CP Violating Effects in $e^+e^- \rightarrow ZH$ and Their Optimization

J. P. Ma and B. H. J. McKellar

Research Center for High Energy Physics
School of Physics
University of Melbourne
Parkville, Victoria 3052
Australia

Abstract: We consider the possibilities of testing CP conservation in the process $e^+e^- \rightarrow ZH$. The easiest is to measure the forward-backward asymmetry of the $Z$ boson, a nonzero value of which signals CP violation. The effects of CP violation are predicted in the two Higgs doublet model. We show here that a choice of optimal observables can improve the possibility of observing CP violation. Although the number of events is not large, useful information about CP violating parameters can be extracted.
1. Introduction

After its discovery in the Kaon system 30 years ago[1], CP violation is still a puzzle. The standard model can accommodate CP violation in the Kaon system, where CP violation is introduced through the imaginary part of the CKM matrix elements. However, because of the special structure of the CP violating interaction[2] in the standard model, predicted CP violating effects in high energy processes, such as high energy scattering, are extremely small. An explicit example is given in [3]. Hence any CP violation found in high energy processes will indicate the new physics beyond the standard model. In any case, the standard model is generally regarded as a low energy effective theory which will be modified or extended at high energies. One of the possible extensions is the two Higgs doublet model with two Higgs doublets instead of one. In this model CP violation can arise not only from the complex CKM matrix elements, but also from the Higgs sector[4], where, for example, the neutral Higgs bosons with different CP eigenvalues can mix. CP violation from this mixing may have sizeable effects in high energy scattering processes, which are studied in [5,6,7,8]. This model also allows a significant electric dipole moment for elementary particles[9].

In this work we will study possible CP violation from the two Higgs doublet model in the process $e^+e^- \rightarrow ZH$, where the initial beams are unpolarized. In this process it turns out that a CP test is possible without the knowledge about the polarization of the final state. A nonzero forward-backward asymmetry of the $Z$ or $H$ boson indicates CP violation. This is in contrast to the case of $e^+e^- \rightarrow$ particle + antiparticle, where a CP test is only possible when one measures the polarization of the final state. However, we will also consider the possible information about CP violation revealed by the polarization of the $Z$ boson, as the leptonic decay of $Z$ can measure its polarization.

In this work we also consider the optimization of CP violating effects. In experiment, if the number of events available for CP test is not large, the statistical error can be the main source of error in the observation. It is possible to reduce the statistical error by a suitable
optimization procedure. A general method for optimization was proposed recently\[10,11\]. We apply the procedure to the present process and compare the effects with and without optimization.

We begin in Sect. 2 with our general formalism and introduce some simple CP odd observables. In Sect. 3 we estimate the effect of CP violation from the two Higgs doublet model and the sensitivity of our simple CP odd observables to CP violating parameters in the model. In Sect. 4 we apply the optimisation procedure mentioned above. Sect. 5 includes a discussion and our conclusion.

CP violation in $e^+e^- \rightarrow ZH$ is also discussed in \[12\] in relation to CP violation in the Higgs decay, where only the real part of possible CP violating form factors is considered. In this case a nonzero forward-backward asymmetry can not be produced.

2. CP odd observables

We consider the following process with the unpolarized initial state:

$$e^+(p_+) + e^-(p_-) \rightarrow Z(k) + H(k_H) \rightarrow \ell^+(k_+) + \ell^-(k_-) + H(k_H)$$  \hspace{1cm} (2.1)

The momenta of the particles in the C.M.S. of the initial state are indicated in the brackets. The lepton pair comes form the $Z$ boson decay. At the tree level, the Higgs boson in the process (2.1) is produced through Bremsstrahlung off the virtual $Z$ boson produced by $e^+e^-$ annilation in the standard model.

