Model-independent Study on Magnetic Dipole Transition in Heavy Quarkonium

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Abstract.

Some new results on magnetic dipole (M1) transitions in heavy quarkonium from nonrelativistic effective field theories of QCD are briefly reported. This model-independent approach not only facilitates a systematic and lucid way to investigate the relativistic corrections, it also clarifies some inconsistent treatment in previous potential model approach. The impact of our formalism on \(J/\psi \rightarrow \eta_c \gamma\), \(\Upsilon(\Upsilon') \rightarrow \eta_b \gamma\) and \(h_c \rightarrow \chi_c \gamma\) are discussed.

Radiative transitions in heavy quarkonium are of considerable experimental and theoretical interest [1]. On the theory side, it provides us with further insight on the dynamics of quarkonium in addition to the knowledge we have gleaned from the spectra of \(c\bar{c}\) and \(b\bar{b}\) families.

Being an old subject, radiative transitions have been extensively studied in phenomenological models, notably the potential model approach [2]. It is certainly desirable to study them from a model-independent perspective. In this talk I will report such a study based on the effective-field-theory (EFT) approach [3]. Since M1 transition is theoretically cleaner and more interesting than E1 transition, I will focus on the former case, despite the fact that the latter is observed more copiously in nature.

In the nonrelativistic limit, the M1 transition rate between two \(S\)-wave onia takes a particularly simple form:

\[
\Gamma[n^3S_1 \rightarrow n'1S_0 + \gamma] = \frac{4\alpha_{\text{em}} e_Q^2}{3m^2} (1 + \kappa_Q)^2 k^3 \gamma \left| \int dr r^2 R_{n'0} R_{n0} \right|^2,
\]

where \(e_Q\) and \(\kappa_Q\) are the electric charge and anomalous magnetic moment of the heavy quark, and \(R_{nl}(r)\) is the radial Schrödinger wave functions. Since the leading dipole operator \(c_F e e_Q/2m \sigma \cdot B^{\text{em}}\) (where \(c_F = 1 + \kappa_Q\)) only flips the spin and doesn’t act on the spatial degrees of freedom, orthogonality of radial wave functions guarantees that: in the allowed transition \((n = n')\), the overlap integral equals 1; in the hindered transition \((n \neq n')\), the overlap integral vanishes.

It is well known that (1) overpredicts the observed \(J/\psi \rightarrow \eta_c \gamma\) transition rate by a factor of \(2 \sim 3\), with a normal input of \(m_c\) and \(\kappa_c\) from perturbative one-loop matching. This clearly indicates large relativistic corrections to (1), as is usually confronted in charmonium system. It has also been speculated that low-energy fluctuations may generate a large negative \(\kappa_c\), so the discrepancy can be reduced.
The $\mathcal{O}(v^2)$ corrections to (1) has been available for a long time from potential model approach [1,2]:

$$\Gamma[nS \to n'S + \gamma] = \frac{1}{2J_n + 1} \frac{4\alpha_{em}^2 Q^2}{m^2} k^2 \sum I_i^2,$$  \hspace{1cm} (2)

where $2J_n + 1$ counts number of polarizations of the parent onium, and

$$I_1 = \langle n'0 \mid (1 + \kappa Q) \left(1 - \frac{k^2 r^2}{24}\right) + (1 + 2\kappa Q) \frac{k \gamma}{4m} \mid n0 \rangle,$$  \hspace{1cm} (3)

$$I_2 = -\langle n'0 \mid (1 + \kappa Q) \frac{p^2}{2m^2} + \frac{p^2}{3m^2} \mid n0 \rangle,$$

$$I_3 = \langle n'0 \mid \frac{\kappa Q r V'_0}{6m} \mid n0 \rangle,$$

$$I_4 = \pm \frac{4}{E^{(0)}_{n0} - E^{(0)}_{n'0}} \langle n'0 \mid (1 + \kappa Q) \frac{V_s}{m^2} \mid n0 \rangle,$$

