Patchy reionization bias on the primordial gravitational wave signal: Better to be sure than sorry

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ABSTRACT

One of the major goals of future cosmic microwave background (CMB) B-mode polarization experiments is the detection of primordial gravitational waves through an unbiased measurement of the tensor-to-scalar ratio $r$. Robust detection of this signal will require mitigating all possible contamination to the B-mode polarization from astrophysical origins. One such extragalactic contamination arises from the patchiness in the electron density during the reionization epoch. Along with the signature on CMB polarization, the patchy reionization can source secondary anisotropies on the CMB temperature through the kinetic Sunyaev-Zeldovich (kSZ) effect. To study this foreground impact, we use a physically motivated model of reionization to evaluate its contribution to the CMB B-mode polarization and temperature anisotropies for upcoming CMB missions. We show that the value of $r$ can bias towards a higher value if the secondary contribution from reionization is neglected. However, combining small-scale kSZ signal, large-scale E-mode polarization, and B-mode polarization measurements, we can put constraints on the patchiness in electron density during reionization and mitigate its impact on the value of $r$. CMB missions such as CMB-S4 and PICO may experience a bias of $>0.17\sigma$ which can go as high as $\sim 0.73\sigma$ for extreme reionization models allowed by the Planck and SPT CMB measurements. As future experiments target to measure $r$ at $5\sigma$, this is likely to affect the measurement significance and hence possibly affect the claim of detection of $r$, if not mitigated properly by using joint estimations of different reionization observables.

Key words: cosmic background radiation, reionization, cosmology : observations

1 INTRODUCTION

Detection of the primordial B-mode polarization signal from the cosmic microwave background (CMB) will be a cornerstone of our understanding of the primordial gravitational waves produced during inflation. For example, the amplitude of this signal will determine the energy scale of inflation as well as give us insights into the nature of the inflaton field (Kamionkowski & Kovetz 2016; Guzzetti et al. 2016, and references therein). This amplitude is usually described in terms of the primordial tensor-to-scalar power spectrum ratio $r$ defined at a wave mode $k_0 = 0.5$ Mpc$^{-1}$. The latest constraint on $r$ was obtained by Keck Collaboration et al. (2022) and is $r < 0.035$ at 95% confidence level. In the next couple of decades, CMB experiments like Simons Observatory (Ade et al. 2019), LiteBIRD (Suzuki et al. 2018), CMB-S4 (Abazajian et al. 2019), PICO (Hanany et al. 2019) are planned to provide sensitive measurements of B-mode polarization at large angular scales and aim to make the first statistically significant constraint on $r$.

The primal challenge in this effort is to correctly factor in the B-mode polarization foregrounds, mostly due to the secondary B-mode polarization signals arising in the post-recombination epochs. There are three primary contributors to this foreground, the galactic B-mode foreground, the lensing B-mode foreground, and the patchy reionization B-mode foreground (Hu & White 1997; Hu 2000; Lewis & Challinor 2006; Smith et al. 2009; Ichiki 2014; Kamionkowski & Kovetz 2016; Mukherjee et al. 2019). The polarized thermal emission from dust within our galaxy and galactic synchrotron contribute to the galactic component (Hu & White 1997; Kamionkowski & Kovetz 2016; Planck Collaboration et al. 2020). The gravitational lensing of the CMB by large-scale structures in the late Universe converts the E-mode signal to B-modes, which constitute the lensing foregrounds (Lewis & Challinor 2006; Ichiki 2014). Finally, the B-mode polarization arising from the patchiness in the reionization process contributes to the patchy reionization foreground (Hu 2000; Mukherjee et al. 2019). While efforts such as multi-wavelength observations and lensing potential reconstruction (Carron et al. 2017; Krachmalnicoff et al. 2018) are in progress to correct for the galactic and lensing foregrounds, relatively less attention is devoted to tackling the reionization foreground. And this neglect is primarily because of our lack in understanding the exact process through which the Universe reionized.

The ionized regions formed during the epoch of reionization around the ionizing sources lead to a picture where reionization is highly inhomogeneous or “patchy”. The inhomogeneous scattering...
of local CMB quadrupole off these patchy free electron population (ionized regions) leads to secondary B-mode polarization (Hu 2000; Baumann et al. 2003; Dvorkin et al. 2009; Mortonson & Hu 2010; Su et al. 2011; Namikawa 2018; Mukherjee et al. 2019; Roy et al. 2021). Mortonson & Hu (2010) found that ionized regions of size ~ 8°Mpc could generate B-mode signal from reionization of comparable amplitude to lensed B-mode signals. More recently, Mukherjee et al. (2019), Paul et al. (2020) and Roy et al. (2021) have found that the amplitude of the secondary B-mode from reionization depends not only on the reionization history of the Universe but also on the morphology of ionized regions. It is also well known that the morphology of these regions depend on the nature of the ionizing galaxies. For a given reionization history, the patchiness in the ionized regions increases with an increase in the minimum mass of haloes $M_{\text{min}}$ which can host ionizing sources and hence, lead to a higher secondary B-mode signal. This implies that reliable knowledge of physical properties of the ionizing sources could play an important role in modelling the B-mode foregrounds, enabling a robust measurement of $r$. It is, therefore, worth carrying out a systematic study to check if reionization histories, as allowed by the available data, can be a significant obstacle in detecting the primordial gravitational waves.

In a recent effort, Choudhury et al. (2020b) constrained a physical model of reionization using only available CMB observables, i.e., the measurements of Thomson scattering optical depth $\tau$ from Planck (Planck Collaboration et al. 2018) and the kinetic Sunyaev-Zeldovich ($kSZ$) temperature anisotropy measurement from SPT (Reichardt et al. 2020). This allowed them to gain insights into the patchiness in the distribution of ionized regions by constraining the properties of ionizing sources and hence put an upper bound on the B-mode power $D_{\ell}^{BB,\text{reion}} < 18\nu K^{2}$ at a multipole $\ell \sim 200$. Additionally, from the constraints on parameters of reionization models, they observed that reionization cannot be driven by extremely rare sources and provided stringent constraints on the duration of reionization at $\Delta z = 1.30_{-0.60}^{+0.19}$. In light of these insights about the process of reionization and lack of understanding of how significant a foreground reionization is to our observations of primordial B-mode signal, it becomes a natural choice to extend the works of Choudhury et al. (2020b) to assess if models of reionization allowed by the current CMB observations of $\tau$ and kSZ signal can bias our inferences on $r$ for the upcoming B-mode observing observatories.

The principal target of this work is therefore twofold: The first is to develop a pipeline to compute reionization observables and CMB anisotropies self-consistently. This would require coupling a model of reionization to codes used for computing the CMB angular power spectra. This has been carried out in the context of the semi-analytical models earlier (Mitra et al. 2011, 2012, 2015, 2018; Chatterjee et al. 2021), here we extend the method to semi-numerical models of patchy reionization. The second aim is to estimate the bias $\Delta r$ on the measurement of $r$ by future CMB experiments arising from an unaccounted-for (or incorrectly modelled) patchy reionization.

Both of these goals can be achieved by a physically motivated model of patchy reionization which can track the evolution of the intergalactic medium (IGM) in a large simulation volume. Furthermore, as we aim to explore the space of unknown parameters in as many details as possible, the model should be numerically efficient. Following our earlier works (Mukherjee et al. 2019; Paul et al. 2020; Choudhury et al. 2020b), we simulate the patchy reionization using an efficient explicitly photon-conserving semi-numerical model of reionization, namely, Semi-numerical Code for ReIonization with PhoTon-conservation (SCRIPT; Choudhury & Paranjape 2018). The advantage of this model is that it allows sufficient flexibility to choose the parameters characterizing the ionizing sources and producing the CMB signals at scales of our interest. We couple the model with an existing Boltzmann solver code for CMB anisotropies, CAMB, and use it to make Bayesian inference on $r$ using a combination of different CMB observables, namely, the simulated $B$-mode, $\tau$, and kSZ measurements corresponding to available and future experiments. The bias $\Delta r$ is then estimated by comparing the inferred $r$ for a model of $B$-mode power spectrum which accounts for reionization correctly with a model which neglects the patchy reionization contribution to $B$-mode. The value of $\Delta r$ thus obtained would provide indications of how crucial the modelling of reionization is going to be for robust measurement of the value of $r$.

