Estimating Latencies of Task Sequences in Multi-Core Automotive ECUs

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Abstract—The computation of a cyber-physical system’s reaction to a stimulus typically involves the execution of several tasks. The delay between stimulus and reaction thus depends on the interaction of these tasks and is subject to timing constraints. Such constraints exist for a number of reasons and range from possible impacts on customer experiences to safety requirements. We present a technique to determine end-to-end latencies of such task sequences. The technique is demonstrated on the example of electronic control units (ECUs) in automotive embedded real-time systems. Our approach is able to deal with multi-core architectures and supports four different activation patterns, including interrupts. It is the first formal analysis approach making use of load assumptions in order to exclude infeasible data propagation paths without the knowledge of worst-case execution times or worst-case response times. We employ a constraint programming solver to compute bounds on end-to-end latencies.

I. INTRODUCTION

Cyber-physical systems (CPS) are an ubiquitous part in today’s connected world. This also spreads to the automotive industry. Application areas in this domain range from in-car entertainment systems over driver assistance to engine control. Many features are implemented by electronic control units (ECU) and have real-time requirements which, if not met, can lead to bad customer experience or possibly even safety risks. Consequently timing analysis is an important part of system engineering and it is considerably complex. Response times for given stimuli can usually be determined in three ways. They can be measured in the end-product or with testbed hardware, determined with simulation or estimated by analytical approaches, as described in [1]. Measurements and simulations have two drawbacks. Firstly, they can only be performed relatively late in the engineering process. Secondly, it cannot be assured that the worst case behavior is captured and in-depth knowledge of the system under investigation is necessary. Analytical approaches on the other hand do not suffer from the problem of a possibly non-exhaustive coverage, but tend to overestimate due to necessary abstractions in the modeling process. Reducing overestimations while not increasing computation time is still becoming more challenging due to the increasing amount of ECUs and software in modern cars. Reacting to this trend, in order to ensure timing analysis, the development cooperation AUTOSAR extended its standards by a formal timing model as described in [2]. These extensions enable AUTOSAR users to give timing restrictions and requirements on different abstraction levels.

The analysis of software timings has consequently been subject to a wide range of scientific research, e.g. [3], [4]. Here, response time is defined as the time between activation and completion of a single task. In current cyber-physical systems the data flow which is needed to provide a function usually passes through multiple task instances possibly being executed on different processing units. These tasks can be activated periodically with different rates, as depicted in Figure 1. The varying relative offsets of the tasks cause different end-to-end timings. In this rather small example there are three different instantiations for one sequence of tasks.

There are different kinds of temporal constraints on such task sequences. The delay between the arrival of an input and the reaction of the last task of the task sequence is called response time. On the contrary, data age describes the fact that outputs may depend on old input values, and there may be some delay before they are updated to reflect later inputs. The differentiation between these concepts was firstly considered in [5]. The data-dependency between two task instances is also referred to as job-level dependencies (cf. [6]). Task sequences with such dependencies are also called cause effect chains. Closely related to them are so-called event chains where the system behavior is described in terms of event models as in [7]. Regardless of the type of constraints, it is preferable to have guarantees about them in early development stages. Therefore the use of formal methods using abstract models, e.g. constraint programming, is promising. Here, we focus on the response time in cause effect chains.

This paper presents an approach for the formal analysis of task chains. The present approach does not require an estimation for the actual execution time of the executables of a task and is therefore applicable in early development stages. We use an intuitive task model which needs very few information about the task set but gives the possibility to add more details at a later stage. The communication between task instances is assumed to happen via signals which are
located in a shared memory. We distinguish between three communication models. In explicit communication values are read and written immediately. Implicit communication means that each task instance creates a local copy of all signals and writes them immediately after termination. In the case of deterministic communication, a task instance only writes at well-defined points of time relative to its activation. In our approach, a system is encoded in a constraint program and a solver is deployed to calculate the worst case response time for a task sequence. Other approaches to complete this task include the compositional (or modular) analysis, exhaustive search and model checking e.g. using hybrid automata. Related work will be presented in the next section. Subsequently, we give a more detailed description of the system that we model and spell out the premises for our approach. Based on this, we define a set of constraints according to our assumptions. Finally, we discuss the results of the analysis of some realistic examples from an automotive supplier and a manufacturer of automobiles.

II. RELATED WORK

Work most related to ours comes from two fields: modeling systems and describing timing behavior, and the estimation of end-to-end latencies. In the automotive domain, a popular representative of the former is AUTOSAR. The AUTOSAR standards’ timing model is based on the Timing Augmented Description Language (TADL2) which was developed in the Timing Model (TIMMO, [8]) project with the goal to standardize timing descriptions in automotive real-time systems as cited in [9]. Another model language which is focused on timing is the time-triggered language for embedded programming Giotto which was introduced in [10] and is the basis for the logical execution time (LET) paradigm. LET considers abstract intervals between the reading and writing of variables instead of the actual execution time of a program and is focused on software only. In the automotive domain a case of application is e.g. the distribution of single-core software on multi-core platforms [11]. It is also a possible basis for new approaches to increase timing predictability of embedded real-time systems [10], [12].

