Model Independent Methods for Determining $\mathcal{B}(\Upsilon(5S) \to B_S^{(*)} \overline{B}_S^{(*)})$

Radia Sia and Sheldon Stone

Department of Physics, Syracuse University, Syracuse, New York 13244-1130
(Dated: August 12, 2009)

We describe a method that provides a model independent measurement of the $B_S$ fraction in $\Upsilon(5S)$ resonance decays, $f_S$, using the relative rates of like-sign versus opposite sign dileptons; the like-sign leptons result from $B^0$ and $B_S$ mixing. In addition, we show that determining the rates of single, double and triple $D_S^{(*)}$ mesons provides an alternative way of finding $f_S$.

The $\Upsilon(5S)$ resonance has long been thought to be a source of $B_S$ mesons, since it is massive enough to produce $B_S\overline{B}_S$ pairs. Recently, the CLEO collaboration established the presence of $B_S$ mesons and made a model dependent measurement that the $B_S$ fraction, $f_S$, of $\Upsilon(5S)$ decays is $(16.0 \pm 2.6 \pm 5.8)\%$, using a theoretical estimate of $(92 \pm 11)\%$ for the inclusive branching ratio $\mathcal{B}(B_S \to D_S X)$ [2]. Explicit $B_S$ final states have also been reconstructed [3]. These results have been confirmed by Belle [4]. Precision measurements of $B_S$ branching fractions are one of the most important goals of such studies. For example, it has been suggested that measurements of $\mathcal{B}(B_S \to D_S^{(*)+} D_S^{(*)-})$ lead to a determination of the lifetime difference between $CP^+$ and $CP^-$ eigenstates [5]. To measure these, however, it is imperative that the number of $B_S$ mesons produced in $\Upsilon(5S)$ decays be well known. The purpose of this paper is to describe techniques that can be used to produce accurate, model independent measurements of $f_S$. (Note that we are not considering $\Upsilon(5S)$ decays to modes that do not contain $B$ mesons.)

Both $B$ and $B_S$ mesons can result from $\Upsilon(5S)$ decays. The possible final states are $B^{(*)-}B^{(*)+}$, $B^{(*)0}\overline{B}^{(*)0}$, $B^{(*)}\pi$, $B\overline{B}\pi\pi$, and $B_S^{(*)0}\overline{B}_S^{(*)0}$, since there is not sufficient energy to produce an extra pion.

Our first method for determining $f_S$ requires the measurement of like-sign versus opposite-sign dileptons. High momentum leptons from $B$ (or $B_S$) decays reflect the flavor of the parent; positive leptons result from $B$ decays, while negative leptons arise from $\overline{B}$ decays. We will assume here that the minimum lepton momentum requirement is large enough so that contamination from the decay sequence $B \to DX$, $D \to Y \ell \nu$ is negligible, or suitable corrections can be applied [6].

This technique relies on the fact that $B_S$ mixing oscillations are very rapid compared to $B_d$. The mass difference for $B^0$ mesons is $\Delta m_d = 0.509 \pm 0.0005$ ps$^{-1}$, while for $B_S$ mesons, $\Delta m_S$ is limited at 90% confidence level to be $> 16.6$ ps$^{-1}$ [7, 8], and recently measured by CDF to be $17.31^{+0.32}_{-0.33} \pm 0.07$ ps$^{-1}$ [9]. $B_S$ mixing, when the meson decays semileptonically, produces a relatively large number of like-sign dilepton events that allows a measurement of $B_S$ production [10]. We must also account for $B_d$ mixing and thus need to know the composition of the $B\overline{B}$ states, since the level of mixing will depend on the Charge Conjugation (C) state of the $B\overline{B}$ system.

Hadrons containing $b$-flavor are produced in pairs in current experiments. If the $B$ and $\overline{B}$ are not in an eigenstate of C, then the probability for a $B^0$ to decay as a $\overline{B}^0$, integrated over time, is given by

$$P(B^0 \to \overline{B}^0) = x = \frac{x^2}{2(1 + x^2)},$$

where the mixing parameter $x = \Delta m_B/\Gamma$.

