Gauge fixing in Higher Derivative Field Theories *

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Abstract

Higher Derivative (HD) Field Theories can be transformed into second order equivalent theories with a direct particle interpretation. In a simple model involving abelian gauge symmetries we examine the fate of the possible gauge fixings throughout this process. This example is a useful test bed for HD theories of gravity and provides a nice intuitive interpretation of the "third ghost" occurring there and in HD gauge theories when a HD gauge fixing is adopted.

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Introduction

Higher Derivative (HD) field theories arise as effective theories in several contexts. Perhaps, the best known example is gravitation, where higher order terms in the curvatures arise from an underlying fundamental string dynamics or from quantizing matter fields in a curved space-time background. The study of the actual dynamical degrees of freedom (d.o.f) of such theories has been faced most fruitfully by bringing them to a lower derivative (LD) equivalent version of second order by means of a Legendre transformation \[1\]. More recently, progress has been made towards a complete diagonalization of these d.o.f in models with quadratic terms in the scalar and Ricci curvatures \[2\].

However in all these studies only invariant theories under general coordinate transformations have been considered which, as such, are unsuited for quantization. In fact, the Green’s functions for the equations of motion are undefined because of the Diff-invariance. The way out of this difficulty in gauge theories is to break the local symmetry by introducing a Gauge Fixing (GF) term, which can be now of the LD or HD type according to computational convenience. Beside the usual gauge ghosts, HD gauge fixings introduce a more subtle “third ghost” \[3\], which calls for further compensating Faddeev-Popov ghosts. Once the GF term has been added to the HD theory the question arises of how does it translate through the Legendre transform down to the LD equivalent theory where the particle contents is apparent. This may also provide an intuitive picture for the third ghost.

In this paper we explore these questions by using a simple HD abelian gauge theory as a test bed. In Section 1 we introduce the model and point out the nature of the states it contains by studying the (tree approximation) propagator. We then perform the Legendre transform in Section 2 to end up with the equivalent Helmholtz LD theory. In Section 3 we test the reliability of the formal covariant treatment by working out explicitly the actual d.o.f of the radiating field. In section 4 we outline the fate of the gauge symmetry(ies) along the process, which is more intriguing when only a LD GF is adopted. Finally we draw some conclusions and consequences of this study. Some details complementary to ref.[4] are given in an Appendix together with the notations used. Through this paper 4D space-time with Lorentzian signature (+ - - - ) is considered.
1 The Model

We consider the simplest fourth-derivative abelian gauge theory quadratic in the gauge fields obtained by extrapolating the QED Lagrangian in a natural way

$$\mathcal{L}_{HD} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4m^2} F_{\mu\nu} \Box F^{\mu\nu} - \frac{\zeta^2}{2} (\partial_\mu A^\mu)^2 - \frac{\zeta^2}{2M^2} (\partial_\mu A^\mu) \Box (\partial_\nu A^\nu) - j_\mu A^\mu$$  \hspace{1cm} (1)

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Here $m^2$ is a dimensional parameter, $\zeta$ and $M^2$ are dimensionless and dimensional gauge parameters respectively.

For $\zeta = 0$ the theory is invariant under the U(1) gauge transformations

$$\delta A_\mu = \partial_\mu \lambda (x)$$  \hspace{1cm} (2)

provided that the source $j^\mu$ is conserved, namely if

$$\partial_\mu j^\mu = 0$$  \hspace{1cm} (3)

The first term in (1) is reminiscent of $\sqrt{-g} R$ in the gravity case, whereas the second term is reminiscent of $\sqrt{-g} R^2$ and $\sqrt{-g} R_{\mu\nu} R^{\mu\nu}$ (usually a term of the form $\sqrt{-g} R^\alpha_{\mu\nu\rho} R^\rho_{\alpha\nu}$ is not considered because of the Gauss-Bonnet identity).

