Chiral Perturbation Theory tests at NA48/2 and NA62 experiments at CERN

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Abstract. Final results from an analysis of about 400 $K^\pm \rightarrow \pi^\pm \gamma\gamma$ rare decay candidates collected by the NA48/2 and NA62 experiments at CERN during low intensity runs with minimum bias trigger configurations are presented. The results include a model-independent decay rate measurement and fits to Chiral Perturbation Theory (ChPT) descriptions. The data support the ChPT prediction for a cusp in the di-photon invariant mass spectrum at the two pions threshold.

1. Introduction
Experimental studies of radiative non-leptonic decays allow crucial tests of Chiral Perturbation Theory (ChPT) describing weak low energy processes. The rare decay $K^\pm \rightarrow \pi^\pm \gamma\gamma$ ($K_{\pi\gamma\gamma}$) was first observed by the BNL E787 experiment in 1997 [1]: 31 $K^+$ decay candidates were reported in the kinematic region $100 \text{ MeV}/c < p^*_\pi < 180 \text{ MeV}/c$, where $p^*_\pi$ is the $\pi^+$ momentum in the $K^+$ rest frame. A related decay mode $K^\pm \rightarrow \pi^\pm \gamma e^+e^-$ ($K_{\pi\gamma ee}$) has been measured from 120 candidates in the kinematic region $m_{ee} > 260 \text{ MeV}/c^2$ at the NA48/2 experiment [2].
A measurement of the $K_{\pi\gamma\gamma}$ decay based on the combination of data collected by the NA48/experiment in 2003 and 2004 and from the NA62 experiment in 2007 is reported here.

2. The CERN Kaon facility
The NA48/2 experiment at CERN, which took data in 2003 and 2004, used simultaneous $K^+$ and $K^-$ beams produced by 400 GeV/c primary SPS protons impinging on a beryllium target. An achromatic system of four dipole magnets selected charged particles with $(60 \pm 3)$ GeV/c momenta, splitting the two beams in the vertical plane and recombining them on a common axis. The beams were directed through collimators and quadrupole magnets and finally entered a 114 m long cylindrical vacuum tank with increasing diameter (from 1.92 to 2.4 m) which contained the decay region. The $K^+/K^-$ flux ratio was 1.79 and the fraction of kaons decaying in the vacuum tank was 22%.
The beam line and setup of the NA48/2 experiment were used for the NA62 data taking in 2007, with different beam parameters. Secondary beams of positive and negative hadrons with

1 On behalf of the NA48/2 Collaboration: Cambridge, CERN, Dubna, Chicago, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien
2 On behalf of the NA62-RK Collaboration: Birmingham, CERN, Dubna, Fairfax, Ferrara, Firenze, Frascati, Mainz, Merced, Moscow, Napoli, Perugia, Pisa, Protvino, Roma I, Roma II, Saclay, San Luis Potosi, Stanford, Sofia, Torino, TRIUMF.
a $(74 \pm 1.4)$ GeV/c momentum were selected. The fraction of $K^\pm$ in these beams was about 6\% and they were delivered, either alternately or simultaneously, into the same vacuum tank. The fraction of beam kaons decaying in the vacuum tank was 18\%.

