Violations of Lorentz Covariance in Light Front Quark Models

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Abstract

Electromagnetic form factors of the nucleon from relativistic quark models are analyzed: results from null-plane projection of the Feynman triangle diagram are compared with a Bakamjian-Thomas model. The magnetic form factors of the models differ by about 15% at spacelike momentum transfer 0.5 GeV², while the charge form factors are much closer. Spurious contributions to electromagnetic form factors due to violations of rotational symmetry are eliminated for both models. One method changes magnetic form factors by about 10%, whereas the charge form factors stay nearly the same. Another one changes the charge form factor of the Bakamjian-Thomas model by more than 50%.

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I. INTRODUCTION

In light-cone quark models wave functions may be boosted kinematically, that is, independent of interactions. Because form factors involve boosted wave functions, this attractive feature has motivated many electroweak form factor calculations \[1,2\] in Dirac’s light-front form of relativistic few-body physics \[3\]. When the quark model is Lorentz invariant and the exact (interaction dependent) current is evaluated, the form factors will be Lorentz invariant. Because most form factor models are based on free quark currents (in impulse approximation, IA, and/or other approximations), the resulting form factors are in fact frame dependent.

The light-front form is obtained from the instant form in the infinite momentum limit, and this amounts to the well-known change of momentum variables \((p^+ = p^0 + p^z, p^- = p^0 - p^z)\) on the light cone \[4\]. Here the conventional choice of the light cone direction is the z-axis and the light cone time is \(x^+ = t + z\). Because in front form both transverse rotation generators become interaction dependent, rotational symmetry is more difficult to implement. Thus light front models include kinematic boosts at the expense of interaction dependent angular momentum operators.

Light front models obtained from quantum field theories usually involve interacting spin operators. There is a unitary (but interaction dependent) transformation to a representation where spins are free and state vectors are specified on a null plane \(\omega \cdot x = 0\) with a lightlike (i.e. \(\omega^2 = 0\)) four-vector \(\omega^\mu\) \[5\]. If a theory is Lorentz invariant and the exact current operator is known, e.g. in quantum field theory given by the Mandelstam prescription, its form factors are expected to be independent of \(\omega^\mu\). In this case the electromagnetic current operator in general has many-body contributions \[6\]. When it is approximated by a sum of free one-body currents, as in the impulse approximation for example, Lorentz invariance of the form factors is violated.

When Lorentz invariance of the form factors is violated, additional \(\omega\)-dependent current components arise along with their form factors which are needed to fully parameterize the current matrix elements that now obey fewer constraints. When Lorentz invariance of the quark model form factors is fully restored by including two-body currents, these spurious currents are canceled and the remaining calculated form factors are modified as well. In other words, achieving \(\omega\)-independence of form factors by eliminating spurious from factors is just one step towards Lorentz invariance but, by itself, it does not guarantee correct form factors yet. In the case of \(\omega\)-dependence the unphysical currents can be removed. As a bonus, the inconsistencies between current matrix elements of different helicities are reduced. This is done here for electromagnetic nucleon form factors of constituent quark models in light front dynamics. These models are kept as simple as possible (e.g. no anomalous magnetic moments of constituent quarks) because we are not attempting to fit the data. Rather we wish to compare their electromagnetic form factors and the size of errors due to violations of the rotational symmetry.

Two light-cone versions of the constituent quark model (NQM), one based on the Bakamjian-Thomas prescription \[7\], called BT in the following, and another on the null-plane projection of a triangle Feynman diagram (see Araújo et al. in \[2\]), called KW (see Konen et al. in \[2\] for \(\alpha = 1/2\), are briefly reviewed in Sect.II. The electromagnetic current matrix elements are given in Sect.III, and the results in Sect.IV and conclusions are
II. LIGHT-CONE QUARK MODELS