$$I_5 = \langle n'0 \mid -\frac{\eta}{m} V_S \mid n0 \rangle,$$

where $V_0$ stands for the static potential, the “+/-” sign in $I_4$ is associated with $^3S_1 \to ^1S_0$ and $^1S_0 \to ^3S_1$, respectively. Notice that $I_4$ accounts for the first-order correction to the wave function due to spin-spin potential (other higher dimensional local potentials cease to contribute in $S$-wave transition), thus is only present in hindered transition. $I_5$ is a prediction specific to the popular assumption in potential models, where one usually decomposes the confining potential into a Lorentz scalar and a vector part, $V_{\text{conf}} = \eta V_S + (1 - \eta) V_V$. This term constitutes the major uncertainty in the potential model predictions, where contradictory claims often appear in the literature [2].

Presence of a hierarchy of scales in quarkonium, $m, mv, mv^2, \Lambda_{\text{QCD}}$, makes the EFT approach an ideal tool to analyze this transition process. One first descends from QCD to NRQCD by integrating out hard modes ($\sim m$) [4], then descends from NRQCD to potential NRQCD (pNRQCD) by further integrating out soft modes ($\sim mv$) [5]. As a result, the inter-quark potentials appear as Wilson coefficients in pNRQCD, and the only dynamical degrees of freedom of pNRQCD are ultrasoft modes ($\sim mv^2$) (for convenience, one can also incorporate the radiated ultrasoft photon into pNRQCD).

A primary task of the EFT approach is to validate/invalidate(2). The real strength of the EFT approach is, however, that it can further answer the following questions that are beyond the scope of potential models:

- Is it possible that a large correction to $\kappa Q$ due to soft modes arises when one descends from NRQCD to pNRQCD?
- Is it possible to reproduce $I_5$ in pNRQCD? If so, how to interpret it?
- Potential model focus exclusively on the $Q\overline{Q}$ Fock-state (with an exception of coupled-channel effects which may not be relevant as long as one excludes those states close to the open-flavor threshold). pNRQCD allows one to include ultrasoft
Invariance (RPI), or essentially Poincare invariance, notably some of the Wilson coefficients are related with each other because of reparameterization and these operators, and these RPI relations can be utilized to condense the expressions.

In fact, the answers to these three questions are all negative \[5\], on which we will elaborate shortly. It turns out that the pNRQCD formalism is able to justify \(\Theta(1/m^3)\) except \(I_5\). In the so-called weak-coupling regime (when \(mv \gg \Lambda_{QCD}\)), this formula is complete; in the so-called strong-coupling regime (when \(mv \sim \Lambda_{QCD}\)), however, \(\Theta(1/m^3)\) is incomplete and further terms are needed.

Before going on to explanations, it is useful to first sketch the derivation of \(\Theta(1/m^3)\) corrections in the framework of pNRQCD. One obvious source is from the contribution of higher dimensional \(M1\) operators. These operators can be identified most conveniently by promoting the color gauge group of NRQCD Lagrangian to a larger gauge group \(SU_c(3) \times U_{em}(1)\). Explicitly, the relevant magnetic operators up to \(\Theta(1/m^3)\) read \[6\]:

\[
\mathcal{L}_{NR} = \psi^\dagger \left( iD_0 + \frac{D^2}{2m} \right) \psi + \frac{c_{F\epsilon e Q}}{2m} \psi^\dagger \mathcal{B}^\text{em} \psi + \frac{i c_{S\epsilon e Q}}{8m^2} \psi^\dagger \mathcal{B}^\text{em} \left[ \nabla \times \mathbf{E}^\text{em} \right] \psi
\]

\[
+ \frac{c_{W_{\epsilon e Q}}}{8m^3} \psi^\dagger \left\{ \nabla^2, \mathcal{B}^\text{em} \right\} \psi - \frac{c_{W_{\epsilon e Q}}}{4m^3} \psi^\dagger \mathcal{B}^\text{em} \nabla_i \psi + \nabla_i \mathcal{B}^\text{em} \nabla_i \psi
\]

