Kinematic Beta-Type Stirling Motor-Driven Compressor

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Abstract. The theoretical functioning of a kinematic one-stage beta-type Stirling motor-driven compressor was analyzed. This machine comprises a kinematic Stirling engine and a reciprocating compressor in one single block. A double acting cylinder houses a piston that plays both the roles of the power piston of the Stirling engine and of the compressor's piston. The maximum performances of this machine are determined through numerical modelling. For a certain compression ratio of the compressor, a functioning regime for which the work produced by the Stirling engine is equal to the absolute value of the work consumed by the compressor can be found.

1. Introduction
A Stirling motor-driven compressor (SMDC) is a combination between a Stirling engine and a compressor. The Stirling engine directly drives the compressor. The SMDC analyzed here has a kinematic beta-type Stirling engine as prime mover. The power piston of the Stirling engine also acts as a compressor piston. It is placed inside a double-acting cylinder. The compression space of the Stirling engine is placed on one side of the shared piston and the working space of the compressor is placed on the other side. Suction and discharge valves and lines are conveniently placed. The working agent of the compressor and the working agent of the Stirling agent could be the same or different.

Several other devices that use Stirling engines for driving pumps and compressors for various uses are known.

Around 1860 W. H. Bailey fabricated at Manchester a Stirling-type hot air engine [1]. This Stirling engine was directly connected through the crankshaft with a water pump. The piston of the pump acts in a separate cylinder. Another Stirling engine, namely Robinson engine, uses a chain drive to transmit mechanical power to a water pump [1].

An “Apparatus for Compressing Gases” that uses a displacer and a constant total volume working space was patented in 1939 by Vannevar Bush [2]. The two variable volume chambers created by the displacer inside the cylinder were connected through a heater, a regenerator and a cooler, hence the resemblance of this device with a Stirling engine. The inventor called the device "direct thermal compressor”.

A direct connection between a Stirling engine working as a prime mover performing as such clockwise thermodynamic cycle and a Stirling refrigerating machine performing a counterclockwise cycle is named a "duplex Stirling engine” [3]. The two machines are connected through a shared power piston, each of them having its own compression space and its line of heat exchangers (heater, regenerator and cooler). Another possibility is to directly connect a Stirling engine prime mover to a heat pump inside which the working agent performs a Rankine cycle [3]. The connection is made
through a double-acting free-piston inertial compressor that shares one of its compression spaces with the buffer space of the Stirling engine.

Dineen [4] obtained in 1982 an US Patent for a hermetic oil-free Stirling engine compressor. An elastic diaphragm serves as power piston. The space outside the power piston form a “hydraulic chamber”, and has another elastic diaphragm that serves as piston for the compressor. Inside this space a linear alternator is housed. The alternator uses some of the work yielded by the Stirling engine.

In 1985 Watanabe et al. obtained an US patent [5] for an “engine compressor having a Stirling cycle engine”. An alpha-type Stirling engine with four cylinders is the prime mover. Each piston of the Stirling engine is directly connected through a guided piston rod to a compressor piston. The compressor cylinders could be single- or double-acting, and are fitted with the corresponding suction and discharge valves.

An US patent issued in 1992 to Momose [6] presents a gamma-type Stirling engine used as prime mover for driving the compressor of an air-conditioning equipment, or an air compressor. The pressure variation inside the Stirling engine is used for producing an axial displacement of a diaphragm. This oil-free compressor has symmetrical construction, and uses four membranes separating several chambers. Suction and discharge valves are placed inside the two working spaces of the compressor.

In 2004 Berkowitz obtained an US patent [7] for a free-piston Stirling engine that drives a heat pump with vapor compression. Both the Stirling prime mover and the compressor are placed inside the same hermetically sealed enclosure. The piston of the compressor is directly connected with the power piston of the Stirling engine, forming a single structural element. The device could use a single working agent, namely CO₂, or two working agents, helium for the Stirling engine and carbon dioxide for the heat pump. For the latter case, helium will leak inside the heat pump circuit, so a gas-liquid phase separator must be used for collecting a helium-rich gas mixture that will go back to the Stirling engine through a return line.

A scheme of a Stirling engine driving a compressor, attributed to William Beale, the inventor of the free-piston Stirling engine, was presented in [8]. A four-cylinder alpha-type Stirling engine was directly connected to four separate single- or double-acting compressor cylinders. The novelty over the Watanabe drawings [5] was that the compressed gas is used for driving a closed cycle gas turbine with intermittent feeding.

