DEVELOPMENT OF THE PHYSICAL AND MATHEMATICAL MODEL OF THE BAKING PROCESS OF THE DOUGH PIECES IN BAKERY OVENS

1. Introduction

Baking bread is the most important process in the production of bakery products. Taste, aroma, porosity, gloss and other quality indicators of finished products is the result of a number of physicochemical changes inside the product during baking, which depend primarily on the thermal regime of baking and steam humidification [1–3]. The oven is the main equipment of the bakery, it determines the type and capacity of the enterprise, the range and quality of products. The oven is not only a thermal, but primarily a technological unit, the main purpose of which is producing high-quality products while ensuring high technical and economic indicators – product output with minimal energy costs. To ensure high-quality performance of the furnace, it is necessary to use models of a real system and conduct experiments based on a mathematical model for analysis, design or redesign, control and forecasting of a specific real process. This is relevant, so many researchers have analyzed the baking process in order to obtain an accurate mathematical model for modeling bread baking. So, the authors of [4] developed a model for the process of baking a yeast dough and assessing the influence of technological parameters. And in [5], the authors made an analysis of heat and mass transfer and changes in product quality during continuous baking of cookies based on modeling of inductive furnaces. The authors of [6] developed a model of bread baking using the direct 3D numerical method at the micro scale based on the analysis of the heat flux field during baking. But the authors of [7] obtained the mathematical dependence of the process of baking a dough biscuit. The authors of [8] described the dependence of the heating of a cylindrical dough piece. Thus, the object of this research is the physical and mathematical models designed to describe the heat and mass transfer inside the porous material during baking. And the aim of research is development of a mathematical model of the process of baking bread in the gas channels of the baking chamber, taking into account radiation-convective heat transfer, mass transfer, taking into account the introduction of water vapor to moisten the dough pieces and turbulence of the multiphase flow.
2. Methods of research

The design of the K-BOM-25 baking oven developed by the author (Fig. 1) includes the following components:

- an all-metal construction assembled from separate modules and insulated from the outside with mineral wool;
- gas channels in which bread is baked;
- a movable bucket conveyor for moving dough pieces (DP) in gas channels;
- input and output nodes of the furnace, including the gas duct for the supply of green gas;
- gas channels in which bread is baked;
- an all-metal construction assembled from separate modules and insulated from the outside with mineral wool;
- lifting volumetric force, N/m3;  $g$ – index of heating gases;
- water vapor index with the inclusion of water droplets;
- – the number of phases in the stream.

That is, in the physical model of the baking oven under consideration, the furnace-igniter unit is not considered, and in accordance with this, the combustion processes of the fuel also.

The environment of the gas channels of the baking oven is considered to be two-phase, selective emitting and absorbing, and is surrounded by diffuse boundaries and consists of green gases and water vapor, which is used to moisten the dough pieces.

![Fig. 1. K-BOM-25 bakery oven.](image)

Dough pieces and bread are considered wet capillary-porous media with effective physical properties.

Dough pieces move with the bucket conveyor.

3. Research results and discussion

In the work, a mathematical model of the process of baking bread in the gas channels of the oven’s baking chamber, taking into account radiation-convective heat transfer, mass transfer by introducing water vapor (with water droplets) to moisten the dough pieces, is baked, the multiphase flow turbulences can be formulated using the system of Euler equations averaged over Reynolds [9, 10].

The equations written for each of the phases: continuity, conservation of momentum, kinetic turbulent energy and its dissipation and energy conservation:

$$
\frac{1}{\rho_i} \left( \frac{\partial (\alpha_i \rho_i)}{\partial t} + \nabla \cdot (\alpha_i \rho_i \vec{V}_i) \right) = \sum_{j=1}^{n} \left( \frac{m_j - m_i}{\rho_i} \right),
$$

where $\rho_i$ – the average density of the $i$-th phase, kg/m3; $\alpha_i$ – the volume fraction of the $i$-th phase in the flow $\sum_{i=1}^{n} \alpha_i = 1$; $\rho_i$ – the density of the $i$-th phase, kg/m3; $\tau$ – the time, s; $\vec{V}_i$ – the vector of the speed of the $i$-th phase averaged beyond Reynolds or Favre, m/s; $\vec{V}$ – Hamilton operator, m/s; $m_i$ and $m_j$ – mass transfer rate from phase $j$ to phase $i$ and vice versa, respectively (moreover), kg/(s·m3); $n$ – the number of phases in the stream.

