CHARACTERISTIC NUMBERS, JIANG SUBGROUP AND NON-POSITIVE CURVATURE

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Dedicated to Professor Boju Jiang on the occasion of his 85th birthday.

ABSTRACT. By refining an idea of Farrell, we present a sufficient condition in terms of the Jiang subgroup for the vanishing of signature and Hirzebruch’s $\chi_y$-genus on compact smooth and Kähler manifolds respectively. Along this line we show that the $\chi_y$-genus of a non-positively curved compact Kähler manifold vanishes when the center of its fundamental group is non-trivial, which partially answers a question of Farrell. Moreover, in the latter case all the Chern numbers vanish whenever its complex dimension is no more than 4, which also provides some evidence to a conjecture proposed by the author and Zheng.

1. INTRODUCTION AND MAIN RESULTS

Unless otherwise stated, all smooth and complex manifolds considered in this article are compact and connected, and the dimension of a complex manifold is referred to its complex dimension.

Let $X$ be a finite CW-complex and $f$ a self-map of $X$. A homotopy $h_t : X \to X$ ($0 \leq t \leq 1$) is called a cyclic homotopy of $f$ if $h_0 = h_1 = f$. In this case the trace $h_t(x_0)$ at any $x_0 \in X$ is a loop at $f(x_0)$ and hence represents an element $[h_t(x_0)] \in \pi_1(X, f(x_0))$. The trace subgroup of cyclic homotopies $J(f, x_0) \leq \pi_1(X, f(x_0))$ is defined by

$$J(f, x_0) := \left\{ \xi \in \pi_1(X, f(x_0)) \mid \text{there exists a cyclic homotopy } h_t \text{ of } f \text{ such that } [h_t(x_0)] = \xi \right\}.$$ (1.1)

This subgroup $J(f, x_0)$ was introduced by Boju Jiang in [Ji64] to compute the Nielsen number of self-maps, which is a refined version of the classical Lefschetz number, and is now called Jiang subgroup and becomes an indispensable tool in Nielsen fixed point theory. We refer the reader to Jiang’s book [Ji83, §2.3-§2.6] for various properties of Jiang subgroup and its applications to Nielsen fixed point theory. The Jiang subgroup $J(f, x_0)$ is indeed a homotopy type invariant and independent of the choice of the basepoint $x_0$ up to an isomorphism which is compatible with $\pi_1(X, x_0)$ ([Ji83, Lemma 3.9]). Therefore we can simply denote it by $J(f)$.

In this article we are concerned with the Jiang subgroup $J(X) := J(\text{id}_X)$ with respect to the identity map. It turns out that $J(X) \leq Z(\pi_1(X))$, the center of $\pi_1(X)$ ([Ji83, Lemma 3.7]), and the Euler characteristic $\chi(X) = 0$ provided that $J(X)$ is non-trivial ([Ji83, Prop. 4.12]). The Jiang subgroup $J(X)$ was also investigated in detail by Gottlieb in [Go65] and many of Jiang’s results concerning $J(X)$ were reproved there. Nevertheless, an important new result obtained by Gottlieb is...
that $J(X) = Z(\pi_1(X))$ whenever $X$ is aspherical ([Go65, p. 848]), i.e., the universal cover of $X$ is contractible.

Combining the above-mentioned facts implies the following useful fact.

**Theorem 1.1** (Jiang, Gottlieb). The Euler characteristic of a finite aspherical CW-complex $X$ vanishes whenever $Z(\pi_1(X))$ is nontrivial.

Farrell showed in [Fa76] that more characteristic numbers of aspherical manifolds vanish if an additional restriction is imposed on the fundamental group. To be more precise, he showed the following theorem ([Fa76, Thms 1.2, 2.1]).

**Theorem 1.2** (Farrell). Let $M$ be an aspherical manifold. If $Z(\pi_1(M))$ is nontrivial and $\pi_1(M)$ is residually finite, then the signature $\text{sign}(M) = 0$. If moreover $M$ is Kähler, then the $\chi_y$-genus $\chi_y(M) = 0$.

More details about signature and the $\chi_y$-genus can be found in Section 2. Recall that a group $G$ is called residually finite if for every $g \in G \setminus \{1\}$, there exists a homomorphism from $G$ to a finite group such that the image of $g$ is not the identity ([CC10, §2]). It is now well-known that the fundamental groups of closed surfaces, 3-manifolds and hyperbolic manifolds are residually finite.

By refining the proof of Farrell in [Fa76], our first result is the following theorem.

**Theorem 1.3.** Let $M$ be a smooth manifold. If the Jiang subgroup $J(M)$ satisfies

\begin{equation}
J(M) \not\subset \bigcap_{H \lhd \pi_1(M) \colon [\pi_1(M) : H] < \infty} H,
\end{equation}

i.e., $J(M)$ is not contained in the intersection of all normal subgroups of finite index of $\pi_1(M)$, then the signature $\text{sign}(M) = 0$. If moreover $M$ is Kähler, then the $\chi_y$-genus $\chi_y(M) = 0$.

**Remark 1.4.**

1. It turns out that $\chi_y(M)\big|_{y=-1} = \chi(M)$ and $\chi_y(M)\big|_{y=1} = \text{sign}(M)$ (see Section 2 for more details). Therefore the vanishing of $\chi_y(M)$ contains more information than those of the Euler characteristic and the signature.

2. One may wonder, under the same hypothesis as in Theorem 1.3 on spin manifolds, if we can deduce the vanishing of the $\mathring{A}$-genus. However, the strategy of the proof in treating the signature and the $\chi_y$-genus can not be carried over to the case of the $\mathring{A}$-genus on spin manifolds, which will be clear in Section 3 (see Remark 3.3).

