ON THE DARK SIDE OF QUASAR EVOLUTION

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ABSTRACT

Recent improved determinations of the mass density /m_0/ of supermassive black holes (SMBHs) in the local universe have allowed accurate comparisons of /m_0/ with the amount of light received from past quasar activity. These comparisons support the notion that local SMBHs are "dead quasars" and yield a value /r_0/ ≥ 0.1 for the average radiative efficiency of cosmic SMBH accretion. BH coalescences may represent an important component of the quasar mass assembly and yet not produce any observable electromagnetic signature. Therefore, ignoring gravitational wave (GW) emission during such coalescences, which reduces the amount of mass locked into remnant BHs, results in an overestimate of /r_0/. Here we put constraints on the magnitude of this bias. We calculate the cumulative mass loss to GWs experienced by a representative population of BHs during repeated cosmological mergers, using loss prescriptions based on detailed general relativistic calculations. Despite the possibly large number of mergers in the assembly history of each individual SMBH, we find that near-equal mass mergers are rare; therefore, the cumulative loss is likely to be modest, amounting at most to a 20% increase in the inferred /r_0/ value. Thus, recent estimates of /r_0/ ≥ 0.1 appear robust. The space interferometer LISA should provide empirical constraints on the dark side of quasar evolution by measuring the masses and rates of coalescence of massive BHs to cosmological distances.

Subject headings: black hole physics — galaxies: nuclei — gravitational waves — quasars: general

1. INTRODUCTION

It is now widely accepted that quasar activity is powered by accretion onto supermassive black holes (SMBHs). From the active phases of accretion that characterize luminous, high-redshift quasars, one expects remnant SMBHs to be present at the centers of nearby galaxies (Lynden-Bell 1967; Soltan 1982; Rees 1990). The evidence for such a population of dead quasars has been growing over the years (see Kormendy & Richstone 1995 for a review) and is now compelling (Mogorrian et al. 1998).

Dynamical studies of nearby massive galaxies indicate that a close link exists between the masses of dead-quasar SMBHs and the properties of their host galaxies, including the spheroid’s mass (Mogorrian et al. 1998; Haering & Rix 2004), velocity dispersion (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002), and the total galactic mass (Ferrarese 2002). These empirically determined correlations allow accurate tests of the idea that the amount of mass locked into SMBHs in nearby dead quasars should be comparable to that inferred from the amount of light received from past quasar activity, with a radiative efficiency /r_0/ ≈ 10%, since the latter is a tracer of BH mass buildup via accretion (Soltan 1982; Chokshi & Turner 1992). Recent comparisons find a good agreement between the mass density in dead-quasar SMBHs and the integrated light from optically bright quasars, provided that the radiative efficiency of BH accretion in luminous quasars is /r_0/ ≥ 0.1 (Yu & Tremaine 2002; Aller & Richstone 2002; Haiman et al. 2004). The luminosity density of the quasar population can also be inferred from the X-ray bands. This has led to suggestions that optical quasar surveys may be missing some of the quasar emission because of obscuration (Fabian & Iwasawa 1999; Barger et al. 2001), which may be indicative of radiatively more efficient accretion onto fast-spinning BHs (Elvis et al. 2002). However, recent work using the soft X-ray luminosity function of Miyaji et al. (2001) has found a low efficiency of /r_0/ ≈ 0.05 (Haiman et al. 2004). Soft X-ray bands miss the most highly obscured sources, but the efficiency is increased further only by a factor of ~2 when hard X-ray sources (with the luminosity function from Ueda et al. 2003) are added in the comparison (Marconi et al. 2004).

In the present study, we investigate the possibility that cumulative mass-energy losses to gravitational waves (GWs) during repeated BH binary coalescences, in the context of standard cosmological hierarchical structure formation models, may significantly reduce the amount of mass currently locked into BHs, and thus effectively bias the comparison between active and dead quasars toward larger values of the radiative efficiency, /r_0/. The role of GW losses for the quasar population has already been considered by Yu & Tremaine (2002), but only with an idealized description of cosmological mergers and for maximized "adiabatic" losses to GWs (see also Ciotti & van Albada 2001; Volonteri et al. 2003; Koushiappas et al. 2004). In a companion paper (Menou & Haiman 2004; hereafter Paper I), we have reconsidered this issue with a more realistic description of cosmological BH mergers. Our results suggest that, while the mass loss in a single merger event is small, after numerous repeated mergers over cosmic times, adiabatic losses can result in a substantial and astrophysically important reduction of the BH mass density. However, the adiabatic assumption provides only an upper bound on the mass-energy carried away by GWs, and thus largely overestimates realistic losses. Here we use an improved prescription for GW losses, based on the latest available general relativistic calculations, to provide more accurate constraints on the possible role of GWs in modifying the mass budget of merging quasars.
2. MODELS

