Brane inflation in tachyonic and non-tachyonic type 0B string theories

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Abstract

We consider the motion of the brane universe moving in a background bulk space of tachyonic and non-tachyonic type 0B string theory. The effective densities are calculated for both cases and they show different power law behavior. Brane inflation for non-tachyonic type 0B background has the same power law behavior as that for type IIB background. The brane inflation under tachyonic background is less divergent than the one without tachyon. The role of tachyonic field in brane inflation scenario is different from that of the ordinary matter field.

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I. INTRODUCTION

Recently the old idea that our universe might be embedded in a higher dimensional space [1] has attracted much interest. The most attractive scenario is the so-called ‘Randall Sundrum (RS) brane world’ [2]. In this scenario our observed universe is embedded in a five-dimensional bulk, in which the background metric is curved along the extra dimension due to the negative bulk cosmological constant. Under this framework everything is confined to live on the brane except for gravity. These models have been studied extensively because they might provide the key to the gauge hierarchy problem and cosmological constant problem [3].

It seems natural that one can generalize these models within a well defined framework such as string theory or M-theory. Many attempts have been made to apply this idea to string theory in the context with D-branes [4]. One of the important issues of the brane world scenario is the cosmological evolution of the early universe. Many cosmological models regarding this have been suggested. These models can be classified into two categories. The first is that the domain walls (branes) are static solution of the underlying theory and the cosmological evolution of our universe is due the time evolution of energy density on the domain wall [5]. The second is that the cosmological evolution of our universe is due to the motion of our brane-world in the background of gravitational field of the bulk [6,7]. One of the interesting among the second category is the the mirage cosmology of Kehagias and Kiritsis [7]. The idea is that the motion of the brane in ambient space induces cosmological expansion (or contraction) on our universe.

There have been studies on how the presence of the various matter fields on the supersymmetric background geometry affects the cosmological evolution of the brane universe in the context of type II theory [7,8]. The brane inflation for the case of non-supersymmetric string background has been studied with the type 0B theory in [9]. However the analysis is incomplete in the sense that their study is restricted to constant dilaton and tachyon field. The running of the tachyon is the key ingredient of the tachyonic type 0B theory [10,11]. In this paper we will study the motion of a three-brane in the background of type 0B theory. We will consider both the tachyonic and non-tachyonic type 0B theories and see how the tachyonic field affects the brane inflation.

The organization of the paper is as follows. In Sec. II we will extend the formalism of brane geodesic in Ref. [7] to the case when tachyon field is present and set up some preliminaries for our calculation. In Sec. III we consider the type 0B background geometries. We will consider the tachyonic 0B theory as well as the non-tachyonic 0B theory. In Sec. IV, using the background solutions of Sec. III, we study the brane inflation under these backgrounds. Finally in Sec. V, we discuss and conclude our results.

II. BRANE GEODESICS

In Ref. [7] it is shown that the motion of the brane in ambient space induces cosmological expansion (or contraction) on our universe simulating various kinds of matter or a cosmological constant. In this section, we extend the formalism to the case when there is tachyon field.
We consider a probe brane moving in a generic static spherically symmetric background. We ignore its back reaction to the ambient space. As the brane moves in a geodesic, the induced world-volume metric becomes a function of time. The cosmological evolution is possible from the brane resident point of view. We will focus on a D3-brane case. For this purpose we parametrize the metric of a D3-brane as

$$ds_{10}^2 = g_{00}(r) dt^2 + g(r)(d\vec{x})^2 + g_{rr}(r) dr^2 + g_S(r) d\Omega_5^2,$$

and we have dilaton $\phi$, tachyon $T(r)$ as well as RR (Ramond-Ramond) background $C(r) = C_{0123}(r)$. In tachyonic type 0B theory the D-brane effective action is determined not only by the dilaton alone, but also by tachyon field. Due to the existence of a tachyon tadpole on the D-brane, the Born-Infeld (BI) part of the effective action of an electric D3-brane, ignoring the fermions, is given by [10,12]

$$S = T_3 \int d^4\xi k(T) e^{-\phi} \sqrt{-\det(\hat{G}_{\alpha\beta} + (2\pi \alpha') F_{\alpha\beta} - B_{\alpha\beta})} + T_3 \int d^4\xi \hat{C}_4 + \cdots,$$

where the induced metric on the brane is

$$\hat{G}_{\alpha\beta} = G_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta}$$

with similar expressions for other fields and

$$k(T) = 1 + \frac{1}{4} T + O(T^2).$$

The reparametrization invariance and T-duality imply that the function $k(T)$ multiplies $\sqrt{-\det(\cdots)}$ in the BI part of the action. The coefficient 1/4 of the tachyon tadpole was found in [14].

