Microwave to optical conversion with atoms on a superconducting chip

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Abstract

We describe a scheme to coherently convert a microwave photon of a superconducting co-planar waveguide resonator to an optical photon emitted into a well-defined temporal and spatial mode. The conversion is realized by a cold atomic ensemble trapped close the surface of the superconducting chip, near the antinode of the microwave cavity. The microwave photon couples to a strong Rydberg transition of the atoms that are also driven by a pair of laser fields with appropriate frequencies and wavevectors for an efficient wave-mixing process. With only several thousand atoms in an ensemble of moderate density, the microwave photon can be completely converted into an optical photon emitted with high probability into the phase matched direction and, e.g. fed into a fiber waveguide. This scheme operates in a free-space configuration, without requiring strong coupling of the atoms to a resonant optical cavity.

1. Introduction

Superconducting quantum circuits, which operate in the microwave frequency range, are promising systems for quantum information processing [1, 2], as attested by the immense recent interest of academia and industry. On the other hand, photons in the optical and telecommunication frequency range are the best and fastest carriers of quantum information over long distances [3, 4]. Hence there is an urgent need for efficient, coherent and reversible conversion between microwave and optical signals at the single quantum level [5]. Here we describe such a scheme, which is compatible with both superconducting quantum information processing and optical quantum communication technologies.

Previous work on the microwave to optical conversion includes studies of optically active dopants in solids [6, 7], as well as electro-optical [8] and opto-mechanical [9] systems. Cold atomic systems, however, have unique advantages over the other approaches. Atomic (spin) ensembles can couple to superconducting microwave resonators to realize quantum memory in the long-lived hyperfine manifold of levels [10, 11]. Using stimulated Raman techniques [12, 13], such spin-wave excitations stored in the hyperfine transition can be reversibly converted into optical photons. Here we propose and analyze an efficient wave-mixing scheme for microwave to optical conversion on a integrated superconducting atom chip. In our setup, the microwave photon is confined in a coplanar waveguide resonator, while a cold atomic ensemble is trapped near the antinode of the microwave cavity mode at a distance of several tens of microns from the surface of the atom chip. We employ a Rydberg transition between the atomic states that strongly couple to the microwave field [14–17]. The coupling strength of the atoms to the evanescent field of the cavity depends on the atomic position, while the proximity of the atoms to the chip surface leads to inhomogeneous Rydberg level shifts and thereby position-dependent detuning of the atomic resonance. This reduces the effective number of atoms participating in four-wave mixing in the presence of a pair of laser fields with appropriate frequencies and wavevectors. Nevertheless, we show that high-efficiency conversion of a microwave photon to an optical photon emitted into a well-defined spatial and...
temporal mode is still possible in this setup. The coplanar waveguide resonator can also contain superconducting qubits, and hence our scheme can serve to interface them with optical photons.

We note a related work [18] on microwave to optical conversion using free-space six-wave mixing involving Rydberg states. The achieved photon conversion efficiency was, however, low, as only a small portion of the free-space microwave field could interact with the active atomic medium. Confining the microwave field in a cavity would be a valuable route to enhance the conversion efficiency. A microwave to optical conversion scheme using a single (Cs) atom that interacts with a superconducting microwave resonator on the Rydberg transition and with an optical cavity on an optical transition was discussed in [19]. The advantage of the single atom approach is that it requires moderate laser power for atom trapping and leads to less light scattering and perturbation of the superconducting resonator. It relies, however, on the technically demanding strong coupling of the single atom to both microwave and optical cavities. Reference [20] discusses the conversion of a microwave photon to an optical telecommunication (E-band) photon employing four-wave mixing in a small ensemble of (Yb) atoms in a copper microwave resonator and a high-finesse optical cavity. In contrast, our present approach uses a large ensemble of atoms with collectively enhanced coupling to the microwave cavity and it leads to a coherent, directional emission of the optical photon even without an optical cavity. In a previous publication [21], we have employed a similar scheme to deterministically produce single photons from a Rydberg excitation of a single source atom coupled to the atomic ensemble via resonant dipole–dipole interaction.

