Mathematical model of decomposition of hydrates in a reservoir

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Abstract. A mathematical model of the process of decomposition of hydrates in a reservoir by reducing pressure is implemented numerically. The dependence of results obtained on main parameters of the problem such as initial hydrate saturation, initial water saturation, depression on the formation, permeability coefficient, porosity has been studied. We compare results of the numerical implementation of the original model with results obtained after linearization of the heat equation, equations of gas flow continuity both separately and simultaneously. It is shown that with relatively low permeability, initial water saturation and depression one can use the model with linearized equations. The relative difference of the calculated hydrate saturation is a few percent. For the distribution of temperature and pressure, this difference is even smaller. The developed algorithm can easily be expanded for a multidimensional problem.

1. INTRODUCTION
The property of natural gas under certain conditions take in the earth’s crust in a solid state and to form gas hydrate deposits, established by V.G. Vasiliev, Yu.F. Makogon, F.A. Trebin, A.T. Trofimuk and N.V. Chersky, was recognized as a discovery in [1]. Studies to determine the physical and mechanical properties of hydrates of natural gases and their fields are summarized in the monographs of Yu.F. Makogon [2], S.Sh.Byk, V.I.Fomina [3]. In the papers of N.V. Chersky and E.A.Bondarev [4], M.Selin and E.Sloyan [5], mathematical models of hydrate dissociation were constructed thermal effects on gas hydrate formation. The modeling of the process of dissociation of a gas hydrate due to a decrease in pressure was performed by Verigin H.H. et al. [6]. Here it was believed that the hydrate saturates the entire pore volume, and in addition it was assumed that the dissociation occurs under isothermal conditions. The mathematical model is reduced to the classical problem of Stefan. A self-similar solution of the linearized system of gas filtration equations was constructed for the case of gas extraction through the gallery. The first attempt at mathematical modeling of wet gas filtration with hydrate dissociation was undertaken in the monograph by E.A. Bondarev et al. [7]. In the above works, the phase transition occurs on a moving surface, as in problems like Stefan. Further development of these models were based on ideas of local thermodynamic equilibrium in the works of E.A.Bondarev, A.M.Maksimov, G.G.Tsyplina [8]. Also noteworthy are the works of Musakayev N. G., Borodin S. L., Belskikh D. S. [9], Khasanov M. K., Stolpovsky M. V., Kildibaeva S. R. [10] and Shagapov V.Sh., Khasanov M.K., Gimaltdinov I.K., Stolpovsky M.V. [11], where mathematical models are proposed and numerically implemented with a phase transition in an extended domain. This paper assesses the contribution of conductive and convective heat transfer to the process of
decomposition of hydrates in near-well zone when extracting gas from gas hydrate deposits through the gallery.

2. PROBLEM STATEMENT

The heat equation contains a hydrate-gas term that takes into account the heat of the phase transition [8-11]:

\[ C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{k(1 - \nu)f_g}{\mu_g} \rho_g c_g \frac{\partial p}{\partial x} \frac{\partial T}{\partial x} + \frac{k(1 - \nu)f_w}{\mu_w} \rho_w c_w \frac{\partial p}{\partial x} \frac{\partial T}{\partial x} + \phi \rho_h q_h \frac{\partial \nu}{\partial t}, \]

\[ 0 < x < l, \ t > 0; \] (1)

gas filtration equation [8-11]:

\[ \phi \frac{\partial}{\partial t} \left( (1 - \nu - \sigma) \frac{\partial P}{\partial x} \right) = \frac{\partial}{\partial x} \left( \frac{k(1 - \nu)f_g}{\mu_g} \frac{\partial p}{\partial x} \right) - \phi \rho_h \frac{\partial \nu}{\partial t} \]

\[ 0 < x < l, \ t > 0; \] (2)

where \( k \) - permeability coefficient of the porous medium; \( \mu_g \) - dynamic viscosity of gas, \( f_g \) - gas phase permeability:

\[ f_g = \begin{cases} 
(1 + 3\sigma) \left( \frac{0.9 - \sigma}{0.9} \right)^{3.5}, & 0 \leq \sigma < 0.9, \\
0, & 0.9 \leq \sigma \leq 1
\end{cases} \]

water filtration equation [8-11]:

\[ \phi \frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x} \left( \frac{k(1 - \nu)f_w}{\mu_w} \frac{\partial p}{\partial x} \right) - \phi \frac{\rho_h}{\rho_w} (1 - \varepsilon) \frac{\partial \nu}{\partial t} \]

\[ 0 < x < l, \ t > 0; \] (3)

