A hybrid method for solving multi-depot VRP with simultaneous pickup and delivery incorporated with Weber basis saving heuristic

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Abstract

Under growing concerns with sustainability in global and changing market, establishing a cooperative and competitive logistic is becoming a keen issue to provide manufacturing systems amenable to sales and operations planning. As a deployment for such practice, we have engaged in the various studies on logistics optimization. Especially, noticing that transportation cost and/or CO\textsubscript{2} emission actually depend not only on distance but also loading weight (Weber basis), we have recently developed a few hybrid meta-heuristic methods for vehicle routing problems (VRP) and shown their effectiveness through numerical experiments. To the best of our knowledge, however, there exist no studies that take the Weber basis into account on VRP except for ours. As a hot interest in this area, we pay our attention on VRP with simultaneous pickup and delivery (VRPSPD). Then, this study attempts to extend the foregoing Weber basis study under single depot to multi-depot problem and intends to reveal some properties of VRPSPD. To work with such concerns, we have developed a novel hierarchical method comprised of a modified tabu search, a graph algorithm for the minimum cost flow problem and a Weber basis saving method. The proposed method is possible to solve various real world applications practically even with large problem sizes. Finally, the effectiveness of the proposed method is validated through numerical experiments taken place from various viewpoints to discuss about some peculiar features of VRPSPD.

Key words: Multi-depot VRPSPD, Hybrid approach, Weber basis saving method, Graph algorithm, Modified tabu search

1. Introduction

Under growing concerns with sustainability in global and changing market, establishing a cooperative and competitive logistic is becoming a keen issue to provide manufacturing systems amenable to sales and operations planning (SOP). As a deployment for such practice, we have engaged in various studies on logistics optimization. Especially, noticing that transportation cost and/or CO\textsubscript{2} emission actually depend not only on distance but also loading weight (tonnage-kilo meter basis), we have recently developed a few hybrid meta-heuristic methods for vehicle routing problem (VRP) and shown their effectiveness through numerical experiments. Though this tonnage-kilo meter basis transportation cost accounting is generally known as Weber basis and popularly applied to facility location problems, it has never been considered in VRP previously. However, we will agree this idea is quite relevant if we only show such facts that reducing weight of vehicle body has been a keen interest in car industry and Japanese government recommend to use the improved tonnage-kilo meter basis, i.e., Weber basis to evaluate the amount of CO\textsubscript{2} emission from vehicles.

As a growing interest of VRP associated with SOP, we recently concerned simultaneous pickup and delivery for single depot VRP (VRPSPD) associated with the Weber basis and its generalized one (Shimizu and Sakaguchi, 2014 a, 2015). To cope with more relevant and realistic scenes in modern logistics systems, this study extends the foregoing concern under single depot VRPSPD to multi-depot problem. The major aim of this study is to reveal some properties of VRPSPD especially interested in comparison with separate transportation and centralized network configuration. To
work with such concerns, we propose a novel hierarchical method that is possible to practically solve various real world applications even with large problem sizes. Besides the computational difficulty referring to permutation for the above single depot VRPSPD, this multi-depot problem comes across another difficulty referring to combination, e.g., selection of client customers for each depot. Due to such integrated computational difficulties, any studies applicable to real world problems have not carried out elsewhere. To the best of our knowledge, this study is the first attempt to deal with this kind of problem in the world. Finally, numerical experiments are provided to validate the effectiveness of the proposed method both for the Weber basis and the generalized one.

The rest of the paper is organized as follows. In Section 2, we briefly describe the problem statement. Then, we formulate the problem in Section 3. Section 4 outlines the proposed solution procedure in a hybrid manner. Numerical experiments are provided in Section 5. Finally, we give some conclusions.

2. Problem Statement

A logistic network design problem known as VRP belongs to an NP-hard combinatorial optimization problem. It tries to minimize total distance traveled by a fleet of vehicles under various constraints such as payload, duration distance, time window, etc. Every transportation from each depot to the respective client customers must take a circular route making the depot as its starting point and destination at the same time. When we consider only one depot, the problem is called single-depot problem (Fig.1(a)). Meanwhile, it is called multi-depot problem when we consider multiple depots (Fig.1(b)).

![Fig. 1. Scheme for solving logistic network design problem concerned here. Before and after of (a) Single-depot VRP, (b) multi-depot VRP.](image)

One of the recent studies on VRP is an extension from the generic customer demand satisfaction and vehicle payload (Liu and Xie, 2013). The other concerns have been paid to multi-depot problem (Chen, Imai and Zhao, 2005; Crevier, Cordeau, and Laporte, 2007; Shimizu and Sakaguchi, 2014 b) and multi-objective formulation (Jozefowiez, Semet and Talbi, 2008). As a special variant of VRP, many researches are recently interested in VRP with pickup and delivery demands of every customer from certain aspects. They are classified into the following three categories (Nagy & Salhi, 2005).

