Soft Colour Interactions and Diffractive DIS

W. Buchmüller
DESY, Notkestrasse 85, D-22603 Hamburg, Germany

Abstract

The basic ideas and some results of the semiclassical approach to diffractive DIS are briefly described. In the production of high-$p_{\perp}$ jets boson-gluon fusion is predicted to be the dominant partonic process. The $p_{\perp}$-spectrum and the two-jet invariant mass distribution provide a clear test of the underlying 'hard' partonic process and the 'soft' mechanism of colour neutralization.

The events with a large gap in rapidity in small-$x$ DIS [1] represent a puzzling phenomenon. The separation of a colour neutral cluster of 'wee' partons from the proton, which then fragments independently of the proton remnant, is a non-perturbative, 'soft' process. On the other hand, in rapidity gap events with high transverse momentum jets also a 'hard' scattering process must take place. To disentangle the 'soft' and the 'hard' aspects of 'hard diffraction' is the main theoretical problem of diffractive DIS [2].

Since diffractive processes are non-perturbative, a purely perturbative approach, similar to ordinary parton model calculations, appears doomed to failure. In the following we shall describe another attempt [3], which is based on a high-energy expansion in the proton rest frame. At small $x$, the proton is treated as a classical colour field localized within a sphere of radius $1/\Lambda$. Partonic fluctuations of the virtual photon, $q\bar{q}$, $gq$, etc., are scattered by this colour field at high energies. Crucial ingredients of this semiclassical approach are light-cone techniques [4] and the description of high-energy scattering processes in terms of Wilson lines [5]. A final state colour singlet partonic configuration is assumed to lead to a diffractive event, since in this case the partonic cluster can fragment independently of the proton remnant. Correspondingly, a colour non-singlet partonic configuration yields an ordinary non-diffractive event.

Without any further ad-hoc assumptions, this simple physical picture leads to a number of predictions which are independent of the details of the proton colour field, as long as it is soft with respect to the energies of the incident partons. The following discussion is closely related to Ref. [6].
Diffractive structure function

In the semiclassical approach inclusive and diffractive cross sections can be expressed in terms of a single non-perturbative quantity, \( \text{tr} W^F_{x_{\perp}}(y_{\perp}) \), where

\[
W^F_{x_{\perp}}(y_{\perp}) = U^\dagger(x_{\perp} + y_{\perp})U(x_{\perp}) - 1
\]

is built from the non-Abelian eikonal factors \( U \) and \( U^\dagger \) of quark and antiquark whose light-like paths penetrate the colour field of the proton at transverse positions \( x_{\perp} \) and \( x_{\perp} + y_{\perp} \), respectively (cf. Fig. 1). The superscript \( F \) is used since quarks are colour triplets. As the colour field outside the proton vanishes \( W^F_{x_{\perp}}(y_{\perp}) \) is essentially a closed Wilson loop through a section of the proton which measures an average of the proton colour field.

In an expansion in the transverse distance between quark and antiquark one has

\[
\int_{x_{\perp}} \text{tr} W^F_{x_{\perp}}(y_{\perp}) = -\frac{1}{4} y_{\perp}^2 C_1 + O(y_{\perp}^4). \tag{2}
\]

The constant \( C_1 \) determines the variation of the inclusive structure function \( F_2(x,Q^2) \) with \( Q^2 \). A comparison with boson-gluon fusion in the parton model shows the connection with the gluon density,

\[
C_1 = 2\pi^2 \alpha_s(x) G(x). \tag{3}
\]

Since \( C_1 \) is constant, \( G(x) \sim 1/x \), which corresponds to a classical bremsstrahl spectrum of gluons.

