Neutron stars and quark stars: Two coexisting families of compact stars?

J. Schaffner-Bielich
Institut für Theoretische Physik/Astrophysik, J. W. Goethe Universität, Max-von-Laue Straße 1, D–60438 Frankfurt am Main, Germany

Abstract. The mass-radius relation of compact stars is discussed with relation to the presence of quark matter in the core. The existence of a new family of compact stars with quark matter besides white dwarfs and ordinary neutron stars is outlined.

1. Introduction

Exotic compact stars dubbed quark stars have been a topic not only of scientific but also of public interest, at least since the press release of NASA of 2002 on the possible existence of compact stars with unusually small radii and rapid cooling due to the presence of quark matter in the dense interior. Indeed, the fundamental theory of strong interactions, quantum chromodynamics (QCD), predicts that at large densities and/or temperatures a phase transition to a plasma of (free) quarks and gluons occurs. The phase diagram of QCD is depicted schematically in Fig. 1. The early universe passed through that phase diagram at (nearly) zero density and high temperatures. Neutron star matter, on the other hand, is located at comparably small temperatures and high densities. Interestingly, a first order phase transition is likely to be present at high densities as present in the core of neutron stars. This phase transition will be probed by the collision of heavy ions at GSI, Darmstadt with the future international facility FAIR.

2. Quark matter and quark stars

Quark matter at high densities is remarkably rich in structure due to the strong effects of pairing quarks to diquarks, the phenomenon of colour-superconductivity. The possible phases have been investigated selfconsistently recently for cold neutron star matter, as well as for hot, neutrino-rich proto-neutron stars, by [Rüster et al. (2005, 2006)] and are shown in Fig. 2. Note, that the phase boundaries have been determined by considering \(\beta\)-equilibrium. The labels in the figure stand for normal (unpaired) quark matter (NQ), two-flavour colour superconducting phase (2SC), gapless 2SC phase (g2SC, etc.), colour-flavour locked phase (CFL), gapless CFL phase (gCFL), which differ in their quark pairing pattern. The phase transition relevant for neutron stars is of first order due to symmetry arguments. Matter changes from the chirally broken phase (\(\chi_{SB}\)), massive quarks, to the (approximate) chirally restored phase with nearly massless quarks. The corresponding order parameter is the (chiral) quark condensate, which generates e.g. the mass of the nucleon besides the small current quark masses.

As a model for cold and dense QCD matter, one can consider basically two possibilities for a first-order chiral phase transition: A weakly first-order chiral transition (or no true phase transition) or a strongly first-order chiral transition. In the former case, there will be just one type of compact star (ordinary neutron stars), in the latter case two types of compact stars, as there exists a new stable and more compact solution with smaller radii.

The essential ingredient is the relation between the pressure and the energy density of the high-density (quark) matter. Usually, the MIT bag model is adopted which assumes that free quarks are confined inside a bag characterised by a vacuum energy density, the MIT bag constant. Interactions up to order \(\alpha_s^2\) of the strong coupling constant have been reconsidered recently by [Fraga et al. (2001)] using perturbative QCD and applied...
to quark stars. Interestingly, higher order corrections seem to mimic the effect of the vacuum energy density of the MIT bag model so that the mass-radius curves of both approaches are surprisingly similar. For one particular choice of one free parameter, the maximum mass and corresponding radius are \( M_{\text{max}} = 1.05 M_\odot \) and \( R_{\text{max}} = 5.8 \) km with a central density of \( n_{\text{max}} = 15 n_0 \). For another one, the values are \( M_{\text{max}} = 2.14 M_\odot, R_{\text{max}} = 12 \text{ km}, \) and \( n_{\text{max}} = 5.1 n_0 \). The latter case corresponds to absolutely stable strange quark matter, i.e. to so called strange (quark) stars (Witten 1984, Haensel et al. 1986, Alcock et al. 1986). Ordinary neutron stars can collapse to this energetically favoured state. Note, that the maximum mass of strange stars is comparable to ordinary stars, in fact it is even slightly larger than two solar masses. In the former case, quark matter is metastable and a surface layer of nuclear matter with a mixed phase is present, which constitutes a so called hybrid (quark) star.

