Grand Unification of Particle Physics and Cosmology: Common Origin of Inflation, Dark Energy, Dark Matter, Baryon Asymmetry and Neutrino Mass

Wei-Min Yang

Department of Modern Physics, University of Science and Technology of China
Hefei 230026, People’s Republic of China
E-mail: wmyang@ustc.edu.cn

Abstract: I propose a grand unified framework of particle physics and cosmology based on both a new extension of the standard particle model and the fundamental principle of the standard cosmology. The unified model can simultaneously account for the common origin of inflation, reheating, baryon asymmetry, dark matter, dark energy, and neutrino mass. The full evolution of the universe from the primordial inflation to the early reheating, to the baryogenesis, to the late hot expansion, to the current CDM condensation into the dark energy is completely and coherently characterized by the model. For each phase of these evolutions, I give its complete dynamical system of equations and solve them by some special techniques, in particular, I establish the inherent relationships between these processes and particle physics. The numerical results clearly show how these things are successfully implemented. This model only employs eight input parameters, but its output results not only perfectly reproduce the measured inflationary data and the current energy density budget, but also finely predict the inflaton mass $M_\Phi \approx 8.3 \times 10^{10}$ GeV, the CDM mass $M_S \approx 0.30$ KeV, the reheating temperature $T_{re} \approx 3.05 \times 10^{11}$ GeV, $\eta_B \approx 6.14 \times 10^{-10}$, $h \approx 0.675$, and so on. Finally, we expect the unified model to be tested by the future experiments.

Keywords: grand unification; inflation; dark energy; dark matter; baryon asymmetry; neutrino mass
I. Introduction

The standard model of particle physics (SM) and the ΛCDM model of cosmology together have successfully accounted for a great deal of the cosmic observations from the BBN era to the present day [1], but they can not address the origin of the hot big bang of the universe [2], namely what happened before the standard hot expansion, and also can not answer the origins of the current dark energy [3], cold dark matter (CDM) [4], and baryon asymmetry [5], in addition, the generation of the sub-eV neutrino mass is yet a puzzle [6]. At present the theoretical and experimental investigations have clearly indicated that the very early universe surely underwent the inflation phase and the followed reheating one [7]. These two processes not only provide the initial conditions of the hot expansion, but also are related to the universe matter genesis [8], nevertheless the relationships between them and particle physics are unknown, so their evolution dynamics have been unestablished as yet. To solve all of the above problems, we have to seek an underlying theory beyond the SM and ΛCDM, therefore this becomes the most challenging research for high energy physics and cosmology. This aspect is currently attracting more and more attentions of theoretical and experimental physicists [9].

In fact, there have been numerous theories about the explanations of the inflation, dark energy, dark matter, baryon asymmetry and neutrino mass, which include many unified particle models [10], some paradigms of the inflation and reheating [11], some special dark energy models [12], even some models based on the non-standard gravity [13], a variety of the CDM candidates [14], the modified Newtonian dynamics [15], all kinds of leptogenesis and baryogenesis mechanisms [16], and a number of neutrino mass models [17]. However, a wide variety of these proposals have a common shortcoming, namely they are only aiming at one or two specific aspects of the universe ingredients rather than considering connections among them in the universe evolution, in other words, the above-mentioned problems are dealt in isolation without regard to the whole evolution of the universe, this is obviously unnatural and inadvisable because the uniqueness of the universe origin destines there are certainly some connections among these ingredients. In fact, the vast majority of these models have now been ruled out by the recent data and analyses [18].

At the present day, by means of the analyses for the power spectra of the anisotropic and polarized temperature of the cosmic microwave background (CMB) [19], we have obtained the following inflationary data, the tensor-to-scalar ratio, the scalar spectral index, the running of the spectral index, and the scalar power spectra. On the other hand, from the global analyses of cosmology which includes CMB, BBN, structure formation, gravitational lenses, particle physic experiments, etc. [20], we have extracted the following universe data, the dark energy density, the CDM density, the ratio of the baryon number density to the photon one, and the neutrino mass sum. The present optimum values of these cosmological data are given as follows [11],

\[
\begin{align*}
    r_{0.05} & < 0.036, \\
    n_s & \approx 0.965, \\
    \frac{dn_s}{d\ln k} & \approx -0.004, \\
    \ln(10^{10}\Delta_R^2) & \approx 3.04, \\
    \Omega_{DE} & \approx 0.685, \\
    \Omega_{CDM} & \approx 0.265, \\
    \eta_B & \approx 6.14 \times 10^{-10}, \\
    \sum_i m_{\nu_i} & \sim 0.1 \text{eV}. \\
\end{align*}
\]

(1)

These key data undoubtedly contain the information of the universe origin and evolution, any one successful theory of particle physics and cosmology has to confront them unavoidably, therefore Eq. (1) severely constrains new model builds [21].

Based on the universe concordance and the nature unification, I attempt to build a grand unified model of particle physics and cosmology, it can naturally relate the above-mentioned
The unified theory is based on the following particle model. I assume that below the GUT breaking scale, the particle contents and symmetries in the universe are showed by Table 1, all kinds of the notations are self-explanatory. Beside the SM particles (but the quarks and gauge bosons are omitted in Table 1), I introduce several dark particles with a global $U_D(1)$ symmetry. The $U_D(1)$ charge is “0” for all the SM particles and “1” for all the dark particles, in other words, the particles of SM are all in the visible sector, while the particles beyond SM (BSM) are all in the dark sector. $\alpha, \beta = 1, 2, 3$ are the fermion family indices. $\nu_L$ and $\nu_R$ will be combined into the super-light Dirac neutrino after the model symmetries are broken. The left-right symmetric doublet $F$ is a super-heavy lepton whose mass is $\sim 10^{10}$ GeV, it can also be regarded as the fourth generation lepton. The doublet scalar $\Phi$ is the inflation field with the same super-heavy mass, its primordial evolution leads to the hot big bang of the universe. The singlet $\phi$ is a

| SM particles (visible sector) | BSM particles (dark sector) |
|-----------------------------|-----------------------------|
| $H = \begin{bmatrix} H^+ \\ H^0 \end{bmatrix}$ | $l_{\alpha} = \begin{bmatrix} \nu_{\alpha L}^0 \\ e_{\alpha L}^- \end{bmatrix}$ |
| $(2,1)$ | $e_{\beta R}^0$ |
| $(2,-1)$ | $(1,-2)$ |
| $0$ | $F_{L/R} = \begin{bmatrix} F_{L/R}^0 \\ F_{L/R}^- \end{bmatrix}$ |
| $(1,0)$ | $(2,-1)$ |
| $(2,1)$ | $\Phi = \begin{bmatrix} \Phi^+ \\ \Phi^0 \end{bmatrix}$ |
| $(1,0)$ | $\phi$ |

Table 1: The particle contents and symmetries of the unified model, the notations are self-explanatory, the second row is the quantum numbers of $SU_L(2) \otimes U_Y(1)$, the third row is the dark charges of $U_D(1)$. $\nu_L$ and $\nu_R$ will be combined into the super-light Dirac neutrino, the left-right symmetric $F$ is the fourth generation lepton with a super-heavy mass, the super-heavy $\Phi$ is the inflation field, the real component of the complex scalar $\phi$ develops a vacuum expectation value to break $U_D(1)$, while its imaginary part component $S^0$ is a light and stable pseudo-scalar boson, $S^0$ will become the cold dark matter and eventually condense into the dark energy. Note that the global $B - L$ number is incidentally conserved in the model.
complex scalar. The \( U_D(1) \) and \( SU_L(2) \otimes U_Y(1) \) breakings are respectively implemented by \( \phi \) and \( H \) developing non-vanishing vacuum expectation values, the former occurs at the scale of \( \sim 10^7 \) GeV, while the latter is at the electroweak scale. After breakings the imaginary part component of \( \phi \) which is denoted by \( S^0 \) is a light and stable pseudo-scalar boson, it gradually cools into the cold dark matter, in the very later stage the CDM becomes super-cool so that it eventually condenses into the current dark energy. In a word, the model has fully and perfectly accommodated all of the ingredients required by the universe evolution.

Based on the particle contents and symmetries in Table 1, the full invariant Lagrangian of the model are

\[
\mathcal{L} = \overline{\nu_R} i\gamma^\mu \partial_\mu \nu_R + \overline{F}(i\gamma^\mu D_\mu - M_F)F + (D^\mu \Phi)^\dagger D_\mu \Phi + \partial^\mu \phi^* \partial_\mu \phi \\
+ \overline{\nu_R} Y_{\alpha\beta} e_{R\beta} H + \overline{F_L} y_{\beta\lambda} e_{R\lambda} \Phi + \overline{\nu_R} Y_{\alpha\beta} \nu_{R\beta}(i\tau_2 \Phi^\dagger) + \overline{F_L} y_{\beta\lambda} \nu_{R\lambda}(i\tau_2 H^\dagger) + y_{\rho} \overline{\nu_R} F_R \phi^* + \text{h.c.} \\
\]

\[
- V_H - V_\phi - \mu_0 (\Phi^1 H \phi + \Phi^T H^* \phi^*) - 2 \lambda_1 H^1 H \phi^* \phi^* - 2 (\lambda_2 H^1 H + \lambda_3 \phi^* \phi) \Phi^1 \Phi, \\
V_H = - \mu_2 H^1 H + \lambda_H (H^1 H^2), \quad V_\phi = \mu_3^2 \Phi^1 \Phi + \lambda_\phi (\Phi^1 \Phi)^2 + \cdots, \quad V_\phi = - \mu_5^2 \phi^* \phi + \lambda_\phi (\phi^* \phi)^2 + \cdots,
\]

where the rest of the SM Lagrangian is omitted, \( D_\mu \) is the SM covariant derivative, \( \tau_2 \) is the second Pauli matrix. Note that the global \( B - L \) number is incidentally conserved in Eq. (2), so any Majorana-type mass or couplings are automatically prohibited. The F mass, which is taken as \( M_F \approx 10^{10} \) GeV, can directly arise from the GUT breaking. \( [Y^e, y^e, Y^\nu, y^\nu, y^l] \) are all Yukawa coupling parameters, the repeated family indices are summed by default. We can individually rotate the flavor spaces of \( l, e_R, \nu_R \) so as to make both real diagonal \( Y^e \) (namely the basis of the charged lepton mass eigenstate) and real \( y^\nu \), then the irremovable complex phases in \( y^e, Y^\nu, y^l \) will become \( CP \)-violating sources in the lepton sector. The self-interacting potentials \( V_\Phi \) and \( V_\phi \) have individually special potential forms, which are very different from the well-known \( V_H \), see the following inflationary potential in Eq. (29), here I only write the quadratic and quartic terms of their series expansions, which are closely related to the \( \Phi \) and \( \phi \) masses, all of the terms whose dimensions are \( \geq 6 \) are suppressed by the power of the squared Plank mass. The triple scalar couplings with the \( \mu_0 \) parameter is very important, it can not only lead to \( \Phi \) developing a tiny small vacuum expectation value, but also give rise to a light mass of \( S^0 \). Finally, I assume \( [\lambda_1, \lambda_2, \lambda_3] \ll 1 \), namely these couplings between two different scalars are all very weak. In short, Eq. (2) completely describes the interactions among all the particles from the primordial inflation to the present universe.

