Second Stage String Fragmentation Model

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Abstract

A string model, advocated by Bowler, provides a physical and intuitive picture of heavy quark fragmentation. When supplemented by an ad hoc factor of \((1 - z)\), to suppress fragmentation near \(z = 1\), it supplies an excellent fit to the data. We extend Bowler’s model by accounting for the further decay of the massive mesonic states produced by the initial string breaking. We find that each subsequent string break and cascade decay beyond the first, introduces a factor of \((1 - z)\). Furthermore we find that including a finite mass for the quarks, which pop out of the vacuum and split the string, forces the first string breaking to produce massive states requiring further decay. This sequence terminates at the second stage of fragmentation where only relatively “light” heavy meson systems are formed. Thus we naturally account for the phenomenologically required factor of \((1 - z)\). We also predict that the ratio of (primary) fragments-vector/(vector plus scalar) should be .61. Our second stage string fragmentation model provides an appealing picture of heavy quark fragmentation.

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1 Introduction

The idea that QCD field configurations resembling a string, or flux tube, are important in regimes where confinement dominates is an old and appealing one. It receives its strongest support from its birthplace – the description of the spectrum of linearly rising Regge trajectories [1]. Its scope was greatly expanded by the proposal of a simple model of string breaking [2]. The string model is the basis for the widely employed Lund model of fragmentation [3]. The application to fragmentation processes has however, a glaring flaw. Data on heavy meson fragmentation clearly indicate that production of heavy mesons near $z = 1$ is highly suppressed, probably like $(1 - z)^a$. The Lund model allows for $(1 - z)^a$ behavior, but $a$ is undetermined. Bowler [4] using a string model originally proposed by Artru and Mennessier [5] derived a string fragmentation model for heavy quarks with no $(1 - z)^a$ suppression factor. Bowler’s distribution peaks at $z = 1$. If a factor of $(1 - z)^\beta$ is arbitrarily appended to Bowler’s fragmentation function an excellent fit to the data is obtained with $\beta = .95 \pm 0.11$ [6].

In this paper we propose an origin for the $(1 - z)$ factor in fragmentation. It arises from allowing the string state to continue fragmenting even after the initial heavy mesonic system has formed. The continuing cascade decays carry away momentum and deplete the population of high $z$, heavy mesons. A crossing of a light and heavy quark worldline naturally defines a series of stages in this cascade decay. Bowler ended his fragmentation at the first crossing. We find that the $(1 - z)$ factor comes from the fragmentation after this crossing. We refer to such a process as second stage fragmentation and in Section III we present a derivation of the corresponding fragmentation function. This second stage fragmentation function is phenomenological equivalent to the modified Bowler function with the benefit that $\beta = 1$ is naturally selected by the physics. The second stage process is distinguished from the modified Bowler function in having a sharper drop off with hadron mass. This leads to a prediction for the fraction of vector to scalar plus vector production of .61 which differs from a value of .67 for Bowler’s modified function.

The second stage fragmentation function is thus seen to be an entirely satisfactory description of heavy quark fragmentation. What is special about the second stage that the bulk of fragmentation should occur here? In an attempt to answer this question we consider effects of finite mass, light quarks and explore a possible weakness of the tunneling model of string breaking. A consequence of tunneling in the string rest frame (lab frame for the first stage) with finite mass, light quarks is that the mesonic state produced is very massive. Its mass is significantly greater than the mass of the lightest “stable” charm mesons. These mesonic states will therefore further fragment i.e. continue their fragmentation into the second stage. The probability of producing a $D_0$ or $D^*$ at
the first stage is small because of the kinematics of the light quark mass. When we apply the same reasoning at the second stage, the first stage dynamics and the second stage kinematics is such that only “light” heavy quark mesons are produced. These “light” states do not need to undergo further fragmentation.

The second stage seems to be special. First stage fragmentation is unlikely to be the final stage. It leads to a massive fireball which further fragments, whereas the second stage is not kinematically forced to continue fragmenting. We do not claim that this is a complete answer, but rather that it is indicative of what might be special about second stage fragmentation. Our arguments on this “specialness” are presented in section IV.

2 First Stage Fragmentation

In this section we review the motion and fragmentation of a heavy quark, -anti-quark pair \((Q, \bar{Q})\) produced in \(e^+e^-\) annihilation, and bound together by a string. Although the kinematics are somewhat simpler in the heavy quark rest frame, for our purposes it will prove useful to work in the laboratory frame. The \(Q\bar{Q}\) are produced with initial momentum \(P_0\), equal to the electron laboratory frame momentum. The equation of motion of \(Q\) (chosen to be moving to the right) is

\[
\frac{dP}{dt} = \frac{d}{dt} \left( \mu \sqrt{v^2 - 1} \right) = -\alpha \tag{2.1}
\]

where \(\alpha\) is the string tension and \(\mu\) is the heavy quark mass. The solutions to the equations of motion are

\[
P_Q(t) = P_0 - \alpha t \tag{2.2}
\]

\[
\alpha x_Q(t) = \sqrt{\mu^2 + P_0^2} - \sqrt{\mu^2 + (P_0 - \alpha t)^2}. \tag{2.3}
\]

