Mixed convection flow of a nanofluid containing gyrotactic microorganisms over a stretching/shrinking sheet in the presence of magnetic field

Fazlina Aman, Wan Nor Hafizah Wan Mohamad Khazim and Syahira Mansur
Faculty of Science, Technology and Human Development, Universiti Tun Hussein Onn Malaysia, 86400 Parit Raja, Johor, Malaysia

E-mail:fazlina@uthm.edu.my

Abstract. Interaction of motile microorganisms and nanoparticles along with buoyancy forces will produce nanofluid bioconvection. Bioconvection happened because of the microorganisms are imposed into the nanofluid to stabilize the nanoparticles to suspend. In this paper, we investigated the problem of mixed convection flow of a nanofluid combined with gyrotactic microorganisms over a stretching/shrinking sheet under the influence of magnetic field. The nonlinear partial differential equations are transformed into a set of five similarities nonlinear ordinary differential equations by using similarity transformation, before being solved numerically. Some of the governing parameters involve in this problem are magnetic parameter, stretching/shrinking parameter, Brownian motion parameter, thermophoresis parameter and Prandtl number. Using tables and graphs, the consequences of numerous parameters on the flow and heat transfer features are examined and discussed. The results indicate that the skin friction coefficient, local Nusselt number, local Sherwood number and local density of the motile microorganisms are strongly affected by the governing parameters.

1. Introduction
Flow and heat transfer in the boundary layer over a continuous solid surface is frequently encountered in many industrial and engineering applications such as materials manufactured by extrusion processes and heat-treated materials traveling between a feed roll and a wind-up roll or on a conveyor belt possess the characteristics of a moving continuous surface. Since [1] initiated the study of boundary layer flow over a flat surface with a constant speed, many researchers successively investigated various kinds of the boundary layer flow due to a continuously moving or stretching/shrinking surface. Recently, flow and heat transfer in nanofluids containing gyrotactic microorganisms have gained several attention due to their great potential in biomedical systems. The uses of combining microorganisms into the suspension add increased mass transfer especially in microvolumes and developed nanofluid stability [2]. A research on nanofluid containing both nanoparticles and gyrotactic microorganisms over a stagnation-point flow over a stretching/shrinking sheet has been done by [3]. While [4] considered a nanofluid containing gyrotactic microorganisms in a porous medium over a solid sphere of a mixed convection flow. The effect of magnetic field in boundary layer problems can be seen in many researches such as in the works done by [5-8]. It is found that magnetic field can be utilized as a good controller of the nanofluid flow field incorporating bioconvection, Brownian motion and motile microorganisms.
The current paper is initiated by the past investigation in the literature. This work analysed the behaviour of some governing parameters on nanofluid containing gyrotactic microorganisms on stretching/shrinking sheet in the presence of magnetic field. Interaction of nanoparticles and microorganisms with magnetic field will presents an interesting phenomenon in fluid dynamics problem.

2. Mathematical Formulation
Consider the steady two-dimensional \((x,y)\) boundary layer flow over a stretching/shrinking sheet immersed in a nanofluid containing gyrotactic microorganisms in the presence of magnetic field. It is assumed that there is no nanoparticle agglomeration and no effect of nanoparticles on the direction of microorganisms’ swimming and on their swimming velocity. This assumption is valid when the nanoparticle suspension is dilute. We consider that the stretching/shrinking velocity is in the form \(U_w = ax\), where \(a\) is a positive constant and \(x\) is the coordinate measured along the stretching/shrinking surface. It is also assumed that the constant mass flux velocity is \(v_0\) with \(v_0 < 0\) for suction and \(v_0 > 0\) for injection or withdrawal of the fluid. The nanofluid is confined to \(y > 0\), where \(y\) is the coordinate measured normal to the stretching/shrinking surface. The flow is subjected to the transverse magnetic field of strength \(B_0\), which is assumed to be applied in the positive \(y\)-direction. The induced magnetic field is also assumed to be small compared to the applied magnetic field; hence, it is neglected.

