AN EVOLVING ENTROPY FLOOR IN THE INTRACLUSTER GAS?
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ABSTRACT
Nongravitational processes, such as feedback from galaxies and their active nuclei, are believed to have injected excess entropy into the intracluster gas, and therefore to have modified the density profiles in galaxy clusters during their formation. Here we study a simple model for this so-called preheating scenario, and ask (1) whether it can simultaneously explain both global X-ray scaling relations and number counts of galaxy clusters, and (2) whether the amount of entropy required evolves with redshift. We adopt a baseline entropy profile that fits recent hydrodynamic simulations, modify the hydrostatic equilibrium condition for the gas by including ≈20% nonthermal pressure support, and add an entropy floor $K_0$ that is allowed to vary with redshift. We find that the observed luminosity-temperature ($L-T$) relations of low-redshift ($z \approx 0.05$) HIFLUGCS clusters and high-redshift ($z \approx 0.80$) WARPS clusters are best simultaneously reproduced with an entropy floor that evolves from $\approx 200\, h^{-1/3}\, \text{keV cm}^2$ at $z \approx 0.8$ to $\approx 300\, h^{-1/3}\, \text{keV cm}^2$ at $z < 0.05$. This evolution may take place predominantly at low redshift ($z \lesssim 0.2$). If we restrict our analysis to the subset of bright ($kT \gtrsim 3\, \text{keV}$) clusters, we find that the evolving entropy floor can mimic a self-similar evolution in the $L-T$ scaling relation. This degeneracy with self-similar evolution is, however, lifted when $0.5\, \text{keV} \lesssim kT \lesssim 3\, \text{keV}$ clusters are included. Using the cosmological parameters from the WMAP 3 yr data, but treating $\sigma_8$ as a free parameter, our model can reproduce the number counts of the X-ray galaxy clusters in the 158 deg$^2$ ROSAT PSPC survey, with a best-fit value of $\sigma_8 = 0.80 \pm 0.05$.

Subject headings: cosmology: theory — galaxies: clusters: general — intergalactic medium — X-rays: galaxies: clusters

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1. INTRODUCTION
Galaxy clusters, the most massive bound objects in the universe, provide several methods to constrain cosmological models, for example, through their abundance (e.g., Evrard 1989; Henry & Arnaud 1991; White et al. 1993; Eke et al. 1996; Viana & Liddle 1999; Mantz et al. 2007), their spatial distribution (Schuecker et al. 2001; Refregier et al. 2002; Hu & Haiman 2003; Blake & Glazebrook 2003; Seo & Eisenstein 2003; Linder 2003), or both (Schuecker et al. 2003). In large future surveys, with tens of thousands of clusters, percent-level statistical constraints are expected to be available on dark energy parameters (Haiman et al. 2001), including constraints on the evolution of the equation-of-state parameter $w_a \equiv -dw/da$ (Weller et al. 2002; Weller & Battye 2003; Wang et al. 2004).

In order to fully realize the cosmological potential of large cluster samples, it is important to understand the cluster mass–observable relations accurately, at least statistically. It is very unlikely that the structure of clusters will be understood from ab initio calculations to the level of precision required for the theoretical uncertainties to not dominate over the exquisite statistical errors (e.g., Levine et al. 2002). However, in principle, when multiple observables depend on the same mass, the mass-observable relations can be accurately determined from the data themselves, simultaneously with cosmological parameters. Several works have proposed and quantified the constraints from such “self-calibration” (Majumdar & Mohr 2004; Wang et al. 2004; Lima & Hu 2005), using parameterized phenomenological relations for the mass-observable relations (e.g., power-law scalings, or arbitrary evolution in prespecified redshift bins). It has been argued recently (Younger et al. 2006) that even if cluster structure is not precisely predictable, parameterized physical models can further improve on such phenomenological self-calibration, especially when multiple observables (such as X-ray flux and Sunyaev-Zel’dovich [SZ] decrement) can be predicted from the same physical model (Younger et al. 2006). In light of this potential, it is important to fit physically motivated cluster models to as many cluster observables as possible; one then hopes that future observations of larger cluster samples will require further fine-tuning of these models, and, at the same time, deliver useful cosmological constraints (Ostriker et al. 2005; Younger et al. 2006).

The gravitational potential of clusters is dominated by dark matter, the behavior of which is determined by gravity alone, and is therefore robustly predictable. The dark matter profiles of galaxy clusters, apart from the innermost regions, are indeed well understood from three-dimensional numerical simulations (Navarro et al. 1997; Moore et al. 1998), and are nearly self-similar, as expected. The physics of gas, on the other hand, involves complicated nongravitational processes such as radiative cooling and star formation, galaxy evolution, and various forms of feedback. If these processes were unimportant, the intracluster gas would trace the self-similar dark matter profile, and its global properties should obey simple scaling relations (Kaiser 1986). Specifically, its X-ray luminosity $L$, if dominated by thermal bremsstrahlung as for clusters with temperature $T > 2\, \text{keV}$, should scale as $L \propto T^2$. This relation is indeed obeyed by clusters in hydrodynamic simulations without nongravitational processes (Evrard et al. 1996; Bryan & Norman 1998). However, the observed $L-T$ scaling relation is significantly steeper than the self-similar prediction, closer to $L \propto T^3$ (Markevich 1998; Arnaud & Evrard 1999). This demonstrates that the effect of nongravitational processes on the intracluster gas is not negligible, even for “bulk” observables.

A long-standing proposal for the dominant such nongravitational effect is that the intracluster gas is heated by some energy

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input (from star formation, supernovae explosion, galactic winds, and/or active galactic nuclei [AGNs]), raising the gas to a higher adiabat before the clusters collapse. Many authors have investigated the effect of such a preheating, and have shown that simply imposing a minimum “entropy floor” for the intracluster gas naturally breaks the self-similarity, and steepens the L-T relation as required by the data (Kaiser 1991; Evrard & Henry 1991; Cavaliere et al. 1997; Tozzi & Norman 2001; Babul et al. 2002; Voit et al. 2002). The preheating idea is further supported by the discovery of excess entropy in the inner regions of low-temperature clusters, which suggests the existence of a universal entropy floor (Ponman et al. 1999; Lloyd-Davies et al. 2000), and by several other independent lines of evidence (for a brief summary and a list of references, see, e.g., Bialek et al. 2001).

A simple model of preheating consists of shifting the entropy profile by an overall additive constant, representing the cumulative effect of nongravitational processes, assumed to be roughly uniform throughout the gas (e.g., Voit et al. 2002). Recent work has tested this simple model, by comparing its predictions with hydrodynamical simulations (Younger & Bryan 2007, hereafter YB07). The model reproduces the simulation results very well (although Borgani et al. [2005] found much larger factors of entropy amplification), but comparisons with observations show that although it can predict the global X-ray scaling relations, the model cannot reproduce the observed entropy profiles (Ponman et al. 2003; Pratt & Arnaud 2005; Pratt et al. 2006) in detail. This requires the model to be further developed, but as far as the global properties are concerned, it appears to be successful, and it is therefore useful to understand the average properties of the intracluster gas.

In this paper we adopt this simple preheating model, and focus on comparisons with both the observed L-T scaling relations in the redshift range 0 ≤ z ≤ 1 and the observed cumulative number counts of the X-ray clusters. A previous study (Bialek et al. 2001) calculated the impact of preheating on the X-ray scaling relations, using a sample of 12 simulated clusters, and found a good fit to the data on local clusters (but they have not explicitly compared the expected evolution to observations, and have not made simultaneous predictions for the number counts). Our work is also somewhat similar to a more recent study by Ostriker et al. (2005), who present a more detailed physical model for the intracluster gas, and show that it can reproduce local X-ray scaling relations (this paper also did not study evolution).

Our goal here is to clarify (1) whether the model can simultaneously explain both the scaling relations and the number counts of galaxy clusters, and (2) whether the amount of entropy required evolves with redshift. In comparing our predictions to the L-T scaling relations and the number counts, we also study the effects of scatter in the L-T and L-M relations, and the corresponding selection biases that arise in a flux-limited survey (Nord et al. 2008). Our first goal is motivated by our earlier study (Younger et al. 2006), in which we found that a similar preheating model, with the entropy adjusted to reproduce observed X-ray and SZ scaling relations, tends to overpredict the number counts of bright clusters, even with a relatively low normalization (σ_g = 0.7) of the power spectrum. A similar discrepancy was found by Ostriker et al. (2005; although they used a higher normalization, σ_g = 0.84, and suggested that agreement can be recovered by lowering this value).

The rest of this paper is organized as the follows. In § 2 we describe in detail the formalism to implement the preheating model. In § 3 we compare the predicted L-T scaling relations to observations, and find the best-fit entropy level K_0 at two different redshifts. In § 4 we further test the model by comparing predictions for the number counts with observations. In § 5 we then study the effect of intrinsic scatter and the corresponding selection effects in flux-limited cluster surveys. In § 6 we discuss our results, and in § 7 we offer our conclusions.

