Quantum secure identification using entanglement and catalysis

Howard Barnum
School of Natural Science and Hampshire College, Amherst, MA 01002, USA
email: hbarnum@hampshire.edu
(May 7, 2019)

I consider the use of entanglement between two parties to enable one to authenticate her identity to another over a quantum communication channel. Exploiting the phenomenon of entanglement-catalyzed transformations between pure states gives a potentially reusable entangled identification token. In analyzing this, I consider the independently interesting problem of the best possible approximation to a given pure entangled state realizable using local actions and classical communication by parties sharing a different entangled state.

I. INTRODUCTION

The protocols to be presented in this paper give Alice and Bob a way of identifying (“authenticating”) each other, using quantum and classical channels. The protocols are such that if Alice and Bob can successfully complete one, Alice is convinced that Bob (or someone who has stolen his identification token) is on the other end of the quantum communication channel. The classical analogue of this can be done by having Bob reveal, over a classical channel, a secret which Alice and Bob had previously securely shared. The quantum protocols use shared entangled states as the counterpart of shared secret key. The quantum version of the protocol differs from the classical protocol in potentially useful ways. At the most elementary level, it authenticates the presence of one quantum laboratory with certain causal relations to another on a network. (This is accomplished using quantum information exchange; lest it be thought impossible, we note that this might also be accomplished via classical information exchange, using quantum nonlocality.) There is protection, via the no-cloning theorem, against copying of the authentication token. Similar ideas may provide theft-detection capability, much as standard quantum cryptography provides security against eavesdropping. Most interestingly, the protocol based on using entangled “catalyst” states may provide reusable authentication tokens, although the security properties when the tokens are reused will require careful analysis.

II. THE TASK

The authentication task considered in this paper is widely performed in practice, with computer login sequences and automatic teller PIN protocols among the most common examples. It arises in settings where the channel is considered sufficiently tamper-proof by the users, but they wish to defend against a certain kind of “terminal” insecurity: the possibility that the workstation (or quantum lab) has been taken over by an impostor. For this to be relevant to the authenticity of a session that follows authentication, there must be some assumption that control of the terminal has a high degree of intertemporal correlation. Perhaps there is some probability per unit time that the control of the terminal will be seized by an impostor. The probability of authorized control of the terminal then decays exponentially with time, but successful login-style authentication at a later time resets the the probability to one, which could provide (with a low enough decay rate) sufficiently increased security for a session bounded in time and starting with the authentication. Or, it may be assumed that once Alice is logged in, transmission remains authentic for the entire session; violations of terminal security are assumed to require stealth on the part of the impostor, and be infeasible (or low-probability) until Alice has left the terminal.

Such protocols are sometimes used as a prelude to further communication, for example in a login sequence for remote use of a computer. Without assumptions such as those of the previous paragraph, these protocols do not provide a solid guarantee of authenticity for the rest of the session. Someone could allow you to log in, then block your access and tap the line for their own purposes. Demonstrating authenticity for the entire session in the presence of such a threat requires methods such as the protocol based on universal hash functions \[\text{\cite{1,2}.} \] (Such protocols are used, for example, to authenticate the classical communication involved in quantum key distribution protocols.) In these protocols the degree of security associated with shared secret key bits may be transferred to the authenticity of the message, by hashing the message and the secret key using a certain type of universal hash function, and sending the hash along with the (unencrypted) message over the public channel. The receiver then hashes the message with his copy of the secret key, and compares to the sent hash; tampering (with either message or hash or both) by someone without knowledge
of the key is overwhelmingly likely to yield a tampered-with hash which is not the valid hash of the authentic message.

Whether the authenticity of the quantum authentication tokens discussed in this paper, which are a counterpart of the classical notion of shared secret key, can be transferred to an entire communication session in the manner of the universal hashing protocol described above, is an extremely interesting question. A simple adaptation of the classical protocol would seem difficult in the case in which the user who logs in may wish to act as a conduit for quantum states unknown to her; since the hashing protocol requires two copies of the message, one to be hashed and sent and the other to be sent unaltered. The preparation-visible case seems more promising, although this is a severe restriction.

