The Cepheid Distance Scale after Hipparcos

Frédéric Pont
Observatoire de Genève, Switzerland

Abstract.

More than two hundred classical cepheids were measured by the Hipparcos astrometric satellite, making possible a geometrical calibration of the cepheid distance scale. However, the large average distance of even the nearest cepheids measured by Hipparcos implies trigonometric parallaxes of at most a few mas. Determining unbiased distances and absolute magnitudes from such high relative error parallax data is not a trivial problem.

In 1997, Feast & Catchpole announced that Hipparcos cepheid parallaxes indicated a Period-Luminosity scale 0.2 mag brighter than previous calibrations, with important consequences on the whole cosmic distance scale. In the wake of this initial study, several authors have reconsidered the question, and favour fainter calibrations of cepheid luminosities, compatible with pre-Hipparcos values.

All authors used equivalent data sets, and the bulk of the difference in the results arises from the statistical treatment of the parallax data. We have attempted to repeat the analyses of all these studies and test them with Monte Carlo simulations and synthetic samples. We conclude that the initial Feast & Catchpole study is sound, and that the subsequent studies are subjected in several different ways to biases involved in the treatment of high relative error parallax data. We consider the source of these biases in some detail. We also propose a reappraisal of the error budget in the final Hipparcos cepheid result, leading to a PL relation – adapted from Feast & Catchpole – of

\[ M_V = -2.81 \text{(assumed)} \log P - 1.43 \pm 0.16 \text{(stat)} \pm 0.03 \text{(syst)} \]

We compare this calibration to recent values from cluster cepheids or the surface brightness method, and find that the overall agreement is good within the uncertainties.

We conclude by commenting on the mismatch between the cepheid parallax distance scale and kinematical determinations, for cepheids as well as RR Lyrae.

1. Introduction

The cepheid distance scale is the central link of the cosmic distance ladder. Because even the nearest cepheids are more than 100 pc away from the Sun
(Polaris at $\sim 130$ pc, $\delta$ Cep at $\sim 300$ pc), no reliable parallax measurements were available for cepheids before the Hipparcos mission. The zero-point of the Period-Luminosity (PL) relation was calibrated by secondary methods, using cepheids in open clusters and associations or Baade-Wesselink techniques. The Hipparcos data for cepheids opened the possibility of a geometric determination of the zero-point of the PL relation.

Soon after the Hipparcos data release, Feast & Catchpole (1997, FC) announced that Hipparcos cepheid data indicated a zero-point about 0.2 mag brighter than previously thought (implying $\mu=18.70$ for the LMC). Adopting the FC notation for the PL relation:

$$M_V = \delta \log P + \rho$$

they find $\rho = -1.43 \pm 0.13$ (for $\delta = -2.81$ assumed). This result attracted considerable attention, since it implied a $\sim 10\%$ downward revision of the value of $H_0$ obtained from galaxy recession velocities. In turn, the higher expansion ages derived could become compatible with the new, lower ages for globular clusters obtained from Hipparcos subdwarfs, resolving the “cosmic age problem” (the fact that globular cluster ages were found to be much older than the expansion age of the Universe).

In the past year however, several authors reconsidered the Hipparcos cepheid data, and voiced criticism against the FC result. All subsequent studies obtain calibrations for the zero-point of the PL relation similar to pre-Hipparcos values, or fainter (see sketch Fig. 1). Szabados (1997) pointed out that known or suspected binaries were abundant in the cepheid sample, and that the PL relation found after removing them was compatible with pre-Hipparcos values. Madore & Freedman (1998) repeated the whole analysis with multi-wavelength data (BVIJHK). They also concluded on a fainter zero-point. Oudmaijer et al. (1998), in a study devoted to the effect of the Lutz-Kelker bias on Hipparcos luminosity calibrations, analyse the cepheid data as an illustration and derive $\rho = -1.29 \pm 0.02$. Finally, Luri et al. (1998) apply the “LM method” (Luri et al. 1996) to the cepheid sample and find a very faint zero-point of $\rho = -1.05 \pm 0.17$.

The situation, one year after the Hipparcos data release, is therefore very perplexing: while before Hipparcos the zero-point of the cepheid PL relation seemed reasonably well determined within 0.1 mag (e.g. Gieren et al. 1998), values derived from the same Hipparcos parallax data cover a range of 0.4 mag!

Fortunately, this distressing situation may be only temporary. As all the studies considered use the same parallax data and similar photometric and reddening values for Hipparcos cepheids, the differences are almost entirely due to the statistical procedures used in the analyses. Contrarily, for instance, to the case of the very metal-poor globular cluster distance scale where observational uncertainties may remain the limiting factor (see review by Cacciari in this volume), it may be hoped that the disagreements about the PL zero-point can be resolved by evaluating the different approaches.

