We study stabilization of moduli in the type–IIB superstring theory on the six-dimensional toroidal orientifold $T^6/\Omega \cdot (-1)^F \cdot Z_2$. We consider background space-filling D9-branes wrapped on the orientifold along with non-Abelian fluxes on its world-volume and demonstrate with two examples that this can stabilize all the complex structure moduli and some of the Kähler moduli.
1 Introduction

Superstring theories live in ten space-time dimensions. Compactifying on a six-dimensional space yields a theory in four dimensions, which are identified with the four dimensions of the real world. Any such consistent compactification yields a vacuum of string theory and there exists plethora of such vacua which have been studied in a variety of contexts over the last few decades. String compactification generically leads to a large number of flat moduli, which should have fixed values in a realistic model. Fixing the values of moduli to obtain a stable vacuum received the appellation \textit{moduli stabilization.}

Certain progress in understanding various mechanisms of moduli stabilization in different string theories has been achieved in the past few years. Among others, there are now schemes for moduli stabilization using background fluxes [1,2] as well as background D-branes with fluxes [3–6], using intersecting brane configurations [7] or branes at angles [8], construction of stable bundles in Heterotic string theories, to mention a few. Moreover, some of these are related through the duality symmetries of string theory. While the question of whether nature has chosen one from \textit{these} nimiet of vacua for us to live in and if so, then whether it was a ‘principled’ choice or a capricious one still awaits a decisive answer, the various constructions of moduli stabilization have interesting features which attracted special attention.

The most explored scheme of moduli stabilization uses background fluxes leading to $\mathcal{N}=1$ supersymmetric vacua, known as flux vacua. In a flux vacuum the NS-NS and RR fluxes in the internal space, through the consistency requirements of string theory, can fix the values of the moduli. In particular, in the type–IIB closed string theory this mechanism can stabilize all the complex structure moduli and the dilaton-axion modulus to fixed values [9–11]. In certain examples [10] it has even been possible to obtain meta-stable vacua akin to the de-Sitter space-time, deemed to be a rather realistic space-time of late. These fluxes, however, do not render themselves to an exact world-sheet description. Moreover, in the type–IIB theory fixing the Kähler moduli using such fluxes calls for non-perturbative means, restricting the viability of such analyses in the effective supergravity theory [10, 11].

Recently, another mechanism of moduli stabilization has been proposed employing constant Abelian magnetic fluxes on the world-volume of a background D-brane wrapped on a six-torus, $T^6$, or its orbifolds [12–15] ( for the compactification of type–I string theory on smooth Calabi-Yau with non-Abelian bundle from a slightly different perspective see [16–18]). The analysis of the Abelian fluxes is exact in the open string theory and thus not restricted to the lowest order in the inverse of the string tension $\alpha'$. These constant magnetic fluxes stabilize many of the complex structure as well as Kähler moduli [12,13]. In this article we study further examples in this class of schemes. We generalize the construction of magnetic branes on toroidal orbifold by turning on constant non-Abelian magnetic fluxes on the two-cycles of the internal space, as opposed to the Abelian fluxes.

The analysis of magnetic branes with non-Abelian fluxes is, again, plagued with the vice of not rendering itself to an exact string theoretic derivation [14,15]. However, in stabilizing the totality of available moduli, the open string moduli are to be coupled with the closed string moduli, with the latter being treated at but the lowest order in any known scheme. Hence, in want of a better method of stabilization, which treats both sides exactly, the present analysis, though limited, is quite relevant.

The paper is structured as follows. In the next section we briefly discuss the salient features of moduli stabilization with magnetic branes. In section 3 we illustrate the stabilization of complex structure moduli and Kähler moduli. We conclude with a discussion in the section 4. Some of the notations and conventions are elaborated in Appendix A.
2 The constraints

Let us begin with a brief discussion of the conditions of supersymmetry preservation for D-branes. Let us consider the type–IIB theory compactified on a six dimensional variety $X$. D-brane configurations are supersymmetric if they are wrapped on supersymmetric cycles of $X$. In the presence of magnetic fluxes in the world-volume of the D-brane this condition gets further modified. For a single space-filling D-brane with magnetic flux one can write a $\kappa$-symmetric action \[12, 13, 20\] from which follows the BPS condition \[20\]
\[(1 - \Gamma)\eta = 0,\] (2.1)
for the fermion $\eta$, with $\Gamma$ given as
\[
\Gamma = \frac{\sqrt{|g|}}{\sqrt{|g + F|}} \sum_{n=0}^{\infty} \frac{1}{2^nn!^2} \gamma_{\mu_1\nu_1...\mu_n\nu_n}^n F_{\mu_1\nu_1...\mu_n\nu_n} J_9^{(n)},
\] (2.2)
where $g$ denotes the metric induced on the world-volume of a D9-brane, $F$ the Abelian field strength on the world-volume and $| \cdot |$ denotes a determinant. Moreover,
\[
J_9^{(n)} = (-1)^n \sigma_3^{n+3} i \sigma_2 \otimes \gamma^{(11)}, \quad \Gamma^{(11)} = \frac{i}{10! \sqrt{|g|}} \epsilon_{\mu_1...\mu_{10}} \gamma_{\mu_1...\mu_{10}},
\] (2.3)
where $\sigma$’s are the Pauli matrices, $\gamma$’s are the ten-dimensional $\gamma$-matrices and $\epsilon_{\mu_1...\mu_{10}}$ denotes the ten-dimensional antisymmetric tensor indicating the choice of the orientation of $X$. This expression contains the perturbative terms of all orders in $\alpha'$ but liable to receive corrections from world-sheet instantons.

In order to obtain a supersymmetric theory upon orientifolding, the supersymmetry preserved by the D-brane configuration needs to be the same as the one preserved by the orientifold plane. It follows from the Dirac-Born-Infeld action \[12, 13, 19\] that supersymmetry is preserved if the following two conditions are met,
\[F_{\mu\nu} = 0,\] (2.4)
\[\text{Im}(e^{i\theta} \int e^{(F_{11} - i\mathcal{J})^3}) = 0,\] (2.5)
where $\mathcal{F}^{(2,0)} = F_{ij}$ and $\mathcal{F}^{(1,1)} = \mathcal{F}_{ij}$ denote the $(2,0)$-forms and $(1,1)$-forms, respectively, representing the constant Abelian magnetic fluxes, while $\mathcal{J}$ denotes the Kähler form and a product of forms is to be interpreted in terms of wedge products. The value of the parameter $\theta$ depends on the kind of orientifolding performed. For O3- and O5-planes, $\theta = 0$ and $\theta = \pi$, respectively. Since the two equations (2.5) and (2.4) depend on the complex structure moduli and Kähler moduli, respectively, they stabilize the respective moduli, as has been demonstrated in models with constant Abelian magnetic fluxes \[12, 13\].