The second category of related work is about performance analysis with regards to end-to-end timing. Different understandings of timing exist and there is a subtle difference in notion. On one hand, end-to-end delays refer to the time data propagation needs to take place on a specific path in a system where tasks are triggered independently [13]. On the other hand, response times refer to the response of the last task in a chain of tasks where task executions are triggered by events, including a triggering from another task [14]. Furthermore, in the context of cyber-physical systems the term output latency is used for the time between stimulus and response of the system [15]. We will stick to the term output latency or latency for short. Regardless of the notation, analysis gets harder when multi-core systems get involved. An approach for the formal estimation of latencies for multi-rate cause effect chains supporting multi-core systems was presented in [5]. Here multi-rate means that the chain contains tasks with different periods. It supports different path semantics yielding different time constraints and is based on an event model of the system. In [6] Becker et al. introduce the notion of job-level dependencies. Building on that they show in [13] how this approach can be used to determine end-to-end data age at different levels of knowledge about the system. The safe estimations however are rather pessimistic when compared to end-to-end data ages determined with given schedules.

Prior to the use of multi-core systems, response time analysis was focused on single tasks, e.g. [16]. Based on event models, Richter and Ernst presented a compositional method in [17] which can be used to connect different analyses via input and output models. It combines different local response time calculations to model the global system behavior. This idea has also been adapted for multi-core systems. For chains which are formed by tasks which trigger their respective successor, a compositional approach based on event models of the system was presented in [7]. A range of industrial tools for the analysis of heterogeneous multiprocessor systems exists [18]–[21]. Some of them can also be used to perform timing analysis on multi-rate cause effect chains. These tools however rely on information which is only available at implementation level, e.g. execution time measurements.

The aforementioned approaches follow an algorithmic approach, e.g. using backtracking. Another approach to estimating end-to-end latencies is based on mixed integer linear programming (MILP) and was presented in [22], [23]. There, the system’s behavior and the timing properties of functional chains are modeled in terms of linear constraints. However, the setting in these papers differs from the one considered here. Firstly, another scheduling model is used. Secondly, the possible interferences of tasks are assumed to be covered in an earlier analysis step, as it makes use of the results of a worst-case response time analysis. Furthermore, the results from these papers suggest that the non-convexity of the problem

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{task-level-chain.png}
\caption{Example of a task-level chain with possible instance-level flows}
\end{figure}
makes it hard for MILP solvers.

To the best of our knowledge no descriptive approach using a constraint modeling language has been presented for the problem of estimating end-to-end latencies for multi-rate cause effect chains. Our approach makes use of a set of constraints for modeling the system and the task sequence for the chain. Thereupon we deploy a constraint solver for estimating latencies. The latencies are safe in the sense that no overload occurs.

III. TASK SEQUENCES AS CONSTRAINT PROGRAM

The presented approach is based on an a declarative description of all possible schedules via a set of constraints. Searching for the worst case end-to-end latency in the satisfying assignments for these constraints simulates an exhaustive search. The end-to-end latency of a sequence of tasks for the scope of this work then is the difference in time between the activation of the first task on the chain and the point of time where the last task on the chain wrote its results. This sequential analysis also suffices for the end-to-end timing analysis of cause-effect chains [13].

The descriptive nature of this approach allows for adapting the model, e.g. with respect to new constraints, without the need of adjusting possibly complex algorithms. Furthermore we can choose from a wide range of solver-backends, some of which support parallel computing.

The contributions of this work are twofold. Firstly, we introduce the constraints needed for a description of a task set in an early stage of the engineering process. Secondly, we show how a constraint solver can be utilized to make use of new formal approaches to end-to-end timing estimations.

In the following we present the system to be modeled and the task chain definition we use. In the main part of this section we are concerned with the formal constraints and the interval we need to consider for safe estimations.

A. System Model

The task set of ECUs in automotive real-time systems commonly consists of two different types of tasks. On the one hand there are periodic tasks which may have an offset. On the other hand, event-based tasks occur sporadically. Instances of tasks communicate to propagate information on specific paths in order to implement system functions. We consider multiprocessor systems deploying a finite set of tasks which communicate via variables which are located in shared memory. We focus on fixed priority preemptive scheduling (FPPS) in this work. Other scheduling policies can be implemented but possibly need further variables and constraints.

Due to our focus on early development stages, we make some assumptions. Firstly, we assume that every task has a minimal execution time, which may equal zero. Secondly, we assume that every task execution finishes before the respective deadline. This appears reasonable as a system should not be overloaded by design. Furthermore, task interferences by e.g. critical sections are handled implicitly, assuming that tasks do not miss their deadline no matter which interferences occur.

Two basic communication paradigms are relevant in the automotive domain: implicit and explicit communication [24]. When implementing explicit communication, every task reads and writes the data it processes immediately from and to shared memory. In implicit communication, each task instance creates a local copy of all data it processes. This data is manipulated while the execution of the task and written back to the global memory after its termination.