Our setup is primarily intended for optical communication between microwave operated quantum sub-registers. As such, we consider the case of at most one microwave photon encoding a qubit state at a time. The conversion of a microwave photon is accompanied by a Rydberg excitation blockade [22, 23] that does not play a role in our scheme, irrespective of whether the atomic ensemble is larger or not than any (irrelevant) blockade distance. This allows us to restrict the analysis to the linear regime of conversion, greatly simplifying the corresponding calculations presented below.

2. The system

Consider the system shown schematically in figure 1. An integrated superconducting atom chip incorporates a microwave resonator, possibly containing superconducting qubits, and wires for magnetic trapping of the atoms. An ensemble of \( N \gg 1 \) cold atoms is trapped near the chip surface, close to the antinode of the microwave cavity field. The relevant states of the atoms are the ground state \( |\text{g}\rangle \), a lower electronically excited state \( |\text{s}\rangle \) and a pair of highly-excited Rydberg states \( |\text{p}\rangle \) and \( |\text{e}\rangle \) (see the inset of figure 1). A laser field of frequency \( \omega_p \) couples the ground state \( |\text{g}\rangle \) to the Rydberg states \( |\text{p}\rangle \) with time-dependent Rabi frequency \( \Omega_p \) and large detuning \( \Delta_p \equiv \omega_p - \omega_{\text{gs}} \gg |\Omega_p| \). The atoms interact non-resonantly with the microwave cavity mode \( \hat{\xi} \) on the strong dipole-allowed transition between the Rydberg states \( |\text{p}\rangle \) and \( |\text{s}\rangle \). The corresponding coupling strength (vacuum Rabi frequency) \( \eta = (\hbar/\hbar) \hat{\xi} \cdot \hat{u}(r) \) is proportional to the dipole moment \( \mu_{\text{gs}} \) of the atomic transition, the field per photon \( \hat{\xi} \) in the cavity, and the cavity mode function \( \hat{u}(r) \) at the atomic position \( r \). The Rydberg transition is detuned from the cavity mode resonance by \( \Delta_p \equiv \omega_p - \omega_{\text{gs}} \gg |\Delta_p| \gg \eta \). A strong driving field of frequency \( \omega_p \) acts on the transition from the Rydberg state \( |\text{s}\rangle \) to the lower excited state \( |\text{e}\rangle \) with Rabi

![Figure 1](image_url)
frequency Ω_and detuning Δ_d = ω_d − ω_m. The transition from the excited state |e⟩ to the ground state |g⟩ is coupled with strengths g_{kσ} to the free-space quantized radiation field modes ˆa_{kσ} characterized by the wave vectors k, polarization σ and frequencies ω_k = ck.

In the frame rotating with the frequencies of all the fields, ω_μ, ω_κ, ω_β, and ω_δ, dropping for simplicity the polarization index, the Hamiltonian for the system reads

\[
\hat{H}/\hbar = -\sum_{j=1}^{N} \left[ \Delta_\nu^{(j)} \hat{\sigma}_\nu^{(j)} + \delta_\nu^{(j)} \hat{\sigma}_\nu^{(j)} + \delta_\epsilon^{(j)} \hat{\sigma}_\epsilon^{(j)} \right. \\
\left. + \Omega_e e^{i \epsilon_r^j \epsilon} e^{-i (\omega_\delta - \omega_\epsilon) t} \hat{\sigma}_\epsilon^{(j)} + \Omega_d e^{i \epsilon_r^j \epsilon} \hat{\sigma}_\epsilon^{(j)} + \hat{H}.c.c. \right],
\]

