DC resistivity near a nematic quantum critical point: Effects of weak disorder and acoustic phonons

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We calculate the resistivity associated with an Ising-nematic quantum critical point in the presence of disorder and acoustic phonons. To perform this analysis, we use the memory-matrix transport theory, which has a crucial advantage compared to other methods of not relying on the existence of well-defined quasiparticles in the low-energy effective theory. As a result, we obtain that by including an inevitable interaction between the nematic fluctuations and the elastic degrees of freedom of the lattice (parametrized by the nemato-elastic coupling $K_{	ext{elas}}$), the resistivity $\rho(T)$ of the system as a function of temperature obeys a universal scaling form described by $\rho(T) \sim T \ln(1/T)$ at high temperatures, reminiscent of the paradigmatic strange metal regime observed in many strongly correlated compounds. For a window of temperatures comparable with $K_{\text{elas}}^{3/2} \varepsilon_F$ (where $\varepsilon_F$ is the Fermi energy of the microscopic model), the system displays another regime in which the resistivity is consistent with a description in terms of $\rho(T) \sim T^\alpha$, where the effective exponent roughly satisfies the inequality $1 \lesssim \alpha \lesssim 2$. However, in the low-temperature limit (i.e., $T \ll K_{\text{elas}}^{3/2} \varepsilon_F$), the properties of the quantum critical state change in an important way depending on the type of disorder present in the system. It can either recover a conventional Fermi liquid described by $\rho(T) \sim T^2$ or it could exhibit yet another non-Fermi liquid regime characterized by the scaling form $\rho(T) - \rho_0 \sim T^2 \ln T$ (implying in the latter case that the system would display a Kondo-like upturn in the resistivity). From a broader perspective, our results emphasize the key role played by both phonon and disorder effects in the scenario of nematic quantum criticality and might be fundamental for addressing recent transport experiments in some iron-based superconductors.

I. INTRODUCTION

Understanding all aspects of quantum critical phenomena [1, 2] in itinerant systems has been a formidable challenge in the field of strongly correlated quantum materials. From a theoretical perspective, this is related to the fact that it is still very hard at the present moment to extract all physical predictions in a completely controlled manner from the corresponding effective theories, especially at low temperatures close to the quantum critical point (QCP). Nevertheless, there has been an accumulation of evidence in recent years of the likely manifestation of quantum criticality in many important systems such as the high-$T_c$ cuprates [3–9], the heavy-fermion compounds [10], and the iron-based superconductors [11–16] (to name only a few systems). One prominent candidate theory for the emergence of this phenomenon in some of the aforementioned materials is related to the onset of Ising-nematic quantum criticality [17]. For tetragonal environments, this order refers to the spontaneous breaking of a point-group discrete rotation symmetry of the lattice from $C_4$ down to $C_2$, while preserving translation symmetry (i.e., it is an order at momentum $\mathbf{q} = 0$). In metallic systems, it can be achieved, e.g., via a Pomeranchuk instability [18] of Landau Fermi-liquid theory in the corresponding charge channel, but it can also appear quite generally in either frustrated or doped incommensurate antiferromagnets once the antiferromagnetism has been destroyed by quantum fluctuations [19].

The original approach to quantum critical phenomena is due to Hertz [20] (and later extended byMillis [21]): It relies on the premise that it is possible to integrate out the fermions to derive an effective action with Landau-damping for the (bosonic) order-parameter field. However, this conventional Hertz-Millis strategy has been challenged due to the fact that it does not treat all excitations of the theory at a given energy scale on equal footing. Despite this cautionary remark, it was thought for some time that such an effective model could be solved exactly [22] at low temperatures in the large-$N_f$ limit, with $N_f$ being the number of fermionic species. Unfortunately, this turned out not to be the case due to strong quantum fluctuation effects that emerge at low energies. Indeed, the problem was declared open again after the work of Lee [23] on a closely related critical Fermi surface state with the $N_f$ fermions coupled to an emergent U(1) gauge field.

Following those important developments, a novel renormalization group (RG) approach up to three-loop order [24] was put forward by Metlitski and Sachdev to address this difficult but general problem of Ising-nematic quantum criticality arising in two-dimensional metals. Their central conclusion was that although a systematic $1/N_f$ approximation fails close to the QCP (and, for this reason, there is still no fully-controlled RG program for this theory), they found interesting renormalizations of several parameters and a clear non-Fermi liquid regime, which emerges inside the corresponding quantum critical region. Other analytical works subsequently confirmed this general picture for the nematic quantum critical scenario [25–27]. On the other hand, from a numerical perspective, there have been also some recent advances in
simulations of this problem using determinantal quantum Monte-Carlo (QMC) methods, where the authors in Refs. [28, 29] confirmed some results obtained previously by the aforementioned analytical methods. Moreover, those numerical works have provided further support to the existence of a non-Fermi liquid phase inside the quantum critical regime at intermediate temperatures and the emergence of a high-$T_c$ superconducting dome in the phase diagram as the temperature is lowered, which have some similarities with the experiment situation [29]. Unfortunately, those QMC simulations are restricted to small lattices and not-too-low temperatures, such that many properties specifically related to the QCP of the model are still not easily accessible.

Transport theories of metallic phases possessing strong quantum critical nematic fluctuations are also of great interest nowadays. Indeed, the resistivity is one of the simplest quantities to be measured experimentally in order to characterize a variety of strongly correlated phases. In this context, a widely employed method for computing non-equilibrium properties is the semi-classical Boltzmann-equation approach [30]. In the Hertz-Millis scenario, this method considers the scattering of the low-energy fermions off the order-parameter bosons, which are assumed to be both in equilibrium and to provide a bath that effectively acts to degrade the fermionic total momentum. For the nematic quantum critical model in two dimensions, this method predicts that the resistivity $\rho(T)$ in the corresponding non-Fermi liquid phase should obey a power-law described by $\rho(T) \sim T^{1/3}$, as a result of considering impurity scattering as the main mechanism for momentum relaxation and by including also multiple bands to avoid special “one-dimensional” geometrical cancellation effects [31, 32]. On the other hand, it has been generally acknowledged that such a conventional Boltzmann-equation approach is eventually expected to fail for the Ising-nematic quantum critical problem because of two profound reasons: Firstly, the quasiparticle excitations are not well-defined in the corresponding non-Fermi liquid that emerges in the quantum critical region of the phase diagram. Secondly, since the $1/N_f$ approximation turns out not to be controlled for the problem of nematic quantum criticality at low enough temperatures [24], the notorious Prange-Kadanoff reasoning [33] for the validity of the Boltzmann equation for such a theory may not work for the present case. Therefore, an alternative transport technique must be used in order to address these aforementioned limitations in a more reliable way.

In this work, we will apply an alternative transport theory to the Ising-nematic quantum phase transition in two-dimensional metals that directly overcomes the first limitation highlighted above and, simultaneously, avoids also the second limitation. As the main source for momentum relaxation in the present theory, we will consider here disorder as the most relevant contribution, following the study by Ref. [34] also in the context of a nematic quantum critical theory. Then, we will proceed to include in the calculation of the corresponding transport coefficient important new effects associated with phonon scattering via a nema-elastic coupling proposed in the literature [35-41], which from an RG point of view turns out to be as relevant as disorder. The technique we will adopt here will be the Mori-Zwanzig memory matrix approach [42], which has the important advantage of not being based on the existence of well-defined quasiparticle excitations at low energies in the
system [43, 44]. We also would like to stress that this method has been employed in recent years with great success by several researchers of the field in many different contexts, such as, e.g., one-dimensional Luttinger liquids [45], two-dimensional quantum critical metals [34, 46–49], a critical (spinon) two-dimensional Fermi surface model coupled to an emergent U(1) gauge field [50], and holographic quantum matter [51, 52]. Here, we will focus on the calculation of the resistivity of the corresponding electronic phase associated with Ising-nematic quantum critical fluctuations and its interplay with both disorder and acoustic phonons. As a consequence, we will be able to investigate this transport coefficient as a function of the nemato-elastic interaction, which emerges inevitably in any realistic nematic QCP model defined on a lattice (see also Figs. 1(a) and 1(b) for an overview of the results obtained in the present work).