We consider a third paradigm which is a variation of the implicit communication as this is only applicable for tasks which are periodically activated. After the termination of a task instance all data is hold in a local buffer and written back only at well-defined points of time which are relative to the activation of the task instance, i.e. before the next instance of the task is activated. We will also refer to this as deterministic communication. Figure 2 and Figure 3 show the differences in communication for two tasks with different activation intervals. In Figure 2 Task A is only allowed to propagate its results at points of time marked with a dashed line (e.g. at \( t = x + 10 \)), Task B is only allowed to do so at points of time marked with a dotted or a dashed line (e.g. at \( t = x + 5 \) or at \( t = x + 10 \)). The dashed arrows show reads and writes which are not possible here but would be possible in the non-deterministic case. Due to these restrictions, the second instance of Task B cannot use the data written by the first instances of Task A, and operates on old data.

In contrary, Figure 3 shows non-deterministic communication. Here data processed by Task A can be read by Task B immediately.

For the analysis of the end-to-end timing of a sequence of tasks it is furthermore necessary to consider all possible relative offsets of instances of these tasks. The relative offsets vary over time with the observable schedules. The amount of possible schedules again depends on several properties of
the tasks, the scheduler, and the mapping of tasks to cores. These key properties hence form the base for the system's encoding as a set of constraints. For a task at least an activation pattern and a deadline are needed. Given this information and a scheduling policy we can represent a superset of all possible schedules as the set of solutions of our constraint program. The analysis will yield more precise results if more details of the system are encoded in the constraints as described below. Within the superset of possible schedules we then search for all data propagation paths between the instances of the task sequence.

Additional details of the system can be used to add more constraints and prune possible schedules. This reduces the set of feasible propagation paths of the task sequence, thus increasing precision. A natural refinement is adding details about the actual core execution times of the task instance and the resulting scheduling decisions. In the following we discuss several ways to decrease the amount of considered schedules and therefore data propagation paths by refining the system model.

**B. Model Refinements**

The first refinement is that a task instance needs a minimum amount of resources to be processed, i.e. a best case execution time (BCET). Consequently its results are not available before some point of time relative to its start. This can result in the infeasibility of some propagation paths, namely the ones where a task cannot propagate its results because it could not have possibly produced it.

Secondly, a task instance might block other tasks or cause the scheduler to pause them due to higher priority. These interferences again might result in the infeasibility of some propagation paths, e.g. when a low-priority task is always delayed and can not possibly propagate its results to the, in terms of activation time, nearest instance of a higher priority task. A benefit of the descriptive approach is that we are able to adjust the level of detail which is provided to the solver depending on the amount of information available in the respective development stage. For example, another refinement which is not considered for now due to our focus on early design stages, is concerned with the software modules of a task. When a task is activated some executable code is processed which typically is assembled from one or more software modules. This is why a promising refinement is adding the exact order of the software modules in each task with best- and worst-case execution times.

**C. Activation Patterns**

An activation pattern describes the temporal nature of how the instances of a task occur.

The first activation pattern is called periodic activation with offset. It describes a pattern which is widely used in embedded systems. A task is activated after a specific and fixed amount of time has passed. We also consider the extended variant with offsets, meaning that the first instance of this task is not activated at time zero but time zero plus offset.

The second activation pattern we consider is chained activation. A task which is chained to an other task is activated when its predecessor triggers its execution. This activation can be used to induce controlled concurrency. Assume a task A fulfils some function on a processing unit which should not interfere with the execution of another task B on a second processing unit. To ensure this, task B can be chained to task A. Thus, task B is activated when task A finishes its execution, which prevents them from running at the same time.

The third activation pattern is taken into consideration to handle angle synchronous tasks. Many time-critical functions in the automotive domain are related with the gearbox and the engine. The engine control unit e.g. computes the right ignition timing. This needs to be done synchronously to the engine speed. The timing properties of these tasks can hardly be modeled with an offset and period without adding too much pessimism. The nature of these activation patterns is rather a minimum and maximum occurrence per time interval, e.g. because the rounds per minute of the engine cannot be arbitrarily high. This can be converted to a minimum and maximum temporal distance of two consecutive activation events as shown in [25]. We call this pattern the bounded activation.

The fourth activation pattern is needed to describe tasks which are activated sporadically with less information about the relative offset between two consecutive activations. These are for example communication interrupts or sensor readings. In these cases the assumption about a specific pattern needs to be relaxed [26]. Since these tasks also require a minimum amount of time to be processed, we need an assumption on the minimum of time between two consecutive occurrences in order to yield a feasible scheduling. This is a valid assumption since usually there is a debouncing time for sensor readings and a maximal sending rate for network messages. Since we only know the minimum time between two task activations but not the maximum time, the theoretical estimation of each chain involving one of these tasks is always infinite. We therefore exclude chains consisting of tasks with this activation pattern from our considerations, except when it is located at the very beginning of the chain. We then add the assumption that the activation happens within a critical interval which will be discussed below.

