Time dependence of the survival probability of an opinion in a closed community

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Abstract

The time dependence of the survival probability of an opinion in a closed community has been investigated in accordance with social temperature by using the Kawasaki-exchange dynamics based on previous study in Ref. [1]. It is shown that the survival probability of opinion decays with stretched exponential law consistent with previous static model. However, the crossover regime in the decay of the survival probability has been observed in this dynamic model unlike previous model. The decay characteristics of both two regimes obey to stretched exponential.

Keywords: Ising Model; Politics; Random Walk; Sociophysics; Sznajd Model.
1 Introduction

Binary models like Ising-type simulation have a long history. They have been applied by Schelling to describe the ghetto formation in the inner cities of the USA, i.e., to study phase separation between black and white [2]. In the sociophysics context, recently, many social phenomena such as election, propagation of information, predicting features of traffic, migration, opinion dynamics and formation in a social group have been successful modelled based on Ising spin systems using models and tools of statistical physics. With this respect, particularly successful models have been developed by Sznajd [3], Deffuant et al. [4] and Hegselmann and Krause [5].

Among those three models, the one developed by Sznajd is the most appropriate for simulation in networks and lattices, since it consider just the interactions between the nearest neighbors. Indeed, the Sznajd model has been successfully applied to model sociophysical and economic systems [6]. On the other hand, several modifications of the Sznajd model have been studied using different rules or topologies starting from different initial opinion densities [6, 7, 8]. All these models are static (i.e. not dynamic) and they allow for consensus (one final opinion), polarization (two final opinion), and fragmentation (more than two final opinions), depending on how tolerant people are to different opinions.

More recently the striking sociophysical model has been suggested by Aydıner [1] in order to explain the time evolution of resistance probability of a closed community in a one-dimensional Sznajd like model based on Ising spin system. It has been shown that resistance probability in this model decay as a stretched exponential with time. In that model spins does not move on the lattice sites during the simulation, so this model was so-called static. However, in a realistic case, spins i.e., people move in the community i.e., in the space. Social or opinion formation formed depend upon dynamics of the system. Because, there must be a direct connection between opinion dynamics and formation in a social system since the social formation is determined by the dynamics. Meyer-Ortmanns [9] studied recent work in which the condition for ghetto formation in a population with natives and immigrants by using Kawasaki-exchange dynamics in a two dimensional Ising model. She showed that ghetto formation can be avoided with a temperature increasing with time. Similarly, Schulze have also generalized Meyer-Ortmanns work to up to seven different ethnic groups to explain ghetto formation in a multi-cultural societies in a Potts-like model [10].

In this study, we have developed a dynamic version of the Aydıner [1] model by combining the Aydıner and Meyer-Ortmanns [9] models based on one-dimensional Ising model.

2 Kinetic Model and Simulation

In one-dimensional static model [1], each site carries a spin which is either spin up (+1) or spin down (-1) randomly. Spin up (+1) represent the host people and spin down (-1) represent the soldier. The host people always against occupation, and, on the other hand, soldier always willing to continue occupation, who always have the opinion opposite of that of the host people. Furthermore, the community member i.e., spins doesn’t also move on the lattice during the process.

In this model, initially, it was assumed that there was a over all consensus among member of the community against occupation even if some exceptions exist. One expects that host people obey to this consensus at least initially. In this sense, community behaves as polarized at zero social temperature [13] against occupation just like Ising ferromagnet at zero temperature.

It was conjectured that host people are influenced by soldiers even though they against occupation owing to they are exposed to intensive biased information or propagation. Soldiers affect the host people and force to change their opinion about occupation. Effected people may change their own opinions depending on resistance probability of the nearest neighbors about occupation. Moreover, effected host people affect neighbors. Such a mechanism depolarize the polarization (resistance probability) of all host people. Hence social polarization destroy.
However, soldiers, unlike host people, have not been influenced by the host people. Their opinion about justifying the occupation does not change during the occupation process, since they may be stubborn, stable or professional etc., who behaves like persistent spins in Ising spin system. It is means that the probability of the against occupation of a soldier is always zero.

