A new method of terrain self-adaptive matching algorithm for autonomous underwater vehicle

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Abstract. As different underwater terrain features will affect the accuracy of unscented Kalman filter terrain matching algorithm, a new terrain self-adaptive matching method is proposed which adjusts sigma point distribution by terrain features. The connection between sigma point distribution distance and three basic terrain features were analysed. Positioning error range were calculated using navigation positioning errors and underwater digital map. Terrain elevation standard deviation was used to characterize the information quantity of decision region. Scaling parameter was adjusted using linear mapping method and sigma point distribution distance was determined by terrain features. The simulation results proved better terrain adaptability and positioning precision of improved self-adaptive matching algorithm.

1. Introduction
Underwater terrain aided navigation has the advantages of low cost, no drift, all-weather working and so on, which can greatly enhances the capabilities of the voyage and high-precision concealed navigation of autonomous underwater vehicle (AUV). At present, many science institutes such as the United States, Britain, Norway, Sweden have carried out a large number of theoretical research and engineering tests for a variety of underwater terrain matching algorithms.[1-4]

Unscented Kalman Filter (UKF) is a kind of nonlinear filtering based on UT transformation and Kalman filter [5]. Compared with the extended Kalman filter, the proposed algorithm has a higher filtering precision and convergence rate [6,7]. Therefore, the terrain matching algorithm based on UKF should theoretically have a better matching effect than SITAN terrain matching algorithm. There are few studies on UKF terrain matching algorithms in the literature. Among them, Andrea.L from the Sweden Institute of Electronic Engineering had some conclusions obtained by simulation that the convergence speed and matching accuracy of UKF are not better than EKF underwater terrain matching algorithm [8]. However, it is found that the UKF terrain matching algorithm of fixed scale factor[9] used in ref.[8] neglected the relation between terrain features and sigma point distribution, which lead to decreasing of convergence speed and matching precision.

This paper analysed the relationship between sigma point distribution distance and terrain features of UKF matching algorithm theoretically. As to the estimation error influenced by the flat underwater terrain and the large variation of depth, a method of self-adaptive UKF underwater terrain matching algorithm based on terrain features is proposed. Firstly, the parameters of the algorithm are initialized by inertial navigation information. Then, the motion equation is used to update the position of the
AUV, the position error reference area [10] and its terrain features are calculated as well. Secondly, the scale factor of UKF is adjusted according to the proportion by calculating the terrain height standard deviation[11] through the underwater digital map. Finally, the UKF terrain matching algorithm with this scale is used to update the prior estimation position and the post estimation position can be obtained at the same time. Simulation analysis show that the improved algorithm makes full use of the navigation error and terrain features which improves the positioning accuracy of AUV.

2. Terrain features and UKF underwater terrain matching algorithm

2.1. UKF underwater terrain matching algorithm

UKF terrain matching algorithm [12] consists of three steps including initialization, time updates and measurement updates. Since the depth of AUV is consistent when performing terrain matching, the navigation plane is only two-dimension (i.e., the state variable $n = 2$, the number of sigma points is $2n + 1$), the system noise $\omega$ and measurement noise $\zeta$ are zero-mean Gaussian white noise $Q$ and $R$. 

(1) Initialization

The sigma point set $\sigma_0 = [X_{i,0}, W_{i,0}]_{i=0}^4$ is calculated by using the AUV initial position $\bar{x}_0 = (\bar{x}_0, \bar{y}_0)$, the initial navigation bias is $P_0$, the UKF scale factor are $\alpha$, $\beta$, $\kappa$ . 

\[
X_i = \bar{x}_0 \quad i = 0 \\
X_i = \bar{x}_0 + (\sqrt{(2+\lambda)} P_0)_i \quad i = 1, 2 \\
X_i = \bar{x}_0 - (\sqrt{(2+\lambda)} P_0)_i \quad i = 3, 4
\]

\[
W_0^m = \frac{\lambda}{2+\lambda} \\
W_0^c = \frac{\lambda}{2+\lambda} + (1-\alpha^2 + \beta)
\]

Where, $W_0^m$, $W_0^c$ are the weights of the mean and variance of the central sigma points $X_0$ respectively, and $W_i^m$, $W_i^c$ are the weights of the mean and variance of the symmetric sigma points $X_i$ respectively.