The upper half of Table I shows the variables we use in the constraints for the activation patterns. Let $T$ be the set of tasks. The lower half of Table I shows the variables needed to describe the task set with the proposed level of detail. We assume that the given core execution times refer to the core given in $core(i)$ for $i \in T$. Let $A = \{ \text{periodic, chained, bound, sporadic} \}$ the set of possible activation types.

**D. Problem Encoding**

Until here we named all information we need to describe the task set on the assumed level of detail. In the following, this information is used to describe the behavior of the task set when instantiated. This leads to a set of schedules and data
flows as described above. The following constraints must be satisfied:

1) Tasks are activated according to their activation pattern as described above (cf. Constraints (1), (2), (3), (4), (5), and (6)).

2) A task instance is enqueued for scheduling after its activation. The task which is scheduled next is selected by the scheduling policy for each processing unit (cf. Constraint (7)).

3) The processing of a task instance is delayed if another task instance is processed on the assigned processing unit which is not preemptable or preferred for execution by the scheduling policy (cf. Constraint (7)).

4) A task instance which gets activated may interrupt a currently processed task instance if the latter is preemptable and the former is preferred for execution by the scheduling policy and both are assigned to the same processing unit (cf. Constraints (8) and (9)).

5) Tasks read and write shared variables according to their communication paradigm which is a fixed one from the above-mentioned (cf. Constraints (14), (16), and (18)).

| Variable | Description | Domain |
|----------|-------------|--------|
| period   | Period of a periodically activated task | N      |
| offset   | Offset of a periodically activated task | N      |
| predecessor | Pointer to the predecessor of a chained task | $\mathcal{T}$ |
| $\Delta t_{\text{min}}$ | Minimum temporal distance of two consecutive activations of a sporadic task (bounded or interrupt) | N      |
| $\Delta t_{\text{max}}$ | Maximum temporal distance of two consecutive activations of a sporadic task with bounded occurrence | N      |
| prio     | The priority of the task for fixed priority scheduling | N      |
| core     | The index of the processing unit on which the task is scheduled (we assume an enumeration.) | N      |
| preemptable | A flag to define whether a task is interruptible | $\{t, f\}$ |
| deadline | The deadline of the task | N      |
| act_type | The activation type of the task | $\mathcal{A}$ |

Table I

In order to model the constraints 1) – 6) each task instance is described with four variables: an activation time $\alpha$, an interrupted time $\iota$, an actual starting time $\sigma$ and a finishing time $\varepsilon$. Figure 4 shows the variables in the context of the lifespan of a task instance. Task $B$ in this figure is a task which delays and interrupts task $A$ for illustrative purposes. Task instances may interact in two ways. Firstly one task instance can delay the execution of instances of other tasks. This is expressed in the differences between $\alpha$ and $\sigma$. Secondly one task instance may pause the execution of instances of other tasks. This is expressed in the $\iota$ of the paused instance. This variable contains the total amount of time for which a task instance was paused after its start.

In the following, we give constraints which must hold for a schedule which starts at time 0, and where a task $i \in \mathcal{T}$ occurs $m_i$ times. The choice of the values $m_i$ will be discussed in detail in Subsection III-F.

The constraints on the $\alpha$s depend on the activation pattern of the respective task. Periodic tasks are activated at multiples of their period, starting at their offset.

\[ \alpha_{i,j} = \text{offset}(i) + j \cdot \text{period}(i) \quad \forall j: 1 \leq j \leq m_i \quad (1) \]

In the case of chained activation, an instance of task $i$ is activated if its predecessor $p(i)$ is terminated. This is,

\[ \alpha_{i,j} = \varepsilon_{p(i),j} \quad \forall j: 1 \leq j \leq m_i \quad (2) \]

for each task $i \in \mathcal{T}$ where $act\_type(i) = \text{chained}$.

For a task $i \in \mathcal{T}$ which is activated non-periodically according to a sporadic activation pattern, the time between two activations is in the interval $[\Delta t_{\text{min}}(i), \Delta t_{\text{max}}(i)]$. Thus, for a task $i$ where $act\_type(i) = \text{bounded}$ the activation time is constrained by

\[ \alpha_{i,1} \geq 0 \quad (3) \]
\[ \alpha_{i,1} \leq \Delta t_{\text{max}}(i) \quad (4) \]
\[ \alpha_{i,j} \geq \alpha_{i,j-1} + \Delta t_{\text{min}}(i) \quad \forall j: 2 \leq j \leq m_i \quad (5) \]
\[ \alpha_{i,j} \leq \alpha_{i,j-1} + \Delta t_{\text{max}}(i) \quad \forall j: 2 \leq j \leq m_i \quad (6) \]

The constraints 3 and 4 assure the consideration of all possible relative offsets with other tasks which we need to ensure a safe estimation.

Lastly, for tasks which occur in a sporadic manner the same constraints except the one in Equation 6 and Equation 4 hold. This means that the first instance might occur in $[0, t^{\text{min}}]$ where $act\_type(i) = \text{sporadic}$.

