The Effect of Supersymmetric Particles on $b \to s\gamma$ Decay in Supergravity Models*

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Abstract

The effect of loops involving charginos with up-type squarks, and gluinos with down-type squarks, on the inclusive decay mode $b \to s\gamma$ is studied in the context of minimal $N = 1$ Supergravity models with a radiatively broken electroweak symmetry group. It is confirmed that the strong constraints imposed by the CLEO upper bound $B(b \to s\gamma) < 5.4 \times 10^{-4}$ on two-Higgs doublets models are much weaker in supersymmetric theories due to partial cancelations from loops involving charginos and up-type squarks. The dependence of the branching ratio and the supersymmetry masses on the top quark mass is explored.

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The decay $b \to s \gamma$ is forbidden at tree level but induced in the Standard Model (SM) at one loop by $W$ and Goldstone bosons together with up-type quarks in the internal lines of the loop\cite{1}. The SM value of the branching ratio of this decay is $B(b \to s \gamma) \approx 4 \times 10^{-4}$ for $m_t = 140$ GeV and increases with $m_t$. In two-Higgs-doublets models, loops involving charged Higgs bosons and up-type quarks have to be added\cite{2}. The contribution from the charged Higgs boson in type II models (where one Higgs doublet couples to the up-type quarks and the other Higgs doublet couples to the down-type quarks) has the same sign as the SM contribution. In type I models (where only one Higgs doublet couples to the fermions) the charged Higgs boson contribution does not have a definite sign.

The latest experimental upper bound on the branching fraction for the inclusive decay mode $b \to s \gamma$, given by $B(b \to s \gamma) < 5.4 \times 10^{-4}$ at 90\% c.l.\cite{3}, sets powerful constraints on the charged Higgs boson mass in two Higgs doublets models of type II\cite{4}. Other corrections that may be important have been calculated recently: next-to-leading logarithmic QCD-corrections\cite{5}, and electroweak corrections in the context of supersymmetry\cite{6} to the charged Higgs mass\cite{7} and to the charged Higgs-fermion-fermion vertex\cite{8}.

In supersymmetry, the contributions from charginos together with up-type squarks and from neutralinos and gluinos together with down-type squarks, have to be included\cite{9,10}. It was stressed that in this case, it is important the effect of loops involving supersymmetric particles\cite{11–13}. Here we study this effect in the context of the radiatively broken Minimal Supersymmetric Model\cite{14}, following ref. [9] and including the effects described in refs. [5,6].

Minimal $N = 1$ supergravity\cite{15} is characterized by the superpotential

$$W = h_{ij}^{ij} Q_i U_j c H_2 + h_{ij}^{ij} Q_i D_j c H_1 + h_{ij}^{ij} E_i E_j c H_1 + \mu \varepsilon_{ab} H_1^a H_2^b \quad (1)$$

where $i, j = 1, 2, 3$ are indeces in generation space, $\varepsilon_{ab}$ with $a, b = 1, 2$ is the antisymmetric tensor in two dimensions, and $\mu$ is the Higgs mass parameter. The different superfields transform under $SU(3) \times SU(2) \times U(1)$ according to: $Q =$
\((3, 2, 1/6), \ U^c = (3, 1, -2/3), \ D^c = (3, 1, 1/3), \ L = (1, 2, -1/2), \ E^c = (1, 1, 1), \ H_1 = (1, 2, -1/2), \) \ and \ \(H_2 = (1, 2, 1/2).\) \ The \(3 \times 3\) \ matrices \(h_U, h_D\) \ and \(h_E\) \ are \ the \ Yukawa \ couplings \ given \ by
\[
h_U = \frac{gV^\dagger}{\sqrt{2}m_Ws_\beta} \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{bmatrix}, \quad h_D = \frac{g}{\sqrt{2}m_Wc_\beta} \begin{bmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{bmatrix}
\] (2)

for the quarks, and for the charged leptons, \(h_E\) \ is \ given \ by \(h_D\) \ when \ the \ quark \ masses \ are \ replaced \ by \(m_e, m_\mu\) \ ans \(m_\tau).\) \ The \ Cabibbo-Kobayashi-Maskawa \ matrix \ is \ represented \ by \(V).\) \ The \ soft \ supersymmetry \ breaking \ terms \ are
\[
L_s = A_{ij}^{ij}U_i \tilde{Q}_i \tilde{U}_j H_2 + A_{ij}^{ij}D_i \tilde{Q}_i \tilde{D}_j H_1 + A_{ij}^{ij}E_i \tilde{E}_i \tilde{L}_j H_1 + B_{\mu\epsilon} \epsilon_{ab} H_1^a H_2^b + h.c. \ (3)
\]