There are a few other nonperturbative approaches on the market which consider pure quark stars within an equation of state of dense QCD matter using e.g. a Schwinger–Dyson model (Blaschke et al. 1999), massive quasi-particles (Peshier et al. 2000), the Nambu–Jona-Lasinio model (Hanauske et al. 2001), hard-dense loop resummation schemes (Andersen & Strickland 2002) and effects from colour superconductivity (Rüster & Rischke 2004). Historically, the first pure quark star was calculated by Itoh (1970) even before the discovery of asymptotic freedom of QCD.

For hybrid stars, one has to match the quark phase to the low-density hadronic phase. There are in principle two possible scenarios. The pressure in the hadronic phase rises strongly with the baryochemical potential (or baryon density) so that it smoothly matches onto that of the quark phase and the phase transition is weakly first order or a cross-over. Or the pressure increases slowly with the baryochemical potential so that there is a large mismatch in the slope of the hadronic and the quark phase and the phase transition is strongly first order. Interestingly, there are hints that asymmetric matter up to \( \sim 2n_0 \) has a relatively small pressure which points to the latter scenario (Akmal et al. 1998).

A strong first order phase transition in compact stars is of particular interest as it allows for compact star twins. Various mass-radius relation for the different scenarios outlined above are depicted in Fig. 3. The solid lines show the mass-radius relation for strange stars, which are selfbound and not bound by gravity so that the curve starts at vanishing small masses. The dashed lines stand for the mass-radius relations of ordinary (hadronic) neutron stars and hybrid stars, where the curve starts at some small mass at large radii. These compact stars are bound by gravity. If a phase transition to quark matter occurs at some critical density, the curve drops off changing its slope. For some smaller values of the onset density of quark matter, the slope changes again at small radii signalling a new stable branch of the mass-radius curve. So, if the phase transition is weak, there exists at most one stable branch of the new class of compact stars has been outlined already for the mass-radius relation of pure quark stars in perturbative QCD (solid lines) and neutron stars and hybrid stars (dashed lines) taken from Fraga et al. (2001).

As there are two other families of compact stars known, white dwarfs and neutron stars, the new solution constitutes a third family of compact stars. The possibility of this new class of compact stars has been outlined already by Gerlach (1968) and has been a subject of discussion.

**Fig. 2.** The possible phases in dense and hot quark matter (taken from Rüster et al. (2005)).

**Fig. 3.** The mass-radius relation of pure quark stars in perturbative QCD (solid lines) and neutron stars and hybrid stars (dashed lines) taken from Fraga et al. (2001).
in the literature since then, see e.g. the historical notes in Schaffner-Bielich (2003). In the context of modern and thermodynamical consistent description of the phase transition, the third family of solutions was rediscovered by Glendenning & Kettner (2000) and Schertler et al. (2000) and found its way into a standard textbook on compact stars (Glendenning 2000). The generic feature of the three families of compact stars is outlined in Fig. 4. The solid line from the right to point A stands for the first family, white dwarfs, which are stabilised by the electron degeneracy pressure. Then follows an unstable branch, until the neutron degeneracy pressure is high enough to support stable neutron stars from point B to C. Finally, the quark pressure allows for stable quark stars from point D to E, which are even more compact than ordinary neutron stars. Note, that the phase transition in neutron stars must be strongly first order to reach the new stable branch, otherwise the mass-radius curve terminates already in an unstable spiral along the points H and I. It is also important to realize that any strong first order phase transition will lead to a new stable branch of solutions, as the only input to the Tolman-Oppenheimer-Volkoff equation is the equation of state, irrespective of the underlying micro-physics (see e.g. Mach & Schaffner-Bielich (2005) for a purely parametric analysis). Indeed, the new solution has been found in a large variety of different approaches to the high-density equation of state, involving quark matter in the MIT bag model (Glendenning & Kettner 2000, Mishustin et al. 2003), massive quasi-particles of quarks (Schertler et al. 2000), interacting quarks in perturbative QCD (Fraga et al. 2001), Kaon condensation (Banik & Bandyopadhyay 2001), hyperon condensation (Schaffner-Bielich et al. 2002), and colour-superconducting quarks (Banik & Bandyopadhyay 2003).