After a task was activated, some time may pass before it is actually started. The following constraints correspond to FPPS, other policies can be implemented in a very similar fashion. In case of FPPS a task instance is delayed when another task instance is currently being processed on the same processing unit which either has a higher priority or is not preemptable. Thus, the time at which a task instance is actually started may either be its activation time or the time at which another task finishes its execution. We therefore consider two sets of termination times with possible impact on the delay of a task instance. For tasks with higher priority, we consider all termination times of task instances which are activated before $\sigma_{i,j}$, as these will be processed first. This set is denoted $D_{i,j}^{\text{HP}}$ and defined as:

\[ D_{i,j}^{\text{HP}} = \{ \varepsilon_{\ell,k} \mid \alpha_{\ell,k} \leq \sigma_{i,j} \land \text{core}(i) = \text{core}(\ell) \land \text{prio}(i) < \text{prio}(\ell) \}. \]
For non-preemptive tasks we consider the set of task instances which are started before the \( j \)-th instance of task \( i \) is activated. These will not be interrupted by the scheduler, and may therefore delay the start of task \( i \). Their execution times are denoted \( D_{i,j}^{NP} \) for \( i, \ell \in T \), \( 1 \leq j \leq m_i \), and \( 1 \leq k \leq m_i \):

\[
D_{i,j}^{NP} = \{ \varepsilon_{\ell,k} \mid \sigma_{\ell,k} \leq \alpha_{i,j} \wedge \begin{align*}
&\text{core}(i) = \text{core}(\ell) \wedge \\
&\text{preemptable}(\ell) = \text{false} \}.
\]

Then the constraints regarding the actual start of a task instance are f.a. \( i \in T \) and \( 1 \leq j \leq m_i \):

\[
\sigma_{i,j} = \max \left( D_{i,j}^{HP} \cup D_{i,j}^{NP} \cup \{ \alpha_{i,j} \} \right).
\] (7)

While running, a task instance might be interrupted by other task instances which are scheduled to the same processing unit. The current instruction pointer is stored and, in the case of FPPS, the higher priority task is processed. After the return of the latter task, the instruction pointer is restored and the remaining instructions of the interrupted task instance are processed, if no further interrupt occurs. To model this behavior we introduce a variable to sum up the time an instance of a task was paused. With FPPS, a task instance might have such paused times when the corresponding task is preemptable and a higher priority task which is mapped to the same processing unit is activated during its lifetime. We therefore define f.a. \( i, \ell \in T \), \( j \in \{1, \ldots, m_i\} \) and \( k \in \{1, \ldots, m_i\} \) the following function:

\[
i_{i,j,\ell,k}^{HP} = \begin{cases}
1 & \text{if } \sigma_{i,k} > \sigma_{i,j} \wedge \varepsilon_{\ell,k} < \varepsilon_{i,j} \wedge \\
&\text{core}(i) = \text{core}(\ell) \wedge \\
&\text{prio}(i) < \text{prio}(\ell) \\
0 & \text{otherwise}
\end{cases}.
\] (8)

This is, \( i_{i,j,\ell,k}^{HP} \) returns 1 if the \( j \)-th instance of task \( i \) was paused by the scheduler because of the \( k \)-th instance of task \( \ell \). Thus the constraints in Equation 9 model the paused time for each task instance. The task instance causing the pause is guaranteed to finish before the paused instance \( j \) of task \( i \). We need to subtract the time the former task instance was paused itself. For all \( i \in T \) and \( 1 \leq j \leq m_i \) we thus define the following constraint:

\[
\iota_{i,j} = \sum_{\ell \in T \setminus \{i\}} \left( \sum_{k=1}^{m_\ell} i_{i,j,\ell,k}^{HP} \cdot (\varepsilon_{\ell,k} - \sigma_{\ell,k} - \iota_{i,k}) \right).
\] (9)

The last variable describing the lifetime of a task instance is \( \varepsilon \) which models the point of time when all software modules of the task instance have been processed. This point of time depends on two factors: the actual starting time of the instance, and it depends on how long the actual code execution for the instance takes. Since we assume an early stage of development, we only assume a best-case execution time for the modules.

This is, we can give an estimation for when a task may terminate and therefore bound the execution time by the following two constraints

\[
\varepsilon_{i,j} \geq \sigma_{i,j} + \text{bcet}(i) + \iota_{i,j} \quad \forall i \in T, 1 \leq j \leq m_i
\] (10)

\[
\varepsilon_{i,j} \leq \sigma_{i,j} + \text{deadline}(i) \quad \forall i \in T, 1 \leq j \leq m_i
\] (11)

where the first constraint enforces that the task instance is considered at least for its best case execution time, and the second constraints limits its execution time by the deadline of the according task.

