Optomechanics with interacting Fermi gases: a new approach to detecting spin–charge separation in one-dimensional ultracold atom systems

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New Journal of Physics 15 (2013) 013013 (11pp)
Received 3 May 2012
Published 9 January 2013
Online at http://www.njp.org/
doi:10.1088/1367-2630/15/1/013013

Abstract. We consider a one-dimensional two-component interacting Fermi gas confined in a cavity. We show that, taking account of the polarization of the cavity field, one can realize an effective cavity optomechanical model with the spin and charge modes playing the role of coupled mechanical resonators, which gives rise to multistability in a steady state. Then, we propose that, by tuning the weak probe laser under a pump field, the signal of spin–charge separation could be probed explicitly via transmission spectra in the sideband resolved regime. Moreover, the spin–charge modes can be addressed separately by designing the probe field configurations, which may be beneficial for future studies of atom-cavity systems.

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1. Introduction

One-dimensional (1D) quantum liquids have been fascinating to condensed-matter physicists for quite a few decades [1]. For a 1D quantum liquid, the low energy behavior of the system lies in a universal class [2], which results in the remarkable phenomenon where a single-particle excitation fractionizes into collective charge and spin parts and separates. However, clear observation of this phenomenon has proven to be challenging in past decades in solid state materials [3]. Recently, low-dimension quantum fluids have been successfully realized in a cold atom context [4]. The unprecedented tunability of interaction and dimensionality make it a powerful tool to explore Luttinger liquid [5] or Tonks gas [6, 7] in 1D. Stimulated by these experimental advances, some authors propose to detect spin–charge separation in ultracold gases by tracking a wave-packet motion or analyzing the spectrum of a single-particle excitation [8–11]. However, because of the small available spatial and limited time scales, the explicit signals have not been observed in cold atom experiments as yet.

Very recently, cavity optomechanics with cold atoms [12, 13] or a Bose–Einstein condensate (BEC) [14] has acquired remarkable achievements, where the low energy collective excitation of cold atoms behaves as a ‘moving mirror’. In such experiments, one observes a clear oscillation with the mechanical modes periodically interrupted by the cavity field. Subsequent to the work by the Esslinger group, some authors [15, 16] extended the cavity optomechanics with BEC to 1D interacting Bose gas or free spinless Fermi gas, and showed that the collective excitations of a 1D quantum gas might be explored by the oscillation frequency or noise spectrum [17]. Stimulated by these works, we propose in this paper to detect the spin–charge separation in 1D quantum liquids based on the cavity optomechanics.

However, the above methods could not be extended directly to probe the spin and charge modes simultaneously. Here, we propose a new procedure by adding a weak tunable and well-controlled probe field to the polarization-degenerate optical cavity [18]. Compared with the conventional Langevin noise, which generally comprises all frequency components in the output spectrum, this probe field has a specific frequency that can be tuned conveniently to excite resonantly the charge/spin modes. Furthermore, the spin configuration of this probe field can be adjusted to be either in-phase or out-of-phase. We shall first derive a new effective cavity optomechanical model with two coupled spin–charge modes [19]\(^4\), which exhibits rich physics of optical multistability. Then we show that, by tuning the weak probe field, the

\(^4\) Recently optomechanics with multiple mechanical modes has attracted much interest, see [19].
Figure 1. (a) A collection of two-component hyperfine fermionic atoms are confined in an effectively 1D trap inside an optical cavity of length $L$. The cavity mode is driven by a ‘pump’ laser of frequency $\omega_L$, and a weak ‘probe’ laser of frequency $\omega_p$ is added to stimulate the system’s optical response. ‘Out’ is the transmission field. Both the fields are polarization dependent, illustrated with red and green arrows in a circle. (b) Internal energy levels of the two-component atoms with detuning $\Delta_\alpha = \omega_L - \omega_a$. Here, $\omega_a$ is the resonant frequency between the ground $|g\rangle$ and excited $|e\rangle$ states.

explicit signal of spin–charge separation could be probed definitely in the sideband regime. Moreover, the spin/charge modes can be addressed separately by designing the probe field configurations. Therefore this technique offers a new feasible scheme beyond the preceding methods.

