Aspects of String unification

Witold Pokorski 1, Graham G. Ross 2

Department of Physics, Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP

Abstract

We consider the phenomenological implications of a class of compactified string theories which naturally reproduces the flavour multiplet structure of the Standard Model. The implications for gauge unification depends on which of three possibilities is realised for obtaining light Higgs multiplets. The more conventional one leads to predictions for the gauge couplings close to that of the MSSM but with an increased value of the unification scale. The other two cases offer a mechanism for bringing the prediction for the strong coupling into agreement with the measured value while still increasing the unification scale. The various possibilities lead to different expectations for the structure of the quark masses.

1E-mail address: W.Pokorski1@physics.ox.ac.uk
2E-mail address: Ross@thphys.ox.ac.uk
1 Introduction

Superstring theory offers us the exciting prospect of a complete unification of all the fundamental interactions including gravity. A preliminary study of its general implications for low-energy structure is quite encouraging. The heterotic string has a gauge symmetry which contains the Standard Model gauge group. In a definite string vacuum the number of massless modes (on the scale of the Planck mass) is determined so there is the prospect of understanding the origin of three families. There are viable predictions for the fundamental couplings, such as gauge couplings, even in the absence of a stage of Grand Unification. However the analysis of superstring phenomenology has been hampered by the profusion of candidate 4-dimensional string theories, corresponding to different string vacua. In this paper we attempt to make some progress by identifying a class of string models that most closely generates the Standard Model structure. While this is certainly not exhaustive of the possible realistic string theories it does, we think, represent a most promising class. Interestingly, within this class one may draw some quite general conclusions about the nature of the low-energy theory and the predictions for the parameters of the Standard Model.

To set the stage for this discussion let us briefly review the most significant features of the Standard Model. A significant feature is that the observed quark and lepton states of the Standard Model transform as singlets or as the fundamental representation of the Standard Model gauge group. Even more noticeable is that the quarks and leptons fill out complete representations of $SU(5)$ (and $SO(10)$?) even though the low-energy gauge group is not $SU(5)$. Moreover the $SU(5)$ assignment of Standard Model states offers an immediate explanation for the left-handed nature of the weak interactions for both the quarks and leptons. However this symmetric view is somewhat spoilt by the sector responsible for spontaneous symmetry breaking. The Higgs doublet is the only representation of matter fields in the Standard Model spectrum that does not have its $SU(5)$ partners (additional $SU(2)$ singlet d quarks).

Let us first consider how these features are accommodated in Grand Unified theories. Clearly the $SU(5)$ assignments strongly suggest an underlying GUT containing $SU(5)$. However a GUT does not by itself explain why only the fundamental representations of the gauge group occur. Moreover in Grand Unified theories the partial GUT representation of the Higgs must be explained and this gives rise to the doublet-triplet splitting problem, namely the need to give the partners of the Higgs a mass while leaving the Higgs doublets light. Solutions to this problem exist but typically require a large number of additional Higgs fields and a somewhat unconvincing and complicated set of interactions. In addition Grand Unified theories (GUTs) must be protected against the radiative coupling of the large Grand Unified scale to the low electroweak breaking scale - the hierarchy problem. Its solution requires a low energy stage of supersymmetry. This leads to the MSSM, the supersymmetric extension of the Standard Model. Grand Unification of couplings plus the radiative corrections of the MSSM lead to the remarkably successful prediction for the ratio of gauge couplings at low energy provided the $U(1)$ normalization is chosen to be that given by $SU(5)$.

How is this unification picture changed in superstring theories [1]? Certainly the string theory has an enlarged symmetry; in the heterotic string $E_8 \otimes E_6$ [2]. Although this symmetry may be broken at the compactification scale a residue of it may persist in the light spectrum even if there is no Grand Unified group. This may explain the pattern of quark and lepton supermultiplets. Moreover, if the string theory is built from level-1 Kač-Moody level theories,
the representation content of the theory is restricted offering an explanation to the question why only low lying representations of quark and leptons are observed. As we shall discuss in more detail shortly the string offers an elegant explanation for the doublet triplet splitting problem in the case there is no stage of Grand Unification below the Planck scale. In this case there is no symmetry reason demanding there should be partners to the Higgs thus finessing the doublet-triplet problem. However the string now has to explain why the quarks and leptons still come as complete $SU(5)$ representations; we shall discuss how this happens naturally in a class of string theories.

Unification of couplings is a prediction of string theories even without a Grand Unified group below the Planck scale. In this case it is a residue of the underlying gauge symmetry before compactification. Taken with a stage of low-energy supersymmetry needed to solve the hierarchy problem this may provide an alternative explanation for the success of gauge coupling unification. Further the string makes an important additional prediction which goes beyond Grand Unification, namely it determines the unification scale in terms of the Planck scale. If this could be checked it would provide the first quantitative test of the unification of the strong, electromagnetic and weak forces with gravity which the superstring provides. Preliminary indications are promising in that the scale of unification predicted by continuing the gauge couplings up in energy from the laboratory scale assuming the minimal supersymmetric Standard Model (MSSM) spectrum gives a very large scale of unification of $O((1 - 3)10^{16}\text{GeV})$ not so far from the Planck scale which sets the scale for the string prediction. In the case of the (weakly coupled) heterotic string the detailed prediction is

$$M_{\text{string}} \approx g_{\text{string}} \times (5.2 \times 10^{17}\text{GeV}) \approx 3.6 \times 10^{17}\text{GeV}$$

(1)

Although this is relatively close to the gauge unification scale it is still a factor of 10-30 too high. One should remember that the Grand Unified scale is the argument of the logarithm and thus, in order to get it correctly, one has to work to very high precision in the coupling constant. Possible problems in determining this scale are:

- **SUSY threshold effects** - It seems that although these could make the factor of 20 difference this only can be achieved with a very peculiar supersymmetry spectrum at low energies in which the gluino is lighter than the Wino.

