Evidence for Kosterlitz-Thouless and 3D-xy critical behavior in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$

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We present reversible magnetization data of a high quality Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ single crystal and explore the occurrence of 3D-xy critical behavior close to the bulk transition temperature $T_c$ and of Kosterlitz-Thouless (KT) behavior. Below and above the presumed Kosterlitz-Thouless transition temperature $T_{KT}$ we observe the characteristic 2D-xy behavior: a downward shift of the crossing point phenomenon towards $T_{KT}$ as the field is decreased and sufficiently below $T_{KT}$ the characteristic 2D-xy relationship between the magnetization an the in-plane magnetic penetration depth $\lambda_{ab}$. In contrast, the measured temperature dependence of the superfluid density does not exhibit the characteristic KT-behavior around the presumed $T_{KT}$. The absence of this feature is traced back to the 2D- to 3D-xy crossover setting in around and above $T_{KT}$. Invoking the Maxwell relation, the anomalous field dependence of the specific heat peak is also traced back to the intermediate 2D-xy behavior. However, close to $T_c$ we observe consistency with 3D-xy critical behavior, in agreement with measurements of $\lambda_{ab}$.

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I. INTRODUCTION

The study of thermal fluctuations received a considerable impetus from the discovery of the cuprate superconductors.\cite{1,2,3,4,5,6} It was realized that in these materials the critical regime where thermal fluctuations dominate can be attained and that some of them are in addition quasi two dimensional (2D)\cite{1} Furthermore, the systematics of the superconducting properties uncovered that the anisotropy is further enhanced with underdoping.\cite{1} In this quasi 2D limit one expects the thermodynamic properties to be close to those of a two-dimensional superconductor, or more precisely of a stack of decoupled two-dimensional superconducting sheets. Although approximate treatments have been invoked to describe the thermodynamic properties of such materials, the essential ingredient, the Kosterlitz-Thouless behavior of the associated zero field transition,\cite{8} has mostly not been taken into account.\cite{9,10,11} Only recently, this behavior was incorporated by combining Kosterlitz-Thouless (KT) renormalization-group flows and explicit computations for plasmas.\cite{12} On this basis the field and temperature dependence of the magnetization density, $m(H_c,T)$, for temperatures $T$ near to and below the Kosterlitz-Thouless (KT) transition temperature $T_{KT}$ was determined for magnetic fields $H_c$ applied perpendicular to the superconducting sheet. These results are interesting on three immediate fronts. First, the resulting trends in the magnetization appear to emerge from the recent Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ data of Lu Li et al. for underdoped and optimally doped samples.\cite{13} Second, by contrast evidence for smeared 3D-xy behavior stems from the measured temperature dependence of the in-plane magnetic penetration depth $\lambda_{ab}$.\cite{14,15,16} Third, the magnetic field dependence of the specific heat peak exhibits, opposite to the generic behavior a shift to higher temperatures with increasing field strength.\cite{17}

In this study we present reversible magnetization data of a Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ single crystal and explore the evidence for intermediate KT- (2D-xy) and 3D-xy critical behavior. Below $T = 89.5$ K $\approx T_{KT}$ we observe consistency with the KT-behavior in terms of the characteristic $m \propto \ln(H_c)$ dependence at fixed temperature. Furthermore, invoking the Maxwell relation $\partial^2 M/\partial T^2|_{H_c} = \partial (C/T)/\partial H_c|_T$ the anomalous field dependence of the specific heat peak is also traced back to 2D-xy behavior. However, close to the bulk transition temperature, $T_c \approx 91.21$ K we observe consistency with 3D-xy critical behavior, consistent with previous measurements of the in-plane magnetic penetration depth $\lambda_{ab}$.\cite{16,17,18} In Section II we sketch the theoretical background including the scaling relations for 2D- and 3D-xy critical behavior. Section III is devoted to the experimental details and in Section IV we present the analysis of the data uncovering the evidence for 2D-xy and 3D-xy critical behavior in the respective temperature regimes. We close with a brief summary and some discussion.

