Problems and stoppers for $\gamma\gamma$, $\gamma\mu$, $\mu p$
colliders using very high energy muons. *

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Abstract

It is well known that at linear $e^+e^-$ (ee) colliders using laser backscattering one can obtain colliding $\gamma\gamma$, $\gamma e$ beams with energy and luminosity comparable to those in $e^+e^-$ collisions. In this paper, it is explained why this cannot be done at high energy muon colliders. Due to several physics reasons the $\gamma\gamma$ luminosity is suppressed here by a factor of $10^{14}$! Another option – $\gamma$’s from a linear collider and muons from a muon collider – is also discussed (and has no sense either). Of course, one can study $\gamma^*\mu$ and $\gamma^*\gamma^*$ interactions at muon colliders in collisions with virtual photons as it is done now at $e^+e^-$ storage rings. Muon-proton colliders are attractive only if the proton beam is cooled and has the same parameters as the muon beam, in which case $L_{\mu p} \sim L_{\mu\mu}$.

1 Introduction

Firstly, I would like to explain the origin of this talk. Two weeks ago the chairman of our workshop Bruce King have sent me e-mail with the request to give a plenary talk on “prospects for very high energy $\gamma\gamma$ or $\gamma\mu$ colliders driven by the muon beams”, he added that “even if it is impractical it would still be nice if you could give a brief explanation”.

I have agreed to give such a talk but only without the word “prospects” in the title because I do not see any prospects here, only stoppers. Nevertheless, this physics is very interesting, and it is pleasure to me to tell briefly about high energy photon colliders based on $e^+e^-$ (ee) linear colliders and explain why such photon colliders are completely impractical with muons.

The third combination of colliding particles, $\mu p$, is also discussed here very briefly.

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2 Photon Colliders based on linear ee colliders

As you know, to explore the energy region beyond LEP-II, linear e+e− colliders (LC) in the range from a few hundred GeV to about 1.5 TeV and higher are under intense study around the world [1, 2, 3, 4].

Beside e+e− collisions, linear colliders provide a unique possibility for obtaining γγ, γe colliding beams with energies and luminosities comparable to those in e+e− collisions [5, 6, 7, 8, 9]. High energy photons for these collisions can be obtained using Compton scattering of laser light on high energy electrons. This idea is based on the following facts:

- Unlike the situation in storage rings, in linear colliders each beam is used only once.
- Using an optical laser with reasonable parameters (flash energy of 1 to 5 J) one can “convert” almost all electrons to high energy photons;
- The energy of scattered photons is close into the energy of initial electrons.

Each one of these items is vital for obtaining γγ, γe collisions at energies and luminosities comparable to those in parental electron-electron collisions.

The physics at high energy γγ, γe colliders is very rich and no less interesting than with pp or e+e− collisions. This option has been included in the pre-conceptual design reports of LC projects [1, 2, 3], and work on full conceptual designs is under way. Reports on the present status of photon colliders can be found elsewhere [11, 12].

Well, can we make similar photon colliders on the basis of muon colliders? What is the difference?

3 γγ, γμ colliders based on high energy μμ colliders

3.1 Multi-pass collisions

At muon colliders two bunches are collided about 1000 times, which is one of the advantages over linear e+e− colliders where beams are collided only once. However, if one tries to convert muons into high energy photons (by whatever means), the resulting γγ luminosity will be smaller than that in μμ collisions at least by a factor of 1000. This argument alone sufficient to give up the idea of γγ colliders based on high energy muon colliders. However, at this workshop F.Zimmermann proposed the idea of one pass muon colliders. So, I will continue enumeration of stoppers.

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1Here we do not discuss “photon colliders” based on collisions of virtual photons. This possibility always exists, however the luminosities and energies are considerably smaller than those in parental ee collisions, see sect. 3.6
3.2 Laser wave length

The required wave length follows from the kinematics of Compton scattering \[6\]. In the conversion region a laser photon with the energy \(\omega_0\) scatters at a small collision angle (head-on) on a high energy electron (muon) with the energy \(E_0\). The maximum energy of scattered photons (in direction of electrons) is given by

\[
\omega_m = \frac{x}{x + 1} E_0; \quad x = \frac{4E_0\omega_0}{m^2c^4}, \quad (1)
\]

where \(m\) is the mass of the charged particle. In order to obtain photons with the energy comparable to that of initial particles, say, 80\%, one needs \(x \sim 4\), or the energy of laser photons

\[
\omega_0 \sim \frac{m^2c^4}{E_0}. \quad (2)
\]

The corresponding laser wave length is then

\[
\lambda \sim \frac{5E_0[TeV]}{[\mu m]} \quad \text{for electron beams;} \quad (3)
\]

\[
\lambda \sim 0.12E_0[TeV] \quad \text{nm} \quad \text{for muon beams.} \quad (4)
\]

So, one can use optical lasers to make high energy photons by means of backward Compton scattering on electron beams, while at muon colliders one would have to use X-ray lasers!

