Parameter Optimization of PSS Using a Novel Simplex-Simulated Annealing Approach

Yanhong Chai\textsuperscript{1}, jingxuan Chai\textsuperscript{2,*}
\textsuperscript{1}Ningxia College of Finance and Economics, Ningxia 750000, China\n\textsuperscript{2}China University of Mining and Technology (Beijing), Beijing 100083, China
\*Corresponding author e-mail: cjx6666@vip.163.com

Abstract. A new simplex-simulated annealing (SSA) approach for optimal design of Power System Stabilizer (PSS) parameters in multi-machine power systems is proposed. This approach formulates the PSS design as a nonlinear programming problem using an eigenvalue-based objective function. For purposes of robust PSS design, several representative operating conditions are applied in construction of the objective function. One of advantages of the SSA approach is that, due to incorporation of the downhill simplex method into the Simulated Annealing (SA) method, it can significantly reduce the heavy computation burden of the SA method. Case studies on the 10-generator 39-bus New England test power system show that the effectiveness and efficiency of PSS optimal design have been tremendously improved by using the SSA approach.

1. Introduction
Many power systems face the problem of troublesome low frequency oscillations in the range of 0.2 to 2.5 Hz. Until now, Power System Stabilizer (PSS) have proved to be an effective and economical tool to damp system oscillations and therefore have been widely used by utilities \cite{1}, \cite{2}.

An important aspect in PSS design is to optimize PSS parameters. Many efforts have been put on this topic. The earliest works on the PSS parameter optimization were based on a single-machine-infinite-bus power system. However the system representation cannot reflect interactions between machines, e.g. inter-area mode oscillations. Sequential methods of PSS design \cite{3} for multi-machine systems were developed to damp out one of the swing modes at a time \cite{4}. The shortcoming of this kind of methods is that the sequential addition of stabilizers may disturb the previously assigned eigenvalues. In order to avoid the eigenvalue drift problem, gradient methods to simultaneously optimize PSS parameters at different operation conditions is presented in \cite{5}-\cite{6}. Unfortunately, analysis of eigenvectors and eigenvalue sensitivity factors is required in carrying out these methods. This gives rise to heavy computational burden and slows the optimization process. In addition, the search process is susceptible to be trapped in local minima. For finding a globally optimal solution, genetic algorithms for PSS design is developed in \cite{7}. However, there exist some structural problems in conventional genetic algorithms such as the premature convergence etc. Simulated Annealing (SA) algorithm for handling the combinatorial optimization problems was proposed to optimize PSS parameters in \cite{8}. It is the advantage that the SA algorithm can always converge to the globally...
optimal solution. In addition, the SA algorithm is robust to the initial conditions. However, SA algorithms suffer from the high computational cost due to their slow convergence rate.

In this paper, a new simplex-simulated annealing (SSA) approach for optimal design of Power System Stabilizer (PSS) parameters in multi-machine power systems is proposed. The approach formulates the design problem of PSS parameter as a constrained optimization problem with an eigenvalue-based objective function [8]. For purposes of robust PSS design, several representative operating conditions are applied in construction of the objective function [5]-[8]. Then a strategy combining a SA algorithm with a downhill simplex method in implementation of the SSA approach is presented for overcoming the problem of SA algorithms. Motivations for developing the SSA approach are to introduce a cost-effective stochastic component into a SA algorithm. The SSA approach inherits merits both of SA algorithms and the simplex method: to be less likely trapped in a local optimum but with faster convergence rate than that of the conventional SA algorithm. Therefore, the SSA approach explores a better tradeoff between computational cost and the solution quality.

The paper is organized as follows: Section II describes formulations used in the PSS design. Section III reviews backgrounds of simplex methods and the SA algorithms, and presents the details in implementation of the SSA approach. Case studies on the 10-generator 39-bus New England test power system are given in Section IV to show that the effectiveness and efficiency of the PSS optimal design have been tremendously improved by using the SSA approach.

2. Formulations for PSS Design

2.1. Power System Model

As shown in Fig. 1, assume that generators considered in this study are equipped by a thyristor type exciter with a conventional lead-lag structure PSS, but the SSA approach does not limit to models of this exciter and PSS.

