On the use of the Gleeble® test as a heterogeneous test: sensitivity analysis on temperature, strain and strain rate

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Abstract. The development of hot and warm forming processes for the manufacture of complex-shaped alloy components has been increasing. At the same time, the knowledge of the effect of certain variables involved in such processes such as temperature and strain rate is essential for an accurate modelling of the materials and processes. This work provides a sensitivity study on the effect of different testing conditions variables on beta titanium-molybdenum (Ti-Mo) alloys. Data from quasi-static uniaxial tensile tests performed on a dog bone shaped specimen using a Gleeble® machine is post-processed with Aramis digital image correlation (DIC) software and numerical models of the tests are developed using Abaqus® finite element analysis (FEA) software. Results considering different temperature conditions, different applied strain rates and different gauge lengths for the determination of the mechanical properties of the material are discussed.

1. Introduction
Titanium alloys have been of high interest for industries such as aeronautical, biomedical, luxury and marine due to their lightweight, high structural properties, excellent corrosion resistance, biocompatibility and also their mechanical properties that allow flexible forming processes for the manufacturing of thin and light parts. However, under certain conditions, such materials may present plastic instabilities (serrated yielding and macroscopic localised bands of plastic strain) associated to the Portevin-Le Chatelier (PLC) effect. Luo et al. [1] have performed an experimental study based on digital image correlation (DIC) to evaluate the effect of temperature and applied strain rate on the PLC bands that appear in Ti-xMo alloys. They performed uniaxial tensile tests using a Gleeble® 3500 system coupled with Aramis digital image correlation (DIC) software [2].

This work intends to provide a first step in the numerical modelling of such experiments initially investigated by Luo et al.. More precisely, an analysis on the use of the Gleeble® test to extract the material properties to be used as inputs in numerical models is here presented. It should be noticed that, at this stage, the focus is not on the PLC effect but the mechanical tests themselves and their interpretation for an accurate numerical modelling.

Gleeble® thermal-mechanical testing systems [3] are commonly used to extract constitutive data of sheet metals that are used in hot and warm forming processes and, therefore, experience complex temperature profiles. Such systems are known for enabling accurate control of temperature and testing speed. However, the heating by Joule effect and the heat loss that occurs due to the water cooling of the grips holding the specimen during the tensile tests may become
a limitation due to the generation of temperature gradients and non-homogeneous deformation in the specimen [4, 10, 5, 6, 7]. At the same time, even performing such tests at constant quasi-static applied strain rates, spatial variations on the strain rate along the specimen are also experimentally observed [8, 9]. This means that the material properties characterisation may be affected as each part of the material deforms differently due to the variation of both temperature and strain rate dependent flow properties. Thus, for an accurate estimation of the material properties, such heterogeneities may need to be taken into consideration. This is particularly important for the case of titanium alloys given their low thermal conductivity and high sensitivity of deformation resistance on temperature when compared with other alloys [7].

A number of works in the literature have reported the experimental flow stress sensitivity of metal alloys to both temperature and strain rate. Typically, for metallic materials such as aluminum alloys [10], magnesium alloys [11], boron steels [12, 13, 6] and dual-phase steels [9] a negative sensitivity to temperature and positive sensitivity to strain rate has been reported. However, the reported flow stress sensitivity of titanium alloys to temperature and strain rate does not follow the ”traditional” behaviour due to the effect of PLC bands [1, 14, 7]. Although in compression tests (not tension) some titanium alloys have presented as well a negative sensitivity to the temperature and positive to the strain rate [14, 7]. Others, in tension, have shown a negative sensitivity to strain rate and a variable sensitivity to the temperature, depending on the chemical composition of the alloy [1].

Considering the flow stress sensitivity to such variables may, however, not be enough to validate the numerical prediction of Gleeble® tests. The heterogeneous spatial distribution of both temperature and strain rate may need to be taken into account [8, 9]. Nonetheless, investigations measuring and quantifying the influence induced by both temperature and strain rate heterogeneities in the testing procedures and in their numerical modelling are still lacking.