III. QUANTUM AUTHENTICATION USING ENTANGLEMENT

Rather than using a shared classical secret, Alice and Bob authenticate via a shared entangled quantum state. Unlike the key in a classical scheme, this quantum key need not be secret, although keeping it secret might make it harder to steal (particularly if it is not maximally entangled), and might have other advantages in a many-user setup. Perhaps the simplest scheme is for Alice and Bob to share some maximally entangled states; Alice, at least, knows which states. To authenticate, Bob sends Alice his half of some of the states; Alice then performs tests on them to assure herself they are indeed the specified Bell states. Say they are all the same Bell state; Bob tests on them to assure herself they are indeed the specified Bell states. Say they are all the same Bell state; Bob sends $N$ of them, and Alice measures them in the Bell basis; by increasing $N$, she can distinguish these states more and more reliably (at a rate which depends on the noise in the overall process) from any states that could come from an impostor which are actually not entangled with her states. In fact, in a $d$-dimensional Hilbert space the maximum matrix element $\langle \psi | \rho | \psi \rangle$ for a separable $\rho$ to pass a test for being a maximally entangled $|\psi\rangle$ is $1/d$ (this occurs where $\rho$ is pure, and is any one of the product states occurring in the Schmidt decomposition). Thus security increases exponentially in the number of qubits $(\log_2 d)$ used.

The quantum scheme differs from its classical counterpart in several potentially useful respects. With a classical secret, an impostor would have to break into Bob’s classical storage area and copy his secret key; it is physically possible to do so without evidence of the break-in. The impostor and Bob could both communicate with Alice, who might not realize anything was wrong for quite a while. Both would be able to authenticate. By contrast, an impostor who stole Bob’s half of a quantum authentication key would have to be able to always divert the authentication portion of a communication session to himself, should he wish to allow Alice and Bob to communicate unaware of the loss of their authentication token. The quantum scheme is also more directed: steal Alice’s classical (symmetric-crypto) key and you have what Alice and Bob share; steal Alice’s half of an entangled state and you still can’t pretend to Alice that you’re Bob. The no-cloning theorem not only makes undetected theft of key more difficult, but also protects a stolen key from dissemination to wide sectors of the underworld. Steal Alice’s classical key, and you can distribute it to your henchmen all over, who can pretend to be Alice when and where they want. Steal her half of an entangled state, and there’s no way of distributing it among your henchpeople (or henchthings, for that matter).

Either the maximally-entangled-state quantum authentication protocol or the classical shared-secret protocol uses up some of the shared key each time authentication takes place. By contrast, a catalysis protocol does not. The fact that the shared key is (or can be) used up each time authentication takes place renders the EPR protocol better for certain purposes: for example, for banknotes it could be good that the state is returned to the bank and destroyed in the authentication process. Such entangled “money” would have the desirable properties of transferability and uncopypability, but not the properties of untraceability or anonymity, nor does the present discussion provide a protocol for verification of its value, let alone nondestructive verification of its value, by third parties; it is far from constituting proper quantum cash. Schemes more suitable for quantum cash were proposed by Wiesner [4] and Bennett, Brassard, Breidbart and Wiesner [5]; they involve nonorthogonality rather than entanglement. For other purposes, however, it may be good to have a reusable i.d. token. The catalysis protocol presented in Section IV may provide such a token. The next section provides background for the understanding of catalysis.