In both experiments the vacuum tank was followed by a magnetic spectrometer contained in a vessel filled with helium at nearly atmospheric pressure, separated from the vacuum by a thin (0.3\%$X_0$) Kevlar\textsuperscript{®} composite window. An aluminium beam pipe of 158 mm outer diameter and 1.1 mm thickness crossed the centre of the spectrometer and all the following detectors, allowing the undecayed beam particles to continue their path in vacuum. The spectrometer was made by four drift chambers (DCH) with a transverse width of 2.9 m, two of them (DCH1 and DCH2) located upstream and two (DCH3 and DCH4) downstream of a dipole magnet providing a horizontal transverse momentum kick of 120 MeV/c (NA48/2) or 265 MeV/c (NA62) for charged particle. The spectrometer momentum resolution was $\sigma_p/p = (1.02 \pm 0.044 \pm p)\%$ (NA48/2) or $\sigma_p/p = (0.48 \pm 0.009 \pm p)\%$ (NA62), where $p$ is expressed in GeV/c. A hodoscope (HOD) consisting of two planes of 64 plastic scintillator strips, each plane divided in four quadrants, was placed downstream of the spectrometer and provided trigger signals and time measurements of charged particles with a resolution of 150 ps. The hodoscope was followed by a quasi-homogeneous liquid krypton electromagnetic calorimeter (LKr) with an active volume of 7 m$^3$, 27 $X_0$ deep and segmented transversally into 13248 projective $2 \times 2$ cm$^2$ cells with no longitudinal segmentation. The LKr energy resolution was $\sigma_E/E = (3.2/\sqrt{E} + 9/E + 0.42)\%$, its spatial resolution for the transverse coordinates $x$, $y$ was $\sigma_x = \sigma_y = (4.2/\sqrt{E} + 0.6)$ mm, with $E$ in GeV. A plane of scintillating fibres located inside the LKr calorimeter volume at a depth of about 9.5$X_0$, close to the maxima of showers initiated by 10 GeV photons, formed the ”neutral hodoscope” (NHOD) which also provided trigger signals. The LKr was followed by a hadronic calorimeter and a muon detector. A detailed description of the detector can be found in [3].

![Figure 1. Beam setup of the NA48/2 and NA62 experiments. The NA48/2 experiment used simultaneous $K^+$ and $K^-$ beams produced by 400 GeV/c primary SPS protons impinging on a beryllium target, the NA62 experiment used $K^+$ and $K^-$ beams either alternately or simultaneously. The achromat system selected charged particles with $(60 \pm 3)$ GeV/c momenta (NA48/2) or $(74 \pm 1.4)$ momenta (NA62).](image1)

![Figure 2. Detector setup for both the NA48/2 and NA62 experiments. Kaons are supposed to come from the bottom right part of the picture.](image2)

3. ChPT description of $K^\pm \to \pi^\pm \gamma\gamma$

The $K^\pm \to \pi^\pm \gamma\gamma$ decay ($K_{\pi\gamma\gamma}$) can be described by two kinematic variables:

$$z = \frac{(q_1 + q_2)^2}{m_K^2} = \left(\frac{m_{\gamma\gamma}}{m_K}\right)^2,$$  

(1)
\[ y = \frac{p (q_1 - q_2)}{m_K^2} \]  

where \( p \) and \( q_{1,2} \) are the four-momenta of the kaon and the two photons, respectively, \( m_{\gamma\gamma} \) is the di-photon invariant mass and \( m_K \) is the charged kaon mass. The allowed region of the kinematic variables is:

\begin{align*}
0 \leq z &\leq z_{\text{max}} = (1 - r_\pi)^2 = 0.515, \quad (3) \\
0 \leq y &\leq y_{\text{max}} = \frac{1}{2} \sqrt{\lambda(1, r_\pi^2, z)} \quad (4)
\end{align*}

where \( r_\pi = m_\pi / m_K \), \( m_\pi \) is the charged pion mass and \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc) \). In the framework of ChPT the first non-trivial contribution to the \( K_{\pi\gamma\gamma} \) decay rate comes from next-to-leading order ChPT, i.e. it has no tree-level \( O(p^2) \) contribution \([4, 5, 6, 7]\). The differential decay rate at leading order \( O(p^4) \) and including \( O(p^6) \) contributions can be written as:

\[ \frac{\partial \Gamma}{\partial y \partial z}(\hat{c}, y, z) = \frac{m_K}{2^9 \pi^3} \left[ z^2 \left( |A(\hat{c}, z, y^2) + B(z)|^2 + |C(z)|^2 \right) + \left( y^2 - \frac{1}{4} \lambda(1, r_\pi^2, z) \right)^2 |B(z)|^2 \right] \]  

The decay rate and spectrum depend on a single parameter \( \hat{c} \) to be determined experimentally; \( A(\hat{c}, z, y^2) \) is an \( O(p^4) \) loop amplitude, \( B(z) \) is a loop amplitude which appears only at \( O(p^6) \) order (and dominates the differential rate at low \( z \)) and \( C(z) \) is a pole amplitude. A cusp is expected in the differential decay rate at the di-pion threshold \( z_{\text{thr}} = 4r_\pi^2 = 0.320 \) generated by the pion loop amplitude. The branching ratio in the full kinematic range is expected to be \( B(K_{\pi\gamma\gamma}) \approx 10^{-6} \) and its dependence on \( \hat{c} \) is shown in Fig. 3. The differential decay rate for different values of the \( \hat{c} \) parameter is shown in Fig. 4 at the \( O(p^4) \) order and in Fig. 5 at the \( O(p^6) \) order.