- Delivery first, Pickup second (VRPB): pickup only after delivered
- Mixed Pickup and Delivery (MVRP): delivery and pickup in any sequence along the routes
- Simultaneous Pickup and Delivery (VRPSPD): simultaneous delivery and pickup

The VRPSPD refers to MVRP when either of pickup demand or delivery demand is placed at each customer. In this study, we concern the VRPSPD since this is the most practical and prospective when we discuss the cooperative and competitive logistic systems in modern society. From this type of transportation, we can expect to realize a higher working rate of vehicle compared with simple one, i.e., delivery alone or pickup alone. In such a special case that every delivery and pickup demand is equal to \( \frac{W}{|K|} \), the working rate of SPD becomes 100%. Here, \( W \) and \( |K| \) denote payload of a vehicle and number of customers, respectively. This type of transportation is frequently encountered in the distribution system of bottled drinks, groceries, LPG tanks, laundry service of hotels, the reverse logistics, etc.

After the first work by Min (1989), growing concerns are paid on this topic, e.g., Cao and Lai, 2007; Ai and Kachitvichyulasak, 2009; Catay, 2010; Lai and Cao2010; Subramanian, et al., 2011; Hezer and Kara, 2011; Jun and Kim, 2012; Goksal, Karaoglan and Altiparmak, 2013; Liu, et al., 2013; Gribkovskaia, Laporte and Vlcek, 2007. These are all single-depot and non-Weber basis problems. Moreover, through heuristic approaches, they solved only small problems with no more than 400 customers to validate the effectiveness of their proposed methods in numerical experiments. Though the Weber bases (Ordinal and Generalized) can describe the correct transportation cost accounting, they have never adopted even in the recent studies except for ours (Shimizu, 2011a, 2011b).

On the other hand, multi-depot VRP is viewed as a variant of location-routing problem since it involves another decision how to allocate the client customers for each depot. Due to such additional complexity, few studies have been known for multi-depot VRPSPD (Gajpal and Abad, 2013, Karaoglan et al., 2012). To the best of our knowledge, this is the first study on multi-depot VRPSPD in terms of the Weber basis cost accounting.
3. Problem Formulation of Multi-Depot VRPSPD

Multi-depot VRPSPD is mathematically formulated by the following combinatorial optimization problem.

\[
(1) \quad \min \sum_{j \in J} H \sum_{v \in V} \sum_{k \in K} g_{j,v} z_{j,v} + \\
\sum_{m \in M} \sum_{m' \in M} \sum_{i \in I} \sum_{j \in J} \left( g_{m,m'} + w_i \right) z_{m,m'} + \sum_{j \in J} F_1 x_j + \sum_{v \in V} F_2 v,
\]

subject to

\[
(2) \quad \sum_{m \in M} z_{m,m'} = 0, \quad \forall m \in M, \forall v \in V
\]

\[
(3) \quad \sum_{j \in J} z_{j,v} = 0, \quad \forall j' \in J, \forall v \in V
\]

\[
(4) \quad \sum_{j \in J} \sum_{k \in K} z_{j,k} = L_y, \quad \forall v \in V
\]

\[
(5) \quad \sum_{j \in J} \sum_{k \in K} z_{j,v} = y_v, \quad \forall v \in V
\]

\[
(6) \quad \sum_{j \in J} \sum_{k \in K} z_{j,v} = y_v, \quad \forall v \in V
\]

\[
(7) \quad \sum_{m \in M} \sum_{m' \in M} \sum_{v \in V} g_{m,m'} z_{m,m'} \leq [2]-1, \quad \forall \Omega \subseteq M \setminus \{1\}, \quad [2] \geq 2, \forall v \in V
\]

\[
(8) \quad \sum_{j \in J} \sum_{k \in K} g_{j,k} z_{j,k,v} \leq U_j x_j, \quad \forall j \in J
\]

\[
(9) \quad g_{m,m'} \leq W_z z_{m,m'}, \quad \forall m,m' \in M, \forall v \in V
\]

\[
(10) \quad \sum_{j \in J} \sum_{k \in K} g_{j,k} = \sum_{k \in K} p_k z_{j,k,v}, \quad \forall j \in J, \forall v \in V
\]

\[
(11) \quad \sum_{j \in J} \sum_{k \in K} g_{m,m'} = \sum_{k \in K} q_k - p_k, \quad \forall k \in K
\]

\[
(12) \quad \sum_{j \in J} \sum_{k \in K} u_{m,m'} = \sum_{k \in K} (u_{m,m'} + s_{m,m'}), \quad \forall m,m' \in M
\]

\[
(13) \quad \sum_{j \in J} \sum_{k \in K} u_{m,m'} - \sum_{k \in K} u_{m,m'} = p_k, \quad \forall k \in K
\]

\[
(14) \quad \sum_{j \in J} \sum_{k \in K} s_{m,m'} - \sum_{k \in K} s_{m,m'} = q_k, \quad \forall k \in K
\]

\[
\begin{align*}
& x_j \in \{0, 1\}, \quad \forall j \in J, \quad y_v \in \{0, 1\}, \quad \forall v \in V \\
& z_{m,m'} \in \{0, 1\}, \quad \forall m, m' \in M, \forall v \in V \\
& g_{m,m'} \geq 0, \forall m, m' \in M, \forall v \in V
\end{align*}
\]