II. THEORETICAL BACKGROUND

When thermal fluctuations dominate and the coupling to the charge is negligible a bulk superconductor is expected to exhibit sufficiently close to $T_c$ 3D-xy critical behavior. In this case the magnetization per unit volume, $m = M/V$, adopts the scaling form\cite{15,16,18}

$$\frac{m}{TH_c^{1/2}} = -\frac{Q^\pm k_B\xi_{ab}}{\phi_0^{3/2}\xi_c} F^\pm(z),$$

$$z = \frac{(\pm)^{1/2} H_c}{\phi_0}.$$ (1)
In this form \( Q^\pm \) is a universal constant and \( G^\pm (z) \) a universal scaling function of its argument, with \( G^\pm (z = 0) = 1 \). In addition \( \gamma = \xi_{ab}/\xi_c \) denotes the anisotropy, \( \xi_{ab} \) the zero-field in-plane correlation length and \( H_c \) the magnetic field applied along the \( c \)-axis. In terms of the variable \( x \) the scaling form \([1,20]\) is similar to Prange’s result for Gaussian fluctuations.\([11]\) Approaching \( T_c \) the correlation lengths diverge as

\[
\xi_{ab,c} = \frac{\xi_{ab,c,0}}{|t|^{-\nu}}, \quad t = T/T_c - 1, \quad \pm = \text{sgn}(t). \tag{2}
\]

Supposing that 3D-\( xy \) fluctuations dominate the critical exponents are given by\([21]\)

\[
\nu \simeq 0.671 \simeq 2/3, \quad \alpha = 2\nu - 3 \simeq -0.013, \tag{3}
\]

and there are the universal critical amplitude relations\([20,21,22]\)

\[
\frac{\xi_{0,ab}}{\xi_{0,c}} \simeq 2.21, \quad \frac{Q^-}{Q^+} \simeq 11.5, \quad \frac{A^+}{A^-} = 1.07, \tag{4}
\]

and

\[
A^-\xi_{0,ab}\xi_{0,c} \simeq A^-\xi_{0,ab}^3/\gamma = (R^{-})^3, \quad R^{-} \simeq 0.815, \tag{5}
\]

where \( A^\pm \) is the critical amplitude of the specific heat singularity, defined as

\[
c = \frac{C}{\sqrt{k_B}} = \frac{A^\pm}{\alpha} |t|^{-\alpha} + B, \tag{6}
\]

where \( B \) denotes the background. The anisotropy is then characterized in terms of

\[
\gamma = \frac{\xi_{ab}}{\xi_c} = \frac{\xi_{ab}}{\xi_{c,0}}. \tag{7}
\]

Furthermore, in the 3D-\( xy \) universality class \( T_c, \xi_{0,c} \) and the critical amplitude of the in-plane magnetic penetration depth \( \lambda_{ab,0} \) are not independent but related by the universal relation\([20,21,24]\)

\[
k_B T_c = \frac{\Phi_0^2}{16\pi^3} \frac{\xi_{ab,0}}{\lambda_{ab,0}^2} = \frac{\Phi_0^2}{16\pi^3} \frac{\xi_{ab,0}}{\gamma \lambda_{ab,0}^2}. \tag{8}
\]

The existence of the magnetization at \( T_c \), of the magnetic penetration depth below \( T_c \) and of the magnetic susceptibility above \( T_c \) imply the following asymptotic forms of the scaling function\([21,24,19]\)

\[
Q^- \left. \frac{1}{\sqrt{z}} \frac{dG^-}{dz} \right|_{z \to -\infty} = Q^+ c^{-}, \tag{9}
\]

\[
Q^- \left. \frac{dG^-}{dz} \right|_{z \to 0} = Q^- c^{-} (ln z + c_1), \tag{9a}
\]

\[
Q^+ \left. \frac{1}{\sqrt{z}} \frac{dG^+}{dz} \right|_{z \to 0} = Q^+ c^+, \tag{9b}
\]

with the universal coefficients

\[
Q^- c^{-} \simeq -0.7, \quad Q^+ c^+ \simeq 0.9, \quad Q^\pm c^\pm \simeq 0.5, \quad c_1 \simeq 1.76. \tag{10}
\]

Noting that Bi2Sr2CaCu2O8+δ is highly anisotropic (\( \gamma >> 1 \))\([22]\) the system is expected to exhibit away from \( T_c \) 2D-\( xy \) behavior. A characteristic property of 2D-superconductors emerges from the magnetic field dependence of the magnetization. Sufficiently below the Kosterlitz-Thouless transition temperature \( T_{KT} \) the magnetization is given by\([23]\)

\[
m = \frac{\pi \rho_s(T)}{2d\Phi_0} \left( 1 - \frac{k_B T}{\pi \rho_s(T)} \right) \ln \left( \frac{\Phi_0}{4\pi H_c a_0^2 \gamma_3} \right) \tag{11}
\]

\[
= -\left( \frac{\Phi_0}{32\pi^2 \lambda_{ab}^2(T)} \right) \ln \left( \frac{\Phi_0}{4\pi H_c a_0^2 \gamma_3} \right) \tag{12}
\]

