Cascaded Composite Turbulence and Misalignment: Statistical Characterization and Applications to Reconfigurable Intelligent Surface-Empowered Wireless Systems

Alexandros-Apostolos A. Boulogeorgos, Senior Member, IEEE, Nestor Chatzidiamantis, Member, IEEE, Harilaos G. Sandalidis, Angeliki Alexiou, Member, IEEE, and Marco Di Renzo, Fellow, IEEE

Abstract—Reconfigurable intelligent surfaces (RISs) empowered high-frequency (HF) wireless systems are expected to become the supporting pillar for several reliability and data-rate hungry applications. Such systems are, however, sensitive to misalignment and atmospheric phenomena including turbulence. Most of the existing studies on the performance assessment of RIS-empowered wireless systems ignore the impact of the aforementioned phenomena. Motivated by this, the current contribution presents a theoretical framework for statistically characterizing cascaded composite turbulence and misalignment channels. More specifically, we present the probability density and cumulative distribution functions for the cascaded composite turbulence and misalignment channels. Building upon the derived analytical expressions and in order to demonstrate the applicability and importance of the extracted framework in different use cases of interest, we present novel closed-form formulas that quantify the joint impact of turbulence and misalignment on the outage performance for two scenarios, namely cascaded multi-RIS-empowered free space optics (FSO) and terahertz (THz) wireless systems. For the aforementioned scenarios, the diversity order is extracted. In addition, we provide an insightful outage probability upper-bound for a third scenario that considers parallel multi-RIS-empowered FSO systems. Our results highlight the importance of accurately modeling both turbulence and misalignment when assessing the performance of such systems.

Index Terms—Optical wireless communications, outage probability, performance analysis, reconfigurable intelligent surfaces, statistical characterization, THz wireless communications.

I. INTRODUCTION

By envisioning unprecedented performance requirements in terms of data-rates, reliability, availability, and security, the sixth generation (6G) wireless era comes with the promise to become the pillar of several “killer-applications”, including but not limited to extended reality, high-speed kiosks, and wireless backhauling [1]–[4]. To achieve these goals, two technology enablers are identified and expected to be exploited, namely high-frequency (HF) communications, in the sub-terahertz, terahertz (THz) [5]–[8] and optical bands [9]–[11], and reconfigurable intelligent surfaces (RISs) [12]–[15]. THz and optical wireless systems can support aggregated data rates that may exceed 1 Tb/s [5], [16]–[19], thus, as envisioned, theoretically investigated and experimentally validated in [3], [20]–[30], they can be used as wireless fiber extenders. Indicatively, in [25], a 1 km wireless fiber extender was demonstrated that operates at 300 GHz, while, in [26], a 500 m THz demonstrator was reported. In [29], a 100 m, FSO demonstrator was presented. In [30], finally, an 800 m outdoor point-to-point optical wireless link was demonstrated.

Despite the paramount importance that HF systems can play as wireless fiber extenders due to their sensitivity to blockages, they are unable to guarantee neither high-reliability nor continuous availability [31]–[34]. On the other hand, RISs are programmable metasurfaces, which are capable of altering the electromagnetic characteristics of the propagation medium, thus transforming conventional communication environments into smart platforms [12], [35]–[37]. In more detail, two of the most commonly examined RIS functionalities are blockage avoidance by creating alternative paths through beam-steering [38]–[40], which ensures uninterrupted connectivity between the source (S) and the final destination (D), and beam focusing in order to extend the system transmission distance [41], [42].

Motivated by these considerations, a great amount of effort has been very recently put on combining the aforementioned technologies and designing RISs that operate either in the THz or the optical band [43]–[49]. In [43] and [44], the authors presented a vanadium dioxide (VO₂)-based multi-functional RIS capable of manipulating THz waves, whereas a large-scale RIS that employs arrays of complementary metal-oxide-semiconductor (CMOS)-based chip tiles and operates at 0.3 THz was documented in [45]. In [46], a THz graphene plasmonic metasurface was reported, while a broadband nonlinear plasmonic metasurface THz emitter operating in the THz band was demonstrated in [47]. Similarly, a graphene-based RIS structure that provides beam steering and focusing capabilities at frequencies around 4.35 THz was discussed.
in [48]. In [49], a micro-electro-mechanical (MEM)-based THz RIS was demonstrated.

In addition, optical RISs have attracted a considerable attention [50]–[56]. In [50], a MEM was employed as an optical RIS in order to provide beam-steering capabilities in an optical wireless communication (OWC) system, while all-dielectric metasurfaces capable of manipulating near infrared waves were reported in [51]. The authors of [52] presented metalens-based metasurfaces that enable diffraction-limited focusing at wavelengths of 405, 532, and 660 nm, whereas an achromatic gallium nitride (GaN)-based optical RIS that operates in the entire visible region was reported in [53]. In [54], an optical RIS, which is capable of controlling the angular dispersions, enabling functionalities that range from perfect mirroring to angular multiplexing, was documented. In [55], in addition, two types of optical RISs were presented based on MEMs and phase array based technologies. Finally, in [56], an achromatic metasurface capable of eliminating the chromatic aberration over a continuous region from 1200 to 1680 nm for circularly-polarized incidence light in a reflection scheme was reported.

From the performance analysis, system design, and optimization points of view, the error performance of RIS-empowered THz wireless satellite systems in the presence of antenna misalignment were studied in [57]. In [58], the outage, error, and capacity performance of a single-RIS-empowered THz wireless system in the presence of α − µ fading and antenna misalignment was analyzed. In [59], a capacity evaluation of RIS-empowered sub-THz wireless systems was conducted. In [60], the authors assessed the joint impact of hardware imperfections and antenna misalignment on RIS-empowered indoor THz wireless systems, whereas the coverage performance of RIS-empowered THz wireless systems were quantified in [61]. In [62], the authors introduced a multi-RIS-empowered THz hybrid beamforming architecture and formulated the design problem of the digital and analog beamforming matrices, assuming that all the intermediate channels, between the transmitter and the receiver, experience neither fading nor misalignment. In [63], in addition, a holographic RIS-empowered THz massive multiple-input multiple-output (MIMO) system accompanied by a low-overhead closed-loop channel estimation scheme was presented. In [64] the authors presented a sum-rate maximization framework for RIS-empowered THz wireless systems, by assuming that the transmitter-RIS and RIS-destination channels have a Rician distribution. In [65], the authors studied the problem of proactive handoff and beam selection in THz drone communication networks assisted with RIS. Similarly, in [66], the joint optimization problem of drone’s trajectory, the phase shift of RIS, the allocation of THz sub-bands, and the power control were investigated aiming to maximize the minimum average achievable rate of all users. In [67], finally, a simultaneous terahertz (THz) information and power transfer system was introduced and the problem of maximizing the information users’ data rate while ensuring the energy users’ and RIS’s power harvesting requirements was formulated and solved. It is worth mentioning that all the aforementioned works assume that the RISs have horizontal and vertical meta-atom periodicity of the order of half of the wavelength.

The performance of RIS-empowered OWC systems was analyzed in [68]–[71]. In [68], the received signal-to-noise-ratio (SNR) in RIS-empowered visible light communication systems were evaluated. In [69], the authors studied the performance of a single-RIS-empowered OWC system in the presence of random obstacles and pointing errors, assuming that the impact of turbulence can be neglected. In [70], the authors considered a single-RIS free-space-optics (FSO) system and quantified its outage performance by devising an equivalent mirror-assisted FSO system that generates a reflected electric field on a mirror, which is identical to that on the source. Although this approach seems very promising, it cannot capture the statistics of the two cascaded links. Aspired by this, a closed-form expression for the outage probability (OP) of a single-RIS-empowered FSO system, in which both links experience different levels of turbulence and misalignment, was derived in [71].

Recently, the concept of using multiple-RISs to provide uninterrupted connectivity between pairs of transmitters and receivers was reported in [72]. Multi-RIS can enable a number of novel functionalities including, but not limited to blockage avoidance, routing, coverage expansion, and beam splitting. Despite the importance and capabilities that this concept can offer, to the best of the authors’ knowledge, a theoretical framework for analyzing the achievable performance of such systems has not been formulated yet. This is due to the lack of a statistical model for the cascaded composite turbulence and misalignment fading channels. Motivated by this, in this paper, we introduce an analytical framework that covers the aforementioned gap as well as its applications in RIS-empowered wireless systems. In particular, the technical contribution of this work is as follows:

- We present closed-form expressions for the probability density function (PDF) and cumulative distribution function (CDF) for cascaded channels that experience different levels of turbulence.
- We derive a novel analytical expression for the PDF of cascaded channels that experience misalignment fading.
- Building upon the aforementioned expressions, we statistically characterize cascaded channels that experience different levels of turbulence and misalignment fading, in terms of PDF and CDF.
- We apply the obtained analytical frameworks for quantifying the outage performance of three relevant application scenarios, namely: i) a cascaded multi-RIS empowered FSO system, ii) a multi-RIS empowered FSO systems, and iii) a cascaded multi-RIS empowered THz wireless system. More precisely, in the first scenario, we introduce an accurate closed-form expression for the system OP that takes into account both the characteristics of the transmitter, receiver and propagation channel. For the second scenario, we report an OP upper-bound, and for the third scenario, we provide a closed-form expression for the OP that accounts for the impact of transceivers hardware imperfections.
- We provide the diversity order of the cascaded multi-RIS empowered FSO and THz systems.
The rest of this contribution is structured as follows: The statistical characterization of cascaded wireless channels that experience turbulence and/or misalignment fading is presented in Section II. Some applications of the analytical framework are discussed in Section III while respective Monte Carlo simulations that verify the theoretical framework, accompanied by insightful discussions, are illustrated in Section IV. Finally, a summary of our main findings and concluding remarks are provided in Section V.

Notations: The absolute value, exponential and natural logarithm functions are respectively denoted by |·|, exp(·), and ln(·), √(·) and ∑(·) respectively return the square root of x, and the product of \( x_1 x_2 \cdots x_L \). Pr(\( A \)) denotes the probability for the event A to be valid. The modified Bessel function of the second kind of order n is denoted as \( K_n(\cdot) \) [73, eq. (8.407/1)]. The Gamma [73, eq. (8.310)] function is denoted by \( \Gamma(\cdot) \) and the error-function is represented by erf(·) [73, eq. (8.250/1)]. The generalized hypergeometric function is denoted by \( \,_pF_q(a;b;z) \). Finally, \( C_{i,m,n}^{\alpha,n_p,n} \left( x \mid a_1, a_2, \cdots, a_p \mid b_1, b_2, \cdots, b_q \right) \) stands for the Meijer G-function [73, eq. (9.301)]. Tables I and II provide the definitions of all the symbols that are employed in this paper.

