Calculation-experimental analysis of the ultimate strength of a material under biaxial tension using the finite element method

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Abstract. The article emphasizes the importance of taking into account the rigidity of the stress-strain state (SSS) when evaluating the static strength of parts and elements of highly loaded structures. The description of the universal characteristic of the SSS allowing to estimate the SSS rigidity in the construction zone is given. The estimation of the SSS rigidity in the zone of destruction of the experimental model of a choke unit with a spherical body under its quasi-static loading by internal pressure is considered. A calculation and experimental technique for estimating the limiting state of spring-steel 50CrV4 steel under conditions of biaxial stretching based on the use of laboratory prismatic samples for evaluating the strength of a material under a biaxial stress state is described. The results of numerical finite element analysis of elastoplastic deformation of prismatic samples performed in the MSC Marc software package are presented. The obtained results show a significant influence of the SSS rigidity on the ultimate characteristics of the material of the steel used in comparison with its traditional strength limit.

1. Introduction

The traditional approach in assessing the strength of deformable structures is based on an analysis of the results of testing specimens of rod-shaped cylindrical shapes under conditions of uniaxial tension, with the further use of classical strength criteria. However, as the practice of studying the strength of parts, such as choke points of high-pressure capacitive apparatus, disks of turbines and compressors, carrying elements of railway and motor transport, and others, in the stress concentration zones in them, the classical criteria can lead to a significant mismatch in the calculated and actual destruction [1]. In Figure 1 shows the distribution of the intensity of deformations of the choke junction of the pressure vessel at the moment of failure.

From Figure 1 it can be seen that the maximum value of the intensity of deformation takes place on the inner surface of the nozzle, with the beginning of the destruction occurring over the outer surface. One of the reasons for the discrepancy between the calculated and real conditions for the destruction of the fitting is the difference in the stiffness (type) of the stress-strain state (SSS) of the material in these zones, caused by geometric structural features (stress concentrators) [2]. To assess the rigidity of SSS, the paper considers the universal characteristic – $P$, defined by the relation

$$ P = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sigma_i}, $$

where $\sigma_1$, $\sigma_2$, $\sigma_3$ are the principal stresses arising at the observation point, $\sigma_i$ is equivalent stress defined by the formula

$$ \sigma_i = \sqrt{\frac{1}{3} \left( \sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right) - \sigma_s^2}, $$

with $\sigma_s$ as the yield strength of the material.
\[ \sigma_i = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}. \] 

Figure 1. Features of the destruction of the pressure vessel choke unit: a - the distribution of deformations at the moment of failure, b - the appearance of the surface of the fracture of the wall

The choice as the characteristic of the SSS type of the quantity \( P \) characterized by the relation (1) is the fact that the coefficient \( P \) is an invariant characteristic of the SSS type – its larger values correspond to (in the terminology of G A Smirnov-Alyaev) a more rigid form of SSS, and its lower values – a softer type of SSS. In addition to physical invariance, the feature and, at the same time, the advantage of criterion \( P \), is its dependence on all three principal stresses in the source of destruction. For example, the uniaxial extension of a standard cylindrical sample corresponds to a value of \( P = 1 \), torsion – \( P = 0 \), compression – \( P = -1 \), biaxial stretching corresponds to \( 1 < P < 2 \).

Analyzing the type of SSS at the time of destruction of the choke unit pressure vessel (Figure 1), we can state that on the inner surface there is uniaxial tension in the circumferential direction and uniaxial compression in the radial. The value of \( P \) in this zone is characterized by a volume state and is equal to \( P = 0.6 \). On the external surface of the union (in the fracture zone), the stress level, as a whole, is less, the value of the SSS stiffness value is \( P = 1.85 \), which corresponds to biaxial stretching. Thus, in places with biaxial stretching, the strength limit of the material decreases. To evaluate the structural strength, in this case, more complex criteria are used, for example, Pisarenko-Lebedev, Yagna-Buzhinsky, Drucker-Prager, Botkin-Mirolyubov and others [3 - 6].

2. Statement of the problem. The proposed solution

The evaluation of the ultimate strength of the material in the case of biaxial stretching is carried out on the basis of a more complicated calculation and experimental technique involving the obtaining of both the ultimate stress-strain state and the strength constants of the material being studied, obtained from the experiment. An obstacle to the use of such a technique is the complexity and complexity in obtaining the values of the components of the principal stresses at the time of destruction and the strength constants of the material. Different types of machines and samples do not allow for effective approximation to the true limit of strength, since not always a purely biaxial stretching is considered. The use of biaxial tearing machines is an extremely expensive and not always effective procedure, depending, one way or another, on the creation of a special sample of the test material intended for testing for biaxial stretching.

In this paper, we propose an approach with the additional use of computer technologies for engineering analysis, which is based on the finite element (FE) solution of the contact problem of the mechanics of a solid deformed body. As a result of the work, on the basis of calculated data, a sample of a prismatic shape was obtained, which makes it possible to obtain a biaxial stress state limiting in the main tensile stresses, with testing it under conditions of a conventional compressive press [7].
Such a laboratory sample allows, by changing its geometric parameters, to simulate a biaxial SSS with any ratio of the principal stresses of a biaxial stretching corresponding to the place of stress concentration in the part, which makes it possible to specify the ultimate strength in this place. The calculated FE model of the prismatic sample is constructed in the MSC Patran software package. The required effect is obtained in the working zone of this sample by solving the contact problem of the mechanics of the elastoplastic behaviour of a solid deformed body.