The paper is organized as follows: In Section 2 we briefly describe the CMB probes of reionization and the simulation of reionization with which we evaluate these probes. We confront our models with existing and upcoming probes of $\tau$ and kSZ signal and obtain constraints on the reionization history, the results of which are presented in 3. In Section 4 we present our framework to compute the $B$-mode angular power spectrum while consistently evaluating power from primary and secondary anisotropy routines. We lay out the parameter estimation framework to estimate $\Delta r$ and present the forecast on the bias of parameter $r$ for upcoming CMB experiments. We also discuss the implications of our findings, with focus on Stage 4 CMB missions, namely, CMB-S4 and PICO. Finally, we summarize our conclusions in Section 5. Throughout the study we have fixed the cosmological parameters to $[\Omega_m, \Omega_b, h, n_s, \sigma_8] = [0.308, 0.0482, 0.678, 0.961, 0.829]$ (Planck Collaboration et al. 2014) which is consistent with Planck Collaborators et al. (2018).
of kSZ signal sourced from patchy reionization era are given as (Ma & Fry 2002; Park et al. 2013; Alvarez 2016)

$$C_{\ell}^{kSZ,\text{reion}} = (\sigma_T \bar{H} T_0)^2 \int c \, dz' \left(1 + z'\right)^4 \frac{H(z')}{\chi^2(z')} \times e^{-2\tau(z')} \frac{P_{q_L}(k) = 1/\chi(z', z')}{2}. \quad (2)$$

Here, $P_{q_L}$ is the power spectrum of transverse component of the Fourier transform of the momentum field $q(k, z)$. The dimensionless momentum field is defined as $q(x, z) \equiv x_e(x, z)v(x, z)/c$ where $v$ is the bulk velocity field. The transverse component of momentum power spectra $P_{q_L}(k)$ at a wave number $k$ receives a contribution from density and velocity auto/cross power spectra at various wave modes. When simulating $P_{q_L}$ for finite box sizes, one misses out on the contribution of wavemodes with wavelengths larger than the size of the box. When integrated along the line of sight, missing wave modes will result in a smaller kSZ estimate when simulating kSZ for smaller box sizes (Iliev et al. 2007; Park et al. 2013). To estimate and correct for the missing power one needs information on large-scale electron density and velocity fields. In this work we include the analytical calculation of the missing power in $P_{q_L}$ for finite box size based on the formulation by Park et al. (2013). Therefore the total power in the transverse component of the momentum field is the sum of the power from the simulation box and the missing power from the analytical estimate. For a more elaborate discussion and exact formulation for missing power used in this work refer to Appendix A. The contribution from patchiness in the electron density during the epoch of reionization depends on the history of reionization, morphology of the ionised bubble and its spatial distribution. All these aspects impact the strength of the power spectrum of the kSZ signal induced during reionization and one needs to capture it from physics-driven reionization models. Several efforts are also made towards such studies (Park et al. 2013; Paul et al. 2020; Gorce et al. 2022; Trac et al. 2022; Chen et al. 2022). The total observed kSZ is an integrated effect which requires the need to account for the kSZ contribution from both reionization $C_{\ell}^{kSZ,\text{reion}}$ and post-reionization $C_{\ell}^{kSZ,\text{postreion}}$ epoch. We account for the post-reionization kSZ using the scaling relations presented in Shaw et al. (2012). Therefore the total kSZ power spectrum is evaluated as $C_{\ell}^{kSZ,\text{tot}} = C_{\ell}^{kSZ,\text{reion}} + C_{\ell}^{kSZ,\text{postreion}}$.

(iii) Finally, the patchy B-mode polarization arises as a result of Thomson scattering of CMB temperature quadrupole off the inhomogeneous ionized field (Hu 2000). Under Limber approximation (valid at $\ell \geq 30$), the B-mode angular power spectra is given by (Hu 2000; Dvorkin & Smith 2009)

$$C_{\ell}^{BB,\text{reion}} = \frac{6\sigma_T^2 H^2}{100} \int c \, dz' \left(1 + z'\right)^4 \frac{H(z')}{\chi^2(z')} \times e^{-2\tau(z')} \frac{P_{ee}(k = l/\chi(z'), z')}{\chi^2(z')} \left(\frac{Q_{RMS}}{2}\right). \quad (3)$$

Here, $P_{ee}$ is the power spectrum of fluctuations in free electron fraction $x_e$. $Q_{RMS}$ is the r.m.s. of the primary quadrupole and is assumed to be constant at a value of 22 $\mu K$ over the redshifts corresponding to the epoch of reionization (Dvorkin & Smith 2009). A computational challenge arises when simulating B-mode power for large angular modes. Under Limber approximation, low-$\ell$ or large angular modes will arise from wave modes of $P_{ee}(k)$ which may be of greater wavelength than the size of the box. In Mukherjee et al. (2019) it was noted that the ionization fluctuations at scales larger than the bubble sizes are determined by fluctuations in the density field of the haloes which produce ionizing photons. The large-scale electron density power spectrum is then a scaled form of dark matter density power spectrum with the scaling term dependent on the halo bias parameter.

As mentioned above, we use Limber approximation to compute the angular power spectra. For a detailed discussion on the applicability of the approximation to our study, refer to Appendix B where we compare the Limber-approximated $B$-mode and kSZ signals to their exact counterparts.

### 2.2 Reionization simulations using SCRIPT

As discussed earlier, simulating and constraining parameter space of reionization using observables requires computationally efficient schemes. For Bayesian methods, physically motivated semi-numerical models are a natural choice as they are numerically efficient in parameter space exploration while allowing us to track relevant astrophysical parameters like Thomson scattering optical depth $\tau$, free electron fraction, and so on.

Considering the above arguments, we employ a semi-numerical scheme SCRIPT to generate ionization maps for this study. SCRIPT (Semi-numerical Code for ReIonization with PhoTon-conservation) is an explicitly photon conserving code for simulating the epoch of reionization (Choudhury & Paranjape 2018). In addition to its efficiency for Bayesian studies, the key advantage of SCRIPT is that the large-scale power spectrum of ionization fields converges across map resolutions. This is useful if one aims to efficiently study the large-scale properties in a simulation box.

As a first step in simulating the CMB signals arising from reionization, we need to simulate the ionization maps given the underlying dark matter density field at redshifts of reionization. The only input parameters needed for this step are the cosmological parameters. Since we do not vary these parameters in this work, this step is needed to be carried out only once. We generate dark matter snapshots for redshifts $5 \leq z \leq 20$ by employing the LPT prescription in MUSIC (Hahn & Abel 2011) for box length of $512 \ h^{-1} \ Mpc$ with $512^3$ particles. The collapsed mass fraction in haloes is computed using a subgrid prescription based on the conditional ellipsoidal collapse model (Sheth & Tormen 2002), see Choudhury & Paranjape (2018) for more details of the method.

To obtain an ionization map at a redshift, SCRIPT requires two input parameters, ionizing efficiency $\zeta$ of star-forming haloes and minimum mass $M_{min}$ of haloes that can host these sources. The output from SCRIPT is the map of ionized hydrogen fraction $x_{HI}(x, z)$. For this study, our parameter of interest is the free electron fraction $x_e(x, z) = \chi_{He} \cdot x_{HI}(x, z) \cdot \Delta(x, z)$, (4)

where, $\chi_{He}$ is the correction factor to account for free electrons from ionized Helium and $\Delta(x, z)$ corresponds to the dark matter overdensity. In our analysis, we consider $\chi_{He} = 1.18$ for $z > 3$ corresponding to contribution from singly-ionized Helium and $\chi_{He} = 1.16$ for $z < 3$ to account for free electron contribution from doubly ionized Helium. To enable us to capture the small-scale inhomogeneities, ionization maps using SCRIPT are generated with the best possible resolution of $2 \ h^{-1} \ Mpc$.

In this study, we operate with the four-parameter physical model of reionization introduced in Choudhury et al. (2020b). For completeness we briefly describe the model here: A power-law dependence of $\zeta$ and $M_{min}$ on redshift is assumed to cover up for the lack of knowledge about the properties of ionizing sources at redshifts corresponding to reionization era. The parameterization for $\zeta$ and $M_{min}$...
3 REVISITING CONSTRAINTS ON REIONIZATION PARAMETERS USING OPTICAL DEPTH AND KSZ

In this section we discuss the parameter constraints obtained on reionization parameters using combinations of $\tau$ and kSZ CMB probes. This will also inform us about the allowed ranges of reionization histories, the knowledge of which will play an important role in the construction of mock observational data with future telescopes (see Section 4).

3.1 Parameter Constraints from current measurements

We first discuss the observational constraints we plan to use in this work. The best parameter on $\tau$ is from Planck (Planck Collaboration et al. 2018) at $\tau = 0.054$ with $\sigma_{\tau}^{\text{obs}} = 0.007$. For the kSZ signal, the first 3$\sigma$ measurement was made by SPT team (Reichardt et al. 2020) at $D_{\ell=3000}^{\text{kSZ,obs}} = (\ell+1)C_{\ell}^{\text{kSZ,obs}} = 3 \mu K^2$ with $\sigma_{\ell=3000}^{\text{kSZ,obs}} = 1 \mu K^2$. Additionally, similar to the study by Choudhury et al. (2020b) we allow only those reionization histories which complete by redshift ($z > 5$), consistent with constraints presented in (McGreer et al. 2011; Kulkarni et al. 2019; Choudhury et al. 2020a; Qin et al. 2020).

We use the publicly available Markov chain Monte Carlo (MCMC) sampler available in the Cobaya framework (Torrisi & Lewis 2021) to sample the set of free parameters $\theta$ and compare the derived parameters ($\tau, D_{\ell=3000}^{\text{kSZ,obs}}$) with the measurement data sets ($\tau^{\text{obs}}, D_{\ell=3000}^{\text{kSZ,obs}}$).

The main input to the MCMC code is the likelihood $L$, which is calculated as

$$ -2 \log L = \left( \frac{\tau - \tau^{\text{obs}}}{\sigma_{\tau}^{\text{obs}}} \right)^2 + \left( \frac{D_{\ell=3000}^{\text{kSZ,obs}} - D_{\ell=3000}^{\text{kSZ,obs}}}{\sigma_{\ell=3000}^{\text{kSZ,obs}}} \right)^2. \tag{6} $$

Apart from constraints on free parameters, we derive constraints on a set of derived parameters, namely, the amplitude of $B$-mode power spectra from patchy reionization $D_{\ell=200}^{BB,\text{reion}}$ at a multipole $\ell = 200$, the redshifts $z_{25}, z_{50}, z_{75}$ corresponding to the mass-averaged ionized fraction $Q_{\text{HI}}(z) = [0.25, 0.50, 0.75]$ respectively and the duration of reionization defined as $\Delta z = z_{25} - z_{75}$ from the model for each sample of free parameters.