This paper is organized as follows. In section 2, we consider a 1D spin-1/2 interacting Fermi gas coupled with an optical cavity dispersively. By taking account of the polarization degree of the light field, we derive an effective two-mode cavity optomechanical model with the collective spin/charge modes playing the role of mechanical oscillators. Then, in section 3, we analyze the steady state of such a model, which shows nontrivial multistability behavior. Based on the steady state solution, we calculate the intracavity field quadrature and transmission spectrum response to a weak probe field in section 4, which encodes the explicit signals of spin/charge separation. We also discuss the experimental-related parameters there. Finally, we give a summary in section 5.

2. The model

The system under investigation is illustrated in figure 1(a), where two-component fermionic atoms of mass $M$ are confined in a 1D trap inside an optical cavity along the cavity axis. The cavity mode of frequency $\omega_0$ is driven by a pump laser, and we also add a weak probe field to the cavity to stimulate the fluctuations of the system. For the following study, we take account of the circular polarization degree of the cavity field, which can be separately driven and probed by the pump and probe field correspondingly. Each circular polarization state of the cavity field couples to an atomic internal state (see figure 1(b)) with resonant energy $\omega_a$ and induces a quantized potential on atoms in the far-off resonance limit. Then, in the dipole and rotating-wave
approximations, the atomic part of Hamiltonian can be written as [20]

\[
\hat{H}_a = \sum_\sigma \int dx \hat{\Psi}_\sigma^\dagger(x) \left[ \frac{\hat{p}_x^2}{2M} + hU_0^\sigma \cos^2(Kx) \hat{c}_\sigma^\dagger \hat{c}_\sigma \right] \hat{\Psi}_\sigma(x) + g_{1D} \int dx \hat{\Psi}_1^\dagger(x) \hat{\Psi}_1(x) \hat{\Psi}_1^\dagger(x) \hat{\Psi}_1(x).
\]

(1)

Here, \( \hat{\Psi}_\sigma(x), \sigma = \uparrow, \downarrow \) is the pseudo-spin atomic field operator for two hyperfine fermionic atoms, \( \hat{c}_\sigma \) is the cavity field operator for left/right polarization, and \( U_0^\sigma = U_0 = g_0^2/\Delta_a \) is the optical dipole potential strength for a single intracavity photon with \( K = 2\pi/\lambda_c \) (\( \lambda_c = 1200 \text{ nm} \)) the wave vector of the cavity mode. \( g_{1D} = \frac{4\pi\hbar_a}{M} \) is the strength of contact interaction between fermions with opposite spin, and \( a_s \) is the effective 1D low-energy s-wave scattering length, which can be tuned by Feshbach resonance.

First, following the standard procedure, we transform the atomic field operator into momentum representation by \( \hat{\Psi}_\sigma(x) = L^{-1/2} \sum_\nu \hat{f}_{k,\sigma} e^{ikx} \), where \( \hat{f}_{k,\sigma} \) is the fermion annihilation operator for a plane wave with wave vector \( k \). Then, Hamiltonian (1) can be rewritten as

\[
\hat{H} = \sum_{k,\sigma} \epsilon(k) \hat{f}_{k,\sigma}^\dagger \hat{f}_{k,\sigma} + g_{1D} \sum_{k_1, k_2, q} \hat{f}_{k_1+q, \uparrow}^\dagger \hat{f}_{k_1, \uparrow} \hat{f}_{k_2-q, \downarrow}^\dagger \hat{f}_{k_2, \downarrow} + \sum_\sigma c_\sigma^\dagger c_\sigma \left[ \hbar \Delta_\sigma + \frac{1}{4} \hbar U_0 \sum_k (\hat{f}_{k+2K, \sigma}^\dagger \hat{f}_{k, \sigma} + \text{h.c.}) \right],
\]

(2)

where \( \epsilon(k) = \hbar^2 k^2/2M \) is the single particle kinetic energy and \( \Delta_\sigma = \omega_0 - \omega_L + U_0 N_\sigma/2 \) is the effective cavity detuning. Here, we concern the spin-balanced case with \( N_\sigma = N \) and \( \Delta_\sigma = \Delta \).