Generally the motion of a probe D3-brane can have a nonzero angular momentum in the transverse directions. We can write the relevant part of the Lagrangian, in the static gauge $x^\alpha = \xi^\alpha$ ($\alpha = 0, 1, 2, 3$), as

$$L = \sqrt{A(r) - B(r) \dot{r}^2 - D(r) h_{ij} \dot{\varphi^i} \dot{\varphi^j} - C(r)},$$

where $h_{ij} \varphi^i \varphi^j$ is the line element on the unit five sphere ($i, j = 5, \ldots, 9$),

$$A(r) = g^3(r) |g_{00}(r)| e^{-2\phi} k^2(T), \quad B(r) = g^3(r) g_{rr}(r) e^{-2\phi} k^2(T), \quad D(r) = g^3(r) g_S(r) e^{-2\phi} k^2(T),$$

and $C(r)$ is the RR background. The momenta of the system are given by

$$p_r = -\frac{B(r) \dot{r}}{\sqrt{A(r) - B(r) \dot{r}^2 - D(r) h_{ij} \dot{\varphi^i} \dot{\varphi^j}}},$$

$$p_i = -\frac{D(r) h_{ij} \dot{\varphi^j}}{\sqrt{A(r) - B(r) \dot{r}^2 - D(r) h_{ij} \dot{\varphi^i} \dot{\varphi^j}}}.$$
Calculating the Hamiltonian and demanding the conservation of energy, we have

\[ H = C(r) - \frac{A(r)}{\sqrt{A(r) - B(r)r^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}} = -E, \]  

where \( E \) is the total energy of the brane. Also from the conservation of the total angular momentum \( h_{ij}p_ip_j = \ell^2 \), we have

\[ h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{\ell^2[A(r) - B(r)r^2]}{D(r)[D(r) + \ell^2]}. \]  

Substituting Eq. (3) into Eq. (8) and solving with respect to \( \dot{r}^2 \), we have the equation for the radial variable

\[ \dot{r}^2 = \frac{A}{B} \left\{ 1 - \frac{A}{(C + E)^2} \frac{D + \ell^2}{D} \right\}. \]  

Plugging Eq. (10) back into Eq. (9), we have the equation for the angular variable

\[ h_{ij}\dot{\varphi}^i\dot{\varphi}^j = \frac{A^2\ell^2}{D^2(C + E)^2}. \]  

The induced four-dimensional metric on the three-brane universe is

\[ ds^2_{4d} = (g_{00} + g_{rr}r^2 + g_ih_{ij}\dot{\varphi}^i\dot{\varphi}^j)dt^2 + g(dx)^2. \]  

Using Eqs. (10) and (11), this reduces to

\[ ds^2_{4d} = -\frac{g_{00}g^3e^{-\phi}k^2(T)}{(C + E)^2}dt^2 + g(dx)^2 \equiv -d\eta^2 + g(r(\eta))(dx)^2, \]  

where we defined, for the standard form of a flat expanding universe, the cosmic time \( \eta \) as

\[ d\eta = \frac{|g_{00}|g^{3/2}e^{-\phi}|k(T)|}{|C + E|}dt. \]  

If we define the scale factor as \( a^2 = g \), we can calculate, from the analogue of the four-dimensional Friedman equation, the Hubble constant \( H = \dot{a}/a \)

