The dynamics of cavitation bubbles in a sealed vessel

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A B S T R A C T

A model of cavitation bubbles is derived in liquid confined in an elastic sealed vessel driven by ultrasound. In this model, an assumption that the pressure acting on the sealed vessel due to bubble pulsations is proportional to total volume change of bubbles is made. Numerical simulations are carried out for a single bubble and for bubbles. The results show that the pulsation of a single bubble can be suppressed to a large extent in sealed vessel, and that of two matched bubbles with same ambient radius can be further suppressed. However, when two mismatched bubbles have the same ambient radii, an interesting breathing phenomenon takes place, where one bubble pulsates inversely with the other one. Due to this breathing phenomenon the suppression effect becomes weak, so the maximum radii of two mismatched bubbles can be larger than that of a single bubble or that of two matched bubbles in sealed vessel. Besides that, for two mismatched bubbles with different ambient radii, the small one in sealed vessel under some certain parameters can pulsate as strong as or even stronger than that of a single bubble in an open vessel.

1. Introduction

When high tensile stress is radiated into liquid, the preexistent nucleation of vapor cavities can be activated, a process called cavitation. Cavitation has begun to be a fascinating topic of research since bubble collapse was found to be the mainly reason of damage of propellers [1]. During the very short period of bubble collapse, high amount of energy is released in very small spatial spots with high velocities which generate high temperature and pressure inside the bubble [2,3]. Due to the extreme conditions [4], cavitation has been utilized across many different industries, such as biomedical domain, sonochemical reactors and ultrasonic cleaning [5,6]. This motivated a large number of studies in free liquid, close to walls or in a tube from both experimental and theoretical aspects [7–10].

However, bubble oscillation in fully sealed vessel, that is, liquid that is surrounded by walls in all directions, has obtained little attention. This events may occur in natural or artificial cells inclusions in quartz [11,12], or in the xylem - the fluidic network of ascending sap in a tree [13–17]. Vincent explored a model of a cavitation bubble when the surrounding liquid is fully sealed in an elastic solid [18]. The results show that unexpectedly fast bubble oscillates in volume and unusually quick damping [19]. Wang’s group studied the fracture of tungsten wire or lesions induced by acoustic cavitation under different hydrostatic pressures up to 10 MPa and driving electric powers up to 9752 W/cm² in a sealed vessel [20,21]. Yasuda studied the efficient generation of the reactive oxygen species generated by cavitation bubbles in a chamber where rose bengal solution was sealed [22]. Rae estimated the temperatures inside of cavitation bubbles induced by ultrasound in aqueous solutions sealed in vials [23]. In the field of high power spallation targets for neutron sources, the erosion damage caused by cavitation severely compromises the structural integrity and reduces the life of the vessels. Futakawa studied the impact erosion damage on the target vessel caused by cavitation in mercury sealed in a chamber which is induced by the pressure wave propagation [24].

In this present work, based on the Lagrangian method, we derive the model of cavitation bubbles in a sealed vessel and examine numerically the effect of the sealed vessel. In this numerical study, our attention is focused on the pulsations of bubbles and on the relationship of matched bubbles or mismatched bubbles in sealed vessel.

2. Model of cavitation bubbles in sealed vessel

Consider gas bubbles in liquid sealed in a vessel. Suppose that the distance between bubbles is always large enough so that the bubbles remain spherical at all times [25]. The pulsations of bubbles cause the volume change of the sealed vessel. The geometry of this system is
shown as Fig. 1:

Let’s say $\varphi$ be the velocity potential, thus, the boundary condition at each bubble surface can be approximately represented as \[25\]

$$\frac{\partial \varphi}{\partial r_i} = \dot{R}_i \quad \text{at} \quad r_i = R_i, \quad i = 1, 2, \ldots, N,$$

(1)

where $R_i$ is the $i$th bubble radius, $r_i$ is the distance from the center of bubble and $\cdot$ denotes the time derivative.

