Aiming at not only economic benefits but also social benefits in university-enterprise cooperation, this paper considers both technological innovation and social responsibility in university-enterprise cooperation. By using differential game theory, this paper provides three models of Nash noncooperative game, Stackelberg game, and cooperative game and discusses the impact of these three models on university-enterprise cooperation. Our results show that the Stackelberg game model can improve the innovation level of the university based on innovation cost-sharing and does not change the social responsibility level of the university without sharing the social cost. Among the three game models, the cooperative game model can bring the greatest innovation level and social responsibility level to the university-enterprise cooperation and make the whole system realize Pareto optimal. Our research can provide a theoretical basis and practical guidance for university-enterprise cooperation.

1. Introduction

The university-enterprise cooperation has been valued by many countries and it is an important strategy. The United States enacted the National Cooperation Research act as early as 1984 to spur the formation of Strategic Research Partnerships (SRP). The SPR includes research collaborations, industry associations, collaborative research and development agreements, strategic alliance networks, technology licensing, university start-ups, and forums for scientists from academia and industry, which contribute to the rapid diffusion of knowledge and technology from universities to businesses [1]. In the Eureka Program, the university-enterprise cooperation is also emphasized, requiring universities and research institutions applying for projects to find corporate partners to jointly apply and undertake [2]. Finland’s science and technology projects require a university-enterprise cooperation model, which makes Finland’s university-enterprise cooperation rank first in the world [3]. As a representative of an emerging developing country, China is establishing a deeply integrated innovation system with university-enterprise cooperation to foster new drivers of economic growth [4]. As a result, most governments have adopted different incentive policies to encourage university-enterprise cooperation and enhance the technological innovation ability of enterprises, so as to improve the core competitiveness of industries.

With the development of economic globalization, it is difficult for enterprises to make innovation and progress by their strength, so they must cooperate with different innovation subjects. Among these partners, universities have become the core of many enterprises because of their unique resource advantages and innovation ability [5, 6]. The university-enterprise collaboration is an extremely complex part of the educational and economic system, which has changed substantially since the 1980s and has led to a series of institutional tools such as contact centers, science parks, incubators, teaching and researcher exchanges, student placements’ programs, joint training programs, and continuing education centers [7, 8]. The university-enterprise collaboration has also been interpreted as industry-university-research cooperation [9–11] which is a teaching form of educational methods and thinking and focuses on cultivating talents with high-level innovation ability and practical skills.
by making full use of the resources of universities and enterprises [12, 13]. At present, there is no unified explanation and definition of university-enterprise cooperation, and some scholars explain it from different perspectives [14–17]. In this paper, we define university-enterprise cooperation as a social and economic activity that promotes industrial development through the combination of various elements by colleges, scientific research institutes, and enterprises relying on their advantages and resources, by cultivating innovative talents and realizing the transformation of scientific and technological achievements. In other words, unlike other cooperation between organizations that only pursue profit, university-enterprise cooperation also assumes social responsibility.

In academic research on university-enterprise cooperation, collaborative innovation has been concerned by many scholars and is a mainstream research subject. This subject mainly includes three aspects: driving factors of cooperative innovation [18–21], innovation network [22–24], and knowledge sharing [25–27]. However, these studies mainly study university-enterprise cooperation from the perspective of economic effect, without considering the social responsibility factor. As enterprises face pressure from different internal and external stakeholders such as consumers, employees, investors, and communities, they have integrated corporate social responsibility (CSR) into their operations and management [28]. The concept of corporate social responsibility was first proposed by Bowen in 1953. He believed that businessmen had the obligation to make policies, make decisions, or take certain actions under the goals and values expected by the society [29]. Since Bowen’s research pointed out that “voluntary social responsibility by businessmen is a feasible way to improve economic problems and better achieve the economic goals we pursue,” more scholars have begun to study CSR [30–35]. From the existing CSR research literature, we can find that the research subjects are mainly for profit enterprises, with few studies on nonprofit organizations. As a nonprofit organization, universities are important members of society, and they are believed to shape the future, to nurture individuals, to nurture ethics, and to make people more responsible for the companies they work for and the society in which they live [36]. At present, there are few studies specifically focusing on university social responsibility (USR). Latif [36] provided a detailed scale to measure university social responsibility (USR). The study found that USR has four types: operational responsibility, research/development responsibility, stakeholder responsibility, and legal responsibility. Alzyoud and Bani-Hani [37] discussed how universities can achieve development, sustainable development, and competitiveness by using the concept of USR. MuivenHeidi [38] suggested that USR can not only be realized through compliance strategy but also needs a strategy to promote the cultural transformation process as a supplement. Based on an empirical assessment of USR levels and complementary factors in BRICS countries (Brazil, Russia, India, China, and South Africa) and based on these literature studies, this paper will focus on the impact of corporate social responsibility and university responsibility on the optimal decision of university-enterprise cooperation.

With the deepening of university-enterprise cooperation and the growing benefits of cooperation, universities and enterprises have begun to pay attention to the cooperation in social responsibility in addition to the pursuit of economic effect. However, through literature review, we find that the research on social responsibility in university-enterprise cooperation has not received enough attention. Gebbels et al. [39] discussed the impact of the joint responsibility of university-enterprise cooperation on students and employees. Smith et al. [40] examined how university-enterprise partnerships change engineering students’ knowledge and attitudes towards social responsibility and found that university-enterprise cooperation can help students gain a more comprehensive understanding of the socio-technical nature of corporate social responsibility and professional engineering practice. However, different from the above studies, this paper establishes three different differential game models to discuss university-enterprise cooperation.