- **Additional heavy states** - A second possibility is that the theory at compactification is not just the Standard Model but is still Grand Unified. In this case the string prediction for the unification scale applies to the unification of couplings of the Grand Unified theory. While possible it limits the predictive power of the string. It may also be that there are additional heavy states which just conspire to change the unification scale. Examples of this are given in [3].

- **String threshold effects** - Another possibility that has been explored are string threshold effects which in a given string theory are calculable and amount to including the effect of the heavy Kaluza Klein modes which are themselves split when the Grand Unified theory is broken. While it is possible to construct examples in which these effects are large, more typically they are too small to explain the discrepancy. However it has recently been shown that threshold effects associated with Wilson line breaking can, for reasonable values of the moduli, generate effects easily large enough to explain
the discrepancy [9]. While such effects typically change the relative evolution of the couplings and hence spoil the success of gauge unification, it has been shown how, for a subset of models, only the scale of unification may be changed as desired. Of course one is left with the question why this subset of models is selected.

• Large gauge coupling - A mechanism which automatically leads to just a change in the scale of unification was proposed by Witten [10]. He observes that, in the absence of large corrections of the type just discussed, the string prediction which is based on the assumption of a weakly coupled heterotic string breaks down. This is because the fit to the values of the low energy gauge couplings and Newton’s constant require the ten dimensional string coupling to be very large. Thus Witten argues one should do the string calculation for the case the 10 dimensional gauge coupling is large in the heterotic string theory (This is the M theory limit [11]). In this case he found the prediction for the gauge unification scale changes and is dependent on a new parameter, the compactification scale associated with the eleventh dimension of M-theory.

In this paper we wish to select from candidate string compactifications those that can explain the features of the effective low-energy theory and the unification predictions just discussed. Given the plethora of candidate string theories this seems a very tall order but we will argue that the most predictive string theories must belong to a very small subclass. Let us first consider the prospects for making a superstring prediction for the parameters including the gauge unification scale of the effective low-energy theory descending from the string. The class of string theory which gives such predictions for the unification scale of the Standard Model gauge couplings is quite restricted. In it there must not be a stage of Grand Unification below the compactification scale because the prediction refers to the unification scale for the couplings of the gauge group at the compactification scale and these must just be those of the Standard Model. Even without such a stage of Grand Unification the gauge couplings are related by the underlying string symmetries [12]. This is readily achieved with the required (SU(5)-like) normalization for the U(1) gauge factor in string theories in which the Kač-Moody level used in the string construction is restricted to be level-1. An advantage of using such level-1 string theories is that they have a restricted multiplet structure with only fundamental representations for the chiral supermultiplets. Encouragingly this is in agreement with the representation content of the Standard Model in which the quarks and leptons transform as singlets or as the fundamental representation of SU(3) ⊗ SU(2). Given this we concentrate on such level-1 theories. (Note that level-1 theories cannot have a stage of Grand Unification with a simple gauge group below the compactification scale because it is necessary to have larger Higgs representations to break such simple Grand Unified groups to the Standard Model.)

We further restrict our string compactifications to those that offer an explanation of the SU(5)-like multiplets of quark and lepton fields even though the gauge group is not-Grand Unified below the compactification scale. To do so we need some relic of an underlying string gauge symmetry to persist in the low energy spectrum. Thus we concentrate on exploring in some generality the low-energy structure of a class of such level-1 string theories which correspond to a compactified heterotic string theory [13]. To make a start in this we note that

\[1\] See [4] for a general discussion of the other possibilities.

\[2\] This does not cover all 4-dimensional string theories for some cannot be viewed as a compactification from 10 dimensions. However in them there is no obvious underlying extended gauge symmetry which requires the
the underlying symmetry of the 10 dimensional heterotic string includes an $E_8 \otimes E_6$ or $SO(32)$ symmetry\footnote{This symmetry has to be broken in the 4 dimensional theory but a residue of the underlying Grand Unification will remain offering an explanation for the $SU(5)$ content of the matter fields of the Standard model. As we mentioned before, the mechanism for breaking the 10 dimensional gauge symmetry cannot be the usual Higgs mechanism because the Higgs content is restricted in a level-1 string theory and is insufficient to break the symmetry down to the Standard Model gauge group. However it has been shown that the necessary breaking can occur through Wilson line breaking\cite{14} in which flux is trapped in non-simply-connected manifolds. Such non-simply-connected manifolds can arise through the modding out of the manifold by a freely acting discrete group with the Wilson lines forming a representation of the group. Indeed such Wilson line breaking can break the gauge symmetry down to just that of the Standard Model. We will consider the class of models in which this is the case as we wish to explore the possibility that the string prediction for the gauge coupling unification scale applies directly to the Standard Model couplings.}. This symmetry has to be broken in the 4 dimensional theory but a residue of the underlying Grand Unification will remain offering an explanation for the $SU(5)$ content of the matter fields of the Standard model. As we mentioned before, the mechanism for breaking the 10 dimensional gauge symmetry cannot be the usual Higgs mechanism because the Higgs content is restricted in a level-1 string theory and is insufficient to break the symmetry down to the Standard Model gauge group. However it has been shown that the necessary breaking can occur through Wilson line breaking\cite{14} in which flux is trapped in non-simply-connected manifolds. Such non-simply-connected manifolds can arise through the modding out of the manifold by a freely acting discrete group with the Wilson lines forming a representation of the group. Indeed such Wilson line breaking can break the gauge symmetry down to just that of the Standard Model. We will consider the class of models in which this is the case as we wish to explore the possibility that the string prediction for the gauge coupling unification scale applies directly to the Standard Model couplings.

