Two-body charmed baryon decays involving decuplet baryon
in the quark-diagram scheme

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Abstract

In the quark-diagram scheme, we study the charmed baryon decays of $B_c \to B^* M$, where $B_c$ is $\Lambda_c^+$ or $\Xi_c^{+(0)}$, together with $B^*$ ($M$) the decuplet baryon (pseudoscalar meson). It is found that only two $W$-exchange processes are allowed to contribute to $B_c \to B^* M$. Particularly, we predict $\mathcal{B}(\Lambda_c^+ \to \Sigma^{*0(+)\pi^{0(0)}}) = (2.8 \pm 0.4) \times 10^{-3}$, which respects the isospin symmetry. Besides, we take into account the $SU(3)$ flavor symmetry breaking, in order to explain the observation of $\mathcal{B}(\Lambda_c^+ \to \Sigma^{*+\eta})$. For the decays involving $\Delta^{++}(uuu)$, we predict $\mathcal{B}(\Lambda_c^+ \to \Delta^{++\pi^-}, \Xi_c^+ \to \Delta^{++K^-}) = (7.0 \pm 1.4, 13.5 \pm 2.7) \times 10^{-4}$ as the largest branching fractions in the singly Cabibbo-suppressed $\Lambda_c^+, \Xi_c^+ \to B^* M$ decay channels, respectively, which are accessible to the LHCb, BELLEII and BESIII experiments.

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I. INTRODUCTION

To determine the mass and lifetime of the $\Lambda_b$ baryon, $\Lambda_c^+$ is often taken as the final state in the $\Lambda_b$ decays [1], which involves the favored Cabibbo-Kobayashi-Maskawa (CKM) matrix elements for bigger branching fractions. With the higher precision in the recent years [2, 3], the subsequent $\Lambda_c^+ \rightarrow pK^-\pi^+$ decay has helped to make more accurate observations for the $\Lambda_b$ decays, which receives the significant contribution from $\Lambda_c^+ \rightarrow \Delta^{++}K^-, \Delta^{++} \rightarrow p\pi^+$. Similarly, one uses $\Xi_b^0 \rightarrow \Xi_c^+\pi^-$ to determine the $\Xi_b^0$ lifetime, whereas we find that the subsequent process $\Xi_c^+ \rightarrow pK^-\pi^+$ and its resonant contribution from $\Xi_c^+ \rightarrow \Delta^{++}K^-, \Delta^{++} \rightarrow p\pi^+$ have not been well studied yet [1]. Therefore, $B_c \rightarrow B^{*}M$ plays the key role in the precision measurements for the multi-body decays of beauty and charm baryons, where $B_c = (\Lambda_c^+, \Xi_c^{++(0)})$, $B^*$ the decouplet baryon and $M$ the meson state, such as $\Lambda_c^+(\Xi_c^+) \rightarrow \Delta^{++}K^-$. The $B_c \rightarrow B^{*}M$ decays are not richly observed. Therefore, it is still unclear how the $B_c \rightarrow B^{*}M, B^* \rightarrow BM'$ decays mix with the non-resonant contributions to $B_c \rightarrow BM'M'$. In addition, $B_c \rightarrow BV, V \rightarrow MM'$ with $V$ the vector meson causes more complicated mixtures [1]. The $SU(3)$ flavor ($SU(3)_f$) symmetry has been widely applied to the charmed baryon decays [4–16]. By well explaining the data, the flavor symmetry does not appear to be severely broken in $B_c \rightarrow BM$ [17], where $B$ denotes the octet baryon. By contrast, $B(\Lambda_c^+ \rightarrow \Sigma^{++}\eta)$ not well interpreted by the $SU(3)_f$ symmetry might hint the broken effect in $B_c \rightarrow B^*M$ [7], which could be as large as that in the $D$ meson decays [18–21].

For a better understanding of the hadronization in $B_c \rightarrow B^{*}M$, there have been some theoretical attempts, which are in terms of the pole model, quark model and irreducible $SU(3)$ flavor ($SU(3)_f$) symmetry [4–7, 22–24]. Particularly, the quark-diagram scheme with the topological $SU(3)_f$ symmetry provides a clear picture for the decay processes [17, 25–27]. Due to the fact that $B^*$ is a spin-3/2 baryon with totally symmetric quark contents, it can be shown that the topological diagrams involving the flavor anti-symmetric quark pair in $B^*$ are forbidden or suppressed. Therefore, we purpose to use the quark-diagram scheme to relate all possible $B_c \rightarrow B^{*}M$ decay channels. With the existing data, we will perform the numerical analysis, and determine different topological contributions. We can hence test the validity of the topological scheme, which involves the $SU(3)_f$ symmetry and its broken effect. Furthermore, we will give predictions for $B(B_c \rightarrow B^{*}M)$ to be compared to the future measurements, which can help to clarify how $B_c \rightarrow B^{*}M, B^* \rightarrow BM'$ mixes
with $B_c \rightarrow BV, V \rightarrow MM'$ and the non-resonant configuration in $B_c \rightarrow BM'M'$.

