$B_{(s)} \to D_{s0}^*(2317)P(V)$ decays in perturbative QCD approach

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Abstract

In this work, we use pQCD approach to calculate 20 $B_{(s)} \to D_{s0}^*(2317)P(V)$ two body decays by assuming $D_{s0}^*(2317)$ as a $\bar{c}s$ scalar meson, where $P(V)$ denotes a pseudoscalar (vector) meson. These $B_{(s)}$ decays can serve as an ideal platform to probe the valuable information on the inner structure of the charmed-strange meson $D_{s0}^*(2317)$, and to explore the dynamics of strong interactions and signals of new physics. These considered decays can be divided into two types: the CKM favored decays and the CKM suppressed decays. The former are induced by $b \to c$ transition, whose branching ratios are larger than $10^{-5}$. The branching fraction of the decay $\bar{B}_s^0 \to D_{s0}^{*+}(2317)\rho^-$ is the largest and reaches about $1.8 \times 10^{-3}$, while the branching ratios for the decay $\bar{B}_s^0 \to D_{s0}^{*+}(2317)K^*$ and other two pure annihilation decays $\bar{B}_s^0 \to D_{s0}^{*+}(2317)K^-, D_{s0}^{*+}(2317)K^-\pi^+$ are only at $10^{-5}$ order. Our predictions are consistent well with the results given by the light cone sum rules approach. These decays are most likely to be measured at the running LHCb and the forthcoming SuperKEKB. The latter are induced by $b \to u$ transition, among of which the channel $\bar{B}_0^0 \to D_{s0}^{*-}(2317)\rho^+$ has the largest branching fraction, reaching up to $10^{-5}$ order. Again the pure annihilation decays $B^- \to D_{s0}^{*-}(2317)\phi, \bar{B}_0^0 \to D_{s0}^{*-}(2317)K^+(K^{*+}), B^- \to D_{s0}^{*-}(2317)K^0(K^{*0})$, have the smallest branching ratios, which drop to as low as $10^{-10} \sim 10^{-8}$.

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I. INTRODUCTION

The charmed-strange meson $D_{s0}^*(2317)$ was first observed by BABAR Collaboration in the inclusive $D_s^+\pi^0$ invariant mass distribution [1, 2], then confirmed by CLEO [3] and Belle Collaboration [4], respectively. Usually, the $D_{s0}^*(2317)$ meson is suggested as a P-wave $\bar{c}s$ state with spin-parity $J^P = 0^+$. However, there exit two divergences between the data and the theoretical predictions: First, the measured mass for this meson is at least $150 \text{MeV}/c^2$ lower than the theoretical calculations from a potential model [5, 6], lattice QCD [7] and so on. For example, the authors [8] obtained $M(D_{s0}^*(2317)) = (2480 \pm 30) \text{MeV}$ by using the standard Borel-transformed QCD sum rule which was higher than the BABAR result by about 160 MeV. While, Narison [9] used the QCD spectral sum rules to get $M(D_{s0}^*(2317)) = (2297 \pm 113) \text{MeV}$ and reached the conclusion that $D_s(2317)$ is a $\bar{c}s$ state. Second, the absolute branching ratio of decay $D_{s0}^*(2317)^- \to D^0 \pi^0$ measured by BESIII Collaboration [10] showed that $D_{s0}^*(2317)^-$ tends to have a significantly larger branching ratio to $\pi^0 D_s^-$ than to $\gamma D_s^+$, which differs from the expectation of the conventional $\bar{c}s$ hypothesis. These puzzles inspired various exotic explanations to its inner structure, such as $DK$ molecule state [11–15], a tetraquark state [16–19], or a mixture of a $\bar{c}s$ state and a tetraquark state [20–22]. In order to further reveal the internal structure of $D_{s0}^*(2317)$, we intend to study the weak production of this charmed-strange meson through the $B_s$ decays, which can serve as an ideal platform to probe the valuable informations on the inner structure of the exotic scalar mesons [23–27]. In the conventional two quark picture the branching ratios of the decays $B_{(s)} \to D_{s0}^*(2317)P(V)$, where $P(V)$ denotes the light pseudoscalar (vector) meson, are expected to be of the same order of magnitude as those of $B_{(s)} \to D_sP(V)$ decays, since the $D_{s0}^*(2317)$ meson decay constant should be close to that of the pseudoscalar meson $D_s$ as required by the chiral symmetry. On the contrary, in the unconventional picture the corresponding decay amplitudes involve additional hard scattering with the participation of four valence quarks. Then the branching ratios are at least suppressed by the coupling constant and by inverse powers of heavy meson masses, such that they are much smaller than those of $B_{(s)} \to D_sP(V)$ decays by one order. So it is meaningful to study the branching ratios of the decays $B_{(s)} \to D_{s0}^*(2317)P(V)$ both in experiment and theory.

$B_{(s)}$ two body nonleptonic decays with $D_{s0}^*(2317)$ meson involved in the final states have been studied in the light cone sum rules (LCSR) approach [23], the relativistic quark model (RQM) [29], and the nonrelativistic quark model (NRQM) [30]. Here we would like to use pQCD approach to study $B_{(s)} \to D_{s0}^*(2317)P(V)$ decays. Studying these decays may shed light on the nature of the $D_{s0}^*(2317)$ meson, explore the dynamics of strong interactions. Further more, the study of these weak decays is important for further improvement in the determination of the Cabibbo-Kobayshi-Maskawa (CKM) matrix elements, for testing the prediction of the Standard Model and searching for possible deviations from theoretical predictions, the so-called "new physics" signals.

