Quantum instability of the de Sitter space

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Abstract

We continue to investigate various instabilities of the fixed backgrounds related to the de Sitter space. It is shown that in many cases the in/in perturbation theory contains IR/UV mixing and thus is non-renormalizable. The application of this result to the global de Sitter space leads to the conclusion that even massive particles generate IR divergence and the huge back reaction. The expanding universe is also unstable but in a weaker sense. We further discuss the strange features of the Gibbons-Hawking radiation and its relation to the above instabilities.

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1 Introduction

In the naive approach to quantum gravity vacuum fluctuations generate a cosmological constant, the size of which is determined by the ultraviolet cut-off, the Planck length. At the same time the cosmic acceleration indicates that the observable cosmological constant is determined by the size of the universe, which can be viewed as an ultimate infrared cut-off. The problem of the cosmological constant can be formulated as following - why \( \Lambda \) is determined by the IR and not by the UV scale? This gives a strong hint that the resolution of the cosmological constant problem must be based on the relevant infrared interactions.

There are many examples in quantum field theory (say the Yang-Mills theory) in which such interactions deform the classical equations of motion beyond recognition. The cosmological term in the Einstein equation may experience the same fate, since it doesn’t contain derivatives of the metric and thus may be relevant in the infrared [1].

Unfortunately, we are still lacking the formal apparatus, needed for this problem. In particular, the scenario in [1] is based on the domains where the vacuum expectation value of the metric \(< g_{\mu\nu} >= 0\). It is not clear how to treat it quantitatively and the problem remains unsolved. However, it is worth mentioning, that the common counter-argument against this approach, namely that the scale factor of the metric can be removed by a gauge transformation and thus is not physical, is incorrect. In the zero metric domain the scale factor can’t be removed, just like the unitary gauge in the standard model can’t be used in the unbroken phase where the expectation value of the Higgs field is zero.

Another approach to the problem (taken in this article) is to study possible instabilities of the backgrounds with positive curvature. Such backgrounds are in many respects similar to the simpler case of the constant electric fields in the flat space (while the negative curvature case is analogous to magnetic field). It is very well known that the electric field makes the empty space unstable by producing pairs of particles. These pairs eventually screen the source of the field and the production stops. The energy needed for the pair production is taken from the electric field which becomes oscillating and decreasing.
If we believe the analogy, we should expect a depletion of the original curvature, due to the back reaction of the particles it has generated. More precisely, the constant curvature means that the space is either expanding or contracting with constant acceleration. As we will see, there is a constant particle creation in this case. The gravitational attraction between the particles will slow down the acceleration, producing something like radiation damping.

The most common objection to this picture is that the de Sitter space seems stable. Indeed, consider a theory of a massive scalar interacting field $\varphi(x)$ in the dS background. The back reaction is defined by the one point functions, like the energy-momentum tensor or simply by the operators like $\langle \varphi^2(x) \rangle$. However, the isometries of the de Sitter space imply that such objects do not depend on $x$ and thus the back reaction never gets large. In contrast, the constant electric field induces the current which grows linearly with time, since the production goes with the constant rate. Let us notice, however, that even in the electric case this blow-up of the current is in the apparent contradiction with the time-translation invariance of the constant electric field.

One can also argue that the field theory can be defined on the Euclidean version of dS (a simple sphere) and then analytically continued to the Lorentzian regime by the Wick rotation. Once again it results, at least perturbatively, in stability, since the theory on a sphere is clearly well defined.

As we will see below, both arguments are incorrect. First of all, as was argued in [4], there is an obstruction to the Wick rotation. Second, we will see that the isometries of the dS being unstable are spontaneously broken in the above case. This fact allows the back reaction to become large.

This situation is very similar to the one we have in the case of the Schwarzschild black hole. In this case the Euclidean field theory is also well-defined in all orders of perturbation theory. However, it misses the black hole evaporation. Similarly, the field theory on a sphere misses the "evaporation" of the positive curvature of the dS space.

Let us mention that the "electric" analogue of the black hole are supercharged nuclei. In this case it decays by creating bounded electrons and run away positrons, until it gets neutralized. Here again the analog of the dS symmetry is time-translation invariance in the presence of time independent electric field. It is spontaneously broken and the back-reaction current becomes proportional to time, thus getting large.

To some extent these questions have been discussed in [2], but many
points remained unclear. In this paper we will concentrate on the origin of the infrared divergences, their unexpected and unusual mixture with the ultraviolet, and on the peculiar mechanism which destroys isometries. As before, we will warm up with the electric fields and then proceed analyzing Poincare patches and finally global de Sitter space. The latter case is especially interesting, since while being classically stable \([15]\) its isometries are destroyed by the IR/UV mixing, mentioned above.

We will stress that the new general phenomenon, responsible for the large back reaction in unstable vacua is the non-commutativity of the UV and IR limits. As it often happens, non-commutative limits are the source of many confusions.

There are large number of different approaches to these problems in the literature and it is sometimes hard to establish connection with the results of this paper. Some, very incomplete, list of references can be found in \([2]\).

