Relativistic treatment of the decay constants of light and heavy mesons

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Novel relativistic expressions are used to calculate the weak decay constants of pseudoscalar and vector mesons within the constituent quark model. Meson wave functions satisfy the quasipotential equation with the complete relativistic potential. New contributions, coming from the negative-energy quark states, are substantial for the light mesons, significantly decrease the values of their decay constants and, thus, bring them into agreement with experiment. For heavy-light mesons these contributions are much less pronounced, but permit to reduce uncertainties of the predicted decay constants. Their values agree with the results of lattice calculations and experimental data.

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The weak decay constants of pseudoscalar and vector mesons belong to their most important characteristics, which enter in various decay rates. Many efforts were undertaken to calculate these constants within lattice QCD (both quenched and unquenched)\cite{1,2,3,4,5,6}, QCD sum rules\cite{7,8,9}, and constituent quark models\cite{10,11,12,13,14,15,16}. At present, the decay constants of light mesons are measured with high precision, while in the heavy-light meson sector only $D$ and $D_s$ meson decay constants are available with rather large errors\cite{17}. Recently, a relatively precise experimental value for the $D$ meson decay constant was presented by the CLEO Collaboration\cite{18}. Therefore it is actual to reconsider the meson decay constants treating quarks, composing the meson, in a consistently relativistic way. Such procedure was formulated and successfully applied for light mesons in the papers\cite{19}. In this letter we evaluate new contributions to relativistic expressions for the meson decay constants coming from the negative-energy quark states both for light and heavy-light mesons. We use the meson wave functions satisfying the quasipotential equation with the complete relativistic potential in order to obtain new, more accurate predictions for the meson decay constants.

In the quasipotential approach a meson is described by the wave function of the bound quark-antiquark state\cite{20}, which satisfies the quasipotential equation of the Schrödinger
type
\[
\left( \frac{b^2(M)}{2 \mu_R} - \frac{P^2}{2 \mu_R} \right) \Psi_M(p) = \int \frac{d^3q}{(2\pi)^3} V(p, q; M) \Psi_M(q),
\]
(1)
where the relativistic reduced mass is
\[
\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3},
\]
(2)
and \(E_1, E_2\) are given by
\[
E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}.
\]
(3)
Here \(M = E_1 + E_2\) is the meson mass, \(m_{1,2}\) are the quark masses, and \(p\) is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads
\[
b^2(M) = \left[ M^2 - (m_1 + m_2)^2 \right] \left[ M^2 - (m_1 - m_2)^2 \right] / 4M^2.
\]
(4)
The kernel \(V(p, q; M)\) in Eq. (1) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. Constructing the quasipotential of the quark-antiquark interaction, we have assumed that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli interaction. The quasipotential is then defined by
\[
V(p, q; M) = \bar{u}_1(p) \bar{u}_2(-p) V(p, q; M) u_1(q) u_2(-q),
\]
(5)
with
\[
V(p, q; M) \equiv V(p - q) = \frac{4}{3} \alpha_s D_{\mu\nu}(k) \gamma_1^\mu \gamma_2^\nu + V^V_{\text{conf}}(k) \Gamma_1^\mu \Gamma_2^\mu + V^S_{\text{conf}}(k),
\]
where \(\alpha_s\) is the QCD coupling constant, \(D_{\mu\nu}\) is the gluon propagator in the Coulomb gauge and \(k = p - q\); \(\gamma_\mu\) and \(u(p)\) are Dirac matrices and spinors. The effective long-range vector vertex is given by
\[
\Gamma_\mu(k) = \gamma_\mu + \frac{i\kappa}{2m} \sigma_{\mu\nu} k^\nu,
\]
(6)
where \(\kappa\) is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to
\[
V^V_{\text{conf}}(r) = (1 - \varepsilon)(Ar + B), \quad V^S_{\text{conf}}(r) = \varepsilon(Ar + B),
\]
(7)
reproducing
\[
V_{\text{conf}}(r) = V^S_{\text{conf}}(r) + V^V_{\text{conf}}(r) = Ar + B;
\]
(8)
\footnote{In our notation, where strong annihilation processes are neglected, antiparticles are described by usual spinors taking into account the proper quark charges.}
**TABLE I: Masses of the ground state light and heavy-light mesons (in MeV).**