The layout of this paper is as follows, we analyze the decay $B_{(s)} \to D_{s0}^*(2317)P(V)$ using the perturbative QCD approach in Section II. The numerical results and discussions are given in Section III, where the theoretical uncertainties are also considered. The conclusions are presented in the final part.
II. THE PERTURBATIVE CALCULATIONS

In the pQCD approach, the only non-perturbative inputs are the light cone distribution amplitudes (LCDAs) and the meson decay constants. For the wave function of the heavy $B_{(s)}$ meson, we take

$$\Phi_{B_{(s)}}(x, b) = \frac{1}{\sqrt{2N_c}}(\not{p}_{B_{(s)}} + m_{B_{(s)}})\gamma_5\phi_{B_{(s)}}(x, b).$$

Here only the contribution of Lorentz structure $\phi_{B_{(s)}}(x, b)$ is taken into account, since the contribution of the second Lorentz structure $\bar{\phi}_{B_{(s)}}$ is numerically small \cite{31} and has been neglected. For the distribution amplitude $\phi_{B_{(s)}}(x, b)$ in Eq.\,(1), we adopt the following model:

$$\phi_{B_{(s)}}(x, b) = N_{B_{(s)}}x^2(1-x)^2\exp\left[-\frac{M_{B_{(s)}}^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right],$$

where $\omega_b$ is a free parameter, we take $\omega_b = 0.4 \pm 0.04(0.5 \pm 0.05)$ GeV for $B(B_s)$ meson in numerical calculations, and $N_B = 101.445 (N_{B_s} = 63.671)$ is the normalization factor for $\omega_b = 0.4(0.5)$. These parameters has been fixed using the rich experimental data on the $B_{(s)}$ decay channels. In this model the significant feature is the intrinsic transverse momentum dependence, which is essential for the $B_{(s)}$ meson. It can provide additional suppression in the large $b$ region, where the soft dynamics dominates and Sudakov suppression is weaker. Considering a small SU(3) breaking, the $s$ quark momentum fraction is a litter larger than that of the $u(d)$ quark in the lighter $B$ meson, because of the heavier mass for the $s$ quark.

From the shape of the distribution amplitude shown in Ref.\,[32], it is easy to see that the larger $\omega_b$ gives a larger momentum fraction to the $s$ quark.

The wave functions of the scalar meson $D_{s0}^*$, we use the form defined in Ref.\,[33]

$$\langle \bar{D}_{s0}^*(2317)^+(p_2)|\bar{c}(z)s_s(0)|0\rangle = \frac{1}{\sqrt{2N_c}} \int dx e^{ip_2z} \left[(\not{p}_2)I_{lj} + m_{D_{s0}}I_{lj}\right] \phi_{D_{s0}^*}.$$  \hspace{1cm} (3)

It is noticed that the distribution amplitudes which associate with the nonlocal operators $\bar{c}(z)\gamma_\mu s$ and $\bar{c}(z)s$ are different. The difference between them is order of $\Lambda/m_{D_{s0}} \sim (m_{D_{s0}} - m_c)/m_{D_{s0}}$. If we set $m_{D_{s0}^*} \sim m_c$, we can get these two distribution amplitudes are very similar. For the leading power calculation, it is reasonable to parameterize them in the same form as

$$\phi_{D_{s0}^*}(x) = \hat{f}_{D_{s0}^*} 6x(1-x)[1 + a(1-2x)]$$

in the heavy quark limit. Here $\hat{f}_{D_{s0}^*} = 225 \pm 25$ MeV is determined from the two-point QCD sum rules, and the shape parameter $a = -0.21$ \cite{28} is fixed under the condition that the distribution amplitude $\phi_{D_{s0}^*}(x)$ possesses the maximum at $\bar{x} = (m_{D_{s0}^*} - m_c)/m_{D_{s0}^*}$ with $m_c = 1.275$ GeV. It is worthwhile to point out that the intrinsic $b$ dependence of this charmed meson’s wave functions has been neglected in our analysis.

1 From now on, we will use $D_{s0}$ to denote $D_{s0}^*(2317)$ for simply in some places.
Since the light cone distribution amplitudes of the pseudoscalar mesons $\pi, K, \eta^{(')}$ and the vector mesons $\rho, K^*, \omega$ have been well constrained in the papers [34–40], and been tested systematically in the work [32], we will use these LCDAs directly listed in that paper [32], together with the corresponding decay constants.

For these processes considered, the weak effective Hamiltonian $H_{\text{eff}}$ can be written as two types:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cq}^* [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)], \quad \text{Type I} \quad (5)$$

where the tree operators are given as:

$$O_1 = (\bar{c}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \quad O_2 = (\bar{c}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \quad (6)$$

for the CKM favored channels, while the effective Hamiltonian for the CKM suppressed decays is written as:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cq}^* [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)], \quad \text{Type II} \quad (7)$$

with

$$O_1 = (\bar{u}_\alpha b_\beta)_{V-A}(\bar{D}_\beta c_\alpha)_{V-A}, O_2 = (\bar{u}_\alpha b_\beta)_{V-A}(\bar{D}_\beta c_\alpha)_{V-A}. \quad (8)$$

Here $D$ represents $d(s)$ quark. The type I channel is induced by $b \to c$ transition, such as $\bar{B}_s^0 \to D^{*+}_{s0} \pi^-(K^-), D^{*+}_{s0} \rho^-(K^{*-}), B^0 \to D^{*+}_{s0} K^-(K^{*-})$. While the type II decay is induced by $b \to u$ transition, such as $\bar{B}_s^0 \to D^{*+}_{s0} \pi^+(K^+), D^{*+}_{s0} \rho^+(K^{*+}), B^- \to D^{*+}_{s0} \eta_0^0(\rho^0, \omega, \phi), D^{*+}_{s0} \eta^{(')}$. For the CKM favored decays, we take the decay $\bar{B}_s^0 \to D^{*+}_{s0} K^-$ as an example, whose Feynman diagrams are given in Fig.1. The first line Feynman diagrams are for the emission type ones, where Fig.1(a) and 1(b) are the factorization diagrams, Fig.1(c) and 1(d) are the nonfactorization ones, their amplitudes can be written...
as:

\[
\mathcal{F}_{B \to D^*_{s0}}^P = 8\pi C_F M_B^4 f_P \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D^*_{s0}}
\]

\[
\times \left\{ \left[ 1 + x_2 - r_{D^*_{s0}}(2x_2 - 1) \right] E_e(t_0) S_i(x_2)(t_0) h_e(x_1, x_2(1 - r_{D^*_{s0}}^2), b_1, b_2) + [(2 - r_{D^*_{s0}}) r_{D^*_{s0}} + r_e(1 - 2r_{D^*_{s0}})] E_e(t_0) S_t(x_1) h_e(x_2, x_1(1 - r_{D^*_{s0}}^2), b_2, b_1) \right\},
\]

\[
\mathcal{M}_{B \to D^*_{s0}}^P = -32\pi C_F m_B^4 / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_3 \phi_B(x_1, b_1)
\]

