Sources of Stellar Energy, Einstein- Eddington Timescale of Gravitational Contraction and Eternally Collapsing Objects

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Abstract

We point out that although conventional stars are primarily fed by burning of nuclear fuel at their cores, in a strict sense, the process of release of stored gravitational energy, known as, Kelvin - Helmholtz (KH) process is either also operational albeit at an arbitrary slow rate, or lying in wait to take over at the disruption of the nuclear channel. In fact, the latter mode of energy release is the true feature of any self-gravity bound object including stars. We also highlight the almost forgotten fact that Eddington was the first physicist to introduce Special Relativity into the problem and correctly insist that, actually, total energy stored in a star is not the mere Newtonian energy but the total mass energy ($E = Mc^2$). Accordingly, Eddington defined an “Einstein Time Scale” of Evolution where the maximum age of the Sun turned out to be $t_E \approx 1.4 \times 10^{13}$ yr. This concept has a fundamental importance though we know now that Sun in its present form cannot survive for more than 10 billion years. We extend this concept by introducing General Relativity and show that the minimum value of depletion of total mass-energy is $t_E = \infty$ not only for Sun but for and sufficiently massive or dense object. We propose that this time scale be known in the name of “Einstein - Eddington”. We also point out that, recently, it has been shown that as massive stars undergo continued collapse to become a Black Hole, first they become extremely relativistic Radiation Pressure Supported Stars. And the life time of such relativistic radiation pressure supported compact stars is indeed dictated by this Einstein -Eddington time scale whose concept is formally developed here. Since this observed time scale of this radiation pressure supported quasistatic state turns out to be infinite, such objects are called Eternally Collapsing Objects (MECO). Further since ECOs are expected to have strong intrinsic magnetic field, they are also known as “Magnetospheric ECO” or MECO.

Key words: Stars: evolution, gravitational collapse, Magnetospheric Eternally Collapsing Object (MECO)

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1 Introduction

A star is a self-luminous self-gravitating object which is evolving all the time, howsoever slow the rate may be. Though it is in quasi-static hydrodynamical equilibrium, because of its constant thermodynamical evolution, in the strictest sense, \( dR_0/\dot{t} = \dot{R}_0 \neq 0 \), where \( R_0 \) is its radius. If the adiabatic index of the stellar fluid is \( \Gamma \), then, virial theorem yields, the internal energy as (Kippenhahn & Weigert 1990, Chandrasekhar 1967)

\[
U = -\frac{1}{3(\Gamma - 1)} \Omega
\]

where

\[
\Omega = -\frac{3}{5-n} \frac{GM^2}{R_0}
\]

is the Newtonian gravitation potential energy of the system. Here \( M \) is the (gravitational) mass of the system. In an approximate manner, the star here is represented by an polytrope of degree \( n \):

\[
p = K \rho^{\frac{1}{1+1/n}}
\]

The foregoing equation refers just to an assumed uniform equation of state \((K, n = constant)\) all over the star and the index \( n \) is not necessarily equal to the ratio of specific heats \( \Gamma \neq 1 + 1/n \). One would have \( \Gamma = 1 + 1/n \) only if the system would be assumed to evolve adiabatically.

The Newtonian total energy of the system is

\[
E_N = U + \Omega = \frac{3\Gamma - 4}{3(\Gamma - 1)} \Omega
\]

If one assumes \( \Gamma \) to remain almost constant during the slow evolution, the rate of change of the (Newtonian energy) of the system is

\[
\frac{dE_N}{dt} = \frac{3\Gamma - 4}{3(\Gamma - 1)} \frac{d\Omega}{dt}
\]

From Eq.(2), we have

\[
\frac{d\Omega}{dt} = \frac{3}{5-n} \frac{GM^2}{R_0^2} \frac{dR_0}{dt}
\]
Here, implicit assumptions are that $n$ and $M$ remain constant during this slow evolution. In the context of Newtonian physics, the latter assumption is perfectly justified. The luminosity of the star due to gravitational contraction, known as Kelvin - Helmholtz process is

$$L_{KH} = -\frac{dE_N}{dt} = \frac{(3\Gamma - 4)}{(5 - n)(\Gamma - 1)} \frac{GM^2}{R_0^2} (-\dot{R}_0)$$

(7)

Since $\dot{R}_0 < 0$ during contraction, $L_{KH} > 0$. If there would not be additional sources of luminosity, this process also defines a natural time scale of contraction (Kippenhahn & Weigert 1990, Chandrasekhar 1967)

$$t_{KH} = \frac{E_N}{L_{KH}} = \frac{\Omega}{d\Omega/dt} = \frac{R_0}{-\dot{R}_0}$$

(8)

Having made this introductory theoretical background, we shall specifically point out the role of both nuclear energy generation and KH-energy generation in Sun. Then we shall highlight the important Special Relativistic concepts introduced into the problem for the first time by Eddington. Following the fundamental concept of a maximal luminosity, developed by none other than Eddington, we shall introduce General Relativity in the problem.

2 Nuclear Fuel Supported Time Scale

The Eq.(7) shows that, even without burning of any nuclear fuel, there could be stars, or in a broad sense, self-gravitating objects of finite temperature and luminosity. This is the reason that the massive primordial clouds have finite pressure and temperature and do not undergo any radiationless catastrophic collapse. On the other hand, they may keep on evolving (contracting) quasistatically for durations much larger than free fall times. Such clouds always have a finite luminosity though the frequency of the emitted radiation would be far below the optical range until the final stages. After the final stages, the central region of the cloud could become hot enough to shine in the visible optical range to appear as “Stars”. Actually, these are “Pre-main-sequence” stars, although, before the development of modern theory of hydrogen burning stars, one would not distinguish between main-sequence and pre-main-sequence stars. The rise of the core temperature of these stars generating energy by purely gravitational process would eventually ignite the central nuclear fuel and give birth to normal main-sequence stars.

On the other hand, very low mass stars, called Brown Dwarfs, would continue to shine exclusively and permanently by the KH-process (Kumar 1962,
Earlier it was believed that Sun too generated energy only by this channel, and, if so, Sun’s age would be (Chandrasekhar 1967)

\[ t_{KH} \approx 1.59 \times 10^7 \times q \text{ yr} \tag{9} \]

where \( q = 3/(5 - n) \). If further, one would assume, \( n = 3/2 \) for Sun, one would have \( t_{KH} = 2.4 \times 10^7 \text{ yr} \). We know it too well that the actual age of Sun, \( t_\odot \), is higher by more than two orders of magnitude and obviously there is at least another important source of energy generation in Sun. This statement is often misinterpreted by stating that “the Kelvin - Helmholtz” contraction hypothesis is incorrect for Sun. In fairness, this hypothesis is incorrect only insofar as it denies presence of other energy generation modes; but the basic process of KH energy generation is no hypothesis and is a fundamental result of astrophysics as seen by Eqs.(1-7). Since, in the strictest sense, Sun is evolving and \( \dot{R}_0 < 0 \), the equation (7) is always operational though the value of actual \( |\dot{R}_0| \ll |\dot{R}_{KH}^0| \) where \( \dot{R}_{KH}^0 \) would be the rate of contraction in the absence of any nuclear energy generation, i.e., the one indicated by Eq.(8). Note that,

\[ \frac{|\dot{R}_0|}{|\dot{R}_{KH}^0|} \sim \frac{t_{KH}}{t_\odot} \sim 10^{-3} \tag{10} \]

3 Enter Eddington

Although the theory of nuclear energy generation in stellar core culminated through the landmark work of Bethe (1939), the very concept that some non-gravitational “sub-atomic” process is responsible for Sun’s energy generation was due to Eddington (Eddington 1920, 1926). Eddington mentioned of “transmutation of elements” and was the first to recognize that such processes involved application of \( E = Mc^2 \) formula. Thus he, for the first time wrote that “stars burn mass” and, “any radiation is radiation of mass”:

\[ L = -\frac{dE}{dt} = -\frac{d(Mc^2)}{dt} \tag{11} \]

With this generic and special relativistic definition of luminosity, the maximum age of the luminous phase, i.e., the time for depletion of entire available energy of a star is

\[ t_E \sim \frac{Mc^2}{L} = \frac{M}{-dM/dt} \tag{12} \]

irrespective of the model and theory of energy generation. As per Bowers and Deeming (1984), this time scale is called the “Einstein Time Scale”. For the
Sun, one can easily see that

\[ t_E \sim \frac{M_\odot c^2}{L_\odot} \approx 1.4 \times 10^{13} \text{ yr} \]  

(13)

The very fact that there could be a natural time scale \( t_E \gg t_{KH} \) strongly gave boost to the theories of stellar energy models different from (purely Newtonian) KH-model. If the “transmutation” process, conceived by Eddington, were operative over the entire domain of Sun (i.e., at any \( R \)) and its efficiency, accumulated over the entire (current) life time of the star would be unity (i.e., entire mass would be converted into energy over the entire life span), the actual value of \( t_\odot \) would have been equal to \( t_E \) as long as one would not invoke GR. The fact that, actually, \( t_\odot \sim 10^{-3}t_E \) only means that the efficiency of the process, averaged over the entire volume, and integrated over the current life span of Sun is accordingly smaller, \( \epsilon \sim 10^{-3} \).

## 4 Termination of Thermonuclear Energy Generation

It is widely mentioned that once Sun would stop thermonuclear energy generation, its internal energy \( U \approx 0 \) until quantum effect would give rise to new source of internal energy at much higher density. Consequently, it is believed that, the pressure would immediately drop significantly, \( p \approx 0 \), and Sun would undergo near free fall to collapse to a point in a time (Kippenhahn & Weigert 1990)

\[ \tau_c = \frac{\pi}{2} \left( \frac{3}{8G\pi \rho(0)} \right)^{1/2} = \frac{\pi}{2} \left( \frac{R^3}{2GM} \right)^{1/2} \]  

(14)

where \( \rho(0) \) is the central density at \( t = 0 \) when the star is assumed to be at rest. Here it is assumed that the star is of uniform density. With its present density, one would have \( \tau_c \approx 28 \) min for Sun. This result is clearly incorrect because we have found that even without any thermonuclear energy support Sun can evolve quasistatically for atleast \( 2.4 \times 10^7 \) yr (Eq.[9])! Therefore the assumption of free fall in this case undermines the contraction time scale by a factor of atleast \( 7 \times 10^{11} \).