\[
- \frac{c_{p'_{\epsilon e Q}}}{8m^3} \left[ \nabla \psi^\dagger \mathcal{B}^\text{em} \cdot \nabla \psi + \nabla_i \mathcal{B}^\text{em} \mathcal{B}^\text{em} \cdot \nabla_i \psi \right] + (\psi \to i \sigma^2 \chi^+) \tag{4}
\]

where \(D_\mu \equiv \partial_\mu + igT^a A_\mu^a + i e e_Q A_\mu^\text{em}\). Various Wilson coefficients can be computed through perturbative matching at the hard scale. For example, at one loop accuracy, \(\kappa_Q = C_F \alpha_s / 2\pi\), is about a few percent for charm and bottom. It is well known that some of the Wilson coefficients are related with each other because of reparameterization invariance (RPI), or essentially Poincare invariance, notably \(c_S = 2C_F - 1\), \(c_{W_1} = C_W - 1\) and \(c_{p'_{\epsilon}} = C_F - 1\) \[3\]. When inherited into pNRQCD, one make replacement \(\nabla \to \mathbf{i} \mathbf{p}\) in these operators, and these RPI relations can be utilized to condense the expressions.

There are other sources of \(\Theta(1/m^3)\) corrections, for instance, multipole expansion of the photon field. One interesting contribution, first pointed out by Groth and Sebastian \[2\], is the Lorentz boost effect due to the final-state recoil. Since the wave function of a moving \(S\)-wave state has a non-vanishing overlap with a \(P\)-wave, spin-flipped state at rest, the \(M1\) transition can be effectively realized by a “E1” transition from the parent to this small component. Some subtlety arises in this effect, namely the recoil correction depends on which “E1” operator one uses, i.e., \(2 e e_Q / m \mathbf{p} \cdot \mathbf{A}^\text{em}\) or \(e e_Q \mathbf{r} \cdot \mathbf{E}^\text{em}\), which are connected by a redefinition of pNRQCD field. The solution to this problem is as following. The matching from NRQCD generates a new \(M1\) operator at \(\Theta(1/m^3)\), which is intimately related to the non-local (depending on the center-of-mass momentum) spin-orbit potential, with a coefficient proportional to \(V_{LS}^\text{CM} = -V_0'/8r\) (Gromes relation). The contribution of this operator also depends on the field redefinition. However, the sum of these two corrections is convention-independent, thus comprises a meaningful relativistic recoil effect.

We are now in a position to discussing general matching of \(M1\) operators from NRQCD to pNRQCD. At \(\Theta(1)\), those operators in \[4\] are trivially inherited to pNRQCD. In general, after integrating out soft modes, new Wilson coefficients will depend on inter-quark separation \(r\).
FIGURE 1. Typical NRQCD diagrams responsible for matching of the dipole operator at \( \mathcal{O}(1/m^0) \). The left diagram represents the vertex correction, and the right one contributes to the wave-function renormalization factor \( Z^s(r) \) due to soft modes, where the cross-cap implies insertion of a unit operator.

Obviously the correction to \( c_F \) can be interpreted as a multiplicative \( \mathcal{O}(1/m^0) \) matching coefficient of the leading \( M_1 \) operator. Dimensional considerations require that this correction is function of \( \log(r) \). We emphasize in passing that it is inappropriate to attribute this new coefficient to the magnetic moment of an individual quark, since it really arises from entangled contributions from both quarks.

The simple answer, no corrections at all, is basically nothing more than the heavy quark spin symmetry and that the \( \sigma \cdot B \) operator behaves like a unit operator in spatial Fock space. These facts are independent of whether the matching is performed perturbatively as in weak-coupling regime, or nonperturbatively, as in strong-coupling regime. In an arbitrary NRQCD diagram depicting transition, as shown in Fig. 1a), we only need to consider the case where the external ultrasoft photon is attached to one of the heavy quark lines. (photon attached to internal light quark loops can be neglected when summing over electric charges of three light flavors). Soft gluons attached to heavy quark lines must be longitudinal to avoid \( 1/m \) suppression. Because all the propagators and vertexes are spin-independent except the \( M_1 \) vertex, such a diagram can factorize into the leading \( M_1 \) operator times minus the wave function renormalization constant \( \delta Z^s(r) \), which can be extracted from the same diagram with the electromagnetic vertex replaced by a unit operator insertion, as shown in Fig. 1b). Therefore, there is no net contribution to this matching coefficient from soft modes.

