Connectivity Matrix Algorithm: A New Optimal Phasor Measurement Unit Placement Algorithm

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Abstract. The maximum benefits of the phasor measurement unit (PMU) in state estimation and other applications in the power system can be achieved if the system is completely observed by PMUs only. The minimum number of PMUs needed to obtain full observability is very important in order to reduce the overall cost. This paper presents a new algorithm for optimal PMU placement (OPP) in transmission and distribution systems. A new concept and vision of the optimization problem is presented. A new approach to implement zero-injection buses is proposed which may be also used in other algorithms. The simulation result of IEEE 14, 24, 30, 57 and 118 bus systems as transmission systems, IEEE 33, 34 and 123 bus systems as distribution systems, and Jordanian power system as an actual system are presented and compared with the existing techniques. The proposed technique is simple to implement and accurate compared to other existing methods. The results confirm the efficiency of the proposed algorithm. The computational time is less than 0.2 seconds for all cases. MATLAB 2016a is used to validate the algorithm.

Keywords. WAMS; PMU; Optimal PMU placement; Observability; Network connectivity.

1. Introduction

In recent years, the new technology of a wide-area monitoring system (WAMS) became one of the most important requirements in modern power system applications; such as control, protection, and operation. The integration of renewable technologies and distributed generations made the power system more complex. Therefore, the real-time dynamic control, operation, and protection based on the real time state estimation model and the development in information and communication technology become the backbone of energy management system (EMS). The integration of these technology with the existing supervisory control and data acquisition (SCADA) can catch some of inter-area oscillation which can’t be damped by local controller perfectly. The maximum benefits of these technologies can be achieved if the measurement and analysis of power system data are based only on PMUs.

PMUs provide synchrophasor voltage, current, frequency and rate of change on frequency (ROCOF). Therefore, linear and dynamic state estimation (SE) models can be implemented. Based on KVL and KCL, few numbers of PMUs in a system may be enough to make the system fully observable. Some researchers introduced different techniques for OPP based on network topology to obtain the network observability.

Different algorithms have been presented in literature. One of the most accurate techniques is presented in [1]. The researchers proposed a method comprised of three stages to provide an optimal placement. In this method, it is assumed that in first and second stages each bus of the system is equipped with a PMU. Iteratively, the less important buses from observability point of view are
determined. In stage three, pruning operation is applied to minimize the number of PMUs in the entire system. IEEE 14, 24, 30, 57, and 118 bus systems are used to validate the method. This method is very effective to obtain optimal location for transmission systems with and without consideration of zero injection buses. However, this method is aimed at investigating the transmission systems only. Integer linear programming (ILP) is an easy way to solve the OPP but may be less accurate. In [2] full coverage of OPP is presented in distribution systems based on concept of ILP technique. IEEE 33, 34, 123 bus systems are simulated in this research. The presented method enhances the accuracy of ILP in solving OPP for distribution systems. Other researchers [3] used Fault-Tolerance Based Approach to solve OPP. It is the most recent research in this field and the researchers validate their algorithm on IEEE 14 and 30 bus systems. The effect of the zero injection bus is considered. The computational time for all cases found to be about 2 seconds on a single processor. ILP is a fast and easy-to-implement algorithm which has been used for long time to solve the OPP problem [4-7]. Some researchers used Intelligent techniques to solve the OPP [8].

Briefly, OPP can be categorized into: Heuristic Methods, Meta-Heuristic Methods and Deterministic Methods [9], where Depth-First Algorithm (DFS) [10] [11] Domination Set [12] [13], and Greedy Algorithm [14] are examples on Heuristic Methods, Genetic Algorithms [15] [16] and Particle Swarm Optimization (PSO) [8] [17] [18] [19] are examples on Meta-Heuristic Methods, and ILP [20-22], and Binary Search Method [23-27] are examples of Deterministic Methods. The enhancement of state estimation by PMUs is addressed in [28].

In our paper a very simple technique is presented to solve OPP for both distribution and transmission systems. The computational time of this algorithm is less than 0.2 seconds for all cases. In section II, the new and simple definition of OPP is presented. This definition is based only on the connectivity matrix of any system. In Section III, the concept of the proposed algorithm and the new vision of zero injection bus implementation based on merging method are discussed. Test validation is done on each of IEEE 14, 24, 30, 33, 34, 57, 118, 123 bus systems and Jordanian Power System. The results obtained from our technique are compared with the existing methods as it is shown in section IV.

