Parity-violating 3-jet observables for massive quarks to order $\alpha_s^2$ in $e^+e^-$ annihilation

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In this talk we discuss the calculation of the QCD corrections to parity-violating 3-jet observables in $e^+e^-$ collisions, keeping the full quark mass dependence \cite{1}.

1. Introduction

The measurement of the forward-backward asymmetry $A_{FB}^b$ for $b$-quark production in $e^+e^-$ collisions at the Z peak provides one of the most precise determinations of the weak mixing angle. It can be determined with an error at the per mille level if the computations of $A_{FB}^p$ are at least as accurate as the present experimental precision of about two percent \cite{2}. To achieve this precision one has to go beyond leading order in perturbation theory. Furthermore given the size of mass effects due to the non-zero $b$-quark mass, it is also necessary to extend existing analyses to account for non-vanishing quark masses. This is mandatory if one studies top-quarks instead of $b$-quarks. Restricting ourselves to perturbative QCD the present knowledge is as follows. The order $\alpha_s$ contributions had been computed first for massless \cite{3} and then for massive quarks \cite{4,5}. These calculations, which used the quark direction for defining the asymmetry were later modified \cite{6–8} to allow the use of the thrust axis to define the asymmetry. The next-to-next-to-leading order (NNLO) coefficient was computed first in ref. \cite{9}, for massless quarks using the quark axis definition. This result was recently corrected by a completely analytical \cite{10} and by a numerical calculation \cite{11}. In ref. \cite{11} the NNLO corrections were also determined for the thrust axis definition. To complete the analyses of $A_{FB}$ at NNLO the corrections for massive quarks are needed. To obtain these corrections it is convenient to compute the individual contributions of the parton jets \cite{9}, in particular to calculate separately the contributions from 2-, 3-, and 4-jet final states involving a heavy quark $Q$. For the 4-jet contribution to $A_{FB}$ one needs only the known Born matrix elements for $e^+e^- \rightarrow 4$ partons involving at least one $QQ$ pair (see, e.g. ref. \cite{12}). The phase-space integration of these matrix elements in order to get \cite{1} for a given jet algorithm can be done numerically.

Compared to \cite{4} for 3-jet the computation of \cite{4} is much more difficult. Here the NLO corrections to the partonic subprocess $e^+e^- \rightarrow Q\bar{Q}g$ are needed. This contribution must be combined with the 3-jet contribution from subprocesses with four partons in the final state. Only by combining the two contributions a result which is free of soft- and mass singularities is obtained. In this talk we will present recent results \cite{13} on the missing ingredients that are needed to calculate \cite{1}.

The computation of \cite{4} for heavy quarks is beyond the scope of this work and remains to be done in the future.

To fix our notation we discuss briefly in section 2 the kinematics and the leading order results. In section 3 we comment on the calculation of the one-loop corrections. In the following section 4 some checks are described. We close with a conclusion in section 5.
2. Kinematics and leading order results

The leading order differential cross section for
\[ e^+ (p_+) e^- (p) \to Q (Q) \bar{Q} (\bar{Q}) g (g) \]

\[ d\sigma \bigg/ d\phi d\cos \theta dx \bigg/ dx = \frac{3}{4} \frac{1}{(4\pi)^3} \sigma_{pt} \left( 1 + \cos^2 \theta \right) F_1 + \left( 1 + 3 \cos^2 \theta \right) F_2 + \cos \theta F_3 + \sin \theta \cos \phi F_4 + \sin^2 \theta \cos 2\phi F_5 + \sin \theta \cos \phi F_6 \]

\[ \text{with} \]
\[ \sigma_{pt} = \sigma (e^+ e^- ! \gamma ! \mu^+ \mu^-) = \frac{4\pi\alpha^2}{3s} \]  

