Review on numerical analysis of electromagnetic characteristics for ferromagnetic wear debris

Tao Chen†, Li-yong Wang†, Yuhai Gu†, Changliang Tang†
†Lab of Modern Measurement and Control Technology, Ministry of Education, Beijing Information and Science Technology University, Beijing, People’s Republic of China
E-mail: chentao@bistu.edu.cn

Abstract: Wear is the main influential reason for the reliability and service life of machinery and the inductive monitoring is the most research and application of online ferromagnetic debris detection for its unique characteristics. In the electromagnetic characteristics analysis of the wear debris, the numerical analysis method can directly solve the problem of electromagnetic field in the form of value, program instead of analytic form. Based on the current research on numerical analysis of electromagnetic characteristics for ferromagnetic debris, the solved problems of numerical analysis are analysed. The core and theoretical basis of numerical analysis of electromagnetic characteristics are studied and assumptions and boundary conditions are provided. Then the present problems and future development trend of the numerical analysis of electromagnetic characteristics for ferromagnetic wear debris are discussed.

1 Introduction

Wear is the main influential reason for the reliability and service life of machinery and wear debris carries a lot of information about friction pair in equipment, which can reflect the internal wear status of equipment (mainly about wear degree, location and type) [1]. Much research work indicates that the solid wear debris in equipment lubrication oil is the main reason for the wear failure of machinery and equipment. The studies of Canada national research council shows that the wear failure induced by debris accounted for 82%, while non-debris induced wear faults only accounted for 18% of the total wear failure of mechanical equipment [2]. The bigger the debris diameter is, the more serious the faults induced by the debris becomes. Among the wear debris in the lubricating oil of equipment, the ferromagnetic metal wear debris plays the most important role. British Professor E.C.Fitch proved through research that the size of ferromagnetic metal wear debris decreases by five times, and the possibility of wear failure is reduced by 50 times [3, 4].

The inductive monitoring is the most research and application of online ferromagnetic debris detection [2, 4–7]. Compared with the magnetic collection, capacitance, resistance monitoring, optical, and acoustic and methods of X ray, the inductive monitoring can count the amount of wear debris and distinguish the material of wear debris. The inductive monitoring is not easy to suffer the interference of external temperature changes and vibrations, not easily affected by the oil bubble and the interference of impurity. Due to the advantages of inductive monitoring, it received wide attention and recognition in the field of oil monitoring technology. In the inductive monitoring of ferromagnetic wear debris, the analysis of electromagnetic characteristics of wear debris plays an important role. The inductive monitoring principle of ferromagnetic wear particle is that due to the effect of electromagnetic induction when the wear debris going into the sensor detecting coil, the coil magnetic field distribution is changed and the magnetic flux is changed by the change of magnetic field, and then the magnetic field changes are changed into the sensor output voltage signal, while the phase and amplitude of the signal, respectively, represent the abrasive material and size. By detecting the electromagnetic characteristics can understand the characteristics of the wear debris [8, 9].

In the electromagnetic characteristics analysis of wear debris, the numerical analysis method can directly solve the problem of electromagnetic field in the form of value, program instead of analytic form. The numerical analysis method usually uses difference instead of differential, finite summation instead of integral, and change the electromagnetic characteristics analysis problem into solving the differential equation or algebraic equation. The numerical analysis can simplify the operation greatly and give the potential distribution of electromagnetic field clearly, while the analytical solving process is very complex, and the analytical solution is ultimately expressed through the series form which could not directly reflect the electromagnetic field distribution. The numerical method has the advantage of universality. The boundary conditions, electrical structure, and excitation characteristics of the wear debris need not to be programmed into the basic program, but input by the user or input through the graphical interface; the characteristics analysis of wear debris can solve practical problems without the knowledge of highly specialised electromagnetic field theory, mathematics, and numerical techniques. The numerical analysis method to analyse the electromagnetic characteristics of the wear debris can obtain the results directly and can provide theoretical basis for the wear debris monitoring and structure optimisation of the coil of the inductive sensor.

2 Solved problem of numerical analysis of electromagnetic characteristics

2.1 Solved problem

The solved problem of numerical analysis of electromagnetic characteristics analysis includes the positive problem and inverse problem. The positive problem of numerical analysis in inductive wear debris monitoring is to reveal the electromagnetic characteristics overtime and space distribution in electromagnetic fields [10–16].

