Limits on temporal variation of fine structure constant, quark masses and strong interaction from atomic clock experiments

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(Dated: January 30, 2022)

We perform calculations of the dependence of nuclear magnetic moments on quark masses and obtain limits on the variation of \((m_q/\Lambda_{QCD})\) from recent atomic clock experiments with hyperfine transitions in H, Rb, Cs, Yb\(^+\), Hg\(^+\) and optical transition in Hg\(^+\). Experiments with Cd\(^+\), deuterium/hydrogen, molecule SF\(_6\), Zeeman transitions in \(^3\)He/Xe are also discussed.

PACS number: 06.20.Jr, 06.30.Ft, 12.10.-r

I. INTRODUCTION

Interest in the temporal and spatial variation of major constants of physics has been recently revived by astronomical data which seem to suggest a variation of the electromagnetic constant \(\alpha = e^2/\hbar c\) at the \(10^{-5}\) level for the time scale 10 billion years, see [1] (a discussion of other limits can be found in the review [2] and references therein). However, an independent experimental confirmation is needed.

The hypothetical unification of all interactions implies that variation of the electromagnetic interaction constant \(\alpha\) should be accompanied by the variation of masses and the strong interaction constant. Specific predictions need a model. For example, the grand unification model discussed in [3] predicts that the quantum chromodynamic (QCD) scale \(\Lambda_{QCD}\) (defined as the position of the Landau pole in the logarithm for the running strong coupling constant) is modified as follows: \(\delta\Lambda_{QCD}/\Lambda_{QCD} = 34\delta\alpha/\alpha\). The variation of quark and electron masses in this model is given by \(\delta m/m \sim 70\delta\alpha/\alpha\). This gives an estimate for the variation of the dimensionless ratio

\[
\frac{\delta (m/\Lambda_{QCD})}{(m/\Lambda_{QCD})} \sim \frac{35}{\alpha} \delta\alpha
\]  

(1)

This result is strongly model-dependent. However, the large coefficients in these expressions are generic for grand unification models, in which modifications come from high energy scales: they appear because the running strong coupling constant and Higgs constants (related to mass) run faster than \(\alpha\). This means that if these models are correct the variation of masses and strong interaction may be easier to detect than the variation of \(\alpha\).

One can measure only variation of the dimensionless quantities, therefore we want to extract from the measurements variation of the dimensionless ratio \(m_q/\Lambda_{QCD}\) where \(m_q\) is the quark mass (with the dependence on the normalization point removed). A number of limits on variation of \(m_q/\Lambda_{QCD}\) have been obtained recently from consideration of Big Bang Nucleosynthesis, quasar absorption spectra and Oklo natural nuclear reactor which was active about 1.8 billion years ago [4, 5, 6, 7] (see also [8, 9, 10, 11, 12]). Below we consider the limits which follow from laboratory atomic clock comparison. Laboratory limits with a time base about a year are especially sensitive to oscillatory variation of fundamental constants. A number of relevant measurements have been performed already and even larger number have been started or planned. The increase in precision is very fast.

It has been pointed out by Karshenboim [13] that measurements of ratio of hyperfine structure intervals in different atoms are sensitive to variation of nuclear magnetic moments. First rough estimates of the dependence of nuclear magnetic moments on \(m_q/\Lambda_{QCD}\) and limits on time variation of this ratio have been obtained in our paper [4]. Using H, Cs and Hg\(^+\) measurements [14, 15], we obtained the limit on variation of \(m_q/\Lambda_{QCD}\) about \(5 \times 10^{-13}\) per year. Below we calculate the dependence of nuclear magnetic moments on \(m_q/\Lambda_{QCD}\) and obtain the limits from recent atomic clock experiments with hyperfine transitions in H, Rb, Cs, Yb\(^+\), Hg\(^+\) and optical transition in Hg\(^+\). It is convenient to assume that the strong interaction scale \(\Lambda_{QCD}\) does not vary, so we will speak about variation of masses. We shall restore \(\Lambda_{QCD}\) in final answers.

