LiDAR-IMU-UWB-Based Collaborative Localization

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Abstract: This article introduced a positioning system composed of different sensors, such as LiDAR, IMU, and ultra-wideband (UWB), for the positioning method in autonomous driving technology under closed coal mine tunnels. First, we processed the LiDAR data, extracted its feature points and merged the extracted feature point clouds to generate a skewed combined feature point cloud. Then, we used the skew combined feature point clouds for feature matching, performed pre-integration processing on the IMU sensor data, and completed the LiDAR-IMU odometer with the LiDAR. Finally, we added UWB data to IMU pose node as a one-dimensional over-edge constraint. By updating the sliding window, the positioning accuracy was further improved. Moreover, we have conducted experiments to verify the proposed positioning system in a simulated roadway. The experimental results showed that the method proposed in this paper is superior to the single LiDAR method and the single UWB method in terms of positioning accuracy.

Keywords: feature extraction; feature matching; pose constrain; localization

1. Introduction

In autonomous driving technology, precise positioning is the basis for ensuring the stability and reliability of the entire system. In the research of positioning technology, multi-sensor fusion has become the best solution to improve the positioning accuracy. The positioning scheme in the urban environment mainly uses visual sensors, IMU, and GPS for fusion positioning, and this scheme can show a good positioning effect. For indoor environments, GPS cannot show good performance, and it is a very good choice to use ultra-wideband (UWB) to replace it [1]. Visual sensors have relatively high requirements in the environment. In order to adapt to the environment more widely, using LiDAR [2] as the main perception sensor is a very popular approach. In tunnels with a harsher environment and fewer feature points, the combination of LiDAR and UWB appears to be more effective [3,4], which has become the main solution to the positioning problem in coal mine tunnels.

In coal mine tunnels, visual sensors are basically not used, mainly because the visible distance is short due to poor light in the environment, and LiDAR can make up for the lack of visual sensors. However, LiDAR sensors are prone to scene degradation in a single-environment roadway, so adding IMU and UWB information can compensate to a certain extent. Most scholars’ studies are related to UWB information, so that it can quickly and accurately respond to location information to reduce errors. In this paper, we focus on processing the laser point cloud and extract features from the point cloud, so that the originally sparse feature point cloud can be better aligned according to the plane features by skewing the point cloud. Then the anchor position and ranging bias in UWB are estimated to update the pose through the bundle adjustment (BA) thread.

In this paper, in order to improve the localization accuracy in the roadway environment, we propose an adaptive estimation method (LIU) based on lidar-IMU odometry and UWB ranging technology. First, we establish a sliding window to estimate the state
of the robot to obtain UWB-related parameters and use BA to update the pose estimation. Second, the data obtained by the sensor is processed, and its point cloud features are obtained by processing the lidar point cloud data. The feature point cloud is extracted and merged to generate a feature skewed point cloud combination, the error generated by the motion state is compensated by the IMU propagation state, and the lidar odometer is established. Third, the road plane in the roadway is mainly segmented, the edge information is fully utilized, and the iterative closest point (ICP) algorithm is used for registration. Finally, the robot’s pose is constrained by the processed UWB information to obtain the accurate positioning of the robot.

The main contributions of this paper are as follows:

1. After processing the lidar point cloud to obtain the feature point cloud, we use the IMU propagation state to compensate the motion distortion generated by it, and finally use the ICP algorithm to perform registration according to the edge feature information. This method can reduce the error caused by the sparse feature points in the coal mine roadway to the greatest extent and improve the point cloud registration effect.

2. Perform average error processing on UWB data to reduce its measurement error, connect it to the IMU pose node as a univariate hyperedge constraint and obtain more accurate positioning results by updating the sliding window.

The balance of the article is organized as follows: Section 2 introduces the relevant work, Section 3 expounds the methods, Section 4 sets out the experimental results, and Section 5 summarizes and analyzes the article.

