In this work we investigate the scaling properties of quasiparticles of Pnictide with "half-Dirac" node under magnetic field in vortex state. By computing the density of states, we aim to find in vortex state the form of non-Simon-Lee scaling predicted for such system by several recent works in non-vortex state. We find by exact diagonalization of the Bogoliubov-de Genne Hamiltonian and finite size scaling a $N(E) \sim \sqrt{E}$ power law in the case without magnetic field which agrees with analytical prediction. We consider the vortex state by first studying the hypothetical situation of uniform magnetic field without vortices and then we properly treat the magnetic field-induced vortex lattice by expressing the BdG Hamiltonian in terms of superfluid velocity and Berry’s gauge fields. The two calculations are shown to agree with each other. We then analyze quantitatively, the effects of anisotropic dispersion to the scaling properties of vortex lattice and show that the vortex lattice spacings will scale as $d'_y \sim s^{-\eta}d'_x$ where $\frac{1}{2} \leq \eta \leq 1$ and $s \sim H^{\frac{2}{3}}$ as compared to $\eta = \frac{1}{2}$ from dimensional scaling analysis of non-vortex state. A very crucial prediction is also made on an upper bound to the value of 'anomalous dimension' $\delta$ of density of states scaling with magnetic field which we find to be $\delta \leq \frac{1}{2}$, a quantity that could not be determined conclusively by previous purely analytical works and a quantity that can be measured experimentally.
I. Introduction

The discovery of the new family of high $T_\text{c}$ superconductors in iron-based compounds [1] has spurred tremendous theoretical and experimental activities to understand the mechanism of superconductivity and its properties. One of the main issues in this case is the pairing gap symmetry. It is believed that the system has $s_\pm$ pairing symmetry [2] with opposite sign between $s$-wave gap in the electron pocket and that in hole pocket. However, it has been suggested from first principle calculations [3] that electron pocket gap function is anisotropic and can have gapless points rather than being just simply isotropic $s$-wave. The existence of gapless point has crucial importance as it can give rise to quasiparticles that have significant implications on the thermodynamic and electronic properties of the superconductor which can be measured in experiment. It is therefore of particular interest to study the implications of this electron pocket gap anisotropy and the associated quasiparticles. In a ground-breaking work, V. Stanev et al. [4] showed that, for a critical case where the anisotropic gap function has gapless point which just touches Fermi surface, the Pnictide superconductor is expected to have nontrivial scaling properties that is distinct from the standard Simon-Lee scaling [5] applicable to $d$-wave superconductor quasiparticle. This prediction was soon verified by R. Fernandes et al. [6] who extended the result to those for more general geometries of electron Fermi pocket and studied in more detail the nature of quantum phase transition which was also predicted in Ref. 4. The Simon-Lee scaling law says that for nodal quasiparticle spectrum of $d$-wave superconductor under magnetic field $H$, the energy eigenvalue scales as $E_n(H) \sim H$ and the density of states scales as $N(E,H) \sim \sqrt{H}$. This is true for nodal quasiparticles of $d$-wave Cuprate superconductors, where the energy spectrum is isotropic around the node, that is, energy varies linearly with momentum away from the node in all directions. In Pnictide, in a two-band model with extended $s_\pm$ wave pairing symmetry, the electron pocket Fermi surface has anisotropic gap function that can be modeled as $\Delta_\text{p} = \Delta_0 + \Delta' \cos(4\theta)$ where $\theta$ is the angle around the $M = (\pi, \pi)$ point of electron pocket in reduced Brillouin zone, corresponding to 2 Fe atoms per unit cell of FeAs lattice. With this gap anisotropy of electron pocket Fermi surface, we can have line of quantum phase transition by varying the parameter ratio $\frac{\Delta_0}{\delta_0}$. This line of QPT occurs at $\frac{\Delta_0}{\delta_0} = 1$ where there is transition from spectrum with node for $\frac{\Delta_0}{\delta_0} > 1$ to spectrum with zero right at the QPT line $\frac{\Delta_0}{\delta_0} = 1$ to nodeless (fully gapped) spectrum for $\frac{\Delta_0}{\delta_0} < 1$. One important finding [4] is that, around the zero, occurring for example at momentum $\mathbf{q} = (p_F, 0)$ measured from the $M = (\pi, \pi)$ point, the Pnictide BdG Hamiltonian without magnetic field

