Measuring Supersymmetry with Heavy Scalars

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Supersymmetry with heavy scalars is a model where at the LHC we have to rely on rate measurements to determine the parameters of the underlying new physics. For this example we show how to properly combine rate measurements with kinematic endpoints, taking into account statistical, systematic and theory uncertainties. Provided we observe a sizeable number of events the LHC should be able to determine many model parameters with small enough error bars to for example test unification patterns.

I. SETUP

Supersymmetry as a prime candidate for new physics at the TeV scale should be discovered at the LHC even with moderate energy and luminosity [1]. The challenge for complex TeV-scale extensions of the Standard Model is to determine as many model parameters as possible at the TeV scale [2–4] and extrapolate them to higher energy scales. This way, we can study the underlying structures and symmetries of an ultraviolet completion of our Standard Model — up to energy scales which might reach for example the scale of grand unification [5–8].

Most studies which focus on understanding new physics at and above the TeV scale rely on a multitude of kinematic observables. In particular at the LHC kinematic measurements are the most powerful, because they can be extracted in the presence of large QCD and top-pair backgrounds and are less prone to huge QCD corrections. Possible limits to such strategies we have seen in LHC studies of supersymmetry with light sleptons. There, the number of kinematic observables is drastically reduced and the remaining kinematic features do not determine the absolute new-physics mass scales well anymore [9]. The question then becomes how much information we can extract from fewer and less robust observables, including production rates of supersymmetric final states. Two aspects of such measurements mean additional complications: first of all, we do not actually measure a total signal cross section, but a matrix of production rates times branching ratios in the presence of backgrounds and possibly relevant kinematic cuts [10]. Secondly, for such measurements the combination of experimental and theory uncertainties becomes the crucial stumbling block which determines if we can for example test gaugino mass unification at the LHC or not. This situation is somewhat similar to Higgs sector analyses at the LHC [11–13].

In supersymmetry with decoupled scalars (DSS), all scalar partners are decoupled from the relevant mass spectrum for the LHC

\[ m_{\tilde{t}} = m_{\tilde{q}} = m_{H,H^\pm,A} \equiv m_S \geq 10^4 \text{ GeV} \] (1)

This scalar mass spectrum might be very large [14, 15] or simply decoupled from LHC production above \( O(10 \text{ TeV}) \) [16]. The observable spectrum consists of the usual Standard Model fields, the gluino \( \tilde{g} \), the wino \( \tilde{W} \), the bino \( \tilde{B} \) and the higgsino components \( \tilde{H}_{u,d} \). Omitting gauge-invariant kinetic terms and non-renormalizable operators, the Lagrangian of the low energy effective theory reads [15, 17, 18]

\[
\mathcal{L} \supset m^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 - \left[ \kappa_u \bar{q}_u \epsilon H^* + \kappa_d \bar{q}_d H + \kappa_e \bar{\ell} e H \\
+ \frac{1}{2} \left( M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} \right) + \mu \tilde{H} \bar{u} \epsilon \tilde{d} \\
+ \frac{H^\dagger}{\sqrt{2}} \left( \tilde{g}_u \sigma \tilde{W} + \tilde{g}_d \tilde{B} \right) \tilde{H} u + \frac{H \tilde{e}}{\sqrt{2}} \left( \tilde{g}_d \tilde{B} - \tilde{g}_u \sigma \tilde{W} \right) \tilde{H} d + \text{h.c.} \right],
\] (2)

where \( \sigma^a \) are the Pauli matrices (\( \epsilon = i \sigma^2 \)). It includes one light Higgs doublet, tuned to have a small mass \( m \) for
\[ H = -\cos \beta e H^*_u + \sin \beta H_u. \] At the scale \( m_S \), the low-energy effective theory is matched to the full MSSM

\[
\lambda(m_S) = \frac{1}{4} \left[ \frac{3}{2} g_1^2(m_S) + g_2^2(m_S) \right] \cos^2 2\beta + \Delta_{\text{th}},
\]

\[
\kappa_u(m_S) = \frac{y_u^2(m_S)}{\cos \beta}, \quad \kappa_d(m_S) = \frac{y_d^2(m_S)}{\cos \beta}
\]

The four parameters \( \hat{g} \) are the Yukawa couplings of the neutralinos and charginos, which are modified with respect to their supersymmetric values. Since it will be impossible to observe these deviations at the LHC [17] we do not include them in our parameter extraction. \( \Delta_{\text{th}} \) is a threshold correction to the quartic Higgs coupling which change the tree-level Higgs mass \( m_H^2 \)

\[
\Delta_{\text{th}} = \frac{3 y_t^4}{8 \pi^2} \left[ \left( 1 - \frac{2 g_1^2 + g_2^2}{8 y_t^2} \right) \frac{X_t^2}{m_S^2} - \frac{X_t^4}{12 m_S^4} \right],
\]

with \( X_t = A_t - \mu / \tan \beta \) in the same range as the gaugino and higgsino masses. The weak-scale parameter \( \tan \beta \) only appears in the boundary conditions and therefore is not a parameter of the low-energy effective theory. It is interpreted as the fine-tuned angle that rotates the two Higgs doublets into one heavy and one light mass eigenstate [19].

New particles entering the renormalization group running at an intermediate scale \( m_S \) \((m_Z < m_S < M_{\text{GUT}})\) contribute identically to the running of the three gauge couplings provided they compose complete representations of the unification group [20]. All sfermions in the MSSM form complete SU(5) representations, so a possible gauge coupling unification scheme in the MSSM is unchanged by heavy scalars. Experimentally establishing such a pattern is one of the main long-term goals of the LHC.

In our analysis we construct the universal gaugino masses at the GUT scale: \( M_i(M_{\text{GUT}}) = m_{1/2} \). They are then evolved down to the scale \( m_S \) based on one-loop renormalization group equations of the MSSM [21, 22]. The higgsino mass term \( \mu \) is provided as an independent input parameter at the scale \( m_Z \). Below \( m_S \) we integrate out all scalars and run the modified renormalization group equations [15] to the desired scale.