To replicate Gleeble® tests through FE models, DIC information as well as the test conditions are used to define the model geometry, boundary conditions and material properties [15, 8, 7]. However, the most adequate procedure on how to extract and use such information is not clearly defined. That is the case, for instance, of (1) the choice of the adequate gauge length to calculate the average strains, (2) the decision of modelling the full specimen or just a part of the gauge area or even (3) the type of numerical model, e.g., as thicker specimens may present temperature gradients in thickness some authors have implemented a coupled electro-thermo-mechanical model considering the full specimen and the grips to capture such gradients [7].

This work suggests a procedure to extract the material properties from full field data considering the smallest and the largest gauge lengths. Simulations of the largest section possible of the Ti-Mo specimen at different conditions of temperature and strain rate are analysed.

2. Experimental procedures and calculation methods
The nominal composition of the tested material is Ti-18Mo and its chemical composition (wt.%) is: Mo 17.87 % and Ti 82.13 %. The material density ($\rho_{\text{Ti-Mo}} = 11.91 \text{ g/cm}^3$) was calculated as the sum of the weighted densities of each component. The tensile specimens are cut from a rolled sheet with 1 mm thickness according to the dimensions provided in figure 1. The longitudinal direction ($x$ direction in Fig. 1) is aligned with the rolling direction of the sheet.

For the uniaxial tensile tests at elevated temperatures, the specimens were heated to 250 and 350 °C (temperature imposed in the centre) using a constant heating rate of 10 °C/s, followed by a holding time of 10 s prior to the test [1]. The temperature during each tensile process is considered constant as the fluctuations measured by three thermocouples, TC1, in the centre of the specimen, TC2 and TC3, both ±5 mm distant from the centre, as shown in Fig. 1, are maintained within 2 °C (see examples of temperature evolution in Fig. 2). As shown in Fig. 2, the first step consists in the heating of the specimen. When the imposed temperature is reached in the centre of the specimen, the mechanical load is applied.
Figure 1: Dimensions of the tested specimens and DIC region of interest (ROI).

Figure 2: Temperature measured by each thermocouple, TC1, TC2 and TC3, and applied load for the tests at a strain rate of $10^{-3}$ s$^{-1}$ and temperatures of 250 $^\circ$C [left] and 350 $^\circ$C [right].

Temperature and load data are recorded by the Gleeble® machine while the DIC full-field displacement and strain data are obtained by Aramis (using two digital cameras with a rate of 10 frames per second). However, Aramis and Gleeble® data are not synchronised and are recorded at different time intervals. Thus, for the synchronisation, the first instant of positive load, ref. time in Fig. 2, is used as reference. Additionally, the Gleeble load-time curves are used to plot the Aramis load-, displacement- and strain-time curves at the same time intervals as the Gleeble® data. This is performed using linear interpolations. The initial time $t_{num} = 0$ s shown in Fig. 3 is defined at the beginning of the load increase observed in Fig. 2.

The variation of temperature along the specimen length (longitudinal direction) is determined by fitting a curve (2nd order polynomial function) to the three points of average temperature measured by the thermocouples during the test, as shown in Fig. 3. In the region where the sectional area of the specimen is constant, i.e. within the 15 mm gauge length, the temperature from the centre to the extremities decreases from 350 to around 250 $^\circ$C, which corresponds to a temperature gradient of 13 $^\circ$C/mm. Variations of temperature in the transversal direction (y direction) are not measured. However, according to the literature, for specimens of such small dimensions, those variations are usually not significant in quasi-static processes [4, 9].

For the DIC analysis on the performed tests, the selected region of interest (ROI) is indicated in Fig. 1 by the coloured region, which has an approximate area of $18 \times 2.5$ mm$^2$. Since this area is manually selected, it slightly varies between tests. For the strain measurement, a subset window or facet size of 9 px and a step size of 4 px is used. A sensitivity study was performed in advance to determine such parameters.