The ChPT description involves a number of external inputs. The \( G_8 \) parameter entering both \( O(p^4) \) and \( O(p^6) \) descriptions is fixed according to \([8]\). The \( O(p^6) \) framework additionally involves 7 parameters of the \( K_{3\pi} \) decay amplitude fixed to those fitted to the experimental data.
Figure 4. $K_{\pi\gamma\gamma}$ Differential decay rate for different values of the $\hat{c}$ parameter in the $\mathcal{O}(p^4)$ approximation of ChPT.

Figure 5. Differential decay rate for different values of the $\hat{c}$ parameter in the $\mathcal{O}(p^6)$ approximation of ChPT.

[9], and 3 polynomial contributions $\eta_i$ ($i = 1, 2, 3$) fixed to $\eta_i = 0$. The parameter $\hat{c}$ enters the $\mathcal{O}(p^6)$ differential decay rate via a linear combination:

$$
\hat{c}^* = \hat{c} - 2 \left( \frac{m_\pi}{m_K} \right)^2 \eta_1 - 2 \eta_2 - 2 \eta_3.
$$

(6)

Therefore setting $\eta_i = 0$ is equivalent to measuring $\hat{c}^*$ and $\hat{c}$ can be computed for any assumed value of $\eta_i$. The values used for the external parameters are listed in Table 1.

Table 1. Values of the external parameters used in this analysis. The notation is introduced in [6, 8, 9].

| Parameter | Value | Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|-----------|-------|
| $G_8 m_K^2 \times 10^6$ | 2.202 | $\beta_1 \times 10^8$ | -27.06 | $\zeta_1 \times 10^8$ | -0.40 |
| $\alpha_1 \times 10^8$ | 93.16 | $\beta_3 \times 10^8$ | -2.22 | $\xi_1 \times 10^8$ | -1.83 |
| $\alpha_3 \times 10^8$ | -6.72 | $\gamma_3 \times 10^8$ | 2.95 | $\eta_i$ | 0 |

4. Data sample and triggers

The NA48/2 experiment collected data during two high intensity runs in 2003 and 2004 (with about $3 \times 10^6 K^\pm$ entering the decay volume per SPS spill of 4.8 s duration), in about 100 days of efficient data taking in total dedicated mostly to collect three-pions decays. The $K_{\pi\gamma\gamma}$ measurement is based on two special $K^\pm$ decay samples collected at about 10% the nominal beam intensity during 12 hours in 2003 and 54 hours in 2004 with a minimum bias trigger condition: a time coincidence of signals in both HOD planes within the same quadrant and an energy deposit of at least 10 GeV in the LKr calorimeter.

The NA62 experiment collected data from about $3.5 \times 10^5$ SPS spills in 4 months of operation in 2007 with low intensity beams at an instantaneous kaon decay rate in the vacuum tank of
about $10^6$ Hz; the total number of $K^\pm$ decays in the vacuum tank was about $2 \times 10^{10}$. About 27% of them were collected with simultaneous $K^+$ and $K^-$ beams, 65% (8%) in single-beam $K^+$ ($K^-$) mode. A downscaled minimum bias trigger was used to collect $K_{\pi\gamma\gamma}$ decays requiring at least one charged track and/or a minimum calorimetric energy deposit. At least one of the following trigger conditions was required:

- a time coincidence of signals in the two HOD planes within the same quadrant combined with a DCH hit multiplicity signal compatible with a single charged track (~20% of the data sample);
- the above condition in coincidence with a LKr energy deposit of at least 10 GeV (~60% of the data sample);
- a signal from the NHOD detector compatible with an electromagnetic shower energy release (~20% of the data sample).