We shall work in the low photon numbers limit and consider only the lowest momentum transfer of 2\( K \) induced by photons. For low temperature and small momentum \( K \ll k_p = \pi N/L \), the particle–hole excitations occur around the Fermi surface (Fermi points in 1D). One may then implement the bosonization procedure [1] by introducing the following bosonic operators:

\[
\hat{a}_{k,\sigma}^\dagger = \sqrt{\frac{2\pi}{Lk}} \hat{\rho}_{k,\sigma}^\dagger(-k), \quad \hat{a}_{k,\sigma} = \sqrt{\frac{2\pi}{Lk}} \hat{\rho}_{k,\sigma}^\dagger(k > 0).
\]

(3)

Here, \( \hat{\rho}_v^\dagger(k) = \sum_\lambda \hat{f}_{k+\lambda v, \sigma}^\dagger \hat{f}_{k, \sigma} \) are density operators for the right and left moving fermions with \( v = R, L \). By further introducing the charge and spin density bosonic operators \( \hat{b}_{k,\lambda}^v = \frac{1}{\sqrt{2}} (\hat{a}_{k,\lambda}^R + \hat{a}_{k,\lambda}^L) \), \( \lambda = c, s \), which physically correspond to the total density and relative density of the two spin components respectively, the Hamiltonian (2) in density operators reads

\[
\hat{H}' = \hbar \nu_F \sum_{\lambda, k > 0} k \left[ \sum_v \left( 1 \pm \frac{g_{1D}}{2\pi \nu_F} \right) \hat{b}_{k,\lambda}^v \hat{b}_{k,\lambda}^v + \frac{g_{1D}}{2\pi \nu_F} (\hat{b}_{k,\lambda}^R \hat{b}_{k,\lambda}^L + \text{H.c.}) \right] + \sum_\sigma \hbar U_0 \frac{KL}{4\pi} \hat{\rho}_\sigma^\dagger (\hat{b}_{2K,\sigma}^\dagger + \hat{b}_{2K,\sigma}^\dagger) + \sum_\sigma \hbar \Delta_\sigma \hat{n}_\sigma.
\]

By performing the Bogoliubov transformations \( \hat{a}_{k,\lambda}^R = \cosh \gamma_\lambda \hat{b}_{k,\lambda}^R + \sinh \gamma_\lambda \hat{b}_{k,\lambda}^L \), \( \hat{a}_{k,\lambda}^L = \sinh \gamma_\lambda \hat{b}_{k,\lambda}^R + \cosh \gamma_\lambda \hat{b}_{k,\lambda}^L \) with tanh 2\( \gamma_\lambda = \frac{\pm \hbar \nu_\lambda}{2\pi \nu_F + g_{1D}} \), we derive an effective cavity optomechanical model with coupled spin–charge modes (other \( q \neq 2K \) modes are irrelevant)

\[
\hat{H}_{\text{eff}} = \sum_{v, \lambda} \hbar \omega_{q,\lambda} \hat{q}_{q,\lambda}^v \hat{a}_{q,\lambda}^v + \sum_{v, \lambda} \hbar \tilde{U}_\lambda \hat{n}_\lambda (\hat{a}_{q,\lambda}^v + \hat{a}_{q,\lambda}^v) + \sum_\sigma \hbar \Delta_\sigma \hat{n}_\sigma.
\]

(4)
Here, the first term describes the charge/spin fluctuations of the 1D interacting Fermi gas, which play the role of mechanical resonators with frequency \( \omega_{q=2K} = 2Ku_\perp \). \( u_\perp = v_F \sqrt{(1 \pm \frac{g_{1D}}{2 \pi v_F})^2 - \left(\frac{g_{1D}}{2 \pi v_F}\right)^2} \) are the sound velocities of the charge and spin excitations for \( g_{1D}/\pi v_F \ll 1 \). The second term is the linear coupling between the mechanical modes and cavity fields with \( \hat{U}_\perp = \frac{4\pi}{\sqrt{2}} \sqrt{\frac{\rho_L}{2}} (\cosh \gamma_\perp - \sinh \gamma_\perp) \) and \( \hat{n}_{c,s} = \hat{n}_\uparrow \pm \hat{n}_\downarrow \).