| Meson | $M^{\text{theor}}$ | $M^{\text{exp}}$ PDG [17] |
|-------|-----------------|------------------|
| $\pi$ | 154             | 139.57           |
| $\rho$ | 776             | 775.8(5)         |
| $K$     | 482             | 493.677(16)      |
| $K^*$    | 897             | 891.66(26)       |
| $\phi$    | 1038            | 1019.46(2)       |
| $D$      | 1872            | 1869.4(5)        |
| $D^*$    | 2009            | 2010.0(5)        |
| $D_s$    | 1967            | 1968.3(5)        |
| $D_s^*$  | 2112            | 2112.1(7)        |
| $B$      | 5275            | 5279.0(5)        |
| $B^*$    | 5326            | 5325.0(6)        |
| $B_s$    | 5362            | 5369.6(2.4)      |
| $B_s^*$  | 5414            | 5416.6(3.5)      |

where $\varepsilon$ is the mixing coefficient.

All the model parameters have the same values as in our previous papers [20, 21]. The constituent quark masses $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV, $m_c = 1.55$ GeV, $m_b = 4.88$ GeV and the parameters of the linear potential $A = 0.18$ GeV$^2$ and $B = -0.3$ GeV have the usual values of quark models. The values of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ and the universal Pauli interaction constant $\kappa = -1$ are specific for our model.

The quasipotential [19] can be used for arbitrary quark masses. The substitution of the Dirac spinors into [19] results in an extremely nonlocal potential in the configuration space. Clearly, it is very hard to deal with such potentials without any additional transformations. In order to simplify the relativistic $q\bar{q}$ potential, we make the following replacement in the Dirac spinors [19, 21]:

$$
\epsilon_{1,2}(p) = \sqrt{m_{1,2}^2 + p^2} \to E_{1,2}.
$$

This substitution makes the Fourier transformation of the potential [19] local, but the resulting relativistic potential becomes dependent on the meson mass in a very complicated nonlinear way. We consider only the meson ground states, which further simplifies our analysis, since all terms containing orbital momentum vanish. The detailed expressions for the relativistic quark potential can be found in Ref. [19]. Here we use these formulas for the calculation of the ground state meson masses. We solve numerically the quasipotential equation with the local fully relativistic potential, which includes both spin-independent and spin-dependent parts. As a result we get the relativistic wave functions of the ground state mesons which depend nonperturbatively on the meson spin (i.e. the pseudoscalar and vector meson wave functions are different). These wave functions are used below for calculating the decay constants of light and heavy mesons. The obtained masses of the pseudoscalar and vector mesons are given in Table I in comparison with the experimental data [17]. The overall good agreement of our predictions with experiment is found.

The decay constants $f_P$ and $f_V$ of the pseudoscalar ($P$) and vector ($V$) mesons parame-
\[ \Psi(M, p) = (1) + (2) + (3) + (4) \]

**Fig. 1:** Weak annihilation diagram of the meson. Solid and bold lines denote the positive- and negative-energy part of the quark propagator, respectively. Dashed lines represent the interaction operator \( V \). Dashed ovals depict the projected wave function \( \Psi^M_K(p) \).

The matrix elements of the weak current \( J^W_\mu = \bar{q}_1 J^W_\mu q_2 = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2 \) between the corresponding meson and vacuum states. They are defined by

\[
\begin{align*}
\langle 0 | \bar{q}_1 \gamma_\mu \gamma_5 q_2 | P(K) \rangle &= i f_P K^\mu, \\
\langle 0 | \bar{q}_1 \gamma_\mu q_2 | V(K, \varepsilon) \rangle &= f_V M_V \varepsilon^\mu,
\end{align*}
\]

where \( K \) is the meson momentum, \( \varepsilon^\mu \) and \( M_V \) are the polarization vector and mass of the vector meson. This matrix element can be expressed through the two-particle Bethe-Salpeter wave function \( \Psi(M, p) \) in the quark loop integral (see Fig. 1)

\[
\langle 0 | J^W_\mu | M(K) \rangle = \int \frac{d^4p}{(2\pi)^4} \text{Tr} \{ \gamma_\mu (1 - \gamma_5) \Psi(M, p) \},
\]

where the trace is taken over spin indices. Integration over \( p^0 \) in Eq. (12) allows one to pass to the Fourier transform of the single-time wave function in the meson rest frame

\[
\Psi(M, p) = \int \frac{dp^0}{2\pi} \Psi(M, p).
\]

This wave function contains both positive- and negative-energy quark states. Since in the quasipotential approach we use the wave function \( \Psi^M_K(p) \) projected onto the positive-energy states, it is necessary to include additional terms which account for the contributions of negative-energy intermediate states. Within perturbation theory the weak matrix element (12) is schematically presented in Fig. 1. The first diagram in the right hand side corresponds to the simple replacing of the wave function (13) \( \Psi(M, p) \) by the projected one \( \Psi^M_K(p) \).\(^2\)

The second and third diagrams account for negative-energy contributions to the first and second quark propagators, respectively. The last diagram corresponds to negative-energy contributions from both quark propagators.