II. FORMALISM

A. Effective Hamiltonian in the flavor symmetry

To study the two-body charmed baryon decays, the corresponding quark-level effective Hamiltonian is given by [28]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=1,2} c_i \left( \lambda_a O^a_i + \lambda_p O^p_i + \lambda_c O^c_i \right),$$

(1)

with $\lambda_{(a,p,c)} \equiv (V^* V_{ud}, V^* V_{up}, V^* V_{us})$ and $p = (d, s)$, where $G_F$ is the Fermi constant, and $c_i$ the Wilson coefficients. The current-current operators $O^{(a,p,c)}_i$ are written as

$$O^a_i = (\bar{u}d)(\bar{s}c), \quad O^p_i = (\bar{u}p)(\bar{p}c), \quad O^c_i = (\bar{u}s)(\bar{d}c),$$

(2)

where $(\bar{q}_1 q_2) = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$, and the subscripts $(\alpha, \beta)$ denote the color indices. With $s_c \equiv \sin \theta_c \simeq 0.22$, where $\theta_c$ denotes the Cabibbo angle for the quark-mixing in the weak interaction, the decays with $|\lambda_{(a,p,c)}| \simeq (1, s_c, s^2_c)$ are regarded as the Cabibbo-allowed (CA), singly Cabibbo-suppressed (SCS) and doubly Cabibbo-suppressed (DCS) processes, respectively.

For the lowest-lying anti-triplet charmed baryon states $\Xi^0_c$, $\Xi^+_c$ and $\Lambda^+_c$ that consist of $(ds - sd)c$, $(su - us)c$ and $(ud - du)c$, respectively, we present them as

$$B_c = \begin{pmatrix} 0 & \Lambda^+_c & \Xi^+_c \\ -\Lambda^+_c & 0 & \Xi^0_c \\ -\Xi^+_c & -\Xi^0_c & 0 \end{pmatrix}. $$

(3)

The pseudoscalar meson states are given by

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}(\pi^0 + c\phi\eta + s\phi'\eta') & \pi^- & K^- \\ \pi^+ & -\frac{1}{\sqrt{2}}(\pi^0 - c\phi\eta - s\phi'\eta') & K^0 \\ K^+ & K^0 & -s\phi\eta + c\phi' \eta' \end{pmatrix},$$

(4)
The decuplet baryons are written as

where \( (\eta, \eta') \) mix with \( \eta_q = \sqrt{1/2(u\bar{u} + d\bar{d})} \) and \( \eta_s = s\bar{s} \). The mixing angle \( \phi = (39.3 \pm 1.0)^\circ \) in \( (s\phi, c\phi) \equiv (\sin \phi, \cos \phi) \) comes from the mixing matrix, given by

\[
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} =
\begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix}
\begin{pmatrix}
\eta_q \\
\eta_s
\end{pmatrix}.
\]

(5)

The decuplet baryons are written as

\[
B^* = \frac{1}{\sqrt{3}}
\begin{pmatrix}
\sqrt{3}\Delta^{++} & \Delta^+ & \Sigma^{*+} \\
\Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\
\Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0}
\end{pmatrix}
\begin{pmatrix}
\Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\
\Delta^0 & \sqrt{3}\Delta^- & \Sigma^{*-} \\
\Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*-}
\end{pmatrix}
\begin{pmatrix}
\Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \\
\Sigma^{*0} & \Xi^{*-} & \sqrt{3}\Omega^-
\end{pmatrix}.
\]

(6)

By neglecting the Lorentz indices, \( H_{\text{eff}} \) for the \( c \rightarrow q_i\bar{q}_j\bar{q}_k \) transition can be presented with the tensor notation, \( H_j^{ki} \), and the nonzero entries are given by

\[
H_2^{31} = \lambda_a, H_2^{21} = \lambda_d, H_3^{31} = \lambda_s, H_3^{21} = \lambda_c.
\]

(7)

B. The quark-diagram scheme

In the quark-diagram scheme, there exist six different topological diagrams for the \( B_c \rightarrow B^{(*)} M \) decays, as drawn in Figs. 1(a,b,c) and 1(d,e,f), parameterized as the topological amplitudes \( (T, C, C') \) and \( (E', E_B, E_M) \), respectively. More explicitly, \( T \) and \( C^{(*)} \) proceed with the \( W \)-boson emission \( (W_{\text{EM}}) \). By exchanging the \( W \) boson \( (W_{\text{EX}}) \), it gives rise to \( E' \) and \( E_B(M) \). Since only \( (T, C) \) can be decomposed of two separate matrix elements based on the factorization, that is, \( (T, C) \propto \langle M|q_1q_2|0\rangle \langle B^{(*)}|\bar{q}_3c|B_c\rangle \) and \( (C', E', E_M, B) \) as the factorizable and non-factorizable amplitudes, respectively.
Furthermore, it is found in Figs. (a,b) that $\mathbf{B}_c$ with $(q_a q_b - q_b q_a)c$ cannot be turned into $\mathbf{B}^*(q_a q_b q_{k(i)})$, where $q_a q_b q_{k(i)}$ are totally symmetric, such that $(T, C)$ give no contributions to $\mathbf{B}_c \rightarrow \mathbf{B}^* M$. Thus, the $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ decays are purely non-factorizable processes. In addition, $C'$ and $E'$ are suppressed in $\mathbf{B}_c \rightarrow \mathbf{B}^* M$ [25], which is in accordance with the Körner-Pati-Woo theorem [31]. With the current-current structure of $(\bar{q}_i q_j)_{V-A}(\bar{q}_k c)_{V-A}$ in Eq. (2), $q_i$ and $q_k$ are color anti-symmetric. When combined as the constituents of the baryon, $q_{i, k}$ are flavor anti-symmetric, such that the topological diagrams ($C', E'$) in Figs. (c,d) contribute to $\mathbf{B}_c \rightarrow \mathbf{B} M$, instead of $\mathbf{B}_c \rightarrow \mathbf{B}^* M$. Consequently, we are left with the $W_{EX}$ topological diagrams ($E_B, E_M$) in Figs. (e,f) for $\mathbf{B}_c \rightarrow \mathbf{B}^* M$.