2. MODIFIED ENTROPY MODEL OF PREHEATING

We adopt the terminology from the literature, and refer to the quantity

\[ K = \frac{P}{\rho_0} \]  

as “entropy.” Here P and ρ_0 are the pressure and density of the gas, and γ is the adiabatic index. For an ideal gas, K is related to the formal thermodynamic entropy per particle s by s = s_0 + ln K, with s_0 a constant. In this paper the baseline entropy profile to be modified is adopted from YB07, and is a fit to that of the clusters in adaptive mesh refinement simulations (Voit et al. 2005) without nongravitational processes. The profile is self-similar when expressed as a function of the gas fraction f_g and normalized by K_vir,

\[ \frac{K(f_g)}{K_{\text{vir}}} = 0.18 + 0.2 f_g + 1.5 f_g^2. \]  

Here f_g(<r) = M_g(<r)/M_vir is the gas mass inside radius r, normalized by the cosmic mass fraction of baryons (f_b = Ω_b/Ω_m), times the total virial mass of the cluster M_vir. We further define \( T_{\text{vir}} = GM_{\text{vir}}m_p/(2r_{\text{vir}}) \), which is the temperature of the corresponding isothermal sphere (Voit et al. 2002). (Throughout this paper, we absorb k_B into T, so temperature is in units of energy.) The mean molecular weight μ = 0.59 is adopted for the intracluster gas, appropriate for a fully ionized H-He plasma with helium mass fraction Y_{He} = 0.25; m_p is the mass of protons, and r_vir is the virial radius. K_vir is then calculated by K_vir = T_{\text{vir}}/(f_gρ_{vir})^{γ}/(μm_p), where ρ_{vir} is the mean density of the cluster within the virial radius [which relates M_vir to r_vir by M_vir = (4π/3)ρ_{vir}r_{vir}^3; see below for its calculation].

The effect of preheating is then realized by adding a constant K_0 to K(f_g),

\[ K^{\text{ph}}(f_g) = K(f_g) + K_0, \]

where the value of K_0 can be determined once the amount of energy injected into the cosmic gas and the density of the gas at the time of the injection are specified. Convective stability requires the specific entropy K to be a monotonically increasing function of radius (Voit et al. 2002), and hence of f_g. The above prescription of preheating may change f_g as a function of r, but it does not change the order of the gas shells. The entropy profile, together with the hydrostatic equilibrium and gas mass conservation equations,

\[ \frac{dP}{dr} = -\eta_{\text{g}} \frac{GM_{\text{tot}}(<r)}{r^2}, \]

\[ \frac{dM_g(<r)}{dr} = 4\pi r^2 \rho_g, \]

3 To examine the sensitivity of our conclusions below to the choice of this baseline profile, we also tried adopting the entropy profile of gas that traces the DM distribution in an NFW halo. We have verified that our main conclusion below, that the entropy floor increases with cosmic time, still holds in this case. In particular, following the procedure in Younger et al. (2006) but assuming f_g = 0.9 and 20% nonthermal pressure support, we find K_0 increases from 363 ± 60 kT cm^2 at z = 0.8 to 507 ± 11 kT cm^2 at z = 0.05 (these numbers include intrinsic scatter, and are to be compared with the values obtained in our fiducial model in § 5.1).
can be used to solve for the pressure and density distribution of
the intracluster gas. Combined with the equation of state for ideal
gases, the temperature profile of the gas also follows from the
solutions. In equation (4), $M_{\text{gas}}(<r) = M_{\text{DM}}(<r) + M_* (<r)$. The dark matter profile $M_{\text{DM}}(<r)$ is known, and is given below. In-
cluding $\eta$ allows deviations from strict hydrostatic equilibrium.
Here we adopt $\eta = 0.8$, the value YBO7 find in their simulations,
suggesting that the remaining support for the gas is provided by
turbulent motions.

The boundary condition for $M_* (<r)$ is naturally chosen to be
zero at the origin (to avoid numerical difficulties, in practice we
give $M_*$ a small value at some small finite radius). The pressure
at the same position is found by giving it a trial value and inte-
grating equations (4) and (5) until the pressure at $r_{\text{vir}}$ matches the
expected momentum flux of infalling gas,

$$P(r_{\text{vir}}) = \frac{1}{3} f_\odot \rho_{\text{NFW}}(r_{\text{vir}}) v_{\text{vir}}^2. \tag{6}$$

Here we assume the accreting gas is cold (Voit et al. 2003), and
that it falls freely from the turnaround radius ($r_{\text{ta}}$) and is shocked
at the virial radius. We assume $r_{\text{ta}} = 2r_{\text{vir}}$, so that the free-fall ve-
locity $v_{\text{vir}}$ from $r_{\text{ta}}$ to $r_{\text{vir}}$ is given by $v_{\text{vir}}^2 = GM_{\text{vir}}/r_{\text{vir}} = 2T_{\text{vir}}/\rho_{\text{vir}} m_p$. The postshock gas density is $f_\odot \rho_{\text{NFW}}$ (see below for the calculation of $\rho_{\text{NFW}}$). Under extreme conditions, the free-fall kinetic en-
tergy is totally transformed into thermal energy, and the postshock gas
has a pressure as given above; this value agrees with that adopted in
YBO7, matching their simulation results. (Besides the difference in
identifying clusters, our boundary pressure has a numerical factor
of 2/3 compared to theirs of 0.7.) These two boundary condi-
tions are sufficient for solving equations (4) and (5). The result is
that the gas fraction $f_\odot$ within the virial radius is 0.88 without pre-
heating, and somewhat less when preheating is turned on.

The matter distribution in virialized clusters is well described
by the NFW (Navarro et al. 1997) model as found from
mass, we neglect its effect on the distribution of dark matter.
(Although it is found that gas will cause the dark matter halo to be
slightly more concentrated, e.g., Lin et al. 2006.) For simplicity,
here we assume the dark matter profile retains the NFW shape,
that is, $\rho_{\text{DM}}(r) = (1 - f_\odot) \rho_{\text{NFW}}(r)$. For a cluster virialized at redshift
$z$ with mass $M_{\text{vir}}$, its NFW density profile is given as

$$\rho_{\text{NFW}}(r) = \frac{\rho_0(r)}{(r/r_s)(1 + r/r_s)^2}, \tag{7}$$

where $\rho_0$, the critical density of the universe, and $\delta_c$ and $r_s$ are pa-
rameters determined from the concentration parameter $c \equiv r_{\text{vir}}/r_s$. We
neglect the weak dependence of $c$ on $M_{\text{vir}}$ and $z$, and simply adopt a constant $c = 5$ in this paper. We identify clusters virialized at redshift $z$ as spherical regions with mean density $\rho_{\text{vir}} = \Delta_\odot \rho_0(z)$, with $\Delta_\odot$ given as a fitting formula by Kuhlen et al. (2005; based on a
spherical collapse model),

$$\Delta_\odot = 18\pi^2 \Omega_m(z)[1 + a\Theta(z)^b], \tag{8}$$

where $\Theta(z) = \Omega_m(z)/\Omega_0 - 1$, $\Omega_m(z)$ is the matter density normal-
ized by $\rho_0(z)$, and $a = 0.432 - 2.001 [w(z)^{0.234} - 1]$, $b = 0.929 - 0.222 [w(z)^{0.727} - 1]$, with $w(z)$ the dark energy equation of state.

### 3. PREHEATING FROM THE L-T SCALING RELATIONS

Once the density, temperature, and pressure profiles of the intra-
cluster gas are specified, global properties, such as the X-ray lumin-
osity, the emission-weighted temperature, and the SZ decrement,
can be readily calculated. Here we compare predictions of the
modified entropy model for the luminosity-temperature scaling
relations with those inferred from X-ray observations. This choice
is motivated by simplicity and robustness: the total luminosi-
ity ($L$) and temperature ($T$) can be inferred from observations
without referring to a model for the intracluster gas. Compar-
sions to relations involving the mass of the cluster (such as the
mass-temperature relation) are somewhat more direct from a
theoretical point of view, but any such comparison would, in any
case, have to rederive cluster masses, using information such as
the observed X-ray surface brightness or temperature profiles, and
using our own model, for a fair comparison with the data. We also
emphasize that similar comparisons with SZ observables will
contain valuable additional information (e.g., McCarthy et al. 2003;
Younger et al. 2006), and should be possible soon with forthcoming
data on cluster profiles from the Sunyaev-Zel’dovich Array (SZA)
survey (Muchovich et al. 2007; Mroczkowski et al. 2007). We postpone
such comparisons to future work.

The X-ray luminosity $L$ of a cluster is calculated as

$$L = \int dV \int d\nu L_\nu(r) \Lambda[T(r), \nu] \tag{9}$$

where $n_e = (1 - Y_{\text{He}} + Y_{\text{He}}/2) (\rho_e/m_p)$ is the number density of
electrons, $n_H = (1 - Y_{\text{He}}) (\rho_e/m_p)$ is the number density of pro-
tons, and $\Lambda$ is the cooling function, calculated by a Raymond-Smith
(Raymond & Smith 1977) code with metallicity $Z = 0.3 Z_\odot$. The
integral is done over the cluster volume $V$ and over frequency $\nu$.
The emission-weighted temperature is calculated as

$$T_{\text{ew}} = \frac{\int dV \int d\nu \nu^2 L_\nu(r) \Lambda[T(r), \nu] T(r)}{\int dV \int d\nu \nu^2 L_\nu(r) \Lambda[T(r), \nu]} \tag{10}$$

The effect of preheating decreases the central density of the gas,
but increases its temperature. The result is a lower luminosity
and a higher $T_{\text{ew}}$; the combined effect at fixed $T_{\text{ew}}$, a decrease in
darkness.