IV. LOCC-CONVERTIBILITY AND CATALYSIS

Nielsen [4] showed that a pure state $|\psi\rangle$ in a Hilbert space $A \otimes B$ is convertible to another, $|\chi\rangle$, by local actions and classical communication (LOCC), if and only if the nonincreasingly-ordered eigenvalues of the reduced density matrix of $|\chi\rangle$ majorize those of $|\psi\rangle$. (These are sometimes called the OSC’s, ordered Schmidt coefficients, of the pure states. I will use the notation $\lambda_i(|\eta\rangle\langle\eta|)$ for the ordered eigenvalues of the reduced density matrix of $|\eta\rangle\langle\eta|$, rather than for those of the density matrix of $|\eta\rangle\langle\eta|$ itself as in more standard notation.) That is, the target state’s reduced density matrix is “less mixed”; it is natural to suppose this means it is less entangled, and the theorem confirms this intuition. I will write this relation (which is a partial ordering on the pure states) $|\chi\rangle \preceq |\psi\rangle$, and write $|\psi\rangle \succ |\chi\rangle$ when nei-
ther $|\psi\rangle \preceq |\chi\rangle$ nor $|\chi\rangle \preceq |\psi\rangle$. (The relation “$\preceq$” may thus be read “is less entangled than.”) I will also use the obvious ordering this induces on one-dimensional projectors on these states, and extend it to density matrices in the manner of Vidal [3]. He defined $d-1$ entanglement measures $E_k$, which are sums of the $k$ lowest Schmidt eigenvalues, and are nonincreasing under LOCC. Define $S_k(|\eta\rangle\langle\eta|) := \sum_{i=1}^d \lambda_i(|\eta\rangle\langle\eta|)$. Also define $E_{k-d+1}(|\eta\rangle\langle\eta|) := 1 - S_k(|\eta\rangle\langle\eta|) \equiv \sum_{k-d+1}^d \lambda_i(|\eta\rangle\langle\eta|)$. Extend this to mixed states via:

$$S_k(\rho) := \max_{\{t_j,|\eta_j\rangle\}} \sum_j t_j S_k(|\eta_j\rangle\langle\eta_j|) .$$

Equivalently, one could define:

$$E_k(\rho) := \min_{\{t_j,|\eta_j\rangle\}} \sum_j t_j E_k(|\eta_j\rangle\langle\eta_j|) .$$

Define

$$\rho_1 \succeq \rho_2 := \forall k, E_k(\rho_1) \geq E_k(\rho_2) .$$

This is the extension of the majorization-based “more entangled than” relation to mixed states. Vidal [3] showed that the $E_{k-d+1} = 1 - S_k$ are entanglement montones, hence cannot be increased by LOCC. Jonathan and Plenio [4] state (indeed, they make a somewhat more general statement; cf. also [5]) that the partial ordering $\succ$ in fact coincides with LOCC-convertability. This is, of course, a generalization of Nielsen’s theorem to mixed states. Jonathan and Plenio [4] have used Nielsen’s Theorem to show the existence of pairs of states $|\psi\rangle \succ |\chi\rangle$, such that if Alice and Bob share a particular entangled state $|\phi\rangle$, they may nonetheless convert $|\psi\rangle$ into $|\chi\rangle$ by LOCC, while retaining $|\phi\rangle$ unchanged at the end of the process. In this paper, this phenomenon is exploited to give a potentially reusable quantum identification token.

V. THE CATALYSIS PROTOCOL

Here, Alice and Bob share a catalyst state $|\phi\rangle$. There are incommensurate states $|\phi_1\rangle$ and $|\phi_2\rangle$ such that in the presence of the catalyst, $|\phi_1\rangle$ can be converted to $|\phi_2\rangle$, while retaining $|\phi\rangle$. For Bob to authenticate himself to Alice, Alice makes $|\phi_1\rangle$ in her laboratory, and sends half of it to Bob. They then go through the steps, involving local measurements, one-way communication of measurement results, and local operations conditional on those measurement results, which convert $|\phi_1\rangle$ to $|\phi_2\rangle$. If Alice’s quantum channel leads to a cheating Derek, who does not possess any subsystem involved in the catalyst state, by Nielsen’s result this conversion cannot succeed. So the protocol continues by having Bob return to Alice the B half of the system which they were to convert into $|\phi_2\rangle$. She then measures the projector onto $|\phi_2\rangle$. If (with the idealization of perfect operations and measurements) she ever gets the result “0”, she knows that Bob was not involved in the protocol (assuming Bob would always implement the protocol correctly were he involved). Usually, even if an impostor is involved, the measurement result will be “1”. But the small probability of “0” in that case can be amplified by repeating the protocol. This has no cost in the stored catalyst state, though it has a cost (polynomial in the desired accuracy) in quantum communication.

