Bell Measurement and Local Measurement in the Modified Lo-Chau Quantum Key Distribution Protocol

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We clarify the argument on how (nonlocal) degenerate Bell measurement can be replaced by local measurements in the modified Lo-Chau quantum key distribution protocol. Discussing security criterion for users, we describe how eavesdropper’s refined information on the final state is not helpful. We argue that current discussions on the equivalence of the Bell and the local measurements are not clear. We show how the problem of equivalence can be resolved using the fact that eavesdropper’s refined information is not helpful for her.

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I. INTRODUCTION

Quantum cryptography, more precisely, quantum key distribution (QKD) [1, 2, 4, 5, 6, 7, 8, 10, 11], is one of the most promising protocols in quantum information processing [11]. Bennett-Brassard 1984 (BB84) QKD protocol [3] had been widely conjectured to be secure based on the quantum no-cloning theorem [13, 14, 15]. However, Refs. 2 and 3 seem to be too complicated to be widely understood while Ref. 4 is relatively simple in that it makes use of tools that are familiar to quantum information scientist, e.g. quantum error correcting code (QECC) and entanglement distillation protocol (EDP). Thus, the approach of Shor and Preskill [4] is being more widely accepted and applied to deal with the security of variations of the BB84 protocols [4, 12].

In Ref. 4, the authors argue that the modified Lo-Chau (LC) protocol based on the EDP is secure. Then, they show that the modified LC protocol reduces to the BB84 protocol. Thus, the security of the BB84 protocol depends on that of the modified LC protocol. In the discussion on security of the modified LC protocol, they use ‘classicalization of statistics’ (or ‘quantum to classical reduction’) [2, 3]. However, in the derivation of the classicalization of statistics, they make use of a (partial) equivalence between (degenerate)Bell measurements and local measurements: Bell measurements can be replaced by Z measurement in the |0⟩, |1⟩ basis and by X measurement in the |0⟩, |2⟩ basis. Here, a Bell measurement is one in the Bell basis |Φ±⟩ = (1/√2)(|0⟩A|0⟩B ± |1⟩A|1⟩B) and |Ψ±⟩ = (1/√2)(|0⟩A|1⟩B ± |1⟩A|0⟩B), where A and B denote two users, Alice and Bob, respectively. |0⟩ = (1/√2)(|0⟩ + |1⟩) and |1⟩ = (1/√2)(|0⟩ − |1⟩). The Y measurement is the one in the basis |0⟩ = (1/√2)(|0⟩ + i|1⟩) and |1⟩ = (1/√2)(|0⟩ − i|1⟩). However, the discussion on the equivalence is not clear as we will see. On the other hand, the security considered so far has been that from Eve’s (eavesdropper’s) point of view. However, what we eventually need is a security criterion for Alice and Bob, the users.

The purpose of this paper is to give a clearer presentation for the equivalence between the Bell and the local measurements in the modified LC protocol. While discussing security criteria for Alice and Bob, we show that Eve’s refined information on the final state is not helpful to her. Then, we clarify the equivalence between Bell measurements and Z and X measurements.

This paper is organized as follows: First, we reformulate the modified LC protocol and arguments for its security in an explicit manner. Next, briefly discussing security criteria for Alice and Bob, we show that Eve’s refined information on the final state is not helpful to her. Then, we argue that the current discussions on the equivalence of the Bell and local measurements are not clear, and we show that the whole equivalence of the quantum state is not helpful to her.

II. MODIFIED LO-CHAU PROTOCOL

The legitimate states in the modified LC protocol are pairs of a Bell state, |Ψ+⟩. However, the whole quantum state that Alice, Bob, and Eve share, after distribution of quantum bits (qubits) and before entanglement distillation, is an arbitrary state |ψABE⟩ that Eve chooses. Let us write down the state |ψABE⟩ in the Bell basis:

|ψABE⟩ = \sum_{\{k\}} C_{\{k\}} \sigma_{k_1(1)} \sigma_{k_2(2)} \cdots \sigma_{k_{2n}(2n)} |Φ+⟩^\otimes 2n |E_{\{k\}}⟩.

