Covariant RPA in Effective Hadronic Field Theory

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Abstract

In an effective hadronic theory constructed to describe long-range nuclear physics, the dynamics of the vacuum can be expanded in terms with zero or a finite number of derivatives acting on the fields. Thus vacuum dynamics can always be absorbed in the (infinite number of) counterterm parameters necessarily present in the effective lagrangian. These finite parameters, which at present must be fitted to data, encode the empirical vacuum physics as well as other short-range dynamics into the effective lagrangian; in practice, only a small number of parameters must be fitted. The strength of the effective field theory (EFT) framework is that there is no need to make a concrete picture of the vacuum dynamics, as one does in a renormalizable hadronic theory. At the one-loop level, the most convenient renormalization scheme requires explicit sums over long-range (“valence”) nucleon orbitals only, thus explaining the so-called “no-sea approximation” used in successful covariant mean-field theory (MFT) calculations of static ground states. When excited states are studied in the random-phase approximation (RPA), the same EFT scheme dictates the inclusion of both familiar particle-hole pairs and contributions that mix valence and negative-energy single-particle Dirac wave functions. The modern EFT strategy therefore justifies and explains the omission of some explicit contributions from the negative-energy Dirac sea of nucleons, as was done to maintain conservation laws in earlier pragmatic calculations of the nuclear linear response.

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I. INTRODUCTION

Whereas RPA calculations of inelastic states in finite nuclei using the simplest version of covariant quantum hadrodynamics (QHD) have long been available \[1, 2, 3, 4, 5, 6, 7, 8, 9\], it is only in recent years that calculations based on accurately calibrated mean-field theories have been performed \[10, 11, 12, 13\]. This renewed interest in covariant RPA, motivated in part by improved measurements of various nuclear compressional modes \[14, 15, 16\], leads us to examine these RPA calculations within the context of effective field theory (EFT) descriptions of nuclear many-body systems, which form the basis for modern QHD \[17, 18\].

The basic issue involves the treatment of the quantum vacuum. In theories where nucleons are described by four-component Dirac spinors, one must consider the role of the Dirac sea of negative-energy states. In older calculations based on renormalizable QHD models, the mean-field (or one-baryon-loop) contributions to the ground state from the Dirac sea could be calculated explicitly, but it was found that these contributions precluded an accurate description of bulk nuclear observables \[19, 20, 21, 22\]. Therefore, for completely pragmatic reasons, these contributions were omitted (in both renormalizable and nonrenormalizable models), resulting in what has historically been called the “no-sea approximation” for the mean-field ground state \[23, 24, 25, 26, 27, 28, 29\]. The “no-sea” hamiltonian, which typically contains sums over occupied valence (positive-energy) Dirac wave functions and polynomials in the mean meson fields, yields accurate results for bulk nuclear observables \[17, 26, 27, 28, 30, 31\].

Questions arose, however, in the treatment of the collective linear response (RPA) of these ground states to external probes. Whereas one might expect (based on the “no-sea approximation”) that this response would contain only the well-known particle-hole contributions \[32, 33, 34\], it has long been known that one must also include contributions from negative-energy basis states, if fundamental principles such as Lorentz covariance and gauge invariance are to be maintained \[7\]. This somewhat confusing situation is resolved in the modern EFT approach \[31, 35\], which shows that the term “no-sea” is in fact a misnomer, and that consistent descriptions of both the mean-field ground state and its linear response follow naturally from the standard rules of quantum field theory. The purpose of this paper is to illustrate these ideas as clearly as possible.

Our most important conclusion is that in the EFT, nothing is omitted in the so-called “no-sea approximation” from either the ground state or the RPA linear response. Although the old-fashioned interpretation discussed above implies that (regulated) negative-energy loop contributions are neglected in the former and that only the “Pauli blocking” corrections to the vacuum response are included in the latter, EFT shows that this interpretation is incorrect. In fact, the negative-energy contributions are always included, as one would expect from the rules of field theory, but they must be combined with the complete set of counterterms present in the QHD lagrangian; only the sum contributes to physical observables. Thus, even though the simple picture of the vacuum as a negative-energy Dirac sea is likely to be incorrect (given the complex nature of the QCD vacuum), it is automatically corrected by combining the baryon loops (which are well defined with a cutoff, for example) with the counterterms and by fitting the resulting (unknown) constants to empirical bulk nuclear properties \[21, 35\]. In principle, these constants could be calculated directly from QCD.

This is the strength of the EFT: by fitting a small number of empirical constants, we encode the correct vacuum dynamics into the mean-field hamiltonian, and there is no need to rely on a specific model for the vacuum dynamics, which is beyond the realm of the low-
energy EFT anyway. Different renormalization/subtraction schemes shift contributions be-
tween baryon loops and counterterms without changing physical observables. A particularly
convenient prescription for nuclear ground states implicitly cancels the sum over negative-
energy states. This procedure is equivalent to the so-called “no-sea approximation”, as we
illustrate below. Precisely the same renormalization scheme (i.e., the counterterm param-
ters remain unchanged) must be applied to the linear response, which leads automatically
to all required terms and maintains all important conservation laws.

II. COVARIANT EFFECTIVE FIELD THEORY

An effective field theory (EFT) describes low-energy physics with low-energy degrees
of freedom. In some theories, like the Standard Model of Electroweak interactions, the
short-range (high-energy) contributions can be explicitly re-expressed as terms in the low-
energy, effective lagrangian. In other effective field theories, like chiral perturbation theory
or QHD, the low-energy lagrangian cannot (yet) be calculated explicitly from QCD, and the
parameters must be fitted to experimental data [31, 35, 36, 37].

Guidance in choosing the form of a hadronic EFT comes from requiring that the la-
grangian maintain the symmetries of the underlying theory of QCD. One also wants to
choose an efficient set of low-energy degrees of freedom ("generalized coordinates") in the
EFT lagrangian, to simplify the treatment of the desired many-body problems. Fortu-
nately, in most applications of chiral perturbation theory or descriptions of bulk nuclear
structure, only a small number of parameters are needed, and predictive power is re-
tained [10, 11, 17, 28, 37, 38]. Thus no attempt is made in these EFTs to construct a
detailed dynamical description of the short-distance or vacuum physics.