## 2 Electric IR/UV mixing

In this section we will discuss instabilities in the strong electric fields. There are hundreds if not thousands of papers devoted to this subject and we are bound to repeat some well known facts. Our reason for this discussion is that the IR/UV mixing, which will be central in the gravitational case, makes an interesting appearance in a much better understood electric case. We will see that there are unusual singularities in the in/ in propagators. Our calculation here is very close to the one performed in the old paper \([5]\) \([7]\) (see also \([2]\), \([8]\)) , but the results seem new. The main goal of this section is to prepare for the gravitational case.

Let us start with the standard set up (see e.g. \([6]\) and references therein) for the homogeneous, time dependent electric field \(E(t) = \dot{A}\), interacting with the complex scalar field \(\varphi\), which satisfies the Klein-Gordon equation. We are interested in the case when the field is turned on adiabatically, stays constant for a while, and turned off. The field \(\varphi\) has the standard mode expansion

\[
\varphi = \sum (a_k f_k^* + b_k^i f_k),
\]

\[
\ddot{f}_k + \omega_k^2(t)f_k = 0
\]
where $\omega_k(t) = \sqrt{(k - A(t))^2 + k_{\perp}^2 + m^2}$. In the adiabatic case we can use the WKB approximation for the modes. Let us assume that $A(t)$ behaves qualitatively as a function $ET\tan(t/T)$ where $T$ is a large adiabatic parameter. In this case we can fix the wave function to be a single exponent in the past, $k \gg A(t)$. At the later time, when $k \simeq A(t)$, the WKB approximation breaks down and the particles start to be created. As the time goes to infinity, the WKB is working again, but the reflected wave appears. So,

$$f_k \rightarrow \frac{1}{\sqrt{2\omega_k}} e^{i\omega_k(t)}, k \gg A(t)$$

$$f_k \rightarrow \frac{1}{\sqrt{2\omega_k}} (\alpha e^{i\omega_k} + \beta e^{-i\omega_k}), k \ll A(t)$$

where $s_k(t) = \int_0^t \omega_k(t') dt'$ and $\alpha, \beta$ are the Bogolyubov coefficients. These coefficients of course depend on $k_{\perp}^2 + m^2$ and, generally speaking, on $k$. However, it is important to realize that if $A(-\infty) \ll k \ll A(\infty)$, we can approximate $A(t) \simeq Et$ and in this case, by the transformation $t \rightarrow t - k/E$ the $k$ can be removed from the wave equation. Thus within this range the Bogolyubov coefficients do not depend on $k$. Outside this range they quickly vanish. These properties are the consequences of the time - translation invariance of the constant electric field.

Now, let us calculate various Green functions. Let us begin with $F = \langle \varphi^*(z, t, x_{1\perp}) \varphi(z, t, x_{2\perp}) \rangle$. This is the correlation between two points, separated in perpendicular direction, but having coinciding projection on the direction of the electric field. Using the above formulae we get

$$F = \int dk dk_{\perp} e^{i k_{\perp}(x_1 - x_2)} |f_k(t)|^2 = F^{(0)} + 2 \int dk_{\perp} |\beta|^2 e^{i k_{\perp}(x_1 - x_2)} \int_0^0 dp \frac{2\omega_p}{2\omega_p}$$

(4)

where $F^{(0)}$ is nonsingular correlator without electric field, $p = k - A(t)$, and the relation $|\alpha|^2 - |\beta|^2 = 1$ has been used. The integral is dominated by $p \gg m, k_{\perp}$, in which case $\omega_p \approx |p|$, and thus we get the following contribution

$$F = \Phi(x_1 - x_2) \log \left( \frac{ET}{m} \right)$$

(5)

where $T$ is the IR cut off - the total time the electric field was on. Here $\Phi(x)$ is a Fourier transform of $|\beta(k_{\perp})|^2$

If we look at the correlators with different $z$ we get

$$F(z_1x_1, z_2x_2) = \Phi(x_1 - x_2) \log \left( \frac{1}{m|z_1 - z_2|} \right)$$

(6)
provided that $\frac{1}{m} \gg |z_1 - z_2| \gg \frac{1}{ET}$. Such a singularity at the non-coinciding points is quite unusual. Even more interesting is the fact that the above formula is applicable only if $ET < \Lambda_{UV}$, where $\Lambda_{UV}$ is the ultraviolet cut-off. When $ET \sim \Lambda_{UV}$ we have a remarkable situation - large time phenomena depend on the ultraviolet completion of the theory, thus explicitly breaking renormalizibility. When interactions are included, we encounter a problem (not yet solved) of summation of leading logarithms.

It must be stressed that these interesting phenomena are lost if we use in/out propagators so they describe non-equilibrium non-renormalizability (NENR). It is easy to give a physical interpretation to the NENR. Imagine two pairs created far away from each other. The electron from one pair moves in the electric field towards the positron of another pair. When they collide, their center of mass energy is of the order of $ET$. If $ET \sim \Lambda_{UV}$, it is clear that the scattering amplitudes must depend on a particular ultraviolet completion of the theory. At the same time, renormalizability is just the statement of independence of the UV completion. It is not working in our case, at least perturbatively. The appearance of the scale $T$ indicates the breakdown of the translation symmetry. Beyond the perturbation theory it is possible that the collisions of created particles will establish an equilibrium without the runaway phenomenon, described above. The outcome depends on the high energy behavior of the cross-sections.