Consider now the production of \( q\bar{q} \) final states. Transverse momenta in the final state vary between \( \Lambda \) and \( Q \). Integration over this range yields the dominant contribution to \( F_2 \) which is proportional to \( \ln Q/\Lambda \). The inclusive structure function \( F_2 \) is linear in \( \text{tr} W^F_{x_{\perp}}(y_{\perp}) \). In contrast, for diffractive final states the structure function \( F_2^D \) is quadratic in \( \text{tr} W^F_{x_{\perp}}(y_{\perp}) \) due to the projection on colour singlet \( q\bar{q} \) final states. Because of Eq. (2), this implies a suppression of large transverse momenta by one power of \( l_{\perp}^2 \). For kinematical reasons, the longitudinal momenta then have to be asymmetric, as in the aligned jet model.
With $\alpha = l_+/q_+ < 1/2$, one has

$$ l_\perp \sim \Lambda , \quad \alpha \sim \frac{\Lambda^2}{Q^2} . \quad (4) $$

For the diffractive structure function one obtains the result

$$ F_D^2 (x, Q^2, \xi) = \frac{\beta}{\xi} \bar{F}(\beta) , \quad (5) $$

where $\xi = x/\beta$ and $\beta = Q^2/(Q^2 + M^2)$. $\bar{F}(\beta)$ can be expressed as an integral over $\text{tr} W_{x\perp} (y_{\perp})$ and it is therefore not calculable perturbatively. Eq. (5) corresponds to a pomeron structure function with $\alpha_P(0) = 1$.

**Jets with large transverse momentum**

One can easily calculate the $p_{\perp}$-spectrum for $q\bar{q}$ final states (cf. Fig. 4). The result reads

$$ \frac{d\sigma_T}{d\alpha dp_{\perp}^2 dt} \bigg|_{t=0} \propto C_1^2 \frac{(\alpha^2 + (1-\alpha)^2)p_{\perp}^2 a^4}{(a^2 + p_{\perp}^2)^6} , \quad (6) $$

where $t = (q - p' - l')^2$ is the momentum transfer to the proton and $a^2 = \alpha(1-\alpha)Q^2$. Since $C_1 \propto xG(x)$, the cross section is proportional to the square of the gluon density. It is in fact identical to the result obtained for two-gluon exchange in leading order [3, 4, 11]. The cross section integrated down to the transverse momentum $p_{\perp}^2$ yields a contribution to the diffractive structure function $F_D^2$ which is suppressed by $\Lambda^2/p_{\perp}^2$ cut.

![Figure 2](image)

Two-jet production with an additional low transverse momentum gluon.

As shown in [3], a ‘leading twist’ contribution with jets of $p_{\perp} \sim Q$ requires at least three partons in the final state, one of which has low transverse momentum. It turns out that the dominant process has a low transverse momentum gluon (cf. Fig. 5),

$$ k_\perp \sim \Lambda , \quad \alpha' \sim \frac{\Lambda^2}{Q^2} \quad (7) $$
and, correspondingly,

\[ k_+ = o'q_+ \sim \frac{\Lambda}{x}, \quad -k_- = -q_- + p_- + l_- \sim \Lambda x, \quad k^2 = -\Lambda^2. \] (8)

One may also view the various diffractive processes in a frame where the proton is fast, e.g., the Breit frame. Note, that in the proton rest frame \( k_+ \sim \Lambda/x \gg -k_- \sim \Lambda x \), whereas in the Breit frame \(-k_- \sim Q \gg k_+ \sim \Lambda^2/Q\). The different cross sections can be written as convolution of ordinary partonic cross sections with diffractive parton densities [11, 12, 13, 14]. The cross section for the process shown in Fig. 2 then corresponds to boson-gluon fusion (cf. Fig. 3),

\[
\frac{d\sigma_T}{d\xi dp'_{\perp}} = \int_x^\xi dy \frac{d\sigma_T^{g+q\bar{q}}(y, p'_{\perp})}{dp'_{\perp}} \frac{dg(y, \xi)}{d\xi}.
\] (9)

The diffractive gluon density describes the probability to extract from the proton

\[ P \rightarrow \gamma^* \rightarrow P' \]