The important point is that while the short-distance, ultraviolet behavior of the effective
theory is (probably) incorrect, it can be corrected systematically by the normalization (or
renormalization) of local operators (“counterterms”), which have at most a finite number of
derivatives acting on the fields. We emphasize that this procedure is not a prescription or a
model for describing the vacuum dynamics; we are truly encoding the appropriate physics by
fitting the unknown constants to data, using a lagrangian that contains all (nonredundant)
terms allowed by the underlying symmetries [17, 31, 37].

The “no-sea approximation” for the static, mean-field nuclear ground state can be un-
derstood as a particularly economical way to define and choose the counterterms, although
other, less efficient prescriptions could be made. We review the arguments underlying this
procedure below [35]. Moreover, because the same counterterm parameters determine the
linear response of the ground state to external, time-dependent perturbations, a framework
that manifests their role will automatically produce a correct treatment of the RPA.

It is convenient to use an effective-action formalism to carry out the EFT program at
finite density and to trace the role of the counterterms. The fundamental object is the
effective action $\Gamma[\phi, V^\mu]$ with spacetime dependent, classical, Lorentz scalar and four-vector
fields $\phi(x)$ and $V^\mu(x)$. $\Gamma[\phi, V^\mu]$ is obtained by a Legendre transformation of the path-
integral generating functional for propagators, which contains sources coupled to the meson
fields [35, 39, 40, 11, 12, 43, 44, 45, 46]. When evaluated with appropriate time-independent
fields, $\Gamma$ is proportional to the ground-state energy [10, 13]. It also generates the one-
particle-irreducible Green’s functions and is therefore related to the linear response of the
system to external probes. Thus we can address the computation of the ground state and
the excited states in the same framework. For simplicity, we show only isoscalar, scalar and
vector fields; the extension to other boson fields is straightforward but not important for our discussion.

We consider only the one-loop effective action, which generates the conventional mean-field or Hartree equations for the ground state and the RPA equations for collective excited states. This would seem to be a severe restriction. Indeed, the successes of QHD mean-field theory are at first somewhat mysterious, since the one-loop approximation is just the finite-density counterpart of the Born approximation at zero density, which is inadequate for a quantitative description of nucleon–nucleon scattering. However, density functional theory (DFT) can explain the successes of these calculations and also provides a basis for understanding the expansion and truncation of the QHD lagrangian.

Conventional DFT is based on an energy functional of the ground-state density of a many-body system, whose extremization yields a variety of ground-state properties. In a covariant generalization of DFT applied to nuclei, the energy and grand potential become functionals of the ground-state scalar density and baryon-number four-current density. Relativistic mean-field theories based on EFT are analogs of the Kohn–Sham formalism of DFT [47, 48, 49, 50], with local scalar and vector fields appearing in the role of Kohn–Sham potentials [17]. They are not Hartree calculations using interactions designed to reproduce free-space nucleon–nucleon observables. Instead, the one-loop energy [see Eq. (21) below] approximates the exact energy functional, which includes all higher-order correlations, using powers and gradients of auxiliary meson fields (or nucleon densities [51, 52, 53]). Multi-loop contributions are implicitly expanded in a generalized local-density approximation plus gradient corrections; the success of this approach in Coulomb systems is well documented [18, 19, 20]. The level of truncation for the desired accuracy is determined by EFT power counting [31, 38]. This approximation is very accurate for the density regime of interest, as verified by the excellent reproduction of nuclear ground-state densities and energies [31, 53, 54, 55]. (For a more complete discussion, see sec. 6.1 of Ref. [17] and Ref. [56].)

The DFT also implies that we have a meaningful power counting for the approximate calculation of the effective action, which allows us to truncate the one-loop energy functional to any desired accuracy. Thus our EFT is systematic to the extent that the one-loop form of the energy functional is flexible enough to be a good approximation to the most important multi-loop corrections. Nevertheless, a fully systematic EFT expansion, in which loop corrections can be included order-by-order in a small expansion parameter, has yet to be developed. We will return at the end to discuss explicit improvements to the energy functional [57].

To carry out the effective-action analysis of vacuum contributions, we start with the following lagrangian (density)

\[
\mathcal{L}(x) = \bar{N}(i\gamma^\mu \partial_\mu - g_s \gamma^\mu V_\mu - M + g_s \phi + \cdots) N - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
+ \frac{1}{2} m_v^2 V^\mu V_\mu + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi) + \cdots,
\]

(1)

where \( g_s \) (\( g_v \)) is the scalar (vector) coupling to the nucleon, the vector field-strength tensor is \( F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu \), and \( U(\phi) \) is an infinite polynomial in \( \phi \). The ellipsis represents contributions from other bosons (e.g., pions), a polynomial in (even powers of) the vector field, a “mixed” polynomial involving both the scalar and vector fields, and terms involving derivatives of the fields, all of which are superfluous for the present illustration. Moreover, the
ellipsis contains the counterterms, which include all possible (nonredundant) terms allowed by the symmetries of the theory; in particular, there are counterterm polynomials in the boson fields with exactly the same form as those mentioned above. There are also Lorentz-covariant counterterms involving the nucleons (e.g., a wave function renormalization), which are needed when one calculates explicitly beyond one-loop order \[13, 57\].

For a given approximation to the effective action, the counterterm parameters can be fixed by any sufficiently complete set of observables, and then the same parameters must apply to all calculations using the effective action. (This is equivalent to the emphasis in conventional RPA discussions on using a consistent interaction for the ground state and excited states \[3, 10, 11\].) Fitting to ground-state properties is predictive for excited states that do not rely on unconstrained parameters or on poorly approximated correlations. Therefore, we expect that collective excited states will be described well.