Therefore, this paper is structured as follows: First, we define the quantum critical nematic model with nemato-elastic coupling that will be our starting point for the analysis of the corresponding transport properties. Then, we will briefly explain the Mori-Zwanzig memory-matrix technique that will be used throughout this work. The analytical and the numerical results that emerge from the integral equations obtained within this method will be presented in the ensuing sections. Lastly, we will end this paper with a summary concerning the present investigation.

II. MODEL

We depart from a standard model that consists of a two-dimensional tetragonal electronic system described by $N_f$ species of fermions coupled to bosonic scalar fields, which represent the Ising-nematic correlations. Those nematic fluctuations are in turn also coupled to the elastic degrees of the freedom of the underlying lattice. The corresponding Hamiltonian in $k$-space is therefore given by

$$H = H_{\text{el-nem}} + H_{\text{nem-latt}}.$$  

The first part of the Hamiltonian ($H_{\text{el-nem}}$) that describes the coupling between the electrons and the nematic fluctuations is given by

$$H_{\text{el-nem}} = \sum_{k,\sigma} \xi_k \psi_{k,\sigma}^\dagger \psi_{k,\sigma} + \frac{N_f}{2} \sum_k s \phi(k)^2$$

$$+ \frac{1}{2} \sum_k |\pi(k)|^2 + \frac{g_{\text{nem}}}{\sqrt{\nu_0}} \sum_{k,q,\sigma} V_{k,q} \psi_{k+q/2,\sigma}^\dagger \psi_{k+q/2,\sigma} \phi_q,$$

where $\psi_{k,\sigma}^\dagger$ and $\psi_{k,\sigma}$ respectively, creates and annihilates electrons with momentum $k$ and spin projection $\sigma \in \{\uparrow, \downarrow\}$; $\xi_k = \varepsilon_k - \mu$ is the energy dispersion relative to the chemical potential of the system. Besides, $s$ is the “distance” of the model to the nematic QCP, $\pi(k)$ is the momentum operator conjugated to $\phi(k)$; $g_{\text{nem}}$ is the nematic interaction that couples the order-parameter field to the electrons, $\nu_0$ is the density of states at the Fermi level, and $V_{k,q} = 2(\cos(k_x) + \cos(q_x) - \cos(k_y) - \cos(q_y))$ is the corresponding $d$-wave form factor. Naturally, the above critical model describes the effects of an electronic structural quantum phase transition from tetragonal ($C_4$) to orthorhombic ($C_2$) symmetry in the system as a function of the control parameter described by $s$.

As for the second part of the Hamiltonian ($H_{\text{nem-latt}}$), which refers to the important coupling between the nematic quantum fluctuations and the elastic degrees of freedom (acoustic phonons) of the underlying two-dimensional lattice, we write

$$H_{\text{nem-latt}} = \frac{1}{2} \sum_{k \neq 0} u_k^\dagger M_k u_k + ig_{\text{latt}} \sum_{k \neq 0} (a_k \cdot u_k) \phi_{-k},$$

where $u_k$ is the Fourier transform of the displacement vector, $a_k = (k_x, -k_y, 0)$ is a two-dimensional vector, $g_{\text{latt}}$ is the nemato-elastic coupling, and $M_k$ is the so-called dynamical matrix [39, 41, 53]. If we constrain the displacement vector $u_k$ to lie effectively in two spatial dimensions, the dynamical matrix $M_k$ can be simplified to

$$M_k = \begin{pmatrix} C_{11}k_x^2 + C_{06}k_y^2 & (C_{12} + C_{06})k_xk_y \\ (C_{12} + C_{06})k_xk_y & C_{66}k_x^2 + C_{11}k_y^2 \end{pmatrix},$$

where the constants $C_{ij}$ refer to the elastic constants in the Voigt notation (see, e.g., Ref. [54]) for a system possessing initially tetragonal symmetry [55].

Regarding the role of the nemato-elastic coupling in the present theory, we will follow an approach similar to the one already explained in Refs. [39, 41, 53]. We first project the displacement vector onto the basis of the polarization vectors of the phonons and diagonalize the dynamical matrix according to $M_k \cdot \hat{e}_p(k) = g_{\text{nw}} \hat{e}_p(k)$, where $\hat{\varphi}$ is the mass density of the ions of the underlying lattice. Then, by integrating out the phonons degrees of freedom, the nematic degrees of freedom become described by the following renormalized propagator $D^{-1}(k, i\Omega_n) = D_0^{-1}(k, i\Omega_n) - \Pi_{\text{latt}}(k, i\Omega_n)$, where

$$\Pi_{\text{latt}}(k, i\Omega_n) = \frac{g_{\text{latt}}}{\omega_n^2} \sum_{\mu} \frac{|a_k \cdot \hat{e}_\mu(k)|^2}{\omega_n^2(k) + \Omega_n^2}.$$  

By defining the angle $\theta = \tan^{-1}(k_y/k_x)$, we obtain that the eigenvalues of $M_k$ can be written as $\omega_{\pm}(k) = \pm \sqrt{\omega_n^2(k) + \Omega_n^2}$, with the sound velocities given by

$$v_{\pm}(\theta) = \frac{1}{\sqrt{2\rho}} \left( \gamma_1 \pm \sqrt{\gamma_1^2 + \gamma_2^2} \right),$$

where the $\gamma_i$'s are given in terms of the elastic constants by $\gamma_1 = C_{11} + C_{06}$, $\gamma_2 = C_{11} - C_{06}$, and $\gamma_3 = C_{12} + C_{06}$. From the above expressions, we can now calculate the “distance” to the nematic quantum critical point $v_0^{-1}s = D^{-1}_{\text{nem}}(0, 0)$ that straightforwardly evaluates to

$$s(\theta) = s_0 - \frac{2g_{\text{latt}}^2 v_0}{\gamma_1^2 - F^2(\theta)} \left[ \gamma_1 + \gamma_2 - (\gamma_2 + \gamma_3) \cos^2(2\theta) \right].$$
We may now calculate \( m^2(T) \) using a standard \( 1/N_f \) expansion that, according to recent theoretical developments (see Refs. [35, 39]), is expected to be a valid approximation within the whole range of temperatures in the present quantum critical model. This special property is due to the fact that the quantum critical theory defined by Eqs. (1) and (2) describing the Ising-nematic fluctuations interacting with acoustic phonons happens to be governed by a Gaussian fixed point in the clean limit. Indeed, according with the Hertz-Millis strategy, we may for this case integrate out the fermions to derive an effective theory for the nematic order-parameter bosons. Consequently, the effective action describing the nematic fluctuations truncated at fourth-order in a expansion in powers of the order-parameter field finally be-

\[
D_0^{-1}(k, i\Omega_n) = \nu_0^{-1} \left[ \Delta s + k^2 + \lambda_{\text{latt}} \cos^2(2\theta) + B \cos^2(2\theta) \frac{\Omega_n}{v_Fk} + C \sin^2(2\theta) \left( \frac{\Omega_n}{v_Fk} \right)^2 \right],
\]

where \( B \) and \( C \) are usually temperature-independent (non-universal) constants that for the present microscopic model are given by \( B = \nu_0 g_{\text{ferm}}^2 \) and \( C = 4\nu_0 g_{\text{ferm}}^2 \). Note that we also included the irrelevant term in the bosonic propagator that is multiplied by the prefactor \( C \), which will be important for some calculations that will appear in this work (see the Appendices A–C).

To proceed further, we will then follow the analysis put forward by Ref. [34] in order to calculate the nematic effective mass \( m^2(T) \) as a function of temperature at the corresponding QCP. In this way, the structure of the bosonic propagator along with the temperature-dependent mass-term at \( \Delta s = 0 \) becomes

\[
D^{-1}(k, i\Omega_n) = \nu_0^{-1} \left[ k^2 + \lambda_{\text{latt}} \cos^2(2\theta) + B \cos^2(2\theta) \frac{\Omega_n}{v_Fk} + C \sin^2(2\theta) \left( \frac{\Omega_n}{v_Fk} \right)^2 + m^2(T) \right].
\]
As for $\Gamma_3(K, -K, 0)$, it evaluates to
\begin{equation}
\Gamma_3(K, -K, 0) = \frac{g_{\text{em}}^4}{\nu_0^2} T \sum_{\nu_n} \int_V V_{q+k/2,q-k/2}^2 V_{q-k/2,q-k/2} \times G_0^2(q + k/2, i\nu_n + i\omega_n) G_0(q - k/2, i\nu_n).
\end{equation}

In all of the above equations, the non-interacting fermionic Green’s function is given by $G_0(k, i\omega_n) = 1/(i\omega_n - \xi_k)$ for the energy dispersion $\xi_k$ and $\langle \cdots \rangle = \int \frac{d^3k}{(2\pi)^3} \langle \cdots \rangle$. Moreover, since we are assuming a second-order nematic QCP in the present model, we will only consider the case in which $U > 0$.