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{(C + E)^2g_se^{2\phi}k^2(T) - |g_{00}||gs^3 + \ell^2e^{2\phi}k^{-2}(T)|}{4|g_{00}|g_{rr}gs^3} \left( \frac{g'}{g} \right)^2, \]  

where the dot denotes the derivative with respect to cosmic time and the prime denotes the derivative with respect to \( r \). The right hand side of Eq. (15) can be interpreted as the effective matter density on the probe brane

\[ \frac{8\pi}{3} \rho_{\text{eff}} = \frac{(C + E)^2g_se^{2\phi}k^2(T) - |g_{00}||gs^3 + \ell^2e^{2\phi}k^{-2}(T)|}{4|g_{00}|g_{rr}gs^3} \left( \frac{g'}{g} \right)^2. \]  

We also have
\[
\frac{\ddot{a}}{a} = \left(1 + \frac{g}{g'} \frac{\partial}{\partial r} \right) \left( (C + E)^2 g_s e^{2\phi} k^{-2}(T) - |g_{00}| g_s g^3 + f^2 e^{2\phi} k^{-2}(T) \right) \left( \frac{g'}{g} \right)^2 - \left(1 + \frac{1}{2}a \frac{\partial}{\partial a} \right) \frac{8\pi}{3} \rho_{\text{eff}}.
\]  

Equating the above to \(-(4\pi/3)(\rho_{\text{eff}} + 3p_{\text{eff}})\), we can find the effective pressure

\[
p_{\text{eff}} = -\rho_{\text{eff}} - \frac{1}{3} \frac{\partial}{\partial a} \rho_{\text{eff}}.
\]

The apparent scalar curvature of the four-dimensional universe is

\[
R_{4d} = 6 \left\{ \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right\} = 8\pi (4 + a\partial_a) \rho_{\text{eff}}.
\]

III. THE TYPE 0B BACKGROUND SOLUTION

A. Tachyonic 0B background solution

The tachyonic type 0B model has a closed string tachyon, no fermions and a doubled set of R-R fields, and thus a doubled set of D-branes \[13\]. With the doubling of R-R fields, the self-dual constraint on the five-form field is relaxed and one can have D3 branes that are electric instead of dyonic. We consider the low energy world volume action of \(N\) coincident electric D3 branes. We start from the action for the tachyonic type 0B theory \[10,14\]

\[
S_0 = -\frac{1}{2\kappa_{10}^2} \int d^{10} x \sqrt{-g} \left[ R - \frac{1}{2} \nabla_n \Phi \nabla^n \Phi - \frac{1}{4} (\nabla_n T \nabla^n T + m^2 e^{(1/2)\Phi} T^2) \right.
\]

\[
- \left. \frac{1}{4 \cdot 5!} f(T) F_{n_1 \ldots n_5} F^{n_1 \ldots n_5} + \cdots \right],
\]

where \(g_{mn}\) is the Einstein-frame metric, \(m^2 = -2/\alpha'\), and the tachyon-R-R field coupling function is

\[
f(T) = 1 + T + \frac{1}{2} T^2.
\]

The equations of motion from this effective action are

\[
\nabla^2 \Phi = \frac{1}{8} m^2 e^{(1/2)\Phi} T^2,
\]

\[
R_{mn} - \frac{1}{2} g_{mn} R = \frac{1}{4} \nabla_m T \nabla_n T - \frac{1}{8} g_{mn} [(\nabla T)^2 + m^2 e^{(1/2)\Phi} T^2] + \frac{1}{2} \nabla_m \Phi \nabla_n \Phi
\]

\[
- \frac{1}{4} g_{mn} (\nabla \Phi)^2 + \frac{1}{4 \cdot 4!} f(T) (F_{mklpq} F_n^{klpq} - \frac{1}{10} g_{mn} F_{sklpq} F^{sklpq}),
\]

\[
(-\nabla^2 + m^2 e^{(1/2)\Phi}) T + \frac{1}{2 \cdot 5!} f'(T) F_{sklpq} F^{sklpq} = 0,
\]
\[ \nabla_m [ f(T) F^{mnkpq} ] = 0. \quad (25) \]