### 3. The steady state

To describe the dynamics of the above driven optomechanical model, we introduce the quadratures of the mechanical oscillators

\[
\hat{X}_\perp = \sum_v \hat{X}^v_\perp, \quad \hat{P}_\perp = \sum_v \hat{P}^v_\perp \quad \text{with} \quad \hat{X}^v_\perp = (\hat{a}^{\dagger} q_\perp + \hat{a} q_\perp)/\sqrt{2}, \quad \hat{P}^v_\perp = i(\hat{d}^{\dagger} q_\perp - \hat{d} q_\perp)/\sqrt{2}.
\]

Then, we arrive at the coupled Heisenberg–Langevin equations

\[
\begin{align*}
\frac{d\hat{X}_\perp}{dt} &= \omega_\perp \hat{P}_\perp, \\
\frac{d\hat{P}_\perp}{dt} &= -\omega_\perp \hat{X}_\perp - \Gamma_\perp \hat{P}_\perp - 2\sqrt{2}\hat{U}_\perp \hat{n}_\perp,
\end{align*}
\]

where \( \kappa \) is the cavity decay rate and \( s_\perp = \bar{s}_\perp + \delta s_\perp \) denotes the total amplitude of the external fields. Here, \( \bar{s}_\perp \equiv \langle s_\perp \rangle \) represents the pump field, and \( \delta s_\perp = s_\perp + \delta s_\perp^\sigma \) is a small perturbation, with \( s_\perp \) the weak probe field and \( \delta s_\perp^\sigma \) the Markovian noise satisfying \( \langle \delta s_\perp^\sigma(t)\delta s_\perp^\sigma(t') \rangle = 2\kappa \delta_{\sigma r}\delta(t-t') \), and \( \langle \delta s_\perp^\sigma(t)\delta s_\perp^r(t') \rangle = \delta_{\sigma r} \theta \). \( \theta \) is the tunable coupling parameter. We also introduce an effective mechanical damping \( \Gamma_\perp \), which is related to the finite lifetime of the spin/charge modes.

This effective cavity optomechanics exhibits rich physics compared with previous works. First, we briefly consider the steady-state behavior of the coupled system. The mean-field solutions of equations (5) are \( \hat{P}_\perp = 0, \hat{X}_\perp = -2\sqrt{2}\hat{U}_\perp \hat{n}_\perp/\omega_\perp \), and

\[
\hat{n}_\sigma = \frac{\eta_\sigma^2}{\kappa^2 + \Delta - 4\langle \hat{U}_\perp^2 \omega_\perp^{-1}\hat{n}_\perp + \sigma \hat{U}_\perp^2 \omega_\perp^{-1}\hat{n}_\perp \rangle^2}
\]

with \( \eta_\sigma = \sqrt{2\theta \kappa \bar{s}_\perp} \). Figures 2(a) and (b) show the mean-field intracavity photon numbers versus the symmetrical pump rate and detuning. We find that, because of the strong coupling between spin and charge modes, there exists exotic optical multistability, see figure 2(c). (i) Besides the expected symmetrical solution (blue line) of \( \hat{n}_\uparrow = \hat{n}_\downarrow \), there exist symmetry-breaking solutions with \( \hat{n}_\uparrow \neq \hat{n}_\downarrow \) (the red and green lines). (ii) For the symmetrical solution, the instability in the bottom channel begins at the crest of the red line not, as expected, of the blue line, see the dashed line. (iii) For the symmetry-breaking solutions, our stability analysis shows that only the thick solid lines with \( \hat{n}_\uparrow \) (red) and \( \hat{n}_\downarrow \) (green) or vice versa are stable. These are unique signatures compared with the usual optical bistability in previous works, and could be explored in experiments via a nonequilibrium pump.

### 4. Intracavity field quadrature response and transmission spectrum

Now, we turn to discuss the system’s optical response to the small perturbation around a steady state. Such a response generally comprises the contribution of the probe field and the noise.
In this paper, we follow the linear response regime, and the contribution of noise can be separated. Thus, to distill the main physics, but without losing generality, in the following we focus on the probe field response. For a symmetrical pump, we consider the stable branch below the threshold of bistability with $\bar{n}_c = \bar{n}_s = \bar{n}$, which gives rise to $\bar{n}_c = 2\bar{n}$, $\bar{X}_c = -4\sqrt{2}\tilde{U}_c\bar{n}/\omega_c$ and $\bar{n}_s = 0$, $\bar{X}_s = 0$. Then, the optical response to the probe field is obtained via a linearization of equations (5) around the steady-state:

$$\frac{d\delta \hat{X}_\lambda}{dt} = \omega_c \delta \hat{P}_\lambda, \quad \frac{d\delta \hat{P}_\lambda}{dt} = -\omega_c \delta \hat{X}_\lambda - \Gamma_c \delta \hat{P}_\lambda - 4\sqrt{2}\tilde{U}_c\sqrt{\bar{n}}\delta \hat{X}_\lambda,$$

$$\frac{d\delta \hat{P}_\lambda}{dt} = \sqrt{2} \kappa \delta \hat{P}_{\lambda,\text{in}} - \tilde{\Delta} \delta \hat{X}_\lambda - \kappa \delta \hat{P}_\lambda - 2\sqrt{2}\tilde{U}_c\sqrt{\bar{n}}\delta \hat{X}_\lambda,$$

$$\frac{d\delta \hat{X}_\lambda}{dt} = \sqrt{2} \kappa \delta \hat{X}_{\lambda,\text{in}} + \tilde{\Delta} \delta \hat{P}_\lambda - \kappa \delta \hat{X}_\lambda,$$

with $\tilde{\Delta} = \Delta - 8\tilde{U}_c^2\bar{n}\omega_c^{-1}$. Here, $\delta \hat{X}_{\sigma,c,s} = (\delta \hat{X}_\uparrow \pm \delta \hat{X}_\downarrow)/\sqrt{2}$, $\delta \hat{P}_{\sigma,c,s} = (\delta \hat{P}_\uparrow \pm \delta \hat{P}_\downarrow)/\sqrt{2}$ represent the cavity-field charge–spin quadratures with $\delta \hat{X}_\sigma = (\delta \hat{c}_\sigma^+ \pm \delta \hat{c}_\sigma)/\sqrt{2}$, $\delta \hat{P}_\sigma = i(\delta \hat{c}_\sigma^+ - \delta \hat{c}_\sigma)/\sqrt{2}$. 

**Figure 2.** Steady-state behavior of the coupled cavity optomechanical model for the symmetrical pump rate $\eta_1 = \eta_2 = \eta$. (a), (b) Mean-field intracavity left (right) circular polarized photon numbers versus $\eta/\kappa$ for $\Delta = (\omega_c + \omega_s)/2$, and versus detuning $\Delta$ for $\eta/\kappa = 4$. (c) The optical multistability shown with left/right circular polarized photon number versus pumping strength $\eta/\kappa$, where the blue and red (green) lines denote the symmetrical and symmetry-breaking solutions respectively, see the main text.
Figure 3. (a) Real and (b) image parts of the intracavity field response versus $\Omega$ (in unit of $\Delta$). The solid line shows the total response $\mathcal{R}_{R}$(↑) to the polarized probe field, and the dash-dotted and dashed lines show the responses $\mathcal{R}_{c}$($\mathcal{R}_{s}$) to the charge (spin) modes respectively.

And $\delta X_{in}^{c,s} = (\delta X_{in}^{\uparrow} \pm \delta X_{in}^{\downarrow})/\sqrt{2}$, $\delta Y_{in}^{c,s} = (\delta Y_{in}^{\uparrow} \pm \delta Y_{in}^{\downarrow})/\sqrt{2}$ denotes the corresponding probing field terms with $\delta X_{in}^{c} = (s_{p}^{c} \sigma_{c}^{s} + s_{p}^{s} \sigma_{c}^{c})/\sqrt{2}$, $\delta Y_{in}^{c} = i(s_{p}^{c} \sigma_{c}^{s} - s_{p}^{s} \sigma_{c}^{c})/\sqrt{2}$.

