Transient thermal analysis of brake disc in c++

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Abstract

This work examines the development of a C++ program to analyze one dimensional thermal conduction for asymmetrical domain using Finite Element Method in C++ environment. Finite Element Library has been used to incorporate various pre-defined Finite Element classes. The various classes have been further grouped into several modules which serve as the fundamental units of any program that encapsulated Finite Element Method. The work strives to illuminate the abstraction behind the computation of the desired unknown solution to a complex real-world problem by capturing the physics of heat conduction and highlights the mathematics required to arrive at an understandable transient solution, which adheres to the conditions of stability, and thus can be relied upon to be implemented in various automotive engineering applications. It also identifies the underlying mechanism through which simple one-dimensional and multi-dimensional partial differential equation problem are generally solved. The elliptic, classic form of partial differential equation has been developed in the form of C++ code. The development begins with elliptic partial differential equations in multi-dimension dimension for steady state heat conduction and then is expanded to included time discretization, which is requirement of a time-dependent problem. Then output of the code is compared to the solution is validated iteratively so as to find the critical time step size for stability, which also validates whether the code actually works for different numerical techniques such as Forward Difference, Backward Difference and the Crank-Nicholson Method, all three distinguished by certain value of a common parameter. The C++ code is both flexible as well as expandable and can be thought of as a mathematical model to solve time-dependent heat conduction problem.

Keywords: Automobile, Analysis, Thermal conduction, programming

1. Introduction

Veiga et al explained the method allows one to solve complicated differential equations. These differential equations capture several physical phenomena. Differential equations (either partial or ordinary) are solved for the unknown field, such as displacement field, heat transfer field, magnetic field etc. [1] In order to solve these complex differential equations, there is a need to integrate these linear or non-linear differential equations, which may be ordinary or partial. Finding the exact solution of complex functions is not easy, hence we try to find an approximate solution using approximation methods such as FEM or Finite Difference Method (FDM). FEM is used as it does not matter what kind of governing equation is being dealt with. Same method can be used to solve a transient heat conduction problem as well as elasticity problem. Simoes et al described a trial solution is substituted back into the equation and the respective boundary/initial conditions are imposed to convert the problem into a simple one,
namely into Ordinary Differential Equations (ODE) or algebraic equation. The trial solution is nothing but a way of approximation and it is all about approximating as close to the exact actual solution as possible, to minimize the error developed in between the two solutions and thus making the trial solution, a reliable one for the engineer to use.[2] In the FEM, the solution region or problem domain of interest is assumed to be divided into, as the name suggests, finite number of elements. Ouyang et al stated these are also known as sub-domains. Each of these elements is connected to one other to engulf the entire continuous domain. Domain with curved boundaries/surfaces/volumes are discretized using Iso-parametric finite elements. To be more specific, the can solve a large palette of real-world engineering problems and hence is a widely used method due to its generality and ease of formulation.[3]

2. Steady state heat conduction

The strong form comprises of an ODE, if the case is one-dimensional, along with the boundary conditions, whether Dirichlet or Neumann. The weak form can be formulated by integrating the strong form by multiplying it with the weighting function. The equation (3.4) is the strong form for a heat conduction in a radial symmetric ring or annulus, which resembles the brake disc in a general sense.[4] The strong form is arrived through the generalized PDE, popularly known as the general energy equation, which is dependent on both: space and time. For a one-dimensional problem, the terms containing any two co-ordinates tend to zero. In the problem statement, where the problem is illustrated in a cylindrical co-ordinate system, the co-ordinates θ and z are eliminated.[5] Similarly, for a steady state problem, the time dependent term tends to zero. Thus, the task remaining is spatial approximation only. Also, the strong form can be arrived from the fact that temperature distribution is the unknown primary variable, so if an annular element at a radius r with a thickness ∆r is taken arbitrarily. It has to be noted here that J, the heat flux is negative if influx of heat is taking place through the domain. In co-ordinate notation for multi-dimensions. The equation is Fourier’s law of heat conduction, which states that flux or rate of flow is directed across the present temperature gradient. Also, κ_ij is the conductivity tensor which is symmetric and positive, semi-definite. It is assumed that the problem statement deals in isotropic heat conduction, where heat flows only in the direction of temperature gradient. Heat flow in one direction does not induce flow in another direction, i.e. in perfect alignment to the direction of temperature gradient.[6]

