Bayesian inference for ammunition demand based on Gompertz distribution

ZHAO Rudong1,2, SHI Xianming1,*, WANG Qian3, SU Xiaobo1,4, and SONG Xing5

1. Department of Equipment Command and Management, Shijiazhuang Campus, Army Engineering University, Shijiazhuang 050003, China; 2. Unit 73127 of the PLA, Fuzhou 350503, China; 3. The Ninth Comprehensive Training Base of Army, Zhangjiakou 075000, China; 4. Army Infantry College of the PLA, Shijiazhuang 050003, China; 5. Unit 68303 of the PLA, Golmud 816000, China

Abstract: Aiming at the problem that the consumption data of new ammunition is less and the demand is difficult to predict, combined with the law of ammunition consumption under different damage grades, a Bayesian inference method for ammunition demand based on Gompertz distribution is proposed. The Bayesian inference model based on Gompertz distribution is constructed, and the system contribution degree is introduced to determine the weight of the multi-source information. In the case where the prior distribution is known and the distribution of the field data is unknown, the consistency test is performed on the prior information, and the consistency test problem is transformed into the goodness of the fit test problem. Then the Bayesian inference is solved by the Markov chain-Monte Carlo (MCMC) method, and the ammunition demand under different damage grades is gained. The example verifies the accuracy of this method and solves the problem of ammunition demand prediction in the case of insufficient samples.

Keywords: ammunition demand prediction, Bayesian inference, Gompertz distribution, system contribution, Markov chain-Monte Carlo (MCMC) method.

DOI: 10.23919/JSEE.2020.000035

1. Introduction

Demand prediction is the basic work of ammunition support decision. Accurately predicting the ammunition demand is the key to the precise ammunition supply, and it is also the focus of current ammunition support research. Many experts have researched on ammunition demand prediction and provided guidance for ammunition support work. Wang et al. [1] established an ammunition effectiveness model and a target damage simulation model, and developed an ammunition consumption prediction model and system simulation platform based on damage to enemy firepower. Zhi et al. [2] proposed a method to calculate the average ammunition consumption required for ship-to-air missile damage targets. It can calculate the average ammunition consumption required for a ship-to-air missile to damage a single target, an evacuated target, or a dense target. Song et al. [3] established a hybrid target optimal firepower allocation scheme based on the most favorable firepower distribution method of a single target, and established a calculation model of the minimum ammunition consumption. The above methods are based on traditional ammunition, and fully grasp the ammunition consumption data. The relevant information on the new ammunition has not been fully grasped, and the demand for new ammunition can only be predicted based on the existing small amount of data.

In the case of insufficient samples of new ammunition demand, this study decides to use Bayesian inference to solve the demand prediction problem. In other areas, Bayesian inferences about the prior distribution of the Gompertz distribution have made some progress. In the study of Shahrastani [4], the E-Bayesian and hierarchical Bayesian of the scalar parameter of a Gompertz distribution under Type II censoring schemes was estimated based on fuzzy data. Alizadeh et al. [5] used the Bayesian method to obtain estimates of Gompertz distribution parameters under three different loss functions. Bayesian inference requires to calculate the posterior estimators, usually expressed as complex multidimensional integrals, which are difficult to find in most practical applications. The Markov chain-Monte Carlo (MCMC) method proposed by scholars at home and abroad has properly solved this problem. Miguel et al. [6] developed a simulation method based on the Dirichlet process and the Gibbs sampler to estimate the posterior distribution of the main parameters of the model. Martino et al. [7] introduced the adaptive in-
dependent sticky MCMC algorithm into the Bayesian inference solution, and effectively extracted samples from the probability density function of any bounded target. The more iterations, the closer the probability density is to the target. Martino et al. [8] also proposed different multiple try metropolis schemes, ensemble MCMC method, particle metropolis-hastings algorithm and delayed rejection metropolis technique to solve the posterior estimator of Bayesian inference.

Based on the research mentioned above, this paper fully considers the distribution rule of ammunition consumption under different damage grades and the lack of new ammunition consumption data. The Bayesian inference model based on Gompertz distribution is established and solved by the MCMC method. And the feasibility and accuracy of this method are verified by examples. It provides methodological guidance for the statistical analysis of data in the case of insufficient samples.

2. Bayesian inference model

2.1 Posterior distribution acquisitions of prior information

In order to ensure that the field test data are not overwhelmed by historical data, this paper uses the credibility of the pre-test information to measure the difference between the two. The degree of consistency between the distribution of simulation data, the distribution of expert experience data and the distribution of parameters to be estimated is reflected by the credibility of prior information.

Suppose there are two subsamples, X and Y. To test whether they belong to the same population, a selection hypothesis is given: \( H_0 \) indicates that X and Y belong to the same population, \( H_1 \) indicates that X and Y do not belong to the same population, and \( A \) indicates the event of adopting \( H_0 \).