Interestingly, if one applies GR for this problem, and assumes free fall, one would obtain exactly the same formula for collapse time; however, in this case \( \tau_c \) would be the proper time of collapse recorded by a comoving observer (Misner, Thorne & Wheeler 1973).
We will see clearly that absence of thermonuclear energy generation in any self-gravitating system does not mean absence of pressure and internal energy. On the other hand, absence of non-gravitational energy generation only means that the system would now be self-consistently dictated by pure gravity. Note that a piece of smouldering ember may be also considered as isolated and self-luminous. But gravity plays no role here and depending on the mass of the ember and other chemical details, the life-time of self-luminosity could be very small or relatively longer. Central thermonuclear energy generation is philosophically akin to the process of burning of the ember once we do not worry about issues like what confines and ignites either the thermonuclear fuel in the Sun or the ember atoms. The piece of ember ceases to shine after the exhaustion of chemical fuel because it is not supported by long range universal attractive self-gravity with a negative specific heat.

On the other hand, the KH process is the true and active signature of gravitational compression and resultant energy generation. Philosophically, one may see the KH process as the conversion of mass into radiation by constant self-gravitational squeezing. When a star is supported by thermonuclear energy generation, the value of $\dot{R}_0$ may be practically considered zero compared to its value in a purely KH-phase. And if the thermonuclear energy generation suddenly stops, definitely, at least, initially, the star would try to collapse. Note that $U$ in Eq.(1) does not, per se, depend on any nuclear energy generation. Moreover $U > 0$ for all physical systems. Then by differentiating Eq.(1), we obtain

$$\frac{dU}{dt} = \frac{1}{\Gamma} \frac{d\Omega}{dt}$$

Using Eq.(6) into Eq.(15), we have,

$$\frac{dU}{dt} = \frac{1}{(\Gamma - 1)} \frac{1}{(5 - n)} \frac{GM^2}{R_0^2} (-\dot{R}_0)$$

By using Eqs. (1) and (2) in the above one, we rewrite

$$\frac{dU}{dt} = U - \frac{\dot{R}_0}{R_0} = U \frac{|\dot{R}_0|}{R_0} > 0$$

Thus as $|\dot{R}_0|$ would suddenly increase due to absence of thermonuclear energy generation, there would be fresh addition of internal energy. This clearly shows that there cannot be any strict pressure free collapse in general and even if one would assume the contrary, fresh supply of internal energy may restore quasistatic contraction. However, there could be almost free fall only for the idealized limiting case of $\Gamma = 4/3$ when the increase in the value of $U$ is exactly
Thus self-gravitation driven KH process causes a negative feedback by creating more pressure and internal energy. In other words, squeezing by self-gravity creates its own antidote and prevents a runaway squeezing in the long run. Hence, if central thermonuclear reaction would stop in Sun, sooner or later, it would enter into a self-gravity driven KH-mode a la a premain-sequence-star or any other self-gravitating object. However, the actual luminosity of Sun, in this phase, need not be at all equal $L_{\odot}$. On the other hand the actual value of $L$ would be self-consistently determined by the solution of coupled collapse equations in tandem with thermodynamics and other physical laws.

In fact, recently it has been shown that there is an approximate generic relationship between radiation and rest mass energy density density of any self-luminous self-gravitating object: $\rho_r/\rho_0 \approx \alpha z$, where $z$ is the surface gravitational redshift, $\rho_0$ is the rest mass energy density and $\alpha = L/L_{ed}$. Here $L$ is the luminosity of the object and $L_{ed}$ is the corresponding Eddington luminosity (Mitra 2006a). Further it has been shown that as the compactness $z$ increases, $\alpha \to 1$ and the object may be supported by self-luminosity because of the KH process.

### 4.1 Physical Mechanisms for Energy Generation

One may wonder how a cold giant molecular cloud at an initial temperature $\sim 10$ K may generate “heat” during gravitational contraction in the absence of liberation of any nuclear or chemical energy. This happens because of excitation/deexcitation of molecular vibrational and rotational levels. Another way of seeing this could be in terms of time dependent (electrostatic) inter/intra molecular interactions. From a gross macroscopic view point, one may understand this by recalling that there is really no “perfect fluid” and heat/radiation must be generated during collapse because of various dissipative processes. More technically, such radiative processes for the cold cloud are due to “bound-bound” interactions.

When the cloud would get partially ionized, there would be, in addition, “free-bound” radiative processes. And if the fluid would become completely ionized, the dominant radiative process would be “free-free” process or Bremsstrahlung. And in case, the fluid would become extremely compact due to self-gravity, the charged particles (as well as non charged particles including neutrinos and photons) would tend to move in nearly circular orbits due to gravita-
tional bending. In such a case, the dominant radiation mechanisms would be \textit{Relativistic Gravitational Bremsstrahlung} (Peters 1970). Along with electromagnetic processes, both weak and strong interactions too may play their respective roles during Bremsstrahlung. At the same time if there would be a strong intrinsic magnetic field in the fluid, radiation will be generated by \textit{Relativistic Gravitational Synchrotron} process (Misner et al. 1972) or cyclotron process.

It may be recalled that, from microphysics point of view, any nuclear or chemical “burning” too is implemented by such basic processes like “free-bound” or “free-free”.

5 Einstein–Eddington Time Scale

Eddington also introduced another fundamental concept in astrophysics: a radiating object has a maximum luminosity at which the outward push of the radiation force on the plasma would just counterbalance the inward pull of gravity. This luminosity, known after his name is defined as

$$L_{\text{ed}}(R) = \frac{4\pi GM(R)c}{\eta \kappa}$$

(18)

where $\eta(R) = L(R)/L$ and $\kappa$ is the average specific opacity for a region $R$. At the boundary of the star, by definition $\eta = 1$ and $M(R) = M$ so that

$$L_{\text{ed}} = \frac{4\pi GMc}{\kappa}$$

(19)

Further if we define a parameter

$$\alpha = \frac{L}{L_{\text{ed}}}$$

(20)

we will have

$$L = \frac{4\pi GMc\alpha}{\kappa}$$

(21)

In case, a Newtonian star would radiate at its maximal limit, i.e., if one would have $\alpha = 1$, then one would obtain the \textit{lowest} possible value of

$$t_{\text{EE}}^{\text{min}} = \frac{\kappa c}{4\pi G}$$

(22)
Further, for propagation of photons within physical matter, the minimum value of $\kappa$ is given by:

$$\kappa = \kappa_T = \frac{\sigma_T}{m_p}$$  \hspace{1cm} (23)

where

$$\sigma_T = \left(\frac{8\pi}{3}\right) \left(\frac{e^2}{m_e c^2}\right)^2$$  \hspace{1cm} (24)

is the Thomson cross-section; $m_e$ is electron rest mass, $m_p$ is the proton rest mass, $e$ is electron charge. The numerical value of $\kappa_T = 0.4$ g cm$^{-2}$ and this is the lowest value of $\kappa$ (Kippenhahn & Weigert 1990). On the other hand, typical stellar opacities could be thousand times larger than this lowest value. Therefore, we find that, the lowest value of

$$t_{EE}^{\min} = \frac{\sigma_T c}{4\pi G m_p}$$  \hspace{1cm} (25)

Using Eq.(24) in (25), we see that

$$t_{EE}^{\min} = \left(\frac{2c}{3Gm_p}\right) \left(\frac{e^2}{m_e c^2}\right)^2$$  \hspace{1cm} (26)

It is interesting that the smallest value of the time to deplete the mass-energy depends only on fundamental constants. Moreover, it is independent of the (gravitational) mass of the source of radiation. The corresponding numerical value of

$$t_{EE}^{\min} \approx 1.5 \times 10^{16} \text{ s} \sim 5 \times 10^8 \text{ yr}$$  \hspace{1cm} (27)

irrespective of the mass of the star. This suggests that, if one would apply Special Theory of Relativity (STR), a self-luminous phase for self-gravitating objects must exist for at least $5 \times 10^8$ yr.

In case the mass energy loss would be because of $\nu$-emission instead of photon emission, the opacity would be extremely small and one may have $t_{EE}^{\min}$ as small as 10 s. Recall that this is indeed the observed time scale of the $\nu$-burst from SN 1987A. On the other hand, the corresponding free fall time scale of the proto neutron star (NS) is as small as $\approx 0.1$ ms. Thus, the free fall assumption is lower, in this case, by a factor of $\approx 10^5$.

The time scale $t_{EE}^{\min}$ actually refers to the mass energy depletion. The NS born in the SN explosion continues to cool (i.e., lose mass-energy) primarily through
photon emission for millions of years. In a strict sense, the cooling time scale
could easily be what is indicated by Eq.(27) eventhough intermediate $\nu$-driven
time scale could be extremely shorter.

