Alpha-decay quantum-tunnelling calculations based on a folded Woods-Saxon potential

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Abstract. Assuming that the $\alpha$ particle is a structureless point particle with two protons and two neutrons, we construct a mean-field-type cluster potential based on the Woods-Saxon potential with a folding factor which is to satisfy the quantization condition of a quasibound cluster state. The folded Woods-Saxon cluster potential has been successfully applied to the calculations of $\alpha$-particle decay in light and superheavy nuclei. The standard values of the Woods-Saxon parameters were used without any adjustment. The calculated $\alpha$-decay widths or lifetimes agree generally with experiment. Such a cluster potential leads to a consistent description of single-particle and cluster motions.

1. Introduction

Alpha-particle emission is the most important decay mode of nuclei heavier than Pb. Also, $\alpha$ decay has been observed widely from the excited states of light nuclei. In light nuclei, $\alpha$-cluster structures are favoured when nuclei are excited to the vicinity of the $\alpha$-decay threshold [1]. Molecular structures with two or more $\alpha$ particles and covalent neutrons in light nuclei have become a hot topic currently in both experiment [2, 3] and theory [4, 5]. In heavy nuclei, $\alpha$ decay can happen from the ground states as well. For superheavy nuclei, detecting $\alpha$ decay is a unique method to identify new superheavy elements.

Theoretically, $\alpha$-particle emission can be considered a process of quantum tunnelling of an $\alpha$ particle through a potential barrier, which is called the Gamow decay model [6]. A reasonable barrier is crucial for the calculation of decay width or lifetime. Several phenomenological $\alpha$-cluster potentials have been proposed for the calculations of $\alpha$-decay half-lives and spectroscopic properties [7, 8, 9]. Double-folding microscopic cluster potentials have also been successfully applied to $\alpha$-decay and $\alpha$-scattering calculations [10, 11, 12, 13, 14].

2. The model

In our previous works [15, 16, 17, 18, 19, 20], we constructed folded mean-field-type cluster potentials based on microscopic Skyrme-Hartree-Fock [15, 16, 17] or phenomenological Woods-Saxon potentials [18, 19, 20]. These cluster potentials have been successfully applied to the calculations of various cluster decays including $\alpha$ and heavier cluster decays [15, 16, 19, 20] and molecular structures as well [16, 18]. In this paper, we review the Woods-Saxon-potential-based calculations with focusing on $\alpha$ decay from excited states of the light nucleus $^{24}$Mg and from the ground states of superheavy nuclei.
In the spherical case, the cluster potential can be decomposed as (e.g., [8])

\[ V(r) = V_N(r) + V_C(r) + \frac{\hbar^2}{2\mu r^2} l(l + 1), \quad (1) \]

which contains the nuclear potential \( V_N(r) \), the Coulomb potential \( V_C(r) \) and the centrifugal potential with \( l \) and \( \mu \) for the angular momentum carried by the \( \alpha \) particle and the reduced mass of the \( \alpha \)-core system, respectively [15]. The general folding procedure to derive a cluster potential can be written as [10]

\[ V_N(r) = \lambda \int \int \rho_1(r_1) \rho_2(r_2) v_{\text{eff}}(|r + r_1 - r_2|) \, d\mathbf{r}_1 \, d\mathbf{r}_2, \quad (2) \]

where \( \lambda \) is the folding factor, \( v_{\text{eff}}(|r + r_1 - r_2|) \) is an effective nucleon-nucleon interaction, and \( \rho_1(r_1) \) and \( \rho_2(r_2) \) are the densities of the daughter and the cluster, respectively. To simplify the equation above, we consider the cluster as a structureless point particle, then obtain

\[ V_N(r) = \lambda \int [Z_c \rho^p_1(r_1) v^p_{\text{eff}}(|r + r_1|) + N_c \rho^n_1(r_1) v^n_{\text{eff}}(|r + r_1|)] \, d\mathbf{r}_1, \quad (3) \]

where the superscripts ‘\( p \)’ and ‘\( n \)’ indicate the proton and neutron, respectively, and \( N_c \) and \( Z_c \) are the neutron and proton numbers of the cluster, respectively. For the \( \alpha \) particle, \( N_c = 2 \) and \( Z_c = 2 \). In Eq. (3) the integral gives the mean-field single-particle potential. Therefore, the nuclear potential between the cluster and the remaining core can be simplified further,