A weak upper bound on the one-shot probability of error (of getting “1” from an impostor) can be gotten by considering the (convex) set of less entangled states (those whose OSC’s majorize $|\phi_1\rangle$’s). Let $\rho^*$ be the closest of these states (in the $L_1$ norm distance which corresponds to error probability) to $|\phi_2\rangle$; then $p_e = \langle \phi_2 | \rho^* | \phi_2 \rangle$. This can be used to obtain, for a specified $\epsilon$, an expression, polynomial in $1/\epsilon$, for a number of repetitions guaranteed to give error probability below $\epsilon$. It is a bound because only the less entangled states are accessible via a protocol between Alice and an impostor Derek unentangled with those of Alice’s systems involved in the protocol. It is a weak bound because choosing the nearest of those states means considering Alice as conspiring with Derek to fool herself. In the actual situation, Alice will not perform her part of a protocol for converting $|\phi_1\rangle$ to $|\phi^*\rangle$, but rather will still perform her part of the protocol for converting $|\phi_1\rangle$ to $|\phi_2\rangle$. As Chris Fuchs pointed out to me, when Alice does the correct protocol in the presence of the impostor, she will of course wind up with the same reduced density matrix she would have if there had been no impostor; i.e., the reduced density matrix of $|\phi_2\rangle$. So, we may now look at the closest state, not merely among the less entangled states, but in the (still convex) set of less entangled states having the same density matrix for Alice as $|\phi_2\rangle$. Since $|\phi_2\rangle$ and local unitary transformations of it are not in the majorized set, the closest such state will be mixed.

Our weak bound on the error probability is given by:

$$\max_{\rho \geq |\phi_1\rangle\langle\phi_1|} \langle \phi_2 | \rho | \phi_2 \rangle .$$

This problem would be very much simplified if we could assume the optimal state $\rho^*$ were pure, but it is not obvious this should be so. It is clear that the majorization constraint on $\rho$ prevents us from attaining 1 in this maximization, but in order to use the protocol in a particular instance, we need an upper bound below 1. One more tractable upper bound comes from considering the $d-1$ maximization problems in which only one of the majorization constraints is imposed, and taking the lowest of these maxima.

$$\min_{k=1,...,d-1} \max_{\rho} f(\rho) := \langle \phi_2 | \rho | \phi_2 \rangle$$

subject to

$$E_k(\rho) \leq \zeta_k .$$

3
Here \( \zeta_k := E_k(\langle \phi_1 | \phi_1 \rangle) \). Because of the definition of \( E_k \) as a minimum over ensembles, we can recast the inner maximization as a maximization of \( f \) over all ensembles of pure states:

\[
\max f(s_1, ..., s_n, |\eta_1\rangle, ..., |\eta_n\rangle) := \sum_j s_j |\langle \phi_2 | \eta_j \rangle|^2
\]

subject to

\[
\sum_j s_j E_k(|\eta_j\rangle|\eta_j\rangle) \leq \zeta_k .
\]

Here \( s_j \) are probabilities, and \( |\eta_j\rangle \) pure states. \( n \) may vary, but there will be a bound from a Davies-type argument. Moreover, there will be a maximum for which all the states \( |\eta_j\rangle \) are Schmidt-codiagonal with \( |\phi_2\rangle \). This follows from two observations. First, Lemma 1 below, which implies that each term in the objective function is maximized (for fixed \( q_j \)), by an \( |\eta_j\rangle \) Schmidt-codiagonal with \( |\phi_2\rangle \). Second, the fact that the local unitary transformation required to get an arbitrary \( |\eta_j\rangle \) into that form has no effect on the value of the constraint function, since \( E_k \) is invariant under local unitaries.

