A pricing model for real-estate business in Bangladesh incorporating the uncertainty in buyer’s readiness: considerations during COVID-19 pandemic

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Abstract
The real-estate business is considered one of the significant contributors to the economy of Bangladesh. However, the sudden occurrence of the COVID-19 pandemic exacerbates the housing business and the owners have been struggling to sell their apartment units. The instability of the income pattern of the customers and dwindling competitions among the real-estate business marketers during and after the COVID-19 pandemic necessitate revising the sales plan of the rental units. This research aims to develop a three-stage optimal selling model considering two random factors that the sellers face exclusively during the COVID-19 pandemic. The number of units sold in each stage is influenced by two random variables: (i) the time span between the initial stage and the final stage and (ii) the income-level inconsistency. This study highlights the randomness of these two factors with the aid of gamma distribution and optimizes the pricing model for three different stages. The findings of this research are illustrated with both numerical and graphical representations considering two different scenarios named identical and independent strategies of pricing. A comparative analysis has been conducted for these two strategies which would help the management to switch decisions depending on the market volatility. The results indicate that sellers can maximize their revenues by selling most of the units at the initial stage; however, sales in the second and third stages are also significant, depending upon the readiness of buyers.

Keywords Pricing model · COVID-19 · Real-estate business · Gamma distribution · Monte Carlo Simulation

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Introduction

The real-estate business is one of the burgeoning sectors of Bangladesh (Islam and Arefin 2009). After the independence of the country in 1971, there were only five registered real-estate firms in this country (REHAB 2021). However, at present, there are more than a thousand registered and unregistered real-estate companies, contributing to 7.96% of the total GDP (BBS 2020) of the nation. With the steady income of the laypeople and expanding middle class, the real-estate industries have been expanding and accumulating gross revenue and high-profit margins.

The residential units of an apartment have been sold in multiple steps in Bangladesh. In most cases, the real-estate agency or developer purchases land from the landlords in contract and build real estate for selling to the buyers (Ahmed et al. 2017). The units of a residential apartment can be sold in three different stages: before construction, during construction, and after the completion of the construction. The primary target of the developer is to sell as many units as possible in the initial stage, and if it is somehow not possible, then they go for the second stage or during the construction stage. However, if the estate developer is unable to sell a significant number of units in both the first and second stages, he has to rely on the third stage, selling units after the construction. From the seller’s point of view, it is often undesirable, since they have to bear a high maintenance cost and the selling price becomes lower than the usual price of the units.

One of the most dominating factors of the real-estate business in Bangladesh is the overall economy of the country which is terrifically altered by the sudden occurrence of the COVID-19 pandemic. A majority of the buyers are withdrawing from investment in purchasing the residential units and others also become incapable to pay the price in time (Dhaka Tribune 2020). Consequently, the real estate and its linkage industries are going through a perplexing situation during this time and facing an economic recession in the entities of this sector. The real-estate owners are now offering compensations, such as lower interest rates, zero equity loans, home loan services, etc., to sell the unsold units (UNB 2021).

One of the primary concerns for real-estate developers is to conduct market research and analyze buyers’ preferences (Rosenthal 2011). Because of the sudden downfall of the economy amid COVID-19, there is some uncertainty in the socio-economic behavior of the buyers (Lin et al. 2020), and marketers need to consider it while designing a marketing plan. In this regard, an optimal pricing policy is required to maximize the profits for the real-estate business. Keeping that in mind, this study aims to develop a three-stage pricing model considering the unpredictability of buyers’ behavior. Two potential sources have been discovered to explain the uncertainty of buyers’ purchasing decisions. They are: (i) if the time horizon between two consecutive stages is large enough, then the plant/location price and availability of raw materials from the linkage industries might be altered which will eventually impact the purchasing decision and (ii) the purchasing capacity of buyers due to income fluctuations as an effect of the COVID-19 pandemic.