2. OPP Problem Definition

PMUs as a synchrophasor measurement device can measure phasor voltages and currents at different geographical locations instantaneously. So, if a PMU is installed at the bus i, the phasor voltage at bus i and the phasor currents in all connected lines are measured directly by the PMU (direct measurements). On the other hand, if a PMU is installed at a bus, the phasor voltage at the adjacent buses can be calculated based on ohm’s law (pseudo measurement).

Any bus in the system without load or generation is called Zero injection bus (ZIB). For such buses, two additional indirect techniques can be applied. First one is based on nodal analysis in which all connected buses are observable, the phasor voltage of ZIB can be computed. The second technique is based on ohm’s law and KCL. In this technique, if ZIB and all connected buses except one are observable then the voltage of unobservable bus can be computed. These two exceptions give the OPP problem additional freedom of solution.

OPP is an optimization problem for finding a minimum number of PMUs needed to make the system fully observable. The proposed algorithm, Connectivity Matrix Algorithm (CMA), is based on the topology network of a power system. The connectivity matrix of the power system is a binary (NXN) square matrix that represents the connections between buses. If bus (i) and (j) are connected, then (ij) element of connectivity matrix will be 1, otherwise 0. From the definition, the main diagonal elements are ones. The first row of the connectivity matrix represents the connections of bus 1. So the element in the first row second column (a12) is 1 if bus 1 and 2 are connected, and so on. Each transmission line is represented by two ones in the matrix. For example, if bus 1 and 5 are connected, then a15 and a51 are ones. It means that the connectivity matrix is a symmetrical, $a_{ij} = a_{ji}$. The topological observability implies that if a PMU is placed at a bus then this bus and all connected buses are observable.

3. The Proposed Algorithm

The proposed algorithm in this paper is depicted in the following flow chart in Figure 1:
The proposed algorithm is based on the mathematical formula (equation 2) for the achievement of minimum number of PMU that are placed for full observable power system. The identification of the current PMU placement is obtained by (1):

\[
\text{optimal} \text{ - bus} = \max \left( \sum_{i=1}^{N} \left( d_{ij} \left( \sum_{k=1}^{N} a_{ik} \right)^{-2} \right) \right), \text{for } j = 1, 2, \ldots
\]  

(1)

where \( d_{ij} \) is a single element in origin connectivity matrix (D) and \( a_{ij} \) is an element in modified connectivity matrix (A) that ignore all observable buses from previous PMUs. Each row in the connectivity matrix represents the number of choices to observe its bus. Each column provides the number of connections for its bus. The summation of the elements of a row is equal to the number of choices in this row. The summation of the elements of a column is equal to the number of connections of its bus. After installing first PMU, each of the observable buses (its rows and columns) will be ignored to identify new critical bus. The algorithm identifies a new matrix (B) based on (A and D). The elements of the new matrix can be represented in (2):

\[
b_{ij} = d_{ij} \times \left( \sum_{k=1}^{N} a_{ik} \right)^{-2}, \text{for } i = 1 \ldots N, j = 1 \ldots N
\]  

(2)

Where, \( b_{ij} \), \( d_{ij} \), and \( a_{ik} \) are elements in matrix B, D, and A, respectively. N: number of buses (dimension of the connectivity matrix).

Based on the new matrix (B), the priority of selection a bus at which the first PMU will be installed can be determined. A column in the matrix (B) which has the maximum elements summation is selected to place the first PMU. By using the concept of matrices, firstly identify the matrix (A and D) which is exactly the connectivity matrix. sum each row in the matrix (A) and compute (sum-2) providing a summation column, this step gives weight to critical buses. Multiply summation column by each column in matrix (D) to compute matrix (B). Finally sum each column in matrix B and select the maximum summation as an optimal placement. The illustration of these steps on IEEE 14 bus
system is shown in Figure 2. From the figure, the optimal first PMU placement is BUS 7. So buses 4, 7, 8, and 9 become observable. From matrix (A) 4, 7, 8, and 9 rows and column is changed to zeros to identify the optimal second PMU placement, and so on.

Figure 2. The proposed algorithm in a matrix form.

Figure 3. Zero injection bus implementation rule 3.
From the previous explanations about ZIB, if the whole system is observable except the ZIB or any bus connected with the ZIB, then the system is considered fully observable. To consider the ZIB in this algorithm, the following five rules to identify a bus which must be removed from the ZIB group:

1) If a bus is connected with the ZIB has only one connection, then this bus will be removed from the system.
2) If the ZIB has only two connections, this bus will be removed.
3) If the ZIB has three connections, then the ZIB will be removed and the two of the three with minimum connections should be connected.
4) If two or more ZIB are connected, then these buses will be merged, Figure 3.
5) If the ZIB has more than three connections, then the bus with minimum connections is removed.

