Violation of Bell’s inequality and postulate on simultaneous measurement of compatible observables

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Abstract

We discuss coupling of violation of Bell’s inequality and non-Kolmogorovness of statistical data in the EPR-Bohm experiment. We emphasize that nonlocality and “death of realism” are only sufficient, but not necessary conditions of non-Kolmogorovness. There can be found other sufficient conditions of non-Kolmogorovness and, hence, violation of Bell’s inequality. We find one important source of non-Kolmogorovness by analyzing axiomatics of quantum mechanics. We pay attention to the postulate (due to von Neumann and Dirac) on simultaneous measurement of quantum observables given by commuting operators. This postulate is criticized as nonphysical. We propose a new interpretation of the Born-von Neumann-Dirac rule for calculation of the joint probability distribution for such observables. It gets a natural physical interpretation by considering conditional measurement scheme. We use this argument (i.e., rejection of the postulate on simultaneous measurement to motivate non-Kolmogorovness of the probabilistic structure of the EPR-Bohm experiment.
1 Introduction

1.1 Nonlocality, “death of reality”

In the physical community violation of Bell’s inequality [1] is typically considered as an evidence of either nonlocality or “death of reality”: local realism is incompatible with predictions of QM. On the other hand, in the quantum logic and quantum probability communities the same problem is often interpreted in a completely different way. As was pointed out in numerous papers (see, e.g., author’s monographs [2], [3] and reviews [4], [5] and see paper [6] in this issue), Bell’s inequality can be violated simply because it is impossible to define a single probability measure which would serve for a few different (incompatible) experimental settings. The existence of such a probability measure was a hidden assumption in Bell’s derivation of his famous inequality. He assumed that all correlations can be represented as averages with respect to the same probability measure \((d\rho(\lambda))\) in his notations).

From the point of view of quantum logic/probability the EPR-Bohm experiment does not have the Kolmogorov probabilistic structure. Hence, it could not be described by a single Kolmogorov probability space. If one works in the non-Kolmogorovian framework, i.e., under the assumption that different experimental settings induce their own probability measures, i.e., instead of a single probability \(d\rho(\lambda)\), one should consider a family of probabilities \(d\rho_{a,b}(\lambda)\) for settings \(a, b\), then only generalizations of Bell’s inequality hold true [2]. Such generalized Bell’s inequalities are not violated by experimental data. However, the idea of non-Kolmogorovness of the EPR-Bohm experiment did not propagate so much in the physical community.

Why was the message from quantum logic/probability ignored in quantum physics?

I think that one of the reasons is that there was not presented a natural physical mechanism of generating non-Kolmogorovness. One should explain why Bell’s identification [1] of the probability \(d\rho(\lambda)\) with the probability distribution of hidden parameters for the initial state can be questioned. One of aims of this paper is to describe the process of generating non-Kolmogorovness in the EPR-Bohm experiment.
1.2 Simultaneous or conditional measurements

The crucial point is that the EPR-measurements should be considered not as *simultaneous measurements on a pair of entangled particles*, but as conditional measurements: first we measure the projection of the polarization vector of the first particle on the axis $a$ and then the projection of the polarization vector of the second particle on the axis $b$ or vice versa. Such a representation of the EPR-Bohm experiment was given in the talk of Aspect at the Conference Foundations of Probability and Physics-3 (Växjö University, June 7-12, 2004), see also [7]. Unfortunately, he considered this shift from the simultaneous measurement picture to the conditional measurement picture just as a metaphor which is convenient for the presentation of Bell’s argument [1]. Moreover, he used the Copenhagen interpretation of the state vector: the $\psi$-function gives the state of an *individual* quantum system (in the EPR-Bohm case photon). Such interpretation immediately induces the picture of nonlocal change of the state of e.g. the second particle under the condition that the state of the first one was collapsed in the process of measurement. In fact, in this situation one gets nonlocality (under the assumption of realism) automatically, without to appeal to Bell’s argument. Alain Aspect mentioned this in his talk; he considered Bell’s inequality merely as a test of the Copenhagen interpretation.

We shall use the so called *ensemble interpretation* of the wave function: the $\psi$-function describes statistical properties of an *ensemble* of identically prepared quantum systems. Therefore our conclusion will be completely different from Aspect’s conclusion.