The model symmetries are spontaneously broken by the following vacuum structures of the three scalar fields,

\[
\phi \rightarrow \phi^0 + \phi_\phi + i S^0, \quad H \rightarrow \frac{0}{h^0 + v_H \sqrt{2}}, \quad \Phi \rightarrow \begin{bmatrix} \phi^+ \\ \Phi^0 + \frac{v_\phi}{\sqrt{2}} \end{bmatrix}, \quad v_\phi \approx 3 \times 10^7 \text{ GeV}, \quad v_H \approx 246 \text{ GeV}, \quad v_\phi \approx 0.1 \text{ eV},
\]

where these three vacuum expectation values are hierarchical and they imply the sequence of breakings. After breakings, \( \phi^0 \) and \( h^0 \) are two real scalars, while \( S^0 \) is a real pseudo-scalar, all of them are bosons with neutral charge. Because \( S^0 \) can not develop a non-vanishing vacuum expectation value, its negative \( CP \) parity is still protected from violation despite of these breakings. From the total potential minimum in Eq. (2), we can derive that these three vacuum
dark Higgs boson mass, note that the mixing between expectation values are actually determined by the following system of equations,

\[ -\mu_H^2 + \lambda_H v_H^2 + \lambda_1 v_\phi^2 = \frac{\mu_0 v_H v_\phi}{\sqrt{2} v_H}, \quad -\mu^2_\phi + \lambda_\phi v_\phi^2 + \lambda_1 v_H^2 = \frac{\mu_0 v_H v_\phi}{\sqrt{2} v_\phi}, \]

\[ \mu^2_\phi + \lambda_2 v_H^2 + \lambda_3 v_\phi^2 = \frac{\mu_0 v_H v_\phi}{\sqrt{2} v_\phi}, \]

where all kinds of the parameter values are chosen as \( \mu_0 \sim 10^2 \text{ GeV}, \ \mu_H \sim 10^4 \text{ GeV}, \ \mu_\phi \sim 10^7 \text{ GeV}, \ |\lambda_H, \lambda_\phi| \sim 0.1, \ |\lambda_1, \lambda_2, \lambda_3| \sim 10^{-5}, \) thus Eq. (4) guarantees the vacuum stability. The symmetry breakings directly give rise to the masses of the scalar particles as follows,

\[ M_{h^0}^2 = \frac{\mu_0 v_H v_\phi}{\sqrt{2} v_H} + 2\lambda_H v_H^2, \quad M_{\phi^0}^2 = \frac{\mu_0 v_H v_\phi}{\sqrt{2} v_\phi} + 2\lambda_\phi v_\phi^2, \quad M_S^2 = \frac{\mu_0 v_H v_\phi}{\sqrt{2} v_\phi}, \quad M_\Phi = \frac{\mu_0 v_H v_\phi}{\sqrt{2} v_\phi}, \]

\[ \implies M_{h^0} \approx 2\lambda_H v_H, \quad M_{\phi^0} \approx 2\lambda_\phi v_\phi, \quad M_S M_\Phi = \frac{\mu_0 v_H}{\sqrt{2}}, \quad \frac{M_{S\phi}}{M_\Phi} = \frac{v_\phi}{v_H}. \]  

\[ M_{h^0} \approx 125 \text{ GeV} \] is exactly the measured mass of the SM Higgs boson. \( M_{\phi^0} \approx 10^7 \text{ GeV} \) is the dark Higgs boson mass, note that the mixing between \( h^0 \) and \( \phi^0 \) is negligible due to \( \lambda_1 \ll 1 \).

The last two equalities in Eq. (5) indicate the seesaw relation between the inflaton mass and the CDM mass, so there are naturally \( M_{S\phi} M_\Phi = \frac{v_\phi^3}{v_H} \approx 2.5 \times 10^4 \text{ GeV}^2 \) and \( \frac{M_{S\phi}}{M_\Phi} = \frac{v_\phi}{v_H} \approx \frac{10^{-17}}{3} \), in fact, we will respectively determine \( M_\Phi \approx 8.3 \times 10^{10} \text{ GeV} \) in Section III and \( M_{S\phi} \approx 0.3 \text{ KeV} \) in Section V.

Below the energy scale of \( v_H \), neutrino masses are required by the experimental data. The second term in \( \nu_\alpha \) can obviously be ignored, by contrast, the second term in \( \nu_\mu \) is \( \sim 0.01 \text{ eV} \), while its first term is \( \sim 0.001 \text{ eV} \), but the second term has only one eigenvalue, while the first term has three eigenvalues, thereby this may lead to \( m_\alpha \approx 0.05 > m_\nu \approx 0.01 \approx m_\nu \approx 0.005 \text{ (eV as unit)}, \) thus we naturally explain that \( \Delta m_{32}^2 \approx 2.4 \times 10^{-3} \text{ eV}^2 \) is much larger than \( \Delta m_{31}^2 \approx 7.5 \times 10^{-5} \text{ eV}^2 \). Under the flavor basis of real diagonal \( Y^e \) and real \( y^\nu \), then \( U_l^e \) is the lepton mixing matrix \( U_{PMNS} \), the \( CP \)-violating phase in \( U_{PMNS} \) purely arises from the complex \( Y^\nu \) and \( y^\nu \); furthermore, we can

\[ \sum_i m_\nu_i = \text{Tr}(U_{PMNS}^L M_e U_{PMNS}^R), \]  

where \( M_\nu \) is diagonalized by the two unitary matrices \( U_L^e \) and \( U_R^e \) which respectively rotate \( \nu_\alpha L \) and \( \nu_\beta R \). Eq. (6) indicates that the most essential difference between \( M_e \) and \( M_\nu \) is an interchange of \( v_H \) and \( v_\phi \). Obviously, this neutrino mass mechanism is a Dirac-type seesaw, which is different from the usual Majorana-type seesaw [22]. All kinds of the Yukawa parameters are chosen as \( [Y^e, Y^\nu] \sim 10^{-2} \) and \( [y^e, y^\nu, y^\nu] \sim 10^{-5} \), in addition, there are \( M_F \approx 10^{10} \text{ GeV}, v_\phi \approx 3 \times 10^7 \text{ GeV}, v_H \approx 246 \text{ GeV}, v_\phi \approx 0.1 \text{ eV} \), thus we can correctly give the charged lepton and neutrino masses required by the experimental data. The second term in \( M_e \) can obviously be ignored, by contrast, the second term in \( M_\nu \) is \( \sim 0.01 \text{ eV} \), while its first term is \( \sim 0.001 \text{ eV} \), but the second term has only one eigenvalue, while the first term has three eigenvalues, thereby this may lead to \( m_\alpha \approx 0.05 > m_\nu \approx 0.01 \approx m_\nu \approx 0.005 \text{ (eV as unit)}, \) thus we naturally explain that \( \Delta m_{32}^2 \approx 2.4 \times 10^{-3} \text{ eV}^2 \) is much larger than \( \Delta m_{31}^2 \approx 7.5 \times 10^{-5} \text{ eV}^2 \). Under the flavor basis of real diagonal \( Y^e \) and real \( y^\nu \), then \( U_l^e \) is the lepton mixing matrix \( U_{PMNS} \), the \( CP \)-violating phase in \( U_{PMNS} \) purely arises from the complex \( Y^\nu \) and \( y^\nu \); furthermore, we can
Figure 1: The tree and one-loop diagrams of $\Phi \rightarrow l^c + \nu_R$. The $CP$ asymmetry of this decay can equally generates the asymmetric anti-lepton and the asymmetric $\nu_R$ although the net lepton number is conserved as zero, the latter is forever frozen out in the dark sector, whereas the former is partly converted into the baryon asymmetry through the SM sphaleron transition.

correctly fit the neutrino mixing angles by choosing a suitable texture of $M_\nu$, here we do not go into it in depth. Based on both the neutrino oscillation experiments and the astrophysics investigations [1], I will take the suitable value $\sum m_\nu \approx 0.06$ eV as an input parameter of the unified model, see the following Table 2.

The dark sector of this model has very important phenomena and implications for cosmology. The $\Phi$ field slowly rolling causes the primordial inflation, see Section III. The $\Phi$ decay and its subsequent decay directly bring about the universe reheating or the hot big bang, among others, the leptogenesis rightly arises from an out-of-equilibrium decay of $\Phi$ in the reheating process, see Section IV. In the light of Eq. (2), the $\Phi$ decay modes have $\Phi \rightarrow e^c + F$, $\Phi \rightarrow l^c + \nu_R$ and $\Phi \rightarrow H + \phi$, but the principal channel is undoubtedly the second one. These two decays of $\Phi \rightarrow l^c + \nu_R$ and $\Phi^* \rightarrow l + \nu_R^c$ have however $CP$ asymmetric widths because of the interference between the tree diagram amplitude and the one-loop diagram one, as shown in Fig. 1. The $\Phi$ decay width and its $CP$ asymmetry are calculated as follows,

$$A_{CP} = \frac{\Gamma(\Phi \rightarrow l^c + \nu_R) - \Gamma(\Phi^* \rightarrow l + \nu_R^c)}{\Gamma_\Phi} = \frac{\text{Im} \text{Tr}[Y_\nu y_3^* Y_\nu^\dagger]}{4\pi \text{Tr}[Y_\nu Y_\nu^\dagger]} (1 - \frac{M_F^2}{M_\Phi^2}),$$

$$\Gamma_\Phi = \frac{M_\Phi}{16\pi} \text{Tr}[Y_\nu Y_\nu^\dagger],$$

(7)

where there must be $M_F < M_\Phi$, though they are of the same order of magnitude, or else the imaginary part of the loop integral factor will be vanishing. In Eq. (7), the $CP$-violating sources purely come from the irremovable complex phases in $y_\nu$ and/or $Y_\nu$, the latter also consists in the neutrino mass matrix in Eq. (6), so it is surely related to the $CP$ violation in the neutrino experiments [22]. Since there are $[Y_\nu, Y_\nu^\dagger] \sim 10^{-2}$ and $y_3^* y_3^\dagger \sim 10^{-9}$ as before, then we naturally obtain $A_{CP} \sim 10^{-10}$, this value is vital for the matter-antimatter asymmetry. In addition, I will take $\text{Tr}[Y_\nu Y_\nu^\dagger] \approx 2 \times 10^{-4}$ as an input to figure out $\Gamma_\Phi$, which is a key quantity in the following reheating. Because the reheating is a non-equilibrium process, the $CP$ asymmetry in Eq. (7) can equally generate the asymmetric anti-lepton and the asymmetric $\nu_R$ although the net lepton number is conserved as zero, the latter will be forever frozen out in the dark sector, whereas the former will be partly converted into the baryon asymmetry through the SM sphaleron transition [24].

The pseudo-scalar boson $S^0$ with $M_{S^0} \approx 0.3$ KeV is the second lightest particle in the dark
Figure 2: The sketch of the universe origin and evolution described by the unified model. The universe energy is step by step released and reduced from the primordial dark energy to the present energy budget, the whole evolution process is just like a cascade of hydropower stations, there is not so-called “cosmological constant problem” in the model.

sector, its decay channel is only $S^0 \to \nu + \nu^c$ in view of the effective couplings in Eq. (6), but this decay is seriously suppressed. The $S^0$ stability can be shown by the following calculation,

$$\Gamma(S^0 \to \nu + \nu^c) = \frac{M_{S^0}^5}{8\pi} \text{Tr} \left[ \frac{M_L M_R^3}{v^2} \right] < H_0 = 2.13 h \times 10^{-42} \text{ GeV},$$

$$\implies \frac{\tau_{S^0}}{\tau_{\text{universe}}} \approx \frac{H_0}{\Gamma} \approx 41.6,$$

where $H_0$ is the present-day Hubble expansion rate with $h \approx 0.674$, and we take $\sum m_{\nu_i}^2 \approx 2.6 \times 10^{-21} \text{ GeV}^2$. The $S^0$ lifetime is obviously much longer than the universe age which is about $1.4 \times 10^{10}$ year, for this reason the light and stable $S^0$ therefore becomes the cold dark matter, and it is gradually condensing into the dark energy, see Section V.

On the basis of the above-mentioned particle model, we can describe the idea framework of the universe origin and evolution by the sketch shown as Fig. 2. In sequence, the universe went through the primordial inflation, the followed reheating, the hot expansion, the transformation from the radiation-dominated to the matter-dominated, and the current supercool CDM condensation into the dark energy. The primordial inflation is implemented by the $\Phi$ field slowly rolling. $\Phi$ has the two physical morphologies or energy forms of $\Phi_{DE}$ and $\Phi_{DM}$ due to its special nature. $\Phi_{DE}$ is an inert condensed state with a negative pressure, it has no kinetic energy and can not interact with the other fields, whereas $\Phi_{DM}$ is an excited massive particle state with vanishing pressure, it has kinetic energy and can take part in couplings to the other particles. Inappropriately, the relationship between $\Phi_{DE}$ and $\Phi_{DM}$ is analogous to ice and vapour, which are merely the two different morphologies of the same material. The same meanings also apply to the $S^0$ field (hereinafter we will omit its superscript “0”), namely its has also the two physical morphologies or energy forms of $S_{DE}$ and $S_{DM}$. In short, I give the above explanations about the unknown physical nature of the dark energy and the dark matter. In what follows, we will
see that the slow-roll inflation is a process of $\Phi_{DE}$ gradually converting into $\Phi_{DM}$, its physical essence is that the superheavy dark matter $\Phi_{DM}$ is slowly growing from the primordial dark energy $\Phi_{DE}$. After the inflation is completed, the decay of $\Phi_{DM}$ into a SM particle and a dark particle is responsible for the reheating and the leptogenesis. When the hot bath is formed, the radiation begins to dominate the universe, at the same time, the asymmetric lepton and the asymmetric right-handed neutrino have equally been generated. In the hot expansion stage, the asymmetric right-handed neutrino is forever frozen out in the dark sector, whereas the asymmetric lepton in the SM sector will be partly converted into the baryon asymmetry through the electroweak sphaleron transition. The two dark particles of $\nu_R$ and $S$ are completely decoupling below the $M_{\phi}$ energy scale. As the universe temperature declining, $S$ will gradually cool into the CDM denoted by $S_{DM}$, in the very later stage, the $S_{DM}$ temperature is more and more approaching to absolute zero, thus the supercool $S_{DM}$ can eventually condense into $S_{DE}$, which is namely the present dark energy, this process is actually that $S_{DE}$ is slowly growing from $S_{DM}$. Therefore, the current condensation is essentially a reverse process of the primordial inflation. Although there is a difference about 106 orders of magnitude between the primordial dark energy of $\Phi_{DE}$ and the present dark energy of $S_{DE}$, the universe energy is step by step released and reduced through the above-mentioned inflation and expansion evolution, this is just like a cascade of hydropower stations, so there is not naturally the so-called “cosmological constant problem” in the model. Finally, I emphasize that all of these assumptions of the unified model are moderate, reasonable and consistent, by which we can completely account for the universe origin and evolution.