3 The Gibbons - Hawking temperature and Jeans instability

It may be worthwhile to discuss quantum instabilities of the de Sitter space from a similar point of view. Let us explore to what extent one can use the following naive ideas. Gibbons and Hawking \[12\] showed that the dS space has intrinsic temperature defined by the area of the horizon, very much like the Schwarzschild black hole. Does it make sense to say that as a result the space simply evaporates? . Also, in a thermal bath we would expect the Jeans instability \[13\]. But our ”bath” is unusual. The statement in \[12\] was that any geodesically moving detector will measure the above temperature. However already these authors noticed that there is something funny about this statement. Unlike the usual heat bath, observers moving relative to each other will not notice the Doppler shift. Also, this temperature, unlike
a physical heat bath, does not destroy the dS isometries.

To clarify the meaning of these notions we will try a different setting of the problem, already touched upon in [2]. Let us examine the space with the warp factor $a(t) = \exp[T \tanh(\frac{t}{T})]$. It starts with the Minkowski space at $t \to -\infty$, then becomes the expanding dS space for a long while, and then returns to Minkowski space once again. When $T$ is large, the quantum field theory on this background can be treated semiclassically most of the time. We will concentrate on the particle production in this situation. The important question is whether the particles, produced in the future, will generate Jeans instability. Another related test is to take the gravitational production of the electrically charged particles and see if they can cause the Debye screening and the plasma waves. Incidentally, the same question can be posed in the case of electric production. It has been addressed long ago in a nice paper [6] which, by the numerical simulations, showed the presence of the plasma oscillation.

Let us find the Green functions near future infinity. The key point is that for the large $T$ we can precisely match the plane waves at $\pm \infty$ with the Bessel-like solutions of the Klein-Gordon equation in the dS space and thus fully determine the reflection coefficient responsible for particle creation. We have the Jost function $f_k(t) \to \frac{1}{\sqrt{2\omega_k}} e^{i\omega_k t}$ as $t \to -\infty$. Here $\omega_k(t) = \sqrt{m^2 + k^2/a^2(t)}$ and the $\pm$ superscripts refer to the limits $t \to \pm \infty$. As we go to the dS region, $|t| \ll T$, this solution matches the Bunch-Davies mode $h_{\mu}(ke^{-t})$. When we proceed to the "cash register" at future infinity we find that the Bogolyubov coefficients

$$f_k(t) \to \frac{1}{\sqrt{2\omega_k}} (\alpha e^{i\omega t} + \beta e^{-i\omega t})$$

(7)

come from the Hankel function and given by $a(\pm \mu)$. As we see they are independent of $k$ (a consequence of dS symmetry). This is asymptotically exact if

$$ma(-\infty) \ll k \ll ma(\infty)$$

(8)

Outside this interval the particle creation quickly disappears. The Green function at future infinity, derived from the above modes, is given by

$$\langle T \varphi(t) \varphi(t') \rangle = \frac{1}{2\omega} [(1 + |\beta|^2)e^{-i\omega|t-t'|} + |\beta|^2e^{i\omega|t-t'|}] + \frac{1}{2\omega} [\alpha\beta^*e^{-i\omega(t+t')} + c.c]$$

(9)
here $\omega = \sqrt{m^2 + k^2/a(\infty)}$. The coefficients $a(\pm \mu)$ satisfy the Wronskian relation $|a(\mu)|^2 - |a(-\mu)|^2 = 1/2\mu$ as well as $|a(-\mu)|^2 = e^{-2\pi \mu}|a(\mu)|^2$ which leads to the formula

$$|\beta|^2 = \frac{1}{e^{2\pi \mu} - 1}$$

We can now interpret the above formula. The first term in ( ) is the thermal propagator at the Gibbons-Hawking temperature ($1/2\pi$ in our units). At the same time the full propagator is NOT thermal - it was obtained by the evolution of the pure state. However, if we coarse-grain this pure propagator, the second term goes to zero due to infinite oscillations and we get the thermal state. It is also clear that the Doppler shift will be absent, since as $a(\infty) \rightarrow \infty$, the momentum dependence disappears from the formulae.

We shall add to it a general comment concerning the purity of the propagator. One might think that the thermal propagator can’t be distinguished from the one describing the $N$-th excited state. Indeed they have the same form except in the thermal state $N$ is given by the Bose distribution. However there is a major difference when we consider the four point function-there is the Wick theorem in thermal state and no such relation in the excited state.

Debye screening means that the photon acquired a mass, while the Jeans instability is an imaginary mass of the graviton. Formally they both appear because the continuity equation for the polarization operator doesn’t force it (except in the vacuum) to be zero at zero momentum [13]. This happens because the one loop polarization operator has singularities at zero energy and momentum. If the particles in the loop have the energy $E_1$ and $E_2$, the polarization operator contains a singularity at the frequency $\omega = E_1 \pm E_2$. The plus sign describes pair creation by the external quantum, while the minus term comes from the collision of the external quantum with the preexisting particle. As a result, when $\omega, q \rightarrow 0$ we have a non-commutative limits, first noticed by A. B. Migdal in the theory of Fermi liquid. For the non-interacting particles the polarization operator has the form

$$\Pi(q, \omega) \sim \int \frac{dp}{\omega_p \omega_{p+q}} \frac{[N(e^{i\omega_{p+q}}) - N(e^{i\omega_p})](e^{i\omega_p} + e^{i\omega_{p+q}})}{\omega^2 - (\omega_p - \omega_{p+q})^2} + ...$$

\footnote{The appearence of the Bose distribution from the Hankel modes was also known to J. Maldacena. (private communication)}
where by dots we denoted the non-singular part and $N$ is the Bose distribution. This term creates real mass for photons (plasma waves) and imaginary mass for gravitons (instability).