In order to present some useful sufficient conditions to the hypothesis condition (1.2), let us recall two related facts in group theory. The residual finiteness of a group $G$ has several equivalent definitions, one of which is that the intersection of all normal subgroups of finite index is trivial ([CC10, Prop. 2.1.11]):

\begin{equation}
\bigcap_{H \lhd G \colon [G : H] < \infty} H = \{1\}.
\end{equation}

Another fact is the so-called Poincaré Theorem in group theory, which states that any subgroup of finite index in $G$ contains a normal subgroup of $G$ which is also of finite index ([CC10, Lemma 2.1.10]). This latter fact, together with (1.3), also implies that the residual finiteness has another equivalent characterization: the intersection of all subgroups of finite index is trivial.

With these facts in mind, The hypothesis condition (1.2) becomes simpler under two extreme cases: the right-hand side (RHS for short) of (1.2) is trivial or its LHS is the whole $\pi_1(M)$. When the RHS
of (1.2) is trivial, i.e., \( \pi_1(M) \) is residually finite, the hypothesis (1.2) means that \( J(M) \) is non-trivial. When \( J(M) = \pi_1(M) \), the hypothesis (1.2) means that \( \pi_1(M) \) contains a proper subgroup of finite index. The hypothesis \( J(M) = \pi_1(M) \) is particularly useful in computing the Nielsen number ([Ji83, Thm 4.2]) and two family of spaces satisfy it: (the homotopy types of) \( H \)-spaces and homogeneous spaces ([Ji83, Thm 3.11]).

We summarize the discussions above into the following consequence.

**Corollary 1.5.** The conclusions in Theorem 1.3 hold true as long as one of the following two situations occurs.

1. The Jiang subgroup \( J(M) \) is non-trivial and \( \pi_1(M) \) is residually finite. In particular for aspherical manifolds this is Farrell’s Theorem 1.2 as in this case \( J(M) = Z(\pi_1(M)) \) by Theorem 1.1.
2. \( J(M) = \pi_1(M) \) and contains a proper subgroup of finite index. In particular, the former holds when \( M \) has the homotopy type of an \( H \)-space or a homogeneous space, and the latter holds when \( \pi_1(M) \) is a non-trivial finite group.

Recall that a non-positive sectional curvature (non-positively curved for short) Riemannian manifold is aspherical, thanks to the Cartan-Hadamard Theorem ([Zhe00, p. 59]). Such a condition is usually regarded as the differential-geometry counterpart to that of asphericity. Farrell conjectured that the condition of \( \pi_1(M) \) being residually finite in Theorem 1.2 may be superfluous ([Fa76, p. 165]). Our second result is to partially answer Farrell’s question in the Kähler setting, but with non-positive sectional curvature in place of asphericity.

The notions of Kähler hyperbolicity and Kähler non-ellipticity were introduced by Gromov et al ([Gr91], [CX01], [JZ00]) to attack the well-known Singer Conjecture ([Lü02, §11]), which include negatively and non-positively curved Kähler manifolds respectively. More details can be found in Section 4. Our second result is the following theorem.

**Theorem 1.6.** Let \( M \) be a Kähler non-elliptic manifold. Then the Euler characteristic of \( M \) vanishes if and only if \( \chi_y(M) = 0 \). In particular, a non-positively curved Kähler manifold \( M \) with non-trivial \( Z(\pi_1(M)) \) has vanishing \( \chi_y \)-genus: \( \chi_y(M) = 0 \).

Indeed we shall show in Proposition 4.6 that the vanishing of the Euler characteristic is equivalent to that of the \( \chi_y \)-genus on those manifolds satisfying the Singer Conjecture, from which Theorem 1.6 follows. The conclusions in Theorem 1.6 can be further strengthened whenever the dimension is small. This is the following result.

**Theorem 1.7.** Let \( M \) be an \( n \)-dimensional non-positively curved Kähler manifold with non-trivial \( Z(\pi_1(M)) \). Then all the Chern numbers of \( M \) vanish when the dimension \( n \leq 4 \).

In [LZ22] the author and F.Y. Zheng obtained some Chern class/number inequalities on numerically effective holomorphic vector bundles and gave some related applications to non-negative and non-positive curvature manifolds in various senses ([LZ22, §3]). Motivated by these results, we proposed the following conjecture ([LZ22, §4]), which can be regarded as the complex analogue to the classical Hopf conjecture.

**Conjecture 1.8 (Li-Zheng).** Let \( M \) be an \( n \)-dimensional Kähler manifold with non-positive holomorphic bisectional curvature whose Ricci curvature is quasi-negative. Then the signed Euler characteristic \( (-1)^n\chi(M) > 0 \).
Conjecture 1.8 is true when \( n = 2 \) ([LZ22, Prop. 4.4]). It is well-known that the sign of holomorphic bisectional curvature is dominated by that of Riemannian sectional curvature ([Zhe00, p. 178]). We shall explain in Section 5 that our proof of Theorem 1.7 indeed leads to the following corollary, which provides some more evidences to Conjecture 1.8.

**Corollary 1.9.** Conjecture 1.8 is true for \( n \)-dimensional non-positively curved Kähler manifolds whose Ricci curvature is quasi-negative when \( n \leq 4 \).

The rest of this article is organized as follows. We briefly collect in Section 2 some basic facts on the signature and \( \chi_y \)-genus in the form we shall use them in this article. Sections 3, 4 and 5 are devoted to the proofs of Theorems 1.3, 1.6 and 1.7 respectively. In the last Section 6 entitled “Appendix”, some non-standard facts mentioned in Section 2 will be proved or further explained for the completeness as well as for the reader’s convenience.