(1)

Here, \{k\} is an abbreviation for k1, k2, ..., k_{2n} with ki \in {0, 1, 2, 3} (i = 1, 2, ..., 2n), and \sigma_0 = I, \sigma_1 = X, \sigma_2 = Y, \sigma_3 = Z are Pauli operators. The \sigma_{k_1(i)} denotes the Pauli operator acting on i-th qubit of Bob. Namely, \sigma_{k_1(i)} is I \otimes \sigma_i acting on i-th qubit pairs that are shared by Alice and Bob. Note that the set of \sigma_{k_1(1)} \sigma_{k_2(2)} \cdots \sigma_{k_{2n}(2n)} |Φ+⟩^\otimes 2n of all \{k\} constitutes the complete Bell basis for the 2n qubit pairs. The C_{\{k\}}’s are coefficients in complex numbers. Eve’s states |E_{\{k\}}⟩ are normalized, but not mutually orthogonal in general. It is notable that the state in Eq. (1) is completely general; thus, it is dealing with all attacks including opaque (intercept-
resend), individual, collective, and joint attacks.

Let us describe the checking method. Consider a measurement $M_Z$ whose projection operators are

$$P_0 = |\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-| = |00⟩⟨00| + |11⟩⟨11|,$$

$$P_1 = |\Psi^+\rangle\langle\Psi^+| + |\Psi^-\rangle\langle\Psi^-| = |01⟩⟨01| + |10⟩⟨10|.$$  

(2)

Also, consider a measurement $M_X$ whose projection operators are

$$P_0 = |\Phi^+\rangle\langle\Phi^+| + |\Psi^-\rangle\langle\Psi^-| = |00⟩⟨00| + |11⟩⟨11|,$$

$$P_1 = |\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+| = |01⟩⟨01| + |10⟩⟨10|.$$  

(3)

The measurements $M_Z$ and $M_X$ are nonlocal. (For example, $P_0|00⟩ = (1/\sqrt{2})P_0(|\Phi^+⟩ + |\Phi^-⟩) = (1/\sqrt{2})|\Phi^+⟩$; that is, a separable state $|00⟩$ is transformed to a nonlocal state $|\Phi^+⟩$.) Thus, obviously they cannot be performed by Alice and Bob who are supposedly separated. However, we assume that they can perform the measurements $M_Z$ and $M_X$ for the time being. We will see later how $M_Z$ and $M_X$ can be replaced by separable measurements $Z$ and $X$.

The error rate in the modified LC protocol is defined as follows: First Alice and Bob randomly choose $n$ pairs of qubits from among the $2n$ pairs. On each of the $n$ chosen pairs, they perform a measurement randomly chosen between $M_Z$ and $M_X$. The error rate $\varepsilon$ is the number of all instances when the measurement outcomes are those corresponding to $P_1$ and $P_1$ divided by the number of samples $n$. From Eqs. (2) and (3), we can see that the legitimate state $|\Phi^+⟩$ has zero probability to give rise to an error, and that other illegitimate states, $|\Phi^−⟩$, $|\Psi^−⟩$, and $|\Psi^+⟩$, have non-zero probabilities, $1/2, 1/2$, and $1$, respectively, to give rise to an error.