The ratio $R$ of mixed events to unmixed events is given by

$$R = \frac{N_{B^0}\overline{B}^0 + N_{\overline{B}^0}B^0}{N_{B^0}\overline{B}^0 + N_{\overline{B}^0}B^0}.$$ \hspace{1cm} (2)

For incoherent states then $N_{B^0}\overline{B}^0 + N_{\overline{B}^0}B^0$ is equal to $2\chi(1 - \chi)$ and $N_{B^0}\overline{B}^0 + N_{\overline{B}^0}B^0$ is equal to $\chi^2 + (1 - \chi)^2$. For $B_S$ mesons the mixing oscillations are so fast that $R$ does depend on the initial $B^0\overline{B}^0$ state. If the initial pair of $B$ mesons is in a C odd configuration then [11]

$$R_- = \frac{x^2}{2 + x^2},$$ \hspace{1cm} (3)

where the mixing parameter $x_B = \Delta m_B/\Gamma$ and is well measured as $0.775 \pm 0.008$ [12]. For $C$ even configurations we have

$$R_+ = \frac{3x^2 + x^4}{2 + x^2 + x^4}. \hspace{1cm} (4)$$

The states containing charged $B$ pairs do not contribute to mixing. We expect that there are an equal number of such states as neutral non-strange $B$‘s, since the mass difference is small and Coulomb corrections are small even at the $\Upsilon(4S)$ [13]. The $B^0\overline{B}^0$ and $B^{*0}\overline{B}^{*0}$ are both produced in an L states one state: thus the $B^0\overline{B}^0$ pair is in a negative C parity state. The $B^0\overline{B}^{*}\gamma$ state,
The fractions in each of these three states are denoted by incoherent, are listed in Table I, and plotted in Fig. 2.

\[ R_{\text{inco}} = \frac{x^2(2 + x^2)}{2 + 2x^2 + x^4}. \]  

The variation of the \( R \) functions with \( x \) is shown in Fig. 1. Note that \( B^0B^0 \pi^+ \) and \( B^+B^- \pi^- \) final states contain only one \( B^0 \) and mix according to Eq. (3) leading to a mixing rate given by Eq. (4) and thus would be treated as an odd \( C \) state for our purpose. The other states can also be classified in terms of \( C \) eigenstates or incoherent states.

CLEO has made the first measurement of \( B \) meson production at the \( \Upsilon(5S) \) \([13]\). They find that \( B^+B^-\pi^0 \) production is largest with \( B^+B^-\pi^0 \) being about 1/3 its rate. Although limits on other decay channels are not small, we will assume that more data will allow the exact composition of the \( B \) decays at the \( \Upsilon(5S) \) to be established.

We now consider like-sign dilepton production coming from neutral \( B \)'s. The yield from \( B^0\overline{B}^0 \) pairs is given by

\[ N_{++} + N_{--} = \sum_i f_i D_i - \pm \pm \pm (x), \]  

where \( N_{\pm \pm} \) is the number of \( \Upsilon(5S) \) events containing a \( B\overline{B} \) pair above continuum background, and \( B_{d-sl} \) is the \( B^0 \) semileptonic branching ratio. The \( D_i - \pm \pm \pm (x) \) functions for the neutral \( B \) pairs in \( C = \pm \) eigenstates, or being incoherent, are listed in Table I and plotted in Fig. 2. The fractions in each of these three states are denoted by \( f_i \); they include the \( B\overline{B}\pi(\pi) \) final states \([16]\). We also list and plot the \( D_i - \pm \pm (x) \) functions for opposite sign dileptons. The fraction of neutral non-strange \( B \) mesons, \( f_u \), is assumed to be equal to that for charged \( B \) mesons \([17]\). Since the sum of neutral, charged, and strange \( B \) mesons is unity, we have

\[ f_u + f_d + f_s = 1 \]  
\[ 2f_d + f_s = 1, \quad \text{and therefore} \]  
\[ f_d = f_u = (1 - f_s)/2. \]  