Notations of this problem are summarized below.

**Variables**

- \( g_{m,m'} \) [ton]: total load of vehicle \( v \) on the path from \( m \in M \) to \( m' \in M \)
- \( s_{m,m'} \) [ton]: delivery load of vehicle \( v \) on the path from \( m \in M \) to \( m' \in M \)
- \( u_{m,m'} \) [ton]: pickup load of vehicle \( v \) on the path from \( m \in M \) to \( m' \in M \)
- \( x_j = 1 \) if candidate depot \( j \) is opened; otherwise 0
- \( y_v = 1 \) if vehicle \( v \) is used; otherwise 0
- \( z_{m,m'} = 1 \) if vehicle \( v \) travels on the path from \( m \in M \) to \( m' \in M \); otherwise 0

**Parameters**

- \( c_v \): transportation cost per unit load per unit distance of vehicle \( v \) [cost unit /ton/km]
- \( d_{m,m'} \) [km]: path distance between \( m \in M \) and \( m' \in M \)
- \( F_1 \): fixed charge of depot \( j \) [cost unit]
- \( F_2 \): fixed charge of vehicle \( v \) [cost unit]
- \( H_z \): handling cost of depot \( j \) [cost unit /ton]
- \( L \): auxiliary constant (Large real number) [-]
Here, objective function is composed of handling cost at depot, routing transportation cost and fixed charges of vehicles and opening depots. On the other hand, each constraint means that every vehicle cannot visit the customer twice by Eq. (1); that coming in vehicle must leave out by Eq. (2); avoiding to make a path between depots by Eq. (3); that vehicle must travel on a certain path by Eq. (4); that each vehicle leaves only one depot and return there by Eqs. (5) and (6); sub-tour elimination by Eq. (7); upper bound load capacity for vehicle by Eq. (8); that the load of returned vehicle must be equal to the total pickup loads by Eq.(10). Equations (11) and (12) are material balances at each depot and path, respectively; load difference from the foregoing visit is described by Eqs.(13) and (14) for pickup and delivery, respectively.

Integrality conditions and positive conditions are imposed on the respective variables.

It is well known that this kind of problem belongs to an NP-hard class, and becomes extremely difficult to obtain an exact optimal solution for large problems. Hence, it is meaningful for applications to provide a practical method that can derive a near optimum solution with acceptable computational efforts.

4. Hybrid Approach for Practical Solution

To practically work with the above problem, we propose a hierarchical procedure outlined in Fig.2 as a flow chart with each solution image in the right side. In this procedure, we apply three major components, i.e., a graph algorithm to solve minimum cost flow (MCF) problems, a Weber basis saving method and a modified tabu search (Shimizu, 2011a). The graph algorithm is used to allocate the client customers for each depot in an effective manner (State (a) in Fig.2). Now the original problem formally refers to multiple single depot problems (sub-problems). Then, the Weber basis saving method is used to derive the initial solution of each sub-problem (State (b)) and the modified tabu search is applied to improve this in the inner loop search (sub-solution) (State (c)). After that, those sub-solutions are to be mixed with each other through the search by the modified tabu in outer loop search (State (d)). In the below, each component characterizing this procedure will be explained more in detail.

Fig 2. Scheme of proposed procedure for multi-depot VRP with solution images: (a) Allocation by MCF: (minimum cost flow), (b) Weber basis saving method (inner), (c) modified tabu search (inner), and (d) modified tabu search (outer).
4.1 Customer allocation by graph algorithm

To decide the client customers to each depot, we give an allocation problem formulated below.

\[
\begin{align*}
(2.2) \quad & \min \sum_{j=1}^{J} \sum_{k \in K} c_{j}d_{jk}g_{jk} + \sum_{j \in J} H_j \sum_{k \in K} g_{jk} \\
& \text{subject to} \\
& \sum_{k \in K} g_{jk} \leq U_j, \quad \forall j \in J \tag{15} \\
& \sum_{j \in J} g_{jk} = \max(q_k, p_k), \quad \forall k \in K \tag{16} \\
& g_{jk} \geq 0, \quad \forall j \in J, \forall k \in K
\end{align*}
\]