\( f_w \) - relative phase permeability of water; \( \mu_w \) - dynamic viscosity of water;

\[ f_w = \begin{cases} 
\left( \frac{\sigma - 0.2}{0.8} \right)^{3.5}, & 0.2 < \sigma \leq 1, \\
0, & 0 \leq \sigma \leq 0.2
\end{cases} \]

equilibrium phase transition condition [2,3,7]:

\[ T = A \ln p + B \]

\[ 0 < x < l, \ t > 0; \] (4)

Initial conditions:

\[ T(0, x) = T_0, \ p(0, x) = A \ln p_0 + B, \ \nu(0, x) = \nu_0, \sigma(0, x) = \sigma_0 \] (5)

Condition on the left border:

\[ T(t, 0) = T_g, \ p(t, 0) = p_g, \] (6)

Condition on the right border:

\[ T(t, l) = T_0, \ p(t, l) = p_0, \nu(t, l) = \nu_0, \sigma(t, l) = \sigma_0 \] (7)
3. SOLUTION ALGORITHM

In order to increase the efficiency of the numerical implementation of the mathematical model (1) - (7), we transform equations (2) and (3). From equation (2) we exclude $\frac{\partial \nu}{\partial t}$ using the equation (1):

$$\phi \frac{\partial}{\partial t} \left( (1 - \nu - \sigma) \frac{p}{TR} \right) = \frac{\partial}{\partial x} \left( \frac{k(1 - \nu) f_g p}{\mu_g RT} \frac{\partial p}{\partial x} \right) - C \frac{\varepsilon A}{\rho_h \rho p} \frac{\partial p}{\partial t} +$$

$$+ \varepsilon \frac{\partial}{\partial x} \left( \frac{A}{\rho} \frac{\partial p}{\partial x} \right) + \frac{k(1 - \nu) f_g}{\mu_g} \frac{\partial p}{\partial T} + \frac{\varepsilon k}{q_h} \frac{1}{\mu_w} \rho_w c_w \frac{\partial p}{\partial x} \frac{\partial T}{\partial x} +$$

Hydrate saturation will be calculated from the equation

$$C \frac{\partial T}{\partial t} - \lambda A \frac{\partial}{\partial x} \left( (1 - \nu) \frac{p}{TR} \right) = \frac{\partial \lambda}{\partial x} \frac{\partial T}{\partial x} - \lambda \frac{\partial K}{\partial p} \frac{\partial p}{\partial x}$$

$$+ \frac{k(1 - \nu) f_g}{\mu_g} \rho_g c_g \frac{\partial T}{\partial x} + \frac{k(1 - \nu) f_w}{\mu_w} \rho_w c_w \frac{\partial T}{\partial x} + \phi \rho_h q_h \frac{\partial T}{\partial t} + \frac{\lambda}{K} \frac{\lambda A}{\rho_h} \frac{\partial \nu}{\partial t},$$

where

$$K = \frac{k(1 - \nu) f_g}{\mu_g} \frac{p}{RT}$$

The equation of water flow continuity (3) is reduced to the form

$$\phi \frac{f_g}{\mu_g} \frac{\partial \sigma}{\partial t} - m \frac{f_w}{\mu_w} \frac{\partial}{\partial x} ((1 - \nu - \sigma) \rho_g) + \phi \frac{f_g}{\mu_g} \rho_g \frac{\partial T}{\partial x} + \frac{f_w}{\mu_w} m \rho_h \frac{\partial \nu}{\partial t}$$

$$= \frac{f_g}{\mu_g} \frac{k}{\mu_w} \frac{\partial (1 - \nu) f_w}{\partial x} \frac{\partial p}{\partial x} - \frac{f_w}{\mu_w} \frac{k}{\mu_g} \frac{\partial f_g \rho_g}{\partial x} \frac{\partial p}{\partial x}$$

obtained by lowering the order of the water filtration equation, (3) using the gas filtration equation (2).

