Note on the Normalization of Predicted GRB Neutrino Flux

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We note that the theoretical prediction of neutrinos from gamma-ray bursts (GRBs) by IceCube overestimates the GRB neutrino flux, because they ignore both the energy dependence of the fraction of proton energy transferred to charged pions and the radiative energy loss of secondary pions and muons when calculating the normalization of the neutrino flux. After correction for these facts the GRB neutrino flux is reduced, e.g., by a factor \( \sim 5 \) for typical GRB spectral parameter, and may be consistent with the present zero event detected by IceCube. More observations are important to push the sensitivity below the prediction and test whether GRBs are the sources of ultra-high energy cosmic rays.

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IceCube has become the most sensitive TeV-scale neutrino telescope that may reach the predicted neutrino flux from gamma-ray bursts (GRBs). The continued non-detection in its 22 (IC22, [1]), 40 (IC40, [2]) and 59 (IC59 [3]) string configuration puts more and more stringent limits on GRB neutrino flux. The IC40 limit is comparable to the theoretical prediction [2], whereas the combined IC40 and IC59 limit is only 0.22 times the prediction [3]. These string configuration puts more and more stringent limits on GRB neutrino flux. The IC40 limit is comparable to the muon neutrino flux from a GRB, \( F_{\nu} \), with neutrino oscillation the electron, muon and tau neutrinos roughly share equal energy [2].

The approach taken by IceCube papers [1,2] is presented in the appendix of the IC22 paper [1]. In their approach, the muon neutrino flux from a GRB, \( F_{\nu} \), (with neutrino oscillation the electron, muon and tau neutrinos roughly share equal energy [2]) is scaled to the proton flux in the GRB, \( F_p \), as

\[
\frac{F_{\nu}}{F_p} = \frac{1}{8} f_{\pi,b}
\]

(this comes from eq [A8] of [1]). Here \( f_{\pi,b} \equiv f_{\pi}(E = E_b) \) is the fraction of energy of protons with \( E_b \) carried by charged pions, and \( E_b \) is the energy of protons that interact with photons with spectral-break energy \( \epsilon_b \) at \( \Delta \) resonance [1],

\[
E_b = 1.3 \times 10^{16} \Gamma_{2.5}^{-2} \epsilon_b^{1 - 1/2} \mathrm{MeV}
\]

where \( \Gamma = 10^{2.5} \Gamma_{2.5} \) is the bulk Lorentz factor of the GRB, and \( \epsilon_b = 1 \epsilon_b \) MeV MeV. The \( f_{\pi,b} \) value depends on the GRB properties, and varies from burst to burst (see eqs [A7] and [A8] of [1]). The factor 1/8 is due to the facts that one half of the \( p\gamma \) interactions produce charged pions, and that each generated neutrino is assumed to carry one fourth of the secondary pion energy. The proton flux can be normalized to gamma-ray flux by \( F_p = (1/f_{\pi})F_{\gamma} \), where \( f_{\pi} \) is the ratio of accelerated proton to electron energy. In what follows we show that the approximation in eq (1) is different from the models of Waxman and Bahcall [1,2] and Guetta et al [3], and leads to overestimate of the neutrino flux.

For a flat proton distribution with index \( p \approx 2 \) \( (E^2dn_p/dE \propto E^{-2}) \), and a typical GRB spectrum with Band-function parameters, \( \alpha_{\gamma} = 1 \) and \( \beta_{\gamma} = 2 \), \( f_{\pi}(E) = f_{\pi,b} \) is valid for protons with \( E > E_b \). However, for \( E < E_b \), \( f_{\pi} \propto E \) reduces with \( E \) decreasing because fewer target photons at high energy [1]. Thus by using \( f_{\pi}(E) = f_{\pi,b} \) in energies \( E < E_b \) eq (1) overestimates the neutrino flux.

Moreover, eq. (1) also ignores the suppression of neutrino production at high energies due to the radiative cooling of secondary pions/muons [1,2]. The synchrotron cooling timescale is shorter than the secondary decay time at energies above the cooling energy [1,3,4],

\[
E_{c,\pi/\mu} = 2 \times 10^{17} (\epsilon_c/\epsilon_B)^{1/2} \Gamma_{4.5}^{-1} \Delta t_{-2}^{-1/2} L_{52}^{-1/2} \times 10^{-2} \left( \frac{\mu^+}{\pi^+} \right) \mathrm{eV}
\]

Here \( L = 10^{52} \) erg s\(^{-1}\) is the GRB (isotropic) luminosity, \( \Delta t = 10^{-2} \Delta t_{-2} \) is the GRB variability time, and \( \epsilon_c \) and \( \epsilon_B \) are the fractions of internal energy carried by postshock electrons and magnetic field, respectively.
Thus the neutrino production is mainly contributed by protons with $E_b < E < E_c$, which is only a fraction of the total accelerated protons in energy. The distribution of the accelerated protons is expected to be a power law between the minimum and maximum energy. For mildly-relativistic GRB internal shocks, the minimum accelerated proton energy might be

$$E_{\text{min}} \approx \Gamma m_p c^2 = 3 \times 10^{11} \Gamma_{300} \text{eV}.$$  (4)