where index j enumerates the atoms at positions \(r_j\), \(\delta_\nu^{(j)} \equiv |\mu\rangle \langle \nu|\) are the atomic projection (\(\mu = \nu\)) or transition (\(\mu \neq \nu\)) operators, \(k_{\mu}^2\) and \(k_{\nu}^2\) are the wave vectors of the corresponding laser fields, \(\delta_\epsilon^{(j)} \equiv \Delta_\nu^{(j)} + \Delta_\epsilon^{(j)} = \omega_\nu + \omega_\epsilon - \omega_\delta - \omega_d\) is the two-photon detuning of level |s⟩, and \(\delta_\epsilon \equiv \Delta_\nu^{(j)} - \Delta_\epsilon^{(j)} = \omega_\nu + \omega_\epsilon - \omega_\delta - \omega_d\) is the three-photon detuning of |e⟩. The energies of the Rydberg levels |i⟩, |j⟩, and thereby the corresponding detunings \(\Delta_\nu^{(j)}\) and \(\delta_\nu^{(j)}\), depend on the atomic distance \((x_0 - x_j)\) from the chip surface at \(x_0\), which may contain atomic adsorbates producing an inhomogeneous electric field [24, 25]. We neglect the level shift of the lower state |e⟩, since it is typically less sensitive to the electric fields and has a large width \(\Gamma_e\) (see below).

We assume that initially all the atoms are prepared in the ground state, \(|G⟩ \equiv |g_0, g_2, \ldots, g_N⟩\), the microwave cavity contains a single photon, |1⟩, and all the free-space optical modes are empty, |0⟩. We can expand the state vector of the combined system as |Ψ⟩ = |b_0⟩ |G⟩ ⊗ |1⟩ ⊗ |0⟩ + \sum_{j=1}^{N} |d_j⟩ e^{i \epsilon_r^j \epsilon} |j⟩ ⊗ |1⟩ ⊗ |0⟩ + \sum_{j=1}^{N} \sum_{k} e^{i \epsilon_k^j \epsilon} |s_j⟩ ⊗ |0⟩ ⊗ |0⟩ + \sum_{j=1}^{N} \sum_{k} e^{i \epsilon_k^j \epsilon} |c_j⟩ ⊗ |0⟩ ⊗ |0⟩ + \sum_{k} |d_k⟩ |l_k⟩, where \(|\mu_j⟩ \equiv |g_0, g_2, \ldots, g_j, \ldots, g_N⟩\) denote the single excitation states, \(\mu_i = i, s, \epsilon, \) and \(|1_j⟩ \equiv \hat{a}_k^\dagger |0⟩\) denotes the state of the radiation field with one photon in mode \(k\). The evolution of the state vector is governed by the Schrödinger equation  ̈|Ψ⟩ = −\frac{i}{\hbar}H |Ψ⟩ with the Hamiltonian (1), which leads to the system of coupled equations for the slowly-varying in space atomic amplitudes,

\[
\partial_t b_0 = i \sum_{j=1}^{N} \Omega_{\nu}^j d_j, \\
\partial_t d_j = i \Delta_\nu^{(j)} d_j + i \Omega_{\nu}^j b_0 - i \eta^j(t_j) c_j, \\
\partial_t c_j = i \delta_\epsilon^{(j)} c_j - i \eta(t_j) d_j + i \Omega_{\epsilon} d_j, \\
\partial_t b_j = i \delta_\epsilon^{(j)} b_j + i \Omega_{\epsilon} c_j + i \sum_{k} \hat{a}_k e^{i (\epsilon_k^j - \epsilon)} \eta^j(t_j) e^{-i (\omega_\delta - \omega_\epsilon) t},
\]

while the equation for the optical photon amplitudes written in the integral form is

\[
\int_{0}^{t} dt' b_j(t') e^{-i (\omega_\delta - \omega_\epsilon) t'}.
\]

The initial conditions for equations (2), (3) are \(b_0(0) = 1, b_j(0), c_j(0), d_j(0) = 0 \forall j, \) and \(a_k(0) = 0 \forall k\).