Though consisting of a decoupled sector in the Abelian case, it may be instructive to consider the Faddeev-Popov (FP) Lagrangian $\mathcal{L}_{FP}$ for the gauge fixing in (1). Together with the gauge-breaking terms it can be expressed as a coboundary in the BRS cohomology

$$\mathcal{L}_g = \bar{\delta} \bar{c} (1 + \frac{\Box}{M^2}) \partial_\mu A^\mu + \frac{1}{2\zeta^2} \bar{c} (1 + \frac{\Box}{M^2}) B'$$  \hspace{1cm} (4)

where the BRS transformation $\bar{\delta}$ is defined by

$$\delta A_\mu = \partial_\mu c$$
$$\bar{\delta} c = 0$$
$$\bar{\delta} \bar{c} = B'$$
$$\delta B' = 0$$

Ghost numbers and mass dimensions are the usual ones:

$$gn(c) = -gn(\bar{c}) = 1, gn(A) = gn(B) = 0; \hspace{0.5cm} [c] = 0, [A] = 1, [\bar{c}] = [B] = 2.$$
Upon the redefinition of the auxiliary field

\[ B' = B - \zeta^2 \partial_\mu A \]

Eq.(4) gets the usual diagonalized form

\[ \mathcal{L}_g = -\zeta^2 \left( \partial_\mu A^\mu (1 + \frac{\Box}{M^2} \partial_\nu A^\nu) - \bar{\epsilon} (1 + \frac{\Box}{M^2}) \Box c + \frac{1}{2\zeta^2} B (1 + \frac{\Box}{M^2}) B \right) \]

where the first term yields the gauge-breaking part in (1), and the last two will be referred to as \( \mathcal{L}_{FP} \) and \( \mathcal{L}_B \) respectively in the following. We now go on with the sector (1) of the theory and come on \( \mathcal{L}_{FP} \) and \( \mathcal{L}_B \) later.

Dropping total derivatives, (1) can be written in the more convenient form

\[ \mathcal{L}_{HD} = \frac{1}{2m^2} A^\mu \Box \left( \Box + m^2 \right) \theta_{\mu\nu} A^\nu + \frac{\zeta^2}{2M^2} A^\mu \Box \left( \Box + M^2 \right) \omega_{\mu\nu} A^\nu - j_\mu A^\mu \]

where

\[ \omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\Box} \]

\[ \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\Box} \]

are the longitudinal and transverse projectors respectively. They are a complete set of orthogonal projectors in the gauge field space. Their properties

\[ \theta_{\mu\nu} + \omega_{\mu\nu} = \eta_{\mu\nu} \]

\[ \theta_{\mu\rho} \theta^\rho_\nu = \theta_{\mu\nu} \]

\[ \omega_{\mu\rho} \omega^\rho_\nu = \omega_{\mu\nu} \]

\[ \theta_{\mu\nu} \omega^\rho_\nu = 0 \]

are fully exploited in what follows.

The form given in (6) is specially suited to work out the gauge field propagator which formally reads

\[ \Delta_{\mu\nu} = \theta_{\mu\nu} \frac{m^2}{\Box (\Box + m^2)} + \omega_{\mu\nu} \frac{1}{\zeta^2 \Box (\Box + M^2)} M^2 \]
and lends itself to a direct reading of the particle contents. In fact (9) may be rewritten as

\[
\Delta_{\mu\nu} = \theta_{\mu\nu} \left( \frac{1}{\Box} - \frac{1}{\Box + m^2} \right) + \omega_{\mu\nu} \frac{1}{\zeta^2} \left( \frac{1}{\Box} - \frac{1}{\Box + M^2} \right) .
\]

(10)

Thus the theory propagates a transverse massless vector field (a photon accounting for 2 d.o.f) and a massive negative-norm transverse vector (a "poltergeist" with 3 d.o.f). Beside this, one has a massless longitudinal d.o.f (the "gauge ghost") and a massive longitudinal "poltergeist" d.o.f (the "third ghost").