The maximum proton energy is determined by the limit of synchrotron cooling for typical GRB parameters [8],

$$E_{\text{max}} = 2.5 \times 10^{20} \Gamma_{300}^{5/2} \alpha^{1/4} \epsilon_{\beta}^{-1/4} g^{-1/2} L_{52}^{-1/4} \text{eV},$$  (5)

where $g \gtrsim 1$ accounts for the uncertainty in particle acceleration time.

Since the neutrino production is mainly contributed by protons with $E_b < E < E_c$ where $f_\pi(E) = f_{\pi,b}$, the muon neutrino flux $\mathcal{F}_\nu$ is estimated to be

$$\mathcal{F}_\nu \approx \int_{E_b}^{E_c} \frac{1}{8} f_{\pi,b} E \frac{d_n}{dE} dE = \frac{\mathcal{F}_\nu^{\text{IC}}}{\mathcal{F}_p} \int_{E_b}^{E_c} E \frac{d_n}{dE} dE \approx \mathcal{F}_\nu^{\text{IC}} \frac{\ln(E_c/E_b)}{\ln(E_{\text{max}}/E_{\text{min}})},$$  (6)

where $\mathcal{F}_p = \int_{E_{\text{min}}}^{E_{\text{max}}} E \frac{d_n}{dE} dE$ and Eq (11) have been used, and the last equation holds for $p \approx 2$. Given $E_{\text{max}}/E_{\text{min}} \sim 10^9$ and $E_{c,\pi}/E_b \sim 10^2$, the correction to Eq (11) is a factor of $\mathcal{F}_\nu/\mathcal{F}_\nu^{\text{IC}} \sim 0.22$.

Below is more detailed calculation about $\mathcal{F}_\nu/\mathcal{F}_p^{\text{IC}}$. For a flat proton distribution, $p = 2$, and a GRB spectrum with Band-function parameters, $\alpha_\gamma$, $\beta_\gamma$ and $\epsilon_b$, the source neutrino spectrum (before neutrino oscillation) is [1, 4–6], for muon neutrinos from secondary pion decay,

$$\frac{dn_\mu}{d\varepsilon} = n_0 \times \begin{cases} \left( \frac{\varepsilon}{\varepsilon_b} \right)^{-\alpha_\mu} & \varepsilon < \varepsilon_b \\ \left( \frac{\varepsilon}{\varepsilon_b} \right)^{-\beta_\mu} & \varepsilon_b < \varepsilon < \varepsilon_{c,\pi} \\ \left( \frac{\varepsilon}{\varepsilon_{c,\pi}} \right)^{-\beta_\mu + 2} & \varepsilon > \varepsilon_{c,\pi} \end{cases}$$  (7)

for electron and muon neutrinos from secondary muon decay,

$$\frac{dn_\mu}{d\varepsilon} = \frac{dn_\mu}{d\varepsilon} = n_0 \times \begin{cases} \left( \frac{\varepsilon}{\varepsilon_b} \right)^{-\alpha_\mu} & \varepsilon < \varepsilon_b \\ \left( \frac{\varepsilon}{\varepsilon_b} \right)^{-\beta_\mu} & \varepsilon_b < \varepsilon < \varepsilon_{c,\mu} \\ \left( \frac{\varepsilon}{\varepsilon_{c,\mu}} \right)^{-\beta_\mu + 2} & \varepsilon_{c,\mu} < \varepsilon < \varepsilon_{c,\pi} \\ \left( \frac{\varepsilon}{\varepsilon_{c,\pi}} \right)^{-\beta_\mu + 2} & \varepsilon > \varepsilon_{c,\pi} \end{cases}$$  (8)

There is no tau neutrino $\nu_\tau$ generated in $p\gamma$ interactions, and the small effect of kaon production on neutrino flux is neglected. Here the power law indices are $\alpha_\gamma = 3 - \beta_\gamma$ and $\beta_\nu = 3 - \alpha_\gamma$, and the normalization is

$$n_0 = \frac{dn_\nu(E = E_b)}{d\varepsilon} = 50 f_{\pi,b} \frac{dn_\nu(E = E_b)}{dE}.  (9)$$

Note the factor $50 = \frac{1}{2} \times 0.2/0.05$ is resulted from the facts that (i) $1/2$ of $p\gamma$ interactions produce charged pions; (ii) since the pion carries $0.2$ of the proton energy, the muon neutrino number (after oscillation) per proton is $f_{\pi,b}/0.2$; and (iii) a single neutrino carries $0.2 \times \frac{1}{6} = 0.05$ of the proton energy, $\varepsilon_b = 0.05 E_b$. Similarly, we assume the other break energies in the neutrino spectrum, $\varepsilon_{c,\pi} \approx 0.05 E_{c,\pi}$, and $\varepsilon_{c,\mu} \approx 0.05 E_{c,\mu}$.