$$
\frac{\partial (\alpha_i \rho_i \vec{V}_i)}{\partial t} + \nabla \cdot \left( \alpha_i \rho_i \vec{V}_i \vec{V}_j \right) = -\alpha_i \vec{V}_i \cdot \vec{\tau} + \alpha_i \rho_i g + \\
+ \sum_{j=1}^{n} \left( K_{ij} (\vec{V}_j - \vec{V}_i) + \vec{m}_j \vec{V}_j - \vec{m}_i \vec{V}_i \right) + F_i + F_{i0} + F_{i,m},
$$

where $p_i$ – the partial pressure of the $i$-th phase, Pa; $\mu_i$ and $\lambda_i$ – the shear and bulk viscosity of phase $i$, respectively, Pa·s;

$$
\tau_i = \alpha_i \mu_i \left( \nabla \vec{V}_i + \nabla \vec{V}_i^t \right) + \\
+ \alpha_i \left( \lambda_i - \frac{2}{3} \mu_i \right) \vec{V}_i \cdot \nabla \vec{V}_i
$$

– stress tensor of the 2nd rank (or the physical equation of state of the medium that relates stress to strain rate), Pa; $I$ – the unit tensor of the 2nd rank; $g$ – the acceleration vector associated with gravity, m/s2; $K_{ij} = K_{ji}$ – coefficient of exchange of momentum between phases, depends on friction, pressure and other factors, kg/(m3·s); $\vec{V}_j / \vec{V}_i$ at $\vec{m}_j > 0$, $\vec{V}_j / \vec{m}_i < 0$ – interphase surface velocity, m/s; $F_i$ – the external mass force related to the volume, N/m3;

$$
F_{i,m} = -0.5 \rho_i \alpha_i \vec{V}_i \cdot \left( \vec{V}_k - \vec{V}_j \right) \\
\times \left( \nabla \times \vec{V}_k - \nabla \times \vec{V}_j \right)
$$

– lifting volumetric force, N/m3; $g$ – index of heating gases;

$$
F_{i,m} = 0.5 \alpha_i \rho_i \left( \frac{\partial \vec{V}_k}{\partial t} - \frac{\partial \vec{V}_j}{\partial t} \right)
$$

– attached volumetric force, N/m3;

$$
\frac{\partial (\alpha_i \rho_i \vec{k}_i)}{\partial t} + \nabla \cdot (\alpha_i \rho i \vec{k}_i) = \nabla \cdot \left( \alpha_i \mu_i / \sigma_i \vec{k}_i \right) + \\
+ (\alpha_i G_{k,i} - \alpha_i \rho_i \vec{e}_i) + \sum_{j=1}^{n} K_{ij} \left( C_{k,j} / C_{k,k} \right) - \\
- \sum_{j=1}^{n} K_{ij} \left( \nabla \cdot \vec{e}_j \right) \left( \frac{\mu_i / \alpha_i}{\sigma_i} \right)
$$

– the number of phases in the stream.
\[
\frac{\partial (\alpha, \rho, \epsilon)}{\partial t} + \nabla \cdot (\alpha, \rho, \nabla \epsilon) = \\
= \nabla \cdot (\alpha, \mu_{ij}, \nabla \epsilon) + \frac{\epsilon_i}{c_p} (C_i \alpha, C_{ee} - C_{e}, \alpha, \rho, \epsilon_0) + \\
+ C_{ee} \frac{\epsilon_i}{k_e} \left[ \sum_{j=1}^{n} K_{ij} (C_{ij} - C_j k) - \\
- \sum_{j=1}^{n} K_{ij} (\nabla_j - \nabla \epsilon) \left( \frac{\mu_{ij}}{\alpha, \sigma_j} \nabla \alpha_j - \frac{\mu_{ij}}{\alpha, \sigma_i} \nabla \alpha_i \right) \right] 
\]