III. Primordial Inflation

The dynamics of the primordial inflation is described as what follows. According to the standard paradigm [25], the inflation field $\Phi$ is considered as spatially uniform distribution, but there are very small fluctuations, which will become sources of the structure formation. Under the flat FLRW Metric, namely $g_{\mu\nu} = \text{Diag}(1, -a^2, -a^2, -a^2)$, where $a(t)$ is the scale factor of the universe expansion, the energy density and pressure of $\Phi$ are given by its energy-momentum tensor as follows,

$$
\mathcal{L}_\Phi = g^{\mu\nu} \partial_\mu \Phi^\dagger \partial_\nu \Phi - V_\Phi, \quad T^\mu_\nu(\Phi) = 2g^{\mu\beta} \partial_\beta \Phi^\dagger \partial_\nu \Phi - \delta^\mu_\nu \mathcal{L}_\Phi,
$$

$$
\Rightarrow T^0_0 = \rho_\Phi = |\dot{\Phi}|^2 + V_\Phi, \quad -\frac{1}{3} \delta^i_j T^0_i = P_\Phi = |\dot{\Phi}|^2 - V_\Phi, \quad (9)
$$

where $\dot{\Phi} = \frac{\partial \Phi}{\partial t}$, and $|\dot{\Phi}|^2 = \dot{\Phi}^\dagger \dot{\Phi} = \dot{\Phi}^+ \dot{\Phi}^- + \dot{\Phi}^0 \dot{\Phi}^0$ and $V_\Phi = V(\Phi^\dagger \Phi) = V(|\Phi|^2)$ are respectively the kinetic energy and potential energy of $\Phi$. Obviously, the potential energy and the kinetic energy together determine $\rho_\Phi$ and $P_\Phi$, and vice versa, however $\rho_\Phi$ is a super-high energy density in the inflation period.

I now introduce the dark energy $\Phi_{DE}$ and the dark matter $\Phi_{DM}$, they are merely two energy forms or components of the same $\Phi$ field, their densities and pressures are determined by the following relations,

$$
\rho_\Phi = \rho_{\Phi_{DE}} + \rho_{\Phi_{DM}}, \quad P_\Phi = P_{\Phi_{DE}} + P_{\Phi_{DM}} = w_\Phi \rho_\Phi, \quad P_{\Phi_{DE}} = -\rho_{\Phi_{DE}}, \quad P_{\Phi_{DM}} = 0,
$$

$$
\Rightarrow \rho_{\Phi_{DE}} = -w_\Phi \rho_\Phi = \frac{-2w_\Phi}{1 - w_\Phi} V_\Phi, \quad \rho_{\Phi_{DM}} = (1 + w_\Phi) \rho_\Phi = |\dot{\Phi}|^2 + \frac{1 + w_\Phi}{1 - w_\Phi} V_\Phi = 2|\dot{\Phi}|^2, \quad (10)
$$
where I employ Eq. (9). \( w_\Phi \) is a parameter-of-state which relates the total pressure to the total energy density, there are generally \(-1 \leq w_\Phi \leq 0\). \( \Phi \) is purely \( \Phi_{DE} \) when \( w_\Phi = -1 \), while \( \Phi \) entirely becomes \( \Phi_{DM} \) when \( w_\Phi = 0 \). Since \( \Phi_{DE} \) is an inert condensed state with negative pressure, it does not carry any of the kinetic energy but shares a part of \( V_\Phi \), whereas \( \Phi_{DM} \) is an excited massive particle state with vanishing pressure, so it carries the total kinetic energy and the rest of \( V_\Phi \), note that the \( \Phi_{DM} \) kinetic energy and its potential energy are always equal to each other. In short, Eq. (10) explicitly shows the inherent relationships among all kinds of the energy forms of the \( \Phi \) field, the physical implications of \( \Phi_{DE} \) and \( \Phi_{DM} \) will further be clear in the following context.

At the beginning of the inflation, the \( \Phi \) field is purely in the \( \Phi_{DE} \) form (or state), then \( \Phi_{DE} \) very slowly converts into \( \Phi_{DM} \), in other words, \( \Phi_{DM} \) is slowly growing from \( \Phi_{DE} \), this process is namely so-called slow-roll inflation. The dynamics of the inflationary evolution are collectively determined by the Friedmann equation, the \( \Phi \) continuity equation, and the \( \Phi_{DM} \) growth equation, they are respectively

\[
\dot{\rho}_\Phi = \rho_{\Phi_{DE}} + \rho_{\Phi_{DM}} - 3\tilde{M}_p^2 H^2, \\
\dot{\rho}_{\Phi_{DE}} + 3H \rho_{\Phi_{DE}}(1 + w_\Phi) = 0 \implies -\dot{\rho}_{\Phi_{DE}} = \dot{\rho}_{\Phi_{DM}} + 3H \rho_{\Phi_{DM}}, \\
\dot{\rho}_{\Phi_{DM}} = -2\eta(t) H \rho_{\Phi_{DM}},
\]

(11)

where \( \tilde{M}_p = \frac{1}{\sqrt{8\pi G}} \approx 2.43 \times 10^{18} \) GeV is the reduced Plank mass, \( H(t) = \frac{\dot{a}(t)}{a(t)} \) is the universe expansion rate, the growth parameter \(-\eta(t) > 0 \) controls the \( \Phi_{DM} \) growth rate, in fact \( \eta \) is exactly one of the slow-roll parameters defined below. Once the evolution of \( \eta(t) \) is specified, Eq. (11) is rightly a closed system of equations, from which we can solve the evolutions of \( \rho_{\Phi_{DM}} \), \( \rho_{\Phi_{DE}} \), \( \rho_\Phi \) and \( H \). The above continuity equation indicates that the \( \rho_{\Phi_{DM}} \) growth in the comoving volume is entirely from the \( \rho_{\Phi_{DE}} \) reduction, therefore the most primordial universe is rightly \( \Phi_{DM} \) originating from \( \Phi_{DE} \). The \( \Phi \) field will completely become the pure \( \Phi_{DM} \) form (or state) at the end of the inflation.

From Eqs. (10) and (11), we can easily derive

\[
\eta(t) = -\frac{d\ln \rho_{\Phi_{DM}}}{2Hdt} = -\frac{d\ln |\Phi|}{Hdt}, \quad \epsilon(t) = -\frac{d\ln \rho_\Phi}{2Hdt} = -\frac{H}{H^2} = \frac{3(1 + w_\Phi)}{2}, \\
0 = \epsilon(0) \leq \epsilon(t) \leq \epsilon(t_{inf}) = \frac{3}{2}, \quad 1 = w_\Phi(0) \leq w_\Phi(t) \leq w_\Phi(t_{inf}) = 0,
\]

(12)

(13)

where we take \( t = 0 \) as the time of inflation begin, and hereinafter use the “inf” subscript to indicate the time of inflation finish. \( \eta \) and \( \epsilon \) are two slow-roll parameters defined as usual, they respectively characterize the \( \rho_{\Phi_{DM}} \) varying rate and the \( \rho_\Phi \) one, of course, they themselves also vary with the inflationary time, or else the inflation will continue on without termination, Eq. (13) gives the inflationary boundary condition. In addition, we can obtain the expansion acceleration equation,

\[
\frac{\ddot{a}}{a} = (1 - \epsilon)H^2 = -\frac{1 + 3w_\Phi}{2} H^2.
\]

(14)

Eq. (14) shows that the accelerating or decelerating expansion only depends on the value of \( \epsilon \) or \( w_\Phi \), there are \( \ddot{a}(t) \geq 0 \) when \( 0 \leq \epsilon \leq 1 \) and \( \ddot{a}(t) < 0 \) when \( 1 < \epsilon \leq \frac{3}{2} \), the former is in the \( \Phi_{DE} \)-dominated universe, whereas the latter is in the \( \Phi_{DM} \)-dominated universe. Note that the
expansion rate and the total energy density are always decreased in the inflation process due to $\dot{H} \leq 0$.

Put Eq. (10) and Eq. (12) together, we can only use $\rho_\Phi$ and $\epsilon$ to express all kinds of the energy forms as follows,

$$\rho_{\Phi_{DE}} = (1 - \frac{2\epsilon}{3})\rho_\Phi, \quad \rho_{\Phi_{DM}} = \frac{2\epsilon}{3}\rho_\Phi, \quad |\dot{\Phi}|^2 = \frac{\epsilon}{3}\rho_\Phi, \quad V_\Phi = (1 - \frac{\epsilon}{3})\rho_\Phi. \quad (15)$$

Eqs. (11), (12) and (15) together make up the fundamental equations of the inflationary evolutions, and Eq. (13) is the boundary condition. In principle, if we can provide the solution of any one of the nine inflationary quantities, $H$, $\rho_\Phi$, $\rho_{\Phi_{DE}}$, $\rho_{\Phi_{DM}}$, $|\dot{\Phi}|^2$, $V_\Phi$, $\epsilon$, $\eta$, $w_\Phi$, then the evolutions of the other inflationary quantities will completely be determined by this system of equations. In what follows, we will find their solutions by a special technique.

One of the inflationary features is that the universe size or the scale factor expands about $10^{25}$ times in an extremely short duration, therefore we use the e-fold number to characterize the inflationary time span instead of the scale factor, it is defined as follows,

$$N(t) = \ln \frac{a(t_{inf})}{a(t)} = \int_{t}^{t_{inf}} H(t') dt' \Rightarrow \dot{N}(t) = -H(t), \quad (16)$$

$$0 = a(0) \leq a(t) \leq a(t_{inf}), \quad +\infty = N(0) \geq N(t) \geq N(t_{inf}) = 0,$$

where the starting point of the inflation is set as $a(0) = 0$ and $N(0) = +\infty$. Eq. (16) now acts as the role of $\frac{a}{H}$ since $N(t)$ replaces $a(t)$ as the time scale, it will frequently be used in the following formula derivations.

By use of Eqs. (11), (12), (15) and (16), we can order by order give the slow-roll parameters by the total energy density $\rho_\Phi$ and its derivative as follows,

$$\rho_\Phi' = \frac{d\rho_\Phi}{dN} = 3\rho_{\Phi_{DM}}, \quad \rho_\Phi'' = \frac{d^2\rho_\Phi}{dN^2} = 3\rho_{\Phi_{DM}}'', \ldots$$

$$\epsilon = \frac{dln\rho_\Phi}{2dN}, \quad \eta = \frac{dln\rho_\Phi}{2dN} = \frac{dln\rho_{\Phi_{DM}}}{2dN}, \quad \theta = \frac{dln(-\rho_\Phi)}{2dN} = \frac{dln(-\rho_{\Phi_{DM}})}{2dN}, \quad \delta = \frac{dln|\rho_{\Phi}^n|}{2dN}, \ldots \quad (17)$$

$$\Rightarrow \frac{dln\epsilon}{2dN} = \eta - \epsilon, \quad \frac{dln|\epsilon|}{2dN} = \theta - \epsilon, \quad \frac{dln|\theta|}{2dN} = \delta - \theta, \ldots \quad (18)$$

where hereinafter the “$'$” superscript denotes a derivative with regard to $N$. These slow-roll parameters in Eq. (17) are closely related to the observable quantities of the inflation, see the following Eq. (27), $\epsilon$ and $\eta$ are relevant to the tensor-to-scalar ratio and the scalar spectral index, $\theta$ is involved in the running of the spectral index, therefore, a key of solving the inflation problem is finding the correct solutions of these slow-roll parameters.

