Movement of liquid droplets in a shear flow

A I Fedyushkin

Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, prospekt Vernadskogo 101-1, Moscow, 119526, Russia

E-mail: fai@ipmnet.ru

Abstract. The problem of the motion of a particle around a particle in a shear flow for the conditions of weightlessness and normal earth gravity is considered. The cases where the particle density is greater or less than the density of the main buffer moving fluid (oil particles in water and water particles in oil) are presented. On the basis of numerical modeling, the nature of the influences of gravitational, viscous, lifting and frontal forces on the trajectory of fluid particles in a shear fluid flow are shown. For the considered liquid particles and stratification of the shear velocity field it is established that the minimum distance of approach of liquid droplets is equal approximately 0.07 of diameter particles.

1. Introduction

Fluid flows with shear velocity fields can be observed in nature and in industrial and technical channels with gas or liquid separation, for example, near-surface atmospheric flow, bottom flow in rivers and seas, in rotating liquid volumes. Such flows are often used for separation of inclusions in multiphase media, as well as for laboratory studies of the properties of various liquids. The flow of liquid and solid particles in the shear velocity field is widely studied in connection with many technical applications for the separation of liquid and solid inclusions in a fluid. In addition, the problem of small particles flowing around a small cylindrical obstacle (thread) is relevant today for studying the protective properties of individual masks in the fight against the spread of viruses.

In this paper, the behavior of small liquid droplets in a slow laminar flow of a buffer liquid is considered. Cases with particle densities greater or less than the density of the main buffer moving fluid (oil particles in water and water particles in oil) are presented. Based on numerical modeling, the nature of the influence of gravitational, viscous, lifting, and drag forces on the trajectory of liquid particles in a shear fluid flow is shown. The minimum distance of approach of liquid small drops and a streamlined drop is shown.

2. Problem statement and mathematical model

It is assumed that the number of particles is too small and they do not affect the main flow, and the shear flow of the buffer fluid affects the movement of particles. In figure 1 a simulation region and the velocity vector field (left) and the particle tracks around the particle with center located in begin of coordinates (right) are shown. On the right, in figure 1 a streamlined particle shows at an enlarged scale.
Figure 1. The scheme of simulation region, field of vector of velocity (at the left) and tracks of particles around a cylindrical particle which there is in the beginning of co-ordinates (on the right).

The particle in the center of calculation region is simulated by not deformable cylindrical surface and rotatable on account of a shear flow. Cases of the task for surface shear stress equal to zero and tasks of a surface tension for water and oil are considered. The calculations have shown that velocity on a cylindrical surface coincides with angular speed \( \omega = \frac{1}{2} \text{rot} V \) (where \( V, \omega \) are vectors of velocity and angular rotation of an element of environment in a point).

Mathematical simulation is performed on the basic of numerical solutions of unsteady 2D Navier-Stokes equations for incompressible laminar fluid flow [1, 2].

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right),
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + F_g,
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

where \( x, y \) - horizontal and vertical Cartesian coordinates; \( u, v \) - components of the velocity vector; \( t \) - time; \( P \) - pressure; \( \rho \) - density and \( F_g = -g \) - gravity force; \( g \) - gravitational acceleration of the earth’s free fall; \( \nu \) - coefficients of kinematic viscosity. Equations (1-3) were solved numerically by the control volume method.

A movement of the particles is calculated in Lagrangian variables.

\[
\frac{d\mathbf{u}_p}{dt} = \mathbf{F}_D + \mathbf{F}_M + \mathbf{F}_L + \mathbf{F}_{pg}
\]

Equation (4) was solving by the Runge-Kutta method of the 4th order of accuracy. In the model of interaction of particles with the fluid flow, the friction forces \( \mathbf{F}_D \), the virtual masses \( \mathbf{F}_M \), and the Saffman forces \( \mathbf{F}_L \) and gravity forces \( \mathbf{F}_{pg} \) were taken into account. The \( \mathbf{F}_D = (\mathbf{u} - \bar{\mathbf{u}}) / \tau_r \) is the drag force per unit particle mass. Here \( \tau_r = \frac{\rho_p d_p^2}{18 \nu p C_d \text{Re}} \) is the droplet or particle relaxation time, where \( C_d \) is the drag factor, \( \bar{\mathbf{u}} \) is the fluid phase velocity, \( \bar{\mathbf{u}}_p \) is the particle velocity of the particle. \( \text{Re} \) is the
relative Reynolds number, which is defined as \( Re = \frac{\rho u \| \vec{u} - \vec{u}_p \|}{\nu} \). The \( p \) index for velocity, density and diameter refers to particles. The “virtual mass” force \( F_M \) can be written as \( F_M = C_{vm} \frac{\rho}{\rho_p} \left( \vec{u}_p \nabla \vec{u} - \frac{d\vec{u}_p}{dt} \right) \), where \( C_{vm} \) is the virtual mass factor in this work was equal to 0.5. The virtual mass force is important when the density ratio \( \rho / \rho_p \) is greater than 0.1. The Saffman’s lift force for small particle Reynolds numbers is \( F_l = -2K \sqrt{\nu} d_{ij} \left( \vec{u} - \vec{u}_p \right) \) where \( K=2.594 \) and \( d_{ij} \) is the deformation tensor [4, 5]. Particle gravity force is \( F_g = \frac{g_p}{\rho} \frac{\rho - \rho_g}{\rho_p} \).