Unlike the second order theories, we see from (5) that also the FP ghosts \( \bar{c}, c \) actually propagate two d.o.f:

\[
\Delta_c = \frac{1}{\Box} - \frac{1}{\Box + M^2} ,
\]

and the auxiliary field B now describes a propagating massive d.o.f

\[
\Delta_B = \frac{\zeta^2 M^2}{\Box + M^2} .
\]

The aim of the Legendre transform is to provide a 2nd-derivative equivalent theory with explicit independent fields for the positive-norm and the poltergeist d.o.f’s. Before doing this we simplify the notation by omitting the indices and write (6) as

\[
\mathcal{L}_{HD} = \frac{1}{2} A \Box (\theta + \zeta^2 \omega) A + \frac{1}{2} A \Box \left( \frac{\theta}{m^2} + \frac{\omega}{M^2} \right) (\theta + \zeta^2 \omega) A - jA
\]

(11)

By defining

\[
\hat{A} = (\theta + \zeta \omega) A
\]

(12)

and making further use of (8), Eq.(11) can be brought to the final condensed form

\[
\mathcal{L}_{HD}[\hat{A}, \Box \hat{A}] = \frac{1}{2} \hat{A} \Box \hat{A} + \frac{1}{2} \hat{A} \Box \left( \frac{\theta}{m^2} + \frac{\omega}{M^2} \right) \hat{A} - j(\theta + \zeta \frac{\omega}{\zeta}) \hat{A} .
\]

(13)
2 Legendre Transform and Helmholtz Lagrangian

In a general HD theory \( \mathcal{L}[\varphi, \partial \varphi] \) the problem arises of finding a function \( f[\partial \varphi] \) of field derivatives of various orders which is suitable to define a canonical conjugate variable

\[
\pi = \frac{\partial \mathcal{L}}{\partial f[\partial \varphi]}. \tag{14}
\]

The condition that this equation be invertible (hyper-regular systems), namely that \( f[\partial \varphi] \) can be worked out as a function of \( \varphi \) and \( \pi \), usually allows just one choice for \( f[\partial \varphi] \). In our case the unambiguous choice is the object \( \hat{\square} \hat{A} \) and (13) has been prepared accordingly. One finds

\[
\hat{\pi} = \frac{\partial \mathcal{L}}{\partial \hat{\square} \hat{A}} = \frac{1}{2} \hat{A} + \left( \frac{\theta}{m^2} + \frac{\omega}{M^2} \right) \hat{\square} \hat{A} \tag{15}
\]

from which

\[
\hat{\square} \hat{A} = (m^2 \theta + M^2 \omega) (\hat{\pi} - \frac{1}{2} \hat{A}) \equiv F[\hat{A}, \hat{\pi}] \tag{16}
\]

The Hamiltonian function is then

\[
\mathcal{H}[\hat{A}, \hat{\pi}] = \hat{A} F[\hat{A}, \hat{\pi}] - \mathcal{L}_{HD}[\hat{A}, F[\hat{A}, \hat{\pi}]]
\]

\[
= \frac{1}{2} \left( \frac{1}{2} \hat{A} - \hat{\pi} \right) (m^2 \theta + M^2 \omega) \left( \frac{1}{2} \hat{A} - \hat{\pi} \right) + j (\theta + \omega) \hat{A} \tag{17}
\]

The canonical equations of motion for \( \hat{A} \) and \( \hat{\pi} \) are the system of (2nd order) equations

\[
\begin{align*}
\hat{\square} \hat{A} &= \frac{\partial \mathcal{H}}{\partial \hat{\pi}} \\
\hat{\square} \hat{\pi} &= \frac{\partial \mathcal{H}}{\partial \hat{A}} 
\end{align*} \tag{18}
\]

which is equivalent to the Euler’s equation from (13). However both Eqs.(18) can also be derived by a variational principle from the so called Helmholtz Lagrangian