Thus in the quasipotential approach this decay amplitude has the form

\[
\begin{align*}
\langle 0 | J^W_\mu | M(K) \rangle = \sqrt{2M} \left\{ \int \frac{d^3p}{(2\pi)^3} \bar{u}_1(p_1) J^W_\mu u_2(p_2) \Psi^M_K(p) + \int \frac{d^3p d^3p'}{(2\pi)^6} \bar{u}_1(p_1) \Gamma_1 \\
\times \frac{\Lambda^-(p'_1) \gamma^0 J^W_\mu \Lambda^+(p'_2) \gamma^0}{M + \epsilon_1(p') - \epsilon_2(p')} \Gamma_2 v_2(p_2) \bar{V}(p - p') \Psi^M_K(p) + (1 \leftrightarrow 2) \right\}
\end{align*}
\]

\(^2\) The contributions with the exchange by the effective interaction potential \( V \) which contain only positive-energy intermediate states are automatically accounted for by the wave function itself.
\[ + \int \frac{d^3p d^3p'}{(2\pi)^6} \bar{u}_1(p_1) \Gamma_1 \frac{\Lambda_1^{(-)}(p'_1) \gamma^0 J_\mu^\pi \Lambda_2^{(-)}(p'_2) \gamma^0}{M + \epsilon_1(p') + \epsilon_2(p')} \Gamma_2 u_2(p_2) \bar{V}(p - p') \Psi_M \kappa(p) \}, \]

where \( p_{1,2} = K/2 \pm p^{(l)}; \epsilon(p) = \sqrt{p^2 + m^2}; \) matrices \( \Gamma_{1,2} \) denote the Dirac structure of the interaction potential \( \Phi \) for the first and second quark, respectively, and thus \( \Gamma_1 \Gamma_2 \bar{V}(p - p') = V(p - p') \). The factor \( \sqrt{2M} \) follows from the normalization of the quasipotential wave function. The positive- and negative-energy projectors have standard definition

\[ \Lambda^{(+)}(p) = \frac{\epsilon(p) \pm (m_\gamma^0 + \gamma^0(\gamma p))}{2\epsilon(p)}. \]

The ground-state wave function in the rest frame of the decaying meson \( \Psi_M(p) \equiv \Psi_{M0}(p) \) can be expressed through a product of radial \( \Phi_M(p) \), spin \( \chi_{ss'} \) and colour \( \phi_{q_1q_2} \) wave functions

\[ \Psi_M(p) = \Phi_M(p) \chi_{ss'} \phi_{q_1q_2}. \]

Now the decay constants can be presented in the following form

\[ f_{P,V} = f_{P,V}^{(1)} + f_{P,V}^{(2+3)} + f_{P,V}^{(4)}, \]

where the terms on the right hand side originate from the corresponding diagrams in Fig. 1 and parameterize respective terms in Eq. (14). In the literature \[10, 14\] usually only the first term is taken into account, since it provides the nonrelativistic limit, while other terms give only relativistic corrections and thus vanish in this limit. Such approximation can be justified for mesons containing only heavy quarks. However, as it will be shown below, for mesons with light quarks, especially for light mesons, other terms become equally important and their account is crucial for getting the results in agreement with experimental data.

The matrix element (14) and thus the decay constants can be calculated in an arbitrary frame and from any component of the weak current \[13\]. Such calculation can be most easily performed in the rest frame of the decaying meson from the zero component of the current. The same results will be obtained from the vector component; however, this calculation is more cumbersome, since then the rest frame cannot be used and, thus, it is necessary to take into account the relativistic transformation of the meson wave function from the rest frame to the moving one with the momentum \( K \). It is also possible to perform calculations in an explicitly covariant way using methods proposed in \[22\].