We consider a generalization of this mechanism to cases with constant non-Abelian magnetic fluxes. The supersymmetry preserved by D-branes with non-Abelian magnetic fluxes on their world-volume has been discussed earlier \[22\] in the context of $\mathcal{N} = 2$ theories, where the internal space $X$ is Calabi-Yau. In this article we consider $X = T^6/\Omega \cdot (-1)^F L \cdot \mathbb{Z}_2$. For the case at hand we need three ingredients from the various facets of the conditions of preservation of supersymmetry. The first ingredient is the requirement that the vector bundle $\mathcal{E}$ on the world-volume of the D-brane, that describes the D-branes with non-Abelian fluxes, is holomorphic. The condition on the fluxes is (2.4), now with a non-Abelian $\mathcal{F}^{(2,0)}$. 

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The second ingredient is the requirement of equality of the phases of supersymmetries preserved by the D-branes and the orientifold plane. The phase of the supersymmetry preserved by a D-brane is given by the grade \( \phi(E) \) of \( E \), which is related to the central charge \( Z[E] \) of the associated D-brane configuration as
\[
\phi(E) = \frac{1}{\pi} \text{Im}(\log Z[E]). \tag{2.6}
\]
Thus, the condition of equality of the phases assumes the form,
\[
\text{Im}(e^{-i\theta} Z[E]) = 0, \tag{2.7}
\]
which generalizes (2.5). In order to see this let us recall that\[22\] the central charge of D-branes wrapped on a six-dimensional compact space \( X \) is determined by their RR charges, which can be obtained from a Wess-Zumino term in the world-volume action \( \int C_{(10-2i)} \wedge \text{ch}_i E \), where \( \text{ch}_i(E) \) denotes the \( i \)-th Chern character of the bundle \( E \) and \( C_i \) denotes an RR \( i \)-form coupled to it. In the large volume limit the central charge for pure D-\(2p\)-branes becomes the volume, given by\[23–26\],
\[
Z[E] = \int_X e^{-iJ \wedge \text{ch}(E)}. \tag{2.8}
\]
For a single brane \( E \) is a line bundle and the central charge reduces to \( Z[E] = \int_X (\mathcal{F}^{(1,1)} - i\partial\bar{\partial})^3 \) and hence (2.7) reduces to (2.5). Using S-duality, the perturbative part of this condition of supersymmetry preservation, namely, that of (2.5) has been shown\[16\] to be related to vanishing of FI terms in compactification of SO(32) Heterotic string theory.

For the case at hand, therefore, the supersymmetry preserving D-brane satisfies (2.4) with non-Abelian fluxes and
\[
\text{Im}(e^{i\theta} \int_X e^{-i\bar{\theta} \wedge \text{ch}(E)}) = 0, \tag{2.9}
\]
obtained from (2.8) which can be expanded to
\[
\tan \theta (J \wedge J \wedge \text{ch}_1(E) - \text{ch}_3(E)) = ( J \wedge J \wedge J \text{ch}_0(E) - J \wedge \text{ch}_2(E) ). \tag{2.10}
\]
Since we shall consider an O3-plane, i.e. \( \theta = 0 \), equation (2.9) reduces to
\[
J \wedge J \wedge J \text{ch}_0(E) = J \wedge \text{ch}_2(E). \tag{2.11}
\]
In addition, as we need \( \theta \) to be the correct phase of supersymmetry, we are also required to abide by
\[
J \wedge J \wedge \text{ch}_1(E) < \text{ch}_3(E). \tag{2.12}
\]

There is an additional condition for the preservation of supersymmetry of the D-brane configuration, which is the third ingredient. It requires that the bundle \( E \) is II-stable. That is, the D-brane configuration described by the vector bundle \( E \) is stable with respect to the decay \( E \to E_1 \oplus E_2 \) if \( \phi(E_i) < \phi(E) \) for \( i = 1, 2 \), where \( \phi(E) \) is the grade defined in (2.6). When \( \phi(E) = \phi(E_1) = \phi(E_2) \), the D-brane configuration is said to be marginally stable with respect to this decay. In the limit of vanishing string length, \( l_s \to 0 \), the II-stability condition reduces to the condition for the existence of solutions to the Hermitian-Yang-Mills equations [22, 23]. In this article we shall refrain from discussing this condition in any further detail, save for a few comments on satisfying it in the final section.
There are further conditions, in addition to the ones arising from supersymmetry preservation, hitherto discussed. These originate from the requirement of tadpole cancellation, that is, the vanishing of the total RR charge of each of the RR fields on the compact manifold. The coupling of a D9-brane to the various RR fields can be obtained from the Wess-Zumino part of the world-volume action, mentioned above, namely,

$$S_{WZ} = \int C_4 \wedge \text{ch}_3(\mathcal{E}) + C_8 \wedge \text{ch}_1(\mathcal{E}).$$  \hspace{1cm} (2.13)

Thus, in the presence of fluxes corresponding to bundles with non-vanishing first and third Chern characters, a D9-brane acquires charges of D7- and D3-branes respectively. Two more restrictions are to be imposed to prohibit this. First, we require that the charge equivalent to that of a D7-brane, as a D9-brane wraps a two-cycle $C_2$ of $X$ vanishes, that is,

$$\int_{C_2} \text{ch}_1(\mathcal{E}) = 0,$$  \hspace{1cm} (2.14)

for all two-cycles of $X$. Similarly, requiring that the equivalent of the charge of a D3-brane vanishes for the brane-orientifold configuration means that the total D3-brane charge on the D9-brane, as the latter wraps $X$, cancels the charges of the sixty four O3-planes positioned at the sixty four fixed points of $T^6/Z_2$, each contributing the equivalent of a quarter of a D3-brane charge with an opposite sign, that is,

$$\int_X \text{ch}_3(\mathcal{E}) = 16.$$  \hspace{1cm} (2.15)

To summarize, we impose the conditions (2.4), (2.11), (2.12), (2.14) and (2.15) on the bundle $\mathcal{E}$. Over and above, there are certain K-theoretic constraints \[34\] to be satisfied for building a consistent model. We shall not delve into a discussion of these conditions, which require the vanishing of the second Chern characters modulo an integer. To keep our discussion simple we shall choose the branes to wrap a cycle only once. The K-theoretic constraints can be taken care of by choosing non-zero wrapping numbers.

Even with this simplicity, we show that it is possible to construct consistent models that has all the complex structure moduli and some of the Kähler moduli stabilized. The axion-dilaton moduli remains unfixed. It may be possible to stabilize it by turning on NS-NS and R-R fluxes \[9\]. While these may be generalized, they are not marred by considering the K-theoretic constraints.

