Direct numerical simulation of MR suspension: the role of viscous and magnetic interactions between particles

Sandris Lācis and Didzis Goško
Department of Physics, University of Latvia, Zellu street 8, Riga, LV-1002
E-mail: sandris.lacis@lu.lv

Abstract. A numerical method is developed with aim to simulate the magnetorheological (MR) suspension taking into account realistic magnetic forces. The MR suspension is described by spherical particles with nonlinear magnetic properties suspended in a shear flow. Inertia effects, Brownian motion and buoyancy forces are neglected. The hydrodynamic interaction between close particles is taken into account approximately. Results of some test simulations are presented.

1. Introduction
The behavior of magnetorheological suspensions is governed by magnetic and hydrodynamic interactions between particles. The direct experimental observation of micron size particles inside a non-transparent suspension is almost impossible. Use of numerical simulation methods seems to be an appropriate tool to solve this problem. However even for present computational resources numerical simulations are still limited to intermediate number of particles (few hundreds or thousands) and use of appropriate simplifications.

Pappas and Klingenberg [1] demonstrated that processes inside MR suspensions could be explained by means of direct numerical simulation of particle system placed in fluid flow. Reasonable simplifications were used to describe the hydrodynamic forces and the effect of the magnetic field. Formation of lamellar structures are observed at the developed flow regimes. Similar numerical simulations were carried out also in [2], [3].

Another recent paper [4] contains similar approach to direct numerical simulation based on use of the Rotne-Prager Yamakawa tensor.

More accurate simulation of a flow of rigid particle suspension [5] is limited to two dimensional case. Three dimensional case requires extremely high computational resources.

This study aims at three dimensional direct numerical simulation of the MR suspension flow under the action of magnetic field. We are following main ideas of [1], concentrating on more accurate account for the hydrodynamic interactions between neighbor particles and the more realistic model of magnetic interactions.
2. Method

Particles reside in a box shaped space enclosed by two opposed walls in \(xy\) planes according to Figure 1. The space between walls is filled with carrier fluid. One wall is still, but the other wall is moving with velocity \(U_0\) in the direction of \(x\)-axis causing a shear flow of a carrier fluid. The walls are considered to be ferromagnetic, this is obtained by introducing ghost particles [1]. The “computational box” with dimensions \(L_x \times L_y \times L_z\) is periodic in \(x\)- and \(y\)-directions.

![Figure 1. Periodic computational domain.](image)

Nondimensional equations are introduced using the reference values. The reference time \(\tau = \frac{L_z}{U_0} = \frac{1}{\dot{\gamma}}\) is introduced by the rate of shear flow \(\dot{\gamma}\). Here \(L_z\) is the length of the box in the \(z\)-direction (perpendicular to shear flow) and \(U_0\) is the velocity of the moving upper wall. So the dimensional time \(t\) is related to the nondimensional one \(\hat{t}\) as \(t = \tau \hat{t}\). In a similar way other nondimensional quantities are introduced. For distance measures \(R = \hat{R} r_0\) and for velocity \(v = \hat{v} v_0\). Here reference quantities are \(r_0\) (the radius of a spherical particle) and the reference velocity \(v_0 = \frac{\dot{r}_0}{r_0} = r_0 \dot{\gamma}\). Nondimensional force is derived from Stokes drag: \(F_0 = 6\pi \eta r_0 v_0\) and thus the dimensional force is \(F = \hat{F} F_0\) (\(\eta\) is the fluid viscosity).

Main magnetic reference value is the saturation magnetization \(M_S\) of magnetizable spheres (particles). Nondimensional magnetization \(\hat{M} = M/M_S\), magnetic field intensity \(\hat{H} = H/M_S\). The Frolich-Kennelly law [6] is used to describe nonlinear magnetization of particles, beside \(M_S\) so called initial magnetic permeability \(\mu_{in}\) characterizes the magnetic material properties. Mason number \(M_n = \frac{\eta \dot{\gamma}}{2 \pi \mu_0 M_S^2}\) characterizes ratio of viscous and magnetic forces, here \(\mu_0\) is magnetic constant.

To calculate the velocity of the \(i\)-th sphere, we use the force balance equation for nondimensional quantities assuming that all inertial effects are negligible:

\[
\sum_{j=1, j\neq i}^{N} \mathbf{F}_{m,ij} + \sum_{i=1, i\neq j}^{N} \mathbf{F}_{c,ij} + \sum_{j=1, j\neq i}^{N} \mathbf{F}_{hd,ij} + \mathbf{F}_{v,i} = 0. \tag{1}
\]

In this equation \(\mathbf{F}_{m,ij}\) is magnetic force between \(i\)-th and \(j\)-th particles, \(\mathbf{F}_{c,ij}\) is collision force (repulsive force, introduced to prevent overlapping of particles [1]), \(\mathbf{F}_{hd,ij}\) is hydrodynamic force and \(\mathbf{F}_{v,i}\) is the Stokes drag. The Stokes drag is in form

\[
\mathbf{F}_{v,i} = -R_i (\mathbf{v}_i - \mathbf{u}_i), \tag{2}
\]

where \(R_i\) is the diameter, \(\mathbf{v}_i\) is the velocity of \(i\)-th particle and \(\mathbf{u}_i\) is the velocity of ambient flow at the location of \(i\)-th particle. Hydrodynamic forces between near particles (as shown in [7])
can be calculated by means of expression

\[ F_{hd} = \eta(X_{11}A - X_{12}A)U. \]

Here \( U \) is half of the relative velocity between spheres. The expressions \( X_{11}A \) and \( X_{12}A \) for different motion modes can be found in [7].