Some scholars have studied the two-party cooperation problem by using the differential game, and the research in this area mainly focuses on R&D. Lamberti and Mantovani [41] discussed the timing of adoption of product and process innovation using a differential game, and they showed that Cournot competition entails lower R&D incentives than Bertrand competition; on this basis, Anton and Greiner [42] presented an R&D differential game of multiproduct oligopolies, and they found that the spillovers may generate a technology lock-in to a situation with a large number of high-quality products. Similar to [42], Liu et al. [43] applied a differential game framework to examine how online customer reviews can be leveraged as external knowledge for duopoly manufacturers R&D, and they found that when the manufacturer utilizes online customer reviews more in R&D, both manufacturers’ profits can either increase or decrease. Recently, Wei and Wang [44] and Deng et al. [45] used a differential game to analyze carbon reduction technology innovation. However, these studies did not address the issue of university-enterprise collaboration. At present, there are few kinds of research on introducing differential game into university-industry cooperation. Yin and Li [46] considered three models which are the Nash noncooperative game model, Stackelberg game model, and cooperative game model in a university-enterprise collaborative innovation system using a differential game. Different from [46], this paper considers not only the profits generated by university-enterprise cooperative innovation but also the social responsibilities of both parties.

In summary, the existing literature on university-enterprise cooperation focuses on the analysis of economic effects, and most of them are a static framework. A few kinds of literature focusing on the dynamic framework only consider the impact of technology innovation on university-enterprise cooperation, and there are few studies on social responsibility in university-enterprise cooperation, especially those that consider both technology innovation and social responsibility. Therefore, this paper considers two factors of technological innovation and social responsibility in university-industry cooperation and establishes three models of Nash noncooperative game, Stackelberg game, and cooperative game by using a differential game.
The rest of our paper is organized as follows. Section 2 describes problems, and Section 3 develops three game models: the Nash noncooperative game model, the Stackelberg game model, and the cooperative game model. Section 4 makes a comparative analysis of the models. Some numerical experiments are given by Section 5, and Section 6 concludes this study and outlines directions for future research.

2. Model Description

Consider a university-enterprise cooperation system composed of an enterprise and a university. In the process of university-enterprise cooperation, the university and enterprise will fulfill their social responsibilities while reaping economic benefits through technological innovation. In other words, the enterprise and university pursue not only economic goals but also social goals. The notations used in our models are presented in Table 1.

In terms of university-enterprise cooperation, the innovation levels invested by the university and enterprise are assumed to be \( h_1(t) \) and \( h_2(t) \), respectively. In the aspect of social responsibility, the social responsibility level invested by the university and enterprise is assumed as \( r_1(t) \) and \( r_2(t) \), respectively. Furthermore, the investment cost of the innovation and social responsibility have a nonlinear increase in \( h_1(t) \) and \( r_1(t) \), respectively \((i = 1, 2)\), and these cost functions at time \( t \) can be assumed as follows:

\[
\begin{align*}
C_{11}(t) &= \frac{1}{2}k_{11}h_1^2(t), \\
C_{21}(t) &= \frac{1}{2}k_{21}h_2^2(t), \\
C_{12}(t) &= \frac{1}{2}k_{12}r_1^2(t), \\
C_{22}(t) &= \frac{1}{2}k_{22}r_2^2(t).
\end{align*}
\]  

(1)

The university-enterprise cooperation is a dynamic process, which is determined by the innovation and social responsibility level of the university and enterprise. On the one hand, the university and enterprise can create new technology through cooperation and improve the level of joint innovation. On the other hand, the university and enterprise can take advantage of their advantageous position and shoulder a wider range of social responsibilities and improve the joint social responsibility. Furthermore, the following differential equations can be used to describe the joint innovation level and social responsibility level changing with time in university-enterprise cooperation:

\[
\begin{align*}
L'(t) &= \frac{dL(t)}{dt} = \lambda_1 h_1(t) + \lambda_2 h_2(t) - \delta L(t), \\
L(0) &= l_0 \geq 0, \\
S'(t) &= \frac{dS(t)}{dt} = \eta_1 r_1(t) + \eta_2 r_2(t) - \delta S(t), \\
S(0) &= s_0 \geq 0.
\end{align*}
\]  

(2)

\( L(t) \) is the level of joint innovation at time \( t \) and \( l_0 \) represents the initial state of joint innovation level. \( S(t) \) is the level of joint social responsibility at time \( t \) and \( s_0 \) represents the initial state of joint social responsibility level.

The investment of the university and enterprise in innovation will directly affect the economic benefits of joint innovation. At the same time, the effort of the university and enterprise in social responsibility will directly affect the level of joint social responsibility and affect social benefits. Therefore, the economic benefits and social benefits of university-enterprise cooperation can be obtained as follows:

\[
\begin{align*}
\pi_1(t) &= \alpha_1 h_1(t) + \alpha_2 h_2(t) + \mu L(t), \\
\pi_2(t) &= \beta_1 r_1(t) + \beta_2 r_2(t) + \xi S(t).
\end{align*}
\]  

(3)

Assuming that the total benefits from joint innovation and social responsibility are distributed among the participants under the preagreed distribution ratio, the total economic benefits and social benefits obtained by the university are \( \varphi_1 \pi_1(t) \) and \( \varphi_2 \pi_2(t) \), respectively, and the total economic benefits and social benefits obtained by the enterprise are \((1 - \varphi_1)\pi_1(t) \) and \((1 - \varphi_2)\pi_2(t) \), respectively. To encourage the university to pay more for innovation in the process of the university-enterprise cooperation, the proportion of innovation cost-shared by the enterprise will be \( \omega (0 < \omega < 1) \). Thus, the objective function of the university and enterprise are, respectively,\n
\[
\begin{align*}
J_1 &= \int_0^\infty e^{-\rho t} \left[ \varphi_1 (\alpha_1 h_1(t) + \alpha_2 h_2(t) + \mu L(t)) + \varphi_2 (\beta_1 r_1(t) + \beta_2 r_2(t) + \xi S(t)) - \frac{1}{2} (1 - \omega) k_{11} h_1(t) - \frac{1}{2} k_{12} r_1(t) \right] dt, \\
J_2 &= \int_0^\infty e^{-\rho t} \left[ (1 - \varphi_1) (\alpha_1 h_1(t) + \alpha_2 h_2(t) + \mu L(t)) + (1 - \varphi_2) (\beta_1 r_1(t) + \beta_2 r_2(t) + \xi S(t)) \right. \\
&\quad \left. - \frac{1}{2} k_{21} h_2(t) - \frac{1}{2} k_{22} r_2(t) - \omega \frac{1}{2} k_{11} h_1(t) \right] dt.
\end{align*}
\]  

