Nonlinear combining of laser beams

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We propose to combine multiple laser beams into a single diffraction-limited beam by the beam self-focusing (collapse) in the Kerr medium. The beams with the total power above critical are first combined in the near field and then propagated in the optical fiber/waveguide with the Kerr nonlinearity. Random fluctuations during propagation eventually trigger strong self-focusing event and produce diffraction-limited beam carrying the critical power.

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The dramatic rise of the output power of fiber lasers in the last 25 years [1,2] resulted in reaching ∼ 10 kW in 2009 [3] for the diffraction-limited beam. Also 20-kW continuous-wave commercial fiber laser was announced in 2013 [4] although the beam quality is not yet specified. However, the growth of power since 2009 has been mostly stagnated because of the encountered mode instabilities [2,5]. The further increase of the total power of the diffraction-limited beam is possible through the coherent beam combining [1,6] where the phase of each laser beam is controlled to ideally produce the combined beam with the coherent phase. However, the beam combining has been successfully demonstrated only for several beams. E.g., Ref. [7] achieved the combining of five 500W laser beam into 1.9kW Gaussian beam with a good beam quality $M^2 = 1.1$. Nonlinearity is expected to be the key issue for further scaling of the coherent beam combining [1].

Here we propose to use nonlinearity to our advantage to achieve combining of multiple laser beams into a diffraction-limited beam by the strong self-focusing in a waveguide with the Kerr nonlinearity. The number of laser beams can be arbitrary but we require that the total power to exceed the critical power of self-focusing. Our estimates below suggest that the commercially available fiber of ∼ 1mm diameter [4] might be a possible choice of the waveguide to achieve the diffraction limited beam with the power of several MWs.

We first consider a stationary self-focusing of the laser beam in the Kerr medium assuming for now that the pulse duration is long enough to neglect time-dependent effects. (We estimate the range of allowed pulse durations below.) The propagation of a quasi-monochromatic beam with a single polarization through the Kerr media is described by the nonlinear Schrödinger equation (NLSE) (see e.g. [8]):

$$i\partial_z \psi + \frac{1}{2k} \nabla^2 \psi + \frac{k n_2}{n_0} |\psi|^2 \psi = 0,$$

where the beam is directed along $z$-axis, $r \equiv (x, y)$ are the transverse coordinates, $\psi(r, z)$ is the envelope of the electric field, $\nabla \equiv \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right)$, $k = 2\pi n_0/\lambda_0$ is the wavenumber in media, $\lambda_0$ is the vacuum wavelength, $n_0$ is the linear index of refraction, and $n_2$ is the nonlinear Kerr index. The index of refraction is $n = n_0 + n_2 I$, where $I = |\psi|^2$ is the light intensity. In fused silica $n_0 = 1.4535$, $n_2 = 3.2 \cdot 10^{-16} \text{cm}^2/W$ for $\lambda_0 = 790$nm and $n_0 = 1.4496$, $n_2 = 2.46 \cdot 10^{-16} \text{cm}^2/W$ for $\lambda_0 = 1070$nm.

NLSE (1) is converted into the dimensionless form

$$i\partial_z \psi + \nabla^2 \psi + |\psi|^2 \psi = 0,$$

by the scaling transformation $(x, y) \rightarrow (x, y)w_0$, $z \rightarrow 2\pi w_0^2$ and $\psi \rightarrow \psi w_0^{1/2}/(2k^2 w_0^2 n_2)^{1/2}$, where $w_0$ is of the order of the waist of each combined laser beam.

NLSE (1) describes the catastrophic self-focusing (collapse) [9,10] of the laser beam provided the power $P$ exceeds the critical value

$$P_c = \frac{N_c \lambda_0^3}{8\pi^2 n_2 n_0} \approx 11.70 \frac{\lambda_0^3}{8\pi^2 n_2 n_0}.$$  

Here $N_c \equiv 2 \pi \int R^2 r dr = 11.7008965 \ldots$ is the critical power for NLSE (2) in dimensionless units and $R(r)$ is the radially symmetric Townes soliton [11] defined as the ground state soliton $\psi = e^{iR(r)}$ of NLSE with $-R + \nabla^2 R + R^3 = 0$, where $R \equiv |r|$. In fused silica $P_c \approx 2$MW for $\lambda_0 = 790$nm and $P_c \approx 4.7$MW for $\lambda_0 = 1070$nm.

Assume that $N$ laser beams are combined in the near field (side-by-side combining) at the entrance $z = 0$ to the optical waveguide (the optical fiber) as shown in Fig.

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Fig. 1. (Color online) A schematic of the nonlinear beam combining. An array of beams with non-correlated phases enters a nonlinear optical fiber at $z = 0$. Inside the fiber the laser field is randomized due to nonlinear interactions (see e.g., a schematic of the typical cross-section at $z = z_1$). A large fluctuation of that random field triggers a strong self-focusing event producing a nearly diffraction-limited hot spot at $z = z_2$ (shown by the long arrow) which carries the critical power $P_c$.