\[
\mathcal{L}_H[\hat{A}, \hat{\pi}] = \hat{\pi} \hat{\square} \hat{A} - \mathcal{H}[\hat{A}, \hat{\pi}] \tag{19}
\]
It depends on $\hat{A}, \hat{\pi}$ and derivatives of $\hat{A}$. The first (derivative) term looks like a kinetic one while the terms in $\mathcal{H}[\hat{A}, \hat{\pi}]$ are of the mass-term type, the problem being that $\hat{A}$ and $\hat{\pi}$ occur mixed. The diagonalization can be trivially performed by defining new tilde fields such that

$$\begin{align*}
\hat{A} &= \tilde{A} + \tilde{\pi} \\
\hat{\pi} &= \frac{1}{2}(\tilde{A} - \tilde{\pi})
\end{align*}$$  

(20)

Eq.(19) becomes

$$\begin{align*}
\mathcal{L}_H[\tilde{A}, \tilde{\pi}] &= \frac{1}{2} \tilde{A} \Box \tilde{A} - \frac{1}{2} \tilde{\pi}[(\Box + m^2)\theta + (\Box + M^2)\omega]\tilde{\pi} \\
&\quad - j(\theta + \frac{\omega}{\zeta})(\tilde{A} + \tilde{\pi})
\end{align*}$$  

(21)

In terms of fields that couple directly to the source, namely

$$\begin{align*}
\mathcal{A} &= (\theta + \frac{\omega}{\zeta})\tilde{A} \\
\Pi &= (\theta + \frac{\omega}{\zeta})\tilde{\pi}
\end{align*}$$  

(22)

Eq.(21) finally gives the desired LD theory

$$\begin{align*}
\mathcal{L}_{LD}[\mathcal{A}, \Pi] &= \frac{1}{2} \mathcal{A} \Box \mathcal{A} - \frac{1}{2} \Pi(\Box + m^2)\theta\Pi + \frac{\zeta^2}{2} \mathcal{A} \Box \omega \mathcal{A} - \frac{\zeta^2}{2} \Pi(\Box + M^2)\omega\Pi \\
&\quad - j(\mathcal{A} + \Pi)
\end{align*}$$  

(23)

The physical meaning is now apparent. Whenever the source emitted a “particle” $A_\mu$ with propagator (10) (or the effective quartic version (9)), on the same line it actually emits a massless transverse particle $A_\mu$ with propagator

$$\theta_{\mu\nu} \frac{1}{\Box},$$  

(24)

a massless longitudinal (gauge ghost) state of $A_\mu$ with propagator

$$\omega_{\mu\nu} \frac{1}{\zeta^2 \Box^2}$$  

(25)
and a massive transverse poltergeist $\Pi_\mu$ with propagator

$$-\theta_{\mu\nu} \frac{1}{\Box + m^2}$$

(26)

together with a massive longitudinal ghost state (or "third ghost") of $\Pi_\mu$ with propagator

$$-\omega_{\mu\nu} \frac{1}{\zeta^2 \Box + M^2} \frac{1}{\Box + M^2}$$

(27)

All of this amounts to the “joint” propagator in (10).

The better asymptotic behaviour of the propagator (9) shows also that the poltergeists can be viewed as regulator fields for an otherwise LD gauge theory.

One should finally notice that the LD Lagrangian (23) contains the non-local term

$$\frac{1}{2}(m^2 - \zeta^2 M^2)\Pi_\mu \partial_\mu \partial_\nu \Pi_\nu$$

(28)

which can be made to vanish by suitably choosing the gauge parameters.

Let us now come to $\mathcal{L}_{FP}$ and $\mathcal{L}_B$. While the latter is already of second order, $\mathcal{L}_{FP}$ is higher-derivative and would in principle deserve the same treatment above. However, $\bar{c}$ and $c$ being independent fields, a Legendre transform cannot be carried out. In any case the eventual diagonalization of the d.o.f described by $\mathcal{L}_{FP}$ is irrelevant in Abelian theories where the FP ghosts are decoupled from the physical sector. Even in the non-Abelian case the massive FP d.o.f does not couple to the physical sector as long as gauge-breaking terms of the type displayed in (1) are considered. In fact $\mathcal{L}_{FP}$ is then

$$-\bar{c}(1 + \frac{\Box}{M^2})\partial_\mu D^\mu c$$

and the (field-independent) operator $1 + \frac{\Box}{M^2}$ can be absorbed by a redefinition of the antighost $\bar{c}$, factorizing a constant functional determinant in Path Integral quantization.