$$H = \left( \frac{p^2}{2m} - \epsilon_F \right) \begin{bmatrix} \Delta & \hat{\Delta} \\ \hat{\Delta}^* & -\frac{p^2}{2m} + \epsilon_F \end{bmatrix} \approx \begin{pmatrix} v_F p_x & \frac{8\Delta_0 p_x^2}{p_F} \\ \frac{8\Delta_0 p_y^2}{p_F} & -v_F p_x \end{pmatrix}$$

where $\Delta = \frac{8\Delta_0 p^2}{p_F}$ leads to anisotropic energy dispersion, $E = \sqrt{(v_F p_x)^2 + \left(\frac{8\Delta_0 p^2}{p_F}\right)^2}$. This anisotropy leads to an entirely new scaling behavior that is distinct from that of Simon and Lee. Following the same procedure used by Simon and Lee, it is shown in [4] that in the absence of magnetic field, the density of states (DOS) scales as $N(E) \sim \sqrt{E}$ rather than $N(E) \sim E$ in Simon-Lee scaling law. The corresponding new scaling laws for other thermodynamics properties, for Pnictide still in the absence of magnetic field, follow accordingly. In the presence of magnetic field applied perpendicular to the plane of superconductor, the situation is much less trivial since there will be vortices. Authors in Ref. 4 showed that even without solving the vortex state Bogoliubov-de Genne(BdG) problem exactly, we can readily get the scaling behavior of spectrum and thermodynamic properties. It is shown that the energy eigenvalues and density of states at low energy (if magnetic field can create quantum states at low energy) at low field scales as $E_n(H \to 0) \sim H^\frac{1}{2}$ and $N(E = 0, H \to 0) \sim H^{\frac{1}{4} + \delta}$ with a fascinating possibility for the existence of nonzero anomalous scaling dimension $\delta$. This possibility of anomalous dimension can supposedly be associated with the fact that the spectrum is anisotropic. The authors in Ref. 4 have used at least two methods to compute the value of $\delta$, one using the semiclassical argument due to G. E. Volovik [7], which gives $\delta = -\frac{1}{12}$ and another method based on the analogy of this problem with the Schrödinger problem of particle inside a box of dimension equal to magnetic length $l = \sqrt{\frac{eB}{\hbar^2}}$ which gives $\delta = \frac{1}{6}$. There is apparently disagreement on the value of this anomalous dimension obtained from the two approaches. It is our goal to use another method to calculate this anomalous scaling dimension. So far the
case where we have vortices in the intermediate state $H_{c1} < H < H_{c2}$ has not been studied and this is what we pursue in this work. We here analyze the problem systematically by first considering a simpler though unphysical situation of Pnictide under magnetic field but without vortices in Section II. We then do the actual vortex state calculation in Section III and discuss the main results in Section IV. We especially focus on the computation of density of states in this paper as it suffices to verify the novel non-Simon-Lee scaling and leave the calculation of other properties, e.g. thermodynamic or magnetic properties, for future work.

II. Density of States of Pnictide half-Dirac Nodal Quasiparticles under Magnetic Field but without Vortices

We consider here a simpler problem of Pnictide under magnetic field but without vortices. We first add magnetic field to the BdG Hamiltonian (1) approximated to the lowest order around the zero at $q = (p_F, 0)$ and choose Landau gauge describing a uniform magnetic field given as $A_x = H y$ and $A_y = 0$ to obtain the following Hamiltonian,

$$H = \left( v_F(p_x + \frac{eH y}{c}) \frac{\Delta}{\hat{\Delta}} - v_F(p_x - \frac{eH y}{c}) \right) \tag{2}$$