Credibility of Bayesian inference results is often judged by the credibility of prior information. The higher the credibility is, the more credible the inference result is. The credibility is expressed as \( P(H_0|A) \). The higher the \( P(H_0|A) \) value, the higher the credibility of the pre-test information, and the more accurate the Bayesian inference results [9]. \( P(H_0|A) \) is obtained by Bayesian formula:

\[
P(H_0|A) = \frac{P(A|H_0)P(H_0)}{P(A|H_0)P(H_0) + P(A|H_1)P(H_1)} = \frac{P(A|H_0)P(H_0)}{P(A|H_0)P(H_0) + P(A|H_1)[1 - P(H_0)]}\]  

(1)

Before the ammunition demand is determined, the target damage grades must be clarified, and the target damage grades can be determined based on changes in its combat effectiveness. Based on the analysis of target physical damage, functional damage, and combat effectiveness damage, classification criteria of damage grades in [10] are used to classify the target damage grades into five categories, as described in Table 1.

| Damage grade | Damage degree description |
|--------------|---------------------------|
| Zero damage \( L_1 \) | The target is intact or slightly damaged, and the overall combat effectiveness of the target is less than 5%. |
| Mild damage \( L_2 \) | The target is relatively lightly damaged. If it is not repaired in time, it will affect the performance of the combat technology, and the overall combat effectiveness of the target will be lost by 5% to 20%. |
| Moderate damage \( L_3 \) | The target is severely damaged, and special repairs and replacement parts are required. The overall combat effectiveness of the target is lost by 20% to 50%. |
| Severe damage \( L_4 \) | The target is severely damaged, and needs to be returned to the factory for overhaul, the repair cycle is long, and the overall combat effectiveness of the target is lost by 50% to 80%. |
| Destruction \( L_5 \) | The target cannot be repaired or has no repair value, and the overall combat effectiveness of the target is lost by more than 80%. |

Suppose \( \theta \) represents the ammunition demand when each damage grade is reached, \( \theta_i \) represents the ammunition demand when the \( i \)th damage grade is reached, and \( \theta^{Γ_i} \) represents the ammunition demand when the \( i \)th data source reaches each damage grade.

Ammunition demands \( \theta_1, \theta_2, \ldots, \theta_n \) at each damage grade are discrete random variables. Multi-source data such as simulation data and expert experience data are used as prior information samples. Under different data sources, the ammunition demands when the target reaches each damage grade are \( \theta^{Γ_1} = (\theta^{Γ_1}_1, \theta^{Γ_1}_2, \ldots, \theta^{Γ_1}_n) \), and \( \theta^{Γ_2} = (\theta^{Γ_2}_1, \theta^{Γ_2}_2, \ldots, \theta^{Γ_2}_n) \), \( \ldots \), \( \theta^{Γ_n} = (\theta^{Γ_n}_1, \theta^{Γ_n}_2, \ldots, \theta^{Γ_n}_n) \) respectively.

The amount of ammunition consumed at each grade of damage is continuously processed, and the points of discrete distribution are fitted to a distribution curve. The distribution of \( \theta^{Γ_1}, \theta^{Γ_2}, \ldots, \theta^{Γ_n} \) can be obtained by the statistical analysis of the ammunition demand \( \theta^{Γ_1}, \theta^{Γ_2}, \ldots, \theta^{Γ_n} \) under each damage grade presented by different source data. Through the simulation data and expert experience data analysis, it is found that the ammunition demand at different damage grades obeys the Gompertz distribution. The growth trend of the ammunition demand when reaching each damage grade is slow in the beginning, fast in the middle, and slow in the later period, \( \theta^{Γ_1}, \theta^{Γ_2}, \ldots, \theta^{Γ_n} \) obey the Gompertz distribution, with parameters \( c_Γ, \lambda_Γ \) respectively, which is expressed as \( \text{Gomp}(c_Γ, \lambda_Γ) \) \( (i = 1, 2, \ldots, n) \).

The prior probability density function of the ammuni-
The combat unit on the comprehensive operational capability is used to solve the weights.

The higher the reliability is, the greater the system contribution degree is the weight of the Bayesian fusion.

The reliability of ammunition demand data is evaluated. The system analyzing the information from different sources, the reliability of ammunition demand is measured by the system contribution to the reliability. This paper introduces the system contribution to the reliability. How to integrate multi-source information with appropriate methods is a hot topic of current research. The posterior probability density with single prior information has been obtained by using the Bayesian formula above. Extend it to the general case and solve the posterior distribution function based on multi-source prior information. This paper introduces the system contribution to the Bayesian fusion.

### 2.2 Weight computing model based on system contribution

The degree of system contribution refers to the influence of the combat unit on the comprehensive operational capability of the combat system. The degree of influence of multi-source prior information on the prediction system of the ammunition demand is measured by the system contribution. The impact is mainly determined by its reliability. By analyzing the information from different sources, the reliability of ammunition demand data is evaluated. The system contribution degree is the weight of the Bayesian fusion. The higher the reliability is, the greater the system contribution degree is. The intuitionistic fuzzy membership function is used to solve the weights. \( \Omega = \{ \Gamma_1, \Gamma_2, \ldots, \Gamma_n \} \) indicates possible sources of ammunition demand data. \( \gamma^d_m \) (\( m = \Gamma_1, \Gamma_2, \ldots, \Gamma_n \)) indicates the membership degree which experts \( \Gamma_d \) (\( d = 1, 2, \ldots, n \)) believe the source of ammunition demand data \( \Gamma_1, \Gamma_2, \ldots, \Gamma_n \) is unreliable. Intuitionistic fuzzy evaluation information from experts \( \Gamma_d \) (\( d = 1, 2, \ldots, n \)) on the reliability of ammunition demand data \( \Gamma_1, \Gamma_2, \ldots, \Gamma_n \) is expressed [12] as

\[
\Phi^d_{\Gamma_i} = \{ \Gamma_i, (\gamma^d_{\Gamma_i}, \eta^d_{\Gamma_i}) \},
\]

among which \( 0 \leq \gamma^d_{\Gamma_i} \leq 1, 0 \leq \eta^d_{\Gamma_i} \leq 1 \), and \( 0 \leq \gamma^d_{\Gamma_i} + \eta^d_{\Gamma_i} \leq 1 \).