It will prove convenient to use light cone variables

\[
x^+ \equiv \frac{(x + t)}{\sqrt{2}} \quad x^- \equiv \frac{(t - x)}{\sqrt{2}} \tag{2.4}
\]

in terms of which the equation of motion (2.3) is

\[
x^+ = \frac{x^- (P_0 + \sqrt{P_0^2 + \mu^2})}{\sqrt{2\alpha x^- + \sqrt{P_0^2 + \mu^2 - P_0}}}. \tag{2.5}
\]

or equivalently

\[
x^- = \frac{(\sqrt{\mu^2 + P_0^2} - P_0)x^+}{\sqrt{\mu^2 + P_0^2} + P_0 - \sqrt{2\alpha x^+}}. \tag{2.6}
\]
The string joining $Q$ with $\overline{Q}$ will eventually break at some point $(x_1, t_1)$ (or equivalently $(x_1^+, x_1^-)$) producing a $q_1\overline{q}_1$ pair, which for the moment, we take to be massless. The $q_1$ and $Q$ trajectories will intersect at point $(x_{m_1}^+, x_{m_1}^-)$ signaling the formation of a heavy quark mesonic system, or fireball, of mass $m_1$. This terminates what we shall refer to as first stage fragmentation (see Fig. 1). The $q_1$ acquires both momentum and energy from being accelerated by the string connecting it to $Q$ so that when it intersects $Q$

\[
P_{q_1} = \alpha(t_{m_1} - t_1) \quad E_{q_1} = \alpha(x_{m_1} - x)
\]

whereas

\[
P_Q(t_{m_1}) = P_0 - \alpha t_{m_1} \quad E_Q(t_{m_1}) = \sqrt{\mu^2 + P_0^2} - \alpha x_{m_1}.
\]

Thus the energy and momentum of meson $m_1$ are

\[
P_{m_1} = P_0 - \alpha t_1 \quad E_{m_1} = \sqrt{\mu^2 + P_0^2} - \alpha x_1
\]

or

\[
(E + P)_{m_1} = P_0 + \sqrt{\mu^2 + P_0^2} - \sqrt{2} \alpha x^+
\]

and

\[
(E - P)_{m_1} = \sqrt{\mu^2 + P_0^2} - P_0 + \sqrt{2} \alpha x^-.
\]

The kinematic invariants we are interested in are the heavy meson mass $m_1$ and $z_1$ the fraction of the total momentum carried by it.

\[
z_1 \equiv \frac{(E + P)_m}{\max(E + P)_Q} = \frac{(E + P)_m}{P_0 + \sqrt{\mu^2 + P_0^2}}
\]

These are determined by the light cone variables $(x_1^+, x_1^-)$ as

\[
z_1 = 1 - \frac{\sqrt{2} \alpha x_1^+}{P_0 + \sqrt{\mu^2 + P_0^2}}
\]

\[
m_1^2 = z_1(\mu^2 + (P_0 + \sqrt{\mu^2 + P_0^2})\sqrt{2} \alpha x_1^-)).
\]

The probability that $m_1$ will form at $x_{m_1}t_{m_1}$ is the probability that no other break occurred in the absolute part of $(x,t)_{m_1}$. Any such break would pre-empt $m_1$ and lead to a different heavy mesonic system. This probability of formation of $m_1$ is

\[
dP = \rho dA e^{-\rho A}
\]

\[\rho\] is the constant probability, per unit 4 space-time volume, that the string will break in $dxdt$. $A$ is the area in the absolute past of $m_1$. A mildly tedious but straightforward calculation leads to

\[
dA = \frac{dm_1^2}{z_1} \frac{dz_1}{2\alpha^2}
\]
\[ dP = \frac{\rho}{2\alpha^2} dm_1^2 dz_1 \exp \left\{ -\rho \mu^2 \left[ \frac{(m_1/\mu)^2}{z_1} \cdot \frac{1}{z_1} - 1 - \ln \left( \frac{(m_1/\mu)^2}{z_1} \right) \right] \right\} \]  

(2.16) 

implying the first stage fragmentation function 

\[ f(m_1^2, z_1) = B \frac{1}{z_1} \exp \left\{ -B \mu^2 \left[ \frac{(m_1/\mu)^2}{z_1} \cdot \frac{1}{z_1} - 1 - \ln \left( \frac{(m_1/\mu)^2}{z_1} \right) \right] \right\} \]  

(2.17) 

where 

\[ B \equiv \frac{\rho}{2\alpha^2}. \]  

(2.18) 

This is the well known result of Bowler. 