Since we expect the vacuum baryon-loop contributions to be largely canceled by the counterterms \[21\], it is efficient to make a reference subtraction to build in this cancellation implicitly and to include (and fit) only the residual counterterms explicitly. We can identify the subtraction by formally considering the effective action at zero temperature and density (which is not meant to describe free-space scattering). The lagrangian enters in an exponential in a path integral over all the fields, so we can start by integrating out the baryon fields. The boson fields act as auxiliary fields and can be redefined (if necessary) to eliminate any terms that are not bilinear in the baryon fields; thus, the result of the integration is a fermion determinant that contributes to the meson action as an additive term given by \[19, 35\]

\[
S_{\text{FD}}[\phi, V_\mu] \equiv \int d^4 x \mathcal{L}_{\text{FD}} = -i \text{Tr} \ln K(0),
\]

where “Tr” indicates a trace over spacetime, spin, and isospin. The kernel \(K(\mu)\) is defined by

\[
-i \text{Tr} \ln K(\mu) \equiv -i \text{Tr} \ln (i\gamma^\mu \partial_\mu + \mu \gamma^0 - \mathcal{M}^* - g_\nu \gamma^\mu V_\mu),
\]

with the shorthand \(\mathcal{M}^* \equiv \mathcal{M} - g_\phi \phi\), and the chemical potential \(\mu\) is introduced in this definition for later convenience. (Baryon counterterms that are needed beyond one-loop order are suppressed.) At present, we are working with \(\mu = 0\), and we will comment on this choice below. Note that no approximation has been made at this point; \(S_{\text{FD}}\) is a functional of the dynamical fields \(\phi\) and \(V_\mu\) that must still be integrated over in the path integral.

The determinant can be evaluated using a derivative expansion of the fields, which takes the form \[11, 42, 58\]

\[
-i \text{Tr} \ln K(0) = -i \text{Tr} \ln (i\gamma^\mu \partial_\mu - \mathcal{M}^* - g_\nu \gamma^\mu V_\mu) \]
\[
= \int d^4 x [ -U_{\text{eff}}(\phi) + \frac{1}{2} Z_{1s}(\phi) \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} Z_{2s}(\phi) (\Box \phi)^2 \]
\[
+ \frac{1}{4} Z_{1v}(\phi) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} Z_{2v}(\phi) (\partial_\alpha F^{\alpha\mu})(\partial^\beta F_{\beta\mu}) + \cdots].
\]
polynomial in $\phi$, and the conservation of baryon number implies that the vector field can enter only in the combination $F_{\mu\nu}$. This expansion was investigated in renormalizable QHD models and found to be rapidly convergent \[41, 42, 58\].

The contributions from $\text{Tr} \ln K(0)$ represent vacuum physics described by the Dirac sea of nucleons, which is most likely incorrect. The inadequacy of such a simple picture for the QCD vacuum is demonstrated by the unnaturalness of the coefficients in $U_{\text{eff}}(\phi)$ under conventional QHD renormalization \[21\]. Nevertheless, the important point is that these vacuum contributions can be written in terms of local products of fields and their derivatives that have the same form as the counterterms already present in the meson lagrangian. Thus the contributions from the fermion determinant can be exactly canceled by the counterterms, leaving only the original polynomial terms shown in Eq. (1). Since the remaining terms contain all possible forms allowed by the symmetries, we can encode the true vacuum dynamics into the lagrangian (or hamiltonian) by fitting the remaining parameters to experimental data.

To see how this works in practice, let us focus on the nonderivative term in Eq. (4), which is obtained by treating the meson fields as constants. (The gradient terms can be handled analogously.) The nonderivative part of $L_{\text{FD}}$ is an infinite polynomial in $\phi$; at the one-loop level, one finds:

$$L_{\text{FD}}[\phi] = \int \frac{d^4k}{(2\pi)^4} \text{tr} \ln G^0(k) + \sum_{n=1}^{\infty} \frac{(-1)^n}{n} [g_\phi]^n \int \frac{d^4k}{(2\pi)^4} \text{tr} [G^0(k)]^n. \quad (5)$$

Here the noninteracting baryon propagator is $G^0(k) = [\gamma^\mu k_\mu - M + i\epsilon]^{-1}$, “tr” denotes a trace over spin and isospin, and we have regularized dimensionally to maintain Lorentz covariance and other symmetries. One could also make these integrals finite by using an explicit cutoff. The point is that this polynomial (which is valid for any values of the background field $\phi$) can be canceled exactly by the implied counterterms in Eq. (1), leaving only the explicit polynomial $U(\phi)$. Indeed, physical observables depend only on the sum of all terms with the same structure, and so in practice, there is no need to compute explicitly either the loop contributions or the counterterms. One simply considers the cancellations to be implicit and takes the original potential $U(\phi)$ to contain finite, renormalized parameters, which can ultimately be fitted to empirical data to encode the true QCD vacuum dynamics into the EFT. Although in principle, an infinite number of meson terms is needed to describe these effects, the principles of EFT power counting and naturalness, which are validated by phenomenological studies \[11, 58\], show that one can truncate the lagrangian to a small number of derivative and nonderivative terms in applications to the structure of nuclei. Thus only a small number of unknown constants must be fitted to data to achieve accurate results, and the predictive power of the QHD effective action is maintained.

Now that the counterterm subtractions have been defined to cancel the vacuum-loop contributions at zero density and temperature, what happens at finite density or in a finite nucleus? For these systems, we invoke the grand canonical ensemble and allow $\mu$ to be nonzero. (For simplicity, we continue to maintain zero temperature, but the generalization is straightforward; see, for example, Ref. \[59\].) The relevant lagrangian density changes to

$$L(x) \rightarrow L'(x) \equiv L(x) + \mu N \gamma_0 N. \quad (6)$$
The effective action of $\mathcal{L}'(x)$ is associated with the grand potential $\Omega$ of the system, instead of the energy. The energy follows from

$$E = \Omega + \mu B, \quad B \equiv -\partial \Omega / \partial \mu,$$  

where $B$ is the baryon number of the system.

We can again integrate over the baryon fields in the path integral, just as at zero density. The result is the fermion determinant at finite density (or chemical potential), $-i \text{Tr} \ln K(\mu)$, to which we can add and subtract the fermion determinant at $\mu = 0$, namely, $-i \text{Tr} \ln K(0)$. The added term $-i \text{Tr} \ln K(0)$ cancels the counterterms exactly as described previously. Note that it contains the same dynamical scalar and vector fields as the fermion determinant at finite $\mu$. The remaining sum

$$-i \text{Tr} \ln K(\mu) + i \text{Tr} \ln K(0)$$

is an explicitly density-dependent contribution (it vanishes for $\mu = 0$), which is finite. (As before, baryon counterterms must be defined when one calculates beyond one-loop order, as we describe in sec. VI.) The scalar potential $U(\phi)$ of Eq. (11) remains intact; the only difference is that the scalar (and vector) fields will acquire different expectation (mean) values due to the presence of valence (positive-energy) nucleons at finite density. We emphasize that Eq. (8) applies to both the ground state and RPA excited states.