Let us now describe the classicalization of statistics. We consider a case $n = 2$, which is simple but illustrative enough. Assume that Alice and Bob’s random choice was to measure the first and the third pairs in the $Z$ and the $X$ bases, respectively. That is, they perform a measurement $M_Z \otimes I \otimes M_X \otimes I$ on a state $|ψ_{AVE}⟩ = \sum_{\{k\}} C(k) |σ_{k1}(1)|σ_{k2}(2)|σ_{k3}(3)|σ_{k4}(4)|Φ^+⟩\otimes4|E(k)⟩$. It is easy to see that, for example, the probability $p_{00} = \sum_{k_1=0,3, k_2=k_3=0,1,k_4} |C(k)|^2$, where $k_2,k_4 = 0,1,2,3$. The resultant state is $|ψ_{00}⟩ = N \sum_{k_1=0,3, k_2,k_3=0,1,k_4} C(k)|σ_{k1}(1)|σ_{k2}(2)|σ_{k3}(3)|σ_{k4}(4)|Φ^+⟩\otimes4|E(k)⟩$, where $N = 1/\sum_{k_1=0,3, k_2,k_3=0,1,k_4} |C(k)|^2$ is the normalization constant. (Note Eqs. (2) and (3) and that $(I \otimes σ_3)|Φ^+⟩ = |Φ^−⟩$ and $(I \otimes σ_3)|Φ^−⟩ = |Φ^+⟩$.) In the same way, we can calculate the probabilities $p_{01}$, $p_{10}$, and $p_{11}$ and the corresponding resultant states. Then, let us consider a case where Eve prepares a mixed state $ρ = \sum_{\{k\}} P(k)|σ_{k1}(1)|σ_{k2}(2)|σ_{k3}(3)|σ_{k4}(4)|Φ^+⟩\otimes4|E(k)⟩$, where $P(k) = |C(k)|^2$. We consider the case where Alice and Bob perform the same checking measurement $M_Z \otimes I \otimes M_X \otimes I$ on the state $ρ$. Then, it is easy to see that, for example, the probability $q_{00}$ that they get $0$ in the $M_Z$ measurement and $0$ in the $M_X$ measurement is the same as the $p_{00}$ given above. However, the resultant state is not the same, but is given by $ρ_{00} = \sum_{k_1=0,3, k_2,k_3=0,1,k_4} P(k)|σ_{k1}(1)|σ_{k2}(2)|σ_{k3}(3)|σ_{k4}(4)|Φ^+⟩\otimes4|E(k)⟩$.

Note a similarity between the states $|ψ_{00}⟩$ and $ρ_{00}$. The difference is that the former is a separable state that is, a separable state $|00⟩$ is transformed to a nonlocal state $|Φ^+⟩$.) Thus, obviously they cannot be performed by Alice and Bob who are supposedly separated. However, we assume that they can perform the measurements $M_Z$ and $M_X$ for the time being. We will see later how $M_Z$ and $M_X$ can be replaced by separable measurements $Z$ and $X$.

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Let us discuss the arguments for security of the modified LC protocol. Assume that Eve distributed the state in Eq. 4 to Alice and Bob. Due to the classicalization of statistics, however, it is sufficient for them to consider a corresponding case where Eve distributed the state in Eq. 4. Thus, we may well separately consider each term

\[
\sigma_{k_1} \cdot \sigma_{k_2} \cdots \sigma_{k_{2n}} \cdot |\Phi^+\rangle^{\otimes 2n} \langle \Phi^+|^{\otimes 2n} \cdot \sigma_{k_1} \cdot \sigma_{k_2} \cdots \sigma_{k_{2n}}
\]

with a particular \(\{k\}\) and then combine them later. Consider the checking measurement where \(n\) randomly chosen pairs are measured along randomly chosen bases. We can see that the error rate \(e\) is statistically proportional to the ratio of the illegitimate states among the \(n\) checked pairs. Alice and Bob abort the protocol if the measured error rate \(e\) is larger than a threshold for checking, \(e_{check}\). The threshold for checking, \(e_{check}\), is set to be a little bit smaller than a threshold for error correction \(e_{cor.}\), to compensate for statistical fluctuations. Let us assume that the number of illegitimate states in the state in Eq. 4 is larger than \((2n)(2e_{cor.})\). Since the probability that each illegitimate state is detected in the checking procedure is equal to or larger than \(1/2\), as seen above, it typically give rise to an error rate larger than \(e_{cor.}\) for the checked pairs. Then, the probability that the state passes the test is negligibly small. In other words, the checking procedure sifts out, with high probability, any state that contains more than \(4ne_{cor.}\) pairs of illegitimate states. Combining this fact with Bayes’s theorem, we get the following: Whatever state in Eq. 4 Eve has prepared, if the state passes the test with a non-negligible probability, then the ratio of illegitimate states of the resultant state is less than \(2e_{cor.}\), with high probability. Therefore, if they choose a QECC that can correct up to an error rate \(2e_{cor.}\), the final state after quantum error correction by using the chosen code will have a high fidelity to the legitimate state.