At the nematic QCP (i.e., $\Delta s = 0$), we must have that $m^2(T = 0) = 0$. This is related to the fact that the theory should be manifestly gauge-invariant at the corresponding nematic QCP. Therefore, we should rewrite the Eq. (11) in a more convenient way
\begin{equation}
m^2(T) = \Delta s + \frac{U}{N_f} \int_k \left[ T \sum_{\omega_n} D(k, i\omega_n) - \int_\omega D_0(k, \omega) \right]_{\Delta s = 0} + \frac{2}{N_f} \int_K [\Gamma_3(K, -K, 0) D(K)]^2,
\end{equation}

where $\sum_{\omega_n} f_{\omega}(\cdots) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \langle \cdots \rangle$. The above equation turns out to be a self-consistent integral equation for $m^2(T)$. We will proceed to solve it (semi-)analytically after the coming section, which in turn contains a brief explanation of the methodology we will employ in the present work.

### III. MEMORY-MATRIX METHOD

The method we will adopt in this work to calculate transport properties of the nematic quantum critical model will be the Mori-Zwanzig memory-matrix formalism [42, 51, 52]. As advertised previously, this technique has the advantage of not relying on the existence of well-defined quasiparticle excitations. According to this formalism, the “generalized” conductivities can be written as follows
\begin{equation}
\sigma(\omega, T) = \frac{\chi^R(T)}{(M - i\omega \chi^R(T))[\chi^R(T)]^{-1}},
\end{equation}

where $\chi^R(T)$ stands for the static retarded susceptibility matrices representing the overlap of the currents of interest with the so-called nearly-conserved operators in the system. These susceptibility matrices can in turn be expressed as
\begin{equation}
\chi_{AB}(i\omega, T) = \int_0^1 d\tau e^{i\omega\tau} \langle T_\tau A^\dagger(0) B(0) \rangle,
\end{equation}

where the corresponding retarded susceptibility is obtained by performing the analytical continuation $i\omega \rightarrow$
\[ \omega + i0^+ \]. In addition, \((\cdots)\) represents the grandcanonical average, \(T_\tau\) is the imaginary-time ordering operator and the volume has been set to unity, for simplicity. The memory matrix denoted by \(\hat{M}\) can then be obtained from the formally exact expression

\[ \hat{M}_{\hat{O}_i\hat{O}_j}(T) = \int_0^{1/T} d\tau \left\{ \hat{O}_j^\dagger(0)Q\frac{i}{\omega - Q\hat{L}Q}\hat{O}_i(\tau) \right\}, \tag{18} \]

where the super-operator \(\hat{L}\) is the Liouville operator, which is conventionally defined as \(\hat{L}\hat{O}_j = [H, \hat{O}_j] = -i\partial\hat{O}_j/\partial\tau\), \(H\) is the Hamiltonian of the model, and \(\hat{O}_j\) refers in principle to an arbitrary operator. For practical reasons though, this method becomes much more useful if the operators \(\hat{O}_j\) turn out to be either conserved or nearly conserved in the transport theory. Consequently, the memory matrix will necessarily become a small quantity, which will allow its calculation via perturbative means. The \(Q\) denotes another super-operator that acts on the operators and it projects out of a formal space spanned by all the conserved (or nearly conserved) operators in the system. Interestingly, the memory matrix defined in Eq. (18) turns out to be a generalization of the collision integral in the Boltzmann equation that is also valid for regimes which possess no well-defined quasiparticles at low-energies. From this perspective, it can be clearly seen that this method emerges as the ideal tool for the investigation of the quantum critical transport that we intend to perform here.

At this point, it is also useful to define the Lagrangian of the system corresponding to Eqs. (1) and (2) with the acoustic phonons being already integrated out in order to apply the Mori-Zwanzig memory-matrix formalism. In position space denoted by \(r\) and imaginary time \(\tau\), it can be written as

\[ \mathcal{L} = \sum_{\sigma} \psi^\dagger_{\sigma} \left( \partial_\tau - \frac{\nabla^2_{\tau}}{2m} - \mu \right) \psi_{\sigma} + \frac{s}{2} N_f \phi^2 + \frac{\epsilon}{2} \left( \partial_\tau \phi \right)^2 - \frac{g_{\text{nem}}}{\sqrt{\nu_0}} \phi \sum_{\sigma} \left( \psi^\dagger_{\sigma} \left[ (\partial^2_x - \partial^2_y) \psi_{\sigma} \right] + \left[ (\partial^2_x - \partial^2_y) \psi^\dagger_{\sigma} \right] \psi_{\sigma} \right). \tag{19} \]

In the above equation, we have included the bosonic kinetic term (multiplied by the parameter \(\epsilon\)) in order to obtain the total momentum of the model. Since this particular bosonic contribution turns out to be irrelevant at low energies due to Landau damping, we could set \(\epsilon \rightarrow 0\) in the end of the present transport coefficient computation, but actually the final result turns out to be independent of \(\epsilon\) [34].

Within the framework of Noether’s theorem, the fact that the Lagrangian of the system is invariant under continuous space translations and global \(U(1)\) symmetry implies that the total momentum \(\mathbf{P}\) and the electric current \(\mathbf{J}\) are conserved at the classical level. These quantities are, respectively, given by

\[ \mathbf{P} = \frac{i}{2} \sum_{\sigma} \left[ (\nabla \psi^\dagger_{\sigma} \psi_{\sigma} - \psi^\dagger_{\sigma} \nabla \psi_{\sigma}) + \epsilon \nabla \phi \partial_\tau \phi \right], \tag{20} \]

\[ J_x = i \left( \frac{1}{2m} + 2 \frac{g_{\text{nem}}}{\sqrt{\nu_0}} \phi \right) \sum_{\sigma} \left( \partial_x \bar{\psi}_x \psi_{\sigma} + \bar{\psi}_x \partial_x \psi_{\sigma} \right), \tag{21} \]

\[ J_y = i \left( \frac{1}{2m} - 2 \frac{g_{\text{nem}}}{\sqrt{\nu_0}} \phi \right) \sum_{\sigma} \left( \partial_y \bar{\psi}_y \psi_{\sigma} + \bar{\psi}_y \partial_y \psi_{\sigma} \right). \tag{22} \]

where the total momentum \(\mathbf{P}\) has both the contribution from the electrons and also the bosonic (drag) term (the latter corresponding to the Ising-nematic fluctuations). As for the electric current \(\mathbf{J}\), we clearly obtain two distinct contributions: one corresponding to the non-interacting part and another that includes the interaction \(g_{\text{nem}}\) between the electron and the nematic fluctuation. This shows that such an interaction affects the transport of charge in the system as well.