The velocity potential, satisfying the Laplace equation $\Delta \varphi = 0$, can be expanded as

$$\varphi = \sum_{i=1}^{N} \varphi_i,$$

(2)

where $N$ is the number of bubbles in sealed vessel, and $\varphi_i$, the scattered potential of the $i$th bubble, is given by \[5,26\]

$$\varphi_i = -\frac{\dot{R}_i R_i^2}{r_i^2}, \quad i = 1, 2, \ldots, N.$$

(3)

The next step is the calculation of the Lagrangian function of the system $L = T - U$. The kinetic energy $T$ of the system is determined by the kinetic energy of the surrounding liquid \[25\]:

$$T = \frac{\rho}{2} \int_{V_s} \|\nabla \varphi\|^2 dV,$$

(4)

with $\rho$ denoting the liquid density and $V_s$ the volume occupied by the liquid.

As mentioned before, the pulsations of bubbles cause the volume change of the sealed vessel. Assume that the additional pressure ($p_s$) acting on the sealed vessel is proportional to volume change ($\Delta V_i$), $p_s = \delta \Delta V_i$, where $\delta$ is a proportional coefficient. Then, the potential energy $U$ of the system can be expressed using Eq. (5)

$$U = -\sum_{i=1}^{N} \int_{V_i} p_s dV_i + \int_{V_s} \rho dV_s$$

$$= -\sum_{i=1}^{N} \int_{V_i} p_s dV_i + \frac{1}{2} \delta \left( \sum_{i=1}^{N} \left( V_i - V_s \right) \right)^2,$$

(5)

where $p_s$ is the scattered pressure at the surface of the $i$th bubble, and $V_i = 4\pi R_i^3/3$ is the time-varying volume of the $i$th bubble.

$$p_s = \left( p_0 + \frac{2\sigma}{R_i} \right) \left( \frac{R_0}{R_i} \right)^3 - 2\sigma \left( \frac{R_0}{R_i} \right)^4 - 4\mu \dot{R}_i - p_0 + p_s \sin \left( \frac{2\pi ft - 2\pi d_i}{\lambda} \right), \quad i = 1, 2, \ldots, N,$$

(6)

where $\lambda$ is the wave length and $p_0$ is the static pressure:

Eq. (4) can be transformed to the sum [25]:

$$T = \sum_{i=1}^{N} T_i, \quad i = 1, 2, \ldots, N,$$

(7)

with

$$T_i = -\pi \rho R_i^2 \int_{-1}^{1} \left( \frac{\partial \varphi}{\partial r_i} \right)_{r_i = R_i} r_i d\cos \theta,$$

(8)

On substitution of Eqs. (1) and (2), Eq. (8) gives

$$T_i = 2\pi \rho \left( R_i \dot{R}_i + R_i \ddot{R}_i + \sum_{j=1, j\neq i}^{N} \frac{R_j \dot{R}_j}{d_{ij}} \right), \quad i = 1, 2, \ldots, N,$$

(9)

where $d_{ij}$ is the distance between bubble $i$ and bubble $j$.

Thus,

$$L = T - U = 2\pi \rho \sum_{i=1}^{N} \left( R_i \dot{R}_i + R_i \ddot{R}_i + \sum_{j=1, j\neq i}^{N} \frac{R_j \dot{R}_j}{d_{ij}} \right) + \sum_{i=1}^{N} \int_{V_i} p_s dV_i$$

$$- \frac{\delta}{2} \left( \frac{4\pi}{3} \sum_{i=1}^{N} \left( R_i^3 - R_s^3 \right) \right)^2,$$

(10)

Considering $R_i$ and $\dot{R}_i$ as generalized coordinates and velocities, the equations of radial and translational motions of the bubbles are obtained in the usual way through the use of the Lagrangian equations

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{R}_i} - \frac{\partial L}{\partial R_i} = 0.$$

(11)

So

$$\frac{\partial L}{\partial R_i} = 2\pi \rho \left( 2R_i \dot{R}_i + 2R_i^2 \sum_{j=1, j\neq i}^{N} \frac{R_j \dot{R}_j}{d_{ij}} \right), \quad i = 1, 2, \ldots, N,$$

(12)

and

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{R}_i} = \frac{d}{dt} \left[ 2\pi \rho \left( 2R_i \dot{R}_i + 2R_i^2 \sum_{j=1, j\neq i}^{N} \frac{R_j \dot{R}_j}{d_{ij}} \right) \right]$$

$$= 2\pi \rho \left( 3R_i \dot{R}_i^2 + R_i \dddot{R}_i + 4R_i \dot{R}_i \sum_{j=1, j\neq i}^{N} \frac{R_j \ddot{R}_j}{d_{ij}} \right) + 2R_i$$

$$\times \sum_{j=1, j\neq i}^{N} \frac{R_j R_i \dot{R}_i}{d_{ij}}, \quad i = 1, 2, \ldots, N,$$