The parameter constraints obtained using Planck (Planck Collaboration et al. 2018) and SPT (Reichardt et al. 2020) are shown in Table 1 with one and two dimensional posterior distribution shown in Figure 1. We note that although the constraints are different to the constraints provided by Choudhury et al. (2020b) the overall conclusions remain the same. The important takeaways are: we see that the data prefer $M_{\text{min,0}} \gtrsim 10^9 M_\odot$ (at 68% C.L.) indicative of suppressed star formation in low mass haloes as a result of radiative feedback at $z \approx 8$ and that $\alpha_\zeta$ prefers a negative constraint. This could be indicative of more efficient cooling and star formation or increased escape fraction at lower redshifts. Finally, we obtain a tight constraint on the width of reionization $\Delta z = 1.19^{+0.27}_{-0.32}$.

The best fit model obtained from the analysis is $[\log(M_{\text{min,0}}) = 9.73, \log(\zeta_0) = 1.58, \alpha_M = -2.06, \alpha_\zeta = -2.01]$. We use this model as the fiducial model of reionization when forecasting for measurements with future probes (see Section 4.3). The value of $\tau$ for this fiducial model is 0.0540. The redshift evolution of the global mass-averaged ionization fraction $Q_{\text{HI}}(z) \equiv Q_{\text{HI}}(x, z, \Delta(x, z))$ is shown in Figure 2 (red curve). In addition, we also use a model, named max-BB, which has the maximum $D_{\ell=200}^{BB,\text{reion}}$ among models that are allowed within the 3$\sigma$ confidence levels (see Section 4.4). This model has parameters $[\log(M_{\text{min,0}}) = 10.39, \log(\zeta_0) = 2.48, \alpha_M = -0.76, \alpha_\zeta = 3.58]$. The value of $\tau$ in this case is 0.0627, the corresponding $Q_{\text{HI}}(z)$ is shown as the blue curve in Figure 2. We find that the max-BB model has an early but more gradual evolution of ionization fraction compared to the fiducial model.

Table 1. Constraints of reionization model parameters obtained from MCMC analysis using current observations from Planck and SPT. The first four rows correspond to the free parameters of the model while the rest of the parameters are the derived parameters. Uniform priors have been assumed for the free parameters in the prior range mentioned in the second column.

| Data Parameter | Prior | Planck + SPT 68% limits |
|----------------|-------|------------------------|
| $\log(\zeta_0)$ | $[0, \infty]$ | $1.70^{+0.49}_{-0.76}$ |
| $\log(M_{\text{min,0}})$ | $[7.0, 11.0]$ | $9.65^{+1.02}_{-0.49}$ |
| $\alpha_\zeta$ | $[-\infty, \infty]$ | $-3.81^{+2.58}_{-2.52}$ |
| $\alpha_M$ | $[-\infty, 0]$ | $> -2.78$ |

Table 2. Noise specification for the upcoming ground-based CMB experiments for their LAT configuration

| Mission     | Frequency (GHz) | $\Delta T$ (µK - arcmin) | Beam (arcmin) | $f_{sky}$ |
|-------------|-----------------|--------------------------|---------------|------------|
| SO LAT(goal) | 145             | 6.3                      | 1.4           | 0.4        |
| CMB-S4      | 150             | 1.8                      | 1.0           | 0.7        |

3.2 Forecasts for future CMB experiments aiming to measure kSZ effect

We extend our analysis of constraints on reionization parameters to future CMB measurements which aim to make high fidelity temperature and polarization power spectra and measure the observables, kSZ signal at $\ell = 3000$ and optical depth $\tau$. Future experiments LiteBIRD
(Suzuki et al. 2018) and PICO (Hanany et al. 2019) aim to measure the reionization bump ($\ell < 10$) of the $E$-mode CMB polarization to constraint $\tau$ with $\sigma_{\tau}^{\text{obs}} = 0.002$. In the future, we also expect to make sensitive measurements of kSZ with Simons Observatory (SO) (Ade et al. 2019) and CMB-S4 (Abazajian et al. 2019). The $\sigma_{kSZ}^{\ell}$ used to forecast constraints in our analysis for upcoming experiments is calculated using LAT configuration noise specification provided in Table 2 along with other sources of noise as specified for Equation (4) of Choudhury et al. (2020b).

Considering the mission timelines we propose the following scenarios of probes for measurements of $\tau$ and kSZ to forecast constraints on the reionization model parameter space:

- **Planck + SO**: $\tau$ measurement using Planck with projected kSZ measurement with the upcoming SO.
- **LiteBIRD + CMB-S4**: projected $\tau$ measurement with LiteBIRD with kSZ using CMB-S4.

We use the fiducial reionization model of the previous section to create mock observations of the upcoming facilities.

Comparative posteriors of free and derived parameters for different combinations of data sets as mentioned in the figure legend has been presented. The posteriors show both 68% and 95% contours in the two-dimensional posterior plots. The dashed magenta lines denote the input values used for forecasting.
ent combinations of $\tau$ and kSZ measuring experiments are shown in Figure 1. The parameter constraints corresponding to the reionization model can be seen in Table 3. It is obvious from the figure that the parameter space gets more constrained with the upcoming more sensitive experiments. In Table 3 we see that, with improved kSZ observations from SO, the error bar on $\tau$ reduces marginally to $\sim 0.005$ from $\sim 0.0065$. While with LiteBIRD measurement of $\tau$ and kSZ measurement of CMB-S4, the error bars on $\tau$ would reduce to $\sim 0.002$ while that on $D_{l=3000}^{kSZ}$ is $\sim 0.06$. Tight constraints for $\tau$ and $D_{l=3000}^{kSZ}$ translate to tighter constraints on the properties of the ionizing sources. This further helps us to gain insight into spatial inhomogeneities and the evolution of global properties of the reionization era. For reference, with each iteration of future experiments, we will be able to achieve tighter constraints on global ionization history through $z_{25}, z_{50}, z_{75}$ and patchiness in electron fraction field along the line-of-sight (LOS) through constraints on $D_{l=200}^{BB,\text{reion}}$.

Figure 2. Redshift evolution of mass-averaged ionized fraction $Q_{HI}(z)$ for the fiducial and max-BB models of reionization. See the text for a description of the models.

4 INFERRING TENSOR TO SCALAR RATIO INCLUDING REIONIZATION FOREGROUNDS

Until now, we have been considering measurements of $\tau$ and $D_{l}^{kSZ}$. We now get to the main aim of this work, i.e., to investigate the effects of patchy reionization on the detection of the B-modes from primordial gravitational waves. Neglecting the contribution of B-mode power generated during patchy reionization will lead to a bias in the mean value of the inferred tensor to scalar ratio $r$. For the upcoming CMB experiments, it is thus essential to study if the bias is sufficient to mislead the estimation of $r$.

4.1 Simulating the B-mode angular spectra from CMB

We first present a self-consistent framework to compute the $B$-mode angular power spectrum for arbitrary reionization histories. The $B$-mode signal has three major contributors, the primordial gravitational wave ($C_{f}^{BB,\text{prim}}$), the lensed scalar modes ($C_{f}^{BB,\text{lens}}$) and finally the $B$-mode arising from the patchiness of ionized fields in the era of reionization ($C_{f}^{BB,\text{reion}}$). The total $B$-mode power observed by us is given as

$$C_{f}^{BB} = C_{f}^{BB,\text{prim}} + A_{\text{lens}} C_{f}^{BB,\text{lens}} + C_{f}^{BB,\text{reion}},$$

(7)

where $A_{\text{lens}}$ is the residual lensing amplitude after delensing the signal. Let us discuss modeling each of these components one by one:

- The patchy reionization component $C_{f}^{BB,\text{reion}}$ is computed using SCRIPT which provides the ionized field power spectrum $P_{\text{reion}} (k, z)$. This serves as an input to Equation (3) for calculating the angular power spectrum. The computation of this component requires knowledge of the free parameters $\zeta_0, M_{\text{min}}, \alpha_\xi, \alpha_M$.

- The primary signal $C_{f}^{BB,\text{prim}}$ is computed using a Boltzmann solver CAMB (Lewis et al. 2000; Howlett et al. 2012). The computation of this component requires the values of parameters related to the primordial tensor power spectrum $r$ and $n_t$ and also $\tau$. While $r$ and $n_t$ are input parameters, the value of $\tau$ depends on the reionization history implied by the parameters related to reionization.

In the default version of CAMB, a user could either specify the midpoint of reionization $z_{\text{mid}}$ or the Thomson scattering optical depth $\tau$ to tune the default tanh ionization history. To ensure consistencies between the primordial and patchy reionization signals, we modify CAMB to account for arbitrary reionization histories. In this Modified CAMB, we use the ionization history of IGM evaluated as per the physical model of reionization with SCRIPT. In particular, we modify the code reionization.f90 which describes the reionization module in CAMB. Additionally, in consistency with SCRIPT to account excess of electrons due to Helium ionization we use, $\chi_{He} = 1.16$ for $z < 3$ and $\chi_{He} = 1.08$ otherwise. Figure 3 we present a comparison of mass-averaged free electron fraction and $B$-mode power spectra from default CAMB and Modified CAMB routines. Note that the two models have the same $\tau$. Interestingly accounting for a general reionization history implied by our fiducial model of reionization leads to a slightly different $D_{l}^{BB}$ at low multipoles (around the reionization “bump”).

![Figure 2](image-url)

Table 3. Forecasts on reionization model and derived parameters for the upcoming CMB experiments. The first four rows correspond to the free parameters of the model while the rest of the parameters are the derived parameters. The free parameters are assumed to have the same priors as Table 1. The second column shows the input value used to construct the mock data based on which forecasts are made.

| Parameter | Model | Planck + SO | LiteBIRD + CMB-S4 |
|-----------|-------|-------------|------------------|
| $\zeta_0$ | 1.58  | $1.85^{+0.41}_{-0.74}$ | $1.69^{+0.42}_{-0.55}$ |
| $M_{\text{min},0}$ | 9.73  | $9.68^{+0.86}_{-0.47}$ | $9.69^{+0.84}_{-0.43}$ |
| $\alpha_\xi$ | $-2.01$ | $-4.24^{+2.91}_{-2.24}$ | $-2.79^{+1.79}_{-1.39}$ |
| $\alpha_M$ | $-2.06$ | $> -2.83$ | $> -3.19$ |

![Table 3](table-url)
The lensing signal $C_{\ell}^{BB,\text{lens}}$ too is computed by the Modified CAMB for any general reionization history.