3. Signals for quark stars

The existence of exotic matter in the core, a strong first order phase transition and the third family of compact stars can be tested by astronomical observations. We flash some of the many signals suggested in the following.

Certainly, the mass-radius of pulsars and non-pulsating compact stars allows for a unique test for the presence of a third family, which is related to the so called phenomenon of the rising twins (Schertler et al. 2000). Standard massive neutron stars have a smaller radius with increasing mass. On the other hand, a new stable branch allows for a pair of compact stars having the exact opposite behaviour, i.e. $M_1 < M_2$ with $R_1 < R_2$! A sizable mixed phase will lead to a spontaneous spin-up of pulsars (Glendenning et al. 1997) as the pulsars passes through the mixed phase during the standard spin-down evolution. The presence of exotic matter in general, be it strange quark matter, kaon condensation or hyperon matter, causes a delayed collapse of a proto-neutron star to a black hole (Pons et al. 2001) as the proto-neutron star allows for a higher maximum mass compared to the cold (exotic) neutron star. The collapse of a neutron star to the third family can lead to a catastrophic rearrangement thereby emitting gravitational waves, $\gamma$-rays, and a burst of neutrinos, see e.g. (Mishustin et al. 2003; Lin et al. 2006). The gravitational waves emitted from colliding neutron stars can exhibit features which are sensitive to the underlying equation of state including quark matter, be it for hybrid star (Oechslin et al. 2004) or strange star collisions (Limousin et al. 2005).

So far several types of compact stars have been discussed. Let us summarise the similarities and difference. Neutron stars with an ordinary hadronic mantle and quark matter in the core are called hybrid stars as there are two completely different phases in the interior of the compact star. I used the term quark star for a special class of hybrid stars which are located in a separate stable branch of the mass-radius diagram constituting a third family of compact stars besides white dwarfs and neutron stars. Those quark stars have for sure a strong first order phase transition and a pure exotic phase in the core, which is most likely but not necessarily restricted to quark matter. Finally, strange (quark) stars are selfbound stars consisting of absolutely stable strange quark matter only. They are purely hypothetical as they rely on the Bodmer-Witten hypothesis (Bodmer 1971, Witten 1984) that there is some exotic matter, strange quark matter, which is more bound than nuclei, so that nuclei are transformed to strange quark matter with releasing energy.

The hypothetical selfbound stars are characterised in general by a vanishing pressure at a finite energy density and a mass-radius relation starting at the origin (ignoring a possible nuclear crust) allowing for arbitrarily small masses and radii down to nuclear scales.

On the contrary, ordinary neutron stars and hybrid stars are bound by gravity where matter has a finite pressure for all energy densities. The corresponding mass-radius relation starts at large radii with a minimum neu-
tron star mass of \( M \sim 0.1M_\odot \) at \( R \sim 200 \) km. This minimum neutron star mass is dictated by the properties of the degenerate low-density gas of neutrons, electrons and nuclei. Note, that white dwarfs can have much smaller masses due to the formation of a lattice of nuclei surrounded by free electrons. The lattice energy for neutron stars is only marginally, if at all, able to support the compact star \cite{Baym1971}.

Selfbound strange stars have similar maximum masses and radii, with a similar nuclear crust, etc. But there are some features which are unique for strange stars. For example, they can have extremely small masses with small radii. White dwarfs with a core of strange star material can exist with unusual mass-radius relation \cite{Glendenning1995}. And bare, hot strange stars allow for super-Eddington luminosity as the material is bound by interactions and not by gravity (see e.g., \cite{Page2002}).

4. Summary

Our present understanding of dense matter in strong interactions opens the possibility of having a first order phase transition from ordinary matter to some exotic (quark) matter at high densities which is connected to chiral symmetry restoration. If the phase transition is strongly first order it generates a new, stable solution for compact stars besides white dwarfs and neutron stars. Note, that the new stable branch in the mass-radius diagram is not constrained by present mass-radius data! The transition to quark matter has not only impacts on the global properties of neutron stars and pulsars, but also on supernovae and proto-neutron star evolution and neutron star mergers. The third family of compact stars, quark stars, coexist with ordinary neutron stars, so it might be worthwhile to check for the possibility of having two different classes of pulsar families!