Keeping one of the supersymmetric Higgs bosons light requires a fine tuning which for this study we accept without offering an explanation. The Higgs mass matrix for the two Higgs scalars

\[
\begin{pmatrix}
|\mu|^2 + m_{H_u}^2 & b \\
-b & |\mu|^2 + m_{H_d}^2
\end{pmatrix}
\]

has eigenvalues \( \langle m_H^2 \rangle \pm \sqrt{\Delta^2 + b^2} \) in terms of \( \langle m_H^2 \rangle = (m_{H_u}^2 + m_{H_d}^2)/2 + |\mu|^2 \) and \( \Delta = (m_{H_u}^2 - m_{H_d}^2)/2 \). Requiring the light Higgs mass to be of the order of the weak scale translates into

\[
\sqrt{\Delta^2 + b^2} - m_{\text{ew}}^2 < \langle m_H^2 \rangle < \sqrt{\Delta^2 + b^2}. \tag{6}
\]

The \( b \) term in the Lagrangian density breaks a Pececi-Quinn symmetry and can therefore be kept small, as opposed to \( \Delta \) and \( \langle m_H^2 \rangle \) which should both be of order \( m_S \). A light Higgs mass means that \( \langle m_H^2 \rangle \) ranges around \( m_{\text{ew}}^2 \). Hence, the fraction of the \( (\langle m_H^2 \rangle, \Delta) \) space that satisfies Eq. (6) corresponds to

\[
\frac{V_{\text{tuned}}}{V_{\text{total}}} \sim \frac{m_{\text{ew}}^2 m_{H_d}^2}{m_S^4} \sim \frac{m_{\text{ew}}^2}{m_S^4}. \tag{7}
\]

Heavy scalars leave the chargino and neutralino spectrum untouched, so the lightest neutralino should still be a good dark matter agent [15, 23]. The measured density of dark matter then imposes a constraint on \( \mu \), which unlike in mSUGRA toy models is not determined by electroweak symmetry breaking. The WMAP measurement \( \Omega_{\text{DM}} h^2 = 0.111^{+0.006}_{-0.008} [24] \) can be reproduced in different parameter regions, which will be represented by our choice of reference parameter points:

- the ‘mixed region’ with \( M_1 \approx \mu \) and a mixed higgsino-gaugino LSP. Here, \( \chi_1^0 \chi_1^0 \) annihilation is enhanced for gauge and/or Higgs bosons or top quarks in the final state.
the ‘pure higgsino’ and ‘pure wino’ regions where the LSP is almost mass degenerate with the $\tilde{\chi}_1^\pm$ and the $\tilde{\chi}_2^0$. This leads to an enhanced co-annihilation. This region generally requires an LSP heavier than 1 TeV.

- the ‘Higgs pole’ region in which the LSP is rather light, $m_{\chi_1^0} \approx \frac{1}{2}m_h$ and the annihilation proceeds via a resonant light Higgs.

If gluinos are lighter than squarks, they will mainly decay through virtual squark exchange into quarks and charginos-neutralinos [25]. For very heavy squarks quantum corrections to the gluino decay processes can be significant, because they are enhanced by the large logarithm $m_{\tilde{g}}/m_S$ [26]. If the scalar mass scale is larger than $10^6$ GeV, the gluino becomes sufficiently stable to form $R$–hadrons [27] which can be analyzed at the LHC without major difficulties [17, 28]. Those are subject to cosmological constraints if they affect nucleosynthesis. A gluino with TeV-scale mass must have a lifetime shorter than 100 seconds to avoid altering the abundances of deuterium and lithium-6. This sets an upper limit of $m_S < 10^9$ GeV [29]. Therefore, in this study we will focus on comparably short-lived gluinos.

To quantitatively study supersymmetry with heavy scalars at the LHC we define the two parameter points shown in Table I. They are in agreement with constraints from both dark matter observations and collider searches. A scalar mass scale of 10 TeV is the lowest value which still qualifies as ‘decoupled’ on LHC energy scales. It limits the amount of fine-tuning in the Higgs sector to one part in $10^4$ and gives us a short enough gluino life time to avoid non-standard phenomenology or undesirable cosmological effects.

The most important model parameters are $\mu$ and $m_{1/2}$. They define the mass spectrum as well as the field content of the neutralinos and charginos and most notably the LSP. Our two choices span most of the LHC-relevant parameter space allowed by LEP and WMAP. DSS1 lies in the Higgs pole region, so the LSP is mostly a bino with a non-vanishing higgsino component. DSS2 lies in the ‘mixed region’. The light LSP leads to an invisible Higgs branching ratio around 1%. In DSS1, the gluino, the $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm$ are fairly light and have correspondingly large LHC cross sections, as shown in the Appendix. For the heavier DSS2 spectrum this is no longer the case.

The large value $\beta = 30$ avoids LEP limits on the light Higgs mass. Its only impact on the low-energy theory is on $m_h$. The effect of $A_t$ on $m_h$ is suppressed by at least $1/m_S^2$, so we can as well set $A_t = 0$.

### II. LHC OBSERVABLES

Throughout our analysis we use leading-order and, where available, next-to-leading-order Monte Carlo generators for signal and background processes. The LO-generated samples are normalized to NLO cross sections using $K$–factors. For SUSY and VV events we rely on HERWIG [31] including initial/final state radiation, spin correlations in the decay of heavy states and angular correlations between jets. The $V + j$ events we obtain from ALPGEN [31] including matrix element and parton shower matching [32]. The NLO normalization of these rates is given by MCFM [33] or PROSPINO2 [34]. Top pairs we simulate with MC@NLO [35], including its NLO normalization. The pure QCD jets background we expect to be heavily suppressed by cutting on, e.g. missing energy. Therefore, we leave it at a leading-order PYTHIA simulation [36]. Detector and reconstruction effects we account for with a standard general purpose detector simulation as described in Ref. [18].

Looking at the DSS signatures we rely on standard observables. They are based on the reconstruction of isolated jets with $R = 0.4$ and leptons with $p_T > 20$ GeV. From the missing energy measurement we can determine the effective mass $M_{\text{eff}}$ as a measure of the total activity in the event. It includes the four hardest jets and all identified jets with $R = 0$. From the missing energy measurement we can determine the effective mass $M_{\text{eff}}$ as a measure of the total activity in the event. It includes the four hardest jets and all identified jets with $R = 0$.
leptons. For SUSY events, $M_{\text{eff}}$ scales with the mass of the heavy particles produced and can be used to quantify the mass scale of SUSY events [37, 38]. The transverse sphericity is defined as $S_T = 2 \lambda_2/(\lambda_1 + \lambda_2)$ where $\lambda_i$ are the eigenvalues of the $2 \times 2$ sphericity tensor $S_{ij} = \sum_k p_{ki} p_{kj}$. This tensor we compute using all jets above $p_T = 20$ GeV and all leptons. SUSY events tend to be relatively spherical ($S_T \sim 1$) since the initial heavy particles are usually produced approximately at rest and their cascade decays emit particles in all directions. In the DSS1 parameter point the sphericities are fairly light, resulting in a uniform distribution between $S_T = 0.1$ and 0.6. For QCD jets or $V+$jets events $S_T$ peaks at zero. A cut on $S_T$ does not reduce the $t\bar{t}$ background as the distribution is very similar to the signal.