In a previous published work from 2005 [9] the authors presented and analyzed five possible schemes for SMDC, based on alpha, beta and gamma configurations. A beta-type Stirling engine could be transformed into a SMDC if the space below the power piston is organized as a working space for a reciprocating compressor, being fitted with suction and discharge valves. The mentioned paper [9] did not develop functioning models for the proposed SMDC.

In 2013 [10] authors developed a physico-mathematical model for a beta-type engine-driven Stirling compressor. This device is a Stirling engine fitted with suction and discharge valves and lines. When the pressure inside the engine rises, the discharge valve opens and some of the working agent leaves the compression chamber. Suction takes place when the pressure inside the machine drops below the pressure inside the suction line. So, it is compulsory that the gas needed to be compressed and the gas serving as working agent for the Stirling engine are the same.

In 2016 Tognarelli et al. obtained an Australian patent for a compressor train with a Stirling engine [11]. The shaft of the reciprocating compressor is connected to the shaft of the Stirling engine through a clutch. A reversible electric machine is connected to the same shaft line. The electric machine operates as an engine when the Stirling engine produces less work than the compressor needs, or as a generator otherwise. The Stirling engine should use the waste heat usually available in refinery plants.

The paper analyzes the functioning of a kinematic one-stage beta-type Stirling motor-driven compressor based on an isothermal physico-mathematical model of the Stirling engine and on a simplified physico-mathematical model for the reciprocating compressor.

2. Scheme and physical model of the beta-type Stirling motor-driven compressor
The beta-type SMDC schematic diagram is presented in figure 1. A SMDC unit comprises the cylinder 7 that houses the displacer piston 14 and the power piston 8. The space delimited by the cylinder heads 10 and 18 and by the cylinder itself is divided by the mentioned pistons in three variable-volume spaces: the expansion space 13 and the compression space 9 of the Stirling engine and the compressor working space 6. The expansion and compression spaces of the of the Stirling engine are connected through the cooler 16, the regenerator 15 and the heater 12. The working space of the compressor is equipped with suction valve 5 and with the discharge valve 19, housed inside the corresponding lines 4 and 20. The linkage between the pistons is made through a rhombic drive comprising four rods 2 and yokes 22 and 23.

![Figure 1. Schematic of a single-stage SMDC of beta-type arrangement:](image)

1 – crankshaft; 2 – connecting rod; 3, 17 – stem; 4 – suction line; 5 – suction valve; 6 – compressor working space; 7 – cylinder; 8 – power piston; 9 – compression chamber; 10 – cylinder head of the Stirling engine; 11 – combustion chamber; 12 – heater; 13 – expansion chamber; 14 – displacer; 15 – regenerator; 16 – cooler; 18 – cylinder head of the compressor; 19 – discharge valve; 20 – discharge line; 21 – gear wheel; 22, 23 – yoke.

The Stirling engine acts as a prime mover, producing the work needed by the compressor. In a pure SMDC the compressor uses completely the work yielded by the Stirling engine. If the Stirling engine is producing more work that needed for the functioning of the compressor, the SMDC could be fitted with an electric generator providing electrical energy.

The Stirling engine functioning is described by the isothermal model with real volume variation. This model is well-known to those skilled in the art [1], [3], [8], and thus superfluous to describe here. The main hypotheses used are: agent is a perfect gas; processes inside each chamber are isothermal and quasistatic; instantaneous pressure inside the machine is the same in all chambers and the regenerator has a temperature that is a mean between heater and cooler temperatures.

The model used for describing the functioning of the reciprocating compressor uses polytropic processes for compression and expansion and isobaric processes for suction and discharge. Mean pressure losses at suction and discharge and mean polytropic indices for compression and expansion were adopted using the recommendations [12].
3. Mathematical model

A mathematical model describes the functioning of the entire SMDC and correlates the work yielded by the Stirling engine as prime mover with the work consumed by the reciprocating compressor. The model allows for calculating all thermodynamic parameters for any cycle point and for calculating the energies exchanged. The model also estimates the performances of the SMDC and calculates the dimensions of the needed flywheel.

The following dimensions needed for the model were mentioned in figure 1: \( r \) - crank radius; \( l \) - connecting rod; \( e \) - crankshaft - wrist pin offset; \( D \) - cylinder bore / power piston and displacer diameter; \( d \) - displacer stem diameter; \( d' \) - power piston stem diameter; \( l_{sd} \) - distance between displacer's yoke and displacer head; \( l_{sp} \) - distance between power piston's yoke and power piston head; \( f \) - distance between cylinder head and displacer head in its top dead center; \( \alpha \) - crank position angle; \( xOy \) - coordinate system.