From the previous fundamental equations and Eq. (16), we can also derive the equation of motion of $\Phi$ and its formal solution, namely

$$\ddot{\Phi} + 3H\dot{\Phi} + \Phi \frac{dV_\Phi}{d|\Phi|^2} = 0 \Rightarrow \Phi'' + (3 - \epsilon)\Phi M_p^2 \frac{dln|V_\Phi|}{d|\Phi|^2} = 0, \quad (19)$$

$$\Rightarrow \Phi_1(N) = \Phi_2(N) = \Phi_3(N) = \Phi_4(N) \Rightarrow |\Phi'| = \sqrt{2} |\Phi'| = \frac{d|\Phi|}{dN},$$

$$d\varphi = \sqrt{2}d|\Phi'|d\Phi \Rightarrow \varphi' = \sqrt{2} |\Phi'| = M_p \sqrt{2}, \quad (20)$$

$$\Rightarrow \frac{\varphi(N)}{M_p} = \sqrt{2} \left[ \frac{|\Phi(N)| - |\Phi(0)|}{M_p} \right] = \int_{0}^{N} \sqrt{2} \varphi(N')dN', \quad (20)$$
where $\Phi' = \frac{d\Phi}{dN}$ and $\varphi' = \frac{d\varphi}{dN}$, and we freely fix $\varphi(0) = 0$. $\Phi_i$ ($i=1,2,3,4$) are four real degree of freedoms of $\Phi$, namely $|\Phi|^2 = \frac{1}{2} \sum_i \Phi_i^2$, they have the same solutions since $V(|\Phi|^2)$ is fully symmetric for them. Above I introduce an auxiliary field $\varphi$ in order to get rid of the multiple-component difficulty. Eq. (20) thus determines the $\varphi$ and $\Phi$ evolution once the $\epsilon(N)$ evolution is provided. $\varphi' > 0$ (namely $\dot{\varphi} < 0$) indicates that $\varphi$ and $\Phi$ are gradually reduced with the time.

By convention, if the inflationary potential $V_\Phi$ is provided, then the conventional slow-roll parameters are given by $V_\Phi$ as follows,

$$\epsilon_V = \frac{M_p^2}{2} \left[ \frac{dV_\Phi}{d\varphi} \right]^2 = \epsilon' \frac{3 - \eta}{3 - \epsilon} \epsilon, \eta \ll 1,$$

$$\eta_V = M_p^2 \left[ \frac{d^2V_\Phi}{V_\Phi d\varphi^2} \right] = \frac{\epsilon + \eta}(3 - \eta) - \frac{\eta'}{3} \epsilon, \eta, \theta, \delta \ll 1,$$

$$\xi_V^2 = M_p^4 \left[ \frac{dV_\Phi}{V_\Phi d\varphi} \right] \left[ \frac{d^3V_\Phi}{V_\Phi d\varphi^3} \right] = \left[ \frac{3 - \eta}{3 - \epsilon} \right]^2 \frac{\eta'(3 - 3\epsilon + 2\eta - 2\theta) - 2\eta\theta'}{3 - \eta}$$

$$\epsilon, \eta, \theta, \delta \ll 1 \quad 4\epsilon \eta + \eta' - \frac{2\eta\theta'}{3}, \quad (21)$$

where $\eta' = \frac{d\eta}{dN}$ and $\theta' = \frac{d\theta}{dN}$. Eq. (21) clearly shows the relations between these two sets of slow-roll parameters, which are derived from the foregoing equations. However, it should be stressed that the above approximations are held only when $\epsilon, \eta, \theta, \delta \ll 1$, this case is only in the early and middle stages of the inflation, when the inflation is close to its end, some slow-roll parameters actually become $\sim 1$, thus these approximations are invalid.

When the inflationary potential is characterized by $V_\Phi(\varphi)$ with $\varphi$ as the argument, we can make Taylor expansion of $V_\Phi(\varphi)$ around $\varphi = 0$ and obtain the following results,

$$V_\Phi(\varphi) = V_\Phi(0) + \frac{dV_\Phi}{d\varphi} \varphi + \frac{d^2V_\Phi}{d\varphi^2} \frac{\varphi^2}{2} + \cdots$$

$$M_\Phi^2 = \frac{d^2V_\Phi}{d\varphi^2} |_{\varphi = 0} = [(3 - \epsilon)\eta V H^2]_{N = 0} \Rightarrow M_\Phi = [\sqrt{(3 - \epsilon)\eta V H}]_{t_{inf}},\quad (22)$$

$$V_\Phi(t_{inf}) = [(1 - \epsilon)\rho_{\Phi}]_{t_{inf}} = V_{\Phi_{min}} = M_\Phi^2 |\Phi(t_{inf})|^2 \Rightarrow \frac{\varphi(t_{inf})}{M_\Phi} = \sqrt{\frac{1}{\eta V(t_{inf})}},\quad (23)$$

where I use Eqs. (21) and (15). $M_\Phi$ is nearly the same size as $H_{inf}$, but $M_\Phi$ is identified as the $\Phi$ mass meaning only when $\eta V$ becomes positive, in the following Fig. 4 we will see how $M_\Phi$ is gradually generated from nothing as $\eta V$ evolving from negative to positive, namely there is a special mechanism generating $M_\Phi$ in the inflation process. Later we will work out $\eta V(t_{inf}) \approx 2.6$, so there is $|\Phi(t_{inf})|/M_\Phi \approx 0.62$, this is a very reasonable value.

A traditional and normal technique of solving the inflation problem is as the following procedure. Firstly, one has to design or guess a function form of $V(|\Phi|^2)$ or $V(\varphi)$. Secondly, one puts $V_\Phi$ into Eq. (19) and ignores the $\epsilon$ term on account of $\epsilon \ll 1$ in the most of the inflation duration, then one can solve the $\Phi$ differential equation to obtain a function of $\Phi(N)$. Thirdly, one makes a derivative of $\Phi(N)$ to derive $\epsilon(N)$ from Eq. (20), thus one can further obtain $\eta(N)$ and $\theta(N)$ by Eq. (18). Lastly, one can calculate $\rho_\Phi, \rho_\Phi_{DE}, \rho_\Phi_{DM}$ and $|\Phi|^2$ by Eq. (15) since $V_\Phi(N)$ and $\epsilon(N)$ are known. Thus far, the inflationary evolutions are completely solved out. Nevertheless, this technique has two serious shortcomings. i) In the later stage of the inflation $\epsilon$ is actually $\sim 1$ rather than $\ll 1$, thereby one neglecting $\epsilon$ in Eq. (19) will lead to the non-rigorous and
incomplete inflationary solutions, in particular, this has great effect on the inflation termination and the subsequent reheating. ii) it is very difficult to fit precisely all of the inflationary data in Eq. (1) by this procedure, in fact, a desirable inflationary potential can not as yet be found although the countless endeavours have been made. Therefore, to solve reliably and completely the inflation problem in the standard gravity framework, we have to find a new approach.

In the system of equations of the inflationary evolution, at least in mathematical sense all of the unknown inflationary quantities have an equal status, therefore we can flexibly choose the η parameter as the starting point to solve the inflation problem instead of the potential $V_\Phi$. In principle, we can employ the following procedure. Firstly, we can design or guess an evolution function of η as below, this amounts to specifying directly the law of the $\Phi_{DM}$ growth in Eq. (11). Secondly, Eq. (11) has now been a closed system of equations, so we can directly solve them to obtain $\rho_{\Phi_{DM}}, \rho_{\Phi_{DE}}, \rho_\Phi$ and $H$. Lastly, we can finally obtain $\epsilon, w_\phi, |\Phi|^2$ and $V_\Phi$ by Eqs. (12) and (15). Thus, all of the inflationary evolutions are completely solved out. By means of this reversal procedure, the inflationary potential is reversely worked out rather than provided. Obviously, this technique is both simple and reliable, and it can overcome the traditional technique’ shortcomings. Whatever technical means is employed, the only criterion is that it is able to fit all of the inflationary data correctly and completely.

After careful analysis and calculation, I find a suitable function form of $\eta(N)$ as follows,

$$\eta(N) = \eta(0)e^{-\alpha(N/N_*)^2} = -\frac{e^{\alpha(1 - N^2/N_*^2)}}{N_* + 6}, \tag{24}$$

where there are two independent parameters $N_*$ and $\alpha$, and I use them to parameterize $\eta(0)$. In fact, $N_*$ is exactly corresponding to the inflationary e-fold number when the pivot scale of $k_*$ $0.05 \text{ Mpc}^{-1}$ exits from the horizon, its value will be calculated as $N_* \approx 51.2$ by the following Eq. (28). $\alpha$ is one of two input parameters in the inflation sector, the other one is $H_{\text{inf}}$ in Eq. (27), we can determine $\alpha \approx 2.93$ by fitting the inflationary data, all of the input parameters of the unified model are later summarized in Table 2 in VI section. From Eq. (24), we can directly derive $\eta'$ and $\theta$ (by use of Eq. (18)), obviously, there are $\eta(0) = \theta(0) = \frac{\alpha N}{N_*^2}$ and $\eta'(0) = 0$ at the end of the inflation.

Starting from Eq. (24), we can easily solve the inflationary evolutions as follows. Firstly, we use the $\eta$ formula in Eq. (17) to obtain $\rho_{\Phi_{DM}}(N)$ by integrating $\eta(N)$. Secondly, we can further integrate $\rho_{\Phi_{DM}}(N)$ to obtain $\rho_\Phi(N)$ by use of the $\rho_\Phi$ formula in Eq. (17). Thirdly, $\epsilon(N)$ is given by the second equality in Eq. (15). The derived results are such as

$$\frac{\rho_{\Phi_{DM}}(N)}{\rho_\Phi(0)} = e^2 \int_0^N \eta(N') dN', \quad \frac{\rho_\Phi(N)}{\rho_\Phi(0)} = 1 + 3 \int_0^N \frac{\rho_{\Phi_{DM}}(N')}{\rho_\Phi(0)} dN', \quad \epsilon(N) = \frac{3}{2} \frac{\rho_{\Phi_{DM}}(N)}{\rho_\Phi(N)}, \tag{25}$$

where there is $\rho_{\Phi_{DM}}(0) = \rho_\Phi(0)$ since $\epsilon(0) = \frac{3}{2}$ at the end of the inflation. Lastly, by use of the relevant relations we can also calculate $w_\phi, \eta_N, |\Phi|^2, V_\Phi$, etc. Note that all kinds of the energy forms are normalized to $\rho_\Phi(0)$, which is namely $\rho_\Phi(t_{\text{inf}}) = 3M_p^2 H_{\text{inf}}^2$.

Now we show the numerical results of this inflation model. Fig. 3 shows the inflationary evolutions of all kinds of the energy forms with $N$ as time scale. In the most phases of the inflation process, these three curves of $\rho_\Phi, \rho_{\Phi_{DE}}, V_\Phi$ almost coincide with each other, moreover,
they nearly keep a constant value, the reason for this is that the growths of $\Phi_{DM}$ or $T_\Phi = |\dot{\Phi}|^2$ are very slow at the early and middle stages, namely there is $\dot{\Phi} \approx 0$, this is so-called slow-roll inflation. In the last phase of the inflation proceeding, the $\rho_{\Phi_{DM}}$ and $T_\Phi$ growths have however been accumulated to a certain amount, thus $\rho_\Phi$, $\rho_{\Phi_{DE}}$ and $V_\Phi$ turn into fall significantly, and their curves are gradually separated each other. Eventually, $\rho_{\Phi_{DM}}$ exceeds $\rho_{\Phi_{DE}}$, the $\Phi_{DE}$-dominated universe is transformed into the $\Phi_{DM}$-dominated one, and the accelerating expansion is also changed into the decelerating one, so the inflation is naturally over. At the time of the inflation finish, namely at $N = 0$, there are $\rho_{\Phi_{DE}} = 0$ and $\rho_\Phi = \rho_{\Phi_{DM}} = 2T_\Phi = 2V_\Phi$. Note that the green curve indicates $\rho_\Phi(\infty)/\rho_\Phi(0) \approx 5.61$, this means that the $\rho_\Phi$ amount only changes about 5.61 times from the inflation beginning to its end. In brief, Fig. 3 clearly shows the full evolution of all kinds of the energy forms in the inflation process, includes their slow-roll features, transformations among these energy forms, and their terminations.

Fig. 4 shows the inflationary evolutions of the slow-roll parameters and the parameter-of-state with $N$ as time scale. One can see the three remarkable features. i) In the most of the inflation duration, $\epsilon \ll 1$ and $w_\Phi \approx -1$ are nearly unvarying, while the other curves can slowly vary. Only when $N(t) \to N(t_{inf}) = 0$, these three parameters of $\epsilon$, $\eta_V$ and $w_\Phi$ sharply rise, thereby this leads to the inflation termination. ii) In the early and middle phases of the inflation, $\eta$ is coinciding with $\eta_V$ due to $\eta \approx \eta_V$, whereas in the last phase of the inflation, $\eta$ is coinciding with $\theta$ due to $\eta \approx \theta$. iii) $\epsilon$ is always positive, while $\eta$ and $\theta$ are always negative, but $\eta_V$ can however change from negative to positive when $N \to 0$. According to Eq. (22), $M_\Phi^2$ is proportional to $\eta_V$, this means that $M_\Phi$ is gradually generated from nothing as $\eta_V$ evolving, certainly, this is closely related to $\Phi_{DM}$ growing, therefore $M_\Phi$ appearing purely arises from the inflationary dynamical evolution. This mass generation mechanism of the inflationary field is very different from that of the SM particles, which simply results from the vacuum spontaneous breaking. In short, these numerical results of Fig. 3 and Fig. 4 can excellently explain the physical implications of the primordial inflation.
Now we set about addressing the observable data of the inflation. In Fig. 4, at $N_\ast \approx 51.2$ the slow-roll parameters and the parameter-of-state are evaluated as follows,

$$
\begin{align*}
\epsilon_V^\ast &\approx \epsilon_\ast \approx 1.013 \times 10^{-8}, \\
\eta_\ast^\ast &\approx \eta_\ast \approx -0.0175, \\
\theta_\ast^\ast &\approx \eta_\ast^\ast - \frac{\alpha}{N_\ast} \approx -0.0747,
\end{align*}
$$

(26)

hereinafter the “$\ast$” subscript specially indicates the $N_\ast$ time.