We describe an algorithm for the numerical solution of problem (8) - (10), (4) - (7). Let the distributions of the sought functions on the $n$-th time layer to be known. To go to the $n + 1$ th temporal layer, we first calculate the pressure distribution. We calculate the temperature distribution using the formula (4). Next, we calculate the distribution of hydrate saturation from equation (9). And at the end we calculate the distribution of water saturation from (10). And we perform simple iterations on nonlinearity.

4. NUMERICAL EXPERIMENTS

The assessment of the contribution of the terms taking into account convective and conductive heat transfer, both separately and jointly, has been carried out. For this, three more variants of equation (8) were considered:

- excluding convective heat transfer

$$\phi \frac{\partial}{\partial t} \left( (1 - \nu - \sigma) \frac{p}{TR} \right) = \frac{\partial}{\partial x} \left( \frac{k(1 - \nu) f_g p}{\mu_g RT} \frac{\partial p}{\partial x} \right) - C \frac{\varepsilon A}{\rho_h \rho p} \frac{\partial p}{\partial t} +$$

- without conductive heat transfer
\( \phi \frac{\partial}{\partial t} \left( (1 - \nu - \sigma) \frac{p}{TR} \right) = \frac{\partial}{\partial x} \left( \frac{k(1 - \nu) f_g}{\mu_g} \frac{p}{RT} \frac{\partial p}{\partial x} \right) - C \frac{\varepsilon}{q_h} \frac{A}{p} \frac{\partial p}{\partial t} + \frac{\varepsilon}{q_h} \frac{k(1 - \nu) f_g}{\mu_g} \rho_g c_g \frac{\partial p}{\partial x} \frac{\partial T}{\partial x} \frac{\partial p}{\partial t} + \frac{\varepsilon}{q_h} \frac{k(1 - \nu) f_w}{\mu_w} \rho_w c_w \frac{\partial p}{\partial x} \frac{\partial T}{\partial x} \frac{\partial p}{\partial t} \)

excluding both terms

\( \phi \frac{\partial}{\partial t} \left( (1 - \nu - \sigma) \frac{p}{TR} \right) = \frac{\partial}{\partial x} \left( \frac{k(1 - \nu) f_g}{\mu_g} \frac{p}{RT} \frac{\partial p}{\partial x} \right) - C \frac{\varepsilon}{q_h} \frac{A}{p} \frac{\partial p}{\partial t} \)

and accordingly three versions of equation (9):

\( C \frac{\partial T}{\partial t} - \frac{\lambda A}{K p} \phi \frac{\partial}{\partial x} \left( (1 - \nu - \sigma) \frac{p}{TR} \right) = \frac{\partial \lambda A}{K p} \frac{\partial T}{\partial x} \frac{\partial x}{\partial x} - \frac{\lambda A}{K p} \frac{\partial K}{\partial x} \frac{\partial p}{\partial t} \)

\( + \phi p h q_h \frac{\partial \nu}{\partial t} + \frac{\lambda A}{K p} \phi p h \frac{\varepsilon}{R} \frac{\partial \nu}{\partial t} \),

\( 0 < x < l, \quad t > 0; \)

\( C \frac{\partial T}{\partial t} = \frac{k(1 - \nu) f_g}{\mu_g} \rho_g c_g \frac{\partial p}{\partial x} \frac{\partial T}{\partial x} + \frac{k(1 - \nu) f_w}{\mu_w} \rho_w c_w \frac{\partial p}{\partial x} \frac{\partial T}{\partial x} + \phi p h q_h \frac{\partial \nu}{\partial t} , \)

\( 0 < x < l, \quad t > 0; \)

\( C \frac{\partial T}{\partial t} = \phi p h q_h \frac{\partial \nu}{\partial t} \)

\( 0 < x < l, \quad t > 0; \)

Also carried out the linearization of equation (2):

\( \phi \left( (1 - \nu_0 - \sigma_0) \frac{1}{T_0 R} \right) \frac{\partial p}{\partial t} = \frac{k(1 - \nu_0) f_g (\sigma_0)}{\mu_g} \frac{p_0}{RT_0} \frac{\partial^2 p}{\partial x^2} - \phi p h \frac{\varepsilon}{R} \frac{\partial \nu}{\partial t} \)

\( 0 < x < l, \quad t > 0; \)

The calculations were carried out at the following values of the parameters:

- \( c_s = 700 \) J/(kg · K); \( \rho_s = 2650 \) kg/m³; \( c_h = 3210 \) J/(kg · K); \( \rho_h = 910 \) kg/m³;
- \( c_w = 4.19 \cdot 10^3 \) J/(kg · K); \( \rho_w = 1000 \) kg/m³; \( c_g = 2093 \) J/(kg · K);
- \( \lambda_s = 2 \) Wt/(m · K); \( \lambda_h = 2, 11 \) Wt/(m · K); \( \lambda_w = 0.58 \) Wt/(m · K); \( \lambda_g = 0.0034 \) Wt/(m · K);
- \( q_h = 5.1 \cdot 10^5 \) J/kg; \( \mu_w = 1.8 \cdot 10^{-3} \) Pa · s; \( \mu_g = 1.3 \cdot 10^{-5} \) Pa · s; \( \varepsilon = 0.129; \phi = 0.1 - 0.3; \)
- \( k = 10^{-13} - 10^{-15} \) m².

The results of numerical calculations are shown in Tables 1-2, and in Figures 1a-1b. Table 1 presents the relative differences in the values of hydrate saturation (in absolute value in percent) obtained as a result of the numerical implementation of the mathematical model under consideration (let’s call model 1) and numerical solution of the following problems: (11), (14), (10), (4) - (7) (model 2); (12), (15), (10), (4) - (7) (model 3); (13), (16), (10), (4) - (7) (model 4) at \( t = 5 \) \( \nu_0 = 0.5, \sigma_0 = 0.3, T_0 = 280, p_0 = 3 \) Mpa, \( \phi = 0.1 \).

The table shows that the effect of conductive heat transfer is relatively high at low permeabilities, and decreases sharply at high. And the effect of convection is more stable and increases with increasing permeability.
Similar results are presented in Table 2, where the difference between the results obtained by the implementation of models 1 and 4 is dependent on the porosity. Note that at high initial pressures, the values shown in the figures and tables are much smaller.

The effects of initial hydrate saturation and water saturation on the results of numerical calculations are small.

| $k$ | $10^{-13}$ | $5 \cdot 10^{-14}$ | $10^{-14}$ | $5 \cdot 10^{-15}$ | $2.5 \cdot 10^{-15}$ | $10^{-15}$ |
|-----|-----------|-------------------|-----------|-------------------|-------------------|-----------|
| model 1-2 | 0.11 | 0.12 | 0.51 | 1.04 | 2.11 | 5.52 |
| model 1-3 | 3.54 | 3.37 | 2.90 | 2.70 | 2.52 | 2.38 |
| model 1-4 | 3.55 | 3.42 | 3.39 | 3.74 | 4.66 | 7.95 |

Table 1: Effect of conductive and convective heat transfer.

| $\phi$ | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 |
|--------|-----|------|-----|------|-----|
| model 1-4 | 7.95 | 3.5 | 2.58 | 2.06 | 1.69 |

Table 2: Dependence on porosity at $k = 10^{-13}$

Figure 1a shows the distribution of hydrate saturation obtained by solving the original problem and problem (16), (17), (10), (4) - (7) with $t = 5$ days, $\nu_0 = 0.5$, $\sigma_0 = 0.3$, $T_0 = 285$, $p_g = 3$ Mpa, $k = 10^{-15}$, $\phi = 0.1$. Figure 1b shows a similar distribution of hydrate saturation at $p_g = 4$ Mpa.

Fig. 1 Distribution of hydrate saturation. at $t = 5$  $\nu_0 = 0.5$, $\sigma_0 = 0.3$, $T_0 = 285$, $k = 10^{-13}$ $p_g = 3$ Mpa – (a), $p_g = 4$ Mpa – (b)

5. CONCLUSION

As shown by numerical studies, conformal and conductive heat transfer does not greatly affect the calculation results. In the case of medium and high penetrability, in the energy conservation equation, the terms taking into account thermal conductivity and convective heat transfer can be neglected. Numerical experiments have shown that the heat capacity coefficient can be considered constant. Then for the approximate calculation of the distribution of hydrate saturation, you can use the formula:

$$\nu = \nu_0 + \frac{C_0}{\phi \rho_h q_h} (T - T_0)$$  (18)
6. ACKNOWLEDGMENTS

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