This is what we are attempting here. From statistical considerations on one hand, Monte Carlo simulations of the different procedures on the other hand, we have tested the robustness and possible biases of the different procedures. The results are presented below in Par. 3 to 8, study by study, beginning with an
imaginary author using the unsophisticated "direct method". It appears that the difficulties involved in the treatment of high relative error parallax data are at the core of the question. On this subject, the review by Arenou in this volume constitute an excellent complement to the present contribution – showing the pitfalls easiest to overlook. As far as cepheids are concerned, we shall contend here that, in fact, the situation is rather clear, and that much of the apparent disagreement is caused by statistical biases that have already been reported in dealing with parallax data.

2. Hipparcos cepheids

Around 200 cepheids were measured with Hipparcos. All but the nearest ones are so remote that $\pi_{\text{real}} < \sigma_\pi$. There are 19 of them with $\sigma_\pi/\pi < 50\%$. The closest cepheid, Polaris, was measured at $\pi = 7.56 \pm 0.48$ mas. $\delta$ Cep itself has $\pi = 3.32 \pm 0.58$ mas. Despite the low accuracy of individual parallax determinations, it is not unreasonable to attempt an accurate determination

---

1 Parallaxes are noted $\pi$ and their uncertainty $\sigma_\pi$
of the PL relation zero-point, because relative distances are precisely known through the PL and PLC relations. Each individual parallax can therefore be seen as a measurement of the PL zero-point. A large number of low-accuracy parallaxes can yield a reliable combined value of the zero-point.

3. Dangers of the “direct method”

The most natural way to infer absolute magnitude from parallax would be to calculate distances from the inverse of the parallax and use Pogson’s law:

\[ m_V - M_V = 5 \log \left( \frac{1}{\pi} \right) - 5 + a_V \]  

(1)

Then, the zero-point of the PL relation could be fit in the Period-\(M_V\) plane, by some kind of weighted least-square.

In fact, this is an extremely biased way to proceed when the relative errors on parallax are high \((\sigma_\pi/\pi > \sim 20\%)\). Fig. 2 shows graphically why the inverse of the parallax is a biased estimator of the distance, and how the magnitude derived from Equ. 1 has a very asymmetrical and skewed distribution if the statistical distribution of the parallax is gaussian. This effect was pointed out by several authors in the wake of the publication of the Hipparcos catalogue (e.g. Luri & Arenou 1997). An essential condition to the use of least-square fit is the symmetry of the error distribution, and if it is not satisfied, first-order biases are to be expected. Moreover, in order to use Equ. 1, negative parallaxes must be ignored and the data selected by some cut in \(\sigma_\pi/\pi\). Both selection criteria introduce further biases. By discriminating against low \(\pi_{\text{meas}}\) at a given \(\sigma_\pi\) and \(\pi_{\text{real}}\), they bias the result towards lower distances and fainter magnitudes.

![Figure 2](image)

Figure 2. Statistical distributions of the parallax, distance computed from \(d = 1/\pi\) and absolute magnitude from Equ. 1 with \(\pi_{\text{real}}=3\) mag and \(\sigma_\pi=1\) mas. The distance and absolute magnitude distributions are biased and asymmetrical.

Monte Carlo simulations with samples similar to the actual Hipparcos cepheid sample show that this “direct method” leads to a bias of \(\sim 0.2\) mag towards a
fainter PL relation zero-point. An important bias indeed, due exclusively to the incorrect statistical treatment of parallax data.

4. Shifting to parallax space: Feast & Catchpole 1997

FC avoid the difficulties of the $\pi \rightarrow M_V$ transformation with a change of variable that allows the parallaxes to be combined linearly. Instead of deriving the PL relation zero-point from $M_V = \delta \log P + \rho$ and Equ. 1, FC compute $10^{0.2\rho}$ from the mathematically equivalent relation

$$10^{0.2\rho} = 0.01 \pi 10^{0.2(m_V - a_V - \delta \log P)}$$

The final value of $\rho$ is recovered from the average of the values of $10^{0.2\rho}$ weighted by the uncertainty on the right term of (2).

At first sight the procedure adopted by FC looks unnecessarily complicated. But we saw in the previous section why a more straightforward approach is not preferable. The FC method removes the statistical biases affecting the direct method: since the parallax appears linearly in (2), negative parallaxes can be kept, no $\sigma_\pi/\pi$ cut is needed, and the uncertainties are symmetrical. However, the condition for the use of this method is that the uncertainties on the exponent $0.2(m_V - a_V - \delta \log P)$ be smaller than the uncertainties on $\pi$. For this reason, the method is only reliable for a group of objects with errors on the relative distances much smaller than the parallax errors, such as the Hipparcos cepheids. Any dispersion of the exponent $0.2(m_V - a_V - \delta \log P)$ will make the distribution of errors on the right term of (2) asymmetrical again, and result in a bias towards brighter magnitudes.