We remind that Einstein had never accepted Bohr’s thesis on completeness of QM [8], [9]. All his life he dreamed of creation of a new fundamental theory of micro phenomena. He was sure that the wave function does not provide the complete description of the state of an individual quantum system. Einstein was the father of the *ensemble interpretation* of the wave function as describing statistical properties of an ensemble of systems created by some preparation procedure. This interpretation was later elaborated by Leslie Ballentine [10]-[12] who used the terminology the *statistical interpretation*. Unfortunately, this terminology is rather misleading, since it was used even by von Neumann: the wave function, although assigned as the state of an individual system, expresses statistics of measurements (but this statistics is coupled to irreducible randomness).
We also remark that the construction in section 2 is similar to so called filtering type measurements which were presented in very detail in Ballentine’s book [11]. (Unfortunately, he did not use our argument in Chapter 20 of his book: he presented only the standard Bell argument.)

2 Simultaneous measurement as an idealization of conditional measurements

In this section we discuss a misconception that has been propagating in the physics literature since the work of von Neumann [13]. As is well known from linear algebra, two diagonalizable matrices $a$ and $b$ can be diagonalized simultaneously if and only if $a$ and $b$ commute. This theorem says, of course, absolutely nothing about measurements.

This misconception is the source of a lot of confusion, wrong statements, and “paradoxes” in the physics literature. In particular, it plays an important role in Bell’s argument [1].

One of the postulates of QM [13] tells that if two observables, say $a$ and $b$, are represented by commutative operators, $\hat{a}$ and $\hat{b}$ : $[\hat{a}, \hat{b}] = 0$, then these observables can be measured simultaneously (and vice versa). The joint probability distribution is given by the Born (Dirac-von Neumann) formula:

$$P(a = \alpha, b = \beta) = |\langle P_{\alpha}^{a} P_{\beta}^{b} \psi \rangle|^2,$$

(1)

where $P_{\alpha}^{a}$ and $P_{\beta}^{b}$ are spectral projectors of operators $\hat{a}$ and $\hat{b}$ corresponding to eigenvalues $\alpha$ and $\beta$. (We restrict considerations to operators with discrete spectra). This postulate has never been questioned and it is commonly accepted (in contrast to e.g. von Neumann’s projection postulate).

I have doubts in validity of this postulate. It seems that in formula (1) there was encoded (by von Neumann and Dirac) the probability distribution for the procedure of conditional measurement. Von Neumann did not present a solid physical motivation of this postulate. He presented a rather long consideration on a possibility to represent two self-adjoint and commutative operators as functions of one fixed operator,

$$\hat{a} = f(\hat{d}), \hat{b} = g(\hat{d}).$$

Then he stressed that a measurement of the observable $d$ provides automatically measurements of observables $a$ and $b$ represented by
operators $\hat{a}$ and $\hat{b}$: the value $d = \gamma$ is transferred into the corresponding values $a = f(\gamma)$ and $b = g(\gamma)$.

The “hidden postulate” of von Neumann which made his construction quite natural from the physical viewpoint is that any self-adjoint operator corresponds to some quantum observable – in the above consideration it was the correspondence $\hat{d} \mapsto d$. However, this hidden postulate by itself is not so natural from the physical viewpoint. We also remark that this postulate plays an important role in von Neumann’s no-go theorem – the first no-go theorem [13].

The main physical reason to reject the postulate on the possibility of simultaneous measurement is impossibility to realize such a measurement in the real experimental setup - at least for measurements on composite systems.

Let us consider the EPR-Bohm experiment for measurement of projections onto axes $a$ and $b$ of polarizations for pairs of entangled photons. It is well known that in real experiments $a$ and $b$ are not measured simultaneously. There is so called time window $\Delta$, see, e.g., [14], [3], which plays the fundamental role in forming of the probabilistic data; cf. [15]–[17]; see also [18] (coupling of time-synchronization in the EPR-Bohm experiment with the use of the projection postulate). The crucial point is that in this experiment $\Delta$ could not be chosen arbitrary small! First of all, if one were so naive to put $\Delta = 0$, then there would be no matched clicks of detectors at all. But even if $\Delta > 0$, but it is small then the majority of entangled pairs (which are generated by a source) disappear. Thus the picture of Alain Aspect is correct: first measurement on one photon and with some nontrivial delay on the second. This is not at all the simultaneous measurement which was discussed by Dirac and von Neumann, see, e.g., [13].