Let us close this section by mentioning the quantization conditions on the fluxes relevant to the D9-brane configuration \[14, 15, 27–31\]. Let us consider the embedding of the brane into the compact space $X$ given by

$$z^i = e^i_\alpha \sigma^\alpha,$$  \hspace{1cm} (2.16)

where $\sigma$ denotes a world-volume co-ordinate of the brane. The matrix $e$ encodes the information of the number of times a brane wraps a cycle of $X$. If the branes wrap the cycles more than once, one has to count the cycles with (integral) multiplicities in evaluating the pairings $\int \text{ch}(\mathcal{E})$ on the cycles. Demanding, then, that the Euler character of $X$ is integral, corresponding to the quantization of fluxes, lets us choose the Chern characters to assume values in the rational cohomology, rather than the integral one. However, for the sake of simplicity, we have chosen $e^i_\alpha = \delta^i_\alpha$, corresponding to unit wrapping number of branes and hence work with integral cohomology.
3 The models

In this section we consider moduli stabilization in an orientifold of the type–IIB theory compactified on a six-torus, $\mathbb{T}^6$. The orientifolding action is given by $\Omega \cdot \mathbb{Z}_2 \cdot (\pm 1)^{F_L}$ where $\Omega$ denotes the world-sheet parity reversal operator, $\mathbb{Z}_2$ flips signs of all the co-ordinates of $\mathbb{T}^6$ and $(\pm 1)^{F_L}$ changes the sign of the Ramond vacuum on the left.

Let us start with a general gauge-theoretic description of the configuration. In the models we discuss below, we consider stacks of D-branes, the number of stacks are two in both the models, but can be more in general [12, 13]. A stack corresponds to a direct summand of the bundle $\mathcal{E}$. Generally, we consider $N$ number of D9-branes wrapped on $X$ in each stack, where $N$ may differ from one stack to another. The configuration corresponding to one stack is described by a $U(N)$ gauge theory, corresponding to a bundle of rank $N$ along with a magnetic flux. Denoting the field strength of the flux by $\mathcal{F}$ in the complex basis of $z$’s and by $F$ in the real basis of $x, y$’s, as given in Appendix A, we can write the components of the field strength in the real basis as

$$
F_{x_ix_j} = f_{ij}, \quad F_{x_iy_j} = g_{ij}, \quad F_{y_ix_j} = h_{ij} \\
= f_{ij} T^a, \quad = g_{ij} T^a, \quad = h_{ij} T^a,
$$

(3.1)

where $T^a, a = 0, 1, \ldots, n^2 - 1$, denote the generators of $U(N)$. The matrices $f^a$ and $h^a$ are anti-symmetric. In this notation the field strengths $\mathcal{F}^{(2,0)}$ and $\mathcal{F}^{(1,1)}$ become

$$
\mathcal{F}_{ij} = [(I - \bar{I})^{-1}]^T (\bar{I}^T f \bar{I} - \bar{I}^T g + g^T \bar{I} + h) (I - \bar{I})^{-1},
$$

(3.2)

$$
\mathcal{F}_{ij} = -[(I - \bar{I})^{-1}]^T (\bar{I}^T f I - \bar{I}^T g + g^T I + h) (I - \bar{I})^{-1},
$$

(3.3)

where $I$ denotes the complex structure matrix and a bar designates complex conjugation. Before constructing a flux configuration with the constraints discussed in the previous section let us impose the simpler restrictions needed to be satisfied. First, we want to stabilize the complex structure and fix it to $I = i \cdot I_3$, where $I_n$ denotes the $n \times n$ identity matrix. In order to obtain the flux configuration that stabilizes the complex structure to this value let us substitute $I = i \cdot I_3$ in the expression for $\mathcal{F}^{(2,0)}$ given in (3.2) and set that equal to zero, thereby guaranteeing the holomorphicity of the two-forms. The restrictions ensuing from this condition are

$$
\begin{align*}
&f_{ij} = h_{ij} \quad g_{ij} = g_{ji},
\end{align*}
$$

(3.4)

Using these and setting the complex structure to $I = i \cdot I_3$ in (3.3) simplifies the expression of $\mathcal{F}^{(1,1)}$ to

$$
\mathcal{F}_{ij} = \frac{1}{2} (f_{ij} + i g_{ij}).
$$

(3.5)

In terms of the basis of $(1, 1)$-forms chosen in Appendix A it becomes $\mathcal{F}_{ij} = [g_{ij} \delta_{ij} + f_{ij} \delta_{ij}]$. In order to avoid unnecessary complications we choose to fix the Kähler $(1, 1)$-form to be a simple one, namely, $\mathcal{J} = (i/2) J \delta_{ij}dz^i \wedge d\bar{z}^j = Jb^+_{kk}$. We shall discuss more about the stabilization of $\mathcal{J}$ to this value shortly.

Let us now present two models and discuss the further issues of moduli stabilization within the context of these specific models only.
3.1 Model I

In this first model we consider two stacks. Although consistent, from stabilization point of view, this model turns out to be of limited use due to the large 3-brane tadpole contribution from just a single pair of stacks. However, this discussion helps set up our notations which we use for more practical use in §§ 3.2. The stacks, represented by two bundles $\mathcal{E}_1$ and $\mathcal{E}_2$, are both $U(2)$ stacks with only an Abelian flux turned on in the former while the latter has a non-Abelian part as well. The fluxes through the various cycles of $X$ are specified completely by specifying the matrices $f$, $g$ and $h$. We shall often suppress the stack indices in these matrices, when the stack they pertain to is clear from the context. The fluxes in the first stack are

$$g_{ij} = \lambda \delta_{ij} \mathbb{1}_2, \quad f_{ij} = f \epsilon_{ijk} \eta_k \mathbb{1}_2, \quad |\eta|^2 = \sum_{i=1}^3 \eta_i^2, \quad (3.6)$$

where $\lambda$, $f$, and $\eta_i$, $i = 1, 2, 3$, are some constant parameters. The moduli are fixed in terms of these parameters. In the basis of $(1,1)$-forms chosen in Appendix A, $\mathcal{F}^{(1,1)}$ can be written as

$$\mathcal{F} = [f \epsilon_{klm} \eta^m \eta_k + \lambda \eta_k^+ \eta_k] \mathbb{1}_2. \quad (3.7)$$

The Chern characters of $\mathcal{E}_1$ for such a choice of magnetic fluxes are

\begin{align*}
\text{ch}_0(\mathcal{E}_1) &= 2, \\
\text{ch}_1(\mathcal{E}_1) &= 2[\lambda \eta_k^+ + f \epsilon_{klm} \eta_m \eta_k^-], \\
\text{ch}_2(\mathcal{E}_1) &= 2[f^2 \eta_k \eta_l \eta_m \eta_l - \lambda^2 \eta_k^- \eta_k^+] , \\
\text{ch}_3(\mathcal{E}_1) &= 2\lambda (\lambda^2 - f^2 |\eta|^2) \mathbb{1}.
\end{align*} \quad (3.8)

Substituting the expression $\mathcal{J} = J \eta_k^+$ of the Kähler two-form and the above Chern characters in (2.11) and (2.12) we obtain the following relations among the parameters,

\begin{align*}
J(3\lambda^2 - f^2 |\eta|^2) &= J^3, \quad (3.9) \\
2\lambda (\lambda^2 - f^2 |\eta|^2 - 3J^2) &> 0. \quad (3.10)
\end{align*}