II. Statistical Characterization of Product Channels

Let \( Z_1 \) be the product of \( N \geq 1 \) independently distributed Gamma-Gamma (GG) random variables (RVs), i.e.,

\[
Z_1 = \prod_{i=1}^{N} r_i,
\]

where \( r_i \), with \( i \in [1, N] \), is the \( i \)-th distributed GG RV. The PDF of \( r_i \) can be written as follows [74, 75]

\[
f_{r_i}(x) = \frac{2}{\Gamma(\alpha_i) \Gamma(\beta_i)} \left( \frac{\alpha_i \beta_i}{\Omega_i} \right)^{\alpha_i+\beta_i-1} x^{\alpha_i-1} \times K_{\alpha_i-\beta_i} \left( \alpha_i - \beta_i, 2 \sqrt{\frac{\alpha_i \beta_i x}{\Omega_i}} \right),
\]

where \( \alpha_i \geq 0 \) and \( \beta_i \geq 0 \) are the shape parameters of the \( r_i \) distribution and \( \Omega_i \) is the corresponding mean, i.e., \( \Omega_i = \mathbb{E}_{r_i}[x] \). Of note, the distribution in (2) is generic, since, for different combinations of \( \alpha_i \) and \( \beta_i \), it returns various models that are usually used in communication systems. For \( \alpha_i \rightarrow \infty \), it approximates the Gamma distribution, which have been widely used to model the fading of radio frequency systems. For \( \alpha_i = 1 \), it returns the K distribution that is suitable for wireless systems with strong line-of-sights components, such as in HF systems. For the special case of \( \alpha_i = \beta_i = 1 \), finally, it reduces to the PDF of the double Rayleigh distribution.

The following theorem provides closed-form expressions for the evaluation of the PDF and the CDF of \( Z_1 \).

**Theorem 1.** The PDF of \( Z_1 \) can be evaluated as

\[
f_{Z_1}(x) = \frac{\left( \frac{\alpha_1 \beta_1}{\Omega_1} \right)^{\alpha_1+\beta_1-1} x^{\alpha_1-1}}{\prod_{i=1}^{N} \Gamma(\alpha_i) \Gamma(\beta_i)} \times \prod_{i=1}^{N} K_{\alpha_i-\beta_i} \left( \alpha_i - \beta_i, 2 \sqrt{\frac{\alpha_i \beta_i x}{\Omega_i}} \right),
\]

whereas, its CDF can be obtained as

\[
F_{Z_1}(x) = \frac{1}{\prod_{i=1}^{N} \Gamma(\alpha_i) \Gamma(\beta_i)} \times G_{1,2N+1}^{2N,1} \left[ \frac{\alpha_1 \beta_1}{\Omega_1} x^{\alpha_1}, \alpha_1, \alpha_2, \beta_2, \cdots, \alpha_N, \beta_N \right].
\]

**Proof:** The product of gamma-gamma RVs is a special case of the product of generalized-gamma RVs derived in [76, eqs. (2) and (9)]. By setting the suitable parameter in [76, eqs. (2) and (9)] and after some algebraic manipulations, we

| Parameters | Definition |
|------------|------------|
| \( p \) | Atmospheric pressure |
| \( T \) | Atmospheric temperature |
| \( P_0 \) | Average power constraint of the FSO signal |
| \( B \) | Bandwidth |
| \( k_B \) | Boltzman’s constant |
| \( A_Q \) | Derivative of the real-part of the refraction index with respect to the relative humidity |
| \( A_T \) | Derivative of the real-part of the refraction index with respect to the temperature |
| \( g_i \) | Deterministic path-gain coefficient of the \( i \)-th THz link |
| \( d_i \) | Distance of the \( i \)-th link |
| \( g_{fs} \) | Free-space path-gain coefficient of the \( i \)-th THz link |
| \( \tau(f, d) \) | Molecular absorption gain coefficient |
| \( \kappa_i \) | Molecular absorption coefficient |
| \( N \) | No. of Gamma-Gamma (GG) distributed random variables (RVs) |
| \( L \) | No. of misaligned links |
| \( f \) | Operation frequency |
| \( G_d \) | Reception antenna gain |
| \( \phi \) | Relative humidity |
| \( \mu_w \) | Saturated water vapor partial pressure |
| \( C_y \) | Structure factor |
| \( G_s \) | Transmission antenna gain |
| \( s_i \) | Transmission signal of the \( i \)-th S at the parallel multi-RIS FSO |
| \( \eta \) | Photodetector (PD) responsivity |
| \( c \) | Speed of light |
| \( \lambda \) | Wavelength of the optical carrier |
| \( \alpha \) | Weather dependent attenuation coefficient |
| \( G_i \) | \( i \)-th FSO link atmospheric gain |
| \( R_i \) | \( i \)-th RIS reflection coefficient |
| \( \rho_i \) | \( i \)-th optical RIS reflection efficiency |
| \( s \) | Intensity of the FSO transmitted signal |
| \( x \) | Transmission signal at the RIS-empowered THz wireless system |
| \( \rho_0 \) | Transmission SNR multiplied by the deterministic path-gain of the multi-RIS FSO system |
| \( r_{th} \) | SNR threshold at the multiple-RIS FSO system |
| \( \rho_{th} \) | SNR threshold at the multiple-RIS FSO system |
| \( \Omega_i \) | Shaping parameters of \( r_i \) |
| \( \Omega_i \) | Parameters and performance indicators definition. |
| \( \xi_i \) | Distribution parameters of \( r_i \) |
| \( \sigma^2_\eta \) | Variance of the additive white Gaussian shot noise |
| \( \sigma^2_\rho \) | Refraction index parameters |
| \( \sigma^2_{R_i} \) | Rytov variance of the \( i \)-th FSO link |
| \( b_i \) | Radius of the circular aperture at the \( i \)-th RIS or D plane |
| \( w_{d_i} \) | Beam waste at the \( i \)-th RIS or D plane |
| \( \mu_{eq,i} \) | Equivalent beam radius at the \( i \)-th RIS or D plane |
| \( \sigma^2_{\alpha_i} \) | Pointing error displacement (jitter) variance |
| \( \sigma^2_{\eta} \) | Noise variance |
| \( \sigma^2_{\eta_i} \) | Variance of \( \eta_i \) |
| \( \kappa_i \) | Error vector magnitude of the S transmitter |
| \( \kappa_{tr} \) | Error vector magnitude of the D receiver |

| Performance indicators |
|-------------------------|
| \( P_{\text{FSO}}^{\text{multi}} \) | Multi-RIS FSO system OP |
| \( P_{\text{FSO}}^{\text{par}} \) | Parallel multi-RIS FSO system OP |
| \( P_{\text{THz}} \) | RIS-empowered THz wireless system OP |
extract (3) and (4). This concludes the proof.

Let $Z_2$ be the product of $L$, $L \geq 1$, independent RVs, i.e.,

$$Z_2 = \prod_{i=1}^{L} l_i,$$

(5)

where $l_i$ is the $i$-th RV and its PDF can be expressed as

$$f_{l_i}(x) = \frac{\xi_i}{A_{o,i}} x^{\xi_i-1},$$

(6)

with $\xi_i$ and $A_{o,i}$ being the distribution parameters of $l_i$. The following theorem provides a closed-form expression for the PDF of $Z_2$.

**Theorem 2.** The PDF of $Z_2$ can be expressed as

$$f_{Z_2}(x) = \frac{1}{(L-1)!} \prod_{i=1}^{L} \frac{\xi_i}{A_{o,i}} x^{\xi_i-1} \left( \ln \left( \frac{\prod_{i=1}^{L} A_{o,i}}{x} \right) \right)^{L-1},$$

(7)

with $0 \leq x \leq \prod_{i=1}^{L} A_{o,i}$.

**Proof:** For brevity, the proof of Theorem 2 is given in Appendix B.

For the special case in which $A_{o,1} = A_{o,2} = \cdots = A_{o,M} = A_o$ and $\xi_1 = \xi_2 = \cdots = \xi_M = \xi$, (7) can be simplified to

$$f_{Z_2}^{\xi}(x) = \frac{1}{(L-1)!} \left( \frac{\xi}{A_o} \right)^L x^{\xi-1} \left( \ln \left( \frac{A_o}{x} \right) \right)^{L-1},$$

(8)

with $0 \leq x \leq A_o^L$.

**Theorem 3.** The PDF and CDF of $Z = Z_1 Z_2$, (9) can be formulated as shown in (10) and (11), given at the top of the next page.

**Proof:** For brevity, the proof of Theorem 3 is given in Appendix C.

It should be noted that the expressions of (10) and (11) can be written in terms of more familiar hypergeometric functions according to (73) Eq. (9.303)). In more detail, for the special, but very realistic, case in which no two elements of the tuple $B = \{\alpha_1, \cdots, \alpha_N, \beta_1, \cdots, \beta_N, \xi_1, \cdots, \xi_L, 0\}$ differ by an integer, (11) can be rewritten as in (12), given at the top of the next page. In (12), $A_i$ with $i = 1, \cdots, L + 1$ refers to the $i$-th element of the tuple $A = \{1, \xi_1 + 1, \cdots, \xi_L + 1\}$. Similarly, $B_i$ with $i = 1, \cdots, 2N + L$ refers to the $i$-th of $B$. According to (77)–(79), since Meijer-G can be expressed as a finite sum of hypergeometric functions, it is considered a closed-form expression. Moreover, notice that nowadays, the Meijer-G function can be directly computed in several software packages and programming languages, including, but not limited to, Mathematica, Maple, Matlab, Python, C++.

The following lemma return the limit of the CDF of $Z$ as $x \rightarrow 0$.

**Lemma 1.** In the case in which no two elements of the tuple $B$ differ by an integer, as $x$ tends to 0, the limit of the CDF of $Z$ can be approximated as

$$F_2^0(x) = \prod_{i=1}^{L} \frac{\xi_i}{A_{o,i}} \left( \ln \left( \frac{\prod_{i=1}^{L} A_{o,i}}{x} \right) \right)^{L-1},$$

(13)

**Proof:** As $x \rightarrow 0$,

$$x^L F_{2N+L} \left( 1 + B_1 - A_1, \cdots, 1 + B_i - A_{L+1}; 1 + B_1 - B_1, \cdots, 1 + B_i - B_i, (1 - 2N)^{-1} \right) \rightarrow 1$$

(14)

By applying (14) into (12), we obtain (13). This concludes the proof.

**III. APPLICATIONS**

This section is focused on presenting some applications of the theoretical framework. In more detail, three scenarios are considered: i) cascaded multi-RIS empowered FSO; ii) parallel multi-RIS empowered FSO; and iii) cascaded multi-RIS empowered wireless THz systems. For each scenario, the corresponding systems models are first presented and then closed-form expressions or upper bounds for the OP are computed.