Intuitive indicators indicate the degree of hesitation that experts believe the source of ammunition demand data \( \Gamma_1, \Gamma_2, \ldots, \Gamma_n \) is reliable or not. Half the degree of hesitation is used to revise the weight. The intuition indicator expression is

\[
\tau^d_{\Gamma_i} = 1 - \gamma^d_{\Gamma_i} - \eta^d_{\Gamma_i}.
\]

The membership degree is expressed as

\[
\varphi_{\Gamma_i} = \frac{1}{n} \sum_{d=1}^{n} \gamma^d_{\Gamma_i} - \frac{1}{n} \sum_{d=1}^{n} \eta^d_{\Gamma_i} + \frac{1}{n} \left( 1 - \frac{1}{n} \sum_{d=1}^{n} \gamma^d_{\Gamma_i} - \frac{1}{n} \sum_{d=1}^{n} \eta^d_{\Gamma_i} \right).
\]

Normalization processing is

\[
\varphi^*_{\Gamma_i} = \frac{\varphi_{\Gamma_i}}{\sum_{i=1}^{n} \varphi_{\Gamma_i}}.
\]

Based on the multi-source information \( \Omega = \{ \Gamma_1, \Gamma_2, \ldots, \Gamma_n \} \), the weight is determined by the contribution degree. The weighted prior probability density function is obtained.

\[
f(\theta; c, \lambda) = \sum_{i=1}^{n} \varphi^*_{\Gamma_i} f(\theta^*_{\Gamma_i}; c, \lambda).
\]

Bayesian fusion formulas are rewritten as

\[
f(c, \lambda | \theta) = \frac{f(\theta^*_{\Gamma_i}; c_{\Gamma_i}, \lambda_{\Gamma_i}) f(\theta; c, \lambda)}{m(\theta^*_{\Gamma_i})},
\]

among which \( \sum_{i=1}^{n} \varphi_{\Gamma_i} = 1 \).

\[
\int \int f(\theta; c, \lambda) f(\theta|c, \lambda)dc\,d\lambda = m(\theta) \text{ is a fixed value and is not affected by the parameters.}
\]
2.3 Bayesian statistical inference output

Combined with (4), (10) and (11), the posterior probability density function is

\[
f(c, \lambda | \theta) = \sum_{i=1}^{n} \varphi_{f_i} m(\theta_{f_i}) f(c, \lambda | \theta_{f_i}) = \sum_{i=1}^{n} \varphi_{f_i} m(\theta_{f_i}) f(c, \lambda | \theta_{f_i}).
\]  

Equation (12) shows that the fusion posterior density is weighted by the posterior density of \( \theta_{f_i} \), \( \theta_{f_2}, \ldots, \theta_{f_n} \). This process fuses field ammunition demand data with prior information.

In order to simplify the calculation, \( \frac{f(\theta | c, \lambda)}{m(\theta)} \) in (4) can be replaced by the likelihood function. The likelihood function [13] is expressed as

\[
L(\theta; c, \lambda) = \lambda^n c \sum_{i=1}^{m} \frac{c^{e_{f_i}}}{\exp\left(-\frac{\lambda}{c} \sum_{i=1}^{n} (c^{e_{f_i}} - 1)\right)}.
\]

The weighted posterior probability density function is

\[
f(c, \lambda | \theta) = \sum_{i=1}^{n} \varphi_{f_i} f(\theta_{f_i}; c_{f_i}, \lambda_{f_i}) L(\theta; c, \lambda).
\]  

3. Model solution

3.1 Consistency tests of prior information

In the testing and appraisal of weapons and equipment, due to the limitations of objective conditions such as test costs, the method of the experimental analysis of completely relying on the field test information can no longer meet the actual needs.

With the maturity of the simulation technology, researchers pay more attention to the use of simulation data for scientific statistical decision-making research in the case of insufficient samples. Since the data are simulation data of the actual situation, the credibility of the results and the difference with the field test data will directly affect the rational use of subsequent test methods. Therefore, before using the simulation data and empirical data, it is necessary to verify the consistency with the field test data, that is, the consistency test. The prior distribution \( F(\theta) \) is completely known, and the field test distribution \( F'(\theta') \) is unknown. At this time, the consistency test problem is transformed into a goodness-of-fit test to verify whether samples \( \theta_1, \theta_2, \ldots, \theta_n \) obey distribution \( F(\theta) \).