If one parametrize the ten-dimensional (10D) Einstein-frame metric as
\[ ds^2_E = e^{(1/2)\xi - 5\eta} d\rho^2 + e^{-(1/2)\xi} (-dt^2 + dx_i dx_i) + e^{(1/2)\xi - \eta} d\Omega_5^2, \quad (26) \]
where \( \rho \) is the radial direction transverse to the three-brane \( (i = 1, 2, 3) \), then the radial effective action corresponding to Eqs. (22) - (25) becomes
\[ S = \int d\rho \left[ \frac{1}{2} \dot{\Phi}^2 + \frac{1}{2} \dot{\xi}^2 - 5\dot{\eta}^2 + \frac{1}{4} \dot{T}^2 - V(\Phi, \xi, \eta, T) \right], \quad (27) \]
\[ V = \frac{1}{2\alpha'} T^2 e^{(1/2)\Phi + (1/2)\xi - 5\eta} + 20e^{-4\eta} - Q^2 f^{-1}(T)e^{-2\xi}. \quad (28) \]
Here the constant \( Q \) is the R-R charge and dot means the derivative with respect to \( \rho \). The resulting set of variational equations
\[ \ddot{\Phi} + \frac{1}{4\alpha'} T^2 e^{(1/2)\Phi + (1/2)\xi - 5\eta} = 0, \quad (29) \]
\[ \ddot{\xi} + \frac{1}{4\alpha'} T^2 e^{(1/2)\Phi + (1/2)\xi - 5\eta} + 2Q^2 f^{-1}(T)e^{-2\xi} = 0, \quad (30) \]
\[ \ddot{\eta} + 8e^{-4\eta} + \frac{1}{4\alpha'} T^2 e^{(1/2)\Phi + (1/2)\xi - 5\eta} = 0, \quad (31) \]
\[ \ddot{T} + \frac{2}{\alpha'} T e^{(1/2)\Phi + (1/2)\xi - 5\eta} + 2Q^2 \frac{f'(T)}{f^2(T)} e^{-2\xi} = 0, \quad (32) \]
should be supplemented by the zero-energy constraint
\[ \frac{1}{2} \dot{\Phi}^2 + \frac{1}{2} \dot{\xi}^2 - 5\dot{\eta}^2 + \frac{1}{4} \dot{T}^2 + V(\Phi, \xi, \eta, T) = 0, \quad (33) \]
which can be used instead of one of the second-order equations.

In the Einstein frame the dilaton decouples from the R-R terms \(|F_5|^2\) and the tachyon mass term plays the role of the source term. It is easy to analyze the case where the \(|F_5|^2\) is large in Eq. (24). Assuming that the tachyon is localized near the extremum of \( f(T) \), i.e., \( T = -1 \), the asymptotic solution in the UV(\( \rho \equiv e^{-y} \ll 1, \ y \gg 1 \)) region (near-horizon) is given by [10]
\[ T = -1 + \frac{8}{y} + \frac{4}{y^2} (39 \ln y - 20) + O\left( \frac{\ln^2 y}{y^3} \right), \quad (34) \]
\[ \Phi = \ln(2^{15} Q^{-1}) - 2 \ln y + \frac{39}{y} \ln y + O\left( \frac{\ln y}{y^2} \right), \quad (35) \]
\[ \xi = \ln(2Q) - y + \frac{1}{y} + \frac{1}{2y^2}(39 \ln y - 104) + O\left(\frac{\ln^2 y}{y^3}\right), \quad (36) \]

\[ \eta = \ln 2 - \frac{1}{y} + \frac{1}{y} + \frac{1}{2y^2}(39 \ln y - 38) + O\left(\frac{\ln^2 y}{y^3}\right). \quad (37) \]

The 10D Einstein frame metric can be written as

\[
\begin{align*}
    ds^2_E &= R_0^2 \left[ \left( 1 - \frac{9}{2y} - \frac{351}{4y^2} \ln y + \cdots \right) \left( \frac{1}{4} dy \right)^2 \\
    &\quad + \left( 1 - \frac{1}{2y} - \frac{39}{4y^2} \ln y + \cdots \right) e^{(1/2)y} dx^\mu dx^\mu + \left( 1 - \frac{1}{2y} - \frac{39}{4y^2} \ln y + \cdots \right) d\Omega_5^2 \right],
\end{align*}
\]

where

\[ R_0^2 = 2^{-1/2} Q^{1/2}. \]