III. GROUND STATE

We can evaluate the sum Eq. (8) explicitly for the ground state at the one-loop level to see how the “no-sea approximation” arises automatically. As we stressed earlier, this “mean-field” calculation should be viewed in the context of Kohn–Sham density functional theory.

The mean-field grand potential is defined in the effective-action formalism by replacing all of the dynamical meson fields by their mean values, resulting in

$$\int dx_0 \Omega = i \text{Tr} \ln K(\mu) - i \text{Tr} \ln K(0) + \int d^4x U_m(x),$$

where the bars indicate that the quantities are to be evaluated with the static scalar and vector mean fields, which we will denote by $\phi_0(x)$ and $V_0(x)$. In particular, the baryon kernel in coordinate space is now

$$\langle x | K(\mu) | y \rangle = \gamma_0 [i \partial_0 + \mu - h(x)] \delta^{(4)}(x-y),$$

with the single-particle Dirac hamiltonian

$$h(x) \equiv -i \alpha \cdot \nabla + g_v V_0(x) + \beta [M - g_s \phi_0(x)],$$

where $\beta = \gamma_0$ and $\alpha = \gamma_0 \gamma$.

The contribution to $\Omega$ from the mean meson fields is defined by

$$U_m(x) \equiv \frac{1}{2} (\nabla \phi_0)^2 + U(\phi_0) - \frac{1}{2} (\nabla V_0)^2 - \frac{1}{2} m^2 V_0^2 + \cdots,$$
where the ellipsis represents any other polynomial or gradient terms (with finite, renormalized constants—as yet unknown) that are retained in the truncated lagrangian to achieve the desired accuracy for the nuclear ground state \([1, 38]\). The equations determining the mean meson fields have not yet been specified, but they will be determined shortly.

First we observe that \(\bar{K}(\mu)\) can be diagonalized by choosing the single-particle basis \(\psi_{\alpha}(x) \equiv \psi_{\alpha}(x)e^{i\omega x_0}\), where \(\{\psi_{\alpha}(x)\}\) are the normalized eigenfunctions of the Dirac equation with eigenvalues \(E_{\alpha}[19, 20, 60]\):

\[
h(x) \psi_{\alpha}(x) = E_{\alpha} \psi_{\alpha}(x) , \quad \int d^3 x \psi_{\alpha}^\dagger(x) \psi_{\beta}(x) = \delta_{\alpha\beta} . \tag{13}
\]

This diagonalization works for any value of \(\mu\) and results in the matrix elements

\[
\langle \alpha\omega | K(\mu) | \beta\omega' \rangle = 2\pi \delta(\omega - \omega') \delta_{\alpha\beta} (-\omega + \mu - E_{\alpha}) . \tag{14}
\]

Applying the appropriate boundary conditions (which reproduce the familiar Feynman boundary conditions for free nucleons) is equivalent to the \(i\epsilon\) prescription \(\omega \to (1 + i\epsilon)\omega\) for evaluating the baryon kernel \([44]\). Note, however, that because \(h(x)\) is to be interpreted as a Kohn–Sham single-particle hamiltonian, the eigenvalues \(E_{\alpha}\) have no directly observable meaning, except at the Fermi surface \((E_{\alpha} = E_F = \mu)\) \([49]\).

Using the result (14) in Eq. (9), we find

\[
\Omega(\phi_0, V_0; \mu) = i \sum_{\alpha} \int \frac{d\omega}{2\pi} \left[ \ln(-\omega + \mu - E_{\alpha}) - \ln(-\omega - E_{\alpha}) \right] + \int d^3 x \bar{U}_m(x) , \tag{15}
\]

where the sum on \(\alpha\) runs over both positive- and negative-energy eigenvalues. To evaluate the integrals, one must take care with the analytic structure of the logarithms, and it is easiest to begin with the computation of the baryon number, as defined in Eq. (7). Contour integration produces

\[
B = -\partial \Omega/\partial \mu = \sum_{\alpha} \left[ \theta(\mu - E_{\alpha}) - 1/2 \right] . \tag{16}
\]

To properly define the normal-ordered baryon number, we can use

\[
\sum_{\alpha} \theta(-E_{\alpha}) = \sum_{\alpha} \theta(E_{\alpha}) = \sum_{\alpha} \frac{1}{2} , \tag{17}
\]

which is valid when \(E_{\alpha} = 0\) separates the valence levels from the Dirac sea.\(^2\) This result is clearly valid for noninteracting nucleons at \(\mu = 0\), and it remains valid when the valence nucleons are added and the interactions are turned on, due to the conservation of baryon number \([19, 31, 35]\).

Using Eq. (17), we can rewrite the baryon number (16) as

\[
B = \sum_{\alpha} \left[ \theta(\mu - E_{\alpha}) - \theta(-E_{\alpha}) \right] = \sum_{\alpha} 1 - \sum_{\alpha} \theta(E_{\alpha}) = \sum_{\alpha} \Theta(E_{\alpha}) = \sum_{\alpha} \Theta(E_{\alpha}) = \sum_{\alpha} 1 \equiv \sum_{\alpha} 1 . \tag{18}
\]

\(^2\) We assume that at \(\mu \neq 0\), as at \(\mu = 0\), the positive-energy (valence) levels are separated in energy from the Dirac sea. This is true in all practical applications of the QHD lagrangian to nuclei and nuclear matter. This is the primary advantage of choosing \(\mu = 0\) to define the counterterms.
where the final sum is over the *occupied* valence orbitals at the given value of $\mu$. This result is precisely what we should expect from the boundary-condition prescription discussed earlier, which leads to the familiar normal ordering of the baryon-number operator \[19\].

To compute $\Omega$, one must also perform contour integrals, using care to orient the contours correctly with respect to the branch points of the logarithms. The procedure is equivalent to a Wick rotation to $\nu = -i\omega$ and produces

$$
\Omega(\phi_0, V_0; \mu) = -\sum_\alpha \int \frac{d\nu}{2\pi} \left[ \ln(-i\nu + \mu - E_\alpha) - \ln(-i\nu - E_\alpha) \right] + \int d^3x \overline{U}_m(x)
$$

$$
= -\sum_\alpha (\mu - E_\alpha) \theta(\mu - E_\alpha) - \sum_\alpha E_\alpha \theta(-E_\alpha) + \frac{1}{2} \sum_\alpha \mu + \int d^3x \overline{U}_m(x)
$$

$$
= -\sum_\alpha (\mu - E_\alpha) \left[ \theta(\mu - E_\alpha) - \theta(-E_\alpha) \right] + \int d^3x \overline{U}_m(x)
$$

$$
\equiv -\sum_\alpha (\mu - E_\alpha) + \int d^3x \overline{U}_m(x) \ , \quad (19)
$$

where we have used Eq. \[17\] to produce the normal-ordered result for the baryon number.