Prior information such as simulation data and expert experience data obeys the Gompertz distribution with parameters \( c \) and \( \lambda \). That is, the amount of ammunition used to reach zero damage, mild damage, moderate damage, severe damage and destruction is a group of samples, hereinafter referred to as the amount of ammunition required to reach each damage grade. The distribution function and the probability density function are

\[
F(\theta_{f_i}; c_{f_i}, \lambda_{f_i}) = 1 - \exp\left\{ -\frac{\lambda_{f_i}}{c_{f_i}} (e^{c_{f_i} \theta_{f_i}} - 1) \right\},
\]

\[
f(\theta_{f_i}; c_{f_i}, \lambda_{f_i}) = \lambda_{f_i} e^{c_{f_i} \theta_{f_i}} \exp\left\{ -\frac{\lambda_{f_i}}{c_{f_i}} (e^{c_{f_i} \theta_{f_i}} - 1) \right\}.
\]

In order to judge whether the field test data, that is, the ammunition demand at each damage grade, satisfies the above distribution, the empirical distribution function (EDF) type goodness-of-fit test is used [14]. The test model is expressed as

\[
\left\{ \begin{array}{ll}
H_0 : F_n(\theta) = F(\theta) \\
H_1 : F_n(\theta) \neq F(\theta)
\end{array} \right.
\]

\( H_0 \) indicates that the simulation data and the expert experience data pass the consistency test on the field test data, and \( H_1 \) is the opposite.

Suppose \( \theta_1, \theta_2, \ldots, \theta_n \) are random samples of the ammunition demand distribution function \( F'(\theta') \) drawn from the field test, and its empirical distribution \( F_n(\theta) \) is defined as

\[
F_n(\theta) = \frac{1}{n} \sum_{i=1}^{n} I\{\theta_i < \theta\} = \frac{\# \{ \theta_i < \theta, i = 1, \ldots, n \} }{n}
\]

where \( I\{\cdot\} \) is an indicative function and \( \# X \) is the number of elements in the set \( X \). \( F_n(-\infty) = 0, F_n(\infty) = 1 \). The empirical distribution function of \( n \) ordered statistics \( \{\theta_i\}_{i=1}^{n} \) is represented [15] by

\[
\overline{F}_n(\theta) = \begin{cases} 
0, & \theta < \theta_1 \\
\frac{k}{n}, & \theta_i \leq \theta < \theta_{i+1}; n = 1, 2, \ldots, n-1 \\
1, & \theta \geq \theta_n 
\end{cases}
\]

To measure the difference between \( F_n(\theta) \) and \( F(\theta) \), the Kolmogorov distance is introduced, indicating the maximum distance between \( F_n(\theta) \) and \( F(\theta) \) in the vertical direction, denoted as \( K_n \) and expressed as

\[
K_n = \sup_{\theta \in \Re} |F_n(\theta) - F(\theta)|.
\]

In the actual calculation, \( K_n \) is expressed as

\[
\overline{K} = \max_{1 \leq i \leq n} |\overline{F}_n(\theta_i) - F(\theta_i)|.
\]
The significance level $\alpha$ of $\bar{K}$ is defined as

$$\alpha = P(K_n > \bar{K}) = \Phi \left( \frac{\sqrt{n} + 0.12 + \frac{0.11}{\sqrt{n}} \bar{K}}{\bar{K}} \right)$$

(21)

where $\Phi(x) = 2 \sum_{i=1}^{\infty} (-1)^{i-1} \exp(-2i^2 x^2)$. When $\alpha$ is less than a given significance level of 0.5, the null hypothesis $H_0$ is rejected, otherwise accepted. Through consistency testing, it provides a reliable source of data for the next Bayesian inference.

3.2 Basic principle of the MCMC method and solution process

The MCMC method [16] belongs to the Monte Carlo method. This method introduces the Markov process into the Monte Carlo simulation, directly simulates the posterior probability distribution, and makes the stable samples that converge to the posterior distribution and analyzes the properties of the posterior distribution function. The MCMC method is applied to the posterior probability distribution function to obtain samples of parameters $c$ and $\lambda$.

The MCMC method is used to sample the posterior distribution function and convenient implementation. The MCMC method is used to sample the posterior distribution in Bayesian inference. The method first extracts the random samples that converge to the posterior distribution and counts them, and then analyzes the properties of the posterior distribution. The posterior estimation of the model parameters obtained by the MCMC method can simplify the calculation to some extent.

For any given stationary distribution $\pi(x)$ of a Markov chain, the MCMC method uses the Metropolis-Hastings algorithm [17–19]. The posterior probability density of the ammunition demand at each damage grade is

$$f(c, \lambda | \theta) = \sum_{i=1}^{n} \varphi_{r_i} f(\theta^{r_i}; c_{r_i}, \lambda_{r_i}) L(\theta; c, \lambda).$$

The MCMC method is applied to the posterior probability density function to obtain samples sequences of $c$ and $\lambda$.

According to the MCMC method, the posterior estimation of the Gompertz ammunition demand prediction model parameters is calculated [20–24]. The specific steps are as follows:

Step 1 $k = 0$.

Step 2 Determine the initial values of $c$ and $\lambda$, and select the estimated values $c^{(0)} = \hat{c}$ and $\lambda^{(0)} = \hat{\lambda}$ of the parameters obtained from the original sample as the initial values.

Step 3 Construct the Markov chain transfer function. The parameters $c$ and $\lambda$ are independent of each other. Under the conditions of $c^{(k)}$ and $\lambda^{(k)}$, the following transfer functions are constructed for $c$ and $\lambda$ respectively:

$$y_c = c^{(k)} + \varepsilon_c,$$

(22)

$$y_\lambda = \lambda^{(k)} + \varepsilon_\lambda$$

(23)

where $\varepsilon_c$ and $\varepsilon_\lambda$ obey the normal distribution $N(0, \sigma_c)$ and $N(0, \sigma_\lambda)$ of parameters $\sigma_c$ and $\sigma_\lambda$ respectively. Determine $\sigma_c$ and $\sigma_\lambda$ during the sampling process by observation.