We compare our predictions to two flux-limited samples of
X-ray clusters. One is the low-redshift Highest X-ray FLUx Galax-

Cluster Sample (HIPFLUGS) presented in Reiprich & Böhringer
(2002), including 63 clusters whose mean redshift is $z = 0.05$.
The other is the high-redshift sample from the Wide-Angle ROSAT
Pointed Survey (WARPS) used in Maughan et al. (2006), including
11 clusters with a mean redshift of $(z) = 0.8$. For each individual
cluster, we predict its observed temperature as the one weighted by
the bolometric emission, and compare the bolometric luminosity,
calculated at this temperature using the preheating model, with the
observed value. To quantify the goodness of fit of this comparison,
we define the usual $\chi^2$ statistic

$$\chi^2 = \sum_{i=1}^N \left[ \frac{\log L_i(T_i, z, K_0) - \log L_i}{\sigma_{\log L_i}} \right]^2 \tag{11}$$

\[4\] We find that the difference of this temperature from that weighted by the
band emission, which is actually observed, is less than 4%. We neglect this
difference. We also checked the bias of the emission-weighted temperature when
comparing to the observed spectroscopic temperatures (see § 6.4).
For reference, we show the sample (z clusters, yields the best-fit entropy floor of a commonly used unit for the entropy defined above is keV cm², a constant factor of (log T/10)²1/3 keV cm² at the average redshift (0.73, 0.13, 0.022, 0.76, 0.96)

Therefore, in this comparison, K₀ is the only free parameter to be determined by the fit (allowing variations in the cosmological parameters is discussed below). We quote the best-fit entropy floor value by multiplying K as defined in equation (1) above, by a constant factor of n_ne/μmp, with n = ρ/μmp. This is equivalent to redefining K as

K = \frac{T}{n_e^2}, \tag{12}

which is the definition widely used in the observational literature (e.g., Ponman et al. 1999, 2003; Pratt & Arnaud 2005). For γ = 5/3, a commonly used unit for the entropy defined above is keV cm², and 1 keV cm² corresponds to ejecting 0.036(1 + δ_0)²/3(1 + z)² × (Ω_b h²/0.022)²3/2 eV per particle to the fully ionized plasma with overdensity δ_0 and redshift z.

Note that the luminosity inferred from observations is cosmology-dependent, and since Reiprich & Böhringer (2002) and Maughan et al. (2006) adopt different values for the cosmological parameters, we rescale their quoted luminosity (by the ratio of the square of the luminosity distances in the two cosmologies) to our fiducial cosmology.

The above procedure, applied to the low-redshift HIFLUGCS clusters, yields the best-fit entropy floor of K₀ = 295 ± 5 h⁻¹/3 keV cm². We find a total χ² = 2293 for this best-fit model, or a χ² per degree of freedom (dof) of 37, indicating that the L-T relation has additional intrinsic scatter (caused, possibly, by a cluster-to-cluster variation in the entropy floor itself; see discussion of scatter in § 5).

For the high-redshift WARPS sample, we find the best-fit K₀ to be 172 ± 133 h⁻¹/3 keV cm². This fit has a total χ² = 7, or a χ² per dof of 0.7.

The L-T scaling relation predicted with the best-fit entropy floor at the average redshift of the HIFLUGCS clusters, z = 0.05, is shown as the solid curve in Figure 1, together with the rescaled low-z data from Reiprich & Böhringer (2002). For reference, the figure shows the predicted L-T relations without an entropy floor (dot-dashed curve) and with the lower K₀ inferred from the high-z sample (dashed curve). The comparison of the data with the K₀ = 0 curve clearly shows the need for the entropy floor, and the comparison with the K₀ = 172 h⁻¹/3 keV cm² curve shows that the observational data, especially of the 1–3 keV clusters, require that the entropy floor at z = 0.05 is higher than the best-fit value at z = 0.8.

The solid curve in Figure 2 shows the model prediction for the L-T scaling relation with the best-fit entropy floor at the average redshift of the WARPS clusters, z = 0.8, together with the rescaled data from Maughan et al. (2006). For reference, the figure again shows the predicted L-T relations without an entropy floor (dot-dashed curve) and with the higher K₀ inferred from the low-z sample (dashed curve). The comparison of the data with the K₀ = 0 curve clearly shows the need for the entropy floor at high-z as well, and the comparison with the K₀ = 295 h⁻¹/3 keV cm² curve shows that the high-z clusters favor an entropy floor smaller than the best-fit value at low z.

A visual inspection of Figures 1 and 2 ("chi by eye") indicates that the preheating model of a universal entropy floor, produced by energy input at an early epoch, cannot fit the scaling relations of the low-redshift and high-redshift clusters simultaneously. (We discuss the significance of the detected evolution quantitatively in § 6.1.) It would be natural, in fact, for the entropy floor to increase with cosmic time, if the energy input is being continuously provided by stars and/or AGNs. Here, in order to obtain a reasonable guess of the entropy floor that best fit
the \( L-T \) scaling relations at an arbitrary redshift, we adopt a power-law form of evolution,

\[
K_0(z) = K_0(z = 0)(1 + z)^{-\alpha};
\]

(13)

from the two best-fit values of \( K_0 \) and their error bars for the two cluster samples at about \( z = 0.05 \) and 0.8, we find \( K_0(z = 0) = 310 \pm 8 \ h^{-1.5} \) keV cm\(^2\) and \( \alpha = 1 \pm 0.4 \).

4. NUMBER COUNTS OF X-RAY CLUSTERS

The preheating model described above, with the evolving entropy floor, can successfully match the observed \( L-T \) scaling relations. This model also predicts a deterministic relation between cluster mass \( M \) and both temperature and luminosity. The mass function of dark matter halos is well understood from both analytic models (Press & Schechter 1974; Bond et al. 1991; Sheth & Tormen 1999) and numerical simulations (Sheth & Tormen 1999; Jenkins et al. 2001). It is therefore natural to compare model predictions to observed clusters counts as a function of either temperature \( T \) or luminosity \( L \) (or, equivalently, flux \( f \)). Here we chose to compare the model predictions to the log \( N \)-log \( f \) relation derived from the 158 deg\(^2\) ROSAT PSPC survey by Vikhlinin et al. (1998). This sample is ideal for our purposes, since it is both large and deep enough to provide a good measurement of the counts to faint fluxes, where the effects of preheating are more pronounced.

We first use the best-fit cosmological model from the \( \text{WMAP} \) 3 yr results, and calculate \( \bar{N}(f) \), the expected surface number density of clusters whose X-ray fluxes exceed \( f \). The counts are calculated as

\[
\bar{N}(f) = \int_0^\infty \frac{d^2V}{dzd\Omega}(z) \int_{M_{180}(f, z)} dM \left( \frac{dn}{dM}(M, z) dM, \right)
\]

(14)

where \( d^2V/dzd\Omega \) is the co-moving volume element and \( dn/dM \) is the cluster mass function. In this paper we use the fitting formula given by Jenkins et al. (2001) for the SO(180) group finder of dark matter halos. The mass limit \( M_{180}(f, z) \) is determined by first finding the virial mass \( M_{\text{vir}}(f, z) \) of the cluster at redshift \( z \) that gives a flux \( f \), then converting it to \( M_{180}(f, z) \) by extending the NFW profile of this cluster until the enclosed matter has a mean density of 180 times the background matter density at that time.

The results by using the average of the power-law approximation are shown as the dashed curve in Figure 3, together with the observational data from Vikhlinin et al. (1998). The figure shows that the \( \text{WMAP} \) 3 yr cosmology, together with the preheating model that fits the \( L-T \) scaling relations, underpredicts the cumulative number counts of X-ray clusters, especially at the low flux limits. Considering the sensitivity of the cluster number counts to \( \sigma_8 \), it is natural to ask whether the discrepancy can be resolved by increasing the value of \( \sigma_8 \) and leaving all other parameters unchanged (clearly, variations in \( \sigma_8 \) will not modify the best-fit \( K_0 \) inferred from the scaling relations). We therefore vary \( \sigma_8 \), and apply a \( \chi^2 \) statistic to the 158 deg\(^2\) ROSAT PSPC data to find its best-fit value. We use

\[
\chi^2 = \sum_i \frac{[\bar{N}(\sigma_8) - N_i] ^2}{\sigma_{\bar{N}}^2 + N_i/A},
\]

(15)

where \( i \) labels independent flux bins, and \( A \) is the survey area. We include a simple Poisson error (uniform sky coverage at all flux limits) in the calculation of the variance in addition to the measurement error. We find the best-fit value of \( \sigma_8 = 0.82 \). Considering the uncertainty on the power-law approximation of the evolving entropy floor obtained in the last section, we obtain approximately an uncertainty of 0.03 on the best-fit \( \sigma_8 \). The result is consistent with the \( \text{WMAP} \) 3 yr best-fit value \( \sigma_8 = 0.76 \pm 0.05 \) (in the presence of scatter, our best fit is reduced to \( \sigma_8 = 0.80 \) with an uncertainty of about 0.05; see below).

