Spinning Radiation from Topological Insulator
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ABSTRACT
We show that thermal radiation from a topological insulator carries a nonzero average spin angular momentum.

Keywords: Thermal radiation, Topological insulator, Spin angular momentum

1. INTRODUCTION
Thermal radiation has a wide range of applications in areas such as thermophotovoltaic systems, radiative cooling, thermal imaging etc. Control over the radiation can be offered by nanolithography techniques through addition of structures with length scales comparable to the characteristic emission wavelength. In particular, photonic crystal having a polarization dependent bandgap or metasurface lacking mirror symmetry have been reported to emit thermal radiation with circular polarization. However the associated design and fabrication complexity makes it desirable to have circularly polarized thermal emission from a material possessing simple planar geometry without any added surface structure.

One solution approach towards this problem can be offered by topological insulators (TI) through its property of mixing electric and magnetic responses. This novel magneto-electric effect gives rise to unusual electromagnetic phenomena such as induction of image magnetic monopole, half-integer quantum Hall effect, Faraday rotation etc. Here we show that this effect also leads to the emergence of spin angular momentum in the thermal emission from TIs.

2. TOPOLOGICAL MAGNETO-ELECTRIC EFFECT
In the presence of a time-reversal symmetry breaking perturbation (which can be realized, for example, through the proximity of a magnetic material or a weak applied magnetic field) an energy gap at the surface of TI is induced which leads to a novel quantum Hall state with the topological magneto-electric effect. The corresponding electrodynamics can still be described by the usual Maxwell equations with the modified constitutive relations
\[ D = \varepsilon E - \kappa B, \]
\[ H = B/\mu + \kappa E \]
where \( \kappa = \alpha \Theta \) is the magnetoelectrical susceptibility with \( \alpha \) being the fine structure constant and \( \Theta \) being the (quantized) axion field. Although this axion model only applies to frequencies below the bulk bandgap, the corresponding cutoff in actual TIs is well above the usual characteristic thermal frequency.

3. THERMAL RADIATION FROM TI
For simplicity of the design, we consider a semi-infinite slab of a TI with permittivity \( \varepsilon \), having a top coating of a thin magnetic film, as shown in Fig. 1(a). Using Rytov’s theory on fluctuational electrodynamics gives us the total spin angular momentum flux in the normal direction (with respect to the TI-air interface) as
\[ J_n = \int d\omega \int d\theta \rho(\omega) \frac{c}{n} \cos \theta' \left( 1 - \frac{|r_+|^2 + |r_-|^2}{2} \right) S_n, \]
where
\[ \rho(\omega) = \frac{\omega^2 n^3}{\pi^2 c^3} \frac{1}{\exp(h\omega/kT) - 1} \]
Figure 1. Spin angular momentum in thermal emission from topological insulator (TI). (a) Geometry of a semi-infinite slab of a TI as a thermal emitter. The top coating of thin magnetic film induces the topological magneto-electric effect. (b) Spin angular momentum per photon along the emission direction, for Re[ε] = 15. The spin angular momentum is nonzero when both the magneto-electric coupling and loss are present. (The density plot shows the functional dependence on the magneto-electric coupling as κ can only assume quantized values - integer multiples of the fine structure constant.) (c) Dependence of the normal component of the per-photon spin angular momentum on the emission angle ϑ.

is the photon density at a particular frequency ω, n(ω) is the real part of refractive index, θ' is the angle of photon incidence corresponding to the emission angle θ, and

\[ r_\pm = \frac{1}{\Delta} \left( \beta \pm \sqrt{\eta^2 + \gamma^2} \right) \]  \hfill (4)

are the two reflection coefficients from TI into the air for the two eigen-polarization states. Here, η = 2κ√ε, γ = 1 + κ^2 - ε, β = ε/ν - ν and Δ = -(1 + κ^2 + ε + ε/ν + ν) with ν = kn,TI/kn,air, and the subscript n corresponds to the normal component of the wavevector in the respective media. And finally, \( S_n \) is the normal component of per-photon (ensemble-averaged) spin angular momentum (which is the same for both eigen-polarization states) given by

\[ S_n = \frac{4\hbar \kappa \text{Im}[ζ]}{4κ^2 + |ζ - 2κ|^2} \cos θ \]  \hfill (5)

with ζ = γ + √(η^2 + γ^2).

Note that both material absorption and magneto-electric susceptibility are required for the presence of spin angular momentum in the thermal emission. This is shown in Fig. 1(b) which illustrates the functional dependence of spin angular momentum \( S \) along the emission direction of an emitted photon on the coupling constant κ for varying amount of loss.

Furthermore, the angle dependence \( S_n \propto \cos θ \) (see Eq. (5)) is shown in Fig. 1(c) for the typical value of κ = α. Even though the spin angular momentum along the direction of propagation is the same for all emission angles, the normal component varies but does not change sign.

4. MAPPING TO A SPIN IN MAGNETIC FIELD

To give a pictorial understanding of these results, we represent the polarization state of a thermal photon as a point on the Poincaré sphere (as shown in Fig. 2) where the three axes X, Y and Z denote the three Stoke’s parameters, with the Y coordinate directly giving the spin angular momentum per photon in units of \( \hbar \) along the direction of propagation. The reflection matrix from TI to air can be represented (in the basis set consisting of p and s polarization states) as

\[ R = \frac{1}{\Delta} (\beta I + \sigma \cdot B) \],  \hfill (6)
Figure 2. Poincaré sphere representation for the eigenstates of thermal photons. (a) The polarization state of light can be completely determined by two angle parameters: $\psi$ and $\chi$ which give the tilt angle and a measure of eccentricity of the polarization ellipse, respectively. (b) The corresponding state is represented as a point on the surface of Poincaré sphere. (c) Evolution of eigenstates of thermal photons in dielectric $\rightarrow$ lossless TI $\rightarrow$ lossy TI and the emergence of spin angular momentum in radiation.

where $B$ is a constant vector $B = (\eta, 0, -\gamma)$, whose components are related to the parameters of our system, and $\sigma$ is the Pauli vector (containing Pauli matrices as its components). Equation (6) then allows us to interpret the eigenstates of $R$ as those of an effective Hamiltonian matrix $H = \sigma \cdot B$ for an electron in a magnetic field $B$.

For a usual dielectric the effective magnetic field simplifies to $B = (0, 0, \epsilon - 1)$ and the associated eigen-polarization states are the two points on the $Z$ axis corresponding to the $p$ and $s$ polarization (see Fig. 2(c)). Introduction of loss or variation of the incidence angle do not change these two polarization coordinates.

For a lossless TI the effective magnetic field $B$ is real and the eigenstates are given by a rotated pair of antipodal points on the equator. They correspond to photons inside TI having polarization states given by a linear combination of $p$ and $s$ states with real coefficients. The radiated photons do not carry any spin angular momentum.

However, for a lossy TI the effective field $B$ becomes complex and the eigenstates are no longer confined in the equator, although they maintain the same $Y$ altitude. This results into each emitted photon carrying an equal amount of spin angular momentum for both eigen-polarizations.

5. CONCLUSION

Thermal radiation from topological insulator carries nonzero spin angular momentum. Through the control over the spin angular momentum of the thermal emission, TIs may therefore find applications in the generation of structured light from thermal sources.

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