Based on the geometry the variations laws of the expansion chamber volume \( V_e \) and compression chamber volume \( V_c \) as functions of crank angle were determined. The volumes occupied by the working agent inside cooler, regenerator and heater are \( V_k \), \( V_{reg} \) and \( V_h \). Usually, these volumes are given as fractions of the volume swept by one of the pistons. The isothermal model of the Stirling engine uses constant temperatures for the agent inside the heat exchangers, \( T_k \), \( T_{reg} \) and \( T_h \), the temperature of the regenerator being calculated as a mean (e.g. a logarithmic one) of the temperatures of the cooler and heater.

Assuming that inside the Stirling engine evolves a constant total mass \( m_T \) of working agent having the specific gas constant \( R \), the instantaneous pressure \( p(\alpha) \) inside the prime mover is determined by the following relation:

\[
p(\alpha) = \frac{m_T R}{V_e(\alpha) + V_k + V_{reg} + V_e(\alpha) + V_h}. \tag{1}
\]

The work yielded by a Stirling engine cycle is given by

\[
W_{se} = \int p(\alpha) \left[ \frac{d}{d\alpha} V(\alpha) \right] d\alpha, \tag{2}
\]

where \( V(\alpha) \) is the total volume of the working agent inside heater, regenerator, cooler, compression and expansion chamber.

The working space of the compressor has a variable volume \( V_{ws}(\alpha) \). Its particular variation law is given by the chosen geometry. Due to the presence of the power piston stem, the maximum value of \( V_{ws} \) is always smaller than the volume swept by the power piston head (adjacent to the compression chamber of the Stirling engine) between its dead centers. The clearance volume \( V_{cl} \) value was adopted as a fraction of the volume swept by the power piston inside the compressor's working space.

The operating regime of the compressor is characterized by the pressure \( p_S \) of the suction tank and by the pressure \( p_D \) of the receiver tank, thus the external compression ratio is \( p_D / p_S \). The suction mean pressure \( p_s \) and the discharge mean pressure \( p_d \) are calculated based on adopting suction and discharge relative pressure drop coefficients [12]:

\[
\delta p_s = \frac{p_s - p_d}{p_s} \quad \text{and} \quad \delta p_d = \frac{p_d - p_D}{p_D}. \tag{3}
\]

Mean values for polytropic indices \( n_c \) for compression and \( n_e \) for expansion were adopted. Temperatures at the end of the suction process and at the end of the discharge process were calculated by adopting the temperature differences \( AT_s \) during the suction gas heating process and \( AT_d \) during the discharge gas cooling process. Temperature at the beginning of the suction process is \( T_A \).
The following functional crank angles were defined: \( \alpha_{es} \) for the end of the suction process, \( \alpha_{ec} \) for the end of the compression, \( \alpha_{ed} \) for the end of the discharge process and \( \alpha_{ee} \) for the end of the expansion process. With these notations, the pressure variation law inside the working space of the compressor is

\[
p_{ws}(\alpha) = \begin{cases} 
  p_s & \text{if } \alpha \in (\alpha_{es}, 2\pi] \text{ and } \alpha \in [0, \alpha_{es}] \\
  p_s \left[ \frac{V_{ws}(\alpha)}{V_{ws}(\alpha_{es})} \right]^{\nu_e} & \text{if } \alpha \in (\alpha_{es}, \alpha_{ec}] \\
  p_d & \text{if } \alpha \in (\alpha_{ec}, \alpha_{ed}] \\
  p_d \left[ \frac{V_{ws}(\alpha_{ed})}{V_{ws}(\alpha)} \right]^{\nu_e} & \text{if } \alpha \in (\alpha_{ed}, \alpha_{ee}]. 
\end{cases}
\] (4)

The suction process is divided in two angle intervals because of the chosen coordinate system and of the origin for measuring the crank angle \( \alpha \).

In accordance with the above statements, the work \( W_{RC} \) consumed by the reciprocating compressor is given by

\[
W_{RC} = \int p_{ws}(\alpha) \left[ \frac{d}{d\alpha} (V_{ws}(\alpha)) \right] d\alpha .
\] (5)

By comparing the work \( W_{RC} \) consumed by the compressor with the work \( W_{SE} \) produced by the Stirling engine three possibilities appears. If \( W_{RC} + W_{SE} < 0 \), the Stirling engine output is not sufficient for powering the compressor. In this case the SMDC could not function by itself. For \( W_{RC} + W_{SE} = 0 \), the device will function as intended, as a Stirling motor-driven compressor. The net work exchange with the surroundings is null in this case. If \( W_{RC} + W_{SE} > 0 \), the Stirling engine produces more than the needs of the compressor. The excess work could be used by an electric generator, as in [11]. The possibility also arises to cope with the excess work by modifying the functioning regime of the Stirling engine, e.g. by reducing the mass of working agent \( m_T \) of the Stirling engine, or by reducing the temperature \( T_h \) of the heater.