5. THE EFFECTS OF INTRINSIC SCATTER

In the above two sections, we assumed that clusters at redshift \( z \) with fixed virial mass \( M_{\text{vir}} \) have temperatures and luminosity exactly as predicted by the preheating model. In reality, deviations from spherical symmetry, as well as cluster-to-cluster variations in nonadiabatic processes, will lead to nonnegligible scatter in these two quantities. For flux-limited surveys, such scatter will cause the observed scaling relations to deviate from the true ones (Stanek et al. 2006; Nord et al. 2008), and the counts to deviate from those of equivalent mass-limited samples without scatter. To make our analysis more realistic, it is necessary to take these effects into account. In this section we repeat the calculations in the above two sections, but we include intrinsic scatter, which we model separately in the \( L-T \) and \( L-M \) relations.

5.1. Scatter in the \( L-T \) Relation

At a given redshift \( z \), the joint probability distribution for \( L \) and \( T \) of a cluster with fixed \( M_{\text{vir}} \) may be conveniently modeled as a bivariate lognormal distribution \( P(L, T|M_{\text{vir}}) \), with the logarithmic means determined by \( M_{\text{vir}} \) (Nord et al. 2008). Convolved with the cluster mass function, this can be used to predict the probability distribution of luminosity for clusters at a fixed temperature \( T \). For a flux-limited sample, the average and variance of \( L \) for these clusters can also be predicted. Here, since we care only about the final \( L-T \) scaling relation, for simplicity we
assume that \( P(L|T) \), the probability distribution function of \( L \) for clusters at fixed \( T \), is lognormal,

\[
P(L|T)dL = \frac{1}{\sqrt{2\pi} \sigma_{ln|L/T}} \exp\left[-\frac{(\ln L - \ln L^*)^2}{2\sigma_{ln|L/T}^2}\right] d\ln L.
\]  

(16)

Given that the lognormal shape of \( P(L, T|M) \) is not particularly well justified to begin with, and that our results are essentially more sensitive to the width of the \( P(L|T) \) distribution than its detailed shape, we regard this as a sensible approach. The logarithmic mean \( \ln L \) in equation (16) is taken to be a constant. Here we choose it to be 0.3, which is close to the value \( \sim 0.4 \) expected for current flux-limited \( \{f(0.1-2.4\, keV) \sim 3 \times 10^{-12} \text{ erg s}^{-1} \text{ cm}^{-2}\} \) samples. In particular, Nord et al. (2008) derive this value by assuming a bivariate lognormal distribution of \( P(L, T|M) \), with intrinsic scatter \( \sigma_{ln|L/T} = 0.59, \sigma_{ln|T/M} = 0.1 \), power-law relations between the means, and a positive correlation between \( \ln L \) and \( \ln T \).

For a flux-limited survey with a threshold \( f_{\text{min}} \) in the observer rest-frame energy band \( [\nu_1, \nu_2] \), the log mean luminosity of detectable clusters at fixed temperature \( T \) is given by

\[
\langle \ln L \rangle(T) = \ln L + \sigma_{\ln|L/T} \sqrt{\frac{2}{\pi}} \text{erfc}(x_{\text{min}}),
\]  

(17)

where erfc is the complementary error function, \( x_{\text{min}} = (\ln L_{\text{min}} - \ln L)/\sqrt{2\sigma_{\ln|L/T}} \), and \( L_{\text{min}} \) is the luminosity corresponding to the flux threshold, \( L_{\text{min}} = 4\pi f_0^2 \phi_0/k_{\text{min}}(T, z) \). Here \( k_{\text{min}}(z) \) is the luminosity distance, and \( K \) is the ratio of the X-ray emission in the energy band \( [\nu_1(1+z), \nu_2(1+z)] \) (cluster rest frame) to the bolometric luminosity, and is calculated by the preheating model for the same cluster when we calculate \( \ln L \). Clusters with luminosity below \( L_{\text{min}} \) are not included in the average. So, \( \langle \ln L \rangle \) is larger than that for the complete sample (the so-called Malmquist bias). The variance for the log of the luminosity for the flux-limited sample can be calculated similarly,

\[
(\langle \ln L - \langle \ln L \rangle \rangle^2(T) = \sigma_{\ln|L/T}^2 + \frac{2}{\pi} \frac{x_{\text{min}} \exp(-x_{\text{min}}^2)}{\text{erfc}(x_{\text{min}})} - \frac{2}{\pi} \frac{(-2x_{\text{min}}^2)}{\text{erfc}(x_{\text{min}})}.
\]  

(18)

Note that equations (17) and (18) have manifestly correct limiting behaviors: in the limit \( L_{\text{min}} \rightarrow 0 \), we have \( \langle \ln L \rangle \rightarrow \ln L \) and \( \langle (\ln L - \langle \ln L \rangle)^2 \rangle \rightarrow \sigma_{\ln|L/T}^2 \), whereas in the limit \( L_{\text{min}} \rightarrow \infty \), we have \( \langle \ln L \rangle \rightarrow L_{\text{min}} \) and \( \langle (\ln L - \langle \ln L \rangle)^2 \rangle \rightarrow \sigma_{\ln|L/T}^2 \) in the intrinsic variance.

To take into account the above, we modify the calculation of the \( \chi^2 \) statistic for the two flux-limited cluster samples. Specifically, in equation (11), we replace the average log \( L(T, z, K_0) \) by \( \langle \ln L \rangle(T, z, K_0) \) for log-gf, and the variance \( (\partial \ln L/\partial \log L) \sigma_{\log L}^2 \) by \( (\ln L - \langle \ln L \rangle)^2/(\langle \ln L \rangle)^2(\log L)^2 \). Measurement errors in the temperature may further modify the average and variance of the \( i \)th cluster’s luminosity; here we include this effect approximately by simply adding a term \( (\partial \ln L/\partial \ln T)\sigma_{\log T}^2 \) to the intrinsic variance.

With these alterations of the \( \chi^2 \) statistic, we find that the best-fit entropy floor \( K_0 \) for the low-redshift HIFLUGCS clusters is increased to \( 327^{+25}_{-15} \text{ h}^{-1/3} \text{ keV cm}^2 \) (with the \( \chi^2 \) per dof of 2.2), and the best-fit \( K_0 \) for the high-redshift WARPS clusters is increased to \( 209^{+66}_{-35} \text{ h}^{-1/3} \text{ keV cm}^2 \) (with a \( \chi^2 \) per dof of 0.5).

Since Malmquist bias shifts the average luminosity to a larger value, more entropy is needed to bring the model prediction into agreement with the observations (see § 6.7 for more discussions on this), but the increase is only \( \sim 10\%-20\% \). More importantly, however, we see that the significance of the difference in \( K_0 \) between the high- and low-redshift samples is reduced, but remains at the interesting level of \( (327 - 209)/(19^+66^-)^1/2 \approx \sqrt{1.7} \). (See § 6.1 for more about this.) Again, in order to obtain a reasonable guess of the entropy floor that best fits the \( L-T \) scaling relations at an arbitrary redshift, we make a power-law assumption, and find \( K_0(z = 0) = 314 \pm 25 \text{ h}^{-1/3} \text{ keV cm}^2, \alpha = 0.83 \pm 0.6 \).

5.2. Scatter in the L-M Relation

In this section, as before, we assume that the bolometric luminosity \( L \) for clusters with virial mass \( M_{\text{vir}} \) at redshift \( z \) has a lognormal probability distribution,

\[
P(L|M_{\text{vir}}, z) dL = \frac{1}{\sqrt{2\pi} \sigma_{\ln|L/M}} \exp\left[-\frac{(\ln L - \ln L^*)^2}{2\sigma_{\ln|L/M}^2}\right] d\ln L.
\]  

(19)

The log mean \( \ln L \) is calculated as the log of the luminosity predicted for the cluster by the preheating model with the evolving entropy floor found from § 5.1. The scatter is taken to be a constant; we adopt the value \( \sigma_{\ln|L/M} = 0.59 \) derived by Stanek et al. (2006) from matching the predicted cluster counts to the REFLEX survey results (Bohringer et al. 2004).