We have tested the FC procedure with Monte Carlo simulations on synthetic samples of various composition. The effect of modifying several assumptions was tested, such as varying the slope of the PL relation, the width of the instability strip or the spatial distribution of cepheids. Samples were also drawn from a larger volume, so that classic Lutz-Kelker biases would be modeled. Representa-

| Sample                      | $\rho$  | $\sigma_\rho$ | N    |
|-----------------------------|---------|---------------|------|
| all (27 stars)              | -1.426  | 0.128         | 1000 |
| without $\alpha$ Umi (26 stars) | -1.433  | 0.197         | 1000 |
| without overtones (20 stars)| -1.436  | 0.232         | 1000 |
| weights < 50 (5 stars)      | -1.448  | 0.267         | 1000 |

Table 1. First test of the FC method on synthetic samples. Starting from samples identical to the real one, each Monte Carlo realization varies the errors on the parallaxes according to the stated $\sigma_\pi$. The input value of $\rho$ is -1.43 in all cases.
Number of stars $<\rho>$ $\Delta M_V$ limit weight
50'000 $-1.431$ 0.2 10
50'000 $-1.438$ 0.5 10
50'000 $-1.461$ 1.0 10
20'000 $-1.430$ 0.2 50

Table 2. Second test of the FC method on synthetic samples. Samples of cepheids filling a volume of space larger than the selection limits are built. The FC procedure is applied to the subsample fulfilling the FC selection criteria. This simulation takes the classic Lutz-Kelker bias into account. The $\Delta M_V$ parameter is the width of the instability strip. The input $\rho$ is $-1.43$ in all cases.


tative results are shown on Tables 1 and 2. The conclusion is that the FC method is sound and robust, and that systematic biases are smaller than 0.03 mag.

However, the dispersion of the results recovered in the simulations is substantially higher than that stated in FC, indicating that the final uncertainty may have been underestimated. We return on this point in Par. 5.

5. Possible effect of binaries: Szabados 1997

A large fraction of known cepheids are confirmed or suspected binaries. Szabados (1997, SZ) has pointed out that binary cepheids showed more scatter in the PL diagram than single cepheids, and suggested that this could be due to the noise induced by binarity on Hipparcos parallax determinations. As an orbit of $\sim$1 AU has the amplitude of the parallax at any distance, unrecognized companions could in principle interfere with the parallax measurement. By fitting a PL relation on single cepheids only (Fig. 3a), SZ recovers a zero-point equivalent to the pre-Hipparcos values (about 0.2 mag fainter than FC).

As shown by Fig. 3a, suspected binaries have larger error bars on average than single cepheids. But given these error bars, the scatter for binary suspects is compatible with the uncertainties (as confirmed by a Kolmogorov-Smirnov test). The question is to understand why binaries have larger error bars, and for this the $\log P - M_V$ plane is not a good representation, as explained in Par. 3, because the same parallax uncertainty can result in greatly different $M_V$ uncertainties.

When the uncertainties are considered in parallax space (Fig. 3b, 3c.), it appears that at a given magnitude the binary cepheids do not have significantly larger parallax uncertainties than the single cepheids. The high dispersion of the binary group on Fig. 3a is simply due to the fact that, on average, suspected binaries are fainter and more remote. Now, this has to be a chance effect due to low-number statistics (unless some weird mechanism weeds out binary cepheids off the solar neighbourhood).

²Similar results were obtained by X. Luri (priv. comm.) who has also extensively tested the FC procedure. These small residual biases are caused by the asymmetrical effect of the dispersion in the exponent $0.2(V_0 - \delta \log P)$ of Equ.

6
Figure 3.  a. Period-luminosity diagram for the nearest Hipparcos cepheids as in Szabados (1997). Known or suspected binaries in white. Error bars from the parallax uncertainties. b. Parallax uncertainties $\sigma_\pi$ as a function of apparent magnitude. c Normalised parallax residuals $(\pi_{\text{observed}} - \pi_{PL})/\sigma_\pi$ as a function of apparent magnitude. $\pi_{PL}$ is the parallax expected with the distance computed from the PL relation. The distribution of parallax residuals, for suspected binaries as well as for single stars, is compatible with a normal distribution.
Another aspect of Fig 3a is that eight single stars look much nearer to the mean relation than their uncertainties would indicate, adding to the visual impression that binary suspects are much more scattered. These stars have $\sigma_\pi = 0.5-0.7$ mas, and much smaller residuals: $\langle \pi_{\text{obs}} - \pi_{PL} \rangle = 0.07$ mas. Again, this can only be due to chance. The alternative is to suggest that, for some reason, Hipparcos parallaxes are a factor 7-10 more precise for single cepheids than for any other star in the catalogue, a rather unreasonable hypothesis. If this sounds like a strange coincidence, one should keep in mind that the cepheid sample was split into two parts under several criteria to check for systematic effects - single and binary, overtone and fundamental pulsator, low and high period, low and high reddening - and that a slightly strange-looking distribution for 8 points according to one of these criteria should not be over-interpreted. A Kolmogorov-Smirnov test indicates that the normalised parallax residuals $(\pi_{\text{obs}} - \pi_{PL})/\sigma_\pi$ (see Fig. 3) for binaries only, for single stars only and for the combined sample, are all compatible with a normal distribution. The lowest KS coefficient, that for single stars, is 0.27.