The resulting expressions for decay constants are given by

\[ f_{P,V}^{(1)} = \sqrt{\frac{12}{M}} \int \frac{d^3p}{(2\pi)^3} \left( \frac{\epsilon_1(p) + m_1}{2\epsilon_1(p)} \right)^{1/2} \left( \frac{\epsilon_2(p) + m_2}{2\epsilon_2(p)} \right)^{1/2} \langle 1 + \lambda_{P,V} \frac{p^2}{(\epsilon_1(p) + m_1)(\epsilon_2(p) + m_2)} \rangle \Phi_{P,V}(p), \]

\[ f_{P,V}^{(2+3)} = \sqrt{\frac{12}{M}} \int \frac{d^3p}{(2\pi)^3} \left( \frac{\epsilon_1(p) + m_1}{2\epsilon_1(p)} \right)^{1/2} \left( \frac{\epsilon_2(p) + m_2}{2\epsilon_2(p)} \right)^{1/2} \left[ \frac{M - \epsilon_1(p) - \epsilon_2(p)}{M + \epsilon_1(p) - \epsilon_2(p)} \right] \left( 1 \leftrightarrow 2 \right) \Phi_{P,V}(p), \]
TABLE II: Different contributions to the pseudoscalar and vector decay constants of light and heavy mesons (in MeV). The notations are taken according to Eqs. (16) and (20).

| Constant | \( f_{M}^{\text{NR}} \) | \( f_{M}^{(1)} \) | \( f_{M}^{(2+3)} + f_{M}^{(4)} \) | \( (f_{M}^{(2+3)} + f_{M}^{(4)})/f_{M}^{(1)} \) | \( f_{M} \) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( f_{\pi} \)  | 1290            | 515             | -391            | -76%            | 124             |
| \( f_{\rho} \)  | 490             | 402             | -183            | -46%            | 219             |
| \( f_{K} \)    | 783             | 353             | -198            | -56%            | 155             |
| \( f_{K^*} \)  | 508             | 410             | -174            | -42%            | 236             |
| \( f_{\phi} \) | 511             | 415             | -170            | -41%            | 245             |
| \( f_{D} \)    | 376             | 275             | -41             | -15%            | 234             |
| \( f_{D^*} \)  | 391             | 334             | -24             | -7%             | 310             |
| \( f_{D_s} \)  | 436             | 306             | -38             | -12%            | 268             |
| \( f_{D_s^*} \)| 447             | 367             | -52             | -14%            | 315             |
| \( f_{B} \)    | 259             | 210             | -21             | -10%            | 189             |
| \( f_{B^*} \)  | 280             | 235             | -16             | -7%             | 219             |
| \( f_{B_s} \)  | 300             | 238             | -20             | -8%             | 218             |
| \( f_{B_s^*} \)| 316             | 264             | -13             | -5%             | 251             |

\[
f_{(4)}^{P,V} = \sqrt{\frac{12}{M} \int \frac{d^3p}{(2\pi)^3} \left( \frac{\epsilon_1(p) + m_1}{2\epsilon_1(p)} \right)^{1/2} \left( \frac{\epsilon_2(p) + m_2}{2\epsilon_2(p)} \right)^{1/2} \frac{M - \epsilon_1(p) - \epsilon_2(p)}{M + \epsilon_1(p) + \epsilon_2(p)} \\
\times \left\{ -\lambda_{P,V} - \frac{p^2}{[\epsilon_1(p) + m_1][\epsilon_2(p) + m_2]} \right\} \\
\times \left\{ \frac{1 - \epsilon}{\epsilon_1^2(p)\epsilon_2^2(p)} + \frac{m_1^2m_2^2}{[\epsilon_1(p) + m_1][\epsilon_2(p) + m_2]} \right\} \Phi_{P,V}(p),
\]

(19)

with \( \lambda_P = -1 \) and \( \lambda_V = 1/3 \). Here \( \epsilon \) is the mixing coefficient of scalar and vector confining potentials (7) and the long-range anomalous chromomagnetic quark moment \( \kappa \) (6) is put equal to \(-1\). Note that \( f_{(2+3)}^{P,V} \) vanishes for pseudoscalar mesons with equal quark masses, such as the pion. The positive-energy contribution (17) reproduces the previously known expressions for the decay constants \( [10] \). The negative-energy contributions (18) and (19) are new and play a significant role for light mesons.

In the nonrelativistic limit \( p^2/m^2 \to 0 \) the expression (17) for decay constants gives the well-known formula

\[
f_{P,V}^{\text{NR}} = \sqrt{\frac{12}{M_{P,V}}} |\Psi_{P,V}(0)|, \tag{20}
\]

where \( \Psi_{P,V}(0) \) is the meson wave function at the origin \( r = 0 \). All other contributions vanish in the nonrelativistic limit.