The second stack, represented by a bundle $\mathcal{E}_2$, is also with a $U(2)$ gauge group. This time, however, we choose the flux to possess an Abelian as well as a non-Abelian $SU(2)$ part. The fluxes in this stack are,

$$g_{ij} = -\lambda \delta_{ij} \mathbb{1}_2 + g \frac{\xi^a}{2} \delta_{ij} T^a, \quad f_{ij} = -f \epsilon_{ijk} \eta_k \mathbb{1}_2 + g \xi^a T^a, \quad |\xi|^2 = \sum_{a=1}^3 (\xi^a)^2 = 4 \quad (3.11)$$

where $T^a$, $a = 1, 2, 3$, denote the three generators of the $SU(2)$ group chosen to satisfy $\text{tr}(T^a T^b) = 2 \delta^{ab}$, so that these can be taken to be the Pauli matrices. We have also introduced new parameters $g$ and $\xi^a$, $a = 1, 2, 3$, for this stack. We have chosen the Abelian part in this second stack to be equal and opposite to that in the first stack, thus ensuring the vanishing of the total D7-brane charge. The restriction given by the third equation in (3.11) on the parameters $\xi^a$ is necessitated by the requirement that both the stacks satisfy (2.11) for the same value of the Kähler modulus $\mathcal{J}$. The magnetic fluxes for this stack can be written in terms of the basis $(1,1)$-cycles as

$$\mathcal{F}^{(1,1)} = -[\lambda \eta_k^+ + f \epsilon_{klm} \eta_m \eta_k^-] \mathbb{1}_2 + g \left[ \frac{\xi^a}{2} h_{kk}^+ + \epsilon_{akl} h_{kk}^- \right] T^a. \quad (3.12)$$
The various Chern characters for the second stack are given by
\[
\begin{align*}
\text{ch}_0(\mathcal{E}_2) &= \text{ch}_0(\mathcal{E}_1), \\
\text{ch}_1(\mathcal{E}_2) &= - \text{ch}_1(\mathcal{E}_1), \\
\text{ch}_2(\mathcal{E}_2) &= \text{ch}_2(\mathcal{E}_1) + g^2 \epsilon_{ijk} \xi^k \delta^i_j, \\
\text{ch}_3(\mathcal{E}_2) &= - \text{ch}_3(\mathcal{E}_1) + 2g^2 f(\xi \cdot \eta) v,
\end{align*}
\]
where \(\xi \cdot \eta = \sum_{i=1}^{3} \xi_i \eta_i\). Substituting these Chern characters in (2.11) leads to (3.9), the same condition as was obtained for the first stack. However, (2.12) now leads to a new restriction on the parameters, namely,
\[
g^2 f(\xi \cdot \eta) - \lambda (\lambda^2 - f^2 |\eta|^2 - 3J^2) > 0.
\]
Comparing (3.10) and (3.14) we see that if we turn off the non-Abelian part of the second stack by setting \(g = 0\), then these two equations cannot be simultaneously satisfied. This has been one of the reasons for introducing the non-Abelian flux in the second stack.

Now combining the equations (3.9), (3.10) and (3.14) we derive
\[
J^2 = 3\lambda^2 - f^2 |\eta|^2, \quad g^2 f(\xi \cdot \eta) > \lambda (\lambda^2 - f^2 |\eta|^2 - 3J^2) > 0.
\]
We still have the tadpole cancellation conditions (2.14) and (2.15) to impose. The contribution of the pair of stacks to the D3-brane tadpole is,
\[
\rho_2 = \text{ch}_3(\mathcal{E}_1) + \text{ch}_3(\mathcal{E}_2) = 2g^2 f(\xi \cdot \eta).
\]
Combined with (3.15) and (3.16) this imposes a condition on the Abelian fluxes, namely,
\[
\rho_2 > 2\lambda (\lambda^2 - f^2 |\eta|^2 - 3J^2) > 0.
\]
Let us now choose a solution to the above equations as \(\lambda^2 = f^2 |\eta|^2\), which implies
\[
J^2 = 2\lambda^2, \quad \rho_2 > 12|\lambda|^3 > 0.
\]
The inequality imposes a restriction on \(|\lambda|\). The contribution of this pair of stacks to the D3-brane tadpole is more than \(12|\lambda|^3\). But the D3-brane tadpole is also bounded above by 16 by (2.15), moreover, as we discuss shortly, the quantization conditions require the value of \(|\lambda|\) to be integral. It follows that the maximum allowed value of \(|\lambda|\) is 1 for this model.

Let us now return to the quantization conditions. In order to keep the quantization simple but capturing all of its non-trivial features, we choose the D9-branes wrapping exactly once around \(X\), by choosing \(e_\alpha = \delta_\alpha^i\) in (2.16), as mentioned earlier. The quantization conditions restrict the fluxes through each of the holomorphic cycles. Denoting the unit flux quanta through the \((1,1)-\) and \((2,2)-\)cycles by \(m\) and \(n\), respectively, we now write down the conditions on the different parameters. For the \((1,1)-\)cycles associated with \(h_{ij}^+\) and \(h_{ij}^-\), respectively, they read
\[
\begin{align*}
2\lambda \delta_{ij} &= m_{ij}^+ = m^+ \delta_{ij}, \\
2f \eta_k \epsilon_{ijk} &= m_{ij}^- = \epsilon_{ijk} m_k^-.
\end{align*}
\]
Similarly, the conditions on the fluxes through the (2, 2)-cycles associated to \( S^*_i \) and \( S^-_i \) are
\[
2[(f \eta_i)(f \eta_j) - \lambda^2 \delta_{ij}] = n^+_i = \epsilon_{ijk} n^+_k,
\]
\[
2\lambda(f \eta_k)\epsilon_{ijk} = n^+_i = \epsilon_{ijk} n^+_k,
\]
\[
g^2\xi^k\epsilon_{ijk} = \hat{n}^+_i = \epsilon_{ijk} \hat{n}^+_k.
\]
Finally, the flux through \( X \) itself is also quantized for each of the stacks and that leads to a pair of further constraints on the parameters, namely,
\[
2\lambda(\lambda^2 - f^2|\eta|^2) = \rho_1, \quad 2g^2f(\xi \cdot \eta) = \rho_2.
\]
The quantization conditions on the fluxes imply that all the quantities \((m^+_i, m^-_i), (n^+_i, n^-_i, \hat{n}^+_i)\) and \((\rho_1, \rho_2)\) are integers.