Therefore, there is no statistically significant indication that the suspected binary cepheids suffer some additional noise on the Hipparcos parallax measurements.

Does the exclusion of suspected binaries change the zero-point derived from Hipparcos parallaxes? SZ uses a straightforward fit in magnitude space to calculate the PL relation of single cepheids. We saw in Par. 3 how biased the results could become. In fact, when analysed with the procedure used by FC, the single cepheids only, the binaries only and the whole sample give similar results ($\rho = -1.51, -1.36$ and $-1.43$ respectively). The zero-point $\sim 0.2$ mag fainter found by SZ is not due to the exclusion of suspected binaries, but to the biases caused by the use of magnitudes calculated from parallaxes.

The implications are that:
1- There is no statistically significant indication that binarity affects Hipparcos parallaxes for cepheids.
2- Keeping or removing the suspected binaries gives essentially the same result for the PL relation zero-point.

6. Multi-wavelength magnitude analysis: Madore & Freedman 1997

Madore & Freedman (1998, MF) reconsider the calibration of Hipparcos cepheids, using data in several visible and infrared wavelengths (BVIJHK) rather than the traditional B and V. This reduces the number of objects available, as only 7 Hipparcos cepheids have been measured in all six wavelengths. MF compute magnitudes from the standard formula

$$m_i - M_i = 5 \log(1/\pi) - 5 + a_i$$

and calculate the PL zero-point $\rho$ by averaging the magnitude residuals with weights $\omega = \pi^2/\sigma^2_\pi$. This procedure yields values for $\rho$ that depend significantly on wavelength (see column two of Table 3), a dependence that MF attribute to
reddening problems. At the longer wavelengths, MF recover the values found by FC, whereas in the infrared fainter values are found, corresponding to the pre-Hipparcos calibration and $\mu_{LMC} \sim 18.5$ mag.

| Filter | $\mu_{LMC}$ | $\Delta \rho$ | $\Delta' \rho$ |
|--------|-------------|---------------|---------------|
| B      | 18.74±0.36  | -0.04         |               |
| V      | 18.67±0.24  | +0.03         | -0.22         |
| I      | 18.71±0.20  | -0.01         |               |
| J      | 18.44±0.24  | +0.26         | -0.26         |
| K      | 18.57±0.14  | +0.13         | -0.15         |

Table 3. Column 2 and 3: values of $\mu_{LMC}$ recovered by MF in the multi-wavelength analysis, with the offset compared to the value of FC. Column 4: offset between the V, J and K calibrations of Laney & Stobie (1994) and the FC method applied to the MF data. In this case, the brighter Hipparcos cepheid scale is recovered and the strong wavelength dependence vanishes.

It should now be clear that the MF procedure is subject to very large biases, as it corresponds to the type of approach outlined in Par. 3 above. The $M_i$ calculated from Equ. 3 are very biased estimators of the absolute magnitude for high values of $\sigma_\pi/\pi$. The introduction of weights depending on the observed $\sigma_\pi/\pi$ ratio only makes matters worse. The effect may be illustrated with a simple example: consider 3 cepheids at the same distance, say 500 pc, measured by Hipparcos with $\sigma_\pi = 1$ mas. The real $\pi$ is 2 mas ($1/500$ pc), and let the measured $\pi$ be 1, 2 and 3 mas respectively. Table 3 shows the magnitudes derived for these 3 objects from the measured parallaxes using Equ. 3 and the weights from MF. A weighted average gives a calibration that is 0.46 mag too faint! A better procedure would be to work in parallax space, in this case averaging the parallaxes weighted by $\sigma_\pi$, to obtain the parallax of the zero-point, a procedure similar in essence to FC.

The biases affecting the MF results were also checked by applying the FC method to the MF multi-wavelength data. The resulting zero-point was compared in V, J and K to the Laney & Stobie (1994) calibrations (Column 4 of Table 3). The resulting zero-point is coherent at all three wavelengths, $\sim 0.2$ mag brighter than Laney & Stobie, and corresponds to the “bright” FC calibration.