The implications of Wilson line breaking for the chiral multiplet content is particularly interesting because such breaking does not reduce the chiral content of the theory. If in the $E_6$ visible sector there are $(n + m)$ left-handed chiral superfields transforming as the 27 representation ( (2,1) harmonic forms) and just $m$ left-handed chiral superfields transforming as the 27 representation ( (1,1) harmonic forms) there is an excess of $n$ “chiral ” superfields with the necessary states to include the (massless) generations of the MSSM. Wilson line breaking does not affect the chiral structure of the theory and so in the compactified theory we are guaranteed to have $n$ left-handed chiral superfields with the same gauge content as occurs in complete 27 representations of $E_6$ even though the gauge group is just that of the Standard Model.

In addition to these supermultiplets there will be further states left light after Wilson line breaking, related to the original $m(27 + \bar{27})$ superfields in the theory without Wilson line breaking. There are two possibilities depending on whether, before modding out by the discrete group, they correspond to the $m_s(27 + \bar{27})$ fields which are singlet representations under the discrete group or they correspond to the $m_{ns}D(27 + \bar{27})$ fields which belong to non-singlet representations of dimension D under the discrete group. In the latter case, after modding out, there are light states with the same gauge content as in $m_{ns}$ complete $(27 + \bar{27})$ representations whether or not there is Wilson line breaking. In the absence of Wilson line breaking these are just the components of the discrete group non-singlet representations which are left invariant under the discrete group transformation. In the case there are non-trivial Wilson loops associated with the singularities introduced by the modding out by the discrete group the light states do not correspond to the same original multiplets in the theory before Wilson line breaking. The reason for this is that the light fields must be singlets under the combined transformation of the Wilson line group element and the discrete group element. Thus the components of the $m_{ns}D(27 + \bar{27})$ which are left invariant by the Wilson line group element must also be left invariant by the discrete group and thus correspond to the components of the corresponding multiplets in the compactification without Wilson line breaking. However the components of the $m_{ns}D(27 + \bar{27})$ which transform non-trivially under the Wilson line group element must also transform under the discrete group in such a way that they are invariant

\footnote{Henceforth we will only consider the phenomenologically interesting $E_8 \otimes E_6$ case where $E_6$ contains the Standard Model group.}
under the combined transformation. Hence they correspond to different multiplets from the case without Wilson line breaking. Rather they correspond to the appropriate fields with non-trivial discrete transformations in the original manifold before modding out. The same is true of the components of the \( n \) 27s family of fields discussed above.

To summarize, it is necessary to break the level-1 heterotic string theory by Wilson lines. Despite the fact this may break the symmetry down to a non-Grand Unified group, perhaps just the Standard Model group, the \( n \) generations have the same gauge content as in complete 27s. The \( SU(5) \) content of a 27 is \( 10 + \bar{\mathbf{5}} + 1 + \mathbf{5} + \bar{\mathbf{5}} + 1 \). The \( (\nu_R) \) singlet and the components of the latter “vectorlike” pair of \( 5 + \bar{\mathbf{5}} \) can acquire \( SU(3) \otimes SU(2) \otimes U(1) \) invariant masses so they are likely to be very heavy. Thus we are left with the conclusion that the light families fill out complete \( SU(5) \) representations just as is observed. Indeed with the right-handed neutrino component, \( \nu_R \), included these families fill a 16-plet of \( SO(10) \). All of this is a relic of the underlying \( E_6 \) symmetry of the heterotic string which persists in the spectrum below the compactification scale. In addition we expect new light states below the compactification scale with gauge representation content equivalent to \( m_{ns} \) complete \( 27 + \bar{27} \) representations. However these too can acquire \( SU(3) \otimes SU(2) \otimes U(1) \) invariant masses so they are likely to be very heavy.

So far we have discussed light fields after compactification which make up complete \( E_6 \) multiplets. Their appearance is encouraging as they neatly explain the existence of GUT multiplets in a compactified theory without a GUT gauge group. However there remains the troublesome question of explaining the existence of the Higgs supermultiplets which do not belong to a light GUT multiplet. With Wilson line breaking there is an excellent mechanism capable of accommodating the Higgs fields. To see this consider now the fate of the \( m_s(27 + \bar{27}) \) fields which are singlet representations of the discrete group. For them only those components which are singlets under the Wilson line group element are left light under Wilson line breaking. Thus there is an immediate reason for “split” multiplets to arise. Moreover, as the Wilson lines must provide a representation of the discrete group there is only a finite number of possibilities for the Wilson line group elements so the possible spectra are limited and the probability it will lead to just the doublet components left light is non negligible. Indeed, as we discuss below, a complete classification of Wilson line breaking shows that there are choices which do just leave the Higgs double components of the \( (27 + \bar{27}) \) light.

Thus we see, in the case of Wilson line breaking, the light multiplet structure after compactification consists of light left-handed chiral superfields which have the same multiplet content as \( (n + m_{ns}) \) complete 27s and \( m_{ns} \) complete 27s of \( E_6 \) even though the gauge group may be much smaller than \( E_6 \) and need not be a Grand Unified group at all. In addition there are \( m_s \) split multiplets consisting of those components of the \( (27 + \bar{27}) \) which are singlets under the Wilson line group elements.

The rest of this paper is devoted developing the implications for low energy physics of this class of superstring theories. In Section 2 we give a more detailed discussion of the implications of Wilson line breaking. In Section 3 we apply Wilson line breaking to the doublet triplet splitting problem and show how it leads to two distinct possibilities. Section 4 considers the phenomenological implications for the class of string model under consideration, both for the unification of gauge couplings and the unification scale and for the third generation masses. Section 5 presents our conclusions.
2 The massless spectrum after Wilson line breaking.