The inverse problem is to optimise the inductive wear debris monitoring system according to the ideal index or parameter [17]. At present, the solution of inverse problem is to change it into a series of positive problems, and then adopt a certain optimisation method to achieve the end optimisation design by iterative solution. In the inverse problem, numerical method makes the analysis of the electromagnetic field problems from the classic analytical method into the numerical analysis method of discrete system. The electromagnetic field of computer-aided analysis of
discrete numerical method makes the difficult complex analytic problems of electromagnetic field easy to be solved with high accuracy. Compare with relative positive problem, the inverse problem is solved with large computation and occupies more computer memory and CPU time.

2.2 Numerical analysis of the solution process

The numerical method can be used to describe the electromagnetic field problem directly in the form of numerical and program. In the numerical method, the difference is usually replaced by differential, and the integral is replaced by finite sum, and the problem is solved by solving the difference equation or algebraic equation. The whole numerical analysis processes of the positive and inverse problem are shown in Figs. 1 and 2 relatively.

From the process above, we can find that the various numerical calculation methods are the core of solved problem. The numerical analysis of electromagnetic characteristics for ferromagnetic wear debris also required electromagnetic field theory, and the solution process still need to adopt the appropriate idealised assumptions, accurate the definite condition (mainly includes initial conditions and boundary conditions).

3 Core and theoretical basis of numerical analysis of electromagnetic characteristics

3.1 Numerical analysis method

At present, the methods commonly used in the numerical analysis of electromagnetic characteristics in wear debris monitoring are finite difference method and finite element method applied to differential equation model. In principle, the two methods can transform a continuous domain into a finite partition and then solve a series of algebraic equations instead of solving differential or integral equations. Using the computer to solve the numerical solution, the accuracy of arbitrary requirements can be achieved theoretically.

Finite difference method also known as difference method is the first method to use in the electromagnetic field numerical calculation method. Finite difference method was put forward at the end of the nineteenth century. The finite difference method is widely used in the field of electromagnetic field numerical analysis, static electromagnetic field, and sine steady-state time-varying electromagnetic field.

The concept of finite difference method is clear and the computation is simple. Finite difference method is an approximate
solution based on differential principle and solving differential equations by approximating with difference equations. When solving the electromagnetic characteristics, first, the solution field is divided into many grids and nodes, and then finite differences approximate the derivatives, so the field of partial differential equation is changed into difference equations with variable of the magnetic vector potential (linear algebraic equations), finally, the discrete nodes and magnetic potential can be obtained by numerical solution of the equations. Moreover, if the selection of discrete point is sufficiently dense, it can obtain the desired calculation precision for the electromagnetic characteristics analysis of the wear debris detection according to the current capacity and speed of the computer.

Finite element method is a method of discretisation to obtain the approximate solution based on the variational principle. By using finite element method, the solved field is divided into a finite number of units, and the variational problem is transformed into the extremum problem of multivariate function by region partition and slice interpolation.

The finite element method adopts the principle of physical discretisation and piecewise polynomial interpolation, which has wide adaptability to material, boundary, and excitation. The mathematical equation is solved by solving algebraic equations, so it is very easy to use and has high accuracy in calculation using the finite element method. The convergence of the method and the symmetry of the coefficient matrix are guaranteed.

Meanwhile, finite element method has drawn and developed the flexibility and the adaptability of the finite difference method on the discrete processing and maintained the sparse of the coefficient matrix in the difference method, which greatly saves the computer capacity in analysing the electromagnetic characteristics of wear debris monitoring.

3.2 Basic theory of numerical analysis of electromagnetic characteristics

3.2.1 Maxwell's equations and its derived equations: In order to analyse the electromagnetic characteristics of wear debris, the mathematical formula of electromagnetic field involved needs to be studied in the wear debris monitoring. The fundamental equations of electromagnetic fields are Maxwell's equations, and Maxwell's equations are the starting point of the numerical analysis of the electromagnetic characteristics of wear debris. The differential form of the electromagnetic field is:

$$\nabla \times \mathbf{H} = J_s + J + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = 0$$

$$\nabla \times \mathbf{D} = \rho$$

In the formula above, $\mathbf{B}$ is the magnetic induction intensity, $\mathbf{H}$ is the magnetic field intensity, $\mathbf{D}$ is the electric displacement vector, $\mathbf{E}$ is the electric field intensity.

The above equation is not complete; there are 16 unknown scalars and only seven independent scalar equations. So the state equation that reflects the structure relationship of the media field needs to be added to get solution. The relationship of the media field is shown in the below formula.

$$D = \varepsilon \mathbf{E}$$

$$B = \mu \mathbf{H}$$

$$J = \sigma \mathbf{E}$$

In the formula above a set of parameters ($\varepsilon, \mu, \sigma$) describing the macroscopic electromagnetic properties of the wear debris are called dielectric constants, magnetic conductivity and conductivity. The above seven equations form the basis of the strict macroscopic description of the electromagnetic phenomenon of wear debris.

