A Non-destructive Ultrasonic Testing Approach for Measurement and Modelling of Tensile Strength in Rubbers

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Abstract

Currently, non-destructive testing is widely used to investigate various mechanical and structural properties of materials. In the present study, non-destructive ultrasonic testing was applied to study the relationship between the tensile strength value and the velocity of longitudinal ultrasonic waves. For this purpose, fourteen specimens of composites with different formulations were prepared. The tensile strength of the composites and the velocity of longitudinal ultrasonic waves inside them was measured. The relevance vector machine regression analysis, as a new methodology in supervised machine learning, was used to define a mathematical expression for the functional relationship between the tensile strength and the velocity of longitudinal ultrasonic waves. The accuracy of the mathematical expression was tested based on standard statistical indices, which proved the expression to be an efficient model. Based on these results, the developed model has the capability of being used for the online measurement of the tensile strength of rubber with the proposed formulation in the rubber industry.

Keywords:
Longitudinal Ultrasonic Waves' Velocity
Relevance Vector Machine
Rubber
Tensile Strength

1. INTRODUCTION

The microstructure of the material shows its macrostructure properties. It is typically believed that the macrostructural properties of a material, such as its physical and mechanical properties, cannot be determined by merely investigating its structural data. One of the conventional approaches to investigate the relationship between the microstructure of materials and some of their macrostructural properties is via the ultrasonic wave velocity through these materials [1]. The characteristics of a pulse traversing among the medium are changed and take information on the medium’s microstructure and macrostructure. The application of the ultrasonic wave velocity measurement technique is not confined only to the measurement of physical and mechanical properties but is also used in non-destructive tests.

There are various experimental methods for evaluating the physical and mechanical properties of materials. The samples were taken from the considered sample port which caused destruction. Non-destructive testing is used as one of the analytical techniques to evaluate the properties of many materials without causing damage. The specimen remains usable for the detection of defects and the determination of material

NOMENCLATURE

| Symbol | Description |
|--------|-------------|
| $K(.,.)$ | Kernel function |
| $N(.)$ | Normal distribution with mean $\mu$ and variance $\sigma^2$ |
| $p(.)$ | Conditional probability distribution function |
| $t$ | Vector of targets |
| $W$ | Vector of weight coefficients |

| Greek Symbols | Description |
|---------------|-------------|
| $\alpha$ | Vector of hyper-parameters |
| $\mu$ | Mean Value |
| $\sigma^2$ | Variance of Gaussian distribution |
properties. With the use of an ultrasonic test, it is possible to determine several properties of a material by measuring the time duration between the emission and reflection of ultrasonic waves [2]. The investigation of quality and the determination of the physical and mechanical properties of materials have been practiced by calculating the changes in the ultrasonic wave velocity for a wide variety of materials and applications. The investigations have ranged from the investigation of fruit quality [3] and the determination of limestone properties in historical monuments [4], to the evaluation of weld quality in friction stir welding of aluminum [5]. In this method, the ultrasonic waves and their reflection are displayed on the monitor, and the required data is obtained by interpretation of these signals, i.e., the initial and the reflected pulses, as shown in Figure 1.

As the ultrasonic test parameters are greatly affected by the material’s microstructure and mechanical properties, this method has been proven as one of the best and most cost-effective non-destructive testing approaches for investigating mechanical material properties. Due to the recent advances in the electronic components used in the testing equipment, the precise measurement of the ultrasonic wave properties has provided the possibility of evaluating various mechanical properties at a reasonable and satisfactory level. Thanks to high test speeds, lack of part damage during the test, and the ability to perform the test on the parts while being manufactured, non-destructive tests provide a suitable substitute for traditional inspection methods.

Remarkable studies have been conducted on the determination of properties by ultrasonic waves for metals. Grain size, the presence of impurities, elasticity modulus, hardness, toughness, and yield strength in metals are among the items that have been measured by ultrasonic testing [6]. Also, the wave-mode-converted principle was used to calculate the shear wave and longitudinal wave velocities of magnesium-based composite samples; to evaluate the relationship among the reinforcement content and the dual-mode ultrasonic velocities. The elastic modulus is also calculated [7]. Based on the relationship between surface roughness and ultrasonic attenuation, an inverse model for the attenuation, using Weaver’s diffuse scattering theory, is fixed to measure grain size in polycrystals [8]. However, only a few surveys have been performed on the application of ultrasonic testing for the determination of the properties of rubbers.