Note that with \( y = 4 \ln u \), one can show that the metric is of the form of AdS\(_5 \times S^5\) at the leading order,

\[
    ds^2_E = R_0^2 \left( \frac{du^2}{u^2} + \frac{u^2}{2R_0^2} dx^\mu dx^\mu + d\Omega_5^2 \right). \quad (39)
\]

The corrections cause the effective radius of AdS\(_5\) to become smaller than that of S\(_5\). One can find the asymptotic freedom from the large \( u \) behavior of the leading effective gravity solution. It is an important question whether it survives the full string theoretic treatment. It has been argued that the solution does survive due to the special structure related to the approximate conformal invariance [10,11]. The crucial fact is that the Einstein metric is asymptotic to AdS\(_5 \times S^5\). This geometry is conformal to flat space, so that the Weyl tensor vanishes in the large \( u \) limit. Furthermore, both \( \Phi \) and \( T \) vary slowly for large \( u \).

One interesting feature of the tachyon RG trajectory is that \( T \) starts increasing from its critical value \( T = -1 \) from condition \( f'(T) = 0 \). The precise form of the trajectory for finite \( \rho \) is not known analytically, but the qualitative feature of the RG equation was analyzed by Klebanov and Tseytlin [11]. Since \( \Phi \), \( \xi \) and \( \eta \) have negative second derivatives [see Eqs. (29) - (31)], each of these fields may reach a maximum at some value of \( \rho \). If, for \( \Phi \), this happens at finite \( \rho \), then one reaches a peculiar conclusion that the coupling is decreasing far in the infrared. However this possibility was not realized. Instead, a different possibility was realized: \( \dot{\Phi} \) is positive for all \( \rho \), asymptotically vanishing as \( \rho \to \infty \). They succeeded in constructing the asymptotic form of such trajectory. It is crucial that, as \( \rho \to \infty \), \( T \) approaches zero so that, as in the UV region, \( T^2 e^{(1/2)\Phi} \) becomes small. For this reason the limiting Einstein-frame metric is again AdS\(_5 \times S^5\). Thus the theory flows to a conformally invariant point at infinite coupling. The shape of the tachyon RG trajectory is that the tachyon starts at \( T = -1 \) at \( \rho = 0 \), and grows according to Eq. (34), then enters an oscillating regime and finally relaxes to zero.

**B. non-tachyonic 0B background solution**

Recently there has been attempts to find a non-tachyonic holographic description of non-supersymmetric gauge theory [13,16]. Here we will consider the background geometry
of non-tachyonic type 0B theory in Ref. [16]. Non-tachyonic 0B theory is based on the framework of 0B strings with a particular orientifold projection [17] to eliminate the tachyon from the bulk and to avoid the problems with the doubling of the R-R sectors. The gauge theory from this orientifold of type 0B model becomes conformal in the planar limit, and several results can be copied from $N = 4$. In particular, the leading (planar) geometry is known to be of the form $\text{AdS}_5 \times X^5$. We consider a dual gravitational description of the gauge theory in the spirit of the AdS/CFT correspondence. The gravity description involves the ten-dimensional metric, the dilaton and the R-R four-form potential (with self-dual field strength) whose dynamic is essentially encoded in the action [16]

$$
S = \frac{1}{(\alpha')^4} \int d^{10}x \sqrt{-g} \left[ e^{-2\Phi} (R - 4\nabla_n \Phi \nabla^n \Phi) + \frac{1}{\alpha'} Ge^{-\Phi} - \frac{1}{4} (\alpha')^4 |F_5|^2 \right],
$$

(40)

or in its equations of motion

$$
e^{-2\Phi} (R + 4 \nabla^2 \Phi - 4 (\nabla \Phi)^2) + \frac{1}{2 \alpha'} Ge^{-\Phi} = 0,
$$

(41)

$$
e^{-2\Phi} (R_{mn} + 2 \nabla_m \nabla_n \Phi) - \frac{1}{4 \alpha'} g_{mn} Ge^{-\Phi} - \frac{1}{4 \cdot 4!} (F_{mklpq} F_{n}^{klpq} - \frac{1}{10} g_{mn} F_{sklpq} F^{sklpq}) = 0,
$$

(42)

$$
\nabla_m (F^{mnkpq}) = 0.
$$

(43)

The term $Ge^{-\Phi} = G g_s^{-1}$ represents the dilaton tadpole expected from half-genus in string perturbation theory. Although the direct computation of this term is difficult, there are plausible arguments in favor of its presence [16]. From now on we set $\alpha' = 1$ for simplicity of calculation.