We substitute equation (3) into the equation for atomic amplitudes \(b_j\), assuming they vary slowly in time, and obtain the usual spontaneous decay of the atomic state |e⟩ with rate \(\Gamma_e\), and the Lamb shift that can be incorporated into \(\omega_{\nu}^j\) [26]. We neglect the field-mediated interactions (multiple scattering) between the atoms [27–29], assuming random atomic positions and sufficiently large mean interatomic distance \(\hat{r}_j \gtrsim \lambda/2\pi\). To avoid the atomic excitation in the absence of a microwave photon in the cavity, we assume that the intermediate Rydberg level |i⟩ is strongly detuned, \(\Delta_\nu^{(j)} \approx -\Delta_\epsilon^{(j)} \gg |\Omega_{\nu}|, \eta, |\delta_\nu^{(j)}|\) for all atoms in the ensemble. In addition, we assume that the variation of \(\Delta_\nu^{(j)} (\Delta_\epsilon^{(j)})\) across the atomic cloud is small compared to its mean value \(\Delta (-\Delta),\) which presumes small enough Rydberg levels shifts in the inhomogeneous electric field. We can then adiabatically eliminate the intermediate Rydberg level |i⟩, obtaining finally

\[
\partial_t b_0 = i \sum_{j=1}^{N} \hat{\eta}_j c_j, \\
\partial_t c_j = i \delta_\epsilon^{(j)} - \Gamma_e/2) c_j + i \hat{\eta}_j b_0 + i \Omega_{\epsilon} b_j, \\
\partial_t b_j = i \delta_\epsilon^{(j)} - \Gamma_e/2) b_j + i \Omega_{\epsilon} c_j,
\]

where \(\hat{\eta}_j \equiv \frac{\eta^j(t_j)}{\Delta} (1 + \frac{\Omega_{\nu}^j}{\Delta})\) is the second-order coupling between \(|g_0⟩ ⊗ |1⟩\) and \(|s_j⟩ ⊗ |0⟩\), while the second-order level shifts of \(|g_0⟩\) and \(|s_j⟩\) are incorporated into the detunings \(\delta_\nu^{(j)} \equiv \delta_\nu^{(j)} + |\Omega_{\nu}^j|^2 - 1 |\eta^j(t_j)|^2\) and

\[
\int_{0}^{t} dt' b_j(t') e^{-i (\omega_\delta - \omega_\epsilon) t'}.
\]
\( \delta_\epsilon = \delta_\epsilon + \frac{1}{2} \Omega_\epsilon \beta \). We have also included the typically slow decay \( \Gamma_\epsilon \) of state \( |s\rangle \) corresponding to the loss of Rydberg atoms \([16, 30]\).

Before presenting the results of numerical simulations, we can derive an approximate analytic solution of the above equations and discuss its implications. We take a time-dependent pump field \( \Omega_p(t) \) (and thereby \( \eta_j(t) \)) and a constant driving field \( \Omega_d < \Gamma_\epsilon / 2 \), which results in an effective broadening of the Rydberg state \( |s\rangle \) by \( \gamma = \frac{1}{2} \frac{\Omega_\epsilon}{\Gamma_\epsilon / 2} \). Assuming \( \gamma \gg \Gamma_\epsilon / 2 \), \( \delta_\epsilon \), we then obtain

\[
\begin{equation}
\begin{aligned}
\eta_j(t) &= -\frac{\gamma}{\Omega_d} \delta_\epsilon \eta_j(t) - b_0(t), \\
\int_0^t dt' \sum_{j=1}^N |\eta_j(t')|^2
\end{aligned}
\end{equation}
\]

(5a)

\[
\begin{equation}
\begin{aligned}
b_0(t) &= b_0(0) \exp \left[-\int_0^t dt' \sum_{j=1}^N |\eta_j(t')|^2 \right].
\end{aligned}
\end{equation}
\]

(5b)