From the cosmological perturbation theory of the structure formation, we know that the above slow-roll parameters are directly related to the following inflationary observable quantities [26],

$$
\begin{align*}
\Delta_R^2(k_\ast) &= \frac{H^2}{8\pi^2 M_p^2 \epsilon} \bigg|_{k_\ast} = \frac{1}{8\pi^2 \epsilon_\ast} \frac{\rho(0)}{\rho_\Phi} \left[ \frac{H_{\text{inf}}}{M_p} \right]^2, \\
n_s(k_\ast) - 1 &= \left. \frac{d \ln \Delta_R^2}{d \ln k} \right|_{k_\ast} = \left. \frac{d \ln \epsilon}{(\epsilon - 1)dN} \right|_{k_\ast} = -4\epsilon_\ast + 2\eta_\ast \approx 6 \left( \frac{\eta_\ast^\prime}{9} - \epsilon_V^\ast \right) + 2\eta_V^\ast, \\
\frac{dn_s}{d\ln k}|_{k_\ast} &= \left. \frac{d n_s}{(\epsilon - 1)dN} \right|_{k_\ast} = 4\epsilon_\ast^\prime - 2\eta_\ast^\prime = 8\epsilon_\ast(\eta_\ast - \epsilon_\ast) - 4\eta_\ast(\theta_\ast - \eta_\ast)
\end{align*}
$$

(27)

where I use Eq. (21). $k_\ast = 0.05 \text{ Mpc}^{-1}$ is the pivot scale exiting from horizon when $N_\ast \approx 51.2$, the relation between them will be given by Eq. (28). $H_{\text{inf}}$ is the expansion rate at the time of the inflation finish, which is the second input parameter in the inflation sector, I take $H_{\text{inf}} \approx 4.2 \times 10^{10}$ GeV to fit correctly the inflationary data. $\rho_\Phi(0) \approx 5.61$ has been calculated out by Eq. (25), also see Fig. 3. Note that the terms with $\frac{\eta_\ast^\prime}{9}$ in Eq. (27) are the same order of magnitude as $\epsilon_V^\ast$, so their contributions to results are actually negligible.

Either of the two sets

Figure 4: The inflationary evolutions of the slow-roll parameters and the parameter-of-state with the e-fold number as time scale, $N_\ast \approx 51.2$ is corresponding to the time of $k_\ast = 0.05 \text{ Mpc}^{-1}$ exiting from horizon.
of slow-roll parameters can be employed to calculate the inflationary data, put Eq. (26) into Eq. (27), we can perfectly reproduce all of the observed data in Eq. (1), the detailed results are summarized in Table 2 in VI Section.

$k_s$ is defined and calculated as follows,

$$
k_s = \frac{a_s H_s}{c} = \frac{H_0}{c} \frac{H_{inf}}{H_0} \frac{H_s}{a_{inf}} \frac{a_{inf} a_{reg} a_{ref}}{a_0} = \left[ \frac{H_0}{c h} \left( \frac{g_s(T_0)}{2} \right) \right]^{-\Omega_s(T_0) \hbar^2} \left[ 4 \frac{H_{inf}}{H_0} \right] \left[ \frac{T_{re}}{T_0} \right] \left[ \frac{\rho_{ph}(N_s)}{\rho_{ph}(0)} \right] \frac{1}{2} e^{-N_s}, \tag{28}
$$

a detailed derivation of Eq. (28) is seen in the Appendix. $c$ is the speed of light and $h$ is the scaling factor for Hubble expansion rate, both $\frac{H_0}{c h} = \frac{100}{3 \times 10^8}$ Mpc$^{-1}$ and $H_0/h \approx 2.13 \times 10^{-42}$ GeV are fundamental constants. Note that $k_s$ is independent of a detailed value of $h$. At the present day, the CMB temperature is $T_0 \approx 2.7255$ K $\approx 2.35 \times 10^{-4}$ eV [1], the effective degrees of freedom is $g_s(T_0) \approx 4.1$ which includes the $\nu_R$ contribution (see the following Eq. (38)), the photon energy density parameter $\Omega_s(T_0) \hbar^2 \approx 2.5 \times 10^{-5}$ will be calculated out by Eq. (50) in Section V. In Section IV we will work out $T_{re} \approx 3.05 \times 10^{11}$ GeV by Eq. (36), which is the reheating temperature at the time of the $\Phi_{DM}$-R equality. Once the above quantities are input into Eq. (28), we can immediately solve out $N_s \approx 51.2$ corresponding to $k_s \approx 0.05$ Mpc$^{-1}$. Therefore, it is worth emphasizing that these fundamental quantities of the inflation, reheating and present-day universe are related together by Eq. (28), this is rightly a characteristic of the unified model.

Finally, we can find out a function form of the inflationary potential $V(|\Phi|^2)$ through fitting its numerical solution. Since the $\epsilon(N)$ solution has been given in Fig. 4, substitute it into Eq. (20) and make a numerical integration, then we can obtain a numerical solution of $|\Phi(N)|/M_p$, on the other hand, the $V_\phi(N)$ evolution has been known in Fig. 3, put the two solutions together, thus we can translate $V_\phi(N)$ with $N$ as variable into $V(|\Phi|^2/M_p^2)$ with $|\Phi(M_p)|^2$ as variable, this is easily achieved by a computer, the calculated results are shown by the black dotted curve in Fig. 5. $|\Phi(t_{inf})|^2/M_p^2 \approx 0.384$ and $|\Phi(t_s)|^2/M_p^2 \approx 6.98$ are respectively corresponding to $N = 0$ and $N_s \approx 51.2$. Apparently, the $V(|\Phi|^2)$ evolution in Fig. 5 is more smooth and steady in comparison with the $V_\phi(N)$ evolution in Fig. 3, in particular, the $|\Phi|^2$ varying amount in the inflation duration is much smaller than the $N$ varying one. Note that when $N > N_s$, there is actually $\dot{\Phi} \approx 0$, namely $\Phi$ is approximately a constant field, therefore there are $|\Phi|^2/M_p^2 \rightarrow 7$ and $V_\phi/\rho_\phi(t_{inf}) \rightarrow 5.61$ for the initial phase of the inflation.

After making a great effort, I eventually find the $V(|\Phi|^2)$ function form such as,

$$
\frac{V(|\Phi|^2)}{\rho_\phi(t_{inf})} = \frac{1}{2} + 5.15 \left[ 1 - e^{-0.013(x-1)^2} \right]^{0.43}, \quad x = \frac{|\Phi|^2}{|\Phi(t_{inf})|^2} = \eta V(t_{inf}) |\Phi(t)|^2 \frac{M_p^2}{3 M_p^2 H_{inf}^2}, \tag{29}
$$

where $\rho_\phi(t_{inf}) = 3 M_p^2 H_{inf}^2$ and I use Eq. (23). The analytical solution of Eq. (29) is shown by the green solid curve in Fig. 5, it can very perfectly fit the black dotted numerical solution. Note that Eq. (29) is different from the Starobinsky-type inflationary potential [27]. If we start from Eq. (29), in principle, we can derive all of the foregoing results by the usual procedure which is employed to deal with the inflation problem, but this potential can not at all be guessed in advance. In short, Fig. 5 clearly shows the inflationary potential evolution, and Eq. (29) surely provides us deep insights into the inflationary potential. Up to now, All of the inflation problem have completely been solved by the unified model.
IV. Reheating and Baryogenesis

At the end of the inflation, the \( \Phi \) field has fully turned into \( \Phi_{DM} \), therefore the universe comes into the \( \Phi_{DM} \)-dominated and decelerating expansion era. Since \( \Phi_{DM} \) is an excited particle state with kinetic energy, it can take part in the interaction couplings in Eq. (2), but \( \Phi_{DM} \) is quite unstable due to its superheavy mass, thus it can shortly decay into one SM particle and one dark particle. The \( \Phi_{DM} \) decay has important cosmological implications, in fact, its decay not only directly produces the hot bath of the universe, namely the universe reheating, but also simultaneously generates the matter-antimatter asymmetry by the following leptogenesis mechanism. In what follows, we will discuss the reheating evolution and the baryon asymmetry genesis.

From now on, we take off the “\( DM \)” subscript for the \( \Phi_{DM} \) particle since \( \Phi_{DE} \) has been vanishing and is not involved in the reheating process. The \( \Phi \) decay has been discussed in Section II, its decay directly produces the earliest radiation of the universe, which are hot plasma consisting of the SM and dark particles. The total energy of the universe now includes the two components of \( \rho_\Phi \) and \( \rho_R \). The dynamical evolution of the reheating process is collectively controlled by Friedmann equation and the continuity equations, namely

\[
\dot{\rho}_\Phi + \frac{3}{2} \tilde{M}_p^2 H^2, \quad \dot{\rho}_R + 3H \rho_R = -\Gamma_\Phi \rho_\Phi, \quad \dot{\rho}_R + 4H \rho_R = \Gamma_\Phi \rho_\Phi, \tag{30}
\]

where \( \Gamma_\Phi \) is the \( \Phi \) decay rate, which has been given in the previous Eq. (7). Eq. (30) physical implications are very clear, it is a closed system of equations, the \( \Gamma_\Phi \) value completely controls the reheating evolution. I take the suitable Tr\[Y\nu Y\nu^\dagger\] \( \approx 2 \times 10^{-4} \) to figure out \( \Gamma_\Phi \) in Eq. (7), see Table 2.

In order to solve the system of equations of Eq. (30), we can define the dimensionless energy
densities and time variable as follows,

$$\tilde{\rho}_i(\tilde{t}) = \frac{\rho_i}{3 M_p^2 \Gamma_{\tilde{t}}}$$, \quad \tilde{t} = (t - t_{\text{inf}}) \Gamma_{\Phi} = \frac{t - t_{\text{inf}}}{\tau_{\Phi}} \quad (0 < \tilde{t} \leq \tilde{t}_{\text{ref}} = \frac{t_{\text{ref}} - t_{\text{inf}}}{\tau_{\Phi}}),$$

Equation (31)

$$\implies \tilde{\rho}_{\Phi}(0) = \left(\frac{H_{\text{inf}}}{\Gamma_{\Phi}}\right)^2, \quad \tilde{\rho}_R(0) = 0,$$

where $i = (\Phi, R)$ and $\tau_{\Phi}$ is the $\Phi$ lifetime, the “ref” subscript specially indicate the time of the reheating finish. The time of the inflation finish is namely the time of the reheating beginning, Eq. (32) is exactly the initial condition of the reheating evolution. Provided $\text{Tr}[Y^\nu Y^{\nu\dagger}] \approx 2 \times 10^{-4}$, the model then gives $\frac{H_{\text{inf}}}{\Gamma_{\Phi}} \approx 1.3 \times 10^5$, therefore the early stage of the reheating is really a severe out-of-equilibrium process. By use of Eq. (31), we can recast Eq. (30) as follows,

$$\tilde{\rho}_\Phi + \tilde{\rho}_R = \left(\frac{H_{\text{inf}}}{\Gamma_{\Phi}}\right)^2, \quad \frac{d\tilde{\rho}_\Phi}{d\tilde{t}} + [3(\frac{H}{\Gamma_{\Phi}}) + 1]\tilde{\rho}_\Phi = 0, \quad \frac{d\tilde{\rho}_R}{d\tilde{t}} + 4(\frac{H}{\Gamma_{\Phi}})\tilde{\rho}_R = 0.$$

Equation (33) shows that the evolutions of $\tilde{\rho}_\Phi$ and $\tilde{\rho}_R$ only depend on the initial values in Eq. (32), so their numerical solutions are easily obtained.