2. Related Work

In research of related positioning algorithms, the first sensor applied is the GPS positioning system. Because it has the earliest positioning, it is also a concern for most researchers. The position error of GPS positioning information in our daily life is roughly between a few meters to tens of meters [5] according to its signal strength factor, which can meet the basic positioning requirements. However, if it is applied to the automatic driving environment, the required accuracy is about 30 cm. Therefore, in the extension of its basic GPS scheme, the differential GPS (DGPS) scheme is proposed in [6]. Specifically, the position error is eliminated according to the position relationship between the fixed GPS unit and the mobile GPS unit. Finally, under this scheme, the positioning accuracy is 1–2 m. In [7], the authors put forward the solution of auxiliary GPS (AGPs), which mainly improves the speed of information acquisition and enhances the signal strength, but its positioning accuracy is slightly lower than DGPS. In [8], the authors put forward a low-cost RTK-GPS positioning solution, which can still have good robustness and positioning accuracy under high-speed road conditions. Although these methods have improved the positioning accuracy compared with GPS solutions, their reliability is still a very serious problem. When we apply them to urban roads or closed scenes, the accuracy of GPS related positioning schemes will face particular challenges. Therefore, in [9], the authors add an IMU sensor to the GPS solution to compensate the GPS signal. This method effectively improves the reliability of the system, but the positioning accuracy is still not ideal. In [10], the authors use two extended Kalman filters to process GPS and IMU signals, which significantly improves positioning accuracy. Some authors directly use visual sensors to replace GPS in order to avoid GPS signal loss. In [11], the authors use a single camera for positioning, mainly through image features for matching positioning, and the positioning accuracy reaches 75 cm. However, the problem is that visual sensors generally have high requirements for the environment (such as light, dust, etc.), so although a single visual sensor improves the positioning accuracy, there is a lack of system stability. In order to improve the stability of the whole system, GPS/IMU positioning information is added to the visual information in [12], and the lane line is recognized through the visual sensor to complete the lateral positioning. The positioning error achieved by the authors through
this method is 0.73 m. In [13], the authors extend the method in [12] to mark roads and lanes, make up for the lack of longitudinal positioning, and further improve their positioning accuracy. In [14], the authors add aerial images on the basis of visual sensor, GPS, and IMU and put forward two positioning methods. The first method is to extract the road features according to the aerial image, and then register the information in the visual sensor with the aerial feature information. The second method is to eliminate the information irrelevant to road features through aerial images, and only retain the key information for feature matching. The position accuracy obtained by the authors in the first method is less than one meter, and the maximum error peaks of the second method are 3.5 m and 7 m. The position accuracy of the two methods in this paper is quite different, so their stability still needs to be improved. In other similar articles [15,16], the authors have also obtained good experimental accuracy by using a visual sensor combined with other sensor information. However, the application effect of a visual sensor is very poor in the coal mine roadway, so the method based on a visual sensor is not used for positioning in this paper.

In the selection of sensors, we note that LiDAR sensors are not vulnerable to the environment, so we focus on LiDAR related methods. In [17], the authors identify and mark the road by using a LiDAR sensor. After adding an RTK-GPS and IMU sensor, the positioning error obtained by this method in an urban environment is 30 cm. In [18], the authors added a LiDAR sensor, GPS, IMU, and wheel odometer to the positioning system. The static map information obtained by the LiDAR sensor and other sensor information is mainly used for fusion positioning. The final positioning error is less than 10 cm, but this method has poor adaptability to the environment. In [19], the dynamic adaptability of the system to the surrounding environment is enhanced. The authors use the LDM method to identify the dynamic object and construct the map according to its intensity value. Finally, the position error of 9 cm is obtained. In the above articles, LiDAR sensors are used to obtain good positioning accuracy, but a GPS signal is added in its information fusion. In [20], the authors’ research shows that the GPS signal will stop working from time to time in the tunnel environment, the accuracy will decrease, and the system stability will be affected. The working scene of this paper is even deeper underground, and the GPS signal cannot reach the mine environment, which makes it unable to work. Therefore, this article uses the UWB positioning unit to replace the GPS information to compensate the lidar odometer information. Next, the method used in this article will be explained in detail.

