Chiral dynamics of the low energy
Kaon-Baryon interactions.

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Abstract

The processes involving $K^- p, KN, \Sigma \pi, \Lambda \pi, \Sigma \eta, \Lambda \eta$ coupled channels are studied in the nonperturbative chiral approach. An effective potential is constructed using a chiral meson-baryon Lagrangian at lowest order. This potential is iterated to all orders with the Lippmann-Schwinger equation. A reasonable fit of the experimental data is obtained. It is pointed out, however, that due to a strong sensitivity of the results to the value of the cut-off, such an approach should be viewed as a rather phenomenological way to fit the experimental data since there is no small expansion parameter allowing for truncation of the chiral expansion of the effective potential at some given order. A possible way to construct the consistent chiral expansion is briefly discussed.

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Chiral perturbation theory (ChPT) has become at present a standard tool to study the low-energy hadronic interaction [1]. ChPT is based on the idea of constructing the most general set of effective lagrangians consistent with the symmetries of QCD and corresponding to the expansion of the scattering amplitudes in increasing powers of mesonic momenta and/or quark masses. An important feature of ChPT is the chiral counting scheme which allows one to estimate the chiral dimension of the scattering amplitude at any given order, endowing the method with more predictability. Initially developed for the meson-meson interaction, ChPT was then generalized to include baryons in a fully relativistic manner [2]. Unfortunately, in this treatment the consistent chiral counting is difficult to implement, since the baryon 4-momenta can never be small in the scale, as is typical for the low-energy meson-meson interactions. In ref. [3] a nonrelativistic formulation of ChPT was proposed, which allows one to reconstruct the power counting rules and thus organize the consistent chiral expansion scheme. This Heavy Baryon Chiral Perturbation Theory (HBChPT), where baryons are assumed to be infinitely heavy in lowest order, has been successful [4] in describing many properties of meson-baryon systems at low energies. However, when applied to the NN interaction ChPT encounters a problem due to the existence of a bound state with an energy close to threshold. This state cannot be reproduced by the standard means of ChPT [5]. Physically, it is rather natural, since in a bound state a particle interacts an infinite number of times, whereas in a perturbative series only a finite number of interaction is taken into account. Weinberg [5] first suggested the formulation of ChPT in nuclear physics, based on the idea that power counting arguments can be applied to the NN (or n-nucleon) potential instead of the scattering amplitude. This potential can then be used to calculate, for example, NN phase shifts by iterating a Lippmann-Schwinger or Schrödinger equation in all orders. However, as it was pointed out in Ref. [6], when the power counting scheme is used for the effective potential, which should then be iterated in all orders, some other problems arise, making the consistent implementation of the above mentioned program rather difficult. The results of the Ref. [6] can be summarized as follows. Firstly, it was shown that there is no small parameter in the chiral expansion of the effective potential,
allowing for the truncation of the series at some given order. Secondly, it was demonstrated that different regularization schemes lead to different physical amplitudes. That means a sensitivity to the short-range physics, that cannot be properly treated in the framework of standard ChPT. These results, obtained for the case of NN scattering, indicate the necessity to consider the other hadronic systems which are dominated by a bound state (or resonance) close to threshold. In this paper we concentrate on the study of the $K^-p$ interaction. The low energy properties of the $K^-P$ system are, to large extent, determined by the $\Lambda(1405)$ resonance, just below $K^-P$ threshold. Since baryons in such an approach can be treated as “heavy” compared to mesons one may hope that the sensitivity to the cut-off parameter will not be so strong and a consistent chiral approach can be formulated. Below we show that this is not the case. One notes, that the $K^-p$ system was earlier considered in Refs. [7] and [8]. In contrast to the $\pi N$ and $K^+N$ cases, the $K^-P$ system is a strongly interacting system with many possible channels, so it is reasonable to choose the scheme of calculations in the spirit of that, suggested in [7], where the effects of coupled channels have been incorporated. We, however, have used an approach, which is somewhat different from that proposed in [7] and, in our opinion, technically easier. That scheme was first developed and successfully implemented in the case of meson-meson interactions in Ref. [9]. We start from the lowest order meson-baryon effective Lagrangian