In the present case the question of the Jeans instability requires further study. The particle distribution $N(q) = |\beta|^2$ is given by (1) and is thermal only for $q \ll ma(\infty)$ while a simple computation of the polarization operator includes the values of $q \sim ma(\infty)$ for which the distribution is $q$ dependent and nonequililibrium. Most likely, the interaction between the particles will equilibrate this distribution at some non-universal temperature and produce the Jeans instability. But this remains to be calculated. It is interesting to notice that in the purely electric case numerical simulations in [6] led to the plasma oscillations.

In the next section we will discuss a different type of instability, related to particles interaction.

4 Unstable symmetry of the Poincaré patch. A conjecture.

The Poincaré patch is a one half of the global de Sitter space, representing expanding or contracting universes. It has some fatal attraction for cosmological theories, from the steady state universe to the inflationary one. It seems well fitted for the observed cosmological expansion. However, in quantum theory one must be careful with the geodesically incomplete spaces, since particles may inadvertently disappear, making havoc of unitarity.

In this section we will show that the standard Bunch-Davies vacuum in the Poincaré patch preserves (in a non-trivial way) the full de Sitter symmetry. At the same time any small variation of this state leads to a complete destruction of symmetry, due to the violent particle production.

The dS space is described by the hyperbolic unit vector $n$, satisfying $n_+ n_- + n_\perp^2 = 1$ with $n_\pm = n_1 \pm n_0$. The Poincaré patch is defined by $n_+ \geq 0$. When we consider a field theory in this space, we must integrate over the position of the vertices in Feynman’s diagrams. In the full space the measure of integration is just $(dn)\delta(n_+ n_- + n_\perp^2 - 1)$. In the Poincaré patch this measure must be multiplied by $\varphi(n_+)$. This factor, generally speaking, breaks the dS symmetry. Indeed, consider a rotation $\delta n_+ = \omega_{+\perp} n \perp$. It obviously changes the measure, since the variation of the step function gives
the factor $n_\perp \delta(n_+)$.

Let us study the effect of this variation. Consider a vertex located at the point $n$ in some generic Schwinger-Keldysh diagram. It is attached to the rest of the diagram by a bunch of the Green functions, $\prod_A G(n_A n)$. One should also remember that each point of the diagram carries $+$ or $-$ signs. The change of the amplitude $F$ is given by

$$
\delta F \sim \sum_{\pm} \int dn_\perp dn_- n_\perp \prod G(n_A n) \bigg|_{n_+ = 0} \tag{11}
$$

We have to analyze this contribution for the various $\pm$ combinations. For that we have to remember that in the Bunch Davies vacuum all Schwinger-Keldysh (SK) functions are various boundary value of the single analytic function, which depends on $z = (nn')$. One also should remember that the ordering of the operators (the arrow of time) could be performed with the respect to $n_+$ or with the respect to $n_0$. These two are equivalent. Indeed, the relation $n'_0 \leq n_0$ doesn’t automatically imply that $n'_+ \leq n_+$ but if the latter is not true, the interval between the two points is space-like and the order of operators is irrelevant. To check this we write $2n_0 = n_+ + n_-$ and notice that the interval $s = (n'_+ - n_+)(n'_- - n_-) + (n' - n)_\perp^2 \geq 0$, if $n'_0 \leq n_0$ but $n'_+ \geq n_+$.

The SK set of the Green functions is given by

$$
G_{++}(n, n') = g(nn' - i0), \quad G_{+-}(n, n') = g(nn' - i\varepsilon \text{sgn}(n'_+ - n_+)), \\
G_{-+}(n, n') = G^*_{+-}, \quad G_{--}(n, n') = G^*_{++}
$$

where the analytic function $g(z)$ in the case of the BD vacuum has a single branch point at $z = 1$, and is real for $z \leq 1$ (the space-like separations). Substituting these functions into the above formula and taking into account that $n_+ = 0$ is the ultimate past, we get

$$
\delta F \sim \text{Im} \int dn_\perp dn_- \int dn_- \prod g(n_{A+} n_- + n_{A\perp} n_\perp - i0) \tag{12}
$$

From this we see that we can close the contour of the $n_-$ integration in the lower half-plane (since $n_{A+} \geq 0$) and the integral, in the case of the BD vacuum, is zero.
However, this symmetry is likely to be unstable. If we change the B.D. Green functions in the past, in such a way that the dS symmetry is preserved on the classical level, the quantum effects will unravel this symmetry. Indeed, let us consider the state with a non-zero occupation numbers. The simplest example, describing the constant occupation numbers, leads to the Green function

\[ G^{(N)}(z) = (1 + N)g(z - i0) + N g(z + i0) \]  

This function is de Sitter symmetric. Notice, however, that it is not one of the Mottola-Allen \( \alpha \)-vacua. The latter describe the Fock vacua, while in our case we average with respect to the excited state. One popular "objection" against such states (and against \( \alpha \)-vacua) is that in the short distance limit these Green functions don’t reduce to the Minkowsky vacuum Green function. This objection is unjustified. In the geodesically incomplete spaces the Green functions are determined by the choices made behind the horizon. As an analogy, consider the Rindler space. Field theory in this space has a well defined vacuum. However, if we assume that the full space is in the ground state, the induced Green function in the Rindler wedge is the thermal one.