The hyperfine structure constant can be presented in the following form

\[
A = \text{const} \times \left[ \frac{m_e e^4}{\hbar^2} \right] \left[ \alpha^2 F_{rel}(Z\alpha) \right] \left[ \mu m_p/m_p \right]
\]  

(2)

The factor in the first bracket is an atomic unit of energy. The second “electromagnetic” bracket determines the dependence on \(\alpha\). An approximate expression for the relativistic correction factor (Casimir factor) for \(s\)-wave electron is the following

\[
F_{rel} = \frac{3}{\gamma(4\gamma^2 - 1)}
\]  

(3)

where \(\gamma = \sqrt{1 - (Z\alpha)^2}\), \(Z\) is the nuclear charge. Variation of \(\alpha\) leads to the following variation of \(F_{rel}\) [14],

\[
\frac{\delta F_{rel}}{F_{rel}} = K \frac{\delta\alpha}{\alpha}
\]  

(4)

\[
K = \frac{(Z\alpha)^2(12\gamma^2 - 1)}{\gamma^2(4\gamma^2 - 1)}
\]  

(5)

More accurate numerical many-body calculations [16] of the dependence of the hyperfine structure on \(\alpha\) have

\[
\frac{\delta F_{rel}}{F_{rel}} = K' \frac{\delta\alpha}{\alpha}
\]  

(4')

\[
K' = \frac{(Z\alpha)^2(12\gamma^2 - 1)}{\gamma^2(4\gamma^2 - 1)}
\]  

(5')
shown that the coefficient $K$ is slightly larger than that given by this formula. For Cs ($Z=55$) $K = 0.83$ (instead of 0.74), for Rb $K = 0.34$ (instead of 0.29), for Hg$^+$ $K = 2.28$ (instead of 2.18).

The last bracket in eq. (2) contains the dimensionless nuclear magnetic moment $\mu$ in nuclear magnetons (the nuclear magnetic moment $M = \mu_{2\hbar c}^\pi$), electron mass $m_e$ and proton mass $m_p$. We may also include a small correction due to the finite nuclear size. However, its contribution is insignificant.

Recent experiments measured time dependence of the ratios of hyperfine structure intervals of $^{199}$Hg$^+$ and H $^{14}$, $^{133}$Cs and $^{87}$Rb $^{17}$ and ratio of optical frequency in Hg$^+$ and $^{133}$Cs hyperfine frequency $^{16}$. In the ratio of two hyperfine structure constants for different atoms time dependence may appear from the ratio of the factors $F_{rel}$ (depending on $\pi$) and ratio of nuclear magnetic moments (depending on $m_q/\Lambda_{QCD}$). Magnetic moments in a single-particle approximation (one unpaired nucleon) are:

$$\mu = (g_\pi + (2j - 1)g_l)/2$$

for $j = l + 1/2$.

$$\mu = j/(2j + 1)(-g_\pi + (2j + 3)g_l)$$

for $j = l - 1/2$. Here the orbital g-factors are $g_l = 1$ for valence proton and $g_l = 0$ for valence neutron. The present values of spin g-factors $g_\pi$ are $g_\pi = 5.586$ for proton and $g_\pi = -3.826$ for neutron. They depend on $m_q/\Lambda_{QCD}$. The light quark masses are only about 1% of the nucleon mass ($m_q = (m_u + m_d)/2 \approx 5$ MeV). The nucleon magnetic moment remains finite in the chiral limit of $m_u = m_d = 0$. Therefore, one may think that the corrections to $g_\pi$ due to the finite quark masses are very small. However, there is a mechanism which enhances quark mass contribution: $\pi$-meson loop corrections to the nuclear magnetic moments which are proportional to $\pi$-meson mass $m_\pi \sim \sqrt{m_q/\Lambda_{QCD}}$; $m_\pi = 140$ MeV is not so small.

According to calculation in Ref. $^{14}$ dependence of the nucleon g-factors on $\pi$-meson mass $m_\pi$ can be approximated by the following equation

$$g(m_\pi) = g(0) 1 + am_\pi + bm_\pi^2$$

where $a = 1.37$ GeV $^2$, $b = 0.452$ GeV $^2$ for proton and $a = 1.85$ GeV $^2$, $b = 0.271$ GeV $^2$ for neutron. This formula is a fit of the numerical calculations performed using the Cloudy Bag Model (pion field coupled to nucleon quarks). This model reproduces leading terms of chiral perturbation theory for nuclear magnetic moments $^{20}$ (small $m_\pi$ limit). The results also agree with lattice calculations $^{19}$ (performed at $m_\pi \sim 500$ MeV). Eq. 8 leads to the following estimates:

$$\frac{\delta g_p}{g_p} = -0.174 \frac{\delta m_\pi}{m_\pi} = -0.087 \frac{\delta m_q}{m_q}$$

$$\frac{\delta g_n}{g_n} = -0.213 \frac{\delta m_\pi}{m_\pi} = -0.107 \frac{\delta m_q}{m_q}$$

Eqs. 9 give variation of nuclear magnetic moments as functions of $m_q/\Lambda_{QCD}$.