2. Preliminaries

2.1. **Signature and the \( \chi_y \)-genus.** We recall in this subsection some basic facts on the signature and Hirzebruch’s \( \chi_y \)-genus.

Let \( M \) be an oriented smooth manifold. Denote by \( \text{sign}(M) \) the signature of \( M \), which is defined to be the index of the intersection pairing on the middle-dimensional real cohomology when the dimension of \( M \) is divisible by 4 and zero otherwise. \( \text{sign}(M) \) is a rationally linear combination of Pontrjagin numbers and hence a characteristic number, thanks to Hirzebruch’s Signature Theorem ([Hi66]). It is well-known that ([AS68, §6]) \( \text{sign}(M) \) can be realized as the index of an elliptic operator \( D \) on \( M \):

\[
\text{sign}(M) = \dim_{\mathbb{C}} \ker(D) - \dim_{\mathbb{C}} \text{coker}(D),
\]

which is called the signature operator and \( \ker(D) \) and \( \text{coker}(D) \) are both subspaces of the (complexified) de-Rham cohomology groups on \( M \). The celebrated Atiyah-Singer index theorem then provides an alternative proof to the above-mentioned Hirzebruch Signature Theorem.

We only briefly recall Hirzebruch’s \( \chi_y \)-genus for compact complex manifolds, which is enough for our purpose in this article. We refer the reader to [Li17] and [Li19, §3] for its definition on general almost-complex manifolds and the summary on its various properties.

Let \( M = (M, J) \) be a complex manifold with (complex) dimension \( n \) and \( \bar{\partial} \) the d-bar operator which acts on the complex vector spaces \( \Omega^{p,q}(M) \) \( (0 \leq p, q \leq n) \) of \( (p, q) \)-type differential forms on \( (M, J) \) in the sense of \( J \). For each integer \( 0 \leq p \leq n \), we have the following Dolbeault-type elliptic complex

\[
0 \to \Omega^{p,0}(M) \xrightarrow{\bar{\partial}} \Omega^{p,1}(M) \xrightarrow{\bar{\partial}} \cdots \xrightarrow{\bar{\partial}} \Omega^{p,n}(M) \to 0,
\]

whose index is denoted by \( \chi^p(M) \) in the notation of Hirzebruch ([Hi66]):

\[
\chi^p(M) := \sum_{q=0}^{n} (-1)^q \dim_{\mathbb{C}} H^{p,q}_{\bar{\partial}}(M) = \sum_{q=0}^{n} (-1)^q h^{p,q}(M),
\]

where \( H^{p,q}_{\bar{\partial}}(M) \) are the Dolbeault cohomology groups and their dimensions \( h^{p,q}(M) \) are the usual Hodge numbers of \( M \). Note that \( \chi^0(M) \) is the Todd genus of \( M \), which is sometimes called the arithmetic genus in algebraic geometry.
The Hirzebruch’s $\chi_y$-genus, introduced by Hirzebruch in [Hi66] and denoted by $\chi_y(M)$, is the generating function of these indices $\chi^p(M)$:

$$\chi_y(M) := \sum_{p=0}^{n} \chi^p(M) \cdot y^p,$$

The general form of the Hirzebruch-Riemann-Roch theorem, which was established by Hirzebruch ([Hi66]) for projective manifolds and by Atiyah and Singer in the general setting ([AS68, §4]), allows us to express these $\chi^p(M)$ and thus $\chi_y(M)$ in terms of Chern numbers of $M$ as follows

$$\chi_y(M) = \int_M \prod_{i=1}^{n} \frac{x_i(1 + ye^{-x_i})}{1 - e^{-x_i}},$$

where $x_1, \ldots, x_n$ are Chern roots of $(M, J)$, i.e., the $i$-th elementary symmetric polynomial of $x_1, \ldots, x_n$, $x_n$ represents the $i$-th Chern class of $(M, J)$. Thus these $\chi^p(M)$ are also characteristic numbers. The formula (2.3) implies that $\chi_y(M)\big|_{y=1} = \chi(M)$, and, together with Hirzebruch’s Signature Theorem, implies that $\chi_y(M)\big|_{y=1} = \text{sign}(M)$. Therefore the vanishing of $\chi_y(M)$ implies those of the Euler characteristic and the signature, as stated in Remark 1.4.

2.2. The $\chi_y$-genus and $L^2$-Hodge numbers. we briefly recall in this subsection that how to express the indices $\chi^p$ in (2.2) in terms of the $L^2$-Hodge numbers for our later purpose. For the reader’s convenience more related details are included in Section 6.1. The reader may also consult [Li19, §4]. A thorough treatment on these materials can be found in [Li02, §1] or [Gr91, §1].

Assume that $M = (M, g, J)$ is an $n$-dimensional complex manifold equipped with a Hermitian metric $g$, and

$$\pi : (\tilde{M}, \tilde{g}, \tilde{J}) \longrightarrow (M, g, J)$$

its universal cover with $\pi_1(M)$ played as an isometric group of deck transformations.