Substituting these into equation (3) and separating the temporal and spatial dependence, we obtain

\[
\begin{equation}
\begin{aligned}
a_k(t) &= -\frac{i}{\Omega_d} A_k(t) \times B_k,
\end{aligned}
\end{equation}
\]

(6)

where

\[
\begin{equation}
\begin{aligned}
A_k(t) &= \int_0^t dt' \Omega_p(t') \exp\left[i(\omega_{\epsilon} - \omega_0)t' + \gamma - i\delta_\epsilon \right],
\end{aligned}
\end{equation}
\]

(7a)

\[
\begin{equation}
\begin{aligned}
B_k &= \delta_\epsilon \sum_{j=1}^N \eta_j(t) \exp[i(k_{\perp} - k_{\parallel})x_j],
\end{aligned}
\end{equation}
\]

(7b)

with \( \beta = \frac{1}{2} \sum_{j=1}^N |\eta_j(t)|^2 \).

Equation (7a) shows that for a sufficiently smooth envelope of the pump field \( \Omega_p(t) \), the optical photon is emitted within a narrow bandwidth \( \beta \Omega_p(t)^2 \) around frequency \( \omega_k = \omega_0 \), which is a manifestation of the energy conservation. The temporal profile of the photon field at this frequency is \( \epsilon(t) = \partial_t A_k(t) = \Omega_p(t) \exp[-\beta \int_0^t dt' |\Omega_p(t')|^2] \), where \( k_0 = \omega_0 / c \). The envelope of the emitted radiation can be tailored to the desired profile \( \epsilon(t) \) by shaping the pump pulse according to \( \Omega_p(t) = \epsilon(t) \left[ 1 - 2 \beta \int_0^t dt' |\epsilon(t')|^2 \right]^{-1/2} \), which can facilitate the photon transmission and its coherent re-absorption in a reverse process at a distant location \([32-34]\). Neglecting the photon dispersion during the propagation from the sending to the receiving node, and assuming the same or similar physical setup at the receiving node containing an atomic ensemble driven by a constant field \( \Omega_d \), the complete conversion of the incoming optical photon to the cavity microwave photon is achieved by using the receiving laser pulse of the shape \( \Omega_d(t) = -\epsilon(t) \left[ 2 \beta \int_0^t dt' |\epsilon(t')|^2 \right]^{-1/2} \).

The spatial profile of the emitted radiation in equation (7b) is determined by the geometry of the atomic cloud, the excitation amplitudes of the atoms at different positions, and the phase matching. We assume an atomic cloud with normal density distribution \( \rho(r) = \rho_0 \exp[-z^2/2z_0^2 - x^2/2x_0^2 - y^2/2y_0^2] \) in an elongated harmonic trap, \( \sigma_x > \sigma_z \). To maximize the resonant emission at frequency \( \omega_k = \epsilon(k_{\perp} - k_{\parallel}) \), into the phase matched direction \( k = k_{\perp} - k_{\parallel} \), we assume the (nearly) collinear geometry \( k_{\perp}, k_{\parallel} \parallel e_z \). In an ideal case of all the atoms having the same excitation amplitude \( b_\parallel \propto 1/S^1/2 \), we have \( B_k \propto \int dr \rho(r) \exp[i(k_{\perp} - k_{\parallel})r] \), the photon would be emitted predominantly into an (elliptic) Gaussian mode \( \mathcal{E}(r) \propto \sum_{k=0} \|k\| B_k \exp[ik_{\perp}r + ik_{\parallel}z] \) with the waists \( w_{x_0,0y} = 2\sigma_{x,y} \), namely

\[
\begin{equation}
\begin{aligned}
\mathcal{E}(x, y, z) = \left( \frac{2 \pi w_{x_0,0y}}{w_{x_0,0y}} \right)^{1/4} e^{ik_{\perp}(x + x_0^2/2z_0^2 + y^2/2y_0^2)}.
\end{aligned}
\end{equation}
\]