Thus one might reject the very idea of simultaneous measurement as nonphysical. In such a case one should provide a reinterpretation of the formula (1) in the conditional probabilistic framework. It can be easily done by using the framework of quantum conditional probability [3].

Let us consider two observables $a$ and $b$ which are represented by self-adjoint operators $\hat{a}$ and $\hat{b}$ with purely discrete spectrum. At the moment commutativity is not assumed. There is the initial state, say $\psi$. The $a$ was measured and the result $a = \alpha$ occurred. Then the initial state $\psi$ is transferred into the post measurement state

$$\psi_\alpha = \frac{P_\alpha \psi}{|P_\alpha \psi|},$$

(2)
Remark. We remark that, although this transformation is commonly used in QM, especially in quantum information theory, its applicability can be questioned, see [20], [21] (especially, for measurements on composite systems; in particular, in the EPR-type experiments). As was pointed out in [20], [21], in the case of observables with degenerate spectra von Neumann did not define the post-measurement state by (2), see [13]. By von Neumann even for a pure initial state $\psi$ the post-measurement state need not be again a pure state again, it can become a mixture, i.e., it can be given not by a state vector, but by a density matrix. Nowadays this von Neumann’s viewpoint is practically forgotten. In [20], [21] was shown that incompatibility of von Neumann’s projection postulate and (2) for observables with degenerate spectra induces an important objection to the standard treatment of the EPR-experiment. We remark that (2) is given by the Lüders projection postulate. In the present paper we do not discuss this delicate point; we proceed as it is usually done in QM (recently the author demonstrated [22] that under sufficiently general experimental conditions one can really proceed with the Lüders projection postulate; it seems that the EPR-Bohm experiment satisfies conditions of [22]).

We now would like to measure the $b$-observable. The crucial point is that it is measured not for the initial state $\psi$ (not for the initially prepared ensemble $S_\psi$), but for the post measurement state $\psi_\alpha$ (for the ensemble of those systems for which the result $a = \alpha$ was obtained). The conditional probability

$$P_\psi(b = \beta | a = \alpha) \equiv P_{\psi_\alpha}(b = \beta) = \frac{||P_\beta P_\alpha^* \psi||^2}{||P_\alpha^* \psi||^2}. \quad (3)$$

We now recall that

$$||P_\alpha^* \psi||^2 = P_\psi(\alpha = a) \quad (4)$$

(the probability to get the value $a = \alpha$ for a system belonging to the initial ensemble $S_\psi$). Thus

$$||P_\beta P_\alpha^* \psi||^2 = P_\psi(a = \alpha) P_\psi(b = \beta|a = \alpha) = Q_\psi(a = \alpha, b = \beta).$$

The latter probability is the probability to get first the value $a = \alpha$ and then the value $b = \beta$. Thus one may interpret Born-Dirac-von Neumann formula (1) as the rule to find joint probability not for simultaneous measurement, but for sequential measurement of $a$ and then $b$. 
Let us now repeat previous consideration by changing the order of measurements of \(a\) and \(b\). First we measure \(b\). The probability to get the value \(b = \beta\) is given by \(P_\psi(b = \beta) = |P_\beta^b \psi|^2\). The occurrence of this result induces a new quantum state:

\[
\psi_\beta = \frac{P_\beta^b \psi}{||P_\beta^b \psi||}
\]

and a new ensemble \(S_{\psi_\beta}\) of systems is created via filtration with respect to this value. We can now perform the \(a\)-measurement for systems belonging \(S_{\psi_\beta}\) and find the conditional probability

\[
P_{\psi}(a = \alpha | b = \beta) \equiv P_{\psi_\beta}(a = \alpha) = \frac{||P_\alpha^a P_\beta^b \psi||^2}{||P_\beta^b \psi||^2}.
\]

Thus

\[
||P_\alpha^a P_\beta^b \psi||^2 = P_\psi(b = \beta) P_{\psi}(a = \alpha | b = \beta) = Q_\psi(b = \beta, a = \alpha),
\]

where by our conditional interpretation the latter probability is the joint probability of the sequential measurement: first \(b = \beta\) and then \(a = \alpha\). If

\[
Q_\psi(a = \alpha, b = \beta) = Q_\psi(b = \beta, a = \alpha),
\]

