Nuclear incompressibility parameters evaluated from isoscalar giant monopole resonance of $N = Z$, $A = 100, 132$ nuclide and Sn isotopes

Shuichiro Ebata

1Faculty of Science, Hokkaido University, Sapporo, 060-0810, Japan

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Abstract

The isoscaler giant monopole resonances (ISGMR) are computed using the canonical-basis time-dependent Hartree-Fock-Bogoliubov theory (Cb-TDHFB) with five kinds of Skyrme parameter sets (SGII, SkM*, SLy4, SkT3 and SkI3). To extract the nuclear matter property from finite system, ISGMRs of $N=Z$ ($Z=20 - 50$), isobar even-even nuclide for $A=100, 132$ and Sn isotopes are analysed systematically. The magnitude relation of nuclear incompressibility-parameter ($K_{\infty}$) among Skyrme parameter sets, can be corresponded to the peak positions of GMR in spherical isotopes over $A=80$. The parameters ($K_{\text{surf}}, K_{\tau}$ and $K_{\text{Coul}}$) which appear in expansion of the finite nucleus incompressibility $K_A$, are determined for each Skyrme parameter. From the comparison experimental data whole mass region and the present results, they indicate that the isospin dependent term $K_{\tau}$ is filtered as $-305 \pm 10$ MeV. The incompressibility parameters of infinite system corresponding to our results are $K_{\tau}^{\infty}=-340 \pm 35$, $K_{\infty}=225 \pm 11$, and $K_{\text{sym}}=-138 \pm 18$ MeV.
I. INTRODUCTION

To extract an equation of state (EOS) for nuclear matter from finite nuclear system, is one of most important task given to nuclear physics. The EOS is a very important topic to connect the nuclear physics to the astrophysical objects such as nucleosynthesis, neutron star, and so on. The EOS is often expressed in the expansion around symmetric matter, as follows.

\[
\frac{E}{A}[\rho, \delta] = \mathcal{E}[\rho, \delta = 0] + \mathcal{S}[\rho, \delta] + \mathcal{O}[\delta^4],
\]

\[
\mathcal{E}[\rho] \equiv \mathcal{E}_0 + \frac{K_\infty}{2} \rho^2 + \frac{Q_0}{6} \rho^3 + \cdots,
\]

\[
\mathcal{S}[\rho, \delta] \equiv \left( J + L \rho + \frac{K_{\text{sym}}}{2} \rho^2 + \cdots \right) \delta^2,
\]  

(1)

where \( \delta = (\rho_n - \rho_p)/\rho \) is an asymmetric parameter which separates a symmetric matter (SM) EOS \( \mathcal{E} \) and symmetry energy \( \mathcal{S} \). They are expressed in \( \rho = (\rho - \rho_0)/3\rho_0 \), which is an expansion around the nuclear saturation density \( \rho_0 \) at which the pressure of nuclear matter is zero.

When the EOS is expanded as Eq. (1), there appear characteristic parameters: the binding energy per nucleon \( \mathcal{E}_0 \), the incompressibility of SM \( K_\infty \), the skewness parameter of SM \( Q_0 \), the symmetry energy \( J \), the slope parameter \( L \) and the symmetry incompressibility \( K_{\text{sym}} \), and so on. The \( \mathcal{E}_0 \) and \( J \) are respectively equal to the \( \mathcal{E} \) and \( \mathcal{S} \) at \( \rho_0 \). The other parameters can be obtained from the density derivation of \( \mathcal{E} \) and \( \mathcal{S} \) as follows.

\[
K_\infty = 9\rho_0^2 \frac{\partial^2 \mathcal{E}}{\partial \rho^2} \bigg|_{\rho = \rho_0},
\]

(2)

\[
Q_0 = 27\rho_0^3 \frac{\partial^3 \mathcal{E}}{\partial \rho^3} \bigg|_{\rho = \rho_0},
\]

(3)

\[
L = 3\rho_0 \frac{\partial \mathcal{S}}{\partial \rho} \bigg|_{\rho = \rho_0},
\]

(4)

\[
K_{\text{sym}} = 9\rho_0^2 \frac{\partial^2 \mathcal{S}}{\partial \rho^2} \bigg|_{\rho = \rho_0}.
\]

(5)

Thus the EOS parameters can be easily obtained at when the energy density functional (EDF) \( E/A[\rho] \) is chosen. Nucleus is a too much small system to extrapolate directly to the infinite nuclear matter \([1]\). Therefore, we evaluate the EOS with the help of effective interaction (Skyrme, Gogny, relativistic mean field) \([2–9]\). However, it is not easy to determine...
acceptable values in nuclear structure and in astrophysics, at same time. Currently also still, a study of the EOS using effective interactions has been progressed theoretically and also experimentally [10–13]. The uncertainties of \( E_0 \) and \( J \) are expected to be small: \( E_0 \approx 16 \text{ MeV} \) which has appeared in the Bethe-Weizäcker mass formula, and \( J=32\pm 3 \text{ MeV} \) [3].