When the displacement current density $\partial \mathbf{D}/\partial t$ in the wear debris monitoring is much less than that of the conduction current density $J$, the effect of the displacement current density is ignored, so the magnetic field is called magneto-quasi-static field (MQS). The MQS field is time-varying and has some properties of static field. The MQS field satisfies the relationship below:

$$\nabla \times \mathbf{H} = J_s + J$$

$$\nabla \times \mathbf{B} = 0$$

In other words, under the premise of ignoring the displacement current, the MQS field has no source and rotation with constant magnetic field.

The numerical analysis of wear debris characteristics, it is not convenient to solve Maxwell's equations directly, and it is usually necessary to introduce different electromagnetic quantities to establish partial differential equations. The magnetic vector potential $A$ and the scalar magnetic potential $\phi_m$ are designed to analyse the convenience of the magnetic field. The vector magnetic potential of $A$ is introduced according to $\nabla \times \mathbf{B} = 0$, and scalar magnetic potential of $\phi_m$ is introduced according to $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$.

With the help of Lorentz gauge and Cullen specification, the constraint equations of the area between wear debris and the coil of the air gap without source current and the induced current is shown below.

$$\Delta \phi_m = 0$$

The constraint equation of coil area with the source current is shown below.

$$\mu^{-1} \Delta \times (\Delta \times \mathbf{A}) = J_s$$

where $J_s$ is the source current density.

The vector magnetic potential of the magnetic field can be calculated by the following equation.

$$\mathbf{B} = \nabla \times \mathbf{A}$$

In the absence of current distribution, Laplace's equation is shown in equation below.

$$\nabla^2 \phi_m = 0$$

The vector magnetic potential of the magnetic field can be calculated by the following equation.

$$\mathbf{B} = - \mu_0 \nabla \phi_m$$

3.2.2 Magnetisation of ferromagnetic wear debris: Since almost all the mechanical friction pair containing iron (Fe), the wear debris are generally divided into ferromagnetic wear debris (such as iron, cobalt, nickel, and the corresponding compounds debris) and non-ferromagnetic wear debris (such as aluminium, copper, lead, and Babbitt metal debris etc.) on the engineering practice of wear debris monitoring. In the application, effects of ferromagnetic wear debris and non-ferromagnetic wear debris on the electromagnetic field are very different as shown in Table 1. The relative permeability of ferromagnetic wear debris is very large, which can form huge magnetisation intensity under the effect of external magnetic field. The relative permeability of non-ferromagnetic abrasive particles is very small (the value is about 1), and the magnetic field has little effect on its magnetisation [18].

In the wear debris monitoring, the ferromagnetic wear debris are in the quasi-static field of the AC coil, which is immediately coupled with the magnetic field, and also produces eddy current. The disturbance of ferromagnetic wear debris to the original field...
is derived from the vortex field of wear debris and the magnetisation intensity of wear debris. As the wear debris is very tiny (usually only tens to hundreds of microns), the disturbance to the original field is very small. In general, for non-ferromagnetic wear debris, its perturbation of magnetic field is mainly on the eddy current, and for ferromagnetic wear debris due to its relative permeability very big, its perturbation of magnetic field is mainly on the internal magnetisation.

The magnetisation of ferromagnetic wear debris is the effect of magnetic field on magnetisation of ferromagnetic wear debris and the magnetoelectrics properties of wear debris have changed by the magnetisation intensity. Magnetisation of ferromagnetic wear debris can be described by magnetic dipole model and its magnetisation degree can be expressed by magnetisation intensity. The relationship between magnetisation intensity $M$ and the external magnetic field intensity $H$ is shown below.

$$M = \chi H$$  \hspace{1cm} (15)

In the formula $\chi$ is the magnetic susceptibility.

The magnetic induction intensity $B$ is connected with the magnetisation intensity $M$ and the magnetic field intensity $H$ as shown below.

$$M = \chi H$$  \hspace{1cm} (16)

$$B = \mu_0(H + M) = \mu_0(1 + \chi)H = \mu_0\mu_H = \mu H$$  \hspace{1cm} (17)

In the formula $\mu$ is permeability of the wear, $\mu_H$ is the relative permeability of the wear, and $\mu_0$ is the permeability in vacuum.