4. Validation Testing
Ensure that In this section, IEEE 14-bus, 24-bus, 30-bus, 57-bus, and 118-bus test systems as transmission systems, IEEE 33-bus, 34-bus, and 123-bus as Distribution systems [2], and Jordanian power system as an actual system will be simulated. All simulations are run at Matlab (R2016a) on a computer with Intel (R) Core(TM) i5-8250U CPU @1.6 GHz and 4 GB 1800MHz DDR3 Memory. All cases are simulated without considering of redundant observability.

Based on the proposed algorithm, the OPP results without considering ZIB are shown in table 1. The implementation of ZIB on the transmission system is shown in table 2. Table 3 shows the comparison of OPP results with available techniques for the normal operating condition without considering ZIB for all cases. The results in table 3, show that the proposed algorithm suggests the minimum PMUs in all cases. Table 4 shows the Comparison of OPP results with available techniques for normal operating conditions considering ZIB for all cases. The CPU time taken by the proposed method under normal operating conditions is given in Table 5 for different test systems. The computation time of the proposed method is compared with the time taken in other algorithms. Performance comparison shows the computational efficiency of the proposed method compared to other methods.

| System | Optimal placement) | number | time(s) |
|--------|---------------------|--------|---------|
| 14-Bus | 2,6,7,9             | 4      | 0.0378  |
| 24-Bus | 2,3,8,10,16,21,23   | 7      | 0.059   |
| 30-Bus | 2 3 6 10 11 12 18 23 25 29 | 10 | 0.0675 |
| 57-Bus | 1 6 9 15 19 21 24 25 28 32 36 38 39 41 46 50 53 | 17 | 0.0858 |
| 118-Bus | 3 5 9 12 13 17 21 23 25 28 34 37 41 45 49 52 56 62 64 68 71 75 77 80 85 87 90 94 101 105 110 114 | 32 | 0.165 |
| 33-Bus | 2 5 8 11 14 17 21 24 27 30 32 | 11 | 0.069 |
| 34-Bus | 1 2 9 10 14 15 21 22 25 26 30 32 | 12 | 0.072 |
| 123-Bus | 1 3 6 8 14 15 19 21 23 25 27 30 31 36 39 41 42 45 47 51 52 55 58 60 64 66 69 71 72 74 78 82 84 87 89 91 93 96 97 100 103 106 108 110 113 | 45 | 0.205 |
| Jordan-68-Bus | 1 4 5 7 18 19 21 23 27 30 33 38 40 41 47 49 52 55 57 60 62 | 22 | 0.105 |

| System | Optimal placement) | number | time(s) |
|--------|---------------------|--------|---------|
| 14-Bus | 2 6 9             | 3      | 0.069   |
| 24-Bus | 2 8 13 15 16 20   | 6      | 0.054   |
| 30-Bus | 2 3 10 12 15 19 28 | 7      | 0.126   |
| 118-Bus | 3 9 11 12 17 21 27 29 32 34 38 41 45 49 52 56 62 72 75 77 80 85 87 90 94 101 105 110 | 28 | 0.143 |
Table 3. Comparison of optimal PMU placement results with available techniques without considering ZIB for transmission system.

| Method             | 14-Bus | 24-Bus | 30-Bus | 57-Bus | 118-Bus | 33-Bus | 34-Bus | 134-Bus | Jordan |
|--------------------|--------|--------|--------|--------|---------|--------|--------|---------|--------|
| proposed           | 4      | 7      | 10     | 17     | 32      | 11     | 12     | 45      | 22     |
| BK, AK [1]         | 4      | 7      | N/A    | 17     | 32      | N/A    | N/A    | N/A     | N/A    |
| Xu, Abur [25]      | 4      | N/A    | 10     | 17     | 32      | 11     | N/A    | N/A     | N/A    |
| Chakrabarti [26]   | 4      | 7      | N/A    | N/A    | 17      | 32     | N/A    | N/A     | N/A    |
| Hurtgen [27]       | 4      | N/A    | N/A    | 17     | 32      | N/A    | N/A    | N/A     | N/A    |
| GA [28]            | N/A    | N/A    | N/A    | 10     | 18      | 33     | N/A    | N/A     | N/A    |
| ILP [2]            | N/A    | N/A    | N/A    | N/A    | N/A     | N/A    | N/A    | N/A     | N/A    |
| Custom [2]         | N/A    | N/A    | N/A    | N/A    | N/A     | N/A    | N/A    | N/A     | N/A    |
| Hybrid Method [2]  | N/A    | N/A    | N/A    | N/A    | N/A     | N/A    | 11     | 12      | 45     |
| Modified SA [29]   | N/A    | N/A    | N/A    | N/A    | N/A     | N/A    | N/A    | N/A     | 45     |
| GTH [29]           | N/A    | N/A    | N/A    | N/A    | N/A     | N/A    | N/A    | N/A     | 25     |