One of the fundamental properties of rubber is its ultimate tensile strength, which is a measure of its ability to withstand a pulling force. The ultimate tensile strength, often shortened to tensile strength, is measured by the maximum stress that a material can withstand while being stretched or pulled before breaking. Although rubber does not reach its ultimate tensile strength, this property is regarded as an index of the quality of the produced rubber. However, online measurement of this property in the production cycle is not possible, and therefore non-destructive testing approaches provide an efficient alternative for this purpose.

In line with previous research by the authors on the use of ultrasonic waves in the rubber industry [9, 10], in this study, an ultrasonic test has been presented as a novel approach for the measurement of the tensile strength of rubber. The proposed method is capable of online measurement of the rubber’s tensile strength while being manufactured. To this end, the tensile strength values in several specimens with different formulations were measured together with the longitudinal ultrasonic wave velocity through them. In addition, the relationship between the tensile strength value and the longitudinal ultrasonic wave velocity was investigated using a novel supervised machine learning algorithm, namely the relevance vector machine (RVM).

Machine learning is a subfield of computer science, in which the study and construction of algorithms that are capable of learning from and making predictions based on a limited set of observed data are explored. Supervised learning is the machine learning task of inferring a function from a set of labeled training data. For this purpose, a model is generated from the dependency of the targets on the inputs based on a set of N observed input vectors \(\{x_n\}_{n=1}^N\) and the corresponding targets \(\{y_n\}_{n=1}^N\), in order to predict the targets for inputs that have not been observed [11].

Supervised learning algorithms can be used to establish a global model from the functional relationship between the outputs and the inputs based on a limited number of measurements [12].

Support vector machines (SVMs) are supervised learning models with associated learning algorithms used for the classification and regression analysis [13], which have proven to be efficient in many practical applications [14]. For SVM-based regression, the input space is mapped into a high dimensional feature space, based on a set of kernel functions and then an optimal linear regression is performed in this space, which can be expressed as follows:

\[
f(x) = y(x, w) = \sum_{i=1}^{N} w_i K(x, x_i) + w_0
g
\]
where \{w_i\} is the model weights, \( K(.,.) \) a kernel function, and \( N \) the number of training samples. The most common formulation of the kernel function is the radial basis function (Gaussian) defined in Equation (2) in which \( \sigma \) is the kernel function parameter. Substituting this kernel function in Equation (1) results in function \( f(x) \) estimated in the form of Equation (3).

\[
K(x, x_i) = \exp \left( -\frac{x-x_i^2}{\sigma^2} \right) \tag{2}
\]

\[
f(x) = \sum_{i=1}^{N} w_i \exp \left( -\frac{x-x_i^2}{\sigma^2} \right) + w_0 \tag{3}
\]

Despite its widespread success, the SVM suffers from some disadvantages, which have been overcome in a newer probabilistic approach named the Relevance Vector Machine (RVM) as proposed by Tipping [15]. RVM is a nonlinear pattern recognition model with a simple structure based on the Bayesian Theory and Marginal Likelihood [15]. The main advantage of the RVM over SVM in our application is the fact that in addition to precision and sparseness, it utilizes a fewer number of kernel functions. Therefore, it is suitable for the development of a formula for an input-output relationship.

In this study, the relevance vector machine regression analysis was used to obtain a mathematical expression for the tensile strength based on the longitudinal ultrasonic wave velocity. The accuracy and generalization capability of the obtained expression are verified based on standard statistical indices, which prove it to a suitable model for the rubber tensile strength based on the ultrasonic wave velocity.

2. EMPIRICAL EXPERIMENTS

2.1. Materials For the fabrication of composites, the following materials have been used: solution caoutchouc styrene-butadiene 1500, butadiene cis caoutchouc, soot, high dispersible silica (HDS), silane, sulphur, sulfonamide accelerator, zinc oxide, stearic acid, zinc stearate.

2.2. Preparation of Composites In the present research, in order to investigate the relationship between the tensile strength of rubber and longitudinal ultrasonic wave velocity, 14 rubber specimens with different formulations (Table 1) were produced.

The composites were prepared in a 2-lit experimental Banbury made by Pomini under similar conditions. The R-E Mccin 305x152 double-roll grind made by Italian Bergamo for material mixing, the experimental 100 tones vulcanizing press made in Japan for the vulcanization of rubber composites, and the Rheometer made by English Alpha co. have been utilized for determination of vulcanization properties. The rotor’s revolution in the stages of adding caoutchouc, chemical materials, and filler was a constant value of 20 rev/min. To ensure complete silanization, the rotor’s revolution was a set variable during the final stages of mixing to maintain the mixing temperature for a long time within a range of 130–150 °C. The overall mixing duration for the composites was set to be six minutes.