### C state

| \( D_i - \pm \pm \pm (x) \) | \( D_i - \pm \pm \pm (x) \) |
|-----------------|-----------------|
| Odd             | \[ \frac{x^2}{2(1 + x^2)} \] | \[ \frac{(2 + x^2)}{2(1 + x^2)} \] |
| Even            | \[ \frac{x^2(3 + x^2)}{2(1 + x^2)^2} \] | \[ \frac{(2 + x^2 + x^4)}{2(1 + x^2)^2} \] |
| Incoherent      | \[ \frac{x^4 + 2x^2}{2(1 + x^2)^2} \] | \[ \frac{2 + 2x^2}{2(1 + x^2)^2} \] |

**TABLE I:** Functions for like-sign and opposite-sign dileptons.

A similar set of expressions exist for like-sign leptons from \( B_S \) decays

\[ N_{++} + N_{--} = N_{5S}f_{5S}B_{S-sl}^2 \sum_i f_i D_i - \pm \pm \pm (x), \]  

where \( B_{S-sl} \) is the semileptonic branching ratio and \( D_{\pm \pm \pm (x)} \) is the function that characterizes the dilepton rate and, in principle, depends on whether or not \( B_S \overline{B}_S \) is in an even or odd eigenstate. The function form is identical to the \( D_i - \pm \pm \pm \pm (x) \) functions listed in Table I but incoherent states are not allowed since there isn’t enough energy at the \( \Upsilon(5S) \) to produce an additional pion. Similarly, the opposite-sign functions for \( B_S \), \( D_{\pm \pm \pm (x)} \), are
identical in form with the $D_{±±}(x)$ functions. Note that all these functions are normalized so that they go to a value of 0.5 as $x$ (or $x_S$) gets large, reflecting the fact that the mixing probability goes to its maximum value of 50%.

The rate of opposite-sign dileptons is given by

\[ N_{+-} + N_{-+} = N_{S} \left[ f_{S} B_{S-}\overline{u} \right] D_{±±}(x) \]

\[ + \frac{1 - f_{S}}{2} B_{u-}\overline{u} \sum f_{i} D_{i-±±}(x) + \frac{1 - f_{S}}{2} B_{u-}\overline{u} \]  \hspace{1cm} \text{(9)}

The last term is due to charged $B_u$ decays which do not mix. We form the ratio of like-sign to opposite-sign dileptons and divide through by the charged $B$ semileptonic branching ratio $B_u^2$. We replace the resulting ratios of semileptonic branching fractions by the lifetime ratios, $B_{d-}/B_{u-}\overline{u} = \tau(B^+)/\tau(B^+)$, and note that this rate is not affected by mixing. Here $\tau$ is either +1 or -1, and whose rate is also not affected by mixing. When both $B$ mesons are produced in almost all $B_S$ decays, while they are produced only at the 10% level in $B$ decays. We define $D_{±±}(x_S) = D_{±±}(x_S) = 0.5$.

Denoting

\[ \rho = \frac{N_{++} + N_{--}}{N_{+-} + N_{-+}} \]

the resulting equation is

\[ \rho = \frac{f_{S} \tau_{S}^2 + (1 - f_{S}) \tau_{0}^2 \sum f_{i} D_{i-±±}(x)}{f_{S} \tau_{S}^2 + (1 - f_{S}) \tau_{0}^2 \sum f_{i} D_{i-±±}(x) + (1 - f_{S})} \]  \hspace{1cm} \text{(10)}

Solving for $f_{S}$ gives

\[ f_{S} = \frac{S_{D} - \rho}{(\rho - 1) \tau_{S}^2 + S_{D} - \rho} \]  \hspace{1cm} \text{(11)}

where

\[ S_{D} = \tau_{0}^2 \sum f_{i} D_{i-±±}(x) - \rho \sum f_{i} D_{i-±±}(x) \]  \hspace{1cm} \text{(12)}

For practical application, we can use Eq. [7] to determine $f_d$ and $f_u$ and then use well measured $B^0$ and $B^+$ branching ratios to normalize the $B_S$ rates.