Following [9], the governing equations can be written as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_f} u, \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_B \frac{\partial C}{\partial y} + D_T \left( \frac{\partial T}{\partial y} \right)^2 \right], \tag{3}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 T}{\partial y^2}, \tag{4}
\]

\[
u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + \frac{bW_c}{(C_w - C_o)} \left[ \frac{\partial}{\partial y} \left( N \frac{\partial C}{\partial y} \right) \right] = D_n \frac{\partial^2 N}{\partial y^2}, \tag{5}
\]

where \(u\) and \(v\) are the velocity components along the \(x\)- and \(y\)-axis respectively, \(T\) is the fluid temperature, \(\omega\) is the kinematic viscosity, \(\alpha\) is the electrical conductivity, \(\alpha\) is the thermal diffusivity, \(B_0\) is the magnetic field, \(D_B\) is the Brownian diffusion coefficient, \(D_T\) is the thermophoresis diffusion coefficient, \(D_n\) is the diffusivity of microorganisms, \(b\) is the chemotaxis constant, \(W_c\) is the maximum cell swimming speed (the product \(bW_c\) is assumed to be constant), \(C\) is the nanoparticle volume fraction and \(N\) is the concentration of microorganisms. Furthermore, \(\tau = \frac{\rho_f}{\rho_p} \frac{(\rho_v)}{(\rho_p)}\) is the ratio between the effective heat capacity of the fluid with \(\rho_f\) and \(\rho_p\) being the density of the fluid and the density of the particles respectively and \(c_f\) and \(c_p\) denote the specific heat of the fluid and the particle at constant pressure, respectively. The subscript \(\infty\) represents the values at large values of \(y\) (outside the boundary layer).

Eqs. (1) – (5) are subjected to the following boundary conditions:

\[
v = v_0, \quad u = \lambda U_w, \quad T = T_{\infty}, \quad C = C_w, \quad N = N_w \quad \text{at} \quad y = 0, \tag{6}
\]

\[
u \to 0, \quad T \to T_{\infty}, \quad C \to C_w, \quad N \to N_w \quad \text{as} \quad y \to \infty, \tag{7}
\]
where $\lambda$ is a constant with $\lambda > 0$ for stretching and $\lambda < 0$ for shrinking. The subscript $w$ denotes the values at the solid surface while the subscript $\infty$ represents the values at the area which is far from the surface. The governing equations (1) – (5) with the boundary conditions (6) and (7) can be expressed in a simpler form by introducing the following transformation:

$$\psi = (aw)^{1/2} \xi(\eta), \quad \eta = \left(\frac{d}{\nu}\right)^{1/2} y, \quad \theta(\eta) = \frac{T - T_w}{T_u - T_w}, \quad \phi(\eta) = \frac{C - C_w}{C_u - C_w}, \quad \chi(\eta) = \frac{N - N_w}{N_u - N_w},$$

(8)

where $\eta$ is the similarity variable and $\psi$ is the stream function defined as $u = \partial \psi / \partial y$ and $v = -\partial \psi / \partial x$, which identically satisfies Eq. (1). By employing the similarity variables (8), Eqs. (2) – (5) can be reduced to the following nonlinear ordinary differential equations:

$$f'' + ff' - f' + Mf' = 0,$$

(9)

$$\frac{1}{Pr} \theta'' + f \theta' + Nb \theta'\phi' + Nt \theta'^2 = 0,$$

(10)

$$\phi'' + \frac{Nt}{Nb} \phi' + Le \phi' = 0,$$

(11)

$$\chi'' + Scf \chi' - Pe [\chi'\phi' + \phi'(\chi + \sigma)] = 0,$$

(12)

and the boundary conditions (6) and (7) become

$$f(0) = S, \quad f'(0) = \lambda, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad \chi(0) = 1,$$

(13)

$$f' = 0, \quad \theta = 0, \quad \phi = 0, \quad \chi = 0 \text{ as } \eta \rightarrow \infty,$$