2.3. Tensile Strength Test To evaluate the tensile strength of the rubber specimens, 14 dumbbell-shaped samples were prepared, as shown in Figure 2,

| No | Sulphur (gr) | CBS (gr) | High Dispersible Silica (gr) | Silane (gr) | Soot N330 (gr) | BR CIS (gr) | SBR1500 (gr) | Other Chemicals (gr) |
|----|-------------|---------|-----------------------------|-------------|----------------|-------------|--------------|---------------------|
| 1  | 5.7         | 4.95    | 0                           | 0           | 386.86         | 0           | 773.71       | 39.43               |
| 2  | 5.7         | 4.95    | 0                           | 0           | 386.86         | 0           | 773.71       | 39.43               |
| 3  | 5.56        | 4.82    | 377.13                      | 30.17       | 0              | 0           | 754.26       | 38.44               |
| 4  | 5.56        | 4.82    | 377.13                      | 30.17       | 0              | 0           | 226.28       | 527.98              |
| 5  | 6.39        | 5.54    | 260.69                      | 26.07       | 0              | 0           | 260.69       | 608.27              |
| 6  | 4.87        | 4.22    | 461.29                      | 46.13       | 0              | 0           | 197.7        | 461.29              |
| 7  | 6.53        | 5.66    | 0                           | 0           | 266.48         | 266.48      | 621.78       | 45.26               |
| 8  | 5.06        | 4.39    | 0                           | 0           | 479.74         | 0           | 205.6        | 479.74              |
| 9  | 6.29        | 5.36    | 377.13                      | 30.17       | 0              | 0           | 226.28       | 527.98              |
| 10 | 7.22        | 6.16    | 260.69                      | 26.07       | 0              | 0           | 260.69       | 608.27              |
| 11 | 5.51        | 4.7     | 461.29                      | 46.13       | 0              | 0           | 197.7        | 461.29              |
| 12 | 7.02        | 5.36    | 377.13                      | 30.17       | 0              | 0           | 226.28       | 527.98              |
| 13 | 8.06        | 6.15    | 260.69                      | 26.07       | 0              | 0           | 260.69       | 608.27              |
| 14 | 6.15        | 4.69    | 461.29                      | 46.13       | 0              | 0           | 197.7        | 461.29              |
based on the ASTM D624 standard [16] employed a
dynamometer made by Hounsfield.

2. 4. Ultrasonic Test

To investigate the
relationship between the tensile strength of rubber and
ultrasonic wave velocity, the longitudinal wave velocity
for each specimen was measured. To this end, a Tru-
Sonic ultrasonic test machine was employed. The
specifications of the machine, specimens, and the probe
are shown in Table 2. The ultrasonic waves, which are
transmitted through the material, always lose a portion of
their energy due to the dispersion at microscopic
interfaces, as well as the effect of internal frictions in the
material. The attenuation effect is actually the drop of the
sonic wave energy during the emission of waves through
the environment. In this empirical study, regarding the
more intense attenuation effect in rubbers [17], a probe
with a frequency of 4 MHz was utilized for
the
determination of the ultrasonic wave’ velocity.

The longitudinal ultrasonic wave emission velocity
for different specimens was measured by recording the
elapsed time between the emission of waves and their
reflex, which is displayed on the monitor by the standard
electronic circuit. The ultrasonic wave measurement
system is shown in Figure 3. The measurement precision
of the ultrasonic wave velocity is 1 m/s. Because
the calculated duration between the transmission and
reception of the wave incorporates the time of the wave’s
traverse across the probe and coupler during each stage
of the wave’s travel, the time carries some errors.
Consequently, it causes an error in the calculation of the
wave emission velocity. First, the travel duration of the
longitudinal ultrasonic wave across the probe and coupler
was measured using the standard block. It was eliminated
from the calculations for the approximation of the
longitudinal wave emission velocity, and then the
longitudinal wave velocity in specimens was calculated.
The measurements were accomplished at a frequency of
4 MHz at the room temperature.