3. Method
3.1. State Estimation

(1) Sliding window: we set the time step to $t_k$. Therefore, the pose of the robot in this time state $x_k$ is defined by the following formula:

$$x_k = (q_k, p_k, V_k, b_k^a, b_k^g)$$  \hspace{1cm} (1)

Among them, $q_k \in \mathbb{Q}$, $p_k \in \mathbb{R}^3$, $V_k \in \mathbb{R}^3$ are the position and velocity of the robot in the w.r.t coordinate system in the local frame $L$ at the quaternion direction and time $t_k$, respectively. Furthermore, $b_k^a$ and $b_k^g \in \mathbb{R}^3$ are the deviations of IMU accelerometer and gyroscope, respectively, and both are applied to the robot framework.

Therefore, the sliding window contains the $M$ state of the last time. We define the state estimation and sliding window size $M$ at each time step as follows:

$$\hat{X}_k = (\hat{q}_k, \hat{p}_k, \hat{v}_k, \hat{b}_k^a, \hat{b}_k^g)$$ \hspace{1cm} (2)

$$\hat{X} = (\hat{X}_w, \hat{X}_{w+1}, ..., \hat{X}_k), w \leq k - M + 1$$ \hspace{1cm} (3)

Moreover, we also try to estimate the inverse depth in the tracking feature of the sliding window. The state estimation is defined as follows:

$$\hat{\lambda}^1, \hat{\lambda}^2, ..., \hat{\lambda}^M$$ \hspace{1cm} (4)

Among them, \(N_k^S\) refers to all the feature points tracked by the sliding window at time \(t_k\).

It should be noted that the symbols \(w\) and \(k\) indicate the first and last time steps in the sliding window. Moreover, we do not consider the errors caused by the external factors of the LiDAR. These can be eliminated by some method, for example, linear interpolation or using IMU odometer information to compensate lidar information.

(2) Global attitude map and UWB parameters: in the process of sliding window optimization, we use marginalized information to construct a global pose map. We define the pose estimation of each key frame \(i\) as \(L_i\). When a cyclic factor or a new key frame appears, the pose estimation is updated in the BA process.

After estimating the pose of the key frame, we also estimate the UWB related parameters:

\[
\psi R = (\psi R, \psi p), \psi R \in SO(3), \psi p \in \mathbb{R}^3; b^r \in \mathbb{R}
\]  

In the above formula, \(\psi R\) represents the transformation between world coordinates and local coordinates. Furthermore, \(b^r\) represents the distance measurement deviation between the machine and the different anchor points of UWB. We understand \(\psi R\) and \(b^r\) as external and internal parameters in the UWB measurement process.

3.2. Data Processing

(1) LiDAR: we need to extract the feature point cloud from the obtained LiDAR point cloud. Then these feature point clouds are synchronized on the time coordinate system, and all the point cloud features are merged to produce a skew combined feature point cloud (SCFC). We denote it as \(\mathcal{F}_m\). For the characteristic skew point cloud combination \(B_{\tau_m}\), which produces motion distortion in SCFC, we use the IMU propagation state to compensate it and then mark it as a combined feature cloud (CFC).

We use the state estimation of the IMU propagation state at \(t_k\) to estimate the obtained CFC \(B_{\tau_k}\) and then define it as \(\bar{x}_k\) and convert it into a local frame. Our previous \(M-1\) CFCs, for example: \(B_{\tau_m}\), \(m = \omega, \omega + 1, \ldots, k - 1\), use the state estimate \(T_m\) obtained by local optimization to convert it into \(L\). The CFCs referenced by \(L\) in \(M\) are matched with the collective local map \(\mathcal{M}\) formed by key frames, and the key frames in this set are selected by searching the KNN of \(\bar{x}_k\). The final feature-to-map matching (FMM) process is a set of coefficients. We define it as \(\{\mathcal{L}_m \leftarrow (f^i, n^i, c^i)\}\), where the amount of time is related to the following data:

\[
r_L(\bar{x}_m, \mathcal{L}_m) = (n^i)^T [\tilde{R}_m f^i + \tilde{p}_m] + c^i
\]

In order to solve the time-consuming problem of the FMM processing process, we can refer to \([21]\) to solve the problem by using parallel threads. This method can effectively save calculation time.