By combining the preceding expressions, we can compute the ground-state energy:

$$
E = \Omega + \mu B = -\mu \sum_\alpha [\theta(\mu - E_\alpha) - \theta(-E_\alpha)] + \mu \sum_\alpha [\theta(\mu - E_\alpha) - \theta(-E_\alpha)]
$$

$$
+ \sum_\alpha E_\alpha \theta(\mu - E_\alpha) - \sum_\alpha E_\alpha \theta(-E_\alpha) + \int d^3x \overline{U}_m(x)
$$

$$
= \sum_{E_\alpha < \mu} E_\alpha - \sum_{E_\alpha < 0} E_\alpha + \int d^3x \overline{U}_m(x) \quad (20)
$$

$$
= \sum_{\text{occ}} E_\alpha + \int d^3x \overline{U}_m(x) \ . \quad (21)
$$

We emphasize that the final sum over occupied valence states only is not the result of a vacuum subtraction, since the trace with $\mu = 0$ [which produces the second sum in Eq. \[20\]] still contains the self-consistent mean fields $\phi_0(x)$ and $V_0(x)$. The true vacuum subtraction was performed earlier when we derived the renormalized (and finite) $U_m$ in Eq. \[12\].

Thus we have arrived at the “no-sea approximation” for the ground state. The energy is determined by a sum over valence-orbital eigenvalues and by a local potential in the meson fields (and their derivatives) with finite, but unknown, constants. How can we see that the vacuum dynamics is still included?

Let us recall how we arrived at Eq. \[21\]. We first showed that at $\mu = 0$, the fermion determinant could be written as a generalized derivative expansion in local terms [Eq. \[4\]] and could therefore be canceled exactly by the counterterms present in the original lagrangian. The physics behind this is that in an EFT dealing with long-range dynamics, the vacuum contributions are so poorly resolved that they can be accurately represented by local terms in a derivative expansion containing the meson fields \[58\]. At finite $\mu$, we then added and subtracted the $\mu = 0$ determinant from the grand potential; the added term cancels exactly against the counterterms (by construction), while the subtracted term was rewritten as a
sum over (negative) eigenvalues to reveal that it exactly removes these contributions from the first term in Eq. (20), which originates from $\text{Tr} \ln K(\mu)$. In principle, we could have argued that the negative-energy part of the first sum in Eq. (20) could indeed be represented by local terms in the fields and derivatives (since such vacuum contributions are not resolved in the low-energy EFT) and simply canceled them away exactly by the local counterterms. Instead, we inserted the intermediate step involving $\text{Tr} \ln K(0)$ to show explicitly the local nature of the second sum in Eq. (20) and that it is equivalent to local counterterms. The analogous procedure that produces the nucleon scalar density is shown schematically in Fig. 1.

All that remains in the energy $E$ is the local meson potential and a sum over valence-orbital eigenvalues; the former has the same form as the counterterms, but with finite, unknown constants that will ultimately get fitted to data to encode the true QCD vacuum dynamics into the energy. As long as we fit the parameters to empirical data, none of the vacuum or short-range QCD dynamics is omitted [17, 31].

The reason these subtraction procedures work is that the local terms in the derivative expansion [e.g., Eq. (4)] have the same form for any values of the fields $\phi$ and $V^\mu$. The counterterm parameters (constants) multiplying the fields are fixed for a given level of approximation to the original lagrangian. Thus at $\mu = 0$, the counterterm subtraction can be implemented while the meson fields are still dynamical (i.e., integration variables in the path integral), while at $\mu \neq 0$, a similar set of counterterm subtractions produces finite results for the grand potential or energy of the system as a function of the mean meson fields. The nuclear energy at the one-loop level automatically reproduces the well-known “no-sea approximation”, as discussed above.

This discussion also reveals why the choice of $\mu = 0$ for the original normalization of the QHD lagrangian is so convenient. Since the positive- and negative-energy eigenvalues are separated by $E^0_\alpha = 0$ at $\mu = 0$ and remain separated by $E_\alpha = 0$ at finite $\mu$ (at least in all nuclear structure applications that we are aware of), the final expression for the energy [Eq. (21)] contains a sum over valence nucleon orbitals only. Thus the “no-sea approximation”, which still allows the inclusion of vacuum dynamics through the fitted parameters in the local meson potential, arises naturally in QHD because of a convenient choice for the normalization of the lagrangian. Other choices ($\mu \neq 0$) for this normalization are certainly possible, but if one chooses a value of $\mu$ for the initial vacuum subtraction that will ultimately lie within the spectrum of positive-energy eigenvalues, subsequent subtractions to define the ground-state energy will be more complicated. Such a renormalization procedure could be implemented in principle, but it would be messy, and the extra complication is unnecessary; the most convenient choice is $\mu = 0$, which leads naturally to familiar results for the one-loop ground-state energy.
To determine the mean meson fields, one extremizes the expression for the energy [Eq. (21)] with respect to these fields. This leads (in general) to nonlinear differential equations for the meson fields, with the nucleon scalar and baryon densities as the sources. (See, for example, Refs. [20, 31, 35].) Thus the mean meson fields and the nucleon wave functions must be determined self-consistently, as is well known [23, 60, 61]. We emphasize, however, that the mean meson fields and the nucleon densities are all local, time-independent functions of a single spatial variable $x$. Moreover, since the meson parameters are fitted to empirical many-body data, the meson fields are to be interpreted as relativistic Kohn–Sham potentials.

IV. LINEAR RESPONSE (RPA)

The renormalization procedure detailed in sec. II also defines the linear response of the ground state to time-dependent fields (RPA). To see the consequences of this procedure, we consider the effective action with fluctuations around the static, ground-state fields:

$$\phi(x) = \phi_0(x) + \tilde{\phi}(x), \quad V_\mu(x) = V_0(x) \delta_{\mu 0} + \tilde{V}_\mu(x).$$  \hspace{1cm} (22)

The fluctuations are denoted with tildes and are explicitly of $O(\hbar^{1/2})$. We will work to leading order in the fluctuations, which turn out to be $O(\hbar)$ and yield the familiar RPA [6, 62, 63, 64, 65].