Where \( V_{ref} \), \( V_i \), and \( E_{\mu} \) are the reference voltage, input and output signals of the exciter; \( \Delta \omega \) and \( V_i \) are the input and output signals of the PSS. In the parameter optimizations of PSS, time constants \( T_a \), \( T_2 \) and \( T_3 \) are usually pre-specified, and the gain coefficient \( K_e \) and time constants \( T_1 \) and \( T_3 \) are the parameters to be optimized [8].

Thus, dynamic behaviors of power systems can be represented by a set of differential-algebraic equations as follows:

\[
\dot{X} = F(X, Y) \\
0 = G(X, Y)
\]

(1)

Where, vectors \( X \) and \( Y \) stand for the state and algebraic variables. Let (2) represent linear incremental model of (1) obtained at an operation point [1].

\[
\Delta \dot{X} = A \Delta Y
\]

(2)
Where, state matrix $A$ is dependent on PSS parameters.

2.2. Formulation of PSS Parameter Optimization

In practice, it is well known that a set of PSS parameters stabilizing a system under a certain operating condition may no longer yield satisfactory results when there is a drastic change in system operating conditions. To insure a sufficient damping for various operation conditions of systems, an eigenvalue based objective function $J$ is applied as that shown in (3).

$$J = \sum_{i=1}^{np} \left[ a_i \sum_{\forall k_i > \sigma_0} (\sigma_{k_i} - \sigma_0) + b_i \sum_{\forall k_i < \zeta_0} (\zeta_0 - \zeta_{k_i}) \right]$$

where $np$ is the number of representative operating conditions considered; $a_i$ and $b_i$ are coefficients taking the value one or zero; $\sigma_{k_i}$ and $\zeta_{k_i}$ are the real part and damping ratio of the $k$th eigenvalue for operating point $i$; $\sigma_0$ and $\zeta_0$ are two pre-assigned thresholds representing the desirable level of system damping and damping ratio. The conditions $\sigma_{k_i} > \sigma_0$, $\zeta_{k_i} < \zeta_0$ are imposed on evaluations of $J$ to consider only the unstable or poorly damped modes which are mainly belonging to the electromechanical ones [8] and to achieve shifting eigenvalues corresponding to them into the shadow areas A, B and C in Fig. 2 for different choices of the parameters $a_i$ and $b_i$.

![Fig. 2. Three desired regions A, B and C in the s-plane for eigenvalues corresponding to swing modes obtained by a different choice of $a_i$ and $b_i$.](image)

Therefore, the PSS design problem can be formulated as the following optimization problem.

$$\begin{align*}
\text{Min} & \quad J(R) \\
\text{s.t.} & \quad R_{\text{min}} \leq R \leq R_{\text{max}}
\end{align*}$$

Where vector $R$ consists of all of PSS parameters considered, that is:

$$R = [K_{11}, T_{11}, T_{12}, K_{12}, T_{12}, \cdots]^{T}$$

The constraints in (4) is equivalent to:

$$\begin{align*}
K_{ij}^{\text{min}} & \leq K_{ij} \leq K_{ij}^{\text{max}} \\
T_{ij}^{\text{min}} & \leq T_{ij} \leq T_{ij}^{\text{max}} & j = 1, \cdots, m
\end{align*}$$

Where $m$ denotes the number of PSS. To transform constrained nonlinear programming (4) into an unconstrained one, a new vector $Z$ is transformed from vector $R$ by (7).
\[
t_{i} = \frac{r_{i}^{\text{max}} + r_{i}^{\text{min}}}{2} + \frac{r_{i}^{\text{max}} - r_{i}^{\text{min}}}{2} \sin(z_{i}) \tag{7}
\]

Where \( r_{i} \) and \( z_{i} \) are the elements of the vectors. Thus, the constrained nonlinear programming (4) becomes unconstrained one (8).

\[
\text{Min } J(Z) \tag{8}
\]

Then SSA algorithm shown in the following sections is employed to solve (8) for an optimal or near optimal solution of (4).