(2) Time update

The sigma point set is updated with the motion equation of the AUV, and the reference points $X_{i,j}\mid_{t-1}$ can be obtained. Then the prior estimate position $\hat{X}_{j\mid_{t-1}}$ and variance $P_{j\mid_{t-1}}$ can be calculated as follows:

\[
X_{i,j\mid_{t-1}} = f(X_{i,j\mid_{t-1}}) \\
\hat{X}_{j\mid_{t-1}} = \sum_{i=0}^4 W_i^m X_{i,j\mid_{t-1}}
\]

\[
P_{j\mid_{t-1}} = \sum_{i=0}^4 W_i^c [X_{i,j\mid_{t-1}} - \hat{X}_{j\mid_{t-1}}] [X_{i,j\mid_{t-1}} - \hat{X}_{j\mid_{t-1}}]^T + Q
\]

(3) Measurement update

The sigma point set is assigned by $\hat{X}_{j\mid_{t-1}}$ with $P_{j\mid_{t-1}}$, then the reference position $X_{i,j}\mid_{t-1}$ can be obtained and the corresponding depth can be found from the digital map. The estimated depth $\hat{z}_{j\mid_{t-1}}$ can be calculated and the variance $P_{j\mid_{t-1}}$ and covariance $P_{j\mid_{t-1}}$ could be figured out finally.

\[
\hat{X}_{i,j\mid_{t-1}} = \left\{ \hat{X}_{j\mid_{t-1}} \pm \left( \sqrt{(2+\lambda)} P_{j\mid_{t-1}} \right) \right\}
\]

\[
\hat{z}_{i,j\mid_{t-1}} = h(\hat{X}_{i,j\mid_{t-1}}) \\
\hat{z}_{j\mid_{t-1}} = \sum_{i=0}^4 W_i^m z_{i,j\mid_{t-1}}
\]
\[ P_{\xi_{\theta}, \eta_{\theta}} = \sum_{i=0}^{4} W_{i}^{(c)} (z_{i, \xi_{\theta}} - \bar{z}_{\theta})^{2} + R \]  
\[ P_{\eta_{\theta}, \xi_{\theta}} = \sum_{i=0}^{4} W_{i}^{(c)} (X_{i, \eta_{\theta}} - \bar{X}_{\theta})^{2} + R \] 

After the gain factor \( K_{\theta} \) calculated, \( \bar{X}_{\theta} \) and \( P_{\theta} \) can be adjusted, then the post estimation of position \( \hat{X}_{\theta} \) and variance \( P_{\theta} \) can be got as well. Where \( z_{t} \) is the measured depth at the moment \( t \), \( z_{t} - \bar{z}_{\xi_{\theta}} \) represents the inconsistent degree of the predicted depth with the measured depth, which is known as measurement residuals.

\[ K_{t} = P_{\xi_{\theta}, \eta_{\theta}} P_{\eta_{\theta}, \xi_{\theta}}^{-1} \hat{X}_{\theta} = \hat{X}_{\theta} + K_{t}(z_{t} - \bar{z}_{\xi_{\theta}}) \]  
\[ \hat{P}_{\theta} = P_{\theta}^{-1} + K_{t}P_{\xi_{\theta}, \eta_{\theta}}K_{t}^{T} \]

Repeat the process of time and measurement update above, we can achieve the estimation position of the AUV route.

2.2. The relationship between terrain features and UKF scale factors

The sigma point set in UKF is similar to the particle swarm of particle filtering. From formulas (1), (2), it can be seen that the state values of the symmetric sigma point is related to the parameter \( \alpha \), \( \kappa \). It is pointed out in Ref. [12] that the distance factor \( \alpha > 0 \) is used to control the distribution distance of the symmetrical sigma point from the center point. The reasonable value of \( \alpha \) selected can reduce the influence of higher moments on the state estimation error, which usually choses a small positive number, such as 0.01;

Ref. [5] only gives the calculation formula of \( \kappa \) under the Gaussian conditions, which is \( \kappa + n = 3 \), while under the non-Gaussian conditions, it should be optimized according to specific problems. It can be seen that the appropriate value chosen for scale factor can adjust the distribution distance of the symmetric sigma point distance, which can improve the convergence speed and positioning accuracy of the UKF terrain matching algorithm as well as the terrain adaptability. From the model of underwater terrain matching problem, the uniform motion of AUV can be expressed by linear equation, so the scale factor adjustment focus on the measurement update stage, as shown in formula (6).