(13)

and

![Fig. 1. Bubbles in a sealed vessel (not to scale).](image-url)
3. Bubble dynamics in a sealed vessel

The calculations are carried out for a single bubble or two bubbles in liquid sealed within a vessel to investigate the effect of parameter $\delta$ on the pulsations of bubbles. Unless otherwise mentioned, the liquid parameters are set to: atmospheric pressure $p_0 = 1$ atm, the density $\rho = 998.2$ kg/m$^3$, the speed of sound $c = 1481$ m/s, the surface tension $\sigma = 0.07275$ N/m, the viscosity $\mu = 0.001$ Pa·s and the heat capacity ratio of gas inside the bubbles $\gamma = 1.4$. The ultrasound parameters are set to: the driving frequency $f = 26.5$ kHz and the amplitude $P_a = 1.275$ atm [27,28].

3.1. Single bubble

Consider a bubble with typical ambient radius $R_0 = 4.5$ $\mu$m [27,28]. The results are shown in Fig. 2 within one acoustic period. It is suggested from this figure that with the increase of the parameter $\delta$, the pulsation of bubble trends to be linear and the maximum radius of bubble decreases.

Fig. 3 shows the maximum expansion ratio of a single bubble under a various of $\delta$. It can be seen that when $\delta$ is larger than 0.1 Pa/µm$^3$, bubble is dramatically suppressed and will be completely suppressed when $\delta$ is high enough.

3.2. Two identical bubbles

In this subsection, the effect of parameter $\delta$ on two bubbles with the same ambient radii, $R_{01} = 4.5$ $\mu$m, $R_{02} = 4.5$ $\mu$m is investigated. In order to compare the results with that of a single bubble, the values of $\delta$ in subsection 3.1 are still used here.

In this and the next subsections, two bubbles in a system are categorized as matched bubbles and mismatched bubbles. Strictly speaking, when the pulsation of one bubble is identical to that of the other one, they are matched bubbles; otherwise, they are mismatched bubbles. According to this definition, there are nearly no matched bubbles. So in this work, matched bubbles mean when the mean volume change in one period $\Delta V = \Delta V_1 + \Delta V_2$ is in a neighborhood of its maximum; similarly, two mismatched bubbles mean when the mean volume change in one period $\Delta V = \Delta V_1 + \Delta V_2$ is in a neighborhood of its minimum. For example, in linear case, if pulsations of two bubbles are $R_1 = R_0 + \text{xsin}(2\pi ft)$, $R_2 = R_0 + \text{xsin}(2\pi ft)$, they are two matched bubbles; if pulsations of two bubbles are $R_1 = R_0 + \text{xsin}(2\pi ft)$, $R_2 = R_0 - \text{xsin}(2\pi ft)$, they are two mismatched bubbles. Although, the

$$\frac{\partial L}{\partial R_i} = 2\pi \rho \left(3R_i^2 \dot{R}_i^2 + 4R_0 \sum_{j=1,j\neq i}^{N} \frac{R_j^2 \dot{R}_j}{d_q} + 4\pi \rho \dddot{R}_i - \frac{16\pi^2 \delta}{3 \rho} \sum_{j=1}^{N} \left(\frac{R_i}{R_0} - 1\right)^3 \right) R_i^2, \quad i = 1, 2, \ldots, N. \tag{14}$$

Inserting Eq.(10) into Eq.(11) yields an ordinary differential equation of second order for each bubble:

$$R_i \dddot{R}_i + \frac{3}{2} \ddot{R}_i \dot{R}_i + \frac{2}{3} R_i - \frac{p_0}{\rho} - \sum_{j=1,j\neq i}^{N} \left(\frac{R_j^2 \dot{R}_j}{d_q} + \frac{R_j}{c} \frac{d \ddot{R}_j}{d\tau} \right) - \frac{4 \delta \pi}{3 \rho} \sum_{j=1}^{N} \left(\frac{R_i}{R_0} - 1\right)^3 \right) \frac{R_i^2}{d_q} \frac{R_j}{c} \frac{d \ddot{R}_j}{d\tau} \right) - \frac{4 \delta \pi}{3 \rho} \sum_{j=1}^{N} \left(\frac{R_i}{R_0} - 1\right)^3 \right) = 1, 2, \ldots, N. \tag{15}$$

where $c$ is the sound speed in liquid.