If we summarize, we can say that none spins does flip fully in the system. Spin up always remains spin up, and spin down always remains spin down. In this respect, the probability of against occupation of host people can be interpreted as a survival probability of opinion of host people about occupation under above considerations. In this sense, the survival probability \(W_i\) of opinion of host people indicate equal to 1 at least initially and, on the other hand, the probability of against occupation of soldier equal to zero, which means that soldier behaves as a trap point lattice which depolarize the survival probability of opinion of host people.

Of course, one may suggest that there are many different number of opinions in society, however, it is possible to find that a society being formed two-state opinion in a real case. Therefore this model is a good example for two-state opinion model as well Galam contrarian model [14] even though it seems that it is very simple. Furthermore, in real social systems, people move on the space, i.e., lattice. Therefore, in this study, we assumed that people i.e., spins randomly move on the lattice through the Kawasaki-exchange dynamics contrary to previous model.

The survival probability \(W_i\) for a people at site \(i\) at the next time \(t+1\) is determined with the survival probability of nearest-neighbors with previous time \(t\) as

\[
W_i(t + 1) = \frac{1}{2}[W_{i+1}(t) + W_{i-1}(t)].
\]  
(1)

We note that the survival probability for all site are calculated as synchronously.

Randomly motion of the spins i.e., people on the lattice through the Kawasaki-exchange dynamics. Firstly, a spin pair is chosen randomly and then it is decided whether spin pair exchange with each other or not. In this approach, the nearest-neighbor spins are exchanged under heat-bath dynamics, i.e., with probability \(p \sim \exp (-\Delta E/k_B T)\), where \(\Delta E\) is the energy change under the spin exchange, \(k_B\) is the Boltzmann constant, and \(T\) is the temperature i.e., social temperature or tolerance. Hence, to obtain probability \(p\) we need to calculate \(E_1\) and \(E_2\) which correspond to energy of the spin pair at first position and after exchange with position of spins, respectively. Energy \(E_1\) and \(E_2\) can be calculated in terms of the survival probability instead of spin value as

\[
E_1(t) = aW_i(t) + bW_{i+1}(t)
\]
(2a)

\[
E_2(t) = aW_{i+1}(t) + bW_i(t)
\]
(2b)

where

\[
a = [W_{i-1}(t) + W_{i+1}(t)]
\]

and

\[
b = [W_i(t) + W_{i+2}(t)]
\]

Energy difference is written as \(\Delta E = E_2 - E_1\) from Eq. 2a and 2b.

In addition, the total survival probability of opinion of host people at the any time \(t\) can be obtained over each person for any \(r\) configuration as

\[
P_r(t) = \frac{1}{m_0} \sum_i W_i(t)
\]
(3)

where \(m_0\) is the initial number of host people. On the other hand, the averaged survival probability at the any time \(t\) can be obtained from Eq. 3 over the independent configuration as

\[
< P(t) > = \frac{1}{R} \sum_{r=1}^R P_r(t)
\]
(4)

where \(R\) is the number of different configurations.
Results and Discussion

We have adopted the Monte Carlo simulation technique to the one-dimensional sociophysical model using the lattice size $L = 1000$ with periodic boundary condition, and independent configuration $R = 1000$ for the averaged results. The simple algorithm for the simulation is as follows: i) at the $t = 0$, Eq. (4) is initially calculated, ii) for $t > 0$ a spin pair is randomly chosen, and then it is decided whether the spin pair exchange or not with the probability $p \sim \exp(-\Delta E/k_B T)$, this step is repeated $L$ times, iii) after ii-steps are completed, Eq. (4) is recalculated again, and to continue this procedure goes to step ii.