The most frequent final states occurring for the DSS1/DSS2 parameter point can be classified by the number of leptons: 0$\ell$ (70%/70%), 1$\ell$ (20%/23%), 2$\ell$ (5%/5%), and 3$\ell$ (3%/1%). The number of decay jets can vary from zero in the case of leptonically decaying $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ pairs to eight in the case of gluino pairs. This does not include jets from the underlying event or initial and final state radiation which have to be carefully studied in addition [39]. The most common channel for both points is the zero-lepton channel $g\bar{g} \to \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp + 4$ jets $\to 8$ jets $+ E_T$. We use it to estimate the discovery potential using standard cuts [38]: One hard jet with $p_T > 100$ GeV and three over 50 GeV, no electrons or muons, an effective mass $M_{\text{eff}} > 800$ GeV, missing transverse energy $E_T > 100$ GeV with $E_T > 0.2 \times M_{\text{eff}}$, sphericity $S_T > 0.2$ and finally a geometric separation between jets and the missing energy vector of $\Delta \phi > 0.2$ for the three leading jets. Both for jet mis-measurement and $b$ decays the $E_T$ vector will be close the direction of one jet, so this cut reduces fake missing energy from QCD. Note that for the various LHC observables discussed below this basic set of cuts will be modified.

Table II shows the number of events remaining after cuts. After all but the $M_{\text{eff}}$ cut $t\bar{t}$ is the dominant background, but there are also significant contributions from $V+$jets. Finally, $M_{\text{eff}} > 800$ GeV reduces the background to below the level of the signal for DSS1.

The systematic uncertainties on the number of background events for 1 fb$^{-1}$ we take to be 50% for QCD multi-jets and 20% for $t\bar{t}$ and $V+$jets, $WW$, $WZ$ and $ZZ$. This corresponds to a combination of data-driven and Monte Carlo methods [38].

A discovery of new physics can then be claimed if the number of observed events exceeds 25 and the significance is larger than five. The significance of the observation of DSS1 with an integrated luminosity of 1 fb$^{-1}$ is 18, so this parameter point will be discovered at the LHC within one year of data taking at low luminosity. For DSS2, the significance is low and, for this set of cuts, does not increase with statistics. However, if we require $M_{\text{eff}} > 1$ TeV the significance for 1 fb$^{-1}$ increases to 6.

**Higgs mass**

With decoupled scalars the light Higgs scalar is essentially equivalent to its Standard Model counterpart at 129 GeV. Its mass depends on $M_S$ and on $\tan \beta$. The $M_S$ dependence arises from the running of $\lambda$ from $m_S$ to the weak scale, while $\tan \beta$ appears as $\cos^2 \beta$ in the matching. This can impact the numerical value of $m_h$ by up to 20 GeV and should — like usually in supersymmetric Higgs studies — allow us to determine $\tan \beta$ at the LHC.
The total next-to-leading order production cross section is 39 pb. It is computed by HIGLU for gluon fusion [40], VV2Hf for weak boson fusion and V2Hf for the production in association with a vector boson. This number includes NLO QCD corrections, and for the first one also the NLO electroweak contributions [41]. The dominant decays are into $b \bar{b}$ (53%), $WW^*$ (29%), $\tau \tau$ (5%), and $ZZ$ (4%) [42, 43]. In addition, the branching ratio into photons reaches its maximum of 0.2%, allowing for a precise mass measurement.

Higgs production through supersymmetric cascades only occurs in $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ decays. The corresponding production rates are small, so their contribution to the total Higgs production is of the order of 100 fb, i.e. negligible compared to SM channels.

Systematic uncertainties on the Higgs mass measurement arise from the electromagnetic energy scale. The calibration of the photon energy scale will be derived from $Z \rightarrow e e$ events and $Z \rightarrow \mu \mu \gamma$ events, with an expected accuracy of 0.1%. The statistical uncertainty for an integrated luminosity 100 fb$^{-1}$ should also range around 0.1% [38]. The theory uncertainty due to higher-order corrections to $m_h$ should not exceed the very conservative limit of 4% [44].

**Di-lepton endpoint**

Kinematic endpoints are usually the main ingredients to supersymmetric parameter analyses, due to their small experimental and theory errors [45-47]. Perfect triangular di-lepton edges in cascade decays occur in successive two-body decays, like $\tilde{\chi}_0^2 \rightarrow \ell \ell \rightarrow \tilde{\chi}_1^0 \ell \ell$. Such a measurement directly constrains the gaugino mass parameters $M_1$ and $M_2$ as well as the higgsino mass parameter $\mu$. In contrast, decays via on-shell $Z$ bosons only give us the $Z$ mass peak with little information on the supersymmetric masses in the decay. For our parameter choices, $\chi_0^3$ (DSS1) or both $\tilde{\chi}_3^{0,4}$ (DSS2) as well as the charginos in DSS2 decay through such an on-shell gauge boson [43, 48].

However, we can also apply endpoint techniques to three-body decays: $\tilde{\chi}_0^3 \rightarrow \tilde{\chi}_0^1 \ell + X$ in both points and $\tilde{\chi}_0^3 \rightarrow \tilde{\chi}_0^3 \ell + X$ in DSS2 lead to two leptons with opposite signs and same flavor (OSSF), as listed in the Appendix. On the production side, the $\tilde{\chi}_1^0 \tilde{\chi}_2^0$ channel has little background due to the small number of jets in the final state. To increase the total rate we also include $\tilde{\chi}_0^2$ production from gluino decays. The total available cross section leading to this decay becomes $\sigma(\tilde{\chi}_0^0 \rightarrow \tilde{\chi}_1^0 \ell + X) \approx 3.5$ pb and 93 fb in DSS1 or DSS2 and $\sigma(\tilde{\chi}_0^0 \rightarrow \tilde{\chi}_2^0 \ell + X) \approx 75$ fb in DSS2.

In addition to the staggered jet cuts ($p_{T,j} > 100, 50, ...$ GeV) we now require at least two OSSF electrons or muons with $p_{T} > 20$ GeV, $|\eta| < 2.5$, and $m_{\ell \ell} < m_{\tilde{\chi}_0^0} - m_{\tilde{\chi}_1^0} + 10$ GeV. Since the true value of the endpoint is a priori unknown, this choice implies that the edge has already been observed. To remove combinatorial as well as top backgrounds we apply flavor subtraction. Many backgrounds cancel in the combination

$$\frac{N(e^+e^-)}{\beta} + \beta N(\mu^+\mu^-) - N(e^+\mu^-)$$

where $\beta$ is an efficiency correction factor equal to the ratio of the electron and muon reconstruction efficiencies.

In the DSS2 parameter point, the lowest endpoint corresponds to the mass splitting $m_{\tilde{\chi}_3^{0,4}}$, while the second one corresponds to $m_{\tilde{\chi}_4^{0,4}}$. We fit its distribution with a superposition of three components, two modeling the decay kinematics plus a Breit-Wigner $Z$ line shape. Table III compares the results with the theoretical values. The extracted values are in good agreement with the input values for DSS1 and in reasonable agreement for DSS2. The statistical errors on the mass differences we can extract from the fit to the reference function. Systematic uncertainties are dominated by the lepton energy scale (0.1%), while theory errors due to unknown higher-order contributions are expected to range around a percent.