$$L_{\text{int}}^{(1)} = \frac{i}{4f^2} Tr[[\phi, \partial_0 \phi], B]$$

where $B$ and $\phi$ are baryon and meson field matrices in their standard form [4]. After straightforward calculations one can show that in the lowest order the tree level amplitude has a form

$$t \simeq \frac{1}{f_\pi^2}(E_i - E_f)$$

where $f_\pi$ is the pion decay constant, and $E_i(E_f)$ is the total energy of the incoming (outgoing) mesons. According to the approach, outlined above, this tree level amplitude should be substituted in the Lippmann-Schwinger equation for the full $T$-matrix.
\[ T = V + VGT \] (3)

where \( V \) is the effective potential, which in our case is given by the tree level amplitude \( t \) and \( G \) is the corresponding propagator, defined by

\[
G_l = (2\pi)^{-4} \int d^4q \frac{i}{(q^2 - m_l^2 + i\epsilon)(p_0 - q_0)}, \tag{4}
\]

where \( q_0(m_l) \) is the energy (mass) of the intermediate meson of the type \( l \). In Ref. [9] it was demonstrated that one can avoid the problem of finding the complete solution of the Lippmann-Schwinger equation with off shell scattering amplitudes since, at least at leading order, only the on-shell information about the tree level amplitudes is really needed. This procedure can easily be extended for the case of meson-nucleon interaction in HBChPT.

We consider for simplicity only the one-loop contribution with a cut-off \( \Lambda \) imposed. The corresponding loop integral consists of meson and baryon propagators and two tree-level amplitudes. The typical expression for the numerator in the one loop term can be decomposed as follows

\[
(p^0 + q^0)^2 = 4p^0q^0 + 4p^0(q^0 - p^0) + (p^0 - q^0)^2. \tag{5}
\]

Putting this expression in the integral part of the Lippmann-Schwinger equation one can represent the loop corrections by the sum of three integrals. First one gives the part of the loop corrections where the rescattering amplitude is taken on-shell. This integral will be calculated below. The other two integrals contain the contributions from the off-shell part of the rescattering amplitude. The expression \( 4p^0(q^0 - p^0) \), when substituted in the loop integral cancels the baryon propagator. One can see that after the integration over energy is done the integral becomes proportional to \( p^0 \int d^3q(2\omega_l(q))^{-1} \approx p^0\Lambda^2 \), which has the structure of the tree level amplitudes and can be combined with them. A similar property holds for the third term in Eq.(5) and for the terms of higher order in the Lippmann-Schwinger equation.

Thus, the corresponding integral equation can be reduced to a system of algebraic equations and the amplitude \( T \) is given by

\[
T = (1 - VG)^{-1}V \tag{6}
\]
One notes that the presence of cut-off parameters is inevitable in an approach of this type. In the ordinary ChPT the divergencies can be removed order by order, but since we have to sum the series to all orders, some regulator should always present. One can still formulate a consistent chiral expansion scheme if the value of the cut off parameter agrees with the chiral symmetry expectations and the dependence of the physical observable on the value of the cut-off parameter is moderate.

There are eight channels \((\pi\Sigma, \pi\Lambda, K^-p, KN, \eta\Lambda, \eta\Sigma)\) which can be involved in the dynamics of the \(K^-P\) interaction at low energies. After imposing the cut off and taking the tree level amplitude out of the integral in the Lippmann-Schwinger equation the only remaining task is to calculate the residual integral. It can be done analytically, using standard field theoretical methods. The result of the calculations is given in the appendix.

The results of our calculations for elastic scattering \(K^-p \rightarrow K^-p\), and for the reactions \(K^-p \rightarrow \pi\Sigma\) are shown in Fig.1-3. In Figs.1-2 the lower lines correspond to calculations using the value of the cut off parameter \(\Lambda=600\) MeV, whereas the upper curves are the results of calculations with \(\Lambda=580\) MeV. The theoretical calculations are in reasonable agreement with the experimental data. We note that, in principle, one can take into account the baryon mass splitting in the SU(3) multiplets when calculating the propagator \(G_l\). The results of such calculations are shown in Fig.3 for the reaction \(K^-p \rightarrow \pi^-\Sigma^-\). At threshold the total cross section gets further enhanced. It could be viewed as a hint of the existence of the resonance. However, it is not completely consistent to include the terms of higher order in the calculations of the scattering amplitude, where only the lowest order lagrangian was used at the beginning. In any case, the “right” behavior of the total cross section, when the effects of the baryon mass splitting are included, may indicate that by including the contributions from the terms of higher order, one can generate a resonance at the right energy. This conclusion is also supported by the results obtained in Ref. \[\text{[7]}\] where the leading and next-to-leading terms of the effective potential were used. One notes, that the calculations for the other possible channels are also in reasonable agreement with the experimental data. They, however, cannot add qualitatively new information about the dynamics involved so we do
not show the corresponding results here. One notes, to avoid confusion, that the phase space factors were, of course, calculated with the physical masses of the corresponding particles.