Biases are also apparent when the MF procedure is applied to synthetic samples. As expected, the recovered luminosity zero-point is systematically too faint, on average 0.25 mag too faint with synthetic samples similar in distance distribution to the actual sample.

---

3MF take individual reddenings from the Fernie et al. (1995) catalogue (and not from the reference given in the article, Fernie, Kamper & Seager 1993, that does not contain reddenings). MF state that they use the same unreddening procedure as FC. However, Fernie et al. use multicolour calibrations for reddenings, whereas FC calculate reddenings from a mean Period-Colour (PC) relation. Thus MF do not benefit from the compensating effect of combining PC reddenings with a PL relation (see for instance Pont et al. 1997 or FC). As a consequence, the biases are amplified, which may explain part of the dependence of their zero-point on wavelength.
Thus the disagreement of MF with FC is not due to the use of multi-wavelength data, but can be entirely attributed to the treatment of parallax data. This confirms the necessity of carefully considering the subtleties involved in deriving magnitude calibrations from high $\sigma_\pi/\pi$ parallax (see Brown et al. 1997, Luri & Arenou 1997).

7. Presence of the Lutz-Kelker bias: Oudmaijer et al. 1998

The Oudmaijer et al. (1998, OGS) study is devoted to showing the presence of the Lutz-Kelker and Malmquist biases in Hipparcos data, and calculating statistical corrections on individual measurements to compensate for these biases. OGS consider the plot of the magnitude residual $\Delta M_V$ versus $\sigma_\pi/\pi$ (see Figure 4a, for the cepheids) as evidence for the presence of such biases. The dependence of $\Delta M_V$ on $\sigma_\pi/\pi$ is indeed impressive.

But this interpretation is incorrect – as detailed by Arenou at this meeting: the features in Fig 4a are primarily due to the fact that the abscissa and ordinate are heavily correlated. Let us suppose that $\sigma_\pi$ is a constant, as is nearly the case for Hipparcos parallaxes, then the abscissa is $\sigma_\pi/\pi \sim 1/\pi$, while the ordinate is

$$\Delta M_V \equiv M_V^{\text{par}} - M_V^{\text{true}} = 5 \log(1/\pi) - 5 - M_V^{\text{true}} \sim -\log(\pi)$$

Both axes strongly depend on the same measured parallax $\pi$, and the relation observed in Fig 4a (and Fig. 2, 3 and 4 of OGS) only reflects this direct correlation, and does not as such reveal any bias.

Fig. 4a would be more useful if the abscissa contained the real $\sigma_\pi/\pi$, and not the observed $\sigma_\pi/\pi$. Unfortunately the real $\pi$ is unknown. For a given real $\pi$, the variations of the observed $\pi_\text{obs}$ due to parallax uncertainties affects $\sigma_\pi/\pi_\text{obs}$ and $\Delta M_V$ in a correlated way and move data points along diagonal lines in the diagram, as illustrated in Fig. 4b. $\sigma_\pi/\pi_\text{obs}$ is a reliable indication of $\sigma_\pi/\pi_\text{real}$ only if $\sigma_\pi \ll \pi$, which is not the case for cepheids.

OGS propose the following correction on individual data

$$\delta M = \left\{1 - \left[\frac{\sigma_M^2}{\sigma^2_{M_0} + 4.715(\sigma_\pi/\pi)^2}\right]\right\}(M_0 - M_\text{obs})$$ (4)

where $M_0$ is the expected magnitude (the “true” magnitude). As this correction depends on the unknown true magnitude $M_0$, it is not determined unless a true
Figure 4.  

a.  $M_{\text{par}} - M_{\text{true}}$ as a function of $\sigma_{\pi}/\pi$ for the 26 cepheids used by OGS. $M_{\text{par}}$ is the $V$ magnitude computed from the parallax with Equ. 1, $M_{\text{true}}$ the magnitude from the PL relation.  

b. Position of an object of fixed true parallax and magnitude when the observed parallax varies of $\pm 1.5\sigma_\pi$, showing the correlation of the abscissa and the ordinate. The arrows illustrate the correction of Equ. 4.  