3. Simplex-Simulated Annealing (SSA) Approach

3.1. Down-hill Simplex Method

A simplex in an \( n \)-dimension space is a geometrical polyhedron of \( n+1 \) vertices. The downhill simplex method developed by Nelder and Mead is used in our SSA algorithm. It is the advantage of the method that no derivative of objective functions is required [9-10]. Starting from an initial simplex, the method is supposed to make its own way downhill in iterations through an unimaginable complexity of \( n \)-dimensional topography, until it encounters a minimum. The implementation of the downhill simplex method is briefly outlined below with the definition of standard error:

\[
sterr = \left\{ \sum_{k=0}^{n} (J(Z_{k}) - J(Z_{0}))^{2} \right\}^{\frac{1}{2}} \tag{9}
\]

1) Initialization: For \( n \)-dimension function, arbitrarily choose a point \( Z_{0} = [z_{0,1}, \ldots, z_{0,n}]^{T} \) as the initial vertex. Then other \( n \) vertices can be generated by (10).

\[
Z_{j} = [z_{0,1}, \ldots, \gamma z_{0,j}, \ldots, z_{0,n}]^{T}; \quad (\gamma > 1) \tag{10}
\]

2) Stopping criterion: The iterations continue until the \( sterr \) becomes smaller than \( \varepsilon \), a small positive number.

3) Reflection: In every iteration, find out \( Z_{h}, Z_{n} \) and \( Z_{l} \) with the largest, second large, and the smallest function values: \( J(Z_{h}) \), \( J(Z_{n}) \) and \( J(Z_{l}) \). Evaluate \( Z_{\text{cen}} \), the centroid of all vertices other than \( Z_{h} \). Then, generate a new vertex \( Z_{r} \) using (11).

\[
Z_{r} = Z_{\text{cen}} + (Z_{\text{cen}} - Z_{h}) \tag{11}
\]

If \( J(Z_{r}) < J(Z_{h}) < J(Z_{n}) \), then substitute \( Z_{r} \) with \( Z_{h} \), and a new iteration begins (go back to step 2)

4) Expansion: if \( J(Z_{r}) < J(Z_{l}) \), expand the simplex using (12), in the hope that more improvement will result from moving further in the same direction, that is:

\[
Z_{e} = Z_{\text{cen}} + (Z_{r} - Z_{\text{cen}}) \tag{12}
\]

If \( J(Z_{e}) < J(Z_{r}) \), then substitute \( Z_{h} \) with \( Z_{e} \); otherwise substitute \( Z_{b} \) with \( Z_{r} \). Then a next iteration begins with the new simplex (go back to step 2).
5) Contraction: If \( J(Z_r) > J(Z_s) \), then contract the simplex using (13) under the assumption that the current move is too far.

\[
Z_e = Z_{cen} + \frac{1}{2}(Z_r - Z_{cen})
\]

(13)

Where \( Z_e \) is either \( Z_r \) or \( Z_s \) (whichever has the smaller objective function value).

If \( J(Z_r) < J(Z_s) \), substitute \( Z_s \) with \( Z_e \). A next iteration begins with the new simplex (go back to step 2).

6) Shrink: If \( J(Z_r) > J(Z_s) \), the contraction has failed, and the entire simplex shrinks. This is done by replacing each point \( Z_i \) by (14).

\[
Z_i = Z_i + \frac{1}{2}(Z_e - Z_i)
\]

(14)

And then, a next iteration begins with the new simplex (go back to step 2).

3.2. Simulated Annealing (SA) Algorithm

The SA algorithm is a technique that has attracted significant attention as suitable for large-scale optimization problems, especially ones where a desired global extremum is hidden among many local extrema [11].

At the heart of the SA algorithm is an analogy with thermodynamics, specifically with the way that liquids freeze and crystallize, or metals cool and anneal. At high temperatures, the molecules of a liquid move freely with respect to one another. If the liquid is cooled slowly, thermal mobility is lost. The atoms are often able to line themselves up and form a pure crystal that is completely ordered over a distance up to billions of times the size of an individual atom in all directions. This crystal is at the state with minimum energy. At any temperature in the cooling process, the Boltzmann probability distribution defined by (15) characterizes the thermal equilibrium state.

\[
p \sim e^{(-E/kT)}
\]

(15)

Where \( k \) is a constant of nature. Boltzmann probability distribution expresses the idea that a system in thermal equilibrium at temperature \( T \) has its energy probabilistically distributed among all different energy states \( E \).