The simulation results are as follow: We have firstly plotted simulation data versus time in Fig. 1 in a several manner. It is explicitly seen from Figs. 1(a)-(c) that there are no power, exponential and logarithmic law dependence in our simulation data, respectively. However, as seen Fig. 1(d), data well fit to the stretched exponential function as

$$< P(t) > \sim e^{-\lambda t^\beta} \quad (5)$$

where $\lambda$ is the relaxation constant, and $\beta$ is the decay exponent of the survival probability. This result indicate that the time evaluation of survival probability of the opinion of the host people in a closed community has stretched exponential character i.e., Kohlraush-William-Watts (KWW) decay law [11, 12].

It should be tested whether Fig. 1(d) satisfies to stretched exponential or not [15]. Because, as noted by Stauffer, the Fig. 1(d) would work as stretched exponential, if pre-factor of Eq. (5) is equal to 1. However, if pre-factor is less than 1, it may give the impression of stretched exponential form, even for $\beta = 1$. Therefore, it can be plotted $-\ln < P(t) >$ versus suitable powers of $t$, like $t^{1/2}, t^{1/3}$, etc., and find out the best straight line among the powers of $t$ for long times. Hence, $-\ln < P(t) >$ was plotted versus powers of $t$ for $\rho = 0.1$ then the best straight fitting line for long times was obtained for $\beta = 0.8$ for $T = 0.0001$, $\beta = 0.7$ for $T = 0.01$, and $\beta = 0.6$ for $T = 0.1$ as seen in Fig. 3(a)-(c) respectively. These results confirm to this method used to find out stretched exponential exponents in Fig. 1(d), and also all figures in Fig. 2 as mentioned. Also, this test indicates that prefactor in Eq. (5) does not effect results presented.
**Fig. 2**: The time dependence of the survival probability of the opinion of host people decays KWW i.e., stretched exponential with time for different social temperature $T$. The time crossover appears in the time evolution of survival probability of the opinion at low social temperatures. The crossover becomes more clear when social temperature decreases.

**Fig. 3**: (a) $-\ln < P(t) >$ versus $t^{0.8}$ for $T = 0.0001$, (b) $-\ln < P(t) >$ versus $t^{0.7}$ for $T = 0.01$, and (c) $-\ln < P(t) >$ versus $t^{0.6}$ for $T = 0.1$. All figures are plotted for fixed $\rho$ value i.e. $\rho = 0.1$, and solid lines represent fitting curves.
in this paper.

It is concluded that results for high temperatures also consistent with static model \[1\]. But, unlike the static model, time crossover has been observed in dynamic model at low temperatures. In order to investigate the transition we have plotted survival probability versus time for different social temperature \( T \) in Fig. 2. It is clearly seen that the time crossover occurs depend on social temperature. When social temperature decreases, the crossover become more clear. Such a behavior was not observed in a static model. We can bridge the short time regime and the long time regime by a scaling function \( f(\frac{t}{t_c}) \)

\[
\langle P(t) \rangle = e^{-(t/\tau)\beta} f(\frac{t}{t_c})
\]  

(6)

where \( t_c \) indicates the time crossover. For our simulation data, the scaling relation (6) can be written for very long and very short time intervals as

\[
\langle P(t) \rangle \sim \begin{cases} 
 e^{-(t/\tau)\beta_1} & \text{if } t << t_c \\
 e^{-(t/\tau)\beta_2} & \text{if } t >> t_c.
\end{cases}
\]  

(7)

On the other hand, in order see how the decay exponent \( \beta \) depend on soldier density \( \rho \), and social temperature \( T \), we have plotted \( \beta \) versus soldier density \( \rho \) in Fig. 4(a) for \( t < t_c \) and \( t > t_c \) in account to taken different social temperatures, and social temperature \( T \) in Fig. 4(b) for a fixed value of density \( \rho \), respectively.