A flowchart diagram summarizing the pipeline for computing the CMB observables for arbitrary reionization histories in a self-consistent manner is shown in Figure 4. Our code requires the cosmological and reionization parameters as user inputs (along with the specifications of the simulation volume and resolution) and produces the resulting reionization history and all the relevant CMB observables. In addition, the code is capable of providing several other high-$z$ observables (e.g., galaxy luminosity function, the 21 cm maps), however, we will not discuss these in this paper.

4.2 Likelihood and Bayesian analysis

To understand the potential impact of reionization models allowed by current CMB measurements on future CMB experiments, we make a Bayesian inference of $r$ for upcoming CMB experiments. In this effort, we use a different combination of $\tau$, kSZ, and $B$-mode power spectra probes based on the current estimates of the timeline for different missions. For each upcoming $B$-mode measuring experiment we position ourselves in time to forecast for the measurement with a choice of best available $\tau$ and kSZ measurement. We propose four combinations for such an analysis:

- Case SO+ : Planck ($\tau$) +SO (kSZ) +SO (BB) [expected availability ~ 2024]
- Case LiteBIRD+ : Planck ($\tau$) +SO (kSZ) +LiteBIRD (BB) [expected availability ~ 2028]
- Case CBMS4+ : LiteBIRD ($\tau$)+CBMS4 (kSZ)+CBMS4 (BB) [expected availability ~ 2030]
- Case PICO+ : LiteBIRD ($\tau$) + CBMS4 (kSZ) + PICO (BB) [expected availability sometime in the next decade]

For choice of combination of probes or cases as discussed above, we sample the parameters $\theta = \{\log(M_{\min,0}), \log(\zeta_0), \alpha_{\ell}, \alpha_M, r\}$ and obtain posteriors using MCMC sampler in the Cobaya framework. The form of likelihood we use to infer $r$ is given as:

$$-2 \log \mathcal{L} = \left( \frac{\tau - \tau_{\text{obs}}}{\sigma_{\tau_{\text{obs}}}} \right)^2 + \left( \frac{D_{\ell=5000} - D_{\ell=3000}}{\sigma_{D_{\ell=5000}}} \right)^2 + \sum_{\ell,\ell' = \ell_{\text{min}}}^{\ell_{\text{max}}} \left( \tilde{C}_{\ell}^{BB} - C_{\ell}^{BB} \right) \Sigma_{\ell\ell'}^{-1} \left( \tilde{C}_{\ell'}^{BB} - C_{\ell'}^{BB} \right)$$

Here $\tilde{C}_{\ell}^{BB}$ represents the mock power spectrum, $C_{\ell}^{BB}$ represents the model data power spectrum, and $\Sigma_{\ell\ell'}$ represents the covariance matrix of the $B$-mode angular power spectrum:

$$\Sigma_{\ell\ell'} = \frac{2}{f_{\text{sky}}(2\ell + 1)} \left( \tilde{C}_{\ell}^{BB} + N_{\ell} \right) \delta_{\ell\ell'}$$

To calculate the elements of the covariance matrix, one must know the mock $B$-mode power $\tilde{C}_{\ell}^{BB}$ and the instrument specifications chiefly noise power spectra $N_{\ell}$ and the fraction of sky accessible to instrument $f_{\text{sky}}$. These instrument specifications are presented in Table 4.

The mock (template) of $C_{\ell}^{BB}$ power spectrum, see Equation (7), has contribution of $B$-mode power spectra from primordial gravitational waves $C_{\ell}^{BB,\text{prim}}$, lensing contribution $C_{\ell}^{BB,\text{lens}}$ and the $B$-mode angular power arising from the patchy reionization $C_{\ell}^{BB,\text{reion}}$.

The generation of the mock data requires one to make choices for the model parameters. We list our choices below:

(i) We explore two values of $r$ while generating the mock $C_{\ell}^{BB,\text{prim}}$, namely, $r = 0.001$ and $r = 5 \times 10^{-4}$. These values are much less than the present upper limits on the parameter and are

Table 4. Specifications of the CMB experiments aiming to target large scale $B$-modes for their SAT configuration

| Experiment | $\Delta_p$ (\muK-arcmin) | $\Theta_{\text{FWHM}}$ (arcmin) | $f_{\text{sky}}$ | Delensing $1 - A_{\text{lens}}$ |
|------------|--------------------------|-------------------------------|-----------------|--------------------------|
| SO         | 2.7                      | 30.0                          | 0.1             | 70%                      |
| LiteBIRD   | 2.4                      | 30.0                          | 1.0             | 70%                      |
| CBMS4      | 1.5                      | 30.0                          | 0.7             | 85%                      |
| PICO       | 0.87                     | 7.9                           | 1.0             | 85%                      |
typical of what the upcoming experiments aim to detect (Abazajian et al. 2019; Hanany et al. 2019). Throughout the analysis, we assume that the spectral index of tensor perturbations, $n_t = 0$.

(ii) The lensing contribution requires the value of $A_{lens}$, which depends on the experiment under consideration. The default values are given in Table 4. In addition to the default values, we also consider a rather optimistic case where we take $1 - A_{lens} = 95\%$ (Diego-Palazuelos et al. 2020). We also consider a case which is idealized with $1 - A_{lens} = 100\%$. This corresponds to a situation where all the lensing signal has been subtracted and is included only to understand the effects of residual lensing signal on our conclusions.

(iii) For the reionization model, our fiducial case is the best-fit model from our analysis in Section 3. In addition, we consider another model which produces the maximum $C_{\ell}^{BB, reion}$ among those allowed within the 3σ limits of the present constraints. We call this variant the “max-BB” reionization model.

In order to understand the importance of patchy reionization while estimating the parameter $r$, we consider two cases while recovering the parameters using the mock spectra:

- In the first case, we assume that the model used for comparing with the mock data does not include the reionization contribution to $B$-mode power, i.e.,
  \[ C_{\ell}^{BB, reion} : C_{\ell}^{BB} = C_{\ell}^{BB, prim} + A_{lens} C_{\ell}^{BB, lens} \]  
  Since the mock data contains the contribution from the patchy reionization while the model does not, this case is expected to lead to a bias in the recovery of $r$.

- In this case, we include the patchy reionization contribution to the model
  \[ C_{\ell}^{BB} = C_{\ell}^{BB, prim} + A_{lens} C_{\ell}^{BB, lens} + C_{\ell}^{BB, reion} \]  
  This case corresponds to a fair comparison between the mock data and the model and should be able to recover the input $r$ (within error bars).
The key idea here is to estimate the bias in the inferred $r$ for the above models given as

$$\frac{\Delta r}{\sigma} = \frac{r_{\text{Template}} - r_{\text{Template}-C_{\ell}^{BB,\text{reion}}}}{\sigma_{r_{\text{Template}}}}, \quad (12)$$

where $r_{\text{Template}}$ is the value of $r$ inferred with full template corresponding to Equation (11), $r_{\text{Template}-C_{\ell}^{BB,\text{reion}}}$ is the one with the template without patchy reionization corresponding to Equation (10) and $\sigma_{r_{\text{Template}}}$ is the statistical error on $r$ for the analysis with the full template. In cases of ground-based observatories like Simons Observatory (SO) and CMB-S4, the low $\ell$ modes are inaccessible, therefore, the multipole range considered in such cases is from $\ell = [52, 252]$. This is not an issue for space-based missions that have access to $\ell$ modes from $\ell_{\text{min}} = 2$, and the multipole range considered are $\ell = [2, 252]$. As was noted, to reduce the time complexity the patchy $B$-mode angular power spectrum is evaluated under Limber approximation which is valid for multipoles of $\ell \geq 30$. This may raise concerns regarding our study of the bias $\Delta r$ with regard to space-based missions for which we evaluate the patchy $B$-mode for multipole as low as $\ell = 2$. In Appendix B we show that the $B$-mode power evaluated through Limber approximation is underestimated at multipoles ($\ell \leq 30$). Therefore, we argue that any bias estimate $\Delta r$ from our study should always be treated as a lower limit in principle. We further note that a substantial difference in the bias estimate is highly unlikely as the amplitude of the primordial signal will be greater than the patchy $B$-mode signal for the concerned range of multipoles by a few orders of magnitude at multipoles $\ell \geq 30$.

As a reference, we show the different $B$-mode power contribution into the mock $B$-mode power spectra in Figure 5. The solid and dashed lines correspond to the power spectra arising from primordial $B$-mode power spectra (in black), lensed $B$-mode power spectra (in cyan), and $B$-mode power from patchiness in the reionization process (in magenta). In addition, we present noise spectra (in dotted lines) of observatories aiming to observe $B$-mode power spectra corresponding to specifications presented in Table 4.

Before moving on to the Bayesian analysis, let us understand the scales where the reionization signal can be important. In Figure 5 we find that the patchy $B$-mode power arising during the reionization era for our fiducial model of reionization (in a solid magenta curve) is comparable or greater than the primordial $B$-mode power ($r = 5 \times 10^{-4}$) for multipoles $\ell \geq 100$. While lensed $B$-modes are a dominant contribution at multipoles $\ell \geq 10$, nevertheless for tensor-to-scalar ratio $r \leq 10^{-3}$ and with improved delensing strategies, patchy $B$-mode may appear a significant foreground.