The average displacements in $x$, $y$ and $z$ were extracted from the boundaries of the ROI, i.e. at $x \approx -9$ mm, here called fixed side, and $x \approx 9$ mm, here called loaded side. The obtained displacement-time curves are provided in Fig. 3. In order to consider zero displacements at the
beginning of the test, the observed initial displacements associated with dilatation effects are removed (see adjusted displacement curves in Fig. 3).

To obtain the true stress-strain curves, the Cauchy stress tensor component $\sigma_{xx}^C$ is calculated by the equation:

$$\sigma_{xx}^C = (F/S_0) \exp(\epsilon_{xx}), \quad (1)$$

where $F$ represents the applied load, $S_0$ the initial cross-sectional area and $\epsilon_{xx}$ the average logarithmic strain. In this work, two different stress-strain curves are obtained by considering two different gauge lengths to calculate the average strain: (1) a larger gauge length of approximately 15 mm, which corresponds to the maximum length with constant cross-sectional area and (2) a smaller gauge length of less than 2 mm, selected where the strain localisation occurs (different for each test) with the aim of capturing larger strain values. Here, the different analysed datasets are mentioned as GLOBAL and LOCAL, for the larger and the smaller localised gauge areas, respectively. Those two different stress-strain curves are used to obtain the material hardening curves by visually fitting a curve between them using the Swift-Voce hardening law that describes the isotropic yield stress as:

$$\sigma_Y = \delta [K(\epsilon_0 + \epsilon_p)^n] + (1 - \delta) [\sigma_{Y0} + R_{sat}(1 - \exp(-\beta \epsilon_p))], \quad (2)$$

where the first and second terms of the equation correspond, respectively, to the Swift and to the Voce law and $\delta$ is the weighting factor. $\epsilon_p$ is the equivalent plastic strain and the parameters $K$, $\epsilon_0$, $n$, $\sigma_{Y0}$, $R_{sat}$ and $\beta$ are material parameters to identify. The plastic strain is derived from the Hook’s law and the total strain decomposition assumption:

$$\epsilon^p = \epsilon - \epsilon^e = \epsilon - \sigma_{xx}^C/E, \quad (3)$$

where $\epsilon$, $\epsilon^p$ and $\epsilon^e$ are, respectively, the total, plastic and elastic strains and $E$ is the Young’s modulus, which is obtained from the slope of the stress-strain curves.

For the calculation of the strain rate, $\dot{\epsilon}_{xx} = \partial \epsilon_{xx}/\partial t$, the forward finite differences method is used considering time intervals of 10 s. Before such calculations, in order to minimise errors from noisy data, the strain-time curves where smoothed using the exponential smoothing method:

$$\epsilon^s_t = \alpha \epsilon_{t-1} + (1 - \alpha) \epsilon^s_{t-1}, \quad (4)$$

where $\epsilon^s_t$ and $\epsilon^s_{t-1}$ represent the forecasted/smoothed strain for the current and previous time period, respectively; $\alpha$ is a smoothing constant between 0 and 1 (here, $\alpha = 0.2$ is used) and $\epsilon_{t-1}$ is the actual value of strain for the previous time period.
3. Numerical modelling: geometry, boundary conditions and material properties

FE simulations using explicit dynamic analysis are performed to study the effect of temperature and strain rate in the replication of the quasi-static tensile tests on Ti-18Mo alloys. For that, a 3D model of the ROI shown in Fig. 1 is developed using Abaqus®/Explicit 2018 [16]. The model geometry is defined by the experimental coordinates \((x, y)\) of the black points located in the vertices of the ROI represented in the DIC mesh of Fig. 4, which corresponds to the coloured region of Fig. 1. For the model thickness, only half is considered (5 mm) and a z-symmetry boundary condition is imposed. The \(x\) and \(y\) displacement curves, adjusted curves of Fig. 3, are imposed to all nodes of the corresponding side of the model as shown in Fig. 4. As a displacement \(\Delta u_x = 1.18\) mm is applied in 115 seconds (0.01 mm/s), a mass scaling factor of \(1 \times 10^6\) is used to speed up the analysis. It should be noticed that the influence of simulating a ROI longer than the constant width area has not been investigated yet. However, it is used only to apply displacement boundary conditions. The average strains, as explained in the previous section, are calculated only over gauge areas with a constant width.