with $\hat{\Delta} = \frac{8\Delta_0 p_F^2}{F_p^2}$ which represents the peculiar half-Dirac node character of Pnictide electron gap function we are considering here. A different choice for the gauge, e.g. symmetric gauge $A = \frac{B}{2}(y, -x)$, will not change the physics and the final conclusions since the system described by Hamiltonian (2) is gauge invariant. We diagonalize this Hamiltonian exactly in plane wave basis on a finite size system with periodic Born-von Karman boundary condition where the wave vectors are given by $k_m = \frac{2\pi n}{L}$ with $L$ as the finite system size. In Fig. 1, we present the density of states $N(E)$ and cumulative density of states defined as $K(E) = \int_0^E N(E')dE'$ of the half-Dirac nodal quasiparticles. If at low energy the density of states $N(E)$ has power law dependence on $E$ as $N(E) \sim E^\alpha$, then the cumulative density of states has power law scaling as $K(E) = \int_0^E N(E')dE' \sim E^{\alpha+1}$. In the case with magnetic field, this generalizes to $K(E, H) = \int_0^E N(E', H)dE'$, where $N(E, H)$ is the corresponding density of states in magnetic field $H$.

The parameters used in producing the results shown in Fig. 1 are $p_F = 2.0, E_F = p_F^2/2m = 2.0, \Delta_0 = 0.0001$ where $p_F$ is the Fermi momentum, $E_F$ is the Fermi energy, and $\Delta_0$ is the gap amplitude. The unit system is chosen in such a way that one unit of mass $m_0$ equals $m_0 = 10^{-31}kg m/s$, one unit of length $l_0$ equals $l_0 = 10^{-6}m = 1\mu m$, one unit of velocity $v_0$ (which fixes one unit of time) equals $v_0 = 10^6 m/s$ which is chosen to be of the same order as typical electron Fermi velocity in metals and from these follows one unit of energy $E_0$ which we define to be $E_0 = \frac{p_0^2}{2m_0} = 0.3125eV$ and one unit of magnetic field $H_0$ which equals $H_0 = 1T$. We observe from Fig. 2 that at zero magnetic field, the density of states increases from zero at zero energy while the one with finite magnetic field increases from some finite value at zero energy. Performing a finite size scaling where the result from finite size system is extrapolated to the infinite size limit as shown in Fig. 2(a), we firmly establish that in the absence of magnetic field, with the Hamiltonian as given by equation (2), the density of states increases as power law $N(E) \sim E^c$ where $c$ is numerically convergent to 0.5. This agrees with the result of theory of non-Simon-Lee scaling [4,6] for the quasiparticles of Pnictide with half-Dirac node which predicts that for the case of superconductor without magnetic field, the density of states should increase with energy as $N(E) \sim \sqrt{E}$. This is a faster increase than that of the $d$-wave Cuprate case and this originates from the parabolic part of the dispersion which turns out to give rise to more quasiparticles to appear in the vicinity of half-Dirac node than
a full-Dirac node does in \(d\)-wave Cuprate superconductor.

\[ N(E, H) = AH^\gamma (E^{\frac{1}{2}} H^{-\gamma} + B) \]  

(3)

where \(A\) and \(B\) are the constants with appropriate dimension to be determined from numerics. This implies \(f(E, H) = A(E^{\frac{1}{2}} H^{-\gamma} + B)\). The reason for this choice is that \(N(E, H)\) correctly reduces to \(N(E, H) \sim E^{\frac{1}{2}}\) at \(H = 0\) and to \(N(E, H) \sim H^\gamma \sim H^{\frac{1}{3} + \delta}\) at low energy as required from dimensional analysis. In Fig. 3, we show the fitting of the numerical results for \(N(E, H)\) with power law scaling function.

With finite magnetic field, where the Hamiltonian is given by equation (2), the density of states at \(E = 0\) acquires finite value, as illustrated in Fig. 1(b). We will verify here whether, as predicted in [4], the low energy weak field density of states \(N(E = 0, H)\) increases with the field \(H\) as power law \(N(E = 0, H) \sim H^\gamma\) and compute precisely the critical exponent \(\gamma\). In Fig. 2 we show \(N(E = 0, H)\) vs. \(H\) and argue that it represents a power law dependence. The half-Dirac nodal quasiparticle density of states as function of energy and magnetic field can be written as \(N(E, H) = H^\gamma f(E, H) = E^{\frac{1}{2}} g(E, H)\) where \(\gamma = \frac{1}{3} + \delta\), \(f(E, H)\) and \(g(E, H)\) are dimensionless universal homogeneous scaling functions. The exponent of \(H\) and \(E\) to lowest order is determined by dimensional analysis and elementary analysis of half-Dirac nodal quasiparticle anisotropic dispersion as was first done in Ref. 4. To obtain the actual critical exponent for power law dependence on magnetic field, we fit the finite zero-energy value of density of states as shown in Fig. 2 with the following scaling function,