III. SECURITY CRITERION AND EVE’S REFINED INFORMATION

Let us now discuss the security criterion. What has been considered so far is Eve’s viewpoint: The higher the probability to pass the test by Alice and Bob, the less the information that Eve gets is. However, what we actually need is the security criterion for Alice and Bob, the users. It is intuitively clear that the security criterion for Eve can be translated to that for Alice and Bob. However, there remain a few difficulties in doing so because Alice and Bob do not know the initial state that Eve knows. Eve’s most general strategy is to prepare a state in Eq. 4 with a probability distribution \(P_{\{k\}}\’s\). Once Alice and Bob know the probability distribution \(P_{\{k\}}\’s\), combined with the Bayes’s theorem, they can calculate the final state. However, the problem is that it is not clear what probability distribution \(P_{\{k\}}\’s\) Eve will choose. However, we can say that Eve will optimize her strategy. That is, she will choose an attack that maximizes her information on the key among those with the same probability to pass the test. Identification of the optimal probability distribution \(P_{\{k\}}\’s\) seems to be at heart of the open problem, to find a clear security criterion for Alice and Bob.

However, a certain probability distribution \(P_{\{k\}}\’s\) has been tacitly assumed in discussions on QKD so far. For example, let us consider the case where Eve chooses an attack that can give her full information on the key once it pass the test, but the probability to pass the test is negligible. That is, she chooses that \(P_{\{k\}} \neq 0\) for only those \(\{k\}\’s\) in which most of \(k\) are nonzero. If Alice and Bob assume that Eve adopts this strategy, combined with the Bayes’s theorem, their conclusion is always that Eve has full information. However, this is not regarded as a strategy that Eve will actually adopt because it blocks communication between Alice and Bob. Here we accept that it is not a good strategy for Eve. Here, we do not try to get a rigorous security criterion. It may be a very subtle problem to find Eve’s optimal strategy because it may depend on the real situations in which Alice, Bob, and Eve find themselves. However, it is reasonable to say that Eve’s optimal strategy is such that Alice and Bob get a state \(\rho_{AB}\) that is almost a legitimate state as a result of the EDP, as assumed so far.

The situation we meet here is that Alice and Bob have only partial information on the state for which Eve has full information. That is, we have

\[
\rho_{AB} = \sum_i p_i \rho_{AB}^i = \sum_i p_i \rho_{AB}^i.
\]

Here, \(\rho_{AB}^i\) denotes a (pure) state inferred from Eve’s classical information \(i\). Consider a case where \(F(\rho_{AB}, |\Phi\rangle^{\otimes k}) \geq 1 - \epsilon\) with \(F\) denoting the fidelity and \(\epsilon\) being a positive real number. Let us see how Eve’s refined knowledge on the state is not so helpful to her by using the following two arguments.

Let us give the first argument. For each \(\rho_{AB}^i\), Eve’s state is given by \(\rho_{E} = tr_{AB}(\rho_{AB}^i)\), and \(S(\rho_{E}) = S(\rho_{AB}^i)\) because \(\rho_{AB}^i\) is a pure state. Then, the mutual information between Eve’s party and Alice and Bob’s party, \(I'(AB; E)\), is bounded by the \(S(\rho_{AB}^i)\). However, we can see that

\[
1 - \epsilon \leq F(\rho_{AB}, |\Phi\rangle^{\otimes k}) = \sum_i p_i F(\rho_{AB}^i, |\Phi\rangle^{\otimes k}) \tag{8}
\]

by using Eq. 7 and the relation \(F(\sum_i p_i |\psi\rangle) = \sum_i p_i F(|\psi\rangle)\). Equation 8 says that the average of the fidelities of \(\rho_{AB}^i\) is bounded by a quantity \(1 - \epsilon\) that bounds the fidelity of the average (mixed) state \(\rho_{AB}\). Here, Eve cannot control the outcome of the classical information \(i\). Thus, it is a meaningful average even if Eve knows the \(i\). More concretely, let us consider a particular \(F(\rho_{AB}^i, |\Phi\rangle^{\otimes k})\). From Eq. 8, we have that
(1 - F(\rho^i_{AB}, |\Phi|^\otimes k))p_i \geq \epsilon, which expresses a reciprocal relation between the closeness of the state to legitimate states and a probability that it happens.