The measured time variations of ratios of hyperfine frequencies depend on two parameters: the ratio of proton spin magnetic moment and proton orbital magnetic moment (this ratio is proportional to $g_p$) and the ratio of proton and neutron spin magnetic moments ($M_p/M_n = g_p/g_n$). According to eqs. 10 the ratio $g_p/g_n$ practically does not depend on $m_q$ and seems to be not sensitive to variation of quark masses and strong interaction. However, this conclusion may be misleading. Magnetic moments depend also on the strange quark mass $m_s$. In the minimal order of the chiral perturbation theory the chiral symmetry is broken by mixing $p - K^+\Lambda$. Similar process involving $K^0$ meson loop and A does not contribute to the neutron magnetic moment since $K^0$ does not carry electric charge. As a result lowest order corrections for proton and neutron magnetic moments are very different: for proton $a(K)/a(\pi) = 0.4$, for neutron $a(K)/a(\pi) = -0.03$ $^{20}$, $^{21}$ (see eq. 8 for definition of the coefficient $a$). The proton (or neutron) g-factor is a ratio of the proton (or neutron) spin magnetic moment to the nuclear magneton $\hbar/2m_e c$ (quantum for proton orbital magnetic moment). Therefore, dependence of $g_p \sim M_p m_p$ and $g_n \sim M_n m_q$ on the strange quark mass may also appear from the relatively large ($\sim 20\%$) contribution of $m_s$ to the proton mass $m_p$. $^{14}$, $^{16}$ In the case of $g_p$ this contribution is of opposite sign to that of the lowest order in the chiral perturbation theory ($a(K) m_K$) and has comparable magnitude. This cancelation means that we cannot reliably determine dependence of $g_p$ on the strange quark mass $m_s$. In this case it would be safer to neglect contribution of the strange quark mass to $g_p$. However, we want to keep dependence on the strange quark mass in the ratio of neutron and proton magnetic moments since in a number of cases this is the only effect which provides dependence on fundamental masses and strong interaction. Using the lowest order chiral perturbation theory results $^{20}$, $^{21}$ presented above we obtain the dependence on the strange quark mass $\frac{\delta g_n}{g_n} = 0.1 \frac{\delta m_s}{m_s}$. This result should possibly be treated as an order of magnitude estimate since $K$-meson mass is not as small as $\pi$-meson mass and accuracy of the chiral theory may be low in this case. Thus, we arrive to the following equations:

$$\frac{\delta g_p}{g_p} = -0.087 \frac{\delta m_q}{m_q}$$

$$\frac{\delta g_n}{g_n} = -0.107 \frac{\delta m_q}{m_q} - \frac{\delta m_s}{m_s}$$

Note that transferring the entire $m_s$-dependence into the neutron g-factor is mostly a matter of convenience, all that we can say is that $g_n/g_p$ depends on $m_q/\Lambda_{QCD}$. Now
we can find variation of nuclear magnetic moments using eqs. \[ \frac{\delta \mu}{\mu} = 0.22 \frac{\delta m_e}{m_e} = 0.11 \frac{\delta m_q}{m_q} \] (13)

For \(^{87}\text{Rb}\) we have valence proton with \(j=3/2, l=1\) and
\[
\frac{\delta \mu}{\mu} = -0.128 \frac{\delta m_e}{m_e} = -0.064 \frac{\delta m_q}{m_q}
\] (14)

As an intermediate result it is convenient to present dependence of the ratio of hyperfine constant \(A\) to atomic unit of energy \(E = \frac{m_e e^4}{2 h^2}\) (or energy of 1s-2s transition in hydrogen equal to \(3/8 E\)) on variation of the fundamental constants. We introduce a parameter \(V\) defined by the realtion
\[
\frac{\delta V}{V} = \frac{\delta (A/E)}{A/E}
\] (15)