Although real-estate pricing models have been suggested by some researchers (Bokhari and Geltner 2011; Crosby et al. 2016), most of these models often
neglect the preparedness of the buyers and are only limited to the sellers’ point of view. In addition, there is hardly any pricing model that can elaborate the optimal pricing decision for the housing business in an emerging economy like Bangladesh. However, the vulnerability of the economic condition of Bangladesh has become prominent after the COVID-19 pandemic, which necessitates a suitable pricing model for the real-estate sector in this region.

With an aim of fulfilling the aforementioned research gaps, we are motivated to develop a real-estate pricing model which would like to address the following research questions:

RQ 1: What are the most crucial factors behind the randomness of the buyers’ purchasing behavior in Bangladesh?

RQ 2: What should be the optimal pricing policy for the rental units that will maximize the profits of the sellers without altering the purchasing decision of the buyers?

By responding to the above research questions, this research contributes to the extant literature by constructing a pricing strategy model for the residential units by considering three different time periods—(i) the initial stage, (ii) the middle of the time span, and (iii) the end, amid the COVID-19. To incorporate the random attitude of the buyers, we consider two gamma-distributed random factors, such that the uncertainty of their purchasing decision can be augmented.

The rest of this article is articulated as follows. In the section “Theoretical background”, a detailed theoretical background of this study is provided. In the section “Problem description”, the description of the problem has been provided along with the notations and assumptions employed in this research. Next, model formulation and solution approach with numerical analysis are presented in the sections “Model formulation” and “Results and discussion”, respectively. In the section “Results and discussion”, graphical analysis and discussions have also been presented for a better interpretation of our result. Finally, the conclusions are outlined along with the prospects that can be included in future research.

**Theoretical background**

**Pricing model**

In the past few decades, researchers have been trying to develop a suitable pricing model considering the uncertainty of buyers’ decisions. However, most of the models assume that the uncertainty of the buyer’s decision can be explained well with the binomial pricing model. Different models have been developed considering binomial pricing strategies. A few instances of such studies include a flexible binomial option pricing model (Lin et al. 2020), and a case for using real options’ pricing analysis to evaluate information technology project investments (Cruz Rambaud and Sánchez Pérez 2016). However, it has been analyzed and found that binomial pricing models are suitable for developing a simple pricing strategy. Therefore,
some authors also suggest using gamma distribution to describe properly the complex nature of pricing models. Some researchers have successfully developed suitable pricing models using gamma distribution (Lam et al. 2002; Zhang and Watada 2018; Febrer and Guerra 2021). Moreover, the randomness of the buyers’ behaviors can also be properly explained with the aid of Gamma distribution (Furman and Landsman 2005; Hughston and Sánchez-Betancourt 2020). Later risk analysis in pricing strategy has been discussed by different researchers (Furman and Landsman 2005; Su and Furman 2016).

The pricing techniques can be segregated into two components—sequential pricing and simultaneous pricing (Jiang and Deng 2014). In sequential pricing, the sellers can set the price of the subsequent products based on the buyer’s response to the initial product (Feng and Shanthikumar 2018). The sequential pricing strategy is preferred when there is a correlation between the products and their number of sales in different stages (Munneke et al. 2019). The simultaneous pricing strategy is the opposite of the sequential pricing strategy where marketers can adjust the selling prices instantaneously (Edalatpour and Mirzapour Al-e-Hashem 2019). The sequential pricing policy may be treated as a tactical decision of marketing and the simultaneous pricing strategy is considered the strategic decision for the management (Zhang et al. 2022).

While selling the products to the customers, there is always some uncertainty associated with the buyers’ decision (Biswas and McHardy 2007; Deng et al. 2012), and marketers need to consider it in their pricing models. Researchers have also shed light in their works on this concern. For instance, Louissot and Ketcham (2014) showed the associated uncertainty in the buyers’ decision from the risks in the business management. Later, the study by Moradian and Soufi (2015) examines the relationship between pricing strategy and market capabilities by considering randomness caused by pricing offerings and incentives. Besides, Liu and Fu (2019) developed a two-stage pricing model to reduce sales risks for fresh products and optimized the mathematical model to maximize the profit.