Let $\mathcal{H}^{p,q}_{(2)}(\tilde{M})$ be the spaces of $L^2$-harmonic $(p, q)$-forms on $L^2\Omega^{p,q}(\tilde{M})$, the squared integrable $(p, q)$-forms on $(\tilde{M}, \tilde{g})$, and denote by

$$\dim_{\pi_1(M)} \mathcal{H}^{p,q}_{(2)}(\tilde{M})$$

the Von Neumann dimension of $\mathcal{H}^{p,q}_{(2)}(\tilde{M})$ with respect to $\pi_1(M)$, which is a nonnegative real number in our situation. Its precise definition is not important in our article and we refer the reader to [Li02] for more details. The $L^2$-Hodge numbers of $M$, denoted by $h_{(2)}^{p,q}(M)$, are defined to be

$$h_{(2)}^{p,q}(M) := \dim_{\pi_1(M)} \mathcal{H}^{p,q}_{(2)}(\tilde{M}) \in \mathbb{R}_{\geq 0}, \quad (0 \leq p, q \leq n).$$

Like the situation of the usual Hodge numbers, it turns out that $h_{(2)}^{p,q}(M)$ are independent of the Hermitian metric $g$ and depend only on $(M, J)$.

The following fact is an application of Atiyah’s $L^2$-index theorem ([At76]).

Lemma 2.1. These $\chi^p(M)$ can be similarly expressed in terms of the $L^2$-Hodge numbers $h_{(2)}^{p,q}(M)$ as follows

$$\chi^p(M) = \sum_{q=0}^{n} (-1)^q h_{(2)}^{p,q}(M) \quad 0 \leq p \leq n.$$

Lemma 2.1 is well-known to experts. Nevertheless, for the reader’s convenience as well as for the completeness, We shall indicate its proof in the Appendix, Section 6.1.
3. Proof of Theorem 1.3

The hypothesis condition (1.2) in Theorem 1.3 leads to the following consequence.

**Lemma 3.1.** Let $X$ be a connected finite CW-complex and its Jiang subgroup $J(X)$ satisfies
\[(3.1) \quad J(X) \not\subset \bigcap_{H \leq \pi_1(X)} [\pi_1(X)/H]_{\infty}\]

Then there exist a finite-sheeted cover $\tilde{X}$ of $X$ and its deck transformation $\tau$ of finite order such that $\tau$ is fixed-point-free and homotopic to the identity map. If moreover $X$ is a smooth (resp. complex) manifold, then this deck transformation $\tau$ is smooth (resp. holomorphic) either.

We first show that how to apply Lemma 3.1 to prove Theorem 1.3, which essentially follows the strategy of Farrell in [Fa76], and postpone its proof to the end of this section.

If $M$ is a compact, connected and oriented smooth (resp. Kähler) manifold with a finite-sheeted cover $\tilde{M} \xrightarrow{f} M$, then $\tilde{M}$ is also a compact connected smooth (resp. Kähler) manifold equipped with the induced orientation. Since characteristic numbers are multiplicative with respect to finite covers, we have
\[
\text{sign}(\tilde{M}) = \deg(f) \cdot \text{sign}(M)
\]
and
\[
\chi_y(\tilde{M}) = \deg(f) \cdot \chi_y(M)
\]
respectively. Together with Lemma 3.1, Theorem 1.3 follows from the following fact.

**Lemma 3.2.** Let $M$ be an oriented smooth (resp. Kähler) manifold equipped with a fixed-point-free smooth (resp. holomorphic) finite cyclic group action. If some generator of this group action is homotopic to the identity map, then $\text{sign}(M) = 0$ (resp. $\chi_y(M) = 0$.)

**Proof.** Let the cyclic group be $G$ and a generator $g \in G$ is homotopic to the identity map.

Since both $\ker(D)$ and $\coker(D)$ of the signature operator $D$ are subspaces of the de-Rham cohomology groups of $M$, homotopy invariance of de-Rham cohomology implies that the equivariant indices of $D$ at $g$ and the identity element are indeed equal:
\[
\text{sign}(g, M) = \text{Trace}(g|_{\ker(D)}) - \text{Trace}(g|_{\coker(D)})
\]
\[
= \text{Trace}(\text{id}|_{\ker(D)}) - \text{Trace}(\text{id}|_{\coker(D)})
\]
\[
= \dim_{\mathbb{C}} \ker(D) - \dim_{\mathbb{C}} \coker(D)
\]
\[
= \text{sign}(M).
\]

On the other hand, the Atiyah-Singer $G$-signature Theorem ([AS68, Thm 6.12]) implies that equivariant index $\text{sign}(g, M) = 0$ as the fixed-point set of this $G$-action is empty, which leads to the desired conclusion that $\text{sign}(M) = 0$.

When $M$ is Kähler, we can define the Lefschetz number $\chi^p(g, M)$ of the elliptic complex $(2.1)$ at $g$. Hodge theory tells us that for Kähler manifolds the Dolbeault cohomology groups $H^{p,q}_\partial(M)$ can be canonically viewed as subspaces of de-Rham cohomology groups and hence have the homotopy
invariance (see [Fa76, Lemma 2.3] for a slightly different treatment at this point). This implies that

\[
\chi^p(g, M) = \sum_{q=0}^{n} (-1)^q \cdot \text{Trace}(g|_{H^p_\partial(M)})
\]

\[
= \sum_{q=0}^{n} (-1)^q \cdot \text{Trace}(\text{id}|_{H^p_\partial(M)}) \quad (g \text{ is homotopic to the identity map})
\]

\[
= \sum_{q=0}^{n} (-1)^q \cdot \dim_{\mathbb{C}} H^p_\partial(M)
\]

\[
= \chi^p(M).
\]

On the other hand, the Lefschetz fixed-point formula of Atiyah-Bott-Singer, which was first treated by Atiyah and Bott in the isolated case in [AB67] and then by Atiyah and Singer in [AS68, §4] in the general setting, yields that \(\chi^p(g, M) = 0\) as this \(G\)-action has no fixed points. This leads to the desired conclusion that these \(\chi^p(M) = 0\) and hence \(\chi_{\hat{g}}(M) = 0\).