In 1953, Metropolis et al. [11] proposed a Monte Carlo method to simulate the process of reaching thermal equilibrium at a fixed temperature. In this method, a simulated thermodynamic system was assumed to change its configuration from energy \( E_1 \) to energy \( E_2 \) with probability \( p = e^{(E_2 - E_1)/kT} \). Notice that if \( E_2 < E_1 \), this probability is greater than unity, in such cases the change is arbitrarily assigned a probability \( p = 1 \). This general scheme, of always taking a downhill step while sometimes taking an uphill step, has come to be known as the Metropolis algorithm. The Metropolis algorithm consists of the following elements:

1. System state \( Z \), a description of possible system configurations.
2. A generator of random changes in the configuration, that is, a procedure for a random step from \( Z \) to \( Z + \Delta Z \).
3. An objective function \( J \) (analog of energy) whose minimization is the goal of the procedure.
4. A control parameter \( T \) (analog of temperature) and an annealing schedule which tells how it is lowered from high to low values.
Kirkpatrick introduced, in 1982, the Metropolis algorithm into optimization procedures and developed the SA algorithm framework for arbitrary numerical optimization problems.

### 3.3. Simplex-Simulated Annealing Approach

A successful application of the SA algorithm for large-scale optimization problems requires an efficient strategy to generate feasible trial solutions. However, the scheme for choosing $\Delta Z$ in the SA algorithm cannot inspire complete confidence [8]. The problem is one of efficiency. The generator of random changes in the SA algorithm is inefficient if, when local downhill moves exist, it nevertheless almost always proposes an uphill move in a long and narrow valley. Moreover, it becomes more and more inefficient as converging a minimum.

The SSA algorithm is an improvement to the SA algorithms. It employs the downhill simplex method as a generator of random changes. This amounts to replacing the single point $Z$ as a description of the system state by a simplex of $n+1$ vertices. The "moves" are the same as those described in subsection A. (in section III), namely reflections, expansions, contractions and shrinks of the simplex. The implementation of Metropolis algorithm is as follows: a positive and logarithmically distributed random variable, proportional to the temperature $T$, is added to the stored function value associated with every vertex of the simplex, and a similar random variable is subtracted from the function value of every new point that is tried as a replacement point. Like the ordinary Metropolis algorithm, this method always accepts a true downhill step, but sometimes accepts an uphill one.

![Fig.3. The strategy in implementation of the SSA approach](image_url)
With a finite value of $T$, the SSA algorithm expands the simplex to a scale that approximates the size of the region that is capable to reach at this temperature $T$, and then executes a stochastic, tumbling Brownian motion within that region, sampling new and approximately random points. The efficiency in exploring a region is independent of its narrowness and orientation. If the decrease in temperature is sufficiently slow, it becomes highly likely that the simplex will shrink into that region containing the lowest relative minimum. In the limit $T \to 0$, the SSA approach is exactly to the downhill simplex method and can quickly converge to a local minimum.

An advantage of the SSA approach is that it provides a convenient way to switch between a local optimization solver and a global optimization solver. If the initial control parameter $T_0$ is set to be zero, the SSA approach becomes the downhill simplex method with fast convergence speed and poor capability in searching a globally optimal solution. By means of increasing $T_0$, the capability of SSA for searching a globally optimal solution has been continuously enhanced, but its computation burden increases. Therefore, a reasonable compromise between above two points can be achieved by adjusting the value of $T_0$ in terms of the nature of the optimization problem.

Flow chart shown in Fig. 3 shows a strategy in implementation of the SSA approach. Here $T_0$ and $T_f$ stand for the initial and final values of variable $T$; $0 < \mu < 1$ is a computation factor for reducing the value of $T$; $L$ and $l$ are the length and counter of each Markov chain; $\epsilon \approx 10^{-5} - 10^{-6}$ is a pre-specified parameter for stopping the procedure; $\text{rand}$ is a random variable in the range of $(0,1]$ generated by a randomizer. As shown in Fig. 3, three criteria are applied to stop the recurrence. To avoid undue and excessive computations, criteria 1 and 3, common-used in SA [11] and downhill simplex methods [9], are employed to terminate the approach if the desired optimal solution does not exist. And criterion 2 is for the case that $Z_i$ has reached at the point corresponding to the PSS optimal solution.

4. Case studies
The 10-generator 39-bus New England power system shown in fig. 4 is used in the studies. Detailed data of generator and system are given in [12]. Although the number and locations of PSS can be studied [13~14], it is assumed that all generators except Generator 1 in the system are equipped with exciters and PSSs as that shown in fig.1 for illustration and comparison purposes. In the studies, the pre-specified parameters are as follows: $T_a = 0.01s$, $K_a = 200$, $T_c = 15.0s$, $T_s = 0.05s$ and $T_i = 0.05s$. $K_{s,i}$, $T_{s,i}$ and $T_{i,i}$ are the parameters to be optimized. To robustly design the PSS using SSA method, the following three operating conditions are considered on the assumption that theses conditions are representative and extremely hard from the stability point of view [8].