In fact, we are much more interested in the energy density parameters and the total parameter-of-state, which are defined by

$$\Omega_i(\tilde{t}) = \frac{\rho_i}{\rho_\Phi + \rho_R}, \quad w_T(\tilde{t}) = \frac{P_R}{\rho_\Phi + \rho_R} = \frac{\Omega_R}{3}.$$
evolution also agrees with this, which is gradually increasing from the initial \( w_T = 0 \) to the final \( w_T = 0.7 \). As a result, the initial \( \Phi \)-dominated universe is transformed into the final R-dominated one. Of course, the universe is always decelerating expansion in the reheating period, and also the expansion rate is continuously declining.

In Fig. 6, the time of the \( \Phi - R \) equality, \( t_{\text{req}} \approx 1.054 \), is a key time point in the reheating process, the universe energy is \( \Phi \)-dominated in \( 0 < t < t_{\text{req}} \) (namely \( t_{\text{inf}} < t \leq t_{\text{req}} \)), whereas the universe enters into the radiation-dominated era once \( t > t_{\text{req}} \), at \( t \approx 10 \) the reheating process is essentially over, the universe will start a evolution of the next phase, namely the hot expansion driven by the radiation. \( t_{\text{req}} - t_{\text{inf}} \approx \tau_\Phi \) means that the most part of the \( \Phi \) decay actually happen at \( t_{\text{req}} \), therefore the radiation temperature \( T_{\text{re}} \) at \( t_{\text{req}} \) is in fact the highest temperature in the reheating process. \( t_{\text{req}} \) and \( T_{\text{re}} \) are determined by the following relations,

\[
\Omega_\Phi(t_{\text{req}}) = \Omega_R(t_{\text{req}}) = \frac{1}{2} \Rightarrow t_{\text{req}} = \frac{t_{\text{req}} - t_{\text{inf}}}{\tau_\Phi} \approx 1.054,
\]

\[
T_{\text{re}} = \sqrt{M_p \Gamma_\Phi \frac{90 \bar{\rho}_R(t_{\text{req}})}{\pi^2 g_*(T_{\text{re}})}} = \sqrt{M_p H(t_{\text{req}}) \frac{45}{\pi^2 g_*(T_{\text{re}})}} = H(t_{\text{req}}) = \Gamma_\Phi \sqrt{2 \bar{\rho}_R(t_{\text{req}})},
\]

where \( g_*(T_{\text{re}}) = 114 \) is the effective number of relativistic degrees of freedom in the model, which includes all of the SM particles and the dark \( \nu_R, \phi^0, S \). The model calculation gives \( T_{\text{re}} \approx 3.05 \times 10^{11} \) GeV and \( \frac{H(t_{\text{req}})}{\Gamma_\Phi} \approx 0.58 \), this completely verifies that the \( \Phi \) decay is indeed out-of-equilibrium in \( t_{\text{inf}} < t < t_{\text{req}} \). The relevant results of the reheating are all listed in the following Table 2. Finally, it should be stressed that the reheating is closely related to the inflation and particle physics, this is another one characteristic of the unified model.

Now we discuss the baryogenesis through the foregoing leptogenesis mechanism. In the light of the discussions in Section II and this Section, the principal decay mode of \( \Phi \), namely \( \Phi \rightarrow l^c + \nu_R \), has the three remarkable features. i) This decay is severely out-of-equilibrium in the reheating process because of \( H_{\text{inf}} \gg \Gamma_\Phi \sim H(t_{\text{req}}) \) (also see Table 2). ii) Its decay rate has the \( CP \) asymmetry in Eq. (7), and also there is \( A_{CP} \sim 10^{-10} \). iii) Although the net lepton number is conserved as zero, the \( CP \) asymmetric decay can equally generate the anti-lepton asymmetry and the \( \nu_R \) one. After the reheating, the interactions between the dark particles and the SM particles are characterized by the effective couplings in Eq. (6), obviously, these Yukawa interactions have seriously been out-of-equilibrium, therefore these two generated asymmetries can not newly be erased by such annihilation as \( l^c + \nu_R \rightarrow \phi^0/S + H \). After \( \phi^0 \) decays, namely when the energy scale is below \( M_{\phi^0} \), the two stable dark particles of \( S \) and \( \nu_R \) are completely decoupling from the hot bath, as a result, the dark sector and the SM sector are forever separated, so the \( \nu_R \) asymmetry is forever frozen out in the dark section, whereas the anti-lepton asymmetry is partly converted into the baryon asymmetry through the SM sphaleron process [24]. In short, although this baryogenesis mechanism does not fully fulfil the Sakharov’s three conditions [28], it is indeed put into effect in the unified model.

After the reheating is completed, the hot bath has essentially been formed, the universe then enters into the hot expansion era. The hot evolution in the SM sector is exactly the well-known hot big bang paradigm, while the evolution in the dark sector will be discussed in Section V. Eventually, the above-mentioned lepton asymmetry is partly converted into the baryon asymmetry through the electroweak sphaleron transition with the \( B - L \) number conservation [29].
The relevant relations are in detail given as follows,

\[
\frac{\bar{n}_l - n_l}{s}|_{T_{re}} = \frac{\bar{n}_{\nu_R} - n\nu_R}{s}|_{T_{re}} = \frac{\bar{n}_\Phi A_{CP}}{s}|_{T_{re}} = \frac{\rho_\Phi A_{CP}}{M_\Phi s}|_{T_{re}} = \frac{\rho_R A_{CP}}{M_\Phi} = \frac{3T_{re}A_{CP}}{4M_\Phi},
\]

\[
\eta_B = \frac{s}{n_\gamma}|_{T_0} Y_B(T_0) = \frac{s}{n_\gamma}|_{T_0} Y_B(T_{ew}) = \frac{s}{n_\gamma}|_{T_0} c_s Y_{B-L}(T_{ew}) = \frac{s}{n_\gamma}|_{T_0} c_s Y_{B-L}(T_{re})
\]

\[
= c_s \frac{s}{n_\gamma}|_{T_0} \frac{\bar{n}_l - n_l}{s}|_{T_{re}},
\]

(37)

where \(s\) is the entropy density, \(\frac{s}{n_\gamma}|_{T_0} \approx 7.38\) (where \(g_*(T_0) \approx 4.1\) includes the \(\nu_R\) contribution, see Eq. (38)), \(Y_B(T) = \frac{[\bar{n}_\nu_B - n\nu_B]}{s}|_{T}\), and \(c_s = \frac{28}{79}\) is the SM sphaleron coefficient. Eq. (37) clearly shows that \(\eta_B\) is collectively determined by the inflaton mass, the reheating temperature, and the \(CP\) asymmetry arising from the particle interaction, this again manifests that the unified model closely relates the inflation, reheating and particle physics together. In Table 2, I take \(A_{CP} \approx 0.85 \times 10^{-10}\) as an input parameter from the particle physics, thus we can naturally figure out \(\eta_B \approx 6.14 \times 10^{-10}\).

V. Present Dark Matter and Dark Energy

The hot expansion evolution in the dark sector is different and independent from that in the SM sector. In any case, the universe temperature which is characterized by the photon temperature is continuously declining as the radiation energy is red-shift. When the temperature falls to \(T < M_{\phi^0}\), the very heavy \(\phi^0\) is depleted by its decay, while the quite light \(S\) and \(\nu_R\) are relativistically decoupling from the hot bath. Since both \(S\) and \(\nu_R\) are stable particles, they are actually two dark radiations in the early universe. The ratios of their effective temperatures to the photon temperature can be worked out by the entropy in the dark sector and that in the SM sector being conserved separately, namely

\[
\frac{a^3(T_{de})}{a^3(T)} = \left(\frac{T_S}{T_{de}}\right)^3 = \left(\frac{T_{\nu_R}}{T_{de}}\right)^3 = \frac{g_{^8s}^{SM}(T)}{g_{^8s}^{SM}(T_{de})}\left(\frac{T}{T_{de}}\right)^3,
\]

\[
\implies \left(\frac{T_S}{T}\right)^3 = \left(\frac{T_{\nu_R}}{T}\right)^3 = \frac{2 + \frac{2}{7} \times 6\left(\frac{T_{\nu_R}}{T}\right)}{106.75} \approx 0.0366,
\]

(38)

where there must be \(T < m_\nu \approx 0.5\) MeV (namely after the electron-positron annihilation), and \(\left(\frac{T_{\nu_R}}{T}\right)^3 \approx \frac{4}{11}\) is the well-known effective temperature of \(\nu_L\) in the SM sector. Eq. (38) implies that the \(S\) and \(\nu_R\) effective temperatures at the present day are only 0.0366\(^*\)\(T_0 \approx 0.9\) K. From Eq. (38), the effective number of neutrinos at the recombination is calculated as \(N_{eff} = 3\left[1 + \left(T_{\nu_R}/T_{\nu_L}\right)^3\right] \approx 3.14\), which is safely within the current limit from the CMB analysis.

The pseudo-scalar boson \(S\) is however a special dark particle, its nature and mass origin have been explained in Section II. We now fix \(M_S \approx 0.3\) KeV as an input value of the unified model so as to fit correctly the current dark matter and dark energy densities. When the \(S\) effective temperature cools to \(T_S = M_S\), then \(S\) is transformed from the dark radiation to the cold dark matter, at this time the corresponding photon temperature is

\[
T_c = \left(\frac{T_c}{T_S}\right)M_S \approx \frac{M_S}{0.0366*} \approx 0.904\ \text{KeV},
\]

(39)
obviously, \( T_c \) is below \( T_{BBN} \approx 0.1 \) MeV but above \( T_{Recom} \approx 0.3 \) eV. In fact, \( T_c \) is both the temperature at which the CDM is formed and the starting point at which the CDM begins to condense into the dark energy.

As the universe expansion and cooling from \( T_c \) to \( T_0 \), the \( S \) effective temperature is continuously reducing from \( M_S \approx 0.3 \) KeV \( \approx 3.5 \times 10^6 \) K to 0.9 K, it is more and more cool and approaching to absolute zero, accordingly the \( S \) kinetic energy is more and more depleted, thus more and more \( S_{DM} \) becomes supercool matter so that it is eventually condensed into the dark energy \( S_{DE} \), this process is exactly a cosmological effect of Bose–Einstein condensate under the extremely low temperature. To some extent, the \( S \) condensation is essentially a reverse process of the \( \Phi \) inflation discussed in III section, namely it is actually a process of the dark energy \( S_{DE} \) slowly growing from the cold dark matter \( S_{DM} \). Note that the \( S \) condensation is very different from the baryon and electron condensing into the usual material (namely the visible world), the former is a pure boson system, whereas the latter is a pure fermion system. In short, the special evolution of \( S \) will eventually lead to the dark matter and dark energy in the present universe.

From now on, we directly use the “\( DM \)” and “\( DE \)” subscripts to indicate \( S_{DM} \) and \( S_{DE} \). The relevant energy density and pressure of the \( S \) field are given as follows,

\[
\rho_S = \rho_{DM} + \rho_{DE}, \quad P_S = P_{DM} + P_{DE} = w_S \rho_S, \quad P_{DM} = 0, \quad P_{DE} = -\rho_{DE},
\]

\[
\implies \rho_{DM} = (1 + w_S)\rho_S, \quad \rho_{DE} = -w_S \rho_S,
\] (40)

where \( w_S \) is the parameter-of-state of \( S \). The physical implications of Eq. (40) is the same as the ones of \( \Phi \) in Eq. (10), see those explanations below Eq. (10). When the universe temperature is below \( T_c \), the universe energy components include the photon \( \rho_\gamma \), the neutrino \( \rho_\nu \) (which contains the energy densities of \( \nu_L \) and \( \nu_R \)), the baryon \( \rho_B \) and the dark \( \rho_S \), their dynamical evolutions are determined by the following system of equations,

\[
\dot{\rho}_\gamma + \dot{\rho}_\nu + \dot{\rho}_B + \dot{\rho}_S = 3M_p^2 H^2,
\] (41)

\[
\dot{\rho}_\gamma + 4H \rho_\gamma = 0, \quad \dot{\rho}_\nu + 3H \rho_\nu (1 + w_\nu) = 0, \quad \dot{\rho}_B + 3H \rho_B = 0, \quad \dot{\rho}_S + 3H \rho_S (1 + w_S) = 0,
\] (42)

\[
\dot{\rho}_{DE} = -\dot{\rho}_{DM} = \kappa(T) H \rho_{DM}.
\] (43)

Eq. (41) is Friedmann equation, which is in charge of the expansion rate, and it relates the SM sector and the dark sector together. Eq. (42) is the continuity equations of the four energy components. Since the neutrino has a sub-eV mass, it can eventually transform from the relativistic state to the non-relativistic one, the turning point will be given by the following Eq. (48), so there is \( w_\nu = \frac{1}{3} \) when the neutrino is relativistic and there is \( w_\nu = 0 \) when the neutrino becomes non-relativistic. The first equality of Eq. (43) is from the \( S \) continuity equation, which indicates the \( S_{DE} \) growth is entirely from the \( S_{DM} \) reduction in the comoving volume. The second equality of Eq. (43) is the growth equation of \( S_{DE} \), the parameter \( \kappa \) characterizes the growth rate (or one can also name it as the condensing rate), \( \kappa \) will rapidly increase as \( T \) is more and more low. Once the \( \kappa(T) \) function is provided, the above system of equations is closed, we can thus solve the evolutions of all kinds of the energy components.