Here, we use the same notations as (p.1) except for \(g_{jk}\) that denotes the amount allocated from depot \(j\) to customer \(k\). This approach is suitable compared with the other methods that relies on a certain geometric reasoning such as Voronoi diagram (Man, Mingyi and Yang, 2012) and cluster divisions (Esnal and Küçükdeniz, 2009), polar angles between the depot and the customers (Gillett and Miller, 1974), etc. These methods just claim their rationality qualitatively, and are far from what is required in the formulation of (p.1). That is to say, they never consider capacity constraint of each depot and the handling cost and the practical transportation cost accounting in the objective function. Against this, since the auxiliary problem (p.2) might partly reflect some conditions of the original problem (p.1), we can assert its rationality even quantitatively. Regarding the present SPD case, however, a certain advantage might be a bit refrained since the demand constraint given by Eq.(16) becomes rather expedient compared with each of the separate cases, i.e., delivery alone or pickup alone.

Actually, we solve the above (linear programming) problem using a graph algorithm of MCF problem to enhance the solution ability. We show the graph structure and its label information in Fig.3 and Table 1, respectively. Here, the depot with no inflow from the source in the MCF graph will not be opened. From those procedures, we can still allocate every customer to each depot very efficiently.

![Fig.3 Minimum cost flow graph for allocation, Label on edge shows (cost, capacity).](image)

| Table 1. Specification of minimum cost flow graph |
|-----------------------------------------------|
| Edge             | Cost       | Capacity       |
|-------------------|------------|----------------|
| Inlet             | -          | \(\sum D = \sum \max(p_j, q_j)\) |
| Source - Depot \(i\) | \(H_i\) | \(U_i\) |
| Depot \(i\) - Customer \(j\) | \(C_{ij}\) | \(U_j\) |
| Customer \(j\) - Sink | 0 | \(D_{ij}=\max(p_j, q_j)\) |
| Outlet           | -          | \(\sum D = \sum \max(p_j, q_j)\) |

\(C_{ij} = \{\text{Weber: } c_{ij}d_{ij}; \text{generalized Weber: } c_{ij} \max(p_j, q_j)^{a-1}d_{ij}^b\}\)

4.2 Inner Loop Search Methods

4.2.1 Weber basis saving method for VRPSPD

Saving method (Clarke and Wright, 1964) is a popularly known heuristic method for solving the conventional VRP. Thereat, saving value that is the reward from integrating the return paths plays a key role to drive the algorithm. It is conventionally calculated only on distance (kilo) basis. Against this, since the delivery transportation cost between customer \(i\) and \(j\) is described by Eq. (17) for the Weber basis, the saving value \(s_{ij}\) is given by Eq. (18) (refer to Fig.4).

\[
C_{\text{max}} = c_{ij}d_{ij}
\]

\[
s_{ij} = d_{ij}(p_j - q_j + w_{ij}) + d_{ij}(-p_i + q_i + w_{ij}) - d_{ij}(p_i + q_j + w_{ij}) \quad \text{for } i, j = 1, 2, \ldots, |K|, i \neq j \tag{18}
\]

where \(d_{ij}\) denotes total load on the path from customer \(i\) to \(j\) and \(|K|\) a total number of customers. Here let the suffix for depot be 0 and \(s_{ij} = 0\). Moreover, for the generalized Weber basis, the foregoing Eqs. (17) and (18) are modified as Eqs. (19) and (20), respectively.
\[ C_{\text{trans}} = c_s d_{ij}^a s_{ij}^b \] (19)

\[ s_{ij} = (c, \gamma) = (q_i + w_i)^a d_{0i}^b + (p_j + w_j)^a d_{0j}^b + (p_j + w_j)^a d_{0j}^b - (q_i + q_j + w_i + w_j)^a d_{ij}^b \] (20)

where \( \alpha \) and \( \beta \) denote the elastic coefficients for the distance and weight, respectively and \( \gamma \) is a constant.

Fig. 4. Scheme to derive Weber basis saving value (Left (before): round path, Middle (transient), right (after): merged path).

For a practical evaluation of economy, it makes sense to account the fixed operating cost of vehicle, \( F_2 \), beside the transportation cost. In this case, it becomes more economical to visit a new customer even if its saving cost \((\gamma c, s_{ij})\) would become negative as long as its absolute value stays within the fixed operating cost of vehicle. Applying these ideas to the original saving method, we developed the Weber basis saving method outlined below (Shimizu, 2011a, b).

Step 1: Create round trip routes from the depot to its client customers. Compute the savings value for every pair of \( i \) and \( j \) in terms of Eq. (18) or Eq. (20).

Step 2: Order these pairs in descending order of savings value.