To proceed, we derive the amplitudes as $A(\mathbf{B}_c \rightarrow \mathbf{B}^* M) = (G_F/\sqrt{2})T(\mathbf{B}_c \rightarrow \mathbf{B}^* M)$. Explicitly, the $T$ amplitudes ($T$-amps) read [17, 25, 27]

$$T(\mathbf{B}_c \rightarrow \mathbf{B}^* M) = E_B^{(s)}(\mathbf{B}_c)_{iab}^a H_j^{ki}(\mathbf{B}^*)_{kab}(M)_{i}^{b} + E_M^{(s)}(\mathbf{B}_c)_{iab}^a H_j^{ki}(\mathbf{B}^*)_{kab}(M)_{i}^{b},$$

where the parameters $E_{B, M}^{(s)}$ correspond to the topological diagrams in Figs. (e) and (f), respectively. The $W_{EX}$ decay process needs an additional quark pair from $g \rightarrow q\bar{q}$, where $q\bar{q}$ could be $u\bar{u}$, $d\bar{d}$ or $s\bar{s}$. To take into account the broken $SU(3)_f$ symmetry, $E_{B(M)}$ with $g \rightarrow s\bar{s}$ can be more specifically denoted by $E_{B(M)}^s$. Under the exact $SU(3)_f$ symmetry, it leads to $E_{B(M)}^s = E_{B(M)}$ [26]. In Tables (I, II) and (III) we present the full expansions of $T(\mathbf{B}_c \rightarrow \mathbf{B}^* M)$ for the CA, SCS and DCS decay modes, respectively. For the branching fractions, we use the equation for the two-body decays, given by [1]

$$B(\mathbf{B}_c \rightarrow \mathbf{B}^* M) = \frac{G_F^2|\bar{p}_{B^*}|\tau_{B_c}|T(\mathbf{B}_c \rightarrow \mathbf{B}^* M)|^2}{16\pi m_{B_c}^2},$$

$$|\bar{p}_{B^*}| = \sqrt{(m_{B_c}^2 - m_+^2)(m_{B_c}^2 - m_-^2)},$$

with $m_\pm = m_{B^*} \pm m_M$, where $\tau_{B_c}$ stands for the $\mathbf{B}_c$ baryon lifetime.

III. NUMERICAL ANALYSIS AND DISCUSSIONS

In the numerical analysis, we adopt the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements as [1]

$$(V_{cs}, V_{ud}, V_{us}, V_{cd}) = (1 - \lambda^2/2, 1 - \lambda^2/2, \lambda, -\lambda),$$

with $\lambda = s_c = 0.22453 \pm 0.00044$ in the Wolfenstein parameterization. Besides, the $\mathbf{B}_c$ and $\mathbf{B}^*$ masses, together with the lifetime for $\mathbf{B}_c$, are adopted from the PDG [1]. We perform
a minimum $\chi^2$-fit with $\chi^2 = \sum(B_{th} - B_{ex})^2/\sigma_{ex}^2$, where $B_{th(ex)}$ represents the theoretical (experimental) input of the branching ratio, and $\sigma_{ex}$ the experimental error. We calculate $B_{th}$ with the equation in Eq. (9), together with $(B_{ex}, \sigma_{ex})$ from Table I. Note that $B(\Xi_c^+ \to \Sigma^* K^0, \Xi^0 \pi^+)$ are not involved in the fit.

We use two scenarios for the global fit. In the first scenario (S1), we take $E_B^s = E_{B(M)}$ under the exact $SU(3)_f$ symmetry. Since $E_B$ and $E_M$ are complex numbers, it leads to three independent parameters, given by

$$|E_B|, |E_M|e^{i\delta_{EM}},$$

where $E_B$ is set to be real, and $\delta_{EM}$ is a relative strong phase. Using the $\chi^2$-fit, we extract that

$$(|E_B|, |E_M|) = (0.41 \pm 0.03, 0.34 \pm 0.03) \text{ GeV}^3,$$

$$\delta_{EM} = (180.0 \pm 35.8)^\circ,$$  \hspace{1cm} (11)

with $\chi^2/n.d.f = 4.5$, where $n.d.f = 1$ is the number of the degrees of freedom. For $\delta_{EM}$, its information is from $B(\Lambda_c^+ \to \Sigma^* \eta)$. Although $\delta_{EM} = 180^\circ$ has induced the largest positive interference between $E_B$ and $E_M$, our result of $B(\Lambda_c^+ \to \Sigma^* \eta) = (5.3 \pm 0.7) \times 10^{-3}$ is still