**Lemma 1:**
For fixed Schmidt coefficients, the pure state \( |\chi\rangle \) which maximizes \( |\langle \phi | \chi \rangle|^2 \) has the same Schmidt basis as \( |\phi\rangle \).

**Proof:**
We show this by mapping the pure states of the \( d^2 \)-dimensional system \( AB \) onto operators on a \( d \)-dimensional Hilbert space, in such a way that the inner product in the tensor product vector space becomes the Hilbert-Schmidt inner product of the operators. Denote such a map by \( \sigma \), and use the notation

\[
\sigma(|\chi\rangle) := G_\chi.
\]

Then

\[
\langle \phi | \chi \rangle \equiv \text{tr} G_\phi^* G_\chi .
\]

We may change to an arbitrary Schmidt basis by local unitaries. In this formalism, and writing the reduced density matrices as \( \phi_1 \) and \( \chi \) respectively, unitaries corresponding to changing \( |\chi\rangle \)'s Schmidt bases in the \( A \) and \( B \) system are mapped, one-to-one, onto unitaries \( U \) and \( W \) such that \( G_\chi = U \chi^{1/2} W \). Let \( \chi \) and \( \phi_1 \) be diagonal in the same basis, for simplicity; by varying over \( U \) and \( W \) we vary over \( G \) operators corresponding to all Schmidt bases, for given eigenvalues. Thus

\[
\max_{\text{unitary}, U, W} |\text{tr} \phi_1^{1/2} U \chi^{1/2} W| = \max_{\text{unitary}, U, W} |\langle \phi_1 | V \otimes Y | \chi \rangle| .
\]

A result of Von Neumann [10,11] states that this maximum occurs where \( \phi_1^{1/2} \) and \( \chi^{1/2} \) are codiagonal, and their eigenvalues are matched in order of size. This proves Lemma 1.

Now we show that there is an ensemble solving (1) which contains just one pure state. As just argued, we may confine our attention to ensembles of states Schmidt-codiagonal with \( |\phi_2\rangle \). Thus, the squared inner products whose average is the objective function in (4) are just squared Bhattacharyya overlaps

\[
B^2(p, q) := \left( \sum_i \sqrt{p_i q_i} \right)^2
\]

between the Schmidt eigenvalues \( p_i \) of \( |\phi_2\rangle \) and those \( (q_i) \) of the states \( |\eta_j\rangle \); since \( B^2 \) is concave in one argument [12], replacing the \( s_j \)-ensemble by the vector with the \( s_j \)-averaged Schmidt coefficients will increase the objective function. Moreover, this will keep the value of the constraint function unchanged. To be explicit, define

\[
|\eta| := \sum_i \sqrt{\sum_j s_j q_{ij}} |i\rangle\langle i|
\]

where \( q_{ij} \) are the ordered Schmidt coefficients of \( |\eta_j\rangle \). Then

\[
|\langle \phi_2 | \eta \rangle|^2 = \left( \sum_i \sqrt{p_i \sum_j s_j q_{ij}} \right)^2 \geq \sum_j s_j \left( \sum_i \sqrt{p_i q_{ij}} \right)^2 = \sum_j s_j |\langle \phi_2 | \eta_j \rangle|^2 .
\]

while (for any \( k \))

\[
S_k(|\eta|) = \sum_{i=1}^k (\sum_j s_j q_{ij}) = \sum_{i=1}^k s_j \left( \sum_{i=1}^k q_{ij} \right) = \sum_{k} s_j S_k(|\eta_j\rangle|\eta_j\rangle) .
\]

This argument doesn’t immediately extend to the multi-constraint problem. Each \( E_k \) is defined by an independent minimization over ensembles; therefore one can’t recast the multiconstraint problem as an single maximization over ensembles.