Extract random number $u$ from uniform distribution $[0, 1]$. The transferred value is determined according to (24).

$$c^{(k+1)} = \begin{cases} y_c, & u \leq \frac{\pi(y_c, y_\lambda)}{\pi(c^{(k)}, \lambda^{(k)})} \\ c^{(k)}, & u > \frac{\pi(y_c, y_\lambda)}{\pi(c^{(k)}, \lambda^{(k)})} \end{cases}$$

(24)

The determination method of $\lambda^{(k+1)}$ is the same as $c^{(k+1)}$.

Step 4 $k = k + 1$.

Step 5 Determine whether the traversal mean $\frac{1}{k} \sum_{i=1}^{k} c^{(i)}$ and $\frac{1}{k} \sum_{i=1}^{k} \lambda^{(i)}$ are converged or not. If they do not converge, jump to Step 3, otherwise turn to Step 6.

Step 6 Continue to iterate $M$ times according to Step 3 and Step 4 to obtain the Markov chain $[c^{(k+1)}, c^{(k+2)}, \ldots, c^{(k+M)}], [\lambda^{(k+1)}, \lambda^{(k+2)}, \ldots, \lambda^{(k+M)}]$ corresponding to parameters $c$ and $\lambda$.

Step 7 On each of the corresponding Markov chains of $c$ and $\lambda$, samples are taken at intervals. Collect $n$ points each, which are $c_i$ and $\lambda_i$, $i \in \{1, \ldots, n\}$. According to (21), since the parameters $c$ and $\lambda$ are independent of each other, the expected value of the parameters can be obtained from the average of the $n$ sample points.

$$E_c[\pi(c, \lambda | \theta)] = \frac{1}{n} \sum_{i=1}^{n} c_i,$$  

(25)

$$E_\lambda[\pi(c, \lambda | \theta)] = \frac{1}{n} \sum_{i=1}^{n} \lambda_i,$$  

(26)

where $i = 1, \ldots, n$.

After obtaining the estimated values of the parameters $c$ and $\lambda$, the posterior probability density function and the posterior distribution function of the ammunition demand amount at each damage grade are determined. In turn, the amount of ammunition consumed at each grade of damage is determined.

3.3 Induction of ammunition demand prediction process

In order to facilitate understanding, this section summarizes the above-mentioned ammunition demand prediction process. The prediction process is shown in Fig. 1 and the specific prediction process is shown in the following example.
4. Application example

In recent years, a large number of new ammunition units have been installed. In order to test the tactical and technical performance of ammunition, the army organized a new ammunition field test at a shooting range. In this case, a new type of suppressed weapon ammunition is selected as the research object. The value of the new ammunition is relatively large, and the number of actual tests is small. The amount of ammunition required to reach each grade of damage is not evident. In the case of insufficient samples, in order to further determine the ammunition demand when each damage grade is reached, this example uses the Bayesian inference method.

According to the analysis of expert experience data, the prior probability density function of the ammunition demand at each damage grade is

\[
f(\theta^X; c^X, \lambda^X) = 0.97 \exp\{-\exp[-0.08(\theta - 20.46)]\}.
\]  

(27)

According to the above data, the origin software can be used to fit the ammunition demand curve when the target reaches different damage grades, as shown in Fig. 2. The image is more intuitive to see the trend of the ammunition demand under different damage grades, and it is verified that it obeys the Gompertz distribution.

![Fig. 2 Ammunition demand curve under different damage grades obtained from expert experience data](image)
When the damage grade is zero damage $L_1$, the OEL of the target equipment is less than 5%, and it is determined that there is no ammunition demand at this time. Through the maximum likelihood estimation of the simulation data, ammunition demand of the target equipment to achieve damage at all grades can be obtained, as shown in Table 4. And the ammunition demand curve is shown in Fig. 3.

$$f(\theta^Y; c_Y, \lambda_Y) = 0.85 \exp\left\{-\exp\left[-0.12(\theta - 18.13)\right]\right\}.$$  \hspace{1cm} (28)

According to the analysis of ammunition demand data of the field test in Table 5, the probability density function of the ammunition demand at each damage grade is

$$f(\theta^S; c_S, \lambda_S) = 0.89 \exp\left\{-\exp\left[-0.1(\theta - 19.02)\right]\right\}.$$  \hspace{1cm} (29)

The ammunition demand curve when the target reaches different damage grades in the field test is shown in Fig. 4.

According to simulation data, the prior probability density function of the ammunition demand at each damage grade is

$$f(\theta^Y; c_Y, \lambda_Y) = 0.85 \exp\left\{-\exp\left[-0.12(\theta - 18.13)\right]\right\}.$$  \hspace{1cm} (28)

According to the analysis of ammunition demand data of the field test in Table 5, the probability density function of the ammunition demand at each damage grade is

$$f(\theta^S; c_S, \lambda_S) = 0.89 \exp\left\{-\exp\left[-0.1(\theta - 19.02)\right]\right\}.$$  \hspace{1cm} (29)

The ammunition demand curve when the target reaches different damage grades in the field test is shown in Fig. 4.