3.3 Flash energy

The probability of Compton scattering for an beam particle in the laser target \(p \sim n\sigma_C l\), where \(n, l, \sigma_C\) are the density of the laser target, its length and the Compton cross section, respectively. The density \(n \sim (A/lS)/\omega_0\), where \(A\) is the laser flash energy and \(S\) is the cross section of the laser beam which should be larger than that of the muon beam. The Compton cross section for muon at \(x = 4\) is about \[3\]

\[
\sigma_C(x = 4) \sim \frac{\pi r_e^2}{m_e} \left(\frac{m_e}{m_\mu}\right)^2, \quad (5)
\]

where \(r_e = e^2/m_ec^2\) is the classical radius of the electron.

From the above relations we get the required laser flash energy (for \(p \sim 1\))

\[
A \sim (S/\sigma_C)\omega_0 = \frac{S}{\pi r_e^2 E_0} \left(\frac{m_\mu}{m_e}\right)^4 \frac{m_e^2 c^4}{1.5 \times 10^{-3} \frac{S[\mu m^2]}{E_0[TeV]} \left(\frac{m_\mu}{m_e}\right)^4} \quad \text{Joule.} \quad (6)
\]

At the muon collider with \(E_0 = 50\) TeV, \(S = 1\) \(\mu m^2\) one needs the X-ray laser with the flash energy \(10^5\) J and the wave length of 6 nm (see eq. \[3\]). This is certainly impossible. Beside this “technical” problem, there are even more fundamental stoppers for photon colliders based on muon beams, see below.

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2In the case of the electron LC, where optical photons are used, the laser spot size is determined by diffraction: \(a_\gamma \sim \sqrt{\lambda/4\pi}\) which is several \(\mu m\) for LC electron beams \[8\]. At muon colliders, the required wave length is much shorter and diffraction can be neglected.
3.4 $e^+e^-$ pair creation in the conversion region

Beside the Compton scattering at the conversion region, at muon colliders there is another competing process: $e^+e^-$ pair creation in collision of laser photons with the high energy muons, $\gamma \mu \rightarrow \mu e^+e^-$. The ratio of the cross sections

$$\frac{\sigma_{\gamma \mu \rightarrow \mu e^+e^-}}{\sigma_{\gamma \mu \rightarrow \gamma \mu}} \sim \frac{28\alpha^2}{9} \ln \frac{4E_{\text{inc}}}{m_e m_\mu c^2} \sim 7 \times 10^{-3} \left(\frac{m_\mu}{m_e}\right)^2 \ln \left(\frac{m_\mu}{m_e}x\right) \sim 2000 \text{ at } x = 4. \quad (7)$$

So, high energy photons are produced with a very small probability, less than $1/1000$! In all other cases muons lose their energy via creation of $e^+e^-$ pairs. This effect alone suppresses the attainable $\gamma\gamma$ luminosity at muon colliders by a factor of more than $10^6$!

3.5 Coherent pair creation

OK, the yield of high energy photons from the conversion region is very small, but this is not the whole story. What happens to the “happy” photons at the interaction region? They will be “killed” by the process of coherent $e^+e^-$ pair creation in the field of the opposing muon beam. This process restricts the luminosity of photon colliders based on electron linear colliders [8, 9, 10]. The effective threshold of this process $\Upsilon = \omega \frac{\mu c^2 B}{B_0} \sim 1$, where $\omega$ is the photon energy, $B$ is the beam field, $B_0 = \alpha e/r_e^2 \sim 4.4 \times 10^{13}$ Gauss. For the “evolutionary” $2E = 100$ TeV muon collider (see the B.King’s tables) with $N = 0.8 \times 10^{12}$, $\sigma_z = 2.5$ mm, $\sigma_{x,y} = 0.2 \mu$m and a photon energy 40 TeV we have $\Upsilon \sim 180$. Using formulae given in ref.[8], one can find the probability of $e^+e^-$ pair creation during the bunch collision: it is very high, about 200. This means that only about 1% of high energy photons will survive in beam collisions and contribute to the $\gamma\gamma$ luminosity.

3.6 Summary on $\gamma\gamma$, $\gamma\mu$ colliders based on high energy muons

1) The laser required for conversion of 50 TeV muons into high energy photons should have flash energy $A \sim 10^5$ J and wave length $\lambda \sim 5$ nm. This is impossible.

2) The achievable $\gamma\gamma$ luminosity

$$L_{\gamma\gamma}/L_{\mu\mu} \sim \frac{1}{1000} \times \left(\frac{1}{2000}\right)^2 \times \left(\frac{1}{100}\right)^2 = 2.5 \times 10^{-14}! \quad (8)$$

Here the first factor is due to the one pass nature of photon colliders, the second one is due to the dominance of $e^+e^-$ creation at the conversion region (instead of Compton scattering), and the third one is due to coherent pair creation at the interaction region.

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For an LC with the energy below about 1 TeV this effects is still not very important and one can obtain, in principle, $L_{\gamma\gamma} > L_{e^+e^-}$
All clear. One can forget about $\gamma\gamma$ (and $\gamma\mu$ too) colliders based on high energy muon beams.