**Gamma distribution**

To deal with the uncertainty in the stochastic process, several parametric models have been utilized and tested by previous researchers (Choi and Yoon 2020). However, Gamma distribution has been employed widely to incorporate the sequential pricing effects in the subsequent stages of the model (Bogomolov and Medvedev 2009; Bellini and Mercuri 2014). The gamma distribution is characterized by two parameters: one is the identical scale parameter ($\theta$) and another one is the time-dependent shape parameter ($k$) (Mahmoodian and Li 2016).

**The Probability Density Function (PDF) of the gamma distribution**

A continuous random variable, $X$, is supposed to follow the gamma distribution if its shape parameter, ($k$) > 0, scale parameter, $\theta$ > 0, and the PDF of the function is demonstrated by the equation as expressed in Eq. (1):
The mean and the variance of the gamma distribution are shown in Eqs. (2) and (3), respectively

\[ E_X(k, \theta)(t) = \frac{k(t)}{\theta}, \quad (2) \]

\[ V_X(k, \theta)(t) = \frac{k(t)}{\theta^2}. \quad (3) \]

The Cumulative Distribution Function (CDF) of the gamma distribution

For the same continuous variable, \( X \), the cumulative distribution function (CDF) can be expressed as both in terms of the lower incomplete gamma function and the upper incomplete gamma function. The lower incomplete gamma function can be defined as

\[ F_X(k, x) = \frac{\gamma(k, \frac{x}{\theta})}{\Gamma(\alpha)}, \quad (4) \]

which can be expressed as an integrated form

\[ F(k, x) = \int_0^x x^{k-1} e^{-x} \, dx. \quad (5) \]

Similarly, the upper incomplete gamma function can be explained as

\[ F(k, x) = \frac{\Gamma(k, \frac{x}{\theta})}{\Gamma(\alpha)}. \quad (6) \]

For numerical purposes, both upper and lower incomplete gamma functions can be used; however, the lower incomplete gamma function is widely used in contemporary literature.

Problem description

To properly understand the context, a case study has been conducted considering the scenario of a real-estate company from Bangladesh. Initially, the prime target of the seller is to sell as many units as possible in the very first stage. However, due to the COVID-19 effect, it is seldom possible to sell all units in the beginning stage. In that case, the remaining units are sold separately at the onward stages. The pricing
for the later stages might be equal to or lower than the initial price depending on two random factors. Since the sales of the onward stages are not simultaneous with respect to the initial stage, we have considered the sequential pricing policy. The sequential pricing strategy can be a suitable tool for marketers to make tactical decisions in the context of the COVID-19 pandemic.

This research deals with a discrete three-stage pricing plan: \( t = 0, t = 1, \) and \( t = 2 \). It is further assumed that \( Y_0, Y_1, \) and \( Y_2 \) denote the prices or amounts that a consumer is ready to pay for the unit at the beginning (\( t = 0 \)) and onward (\( t = 1 \) and \( t = 2 \)) stages, respectively. \( Y_0, Y_1, \) and \( Y_2 \) are stochastic price factors for the residential unit seller and the seller will set his final price accordingly considering the random nature of the variables. This model is formulated to determine the optimal price \( p_0 \) for the first selling stage, and price, \( p_1, p_2 \) for the following stages. The random nature of the variables will be explained by the gamma distribution with two parameters: (i) shape parameter and (ii) scale or rate parameter. Although this model is adaptive to different types of parametric distribution techniques, many recent studies on pricing models have used gamma distribution for developing the models. Inspired by those recent works, we are encouraged to develop the pricing model, assuming that both random variables will follow the gamma distribution.

### Notation

The following notation is considered to develop the model:

| Symbol | Description |
|--------|-------------|
| \( T \) | Time period for the pricing model (i.e., \( t = 0 \) or \( t = 1 \) or \( t = 2 \)) |
| \( P \) | Probability of any particular event; \( 0 \leq P < 1 \) |
| \( Y_0 \) | The preferable price of the buyer that he likes to offer at the initial stage |
| \( Y_1 \) | The preferable price of the buyer that he likes to offer at the second stage |
| \( Y_2 \) | The preferable price of the buyer that he likes to offer at the third stage |
| \( P_{r1} \) | Random variable due to the length of time interval |
| \( P_{r2} \) | Random variable due to the fluctuation of economic stability |
| \( p_0 \) | Price at the first stage set by the marketer considering \( P_{r1} \) and \( P_{r2} \) |
| \( p_1 \) | Price at the second stage set by the marketer considering \( P_{r1} \) and \( P_{r2} \) |
| \( p_2 \) | Price at the third stage set by the marketer considering \( P_{r1} \) and \( P_{r2} \) |
| \( k \) | The shape parameter for the gamma distribution |
| \( \theta \) | The rate parameter for the gamma distribution |
| \( f_{\lambda}(k, \theta)(t) \) | Probability density function (PDF) of the gamma distribution |
| \( F_{\lambda}(k,y) \) | Cumulative density function (CDF) of the gamma distribution |
| \( \Gamma(k,y) \) | Lower incomplete gamma function |
| \( \delta \) | Initial cost that will be added to the total price of the product |

### Assumptions

We have considered the following assumptions to formulate the pricing model:
(i) The time horizon is finite and selling will be started at the initial stage \((t = 0)\).
(ii) The pricing of the model will follow the sequential pricing strategy.
(iii) No second-tier contractors are involved in the decision-making policy.
(iv) Two random variables are independent of each other.
(v) Random variables will follow a gamma distribution.
(vi) For each of the random variables, the shape parameter and the rate parameter are known by the top management.

**Model formulation**

Here, for sequential pricing, a three-stage pricing model has been developed. If the marketer plans to set the prices \(p_0, p_1, p_2\), respectively, he needs to set them, such that gross profit for each stage will reach the maximum value. The maximum value of the profit is mathematically denoted as

\[
\Pi_0(p_0) = P(Y_0 \geq p_0)p_0, \quad (7)
\]

which will reach maximum value for a specific value of \(p_0\) which will be denoted as \(p_{0,\text{max}}\).

The net amount of profit during the second stage can reach the maximum when the value of the profit function of the second stage is maximum. The profit function for the second stage can be denoted as

\[
\Pi_1(p_1) = P(Y_0 < p_{0,\text{max}}, Y_1 \geq p_1)p_1, \quad (8)
\]

which will be maximum for a certain value of \(p_1\) and denoted as \(p_{1,\text{max}}\).

Similarly, for the third stage, marketers will make the maximum gross profit when the profit function

\[
\Pi_2(p_2) = P(Y_1 < p_{1,\text{max}}, Y_2 \geq p_2)p_2, \quad (9)
\]

reaches at a maximum value at a defined value \(p_2\) and denoted as \(p_{2,\text{max}}\). In sequential pricing, the seller will try to sell in such a way that each stage reaches its maximum level.

**Gamma-distributed pricing formulation**

The price that the customers are willing to pay in each stage is considered a random variable that can be resulted from two random variables. The random nature of each of these variables is mainly for two factors \(P_{r1}\) and \(P_{r2}\) which exhibit randomness in an emerging economy like Bangladesh. In particular, the random traits of \(P_{r1}\) and \(P_{r2}\) can be well explained by taking the COVID-19 pandemic time into account. For instance, when the sellers are not capable to complete the construction works of the rental units within a certain time interval due to the disruptions primarily caused by COVID-19 in the internal linkage industries, consumers’ purchasing decisions become altered; which is denoted by \(P_{r1}\). Similarly,
the steadiness level of the buyers’ income which is strongly influenced by the pandemic can also fluctuate the buying decision as indicated by $P_{r2}$. The offered price from the consumer for the residential unit can be expressed as the combined effect of $P_{r1}$ and $P_{r2}$. Mathematically, we can express the price offered by the customer at any stage as

$$Y = P_{r1} + P_{r2},$$

where both $P_{r1}$ and $P_{r2}$ follow the gamma distribution. According to Moschopoulos (1985), if two random variables are gamma distributed, then their combination will also follow the gamma distribution. Therefore, it can be said that the price offered by the consumer also follows gamma distribution for each of the three stages. For the simplicity of the model, it is assumed that $P_{r1}$ and $P_{r2}$ are statistically independent, which means that the occurrence of one will not have any effect on another.