For the simulations requiring to account with the longitudinal temperature profiles, a temperature analytical field using the equation provided in Fig. 3 is also imposed as an initial boundary condition to all nodes of the model. It should be noticed that Abaqus® only takes into account this boundary condition when temperature-dependent material properties are provided. Given the reduced thickness of the specimen, the variation of the temperature along the \(y\) axis is assumed constant. The generated mesh is composed of eight-node linear brick reduced integration elements (C3D8R) with enhanced hourglass control (default) and an average size of 0.14 mm, which is approximate to the dimensions of the mesh from the DIC analysis, as can be seen in Fig. 4, in order to obtain a similar number of measurement points as in the experiments.

Isotropic elasto-plasticity is assumed for the material mechanical behaviour. The material properties to be used as input in the numerical model include the material density, the Young’s modulus, the Poisson’s ratio and the hardening curves. Both the density and the Poisson’s ratio \((\nu = 0.33)\) are considered to be the same for different temperatures and strain rates. The Young’s modulus at a certain temperature and strain rate is determined by the slope of the Cauchy stress - logarithmic strain curve of the test performed under such conditions. The same for the hardening curve at each specific condition of temperature and strain rate. Considering such properties, four different scenarios are simulated in this work:
num: a material with no temperature nor strain rate dependency, so a single value for the Young’s modulus and a single hardening curve are considered;

num_SR: a strain rate dependent material, where an additional hardening curve obtained from a test at a lower strain rate is considered;

num_T: a temperature dependent material, where an additional value for the Young’s modulus and an additional hardening curve obtained from a test performed at a lower temperature are considered; and

num_T_SR: a temperature and strain rate dependent material, which includes an additional Young’s modulus at a lower temperature and 3 additional hardening curves, 1 at a lower temperature, same strain rate, and 2 at a lower strain rate and two different temperatures.

To define the isotropic hardening model in Abaqus®, the yield stress $\sigma_Y$ is given as a tabular function of the plastic strain and, depending on the scenario, as a function of temperature and/or strain rate. The yield stress at a given state is simply interpolated from this table of data, and it remains constant for plastic strains exceeding the last value given as tabular data.

For strain rate dependent materials, the hardening curve at static strain rate is also a required input. Since such data is not available for the studied material, the same hardening curve obtained at the lowest strain rate is used, meaning the material is considered to not change its plastic mechanical behaviour for lower applied strain rates.

4. Results and discussion

In this section, the material properties extracted from tests performed at two different temperatures (250 and 350 °C) and two different strain rates ($10^{-4}$ and $10^{-3}$ s$^{-1}$) are provided. After that, the results of the numerical simulation of the tensile test performed at 350 °C and $10^{-3}$ s$^{-1}$ considering the four different scenarios described in section 3 are provided and compared with the experimental results. It should be noticed that the presented results correspond to selected tests at each different condition of temperature and strain rate, however, the analysis of two tests at each different condition was performed to verify the repeatability of the results.

Fig. 5 provides the Cauchy stress - logarithmic strain curves obtained from different average strains according to the selected gauge length (GLOBAL/LOCAL). It can be observed that smaller gauge lengths allow to significantly obtain larger values of strain. However, smaller gauge lengths, when centred in the localised strain, also increase the noise in data. For this reason, the Young’s modulus is calculated from the slope of the GLOBAL stress-strain curves. The flow stress curves also demonstrate that the studied Ti-18Mo alloy has a negative sensitivity to both temperature and strain rate, i.e., the stress decreases when the temperature increases from 250 to 350 °C and/or when the strain rate increases from $10^{-4}$ to $10^{-3}$ s$^{-1}$.