Note that the ground-state mean fields are determined by extremizing the energy [Eq. (21)], which implies that we are working in the canonical ensemble at fixed baryon number (or density). The desired density can be imposed by applying the appropriate infinitesimal boundary conditions on the baryon propagator [6, 11, 19]. A careful analysis based on the grand (thermodynamic) potential produces identical results, when one works at the level of the lowest-order RPA, as we are here [63, 66]. In particular, the RPA propagators are to be evaluated in the presence of the ground-state mean fields $\phi_0$ and $V_0$.

Since the effective action $\Gamma$ is the generator of one-particle-irreducible (1PI) Green’s functions, all we need to describe the linear response are the terms in $\Gamma$ that are quadratic in the fluctuation fields. These terms give us the meson polarization insertions, expressed in terms of baryons propagating in the static mean fields. (Recall that terms linear in the fluctuation fields vanish identically, since their coefficients are zero by virtue of the ground-state field equations.) In particular, $\partial^2 \Gamma / \partial \phi(x) \partial \phi(y)$ evaluated at $\phi_0(x)$ and $V_0(x)$ is the inverse scalar meson propagator in the presence of the mean fields. The inverse vector meson propagator and other response functions follow similarly.

It is critical to recognize that the fermion determinants $i \text{Tr} \ln K(\mu)$ and all of the local meson terms retain exactly the same form as in the ground-state calculation. In particular, the numerical constants in these counterterm contributions are determined from fits to bulk nuclear properties, and the same constants are used in the RPA calculation; all that changes are the values of the meson fields due to the fluctuations.

Thus we can make the same substitutions and subtractions as we made in considering the ground state. We therefore carry out the expansion of $\Gamma$, focusing on the $i \text{Tr} \ln K(\mu)$ term and substituting $-i \text{Tr} \ln K(0)$ for the local counterterms. [Note that there are no “bars” on these $K$’s, since they contain the modified fields of Eq. (22).] If we consider the $\tilde{\phi}$ terms in a schematic notation, we find that the quadratic contribution

$$\ln(G_{KS}^{-1} + g_s \tilde{\phi}) - \ln(G_{KS}^{-1}) = -\frac{1}{2} G_{KS} g_s \tilde{\phi} G_{KS} g_s \tilde{\phi} + \ldots$$  \hspace{1cm} (23)
FIG. 2: Spectral content of the Kohn–Sham propagator, which is the same as the nucleon propagator in a relativistic Hartree approximation [19]. Single-particle states with energies between $-M$ and $+M$ are bound, while those with energies less than the chemical potential $\mu$ are occupied.

gives the lowest order RPA ring diagram $[\text{tr} \ G_{KS}(x,y)G_{KS}(y,x)]$ for the scalar meson polarization insertion, where “tr” denotes a trace over spin and isospin. Here the ring is formed from the ground-state baryon propagators $G_{KS}$, which involve the mean meson fields and the resulting self-consistent Kohn–Sham (KS) baryon wave functions from Eq. (13), as discussed earlier. The final RPA equations will be the same as in Refs. [10, 11], so we focus here on the spectral content of the nucleon part rather than the details of the derivation.

Since the baryon propagator can be written as a bilinear form using the Kohn–Sham wave functions (and their adjoints), we can use the eigenvalue equation (13) to convert the propagators to frequency space. They take the form

$$G_{KS}(x,x';\omega) = \sum_{\alpha} \left[ U_{\alpha}(x)U_{\alpha}(x') \left( \frac{\theta(E_{\alpha}^+ - \mu)}{\omega - E_{\alpha}^+ + i\eta} + \frac{\theta(\mu - E_{\alpha}^-)}{\omega - E_{\alpha}^- - i\eta} \right) + V_{\alpha}(x)V_{\alpha}(x') \right],$$

(24)

where $U_{\alpha}(x)$ and $V_{\alpha}(x)$ are positive- and negative-energy energy solutions, respectively, of a Dirac equation with background scalar $\phi_0(x)$ and vector $V_0(x)$ fields:

$$\left\{ -i \alpha \cdot \nabla + \beta [M - g_\phi \phi_0(x)] + g_\omega V_0(x) \right\} \left\{ \begin{array}{c} U_{\alpha}(x) \\ V_{\alpha}(x) \end{array} \right\} = \left\{ \begin{array}{c} E_{\alpha}^+ U_{\alpha}(x) \\ E_{\alpha}^- V_{\alpha}(x) \end{array} \right\},$$

(25)

and $\eta$ is a positive infinitesimal. Note that including additional background fields does not affect the present discussion. The pole structure of Eq. (24) is illustrated in Fig. 2. (The infinitesimals that enforce the appropriate boundary conditions are defined at both zero and finite density in the usual way [6]).

The frequency integration over the two $G_{KS}$ propagators picks up contributions from both particle-hole ($ph$) pairs and particle-negative-energy ($p/-$) pairs for the determinant with $\mu > 0$, where “particle” implies a Dirac state that is unoccupied in the ground state, while “hole” implies an occupied state. In contrast, for $\mu = 0$, we get contributions from all positive-negative-energy ($+/-$) terms, which resemble the usual “vacuum polarization”. Remember, however, that all of these terms involve the Kohn–Sham background fields, since the true vacuum subtraction was made back in sec. II.

When all of these ring contributions are combined, the net result: ($ph$) + ($p/-$) − ($+/-$) is just the difference ($ph$) − ($h/-$), as illustrated in Fig. [3]. Thus, by carrying out our consistent normalization and renormalization procedures in the QHD EFT, the RPA response contains both familiar particle-hole pairs and mixing between occupied particle states and negative-energy states in the single-particle (Kohn–Sham) basis. Because this second term forces the inclusion of negative-energy states that complete the Dirac basis, it is crucial for maintaining
FIG. 3: The subtraction procedure for RPA in the “no-sea” prescription (represented diagrammatically) yields particle-hole (ph) pairs minus mixing terms between occupied positive-energy and negative-energy basis states (h/−). The negative-energy states are important for maintaining the completeness of the Dirac single-particle basis.