For \( z_1 \) close to one \((1 - z_1) \ll 1\), 

\[ \int f(m_1^2, z_1) dm_1^2 \approx \sqrt{\pi B \mu^2} \frac{B \mu^2}{z_1}(1 - z_1). \]  

(2.19) 

while for larger \( z_1 \) values 

\[ \int f(m_1^2, z_1) dm_1^2 \approx \frac{z_1}{1 - z_1} e^{-B \mu^2 (\frac{1 - \mu}{z_1})^2}. \]  

(2.20) 

3 Second Stage Fragmentation 

It will prove kinematically convenient to perform this calculation in the initial rest frame of the heavy quark. We refer to Figure 2 for the definition of the important spacetime points involved in the fragmentation. The heavy quark \( Q_0 \) is formed at the origin. A \( q_1 \bar{q}_1 \) pair forms at \((x_1, t_1)\) and the \( \bar{q}_1 \) crosses the \( Q_0 \) trajectory at \( m_1 \). Between \( t_1 \) and \( t_{m_1} \), the \( q_1 Q \) system is an extended mesonic system or fireball. It is not necessarily a stable or even semi-stable meson. Although the actual crossing point \( m_1 \) is of little physical significance it is a convenient distinguishing point. Fragmentations occurring before \( t_{m_1} \) we refer to as first stage fragmentation. The events occurring between \( t_{m_1} \) and \( t_{m_3} \) are referred to as second stage fragmentation. Higher order stages are defined similarly. During the second stage process another \( q_2 \) tunnel into existence at \((x_2, t_2)\). This leads to the observed heavy meson fireball forming at \( m_3 \), while another light quark state forms at \( m_2 \).

The energy-momentum of \( m_3 \) is the sum of the energy-momentum of \( Q_0 \) at space-time point \((m_3)\), and the energy-momentum of \( \bar{q}_2 \). The energy-momentum of \( Q_0 \) at space-time point \((m_3)\), evaluated in the initial rest frame of \( Q_0 \), is 

\[ E_{Q_0}(m_3) = \mu + \alpha(x_{m_3} - 2x_{m_1}) \]  

(3.1)
\[ P_{Q_0}(m_3) = \alpha(t_{m_3} - 2t_{m_1}) \] (3.2)

We have used as a starting point the energy-momentum of \( Q_0 \) at point \( m_1 \). The energy-momentum of \( \overline{q}_2 \) is

\[ E_{\overline{q}_2}(m_3) = \alpha(x_2 - x_{m_3}) \] (3.3)
\[ P_{\overline{q}_2}(m_3) = \alpha(t_2 - t_{m_3}) \] (3.4)

leading to the energy-momentum of meson \( m_3 \)

\[ E_{m_3} = \mu + \alpha(x_2 - 2x_{m_1}) \] (3.5)
\[ P_{m_3} = \alpha(t_2 - 2t_{m_1}) \] (3.6)

The relevant kinematic invariants are \( z_3 \) the fractional momentum carried by the heavy meson \( m_3 \) and its mass squared \( m_3^2 \). These can be expressed in the coordinates of Fig. 2 as

\[ z_3 \equiv \frac{E_{m_3} + P_{m_3}}{\mu} = 1 + \frac{\sqrt{2} \alpha}{\mu} (x_2 - 2x_{m_1}) = z_1 - \frac{\sqrt{2} \alpha}{\mu} (x_{m_2} - x_2) \] (3.7)

\[ m_3^2 \equiv (E_{m_3} + P_{m_3})(E_{m_3} - P_{m_3}) = \mu^2 z_3 (1 - \frac{\sqrt{2} \alpha}{\mu} (x_2^+ - 2x_{m_1}^+)) = \frac{m_1^2 z_3}{z_1} - \frac{m_2^2 z_3}{z_2}. \] (3.8)

The last expression for \( m_3^2 \) is easily derived using the conservation of energy-momentum \( P_\mu, P_\mu^{m_3} = P_\mu^{m_1} - P_\mu^{m_2} \).

The probability \( dP_2 \), of forming a meson at \( m_3 \) in the second stage, is

\[ dP_2 = (\text{Probability of forming } m_1) \cdot (\text{probability of no break in absolute past of } m_3) \]

\[ = (\rho dA_1 e^{-\rho A_1}).(\rho dA_2 e^{-\rho A_2}) \equiv dz_1 dm_1^2 dz_3 dm_3^2 f(z_1, m_1^2, z_3, m_3) \] (3.9)

so

\[ f(z_1, m_1^2, z_3, m_3) = \left( \frac{\rho}{2\alpha^2} \right)^2 \frac{e^{-\rho (A_1 + A_2)}}{z_1 z_3} \] (3.10)

The calculation of the second stage fragmentation function is now reduced to the geometrical calculation of the area \( A_2 \). In order to calculate \( A_2 \) we need the equation of motion for \( Q_1 \) after it passes the crossover point \( m_1 \).