where \(k_i\) – the mass turbulent kinetic energy of the i-th phase, \(J/\text{kg} \cdot \mu_i = \rho_i C_i k_i / \epsilon_i\) – turbulent viscosity of the i-th phase, \(P_a / \epsilon_i\) – the dissipation rate of the turbulent kinetic energy of the i-th phase, \(J/(\text{kg} \cdot \text{s})\); \(G_i = \mu_i \nabla \cdot (\nabla \nabla \cdot \nabla \nabla)\); \(P_a / \epsilon_i\); \(C_{ij} = 2 \eta_{ij} (1 + \eta_j)\) – coefficient associated with the dispersion of drops [2]; \(s_0\) – a constant; \(s_{ij}, \sigma_i, C_i, C_{1i}, C_{2i}, C_{3i}\) – the parameters (constants) of the \(k-e\) model;

\[
\frac{\partial (\alpha, \rho, h)}{\partial t} + \nabla \cdot (\alpha, \rho, \nabla h) = \alpha, \rho, \nabla, \nabla, - \nabla \cdot \nabla, q_i + \\
+ \sum_{j=1}^{n} (Q_{ij} + \tilde{m}_i h_j - \tilde{m}_j h_i) + \epsilon_i, T_i, + S_i, 
\]

where \(\cdot \cdot \cdot \cdot \) – the double scalar product operator;

\[
h = \sum_{i} c_{ij} dT_i
\]

– mass phase of the mixture, \(J/\text{kg}\); \(c_{ij}\) – the mass isobaric heat capacity, \(J/(\text{kg} \cdot \text{K})\); \(T_{ref}\) – reference temperature, \(K\); \(Q_{ij} = -\epsilon_i, T_i\) – the heat flux density vector of phase j (or the physical equation of state of the mixture connecting q with \(\nabla T\)), which takes into account heat transfer for flow turbulence account, \(W/\text{m}^2\); \(\lambda_i\) – the thermal conductivity of phase i, \(W/(\text{m} \cdot \text{K})\); \(Q_{ij} = -\epsilon_i, T_i\) – the heat transfer intensity between phases j and i (with \(Q_{ij} = 0\)), \(W/\text{m}^2\); \(h_i\) – the enthalpy on the interphase surface (during evaporation, this may be the vapor enthalpy at the temperature of water droplets), \(J/\text{kg}\); \(S_i\) – volumetric heat source from chemical reactions, \(W/\text{m}^3\);

\[
E_i(T) = \int \Omega_{i} (I_{i} \Omega - 4 \pi n_i^2 l_{i} \Omega (T) \Omega) \text{d}v
\]

– bulk density of the radiation heat flux of the selective emitting and absorbing medium of the i-th phase, \(W/\text{m}^3\); \(\nu\) – the radiation frequency, Hz; \(k_{ij}\) and \(n_{ij}\) – the selective absorption coefficient (\(m^2\)) and the refractive index of the i-th phase, respectively; \(\Omega\) – the solid angle, sr; \(l_{i} \Omega_{i}\) – Planck function, \(W/\text{m}^3\). 

The spectral radiation intensity of the i-th phase \((W/\text{m}^3\cdot\text{sr})\) \(I_{i} \Omega\) for the direction \(s\) in the solid \(d\Omega\) can be represented as the dependence:

\[
I_{i} (s) = I_{i} (s_0) \exp \left( - \int_{s_0}^{s} K_{i} ds \right) + \\
+ \int_{s_0}^{s} n_i^2 l_{i} K_{i} \exp \left( - \int_{s_0}^{s} K_{i} ds \right) ds',
\]

where \(s_0\) corresponds to the boundary of the medium.

For solid structural elements of the furnace and dough pieces move together with a bucket conveyor, the system of equations (1)–(5) is simplified to the heat equation of the form:

\[
\frac{\partial \rho h}{\partial t} = \nabla \left[ \lambda(T) \nabla T \right] + q_i,
\]

\[
X_{max} (x, y, z, \tau) \cup X_{min} (x, y, z) \in X(x, y, z) \in \Omega,
\]

where \(q_i\) – the volumetric heat source associated with loading the DP and unloading the baked bread (BB) from the oven, \(W/\text{m}^3\), the specific power of which is determined from the system of equations:

\[
q_i = \frac{\int_{T} c_{i} (\Omega | T) \text{d}T}{V_{sp}},
\]

\[
\frac{\int_{T} c_{i} (\Omega | T) \text{d}T}{V_{bb}},
\]

where \(\hat{m}_{sp}, V_{sp}\) – the mass flow rate of the DP/BB in the loading and unloading unit, respectively, \(\text{kg} / \text{s}\); \(c_{i} (\text{DP}, \text{BB})\) – mass heat capacity of the DP/BB, respectively, \(J/(\text{kg} \cdot \text{K})\); \(V_{dp}, V_{bb}\) – volume of DP/BB, respectively, \(\text{m}^3\); \(X(y, z)\) – the Cartesian coordinate system of the solid elements of the furnace, including the fixed – \(X_{max} (x, y, z) \) and the moving part, which refers to the bucket conveyor and dough pieces – \(X_{min} (x, y, z) \); \(V_{sp}\) – the conveyor speed vector along with the bread that is baked, \(m/s\); \(\Omega\) – the calculated area of the baking oven.