2.1. Merger History of Massive Black Holes

Our description of the cosmological merger history of massive BHs follows very closely that presented in Paper I (see also Menou et al. 2001 for details). We use a dark matter halo merger tree with a standard ΛCDM cosmology to evolve the population of massive BHs and their host galaxies. Only galaxies with a total mass exceeding a virial temperature equivalent of 10^4 K are described by the tree, since these are the galaxies with baryon cooling efficient enough to allow BH formation (smaller objects rely on H2 cooling and are subject to disruptive feedback; Oh & Haiman 2003). It is assumed by default in our models that all potential host galaxies harbor a massive BH, although we have also explored models in which massive BHs are 10 times rarer and are initially confined to the most massive 10% of galaxies (as described in Paper I).

Recent quasar evolutionary studies indicate that the majority of the mass currently locked into SMBHs was accreted between redshifts \( z \approx 3 \) and \( z = 0 \) (e.g., Yu & Tremaine 2002; Marconi et al. 2004). Since most of the losses due to mergers are expected to occur when most of the BH mass is being assembled, in order to estimate the GW losses due to mergers, there is no need to extend our models much beyond redshifts \( z \approx 3 \). We assume that the relation between SMBHs and their host galaxy velocity dispersion is the same at \( z \approx 3 \) as locally (Shields et al. 2003), and we adopt a mass ratio between BHs and their parent halos given by Ferrarese (2002) and Wyithe & Loeb (2004):

\[
M_{\text{BH}} = 10^9 M_\odot \left( \frac{M_{\text{halo}}}{1.5 \times 10^{12} M_\odot} \right)^{5/3} \left( \frac{1 + z}{7} \right)^{5/2},
\]

where \( M_{\text{halo}} \) is the mass of the dark matter halo associated with each galaxy. This relation may result from the BH mass being limited (at least initially, during the luminous quasar phase) by feedback from the quasar’s radiation (Silk & Rees 1998; Wyithe & Loeb 2003). We have found in exploratory models (see Paper I) that the shape of this \( M_{\text{BH}} \)-\( M_{\text{halo}} \) relation is not strongly modified by the redistribution of BHs in galaxies as a result of cosmological mergers. This provides additional motivation for setting up BH masses according to equation (1) at \( z \approx 3 \) and neglecting the role that accretion may subsequently have in modifying them over cosmic times (modulo an overall scaling factor). Below, we express all mass deficits in evolutionary models with GW losses relative to a no-loss model, thus effectively scaling out the exact \( \rho_{\text{BH}} \) value from the loss problem.

Starting at \( z \approx 3 \), we let the BH population evolve through a series of cosmological mergers until \( z = 0 \). We assume that each time two galaxies merge, the two BHs they were hosting coalesce rapidly. In doing so, we ignore complications related to the “last parsec” problem for BH coalescences (Begelman et al. 1980; see Milosavljevic & Merritt 2003 for a recent discussion) and effectively maximize BH merger rates in our models. Rapid coalescences may be induced by effects due to the triaxility of galaxies (Yu 2002) or the presence of surrounding gas (Gould & Rix 2000; Armitage & Natarajan 2002; Escala et al. 2004). We do, however, account for the inefficiency of dynamical friction in initially bringing the two BHs together by assuming, following Yu (2002), that BH binaries do not form for mass ratios \( q < 10^{-3} \) (see also Paper I). Finally, we emphasize that the above model, which ignores gas accretion, is not intended to yield a realistic description of the quasar BH population. Rather, our limited goal here is to quantify the effect of mergers alone on the remnant BH mass budget.

2.2. Mass Loss to Gravitational Waves

In their final stages of coalescence, energy and momentum are extracted from massive BH binaries by emission of GWs. As a result, the BH merger remnant has a mass that is less than that of its two progenitors. This mass loss to GWs, and its cumulative effect on the global BH mass density through repeated cosmological mergers, is the main focus of our study.

Rather than adopting simple GW loss prescriptions, as in Paper I, we wish to obtain more accurate constraints based on the latest available general relativistic calculations. This is no easy task, however, because the general relativistic BH coalescence problem has not been solved in full generality (see Baumgarte & Shapiro [2003] for a review of numerical progress), and approximate analytical solutions exist only for some limiting cases. Here we will use such approximate solutions and extrapolate them whenever necessary.