\[ \text{FIG. 2 (a) A finite size scaling for the critical exponent } c \text{ of scaling law } N(E) \sim E^c \text{ with } N \text{ as the system size for a Pnictide with half-Dirac node in the absence of magnetic field (zero field) (b) The density of states vs. magnetic field for a Pnictide with half-Dirac node under magnetic field but without vortices, obtained from exact diagonalization of Hamiltonian in equation (2).} \]

\[ \text{FIG. 3 The power law fitting of density of states as function of magnetic field without vortices} \]

Fig. 3 shows that the best fitting gives a power law scaling exponent \(\gamma = \frac{1}{3} + \delta \simeq 0.84\) which implies an ‘anomalous dimension’ of \(\delta = 0.5066\) which is relatively close to \(\delta = 0.50\). This is to be compared with computation of the anomalous dimension using Schrödinger particle in box argument which yields \(\delta = \frac{1}{6}\) or \(\gamma = \frac{1}{2}\) and using semiclassical argument ala Volovik which yields \(\delta = -\frac{1}{12}\) or \(\gamma = \frac{1}{4}\) as stated in Ref. 4. This value of \(\delta = \frac{1}{2}\) is one of the main results in this paper that can be of particular interest to experimentalists to measure directly from experiment. It is to be seen later that this prediction from the hypothetical case of the strongly type II Pnictide in magnetic field but without vortices turns out to very remarkably agree with the actual vortex state analysis presented in the next section.
III. Density of States of Half-Dirac Nodal Pnictide Quasiparticles in Vortex State

In the actual vortex state of superconductor under magnetic field, we are dealing with spatially dependent order parameter as it acquires complex phase factor whose spatial variation describes the vortex lattice configuration. The phase $\phi(r)$ of the order parameter in real space $\Delta(r) = |\Delta(r)|e^{i\phi(r)}$ has to satisfy the vorticity condition $\nabla \times \nabla \phi(r) = 2\pi \sum_{n=1}^{N} \delta(r - r_i)$ and is in general spatially varying function in both amplitude and phase parts. However, for strongly type II Pnictide superconductors, where inter vortex separation is much larger than vortex core size, we can assume that the amplitude is constant and we are left with spatially varying phase function $\phi(r)$. The Hamiltonian will also contain gauge field which can be fixed with spatially varying phase function $\phi(r)$. The definition (8) alone already represents the periodicity of vortex lattice. The spectrum of the velocity field is thus given by $\textbf{V}_\phi^\mu(\textbf{Q}) \sim \frac{\textbf{Q} \times \textbf{z}}{Q^2} e^{-iQ \cdot \textbf{R}_0^\mu}$. The magnitude of which decreases with wave vector $Q$. From this Fourier representation, we can easily derive the Fourier representation of velocity and Berry gauge fields and hence the Hamiltonian in Fourier space.

The magnetic field is not uniform and vanishes everywhere except at the centers (the cores) of the vortices where the magnetic field penetrates through the superconductor and the order parameter vanishes. We treat this periodic arrangement of vortices by rewriting the BdG Hamiltonian in terms of superfluid velocity and gauge fields and are thus weak with respect to the ‘unperturbed’ Hamiltonian. Upon performing

$$
H = \left(\frac{(p - D^2)}{2m} - \epsilon_F \right) + \frac{\hat{\Delta}}{2m} \phi(r)$$  (4)

where $\hat{\Delta} = \frac{8\Delta}{p_F} \left\{ \partial_y, \partial_y, \Delta e^{i\phi(r)} \right\} + \frac{2i}{p_F} \Delta(r) \partial_y^2 \phi(r)$ is the gauge-invariant gap operator, we perform Franz-Tesanovic transformation [8,9,10] on this Hamiltonian, defined as