When the power piston goes down (in figure 1), the total volume \( V(\alpha) \) occupied by the working agent inside the Stirling engine increases, and the prime mover produces work. In the same time, inside the working space of the compressor the volume \( V_{ws}(\alpha) \) decreases and the compression and discharge processes are taking place. So, it is compulsory to check if the Stirling engine delivers the right amount of work at the very moments when the compressor needs it. If a good compatibility between the work produced and the work consumed is achieved, there is a certain chance that the SMDC does not need a separate flywheel. If not the case, the SMDC needs a flywheel for accumulating appropriate energy during the excess work production periods and returning it during the periods when the device is producing less work than needed, i.e. during compression.

The torque variation of the SMDC against crank angle \( \alpha \) could be calculated with a method using the pressure forces, the inertial forces of the reciprocating parts, masses and linear dimensions, and [13]. The algorithm used calculates the pressure forces exerted by the working agents on the displacer and on the power piston, the forces of inertia for all parts performing reciprocating movements, the resultant forces that act on displacer and power piston and the component forces acting on the crankpin journal, perpendicular to the crank web. Torque \( M \) is finally expressed as product between crank radius and the mentioned component forces.

The masses of the main parts of the SMDC - displacer, power piston, connecting rods, stems, yokes, wrist pins - must be known, in order to calculate the inertial forces in the reciprocating parts. The mass of a connecting rod was divided in two, the first partial mass performing reciprocating movement together with the yoke, stem and piston, and the second partial mass performing a rotational movement with the crankshaft. Only the first partial mass is useful for torque calculations,
and is usually expressed as a fraction of the connecting rod total mass [13]. The two-masses system has a dynamic behavior similar to the one of the connecting rod.

4. Results of the numerical simulation of the SMDC

A SMDC based on the configuration presented in figure 1 and described by the following particular values: \( r = 0.0175 \) m, \( l = 0.0525 \) m, \( e = 0.024 \) m, \( l_{ol} = 0.310 \) m, \( D = 0.040 \) m, \( d = 0.008 \) m, \( d' = 0.015 \) m, \( f = 0.0005 \) m was analyzed through numerical simulation.

Volumes of the heat exchangers of the Stirling engine are \( V_h = V_k = 0.05 \) \( V_{c_{\text{max}}} \); \( V_{\text{reg}} = 1.2 \) \( V_{c_{\text{max}}} \), where \( V_{c_{\text{max}}} \) is the maximum volume of the compression chamber. Temperatures of the heat exchangers are \( T_h = 823 \) K and \( T_k = 310 \) K. Regenerator mean temperature is \( T_{\text{reg}} = 525.4 \) K, the logarithmic mean of \( T_h \) and \( T_k \).

The mass of working agent performing inside the Stirling engine is \( m_T = 0.000667 \) kg. This particular value was obtained by imposing a null net work output of the SMDC. Each crankshaft has a rotational speed of \( n = 1000 \) rpm.

The working agent of the Stirling engine and the working agent of the compressor is air, considered to be a perfect gas, with specific gas constant \( R_{\text{air}} = 287 \) J kg\(^{-1}\) K\(^{-1}\).

The functional parameters of the compressor are \( p_S = 0.1 \) MPa, \( p_D = 0.9 \) MPa, \( \Delta p_r = 0.05 \), \( \delta p_s = 0.10 \), \( n_s = 1.38 \), \( n_d = 1.34 \), \( \Delta T_s = 10 \) K, \( \Delta T_d = 15 \) K, \( T_S = 288 \) K, \( V_S = 0.0124 V_{Sc} \), where \( V_{Sc} \) is the volume swept by the power piston inside the working space of the compressor.

The mass performing a reciprocating movement with the displacer (including the mass of the displacer, stem, yoke, wrist pins, and first portions of the mass of the two connecting rods) is 0.763 kg. The corresponding mass for the power piston is 0.791 kg. These masses were calculated using the particular design made for a small Stirling engine by the authors.