### TABLE I. Cabibbo-allowed $B_c \to B^*M$ decays.

| Decay modes | $T$-amp | $10^3 B$ (S1, S2) [our work] | $10^3 B$ (Spm, Sem) [7] | $10^3 B_{ex}$ [1, 32] |
|-------------|---------|-------------------------------|--------------------------|-------------------|
| $\Lambda_c^+ \to \Delta^{++}K^-$ | $-\lambda_4 E_M$ | $(12.0 \pm 2.2, 11.7 \pm 2.3)$ | $(15.3 \pm 2.4, 12.4 \pm 1.0)$ | $10.8 \pm 2.5$ |
| $\Lambda_c^+ \to \Delta^+K^0$ | $-\lambda_6 E_M$ | $(4.0 \pm 0.7, 3.9 \pm 0.8)$ | $(5.1 \pm 0.8, 4.1 \pm 0.3)$ | — |
| $\Lambda_c^+ \to \Sigma^{*0}\pi^+$ | $-\lambda_7 \sqrt{6} E_B$ | $(2.9 \pm 0.4, 2.8 \pm 0.4)$ | $(2.2 \pm 0.4, 2.1 \pm 0.2)$ | — |
| $\Lambda_c^+ \to \Sigma^{*+}\pi^0$ | $-\lambda_8 \sqrt{6} E_B$ | $(2.0 \pm 0.4, 2.8 \pm 0.4)$ | $(2.2 \pm 0.4, 2.1 \pm 0.2)$ | — |
| $\Lambda_c^+ \to \Sigma^{*+}\eta$ | $-\lambda_7 \sqrt{6} (E_B c\phi - \sqrt{2} E_M^{(s)} s\phi)$ | $(5.3 \pm 0.8, 7.3 \pm 1.5)$ | $(3.1 \pm 0.6, 6.2 \pm 0.5)$ | $9.1 \pm 2.0$ |
| $\Lambda_c^+ \to \Sigma^{*+}\eta'$ | $-\lambda_8 \sqrt{6} (E_B s\phi + \sqrt{2} E_M^{(s)} c\phi)$ | $(0.0)$ | — | — |
| $\Lambda_c^+ \to \Xi^{*0}K^+$ | $-\lambda_9 \sqrt{6} E_M$ | $(3.9 \pm 0.6, 3.9 \pm 0.6)$ | $(1.0 \pm 0.2, 4.1 \pm 0.3)$ | $4.3 \pm 0.9$ |
| $\Xi^0 \to \Sigma^{*+}K^-$ | $\lambda_1 \sqrt{3} E_M$ | $(1.8 \pm 0.3, 1.7 \pm 0.3)$ | $(3.1 \pm 0.5, 2.3 \pm 0.2)$ | — |
| $\Xi^0 \to \Sigma^{*0}K^0$ | $\lambda_1 \sqrt{3} E_M$ | $(0.9 \pm 0.2, 0.9 \pm 0.2)$ | $(1.6 \pm 0.2, 1.2 \pm 0.1)$ | — |
| $\Xi^0 \to \Sigma^{*0}\pi^+$ | $\lambda_1 \sqrt{3} E_B$ | $(2.6 \pm 0.4, 2.5 \pm 0.4)$ | $(2.8 \pm 0.5, 2.3 \pm 0.2)$ | — |
| $\Xi^0 \to \Xi^{*0}\pi^0$ | $\lambda_2 \sqrt{6} E_B$ | $(1.3 \pm 0.2, 1.3 \pm 0.2)$ | $(1.4 \pm 0.2, 1.2 \pm 0.1)$ | — |
| $\Xi^0 \to \Xi^{*0}\eta$ | $\lambda_3 \sqrt{6} (E_B c\phi - \sqrt{2} E_M^{(s)} s\phi)$ | $(2.4 \pm 0.4, 3.4 \pm 0.7)$ | $(2.1 \pm 0.4, 3.5 \pm 0.3)$ | — |
| $\Xi^0 \to \Xi^{*0}\eta'$ | $\lambda_3 \sqrt{6} (E_B s\phi + \sqrt{2} E_M^{(s)} c\phi)$ | $(0.01 \pm 0.04, 0.08 \pm 0.10)$ | — | — |
| $\Xi^0 \to \Omega^{-}K^+$ | $\lambda_4 E_B$ | $(4.8 \pm 0.7, 4.8 \pm 0.7)$ | $(2.3 \pm 0.5, 7.0 \pm 0.6)$ | $4.2 \pm 1.0$ |
| $\Xi^+ \to \Sigma^{*0}K^0$ | 0 | 0 | 0 | $28.6 \pm 16.8$ |
| $\Xi^+ \to \Xi^{*0}\pi^+$ | 0 | 0 | 0 | < 4.0 |
shown to be in tension with the observation of \((9.1 \pm 2.0) \times 10^{-3}\). Sizeably, it adds 3.6 to the total \(\chi^2\) value.