Since the continuum model defined in Eq. (19) conserves total momentum, the calculation of its resistivity \(\rho(T)\) yields zero for any temperature \(T\). Therefore, in order for the electric current to decay in the system, we must specify a relaxation mechanism for the total momentum of the model. Here, we consider disorder effects as the main mechanism for relaxing the momentum of the system. Therefore, we must include the following impurity terms that couple to both the electrons and the nematic fluctuations in the Lagrangian

\[ \mathcal{L}_{\text{imp}} = \sum_{\sigma} V(r)\bar{\psi}_\sigma(r)\psi_\sigma(r) + h(r)\phi(r), \tag{23} \]

which should satisfy Gaussian disorder averages:

\[ \langle \langle V(r) \rangle \rangle = \langle \langle h(r) \rangle \rangle = 0, \langle \langle V(r)V(r') \rangle \rangle = V_0^2 \delta^2(r - r'), \]

\[ \langle \langle h(r)h(r') \rangle \rangle = h_0^2 \delta^2(r - r'), \]

where \(V_0\) is a random potential for the electron field and the parameter \(h_0\) is the so-called random field disorder that couples to the nematic fluctuation \(\phi\). Both contributions were considered in an earlier work [34] for the case of \(\lambda_{\text{latt}} = 0\). As depicted in Fig. 4, the leading contributions to the

\[ \text{FIG. 4. Feynman diagrams for the calculation of the leading contributions to the memory matrix in the present theory: (a) the random-field-disorder contribution where the red squares represent the } E_0 \text{ coupling, and (b) the random potential contribution coupled to the fermions where the blue circles stand for the } V_0 \text{ coupling. The dashed lines in both diagrams refer to the impurity lines and carry only internal momentum and external energy } \omega. \]
corresponding resistivity are given by

$$
\rho(T) = \frac{1}{\chi_{JP}^2} \lim_{\omega \to 0} \int \frac{d^2 k}{(2\pi)^2} k^2 \cos^2(\theta - \varphi) \left[ h_0^2 \text{Im} D_R(k, \omega) \right] + V_0^2 \text{Im} \Pi_R(k, \omega),
$$

(24)

where $D_R(k, \omega) = D(k, i\omega \to \omega + i0^+)$ is the retarded nematic propagator, $\Pi_R(k, \omega)$ is the corresponding retarded free-fermion polarizability, $\chi_{JP}$ is the susceptibility given by Eq. (17), and we are considering, for the sake of generality, transport of current along an arbitrary angle $\varphi$.

To keep our discussion simple, we will first compute here the effect of the random-field disorder contribution to the resistivity here and postpone the analysis of the $V_0$ contribution to the resistivity to a later section in this work. In this way, by calculating analytically only the $h_0^2$ term in Eq. (24), we obtain that it becomes

$$
\rho_{h_0}(T) = \frac{B_0 h_0^2}{8\pi v_F \chi_{JP}^2 \lambda_{latt}^2} \left[ \lambda_{latt} + m^2(T) \right] \times \left[ \frac{\lambda_{latt}}{\lambda_{latt} + m^2(T)} \right] - m^2(T) K \left[ \frac{\lambda_{latt}}{\lambda_{latt} + m^2(T)} \right].
$$

(25)

where $E(x)$ and $K(x)$ are hyperbolic functions defined, respectively, according to $E(x) \equiv \int_0^{\pi/2} [1 - x \sin^2(\theta)]^{1/2} d\theta$ and $K(x) \equiv \int_0^{\pi/2} [1 - x \sin^2(\theta)]^{-1/2} d\theta$.

In order to have an idea of the low-temperature dependence of the resistivity, we can expand it to first order in

$$
m^2(T) = \Delta s + \frac{U}{N_f} \int \frac{d^2 k}{(2\pi)^2} \left[ T \sum_{\omega_n} k^2 + \lambda_{latt} \cos^2(2\theta) + B \cos^2(2\theta) |\omega_n| / (v_F k) + C \sin^2(2\theta) \omega_n^2 / (v_F k)^2 + m^2(T) \right] \nu_0 \int_0^\nu_0 \frac{d\omega}{2\pi} \frac{k^2 + \lambda_{latt} \cos^2(2\theta) + B \cos^2(2\theta) |\omega| / (v_F k) + C \sin^2(2\theta) \omega^2 / (v_F k)^2}{\lambda_{latt} + m^2(T)}.
$$

(27)

where we substituted the nematic propagators in Eqs. (8) and (9). The details regarding the evaluation of the momentum-frequency integrals and the Matsubara sum on the right-hand side of Eq. (27) are described in Appendix A. By following the analysis developed there, we obtain that the equation for $m^2(T)$ evaluates to

$$
m^2(T) = \frac{\Delta s}{1 + \gamma} + \frac{\nu_0 U T}{2\pi(1 + \gamma) N_f} \times \left[ \frac{m(T)}{(2\pi B T / v_F)^{1/3}} \right] \left[ \frac{\lambda_{latt}^{1/2}}{(2\pi B T / v_F)^{1/3}} \right] \Xi, \quad \text{(28)}
$$

where the function $\Xi(x, \tau)$ is defined in Eq. (A12) and the parameter $\gamma$ refers to

$$
\gamma = \frac{v_F \nu_0 U}{4\pi^3 \sqrt{C} N_f} \int_0^{\lambda_{latt}^{1/2} / B} d\rho \int_0^{2\pi} d\theta \int_0^\infty dv \rho^5 \left[ \rho^4 + 4\kappa_{latt} \rho^2 \cos^2(\theta) + \rho v \cos^2(\theta) + v^2 \sin^2(\theta) \right]^2.
$$

(29)

Here, $\Lambda$ denotes an ultraviolet momentum cutoff and $\kappa_{latt} \equiv \lambda_{latt} / (\nu_0 g_{\text{hem}}^2) = 1 / 2 \omega_0 \left( 1 + \frac{1}{C_{11}} \right) g_{\text{hem}}^2$ is a dimen-
In fact, as described in Appendix B, we obtain

\[ \text{energy, which corresponds to the low-temperature limit.} \]

For simplicity, we will consider here only the forward scattering channel, since it is expected to be the most relevant contribution in the present model. Other types of contributions (backscattering, large-angle scattering, Umklapp) will not be considered here, and are left for a future investigation. The nematic fluctuations interacting with the fermionic degrees of freedom in a single patch on the Fermi surface will give only subleading corrections to the free fermion polarizability, whose imaginary part is given by \( \Im \Pi^R(k, \omega) = c_\theta \frac{\omega}{\varepsilon_{F} k^2} \) with \( c_\theta = 1/(4\pi) \), as also obtained in Refs. [24, 56–58]. As a consequence, by applying the generalization of that result to our model with a

\[ \text{interaction } \kappa_{\text{latt}}. \]

\[ \text{In fact, the higher the coupling } \kappa_{\text{latt}} \text{ of the nematic fluctuations to the acoustic phonons the larger the critical temperature associated with the onset of nematic order in the model.} \]

\[ \text{This confirms that such an unavoidable coupling to the lattice turns out to be beneficial for the enhancement of nematic fluctuations in the system [39].} \]

V. CONTRIBUTIONS TO THE DC RESISTIVITY

From Eq. (24), one can see that in order to obtain the full profile of the temperature dependence of \( \rho(T) \), one still have to calculate the temperature dependence of \( \chi_{JP} \). The details of this calculation are shown in the Appendix C. We will now proceed to discuss the contributions to the resistivity coming from the two types of disorder considered in the present work.

A. Random potential coupled to the fermions

First, we analyze the random potential contribution (proportional to \( V_0^2 \) in Eq. (24)) coupled to the fermions. For simplicity, we will consider here only the forward scattering channel, since it is expected to be the most relevant contribution in the present model. Other types of contributions (backscattering, large-angle scattering, Umklapp) will not be considered here, and are left for a future investigation.

\[ \text{FIG. 5. (a) Nematic effective mass (in units of } U \text{) as a function of temperature } T \text{ for several choices of the nemato-elastic coupling } \lambda_{\text{latt}} \text{ at } \Delta s = 0. \]

\[ \text{(b) Same quantity (in units of } U \text{) as a function of } T \text{ for several choices of the tuning parameter } \Delta s \text{ (for this latter plot, we set the nemato-elastic coupling equal to } \lambda_{\text{latt}} = 0.5).} \]

\[ \text{sionless parameter that measures the strength of the lattice coupling with respect to the nematic interaction [41]. The numerical solution of the Eq. (28) is displayed in Fig. 5. In Fig. 5(a), we can see that if the effective nemato-elastic coupling } \kappa_{\text{latt}} \text{ is equal to zero (i.e., with no phonons included), the effective mass obeys the following scaling form} \]

\[ m^2(T) \sim T \ln(1/T) \text{ (see Appendix B), in agreement with Ref. [34]. As the nemato-elastic coupling becomes finite, this effective mass continues to be described by a } T\text{-linear contribution at high enough temperatures, but its dependence evolves into } T^2 \text{ at temperatures } T \ll \kappa_{\text{latt}}^3/\varepsilon_{F}, \text{ where } \varepsilon_{F} = v_F k_F \text{ is the Fermi energy, which corresponds to the low-temperature limit.} \]

