What is the Reynolds number of the Reynolds’ Layer?†

By ROBERT A. BENJAMIN1‡

1Department of Physics, University of Wisconsin-Madison, 1150 University Ave, Madison, WI 53706, USA

Several authors have now suggested that some interstellar clouds above the plane of the Galaxy are interacting with the Reynolds’ layer, the warm ionized gas extending well above ($H \geq 910$ pc) the Galactic plane (Reynolds 1993). Characterizing the interaction between these clouds and their surroundings should be useful in understanding one source of interstellar turbulence: vertical shear flows. This paper discusses how studies of the morphology and drag coefficient of falling clouds might be used to constrain the Reynolds number for the flow, and hence the effective viscosity of the warm ionized medium. If arguments based on morphology are correct, the effective viscosity of the warm ionized medium is significantly higher than the classical values. Possible resolutions to this problem are suggested.

1. Turbulence from Vertical Flows

The spectrum of density and velocity fluctuation in the ionized interstellar medium (ISM) measured by scintillation of pulsars suggests that on small scales much of the structure of the diffuse ionized ISM may arise as the result of turbulent processes. Turbulence arises in regions of viscous shear flows. In the Galaxy, such flows have a large range of outer length scales, and include galactic rotational shear in both the radial and vertical (c.f. Walterbos 1998, this volume) directions, spiral density waves, stellar mass outflows (jets, winds, and explosions), and photoionization-driven flows. The structures formed contain energy over a range of length scales which is ultimately dissipated via viscous (hydrodynamical) and resistive (magneto-hydrodynamical) processes.

An additional source of interstellar turbulence is the shear flow generated by buoyant motions of rising bubbles (Parker 1992) or falling clouds. Since the sizes and velocities of at least some clouds are known, they provide a good environment to study one mechanism for the production of interstellar turbulence and constrain some relevant physical parameters.

2. How do clouds fall...

...At terminal velocity? Classical dynamics has conditioned us to think that objects speed us as they fall and that the acceleration is independent of the mass of the object. But experience shows that this is not always the case. A hammer, for example, falls faster than a feather. This is because of the presence of an external fluid medium. If the viscosity of the fluid is non-zero, there will be momentum transferred from the object to the surrounding fluid. Drag is a tricky thing to calculate, and counterintuitive things can happen. For instance, if the ambient fluid has a density gradient, as in a gravitationally stratified atmosphere, sufficiently “light” objects will slow down as they fall rather than speed up, because the drag force decelerates the object to terminal velocity, and drag

‡ Previous address: Minnesota Supercomputer Institute, 1200 Washington Ave. South, Minneapolis, MN 55415

† Grammar afficianados will note that while objects, such as the warm ionized layer, take the possessive form, dimensionless numbers do not (Vogel 1983).
increases as the object falls down the density gradient. This effect is seen in raindrops (Foote & du Toit 1969).

In Benjamin & Danly (1997) (BD97), it was suggested that the same effect, where the Reynolds' layer plays the role of the Galactic atmosphere, produces a velocity stratification of neutral hydrogen clouds with height. Measurements of the column density and velocity of these clouds, along with models of the gravitation field and external density, were used to predict cloud distances, which could be checked with absorption lines studies. There was good agreement with the available data, including correctly predicting the distance to the high velocity cloud Complex M. It also provides a satisfactory explanation for why high velocity clouds are high velocity, i.e. they exist in distant, low drag environment. Just how distant remains a point of some debate (see Wakker & van Woerden 1997).

The terminal velocity is given by $v_T(N, z) = \sqrt{2N_{HI}g(z)/n_h(z)f_cC_D}$, where $f_c$ is
the cloud neutral fraction, and $C_D$ is the drag coefficient. Clouds falling at a terminal velocity obey the relation

$$\frac{v}{\sqrt{N_{HI}}} = \sqrt{\frac{2g(z)}{n_h(z)f_cC_D}} = A(z)$$

which isolates the observable quantities on the left hand side, and the model assumptions for gravity and gas density on the right. We can then ask whether the normalized velocity, $v/\sqrt{N_{HI}}$, of observed clouds depend upon height. This is shown in Figure 1, which plots the normalized velocity vs. the distance brackets on clouds. Most clouds have $|b| > 50^\circ$ to minimize the contribution of galactic rotation to the observed velocity. The curve shows a “best guess” $A(z)$ from BD97, where the density includes the H I, warm ionized medium, and a hot halo. We have also added three cloud with distances determined subsequent to this paper, although they are at less than $b = 50^\circ$: Draco at $b = 38$, G86.0+38.3 (Gladders 1998), and high velocity cloud complex A at $b = 43$ (Wakker 1998). 

Note that the terminal velocity formula successfully predicted the distance to these clouds. These data show a trend for $v/\sqrt{N}$ to increase with height. In an ideal situation, the curve $A(z)$ could be shifted to solve for the best value of $C_D$. Right now, all one can say is that $C_D < 20$ and is consistent with being one, expected theoretically (Jones et al 1996).