A. Cascaded multi-RIS empowered FSO systems

1) System model: As depicted in Fig. 1 an FSO system, $N-1$ optical RISs, and a destination $D$ equipped with a lens and a photo-detector (PD), is considered. The LS transmits a Gaussian beam towards the RIS. The $i$-th RIS reflects the incident beam towards the $(i+1)$-th RIS until the $(N-1)$-th RIS. The $(N-1)$-th RIS reflects the incident beam towards the lens $D$, which focuses it to the PD. Of note, as reported in [53], the RIS is capable of changing the beam shape. We assume that the LS, RISs, and PD are located in fixed positions and that the $i$-th RIS shapes the beam in an optimal manner, i.e., in a way that its footprint at the $i+1$-th RIS and the PD planes are a circle. Finally, it is assumed that the direct $S - D$ link is blocked, while all the intermediate RIS-empowered and RIS-D links are in line-of-sight (LoS).

The turbulence coefficient of the $i$-th link is represented by $h_i$, which can be modeled as a GG RV. Each link may or
may not experience misalignment fading. Let us assume that 
$L$ out of $N$ links suffer from misalignment fading and let us 
model its impact on the $i$–th link through a coefficient $h_{p,i}$. 
Thus, the received signal at the PD of $D$ can be written as 
$$r = As + n,$$ 
where $s \in \mathbb{R}^+$ represents the intensity of the transmitted signal with $\mathbb{E}[s] \leq P$ with $P$ being the average power constraint, $n \in \mathbb{R}$ stands for the additive white Gaussian shot noise with variance $\sigma_n^2$, which is caused by the ambient light at the PD, and $A \in \mathbb{R}^+$ is the end-to-end (e2e) channel. The e2e channel coefficient can be expressed as 
$$A = \eta \prod_{i=1}^{L} h_{p,i} \beta_i G_i \prod_{i=L+1}^{N} h_i G_i,$$ 
where $\eta$ represents the PD responsivity, while $G_i$ stands for the $i$-th link atmospheric gain and can be written as 
$$G_i = \rho 10^{-\alpha(d_i-1) + \beta_i d_i}/10.$$ 
In $\{17\}$, $\rho$ is the $i$–th RIS reflection efficiency that typically ranges in $[0.7, 1]$ $\{60, 61\}$. $\alpha$ is a weather-dependent attenuation coefficient, $d_{i-1}$ and $d_i$ are the distances between the $i-1$ and $i$ links, respectively. Furthermore, the parameters $h_i$ follow the GG distribution with atmospheric parameters $\alpha_i$ and $\beta_i$, calculated based on $d_{i-1}$ or $d_i$, the refraction index parameter $C_n^2$ and the wavelength of the optical carrier $\lambda$ $\{82\}$ according to the following formulas 
$$\alpha_i = \left( \exp \left\{ \frac{0.49 \sigma_{R_i}^2}{1 + 1.11 \sigma_{R_i}^2} \right\} - 1 \right)^{-1},$$ 
and 
$$\beta_i = \left( \frac{0.51 \sigma_{R_i}^2}{1 + 0.69 \sigma_{R_i}^2} \right)^{-1}.$$ 
Moreover, $\sigma_{R_i}^2$ is the Rytov variance given by 
$$\sigma_{R_i}^2 = 1.23 C_n^2 \left( \frac{2\pi}{\lambda} \right)^{\frac{2}{3}} \frac{d_i^{\frac{4}{3}}}{\bar{d}_i^{\frac{4}{3}}},$$ 
with $C_n^2$ being the reflective index structure parameter, which is used to characterize the atmospheric turbulence.

Finally, the PDF $h_{p,i}$ can be obtained as in $\{6\}$. In this scenario, 
$$A_{o,i} = \left[ \text{erf} \left( v_i \right) \right]^2$$ 
stands for the fraction of the collected power in the ideal case of zero radial displacement. Moreover, 
$$v_i = \frac{\sqrt{\pi} b_i}{\sqrt{2} w_o},$$ 
with $b_i$ and $w_o$ representing the radius of the circular aperture at the $i$–th RIS, with $i \in [1, N-1]$, or $D$’s side, for $i = N$, respectively and the beam waste on the corresponding plane. Likewise, $\xi_i$ is the equivalent beam radius, $w_{eq,i}$, to the pointing error displacement standard deviation square ratio and can be calculated as 
$$\xi_i = \frac{w_{eq,i}^2}{4 \sigma_{s,i}^2},$$ 
with $\sigma_{s,i}^2$ denoting the pointing error displacement (jitter) variance. Finally, 
$$w_{eq,i}^2 = w_o^2 \frac{\sqrt{\pi} \text{erf} \left( v_i \right)}{2 v_i \exp \left( - v_i^2 \right)},$$ 
2) Performance assessment: The SNR at the RX can be expressed as 
$$\rho_{FSO} = A^2 \rho_s,$$ 
where $\rho_s$ stands for the transmission SNR multiplied by the deterministic path-gain. Thus, the OP is defined as 
$$P_{\alpha}^{FSO} = \Pr \left( \rho_{FSO} \leq \tau_{\alpha} \right),$$ 
or, from $\{25\}$, 
$$P_{\rho_s}^{FSO} = \Pr \left( A \leq \sqrt{\frac{\tau_{\rho_s}}{\rho_s}} \right),$$ 
which, by applying $\{11\}$, can be written in a closed-form expression. For the special case, in which $d_1 \neq d_2 \neq \cdots \neq d_N$, $\{12\}$ can be employed in order to assess the OP of the cascaded multi-RIS empowered FSO system. Of note, for the special case in which $N = L = 2$, $d_1 = d_2$, $b_1 = b_2$, 

![Fig. 1: Cascaded multi-RIS empowered FSO system model.](image-url)
Fig. 2: Parallel Multi-RIS FSO system model.

Let $w_{d_1} = w_{d_2}$ and $\sigma_{s,1} = \sigma_{s,2}$ as in (27) 

The following lemma returns the diversity order of the cascaded multi-RIS empowered FSO system.

**Lemma 2.** The diversity order of the cascaded multi-RIS empowered FSO system can be obtained as

$$D_{FSO} = \min_{i=1,\ldots,2N+L} B_i/2.$$  \hfill (28)

**Proof:** For $\rho_s \to \infty$, the term $\frac{\sigma_{s,2}}{\rho_s}$ tend to 0; thus, based on (13), the outage probability can be approximated as

$$p_{FSO} = \frac{\prod_{i=1}^{L} \xi_i}{\prod_{i=1}^{N} \Gamma(\alpha_i) \Gamma(\beta_i)} \times \sum_{i=1}^{2N+L} \prod_{j=1}^{2N+L+1} \Gamma(B_j - B_i) \Gamma(A_j - B_i) \prod_{j=2}^{L+1} \Gamma(A_j - B_i) \times \left(\frac{\rho_s}{\rho_s}\right)^{B_i/2}.$$  \hfill (29)

It is evident that in (29), $\left(\frac{\rho_s}{\rho_s}\right)^{B_i/2}$ contributes with diversity order $B_i/2$ in the asymptotic OP. Hence, the diversity order can be obtained as in (28). This concludes the proof. \hfill \blacksquare

**B. Parallel Multi-RIS FSO systems**

1) **System model:** As presented in Fig. 2, we consider a multi-RIS FSO system that consists of a $S$, $N$ RIS, and a $D$. The $S$ is equipped with $N$ transmit apertures, each one at a different RIS. The $i$-th RIS steers the incident beam towards $D$, which is equipped with a single PD. Thus, the received signal at the destination can be expressed as

$$r_p = \sum_{i=1}^{N} B_i s_i + n,$$  \hfill (30)

where $s_i$ is the transmission signal by the $i$-th S’s aperture, while

$$B_i = h_{p_1,i}h_{p_2,i}h_{1,i}h_{2,i}G_i,$$  \hfill (31)

represents the $S-i$-th RIS-D channel coefficient. In more detail, in (31), $h_{p_1,i}$ and $h_{p_2,i}$ denote the fading misalignment coefficient of the $S-i$-th RIS and $i$-th RIS-D links, respectively, whereas $h_{1,i}$ and $h_{2,i}$ are the turbulence coefficients of the $S-i$-th RIS and $i$-th RIS-D links, respectively.

2) **Performance assessment:** By assuming that the total transmission power, $P_s$, is equally divided into the $N$ transmission apertures, the instantaneous SNR at $D$ can be obtained as

$$\rho_{par} = \left(\sum_{i=1}^{N} B_i\right)^2 P_s / NN_0,$$  \hfill (32)

or equivalent

$$\rho_{par} = NS^2 \rho_s,$$  \hfill (33)

where

$$S = \frac{1}{N} \sum_{i=1}^{N} B_i.$$  \hfill (34)

From (33), the OP can be expressed as

$$P_{par}^o = Pr(\rho_{par} \leq \rho_{th}),$$  \hfill (35)

or equivalently

$$P_{par}^o = Pr\left(S \leq \sqrt{\frac{\rho_{th}}{N\rho_s}}\right),$$  \hfill (36)

and

$$P_{par}^o = F_S\left(\sqrt{\frac{\rho_{th}}{N\rho_s}}\right),$$  \hfill (37)

with $F_S(\cdot)$ being the CDF of $S$. According to [83, eq.(14)], the OP of the parallel multi-RIS FSO scenario can be upper bounded as

$$P_{par}^o \leq \left(\sqrt{\frac{\rho_{th}}{N\rho_s}}\right)^N.$$  \hfill (38)

C. **RIS-empowered THz wireless systems**

In this section, the system model and theoretical framework that quantifies the outage performance of multi-RIS-empowered outdoor THz wireless systems is provided. Note that this analysis can be also and straightforwardly extended to millimeter wave systems by replacing the term that describes the deterministic path-gain of the THz system with the corresponding one for the millimeter wave system.

1) **System model:** A multi-RIS-empowered outdoor THz wireless systems is considered, in which $S$ and $D$ are equipped with high-directional antennas. This system can be used a wireless fiber extender; thus, both the $S$ and $D$ antennas are placed high, e.g., in rooftops. Again, it is assumed that due to static obstacles, e.g., buildings, no direct link between the $S$ and $D$ can be established; as a result, $N$ RISs are used. Each RIS can be seen as a reflector that steers and/or focuses the beam towards the desired direction. Thus, the received signal at $D$ can be obtained as

$$y = A_x (x + \eta_s) + \eta_d + w,$$  \hfill (39)

where $x$ is the transmission signal, while $w$ stands for the AWGN. Moreover, $\eta_s$ and $\eta_d$ are the $S$ and $D$ distortion noises, due to transmitter’s and receiver’s hardware imperfections, respectively. According to [84-87], $\eta_s$ and $\eta_d$ can be modeled as two independent RVs that, for a given channel realization, have zero-mean complex Gaussian distributions with variances

$$\sigma_s^2 = \kappa_t^2 P_s$$

and

$$\sigma_d^2 = \kappa_r^2 A^2 P_s,$$  \hfill (40)

respectively. In (40), $\kappa_t$ and $\kappa_r$ stand for the error vector magnitude of the S’s transmitter and the D’s receiver, while $P_s$ denotes the transmission power.