\[ \text{In fact, as described in Appendix B, we obtain} \]

\[ m^2(T) = \frac{16\pi^2 k_F^2}{\eta^2 \kappa_{\text{latt}}^2} W_0^2 \left( \frac{\eta}{2} \sqrt{\frac{\alpha \Gamma}{1 - \beta \Gamma}} \right) \left( \frac{T}{\varepsilon_{F}} \right)^2, \]

\[ \text{where } \alpha \approx 0.156, \beta \approx 1.698, \eta \approx 1.277, W_0(z) \text{ is the first branch of the Lambert function, and } \Gamma \text{ is a model parameter given in Eq. (B11).} \]

\[ \text{In addition, we show in Fig. 5(b) that the effect of varying the tuning parameter } \Delta s \text{ for positive values does not change this } T^2 \text{ contribution at lower temperatures.} \]

\[ \text{By contrast, in the situation where the control parameter satisfies } \Delta s < 0, \text{ the nematic effective mass } m^2(T) \text{ vanishes for } T < T_\kappa(s) \text{ (see again Fig. 5(b)). Therefore, our calculation departing from the tetragonal phase successfully predicts that there is a finite temperature transition, which signals the onset of Ising-nematic order in the present model. We have also checked numerically that the dependence of } T_\kappa(s) \text{ is consistent with the scaling form } T_\kappa(s) \sim |\Delta s| \ln(1/|\Delta s|), \text{ which is displayed in Fig. 6.} \]

\[ \text{This scaling form is universal and independent of } \kappa_{\text{latt}} \text{ (see also Figs. 1(a) and 1(b) for the phase diagram corresponding to the two scenarios obtained in this work).} \]

\[ \text{On the other hand, we note that the prefactor associated with the aforementioned functional dependence of } T_\kappa(s) \text{ is non-universal and strongly depends on the interaction } \kappa_{\text{latt}}. \]
Fermi surface of circular shape, the fermionic contribution to the resistivity yields

$$\rho_{c}(T) = c_{A} \frac{V_{F}^{2}}{\chi_{JF}(T)},$$

where $c_{A}$ is an ultraviolet cutoff-dependent prefactor. This gives a $T$-independent residual resistivity $\rho_{c}^{(0)} = c_{A} \frac{V_{F}^{2}}{\chi_{JF}}$ plus an additional term, from which comes the temperature dependence of $\chi_{JF}$. As indicated in Appendix C, the susceptibility $\chi_{JF}(T)$ in the limit of low temperatures ($T \ll \kappa_{l_{\text{att}}}^{3/2} \varepsilon_{F}$) becomes naturally described at $s = s_{c}$ by

$$\chi_{JF}(T) = \chi_{JF}^{(0)} - \frac{16\pi^{2}C_{0}N_{F}k_{F}^{2}}{\eta^{2}Uk_{F}^{2}} \left( \frac{\eta}{2} \sqrt{\frac{\alpha\Gamma}{1 - \beta T}} \right) \left( \frac{T}{\varepsilon_{F}} \right)^{2},$$

where $C_{0}$ is a temperature-independent constant. Consequently, the resistivity in this regime behaves as

$$\rho_{c}(T) - \rho_{c}^{(0)} = \frac{32\pi^{2}C_{0}N_{F}k_{F}^{2}}{\eta^{2}U\chi_{JF}^{(0)}k_{F}^{2}} \left( \frac{\eta}{2} \sqrt{\frac{\alpha\Gamma}{1 - \beta T}} \right) \left( \frac{T}{\varepsilon_{F}} \right)^{2},$$

which clearly describes a Fermi-liquid-like behavior. This result is in qualitative agreement with some recent experimental measurements on the compound FeSe$_{0.88}$S$_{0.11}$ [59, 60], which show a Fermi-liquid dependence of the resistivity as a function of temperature in the vicinity of a putative nematic QCP.

In the high-temperature limit, the resistivity becomes nearly $T$-linear with a coefficient proportional to $U$ (for more details, see Appendix B). This behavior is of course reminiscent of the paradigmatic strange metal regime observed in many strongly correlated compounds. At intermediate temperatures, it can be verified only numerically that there is a window in which the resistivity of the present model is consistent with a description in terms of $\rho(T) \sim T^{\alpha}$ (with the effective exponent $\alpha$ roughly satisfying the inequality $1 \leq \alpha \leq 2$). This intermediate regime naturally interpolates between the nearly $T$-linear resistivity that emerges at high temperatures and the $T^{2}$ behavior that appears at low temperatures (see Fig. 1(a) for the corresponding phase diagram obtained in this scenario). Therefore, our results indeed share some similarities with recent experimental data reported by Coldea and collaborators [59, 60], where it was observed for the FeSe$_{0.88}$S$_{0.11}$ compound under applied external pressure that there is a clear regime where the corresponding resistivity displays a $T^{\alpha}$-behavior (with a seemingly temperature-dependent $\alpha$ exponent, which also satisfies the inequality $1 \leq \alpha \leq 2$).

On the other hand, in the low-temperature limit, it can be shown via a power-counting scaling analysis that the random potential $V_{0}$ coupling turns out to be less relevant (from an RG point of view) than the random field disorder $h_{0}$ coupling. Therefore, at low temperatures, the random-field disorder (if present in the system) will likely dominate the transport properties near the nematic QCP in a model with acoustic phonons (this conclusion also agrees with Ref. [34] who investigated a nematic quantum critical model, but with no phonons included). For this reason, we now turn our attention to this potentially relevant contribution for the present model in the low-temperature regime.

![FIG. 6. The nematic transition temperature $T_{n}(s)$ versus the tuning parameter $\Delta s < 0$ for some choices of $\kappa_{latt}$. The lines refer to the best fitting curves described by the family of functions given by $T_{n}(s) = \alpha_{s}|\Delta s| \ln(b_{s}/|\Delta s|)$, where $\alpha_{s}$ and $b_{s}$ are fitting parameters that depend on the nemato-elastic coupling $\kappa_{latt}$.]

B. Random-field disorder coupled to the nematic fluctuations

Here, we analyze the contribution in the present quantum critical model coming from another possible type of disorder that some iron-based superconducting materials could also potentially host in the system (i.e., the random-field disorder coupled to the nematic order parameter defined in Eq. (23)). In the limit of low enough temperatures ($T \ll \kappa_{l_{\text{att}}}^{3/2} \varepsilon_{F}$), we obtain from Eqs. (26) and (32) that the resistivity associated with the nematic quantum critical phase as a result of the random field disorder effects in the presence of acoustic phonons is given, to leading order, by

$$\frac{\rho_{n}(T) - \rho_{n}^{(0)}}{\rho_{n}^{(0)}} = \frac{C_{1}}{4} \left( \frac{T}{\varepsilon_{F}} \right)^{2} \ln \left[ C_{1} \left( \frac{T}{\varepsilon_{F}} \right)^{2} \right] + C_{2} \left( \frac{T}{\varepsilon_{F}} \right)^{2},$$

where $\rho_{n}^{(0)}$ is, as we have discussed before, a temperature-
independent residual resistivity and we have also defined

\[ C_1 = \frac{16\pi^2 k_F^2}{\eta^2 \nu_0 g_{nem}^2 \kappa_{latt}^2} W_0^{\frac{\eta}{2}} \left( \frac{\alpha \Gamma}{2 \left( 1 - \beta \right)^2} \right), \tag{35} \]

\[ C_2 = \frac{4\pi^2 k_F^2}{\eta^2 \kappa_{latt}^2} \left[ \frac{8C_0 N_f}{U \chi'_{p0}} - \frac{4 \ln(2) - 1}{\nu_0 g_{nem}^2 \kappa_{latt}^2} \right] W_0^{\frac{\eta}{2}} \left( \frac{\alpha \Gamma}{2 \left( 1 - \beta \right)^2} \right). \tag{36} \]

Consequently, the resistivity of the present model at low temperatures evolves into an approximate \( T^2 \)-behavior, which one could naively think that a Fermi liquid behavior would also be recovered for this regime as a result the coupling of the nematic fluctuations to the lattice. However, it is important to point out that, in the low-temperature limit, the logarithmic correction appearing in Eq. (34) will eventually dominate the transport properties of the model. This means that in the vicinity of the nematic QCP a non-Fermi-liquid with a Kondo-like upturn is obtained due to the random-field disorder effects considered in the present case. Despite this, apart from this very important difference, other transport properties turn out to be similar to the ones discussed so far: a nearly \( T \)-linear resistivity at high-temperatures and a regime consistent with \( \rho(T) \sim T^\alpha \) at intermediate temperatures with the effective exponent \( \alpha \) roughly satisfying the inequality \( 1 \lesssim \alpha \lesssim 2 \) (see Fig. 1(b) for the corresponding phase diagram obtained in this scenario). In this way, our result implies from a broad perspective that the low-temperature behavior associated with Ising-nematic quantum criticality in the presence of acoustic phonons depends in a crucial way on the type of disorder that a given material hosts in the system. This could provide a unified explanation of different behaviors in the resistivity observed at low temperatures in many iron-based superconducting compounds available in the literature.