We start from the hyperfine structure of \(^{133}\text{Cs}\) which is used as a frequency standard. Using eqs. (2,13) we obtain
\[
V(^{133}\text{Cs}) = \alpha^{2.83} \left( \frac{m_q}{\Lambda_{QCD}} \right)^{0.11} \left( \frac{m_e}{\Lambda_{QCD}} \right)
\] (16)

Here we have taken into account that the proton mass \(m_p \propto \Lambda_{QCD}\). For hyperfine transition frequencies in other atoms we obtain
\[
V(^{87}\text{Rb}) = \alpha^{2.34} \left( \frac{m_q}{\Lambda_{QCD}} \right)^{-0.06} \left( \frac{m_e}{\Lambda_{QCD}} \right)
\] (17)
\[
V(^{1}\text{H}) = \alpha^{2} \left( \frac{m_q}{\Lambda_{QCD}} \right)^{-0.09} \left( \frac{m_e}{\Lambda_{QCD}} \right)
\] (18)
\[
V(^{2}\text{H}) = \alpha^{2} \left( \frac{m_q}{\Lambda_{QCD}} \right)^{-0.04} \left( \frac{m_e}{\Lambda_{QCD}} \right)^{-0.23} \left( \frac{m_e}{\Lambda_{QCD}} \right)
\] (19)
\[
V(^{199}\text{Hg}^+) = \alpha^{4.3} \left( \frac{m_q}{m_q} \right)^{0.11} \left( \frac{m_e}{\Lambda_{QCD}} \right)
\] (20)
\[
V(^{171}\text{Yb}^+) = \alpha^{3.5} \left( \frac{m_q}{m_q} \right)^{0.11} \left( \frac{m_e}{\Lambda_{QCD}} \right)
\] (21)
\[
V(^{111}\text{Cd}^+) = \alpha^{2.6} \left( \frac{m_q}{m_q} \right)^{0.11} \left( \frac{m_e}{\Lambda_{QCD}} \right)
\] (22)

Now we can use these results to find limits on variation of the fundamental constants from the measurements of the time dependence of hyperfine structure intervals. The dependence of ratio of frequencies \(A(^{133}\text{Cs})/A(^{87}\text{Rb})\) can be presented in the following form
\[
X(CsRb) = \frac{V(Cs)}{V(Rb)} = \alpha^{0.49} \left( \frac{m_q}{\Lambda_{QCD}} \right)^{0.17}
\] (23)

Therefore, the result of the measurement \[17\] may be presented as a limit on variation of the parameter \(X\):
\[
\frac{1}{X(CsRb)} \frac{dX(CsRb)}{dt} = (0.2 \pm 7) \times 10^{-16}/\text{year}
\] (24)

Note that if the relation \[11\] is correct, variation of \(X(CsRb)\) would be dominated by variation of \([m_q/\Lambda_{QCD}]\). The relation \[11\] would give \(X(CsRb) \propto \alpha^7\).

For \(A(^{133}\text{Cs})/A(\text{H})\) we have
\[
X(CsH) = \frac{V(Cs)}{V(H)} = \alpha^{0.83} \left( \frac{m_q}{\Lambda_{QCD}} \right)^{0.2}
\] (25)

Therefore, the result of the measurements \[15\] may be presented as
\[
\frac{1}{X(CsH)} \frac{dX(CsH)}{dt} < 5.5 \times 10^{-14}/\text{year}
\] (26)

For \(A(^{199}\text{Hg})/A(\text{H})\) we have
\[
X(HgH) = \frac{V(Hg)}{V(H)} \approx \alpha^{2.3} \left( \frac{m_e}{\Lambda_{QCD}} \right)^{0.1}
\] (27)

The result of measurement \[14\] may be presented as
\[
\frac{1}{X(HgH)} \frac{dX(HgH)}{dt} < 8 \times 10^{-14}/\text{year}
\] (28)

In Ref. \[13\] the limit on variation of the ratio of hyperfine transition frequencies \(^{171}\text{Yb}^+/^{133}\text{Cs}\) has been obtained (this limit is based on measurements \[22\]). Using eqs. \[10\ 21\] we can present the result as a limit on \(X(YbCs) = \alpha^{0.7} \left( \frac{m_e}{\Lambda_{QCD}} \right)^{0.11}:
\[
\frac{1}{X(YbCs)} \frac{dX(YbCs)}{dt} \approx -1(2) \times 10^{-13}/\text{year}
\] (29)