The hardening curves obtained from both the GLOBAL and LOCAL average strains are shown in Fig. 6. As can be observed, the Swift-Voce law, adjusted by a trial-and-error visual fitting approach, provides a satisfactory fit with the experimental data. At a first stage of this work, the use of just the LOCAL data to get the hardening curve was tested. However, it demonstrated not to be the best approach as the numerical flow stress curves would present lower values of stress compared with the real/experimental curves. The same was observed when using just the GLOBAL data, with resulting stress values higher than expected and lower strain ranges. Adjusting a curve between the GLOBAL and LOCAL experimental curves allow to obtain the closest approximation to the real material behaviour as it is demonstrated ahead. The Swift-Voce parameters of each curve, as described in equation 2, are provided in Table 1, where
Figure 5: Cauchy stress - logarithmic strain curves for the selected tests using the average strain from a large (GLOBAL) and localised (LOCAL) gauge lengths.

10-3, 350, 10-3, 250, 10-4, 350 and 10-4, 250 correspond to the tests at the imposed conditions of 10^{-3} \ s^{-1}/350 \ ^\circ C, 10^{-3} \ s^{-1}/250 \ ^\circ C, 10^{-4} \ s^{-1}/350 \ ^\circ C \ and \ 10^{-4} \ s^{-1}/250 \ ^\circ C, \ respectively.

Figure 6: Hardening curves obtained experimentally and fitted Swift-Voce hardening for the selected tests performed at 4 different conditions of temperature and strain rate.

Table 2 presents the material properties extracted from the analysed tests. The values of the Young’s modulus and the initial yield stress correspond to the average of two tests performed under the same conditions of temperature and strain rate. The maximum logarithmic strains before rupture correspond to the values obtained for the selected tests.

To analyse the influence of temperature and strain rate in the FE replication of the tensile tests, 4 different scenarios are simulated. Fig. 7 shows the material inputs (hardening curves and values of Young’s modulus) considered in each different scenario. For the scenarios considering material strain rate dependency (SR), the 10^{-4} \ s^{-1} curves are used for the static strain rate.

The validation of the numerical simulations with the experimental results obtained by DIC is presented in Fig. 8 through the representation of the nominal stress - relative elongation curves. Fig. 9 shows, for each case (the experimental and the four numerical scenarios), the strain maps in the longitudinal direction (\(\epsilon_{xx}\)) at two different stages of the tensile tests: (1) at
Table 1: Swift-Voce hardening parameters for the tests performed at 4 different conditions.

| Param | Test 10-3 | 10-4 | 10-3 | 10-4 |
|-------|-----------|------|------|------|
| δ [-] | 0.88      | 0.96 | 0.40 | 0.50 |
| K [MPa] | 840      | 700  | 800  | 990  |
| ε₀ [-] | 0.050     | 0.030| 0.015| 0.040|
| n [-]  | 0.28      | 0.19 | 0.20 | 0.18 |
| σ₀ [MPa] | 450      | 500  | 550  | 500  |
| Rₘₐₜ [MPa] | 250      | 230  | 500  | 80   |
| β [-]  | 200       | 1000 | 4    | 400  |

Table 2: Young’s modulus $E$, initial yield stress $σ₀$, and maximum logarithmic strain before rupture $ε_{max}$ obtained for the 4 different testing conditions. $E$ and $σ₀$ are average values from two tests.

| Test | $E$ [GPa] | $σ₀$ [MPa] | $ε_{max}$ [-] |
|------|-----------|------------|---------------|
| 10-3,350 | 74      | 409       | 0.54          |
| 10-3,250 | 64      | 450       | 0.59          |
| 10-4,350 | 65      | 413       | 0.62          |
| 10-4,250 | 63      | 514       | 0.46          |

Figure 7: Difference in plastic and elastic properties considered in each simulated scenario.

Figure 8: Experimental and numerical results for the nominal stress - relative elongation curves.