\[ V_N(r) = \lambda [N_c v_n(r) + Z_c v_p(r)], \quad (4) \]

where \( v_n(r) \) and \( v_p(r) \) are single-neutron and single-proton potentials (excluding the Coulomb potential) respectively, generated by the core. Single-particle potentials can be obtained by mean-field models, such as the Skyrme-Hartee-Fock (SHF) [15, 16, 17] or the simple Woods-Saxon potential [18, 19, 20]. The Coulomb potential \( V_C(r) \) need not be calculated by a folding procedure; we take its usual form [21] with assuming a homogeneous charge distribution in the daughter.

In this paper, we adopt the Woods-Saxon potential for the single-particle mean-field, that is

\[ v(r) = \frac{V_0}{1 + e^{-\kappa r}}, \quad (5) \]

with

\[ V_0(r) = -V_{00} \left( 1 \pm \kappa \frac{N_d - Z_d}{N_d + Z_d} \right), \quad (6) \]

where the sign is + (−) for the proton (neutron). The index ‘\( d \)’ indicates the daughter.

The folding factor \( \lambda \) is determined by the Bohr-Sommerfeld quantization condition, which, for the ground state, looks like

\[ \int_{r_1=0}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}} |Q_0 - V(r)| \, dr = (2n + 1) \frac{\pi}{2} = (G + 1) \frac{\pi}{2}, \quad (7) \]

where \( r_1, r_2 \) (and \( r_3 \) later) are classical turning points obtained from \( V(r) = Q_0^* \) (the decay energy). The global quantum number \( G = 2n \) (\( n \) is the node number in the radial wave function of the cluster tunnelling motion) is determined by the Wildermuth rule [22], giving \( G = \sum_i A_i g_i \), where \( A_i \) is the nucleon number of the cluster and \( g_i \) is the oscillator quantum number of a
cluster nucleon orbiting the core. The $g_i$ numbers are those of the single-particle states occupied in the parent nucleus by the nucleons constituting the $\alpha$ particle to be emitted.

The partial $\alpha$-decay width is calculated by [8, 16, 19]

$$\Gamma = P^2 \frac{\hbar^2}{4\mu} \exp \left[ -2 \int_{\kappa}^{\kappa_0} k(r) dr \right]$$

where $k(r) = \sqrt{(2\mu/\hbar^2)|Q_l^* - V(r)|}$ is the wave number, and $P$ is the preformation factor of the $\alpha$-particle being formed in the mother. For even-even nuclei, it has been well established that the $P = 1$ assumption under the use of the Bohr-Sommerfeld condition can well reproduce the experimental half-lives of various cluster decays [8, 15]. The decay half-life is calculated by $T_{1/2} = \hbar \ln 2/\Gamma$. In the equations above, $Q_l^*$ is the $\alpha$-decay energy from an excited state,

$$Q_l^* = Q_0 + E_{i,l}^* - E_{J_f}^*$$

where $Q_0$ is the $\alpha$-decay energy of the ground state, and $E_{i,l}^*$ and $E_{J_f}^*$ are the excitation energies of the mother with spin $J_i$ and the daughter with spin $J_f$. The orbital angular momentum $l$ carried by the $\alpha$ particle can be determined from the vector coupling of $l$ and $J_f$ to $J_i$ and from parity conservation. Since the decay calculation is very sensitive to the $Q_l^*$ value, experimental values have been used for $Q_0$ and the excitation energies.