4.3 Estimation of bias in $r$: fiducial reionization model

We infer the parameter $r$ for models of Template and Template $-C_{\ell}^{BB,\text{reion}}$ for different cases. The inferred $r$ and bias $\Delta r/\sigma$ are shown in the Table 5. For completeness, we show the limits on the free and derived parameters of the model in Appendix C. Additionally, for reference, we have shown the posteriors of $r$ for input $r$ of $10^{-3}$ in Figure 6.

From Table 5, the first obvious point to note is that the error bars on the measurement of $r$ also tighten with increased sensitivity and consequent improvement in the instrumental noise. We are more interested in the effect of reionization of inferred $r$, so to this end we note that for the choice of model Template $-C_{\ell}^{BB,\text{reion}}$ the mean of the parameter $r$ is always overestimated when compared to the estimate of $r$ obtained for the model Template. This increase in the inferred mean value is intuitive as the model neglecting the patchy $B$-mode contribution has to compensate with a higher estimate of the parameter $r$.

When comparing the response of the observatories to the two models Template and Template $-C_{\ell}^{BB,\text{reion}}$, the relevant quantity of interest is $\Delta r/\sigma$ a measure of the significance of the bias introduced as a result of incorrect modeling of $B$-mode power spectra. For a fiducial choice of $r = 10^{-3}$, we find that LiteBIRD’s inference on $r$ will suffer a bias of $\sim 0.03 \sigma$ but as the sensitivity of CMB experiments improves we may observe a bias of $\sim 0.2 \sigma$ with space-based experiments like PICO. The bias is even more significant if the true value of $r$ is even lower at $5 \times 10^{-4}$. In such a Universe, even with ground-based experiments like CMB-S4 will always suffer an $\sim 0.18 \sigma$ and for PICO we will observe a bias of $\sim 0.22 \sigma$.

We present two more cases of bias estimation with CMB-S4 and PICO at higher delensing in the last two rows of Table 5 at 95% and 100% for an input $r = 5 \times 10^{-4}$. While the 100% delensing corresponds to a hypothetical case where we would have correctly reconstructed the lensing potential through the large-scale structure surveys, the 95% delensing corresponds to the best delensing possible allowed by instrumental beam and sensitivity of PICO (Diego-Palazuelos et al. 2020). As our ability to delens improves, our inference of $r$ becomes even more susceptible to confusion from patchy $B$-mode signal. With 95% delensing the bias estimate for PICO is $\sim 0.31 \sigma$ while for 100% delensing it increases to $\sim 1.59 \sigma$. For CMB-S4, 95% and 100% delensing translates to biases of $\sim 0.25 \sigma$ and $\sim 0.42 \sigma$. We thus conclude that, for a given reionization history, the bias on $r$ will increase when the true value of $r$ is smaller and/or when the sensitivity of the instrument is better and/or when the delensing is more efficient.

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1 The case with 100% delensing is a hypothetical scenario considered in the analysis to show the maximum impact on the $B$-mode signal.
Table 5. Constraints on parameter $r$ presented as $(\hat{r}^{\text{mock}})^{\times 10^3}$ obtained from the MCMC analysis of models Template and Template - $C_{\ell}^{BB,\text{reion}}$ corresponding to different observatory cases have been presented. Here, $\hat{r}$ refers to the mean of the $r$ posterior while $\sigma_r$ and $\sigma_{\hat{r}}$ refers to the 68% limits of the posterior. The constraints for the two choices of the mock value of $r$ used in this analysis are $10^{-3}$ and $5 \times 10^{-4}$ presented separately in this Table. $\Delta r/\sigma r$ is a measure of the significance of the bias with respect to the error on the measurement and is presented in the fourth column.

| Observatory case | Model                        | 68% limits $\Delta r/\sigma r$ |
|------------------|------------------------------|---------------------------------|
| $10^3 \times r = 1$ | Template                     | $< 3.53$                        |
|                  | Template - $C_{\ell}^{BB,\text{reion}}$ | $< 3.54$                        |
|                  | Template                      | $0.859^{+0.391}_{-0.601}$       |
|                  | Template - $C_{\ell}^{BB,\text{reion}}$ | $0.874^{+0.404}_{-0.560}$       |
|                  | CMBS4+ Template               | $0.986^{+0.185}_{-0.185}$       |
|                  | Template - $C_{\ell}^{BB,\text{reion}}$ | $1.019^{+0.185}_{-0.186}$       |
|                  | Template                       | $0.948^{+0.104}_{-0.105}$       |
|                  | Template - $C_{\ell}^{BB,\text{reion}}$ | $0.969^{+0.105}_{-0.105}$       |

| $10^3 \times r = 0.5$ | Template                      | $0.497^{+0.182}_{-0.182}$       |
|                      | Template - $C_{\ell}^{BB,\text{reion}}$ | $0.530^{+0.182}_{-0.181}$       |
|                      | Template                       | $0.457^{+0.097}_{-0.096}$       |
|                      | Template - $C_{\ell}^{BB,\text{reion}}$ | $0.479^{+0.097}_{-0.097}$       |

| $10^3 \times r = 0.5$ | Delensing at 95%              | $0.502^{+0.122}_{-0.122}$       |
|                      | Template                      | $0.532^{+0.122}_{-0.122}$       |
|                      | Template - $C_{\ell}^{BB,\text{reion}}$ | $0.469^{+0.066}_{-0.064}$       |
|                      | Template                       | $0.489^{+0.065}_{-0.064}$       |

| $10^3 \times r = 0.5$ | Delensing at 100%             | $0.493^{+0.091}_{-0.091}$       |
|                      | Template                      | $0.531^{+0.091}_{-0.091}$       |
|                      | Template - $C_{\ell}^{BB,\text{reion}}$ | $0.493^{+0.016}_{-0.017}$       |
|                      | Template                       | $0.520^{+0.017}_{-0.017}$       |

4.4 Estimation of bias in $r$: max-BB reionization model

The best-fit model of reionization, as inferred from the present CMB observations, may not necessarily correspond to the true model of reionization of the Universe. In order to study a case of extreme bias, we choose a model of reionization corresponding to the maximum allowed $B$-mode power spectra contribution allowed by $3\sigma$ contours of the MCMC chains of the Planck+SP + type. This model was introduced in Section 3.1 and the corresponding reionization history can be found in Figure 2. Corresponding to the model of reionization $D_{BB,\text{reion}}^{BB,\text{reion}}$ contribution is $18.41 \times 10^2 K^2$ (for comparison, $D_{BB,\text{reion}}^{BB,\text{reion}} = 7.03 \times 10^2 K^2$ for the fiducial model). The bias estimate for the max-BB model of reionization for the case of CMBS4+ and PICO+ with an input $r = [1 \times 10^{-3}, 5 \times 10^{-4}]$ has been presented in Table 6. With an increased contribution from reionization, we find that for both choices of input $r$ we obtained a higher bias as expected. With 95% delensing we begin to see the bias increase to $\sim 0.5\sigma r$ for both the choice of experiments.

Table 6. Same as Table 5 but with max-BB model of reionization in the mock $B$-mode power spectra.

| Observatory case | Model                        | 68% limits $\Delta r/\sigma r$ |
|------------------|------------------------------|---------------------------------|
| $10^3 \times r = 1$ | Delensing at 85%             | $1.010^{+0.186}_{-0.187}$       |
|                  | Template                      | $1.070^{+0.187}_{-0.187}$       |
|                  | Template - $C_{\ell}^{BB,\text{reion}}$ | $0.956^{+0.105}_{-0.105}$       |
|                  | Template                       | $0.995^{+0.105}_{-0.106}$       |

| $10^3 \times r = 0.5$ | Delensing at 95%             | $1.011^{+0.131}_{-0.131}$       |
|                      | Template                      | $1.081^{+0.132}_{-0.131}$       |
|                      | Template - $C_{\ell}^{BB,\text{reion}}$ | $1.021^{+0.071}_{-0.071}$       |
|                      | Template                       | $0.976^{+0.072}_{-0.072}$       |

| $10^3 \times r = 0.5$ | Delensing at 85%             | $0.510^{+0.184}_{-0.185}$       |
|                      | Template                      | $0.578^{+0.185}_{-0.185}$       |
|                      | Template - $C_{\ell}^{BB,\text{reion}}$ | $0.454^{+0.098}_{-0.099}$       |
|                      | Template                       | $0.496^{+0.099}_{-0.099}$       |

| $10^3 \times r = 0.5$ | Delensing at 95%             | $0.514^{+0.125}_{-0.125}$       |
|                      | Template                      | $0.583^{+0.125}_{-0.127}$       |
|                      | Template - $C_{\ell}^{BB,\text{reion}}$ | $0.467^{+0.067}_{-0.068}$       |
|                      | Template                       | $0.516^{+0.068}_{-0.068}$       |

4.5 Caution for $5\sigma$ measurement of tensor-to-scalar ratio by upcoming CMB missions

The stage-4 CMB experiments are targeting a $5\sigma$ measurement of the tensor to scalar ratio $r$. A huge effort is underway to observe the pristine primordial $B$-mode power spectra by mitigating the instrument
noise, galactic foregrounds, and extra-galactic foregrounds. One of the key aspects which makes it possible to distinguish between CMB and foregrounds is their distinguishable frequency spectrum between a few tens of GHz to nearly THz frequency range. However, the effect of patchy reionization which is discussed in this paper leads to the same kind of spectrum as CMB, hence cannot be distinguished from the actual CMB signal using the frequency spectrum. This makes the extragalactic contamination due to patchy reionization particularly critical. As the model of reionization is not well known and large fluctuations in the electron density during reionization can cause large fluctuations in B-mode polarization, we need to make sure that the inferred value of tensor to scalar ratio is due to primordial gravitational waves and not due to patchy reionization. This is significant in particular as CMB measurements are the only observational probes to measure primordial gravitational waves over these frequencies.