Returning to the case above, we can’t close the contour and the dS symmetry is badly broken. Moreover, as we will see below, the breaking is amplified by the logarithmic IR divergence. In this sense the Poincare patch symmetry is likely to be unstable, at least in the linear approximation. We will consider now the general variation of particle numbers. To simplify notations, we will discuss a \( \varphi^3 \) theory.

In the previous papers \cite{2}, \cite{4} we introduced various production amplitudes. They describe the following allowed decays and fusions. We can have the process \( \text{vac} \Rightarrow (q) + (k) + (-k - q) \), with the amplitude \( A(q, k) \) and the inverse process with the amplitude \( A^*(q, k) \). (the notations mean that the vacuum decays onto first excited states with the momenta \( q, k, -k - q \)). We can also have the particle decay, \( (q) \Rightarrow (k) + (q - k) \) with the amplitude \( B(q, k) \), and most importantly, the bremsstrahlung radiation, \( (k) \Rightarrow (k - q) + (q) \), with the amplitude \( C(q, k) \). Using the standard mode expansion, one expresses
these amplitudes in terms of overlap integrals [4]

\[ A(q, k) = \int_{\tau}^{\infty} d\tau \tau^{d-1} h(q\tau) h(k\tau) h(|k + q|\tau) \]  

(14)

\[ B(q, k) = \int_{\tau}^{\infty} d\tau \tau^{d-1} h^*(q\tau) h(k\tau) h(|q - k|\tau) \]  

(15)

\[ C(q, k) = \int_{\tau}^{\infty} d\tau \tau^{d-1} h(q\tau) h^*(k\tau) h(|k + q|\tau) \]  

(16)

where \( h(x) \) are proportional to the Hankel functions \( H^{(1)}_{\mu}(x) \). The self-energy corrections to the Schwinger - Keldysh propagators are expressed in terms of the products of these amplitudes, for example the +/- correction to the two point function \( \langle \varphi(q\tau)\varphi(-q\tau) \rangle \) is proportional to the integral \( \int d^d k |A(q, k)|^2 \) (see [2] for the details). The infrared corrections (not divergences) appear because for \( k \gg q \), \( |A(q,k)|^2 \sim k^{-d} \), which generates a logarithmic correction \( \log(\frac{1}{q\tau}) \). These corrections, in agreement with the above theorem, do not violate the de Sitter symmetry. Another reason for the absence of the divergence in this case follows from the observation [9] that the one point function in the Poincare patch is the same as in the static patch. However, in the latter case we have thermal equilibrium and the collision integral is equal to zero. This is also in agreement with the absence of corrections to the self-energy part [10].

The situation changes drastically if we start with the non-zero occupation numbers. Consider a bare Green function

\[ \langle \varphi(q\tau)\varphi(-q\tau') \rangle = (1 + n(q)) h^*(q\tau_+) h(q\tau_-) + n(q) h^*(q\tau_-) h(q\tau_+) \]  

(17)

which corresponds to the averaging with the respect to the excited states with the occupation numbers \( n(q) \). The major difference comes from the \( C \) - amplitude (which doesn’t appear when \( n(q) = 0 \)). As is seen from the above formulae, the integrand for this amplitude is not oscillating at very large \( k \) and this changes the asymptotic behavior of \( C \). As \( k \to \infty \) we have

\[ C \approx \frac{1}{|k|} \int d\tau \tau^{d-2} h(q\tau)e^{iq\cos \theta \tau} \]  

(18)

where \( \theta \) is the angle between \( k \) and \( q \). We can now evaluate the contribution of the \( C \)-process to the occupation numbers. Keeping only the linear term in \( n(k) \) in the collision integral, we obtain

\[ \Delta n(q) \propto \int d^d k |C(q, k)|^2 n(k) \propto \frac{1}{q^{d-2}} \int \frac{d^d k}{k^2} n(k) \]
This integral can become divergent for the perturbations concentrated at large \( k \). This is an indication of the linearized instability of the Poincare patch. Indeed, even if we assume that the original perturbation \( n(k) \) is vanishing so fast that the integral converges, in the second iteration one gets \( \Delta n(k) \sim 1/k^{d-2} \) which leads to the IR divergence. One needs to study the collision integral in full to make a definite conclusion beyond the linear approximation. We will not proceed with it here, since our main interest is in the geodesically complete case. Notice, however, that the contribution of \( A \) and \( B \) are suppressed by the factor \( (q/k)^{d-2} \).

Let us stress the difference between the dS and the Minkowski space. In the latter case we can also change the occupation numbers, breaking the symmetry at the initial moment. The system then will tend to thermal equilibrium. The temperature for the small initial perturbations will remain small. In the dS case the conserved energy is not positive definite, there is no H-theorem, and we may experience the instability.