In Table II we present our predictions for the light and heavy-light meson decay constants calculated using the meson wave functions which were obtained as the numerical solutions of the quasipotential equation.\(^3\) The values \(^4\) of \( f_{M}^{\text{NR}} \), obtained from the nonrelativistic

\(^3\) We roughly estimate the uncertainties in our calculations to be about ten MeV for light mesons and of a several MeV for heavy-light mesons.

\(^4\) For the evaluation of \( f_{M}^{\text{NR}} \) the relativistic wave functions were used. Thus the difference of the pseudoscalar...
**TABLE III: Pseudoscalar and vector decay constants of light mesons (in MeV).**

| Constant | this work | [10] | [11] | [12] | [16] | Lattice [2] | Lattice [3] | Experiment [17] |
|----------|-----------|------|------|------|------|-------------|-------------|-----------------|
| $f_\pi$  | 124       | 180  | 131  | 219  | 138  | 126.6(6.4)  | 129.5(3.6)  | 130.70(10)(36)  |
| $f_K$    | 155       | 232  | 155  | 238  | 160  | 152.0(6.1)  | 156.6(3.7)  | 159.80(1.4)(44) |
| $f_\rho$ | 219       | 220  | 207  | 238  | 239  | 239.4(7.3)  | 220(2)*     | 209(4)**        |
| $f_{K^*}$| 236       | 267  | 241  | 241  | 255  | 255.5(6.5)  | 217(5)†      | 229(3)‡         |
| $f_\phi$ | 245       | 336  | 259  |       |       | 270.8(6.5)  |             |                 |

* derived from the experimental value for $\Gamma_\rho^{\mu+e^-}$.
** derived from the experimental value for $\Gamma_\tau^{\mu+e^-}$.
† derived from the experimental value for $\Gamma_{\tau^{K^*\mu+e^-}}$.
‡ derived from the experimental value for $\Gamma_{\phi^{ee}}$.

expression (20), as well as the values of different contributions $f_M^{(1,2,3,4)}$ (17)–(19) and the complete relativistic values of $f_M$ (18) are given. In Table III we compare our results for the decay constants $f_M$ of light mesons with other quark model predictions [10, 11, 12, 16], recent values from two- [2] and three-flavour [3] lattice QCD and available experimental data [17]. It is clearly seen that the nonrelativistic predictions are significantly overestimating all decay constants, especially for the pion (almost by a factor of 10). The partial account of relativistic corrections by keeping in Eq. (16) only the first term $f_M^{(1)}$ (17), which is usually used for semirelativistic calculations, does not substantially improve the situation. The disagreement is still large. This is connected with the anomalously small masses of light pseudoscalar mesons exhibiting their chiral nature. In the semirelativistic quark model [10] the pseudoscalar meson mass is replaced by the so-called mock mass $M_\pi$, which is equal to the mean total energy of free quarks in a meson, and with our wave functions: $M_\pi = 2\langle \epsilon_1(p) \rangle \approx 1070$ MeV ($\sim 8M_\pi$) and $M_K = \langle \epsilon_q(p) \rangle + \langle \epsilon_s(p) \rangle \approx 1232$ MeV ($\sim 2.5M_K$). Such replacement yields values of $f_P^{(1)}$ which are still $\approx 1.4$ times larger than experimental ones (cf. [10]). As we see from Table III it is not justified to neglect contributions of the negative energy intermediate states for light meson decay constants. Indeed, the values of $f_M^{(2+3)} + f_M^{(4)}$ are large and negative (reaching $-76\%$ of $f_\pi^{(1)}$ for the pion) thus compensating the overestimation of decay constants by the positive-energy contribution $f_M^{(1)}$. This is the consequence of the smallness of the light pseudoscalar meson masses compared to the energies of their constituents. The negative-energy contributions [18], [19] are proportional to the ratio of the meson binding energy $M - \epsilon_1(p) - \epsilon_2(p)$ to its mass and quark energies. For mesons with heavy quarks this factor is small and leads to the suppression of negative-energy contributions. This results in the dominance of the positive-energy term $f_M^{(1)}$. Indeed the negative-energy terms for heavy-light $D$ and $B$ mesons give $10 - 15\%$ contributions (see

and vector decay constants in this limit results from the difference of the corresponding relativistic wave functions.