(4)

This paper assumes that the university and enterprise are faced with the same game at any time point in the university-enterprise system. For analysis purposes, \( t \) will be omitted below.
3. Model Construction and Solution

3.1. Nash Noncooperative Game. In the Nash noncooperative game, the university and the enterprise are independent and have an equal footing, i.e., \( \omega = 0 \). The benefit functions of the two sides are \( V_1(L, S) \) and \( V_2(L, S) \), respectively. Since \( V_1(L, S) \) and \( V_2(L, S) \) are continuously bounded and differentiable for \( L, S \geq 0 \), the HJB equations are as follows:

\[
\rho V_1(L, S) = \max_{h_1, r_1} \left[ \phi_1 \left( \alpha_1 h_1 + \alpha_2 h_2 + \mu L \right) + \phi_2 \left( \beta_1 r_1 + \beta_2 r_2 + \xi S \right) - \frac{1}{2} k_{11} h_1^2 - \frac{1}{2} k_{21} h_2^2 + \frac{\partial V_1}{\partial L} \left( \lambda_1 h_1 + \lambda_2 h_2 - \delta L(t) \right) + \frac{\partial V_1}{\partial S} \left( \eta_1 r_1(t) + \eta_2 r_2(t) - \theta S(t) \right) \right],
\]

(5)

\[
\rho V_2(L, S) = \max_{h_2, r_2} \left[ \left(1 - \phi_1\right) \left( \alpha_1 h_1 + \alpha_2 h_2 + \mu L \right) + \left(1 - \phi_2\right) \left( \beta_1 r_1 + \beta_2 r_2 + \xi S \right) - \frac{1}{2} k_{21} h_1^2 - \frac{1}{2} k_{22} h_2^2 + \frac{\partial V_2}{\partial L} \left( \lambda_1 h_1 + \lambda_2 h_2 - \delta L(t) \right) + \frac{\partial V_2}{\partial S} \left( \eta_1 r_1(t) + \eta_2 r_2(t) - \theta S(t) \right) \right].
\]

(6)

**Proposition 1.** In a Nash noncooperative game, the optimal strategies of university and enterprise are

\[
\left( h_1^N, r_1^N \right) = \left( \frac{\phi_1 \left( \alpha_1 h_1 + \alpha_2 h_2 + \mu L \right)}{\rho + \delta k_{11}}, \frac{\phi_2 \left( \beta_1 r_1 + \beta_2 r_2 + \xi S \right)}{\rho + \delta k_{12}} \right).
\]

(7)

\[
\left( h_2^N, r_2^N \right) = \left( \frac{\left(1 - \phi_1\right) \left( \alpha_1 h_1 + \alpha_2 h_2 + \mu L \right)}{\rho + \delta k_{21}}, \frac{\left(1 - \phi_2\right) \left( \beta_1 r_1 + \beta_2 r_2 + \xi S \right)}{\rho + \delta k_{22}} \right).
\]

(8)
Proof. It is easy to judge that equations (5) and (6) are concave functions with respect to \((h_1, r_1)\) and \((h_2, r_2)\), respectively. Based on the first-order conditions, the optimal decisions of the university and the enterprise can be obtained as follows:

\[
(h_1, r_1) = \left( \frac{\varphi_1 \alpha_1 + \lambda_1 \partial V_1 / \partial L}{k_{11}}, \frac{\varphi_2 \beta_1 + \eta_1 \partial V_1 / \partial S}{k_{12}} \right),
\]

\[
(h_2, r_2) = \left( \frac{(1 - \varphi_1) \alpha_2 + \lambda_2 \partial V_2 / \partial L}{k_{21}}, \frac{(1 - \varphi_2) \beta_2 + \eta_2 \partial V_2 / \partial S}{k_{22}} \right).
\]

Substituting the above two equations into equations (5) and (6), respectively, it can be concluded that the revenue functions of university and enterprise are binary quadratic functions about \(L\) and \(S\). Thus, the following function form can be set:

\[
\begin{align*}
V_1 (L, S) &= f_{11} L + f_{21} S + f_{31}, \\
V_2 (L, S) &= g_{11} L + g_{21} S + g_{31}.
\end{align*}
\]

Furthermore, by the method of undetermined coefficients, we can obtain

\[
\begin{align*}
f_{11} &= \frac{\varphi_1 \mu}{\rho + \delta}, \\
f_{21} &= \frac{\varphi_2 \xi}{\rho + \theta}, \\
f_{31} &= \frac{(\varphi_1 \alpha_1 + \lambda_1 f_{11})^2 + (\varphi_2 \beta_1 + \eta_1 f_{21})^2 + (\varphi_1 \alpha_2 + \lambda_2 f_{11})(1 - \varphi_1) \alpha_2 + \lambda_2 g_{11} + (\varphi_2 \beta_2 + \eta_2 g_{21})(1 - \varphi_2) \beta_2 + \eta_2 g_{21})}{\rho k_{21}}, \\
g_{11} &= \frac{(1 - \varphi_1) \mu}{\rho + \delta}, \\
g_{21} &= \frac{(1 - \varphi_2) \xi}{\rho + \theta}, \\
g_{31} &= \frac{(\varphi_1 \alpha_1 + \lambda_1 f_{11})(1 - \varphi_1) \alpha_1 + \lambda_1 g_{11} + (\varphi_2 \beta_1 + \eta_1 f_{21})(1 - \varphi_2) \beta_1 + \eta_1 g_{21} + ((1 - \varphi_1) \alpha_2 + \lambda_2 g_{11})^2 + ((1 - \varphi_2) \beta_2 + \eta_2 g_{21})^2}{2 \rho k_{21}}.
\end{align*}
\]

From equation (11), equations (7) and (8) can be obtained. Thus, this proposition is proved.