\[
\times \phi_{D^*_{s0}}(x_2) \phi_P(x_3) \left[ (r_{D^*_{s0}} x_2 - x_3) E_{en}(t_0) h_{en}^c(x_1, x_2, x_3, b_1, b_3)
\right.
\]

\[
+ \left. (1 + x_2 - x_3 - r_{D^*_{s0}} x_2) E_{en}(t_4) h_{en}^d(x_1, x_2, x_3, b_1, b_3) \right],
\]

where \( P \) denotes a pseudoscalar meson, \( r_{D^*_{s0}} = m_{D^*_{s0}} / M_B \), \( r_c = m_c / M_B \) and \( f_P \) is the decay constant of the pseudoscalar meson. The evolution factors evolving the scale \( t \) and the hard functions for the hard part of the amplitudes are listed as:

\[
E_e(t) = \alpha_s(t) \exp[-S_B(t) - S_{D^*_{s0}}(t)],
\]

\[
E_{en}(t) = \alpha_s(t) \exp[-S_B(t) - S_P(t) - S_{D^*_{s0}}(t)|_{b_1 = b_2}],
\]

\[
h_e(x_1, x_2, b_1, b_2) = K_0(\sqrt{x_1 x_2 m_B b_1}) [\theta(b_1 - b_2) K_0(\sqrt{x_2 m_B b_1}) I_0(\sqrt{x_2 m_B b_2})
\]

\[
+ \theta(b_2 - b_1) K_0(\sqrt{x_2 m_B b_2}) I_0(\sqrt{x_2 m_B b_1})],
\]

\[
h_{en}^c(x_1, x_2, x_3, b_1, b_3) = \left[ \theta(b_1 - b_3) K_0(\sqrt{A^2 m_B b_1}) I_0(\sqrt{A^2 m_B b_3})
\right.
\]

\[
\left. + (b_1 \leftrightarrow b_3) \right] \begin{pmatrix} K_0(A_j m_B b_3) & \text{for } A_j^2 \geq 0 \\ \frac{A_j}{2} \mu H_0^{(1)}(\sqrt{A_j^2 m_B b_3}) & \text{for } A_j^2 \leq 0 \end{pmatrix}_{j=c,d},
\]

with the variables

\[
A^2 = x_2 x_1,
\]

\[
A^2_0 = x_2(x_1 - x_3(1 - r_{D^*_{s0}}^2)),
\]

\[
A^2_0 = x_2 x_1 - (1 - x_3)(1 - r_{D^*_{s0}}^2)).
\]

The hard scales \( t \) and the expression of Sudakov factor in each amplitude can be found in Appendix. As we know that the double logarithms \( \alpha_s \ln^2 x \) produced by the radiative corrections are not small expansion parameters when the end point region is important, in order to improve the perturbative expansion, the threshold resummation of these logarithms to all order is needed, which leads to a quark jet function

\[
S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c,
\]

with \( c = 0.5 \). It is effective to smear the end point singularity with a momentum fraction \( x \to 0 \). This factor will also appear in the factorizable annihilation amplitudes.

As to the amplitudes for the second line Feynman diagrams can be obtained by the
Feynman rules and are given as:

\[ \mathcal{M}_{\text{ann}}^{D^0} = 32\pi C_f m_B^4 \sqrt{2NC} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 db_3 \phi_B(x_1, b_1) \phi_{D^0}(x_2) \times \left\{ -[r_{D^0} r_P((x_2 - x_3 + 3) \phi_p^T(x_3) - (x_2 + x_3 - 1) \phi_p(x_3)) + x_2 \phi_p(x_3)] E_{\text{an}}(t) \right\} h_{an}^i (x_1, x_2, x_3, b_1, b_3) + \left[ (1 - x_3) \phi_P(x_3) + r_{D^0} r_P((x_2 - x_3 + 1) \phi_p^T(x_3) + (x_2 + x_3 - 1) \phi_p(x_3))] \times E_{\text{an}}(t) h_{an}^f (x_1, x_2, x_3, b_1, b_3) \right\}, \]

(19)

\[ \mathcal{F}_{\text{ann}}^{D^0} = -8\pi C_f m_B^4 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_3 db_3 \phi_D^0 (x_2) \{ [4r_P r_{D^0} (1 - x_3) \phi_p^T(x_3) + r_P (r_c + 2r_{D^0} x_3) (\phi_p^T(x_3) + \phi_p(x_3)) + (1 + 2r_{D^0} x_3) \phi_p(x_3)] E_{\text{af}}(t) \times S_{t}(x_3) h_{af} (x_2, x_3 (1 - r_{D^0}^2), b_2, b_3) - \left[ x_2 \phi_p(x_3) + 2r_P r_{D^0} (1 + x_2) \phi_p^T(x_3) \right] \times E_{\text{af}}(t) S_{t}(x_3) h_{af} (x_2, x_3 (1 - r_{D^0}^2), b_3, b_2) \}. \]

(20)

Here \( \mathcal{F}_{\text{ann}}^{D^0} (\mathcal{M}_{\text{ann}}^{D^0}) \) are the (non)factorizable annihilation type amplitudes, where the evolution factors \( E \) evolving the scale \( t \) and the hard functions of the hard part of factorization amplitudes are listed as:

\[ E_{\text{an}}(t) = \alpha_s(t) \exp [-S_B(t) - S_{D^0}(t) - S_P(t)]_{b_2 = b_3}, \]

(21)

\[ E_{\text{af}}(t) = \alpha_s(t) \exp [-S_{D^0}(t) - S_P(t)], \]

(22)

\[ h_{an}^j (x_{i=1,2,3}, b_1, b_3) = \frac{i\pi}{2} \left[ \theta(b_1 - b_3) H_0^{(1)}(\sqrt{x_2 x_3 (1 - r_{D^0}^2) m_B b_1}) J_0(\sqrt{x_2 x_3 (1 - r_{D^0}^2) m_B b_3}) + (b_1 \leftrightarrow b_3) \right] L_j \geq 0, \]