In this work, we comprehensively studied the nature of this contamination on tensor to scalar ratio for different scenarios of reionization and formulated a framework that can self-consistently mitigate the contamination from patchy reionization by combining multiple CMB probes such as E-mode polarization, kSZ, and B-mode polarization. The joint study enables to construct of a data-driven model of reionization and can make it feasible to obtain an upper bound (or a measurement) of patchiness in electron density during the epoch of reionization and hence can also provide an upper bound (or measurement) of the B-mode signal arising from this effect.

Using a self-consistent analysis framework proposed in this work, it is quite evident that with planned sensitivities of B-mode observations by CMB-S4 and PICO we will observe a bias of $\sigma$ > 0.15$r$ in the inference in $r$. For the allowed scenarios of reionization that are consistent with the kSZ temperature fluctuation detected by SPT (Reichardt et al. 2020), neglecting the reionization contribution can lead to a $\sim 18\%$ bias with respect to the standard deviation from CMB-S4 when $r = 10^{-3}$. We can generalize this inference that a $5\sigma$ detection with CMB-S4 would rather be a $\sim 4.82\sigma$ detection when the patchy B-mode is neglected. This scenario worsens if the true $r$ were smaller in such a case the contribution of reionization will become even more dominant.

Following the same argument, a probable $5\sigma$ detection with a more sensitive probe like PICO would rather be a $\sim 4.8\sigma$ detection. Additionally, for completeness, we chose a model of reionization allowed by current CMB measurements with maximum possible patchy B-mode signal amplitude. In such a case we find that for a Universe with $r = 5 \times 10^{-4}$ we will achieve a maximum bias of $\sim 0.55\sigma$ with CMB-S4 and $\sim 0.73\sigma$ with PICO further affecting our chance of claiming the detection of the actual value. In an extrapolation to our results, we additionally would want to claim that when and if more sensitive experiments come up and if true $r$ than $5 \times 10^{-4}$ were even lower, dealing with patchy reionization will be challenging and to improve our understanding of patchy B-modes any further than this analysis would require redshift based information of galaxy evolution information which the line-of-sight integrated CMB observables lack. We also learned that B-mode observations even with the choice of the correct model may not help with exploring reionization any more than we will learn with the optical depth $\tau$ and kSZ measurement due to the monotonic shape of the power spectrum of $C^{BB}_\ell$ with the angular multipoles $\ell$ for scales larger than about a degree and degeneracy with other contamination such as lensing.

While, on one side of the coin we have been discussing how the bias may impact future observations of $r$, on the flip side, a bias of such high significance also points to the opportunity of making an independent detection of patchy B-mode signal from reionization by the upcoming CMB telescopes using the correlation between different angular multipoles $\ell$. This detection will be complementary to other upcoming probes of reionization, e.g., the 21 cm emission (Mellema et al. 2013, 2015; Choudhury et al. 2016). We would like to explore such a possibility with the upcoming B-mode by observing experiments of CMB-S4 and PICO in our future work using simulations.

5 CONCLUSION

Sensitive observations of the B-mode polarization signal of the CMB in the future would enable the first detection of tensor-to-scalar ratio parameter $r$. This would help us to reject a large class of inflationary models, enabling us to gain insights into the mechanisms that laid the seeds of structure formation. Unbiased detection of $r$ is hence of critical importance. In this effort, we attempted to estimate the bias that might be introduced in the inference of $r$ when and if the contribution from patchy B-modes is neglected. We propose a framework that can mitigate the contamination from patchy reionization on tensor to scalar ratio by estimating an upper bound (or a measurement) on it by using a data-driven model of reionization constructed using different observables to reionization such as optical depth and kSZ signal.

If the patchy reionization effect is not included, then for the allowed range of reionization models considered in this analysis, we find that with ground-based observatories like CMB-S4 the inference on $r$ would be biased at $> 0.17\sigma$. This bias becomes more significant when PICO starts B-mode observations. We showed a case of extreme bias of $\sim 0.73\sigma$ which observations with PICO might experience with aggressive delensing of 95%. Further, we showed how this bias may impact the claim of the $5\sigma$ detection of $r$ by the Stage-4 CMB experiments if reionization is significantly inhomogeneous. While exploring the bias in $r$ we additionally found that even with the correct choice of model of B-mode the constraints on reionization parameters were governed by the sensitivity of $r$ and kSZ measurement. However, one important caveat that needs to be kept in mind is that the expected bias which is estimated depends on the models of reionization and the current constraints from Planck (Planck Collaboration et al. 2018) and SPT (Reichardt et al. 2020). In reality, reionization can be far more complicated and the impact on the B-mode can be significant. As a result, the proposed technique needs to be implemented to make a robust interpretation of an observed B-mode polarization signal.

The imprints of reionization can also lead to additional correlations between different angular multipoles for temperature, and polarization. This can potentially further improve our understanding during the epoch of reionization. In the future, we would like to explore the possibility of measuring such signatures beyond the power spectrum. Also, in addition, 21-cm measurements will open a complementary probe to understand the epoch of reionization, which can help in further understanding the history of reionization and the morphology of the inhomogeneities in electron density during reionization. This will further improve in limiting any contamination from patchy reionization on B-mode polarization and can measure primordial gravitational wave signal immune from at least one extra-galactic foreground contamination having the same frequency spectrum as CMB.
DATA AVAILABILITY

A basic version of the semi-numerical code SCRIPT for generating the ionization maps used in the paper is publicly available at https://bitbucket.org/rctirthankar/script. Any other data related to the paper will be shared on reasonable request to the corresponding author (DJ).

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APPENDIX A: MISSING POWER IN MOMENTUM FIELD FOR FINITE SIZED SIMULATION BOXES

The transverse component of the momentum field power spectrum is given as (Park et al. 2013)

\[
P_{q_L}(k, z) = \int\frac{d^3 k'}{(2\pi)^3} \left( 1 - \mu^2 \right) P_{ee}(k-k') P_{vv}(k') (A1)
\]

\[
- \frac{k'}{|k-k'|} P_{ev}(k-k') P_{ev}(k') , (A2)
\]

where \( P_{ev}(k) \) is the velocity power spectrum and \( P_{ev} \) is the cross power spectrum of the ionization and velocity fields. The missing power arises as we do not have information about the power spectra \( P_{ee}(k), P_{vv}(k), P_{ev}(k) \) for \( k < k_{box} = 2\pi/L_{box} \) where \( L_{box} \) is the...
The missing power is thus given by

\[ P_{q_{\perp}}^{\text{missing}}(k,z) = \int_{k' < k_{\text{box}}} \frac{d^3 k'}{(2\pi)^3} (1 - \mu'^2) P_{ee}(|k - k'|,z) P_{VV}(k',z), \quad (A3) \]

where \( \mu' = \hat{k} \cdot \hat{k}' \).

It is possible to calculate the expected missing power analytically at very large scales. At scales larger than the bubble sizes, we know that the electron density power spectrum is a scaled version of dark matter power spectra given as

\[ P_{ee}(k,z) = R^2(k,z)D^2(z)P_{\delta \delta}(k) \quad \text{(Mukherjee et al. 2019)} \]

where \( R(k,z) \equiv \chi_{HeQHII}(z)b_{hi}(k,M_{\text{min}},z) \) and \( D(z) \) is the linear growth factor. At very large scales \( \lim_{k \to 0} b_{hi}(k,z) = b(z) \) and hence we denote

\[ R(z) = b(z)
\chi_{HeQHII}(z). \]

Further, in the linear regime, we can relate the velocity power spectrum to the density power spectrum as

\[ P_{VV}(k) = \left( \frac{f_{a}^2}{k} \right)^2 P_{\delta \delta}(k) \quad (A4) \]

length of the simulation box. As argued by Park et al. (2013), for \( k' < k_{\text{box}} \), most of the missing power is contributed by \( P_{ee}(|k - k'|)P_{VV}(k') \) because of the way the velocity field scales at large scales. The missing power is thus given by

\[ P_{q_{\perp}}^{\text{missing}}(k,z) = \int_{k' < k_{\text{box}}} \frac{d^3 k'}{(2\pi)^3} (1 - \mu'^2) P_{ee}(|k - k'|,z) P_{VV}(k',z), \quad (A3) \]

where \( \mu' = \hat{k} \cdot \hat{k}' \).

It is possible to calculate the expected missing power analytically at very large scales. At scales larger than the bubble sizes, we know that the electron density power spectrum is a scaled version of dark matter power spectra given as \( P_{ee}(k,z) = R^2(k,z)D^2(z)P_{\delta \delta}(k) \quad \text{(Mukherjee et al. 2019)} \) where \( R(k,z) \equiv \chi_{HeQHII}(z)b_{hi}(k,M_{\text{min}},z) \) and \( D(z) \) is the linear growth factor. At very large scales \( \lim_{k \to 0} b_{hi}(k,z) = b(z) \) and hence we denote

\[ R(z) = b(z)
\chi_{HeQHII}(z). \]

Further, in the linear regime, we can relate the velocity power spectrum to the density power spectrum as

\[ P_{VV}(k) = \left( \frac{f_{a}^2}{k} \right)^2 P_{\delta \delta}(k) \quad (A4) \]
where, \( f = d \log D / d \log a \). The missing power is then

\[
P_{q_z}^{\text{missing}}(k, z) = R^2 (z) D^2 (z) \left( \frac{f d}{(2 \pi)^{3/2}} \right)^2 \int_{k^i < k_{\text{box}}} \frac{d^3 k'}{k'^2} (1 - \mu^2) P_{\delta \delta}(|k - k'|) P_{\delta \delta} (k'),
\]

which can be computed analytically.