![Figure 6](image1.png) **Figure 6.** NA48/2 data: invariant mass distributions of $\pi^\pm\gamma\gamma$ compared with the sums of estimated signal and background components. The estimated $K_{\pi\gamma\gamma}$ signal corresponds to the result of a ChPT $O(p^6)$ fit. The limits of the signal region are indicated with vertical arrows.

![Figure 7](image2.png) **Figure 7.** NA48/2 data: invariant mass distributions of $\pi^\pm\pi^0$ compared with the sums of estimated signal and background components. The limits of the signal region are indicated with vertical arrows.

5. Measurement method

The $K_{\pi\gamma\gamma}$ decay rate is measured with respect to the normalization decay chain collected simultaneously with the same trigger logic: $K^\pm \rightarrow \pi^\pm\pi^0$ decay ($K_{2\pi}$) followed by $\pi^0 \rightarrow \gamma\gamma$ decay ($\pi^0_{\gamma\gamma}$). The kaon beam flux and downscaling factors cancel in the ratio as well as several systematic effects at first order.

The normalization mode branching ratio $B(K_{2\pi})B(\pi^0_{\gamma\gamma}) = 0.204 \pm 0.001$ is large and known to a good precision [10].
6. Data selection

The signal \((K_{\gamma\gamma})\) and normalization \((K_{2\pi}, \pi_{\gamma\gamma}^0)\) decay modes are characterised by the same set of particles in the final states. Therefore the following selection criteria are common for the two modes leading to cancellation of systematic effects.

- A \(\pi^\pm\) candidate track consistent with originating from a beam \(K^\pm\) is required. The decay vertex, reconstructed at the point of closest approach of the track and the beam axis, should be located within a 98 m long fiducial volume contained in the vacuum tank.
- Track impact points in the DCH, HOD and LKr calorimeter should be within their fiducial acceptances.
- The reconstructed track momentum is required to be between 10 and 40 GeV/c (NA48/2) or between 8 and 50 GeV/c (NA62). The upper momentum cut is equivalent to a lower limit on the total energy of the two photons ensuring the high efficiency of the LKr trigger conditions.
- The \(\pi^\pm\) is identified by the ratio of the energy release in the LKr calorimeter to momentum measured by the spectrometer: \(E/p < 0.85\).
- Clusters of energy deposition in the LKr calorimeter in time with the track are considered as photon candidates. Exactly two photon candidates are required, with a minimum energy of 3 GeV, a minimum separation of 20 cm and a distance from the track impact point of at least 25 cm.
- The reconstructed \(\pi^\pm\gamma\gamma\) momenta are required to be between 55 and 65 GeV/c (NA48/2) or between 70 and 78 GeV/c (NA62), while its transverse component relative to the beam axis should be \(p_T^2 < 0.5 \times 10^{-3}\) (GeV/c)².
- The reconstructed \(\pi^\pm\gamma\gamma\) \((\pi^\pm\pi^0)\) invariant mass should be between 480 and 510 MeV/c².

Figure 8. NA62 data: invariant mass distributions of \(\pi^\pm\gamma\gamma\) compared with the sums of estimated signal and background components. The estimated \(K_{\pi\gamma\gamma}\) signal corresponds to the result of a ChPT \(O(p^6)\) fit. The limits of the signal region are indicated with vertical arrows.

Figure 9. NA62 data: invariant mass distributions of \(\pi^\pm\pi^0\) compared with the sums of estimated signal and background components. The limits of the signal region are indicated with vertical arrows.
The corresponding mass resolutions in NA48/2 are 5.9 (3.9) MeV/c² and in NA62 are 5.4 (3.3) MeV/c² for the $K_{\pi\gamma\gamma}$ ($K_{2\pi}$) decays.

The $K_{\pi\gamma\gamma}$ and $K_{2\pi}$ selections differ only in the di-photon invariant mass requirement.