(23)

\[ h_{af} (x_2, x_3, b_2, b_3) = (i\frac{\pi}{2})^2 H_0^{(1)}(\sqrt{x_2 x_3 m_B b_2}) \times \theta(b_2 - b_3) H_0^{(1)}(\sqrt{x_2 x_3 m_B b_2}) J_0(\sqrt{x_3 m_B b_3}) + (b_2 \leftrightarrow b_3), \]

(24)

where the definition of \( L_j^2 \) are written as:

\[ L_e^2 = r_{D^0}^2 - (1 - x_2) \left[ x_3 (1 - r_{D^0}^2) + r_{D^0}^2 - x_1 \right], \]

(25)

\[ L_f^2 = x_2 \left[ x_1 (1 - x_3) (1 - r_{D^0}^2) \right]. \]

(26)

The functions \( H_0^{(1)}, J_0, K_0, I_0 \) in the upper hard kernels \( h_e, h_{en}^j, h_{an}^j, h_{af} \) are the (modified) Bessel functions, which can be obtained from the Fourier transformations of the quark and gluon propagators.
Similarly, we can also give the amplitudes for the CKM suppressed decay channels,

\[
\mathcal{F}_{B \rightarrow P}^{D^*_{s0}} = 8\pi C_F M^4_{F} f_{D^*_{s0}} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_2 db_3 \phi_B(x_1, b_1) \\
\times [(1 + x_3)\phi_p(x_3) - r_p(2x_3 - 1)(\phi^T_p(x_3) + \phi^T_{P}(x_3))] \\
\times E_e(t_a) S_t(x_2) h_e(x_1, x_3, 3 - r^2_{D^*_0}, b_1, b_3) \\
+ [2r_p \phi_p(x_3)] E_e(t_b) S_t(x_1) h_e(x_3, x_3(1 - r^2_{D^*_0}), b_3, b_1),
\]

(27)

\[
\mathcal{M}_{B \rightarrow P}^{D^*_{s0}} = 32\pi C_F m^4_{B} / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D^*_{s0}}(x_2) \\
\times \{ [x_2 \phi_p(x_3) + r_p(x_3 + 2)(\phi^T_p(x_3) - \phi^T_p(x_3)) \}
\]

\[
\times E_e(t_c) h^c_{en}(x_1, x_2, 3 - r^2_{D^*_0}, b_3) \\
- [(1 - 3x_3)\phi_p(x_3) - 3r_p(x_3 + 2)(\phi^T_p(x_3) - \phi^T_p(x_3)) \\
\times E_e(t_c) h^c_{en}(x_1, x_3, 3 - r^2_{D^*_0}, b_3) \},
\]

(28)

where these two amplitudes are factorizable and nonfactorizable emission contributions, respectively. The amplitudes \(\mathcal{F}_{B \rightarrow D^*_{s0}}^{P}, \mathcal{M}_{B \rightarrow D^*_{s0}}^{P}\) are the color allowed amplitudes, while \(\mathcal{F}_{B \rightarrow P}^{D^*_{s0}}, \mathcal{M}_{B \rightarrow P}^{D^*_{s0}}\) are the color suppressed ones. The annihilation type amplitudes are listed as:

\[
\mathcal{M}_{ann}^{P} = 32\pi C_F m^4_{B} / \sqrt{2N_C} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_{D^*_{s0}}(x_2) \\
\times \{ [r_{D^*_{s0}} r_p((x_2 + 3x_3 + 2)\phi^T_p(x_3) - (x_2 - 3x_3)\phi^T_p(x_3))] \\
-x_3\phi_p(x_3) \}
\times E_{an}(t_c) h^c_{an}(x_1, x_2, 3 - r^2_{D^*_0}, b_3) \\
+ [ -r_{D^*_{s0}} r_p((x_2 + 3x_3)\phi^T_p(x_3) + (x_2 - 3x_3)\phi^T_p(x_3)) + x_2\phi_p(x_3)]
\times E_{an}(t_c) h^c_{an}(x_1, x_2, 3 - r^2_{D^*_0}, b_3),
\]

(29)

\[
\mathcal{F}_{ann}^{P} = 8\pi C_F f_{B} m^4_{B} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_{D^*_{s0}}(x_2) \\
\times \{ [2r_{D^*_{s0}} r_p(1 + x_3)\phi^T_p(x_3) - x_2\phi_p(x_3)] E_{af}(t_a) S_t(x_2) h_{af}(x_2, 3 - r^2_{D^*_0}, b_2, b_3) \\
+ [(x_3 - r_{D^*_{s0}}(2r_p - 3r_{D^*_{s0}}))\phi_p(x_3) + r_p(r_c - 3r_{D^*_{s0}}(1 + x_3))\phi^T_p(x_3) \\
-r_p(r_c - 3r_{D^*_{s0}}(1 - x_3))\phi^T_p(x_3)] E_{af}(t_b) S_t(x_2) h_{af}(x_2, 3 - r^2_{D^*_0}, b_2, b_3) \}
\]

(30)

The definitions for the evolution factors, the hard functions and the jet function \(S_t(x)\) in Eqs.\((27) \sim (30)\) can be found in Eqs.\((9), (10)\) and Eqs.\((19), (20)\) with the different parameters in the hard function \(h^c_{en, an}^{f}\), which are listed as:

\[
A \rightarrow A^2 = x_1 x_3 (1 - r^2_{D^*_0}),
\]

(31)

\[
A_c^2 \rightarrow A_c^2 = (x_1 - x_2) x_3 (1 - r^2_{D^*_0}),
\]

(32)

\[
A_d^2 \rightarrow A_d^2 = r^2_c - r^2_{D^*_0} + (x_1 + x_2) r^2_{D^*_0} - (1 - x_2 - x_1) x_3 (1 - r^2_{D^*_0}),
\]

(33)

\[
L_e \rightarrow L_e^2 = r^2_b - (1 - x_2) \left[ 1 - x_3 (1 - r^2_{D^*_0}) - x_2 r^2_{D^*_0} - x_1 \right],
\]

(34)

\[
L_f \rightarrow L_f^2 = x_2 \left[ x_1 - x_3 (1 - r^2_{D^*_0}) - x_2 r^2_{D^*_0} \right].
\]

(35)
For the decays $B_s \to D^{*0} V$, their amplitudes can be obtained from the ones of decays $B_s \to D^{*0} P$ with following substitutions:

$$
\phi_p \to \phi_V, \phi_P^1 \to \phi_V^s, \phi_P^t \to \phi_V^t, \tau_p \to -\tau_V, f_P \to f_V.
$$