3. Alpha-decay calculations for the excited states of $^{24}\text{Mg}$ and the ground states of superheavy nuclei

In $\alpha$-decay calculations for $^{24}\text{Mg}$, we take the Chepurnov parameters [23] of the Woods-Saxon potential, which work well for light nuclei. For the $^{24}\text{Mg}$ ground state, nucleons belonging to the $\alpha$ particle should occupy orbits immediately above the Fermi surface of the daughter $^{20}\text{Ne}$, i.e., the $d_{5/2}$ shell, which gives a value of $G = 8$ [19]. Using the Bohr-Sommerfeld condition with taking the experimental $Q_0 = -9.316$ MeV for the $\alpha+^{20}\text{Ne}$ channel, we obtain a folding factor of $\lambda = 0.608$ for the $\alpha$-cluster potential in $^{24}\text{Mg}$. It has been known experimentally that the $^{24}\text{Mg}$ excited states in the energy range of $\approx 10 - 15$ MeV decay dominantly by $\alpha$ emission [24]. The occurrence of $\alpha$ decay requires decay energies satisfying $Q_l^* = Q_0 + E_{i,l}^* - E_{J_f}^* > 0$. Therefore, the excited states in this energy range decay mainly into the $0_1^+$ (g.s.) and $2_1^+$ (1.63 MeV) states of $^{20}\text{Ne}$. Assuming a preformation factor of $P \approx 1$ for every $\alpha$-decay transition, we calculated the $\alpha$-decay widths of some excited states of $^{24}\text{Mg}$. The widths obtained agree, in most cases, with the experimental data within two orders of magnitude, see [19] for detailed results. The preformation probabilities of an $\alpha$ particle in the excited states of $^{24}\text{Mg}$ range from $10^{-2}$ to 1 [25], which might explain the discrepancies between the calculated and experimental widths.

The folded Woods-Saxon cluster potential has also been applied in the $\alpha$-decay calculations of the ground states of superheavy nuclei [20]. For superheavy nuclei, however, we took another set of the Woods-Saxon parameters, which has been widely used in cranking shell-model calculations of high-spin states in heavy and superheavy nuclei (see, e.g., [26] and references therein). They are

$$V_{00} = 53.754 \text{ MeV},$$
$$\kappa = 0.791,$$
$$a = 0.637 \text{ fm},$$
$$r_0 = 1.19 \text{ fm}.$$  

The main difference between this set of parameters and the Chepurnov parameters for light nuclei is in the radius parameter $r_0$. The Chepurnov parameterization takes $r_0 = 1.24 \text{ fm}$ [23].
Table 1 lists the calculated half-lives of α decays for even-even superheavy nuclei. They agree with the experimental values within one order of magnitude. For superheavy nuclei, we took the global quantum number $G = 22$ [17], which is consistent with the Wildermuth rule. The discrepancies between calculations and data might come from two main factors: 1) deformation effects (most superheavy nuclei are deformed, while our calculations were limited to spherical shapes); 2) possible large uncertainties in experimental half-lives due to poor statistics of decay events.

Table 1. Calculated half-lives of observed α decays for even-even superheavy nuclei. Data are from [27, 28, 29].

| Nuclei | $\lambda$ | $T_{1/2,\alpha}^{\text{calc}}$ (second) | $T_{1/2,\alpha}^{\text{expt}}$ (second) |
|--------|-----------|---------------------------------|---------------------------------|
| $^{246}$Fm | 0.764 | 7.33×10$^{-1}$ | 1.1×10$^{+0}$ |
| $^{248}$Fm | 0.763 | 1.28×10$^{+1}$ | 3.6×10$^{+1}$ |
| $^{250}$Fm | 0.762 | 5.16×10$^{+2}$ | 1.8×10$^{+3}$ |
| $^{252}$Fm | 0.761 | 2.02×10$^{+4}$ | 9.1×10$^{+4}$ |
| $^{254}$Fm | 0.757 | 4.32×10$^{+3}$ | 1.2×10$^{+4}$ |
| $^{256}$Fm | 0.755 | 5.99×10$^{+4}$ | 9.5×10$^{+3}$ |
| $^{252}$No | 0.758 | 1.01×10$^{+0}$ | 2.4×10$^{+0}$ |
| $^{254}$No | 0.757 | 1.13×10$^{+1}$ | 5.1×10$^{+1}$ |
| $^{256}$No | 0.751 | 6.95×10$^{−1}$ | 2.9×10$^{+0}$ |
| $^{256}$Rf | 0.755 | 3.30×10$^{−1}$ | 3.6×10$^{−1}$ |
| $^{260}$Sg | 0.747 | 2.27×10$^{−3}$ | 3.6×10$^{−3}$ |
| $^{266}$Sg | 0.744 | 1.94×10$^{+0}$ | 2.6×10$^{+1}$ |
| $^{264}$Hs | 0.742 | 1.88×10$^{−4}$ | 1.0×10$^{−4}$ |
| $^{266}$Hs | 0.740 | 7.59×10$^{−4}$ | 2.3×10$^{−3}$ |
| $^{270}$Hs | 0.747 | 5.60×10$^{−1}$ |                          |
| $^{270}$Ds | 0.734 | 2.55×10$^{−5}$ | 1.0×10$^{−5}$ |
| $^{284}$Cn | 0.730 | 9.39×10$^{+0}$ | 9.8×10$^{+0}$ |
| $^{288}$114 | 0.725 | 4.37×10$^{−1}$ | 1.9×10$^{+0}$ |
| $^{292}$116 | 0.720 | 1.82×10$^{−2}$ | 3.3×10$^{−2}$ |