Since \(\Lambda_c^+ \to \Sigma^+ \eta\) is in association with \(|E_M^s|\), the tension hints the broken \(SU(3)_f\) symmetry, where \(|E_M^s|\) is not equal to \(|E_M|\). On the other hand, \(B(\Xi_c^0 \to \Omega^- K^+)\) is fitted to agree with the data, indicating that \(|E_B^s|\) is not deviating from \(|E_B|\). Currently, the data points are not sufficient for an independent extraction of \(|E_M^s|\). We hence adopt the numerical results from the two-body \(D\) meson decays, where the similar \(W_{EX}\) contributions have been found to induce the severe \(SU(3)_f\) symmetry breaking \([18, 21]\). In the second scenario (\(S2\)), we take \(|E_M^s| = n_q \times |E_M|\) and \(|E_B^s| \simeq |E_B|\), with \(n_q = 1.4\) adopted from \([21]\). Consequently, we obtain

\[
(|E_B|, |E_M|) = (0.40 \pm 0.03, 0.34 \pm 0.03) \text{ GeV}^3, \\
\delta_{E_M} = (180.0 \pm 46.8)^0,
\]

where \(\chi^2/n.d.f\) is reduced as 1.3. As the demonstration, we obtain \(B(\Lambda_c^+ \to \Sigma^+ \eta) = (7.3 \pm 1.5) \times 10^{-3}\), which alleviates the deviation from the observation. With the fit values of \((|E_B|, |E_M|, \delta_{E_M})\) in \(S1\) and \(S2\), we present the branching ratios of the \(B_c \to B^* M\) decays in Tables I, II and III along with the recent theoretical results for comparison.

We get some useful relations in the quark-diagram scheme. For example, we find out three triangle sum rules for \(B_c \to \Delta \pi\), given by

\[
T(\Lambda_c^+ \to \Delta^0 \pi^+) - T(\Lambda_c^+ \to \Delta^{++} \pi^-) - \sqrt{6} T(\Lambda_c^+ \to \Delta^+ \pi^0) = 0, \\
T(\Xi_c^+ \to \Delta^0 \pi^+) - T(\Xi_c^+ \to \Delta^{++} \pi^-) - \sqrt{6} T(\Xi_c^+ \to \Delta^+ \pi^0) = 0, \\
T(\Xi_c^0 \to \Delta^+ \pi^-) - T(\Xi_c^0 \to \Delta^- \pi^+) - \sqrt{6} T(\Xi_c^0 \to \Delta^0 \pi^0) = 0.
\]

(14)

Besides, we obtain

\[
T(\Lambda_c^+ \to \Delta^+ K^0, \Delta^0 K^+) = 0, \\
T(\Xi_c^+ \to \Sigma^+ K^0, \Sigma^0 \pi^+) = 0,
\]

(15)

as the consequence of \(C'\) being set to give no contribution to \(B_c \to B^* M\). Indeed, \(C'\) is the only topology that takes part in the decays in Eq. (15), but suppressed due to the Körner-Pati-Woo theorem \([31]\). According to the other theoretical calculations \([6, 7, 22, 24]\), \(B(\Xi_c^+ \to \Sigma^+ K^0, \Xi^0 \pi^+) = 0\) is also predicted, which supports that \(C' = 0\). Experimentally, \(B_{ex}(\Xi_c^+ \to \Sigma^+ K^0, \Xi^0 \pi^+)\) in Table III can be used to test the suppression. With \(B(\Xi_c^+ \to \Sigma^+ K^0, \Xi^0 \pi^+)\)
which leads to an additional weight factor of $\lambda_\phi$, we need more accurate observations to test if $C'(E') = 0$.

Uniquely, the decuplet baryon can contain three identical quarks, denoted by $B^\ast(qqq)$, which leads to an additional weight factor of $\sqrt{3}$ among the decuplet baryons in Eq. (3). The factor can be considered as the main reason why $\Lambda_c^+ \to \Delta^{++} K^-$ and $\Xi^0 \to \Omega^- K^+$ are measured with the largest branching fractions in the CA decay channels of $\Lambda_c^+, \Xi_c^0 \to B^* M$, respectively. Accordingly, the $T$-amps with $B^\ast(qqq)$ are listed as

$$T(\Lambda_c^+ \to \Delta^{++} K^-, \Delta^{++} \pi^-) = -(\lambda_a, \lambda_d) E_M,$$