Wilson lines may arise in theories compactified on non-simply-connected manifolds. The usual way of constructing non-simply-connected spaces is to mod out an initial simply connected manifold, $M$, by some discrete symmetry group, $D$. In the absence of Wilson lines, on $M/D$, fields must satisfy

$$\psi(x) = \psi(fx), \quad (2)$$

where $fx$ is the image of $x$ under the action of $D$. This of course means that only the discrete group singlet fields remain after such modding out. Wilson lines, $U_f$, correspond to flux lines trapped in non-contractible loops $\gamma$ in the non-simply-connected manifold \[14\]

$$U_f = P \exp \left( -i \int_\gamma T^a A^a_m dx^m \right) \quad (3)$$

The Wilson line depends on the vacuum configuration of the gauge fields $A^a_m$ in the compactified six dimensions (for $m = 4...9$). Thus $U_f$ will be a group element of the underlying gauge group $G$ and the set of $U_f$s give a representation of $D$ in $G$. In their presence the implication for the light spectrum is much richer. When we go around the singularity associated with $\gamma$ the wave function acquires a phase $U_f$ due to the non trivial configuration of the gauge field. In such a case the condition (2) becomes

$$U_f \psi(x) = \psi(fx). \quad (4)$$

The implications for the light spectrum follow immediately. All of the components of a chiral supermultiplet on $M$ transforms in a given way under the discrete group. On the other hand the Wilson line gives different phases to the different components (different representations in the decomposition under the unbroken group) of the initial supermultiplet. As result, if the state on $M$ is a discrete group singlet, only the component which does not acquire a phase from the Wilson line remains massless. If the state on $M$ transforms non-trivially under $D$ picking up an overall phase one immediately sees from eq(4) there will be left light only that part of it which acquires the same phase from the Wilson line.

There is a very important implication of this. States on $M$ which transform as a representation $R$ under $G$ and which make up a non-singlet representation of $D$ will give rise to light states on $M/D$ which fill out a complete representation $R$ of $G$ even though the gauge group on $M/D$ is the smaller group $G/D$. Thus if on $M$ the discrete non-singlet states transform as a 27s under $E_6$ there will be 27 states left light after Wilson line breaking with the gauge quantum numbers under $G/D$ as if they belonged to a complete 27 of $E_6$. This follows because for each component of $R$ it is possible to pick a component of the discrete group representation with the correct transformation property to satisfy eq(4). Of course these residual light components will correspond to different multiplets on $M$ so their couplings will not be related by the underlying gauge symmetry but the overall multiplet content does keep a memory of the underlying symmetry.

There is another extremely important factor determining the low energy spectrum. Wilson line breaking does not affect the index of the Dirac operator \[1\] which, in turn, does not depend on the representation of the unbroken group $G/D$. As a result if there are $n_l(n_r)$ left- (right-) handed chiral superfields transforming as $R$ on the manifold $M/D$ without Wilson lines there must be left light left-handed fields making up $n = (n_l - n_r)$ complete representations $R$. As
discussed above, the components of $R$ having different transformation properties under $U_f$ will correspond to different fields on $M$ having different transformation properties under $D$.

The appearance of these $n$ chiral representations coming in complete $E_6$ representations, even in the theory in which the gauge group is broken, gives a natural explanation for the pattern of states of the observed families as we discussed in the previous section. In addition there will be $m_{ns}$ copies of $(27 + \bar{27})$ corresponding to the discrete group non-trivial representations. However note that there is one further possibility for light fields not yet discussed. On $M$ we may also have states which belong to the singlet representation of $D$. From eq(4) we see that on $M/D$ the light states must transform trivially under the Wilson lines and thus do not make up complete multiplets under $G$. These are the states that give a natural explanation to the doublet-triplet splitting problem because if the states left light are Higgs doublets we see there are no associated colour triplets and the doublet-triplet splitting problem never arises.

To illustrate this we consider specific examples of Wilson loops which are relevant for realistic theories. We focus on Wilson loops embedded in an $E_6$ factor which contains an $SU(3) \otimes SU(2) \otimes U(1)$ subgroup. This case has been explored in detail by Breit, Ovrut and Segre [15]. As long as the discrete group by which we mod out is Abelian ($Z_N$), the Wilson loop can be written down as a linear combination of the elements of the Cartan subalgebra $H^i$ of $E_6$ [16]

$$U = \exp(i \sum \bar{\lambda}_i H^i).$$

Using the method of Weyl weights, we determine the symmetry breaking direction in the weight space which leaves the Standard Model gauge group unbroken. We know that the elements of the mass matrix for gauge fields in four dimensions are proportional to

$$(\sum_i \bar{\lambda}_i c_i)^2,$$  

where $\bar{\lambda}_i$ are dual components of the symmetry breaking axis, and $c_i$ are Dynkin components of the root of a given generator (i.e. a given gauge field).

We want to leave the Standard Model gauge group, $SU(3) \otimes SU(2) \otimes U(1)$, unbroken so we need to have these gauge fields massless. The corresponding $E_6$ roots are: (000001), (0100-10), (0-10011), for color and $\frac{1}{2}(10001-1)$ for $I^W_3$ axis. Using eq(1) shows that $\bar{\lambda}$ must have the following form: $[-c, c, a, b, c, 0]$. Because the Wilson line is a representation of the discrete group $Z_N$, each of the numbers $a, b, c$ must be of the form

$$k \frac{2\pi}{N},$$

where $k = 0, ..., N - 1$ (k can be a priori different for $a, b$ and $c$ respectively).