However, if worst case execution times (WCET) or worst case response times (WCRT) are given, these would enable us to consider sporadic overload. These information can easily be incorporated using the following constraints:

\[
\varepsilon_{i,j} \leq \sigma_{i,j} + \text{WCET}(i) \quad \forall j: 1 \leq j \leq m_i
\]

\[
\varepsilon_{i,j} \leq \sigma_{i,j} + \text{WCRT}(i) + \iota_{i,j} \quad \forall j: 1 \leq j \leq m_i
\]

Again, due to the focus on early development stages these constraints are not considered in the tests of Section IV but the implications of their usage are discussed in Section V.

E. Task Sequences

A task sequence here is a finite, ordered sequence of tasks where a signal is interchanged from one task to another to implement a system function. It is important that these signals are pairwise causally related. On a more detailed level, an intra-ECU cause effect chain can be described in terms of an ordered sequences of so-called Executable Entities [2]. However these entities are used as an abstraction for executable code and are assignable to a task, which means that, in order to analyze cause-effect chains on this level of detail, it is sufficient to be able to analyze task sequences on instantiation-level. The timing properties of such a communicating task sequence again can be characterized by an ordered sequence of points of time which describe when the value of a variable is possibly processed at the corresponding task and when a response was calculated (cf. [5]). The response time of such a sequence then is the difference in time between the point of time at which an instance of the first task could possibly process the first signal and the point of time at which an instance of the last task provides a response. For functionalities which are computed e.g. only every second task instance, we need to add a task a second time to the chain in order to obtain safe bounds. Since a maximum delay between two consecutive activations of the first task on the chain is given, we can give a bound on the time between an update of a variable and its next processing in a task instance. Therefore we are able to change the problem to finding the difference in time between the activation of the first task and the point of time at which the last task calculated the response. To obtain a safe bound when starting at an arbitrary point of time, we add the maximum temporal gap between two consecutive task instances of the first task on the chain.

Corresponding to this definition, in our set of constraints chains are sequences \( p_k^\ell \) where \( p_k \in T \) f.a. \( k \in \{1, \ldots, \ell\} \)
and $\ell \in \mathbb{N}_{\geq 1}$. Their encoding in the set of constraints is discussed below. The worst case end-to-end latency for such a chain is the maximum difference between the activation of the first task and the response of the last task under the above-mentioned constraints. This difference mainly depends on the relative offsets of the task pairs $p_k$ and $p_{k+1}$ f.a. $k \in \{1, \ldots, \ell - 1\}$. To get a safe upper bound for the response time we therefore need to ensure that all possible relative offsets are considered. We need to give an estimation for the lower bound of the length of the timespan in which what we call the critical offsets will certainly occur. This timespan is discussed in Subsection III-F. For now let $p_{\text{mn}}$ denote the length of this interval when starting at time zero. Given this interval the amount of instances for each task can be bounded with the minimum distance between two occurrences. Now, a bound for two values can be given: the indices which need to be considered for each task and the points of time at which accesses on the shared variables happen. Since all possible execution times are considered the latter is guaranteed to happen within $[0, p_{\text{mn}}]$.

For the problem of estimating the end-to-end latency of a task chain $p_{k+1}$ we add two additional sets of variables to our constraint program: $n$ and $x$. We use $n$ for the index of the task instance which participates in the chain at index $k$ and $x$ to hold the point of time at which the same instance has written its results certainly f.a. $k \in \{1, \ldots, \ell\}$. The constraints on $n$ and $x$ $n_1 \geq 1$, $x_1 \geq 0$ and are f.a. $k \in \{2, \ldots, \ell\}$ where $i$ is the index of $p_k$ and $i$ communicates recording the the implicit communication:

$$n_k = \min \left( \{ j \mid \sigma_{i,j} \geq x_{i-1} \} \right)$$  \hspace{1cm} (12)

$$x_k \geq \sigma_{i,n_k}$$  \hspace{1cm} (13)

$$x_k \leq \varepsilon_{i,n_k}$$  \hspace{1cm} (14)

Equation 14 shows that in the case of implicit communications the consideration of more details regarding the actual execution times is promising, since here only the delay is relevant. However, in the case of explicit communication the BCET also gets relevant. Here, the constraints change f.a. $k \in \{2, \ldots, \ell\}$ where $i$ is the index of $p_k$ to:

$$n_k = \min \left( \{ j \mid \sigma_{i,j} \geq x_{i-1} \} \right)$$  \hspace{1cm} (15)

$$x_k = \varepsilon_{i,n_k}$$  \hspace{1cm} (16)

Lastly, in the case of deterministic communication the constraints f.a. $k \in \{2, \ldots, \ell\}$ where $i$ is the index of $p_k$ change to:

$$n_k = \min \left( \{ j \mid \alpha_{i,j} \geq x_{i-1} \} \right)$$  \hspace{1cm} (17)

$$x_k = (n_k + 1) \cdot \text{period}(p_k)$$  \hspace{1cm} (18)

Before we discuss the interval which needs to be considered for safe estimations, we want to give a motivational example for the consideration of the actual processing times of each task. Therefore, Figure 5 and Figure 6 give an example on the impact of neglecting the actual possibly execution times of the task instances. The priorities are: $\text{prio}(\text{Task A}) > \text{prio}(\text{Task B}) > \text{prio}(\text{Task C})$. We assume implicit communication and the following sequence: (Task A, Task B, Task C, Task A). In Figure 5 the actual execution times are unknown to the solver. We see that the it can not safely say that the second instance of Task C has not started before the second instance of Task B has written its results. Figure 6 again shows the relative offsets of the tasks in Figure 5 but with respect to actual execution times which are shown darker. Since the priority of Task B is higher than the priority of Task C it will always finish before the scheduler selects Task C for processing. Therefore the solver can safely reason that the second instance of Task C will always process the data of the second instance of Task B. This makes the worst-case path depicted in Figure 5 infeasible.