Without more data, it is hard to be sure whether the terminal velocity hypothesis is true for some or all of the diffuse galactic clouds. Our ignorance of how clouds relate to the intercloud medium is marked. Other suggestions that can be found in the literature are the following:

...Ballistically? In the originally envisioned galactic fountain model (Shapiro & Field 1976; Bregman 1980), clouds condensed out of thermal instabilities and were assumed to fall ballistically back to the disk. This assumption was in keeping with the ballistic cloud models of Oort (1954) and Spitzer (1978). Although I have found that most people personally disavow the ballistic option, discussion of cloud “trajectories” and “orbits” which neglect the effects of drag continue to be promulgated through the literature.

...Apart? The timescale for the Rayleigh-Taylor and Kelvin-Helmholtz instability to develop in falling clouds is shorter than the free-fall time. However, such instabilities may be inhibited by incorporation of radiative cooling (Vietri et al 1996) or magnetic fields (MacLow et al 1994). This point of view is supported by the fact that the internal velocity dispersion of high velocity clouds is sufficiently small that the clouds should not disperse before hitting the plane. Note also that raindrops are also subject to the same instabilities. In that case, surface tension saves the day (and the crops).

... They don’t? There are several possible forces that might act to prevent a cloud from falling: photolevitation (Franco et al 1991), galactic winds, or magnetic tension (Franco 1998, this volume). Of course, if these are always operative, there would be no vertical mass circulation. So although these mechanisms might act frequently, there must be times when they break down.

What do you mean “cloud”? The “interstellar cloud” as a discrete and time-evolving entity may be a misleading concept (c.f., Scalo 1990). If clouds are just short-lived density concentrations in the interstellar fluid, talking about the time history of an coherent fluid element may be nonsensical.
3. Viscosity

Here, I will take the point of view that clouds do fall due to lack of buoyancy and interact with the intercloud medium. This point of view can be supported by the velocity-distance correlation discussed above, the work of Odenwald (1988) and Reach, Wall, & Odegard (1998) who show several Galactic “cometary” clouds from IRAS and DIRBE surveys; Howk & Savage (1997), who show cometary clouds 1 kpc above the plane of NGC 891; Kerp et al (1996), who argue for a positional correlation between H I HVC and X-ray emission; Pietz et al (1996) who show velocity bridges between low and high velocity gas, and so on. These papers suggest that at least some clouds are being shaped by interaction with the ambient medium. What physical information can we glean from these environments?

The fundamental parameter that characterizes a flow of velocity $V$ around an object of length $L$ is the Reynolds number, $Re = LV/\nu_{eff}$, where $\nu_{eff}$ is the “effective” kinematic viscosity. I say “effective”, because what is called turbulent viscosity and magnetic effects may and probably do come into play. In the absence of magnetic fields, the classical kinematic viscosity ($cm^2 s^{-1}$) of the ISM is $\nu = 6 \times 10^{19} T^{5/4} n^{-1}$, In the presence of a magnetic field, momentum transfer perpendicular to the direction of the magnetic field is decreased, and the viscosity decreases by twelve (!) orders of magnitude to $\nu = 2.8 \times 10^7 B_{10}^{-2} T_{4}^{-1} n_{-2}$. Flows of similar Reynolds numbers (all other things, like Mach number, being equal) will have similar (1) morphological characteristics, (2) drag coefficients, and (3) energy dissipation rates. All three of these quantities may be constrained observationally, giving an estimate for the Reynolds number. Here, I concentrate on the Draco molecular cloud. This cloud shows a “cometary” morphology (see Odenwald & Rickard 1987). Assuming purely vertical motion, its downward velocity is $V_z = -34 km s^{-1}$ with a column density of $N(HI) \approx 1.8 \times 10^{20} cm^{-2}$. Using the recently determined distance of $240 < z < 390$ pc (Gladders 1998), its size is $L = 3.4 - 5.4$ pc. Assuming that $T = 8000 K$ and $n_e = 0.02 cm^{-3}$ in the above formulae, the Reynolds number for the cloud should be $Re = 2 \times 10^6$ or $Re = 6 \times 10^{17}$, for the viscosity without or with magnetic fields, respectively. How do our empirical measurement compare?

Morphology: Odenwald (1988) identified 15 IRAS 100 $\mu$m clouds with a “head-tail” or cometary appearance, suggesting that the morphology of these clouds arose from cloud-ISM interaction. (Interestingly, several of these clouds also showed evidence of star-formation, possibly triggered by the same interaction.) Under the assumption that the morphology of the tail was due to the ”wake” produced by the cloud, Odenwald associated a Reynolds number with the morphological characteristics of the cloud. Flows of extremely low $Re$ (high viscosity) will have smooth, laminar flow patterns; flows of intermediate $Re$ will produce irregular clumps and vortices that will detach from the back of the cloud; very high $Re$ will produce fully turbulent flow, in which the wake will appear smooth again. Based on an elongated and clumped morphology, Odenwald assigned the flow around the Draco cloud with $Re < 50$.