For $E_6$ the most convenient way of presenting the Wilson loop is to write it in the maximal subgroup basis, $[SU(3)]^3$. This gives

$$\left( \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right)_c \exp i \left( \begin{array}{cc} -c & -c \\ -c & 2c \end{array} \right)_L \exp i \left( \begin{array}{ccc} a-b & c-a \\ & & b-c \end{array} \right)_R$$

Using the decomposition of the 27 dimensional $E_6$ fundamental representation

$$27 = (1, \bar{3}, 3) + (3, 3, 1) + (\bar{3}, 1, \bar{3}) = \chi + Q + Q^c$$

$$7$$
Table 1: Transformation of the components of a 27 dimensional $E_6$ representation under the
Wilson line discrete group element.

| $\ell$  | $e^c$  | $\nu^c$  | $H_u$  | $H_d$  | $N$  | $q$  | $D$  | $D^c$  | $d^c$  | $u^c$  |
|---------|--------|----------|--------|--------|------|------|------|--------|--------|--------|
| $b$     | $a-b-2c$ | $-c-a$  | $a-b+c$ | $2c-a$  | $b-3c$ | $-c$ | $2c$ | $c-b$  | $a-c$  | $b-a$  |

we can determine how the Wilson line will act on different elements of the spectrum. We have

$$\chi \equiv \begin{bmatrix} H^0 & H^+ & e^c \\ \tilde{H}^- & \tilde{H}_0 & \nu^c \\ e^- & \nu & N \end{bmatrix} = \begin{bmatrix} H_u & e^c \\ H_d & \nu^c \\ \ell & N \end{bmatrix}; Q \equiv \begin{bmatrix} u \\ d \\ D \end{bmatrix}; Q^c \equiv \begin{bmatrix} u^c \\ d^c \\ D^c \end{bmatrix}$$ (10)

where we have shown the assignment of the first generation of quarks and leptons and the fields $H_u$, $H_d$ have the quantum numbers of the Higgs supermultiplets in the MSSM and $D$ and $D^c$ are new colour triplets with the same charge as the $d$ quarks. The transformation properties of these fields under the action of the Wilson line is given by Table 1. From it we can readily determine for any model the conditions necessary to leave a given particle massless.

### 2.1 Light Higgs doublets.

As we have discussed, after compactification there are $(n+m_{ns})$ complete 27s and $m_{ns}$ complete 27s and $m_s$ split multiplets. Let us consider how this can lead to the light Higgs doublets needed in the MSSM. Consider first the $(n+m_{ns})$ 27s. As may be seen from eq(10) the field content of a 27 dimensional representation of $E_6$ contains states with the quantum numbers of the two Higgs doublets needed in the MSSM. However it also contains the colour triplet states, $D$, the partners of the Higgs multiplets making up the $5 + \bar{5}$ $SU(5)$ vectorlike representations. There must be a stage of spontaneous symmetry breaking giving the $D$s a large $SU(3) \otimes SU(2) \otimes U(1)$ invariant mass. This can arise through a coupling of $D, D^c$ to the $N$ field in eq(10), if $N$ acquires a large vev. In a GUT arranging for this to happen while keeping their Higgs partners light is difficult because their couplings are related by the GUT gauge symmetry. In the string case without Grand Unification the situation is quite different because, as noted above, the D components and the Higgs components do not come from the same 27 in the theory before modding out. For this reason their couplings are not related and it is quite possible for the $D$s to obtain a large mass coupling to the $N$ field while the Higgs remain massless because (presumably due to a symmetry of the theory) they do not couple to the $N$ field. The need for such a symmetry is generic to supersymmetric extensions of the Standard Model. In the MSSM the “$\mu$” term in the superpotential $\mu H_1 H_2$ must have the coefficient $\mu$ of order the electroweak breaking scale. Since the unbroken Standard Model gauge group allows such a mass term its smallness must be due to an additional discrete symmetry left unbroken by the large scale breaking. We refer to this symmetry as the “$\mu$” symmetry.

A second possibility for generating a light Higgs arises because, as we have already noted, the Wilson line breaking mechanism provides us a very natural way of generating some extra states which come as incomplete representations of the initial ($E_6$) gauge group. If there are any discrete group singlet representations in the original manifold then necessarily there will
be such split multiplets. Indeed in Calabi Yau compactification we know there is at least
one pair of such fields transforming as $27 + \bar{27}$, the $\bar{27}$ corresponding to the harmonic $(1,1)$
Kähler form so it is a prediction of the theory that there should be at least one multiplet below
compactification that does not fit into a complete $SU(5)$ representation! If the split multiplet
should contain just the Higgs doublets it solves the doublet triplet splitting problem. That
this can happen with Abelian Wilson loops follows from Tables 1 and 2. One may see that
the choice $b = a + c$, $a, c \neq 0$, $a \neq \pm c$, $a \neq 2c$ leaves only the component $H_u$ light in the
original $27$ $(2,1)$ form and breaks the gauge symmetry down to that of the Standard Model.
Note that there is also left light a Higgs doublet, $\bar{H}_u$, in the conjugate representation these
fields originating in the harmonic $(1,1)$ forms corresponding to the $\bar{27}$ representations on the
original manifold. The field $\bar{H}_u$ has the correct quantum numbers to play the role of $H_d$ but,
if so, there is an interesting implication for fermion masses as discussed below. From eq(8) one
may see that in this case the group left unbroken by Wilson lines is $SU(3) \otimes SU(2) \otimes U(1)$
provided the discrete group is $Z_N$ with $N > 3$. Thus we have a straightforward mechanism
capable of leaving just a doublet Higgs pair of fields light after compactification. Note that
any large scale breaking such as is necessary to give mass to the vectorlike $5 + \bar{5}$ representation
must not generate a mass term for $H_u$, $\bar{H}_u$ otherwise the mechanism for keeping Higgs doublets
light fails. This should follow from some symmetry of the theory. This is just the general “µ”
symmetry discussed above for this case.