**F. Relevant Period**

The constraints stated in the previous sections describe a schedule in which each task $i$ is considered with $m_i$ task instances, i.e. a schedule in a certain interval of time is described. It is crucial that this interval is chosen sufficiently large to ensure a sound computation of upper bounds on the response times of task chains. On the other hand, the solving time increases with larger intervals, thus it is preferable to consider intervals which are not larger than necessary. Figure 7 sketches the interval we used for our analyses. We first describe its derivation, and afterwards discuss that not all tasks must actually be considered, i.e. the tasks which are relevant for the size of the interval.

Let $\text{LCM}$ denote the least common multiple of periods of tasks which are relevant for the size of the interval. A worst-case occurrence of a task chain will start at some point of this so-called hyper period, thus the interval for the analysis must contain a full hyper period. Next, the interval must be large enough such that this occurrence fits into it, independent of its starting time. We thus extend the interval by UB, a trivial upper bound on the length of the chain, as shown on the right hand side of Figure 7. Such bounds can be derived by summing up trivial upper bounds on the worst case response time of every task on the chain. As the execution of task instance may be delayed due to the execution of other tasks, we add an offset to the interval size (on the left-hand side of Figure 7) such that every task can be started at least once before this interval. For periodic tasks, this requires an offset of $O_p = \max \{ \text{offset}(i) + \text{period}(i) \mid i \in T \}$. In the case of sporadic tasks, let $O_s$ represent the maximum offset between two occurrences of these tasks. Then we choose the relevant interval as the interval $[0, T]$ with $T = O + \text{LCM} + \text{UB}$ and $O = \max\{O_s, O_p\}$.

Assume this interval was not sufficiently large, i.e. there was a $T' > T$ such that choosing the interval $[0, T']$ would yield a larger end-to-end delay, and that $T'$ is minimal with this property. Then, the chain must start at a time larger than $O + \text{LCM}$. Subtracting $\text{LCM}$ from all values for the variables $\alpha, \sigma, \varepsilon$ and adjusting the right-hand side of the constraints accordingly yields a shifted solution. Now, all task instance occurring at a negative point of time can be ignored. If
they had a influence on the solution, using the offset on
the left-hand side allows for simulating the according system
load. This yields a new solution within a smaller interval,
contradicting the assumption that $T'$ was minimal.

Real-world problem instances often contain tasks which run
with a large period, e.g. 1 second, and a very low priority. As
such tasks significantly increase the hyper period, we seek
target to focus on relevant tasks, i.e. tasks which actually have an
influence on the end-to-end delay of a task chain.

We therefore define the set of relevant tasks $T_{rel}$ for a chain
$p_{k=1}^{\ell}$ as the smallest subset of $T$ such that:

1) All tasks which occur on the chain are relevant: For all
t $t \in T$, if exists $k \in \{1, \ldots, \ell\}$ with $p_k = t$, then $t \in T_{rel}$.
2) Tasks which have a higher priority than a relevant task
which runs on the same core also become relevant: For all
t $t \in T$, if exists $t' \in T_{rel}$ such that $core(t') = core(t)$
and $prio(t') > prio(t)$, then $t \in T_{rel}$.
3) Non-preemptable task may have an influence on the task
chain. For all $t \in T$, if exists $t' \in T_{rel}$ such that $core(t') = core(t)$
and $preemptable(t') = \text{false}$, then $t \in T_{rel}$.
4) Tasks which activate relevant tasks must also be consid-
ered. For all $t \in T$ where $act\_type(t) = \text{chained}$, if
exists $t' \in T_{rel}$ such that $t'$ is chained to $t$, then $t \in T_{rel}$.

This is, $T_{rel}$ contains the tasks which occur on the chain, and
every task which may influence a relevant task.

As we will show in the next section, considering only
relevant tasks for the computation of the interval size can
reduce its size by an order of magnitude.

IV. EVALUATION

We encoded the constraints described in the previous
section in MiniZinc [27]. This allows for both a structured
representation of our problem and for using a wide range of
solvers as backend. MiniZinc models together with data
describing a particular problem instance are translated to a
FlatZinc formula, which is then solved by a solver backend.
In our experiments we chose CHUFFED with parallelization as
presented in [28] as the solver.