6 Enter General Relativity

The physics of Sun or anything else having associated mass energy, in the
strict classical sense, is determined by not Special Relativity, but by GR. The
formal entry of GR counterpart of Eq.(11) was facilitated with the discovery
of the spacetime structure exterior to a radiating star by Vaidya (1951). At
the boundary of the star, one has

$$ds^2 = (1 - 2GM/R_0c^2)du^2 + 2du dR - R_0^2(\theta^2 + \sin^2 \theta d\phi^2)$$  (28)

where $u$ is the retarded time, $\theta$ is the polar and $\phi$ is the azimuth angle. As the
collapse/contraction proceeds both $M(R_0)$ and $R_0$ decrease. The luminosity
of the star as seen by a distant observer is

$$L^\infty = -\frac{d(Mc^2)}{du}$$  (29)

It may be reminded here that gravitational mass $Mc^2$ is the total mass-energy
content seen by a distant observer and not by a local observer. On the other
hand, in GR, the spacetime around a strictly static object is given by the
radiationless vacuum Schwarzschild metric which corresponds to both

$$L_{\text{local}} = L^\infty = 0$$  (30)

This very property should disqualify strictly static objects to acquire the
nomenclature of a ”Star”, and it would be more reasonable to call them as
”Objects” only. Since there is mass-energy associated with any radiation, by
definition, strictly static GR objects must not be self-luminous, and, one must
have temperature $T = T_s = 0$ at the boundary. But if $T$ is finite just beneath
the boundary, the associated radiation field must penetrate the boundary too
(unless the internal region is a trapped one) and hence strictly static GR ob-
jects must have $T = 0$ everywhere. Hence, there could be strictly static GR
objects only for perfectly degenerate and absolutely cold material.

For relativistic objects, one important observable measure of the grip of gravity
on the surface is the surface gravitational redshift (of spectral lines emanating
from the surface)

$$z = (1 - 2GM/R_0c^2)^{-1/2} - 1$$  (31)
For this latter external metric, a strictly static self-gravitating object has a fundamental constraint $z < 2$ (Buchdahl 1959). However, when the external spacetime contains radiation, i.e., when it is to be described by Vaidya metric, there is no such constraint: $z \leq \infty$. This infinite difference in the limiting value of $z$ depending on whether the external spacetime is luminous or not is indeed a very subtle point reminding one of the epithet “Subtle is the Lord” in the context of GR. Let us recall here that very massive stars or any sufficiently dense object would undergo continued gravitational collapse to $z = \infty$ Black Hole (BH) stage.

As discussed earlier, the minimum self-luminous time scale is obtained when $L = L_{ed}$. And GR causes the local value of $L_{ed}$ to increase from its Newtonian value by a factor of $(1 + z)$ (Mitra 1998a, Mitra 2006a):

$$L_{ed} = \frac{4\pi GMc}{\kappa}(1 + z) \quad (32)$$

However, because of the joint effect of gravitational redshift and gravitational time dilation, the distant observer sees a reduced Eddington luminosity:

$$L_{\infty}^{ed} = \frac{L_{ed}}{(1 + z)^2} = \frac{4\pi GMc}{\kappa(1 + z)} \quad (33)$$

Consequently, the minimum value of previously defined Einstein- Eddington Time Scale, in GR, becomes

$$u_{\text{EE}}^{\min} = \frac{M}{L_{\infty}^{ed}} = \frac{k c}{4\pi G}(1 + z) \quad (34)$$

Since in principle, during continued collapse, $z \to \infty$, clearly, the Einstein- Eddington time scale for depletion of mass energy becomes infinite for arbitrary value of the opacity $\kappa$:

$$u_{\text{EE}}^{\min} = \infty \quad (35)$$

This immediately shows that irrespective of the actual value of mass of the self-gravitating object, the phenomenon of self-luminosity becomes eternal.

Considering the minimum value of photon opacity $\kappa = \sigma_T/m_p$ in equation (34), we have

$$u_{\text{EE}}^{\min} = \frac{\sigma_T c}{4\pi Gm_p}(1 + z) \quad (36)$$
In terms of fundamental constants, we eventually obtain

\[
u_{EE}^{\min} = \left( \frac{2c}{3Gm_p} \right) \left( \frac{e^2}{m_e c^2} \right)^2 (1 + z) \approx 5 \times 10^8 (1 + z) \text{ yr} \quad (37)
\]

As discussed in the previous section if there would be a GR collapse and formation of a compact object, it is likely that the opacity would be determined by \( \nu \)-emission. In such a case, in Eq.(34), one should use appropriate \( \nu \) opacities rather than \( \sigma_T \). The resultant time scale in such a case would be \( \nu_{EE}^{\min} > 10(1 + z) \) s. However since it is much easier to maintain a photon mediated Eddington luminosity \( \sim 10^{38} \) erg/s instead of a \( \nu \)-Eddington luminosity of atleast \( 10^{54} \) erg/s (for \( 1M_\odot \) object) and the system would like to spend minimum energy, it is likely that the eventual long term energy depletion time scale would be still governed by Eq.(37). Note again that, in any case, for BH formation \( (z \to \infty) \), \( \nu_{EE}^{\min} \to \infty \) irrespective of the value of opacity \( \kappa \).

7 Physical Implications and Applications

There have been some recent developments in the study of GR gravitational collapse which show that GR collapse time scale is indeed determined by the Einstein-Eddington time scale developed above. Note that the massive objects tend to collapse inexorably to BHs having an Event Horizon (EH) with \( z = \infty \). If so, the object must pass through states having arbitrarily large but finite \( z \) states to reach the \( z = \infty \) state. It has been shown that as the object would become more and more compact (i.e., \( z \) would increase), the object would become radiation energy dominated, \( \rho_r \gg \rho_0 \) (Mitra 2006a). This happens because collapse generated radiation quanta (i) Spend more time within the body because of intense matter-radiation interaction (diffusion) and also because (ii) they get trapped within the body because of the extremely strong self-gravity. The density of trapped radiation increases as \( \rho_r \sim R^{-3}(1 + z)^2 \) (Mitra & Glendenning 2006). The corresponding heat flux grows in a similar fashion

\[
q_{\text{trap}} \sim R^{-3}(1 + z)^2 \quad (38)
\]

The GR local Eddington luminosity (Eq.[32]) corresponds to a critical outward heat flux of

\[
q_{ed} = \frac{L_{ed}}{4\pi R^2} = \frac{GMc}{\kappa R^2} (1 + z) \quad (39)
\]
The ratio of the actual heat flux to the critical Eddington flux grows as

\[
\alpha = \frac{q_{\text{trap}}}{q_{\text{ed}}} \sim \frac{1 + z}{RM}
\]  

(40)

Initially, of course, the value of \( \alpha \ll 1 \). But as \( z \to \infty \) during the BH formation, the value of \( \alpha \) starts increasing dramatically \( \sim (1 + z) \). Sooner or later, at an appropriate range of finite value of \( z \), one must attain a state \( \alpha \approx 1 \) when the outward radiation flux would attain its critical “Eddington value”. By the very definition of an “Eddington Luminosity”, catastrophic collapse would then degenerate into a secular quasistatic contraction supported by radiation pressure (Mitra & Glendenning 2006).

Even in Newtonian gravitation, there could be stars supported entirely by radiation pressure. However, in Newtonian gravitation, such Radiation Pressure Supported Stats (RPSSs) turn out be extremely massive. Recently, it has been explained that this requirement of an excessive large mass for Newtonian RPSSs stems from the fact that they have weak gravity \( (z \ll 1) \) and the only way radiation pressure can still dominate would be if \( M > 7200M_\odot \) (Mitra 2006b). On the other hand, if there would be objects with appropriately large \( z \gg 1 \), one will have an RPSS at arbitrary low or high value of \( M \) because in such a case, \( \rho_r \gg \rho_0 \) as \( z \gg 1 \) (Mitra 2006b). In otherwords, during continued collapse, the collapsing object first becomes an extremely relativistic \( (z \gg 1) \) RPSS before becoming a true BH with \( (z = \infty) \). Then as explained by Eq.(37), as the RPSS tends to become a true BH with \( z \to \infty \), the lifetime of the PRSS phase becomes infinite:

\[
u_{EE}^{\text{min}} = 5 \times 10^8 (1 + z) \text{ yr} \to \infty
\]  

(41)

Hence such objects have been termed as “Eternally Collapsing Objects” (ECO) (Mitra 1998b, Mitra 2000, Mitra 2002a,b, Mitra 2006b,c,d,e,f,g,h, Mitra & Glendenning 2006). Any astrophysical plasma is always endowed with microscopic currents and some intrinsic magnetic field \( (B) \). When such a plasma contracts, in a crude picture, its magnetic field gets compressed as \( B \sim R^2 \). And this is the basic reason that a compact Neutron Star has strong intrinsic magnetic field. Based on this simple argument, it was postulated by the present author that the compact and non-singular ECOs thus must possess strong intrinsic magnetic field (Mitra 1998b Mitra 2000, Mitra 2002a,b). It was thus natural that a spinning ECO would have a magnetosphere like a pulsar and accordingly, the phrase Magnetospheric ECO or MECO first coined by Leiter, Mitra & Robertson (2001). Subsequently Mitra withdrew his name from revisions of this preprint which attempted to make a particular version of MECO with assumed equipartition magnetic field (Leiter & Robertson 2003).
Thus the Einstein–Eddington time scale developed here is realized in the arena of GR.

7.1 Difference with Traditional BH Picture

The only exact analytical model of BH formation involves the collapse of a homogeneous spherical ball of dust (Oppenheimer & Snyder 1939, OS). It is known that, in this case, the time of formation of the Event Horizon \((z = \infty)\), as seen by a distant observer, is \(t_{EH} = \infty\). On the other hand, the comoving proper time of collapse (upto the central singularity) is still given by the Newtonian expression (14). One may initially think that Eq.(37) exactly corresponds to the ideal Oppenheimer-Snyder collapse yielding \(t_{EH} = \infty\). But we discuss below that such an impression is far from correct.