\[\square\]

**Remark 3.3.**

1. We can see from the proof of this lemma that if a compact connected Lie group \(G\) acts smoothly (resp. holomorphically) on an oriented smooth (resp. Kähler) manifold, then the equivariant index \(\text{sign}(g, M)\) (resp. \(\text{sign}^p(g, M)\)) is a constant:

\[
\text{sign}(g, M) \equiv \text{sign}(M), \quad \text{ resp. } \text{sign}^p(g, M) \equiv \chi^p(M), \quad \forall g \in G.
\]

This observation is indeed the starting point of the rigidity phenomena of the Dolbeault complexes (2.1) on general complex manifolds ([Ko70]) and of the Dirac operator on spin manifolds ([AH70]). The latter in turn motivates the rigidity of the elliptic genera ([Ta89], [BT89], [Liu96], [Liu95]).

2. On the other hand, if a spin manifold admits a diffeomorphism which is homotopic to the identity map, its equivariant index may not equal to the index of the Dirac operator as in this situation the kernel and cokernel of the Dirac operator in general have no properties of homotopy invariance. So even if assuming the same hypotheses as in Theorem 1.3 on spin manifolds, we cannot deduce the vanishing of the \(\hat{A}\)-genus.

3.1. **Proof of Lemma 3.1.** The proof is based on some standard facts in covering space theory and a standard reference is [Ha02, §1.3].

**Proof.** Let us choose once for all a basepoint \(x_0\) in \(X\). By the hypothesis (3.1), there exist an \(\alpha \in \pi_1(X, x_0)\) and a normal subgroup \(H\) of finite index in \(\pi_1(X, x_0)\) such that \(\alpha \notin H\). Let \((\tilde{X}, \tilde{x}_0) \xrightarrow{p} (X, x_0)\) be the finite-sheeted covering map which corresponds to \(H\) and thus \(p_*(\pi_1(\tilde{X}, \tilde{x}_0)) = H\) ([Ha02, Prop. 1.36]). By the definition of (1.1), there exists a homotopy \(h_t : X \to X\) such that \(h_0 = h_1 = \text{id}_X\) and \([h_t(x_0)] = \alpha\). By the homotopy lifting property ([Ha02, Prop. 1.30]), there exists a unique homotopy \(\tilde{h}_t : \tilde{X} \to \tilde{X}\) with \(p \circ \tilde{h}_t = h_t \circ p\) and \(\tilde{h}_0 = \text{id}_{\tilde{X}}\), i.e., we have the following commutative diagram:

\[
\begin{array}{ccc}
\tilde{X} & \xrightarrow{\tilde{h}_t} & \tilde{X} \\
p \downarrow & & \downarrow p \\
X & \xrightarrow{h_t} & X
\end{array}
\]

\(\tilde{h}_0 = \text{id}_{\tilde{X}}\).

Note that the path \(\tilde{h}_t(\tilde{x}_0)\) in \(\tilde{X}\) is a lift of the path \(h_t(x_0)\) in \(X\):

\[
p \circ \tilde{h}_t(\tilde{x}_0) = h_t \circ p(\tilde{x}_0) = h_t(x_0).
\]

\[
\]
Since
\[ [h_t(x_0)] = \alpha \notin H = p_*\left( \pi_1(X, x_0) \right), \]
the lifting path \( \tilde{h}_t(\tilde{x}_0) \) is not a loop ([Ha02, Prop. 1.31]) and hence
\[ (3.2) \quad \tilde{x}_1 := \tilde{h}_1(\tilde{x}_0) \neq \tilde{h}_0(\tilde{x}_0) = \tilde{x}_0. \]

The subgroup \( H \) is normal in \( \pi_1(X, x_0) \) and thus the action of deck transformation group on each fiber of \( p \) is transitive ([Ha02, Prop. 1.39]), which means that there exists a deck transformation \( \tau \) on \( \tilde{X} \) such that \( \tau(\tilde{x}_0) = \tilde{x}_1 \). On the other hand \( \tilde{h}_1 \) is also a lift of \( p \) as \( p \circ \tilde{h}_1 = h_1 \circ p = p \). Then the unique lifting property ([Ha02, P. 1.34]) implies that \( \tilde{h}_1 = \tau \) as both of them send \( \tilde{x}_0 \) to \( \tilde{x}_1 \).

Now this \( \tau = \tilde{h}_1 \) is the desired deck transformation. Firstly \( \tau \neq \text{id}_{\tilde{X}} \) by (3.2) and hence \( \tau \) is fixed-point free, still due to the unique lifting property. Secondly, \( \tau \) is of finite order as the deck transformation group is isomorphic to the quotient group \( \pi_1(X, x_0)/H \) ([Ha02, Prop. 1.39]), which is finite as \( H \) is of finite index.

If \( X \) is moreover a smooth (resp. complex) manifold, then the covering map \( p \) is smooth (resp. holomorphic) and so is \( \tau \) as locally \( p \) is a diffeomorphism (resp. biholomorphism).

\[ \square \]

4. PROOF OF THEOREM 1.6

Denote by \( b_i^{(2)}(M) \) the \( L^2 \)-Betti numbers of a smooth manifold \( M \) (see Section 6.2 for more details on their definition and basic properties). The following well-known conjecture is usually attributed to Singer ([Lü02, §11], [Do79]).

Conjecture 4.1 (Singer Conjecture). Let \( M \) be a \( 2n \)-dimensional aspherical smooth orientable manifold. Then \( b_i^{(2)}(M) = 0 \) whenever \( i \neq n \). If moreover \( M \) admits a negatively curved Riemannian metric, then \( b_n^{(2)}(M) > 0 \).