According to Proposition 1, in Nash noncooperative game, the level of joint innovation and social responsibility can be obtained as follows:

\[
L^N = \frac{\lambda_1 h_1^N + \lambda_2 h_2^N}{\delta} + \left( l_0 - \frac{\lambda_1 h_1^N + \lambda_2 h_2^N}{\delta} \right) e^{-\delta t},
\]

\[
S^N = \frac{\eta_1 r_1^N + \eta_2 r_2^N}{\theta} + \left( s_0 - \frac{\eta_1 r_1^N + \eta_2 r_2^N}{\theta} \right) e^{-\theta t}.
\]

Furthermore, the optimal revenue functions of the university and enterprise are obtained as follows:

\[
\begin{align*}
V_1^N &= \frac{\varphi_1 \mu}{\rho + \delta} L^N + \frac{\varphi_2 \xi}{\rho + \theta} S^N + f_{31}, \\
V_2^N &= \frac{(1 - \varphi_1) \mu}{\rho + \delta} L^N + \frac{(1 - \varphi_2) \xi}{\rho + \theta} S^N + g_{31}.
\end{align*}
\]

Then, the optimal revenue function of the university-enterprise cooperation system under this game model can be obtained:
\[ V^N = \frac{\mu}{\rho + \delta} L^N + \xi \frac{\rho}{\rho + \theta} S^N + f_{11} + g_{12}. \]  

(16)

3.2. Stackelberg Game. In the Stackelberg game, the enterprise is the leader and the university is the follower, and both aim to maximize their own benefits. To encourage the university to improve the level of innovation, the enterprise will share the innovation costs of the university in a certain proportion \( \omega \). Thus, the HJB equations are as follows:

\begin{align*}
\rho V_1(L,S) &= \max_{h_1,r_1} \left[ \varphi_1(\alpha_1 h_1 + \alpha_2 h_2 + \mu L) + \varphi_2(\beta_1 r_1 + \beta_2 r_2 + \xi S) - \frac{1}{2} (1 - \omega) k_{11} h_1^2 - \frac{1}{2} k_{12} r_1^2 - \frac{1}{2} k_{12} r_1 \right. \\
& \quad \left. + \frac{\partial V^1}{\partial L} (\lambda_1 h_1 + \lambda_2 h_2 - \delta L(t)) + \frac{\partial V^1}{\partial S} (\eta_1 r_1 (t) + \eta_2 r_2 (t) - \theta S(t)) \right], \\
\rho V_2(L,S) &= \max_{h_1,r_1} \left[ (1 - \varphi_1)(\alpha_1 h_1 + \alpha_2 h_2 + \mu L) + (1 - \varphi_2)(\beta_1 r_1 + \beta_2 r_2 + \xi S) \\
& \quad - \frac{1}{2} k_{21} h_2^2 - \frac{1}{2} k_{22} r_2^2 - \frac{1}{2} k_{12} r_1^2 + \frac{\partial V^2}{\partial L} (\lambda_1 h_1 + \lambda_2 h_2 - \delta L(t)) + \frac{\partial V^2}{\partial S} (\eta_1 r_1 (t) + \eta_2 r_2 (t) - \theta S(t)) \right]. 
\end{align*}

(17) \hspace{1cm} (18)

The game process is divided into two stages: the enterprise first decides the optimal strategy and the proportion of sharing, and the university chooses its optimal strategy after observing the enterprise’s decision. Therefore, Proposition 2 can be obtained.

\begin{align*}
(h_1^*, r_1^*) &= \left( \frac{(2 - \varphi_1)((\rho + \delta) \alpha_1 + \mu \lambda_1)}{2(\rho + \delta) k_{11}}, \frac{\varphi_2((\rho + \theta) \beta_1 + \xi \eta_1)}{(\rho + \theta) k_{12}} \right), \\
(h_2^*, r_2^*) &= \left( \frac{(1 - \varphi_1)((\rho + \delta) \alpha_2 + \mu \lambda_2)}{(\rho + \delta) k_{21}}, \frac{(1 - \varphi_2)((\rho + \theta) \beta_2 + \xi \eta_2)}{(\rho + \theta) k_{22}} \right), \\
\omega^* &= \frac{2 - 3 \varphi_1}{2 - \varphi_1}.
\end{align*}

(19) \hspace{1cm} (20) \hspace{1cm} (21)

Proof. It is easy to judge that equation (17) is a concave function with respect to \((h_1,r_1)\). Based on the first-order conditions, the optimal reaction function of the university can be obtained:

\[ (h_1, r_1) = \left( \frac{\varphi_1 \alpha_1 + \lambda_1 \partial V_1/\partial L}{(1 - \omega) k_{11}}, \frac{\varphi_2 \beta_1 + \eta_1 \partial V_1/\partial S}{k_{12}} \right). \]

(22)

Substituting equation (22) into equation (18), the optimal decisions of the enterprise can be obtained as follows:

\begin{align*}
(h_2, r_2) &= \left( \frac{(1 - \varphi_1) \alpha_2 + \lambda_2 \partial V_2/\partial L}{k_{21}}, \frac{(1 - \varphi_2) \beta_2 + \eta_2 \partial V_2/\partial S}{k_{22}} \right), \\
\omega &= \frac{(2 - 3 \varphi_1) \alpha_1 + \lambda_1 (2 \partial V_1/\partial S - \partial V_1/\partial L)}{(2 - \varphi_1) \alpha_1 + \lambda_1 (\partial V_1/\partial L + 2 \partial V_1/\partial S)}. 
\end{align*}

(23) \hspace{1cm} (24)

Substituting equations (22)–(24) into equations (17) and (18), respectively, it can be concluded that the revenue functions of university and enterprise are binary quadratic functions about \( L \) and \( S \). Thus, the following function form can be set:
Discrete Dynamics in Nature and Society

\[ V_1(L,S) = f_{11}L + f_{22}S + f_{32}, \]
\[ V_2(L,S) = g_{11}L + g_{22}S + g_{32}. \]  

(25)  