The solver initially has no indications on how to branch on
the different variables. Therefore, in order to decrease the time
needed to find a result, we use so-called search annotations to
impose a search strategy by influencing branching decisions.
Generally, we add annotations telling the solver to determi-
n the activation of a task before trying to determine its termina-
tion by branching on a small value for $\sigma$ and a large value for
$\varepsilon$. Furthermore we ensure that periodic tasks are put in order
first and chained tasks last. In this way, the solver is led to
parts of the search space which describe situations with high
system load first, as it is more likely to find a large response
time here.

Furthermore, we use a preprocessing step to simplify many
constraints. For example, in the sum in Equation 9 it is
sufficient to consider only task instances which might possibly
interact with the task instance $(i, j)$. In many cases, this
significantly reduces the number of summands and the time
required for translating the MiniZinc problem into FlatZinc.
Additionally, with an eye to increasing complexity due to an
increased level of detail, we introduce two variations of the
problem. Firstly, computing time can be reduced by splitting
the relevant period into smaller, overlapping intervals and
estimate response times for each of these intervals separately.
The results of this analysis are not safe but they give a
lower bound. Secondly, an upper bound for the maximum
response time of a task chain can be computed by relaxing
the problem. This might be interesting when adding more complexity to the task set or when a very fast evaluation of different configurations needs to be performed. Ignoring the constraints 7 and 9 massively simplifies the problem, and yields interesting bounds in our experiments. However, for the considered level of detail the full period with all constraints can be solved on a general-purpose computer.

This has been tested with three different industrial-scale task sets. Two of them model control units which can be found in the Daimler powertrain (PTC A and PTC B). The third one was taken from the FMTV 2016 Verification Challenge of Bosch (ECM) with small adjustments. It is a realistic example of an engine control unit. The details of task sets are:

**PTC A** Is taken from an ECU with two cores which runs 17 tasks. Nine tasks are activated periodically on the first core, eight run the second core and are chained to a counterpart on core one. The chain is an example of a chain from a networking task via middleware to application and back.

**PTC B** Is taken from an ECU with four cores which runs 39 tasks. 23 tasks are activated periodically with activation periods between 1ms and 1000ms. The other tasks are executed sporadically. We made assumptions for the minimum time distance of two consecutive occurrences. Again the chain is an example of a chain from a networking task via middleware to application and back.

**ECM** The task set of the ECU described in [29]. Unfortunately the task-level chains in the provided data are rather short. Thus, we designed a more complex chain here. We assumed functionalities for the tasks and then created a chain which spans over six tasks. It is intended to represent a path from angle synchronous software to an application software and back.

The calculations were performed on a notebook equipped with an Intel Core i7-3740QM CPU, 16GB of RAM, running Ubuntu 16.04 LTS. We used the parallel version of chuffed on 4 cores. The results are shown in Table II. For the "Full period" we considered an interval of 111ms for the PTC A, 411ms for the PTC B, and 400ms for the ECM. For the decomposition of the PTC A we then used the same interval also for the decomposition, resulting in only one interval to check. For the PTC B and ECM we split the relevant period in 2 equal-sized time slices of 30000µs. The computational resources needed until the result from "Full period" was found are shown in the rows "Decomposition". For the benchmarks PTC B and ECM with the analyzed chains, this always happened within the first time slice. However, this is in general not the case, e.g. when size or overlapping of the intervals are chosen disadvantageously. Lastly, in the case of differences in the results of the relaxed and the detailed model, the precision of latency estimation without the knowledge of execution times has been improved. In our experiments we saw differences of up to 5ms which means a clear improvement of the quality of the estimation as it is a whole period of a task in the analyzed task sequence.

### V. Future Work

In this paper, we presented an approach focused on early development stages. Future work takes aim at later development stages, where it is possible to integrate further constraints based on data which usually becomes available in later development stages. Worst case execution times or worst case response times may be available then. This would on the one hand enable us to consider overload situations, on the other hand intervals in which data propagation is possible could be bounded more precisely. Furthermore, one may obtain tighter bounds on the response time of the last task in the chain. These possible advantages are in contrast with new challenges. Some overload situations may be infeasible, e.g. in cases where the WCETs of certain tasks cannot happen simultaneously. Additionally, overload situations need to be bound somehow, otherwise the solver will use them to stretch response times to infinity. Hence, going one level deeper to do module-level analyses seems promising although not trivial.

### VI. Conclusions

In this paper we presented a template for a constraint program to estimate end-to-end latencies of task sequences in task sets of multi-core ECUs. This can be used to obtain a safe estimation for the response time of task sequences within the task set. For large scale systems, relaxation and decomposition can be used to compute lower and upper bounds on the response time. For the basic approach very few information about the system to analyze is needed, nevertheless our approach would allow to add more details about the task containers, if available. The results can be used to express the abstract timing behavior of an ECU for arriving network
messages. This is done by analyzing the part of the system-level cause effect chain which starts at the receiving network task of the ECU and ends at the corresponding network task to propagate the stimulus or a task controlling an actuator. We showed how to utilize a parallel solver for this specific problem and the practical application shows that the approach scales for industrial-size problems while running on a general-purpose computer.

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