If we nevertheless assume the solution is pure in the original multi-constraint problem, it reduces to:

\[
\max_{q_1, ..., q_d} \sum_i \sqrt{p_i q_i} \\
\text{subject to :} \\
\zeta_k - \sum_{i=1}^k q_i \leq 0 \quad (k = 1, ..., d), \\
-\sum_{i} q_i - 1 \leq 0 ,
\]

where \( p_i \), \( q_i \), \( r_i \) are the reduced density matrix eigenvalues of \( |\phi_2\rangle \), \( |\chi\rangle \), and \( |\phi_1\rangle \) respectively, and \( \zeta_k := \sum_{i=1}^k r_k . \)
Since the objective function is concave in $q$ and the feasible set (since it is given by a conjunction of linear inequalities $g_i(q) \leq 0$) is convex, the Kuhn-Tucker conditions are necessary and sufficient for a maximum as long as constraint qualification holds. These are:

$$\frac{\partial f(q)}{\partial q_i} = \sum_j \lambda_i \frac{\partial g_i(q^*)}{\partial q_j}.$$  \hspace{1cm} (15)

Here $\lambda_i \geq 0$, with $\lambda_i = 0$ for slack constraints.

In our problem the objective function $f(q) := \sum_i \sqrt{p_i} q_i$ has derivatives

$$\frac{\partial f}{\partial q_i} = (1/2) \sqrt{p_i/q_i}.$$  \hspace{1cm} (16)

The positivity constraints on the $q_i$ will be slack because the partial derivatives go to $+\infty$ at $q_i = 0$.

Here is an example of the weak bound, using states from $\mathcal{B}$. These are:

$$|\phi_1\rangle = \sqrt{0.4}|00\rangle + \sqrt{0.4}|11\rangle + \sqrt{0.1}|22\rangle + \sqrt{0.1}|33\rangle$$

$$|\phi_2\rangle = \sqrt{0.5}|00\rangle + \sqrt{0.25}|11\rangle + \sqrt{0.25}|22\rangle.$$  \hspace{1cm} (17)

We will make the assumption that the optimal state is pure, which will turn out to be correct in this particular case. If we start with $|\phi_1\rangle$ and reallocate Schmidt coefficients from lower-weight to higher-weight basis states (so that the resulting state “majorizes” $|\phi_1\rangle$), it is clear we should take all of the weight from $|33\rangle$ and move it to a higher-probability state, say $|22\rangle$ to get its probability closer to $p_2$’s. I chose to allocate it to $q_2$ because the derivative of the objective function was largest with respect to $q_2$, even after the reallocation of all of $q_2$ to it. No further increase in $q_2$ is possible, since the weight would have to come from $q_0$ or $q_1$, and that would violate the majorization constraint $q_0 + q_1 = 0.8$. Reallocation that 0.8 optimally between the $q_0$ and $q_1$ (by equating derivatives of the objective function with respect to them) yields $q_0 = \sqrt{8/15}, q_1 = \sqrt{4/15}$. There is (of course) still no advantage to reallocating $q_2$ to either of the larger probabilities, since $\partial f/\partial q_2$ is still the largest derivative. Thus

$$|\chi^*\rangle = \sqrt{8/15}|00\rangle + \sqrt{4/15}|11\rangle + \sqrt{0.2}|22\rangle.$$  \hspace{1cm} (18)

Hence $p_e = |\langle \chi | \phi_2 \rangle|^2 = 0.9964102\ldots$. Moreover, here the only binding majorization constraint is $q_0 + q_1 \geq 0.8$.