The fraction of clusters with flux above \( f \), or luminosity above \( L_{\text{min}} \), is then simply

\[
P(>f|M_{\text{vir}}) = \frac{1}{\sqrt{2\pi} \sigma_{\ln|L/M}} \text{erfc}(x_{\text{min}}),
\]  

(20)

where \( L_{\text{min}} \) and \( x_{\text{min}} \) are calculated as in § 5.1. Finally, the number counts are given by

\[
N(>f) = \int dV \int dm(\ln M, z)P(>f|M_{\text{vir}}, z) dm.
\]  

(21)

Note that \( M \), the mass of the cluster employed in the mass function, is defined by an overdensity of 180 of the background matter density, different from \( M_{\text{vir}} \). As before, the NFW profile is used to convert \( M_{\text{vir}} \) to \( M \). The counts predicted with scatter \( \sigma_{\ln|L/M} \) and the average power-law approximation for the evolving entropy expected in the above subsection are shown as the dotted curve in Figure 3. The difference from the original calculation, assuming no intrinsic scatter (dashed curve), is relatively small. Although a nonzero \( \sigma_{\ln|L/M} \) by itself tends to significantly increase the number counts, we are also allowing the log mean luminosity \( \ln L \) (at fixed \( M \)) to change. As explained in the preceding subsection, a nonzero \( \sigma_{\ln|L/T} \) necessitates more entropy in order to match the \( L-T \) scaling relations, and tends to reduce \( \ln L \) (at fixed \( T \), and also at fixed \( M \)), and hence to decrease the number counts. The combination of these two effects is that \( N(>f) \) increases, but only by a relatively small factor (\( \sim 20\% \)).

By repeating the analysis as is done at the end of § 4, we find that when all other cosmological parameters are kept fixed at the

5 Two clusters in the high-redshift WARPS sample are removed here because they fall below the nominal flux threshold given in Maughan et al. (2006).
6. DISCUSSION

In this section we discuss, quantitatively, a range of issues that should help us understand our results and assess their robustness.

6.1. Significance of the Inferred Entropy Evolution

Perhaps our most interesting result is the increase in the entropy floor from the \( z \sim 0.8 \) to the \( z \sim 0.05 \) cluster sample, and therefore, here we discuss the statistical significance of this difference. In our analysis above, we have assumed a constant (not evolving) intrinsic scatter \( \sigma_{ln/LF} \), adapted from the work of Nord et al. (2008), resulting in a \( \sim 1.7 \) detection for the difference in the entropy floor values at \( z = 0.8 \) and 0.05 (see § 5.1). In reality, the measurement errors of the low-z cluster sample are much smaller than those of the high-z sample, and the intrinsic scatter can, in fact, be inferred self-consistently from the \( L-T \) relation we fit. Here we repeat the analysis in § 5.1, but now the scatter \( \sigma_{ln/LF} \) to vary, and attempt to adjust its value to find a \( \chi^2 \) per degree of freedom, for both cluster samples. We find that the low-z sample then requires a scatter of \( \sigma_{ln/LF} = 0.49 \), which is larger than our adopted value. Using this larger scatter shifts the best-fit entropy level to \( 372^{+15}_{-309} \) \( h^{-1/3} \text{ keV cm}^2 \). For the high-z sample, we find that the measurement errors are so large that the best-fit model has a \( \chi^2 \) per degree of less than 1 (\( \approx 0.7 \)) even in the absence of any intrinsic scatter. We conclude that the current data cannot yet be used to establish evidence for any intrinsic scatter in the high-z sample. The best-motivated statistical comparison, then, is between the best-fit \( K_0 \) we obtain for the low-z sample with \( \sigma_{ln/LF} = 0.49 \) and the best-fit value for the high-z sample obtained with \( \sigma_{ln/LF} = 0 \) (\( 172^{+237}_{-333} \) \( h^{-1/3} \text{ keV cm}^2 \); see § 3). This implies a significance of the difference between the best-fit values of \( (372 - 172)/(372 + 333)^{1/2} \approx 4 \sigma \). Clearly, better temperature measurements for the high-z clusters would help determine whether the intrinsic scatter evolves, which would be important to validate this result.

The entropy floor has a larger impact on the smallest clusters, and one may wonder to what extent the inferred entropy floor is driven by the two low-temperature clusters in Figure 1 that lie visibly below the best-fit relation. When we omit these two clusters and repeat our analysis with the rest of the HI-FUFLGCS sample, we find that the best-fit entropy floor decreases by 7\%, from \( 295^{+55}_{-55} \) to \( 273^{+37}_{-37} \) \( h^{-1/3} \text{ keV cm}^2 \), when ignoring intrinsic scatter in the analysis; by 12\%, from \( 327^{+237}_{-333} \) to \( 287^{+220}_{-220} \) \( h^{-1/3} \text{ keV cm}^2 \), when including intrinsic scatter (\( \sigma_{ln/LF} = 0.3 \)); and also by 12\%, from \( 372^{+15}_{-309} \) to \( 329^{+40}_{-30} \) \( h^{-1/3} \text{ keV cm}^2 \), when including a larger intrinsic scatter (\( \sigma_{ln/LF} = 0.49 \)). The 4 \( \sigma \) significance of the difference claimed above now reduces to 3 \( \sigma \).

Finally, we use an alternative statistic to assess the significance of the difference between the high-z and low-z entropy floors. We derive \( K_0 \) for each individual cluster in the two samples by simply setting \( L(T_i, z, K_0) = L_i \) (following the notation in § 3). This results in a range of \( K_0 \) values, shown by the symbols in Figure 4, which can be used to construct two separate \( K_0 \) distributions for the high-z and low-z samples. We then apply a Kolmogorov-Smirnov (K-S) test to the two \( K_0 \) distributions. We find \( D = 0.4 \) and a \( P \)-value of 0.07, which makes it unlikely that the two sets of \( K_0 \) values were drawn from the same underlying distribution. Unfortunately, this test remains inconclusive at present, since, as mentioned above, the observational errors on the temperature are much larger in the high-z sample than in the low-z sample, and this difference alone introduces a difference in the inferred \( K_0 \) distributions. Furthermore, the intrinsic scatter may evolve between the two redshifts due to reasons unrelated to the entropy floor. Indeed, this is suggested by the presence of negative \( K_0 \) values in the low-z sample, which presumably arises from unmodeled processes that brighten some clusters’ X-ray emission (e.g., cooling cores). In order to conclude that the K-S tests detect a true evolution (either in entropy or in some other process modifying the luminosity distribution at fixed \( T \) ), we would have to explicitly model the observational errors, which is not yet warranted, given the large errors in the high-z sample.

6.2. The Impact of Redshift Binning

In our analysis so far, we have combined the entropy floor of all 63 low-redshift clusters into a single value (for a mean redshift of \( \langle z \rangle = 0.05 \)), and derived a single best-fit value for the range \( 0 < z \leq 0.2 \). Likewise, the 11 high-redshift clusters at \( 0.6 \leq z \leq 1 \) were binned together for a single point at \( \langle z \rangle = 0.8 \). Given that we infer a significant evolution using these two binned points, a natural question is whether the entropy floor evolves within the finite \( z \)-range in both bins, and whether this may bias the mean \( K_0 \) inferred from each bin.

To address this question directly, we computed the average value of \( K_0 \) (as we did earlier in § 5.1) within discrete subbins in the \( 0 < z \leq 0.2 \) range (the number of clusters in the high-z bin is insufficient for subbining). The subbins at \( z \leq 0.2 \) were chosen to contain roughly the same number of clusters each. For this exercise, we excluded the two clusters at \( z = 0.15 \) and 0.2, because they are the only two clusters in the sample around this redshift, and A2204 at \( z = 0.15 \) has cooling flows, while A2163 at \( z = 0.2 \) is known to be affected by recent or ongoing mergers. (Including these two objects, however, would not significantly
The core entropy of the system, according to the preheating model, is defined as the gravitational entropy measured at the radius 0.1$R_{\Delta}$, which is the Hubble parameter normalized by its present-day value, denoted by $H_0$. Here, $R_{\Delta}$ is the virial radius at all redshifts, and $z$ is the redshift-dependent overdensity of a spherical top-hat perturbation at virialization. The model for the same quantity, $K F_{T}^{4/3}$, is expressed as $T_{\text{vir}}$ (in keV), with $K$ denoting the total (preheating + gravitational) entropy measured at the radius 0.1$R_{\Delta}$, with $R_{\Delta}$, the radius defined by an enclosed mean overdensity of $\Delta_c = 200[\Delta_c(z)/\Delta_c(\text{EdS})]$, with respect to $\rho_v$. Here, $\Delta_c(z)$, as before, is the redshift-dependent overdensity of a spherical top-hat perturbation at virialization, and $\Delta_c(\text{EdS}) = 18\pi^2$ is the same quantity in an Einstein-de Sitter universe (the latter is redshift-independent). With this definition, $R_{\Delta}$ is a fixed fraction of the virial radius at all redshifts. In the absence of nongravitational processes, the entropy measured at this radius will scale with the virial gas temperature $T$ as $K \propto T^{4/3}$, with $F_5 = E_c[\Delta_c(z = 0)z^{1/2}$ (note $\Delta_c \propto \Delta$). Here, $E_c$ is the Hubble parameter normalized by its present-day value. E04b present their results for the entropy with this self-similar evolution factored out; i.e., fixing the scaling relation to be in the form of $K F_{T}^{4/3} \propto CT^{4}(1 + z)^{6}$. With $C$ and $A$ taken from Ponman et al. (2003), the redshift evolution power-law slope is found to be $B = 0.68$. Their results are shown by the pair of dashed curves in Figure 5 at the two redshifts we are interested in, $z = 0.05$ (lower dashed curve) and $z = 0.8$ (upper dashed curve). The figure also shows the predictions of our preheating model for the same quantity (upper and lower solid curves at $z = 0.8$ and 0.05, respectively). For reference, the dash-dotted line shows the self-similar case, without any entropy floor (which is independent of redshift, by construction).