c. Parallax residuals ($\pi_{\text{obs}} - \pi_{\text{true}}$) as a function of $\sigma_{\pi}/\pi_{\text{true}}$ for the 26 cepheids and the imaginary object as in (b). $\pi_{\text{true}}$ is the parallax expected from the PL relation distance. The feature observed in the upper and middle panels is only due to the correlated nature of the abscissa and the ordinate. On the uncorrelated last panel, a correction such as Equ. 4 becomes unnecessary.
magnitude $M_0$ is assumed. In the case of cepheids, OGS chose the procedure of trying different values of $\rho$ until the residuals of the magnitudes corrected by Equ. 4 reach a minimum, and recover a value of $\rho$ similar again to the pre-Hipparcos calibrations. Their procedure is illustrated by the arrows in Fig. 4b: a large correction depending on $M_0$ and $\sigma_\pi/\pi$ is applied (Equ. 4), and different $\rho$ are tried until the residuals are minimum. OGS take as a proof that their procedure has corrected for the biases the fact that, after correction, the $\sigma_\pi/\pi$ vs. $\Delta M_V$ plot looks very tidy. A closer look at Equ. 4 shows that this has nothing to do with bias correction: because the correction itself tends to $(M_0 - M_{\text{obs}})$ as $\sigma_\pi/\pi$ becomes high, the corrected $M_{\text{obs}}$ is forced to $M_0$ ($M_{\text{obs}} + \delta M \simeq M_{\text{obs}} + (M_0 - M_{\text{obs}}) = M_0$), so that the residuals artificially tend to zero! If an abscissa independent of the ordinate is chosen, e.g. $\sigma_\pi/\pi_{\text{true}}$, where $\pi_{\text{true}}$ is the parallax expected from the PL relation, the largest part of the apparent bias vanishes (Fig. 4c), and correction Equ. 4 becomes unnecessary.

We have tested the OGS procedure for cepheids on synthetic samples and find that it gives systematically too faint results by $\sim 0.17$ mag. It also fails the “3 cepheid” test of the previous section (giving a bias of 0.36 mag). The crux of the matter here is that, as the relative error on parallax increases, the measured $\sigma_\pi/\pi$ becomes an increasingly bad estimator of the real $\sigma_\pi/\pi$. In our small “3 cepheid” example, the measured $\sigma_\pi/\pi$ is 33%, 50% and 100%, while the true $\sigma_\pi/\pi$ is 50%. As a rule of thumb, $\sigma_\pi/\pi$ should no longer be used, even in statistical corrections, for values higher than about 20%. Due to its indirect nature (the true magnitude must first be assumed and then corrected residuals are minimised), the OGS method is also quite unstable. One can get a feeling of this instability by comparing the results of the OGS method with that of the FC method on several cepheid subsamples (Table 5).

| Sample | $\rho$ with OGS method | $\rho$ with FC method |
|--------|-------------------------|-----------------------|
| 26 stars | -1.34 | -1.42 |
| without $\alpha$UMi | -1.16 | -1.44 |
| without overtones | -1.29 | -1.49 |
| without binaries | -1.35 | -1.36 |
| only binaries | -0.84 | -1.51 |

Table 5. The OGS and FC procedure compared on several subsamples of the Hipparcos cepheid data. Note the instability of the OGS results.

We conclude that while the OGS method might be justified to estimate the absolute magnitude of individual objects from a high $\Delta M_V$ population with low $\sigma_\pi/\pi$ and about which nothing is known, it is far from optimal in the case of the cepheids, which have high $\sigma_\pi/\pi$ and low $\Delta M_V$. In that case, $\pi_{\text{true}}$ is better known from the PL relation itself than from $\pi_{\text{meas}}$, and the use of $\sigma_\pi/\pi$ can be avoided by working in parallax space.

4The remaining part of the bias is the much smaller classical Lutz-Kelker bias, due to the fact that different parallax intervals cover very different space volumes. In the case of Hipparcos cepheids selected as in FC, it amounts to $\sim 0.02$ mag.
8. Maximum likelihood method: Luri et al. 1998

Luri et al. (1996) have devised a maximum-likelihood method of determination of absolute magnitudes from Hipparcos data that takes all available data into account, including proper motion and radial velocity. Luri et al. (1998, LGT) apply this method to the zero-point of the cepheid PL relation and derive $\rho = 1.05 \pm 0.17$ mag for a fixed slope of $\delta \equiv -2.81$, a value 0.38 mag fainter than FC using exactly the same sample.

First, it should be noted that LGT use the global reddening model of Arenou et al. (1992), that gives redenings on average 0.05 mag higher than those usually adopted for cepheids. If the reddening scale of FC is adopted instead, the result of the LM method becomes $\rho = -0.89$ mag, an even fainter value.

In collaboration with X. Luri, we tested the method with synthetic samples, and no significant bias was found. The key to this puzzling problem was pointed out by F. van Leeuwen at this meeting: further tests showed that the LGT solution is much more sensitive to kinematical data (proper motions and radial velocities) that to the parallaxes. In fact, to the first order, the parallaxes have no influence on the solution, so that the LM method becomes similar in principle to a statistical parallax analysis. It cannot be directly compared to the geometrical distance determinations considered above.

The large disagreement of the LM method with FC remains a question that deserves detailed study, as it may contain precious hints on cepheid distances or kinematics (see Par 12 below).