Similarly to the inflation process, we can introduce the e-fold number of the condensation process with \( T_c \) as the starting point as follows,

\[
N(T) = \ln \frac{a(T)}{a(T_c)} = \ln \frac{T}{T_c} \implies \dot{N}(t) = H(t),
\] (44)

\[
a(T_c) \leq a(T) \leq a(T_0) = a_0, \quad 0 = N(T_c) \leq N(T) \leq N(T_0) = N_0,
\]
where I make use of the entropy conservation to obtain the $N$ evolution with the temperature. Note that here $\dot{N}(t)$ is positive, namely $N$ is increasing with the time, it is different from the negative $\dot{N}(t)$ defined in Eq. (16), one should not confuse them.

Now we use $N$ as the time variable and normalize all kinds of the energy densities to $\rho_{\nu}(N(T_c)) = \rho_{\gamma}(0)$, then we can easily derive the following initial relations,

$$
\frac{\rho_{\nu}(0)}{\rho_{\gamma}(0)} = \frac{21}{8} \left( \frac{T_{\nu L}}{T_c} \right)^4 + \left( \frac{T_{\nu R}}{T_c} \right)^4, \quad \frac{\rho_B(0)}{\rho_{\gamma}(0)} = \frac{M_B n_B(0) n_{\gamma}(0)}{n_{\gamma}(0) \rho_{\gamma}(0)} = \frac{36 M_B \eta_B}{\pi^2 T_c},
$$

$$
\frac{\rho_{S}(0)}{\rho_{\gamma}(0)} = \frac{M_S n_S(0)}{\rho_{\gamma}(0)} = \frac{18 T_S}{\pi^4 T_c}, \quad \rho_{DM}(0) = \rho_{S}(0), \quad \rho_{DE}(0) = 0, \quad w_{S}(0) = 0, \quad (45)
$$

where I employ $\frac{n_{\nu}(0)}{n_{\gamma}(0)} = \frac{n_{\nu}(N_0)}{n_{\gamma}(N_0)} = \eta_B$. These ratios of $\frac{T_{\nu L}}{T_c}, \frac{T_{\nu R}}{T_c}, \frac{T_{S}}{T_c}$ have been given by Eq. (38), while $\eta_B$ and $T_c$ have been determined by the model. In addition, Eq. (45) indicates $\frac{\rho_{B}(0)}{\rho_{S}(0)} = \frac{2M_B n_B(T_{\nu L})}{M_S n_S(T_{\nu L})} - 3 \approx 0.105$, which is approximately two times the present-day ratio $\frac{\rho_{B}(N_0)}{\rho_{S}(N_0)} \approx 0.052$.

In order to solve Eqs. (41)-(43), I assume that $\kappa(T)$ is given by the following $F$ function,

$$
\kappa(N(T)) = \frac{dF(N)}{dN} \Leftrightarrow \int^N_0 \kappa(N') dN' = F(N) = b e^{a(1 - \frac{N^2}{N^2})}, \quad (46)
$$

where $a \approx 31.0$ and $b \approx 0.574$ are two input parameters in the $S$ condensation process, which are determined by fitting the current energy budget of the dark matter and dark energy, see Table 2. The role of Eq. (46) is very similar to that of Eq. (24) in the $\Phi$ inflation, the former controls $S_{DE}$ growing from $S_{DM}$, whereas the latter is in charge of $\Phi_{DM}$ growing from $\Phi_{DE}$.

Make use of Eqs. (44)-(46), then the solutions of the relevant energy densities in Eqs. (41)-(43) are analytically given as follows,

$$
\frac{d\ln \rho_{\nu}}{dN} = -4 \Rightarrow \ln \frac{\rho_{\nu}(N)}{\rho_{\gamma}(0)} = -4 N, \quad (47)
$$

$$
\frac{d\ln \rho_{\nu}}{dN} = -3(1 + w_{\nu}) \Rightarrow \ln \frac{\rho_{\nu}(N)}{\rho_{\gamma}(0)} = -3 N + \ln \frac{\rho_{\nu}(0)}{\rho_{\gamma}(0)} (0 \leq N \leq N_\nu),
$$

$$
\ln \frac{\rho_{\nu}(N)}{\rho_{\gamma}(0)} = -3(N - N_\nu) - 4N + \ln \frac{\rho_{\nu}(0)}{\rho_{\gamma}(0)} (N > N_\nu),
$$

$$
\frac{d\ln \rho_B}{dN} = -3 \Rightarrow \ln \frac{\rho_B(N)}{\rho_{\gamma}(0)} = -3 N + \ln \frac{\rho_B(0)}{\rho_{\gamma}(0)},
$$

$$
\frac{d\ln \rho_{DM}}{dN} = -3 - \kappa \Rightarrow \ln \frac{\rho_{DM}(N)}{\rho_{\gamma}(0)} = -3 N - F(N) + \ln \frac{\rho_S(0)}{\rho_{\gamma}(0)},
$$

$$
\frac{d\rho_{DE}}{dN} = \kappa \rho_{DM} \Rightarrow \ln \frac{\rho_{DE}(N)}{\rho_{\gamma}(0)} = \ln \left[ \int^N_0 \kappa(N') e^{-3N' - F(N')} dN' \right] + \ln \frac{\rho_S(0)}{\rho_{\gamma}(0)},
$$

where $N_\nu$ is the time point when the neutrino is transformed from relativistic state to non-relativistic state, there is $w_{\nu} = \frac{1}{3}$ when $0 \leq N \leq N_\nu$ and $w_{\nu} = 0$ when $N > N_\nu$. Since the initial conditions have been given by Eq. (45), we immediately calculate the numerical results from Eq. (47) once input the model parameters.
Figure 7: The evolutions of the relevant energy densities from $T_c \approx 0.904$ KeV to the present day with $N(T)$ as time scale. $N(T_c) = 0$ is both the time of the CDM being formed and the starting point of the dark energy growing, several key time points are also shown. The SM sector carries out the normal hot evolution, while in the dark sector the current CDM is condensing into the dark energy, which is essentially a reverse process of the primordial slow-roll inflation when the evolutions of $\rho_{DM}$ and $\rho_{DE}$ are compared with that of $\rho_{\Phi_{DE}}$ and $\rho_{\Phi_{DM}}$ in Fig. 3. First of all, we can calculate out the following key time points in the universe evolution,

$$
\begin{align*}
N_{eq} &= \ln \frac{T_c}{1.31 \text{ eV}} \approx 6.54, \\
N_{\nu} &= \ln \frac{T_c}{0.1555 \Sigma m_{\nu}} \approx 11.48, \\
N_{DE} &= \ln \frac{T_c}{0.00027 \text{ eV}} \approx 15.02, \\
N_0 &= \ln \frac{T_c}{T_0} \approx 15.16.
\end{align*}
$$

(48)

$N_{eq}$ is the time of the matter-radiation equality, it occurs at $T \approx 1.31$ eV, namely the universe age is about $4.5 \times 10^4$ years. $N_{\nu}$ is the time of the neutrino turns into non-relativistic state, its corresponding temperature is $0.1555 \Sigma m_{\nu}$, where 0.1555 is fixed by the second equality in Eq. (50), and $\Sigma m_{\nu} \approx 0.06$ eV is an input parameter of the unified model, see Table 2. $N_{DE}$ is time of the DE-matter equality, it takes place at $T \approx 0.00027$ eV $\approx 3.13$ K, which is very close to the present-day $T_0 \approx 0.000235$ eV $\approx 2.7255$ K, namely the $N_{DE}$ time is about three billion years ago.

Fig. 7 numerically shows the evolutions of the relevant energy densities from the $T_c$ temperature to the present day. $\rho_{\gamma}$ (the yellow curve), $\rho_{\nu}$ (the blue dotted curve) and $\rho_B$ (the brown curve) carry out the normal hot evolutions in the SM sector, by contrast, $\rho_S$ (the green curve) implements a distinctive evolution in the dark sector. As the temperature is more and more cool, the supercool $\rho_{DM}$ (the pink curve) is more and more condensing into $\rho_{DE}$ (the black curve) once its kinetic energy is depleted, namely $\rho_{DE}$ is slowly growing from $\rho_{DM}$, these three curves clearly show this condensing evolution. At $N_{eq} \approx 6.54$, the total matter energy exceeds the radiation energy, the R-dominated universe is transformed into the M-dominated one. At $N_{\nu} \approx 11.48$, the neutrino turns into the non-relativistic state, so its evolution departs from the radiation, eventually the radiation is only left with the photon. At $N_{DE} \approx 15.02$ (which is not
shown in Fig. 7 due to too close), the dark energy eventually exceeds the total matter energy so that it begins to dominate the universe, thus the universe is newly transformed from the decelerating expansion to the accelerating one. At $N_0 \approx 15.16$, the CDM effective temperature is very close to absolute zero, so the supercool CDM is more and more condense into the dark energy. Once all of the CDM are completely condensed into the dark energy, then the dark energy will become a constant energy. When the evolution of $\rho_{DM}$ and $\rho_{DE}$ are compared with that of $\rho_{\Phi,DE}$ and $\rho_{\Phi,DM}$ in Fig. 3, one can see that the current $\rho_{DM}$ condensing into $\rho_{DE}$ is essentially a reverse process of the primordial slow-roll inflation, the reason for this is of course that the two fields of $S$ and $\Phi$ have same nature and similar dynamics.

Make use of the results of Eq. (47), we can further calculate the density parameters of each energy component, the total parameter-of-state and the value of $h$ as follows,

$$\Omega_i(N) = \frac{\rho_i(N)}{\sum_i \rho_i(N)}, \quad w_T(N) = \frac{\sum_i P_i(N)}{\sum_i \rho_i(N)} = \frac{\Omega_R(N)}{3} - \Omega_{DE}(N),$$

$$\sum_i \rho_i(N_0) = 3 \tilde{M}_p^2 H_0^2 \implies h \approx 0.675,$$

where $3 \tilde{M}_p^2 H_0^2$ is the present-day critical energy density with $H_0 \approx 2.13 \times 10^{-42}h$ GeV. Note that $h \approx 0.675$ is an output value of the unified model rather than an input parameter. The evolutions of the relevant density parameters are shown in Fig. 8. These curves more clearly show the standard hot evolution in the visible sector and the supercool DM condensing into the DE in the dark sector, this further confirms our previous discussions and results. When $N < N_{eq}$, the universe is R-dominated. When $N_{eq} < N < N_{DE}$, the universe is M-dominated. When $N > N_{DE}$, the dark energy begins to dominate the universe, moreover, it will eventually
\[ w_T = \Omega_R / 3 - \Omega_{\text{DE}} \]

\[ w_S = -\rho_{\text{DE}} / \rho_S \]

\[ N_{\text{eq}} = 6.54 \]

\[ N_0 = 15.16 \]

Figure 9: The evolutions of the relevant parameters-of-state from \( T_c \approx 0.904 \text{ KeV} \) to the present day with \( N(T_c) \) as time scale. \( N^*(T_c) = 0 \) is both the time of the CDM being formed and the starting point of the dark energy growing. When \( w_T < -1/3 \), the universe is newly transformed from the decelerating expansion to the accelerating one. The current CDM condensation into the dark energy is essentially a reverse process of the primordial slow-roll inflation when the evolution of \( w_S \) is compared with that of \( w_\Phi \) in Fig. 4. The future fate of the universe will newly become De Sitter universe which is the same state as the primordial universe, but their energy densities differ by about 106 orders of magnitude.

Finally, from Eq. (47) we can analytically derive the following ratio relations among the present-day density parameters,

\[ \Omega_\gamma(N_0) = \frac{\pi^2 T_0^4}{45 M_p^2 H_0^2}, \quad \frac{\Omega_\nu(N_0)}{\Omega_\gamma(N_0)} = \frac{27 \Sigma m_\nu}{\pi^4 T_0} \left[ (\frac{T_{\nu E}}{T_c})^3 + (\frac{T_{\nu H}}{T_c})^3 \right], \]

\[ \frac{\Omega_B(N_0)}{\Omega_\gamma(N_0)} = \frac{36 M_B \eta_B}{\pi^4 T_0}, \quad \frac{\Omega_{\text{DM}}(N_0)}{\Omega_B(N_0)} = \frac{M_S}{2 M_B \eta_B} \left( \frac{T_S}{T_c} \right)^3 e^{-F(N_0)}. \]  

(50)

Eq. (50) shows that the current energy budget in fact depends on these fundamental quantities \( h, T_0, \Sigma m_\nu, M_B, \eta_B, M_S \) and \( F(N_0) = b \), by use of the input values of the unified model in Table
2, Eq. (50) is able to predict the present-day energy density budget very well, the calculated results are all listed in Table 2.