For the metric solution, we make the ansatz compatible with four-dimensional Poincaré invariance

$$
d s^2 = d\tau^2 + e^{2\lambda(\tau)} (-dt^2 + dx_i dx_i) + e^{2\nu(\tau)} d\Omega_5^2,
$$

(44)

where $\tau$ is the radial direction transverse to the three-brane ($i = 1, 2, 3$). The five-form field strength has $N - 1/2$ units of flux induced by the $N$ D3 branes and the $O'3$ plane. We shall ignore the contribution of the orientifold plane, manifestly suppressed for large $N$. Inserting the above ansatz in Eqs. (41) and (42) and allowing for a $\tau$ dependence of the various fields, we have

$$
2 \ddot{\Phi} - 4 \ddot{\lambda} - 5 \ddot{\nu} - 4 \dot{\lambda}^2 - 5 \dot{\nu}^2 + \frac{1}{2} N^2 e^{2\phi - 10\nu} - \frac{1}{4} Ge^{\Phi} = 0,
$$

(45)

$$
\ddot{\lambda} - (2 \dot{\Phi} - 4 \dot{\lambda} - 5 \dot{\nu}) \dot{\lambda} - \frac{1}{2} N^2 e^{2\phi - 10\nu} + \frac{1}{4} Ge^{\Phi} = 0,
$$

(46)

$$
\ddot{\nu} - (2 \dot{\Phi} - 4 \dot{\lambda} - 5 \dot{\nu}) \dot{\nu} - 4 e^{-2\nu} + \frac{1}{2} N^2 e^{2\phi - 10\nu} + \frac{1}{4} Ge^{\Phi} = 0,
$$

(47)
\[ 4\dot{\Phi}^2 + 12\dot{\lambda}^2 + 20\dot{\nu}^2 - 16\ddot{\lambda} - 20\nu\dot{\Phi} + 40\dot{\nu}\dot{\Phi} - 20e^{-2\nu} - Ge^\Phi + N^2e^{2\phi-10\nu} = 0, \quad (48) \]

where dot denotes the derivative with respect to \( \tau \). Note that the \( \tau\tau \) component (Eq. (48)) of the Einstein’s equation is not independent. This corresponds to the zero energy constraint for the associated mechanical system. If we redefine \( \Phi \rightarrow \Phi - \ln N \) to see the role of the different contributions of the low-energy action, the independent field equations are

\[ 2\ddot{\Phi} - 4\dot{\Phi}^2 + 8\dot{\Phi}\dot{\lambda} + 10\dot{\Phi}\dot{\nu} + 3\frac{G}{N}e^{\Phi} = 0, \quad (49) \]

\[ \ddot{\lambda} - (2\dot{\Phi} - 4\dot{\lambda} - 5\dot{\nu})\dot{\lambda} - \frac{1}{2}e^{2\phi-10\nu} + \frac{1}{4}G e^\Phi = 0, \quad (50) \]

\[ \ddot{\nu} - (2\dot{\Phi} - 4\dot{\lambda} - 5\dot{\nu})\dot{\nu} - 4e^{-2\nu} + \frac{1}{2}e^{2\phi-10\nu} + \frac{1}{4}G e^\Phi = 0. \quad (51) \]