Thouless’ theorem and various conservation laws [6, 10, 11, 67]. We have therefore succeeded in showing that the standard rules of quantum field theory (and a convenient choice for the normalization of the QHD lagrangian) lead to the RPA contributions in Fig. 3.

Why should we expect that this RPA framework yields reasonable results for nuclear collective excitations? The first thing to remember is that the Kohn–Sham energy functional, while an approximation to the exact energy functional, is an excellent approximation over the density regime of interest and is fit not only at the equilibrium point but also in the vicinity of this point. This (approximate) energy functional goes beyond simple Hartree theory and implicitly includes contributions from nucleon exchange, correlations, hadronic structure, short-distance physics, and the quantum vacuum into the description of nuclear densities and energies [68]. This description is very accurate, based on the agreement between calculated ground-state results and the data [28, 31, 35]. Since the ground-state energy functional is fitted to the empirical bulk properties of nuclei, it includes all of the relevant long-range physics. Just as we would expect a modest extrapolation in density or proton fraction away from the nuclear matter equilibrium point to be accurately described by the fitted energy functional, we expect that low-lying (acoustic) collective excitations of the mean-field ground state should be described accurately as well. Nevertheless, one must include all of the states in the complete Dirac basis to achieve this accuracy.

Although recent RPA calculations [10, 11, 12, 13, 29] include only a subset of the local meson terms (cubic and quartic non-derivative scalar terms) present in the full effective potential, power counting [17, 31] implies that other terms, such as mixed scalar–vector cubic and vector quartic terms, are equally important. Calculations with these additional terms have yet to be done, and data on excited nuclear states may provide new constraints that can help determine the numerical coefficients of these terms in the energy functional.

V. RELATIONSHIP BETWEEN EFT AND EARLIER APPROACHES

As we have seen in the preceding EFT discussion, nucleons in the occupied Fermi sea modify the QCD vacuum, which in turn acts back on the valence nucleons through the fitted terms in the local meson potential [31]. If one constructs a renormalizable hadronic field theory based on low-energy degrees of freedom, one is making an explicit model for this vacuum dynamics; namely, that loop integrals involving these hadronic degrees of freedom can adequately describe the vacuum [19, 69]. For example, in the relativistic Hartree approximation (RHA), the contribution of the Dirac sea for a uniform system is a Casimir energy at finite density. This arises from the nonzero scalar mean field φ₀, which changes
the sum over energies in the Dirac sea:

$$\delta H = - \sum_{k, \lambda} \left[ (k^2 + M^{*2})^{1/2} - (k^2 + M^2)^{1/2} \right], \quad (26)$$

where $M^{*} \equiv M - g_s \phi_0$, and $\lambda$ is the spin-isospin degeneracy. After renormalization, the resulting contribution to the ground-state energy is an (infinite) polynomial in $\phi_0$. The phenomenological consequences (after conventional renormalization) include smaller mean fields than those found in best-fit mean-field theories; the smaller fields lead to incorrect spin-orbit splittings in nuclei [19].

In phenomenologically successful covariant mean-field models [25, 26, 27, 28], these Dirac sea contributions are neglected by fiat. This means that the nucleon scalar density, for example, is computed from a sum over self-consistent, positive-energy, single-particle states only. Yet the self-consistent Hartree propagator must be constructed from a complete basis of Dirac single-particle wave functions; otherwise, it fails to satisfy the appropriate differential equation [10]. Thus the single-particle Hartree propagator takes the form of Eq. (24) [1, 2], in which the sum over states in Eq. (24) includes negative-energy solutions to the Dirac equation.

For ground-state quantities, however, the “no-sea approximation” implies that one should drop the contributions from the negative-energy poles of this Green’s function. Although the “no-sea approximation” is known to be both covariant and thermodynamically consistent [19], its early use lacked any formal justification, and it was not clear how to generalize it [26, 27, 28]. Moreover, its accuracy was suspect, especially in view of the unnaturally large contributions from the Casimir energy [21]. Our discussion in the preceding sections (based on modern EFT) shows how this approximation arises naturally from the standard rules of quantum field theory, without an explicit model for the dynamics of the vacuum and without dropping any poles. Extension to higher-order approximations is straightforward, but tedious [27], as we will comment on in the Discussion section below.

When extended to linear-response theory, the “no-sea approximation” leads to the naive expectation that only particle-hole pairs should be used in the RPA configuration space [32, 33, 34]. It has been known for fifteen years [4, 67], however, that the consistent linear response to a “no-sea” ground state must include, in addition to the conventional particle-hole ($ph$) pairs, contributions that mix occupied states or holes ($h$) and negative-energy ($-$) basis states from the Hartree ground-state calculation. This requirement has been discussed using perturbation theory and Green’s functions for the elastic case [67], a self-consistent, conserving-approximation functional approach [4], and a time-dependent Hartree approximation [29]. Neglecting these contributions has disastrous phenomenological consequences. Chief among them is the failure to fully remove the spurious center-of-mass strength from the isoscalar dipole response as well as the violation of electromagnetic current conservation.

The inclusion of ($h/-$) contributions in an RPA calculation can be accomplished explicitly by expanding the RPA matrix to include these additional configurations, in what is called the spectral RPA approach [4], or by using a more efficient non-spectral approach (and looking for poles in the response function) [10, 11]. A formal solution to the consistent use of Eq. (24), proposed in Ref. [4], is to shift (by fiat) the negative-energy poles to the lower-half plane (see also Ref. [29]). Then one picks up the desired poles for both ground states and excited states. This shift is equivalent to normal ordering the single-particle density operator.
The explanations for including \((h/-)\) contributions are not at all satisfying when based on earlier approaches that either modeled the QCD vacuum as an interacting Dirac sea of nucleons or that neglected the Dirac sea completely in the computation of the Hartree ground state. The primary motivation for including these \((h/-)\) terms was that they were necessary to maintain the conservation laws and to agree with the observed phenomenology. Within the modern EFT/DFT/KS approach, however, one realizes that the long-range (nucleon and meson) degrees of freedom cannot adequately describe the QCD vacuum. Nevertheless, since the vacuum contributions from these terms can always be represented in the EFT by local terms in a meson potential, they can be combined with other terms in the QHD lagrangian (which contains all nonredundant terms consistent with the symmetries), leading to a meson effective potential that contains all vacuum effects when it is fitted to empirical data.