The optical clock transition energy \(E(Hg) (\lambda=282\text{ nm})\) in Hg\(^+\) ion can be presented in the following form:
\[
E(Hg) = \text{const} \times \left( \frac{m_e e^4}{\hbar^2} \right) F_{rel}(Z\alpha)
\] (30)

Numerical calculation of the relative variation of \(E(Hg)\) has given \[16\] :
\[
\frac{\delta E(Hg)}{E(Hg)} = -3.2 \frac{\delta \alpha}{\alpha}
\] (31)

This corresponds to \(V(HgOpt) = \alpha^{-3.2}\). Variation of the ratio of the Cs hyperfine splitting \(A(Cs)\) to this optical transition energy is described by
\[
X(\text{Opt}) = \frac{V(Cs)}{V(HgOpt)} = \alpha^6 \left( \frac{m_q}{\Lambda_{QCD}} \right)^{0.11} \left( \frac{m_e}{\Lambda_{QCD}} \right)
\] (32)

The work \[12\] gives the limit on variation of this parameter:
\[
\frac{1}{X(\text{Opt})} \frac{dX(\text{Opt})}{dt} < 7 \times 10^{-15}/\text{year}
\] (33)
Molecular rotational transitions frequencies are proportional to $m_n/m_p$, therefore one should assume $V = (m_n/\Lambda_{QCD})^{1/2}$. For vibrational molecular transitions $V = (m_n/\Lambda_{QCD})^{1/2}$. Comparison of Cs hyperfine standard with $\text{SF}_6$ molecular vibration frequencies was discussed in Ref. [23]. In this case $X(\text{CsVibrations}) = \alpha^2 [m_n/\Lambda_{QCD}]^{0.3}[m_q/\Lambda_{QCD}]^{0.1}$.

The measurements of hyperfine constant ratio in different isotopes of the same atom depends on the ratio of magnetic moments and is sensitive to $m_q/\Lambda_{QCD}$. For example, it would be interesting to measure the rate of change for hydrogen/deuterium ratio where $X(\text{HD}) = [m_q/\Lambda_{QCD}]^{-0.05}[m_n/\Lambda_{QCD}]^{0.23}$.

R. Walsworth suggested to measure the ratio of the Zeeman transition frequencies in noble gases which gives us time dependence of ratio of nuclear magnetic moments. Consider, for example $^{129}\text{Xe}/^{3}\text{He}$. For $^{3}\text{He}$ the magnetic moment is very close to that of neutron. For other noble gases nuclear magnetic moment is also given by valence neutron, however, there are significant many-body corrections. For $^{129}\text{Xe}$ the valence neutron is in $s_{1/2}$ state which corresponds to the single-particle value of nuclear magnetic moment $\mu = \mu_n = -1.913$. The measured value is $\mu = -0.778$. The magnetic moment of the nucleus changes most efficiently due to the spin-spin interaction because valence neutron transfers a part of its spin $<s_z>$ to core protons and proton magnetic moment is large and has opposite sign. In this approximation $\mu = (1 - b)\mu_n + b\mu_p$. This gives $b=0.24$ and the ratio of magnetic moments $Y \equiv \mu(^{129}\text{Xe})/\mu(^{3}\text{He})= 0.76 + 0.24\mu_p/\mu_n$. Using eqs. (11,12) we obtain an estimate for the variation of this ratio $\delta Y/Y \approx 0.1\delta m_q/m_s$. Therefore, in this case one can measure variation of $\mu(^{129}\text{Xe})/\mu(^{3}\text{He})$ corresponding to variation of $X = [m_s/\Lambda_{QCD}]^{0.1}$.

Note that an accuracy of the results presented in this paper depends strongly on fundamental constant we are studying. The accuracy for the dependence on $\alpha$ is few percent. The accuracy for $m_q/\Lambda_{QCD}$ is about 30\%, it is limited by the accuracy of a single-particle approximation for nuclear magnetic moments. The results for $m_s/\Lambda_{QCD}$ should possibly be treated as order of magnitude estimates. However, the accuracy here may be improved using proper QCD calculation [24]. Relation (11) between variation of $\alpha$ and $m/\Lambda_{QCD}$ has been used as an illustration only.

The author is grateful to C. Chardonnet, S. Karshenboim, D.B. Leinweber, A.W. Thomas and R. Walsworth for valuable discussions. This work is supported by the Australian Research Council.

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