Meanwhile, in (39), $A_x$ represents the e2e channel, which

1Note that, since the transmission and reception antennas as well as the RISs are placed high, no dynamic blockage is expected.
can be expressed as

\[ A_i = \prod_{i=1}^{L} h_{p,i}^{\text{THz}} h_{t,i}^{\text{THz}} g_i = \prod_{i=L+1}^{N} h_{t,i}^{\text{THz}} g_i. \]  

(41)

In (41), \( h_{p,i}^{\text{THz}} \) and \( h_{t,i}^{\text{THz}} \) are the misalignment fading and turbulence coefficients that are distributed according to (3) and (2), respectively. Note that the turbulence model that is employed has been experimentally validated in [4], theoretically analyzed in [89], and supported by ITU-R [90]. The \( \alpha_i \) and \( \beta_i \) parameters of (2) can be evaluated according to [89] as

\[ \alpha_i^{\text{THz}} = \left( \exp \left( \frac{\sigma_{R_i}^2}{1 + 0.65 D_i^2 + 1.11 \sigma_{R_i}^{12/5}} \right)^{7/6} \right) - 1 \]  

(42)

and

\[ \beta_i^{\text{THz}} = \left( \exp \left( \frac{0.51 \sigma_{R_i}^2}{1 + 0.9 D_i^2 + 0.62 D_i^2 \sigma_{R_i}^{12/5}} \right)^{-5/6} \right) - 1 \]  

(43)

while \( \sigma_{R_i}^2 \) can be obtained as in (20). Additionally,

\[ D_i = \sqrt{\frac{\pi b_i^2}{2 \lambda_i}}. \]  

(44)

According to ITU-T, in the THz band, the reflection index structure parameter can be formulated as [91]

\[ C_n^2 = \frac{C_T^2}{T} \left( A_T^2 (\lambda) + 10^4 A_R^2 (\lambda) + 200 A_T (\lambda) A_R (\lambda) \right), \]  

(45)

where \( T \) stands for the temperature, \( C_T \) denotes the structure factor for the temperature, while \( A_T \) and \( A_R \) are the derivatives of the real part of the refractive index with respect to the temperature and humidity, respectively. Finally, \( g_i \) represents the deterministic path-gain coefficient of the \( i \)-th link and can be written as

\[ g_i = g_{f,i} (f, d_i) \tau (f, d_i), \]  

(46)

where \( g_{f,i} \) denotes the free space path loss coefficient, which based on the Friis transmission equation, can be expressed as

\[ g_{f,i} (f, d_i) = \begin{cases} \frac{\pi f d_i}{2 \lambda_i}, & \text{for } i = 1, \\ \frac{\pi f d_i}{R_{i-1}}, & \text{for } i \in [2, N-1], \\ \frac{\pi f d_i}{R_d}, & \text{for } i = N \end{cases}, \]  

(47)

whereas \( \tau (f, d_i) \) denotes the molecular absorption coefficient and can be obtained as

\[ \tau (f, d_i) = \exp \left( -\frac{1}{2} \kappa (f) d_i \right). \]  

(48)

In (47), \( f \) and \( c \) stand for the transmission frequency and the speed of light, respectively, while \( G_s \), \( R_i \) and \( G_d \) represent the S transmission antenna gain, the \( i \)-th RIS reflection coefficient, and the D reception antenna gain, respectively. Likewise, in (48), \( \kappa (f) \) denotes the molecular absorption coefficient, which depends on the atmospheric temperature \( (T) \), pressure \( (p) \), as well as relative humidity \( (\phi) \) and can be evaluated as in [92], [93] and [94 eq. (8)],

\[ \kappa (f) = g(f) + g_1 + g_2, \]  

(49)

where

\[ g(f) = \sum_{i=0}^{3} p_i f^i, \]  

(50)

with \( p_0 = -6.36, p_1 = 9.06, p_2 = -3.94 \) and \( p_3 = 5.54 \).

Moreover,

\[ g_1 = \frac{G_A (\mu_w)}{G_B (\mu_w) + \left( \frac{\mu_w}{100} - c_0 \right)^2}, \]  

(51)

where \( G_A (\mu_w) = 0.2205 \mu_w (0.1303 \mu_w + 0.0294) \), \( G_B (\mu_w) = (0.4093 \mu_w + 0.0925)^2 \), \( c_0 = 10.835 \) with \( \mu_w \), being the volume mixing ratio of the water vapor and can be computed as

\[ \mu_w = \frac{\phi}{100} p_w (T, p). \]  

(52)

In (52), \( p_w (T, p) \) is the saturated water vapor partial pressure and can be expressed as

\[ p_w (T, p) = k_1 (k_2 + k_3 \phi) \exp \left( \frac{k_4 (T - T_1)}{T - T_2} \right), \]  

(53)

where \( k_1 = 6.1121, k_2 = 1.0007, k_3 = 3.46 \times 10^{-6} \text{ hPa}^{-1}, k_4 = 17.502, T_1 = 273.15^0 \text{K}, \) and \( T_2 = 32.18^0 \text{K}. \) Finally,

\[ g_2 = \frac{G_D (\mu_w)}{G_D (\mu_w) + \left( \frac{100 \sigma}{c_1} - 1 \right)^2}, \]  

(54)

where \( G_D (\mu_w) = (2.014 \mu_w (0.1702 \mu_w + 0.0303), G_D = (0.537 \mu_w + 0.0956)^2 \) and \( c_1 = 12.664. \)

2) Performance assessment: From (39), the signal-to-distortion-plus-noise (SDNR) can be obtained as

\[ A_t^2 P_s \]  

(55)

where \( N_o \) is the noise variance. Moreover, the OP is defined as

\[ P_{\text{THz}}^o = \Pr (\gamma \leq \gamma_{th}), \]  

(56)

where \( \gamma_{th} \) represents the SNR threshold. With the aid of (55), (56) can be rewritten as

\[ P_{\text{THz}}^o = \begin{cases} \Pr \left( A_t^2 \leq \frac{1}{1 - \gamma_{th}(\kappa_1^2 + \kappa_2^2)} \gamma_{th} \right), & \text{for } \gamma_{th} \leq \frac{1}{\kappa_1^2 + \kappa_2^2}, \\ 1, & \text{for } \gamma_{th} > \frac{1}{\kappa_1^2 + \kappa_2^2}. \end{cases} \]  

(57)

Finally, by applying (41) in (57), we get (58), given at the top of the next page. In (58), \( \gamma_{th} \) is the transmission SNR multiplied by the deterministic path-gain, i.e.,

\[ \gamma_s = \frac{P_s \prod_{i=1}^{N} g_i}{N_o}. \]  

(59)

Note that from (58), it becomes evident that hardware imperfections set a limit to the maximum allowed spectral efficiency of the transmission scheme. In more detail, since the spectral efficiency, \( p \), is connected with the SNR threshold through \( p = \log_2 (1 + \gamma_{th}) \), choosing a \( p \) greater than \( \log_2 (1 + 1/\kappa_1^2 + \kappa_2^2) \) would lead to an OP equal to 1.

The following lemma returns the diversity order of the multi-RIS empowered THz system.

Lemma 3. The diversity order of the multi-RIS empowered THz system can be obtained as

\[ D_{\text{THz}} = \min_{i=1,\cdots,2N+L} B_{i}^{\text{THz}} / 2, \]  

(60)

where \( B_{i}^{\text{THz}} \) is the \( i \)-th element of the tuple \( \{ \alpha_1^{\text{THz}}, \cdots, \alpha_N^{\text{THz}}, \beta_1^{\text{THz}}, \cdots, \beta_L^{\text{THz}}, \xi_1, \cdots, \xi_L, 0 \} \)

\[ \text{Proof: This lemma can be proven by following the same steps as the proof of Lemma 2.} \]

IV. RESULTS & DISCUSSION

This section focuses on verifying the theoretical framework presented in Section II by means of Monte Carlo simulations. To this end, lines are used to denote analytical results, while
markers are employed for simulations. The rest of this section is organized as Section IV-A focuses on presenting numerical results that quantify the outage performance of cascaded multi-RIS-empowered FSO systems, whereas Section IV-B verifies the outage performance bounds of parallel multi-RIS-empowered FSO systems. Finally, in Section IV-C the outage performance of cascaded multi-RIS-empowered THz wireless systems is assessed.

A. Cascaded multi-RIS-empowered FSO systems

Figure 3 quantifies the impact of turbulence in a multi-RIS-empowered FSO system, where \( N = 2 \) and \( L = 0 \). In more detail, the OP is plotted against \( \rho_s/\rho_{th} \) for different levels of turbulence. The following scenarios are considered: i) both the S-RIS and RIS-D links experience weak turbulence, i.e., \( \alpha_1 = \alpha_2 = 10.02 \) and \( \beta_1 = \beta_2 = 2.98 \), ii) either the S-RIS or the RIS-D link experience weak (\( \alpha_1 = 10.02 \) and \( \beta_1 = 2.98 \)), while the other suffers from strong turbulence (\( \alpha_2 = 4.942 \) and \( \beta_1 = 1.231 \)), iii) both links experience strong turbulence, iv) one link suffers from strong, while the other from moderate (\( \alpha_2 = 2.53 \) and \( \beta_1 = 3.02 \)), and v) both links experience from moderate turbulence. This figure reveals that for fixed turbulence conditions, the RIS-empowered FSO system outage performance improves, as \( \rho_s/\rho_{th} \) increases. For example, for \( \alpha_1 = \alpha_2 = 10.02 \) and \( \beta_1 = \beta_2 = 2.98 \), as \( \rho_s/\rho_{th} \) changes from 25 to 35 dB, the OP decreases by about 10 times. Additionally, we observe that, for a fixed \( \rho_s/\rho_{th} \), as the intensity of turbulence increases in either one of the S-RIS and RIS-D links, the outage performance degrades. For instance, for \( \rho_s/\rho_{th} = 35 \) dB, the OP increases for more than an order of magnitude as turbulence conditions changes from scenario i to ii. Interestingly, for a fixed \( \rho_s/\rho_{th} \), the systems in which both links suffer from moderate turbulence may outperform the ones in which one link suffers from weak and the other from strong turbulence. This highlights the importance of accurately characterizing turbulence intensity when assessing the performance of RIS-empowered FSO systems.

Figure 4 assesses the impact of turbulence in FSO systems that employ multiple RISs. In particular, the OP is depicted as a function of \( \rho_s/\rho_{th} \), for different values of \( N \) and levels of turbulence, assuming \( L = 0 \).