Combining these amplitudes, one can ease to write out the total decay amplitude of each considered channel:

$$
A(B_s^0 \to D_{s0}^{*+} \pi^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* (F_{B_s^0 \to D_{s0}^*} a_1 + M_{B_s^0 \to D_{s0}^*} C_1),
$$

$$
A(B_s^0 \to D_{s0}^{*+} K^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (F_{B_s^0 \to D_{s0}^*} a_1 + M_{B_s^0 \to D_{s0}^*} C_1 + M_{ann}^2 C_2 + M_{ann}^2 a_2),
$$

$$
A(B_s^0 \to D_{s0}^{*+} \eta^-) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^* (M_{ann}^2 C_2 + M_{ann}^2 a_2),
$$

where $\eta_{nn} = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $\eta_{ss}$. The physical states $\eta$ and $\eta'$ can be related to these two flavor states $\eta_{nn}$ and $\eta_{ss}$ through the following mixing mechanism:

$$
\begin{pmatrix}
|\eta\rangle \\
|\eta'\rangle \\
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi \\
\end{pmatrix} \begin{pmatrix}
|\eta_{nn}\rangle \\
|\eta_{ss}\rangle \\
\end{pmatrix},
$$

with the mixing angle $\phi = 39.3^\circ \pm 1.0^\circ$ \cite{41}. The formulae for the $B_s \to D^{*0} V$ can be obtained through the following substitutions in Eqs.\cite{37}-\cite{45},

$$
\pi^\pm \to \rho^\pm, \quad \pi^0 \to \rho^0, \omega, \quad K \to K^*, \quad \eta_{ss} \to \phi.
$$

### III. THE NUMERICAL RESULTS AND DISCUSSIONS

We use the following input parameters in the numerical calculations \cite{28, 42}:

$$
f_B = 190MeV, f_{B_s} = 230MeV, M_B = 5.28GeV, M_{B_s} = 5.37GeV,
$$

$$
\tau_B^\pm = 1.638 \times 10^{-12}s, \tau_{B^0} = 1.519 \times 10^{-12}s, \tau_{B_s} = 1.512 \times 10^{-12}s,
$$

$$
M_W = 80.42GeV, M_{D^{*0}} = 2.3177GeV, f_{D^{*0}} = (225 \pm 25)MeV.
$$
TABLE I: Branching ratios ($\times 10^{-4}$) of the CKM favored (Type I) decays obtained in the pQCD approach (This work), where the errors for these entries correspond to the uncertainties in the $w_0 = 0.4 \pm 0.04(0.5 \pm 0.05)$ for $B(B_s)$ meson, the hard scale $t$ varying from 0.75$t$ to 1.25$t$, and the CKM matrix elements. In Ref. [28], the branching ratios are calculated in the factorization assumption (FA) with the form factors obtained in the light cone sum rules (LCSR). We also list the results given by the relativistic quark model (RQM) [29] and the nonrelativistic quark model (NRQM) [30], respectively.

| Modes                  | This work     | LCSR [28] | RQM [29] | NRQM [30] |
|------------------------|---------------|-----------|----------|-----------|
| $\bar{B}_s \rightarrow D^{*+}_{s0}(2317)\pi^-$ | $5.49^{+2.64+0.41+0.35}_{-1.68-0.27-0.35}$ | $5.22^{+2.5}_{-2.1}$ | 9 | 10 |
| $\bar{B}_s \rightarrow D^{*+}_{s0}(2317)^+K^-$ | $0.51^{+0.06+0.01+0.01}_{-0.04-0.01-0.01}$ | $0.44^{+0.2}_{-0.2}$ | 0.7 | 0.9 |
| $\bar{B}_s \rightarrow D^{*+}_{s0}(2317)^+\rho^-$ | $17.7^{+8.5+1.3+1.2}_{-5.3-0.8-1.1}$ | $13^{+6}_{-5}$ | 22 | 27 |
| $\bar{B}_s \rightarrow D^{*+}_{s0}(2317)^+K^{*-}$ | $1.01^{+0.44+0.06+0.05}_{-0.31-0.06-0.07}$ | $0.8^{+0.4}_{-0.3}$ | 1.2 | 16 |
| $\bar{B}_s \rightarrow D^{*+}_{s0}(2317)^+K^{-}$ | $0.18^{+0.06+0.01+0.01}_{-0.04-0.01-0.01}$ | $0.14^{+0.02+0.01}_{-0.07+0.02+0.01}$ | – | – |
| $\bar{B}_s \rightarrow D^{*+}_{s0}(2317)^+\pi^0$ | $0.25^{+0.06-0.02}_{-0.06-0.02}$ | – | – |

For the CKM matrix elements, we adopt the Wolfenstein parametrization with values from Particle Data Group (PDG) [42] $A = 0.811 \pm 0.026$, $\lambda = 0.22506 \pm 0.00050$, $\rho = 0.124^{+0.019}_{-0.018}$ and $\eta = 0.356 \pm 0.011$.

In the $B(s)$-rest frame, the decay rates of $B(s) \rightarrow D^{*+}_{s0}(2317)P(V)$ can be written as

$$BR(B(s) \rightarrow D^{*+}_{s0}(2317)P(V)) = \frac{\tau_{B(s)}}{16\pi M_B} (1 - r^2_{D^{*+}_{s0}}) A,$$

where $A$ is the total decay amplitude of each considered decay, which has been listed in Eqs. (37)- (45). The branching ratios for the CKM favored (Type I) decays are given in Table I where one can find that our predictions are consistent well with those calculated in the light cone sum rules approach within errors. While our predictions are smaller than the results given by the relativistic quark model (RQM) [29] and the nonrelativistic quark model (NRQM) [30], respectively. Especially, for the pure annihilation decay $\bar{B}_s \rightarrow D^{*+}_{s0} K^{-}$, whose branching fraction reaches up to $10^{-3}$ predicted by NRQM approach, it seems too large to be acceptable.