Remark 4.2. Conjecture 4.1 particularly implies that \((-1)^n \chi(M) \geq 0\) for aspherical manifolds and \((-1)^n \chi(M) > 0\) when admitting a negatively curved Riemannian metric (see formula (6.3)), which is the assertion of another well-known Hopf Conjecture.

In order to attack the Singer Conjecture for Kähler manifolds, Gromov introduced the notion of Kähler hyperbolicity ([Gr91]) and it was further extended to Kähler non-ellipticity by Cao-Xavier and Jost-Zuo independently ([CX01], [JZ00]). For a very recent development around this notion we refer to [BDET22]. Recall that a Kähler manifold \( (M, \omega) \), where \( \omega \) is the Kähler form, is called Kähler hyperbolic (resp. Kähler non-elliptic) if \( \pi^*(\omega) = d\beta \), where \( \tilde{M} \xrightarrow{\pi} M \) is the universal cover, such that \( \beta \) is a bounded (resp. sub-linear) one-form on \( (\tilde{M}, \pi^*(\omega)) \). The following example illustrates the ampleness of these two notions.

Example 4.3. (1) Typical examples of Kähler hyperbolic manifolds include ([Gr91, p. 265]) Kähler manifolds homotopy equivalent to negatively curved Riemannian manifolds and compact quotients of the bounded homogeneous symmetric domains in \( \mathbb{C}^n \) are Kähler hyperbolic. Submanifolds and product manifolds of Kähler hyperbolic manifolds are still Kähler hyperbolic.

(2) Typical examples of Kähler non-elliptic manifolds include ([JZ00, p. 4]) Kähler hyperbolic manifolds, non-positively curved Kähler manifolds and holomorphic immersed submanifolds of complex tori.
The following vanishing-type results due to Gromov, Cao-Xavier and Jost-Zuo ([Gr91], [CX01], [JZ00]) provide an affirmative solution to the Singer Conjecture and hence the Hopf conjecture for non-positively curved and negative curved Kähler manifolds.

**Theorem 4.4** (Gromov, Cao-Xavier, Jost-Zuo). Let $M$ be an $n$-dimensional Kähler non-elliptic manifold. Then the $L^2$-Betti number $b_i^{(2)}(M) = 0$ whenever $i \neq n$. If moreover $M$ is Kähler hyperbolic, then $b_n^{(2)}(M) > 0$.

**Remark 4.5.** By combining Gromov’s idea with some special properties of the $\chi_y$-genus, the author deduced in [Li19, Thm 2.1] that Kähler hyperbolic manifolds indeed satisfy a family of optimal Chern number inequalities and the first one is exactly $(-1)^n c_n \geq n + 1$, which is an improved inequality expected by the Hopf conjecture.

With the facts above and in Section 6.2 in hand, we can now show the following fact.

**Proposition 4.6.** Let $M$ be an $n$-dimensional Kähler manifold satisfying $b_i^{(2)}(M) = 0$ whenever $i \neq n$. Then the Euler characteristic of $M$ is zero if and only if $\chi_y(M) = 0$.

**Proof.** One direction is obvious as $\chi_y(M)|_{y=-1}$ is the Euler characteristic. For the converse direction, we first note that the hypotheses $b_i^{(2)}(M) = 0$ whenever $i \neq n$, together with the $L^2$-Hodge decomposition (6.4), imply that

$$h^p,q_{(2)}(M) = 0, \quad \text{whenever } p + q \neq n. \quad (4.1)$$

On the other hand,

$$\chi(M) = \sum_{i=0}^{2n} (-1)^i b_i^{(2)}(M) \quad (\text{by } (6.3))$$

$$= (-1)^n b_n^{(2)}(M) \quad (\text{by hypothesis condition})$$

$$= (-1)^n \sum_{p=0}^{n} h^{p,n-p}_{(2)}(M) \quad (\text{by } L^2\text{-Hodge decomposition } (6.4)). \quad (4.2)$$

Thus under the hypothesis, the Euler characteristic $\chi(M) = 0$ implies that

$$h^{p,n-p}_{(2)}(M) = 0, \quad \text{for all } 0 \leq p \leq n. \quad (4.3)$$

Combining (4.1) with (4.3) yields that all the $L^2$-Hodge numbers vanish, and hence so are $\chi^p(M)$ due to Lemma 2.1. \qed

Now we are ready to prove the following consequence, which is exactly Theorem 1.6.

**Corollary 4.7** (=Theorem 1.6). Let $M$ be a Kähler non-elliptic manifold. Then the Euler characteristic of $M$ vanishes if and only if $\chi_y(M) = 0$. In particular, a non-positively curved Kähler manifold $M$ with non-trivial $Z(\pi_1(M))$ has vanishing $\chi_y$-genus: $\chi_y(M) = 0$.

**Proof.** The first assertion follows from Theorem 4.4 and Proposition 4.6. For the second assertion, only note that a non-positively curved Kähler manifold $M$ is both non-elliptic (Example 4.3) and aspherical. Asphericity and Theorem 1.1 imply that $\chi(M) = 0$ and hence $\chi_y(M) = 0$. \qed
Various notions and basic facts in algebraic geometry used in this section can be found in [La04-1] and [La04-2].

In [DPS94] Demailly, Peternell and Schneider systematically investigated numerically-effective (nef for short) vector bundles over Kähler manifolds and paid special attention to those Kähler manifolds whose (holomorphic) tangent bundles are nef. Among other things, they obtained that ([DPS94, Coro. 5.5]) a Kähler manifold $M$ with nef tangent bundle $T_M$ is Fano, i.e., the anti-canonical line bundle $K_M^{-1}$ is ample, if and only if its Todd genus $\chi^0(M) > 0$. For related applications of this result, we refer to [Ya17, Thm 1.2] and [LZ22, Thm 3.2] and the comments therein.