Drag coefficient: The recently determined distance to the Draco cloud matches the terminal velocity prediction of BD97 assuming that $f_c = 1$ and provided that $0.9 < C_D < 1.2$. Unfortunately, no study exists characterizing how the drag coefficient of interstellar clouds should vary with Reynolds number. This is partially because clouds, unlike solid objects, may deform and even fragment depending upon the initial conditions and constituent physics, both of which are poorly known. Numerical simulations of Jones et al 1996, among others, suggest that $C_D = 1$ for simulations which have an effective $Re \approx$ few hundred. Based on flows around solid objects, $C_D \rightarrow 1$ for large enough $Re$. 
Comparing our derived range of drag coefficient to the value for flow around a sphere (Landau & Lifschitz 1987, Fig 34) yields $Re > 90$.

4. The Reynolds number

The alert reader will have noticed that the two estimates of the Reynolds number for the same halo cloud are disjoint. However, there is sufficient uncertainty in both of them that they may both be reconciled. Given that both are estimated using information for solid objects moving through a fluid medium, there will need to be some modification in the above numbers. The possibility of a bow shock will also be important. However, the difference between the Reynolds number based on morphology and the classical value differ by more than four orders of magnitude. Either the morphological interpretation is in error or the viscosity is significantly higher than the classical value. Both of these are possible, but it is difficult to distinguish between them. Those experienced in accretion disk physics are well aware of the possibility of anomalous viscosity; the magneto-rotational instability discussed by Gammie (1998, this volume) is one mechanism. But even shakier still is our ability to convert morphology into a physical scenario. For instance, rather than the tail being produced in a turbulent flow, it could be material ablated from the cloud via a Kelvin-Helmholtz instability. Another example is G110-13 (Odenwald et al 1992), a cometary cloud which turned out to be a probable cloud-cloud collision. The lesson here is interpreting morphology in the absence of other information is dangerous. Numerical simulations of this process will probably be needed to build some intuition as to what such processes should actually look like.

I would like to thank Pepe Franco and the other organizers for a most original and stimulating meeting and Don Cox (and NASA grant NAG5-3155) for sending me there. Some of the work done here was done using the facilities of the Minnesota Supercomputer Institute.

REFERENCES
Benjamin, R.A. & Danly, L. 1997, ApJ, 481,764
Bregman, J.N. 1980, ApJ, 236, 577
Foote, G.B. & du Toit, P.S. 1969, J. Appl. Meteor., 8, 249
Franco, J., Ferrini, F., Barsella, B., & Ferrara, A. 1991, Ap J, 366, 443
Gladders, M., 1998, priv. communication
Howk, J.c. & Savage, B.D. 1997, AJ, 114, 2463
Jones, T.W., Ryu, E., & Tregillis, I.L. 1996, ApJ, 473, 365
Kerp, J. et al 1996 A & A 312, 67
Landau, L.D. & Lifshitz, E.M. 1987. Fluid Mechanics, 2nd edition. Butterworth-Heinemann, 181.
Oort, J.H. 1954 Bull. Astron. Inst. Netherlands, 12, 177
Mac Low, M-M., McKee, C.F., Klein, R.I., Stone, J.M., & Norman, M.L. 1994, ApJ, 433, 757
Odenwald, S.F. 1988, ApJ, 325, 320
Odenwald, S.F. & Rickard, L.J. 1987, ApJ, 318, 702
Odenwald, S.F., Fischer, J., Lockman, F.J., & Stemwedel, S. 1992, ApJ, 397, 174
Parker, E.N. 1992 ApJ, 401, 137
Pietz, J. et al 1996, A & A 308, 37
Reach, W.T., Wall, W.F., & Odegard, N. 1998, ApJ, submitted
Reynolds number of the Reynolds' layer

Reynolds, R.J. 1993 in AIP Conf. Series 278, Back to the Galaxy: Proc. Third Annual Astrophys. Conf. in Maryland, ed. S.S. Holt & F. Verter (New York: AIP), 156
Scalo, J. 1990 In Physical Processes in Fragmentation and Star Formation (eds. R. Capuzzo-Dolcetta, C. Chiosi, & A. deFazio), Kluwer, 151
Shapiro, P.R. & Field, G.B. 1976, ApJ, 205, 762
Spitzer, L., Jr. 1978 Physical Processes in the Interstellar Medium. John Wiley & Sons, 230
Vietri, M., Ferrara, A, & Miniati, F. 1997 ApJ, 483, 262
Vogel, S. 1981 Life in moving fluids: The physical biology of flow. Princeton University Press.
Wakker, B. P. 1998, priv. communication
Wakker, B.P. & van Woerden, H. 1997, ARAA, 35, 217