To see this, in a first approximation, we may transform, \(\dot{u}_{EE}^{\text{min}}\) in Eq.(37) into local proper time scale which would be lower by a factor of \((1 + z)\) because of GR time dilatation:

\[
\tau_{EE}^{\text{min}} = \frac{\sigma T c}{4\pi Gm_p} \approx 5 \times 10^8 \text{ yr} \tag{42}
\]

In contrast, for the dust collapse, the proper comoving time scale is essentially the free fall time scale

\[
\tau_{ff} \sim (G\rho)^{-1/2} \tag{43}
\]

which has close similarity with Eq.(14). Now suppose we are considering the continued collapse of a \(10M_\odot\) mass star into a BH. During pressureless freefall, \(\rho\) would keep on increasing as \(R^{-3}\) and at one stage density would attain nuclear value \(\rho \sim 10^{14} \text{ g cm}^{-3}\). At this stage one would have \(\tau_{ff} \sim 10^{-4} \text{ s}\). In such a case, we find that

\[
\frac{\tau_{EE}^{\text{min}}}{\tau_{ff}} \sim 10^{20} \rho_{14}^{1/2} \tag{44}
\]

where \(\rho_{14}\) is the density in units of \(10^{14} \text{ g cm}^{-3}\).

- Therefore, clearly, even though, formally, both, \(u_{EH} = t_{EH} = \infty\), there are extreme latent physical differences between the two cases; the actual duration of the corresponding proper time scales in the two cases could differ by a factor of \(10^{20}\) or even more (actually \(\infty\)). We recall here that even when we were considering a purely Newtonian case, the free fall assumption undermined
actual contraction time scale by a factor of at least $7 \times 10^{11}$ for the case of would be collapse of Sun.

Further, suppose, we are considering the actual observational aspect of the contracting star at the late stages of contraction, i.e., $z \gg 1$. To be more specific, suppose we are considering a stage with $z = 10^8$. Recall that, in comparison, for a Neutron Star, we have $z = 0.1 - 0.2$. The minimum time scale of this stage would be

$$u_{EE}^{\text{min}} \approx 1.5 \times 10^{24} \text{s} \sim 3 \times 10^6 \text{ Hubble Time}$$

(45)

In contrast, for Free-Fall, one may also tentatively write the observed time scale as

$$t_{ff} \sim (1 + z)\tau_{ff}$$

(46)

so that, in this case, we have,

$$t_{ff} \sim (1 + z)10^{-5} \sim 10^3 \text{ s}$$

(47)

because, in this case, the density would be $\rho \sim 10^{16} \text{ g cm}^{-3}$. However, the free-fall time scale, in a strict sense, is actually devoid of any physical meaning because a dust collapse is radiationless, and the distant observer, in a strict sense, would never see the collapse in the absence of any emitted radiation:

$$L_{ff} = L_{ff}^{\infty} \equiv 0$$

(48)

• In contrast even at this extreme late stage of radiative collapse ($z = 10^8$), a distant astronomer would measure a luminosity of

$$L_{ed}^{\infty} = 1.3 \times 10^{31} \text{ erg/s}$$

(49)

In principle, this luminosity is measurable for a duration of $\sim 3 \times 10^6$ Hubble Time (see Eq.[36]). In fact, the quiescent luminosity of stellar mass BH candidates could indeed lie in this range. However because of extreme redshift of MECO surface and its photosphere, the observed radiation would certainly not be in the X-ray band. On the other hand, it could be in infrared, millimeter or microwave band. Nonetheless, the Robertson & Leiter model of MECO (2006a) has some specific prediction on this based on several assumptions and simplifications.

Despite such extreme physical differences between the scenarios involving idealized free fall and realistic radiative cases, it is often assumed that, the real
collapsing stars may obey the rules of pressureless dust collapse. If so, then the luminosity of the star at late stages fall off exponentially in a fashion (Misner, Thorne & Wheeler 1972)

\[ L_{\infty, ff} = L_0 \exp \left[ -\frac{2}{3\sqrt{3}} \frac{c t_{ff}}{R_g} \right] \]  

(50)

where \( R_g = \frac{2GM}{c^2} = 3 \times 10^6 \text{cm} \) is the gravitational radius of the star and is constant in the pressureless/radiationless case. Let the value of initial luminosity of the star be \( L_0 \sim 10L_\odot \sim 4 \times 10^{34} \text{ erg/s} \). For the 10\( M_\odot \) mass star, the time constant in the exponential function is

\[ \frac{3\sqrt{3} R_g}{2c} \sim 10^{-4} \text{ s} \]  

(51)

Then combining Eqs. (43), (50), and (51), it follows that, for the given case, we will have

\[ L_{\infty, ff} \sim L_0 \exp\left[-(1+z)\right] \sim L_0 \exp[-10^8] \]  

(52)

• It is very clear then that even when the star will have \( z \sim 10 \), let alone, \( z = 10^8 \), for all practical purpose the observed luminosity \( L_{\infty, ff} \approx 0 \). Further, for \( z \gg 1 \), the luminosity would be infinitesimal \( L_{\infty} \rightarrow 0 \). Let us clarify that here we took \( z = 10^8 \) only as a likely case of extreme large value of \( z \). In no way do we insist that the actual value of \( z \) will be precisely this. Just like the local temperature \( T \) of a MECO depends uniquely on \( M \) (Mitra, 2006b, Mitra & Glendenning 2006):

\[ T \approx 600 \text{MeV} \left( \frac{M}{M_\odot} \right)^{-1/2} \]  

(53)

it is possible that \( z \) too depends uniquely on \( M \). As the ECO keeps on radiating indefinitely, \( M \) keeps on decreasing indefinitely and \( z \) must be evolving (increasing) all the time to attain the \( z = \infty \) BH stage. The above qualitative discussion would remain valid for any large value of \( z \gg 1 \). Thus in the present paper, neither do we really assume any specific value of \( z \) nor do we purport to justify any specific \( z - M \) relationship. On the other hand, this could be the topic of a future study on the fundamental property of ECOs.

Even if the value of \( L_0 \) would be extremely larger than what has been considered here, the foregoing conclusion would remain unchanged for \( z \gg 1 \). Thus while the assumption of free fall collapse leads a scenario where the luminosity of the contracting star practically becomes zero in a few \( \tau_{ff} \), for the radiative
collapse, it is possible that the star remains observable for practically infinite time scale even though ultimately it would eventually (mathematically) become a BH with $L^\infty = 0$. In other words, while the effective duration of the luminous phase of a contracting star, in the free fall paradigm, is only few free fall time scales and thus insignificant, for radiative collapse the same could easily be more than the age of the Universe.

Thus the assumption of free fall clearly leads to an incorrect picture for physical radiative collapse even though one obtains $u_{EH} = t_{EH} = \infty$ in either case. Again we recall that even when we do not consider any relativity, special or general, and would naively consider the present internal energy of the Sun as its ultimate energy source, the free fall assumption undermines the duration of the active luminous phase by a factor of at least $7 \times 10^{11}$.

8 Comparison With Other Works

Since $u_{EE}$ refers to a depletion of mass energy, one expects $M = 0$ for the end product. Thus the BH formed asymptotically should have a unique mass $M = 0$. And it has indeed been shown that the integration constant which appeared in the so-called vacuum Schwarzschild solution has the unique value of zero: $\alpha_0 = 2M = 0$ (Mitra 2005, 2006b,c,d,e). Physically, as the continued collapse process becomes eternal, all available mass energy is radiated away. However, a BH with $M = 0$ does not necessarily mean absence of matter or violation of any baryon/lepton number. All it means is that total positive mass energy associated with baryons, leptons and radiation gets exactly offset by negative self-gravitational energy. We may see now that this profound conclusion is indeed consistent with the phenomenon of GR collapse.

A vacuum Schwarzschild solution, actually Hilbert solution (Mitra 2006f), with a supposed $M > 0$ yields the traditional BH paradigm. On the other hand, a solution with supposed $M < 0$ is equally valid from pure mathematical perspective and would yield a “Naked Singularity” (NS) without an Event Horizon (EH). Thus, a $M = 0$ BH is a borderline case between the two and it is no spacetime singularity at all because it requires infinite comoving proper time to form: $\tau = \infty$.

8.1 Physical Interpretation of Pressureless Collapse

First, note that, for the assumed pressureless case, one obtains exactly the same value of collapse (proper) time scale for both the Newtonian and the GR case (the same Eq.(14) remains valid in either case). But from a strict
GR viewpoint, a purely Newtonian result is valid only in a completely flat spacetime, i.e., when gravitation is infinitely weak. In other words, at least for isolated objects, GR would approach the exact Newtonian limit if the mass-energy density $\rho c^2 \to 0$ everywhere. Thus we get a hint that a strict denial of pressure probably is tantamount to denial of mass-energy, and hence, denial of gravity itself! In the Newtonian case, one can see this from the static virial theorem:

$$\Omega + 3 \int_0^{R_0} p \, 4\pi R^2 \, dR = 0$$

which shows that if $p \equiv 0$, one must have $\Omega \equiv 0$. Recently, GR version of such virial theorem has been obtained for the first time (Mitra 2006c) and it shows that, in GR too, *denial of pressure for a fluid means denial of self-gravity*. Actually, whether in Newtonian or in Einstein or in any other gravity, it must be so because of sheer thermodynamical reasons:

By thermodynamics, the equation of state (EOS) of the fluid under consideration may be represented as $p \propto \rho^\Gamma$ and conversely $\rho \propto p^{1/\Gamma}$. Then clearly a $p \to 0$ situation implies $\rho \to 0$. And if $\rho = 0$, we would have $M = 0$ and so would be the self-gravity; $\Omega = 0$. Thus from physics point of view, if the only *exact* solution of collapse of a uniform dust is really to be considered as a mathematical physics problem, the mass of the BH formed therein is $M = 0$. Since the proper comoving proper time of its formation is $\tau \propto M^{-1/2}$, clearly, the actual value of $\tau_{EH} = \tau_{\text{singularity}} \to \infty$ in this problem. This understanding has several ramifications. When one erroneously assumes $M > 0$ for this idealized collapse, one obtains the traditional BH paradigm where the finite mass BH is formed in prompt free fall time scale while the time scale of its formation as recorded by an astronomer is infinite ($t_{EH} = \infty$). In other words although an astronomer would never see a BH, it is supposed to be formed only w.r.t. a local observer outside the realm of observed physical universe. This is at variance with the principles of all theories of relativity, Galilean, Special or General. The spirit of relativity is that “rulers” and “clocks” could indeed be different for different observers and so could be the measured numerical values of relevant physical quantities. However, if the physical phenomenon is observable to one observer, it must be observable to all, if it is not observable to a given observer, it must not be observable to any other observer either. In other words, all observers are on equal fundamental footing and there is no favoured or disfavoured observer. Physically, it means that all physically valid coordinate systems must be connected by *non singular* transformations. This fundamental spirit of relativity and physical rationality gets violated in the traditional BH paradigm. This basic inconsistency associated with the supposed traditional BH formation picture gets resolved by realizing that actually $M_{BH} = 0$ and $\tau_{EH} = \tau_{\text{singularity}} = t_{EH} = t_{\text{singularity}} = \infty$. In other
words, neither the comoving observer nor the astronomer sees either the EH or the central singularity formation as they indeed happen asymptotically.