$$
H \to U^{-1} H U \\
$$  (5)

where

$$
U = \begin{pmatrix}
e^{i\phi_A(r)} & 0 \\
0 & e^{-i\phi_B(r)}
\end{pmatrix}
$$

and expansion around $\textbf{q} = (p_F, 0)$, we have

$$
H' = \left( v_F(p_x + a_x) + mp_F v_{sx} \right) + \frac{8\Delta}{p_F} (p_y(p_y + 2mv_{sy} + 3a_y)) - v_F(p_x + a_x) + mp_F v_{sx}
$$

where the superflow and gauge fields are defined respectively as $v_s = \frac{1}{2}(v_s^A + v_s^B)$ and $a = \frac{m}{2} (v_s^A - v_s^B) = \frac{\hbar}{2} (\nabla \phi_A - \nabla \phi_B)$ with $v_s^\mu = \frac{1}{m} (h \nabla \phi_\mu - \frac{e\textbf{A}}{c})$. We can write $H'$ in terms of Pauli matrices,

$$
H' = mp_F v_{sx} \sigma_0 + v_F(p_x + a_x) \sigma_3 + \frac{8\Delta_0}{p_F} (p_y(p_y + 2mv_{sy} + 3a_y)) \sigma_1
$$

We progress by writing the Fourier representation for superflow velocity field

$$
\textbf{V}_s^\mu(\textbf{Q}) = \frac{2\pi}{md^2} \sum_{Q \neq 0} \frac{i\textbf{Q} \times \textbf{z}}{Q^2} e^{-i\textbf{Q} \cdot \textbf{R}_0^\mu} (\textbf{v}_s^0 + i\textbf{Q} \cdot \textbf{r})
$$

where $\mu = A, B$. We define our vortex lattice as a square vortex lattice with sublattices A and B in such a way that $\textbf{R}_0^A = -\textbf{R}_0$ and $\textbf{R}_0^B = \textbf{R}_0$. The dependence on magnetic field $H$ enters through lattice spacing of the vortex lattice which in general is of the order magnetic length $d \sim l_B = \sqrt{\frac{\pi \hbar}{eH}}$. The definition (9) alone already represents the periodicity of vortex lattice. The unperturbed Hamiltonian (7) into two parts. The first part, the unperturbed one, is

$$
H_0 = \left( v_Fp_x \frac{8\Delta_0p_y^2}{p_F^2} \right)
$$

While the second part, the perturbed Hamiltonian, is

$$
H'_0 = \left( v_Fa_x + mp_F v_{sx} \right) + \frac{8\Delta}{p_F} (p_y(2mv_{sy} + 3a_y)) - v_Fa_x + mp_F v_{sx}
$$

We include the terms with $p_y$ in the off-diagonal elements here because they are first order in the weak velocity and gauge fields and are thus weak with respect to the 'unperturbed' Hamiltonian. Upon performing
Fourier transformation using (9) and with the definition of full lattice superflow velocity and gauge fields, the BdG equation in vortex state is

\[
H_0(q) \left( \begin{array}{c} \Psi_{1q} \\ \Psi_{2q} \end{array} \right) + \sum_{q \neq Q} H'_f(q + Q) \left( \begin{array}{c} \Psi_{1q+Q} \\ \Psi_{2q+Q} \end{array} \right) = E \left( \begin{array}{c} \Psi_{1q} \\ \Psi_{2q} \end{array} \right) \tag{12}
\]

where $\Theta = Q \cdot R_0$. For Bravais lattice, as is the case for our square vortex lattice, $R_0 = \left( \frac{a_1}{4} + \frac{a_2}{4} \right)$ and with $Q = (\frac{m_x}{d_x} \hat{b}_1 + \frac{m_y}{d_y} \hat{b}_2)$ and $a_i b_j = 2\pi \delta_{ij}$. This implies that the only reciprocal lattice wave vectors that contribute to the quasiparticles dynamics in vortex state are given by $(Q_x, Q_y) = 2\pi (\frac{m_x}{d_x}, \frac{m_y}{d_y})$. Notice carefully that we have to assign different variables for the lattice spacing in $x$ and $y$ directions $d_x$ and $d_y$ respectively to anticipate the anisotropy of the system that will manifest also in vortex state. This is a crucial difference from the $d$-wave case of Cuprate where the $x$ and $y$ directions are scaled isotropically and so we can use the same lattice spacing $d$ if we assume a square vortex lattice. This again just reflects the anisotropy of the system that we have. We therefore have $\Theta = 2\pi (\frac{m_x}{d_x}, \frac{m_y}{d_y}) \frac{1}{4} (d_x, d_y) = \frac{\pi}{2} (m_x, m_y)$. The original Schrödinger equation for this problem is then eventually equivalent to Bloch equation [11] which has the following generic form,