There might occur doubts if the second edge giving $m_{\tilde{\chi}_4^{0,4}}$ is actually visible, so we check that the curve between the $m_{\tilde{\chi}_3^{0,4}}$ edge and the onset of the $Z$ peak indeed lies 5σ above the background-only prediction.

Assigning these measured values to sparticate mass differences necessitates a few assumptions. In DSS1, lepton pairs are quite frequent with respect to the overall SUSY production. This suggests that the neutralino triggering this decay is somewhat light. In addition, a decay through a 296 GeV $\tilde{\chi}_0^3$ is rather unlikely, and in such a case additional

| theory | fit value | statistical error |
|--------|-----------|-------------------|
| DSS1   | $m_{\tilde{\chi}_0^3} - m_{\tilde{\chi}_1^0}$ | 55.1 | 55.2 | $\pm 0.6 / 10$ fb$^{-1}$ |
| DSS2   | $m_{\tilde{\chi}_0^3} - m_{\tilde{\chi}_1^0}$ | 60.7 | 60.2 | $\pm 2 / 100$ fb$^{-1}$ |
|        | $m_{\tilde{\chi}_0^3} - m_{\tilde{\chi}_1^0}$ | 81.9 | 79.0 | $\pm 3 / 100$ fb$^{-1}$ |

**TABLE III**: Results of the fit to the invariant mass distribution. All values given in GeV.
structure would be seen. In DSS2, with two endpoints and a $Z$ peak, the interpretation is more complicated. In addition to the assumption that the endpoints arise from $\tilde{\chi}_2^0$ decays we have to assume that $\tilde{\chi}_3^0$ decays preferably to $\tilde{\chi}_1^0$. Otherwise, the largest endpoint could correspond to $m_{\tilde{\chi}_{3,2}^0}$ and the $Z$ peak to the decay $\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0 Z$.

\textit{Di-jet endpoint}

Unfortunately, the technique described above is only applicable to decays involving $b$ jets, but not light-flavor jets. In DSS1, 1.7% of gluinos decay to the LSP with two bottom quarks. We select these events by requiring four jets with $E_T > 50$ GeV, no leptons and $E_T > 100$ GeV. For two $b$-tagged jets we compute the invariant mass $m_{bb}$. The background is dominated by $t\bar{t}$ events, as well as combinations due to decays other than $\tilde{g} \rightarrow \tilde{\chi}_1^0 bb$. The fit output from $m_{bb}$ we compare to the theoretical values: for $10$ fb$^{-1}$ the fit value of 380.6 GeV corresponds to the input of 383.0 GeV within the statistical error of 5.2 GeV. This measurement is the basis for the determination of $M_3$ from the gluino mass $M_{\tilde{g}}$.

\textit{Tri-lepton cross section}

In contrast to the usual and more optimistic scenarios \cite{2, 46, 49}, decoupling all scalars at the LHC implies that we will not have enough kinematic information to extract masses and model parameters of the underlying new-physics model. Therefore, we need to rely on cross section measurements, in spite of their larger experimental and theory errors. The main purpose of this analysis is to show how such rate measurements can indeed be used as input to new physics measurements.

As usually, signatures involving leptons have lower LHC backgrounds and increase the precision of the measurement. For heavy supersymmetric scalars charginos and neutralinos will give different final states with numerous isolated leptons. The tri-lepton final state allows for background rejection by requiring two OSSF leptons. It arises from chargino and neutralino production with subsequent decays $\tilde{\chi}_2^+ \rightarrow \tilde{\chi}_1^0 \ell$, $\tilde{\chi}_2^- \rightarrow \tilde{\chi}_1^0 \ell$, $\tilde{\chi}_3^0 \rightarrow \tilde{\chi}_1^0 \ell\nu$. In Table IV we show the composition of the tri-lepton signal in DSS1. Pairs of $\tilde{\chi}_{1,2}^\pm \tilde{\chi}_1^0$ are produced directly as well as in gluino decays.

We select these events requiring at least one OSSF pair and exactly three leptons. In case of direct production, two LSPs will be emitted essentially back-to-back, hence canceling the missing transverse energy, so we lower our cut to $E_T > 50$ GeV. An optional jet veto above $p_T \sim 20$ GeV (dependent on detailed QCD studies) can be applied in order to select events from direct $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ production rather than $\tilde{g}$ pair decays. However, its effect on signal and background rates is hard to predict, so the results should be taken with a grain of salt. To reject $Z$ decays we veto $m_{\ell\ell} = 81.2\ldots102.2$ GeV. For our signal, we expect $m_{\ell\ell} \lesssim 56$ GeV due to the $\tilde{\chi}_2^0$ mass splitting.

In Table V we present the number of events after cuts. Only $t\bar{t}$ and $WW/ZZ$ are significant, where the latter is already partly removed by the $m_{\ell\ell}$ cut while the former can be removed by a jet veto.

An as precise as possible extraction of the number of tri-lepton signal events relies on our knowledge of the backgrounds and a complete understanding of detector effects, luminosity, parton distributions, and finally cut efficiencies. The systematic uncertainty on $\sigma(SUSY \rightarrow 3\ell)$ is certainly bounded from below by the knowledge of the luminosity $L$, \textit{i.e.} of the order of 5% \cite{50}. In order to take into account additional systematic errors we consider cases of 5%, 10% and 20%. The theory uncertainty due to QCD effects we estimate to be of the order of 12% \cite{34}.

\textit{Gluino pair cross section}

The gluino pair channel constitutes a large fraction of the signal events in both parameter points (77% in DSS1 and 22% in DSS2). Different strategies should allow us to select only $\tilde{g}\tilde{g}$ events. However, they will be very model dependent, as they require at least a guess of the gluino decays branching fractions. We can for instance take

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
DSS1 & $\sigma$ & BR & $\sigma(3\ell)$ \\
\hline
direct & 11.7 fb & 1.54% & 180 fb \\
via $\tilde{g}\tilde{g}$ & 10.4 fb & 160 fb \\
\hline
\hline
DSS2 & $\sigma$ & BR & $\sigma(3\ell)$ \\
\hline
direct & 1390 fb & 1.54% & 21.4 fb \\
via $\tilde{g}\tilde{g}$ & 166 fb & 2.6 fb \\
\hline
\end{tabular}
\caption{Cross sections contributing to the tri-lepton signal for the LHC running at 14 TeV.}
\end{table}
advantage of the very short zero-lepton cascade $\tilde{g}\tilde{g} \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 q\bar{q}$ to remove SUSY background from chargino and neutralino channels. Again, the systematic uncertainty is bounded from below by the knowledge of the luminosity, so we consider systematic errors of 5%, 10% and 20%. Theory uncertainties from higher order contributions can be 30%, estimated from a scale variation by a factor $1/4\ldots4$ at next-to-leading order [34]. It is meant to be conservative to also accommodate additional sources of uncertainties, like parton densities and the strong coupling. It can be reduced once we systematically include higher-order QCD corrections to the production of heavy particles [51]. This choice of theory errors will give us a conservative estimate if including rate information at the LHC should become a part of supersymmetric parameter analyses.