The matching at \( \mathcal{O}(1/m^2) \) becomes much more cumbersome. We will not dwell on the explicit expressions for these case. However, it should be kept in mind that they may be neglected in the weak-coupling regime because of suppressions by higher pow-
ers of $\alpha_s \sim v$. In the strong-coupling regime, since $\alpha_s \sim 1$, these operators might be as important as other $\mathcal{O}(v^2)$ corrections. These corrections involve some unknown nonperturbative Wilson loop amplitudes, so that predictive power is unavoidably damaged.

The color-octet effect to radiative transitions was first envisaged by Voloshin long time ago, but without a detailed study [7]. pNRQCD allows a systematic treatment of this effect. The corresponding diagrams are shown in Fig. 2. It is not surprising that the color-octet effect is nearly absent in $M_1$ transition, because of the exactly same reason as before. It is interesting to note this bears some resemblance as absence of leading nonperturbative correction to $B \to D^*$ form factor at the zero recoil, known as Luke’s theorem [8]. Here we only give a heuristic argument based on the quantum-mechanical perturbation theory. Treating the chromo-E1 interaction as perturbation up to second order, one can schematically express a “true” quarkonium state as superpositions of the following Fock components (stripping off spin d.o.f.):

$$|N\rangle = \sqrt{Z_{us}^n}|Q_1(n)\rangle + |Q_8 g\rangle + \sum_{m \neq n}|Q_1(m)\rangle \ldots,$$

where $|Q_1(n)\rangle$ is the unperturbed state, and $Z_{us}^n$ is the wave function renormalization factor for state $n$ due to ultrasoft gluons. Spin-independence of chromo-E1 interaction, plus $M_1$ operator being a unit operator in coordinate space, imply that the full $M_1$ transition amplitude is nothing but the inner product $\langle N'|N\rangle$. Since orthogonality condition is preserved in perturbed states, it is equal to $\langle n'|n\rangle$, the color-octet contribution thus vanishes. We emphasize that sizable color-octet effect might arise in $E_1$ transition, since the $E_1$ operator is no longer a unit operator in coordinate Fock space.

Finally we turn to the phenomenological implication of (2), with the understanding that we restrict only to the weakly-coupled system for consistency and the unphysical $I_5$ term has been dropped. Thus far, the only observed $M_1$ transitions are $J/\psi(\psi') \to \eta_c \gamma$, and upper bounds on $\Upsilon(\Upsilon'') \to \eta_b \gamma$ have recently been set by CLEO [10]. Empirically, $\Upsilon(1S)$ and $\eta_b$ are believed to lie in the weak-coupling regime, whereas $J/\psi$, $\eta_c$ fit in this regime to a less extent. $\Upsilon'$ and $\Upsilon''$ are usually regarded as strongly-coupled system. As for $\psi'$ and $\eta_c'$, they are too close to the open-flavor threshold and cannot be correctly described by current formulation of pNRQCD, therefore we exclude them in our analysis.

For a weakly-coupled system, the dynamics is largely governed by the perturbative static potential, i.e., $V_0 \approx -C_F\alpha(\mu)/r$, where the natural choice of $\mu$ is around the

**FIGURE 2.** pNRQCD diagrams for color-octet contribution to radiative transition. Single and double lines correspond to the singlet and octet fields. The singlet-octet-gluon vertex is of chromo-E1 type.
typical three-momentum scale. One also needs to specify the quark mass in (2). A naive input is to use \( \hat{m} \), half of the center-of-gravity ground state mass. However, this simple procedure may induce an error of order \( v^2 \), which shouldn’t be neglected according to power counting. A more consistent way is to choose the 1S mass \( m \), which is defined implicitly through \( \hat{m} = m - \langle 10 | \vec{p}^2 / 2m | 10 \rangle \approx m(1 - C^2_F \alpha_s(\mu)^2 / 8) \).