Step 3: Merge the path for the pair, following the order obtained from Step 2 as long as it is feasible and the savings value is greater than \(-F_2/(\gamma c)\). Through this method, we can derive the initial routes more practically compared with the conventional method that ignores the loading and unladen weights and the fixed operating cost of vehicles. Moreover, the treatments in Sec. 4.1 and 4.2.1 may accelerate the solution efficiency in a sense the better initial solution we have, the more efficiently we can solve the problem by evolutionary method like the modified tabu search employed here.

4.2 Modified tabu search

Since the Weber basis saving method derives only an approximated solution, we try to improve it by applying the modified tabu search. The tabu search is a simple but powerful heuristic method that refers to a local search with certain memory structure. In the present local search, we generate a neighbor solution from either of insert, swap, cross or 2-opt operations selected randomly. To avoid trapping into a local minimum, our modified method allow even a degraded neighbor solution to be a new tentative solution as long as it would be feasible and not be involved in the tabu lists. Such decision is made in terms of the probability \( p \) whose distribution obeys the following Maxwell-Boltzmann function and used in simulated annealing.

\[
p = \begin{cases} 
1 & \text{if } \Delta e \leq 0 \\
\exp(-\Delta e / T) & \text{if } \Delta e < \varepsilon \\
0 & \text{if } \Delta e > \varepsilon 
\end{cases}
\] (21)

where \( \Delta e \) denotes difference of objective function from the present best value, and \( \varepsilon \) a small positive number. Moreover, \( T \) is the temperature that will decrease along with the iteration \( k \) geometrically, i.e., \( T^{k} = \lambda T^{k-1}, \lambda < 1 \).

4.3 Outer loop search methods

To update the sub-solutions, we also apply the above modified tabu search in the outer loop search. Thereat, the neighbor solution is generated in the limited extent as outlined below (refer to Fig. 5). Such a limitation means that we weigh the practice more than the risk to miss better solutions in computation.

Step 1: Select randomly two depots, i.e., an outgoing depot \( i \) and an incoming depot \( j \).

Step 2: Select randomly \( n \) emigrant customers in depot \( i \) and \( m \) immigrant ones in depot \( j \). To reserve the near optimum structure composed of sub-solutions besides the computation practice, those numbers are restricted within narrow ranges, i.e., \( n \in [1, 2] \) and \( m \in [0, 1] \). Moreover, if the conditions mentioned below on the increments regarding distance or capacity constraints of depot are not satisfied, go back to Step 1 after cancelling this migration. Otherwise go to the next step.

\[ \text{Constraints:} \]
\[ \delta d_{ij} \leq \alpha \]
\[ \delta w_{ij} \leq \beta \]
\[ \delta c_{ij} \leq \gamma \]

where \( \delta d_{ij}, \delta w_{ij}, \delta c_{ij} \) denote increments of distance, weight, and capacity, respectively.
Step 3: If the above selection in Step 1 and 2 is not involved in the tabu list, go to the next step while adding this in the tabu list. Otherwise go back to Step 1.

Step 4: Replace the costs on the edges of MCF graph concerned here as shown in Fig. 3. Then, we can induce the flow to the edge with cost \(-L\) while prevent it to that with cost \(L\). Here, \(L\) denotes the large number. Through such re-labeling on a few edges of the MCF graph, we can realize the intended migration substantially while keeping solution efficiency in terms of sensitivity analysis of the graph algorithm.

In Step 2, we can calculate the increments of distance and capacity accompanying the migration as follows.

1. when \(n = 1, m = 0\):
   \[
   \Delta d = d(j, i_j) - d(i, i_i), \quad \Delta Q_j = \max(p_j, q_j)
   \]

2. when \(n = 1, m = 1\):
   \[
   \Delta d = d(j, i_j) + d(i, i_i) - (d(i, i_i) + d(j, j_i))
   \]
   \[
   \Delta Q_j = \max(p_j - p_i, q_j - q_i), \quad \Delta Q_i = -\Delta Q_j
   \]

3. when \(n = 2, m = 1\):
   \[
   \Delta d = d(j, i_j) + d(i, i_i) + d(j, i_2) - (d(i, i_i) + d(i, i_2) + d(j, j_i))
   \]
   \[
   \Delta Q_j = \max(p_j - (p_i + p_2), q_j - (q_i + q_2)), \quad \Delta Q_i = -\Delta Q_j
   \]

If we might ignore a slight possibility missing the better solution in Step 2, it makes sense not to take such a migration that anyone of the following relation holds, i.e., \(\Delta d \geq d_{\text{max}}, \Delta Q_j \geq \overline{Q}, \Delta Q_i \geq \overline{Q}\). Here, \(d_{\text{max}}\) denotes the threshold of allowable increase in distance. On the other hand, \(\overline{Q_j}\) and \(\overline{Q_i}\) represent the margin of capacity at depot \(i\) and \(j\), respectively.