VI. SUMMARY

In this work, we have analyzed the resistivity associated with an Ising-nematic quantum critical point in the presence of both disorder and acoustic phonons in the corresponding lattice model. We have employed the Mori-Zwanzig memory-matrix transport theory, which does not rely on the existence of well-defined quasiparticles in the effective theory. As a result, we have obtained that by including the unavoidable interaction between the nematic fluctuations and the acoustic phonons, those fluctuations clearly become enhanced, and a higher transition critical temperature associated with the onset of nematic order appears. Moreover, the resistivity \( \rho(T) \) in the tetragonal phase as a function of temperature becomes described by a universal scaling form given by \( \rho(T) \sim T \ln(1/T) \) at high temperatures, reminiscent of the strange metal regime observed in many quantum critical materials. By contrast, for a window of temperatures comparable with \( \kappa_{latt} \xi_F \) (where \( \xi_F \) is the Fermi energy of the microscopic model), the system exhibits a regime in which the resistivity is consistent with a description in terms of \( \rho(T) \sim T^\alpha \), where the effective exponent roughly satisfies the inequality \( 1 \lesssim \alpha \lesssim 2 \). Lastly, towards the low-temperature limit, we have shown that the properties of the quantum critical state change in an important way depending on the type of disorder present in the system: It can either turn into a conventional Fermi liquid described by \( \rho(T) \sim T^2 \) for the case where only the random potential \( V_0 \) coupling is included in the model, or it could display a crossover to yet another non-Fermi liquid phase characterized by the scaling form \( \rho(T) - \rho_0 \sim T^2 \ln T \) due to the effects of the random-field disorder coupling \( h_0 \) (implying in the latter case that the system would exhibit a Kondo-like upturn in the resistivity). We have argued that our present results could be important for the interpretation, e.g., of recent transport experiments performed in the FeSe\(_{0.85}\)S\(_{0.11}\) compound under applied pressure, and potentially in many other iron-based superconducting compounds as well.

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Appendix A: Derivation of the nematic-mass equation

To evaluate the Matsubara sum and the integrals on the right-hand side of Eq. (27), we proceed as in Ref. [34] and write Eq. (27) as a sum of the two terms

\[ I_1 = \int \frac{d^2k}{(2\pi)^2} \left[ T \sum_{\omega_n} k^2 + \lambda_{latt} \cos^2(2\theta) + B \cos^2(2\theta) |\omega_n|/(v_F k) + C \sin^2(2\theta) \omega_n^2/(v_F k)^2 + m^2(T) \right] \]

\[ - \int \frac{d\omega}{2\pi} k^2 + \lambda_{latt} \cos^2(2\theta) + B \cos^2(2\theta) |\omega|/(v_F k) + C \sin^2(2\theta) \omega^2/(v_F k)^2 + m^2(T) \] \]

\tag{A1}
\[ I_2 = \int \frac{d^2 k}{(2\pi)^2} \left[ \int \frac{d\omega}{2\pi} \frac{\nu_0}{k^2 + \lambda_{\text{latt}} \cos^2(2\theta) + B \cos(2\theta) |\omega|/(v_F k) + C \sin^2(2\theta) \omega^2/(v_F k)^2 + m^2(T)} \right. \\
- \left. \int \frac{d\omega}{2\pi} \frac{\nu_0}{k^2 + \lambda_{\text{latt}} \cos^2(2\theta) + B \cos(2\theta) |\omega|/(v_F k) + C \sin^2(2\theta) \omega^2/(v_F k)^2} \right]. \]  

(A2)

In order to find a convergent expression for \( I_1 \), it is not necessary to keep the formally irrelevant \( \omega^2 \) term in the nematic propagator. Indeed, to prove that, we first recall that the bosonic Matsubara frequency is given by \( \omega_n = 2\pi nT \) and then define the frequency cutoff \( \Omega = 2\pi NT \), where \( N \) is a very large integer number. Having this in mind, we write \( I_1 \) as the limit

\[ I_1 = \int \frac{d^2 k}{(2\pi)^2} \lim_{\Omega \to \infty} \left[ T \sum_{n=-N}^{N} \frac{\nu_0}{k^2 + \lambda_{\text{latt}} \cos^2(2\theta) + m^2(T) + 2\pi BT \cos^2(2\theta)|n|/(v_F k)} \\
- \int_{-\Omega}^{\Omega} \frac{d\omega}{2\pi} \frac{\nu_0}{k^2 + \lambda_{\text{latt}} \cos^2(2\theta) + m^2(T) + B \cos^2(2\theta)|\omega|/(v_F k)} \right]. \]  

(A3)

Next, we can proceed with the evaluation of the Matsubara sum for \( N \) finite by utilizing the following mathematical identity

\[ \sum_{n=-N}^{N} \frac{1}{a + b|n|} = \frac{2}{b} \left[ \psi^{(0)} \left( \frac{a}{b} + N + 1 \right) - \psi^{(0)} \left( \frac{a}{b} + 1 \right) \right] \\
+ \frac{1}{a}. \]  

(A4)

Here, the parameters \( a \) and \( b \) correspond to positive real numbers and \( \psi^{(0)}(z) \) denotes the digamma function. Thus, using the set of definitions

\[ a \equiv k^2 + \lambda_{\text{latt}} \cos^2(2\theta) + m^2(T), \]  
\[ b \equiv \frac{2\pi BT}{v_F k} \cos^2(2\theta), \]  

(A5)  

(A6)

one can show that

\[ I_1 = \nu_0 \int \frac{d^2 k}{(2\pi)^2} \left\{ \frac{T}{a} + \frac{2T}{b} \lim_{\Omega \to \infty} \left[ \psi^{(0)} \left( \frac{a}{b} + \frac{\Omega}{2\pi T} + 1 \right) \right] \\
- \ln \left( 1 + \frac{b \Omega}{2\pi a T} \right) \right\} - \left[ \frac{2T}{b} \psi^{(0)} \left( \frac{a}{b} + 1 \right) \right], \]  

(A7)

Finally, by substituting the expressions for \( a \) and \( b \) and then rescaling the momentum integral according to

\[ k = \frac{(2\pi BT)^{1/3}}{v_F y}, \]  

(A10)

one finds that Eq. (A9) simplifies to

\[ I_1 = \nu_0 T \frac{2\pi}{m(T)} \Xi \left[ \frac{m(T)}{(2\pi BT/v_F)^{1/3}} \cdot \left( \frac{\lambda_{\text{latt}}^{1/2}}{2(\pi BT/v_F)^{1/3}} \right) \right]. \]  

(A11)

where

\[ \Xi(x, \tau) = \int_0^\infty dy \int_0^{2\pi} \frac{d\theta}{2\pi} \left( \frac{y}{y^2 + x^2 + \tau^2 \cos^2(2\theta)} + \frac{2y^2}{\cos^2(2\theta)} \right) \ln \left[ \frac{y^3 + [x^2 + \tau^2 \cos^2(2\theta)]y}{\cos^2(2\theta)} \right] \]  

(A12)

In order to determine \( I_2 \), we expand the integrand on the right-hand side of Eq. (A2) in powers of \( m(T) \).
where the parameters on the right-hand side are given by $\alpha \approx 0.156067$, $\beta \approx 1.69823$, and $\eta \approx 1.27679$.