Considering neutrino oscillation, the muon neutrino spectrum (including $\nu_\mu$ and $\bar{\nu}_\mu$) detected on the Earth is approximated as [7]

$$\frac{dn_{\mu,\nu}}{d\varepsilon} \approx 0.2 \frac{dn_{\mu,0}}{d\varepsilon} + 0.4 \frac{dn_{\mu,0}}{d\varepsilon} + 0.4 \frac{dn_{\mu,0}}{d\varepsilon} = 0.2 \frac{dn_{\mu}}{d\varepsilon} + 0.4 \left( \frac{dn_{\mu}}{d\varepsilon} + \frac{dn_{\mu}}{d\varepsilon} \right).  (10)$$

Note here $dn_{\nu,\tau}/d\varepsilon = 0$ since no tau neutrino produced. The muon neutrino flux is calculated as

$$\mathcal{F}_\nu/\mathcal{F}_p = \int_{0.05E_{\text{min}}}^{0.05E_{\text{max}}} \varepsilon \frac{dn_{\mu}}{d\varepsilon} d\varepsilon \int_{E_{\text{min}}}^{E_{\text{max}}} E \frac{dn_{\pi}}{dE} dE.$$  (11)
For various values of GRB spectral parameters, \( \alpha_r \), \( \beta_r \), and \( \epsilon_b \), we calculate the value of \( F_\nu/F_\nu^{IC} \) with eqs. (7-11), and show the \( F_\nu/F_\nu^{IC} \) value in Table I. We find that the approximation of eq (11) overestimates GRB neutrino flux by a few. Typically for GRBs with \( \alpha_r = 1 \), \( \beta_r = 2 \), and \( \epsilon_b^{obs} = \epsilon_b/(1+z) = 0.2 \) MeV, the correction factor is \( F_\nu/F_\nu^{IC} = 0.2 \), i.e., the neutrino flux is overestimated by a factor of 5 in [1, 3].

Note the neutrino spectrum used here (eqs. 7 and 8) is different from the one by IceCube papers [1, 3] at high energy; they assume a simple steeping \( d\nu/d\varepsilon \propto \varepsilon^{-(\beta_r+2)} \) at \( \varepsilon > \varepsilon_{c,\nu} \) (see eq. [A3] in IC22 paper [1]), which underestimate the neutrino emission from pion decay. Using their spectral shape, the neutrino flux is even smaller than using ours, thus the correction factor is even smaller.

Some comments should be made here. In this brief note we point out that when trying to test the models of [4–6] with data, the IceCube papers [1, 3] actually take an approach different from the former models in calculating the neutrino flux, leading to underestimate. However, the models of [4–6] consider only \( \Delta \) resonance and neglect the multi-pion production [9, 10], the kaon production [11] and the possible secondary particle acceleration [12]. All these facts may increase the neutrino flux and somewhat compensate the overestimate. It is also worth mentioning that if UHECRs are produced by the decay of neutrons that escape from the GRB outflow, it is straightforward to relate the neutrino flux with the observed UHECR flux, and avoid the uncertainties in parameters, e.g., \( f_p \) and \( f_\nu \) [13]. If the assumption that only neutrons escape is true, the neutrino flux will also increase.

Finally it should be stressed that there are large uncertainties in the GRB neutrino models, which may cause lower \( f_\nu \) and hence lower neutrino flux, e.g., larger emission size or larger bulk Lorentz factor. More observations by IceCube are required to push the sensitivity below the uncertainties [14], and test the assumption of GRBs as the UHECR sources.

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TABLE I: The value of \( F_\nu/F_\nu^{IC} \) for various spectral parameter values.

| \( \alpha_r \) | \( \beta_r \) | \( \epsilon_b^{obs}/\text{MeV} \) | \( F_\nu/F_\nu^{IC} \) |
|---|---|---|---|
| 1 | 2 | 0.2 | 0.20 |
| 0.5 | 2 | 0.2 | 0.12 |
| 1.5 | 2 | 0.2 | 0.56 |
| 1 | 3 | 0.2 | 0.25 |
| 1 | 2 | 0.05 | 0.18 |
| 1 | 2 | 2 | 0.14 |
| 1 | 2 | 2 | 0.30 |

Note–The other parameters are \( \Gamma = 300 \), \( \Delta t = 10^{-2}s \), \( L = 10^{52}\text{erg s}^{-1} \), \( z = 1.5 \) and \( E_{\text{max}} = 10^{21}\text{eV} \). The redshift \( z \) only has effect on the GRB spectral break, \( \epsilon_b = \epsilon_b^{obs}(1+z) \).