One can understand the role of the dilaton tadpole from the above equations: it represents a \( 1/N \) correction to the leading type IIB supergravity equations. In the large \( N \) limit, we can solve the equations by iterations. The solutions up to the next-leading-order are

\[ \Phi = \varphi - \frac{3}{8 \cdot 2^{3/8}} \frac{G}{N} e^{(5/4)\varphi} \tau, \quad (52) \]

\[ \lambda = 2^{3/8}e^{-(1/4)\varphi} \tau + \frac{3}{64 N} e^{\varphi} \tau^2, \quad (53) \]

\[ \nu = \frac{3}{8} \ln 2 + \frac{1}{4} \varphi - \frac{3}{32} \frac{G}{N} e^{(5/4)\varphi} \tau, \quad (54) \]

and the metric tensor is

\[ ds^2 = d\tau^2 + \exp \left[ 2 \cdot 2^{3/8}e^{-(1/4)\varphi} \tau + \frac{3}{32 N} e^{\varphi} \tau^2 \right] (-dt^2 + dx^2) \]

\[ + \exp \left[ -\frac{3}{8} \ln 2 + \frac{1}{2} \varphi - \frac{3}{32} \cdot 2^{3/8} \frac{G}{N} e^{(5/4)\varphi} \tau \right] d\Omega_5^2, \quad (55) \]

where \( \varphi \) is a constant. Note that the introduction of a dilaton tadpole results into a running of dilaton and running radii for the AdS\(_5\) and \( \text{RP}^5 (= \text{S}^5/\mathbb{Z}_2) \) [10], where \( \mathbb{Z}_2 \) the non-tachyonic involution associated to \( O'3 \).

**IV. BRANE COSMOLOGY**

In this section we will consider the cosmology of probe D3-brane when it is moving along a geodesic in the background of two type 0B solutions of the previous section.
A. brane inflation under the tachyonic 0B background

The metric of D3-brane using the background solution (38) can be written as

\[ g_{00}(y) = g(y) = \left(1 - \frac{1}{2}y - \frac{39}{4y^2} \ln y + \cdots \right) e^{y/2} \]
\[ g_{yy}(y) = \left(1 - \frac{9}{2}y - \frac{351}{4y^2} \ln y + \cdots \right) \frac{R_0^2}{16} \]
\[ g_S(y) = \left(1 - \frac{1}{2}y - \frac{39}{4y^2} \ln y + \cdots \right) R_0^2 \]

(56)

To apply the formalism of Sec. II we also need to express RR field in terms of \( y \). From the ansatz for the RR field

\[ C_{0123} = C(y), \quad F_{0123} = \frac{dC(y)}{dy}, \]

Eq. (57) can be integrated once

\[ \frac{dC(y)}{dy} = C_1 g^2 g_{-5/2} g_{yy} f^{-1}(T) \]

(58)

where \( C_1 \) is a constant. One can easily check that \( C_1 = 2Q \) is the right choice to match the parameters which we already used in Sec. III. Using the solution of the metric in (56), the RR field can be integrated with appropriate normalization

\[ C(y) = \frac{1}{Q} e^y \left(1 - \frac{2}{y} \right) + Q_1, \]

(59)

where \( Q_1 \) is another integration constant. This constant can be absorbed in the redefinition of energy \( E' \equiv E + Q_1 \)

Now we can calculate the effective density on the brane using Eqs. (34), (35), (56), and (59)

\[ \frac{8\pi}{3} \rho_{\text{eff}} = \left(\frac{8}{Q}\right)^{1/2} \left(1 + \frac{7}{y} \right) \]
\[ \times \left[ \left(\frac{1}{Q} \left(1 - \frac{2}{y} \right) + \frac{E'}{e^y} \right)^2 \frac{2^{31}}{y^4} \left(1 - \frac{1}{2}y \right) + \cdots \right] k^{-2}(T) \]
\[ + \left(1 - \frac{2}{y} \right) \left[ \left(1 - \frac{3}{2}y \right) + \cdots \right] \left(\frac{\ell^2}{Q} \frac{2^{32}}{y^4} e^{-(3/2)y} k^{-2}(T) \right) \].