On-shell vs off-shell $Z$ bosons

If the mass difference between two neutralinos or two charginos is larger than $m_Z$, the invariant mass distribution of its decay products will exhibit a very sharp peak. In contrast, if the mass difference between the two sparticles is too small, the $m_{\ell\ell}$ distribution will show a triangular shape with a sharp endpoint below $m_Z$. This effect can provide valuable information about the neutralino and chargino masses as well as their couplings to the $Z$ boson.

The measurement of the $\sigma(\text{SUSY} \rightarrow Z)$ rate requires a good knowledge of the luminosity, lepton efficiencies and background rates. Some of these source of systematic uncertainties cancel from the ratio

$$R_Z = \frac{N(m_{\ell\ell} > \text{endpoint})}{N(m_{\ell\ell} < \text{endpoint})}. \quad (9)$$

For our two reference points we find $R_Z < 0.004 \sim 0$ (DSS1) and 0.196 (DSS2), respectively. Systematic errors arise if the $p_T$ spectra of leptons from the $Z$ peak and from the triangular shape are different or if the identification efficiencies for these two spectra are different. However, electron identification efficiencies are reasonably flat for $p_T > 25$ GeV. Leptons from the $Z$ peak will have transverse momenta around 45 GeV, while those under the triangle curve we cut to $p_T > 20$ GeV. We therefore include an overall 1% systematic error to mainly account for lepton identification uncertainties. The theory uncertainty due to the prediction of branching ratios should be of the order of 1% [43, 48].

III. PARAMETER DETERMINATION

Once new physics (e.g. supersymmetry) will be discovered at the LHC, we have to turn our focus towards understanding the corresponding signatures. On the one hand, we will have to test different types of TeV-scale models with the available data, while on the other hand we have to consistently determine the parameters of the underlying theory for each of these model hypotheses. This might well include combining LHC observables with other measurements such as the relic density of dark matter, the magnetic moment of the muon, or flavor physics. Note that a consistent approach does not allow for the replacement of some measurements by top-down predictions in someone’s favorite model. Instead, we need to see how far we can get, for example with the limited set of observables described in the last section.

SFitter [2, 12] is designed to map up to 20-dimensional highly complex parameter spaces onto a large set of observables of varying quality, which can be highly correlated. It can be used to estimate the reach in terms of a given model for any experiment, but also to realize a proper bottom-up approach to determine the parameters of a fundamental theory.

The determination of the parameters then proceeds in two steps. First, we maximize the exclusive log-likelihood using a weighted Markov chain [2] to identify the best-fitting point in parameter space. The starting point of this Markov chain is arbitrary, and we repeat the search several times to ensure our procedure converges well. This

|                          | after cuts | add’l jet veto |
|--------------------------|------------|----------------|
| DSS1                     | 681        | 43             |
| DSS2                     | 87         | 4              |
| tt                       | 1,106      | 59             |
| QCD                      | 0          | 0              |
| W/Z + jets               | 14         | 0              |
| WW/WZ/ZZ                 | 235        | 73             |

TABLE V: Number of tri-lepton events remaining after the cuts discussed in the text (assuming 10 fb$^{-1}$).
minimum then serves as starting point for a MINOS hill-climbing minimization to improve the resolution and to estimate the errors.

In models with decoupled scalars we can use the different LHC observables discussed above to determine the parameters of the model. Table VI summarizes them along with their expected uncertainties for both parameter points. For the mass differences we use the result of detailed experimental analyses, while for the rate-related observables we rely on the theoretical central value, lacking the complete experimental analysis.

By definition, no information on the squark and slepton sector is available except for its explicit absence. Consequently, we fix $m_S$ and $A_t$ to large (nominal) values. The three gaugino mass parameters we fit independently, to allow for a bottom-up experimental test of gaugino mass unification. Technically, we know that for scalar masses a consistent bottom-up approach does not reproduce the usual top-down results, which means we would have to evolve all parameters strictly from the weak scale to the high scale $M_{1067}$ GeV. For gauginos the differences between the two methods are not as large, so for illustration purposes we use a top-down running for the renormalization group equations.

Four scenarios illustrate well the precision we can reach on the determination of supersymmetric model parameters with decoupled scalars. We use about 1000 toy experiments for each scenario and each benchmark point. The toy experiments are generated by smearing the observables according to the expected experimental and/or theoretical errors, depending on the scenarios. Correlations among the measurements are taken into account separately for the energy scale of leptons, jets and the luminosity measurements. For each toy experiment the best-fit parameter set is determined. From the distribution of the best-fit parameter we read off the error as the RMS (Root-Mean-Square) of all parameters, including

$$
\Delta \tan\beta \approx 300 \text{ fb}^{-1}
$$

We see that with roughly a 50% error on $\tan\beta$ we can hardly determine this parameter in both reference points. Because all heavy Higgs bosons are decoupled our only leverage is the light Higgs mass which depends on several parameters, including $m_S$ in the (s)top sector. The better way to study the Higgs sector, possibly including $\tan\beta$, would be a dedicated Higgs analysis at the LHC [12, 13].

The second Higgs-sector parameter $\mu$ is determined to better than 1% in DSS1 and 4% in DSS2. In the former, this is due to large production rates for neutralinos and charginos. In the latter, the higgsino fractions are well spread over all neutralinos, so neutralino mass differences include this information. Thanks to very small statistical uncertainties on all mass differences in DSS1, also the gaugino mass parameters are determined within 2%. This is not the case in DSS2, implying a deterioration to $\sim 10\%$. As discussed above, the observability of the second neutralino edge is arguable. We perform the DSS2 fit with and without this observable and find

| $\Delta \%$ | $\Delta \%$ | $\Delta \%$ | $\Delta \%$ |
|---|---|---|---|
| $\Delta \tan\beta$ | 30 | 40 | 50 | 60 |

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- **EXPerimental errors.** In order to evaluate the impact of the pure experimental uncertainties on the determination of the DSS parameters for a given luminosity of 100 fb$^{-1}$, we take into account the statistical and systematic errors, but we do not include theory errors.

We see that with roughly a 50% error on $\tan\beta$ we can hardly determine this parameter in both reference points. Because all heavy Higgs bosons are decoupled our only leverage is the light Higgs mass which depends on several parameters, including $m_S$ in the (s)top sector. The better way to study the Higgs sector, possibly including $\tan\beta$, would be a dedicated Higgs analysis at the LHC [12, 13].