Figure 5 depicts the OP as a function of the \( \sigma_s/\beta \) for different values of \( \beta/w_d \) and \( N = L \).
both RIS’s and D lens radius are (i) half, (ii) equal to, and (iii) twice the radiation beam footprint. As a benchmark, the OP for the case in which there is no misalignment is also presented. As expected, for given $N = L$ and $b/w_d$, as $\sigma_s/b$ increases, i.e., the misalignment intensity becomes more severe, the outage performance degrades. For example, for $N = L = 2$ and $b/w_d = 1$, the OP increases for more than one order of magnitude, as $\sigma_s/b$ changes from 0.1 to 0.5. Likewise, for fixed $\sigma_s/b$ and $b/w_d$, as $N = L$ increases, the joint effect of misalignment and turbulence becomes more severe; thus, the OP increases. Finally, for given $M = N$ and $\sigma_s/b$, as $b/w_d$ increases, the beam-waist is constrained; as a result, an outage performance improvement is observed. Interestingly, even in the low-$\sigma_s/b$ regime, if $b/w_d \leq 1$, there exists an OP gap between the two systems that consider and ignore the misalignment fading. This is because RIS does not account for the beam-waist. On the other hand, for $b/w_d > 1$, in the low-$\sigma_s/b$ regime, the OP of the FSO system that suffers from misalignment fading tends to the one with misalignment fading immunity. This indicates the importance of taking into account $b/w_d$, when assessing the outage performance of RIS-empowered FSO systems.

Figure 6 demonstrates the impact of different levels of misalignment fading on the OP of a multi-RIS-empowered FSO system with $N = L = 2$. Specifically, the OP is presented as a function of $\sigma_{s1}/b$ and $\sigma_{s2}/b$, assuming $\rho_s/\rho_{th} = 40$ dB, $\alpha_1 = \alpha_2 = 10.02$ and $\beta_1 = \beta_2 = 2.98$. As expected, for a given $\sigma_{s1}/b$, as $\sigma_{s2}/b$ increases, the outage performance degrades. For example, for $\sigma_{s1}/b = 0.1$, as $\sigma_{s2}/b$ changes from 0.1 to 0.6, the OP increases for approximately one order of magnitude. Similarly, for a fixed $\sigma_{s2}/b$, as $\sigma_{s1}/b$ increases, the OP increases. This indicates that the performance of the multi-RIS-empowered FSO system is determined by the performance of its worst link. Finally, from this figure, we observe that a multi-RIS-empowered FSO system in which $\sigma_{s1}/b = \sigma_{s1}/b = v_1$ and $\sigma_{s1}/b = v_2$ achieves the same performance as the one with $\sigma_{s1}/b = v_2$ and $\sigma_{s1}/b = v_1$.

Figure 7 depicts the OP as a function of $\rho_s/\rho_{th}$ for different values of $N$ and $L$, assuming $\alpha_1 = \alpha_2 = \cdots = \alpha_N = 10.02$ and $\beta_1 = \beta_2 = \cdots = \beta_N = 2.98$. As expected, for fixed $N$ and $L$, as $\rho_s/\rho_{th}$ increases, the OP decreases. For instance, for $N = 3$ and $L = 2$, the outage performance improves for about one order of magnitude, as $\rho_s/\rho_{th}$ increases from 20 to 35 dB. Additionally, for given $L$ and $\rho_s/\rho_{th}$, performance degradation is observed as $N$ increases. Finally, for given $N$ and $\rho_s/\rho_{th}$, as $L$ increases, the impact of misalignment becomes more severe; hence, the OP increases.

B. Parallel Multi-RIS FSO System

In Fig. 8 the OP of the parallel multi-RIS FSO system is presented as a function of $\rho_s/\rho_{th}$, for different turbulence conditions, assuming $N = L = 2$, $\sigma_{s1}/b = \sigma_{s2}/b = 0.1$ and $b/w_d = 0.5$. In this figure, continuous lines are used to denote results extracted from Monte Carlo simulations, while dashed-ones are obtained according to (38). It is observed that for a given turbulence condition, as $\rho_s/\rho_{th}$ increases, the OP decreases. Moreover, from this figure, the upper bound is verified. Finally, it becomes apparent that, for a fixed $\rho_s/\rho_{th}$, as the turbulence intensity increases, the error between the simulation and upper bound increases. Despite this fact, the OP upper bound presented in (38) can become a useful tool for the design of parallel multi-RIS FSO systems.

Figure 9 illustrates the OP of parallel multi-RIS FSO system as a function of $\rho_s/\rho_{th}$, for different values of $N = L$, assuming $\alpha_i = 10.02$, $\beta_i = 2.98$, $\Omega_i = 1$, $\sigma_{si}/b_i = 0.1$ and $\beta_i/b_i = 0.5$ for $i \in [1, N]$. Again, continuous lines are used to denote results, which are extracted from simulations, while dashed-ones represent the OP upper bound, as derived from (38). As expected, for a fixed $N = L$, both the OP and its bound decreases, as $\rho_s/\rho_{th}$ increases. Additionally, it is observed that for a given $\rho_s/\rho_{th}$, as $N = L$ increases, the diversity order increases; thus, outage performance improves. Meanwhile, as $N = L$ increases, the error between the simulations and the OP upper bound increases. Finally, from this figure, it becomes evident that the bound accurately
follows the simulations; as a consequence, it can be used to assess the outage performance ceiling of the parallel multi-RIS-empowered FSO system.

C. Cascaded RIS-empowered THz wireless systems

For the cascaded RIS-empowered THz wireless systems, we consider the following insightful scenario. Unless otherwise stated, $G_s = G_t = 50$ dBi, $R_i = 1$ for $i \in \{1, N - 1\}$ and $f = 300$ GHz. Additionally, standard atmospheric conditions, i.e., relative humidity, temperature, and pressure respectively equal to 50%, 296 K, and 101325 Pa, are assumed. Under these atmospheric conditions, a realistic value for $C_0^2$ is $2.3 \times 10^{-9} \text{ m}^{-2/3}$ [95] and $\kappa(f) = 5.8268 \times 10^{-4}$.

Figure 10 quantifies the impact of turbulence on the outage performance of multi-RIS-empowered THz wireless systems in the absence of misalignment and hardware imperfections, assuming $N = 2$, $\gamma_s/\gamma_{th} = 25$ dB. In more detail, the OP is illustrated as a function of $d_1$ and $d_2$. In this figure, for the sake of convenience, the corresponding values of $\sigma_{R1}$ and $\sigma_{R2}$ are respectively provided in the top horizontal and right vertical axes. Of note, according to [89], $\sigma_{R1} < 1$, $1 < \sigma_{R1} < 2$ and $\sigma_{R1} > 2$ respectively correspond to weak, moderate, and strong turbulence conditions. As expected, for a fixed $d_1$, as $d_2$ increases, $\sigma_{R2}$ increases; thus, an outage performance degradation is observed. For example, for $d_1 = 100$ m, the OP increases by approximately one order of magnitude, as $d_2$ changes from 110 to 150 m. Similarly, for a given $d_2$, as $d_1$ increases, $\sigma_{R1}$ also increases, i.e., turbulence intensity increases; in turn, the OP increases. For instance, for $d_2 = 100$ m, the OP increases by about 10 times, as $d_2$ changes from 110 to 150 m. The aforementioned examples reveal that the system with $d_1 = v_1$ and $d_2 = v_2$ achieves the same outage performance as the one with $d_1 = v_2$ and $d_1 = v_1$, where $v_1$ and $v_2$ are independent variables. Finally, from this figure, we observe that for a given e2e transmission distance, $d_1 + d_2$, the worst outage performance is observed for $d_1 = d_2$. For example, for $d_1 + d_2 = 300$ m, the OP for the case in which $d_1 = 100$ m and $d_2 = 200$ m is equal to $8.58 \times 10^{-4}$, while, for $d_1 = d_2 = 150$ m, it is $8.8 \times 10^{-4}$. Of note, this comes in line with the majorization theory [96].

Figure 11 illustrates the joint effect of molecular absorption and turbulence in terms of OP. In particular, the OP is plotted against the transmission frequency and temperature. The following insightful scenario is considered. The transmission power, bandwidth, $B$, and spectral efficiency of the transmission scheme are respectively set to 0 dBW, 50 GHz, and 2 bit/s/Hz, while $N$ and $L$ are respectively equal to 2 and 0. Moreover, $d_1 = d_2 = 100$ m and $\kappa_t = \kappa_r = 0$. At the destination side, we assumed that the low noise amplifier’s gain and noise figure (NF) are 35 dB, and 1 dB, respectively. The mixer and miscellaneous losses are respectively 5 and 3 dB, whereas, the mixer’s NF is 6 dB. The thermal noise power is evaluated as $N_t = k_B T B$, where $k_B$ is the Boltzmann’s constant. Note that the simulation parameters that we used in this work are inline with [7]. This figure shows that two molecular absorption “walls” exist around 325 and 380 GHz. Within the molecular absorption walls, the OP is higher than $10^{-2}$. The length of these walls depends on the temperature. Specifically, as the temperature increases, the molecular absorption becomes more severe; as a result, the length of the molecular absorption walls also increases. Outside the aforementioned walls, in the 100 to 500 GHz band, there exist three transmission windows. Within these windows, for a fixed temperature, as the transmission frequency increases, the system outage performance degrades. Similarly, for a given transmission frequency, as the temperature increases, the OP also increases. It is worth noting that, within the transmission windows, the dominant phenomenon that affects the losses is free space propagation, while molecular absorption plays a secondary role.

Figure 12 quantifies the outage performance of multi-RIS empowered THz wireless systems in the presence of turbulence and misalignment, assuming that its transceivers are equipped with ideal RF front-end, i.e., $\kappa_t = \kappa_r = 0$, and the transmission distances for all the links are set to 100 m. Specifically, the OP is given as a function of $\sigma_s$, for different values of $N = L$ and $\gamma_s/\gamma_{th}$. Of note, in this

---

**Fig. 9**: OP vs $\rho_s/\rho_{th}$, for different values of $N = L$.

**Fig. 10**: OP vs $d_1$ and $d_2$.

**Fig. 11**: OP vs $f$ and $T$. 

**Fig. 12**: OP vs $\sigma_s$, for different values of $N = L$ and $\gamma_s/\gamma_{th}$. Of note, in this
For instance, for $\sigma_s$ as $N$ fading is also plotted. As expected, for given $\sigma_s$, the outage performance degrades, as the intensity of misalignment fading, i.e., $\sigma_s$, becomes more severe. For example, for $N = L = 3$ and $\gamma_s/\gamma_{th} = 30$ dB, the OP increases for about four orders of magnitude, as $\sigma_s$ changes from 1 cm to 1 dm. Meanwhile, for fixed $N = L$ and $\sigma_s$, as $\gamma_s/\gamma_{th}$, the OP decreases. On the other hand, for given $\sigma_s$ and $\gamma_s/\gamma_{th}$, as $N = L$ increases, the OP also increases. For instance, for $\sigma_s = 4$ cm and $\gamma_s/\gamma_{th} = 25$ dB, the OP increases by approximately 10 times, as $N = L$ changes from 2 to 3. Finally, from this figure, the detrimental impact of misalignment fading becomes apparent by comparing; thus, the importance of accurately characterizing the channels of the RIS-empowered THz wireless systems is highlighted.