Inspired by this result, Qi Zhang conjectured a dual version ([Zha97, p. 779]): the canonical line bundle $K_M$ of a Kähler manifold $M$ with nef cotangent bundle $T^*_M$ is ample if and only if the signed Todd genus $(-1)^n\chi^0(M) > 0$. He showed the “if” part for all dimensions $n$ and the “only if” part when $n \leq 4$ ([Zha97, Thm 4]). For our purpose, we state a slight variant of Zhang’s result as follows and indicate a proof for the reader’s convenience.

**Theorem 5.1** (Qi Zhang). Let $M$ be a Kähler manifold with nef $T^*_M$ and dimension $n \leq 4$. If the Chern number $(-c_1)^n > 0$, then the signed Todd genus $(-1)^n\chi^0(M) > 0$.

**Proof.** The ideas of the proof are essentially scattered in [Zha97], although not stated as above. Below some points of our proof are different from those in [Zha97].

**Claim 1:** $M$ is of general type and projective. Indeed, the nefness of $T^*_M$ implies that of $K_M$ ([La04-2, p. 24]), which, together with the condition $(-c_1)^n > 0$, implies that $K_M$ is big ([DP04, Thm 0.5]). Then $M$ is Moishezon and hence projective as it is Kähler ([MM07, p. 95]).

**Claim 2:** $K_M$ is ample, i.e., the first Chern class $c_1(M) < 0$. The nefness of $T^*_M$ and the projectivity of $M$ implies that it contains no rational curves. Hence $K_M$ is ample ([Zha97, p. 785], [De01, p. 219], [CY18, p.1481-1482]).

For simplicity we write $c_i := c_i(M)$. Note that the ampleness of $K_M$ indeed holds for all $n$. Then the Miyaoka-Yau Chern number inequality ([Ya77]) reads

$$c_2(-c_1)^{n-2} \geq \frac{n}{2(n+1)}(-c_1)^n > 0. \tag{5.1}$$

When $n = 2$ or $3$, $(-1)^n\chi^0(M) = \frac{1}{12}(c_1^2 + c_2)$ or $-\frac{1}{24}c_1c_2$ respectively ([Hi66, p. 14]). Its positivity follows from (5.1) directly.

When $n = 4$, (5.1) reads

$$5c_2c_1^2 \geq 2c_4^2. \tag{5.2}$$

On the other hand, we have (cf. [LZ22, Thm 2.9])

$$c_1c_3 - c_4 \geq 0, \quad c_2^3 \geq 0. \tag{5.3}$$
Therefore,
\[
(-1)^4 \chi^0(M) = \frac{1}{720}(-c_4 + c_1c_3 + 3c_2^2 + 4c_1^2c_2 - c_4^2) \quad ([Hi66, p. 14])
\]
\[
= \frac{1}{720}[(c_1c_3 - c_4) + 3c_2^2 + \frac{1}{2}(5c_1^2c_2 - 2c_4^2) + \frac{3}{2}c_1^2c_2]
\]
\[
\geq \frac{1}{480}c_1^2c_2 \quad \text{(by (5.2) and (5.3))}
\]
\[
\geq \frac{1}{1200}c_1^4 \quad \text{(by (5.2))}
\]
\[
> 0. \quad \text{(by the hypothesis condition)}
\]

We now arrive at the proof of Theorem 1.7.

**Theorem 5.2** (=Theorem 1.7). Let \( M \) be an \( n \)-dimensional non-positively curved Kähler manifold with non-trivial \( Z(\pi_1(M)) \). Then all the Chern numbers of \( M \) vanish whenever \( n \leq 4 \).

**Proof.** By [LZ22, Coro. 3.5], in this situation all the signed Chern numbers are non-negative and bounded above by \((-c_1)^n\). Thus it suffices to show \((-c_1)^n = 0\). Suppose on the contrary that \((-c_1)^n > 0\). We have mentioned in Section 1 that the sign of holomorphic bisectional curvature is dominated by that of Riemannian sectional curvature, while the non-negativity of the former famously implies the nefness of the cotangent bundle. Then Theorem 5.1 implies that the signed Todd genus \((-1)^n \chi^0(M) > 0\) and hence the \( \chi_y \)-genus \( \chi_y(M) \neq 0 \). Therefore Proposition 4.6 tells us that the Euler characteristic of \( M \) is nonzero, which by Theorem 1.1 contradicts to the hypothesis of \( Z(\pi_1(M)) \) being non-trivial. □

Now we explain that why the proof of Theorem 5.2 leads to Corollary 1.9. As remarked in [LZ22, Remark 4.2], Conjecture 1.8 is equivalent to the simultaneous positivity and vanishing of all signed Chern numbers for Kähler manifold with non-positive holomorphic bisectional curvature. Thus we have

**Corollary 5.3** (=Corollary 1.9). Let \( M \) be an \( n \)-dimensional non-positively curved Kähler manifold with \( n \leq 4 \). Then all the signed Chern numbers of \( M \) are either positive or zero. Thus Conjecture 1.8 holds true for these manifolds.