### TABLE II. Singly Cabibbo-suppressed $B_c \to B^* M$ decays.

| Decay modes                  | $T$-amp                        | $10^4 B(S1, S2)$ [our work] | $10^4 B(S_{pm}, S_{ms})$ [7] |
|------------------------------|--------------------------------|-----------------------------|-------------------------------|
| $\Lambda_c^+ \to \Delta^{++} \pi^-$ | $-\lambda_d E_M$              | $7.2 \pm 1.3, 7.0 \pm 1.4$ | $12.5 \pm 2.0, 6.6 \pm 0.6$ |
| $\Lambda_c^+ \to \Delta^{++} \pi^0$  | $-\lambda_d \frac{1}{\sqrt{3}}(E_B - E_M)$ | $5.8 \pm 0.9, 5.6 \pm 1.1$ | $8.3 \pm 1.3, 4.4 \pm 0.4$ |
| $\Lambda_c^+ \to \Delta^{++} \pi^+$  | $-\lambda_d \frac{1}{\sqrt{3}} E_B$  | $3.5 \pm 0.5, 3.3 \pm 0.5$ | $4.2 \pm 0.7, 2.2 \pm 0.2$ |
| $\Lambda_c^+ \to \Delta^+ \eta^-$  | $-\lambda_d \frac{1}{\sqrt{6}}(E_B + E_M)c\phi$ | $(0.03 \pm 0.28, 0.02 \pm 0.45)$ | — |
| $\Lambda_c^+ \to \Delta^+ \eta'$  | $-\lambda_d \frac{1}{\sqrt{6}}(E_B + E_M)s\phi$ | $(0.01 \pm 0.09, 0.01 \pm 0.15)$ | — |
| $\Lambda_c^+ \to \Sigma^0 K^0$    | $-\lambda_d \frac{1}{2} E_M^{(s)}$ | $(1.8 \pm 0.4, 3.5 \pm 0.7)$ | $(1.3 \pm 0.2, 2.2 \pm 0.2)$ |
| $\Lambda_c^+ \to \Sigma^- K^+$    | $-\lambda_d \frac{1}{\sqrt{3}} E_B^{(s)}$ | $(1.3 \pm 0.2, 1.3 \pm 0.2)$ | $(0.7 \pm 0.1, 1.1 \pm 0.1)$ |
| $\Xi_c^0 \to \Delta^+ K^-$       | $-\lambda_d \frac{1}{\sqrt{3}} E_M$ | $(1.1 \pm 0.2, 1.0 \pm 0.2)$ | $(3.0 \pm 0.5, 1.2 \pm 0.1)$ |
| $\Xi_c^0 \to \Delta^0 \bar{K}$   | $-\lambda_d \frac{1}{\sqrt{3}} E_M$ | $(1.1 \pm 0.2, 1.0 \pm 0.2)$ | $(3.0 \pm 0.5, 1.2 \pm 0.1)$ |
| $\Xi_c^0 \to \Sigma^{+} \pi^+$   | $-\lambda_d \frac{1}{\sqrt{3}} (E_B - \lambda_s E_B)$ | $(6.1 \pm 0.9, 5.8 \pm 0.9)$ | $(9.9 \pm 1.6, 4.9 \pm 0.4)$ |
| $\Xi_c^0 \to \Sigma^{+} \pi^-$   | $-\lambda_a \frac{1}{\sqrt{3}} E_M$ | $(1.0 \pm 0.2, 1.0 \pm 0.2)$ | $(2.5 \pm 0.4, 1.2 \pm 0.1)$ |
| $\Xi_c^0 \to \Sigma^{+} \pi^0$   | $-\frac{1}{\sqrt{12}} [\lambda_d (E_B - E_M) - \lambda_s E_B]c\phi$ | $(3.1 \pm 0.4, 2.9 \pm 0.5)$ | $(5.6 \pm 0.9, 2.8 \pm 0.2)$ |
| $\Xi_c^0 \to \Sigma^{0} \eta^-$  | $-\frac{1}{\sqrt{12}} [\lambda_d (E_B + E_M)c\phi + \lambda_s (\sqrt{2} E_M^{(s)} s\phi - E_B c\phi)]$ | $(0.9 \pm 0.2, 1.2 \pm 0.3)$ | $(1.1 \pm 0.2, 0.9 \pm 0.1)$ |
| $\Xi_c^0 \to \Sigma^{0} \eta'$   | $-\frac{1}{\sqrt{12}} [\lambda_d (E_B + E_M)s\phi - \lambda_s (\sqrt{2} E_M^{(s)} c\phi + E_B s\phi)]$ | $(0.004 \pm 0.120, 0.050 \pm 0.250)$ | — |
| $\Xi_c^0 \to \Xi^0 K^0$          | $\lambda_d E_M^{(s)}$ | $(0.8 \pm 0.2, 1.6 \pm 0.4)$ | $(0.9 \pm 0.2, 1.2 \pm 0.1)$ |
| $\Xi_c^0 \to \Xi^- K^+$          | $\lambda_d E_B^{(s)}$ | $(4.6 \pm 0.7, 4.6 \pm 0.7)$ | $(3.6 \pm 0.6, 4.9 \pm 0.4)$ |
| $\Xi_c^+ \to \Delta^{++} K^-$    | $-\lambda_d E_M$ | $(13.8 \pm 2.5, 13.5 \pm 2.7)$ | $(35.0 \pm 5.7, 14.6 \pm 1.2)$ |
| $\Xi_c^+ \to \Delta^{+} \bar{K}$ | $-\lambda_s \frac{1}{\sqrt{3}} E_M$ | $(4.6 \pm 0.8, 4.5 \pm 0.9)$ | $(11.7 \pm 1.9, 4.9 \pm 0.4)$ |
| $\Xi_c^+ \to \Sigma^{+} \pi^+$   | $-\lambda_s \frac{1}{\sqrt{3}} E_B$ | $(3.4 \pm 0.5, 3.2 \pm 0.5)$ | $(4.8 \pm 0.8, 2.4 \pm 0.2)$ |
| $\Xi_c^+ \to \Sigma^{0} \pi^0$   | $-\lambda_s \frac{1}{\sqrt{3}} E_B$ | $(3.4 \pm 0.5, 3.2 \pm 0.5)$ | $(4.8 \pm 0.8, 2.4 \pm 0.2)$ |
| $\Xi_c^+ \to \Sigma^{+} \eta^-$  | $-\lambda_s \frac{1}{\sqrt{3}} (E_B c\phi - \sqrt{2} E_M^{(s)} s\phi)$ | $(6.4 \pm 1.0, 9.1 \pm 1.8)$ | $(8.7 \pm 1.4, 7.3 \pm 0.6)$ |
| $\Xi_c^+ \to \Sigma^{+} \eta'$  | $-\lambda_s \frac{1}{\sqrt{3}} (E_B s\phi + \sqrt{2} E_M^{(s)} c\phi)$ | $(0.1 \pm 0.3, 0.6 \pm 0.8)$ | — |
| $\Xi_c^+ \to \Xi^0 K^+$          | $-\lambda_s \frac{1}{\sqrt{3}} E_B^{(s)}$ | $(5.0 \pm 0.8, 5.0 \pm 0.8)$ | $(3.5 \pm 0.6, 4.9 \pm 0.4)$ |
\[ T(\Xi_c^+ \to \Delta^{++} K^-, \Delta^{++} \pi^-) = -(\lambda_s, \lambda_c) E_M, \]
\[ T(\Xi_c^0 \to \Omega^- K^+, \Delta^- \pi^+) = (\lambda_s, -\lambda_c) E_B. \]  