As an illustration we consider the momentum field power spectra for two boxes, one with length \( L_{\text{box}} = 512 h^{-1} \text{Mpc} \) and another with \( L_{\text{box}} = 1024 h^{-1} \text{Mpc} \). We plot the corresponding power spectrum \( P_{q_z}^{512, \text{sim}} (k) \) and \( P_{q_z}^{1024, \text{sim}} (k) \) (black), uncorrected for the missing power, in Figure A1. It is clear that at large scales \( k \leq 0.03 h \text{Mpc}^{-1} \), the power spectra of the smaller box are lower than that of the larger box which is a clear indication of the missing modes. We then add the power contributed by missing velocity modes in the range \( 2 \pi / 1024 h^{-1} \text{Mpc} \leq k \leq 2 \pi / 512 h^{-1} \text{Mpc} \) using the Equation (A5) to \( P_{q_z}^{512, \text{sim}} (k) \), let us call the corrected power spectrum as \( P_{q_z}^{512, \text{cor}} (k) \). It is evident that the corrected power spectrum (red) agrees with the larger box quite well. The amplitude of the missing power is shown by the magenta curve and denoted here as \( P_{q_z}^{512, \text{miss}} (k) \).

The consequential convergence of \( P_{q_z} \) for box of length \( 512 h^{-1} \text{Mpc} \) with that for a box of length \( 1024 h^{-1} \text{Mpc} \) upon adding the missing power is prominent in Figure A2. Here, we plot the ratio of \( P_{q_z}^{512, \text{cor}} \) to \( P_{q_z}^{1024, \text{sim}} \) (in red) and the ratio of \( P_{q_z}^{512, \text{sim}} \) to \( P_{q_z}^{1024, \text{sim}} \) (in green). We find that at \( k \geq 0.1 \) the ratio corresponding to \( P_{q_z}^{512, \text{cor}} \) is close to 1 while that corresponding to \( P_{q_z}^{512, \text{sim}} \) is \(~0.9\).

Finally, after correcting for missing power at each redshift corresponding to our model of reionization, we plot the kSZ angular power spectra corresponding to both the boxes as shown in Figure A3. We denote kSZ angular power spectra for \( 512 h^{-1} \text{Mpc} \) box with corrected power as \( C_{BB}^{512, \text{cor}} (\ell) \) (in red) while the other one as \( C_{BB}^{512, \text{sim}} (\ell) \) (in green). In the dashed line we denote the kSZ power corresponding to the simulation box of length \( 1024 h^{-1} \text{Mpc} \). We find that post correcting for missing power the kSZ angular power for both the boxes is fairly convergent.

In this spirit, for our analysis throughout the paper we consider to correct for missing velocity modes ranging from \( 2 \pi / 8192 h^{-1} \text{Mpc} \leq k \leq 2 \pi / 512 h^{-1} \text{Mpc} \) to account for missing power in \( P_{q_z} \) at even larger scales. In Figure A4 we show the convergence of kSZ angular power spectra evaluated for boxes with length \( 512 \) and \( 1024 h^{-1} \text{Mpc} \) when missing power is corrected for wave-mode range of \( 2 \pi / 8192 h^{-1} \text{Mpc} \leq k \leq 2 \pi / L_{\text{box}} h^{-1} \text{Mpc} \).

### Appendix B: Applicability of the Limber Approximation

The patchy B-mode angular power spectra in this study are evaluated under the assumption of Limber approximation with an intent to relax the complexity of the code during MCMC sampling. Let us discuss the validity of the approximation in more detail. In general, the B-mode arising from Thomson scattering of CMB photons with the patchy spatial distribution of free-electrons in the reionization

\[
C_{BB}^{\text{kSZ}, \text{sim}}(\ell) = \frac{(\ell + 1)^2}{\pi} \left( \sigma_T \langle \bar{n} H \rangle \ell_0 \right)^2 \int dk (1 + z)^2 e^{-\tau (\chi)}
\]

\[
\times \int d\chi' (1 + z')^2 e^{-\tau (\chi')} \int k^2 dk' \frac{j_\ell (k \chi)}{k \chi} \frac{j_\ell (k' \chi')}{k' \chi'} P_{q_z} (k, \chi)
\]

where the power spectrum of transverse component of the

\[
\begin{align*}
\text{Figure B1. The B-mode angular power spectrum arising from patchy reionization has been shown for the exact method of evaluation using Equation (B1) and the Limbers method of evaluation using Equation (3)}
\end{align*}
\]
Fourier transform of momentum field $q_L(k, z)$ is written as $\langle q_L(k, z) q_L^\prime(k', z) \rangle \equiv \Psi_L(k, z) \delta_D(k - k')$ and $j_L(k \chi)$ are the spherical Bessel function. Just like the integrals related to $B$-mode power spectrum, the evaluation of the kSZ spectrum using the exact formulation is similarly time-complex. However, unlike the approximated $B$-mode, the Limber approximated kSZ spectrum is rather consistent with the exact evaluation for the typical scales we are interested in ($\ell \geq 1000$). In Figure B3 we plot the kSZ spectrum evaluated using the exact formulation using Equation (B2) and the kSZ spectrum evaluated from the Limber approximation using Equation (2). We find that at all angular scales of our interest, the Limber approximated kSZ overlaps with the exact kSZ evaluation. We have checked and found that the match between the two power spectra is within numerical errors.

Based on the convergence of the two signals, we treat the Limber approximated kSZ to be a proxy for the exact evaluation with no visible effect on our bias studies.

**APPENDIX C: CONSTRAINTS ON FREE AND DERIVED PARAMETERS FROM MCMC ANALYSIS USING SIMULATED OBSERVATIONS FROM FUTURE CMB EXPERIMENTS**

In Tables C1 and C2 we have listed the constraints on free and derived parameters corresponding to the cases presented in Section 4.2. We note that even after adding the additional signal of patchy $B$-mode through the model Template the constraints of both free and derived reionization parameters are surprisingly similar to the constraints obtained for the model Template - $C_{l}^{BB,\text{reion}}$. We find this trend consistent even with improved delensing ability. The probable cause for such a trend is that the constraints on reionization parameters are largely governed by error bars on Thomson scattering optical depth $\tau$ and the kSZ signal.

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.
Table C1. Parameter Constraints for the free parameters obtained from the MCMC analysis of models Template and Template - $C_l^{BB, reion}$ corresponding to different observatories have been presented. The constraints for the two choices of mock value of $r$ used in this analysis are $10^{-3}$ and $5 \times 10^{-3}$ presented separately in this Table. The constraints on $r$ are presented in form of $(\sigma_r^2) \times 10^3$

| Observatory Case | Model | $\log (M_{min,0})$ | $\log (\xi_0)$ | $\sigma_{\xi}$ | $\sigma_M$ | $10^3 \times r$ |
|------------------|-------|---------------------|----------------|---------------|-----------|----------------|
| Input            | 9.73  | 1.57                | -2.01          | -2.06         | 1         |                |
| SO+              | Template | 9.67$\pm$0.98  | 0.43            | -0.71         | -4.30$^{+0.89}_{-0.71}$ | -2.19      | -2.84         | < 3.53 |
|                  | Template - $C_l^{BB, reion}$ | 9.69$\pm$0.95  | 0.38            | -0.75         | -4.25$^{+0.86}_{-0.75}$ | -2.27      | -2.82         | < 3.54 |
| LiteBIRD+        | Template | 9.64$\pm$0.46  | 0.42            | -0.67         | -3.95$^{+2.15}_{-0.67}$ | -2.15      | -2.94         | 0.859$^{0.391}_{0.601}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.66$\pm$0.43  | 0.41            | -0.71         | -3.95$^{+2.88}_{-0.71}$ | -2.16      | -2.85         | 0.874$^{0.404}_{0.560}$ |
| CMBS4+           | Template | 9.77$\pm$0.79  | 0.39            | -0.53         | -2.25$^{+1.87}_{-0.53}$ | -1.99      | -3.24         | 0.986$^{0.185}_{0.185}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.74$\pm$0.48  | 0.39            | -0.53         | -2.22$^{+1.59}_{-0.53}$ | -1.29      | -3.26         | 1.019$^{0.185}_{0.186}$ |
| PICO+            | Template | 9.75$\pm$0.79  | 0.39            | -0.54         | -2.08$^{+1.52}_{-0.54}$ | -1.37      | -3.25         | 0.948$^{0.104}_{0.105}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.73$\pm$0.45  | 0.38            | -0.55         | -2.02$^{+1.51}_{-0.55}$ | -1.24      | -3.78         | 0.969$^{0.105}_{0.106}$ |

| CMBS4+           | Template | 9.75$\pm$0.23  | 0.38            | -0.48         | -2.20$^{+1.50}_{-0.48}$ | -1.04      | -2.91         | 0.497$^{0.182}_{0.182}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.76$\pm$0.34  | 0.37            | -0.45         | -2.12$^{+1.40}_{-0.45}$ | -0.92      | -2.72         | 0.530$^{0.182}_{0.181}$ |
| PICO+            | Template | 9.70$\pm$0.84  | 0.43            | -0.54         | -2.10$^{+1.57}_{-0.54}$ | -1.28      | -3.24         | 0.457$^{0.096}_{0.097}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.74$\pm$0.44  | 0.38            | -0.57         | -2.08$^{+1.57}_{-0.57}$ | -1.33      | -3.30         | 0.479$^{0.097}_{0.097}$ |