Now a few remarks about the contributions involving $\eta$ mesons are in order. The channels with $\eta$ mesons give moderate, but non-negligible contributions. In the case of low energy $K^-P$ interactions the tree level amplitudes with $\eta$’s in the final state can only be off-shell by 250-300 MeV, since there is not enough energy to generate the real $\eta\Lambda$ or $\eta\Sigma$ pairs. However, in lowest order of HBChPT, when all baryons are assumed to be infinitely heavy, the degree of “off-shellness” becomes much less (about 50 MeV) and negligible in lowest order. So we formally put the amplitudes on shell for the processes like $K^-P \to \eta\Lambda$ and $K^-P \to \eta\Sigma$ and use the expression

$$t_{K^-P \to \eta\Lambda(\Sigma)} \simeq \frac{2}{f_\pi^2} E^2_i$$

We note, however, that in the next orders, when the baryon mass difference should be taken into account, the contributions of the channels including $\eta$’s may be considerably reduced, due to increased degree of the ”off-shellness”. In fact, in Ref [7] a good fit of the experimental data was obtained without channels with $\eta$’s. So we conclude, that at the lowest order its contribution is enhanced somewhat artificially.

The other interesting physical quantities in low-energy KN scattering are the following branching ratios

$$\gamma = \frac{\Gamma(K^-p \to \pi^+\Sigma^-)}{\Gamma(K^-p \to \pi^-\Sigma^+)} = 2.36 \pm 0.04$$

$$R_c = \frac{\Gamma(K^-p \to \text{charged particles})}{\Gamma(K^-p \to \text{all})} = 0.664 \pm 0.011$$

$$R_n = \frac{\Gamma(K^-p \to \pi^0\Lambda)}{\Gamma(K^-p \to \text{all neutral states})} = 0.189 \pm 0.015$$

The experimental values are taken from Refs. [10] and [11]. They should be compared with the theoretical calculations, which give the following numerical results

$$\gamma = 2.01, R_c = 0.58, R_n = 0.13.$$
One can see that the leading order terms give the dominant contribution, although higher order corrections are apparently required.

One can see from Figs. 1-3 that, although the results of the calculations are in fairly good agreement with the experimental data, they are extremely sensitive to the value of the cut-off parameter used. This means, that the loop momenta in the vicinity of the cut-off may give a significant contribution. Similar results were obtained and discussed in Ref. [6] for the case of $NN$ scattering. That, in turn, indicates that there is no small parameter, providing convergence of the chiral expansion of the effective potential which is the sum of the tree level diagrams of increasing order, so that the chiral expansion cannot be arbitrarily truncated. One could hope that taking into account the next-to-leading corrections to the tree level amplitude might decrease of the sensitivity of the calculations to the value of the cut-off. However, in the calculations in the Ref. [7] it was demonstrated that inclusion of the next-to-leading terms does not significantly affect the numerical value of the effective cut-off so the strong sensitivity to the value of $\Lambda$ is still there even if next-to-leading order corrections to the tree level amplitude are included. One notes that, as long as the factorization property holds at lowest order, one can consider the processes where the external momenta are small compared to the cut-off. However, one can see that the factorization property breaks down when the terms of the next order are taken into account. Then, in order to have a reliable scheme for the calculations one needs to estimate the effective potential in the region of momenta close to the value of the cut-off, where, due to the absence of the small parameter of the order $m_\pi/\Lambda$, the chiral expansion cannot be applied. In other words, using the cut-off introduces an additional length scale, which makes it rather difficult to implement the chiral expansion in the standard sense. It is again worth mentioning that in the standard perturbative procedure the dependence of the physical amplitudes on the cut-off can be removed order by order by using counterterms corresponding to the tree level terms of higher order. One notes, that in our case we have the power-law divergencies, so the situation is again identical to that existing in nucleon-nucleon scattering, where the divergencies also have the power-law character so that dimensional regularization and a cut-off scheme give
different results for the physical observables [6]. So one can expect that the same problem exists in the other systems, where the dominated dynamical feature is the existence of the bound state near threshold. This, in turn, means strong sensitivity to the short range effects similar to that found for the case of NN scattering [6].