We start with a brief description of the light front notation. The longitudinal quark momentum fractions are defined as $x_i = p_i^+ / P^+$ with the total nucleon momentum $P^+ = \sum_i p_i^+$ so that $\sum_i x_i = 1$ and $0 \leq x_i \leq 1$, where $p_i$ are the quark momentum variables. The $i$th quark momentum in the nucleon rest frame is given by

$$\vec{k}_i^\perp = \vec{p}_i^\perp - x_i \vec{P}_\perp,$$

(1)

with $\vec{k}^\perp = (k_x, k_y)$ etc. so that these internal variables satisfy $\sum_i \vec{k}_i^\perp = 0$ and $k_i^+ = 0$. For a three-quark bound state the relative four-momentum variables are the space-like Jacobi momentum variables in which the kinematic invariants $x_i$ play the role of masses. For example, $q_3$ is the relative quark momentum between the up quarks of the proto n in the uds-basis and $Q_3$ between the down quark and the up quark pair, so that for the $+ \quad \perp = (x, y)$ components

$$q_3 = \frac{x_1 p_2 - x_2 p_1}{x_1 + x_2}, \quad Q_3 = (x_1 + x_2) p_3 - x_3 (p_1 + p_2),$$

(2)

e tc. Because $q_3^+ = 0 = Q_3^+$ both relative momentum variables are space-like. In the uds-basis [8] quarks are treated as distinguishable, and up and down quarks are symmetrized explicitly. The antisymmetric color wave function is suppressed. For the proton (neutron) the down (up) quark carries the label 3.

In light front dynamics the total momentum motion rigorously separates from the internal motion. Therefore, the internal nucleon wave function $\psi(x_i, q_3, Q_3, \lambda_i)$ does not change under kinematic Lorentz transformations or translations. Here the $\lambda_i$ are the quark helicities. Thus, if the wave function is known in the nucleon rest frame it is known everywhere because the seven kinematic generators are transitive in the null plane.

Three-quark wave functions for baryons have been constructed in light cone versions of the constituent quark model (LCQM) to study the static properties of the nucleon [1,2] and electromagnetic transition form factors for $N \rightarrow N^*$ and $N \rightarrow \Delta$ processes [9,10]. Such relativistic quark models significantly improve many predictions of the NQM of which the nucleon weak axial charge $g_A$ is the best known example. For a recent analysis of the static observables and nucleon electroweak form factors see ref. [12].

The three-quark wave function for a nucleon is the product of a totally symmetric momentum wave function and a nonstatic spin wave function $\chi_\lambda$ which is an eigenfunction of the total angular momentum (squared) and its projection on the light cone axis. The spin wave function can be represented as a linear combination of products of matrix elements between valence quark light-cone spinors coupled by appropriate $\gamma$ (and isospin)-matrices to the spin and isospin of the nucleon. Free Melosh rotations [11] are of central importance in LCQMs for the construction of relativistic many-body spin-flavor wave functions for hadrons from those of the nonrelativistic quark model (NQM). Instant Dirac-spinors are transformed into light-front helicity eigenstates (light-cone Dirac-spinors) by Melosh transformations which include the kinematic quark boosts. Direct coupling of Pauli spinors by
SU(2) Clebsch-Gordan coefficients leads to a set of spin-flavor wave functions that are in one-to-one correspondence with the NQM states. (For details, see ref. [13].)

The wave function of the nucleon with nucleon helicity $\lambda$ is here taken to be

$$\psi_N(\lambda) = N \phi(x_3, q_3, Q_3) \chi_\lambda,$$

where $N$ is a normalization constant fixed by the proton charge (electric form factor at $q^2 = 0$). The Gaussian momentum wave function is written in terms of the relative momentum variables according to the Brodsky-Huang-Lepage prescription [14]

$$\phi(x_i, q_3, Q_3) = e^{-M_3^2/6\beta^2}$$

with a size parameter $\beta$ and the free mass operator $M_3$, i.e. the sum of the quark light-cone energies in the nucleon rest frame,

$$M_3^2 = \sum_{i=1}^3 \frac{k_{i\perp}^2 + m_i^2}{x_i} = -q_3^2 \frac{1 - x_3}{x_1 x_2} - \frac{Q_3^2}{x_3(1 - x_3)} + \sum_{i=1}^3 \frac{m_i^2}{x_i}.$$  \hfill (5)