As to the underwater terrain matching algorithms, the terrain features consists of three kinds approximately: flat, slope and steep. The slope topography contains much useful information of terrain, which can quickly reduce the algorithm matching error, while flat topography and abrupt terrain cause the position error fluctuations easily, even lead to matching failure[11]. In this section, we will discuss the relationship between the terrain and the sigma points distribution. In order to facilitate discussion, one single direction of the matching problem is discussed, which is \( n = 1 \), \( i = 0, 1, 2 \), the two-dimensional case is similar with this situation.

The relationship between different terrain features and sigma point distribution is shown in Fig.1.

![Fig.1 Relationship between terrain features and distribution of distance](image-url)
Where $\chi_0$ is the central point of sigma, $\chi_1$, $\chi_2$ and $\chi_1'$, $\chi_2'$ represent the distribution of symmetric sigma points under the two scale factors respectively. $z_i$ and $z_i'$ ($i = 0, 1, 2$) represent the corresponding depth value of sigma points on the digital map. The real position of the AUV $X$ is indicated by "★", and the corresponding measured depth is $z$.

Fig.1-(a) shows in the case of flat topography, the corresponding depth $\{z_i, z_2, z_3\}$, $\{z_i', z_2', z_3'\}$ of sigma point $\{\chi_0, \chi_1, \chi_2\}$ is close to the depth $h$ at the true position $X$.

The main component of the residual error $z_i - z$ is measurement noise $\xi_i$. From formulas (8)-(10), a larger gain $K_i$ can be calculated, which indicates that the measurement result $z$ has a great influence on the post estimation $\hat{X}_{\theta}$. From formulas (9), it can be seen that the post position estimation error $\hat{X}_{\theta}$ will increase. By increasing the scale factor, the symmetric sigma point set $\{\chi_0, \chi_1', \chi_2'\}$ has a larger distribution distance, the difference of corresponding depth $\{z_i, z_2, z_3\}$ and the actual depth $h$ increase as well. The gain $K_i$ reduces and the effect of the measurement $z$ on the post estimation $\hat{X}_{\theta}$ is restrained, the information validity of the residual error $z_i - z$ (signal-to-noise ratio) is improved, meanwhile the influence of the measurement error on the prior estimation $\hat{X}_{\theta-1}$ is suppressed better, which makes the post position $\hat{X}_{\theta}$ more accurate.

Fig.1-(b) shows in the case of abrupt topography, the corresponding depth $\{z_i, z_2, z_3\}$ of the symmetric sigma point set $\{\chi_0, \chi_1', \chi_2'\}$ is quite different from the true position $X$ with the depth of $h$, and the residual error $z_i - z$ contains abundant terrain information. From formulas (8) to (10), we get a smaller gain $K_i$, which shows that the effect of the measurement $z$ on the post estimation $\hat{X}_{\theta}$ is not obvious. From formula (9), it can be seen that the impact of measurement update on the prior position $\hat{X}_{\theta-1}$ decreases and the accuracy of $\hat{X}_{\theta}$ is reduced. By reducing the scale factor, the sigma point set $\{\chi_0, \chi_1', \chi_2'\}$ has a smaller distribution distance, the gain $K_i$ becomes larger, which enhances the effect of the measurement $z$ on the post estimation $\hat{X}_{\theta}$. At the same time, the terrain information of residual error $z_i - z$ is not affected which ensures measurement information to adjust the prior estimation $\hat{X}_{\theta-1}$ adequately.

Fig.1-(c) shows in the case of slope topography, the corresponding depth $\{z_i, z_2, z_3\}$, $\{z_i', z_2', z_3'\}$ of sigma points $\{\chi_0, \chi_1, \chi_2\}$, $\{\chi_0, \chi_1', \chi_2'\}$ is difference. However, since the sigma points have a character of symmetrical distribution, which makes they have similar depth mean $\bar{z}$ and gain $K_i$. Therefore, although the different distribution distances exist, the measurement value $z$ has almost the same effect on post estimation $\hat{X}_{\theta}$.

It can be seen that the UKF underwater terrain matching algorithm has the best matching effect on the moderate slope topography, while for the flat topography and abrupt topography, it is necessary to adjust the distribution distance to suppress the fluctuation of matching error.
3. Improved UKF underwater terrain matching algorithm

3.1. Algorithm improvement ideas

In the previous section, the relationship between the sigma point distribution and the underwater terrain features is analyzed both from theorem and simulation data. It is found that the distribution distance should be reduced when the terrain is abrupt while the distribution distance should be increased when terrain is flat.