(2) IMU: the pre-integrated observation equation \(\tilde{\mathcal{H}}_m \triangleq (\tilde{a}_m, \tilde{\omega}_m, \tilde{y}_m)\) coupled under the continuous state \(X_{m-1}\) and \(X_m\) is established in the IMU data within the time \([t_{m-1}, t_m]\). Among them, \(\tilde{a}_m, \tilde{\omega}_m, \tilde{y}_m\) are expressed as follows:

\[
\begin{align*}
\tilde{y}_m & \triangleq \int_{t_{m-1}}^{t_m} \left( \gamma_s \circ \int_{t_{m-1}}^{\gamma_s} \sigma_s - \tilde{b}_s \right) ds
da_m & \triangleq \int_{t_{m-1}}^{t_m} \left( \int_{t_{m-1}}^{u_s} m^{-1} \tilde{R}_s \left( \int_{t_{m-1}}^{u_s} \tilde{a}_s - \tilde{b}_s \right) ds \right) du
\end{align*}
\]

Among them, \(\tilde{a}_s\) and \(\tilde{\omega}_s\) represent the acceleration and angular velocity in IMU \(\tilde{y}_m = q_p, \tilde{\omega}_{t_{m-1}} = 0, a_{t_{m-1}} = 0\). For the sample discrete-time pre-integration formula and residual error in IMU, we can refer to \([22]\).

The above is the pre-integration operation on the IMU; we also need to bring the state estimation of the IMU to the current moment and the next moment. In Figure 1, we substitute the state estimation result \(\bar{x}_{k-1}\) into \(t_k\) and \(t_k\) to obtain the conversion \(\psi_k^T T\) and
\( t_k^F \) converts \( B_F^{"F} \) to \( L_{F+k} \) and \( t_k^P \) through the correction of SCFC \( \tilde{F}_k \), CFC \( B_F^k \) is obtained. In addition to the above usage, we also applied it to the relative change \( T_{k+i}^i \), where \( T_i^i \) represents the time base of UWB data, and this transformation is applied to the BA process. Refer to [23] for the specific process of BA.

![Figure 1. Synchronization between the CFC, UWB, and IMU.](image)

(3) UWB: from the brown part in Figure 2, we can see the process of UWB positioning. We can get the distance between anchor to anchor and anchor to robot from UWB ranging and communication network. After we have arranged the anchor points, we can first realize the self-positioning information of the anchor points. In order to reduce the error of measuring the distance between the anchor points, we take the distance information between 100 anchor points as the average value, and then obtain the anchor point. Since the anchor point has been fixed, it can be regarded as the known position coordinate information \( r_{101}, r_{102}, r_{112} \) in the map. After fixing the coordinates of the anchor point, the distance information between the robot and the anchor point has a range, and we will mark it. In this way, the information in UWB can be checked, and it can be detected whether it is a valid value, because if the distance information exceeds the range of the mark, it must be an abnormal value. After confirming that the UWB information is valid, the time step of the UWB sample is synchronized with the time step on the sliding window. The abnormal value measured by UWB is eliminated through IMU propagation state detection. In each time interval \( (t_{m-1}, t_m) \), select \( N^i_u \) UWB measurement values \( \{u^i_{m}\}_{i=1} \) to set the final result; \( u^i_{m} \) mainly consist of the following:

\[
\begin{align*}
\mathbf{U}^i_{m} &= (\mathbf{d}^i, \mathbf{x}^i, y^i, t^i_m, t_{m-1}, t_m), i = 1, 2, \ldots, N^i_u
\end{align*}
\]

where \( \mathbf{d}^i \) is the distance measurement value, \( \mathbf{x}^i \) is the position information of the anchor point in the world coordinate system w.r.t. \( W \), \( y^i \) is the ranging node when the robot is in the \( B_{m}^{n} \) frame, \( T^i_m \) is the time information when the message is sent, \( t_{m-1} \) and \( t_m \) are a time step of \( T^i_m \).