Figure 3: Interpretation of the process of Fig. 2 in terms of boson-gluon fusion in a frame where the proton is fast, e.g., the Breit frame.

a colour neutral pair of gluons: a virtual gluon with momentum fraction \( y \) which participates in boson-gluon fusion and a real gluon with momentum fraction \( \xi - y \) which contributes to the diffractive final state. The diffractive gluon density is determined by the proton colour field [14],

\[
\frac{dg(y, \xi)}{d\xi} = \frac{1}{8y(\xi - y)} \frac{d^2k_{\perp} (k'_{\perp})^2}{(2\pi)^4} \int_{x_\perp} \left| \frac{d^2k_\perp}{(2\pi)^2} \text{tr}[\hat{W}^A_{k_\perp} (k'_{\perp} - k_\perp)] t^{ij} \right|^2,
\] (10)
where $t^{ij}$ is a tensor involving the transverse momenta $k_\perp$ and $k'_\perp$, and $u = (\xi - y)/y$. The function $\tilde{W}_A^{x \perp}$ is the Fourier transform of $W_A^{x \perp}$, which is defined as in Eq. (1), but with the $\hat{U}$-matrices in the adjoint representation.

From Eqs. (9) and (10) one obtains for the differential cross section in the leading-$\ln(1/x)$ approximation,

$$\frac{d\sigma_T}{d\alpha dp_{\perp}^2} \propto \alpha_s \frac{(\alpha^2 + (1-\alpha)^2) (p_{\perp}^2 + a^4)}{(a^2 + p_{\perp}^2)^4} \ln(1/x).$$

(11)

A comparison of Eqs. (6) and (11) shows that the $p_{\perp}$-spectrum for the $q\bar{q}g$ configuration is much harder than that for the $q\bar{q}$ configuration. This is expected since in boson-gluon fusion $p_{\perp}$ is distributed logarithmically between $\Lambda$ and $Q$, thus resulting in a significant high-$p_{\perp}$ tail. The quantitative differences are particularly pronounced in the integrated cross section with a lower cut on transverse momentum $p_{\perp,\text{cut}}^2$. The shape of the momentum distribution is shown in Fig. 4. Each curve is normalized to its value at $p_{\perp,\text{cut}}^2 = 5 \text{ GeV}^2$.

The $q\bar{q}$ and $q\bar{q}g$ final states also differ with respect to the invariant mass distribution of the two jets. The additional wee gluon contributes significantly
to the diffractive mass of the final state \( m \). Rather similar to the diffractive production of high \( p_T \)-jets are the qualitative features of diffractive open charm production \( \).

**Comparison with other approaches**

It is instructive to compare the described results with those of other approaches to diffractive DIS. The phenomenology of the semiclassical approach is qualitatively very similar to ‘soft’ pomeron models \( \), if the pomeron is ‘gluonic’. In leading order it corresponds to \( \alpha_s(0) = 1 \). The projection on colour singlet final states for diffractive DIS yields asymmetric parton configurations in the proton rest frame, which is the basis of the aligned jet model \( \).

A qualitative difference with respect to ‘hard’ pomeron models, so far mostly two-gluon exchange \( \), is the \( p_T \)-spectrum. It will be interesting to see how important \( q\bar{q}g \) final states are in the two-gluon exchange model. In the semiclassical approach the dominant partonic process is boson-gluon fusion, as in the boson-gluon fusion model for diffraction \( \). However, in contrast to this model, the semiclassical approach predicts an additional low transverse momentum gluon in the final state. Similar in spirit is the soft colour interaction model \( \), where some soft non-perturbative gluon exchange is incorporated in a Monte Carlo event generator.

Finally, important questions which remain to be studied in the semiclassical approach are: the evolution in \( Q^2 \), the effect of higher order corrections on the \( \xi \)-dependence, the treatment of further low transverse momentum gluons in the diffractive final state and, on the theoretical side, the question of the validity of the semiclassical approximation.
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