Therefore, as to the AUV terrain matching navigation, the sigma point distribution can be adjusted in real time according to the inertial navigation system, the positioning error provided by the digital map and the terrain features around the estimated position. Thus, the matching precision and terrain adaptability of the algorithm can be improved at the same time.

The algorithm improvement ideas can be summarized as follows: Taking the prior position as the estimation center, the terrain feature reference region is determined based on the inertial deviation and the digital map, the sigma point scale factor of the post estimation is adjusted by the terrain feature of the reference region. By choosing the appropriate sigma point distribution distance, the purpose of reducing the positioning error of UKF underwater terrain matching algorithm can be realized. There are three key problems in the method: reference region size determination, terrain features measurement and the adjustment of sigma point distribution.

3.2. Reference region size determination

It is assumed that the inertial navigation positioning error obeys the standard normal distribution. \(a, b\) represent long and short axle of ellipse respectively [13], \(\varphi\) represents the angle between the ellipse long axis and perpendicular to the north. The error circumscribed rectangle calculation formulas are deduced in Ref. [13] as follows:

\[
\begin{align*}
x_m &= 2\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi} \\
y_m &= 2\sqrt{a^2 \cos^2 \varphi + b^2 \sin^2 \varphi}
\end{align*}
\]

\[
\varphi = \frac{\pi}{2} - \frac{1}{2} \arctan \left( \frac{2\sigma_{xy}}{\sigma_x - \sigma_y} \right)
\]

(11)

The reference region size \(E_t\) can be determined by estimating the \(t\) moment navigation position and the length of the rectangle \(x_m, y_m\).

3.3. Terrain height standard deviation calculation

Terrain height standard deviation is usually used to describe the features of terrain roughness, it is irrelevant with the terrain elevation order in digital map as well as the observation direction [14]. As underwater terrain can be reflected by the depth indirectly, the formula can be calculated as follows:

\[
\sigma_M = \sqrt{\frac{1}{N_x} \sum_{j=1}^{N_x} (h_j - \bar{h})^2}
\]

(12)

Where, \(h_j\) represents the depth represented by each grid point on the digital map, \(\bar{h}\) represents the mean depth of the water. \(N_x\) is the total number of depth measurement positions in digital maps.

3.4. Sigma point distribution adjustment

The formulas (1) - (2) show that the direct method to optimize the sigma point distribution distance is to adjust the scale factors. Ref. [12] pointed out that the scale factor \(\alpha\) and the distribution distance are contacted closely, so sigma point distribution can be optimized by adjusting \(\alpha\) properly.

The inner relationship between underwater terrain features and the sigma point distribution distance can be obtained by a large number of simulations or experiments. In this paper, the underwater digital maps were produced by using six groups of real depth data from Yellow Sea and South China Sea. 500 randomly matching simulation paths were generated and the appropriate range of \(\alpha \in [0.5, 1.7]\)
was determined. For engineering practice, a simple linear mapping is used to express the relationship between the terrain height standard deviation of and the scale factor

\[
\alpha ; \alpha_t = 0.5 + \frac{1.2 \times (\sigma_t - \sigma_{\text{min}})}{\sigma_{\text{max}} - \sigma_{\text{min}}}
\]  

(13)

Where, \( \sigma_t \) represents the terrain height standard deviation of the reference area at the measured point, \( \sigma_{\text{max}}, \sigma_{\text{min}} \) represent the maximum and minimum variance value in terrain standard variance map respectively.

3.5. The algorithm improvement steps

(1) The UKF underwater terrain matching algorithm is used to obtain the a priori estimation position of the AUV \((\hat{x}_{\text{IP}-1}, \hat{y}_{\text{IP}-1})\).

(2) The size of the reference region \( E_x \) is determined by the Navigation bias and estimated position \((\hat{x}_{\text{IP}-1}, \hat{y}_{\text{IP}-1})\).

(3) Calculate the terrain height standard deviation \( \sigma_H \) of the reference region by formulas (12).

(4) The scale factor \( \alpha \) is calculated by formulas (13).

(5) The scale factor is used to adjust the sigma points, and the measurement is updated to obtain the a posterior estimation position \((\hat{x}_{\text{IP}}, \hat{y}_{\text{IP}})\).