**Scenarios**

**Scenario I: identical $Y_0, Y_1, Y_2$**

Whether the marketer sells the whole unit at a single stage or multiple stages, a buyer might not bargain to lower the prices in the later stages if his income is not affected by the pandemic situation, which means that he will agree to pay the same that he would have to pay at the initial stage. Therefore, we can define all the three different offered prices by the buyer as $Y$. In other terms, all the random variables $Y_0$, $Y_1$, $Y_2$ are identical now, which means that all are equal to $Y$, which is set to be

$$Y = P_{r1} + P_{r2}. \tag{10}$$

Hence, under the scenario I, using Eq. (1), the pricing function can be written for the initial stage as

$$\Pi_0(p_0) = (1 - \frac{\gamma(k, \theta(p_0 - \delta))}{\Gamma(\alpha)})p_0. \tag{11}$$

For calculating maximum profit at the second stage, Eq. (2) can be written as

$$\Pi_{1}(p_{1}) = \frac{\gamma(k, \theta(p_{0,\text{max}} - \delta)) - \gamma(k, \theta(p_{1} - \delta))}{\Gamma(\alpha)}p_{1}. \tag{12}$$

To calculate the maximum value function at the third stage, Eq. (3) can be written as

$$\Pi_{2}(p_{2}) = \frac{\gamma(k, \theta(p_{1,\text{max}} - \delta)) - \gamma(k, \theta(p_{2} - \delta))}{\Gamma(\alpha)}p_{2}. \tag{13}$$

Therefore, we need to simultaneously solve Eqs. (11), (12), and (13) to obtain the optimal prices that need to be set by the marketers.
Scenario II: independent $Y_0, Y_1, Y_2$

For the second scenario, it has been assumed that prices are independent, which means that prices offered by the buyers are not equal at three different stages, which is different from Scenario I. Nonetheless, the price from the consumer at each stage will follow the gamma distribution in this stage as well. Theoretically, every three prices should converge at the same point similar to scenario I, as all of them follow the same distribution. The profit function for the first stage will remain the same as the identical one. Therefore, $p_{0, \text{max}}$ will be the same as in Scenario I. However, for the second stage, the profit function can be written as (Hong and Shum 2006)

$$\Pi_1(p_1) = P(Y_0 < p_{0\text{max}}) \times P(Y_1 \geq p_1)p_1,$$

which can be rearranged as

$$\Pi_1(p_1) = \frac{\gamma(k, \theta(p_{0\text{max}} - \delta))}{\Gamma(\alpha)} \times (1 - \frac{\gamma(k, \theta(p_1 - \delta))}{\Gamma(\alpha)})p_1. \quad (15)$$

Similarly, for the third stage profit function will be

$$\Pi_2(p_2) = P(Y_0 < p_{0\text{max}}) \times P(Y_1 < p_{1\text{max}}) \times P(Y_2 \geq p_2)p_2. \quad (16)$$

For the third stage, using the gamma distribution function, we get

$$\Pi_2(p_2) = \frac{\gamma(k, \theta(p_{0\text{max}} - \delta))}{\Gamma(\alpha)} \times \frac{\gamma(k, \theta(p_{1\text{max}} - \delta))}{\Gamma(\alpha)} \times (1 - \frac{\gamma(k, \theta(p_2 - \delta))}{\Gamma(\alpha)})p_2. \quad (17)$$

The marketers will target to set the prices at the second and third stages in such a way that will maximize the function value of Eqs. 11, 15, and 17.

Results and discussion

Numerical analysis

This section outlines the implementation of the model that has been formulated in the preceding section. The proposed model is solved with the aid of a suitable solver that will provide the target setting prices as well.