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boundary between matched bubbles and mismatched bubbles is not very clear because of the nonlinear characteristics of bubble pulsations, just like that between elastic ball and inelastic ball in mechanics, we still can study them in a relative safety range. Some parameters that make two bubbles be matched bubbles or mismatched bubbles are the bubble ambient radii, the bubble translations (beyond the scope of this work), driving frequencies and the driving amplitudes acting on two bubbles [29].

**Matched bubbles**

First, a system of matched bubbles is investigated. The pulsations of matched bubbles are shown in Fig. 4 under three chosen \( \delta \).

Comparing Fig. 4 with Fig. 2, one can find for the same \( \delta \) (except \( \delta = 0 \)) that the maximum radius of one bubble is smaller than that in single bubble system.

**Mismatched bubbles**

Second, a system of mismatched bubbles is investigated. The pulsations of mismatched bubbles are shown in Fig. 5 under three chosen \( \delta \).

Comparing Fig. 5 with Fig. 2 and Fig. 4, one can find for the same \( \delta \) (except \( \delta = 0 \)) that the maximum radius of one bubble is bigger than that both in single bubble system and in two-matched-bubble system. This means that one of mismatched bubbles is more easier to expand (to a bigger radius) than both a single bubble and one of matched bubbles in sealed vessel.

Comparing two dash-dotted lines in Fig. 5, it is suggested that the radius of one bubble varies inversely with that of the other bubble, which represents the breathing phenomenon (one bubble expands and the other one compresses at the same moment) occurring between two mismatched bubbles. This is because when two bubbles are mismatched, which means one bubble is in the rarefaction phase of ultrasound to expand, and the other one is in the compression phase to shrink. And when one bubble gets its maximum radius, the other one gets its minimum radius at the same moment. Due to this breathing phenomenon, the maximum radius of one bubble can be larger than that in single bubble system in sealed vessel. But this breathing phenomenon can not be found in two-matched-bubble system (see Fig. 4).

### 3.3. Two different bubbles

In cavitation field, there are many bubbles with different ambient radii [30], so it is worth to discuss the effect of parameter \( \delta \) on two bubbles which have different ambient radii. In this subsection, the ambient radius of the studied bubble (bubble 1) is still set to be 4.5 \( \mu m \) and the ambient radius of the other one (bubble 2) is larger than 4.5 \( \mu m \).

**Matched bubbles**

In this part, matched bubbles with different ambient radii are investigated firstly.

In Fig. 6, the pulsations of matched bubble are shown as a function of acoustic period within one period. This figure shows that with the increase of the ambient radius of bubble 2, the expansion ratios \( R_{\text{max}}/R_0 \) of both bubbles decrease because of the limited space in sealed vessel.

In Fig. 7, the maximum expansion ratio of bubble 1 \( (R_{10} = 4.5 \mu m) \) is shown as a function of ambient radius of bubble 2 for \( \delta = 100 \text{ Pa}/\mu m^3 \). In this figure, it can be seen that with the increase of the ambient radius of bubble 2, the maximum radius that bubble 1 can pulsate decreases generally within a small range.

**Mismatched bubbles**

In this part, mismatched bubbles with different ambient radii are investigated secondly.

In Fig. 8, the pulsations of mismatched bubbles are shown as a
function of acoustic period within one period. This figure shows that generally when bubble 1 expands at the rarefaction phase of ultrasound wave, that of bubble 2 corresponds to shrink at the compression phase of ultrasound wave. As the increase of ambient radius of bubble 2, the maximum radius of bubble 1 increases and the nonlinearity of the pulsation of bubble 1 is growing obviously.

In Fig. 9, the maximum expansion ratio of bubble 1 ($R_{1\text{max}}/R_{10}$) as a function of ambient radius of bubble 2; ambient radius of bubble 1 $R_{10} = 4.5 \, \mu m$.

In Fig. 10, the pulsation of two bubbles. The frequency $f = 172$ kHz; the ambient radii $R_{10} = 4.5 \mu m$, $R_{20} = 50 \mu m$, $\delta = 1$ Pa/μm$^3$.