Acknowledgements. I thank the organisers for their kind invitation and especially Slavko Bogdanov, Okkie de Jager, Hovik Gregorian, Frank Haberland, Matthias Hempel, Michael Kramer, Wolfgang Kudritzki, Harald Lesch, Aristeidis Noutsos, Dany Page, Mal Ruderman, Irina Sagert, Morten Stejner, Joachim Trümper, Roberto Turolla, Christo Venter, Fridolin Weber, Slava Zavlin for the numerous and enjoyable discussions during the meeting.

References

Akmal, A., Pandharipande, V. R., & Ravenhall, D. G. 1998, Phys. Rev. C, 58, 1804
Alcock, C., Farhi, E., & Olinto, A. 1986, Astrophys. J., 310, 261
Andersen, J. O. & Strickland, M. 2002, Phys. Rev., D66, 105001
Banik, S. & Bandyopadhyay, D. 2001, Phys. Rev. C, 64, 055805
Banik, S. & Bandyopadhyay, D. 2003, Phys. Rev. D, 67, 123003
Baym, G., Pethick, C., & Sutherland, P. 1971, Astrophys. J., 170, 299
Blaschke, D., Grigorian, H., Poghosyan, G., Roberts, C. D., & Schmidt, S. M. 1999, Phys. Lett. B, 450, 207
Bodner, A. R. 1971, Phys. Rev. D, 4, 1601
Fraga, E. S., Pisarski, R. D., & Schaffner-Bielich, J. 2001, Phys. Rev. D, 63, 121702(R)
Gerlach, U. H. 1968, Phys. Rev., 172, 1325
Glendenning, N. K. 2000, Compact Stars — Nuclear Physics, Particle Physics, and General Relativity, 2nd edn. (New York: Springer)
Glendenning, N. K. & Kettner, C. 2000, Astron. Astrophys., 353, L9
Glendenning, N. K., Kettner, C., & Weber, F. 1995a, Astrophys. J., 450, 253
Glendenning, N. K., Kettner, C., & Weber, F. 1995b, Phys. Rev. Lett., 74, 3519
Glendenning, N. K., Pei, S., & Weber, F. 1997, Phys. Rev. Lett., 79, 1603
Haensel, P., Zdunik, J. L., & Schaeffer, R. 1986, Astron. Astrophys., 160, 121
Hanauske, M., Satarov, L. M., Mishustin, I. N., Stöcker, H., & Greiner, W. 2001, Phys. Rev. D, 64, 043005
Itoh, N. 1970, Prog. Theor. Phys., 44, 291
Lin, L.-M., Gondek-Rosinska, D., & Gourgoulhon, E. 2005, Phys. Rev., D71, 064012
Lin, L.-M., Cheng, K. S., Chu, M. C., & Su, W.-M. 2006, Astrophys. J., 639, 382
Mach, J. & Schaffner-Bielich, J. 2005, Eur. J. Phys., 26, 341
Mishustin, I. N., Hanauske, M., Bhattacharyya, A., et al. 2003, Phys. Lett. B, 552, 1
Oechslin, R., Uryū, K., Poghosyan, G., & Thielemann, F. K. 2004, Mon. Not. R. Astron. Soc., 349, 1469
Page, D. & Usov, V. V. 2002, Phys. Rev. Lett., 89, 131101
Peshier, A., Kämpfer, B., & Soff, G. 2000, Phys. Rev. C, 61, 045203
Pons, J. A., Steiner, A. W., Prakash, M., & Lattimer, J. M. 2001, Phys. Rev. Lett., 86, 5223
Rüster, S. B. & Rischke, D. H. 2004, Phys. Rev. D, 69, 045011
Rüster, S. B., Werth, V., Buballa, M., Schovkov, I. A., & Rischke, D. H. 2005, Phys. Rev., D72, 034004
Rüster, S. B., Werth, V., Buballa, M., Schovkov, I. A., & Rischke, D. H. 2006, Phys. Rev., D73, 034025
Schaffner-Bielich, J. 2005, J. Phys. G, 31, S651
Schertler, K., Greiner, C., Schaffner-Bielich, J., & Thoma, M. H. 2000, Nucl. Phys., A677, 463
Witten, E. 1984, Phys. Rev. D, 30, 272