Condition 1: Basic operating case;
Condition 2: Outage of line 21-22 on the basic case;
Condition 3: Outage of line 14-15 on the basic case.
For comparison purpose, eigenvalues of the three conditions for the case without PSSs are shown in Figs. 5, 6 and 7. It is clear that some swing modes are poorly damped or unstable in the case. In the studies, parameters $\sigma_0 = 1.0$ and $\zeta_0 = 0.02$ are used to evaluate the objective functions (3). According experiences, a set of initial PSS parameter: $K_i = 7, T_i = 0.2, (i = 2, \cdots, 10)$, is used to form the initial vertex $Z_0$ by (7). And the rest vertices to form the initial simplex for the SSA approach are generated by (10).

Three objective functions for different choice of $a_i$ and $b_i$, called cases A, B and C for simple presentation, as that shown in Fig. 2 are used in the SSA approach in the studies. The results of the PSS parameter optimization are shown in Table I, and the eigenvalues corresponding to the three cases are shown in Figs. 5, 6 and 7. It is clearly indicated that all the eigenvalues to the swing modes are successfully moved to the desired regions.

| Case | Generator | $K_i$ | $T_1$ | $T_3$ |
|------|-----------|------|------|------|
| A    | G2        | 6.7748 | 0.2222 | 0.0874 |
|      | G3        | 10.7619 | 0.1758 | 0.2428 |
|      | G4        | 13.5737 | 0.1146 | 0.1373 |
|      | G5        | 10.2986 | 0.1389 | 0.0677 |
|      | G6        | 22.6656 | 0.3329 | 0.2101 |
|      | G7        | 14.0001 | 0.0208 | 0.2449 |
|      | G8        | 11.1500 | 0.1676 | 0.0812 |
|      | G9        | 16.8928 | 0.1954 | 0.1301 |
|      | G10       | 18.2538 | 0.1148 | 0.0421 |
| B    | G2        | 7.6482 | 0.1216 | 0.2645 |
|      | G3        | 14.3054 | 0.1344 | 0.0722 |
|      | G4        | 14.9945 | 0.0629 | 0.1357 |
|      | G5        | 13.6018 | 0.1357 | 0.0781 |
|      | G6        | 18.1100 | 0.3379 | 0.1207 |
|      | G7        | 18.3008 | 0.2627 | 0.2314 |
|      | G8        | 12.1428 | 0.1435 | 0.1273 |
|      | G9        | 7.80808 | 0.1876 | 0.3163 |
|      | G10       | 19.0324 | 0.0322 | 0.0905 |
| C    | G2        | 37.5602 | 0.0569 | 0.5129 |
|      | G3        | 14.9798 | 0.0843 | 0.1818 |
|      | G4        | 17.0323 | 0.1312 | 0.0217 |
|      | G5        | 18.3638 | 0.0645 | 0.0688 |
|      | G6        | 33.7239 | 0.0331 | 0.2095 |
|      | G7        | 30.9195 | 0.0889 | 0.1957 |
|      | G8        | 15.5505 | 0.0719 | 0.0293 |
|      | G9        | 1.35112 | 0.4513 | 0.3264 |
|      | G10       | 28.1214 | 0.0806 | 0.0452 |
To compare the efficiency between SSA and SA approaches, a SA program [11] is also implemented and performed for the same PSS parameter optimal problem under the same computation conditions and the same initial PSS parameters. To show the convergence process, curves of objective function of \((a_i = 1, b_i = 1)\) versus the numbers of Markov chains for the two approaches are plotted in Fig. 8. The computer time to carry out the SSA and SA programs to obtain the last optimal results is shown in Table II.

**Table 2. Computer Time of SSA and SA in Case A, B, C**

| Case      | SSA (minutes) | SA (minutes) |
|-----------|---------------|--------------|
| Case A    | 5-6           | 211-212      |
| Case B    | 4-5           | 198-199      |
| Case C    | 9-10          | 225-226      |

**Fig 5.** Eigenvalues associated with swing modes on the three operation conditions for the objective function of \((a_i = 1, b_i = 0)\).