We note that, although both the mechanical modes are coupled nonlinearly with the cavity field in equations (5), the fluctuations of spin and charge modes in the above equations (7) can be excited independently, which encodes the explicit signal of spin–charge separation. To see this, we transform equations (7) into frequency space in a rotating frame. Here both the fluctuations of mechanical and cavity field variables oscillate at frequencies $\pm \Omega$ around the steady-state, with $\Omega = \omega_{p} - \omega_{L}$ being the frequency difference between the probe and pump fields. Then, the intracavity field amplitude can be derived as

$$A_{\lambda}[\Omega] = \frac{1 - i f_{\lambda}(\Omega)}{-i(\Omega - \Delta) + \kappa + 2\Delta f_{\lambda}(\Omega)} \sqrt{2\kappa s_{p}^{c}}$$

with

$$f_{\lambda}(\Omega) = \frac{4\tilde{U}_{c,s}^{2}n_{c}^{2} + 1}{\kappa - i(\Omega + \Delta)} \frac{\Omega^{2} - \omega_{c,s}^{2} + i\Omega}{\omega_{c,s}^{2} + i\Omega}$$

Here, $s_{p}^{c} = (s_{p}^{\uparrow} \pm s_{p}^{\downarrow})/2$ represents the input charge/spin probe field amplitudes.

Before proceeding, we consider the following parameters [16]: $L \sim 100 \mu m$, $U_{0} \simeq 2\pi \times 20$ kHz, and $N \simeq 5000$ alkali metal atoms. For $^{6}$Li, $M \sim 1 \times 10^{-26}$ kg. So that $\tilde{U}_{c,s} \simeq 2\pi \times (0.39, 0.42)$ MHz and $\omega_{c,s} \simeq 2\pi \times (7.14, 6.08)$ MHz for $g_{1D} = 0.5v_{F}$. Here we should mention

5 We note that, because the frequency of the probe field is $\omega_{p} = \omega_{c} + \Omega$, the transmission depends only on the $+\Omega$ part of the cavity field amplitudes.

6 The effective mechanical frequency $\omega_{c,s}$ is intimately related to the atom number $N$, atom mass $M$ and the cavity wave vector $K$.}

New Journal of Physics 15 (2013) 013013 (http://www.njp.org/)
Figure 4. Normalized transmission spectra for different values of detuning $\tilde{\Delta}$ and three kinds of probe fields with polarized (solid), in-phase (dashed) and out-of-phase (dash-dotted) configurations.

that this theoretical atom number is in fact a little bit harder in current experiments because the cooling of the fermionic gas is not as easy as bosonic gas due to the fermionic Pauli exclusion principle. However, with the foreseeable advancement in the state of the art, we expect that this number can be realized in the foreseeable future. To demonstrate the main physics of our work, here we choose more optimistic parameters for better illustration. Correspondingly, the typical cavity damping $\kappa$ in experiments is about $\kappa = 2\pi \times 1$ MHz. Such parameters satisfy $\kappa \ll \omega_\lambda$ and place the system well in the resolved sideband regime, in which the $-\Omega$ part of cavity fluctuations can be neglected [21, 22]. In this regime, when the coupling between the cavity mode and the mechanical mode is strong, i.e. $U_{\lambda} \sqrt{n} \gg \kappa$, there exists normal mode splitting [21]. Here, we consider the opposite $U_{\lambda} \sqrt{n} \ll \kappa$ [22], where the signal of spin–charge separation could be probed explicitly, see below. Also for cold atom systems, the effective mechanical damping $\Gamma_\lambda$ (i.e. the lifetime of the collective excitations), which broadens the width of the peaks in the transmission spectrum [7], is estimated to be $\Gamma_{\lambda} \sim (k_B T/\epsilon_{kr})^2 \omega_\lambda \ll \omega_\lambda$ [8], so we neglect the damping effect in the following discussion.

\footnote{In the sideband resolved regime and weak coupling case ($U_{\lambda} \sqrt{n}, \Gamma_\lambda \ll \kappa$), the peaks around $\omega_\lambda$ in the transmission spectrum can be described well by a Lorentzian form of $\sim \frac{1}{(U_{\lambda} \sqrt{n}/\kappa + 1)^2 + 4(\Omega - \omega_\lambda)^2}$. It can be seen that}
Figure 5. Frequencies of the charge ($\omega_c$) and spin ($\omega_s$) modes versus interacting parameter $g_{1D}$ for giving $K$. The dotted line marks the parameter used in figures 3 and 4.