Let us now consider the solution \( \lambda^2 = f^2|\eta|^2 \). The quantization conditions impose the following restrictions,
\[
2\lambda = m^+, \quad 2f\eta_k = m^+_k,
\]
\[
2\lambda(f \eta_k) = n^+_k, \quad g^2\xi^k = \hat{n}^+_k, \quad \rho_1 = 0, \quad 2g^2f(\xi \cdot \eta) = \rho_2,
\]
and,
\[
2\left(-\lambda^2 + (f\eta_i)^2\right) - \lambda^2 + (f\eta_j)^2 = \left(n^+_i n^+_j n^+_k \right) = \left(n^-_1, n^-_2, n^-_3\right)
\]
\[
(3.23)
\]
One simple choice that satisfies all these quantization condition is \( \lambda^2 = f^2, \eta_k = (1, 0, 0) \), and \( \xi^k = (2, 0, 0) \). For this choice the quantized fluxes are,
\[
\lambda = \frac{1}{2}m^+, \quad f = \frac{1}{2}m^-_1,
\]
\[
(3.24)
\]
\[
\left(n^-_i\right)^2 = \left(m^+_i\right)^2, \quad m^+_2 = m^-_3 = 0,
\]
\[
(3.25)
\]
\[
n^+_i = \begin{cases} 
\frac{1}{4}(m^+_i)^2, & \text{for } i = 1 \\
0, & \text{otherwise.}
\end{cases}
\]
\[
(3.26)
\]
\[
\hat{n}^+_i = \begin{cases} 
2g^2 & \text{for } i = 1, \\
0, & \text{otherwise}
\end{cases}
\]
\[
(3.27)
\]
\[
4g^2f = \hat{n}^+_1 m^-_1 = \rho_2.
\]
\[
(3.28)
\]
We also choose \( \lambda = -f \), which implies \( m^+ = m^-_1 \). The equation (3.28) exhibits the contribution to the D3-brane tadpole for each such pair of stacks.

Although this model is consistent, in order to stabilize all the Kähler moduli one needs to consider a few more of such stacks. However, there is a rather stringent restriction on the number of such pairs of stacks arising from the D3-brane tadpole cancellation condition. The condition (3.25) dictates \( m^+ \) to
be an even integer and so its minimum value is \( m^+ = 2 \). From (3.18) and (3.24), on the other hand, we derive, \( \rho_2 > (3/2)(m^+)^3 \), which implies that the contribution to the D3-brane tadpole is \( \rho_2 > 12 \). Since the maximum contribution to the D3-brane tadpole is 16 we have only one such pair of stacks. There exists a solution with \( \lambda^2 = f^2 \) and \( \eta = (1, 1, 0) \), where one can introduce two such pairs of stacks which is more suitable, though not sufficient, for further moduli stabilization. We would like to point out that this restriction on the number of stacks depends on the particular model. In fact, in next subsection we introduce appropriate modifications to allow a larger set of brane with fluxes to stabilize several complex and Kähler moduli.

### 3.2 Model II

We now go on to present another model, again with two \( U(2) \) stacks. However, this models has constant non-Abelian fluxes in both the stacks. Let us denote the bundles corresponding to the fluxes in the first and the second stack by \( \mathcal{E} \) and \( \hat{\mathcal{E}} \), respectively. The fluxes are chosen as

\[
\begin{align*}
g_{ij} &= \lambda \delta_{ij} \mathbb{I}_2 + \frac{\xi^a}{2} g \delta_{ij} T^a, & f_{ij} &= f \epsilon_{ijk} \eta_k + g \epsilon^a_{ij} T^a, \quad (3.29) \\
g_{ij} &= -\lambda \delta_{ij} \mathbb{I}_2 + \frac{\xi^a}{2} \hat{g} \delta_{ij} T^a, & f_{ij} &= -f \epsilon_{ijk} \eta_k + \hat{g} \epsilon^a_{ij} T^a, \quad (3.30)
\end{align*}
\]

which in terms of the basis \((1,1)\)-cycles read

\[
\begin{align*}
\mathcal{F}(\mathcal{E}) &= \left[ \lambda h^+_{kk} + f \epsilon_{klm} \eta_m h^-_{kl} \right] \mathbb{I}_2 + g \left[ \frac{\xi^a}{2} h^+_{kk} + \epsilon_{akt} h^-_{kl} \right] T^a, \\
\mathcal{F}(\hat{\mathcal{E}}) &= -\left[ \lambda h^+_{kk} + f \epsilon_{klm} \eta_m h^-_{kl} \right] \mathbb{I}_2 + \hat{g} \left[ \frac{\xi^a}{2} h^+_{kk} + \epsilon_{akt} h^-_{kl} \right] T^a
\end{align*}
\]  

(3.31)

The Chern characters of the two bundles following from these choices are,

\[
\begin{align*}
\text{ch}_0(\mathcal{E}) &= 2, \\
\text{ch}_0(\hat{\mathcal{E}}) &= 2, \\
\text{ch}_1(\mathcal{E}) &= 2[\lambda h^+_{kk} + f \epsilon_{klm} \eta_m h^-_{kl}], \\
\text{ch}_1(\hat{\mathcal{E}}) &= -\text{ch}_1(\mathcal{E}), \\
\text{ch}_2(\mathcal{E}) &= 2 \left[ -\lambda^2 h^-_{kk} + f^2 \eta_k \eta \delta^-_{kl} + f \lambda \epsilon_{klm} \eta_m \delta^+_{kl} + \frac{g^2}{2} \epsilon_{ijk} \zeta \delta^+_{ij} \right], \\
\text{ch}_2(\hat{\mathcal{E}}) &= 2 \left[ -\lambda^2 h^-_{kk} + f^2 \eta_k \eta \delta^-_{kl} + f \lambda \epsilon_{klm} \eta_m \delta^+_{kl} + \frac{\hat{g}^2}{2} \epsilon_{ijk} \zeta \delta^+_{ij} \right], \\
\text{ch}_3(\mathcal{E}) &= [2 \lambda (\lambda^2 - f^2 |\eta|^2) + 2g^2 f(\xi, \eta) |\nu|], \\
\text{ch}_3(\hat{\mathcal{E}}) &= -[2 \lambda (\lambda^2 - f^2 |\eta|^2) + 2\hat{g}^2 f(\xi, \eta) |\nu|],
\end{align*}
\]

(3.32)

Substituting these expressions and \( \mathcal{J} = J h^+_{kk} \) (2.11) we obtain

\[
2J(3\lambda^2 - f^2 |\eta|^2) = 2J^3.
\]  

(3.33)

Similarly, upon substitution of the same, (2.12) leads to the following pair,

\[
\begin{align*}
g^2 f(\xi \cdot \eta) + \lambda (\lambda^2 - f^2 |\eta|^2 - 3J^2) &> 0, \\
\hat{g}^2 f(\xi \cdot \eta) + \lambda (\lambda^2 - f^2 |\eta|^2 - 3J^2) &< 0.
\end{align*}
\]  

(3.34) 

(3.35)
Finally, the contribution to the D3-brane tadpole turns out to be

\[ \hat{\rho}_2 = 2g^2 f(\xi \cdot \eta) - 2\hat{g}^2 f(\hat{\xi} \cdot \eta). \]  

(3.36)

As in Model I, we choose the simple solution \( \lambda^2 = f^2 \eta^2 \). Substituting this in (3.33) we fix the value of \( J \) to

\[ J^2 = 2\lambda^2. \]  