Overall, the FE models approximate well the DIC results up to the maximum load. After that, the numerical model is not capable of capturing the material softening.

Comparing the most simple model (num) with the models assuming a temperature dependent material (num_T and num_T_SR), it is clear that adapting the boundary conditions (specifically, in this case, considering the temperature spatial variation), allows to optimise the simulation of the mechanical test. In fact, only the temperature dependent models (T) achieve the expected strain localisation, as can be observed in Fig. 9. At the same time, only these two models result in a wider range of strain. However, the maximum strain value is only achieved by the model with both temperature and strain rate dependency (num_T_SR) and uniquely at the instant of...
maximum load ($\varepsilon_{\text{max}} \approx 0.06$). The experimental value of $\varepsilon_{\text{max}}$ indicated in Fig. 9 is 0.144 but this corresponds to a measurement in two specific points in the specimen containing a defect in the speckle pattern (red-yellow points observable in the figure). Indeed, the localised region is represented by the green colour which corresponds to $\varepsilon_{\text{max}} \approx 0.06$. At the same time, for the experimental scenario before rupture, the indicated value of maximum strain, in the red region, is 0.541 but the green region presents an approximate value of 0.26.

A more detailed description of the DIC and numerical (num_T_SR) strain profiles along the specimen during the tensile test is shown in Fig. 10. The presented data is extracted from a line parallel to the tensile direction $x$ in the middle of the specimen ($y = 0$). The superimposed experimental and numerical data up to the maximum load demonstrates that the numerical model is capable of predicting not only the maximum value of strain, as previously observed, but also the shape of spatial variation of strain along the specimen length.

Figure 10: Strain spatial distribution obtained by DIC (exp) and superimposed numerical results from the two temperature dependent models up to the instant of maximum load.

Since the strain rate gives a more accurate picture of the instantaneous evolution of the structure, its spatial distribution at different stages of the tensile test up to the maximum load obtained experimentally (exp) and by the two numerical models assuming a temperature dependent material (num_T and num_T_SR) is presented in Fig. 11. The num_T_SR is capable of predicting well the maximum strain rate that occurs in the middle of the specimen ($x = 0$ mm) when reaching the maximum load. However, the strain rate decreases faster than desired when going far away from the centre. Surprisingly, the num_T model is capable of predicting, at the instants near the maximum load, the (almost) constant strain rate from the centre up to around $\pm 3$ mm but at a lower value of strain rate as expected. On the other hand, the results for this particular model present not only a single strain rate "peak" in the centre of the specimen but also two additional smaller "peaks" that start near the centre of the ROI (even before the main "peak" starts appearing at 40-50 s) and move in the direction of the extremities of the ROI as the deformation proceeds (similar with the numerical observations reported in [8]).
This also demonstrates the importance of modelling a larger region of the specimen than the commonly used small lengths where the temperature remains approximately constant. The latter may omit important information concerning the accuracy in predicting the evolution of the structure. Such additional peaks are not observed by the num_T_SR model, which makes it a better approximation of the DIC results.

Figure 11: Evolution of the strain rate profiles along the specimen length (at y=0) up to the instant of maximum load obtained by DIC (exp) and by the two temperature dependent models.

5. Conclusions

The present work demonstrates that the temperature gradients and the strain rate spatial distribution along the specimens in Gleeble® tensile tests that are observed experimentally have a significant impact in the deformation behaviour of the material and should be taken into consideration in the numerical simulations to ensure an adequate replication of the tests. Moreover, the presented results show that the use of the Gleeble® test, if performed at two different conditions of both temperature and strain rate, can provide rich information for a satisfactory numerical prediction of the material behaviour over a large gauge length. However, specimens of different materials, size and shapes (as suggested by [4] and [5]), different gauge lengths (also suggested by [4] and [6]), as well as different testing conditions (such as higher temperatures and different strain rates) should be analysed in order to validate any suggestion for a standardisation of the Gleeble tensile testing for mechanical properties measurement.

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