Table 4. Optimal PMU placement results with available techniques considering ZIB for transmission system.

| Method             | 14-Bus | 24-Bus | 30-Bus | 57-Bus | 118-Bus | 33-Bus | 34-Bus | 134-Bus | Jordan |
|--------------------|--------|--------|--------|--------|---------|--------|--------|---------|--------|
| proposed           | 3      | 6      | 7      | 28     |         |        |        |         |        |
| BK, AK* [1]        | 3      | 6      | 7      | 28     |         |        |        |         |        |
| Xu, Abur [25]      | 3      | N/A    | 7      | 28     |         |        |        |         |        |
| Chakrabarti [26]   | 3      | 6      | 7      | 28     |         |        |        |         |        |
| Hurtgen [27]       | 3      | N/A    | 7      | 28     |         |        |        |         |        |
| GA [28]            | 3      | N/A    | 7      | 28     |         |        |        |         |        |
| Modified SA [29]   | 3      | N/A    | 7      | 33     |         |        |        |         |        |
| GTH [29]           | 3      | N/A    | 8      | 40     |         |        |        |         |        |

Table 5. Comparison of computation time in seconds for obtaining optimal solution by the proposed method with other simulated methods.

| Method             | 14-Bus | 24-Bus | 30-Bus | 57-Bus | 118-Bus | 33-Bus | 34-Bus | 134-Bus | Jordan |
|--------------------|--------|--------|--------|--------|---------|--------|--------|---------|--------|
| proposed           | 0.038  | 0.059  | 0.067  | 0.085  | 0.165   | 0.069  | 0.072  | 0.205   | 0.105  |
| BK, AK* [1]        | 0.66   | 0.76   | 0.83   | 0.87   | 1.34    | 11     | N/A    | N/A     | N/A    |
| BIP* [1]           | 1.16   | 1.34   | 1.24   | 1.94   | 1.55    | 11     | N/A    | N/A     | N/A    |
| Exhaustive search  | 1.3    | 75.4   | N/A    | N/A    | 62.13   | 0.26   | N/A    | N/A     | N/A    |
| ** [1,2]           |        |        |        |        |         |        |        |         |        |
| Hybrid Method**    | N/A    | N/A    | N/A    | N/A    | N/A     | 3.61   | 3.54   | 12.77   | N/A    |
| Modified SA **     | 0.91   | 16.52  | 157.5  | 920.1  | N/A     | N/A    | N/A    | 2.79    |        |
| GTH ** [29]        | 0.024  | N/A    | 0.112  | 0.472  | 1.3     | N/A    | N/A    | 0.493   |        |

(*) Run at Intel Pentium 4, 3.0-GHz CPU with 512 MB of RAM
(**) Run at MacOS, 2.7 GHz i7 CPU and 16GB 1600MHz DDR3 Memory

5. Conclusions
A new algorithm for the optimal placement of PMU is presented in this paper. The algorithm provides a global solution for any power system based on connectivity matrix.

The new technique to implement the ZIB is also presented. The proposed algorithm is tested on IEEE 14-bus, 24-Bus, 30-Bus, 57-Bus, 118-Bus systems as transmission systems, IEEE 33- Bus, 34-Bus, and 123-Bus systems a distribution system and Jordanian power system as an actual system. The breakers state in distribution systems is selected in a way that covers all operation cases, so the optimal
placement in distribution systems ensures the full observability of the system at any normal operating conditions. The results obtained are compared with some existing methods. The comparison confirms the ability of proposed algorithm to obtain a global solution with high stability. The concept of new implementation of ZIB can be used in any other algorithms which will reduce the overall structure of the network. The computation time of the proposed algorithm which indicates the method complexity is lower than 10% of faster existing algorithm time. No iteration is required. The simulation results for different networks prove the effectiveness of the proposed method in obtaining the minimum number of PMUs required for complete observability of power systems.

6. References

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