plus a set of scalar and gaugino mass terms, which at the unification scale are
\[
L_m = m_0^2 \sum_i |S_i|^2 + \left[\frac{1}{2}M_{1/2} (\lambda_1 \lambda_1 + \lambda_2 \lambda_2 + \lambda_3 \lambda_3) + h.c.\right] \ (4)
\]

where \(S_i\) \ are \ all \ the \ scalars \ of \ the \ theory \ and \(\lambda_i, i = 1, 2, 3\) \ are \ the \ gauginos \ corresponding \ to \ the \ groups \(U(1), SU(2)\) \ and \(SU(3)\) \ respectively. \ In \ eq. \ (3) \ all \ the \ fields \ are \ scalar \ components \ of \ the \ respective \ superfields. \) \ The \ mass \ parameters \(A\) \ and \(B\) \ are \ of \ \(O(m_0)\) \ and \ in \ some \ supergravity \ models \ they \ satisfy \ the \ following \ relation \ at \ the \ scale \(M_X: \ A = B + m_0\) \ where \(A\) \ is \ the \ common \ value \ for \ the \(A_a^{ij} (a = U, D, E)\) \ parameters \ at \ the \ unification \ scale: \(A_U^{ij} = A_D^{ij} = A_E^{ij} = \delta^{ij}.\)

At the weak scale, the tree level Higgs potential is given by
\[
V = m_{1H}^2 |H_1|^2 + m_{2H}^2 |H_2|^2 - m_{12}^2 (H_1 H_2 + h.c.) + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g^2 |H_1^* H_2|^2 \ (5)
\]

where \(m_{1H}^2 = m_i^2 + |\mu|^2 \ (i = 1, 2)\) \ and \(m_{12}^2 = -B\mu.\) \ The \ two \ Higgs \ doublets \ mass \ parameters \(m_1\) \ and \(m_2\) \ satisfy \(m_1 = m_2 = m_0\) \ at \ the \ unification \ scale \(M_X.\) \ The
three mass parameters in eq. (5) can be replaced by the $Z$ boson mass $m_Z$, the CP-odd Higgs mass $m_A$, and the ratio between the vacuum expectation values of the two Higgs doublets $\tan \beta \equiv v_2/v_1$, according to the formulas

\begin{align*}
m_{1H}^2 &= -\frac{1}{2}m_Z^2c_{2\beta} + \frac{1}{2}m_A^2(1 - c_{2\beta}) \\
m_{2H}^2 &= \frac{1}{2}m_Z^2c_{2\beta} + \frac{1}{2}m_A^2(1 + c_{2\beta}) \quad (6) \\
m_{12}^2 &= \frac{1}{2}m_A^2s_{2\beta}
\end{align*}

where $s_{2\beta}$ and $c_{2\beta}$ are sine and cosine functions of the angle $2\beta$. The previous relations are valid at tree level. The effects of the one-loop corrected Higgs potential may be important in some cases\[16\], especially near $\tan \beta = 1$ when $m_{1H} = m_{2H} = m_{12}$ and the lightest neutral Higgs mass comes only from radiative corrections\[17\].

In Fig. 1 it is plotted a typical solution of the renormalization group equations (RGE) in the spirit of ref. [18], but including the trilinear $A$ parameters and other Higgs mass parameters as well. The effects of the supersymmetric threshold are neglected. Also, the squark and slepton mass parameters $M_i, i = Q, U, D, L, E$ and the $A_i, i = U, D, E$ parameters are $3 \times 3$ matrices in generation space and the third diagonal element is plotted. The RGE used are given in ref. [9] with the exception of the $A$ parameters, whose equations are taken from ref. [19]. The set of independent parameters is chosen to be $m_t, m_A$ and $\tan \beta$ at the weak scale, $M_{1/2}$ at the unification scale, and the sign of $\mu$ as a discrete parameter. Several features can be seen from Fig. 1. The sleptons are lighter than the squarks, the reason being that there is no contribution from the strong coupling constant to their RGE. Besides, $M_L$ is larger than $M_E$ at the weak scale because the gauge coupling constant $g$ is larger than $g'$. In the squark mass parameters, the effect of the gauge couplings is counteracted by the Yukawa couplings, especially from the top quark. $M_D$ is the largest because it does not receive an opposite effect from the top quark Yukawa; on the other hand, $M_U$ is the smaller of the three squark mass parameters because in its RGE the top quark Yukawa coupling has a larger coefficient. The evolution of the mass parameters in the gaugino sector is trivial since it is governed by the respective gauge coupling. In the Higgs sector, since the squared masses
may become negative, \(-\sqrt{-m_i^2}\) is plotted when \(m_i^2 < 0\), where \(i = 1H, 2H, 12\). The evolution of \(m_1\) is dominated by the gauge coupling constants, but since there is no contribution from the strong coupling, its dependence on the scale is weak. On the contrary, the evolution of \(m_2\) is dominated by the top Yukawa coupling, producing a large splitting \(m_1^2 - m_2^2\) at the weak scale. The value of this splitting is a necessary ingredient to produce the correct electroweak symmetry breaking.