(3.37)

Substituting this value of \( J \) in (3.34) and (3.35) we derive

\[ g^2 f(\xi \cdot \eta) > 6\lambda^3, \]
\[ \hat{g}^2 f(\hat{\xi} \cdot \eta) < 6\lambda^3. \]  

(3.38)

The quantization conditions restrict each of the three quantities \( g^2 f(\xi \cdot \eta), \hat{g}^2 f(\hat{\xi} \cdot \eta) \) and \( 6\lambda^3 \) to be integral. Let us now consider the quantization conditions in some detail. The fluxes through the \((1,1)\)-cycles are quantized as

\[ 2\lambda \delta_{ij} = m^+_{ij} = m^-_{ij}, \]
\[ 2f \eta_k = m^-_k, \]
\[ 2([f \eta_k](f \eta_j) - \lambda^2 \delta_{ij}) = n^-_{ij}, \]
\[ 2\lambda(f \eta_k) + g^2 \xi^k = n^+_k, \]
\[ 2\lambda(f \eta_k) + \hat{g}^2 \hat{\xi}^k = \hat{n}^+_k, \]
\[ 2\lambda(\lambda^2 - f^2 |\eta|^2) + 2g^2 f(\xi \cdot \eta) = \rho_1, \]
\[ 2\lambda(\lambda^2 - f^2 |\eta|^2) + 2\hat{g}^2 f(\hat{\xi} \cdot \eta) = -\hat{\rho}_1. \]  

(3.39)

These conditions lead to fixing the different parameters to discrete values as,

\[ \lambda = \frac{1}{2} m^+, \]
\[ f \eta_k = \frac{1}{2} m^-_k, \]
\[ g^2 \xi^k = n^+_k - \frac{1}{2} m^+ m^-_k, \]
\[ \hat{g}^2 \hat{\xi}^k = \hat{n}^+_k - \frac{1}{2} m^+ m^-_k, \]
\[ n^-_{ij} = \frac{1}{2} [m^-_i m^-_j - (m^+)^2 \delta_{ij}] \]
\[ \rho_1 = \frac{1}{4} m^+ [(m^+)^2 - (m^-)^2] + m^-_k [n^+_k - \frac{1}{2} m^+ m^-_k] > 0, \]
\[ -\hat{\rho}_1 = \frac{1}{4} m^+ [(m^+)^2 - (m^-)^2] + m^-_k [\hat{n}^+_k - \frac{1}{2} m^+ m^-_k] < 0. \]  

(3.40)

We have already chosen a solution, namely, \( \lambda^2 = f^2 \eta^2 \). Let us also choose \( \eta_k = (1,0,0) \) and \( \xi^k = \hat{\xi}^k = (2,0,0) \). We then have \( m^+ = m^-_3 = 0, 2g^2 = n^+_1 - \frac{1}{2}(m^+)^2 \) and \( 2\hat{g}^2 = \hat{n}^+_1 - \frac{1}{2}(m^+)^2 \). Since each non-zero entry of the matrix \( n^-_{ij} \) is an integer, \( m^+ = m^-_3 \) has to be an even integer too. Therefore it follows from the above equations that both \( g^2 \xi^k \) and \( \hat{g}^2 \hat{\xi}^k \) are integers. Further, substituting the values of \( \lambda, f, \xi, \hat{\xi} \) and \( \eta \) in (3.38) we obtain,

\[ \rho_1 = 2m^+ g^2 > \frac{3}{2}(m^+)^3, \]
\[ \hat{\rho}_1 = -2m^+ \hat{g}^2 > -\frac{3}{2}(m^+)^3. \]  

(3.41)
so that the minimal choice is
\[ \rho_1 = \frac{3}{2}(m^+)^3 + 2, \quad \hat{\rho}_1 = -\frac{3}{2}(m^+)^3 + 2. \] (3.42)

Thus, the contribution to the D3-brane tadpole is
\[ \rho_2 = \rho_1 + \hat{\rho}_1 = 4. \] (3.43)

Now from (3.41) and (3.42) we can write down the value of \( g^2 \) and \( \hat{g}^2 \) as,
\[ 2g^2 = \frac{3}{4}(m^+)^3 + \frac{2}{m^+}, \quad 2\hat{g}^2 = \frac{3}{4}(m^+)^3 - \frac{2}{m^+}. \] (3.44)

Since \( 2g^2 \) and \( 2\hat{g}^2 \) are integers, and \( m^+ \) is an even integer, the only possible solution is
\[ m^+ = 2, \quad g^2 = \frac{7}{2}, \quad \hat{g}^2 = \frac{5}{2}. \] (3.45)

As the maximum allowed value of D3-brane tadpole is 16 from (3.43) we conclude that this model can accommodate 4 such pairs of stacks.

We now demonstrate that three such pairs of stacks can stabilize all the complex structure moduli. Let us consider three pairs of stacks with fluxes such that
\[ \xi^{(1)} = \xi^{(2)} = 2\eta^{(1)}(0, 0, 0), \]
\[ \xi^{(2)} = \xi^{(3)} = 2\eta^{(2)}(0, 2, 0), \]
\[ \xi^{(3)} = \xi^{(1)} = 2\eta^{(3)}(0, 0, 2), \] (3.46)
where \( \eta^{(a)} \) denotes the value of the parameter \( \eta \) in the \( a \)-th pair. Substituting these in the expression of \( F(2,0) \) given in (3.2) and demanding that it vanishes, it turns out that the Abelian part is restrictive enough to fix all the complex structure moduli and so we consider only an Abelian flux. Thus we obtain,
\[ f_{\epsilon_klm}\eta_{m}\bar{I}_{ki}\bar{I}_{lj} + f_{\eta_{m}\epsilon_{jm}} = \lambda(\bar{I}_{ji} - \bar{I}_{ij}). \] (3.47)

Since \( \bar{I} \) is an invertible matrix, we can rewrite the first term in this equation as,
\[ \epsilon_{kml}\bar{I}_{ki}\bar{I}_{lj}i = \epsilon_{kln}\bar{I}_{ki}\bar{I}_{lj}(\bar{I}_{np}\bar{I}_{pm}^{-1}), \]
\[ = \Delta \bar{I}_{i} \epsilon_{ijp}\bar{I}_{pm}^{-1}, \] (3.48)
where \( \Delta = \det \bar{I} \) to simplify (3.47) to,
\[ f_{\epsilon_{in}}[\delta_{nm} + \Delta \bar{I}_{nm}^{-1}]\eta_{m} = \lambda(\bar{I}_{ni} - \bar{I}_{il}). \] (3.49)