While the scheme just considered is very attractive it is not the only natural way a light
pair of Higgs doublets may occur. An alternative is for the $(2,1)$ split multiplet to contain
just the $D$ states of eq(10). We will denote these states by $D_S$. The reason this apparently
bizarre suggestion is viable follows because these states, left light after compactification, may
acquire a large mass through a stage of spontaneous symmetry breaking by coupling to the the $D^c$
components of one of the complete 27 dimensional set of fields needed to accommodate the
families as discussed above. Of course, just as in the previous example, there will also be $\bar{D}_S$
states left light in the $(1,1)$ form and this should couple to the $D$ component of the 27. These
masses follow from the couplings

$$<\phi> D_S D^+ < N > D_S D^c$$

where $\phi$ is the $E_6$ singlet field whose vev is responsible for generating the masses in the additional
vectorlike $m_{ns}(27 + \bar{27})$ fields via the coupling $\phi 27 \bar{27}$. In this case the “µ” symmetries of the
theory should forbid the $N5\bar{5}$ mass term of the non-MSSM states in one of the family 27s.

The net result is that, in addition to the Standard Model states, only the Higgs doublets $H_u$ and
$H_d$ are left light from this family 27. We shall discuss in the next section the different
phenomenological implications of the three mechanisms we have identified capable of giving
rise to the light Higgs doublets.

\footnote{The case $b = 3c$, $a = 2c$ leaves both $H_u$ and $H_d$ light in the $(2,1)$ form but has also a further pair of Higgs
doublets in the conjugate representation. Furthermore in this case the gauge group is $SU(3) \otimes SU(2) \otimes SU(2) \otimes
U(1) \otimes U(1)$. However if the $SU(3) \otimes SU(2) \otimes U(1) \otimes U(1)$ singlet fields $\nu_R$ and $N$ acquire vevs the gauge group will be
further be broken down to just the Standard Model. Indeed there exist compactifications in which these fields
acquire Planck scale vevs, effectively breaking the initial gauge symmetry to just $SU(5)$.}

\footnote{This corresponds to the case $b = c$ in Table 2.}
3 Unification predictions

As we have discussed, even in string theories without a Grand Unified group below the compactification scale, the effect of the underlying extended gauge symmetry remains in the light spectrum. Even if the gauge group is just that of the Standard Model the gauge couplings are related and the unification scale determined close to the compactification scale. Thus such theories can lead to predictions for the low-energy gauge couplings of a similar nature to those obtained in Grand Unification. Particularly since such predictions provide the main evidence for a stage of Grand Unification, it is of interest to examine these predictions in some detail, for the string unification case in which the gauge group is not Grand Unified below the compactification scale.

Thus we consider level-1 string theories in which the couplings are related at the unification scale in the same way as they would be in an $SU(5)$ theory but, due to Wilson line breaking, the gauge group below the compactification scale is just that of the Standard Model. The multiplet structure is just that of the standard model plus the additional heavy states which acquire $SU(3) \otimes SU(2) \otimes U(1)$ invariant masses below the unification (or compactification) scale. Thus we must allow for additional multiplets with Standard Model gauge content equivalent to “vectorlike” representations coming in $(5 + \bar{5})$ or $(10 + \bar{10})$ representations of $SU(5)$ plus any states associated with the appearance of the Higgs “split” multiplets in the MSSM. The latter fall into two main categories. In the more “conventional” case there are just two split multiplets containing the MSSM Higgs supermultiplets and so there are no further massive multiplets to consider. In the unconventional case the pair of light Higgs at low energy comes from a $5 + \bar{5}$ plus a $D_S + D_S^c$ split multiplet, the new states obtaining intermediate scale masses. The third possibility is that the Higgs came from a 5 associated with one of the families and in this case there will be additional $D + D^c$ states at the intermediate scale. The implications for unification of couplings of this case is similar to the unconventional case as discussed below but, as we shall see, the implications for fermion masses do vary.

Let us first consider the “conventional” split multiplet case. In addition to the MSSM states, the only flexibility in the spectrum is the appearance of additional vectorlike states filling out complete 5 or 10 dimensional $SU(5)$ representations together with some Standard Model singlet states. The expectation is that these vectorlike states will acquire large masses. Since they come in vectorlike combinations, we can write down $SU(3) \otimes SU(2) \otimes U(1)$ invariant mass terms $\mu_{\psi} \bar{\psi} \psi$ where the mass $\mu_{\psi}$ can be far above the electroweak breaking scale. This term will arise from a stage of spontaneous symmetry breaking through the coupling $\lambda_{\psi} \phi \bar{\psi} \psi$, when the $SU(3) \otimes SU(2) \otimes U(1)$ invariant scalar field $\phi$ acquires a vacuum expectation value (vev). There is a very natural explanation for the origin of this vev, because the mass squared of the fields $\phi$ will be driven negative by the radiative corrections involving the coupling $\lambda_{\psi}$, in the usual radiative breaking mechanism.

The unification predictions of this class of model differs from the minimal unification predictions due to the radiative effects of these new states which, while expected to be heavy due to intermediate scale breaking are lighter than the unification scale. The phenomenological effects of such a non-standard spectrum has been discussed in detail in [17]. This analysis uses the measured values of $\alpha_{1,2}(M_Z)$ to predict the value of $\alpha_3(M_Z)$ in terms of the unified coupling, assuming string threshold corrections are negligible. At one loop the predictions for the unifi-
cation scale and $\alpha_3(M_Z)$ do not change. This follows because complete SU(5) multiplets give the same additive contribution to the SU(3), SU(2) and U(1) beta functions. The predictions for the unification scale and $\alpha_3(M_Z)$ follow from the differences of the beta functions, hence this result. However the magnitude of the gauge coupling at unification does change due to the extra light matter, causing it to increase. At two loop order there are two effects. The first is the usual two-loop contribution to the beta functions. The second is the radiative splitting of the masses of the new intermediate scale mass states. This splitting occurs at one loop order but its effect on the gauge couplings occurs only via the one loop beta function and hence the net effect is at two loop order. Remarkably the two effects largely cancel [17, 18] leaving the predictions for $\alpha_3(M_Z)$ and $M_X$ very close to those of the MSSM even in the case there is a large number of additional vectorlike states left light after compactification. At this order the predictions depend only on the combination $p = n_5 + 3n_{10}$ where $n_5$ and $n_{10}$ denote the number of additional 5s and 10s. For $\alpha_i(M_X)$ in the perturbative domain the value of $\alpha_3(M_Z)$ and $M_X$ increases by a factor 3.5. For $\alpha_3(M_Z) = 0.13$ and $M_X$ increases by a factor 7.5. The value of $\alpha_3$ is uncomfortably large compared to the experimental mean of 0.118 ± 0.003, but the increase in $M_X$ for $p = 5$ takes it closer to the (weakly coupled) heterotic string value.