Furthermore, by the method of undetermined coefficients, we can obtain

\[
\begin{aligned}
    f_{12} &= \frac{\varphi_1 \mu}{\rho + \delta} \\
    f_{22} &= \frac{\varphi_2 \xi}{\rho + \theta} \\
    f_{32} &= \frac{(\varphi_1 a_1 + \lambda_1 f_{12})((2 - \varphi_1) a_2 + \lambda_1 (f_{12} + 2g_{12}))}{4pk_{11}} + \frac{(\varphi_1 a_1 + \lambda_1 (f_{12} + 2g_{12}))^2}{2pk_{12}} + \frac{(\varphi_1 a_2 + \lambda_2 f_{12} + \lambda_3 g_{12})^2}{2pk_{21}} + \frac{(\varphi_2 a_2 + \eta_1 f_{32})((1 - \varphi_1) a_2 + \lambda_2 g_{32})^2}{2pk_{22}} \nonumber \\
    g_{12} &= \frac{(1 - \varphi_1) \mu}{\rho + \delta} \\
    g_{22} &= \frac{(1 - \varphi_2) \xi}{\rho + \theta} \\
    g_{32} &= \frac{(2 - \varphi_1) a_1 + \lambda_1 (f_{12} + 2g_{12}))^2}{8pk_{11}} + \frac{(\varphi_1 a_1 + \lambda_1 f_{12})((1 - \varphi_1) a_2 + \lambda_1 (f_{12} + 2g_{12}))}{2pk_{12}} + \frac{(\varphi_1 a_2 + \lambda_2 f_{32} + \lambda_3 g_{32})^2}{2pk_{21}} + \frac{(1 - \varphi_1) a_2 + \lambda_2 g_{32})^2}{2pk_{22}}. 
\end{aligned} \]  

(26)  

From equation (26), equations (19)–(21) can be obtained. Thus, this proposition is proved.

According to Proposition 2, in a Stackelberg game, the level of joint innovation and social responsibility can be obtained as follows:

\[ L^S = \frac{\lambda_1 h_1^S + \lambda_2 h_2^S}{\delta} + (I_0 - \frac{\lambda_1 h_1^S + \lambda_2 h_2^S}{\delta})e^{-\delta t}, \]  

(27)  

\[ S^N = \frac{\eta_1 r_1^S + \eta_2 r_2^S}{\theta} + (s_0 - \frac{\eta_1 r_1^S + \eta_2 r_2^S}{\theta})e^{-\theta t}. \]  

(28)  

Furthermore, the optimal revenue functions of the university and enterprise are obtained as follows:

\[ V_1^S = \frac{\varphi_1 \mu}{\rho + \delta} L^S + \frac{\varphi_2 \xi}{\rho + \theta} S^S + f_{32}, \]  

(29)  

\[ V_2^S = \frac{(1 - \varphi_1) \mu}{\rho + \delta} L^S + \frac{(1 - \varphi_2) \xi}{\rho + \theta} S^S + g_{32}, \]  

(30)  

Then, the optimal revenue function of the university-enterprise cooperation system under this game model can be obtained:

\[ V^S = \frac{\mu}{\rho + \delta} L^S + \frac{\xi}{\rho + \theta} S^S + f_{32} + g_{32}. \]  

(31)  

3.3. Cooperative Game. In the cooperative game, as a unified whole, the university and enterprise aim to maximize the overall system and work together to enhance comprehensive innovation capacity and fulfill social responsibility. Since \( V(L,S) \) is continuously bounded and differentiable for \( L,S \geq 0 \), the HJB equation is as follows:

\[
\begin{aligned}
\rho V(L,S) &= \max_{k_1,k_2,r_1,r_2} \left[ \alpha_1 h_1 + \alpha_2 h_2 + \mu L + \beta_1 r_1 + \beta_2 r_2 + \xi S - \frac{1}{2}k_{11}h_1^2 - \frac{1}{2}k_{12}r_1^2 - \frac{1}{2}k_{22}h_2^2 - \frac{1}{2}k_{32}r_2^2 \right. \\
&\quad + \frac{\partial V}{\partial L} (\lambda_1 h_1 + \lambda_2 h_2 - \delta L(t)) + \frac{\partial V}{\partial S} (\eta_1 r_1(t) + \eta_2 r_2(t) - \delta S(t)) \bigg].
\end{aligned} \]  

(32)
Proposition 3. In a cooperative game, the optimal strategies of the university and enterprise system are
\[
(h_1^c, r_1^c) = \left(\frac{(\rho + \delta)\alpha_1 + \mu\lambda_1}{(\rho + \delta)k_{11}}, \frac{(\rho + \theta)\beta_1 + \xi_1}{(\rho + \theta)k_{12}}\right),
\]
(33)
\[
(h_2^c, r_2^c) = \left(\frac{(\rho + \delta)\alpha_2 + \mu\lambda_2}{(\rho + \delta)k_{21}}, \frac{(\rho + \theta)\beta_2 + \xi_2}{(\rho + \theta)k_{22}}\right).
\]
(34)

Proof. It is easy to judge that equations (32) is a concave function with respect to \((h_1, r_1)\) and \((h_2, r_2)\). Based on the first-order conditions, the optimal decisions of the university and enterprise can be obtained as follows:

\[
\begin{align*}
  m_1 &= \frac{\mu}{\rho + \delta}, \\
  m_2 &= \frac{\xi}{\rho + \theta}, \\
  m_3 &= \frac{(\alpha_1 + \lambda_1m_1)^2 + (\beta_1 + \eta_1m_2)^2}{2\rho k_{11}} + \frac{(\alpha_2 + \lambda_2m_1)^2 + (\beta_2 + \eta_2m_2)^2}{2\rho k_{21}}.
\end{align*}
\]

From equation (38), equation (34) can be obtained. Thus, this proposition is proved.

According to Proposition 3, in a cooperative game, the level of joint innovation and social responsibility can be obtained as follows:

\[
L^c = \frac{\lambda_1h_1^c + \lambda_2h_2^c}{\delta} + \left(l_0 - \frac{\lambda_1h_1^c + \lambda_2h_2^c}{\delta}\right)e^{-\theta},
\]
(39)
\[
S^c = \frac{\eta_1r_1^c + \eta_2r_2^c}{\theta} + \left(s_0 - \frac{\eta_1r_1^c + \eta_2r_2^c}{\theta}\right)e^{-\theta}.
\]
(40)

Furthermore, the optimal revenue function of the whole system can be obtained:

\[
V^c = \frac{\mu}{\rho + \delta}L^c + \frac{\xi}{\rho + \theta}S^c + m_3.
\]
(41)

4. Model Analysis

In this section, we compare and analyze the optimal decision and the optimal revenue under the three kinds of game models and get the following propositions.