A dedicated program for the SMDC was developed in Mathcad. At the working regime characterized by the chosen values of \( p_S \) and \( p_D \) the following main results were obtained: \( W_{SE} = 12.83 \) J/cycle, \( W_{RC} = -12.83 \) J/cycle. The power produced by the Stirling engine and consumed by the compressor is 213.9 W. The volumetric flow rate at the end of the suction process state is \( q = 0.041 \) m\(^3\) min\(^{-1}\), and the mass flow rate is \( \dot{m} = 0.76 \times 10^{-3} \) kg s\(^{-1}\).

![Figure 2](image.png)

**Figure 2.** SMDC cyclical behavior:

(a) – variation of the pressure with crank angle inside the Stirling engine and inside the reciprocating compressor; (b) – torque variation with crank angle for an one-unit SDMC and for a SDMC with two-units with twin-cylinder with cranks at 180° apart configuration.

Figure 2 (a) shows the cyclical variations of the working agent pressure inside the Stirling engine and inside the working space of the compressor. The compression, discharge, expansion and suction processes inside the compressor are clearly visible. When compression and discharge processes take place the pressure inside the Stirling engine decreases. This confirms the assertion made in the previous chapter, that in the phase when the prime mover delivers work, the compressor uses some of the work directly.
In figure 3 (b) curve $M_1(\alpha)$ was calculated for a SMDC based on the scheme in figure 1. The curve shows that there is a part of the SMDC cycle when the torque is positive, and part when the torque is negative. I.e., there is a part of the cycle when SMDC produces work (the Stirling engine produces more that the compressor could consume), and a part of the cycle when SMDC needs work for the functioning. From here the necessity results of accumulating the excess work in a flywheel during the period when SMDC produces work. The flywheel restitutes the accumulated energy to the SMDC during the other period. The dimensions of the needed flywheel can be calculated by choosing a value for the coefficient of crankshaft speed non-uniformity $\delta$. E.g., for $\delta = 1 / 200$ the flywheel must have a moment of inertia $I = 0.622$ kg m$^2$. A flywheel satisfying this condition and made of steel could have: internal radius $r_i = 0.250$ m, external radius $r_e = 0.2625$ m, width of 0.060 m and a total mass of 9.46 kg. The flywheel could be mounted on one of the crankshafts of the rhombic drive mechanism, or could be divided in two smaller flywheels, one for each crankshaft.

For obtaining a better value of the coefficient $\delta$ the SMDC could be realized with multiple units on the same crankshaft. In figure 3 (b) curve $M_2(\alpha)$ was calculated for a SMDC having two functional units, in a twin-cylinder with cranks at 180° apart configuration. In comparison with the other curve in figure 3 (b), the reduction of the torque variation interval is clearly visible, so the dimensions of the flywheel will be also reduced. The needed moment of inertia in this case is $I = 0.223$ kg m$^2$.

5. Conclusions
A scheme for a motor-driven compressor comprising a Stirling engine directly coupled to a reciprocating compressor, in which the compressor is using in its construction the power piston of the Stirling engine, was proposed.

By applying an isothermal physico-mathematical model to the Stirling engine and a physico-mathematical model based on an equivalent p-V indicator diagram to the reciprocating compressor, it was found that, for a certain compression ratio of the compressor, there is a single functioning regime for which the net work exchanged with the surroundings by the SMDC is null.

The equality between the absolute value of the cyclical work required by the compressor and the cyclical work yielded by the Stirling engine could be achieved by various means, e.g. by regulating the total mass of working agent inside the Stirling engine.

Due to the nature of the physico-mathematical models used, the calculated performances of the SMDC have the character of maximum achievable performances.

A numerical example was used for obtaining the cyclical variations of the pressure inside the Stirling engine and inside the compressor, and the cyclical variation of the SMDC torque, as diagrams. The analysis of these diagrams shows that there is a period of the cycle in which the Stirling engine is producing work, and a period when it needs work. The same conclusion is valid also for the SMDC as a whole. This situation imposes the presence of an accumulator of mechanical energy into the component of the SMDC - a flywheel. The flywheel stores the excess energy produced by the SMDC during the expansion of the working agent of the Stirling engine, and returns it to the SMDC when needed.

The numerical example allowed for establishing the maximum performances that a SMDC is capable of.

The SMDC could be an attractive solution for several applications, especially when conventional primary energy sources cannot be accessed. Possible uses of this motor-driven compressor are as:
- portable air compressor, for off-grid applications;
- for replacing a regular engine-driven compressor powered by an internal combustion engine;
- for recovering the waste heat of hot fluids from various industries;
- for harnessing the solar energy.

When properly designed, the SMDC could be also fitted with an electric generator, so the output could be compressed gas, electrical energy or a combination of both.

In our opinion, the SMDC is an unconventional heat engine worth to be further studied.
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