- For $K_{\pi\gamma\gamma}$ the signal kinematic region is defined as $z > 0.2$. The low $z$ region is dominated by the $\pi^\pm$ monochromatic line at $z = (m_{\pi^0}/m_K)^2 = 0.075$. The resolution is the $z$ variable increases from $\delta z = 0.005$ at $z = 0.2$ to $\delta z = 0.003$ at $z_{\text{max}} = 0.515$.
- For $K_{2\pi}$ it is required that $|m_{\pi\gamma} - m_{\pi\gamma}| < 10$ MeV/c² ($0.064 < z < 0.086$). The mass resolution is $\delta m_{\pi\gamma} = 1.6$ MeV/c² (corresponding to $\delta z = 0.002$).

The $\pi^\pm\gamma\gamma$ and $\pi^\pm\pi^0$ invariant mass spectra of the selected signal and normalization candidates are shown in Fig. 6 and Fig. 7 for NA48/2 and in Fig. 8 and Fig. 9 for NA62 together with the expectations for the signal and background contributions evaluated with MC simulations.

In the NA48/2 data the number of reconstructed $K_{\pi\gamma\gamma}$ candidates is $N_{\pi\gamma\gamma} = 149$, of which 97 (52) are $K^+$ ($K^-$) decay candidates. The number of reconstructed $K_{2\pi}$ candidates is $N_{2\pi} = 3.628 \times 10^7$, of which 2.321 (1.307) $\times 10^7$ are $K^+$ ($K^-$) decay candidates.

In the NA62 data the number of reconstructed $K_{\pi\gamma\gamma}$ candidates is $N_{\pi\gamma\gamma} = 232$, of which 179 (53) are $K^+$ ($K^-$) decay candidates. The number of reconstructed $K_{2\pi}$ candidates is $N_{2\pi} = 5.488 \times 10^7$, of which 4.431 (1.057) $\times 10^7$ are $K^+$ ($K^-$) decay candidates.

The reconstructed $z$ spectrum of the $K_{\pi\gamma\gamma}$ candidates is displayed in Fig. 10 for NA48/2 and in Fig. 11 for NA62.

**Figure 10.** NA48/2 data: reconstructed $z = (m_{\pi\gamma}/m_K)^2$ distribution for the $K_{\pi\gamma\gamma}$ candidates and estimated signal and background contributions. The estimated signal corresponds to the result of a ChPT $O(p^6)$ fit. The limits of the signal region are indicated with vertical arrows.

**Figure 11.** NA62 data: reconstructed $z = (m_{\pi\gamma}/m_K)^2$ distribution for the $K_{\pi\gamma\gamma}$ candidates and estimated signal and background contributions. The estimated signal corresponds to the result of a ChPT $O(p^6)$ fit. The limits of the signal region are indicated with vertical arrows.

**7. Backgrounds**

The main background to the normalization mode ($K_{2\pi}, \pi^0_{\gamma\gamma}$) comes from $K^\pm \to \pi^0\mu^\pm\nu$ decays ($K_{\mu3}$) followed by $\pi^0 \to \gamma\gamma$ decays and it corresponds to 0.13% (NA48/2) or 0.115% (NA62),
where the branching ratios $B(K_{2\pi})$ and $B(K_{\mu3})$ are taken from [10]. The acceptances have been evaluated with MC simulations and are 19.18\% (NA48/2) or 16.88\% (NA62) for $K_{2\pi}$ decays, 0.15\% (NA48/2) or 0.12\% (NA62) for $K_{\mu3}$ decays.

The product $N_K$ of the number of $K^{\pm}$ decays in the fiducial volume in the analysed data set and the trigger efficiency for $K_{2\pi}$ sample is computed as:

$$N_K = (0.925 \pm 0.004) \times 10^9 \quad [\text{NA48/2}]$$  \hspace{1cm} (7)

$$N_K = (1.592 \pm 0.006) \times 10^9 \quad [\text{NA62}]$$  \hspace{1cm} (8)

where the uncertainty is dominated by the limited precision on the external input $B(K_{2\pi})$.

The principal background in the $K_{\pi\gamma\gamma}$ sample comes from $K^{\pm} \rightarrow \pi^{\pm}\pi^0\gamma$ decays followed by $\pi^0_\gamma$ decays. It is due to the merging of LKr energy deposition clusters produced by a photon from the $\pi^0$ decay and a photon from the parent $K^{\pm}$ decay. The total background from $K^{\pm} \rightarrow \pi^{\pm}\pi^0\gamma$ decays is estimated to be 11.4 ± 0.6 events (NA48/2) or 15.3 ± 1.1 events (NA62), where the uncertainty comes from MC simulation statistics.