The Belle Collaboration has measured the product of the branching fractions $Br(\bar{B}^0 \rightarrow D^{*+}_{s0}(2317)^+K^-) \times Br(D^{*+}_{s0}(2317)^+ \rightarrow D^+_s\pi^0)$ $^2$, which is given as $(5.3^{+1.5}_{-1.3} \pm 0.7 \pm 1.4) \times 10^{-5}$ [43]. After rescaling the branching ratio of the decay $D^+ \rightarrow \phi\pi^+$, PDG reported $Br(\bar{B}^0 \rightarrow D^{*+}_{s0}(2317)^+K^-) \times Br(D^{*+}_{s0}(2317)^+ \rightarrow D^+_s\pi^0) = (4.2^{+1.4}_{-1.3} \pm 0.4) \times 10^{-5}$ [42]. Then the Belle Collaboration improved the measurement for the decay $\bar{B}^0 \rightarrow D^{*+}_{s0}(2317)^+K^-$ and renewed the branching ratio as $(3.3^{+0.6}_{-0.7} \pm 0.7) \times 10^{-5}$ [44], where the authors concluded that the branching ratio for this pure annihilation decay is of the same order of magnitude as $Br(\bar{B}^0 \rightarrow D^+_sK^-)$, which is measured as $(2.7 \pm 0.5) \times 10^{-5}$ [42]. Although the decay

$^2$ Recently, the absolute branching fraction of $D^{*+}_{s0}(2317)^+ \rightarrow D^+_s\pi^0$ has been measured by the BESIII Collaboration as $1.00^{+0.00}_{-0.14} \pm 0.14$ [10].
$\bar{B}^0 \to D^*_{s0}(2317)^+K^-$ has not been measured accurately by experiment, we believe that our prediction is reasonable.

It is helpful to define the following ratios based on the factorization assumption:

$$R_1 = \frac{\text{Br}(\bar{B}_s^0 \to D^{* +}_{s0}\pi^-)}{\text{Br}(B^0_s \to D^{* +}_{s0}K^-)} \approx \frac{|V_{ud}f_{\pi}|}{|V_{us}f_K|}^2 \approx 12.4,$$  \hspace{1cm} (52)

$$R_2 = \frac{\text{Br}(\bar{B}_s^0 \to D^{* +}_{s0}\rho^-)}{\text{Br}(B^0_s \to D^{* +}_{s0}K^*)} \approx \frac{|V_{ud}f_{\rho}|}{|V_{us}f_{K^*}|}^2 \approx 17.4,$$  \hspace{1cm} (53)

which are consistent with the results given by our predictions.

In the following, we list the branching ratios for the CKM suppressed decays $B_s \to D^*_{s0}(2317)P$ as following:

$$\text{Br}(\bar{B}_s^0 \to D^*_{s0}(2317)^-K^+) = (6.86^{+2.60+0.29+0.45}_{-1.79-0.20-0.43}) \times 10^{-6},$$  \hspace{1cm} (54)

$$\text{Br}(\bar{B}_s^0 \to D^*_{s0}(2317)^-\pi^+) = (6.91^{+2.93+0.45+0.43}_{-1.95-0.44-0.30}) \times 10^{-6},$$  \hspace{1cm} (55)

$$\text{Br}(B^- \to D^*_{s0}(2317)^-\pi^0) = (3.72^{+1.59+0.23+0.25}_{-1.04-0.16-0.23}) \times 10^{-6},$$  \hspace{1cm} (56)

$$\text{Br}(B^- \to D^*_{s0}(2317)^-\eta) = (6.30^{+2.17+0.41+0.44}_{-1.52-0.40-0.29}) \times 10^{-7},$$  \hspace{1cm} (57)

$$\text{Br}(B^- \to D^*_{s0}(2317)^-\eta') = (4.17^{+1.50+0.33+0.27}_{-1.05-0.18-0.27}) \times 10^{-7},$$  \hspace{1cm} (58)

$$\text{Br}(\bar{B}_s^0 \to D^*_{s0}(2317)^-K^+) = (5.99^{+0.56+0.33+0.30}_{-0.60-0.31-0.38}) \times 10^{-9},$$  \hspace{1cm} (59)

$$\text{Br}(B^- \to D^*_{s0}(2317)^-K^0) = (0.82^{+0.17+0.15+0.06}_{-0.15-0.09-0.05}) \times 10^{-9},$$  \hspace{1cm} (60)

where the first uncertainty comes from the $w_b = 0.4 \pm 0.04(0.5 \pm 0.05)$ for $B(B_s)$ meson, the second error is from the hard scale-dependent uncertainty, which we vary from 0.75$t$ to 1.25$t$, and the third one is from the CKM matrix elements. For these CKM suppressed decays, the factorizable emission diagrams (where $D^*_{s0}(2317)$ meson is emitted from the weak vertex) are the color favored ones with the Wilson coefficients $a_1 = C_2 + C_1/3$, while the nonfactorizable emission diagrams are highly suppressed by the Wilson coefficient $C_1/3$. This means that the dominant amplitudes are nearly proportional to the product of $D^*_{s0}(2317)$ meson decay constant and a $B$ to light meson form factor. Unfortunately, the decay constant of the scalar meson for vector current is small, which is defined as $\langle 0|s\gamma_{\mu}c|D^*_{s0}(P)\rangle = f_{D^*_{s0}}P_{\mu}$. This vector current decay constant $f_{D^*_{s0}}$ can be related with the scale-dependent scalar one $\tilde{f}_{D^*_{s0}}$ by equation of motion

$$f_{D^*_{s0}} = \frac{(m_c - m_s)}{m_{D^*_{s0}}}\tilde{f}_{D^*_{s0}},$$  \hspace{1cm} (61)

where $m_c(s)$ is the current quark $c(s)$ mass and $\tilde{f}_{D^*_{s0}}$ defined as $\langle |\bar{s}c|D^*_{s0}(P)\rangle = m_{D^*_{s0}}\tilde{f}_{D^*_{s0}}$. $\tilde{f}_{D^*_{s0}} = (225\pm 25)$ MeV has been determined from the two-point QCD sum rules. If taking $m_c = 1.275$ GeV, $m_s = 0.096$ GeV, $m_{D^*_{s0}} = 2.3177$ GeV \cite{12}, one can find that $f_{D^*_{s0}} = 0.11$ GeV. So we can speculate that the branching ratio of the decay $\bar{B}^0 \to D_s^0(2317)^-\pi^+$ should be much larger than that of $\bar{B}^0 \to D^*_{s0}(2317)^-\pi^+$. This is indeed the case: If we replace the decay constant, the mass and the wave functions of the scalar meson $D^*_{s0}$ with those of the pseudoscalar meson $D^-$ in the calculation program, we find that the branching ratio $\text{Br}(\bar{B}^0 \to D_s^0(2317)^-\pi^+) = (27.6^{+8.24}_{-6.65}) \times 10^{-5}$, which is consistent with the current experimental value $(21.6 \pm 2.6) \times 10^{-5}$ \cite{42} within errors. Since the form factors of $B \to V$ are a litter
TABLE II: The amplitudes from the nonfactorizable and factorizable annihilation Feynman diagrams, which denote as NFAA and FAA, respectively. For each amplitude, the value has been given, together with the corresponding Wilson coefficient (WC).