(60)

If we substitute the scale \( a^2 = g(y) \) and Eq. (4) to see the leading power behavior near the horizon region, we have

\[ \frac{8\pi}{3} \rho_{\text{eff}} = \left(\frac{8}{Q}\right)^{1/2} \left[ \left(\frac{1}{Q} + \frac{E' \ell^2}{2Q a^4} \right)^2 \frac{2^{27}}{9 (\ln a)^4} - \left\{1 + \frac{\ell^2}{9\sqrt{2} Q^{5/2}} \frac{2^{27} (\ln a)^4}{a^6} \right\} \right]. \]

(61)

Near the black brane, one can see that \( \rho_{\text{eff}} \sim 1/[e^{8(\ln a)}] \). Without the logarithmic term, the cosmological expansion due to the brane motion is indistinguishable from that of type IIB background.
B. brane inflation under non-tachyonic 0B background

We can repeat the same procedure for non-tachyonic case by setting \( k(T) = 1 \). The dilaton and metric background solution for this case are

\[
\Phi = \varphi - \frac{3}{8 \cdot 2^{3/8}} \frac{G}{N} e^{(5/4) \varphi} \tau, \tag{62}
\]

\[
|g_{\tau 0}(\tau)| = g(\tau) = \exp \left[ 2 \cdot 2^{3/8} e^{-(1/4) \varphi} \tau + \frac{3}{32} \frac{G}{N} e^{(5/4) \varphi} \tau^2 \right], \tag{63}
\]

\[
g_{\tau \tau} = 1, \tag{64}
\]

\[
g_S(\tau) = \exp \left[ -\frac{3}{8} \ln 2 + \frac{1}{2} \varphi - \frac{3}{32} \cdot 2^{3/8} \frac{G}{N} e^{(5/4) \varphi} \tau \right]. \tag{65}
\]

Also from the ansatz for the RR field

\[
C_{0123} = A(\tau), \quad F_{0123} = \frac{dA(\tau)}{d\tau}, \tag{66}
\]

Eq. (43) can be solved as

\[
A(\tau) = \frac{N}{\sqrt{2}} e^{-\varphi} \exp \left[ 4 \cdot 2^{3/8} e^{-(1/4) \varphi} \tau + O(1/N) \right] + A_1, \tag{67}
\]

where \( A_1 \) is an integration constant which can be absorbed in the redefinition of energy \( E' = E + A_1 \). If we calculate the effective density on the brane, in the large \( N \) limit, we have

\[
\frac{8\pi}{3} \rho_{\text{eff}} = 2^{3/4} e^{-\varphi/2} \left[ \left( \frac{N}{\sqrt{2}} + \frac{E'^{2 \varphi}}{\bar{a}^4} \right)^2 - (1 + \ell^2 2^{3/4} e^{(3/2) \varphi} a^{-6}) \right]. \tag{68}
\]

In the limit \( a \to 0 \), we have \( \rho_{\text{eff}} \sim a^{-8} \). This power behavior agrees with the result of type II case [7].

V. DISCUSSION

We considered the motion of a three-brane moving in a background bulk space of type 0B string theory. Both the tachyonic and non-tachyonic backgrounds are considered. The cosmological pictures of two non-supersymmetric backgrounds are expected to be different. This is because the Born-Infeld action, which determine the motion of the brane, in the presence of the tachyonic field is different from the one without it. We set up the the modified formalism of brane inflation when there is tachyon field in the background.

The effective density for non-tachyonic 0B background is proportional to \( a^{-8} \) as \( a \to 0 \). This is the same power law behavior as that for type IIB background. The effective density for tachyonic 0B background is \( \rho_{\text{eff}} \sim 1/[a^8 (\ln a)^3] \). Comparing the two results we conclude that the brane inflation with tachyonic field in the background is less divergent than the one without tachyon. At first sight, this seems contradiction to our physical sense. If we add any matter field in the ambient space the brane universe will inflate faster, compared
with the case without that field, due to the increased effective density. This phenomenon
was shown in type IIB theory by turning on the electromagnetic energy on the brane [7] and
by turning on the axion field [8]. It seems that this is true for any ordinary matter field.
However, this might not be true when the field is tachyonic whose mass squared is negative.
The presence of the tachyonic field decreases the effective density on the brane and slows
down the inflation. So we conclude that the role of tachyonic field in brane inflation scenario
is opposite to the ordinary matter field.

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