The \( K_{\infty} \) does not have so large uncertainty due to the consistency among experiments and theoretical prediction: \( K_{\infty}=230\pm 30 \text{ MeV} \) [6]. The slope parameter \( L \) which will be strongly related to nuclear dipole mode, has been well studied from the many points of view, for instance the relation among the neutron-skin thickness [13, 15–18], pygmy dipole resonance [16], giant dipole resonance (GDR) [19] and polarizability [13, 18, 20], although it has been yet floated: \( L=58\pm 18 \text{ MeV} \) [3, 4]. These untiring studies narrow downs the range of the EOS parameters, however the \( K_{\text{sym}} \) especially has a large range of values. Basically, it is difficult to connect between EOS parameters and experimental values directly, therefore we usually take the procedure: to search an EDF to reproduce experiments and then to extract the EOS parameters from the EDF. In a present work, we evaluate incompressibility parameters from the isoscalar giant monopole resonance (ISGMR) of finite nuclear system. In order to estimate the parameters independently of the speciality of each nucleus, we use the finite nuclear incompressibility \( K_A \)-expansion to analyse them, although we can compare the ISGMRs in theory and in experiments.

If the energy of ISGMR \( E_{\text{GMR}} \) is represented in the root mean square radius of the nucleus and the GMR can be regarded as a single phonon mode, \( E_{\text{GMR}} \) is written in

\[
E_{\text{GMR}} = \sqrt{\frac{\hbar^2 K_A}{m \langle r^2 \rangle}},
\]

where \( K_A \) is defined as an incompressibility of finite nuclear system [1]. Although the \( K_A \) can not directly equal to \( K_{\infty} \) at a limit of \( A \to \infty \), it will bring the relation between the parameters of finite and infinite system. The \( K_A \) is expanded around \( K_{\infty} \) as

\[
K_A = K_{\infty} + K_{\text{surf}} A^{-1/3} + K_r \left( \frac{N-Z}{A} \right)^2 + K_{\text{Coul}} \frac{Z^2}{A^{4/3}}.
\]

When incompressibility parameters are extracted from the experiments, the expansion (7) is often used [10–12, 14]. The surface and Coulomb term (\( K_{\text{surf}}, K_{\text{Coul}} \)) are estimated by infinite EOS parameters in previous studies, which will be confirmed how available in this
work. The isospin term \( K_\tau \) has not been clear, which is a most important key to know the information of nuclear matter incompressibility using Eq.(7).

This paper is organized as follows. In Sec.II, we present the methods to calculate ISM mode and to evaluate the centroid energy of ISGMR \( \bar{E}_{\text{GMR}} \). In Sec.III, at first, we show partially the strength functions of \( N = Z \) nuclide to check the behaviour of them including the results of deformed nuclei. After that, we extract the EOS parameters of finite nuclear system by the chi-square fitting with Eq.(7), from the \( K_A \) evaluated with Eq.(6) and \( \bar{E}_{\text{GMR}} \). In particular, we investigate \( N = Z \) nuclide for \( K_{\text{surf}} \) and \( K_{\text{Coul}} \), and isobar nuclide for \( K_\tau \). Furthermore, to confirm the expansion itself and with our coefficients, we compare them with the experimental \( E_{\text{GMR}} \) and calculated \( \bar{E}_{\text{GMR}} \) for Sn isotopes. In Sec.IV, we also compare the experimental measurements for a whole mass region with \( A = 24 - 238 \), in order to narrow down the candidates of effective interaction. And we mention the mass dependence of \( E_{\text{GMR}} \) in experiments. The present work is summarized in Sec.V.

II. FORMULATION

To access the GMR, we apply the Canonical-basis time-dependent Hartree-Fock-Bogoliubov (Cb-TDHFB) theory \[21\] in three-dimensional (3D) coordinate space which can be successfully applied to the study of the dipole \[21, 22\] and quadrupole \[23, 24\] modes of many isotopes, systematically. The Cb-TDHFB can describe the dynamical effects of pairing correlation in fully self-consistently. The Cb-TDHFB equations are derived from the full TDHFB equation represented in the canonical basis \( \{ \phi_l(t), \phi_l(t) \} \) which diagonalize a density matrix, and by assuming the diagonal form of pairing functional. The Cb-TDHFB equations compose the time-evolution equations for the canonical pair \( \{ \phi_l(t), \phi_l(t) \} \), its occupation probability \( \rho_l(t) \) and pair probability \( \kappa_l(t) \),

\[
\begin{align*}
&i \frac{d}{dt} \rho_l(t) = \eta_l(t) \Delta^*_l(t) - \kappa^*_l(t) \Delta_l(t), \\
&i \frac{d}{dt} \kappa_l(t) = (\eta_l(t) + \eta^*_l(t)) \kappa_l(t) + \Delta^*_l(t) (2\rho_l(t) - 1),
\end{align*}
\]

where the phase of canonical basis is chosen as \( \eta_l(t) \equiv \langle \phi_l(t) | h(t) | \phi_l(t) \rangle \), and the \( h(t) \) and \( \Delta_l(t) \) are the single-particle Hamiltonian and the gap energy, respectively.
Due to the appearance of deformed ground state in our subjective nuclide, we should choose a flexible calculation space. We use the 3D Cartesian coordinate-space representation for canonical basis, $\phi_l(\mathbf{r}, \sigma; t) = \langle \mathbf{r}, \sigma | \phi_l(t) \rangle$ with spin $\sigma = \pm 1/2$. The condition of calculation space is discretized in a square mesh of 1.0 fm inside of a sphere of radius 15 fm for all nuclide in present work.