| Modes                  | NFAA | FAA      | Total |
|------------------------|------|----------|-------|
|                        | WC   | value    | WC    | value   |       |
| $B^- \to D_{s0}^+ K^0 (\times 10^5)$ | C1   | $-3.99 - i2.75$ | $C_1 + C_1/3$ | $1.72 + i0.57$ | $-2.13 + i3.31$ |
|                        |      | $-1.02 - i4.60$ |        | $1.38 + i0.87$ | $-0.50 + i3.72$ |
| $\bar{B}^0 \to D_{s0}^+ K^+ (\times 10^5)$ | C2   | $8.50 - i8.22$ | $C_1 + C_2/3$ | $-0.50 - i0.35$ | $8.00 - i8.57$ |
|                        |      | $-8.80 - i13.54$ |        | $0.07 - i0.11$ | $-0.26 - i1.15$ |
| $\bar{B}^0 \to D_{s0}^+ K^+ (\times 10^5)$ | C2   | $4.46 - i4.44$ | $C_1 + C_2/3$ | $-0.32 - i0.16$ | $4.14 - i4.60$ |
|                        |      | $4.83 - i5.47$ |        | $-0.18 - i0.06$ | $4.65 - i5.53$ |

large, one can expect that these tree operator dominant decays $B \to D_{s0}^+ V$ have a larger branching ratios than those of $B \to D_{s0}^- P$ decays. While this conclusion is not satisfied to the pure annihilation type decays.

For the decays $\bar{B}^0 \to D_{s0}^+ \pi^+$, $B^- \to D_{s0}^- \pi^0$, their branching ratios are sensitive to the form factor $B \to \pi$. If using the Gegenbauer coefficients $a_3^\pi = 0.44, a_4^\pi = 0.25$, we will get a reasonable form factor $F^{B \to \pi}(0) = 0.22$, which is larger than $F^{B \to \pi}(0) = 0.18$ obtained by using the updated Gegenbauer coefficients $a_3^\pi = 0.115, a_4^\pi = -0.015$. Corresponding to the smaller form factor, the branching ratios of the decays $\bar{B}^0 \to D_{s0}^- \pi^+, B^- \to D_{s0}^- \pi^0$ will have a noticeable decrement and become $Br(\bar{B}^0 \to D_{s0}^- \pi^+) = 3.78 \times 10^{-6}$, $Br(B^- \to D_{s0}^- \pi^0) = 2.05 \times 10^{-6}$. It is similar for the decays $B^- \to D_{s0}^- \eta(0)$. While for the CKM favored decay $\bar{B}^0 \to D_{s0}^+ \pi^-$, the branching ratio is not sensitive to the Gegenbauer coefficients for $\pi$ meson wave functions. The difference of the branch ratios by using these two group Gegenbauer coefficients is only about 4%.

Similarly, the branching ratios of the decays $B_{(s)} \to D_{s0}^*(2317)^- V$ are calculated as:

$$Br(\bar{B}_{s}^0 \to D_{s0}^*(2317)^- K^{*+}) = (7.97_{+2.56+0.49+0.52}^{+2.56+0.49+0.52} \times 10^{-6})$$

$$Br(B_{s}^0 \to D_{s0}^*(2317)^- \rho^+) = (1.61_{+0.54+0.51}^{+0.54+0.51} \times 10^{-5})$$

$$Br(B^- \to D_{s0}^*(2317)^- \rho^-) = (8.70_{+3.42+0.51}^{+3.42+0.51} \times 10^{-6})$$

$$Br(B^- \to D_{s0}^*(2317)^- \omega^-) = (5.44_{+1.86+0.36}^{+1.86+0.36} \times 10^{-6})$$

$$Br(B^- \to D_{s0}^*(2317)^- \phi^-) = (1.74_{+0.50+0.11}^{+0.50+0.11} \times 10^{-8})$$

$$Br(B_{s}^0 \to D_{s0}^*(2317)^- K^{*+}) = (6.38_{+1.25+0.41}^{+1.25+0.41} \times 10^{-9})$$

$$Br(B^- \to D_{s0}^*(2317)^- K^{*0}) = (0.73_{+0.19+0.04}^{+0.19+0.04} \times 10^{-9})$$

where the errors are the same as ones given in Eqs.(64)-(66).

The pure annihilation decays have the smallest branching ratios both for the CKM allowed and the CKM suppressed ones. In Table II, we list the contributions from the nonfactorizable annihilation amplitudes (NFAA) and the factorizable annihilation amplitudes (FAA), where the Wilson coefficients have been included. One can find that the nonfactorizable contributions are more important than the factorizable ones. Even the FAA with the large Wilson Coefficient ($a_1 = C_2 + C_1/3$) also has smaller value...
because of the destructive interference between the pair of factorizable annihilation Feynman diagrams in each channel, such as Fig.1(g) and Fig.1(h). For example, in the decay $B^− \rightarrow D_{s0}^* K^0$ both of the two factorization annihilation amplitudes have large imaginary parts in magnitude but with opposite signs: One is $3.71 \times 10^{-5}$, the other is $-4.28 \times 10^{-5}$, so the imaginary part of the total FAA becomes $-5.7 \times 10^{-6}$ given in Table II.