\[ x^+(x^-) = \frac{\mu^2}{2\alpha^2 [2x_1 + \mu - x^-]} + 2x_{m_1}^+ - \frac{\mu}{\sqrt{2} \alpha} \] (3.11)

valid in the range

\[ x_1^- \leq x^- \leq x_{m_3}^- \] (3.12)

Recall we are now in the heavy quark initial rest frame, where

\[ x_{m_1}^+ = \frac{\mu x_1^-}{(\mu + \sqrt{2} \alpha x_1^+)} = \frac{\mu}{\sqrt{2} \alpha} \left( 1 - \frac{\mu^2 z_1^2}{m_1^2} \right). \] (3.13)
We also need

\[ x_2^+ = \frac{\mu}{\sqrt{2\alpha}} \left\{ z_3 + 1 - 2\frac{\mu^2 z_1}{m_1^2} \right\} \]  \hspace{1cm} (3.14)

and

\[ x_3^- = \frac{\mu}{\sqrt{2\alpha}} \left( \frac{2m_1^2}{\mu^2 z_1} - 1 - \frac{1}{z_3} \right). \]  \hspace{1cm} (3.15)

Putting this together with the geometry of the shaded area in Fig. 2 we find

\[ A_2 = \frac{\mu^2}{2\alpha^2} \left[ m_1^2 z_3 - 1 - \ln \left( \frac{m_1^2}{\mu^2 z_1} z_3 \right) \right]. \]  \hspace{1cm} (3.16)

It is notationally convenient to express the fragmentation function as a function of the variables

\[ X_1 \equiv \frac{\rho}{2\alpha^2} \frac{m_1^2}{z_1} = \frac{B m_1^2}{z_1} \]

and

\[ X_0 = B \mu^2 \]

\[ f(z_3, m_3^2, z_1, X_1) = \frac{B^2}{z_3 z_1} \left( \frac{X_1^2}{X_0^2} \right)^{X_0} e^{2X_0 - X_1 (1 + z_3)}. \]  \hspace{1cm} (3.17)

The experimentally relevant function involves only the mass \( m_3 \) and fractional momentum \( z_3 \) carried by the “meson” \( m_3 \). It is obtained by integrating over \( z_1 \) and \( m_1^2 \)

\[ f(z_3, m_3^2) = \int \int dz_1 dm_1^2 f(z_3, m_3^2, z_1, X_1). \]  \hspace{1cm} (3.18)

We now need the limits of integration. For fixed \( z_3 \) and \( m_3^2 \)

\[ z_3 < z_1 < 1 \]

\[ \frac{m_3^2 z_1}{z_3} < m_1^2 < 4P_0^2 \approx \infty. \]

Changing integration variables from \( m_1^2 \) to \( X_1 \), and defining \( X_3 \equiv \frac{B m_3^2}{z_3} \) we find for the second stage fragmentation function

\[ f(z_3, m_3^2) = \frac{1}{z_3} \int_{z_3}^1 dz_1 \int_{X_1}^\infty dX_1 \left( \frac{X_1^2}{X_0^2} \right)^{X_0} e^{2X_0 - X_1 (1 + z_3)} \]

\[ = \frac{1 - z_3}{z_3} \int_{X_3}^\infty \left( \frac{X_1^2}{X_3^2} \right)^{X_0} e^{2X_0 - X_1 (1 + z_3)} dX_1. \]  \hspace{1cm} (3.19)

This is our key result. A factor of \( (1 - z_3) \) naturally arises from the string model. Physically it comes from the fact that the light meson, \( m_2 \), produced in second stage fragmentation
(see Fig. 2) carries off part of the initial momentum, leaving \( m_3 \) with a smaller fraction of \( z_1 \). The integrand in 3.20 carries with it the sharp falloff in \( \frac{1}{z_1} \) and \( m_3 \) that characterized first stage fragmentation.

It is illuminating to compare this second stage fragmentation function to the first stage fragmentation by considering the limit \( X_3 \gg X_0 \) (i.e. \( \frac{m_3^2}{z_3} \gg \mu^2 \)). Equation (3.20) can then be approximated as

\[
\left( \frac{1-z_3}{z_3} \right) \left( \frac{X_3}{X_0} \right)^{X_0 - X_3} \left( \frac{m_3^2}{\mu^2} \right)^{X_0} e^{B(\mu^2 - m_3^2)} \left( 1 + \frac{\mu^2}{m_3^2} \left( \frac{2z_3}{1+z_3} \right) + ... \right)
\]

(3.21)

The first term in this expression is identical to the Bowler expression with the crucial addition of a factor \( 1 - z_3 \). At fixed value of \( m_3^2 \) the factor in square brackets is a weakly varying function of \( z_3 \) that changes the normalization of the Bowler distribution. At fixed \( z_3 \) the square brackets provides an additional fall off with \( m_3^2 \) when compared to the Bowler function.