To apply our method to systematic investigation, we choose the Skyrme EDF to $ph$-channel and the simple pairing functional form to $pp(hh)$-channel: $\Delta_l(t) \equiv \sum_k G_{kl} \kappa_k(t)$ where $G_{kl}$ is constant in real-time evolution, as same as Ref. [21]. Our choice of five Skyrme parameter sets is SGII [25], SkM* [26], SLy4 [27], SkT3 [28] and SkI3 [29]. The reason to choose them is not only their usefulness, also corresponds to the limitation of $K_\infty = 230 \pm 30$ MeV indicated in Ref. [6].

A. Linear response calculation with Cb-TDHFB

In order to induce monopole responses, we add a weak instantaneous external field $V_{\text{ext}}(\mathbf{r}, t) = \xi \hat{F}(\mathbf{r}) \delta(t)$ to initial states of the time evolution. Here the isoscalar monopole operator acting on nucleus is given as $\hat{F}_E^{IS} \equiv r^2 Y_{00}$. The amplitude of the external field is so chosen to be a small number $\xi = 2 \times 10^{-3}$ fm$^{-2}$ to guarantee the linearity. The strength function $S(E; E_0)$ can be obtained through the Fourier transformation of $\mathcal{F}(t) \equiv \langle \Psi(t) | \hat{F}_E^{IS} | \Psi(t) \rangle$:

$$S(E; E_0) \equiv \sum_n |\langle \Psi_n | \hat{F}_E^{IS} | \Psi_0 \rangle|^2 \delta(E_n - E) = \frac{-1}{\pi \xi} \text{Im} \int_0^\infty [\mathcal{F}(t) - \mathcal{F}(0)] e^{i(E + \Gamma/2)t} dt,$$

where $|\Psi_0\rangle$ and $|\Psi_n\rangle$ are the ground and excited states, respectively. $\Gamma$ is a smoothing parameter set to 1 MeV for whole nuclide in present.

B. Evaluation of mean energy for GMR

We need a procedure to compute the mean energy of GR, in common among subjective nuclide. To compute the centroid energy $E$ of GR, we use $m_1/m_0$, although there are some evaluations ($\sqrt{m_3/m_1}$ or $\sqrt{m_1/m_{-1}}$).
The \( m_1/m_0 \) is computed as follows in present work.

\[
\bar{E}_{\text{GMR}} \equiv \frac{m_0}{m_1} \simeq \frac{\int_{<e} dE \, ES(E; E_0)}{\int_{>e} dE \, S(E; E_0)}, \tag{10}
\]

where \( <e \) and \( >e \) are an upper and under cut-off energy respectively. They should be decided with more carefully, because the \( \bar{E}_{\text{GMR}} \) is sensitive to them. In this work, we decide them as: \( <e = E_C + 7.5 \) and \( >e = E_C - 7.5 \) MeV, where \( E_C = 80A^{-1/3} \). The empirical formula \( E_C \) is found in a droplet model [30]. Purposely we chose this way, because Eq.(6) to relate the \( K_A \) with nuclear response, which is based on the one phonon picture in other words the GMR is assumed as one mode.

III. RESULT

We evaluate the mean energy of GMR using Eq.(10), and from them the finite incompressibility \( K_A \) is also evaluated according to Eq.(6) with using calculated \( \langle r^2 \rangle \) in Table I. To determine the expression parameters of \( K_A \) according to Eq.(7), we proceed a following way step by step. First, we determine the \( K_{\text{surf}} \) and \( K_{\text{Coul}} \) using the results of \( N = Z \) nuclide in which the isospin term \( K_\tau \) does not contribute to \( K_A \). Second, we determine the \( K_\tau \) while using the \( K_{\text{surf}} \) and \( K_{\text{Coul}} \) fixed in the first step. To obtain the \( K_\tau \) for each Skyrme interaction, we analyse isobar nuclide for both \( A =100 \) and 132 at same time. Lastly, to confirm that the expansion of \( K_A \) with determined parameters reproduces the results of Sn isotopes, we calculate the centroid energies \( E_{\text{GMR}} \) according to Eq.(6) with \( K_A \) and \( \langle r^2 \rangle \) in Table II and compare them with actually calculated \( \bar{E}_{\text{GMR}} \).