Here \( d \) is the thickness of the independent superconducting sheets, \( \gamma_3 \) is a parameter that vanishes as \( T \) approaches \( T_{KT} \), and \( a_0 \) the microscopic short-distance cutoff length. Moreover, \( \rho_s(T_{KT}) \) and \( T_{KT} \) are related by

\[
\rho_s(T_{KT}) = \frac{2}{\pi} k_B T_{KT}, \tag{13}
\]

while \( \rho_s(T) = 0 \) above \( T_{KT} \). Below this universal Nelson-Kosterlitz jump \( 1/\lambda_{ab}^2(T) \) increases as\([23,24]\)

\[
\lambda_{ab}^2(T_{KT})/\lambda_{ab}^2(T) = \frac{T_{KT}}{T_{KT}} \left[ 1 + \hat{b} (T_{KT} - T)^{1/2} \right]. \tag{14}
\]

Note that Eq. (11) gives a simple relation between the superfluid density and the derivative of the magnetization, namely

\[
\frac{dm}{d\ln(H_c)} = g(T) = \frac{\Phi_0}{32\pi^2 \lambda_{ab}^2(T)} \frac{k_B T}{d\Phi_0}. \tag{15}
\]

Furthermore, at criticality \( m \) depends on \( H \) in terms of \([22]\)

\[
m = -\frac{k_B T_{KT}}{d\Phi_0} \ln \left( \frac{\gamma_1 \ln \left( \frac{\Phi_0}{4\pi H_c a_0^2 \gamma_2} \right)}{\gamma_1} \right), \tag{16}
\]

where \( \gamma_1 \) and \( \gamma_2 \) are constants. Correspondingly, plots of \( M \) vs. \( T \) at different fields should then exhibit a systematic drift of the “crossing phenomenon”. Above \( T_{KT} \) and for asymptotically small fields the magnetization is given by\([22,20,21]\)

\[
m \simeq -\frac{k_B T}{2d\Phi_0} \xi_{ab} \xi_{ab} H_c, \quad \xi_{ab} = \xi_{ab,0} \exp \left( \frac{-\tilde{b}}{(T/T_{KT} - 1)^{1/2}} \right), \tag{17}
\]

\[
\tilde{b} = \hat{b} \frac{1}{2} (T_{KT} - T)^{1/2}. \tag{18}
\]
easily cleaved from the crystallized boule and annealed
25 can be found elsewhere.
stable for several days. A review of the growth technique
growth conditions at the flat zone interface were kept
star), equipped with two 1000 W halogen lamps. The
rotating at 18 rpm. The FZ growth was performed in
melting at a rate of 24 mm/h in Ar, the crystal growth
with a previously crystallized rod. After a first fast FZ
and about 7 mm in diameter. The seed rod was made
after pressing and sintering to a shape of 7 cm in length
was performed at a slow rate of 0.2 mm/h in a 7%
O2−93% Ar atmosphere, while both rods were counter-
rotating at 18 rpm. The FZ growth was performed in
a commercial two-mirror vertical furnace (from Cyber-

FIG. 1: (color online) Temperature dependence of the
measured reversible magnetic moment of the studied
Bi2Sr2CaCu2O8+δ single crystal at various magnetic fields
applied along the c-axis. The inset depicts susceptibility
measurements in 1 Oe in the ZFC and FC mode. The sharp onset
of superconductivity points to a transition temperature close
to $T_c \approx 91.2$ K.

where $\xi_{ab}$ is the Kosterlitz-Thouless correlation length.36
The parameters $b$ and $\tilde{b}$, determining the temperature
dependence of the magnetic penetration depth below the
jump (Eq. (14)), are related by:

$$\tilde{b}b = \pi / \left(2T_{KT}^{1/2}\right) \approx 0.17 \, \text{K}^{-1/2}.$$ (18)

III. EXPERIMENTAL DETAILS

Single crystals of Bi2Sr2CaCu2O8+δ were grown with the
Floating Zone (FZ) method, from direct crystallization
from the melt (no solvent used). The feed rod was
obtained from a commercial Bi2Sr2CaCu2O8+x powder
after pressing and sintering to a shape of 7 cm in length
and about 7 mm in diameter. The seed rod was made
with a previously crystallized rod. After a first fast FZ
melting at a rate of 24 mm/h in Ar, the crystal growth
was performed at a slow rate of 0.2 mm/h in a 7%
O2−93% Ar atmosphere, while both rods were counter-
rotating at 18 rpm. The FZ growth was performed in
a commercial two-mirror vertical furnace (from Cyber-

FIG. 2: (color online) $M/H_{c}^{1/2}$ vs. $T$ yielding the estimate
$T_c \approx 91.21$ K in terms of the crossing point at $M/H_{c}^{1/2} \approx
−2.13 \times 10^{-7}$ emuOe$^{-1/2}$. 