In GR collapse equations, physics appears when one considers (i) an EOS of the collapsing fluid, (ii) the evolution of the EOS at arbitrary high pressure, temperature or other relevant parameters, (iii) generation of heat/radiation within the body due to dissipation and (iv) propagation of the outward radiation and its effect on the collapsing fluid in a self-consistent manner. Without such physical aspects, the GR collapse problem becomes a mere exercise in applied/numerical mathematics something like the dissection of a corpse without any flow in the veins and a throbbing heart. Yet starting from OS, even now, many authors avoid all such intractable physical aspects by setting \( p \equiv U \equiv 0 \) in the problem. Technically, this is known as “dust approximation”. Formally, one can also assume the “dust” to be inhomogeneous instead of the homogeneous case considered by OS. But once one assumes inhomogeneity, infinite forms of inhomogeneity (even though density would be assumed to decrease with \( R \)) can be assumed. Then complexity of the GR collapse equations offer infinite scope for various additional applied mathematics exercises. In particular, in some cases, it is claimed that, instead of a BH with an EH, there may be “Naked Singularity” where the central singularity forms before the central region gets trapped and hence, the central singularity may be visible either momentarily or permanently (Joshi 2004). If truly so, this phenomenon would be in the arena of observable astrophysics. However such discussions never consider physical questions such as (i) the value of \( z \) at the moment of formation/eventual of the supposed Naked Singularity, (ii) 3-speed and acceleration of the fluid and (iii) Mass of the Singularity though in some cases it is believed that it would be have \( M = 0 \). Even if one would assume, \( M = 0 \) for such Naked Singularities, one wonders why they must not hurtle to \( M = -\infty \), in the absence of an EH. In other words, why formation of a supposed global naked singularity of any mass must not be followed by an infinitely strong and visible energy outburst. Such physical questions are rarely even posed, let alone answered, in the context GR cum applied mathematics research (Joshi 2004).

We may point out that such questions too get resolved by realizing that \( p \equiv 0 \) actually implies \( \rho \equiv 0 \) so that all dust collapse problems must correspond to uniform density \( \rho = 0 \) OS problem which uniquely and mathematically, results in a mathematical BH of \( M = 0 \) (physically, a \( M = 0 \) BH is never formed as the collapse process becomes eternal with \( \tau = \infty \)). Since the sound speed within a fluid is \( c_s = \sqrt{dp/d\rho} \), in order that \( c_s \) is finite, it is necessary that \( dp = 0, \) if \( d\rho = 0 \). Hence from, this consideration too, there cannot be any true “inhomogeneous dust” and no Naked Singularity in dust collapse.
To settle this question permanently, let us recall the GR Poisson equation for a static fluid (Ehlers, Oszavath and Schucking 2006):

\[ \nabla^2 \sqrt{g_{00}} = 4\pi G \sqrt{g_{00}} \left( \rho + 3p \right) \]  

(55)

where \( g_{00} \) is the time-time component of the metric coefficient defining both gravitational redshift and relativistic potential. If the fluid would tend to become pressureless dust, it would tend to undergo free fall where it would be possible to set \( g_{00} = \text{uniform} \) (actually unity) over the entire fluid. Thus, as \( p \to 0 \), \( \nabla^2 \sqrt{g_{00}} \to 0 \) too.

Then it follows from the foregoing equation that \( \rho \to 0 \) as \( p \to 0 \). Thus all dust collapse, in reality, correspond to the OS collapse which in turn corresponds to the formation of a \( M = 0 \) BH.

8.2 Physical Interpretation of Adiabatic Collapse

Many authors attempt to inject partial physics into the GR collapse problem by considering the fluid to possess a pressure. However, tracking of the actual evolution of the EOS, particularly at arbitrary high density and temperature is impossible both in principle and practice. Thus usually some mathematically amenable fixed EOS or modestly changing EOS is considered for analytical/numerical studies. Further, in many cases, propagation of radiation through the fluid is not considered at all by setting the heat flow flux \( q \equiv 0 \). Yet, one can have infinite variations of input parameters here to obtain various results such as finite mass BHs or Naked Singularities as the end product. But do such results have any physical validity in the absence of consideration of radiation generation and its transport?

The recently developed GR virial theorem (Mitra 2006c) shows that, for slow gravitational contraction

\[ dQ = \frac{(3\Gamma - 4)}{3(\Gamma - 1)} |d\Omega| \]  

(56)

where, \( dQ \) is the amount of heat radiated by the collapsing body and \( |d\Omega| \) is the magnitude of the change of the self-gravitational energy of the fluid.

For strictly adiabatic radiationless collapse, \( dQ \equiv 0 \), and thus \( \Omega \) must remain fixed during such contraction. But in spherical geometry, \( \Omega \) cannot remain fixed during collapse unless it is pegged fixed at \( \Omega \equiv 0 \). Thus despite detail applied mathematics along with some semblance of physical considerations, a strict adiabatic collapse implies \( \Omega \equiv 0 \), a condition which is satisfied only
for $p = M = 0$. In such a case, despite the mathematical consideration of
a pressure, actually, $\rho = 0$. Hence despite many apparent richness of pressure
aided adiabatic collapse, physics wise it is equivalent to the dust collapse if one
must pretend a “collapse” in such a case. Therefore, despite the appearances
of either finite mass BHs or Naked Singularities through applied mathemat-
ics exercises, adiabatic collapse should uniquely result in a zero mass BH.
Technically a zero mass BHs is a borderline case between a BH and a Naked
Singularity. Physically it is never formed since $\tau = \infty$. Even more physically,
there is no strict adiabatic collapse and all related applied mathematical re-
results are just a chimera devoid of physics.

If a given collapse problem would claim to find emission of radiation, then, by
deinition, one must use the formalism of radiative collapse which necessarily
involves heat flux $q$. The local value of luminosity would be $L = 4\pi R^2 q$.
Conversely, any (adiabatic) collapse study which sets $q = 0$ beforehand, must
not find any finite value of $L$ (by definition). However, in a peculiar case of
mathematical treatment, by introducing negative pressure and fudging of the
boundary conditions, one might claim to predict “strong burst of radiation”
(i.e., very large $L$) in a purely adiabatic collapse with $q \equiv 0$ (Joshi & Goswami
2005, Bojowald, Goswami, Maartens and Singh 2005, Goswami, Joshi and
Singh 2006)! Such examples show that a mere applied mathematics treatment
of the GR collapse equations can not only be physically vacuous but could be
completely misleading too.

8.3 Physical Interpretation of Radiative Collapse

The amount of energy radiated in a continued or any collapse, $Q$, is to be
determined from self-consistent solution of GR collapse equations. However
the GR virial theorem could roughly indicate this amount from overall ther-
modynamical considerations. And when one is purporting to probe a likely
singular state with infinite density, pressure and temperature, one must not
make crucial simplifying assumptions about the value of $Q$. However, many
radiative studies of GR collapse, either for numerical or analytical simplicity,
implicitly or explicitly assume beforehand $Q \ll M_0 c^2$. From the point of view
of final stages of collapse, such an assumption effectively reduces the problem
to the adiabatic case, and in turn, to the dust collapse case. To confirm this we
recall a recent generic result on contraction of self-gravitating conﬁgurations
(Mitra 2006a):

$$\frac{\rho_r}{\rho_0} \sim z \gg 1; \text{ when } z \gg 1$$

(57)
Note that the body becomes a radiation dominated early Universe like fireball even before any EH would form: $\rho_r \gg \rho_0$ for $z \gg 1$. The heat flux $q \propto \rho_r$ accordingly becomes extremely large and the associated outward radiation force on the plasma halts the collapse to form an ECO. Usually numerical radiative collapse studies, on the contrary, assume $\rho_r \ll \rho_0$. In view of the foregoing equation, this latter assumption can be valid only in the regime of $z \ll 1$. As far as the regime $z \gg 1$ is concerned, at best this assumption would imply $\rho_0 = \rho_r = 0$. Thus true physical interpretation of such simplified continued radiative collapse studies is that they correspond to $\rho = 0$ and a BH of $M = 0$ though, by means of mere applied mathematics, bereft of consistent physics and thermodynamics, they may talk of generating finite mass BHs or Naked Singularities.

Thus even if one would consider idealized fluids such as a “Scalar Field”, thermodynamics demands that there is dissipation and heat transport, uninhibited by any simplifying favorable assumptions, if the fluid must collapse. And if such dissipative processes and heat/radiation transport mechanisms would be denied there would be no collapse at all though by means of applied mathematics/numerical computation one may arrive at variety of “results”.

