Computing the static potential using non-string-like trial states

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Motivation
- For calculating the static potential with a high resolution we have to work with off axis separated quarks.

- e.g. matching the lattice QCD potential with the perturbative potential to determine $\Lambda_{\overline{MS}}$ in Fourier space. [F. Karbstein, A. Peters and M. Wagner, JHEP 1409, 114 (2014) [arXiv:1407.7503 [hep-ph]]]
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- The quantity of interest is the Wilson loop, which connects the two quarks like a string.
To compute the spatial part of the Wilson loop one has to go over stair-like paths through the lattice.
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- These **stair-like paths** are causing a big **computational** effort for a large number of lattice points.

- Idea: Substitute the spatial part of the Wilson loop by an other object to **avoid** the calculation of **stair-like paths**.
The Technical Part
The spatial Wilson line is needed to ensure gauge invariance of the $q\bar{q}$ trial state.
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- We explore an idea, which has been used in the context of Polyakov loops and the static potential at finite temperature.

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- Consider the covariant lattice Laplace operator:

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\Delta f = \frac{1}{a^2} \left( U_1^\dagger(x - a, y, z) f(x - a, y, z) - 2f(x) + U_1(x, y, z) f(x + a, y, z) \right) \\
+ \frac{1}{a^2} \left( U_2^\dagger(x, y - a, z) f(x, y - a, z) - 2f(x) + U_2(x, y, z) f(x, y + a, z) \right) \\
+ \frac{1}{a^2} \left( U_3^\dagger(x, y, z - a) f(x, y, z - a) - 2f(x) + U_3(x, y, z) f(x, y, z + a) \right)
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- Transformation behavior: $\Delta' = G(x)\Delta G^\dagger(x)$
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- Apply an gauge transformation on the eigenvector-equation.
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- Apply a gauge transformation on the eigenvector-equation.

- We see: $G^\dagger(x)f'(x)$ is again eigenvector to the covariant Laplace operator.
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- In SU(3) the eigenvalues are in general **nondegenerate**. This means:

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- In SU(2) however, the eigenvalues are always two fold degenerate. This means:

\[
\alpha f_1(x) + \beta f_2(x) = G^\dagger(x)f'(x)
\]

- Where \( f_1 \) and \( f_2 \) are an orthonormal basis of the corresponding eigenspace.
Transformation law for SU(3):
\[ f(x)e^{i\phi} = G^\dagger(x)f'(x) \]

Transformation law for SU(2):
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Wilson Line:
\[ U'(x, y) = G(x)U(x, y)G^\dagger(y) \]

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SU(3) - Case:
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SU(2) - Case:
\[ \sum_{i=1}^{2} f'_i(x)f'^\dagger_i(y) = G(x)\left(\sum_{i=1}^{2} f_i(x)f^\dagger_i(y)\right)G^\dagger(y) \]

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- Where \( f_1 \) and \( f_2 \) are an orthonormal basis of the corresponding eigenspace.
We found objects with the required transformation behavior given by
\[ f(x)f^\dagger(y) \text{ for SU(3) and } \sum_{i=1}^{2} f_i(x)f_i^\dagger(y) \text{ for SU(2)}. \]

- With these new objects it is not necessary to distinguish a certain path between \( x \) and \( y \).

Advantages:
- The computation of stair-like paths is not longer needed.

Price to pay:
- One has to compute the eigenvectors of \( \Delta \) first.
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Figure: Runtime of the eigenvector calculation and the remaining computations using the new method
Results
Figure: Effective mass in units of the lattice spacing for the ordinary Wilson loop - using 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089fm$) on a 24x24 Lattice
Figure: Effective mass in units of the lattice spacing for the new method - using 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5 \ (\approx 0.089\text{fm})$ on a 24x24 Lattice
Figure: Comparison of the effective masses from the ordinary Wilson loop and the new method - using 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089\text{fm}$) on a 24x24 Lattice.
Figure: Potential for the static $q\bar{q}$ pair in units of the lattice spacing - using the ordinary Wilson loop on 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089 \text{fm}$) on a 24x24 Lattice.
Figure: Potential for the static $q\bar{q}$ pair in units of the lattice spacing - using the new method on 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089\text{fm}$) on a 24x24 Lattice
Figure: Off-axis potential for the static $q\bar{q}$ pair in units of the lattice spacing - using the new method on 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089\, fm$) on a 24x24 Lattice.
Figure: On-axis potential for the static $q\bar{q}$ pair in units of the lattice spacing - using the new method on 60 basically independent SU(3) gaugelink configurations with $\beta = 3.9$ on a 48x24 Lattice
Summary

- In the ordinary approach the calculation of the static $q\bar{q}$-potential, for off-axis separations, requires the computation of time consuming stair-like paths.

- These stair-like paths come from the Wilson loop, an object that ensures gauge invariance of the used $q\bar{q}$ trial state.

- By using the eigenvectors of the covariant Laplace operator we were able to substitute the spatial part of the Wilson loop by a new object.

- Advantages: Fast computation times for off axis calculations and nearly similar quality of the results (error bars).

- Possible application: The potential with fine resolution can be used for better modeling and comparison with perturbative theories ($\Lambda_{\overline{MS}}$-determination, $b\bar{b}$-spectrum).