The Bi2Sr2CaCu2O8+δ sample used in this work, a
$V \approx 4.6 \times 10^{-5}$ cm$^3$ single crystal, was chosen by its sharp
low-field Meissner transition from several high quality
single crystals. The magnetization was measured in a
Quantum Design DC-SQUID magnetometer MPMS XL
with an installed Reciproting Sample Option. The inset
in Fig. 1 shows the measured susceptibility at $H_c = 1$ Oe
applied along the c-axis. It reveals a rather sharp transition
at $T_c \approx 91.2$ K and a well saturated Meissner state,
pointing to excellent quality. The volume of the sample
was estimated by susceptibility measurements below $T_c$
in the Meissner state with a magnetic field applied along the
$ab$-plane to minimize demagnetization effects. The
extracted volume of $V \approx 4.6 \times 10^{-5}$ cm$^3$ compares well
with that estimated with an optical microscope. Fig.
1 summarizes the measured temperature dependence of
the magnetic moment at fields ranging from 25 Oe to
15000 Oe applied along the c-axis. After applying the
magnetic field, well below $T_c$ it was kept constant and
the magnetic moment of the single crystal was measured
at a stabilized temperature by moving the sample with a
frequency of 0.5 Hz through a set of detection coils.
The reversible superconducting diamagnetic magnetization,
$M = nV$, was then obtained by comparing field
cooled (FC) and zero-field cooled (ZFC) data. Due to a
substantial pinning contribution at low magnetic field
IV. DATA ANALYSIS

We are now prepared to analyze the magnetization data. To estimate the bulk $T_c$ we invoke Eqs. (1), (5) and (10), revealing that the plot $m/H_{c}^{1/2} \text{ vs. } T$ should exhibit a crossing point at $T_c$. Here $m/(TH_{c}^{1/2})$ adopts with Eq. (7) the value $m/(T_{c}H_{c}^{1/2}) = -0.5k_{B}\gamma_{0}^{-3/2}$. According to Fig. 2, showing $M/H_{c}^{1/2} \text{ vs. } T$ there is a crossing point at $T_{c} \approx 91.21$ K where $M/H_{c}^{1/2} \approx -2.13 \times 10^{-7}$ emuOe$^{-1/2}$. With $V \approx 4.6 \times 10^{-5}$ cm$^3$, where $m = M/V$, it yields for the anisotropy (Eq. 7) the estimate

$$\gamma = \frac{\xi_{ab}}{\xi_{c}} \approx 69,$$

compared to $\gamma \approx 133$ for an underdoped sample with $T_{c} \approx 84.2$ K and in reasonable agreement with earlier estimates for optimally doped samples$^{22,26}$.

This rather large anisotropy the 2D- to 3D-$xy$ crossover is expected to occur rather close to the bulk $T_{c}$. Indeed, there is as predicted a downward shift of the “crossing point” towards $T_{KT}$ from above as the field is decreased$^{12}$. The same behavior was also observed in the highly anisotropic Tl-1223, Bi-2201, and underdoped La$_{2-x}$Sr$_x$CuO$_4$ single crystals$^{22,27,28,29}$.

To substantiate 2D-$xy$ behavior further we invoke Eq. (11) in terms of the plot $M \text{ vs. } H_{c}$ shown in Fig. 4.

The solid lines in Fig. 4 indicate that sufficiently below $T_{KT} \approx 89.5$ K the 2D relation (15) between the superfluid density and the derivative of the magnetization with respect to the logarithm of the field is for small fields well obeyed, so in this temperature regime the system indeed behaves as a stack of essentially decoupled two-dimensional layers.

FIG. 3: (color online) $M$ vs. $T$ at various magnetic fields exhibiting a crossing phenomenon shifting towards $T_{KT} \approx 89.5$ K from above as the field is decreased.