The waveguide can be either multimode optical fiber or any type of waveguide structure with the Kerr nonlinearity (e.g., it can be a capillary with the reflective internal walls, filled by a gas or a liquid with the dominated Kerr nonlinearity). We assume that the diameter of waveguide is large enough for the applicability of NLSE (2). The single polarization is ensured e.g., by the use of the polarization-maintaining optical fiber. We note that a generalization to a case of arbitrary polarization is possible but is beyond the scope of this Letter.

The properties of the waveguide in our simulations are taken into account through the boundary conditions in NLSE along $x$ and $y$. Example is the multimode optical fiber with the diameter in the range between hundreds of $\mu$m to several mm. At $z = 0$ we approximate each beam to have the Gaussian form with the plane wavefront so that the initial condition for NLSE (2) is $\psi\left(x, y, z = 0\right) = \sum_{n=1}^{N} \psi_n = A_n \exp\left(-\frac{(x-x_n)^2+(y-y_n)^2}{r_n^2}\right) + i\phi_n$, where $r_n$, $A_n$, $\phi_n$ and $(x_n, y_n)$ are the width, the amplitude, the phase, and the location of the center of the $n$th beam, respectively. In simulation we assume the same amplitudes $A = A_n$ and widths $r_n = r_0$ for all $N$ beams, but phases $\phi_n$ are randomly distributed at $[0, 2\pi]$. Randomness of phases $\phi_n$ reflects the randomness in environmental fluctuations and fiber amplifiers of lasers.

Fig. 2 shows the typical result of NLSE (2) simulation. We took the square array of $N = 10 \times 10$ beams at $z = 0$ uniformly located in the domain $0 < x < L$, $0 < y < L$, $L = 25.6$. Each beam had the radius $r_0 = 1.13$ and carried the power $0.1P_c$ (i.e., the total power is $10P_c$). A typical evolution of the system along $z$ is shown in Figure 2 for simulations with the periodic boundary conditions in $x$ and $y$.

The middle column of Fig. 2 ($z = z_2$) shows that the amplitudes and phases become random after propagation of the nonlinear distance $z_{nl} \equiv 1/\langle|\psi|^2\rangle$, where $\langle|\psi|^2\rangle = P/S$ is the spatial average of the light intensity in the cross-section area $S$ at $z = const$. For $z > z_{nl}$ the amplitude and phase experience fluctuations along $z$ (optical turbulence) until a large fluctuation at $z \simeq 15$ in...
beams (at \optical turbulence dominated by collapses \cite{14–17} that \transient propagation), the fluctuations of the intensity \langle |\psi|^2 \rangle dependence was found in Ref. \cite{13}. It means that the launching of \psi|^2 \psi - a_1|\psi|^4 = 0, \quad (5)

where 0 < a_1 \ll 1. This type of saturated nonlinearity was found e.g. in chalcogenide glasses with the negative fifth order nonlinearity \n = n_0 + n_2 I + n_4 I^2, n_4 < 0 \cite{18}. The dashed in Fig. 3 shows the z-dependence of the maximum amplitude \max |\psi| for the solution of (4) with a_1 = 10^{-3} and the same initial condition as for the solid curve of Fig. 3. It is seen that instead of the catastrophic collapse near \zsf = 20 as in NLSE (2), we observe the periodic oscillations with the maximum amplitude roughly estimated as \langle |\psi|^2 \rangle \simeq 1/a_1^{1/3}.

Another type of the collapse regularization is the multi-photon absorbtion described by the term \i \beta(K) |\psi|^{2K-2}\psi added to the left-hand side (l.h.s.) of NLSE (1). Here K is the number of photons absorbed by the electron in each elementary process (K-photon absorption) and \beta(K) is the multiphoton absorption coefficient. For fused silica with \lambda_0 = 790nm a dominated nonlinear absorption process for this wavelength is K = 5 with \beta(5) = 1.80 \times 10^{-5}cm^7W^{-4} \cite{5} which leads to the formation of plasma and optical damage. Thus the special measures must be taken to prevent the damage of the waveguide. The detailed discussion of that topic is outside the scope of this Letter and we only highlight below several possible ways to overcome that difficulty. First and perhaps simplest way would be to use the waveguide short enough to avoid a full development of the catastrophic collapse. Second possible choice is to use a waveguide filled with a gas and ultrashort pulses such that the multiphoton ionization produces plasma which results in the plasma defocusing and clamping of the collapsing filament. Such type of clamping has been demonstrated experimentally to allow a formation of filaments of up to several meters in length \cite{8} for the propagation of ultrashort pulses in air. The drawback of that approach is that it would allow beam combining to short pulses only limiting the total energy of the combined beam. Third option is to use chalcogenide glasses with the negative fifth order nonlinearity as described in Eq. (5) \cite{18}. Fourth choice is to use of the waveguide with the specially chosen transverse profile of \n_0(x,y) and \n_2(x,y) such that the collapse starts near the center of the waveguide because of the larger value of \n_0 there while the catastrophic collapse is stopped.
by the decrease of $n_2$ is that region [19]. Firth choice is nonlinearity management [20] when $n_2$ is periodically modulated along $z$ to prevent the collapse. Sixth choice is to form a ring cavity from the waveguide such that the length of the single round trip along cavity (i.e., along $z$) is not sufficient to achieve catastrophic collapse while the optical switching is used to remove from the cavity the nearly collapsed diffraction-limited beam. The power depletion from such removal can be compensated by the coupling of the cavity to the laser beams.