A. \( N = Z \)

ISM strength functions of even-even \( N = Z \) nuclide from \(^{40}\text{Ca}\) to \(^{100}\text{Sn}\) with SkM* are shown in Fig.1. Chain, dashed, dotted and thick lines show the results of \( Z =20, 30, 40 \) and 50, respectively. We can see the broad strength distribution of \(^{40}\text{Ca}\) in high energy around 21 MeV. The distribution becomes localized and its centre shifts to low energy, as mass number increase. In these strength, split distributions can be seen in thin lines which are corresponding to \(^{48}\text{Cr}\) and \(^{72}\text{Kr}\). The split is caused by the coupling monopole with
quadrupole excitations due to the deformation. Typically the quadrupole GR appear in lower energy than GMR, thus the $E_{\text{GMR}}$ of a well deformed nucleus shifts to low energy.

There are some strengths in vicinity of zero energy, which corresponds to numerical spurious mode due to the detail of mesh size and of time step. They are excluded from the estimation of $E_{\text{GMR}}$ in Eq. (10).

![Graph](image1)

**FIG. 1:** (Colour online) Strength functions of isoscalar monopole vibrational modes of $N = Z$ even-even nuclide from $^{40}\text{Ca}$ to $^{100}\text{Sn}$.

Figure 2 shows the $E_{\text{GMR}}$ for $N = Z$ nuclide with five Skyrme parameters. Filled symbols means results of spherical nuclei or the nuclei which have small deformation ($|\beta| < 0.1$), open ones means those in deformed nuclei. Over $A = 80$, the trend and relation among the results of each Skyrme parameter become clarified. The behaviour of $E_{\text{GMR}}$ in deformed nuclei diverges from the trend of spherical. The order of $E_{\text{GMR}}$ can be almost corresponded to the order of $K_\infty$ magnitude (refer to Tab. III).

![Graph](image2)

**FIG. 2:** (Colour online) Mean energies of ISGMR for $N = Z$ even-even nuclide, which are computed by Eq. (10) using the strengths with five Skyrme parameter sets.
Figure 3 shows $K_A$ obtained by Eq.(6). Same as Fig.2 the filled and open symbols correspond to the results of spherical and deformed nuclei, but we use the star symbol for double magic (DM) nuclei ($^{40}\text{Ca}$, $^{56}\text{Ni}$, $^{100}\text{Sn}$). The behaviour of $K_A$ well corresponds to that of $\bar{E}\text{GMR}$, thus the trend and the relation among interactions are stable over $A=80$, and the results of deformed nuclei clearly have difference from spherical nuclei.

To obtain the expansion coefficients ($K_{\text{surf}}$, $K_\tau$, $K_{\text{Coul}}$), we analyse our results according to the $K_A$ expansion in Eq.(7). The results of deformed nucleus are excluded from our analysis, because they have clearly a different trend from those of spherical nuclide. If they can be included into the analysis, we will need the way to separate quadrupole and monopole modes. Here, two cases are considered: (i) excluding DM nuclei and (ii) including them. Our purpose is to extract nuclear matter properties from nucleus. The effects due to the special nuclear structure such as a modes coupling in deformed nuclei, should be excluded from the matter property analysis. As mentioned in Ref.[32], the magicity effects in the incompressibility will appear, which should be confirmed in the comparison the (i) and (ii).

In this section, we fix the $K_{\text{surf}}$ and $K_{\text{Coul}}$ expansion coefficients which are listed in the Table III. The surface term $K_{\text{surf}}$ is often estimated as an opposite sign of $K_\infty$ [2, 6, 10–12], and the Coulomb term $K_{\text{Coul}}$ is estimated in Ref.[1, 2] as

$$K_{\text{Coul}} = -\frac{3}{5} \frac{e^2}{R'} \left( \frac{Q_0}{K_\infty} + 8 \right), \quad R' \equiv \left( \frac{3}{4\pi\rho_0} \right)^{1/3}. \quad (11)$$

Our $|K_{\text{surf}}|$ are larger than $|K_\infty|$, which have been mentioned already in Ref.[2]. Although an assumption $K_{\text{surf}} \equiv -K_\infty$ is sometimes used for non-relativistic models in previous analyses.
It is not suitable actually and the difference over 30% from it might cause a serious missing in nuclear property. In both (i) and (ii), $K_{\text{Coul}}$ closes to the $K_{\text{Coul}}$. $K_{\text{surf}}$ and $K_{\text{Coul}}$ in (ii) are a little weaker and stronger than those of (i), respectively in most interaction-cases. The effect of DM on $K_A$ is regarded as small as shown in Fig.8 although small kinks appear at $^{56}\text{Ni}$. The nuclear magicity is not so sensitive to the $K_A$ excluding light DM nuclei.

B. $A = 100, 132$

In this section, we determine the isospin term $K_\tau$ from isobar nuclide $A=100$ and 132, while using $K_{\text{surf}}$ and $K_{\text{Coul}}$ fixed in previous section. In same as Sec. III A, we exclude the deformed nuclei from the analysis. Figure 4 shows the $K_A$ of the spherical isobars with $A=100$ and 132 with respect to isospin asymmetry $(N - Z)/A$, in which the vertical chain line separates $A=100$ and 132. The root mean square radii and quadrupole deformations of the isobars are listed in Table I.