(8)

where \( w_{x_0,0y} = w_{x_0,0y} \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right]^{1/2} \) and \( g_{x_0,0y} = z - \zeta_{x_0,0y} \). The corresponding angular spread (divergence) of the beam is \( \Delta\theta_{x_0,0y} = \frac{\lambda_0}{2w_{x_0,0y}} = \frac{1}{k_0w_{x_0,0y}} \), which spans the solid angle \( \Delta\Omega = \pi \Delta\theta_{x_0,0y} \). The probability of the phase-matched, cooperative photon emission into this solid angle is \( P_{\Delta\Omega} \propto N \Delta\Omega \), while the probability of spontaneous, uncorrelated photon emission into a random direction is \( P_{\Delta\Omega} \propto 4\pi \). With \( P_{\Delta\Omega} + P_{4\pi} = 1 \), we obtain \( P_{\Delta\Omega} \sim N \Delta\Omega / (N \Delta\Omega + 4\pi) \) which approaches unity for \( N \Delta\Omega \gg 4\pi \) or \( N \gg 4k_0^2 \sigma_x \sigma_y \). Hence, for the product \( N \Delta\Omega \), and thereby \( P_{\Delta\Omega} \), to be large, we should take an elongated atomic cloud with large \( \sigma_z \) (to have many atoms \( N \) at a given atom density) and small \( \sigma_x, \sigma_y \) (to have large solid angle \( \Delta\Omega \)).

In our case, however, not all the atoms participate equally in the photon emission, since the atomic amplitudes \( b_\parallel \propto \eta_j(t) \) depend strongly on the distance \( (x_0 - x_j) \) from the chip surface via both the...
atom–cavity coupling strength $\eta(r) = \eta_0 e^{-i\omega t - |x-x_c|/D}$, and, more sensitively, the Rydberg state detuning $\delta_\text{R}(x) = \alpha x$ (see below). This detuning results in a phase gradient for the atomic amplitudes in the $x$ direction, which will lead to a small inclination of $k$ with respect to $k_p - k_3$ in the $x-z$ plane. More importantly, for strongly varying detuning, $\alpha > 2\gamma/\sigma_x$, only the atoms within a finite-width layer $\Delta x < \sigma_z$ are significantly excited to contribute to the photon emission. This reduces the cooperativity via $N \rightarrow \xi N$ with the effective participation fraction $\xi \approx \Delta x/\sigma_z < 1$, but also leads to larger divergence $\Delta k_z \approx 1/(k_c \Delta x)$ in the $x-z$ plane.