Instead of the momentum $k_+$ in Eq. (2.1), we will use the momentum $q_+$ of $\ell^+$ in the $Z$ rest frame, which is of course related to $k_+$ by a Lorentz boost. We use the notation $\hat{k}$ to indicate a unit vector in the direction of $k$. We can introduce a density matrix $R(p_+, k, q_+)$ for the process in Eq. (2.1), which we normalize so that

$$\frac{\sigma(e^+e^- \rightarrow ZH \rightarrow \ell^+\ell^-H)}{\text{Br}(Z \rightarrow \ell^+\ell^-)} = \frac{1}{2s} \cdot \frac{|k|}{4\pi\sqrt{s}} \cdot \frac{1}{4\pi} \int d\Omega \cdot \frac{1}{4\pi} \int d\Omega_+ R(p_+, k, q_+)$$  \hspace{1cm} (2.2)

Here $s = (p_+ + p_-)^2$ and $d\Omega(d\Omega_+)$ is the solid angle of the vector $k(q_+)$. The quantity $R$
can always be written in the form:

\[ R(p_+, k, q_+) = A(p_+, k) + \hat{q}_+ \cdot B(p_+, k) + (\hat{q}_+ \hat{q}_{+j} - \frac{1}{3} \delta_{ij}) C_{ij}(p_+, k) \]  

(2.3)

If CP invariance holds, we have:

\[ A(p_+, k) = A(p_+, -k), \quad B(p_+, k) = B(p_+, -k), \quad C_{ij}(p_+, k) = C_{ij}(p_+, -k) \]  

(2.4)

To test CP conservation one therefore has to verify the equations in Eq. (2.4). In Eq. (2.3) the quantities \( B \) and \( C_{ij} \) contain the information about the polarization of the \( Z \) boson. Without measuring the \( Z \) polarization one can still test CP invariance by checking the CP constraint for \( A \) in Eq. (2.4). We propose the following CP odd observables for a CP test:

\[ O_1 = x = \hat{p}_+ \cdot \hat{k}, \quad O_2 = (\hat{q}_+ \cdot \hat{p}_+) (\hat{q}_+ \cdot \hat{k}), \quad O_3 = (\hat{q}_+ \cdot \hat{p}_+) \hat{q}_+ \cdot (\hat{p}_+ \times \hat{k}) \]  

(2.5)

A nonzero expectation value of any observable \( O_i \) in Eq.(2.5) signals CP violation. We will call these observables simple CP odd observables. Note that the \( \langle O_1 \rangle \) can only be nonzero if the quantity \( A \) does not satisfy its CP constraint. The forward-backward asymmetry of the \( Z \) boson can be defined through \( O_1 \):

\[ a_{CP} = \langle \theta(O_1) - \theta(-O_1) \rangle = \frac{N(O_1 > 0) - N(O_1 < 0)}{N(O_1 > 0) + N(O_1 < 0)} \]  

(2.6)

where \( N(O_1 > 0) \) and \( N(O_1 < 0) \) is the number of events number with \( O_1 > 0 \) and \( O_1 < 0 \) respectively.

3. CP violation from the two Higgs doublet model

We consider here a two Higgs doublet model as a possible extension of the standard model. In this model CP can be violated due to the complex expectation values of the Higgs doublets. However, in general CP violation occurs together with the presence of flavour changing neutral currents at the tree level. The flavour changing neutral currents can be eliminated by imposing some discrete symmetry on the model. This discrete symmetry is
softly broken in the Higgs potential[4]. In this way it is possible to have CP violation in
the model with the absence of the flavour changing neutral currents at the tree level.

In this model, there are three neutral Higgs bosons, two of them have the CP eigen-
value +1, and the other one has −1 as its CP eigenvalue, if CP invariance holds. In this
case, the neutral Higgs boson with the CP eigenvalue −1 can not be produced in the pro-
cess (2.1) at the tree level, because it decouples from two Z bosons. If CP is violated, the
neutral Higgs bosons do not have any definite CP eigenvalue, but are mixtures of the CP
eigenstates. In the process we consider the effect of CP violation can occur at one loop
level only through the type of the diagrams given in Fig.1, where the loop is a fermion
loop. If the mass of the light flavours is neglected compared to $m_t$, only the top quark
contributes. In the two Higgs doublet model we write the relevant interaction between the
neutral Higgs boson $H$ and the top quark and the interaction between two $Z$ boson and
the Higgs boson $H$ as:

$$L_{Ht\bar{t}} = -iB_t\bar{t}\gamma_5 t H, \quad L_{ZZH} = \frac{eM_Z}{2\sin\theta_W\cos\theta_W}d \cdot Z^\mu Z^\nu H$$

(3.1)

The parameters $B_t$ and $d$ are real. Furthermore $|d| < 1$. If $B_t$ and $d$ are simultaneously
nonzero, CP is violated.