Proposition 4. (i) \(h_1^c > h_1^s > h_1^N\) and \(r_1^c > r_1^s = r_1^N\); (ii) \(h_2^c = h_2^s = h_2^N\) and \(r_2^c = r_2^s = r_2^N\).

Proof. (i) From equations (7), (19), and (33), we have

\[
(h_1, r_1) = \left(\frac{\alpha_1 + \lambda_1\partial V/\partial L}{k_{11}}, \frac{\beta_1 + \eta_1\partial V/\partial S}{k_{12}}\right),
\]
(35)
\[
(h_2, r_2) = \left(\frac{\alpha_2 + \lambda_2\partial V/\partial L}{k_{21}}, \frac{\beta_2 + \eta_2\partial V/\partial S}{k_{22}}\right).
\]
(36)

Substituting the above two equations into equation (32), it can be concluded that the revenue function of university-enterprise is a binary quadratic function about \(L\) and \(S\). Thus, the following function form can be set:

\[
V(L, S) = m_1L + m_2S + m_3.
\]
(37)

Furthermore, by the method of undetermined coefficients, we can obtain

\[
\begin{align*}
  h_1^c - h_1^s &= \frac{\varphi_1((\rho + \delta)\alpha_1 + \mu\lambda_1)}{2(\rho + \delta)k_{11}} > 0, \\
  h_1^s - h_1^N &= \frac{2 - 3\varphi_1((\rho + \delta)\alpha_1 + \mu\lambda_1)}{2(\rho + \delta)k_{11}} > 0, \\
  r_1^c - r_1^s &= \frac{1 - \varphi_2((\rho + \theta)\beta_1 + \xi_1)}{(\rho + \theta)k_{12}} > 0, \\
  r_1^s - r_1^N &= 0. \\
\end{align*}
\]
(42)

(ii) From these equations (8), (19), and (34), we have

\[
\begin{align*}
  h_1^c - h_1^s &= \frac{\varphi_1((\rho + \delta)\alpha_1 + \mu\lambda_2)}{(\rho + \delta)k_{21}} > 0, \\
  h_2^s - h_2^N &= 0, \\
  r_2^c - r_2^s &= \frac{\varphi_2((\rho + \theta)\beta_2 + \xi_2)}{(\rho + \theta)k_{22}} > 0, \\
  r_2^s - r_2^N &= 0.
\end{align*}
\]
(43)

Taken together, the proposition is proved.

Proposition 4. This proposition indicates that whether it is a Nash noncooperative game or is a Stackelberg game, the enterprise’s efforts in innovation and social responsibility remain unchanged. However, in the Stackelberg game, the enterprise shares the innovation input of the university, thus improving the innovation level of the university. Proposition 4
also shows that, under the cooperative game model, the university and enterprise have the highest level of effort in innovation and social responsibility, which is better than the other two models.

Proposition 5. \( L^C > L^N > L^S \) and \( S^C > S^S = S^N \).

Proof. From equations (12), (27), and (39), we have
\[
L^C - L^S = \frac{\lambda_1(h_1^C - h_1^S) + \lambda_2(h_2^C - h_2^S)}{\delta}(1 - e^{-\delta t}) > 0,
\]
\[
L^S - L^N = \frac{\lambda_1(h_1^S - h_1^N) + \lambda_2(h_2^S - h_2^N)}{\delta}(1 - e^{-\delta t}) > 0.
\]

From equations (13), (28), and (40), we have
\[
S^C - S^S = \frac{\eta_1(r_1^C - r_1^S) + \eta_2(r_2^C - r_2^S)}{\delta}(1 - e^{-\delta t}) > 0,
\]
\[
S^S - S^N = \frac{\eta_1(r_1^S - r_1^N) + \eta_2(r_2^S - r_2^N)}{\delta}(1 - e^{-\delta t}) = 0.
\]

Thus, the proposition is proved.

Proposition 5. The enterprise shares the innovation cost of the university, which can promote the innovation level of the university-enterprise cooperation. As the enterprise does not share the social responsibility input of the university, the level of social responsibility of the university is not high. Proposition 5 also shows that the joint innovation and social responsibility level of the university and enterprise are the highest under the cooperative game mode, which is better than the other two game modes.

Proposition 6. \( V_1^S > V_1^N, V_2^S > V_2^N, \) and \( V^C > V^S > V^N \).

Proof. From equations (14) and (29), we have
\[
V_1^S - V_1^N = \frac{(2 - 3\phi_1)(\rho + \delta)\alpha_1 + \lambda_1\mu^2}{4\rho(\rho + \delta)^2k_{11}} > 0.
\]

From (15) and (30), we have
\[
V_2^S - V_2^N = \frac{(2 - 3\phi_1)^2((\rho + \delta)\alpha_1 + \lambda_1\mu)^2}{8\rho(\rho + \delta)^2k_{11}} > 0.
\]

Thus, we can get \( V^S > V^N \). Furthermore, according to Propositions 4 and 5 and combined with equations (39) and (50), we can obtain \( V^C > V^S \). As a result, the proposition is proved.

Proposition 6. In the Stackelberg game, the optimal revenue of the university and enterprise and the revenue of the university-enterprise system are both higher than the corresponding value of the Nash noncooperative game. In other words, the Stackelberg game can achieve Pareto improvement. Proposition 6 also shows that the overall profit of the university-enterprise cooperative innovation system is the highest in the cooperative game.