In the projection of the triangle Feynman diagram to the null plane the totally symmetric nonstatic proton spin wave function $\chi_\lambda$ (see [13] for more details) originates from the quark-nucleon Lagrangian [12]

$$L_{q^2-N} = \left( \bar{q}_1 [i \partial N] \gamma_5 i \tau_2 C \bar{u}_2^T \right) \cdot \bar{q}_3 \Psi_N + (23)1 + (31)2,$$

where $q_i$ and $\Psi_N$ are the quark and nucleon fields, $C = i \gamma^2 \gamma^0$ is the charge-conjugation operator, and $\gamma_5$ is characteristic of the spin 0 of the quark pair [16]. The term $[P] \equiv \frac{\gamma P + m_N}{2m_N}$ with $P$ the total momentum and $m_N$ the nucleon mass for the null-plane projection model (KW), while for the Bakamjian-Thomas model (BT) the nucleon mass in $[P]$ is replaced by the free three-quark mass $M_3$ of Eq. 5. In the BT-model $[P]$ derives from the Melosh rotation of the quark Pauli spinors. The nucleon positive-energy projection operator $[P]$ corresponds to the mixing parameter $\alpha = 1/2$ in Araújo et al. of [2] of two of the three linearly independent three-quark-nucleon couplings that reduce to the NQM S-wave ground state in the static limit [3,13]. We restrict ourselves to the case $\alpha = 1/2$ because it can be directly compared with the BT-model.

In the triangle diagram for the form factors the quark fields are replaced by light-cone spinors $u_i$, quark 3 being the down quark of the proton in the $uds$-basis, so that the conventional nonstatic spin wave function

$$\chi_N = \bar{u}_2[P] \gamma_5 \bar{C} \bar{u}_3^T \cdot \bar{u}_1 u_N - \bar{u}_3[P] \gamma_5 C \bar{u}_1^T \cdot \bar{u}_2 u_N$$

results from Eq. 5. The momentum wave function $(M_3^2 - m_N^2)\phi$ in Eqs. 3,4 is proportional to the vertex function of $L_{q^2-N}$ of Eq. 5.

These simple relativistic quark models depend on two parameters, the common $u, d$ constituent quark mass $m_q$ and the size constant $\beta$ of the confinement potential. The proton mean square radius determines $\beta$ so that $1/\beta \sim \langle r^2 \rangle_p^{1/2}$ up to relativistic corrections.
III. ELECTROMAGNETIC FORM FACTORS OF THE NUCLEON

If $J^\mu$ is the full electromagnetic current, the current matrix element of a nucleon state consists of the Dirac and Pauli conserved currents,

$$\langle N(P', \lambda') | J^\mu | N(P, \lambda) \rangle = \bar{u}_N(P', \lambda') \left[ \gamma^\mu F_1(q^2) + i \sigma_{\mu\nu} q_\nu \right] u_N(P, \lambda),$$

where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$. We will also use the standard electric and magnetic Sachs form factors

$$G_E(q^2) = F_1(q^2) - \eta F_2(q^2), \quad G_M = F_1 + F_2,$$

where $\eta \equiv -q^2/4m_N^2 \geq 0$.