\[
\left( \frac{\hbar^2}{2m} (\mathbf{k} - \mathbf{K})^2 - \epsilon \right) c_{\mathbf{k} - \mathbf{K}} + \sum_{\mathbf{K}'} U_{\mathbf{K}' - \mathbf{K}'} c_{\mathbf{K}' - \mathbf{K}'} = 0 \tag{13}
\]

Here $\mathbf{k}$ is plane wave vector inside the first Brillouin zone satisfying Born-von Karman (periodic) boundary condition while $\mathbf{K}$ is the reciprocal vortex lattice vectors which are infinite in number if we consider infinite system. Bloch equation describes a system with periodic potential and here the periodic potential is given by the superflow velocity and Berry’s phase gauge fields arising from the periodic vortex lattice. It needs to be pointed that such periodic potential representation is valid in this problem only if the average magnetic field is zero as is the case here. Solving Bloch equation simply means diagonalizing the Hamiltonian in momentum space expanded with respect to the Bloch wave vectors in the first Brillouin zone of vortex lattice which is accomplished here numerically.

IV. Scaling Analysis of Low Energy Density of States Dependence on Magnetic Field in Vortex State

In this section, we will describe the effects of anisotropy to the quasiparticles scaling in vortex state. Looking at the approximated Hamiltonian in equation (2),

\[
H = \left( \begin{array}{c} v_F (p_x + \frac{eHy}{c}) \\ -v_F (p_x - \frac{eHy}{c}) \end{array} \right) \hat{\Delta} \tag{14}
\]

with $\hat{\Delta} = \frac{s \Delta_0 r^2}{\rho_F}$, we see that the Hamiltonian will be homogeneously-scaled if we rescale the variables as follows,

\[
\begin{align*}
p_x &= s p_x' \\
p_y &= s^{1/2} p_y' \\
y &= s y' \\
x &= r x'
\end{align*} \tag{15}
\]

where $r$ is yet undetermined and is to be set later. This way, the Hamiltonian can be written as $H = s^n H'$ for some exponent $n$. This suggests that in all later calculations, a proper treatment should keep the Hamiltonian writable in such form and so transformations (15) should be used. The eigenvalues will correspondingly transform as $E = s^n E'$. We can see here that the momentum space is anisotropically-scaled and so is the real space, in general. This just reflects the anisotropy of the system. This has crucial importance to the vortex state calculation since it now implies that the reciprocal vortex lattice should also obey (15). That is, the momenta in $p_x$ and $p_y$ directions in reciprocal lattice are not scaled the same way and so this implies the vortex lattice spacings in $x$ and $y$ directions are also rescaled differently. Let us consider again the vortex state Hamiltonian (7) which basically describes the quasiparticles dynamics under periodic superfluid velocity and Berry’s phase gauge fields to the lowest (1st) order in magnetic field strength. Looking at the definition of the fields given just after equation (7), considering $v_{sx}$, we would naively expect that the $x$ coordinate transform as $x = r x'$ but the off-diagonal terms of the vortex state Hamiltonian requires us to have for $a_y$ and $v_{sy}$, $a_y = s^{1/2} a_y$, $v_{sy} = s^{1/2} v_{sy}$ in order to have $H = s^n H'$. Let us now consider the explicit definition of superfluid velocity component $v_{sy}$ as given by its Fourier series representation in equation (9) which can be rewritten as follows,
\[
\nu_{sp}(r) = \frac{2\pi}{md^2} \sum_{Q \neq 0} \frac{-iQ_x}{Q_x^2 + Q_y^2} e^{-iQ \cdot R_0} + iQ \cdot r
\]

\[
= \frac{2\pi}{md^2} \sum_{Q \neq 0} \frac{-isQ'_x}{s^2Q_x'^2 + sQ_y'^2} e^{-iQ \cdot R_0} + iQ \cdot r
\]