VI. Summary for Numerical Results

Now we summarize all kinds of the important numerical results given by the unified model, and then compare them with the measured experimental data. The used physical constants only include

\[
\tilde{M}_p = 2.43 \times 10^{18} \text{ GeV}, \quad M_B = 0.9383 \text{ GeV},
\]
\[
T_0 = 2.7255 \text{ K} = 2.35 \times 10^{-4} \text{ eV}, \quad \frac{H_0}{h} = 2.13 \times 10^{-42} \text{ GeV}.
\]  

Table 2 in detail lists the eight input parameters in the unified model, by which the relevant cosmological quantities are calculated as the outputs.

The \( \Phi \) inflation has only the two input parameters of \( H_{\text{inf}} \) and \( \alpha \), but its output results excellently fit all of the inflationary data, in particular the model predicts that \( r_{0.05} \) is too small to be detected, and the inflaton mass \( M_{\Phi} \approx 8.3 \times 10^{10} \text{ GeV} \) purely arises from the inflationary dynamical evolution. The \( S \) condensation is controlled by the two input parameters of \( a \) and \( b \), their values are determined by fitting the densities of the current CDM and dark energy. The remaining four input parameters are provided by the particle physics, \( \text{Tr}[Y_Y^\dagger Y_Y] \) and \( A_{CP} \) are respectively responsible for the reheating outputs and the baryon asymmetry, \( \sum m_{\nu_i} \) is in charge of the relic density of the cosmic neutrino, \( M_{S} \) entirely fixes the value of \( T_c \) which is the starting temperature of the \( S \) condensation. Although there are no observable data of the universe reheating as yet, these output results of the reheating are very reasonable and believable. However, we again stress that the \( \Phi \) decay simultaneously brings about the universe reheating and the matter-antimatter asymmetry. In the last panel, the current energy density
budget is perfectly reproduced, at the same time, the model finely predicts \( h \approx 0.675 \) and \( \eta_B \approx 6.14 \times 10^{-10} \). In short, all of the numerical results are consistent, reasonable and without any fine-tuning, they are very well in agreement with the present measured data in Eq. \((1)\).

All of the energy scales in Table 2 are below the GUT scale, the primordial inflation really occurs after the GUT phase transition, so the magnetic monopole problem is naturally eliminated. In the unified model, there is not so-called the cosmological constant problem at all, although there is difference about 106 orders of magnitude between the primordial super-high dark energy which drives the inflation and the current super-low dark energy which drive the universe accelerating today, the model describes how the universe energy is step by step released and reduced in the evolution history of 13.8 billion years, the whole evolution process is just like a cascade of hydropower stations, see Fig. 2. All of these fully manifest that the model is highly of self-consistence, concordance and unification. Up to now, we have completely demonstrated that the unified model is very successful, it is indeed able to account for the universe origin and evolution elegantly and excellently. Finally, we expect that the future experiments test this unified model.

VII. Conclusions

I put forward to a grand unified framework of particle physic and cosmology based on both a new extension of the standard particle model and the fundamental principle of the standard cosmology. This new theory covers both the SM (visible sector) physics and the BSM (dark sector) one, it can completely account for the full process of the universe origin and evolution in the unified and coherent ways, which includes the primordial inflation, the early reheating, the late hot expansion, and the current CDM condensation into the dark energy, simultaneously, the model can elegantly explain the origins of the neutrino mass and the baryon asymmetry. In the unified model, I introduce the two special scalar fields of \( \Phi \) and \( S \), which have the same nature and similar dynamics. The primordial inflation is \( \Phi_{DM} \) slow growing from \( \Phi_{DE} \), whereas the current condensation is \( S_{DE} \) slow growing from \( S_{DM} \), the latter is essentially a reverse process of the former. After the inflation finishes the \( \Phi_{DM} \) decay not only causes the universe reheating, but also leads to the matter-antimatter asymmetry through the leptogenesis in Fig. 1. For each phase of the above evolutions, I give its complete dynamical system of equations and solve them by some special techniques, in particular, I establish the inherent relationships between these evolution processes and particle physics. All kinds of the numerical results have clearly been shown by the figures and Table 2.

From those numerical solutions of the model, we have clearly seen how the slow-roll inflation is implemented, how the inflaton mass is generated from the inflationary evolution, what the inflationary potential shape is really, the details of reheating process, the mechanism of matter genesis, the CDM formation and condensation, the current dark energy genesis. From the primordial super-high dark energy driving inflation to the current supercool CDM condensing into the super-low dark energy, the universe energy is step by step released and reduced in the evolution history of 13.8 billion years, this is just like a cascade of hydropower stations, so there is no the cosmological constant problem.

This unified model only employs eight input parameters, but it can excellently fit all of the cosmological observations. It not only perfectly reproduces the measured inflationary data and the current energy density budget, but also finely predicts some important cosmological quantities, for instance, the tensor-to-scalar ratio \( r_{0.05} \approx 1.62 \times 10^{-7} \), the inflaton mass \( M_\Phi \approx \)
8.3 \times 10^{10} \text{ GeV}, the CDM mass $M_S \approx 0.30 \text{ KeV}$, the reheating temperature $T_{re} \approx 3.05 \times 10^{11} \text{ GeV}$, the baryon asymmetry $\eta_B \approx 6.14 \times 10^{-10}$, the scaling factor of expansion rate $h \approx 0.675$, etc. In short, the model truly achieves the unification of particle physics and cosmology, it is very successful and believable, therefore we expect that it is tested in the near future.

Acknowledgements

I would like to thank my wife for her great helps. This research is supported by the Fundamental Research Funds for the Central Universities of China under Grant No. WY2030040065.

Appendix

The derivation of Eq. (28) is as follows,

$$k_s = \frac{a_s H_s}{c} = \frac{H_0}{c H_0} H_{inf} = \frac{\rho_{\Phi}(N_*)}{\rho_{\Phi}(0)} \left[ e^{-N_*} \frac{\rho_{\Phi}(t_{req})}{\rho_{\Phi}(t_{inf})} \right] \left[ \frac{\rho_R(t_{req})}{\rho_R(t_{inf})} \right] \frac{s(T_0)}{s(T_{ref})} \frac{\gamma_c(T_0) T_0}{g^*_s(T_{ref}) T_{ref}} \frac{g^*_s(T_0) T_0}{g^*_s(T_{ref}) T_{ref}} \frac{1}{\gamma_c(T_0) T_0} \frac{1}{g^*_s(T_{ref}) T_{ref}} \frac{1}{g^*_s(T_0) T_0}$$

where I use $a \propto \rho^{-\frac{1}{4}}$ for the $\Phi$-dominated phase in $t_{inf} < t < t_{req}$ and $a \propto \rho^{-\frac{1}{4}}$ for the $R$-dominated phase in $t_{req} < t < t_{ref}$, in addition, I employ $\rho_{\Phi}(t_{req}) = \rho_{\Phi}(t_{inf})$ and $g^*_s(T_{req}) = g^*_s(T_{ref})$.

References

[1] Refer to the relevant reviews in PDG, R.L. Workman et al., Prog. Theor. Exp. Phys. 2022, 083C01 (2022).

[2] E. W. Kolb and M. S. Turner, The Early universe, Front. Phys. 69, 1 (1990);
D. S. Gorbunov and V. A. Rubakov, Introduction to The Theory of The Early Universe: Hot Big Bang Theory (World Scientific Publishing Co. Pte. Ltd, 2018).

[3] M. Bartelmann, Rev. Mod. Phys. 82, 331 (2010).

[4] A. Arbey and F. Mahmoudi, Progress in Particle and Nuclear Physics 119, 103865 (2021);
J. S. Bullock and M. Boylan-Kolchin, Annu. Rev. Astron. Astrophys. 55:343–387 (2017).

[5] Michael Dine, Rev. Mod. Phys. 76, 1 (2004).
[6] G. Senjanovic, Int. J. Mod. Phys. A 36, 2130003 (2021).

[7] A. H. Guth, Phys. Rev. D 23, 347 (1981);
A. D. Linde, Phys. Lett. B108, 389 (1982);
J. Ellis and D. Wands, the review of inflation in PDG, R.L. Workman et al., Prog. Theor. Exp. Phys. 2022, 083C01 (2022);
D. S. Gorbunov and V. A. Rubakov, Introduction to The Theory of The Early Universe: Cosmological Perturbations and Inflationary Theory (World Scientific Publishing Co. Pte. Ltd, 2011).

[8] P. Di Bari, Progress in Particle and Nuclear Physics 122, 103913 (2022);
James M. Cline, arXiv:1807.08749.

[9] Pran Nath, Int. J. Mod. Phys. A 33, 1830017 (2018);
D. H. Lyth and A. Riotto, Phys. Reps. 314, 1-146 (1999);
A. Mazumdar and G. White, Rep. Prog. Phys. 82, 076901 (2019).

[10] R. N. Mohapatra and N. Okada, Phys. Rev. D 105, 035024 (2022);
W. M. Yang, J. High Energy Phys. 01, 148 (2020);
W. M. Yang, Nucl. Phys. B944, 114643 (2019);
W. M. Yang, J. High Energy Phys. 03, 144 (2018).

[11] Konstantinos Dimopoulos, Introduction to Cosmic Inflation and Dark Energy (CRC Press, 2021);
Kaloian Lozanov, Reheating After Inflation (SpringerBriefs in Physics, 2020).

[12] B. A. Bassett, S. Tsujikawa, D. Wands, Rev. Mod. Phys. 78, 537 (2006);
R. Allahverdi, R. Brandenberger, Francis-Yan Cyr-Racine, A. Mazumdar, Annu. Rev. Nucl. Part. Sci. 60:27–51 (2010).

[13] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. 82, 451 (2010).

[14] L. Roszkowski, E. M. Sessolo, S. Trojanowski, Rep. Prog. Phys. 81, 066201 (2018);
P. H. Frampton, Int. J. Mod. Phys. A 33, 1830030 (2018);
H. Baer, Ki-Young Choi, J. E. Kimc, L. Roszkowski, Phys. Reps. 555, 1–60 (2015).

[15] Eugene Oks, New Astron. Rev. 93, 101632 (2021).

[16] D. Bodeker and W. Buchmuller, Rev. Mod. Phys. 93, 035004 (2021);
Neil D. Barrie, Chengcheng Han, H. Murayama, Phys. Rev. Lett 128, 141801 (2022).

[17] Andre de Gouvea, Annu. Rev. Nucl. Part. Sci. 66:197–217 (2016).

[18] D. Chowdhury, J. Martin, C. Ringeval, V. Vennin, Phys. Rev. D 100, 083537 (2019);
J. Martin, C. Ringeval, R. Trotta, V. Vennin, J. Cosmology and Astroparticle Phys. 1403, 039 (2014).
[19] D. Scott and G. F. Smoot, the review of Cosmic Microwave Background in PDG, R.L. Workman et al., Prog. Theor. Exp. Phys. 2022, 083C01 (2022).

[20] Y. Akrami et al., [Planck Collaboration], Planck 2018 results. X. Constraints on inflation, arXiv:1807.06211.

[21] A. Joyce, B. Jain, J. Khoury, M. Trodden, Phys. Reps. 568, 1–98 (2015).

[22] M. Gell-Mann, P. Ramond, R. Slansky, in Supergravity, eds. P. Van Niewenhuizen and D. Z. Freeman (North-Holland, Amsterdam, 1979);
T. Yanagida, in Proc. of the Workshop on Unified Theory and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto (Tsukuba, Japan, 1979);
R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[23] G. C. Branco, R. G. Felipe, F. R. Joaquim, Rev. Mod. Phys. 84, 515 (2012).

[24] V. A. Kuzmin, V. A. Rubakov, M. A. Shaposhnikov, Phys. Lett. B 155, 36 (1985).

[25] S. Dodelson and F. Schmidt, Modern Cosmology (Academic Press, London, 2020);
O. Piattella, Lecture Notes in Cosmology (Springer, Berlin, 2018).

[26] V. Mukhanov, Principles of Physical Cosmology (Cambridge University Press, 2005);
K. A. Malik and D. Wands, Phys. Reps. 475, 1-51 (2009).

[27] A. A. Starobinsky, Phys. Lett. B 91, 99 (1980).

[28] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5, 32 (1967), JETP Lett. 5, 24 (1967), Sov. Phys. Usp. 34, 392 (1991), Usp. Fiz. Nauk 161, 61 (1991).

[29] W. Buchmuller, R. D. Peccei and T. Yanagida, Annu. Rev. Nucl. Part. Sci. 55:311–355 (2005).