We can estimate the error in $f_{S}$ by taking the C odd contribution as 75%, the C even component as 25%, no incoherent $B^0\overline{B}$ contribution and $f_{S}$ equal to 16% from the CLEO model dependent determination that used $B(D_S \rightarrow \phi \pi^+)$ = (4.4 $\pm$ 0.5)% which is an average between the PDG value [7] and a recent BaBar measurement [18]. In this case $\rho$=0.25. Taking into account the semileptonic branching (10.5%), the fraction of high momentum leptons above the minimum lepton momentum cut (1/3), and the lepton efficiency (0.8), we estimate that an error of $\pm$4% on $f_{S}$ can be achieved with 30 fb$^{-1}$ of data. (We find that the fractional error in $f_{S}$ is about twice the fractional error in $\rho$.) Interestingly, a new preliminary value of $B(D_S \rightarrow \phi \pi^+)$ (3.5 $\pm$ 0.4)% based on CLEO-c data [19] raises $f_{S}$ to 21%, changes $\rho$ to 0.29, and consequently improves the sensitivity to about $\pm$3%. Thus, this method can lead to a model independent determination of $f_{S}$ once the C states of the $B^0$ decays are experimentally determined more precisely.

We next discuss another method for finding the $B_S$ fraction that uses $B$ mixing as input, but is not the main ingredient. Rather, we make use of the fact that $D_S^{±}$ mesons are produced in almost all $B_S$ decays, while they are produced only at the 10% level in $B$ decays. We define $S_{1} = B(B \rightarrow D_{S}X)$, $S_{2} = B(B \rightarrow D_{S}^{±}D_{S}^{±}X)$, and $S_{3} = B(B \rightarrow D_{S}^{±}D_{S}^{±}X)$. Here $S_{1}$ is the rate for the $B_S$ to decay into one and only one $D_{S}^{±}$ meson, $S_{2}$ is the rate for the $B_S$ to decay into a $D_{S}^{±}$ meson pair. The inclusive rate $B(B_S \rightarrow D_{S}X) = S_{1} + 2S_{2}$. (Note that the probability that a $B_S$ does not decay into a $D_{S}$ meson is given by $1 - S_{1} - S_{2}$.)

In this analysis there are two sources of $D_{S}$ production, the $B_S$ again with fraction $f_{S}$, and $B$ decays, where we take the mixture of $B^0$ and $B^+$ events to have the same $D_{S}$ yields as on the $\Upsilon(4S)$. Since we do not expect the charge of the $B$ to effect the $D_{S}$ rates, this method is insensitive to their relative contribution. Thus, the $B_S$ fraction is taken as (1-$f_{S}$).

Consider now the production of multiple $D_{S}$ candidates in single $\Upsilon(5S)$ events, taking into account mixing. $\Upsilon(5S)$ decays can produce events with 4 $D_{S}$ mesons, when both $B_S$ mesons decay into $D_{S}^{±}D_{S}^{±}X$. We denote as $N^{±±±±}$ the observed number of such 4 $D_{S}$ events and note that this rate is not affected by mixing. $N^{±±±±}$ refers to events with 3 observed $D_{S}$, whose charge sum is either +1 or -1, and whose rate is also not affected by mixing. $N^{±±}$ denotes finding an event with oppositely charge $D_{S}^{±}D_{S}^{±}$ mesons, whose rate is changed by both $B_S$ and $B$ mixing, so we introduce the parameters $f_{mix}^{S}$ and $f_{mix}^{D}$, where $f_{mix}^{S}$ = 1/2 and $f_{mix}^{D}$ equals average mixing rate over the C even, odd, incoherent mixtures and charged $B$ decays defined above. $N^{±±±±}$ denotes a $D_{S}^{±}D_{S}^{±}$ or $D_{S}^{±}D_{S}^{±}$ pair, while $N^{±±}$ indicates the detection of a single $D_{S}^{±}$ meson. Here, the single rate is inclusive of all double, triple and quadruple rates, etc.. The resulting equations relating the observed numbers to the branching ratios and $f_{S}$ are

\[ N^{±±±±}/e^{2}N_{5S} = S_{D}^{2}f_{S} \]

\[ N^{±±±±}/e^{3}N_{5S} = (2S_{1}S_{2} + 4S_{2}^{2})f_{S} \]