Equations (1) are solved iteratively for all system of particles. Iteration scheme is build expressing velocity \( \vec{v}_i \) from the equation (1). Knowing velocities \( \vec{v}_i \) positions of all particles are advanced.

At dipolar approximation magnetic forces are calculated using the following expression:

\[
\vec{F}_{m,ij} = \frac{1}{M_n R^5} \left[ 2(\vec{m} \cdot \vec{R})\vec{m} + (\vec{m} \cdot \vec{m})\vec{R} - 5(\vec{m} \cdot \vec{R})(\vec{m} \cdot \vec{R})\vec{R} \right].
\] (3)

Here \( \vec{m} = \vec{M}/M \) (unit vector) and \( \vec{R} \) is nondimensional vector between centers of both particles. Denoting angle between vectors \( \vec{m} \) and \( \vec{R} \) as \( \theta \) we have \( \vec{R} = \vec{R}(\cos \theta \vec{m} + \sin \theta \kappa) \), where \( \kappa \) is the unit vector perpendicular to \( \vec{m} \). Thus we arrive at

\[
\vec{F}_{m,ij} = -\frac{1}{4M_n R^4} \left[ (3A_1 \cos \theta + 5A_3 \cos 3\theta) \vec{m} + (B_1 \sin \theta + 5B_3 \sin 3\theta) \kappa \right],
\] (4)

where \( A_1 = A_3 = B_1 = B_3 = 1 \). If nonlinear magnetization is taken into account the expression (4) provides good approximation, but values of \( A_1, A_3, B_1, B_3 \) depend on field value and distance \( d = \vec{R} - 2 \) between particles, so they are calculated by finite element method, solving separate magnetic field problem for two spherical particles. Here we discuss only the case of \( \mu_{in} = 100 \) and \( \vec{H}_0 = 0.1 \), values of coefficients are given in Table 1.

In the simplest case, ambient flow velocity can be given as linear shear flow profile depending on the distance from the walls. However in more general situations the Stokes flow problem should be solved, taking into account volume force created by Stokes drag of particles. So called SIMPLE algorithm [8] was selected as fluid flow solution method.

3. Results

| Table 1. Coefficients \( A_1, A_3, B_1, B_3 \) at the magnetic field value \( \vec{H}_0 = 0.1 \). |
| --- | --- | --- | --- | --- |
| \( d \) | \( A_1 \) | \( A_3 \) | \( B_1 \) | \( B_3 \) |
| 0.00 | 11.67 | 2.53 | 19.28 | 3.33 |
| 0.02 | 10.53 | 2.49 | 16.60 | 3.11 |
| 0.04 | 9.39 | 2.44 | 13.93 | 2.89 |
| 0.08 | 7.61 | 2.31 | 10.71 | 2.51 |
| 0.10 | 6.89 | 2.21 | 9.58 | 2.31 |
| 0.14 | 5.76 | 2.01 | 8.03 | 2.07 |
| 0.20 | 4.67 | 1.79 | 6.60 | 1.82 |
| 0.30 | 3.57 | 1.57 | 5.13 | 1.58 |
| 0.50 | 2.47 | 1.32 | 3.54 | 1.32 |
| 1.00 | 1.63 | 1.13 | 2.16 | 1.13 |
Calculations were carried out for different volume fractions $\varphi$ of particles inside MR suspension at $M_r = 0.00003$. If hydrodynamic interaction is limited only to Stokes drag, formation of lamellar structures is reproduced like in [1], see Figure 2, 3.

Effect of using both improved hydrodynamic and magnetic forces is depicted on Figures 4, 5, 6. If magnetic field is perpendicular to the gap between walls, the concentration of particles near both wall is observed (Figure 4). Once some particle comes close to the wall, it is attracted by ghost particle (ferromagnetic wall) and there is no force to tear it away. Structure inside the central region of the gap is quite complex, formation of long chains of particles is not present. If the orientation of the magnetic field is parallel to the walls, particles are slightly repulsed from walls due to the dipolar nature of magnetic forces, what could be observed in Figure 5 and Figure 6. Both figures certify that in this case there is no formation of long chains of particles in MR suspension. If ambient fluid flow calculation is included, rise of effective viscosity of MR suspension is observed. The accuracy of viscosity calculation is still not sufficiently good due to some numerical instabilities in the computational algorithm which should be fixed.

4. Conclusions
The results of simulation show that introduction of more realistic magnetic interaction force between particles change particle behavior in the flow. The hydrodynamic force between close particles prevents particles from fast movements caused by strong magnetic forces. In such a way the hydrodynamic interaction exerts a stabilizing influence on the numerical algorithm suppressing oscillations due to the repulsive force. Numerical method is considerably fast, after small improvements it could be used also for systems of few thousand particles.

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