5. Numerical Experiment

Numerical experiments were carried out to validate the effectiveness of the proposed method. Since we have never found even the studies of VRP concerned with Weber basis anywhere, we prepared the benchmark problems by ourselves. First, we randomly generated the prescribed numbers of customers within the entire rectangular region (aspect ratio=1.4) while the depots within a smaller region involved in it (refer to Fig.6). The distances between the depot and customers and also those between every customer are given by Euclidian basis. Moreover, demands of each customer for delivery are randomly given within the prescribed ranges. Then, the pickup demands are set like \(p_i=(0.3+0.7)\text{ rand}()q_k\) so that \(p_i \leq q_k, \forall k \in K\) will be satisfied for the simplicity of the computer program implementation. However, it is not difficult to remove such condition. These parameter settings are summarized in Table 2.

![Fig. 5. A scheme to generate neighbor solution in tabu search (a) \(n = 1, m = 0\), (b) \(n = 1, m = 1\), and (c) \(n = 2, m = 1\).](image)

![Fig. 6. A scheme to generate logistic members, Customers located within outer box with aspect ratio 1.4 while depot within inner box.](image)

| Table 2. Outline of parameter setting. |
|---------------------------------------|
| Distance between customers | [180, 860, 10] |
| Distance between depot & customers | [2, 516, 2] |
| Capacity of depot | [2, 50] |
| Handling cost at depot | [20, 60, 2] |
| Fixed-charge at depot | [200, 600, 10] |
| Delivery demand \(q_i\) | [5, 100, 5] |
| Pickup demand \(p_i\) | [0.7, 1.0] |

Fixed-charge of vehicle=50000, Unladen weight=Payload
On the other hand, size of tabu list is changed in the range depending on the problem size. Convergence condition is given by either total number of generation or number of successive failures in local search. Those settings are also changed with problem size. Moreover, $d_{\text{max}}$ is set at one thirds of the average distance between depots and customers.

We used a PC with CPU: Intel® Core®2 Quad Processor Q6600 2.4GHz, and RAM: 3GB. The following discussions are made by averaging the results over 10 samples.

5.1 Results of major concern

As an alternative method to satisfy both demands regarding delivery and pickup for every customer, we think about doing them separately. In this case, we need to solve the delivery (D) and pickup (P) problems independently, and those results must be summed up to evaluate the total performance. For this purpose, we can apply the same procedure mentioned in Sec.4 just by using the Weber basis saving value suitable for the respective problem (refer to Appendix) and rewrite the right hand side of Eq.(16) with $q_k$ or $p_k$ for delivery or pickup problem, respectively. Actually, we compared the results between SPD and such step-wise dealing (P&D) through the procedure depicted in Fig.7.

Due to a weaker ground on the client customer allocation problem mentioned in Section 4.1 (the right hand side value of Eq.(16)), we preliminarily examined the solution abilities for the four plausible procedures in our hybrid method.

Actually, we evaluated the following cases to select the most suitable one.

1. Elaborate inner-loop search from the beginning of the outer-loop search (as same as the individual case)
2. Gradually increasing such elaborateness along with the outer-loop search
3. Elaborate inner-loop search only at the end of outer-loop search
4. The outer-loop search is taken place without reallocation in terms of the MCF problem.

Through these preliminary experiments, we confirmed the first one is still superior to the others. So we adopted it for the following considerations.

In Table 3, we summarize the results of six benchmark problems with same scale factor but with different sizes each of which is shown in ‘Size’ column. Here scale factor means the apparently averaged number of client customers for each DC, i.e., $|K|/|J|$. Threat, figures in the parenthesis denote the numbers of depot and customer, respectively. It involves such large problems that have not been solved elsewhere. Thereat, a major interest is to examine a rate meaning how much we can save the cost by the simultaneous transport compared with the separate case. It is evaluated by $(\text{Cost(P&D)} - \text{Cost(SPD)})/\text{Cost(P&D)}$ and shown in “Improved rate” column.

In any cases, we know it more than 30% and gradually but slowly increases along with the DC number as shown in Fig.8. This is because it becomes easier to choose the relevant combination of opening DCs and their client customers than the separate case along with globalization.

On the other hand, we can examine the solution ability in ‘Rate’ and ‘Rate2’ columns that denote improved rates of the final solution in the outer loop search and from the initial one (result from the Weber basis saving method), respectively. Accordingly, it always satisfy the relation such that ‘Rate2’ > ‘Rate’. As supposed from the reason pointed out in Sec.4.1, we can obtained pretty greater values of ‘Rate2’ than those of the independent problems whose values stayed at most around 0.05. On the other hand, it seems value of ‘Rate’ is a bit smaller since the inner-loop search can give a near optimum solution every time and may left only a small margin for improvement.