IV. CONCLUSION

In summary, we investigate the branching ratios of the decays $B_s \rightarrow D_{s0}^*(2317)\rho(V)$ within pQCD approach by assuming $D_{s0}^*(2317)$ as a $\bar{c}s$ scalar meson. For the CKM favored decays, their branching fractions are larger than $10^{-5}$, even for the pure annihilation type channels. Our predictions are consistent well with the results given by the light cone sum rules approach. So we consider that these decays can be measured at the running LHCb and the forthcoming SuperKEKB. We may shed light on the nature of the meson $D_{s0}^*(2317)$ by combining with the future data and the theoretical predictions: If they are consistent with each other, one can conclude that this charmed-strange meson is composed (mainly) of $\bar{c}s$. Otherwise, some other component or the $DK$ threshold effect in meson-meson scattering may be important to the dynamic mechanism for the $D_{s0}^*(2317)$ production. As for the CKM suppressed decays, their branching ratios are usually at $10^{-6}$ order. While the branching fraction for the decay $B^0 \rightarrow D_{s0}^*(2317)^-\rho^+$ reaches up to $1.61 \times 10^{-5}$. As to the pure annihilation type decays $B^- \rightarrow D_{s0}^*(2317)^-\phi$, $B^0 \rightarrow D_{s0}^*(2317)^-K^+(K^{*+})$ and $B^- \rightarrow D_{s0}^*(2317)^-K^0(K^{*0})$, their branching fractions drop to as low as $10^{-10} \sim 10^{-9}$. Here the decay $B^- \rightarrow D_{s0}^*(2317)^-\phi$ has the larger branching ratio because of the large CKM matrix element $V_{cs}$. The branching ratios of the decays $\bar{B}^0 \rightarrow D_{s0}^*(2317)^-K^+(K^{*+})$ are larger than those of $B^- \rightarrow D_{s0}^*(2317)^-K^0(K^{*0})$ because of owning the larger nonfactorizable annihilation amplitudes. For these pure annihilation type decays, the magnitudes of the nonfactorizable amplitudes are generally larger than those of factorization amplitudes. It is because there exists the destructive interference between the pair of factorization amplitudes in each decay mode. If this type of pure annihilation decay is observed by the future experiments with larger branching fractions than our predictions, it may indicate that some new physics contributes to these decays.

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Appendix A

\[ t_a = \max(\sqrt{x_2 m_B}, 1/b_1, 1/b_2), \]  
\[ t_b = \max(\sqrt{x_1 m_B}, 1/b_1, 1/b_2), \]  
\[ t'_a = \max(\sqrt{x_3 (1 - r_{D_{so}}^2)} m_B, 1/b_1, 1/b_3), \]  
\[ t'_b = \max(\sqrt{x_1 (1 - r_{D_{so}}^2)} m_B, 1/b_1, 1/b_3), \]  
\[ t_{c,d} = \max(\sqrt{x_1 x_2 m_B}, \sqrt{|A_{c,d}^2|} m_B, 1/b_1, 1/b_2), \]  
\[ t'_{c,d} = \max(\sqrt{x_1 x_3 (1 - r_{D_{so}}^2)} m_B, \sqrt{|A_{c,d}^2|} m_B, 1/b_1, 1/b_3), \]  
\[ t_{e,f} = \max(\sqrt{x_2 (1 - x_3) (1 - r_{D_{so}}^2)} m_B, \sqrt{|L_{e,f}^2|}, m_B, 1/b_1, 1/b_3), \]  
\[ t'_{e,f} = \max(\sqrt{x_2 x_3 (1 - r_{D_{so}}^2)} m_B, \sqrt{|L_{e,f}^2|}, m_B, 1/b_1, 1/b_3), \]  
\[ t_g = \max(\sqrt{(1 - x_3) (1 - r_{D_{so}}^2)} m_B, 1/b_2, 1/b_3), \]  
\[ t_h = t'_g = \max(\sqrt{x_2 (1 - r_{D_{so}}^2)} m_B, 1/b_2, 1/b_3), \]  
\[ t'_h = \max(\sqrt{x_3 (1 - r_{D_{so}}^2)} m_B, 1/b_2, 1/b_3), \]  

where the definitions of \( A_{c,d}^{(i)} \), \( L_{e,f}^{(i)} \) are listed in Eqs. (16), (17), (25), (26), (32)-(35). And the \( S_j(t) (j = B, D_{so}, P) \) functions in Sudakov form factors in Eq. (11), Eq. (12), Eq. (21) and Eq. (22) are given as

\[ S_B(t) = s(\frac{x_1 m_B}{\sqrt{2}}, b_1) + 2 \int_{1/b_1}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \]  
\[ S_{D_{so}}(t) = s(\frac{x_3 m_B}{\sqrt{2}}, b_3) + 2 \int_{1/b_3}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \]  
\[ S_P(t) = s(\frac{x_2 m_B}{\sqrt{2}}, b_2) + s((1 - x_2) \frac{m_B}{\sqrt{2}}, b_2) + 2 \int_{1/b_2}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \]

where the quark anomalous dimension \( \gamma_q = -\alpha_s/\pi \), and the expression of the \( s(Q, b) \) in one-loop running coupling constant is used

\[ s(Q, b) = \frac{A^{(1)}_{Q, b}}{2\beta_1} \ln(\frac{\hat{Q}}{\hat{b}}) - \frac{A^{(1)}_{Q, b}}{2\beta_1} (\hat{Q} - \hat{b}) + \frac{A^{(2)}_{Q, b}}{4\beta_1^2} (\frac{\hat{Q}}{\hat{b}} - 1) \]

\[ - \left[ \frac{A^{(2)}_{Q, b}}{4\beta_1^2} - \frac{A^{(1)}_{Q, b}}{4\beta_1} \ln(\frac{e^{2\gamma_q - 1} - 1}{2}) \right] \ln(\frac{\hat{Q}}{\hat{b}}), \]  

(A15)
with the variables are defined by $\hat{q} = \ln\left[\frac{Q}{(\sqrt{2}\Lambda)}\right], \hat{q} = \ln\left[\frac{1}{(b\Lambda)}\right]$ and the coefficients $A^{(1,2)}$ and $\beta_1$ are

$$\beta_1 = \frac{33 - 2n_f}{12}, A^{(1)} = \frac{4}{3};$$

$$A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln\left(\frac{1}{2e^{\gamma_E}}\right),$$

(A16)

(A17)

here $n_f$ is the number of the quark flavors and $\gamma_E$ the Euler constant.

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