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- **SYSTematic errors.** Here we assume that a very large number of events has been gathered at the LHC ($\geq 300$ fb$^{-1}$) and that statistical uncertainties are negligible. By that time, we might assume that theoretical
predictions of \(m_h\) and the different cross sections will have rendered the theory error negligible as well. This idealized scenario is useful to measure the impact of systematic errors and the ultimate precision of the parameter determination.

In Table VII we observe that with only systematic errors all parameters are determined within 1%, except for \(\tan \beta\) which still suffers from an invisible extended Higgs sector. Systematic errors on cross section measurements we vary between of 5 and 20%. In DSS1, this hardly affects the parameter determination as all model parameters are already well constrained by mass measurements. In DSS2, a 20% systematic error on \(\sigma(\tilde{g}\tilde{g})\) doubles the uncertainty on \(M_3\) as compared to the case where we are dominated by luminosity measurement (5% systematic error). In contrast, a 20% systematic error on \(\sigma(3\ell)\) again does not affect the parameter determination, because the weak masses are constrained by kinematics measurements.

Compared to the first case including statistical and systematic uncertainties we observe a gain of at least a factor 4 in the errors for DSS2. This shows that for the integrated luminosity of 100 fb\(^{-1}\) assumed in the scenario EXP the statistical error still dominates.

- THeory errors. Again, we assume very large statistics but include theory errors, thus taking into account theoretical and systematic errors. This scenario gives us a flavor of what is achievable after at least five years of operation of the LHC at full luminosity.

With a 4% theory uncertainty on \(m_h\), \(\tan \beta\) is now practically undetermined. The interplay of neutralino mass splittings, \(R_Z\) and the tri-lepton cross section provides enough constraints on the neutralino and chargino sector to allow for a determination of \(M_1\), \(M_2\) and \(\mu\) to better than 4%. In DSS1, \(M_3\) is very well determined by the mass splitting \(m_{\tilde{g}} - m_{\tilde{g}}\). However, in DSS2 the only available handle on \(M_3\) comes from the gluino pair cross section which suffers from a large theoretical uncertainty which we set conservatively to 30%. As the production cross section strongly decreases as function of \(M_3\), the distribution of the toy experiments resulting from the symmetric theoretical error is highly asymmetric with a long tail to large values of \(M_3\) as expected. In spite of the large theoretical error on the cross section prediction, a \(M_3\) measurement significantly better than at the 10% level is feasible.

- FULL errors. In this scenario we combine the experimental errors (EXP) with the theoretical errors (TH) on the observables for 100 fb\(^{-1}\). In DSS1, the errors on the gaugino masses are slightly larger than the errors for the TH scenario. In DSS2 the errors on \(M_3\) and \(M_2\) are essentially the same as the EXP errors, the corresponding mass difference measurements are dominated by the statistical error. For \(M_3\) the experimental errors are fairly small compared to the theoretical error. Nevertheless, the determination remains at the level of several percent due to the steep descent of the cross sections as a function of the gluino mass. Including further theoretical developments is necessary to increase the precision of the parameter determination by reducing the theoretical error [51].

IV. OUTLOOK

Supersymmetry with heavy scalars is a variation of the MSSM which more naturally accommodates for example flavor constraints, at the expense of the solution of the hierarchy problem. For the LHC, it is irrelevant as which energy scale the scalars reside, as long as they are at least of the order of \(10^3\) GeV.

Such a model is a serious challenge to any kind of supersymmetric parameter analysis, because it severely reduces the number of LHC observables, in particular from cascade decay kinematics. Instead, we need to rely for example on rate measurements, including their complex systematic and theory error structure. Of our two parameter points the first one should be discoverable within a year of data-taking at the LHC, due to very large SUSY cross sections. The second point has lower rates but should be discovered within a few years. For both scenarios we establish a set of observables, including statistical, systematic, and theory uncertainties.

For a model with heavy scalars we have shown that a global fit of the model parameters to the experimental observables is still able to determine the correct central values and corresponding errors for all parameters. The weakly interacting sector \((M_1, M_2 \text{ and } \mu)\) can be fairly well measured at the LHC, with conservative accuracies of the order of a few percent after including all error sources. Most notably, this includes all uncertainties related to rate measurements at the LHC. The Higgs sector suffers from the fact that we will only have one observable at hand, namely the light Higgs mass. For such a situation we will have to resort to a dedicated Higgs sector analysis at the LHC [12, 13].

Even for optimistic LHC luminosities and energies the impact of systematic uncertainties is likely not dominant, even though an estimate of these errors prior to a full-fledged analysis on real data should be taken with a grain of
salt. For all theory uncertainties we have relied on particularly conservative estimates, which means that the positive outcome of our study is generally dependable.

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Appendix A: Production rates and branching ratios

In contrast to most supersymmetric parameter analyses for the parameter point SPS1a \[49\], the analysis presented in this paper heavily relies on rate information. Note that ‘rate’ refers to cross sections times branching ratios for signal plus background. Therefore, we list all cross sections as well as all branching ratios computed at NLO in this appendix.

Table VII lists the next-to-leading order cross sections for direct production of supersymmetric particles at the LHC for DSS1 and DSS2 as computed by Prospino2 \[34\]. The individual rate add to 81.6 pb (DSS1) and 4.3 pb (DSS2).

|        | DSS1 | DSS2 |
|--------|------|------|
| $gg$   | 62.8 pb | 954 fb |
| $\tilde{\chi}_1^0$ $\tilde{\chi}_2^0$ | 5.9 pb | 642 fb |
| $\tilde{\chi}_1^-$ $\tilde{\chi}_1^+$ | 18 fb | 38 fb 827 fb |
| $\tilde{\chi}_1^0$ $\tilde{\chi}_2^0$ | 56 fb | 147 fb |

Table VIII lists the higher-order branching fractions of the supersymmetric particles involved. Most gluino decays contain three particles in the final state, proceeding through a very virtual squark. Due to its wino-like nature, the $\tilde{\chi}_2^0$ decays like a $Z$ boson plus missing energy. The larger higgsino component in the DSS2 parameter point brings in decays to charginos. Again, the light chargino decays like a $W$ boson plus missing energy.
Appendix B: Error treatment

As discussed in this paper, the focus on rate measurements in the case of heavy scalars forces us to generally treat the theory uncertainties on rate observables as possibly the dominant error at the LHC. At the same time, some of the production rates are small, so our Markov chain might run into parameter regions with small event numbers which have to be treated using Poisson instead of Gaussian statistics.