3. THEORY of RELEVANCE VECTOR MACHINE

In RVM-based regression, to predict a function based on
a set of N input-target pairs \( \{(x_i, t_i)\}_{i=1}^N \), each target is
modeled as a function of the corresponding inputs with
additive white Gaussian noise to accommodate
measurement error on the target:

\[
t_i = y(x_i, w) + \epsilon_i
\]

\( \epsilon_i \) is assumed to be mean-zero Gaussian with variance \( \sigma^2 \)
and similar to the SVM, \( y(x, w) \), is considered as a linear
combination of \( N \) kernel functions centered at the
training samples inputs, in the form of Equation (3). Therefore, with the assumption that we know \( y(x_n) \), each
target is of normal independent distribution with the
mean \( y(x_n) \) and variance \( \sigma^2 \), expressed as [17]:

\[
p(t_n|x) = N(t_n|y(x_n), \sigma^2)
\]

Due to the assumption of independence of the targets, the
likelihood function of whole samples can be obtained by
the multiplication of the probability distributions as

\[
p(t|w, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{t^\top t}{2\sigma^2}\right)
\]

where

\[
t = (t_1 \ldots t_N)^T
\]

\[
w = (w_0 \ldots w_N)^T
\]

and \( \phi \) is an \( N \times (N+1) \) matrix, calculated as follows:
The iterative calculation of the parameters $\alpha_i$ and $\sigma^2$ from Equations (18) and (19) concurrent with updating of the following statistics $\Sigma$ and $\mu$ from Equations (12) and (14) is repeated until some suitable convergence criteria have been satisfied.

At the end of this procedure, the maximizing values $\alpha_{\text{MAP}}$ and $\sigma^2_{\text{MAP}}$ are obtained, and the predictions for the new samples are made based on the posterior distribution over the weights conditioned on them. For a new sample $x^*$, a Gaussian predictive distribution is assumed for the output, expressed as [19]:

$$p(t^*|t) = N(t^*|\mu^*, \sigma^2)$$

where

$$\mu^* = \mu^T \varphi(x^*)$$

$$\sigma^2 = \sigma^2_{\text{MAP}} + \varphi(x^*)^T \Sigma \varphi(x^*)$$

$$\varphi(x^*) = [K(x^*, x_1), ..., K(x^*, x_N)]^T$$

$$N = 1, ..., N$$

The mean value of the distribution $\mu^*$ is considered as the predicted output value and the variance, $\sigma^2$, provides an index of uncertainty in prediction.

In the iterative calculation of hyper-parameters $\alpha_i$, many of them tend to infinity. This means that the distribution $w_i$ is peaked at zero and they are estimated to be zero; thus, many of them tend to infinity. This means that the index of uncertainty in prediction.

In this formulation, $\Sigma$ is the variance and is calculated as

$$\Sigma = (\sigma^2 \varphi \varphi^T + A)^{-1}$$

wherein $A$ is a diagonal matrix formed as

$$A = \text{diag}(\alpha_0, \alpha_1, ..., \alpha_N)$$

The mean value $\mu$ can be obtained as

$$\mu = \sigma^{-2} \Sigma \varphi^T t$$

It can also be concluded based on this formulation that when $\alpha_i \rightarrow \infty$, $\mu_i \rightarrow 0$.

Integrating $p(w|\alpha, \sigma^2)$ over the weights $w$, the marginal likelihood [13] for the hyper-parameters is calculated as

$$p(t|\alpha, \sigma^2) = \int p(t|w, \sigma^2) p(w|\alpha) \, dw$$

The above integral is a convolution of Gaussians, which can be calculated as

$$p(t|\alpha, \sigma^2) = (2\pi)^{-\frac{N}{2}} |\Omega|^{-\frac{1}{2}} e^{-\frac{\sigma^2 t^T \Omega^{-1} t}{2}}$$

The matrix $\Omega$ in the marginal likelihood can be obtained as

$$\Omega = \sigma^2 I + \varphi A^{-1} \varphi^T$$

The optimal parameters $\alpha$ and $\sigma^2$ can be obtained by maximizing the marginal likelihood $p(t|\alpha, \sigma^2)$ over the training dataset. They are estimated in an iterative re-estimation procedure in the learning process of RVM. Following the approach of MacKay [18], the following iterative relationships were proposed for this purpose:

$$\alpha^\text{new} = \frac{\alpha - n \Sigma w}{\sigma^2}$$

$$\sigma^2 = \frac{t - \mu^* \mu^T}{N - \sum_{i=1}^{N} (1 - \alpha_i \Sigma w)}$$