If free quark currents are used as in the impulse approximation, the possible $\omega$-dependence of the electromagnetic current matrix elements between baryon states leads to three additional conserved components with form factors $B_i$ so that Eq. 8 becomes

$$\langle N(P', \lambda') | \sum_j e_j \bar{q}_j \gamma^\mu q_j | N(P, \lambda) \rangle = \bar{u}_N(P', \lambda') \left( F_1 \gamma^\mu + i \frac{F_2}{2m_N} \sigma^{\mu\nu} q_\nu \right) u_N(P, \lambda)$$

$$+ \bar{u}_N(P', \lambda') B_1 \left( \frac{\omega \cdot \gamma}{\omega \cdot P} - \frac{1}{(1+\eta)m_N} \right) (P^\mu + P^\mu) u_N(P, \lambda)$$

$$+ \bar{u}_N(P', \lambda') \left( B_2 \frac{m_N}{\omega \cdot P} \omega^\mu + B_3 \frac{m_N^2}{(\omega \cdot P)^2} \omega \cdot \gamma^\mu \right) u_N(P, \lambda),$$

where $\omega \cdot q = q^+ = 0$ for the standard choice of light cone axis.

In front form all form factors are usually extracted from matrix elements of $J^+$ in the light-cone analog of the Breit frame (where $q^+ = 0$), the so-called 'good' current component with

$$eF_1(q^2) = \frac{m_N}{P^+} \langle N(P') | J^+ | N(P) \rangle,$$

$$\frac{eL}{2m_N}F_2(q^2) = -\frac{m_N}{P^+} \langle N(P') | J^+ | N(P) \rangle,$$

with $q^L = q_x - iq_y$. This is no longer possible when $\omega$-dependent currents are present. We therefore define the more general current matrix elements

$$J_{X,\lambda}^\mu \equiv \langle N', \lambda' | J^\mu | N, \lambda \rangle$$

in the rest frame of the nucleon. Just as Eq. 11 follows from Eq. 8 we obtain

$$J_{\uparrow\uparrow}^+ = P^+ \left( \frac{F_1}{m_N} + 2B_1 \left[ \frac{1}{m_N} - a \right] \right),$$

$$J_{\downarrow\uparrow}^+ = P^+ q^L \left( -\frac{F_2}{2m_N^2} - \frac{aB_1}{m_N} \right),$$

$$J_{\uparrow\downarrow}^L = q^L \left( \frac{F_1}{m_N} + \frac{F_2}{2m_N} + B_1 \left[ \frac{1}{m_N} - a \right] \right),$$

$$J_{\downarrow\downarrow}^L = \frac{eL}{2m_N} \left( -\frac{m_N}{P^+} \right).$$
\[ J_{\uparrow\downarrow}^L = (q^L)^2 \left( -\frac{F_2}{4m_N^2} - \frac{B_1a}{2m_N} \right), \]
\[ J_{\uparrow\uparrow}^L = -q^2 \left( \frac{F_2}{4m_N^2} + \frac{B_1a}{2m_N} \right), \]
\[ J_{\downarrow\downarrow}^L = q^L \left( -\frac{F_2}{2m_N} + B_1 \left[ \frac{1}{m_N} - a \right] \right) \]

from Eq. 10 and 13, where \( a = 1/(1 + \eta)m_N \). It is straightforward to check the symmetries

\[ J_{\uparrow\downarrow}^+ = J_{\uparrow\uparrow}^+, \quad \frac{J_{\uparrow\downarrow}^+}{q^L} = -\frac{J_{\uparrow\uparrow}^+}{q^L}, \]

etc. From Eq. 10 in conjunction with Eq. 13 we find two solutions for the Dirac and Pauli form factors

\[ F_1 = m_N \{ (2am_N - 1) \frac{J_{\uparrow\downarrow}^+}{P^+} + 2m_N^2 \left[ \frac{1}{m_N} - a \right] \frac{J_{\uparrow\downarrow}^+}{P^+q^L} + 2(1 - am_N) \frac{J_{\uparrow\downarrow}^L}{q^L} \}, \]
\[ F_2 = -m_N^3 \left[ \frac{2a}{m_N} J_{\uparrow\downarrow}^+ + 2 \left[ \frac{1}{m_N} - a \right] \frac{J_{\uparrow\downarrow}^+}{P^+q^L} - \frac{2a}{m_N} \frac{J_{\uparrow\downarrow}^L}{q^L} \right], \]