Let us give the second argument. Consider a purification of the state \rho_{AB}

$$|\psi_{AB}\rangle = \sum_i \sqrt{p_i}|i\rangle|\psi^i_{AB}\rangle$$

(9)

that is compatible with Eq. 7. Here, |\psi^i_{AB}\rangle\langle\psi^i_{AB}| = \rho^i_{AB}, and the qubits storing the classical information are at Eve’s hands. If Eve first performs a measurement on qubits storing the i, then the state reduces to \rho^i_{AB} with probability p_i. The situation now is equivalent to the case above where Eve has the state \rho^i_{AB} with knowledge of the i. However, in this case the bound in Eq. 10 is valid for the pure state |\psi_{AB}\rangle. Therefore, we can say that the bound in Eq. 5 applies to the case where Eve has more refined information i about the final state.

IV. EQUIVALENCE OF THE BELL AND THE LOCAL MEASUREMENTS

Now let us discuss the problem of partial equivalence. So far, we have assumed that Alice and Bob can perform M_Z and M_X. However, they are nonlocal measurements that cannot be actually done by Alice and Bob, as seen above. Their argument for this problem is the following [4, 5, 6, 7]: Let us consider the actual situation where Alice and Bob each perform Z measurements on a pair of qubits. The basis for this measurement is |00\rangle, |01\rangle, |10\rangle, and |11\rangle. By Eq. 6, however, we can estimate the probabilities involved with the M_Z measurement solely from the outcomes of the Z measurement: For example, tr(\rho(|\Phi^+\rangle\langle\Phi^+| + |\Phi^−\rangle\langle\Phi^−|)) = tr(\rho(00\langle00| + 11\langle11|)) = tr(\rho(00\langle00|) + tr(\rho(11\langle11|). The same thing can be said for the M_X and the X measurements. However, the measured (or checked) n pairs are not further used by Alice and Bob. Only the remaining n information pairs are used for key generation. That is, the checked and the information pairs are different. Therefore, the density operator for the information pairs is invariant to whatever measurement they do on the checked pairs, provided that they do not make use of the information on their measurements outcomes. (One’s quantum state can depend on his/her knowledge about the measurement outcomes on the other side when the shared initial state is entangled, as is well known.) Alice and Bob may not make use of the information on the measurement outcomes. Therefore, if the error rate e_L estimated by the local measurements Z and X is below the threshold for checking e_{check}, the EDP will be successful with high probability: The fact that e_L is less than e_{check} implies that if they had estimated the error rate e_B by performing the nonlocal measurements M_Z and M_X, then they also would have found that e_B was less than e_{check} with high probability. However, as we have seen above, if e_B is less than e_{check}, then the EDP is successful with high probability.

However, the measurement outcomes are publicly announced in the protocol; thus, Eve knows it. The problem is that Eve can make use of the information on the measurement outcomes. Our solution to this problem is the following: The best thing that Eve can get is the refined knowledge on the final state, however, and we have seen above that such refined information is not so helpful to Eve. Therefore, now we can safely say that the local Z and X measurements are equivalent to the nonlocal M_Z and M_X measurements as far as the security of BB84 protocol is concerned.

V. CONCLUSION

To summarize, we reformulated the modified LC protocol and arguments for its security in an explicit manner. While discussing the security criterion for Alice and Bob briefly, we showed that Eve’s refined information on the final state is not helpful to her. Then, we argued that current discussions on the equivalence of the Bell and the local measurements are not clear, and we showed that the problem of the equivalence can be resolved using the fact that Eve’s refined information is not helpful to her.

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