In fitting the experimental \( D^* \) and \( D^0 \) fragmentation functions a modified form of the Bowler fragmentation function has been employed [6]. A factor of \( (1 - z)^\beta \) was arbitrarily appended to the function of (2.17), and \( \rho \) was treated as a parameter. The so called modified Bowler parametrization is

\[
(1 - z)^{\beta} e^{B(\mu^2 - m_3^2)} \left( \frac{m_3^2}{\mu^2} \right)^{B\mu^2}
\]

(3.22)

where \( \beta \) was determined by fitting the data as \( \beta = .95 \pm .11 \).

We propose the second stage string fragmentation function (3.20) as a substitute for the modified Bowler form. The \( (1 - z) \) factor, i.e. \( \beta = 1 \) is a direct consequence of the second stage fragmentation. The similarity between Eqs. (2.17) and (3.21) generates an equally good fit. In Fig. 3 we compare the second stage function (3.20) (with \( B = .65, \mu = 1.5 \)) to the modified Bowler function used in CLEO collaboration fits (\( \beta = .95, B = .63 \)). They are indistinguishable. The second stage fragmentation function thus provides a two parameter (overall normalization factor, and \( B \)) fit to the full \( z \) range of fragmentation for both \( D^0 \) and \( D^* \).

Given that \( D^0 \) and \( D^* \) distributions are in accord with the 2nd stage predictions we naturally ask whether their relative production rates are given by our model. String models predict a suppression of large mass states with respect to lighter states. This is evidenced in the exponential drop off with mass in Eqs. (2.17) and (3.20). (Even though the mass difference between \( D^* \) and \( D^0 \) is usually attributed to single gluon exchange, and not to any string dynamics our prediction of exponential suppression in \( m_3^2 \) is robust. This is because, whatever the origin of the extra mass, in our model this mass must be paid for by a longer string segment. The energy needed to form more massive states has to come from the string energy. How that piece of string re-arranges itself to form the final meson, is not relevant to the accounting of how much string needs to be allotted...
to the state. Therefore the mass dependency will be valid since it depends only on how much string (energy) is transferred to the produced meson.)

The experiments cannot distinguish between primordially produced $D^*$, and those that are the end products of decay chains. What can be unraveled is the ratio.

$$P_v \equiv \frac{N_v}{N_v + N_s}$$

where $N_{v(s)}$ refers to the number of primordial vectors (scalars) produced. Naively this ratio should be $3/4$ since $N_v = 3N_s$ from spin state counting. In string models the more massive vectors are produced less frequently than the lighter mass $D^0$, so $N_v < 3N_s$.

Integrating the second stage fragmentation function (3.20) over $z$ for both $D^0$ and $D^*$ we find

$$P_v = .61$$

(3.24)

A similar prediction can be made for $D_s^*$ to $D_s^0$ production, and we find an identical result

$$P_v = .61$$

(3.25)

The second stage fragmentation has a sharper drop-off in mass than the modified Bowler form (3.22). Therefore the results of (3.24) and (3.25) are smaller than we would get from (3.22). The modified Bowler prediction is

$$P_v = .67$$

The ratio $P_v$ thus offers an experimentally measurable distinction between the second stage fragmentation and the modified Bowler fragmentation function.

In this section we have calculated the heavy quark fragmentation function at the end of the second stage of fragmentation. The heavy meson which appears is the result of two events of string breaking. The fragmentation function automatically incorporates the factor of $(1 - z)$ which is required by experiment. It is phenomenologically indistinguishable from the modified Bowler function. The experimental data thus strongly support the string fragmentation model whereby heavy quark mesons are the end result of a two stage, string breaking cascade decay.

We can continue the cascade process by considering a third stage of fragmentation. An additional factor of $(1 - z)$ arises leading to a $(1 - z)^2$ behavior near $z = 1$. Even more rapid $m^2$ damping is also present. It appears the trend will continue for ever higher cascades. Thus while the data could certainly accommodate a small admixture of third or higher stages, the preponderance of fragmentation seems to come from second stage string breaking.
4 Finite, Light Quark Mass, Tunneling and Something Special about the 2nd Stage

There are no massless quarks in nature. The $u, d$ quarks have a constituent quark mass of $\sim 300$ MeV. We also know that hadron fragmentation is not purely 1 space-dimensional. Strings have finite thickness and particles are produced with momentum transverse to the jet direction. It should be possible to incorporate some of these transverse momentum effects into an effective mass.

$$m_{\text{eff}} = \sqrt{m^2 + P^2_\perp} \equiv m_\ell$$  \hspace{1cm} (4.1)

It therefore behooves us to re-examine the simple model of section II and III for any significant effects that might arise from the finite mass $m_\ell$ of $q$.

One immediate consequence of finite mass is that a finite segment $\ell$, of string, with $\ell = \frac{2m_\ell}{\alpha}$ must disappear in order for the quarks to materialize when the string breaks. There will be a gap between the $q$ and $\bar{q}$ which are produced. We work in the laboratory rest frame. (See Fig.4.)