In summary, we have proposed a folded Woods-Saxon α-cluster potential. The advantage of this cluster potential is that no new free parameter is to be introduced, which makes the predictions reliable. Moreover, it leads to a consistent description of single-particle and cluster motions. Within the framework of the quantum-tunnelling picture, the potential has been successfully applied to the α-decay of some excited states of $^{24}$Mg with excitation energies of $E_x \approx 10$–15 MeV and to the ground states of superheavy nuclei. The calculated decay widths or half-lives agree reasonably with experimental values.

Acknowledgments
This work has been supported by the National Key Basic Research Program of China under Grant 2013CB834400, and the National Natural Science Foundation of China under Grant Nos. 11235001 and 10975006.
References

[1] Ikeda K, Takigawa N and Horiuchi H 1968 Prog. Theor. Phys. Suppl. (Japan) Extra Number 464
[2] Freer M et al 1999 Phys. Rev. Lett. 82 1383
[3] Itoh M et al 2011 Phys. Rev. C 84 054308
[4] Kanada-En’yo Y and Horiuchi H 2003 Phys. Rev. C 68 014319
[5] Ito M, Itagaki N, Sakurai H and Ikeda K 2008 Phys. Rev. Lett. 100 182502
[6] Gamow G 1928 Z. Phys. 51 204
[7] Buck B, Merchant A C and Perez S M 1992 Phys. Rev. C 45 2247
[8] Buck B, Merchant A C and Perez S M 1993 At. Data Nucl. Data Tables 54 53
[9] Xu C and Ren Z 2004 Phys. Rev. C 69 024614
[10] Mohr P 2000 Phys. Rev. C 61 045802
[11] Atzrott U, Mohr P, Abele H, Hillemeyer C and Staudt G 1996 Phys. Rev. C 53 1336
[12] Xu C and Ren Z Z 2005 Nucl. Phys. A 760 303
[13] Ren Z Z, Xu C and Wang Z 2004 Phys. Rev. C 70 034304
[14] Chowdhury P R, Samanta C and Basu D N 2006 Phys. Rev. C 73 014612
[15] Xu F R and Pei J C 2006 Phys. Lett. B 642 322
[16] Pei J C and Xu F R 2007 Phys. Lett. B 650 224
[17] Pei J C, Xu F R, Lin Z J and Zhao E G 2007 Phys. Rev. C 76 044326.
[18] Xu C, Qi C, Liotta R J, Wyss R, Wang S M, Xu F R, and Jiang D X 2010 Phys. Rev. C 81 054319
[19] Wang S M, Xu C, Liotta R J, Qi C, Xu F R and Jiang D X 2011 Sci. China Ser. G-Phys. Mech. Astron. 54 s130
[20] Lin Z J, Pei J C and Xu F R 2012 Phys. Scr. T150 014022
[21] Buck B, Hopkins P D B and Merchant A C 1990 Nucl. Phys. A 513 75
[22] Wildermuth K and Tang Y C 1977 A Unified Theory of the Nucleus (New York: Academic Press)
[23] Chepurnov V A 1967 Yad. Fiz. 6 955
[24] Firestone R B 2007 Nucl. Data Sheets 108 2319
[25] Hodgson P E and Betak E 2003 Phys. Rep. 374 1
[26] Xu F R, Satula W and Wyss R 2000 Nucl. Phys. A 669 119
[27] Oganessian Y T et al 2004 Phys. Rev. C 70 064609
[28] Oganessian Y T et al 2004 Phys. Rev. C 74 044602
[29] Audi G et al 2003 Nucl. Phys. A 729 3