Figure 13 illustrates the impact of hardware imperfections in multi-RIS-empowered THz wireless systems, assuming $L = M = 2$, $d_1 = d_2 = 100$ m, $\sigma_t = \sigma_r = 1$ mm and $\gamma_s/\gamma_{th} = 25$ dB. Of note is that, $\kappa_t = \kappa_r = 0$ corresponds to the best-case scenario, in which both the S and D are equipped with ideal RF front-ends. We observe that, for a given $\kappa_t$, as $\kappa_r$ increases, the OP also increases. Similarly, for a fixed $\kappa_t$, an outage performance degradation occurs, when $\kappa_r$ increases. Additionally, for a constant $\kappa_t + \kappa_r$, we observe that the OP is maximized for $\kappa_t = \kappa_r$. Finally, it is verified that systems with the same $\kappa_t^2 + \kappa_r^2$ achieve the same outage performance.

In Fig. 14, the OP is presented as a function of $\gamma_s$ for different values of $\kappa_t = \kappa_r = \gamma_s$, as well as $\kappa_t$ and $\kappa_r$, assuming $N = L = 2$ and $d_1 = d_2 = 100$ m. In particular, the ideal ($\kappa_t = \kappa_r = 0$), best ($\kappa_t = \kappa_r = 0.1$) and worst ($\kappa_t = \kappa_r = 0.4$) practical cases from the hardware imperfections point of view are illustrated. As expected, for given $\gamma_s$, as $\kappa_t = \kappa_r$ increases, the OP decreases. For example, for ideal RF front-end, $\gamma_s = 0$ dB and $\sigma_s = 1$ mm, the OP decreases by 100 times, as $\gamma_s$ changes from 20 to 30 dB. On the other hand, as predicted by (58), the OP is equal to 1. Meanwhile, for fixed $\gamma_s$, as $\kappa_t = \kappa_r$ increases, the OP also increases. For instance, for $\kappa_t = \kappa_r = 0.1$, $\gamma_s = 0$ dB and $\gamma_s = 30$ dB, the outage performance degrades by about one order of magnitude, as $\sigma_s$ increases from 1 mm to 0.1 m. This example highlights the significance of the impact of misalignment fading in multi-RIS-empowered THz wireless systems. Likewise, for given $\sigma_s$, $\kappa_t$, $\kappa_r$ and $\gamma_s$, a $\gamma_s$ increase causes an OP increase. Finally, the impact of hardware imperfections in systems operating at higher $\gamma_s$ is more severe than the ones working in lower $\gamma_s$. Since $\gamma_s$ is connected with the transmission scheme’s spectral efficiency, the impact of hardware imperfections can be constrained by selecting transmission schemes of low-spectral efficiency.

V. CONCLUSIONS

In this paper, we introduced a theoretical framework for the statistical characterization of cascaded composite turbulence and misalignment channels in terms of PDF and CDF. Moreover, we assessed the outage performance of cascaded multi-RIS FSO and THz systems in the presence of misalignment and turbulence by computing novel closed-form expressions for their OP. Additionally, we provided an insightful upper bound for parallel multi-RIS FSO systems. Our results revealed that in cascaded multi-RIS FSO and THz systems, as the number of RIS and thus the distance between S and D increases, the turbulence and misalignment fading intensity becomes more severe; as a result, the outage performance degrades. On the other hand, in parallel multi-RIS FSO systems, as the number of links increases, the diversity order also increases; hence, the OP decreases. Likewise, the impact of unavoidable transceivers’ hardware imperfections on the outage performance of cascaded multi-RIS THz systems was quantified. Finally, the importance of accurately characterizing the transmission medium was emphasized.

APPENDICES

APPENDIX A

PROOF OF THEOREM 2

In order to prove that the PDF of $Z_2$ can be expressed as in (7), we use the induction method [98]. First, we prove
that (7) holds for $L = 2$. In this case, since $l_1$ and $l_2$ are independent, the PDF of $Z_2$ can be evaluated as

$$f_{Z_2}(x; L = 2) = \int_{\frac{A_{o,2}}{x}}^{A_{o,2}} \frac{1}{y} f_{l_1}(x \ y) f_{l_2}(y) \ dy.$$  \hfill (62)

By applying (6) in (62), we obtain

$$f_{Z_2}(x; L = 2) = \frac{\xi f_2}{A_{o,1} A_{o,2}} x^{\xi - 1} \int_{\frac{A_{o,2}}{x}}^{A_{o,2}} y^{-1} \ dy.$$  \hfill (63)

which, according to [73, eq. (2.01/2)], can be rewritten as

$$f_{Z_2}(x; L = 2) = \frac{\xi f_2}{A_{o,1} A_{o,2}} x^{\xi - 1} \ln \left( \frac{A_{o,2}}{x} \right).$$  \hfill (64)

for $0 \leq x \leq A_{o,1} A_{o,2}$. By comparing (64) with (7), it becomes evident that (7) is true for $L = 2$.

Next, for $L = 3$, since $l_1$, $l_2$, and $l_3$ are independent, the PDF of $Z_2$ can be obtained as

$$f_{Z_2}(x; L = 3) = \int_{\frac{A_{o,3}}{x}}^{A_{o,3}} \frac{1}{y} f_{Z_2}(y; L = 2) f_{l_3}(x \ y) \ dy,$$  \hfill (65)

which, with the aid of (6) and (64), can be rewritten as

$$f_{Z_2}(x; L = 3) = \frac{\xi f_3}{A_{o,1} A_{o,3}} x^{\xi - 1} \times \int_{\frac{A_{o,3}}{x}}^{A_{o,3}} y^{-1} \ ln \left( \frac{A_{o,3}}{x} \right) \ dy.$$  \hfill (66)

Next, by applying [73, eq. (2.721/3)] in (66), we get

$$f_{Z_2}(x; L = 3) = \frac{1}{2} \left( \frac{\xi f_3}{A_{o,1} A_{o,3}} x^{\xi - 1} \left( \ln \left( \frac{A_{o,3}}{x} \right) \right)^2 \right),$$  \hfill (67)

with $0 \leq x \leq A_{o,3}$.

Let us assume that (7) holds for $L = M$, i.e.,

$$f_{Z_2}(x; L = M) = \frac{1}{(M-1)!} \left( \frac{\xi f_M}{A_{o,1}} \right) x^{M - 1} \ln \left( \frac{A_{o,1}}{x} \right).$$  \hfill (68)

Then, for $L = M + 1$, the PDF of $Z_2$ can be evaluated as

$$f_{Z_2}(x; L = M + 1) = \int_{\frac{A_{o,M+1}}{x}}^{A_{o,M+1}} \frac{1}{y} f_{Z_2}(y; L = M) f_{l_{M+1}}(x \ y) \ dy,$$  \hfill (69)

which, by applying (6) and (68), can be rewritten as

$$f_{Z_2}(x; L = M + 1) = \frac{\xi f_{M+1}}{A_{o,1}^{M+1}} x^{M - 1} \times \left( \ln \left( \frac{A_{o,1}}{x} \right) \right)^{M - 1}. \hfill (70)$$

By applying [73, eq. (2.721/1)], (70) can be written in a closed-form as

$$f_{Z_2}(x; L = M + 1) = \frac{1}{M!} \left( \frac{\xi f_{M+1}}{A_{o,1}^{M+1}} \right) x^{M - 1} \times \left( \ln \left( \frac{A_{o,1}}{x} \right) \right)^{M - 1}.$$  \hfill (71)

with $0 \leq x \leq x^{M+1}$. Notice, that from (71), it becomes apparent that if (7) holds for $L = M$, it also holds for $L = M + 1$. Since it also holds for $N - 2$, then it stands for each $N \geq 2$. This concludes the proof.

### APPENDIX B

#### PROOF OF THEOREM 3

Since $L \leq N$, with the aid of (3) and (5), (9) can be rewritten as

$$Z = Y_1 Y_2,$$  \hfill (72)

where

$$Y_1 = \prod_{i=1}^{L} r_i l_i \quad \text{and} \quad Y_2 = \prod_{i=L+1}^{N} r_i.$$  \hfill (73)

To extract a closed-form expression for the PDF of $Z$, we need first to evaluate the PDF of $Y_1$. In this direction, let us assume that $L = 1$; then, the PDF of $Y_1$ yields as

$$f_{Y_1}(x; L = 1) = \int_{\frac{1}{y}}^{\infty} \frac{1}{y} f_{r_1}(y) f_{l_1} \left( \frac{x}{y} \right) \ dy,$$  \hfill (74)

which, by applying (7) and (6), can be rewritten as

$$f_{Y_1}(x; L = 1) = 2 \left( \frac{\alpha_1 \beta_1}{\Omega_1} \right)^{\frac{x + \beta_1 - 1}{\alpha_1 - 1} - 1} \xi \ \Gamma \left( \frac{(\alpha_1 - 1)(\beta_1 - 1)}{\Omega_1} \right) \ dy.$$  \hfill (75)

By employing [99, ch. 2.6], (75) can be written as in (76), given at the top of the next page. Additionally, by applying [77, eq. (2.24.5/3)], (76) can be analytically expressed as

$$f_{Y_1}(x; L = 1) = \left( \frac{\alpha_1 \beta_1}{\Omega_1} \right)^{\frac{x + \beta_1 - 1}{\alpha_1 - 1} - 1} \xi \ \Gamma \left( \frac{(\alpha_1 - 1)(\beta_1 - 1)}{\Omega_1} \right) \ dy.$$  \hfill (77)

which, with the aid of [100,101], can be rewritten as

$$f_{Y_1}(x; L = 1) = \xi \Gamma \left( \frac{(\alpha_1 - 1)(\beta_1 - 1)}{\Omega_1} \right) \ dy.$$  \hfill (78)

For $L = 2$, the PDF of $Y_1$ can be obtained as

$$f_{Y_1}(x; L = 2) = \int_{\frac{1}{y}}^{\infty} \frac{1}{y} f_{Y_1}(y; L = 1) f_{y_1} \left( \frac{x}{y} ; L = 1 \right) \ dy,$$  \hfill (79)

which, by applying (78), can be rewritten as in (80), given at the top of the next page. By employing [73, eq. (9.31/1)], we can write (80) as in (81), given at the top of the next page. Finally, by applying [77, eq. (2.24.5/3)] in (81), we get

$$f_{Y_1}(x; L = 2) = \prod_{i=1}^{L} \xi f_{Y_1} \left( \frac{x}{y} \right) \ dy.$$  \hfill (82)

By recursively conducting this procedure, for $L = 3, 4, \ldots$, we prove that $f_{Y_1}(x)$ can be obtained as in (83), given at the top of the next page.