By performing the substitutions $k = B\eta^{-1/2}\rho$ and $\omega = v_F B^2 C^{-1/2}v$, we arrive at the result

$$I_2 = -\frac{\nu_0 v_F m^2(T)}{4\pi^3 \sqrt{C}} \int_0^{\Lambda/\sqrt{C}} d\rho \int_0^{2\pi} d\theta \int_0^\infty dv$$

$$\times \left[ \rho^4 + 4\kappa_l \rho^2 \cos^2(2\theta) + 5\rho v \cos^2(2\theta) + v^2 \sin^2(2\theta) \right]^2,$$

where the dimensionless parameter $\kappa_l = \lambda_{latt} / (\nu_0 \eta_{nem})$ measures the strength of the nemato-elastic coupling with respect to the nematic interaction. Upon substituting the expressions for $I_1$ and $I_2$ obtained here into Eq. (27), we arrive at the result given in Eq. (28).

$$\Xi \left[ \frac{m(T)}{(2\pi B T/v_F)^{1/3}} \right] = n_c \left[ \frac{\lambda_{latt}^{1/2}}{(2\pi B T/v_F)^{1/3}} \right]^{-3} \Xi \left[ \left( \frac{\lambda_{latt}^{1/2}}{(2\pi B T/v_F)^{1/3}} \right)^2 \frac{m(T)}{(2\pi B T/v_F)^{1/3}} \right],$$

where

$$\Xi(x) = \frac{1}{2\pi} \int_0^\infty dy \int_0^\infty dz \left( \frac{y}{x^2 + y^2 + z^2} + \frac{2y^2}{z^2} \right) \left\{ \ln \left[ \frac{y^2 + (x^2 + z^2)}{z^2} \right] - \psi(0) \left[ \frac{y^2 + (x^2 + z^2)}{z^2} + 1 \right] \right\}.$$

Here, we have sent the upper cutoff $z_0 = \frac{\theta_0 v_F \lambda_{latt}^{1/2}}{(2\pi B T/v_F)}$ in the integration over the $z$ variable to infinity, because we are interested in the limit of lower temperatures. As a result, if we consider the interval $0 < x \ll 1$, $\Xi(x)$ can be well approximated by

$$\Xi(x) = \alpha e^{-\eta x} + \beta x^2,$$

where the parameters on the right-hand side are given by $\alpha \approx 0.156067$, $\beta \approx 1.69823$, and $\eta \approx 1.27679$ (see Fig. 7).

Next, the substitution of Eqs. (B5) and (B7) into Eq. (28) yields

$$\left[ 1 - \frac{\beta n_c v_F \nu_0 \lambda_{latt}^{1/2}}{4\pi^2 (1 + \gamma) B N_f} \right] m^2(T) = \frac{\Delta s}{1 + \gamma} + \frac{\alpha n_c v_F U T^2}{(1 + \gamma) v_F \lambda_{latt}^{3/2} N_f} \times \exp \left[ - \frac{\eta v_F \lambda_{latt}^{1/2} m(T)}{2\pi B T} \right].$$

At the nematic QCP, the above equation can be solved exactly. Indeed, by writing it as $m^2(T) = \alpha e^{-\eta x} + \beta x^2$,

Appendix B: Asymptotic solution of the nematic-mass equation

1. Low-temperature behavior

This section is aimed at determining the asymptotic dependence of $m^2(T)$ on both $T$ and $\lambda_{latt}$ when the effective coupling

$$\lambda_{latt}(T) = \frac{\lambda_{latt}^{1/2}}{(2\pi B T/v_F)^{1/3}}$$

becomes larger than any energy scale of the model [see Eq. (28)]. In other words, this is equivalent to considering the temperature regime $T \ll \kappa_{latt} \varepsilon_F$, with $\varepsilon_F$ being the Fermi energy of the system.

To begin with, we linearize the cosine function $\cos(2\theta)$ on the right-hand side of Eq. (A12) around the cold spots $\theta_n = (2n + 1)\pi/4$. As a result, we are able to write

$$\int_0^{2\pi} \frac{d\theta}{2\pi} \approx \sum_{n=1}^{n_c} \int_{\theta_n - \theta_0}^{\theta_n + \theta_0} \frac{d\theta}{2\pi},$$

where $n_c = 4$ refers to the number of cold spots in the interval $[0, 2\pi)$, and $\theta_0$ is an angular cutoff that sets the validity of the linear approximation for $\cos(2\theta)$. By performing the following substitutions

$$y \rightarrow \frac{y}{\lambda_{latt}^{2/3}(T)},$$

$$\theta \rightarrow \theta_n + \frac{\gamma}{2 \lambda_{latt}^3(T)},$$

one obtains

$$\left[ 1 - \frac{\beta n_c v_F \nu_0 U}{4\pi^2 (1 + \gamma) B N_f} \right] m^2(T) = \frac{\Delta s}{1 + \gamma} + \frac{\alpha n_c v_F U T^2}{(1 + \gamma) v_F \lambda_{latt}^{3/2} N_f} \times \exp \left[ - \frac{\eta v_F \lambda_{latt}^{1/2} m(T)}{2\pi B T} \right].$$

FIG. 7. Comparison between the dependence of both functions $\Xi(x)$ and the fit given by $f(x) = \alpha e^{-\eta x} + \beta x^2$ for the interval $0 < x < 1$. The parameters in the latter function are determined exactly. Indeed, by writing it as $m^2(T) = \alpha e^{-\eta x} + \beta x^2$. 

By performing the substitutions $k = B\eta^{-1/2}\rho$ and $\omega = v_F B^2 C^{-1/2}v$, we arrive at the result

$$I_2 = -\frac{\nu_0 v_F m^2(T)}{4\pi^3 \sqrt{C}} \int_0^{\Lambda/\sqrt{C}} d\rho \int_0^{2\pi} d\theta \int_0^\infty dv$$

$$\times \left[ \rho^4 + 4\kappa_l \rho^2 \cos^2(2\theta) + 5\rho v \cos^2(2\theta) + v^2 \sin^2(2\theta) \right]^2,$$

where the dimensionless parameter $\kappa_l = \lambda_{latt} / (\nu_0 \eta_{nem})$ measures the strength of the nemato-elastic coupling with respect to the nematic interaction. Upon substituting the expressions for $I_1$ and $I_2$ obtained here into Eq. (27), we arrive at the result given in Eq. (28).
\[ m_0 \exp[-\kappa_0 m(T)], \] its solution evaluates to

\[ m^2(T) = \frac{4}{\kappa_0} W_0^2 \left( \frac{\sqrt{m_0 \kappa_0}}{2} \right), \quad (B9) \]

with \( W_0(z) \) being the principal branch of the so-called Lambert W function, i.e., \( W_0(0) = 0 \). By employing the definitions of \( m_0 \) and \( \kappa_0 \) in terms of the original model parameters, the expression for the nematic mass becomes

\[ m^2(T) = \frac{16 n^2 B^2}{\eta^2 v_F \lambda_{\text{latt}}^2} W_0^2 \left( \frac{\eta}{2} \sqrt{\frac{\alpha \Gamma}{1 - \beta \Gamma}} \right) T^2, \quad (B10) \]

where we have set

\[ \Gamma = \frac{n_c v_F \nu_0 U \lambda_{\text{latt}}^{1/2}}{4 \pi^2 (1 + \gamma) \beta N_f}. \quad (B11) \]

By making the substitutions \( B = \nu_0 \gamma_{\text{nem}} \) and \( \varepsilon_F = v_F k_F \) into Eq. (B10), one finds the expression for \( m^2(T) \) given in the main text [see Eq. (30)].