However we may consider more general gauges of the form

$$(\partial_\mu A^\mu)(1 + \frac{\Box}{M^2} + f(A))(\partial_\nu A^\nu)$$

(29)
where \( f(A) \) is a function of the quantum gauge field and/or generally of background fields, so that also the massive FP ghost d.o.f couples to the gauge field. Then the (propagating) auxiliary field \( B \) gets coupled to both the gauge and the FP fields as well.

## 3 Physical degrees of freedom

We perform here a canonical analysis of the phase space along the lines of ref.[4].

We consider now the non FP sector of the model and assume the conservation of the matter source. Then (13) describes a higher-derivative theory for the four functional d.o.f in \( \hat{A} \) (notice that they are the same d.o.f contained in \( A \) as long as \( \zeta \neq 0 \)).

Already in the higher derivative version the theory can be seen to contain less physically meaningful configuration d.o.f than the eight d.o.f one could expect from the doubling (caused by the fourth differential order of the theory) of the four quoted above.

Consider first the equation of motion stemming from (1), namely

\[
\Box (1 + \frac{\Box}{M^2}) A^\mu - [(1 - \zeta^2) + \Box (\frac{1}{m^2} - \frac{\zeta^2}{M^2})] \partial^\mu (\partial_\nu A^\nu) = j^\mu
\]

(30)

Taking the divergence of both sides one gets

\[
\Box (1 + \frac{\Box}{M^2}) \partial_\nu A^\nu = 0
\]

(31)

which shows that \( \partial_\nu A^\nu \) actually describes two decoupled free scalars (one massless and one with mass \( M \)), already identified in (10). They amount to two configuration variables or equivalently to four phase-space variables.

On the other hand, a further (non-covariant) d.o.f can be absorbed by a redefinition of the matter fields. This can be seen by writing (1) in an equivalent lower-order form in phase-space variables which is the analogous of the first order (43) for the (second order) QED. To simplify matters we get rid of the scalar d.o.f above by considering only the Lorentz-transverse part of \( A \). This is accomplished by omitting the gauge fixing terms and remembering that \( \partial_\nu A^\nu = 0 \) when necessary. Thus we consider the Lagrangian

\[
\mathcal{L}^{(4)} = -\frac{1}{4} F_{\mu\nu} \vec{\partial} F^{\mu\nu} - j_\mu A^\mu
\]
\[
\frac{1}{2}(-\partial_0 \vec{A} - \vec{\nabla} A^0)\bar{\psi} (-\partial_0 \vec{A} - \vec{\nabla} A^0) - \frac{1}{2} (\vec{\nabla} \times \vec{A})\bar{\psi} (\vec{\nabla} \times \vec{A}) - j_\mu A^\mu
\]

where \( \bar{\psi} \) stands for the Klein-Gordon operator \((1 + \frac{\Box}{m^2})\) and the remaining notations are given in the Appendix.

Then the lower-order Lagrangian is

\[
L^{(3)} = \vec{E} \bar{\psi} (-\partial_0 \vec{A} - \vec{\nabla} A^0) - \frac{1}{2} [\vec{E} \bar{\psi} \vec{E} + (\vec{\nabla} \times \vec{A})\bar{\psi} (\vec{\nabla} \times \vec{A})] - A^0 \rho + \vec{A} j
\]

Notice that solving the equation of motion for \( \vec{E} \) yields the same result (41) so that recovering (32) from (33) is trivial.