one could forget about the order of measurements. Commutativity of operators \(\hat{a}\) and \(\hat{b}\) is a sufficient condition of such a coincidence, \(\text{[2]}\). It seems that it was the main reason for invention of the Dirac-von Neumann postulate on simultaneous measurement of observables which are represented by commutative operators. Commutative induced impression that, since one need not take care of the order of measurements, it is possible to interpret sequential probabilities \(Q_\psi(a = \alpha, b = \beta)\) and \(Q_\psi(b = \beta, a = \alpha)\) as just a single probability

\[
P_{\psi}(a = \alpha, b = \beta) \equiv Q_\psi(a = \alpha, b = \beta) = Q_\psi(b = \beta, a = \alpha),
\]

i.e., that there exists a probability measure \(P_{\psi}\) which does not depend on measured observables \(a\) and \(b\) represented by commutative operators and such that all bi-measurement probabilities can be represented on the basis of this single measure.
3 EPR-Bohm experiment

If we take another pair of observables, say $c$ and $d$, represented by commutative operators $\hat{c}$ and $\hat{d}$, we might misleadingly operate with the probability of simultaneous measurement, $P_\psi(c = \gamma, d = \epsilon)$.

This induces the impression that all such probability distributions are related to the same Kolmogorov probability measure $P_\psi$. And it would be correct if one uses the simultaneous measurement interpretation of probabilities under consideration. However we use the conditional measurement interpretation. Here the probability $Q_\psi(a = \alpha, b = \beta)$ is based not only on probability with respect to the original state $\psi$, namely, $P_\psi(a = \alpha)$, but also on probability with respect to a completely different state, namely, $P_{\psi\alpha}(b = \beta)$.

In the same way $Q_\psi(c = \gamma, d = \epsilon)$ is based not only on the $\psi$-probability $P_\psi(c = \gamma)$, but also on the $\psi_\gamma$-probability $P_{\psi\gamma}(d = \epsilon)$.

Probabilities $P_{\psi\alpha}$ and $P_{\psi\gamma}$ need not coincide. Therefore $Q_\psi(a = \alpha, b = \beta)$ and $Q_\psi(c = \gamma, d = \epsilon)$ could not be represented as probability distributions with respect to a single probability measure.

4 The conditional probabilistic structure of the EPR-Bohm experiment

In the derivation of Bell’s inequality $[1]$

$$|\langle a^{(1)}, b^{(2)} \rangle - \langle b^{(1)}, c^{(2)} \rangle| \leq 1 - \langle a^{(1)}, c^{(2)} \rangle$$

Bell used a single probability measure $d\rho(\lambda)$. Here the indexes 1 and 2 are related to measurements on the first and the second particle, respectively, in the EPR pair of photons; $a, b, c$ are orientations of polarization beam splitters. By Bell’s “hidden assumption” $[1]$:

$$\langle a^{(1)}, b^{(2)} \rangle = \int_{\Lambda} a^{(1)}(\lambda)b^{(2)}(\lambda)d\rho(\lambda), \ldots, \langle a^{(1)}, c^{(2)} \rangle = \int_{\Lambda} a^{(1)}(\lambda)c^{(2)}(\lambda)d\rho(\lambda).$$

To compare classical correlations with quantum mechanical correlations, J. Bell assumed the validity of the Dirac-von Neumann postulate on the simultaneous measurement. We can say that this postulate was “super-hidden assumption”. People never paid attention on its crucial role in Bell’s argument.
Now we consider the same EPR-Bohm experiment not from the viewpoint of simultaneous measurements of projections of polarizations (which is evidently nonphysical), but from the viewpoint of conditional (sequential) measurements (which corresponds to the real experimental situation).

Consider the \((a^{(1)}, b^{(2)})\)-measurement. The results of measurements can be divided into three groups:

\[ G_{12}(a^{(1)}, b^{(2)}) \] (first \(a\) clicks for the first particle and only then \(b\) clicks for the second one, \(G_{21}(a^{(1)}, b^{(2)})\) - vice versa, \(G(a^{(1)}, b^{(2)})\) - simultaneous clicks.

Since the number of simultaneous clicks is negligible we can forget about \(G(a^{(1)}, b^{(2)})\) and operate with only \(G_{12}(a^{(1)}, b^{(2)})\) and \(G_{21}(a^{(1)}, b^{(2)})\).

The group \(G_{12}(a^{(1)}, b^{(2)})\) can be split into two subgroups \(G_{\alpha_{12}}(a^{(1)}, b^{(2)})\), \(\alpha = \pm 1\), corresponding to results of measurements on the first photon: \(a^{(1)} = \alpha\). We should associate with each such subgroup its own probability measure \(dP_{\alpha}(\lambda)\), \(\alpha = \pm 1\).