which we label set 1, while

\[ F_1 = m_N \{ \frac{J_{\uparrow\uparrow}^+}{P^+} + 4m_N^2 \left[ \frac{1}{m_N} - a \right] \left( \frac{-J_{\downarrow\downarrow}^L}{2m_Nq^L} + \frac{J_{\uparrow\downarrow}^L}{(q^L)^2} \right) \}, \]
\[ F_2 = -4m_N^2 \{ [1 - am_N] \frac{J_{\uparrow\downarrow}^+}{(q^L)^2} + \frac{a}{2} \frac{J_{\uparrow\downarrow}^L}{q^L} \}. \]

is the set 2.

Subtracting the spurious form factors according to the prescriptions of Eqs. 15, 16 will be called corrected form factors in the following.

**IV. NUMERICAL RESULTS**

The form factor calculations involve matrix elements of products of up to ten Dirac spinors which are evaluated and summed using symbolic codes generated by Mathematica. For the KW-model the transverse momentum integrations are carried out analytically, so that only the integrations over the longitudinal momentum fractions \( x_1, x_2 \) are evaluated numerically. The BT-model involves six-dimensional numerical integrations which are performed by Monte Carlo routines [20].

Although relativistic models yield predictions for any value of the momentum transfer, the constituent quark concept is expected to lose validity (i.e. the momentum dependence of the quark selfenergy becomes less important) at the spontaneous chiral symmetry breaking scale \( \chi = 4\pi f_\pi \sim 1\text{GeV} \), where current quarks are expected to become the relevant degrees of freedom. This limits the validity of the models to fairly small values of \( Q^2 = -q^2 \). Both of our light-cone quark models are based on pointlike constituent quarks. (Better fits can
be obtained using small anomalous magnetic moments and a finite size form factor, but this is not our objective here. See Cardarelli et al. in [1].

In figures 1 to 4 we compare the electromagnetic form factors of the KW and BT models (uncorrected and corrected) for the same parameters $\beta = 0.32$ GeV and $m_q = 0.32$ GeV. Results are shown relative to the dipole form factor $F_D = (1 - q^2/0.71 \text{GeV}^2)^{-2}$ with $q^2$ in GeV$^2$ except for Fig. 3 (neutron charge form factor). While the charge form factors of KW and BT (both uncorrected) for the proton in Fig. 1 are close to each other below 0.7 GeV$^2$, both models’ magnetic form factors in Fig. 2 differ by a roughly constant amount 0.2, i.e. 20% at 1 GeV$^2$. The corrected magnetic proton and neutron form factors BT1 and BT2 in Figs. 2 and 4 are close to each other but differ by about 20% from the uncorrected BT-model. This we interpret to mean that the corrected magnetic form factor of the BT-model is nearly unambiguous and correct. The corrected KW1 and KW2 form factors differ by a few percent (Figs. 2 and 4), however the correction itself is smaller than in the BT case. The charge form factors KW1 and KW coincide, so that the dotted line is not visible in the figures and KW2 leads to a slight correction in the proton case and a larger correction for the neutron. For the proton charge form factor BT1 and BT2 differ much more, so that it is poorly predicted by the BT-model in contrast to the KW-model.

Better fits can be obtained for both models, but for different parameters sets, and samples are shown in Figs. 5-9. In contrast to the NQM, the proton charge form factor $G_E^p$ falls faster than $G_M^p$ in both relativistic quark models. This is a general feature of LCQMs, which is in qualitative agreement with the recent data from Jefferson Laboratory [22]. Yet the proton charge form factors of both models fall off too fast compared to the JLab data in Fig. 5.