The $K_A$ in Fig.4 has a parabolic shape in $(N - Z)/A$ which corresponds to the expansion Eq.7, however the centre of the parabolic function is not always at $N=Z$. In this analysis, we also consider the two cases: (i) with DM and (ii) without DM. The isospin term $K_\tau$ obtained in the cases are listed in Table III. The effects of DM are not so large also in $K_\tau$ excluding the results of SkI3. When we exclude the result of DM from SkI3 results, the points which can be used in the analysis are only four, therefore the analysis ambiguity becomes large. To determine the $K_\tau$, the number of isobar nuclide is essential.

In several papers, an isospin dependence of incompressibility for nuclear matter is estimated at saturation density $\rho_0$ with a small isospin asymmetry $4, 6, 14$, which can be written in

$$K_\tau^\infty = K_{\text{sym}} - 6L - \frac{Q_0}{K_\infty^\infty}L. \quad (12)$$

The $K_\tau^\infty$ can not be regarded as the finite incompressibility $K_\tau$, which is mentioned also in Ref. 11, although the strong correlation between $K_\tau$ and $K_\tau^\infty$ can be expected naively. The $K_\tau^\infty$ is often used to expand the $K_\tau$: $K_\tau = K_\tau^\infty + K_\tau^{\text{surf}} A^{-1/3}$. The $K_\tau^{\text{surf}}$ could not be estimated because the number of isobar-chain sample is only two. The absolute value of $K_\tau$ is usual smaller than $K_\tau^\infty$ in present work and the correlation among them does not seem
FIG. 4: (Colour online) Same as Fig.3 but for $K_A$ of spherical $A = 100, 132$ isobar with respect to $(N - Z)/A$.

C. Sn isotope

We obtain the expansion coefficients of $K_A$ in previous sections. To confirm the coefficients and the expansion Eq. (7) itself, we compare the $E_{\text{GMR}}$ in Eq. (6) with the coefficients ($K_{\text{surf}}, K_{\tau}, K_{\text{Coul}}$), in the experiment and $\tilde{E}_{\text{GMR}}$ by the linear response calculation with Eq. (10), for Sn isotopes ($A = 100 - 132$). Figure 5 shows the $E_{\text{GMR}}$ for Sn isotopes. Solid lines and filled symbols mean the results directly calculated with Eq. (10), dashed lines correspond to the $E_{\text{GMR}}$ in Eq. (6) with the coefficients of case (i) for each interaction, and open square symbols are experimental data at RCNP [10]. Furthermore, to show the pairing effects we add the $\tilde{E}_{\text{GMR}}$ with TDHF only for SkM* which are symbolized by open circles and dotted line.

The dashed lines well reproduce whole $\tilde{E}_{\text{GMR}}$ of Sn isotopes within a smaller than 0.3 MeV. It means the expansion Eq. (7) is an effective procedure and the coefficients are suitable. The comparison our results and he experimental data may recommend SkM* and SGII parameters as a candidate of the “answer”.

The pairing effects to the trend on Sn isotope ISGMR can be discussed in the comparison between our results obtained by TDHF (open) and by Cb-TDHF (filled) with SkM*. The small difference between them appears in whole isotopes, which can be expected due to the small deformation of HF ground states. While the effect to soften EOS slightly in a surface-
type pairing functional was reported \cite{32, 35, 36}, our results indicate the opposite effects, however whose mechanism is different from the previous studies. The HF ground states in Sn isotopes have some deformed aspects as Table II. As the explanation in Sec. IIIA the centroid energy of ISGMR is estimated at lower than that of spherical nucleus due to the coupling with other modes, while using the summation analysis such as Eq.(10).

![Graph](image)

**FIG. 5:** (Colour online) Same as Fig.2 but for Sn isotopes from $A=100$ to 132 with respect to $A$. Filled and open circle means the result calculated by Cb-TDHF and by TDHF with SkM*, respectively. Square symbols shows experimental data at RCNP \cite{10}.

**IV. DISCUSSION**

We obtain the expansion coefficients ($K_{\text{surf}}, K_\tau, K_{\text{Coul}}$) through the Sec.III A and III B, and confirm them in Sn isotopes. SkM* and SGII parameters might be likely candidates in Sec.III C however it is not clear that they can approach to more heavier system, at least in present results. The many GMR distributions for whole mass region have been measured in the past, which are summarised in Ref.\cite{34}. We extend the mass region to apply Eq.(6), and attempt to narrow down the parameters.