The coefficient of exchange of momentum between the phases in equations (2)–(5) depends on the selected model of hydraulic resistance, Reynolds number, viscosity, etc. and is determined depending on the physical state of the phases interacting: liquid-liquid or liquid-gas, or gas-gas. So, for example, the coefficient of exchange of momentum between water droplets \(p\) and gas \(g\) can be determined as [4]:

\[
K_{pg} = \frac{\alpha_p \alpha_g \rho_p}{\tau_p},
\]

where

\[
\tau_p = \frac{\rho_p d_p^2}{18 \mu_{eff}}
\]

– the relaxation time of water droplets, \(s\); \(d_p\) – diameter of water droplets, \(m\); \(\mu_{eff} = \mu_g (1 - \alpha_p)^{2.5}\) – effective dynamic viscosity of a gas-liquid droplets, \(\text{Pa} \cdot \text{s}\); \(\mu_{eff} = \mu_g (1 - \alpha_p)^{2.5}\) – hydraulic resistance function; \(\alpha_p\) – the coefficient of hydraulic resistance.
The Reynolds number of a two-phase medium is determined by the ratio:
\[
Re = \frac{\rho \|\vec{V}_i - \vec{V}_f\|}{\mu_{eff}}.
\]  
(10)

The coefficient of hydraulic resistance is determined depending on the Reynolds number:
\[
C_D = \begin{cases} 
\frac{24}{Re} & \text{at } Re < 1; \\
\frac{24}{Re} \left(1 + 0.1Re^2\right) \left(1 \leq Re \leq 1000; \right) \\
\frac{2}{3} \lambda_{RT} \left[\frac{1 + 17.67(f')^2}{18.67f'}\right] & \text{at } Re > 1000,
\end{cases}
\]  
(11)

where \( f' = (1 - \alpha_0)^2; \lambda_{RT} = (\sigma/(\kappa g \Delta \rho_g))^{0.5} \) — wavelength of the Rayleigh-Taylor instability, \( \mu; \sigma \) — the coefficient of surface tension, \( \kappa/m; g \) — the acceleration of gravity, \( \kappa/s^2; \Delta \rho_g \) — the density difference between the phases \( p \) and \( g, \kappa/g^m \).

The initial conditions for the system of equations (1)–(6) at \( t = 0 \):
\[
\begin{align*}
\vec{V}_i &= 0; & p_i &= 0; & T_i &= T_0; \\
\rho_i &= \rho_i^0; & \epsilon_i &= \epsilon_i^0; \\
\vec{l}_p &= \vec{l}_p^0; & \alpha_i &= \alpha_i^0,
\end{align*}
\]  
(12)

The boundary conditions for the system of equations (1)–(8) with:
– in the inlet sections of the furnace, parameters are set for green gas \((i = g)\) and water vapor \((i = p)\), mass flow rate and DP temperature:
\[
\begin{align*}
\vec{V}_i &= V_i^*; & T_i &= T_i^*; & \rho_i &= \rho_i^*; & T_D &= T_D^*; \\
\rho_i &= \rho_i^0; & \epsilon_i &= \epsilon_i^0; & \alpha_i &= \alpha_i^0,
\end{align*}
\]  
(13)