5 In our model $\rho$ and $\omega$ mesons are degenerate, therefore their decay constants are equal. The experimental value for the decay constant of the $\omega$ meson, derived from $\Gamma_{\omega^{\mu+e^-}}$ [17], is $f_\omega = 195(3)$ MeV.
TABLE IV: Pseudoscalar and vector decay constants of heavy mesons (in MeV).

| Constant | Quark models | Lattice QCD | QCD sum rules | Experiment |
|----------|--------------|-------------|---------------|------------|
|          | this work [15] |  [2] | [4, 5] | [7] | [8] | [9] | [17, 18] |
| \( f_D \) | 234 | 230(25) | 225(14)(40) | 201(3)(17) | 203(20) | 195(20) | 222.6(16.7)(3^8_{14}) |
| \( f_{D_s} \) | 268 | 248(27) | 267(13)(48) | 249(3)(16) | 235(24) | 266(32) |
| \( f_{D_s}/f_D \) | 1.15 | 1.08(1) | 1.24(1)(7) | 1.15(4) |
| \( f_B \) | 189 | 196(29) | 208(10)(29) | 216(9)(19)(6) | 203(23) | 206(20) | 210(19) |
| \( f_{B_s} \) | 218 | 216(32) | 250(10)(35) | 259(32) | 236(30) | 244(21) |
| \( f_{B_s}/f_B \) | 1.15 | 1.10(1) | 1.20(3)(1) | 1.16(4) | 1.16 |

Table IV which have the typical magnitude of the heavy quark corrections. This explains the closeness of the obtained values of constants to our previous results [14]. On the other hand, for light mesons, especially for the pion and kaon, the binding energies are not small on the meson mass and quark energy scales and, thus, such factor gives no suppression. The complete relativistic expression [10] for decay constants \( f_M \) brings theoretical predictions for light mesons in good agreement with available experimental data.

The comparison of our values of the decay constants of light mesons with other predictions in Table III indicate that they are competitive even with the results of more sophisticated approaches (e.g. [11]) which are based on the Dyson-Schwinger and Bethe-Salpeter equations. On the other hand, our model is more selfconsistent than some other approaches [10, 16]. We calculate the meson wave functions by solving the quasipotential equation in contrast to the models based on the relativistic Hamilton dynamics [16] where various ad hoc wave function parameterizations are employed.

In Table IV we confront our results for pseudoscalar decay constants of the heavy-light mesons as well as their ratios with the recent predictions based on the Salpeter equation [15], values from the unquenched two- [2] and three-flavour [4, 5] lattice QCD, QCD sum rules [7, 8, 9] and available experimental data [17]. Reliable experimental data, up till recently, existed only for \( f_{D_s} \), which was measured by several experimental collaborations (ALEPH, DELPHI, L3, OPAL, Beatrice, CLEO, E653, WA75, BES) both in the \( D_s \rightarrow \mu \nu \) and the \( D_s \rightarrow \tau \nu \) decay channels. At present, experimental errors are still rather large for this constant. Very recently, the CLEO Collaboration [18] published a relatively precise value for the decay constant \( f_D \) measured in \( D \rightarrow \mu \nu \) decay. We see from Table IV that there is a good (within error bars) agreement between all presented theoretical predictions as well as with available experimental data.

In summary, the weak decay constants of pseudoscalar and vector light and heavy-light mesons were investigated with the special emphasize on the role of relativistic effects. For our calculations we used the meson wave functions which were obtained by the numerical solution of the quasipotential equation with the nonperturbative treatment of all spin-dependent and spin-independent relativistic contributions to the quark interaction potential. It was argued that both positive- and negative-energy parts of the quark propagators in the weak annihilation loop should be taken into account. The positive-energy contributions, which are

6 The recent quenched lattice QCD values [6] for the pseudoscalar decay constants are \( f_K = 152(6)(10) \) MeV, \( f_D = 235(8)(14) \) MeV and \( f_{D_s} = 266(10)(18) \) MeV.
usually considered in the semirelativistic quark models, significantly overestimate the decay constants of light mesons. We showed that the negative-energy contributions to the light meson decay constants are large and negative. Their account is necessary to bring theoretical predictions in agreement with experimental data. On the other hand, these negative-energy contributions are considerably smaller for decay constants of heavy-light mesons and have the order of magnitude of the lowest correction in the heavy quark expansion. The consistent inclusion of relativistic effects coming both from the quark propagators and the meson wave functions considerably improve the accuracy and reliability of the obtained predictions.

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