Therefore, this also solves the problem with that constraint alone, and so provides a genuine upper bound on the protocol’s error probability. This is, of course, quite high, but since the error is one-sided it can be exponentially suppressed by repetition. For example, after two thousand repetitions we get $p_e = .0007522\ldots$. Still, computation of the stronger bound obtained from consideration of Alice performing the actual protocol is obviously desirable. For practical purposes, it might be very useful to find examples of catalyzable transformations $|\phi_1\rangle \rightarrow |\phi_2\rangle$ for which the error probability of any local approximation of the transformation is bounded much further below 1. It is natural to look in higher-dimensional systems for such examples. Such pairs of states must be incommensurable, and Nielsen has conjectured that incommensurability approaches being generic in high-dimensional systems. Whether catalyzability is equally common (or whether it is measure-zero even for finite-dimensional systems) is an interesting question. The distinguishability of a maximally entangled state from the separable states increases with dimensionality, with error-probability approaching zero. It is natural to ask whether the error probability for distinguishing states locally produced from a given state from some state locally producible only with the aid of a catalyst may be made to approach zero in some sequence of examples of increasing dimension.

Whatever the ultimate judgement on the catalyzed protocol from the cryptographic standpoint, the optimization problem discussed in this section, of finding the closest approximation to $|\phi_2\rangle$ obtainable by local quantum operations and quantum communication, given the state $|\phi_1\rangle$, is of independent interest.

VI. SECURITY

How secure is this protocol? The error probability above gives a measure. Since Alice may destroy her test states (the ones to be turned from $|\phi_1\rangle$ into $|\phi_2\rangle$) after each use, coherent eavesdropping involving these states may seem unlikely to be of any use in corrupting later uses of the protocol. However, one must investigate whether a Derek able to divert some of the quantum communications, and possibly also impersonate Alice or Bob during the classical discussion could redirect the catalyst state to himself and use it in later rounds. If Derek is a true man-in-the-middle, he could receive Alice’s test state, keep it, and send whatever he wanted to Bob. (He could even send half of a $|\phi_1\rangle$ state.) If he could also impersonate Alice and Bob on the classical channel, then he could in fact do the protocol with Bob (who would be using half of a Derek-supplied $|\phi_1\rangle$ state) and then send the resulting $|\phi_2\rangle$ state on to Alice. But that’s not “really” a problem with the protocol: Alice is indeed identifying herself. (Of course, Derek can jam her ability to i.d. herself, if he has this setup, but men-in-the-middle can always do that.) But can Derek steal the catalyst?

$^1$Classical hashing techniques could be used to prevent this, but this would require using up classical secret key, and, if crucial to security, might obviate some advantages conferred by the quantumness of the remainder of the protocol.
It’s somewhat plausible that he could. For, he can send whatever state he wants to Bob (even one entangled with a system Derek keeps), while keeping the state Alice sent him. Bob will measure this other state jointly with his end of the catalyst, and broadcast the result. Bob will even send the noncatalyst part of the measured system back to Derek. The basic elements present (entanglement between Derek and Bob, joint measurement by Bob of the system entangled with Derek and Bob’s part of the catalyst, and broadcast by Bob of the measurement result) are also present in a protocol for teleporting Bob’s part of the catalyst state to Derek. There is of course no guarantee (and probably no particular reason to think) that this is such a teleportation protocol. Quite possibly, anything Derek does to steal the catalyst will be likely to make the conversion procedure fail (after all, the simplest procedure for ensuring its success, which is for Derek to just act as a conduit for $|\psi_1\rangle$, cannot steal the catalyst). But a full analysis is required before claiming that the catalysis protocol is secure when reused (and reuse is necessary just to get the error probability acceptably low, in the example given above). It seems almost too good to be true that a phenomenon such as catalysis, closely linked to properties (e.g. incommensurability, in the operational sense of non-interconvertibility via LOCC) of finite-dimensional quantum states that disappear when the tasks defining them are defined asymptotically on large numbers of copies, should nevertheless be usable by repetition to achieve asymptotically useful results in a cryptographic task. But quantum information has surprised us before. And even if repeated authentication via catalysis turns out not to be secure, knowing this (and knowing why) will shed light on the phenomenon of catalysis and related information-theoretic concepts and tasks.