3. Results

We have verified these arguments via exact numerical simulations of the dynamics of the system. We place $N$ ground state $|g\rangle$ atoms in an elongated volume at random positions $r_i$ normally distributed around the origin, $x,y,z=0$, with standard deviations $\sigma_x \gg \sigma_y, \sigma_z$. With the peak density $\rho_0 = 2.35 \, \mu m^{-3}$ and $\sigma_y, \sigma_z = 24 \, \mu m$, we have $N = 15,000$ atoms in the trap interacting with the co-planar waveguide resonator at position $x_0 \approx 40 \, \mu m$ (see figure 1). Taking the strip–line length $L = 10.5 \, \mu m$ and the grounded electrodes at distance $D = 10 \, \mu m$, the effective cavity volume is $V_e \approx 2\pi D^2 L [10]$, yielding the field per photon $\xi_i = \sqrt{\hbar \omega_c/\epsilon_0 V_e} \approx 0.37 \, V \, m^{-1}$ for the full-wavelength cavity mode of frequency $\omega_c/2\pi = c/L_0 = 12 \, GHz$, and dipole moment $\mu_{ij} \approx 2185 a_0 e$. This results in the vacuum Rabi frequency $\eta(0)/2\pi \approx 190 \, kHz$ at the cloud center $r = 0$. We take a sufficiently large intermediate state detuning $\Delta/2\pi \approx 10 \, MHz$, and time-dependent pump field $\Omega_p(t) = \Omega_0 \left(1 + \frac{1}{2} \text{erf}\left(\frac{t-t_0}{\sigma_\text{t}}\right)\right)$ of duration $t_\text{end} \approx 10 \, \mu s$ with $t_0 = t_\text{end}/3, \sigma_\text{t} = t_\text{end}/8$ and the peak value $\Omega_0/2\pi \approx 200 \, kHz$ (see figure 2(a)). The wavelength of the pump field is $\lambda_\text{p} \approx 297 \, nm$ corresponding to a single-photon transition from $|g\rangle = |5S_{1/2}, m_f = 1/2\rangle$ and $|i\rangle = |69S_{1/2}, m_f = 1/2\rangle$ of Rb with the quantum defects $\delta_g = 2.651$ and $\delta_i = 3.131$ [36], leading to the transition frequency $\omega_{gi}/2\pi \approx 12.1 \, GHz$ and dipole moment $\mu_{gi} \approx 2185 a_0 e$. This results in the Rabi frequency $\Omega_{gi} = \Omega_0 = 0.019 \, MHz$ and $\gamma_{gi} \approx 2 \times 10^{-4} a_0 e$, and the required peak intensity of the UV field to attain the Rabi frequency $\Omega_0$. We can estimate the absorption of the laser fields by the superconducting electrodes of the microwave resonator, which would break Cooper pairs and reduce the cavity quality factor resulting in microwave absorption. The intensity of the focused UV pulse at the chip surface is reduced by a factor of $\mu_{gi} e^{-16}$ from its peak value at the cloud center, which means that about 3000 photons will hit the chip surface above the atomic cloud. In addition, the atoms in the cloud will scatter the UV photons in all 4 directions, and $\mu_{gi} e^{-16}$ of the photons will be absorbed per pulse. Assuming the surface reflectivity of 0.999 (the field propagation direction is parallel to the surface), we have only a few absorbed photons per pulse, which is negligible compared to the cooling rate of the cryogenic environment. Similar estimates for the driving field show that only about 700 photons will hit the surface of the atom chip, and less than one will be absorbed during the conversion cycle, while the scattering from the cloud is negligible since at most only one atom can be excited to state $|s\rangle$ at a time.

In figure 2 we show the results of our numerical simulations of the dynamics of the system and compare them with the analytical solutions. In the inset of figure 2(b) we show the time dependence of the total population $p_l(t) = \sum_{j=1}^N |b_j(t)|^2$ of the atoms in the excited state $|e\rangle$. As atoms decay from state $|e\rangle$ to the ground state $|g\rangle$, they emit a photon with rate $\Gamma_e |b_e(t)|^2$. The spatial distribution of time-integrated photon emission probability (in any direction) $P(\tau) = \Gamma_e \int_{t_\text{end}}^{t_\text{end}} |b_e(t)|^2 dt$ is shown in figure 2(c). This probability follows the Gaussian density profile of the atoms along the $y$ and $z$ directions, but in the $x$ direction it is modified by an approximate Lorentzian factor $\frac{1}{\gamma^2 + \delta_\text{L}(x)^2}$ (if we neglect the $x$ dependence of $\delta_\text{L}(x)$) due to the position-dependent detuning $\delta_\text{L}(x)$. Only part of the radiation is coherently emitted into the phase-matched direction $k = k_p - k_3$,
with probability $|a_k(t)|^2$ of photon emission into the resonant $\omega_k = \omega_{eg}$ mode shown in figure 2(b). Note that for non-resonant modes $\omega_k \neq \omega_{eg}$ with the rapidly oscillating phase factor $e^{i\omega_k t}$ in equation (3) or (7a), the photon amplitude $a_k(t) \propto \sum_j b_j(t)$ tends to zero at large times $t_{\text{end}}$ (as do $b_j(t)$'s), even for the phase-matched direction $k \simeq k_0 - k_d$.