(16)

While \( B(\Lambda_c^+ \to \Delta^{++} K^-) \) and \( B(\Xi_c^0 \to \Omega^- K^+) \) have been observed, the other branching fractions are given by

\[ B(\Lambda_c^+ \to \Delta^{++} \pi^-, \Xi_c^+ \to \Delta^{++} K^-) = (7.0 \pm 1.4, 13.5 \pm 2.7) \times 10^{-4}, \]
\[ B(\Xi_c^+ \to \Delta^{++} \pi^-, \Xi_c^0 \to \Delta^- \pi^+) = (7.8 \pm 1.6, 2.5 \pm 0.4) \times 10^{-5}, \]

(17)

which are predicted as the largest branching fractions in the SCS \( \Lambda_c^+ (\Xi_c^+) \) and DCS \( \Xi_c^{(0)} \) decay channels, respectively. Here, we present our predictions of \( S2 \), which is favored by the \( \chi^2 \)-fit. The equality of \( T(\Lambda_c^+ \to \Sigma^{0+} \pi^+) = T(\Lambda_c^+ \to \Sigma^{*+} \pi^0) \) corresponds to the isospin symmetry. The branching fraction, given by

\[ B(\Lambda_c^+ \to \Sigma^{*0(+)} \pi^{+(0)}) = (2.8 \pm 0.4) \times 10^{-3}, \]

(18)

can be used to test the broken effect. The decays \( B_c \to B^* \eta^{(l)} \), \( \Lambda_c^+ \to \Sigma^{*+} \eta, \Xi_c^0 \to \Xi^{*0} \eta \) and \( \Xi_c^+ \to \Sigma^{*+} \eta \) have sizeable branching fractions, which is due to the constructive interferences between \( E_B \) and \( E_M \). However, the other branching fractions of \( B_c \to B^* \eta^{(l)} \) are typically small with the destructive interferences. Moreover, we find that \( B(\Lambda_c^+ \to \Sigma^{*+} \eta') = 0 \) with \( m_{\Lambda_c^+} < m_{\Sigma^{*+}} + m_{\eta'} \).

| Decay modes  | \( T\)-amp | \( 10^3 B \) \( \text{(S1, S2)} \) [our work] | \( 10^3 B \) \( \text{(S_{em}, S_{sem})} \) [2] |
|--------------|------------|---------------------------------|---------------------------------|
| \( \Lambda_c^+ \to \Delta^{+} K^0 \) | 0          | 0                               | 0                               |
| \( \Lambda_c^+ \to \Delta^{0} K^+ \) | 0          | 0                               | 0                               |
| \( \Xi_c^0 \to \Delta^{+} \pi^- \) | \( -\lambda_c \frac{1}{\sqrt{3}} E_M \) | \( 0.6 \pm 0.1, 0.6 \pm 0.1 \) | \( 2.2 \pm 0.4, 0.7 \pm 0.1 \) |
| \( \Xi_c^0 \to \Delta^{0} \pi^0 \) | \( -\lambda_c \frac{1}{\sqrt{3}} (E_B - E_M) \) | \( 1.5 \pm 0.2, 1.4 \pm 0.3 \) | \( 4.3 \pm 0.7, 1.3 \pm 0.1 \) |
| \( \Xi_c^0 \to \Delta^{0} \pi^- \) | \( -\lambda_c E_B \) | \( 2.7 \pm 0.4, 2.5 \pm 0.4 \) | \( 6.5 \pm 1.1, 2.0 \pm 0.2 \) |
| \( \Xi_c^0 \to \Delta^{0} \eta \) | \( -\lambda_c \frac{1}{\sqrt{3}} (E_B + E_M) c\phi \) | \( 0.01 \pm 0.07, 0.01 \pm 0.12 \) | --- |
| \( \Xi_c^0 \to \Delta^{0} \eta' \) | \( -\lambda_c \frac{1}{\sqrt{3}} (E_B + E_M) s\phi \) | \( 0.003 \pm 0.034, 0.003 \pm 0.055 \) | --- |
| \( \Xi_c^0 \to \Sigma^{*0} K^0 \) | \( -\lambda_c \frac{1}{\sqrt{3}} E_M^{(i)} \) | \( 0.2 \pm 0.1, 0.5 \pm 0.1 \) | \( 0.4 \pm 0.1, 0.3 \pm 0.0 \) |
| \( \Xi_c^0 \to \Sigma^{*0} K^+ \) | \( -\lambda_c \frac{1}{\sqrt{3}} E_B^{(i)} \) | \( 0.7 \pm 0.1, 0.7 \pm 0.1 \) | \( 0.9 \pm 0.1, 0.7 \pm 0.1 \) |
| \( \Xi_c^+ \to \Delta^{+} \pi^- \) | \( -\lambda_c E_M \) | \( 8.0 \pm 1.5, 7.8 \pm 1.6 \) | \( 25.5 \pm 4.4, 7.8 \pm 0.7 \) |
| \( \Xi_c^+ \to \Delta^{+} \pi^0 \) | \( -\lambda_c \frac{1}{\sqrt{3}} (E_B - E_M) \) | \( 6.5 \pm 1.0, 6.3 \pm 1.2 \) | \( 17.0 \pm 2.9, 5.2 \pm 0.4 \) |
| \( \Xi_c^+ \to \Delta^{0} \pi^+ \) | \( -\lambda_c \frac{1}{\sqrt{3}} E_B \) | \( 3.9 \pm 0.6, 3.7 \pm 0.7 \) | \( 8.5 \pm 1.5, 2.6 \pm 0.2 \) |
| \( \Xi_c^+ \to \Delta^{+} \eta \) | \( -\lambda_c \frac{1}{\sqrt{3}} (E_B + E_M) c\phi \) | \( 0.03 \pm 0.32, 0.03 \pm 0.52 \) | --- |
| \( \Xi_c^+ \to \Delta^{+} \eta' \) | \( -\lambda_c \frac{1}{\sqrt{3}} (E_B + E_M) s\phi \) | \( 0.01 \pm 0.15, 0.01 \pm 0.24 \) | --- |
| \( \Xi_c^+ \to \Sigma^{*+} K^0 \) | \( -\lambda_c \frac{1}{\sqrt{3}} E_M^{(i)} \) | \( 2.1 \pm 0.4, 4.2 \pm 0.8 \) | \( 3.5 \pm 0.6, 2.6 \pm 0.2 \) |
| \( \Xi_c^+ \to \Sigma^{*0} K^+ \) | \( -\lambda_c \frac{1}{\sqrt{3}} E_B^{(i)} \) | \( 1.5 \pm 0.2, 1.5 \pm 0.2 \) | \( 1.7 \pm 0.3, 1.3 \pm 0.1 \) |
The approach of the irreducible $SU(3)_f$ symmetry has been widely used in the hadron weak decays $[4,16]$. For $B_c \to B^* M$, there exist four parameters $a_8$ and $a_{9,10,11} [4,6]$, which correspond to the decomposition of $H_{\text{eff}} = H(6) + H(\overline{15})$ in the $SU(3)_f$ representation of 6 and $\overline{15}$, respectively. By comparison, we derive that