Another source of background in the $K_{\pi\gamma\gamma}$ sample is due to $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$ decays followed by $\pi^0_\gamma$ decays. They contribute via photon missing the LKr acceptance as well as LKr cluster merging. This background is estimated to be 4.1 ± 0.4 events (NA48/2) or 2.1 ± 0.3 events (NA62), where the uncertainty comes from MC simulation statistics.

### 8. Model-independent branching ratio measurement

Partial $K_{\pi\gamma\gamma}$ branching fractions $B_j$ in 8 bins of the $z$ variable are evaluated as:

$$B_j = \left( \frac{N_j - N_j^B}{N_K A_j} \right)$$  \hspace{1cm} (9)

where $N_j$ is the number of reconstructed $K_{\pi\gamma\gamma}$ candidates, $N_j^B$ is the estimated number of background events, $A_j$ is the signal acceptance in bin $j$ and $N_K$ is the number of $K^{\pm}$ decays in the fiducial volume defined in the previous section. The first 7 bins of $z$ are defined in the range 0.20 < $z$ < 0.48 with $\Delta z = 0.04$, the last bin is for 0.48 < $z$ < $z_{max}$ due to the lack of statistics induced by the small acceptance for $\pi^{\pm}$ very close to the beam pipe.

The $y$-dependence of the differential decay rate expected within the ChPT framework [6, 7] is weak and the $y$-dependence of acceptance is also weak. The model-independent branching ratio in the kinematic region $z > 0.2$ is computed by summing over the $z$ bins:

$$B_{MI} (z > 0.2) = \sum_{j=1}^{8} B_j$$  \hspace{1cm} (10)

and it is:

$$B_{MI} (z > 0.2) = (0.877 \pm 0.087_{\text{stat}} \pm 0.017_{\text{syst}}) \times 10^{-6} \quad [\text{NA48/2}]$$  \hspace{1cm} (11)

$$B_{MI} (z > 0.2) = (1.088 \pm 0.093_{\text{stat}} \pm 0.027_{\text{syst}}) \times 10^{-6} \quad [\text{NA62}]$$  \hspace{1cm} (12)

where the first error is statistical and the second systematic, dominated by uncertainties on the background subtraction. The $B_j$ measurement of NA48/2 and NA62 have been combined and then summed, providing the combined result:

$$B_{MI} (z > 0.2) = (0.965 \pm 0.061_{\text{stat}} \pm 0.014_{\text{syst}}) \times 10^{-6} \quad [\text{NA48/2 and NA62}]$$  \hspace{1cm} (13)

The measurements performed separately for $K^+$ and $K^-$ decays are consistent in the NA48/2 data sample:

$$B_{MI}^+ (z > 0.2) = (0.881 \pm 0.107_{\text{stat}}) \times 10^{-6} \quad [\text{NA48/2}]$$  \hspace{1cm} (14)

$$B_{MI}^- (z > 0.2) = (0.868 \pm 0.147_{\text{stat}}) \times 10^{-6} \quad [\text{NA48/2}]$$  \hspace{1cm} (15)
and in the NA62 data sample:

\[
\begin{align*}
\mathcal{B}_{+}^{MI}(z > 0.2) &= (1.010 \pm 0.098_{\text{stat}}) \times 10^{-6} \text{ [NA62]} \\
\mathcal{B}_{-}^{MI}(z > 0.2) &= (1.417 \pm 0.256_{\text{stat}}) \times 10^{-6} \text{ [NA62]}
\end{align*}
\]

The combined \( \mathcal{B}^{\pm}_{MI}(z > 0.2) \) measurements obtained by averaging the NA48/2 and NA62 results are:

\[
\begin{align*}
\mathcal{B}_{+}^{MI}(z > 0.2) &= (0.951 \pm 0.072_{\text{stat}}) \times 10^{-6} \text{ [NA48/2 and NA62]} \\
\mathcal{B}_{-}^{MI}(z > 0.2) &= (1.004 \pm 0.127_{\text{stat}}) \times 10^{-6} \text{ [NA48/2 and NA62]}
\end{align*}
\]

These values are consistent and the charge asymmetry of the decay rate is:

\[
\Delta (K_{\pi\gamma\gamma}) = \frac{\mathcal{B}_{+}^{MI} - \mathcal{B}_{-}^{MI}}{\mathcal{B}_{+}^{MI} + \mathcal{B}_{-}^{MI}} = -0.03 \pm 0.07
\]

where the sub-dominant systematic uncertainties are neglected.