In figure 3 we show the angular probability distribution of the emitted photon. The beam divergence $\Delta \theta_x = \frac{1}{k_0 \Delta x} \approx 0.015\pi$ in the $x - z$ plane is almost twice larger than that $\Delta \theta_y = \frac{1}{k_0 \sigma_y} \approx 0.008\pi$ in the $y - z$ plane, consistent with the narrower spatial distribution $\Delta x \simeq 2.6 \mu m < \sigma_x = 4 \mu m$ of the atomic excitation (or emission) probability $P(r)$, as discussed above. In the collinear geometry, $k_x, k_y, k_d || \hat{z}$, the radiation is emitted at a small angle $\theta_{\text{end}} = \theta_{0} \simeq 0.014\pi$ due to the detuning induced phase gradient of the atomic amplitudes $b_j$ along $x$. With a small angle $k_x / k_y = 0.009\pi$ between the drive and the pump fields, the latter still propagating along $z$, we can compensate this phase gradient, resulting in the photon emission along $z$ ($\theta_{\text{end}} = 0$). We may approximate the angular profile of the emitted radiation with a Gaussian function

$$B_k \propto B(\theta_{\text{xx}}, \theta_{\text{yy}}, \theta_{\text{zz}}) = e^{-\left(\theta_x - \theta_{0x}\right)^2 / 2\Delta \theta_x^2} e^{-\left(\theta_y - \theta_{0y}\right)^2 / 2\Delta \theta_y^2}. \quad (9)$$

We then see from figure 3 that in the $y - z$ plane the angular profile corresponds to a Gaussian mode with $\theta_{\text{end}} = 0$, but in the $x - z$ plane the angular profile deviates from the Gaussian, the more so for the case of the corrected emission angle $\theta_{\text{end}} = 0$. To fully collect this radiation, we thus need to engineer an elliptic lens with appropriate non-circular curvature along the $x$ direction.

The total probability of radiation emitted into the free-space spatial mode $\mathcal{E}(r)$ subtending the solid angle $\Delta \Omega = \pi \Delta \theta_x \Delta \theta_y$ is $P_{\Delta \Omega} \simeq 0.74$. This probability can be increased by optimizing the geometry of the sample, e.g. making it narrower and longer, as discussed above. Alternatively, we can enhance the collection efficiency of the coherently emitted radiation by surrounding the atoms by a moderate finesse, one-sided optical cavity. Assuming a resonant cavity with frequency $\omega_{cav}$, mode function $u_n(r)$ and length $L_0$, the overlap $V = \frac{1}{n} \int |r| \mathcal{E}(r) u_n^*(r) \, dr$ determines the fraction of the radiation emitted by the atomic ensemble into the cavity mode, while the cavity finesse $F$ determines the number of round trips of the radiation, $n \approx F/2\pi$, and thereby the number of times it interacts with the atoms, before it escapes the cavity. The probability of coherent emission of radiation by $N$ atoms into the cavity output mode is then $P_{\text{out}} \simeq \frac{|V|^2 n N}{|\mathcal{F}| nN + 4\pi}$. 

![Figure 2](figure2.png)
4. Conclusions

We have proposed a scheme for coherent microwave to optical conversion of a photon of a superconducting resonator using an ensemble of atoms trapped on a superconducting atom chip. The converted optical photon with tailored temporal and spatial profiles can be fed into a waveguide and sent to a distant location, where the reverse process in a compatible physical setup can coherently convert it back into a microwave photon and, e.g. map it onto a superconducting qubit.

In our scheme, the atoms collectively interact with the microwave cavity via a strong, dipole-allowed Rydberg transition. We have considered the conversion of at most one microwave photon to an optical photon, for which the interatomic Rydberg–Rydberg interactions are absent. In the case of multiple photons, however, the long-range interatomic interactions will induce strong non-linearities accompanied by the suppression of multiple Rydberg excitations within the blockade volume associated with each photon \[39, 40\]. This can potentially hinder the microwave photon conversion and optical photon collection due to distortion of the temporal and spatial profile of the emitted radiation.

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