Figure 6 shows experimental $E_{\text{GMR}}$ (square) which are measured at Texas A&M, Grenoble, Groningen \cite{34} and RCNP \cite{10, 11}, and the centroid energy (solid line) by Eq.(10) for each Skyrme parameter but with $\langle r^2 \rangle$ expressed as $\sqrt{\langle r^2 \rangle} = 0.895 A^{1/3} + 0.321$ fm which is obtained by fitting systematic mean radii calculated in SkM*. The experimental data cover a wide mass region $A=24–238$ with $Z=12–92$. In $A<50$ mass region, the mass dependence of experiment values is not clear apparently. Over $A=100$, they make a visible trend, though
FIG. 6: (Colour online) The peak positions of GMR in experiment (square) and the lines obtained from Eq.(6) with the $K_A$ for each Skyrme parameter, where the the mean radii $\langle r^2 \rangle$ are calculated with SkM* parameter, are shown.

some values around $A=90$ deviate from the trend.

Whole mass region, the solid lines keep their magnitude order which is same one in Fig.5 thus namely SkI3, SkT3, SLy4, SGII and SkM* in decreasing order. For SkI3, the results overestimate experiments. For SGII and SkM* they underestimate over $A=160$, while they can well reproduce those around $A=110$. If we emphasize the agreement with a heavy system, SGII and SkM* can not reach to the conclusion as the best choice. SkI3 should be excluded from the candidate, due to the missing over $A=100$. From the agreements with $E_{GMR}$ over $A=140$, SLy4 and SkT3 are good candidates. Therefore our results indicate that the range of $K_\tau$ is $-305\pm10$ MeV, and the incompressibility parameters for infinite system corresponding to the $K_\tau$ are $K_\tau^{\infty}=-340\pm35$, $K_\infty=225\pm11$ and $K_{\text{sym}}=-138\pm18$ MeV, which are expected from SGII, SkM*, SLy4 and SkT3 parameters.

In this paper, our analysis has constructed within the assumption Eq.(6) and (7). As Fig.6 shown that, the expansion Eq.(7) well work to connect between the infinite nuclear properties and finite nuclear excitation mode in Eq.(6). However, the mass dependence of $E_{GMR}$ in experiment is apparently different from that scaled by $A^{-1/3}$, which has been mentioned also in Ref.[8, 36, 37]. The small panel in Fig.6 shows the fitting experimental data for $A=90$–238, by $A^{-1/3}$ (dashed) and by $A^{-1/3}+A^{-1/6}$ (solid). The coefficient of $A^{-1/3}$ is consistent with the ordinary value [30], but it underestimates experiment in heavy mass region. The solid line reproduces experimental data in a whole mass region. The $A^{-1/3}$ and
$A^{-1/6}$ scaling are well known on the GDR as Steinwedel-Jensen (SJ) and Goldhaber-Teller (GT) model [38], respectively in which the dipole moments are caused by polarization and by proton-neutron displacement. If we adapt the models to GMR, the SJ and GT-model will respectively correspond to the density wave oscillation keeping nuclear size, and to the vibration between expansion and contraction of the size.

V. CONCLUSION

We have performed a systematic investigation of the ISGMR to extract the isospin dependent compression EOS parameter from finite nuclear system, using the linear response calculation with Cb-TDHFB represented in 3D coordinate space. The expansion coefficients ($K_{\text{surf}}$, $K_{\tau}$, $K_{\text{Coul}}$) of finite incompressibility $K_A$ are determined from $N=Z$ and isobar $A=100, 132$ nuclide for each Skyrme interaction (SGII, SkM*, SLy4, SkT3, SkI3). Furthermore in Sn isotopes ($A=100-132$), it is confirmed that the coefficients are suitable and the $K_A$ expansion is available.

The absolute values of $K_{\text{surf}}$ are larger than those of $K_{\infty}$, thus the analysis assuming $K_{\text{surf}}=-K_{\infty}$ is not effective, which are often used in previous study for non-relativistic interaction. The $K_{\text{Coul}}$ estimation Eq.(11) works well. The isospin term $K_{\tau}$ in the finite system is smaller than the $K_{\tau\infty}$. The magicity in a present work does not affect the conclusion. And the pairing effect in Sn isotopes are not so large, although the deformation due to the lack of pairing causes the coupling quadrupole and monopole modes, and disturbs the position of GMR.

We narrow down the values from the comparison experimental data with our results in a whole mass region, which indicates the range of $K_{\tau}$ is $-305\pm10$ MeV. The incompressibility parameters corresponding to the range have $K_{\infty}=225\pm11$ and $K_{\text{sym}}=-138\pm18$ MeV, which is consistent with the previous study of slop parameter $L$ [7]. As an indication of Eq.(12), the $L$ has an important role to decide $K_{\text{sym}}$. Thus the studies related to $L$ should be also pressed forward in parallel. The ordinary mass scaling of GMR has underestimation in heavy mass system, which may indicate the needs to reconsider the expression of incompressibility.

To determine the isospin term $K_{\tau}$, the systematic ISGMR data in isobar nuclide are very effective. They should be heavy system over $A=100$ at least, because in light system the nuclear speciality is showing up. When the heavy nucleus is used to analyse GMR, we should
note the deformation of open shell nuclei. If the monopole and other modes in deformed nucleus can be separated, the analysis data increases and the relation EOS parameter and finite system will be more robust.