So we have to conclude that the nonperturbative approach, described above, should be considered as a rather phenomenological method to fit the experimental data. The problem of the strong dependence of the physical amplitudes on both the numerical parameters of the regulator and the way the regularization is done will inevitably arise for the approaches of this type, when, due to the presence of a subthreshold bound state, the chiral expansion is applied to the effective potential, iterated then to infinite order. It seems, therefore that there is no way in this case to formulate an effective chiral theory because of the absence of a small parameter in the chiral expansion of the effective potential. One needs to stress that we have discussed one particular type of a chiral approach. Effective field theory is based on the very general principles and should, therefore, work. The practical realization of those principles may, however, be somewhat complicated due to existence of a subthreshold bound state. The similar conclusion was made in Ref. [6] for the case of NN scattering.

One notes that a possible way out was suggested in Ref. [8] where the Λ(1405) resonance was treated as an explicit degree of freedom. It was shown in [6] that, once the Λ(1405) state is imbedded in the lagrangian as an elementary field, the leading order calculations can explain most of the branching ratios. A similar idea was demonstrated to be very successful [12] for the NN scattering. Since in this case one can formulate a consistent chiral expansion with power counting rules, this direction clearly deserves further study.

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FIGURES CAPTIONS

FIG.1 The results of the theoretical calculations compared with experimental data [10] for \( K^-p \) elastic scattering. The upper curve corresponds to the calculations with \( \Lambda=580 \text{ MeV} \) and the lower curve is the result obtained with \( \Lambda=600 \text{ MeV} \).

FIG.2 The same as in Fig.1 but for the \( K^-p \rightarrow \Sigma^0\pi^0 \) reaction.

FIG.3 The results of the theoretical calculations for the \( K^-p \rightarrow \Sigma^-\pi^+ \) reaction. The dashed line represents the results of the calculations when the baryon mass difference is taken into account in the propagator \( G_l \). The solid curve is the result of the calculations in the strict “heavy baryon” limit. Both curves are obtained with \( \Lambda=580 \text{ MeV} \).
APPENDIX

Here we present the results of the analytical calculation of the main value integral appearing when calculating the propagator \( G_l \). After the angular integration and using the standard expression \( \frac{1}{a + i\epsilon} = \frac{1}{a} - i\pi\delta(a) \) the mean value integral can be defined as follows

\[
I = \int dq q^2 \frac{i}{(k - \omega(q))\omega(q)},
\]

where \( \omega(q) = \sqrt{q^2 + m^2} \). The result of computation is \( I = i(I_1 + I_2 + I_3) \), where

\[
I_1 = -k \ln(\Lambda + \sqrt{\Lambda^2 + m^2}) + (\text{arctanh}(\Lambda\sqrt{k^2 - m^2 + m^2}) \frac{1}{2k\sqrt{\Lambda^2 + m^2}})\sqrt{k^2 - m^2},
\]

\[
I_2 = -(\text{arctanh}(-\Lambda\sqrt{k^2 - m^2 + m^2}) \frac{1}{2k\sqrt{\Lambda^2 + m^2}})\sqrt{k^2 - m^2} - \Lambda,
\]

\[
I_3 = k \ln(m) + \sqrt{-k^2 + m^2} \text{arctanh} \frac{1}{-k^2 + m^2}.
\]

The calculation of the part with delta function is rather trivial so we do show the corresponding result here.
Fig. 1

Total cross section (mb) vs. Plab (MeV)

- Data points with error bars indicate the measured cross section values along with their uncertainties.
- The solid line represents a theoretical model or fit to the data.

The graph illustrates the relationship between the total cross section and the laboratory momentum (Plab) for a specific reaction or process.