5 Hyperbolic motion, an analogy

The picture described above is analogous to the classic problem of radiation of the accelerated charge. We will briefly describe it to stress the analogy. The probability of radiation is given by the "golden rule"

\[
w \sim \int |J_{\mu}(k)|^2 \theta(k_0) \delta(k^2) d^4k
\]  

(19)

where the current (or vertex operator) is given by \( J_{\mu}(k) = \int ds x'_{\mu}(s)e^{ikx(s)}. \), with \( x(s) \) being a trajectory of the charge. It is instructive to perform the \( k \)-integration and obtain the expression for the probability

\[
w \sim \int \int ds_1 ds_2 (x'(s_1)x'(s_2)) \frac{1}{(x(s_1) - x(s_2) + i\epsilon)^2}
\]

(20)

and for the radiated energy (obtained by adding \( k_{\mu} \) factor in (19 ))

\[
P_{\mu} \sim \int \int ds_1 ds_2 (x'(s_1)x'(s_2))(x_{\mu}(s_1) - x_{\mu}(s_2)) \frac{1}{(x(s_1) - x(s_2) + i\epsilon)^4}
\]

(21)

Here \( \epsilon \) is an infinitesimal time-like vector. The first expression is an interesting version of the Wilson loop (different from the standard one by the \( i\epsilon \)
prescription). This prescription makes (21) non-zero. Using the antisymmetry of this expression we get the standard Lorentz-Dirac formula

\[ P_\mu \sim \int ds[v''_\mu - (vv'')v_\mu] \]

(22)

\[ (v = x', v^2 = 1) \]

(23)

The condition of the constant rate can be formulated as a symmetry of the trajectory. In the case of the inertial motion of the charge, the symmetry is simply \( x(s + t) = x(s) + c \) and we have no radiation. In the hyperbolic case the we have

\[ x_\mu(s + t) = \Lambda_{\mu\nu}(t)x_\nu(s) \]

(24)

where \( \Lambda \) is a Lorentz transformation. This is the analogue of the dS symmetry. The Lorentz-Dirac equation has the form

\[ v'_\mu = \varepsilon_{\mu\nu}v_\nu + \frac{2\alpha}{3}[v''_\mu - (vv'')v_\mu] \]

(25)

and the last term vanishes for the hyperbolic motion. This is analogous to the preservation of the dS symmetry in the preceding section. It means that there is no radiation from the hyperbolic motion. However this solution is unstable. It is well known that the equation has runaway solutions violating the above symmetry. Under a small perturbation the particle begins to radiate. Dirac tried to impose initial and final conditions together in order to avoid the runaway. In our language this amounts to considering in/out matrix elements. As we mention above, there is no IR divergences in these amplitudes.

6 Upside-down description of the global dS space

We shall now apply the above methods to the global (geodesically complete) space. This space is known to be classically stable [15] in the sense that the Cauchy problems at past infinity are well posed. In a vulgar language of a field-theorist, this means that the tree S-K diagrams (representing a perturbative solution of the Cauchy problem) are not IR divergent. However, this is not true for quantum corrections.
Once again we will consider an interacting massive field on the dS background and assume that the interaction is adiabatically turned on near the past infinity. If we are interested in the one point functions, we can achieve a considerable simplification by the following trick. Suppose that the point in consideration is given by a hyperbolic vector $n^*$, which we can always choose to have $n^*_− ≥ 0$. In the Schwinger-Keldysh approach all interactions take place inside the past light cone, defined by $(n − n^*)^2 ≤ 0$ and $n_0 ≤ n^*_0$. Rewriting this as $(n_− − n^*) (n_+ − n^*_+) ≤ 0$ and $n_+= n_− ≤ n^*_− − n^*_+$ we conclude that $n_− ≡ 1/τ ≥ 0$. Therefore, as far as one point functions are concerned, the global dS space is identical to the Poincare patch, turned upside down, with $τ = 0$ being past infinity and the time development defined by the increasing $τ$ (which we may call the ”Anti-Poincare patch”).

This statement may looks strange, since we ended up with the contracting patch, while the global dS contains both contraction and expansion. It should be remembered, however, that we are dealing here with the one-point functions and it is impossible to distinguish contraction from expansion while staying at one point. Moreover, the standard definition of the red shift refers to two static observers (to eliminate the Doppler effect from the peculiar velocities). But in the case of the dS space the ”static observer” in the Poincare coordinates is generally non-static in the global coordinates.

The global two point function can also be described in the above patch, provided that the two points are lying inside the horizon. It is easy to see that this restricts the positions so that $(n_1 n_2) ≥ −1$. As we cross this boundary, the Green function can be non-analytic. Yet another interesting feature of the Anti-Poincare patch is that the derivation of the dS symmetry given in the preceding section is not applicable. Technically this happens because we reverse the arrow of time. This leads to a different $iε$ prescription in the formula ( ) and this integral isn’t zero anymore.