Moreover, by following the same steps as in Appendix B, it can be proven that the PDF of $Y_2$ can be obtained as in (84), given at the top of the next page. Thus, since $Y_1$ and $Y_2$ are independent, the PDF of $Z$ occurs as

$$f_Z(x) = \int_{\frac{1}{y}}^{\infty} \frac{1}{y} f_{Y_1}(y) f_{Y_2} \left( \frac{x}{y} \right) \ dy,$$  \hfill (85)

which by substituting (83) and (84), can be written as in (86), given at the top of the next page. By employing [100,101], (86) can be rewritten in a closed-form as in (87), given at the top of the next page. Finally, by employing [77, eq. (2.24.5/3)],
\[ f_{Y_1}(x; L = 1) = \left( \frac{\alpha_1 \beta_1}{\Omega_1} \right)^{\frac{1}{\Omega_1}} \frac{1}{\Gamma(\alpha_1)} \frac{1}{\Gamma(\beta_1)} \frac{\xi}{A_{\alpha_1 \beta_1}} x^{\xi - 1} \int_0^{\infty} y^{-1} G_0,2(\xi) \left( \frac{1}{y} \right) \left( G_0,2 \left( \frac{\alpha_1 \beta_1 y}{\Omega_1} \left| \frac{1}{2} \right| - \frac{\alpha_1 - \beta_1}{2} \right) \right) dy \]

\[ f_{Y_2}(x; L = 1) = \frac{\prod_{i=1}^{L} \xi_i}{\prod_{i=1}^{L} \Gamma(\alpha_i) \Gamma(\beta_i)} x^{\sum_{i=1}^{L} \xi_i - 1} \int_0^{\infty} y^{-1} G_{3,0} \left( \frac{\alpha_1 \beta_1 y}{\Omega_1} \left| \frac{1}{3} \right| - \frac{\alpha_1 + \beta_1 + 1}{3} \right) \left( G_{3,0} \left( \frac{\alpha_2 \beta_2 x}{\Omega_2} \left| \frac{1}{3} \right| - \frac{\alpha_2 + \beta_2 + 1}{3} \right) \right) dy \]

we get (10).

The CDF of \( Z \) can be evaluated as

\[ F_Z(x) = \int_0^x f_Z(y) \, dy, \]

which, by applying (77) eq. (2.24.5/3), returns (11). This concludes the proof.

**References**

[1] M. Latva-aho, K. Leppänen, F. Clazzer, and A. Munari, “Key drivers and research challenges for 6G ubiquitous wireless intelligence,” White Paper, 2020.

[2] L. Bariah, L. Mohjazi, S. Muhaidat, P. C. Sofotasios, G. K. Kurt, H. Yanikomeroglu, and O. A. Dobre, “A prospective look: Key enabling technologies, applications and open research topics in 6G networks,” IEEE Access, vol. 8, pp. 174792–174820, Aug. 2020.

[3] A.-A. A. Boulogeorgos, A. Alexiou, D. Kriharris, A. Katsiotis, G. Moutou, J. Kokkonen, J. Lethomaki, M. Juntti, D. Yankova, A. Mokhtar, H. C. Point, J. Machtrod, R. Elschnier, C. Schubert, T. Merkle, R. Ferreira, F. Rodrigues, and J. Lima, “Wireless terahertz system architectures for networks beyond 5G,” TERRANOVA CON-SORTIUM, White paper 1.0, Jul. 2018.

[4] S. Deng, O. Amin, B. Shihada, and M.-S. Alouini, “What should 6G be?” Nature Electronics, vol. 3, no. 1, pp. 20–29, Jan. 2020.

[5] A.-A. A. Boulogeorgos, A. Alexiou, T. Merkle, C. Schubert, R. Elschnier, A. Katsiotis, P. Stavrianos, D. Kriharris, P. K. Chrtias, J. Kokkoneni, M. Juntti, J. Lethomaki, A. Teixeir, and F. Rodrigues, “Terahertz technologies to deliver optical network quality of experience in wireless systems beyond 5G,” IEEE Commun. Mag., vol. 56, no. 6, pp. 144–151, Jun. 2018.

[6] Z. Zhang, Y. Xiao, Z. Ma, M. Xiao, Z. Ding, X. Lei, G. K. Karagianidis, and P. Fan, “6G wireless networks: Vision, requirements, architecture, and key technologies,” IEEE Veh. Technol. Mag., vol. 14, no. 3, pp. 28–41, sep 2019.

[7] I. F. Akylidz, A. Kak, and S. Nie, “6G and beyond: The future of wireless communications systems,” IEEE Access, vol. 8, pp. 133995–134030, Jul. 2020.

[8] A.-A. A. Boulogeorgos and A. Alexiou, “Error analysis of mixed THz-RF wireless systems,” IEEE Commun. Lett., vol. 24, no. 2, pp. 277–281, feb 2020.

[9] N. Chi, Y. Zhou, Y. Wei, and F. Hu, “Visible light communication in 6G: Advances, challenges, and prospects,” IEEE Veh. Technol. Mag., vol. 15, no. 4, pp. 93–102, Dec. 2020.

[10] P. Porambage, G. Gur, D. M. P. Osoiro, M. Liyanage, A. Gurtov, and M. Ylianttila, “The roadmap to 6G security and privacy,” IEEE Open Journal of the Communications Society, vol. 2, pp. 1094–1122, May 2021.

[11] M. A. Arefe, M. D. Soliani, I. Tavakkolinia, A. Ghreyab, M. C. Assi, M. Safari, and H. Haas, “Measurements-based channel models for indoor LiFi systems,” IEEE Trans. Wireless Commun., vol. 20, no. 2, pp. 827–842, Feb. 2021.

[12] M. Di Renzo, M. Debbah, D.-T. Phan-Huy, A. Zappone, M.-S. Alouini, C. Yuen, V. Sciancalepore, G. C. Alexandropoulos, J. Hoydis, H. Gacanin, J. d. Rosny, A. Bounceur, G. Lerosey, and M. Fink, “Smart radio environments empowered by reconfigurable meta-surfaces: An idea whose time has come,” EURASIP Journal on Wireless Communications and Networking, vol. 2019, no. 1, pp. 1–20, May 2019.

[13] M. Di Renzo, A. Zappone, M. Debbah, M.-S. Alouini, C. Yuen, J. de Rosny, and S. Tretjakov, “Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and the road ahead,” IEEE J. Sel. Areas Commun., vol. 38, no. 11, pp. 2450–2425, nov 2020.

[14] O. Tsilipakos, A. C. Tsolamprou, A. Pitilakis, F. Li, X. Wang, M. S. Mirmoosa, D. C. Tzoraonich, S. Badal, H. Taghvaei, C. Liaskos, A. Tsioliaridou, J. Georgiou, A. Cabelllos-Aparicio, A. Alarcn, S. Ioanidis, A. Pitsilides, I. F. Akyildiz, N. V. Kautzartz, E. N. Economou, C. M. Soukoulis, M. Kafesaki, and S. Tretjakov, “Toward intelligent metasurfaces: The progress from globally tunable metasurfaces to software-defined metasurfaces with an embedded network of controllers,” Adv. Opt. Mater., vol. 8, no. 17, p. 2000783, Jul. 2020.

[15] A.-A. A. Boulogeorgos and A. Alexiou, “Performance analysis of reconfigurable intelligent surface-assisted wireless systems and comparison with relaying,” IEEE Access, vol. 8, pp. 94463–94483, 2020.

[16] S. Koenig, D. Lopez-Diaz, J. Antes, F. Boes, R. Henneberger, A. Leuther, A. Tессmann, R. Schmogrow, D. Hiellerkus, R. Palmer, T. Zwick, C. Koos, W. Freude, O. Ambacher, I. Leuthold, and I. Kallfass, “Wireless sub-THz communication system with high data rate,” Nat. Photonics, vol. 7, pp. 977 EP–, Oct. 2013.

[17] G. Parca, “Optical wireless transmission at 1.6-Tbit/s (10×100 Gbit/s) for next-generation convergent urban infrastructures,” Opt. Eng., vol. 52, no. 11, p. 116102, Nov. 2013.

[18] M. A. Esmail, A. Ragheb, H. Fatehallah, and M.-S. Alouini, “Experimental demonstration of outdoor 2.2 Tbps super-channel FSO transmission system,” in IEEE International Conference on Communications Workshops (ICC), IEEE, May 2016.

[19] M. V. Mazurczyk, J.-X. Cai, M. Paskov, W. W. Patterson, O. V. Sinkin, Y. Hu, C. R. Davidson, P. C. Corbett, T. E. Hammon, M. A. Bohlynsky, D. G. Foursa, and A. N. Pilipetski, “Demonstration of 3.010 km WDM transmission in 3.83 THz bandwidth using SOAs,” in Optical Fiber Communications Conference and Exhibition (OFC), Mar. 2020, pp. 1–3.

[20] C. Jastrzembski, A. Boes, T. Messinger, J. Antes, A. Inam, U. Lewark, A. Tessmann, and R. Henneberger, “64 Gbit/s transmission over 850 m fixed wireless link at 240 GHz carrier frequency,” in Infrared Millim. Terahertz Waves, vol. 36, no. 2, pp. 221–233, Jan. 2015.

[21] C. Castro, R. Elschnier, T. Merkle, C. Schubert, and R. Freund, “Experimental demonstrations of high-capacity THz-wireless transmission systems for beyond 5G,” IEEE Commun. Mag., vol. 58, no. 11, pp. 41–47, Nov. 2020.