To find out the desired solution for both scenario I and scenario II, we need to find out the parameters of the gamma-distributed random price offered by the customer which is the summation of two gamma-distributed variables, $P_{r1}, P_{r2}$. It has been observed from the previous records that $P_{r1}$ follows a gamma distribution with shape parameter $(k)=15$ and scale parameter $(\theta)=4$. The accuracy and adequacy of the solution depend on the number of simulated data. The higher the number of solutions, the more accurate our final result will be. Using the values of these two parameters, we have generated 100 data by simulation. Simulated data are generated
using the inverse gamma function. At first, 100 sets of random numbers are generated, and then corresponding to those random numbers, 100 sets of gamma-distributed variables have become available. Using any software, random numbers and gamma random variables can be generated. However, we have used `gamrnd` function in ‘MATLAB R2018 a’ to generate 100 sets of data, as depicted in Table 1.

Similarly, for the fluctuation of economic stability of the consumers, we have introduced another gamma-distributed random variable, $P_{r_2}$ with shape parameter 12 and scale parameter 4.55 which is assumed to be known from past data available to the company. After that, we generated another 100 sets of data as presented in Table 2.

Therefore, when the buyer sets his final price, he will consider the above-mentioned random variables. Hence, the seller has to set a price considering both of these random price variations and also a fixed expense as the reserve price which is the smallest price having zero profit and just considering the expenses associated with the product. Using Monte Carlo Simulation (MCS) technique, we have generated 100 sets of data to estimate our final model parameters. In Table 3, 100 sets of data for the overall gamma-distributed price are enlisted.

To calculate gamma-distributed probability, we need to estimate the parameters of the above 100 data mentioned in Table 3. There are many parameter estimation techniques like the maximum-likelihood method, method of moment, Bayesian minimum mean squared error, etc. In addition, many software packages are also available now to estimate the parameters of different statistical distributions. In our research, the estimation criteria have been done in ‘MATLAB R2018 a’ using `gamfit` function. After

| Table 1 | Gamma-distributed random variables ($P_{r_1}$) with $k = 15, \theta = 4$ |
|---------|------------------|------------------|------------------|------------------|
| 3.2401  | 1.6734           | 2.7877           | 2.9194           | 3.8776           |
| 4.3339  | 2.0719           | 2.6034           | 1.445            | 2.5204           |
| 1.593   | 2.3223           | 2.0807           | 4.3511           | 2.3021           |
| 3.2076  | 2.8962           | 1.7409           | 3.0729           | 3.482            |
| 2.3079  | 3.5853           | 1.6102           | 2.8257           | 2.9396           |
| 2.6748  | 2.8648           | 3.4277           | 2.0891           | 2.3642           |
| 3.413   | 1.8182           | 3.038            | 2.3139           | 2.1974           |
| 1.7603  | 2.0905           | 1.0284           | 4.6326           | 2.4491           |
| 3.1625  | 1.4701           | 2.8756           | 2.4493           | 2.7247           |
| 1.8942  | 2.7148           | 1.9987           | 3.2217           | 2.6442           |
| 1.2305  | 3.0564           | 2.4851           | 1.9624           | 2.4898           |
| 2.1572  | 2.1527           | 1.5941           | 3.7006           | 2.6934           |
| 2.4482  | 1.5362           | 3.3483           | 2.5497           | 2.3397           |
| 2.0941  | 2.1998           | 2.2803           | 2.3722           | 1.7686           |
| 2.3776  | 5.2397           | 2.8654           | 2.2779           | 2.0211           |
| 2.6916  | 2.3615           | 4.0821           | 1.3232           | 3.0907           |
| 2.9036  | 4.1315           | 3.9137           | 2.529            | 3.1805           |
| 2.3254  | 2.9115           | 2.3793           | 1.3896           | 2.2176           |
| 2.6382  | 2.0196           | 4.6405           | 2.2229           | 1.6255           |
| 4.5631  | 1.9972           | 2.9329           | 1.8982           | 2.4391           |
Table 2  Gamma-distributed random variables \((P_{12})\) with \(k = 12, \theta = 4.55\)