We refer to the breaking point $x_1, t_1$, as the point midway between the $q$ and $\bar{q}$ which have tunneled out of the vacuum, see Fig. 4. Except if breaking occurs very near the turning point of $Q$, the $\bar{q}$ will have ample time for its trajectory to asymptote to the trajectory of a massless $\bar{q}$ produced at $x_1$. This is indicated by the dotted line in Fig. 4. Therefore the intersection point $m_1$ is the same as in section II for string breaking at $x_1$ and

$$P_{m_1} = \alpha(t_{m_1} - t) \quad E_{m_1} = \alpha(x_{m_1} - x_{\bar{q}}) + m_\ell = \alpha(x_{m_1} - x_1)$$  \hspace{1cm} (4.2)

since $m_\ell = \alpha(x_{\bar{q}} - x_1)$. The kinematics are identical to the massless case for breaking at $x_1$ where $x_1$ and $x_{\bar{q}}$ are defined as in Fig. 4. An important difference is that $x_1$ must be a finite distance away from the $Q$ trajectory. It cannot get closer than $\Delta x \equiv \frac{m_\ell}{\alpha}$ and must therefore produce a mesonic system with mass $M > \mu$.

The simplest way to find the minimum mass is to consider the case where $\bar{q}$, with mass $m_\ell$, materializes, at rest, immediately next to the moving $Q$

$$M^2 = (E_Q + m_\ell + P_Q)(E_Q + m_\ell - P_Q) = \mu^2 + 2m_\ell E_Q + m_\ell^2 \sim \mu^2 + 2m_\ell E_Q.$$  \hspace{1cm} (4.3)

$z$ for this state is

$$z = \frac{E_Q + P_Q + m_\ell}{P_0}$$  \hspace{1cm} (4.4)

implying

$$M^2 = \mu^2 + 2m_\ell \sqrt{P_0^2 z^2 + \frac{\mu^2}{2}}$$  \hspace{1cm} (4.5)

Except for very small $z$, $M$ will be large, significantly greater than $\mu^2$. 9
Large mass states, in general, are unstable and readily decay, emitting pions until the lowest mass, heavy quark meson is produced. Thus the fragmentation considered by Bowler and described in section II is not the final result, but the first step in a cascade process. Only after several steps or stages will a relatively stable, heavy quark meson be produced terminating the cascade and producing the experimentally observed fragment.

To make these observations concrete consider charm quark fragmentation at CESR and ARGUS with $P_0 \approx \frac{10.55}{2}$ GeV/c, $\mu = 1.5$ GeV.

We chose $m_\ell \approx 350 - 400$ MeV, which is a reasonable value for the constituent quark mass described in the discussion surrounding (4.1). We now ask; for what values of $z$ will the fireball produced by the (first stage) fragmentation of Fig. 4 continue to decay? We take the minimal mass $M$ of this fireball to be 2.15 GeV. This is above $(m_\pi + m_{D^*})$ providing phase space to decay into the “stable” minimal mass heavy charmed mesons $D^0(1869)$ and $D^*(2010)$. From Eq. (4.5)

$$z \geq \sqrt{\left(\frac{M^2 - \mu^2}{2m}\right)^2 - \frac{\mu^2}{2P_0}} \approx .52 - .61$$

(4.6)
depending on our choice of $m_\ell$. This result is also sensitive to the value of $\mu$, varying from .44 to .52 for $\mu = 1.6$ GeV.

For all $z$ greater than .5 to .6, massive fireballs will be produced, with mass $\geq 2.15$ GeV. Such massive states require further cascade decays before producing stable states. Since the original, first stage, fragmentation function is strongly peaked near $z = 1$, (see Eq. (2.17) and (2.19)) between 95% and 75% of all states produced by first stage string breaking will proceed to second or higher stages. Only the 5-25% produced with small $z$ ($z \leq .5$ to .6) will be observable as $D$ or $D^*$ products of first stage fragmentation. All high $z$ $D_0$ or $D^*$ must be the result of second, or higher, stage fragmentation and will be described by e.g. (3.20), or its higher iterations. The finiteness of the light quark mass and the kinematics of string breaking force us into a multi-cascade picture of fragmentation. As we showed in Section 3 cascade decay automatically suppresses fragmentation near $z = 1$.

Our result for the minimum mass of the fireball is strongly frame dependent since it arises from the relative momentum of $Q$ with respect to the $\bar{7}$ which has tunneled from the vacuum. The tunneling phenomena itself is frame dependent and seems to us most reliable in the string rest frame.