One of the most important observable quantities in experiments is the circular polarized intracavity field response to the probe field $R_\sigma[\Omega] \equiv \sqrt{2\theta \kappa} A_\sigma[\Omega]/s_\sigma$, which reads

$$R_\sigma[\Omega] = \alpha R_c[\Omega] + \sigma \beta R_s[\Omega],$$

with $R_\sigma[\Omega] = \sqrt{2\theta \kappa} A_\sigma[\Omega]/s_\sigma$ and $\alpha = s^c_p/s^\sigma_p$, $\beta = s^s_p/s^\sigma_p$ determined by the configuration of the probe field. By numerically solving equation (8) in the sideband regime, we show the main results of the intracavity field responses to the circular polarized probe field in figure 3 with $\alpha = \beta = 1/2$. A remarkable feature of the spectrum $R_{\uparrow(\downarrow)}$ is that it demonstrates a well-defined double dip centered at the charge/spin modes. The underlying mechanism can be understood as follows: when we input a small probe field to disturb the system around the steady state stabilized by the pump field, the corresponding intracavity field response sets up, which is generally comprised of the response of charge/spin modes. When the frequency difference between the probe and pump fields $\Omega$ is tuned to be one of the mode frequencies, the corresponding collective mode would be excited resonantly, and accordingly the cavity response would drop dramatically. Therefore in this scheme, one does not need to track the motions of spin/charge wave-packets [8], which have different velocities; we only need to detect the resonant frequencies of the collective spin/charge excitations by tuning the probe fields.

To clarify it further, we turn to the normalized probe power transmission $|\tilde{t}_\sigma|^2 = |\tilde{t}^\sigma - t_\sigma^r|^2$, which is independent on the coupling parameter $\theta$. Here, transmission $t_\sigma^\sigma \equiv 1 - R_\sigma$ and $t_\sigma^r = t_\sigma^\sigma (\Omega = \bar{\Delta}, \bar{n} = 0)$ is the residual on resonance transmission in the absence of pumping lasers. In figure 4, we plot the transmission $|\tilde{t}_\sigma|^2$ for different probe field configurations. It can be seen that the charge/spin modes are clearly resolved (solid line). Generally, the probe field has the form of $(s^\uparrow_p + e^{i\phi}s^\downarrow_p)$, where $\phi$ is the relative phase and $s^\sigma_p$ are the amplitudes. We see that, by the main effect of the damping is to broaden the width of the peaks in the transmission spectrum.
adjusting the probe field configuration to be in-phase ($\phi = 0$, dashed line) with equal amplitudes or out-of-phase ($\phi = \pi$, dash-dotted line) with equal amplitudes, the charge/spin modes can be addressed separately. In figure 4, We also investigate the impact of detuning $\Delta$ on the spectra. It can be seen that although the transmission is generally modified, the peaks of the probe spectra always occur at the charge/spin modes, which are independent of detuning $\Delta$. The frequencies of the charge and spin modes versus the interacting parameter $g_{1D}$ for $g_{1D}/\pi v_F < 1$ are shown in figure 5, which could also be observed in experiments.

Until now, we have mainly focused on the fermionic gas. Experimentally, the above scheme could also be realized in two-component Bose gas by implementing the hydrodynamical theory. The velocities of the charge/spin excitations are $u_{c,s} = u_0 \sqrt{1 \pm g_{12}/g}$, with $g$ and $g_{12}$ the intraspecies and interspecies interactions [11]. We find that the effective frequencies of the collective modes in the Luttinger liquid regime also lie well within the resolved sideband limit. Further studies will consider trapping potentials, where the frequency of collective modes have to be time-averaged in a period because of the position-dependent velocities of the excitations [8].

5. Conclusions

In summary, we have shown that, by tuning the weak probe laser under a pump field, the intriguing spin–charge separation in 1D quantum liquids can be probed definitely via the optomechanical coupled atom–cavity system. Such experiments allow us to determine the spin and charge modes simultaneously by designing the weak probe field. Furthermore, the two-mode optomechanics itself, which exhibits optical multistability, may be of interest for future studies of quantum physics.

Acknowledgments

We acknowledge T Esslinger for private communications and Hui Hu for helpful discussions. This work is supported by NCET, NSFC under grants numbers 11704175, 10934010, 60978019, and NKBRSFSC under grants numbers 2011CB921502, 2012CB821305.

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