To ascertain the merit of SPD besides the lack of redundant paths seen in the separate transportations, we roughly estimated the working rate of vehicle by the averaged and uniform basis on amount and distance for every travel (Refer to Appendix 2). Such comparison shown in Table 4 reveals greater advantage of SPD than the separate transportations.

![Flow chart](https://example.com/flowchart.png)

Fig.7. Flow chart of the evaluation process.
Table 3. Result under various problem sizes (Weber/SPD; Scale factor 100, 2 Ton)

| Size* | Improved rate [-] | Rate [-] | Rate2 [-] | Cost [cost unit] | CPU [s] |
|-------|-------------------|----------|-----------|-----------------|--------|
| (3, 300) | 0.317 | 0.041 | 0.588 | 3.88E+07 | 57.9 |
| (5, 500) | 0.347 | 0.093 | 0.580 | 6.36E+07 | 158.7 |
| (10, 1000) | 0.379 | 0.143 | 0.621 | 1.15E+08 | 309.2 |
| (15, 1500) | 0.381 | 0.157 | 0.654 | 1.76E+08 | 589.2 |
| (20, 2000) | 0.388 | 0.144 | 0.641 | 2.29E+08 | 747.7 |
| (25, 2500) | 0.390 | 0.150 | 0.640 | 2.82E+08 | 986.0 |

*(Depot#, Customer#).

Table 4. Factors to evaluate the average working rate of vehicle (Weber/SPD; Scale factor 100, 2 Ton)

| Size [-] | Open DC number [-] | Vehicle Number [-] | Working rate: SPD (D, P) [-] |
|----------|---------------------|---------------------|-------------------------------|
| (3, 300) | 2.0 | 4.00 | 0.740 (0.464, 0.381) |
| (5, 500) | 4.1 | 3.50 | 0.705 (0.440, 0.410) |
| (10, 1000) | 7.0 | 3.79 | 0.746 (0.455, 0.407) |

Fig. 8. Effect of number of depots on the improved rate; increase slowly and saturated at last.

Fig. 9. Profile of the decentralized gain with scale factor; increases almost linearly.

Fig. 10. Profiles of convergence for outer-loop search; rather rapidly converged.

Fig. 11. Profiles of convergence for inner-loop search; adequately converged.
Moreover, it is interesting to confirm the effect of decentralization or replacing a single-depot (centralized) logistics with a multi-depot (decentralized) logistics. Actually, we evaluated this gain by (Cost(Single) - Cost(Multi))/Cost(Single). For fair comparison, we put the single depot at the center of the multiple depots. Capacity of the depot is set total sum in the multi-depot problem. Moreover, we gave the handling cost and the fixed charge of depot by applying two-thirds law of scale to the total sum of the multi-depot problem. Figure 9 shows a profile of the average values over 3, 5 and 10 DC problems. The profile reflects a merit of decentralization by virtue of shorter total Ton-Kilo value over the loss of scale merit according to globalization.

Convergence profile of the outer loop search is shown in Fig.10 for the problem with |J|=10 and |K|=1000. From this, we know fairly plausible convergence is attained quickly. On the other hand, Fig.11 shows that of the inner loop search for a certain route from a certain depot at the first stage of the outer-loop search. From this, we know sufficient convergence is attained by 10000-th iteration.

5.2 Miscellaneous results
First, the effect of the scale factor on the improved rate is shown in Fig.12 (2-ton vehicle). For every DC number, the improved rate slowly decreases with the scale factor. This is because the separate transportations have more flexibility regarding the selection of clients than SPD according to globalization.

The following discussions are made on the problems with scale factor 100. First, the effect of payload of vehicle on the cost is shown in Fig.13. As supposed easily, the greater the payload becomes, the lower the working rate of vehicle will be resulted in besides the increase in unladen weight. Hence, necessary cost increases along with the payload in every cases.

Looking at Fig.14 (a), we know CPU time expands almost linearly along with the problem size when the scale factor is fixed. At a glance, it seems a bit strange as a usual nature of evolutionary methods. However, we can observe a popular profile in Fig.14 (b) when we take the scale factor for horizontal axis. This means the scale factor must be a size factor causing NP-hardness in the present problem. All over the problems, however, we can obtain the results within an acceptable time.

Finally, in Table 5, we show the results for the generalized model. Regardless of the high non-linearity, we know the similar results are obtained except for the greater CPU times. Thereat, the cost itself is less than that of the Weber basis due to the elastic coefficients (α=0.894, β=0.750; γ=1.726). This outcome also supports the effectiveness of the proposed approach.