(16)

Focusing on the Fourier coefficient given by term in the fraction and using the value of reciprocal wave vector for a periodic boundary condition given after equation (2), \((Q_x, Q_y) = 2\pi \left( \frac{\partial n}{\partial x}, \frac{\partial n}{\partial y} \right)\), the denominator immediately suggests that the vortex lattice spacings satisfy \(d'_y \sim s^{-\frac{1}{2}} d'_x\). We aim to have a rescaling of the form \(v_{sy} \sim s^{1/2} v_{sy}'\) and this will require spacing \(d\) to satisfy \(d' \sim s^{1/4} d\) while imposing condition \((a_x, v_{sx}) \sim s(a'_x, v_{sx}')\) on \((a_x, v_{sx})\) appearing on the diagonal elements will lead to another condition of the form \(d'_y \sim s^{-1} d'_x\). We see that we have two different scaling conditions for the same quantities; \(d'_y \sim s^{-\frac{1}{2}} d'_x\) and \(d'_y \sim s^{-1} d'_x\). Apparently, to this lowest order of approximation for vortex state Hamiltonian, there is no a set of variable scaling transformations that can make the Hamiltonian homogeneously scaled. We interpret this as a new requirement that the actual scaling in the true vortex state should eventually take a form \(d'_y \sim s^{-\eta} d'_x\) where \(\frac{1}{2} \leq \eta \leq 1\). This is the precise rescaling that we have to use in doing vortex state calculation. This result is to be compared with that of dimensional scaling analysis for non-vortex state of Ref. 4 where redefinition of coordinate variables \(x' = x/d\) and \(y' = yd/l^2\) (using Ref. 4’s notations, \(d = (l^4/a)^{\frac{1}{2}}, a = \frac{8\Delta}{v_F v_F'}\) and \(l = \sqrt{\frac{eF}{\mu}}\)) implies coordinate rescaling of the form \(x' = sx\) and \(y' = s^{1/2} y\) and this corresponds to \(\eta = \frac{1}{2}\). The difference in the exponents from those of our vortex state is to be noted here. Thus the dimensional scaling analysis exponent from Ref. 4 actually corresponds to the lower bound of the true vortex state exponent. This point is the key distinction between dimensional scaling analysis and true vortex state scaling. It is to be noted that the vortex lattice in real space, assuming a square lattice, is not physically deformed, e.g. into a rectangular lattice due to this anisotropic scaling. This peculiar result nevertheless takes effect (and thus must be correctly taken into account) instead internally in calculations involving the Hamiltonian such as density of states or band structure calculations. One interesting open problem is then what will happen if the vortex lattice is infinitely anisotropically rescaled, i.e. the dimension in one direction is infinitely larger than that in the perpendicular direction with regard to power law of those thermodynamic or electronic properties.

We present in Fig. 4 the result of calculations for density of states of half-Dirac nodal Pnictide quasiparticles in vortex state. In the plots that follow, we use \(v_F = 100.0\) and \(v_\Delta = 100.0\) where \(v_F\) is the Fermi velocity, \(v_\Delta = \frac{\Delta_0}{v_F}\) is the "gap velocity". The unit system is the same as that used in Section II. In the following plots, \(d\) represents the vortex lattice spacing which is taken to be of order of magnetic length.

FIG. 4 (a) The density of states and (b) the (low energy) cumulative density of states of half-Dirac nodal Pnictide quasiparticles in vortex state with \(v_F = 100.0, v_\Delta = 100.0\), and \(d = 10\) (c) The critical exponent \(\beta\) defined in equation (18) as function of lattice spacing \(d\).