The internal magnetisation field is directly proportional to the magnetisation intensity of the wear debris in the case of the wear volume is fixed. The internal magnetic field of the abrasive particles is shown below.

$$H_{in} = -NM$$  \hspace{1cm} (18)

$$B_{in} = \mu_HH_{in} + \mu_0M$$  \hspace{1cm} (19)

In the formula, $H_{in}$ is the magnetic field intensity inside the wear debris; $B_{in}$ is the magnetic induction intensity inside the wear debris; $N(0 < N < 1)$ is the demagnetised factor of wear Debris. The magnetisation field is determined by demagnetisation factor and magnetisation intensity.

### 4 Assumptions and boundary conditions

#### 4.1 Assumptions of the characteristics analysis

In numerical analysis of electromagnetic characteristics for ferromagnetic wear debris, the alternating magnetic field in the sensor coil satisfies the condition of quasi-static field. In general sensor of inductive coil driven by sine excitation, and the driving frequency and the sensor size satisfy the MQS condition. That is the influence of the displacement current can be ignored when the time interval of the source change on the spot is much more than electromagnetic disturbance across by the physical wear debris monitoring system. Therefore, in time harmonic field, MQS field conditions can be expressed as:

$$L \times f = c$$  \hspace{1cm} (20)

In the formula, $L$ is the size of the physical system; $f$ is the frequency of the harmonic field, $c$ is the speed of light.

### 4.2 Boundary condition

In numerical analysis of electromagnetic characteristics for wear debris, Maxwell equations and definite condition (mainly including initial conditions and boundary conditions) constitute the solution problem and make the solution definite exist with existence, uniqueness of uniqueness and stability of stability.

The boundary condition is the condition that the electromagnetic field vectors on the interface of different media needs to satisfy and it is also the basic properties of electromagnetic field on the interface of different media. The analysis of the electromagnetic characteristics of the actual wear debris is in a certain physical space, which is composed of a variety of different media. Due to the mutation of the characteristic parameters on the boundary surface, the field also mutates on both sides of the interface. The differential form of Maxwell's equations is meaningless on both sides of the boundary, and boundary conditions must be applied.

For wear debris characteristics analysis of partial differential equation, the definite conditions can be divided into two kinds, one kind is to determine the field of initial state, known as the initial conditions, the other is to express field place in physical situation, known as boundary conditions. A universal law of differential equation without additional definite condition is called the fundamental equation. The fundamental equation is the basis to solve the problem, but cannot determine the specific physical process. The fundamental equations and the conditions of definite solutions as a whole are known as definite solutions.

For Laplace equation, the electrostatic field of magnetic potential, the boundary conditions can be divided into the following three categories as shown in Fig. 3.

(i) Dirichlet boundary condition is also known as the first boundary condition. In this condition the constant value of the position function is given on the entire boundary.

(ii) Newman boundary condition is also known as the second boundary condition. In this boundary the normal derivative of each point function is given, namely the rate of the boundary change is given.

(iii) Mixed boundary condition is the mixture of The Dirichlet boundary and the Newman boundary condition. In this condition the potential of each point is given a constant value of the position function and on a given part of the boundary and the normal derivative of each point function.
obtained a lot of application research results [19–21]. There are high precision and high efficiency.

5 Present problems and future development trend

The development of computer technology, the advent of high speed and large capacity computer as well as the enrichment and perfection of numerical calculation of electromagnetic fields have laid a foundation for the numerical analysis of the electromagnetic characteristics for wear debris in lubricating oil with high precision and high efficiency.

After development of more than 30 years, the numerical analysis of electromagnetic field for ferromagnetic wear debris has obtained a lot of application research results [19–21]. There are many problems in the numerical analysis at present, the main problems is summarised in the following aspects.

(i) At present in order to simplify the calculation, the analysis of electromagnetic characteristics for ferromagnetic debris is mostly based on the equivalent sphere radius, and analysis of most actual non-spherical wear debris (such as flake, ellipsoid etc.) is much less.

(ii) The actual factors such as the speed of abrasive movement, temperature, and vibration are less involved in the analysis of the electromagnetic characteristics for ferromagnetic wear debris.

(iii) The randomness characteristic of wear debris production and concentration distribution, and the location of wear debris in lubricating oil are less considered in the analysis of the electromagnetic characteristics of ferromagnetic wear debris at present.

In order to better reveal the electromagnetic characteristics of ferromagnetic wear debris, the future development trend of the numeric analysis includes:

(i) The electromagnetic characteristics analysis of debris with different length diameters (sheet, ellipsoid etc.) need to be studied by considering the difference of wear debris produced by different wear faults.

(ii) Electromagnetic field analysis need to be coupled with particle movement, vibration and thermal analysis to conduct wear debris analysis close to the actual conditions using the powerful coupling function of electromagnetic analysis software.