Let us denote by \( m \) and \( M \) the masses of the two BHs involved in a coalescence, with \( m \leq M \). The mass ratio is \( q = m/M \leq 1 \), and the reduced mass is defined as \( \mu = mM/(m + M) \). In the test-particle limit (\( q \ll 1 \)), the coalescence can be decomposed into three successive phases: (1) a slow inspiral phase during which the two BHs spiral in quasi-adiabatically on nearly circular orbits, (2) a plunge phase due to the existence of an innermost stable circular orbit (ISCO) past which the two BHs are brought together via a dynamical instability, and (3) a final ring-down phase during which the perturbed merger remnant relaxes to a stationary Kerr BH. The separation between the plunge and ring-down phases is somewhat arbitrary. In addition, it is possible that no ISCO exists for some combinations of BH masses and spins when \( q \rightarrow 1 \). Clearly, the decomposition into three successive phases must be used with caution. It is useful, however, in that approximate solutions for GW losses have been derived in some limiting cases for some of these phases.

We consider the two extreme limits for the spins of BHs involved in coalescences. In the slow-spin limit, BHs are assumed to have no spin. In the fast-spin limit, BHs are assumed to be maximally rotating (with a spin parameter \( a \equiv J_{\text{BH}}/M_{\text{BH}} = 1 \), in \( c = G = 1 \) units). In a given evolutionary model, we assume for simplicity that all the BHs involved satisfy one or the other spin limit. If we generically write a mass loss from the BH binary as \( \Delta(m + M) \), then the losses we have adopted in our models for the inspiral, plunge, and ring-down phases are, respectively,

\[
\Delta(m + M)_{\text{ins}} = -\alpha_{\text{ins}} \mu, \quad \Delta(m + M)_{\text{plu}} = -\alpha_{\text{plu}} M q^2, \quad \Delta(m + M)_{\text{rin}} = -\alpha_{\text{rin}} M \cos q^2, \quad \text{(2-4)}
\]

where the “coalesced” mass (before ring-down starts) is \( M_{\text{coa}} = m + M - \Delta(m + M)_{\text{ins}} - \Delta(m + M)_{\text{plu}} \). The values of the loss coefficient \( \alpha \) for the two spin limits are given in Table 1. Justifications for these prescriptions follow.

Losses to GWs during the inspiral phase have been discussed extensively. They involve calculating the binding energy at the location of the ISCO, since the quasi-adiabatic inspiral experienced by the binary means an efficient loss of this binding energy to GWs via a succession of nearly circular orbits. In the test-particle limit (\( m \ll M \)), it is well known that
the loss during inspiral is ~6% of \(mc^2\) in the slow-spin limit and ~42% of \(mc^2\) in the fast- (prograde-) spin limit (as is the case for accretion efficiency; see, e.g., Shapiro & Teukolsky 1983). In the equal-mass binary limit \((m = M)\), results have been derived under a variety of approximations. For nonspinning, equal-mass BHs, the binding energy per unit reduced mass at the ISCO is roughly consistent with the test-particle result (see Table 1 in Gammie et al. 2004). For spinning, equal-mass BHs, the analysis of Pfeiffer et al. (2000; their Table 1) indicates somewhat larger binding energies per unit reduced mass at the ISCO than for the test-particle case, for a few moderate-spin configurations. On the other hand, post-Newtonian calculations (e.g., Blanchet 2002) suggest somewhat lower binding energies for distributions of BH mass ratios comparable to those found in our models (see eq. [2] and Table 1).

Losses to GWs during the plunge phase are much less well understood. An exact result for the combined plunge + ring-down phase exists for the test-particle case in the absence of any spin or orbital angular momentum (Davis et al. 1971): it amounts to a loss of \(\approx 0.01Mq^2c^2\). We adopt this minimal loss for the plunge phase in our slow-spin models. Orbital angular momentum should always be important in astrophysical BH coalescences, and it is likely that plunge losses will then become substantially larger. For definiteness, we adopt 10 times larger losses during the plunge for the fast-spin models (see, e.g., Nakamura et al. 1987). In the absence of analytical results on the plunge phase for equal-mass binaries, we further assume that the above test-particle \(q^2\) mass scaling is valid for any BH mass combination (see eq. [3]). Finally, we add a contribution to GW losses from the ring-down phase. Our prescription is adapted from the results of Khanna et al. (1999; extrapolated at large spin values), with the same assumed \(q^2\) mass scaling as before (see eq. [4]).