Let us begin with the processes described by the $A$ - amplitude. In the Anti-Poincare case it is given by the same expression as in the usual case, except that the integral over $τ$ is going from zero (past infinity) to the present moment. We also have to introduce the IR-cutoff $ε$, defined as the time in the past when the interaction was turned on. We have

\[ A(q, k) = \int_ε^τ \frac{dτ_1}{τ_1} τ_1^{d/2} h(qτ_1) h(kτ_1) h(|k − q|τ_1) \quad (26) \]

and analogous formulae for $B$ and $C$ amplitudes. The dominant contribution comes from $k ≫ q$. Just as in [2], using the expansion at $x → 0$, $h(x) =$
for the Hankel’s functions, normalized by $h(x) \to (2x)^{-\frac{1}{2}} e^{ix}$ as $x \to \infty$, we obtain

$$A(q, k) \to k^{-\frac{d}{2}} [a(\mu)g(\mu)\left(\frac{q}{k}\right)^{i\mu} + (\mu \to -\mu)]$$

(27)

$$g(\mu) = \int_{0}^{\infty} dx x^{d/2-1+i\mu} h^2(x)$$

(28)

This asymptotics assumes that

$$\frac{1}{\varepsilon} \gg k \gg \max(q, 1/\tau)$$

(29)

since under these conditions we can drop the limits in the above integral. The contribution to the Green function can be calculated using the Schwinger-Keldysh approach and in this order we have

$$G^{(1)}_{++}(q, \tau) = G_{++} \Sigma_{++} G_{++} - G_{+-} \Sigma_{+-} G_{++} + (\leftrightarrow -)$$

$$G_{++}(q, \tau_1, \tau_2) = (\tau_1 \tau_2)^{d/2} h^*(q\tau_>) h(q\tau_<)$$

(30)

Combining these formulae, we get the general structure of the correction to the Green function (for simplicity we look at the coinciding times)

$$G^{(1)} = 2N(q, \tau)|h(q\tau)|^2 + \text{Re}(\alpha^*(q, \tau)h^2(q\tau))$$

(31)

The same structure appeared in [2], however in the global dS space it has very different meaning. Let us look at the $\Sigma_{+-}$ term which contains a square of the $A$ amplitude. It gives a contribution to $N$

$$N(q, \tau) \sim \int |A(q, k)|^2 d^d k$$

(32)

In the Poincare patch it gave a contribution of the order of $\log(1/q\tau)$ which was important only if $q\tau \ll 1$. These logarithms modify the large distance behavior of the Green function but do not contain the infrared divergence, responsible for the secular change of the vacuum (and for the irreversible time in the Boltzmann equation). This fact is also a consequence of the above theorem on the symmetry of the Poincare patch. It all changes in the global case. Using ( ) we have

$$|A(q, k)|^2 = (|ag|^2 + |\tilde{a}g|^2)k^{-d} + ...$$

(33)
where tilde means the change $\mu \rightarrow -\mu$ and the dots stand for the oscillating terms. As a result we get

$$N(q, \tau) \propto (|ag|^2 + |	ilde{a}g|^2) \log(\frac{\tau}{\varepsilon})$$  \hspace{1cm} (34)

in the region $q\tau \sim 1$. This logarithm is equal to the proper time passed since the interaction was switched on, up to the present moment and thus we have a linear divergence in time, just as we have one in the Boltzmann equation when the collision integral is non-zero (we mean the kinetic equation $\frac{dn}{dt} = C[n]$, being iterated with the non-equilibrium initial conditions). The above result comes from the $\Sigma^+$ term. The terms with $\Sigma^{++}$ and $\Sigma^{--}$ generate the contributions proportional to $h^2(q\tau)$ and its complex conjugate. As was explained in [2], they contain a product of $A$ and $B$ amplitudes, leading to

$$\alpha(q, \tau) \sim \int d^dk A(q,k)B^*(q,k) \sim \tilde{a}\tilde{a}(|g|^2 + |\tilde{g}|^2) \log(\frac{\tau}{\varepsilon})$$  \hspace{1cm} (35)

The $C$ amplitude doesn’t appear in this order, since we started from the vacuum state and the $C$ - process requires non-zero occupation numbers.

It is easy now to evaluate the one point function in this approximation (see also [2]). We have

$$\langle \varphi^2(n) \rangle \sim \tau^d \int d^dq |h(q\tau)|^2 \int d^dk |A(q,k)|^2$$  \hspace{1cm} (36)

The integral is divergent at large physical momentum $p = q\tau$. It is cut off in two different ways. First, the physical momentum must satisfy $p \ll \Lambda_{UV}$, where $\Lambda_{UV}$ is the ordinary ultraviolet cut-off. Notice, that without interaction the second integral in this formula is absent and we have the standard flat space result $\langle \varphi^2(n) \rangle^{(0)} \sim (\Lambda_{UV})^{d-1}$, since $|h(p)|^2 \sim 1/p$ as $p \rightarrow \infty$. This is just an uninteresting UV correction to the cosmological constant.