We turn now to the “unconventional” split multiplet case in which, below the compactification scale, there are additional light colour triplets. At one loop order this affects the running of the couplings and hence the unification predictions

$$\alpha_i^{-1}(Q) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi} \log \frac{Q}{M} + \frac{b_1'}{2\pi} \log \frac{M}{M'_X}$$

(12)

where $M'_X$ is the unification scale and $M$ is the mass of the additional colour triplet states. Below $M$ the one loop beta function, $b_i$, is just that of the MSSM. Above $M$ the beta function changes to $b_i'$. We wish to determine the change in the unification scale and strong coupling in this scheme relative to the MSSM in which

$$\alpha_{MSSM,i}^{-1}(Q) = \alpha_{MSSM,GUT}^{-1} + \frac{b_i}{2\pi} \log \frac{Q}{M_X}$$

(13)

where $M_X$ is the unification scale in the MSSM. In both cases $\alpha_{1,2}(M_Z)$ are input as their measured values. The unification scale is found from the relative evolution of $\alpha_{1,2}$. In the model with $D$s above $M$ the relative evolution of $\alpha_2$ speeds up causing $\alpha_{1,2}$ to meet sooner lowering the unification scale. On the other hand due to the $D$s, the relative evolution of $\alpha_3$ and $\alpha_2$ is decreased above $M$ so the value of $\alpha_3(M_Z)$ must be reduced to allow the couplings to meet. Quantitatively, combining eqs(12,13), we find

$$\log \left( \frac{M_Z}{M_X} \right) = \log \left( \frac{M_Z}{M} \right) + \frac{b_2' - b_1'}{b_2 - b_1} \log \left( \frac{M}{M'_X} \right),$$

(14)

or

$$\log \left( \frac{M}{M_X} \right) = \frac{b_2' - b_1'}{b_2 - b_1} \log \left( \frac{M}{M'_X} \right).$$

(15)
and
\[ \Delta \alpha^{-1}_{3} = -\frac{1}{b'_1 - b'_2} \left[ (b'_2 - b'_3) \Delta b'_1 + (b'_3 - b'_1) \Delta b'_2 + (b'_1 - b'_2) \Delta b'_3 \right] \frac{1}{2\pi} \log \left( \frac{M}{M_X} \right), \] (16)

where \( \Delta b_i \) are defined to be
\[ \Delta b_i = b'_i - b_i. \] (17)

The MSSM prediction for \( \alpha_3 \) is approximately 0.006 larger than the experimental measurement. This discrepancy can be eliminated by choosing \( M'_X \) appropriately. This gives
\[ \frac{M'_X}{M_X} \approx 0.9 \] (18)

Note that the effect of this scheme is largely confined to a change in \( \alpha_3 \), the change in the unification scale being very small.

So far we have neglected the possible effect of additional vectorlike states on our prediction. At one loop these do not change the expectation for the unification scale or the strong coupling. However they do cause the coupling at unification to increase, increasing the potential importance of two loop effects. These are straightforward to compute and have been considered in detail in \[17\] (the effect is that both \( M_X \) and \( \alpha_3 \) increase). In this case however the spectrum including the \( D \) quark split multiplets reduces \( \alpha_3 \) at one loop so the effects together lead to a net reduction which may be adjusted to give the experimentally measured value through choice of \( M_X \). However, as we have seen, the new one loop effects leave \( M_X \) essentially unchanged so, for it, the net effect is an increase in the value of \( M_X \) from the MSSM value. For a perturbative value of the unified coupling the increase in \( M_X \) is only a factor of \((3.5-7.5)\), the upper value reducing, but not eliminating, the discrepancy with the string prediction in eq(1).

We have considered the gauge unification predictions for two of the three possible mechanisms we identified as being able to give light Higgs doublets. The third possibility is that the Higgs came from a 5 associated with one of the families and in this case there will be an additional \( D + D^c \) states at the intermediate scale. This possibility is intermediate between the first two because in the conventional split multiplet case there are no additional states and in the unconventional split multiplet case there are two pairs of \( D + D^c \) states at the intermediate scale. Thus the prediction for this third possibility follows that of the unconventional case provided one adjusts the intermediate mass scale to take account of the reduced number of \( D \) states. In this latter case there is a further ambiguity in the intermediate scale spectrum corresponding to the split multiplets which must acquire intermediate scale masses. We will not consider their effect here.

3.1 Fermion masses

We close this section with a brief discussion of the expectation for fermion masses in the class of string theory of interest here. As we noted above the case of unification at strong coupling is particularly interesting and is motivated by the appearance of the additional vectorlike states in the string theory. In this case the ratios of fermion masses are determined by infra-red fixed points. The details depend on which Yukawa couplings are allowed by the symmetries of the model and have been studied in \[17\]. There it is found, for example, that the ratio \( m_b/m_\tau \), may be driven to an initial value close to 1 leading to an excellent prediction for the \( b \) mass at
low energies. This prediction does not rely on a GUT relation between the couplings. Rather it follows because the multiplet structure carries the underlying GUT structure allowing for a similar set of couplings involving the $b$ and the $\tau$ and thus leading to fixed points for these couplings which are nearly the same for the $b$ and the $\tau$.