The equations of plastic flow with isotropic hardening (the Prandtl-Reuss equations [8]) are used as a mathematical model of the process of development of elastoplastic deformations arising in the process of deformation of the material of the structures under study. The choice of this theory is due to the fact that the application of the equations of plastic flow allows physically and geometrically correctly to take into account the loading history of the tested numerical models. Numerical interpretation of the equations of this theory is realized in the program complex MSC Marc. The equations of this theory are presented in [8].

The solution of the corresponding problem of flow theory is constructed with the help of the stepwise loading method [7, 8]. The loading of the numerical model is carried out in this case in several steps, the magnitude of which is determined by the necessary accuracy of the approximation of the process under consideration. At each step, the boundary conditions in their totality form a nonlinear boundary value problem. This problem is formulated in increments of displacements, its solution exists and can be determined by known loading methods [9]. The construction and numerical analysis of the model of the prismatic sample with certain values of the geometric parameters that create the required value of the value of $P$ are intended to obtain the limiting characteristics of the material's SSS at the time of its destruction.

3. Results of experimental studies

Consider the destruction of a prismatic sample made of 50CrV4 steel with a tensile strength of 1270 MPa. The prismatic samples were tested before failure and the destructive load was determined on an electromechanical testing machine Instron 5989. A series of samples were chosen for the studies, in which the hardness of the SSS in the working area was determined by $P = 1.8$. The possibility of such a choice is provided by previously performed computational variant studies, which made it possible to establish the dependence of the value of $P$ and the coefficient of concentration of equivalent stresses $K_{\sigma}$ in the working zone on the geometric parameters of prismatic samples. The averaged force diagram for the five destroyed prismatic samples is shown in Figure 2. Quasistatic loading was carried out with the speed of the pusher displacement of the testing machine 2 mm / min.

![Figure 2](image.png)

_Figure 2. The diagram of the change in the force of the testing machine in the process of loading prismatic sample depending on the displacement of the pusher_

The diagram in Figure 2 shows that before the appearance of a crack (point 1 in Figure 2) the test specimen resists the load, then, when a crack appears in the working zone (the moment of destruction),
the pusher moves with some constant force, leading to an increase in the size of the crack. At the moment of failure, the breaking force of the prismatic sample $F_{des} = 205$ kN was recorded.

4. Results of computational modeling
In accordance with the procedure [3, 4], the fracture force $F_{des} = 205$ kN is used in the analysis of the elastoplastic behavior of the FE model of the samples. The results of a numerical analysis of elastoplastic deformation of prismatic samples with a SSS stiffness in the working area of $P = 1.8$ at the time of their destruction (the moment of the crack occurrence) performed in the MSC Marc software are shown in Figure 3 (the deformed state is shown on an enlarged scale). In Figure 3 shows the calculated distribution of the stress intensity according to Von Mises, in Figure 4 – the calculated distribution of the component of the first main voltage (in Figure 4 the first main area is parallel to the longitudinal axis of the sample).

**Figure 3.** The calculated stress intensity distribution at the moment of destruction of a prismatic sample with $P = 1.8$ (color scale on the right – equivalent stresses Von Mises)

**Figure 4.** The calculated distribution of the first principal stress at the moment of failure prismatic sample with $P = 1.8$ (a kind of quarter of the sample carved from it by two planes of symmetry, color scale on the right – equivalent stresses Von Mises)
Based on the results of a numerical analysis of the prismatic samples shown in Figure 3 and Figure 4 that the maximum stress intensity at the moment of destruction in the working zone of samples with \( P = 1.8 \) is 917 MPa. Numerical simulation made it possible to establish that at the moment of sample destruction in its working zone, the maximum value of the first principal voltage is 1050 MPa (in Figure 4 the first main area is parallel to the longitudinal axis of the sample). Inspection of the destroyed samples showed that in their working area the surface of the samples had a visible toroidal shape, which coincides with the pattern of deformation revealed by numerical simulation (Figure 4).

5. Conclusions

The described calculation-experimental technique for estimating the limiting state of a material under conditions of biaxial stretching based on the use of prismatic samples tested on standard test equipment has made it possible to establish the ultimate strength characteristics of the material with the rigidity of the SSS corresponding to biaxial stretching. This allows us to apply the developed design models of deformation to determine the strength characteristics of the material for different degrees of biaxial stretching and to perform an appropriate calculation for the strength of various structures taking into account the real SSS.

The obtained experimental results show that the influence of the SSS rigidity in the failure site can be significant – for the tested samples with \( P = 1.8 \) the limiting value of the stress intensity and the magnitude of the first principal stress corresponding to the moment of their destruction decrease by an average by almost a quarter in comparison with the traditional the tensile strength of the steel under study, for which \( \sigma_0 = 1270 \) MPa, determined under conditions of uniaxial tension (\( P = 1 \)). The central element of the practical importance of the work is the simplification of the possibility of refinement (estimation) of the limiting state for the strength of parts that have stress concentrators in their design.

6. References

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