|       |       |       |       |       |
|-------|-------|-------|-------|-------|
| 3.4119 | 3.9691 | 4.1824 | 2.3631 | 3.0446 |
| 3.4070 | 2.8989 | 2.8371 | 5.4267 | 4.9305 |
| 2.0321 | 4.4778 | 3.5286 | 3.7064 | 3.6433 |
| 2.9333 | 1.8578 | 5.5245 | 3.6372 | 4.7331 |
| 4.1748 | 3.9953 | 3.2227 | 4.0902 | 3.6678 |
| 3.6988 | 4.1163 | 2.6534 | 3.4449 | 1.7642 |
| 4.8556 | 3.8454 | 3.4105 | 3.9994 | 1.9427 |
| 3.3877 | 4.5555 | 3.4334 | 2.293  | 3.7437 |
| 3.4215 | 4.1198 | 5.4261 | 3.4068 | 4.0749 |
| 3.3999 | 3.3553 | 3.4512 | 2.2888 | 4.5507 |
| 2.8068 | 4.7542 | 3.2572 | 2.6237 | 4.1140 |
| 3.1792 | 5.0109 | 3.9374 | 3.6625 | 4.5158 |
| 4.6695 | 3.6021 | 1.4078 | 3.9888 | 4.5953 |
| 3.6475 | 3.4622 | 2.5949 | 4.7167 | 3.5275 |
| 2.9543 | 3.6888 | 3.0019 | 3.9183 | 1.9861 |
| 3.5406 | 4.5158 | 3.3682 | 2.5412 | 2.5797 |
| 5.1187 | 4.1321 | 1.4639 | 3.6667 | 4.3235 |
| 3.1311 | 4.2984 | 4.9895 | 4.6100 | 2.7782 |
| 2.9133 | 2.7665 | 4.6353 | 4.0123 | 3.7994 |
| 6.6558 | 3.9686 | 3.639  | 4.6309 | 3.9633 |

Table 3  Gamma-distributed random variables \((Y)\) with \(k, \theta\)

|       |       |       |       |       |
|-------|-------|-------|-------|-------|
| 6.652 | 5.6425 | 6.9701 | 5.2825 | 6.9222 |
| 7.7409 | 4.9708 | 5.4405 | 6.8717 | 7.4509 |
| 6.3251 | 8.0011 | 5.6093 | 8.0575 | 5.9454 |
| 6.1409 | 4.754  | 7.2654 | 6.7101 | 8.2151 |
| 6.4827 | 7.5806 | 4.8329 | 6.9159 | 6.6074 |
| 6.3736 | 6.9811 | 6.0761 | 5.534  | 4.1284 |
| 8.2686 | 5.6636 | 6.4485 | 6.3133 | 4.1401 |
| 5.148 | 6.646  | 4.4618 | 6.9256 | 6.1928 |
| 6.584 | 5.5899 | 8.3017 | 5.8561 | 6.7996 |
| 5.2941 | 6.0701 | 5.4499 | 5.5105 | 7.1949 |
| 4.0373 | 7.8106 | 5.7423 | 4.5861 | 6.6038 |
| 5.3364 | 7.1636 | 5.5315 | 7.3631 | 7.2092 |
| 7.1177 | 5.1383 | 7.4561 | 6.5385 | 6.935 |
| 5.7416 | 5.662  | 4.8752 | 7.0889 | 5.2961 |
| 5.3319 | 8.9285 | 5.8673 | 6.1962 | 4.0072 |
| 6.2322 | 6.8773 | 7.4503 | 3.8644 | 5.6704 |
| 8.0223 | 8.2636 | 5.3776 | 6.1957 | 7.504 |
| 5.4565 | 7.2099 | 7.3688 | 5.9996 | 4.9958 |
| 5.5515 | 4.7861 | 9.2758 | 6.2352 | 5.4249 |
| 11.2189 | 5.9658 | 6.5719 | 6.5291 | 6.4024 |
estimating, we have finally found that our final model’s shape parameter, $k = 24.89$, and scale parameter, $\theta = 4.125$. To calculate the final price, a threshold selling price is set to 180 thousand USD, which is the zero-profit margin for the seller.

To solve an equation that is involved by gamma-distributed function, we employed the Nelder–Mead solution technique. Obtained solution for Scenario I is reported in Table 4 and graphically illustrated in Fig. 1.

Also, for Scenario II, the obtained result is tabulated in Table 5 and graphical representations are shown in Fig. 2.