Like in all SFitter analyses \[2, 12\] we follow the Rfit scheme \[52\] to combine Gaussian experimental and flat theory errors. This scheme interprets theory errors as a lack of knowledge on a parameter. As long as the deviation between theory and experiment is within the theory error, this must not have any influence on the total likelihood. Once the difference becomes larger the (perturbative) theory is simply ruled out instead of just very, very unlikely. This means that we cannot simply convolute some kind of theory likelihood distribution with a Gaussian experimental error. Even assuming a flat theory likelihood curve would give the difference of two one-sided error functions, i.e. a peaked likelihood. Instead, the combined log-likelihood which can also be derived using a profile likelihood ansatz is given by

\[
-2 \log L = \chi^2 = \begin{cases} 
0 & \text{for } |d_i - \overline{d}_i| < \sigma_i^{th} \\
\left(\frac{|d_i - \overline{d}_i| - \sigma_i^{th}}{\sigma_i^{exp}}\right)^2 & \text{for } |d_i - \overline{d}_i| \geq \sigma_i^{th} \end{cases} \quad (B1)
\]

For large enough event numbers the experimental error is a combination of three different sources, all Gaussian and summed in quadrature. The statistical error is uncorrelated between different measurements. A first systematic error originates from the lepton energy scale and the second from the hadronic energy scale. They are treated separately. Each is taken as 99% correlated between different observables.

For smaller signal numbers systematic and statistical uncertainties are incorporated by convoluting a Poisson probability with a Gaussian probability, where the number of background events \( N_b \) is the mean and \( \delta_b \) (systematic uncertainties) the standard deviation \[53\]. The probability that the background fluctuates at least to the observed number of events \( N_{\text{obs}} = N_{\text{signal}} + N_b \) is

\[
p = A \int_0^{\infty} db \text{ Gauss}(N_b, \delta_b) \sum_{j=0}^{N_{\text{obs}}} \frac{e^{-b}b^j}{j!} \quad (B2)
\]

where \( A \) normalizes the integral. Then the significance of the signal reads \( Z_0 = \sqrt{2} \text{erf}^{-1}(1 - 2p) \). If \( N_{\text{obs}} \) is very large compared to \( N_b \), this significance is approximated by \[54\]

\[
Z_0 = \frac{2}{\sqrt{1 + \delta_b^2/N_b}} \left( \sqrt{N_{\text{obs}} + \frac{3}{8}} - \sqrt{N_b + \frac{3 \delta_b^2}{8 N_b}} \right). \quad (B3)
\]

| \(\bar g\rightarrow\) | \(\chi_1^q\) | \(\chi_2^q\) | \(\chi_3^q\) | \(\chi_4^q\) |
|---|---|---|---|---|
| \(\chi_1^q\) | \(\chi_1^q\) | \(\chi_1^q\) | \(\chi_1^q\) | \(\chi_1^q\) |
| \(\chi_2^q\) | \(\chi_2^q\) | \(\chi_2^q\) | \(\chi_2^q\) | \(\chi_2^q\) |
| \(\chi_3^q\) | \(\chi_3^q\) | \(\chi_3^q\) | \(\chi_3^q\) | \(\chi_3^q\) |
| \(\chi_4^q\) | \(\chi_4^q\) | \(\chi_4^q\) | \(\chi_4^q\) | \(\chi_4^q\) |