We can draw several conclusions from Figure 5. First, in the observed temperature range $3 \text{ keV} \leq T \leq 11 \text{ keV}$, from which the $E04b$ results are derived, our model predicts a factor of $\approx 2$ larger entropy than E04b finds. The reason for this overall discrepancy is the difference in our adopted model profiles. As mentioned in § 1, the findings of YB07 already suggest that a constant (radius-independent) additive entropy can reproduce global scaling relations, but not the radial profiles. The density at the inner radius of 0.1$R_{\Delta}$ in our model, in particular, is known to be too low compared to that derived from observations (i.e., of the X-ray.

---

**TABLE 1**

| Bin No. | $K_0$ | $z$ Range |
|---------|------|----------|
| 1................. | 339 ± 23 | 0.000 < $z$ ≤ 0.035 |
| 2................. | 332 ± 44 | 0.035 < $z$ ≤ 0.056 |
| 3................. | 232 ± 69 | 0.056 < $z$ ≤ 0.201 |

**NOTES.**—The redshift range of each subbin is chosen so that they contain roughly equal numbers of clusters. The upper half of the table shows results for three subbins (of ~20 clusters each); the lower half shows the results for six subbins (of ~10 clusters each).
The problem is inherent in the assumption of a constant additive entropy, and can only be addressed by changing our underlying model (e.g., by allowing the additive entropy to vary with radius, or allowing for cooling or other radius-dependent, nonadiabatic processes). We intend to examine such improved models in future work. For now, we note that the observational results of E04b are based on the measurement of the total entropy by fitting a $\beta$-model to the X-ray surface brightness profile, and inferring the gas density $n$ at 0.1$R_\Delta$ (the entropy then follows from $K = T/n^{2/3}$). In principle, a correction for this difference could be made; for example, using our model to explicitly fit the X-ray surface brightness at 0.1$R_\Delta$, instead of the total X-ray luminosity (the latter is dominated by 2–3 times larger radii; see Fig. 10). Given the level of agreement between our bottom-line conclusions, this laborious comparison is not warranted.

Second, our model prediction for $K$ at a given redshift scales as $\propto T^0$ at the low-$T$ end, steepening to $\propto T$ at the high-$T$ end (this, of course, is a generic feature of preheating preferentially affecting low-$T$ clusters). In their fitting procedure, E04b use an intermediate, $T$-independent power-law slope, $K \propto T^{0.6}$ (taken from Ponman et al. [2003]; this is accurate in the temperature range 0.5–17 keV for their sample of local clusters).

Third, our model predicts an evolution of $K(0.1R_\Delta)$ that is also $T$-dependent. This is because the constant-entropy floor represents a different fraction of the total entropy $K(0.1R_\Delta)$ at different temperatures (furthermore, the total entropy is not a simple sum of the entropy floor and the gravitational entropy in the absence of preheating). The constant fractional change in the entropy floor likewise generically introduces a $T$-dependence evolution in the total entropy. When scaled by the factor of $F^{2/3}$, our entropy floor, i.e., $K_0F^{2/3}$, decreases from $z = 0.8$ to 0.05 by 25% (note that in our cosmology $\Delta_5 = 100$, $E_z = 1.02$ at $z = 0.05$, and $\Delta_5 = 144$, $E_z = 1.47$ at $z = 0.8$). At the low-$T$ end, the total entropy $KF_z^{2/3}$ evolves by the same amount, whereas clusters at the high-$T$ end approach the self-similar case, with no evolution. In the temperature range studied by E04b, $3 \text{ keV} \leq T \leq 11 \text{ keV}$, the scaled entropy decreases by 9%–15%. In contrast to our model, E04b force the form of the evolution to be $T$-independent. They find that the evolution has the same sign as we inferred, but they find a larger factor (of about 30%). E04b also present the theoretically expected evolution, obtained for clusters found in hydrodynamic simulations with $2 \text{ keV} \leq T \leq 7 \text{ keV}$. These simulations predict a weaker evolution, i.e., a 13% decrease from $z = 0.8$ to 0.05 (corresponding to $B = 0.26$), which is closer to our result in the same temperature range (10%–17%), although the evolution in the simulations is also forced to be $T$-independent.

Another theoretical study of the evolution of the $K$-$T$ scaling relation is presented by Nath (2004). By using his preheating model with a constant entropy floor, he finds that at a given redshift, the $K$-$T$ scaling relation, with $K$ measured at 0.1$R_{500}$, is flat; i.e., the effect of the entropy floor can be seen over a much higher temperature range (within 1–10 keV) than ours ($<1 \text{ keV}$). This is mainly caused by the different preheating prescription (his eq. [4]) and unmodified entropy profile (which again appears to have a much smaller central entropy than ours) used by Nath (2004). The evolution is then just $E_z^{2/3}$, steeper than that found by E04a (his Fig. 2). Nath (2004) also included radiative cooling from the time of formation of the cluster—the same epoch that preheating is added—to the time of observation. Since massive clusters are found to lose entropy at a faster rate, the result is that the flat $K$-$T$ scaling relations is tilted with a negative slope. The evolution becomes even steeper because lower redshift clusters have a longer time to lose their entropy. Allowing for a time-evolving entropy floor as we find here, decreasing with redshift, may help his model give a shallower evolution, in better agreement with E04a.

Next, we compare our predictions for the evolution of $L$-$T$ scaling relations with observations. Our high-$z$ sample is taken from Maughan et al. (2006), who also analyzed the evolution of the $L$-$T$ relation, and found that this evolution is consistent with the expectation in self-similar models (with no preheating). How can this be reconciled with our results? We first note that the high-$z$ sample in Maughan et al. (2006) includes only clusters with $T > 3 \text{ keV}$, and that their inferred evolution relates to the normalization of the best-fit power-law relations (whereas our $L$-$T$ relations are not power laws). For a clear illustration of how the two results can be reconciled, we return to our calculations without intrinsic scatter. In Figure 6 we reproduce the mean $L$-$T$ relations from Figures 1 and 2, and overlay the six model curves in a single figure. Note that the lowest solid curve and the middle dashed curve are predicted at our best-fit entropy levels for the low-$z$ and high-$z$ sample, respectively. Comparing these two curves with those predicted with $K_0 = 0$, we find that our evolving entropy floor predicts a self-similar-like evolution for the $L$-$T$ scaling relations when $T > 3 \text{ keV}$, in agreement with Maughan et al. (2006). This figure also clearly shows that a constant but nonzero $K_0$ cannot mimic a self-similar-like evolution.

This can be explained by the effect mentioned in the preceding paragraphs: for clusters at the same redshift, the same entropy floor ($K_0$) affects the low-temperature clusters more than it does the high-temperature ones, because the latter have larger characteristic (gravitationally heated) entropy. (This, of course, is well known, and it is the effect that leads to a larger fractional reduction in the luminosity for the low-$T$ systems, steepening the $L$-$T$ scaling relations.) Similarly, for clusters with the same $T$ but at different redshifts, a constant entropy level leads to a larger fractional reduction in the luminosity for the higher redshift clusters, because they have a larger characteristic density and a smaller entropy. As a result, maintaining the self-similar-like evolution requires less entropy at higher redshift. Provided $T \propto M_{\text{vir}}^{2/3} \rho_{\text{vir}}(z)^{1/3}$, we have $K_{\text{vir}} \propto T\rho_{\text{vir}}(z)^{-2/3}$; to maintain self-similar evolution of $L$ at fixed $T$, we would need
$K_0 \propto \rho_{\text{vir}}(z)^{-2/3}$. Taking $K_0 = 295$ at $z = 0.05$, this requires $K_0 = 136$ at $z = 0.8$, 20% smaller than our best-fit value at this redshift. This indicates that the evolution of our $L$-$T$ scaling relation is not exactly self-similar, but a little shallower. Figure 4 in Maughan et al. (2006) is indeed consistent with this small deviation from self-similarity.

6.4. Bias of the Emission-Weighted Temperature

In our analysis above, we have compared the predicted emission-weighted temperature $T_{\text{ew}}$ for a cluster to its observational counterpart. Since the latter is generally obtained by fitting a thermal model to the observed spectrum, in general the former is a biased estimator. In particular, Mazzotta et al. (2004) have demonstrated that $T_{\text{ew}}$ always overestimates the spectroscopic temperature if the cluster has a complex multitemperature thermal structure. They proposed alternatively using a so-called spectroscopic-like temperature $T_{\text{sl}}$, which they found to be within a few percent of the actual spectroscopic temperature, measured for simulated clusters hotter than 2 – 3 keV. To quantify how the bias in $T_{\text{ew}}$ affect our results, we adopted the formula for $T_{\text{sl}}$ from Mazzotta et al. (2004) and repeated our calculations. We find that $T_{\text{ew}}$ is larger than $T_{\text{sl}}$ by around 10%. The result is that the best-fit entropy level shifts to a higher value: from 327 to 420 $h^{-1/3}$ keV cm$^2$ for the HIFLUGCS sample, and from 209 to 287 $h^{-1/3}$ keV cm$^2$ for the WARPS sample. (The effects of intrinsic scatter in $\sigma_{\text{in,}\text{L/T}}$ and Malmquist bias are included in these results, as in § 5.1.)