(iii) The lubricating oil environment need to be considered to conduct fluid-solid coupling analysis as the wear debris are located in field of the wear debris and the flow of the lubricating oil so as to reveal the electromagnetic characteristic of wear debris and provide basis for ferromagnetic wear debris monitoring.

6 Acknowledgments

The authors thank the co-author for the valuable discussion and recommendation. The paper is supported by fund program of KZ201611232032.

7 References

[1] Lee, J., Ni, J., Djurdjanovic, D.: ‘Intelligent prognostics tools and e-maintenance’, Comput. Ind., 2006, 57, pp. 476–489
[2] Miller, J.L., Kitalyevich, D.: ‘In-line oil debris monitor for aircraft engine condition assessment’. IEEE Aerospace Conf., Big Sky, MT, USA, 2000, pp. 49–56
[3] El-Thalji, I., Jantunen, E.: ‘Dynamic modelling of wear evolution in rolling bearings’, Tribol. Int., 2015, 84, pp. 90–99
[4] Schalcosky, D.C., Byington, C.S.: ‘Advances in real time oil analysis’, Pract. Oil Anal. Mag., 2000, 11, (2), pp. 28–34
[5] Lijuan, Q., Zhengnan, X., Zhang, X.: ‘Research progress of oil on-line monitoring technology’, Transducer Microsyst. Technol., 2015, 34, (4), pp. 4–7
[6] Poley, J.: ‘Oil analysis sensors’, Tribol. Lubr. Technol., 2008, 64, (3), pp. 78–79
[7] Xinping, Y., Yuelei, Z., Junhong, M.: ‘Development status and research spots of on-line oil monitoring technology’, Lubr. Eng., 2011, 36, (10), pp. 1–4
[8] Iwai, Y., Honda, T., Miyajima, T., et al.: ‘Quantitative estimation of wear amounts by real time measurement of wear debris in lubricating oil’, Tribol. Int., 2010, 43, (1–2), pp. 388–394
[9] Zha, X., Zhong, C., Zhu, J.: ‘Lubricating oil conditioning sensors for online machine health monitoring: a review’, Tribol. Int., 2017, 109, pp. 473–484
[10] Henneberg, M., Eriksen, R.L., Fich, J.: ‘Modelling and measurement of wear particle flow in a dual oil filter system for condition monitoring’, Wear, 2016, 362–363, pp. 153–160
[11] Li, D., Jiang, Z.: ‘A high throughput inductive pulse sensor for online oil debris monitoring’, Tribol. Int., 2011, 44, (2), pp. 175–179
[12] Yonghui, Y.: ‘Study on the online monitoring technology based on the method of inductance and optic-fiber transducer’ (Wuhan University of Technology, Wuhan, China, 2002)
[13] Xingming, Z.: ‘Study on metal particle magnetization in harmonic field and mechanism of microfluidic oil detection’ (Dalian Maritime University, Dalian, China, 2014)
[14] Chao, W., Changsong, Z., Biao, M.: ‘Simulation study on the characteristic of ferromagnetic wear debris in inductive wear debris’, Chin. J. Sci. Instrum., 2011, 32, (12), pp. 2774–2780
[15] Xingming, Z., Hongpeng, Z., Yuqing, S., et al.: ‘Research on output regularities of oil detection microfluidic chip based on FEM’, Ship Eng., 2014, 36, (1), pp. 59–62
[16] Yan, H., Zhang, Y.: ‘The design of an on-line monitoring sensor of wear mental particles and the analysis of its characteristic’, Chin. J. Sens. Actuator, 2002, 4, pp. 333–338
[17] Liu, C., Liang, M.: ‘Enhancement of oil debris sensor capability by reliable debris signature extraction via wavelet domain target and interference signal tracking’, Measurement, 2013, 46, pp. 1442–1453
[18] Daosheng, D., Kunming, Q.: ‘Ferromagnetism’ (Science Press, Beijing, China, 2002)
[19] Hongbo, F., Zhongbo, H., Yingtang, Z.: ‘Online monitoring sensor technology for equipment wear debris’ (National Defence Industry Press, 2013)
[20] Harvey, T.J., Wood, R.J.K., Powrie, H.E.G.: ‘Electrostatic wear monitoring of rolling element bearing’, Wear, 2007, 263, (7), pp. 1492–1501
[21] Chamber, K.W., Areb, M.C., Waggoner, C.A.: ‘An on-line ferromagnetic wear debris sensor for machinery condition monitoring and failure detection’, Wear, 1988, 128, (3), pp. 325–337

Fig. 3 Three types of boundary condition
(a) Dirichlet boundary condition, (b) Newman boundary condition, (c) Mixed boundary condition