**Fig 6.** Eigenvalues associated with swing modes on the three operation conditions for the objective function of \((a_i = 0, b_i = 1)\).
To demonstrate the effectiveness of the SSA approach, swing angle simulations on the test system subjected to a fault of 3-phase short circuit on bus 16 occurring at 0.2s and disappearing at 0.3s is performed for the PSS parameters optimized using the objective function with $a_i=1$ and $b_j=1$. 

Fig 7. Eigenvalues associated with swing modes on the three operation conditions for the objective function of $(a_i=1, b_j=1)$.

Fig 8. Objective function variations of case C

Fig 9. Angle swing curves of generator 9 (with respect to center of inertia) in the test power system for three PSS conditions.
Simulations for the same fault conditions in the system are also conducted on the cases with unoptimized PSSs (parameter before optimization) and without PSSs. Fig. 9 shows the angle swing curves of generator 9 with respect to center of inertia. It is clear that designed PSSs using the SSA approach provide very good damping effect to system oscillations and enhance greatly the dynamic stability of power systems.

5. Conclusion
In the paper, a simplex-simulated annealing approach is applied to problem of robust PSS parameter optimization. The SSA approach inherits the advantages of the SA method, such as the globally optimal PSS solution, robustness to the initial PSS parameter settings and effectiveness to variations of system operating conditions. In addition, the SSA approach provides the convenient way to switch between the simplex approach, a local optimization solver, and the SA method, a global optimization solver. Case studies on the 10-generator test system show that the SSA can tremendously enhance the efficiency in PSS design.

Acknowledgments
Ningxia Higher Education Research Project (No.: NGY20140145)

References
[1] P. Kundar, Power System Stability and Control, New York: McGraw-Hill, 1994.
[2] E.V. Larsen, D.A. Swann, "Applying Power System Stabilizers," IEEE Trans. Power App. Sys., vol. PAS-100, pp. 3017-3046, 1981.
[3] C.T. Tse, S.K. Tso, "Approach to the Study of Small-perturbation Stability of Multi-machine Systems," Proc. Inst. Elect. Eng.--Gen. Transm. Dist., vol. 135, no. 5, pp. 406-415, 1988.
[4] C.T. Tse, S.K. Tso, "Design Optimization of Power System Stabilizers Based on Modal and Eigenvalue-sensitivity Analysis," Proc. Inst. Elect. Eng.--Gen. Transm. Dist., vol. 135, no. 5, pp. 396-405, 1988.
[5] C.D. Vournas, B.C. Papadias, "Power System Stabilization Via Parameter Optimization Application To the Hellenic Interconnected System," IEEE Trans. Power Systems, vol. 2, pp. 615-623, 1987.
[6] V.A. Maslennikov, S.M. Ustinov, "Method and Software for Coordinated Tuning of Power System Regulators," IEEE Trans. Power Systems, vol. 12, pp. 1419-1424, 1997.
[7] Y.L. Abdel-Magid, M.A. Abido, "Optimal Multi-objective Design of Robust Power System Stabilizers Using Genetic Algorithms," IEEE Trans. Power Systems, vol. 18, pp. 1125-1132, 2003.
[8] M.A. Abido, "Robust Design of Multimachine Power System Stabilizers Using Simulated Annealing," IEEE Trans. Energy Conversion, vol., 15, pp. 297-304, 2000.
[9] Rao, S. S., Optimization: theory and applications, 2nd. ed., New York: Halsted Press, 1984.
[10] William H. Press, Saul A. Teukolsky, William T. Vetterling, Brian P. Flannery, Numerical recipes in C, 2nd. ed., UK: Cambridge university press, 1988.
[11] E. Aarts and J. Korst, Simulated Annealing and Boltzmann Machines: A stochastic Approach to combinatorial Optimization and Neural Computing, New York: John Wiley & Sons, 1989.
[12] M. A. Pai, Energy Function Analysis for Power System Stability, USA: Kluwer Academic Publisher, 1989.
[13] F.P. Demello, P.J. Nolan, T.F. Laskowski, J.M. Undrill, "Coordinated Application of Stabilizers in Multi-machine Power Systems," IEEE Trans. Power App. Sys., vol. 99, pp. 892-901, 1980.
[14] Y.Y. Hsu and C.L. Chen, "Identification of Optimum Location for Stabilizer Application Using Participation Factors," Proc. Inst. Elect. Eng.--Gen. Transm. Dist., vol, 134, no. 3, pp. 238-244, 1987.