9. A note on overtone pulsators

Overtone pulsators, cepheids that pulsate in the first harmonic rather than the fundamental mode, are usually identified by their low amplitude and sinusoidal light curve. Their period is about 30% shorter than a fundamental pulsator of the same luminosity. In a luminosity calibration, detected overtones can either be removed or their period adjusted by 30%, but the presence of undetected overtones may bias the result.

The possible presence of undetected overtones was tested by repeating the zero-point determination on subsamples selected by period intervals. Because overtones usually have small periods, their undetected presence should appear as a period dependence of the solution. No significant trend was found, but the sample is not large enough to exclude such a trend below the $\sim 0.1$ mag level.
10. Value and uncertainty of the Hipparcos cepheid PL zero-point

Fig. 5 graphically summarizes our conclusions: the Hipparcos parallaxes for cepheids do indeed indicate a magnitude calibration brighter than previously accepted ($\rho = -1.43 \pm 0.16$ for a fixed PL slope $\delta \equiv -2.81$), as found by FC.

All other subsequent analyses that we considered suffer from strong systematic biases due to the procedures used to infer magnitude calibrations from parallax data. The impression one could infer by “weight of numbers”, from an uncritical list of all calibrations, namely that after all Hipparcos cepheid data confirm previous magnitude calibrations, is therefore misleading. On the contrary, we confirm the conclusion by FC that a brighter cepheid PL calibration is implied by the Hipparcos parallaxes.

Let us now consider the uncertainty on this value. In our simulations, the dispersion in the recovered $\rho$ came out 15% to 50% higher than the uncertainties derived in FC. In fact, the error in FC is lower than the uncertainty caused by the propagation of Hipparcos parallax uncertainties alone. This is due to the fact that FC calculate the uncertainty from the residuals, not from the Hipparcos parallaxes.

Figure 5. Calibrations of the cepheid PL relation zero-point as in Fig 1, with the bias corrections discussed in this paper. The error bars of FC are increased. The other results are modified to account for the biases in the treatment of parallaxes, except LGT, for which the reddening scale has been adjusted.

Let us now consider the uncertainty on this value. In our simulations, the dispersion in the recovered $\rho$ came out 15% to 50% higher than the uncertainties derived in FC. In fact, the error in FC is lower than the uncertainty caused by the propagation of Hipparcos parallax uncertainties alone. This is due to the fact that FC calculate the uncertainty from the residuals, not from the Hipparcos parallaxes.

5With the interesting exception of LGT, that, as explained in Par. 8., cannot be strictly considered as a parallax calibration, but should rather be seen as a kinematical calibration.
\( \sigma_\pi \), and that the residuals are on average lower than the uncertainties:

\[
\left< \frac{\pi_{\text{observed}} - \pi_{\text{PL}}}{\sigma_\pi} \right> = 0.87
\]

Obtaining normalised residuals lower than unity can have two causes: either the uncertainties were overestimated, or the effect is due to chance and low-number statistics.

It is unlikely that Hipparcos parallax errors were overestimated for cepheids, and we shall rather assume that a statistical fluctuation causes the residuals to be smaller than the uncertainties. In that case, the final error to be used is not the one from the residuals, but the one propagated from the uncertainties on the parallaxes. With this consideration, we modify the error in FC upwards to 0.16 mag, noting that this uncertainty is due only to the \( \sigma_\pi \) and than any other source of uncertainty (e.g. reddening scale, PL slope) would be additional. Our Monte Carlo simulations (Table 1) show typical scatters as high as 0.20 mag in the recovered \( \rho \) for the sample without \( \alpha \) UMi.

Recalling the discussion of section 4, we add a possible systematic bias of \( +0.03 \) mag, for a final Hipparcos cepheid PL relation zero-point modified from FC of

\[
M_V = -2.81 \log P - 1.43 \pm 0.16 \text{ [stat]} + 0.03 \text{ [syst]} \tag{5}
\]

If our rediscussion of the uncertainties of the FC result is correct, the Hipparcos calibration is not incompatible with previous calibrations from cluster cepheids or from surface brightness techniques. The uncertainty on the geometrical calibration remains high and does not force a shift of the distance scale. The Hipparcos parallax data do however indicate that the real PL relation is probably situated near the bright end of previous uncertainty intervals.