However, $\gamma\gamma, \gamma\mu$ interactions can be studied at muon colliders in collisions of virtual photons (without $\mu \to \gamma$ conversion). The luminosities in such collisions are

\begin{align*}
L_{\gamma^*\gamma^*} & \sim 10^{-2} L_{\mu\mu} \quad W_{\gamma\gamma} > 0.1 \times 2E_0 \\
L_{\gamma^*\gamma^*} & \sim 10^{-4} L_{\mu\mu} \quad W_{\gamma\gamma} > 0.5 \times 2E_0 \\
L_{\gamma^*\mu} & \sim 0.15 L_{\mu\mu} \quad W_{\gamma\mu} > 0.1 \times 2E_0 \\
L_{\gamma^*\mu} & \sim 0.05 L_{\mu\mu} \quad W_{\gamma\mu} > 0.5 \times 2E_0.
\end{align*}

(9) \quad (10) \quad (11) \quad (12)

\section{$\gamma\mu$ collisions at LC–muon colliders}

One can also consider $\gamma\mu$ colliders where high energy photons are produced at LC (on electrons) and then are collided with high energy muon beams. This option also has no sense for several reasons:

a) $N_e \sim 10^{-2} N_\mu$;

b) loss of photons at the IP due to coherent $e^+e^-$ pair creation;

c) none of the LCs have the pulse structure of muon colliders (almost uniform in time), which results in a factor 100 times loss in luminosity.

All factors combined give $L_{\gamma\mu} < 10^{-5} L_{\mu\mu}$. Such $\gamma\mu$ collider has no sense; besides, $\gamma\mu$ collisions can be studied for free with much larger luminosities in $\gamma^*\mu$ collisions (see the end of the previous section).

\section{$\mu p$ colliders}

Let us first consider collisions of the LHC proton beams with muon beams of a 100 TeV muon collider. Without special measures, the luminosity in such collisions is lower than that in $pp$ collisions at LHC due to larger distance between bunches at muon colliders (smaller collision rate).

\begin{equation}
L_{\mu p} \sim L_{pp} \times (\nu_\mu/\nu_p) \sim 10^{-3} L_{pp} \sim 10^{31} \text{ cm}^{-2}\text{s}^{-1}.
\end{equation}

(13)

This is too small for study of any good physics.

However, one can think about a special source of proton with several stages of electron cooling. If parameters of the proton beam are the same as those of the muon beam, then the luminosity at the 100 TeV $\mu p$ collider $L_{\mu p} = L_{\mu\mu} \sim 10^{36} \text{ cm}^{-2}\text{s}^{-1}$. That is not easy to achieve, but such possibility is not excluded.

One of problems at such colliders is hadronic background. At $L_{\mu p} = 10^{36}$ and $\nu = 10^4$ the number of background $\gamma p$ reactions is about 5000/crossing. One can decrease backgrounds by increasing the collision rate (up to a factor of 5–10). It is not excluded that
even with such backgrounds one can extract interesting physics. This option certainly makes sense, if a very high energy $\mu\mu$ collider is to be built. Its feasibility and potential problems should be studied in more detail.

References

[1] Zeroth-Order Design Report for the Next Linear Collider LBNL-PUB-5424, SLAC Report 474, May 1996.

[2] Conceptual Design of a 500 GeV Electron Positron Linear Collider with Integrated X-Ray Laser Facility DESY 97-048, ECFA-97-182; R.Brinkmann et al., Nucl. Instr. & Meth. A 406 (1998) 13.

[3] JLC Design Study, KEK-REP-97-1, April 1997; I. Watanabe et. al., KEK Report 97-17.

[4] J.P. Delahaye et al., Acta Phys. Polon. B30:2029-2039, 1999; R. Bossart et al. CERN-PS-99-005-LP, Apr 1999.

[5] I.Ginzburg, G.Kotkin, V.Serbo, V.Telnov, Pizma ZhETF, 34 (1981)514; JETP Lett. 34 (1982) 491 (Prepr. INP 81-50, Novosibirsk, Febr. 1981).

[6] I.Ginzburg, G.Kotkin, V.Serbo, V.Telnov, Nucl. Instr. & Meth. 205 (1983) 47.

[7] I.Ginzburg, G.Kotkin, S.Panfil, V.Serbo, V.Telnov, Nucl. Instr.&Meth. 219(1984)5.

[8] V.Telnov, Nucl. Instr.&Meth.A 294 (1990)72.

[9] V.Telnov, Nucl. Instr.&Meth.A 355(1995)3.

[10] V.Telnov, Proc. of ITP Workshop “Future High energy colliders” Santa Barbara, USA, October 21-25, 1996, AIP Conf. Proc. No 397, p.259; e-print: physics/9706003.

[11] V.Telnov, Proc. of the International Conference on the Structure and Interactions of the Photon (Photon 99), Freiburg, Germany, 23-27 May 1999, to be published in Nucl. Phys. Proc. Suppl. B, e-print: hep-ex/9908003.

[12] V.Telnov, Proc. of World-Wide Study of Physics and Detectors for Future Linear Colliders (LCWS 99), Sitges, Barcelona, Spain, 28 Apr–5 May 1999, e-print: hep-ex/9910010, hep-ex/9910011.