Corresponding to the three choices of \( \eta \), that is, \( \eta^{(1)}, \eta^{(2)} \) and \( \eta^{(3)} \) in (3.46), equation (3.49) gives rise to three equations, namely,
\[ \delta_{n1} + \Delta \bar{I}_{n1}^{-1} = \frac{1}{2}f_{\epsilon_{in}}(\bar{I}_{ni} - \bar{I}_{il}), \]
\[ \delta_{n2} + \Delta \bar{I}_{n2}^{-1} = \frac{1}{2}f_{\epsilon_{in}}(\bar{I}_{ni} - \bar{I}_{il}), \]
\[ \delta_{n3} + \Delta \bar{I}_{n3}^{-1} = \frac{1}{2}f_{\epsilon_{in}}(\bar{I}_{ni} - \bar{I}_{il}). \] (3.50)
Using these equations we construct the $3 \times 3$ matrix $\bar{I}^{-1}$ as

$$\bar{I}^{-1} = \begin{pmatrix}
 a - \Delta^{-1} & a & a \\
 b & b - \Delta^{-1} & b \\
 c & c & c - \Delta^{-1}
\end{pmatrix},$$

(3.51)

where $a$, $b$, and $c$ are defined as,

$$(\bar{I}_{32} - \bar{I}_{23}) = a \Delta, \quad (\bar{I}_{13} - \bar{I}_{31}) = b \Delta, \quad (\bar{I}_{21} - \bar{I}_{12}) = c \Delta.$$  \hspace{1cm} (3.52)

Equating the determinant of the matrix on the right side of (3.51) to $\Delta - \Delta^{-1}$ we derive a relation among $a$, $b$, $c$, as

$$a + b + c = \Delta + \Delta^{-1}.$$ \hspace{1cm} (3.53)

Now we invert the matrix in (3.51) to obtain the conjugate of the complex structure matrix,

$$\bar{I} = \begin{pmatrix}
 a - \Delta & a & a \\
 b & b - \Delta & b \\
 c & c & c - \Delta
\end{pmatrix},$$

(3.54)

where we have the relation used (3.53). Upon substituting the entries of the matrix $\bar{I}$ from (3.54) into (3.52), self-consistency of these expressions leads to three equations as,

$$\begin{pmatrix} 1 & -1 & \Delta \\ \Delta & 1 & -1 \\ -1 & \Delta & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0.$$ \hspace{1cm} (3.55)

Summing the three equations we derive $a + b + c = 0$ and therefore, by (3.53), we get

$$\Delta + \Delta^{-1} = 0.$$ \hspace{1cm} (3.56)

Thus, we have two possible values of $\Delta$, viz. $\Delta = \pm i$. Noting that the only solution to the equation (3.55) is $a = b = c = 0$, we derive from (3.54) that $\bar{I}_{ij} = \mp i \delta_{ij}$. Thus, all the complex structure moduli are stabilized up to a sign in this model. Between these, only $\bar{I} = i \mathbb{1}_3$ is physically acceptable.

Let us now examine how many of the Kähler moduli can be stabilized in this model. For this, let us turn on fluxes with the parameters chosen above in the two stacks $\mathcal{E}$ and $\tilde{\mathcal{E}}$. Substituting (3.31) with these parameters in (2.11), we find two equations from the stacks $\mathcal{E}$ and $\tilde{\mathcal{E}}$, respectively.

$$\lambda^2 \bar{d}_{ii} - f^2 (\bar{d}_{ij} \eta_i \eta_j) = \det \mathcal{J} + (\lambda f \eta_k + g^2 \xi^k) \epsilon_{ijk} \bar{d}_{ij} = 0,$$ \hspace{1cm} (3.57)

$$\lambda^2 \bar{d}_{ii} - f^2 (\bar{d}_{ij} \eta_i \eta_j) = \det \mathcal{J} + (\lambda f \eta_k + \hat{g}^2 \tilde{\xi}^k) \epsilon_{ijk} \bar{d}_{ij} = 0,$$ \hspace{1cm} (3.58)

to be simultaneously satisfied to fix the Kähler form $\mathcal{J}$. Subtracting one from the other of these equations we obtain,

$$\left(g^2 \xi^k - \hat{g}^2 \tilde{\xi}^k\right) \epsilon_{ijk} \bar{d}_{ij} = 0.$$ \hspace{1cm} (3.59)

We now substitute the values of the parameters, determined above, $g^2 = 7/2$, $\hat{g}^2 = 5/2$ and the following three values of $\xi$ corresponding to the three pairs of stacks introduced in (3.46),

$$\xi^{(1)} = (2, 0, 0), \quad \xi^{(2)} = (0, 2, 0), \quad \xi^{(3)} = (0, 0, 2).$$ \hspace{1cm} (3.60)
in equation (3.59) and derive, $\mathcal{J}_{12} = \mathcal{J}_{21}, \mathcal{J}_{23} = \mathcal{J}_{32}, \mathcal{J}_{31} = \mathcal{J}_{13}$. Thus, $\mathcal{J}$ is fixed to be a symmetric matrix. Upon setting the antisymmetric part of $\mathcal{J}$ equal to zero reduce (3.57) and (3.58) into a single equation,

$$\lambda^2 \mathcal{J}_{ii} - f^2 \mathcal{J}_{ij} \eta_i \eta_j = \det \mathcal{J}. \tag{3.61}$$

Finally, substituting $\lambda^2 = f^2 |\eta|^2$ in (3.61), along with the following three values of $\eta$ corresponding to the three pairs of stacks (3.46),

$$\eta^{(1)} = (1, 0, 0), \quad \eta^{(2)} = (0, 1, 0), \quad \eta^{(3)} = (0, 0, 1), \tag{3.62}$$

we get equations:

$$\lambda^2 (\mathcal{J}_{22} + \mathcal{J}_{33}) = \lambda^2 (\mathcal{J}_{33} + \mathcal{J}_{11}) = \lambda^2 (\mathcal{J}_{11} + \mathcal{J}_{22}) = \det \mathcal{J}, \tag{3.63}$$

which implies, in turn, that

$$\mathcal{J}_{11} = \mathcal{J}_{22} = \mathcal{J}_{33} \equiv \mathcal{J}. \tag{3.64}$$

This exhausts all the relations available for fixing the moduli. Thus, in this model three parameters from the symmetric part of $\mathcal{J}$ are not fixed.

Let us indicate a possible set up for overhauling this stabilization scheme by fixing the symmetric part of $\mathcal{J}$ too. However, let us point out that tadpole bounds can not be met in trying to fix the Kähler moduli in totality. We can stabilize the off-diagonal terms of the symmetric matrix $\mathcal{J}$ to zero by introducing three further stacks of branes. Taking the liberty of introducing three more stacks with $\eta$ given by,

$$\eta^{(4)} = (1, 1, 0), \quad \eta^{(5)} = (1, 0, 1), \quad \eta^{(6)} = (0, 1, 1), \tag{3.65}$$

perhaps at the risk of violating the the tadpole conditions, and substituting these in (2.11) we derive,

$$\lambda^2 (2J - \mathcal{J}_{12}) = \lambda^2 (2J - \mathcal{J}_{23}) = \lambda^2 (2J - \mathcal{J}_{31}) = \det \mathcal{J}, \tag{3.66}$$

where we have used equation (3.64) and the fact that $\mathcal{J}$ is symmetric. These equations set all the off-diagonal terms to zero upon using $\det \mathcal{J} = 2\lambda^2 J$. Thus, these three additional pairs of stacks stabilize all the Kähler moduli $\mathcal{J} = iJ_{ij} dz^i \wedge d\bar{z}^j$ to $\mathcal{J} = \mathcal{J}^+_k l_k$.