The major disagreement involves the neutron charge form factor that is low by a factor of more than 2. However, this is not surprising because it is known to be sensitive to the spin force and pion cloud effects that are beyond the scope of the simple relativistic quark models considered here.

The trends of uncorrected form factors persist when spurious form factors are subtracted according to Eqs. [15],[16] as shown in Figs. 1 to 9. From Fig. 1 we see that the proton charge form factors of the KW-model and the BT model for set 1 are hardly affected by the spurious contributions, while the corrections for the BT-model for set 2 are more than 50%. For the magnetic nucleon form factors in Figs. 2, 4 the corrections are similar for both models.

V. SUMMARY AND CONCLUSION

The impulse approximation leads to violations of gauge invariance and rotational symmetry by LCQMs, i.e. spurious dependence on the choice of light-cone axis. Depending on which helicity matrix elements are used to extract electromagnetic form factors, there are different schemes of subtracting spurious components from these form factors. Subtracting spuriousities reduces inconsistencies between helicity matrix elements. When electromagnetic form factors roughly agree that are obtained from different correction schemes, then we consider them as essentially correct and reliably predicted by the LCQM in question. This is the case for the magnetic proton and neutron form factors of the KW- and BT-models. The KW-model also predicts $G_E^p$ reliably in this sense, in contrast to the BT-model that predicts $G_E^p$ poorly. The typical corrections for spuriousities are larger for the BT-model as a consequence of its larger high-momentum components that enter via the three-quark mass
$M_3$ in the quark-nucleon coupling in Eq. 6. (Note that $M_3$ of Eq. 5 depends on the internal quark-momentum variables.) For $-q^2 < 1$ GeV$^2$, the corrected form factors differ from the uncorrected ones by about 20% for the KW-model and up to 50% for the BT-model. These are also the typical differences between reliably predicted form factors of these models. Thus, the theoretical errors of the form factors of the models are of roughly the same size as the differences between reliably predicted form factors of the models, so that it is difficult to distinguish them. These findings suggest that it is important for LCQMs to improve the impulse approximation and use conserved electromagnetic currents that are consistent with the interactions of the models.

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FIG. 1. Proton charge form factor $G_E^p/F_D$ relative to dipole shape $F_D$ for $m_q = \beta = 0.32$ GeV: The solid line is KW uncorrected while dashed and dotted lines KW1, KW2 are the corrected set 1,2 cases, respectively. The BT-model curves are similarly denoted by BT, BT1, BT2.
FIG. 2. Proton magnetic form factor $G_M^p/\mu_p F_D$ relative to dipole shape $\mu_p F_D$. The notation and parameters are as in Fig. 1.
FIG. 3. Neutron charge form factor $G_E^n$ with the notation and parameters of Fig. 1.
FIG. 4. Neutron magnetic form factor $G_M^n / \mu_n F_D$ relative to dipole shape $\mu_p F_D$ with the notation and parameters of Fig. 1.
FIG. 5. Fit of $G_E^p/Q^2/F_D$ for the KW-model (KW) with $m_q = 0.34$ GeV, $\beta = 0.32$ GeV and BT-model (BT) with $m_q = 0.26$ GeV, $\beta = 0.36$ GeV in the notation of Fig. 1. The results corrected for spurious form factors according to sets 1, 2 are denoted by KW1, KW2 and BT1, BT2. The full circle data are from ref. [21], the full squares are from [22].
FIG. 6. Fit of $G_M^p/F_D$ with the notation, parameters of Fig. 5. Data are from ref. [21].
FIG. 7. Fit of $G_n$ with the notation and parameters of Fig. 5. Data are from ref. [23].
FIG. 8. Fit of $G_{\pi}^m/F_D$ with the notation and parameters of Fig. 5. The full circle data are from ref. [24], the diamonds from [25].
FIG. 9. Fit of \( \mu_p G_E^p/G_M^p \) without spurious form factors with the notation and parameters of Fig. 5. The data are from JLab. [22]