We parameterized the CP violating part of the $VHZ$ vertex with a form factor $F_V$ as

$$i\Gamma^\mu_\nu (k, q) = \frac{ie}{2\sin\theta_W\cos\theta_W}e^{\mu\nu\rho\sigma}k_\rho q_\sigma F_V(q^2)$$

(3.2)

where the indices $\mu$ and $\nu$ are the Lorentz indices for the polarization of the virtual vector
boson $V$ and the real $Z$ boson respectively. $V$ stands for a virtual photon or a virtual
$Z$ boson and has the momentum $q$ with $q^2 = s$, where $s > (M_H + M_Z)^2$. The on-shell
$Z$ boson carries the momentum $k$. Calculating the relevant diagrams, one finds that the
real part of the form factor $F_V$ has a complicated form, expressed with Spence functions.

Since we will take $\sqrt{s} = \sqrt{q^2} = 500\text{GeV}$, and the mass of the top quark will be taken as
$175\text{GeV}$, as suggested by recently in experiment[13], the absorbive part of the vertex, i.e.
the imaginary part of the form factor, is always nonzero in the $s$ channel. We calculate the imaginary part first, the real part can then be obtained from the dispersion relation.

For $V = Z$ the imaginary part is:

$$\text{Im}F_Z(s) = \frac{6em_tB_t}{\sin \theta_W \cos \theta_W}\{(g^t_V)^2I_V(s) + (g^A_V)^2I_A(s)\}$$

$$I_V(s) = \frac{D_L}{16\pi \beta_Z E_Z \sqrt{s}} \quad I_A(s) = \frac{E_Z - \sqrt{s}}{16\pi \beta_Z^2 E_Z^2 s}\{(\sqrt{s} - E_Z + \beta_Z^2 E_Z)D_L + 2\beta_t \sqrt{s}\} \quad (3.3)$$

$$D_L = \ln \left(\frac{(s - M_H^2 - M_Z^2)}{(s - M_H^2 - M_Z^2 - (s - M_H^2 + M_Z^2)\beta_t \beta_Z)}\right)$$

Here $\beta_t$ and $\beta_Z$ are the velocity of the top quark and the $Z$ boson in the rest frame of the virtual $Z$, and $E_Z$ is the energy of the real $Z$ boson. $\text{Im}F_\gamma(s)$ can be obtained from Eq. (3.3) by the substitution: $(g^t_V)^2 \rightarrow 4 \frac{4}{3} \sin \theta_W \cos \theta_W g^t_V$ and $(g^A_V)^2 \rightarrow 0$. From the imaginary part we can obtain the real part:

$$\text{Re}F_V(s) = \frac{1}{\pi}\int_{s_0}^{\infty} ds' \frac{\text{Im}F_V(s')}{s' - s}, \quad s_0 = \max\{4m_t^2, (M_H + M_Z)^2\} \quad (3.4)$$

Note that there is also another contribution to the imaginary part of the form factor, if the mass of the Higgs boson is larger than $2m_t$. We will not consider this case. The CP violating part of the density matrix $R$ is generated by the interference between the amplitude at tree level and the amplitude including the vertex in Eq. (3.2). The result for $R$ is:

$$R(p_+, k, q_+) = \left(\frac{ed}{\sin^2 \theta_W \cos^2 \theta_W}\right)^2 \frac{3s}{64(s - M_Z^2)^2}\left\{E_Z^2(1 - \beta_Z^2 x^2) + M_Z^2(p_+ \cdot \hat{q}_+)\right\}$$

$$+ 2M_Z(E_Z - M_Z)x(p_+ \cdot \hat{q}_+)(\hat{k} \cdot \hat{q}_+) + (\hat{k} \cdot \hat{q}_+)^2(-\beta_Z^2 E_Z^2 + x^2(E_Z - M_Z)^2)$$