**Frequency effect for mismatched bubbles**

In subSection 3.2, considering two bubbles with the same ambient radii, their responses to the driving frequency are the same because of their same nature frequencies. As a result, the linear pulsations of two bubbles with distances of half wavelength can match inversely better than that with other distances under suitable $\delta$ in terms of the maximum radius that one bubble can expand. But in this subsection, for two bubbles with different ambient radii, because of the difference of their nature frequencies, the distance may be not half wavelength in terms of the maximum radius that bubble 1 can expand.

**Fig. 9.** The maximum expansion ratio of bubble 1 ($R_{1\text{max}}/R_{10}$) as a function of ambient radius of bubble 2; ambient radius of bubble 1 $R_{10} = 4.5 \, \mu m$.
between two bubbles.

4. Conclusion

In this work, a model of cavitation bubbles in a sealed vessel is derived using the Lagrangian method by introducing a pressure proportioned (proportional coefficient is $\delta$) to volume change due to bubble pulsations. The results suggest that when the $\delta$ is big enough, the pulsation of a single bubble or that of two matched bubbles can be suppressed to a large extent. For two mismatched bubbles with the same ambient radii, this suppression effect becomes weak, so two bubbles can be suppressed to a large extent. For two mismatched bubbles with the same or different pulsation of a single bubble or that of two matched bubbles can be proportioned (proportional coefficient is derived using the Lagrangian method by introducing a pressure phenomenon in this work. Because the number of bubbles is two, the weakness of suppression effect due to breathing phenomenon is not large in this work. Breathing phenomenon among many bubbles will be studied in the further. But for two mismatched bubbles with different ambient radii in the sealed vessel, the pulsation of smaller bubble can be as strong as or even stronger than that of single bubble in an open vessel ($\delta = 0$) when the second one has a big enough ambient radius. The reason causing this phenomenon is the phase difference between two mismatched bubbles and thus the small bubble in expansion phase can take advantage of the big bubble which is in contraction phase.