4. Simulation analysis and Discussions

To verify the conclusions of the previous section, a two-dimensional mathematical mode for underwater terrain matching was established. Suppose that the depth of AUV remains unchanged, only the sonar sounder error is considered.

\[
\begin{align*}
\dot{x}_{t+1} &= x_t + v'_t \cos \phi'_t \\
\dot{y}_{t+1} &= y_t + v'_t \sin \phi'_t \\
\dot{z}_t &= h(x_t, y_t) + \xi_t
\end{align*}
\]

(14)

(15)

Where, \( x_t, y_t \) represent for the position of coordinates at the time \( t \); \( \dot{z}_t, v'_t, \phi'_t \) represent the measurement depth, measurement speed and heading of the AUV respectively at the time \( t \). Depth measurement error is \( \xi_t \sim N(0, \sigma^2_z) \), \( v_t, \phi_t \) represent for the actual speed and heading respectively, \( \alpha_v \sim N(0, \sigma^2_v) \) is the speed error and \( \alpha_\phi \sim N(0, \sigma^2_\phi) \) is the heading error.

![Fig.2 Relationship between terrain features and distribution of distance](image)

(a) underwater terrain of simulation  (b) unimproved algorithm  (c) self-adaptive algorithm
The computer simulations are performed on the unimproved algorithm and the improved self-adaptive terrain matching algorithm as the contradistinction. The algorithm parameters setting are as follows: the UKF scale factors of unimproved algorithm are supposed as: \( \alpha = 0.6 \), \( \beta = 2 \), \( \kappa = 1 \); \( \alpha = 1.5 \), \( \beta = 2 \), \( \kappa = 1 \), the sliding window of self-adaptive algorithm is \( 150 \times 150 \) m\(^2\) and the parameters are \( \sigma_{\text{un}} = 0.04 \), \( \sigma_{\text{uv}} = 7.07 \), \( \alpha \in [0.5, 1.7] \). 200 Monte Carlo simulations are taken under the same underwater model and digital map with the resolution of 6m, as shown in Figure 2-(a). the AUV initial position deviation is (50m, 50m), with a speed of 10m/s. The value of depth variance \( \zeta \), velocity variance \( \sigma_v \), heading variance \( \sigma_\omega \) are \( 0.1/\text{m}^2 \), \( \text{diag}(0.1^2, 0.1^2) / \text{m}^2 \), \( \text{diag}(0.3^2, 0.3^2) / \text{degree}^2 \), which satisfy Gaussian white noise. The simulation result of unimproved algorithm is shown in Fig.2.

As shown in Fig.2-(a), the selected navigation route has travelled through two kinds of water areas, which are moderate slope terrain A and flat terrain B. In Fig.2-(b), during the anterior half segment of route, the UKF terrain matching algorithm can converge fast under two different scale conditions due to the rich terrain features and moderate slope, and the positioning error drops below 10m quickly. However, with the rapid increase of terrain slope, algorithm positioning error of \( \alpha = 0.6 \) can reduce continuously and kept within 6m, while algorithm positioning error of \( \alpha = 1.5 \) is no longer reducing. The error curve appears large fluctuations at point A and the maximum error is 11.4m. During the bottom half segment of route, the terrain is flattened gradually, the underwater terrain information is relatively lacked, and the algorithm positioning error of \( \alpha = 0.6 \) increases dramatically, reaching at 15m, while the error can be kept within 8m on condition of \( \alpha = 1.5 \).

In Fig.2-(c), the improved algorithm converges faster, and the positioning error always keeps a small fluctuation below 9m. Comparing the Fig. 2-(b) with the Fig. 2-(c), it can be found that the self-adaptive algorithm suppresses the increase of positioning error at area A and C with depth drastic change, and also has a better error correction effect at area B.

5. Conclusion
Based on theoretical analysis and simulation, the relationship among underwater terrain features, sigma point distribution distance and algorithm matching error are discussed in this paper. An improved matching method by making use of the inertial navigation error, underwater height standard deviation and scale factor adjustment is put forward. The improved steps and the key problems solution are given out as well. The simulation results show that the improved self-adaptive algorithm can adjust the sigma point distribution distance according to different terrain features, and can suppress the matching error increasing caused by the flat terrain and depth drastic change, which has a better terrain adaptability and matching robustness.

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