$$+ \frac{4E_Z \sqrt{s}}{M_Z} \beta_Z \sin \theta_W \cos \theta_W (1 - \frac{M_Z^2}{s})\frac{1}{d}\text{Im}F_\gamma(s)[(p_+ \cdot \hat{q}_+)(\hat{k} \cdot \hat{q}_+)M_Z E_Z$$

$$+ (\hat{k} \cdot \hat{q}_+)^2 xM_Z(M_Z - E_Z) + xM_Z^2)$$

$$- \frac{\sqrt{s}}{d}\text{Re}F_Z(s)\beta_Z E_Z \hat{q}_+ \cdot (p_+ \times \hat{k})[M_Z(p_+ \cdot \hat{q}_+) + x(E_Z - M_Z)(\hat{k} \cdot \hat{q}_+)])\right\} \quad (3.5)$$

Here we used the fact that the coupling constant $g^t_V(= -\frac{1}{2} + 2 \sin^2 \theta_W)$ of the $Z$ boson to the neutral vector current is much smaller than the coupling constant $g^A_V$ to the neutral
axial current and neglected $g_V^{\ell}$. The CP conserving part of $R$ at the tree level is also given in Eq. (3.5). With the density matrix we can now calculate the expectation value of the simple CP odd observables and the CP asymmetry in Eq. (2.6). These quantities are proportional to $\frac{B_t}{d}$:

$$< O_i > = w_i \frac{B_t}{d}, \quad a_{CP} = y \frac{B_t}{d}$$

(3.6)

Further, $< O_{1,2} >$ receive contributions only from the imaginary part of the form factors, while the real part contributes to $< O_3 >$. We take $\sqrt{s} = 500\text{GeV}$ and $m_t = 175\text{GeV}$ and obtain the numerical results for the coefficients in Eq. (3.6):

$$w_1 = 5.6 \times 10^{-3}, \quad w_2 = 3.8 \times 10^{-3}, \quad w_3 = -4.4 \times 10^{-4}, \text{ for } M_H = 100\text{GeV}$$

$$w_1 = 6.7 \times 10^{-3}, \quad w_2 = 4.3 \times 10^{-3}, \quad w_3 = -4.6 \times 10^{-4}, \text{ for } M_H = 200\text{GeV}$$

(3.7)

For the density matrix $R$ given in Eq. (3.5), $y$ has a simple relation to $w_1$: $y = \frac{3}{2} w_1$. The result for $y$ as a function of the Higgs mass $M_H$ is plotted in Fig.2. From Fig.2 note that $y$ increases as $M_H$ increases, mainly due to the decreasing cross section with the increasing Higgs mass.

To study how sensitive our observables are to the CP odd parameter $\frac{B_t}{d}$, we introduce the sensitivity $\rho_i$ of a observable $O_i$:

$$\rho_i = \frac{\sqrt{< O_i^2 >}}{|w_i|}$$

(3.8)

Remembering that the statistical error of a observable $O_i$ is given by:

$$\delta < O_i > = \sqrt{\frac{< O_i^2 > - < O_i >^2}{N}} \approx \sqrt{\frac{< O_i^2 >}{N}},$$

(3.9)

where $N$ is the number of the available events for measuring the observable $O_i$, the sensitivity $\rho_i$ has the meaning that the parameter $\frac{B_t}{d}$ must be at least larger than $\frac{\rho_i}{\sqrt{N}}$ to have a measurable effect on $O_i$. The results for $\rho_i$ are:

$$\rho_1 = 88, \quad \rho_2 = 80, \quad \rho_3 = 558 \quad \text{for } M_H = 100\text{GeV}$$

$$\rho_1 = 75, \quad \rho_2 = 72, \quad \rho_3 = 532 \quad \text{for } M_H = 200\text{GeV}$$

(3.10)
\(\rho_1\) as a function of \(M_H\) is again plotted in Fig.3. From Eq.(3.10) we see that the observable \(O_{1,2}\) are much more sensitive than \(O_3\). The sensitivity for \(a_{CP}\) is given simply by \(y^{-1}\), and it is about 30% larger than \(\rho_1\).

