskii and A. Gazizov, Sov. J. Nucl. Phys. 29, 816 (1979)

[10] F. Stecker et al., Phys. Rev. Lett. 66, 2697 (1991); 69, 2738(E) (1992). For a recent estimate see F. Stecker and M. Salamon, High Energy Neutrinos from Quasars, NASA preprint astro-ph/9501064, to appear in Space Science Review.

[11] L. Nellen, K. Mannheim and P. Biermann, Phys. Rev. D47, 5270 (1993)

[12] D. Kazanas, in NESTOR, 3rd NESTOR International Workshop, Pylos, 1993, edited by L. Resvanis, p. 29

[13] V. Berezinskii et al., Sov. J. Nucl. Phys. 43, 406 (1986)
[14] L. Volkova and G. Zatsepin, Acad. Sci. USSR, Bull. Phys. Ser. **38**, 151 (1974)

[15] A. de Rujula et al., Phys. Rep. **99**, 341 (1983)

[16] M. Reno and C. Quigg, Phys. Rev. **D37**, 657 (1988)

[17] L. Bergström, R. Liotta and H. Rubinstein, Phys. Lett. **B276**, 231 (1992)

[18] C. Kuo et al., Earth and Planetary Sci. Lett. **133**, 95 (1995)

[19] A. Mann, University of Pennsylvania UPR 0232E preprint (1995)

[20] For reviews see L. Gribov, E. Levin and M. Ryskin, Phys. Rep. **100**, 1 (1983); E. Laenen and E. Levin, Annu. Rev. Nucl. Part. Sci. **199** (1994); A. Martin, in The heart of the matter, Editions Frontières, p. 157 (1994)