## 4 Discussion

In this article we have discussed a scheme for moduli stabilization in an orientifold $T^6 / \Omega : \mathbb{Z}_2 \cdot (-1)^F_L$ of the type–IIB theory using D9-branes with a non-Abelian magnetic flux on the world-volume of the brane. Unlike its Abelian counterpart, derivation of the supersymmetry condition, though possible [19], has not been done explicitly from open string theory. We use the supersymmetry condition proposed in the context of BPS branes in an $\mathcal{N} = 2$ theory. We use this supersymmetry preserving condition in the toroidal orientifold. We have presented two simple models and demonstrated that properly chosen non-Abelian fluxes stabilize most of the moduli. As is well known, there is a restriction ensuing from D3-brane tadpole cancellation condition which limits the number of allowed D-brane severely. Therefore, although in the second model all the complex structure moduli are stabilized, the only some of the Kähler moduli could be stabilized in compliance with the tadpole constraints, a complete stabilization being prevented by the dearth of branes allowed.

In both the models that we analyze we obtain the value of the Kähler volume to be of the order of the string scale. In these models the arbitrarily small values of the Kähler volume can be attained by scaling
the wrapping numbers. Though we have not studied it, a natural candidate to obtain large Kähler volume seems to be the T-dual of the present version, namely the type–I string theory on the orientifold [12, 18].

We have not considered the issues regarding the stability of the bundles used for stabilization. As the volume turns out to be small it will be prudent to check the Π-stability of the bundles which is valid all over the Kähler moduli space. However, in order to check Π-stability one needs exact expressions of the central charge, which is not available in the present case at the moment. For some of the Calabi-Yau manifolds, though, one can write down exact expressions of central charges associated with the D-brane configuration from the periods of the mirror manifolds. Thus, Calabi-Yau manifolds are interesting candidates for generalizations of this scheme of moduli stabilization. As we have explored here only a fraction of the different possibilities, it may be possible, even within the current set up, to find stable bundles that stabilize the Kähler volume to a large value.

Though we showed the stabilization of complex structure moduli, the axion-dilaton modulus remains unfixed. In addition to the present configuration, one can [9] turn on NS-NS and RR fluxes through three-cycles and thus stabilize the axion-dilaton modulus as has been done in the context of Abelian fluxes. However, turning on three-form fluxes in the presence of brane may involve Freed-Witten anomaly [25] which need to be taken care of. For an analysis of Supersymmetric D-branes in presence of background RR fluxes see [35].

A natural extension of this work is to study supersymmetry breaking in this formulation. A mechanism of supersymmetry breaking by generating D-terms has already been discussed [36,37] for D-branes with Abelian magnetic fluxes. It is interesting to study this (and other) supersymmetry breaking mechanism to the case of non-Abelian fluxes. A statistical measure of the realistic vacua for this class of moduli stabilization along the line of [38, 39], as well as a study of open string moduli [40] will also be of relevance for a better understanding of such string backgrounds.

A Appendix

In this Appendix we lay down the conventions and notations we used in the main text. Complex co-ordinates of $X$, namely, $z^i$, $\bar{z}^\bar{i}$ are related to the real co-ordinates $x^i$, $y^i$ through $z^i = (x^i + I_{ij}y^j)$, $i, j = 1, 2, 3$. The $3 \times 3$ matrix $I_{ij}$ with complex entries stands for the complex structure of $X$. We use unbarred indices such as $i, j, \cdots$ for the holomorphic co-ordinates and barred indices such as $\bar{i}, \bar{j}, \cdots$ for the antiholomorphic ones.

In the complex basis of co-ordinates, the two-form field strengths are denoted $\mathcal{F}^{(2,0)} = \mathcal{F}_{ij}dz^i \wedge dz^j$, $\mathcal{F}^{(1,1)} = \mathcal{F}_{ij}dz^i \wedge d\bar{z}^j$, etc., while in the basis of real forms, they are denoted $F_{x^ix^j}$. The Kähler two-form of the compactification manifold $X$ is written as $\mathcal{J} = \mathcal{J}_{ij}dz^i \wedge d\bar{z}^j$. We use the following notation for the basis of $(1,1)$-, $(2,2)$- and $(3,3)$-forms, respectively, in space-time co-ordinates,

$$h_{ij} = \frac{1}{2} dz^i \wedge d\bar{z}^j,$$

$$\mathcal{H}_{ij} = -\frac{1}{4}(\frac{1}{2}\epsilon_{ikm}dz^k \wedge dz^m) \wedge (\frac{1}{2}\epsilon_{jln}d\bar{z}^l \wedge d\bar{z}^n),$$

$$v = -\frac{i}{8} \Pi_{i=1}^3 (dz^i \wedge d\bar{z}^i)$$

(A.1)

(A.2)

(A.3)

The elements of the above basis for $(1,1)$-forms and $(2,2)$-forms are complex. We introduced another
basis in which the even dimensional forms are also hermitian,

\[ h_{ij}^- = \frac{1}{2} [h_{ij} - h_{ji}] , \]  
\[ h_{ij}^+ = \frac{i}{2} [h_{ij} + h_{ji}] , \]  
\[ \delta h_{ij}^- = \frac{1}{2} [\delta h_{ij} + \delta h_{ji}] , \]  
\[ \delta h_{ij}^+ = \frac{i}{2} [\delta h_{ij} - \delta h_{ji}] . \]  

The \((1, 1)\)- and \((2, 2)\)-forms are related as,

\[ h_{ij}^- \wedge h_{kl}^- = \frac{1}{2} (\epsilon_{ikm} \epsilon_{jln} - \epsilon_{ilm} \epsilon_{jkn}) \delta h_{mn}^- , \]  
\[ h_{ij}^- \wedge h_{kl}^+ = \frac{1}{2} (\epsilon_{ikm} \epsilon_{jln} + \epsilon_{ilm} \epsilon_{jkn}) \delta h_{mn}^+ , \]  
\[ h_{ij}^+ \wedge h_{kl}^+ = -\frac{1}{2} (\epsilon_{ikm} \epsilon_{jln} + \epsilon_{ilm} \epsilon_{jkn}) \delta h_{mn}^- . \]

The intersections of these forms are,

\[ h_{ij}^- \wedge \delta h_{kl}^- = 0 = h_{ij}^- \wedge \delta h_{kl}^+ , \]  
\[ h_{ij}^+ \wedge \delta h_{kl}^- = -\frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) v , \]  
\[ h_{ij}^- \wedge \delta h_{kl}^+ = -\frac{1}{2} (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}) v . \]

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