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TABLE I: Root mean square radius $\tilde{R} = \sqrt{\langle r^2 \rangle}$ [fm] and absolute value of quadrupole deformation parameter $\beta$ in ground state for $N = Z$ and $A = 100, 132$ nuclide.

|       | SGII $\tilde{R}$ | SkM$^*$ $\tilde{R}$ | SLy4 $\tilde{R}$ | SkT3 $\tilde{R}$ | SkI3 $\tilde{R}$ |
|-------|------------------|----------------------|------------------|------------------|------------------|
| $^{40}$Ca | 3.35 0.00       | 3.40 0.00           | 3.39 0.00       | 3.40 0.00       | 3.36 0.00       |
| $^{44}$Ti  | 3.44 0.00       | 3.48 0.00           | 3.48 0.00       | 3.47 0.00       | 3.46 0.00       |
| $^{48}$Cr  | 3.56 0.26       | 3.58 0.21           | 3.60 0.26       | 3.53 0.03       | 3.58 0.29       |
| $^{52}$Fe  | 3.58 0.00       | 3.61 0.00           | 3.62 0.00       | 3.60 0.00       | 3.60 0.00       |
| $^{56}$Ni  | 3.64 0.00       | 3.66 0.00           | 3.67 0.00       | 3.65 0.00       | 3.65 0.00       |
| $^{60}$Zn  | 3.77 0.20       | 3.75 0.00           | 3.77 0.02       | 3.74 0.00       | 3.76 0.02       |
| $^{64}$Ge  | 3.83 0.00       | 3.84 0.00           | 3.86 0.00       | 3.82 0.00       | 3.86 0.00       |
| $^{68}$Se  | 3.95 0.22       | 3.94 0.09           | 3.97 0.17       | 3.91 0.01       | 3.97 0.23       |
| $^{72}$Kr  | 4.02 0.19       | 4.07 0.26           | 4.05 0.17       | 4.03 0.23       | 4.08 0.31       |
| $^{76}$Sr  | 4.06 0.01       | 4.12 0.00           | 4.08 0.01       | 4.15 0.39       | 4.08 0.13       |
| $^{80}$Zr  | 4.12 0.00       | 4.15 0.00           | 4.15 0.00       | 4.12 0.00       | 4.12 0.00       |
| $^{84}$Mo  | 4.17 0.00       | 4.20 0.00           | 4.20 0.00       | 4.17 0.00       | 4.18 0.00       |
| $^{88}$Ru  | 4.22 0.00       | 4.25 0.00           | 4.25 0.00       | 4.23 0.00       | 4.23 0.00       |
| $^{92}$Pd  | 4.27 0.00       | 4.29 0.00           | 4.30 0.00       | 4.27 0.00       | 4.28 0.00       |
| $^{96}$Cd  | 4.32 0.00       | 4.34 0.00           | 4.34 0.00       | 4.32 0.00       | 4.32 0.00       |
| $^{100}$Sn | 4.36 0.00       | 4.38 0.00           | 4.39 0.00       | 4.36 0.00       | 4.36 0.00       |
| $^{100}$Kr | 4.50 0.23       | 4.54 0.23           | 4.54 0.22       | 4.50 0.21       | 4.57 0.24       |
| $^{100}$Sr | 4.54 0.39       | 4.56 0.38           | 4.57 0.39       | 4.52 0.36       | 4.59 0.40       |
| $^{100}$Zr | 4.52 0.38       | 4.52 0.36           | 4.54 0.36       | 4.48 0.33       | 4.59 0.43       |
| $^{100}$Mo | 4.40 0.00       | 4.42 0.00           | 4.43 0.00       | 4.38 0.00       | 4.46 0.20       |
| $^{100}$Ru | 4.39 0.00       | 4.40 0.00           | 4.42 0.00       | 4.37 0.00       | 4.43 0.19       |
| $^{100}$Pd | 4.38 0.00       | 4.39 0.00           | 4.40 0.00       | 4.36 0.00       | 4.39 0.01       |
| $^{100}$Cd | 4.37 0.00       | 4.38 0.00           | 4.39 0.00       | 4.35 0.00       | 4.38 0.00       |
| $^{132}$Sn | 4.78 0.00       | 4.80 0.00           | 4.80 0.00       | 4.79 0.00       | 4.82 0.00       |
| $^{132}$Te | 4.79 0.00       | 4.80 0.00           | 4.81 0.00       | 4.78 0.00       | 4.82 0.00       |
| $^{132}$Xe | 4.79 0.00       | 4.81 0.00           | 4.81 0.00       | 4.78 0.00       | 4.82 0.00       |
| $^{132}$Ba | 4.81 0.15       | 4.83 0.16           | 4.84 0.15       | 4.79 0.14       | 4.83 0.16       |
| $^{132}$Ce | 4.83 0.21       | 4.85 0.23           | 4.86 0.21       | 4.82 0.23       | 4.84 0.21       |
TABLE II: Same as Table I but for Sn isotopes with from $N = 52$ to 80. Although the HF+BCS ground states take only a spherical shape in present work, the HF results in SkM* have some deformation.