Since the quark and anti-quark which tunnel out of the vacuum have mass $m$, a finite segment of string $\ell$, determined by $\ell = \frac{2m}{a}$ will disappear. This phenomena produces the quark mass dependence $e^{-m}$ characteristic of tunneling. It also provides problems for the tunneling model. The quark and its anti-quark partner will materialize at different space-time points, hence the tunneling is non-local. (By contrast, massless quarks will always pop out simultaneously at the same spot. Thus string breaking by massless quarks is local.) An immediate consequence of this is that the tunneling looks dramatically different in different Lorentz Frames. The semi-classical model works best in the string rest frame.
The quark and antiquarks materialize semiclassically, simultaneously, at rest, equidistant from the breaking point. Tunneling is completely symmetric between \( q \) and \( \bar{q} \). This symmetry is lost in any other Lorentz frame. If the materialization of \( q \) and \( \bar{q} \) are no longer simultaneous, the \( q \) and \( \bar{q} \) will be moving, possibly at different speeds. Not only will the tunneling process be more complicated (e.g. quarks have to absorb kinetic energy in addition to rest mass energy from the field) but it might not even make sense. If the \( q \) and \( \bar{q} \) do not materialize simultaneously we will have unshielded, dangling color fields. [If \( q \) pops out first it will saturate the color field from \( Q \), leaving \( Q \)'s color naked while it waits for the \( \bar{q} \) to appear!]. For these reasons, we feel that the tunneling model is a reliable physical model for string breaking primarily in the string rest frame. This has consequences for the fragmentation.

The tunneling model for string breaking when applied in the string rest frame, indicates that fragmentation must continue beyond the first stage. Does the same argument force the fragmentation to proceed to third or higher stages? As we shall see, the kinematics are significantly different at the second stage, so that there is no necessity of higher stage fragmentation. In this sense there is something special about the second stage.

According to our arguments the tunneling model for string breaking at the second stage should be applied in the rest frame of the fireball of mass \( m_1 \). We must therefore make a Lorentz transformation from the lab frame to the rest frame of \( m_1 \). This Lorentz transformation is determined by

\[
g = \frac{E_{m_1}}{m_1} \quad \gamma \beta = \frac{P_{m_1}}{m_1}
\]  

(4.7)

The role of \( P_0 \) will now be played by Lorentz transformation of \( P_Q, P_Q^1 \)

\[
P_Q^1 = \lambda P_Q
\]  

(4.8)

where \( \lambda \) is the Lorentz Transformation of \( (4.7) \). From \( (2.9) \)

\[
P_Q = P_0 - \frac{\alpha}{\sqrt{2}} (x_+ + x_-) m_1.
\]  

(4.9)

For all cases of interest the \( \bar{q} \), which has tunneled out of the vacuum, will have asymptoted to the light cone of a massless \( \bar{q} \) produced at \( x_1, t_1 \) (See Fig.4). So \( x_{m_1} = x^- \). \( x^- \) is determined in terms of \( m_1^2 \) and \( z_1 \), (see \( (2.14) \))

\[
\frac{m_1^2}{z_1} - \mu^2 = (\sqrt{P_0^2 + \mu^2}) \sqrt{2} \alpha x^-.
\]  

(4.10)

\( x^- \) is fixed by the equation of motion Eq.\( (2.3) \)

\[
x_{m_1}^+ = \frac{x^- (P_0 + \sqrt{P_0^2 + \mu^2})}{m_1^2 (P_0 + \sqrt{P_0^2 + \mu^2}) z_1} \approx \frac{1 - \frac{\mu^2 z_1}{m_1^2}}{2\alpha} (2P_0)
\]  

(4.11)
where we assume $P_0 \gg \mu$. Hence

$$P_Q = P_0 z_1 \left( \frac{\mu^2}{m_1^2} \right) \quad (4.12)$$

The kinematic region of greatest interest with $z_1$ relatively large corresponds to

$$\frac{z_1 P_0 \mu}{m_1^2} \geq 1 \quad (4.13)$$

In this regime

$$P_Q^1 \simeq \frac{1}{2} \left( 1 + \frac{\mu^2}{m_1^2} \right) m_1 \quad (4.14)$$

which is much smaller than $P_0$ for all relevant values of $m_1$. From Eq. (4.15) we see that it is the large value of $P_0$ which drives the fragmentation to large mass states, requiring further fragmentation. $P_Q$ (4.14) will not become large unless $m_1$ becomes large. But because of the exponentially rapid falloff with $m_1^2$ of the first stage fragmentation function (2.17) very little fragmentation occurs with large $m_1^2$.

At the second stage the minimal mass $M^2$ (4.5) becomes

$$M^2 = \mu^2 + 2m_\ell \sqrt{(P_Q^1)^2 z_2^2 + \frac{\mu^2}{2}} = \mu^2 + 2m_\ell \sqrt{\left( \frac{1 + \frac{\mu^2}{m_1^2}}{4} \right) m_1^2 z_2^2 + \frac{\mu^2}{2}} \quad (4.15)$$

Again requiring that $M \geq 2.15 \text{ GeV} = 1.43 \mu$ we find that we need

$$m_1 \geq (3 - 5) \mu$$

to form a fireball at the second stage, sufficiently massive to force a third stage of the fragmentation process. These massive, first stage states, are almost never produced because of the $e^{-m_1^2/\mu^2}$ factor in the first stage fragmentation function.