FIG. 4: (color online) $M$ vs. $H_{c}$ at various fixed temperatures. a) from $T = 82$ to 90.7 K. The straight lines indicate the characteristic 2D relation between the superfluid density and the derivative of the magnetization with respect to the logarithm of the field (Eq. (15)), valid in the low field regime as long as pinning contributions are negligible. b) $M$ vs. $H_{c}$ at $T = 91, 91.25$ and 91.5 K. The solid line is $M = -2.13 \times 10^{-7} \cdot H_{c}^{1/2}$ emu indicating the characteristic 3D-$xy$ behavior at $T_{c}$ in terms of Eq. (20).
FIG. 5: (color online) $1/\lambda_{ab}^2(T)$ vs. $T$.  $\cdot$ derived from the magnetization with the aid of Eq. (11) rewritten as $M(H_c, T) = f(T) + g(T) \cdot \ln(H_c)$ and $d = 30 \AA$; $\blacksquare$: Experimental data derived from Lee et al. with $\lambda_{ab} (T = 0) = 1350 \AA$. The solid line is $1/\lambda_{ab}^2(T) = 1/\lambda_0^2(1 - T/T_c)^{2/3}$ with $1/\lambda_{ab}^2 = 5 \times 10^9 \text{cm}^{-2}$ and $T_c = 91.21 \text{K}$ indicating 3D-xy critical behavior. The dashed line is the KT-line $1/\lambda_{ab}^2(T) = 3.64 \times 10^6 \cdot T \text{ cm}^{-2}$ (Eq. (13)), the dotted one $1/\lambda_{ab}^2(T_{KT}) = 3.26 \times 10^8 \text{ cm}^{-2}$ with $T_{KT} = 89.5 \text{K}$ and the dash-dot-dot-one is Eq. (14) with $\delta \approx 1 \text{K}^{-2/3}$.

dimensional Kosterlitz-Thouless films with $T_{KT} \approx 89.5 \text{K}$. However, close to the bulk $T_c \approx 91.21 \text{K}$ one expects 3D-xy critical behavior. In this case and in the magnetic field range considered here the limit $z \to \infty$ is then approached. Here Eqs. (1), (9) and (10) imply the limiting behavior

$$m = - \frac{Q^+ c_\infty k_B \xi_{ab} T}{\Phi_0^{3/2} \xi_c} H_c^{1/2} \simeq - \frac{0.5k_B \xi_{ab0} T}{2\Phi_0^{3/2} \xi_c} H_c^{1/2}.$$  (20)

Fig. 4b, depicting $M$ vs. $H_c$ close to $T_c \approx 91.21 \text{K}$ at $T = 91, 91.25$ and $91.5 \text{K}$, shows that this expectation is well confirmed. According to this, approaching $T_c$ the system undergoes a 2D-to 3D-xy crossover. It implies that the characteristic 2D-xy critical behavior, including the jump of the superfluid density at $T_{KT}$ (Eqs. (13) and (14)), are removed.

To clarify this point we invoke Eq. (11) to determine the temperature dependence of $1/\lambda_{ab}^2$ sufficiently below $T_{KT}$. Setting $M(H_c, T) = f(T) + g(T) \cdot \ln(H_c)$, $g(T)$ is then given by Eq. (15) and follows from plots as shown in Fig. 4b in terms of the slope of the straight lines. For $d = 30 \AA$ we obtain for $1/\lambda_{ab}^2(T)$ the data points shown in Fig. 5. For comparison we included the ab-plane microwave surface impedance data of Lee et al. for a high-quality Bi$_2$Sr$_2$CaCu$_2$O$_8$ single crystal, plotted in terms of the superfluid density, assuming $\lambda_{ab}(0) = 1350 \AA$. Sufficiently below $T_{KT}$ we observe reasonable agreement with the 2D-xy prediction (●) and the measured $1/\lambda_{ab}^2(T)$ (■). Accordingly, in this regime the expulsion of vortices from the KT phase dominate. Here we also observe consistency with the characteristic KT-behavior indcated by the dash-dot-dot line (Eq. (14)). However, around $T_{KT}$, indicated by the horizontal and KT-line, the measured $1/\lambda_{ab}^2(T)$ does not exhibit any evidence for the characteristic jump from $1/\lambda_{ab}^2(T_{KT})$ to zero for $T > T_{KT}$. In contrast agreement with the leading 3D-xy critical behavior is observed (solid line), revealing the removal of the characteristic jump in $1/\lambda_{ab}^2(T)$ at $T_{KT}$ due to the 2D- to 3D-xy crossover. To explore the effect of the 3D-xy fluctuations further, we use the critical amplitude $1/\lambda_{ab0} = 5 \times 10^9 \text{cm}^{-2}$ and $T_c = 91.21 \text{K}$ to obtain from the universal relation (3) the estimate

$$\xi_{c0}^- \approx 2.9 \AA,$$  (21)

for the amplitude of the $c$-axis correlation length. The occurrence of 3D-xy critical behavior then requires that

$$\xi_{c}(T) = \xi_{c0} |t|^{-2/3}$$

exceeds $d$ considerably, yielding with $d = 30 \AA$ and $T_c = 91.21 \text{K}$ for the onset of 3D fluctuations the lower bound $T > 0.24 \text{K}$. Furthermore, around and above $T = 91 \text{K}$ is the 3D-xy critical regime is attained (see Fig. 4b). So the occurrence of KT-behavior in $1/\lambda_{ab}^2(T)$ is restricted to temperatures below $T_{KT} \approx 89.5 \text{K}$.