As a nature of evolutionary method, it is unable to guarantee the optimality of the solutions obtained here. Moreover, we cannot compare the performance with other methods since any method relied on the Weber basis has never been known elsewhere. Then, through numerical experiments, we have shown a well-approximated solution is surely derived by the proposed method within an acceptable computation time even for large problems. In addition to this fact, by relying on the previous experiences that revealed the high performance for our variant approaches cited already (Shimizu, 2011a, 2011b, Shimizu and Sakaguchi, 2014a, 2014b, 2015), we can still claim the effectiveness of our approach. This is of special importance for real world application.

Fig.12. Effect of the scale factor on the improved rate; Every case decreases almost linearly.

Fig.13. Effect of payload of the employed vehicle on cost.; Every case increases almost linearly.
Table 5. Result under various problem sizes (Generalized Weber/SPD; Scale factor 100, 2 Ton)

| Size    | Improved rate [-] | Rate [-] | Rate2 [-] | Cost [cost unit] | CPU [s] | Vehicle# [-] |
|---------|------------------|---------|-----------|-----------------|---------|--------------|
| (3, 300)| 0.369            | 0.022   | 0.588     | 9.78E+06        | 306.5   | 4.00         |
| (5, 500)| 0.357            | 0.057   | 0.586     | 1.74E+07        | 610.1   | 3.45         |
| (10, 1000)| 0.385          | 0.075   | 0.601     | 3.13E+07        | 1986.6  | 3.86         |

Fig.14. Profile of CPU time with number of (a) depot; increase linearly and (b) scale factor; increase exponentially.

6. Conclusion

To promote a sustainability in modern global and changing logistics system, this study concerned with a multi-depot VRPSPD. To the best of our knowledge, any methods have not been known previously due to the great difficulty of solution. To cope with the problem in practice, we extended our foregoing idea on single depot in a similar framework. Actually, we have developed a practical hierarchical solution method comprised of the Weber basis saving method, the modified tabu search and the graph algorithm of MCF problem. It is not only practical but also powerful enough to cope with various real world applications. Finally, applying this approach, we discussed on a few key issues associated with the characteristics of VRPSPD through numerical experiments. Thereat we have validated the advantages of SPD through comparison with an expedient approach besides high solution ability and performance.

In future studies, we aim at extending the idea so that we can cope with the problems more in general framework and consider various practical conditions. It is also meaningful to turn our interests toward multi-objective optimizations that manage the trade-off among economics, risks, services, environment issues and so on.

Appendix 1

The Weber basis saving values for the other types are summarized in Table A-1. Regardless of using the ordinal or the generalized models, the proposed method is available within the same framework. It only requires replacing the saving value used thereat. In contrast, the foregoing mixed-integer bi-linear formulation turns to the non-linear one for the generalized model. This will expand the difficulty of solution greatly.

Table A-1. The Weber basis saving values.

| Type   | Weber model                                                                 | Generalized Weber model                                                                 |
|--------|----------------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| Delivery| \(q_i(d_{ij} - d_{ij} - d_{ij}) + w_i(d_{ij} + d_{ij} - d_{ij})\) + \((w_i + q_i)^{\alpha} - (w_i + q_i)^{\alpha} + w_i d_{ij}^{\beta}\) + \((w_i + q_i)^{\alpha} - (w_i + q_i)^{\alpha} + w_i d_{ij}^{\beta}\) | \((w_i + p_i)^{\alpha} - (w_i + p_i)^{\alpha} + w_i d_{ij}^{\beta}\) + \((w_i + p_i)^{\alpha} - (w_i + p_i)^{\alpha} + w_i d_{ij}^{\beta}\) |
| Pick up| \(p_i(d_{ij} - d_{ij} - d_{ij}) + w_i(d_{ij} + d_{ij} - d_{ij})\) + \((w_i + p_i)^{\alpha} - (w_i + p_i)^{\alpha} + w_i d_{ij}^{\beta}\) + \((w_i + p_i)^{\alpha} - (w_i + p_i)^{\alpha} + w_i d_{ij}^{\beta}\) | \((w_i + p_i)^{\alpha} - (w_i + p_i)^{\alpha} + w_i d_{ij}^{\beta}\) + \((w_i + p_i)^{\alpha} - (w_i + p_i)^{\alpha} + w_i d_{ij}^{\beta}\) |

Appendix 2

Let \(n\) be an actual number of the average client customers, i.e., \(|K|/\text{number of opening DCs/number of working vehicles per DC}\). From the assumptions mentioned in the body, pickup (delivery) amount will expand by the rate of \(1/n\) along with each visit to a new customer like \(1/n, 2/n, \ldots, n/n\). Totally it becomes \((n+1)/2\). Since the average amount on the path is given by \(\Sigma p_i/|K|\), working rate of vehicle is calculated by \((n+1) \Sigma p_i/(2|K|W_i)\).
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