| Delensing at 95% | CMBS4+ | Template | 9.71$\pm$0.81  | 0.40            | -0.52         | -2.29$^{+1.66}_{-0.52}$ | -1.19      | -3.29         | 0.502$^{0.122}_{0.122}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.73$\pm$0.40  | 0.40            | -0.52         | -2.22$^{+1.68}_{-0.52}$ | -1.23      | -3.21         | 0.532$^{0.121}_{0.122}$ |
| PICO+            | Template | 9.73$\pm$0.80  | 0.41            | -0.52         | -2.18$^{+1.60}_{-0.52}$ | -1.32      | -3.30         | 0.469$^{0.066}_{0.064}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.74$\pm$0.39  | 0.41            | -0.55         | -2.11$^{+1.55}_{-0.55}$ | -1.25      | -3.24         | 0.489$^{0.065}_{0.074}$ |

| Delensing at 100% | CMBS4+ | Template | 9.79$\pm$0.42  | 0.38            | -0.58         | -2.17$^{+1.59}_{-0.58}$ | -1.37      | -3.18         | 0.493$^{0.091}_{0.091}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.71$\pm$0.42  | 0.39            | -0.55         | -2.27$^{+1.61}_{-0.55}$ | -1.29      | -3.25         | 0.531$^{0.091}_{0.091}$ |
| PICO+            | Template | 9.79$\pm$0.87  | 0.42            | -0.58         | -2.03$^{+1.54}_{-0.58}$ | -1.28      | -3.20         | 0.493$^{0.016}_{0.017}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.82$\pm$0.36  | 0.41            | -0.49         | -2.25$^{+1.55}_{-0.49}$ | -1.39      | -3.39         | 0.520$^{0.017}_{0.017}$ |

| Delensing at 85% | CMBS4+ | Template | 9.75$\pm$0.42  | 0.38            | -0.56         | 0.96$^{+1.07}_{-1.85}$ | -3.28      | 0.510$^{0.184}_{0.185}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.79$\pm$0.68  | 0.38            | -0.53         | 0.93$^{+1.08}_{-1.85}$ | -3.25      | 0.578$^{0.185}_{0.185}$ |
| PICO+            | Template | 9.81$\pm$0.39  | 0.39            | -0.80         | 1.09$^{+1.02}_{-2.05}$ | -3.13      | 0.454$^{0.098}_{0.099}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.80$\pm$0.43  | 0.36            | -0.55         | 1.04$^{+1.05}_{-1.92}$ | -3.21      | 0.496$^{0.099}_{0.099}$ |

| Delensing at 95% | CMBS4+ | Template | 9.74$\pm$0.38  | 0.38            | -0.52         | 0.82$^{+1.06}_{-1.78}$ | -3.31      | 0.514$^{0.125}_{0.125}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.79$\pm$0.41  | 0.38            | -0.54         | 0.97$^{+1.00}_{-1.97}$ | -3.23      | 0.583$^{0.125}_{0.127}$ |
| PICO+            | Template | 9.85$\pm$0.34  | 0.38            | -0.50         | 1.14$^{+1.13}_{-1.96}$ | -2.98      | 0.467$^{0.067}_{0.067}$ |
|                  | Template - $C_l^{BB, reion}$ | 9.79$\pm$0.41  | 0.39            | -0.53         | 0.96$^{+1.06}_{-1.92}$ | -3.39      | 0.516$^{0.068}_{0.068}$ |
Table C2. Same as Table C1 but for the derived parameters

| Observatory | Model | \( \tau \) | \( D_{t=3000}^{KZ} \) | \( D_{t=200}^{BB,\text{reion}} \) | \( \xi_{25} \) | \( \xi_{50} \) | \( \xi_{75} \) | \( \Delta \xi \) |
|-------------|-------|-------------|----------------|----------------|-------------|-------------|-------------|-------------|
| Input       | 0.0540 | 3.00        | 7.03           | 8.09           | 7.27         | 6.78         | 1.31        |
| \( 10^{-3} \times r = 1.0 \) |
| SO+         | Template | 0.0579\( \pm 0.0055 \) | 2.94\( \pm 0.009 \) | 6.39\( \pm 0.06 \) | 8.43\( \pm 0.46 \) | 7.72\( \pm 0.64 \) | 7.31\( \pm 0.79 \) | 1.12\( \pm 0.24 \) |
| LiteBIRD+   | Template | 0.0572\( \pm 0.0055 \) | 2.93\( \pm 0.09 \) | 6.46\( \pm 1.04 \) | 8.37\( \pm 0.52 \) | 7.63\( \pm 0.65 \) | 7.19\( \pm 0.81 \) | 1.18\( \pm 0.25 \) |
| CMBS+       | Template | 0.0538\( \pm 0.0019 \) | 2.95\( \pm 0.07 \) | 7.28\( \pm 0.80 \) | 8.08\( \pm 0.81 \) | 7.22\( \pm 0.27 \) | 6.71\( \pm 0.43 \) | 1.37\( \pm 0.20 \) |
| PICO+       | Template | 0.0532\( \pm 0.0018 \) | 2.95\( \pm 0.06 \) | 7.39\( \pm 0.78 \) | 8.04\( \pm 0.18 \) | 7.16\( \pm 0.20 \) | 6.64\( \pm 0.33 \) | 1.40\( \pm 0.44 \) |
| \( \text{Input} \) | 0.0540 | 3.00        | 7.03           | 8.09           | 7.27         | 6.78         | 1.31        |
| \( 10^{-3} \times r = 0.5 \) |
| CMBS+       | Template | 0.0539\( \pm 0.0018 \) | 2.95\( \pm 0.07 \) | 7.17\( \pm 0.69 \) | 8.09\( \pm 0.17 \) | 7.24\( \pm 0.28 \) | 6.75\( \pm 0.42 \) | 1.33\( \pm 0.18 \) |
| PICO+       | Template | 0.0533\( \pm 0.0018 \) | 2.95\( \pm 0.06 \) | 7.39\( \pm 0.81 \) | 8.05\( \pm 0.21 \) | 7.17\( \pm 0.25 \) | 6.65\( \pm 0.31 \) | 1.40\( \pm 0.45 \) |
| \( 10 \times r = 0.5 \) |
| Delensing at 95% |
| CMBS+       | Template | 0.0538\( \pm 0.0020 \) | 2.95\( \pm 0.07 \) | 7.24\( \pm 0.79 \) | 8.09\( \pm 0.20 \) | 7.23\( \pm 0.27 \) | 6.72\( \pm 0.43 \) | 1.37\( \pm 0.18 \) |
| PICO+       | Template | 0.0536\( \pm 0.0019 \) | 2.95\( \pm 0.06 \) | 7.33\( \pm 0.84 \) | 8.07\( \pm 0.18 \) | 7.20\( \pm 0.26 \) | 6.69\( \pm 0.40 \) | 1.38\( \pm 0.20 \) |
| \( 10 \times r = 0.5 \) |
| Delensing at 100% |
| CMBS+       | Template | 0.0539\( \pm 0.0020 \) | 2.96\( \pm 0.06 \) | 7.32\( \pm 0.74 \) | 8.10\( \pm 0.18 \) | 7.23\( \pm 0.28 \) | 6.73\( \pm 0.43 \) | 1.37\( \pm 0.18 \) |
| PICO+       | Template | 0.0535\( \pm 0.0018 \) | 2.95\( \pm 0.06 \) | 7.47\( \pm 1.17 \) | 8.04\( \pm 0.18 \) | 7.20\( \pm 0.24 \) | 6.71\( \pm 0.29 \) | 1.34\( \pm 0.43 \) |
| \( \text{Input: max-BB} \) \( (3\sigma) \) | 0.0627 | 4.03        | 18.41          | 9.22           | 8.12         | 7.38         | 1.84        |
| \( 10 \times r = 0.5 \) |
| Delensing at 85% |
| CMBS+       | Template | 0.0623\( \pm 0.0019 \) | 3.99\( \pm 0.06 \) | 14.52\( \pm 0.97 \) | 9.26\( \pm 0.16 \) | 7.89\( \pm 0.30 \) | 7.03\( \pm 0.48 \) | 2.23\( \pm 0.25 \) |
| PICO+       | Template | 0.0612\( \pm 0.0018 \) | 3.99\( \pm 0.06 \) | 14.50\( \pm 1.94 \) | 9.23\( \pm 0.16 \) | 7.87\( \pm 0.24 \) | 7.01\( \pm 0.45 \) | 2.22\( \pm 0.24 \) |
| \( 10 \times r = 0.5 \) |
| Delensing at 95% |
| CMBS+       | Template | 0.0623\( \pm 0.0020 \) | 3.99\( \pm 0.06 \) | 14.40\( \pm 1.06 \) | 9.26\( \pm 0.16 \) | 7.89\( \pm 0.29 \) | 7.03\( \pm 0.48 \) | 2.23\( \pm 0.24 \) |
| PICO+       | Template | 0.0621\( \pm 0.0018 \) | 3.99\( \pm 0.06 \) | 14.74\( \pm 1.89 \) | 9.24\( \pm 0.17 \) | 7.89\( \pm 0.29 \) | 7.02\( \pm 0.25 \) | 2.22\( \pm 0.23 \) |
| \( 10 \times r = 0.5 \) |
| Delensing at 95% |
| CMBS+       | Template | 0.0623\( \pm 0.0020 \) | 3.99\( \pm 0.06 \) | 15.00\( \pm 1.10 \) | 9.22\( \pm 0.16 \) | 7.90\( \pm 0.28 \) | 7.05\( \pm 0.45 \) | 2.17\( \pm 0.19 \) |
| PICO+       | Template | 0.0618\( \pm 0.0018 \) | 3.99\( \pm 0.06 \) | 14.76\( \pm 0.90 \) | 9.20\( \pm 0.17 \) | 7.84\( \pm 0.29 \) | 6.97\( \pm 0.46 \) | 2.24\( \pm 0.24 \) |

Reionization bias of primordial GWs

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