We point out that the probability \(P_{\alpha}(\lambda)\) does not depend on \(b^{(2)}\). This is the condition of locality. Quantum mechanics is local in the conditional measurement framework.

Finally, as J. Bell did, we also consider the initial distribution of hidden variables \(dP^0(\lambda)\) corresponding the initial state preparation. The crucial point is that the covariance \(\langle a^{(1)}, b^{(2)} \rangle\) could not be expressed in terms of only \(dP^0(\lambda)\), the probabilities \(dP^{\alpha}(\lambda)\) should be involved:

\[
\langle a^{(1)}, b^{(2)} \rangle = P^0(\lambda \in \Lambda: a^{(1)}(\lambda) = +1) \int_{\Lambda} b^{(2)}(\lambda) dP_{+}^{\alpha}(\lambda) - P^0(\lambda \in \Lambda: a^{(1)}(\lambda) = -1) \int_{\Lambda} b^{(2)}(\lambda) dP_{-}^{\alpha}(\lambda).
\]

If we repeat the previous considerations for the pair of settings \((b^{(1)}, c^{(2)})\) we obtain:

\[
\langle b^{(1)}, c^{(2)} \rangle = P^0(\lambda \in \Lambda: b^{(1)}(\lambda) = +1) \int_{\Lambda} c^{(2)}(\lambda) dP_{+}^{b^{(1)}}(\lambda) - P^0(\lambda \in \Lambda: b^{(1)}(\lambda) = -1) \int_{\Lambda} c^{(2)}(\lambda) dP_{-}^{b^{(1)}}(\lambda).
\]

Probabilities \(P_{+}^{\alpha}(\lambda)\) and \(P_{-}^{\alpha}(\lambda)\) can differ. Therefore one is not able to repeat manipulations which had been done by J. Bell. There is no
Bell’s inequality and, hence, no problems at all.

5 Discussion

We demonstrated that Bell’s arguments were fundamentally based on the Dirac-von Neumann postulate on the possibility of simultaneous measurement of observables represented by commutative operators. This QM-postulate was projected onto a prequantum model with hidden variables. Consequently Bell assumed that (classical) observables in all pairs:

\[(a^{(1)}(\lambda), b^{(2)}(\lambda)), (b^{(1)}(\lambda), c^{(2)}(\lambda)), (d^{(1)}(\lambda), c^{(2)}(\lambda)),\]

can be measured simultaneously. This assumption induces the illusion that covariations for all these pairs of classical observables, namely,

\[\langle a^{(1)}, b^{(2)} \rangle, \langle b^{(1)}, c^{(2)} \rangle, \langle a^{(1)}, c^{(2)} \rangle\]

can be written with respect to a single probability measure: \(d\rho(\lambda)\) in Bell’s notations \((dP^0(\lambda)\) in our notations) corresponding the initial state \(\psi\). This assumption induces the derivation of Bell’s inequality. Violation of the latter implies the revolutionary conclusion that QM is incompatible with local realism.

We propose to reject the Dirac-von Neumann postulate on simultaneous measurement. We propose to interpret the Born-Dirac-von Neumann formula \(1\) for the probability distribution for simultaneous measurement as the formula for the joint probability distribution in the sequential measurement. This formula can be applied even in the case of observables represented by noncommutative operators. The main motivation of our substitution of the postulate on conditional measurements in the place of the postulate on simultaneous measurement is the evident experimental fact that simultaneous measurement is really impossible in experiments with composite systems, e.g., pairs of entangled photons. The time window is always nontrivial. Measurements are always conditional (sequential). Therefore conditional probabilistic formalism should be applied. Finally, we remark that the rejection of the Dirac-von Neumann postulate on simultaneous measurement induces just minority reconsideration of foundations of

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1 By operating with a family of probabilities one can derive generalized Bell’s inequalities \(2\). But such inequalities do not contradict to probabilistic predictions of QM.
physics comparing with rejection of local realism (or as an alternative - questioning of the validity of the mathematical formalism of QM).