$$(E_B, E_M) = (-2a_8 + a_9, 2a_8 + a_9), \ (E', C') = (-2a_9 - 2a_{10}, -2a_{11}), \quad (19)$$

such that $a_i$ are found to correspond to the topologies. Since $(E', C')$ have been the vanishing topological parameters, one has $a_9 = -a_{10}$ and $a_{11} = 0$. Moreover, our global fits for $E_{B,M}$ indicate that $a_{9(10)}$ from $H(\overline{15})$ has a non-zero value. By contrast, the numerical analysis performed with the irreducible $SU(3)_f$ symmetry neglects the contributions from $H(\overline{15}) \ [7]$, whose results are given in the tables. In the physical mass scenario ($S_{pm}$) for the global fit in Ref. $[7]$, where $m_{B_c}, m_{B^*}$ and $m_M$ are taken from the physical values in Ref. $[1]$, $B(\Lambda_c^+ \to \Sigma^{*+}\eta, \Xi^{*0}K^+)$ and $B(\Xi_c^0 \to \Omega^- K^+)$ are fitted to be a few times smaller than the observations. Instead of considering the $SU(3)_f$ symmetry breaking, one performs another fit in the equal mass scenario ($S_{em}$), where $m_{\Lambda_c} = m_{\Xi_c}, m_\Delta = m_{\Sigma^*} = m_\Omega$ and $m_\pi = m_\eta = m_K$, resulting in the raised values of the above branching fractions.

**IV. CONCLUSIONS**

In summary, we have studied the $B_c \to B^* M$ decays in the quark-diagram scheme. We have found that only two $W$-exchange diagrams, $E_B$ and $E_M$, could give contributions to the observed branching fractions of $\Lambda_c^+ \to (\Delta^{++}K^-, \Sigma^{*+}\eta, \Xi^{*0}K^+)$ and $\Xi_c^0 \to \Omega^- K^+$. In addition, we have predicted $B(\Lambda_c^+ \to \Sigma^{*0(+)}\pi^{+(0)}) = (2.8 \pm 0.4) \times 10^{-3}$, which respects the isospin symmetry. To interpret the observation of $B(\Lambda_c^+ \to \Sigma^{*+}\eta)$, we have taken into account the $SU(3)_f$ symmetry breaking. In particular, $B(\Lambda_c^+ \to \Delta^{++}\pi^-, \Xi_c^+ \to \Delta^{++}K^-) = (7.0 \pm 1.4, 13.5 \pm 2.7) \times 10^{-4}$ and $B(\Xi_c^+ \to \Delta^{++}\pi^-, \Xi_c^0 \to \Delta^-\pi^+) = (7.8 \pm 1.6, 2.5 \pm 0.4) \times 10^{-5}$ have been predicted as the largest branching fractions in the SCS $\Lambda_c^+(\Xi_c^+)$ and DSC $\Xi_c^{+(0)}$ decay channels, respectively.
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