9. Measurement of the ChPT parameters

The \( \hat{c} \) parameter described in section 3 is measured in the ChPT \( \mathcal{O}(p^4) \) and \( \mathcal{O}(p^6) \) framework [6] by fitting the reconstructed \( z \) spectrum shown in Fig. 10 for NA48/2 and in Fig. 11 for NA62 with a binned log-likelihood. The fit is performed for \( 0.2 < z < 0.54 \) with a bin width of \( \Delta z = 0.02 \), with the \( \mathcal{O}(p^4) \) and \( \mathcal{O}(p^6) \) description separately:

\[
\begin{align*}
\hat{c}_4 &= 1.37 \pm 0.33_{\text{stat}} \pm 0.14_{\text{syst}}, \quad \hat{c}_6 = 1.41 \pm 0.38_{\text{stat}} \pm 0.11_{\text{syst}} \text{ [NA48/2]}, \\
\hat{c}_4 &= 1.93 \pm 0.26_{\text{stat}} \pm 0.08_{\text{syst}}, \quad \hat{c}_6 = 2.10 \pm 0.28_{\text{stat}} \pm 0.18_{\text{syst}} \text{ [NA62]}
\end{align*}
\]

The data are consistent with both ChPT descriptions. The \( z \) spectrum corresponding to the \( \mathcal{O}(p^6) \) fit result is shown in Fig. 10 for NA48/2 and in Fig. 11 for NA62. The measured values of \( \hat{c}_6 \) translates into the following model-dependent branching ratio in the full kinematic range, obtained by integration of the ChPT \( \mathcal{O}(p^6) \) differential decay rate:

\[
\begin{align*}
\mathcal{B}_{ChPT} &= (0.910 \pm 0.072_{\text{stat}} \pm 0.022_{\text{syst}}) \times 10^{-6} \text{ [NA48/2]}, \\
\mathcal{B}_{ChPT} &= (1.058 \pm 0.066_{\text{stat}} \pm 0.044_{\text{syst}}) \times 10^{-6} \text{ [NA62]}
\end{align*}
\]
The combination of the previous results from NA48/2 and NA62 is:

\[ \hat{c}_4 = 1.72 \pm 0.20_{\text{stat}} \pm 0.06_{\text{syst}}, \quad \text{[NA48/2 and NA62]}, \]
\[ \hat{c}_6 = 1.86 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}}, \quad \text{[NA48/2 and NA62]}, \]
\[ B_{\text{ChPT}} = (1.003 \pm 0.051_{\text{stat}} \pm 0.024_{\text{syst}}) \times 10^{-6} \quad \text{[NA48/2 and NA62]}. \]

The corresponding model dependent values of \( B_{\text{ChPT}} \) in \( z \) bins are displayed in Fig. 12.

10. Conclusions
A model-independent measurement of the \( K_{\pi\gamma\gamma} \) decay rate and fits to the ChPT description have been performed by the NA48 experiment and the NA62 experiment and the results are published in [11] and [12], respectively, and have been combined. The model independent branching ratio in a limited kinematic range is \( B_{MI}(z > 0.2) = (0.965 \pm 0.063) \times 10^{-6} \). The observed decay spectrum agrees with the ChPT description and ChPT parameters measured within the considered formulation are \( \hat{c}_4 = 1.72 \pm 0.21 \) and \( \hat{c}_6 = 1.86 \pm 0.25 \). The branching ratio in the full kinematic range assuming the \( O(p^6) \) description is \( B_{\text{ChPT}} = (1.003 \pm 0.056) \times 10^{-6} \). The uncertainties are dominated by the statistical errors while including a small experimental systematic contribution.

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