### 3. RESULTS

The evolution of the distributions of BH and galaxy masses in our evolutionary models has been discussed extensively in Paper I. We use Monte Carlo realizations of the merger tree of dark matter halos, starting with \(N \approx 4.6 \times 10^4\) halos at \(z = 3\) with masses in the range \(10^{8.6}-10^{12.1}\) \(M_\odot\). Our merger tree database effectively describes a fixed comoving volume \(\approx 1.7 \times 10^4\) Mpc\(^3\). It is then straightforward to calculate the comoving mass density in BHs, \(\rho_{\text{BH}}\), and to monitor its evolution: we simply follow the merger history of BHs and subtract, at each merger event, the mass-energy lost to GWs.

| BH Spin Limit | Inspiral \(\alpha_{\text{in}}\) | Plunge \(\alpha_{\text{pl}}\) | Ring-down \(\alpha_{\text{rd}}\) |
|---------------|-----------------|-----------------|-----------------|
| Slow spin     | 0.06            | 0.01            | \(10^{-5}\)      |
| Fast spin     | 0.42            | 0.10            | 0.03             |

\(^3\) A mass scaling with the “reduced mass ratio,” \(\eta = \mu/(m + M)\), replacing \(q\) in eqs. (3) and (4) may be more accurate in the limit \(q \to 1\), according to post-Newtonian calculations (S. Hughes 2004, private communication). This would reduce the importance of plunge and ring-down losses in our models, since \(\eta \to q\) when \(q \to 0\) but \(\eta \to 1/4\) when \(q \to 1\).

In models without any GW losses, \(\rho_{\text{BH}} \approx 1.4 \times 10^5\) \(M_\odot\) Mpc\(^{-3}\), and it does not evolve with cosmic times. In models including GW losses, however, a small fraction of \(\rho_{\text{BH}}\) is lost each time two BHs coalesce, leading to a growing cumulative deficit.

Figure 1 shows the evolution of the deficit in \(\rho_{\text{BH}}\) from \(z = 3\) to 0 in the slow- and fast-spin models of Table 1. The cumulative deficit is relatively small in the slow-spin model (~3% of the initial \(\rho_{\text{BH}}\) value), but it reaches ~20% in the fast-spin model. Models with a 10 times rarer population of massive BHs (initially confined to the most massive 10% of galaxies) give essentially identical results (dotted lines in Fig. 1). This is because most of the BH mass loss occurs at the largest masses (see Paper I). The cumulative deficit does, however, depend on the value of the redshift at which cosmological evolution is initiated, as shown by the dashed line in Figure 1 (a fast-spin model starting at \(z = 2\)). This is a consequence of the reduced total number of BH mergers. We also note that the cumulative \(\rho_{\text{BH}}\) deficits shown in Figure 1 are slightly smaller than the corresponding values for the simpler slow- and fast-spin models discussed in Paper I. This results simply from the more realistic GW loss prescription adopted here.

Figure 2 shows, for the fast-spin model, how various subcategories of GW losses contribute to the \(\rho_{\text{BH}}\) deficit. Little evolution with redshift is seen except early on, when the initial BH masses are redistributed in galaxies. Inspiral losses largely dominate the overall mass-loss budget, while plunge and ring-down losses contribute little. A combination of the adopted BH masses and the cosmological merger history experienced by BHs results in most of the inspiral mass loss originating from BH binaries with mass ratios \(q < 0.5\) (compare solid and long-dashed lines in Fig. 2; see also Fig. 5 in Menou [2003] for distributions of BH mass ratios comparable to those found in our models). This is important because inspiral losses in the
limit $q \ll 1$ are the best known of all. Since the low $q$ limit is still relatively accurate up to mass ratios $q \lesssim 0.5$, according to post-Newtonian calculations (see, e.g., discussion in Hughes & Blandford 2003), this indicates that our results may not be critically sensitive to various uncertainties affecting our GW loss prescriptions for the other regimes (see §2.2). Cumulative losses at $z = 0$ correspond to fractions $\sim \alpha_{\text{ins}}/2$ in both the slow- and fast-spin models (compare Fig. 1 and Table 1). This shows that a substantial fraction of the final mass density has been assembled through mergers of BH binaries with $q < 0.5$. The exact contribution to the mass assembly is difficult to estimate from losses alone, however, because our prescription for inspiral losses (written in units of reduced mass in eq. [2]) effectively reduces the losses per unit “real” mass for large mass ratios ($\mu \to m/2$ in the limit $q \to 1$).