Now the three-vectors in (33) can be decomposed in (3-space) longitudinal and transverse parts, namely

\[
L^{(3)} = -\vec{E}_T \bar{\psi} \partial_0 \vec{A}_T - \vec{E}_L \bar{\psi} \partial_0 \vec{A}_L + A^0 (\bar{\psi} \vec{\nabla} \vec{E}_L - \rho)
\]

Thus \( A^0 \) yields a constraint that can be solved giving

\[
\vec{E}_L = \bar{\psi}^{-1} \Delta^{-1} \vec{\nabla} \rho
\]

Then the term

\[
-\vec{E}_L \bar{\psi} \partial_0 \vec{A}_L = - (\Delta^{-1} \vec{\nabla} \rho) \partial_0 \vec{A}_L
\]

can be absorbed by the same redefinition of the fermion fields given in ref.[4], whilst one has

\[
\vec{E}_L \bar{\psi} \vec{E}_L = - \rho \bar{\psi}^{-1} \Delta^{-1} \rho
\]

Because of the occurrence of the differential operator \( \bar{\psi} \), it is not easy to read the remaining actual d.o.f directly out of (34). This analysis just shows that the phase-space is further deprived of two (non-covariant) variables.

To work out the remaining phase space we go back momentarily to the covariant treatment and perform first the Legendre transformation that leads to (19) and then to (21), though we already know that they contain some physically irrelevant d.o.f. The diagonalization (20) just rearranges the d.o.f without altering them, and for a conserved source there is no need of further field redefinitions.
Now we consider the $\tilde{A}$ and $\tilde{\pi}$ sectors of (21) (equation (23) is equally suited, so the choice is a matter of taste). Both fields feature the well-known Lorentz-longitudinal parts that are decoupled from the conserved matter source. Thus for the massless field $\tilde{A}$ one is left with its Lorentz-transverse part, described by an ordinary gauge-invariant Lagrangian as in (40). In ref.[4] the constraint was solved showing that one finally has the two d.o.f of a photon. For the remaining massive transverse poltergeist $\tilde{\pi}$ the analogous analysis is even simpler. We discuss this case in the Appendix where we will find three d.o.f as expected.

We finally stress that when the source is not conserved, as it would be the case of a non-Abelian quantum theory, the two Lorentz-longitudinal parts do couple to the matter, but this is compensated by the (higher-derivative) FP sector of the theory.

## 4 Gauge invariance and gauge fixings

For $\zeta = 0$ and conserved source the starting HD theory (1) is exactly invariant under the $\text{U}(1)$ gauge transformations (2).

For arbitrary $\zeta$ the variation (2) induces the following ones in the intermediate field variables:

$$
\delta \hat{A}_\mu = \zeta \partial_\mu \lambda \\
\delta \hat{\pi}_\mu = \zeta (\frac{1}{2} + \frac{\Box}{m^2}) \partial_\mu \lambda \\
\delta \tilde{A}_\mu = \zeta (1 + \frac{\Box}{m^2}) \partial_\mu \lambda \\
\delta \tilde{\pi}_\mu = -\frac{\zeta}{m^2} \Box \partial_\mu \lambda 
$$

(35)

and in the final variables

$$
\delta A_\mu = (1 + \frac{\Box}{m^2}) \partial_\mu \lambda \\
\delta \Pi_\mu = -\frac{\Box}{m^2} \partial_\mu \lambda 
$$

(36)

so that

$$
\delta (A_\mu + \Pi_\mu) = \partial_\mu \lambda
$$

(37)

For $\zeta = 0$ the final LD theory (23) is therefore invariant under the induced variations (36). Notice that the symmetric limit poses no problem with the potentially troublesome source terms in (13),(17) and (21). Then they
actually are $j\hat{A}$, $\tilde{j}\hat{A}$ and $j(\tilde{A} + \tilde{\pi})$ respectively because of source conservation, and the redefinition (22) is not needed. However the theory is also invariant under the independent variations

$$
\begin{align*}
\delta A_\mu & = \partial_\mu \Lambda \\
\delta \Pi_\mu & = \partial_\mu \Lambda'
\end{align*}
$$

(38)

This shows that in the LD theory one actually has a larger accidental $U(1) \times U'(1)$ symmetry, which is hidden in the HD theory. If the matter source is not conserved, the diagonal subgroup of transformations

$$
\begin{align*}
\delta A_\mu & = \partial_\mu \Lambda \\
\delta \Pi_\mu & = -\partial_\mu \Lambda
\end{align*}
$$

(39)

still survives. The original symmetry, as given by (36), appears also as a subgroup.