According to simulation data, the prior probability density function of the ammunition demand at each damage grade is

$$f(\theta^Y; c_Y, \lambda_Y) = 0.85 \exp\left\{-\exp\left[-0.12(\theta - 18.13)\right]\right\}.$$  \hspace{1cm} (28)

According to the analysis of ammunition demand data of the field test in Table 5, the probability density function of the ammunition demand at each damage grade is

$$f(\theta^S; c_S, \lambda_S) = 0.89 \exp\left\{-\exp\left[-0.1(\theta - 19.02)\right]\right\}.$$  \hspace{1cm} (29)

The ammunition demand curve when the target reaches different damage grades in the field test is shown in Fig. 4.

According to simulation data, the prior probability density function of the ammunition demand at each damage grade is

$$f(\theta^Y; c_Y, \lambda_Y) = 0.85 \exp\left\{-\exp\left[-0.12(\theta - 18.13)\right]\right\}.$$  \hspace{1cm} (28)

According to the analysis of ammunition demand data of the field test in Table 5, the probability density function of the ammunition demand at each damage grade is

$$f(\theta^S; c_S, \lambda_S) = 0.89 \exp\left\{-\exp\left[-0.1(\theta - 19.02)\right]\right\}.$$  \hspace{1cm} (29)

The ammunition demand curve when the target reaches different damage grades in the field test is shown in Fig. 4.

According to simulation data, the prior probability density function of the ammunition demand at each damage grade is

$$f(\theta^Y; c_Y, \lambda_Y) = 0.85 \exp\left\{-\exp\left[-0.12(\theta - 18.13)\right]\right\}.$$  \hspace{1cm} (28)

According to the analysis of ammunition demand data of the field test in Table 5, the probability density function of the ammunition demand at each damage grade is

$$f(\theta^S; c_S, \lambda_S) = 0.89 \exp\left\{-\exp\left[-0.1(\theta - 19.02)\right]\right\}.$$  \hspace{1cm} (29)

The ammunition demand curve when the target reaches different damage grades in the field test is shown in Fig. 4.
\( \alpha = 0.500 \text{ 162.} \) When it fits with simulation data, the significance level is \( \alpha = 0.661 \text{ 42.} \) Both are more than 0.5, so the null hypothesis \( H_0 \) is accepted, the field test data are subject to the Gompertz distribution, and the sample satisfies the consistency.

The current data on ammunition demand data are of two types: one is the expert experience value obtained from a small amount of the past field test; the other is the simulation data obtained by computer software. Next, use the system contribution to determine the fusion weight of the two. The specific practices are shown in Table 6 and Table 7.

| Number | \( \gamma_X^d \) | Membership degree | \( \eta_X^d \) | Membership degree |
|--------|-----------------|-------------------|-----------------|-------------------|
| 1      | \( \gamma_X^1 \) | 0.5               | \( \eta_X^1 \) | 0.2               |
| 2      | \( \gamma_X^2 \) | 0.6               | \( \eta_X^2 \) | 0.3               |
| 3      | \( \gamma_X^3 \) | 0.5               | \( \eta_X^3 \) | 0.2               |
| 4      | \( \gamma_X^4 \) | 0.4               | \( \eta_X^4 \) | 0.5               |
| 5      | \( \gamma_X^5 \) | 0.3               | \( \eta_X^5 \) | 0.5               |
| 6      | \( \gamma_X^6 \) | 0.5               | \( \eta_X^6 \) | 0.2               |
| 7      | \( \gamma_X^7 \) | 0.6               | \( \eta_X^7 \) | 0.3               |
| 8      | \( \gamma_X^8 \) | 0.7               | \( \eta_X^8 \) | 0.4               |
| 9      | \( \gamma_X^9 \) | 0.4               | \( \eta_X^9 \) | 0.5               |
| 10     | \( \gamma_X^{10} \) | 0.7               | \( \eta_X^{10} \) | 0.1               |

Table 6: Expert rating of empirical data reliability

| Number | \( \gamma_Y^d \) | Membership degree | \( \eta_Y^d \) | Membership degree |
|--------|-----------------|-------------------|-----------------|-------------------|
| 1      | \( \gamma_Y^1 \) | 0.5               | \( \eta_Y^1 \) | 0.3               |
| 2      | \( \gamma_Y^2 \) | 0.4               | \( \eta_Y^2 \) | 0.5               |
| 3      | \( \gamma_Y^3 \) | 0.7               | \( \eta_Y^3 \) | 0.2               |
| 4      | \( \gamma_Y^4 \) | 0.5               | \( \eta_Y^4 \) | 0.4               |
| 5      | \( \gamma_Y^5 \) | 0.7               | \( \eta_Y^5 \) | 0.2               |
| 6      | \( \gamma_Y^6 \) | 0.6               | \( \eta_Y^6 \) | 0.3               |
| 7      | \( \gamma_Y^7 \) | 0.8               | \( \eta_Y^7 \) | 0.2               |
| 8      | \( \gamma_Y^8 \) | 0.4               | \( \eta_Y^8 \) | 0.5               |
| 9      | \( \gamma_Y^9 \) | 0.6               | \( \eta_Y^9 \) | 0.4               |
| 10     | \( \gamma_Y^{10} \) | 0.5               | \( \eta_Y^{10} \) | 0.2               |

Table 7: Expert rating of simulation data reliability

According to (13), (14) and normalization, the system contribution of the two can be determined. Fusion weights are \( \varphi_X = 0.52, \varphi_Y = 0.48. \)

The weighted prior probability density is

\[
f(\theta; c, \lambda) = 0.52 \times 0.97 \exp\{- \exp[-0.08(\theta - 20.46)]\} + 0.48 \times 0.85 \exp\{- \exp[-0.12(\theta - 18.13)]\}.
\] (30)

According to the Bayesian formula, the probability density of the fusion is obtained by means of the MCMC method:

\[
f(c, \lambda | \theta) = \frac{1}{\Gamma(c, \lambda)} \exp\{- \Gamma(c, \lambda) \theta\} L(\theta; c, \lambda) = 0.52 \times 0.97 \exp\{- \exp[-0.08(\theta - 20.46)]\} + 0.48 \times 0.85 \exp\{- \exp[-0.12(\theta - 18.13)]\}.
\]

According to the ammunition demand obtained by the Bayesian inference, the ammunition demand curve in this case can be determined, as shown in Fig. 5.