Many of terms in (2) are practically negligible. Since \( \kappa_Q \) retains its NRQCD value and is only a few per cent for charm and bottom, we may simply put it to be zero in those \( \mathcal{O}(v^2) \) terms (as a result, \( I_3 \) can be dropped in both allowed and hindered transitions). In fact, only \( I_2 \) in (2) accounts for genuine \( \mathcal{O}(v^2) \) correction for allowed transition, and we end up with a simple expression

\[
\Gamma[J/\psi \to \eta_c \gamma] \approx \frac{4\alpha_em_e^2 \alpha_s^2(\mu)}{3\hat{m}^2_c} \left[ 1 + 2\kappa_c - \frac{2C^2_F \alpha_s^2(\mu)}{3} \right].
\]  

Fig. 3 shows a comparison between this formula and the data. It seems that (6) is compatible within the error to the data when \( \mu \) is about 0.8 GeV. Note this value is consistent with the empirical \( mv \) value for \( J/\psi \). Therefore, our reasonable success in describing \( J/\psi \to \eta_c \gamma \) may be viewed as \textit{a posteriori} support for the weak-coupling assignment. However, rather sharp \( \mu \) dependence may suggest that \( J/\psi \) is not far from the strong-coupling regime. One can employ (6) to \( \Upsilon \to \eta_b \gamma \) with more confidence, and finds a smaller \( \mathcal{O}(v^2) \) correction with flatter scale dependence. Unfortunately, very narrow width of 2 ∼ 3 eV makes this transition unlikely to be detected.

For hindered transitions, \textit{e.g.} \( \Upsilon' (\Upsilon''') \to \eta_b \gamma \), the experimental upper bounds are already rather tight and many model predictions have been ruled out \textit{[10]}. To be conservative, one would not expect (2) to reliably describe these transitions if excited bottomonia are indeed in the strong-coupling regime. Nevertheless, it may not be too optimistic to treat, at least \( \Upsilon' \), as being in weak-coupling regime. Proceeding along this line, we find that the \( I_4 \) term is dominating over \( I_1 \) and \( I_2 \), and the latter two nearly cancel each other. As a result, the predicted width is about an order-of-magnitude larger than the experimental upper bound! Even though there are lots of uncertainty associated with \( I_4 \), this alarmingly large discrepancy seems to indicate that the weak-coupling assignment of \( \Upsilon' \) is problematic and a strong-coupling analysis might be more appropriate.
We end with a brief discussion on the $P$-wave $M1$ transitions, which have received less attention in the past, due to lack of phenomenological impetus. Recent discovery of the $h_c$ state [11] may arouse the interest to study these processes. Color-octet effect again vanishes because of the exactly same reason. It is straightforward to apply the pNRQCD formalism to derive the $\mathcal{O}(v^2)$ corrections. For simplicity, we just quote the weak-coupling formula for the allowed transition:

$$\Gamma[n^3P_J \rightarrow n^1P_1 + \gamma] = \frac{4\alpha_{em}e_Q^2}{3m^2} k^3_{\gamma} \left[1 + \kappa_Q - c_J \langle n^1 \left| \frac{p^2}{m^2} \right| n^1 \rangle \right]^2,$$

(7)

where $c_J = 1/2, 1, 4/5$ for $J = 0, 1, 2$. For the $n^1P_1 \rightarrow n^3P_J \gamma$ transition, one simply multiplies the right side of (7) by a statistical factor of $(2J+1)/3$. It should be understood that this formula may be of limited use, since $P$-wave onia may necessarily live in the strong-coupling regime.

The fine splittings between $h_c$ and $\chi_{c0}$, $\chi_{c1}$ and $\chi_{c2}$ are about 110, $-13$, $-32$ MeV, respectively. It seems that only $h_c \rightarrow \chi_{c0}\gamma$ has a serious chance to be observed, with a width comparable to $\Gamma[\psi/\eta_c \rightarrow \eta_c\gamma]$. Future experimental input will enable us to infer the size of relativistic corrections, thus enriching our understanding of $P$-wave onia.

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