To estimate the parameters for a potential experimental realization of the nonlinear beam combining, we assume that the typical intensity from the combined beams in the waveguide is $I_0 = 10^9$ W/cm$^2$ which allows continuous-wave (cw) operation without optical damage [21]. Consider the case of $(z_s f) = 31.30$ for $10 \times 10$ combined beams as in Fig. 4. Using the parameters $n_0 = 1.4496$, and $n_2 = 2.46 \cdot 10^{-16}$ cm$^2$/W of fused silica at $\lambda_0 = 1070$ nm (correspond to the wavelength of the commercially available 50kW cw fiber laser [4]) we obtain in dimensional units the typical required length of the waveguide $l \sim (z_s f) = 4m$ and the waveguide thickness $\sim 2mm$ which is comparable with the commercially available fiber of the 1mm diameter [4]. Thus we estimate that the combining of several hundred beams from 50kW cw fiber laser [4] may allow to produce a nearly diffraction-limited combined beam with the power $\simeq P_c = 4.7MW$. We also note that the high beam quality is not required for each of the combing beams because the self-focusing collapse spontaneously produces the near diffraction-limited beam from the generic superpositions of combined beams.

For the pulsed operations, the optical damage threshold is higher than for cw which would allow to achieve nonlinear beam combining in a smaller settings. E.g., typical experimental measurements of the optical damage threshold in fused silica give the threshold intensity $I_{\text{thresh}} \sim 5 \cdot 10^{11}$ W/cm$^2$ for 8 ns pulses and $I_{\text{thresh}} \sim 1.5 \cdot 10^{12}$ W/cm$^2$ for 14ps pulses [22]. Thus the short pulse operations might allow to scale down the typical lengths $l$ in $z$ and the waveguide cross section in 2-3 orders of magnitude for the same optical power. However, for such short pulse durations, $t_0$, we generally might need to take into account a group velocity dispersion (GVD). Its contribution is described by the addition of the term $-\frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} \hat{\psi}$ into the left-hand side of equation (1). Here $\beta_2 = 3705$ fs$^2$/cm is the GVD coefficient for fused silica at $\lambda_0 = 790$ nm and $t$ is the retarded time $t = T - z/c$, where $T$ is the physical time and $c$ is the speed of light. At fiber lengths in several meters, the linear absorption of optical grade fused silica is still negligible. The GVD distance $z_{\text{GVD}} \equiv 2\beta_2/\beta_2$ must exceed $l$ for NLSE applicability, which gives $t_0 \gtrsim 0.3ps$ for $l = 4m$.

Another possible effects beyond NLSE include a stimulated Brillouin scattering (can be neglected for the pulse duration $\lesssim 10$ ns [23] or, similar, if the linewidth of the lasers is made large enough) and a stimulated Raman scattering (SRS). The threshold of SRS for a long pulse in fused silica was estimated from a gain exponent $g_{\text{fsl}} \approx 16$, where $g \approx 10^{-11}$ cm/W is the Raman gain constant [23]. This estimate was obtained assuming that the spontaneous emission is amplified by SRS (with the amplification factor $e^{\lambda I_0} = e^{16}$) up to the level of the average light intensity $I_0$ in the waveguide. Taking $l = 4m$ and $I_0 = 10^9$ W/cm$^2$ we obtain the gain exponent $g_{\text{fsl}} \approx 4 \approx 16$, i.e., we still operate well below the SRS threshold and can neglect SRS. This SRS threshold estimate is true for relatively long pulses $\gtrsim 10$ ps [24]. For pulses of shorter duration, SRS is additionally suppressed because the laser beam and the SRS wave move with different group velocities.

In conclusion, we demonstrated the possibility to achieve a nonlinear beam combining by propagating multiple laser beams in the waveguide with the Kerr nonlinearity. Large fluctuations during propagation seed the collapse event resulting in the formation of near diffraction-limited beam.

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