6.5. Parameter Degeneracies

An obvious issue, even within the context of the simple model adopted in our study, is that of parameter degeneracies. A full multidimensional degeneracy study is left for future work; here we examine only the variations between parameters that we expect may have the largest effect on our conclusions.

Overall degeneracy between $\eta$, $K_0$, and $\sigma_8$.—In our fiducial model, we have included 20% nonthermal pressure support (i.e., $\eta = 0.8$). This choice is motivated by simulations that reveal turbulent motions in the intracluster gas (Norman & Bryan 1999; Faltenbacher et al. 2005; YB07). Including turbulent support in the analytical model is indeed necessary in order to reproduce in detail the density and temperature profiles for the intracluster gas in simulations with preheating (YB07). There is also direct observational evidence for turbulence in the Coma Cluster (Schuecker et al. 2004). In addition to turbulence, however, relativistic particles accelerated by cosmic shocks or other mechanisms can provide further pressure support for the intracluster gas (Miniati 2005). In order to account for the possibility of such an additional pressure component, we repeated the analysis of the previous sections, but changed the value of $\eta$ from 0.8 to 0.7. This new calculation serves, more generally, to quantify the impact of uncertainty in the nonthermal pressure component on our result.

We find that more nonthermal pressure support decreases both the density and the temperature for a cluster at fixed mass, and decreases both its $L$ and $T_{\text{ew}}$. However, at a fixed $T_{\text{ew}}$ we find that $L$ is slightly increased. As a result, in order to reproduce the observed $L$-$T$ scaling relations, more entropy is needed (both at low and high redshift). We find that the average of the power-law approximation of the evolving entropy floor is changed to $K_0(z) = 381(1+z)^{-0.84} h^{-1/3}$ keV cm$^2$. After preheating is included, keeping the WMAP 3 yr cosmological parameters fixed, the model underpredicts the number counts even more, as a result of the decreased luminosity at fixed virial mass. Treating $\sigma_8$ as a free parameter, we find that the best-fit value is increased to $\sigma_8 = 0.86$. (Intrinsic scatters are included in the analysis, as in § 5.)

According to this analysis, nonthermal pressure support is degenerate with both the entropy floor and the normalization of the power spectrum: a 50% increase in nonthermal pressure results in a 8% increase in $\sigma_8$ and an $\approx 12\%$ increase in $K_0$ (with virtually no effect on the slope of the entropy evolution).

Degeneracy between $K_0$ and $\sigma_8$ from $dN/dm$.—We found above that if the entropy floor $K_0$ is fitted from the scaling relations alone, then the best-fit $\sigma_8$ is somewhat higher than the preferred WMAP 3 yr value. It is interesting to quantify the degeneracy between $K_0$ and $\sigma_8$ from the counts alone—in particular, to see how large a change in $K_0$ is required if one insists on the preferred WMAP 3 yr value of $\sigma_8 = 0.76$. We fix the assumed power-law approximation of the evolution, $K_0(z) = K_0(z = 0)(1+z)^{-0.83}$ (and include a scatter $\sigma_{\text{in,}\text{L/M}} = 0.59$, as before), and compute the $\chi^2$ statistic from the number counts, varying $K_0(z = 0)$ and $\sigma_8$ simultaneously. The results are shown in Figure 7. As this figure reveals, the best-fit $K_0$ varies monotonically with $\sigma_8$, by a factor of $\approx 2$ over the range $0.7 < \sigma_8 < 0.85$. Also, by decreasing $K_0$ by $\approx 20\%$, the best-fit value for $\sigma_8$ from the $L$-$T$ relation 0.8 can be brought down to the WMAP 3 yr value of $\sigma_8 = 0.76$.

Degeneracy between $\Omega_m$ and $\sigma_8$.—Cluster number counts produce a well-known degeneracy between $\Omega_m$ and $\sigma_8$, approximately of the form $\sigma_8 \Omega_m^{0.5} = \text{constant}$ for shallow X-ray counts (e.g., Eke et al. 1996; Bahcall & Fan 1998).

To examine the impact of uncertainty in $\Omega_m$ on our results, we changed $\Omega_m$ from 0.24 to 0.30 (corresponding to changing $\Omega_m h^2$ from 0.13 to 0.16). We otherwise fix the WMAP 3 yr cosmological parameters, and refit the $L$-$T$ scaling relations. We find that the best-fit entropy floor is decreased significantly, by $\sim 40\%$, with the average of the assumed power-law evolution being $K_0(z) = 194(1+z)^{-0.72} h^{-1/3}$ keV cm$^2$. This can be understood easily: increasing $\Omega_m$ decreases the cosmic baryon fraction. For a cluster with fixed $(M_{\text{vir}}, z)$, the baryon content is therefore decreased. This reduces its luminosity with the same entropy floor. On the other hand, the temperature is essentially unchanged. The net result is that the normalization of the $L$-$T$ relation is reduced, and less entropy is needed to bring it into agreement with the observations.

The model with the best-fit entropy floor is then found to overpredict the X-ray cluster counts. This is mostly due to the
increase in the underlying mass function $dn/dM$, although we also find an increased detection probability for the low-mass clusters at a given flux limit, which may be caused by increased mean luminosity, reduced luminosity distance, etc. Allowing $\sigma_8$ to vary, we find a best-fit value of $\sigma_8 = 0.66$. This value is smaller than $\sigma_8 = 0.72$, the value expected from the usual degeneracy $\sigma_8 \Omega_m^{0.5}$. (Intrinsic scatters are included in the analysis, as in § 5.)

6.6. Which Clusters are Responsible for the Number Counts Constraints?

It is useful to know, within our model, the masses and redshifts of clusters that dominate the number counts. In Figure 8 we show $dn(>f)/dz$ and $M_{500}(f,z)$ at the four different flux thresholds we utilized from Vikhlinin et al. (1998). The constant intrinsic scatter of $\sigma_{ln L/T} = 0.3$ as before, but we do not apply any flux limit (this corresponds to setting $x_{min} \rightarrow -\infty$ in § 5.1). The best-fit entropy floor is found to be $333 \ h^{-1/3} \ keV \ cm^2$ for the HIFLUGCS clusters and $174 \ h^{-1/3} \ keV \ cm^2$ for the WARPS clusters (the two clusters with fluxes below the claimed flux limit are excluded, for the purpose of fairly comparing with the results that take into account of the effect of the flux limit).

Compared with the results with no intrinsic scatter, the entropy levels favored by these two cluster samples both increase. This increase is caused by the constant intrinsic scatter added to the denominator in the calculation of the $\chi^2$ analysis, which changes the relative weight of each cluster (more specifically, reducing the down-weighting of the [small] clusters that require a higher entropy floor).

Compared with the results that include the intrinsic scatter and also apply the survey flux limits, the entropy level for the HIFLUGCS sample increases a little, while for the WARPS sample it decreases. Overall, the impact of the flux limit is surprisingly modest. One naively expects that the clusters that are most important for determining the best-fit value for the entropy floor are the smallest ones, i.e., those just above the detection threshold, which are most susceptible to bias effects. In particular, a naive expectation is that this bias will increase the average luminosity, and will require a larger entropy floor. It is therefore worth understanding the relative insensitivity of our results to imposing a flux limit.

The effects of applying the flux limits have been analyzed in § 5.1: in addition to increasing the average luminosity, it also decreases the intrinsic scatter. The former effect shifts the best-fit entropy to a higher value, while the latter preferentially increases the value of $\chi^2$ at a larger entropy floor, and effectively shifts the best-fit entropy level to a lower value. Depending on the competition between these two effects, the net result may be either a larger or a smaller value for the best-fit entropy floor. To clarify this competition, we perform an intermediate calculation, in which the effect of the flux limit is included only on the average luminosity (i.e., artificially ignoring the corresponding reduction in the scatter). We find that the HIFLUGCS clusters now favor $K_0 = 411 \ h^{-1/3} \ keV \ cm^2$, and the WARPS clusters favor $K_0 = 251 \ h^{-1/3} \ keV \ cm^2$. These value are much larger than the values obtained by assuming there are no flux limits, demonstrating that the robustness of the inferred entropy floor results from the above-mentioned cancellation. We conclude that provided the intrinsic scatter is known a priori (before a flux limit is applied), the effect of Malmquist bias on the inferred entropy floor is small.