**Discussions and implications**

The numerical and graphical illustrations as depicted in the previous section have several implications for real-estate business owners. As depicted in Tables 4 and 5,
for identical pricing, the maximum offering prices for the second and third stages are higher than the prices offered in the independent pricing policy. Since customers are determined to be fixed on their pricing decisions for identical pricing in all stages, the sellers need to compromise their profits in comparison to the independent pricing scheme. Nonetheless, the total amount of profits from all three stages do not differ significantly for these two different pricing strategies which implies that the sellers can switch to any pricing policy depending on the purchasing behavior of the buyers.

We can see from both Tables 4 and 5 that, among the total profit, a significant amount has come from the initial stage sales, which is the ultimate target of the marketers. However, profits in the second and third stages are also not negligible. A decrease in the maximum price offered in the first stage and the second stage suggests that a

| Table 5 Different prices and profits at three different stages for independent pricing |
|---|---|---|---|---|---|---|---|---|
| $K$ | $\theta$ | $p_{0\text{max}}$ | $\Pi_0(p_0)$ | $p_{1\text{max}}$ | $\Pi_1(p_1)$ | $p_{2\text{max}}$ | $\Pi_2(p_2)$ | $\Pi(p_{0\text{max}}, p_{1\text{max}}, p_{2\text{max}})$ |
| 24.89 | 4.125 | 204.44 | 119.9097 | 194.75 | 65.113 | 186.36 | 40.094 | 225.1167 |

![Fig. 2 Prices and profits for three different stages at independent pricing](image-url)
significant number of buyers are deferring their purchasing decisions to the later stages due to the impacts of COVID-19. Marketers have to keep themselves aware of the purchasing ability of the buyers during the COVID situation and set their prices accordingly while selling in the second and third stages, respectively.

Our study intends to formulate a pricing model incorporating the stochastic buying pattern of the customers amidst the COVID-19 outbreak. The need for a suitable pricing model in this sector becomes paramount during the pandemic because of the larger economic shift of the nation. However, the housing industry can adopt our proposed pricing model beyond the COVID-19 era to set the desired pricing value of each rental unit and maximize their business gains. The sources of uncertainty in buyers’ purchasing decisions identified in this study using two random factors are not limited to the COVID-19 pandemic period. For instance, the timeframe of the construction will remain random due to its dependence on other external factors, such as loan sanctions and interest rate, asset value, time value of money, etc. Moreover, by considering the economic affordability of the buyers, this model may help the real-estate owners and managers to focus on different income-level people of Bangladesh and provide them with effective housing solutions.

Conclusions and recommendations

Conclusions

The COVID-19 effect significantly impacts the real-estate sector of Bangladesh and necessitates a proper pricing plan to maximize the profit for the business proprietors. Motivated by their needs, we have formulated a three-stage optimum pricing model and investigated different pricing policies for some random variations. Buyers’ readiness is considered dependent on two random factors such as the time interval of the sales and income fluctuation during the pandemic. This model includes the influences from such practical scenarios as the facts of buyers’ alteration which are uncertain to the real-estate marketers. Although in the real situation, the three stages might be dependent or independent and so the pricing random variables, in this research, the two random variables or factors are assumed to be independent which follow the gamma distribution. The model also considers two different types of pricing: identical and independent pricing to show the generalizability of the model. The theoretical findings of the project are explained both numerically and with graphical representation. It has been observed that the seller will gain maximum profit if he can sell most of his selling units in the initial stage. However, a significant number of profits can also be attained if he can successfully revise his plans for the onward stages. For solving the problem and better interpretation, both numerical data and graphical schemes help one to understand the literature quite easily.
Recommendations

There are some possible extensions that can be integrated into future research. For instance, the model can be further tested with other distributions like beta distribution or Weibull two-parameter distribution to check the adequacy of the model. The random variables might be considered dependent on each other, and if that is the case, how the optimal solution will be changed may be examined in future studies. Furthermore, if anyone wants to develop a model considering additional random factors such as the price volatility of the housing units, location, etc., it can be incorporated into future research.

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Declarations

Conflict of interest  On behalf of all authors, the corresponding author states that there is no conflict of interest. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

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