|       | SGII | SkM* | SLy4 | SkT3 | SkI3 |
|-------|------|------|------|------|------|
|       | $\tilde{R}$ | $\tilde{R}$ | $\tilde{R}_{\text{HF}}$ | $|\tilde{\beta}_{\text{HF}}|$ | $\tilde{R}$ | $\tilde{R}$ | $\tilde{R}$ |
| $^{102}\text{Sn}$ | 4.39 | 4.405 | 4.401 | 0.04 | 4.42 | 4.38 | 4.40 |
| $^{104}\text{Sn}$ | 4.43 | 4.434 | 4.426 | 0.05 | 4.45 | 4.40 | 4.43 |
| $^{106}\text{Sn}$ | 4.46 | 4.462 | 4.448 | 0.00 | 4.48 | 4.43 | 4.47 |
| $^{108}\text{Sn}$ | 4.48 | 4.491 | 4.488 | 0.07 | 4.51 | 4.46 | 4.50 |
| $^{110}\text{Sn}$ | 4.51 | 4.521 | 4.527 | 0.10 | 4.54 | 4.49 | 4.53 |
| $^{112}\text{Sn}$ | 4.54 | 4.551 | 4.558 | 0.10 | 4.57 | 4.52 | 4.56 |
| $^{114}\text{Sn}$ | 4.57 | 4.581 | 4.596 | 0.11 | 4.60 | 4.54 | 4.59 |
| $^{116}\text{Sn}$ | 4.59 | 4.609 | 4.632 | 0.15 | 4.62 | 4.57 | 4.61 |
| $^{118}\text{Sn}$ | 4.62 | 4.636 | 4.699 | 0.28 | 4.65 | 4.60 | 4.64 |
| $^{120}\text{Sn}$ | 4.64 | 4.662 | 4.677 | 0.17 | 4.67 | 4.63 | 4.67 |
| $^{122}\text{Sn}$ | 4.67 | 4.688 | 4.703 | 0.14 | 4.70 | 4.66 | 4.70 |
| $^{124}\text{Sn}$ | 4.69 | 4.712 | 4.724 | 0.10 | 4.72 | 4.68 | 4.72 |
| $^{126}\text{Sn}$ | 4.71 | 4.736 | 4.743 | 0.00 | 4.74 | 4.71 | 4.75 |
| $^{128}\text{Sn}$ | 4.74 | 4.759 | 4.766 | 0.06 | 4.76 | 4.74 | 4.77 |
| $^{130}\text{Sn}$ | 4.76 | 4.781 | 4.784 | 0.04 | 4.79 | 4.76 | 4.80 |
TABLE III: Parameters related to EOS ($\rho_0$[fm$^{-3}$], $K_\infty$, $Q_0$, $L$, $K_{\text{sym}}$, $K_\infty^\tau$, $\tilde{K}_{\text{Coul}}$ [MeV]) and finite incompressibility ($K_{\text{surf}}$, $K_\tau$, $K_{\text{Coul}}$ [MeV]) for each Skyrme interaction. The $K_\infty^\tau$ and $\tilde{K}_{\text{Coul}}$ are obtained by Eq.(12),(11). Nucleon mass $m c^2=938.9187$ MeV, $\hbar c=197.327$ MeV fm and $\alpha^{-1}=137.036$ are used [31].

| Int. | $\rho_0$ | $K_\infty$ | $Q_0$ | $L$ | $K_{\text{sym}}$ | $K_\infty^\tau$ | $\tilde{K}_{\text{Coul}}$ | $K_{\text{surf}}$ | $K_\tau$ | $K_{\text{Coul}}$ | $K_{\text{surf}}$ | $K_\tau$ | $K_{\text{Coul}}$ |
|------|----------|-------------|------|----|-----------------|-----------------|----------------|----------------|--------|----------------|----------------|--------|----------------|
| SGII | 1.583    | 214.6       | -380.9 | 37.63 | -145.9 | -304.9 | -4.69 | -273.7 | -295.9 | -4.52 | -260.4 | -283.5 | -5.10 |
| SkM$^*$ | 1.603   | 216.6       | -386.1 | 45.78 | -155.9 | -349.0 | -4.70 | -285.3 | -282.0 | -4.76 | -276.7 | -285.0 | -5.09 |
| SLy4  | 1.595    | 229.9       | -363.1 | 45.96 | -119.7 | -322.9 | -4.85 | -318.4 | -314.3 | -4.73 | -308.3 | -305.4 | -5.13 |
| SkT3  | 1.610    | 235.7       | -382.7 | 55.31 | -132.1 | -374.1 | -4.83 | -326.1 | -305.1 | -4.54 | -325.0 | -305.2 | -4.51 |
| SkI3  | 1.598    | 258.2       | -303.8 | 100.5 | 73.03 | -411.8 | -5.13 | -372.6 | -352.0 | -4.75 | -364.8 | -332.6 | -4.99 |