3. Transient state heat conduction

Explicit Time-Stepping: The method essentially comes from Taylor’s series, neglecting various HOT. The series in particular is a variation of Maclaurin’s Series. If a curve is shifted from origin along the horizontal axis, as denoted by above, Taylor’s series in obtained. It is the simplest numerical technique in the Euler family and is a simplified version of the more elaborate Runge-Kutta method. The Forward Euler method is a first-order method due to the fact that global truncated error or GTE is proportional to the step size. However, in spite of all its simplicity, the method has stability issues for stiff problems. Hence the method is only conditionally stable. Implicit Time-Stepping: Implicit method or Backward Difference Method is a form of implicit Runge-Kutta method. Implicit methods are much harder to implement and require high level of computation. Using explicit methods require extremely small-time steps such that the error in the result remains bounded. So, in these scenarios, the time taken to compute is lesser with higher time steps. Also, the method is unconditionally stable. Crank-
Nicholson Method: It a second order method in time due to the fact that global error is proportional to the square of step size. The gradient with respect to the assumed step size is carried forward until half of that step size is reached horizontally. After that the existing gradient is used to approximate to the gradient of the curve from the midpoint to the end node, thus significantly reducing error. Out of all the three methods, the LTE is much less and so does the GTE. The method is unconditionally stable, however is not completely immune to oscillations.

Figure 1: curve translation for euler method derivation

4. Meshing
Various meshing classes were called and iterated simultaneously. An initial idea of 2-D mesh was formulated and various iterations to refine the mesh further using for loops were carried out. All this was done considering that in comparison to the radial dimension, the thickness of the rotor is very small and thus can be neglected. However, this was decided to take into account as well and thus a 3-D hollow cylinder was chosen with reduced height. This was later refined, thus increasing the total number of elements. Triangulation. Execute coarsening and refinement under triangulation class was used for the same, where triangulation object was defined to call the function of the class.
5. Time stepping methods

The semi-discrete matrix vector problem is said so because of the fact that the temporal discretization has yet to be carried out. This is when the concept of time-stepping steps in. The time domain is divided into numerous sub-domains or sub-intervals and the primal unknown variable of interest, which is also a function of time, is computed while jumping from one interval to another. In the table 6.1, the variation of temperature versus time is listed for $\alpha$ being 0, 1 and 0.5, i.e. the three common methods in the Euler family. It can be seen for supposed time step of 1 second, the Backward Method and the C-N Method both reach the specified temperature of 310 degree C on the outer radius. This shows that they are stable and on further iterations involving variation in time step size, it is proved that they are unconditionally stable. The Forward Method, on the other hand, shows instability for the supposed time step right away, establishing that the method is conditionally stable and the overall error level does not keep increasing with respect to time until the chosen time-stepping size adheres to the critical size. Similar trend can be seen in the $l_2$ norm of the difference between steady state and transient state solution for each time step. The Forward Method gives result as "Nan", i.e. Not a Number, indicating that extremely high instability has been obtained. The other two methods show stable solutions for the time steps. The $l_2$ norm of error indicates a similar trend as well.

6. Steady state solution

The steady state solution has found out in the direction of positive x-axis of the rotor with the origin as the specified three-dimensional point (0, 0, 0). The steady state solution is obviously independent of time and is obtained solely using spatial approximation.
7. Transient state solution

The transient solution is evaluated at three different values of alpha to emulate the three different methods of Euler Family. The time step chosen is 1 sec and the total number of time steps is 3000, i.e. from 1 second to 3001 second. The output is stored in a .vtk file every 100 seconds using if-else condition. Open-source software Para View is used to visualize the result. As has been confirmed earlier, the solution at alpha=0 vents out unacceptable visualization due to its conditional stability. However, the transient response of scalar temperature field can be confirmed properly for the other two values of alpha.

8. Conclusion

This work successfully gives an insight into the abstracted world of computer-based analysis by exploring the entire process of solving a entirely, right from the governing equation to the final matrix-vector weak form. In that process, it also gives a fair idea about how to capture the
physics behind a particular engineering based real-world problem. Capturing physics of a problem translates to recognizing the constitutive relationship of the problem, the type of problem and its conditions such as boundary conditions, initial conditions etc. The work then translates these fundamental ideas to a C++ program, which is highly flexible and the content of functions inside the program can be tweaked according to the needs of the problem. The function governing the mesh generation can be modified to emulate any automotive component, be it suspension links, tie rods, drive shafts, gears etc. The work successfully finds that the established facts surrounding a transient physical problem have been cemented. It is highlighted that the stability of the solution might be conditional depending upon the scheme used. The implicit scheme allows for greater jumps in time, and is stable irrespective of choice of step size. The explicit scheme comes out to be unstable for high step size. This fact validates the C++ program. The output for different time-steps adhere to these very facts. The critical time-step, beyond which the explicit scheme results in an unstable temperature variation with respect to time, comes out to be 0.25 seconds for the problem. The scheme is on the verge of instability at that time step. The implicit scheme remains unaffected throughout, and remains stable even for time step= 1 second. Thus, the C++ program in the work is validated using iterations for different step sizes and values of alpha and then relative comparisons to one another to establish the above conclusion, which have also been visually validated, in another open-source software: Para View. The solution files that are acquired at every 100 second interval up to 3000 seconds, are visualized in the software and the transition can be clearly seen.

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