\[ N^{±±}/e^{2}N_{5S} = \left[(1 - f_{mix}^{S})S_{D}^{2} + 2(1 - f_{mix}^{S})S_{1}S_{2} \right]f_{S} \]

\[ + 2S_{2}(1 - S_{1} - S_{2} + 4S_{2}^{2})f_{S} + \left(1 - f_{S} \right)(1 - f_{mix}^{D})B_{D} \]

\[ N^{±±}/e^{2}N_{5S} = \left[f_{mix}^{S}(S_{D}^{2} + 2S_{1}S_{2} + 2S_{2}^{2})\right]f_{S} \]  \hspace{1cm} \text{(17)}
\[ N^\pm /\epsilon N_{5S} = 2(S_1 + 2S_2) f_S + 2(1 - f_S) B, \]

where \( \epsilon \) indicates the detection efficiency of a single \( D_S \); it is the sum of the branching ratio times efficiency for each decay mode that is used. We ignore two small corrections; first of all we take the rate for a single \( B \) meson to produce a \( D_S^* D_S^- \) pair to be negligibly small, and secondly we don’t account properly for the charge correlations resulting from the small production of “wrong-sign” \( D_S \) production from \( B_S \) decays that can occur via a \( b \to u \) transitions. The latter effects only equations containing \( f_{mix} \) [21].

We expect that \( \epsilon \) could be made as large as 10% by including many modes. We use \( S_1 = 0.8 \) and \( S_2 = 0.1 \), which allows estimates of the various rates. We do not expect to be able to observe a significant quadruple \( D_S \) rate. On the other hand, we estimate that in a 50 fb\(^{-1} \) data sample, there would be \( \approx 500 \) observable triple \( D_S \) events allowing the use of Eq. 17. Thus, we would have four equations, in this case Eqs. 15-18 relating our three unknowns: \( f_S, S_1 \) and \( S_2 \). The best values for the unknowns can be obtained by using a constrained fit to find the solution. Note that Eq. 16 can be simplified as

\[
N^\pm /\epsilon^2 N_{5S} = \left[ (1 - f_{mix}^S)S_1^2 - 2f_{mix}^S S_1 S_2 + 2S_2 + 2S_2^2 \right] f_S + (1 - f_S)^2 (1 - f_{mix}) B^2.
\]

Should not enough triple \( D_S \) events be found in the data sample, Eqs. 16-18 could be solved for the three unknowns. Another possibility, if precise measurements on the \( B^0 \) C parity contributions are not available, is to add Eqs. 16 [17] and 17 since the resulting equation is not a function of either \( B^0 \) or \( B_S \) mixing. In this case, however, measurement of the triple \( D_S \) rate is necessary.

In conclusion, knowledge of the number of \( B_S \) mesons is essential for all branching fraction determinations at the \( \Upsilon(5S) \). We present a model independent method of determining the fraction of \( B_S \) mesons at the \( \Upsilon(5S) \), and hence \( B \left[ \Upsilon(5S) \to B_S^{(*)} \overline{B}_S^{(*)} \right] \), using the complete mixing of the \( B_S \) via dileptons. The amount of luminosity required to make an accurate measurement of the \( B_S \) fraction will depend on the actual compositions of the \( B^0 \) final states, but it is likely to require several tens of fb\(^{-1} \). We also suggest another technique using the counting of single, double and triple \( D_S \) mesons in \( \Upsilon(5S) \) events [21].

This work was supported by the National Science Foundation under grant #0553004. We thank Remi Louvot for useful comments.

[1] S. Ono et al., Phys. Rev. Lett., 55, 2938 (1985); A. D. Martin and C.-K. Ng, Z. Phys. C 40, 139 (1988); N. Beyers and E. Eichten, Nucl. Phys. B (Proc. Suppl.) 16, 281 (1990); N. Beyers [hep-ph/9412292] (1994).

[2] M. Artuso et al. (CLEO) Phys. Rev. Lett. 95, 261801 (2005) [hep-ex/0508047].

[3] G. Bonvicini et al. (CLEO), Phys. Rev. Lett. 96 022002 (2006) [hep-ex/0510034].