**Proof.** By [LZ22, Coro. 3.5], it suffices to show that when the Chern number \((-c_1)^n > 0\), then the signed Euler characteristic \((-1)^n \chi(M) > 0\). This has exactly been done in the proof of Theorem 5.2. □

6. **Appendix**

6.1. **Proof of Lemma 2.1.** We first explain that the elliptic complexes (2.1) can be made into more compact two-step elliptic operators as follows. The choice of a Hermitian metric on the complex manifold \((M, J)\) enables us to define the Hodge star operator \(*\) and the formal adjoint operator \(\bar{\partial}^* = -*\bar{\partial} \ast\) of the \(\bar{\partial}\)-operator. Then for each \(0 \leq p \leq n\), we have the following first-order elliptic operator \(D_p\):

\[
D_p := \bar{\partial} + \bar{\partial}^* : \bigoplus_{q \text{ even}} \Omega^{p,q}(M) \longrightarrow \bigoplus_{q \text{ odd}} \Omega^{p,q}(M),
\]

(6.1)
whose index by definition is
\[
\text{ind}(D_p) = \dim \mathbb{C} (\ker D_p) - \dim \mathbb{C} (\text{coker} D_p)
= \dim \mathbb{C} \bigoplus_{q \text{ even}} \mathcal{H}^{p,q}_\partial(M) - \dim \mathbb{C} \bigoplus_{q \text{ odd}} \mathcal{H}^{p,q}_\partial(M)
= \sum_{q \text{ even}} h^{p,q}(M) - \sum_{q \text{ odd}} h^{p,q}(M)
= \chi^p(M),
\]
where \(\mathcal{H}^{p,q}_\partial(M)\) are the spaces of complex-valued \(\bar{\partial}\)-harmonic forms and their dimensions are famously equal to \(h^{p,q}(M)\).

The elliptic operators \(D_p\) in (6.1) on the Hermitian manifold \((M, g, J)\) can be lifted to its universal cover \((\tilde{M}, \tilde{g}, \tilde{J})\):
\[
\tilde{D}_p : \bigoplus_{q \text{ even}} L^2 \Omega^{p,q}(\tilde{M}) \longrightarrow \bigoplus_{q \text{ odd}} L^2 \Omega^{p,q}(\tilde{M}),
\]
and one can define the \(L^2\)-index of the lifted operators \(\tilde{D}_p\) by
\[
\text{ind}_{\pi_1(M)}(\tilde{D}_p) := \dim_{\pi_1(M)}(\ker \tilde{D}_p) - \dim_{\pi_1(M)}(\text{coker} \tilde{D}_p)
= \dim_{\pi_1(M)} \left[ \bigoplus_{q \text{ even}} \mathcal{H}^{p,q}_\partial(\tilde{M}) \right] - \dim_{\pi_1(M)} \left[ \bigoplus_{q \text{ odd}} \mathcal{H}^{p,q}_\partial(\tilde{M}) \right]
= \sum_{q \text{ even}} \dim_{\pi_1(M)} \mathcal{H}^{p,q}_\partial(\tilde{M}) - \sum_{q \text{ odd}} \dim_{\pi_1(M)} \mathcal{H}^{p,q}_\partial(\tilde{M})
= \sum_{q=0}^n (-1)^q h^{p,q}_{(2)}(\tilde{M}).
\]

The \(L^2\)-index theorem of Atiyah ([At76]) asserts that
\[
\text{ind}(D_p) = \text{ind}_{\pi_1(M)}(\tilde{D}_p)
\]
and so we have the following crucial identities among \(\chi^p(M)\) and the \(L^2\)-Hodge numbers \(h^{p,q}_{(2)}(M)\) stated in (2.4):
\[
\chi^p(M) = \sum_{q=0}^n (-1)^q h^{p,q}_{(2)}(M).
\]

**Remark 6.1.** Note that the \(\bar{\partial}\)-operator \(\bar{\partial}\) can also be defined for almost-complex manifolds \((M, J)\), and the almost-complex structure \(J\) is integrable if and only if \(\bar{\partial}^2 \equiv 0\), i.e., (2.1) are complexes. So another advantage of using (6.1) rather than (2.1) is that (6.1) and hence the \(\chi_g\)-genus can be defined for general almost-complex manifolds.

### 6.2. \(L^2\)-Betti numbers and \(L^2\)-Hodge decomposition.

As in the discussions in Section 2.2, we can similarly define the \(L^2\)-Betti numbers \(b^{(2)}_i(M)\) of a general \(k\)-dimensional compact smooth orientable manifold \(M\) to be the Von Neumann dimension of the space of \(L^2\) harmonic \(i\)-forms \(\mathcal{H}^{i}_{(2)}(\tilde{M})\) with respect to \(\pi_1(M)\) under an arbitrarily chosen Riemannian metric:
\[
b^{(2)}_i(M) := \dim_{\pi_1(M)} \mathcal{H}^{i}_{(2)}(\tilde{M}) \in \mathbb{R}_{\geq 0}, \quad 0 \leq i \leq k.
\]
The Euler characteristic $\chi(M)$ is the index of the following elliptic operator
\begin{equation}
(6.2) \quad d + d^* : \bigoplus_{i \text{ even}} \Omega^i(M) \to \bigoplus_{i \text{ odd}} \Omega^i(M).
\end{equation}

Parallel to the discussions in the above subsection, we can lift (6.2) to the universal cover $\tilde{M}$ and apply the $L^2$-index theorem of Atiyah ([At76]) to yield
\begin{equation}
(6.3) \quad \chi(M) = \sum_{i=0}^{k} (-1)^i b_i^{(2)}(M).
\end{equation}

When $M$ is Kähler with complex dimension $n$, the $L^2$-Hodge-decomposition (cf. [Gr91, §1.2]) implies that
\begin{equation}
(6.4) \quad b_i^{(2)}(M) = \sum_{p+q=i} h^{p,q}_{(2)}(M).
\end{equation}

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