4. DISCUSSION AND CONCLUSION

Cumulative mass loss to GWs during repeated cosmological BH coalescences from $z \approx 3$ to $z = 0$ could reduce the amount of BH mass locked into nearby dead quasars by up to $\sim 20\%$, according to our fast-spin model (Fig. 1). This reduction in the local BH mass density would effectively lead to a similar fractional increase in the value of the radiative efficiency for cosmic BH accretion, $\epsilon$, if a comparison between dead and active quasars is attempted without accounting for GW losses. Each individual SMBH experiences numerous repeated mergers in its assembly history (especially BHs in the most massive halos). However, our detailed study of the merger history shows that the majority of these mergers have small mass ratios for which losses to GWs are equally small (see eqs. [2]–[4]; note that the fraction of mergers with $q \sim 1$ can be significantly higher at $z \gtrsim 6$, where the effective slope of the power spectrum at the mass scale of collapsing objects is shallower; Haiman 2004).

It is important to emphasize that our models are highly idealized and that a number of effects ignored in our calculation are likely to mitigate the already small magnitude of the $\rho_{\text{BH}}$ deficit. Except for the role of inefficient dynamical friction, we have assumed maximally efficient BH coalescences and thus maximized GW losses in our models. The “last parsec” problem and gravitational radiation recoil effects (Milosavljevic & Merritt 2003; Favata et al. 2004), for example, will only make BH coalescences less frequent than assumed here.\footnote{Note that, by displacing or ejecting BHs from galactic centers (e.g., Madau & Quataert 2004), gravitational radiation recoil could also lead to an underestimate of the mass density in quasar remnants.} We have also neglected the role of orbital configurations in our loss prescriptions. For randomly oriented BH encounters, some will be retrograde spin-orbit configurations and lead to smaller inspiral losses than assumed in our slow-spin model, even when BHs are spinning fast (e.g., $\alpha_{\text{ins}} \approx 0.04$ for a maximally rotating retrograde configuration in the test-particle limit; see also Koijma & Nakamura 1984). A more accurate calculation should therefore account for the distribution of orbital parameters of coalescing BHs and would probably find losses intermediate between those predicted by our slow- and fast-spin models. A proper calculation should also account for the growth of $\rho_{\text{BH}}$ with cosmic time due to accretion. We have effectively maximized fractional losses by assuming that a given mass density is in place at $z = 3$ and that losses occur after this redshift without any subsequent increase in $\rho_{\text{BH}}$. Finally, typical BH spins may be moderated by coalescences and accretion (Hughes & Blandford 2003; Gammie et al. 2004), and this could easily bring losses closer to the predictions of our slow-spin model.

Another model uncertainty is the limited range of masses described by our merger tree. Losses in our models are dominated by the few most massive BHs that happen to be present in our Monte Carlo realizations of the cosmological merger tree. These massive BHs still have lower masses than the $>10^8 M_{\odot}$ BHs of interest when comparing dead and active quasars (see Paper I and, e.g., Yu & Tremaine 2002). We have argued in Paper I that including more massive BHs would increase the value of the cumulative $\rho_{\text{BH}}$ deficit, but this increase is likely to remain modest. For example, a simple extrapolation with BH mass of the $\rho_{\text{BH}}$ deficit value predicted at $z = 0$ shows a small ($\approx 2 \times$) increase in the fractional loss (shown in Fig. 1) up to BH masses $\sim 10^8 M_{\odot}$.

Given the above arguments, the magnitude of $\rho_{\text{BH}}$ deficits shown in Figure 1 cannot be taken at face value, and it appears likely that the losses amounting to $\sim 10\%$–20\% of $\rho_{\text{BH}}$ are only conservative upper limits to more realistic values. The corresponding bias on the value of the radiative efficiency of cosmic BH accretion, $\epsilon$, would also be $\lesssim 10\%$–20\%, thus well within error bars of current estimates (e.g., Aller & Richstone 2002; Elvis et al. 2002; Yu & Tremaine 2002). Therefore, inferences that $\epsilon \gtrsim 0.1$ appear robust and may indeed indicate radiatively efficient accretion onto fast-spinning BHs. In the future, it is likely that the space interferometer $\text{LISA}$ will offer us some of the best empirical constraints on the dark side of quasar evolution. Even though the typical BH masses probed by $\text{LISA}$ are smaller than those of luminous quasars (e.g., Hughes 2002; Menou 2003), measurements of the cosmological rate of massive BH coalescences and constraints on the masses, and
perhaps the spins, of these BHs will prove very useful in clarifying many of the uncertainties we have highlighted above. A pulsar timing array may also put interesting constraints on the magnitude of the stochastic GW background generated by cosmological BH mergers (e.g., Jaffe & Backer 2003).

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