11. Distance of the LMC

The Hipparcos cepheid distance scale can be compared to that obtained from other methods, taking the distance of the LMC as a point of comparison. Hipparcos cepheids give:

\[
\mu_{\text{LMC}} = 18.70 \pm 0.16 \text{ [ } -0.03 \text{ ] (Hipparcos parallaxes, modified from FC) } \tag{6}
\]

The other two main calibrations of the cepheid PL zero-point are the surface brightness technique and cepheids in clusters and associations. From the surface brightness method, Gieren, Fouqu & Gomez (1998) obtain:

\[
\mu_{\text{LMC}} = 18.46 \pm 0.02 \text{ [ } +0.06 \text{ ]}
\]

where the 0.02 term is the internal statistical uncertainty, and the [+0.06] term is a metallicity correction that the authors chose not to implement. In order to put this value on the same scale as (6), we force the slope of the PL relation to \( \delta \equiv -2.81 \) as in FC, which yields \( \mu_{\text{LMC}} = 18.52 \) (P. Fouqu, priv. comm.). In the absence of an external error budget we increase the uncertainty to 0.1 mag:

\[
\mu_{\text{LMC}} = 18.52 \pm 0.17 \text{ [ } +0.06 \text{ ] (surface brightness, modified from Gieren et al. 1998)}
\]
A recent cluster cepheid calibration is Laney & Stobie (1994) who find:

$$\mu_{LMC} = 18.49 \pm 0.04 \ [+0.04]$$

where the last term is a metallicity correction. We adjust this result to the new Hipparcos Hyades parallax of $\mu = 3.33$:

$$\mu_{LMC} = 18.55 \pm 0.04 \ [+0.04] \text{(cluster cepheids, modified from Laney & Stobie 1994)}$$

It is noticeable that the final agreement between the three different calibration is compatible with the statistical uncertainties. A weighted mean, applying half the systematic corrections given in brackets, yields for the distance modulus of the LMC:

$$\mu_{LMC} = 18.58 \pm 0.05$$

We conclude that while the Hipparcos parallax calibration does indicate that the PL zero-point may be at the bright end of previous uncertainty intervals, it is not incompatible with other determinations. Or, depending on our viewpoint, that while the Hipparcos PL zero-point is within one sigma of previous calibrations, it does indicate that the PL zero-point is near the bright end of previous uncertainties.

12. A note on kinematical determinations

Feast et al. (1998) have calculated the cepheid PL zero-point $\rho$ from kinematical data, by comparing Hipparcos proper motions and radial velocities. They adjust $\rho$ in order to obtain the same value for the Oort constant $A$ from proper motions and from radial velocities. They find that this approach favours a brighter zero-point similar to FC. A brighter zero-point would also decrease the mismatch between the rotation curve of cepheids and HII regions in the outer disc (see Pont et al. 1997). One could then conclude that the kinematics favours a longer distance scale. It is striking however that the “LM method”, also based on kinematics, finds such a faint zero-point for cepheids (see Par. 8). In addition, the RR Lyrae statistical parallax analyses (such as Fernley et al. 1998), also kinematical, yield a much fainter distance modulus for the LMC ($\mu \sim 18.3$). This could lead us to wonder whether all assumptions implicit in the kinematical methods about the cepheid or RR Lyrae velocity field are really fulfilled. If not, it could turn out to be the key to understanding the puzzling disagreement between the Hipparcos cepheid zero-point on one hand, the LM method for cepheids and RR Lyrae statistical parallaxes on the other.

Acknowledgments. This study has benefited a lot from enlightening discussions with Laurent Eyer, Michael Feast, Pascal Fouqu and Xavier Luri. I also thank Martin Groenewegen and Barry Madore for useful exchanges.
References

Arenou F., Grenon M. & Gómez A.E. 1992, A&A 258, 104
Brown A., Arenou F., van Leeuwen F., Lindegren L. & Luri X. 1997, ESA SP-402, Hipparcos Venice’97, 63
Feast M.W., Catchpole R.M. 1997, MNRAS 286, L1 [FC]
Feast M.W., Pont F. & Whitelock P. 1998, MNRAS 298, L43
Fernie J.D., Evans N.R., Beattie B., Seager S. 1995, IBVS 4148, 1
Fernley J., Barnes T.G., Skillen I. et al. 1998, A&A 330, 515
Gieren W., Fouqu P. & Gomez M. 1998, ApJ 496, 17
Laney C.D. & Stobie R. S. 1994, MNRAS 266, 441
Luri X. & Arenou F. 1997, ESA SP-402, Hipparcos Venice’97, 449
Luri X., Mennessier M.O., Torra J. & Figueras F. 1996, AAS 117, 405
Luri X., Gomez A.E., Torra J., Figueras F. & Mennessier M.O. 1998, AA 335, L81 [LGT]
Madore B.F. & Freedman W.L. 1998, ApJ 492, 110 [MF]
Oudmaijer R., Groenewegen M. & Schrijver H. 1998, MNRAS 294, L41 [OGS]
Pont F., Queloz D., Bratschi P. & Mayor M., 1997, A&A 318, 416
Szabados L. 1997, ESA SP-402, Hipparcos Venice ’97, 657 [SZ]