It is found that in most range of the strengths of magnetic field, the density of states in vortex case vanishes at zero energy despite relatively strongly fluctuation at finite energy. This behavior agrees perfectly with all past works on nodal \(d\)-wave Cuprate superconductor [8, 9, 10]. With such fluctuating density of states, it is
crucial to define a correct scaling law for the density of states that we propose to be valid even in this vortex phase. In this case, we assume the global trend or average of the fluctuating density of states at any given energy and magnetic field to obey a power law. We then write the power law as follows,

\[ N(E, H) = \alpha E^{\frac{2}{3}} (H^{\frac{2}{3}} E^{-\frac{1}{3}})^{\beta} \] (17)

which perfectly agrees with the observed behavior of \( N(E, H) \) shown in Fig. 4(a). The justification for this scaling form is that \( N(E, H) \) correctly reduces to the dimensional analysis result for \( \gamma = 1 \). As before, we better consider the cumulative density of states which will give entirely equivalent conclusion as the density of states does. The cumulative density of states from (17) is then given by

\[ K(E, H) = \int_0^E N(E', H) dE' = \frac{2\alpha}{3 - \beta} E^{2(3 - \beta)} H^{\frac{\beta}{3}} \] (18)

and in this form the critical exponent \( \beta \) is now what we are looking for. In Fig. 4(c) we show the exponent \( \beta \) as a function of lattice spacing \( d \). Since as mentioned before the lattice spacing of the order of magnetic length \( d \sim l_B = \sqrt{\frac{e}{\pi n}} \), we see that the critical exponent is very small at very strong magnetic field. But then, as we decrease the magnetic field, the critical exponent increases and finally, at the limit of very large \( d \sim l_B = \sqrt{\frac{e}{\pi n}} \), or very weak magnetic field, the density of states critical exponent settles down to a stable value \( \beta \sim 2.5 \). The equilibrium value of critical exponent \( \beta \sim 2.5 \), looking at the form of scaling law in equation (17), implies a power law dependence on magnetic field with critical exponent \( \gamma = \frac{\beta}{3} \sim 0.8333 \). This comes in remarkable agreement with the result from Section II where we considered the hypothetical situation of Pnictide under magnetic field but without vortices and obtained \( \gamma \sim 0.84 \). One can say that the calculation in Section II and that in Sections III and IV represent two different levels of solution to the same vortex state problem which give correction to the dimensional scaling analysis and which turn out to give scaling law which clearly agree with each other between the first two. This eventually supports the 'hand waving' argument that the authors in Ref. 4 used to derive the proposed non-Simon-Lee scaling law. This striking agreement between the two values obtained from two completely different methods provides yet another support for the validity of our result for this experimentally measureable and theoretical crucial 'anomalous dimension' that accompanies the novel non-Simon-Lee scaling law for half-Dirac nodal quasiparticles. This result for anomalous dimension which gives correction to the critical exponent obtained from simple dimensional analysis seems to be rather large, but since in this work we treat the vortex state properly, we can argue that the value of 'anomalous dimension' predicted here should be more valid than the alternative results estimated in Ref. 4. Since the critical exponent computed in this work converges to a value that is smaller in the infinite size limit than the value for a finite size system, as illustrated in Fig. 2(a), it is more accurate to say that the anomalous dimension \( \delta \) obtained here is an upper bound to the actual value one might obtain in experiment while the lower bound is \( \delta = -\frac{1}{3} \). This result has direct relevance to experiment where this quantity can be measured directly. However, since half-Dirac node is a critical point that can only be obtained by fine tuning, it is highly dependent on the experimental feasibility to adjust control parameters in order to be able to verify the predictions made in this and previous works.

V. Conclusions

In conclusion, we have verified numerically the non-Simon-Lee scaling law predicted for Pnictide with half-Dirac-Lee scaling law predicted for Pnictide with half-Dirac quasiparticles. This result for anomalous dimension which gives correction to the critical exponent from of dimensional scaling analysis in terms of 'anomalous dimension'. The computed density of states in vortex state with proper treatment of the superfluid velocity and Berry's phase gauge fields of the vortices follows a power law that also leads to a prediction of the value of anomalous dimension and serves as the next level of correction to the dimensional scaling analysis result. The difference between scaling anisotropy in true vortex state and that from dimensional scaling analysis is elucidated. In summary, this work provides another verification and further progresses the investigation of the hypothesized non-Simon-Lee scaling proposed for such critical half-Dirac nodal Pnictide.
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