Moreover, a straightforward extension of the ground-state calculation to excited states, which is performed by allowing the meson fields to fluctuate around their mean values, naturally leads (again from the standard procedures of quantum field theory) to all the necessary loop contributions in the RPA calculation. This derivation shows that the vacuum has not been neglected; the parts of the vacuum that are beyond the limits of description by the EFT are parametrized in terms of the meson potential, and the long-range vacuum modifications arising from the \((h/-)\) mixing are included explicitly, as they should be. Thus the modern approach not only vindicates the inclusion of the correct terms in the RPA linear response, but it also shows why certain terms (the so-called \(NN\) pairs) should not be calculated explicitly, because they are beyond the range of validity of the EFT, and they will always be included implicitly in the meson potential present in the mean-field hamiltonian.

VI. DISCUSSION AND SUMMARY

The purpose of this paper is to show how the modern EFT/DFT approach to QHD \cite{17,18} can be implemented straightforwardly at the mean-field level using standard quantum-field-theory procedures. The results imply that the so-called “no-sea approximation” for the nuclear ground state and its generalization to the RPA linear response (which includes long-range contributions from negative-energy states in the complete Dirac single-particle basis) are justified by modern field-theoretic approaches to the nuclear many-body problem. Since one expects that QCD vacuum physics cannot be adequately described by low-energy hadronic degrees of freedom alone, vacuum contributions from these terms must be subtracted away by the counterterms present in the QHD lagrangian and combined with local meson terms that are explicitly fitted to empirical data. As we emphasized, only the sum of all of these terms is constrained by experimental results, and by fitting to data we can implicitly include the vacuum effects, as well as other short-range and many-body effects \cite{31}.

The implicit “no-sea” subtraction procedure removes contributions from explicit sums over the entire Dirac sea of nucleons, which are beyond the realm of the low-energy EFT anyway. This means that no explicit calculations of loop integrals or counterterms must be made (unlike the Relativistic Hartree Approximation \cite{19,20}). In principle, an infinite set of counterterms is needed to describe the vacuum dynamics, but in practice, the finite residual parts are under-determined, and naturalness implies that most are numerically unimportant \cite{38}. Moreover, the mean-field computation of the single-particle Dirac wave functions can be related to relativistic Kohn–Sham theory using density-functional arguments, which show that more than simple single-particle (“Hartree”) physics is included in these wave
functions. The key to these arguments is that the local meson potentials provide an excellent approximation to the exact ground-state energy functional in the density regime of interest [31, 68]. Thus mean-field predictions for bulk nuclear properties will be accurate, and the theory is predictive when the most important (dominant) local terms have been fitted to data.

In summary, we regard these as our most important conclusions:

1. The strength of the EFT is that while the short-distance behavior of the theory is (probably) incorrect, it can be corrected systematically by the counterterms. The well-known “no-sea approximation” is a particular, yet convenient, prescription for performing this renormalization. It is therefore incorrect to view the “no-sea approximation” as an “empty” Dirac sea. Indeed, not only have vacuum effects been included, but the true vacuum dynamics becomes encoded in a small number of empirical constants that define the local meson potential.

2. The successful phenomenology of the “no-sea approximation” is justified by density-functional theory. The self-consistent Kohn–Sham equations contain Hartree theory as a particular limit, yet they go beyond Hartree theory by implicitly including many-body correlations, nucleon exchange, and short-range effects. This is achieved by fitting the empirical constants in the model to bulk properties of nuclei, rather than to two-body data.

3. The consistent linear response of the “no-sea” ground state must include contributions that mix (positive-energy) holes and and negative-energy states, in addition to the familiar particle-hole excitations. This result is dictated by the EFT that demands the same renormalization scheme for the ground state as for the linear response (excited states). Moreover, when the Kohn–Sham fields determined for the ground state are also used for the computation of the excited states, fundamental conservation laws are maintained, thereby guaranteeing the phenomenological success of the RPA. Equivalently, by demanding that the particle-hole interaction driving the RPA response be consistent with the accurately calibrated particle-particle interaction used to generate the ground state, the small fluctuations about equilibrium are guaranteed to be accurately reproduced. This is true even though existing RPA calculations within the EFT/DFT approach include only scalar cubic and quartic terms, which are just a subset of all allowed terms, but which are sufficient for quite a good description of ground-state nuclear properties.

Finally, although our discussion has thus far been entirely in the context of the one-loop approximation to the QHD effective action (or density functional), the underlying principles are more general. One can improve the analysis to include long-range correlations more explicitly within an EFT/DFT framework, by exploiting the effective-action formalism. Explicit, dynamical, long-range terms are expected to introduce new nonanalytic functions of the nucleon densities, which should improve the approximation to the exact energy functional and allow for extrapolation outside the density regime described accurately by the mean-field treatment discussed above.

Calculations beyond one-loop order have been studied by Hu [57]. Several important new features arise: first, one must retain the baryon counterterms in the lagrangian, since many of the vacuum contributions involve expansions in local terms with these forms. Second, one must use care in removing redundant terms from the lagrangian [17, 31] and develop a
systematic procedure for redefining the fields, so that order-by-order in the relevant expansion parameter ($\hbar$, or “hole-lines”, etc.), the lagrangian can always be recast in a standard (“canonical”) form. This allows vacuum subtractions to be made unambiguously as one proceeds to more and more complex approximations. Third, as shown by Hu for a wide class of approximations beyond one-loop order, it is always possible to separate the long-range nucleon loops from the short-range and vacuum parts of the loop integrals. The latter can be written in the same form as local counterterms and treated analogously to the subtracted terms described above; thus, one never has to calculate explicitly either the short-range terms or the counterterms that cancel them. The long-range terms must be retained and calculated explicitly; these resemble familiar nuclear many-body integrals, like nucleon exchange, rings, ladders, etc. [62].

Thus, in the end, we have a systematic way to generalize the relativistic nuclear many-body problem beyond one-loop order [17, 57]. Long-range nuclear terms can be organized and calculated in much the same way as in conventional nuclear structure physics; the only differences are that we now have four-component Dirac wave functions and propagators, the meson propagators are retarded and mixed together by various terms in the meson potential, and there are a small number of unknown constants that specify the short-range and vacuum QCD behavior, which must be determined by fitting to many-body (or nuclear matter) data. These more sophisticated analyses of effective QHD lagrangians will provide the basis for future investigations.

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