4. RESULTS AND DISCUSSION

In this survey, non-destructive tests using ultrasonic waves were used to examine the tensile strength of rubber composites, and a database of fourteen values of tensile strength and the corresponding longitudinal ultrasonic wave speed was obtained, as shown in Table 3. The RVM model was trained by eleven values of the measurements listed in Table 3, and it was tested by three of them, marked in bold. The Sparse Bayes package for Matlab [20] was used for the implementation of the model. Using the Gaussian kernel function formulated as Equation (25), with the parameter of $\sigma = 45$, the relevance vector contains only two of the training samples. Therefore, based on the calculated weights, the functional relationship between the tensile strength and the longitudinal wave speed can be defined as

$$\hat{y} = 1.483 \times \exp\left(-\frac{(x-1489.5)^2}{2025}\right) - 8.796 \times \exp\left(-\frac{(x-1489.2)^2}{2025}\right) + 16.153$$

where $x$ is the longitudinal wave speed.
### TABLE 3. The resilience and longitudinal ultrasonic waves' velocity of the samples used

| No | Longitudinal ultrasonic waves' velocity (m/s) | Tensile Strength (MPa) |
|----|---------------------------------|------------------|
| 1  | 1717                            | 17.04            |
| 2  | 1537.5                          | 15.25            |
| 3  | 1749.5                          | 18.46            |
| 4  | 1494                            | 7.53             |
| 5  | 1469.667                        | 7.47             |
| 6  | 1536.75                         | 14.9045          |
| 7  | 1501.1667                       | 7.63             |
| 8  | 1573.75                         | 15.34            |
| 9  | 1559.833                        | 13.65            |
| 10 | 1500.8                          | 10               |
| 11 | 1483.2                          | 7.67             |
| 12 | 1599                            | 16.94            |
| 13 | 1467                            | 10.03            |
| 14 | 1467.125                        | 6.06             |

The accuracy of the proposed expression was evaluated based on root-mean-square error (RMSE) and the coefficient of determination ($R^2$) of statistical indices, defined as follows:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \bar{y}_i)^2} \quad (25)$$

$$R^2 = 1 - \frac{\sum_{i=1}^{N} (y_i - \bar{y}_i)^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2} \quad (26)$$

In these equations, $y_i$ and $\bar{y}_i$ are the measured and the predicted outputs, respectively. $N$ is the number of training samples, $y_{\text{max}}$ and $y_{\text{min}}$ are the maximum and minimum values of the measured outputs, and $\bar{y}$ is the mean value of the measured output, calculated as follows:

$$\bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i \quad (27)$$

The calculated values of the indices are listed in Table 4. As it can be observed, the RVM method provides the possibility of defining an explicit mathematical expression along with reasonable accuracy and a generalization capability. The measured outputs, together with the outputs predicted by the RVM method, are depicted in Figure 4, showing a good agreement between them.

### Table 4. Statistical indices for evaluation of the RVM model

| Database   | RMSE   | $R^2$  |
|------------|--------|--------|
| Training   | 1.2957 | 0.9023 |
| Testing    | 0.6259 | 0.9768 |

**Figure 4.** The measured outputs and the outputs predicted by RVM

### 5. CONCLUSION

In this paper, the application of ultrasonic testing for the measurement and modeling of the tensile strength of rubber has been proposed. For this purpose, the tensile strength of a set of rubber samples together with longitudinal ultrasonic wave velocity is measured. Based on these measurements, the relevance vector machine regression analysis is used to define an explicit mathematical expression to model the relationship between the tensile strength and the wave velocity, which is proven to provide reasonable accuracy and generalization capability. Based on these results, the developed model has the capability of being used for the online measurement of the tensile strength of rubber with the proposed formulation in the rubber industry.
6. REFERENCES

1. Stegmann, D., Raj, B., and Bhaduri, A. “NDT for Analysis of Microstructures and Mechanical Properties of Metallic Materials.” In Reference Module in Materials Science and Engineering. Elsevier, 2016. https://doi.org/10.1016/B978-0-12-803581-8.03429-9

2. Yilmaz, T., Erkildi, B., Karaman, K., and Kulekci, G. “Assessment of strength properties of cemented paste backfill by ultrasonic pulse velocity test.” *Ultrasonics*, Vol. 54, No. 5, (2014), 1386–1394. https://doi.org/10.1016/j.ultras.2014.02.012

3. Morrison, D. S., and Abeyratne, U. R. “Ultrasonic technique for non-destructive quality evaluation of oranges.” *Journal of Food Engineering*, Vol. 141, (2015), 107–112. https://doi.org/10.1016/j.jfoodeng.2014.05.018