On the other hand, the small amplitude of the $c$-axis correlation length leads with $\gamma = 69$ (Eq. (19)) to a rather large amplitude of the in-plane correlation length, $\xi_{ab0} = \gamma \xi_{c0} \approx 200 \AA$ and with the universal relation (5) to a very small amplitude of the specific heat singularity,

$$A^- = \frac{0.815}{\gamma^2 (\xi_{c0}^-)^3} \approx 4.7 \times 10^{-6} \approx 3,$$  (22)

in comparison with $A^- \approx 6.8 \times 10^{-4} \approx 3$ in optimally doped YBa$_2$Cu$_3$O$_{6.5}$ and $1.7 \times 10^{-3} \approx 3$ in 4He. This small critical amplitude renders it difficult to observe 3D-xy critical behavior in the specific heat of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. Considering Eqs. (1), (9) and (10) consistency with 3D-xy critical behavior also requires that the data scales below $T_c$ as

$$|t|^{-2/3} \frac{m}{T} = - \frac{Q^- c_\infty k_B}{\Phi_0 \xi_{c0}} \left( \ln \left( \frac{\xi_{ab0}}{\Phi_0} H_c |t|^{-4/3} \right) + c_1 \right), \quad z \to 0,$$

and

$$|t|^{-2/3} \frac{m}{T} = - \frac{Q^- c_\infty k_B \xi_{ab0}}{\Phi_0^{3/2} \xi_{c0}^{3/2}} \left( H_c |t|^{-4/3} \right)^{1/2}, \quad z \to \infty,$$  (24)
and the dashed one. In Fig. 6 we plot \(-|t|^{-2/3} M/T\) vs. \(H_c |t|^{-4/3}\) for \(T < T_c\) at various applied magnetic fields. The solid line is Eq. \(25\) and the dashed one Eq. \(26\).

\[
-|t|^{-2/3} \frac{M}{T} = -3.32 \times 10^{-6} \left( \ln(H_c |t|^{-4/3}) - 11.23 \right) \text{ (emuK}^{-1})
\]

and the dashed one

\[
-|t|^{-2/3} \frac{M}{T} = 2.3 \times 10^{-9} (H_c |t|^{-4/3})^{1/2} \text{ (emuK}^{-1})
\]

represents the limit \(24\). While the \(H_c |t|^{-4/3}\) \(\propto \) \(z \to \infty\) limiting behavior (dashed curve) is well confirmed we observe with decreasing \(z\) substantial deviations from the data collapse on a single curve.

From Eq. \(26\) we derive with \(V = 4.6 \times 10^{-5} \text{ cm}^3\), where \(m = M/V\), the estimate

\[
\gamma = \xi_{ab0}/\xi_0 \simeq 68,
\]

in reasonable agreement with \(\gamma = \xi_{ab0}/\xi_0 \simeq 69\) (Eq. \(19\)), derived from the crossing point in \(M/H_c^{1/2}\) vs. \(T\) shown in Fig. 2. The apparent failure of 3D-\(xy\) scaling outside the limit \(H_c |t|^{-4/3} \propto z \to \infty\) is then attributable to the 3D- to 2D-\(xy\) crossover. In particular, very small magnetic fields would be required to attain the limit \(H_c |t|^{-4/3} \propto z \to 0\) with reduced temperatures \(t\) in the range where 3D-\(xy\) fluctuations dominate. Indeed, given our evidence for 2D-\(xy\) behavior above \(T_{KT} \simeq 89.5\) K in terms of the crossing phenomenon (Fig. 3) and

\[
M/(TH_c^{1/2}) \propto |t| H_c^{-3/4}
\]