(14)

where primes denote differentiation with respect to $\eta$. Further, $Pr$ is the Prandtl number, $Nb$ is the Brownian motion parameter, $Nt$ is the thermophoresis parameter, $Le$ is the Lewis number, $Sc$ is the Schmidt number, $Pe$ is the bioconvection Péclet number, $S$ is the mass flux parameter with $S > 0$ for suction and $S < 0$ for injection, $M$ is the magnetic parameter number, and $\sigma$ is a dimensionless constant, which are defined as

$$Pr = \frac{\nu}{\alpha}, \quad Nb = \frac{\tau D_B (C_u - C_w)}{\nu}, \quad Nt = \frac{\tau D_I (T_u - T_w)}{\nu \tau}, \quad Le = \frac{\nu}{D_B}, \quad Sc = \frac{\nu}{D_n}, \quad S = -\frac{V_0}{\sqrt{aw}},$$

$$M = \frac{\sigma B_w^2}{\alpha \rho_f}, \quad Pe = \frac{b W_c}{D_n}, \quad \sigma = \frac{N_w}{N_u - N_w},$$

(15)

The physical quantities of interest are the skin friction coefficient $C_f$, the local Nusselt number $Nu_x$, the local Sherwood number $Sh_x$ and the local density of the motile microorganisms $Nn_x$ which are defined as

$$C_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{x q_w}{k(T_w - T_0)}, \quad Sh_x = \frac{x q_m}{D_B (C_w - C_u)}, \quad Nn_x = \frac{x q_n}{D_B (N_w - N_u)}.$$

(16)

where $\tau_w$, $q_w$, $q_m$ and $q_n$ are the surface shear stress, the wall heat flux, the wall mass flux and the wall motile microorganisms flux respectively, which are given by
\begin{equation}
\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad q_m = -D_p \left( \frac{\partial C}{\partial y} \right)_{y=0}, \quad q_n = -D_n \left( \frac{\partial N}{\partial y} \right)_{y=0}
\end{equation}

Using the similarity variables (8), we obtain
\begin{equation}
C_y Re_x^{1/2} = -f'(0), \quad Nu_x Re_x^{1/2} = -\theta'(0), \quad Sh_x Re_x^{1/2} = -\phi'(0), \quad Nn_x Re_x^{1/2} = -\chi'(0),
\end{equation}

where \( Re_x = \frac{U_w x}{\nu} \) is the local Reynolds number.

3. Results and Discussion

Numerical solutions to the nonlinear ordinary differential equations (9)-(12) subjected to the boundary conditions (13) and (14) are obtained using shooting method with the help of shootlib function in Maple software. The effects of the governing parameters on the flow field and heat transfer characteristics are analysed for both stretching and shrinking cases. The value of Prandtl number is fixed at 6.2 (water). Using this method, solutions are obtained using different initial guess for the missing values of \( f(0), f'(0), \theta(0), \theta'(0), \phi(0), \phi'(0), \chi(0), \chi'(0) \). Further, the values of the skin friction coefficient \( C_y Re_x^{1/2} \), the local Nusselt number \( Nu_x Re_x^{1/2} \), the local Sherwood number \( Sh_x Re_x^{1/2} \) and the local density of the motile microorganisms \( Nn_x Re_x^{1/2} \) are shown in table 1 and table 2, respectively for some values of stretching/shrinking parameter and magnetic parameter. It can be seen from table 1 that the skin friction coefficient and the local Nusselt number increase, while the local Sherwood number and the local density of the motile microorganisms decrease in the shrinking case. The opposite behaviour is observed for the stretching case. The effect of magnetic field is significant on the skin friction coefficient as depicted in table 2, where the shear stress at the surface decreases as \( M \) increases.