In Fig. 6, the circle represents the ammunition demand when the target reaches different damage grades in the field test; the triangle represents the ammunition demand at different damage grades obtained from the simulation data; the square represents the ammunition demand at different damage grades obtained from expert experience data; the five-pointed star represents the ammunition demand when the target reaches different damage grades under the Bayesian inference.

In order to more intuitively compare the similarity between expert experience data, simulation data, Bayesian inference data and field test data, we summarize the contents of Fig. 2 to Fig. 5 in Fig. 6. It can be seen from Fig. 6.
that compared with the expert experience data and the computer simulation data, the Bayesian inference value of the ammunition demand at different damage grades is closer to the actual value.

It is not difficult to find through Fig. 7 that the Bayesian inferred value of the ammunition demand is less distinct from the true value, and the relative error can be within a reasonable range. The relative errors are detailed in Table 8. The Bayesian inference method for the ammunition demand based on Gompertz distribution and MCMC is effective and feasible. The advantage of this method is the fact that it can solve the problem of ammunition demand prediction in the case of insufficient samples.

![Graph](image)

**Fig. 7 Comparison of the actual value of ammunition demand with the Bayesian inference value**

With a small amount of real ammunition consumption data, the ammunition demand when the target reaches each damage grade can be obtained. This method is particularly suitable for the demand prediction of two types of ammunition. One is the new type of ammunition for the newly installed troops, and the other is the expensive ammunition.

| Damage grade     | Real ammunition demand | Ammunition demand from Bayesian inference | Relative error |
|------------------|------------------------|------------------------------------------|----------------|
| Zero damage $L_1$ | 0                      | 0                                        | 0              |
| Mild damage $L_2$ | 6.752                   | 6.494 0.752                              | 0.038 25       |
| Moderate damage $L_3$ | 15.401 8                | 15.596 0.105                             | 0.012 661      |
| Severe damage $L_4$ | 24.230 3                | 24.852 0.628                             | 0.025 691      |
| Destruction $L_5$  | 41.536 0                | 42.653 0.117                             | 0.026 904      |

**Table 8 Comparison of actual ammunition demand with Bayesian inference**

5. Conclusions

The Bayesian inference method based on Gompertz distribution and MCMC for the ammunition demand proposed in this paper has fully considered the ammunition consumption rule under different damage grades and the characteristic of fewer modern ammunition consumption data. It will provide a reference for the statistical analysis of data in the case of deficient samples. The main tasks are as follows:

(i) The consumption rule of new ammunition under different damage grades has been studied. It has been found that the ammunition demand obeys the Gompertz distribution. The growth trend of the ammunition demand when reaching each damage grade is slow in the beginning, fast in the middle, and slow in the later period. From mild damage to moderate damage, moderate damage to severe damage, the ammunition demand increases significantly.

(ii) The fusion weight calculation model based on the system contribution degree has measured the influence of multi-source information on the ammunition demand under different damage grades, and the weight has been given according to the reliability degree of multi-source information.

(iii) Through the goodness-of-fit test of prior information, the field test data has not been overwhelmed by a large amount of prior data. The step has provided a reliable source of data for the next Bayesian inference.

(iv) The Bayesian fusion formula has been used to fuse the ammunition demand information from diverse sources. The MCMC method has been used to determine the posterior probability density of the ammunition demand. Thus, the amount of ammunition consumed when the target reaches each damage grade has been determined.

In the next study of ammunition demand prediction, it is necessary to fully understand the tactical and technical performance of new ammunition and uncertainties affecting the ammunition consumption. The next step is to properly classify the target damage grade based on the consideration of various indicators of the fresh ammunition. It is expected that the proposed method will be more suitable for different ammunitions and achieves the purpose of precise guarantee of ammunition.

**References**

[1] WANG G Y, SHI Q, YOU Z F. Study on the simulation method of target damage of ground artillery fire strike cluster equipment. Acta Armamentari Sinica, 2016, 37(1): 36 – 43. (in Chinese)

[2] ZHI Y L, DOU J H. Evaluation of the required bomb consumption for air missile damage targets. Command Control & Simulation, 2016, 38(5): 82 – 84. (in Chinese)

[3] SONG X E, SONG W D, ZHAO C W, et al. A method for solving hybrid target fire distribution and ammunition consump-
SHAHRASTANI Y S. Estimating E-Bayesian and hierarchical Bayesian of scalar parameter of Gompertz distribution under type II censoring schemes based on fuzzy data. Communications in Statistics — Theory and Methods, 2019, 48(4): 831–840.

ALIZADEH M, BENKHelifA L, RASEKH M, et al. The odd log-logistic generalized Gompertz distribution: properties, applications and different methods of estimation. Communications in Mathematics and Statistics. DOI:10.1007/s40304-018-00175-y.