at various applied magnetic fields. The dashed line is Eq. \(30\).
Noting again that the occurrence of 3D behavior requires that the c-axis correlation length $\xi_c = \xi_0^{+}|t|^{-2/3}$ exceeds the interlayer spacing $d$ it is clear that the 3D- to 2D-xy crossover is not restricted to temperatures below $T_c$. Given the universal ratio $\xi_c^+ \sim \xi_0^{+}/2.21$ (Eq. 4) it even follows that above $T_c$ the 3D-xy critical regime is even much narrower. Considering then the limits $H_c |t|^{-1/3} \propto z \to 0$ and $z \to \infty$ above $T_c$, Eqs. (1), (9) and (10) yield the scaling forms

$$|t|^{-2/3} \frac{M}{T} = -\frac{Q^+ c_0^+ k_B}{\Phi_0^{2/3} \xi_0^{+}} (H_c |t|^{-1/3})^{1/2}, z \to \infty.$$  

and

$$|t|^{-2/3} \frac{M}{T} = -\frac{Q^+ c_0^+ k_B}{\Phi_0^{2/3} \xi_0^{+}} H_c |t|^{-1/3}, z \to 0$$  

In Fig. 7 we depicted $-|t|^{-2/3} M/T$ vs. $H_c |t|^{-1/3}$ for $T > T_c$ at various applied magnetic fields. The solid line is Eq. (28) in terms of

$$-|t|^{-2/3} \frac{M}{T} = 2 \times 10^{-13} H_c |t|^{-1/3} \text{ (emuK}^{-1}$$  

and the dashed one

$$-|t|^{-2/3} \frac{M}{T} = 2.3 \times 10^{-9} (H_c |t|^{-1/3})^{1/2} \text{ (emuK}^{-1}$$  

corresponding to the limit (29). Note that Eqs. (26) and (31) fully agree with each other and with that confirm $Q^+ c_0^+ = Q^+ c_0^-$ (Eq. 10). In analogy to Fig. 6 we observe that the $z \to \infty$ limiting behavior is well confirmed, while substantial deviations from a data collapse on a single curve set in for $H_c |t|^{-1/3} < 2 \times 10^6$ Oe. Here the crossover to 2D-xy behavior sets in and 3D-xy scaling fails. Nevertheless, in the limit $H_c \to 0$ and reduced temperatures $t$ in the range where 3D-xy fluctuations dominates, the limit $H_c |t|^{-1/3} \propto z \to 0$ should be attainable. It is indicated by the solid line (Eq. (30)) in Fig. 7.

According to Eq. 1 one expects that the data plotted as $M/(TH_c^{1/2})$ vs. $|t|H_c^{-3/4}$ should fall on two branches. An upper branch corresponding to $T > T_c$ and a lower one for $T < T_c$. A glance at Fig. 8 depicting this plot, clearly reveals the flow to 3D-xy critical behavior by approaching $T_c$ (at $|t| = 0$) and in particular consistency with the leading 3D-xy critical behavior below $|t|H_c^{-3/4} = 10^{-3}$ Oe$^{-3/4}$.

Another property where KT-behavior in terms of the 2D- to 3D-xy crossover should be observable is the magnetic field dependence of the specific peak. Indeed, in Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ this peak shifts[13] opposite to the generic behavior[12]. Given the Maxwell relation,

$$\frac{\partial^2 M}{\partial T^2} |_{H_c} = \frac{\partial}{\partial H_c} (C/T) |_{T},$$  

relating magnetization and specific heat, this abnormality should also be observable in $\partial^2 M/\partial^2 T |_{H_c}$. A glance at Fig. 9 depicting $\partial^2 M/\partial^2 T |_{H_c}$ vs. $T$ for various magnetic fields reveals consistency with the anomalous shift of the specific heat peak. Indeed the dip shifts towards higher temperatures.

Noting that the anomalous shift occurs slightly above $T_{KT} \simeq 89.5$ K it is indeed expected to reflect 2D-xy behavior. To check this conjecture we invoke expression[17] for the magnetization valid above $T_{KT}$ in the limit $H_c \to 0$. Although this limit is not attained in the field range considered here, $m$ is expected to scale as $m \propto - (k_BT / (2d\Phi_0^c)) \xi_{ab} f(H_c)$ with $f(H_c) = H_c$ for $H_c \to 0$[23] Accordingly, $\partial^2 M/\partial^2 T |_{H_c}$ diverges at $T_{KT}$ However, there is the magnetic field induced finite size effect preventing the correlation length $\xi_{ab}$ to grow beyond the limiting magnetic length $L_{H_c} = (\Phi_0 / (aH_c))^{1/2}$ where $a \simeq 3.12$[14,16] As a consequence, in finite fields the divergence is removed by a dip. Its minimum occurs at $T_p$ given by