2. High-temperature behavior

Now we turn our attention to determining the asymptotic behavior of \( m^2(T) \) for the interval of high temperatures \( \kappa_{3/2} \varepsilon_F \ll T \ll \varepsilon_F \). In that case, Eq. (28) can be approximated as

\[ m^2(T) = \frac{\Delta s}{1 + \gamma} + \frac{\nu_0 UT}{2 \pi (1 + \gamma) N_f} \Xi \left[ \frac{m(T)}{(2 \pi B T / v_F)^{1/3}} \right]. \quad (B12) \]

One should notice that as \( x \to 0 \), \( \Xi(x, 0) \) has the asymptotic form

\[ \Xi(x, 0) = \ln(1/x) - 1.0747. \quad (B13) \]

So within logarithmic accuracy and also considering that the system is exactly at its nematic QCP, we are able to write Eq. (B12) as

\[ m^2(T) = \frac{\nu_0 UT}{2 \pi (1 + \gamma) N_f} \ln \left[ \frac{(2 \pi B T / v_F)^{1/3}}{m(T)} \right]. \quad (B14) \]

The solution of the above equation can be expressed as

\[ m^2(T) = \kappa_0^2 \exp \left[ -W_0 \left( \frac{2 \kappa_0}{m_0} \right) \right], \quad (B15) \]

where

\[ \kappa_0 \equiv \frac{\nu_0 UT}{2 \pi (1 + \gamma) N_f}, \quad (B16) \]

\[ \tilde{\kappa}_0 \equiv \left( \frac{2 \pi B T}{v_F} \right)^{1/3}. \quad (B17) \]

Since for \( x \gg 1 \), the Lambert function \( W_0(x) \) can be well described to first order approximation by

\[ W_0(x) \approx \ln \left( \frac{x}{\ln x} \right), \quad (B18) \]

we come to the conclusion that the mass of the nematic fluctuations for \( \kappa_{3/2} \varepsilon_F \ll T \ll \varepsilon_F \) is asymptotically given by

\[ m^2(T) = \frac{k_F^2 U}{12 \pi^2 (1 + \gamma) N_f} T \ln \left[ \frac{256 \pi^6 (1 + \gamma)^3 \gamma_{\text{nem}}^4 N_f^3} {U^2 \varepsilon_F^2} \right], \quad (B19) \]

where we have made the substitutions \( B = \nu_0 \gamma_{\text{nem}} \) and \( \varepsilon_F = v_F k_F \).

Appendix C: Susceptibility \( \chi_{JP} \)

In this appendix, we calculate the temperature dependence of the susceptibility \( \chi_{JP}(T) \). To do that, we will follow a similar treatment as discussed in Ref. [34]. For conciseness, only the relevant Feynman diagrams for this calculation are displayed in Fig. 8.

The first diagram represented in Fig. 8(a) readily evaluates to

\[ \mathcal{D}_a = -T \sum_{\nu_n} \int \frac{d^2 q}{(2 \pi)^2} \nu^2_{k+q,q} G_0(k+q, i \omega_n + i \nu_n) \times G_0(q, i \nu_n) \]

\[ = -\int \frac{d^2 q}{(2 \pi)^2} \nu^2_{q} \left[ n_F(\xi_{q+k/2}) - n_F(\xi_{q-k/2}) \right] \]

\[ \approx -\int \frac{d^2 q}{(2 \pi)^2} \nu^2_{q} n_F(\xi_{q}) \equiv \chi_0, \quad (C1) \]

where \( n_F(\omega) = 1/(e^{\omega/T} + 1) \) is the Fermi distribution function. Since we are working with the continuum theory described by Eq. (19), the \( d \)-wave form factor may be written as \( V_{q,q} = -2(q_x^2 - q_y^2) \). The above integral naturally yields a constant contribution (denoted by \( \chi_0 \)) to \( \chi_{JP}(T) \).

As for the diagram represented in Fig. 8(b), we obtain

\[ \mathcal{D}_b = 2T \sum_{\Omega_m} \int \frac{d^2 k}{(2 \pi)^2} \Sigma_1(k, i \Omega_m) D(k, i \Omega_m), \quad (C2) \]

where \( \Sigma_1 \) is a bosonic self-energy on one fermion line (the factor 2 is related to the fact that there are two diagrams that give exactly the same result). The corresponding bosonic self-energy \( \Sigma_1 \) is given by

\[ \Sigma_1 = -\frac{\gamma_{\text{nem}}^2}{\nu_0 m} T \sum_{\nu_n} \int \frac{d^2 q}{(2 \pi)^2} \nu^2_{q} \nu_{k+q,q} G_0(k + q, i \Omega_m + i \nu_n) \times G_0(q, i \nu_n). \quad (C3) \]

In addition, for the diagram in Fig. 8(c), we have

\[ \mathcal{D}_c = T \sum_{\Omega_m} \int \frac{d^2 k}{(2 \pi)^2} \Sigma_2(k, i \Omega_m) D(k, i \Omega_m), \quad (C4) \]
where we defined the quantity $\Sigma_2$ (despite the notation, this is not a self-energy insertion), which is given by

$$
\Sigma_2 = -\frac{g^2_{\text{nem}}}{\nu_0 m} T \sum_{\nu_n} \int \frac{d^2 q}{(2\pi)^2} q_x (q_x + k_x) V^2_{k+q,q} x 
\times G_0^2(k + q, i\Omega_m + i\nu_n) G_0^2(q, i\nu_n)
= \Sigma_2^{(a)} + \Sigma_2^{(b)},
$$

(C5)

where $\Sigma_2^{(a)}$ is the term in the integral proportional to $q_x^2$ and $\Sigma_2^{(b)}$ is the term in the integral proportional to $q_x k_x$. In a similar way, for the diagram in Fig. 8(d), we obtain

$$
D_d = T \sum_{\Omega_m} \int \frac{d^2 k}{(2\pi)^2} \Sigma_3(k, i\Omega_m) D(k, i\Omega_m),
$$

(C6)

where we defined $\Sigma_3$, which is given by

$$
\Sigma_3 = -8\frac{g^2_{\text{nem}}}{\nu_0} T \sum_{\nu_n} \int \frac{d^2 q}{(2\pi)^2} q_x \left( q_x + \frac{k_x}{2} \right) V_{k+q,q}
\times G_0(k + q, i\Omega_m + i\nu_n) G_0^2(q, i\nu_n).
$$

(C7)

Collecting all the above terms, it can be shown that $\Sigma_1 + \Sigma_2 + \Sigma_3 \equiv \mathcal{C}_0$, where $\mathcal{C}_0$ is a constant given approximately by

$$
\mathcal{C}_0 \approx -\frac{g^2_{\text{nem}}}{\nu_0} \int \frac{d^2 q}{(2\pi)^2} \left[ \frac{g^2}{2m} V^2_{\mathbf{q}, \mathbf{q}, \Omega_{\mathbf{q}}^p}(q_x) + 8q^2 V_{\mathbf{q}, \mathbf{q}, \Omega_{\mathbf{q}}^p}(q_x) \right].
$$

(C8)

Therefore, we can write

$$
\chi_{\mathcal{J}P}(T) = \chi_{\mathcal{J}P}^{(0)} - \frac{\mathcal{C}_0 N_F}{U} [m^2(T) - \Delta s].
$$

(C10)

As a result, if we consider the temperature regime $T \ll \eta^3/\kappa_{\text{m}}^2 \varepsilon_F$ and also suppose that the system is at its nematic QCP (see Appendix B), the susceptibility $\chi_{\mathcal{J}P}(T)$ evaluates to

$$
\chi_{\mathcal{J}P}(T) = \chi_{\mathcal{J}P}^{(0)} - \frac{16\pi^2 C_0 N_F k_F^2 W_0^2}{\eta^2 U \kappa_{\text{m}}^2} \left( \frac{\eta}{2} \sqrt{\alpha \Gamma} \right) \left( \frac{T}{\varepsilon_F} \right)^2
$$

(C11)

On the other hand, for the interval of temperature $\kappa_{\text{m}}^3 \varepsilon_F \ll T \ll \varepsilon_F$ the nemato-elastic coupling can be dropped and, as a result, the susceptibility $\chi_{\mathcal{J}P}(T)$ becomes

$$
\chi_{\mathcal{J}P}(T) = \chi_{\mathcal{J}P}^{(0)} - \frac{C_0 k_F^2}{12\pi^2 (1 + \gamma)} \frac{T}{\varepsilon_F}
\times \ln \left[ \frac{256\pi^2 (1 + \gamma)^{3/2} \kappa_{\text{m}}^2 \varepsilon_F}{U^3} \right].
$$

(C12)

The last two results are the limits of $\chi_{\mathcal{J}P}(T)$ discussed in the main text.
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