MIGUEL G, CRISTINA G, RODRIGO M. Non-parametric Bayesian inference through MCMC method for Y-linked two-sex branching processes with blind choice. Journal of Statistical Computation and Simulation, 2018, 88(18): 1–23.

MARTINO L, CASARIN R, LEISEN F, et al. Adaptive independent sticky MCMC algorithms. EURASIP Journal on Advances in Signal Processing, 2018, 2018: 5.

MARTINO L. A review of multiple try MCMC algorithms for signal processing. Digital Signal Processing, 2018, 75: 134–152.

FENG J, PAN Z Q, SUN Q, et al. Reliability information fusion method for small sub-complex systems and its application. Beijing: Science Press, 2015. (in Chinese)

CHEN X, LI L. Evaluation of target damage effect under uncertain information environment. Systems Engineering and Electronics, 2013, 35(4): 777–780. (in Chinese)

XU T X, LIU Y, ZHAO J Z, et al. Fusion method of maintainability prior information. Systems Engineering and Electronics, 2014, 36(9): 1887–1892. (in Chinese)

MEN G T. Application of intuitionistic fuzzy reasoning for armed police equipment support command decision. Journal of Armed Police University, 2016, 32(2): 41–45. (in Chinese)

ADAMM L. The moments of the Gompertz distribution and maximum likelihood estimation of its parameters. Scandinavian Actuarial Journal, 2014, 2014(3): 255–277.

YANG Z H, CHENG W H, ZHANG J J. Goodness of fit test. Beijing: Science Press, 2011. (in Chinese)

ZHAO Z J, QIANG F F, LI W, et al. Blind estimation of pseudo-code of NPLSC-DSSS signal using goodness of fit test. Journal of Electronics & Information Technology, 2017, 39(3): 749–753. (in Chinese)

SOLIMAN A A, ABD ELLAH A H, ABOU-ELHEGGAG N A, et al. A simulation-based approach to the study of coefficient of variation of Gompertz distribution under progressive first-failure censoring. Indian Journal of Pure & Applied Mathematics, 2011, 42(5): 335–356.

ADAMM L, TRIFON I M. Goodness-of-fit tests for the Gompertz distribution. Communications in Statistics — Theory and Methods, 2016, 45(10): 2920–2937.

WU J W, HUNG W L, TSAI C H. Estimation of the parameters of the Gompertz distribution under the first failure-censored sampling plan. Statistics: A Journal of Theoretical and Applied Statistics, 2010, 37(6): 517–525.

ISMAILA A. Bayes estimation of Gompertz distribution parameters and acceleration factor under partially accelerated life tests with type-I censoring. Journal of Statistical Computation and Simulation, 2010, 80(11): 1253–1264.

LI J, ZHAO Y J, LI D H. A time delay estimation algorithm based on Markov chain Monte Carlo. Acta Physica Sinica, 2014, 63(1): 1–7. (in Chinese)

LIU J S, XIA Q. Bayesian statistical method based on MCMC algorithm. Beijing: Science Press, 2016. (in Chinese)

DOSS H, PARK Y. An MCMC approach to empirical Bayes inference and Bayesian sensitivity analysis via empirical processes. The Annals of Statistics, 2018, 46(4): 1–35.

BLASCO A. A scope of the possibilities of Bayesian inference + MCMC. BLASCO A. Bayesian data analysis for animal scientists. Berlin: Springer, 2017: 167–192.

NEMETH C, SHERLOCK C. Merging MCMC subposteriors through Gaussian-process approximations. Bayesian Analysis, 2018, 13(2): 507–530.

SHI X M, ZHAO R D, LUO X Y, et al. Research on ammunition conversion coefficient of plateau cold region based on fuzzy BP network. Fire Control & Command Control, 2020, 45(2): 59–63. (in Chinese)

**Biographies**

**ZHAO Rudong** was born in 1995. He received his Bachelor’s degree in management from Changchun Institute of Technology in 2017 and Master’s degree in management from the Army Engineering University in 2019. His research interests are ammunition support and equipment management.

E-mail: zrd13376475476@126.com

**SHI Xianming** was born in 1975. He received his B.E. degree from the Academy of Ordnance Engineering in 1996, M.E. degree from the National University of Science and Technology in 2002, and Ph.D. degree from National University of Science and Technology in 2006. He is currently an associate professor in Shijiazhuang Campus of Army Engineering University. His research interests are equipment support and systems engineering.

E-mail: x.m.shi@126.com

**WANG Qian** born in 1989. He received his B.E. degree from the University of Defense Science and Technology in 2011 and M.S. degree in military science from the Army University of Engineering in 2019. He is a lecturer in the Ninth Comprehensive Training Base of Army. His research interest is equipment support.

E-mail: 18003131595@163.com

**SU Xiaobo** was born in 1978. He received his B.E. degree from the Armored Corps College of Engineering in 2001, and M.S. degree in military science from the Academy of Ordnance Engineering in 2009. He is pursuing his Ph.D. degree at the Army Engineering University. He is a lecturer in the Army Infantry College of the PLA. His research interest is equipment support.

E-mail: giantsu030700@sina.com
SONG Xing was born in 1991. He received his B.E. degree from the Academy of Ordnance Engineering in 2013, and M.S. degree in military science from the Army University of Engineering in 2019. His research interest is equipment management. E-mail: flying506@163.com