4. Vasaneli, E., Colangiuli, D., Calia, A., Silio, M., and Aiello, M. A. “Ultrasonic pulse velocity for the evaluation of physical and mechanical properties of a highly porous building limestone.” *Ultrasonics*, Vol. 60, (2015), 33–40. https://doi.org/10.1016/j.ultras.2015.02.010

5. Hynes, N. R. J., Nagaraj, P., and Sujana, J. A. J. “Ultrasonic evaluation of friction stud welded AA 6063/AA1030 steel joints.” *Materials and Design*, Vol. 62, (2014), 118–123. https://doi.org/10.1016/j.matdes.2014.05.017

6. Agrawal, M., Prasad, A., Bellare, J. R., and Seshia, A. A. “Characterization of mechanical properties of materials using ultrasound broadband spectroscopy.” *Ultrasonics*, Vol. 64, (2016), 186–195. https://doi.org/10.1016/j.ultras.2015.09.001

7. Liu, Y., Song, Y., Li, X., Chen, C., and Zhou, K. “Evaluating the reinforcement content and elastic properties of Mg-based composites using dual-mode ultrasonic velocities.” *Ultrasonics*, Vol. 81, (2017), 167–173. https://doi.org/10.1016/j.ultras.2017.07.001

8. Li, X., Han, X., Arguelles, A. P., Song, Y., and Hu, H. “Evaluating grain size in polycrystals with rough surfaces by corrected ultrasonic attenuation.” *Ultrasonics*, Vol. 78, (2017), 23–29. https://doi.org/10.1016/j.ultras.2017.02.018

9. Foorginejad, A., Taheri, M., and Mollayi, N. “Measurement and Modelling of the Rubber Resilience based on Ultrasonic Non-destructive Testing in Tires.” *AUT Journal of Modeling and Simulation*, Vol. 50, No. 2, (2018), 165–170. http://doi.org/10.22061/jms.2018.14543.5109

10. Foorginejad, A., Taheri, M., Mollayi, N., and Shiva, M. “Tire Hardness Modeling Based on Longitudinal Ultrasonic Velocity Using the Gaussian Process Regression.” *Amirkabir Journal of Mechanical Engineering*, Vol. 51, No. 5, (2019), 317–318. https://doi.org/10.22061/mej.2018.13494.5649

11. Provost, F. “Glossary of Terms.” *Journal of Machine Learning*, Vol. 30, No. 2–3, (1998), 271–274. Retrieved from http://robotics.stanford.edu/~ronnyk/glossary.html

12. Bishop, C. M. Pattern Recognition and Machine Learning. Springer, 2006. Retrieved from https://cds.cern.ch/record/998831

13. Cortes, C., and Vapnik, V. “Support-vector networks.” *Machine Learning*, Vol. 20, No. 3, (1995), 273–297. https://doi.org/10.1007/bf00994018

14. Meyer, D., Leisch, F., and Hornik, K. “The support vector machine under test.” *Neurocomputing*, Vol. 55, No. 1–2, (2003), 169–186. https://doi.org/10.1016/S0925-2313(03)00434-4

15. Tipping, M. E. “The relevance vector machine.” In NIPS’99: Proceedings of the 12th International Conference on Neural Information Processing Systems, (1999), 652–658. Retrieved from https://dl.acm.org/doi/abs/10.5555/3009657.3009750

16. ASTM 624-00: Standard test method for tear strength of conventional vulcanized rubber and thermoplastic elastomers. (2007).

17. Tipping, M. E. “Sparse Bayesian Learning and the Relevance Vector Machine.” *Journal of Machine Learning Research*, Vol. 1, No. Jun, (2001), 211–244. Retrieved from https://www.jmlr.org/papers/v1/tipping01a.html

18. Mackay, D. J. C. “Introduction to Gaussian processes.” *NATO ASI Series F Computer and Systems Sciences*, Vol. 168, (1998), 133–166. Retrieved from http://www.cs.toronto.edu/~radford/

19. Tipping, M. E. “Bayesian inference: An introduction to principles and practice in machine learning.” *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, Vol. 3176, (2004), 41–62. https://doi.org/10.1007/978-3-540-26650-9_3

20. Tipping, M. E. “SPARSEBAYES V1.1: A Baseline Matlab Implementation of ‘Sparse Bayesian’ Model Estimation”, (pp. 1–5). Retrieved from https://usermanual.wiki/Pdf/sb1manual.1468011241/view