However one may choose to put only the HD GF term in (1) off. This is accomplished by taking the limit $M \to \infty$. What happens at the level of the propagators is now clear in either (9) or (10): the propagation of the "third ghost", of the massive FP d.o.f, and of the $B$ field fade away. This is equivalent to suppressing the term corresponding to the third ghost in (23). In that case the symmetry $U'(1)$ is preserved.

We conclude that the LD and HD GF terms are associated to the breaking of the $U'(1)$ and $U(1)$ symmetries respectively. However we cannot adopt a pure HD GF because then the Legendre transform becomes singular, as it does happen for a theory with only HD terms.

These results are of quite a limited relevance since they are characteristic to the Abelian theory. In fact the model we have considered describes a theory which, except for a spectator interaction with external source fields, is essentially free and hence trivial. Because of the occurrence of self-interactions, the second “hidden” symmetry is absent as soon as a non-Abelian generalization of the model is considered. However the results above still hold for the trivial Abelian symmetry of the free parts.
5 Conclusions

From a HD theory of one vector field $A_\mu$ with quartic propagator we have obtained an equivalent LD theory with quadratic propagators for positive norm and poltergeist states. Except for the gauge fixing the original theory had a U(1) gauge symmetry.

Exploring the symmetries of the LD theory we have deduced that the possible LD and HD gauge fixing terms in the HD theory actually fix separate $U(1)$ and $U'(1)$ hidden symmetries of the free theory. The family of compensating Faddeev-Popov ghosts includes, also in the non-Abelian case, a further massive FP d.o.f and a propagating massive commuting field. The loss of unitarity due to the negative-norm poltergeists is instead unavoidable at this level.

The above procedures extrapolate naturally to the theory of gravitation with little modification up to technical details. Beside the graviton there we have a massive spin-two poltergeist, and a physical scalar field in the HD Diff-invariant theory together with a richer set of orthogonal projectors. They include a spin one and a further scalar components when the gauge is fixed. The findings above regarding the symmetries of the free parts, may help to understand how a Fierz-Pauli kinetic term arises for the massive spin-two poltergeist in the LD equivalent theory. Work is in progress in that direction.

The example we have worked out in this paper is interesting also by itself: it contains a sort of regularization that allows an elegant proof of the Adler-Bardeen theorem. In fact this regularization, extended to the fermion propagators, works for the potentially anomalous graphs of two or more loops and preserves the chiral gauge invariance.

Appendix

We use the notations

\[ j^\mu = (\rho, \vec{j}) \]
\[ A^\mu = (A^0, \vec{A}) \]
\[ \partial_\mu = (\partial_0, \vec{\nabla}) \]
\[ \Delta \equiv \nabla^2 \]
\[ \Box = \partial_0^2 - \Delta \]
The (second order) electromagnetic Lagrangian
\[ L^{(2)} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu A^\mu \]
\[ = \frac{1}{2} \left[ (-\partial_0 \vec{A} - \nabla A^0)^2 - (\vec{\nabla} \times \vec{A})^2 \right] - j_\mu A^\mu \]  
(40)
can be cast in a first order equivalent form by a Legendre transformation. In fact one may define the conjugate variable
\[ -\vec{E} = \frac{\partial L^{(2)}}{\partial \partial_0 \vec{A}} = \partial_0 \vec{A} + \vec{\nabla} A^0 \]  
(41)
The Hamiltonian is then
\[ H = \frac{1}{2} \vec{E}^2 + \vec{E} \vec{\nabla} A^0 + (\vec{\nabla} \times \vec{A})^2 + j_\mu A^\mu \]  
(42)
so that the first order (Helmholtz) Lagrangian becomes
\[ L^{(1)} = \vec{E} (-\partial_0 \vec{A} - \vec{\nabla} A^0) - \frac{1}{2} \left[ \vec{E}^2 + (\vec{\nabla} \times \vec{A})^2 - m^2 (A^2_0 - \vec{A}^2) \right] - j_\mu A^\mu \]  
(43)
which is equation (15a) in ref.[4], with \( \vec{B} = \vec{\nabla} \times \vec{A} \). Equation (40) can be recovered by solving the equation of motion for \( \vec{E} \) in (43), obtaining (41), and then substituting it back in (43). The field \( A^0 \) gives rise to a constraint that can be solved as shown in ref.[4]. This eliminates \( \vec{A}_L \) and \( \vec{E}_L \) from the radiating field so that only the two d.o.f of the photon survive.