However, there may be important differences in the prediction for the quark masses in the various versions of our string compactifications. This follows from the structure of the light Higgs sector discussed above. Let us consider both the conventional and unconventional split multiplet cases. In this the Higgs doublets $H_u$ and $\bar{H}_u$ are $(2,1)$ and $(1,1)$ harmonic forms respectively. However the chiral matter fields are $(2,1)$ harmonic forms. The couplings leading to fermion masses come from the coupling of three $(2,1)$ forms there being no coupling of two $(2,1)$ forms to a $(1,1)$ form. Thus we have the prediction that only the up quark masses are non-zero. While this is a good starting point, explaining why the top mass is much larger than the bottom (and $\tau$) mass, it is only the first approximation. The down quark and lepton masses will be generated through mixing of the $(1,1)\bar{H}_u$ with the $(2,1)H_d$ states in the spectrum below the compactification scale. As discussed above these states appear as part of the fields making up the 27 dimensional representations the families belong to and as part of the additional vectorlike states filling out complete 27 plus $\bar{27}$ representations. This mixing is generated through the couplings needed to give the non-MSSM states a mass. Suppressing Yukawa couplings we have

$$\phi H_u \bar{H}_u + NH_u H_d$$

If $\phi$ and $N$ acquire vevs the combination $<\phi>\bar{H}_u + <N>H_d$ is heavy leaving the combination $<N>H_u - <\phi>\bar{H}_d$ light. The component $H_d$ can couple to down quarks (and leptons) allowing for the generation of down (and lepton) quark masses. The important point is that the ratio of up to down quark masses is determined by the ratio $<\phi>/ <N>$ offering a simple explanation for the difference between the top and bottom (and $\tau$) masses.

Finally we consider the third case in which the Higgs originate in one of the family $27$s, i.e. both Higgs are $(2,1)$ forms. In this case both can couple directly to the quarks and so there is no immediate reason for the top Yukawa to be enhanced relative to the bottom Yukawa. This case fits most easily with the large $\tan\beta$ solution in which the difference between the top and bottom masses follows because of the asymmetry between the two Higgs vacuum expectation values.

## 4 Summary and Conclusions

While there has been much progress in understanding and constructing four dimensional string theories there has been relatively little progress in classifying their phenomenological implications. This is mainly because the vast number of candidate string vacua makes it difficult to draw general conclusions. In this paper we have attempted to make progress in this direction by identifying what we consider to be a particularly promising class of string compactifications and studying some general phenomenological features they display. The class of string theories we consider is based on compactification of the level-1 heterotic string with Wilson loops. This gives a low-energy structure remarkably close to that observed. The families fill out complete

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6 We assume the chiral multiplet excess is in the $(2,1)$ sector. If it is in the $(1,1)$ sector the same discussion applies with $(2,1)$ and $(1,1)$ interchanged.
$SU(5)$ representations even though the Gauge Group is not Grand Unified. The quarks and leptons must belong to the fundamental representation of the Standard Model group, higher representations are not allowed. The gauge couplings have the $SU(5)$ relations at the string unification scale giving rise to the good unification predictions for the couplings at low energies even though there is no stage of Grand Unification. The problem encountered in GUTs of splitting the light Higgs doublets from their colour triplet partners is evaded because the Higgs states are not related by a GUT to colour triplets.

Given this motivation we considered in some detail the phenomenological implications of this class of theory. The first important observation is that the spectrum will be that of the MSSM plus additional states which are left light at the compactification scale but which will acquire mass at an intermediate scale. The nature of this additional matter is quite constrained. The light multiplet structure after compactification consists of light left-handed chiral superfields which have the same multiplet content as $(n + m_{ns})$ complete $27s$ and $m_{ns}$ complete $\bar{27}s$ of $E_6$ even though the gauge group is just that of the Standard Model. In addition there are $m_s$ split multiplets consisting of those components of the $(27 + \bar{27})$ which are singlets under the Wilson line group elements.

We identified three mechanisms for giving rise to the light Higgs doublets of the MSSM and analyzed their predictions for the unification scale and the strong coupling, assuming string threshold corrections are negligible. The first has the light Higgs doublets coming from the split multiplets. It gives a prediction for $\alpha_3(M_Z)$ larger than that of the MSSM, perhaps unacceptably so for the case when unification occurs for a large value of the gauge coupling. In this case $M_X$ is also increased, by a factor (3.5-7). The second mechanism to get light Higgs doublets also involved the split multiplets but in this case they contain $D$ quarks. Thus the spectrum above the intermediate scale is modified. This has the interesting effect of lowering $\alpha_3$, easily accommodating the experimental value. For the case of large unified coupling $M_X$ is still increased by a factor (3-6). The third case in which the light Higgs originate from one of the family multiplets gives predictions quite similar to this case.

We find it remarkable how close to the Standard Model structure this class of string theory is. Moreover it illustrates quite clearly how the unification predictions which have led many to conclude there is a stage of Grand Unification may follow equally well without Grand Unification and without many of the complications that are required to build a viable Grand Unified theory. The precise predictions are sensitive to the mechanism giving light Higgs doublets and it is interesting that two of the possible mechanisms offer an explanation for the discrepancy between the MSSM prediction for the strong coupling and its precision measurement. String unification without Grand Unification leads to a quantitative test of the idea of unification including gravity for the gauge unification scale of the Standard Model couplings is also the string unification scale. The initial comparison is encouraging. In the case of unification at large coupling the gauge unification scale is found to be $(0.9 - 1.8)10^{17}\text{GeV}$ to be compared with the string prediction following from eq(1) of $1.510^{17}\text{GeV}$ in the 10D weakly coupled heterotic string case and somewhat less in the strongly coupled case.

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