### 8.4 Physical Gravitational Collapse

Gravitational collapse must be accompanied by emission of radiation/heat flow from the collapsing fluid. The density of radiation within the fluid gets enhanced by (a) matter-radiation interaction (diffusion) by which $\rho_r$ increases from the value one would obtain using free streaming assumption (Mitra 2006a). And in the $z \gg 1$ regime $\rho_r$ also increases (b) due to trapping of the radiation by self-gravitation of the fluid (Mitra & Glendenning 2006). The process (a) has recently been specifically considered. In a very important paper Herrera & Santos (2004) have shown that the effect of outward flow of heat can stall the GR continued collapse. This pioneering suggestion has been confirmed by Herrera, Di Prisco and Barreto (2006) by means of a specific numerical modeling. In another related work, Cuesta, Salim and Santos (2005) have found that Newtonian supermassive stars undergo collapse to form a hot quasistatic ECO rather than a static cold BH. Such works however have not considered the generic mechanism (b) of self-gravitational trapping of radiation/heat. On the other hand if this mechanism would indeed be implemented in an appropriate numerical scheme, it would be found that ECOs rather than BHs are formed for arbitrary initial mass of the fluid provided it is dense enough to undergo continued collapse to $z \to \infty$. 
8.5 Comments on Some Numerical Works

Following the suggestion by one of the referees, we shall specifically point out why some numerical works lead to conclusions completely different from what has been found here.

- 1. Let us consider the paper “Collapse of a rotating supermassive star to a supermassive black hole: fully relativistic solution” by Shapirao & Shibata (2002). GR is meant to be a physical theory where all forms of mass energy couple with one another and one may have the exact realization of the $E = Mc^2$ formula whereby entire passive initial gravitational mass may actually be transformed into pure energy (radiation). Normally the nomenclature “fully relativistic” would mean that the concerned study has incorporated all physical aspects of of the problem without making any simplifying crucial assumption in the framework of strongest possible gravity. But the actual reality is far from this. Any number of crucial assumptions and simplifications are often made in such studies and the nomenclature “fully relativistic” is used from purely applied mathematic point of view, i.e., the calculations are no “Post Newtonian” ones. From this this definition of “fully relativistic”, the Oppenheimer Snyder (1939) study which suppressed all physics by assuming $p = U ≡ 0$ even at the singularity is also “fully relativistic” calculation. Ironically, despite this, it is indeed the only exact “fully relativistic” calculation because here $p = ρ = q = L = M = M_0 = 0$.

Referring back to this work by Shapirao & Shibata (2002), it does not consider any radiation transport at all, $q ≡ 0$. Thus, as discussed before, from thermodynamics point of view, eventually in the regime of $z \gg 1$, it is just the Oppenheimer Snyder (1939) collapse.

- 2. “Collapse of a magnetized star to a black hole” by Baumgarte and Shapiro (2003). While this paper considers magnetic field in the fluid, it does not consider any radiation transport. Thus it has no relevance for final stages of physical continued collapse. Further, Eqs.(1-3) used by this paper are the equations for free fall which are strictly valid when magnetic field $B = 0$, pressure $p = 0$, heat flux $q = 0$! One definitely cannot expect formation of hot ECO in such a study.

- 3. “Collapse of uniformly rotating stars to black holes and the formation of disks” by Shapiro (2004). This is a Newtonian cum Post Newtonian study and does not consider any radiation transport at all. Thus, in reality, it too has got no relevance for final stages of physical radiative continued collapse.
9 More Fundamental Reasons

There are much more fundamental reasons for the GR continued contraction time scale to be $t = \infty$ than what has been elaborated above.

9.1 Speed at the Event Horizon

STR is founded on the principle “nothing can move faster than light” (actually faster than a certain limiting speed). GTR asserts that this principle is valid even in curved spacetime, in the presence of gravity, for an observer with arbitrary acceleration. It is known for long that if a test particle would approach the EH, its 3-speed would approach the speed of light, $v \to c$ and in fact, if one uses the so-called Schwarzschild coordinates, one has $v = c$ on the EH (Mitra 2000, 2002a,b). If the BH would have, $M > 0$, $R_g > 0$, one would, in this case, have $v = c$ at $R = R_g > 0$. If so, the speed of the particle inside the EH would exceed the speed of light, $v > c$. Many GR “experts” try to fudge this real fundamental problem by insisting that, in suitable coordinates, one may have $v_{EH} < c$. This is actually impossible if the physics would be treated self-consistently because, STR velocity addition law ensures that once $v \to c$ w.r.t. a certain observer, it must be so w.r.t. any other observer. Thus, at best one may assume that $v \to c$ asymptotically without ever exceeding it. This would be possible only if $M = 0$ and proper time to approach the EH, $\tau \to \infty$. (Mitra 2000, Mitra 2002a,b). In technical parlance, existence of a finite mass BH would violate the condition that the worldline of the infalling particle must be timelike (Mitra 2000, 2000a,b, Leiter & Robertson 2003).

It may be recalled that astrophysicists claiming to study the problem of accretion around supposed finite mass BHs indeed (correctly) insist that one must have $v = c$ at the EH (Chakrabarti 1996, 2001) unmindful of the fact that this is not allowed in finite mass BH paradigm and many BH “experts” struggle hard to suppress this fact! In an ironical situation, such astrophysical works which find $v = c$ at the EH, in blatant violation of the BH paradigm claim to have found “evidences for astrophysical BHs”! But no “BH expert” would ever point out that such works are in violation of the BH paradigm and on the other hand claim that the BH paradigm has been confirmed by astrophysical studies!

9.2 Acceleration at the Event Horizon

It is also known for long that radial component of the 3-acceleration $a^R$ of the test particle blows up at the EH. This profound physical result, inconvenient
for the BH paradigm, has traditionally been brushed aside by mentioning that suitable coordinate transformations may remove this singular acceleration by ignoring the simple STR result that if acceleration is infinite in one frame, it is so in any other frame. Later it was pointed out that one can construct an acceleration scalar which too behaves in exactly the same singular way. This unequivocally showed the physically singular nature of the EH, which implies that the EH itself is the central singularity (i.e., \( M = 0 \)) (Mitra 2002a,b). However “BH experts” found such conclusions inconvenient and pretended to be unaware of it. Recently MacCallum (2006) has admitted that the acceleration scalar indeed blows up not only at the horizon of only Schwarzschild BH but for all BHs, for instance, spinning Kerr BH. However, in order to still uphold the BH paradigm, he has suggested new mathematical criterion for the definition of spacetime singularities completely ignoring that any such new rule would not change the basic fact that the physically measurable acceleration would blow up at the horizon in a coordinate independent manner. Recall, the original claim for supposed spacetime regularity at the EH was that “no singular unusual physics happens there”. Thus clearly this attempt by MacCallum to uphold the BH paradigm is unjustified and inconsistent from the point of view of the accepted notion about a “regular event horizon”. At this rate, one can claim that attainment of speed of light by a material particle is no violation of GR. In reality, at least for a material test particle attainment of infinite (scalar) acceleration and infinite Lorentz factor \((v = c, \gamma = \infty)\) are closely related phenomenon.

9.3 Non occurrence of Trapped Surfaces

The singularity theorems buttressing the BH and Naked Singularity paradigms are based on the assumption that trapped surfaces are formed in continued spherical GR collapse. But it was shown very transparently that trapped surfaces do not form in GR collapse (Mitra 2004a, 2006f):

\[
\frac{2GM(r)}{R} \leq 1
\]

(58)

where \( r \) is the comoving radial coordinate. In contrast, formation of trapped surfaces demand \( 2GM(r)/R > 1 \) and the equality sign denotes formation of an “apparent horizon”. Eq.(58) shows that under the condition of positivity of mass, one must have \( M \to 0 \) as the singularity would be approached \( R \to 0 \). If one would have \( 2GM/R < 1 \) at the singular state, there would be no horizon, and \( M \) should wander towards the negative branch. Thus one must approach the equality limit of a zero mass BH: \( 2GM/R \to 1 \) as \( R \to 0 \). However since the worldlines of the fluid must always be timelike, this limit which corresponds to a null condition must not ever be attained. In other words, one
must have \( \tau \to \infty \) as \( R \to 0 \). We saw this categorically for the dust collapse.

The physical reason for non-occurrence of trapped surfaces is again “nothing can move faster than light” because it was shown that occurrence of a trapped surface would mean the local 3-speed of the collapsing fluid would exceed the speed of light \( v > c \) (Mitra 2004a, 2005e). In fact, at the “apparent horizon”, one should find the acceleration of the fluid to blow up. Therefore, an apparent horizon cum EH may only asymptotically form as \( R \to 0 \) and \( M \to 0 \). Unfortunately the BH and singularity “experts”, having already made too much commitment and investment in the BH/singularity paradigm, found both the exact derivation of the theorem of non-occurrence of trapped surfaces and its physical interpretation to be most inconvenient and chose to quietly ignore them. Since an EH is never formed, there is no trapping of any quantum information within any EH. Thus actually, there was never any Hawking radiation or Quantum Information paradox. And obviously there need not be any resolution of this paradox either because, in reality, it was never there because BH mass \( M \equiv 0 \) (Mitra 2006f).

9.4 Ultimate Result

By using the basic rule of differential geometry and curvilinear coordinate transformation that the \textit{proper 4-volume} for a Schwarzschild BH must remain same in all coordinates, it has been directly shown that the Schwarzschild BHs have the unique mass \( M \equiv 0 \) (Mitra 2005, 2006d,e,f). It has also been shown that, the rotating Kerr BH has the same fate (Mitra 2004b,c). Thus one can physically understand the result of McCllum (2006) that acceleration scalar blows up at the horizon in the latter case too.