[22] A.-A. A. Boulogeorgos, J. M. Jornaet, and A. Alexiou, “A quantitative look at directional terahertz communication systems for 6G: Fact check,” IEEE Veh. Technol. Mag., vol. 16, no. 4, Oct. 2021.
\begin{align}
    f_{Y_2}(x) &= \frac{G_{0,2(N-L)}^{(N-L)} \left[ \frac{x}{\prod_{i=L+1}^{N} \frac{\alpha_i}{\beta_i} \Gamma(\alpha_i / \beta_i) \beta L_1 + 1, \beta L_2, \cdots, \alpha N, \beta N} \right]}{\prod_{i=L+1}^{N} \frac{\alpha_i}{\beta_i} \Gamma(\alpha_i / \beta_i) \beta L_1 + 1, \beta L_2, \cdots, \alpha N, \beta N} \left( \frac{y}{\prod_{i=L+1}^{N} \frac{\alpha_i}{\beta_i} \Gamma(\alpha_i / \beta_i) \beta L_1 + 1, \beta L_2, \cdots, \alpha N, \beta N} \right) \right]}
    \nonumber
\end{align}

\begin{align}
    f_Z(x) &= \frac{\prod_{i=1}^{L} x_i}{\prod_{i=1}^{N} \frac{\alpha_i}{\beta_i} \Gamma(\alpha_i / \beta_i) \beta L_1 + 1, \beta L_2, \cdots, \alpha N, \beta N} \left( \frac{y}{\prod_{i=L+1}^{N} \frac{\alpha_i}{\beta_i} \Gamma(\alpha_i / \beta_i) \beta L_1 + 1, \beta L_2, \cdots, \alpha N, \beta N} \right) \right]}
    \nonumber
\end{align}

\begin{align}
    f_Z(x) &= \frac{\prod_{i=1}^{L} x_i}{\prod_{i=1}^{N} \frac{\alpha_i}{\beta_i} \Gamma(\alpha_i / \beta_i) \beta L_1 + 1, \beta L_2, \cdots, \alpha N, \beta N} \left( \frac{y}{\prod_{i=L+1}^{N} \frac{\alpha_i}{\beta_i} \Gamma(\alpha_i / \beta_i) \beta L_1 + 1, \beta L_2, \cdots, \alpha N, \beta N} \right) \right]}
    \nonumber
\end{align}

[24] A.-A. A. Boulogeorgos, E. N. Papasotiriou, J. Kokkoniemi, J. Lehtomaki, A. Alexiou, and M. Juntti, “Performance evaluation of THz wireless systems operating in 275–400 GHz band,” IEEE Vehicular Technology Conference (VTC), 2018.

[25] R. Elschner, C. Castro, R. Ferreira, T. Merkle, A. Alexiou, D. Kritidis, J. Kokkoniemi, and A.-A. A. Boulogeorgos, “D6.2-THz high-capacity demonstratorimplementation report,” TERRANOVA, Tech. Rep., 2020.

[26] F. Rodrigues, R. Ferreira, C. Castro, G. Ntouri, R. Elschner, T. Merkle, A.-A. A. Boulogeorgos, J. Kokkoniemi, and A. Alexiou, “D6.3-TERRANOVA proof-of-concept test andvalidation report,” TERRANOVA, resreport, Mar. 2020.

[27] F. Elzanaty and M.-S. Alouini, “Adaptive coded modulation for IM/DD free-space optical backhauling: A probabilistic shaping approach,” IEEE Trans. Commun., vol. 10, no. 1, pp. 1–12, feb 2017.

[28] W. Tang, J. Y. Dai, M. Z. Chen, K.-K. Wong, X. Li, X. Zhao, S. Jin, Q. Cheng, and T. J. Cui, “MIMO transmission through reconfigurable intelligent surfaces vs. relaying: Differences, similarities, and performance comparison,” IEEE Open Journal of the Communications Society, pp. 1–1, 2020.

[29] C. Liaskos, Internet of materials. S.J: CRC PRESS, 2020.

[30] M. Di Renzo, K. Ntontin, J. Song, F. H. Danufane, X. Qian, F. Lazarakis, J. de Rosny, D.-T. Pham-Huy, O. Simeone, R. Zhang, M. Debbah, G. Le Rosey, M. Fink, S. Tretyakov, and S. Shamai, “Reconfigurable intelligent surfaces vs. relaying: Differences, similarities, and performance comparison,” IEEE Open Journal of the Communications Society, pp. 1–1, 2020.
M. Khorasaninejad, W. T. Chen, R. C. Devlin, J. Oh, A. Y. Zhu, and F. Capasso, “Metals at visible wavelengths: Diffraction-limited focusing and subwavelength resolution imaging,” Science, vol. 352, no. 6290, pp. 1190–1194, Jun. 2016.

S. Wang, P. C. Wu, V.-C. Su, Y.-C. Lai, M.-K. Chen, H. Y. Kuo, B. H. Chen, Y. H. Chen, T.-T. Huang, J.-H. Wang, R.-M. Lin, C.-H. Kuo, S.-T. Li, Z.-J. Sun, and D. P. Tsai, “A broadband achromatic metalens in the visible,” Nat. Nanotechnol., vol. 13, no. 3, pp. 227–232, Jan. 2018.

X. Zhang, Q. Li, F. Liu, M. Qiu, S. Sun, Q. He, and L. Zhou, “Controlling angular dispersions in optical metasurfaces,” Light: Science & Applications, vol. 9, no. 1, May 2020.

H. Du, J. Zhang, K. Guan, B. Ai, and T. Kurner, “Reconfigurable metasurfaces for THz communication in LEO satellite networks,” ArXiv, Apr. 2021.

V. K. Chapala and S. M. Zafaruddin, “Exact analysis of RIS-aided THz wireless systems over α-μ fading with pointing errors,” IEEE Commun. Lett., pp. 1–1, 2021.

E. N. Papasotiriou, A.-A. A. Boulogeorgos, A. Stratakou, and A. Alexiou, “Performance evaluation of reconfigurable intelligent surface assisted d-band wireless communication,” in IEEE 3rd 5G World Forum (5GWf), IEEE, Sep. 2020.

H. Du, J. Zhang, K. Guan, B. Ai, and T. Kürmer, “Reconfigurable intelligent surface aided terahertz communications under misalignment and hardware impairments,” ArXiv, Dec. 2020.

A.-A. A. Boulogeorgos and A. Alexiou, “Coverage analysis of reconfigurable intelligent surface assisted THz wireless systems,” IEEE Open Journal of Vehicular Technology, vol. 2, pp. 94–110, Jan. 2021.

C. Huang, Z. Yang, G. C. Alexandropoulos, K. Xiong, L. Wei, C. Yuen, Z. Zhang, and M. Debbah, “Multi-hop RIS-empowered terahertz communications: A DRL-based hybrid beamforming design,” IEEE J. Sel. Areas Commun., vol. 39, no. 6, pp. 1663–1677, Jun. 2021.

Z. Wan, Z. Gao, F. Gao, M. Di Renzo, and M.-S. Alouini, “Terahertz massive MIMO with holographic reconfigurable intelligent surfaces,” IEEE Trans. Commun., pp. 1–1, Mar. 2021.

Y. Lu and L. Dai, “Reconfigurable intelligent surface based hybrid precoding for THz communications,” ArXiv, Dec. 2020.

M. Khorasaninejad, W. T. Chen, R. C. Devlin, J. Oh, A. Y. Zhu, and F. Capasso, “Using any surface to realize a new paradigm for wireless communications,” Science, vol. 370, no. 6514, pp. 1010–1013, May 2020.

S. Wang, P. C. Wu, V.-C. Su, Y.-C. Lai, C. H. Chiu, J.-W. Chen, S.-H. Lu, J. Chen, B. Xu, C.-H. Kuan, T. Li, S. Zhu, and D. P. Tsai, “Broadband achromatic optical metasurface devices,” Nat Commun., vol. 8, no. 1, Aug. 2017.

M. Tekbıyık, G. K. Kurt, A. R. Ekti, A. Görçün, and H. Yanıkomeroglu, “Reconfigurable intelligent surfaces empowered THz communication in LEO satellite networks,” ArXiv, Apr. 2021.

Y. K. Chapala and S. M. Zafaruddin, “Exact analysis of RIS-aided THz wireless systems over α-μ fading with pointing errors,” IEEE Commun. Lett., pp. 1–1, 2021.

A.-A. A. Boulogeorgos, A. Papasotiriou, A. Alexiou, and M. Alrabeiah, “How much do hardware imperfections affect the performance of reconfigurable intelligent surface-assisted systems?” IEEE Open Journal of the Communications Society, vol. 1, pp. 1185–1195, 2020.

A.-A. A. Boulogeorgos, N. D. Chatzdiamantis, and G. K. Karagiannidis, “Energy detection spectrum sensing under RF imperfections,” IEEE Trans. Commun., vol. 64, no. 7, pp. 2754–2766, Jul. 2016.

T. Schenk, RF Imperfections in High-Rate Wireless Systems. The Netherlands: Springer, 2008.

A.-A. A. Boulogeorgos and G. K. Karagiannidis, “Energy detection in full-duplex systems with residual RF impairments over fading channels,” IEEE Wireless Commun. Lett., vol. 7, no. 2, pp. 246–249, Apr. 2018.

M. Taherkhani, Z. G. Kashani, and R. A. Sadeghzadeh, “On the performance of THz wireless LOS links through random turbulence channels,” Nano Commun. Networks, vol. 23, p. 100282, Feb. 2020.

ITU-R, “Attenuation by atmospheric gases,” ITU-R, Tech. Rep., Sep. 2013.

H. Hill, R. Bohlander, S. Clifford, R. McMillan, J. Priestly, and W. Schoenfeld, “Turbulence-induced millimeter-wave scintillation compared with micrometeorological measurements,” IEEE Trans. Geosci. Remote Sens., vol. 26, no. 3, pp. 330–341, May 1988.

J. Kokkonemi, J. Lehtomäki, and M. Juntti, “A line-of-sight channel model for the 100-450 gigahertz frequency band,” EURASIP Journal on Wireless Communications and Networking, vol. 2021, no. 1, apr 2021.

J. Kokkonemi, J. M. Jornet, V. Petrov, Y. Koucheryavy, and M. Juntti, “Channel modeling and performance analysis of airplane-satellite terahertz band communications,” IEEE Trans. Veh. Technol., vol. 70, no. 3, pp. 2047–2061, mar 2021.

A.-A. A. Boulogeorgos, E. N. Papasotiriou, and A. Alexiou, “Analytical performance assessment of THz wireless systems,” IEEE Access, vol. 7, no. 1, pp. 1–18, Jan. 2019.

A. A. Taherkhani, R. A. Sadeghzadeh, and Z. G. Kashani, “Attenuation analysis of THz/IR waves under different turbulence conditions using Gamma-Gamma model,” in Iranian Conference on Electrical Engineering (ICEE). IEEE, May 2018.

A. W. Marshall, I. Olkin, and B. C. Arnold, Inequalities: Theory of Majorization and Its Applications. Springer-Verlag GmbH, 2010.

A.-A. A. Boulogeorgos, J. M. Jornet, and A. Alexiou, “Directional terahertz communications systems for 6G: Fact check: A quantitative look,” IEEE Veh. Technol. Mag., pp. 2–10, Oct. 2021.

D. Gunderson, Handbook of mathematical induction : Theory and applications. Boca Raton, FL: CRC Press, 2011.

A. M. Mathai and R. K. Saxena, “Particular cases of Meijer’s G-function,” in Lecture Notes in Mathematics. Springer Berlin Heidelberg, 1973, ch. 2, pp. 41–68.

W. R. Inc., “The Wolfram functions site,” [http://functions.wolfram.com] accessed: 2021-06-08.