In robot movement, we think that the robot is traveling at a constant speed, so its speed and direction change rate are constant. At the time \( T^i_m \), the positional relationship between the ranging node \( y^i \) and the anchor point \( X^i \) is as follows:

\[
\begin{align*}
I\mathbf{d}^i &= d(X_{m-1}, X_{m-1}, W^iT, \mathbf{U}^i_{m})
\triangleq p_m + R_{m-1}Exp(s^i\log(R_{m-1}^{-1}R_m))y^i \\
&-a^i\nu_{m-1} - b^i\nu_m - q^i Rx^i - \partial^i t^i
\end{align*}
\]

where \( s^i = \frac{\partial t^i}{\Delta t_m}, a^i = \frac{\Delta t^i_m - \partial t^i_m}{2\Delta t_m}, b^i = \frac{(\Delta t_m - \partial t^i_m)^2}{2\Delta t_m}, \delta^i t^i = t^i_m - t_{m-1}, \Delta t_{m} \delta t_{m-1}. \)
We consider the distance information $\tilde{d}_i$ measured at time $t_k + \delta t^{il}$ as the norm of the vector $w_d^i$, which is the zero mean Gaussian noise error $\eta$ of $\delta^k$ and (10) is destroyed by the ranging deviation, namely:

$$\tilde{d}_i = \|d_i^k\| + \eta^i + b^r$$

Therefore, the obtained UWB ranging residual is:

$$r_i(\tilde{x}_{m-1}, \tilde{x}_m, \tilde{w}_m^t, \tilde{b}^r, u^i_m) = \|d(\tilde{x}_{m-1}, \tilde{x}_m, \tilde{w}_m^t, u^i_m)\| + \tilde{b}^r - \tilde{d}_i$$

### 3.3. LiDAR-IMU Odometer

In the experiment of this thesis, the LiDAR we used is a Velodyne type; in the point cloud registration stage, through its multiple laser beams to obtain the surrounding environment point cloud, we mainly use the ICP algorithm for registration. Since the scene in this article is in a mine tunnel, in order to make full use of the scene information in the tunnel, we perform curb detection on the road surface in the tunnel, and then segment the road plane in the tunnel [24]. In order to improve the detection efficiency of the surrounding environment, we make full use of the edge feature information for registration [25]. By defining the convex surface in the point cloud, and then calculating the smoothness of all the point clouds, if the calculation result is large, it is proven to be an edge feature, and if the calculation result is small, it is proven to be a surface feature. Point cloud matching is performed to find the minimum distance from the point to the line $d_L$ and the minimum distance from the point to the surface $d_P$. We describe this problem with the following formula:

$$T = \arg \min_T \sum_{i \in \mathcal{E}} d_L + \sum_{i \in \mathcal{S}} d_P^i + \sum_{i \in \mathcal{G}} d_P^i$$

In the above formula, $\mathcal{E}$, $\mathcal{S}$, and $\mathcal{G}$ refer to the collection of edge points, surface points, and ground points, respectively, and $T$ is the pose information of the vehicle. As in the above formula, the pose information of the vehicle can be calculated. We obtain the LiDAR-IMU odometer by coupling the LiDAR-constrained vehicle pose $T$ and IMU pre-integrated data.
3.4. UWB Constraints

In our UWB measurement unit, we can only extract the position information of the robot, but cannot obtain the direction information of the robot, so in order to obtain the position information of the vehicle, we bind the UWB tag with the IMU and pass the direction of the IMU. The information compensates for the direction information of UWB. For this we define the following residual function:

\[ \sum_{i \in P} \| t_i - p \|^2 \]  

(13)

Among them, \( P \) represents the synchronization set of UWB position and IMU pose, \( t \) represents the translation component in the IMU, and \( p \) is the location information of the UWB tag.

In order to better constrain the vehicle positioning information, we connect the UWB information as a univariate hyper-edge constraint to the IMU pose node, as shown in Figure 3, and then update the sliding window to obtain more accurate positioning results.

![Pose graph illustration](image)

Figure 3. Pose graph illustration.