As seen from Fig. 4(a) that \( \beta_1 \) and \( \beta_2 \) are linearly depend on soldier density both of two regimes at low social temperature. On the other hand, the decay exponent has two different character for \( t < t_c \) and \( t > t_c \) depend on social temperature \( T \) in Fig. 4(b), the decay exponent \( \beta_1 \) decreases with increasing temperature \( T \) for \( t < t_c \), whereas \( \beta_2 \) increases with increasing temperature \( T \) for \( t > t_c \) at low temperatures. However, for relatively high temperatures we roughly say that \( \beta_1 \) approach to \( \beta_2 \) for both two regimes obey to Eq. (7).

Finally, to understand the social temperature and soldier density dependence of the time crossover \( t_c \), we have plotted \( t_c \) versus social temperature \( T \) in Fig. 5(a) for a fixed soldier density.
density $\rho$, and versus soldier density in Fig. 5(b) for fixed social temperature $T$, respectively. It seems from Fig. 5(a) that the crossover transition $t_c$ quite rapidly decrease with increasing $T$, on the other hand, it seems from Fig. 5(b) that it slowly decrease with increasing soldier density $\rho$. We note that as seen inserted figure in Fig. 5(b) the crossover transition $t_c$ depends on soldier density with power law for fixed social temperature.

4 Conclusions

We suggest that the stretched exponential behavior of decay must be originated from model system. The persistent spins i.e., the soldiers doesn’t flip during simulation, therefore they behave as a trap in the system. Hence they play a role diminishing the survival probability of the neighbor spins in the system. Consequently, decay characteristic of the system can be explain due to the trapping states. Another say, this characteristic behavior doesn’t depend on either diffusion dynamics of spins or interaction rules between spins.

Another unexpected behavior is the time crossover in contrast to previous model [1]. We supposed that this amazing result originated from opinion dynamics depend on social temperature. Model allows to the opinion formation with time. Indeed, there is a direct connection between opinion dynamics and formation in a social system since the social formation is determined by the dynamics as depend on the social temperature. For example, in a real spin system, decreasing temperature phase separation may occur in the system. In the sociophysical sense, it means that people who have different opinion are separated each other with decreasing social tolerance, and therefore the ghetto formation or polarization may occur in the system.

It is expected that interactions between soldier and host people is maximum when soldiers are randomly distributed in the community. As social temperature, i.e., tolerance is decreased, however, phase separation occur with time, so this leads to decreasing of the interactions.

In our opinion, the ghetto formation in the system doesn’t leads crossover transition in time because of the ghetto formation is randomly distributed relatively. On the other hand, the time average of survival probability over different configuration effect of ghetto formation.
may probably destroy. So we don’t hope that ghetto formation is not responsible crossover transition. However, polarization must be occurred at low temperature leads to meaningful phase separation in the system. Such a polarization may leads to crossover transition in time.

Stretched exponential behavior indicates mathematically that decay for the relatively short times is fast, but for relatively long times it is slower. One can observe that this mathematical behavior corresponds to occupation processes in the real world. In generally, a military occupation is realized after a hot war. The community does not react to occupation since it occurs as a result of defeat. People are affected easily by propaganda or other similar ways. Therefore, it is not surprised that resistance probability decrease rapidly at relatively short times. On the other hand, spontaneous reaction may begin against occupation in the community after the shock. Hence, community begins by regaining consciousness and more organized resistance may display difficulties for occupants. For long times, the resistance probability decreases more slowly. This means that resistance against occupation extends to long times in practice. At this point, the number of soldiers is also important, because the density of soldiers determines the speed of decaying.

The different regimes have been observed in the decay of the survival probability. These regimes clearly appear particularly at low temperatures. In the case of the social temperature is very low, \( \beta_1 \) is bigger than \( \beta_2 \) which indicates the decay of the survival probability for relatively short time is slower than for relatively long time. This can be interpreted that the resistance of host people against occupation may be broken spontaneously if soldier can wait enough time.

Of course, the mechanism considered in this work can be regarded as simple, but, it would be useful to understand the time evolution of the resistance probability of the community against to occupation in the one-dimensional model under some considerations. We remember that simple social rules lead to complicated social results, hence we believe that the obtained results and model can be applied the real social phenomena in the societies to understand the basis of them.

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