$$\xi_{ab}(T_p) = L_{H_c} = (\Phi_0 / (aH_c))^{1/2}$$  

In Fig. 10 we show $T_p$ vs. $H$ derived from the data depicted in Fig. 9. For comparison we included

$$T_p(H_c) = T_{KT} \left(1 + \sqrt{b/\ln(\frac{\Phi_0}{a\xi_{ab}^2 H_c})} \right)$$  

resulting from Eqs. (17) and (33), for a realistic set of parameters. Apparently, the characteristic shift towards higher temperatures and the $\ln(H_c)$ behavior are
FIG. 10: (color online) \textit{T}_p \textit{vs.} \textit{H}_c determined from the data depicted in Fig. 9. The solid line is Eq. (34) with \textit{T}_{KT} = 89.5 K, \( \tilde{b} \approx 0.3 \), \( a = 3.12 \), and \( \xi_{ab0} = 84 \) Å.

well confirmed and the values for \( \tilde{b} \) (Eq. (17)) and \( \hat{b} \) (Eq. (14)) are reasonably consistent with the relation \( \tilde{b} \hat{b} = \pi / (2T_{KT}^{1/2}) \approx 0.17 \) K\(^{-1/2} \) (Eq.(18)). Furthermore, the observed \( H_c \) dependence of \( T_p \) down to 25 Oe also implies that the lateral extent \( L_{ab} \) of the homogenous regions exceeds \( L_{H_c} \approx 3640 \) Å. Indeed, in the opposite case (\( L_{ab} < L_{H_c} \)) \( T_p \) would be independent of \( H_c \). Thus, \( L_{ab} > 3640 \) Å uncovers a remarkable sample homogeneity. Nevertheless it appears to be unlikely that the asymptotic \( H_c \rightarrow 0 \) is experimentally attainable. Here \( T_p(H_c) \) follows from \( \xi_{ab} = \xi_{ab0}[T_p]^{-2/3} = L_{H_c} \), so

\[
T_p = T_c \left( 1 - \frac{aH_c (\xi_{ab0}^{-2})}{\Phi_0} \right)^{3/4}, \tag{35}
\]

whereupon \( T_p \) shifts with increasing field to lower temperatures, as observed in a variety of less anisotropic cuprates.

\section*{V. SUMMARY AND DISCUSSION}

Even though the mechanism of superconductivity in the cuprates remains mystery the associated phase tran-

sition properties can be understood as consequences of thermal fluctuations within the framework of the theory of critical phenomena. In this work we presented and analyzed reversible magnetization data of the highly anisotropic \( \text{Bi}_{2}\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) for magnetic fields applied along the \( c \)-axis of the high quality single crystal. We examined the occurrence of 3D-\( xy \) critical behavior close to the bulk transition temperature \( T_c \) and of Kosterlitz-Thouless behavior. Below \( T_c \) and above the presumed Kosterlitz-Thouless transition temperature \( T_{KT} \) we observed, in agreement with the theoretical prediction\(^{12} \) a downward shift of the “crossing point” towards \( T_{KT} \) from above as the field is decreased. Sufficiently below \( T_{KT} \) we verified the characteristic 2D-\( xy \) relationship between the magnetization an the in-plane magnetic penetration depth\(^{13} \). In contrast, we have seen that the measured temperature dependence of the superfluid density does not exhibit the characteristic KT-behavior (Nelson-Kosterlitz jump) around the presumed \( T_{KT} \). The absence of this feature was traced back to the 2D- to 3D-\( xy \) crossover setting in around and above \( T_{KT} \). Indeed, in the limit \( H_c |t|^{-4/3} \rightarrow \infty \) we established clear evidence for 3D-\( xy \) critical behavior above and below \( T_c \), while in the opposite limit \( (H_c |t|^{-4/3} \rightarrow 0) \) its failure was attributed to the dimensional crossover. Invoking the Maxwell relation \( \partial^2 M / \partial^2 T \big|_{|J_f|} = \partial (C/T) / \partial H \big|_T \), the anomalous field dependence of the specific heat peak was also traced back to the intermediate 2D-\( xy \) behavior\(^{15} \). Implications include: First, sufficiently below \( T_{KT} \) the isotope and pressure effects on \( dm/d\ln(H_c) \) at fixed temperature \( T \) and the in-plane magnetic penetration depth \( \lambda_{ab}(T) \) are not independent but related by Eq. (15). Second, the bulk transition temperature \( T_c \) and the critical amplitudes of the \( c \)-axis correlation length and in-plane magnetic penetration depth are related by the universal relation\(^{14} \), so the isotope and pressure effects on these properties are related\(^{15} \).

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