In Eq. (3), \( \theta \) denotes the angle between the direction \( p \) of the incoming electron \( e^- \) and the direction \( k_Q \) of the heavy quark \( Q \). The angle \( \phi \) is the oriented angle between the plane defined by \( e^- \) and \( Q \) and the plane defined by the quark anti-quark pair. The functions \( F_i \) depend only on the scaled c.m. energies
\[ x = \frac{2k}{s} \text{ and } \bar{x} = \frac{2k}{s} \; \] (4)
and on the scaled mass square
\[ z = \frac{m^2}{s} \] (5)
of the heavy quark and anti-quark, where \( k = p_+ + p \) and \( s = k^2 \). Note that the leading order differential cross section for a final state with \( n = 4 \) partons can be written in an analogous way in terms of the two angles \( \theta, \phi \) and \( 3n = 7 \) variables that involve only the final state momenta. In higher orders absorptive parts give rise to three additional functions \( F_{1,2,3} \) (cf. ref. [13]). The four functions \( F_{1,2,3,6} \) are parity even. In particular, the 3-jet production rate is determined by \( F_1 \). In the case of massive quarks next-to-leading order results for \( F_1 \) were given in references [14–21]. The two functions \( F_1 \) and \( F_6 \) are induced by the interference of a vector and an axial-vector current. In particular, using the quark-axis, the 3-jet forward-backward asymmetry is related to \( F_3 \) in leading order in the following way:

\[ \langle A_{\text{FB}} \rangle_{3\text{-jet}} = \frac{R}{\sigma} \left( \frac{d\sigma_{\text{pt}}}{d\phi} \right) \sin \phi \cos \theta \]

\[ = \frac{3}{8} \frac{dxd\bar{x}}{d\phi} \theta_{\text{cut}} (\langle x_{\text{cut}} \rangle) F_3 (\langle x_{\text{cut}} \rangle - \langle x_{\text{cut}} \rangle) \] (6)

where \( \theta_{\text{cut}} (\langle x_{\text{cut}} \rangle) \) defines, for a given jet finding algorithm and jet resolution parameter \( y_{\text{cut}} \), a region in the \( (x, \bar{x}) \) plane. The electroweak couplings appearing in \( F_{3,6} \) can be factored out as follows:

\[ F_{3,6} = \left| g_3^f (1, \lambda \chi) \right| Q \left| Q \right| R \left( f, \lambda \chi \right) \left| g_6^f \right| \]

\[ \left( f, \lambda \chi \right) \left| g_6^f \right| F_{3,6} (x) \]

In Eq. (7),

\[ g_v^f = T_3 f 2Q_f \sin^2 \theta_W ; \]

\[ g_u^f = T_3 f ; \]

\[ \chi = \frac{1}{4} \frac{\sin^2 \theta_W \cos^2 \theta_W}{s} \frac{s}{m^2 + \text{i}m_2 \Gamma_2} ; \]

\[ f (\lambda \chi) = \frac{\lambda}{1 \lambda \lambda^*} ; \]

where \( T_3^f \) is the third component of the weak isospin of the fermion \( f, \theta_W \) is the weak mixing angle, and \( \lambda \) denotes the longitudinal polarization of the electron (positron). In next-to-leading order in \( \alpha_s \), additional contributions to \( F_{3,6} \), with electroweak couplings different from those in Eq. (7), are induced which we will not consider here. They are either proportional to\( \text{i}m_1 \chi \) and thus suppressed formally in the electroweak coupling or generated by the triangle fermion loop diagrams. The contribution of the triangle fermion loop is gauge independent and UV and IR finite. It was calculated some time ago in ref. [23].

The functions \( F_{3,6} (x, \bar{x}) \) may be expressed in terms of functions \( h_6, h_7 \) which appear in the decomposition of the so-called hadronic tensor as performed for example in references [24, 25].

\[ F_3 = \frac{1}{2} x^2 \left( 4zh_6 (x, \bar{x}) \right. \]

\[ + \left. 4z \cos \theta Q Q \right) ; \]

\[ F_6 = \frac{1}{2} x^2 \left( 4z \sin \theta Q Q \right) ; \]

\[ (9) \]
where $\vartheta_{Q\bar{Q}}$ is the angle between $Q$ and $\bar{Q}$ in the c.m. frame and we have

$$\cos \vartheta_{Q\bar{Q}} = \frac{2 (l^2 x + \bar{x} + 2z) + x\bar{x}}{4z^2 x^2 4\bar{x}^2} ;$$ (10)

It would seem pointless to trade $F_{3,B}$ for $h_{6,7}$ if it were not for the relation (which follows from CP invariance):

$$h_{7}(x;\bar{x}) = h_{6}(\bar{x};x) ;$$ (11)