CRediT authorship contribution statement

Yang Shen: Conceptualization, Methodology, Software, Data curation, Writing - original draft. Weizhong Chen: Methodology, Funding acquisition, Supervision. Lingling Zhang: Validation, Writing - review & editing. Yaorong Wu: Writing - review & editing. Shaoyang Kou: Writing - review & editing. Guoying Zhao: Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] C.E. Brennen, Cavitation and bubble dynamics, Cambridge University Press, 2014.
[2] Y. Shen, K. Yasui, Z. Sun, B. Mei, M. You, T. Zhu, Study on the spatial distribution of the liquid temperature near a cavitation bubble wall, Ultrason. Sonochem. 29 (2016) 394–400.
[3] Y. Shen, L. Zhang, Y. Wu, W. Chen, The role of the bubble-bubble interaction on radial pulsations of bubbles, Ultrason. Sonochem. 73 (2021), 105535.
[4] K.S. Sanlack, N.C. Eddinsaas, D.J. Flameign, S.D. Hopkins, H. Xu, Extreme conditions during multibubble cavitation: Sonoluminescence as a spectroscopic probe, Ultrason. Sonochem. 18 (4) (2011) 942–946.
[5] K. Yasui, Acoustic Cavitation and Bubble Dynamics, Springer, Cham, Switzerland, 2018.
[6] T.J. Mason, Ultrasonic cleaning: an historical perspective, Ultrason. Sonochem. 29 (2016) 519–523.
[7] R. Pflieger, T. Chave, G. Vite, L. Jouve, S.I. Nikitenko, Effect of operational conditions on sonoluminescence and kinetics of H2O2 formation during the sonolysis of water in the presence of Ar/O2 gas mixture, Ultrason. Sonochem. 26 (2015) 169–175.
[8] K. Manni, Q. Wang, Acoustic microbubble dynamics with viscous effects, Ultrason. Sonochem. 36 (2017) 427–436.
[9] B. Gieden, S. Marchal, J. Jordens, L.C. Thomassen, L. Braeken, T. Van Gerven, Influence of dissolved gases on sonochemistry and sonoluminescence in a flow reactor, Ultrason. Sonochem. 31 (2016) 463–472.
[10] N. Pokhrel, F.K. Vabbinia, N. Pala, Sonochemistry: science and engineering, Ultrason. Sonochem. 29 (2016) 104–128.
[11] Q. Zheng, D.J. Durben, G.H. Wolf, C.A. Angeli, Liquids at large negative pressures: water at the homogeneous nucleation limit, Science 254 (5033) (1991) 829–832.
[12] A coherent picture of water at extreme negative pressure, Nat. Phys. 9 (1) (2013) 38–41.
[13] M.T. Tyree, J.S. Sperry, Vulnerability of Xylem to Cavitating and Embolism, Annu. Rev. Plant Physiol. Plant Mol. Biol. 40 (1) (1989) 19–38.
[14] M.T. Tyree, M.H. Zimmermann, Xylem Structure and the Ascent of Sap, Springer Science & Business Media, 2013.
[15] H. Cockhard, Cavitation in trees, C.R. Phys. 7 (9–10) (2006) 1018–1026.
[16] O. Vincent, P. Marmottant, P.A. Quinto-Su, C.D. Ohl, Birth and growth of cavitation bubbles within water under tension confined in a simple synthetic tree, Phys. Rev. Lett. 108 (18) (2012) 1–5.
[17] C. Scognaniglio, F. Magaletti, Y. Izmaylov, M. Gallo, C.M. Canciola, X. Noblin, The detailed acoustic signature of a micro-confined cavitation bubble, Soft Matter 14 (39) (2018) 7967–7975.
[18] O. Vincent, P. Marmottant, S.R. Gonzalez-Avila, K. Ando, C.D. Ohl, The fast dynamics of cavitation bubbles within water confined in elastic solids, Soft Matter 10 (10) (2014) 1455–1461.
[19] O. Vincent, P. Marmottant, On the statics and dynamics of fully confined bubbles, J. Fluid Mech. (827) (2017) 194–224.
[20] Y. Zhang, Z. Zhang, J. Wu, Y. Liu, M. Zhang, C. Yang, M. He, X. Gong, Z. Zhang, Z. Wang, F. Li, Study on fracture of tungsten wire induced by acoustic cavitation at different hydrostatic pressures and driving electric powers, Ultrason. Sonochem. 68 (April) (2020), 105232.
[21] M. He, Z. Zhong, X. Li, X. Gong, Z. Wang, F. Li, Effects of different hydrostatic pressure on lesions in ex vivo bovine livers induced by high intensity focused ultrasound, Ultrason. Sonochem. 36 (2017) 36–41.
[22] J. Yasuda, S. Yoshizawa, S.I. Umemura, Efficient generation of reactive oxygen species sonochemically generated by cavitation bubbles, Jpn. J. Appl. Phys. 54 (2015) 071021.
[23] J. Rae, M. Ashokkumar, O. Eulaerts, C. Von Sonntag, J. Reisse, F. Grieser, Estimation of ultrasound induced cavitation bubble temperatures in aqueous solutions, Ultrason. Sonochem. 12 (5) (2005) 325–329.
[24] J. Meier, Z. Zhong, J. Wu, Y. Liu, M. Zhang, C. Yang, M. He, X. Gong, Z. Zhang, Z. Wang, F. Li, Study of the repetitive imiplosion of a sonoluminescing bubble, Phys. Rev. Lett. 69 (26) (1992) 2924–2927.
[25] R. Mettin, I. Akhatov, U. Parlitz, C.D. Ohl, Bjerknes forces between small cavitation bubbles in a strong acoustic field, Phys. Rev. E 56 (3) (1997) 2840–2842.
[26] J. Yasuda, T. Naoe, H. Kogawa, Y. Ikeda, Damage diagnostic of localised impact erosion by measuring acoustic vibration, J. Nucl. Sci. Technol. 41 (11) (2004) 1059–1064.
[27] T.P. Barber, S.J. Putterman, Light scattering measurements of the repetitive supersonic implosion of a sonoluminescing bubble, Phys. Rev. Lett. 69 (26) (1992) 3839–3842.
[28] K. Yasui, Variation of Liquid Temperature at Bubble Wall near the Sonoluminescence Threshold, J. Phys. Soc. Jpn. 65 (9) (1996) 2830–2840.
[29] L. Zhang, W. Chen, Y. Shen, Y. Wu, G. Zhao, The nonlinear characteristics of the liquid temperature near a cavitation bubble wall, Ultrason. Sonochem. 29 (2016) 104–128.
[30] Q. Zheng, D.J. Durben, G.H. Wolf, C.A. Angeli, Liquids at large negative pressures: water at the homogeneous nucleation limit, Science 254 (5033) (1991) 829–832.