But independent of the above result, following the field theoretic analysis of Arnowitt, Deser and Misner (1962), one can find that \( M = 0 \) for a neutral BH. The vacuum Schwarzschild solution describes the spacetime structure of a neutral \textit{point mass}. And the “clothed mass” of a sufficiently small static neutral sphere of radius \( R_0 \) is:

\[
M = G^{-1} \left( -R_0 + [R_0^2 + 2M_b R_0 G]^{1/2} \right) \tag{59}
\]

where \( M_b \) is the “bare mass”. When \( R_0 > 0 \), of course, \( M > 0 \). But as \( R_0 \to 0 \) to become a “point mass”, \( M \to 0 \) due to negative self-gravity in exact accordance with our results.
A Schwarzschild BH is just a *point mass* and its all vacuum outside it even if we momentarily admit of the (incorrect) possibility \( M > 0 \). In pure classical vacuum, electromagnetic waves can of course propagate. In such a case, one obtains a “vacuum impedance” of \( I \sim 377 \text{ Ohms} \). But this “impedance” is no measure of vacuum resistance \( R \neq \frac{1}{I} \) and the latter is always \( \infty \) in the absence of any free charge. Thus while electromagnetic waves can propagate in a perfect classical vacuum, *no current can pass through* the same. One may ask then how does there is an electric discharge between two parallel *conducting* plates if sufficiently high external electric field is applied between them. This is so because a sufficiently strong electric field would *pulls out free electrons, ions* from the *conducting plate*. It is these released free electrons which modify the vacuum and become the carrier of the current. However not a single electron is added to the discharge by the original vacuum.

If on the other hand, one would replace the *conducting* electrodes by two *perfect insulator*, there will be no vacuum current howsoever strong the applied field may be because \( R = \infty \). Further, in the previous case, even when the plates are physical conductors with abundant source of free charges, there will be no current if the *intervening region would be trapped from which nothing can escape*. In such a case, there can be only inward flow of electrons inside the *trapped region* and there cannot be any outward flow of charge by the very definition of Event Horizon and *trapped surfaces*. This inward discharge would continue until the entire source of free electrons gets depleted. Therefore there cannot be any steady or even momentary current outflow from a BH.

Again going back to the previous example, even if there would be no trapped region between the conducting electrodes and the free electrons of the *electrodes and not of the vacuum* would constitute a steady current, there would be no electromagnetic coupling of the intervening media (vacuum) with the electrodes. Thus there is no question of energy extraction from a neutral non-conducting “point particle” sitting at the centre of the vacuum. On the other hand, such a coupling would be established only when the intervening medium is indeed a medium with either an intrinsic magnetic field or an induced magnetic field due to some unipolar induction mechanism. The latter would require that the intervening medium is a spinning conductor and then, indeed, there can be a real electromagnetic coupling between a conducting Neutron Star or any star with its accretion disk.

Only quantum electrodynamical processes may lend a pure vacuum some electrodynamic properties as it happens for the Cashimir effect between two close by glass plates. And this happens without the aid of external electric or magnetic field. A horizon can only suppress all electro dynamical properties by
screening the central matter/plasma.

- However, the astrophysical BH paradigm was built by blurring the physics and GR by means of wishful artifacts like imagining the effective existence of a physical conducting “membrane” having finite redshift in place of an EH, an imaginary 2-sphere without any physical surface or conducting properties and having infinite redshift. This was done by “stretching” the EH to suppress the physically singular behavior of the EH (Thorne, Price & Macdonald 1986). In fact, these authors admitted that

(i) “the velocity with which these FFOs see the FIDO ..becomes the velocity of light at the horizon” (pp. 22)

without realizing that this is a singular behavior.

(ii) “A FIFO at the horizon measures a divergently large gravitational attraction $g$... but he normalizes $g$ to a per-unit-universal-time basis, he obtains a finite acceleration” (pp. 97)

This is nothing but distortion of physics by dividing one $\infty$ by another $\infty$ to pretend that physics is regular at the horizon.

(iii) “The mental deceit of stretching the horizon is made mathematically viable, indeed very attractive, by the elegant set of membrane-like boundary conditions to which it leads at the stretched horizon” (pp. 46)

The phrase mental deceit says it all. In one of the crucial foundational papers of this paradigm where it is claimed that a spinning BH immersed in an external electromagnetic field would develop “eddy currents”, Damour (1978) admits that

“From a phenomenological point of view it is convenient to introduce a surface charge density and a current on the horizon. The heuristic justification” is

“The therefore if we wish to keep the charge and current conserved...”

Essentially, it is presumed beforehand that a BH, as the supposed central engine of quasar must electromagnetically interact with magnetic field of the accretion disk, the way a truly magnetized pulsar would do. Having made this presumption, the rest of the scholarly theory is built based on “phenomenological” and “heuristic” means. In the same vain, if one would demand that “there must be a conserved current threading any perfectly insulating sphere”, one would obtain a “surface current”, a “surface charge” and all other attributed properties of a neutral BH. And one would eventually obtain the magnetic spin down luminosity of any spinning non-magnetized insulator sphere! Further, subsequent researchers might think that the novel concepts like “surface
charge” and “surface current” for a perfect insulator were derived from the first principles.

Later since most authors would require a framework to explain astrophysical phenomenon, they would use such “results” ever delving deep into their roots. In the absence of alternative physical theories, the authors will not have any option either. But now that we know that $M = 0$ for Schwarzschild BHs, it may be realized that this approach was incorrect even if one would naively accept that $\mathbb{R} = 377$ Ohm when for a BH or any pure vacuum, actually, $\mathbb{R} = \infty$.

In contrast a MECO as a hot ball of magnetized plasma and without a horizon would be the ideal central engine for most of the high energy processes in astrophysics.

11 Conclusions

Since the basic cause of energy liberation in astrophysics is self-gravitation, a star or any other self-gravitating body, upon exhaustion of thermonuclear or any other specific external source of energy, cannot be completely devoid of supply of energy. Further, as first correctly shown by Eddington, in the ultimate analysis, the reservoir of energy of a star or any self-gravitating object is its total mass energy. When one applies only special theory of relativity, it appears that the minimum time scale for depletion of this reservoir is a finite number determined only by fundamental constants. This time scale may aptly be called “Einstein- Eddington” time scale. However once we consider GR, in principle, even the minimum value of Einstein - Eddington time scale could be infinite. We found that it is this Einstein -Eddington time scale rather than the pressureless free fall time scale (whose comoving value remains same in both Newtonian and GR cases) is a measure of the contraction time scale of self-gravitating bodies. The extremely large value of the radiative time scale (see Eq.[37]) corresponding to a measureable luminosity has got important physical significance.

Recently, it has been shown that during the final stages of BH formation ($z \gg 1$), the collapsing object would be dominated by radiation energy rather than by rest mass energy density (Mitra 2006a). For such a self-gravitating ball of radiation, the luminosity would indeed be maximal, i.e., $L \to L_{ed}$ or $\alpha \approx 1$. Consequently, the observed duration of such final stages would indeed be determined by Einstein -Eddington time scale as obtained in this paper (Mitra 2006b, Leiter & Robertson 2003, Mitra & Glendenning 2006). The fundamental reason that both the observed Einstein Eddington time scale as well as the comoving proper time scale for formation of the eventual zero mass
BH is infinite is that trapped surfaces are not allowed in collapse of isolated bodies and, at least for isolated bodies, GR is a singularity-free theory even at the classical non-quantum level as cherished by its founder Einstein. The physical reason for the non-occurrence of both trapped surfaces and finite mass BHs is the same relativistic adage “nothing can move faster than light”. One cannot but recall at this juncture that Einstein (1939) too attempted to disprove the existence of BHs by using the same adage. However he failed to properly recognize that the vacuum Schwarzschild (actually Hilbert) solution indeed suggests formation of unique zero mass BHs. On the other hand, since he tried to be aloof towards both the implications of this important solution and also the exact OS solution, his attempted disapproval of BH looked inconsistent and suspicious. Many BH/singularity “experts” of present epoch however take unkind advantage of this situation and often try to portray Einstein as a scientist who lacked sufficient appreciation of GR (Baez & Hillman 2000):

“In 1939, Einstein publishes a paper which presents a rather desperate (and entirely incorrect) argument that no body could collapse past its Schwarzschild radius. The nature of the conceptual errors in this paper show that Einstein still did not understand either the distinction between a coordinate singularity (the boundary of a coordinate chart) and a geometric singularity, nor the distinction between local and global structure. (Indeed, there is no evidence that Einstein ever understood correctly the geometry of all exact solutions to his field equations).”

However the present paper and other relevant papers have shown that Einstein was actually correct contrary to the presently accepted view. And we are certain that with the development of astronomical observational techniques, probably in next 10-20 years, it would be recognized by all that

- Einstein’s physical intuition about non-existence of (finite mass) BHs was correct though he could not see (zero mass) BHs as the asymptotical solutions of physical continued gravitational collapse of a chargeless fluid. However, with regard, to a point particle possessing a charge, Einstein & Rosen (1935) wrote that

“It also turns out that for the removal of the singularity it is not necessary to take the ponderable mass $m$ positive. In fact, as we shall show immediately, there exists a solution free from singularities for which the mass constant $m$ vanishes. Because we believe that these massless solutions are the physically important ones we will consider here the case $m = 0$” (emphasis is due to the author).

- Most of the present day BH/Singularity “experts” and many of the GR experts having, in some cases, more mathematical/numerical skill than Einstein
were actually experts on either Differential Geometry, or Applied Mathematics relevant for GR studies or Numerical Computations riding on GR and not necessarily on the intricate and subtle physics lying at the throbbing heart of GR.

Also it would be recognized that such experts sustained the BH paradigm by ignoring/avoiding consideration of physically measurable 3-speed and acceleration at EH or apparent horizons and by blurring the physics/thermodynamics/radiation transport aspects in the gravitational collapse problem.