To calculate the functions $F_{3,B}$ it is thus sufficient to determine the function $h_{6}(x;\bar{x})$. In the one-loop corrections to $h_{6}$ one encounters both ultraviolet (UV) and infrared (IR) singularities. Regulating the UV as well as the IR singularities by continuation to $d = 4 - 2\epsilon$ space-time dimensions, we need the Born result in $d$ dimensions:

$$h_{6}^{1,0}(x;\bar{x}) = 16\pi\alpha_{s}(N^2 - 1)B$$

$$2x (x\bar{x} + \bar{x}^2 + 2 \bar{x}\epsilon) - \frac{4x\bar{x}}{1 - x}$$ (12)

with

$$B = \frac{1}{(l - x)(l - \bar{x})} ;$$ (13)

and the scaled gluon energy

$$x_{e} = \frac{2k_{s}}{s} = 2x \bar{x} ;$$ (14)

Note that the terms proportional to $\epsilon$ in Eq. (12) depend on the prescription used to treat $\gamma_{s}$ in $d$ dimensions. To derive the above equation we have used the prescription

$$\gamma_{s} = \frac{\alpha_{s}}{\pi} C_{F} \frac{1}{3!} \epsilon^{\rho\sigma\tau\nu} \gamma^{\rho} \gamma^{\sigma} \gamma^{\tau} \gamma^{\nu}$$ [26,27] ; (15)

3. Virtual corrections

Given the vast knowledge on how to perform one-loop calculations the derivation of the one-loop corrections to $h_{6}$ is in principle straightforward. So we restrict our discussion of the virtual corrections to some technical remarks and the final results.

We used the background field gauge [28,29] with the gauge parameter set to one. In this gauge the three-gluon vertex is simplified which leads to a reduction of the number of terms encountered in intermediate steps of the calculation. Furthermore we used the Passarino-Veltman reduction procedure [30] to reduce the one-loop tensor integrals to scalar one-loop integrals. As mentioned earlier we regularized both UV and IR singularities by continuation to $d = 4 - 2\epsilon$ space-time dimensions. Note that the \textquoteleft t \textquoteleft t Hooft-Veltman prescription to treat $\gamma_{s}$ in $d$ dimensions give rise to an additional \textquoteleft finite renormalization\textquoteleft (see Eq. (15)) to restore the chiral Ward identities.

The finite contributions of the loop-diagrams are too long to be reproduced here. They are given explicitly in ref. [1]. Here we give only the results for the UV and IR divergences and the contribution from the renormalization procedure. In the following we denote with $m$ always the mass parameter renormalized in the on-shell scheme and with $\alpha_{s}$ the strong coupling in the usual $\overline{MS}$ scheme evaluated at the renormalization scale $\mu$.

Keeping only the pole-part of the integrals we obtain the following result for the UV singularities:

$$h_{6,UV}^{\text{virt.}} = \frac{1}{\epsilon} \frac{m^2}{4\pi^2} \frac{\alpha_{s}}{2\pi} C_{F} h_{6}^{1,0}$$

$$+ 24\alpha_{s}^2 (\bar{s}^2 - 1)C_{F} \delta_{6}+ 8\alpha_{s}^2 (\bar{s}^2 - 1)C_{F} B \epsilon \ 1 + x$$

$$2\bar{x} + (l - x)(\bar{x}) + z \bar{B}(\bar{s}^2 - 2\bar{x})(l - x)$$

$$+ (l - 3x)(l - \bar{x}) + g\frac{\bar{x}^2}{x} + \frac{4z}{x} + x_{e} \bar{x}$$

$$+ 4x\bar{x} \bar{B}(\frac{\bar{x}}{x})^2 + xg_{1}$$

$$+ 16\alpha_{s}^2 C_{F} B \epsilon \frac{1}{x_{e}} (\bar{x} + 4\bar{x}^2 + 2\bar{x})$$

$$2x_{e} \bar{B}(\bar{x} - \bar{x}) + O(\bar{x}^2)$$ (16)

with

$$\delta_{6} = B^2 + 2\bar{x}^2 + 3\bar{x}^2 + 3x\bar{x} + 2\bar{x}^2$$

$$4x + 2) \ x_{e} (\bar{x}^2 + x\bar{x} + 4x + 2\bar{x}) \epsilon \ z$$

$$4 \ \frac{1}{1} (\bar{x} - 3\bar{x} + 2\bar{x}^2 + 4 + x^2 + x\bar{x})^2$$ (17)

and

$$g_{1} = \frac{1}{x_{e}} (\bar{x} - 2\bar{x})(\bar{x}(\bar{x} + 2\bar{x}) - 2(l - x_{e})) ;$$ (18)
The usual one-loop factor $r_T$ is given by:

\[
    r_T = \frac{\Gamma(l + \epsilon)\Gamma^2(l - \epsilon)}{\Gamma(l - 2\epsilon)};
\]