4. Results and Discussion

The environment used in this research is a mine roadway, but because the equipment used has not been subjected to explosion-proof treatment for the time being, this paper adopts the simulated roadway as the experimental environment. The main structural parameters of the vehicle used in this experiment are shown in Table 1, and the equipment and model information used in the experiment are shown in Table 2. In the experiment, we chose LeGo-LOAM [22] as the LiDAR-IMU data fusion scheme. The authors also obtained good positioning accuracy in the urban environment, and we fused the LiDAR-UWB sensor data. Finally, the above two methods are evaluated with the LiDAR-IMU-
UWB (LIU) method proposed in this paper for accuracy. In order to avoid accidental errors caused by the experimental environment, we chose two roadways with different lengths for the experiment. The trajectories obtained by the sensors are shown in Figures 4a and 5a. For the convenience of comparison, its x-axis position error and y-axis position error are shown in Figures 4b,c and 5b,c. In order to show the error between different schemes and the benchmark, we have enlarged the details in the figures. From the trajectory curve, it can be seen that the scene degradation problem of LeGo-LOAM in the roadway environment is very serious, and the LiDAR-UWB scheme has a significant improvement in positioning because the UWB sensor can provide a more accurate position information. The LIU method is superior to the previous two in terms of positioning accuracy. In Figures 4d and 5d, this paper uses a violin histogram to represent the distribution state of the absolute pose error (APE). From the figures, we can see that the method in this paper (LIU) has also improved relatively in terms of system stability. Finally, by calculating the root mean square error (rmse) generated by the three methods, and the comparison diagrams shown in Figures 4e and 5e, it is concluded that the method proposed in this paper (LIU) 100-m rmse is less than 10 cm, while LiDAR-UWB is about 12 cm, and LeGo-LOAM is about 250 cm.

In two environments with different lengths, the tests show that the LIU method proposed in this paper can solve the scene degradation problem of LiDAR sensor well and is also more accurate than the LiDAR-UWB method. Therefore, it can be concluded from the experiments that the LIU method proposed in this paper can complete accurate positioning in the mine roadway.

Table 1. Vehicle body structure parameters.

| Name            | Parameters                                      |
|-----------------|-------------------------------------------------|
| Shape           | 2490 × 1550 × 616 mm                            |
| Bearing spacing | 1900 mm                                         |
| Wheel spacing   | 1355 mm                                         |
| Frame material  | High strength aluminum alloy                    |
| Suspension type | Front and rear double wishbone independent suspension |
| Tire specifications | 205/45R17                                     |
| Ground clearance| 200 mm                                          |
| Energy type     | Pure electric                                   |
| Maximum load    | 1000 KG                                         |
| Maximum speed   | 40 Km/h                                         |
| Maximum grade   | 20%                                             |
| Steering Type   | Four-wheel steering (mode adjustable)           |
| Recharge mileage| 70 KM                                           |
| Braking method  | Electro-hydraulic brake                         |
| Braking type    | Four-wheel disc brake                           |

Table 2. Information about experimental equipment.

| Equipment          | Model            |
|--------------------|------------------|
| Industrial Personal Computer | Nuvo-6108gc       |
| RTK                | CGI-410          |
| LiDAR              | Velodyne (VLP-16) |
| IMU                | N100             |
| UWB                | DWM100           |
Figure 4. Experiment in a roadway with a length of about 85 m. (a) Trajectory curve; (b) position error along x-axis; (c) position error along y-axis; (d) APE violin histogram; (e) error value comparison.
5. Conclusions

In this article, we mainly propose the LIU method for the complex environment (mainly light and dust, etc.) in the closed coal mine roadway and the GPS rejection environment. Because a single LiDAR is prone to feature loss in a roadway with a simple structure, we added IMU to complete the radar inertial odometer after the feature establishment of the LiDAR skewed point cloud. In order to further improve the positioning accuracy of the system, we use the UWB positioning unit. After processing the UWB information, the UWB information is added to the IMU pose node as a univariate over-edge...
constraint, and then the positioning is completed by updating the sliding window. On the basis of the method proposed in this paper, we have carried out experimental verification on the proposed method in simulating coal mine roadway. In the single LeGo-LOAM and LiDAR-UWB positioning methods compared, the method in this paper effectively solves the roadway problem compared to these two methods. Insufficient features and missing signals show good positioning accuracy and stability.

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