where \( \vec{n} \) — the vector of the external normal to the surface of the input furnace sections; \( V_i^* \) — normal speed, \( m/s; \) \( T_i^* \) — temperature of gas phases, \( \kappa; \) \( m_i^0 \) — mass flow rate, \( \kappa/g; \) \( T_D^0 \) — DP temperature, \( \kappa; \) \( \alpha_i^0 \) — mass turbulent kinetic energy, \( \kappa/(\kappa/g)\); \( \epsilon_i^0 \) — rate of its dissipation, \( \kappa/(\kappa/g)\); \( \alpha_i^0 \) — volumetric part of the \( i \)-th phase at the entrance to the furnace;
– in the outgoing sections of the oven, parameters are set for the mixture (14), gas phases and mass flow rate, as well as the temperature of the baked bread (15):
\[
\begin{align*}
p_{mix} &= 0, \\
\{T_i = T_i^*; & \alpha_i = \alpha_i^*; & m_i^0 = m_i^0; & T_D = T_D^*; \\
\rho_i &= \rho_i^0; & \epsilon_i &= \epsilon_i^0
\end{align*}
\]  
(14)

where \( p_{mix} \) — the overpressure of the gas mixture, \( P; \) \( T_i^*; \) \( \alpha_i^*; \) \( m_i^0; \) \( T_D^*; \) \( \epsilon_i^0; \) \( \alpha_i^0 \) — temperature (\( \kappa \)), volume fraction of the \( i \)-th phase, BB mass flow rate (\( \kappa/g; \) ), BB temperature (\( \kappa \)), mass turbulent kinetic energy (\( \kappa/(\kappa/g) \)) and its dissipation rate (\( \kappa/(\kappa/g) \));
– at the contact boundary between solids, the conditions for absolute contact are set:
\[
\begin{align*}
\{T &= 0; \} \\
\{\vec{n} \cdot \vec{q} &= 0, \}
\end{align*}
\]  
(16)

where \( \{T = T^* - T; \}\{\vec{n} \cdot \vec{q} = n \cdot \vec{q} - n \cdot \vec{q}^* \}; \) \( \leftrightarrow \leftrightarrow \) means to the left and right of the contact boundary; \( q_{\alpha} = -\lambda(T)\vec{V}T \) — the heat flux density vector; \( n \) — the vector of the external normal to the surface of the body;
– at the contact boundary between the multiphase medium of the furnace, DP and the fencing of the gas channels, the adhesion conditions for each of the gas phases and the interface conditions in terms of temperature and heat flux density are set:
\[
\begin{align*}
\vec{V}_i &= 0; & \{T_i &= 0; \} \\
\{\vec{n} \cdot \vec{q} &= 0, \}
\end{align*}
\]  
(17)

where \( n \) — the number of phases in the stream;
\[
q_{\alpha} = \alpha \int_{\Omega} [n^2 \epsilon_i^0 L_{mov} - \epsilon_i \int_{\Omega} I_{mov}(s \vec{n}) d\Omega] d\Omega
\]  
– the resulting radiation flux of the \( i \)-th phase, \( \kappa/m^2; \) \( \epsilon_i \) — the spectral blackness of the surface of the gas channels of the furnace;
\[
q_{\beta} = \frac{\lambda_{mov}(T_i) - m_{mov}(T_i)}{F_{DP(BB)}}
\]  
– surface heat flux density associated with mass transfer on the DP surface (moisture exchange due to condensation/evaporation of moisture), \( \kappa/m^2; \) \( \lambda_{mov} \) — the heat of the first-order phase transition for water (condensation/evaporation), \( \kappa/g; \) \( m_{mov} \) — mass flow rate of water during condensation and evaporation, respectively, \( \kappa/g; \) \( T_i \) — average integral DP/BB surface temperature, \( \kappa; \) \( F_{DP(BB)} \) — DP/BB surface area, \( \kappa^2; \) \( l_{eff} \) — the effective thermal conductivity of the DP/BB, \( \kappa/(\kappa/m \cdot K) \); 
– at the boundary of the contact of the fencing of the furnace with the surrounding air, boundary conditions of convective type are set:
\[
\{\vec{n} \cdot [-\lambda(T)\vec{V}T] = \alpha(T)(T - T_a) \}
\]  
(18)

where \( \alpha \) — the heat transfer coefficient; \( T_a \) — the ambient temperature.

4. Conclusions

A mathematical model of the process of baking bread in the gas channels of the baking chamber is developed.
taking into account radiation-convective heat transfer, mass transfer taking into account the introduction of water vapor to moisten the dough pieces and turbulence of the multiphase flow. It is theoretically grounded that this model will allow with sufficient accuracy and detail to take into account all the operational and design features of modern conveyor baking ovens. And it will also allow for extensive parametric studies of conjugate heat transfer in them with access to the final indicator – the quality of finished products. But this theoretical justification requires empirical evidence.

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