5. Numerical Analysis
We illustrate our theoretical results by using some numerical examples and discuss some related issues. Let \( k_{11} = 1, \)
\[ k_{12} = 2, k_{21} = 2, k_{22} = 1, \lambda_1 = 0.6, \lambda_2 = 0.4, \delta = 0.3, \eta_1 = 0.4, \eta_2 = 0.3, \theta = 0.2, \alpha_1 = 4, \alpha_2 = 3, \mu = 2, \beta_1 = 3, \beta_2 = 2, \xi = 1, \rho = 0.1, l_0 = s_0 = 0, \phi_1 = 0.3, \text{ and } \phi_2 = 0.2; \text{ then, we use maple 2020 and can get Figures 1–3.} \]

As shown in Figure 1, in the early stage of university-enterprise cooperation, the level of joint innovation and social responsibility grow rapidly, that is, exponentially. However, with the increase of cooperation span, the level of joint innovation and social responsibility grow steadily and converge to a certain value. Figure 1 also shows that the cooperative game model will bring the greatest innovation level and social responsibility level to the university-enterprise cooperation.
enterprise cooperation, while the noncooperative game model will lead to a lower level.

As shown in Figures 2 and 3, with the continuous promotion of university-enterprise cooperation, the revenue of both sides and the whole system is steadily increasing and tends to a certain stable value. It can be easily observed from Figures 2 and 3 that the cooperative game mode is superior to the Stackelberg game mode, while the Stackelberg game mode is superior to the Nash noncooperative game mode. In other words, the cooperative game model can make university-enterprise cooperation reach Pareto optimal.

6. Conclusions

In this paper, we used a differential game theory to examine the cooperation problem of a system composed of a university and an enterprise in the field of joint innovation and social responsibility. We have developed three game models of Nash noncooperative game, Stackelberg game, and cooperative game to discuss the important influence of the innovation level and social responsibility level in university-enterprise cooperation.

Our results showed that the cost-sharing strategy for innovation input can significantly improve the innovation level of the university and can keep the enterprises’ innovation level unchanged, while the cost-sharing strategy did not affect the social responsibility level of the university and enterprise. In addition, if the university and enterprise adopt the cooperative game model, comparing with the Nash noncooperative game and Stackelberg game, the innovation level and social responsibility level of the university and enterprise are the highest. Our results also showed that the Stackelberg game for innovation input can effectively improve optimal revenue of the university, the enterprise, and the whole system, which is Pareto efficient. More importantly, in the cooperative game mode, the level of innovation and social responsibility of the university and enterprise will reach the maximum, and the whole system benefits are higher than those of the other two game modes, so as to achieve Pareto optimization.

Our study also has some limitations. We assumed all information is known such as the cost coefficients of the university and enterprise for innovation and social responsibility. However, this information may be asymmetric. Moreover, it may be useful to develop an equilibrium model in the presence of multiple universities or enterprises.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The authors thank the sponsor for this research from the National Social Science Foundation of China (no. RZ1900001103).

References

[1] D. S. Siegel and V. Zervos, “Strategic research partnerships and economic performance: empirical issues,” *Science and Public Policy*, vol. 29, no. 5, pp. 331–343, 2002.
[2] C. Bayona-Sáez and T. García-Marco, “Assessing the effectiveness of the Eureka program,” *Research Policy*, vol. 39, no. 10, pp. 1375–1386, 2010.
[3] T. Lemola, “Convergence of national science and technology policies: the case of Finland,” *Research Policy*, vol. 31, no. 8-9, pp. 1481–1490, 2002.
[4] H. Xia and J. Weng, “Effectiveness of industry-university-research cooperation in China: impact of innovation input and open innovation environment,” *International Journal of Innovation Science*, 2021.
[5] E. Giannopoulou, P.-J. Barlatier, and J. Pénin, “Same but different? research and technology organizations, universities and the innovation activities of firms,” *Research Policy*, vol. 48, no. 1, pp. 223–233, 2019.
[6] R. Al-Hanakta, B. C. Illés, A. Dunay, G. S. Abdissa, and M. Abdi Khalife, “The effect of innovation on small and medium enterprises: a bibliometric analysis,” *Visegrad Journal on Bioeconomy and Sustainable Development*, vol. 10, no. 1, pp. 35–50, 2021.
[7] E. Prosser, “University-enterprise cooperation,” *Industry and Higher Education*, vol. 11, no. 3, pp. 153–159, 1997.
[8] A. Gibb, G. Haskins, and I. Robertson, “Leading the entrepreneurial university: meeting the entrepreneurial development needs of higher education institutions,” in *Universities In Change*, pp. 9–45, Springer, New York, NY, USA, 2013.
[9] D. O. Gray, “Government-sponsored industry-university cooperative research: an analysis of cooperative research center evaluation approaches,” *Research Evaluation*, vol. 9, no. 1, pp. 57–67, 2000.
[10] D. O. Gray, M. Lindblad, and J. Rudolph, “Industry–university research centers: a multivariate analysis of member retention,” *The Journal of Technology Transfer*, vol. 26, no. 3, pp. 247–254, 2001.
[11] H. Xu, C. Wang, K. Dong, R. Luo, Z. Yue, and H. Pang, “A study of methods to identify industry-university-research institution cooperation partners based on innovation chain theory,” *Journal of Data and Information Science*, vol. 3, no. 2, pp. 38–61, 2018.
[12] M. R. Luca and L. T. David, *Tuning Research on the University-Enterprise Partnership in Training Entrepreneurship*, Eikon, London, UK, 2010.
[13] S. Zhang, H. Duan, T. Wei, and G. Shane, “A review of university-enterprise-cooperation research in China: a quantitative perspective,” *Construction Economics and Building*, vol. 20, no. 3, pp. 62–77, 2020.
[14] A. K. Agrawal, “University-to-industry knowledge transfer: literature review and unanswered questions,” *International Journal of Management Reviews*, vol. 3, no. 4, pp. 285–302, 2001.
[15] T. Bjerregaard, “Industry and academia in convergence: micro-institutional dimensions of R&D collaboration,” *Technovation*, vol. 30, no. 2, pp. 100–108, 2010.
[16] S. Ankrath and A. T. Omar, "Universities–industry collaboration: a systemic review," Scandinavian Journal of Management, vol. 31, no. 3, pp. 387–408, 2002.

[17] A. Scandura, "University-industry collaboration and firms’ R&D effort," Research Policy, vol. 45, no. 9, pp. 1907–1922, 2016.