[4] A. Drutskoy, “Results of the \( \Upsilon(5S) \) Engineering Run,” presented at Rencontres De Moriond, March 2006, La Thaille, Italy [http://moriond.in2p3.fr/EW/2006/Transparencies/A.Drutskoy.pdf].

[5] A. Drutskoy, “Determining \( \Delta\Gamma_{S1}/\Gamma \) from \( B(B_S \to D_S^{(*)}\overline{D}_S^{(*)}) \) Measurements at \( \Upsilon(5S) \) Resonance,” [hep-ph/0604061].

[6] Dilepton measurements done on the \( \Upsilon(4S) \) used minimum momentum criteria near 1.5 GeV/c. See for example, J. Bartelt et al. (CLEO), Phys. Rev. Lett. 71, 1680 (1993).

[7] S. Eidelman et al. (PDG), Phys. Lett. B 592, 1 (2004).

[8] V. Abazov et al. (D0), “First Direct Two-Sided Bound on the \( B_S^0/\overline{B}_S^0 \) Oscillation Frequency,” (2006) [hep-ex/0603029].

[9] A. Abulencia et al. (CDF), “Measurement of the \( B^0 \) \( \to \overline{B}^0 \) Oscillation Frequency,” (2006) [hep-ex/0600027].

[10] Similar techniques have been used at LEPI and Tevatron experiments to help improve the accuracy of the determination of \( b \)-hadron fractions. See O. Schneider, “\( B^0/\overline{B}^0 \) Mixing,” in ref. [8], and references contained therein.

[11] I. I. Bigi and A. I. Sanda, CP Violation, Cambridge Univ. Press (Cambridge), p157 (2000).

[12] E. Barberio et al., “Averages of b-hadron Properties at the End of 2005,” [hep-ex/0603003] (2006).

[13] D. Atwood and W. Marciano, Phys. Rev. D41, 1736 (1990).

[14] H. Schröder, “\( B^0/\overline{B}^0 \) Mixing,” in B Decays Revised 2nd edition, ed. by S. Stone, World Scientific, Singapore (1994) p449; P. Krawczyk, D. London and H. Steger, Nucl. Phys. B321, 1 (1989).

[15] O. Aquences et al. CLEO, “First Measurements of the Exclusive Decays of the \( \Upsilon(5S) \) to \( B \) Meson Final States and Improved \( B^0_S \) Mass Measurement.” [hep-ex/0601044].

[16] For better accuracy, final states with a charged and neutral \( B \) such as \( B^* \overline{B}^0 \) - \( \pi^{-} \) should be put in a separate sum where the coefficient is \( B_{s\to sl} B_{s\to sl} \) instead of \( B_{s\to sl}^2 \). These states are expected to constitute a small fraction of the decays, so we have not done this here.

[17] In principle, the ratio of \( B^- \) to \( B^0 \) production can be independently determined by measuring exclusive decays and using branching ratios measured at the \( \Upsilon(4S) \). A change in this ratio from unity will affect the determination of \( f_S \) using mixing but not using inclusive \( D_S \) decays.

[18] B. Aubert et al. Phys. Rev. D 71, 091104(R) (2005).

[19] S. Stone, “Hadronic Charm Decays and D Mixing,” invited talk at Flavor Physics and CP Violation Conference, April 6-12, 2006, Vancouver Canada [http://fpcp2006.triumf.ca/agenda.php?hep-ph/0605134].

[20] In addition, to take precisely into account any “wrong-sign” \( B \) decays, e. g. \( B^- \to D_S^+ X \), we advocate measuring \( f_{mix} \) on the \( \Upsilon(4S) \) using the ratio of like-sign to opposite-sign \( D_S \) events.

[21] It may also be possible to determine \( \overline{B}^0 \) and \( B^- \) production cross-sections by reconstructing exclusive decays. All possible modes \( B^+ \overline{B}^- \), \( B^+ \overline{B}^- \pi^- \) and \( B^+ \overline{B}^- \pi^+ \) must be included. The sum of these cross-sections with the \( B_S \) yield found using the techniques described above should
equal the total $\Upsilon(5S)$ cross-section. Any short fall would be indicative of decays of the $\Upsilon(5S)$ to final states not containing $b$ and $\bar{b}$ quarks.