Table 1. Values of \( C_y Re_x^{1/2}, Nu_x Re_x^{1/2}, Sh_x Re_x^{1/2} \) and \( Nn_x Re_x^{1/2} \) for some values of stretching/shrinking parameter \( \lambda \) when \( M = 0 \), \( Pr = 6.2 \), \( Nb = 0.5 \), \( Nt = 0.5 \), \( Le = 2 \), \( \sigma = 1 \), \( Sc = 1 \), \( Pe = 1 \) and \( S = 2.5 \).

| \( \lambda \) | \( C_y Re_x^{1/2} \) | \( Nu_x Re_x^{1/2} \) | \( Sh_x Re_x^{1/2} \) | \( Nn_x Re_x^{1/2} \) |
|-------------|-----------------|-----------------|-----------------|-----------------|
| -1          | 0.4815          | 4.0131          | 0.0969          | 2.7643          |
| -0.6        | -0.0933         | 3.8494          | 0.5803          | 3.7463          |
| -0.4        | -0.4712         | 3.7940          | 0.7658          | 4.1249          |
| 0           | 0               | 3.5583          | 1.4417          | 5.3835          |
| 1           | -2.8508         | 3.4822          | 1.9150          | 6.4107          |
| 1.5         | -4.5000         | 3.4589          | 2.1098          | 6.8369          |
| 2           | -6.2749         | 3.4415          | 2.2860          | 7.2236          |

Table 2. Values of \( C_y Re_x^{1/2}, Nu_x Re_x^{1/2}, Sh_x Re_x^{1/2} \) and \( Nn_x Re_x^{1/2} \) for some values of magnetic parameter \( M \) when \( Pr = 6.2 \), \( Nb = 0.1 \), \( Nt = 0.1 \), \( Le = 10 \), \( Sc = 1 \), \( Pe = 0.3 \), \( \sigma = 0.2 \), \( S = 1 \) and \( \lambda = 0.1 \).

| \( M \)   | \( C_y Re_x^{1/2} \) | \( Nu_x Re_x^{1/2} \) | \( Sh_x Re_x^{1/2} \) | \( Nn_x Re_x^{1/2} \) |
|-----------|------------------|------------------|------------------|------------------|
| 0         | -0.1093          | 3.4658           | 6.6933           | 3.4463           |
| 1         | -0.1662          | 3.4630           | 6.6826           | 3.4359           |
| 1.5       | -1.8860          | 3.4621           | 6.6793           | 3.4330           |
| 2         | -2.0333          | 3.4614           | 6.6766           | 3.4307           |
Figures 1-4 show the effects of stretching/shrinking parameter $\lambda$ on the velocity, temperature, nanoparticle volume fraction and the density of motile microorganisms profiles, respectively. It can be seen from Figure 1 that the boundary layer thickness of the stretching case is smaller compared to the shrinking case for the velocity distribution. The effect of $\lambda$ on the other three physical quantities is to reduce the boundary layer thickness, hence increase $Sh Re^{-1/2}$ and $Nu Re^{-1/2}$, while $Nu Re^{-1/2}$ decreases in the stretching case. Contrarily, in the shrinking case, the opposite behavior is observed.

Figure 5 presents the effect of magnetic parameter on the velocity distribution. It is found that an increase in $M$ leads to reduction of the velocity profiles. The physical meaning of the behaviour is due to the Lorentz force which results from the presence of a magnetic field in the nanofluid and it works to slow down the velocity.
Figures 5 - 8 are plotted to show that as $M$ increases, the local Nusselt number, the local Sherwood number and the density of the motile microorganisms is slightly decreasing.

4. Conclusions
The mixed convection flow of a nanofluid containing microorganisms over a stretching/shrinking sheet in the presence of magnetic field is examined. The main points the above analysis have mentioned are as follows:

- The skin friction coefficient and the local Nusselt number increase, while the local Sherwood number and the local density of the motile microorganisms decrease as $\lambda$ increases in the shrinking case.
- The skin friction coefficient and the local Nusselt number decrease, while the local Sherwood number and the local density of the motile microorganisms increase as $\lambda$ increases in the stretching case.
- The effect of magnetic field is to decrease all the physical quantities, where it affected the velocity profiles significantly.
Acknowledgements
This work was supported by a research grant (Project Code: GPPS/U811) from Universiti Tun Hussein Onn Malaysia.

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