4. Optimization

In [9,10], it is discussed how to construct the most sensitive observable to the value of a physical parameter in a reaction, where the distribution is linear in the physical parameter. We give here briefly the result from [10]. Let us consider a scattering process where the differential cross section has the form:

\[
\Sigma(\phi)d\phi = (\Sigma_0(\phi) + g \cdot \Sigma_1(\phi))d\phi
\]  

(4.1)

Here \(\phi\) denotes the sample of the relevant phase-space variables, with which we construct observables. The observable \(T\) which is most sensitive to the parameter \(g\) in Eq. (4.1) takes form:

\[
T = T(\phi) = \frac{\Sigma_1(\phi)}{\Sigma_0(\phi)}
\]  

(4.2)

This result is generalized in [11] for more complicated cases. Such types of the observables may be called optimal observables, following [11].

In our case, with the distribution, i.e. the density matrix \(R\) given in Eq.(3.5) we construct according to Eq. (4.2) the optimal observables for the form factors in Eq.(3.2) which we want to measure. The observables are:

\[
T_2 = \frac{(\hat{p}_+ \cdot \hat{q}_+)(\hat{q}_+ \cdot \hat{k})M_ZE_Z + (\hat{q}_+ \cdot \hat{k})^2xM_Z(M_Z - E_Z) + xM_Z^2}{D_0}
\]

\[
T_3 = \frac{-\beta ZE_Z\hat{q}_+ \cdot (\hat{p}_+ \times \hat{k})(M_Z(\hat{p}_+ \cdot \hat{q}_+) + x(E_Z - M_Z)(\hat{q}_+ \cdot \hat{k}))}{D_0}
\]

\[
D_0 = E_Z^2(1 - \beta_Z^2x^2) + M_Z^2(\hat{p}_+ \cdot \hat{q}_+)^2 + 2M_Z(E_Z - M_Z)x(\hat{p}_+ \cdot \hat{q}_+)(\hat{q}_+ \cdot \hat{k})
\]

\[
+ (\hat{q}_+ \cdot \hat{k})^2(-\beta_Z^2E_Z^2 + x^2(E_Z - M_Z)^2)
\]  

(4.3)

The optimal observables \(T_2\) and \(T_3\) correspond to the simple observables \(O_2\) and \(O_3\). To
present the numerical results we define in analogy to \( w_i \) and \( \rho_i \) the quantities:

\[
<T_i> = z_i \frac{B_t}{d}, \quad \rho_{T_i} = \sqrt{\frac{<T_i^2>}{|z_i|}}
\]

(4.4)

The numerical results are:

\[
z_2 = 4.2 \times 10^{-3}, \quad z_3 = 1.0 \times 10^{-3}, \quad \rho_{T_2} = 67, \quad \rho_{T_3} = 401 \text{ for } M_H = 100\text{GeV}
\]

\[
z_2 = 4.2 \times 10^{-3}, \quad z_3 = 9.3 \times 10^{-4}, \quad \rho_{T_2} = 60, \quad \rho_{T_3} = 414 \text{ for } M_H = 200\text{GeV}
\]

(4.5)

Comparing the sensitivity given in Eq.(3.10) for the simple observables the sensitivity of the optimal observables is from 20% to 40% better. However, the optimal observable \( T_3 \) is much less sensitive than \( T_2 \), and even much less sensitive than \( O_{1,2} \).

To construct the optimal observable \( T_1 \) corresponding to \( O_1 \), we integrate out the solid angle of the lepton momentum, i.e. extract \( A(p_+, k) \) in Eq. (2.3):

\[
A(p_+, k) = \left( \frac{e^2}{\sin^2 \theta_W \cos^2 \theta_W} \right)^2 \frac{d^2}{32 (s - M_Z^2)^2} \frac{E_Z s}{s} \cdot \left\{ 2 - \beta_Z^2 - \beta_Z^2 x^2 \right. \\
+ 8(1 - \frac{M_Z^2}{s}) \sin \theta_W \cos \theta_W \frac{M_Z \beta_Z \sqrt{s}}{E_Z \text{Im} F_\gamma(s) \cdot x} \right\}
\]

(4.6)

In this simple case we find that the effect of the improvement with the optimal observable is tiny, as also claimed in [10]. Hence we do not present our numerical results for \( T_1 \).