The QCD uncorrected amplitude for the decays \(b \to s\gamma\) and \(b \to sg\) are

\[
A^{\gamma,g}(m_W) = A^{\gamma,g}_{SM} + (f^+ f^-)A^{1\gamma,g}_{H^\pm} + (f^-)^2 \cot^2 \beta A^{2\gamma,g}_{H^\pm} + A^{\gamma,g}_{\chi^\pm} + A^{\gamma,g}_{\tilde{g}} \tag{7}
\]

where the form factors \(f^\pm\) come from the renormalization of the charged Higgs boson coupling to a pair of fermions\(^8\). The different amplitudes \(A\) can be found in ref. [9]. If we now run the scale from \(m_W\) to \(m_b\) and introduce the QCD corrections, we get

\[
A^{\gamma}(m_b) = \eta^{\frac{16}{23}} \left[ A^{\gamma}(m_W) + \frac{8}{3} A^g(m_W) (\eta^{\frac{2}{23}} - 1) \right] + CA_0^\gamma \tag{8}
\]

where \(\eta = \alpha_s(m_b) / \alpha_s(m_W) \approx 1.83\) and \(A_0^\gamma\) is given by

\[
A_0^\gamma = \frac{\alpha_W \sqrt{\alpha} V_{ts}^* V_{tb}}{2\sqrt{\pi} m_W^2} \tag{9}
\]

with \(C = 0.177\), \(\alpha_W = g^2 / 4\pi\), and \(\alpha = e^2 / 4\pi\). This last term proportional to \(C\) comes from mixing of four quark operators\(^{12}\).

In fig. 2 the top quark mass is taken as a variable, keeping all the other parameters of fig. 1 unchanged. The dependence on the top quark mass of the branching ratio \(B(b \to s\gamma)\) can be seen in fig. 2(a). In the SM this branching ratio grows with the top quark mass and remains below the CLEO bound in the hole range of \(m_t\). In the SUSY–GUT model, for the parameters considered here, the branching ratio decreases with the top quark mass except for very large values of \(m_t\). This effect is due to a faster growing (and with opposite sign) chargino contribution compared
to the charged Higgs contribution [see fig. 2(b)]. This can be seen also in ref. [12] taking into account that in the SUSY–GUT model, a change in \( m_t \) implies a change in \( m_0 \) [see fig. 2(e)], in opposition to the treatment in ref. [12] where both parameters are independent. If the top quark mass increases, the splitting \( m_1^2 - m_2^2 \) at the weak scale increases also, and to keep it constant \( m_0 \) must decrease. This in turn will decrease the absolute values of \( m_1^2 \) and \( m_2^2 \), so \( \mu \) will grow in order to keep \( m_{1H}^2 \) and \( m_{2H}^2 \) unchanged. Finally, in figs. 2(c) and 2(d) are displayed the masses of the different particles as a function of the top quark mass. As a final comment, it is clear that the decay \( b \to s\gamma \) does not strongly constrain supersymmetry, and in the case of minimal \( N = 1 \) supergravity models with radiatively broken electroweak symmetry, the branching ratio \( B(b \to s\gamma) \) lies below the CLEO bound and even below the SM value for reasonable values of the free parameters.

Note added: When this work was completed, we received two preprints\(^{[20]}\) that calculate the effect on \( B(b \to s\gamma) \) due to loops involving charginos and up-type squarks.

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FIGURE CAPTIONS

1) Evolution of the different parameters of the model from the unification scale $M_X = 10^{16}$ GeV to the weak scale $m_Z$.

2) Dependence on the top quark mass of: (a) Branching ratio of the inclusive decay $B(b \rightarrow s\gamma)$ for the SM and for the SUSY-GUT model. (b) Relative size of the chargino, charged Higgs and gluino contributions to the $b \rightarrow s\gamma$ amplitude with respect to the SM amplitude. The QCD corrections are not implemented. Note that the sign of the chargino and gluino contributions is changed, and that the gluino contribution is amplified by a factor of 10. (c) Masses of the lightest and the heaviest up-type squark (solid) and charginos (dashes), the charged Higgs (dotdash) and the gluino (dots). (d) Masses of the lightest and the heaviest down-type squark (solid), charged slepton (dashes) and neutralino (dotdash), and the lightest of the sneutrinos (dots). (e) Common scalar mass parameter $m_0$ at the unification scale and the Higgs mass parameters $B$ and $\mu$ at the weak scale.