Our conclusion is that there is something special about the second stage of the string fragmentation process. The Bowler function which describes first stage fragmentation is sharply peaked near $z = 1$, and $m_1 \approx \mu$. When we account for the finite mass of the constituent quarks that pop out of the vacuum to break the string we find that only massive states, with $z > 1/2$, are abundantly produced. These states must continue to fragment at the second stage. Except for extremely massive states, fragmentation will end (i.e. produce either $D_0$ or $D^+_0$) at this stage. Third stage fragmentation can only be populated by first stage fragments of such high masses that we expect very few of them to be produced. The vast majority of $D$ and $D^*$ observed will have been produced by two stages of breaking and will therefore be described by the phenomenologically successful Eq. (3.20).
5 Conclusions

The string model is widely invoked to provide physical insight into the physics of confinement in QCD. Its relevance to hadrons production in $e^+, e^-$ collisions has long been appreciated. It has therefore been disappointing that a characteristic feature of the experimental data, a suppression of heavy meson production near $z \to 1$ has defied simple explanation in the string model. Our main accomplishment in this paper is the demonstration that the string model of Artru, Mennessier and Bowler, when properly extended, gives rise in a very natural way to a $(1 - z)$ factor in heavy quark fragmentation. In the heavy quark fragmentation studies he pioneered, Bowler somewhat arbitrarily cut off the fragmentation process after a heavy quark mesonic system formed. We removed this restriction and studied in detail the subsequent fragmentation of this mesonic system. The mesonic system decays in a cascade pattern, spitting out light mesons while degrading the fractional momentum carried by the heavy quark meson. Thus, at each stage of fragmentation, it is increasingly less likely that the heavy meson will carry all the initial momentum. It can not have $z = 1$. The calculation makes this explicit, manifesting the degradation as a factor of $(1 - z)$ for each stage beyond the first. We found that the second stage fragmentation function provided an excellent fit to experimental charm fragmentation data and provides the rational for the heretofor mysterious $(1 - z)$ factor appended to the Bowler fragmentation function. We then attempted to justify the predominance of the second stage of the fragmentation process.

An examination of the fundamental string breaking mechanism, quark anti-quarks tunneling from the vacuum in a strong QCD Field, revealed the non-Lorentz invariant nature of this process. A preferred reference frame, the rest frame of the string, emerges as the natural stage on which to perform this quintessential quantum act. Redoing the fragmentation analysis in this frame, which corresponds to the laboratory frame, the existence of a non-zero effective quark mass for the popped quarks imposes a minimal mass for the heavy meson fragments produced. For all, except the smallest $z$ values where fragmentation is unlikely in any event, this minimal mass is well above the mass of stable heavy quark mesons, implying that at least one more fragmentation stage is necessary. This is the physical reason why the first stage fragmentation process is not relevant for the experimental observation.

This minimal mass effect is much less robust at higher stages of fragmentation. The Lorentz Transformation to the rest frame of the newly produced heavy quark mesonic system greatly deflates the strength of this effect. The minimal mass system produced at the second stage is within the range of stable heavy meson states. Thus the second stage fragmentation function is special. Fragmentation must proceed to at least this stage, but can end at this stage. The phenomenological success of the modified Bowler function can
now be recognized as a success of the second stage of string fragmentation.

Acknowledgements

We wish to acknowledge partial support from DOE under contract number DE-FG02-85-ER40231. We have benefitted from conversations with encouragements from N. Horwitz and G.G. Moneti

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Figure Captions

Fig. 2 First stage heavy quark fragmentation in the laboratory frame. The string joining $Q_0$ to $\overline{Q}_0$ breaks at $(x_1, t_1)$ by producing a massless $q\overline{q}$ pair. The $\overline{q}$ is then accelerated by the string attaching it to $Q_0$. The $\overline{q}$ and $Q_0$ trajectories intersect at point $m_1$ terminating first stage fragmentation with the production of a fireball of mass $m_1$.

Fig. 2 Second stage fragmentation in the heavy quark rest frame. The first stage terminated at point $m_1$. Second stage fragmentation is caused by the string breaking at $(x_2, t_2)$ and terminates when $\overline{q}_2$ intersects the world line of $Q_0$ at $m_3$, producing a heavy quark mesonic state of mass $m_3$. The shaded region in the area we call $A_2$ in section 2.

Fig. 3 A comparison between the modified Bowler function of Ref. 6 to our second stage fragmentation function. By a slight shift in a phenomenologically determined parameter ($B = .65$ rather than .63) we see that the two functions are essentially indistinguishable.

Fig. 4 String breaking by the production of a massive $q, \overline{q}$ pair. The $q$ and $\overline{q}$ no longer appear at the same location. As the $q$ and $\overline{q}$ accelerate their worldline eventually asymptote to the light like trajectories of a massless $q$ (or $\overline{q}$) produced at $x_1 t_1$. 
This figure "fig1-1.png" is available in "png" format from:

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