We recall that our representation of the EPR-Bohm experiment as conditional measurement recalls the original consideration of EPR\textsuperscript{19}. There was nothing about simultaneous measurement in the original EPR-framework. J. Bell by introducing simultaneous measurements changed the original problem. He analyzed the EPR-Bohm experiment under the additional assumption of validity of the Dirac-von Neumann postulate. He did not recognized the fundamental role of this assumption in his model of prequantum reality. Of course, the realization of “Bell’s project” has a fundamental consequence that the Dirac von Neumann postulate should be rejected. However, Bell did not recognize this and he used this argument to support the hypothesis on nonlocality. (Bell was “nonlocal realist”.)

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1. C. D. Scott and R. E. Smalley, J. Nanosci. Naotech. 3, 75 (2003) 2. J. M. Cowley, Diffraction Physics, Elsevier, Amsterdam (1995)

3. W. Heimbrodt and P. J. Klar, in Magnetic Nanostructures, Edited by H. S. Nalwa, American Scientific Publishers, Los Angeles (2002), Chapter 1, pp.1-58

4. J. P. Turner and P. C. Chu, in Microelectronics, T. J. Kern, Ed., Materials Research Society, Warrendale, PA (1995), Vol. 143, p.375.

\textsuperscript{2}We remark that the Dirac-von Neumann postulate does not belong to the mathematical domain of QM. It belongs to the interpretation of the mathematical formalism of QM. We proposed another interpretation for the mathematical formula \textsuperscript{11}. 
References

[1] J. S. Bell, Speakable and Unspeakable in Quantum Mechanics, Cambridge Univ. Press, Cambridge (1987)

[2] A. Khrennikov, Interpretations of Probability, VSP International Science Publishers, Utrecht (1999); the second addition (completed): De Gruyter, Berlin (2009)

[3] A. Khrennikov, Contextual approach to quantum formalism (Fundamental Theories of Physics), Springer, Heidelberg-Berlin-New York (2009)

[4] A. Khrennikov, Theor. and Math. Phys. 157, 1448 (2008)

[5] A. Khrennikov, J. Russian Laser Research 28, 244 (2007)

[6] H. De Raedt, K. Hess, K. Michielsen, J. Comp. Theor. Nanoscience, this issue (2010)

[7] A. Aspect, Bell’s Theorem: The Naive View of an Experimentalist. [http://arxiv.org/abs/quant-ph/0402001](http://arxiv.org/abs/quant-ph/0402001)

[8] A. Einstein, The Collected Papers of Albert Einstein, Princeton Univ. Press, Princeton (1993).

[9] A. Einstein and L. Infeld, The evolution of Physics, From Early Concepts to Relativity and Quanta, Free Press, London, (1967)

[10] L. E. Ballentine, Rev. Mod. Phys. 42, 358 (1970)

[11] L. E. Ballentine, Quantum Mechanics: A Modern Development, Englewood Cliffs, New Jersey (1989)

[12] L. E. Ballentine, in Foundations of Probability and Physics, A. Yu. Khrennikov, Ed., ser Q. Prob. White Noise Anal., WSP, Singapore (2001), Vol. 13, pp. 71-84

[13] J. von Neumann, Mathematical foundations of quantum mechanics, Princeton Univ. Press, Princeton, N.J. (1955)

[14] G. Weihs, in Foundations of Probability and Physics-4, American Institute of Physics, Melville, NY (2007), Vol. 889, pp. 250-262

[15] K. De Raedt, K. Keimpema, H. De Raedt, K. Michielsen, and S. Miyashita, European Phys. J. B 53, 139 (2006)

[16] H. De Raedt, K. De Raedt, K. Michielsen, K. Keimpema, and S. Miyashita, The Phys. Soc. of Japan 76, 104005 (2007)

[17] S. Zhao, H. De Raedt, K. Michielsen, Found. Phys. 38, 322 (2008)
[18] A. Khrennikov, Int. J. Quant. Inf. 7, 71 (2009)
[19] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47, 777 (1935)
[20] A. Khrennikov, J. Russian Laser Research 29, 296 (2008)
[21] A. Khrennikov, Laser Physics 19, 346 (2009)
[22] A. Khrennikov, Int. J. Quant. Inf. 7, 1303 (2009)
[23] W.M. de Muynck, in Foundations of Probability and Physics - 5, L. Accardi, G. Adenier, C.A. Fuchs, G. Jaeger, A. Yu. Khrennikov, J.-A. Larsson, and S. Stenholm, Eds., American Institute of Physics, Melville, NY (2009), Vol. 1101, pp. 37-41