- Despite Eddington’s unjustified public denouncement of Chandrasekhar’s correct result on upper limit of cold self-gravitating objects, Eddington’s physical intuition and insight were far superior to that of Chandrasekhar; he was the first to correctly visualize the unphysical Nature of (finite mass) BHs and insisted that

“I think there should be a law of nature to prevent a star from behaving in this absurd way”

And as emphasized by Mitra (2006b) and Mitra & Glendenning (2006), this “law of Nature” is nothing but the bending of radiation due to strong self-gravity and consequent attainment of a critical Eddington luminosity.

Of course, at that time, Eddington too failed to recognize that the gravitational contraction process must be radiative and a BH (with \( M = 0 \)) should indeed be the asymptotical solution of the continued collapse process. It would be recognized much later that Chandrasekhar’s result about upper limit of cold objects was almost universally misinterpreted, most notably by Chandrasekhar himself, as an upper limit on mass of all compact objects, hot or cold. Thus it would be recognized that Chandrasekhar’s discovery had a profound retrograde effect on the development of the physical theory of continued gravitational collapse and relativistic astrophysics in general. Probably this misinterpretation along with the misinterpretation that the OS collapse was physical and suggested formation of finite mass BHs (when in reality, there is no collapse without finite pressure and heat flux, or, mathematically, \( M = 0 \) in such a case), put the clock back by 60 years as far as the question of the final state of continued collapse is concerned.

It may be also recalled that the original idea of “Relativistic Degeneracy” was due to Anderson (1929); the original (crude) calculations about the upper mass limit of a (cold) White Dwarf was due to Stoner (1930); and the basic idea that White Dwarfs are supported by (cold) quantum degeneracy pressure was due to Fowler (1926). And Chandrasekhar jelled together such ideas with considerable mathematical rigor within the framework of Newtonian gravity.

Finally, if a body is not undergoing continued collapse its lifetime against
collapse is infinite. And if the body of arbitrary mass would be undergoing continued collapse because of sufficient density, its collapse time scale would be determined by Einstein-Eddington time scale rather than by any free fall time scale. During the course of the collapse the object must become a hot throbbing dynamic ECO and continue to collapse eternally by avoiding the static cold BH stage as per the correct intuition of Einstein and Eddington.

Already there are tentative evidences that the stellar mass BH candidates have strong intrinsic magnetic field in lieu of an EH (Robertson & Leiter 2002, 2003, 2004, 2006a) as was predicted earlier (Mitra 1998, 2000a,b, 2002). There is also an direct evidence that the central compact object of one of the most well studied quasar Q0957+561 is a strongly magnetized ECO rather than a BH (Schild, Leiter and Robertson 2006). Further, most of the observations of the Sgr A*, the BH Candidate at the center of the milkyway may be explained by considering it a strongly magnetized ECO instead of a BH (Robertson & Leiter 2006b).

11.1 Epilogue

General Relativistic continued collapse is an eternal story of all objects trying to “burn” away their complete stock of mass due to the grip of self-gravity in ultimate realization of the \( E = Mc^2 \) formula. As they “burn mass” they eventually become hot ECOs/MECOs having neither any lower (except zero) nor any upper mass limit.

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References

[1] Anderson, W., Zs.f. Phys., 54, 433 (1929)

[2] Arnowitt, R., Deser, S. & Misner, C.W., 1962, “The Dynamics of General Relativity” in *Gravitation: An Introduction to Current Research* (ed. L. Witten, Wiley, NY), (gr-qc/0405109)
[3] Baez, J. & Hillman, C, 2000, see http://math.ucr.edu/home/baez/RelWWW/history.html

[4] Baumgarte, T.W. & Shapiro, S.L., 2003, ApJ, 585, 930

[5] Bethe, H. A., 1939, Phys. Rev., 55, 434

[6] Bojowald, M., Goswami, R., Maartens, R. and Singh, P., 2005, PRL, 95, 091302

[7] Bowers, R.L. & Deeming, T., 1984, Astrophysics I, Stars, (Jones and Barlett, Boston)

[8] Buchdahl, H.A., 1959, Phys. Rev., 15, 1027

[9] Chakrabarti, S.K., 1996, MNRAS, 283, 325

[10] Chakrabarti, S.K., 2001 (Private Communication)

[11] Chandrasekhar, S., 1967 An Introduction to the Study of Stellar Structure (Dover, New York, 1967)

[12] Cuesta, H.J.M., Salim, J.M. and Santos, N.O., 2005, Paper Presented in 100 Years of Relativity, Sao Paulo, Brazil, see http://www.biblioteca.cbpf.br/apub/nf/NF-2005.html

[13] Damour, T., 1978, PRD, 18(10), 3598

[14] Eddington, A., 1920, Brit. Assoc. Repts., 45

[15] Eddington, A., 1926, The Internal Constitution of Stars, (Cambridge Univ. Press, Cambridge, 1926)

[16] Ehlers, J., Oszvath, I. & Schuking, E.L., 2006, Am.J. Phys., 74(7), 607

[17] Einstein, A. & Rosen, N., 1935, Phys. Rev., 48, 73

[18] Einstein, A., 1939, Ann. Math., 40, 922

[19] Fowler, 1926, R.H., MNRAS, 87, 114

[20] Goswami, R., Joshi, P.S. and Singh, P., 2006, PRL, 96, 031302

[21] Herrera, L. & Santos, N.O., 2004, PRD, 70, 084004 (gr-qc/0410014)

[22] Herrera, L., Di Prisco, A. and Barreto, W., 2006, PRD, 73, 024008 (gr-qc/0512032)

[23] Joshi, P.S., 2004, preprint (gr-qc/0412082)

[24] Joshi, P.S. and Goswami, R., 2005, preprint (gr-qc/0504019)

[25] Kippenhahn, A. & Weigert, A., 1990, Stellar Structure and Evolution, (Springer, Berlin)

[26] Kumar, S.S., 1962, AJ, 67, 579
[27] Kumar, S.S., 1963a, ApJ, 137, 1121
[28] Kumar, S.S., 1963b, ApJ, 137, 1126
[29] Leiter, D.J., Mitra, A. & Robertson, S.L., 2001 (astro-ph/0111421) (the original version)
[30] Leiter, D.J. & Robertson, S.L., 2003, Found. Phys. Lett., 16, 143
[31] McCallum, M.A.H., 2006, Preprint (gr-qc/0608033)
[32] Misner, C.W. et al., 1972, PRL, 28, 998
[33] Misner, C. W., Thorne, K.S., and Wheeler, J.A., 1972, Gravitation (W.H. Freeman, New York, 1973)
[34] Mitra, A. 1998a, Preprint (astro-ph/9811402)
[35] Mitra, A., 1998b, Preprint (astro-ph/9803014)
[36] Mitra, A., 2000, Found. Phys. Lett., 13, 543, (astro-ph/9910408)
[37] Mitra, A., 2002a, Found. Phys. Lett., 15, 439, (astro-ph/0207056)
[38] Mitra, A., 2002b, Bull. Astron. Soc. India, 30, 173 (astro-ph/0205261) (Conf. Proc., Invited Talk)
[39] Mitra, A., 2004a, Preprint, (astro-ph/0408323)
[40] Mitra, A., 2004b, Preprint, (astro-ph/0407501)
[41] Mitra, A. 2004c, Preprint, (astro-ph/0409049)
[42] Mitra, A., 2005, Preprint (physics/0504076)
[43] Mitra, A., 2006a, MNRAS Lett., 367, L66 (gr-qc/0601025)
[44] Mitra, A., 2006b, MNRAS, 369, 492, gr-qc/0603055
[45] Mitra, A., 2006c, Phys. Rev., D74, 0605066, gr-qc/0605066
[46] Mitra, A., 2006d, Advances in Space Research (in press), (astro-ph/0510162)
[47] Mitra, A., 2006e, Oral Presentation, 11th Marcel Grossmann Meeting on General Relativity (Berlin)
[48] Mitra, A., 2006f, “Black Holes or Eternally Collapsing Objects: A Review of 90 Years of Misconceptions” in Focus on Black Hole Research, (ed. P.V. Kreitler, Nova Sc. Publishers, NY)
[49] Mitra, A., 2006g, Proc. 29th Int. Cos. Ray Conf., Vol. 13, p.125 (physics/0506183)
[50] Mitra, A., 2006h, Proc. 29th Int. Cos. Ray Conf., Vol. 4, p.187 (astro-ph/0507697)
[51] Mitra, A. & Glendenning, N.K., 2006, Oral Presentation, 11th Marcel Grossmann Meeting On General Relativity (Berlin, 2006)

[52] Oppenheimer, J.R. & Snyder, H., 1939, Phys. Rev., 56, 455

[53] Peters, P.C., 1970, PRD, 1(6), 1559

[54] Robertson, S. & Leiter, D., 2002, ApJ, 565, 447

[55] Robertson, S. & Leiter, D., 2003, ApJL, 596, L203

[56] Robertson, S. & Leiter, D., 2004, MNRAS, 350, 1391

[57] Robertson, S. & Leiter, D., 2006a, “The Magnetospheric Eternally Collapsing Object (MECO) Model of Galactic Black Hole Candidates and Active Galactic Nuclei” in New Directions in Black Hole Research, ed. P.V. Kretler (Nova Sc. Publishers, NY, 2005)

[58] Robertson, S. & Leiter, D., 2006b, astro-ph/0603746

[59] Schild, R.E., Leiter, D.J., and Robertson, S.L., 2006, AJ, 132, 420, astro-ph/0505518

[60] Shapiro, S.L., 2004, ApJ, 610, 913

[61] Shapiro, S.L. & Shibata, M., 2002, ApJ, 577, 904

[62] Stoner, E.C., 1930, Phil. Mag., 9, 944

[63] Thorne, K.S., Price, R.H., and, Macdonald, D.A., 1986, Black Holes: The Membrane Paradigm, (Yale University Press, New Haven)

[64] Vaidya, P.C., 1951, Proc. Ind. Acad. Sc., A33, 264