**TABLE VIII:** Branching fractions for both scenarios computed by SDECAY \[43, 48\]. Values rounded to the full percentage.
[1] D. E. Morrissey, T. Plehn and T. M. P. Tait, arXiv:0912.3250 [hep-ph].
[2] R. Lafaye, T. Plehn, M. Rauch and D. Zerwas, Eur. Phys. J. C 54, 617 (2008) arXiv:0709.3985 [hep-ph]; for earlier versions of SFitter see: R. Lafaye, T. Plehn and D. Zerwas, arXiv:hep-ph/0404282; R. Lafaye, T. Plehn and D. Zerwas, arXiv:hep-ph/0512028.
[3] P. Bechtle, K. Desch and P. Wienemann, arXiv:hep-ph/0412012; P. Bechtle, K. Desch, W. Porod and P. Wienemann, Eur. Phys. J. C 46, 533 (2006) arXiv:hep-ph/0511006; P. Bechtle, K. Desch, M. Uhlenbrock and P. Wienemann, arXiv:0907.2559 [hep-ph].
[4] S. S. AbdusSalam, B. C. Allanach, F. Quevedo, F. Feroz and M. Hobson, arXiv:0904.2548 [hep-ph].
[5] G. A. Blair, W. Porod and P. M. Zerwas, Phys. Rev. D 63, 017703 (2001); arXiv:hep-ph/0007107. B. C. Allanach, G. A. Blair, S. Kraml, H. U. Martyn, G. Polesello, W. Porod and P. M. Zerwas, arXiv:0403133; G. A. Blair, A. Freitas, H. U. Martyn, G. Polesello, W. Porod and P. M. Zerwas, Acta Phys. Polon. B 36, 3445 (2005) arXiv:hep-ph/0512084.
[6] J. L. Kneur and N. Sahoury, Phys. Rev. D 79, 075010 (2009) arXiv:0808.0144 [hep-ph].
[7] C. Adam, J. L. Kneur, R. Lafaye, T. Plehn, M. Rauch and D. Zerwas, arXiv:1007.2190 [hep-ph].
[8] F. Brummer, S. Fichet, A. Hebecker and S. Kraml, JHEP 0908, 011 (2009) arXiv:0906.2557 [hep-ph].
[9] C. G. Lester, M. A. Parker and M. J. White, JHEP 0601, 080 (2006) arXiv:hep-ph/0508143.
[10] R. K. D. R. Sinha, M. Kröner, J. M. Lindert and B. Over, JHEP 1004, 109 (2010) arXiv:1003.2648 [hep-ph].
[11] R. Kimmunen, S. Lehti, F. Mertig, A. Nijsen and M. Spira, Eur. Phys. J. C 40N. 5, 23 (2005) arXiv:hep-ph/0503075.
[12] R. Lafaye, T. Plehn, M. Rauch, D. Zerwas and M. Duhrsen, JHEP 0908, 009 (2009) arXiv:0904.3866 [hep-ph].
[13] S. Bock, R. Lafaye, T. Plehn, M. Rauch, D. Zerwas and P. M. Zerwas, arXiv:1007.2616 [hep-ph].
[14] N. Arkani-Hamed and S. Dimopoulos, JHEP 0506, 073 (2005) arXiv:hep-th/0405159; N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice and A. Romanino, Nucl. Phys. B 709, 3 (2005) arXiv:hep-ph/0409232.
[15] G. F. Giudice and A. Romanino, Nucl. Phys. B 699, 65 (2004) Erratum-ibid. B 706, 65 (2005) arXiv:hep-ph/0406088.
[16] J. D. Wells, Phys. Rev. D 70, 075006 (2004) arXiv:hep-ph/0406144.
[17] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 170, 377 (2007) arXiv:astro-ph/0603449.
[18] A. Djouadi and Y. Mambrini, Phys. Lett. B 493, 120 (2000) arXiv:hep-ph/0007174; M. Toharia and J. D. Wells, JHEP 0602, 015 (2006) arXiv:hep-ph/0503175.
[19] P. Gambino, G. F. Giudice and P. Slavich, Nucl. Phys. B 726, 35 (2005) arXiv:hep-ph/0506214.
[20] G. R. Farrar and P. Fayet, Phys. Lett. B 76, 575 (1978).
[21] A. C. Kraan, Eur. Phys. J. C 37, 91 (2004) arXiv:hep-ex/0404001; J. L. Hewett, L. Lillie, M. Masip and T. G. Rizzo, JHEP 0409, 070 (2004) arXiv:hep-ph/0408248; M. Fairbairn, A. C. Kraan, D. A. Milstead, T. Sjostrand, P. Skands and T. Sloan, Phys. Rept. 438, 1 (2007) arXiv:0611040.
[22] A. Arvanitaki, C. Davis, P. W. Graham, A. Pierce and J. G. Wacker, Phys. Rev. D 72, 075011 (2005) arXiv:hep-ph/0504210.
[23] G. Corella et al., arXiv:0201201.
[24] M. L. Mangano, M. Moretti, F. Piccinini, R. Pittau and A. D. Polosa, JHEP 0307, 001 (2003) arXiv:hep-ph/0206293.
[25] J. Alwall et al., Eur. Phys. J. C 53, 473 (2008) arXiv:hep-ph/0706.2550 [hep-ph].
[26] J. M. Campbell and R. K. Ellis, Phys. Rev. D 60, 113006 (1999) arXiv:hep-ph/9905386.
[27] W. Beenakker, R. Höcker, M. Spira and P. M. Zerwas, Nucl. Phys. B 492, 51 (1997); arXiv:hep-ph/9610490. W. Beenakker, M. Krämer, T. Plehn, M. Spira and P. M. Zerwas, Nucl. Phys. B 515, 3 (1998); arXiv:hep-ph/9710451. W. Beenakker, M. Klasen, M. Krämer, T. Plehn, M. Spira and P. M. Zerwas, Phys. Rev. Lett. 83, 3780 (1999); arXiv:hep-ph/9906298. T. Plehn, arXiv:hep-ph/9809319.
[28] S. Frixione and B. R. Webber, arXiv:hep-ph/0612227.
[29] T. Sjostrand, S. Mrenna and P. Z. Skands, JHEP 0605, 026 (2006) arXiv:hep-ph/0603175.
[30] D. R. Tovey, Eur. Phys. J. direct C 4, N4 (2002).
[31] G. L. Bayatian et al. [CMS Collaboration], J. Phys. G 34, 995 (2007); G. Aad et al. [The ATLAS Collaboration], arXiv:0901.0512 [hep-ex].
[32] T. Plehn, D. Rainwater and P. Skands, Phys. Lett. B 645, 217 (2007) arXiv:hep-ph/0510144; T. Plehn and T. M. P. Tait, J. Phys. G 36, 075001 (2009) arXiv:0810.3919 [hep-ph]; J. Alwall, S. de Visscher and F. Maltoni, JHEP 0902, 017 (2009) arXiv:0810.5550 [hep-ph].
[33] M. Spira, arXiv:hep-ph/9510347 M. Spira, Fortsch. Phys. 46, 203 (1998) arXiv:hep-ph/9705337.
[34] S. Actis, G. Passarino, C. Sturm and S. Uccirati, Nucl. Phys. B 811 (2009) 182 arXiv:0809.3667 [hep-ph]; Phys. Lett. B 670 (2008) 12 arXiv:0809.1301 [hep-ph]; Phys. Lett. B 669 (2008) 62 arXiv:0809.1302 [hep-ph].
[35] A. Djoudi, J. Kalinowski and M. Spira, Comput. Phys. Commun. 108, 56 (1998) arXiv:hep-ph/9704448.
[43] A. Djouadi, M. M. Mühlleitner and M. Spira, Acta Phys. Polon. B 38, 635 (2007) [arXiv:hep-ph/0609292].
[44] H. E. Haber, R. Hempfling and A. H. Hoang, Z. Phys. C 75, 539 (1997); G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich and G. Weiglein, Eur. Phys. J. C 28, 133 (2003); T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, G. Weiglein and K. Williams, Pramana 69, 861 (2007); P. Kant, R. V. Harlander, L. Mihaila and M. Steinhauser, JHEP 1008, 104 (2010).
[45] H. Bachacou, I. Hinchcliffe and F. E. Paige, Phys. Rev. D 62, 015009 (2000); [arXiv:hep-ph/9907518]; B. C. Allanach, C. G. Lester, M. A. Parker and B. R. Webber, JHEP 0009, 004 (2000) [arXiv:hep-ph/0007009]; B. K. Gjelsten, D. J. Miller and P. Osland, JHEP 0412, 003 (2004); [arXiv:hep-ph/0410303]; B. K. Gjelsten, D. J. Miller and P. Osland, JHEP 0506, 015 (2005) [arXiv:hep-ph/0501033]; C. G. Lester, Phys. Lett. B 655, 39 (2007) [arXiv:hep-ph/0603171].
[46] B. K. Gjelsten, D. J. Miller and P. Osland, JHEP 0506, 015 (2005) [arXiv:hep-ph/0501033].
[47] R. Horsky, M. I. Krämer, A. Mück and P. M. Zerwas, Phys. Rev. D 78, 035004 (2008) [arXiv:0803.2603 [hep-ph]].
[48] M. Mühlleitner, Acta Phys. Polon. B 35, 2753 (2004) [arXiv:hep-ph/0409200].
[49] B. C. Allanach et al., Eur. Phys. J. C 25, 113 (2002) [arXiv:hep-ph/0202233]; G. Weiglein et al., [LHC/LC Study Group], Phys. Rept. 426, 47 (2006) [arXiv:hep-ph/0410364].
[50] CERN-LHCC-2008-004
[51] A. Kulesza and L. Motyka, Phys. Rev. Lett. 102, 111802 (2009) [arXiv:0807.2405 [hep-ph]]; U. Langenfeld and S. O. Moch, Phys. Lett. B 675, 210 (2009) [arXiv:0901.0802 [hep-ph]]; A. Kulesza and L. Motyka, Phys. Rev. D 80, 095004 (2009) [arXiv:0905.4749 [hep-ph]]; M. Beneke, P. Falgari and C. Schwinn, arXiv:0909.3488 [hep-ph]; W. Beenakker, S. Brensing, M. Krämer, A. Kulesza, E. Laenen and I. Niessen, JHEP 0912, 041 (2009) [arXiv:0909.4418 [hep-ph]].
[52] A. Hocker, H. Lacker, S. Laplace and F. Le Diberder, Eur. Phys. J. C 21, 225 (2001) [arXiv:hep-ph/0104062].
[53] R. D. Cousins and V. L. Highland, Nucl. Instrum. Meth. A 320, 331 (1992); J. T. Linnemann, [arXiv:physics/0312059].
[54] S. N. Zhang and D. Ramsden, Experimental Astronomy 1, 145 (1990).