(19)

The IR divergent contributions from the loop-integrals are given by

\[
    h_{6\,\text{virt.},\text{IR div.}} = \frac{\alpha_s}{2\pi} \frac{4\pi\mu^2}{m^2} \frac{\epsilon}{\epsilon - 1} \Gamma^{1\,\text{LO}}^{6\,\text{LO}} + \frac{1}{N} \frac{1}{\epsilon - 1} \left( \frac{\epsilon}{\epsilon - 1} \ln \frac{t_{ij}}{m^2} \right) + \frac{1}{N} \frac{1}{\epsilon - 1} \ln (\omega);
\]

(20)

with $t_{ij} = 2k_i \cdot k$ and

\[
    \omega = \frac{1}{\epsilon - 1} \left( \frac{\epsilon}{\epsilon - 1} + \frac{\epsilon}{\epsilon - 1} \right);
\]

(21)

Finally, the entire contribution from renormalization (including the Lehmann–Symanzik–Zimmermann residue and the ‘γ6-counterterm’) is given by:

\[
    h_{6\,\text{virt.},\text{ren.}} = \frac{\alpha_s}{2\pi} r_T C_F \frac{m^2}{4\pi\mu^2} \frac{\epsilon}{\epsilon - 1} h_{6\,\text{LO}} + 24\alpha_s^2 N^2 1) C_F r_T \frac{m^2}{4\pi\mu^2} \frac{\epsilon}{\epsilon - 1} h_{6\,\text{LO}} + \frac{\alpha_s}{4\pi} \left( \frac{m^2}{4\pi\mu^2} \frac{\epsilon}{\epsilon - 1} \right) 4 C_F \frac{2}{3} n_f \frac{N}{3} h_{6\,\text{LO}} + \frac{\alpha_s}{4\pi} \left( \frac{m^2}{\mu^2} \frac{\epsilon}{\epsilon - 1} \right) 2 \sum_i \ln \frac{m_i^2}{m^2} 8 C_F \frac{1}{6} h_{6\,\text{LO}} + 32\alpha_s^2 N^2 1) C_F \delta h_6 \frac{m^2}{4\pi\mu^2} \frac{\epsilon}{\epsilon - 1} h_{6\,\text{LO}}.
\]

(22)

Note that the singularity in the third line of Eq. (22) is a collinear singularity and not a UV singularity.

4. Checks

In this section we discuss some checks to assure the correctness of our results. Given the fact that the finite results are given in the form (coefficients) (one-loop integrals) and that the loop integrals are known, it is sufficient to check the coefficients. As one can see from the comparison of Eq. (16) and Eq. (22), the UV divergences from the loop diagrams are cancelled exactly by the UV singularities from the renormalization procedure. This cancellation is thus an excellent check of the coefficients of the UV divergent integrals, namely the one- and two-point integrals. We checked also that the finite renormalization due to the ‘t Hooft-Veltman prescription restores the chiral Ward identities.

According to the Kinoshita-Lee-Nauenberg theorem [31,32] the remaining IR divergences must be cancelled by the real corrections. To check this cancellation we have calculated the singular contributions from real emission by using a modified version of the phase space slicing method [34,35]. By this cancellation the coefficients of the IR divergent triangle integrals and the box integrals are checked. The explicit results for the singular contributions of the real corrections are given in ref. [1].

Furthermore we have verified that we are able to reproduce the coefficients of the loop-integrals in the massless result [23,25]. We found agreement with Eq. (4.10) of ref. [24]. In the case of Eq. (4.9) of ref. [24] we found agreement only up to an overall factor (1). This is most probably caused by a typographical error which is also present in ref. [23], but not in ref. [25] with which we fully agree.

5. Conclusions

In this talk we have presented the necessary ingredients for calculating virtual corrections to parity-violating three jet observables. Together with the existing results on the real contribution [2], in particular the singular contributions [1], these results allow for the calculation of parity-violating three-jet observables to next-to-leading order accuracy.

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