5. Conclusion

In this work we studied CP violating effects from the two Higgs doublet model in \( e^+e^- \to ZH \). The effects depend on a unknown parameter \( \frac{B_t}{d} \) of the model, defined in Eq. (3.1). To detect these effects we propose two sets of CP odd observables. One set contains the simple CP odd observables, the other one contains the so-called optimal observables, which are designed specially to maximize the CP violating effects from the two Higgs doublet model, or more generally from the form factors in Eq. (3.2). The sensitivity of these observables to the parameter \( \frac{B_t}{d} \) is estimated. From our results, however, the optimal observables do not improve the sensitivity to the CP violating effects very much,
the improvement is about 20% to 40%, the price for this improvement is that the optimal observables take a more complicated form than the simple observables.

One of the simple observables, i.e. the forward-backward asymmetry $a_{CP}$ of the $Z$ boson or the corresponding mean value $< O_1 >$ should be readily accessible in experiments. In principle, since the lepton momentum is not involved here, more events should be available for measuring $O_1$ and $a_{CP}$ than for the other observables $O_{2,3}$ and $T_{2,3}$. Comparing the results obtained in the previous sections, we conclude that the observable $O_1$ or $a_{CP}$ are at most sensitive to the CP odd parameter $\frac{B_t}{d}$. However, the other observables should also be studied in experiments, because our conclusion is specific to two Higgs doublet model, and it is possible in other possible extensions of the standard model that $< O_1 >$ and $a_{CP}$ both vanish, but others do not vanish.

Finally, we note that one can determine how large $B_t$ should at least be, independent of the parameter $d$, to have a possibly observable effect, if one knows the luminosity of the $e^+e^-$ collider. Assuming the luminosity to be $10\text{fb}^{-1}$ per year and requiring CP violation to be such that the expectation value of $O_1$ is larger than twice of its statistical error, then we require for $M_H = 100\text{GeV}$:

$$B_t > 7.2$$

for the observability of the effect.

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Reference

[1] J.H. Christensen, J. Cronin, V.F. Fitch and R. Turlay, Phys. Rev. Lett. 13 (1964) 138
[2] C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039, Z. Phys. C29 (1985) 491
[3] A. Brandenburg, J.P. Ma and O. Nachtmann, Z. Phys. C55 (1992) 115
[4] G.C. Branco and M.N. Rebelo, Phys. Lett. B160 (1985) 117
    J. Liu and L. Wolfenstein, Nucl. Phys. B289 (1987) 1
    S. Weinberg, Phys. Rev. D42 (1990) 860
[5] C.R. Schmidt and M. Peskin, Phys. Rev. Lett. 69 (1992) 410
    W. Bernreuther and A. Brandenburg, Phys. Lett. B314 (1993) 104, Phys. Rev. D49 (1994) 4481
[6] J.P. Ma and B.H.J. McKellar, Phys. Lett. B319 (1993) 533
[7] B. Grzadkowski and J.F. Gunion, Phys. Lett. B294 (1992) 361
[8] W. Bernreuther, O. Nachtmann, P. Overmann and T. Schröder, Nucl. Phys. B388 (1992) 53 B406 (1993) 516(E)
[9] X.G. He, B.H.J. McKellar and S. Pakvasa, Int. J. Mod. Phys. A4 (1989) 5011
[10] D. Atwood and A. Soni, Phys. Rev. D45 (1992) 2405
[11] M. Diehl and O. Nachtmann, Heidelberg–Preprint, HD–THEP–93–37
[12] D. Chang, W.Y. Keung and I. Phlips, Phys. Rev. D48 (1993) 3225
[13] F.Abe et al (CDF Collaboration) Preprint, FERMILAB-PUB-94/116-E

Figure Caption

Fig.1: One of the Feynman diagrams contributing to the form factor in Eq. (3.2)
Fig.2: The parameter $y$ for the CP asymmetry $a_{CP}$ as a function of the Higgs mass.
Fig.3: The sensitivity $\rho_1$ for the observable $O_1$ as a function of the Higgs mass.
This figure "fig1-1.png" is available in "png" format from:

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