The massive case can be treated along the same lines. The second order Proca Lagrangian
\[ L^{(2)}_m = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu - j_\mu A^\mu \]  
(44)
only adds non-derivative terms to the massless case above and contains also a decoupled Lorentz-longitudinal d.o.f. Now the first order Lagrangian is
\[ L^{(1)}_m = \vec{E} (-\partial_0 \vec{A} - \vec{\nabla} A^0) - \frac{1}{2} \left[ \vec{E}^2 + (\vec{\nabla} \times \vec{A})^2 - m^2 (A^2_0 - \vec{A}^2) \right] - A^0 \rho + \vec{A}^\mu \vec{j} \]  
(45)
Again the time derivative of \( A^0 \) is absent but this does not yield a constraint. In fact, dropping total derivatives, the terms containing \( A^0 \) are
\[ A^0 (\vec{\nabla} \vec{E}_L - \rho) + \frac{m^2}{2} A^0^2 \]  
(46)
Then $A^0$ can be solved in terms of $\bar{E}_L$ and $\rho$. Substituting it back in (45) one obtains
\[
\mathcal{L}^{(1)}_m = -\bar{E}_0 \bar{A} - \frac{1}{2m^2}(\nabla \bar{E}_L - \rho)^2 - \frac{1}{2}(\bar{E}^2 + (\nabla \times \bar{A})^2 - m^2 \bar{A}^2) + \bar{A}^j_j \quad (47)
\]
One sees that now $\bar{E}_L$ is an independent field and $\bar{A}_L$ cannot be absorbed into a redefinition of the fermion fields in the matter source. Thus the space-longitudinal d.o.f survives and consequently we are left with 3 d.o.f.

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**References**

[1] G. Magnano, M. Ferraris, M. Francaviglia, Gen. Rel. Grav. **19** (1987) 465.
A. Jakubiec, J. Kijowsky, Phys. Rev. **D37** (1988) 1406.
G. Magnano, M. Ferraris, M. Francaviglia, J. Math. Phys. **31** (1990) 378.

[2] J. C. Alonso, J. Julve, “Particle contents of Higher Order Gravity”, to be published in the Proceedings of the First Iberian Meeting on Gravity (Evora, Portugal, 1992), World Sci. Pub.
J. C. Alonso, F. Barbero, J. Julve, A. Tiemblo, “Particle contents of Higher Derivative Gravity”, preprint IMAFF 93/9. To appear in Class. Quantum Grav.

[3] E. S. Fradkin, A. A. Tseytlin, Nucl. Phys. **B201** (1982) 469.

[4] L. Faddeev, R. Jackiw, Phys. Rev. Lett. **60** (1988) 1692.

[5] R. Jackiw, S. B. Treiman, D. J. Gross, “Lectures on Current Algebra and its applications”, Princeton Univ. Press 1972.
J. C. Taylor, “Gauge Theories of Weak Interactions”, Cambridge Univ. Press 1976.
[6] A.A. Slavnov, Teor. Math. Phys. 13 (1972) 1064.