[18] M. D. Santoro and A. K. Chakrabarti, "Corporate strategic objectives for establishing relationships with university research centers," IEEE Transactions on Engineering Management, vol. 48, no. 2, pp. 157–163, 2001.

[19] E. G. Carayannis, J. Alexander, and A. Ioannidis, "Leveraging knowledge, learning, and innovation in forming strategic government-university-industry (GUI) R&D partnerships in the US, Germany, and France," Technovation, vol. 20, no. 9, pp. 477–488, 2000.

[20] J. C. Hayton, S. Sehili, and V. Scarpello, "Why do firms join consortial research centers? An empirical examination of firm, industry and environmental antecedents," The Journal of Technology Transfer, vol. 35, no. 5, pp. 494–510, 2010.

[21] D. Mahfoudh, Y. Boujelbene, and J. P. Mathieu, "University-enterprise cooperation: determinants and impacts," in Mediterranean Symposium on Enterprise-New Technology Synergy, pp. 91–121, Springer, Berlin, Germany, 2019.

[22] T. Thune, "University-industry collaboration: the network embeddedness approach," Science and Public Policy, vol. 34, no. 3, pp. 158–168, 2017.

[23] R. Huggins and D. Prokop, "Network structure and regional innovation: a study of university-industry ties," Urban Studies, vol. 54, no. 4, pp. 931–952, 2017.

[24] L. Lyu, W. Wu, H. Hu, and R. Huang, "An evolving regional innovation network: collaboration among industry, university, and research institution in China’s first technology hub," The Journal of Technology Transfer, vol. 44, no. 3, pp. 659–680, 2019.

[25] D. S. Siegel, D. A. Waldman, L. E. Atwater, and A. N. Link, "Commercial knowledge transfers from universities to firms: improving the effectiveness of university-industry collaboration," The Journal of High Technology Management Research, vol. 14, no. 1, pp. 111–133, 2003.

[26] S. H. Ting, S. Yahya, and C. L. Tan, "The influence of researcher competence on university-industry collaboration," Journal of Entrepreneurship in Emerging Economies, vol. 11, no. 2, pp. 277–303, 2019.

[27] W. Wang and S. Lu, "University-industry innovation community dynamics and knowledge transfer: evidence from China," Technovation, vol. 106, p. 102305, 2021.

[28] M. S. Sodhi, "Conceptualizing social responsibility in operations via stakeholder resource-based view," Production and Operations Management, vol. 24, no. 5, pp. 1375–1389, 2015.

[29] H. R. Bowen, Social Responsibilities of the Businessman, Harpor & Row, New York, NY, USA, 1953.

[30] D. Matten and J. Moon, "Corporate social responsibility," Journal of Business Ethics, vol. 54, no. 4, pp. 323–337, 2004.

[31] H. Aguinis and A. Glavas, "What we know and don’t know about corporate social responsibility," Journal of Management, vol. 38, no. 4, pp. 932–968, 2012.

[32] H. Y. Li, W. Xu, Y. Cui, Z. Wang, M. Xiao, and Z. X. Sun, "Preventive maintenance decision model of urban transportation system equipment based on multi-control units," IEEE Access, 2020.

[33] A. L. Awaga, W. Xu, L. Liu, and Y. Zhang, "Evolutionary game of green manufacturing mode of enterprises under the influence of government reward and punishment," Advances in Production Engineering & Management, vol. 15, no. 4, pp. 416–430, 2020.

[34] M. Tamvada, "Corporate social responsibility and accountability: a new theoretical foundation for regulating CSR," International Journal of Corporate Social Responsibility, vol. 5, no. 1, pp. 1–14, 2020.

[35] P. Velte, "Meta-analyses on corporate social responsibility (CSR); a literature review," Management Review Quarterly, pp. 1–49, 2021.

[36] K. F. Latif, "The development and validation of stakeholder-based scale for measuring university social responsibility (USR)," Social Indicators Research, vol. 140, no. 2, pp. 511–547, 2018.

[37] S. A. Alzyoud and K. Bani-Hani, "Social responsibility in higher education institutions: application case from the Middle East," European Scientific Journal, vol. 11, no. 8, pp. 122–129, 2015.

[38] H. S. MuijenHeidi, "Corporate social responsibility starts at university," Journal of Business Ethics, vol. 53, no. 1, pp. 235–246, 2004.

[39] S. Gebbels, S. M. Evans, and J. E. Delany, "Promoting environmental citizenship and corporate social responsibility through a school/industry/university partnership," Journal of Biological Education, vol. 45, no. 1, pp. 13–19, 2011.

[40] N. M. Smith, J. M. Smith, L. A. Battalora, and B. A. Teschner, "Industry-university partnerships: engineering education and corporate social responsibility," Journal of Professional Issues in Engineering Education and Practice, vol. 144, no. 3, p. 04018002, 2018.

[41] L. Lamberti and A. Mantovani, "Process and product innovation: a differential game approach to product life cycle," International Journal of Economic Theory, vol. 6, no. 2, pp. 227–252, 2010.

[42] B. Anton and A. Greiner, "Catching-up and falling behind: effects of learning in an R&D differential game with spillovers," Journal of Economic Dynamics and Control, vol. 91, pp. 134–156, 2018.

[43] W. Liu, K. Xu, R. Chai, and X. Fang, "Leveraging online customer reviews in new product development: a differential game approach," Annals of Operations Research, pp. 1–24, 2020.

[44] J. Wei and C. Wang, "Improving interaction mechanism of carbon reduction technology innovation between supply chain enterprises and government by means of differential game," Journal of Cleaner Production, vol. 296, p. 126578, 2021.

[45] Y. Deng, D. You, and Y. Zhang, "Research on improvement strategies for low-carbon technology innovation based on a differential game: the perspective of tax competition," Sustainable Production and Consumption, vol. 26, pp. 1046–1061, 2021.

[46] S. Yin and B. Li, "Transferring green building technologies from academic research institutes to building enterprises in the development of urban green building: a stochastic differential game approach," Sustainable Cities and Society, vol. 39, pp. 631–638, 2018.