6.8. Predictions for the SZ Decrement

As mentioned above, our model fully determines other possible observables, such as those that can be measured with the SZ effect. In Figure 9 we plot predictions for the $Y_{2500}$ scaling relation, together with the data from Bonamente et al. (2008; see also Reese et al. 2002; McCarthy et al. 2003; Bonamente et al. 2006; LaRoque et al. 2006 for further discussions of SZ decrements). Here $Y_{2500}$ is the integration of the usual Compton parameter over the solid angle extended by the cluster within the projected radius of $r_{2500}$ (the radius that gives a mean enclosed density of 2500 times of the critical density), and $T$ is the (X-ray) emission-weighted temperature, as before. The solid curve corresponds to our preheating model with the average power-law approximation for the evolving entropy floor, and the dashed curve, for reference, shows the prediction in a model without preheating. Both are made at the mean redshift of the data, $z = 0.2$. While a detailed quantitative comparison with the present SZ data would be somewhat premature, a visual inspection of the dashed and
solid curves (“chi by eye”) indicates these data do require preheating, and that the entropy level we found from the X-ray scaling relations roughly agrees with the data. A thorough investigation of relationships roughly agrees with the data. A thorough investigation of these relations roughly agrees with the data. A thorough investigation of these relations roughly agrees with the data.

6.9. Moore vs. NFW Dark Matter Profiles

High-resolution numerical simulations suggest that the dark matter distribution in the central regions of virialized halos is significantly steeper than the NFW shape (Moore et al. 1998; Klypin et al. 2001). To examine the dependence of our results on possible variations of the dark matter profile, we here adopt

$$\rho(r) = \frac{\delta_c \rho_0(z)}{(r/r_s)^{1.5}(1 + r/r_s)^{0.5}}. \quad (22)$$

with a fixed concentration parameter $c = 4$, and recompute our results. We find that the steeper dark matter profile gives a higher central density and temperature for the intracluster gas, so at a fixed $M_{\text{vir}}$, both $L$ and $T$ are increased, but at a fixed $T$ the luminosity is decreased. As a result, less entropy is needed for the preheating model to agree with the observed $L-T$ scaling relations. We find that the power-law approximation for the evolving entropy floor has an average of $K_0(z) = 341(1 + z)^{-0.83}$ $h^{-1/3}$ keV cm$^2$ (solid curve), and without an entropy floor (dashed curve) at redshift $z = 0.2$. The points with error bars are data from Bonamente et al. (2008) for clusters within the redshift range of $[0.1, 0.3]$. [See the electronic edition of the Journal for a color version of this figure.]

![Figure 9](image-url) - $Y_{2500}$-$T$ SZ scaling relations predicted by the preheating model with the average of the power-law approximation for the evolving entropy floor given by $K_0(z) = 341(1 + z)^{-0.83}$ $h^{-1/3}$ keV cm$^2$ (solid curve), and without an entropy floor (dashed curve) at redshift $z = 0.2$. The points with error bars are data from Bonamente et al. (2008) for clusters within the redshift range of $[0.1, 0.3]$. [See the electronic edition of the Journal for a color version of this figure.]

L and $Y_{2500}$ also get smaller (from $L = 8.02 \times 10^{44}$ to $7.25 \times 10^{44}$ erg s$^{-1}$, and from $Y_{2500} = 5.25 \times 10^{-11}$ to $4.65 \times 10^{-11}$). However, $\gamma_0$ must be more sensitive to this steeper dark matter profile than the other two observables to finally get an increase (from $\gamma_0 = 5.84 \times 10^{-5}$ to $7.18 \times 10^{-5}$). In Figure 10 we explicitly show the contributions to $L$, $\gamma_0$, and $Y_{\text{vir}}$ (similar to $Y_{2500}$, except the integration is done within the projected radius of $r_{\text{vir}}$) from logarithmic radial bins for both the NFW case (solid curves) and Moore et al. case (dotted curves). This figure clearly shows that $\gamma_0$, $L$, and $Y$ are dominated by increasingly larger logarithmic radius bins. This behavior can be explained by the fact that the X-ray luminosity and the integrated SZ decrement are integrations over volume ($\propto r^3$), whereas the central SZ decrement is integration over the line of sight ($\propto r$).

6.10. Comparison with Younger et al. (2006)

With our best-fit preheating model, adjusted to satisfy the X-ray scaling relations, and with the WMAP 3 yr cosmology, we found that the cumulative number counts of the X-ray clusters were underpredicted. This is different from the conclusions in earlier work (Younger et al. 2006), which found an overprediction in a similar model (Ostriker et al. [2005] also found an

We show $Y_{2500}$ instead of $Y_{2500}$ in order to remove the additional geometrical weighting of different radial bins. For reference, $Y_{2500}$ decreases from $1.8 \times 10^{-10}$ in the NFW case to $1.2 \times 10^{-10}$ in the Moore et al. (1998) case.
overprediction, using a higher $\sigma_8$ and a more elaborate cluster structure model). By comparing our prediction (without intrinsic scatter) with that of Younger et al. (2006), we find that the discrepancy can be attributed to four differences between our calculation and theirs. First, we use a larger value of the entropy floor in the redshift range where the clusters dominate the number counts, compared with the constant-entropy floor of $194 h^{-1/3}$ keV cm$^2$ adopted by Younger et al. (2006). Second, we use the WMAP 3 yr cosmological model with $\sigma_8 = 0.76$ instead of the WMAP 1 yr cosmological model with $\sigma_8 = 0.7$. Third, in our preheating model, we use the fitting formula for the baseline entropy profile from hydrodynamic simulations, which is higher in the central regions than that adopted by Younger et al. (2006); and fourth, we also include 20% nonthermal pressure support. All of these differences (except for our larger $\sigma_8$) lead to reductions in the number density, and the amount of reduction is larger than the increase caused by $\sigma_8$, leading to a net decrease in the predicted counts.

6.11. Expected Entropy Evolution

Since we find evidence for a significant increase in the entropy floor from the $z \sim 0.8$ to the $z \sim 0.05$ cluster sample, it is interesting to ask whether such an evolution is indeed expected if energy is continuously being injected into the intracluster gas. It is possible to estimate the entropy history of the IGM from the known global evolution of the AGN and star formation rate. For example, Valageas & Silk (1999) find that the mean entropy level of the IGM is increasing with time in both scenarios. In this case, clusters that form at earlier times will indeed contain gas with a lower entropy floor. Assuming the resulting entropy floor can be represented by the background entropy at the formation redshift, we find the entropy floor for clusters at $z = 0.8$ evolves to $z = 0.05$ by an increase of a factor of $\sim 2$, according to the calculation of Valageas & Silk (1999) for the AGN heating scenario (see their Fig. 2; in the stellar heating case, the evolution is much steeper, but it is unclear whether stars can provide the necessary amount of heat). This increase is comparable to our findings:

$\sim 70\%$ when we assume no intrinsic scatter, and $\sim 60\%$ when we include an intrinsic scatter. Of course, this comparison is based on a simple assumption, and the heating sources in (proto)clusters may also be different from the general average population. Furthermore, the discussion in § 6.2 suggests that the bulk of our inferred evolution takes place at low redshift ($z \leq 0.2$); the evolution that is observed in this low-$z$ range can, of course, reflect the entropy increase at higher redshifts, at or before the time these clusters form. We leave a more serious comparison to future work.

7. CONCLUSIONS

There is ample evidence that nongravitational processes, such as feedback from stars and black holes in galaxies, have injected extra entropy into the intracluster gas, and therefore have modified its density profile. In the simplest scenario, the excess entropy is injected instantly at a high redshift, well before clusters actually form, and results in a universal entropy floor in galaxy clusters. A more realistic expectation is that the amount of extra entropy evolves with the cosmic epoch, tracking ongoing star and black hole formation.

Here we studied a simple model of this preheating scenario, and found that it can simultaneously explain both global X-ray scaling relations and number counts of galaxy clusters. The level of entropy required between $z = 0$ and 1 is $\sim 200–300$ keV cm$^2$, corresponding to $\approx (0.6–0.9)\frac{(1 + \delta)}{100^{2/3}}\frac{(1 + z)^2}{2} h^{-1/3}$ keV per particle if the energy is deposited in gas at overdensity $\delta$ at redshift $z$. This overall level of enrichment is in agreement with previous studies. Here we find, additionally, evidence that the entropy floor evolves with redshift, increasing by about $\sim 60\%$ from $z = 0.8$ to 0.05. This fractional increase is in rough agreement with the evolution expected for the IGM if the heating rate follows the global evolution of the AGN. The normalization $\sigma_8 = 0.8$ preferred when X-ray cluster number counts are fit with our model is somewhat higher than, but still consistent with, the best-fit value from the WMAP 3 yr data. For a flux-limited cluster catalog, we also find that including an intrinsic scatter in log luminosity at both fixed temperature and at fixed mass does not have a big effect on the results.

The models presented in this paper should be improved in future work, being refined to fit detailed cluster profiles (like the entropy-temperature scaling relations at both inner and outer radius and different redshifts), in addition to the scaling relations between global observables and their evolution, allowing a cluster-to-cluster variation of the level of heating, with a possible systematic dependence on cluster mass, in addition to redshift, and/or allowing a radius-dependent additive entropy. It will soon be possible to confront the simple model in this paper or the above type of more detailed modeling with forthcoming SZ observations. We expect this comparison to securely establish whether the level of entropy is indeed increasing with cosmic epoch, and to place further interesting constraints on both cluster structure models and cosmology.

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