The teleportation attack scenario is not so worrisome in the case in which only “terminal security” is at issue, which is the one in which using the login protocol as a prelude to further communication makes the most sense. Nevertheless, it is worth investigating, if any level of channel insecurity exists: it would enable channel insecurity to become terminal insecurity over time. Also, in a model with no channel insecurity, it is questionable why one would want to use the catalysis protocol instead of the maximally-entangled protocol. For then the problem of using up the supply of entangled key may be solved by sharing more entangled states over the (supposedly secure) quantum channel during the period of high terminal security initiated by authentication. With any terminal insecurity, the security of the currently-used entangled states will decay with repeated sessions (some of the new entangled states may occasionally be transmitted to an impostor). Possibly this is not so for the catalyst states even with some terminal insecurity during a communication session, since once shared the catalyst states are never retransmitted; here, again, teleporting attacks are the issue.

VII. CONCLUSION

I have given some protocols with which two parties may share an entangled quantum state, and use it as a secure identification token. The simplest protocols just involve sharing a maximally entangled state of high-dimensional quantum systems; authentication is accomplished by one party’s sending the other her half of the state, which can then be distinguished by measurement from anything a disentangled impostor could present. Advantages over classical shared secrets might include theft detectability, uncopiability, and of course the ability to authenticate a quantum laboratory’s presence on a quantum network. The possibility of a protocol based on the ability of some shared entangled states to catalyze certain transformations between other shared entangled states (which would otherwise be impossible by local actions and classical communication) was also introduced. This occasioned some analysis of the problem, interesting in itself, of the best LOCC approximation to such a transformation. An interesting potential advantage of the catalysis protocol is repeatability without using up the identification token. Showing the security, or lack of it, of repeated use of this protocol could illuminate several interesting areas of the theory of quantum information and entanglement, in addition to shedding light on nature of the curious phenomenon of catalysis.

ACKNOWLEDGMENTS

Supported in part by NSF grant #PHY-9722614, and by a grant from the ISI Foundation, Turin, Italy, and Elsag-Bailey, a Finmeccanica company. Thanks to C. Bennett, C. Fuchs and L. Hardy for discussions and encouragement.

[1] M. N. Wegman and J. L. Carter, J. Computer and System Sciences 22, 265 (1981).
[2] J. L. Carter and M. N. Wegman, J. Computer and System Sciences 18, 143 (1979).
[3] S. Wiesner, SIGACT News 15, 78 (1983).
[4] C. H. Bennett, G. Brassard, S. Breidbart, and S. Wiesner, in Advances in Cryptology: Proceedings of Crypto '82 (Plenum Press, New York, 1982), pp. 267–275.
[5] M. Nielsen, LANL ArXiv quant-ph/9811053 (1999).
[6] G. Vidal, LANL ArXiv quant-ph/9902033 v2 (1999).
[7] D. Jonathan and M. Plenio, LANL ArXiv quant-ph/9903052 v2 (1999).
[8] L. Hardy, LANL ArXiV quant-ph/9903001 v2 (1999).
[9] D. Jonathan and M. Plenio, LANL ArXiV quant-ph/9905074 (1999).
[10] J. von Neumann, Tomsk. Univ. Rev. 1, 286 (1937).
[11] R. A. Horn and C. R. Johnson, *Matrix Analysis* (Cambridge University Press, Cambridge, 1985).
[12] C. Fuchs and J. van de Graaf, LANL e-print quant-ph/9712042 (1997), submitted to IEEE Transactions on Information Theory.
[13] H. R. Varian, *Microeconomic Analysis* (Norton, New York, 1978).