Initial distances of particles were 2d from the center of region and they regular intervals located on a vertical on a range from \( y=0 \) to \( y=0.5d \). Initial speeds of particles corresponded to velocities of environment in the given points.

In given paper results of simulation are presented for following values of parameters: a diameter of particles \( d=0.0001m \), \( X = 0.0025m \), \( Y=0.001m \), \( \alpha = 300 \text{sec}^{-1} \). Boundary conditions for the velocity at the input and on upper and lower walls corresponding to the following expressions for velocity components \( u = \alpha y \), \( v = 0 \), at the output boundary the pressure \( p=0 \) was set.

Two variants of particles and a buffer liquids are considered with following properties:
1) the drops of diesel oil (a density is 730 kg/m\(^3\)) in water (a density is 875 kg/m\(^3\), a viscosity is 0.000589 kg/m sec),
2) the drops of water in diesel oil (a density is 730 kg/m\(^3\), a viscosity is 0.0024 kg/m sec).

3. Results
The problem of particle motion around a particle in a shear flow is considered for conditions of weightlessness and normal earth gravity. Cases with particle densities greater or less than the densities of the main buffer moving fluid (oil particles in water and water particles in oil) are presented. The time dependences of position (y co-ordinate) of ten oil particles in water shear flow are shown in figure 2 for normal and weightlessness gravity conditions.

The time dependences of position (y co-ordinate) of ten water particles in oil shear flow are shown in figure 3 for normal and weightlessness gravity conditions. In all cases, at the initial moment, the particles were equidistant on the x axis at a distance (0.5 d) from center of the streamlined particle and were located at the heights shown in figures 2 and 3 on the ordinate axis at \( t=0 \) sec.

Despite the fact that the initial vertical location of the particles differed by a small amount, the trajectories of the particles changed significantly over time. Some particles did not reach the streamlined particle and turned in the opposite direction. On the trajectories of the particles, you can see the areas of their approach to the streamlined particle and their departure from it.
Figure 2. Y co-ordinate position of the oil particles in water from time for Ground (a) - $g=9.81\text{m/s}^2$) and weightlessness conditions (b) - $g=0\text{ m/s}^2$).
Figure 3. Y co-ordinate position of the water particles in oil from time for Ground (a) - $g=9.81 \text{m/s}^2$ and weightlessness conditions (b) - $g=0 \text{ m/s}^2$.

Time dependences of distances of two oil running particles to a surface of particle from x co-ordinate are presented in figure 4. The given two oil particles (in figure 4) are chosen, as the coming most nearer to the particle for the given series of calculations.

The minimum distance of closeness with the cylindrical particle approximately is 0.07$d$. The minimum approach distance of liquid droplets is important to know when designing shear flow devices for separating petroleum products in an electric field.
4. Conclusions

The results of numerical simulation have shown character of influence of gravity, viscosity, lift and drag forces on trajectories of a movement of water and oil particles in a shear flow. The minimum distance between liquid small drops and a streamlined drop is shown. The minimum distance of approach of liquid drops to a liquid streamlined drop is found to be equal approximately 0.07 d.

Acknowledgements

The study was supported by the Government program (contract # AAAA-A20-120011690131-7) and was funded by RFBR, project number 20-04-60128.

References

[1] Landau L D and Lifshitz E M 1987 Fluid Mechanics 6 (Oxford: Pergamon Press 2nd ed.) p 556
[2] Batchelor G K 2002 An Introduction to Fluid Dynamics (Cambridge: University Press) p 615
[3] Saffman P G 1965 The lift on a small sphere in a slow shear flow J. Fluid Mech. 22 385–400
[4] Li A and Ahmadi G 1992 Dispersion and Deposition of Spherical Particles from Point Sources in a Turbulent Channel Flow Aerosol Science and Technology 16 209–226