Let us evaluate the above expression. For that we rewrite the formula (17) taking into account the domain of the integration. We have

$$g(\mu, \tau, \varepsilon) = \int_{k\varepsilon}^{k\tau} dxd^{d-1}x \cdot h^2(x)$$  \hspace{1cm} (37)

We see that the maximal contribution to the one point function comes from the region

$$m \ll q\tau = p \ll k\tau \ll \min(m\frac{\tau}{\varepsilon}, \Lambda_{UV})$$  \hspace{1cm} (38)
here we introduced explicitly the mass $m$ of our scalar field, which was a dimensionless number before (measured in the units of the Hubble constant). We see the same non-commutativity of limits as for the electric case, namely, if $\Lambda_{UV} \gg m\tilde{\varepsilon}$ we get

$$\langle \varphi^2(n) \rangle^{(1)} \sim m^{d-1}\left(\frac{\tau}{\varepsilon}\right)^{d-1} + O(\log\left(\frac{\tau}{\varepsilon}\right))$$

where the second term represents the contributions of the $B-$ amplitude, proportional to $h^2(p)$ and its complex conjugate. These functions oscillate at the large $p$ and thus do not produce the power UV divergence.

A natural interpretation of the above formulae is the large blue shift of the virtual particles. However, one must be careful with such an interpretation, since, as was already mentioned, this notion is coordinate-dependent due to the Doppler shift. Although on a technical level we are working with the universe contracting to zero and having large blue shifts, we apply these results to describe a non-singular global dS space.

A question, arising at this stage, is the summation of the leading logarithms, the contributions of the order $(\lambda \log(\frac{\tau}{\varepsilon}))^n$ where $\lambda$ is a coupling constant. The answers to this question is very different in the cases of the Poincare patch and the global AdS. In both cases we begin in the lowest order with the contributions to the self-energy part (written schematically) $\Sigma(q) \sim \int G(k)G(q-k)d^dkd\tau_1d\tau_2$. The lowest logarithmic contribution, discussed above, is coming from the domain $k \sim 1/\tau_1,2 \gg q$. The next correction comes from the correction to $G$ in the above integral, $G^{(1)}(k) \propto \int G^2(k')d^dk'd\tau_3d\tau_4$.

In the Poincare patch this correction is negligible, since as we indicated before, the logarithms in $G(k)$ arise only when $k\tau \ll 1$. As a result, in this case higher order corrections to the self-energy part are not logarithmic since they involve the Green functions at the momenta $k \sim 1/\tau$. This conclusion agrees with the work [10].

In the global AdS we found that the logarithmic contribution to the Green function $G(q,\tau)$ is present for any $q\tau$, provided that $q\varepsilon \ll 1$ and $\tilde{\varepsilon} \gg 1$. Hence, in this case the self-energy part receives non-trivial corrections.

Qualitatively they can be viewed as a cascade production and absorption of particles with the momenta $q \ll k \ll k' \ll ...$. With each generation the momentum increases. This is very similar to the direct cascade in the theory of turbulence.
7 An even-odd curiosity and four-valued time

It was already noticed [4] that in the lowest order of perturbation theory the amplitudes for particle creation are zero. This has an interesting consequence for the infrared divergences. Let us repeat the calculations of the previous section by keeping explicitly the contracting phase [2] and using the global eigenmodes. The Schwinger - Keldysh rules are modified, making the flow time four-valued (forward / backward and contracting / expanding). As was explained in [2] the logarithmic corrections arise in the region where the global modes, which are the Legendre functions can be approximated by the Hankel functions as following

\[ f_q(t) \rightarrow \{ \begin{array}{ll} \tau^{d/2} h(q\tau) \\ (-\tau')^{d/2} h(-q\tau') \end{array} \] (40)

Here \( \tau = e^{-t} \) and \( t \rightarrow +\infty \) in the upper expression. This term describes the expanding stage. In the lower expression \( \tau' = e^t \) and \( t \rightarrow -\infty \) and thus describes the contracting phase. In the Feynman rules for the interacting theory each vertex contains not only the usual Schwinger - Keldysh but also a summation over expanding/contracting locations, making time a four-valued quantity.

In the present case the contributions to the \( A \) amplitude come from the both stages. We have the total contribution in the form

\[ A = A^{(e)} + (-1)^{d+1} A^{(c)} \] (41)

where the amplitudes are given by the overlap integral (14) with the functions (40). Namely

\[ A^{(e)} = \int_{\tau_1}^{\infty} \frac{d\tau_1}{\tau_1^{d+1}} \tau_1^{3d/2} hh  \] (42)

\[ A^{(c)} = \int_{-\infty}^{-\varepsilon} \frac{d\tau_1}{\tau_1^{d+1}} \tau_1^{3d/2} hh  \] (43)

The sign factor in (41) comes from the \((-\tau)^{d+1}\) in the phase volume. When the dimension \( D = d + 1 \) is even, there is a destructive interference between contraction and expansion. This is seen from the identity

\[ \int_{-\infty}^{\infty} \frac{d\tau}{\tau^{d+1}} \tau^{3d/2} hh = 0 \] (44)
since we can close the contour in the upper half-plane. In the previous section we obtained the scale invariant behavior of the amplitudes in the domain where the lower limits of the integral (42) can be set to zero. If we can set to zero the upper limit of (43) also, we will get the cancellation in the even dimension and doubling in the odd. The condition for this to happen is to have $\tau_1 \sim 1/k \gg \tau$ in the first integral and $\tau_1 \sim 1/k \gg \varepsilon$ in the second. As before, we also need $k \gg q$. With these conditions we obtain the first correction to the occupation numbers in the form

$$N(q, \tau) \propto \log\left(\frac{1}{q\varepsilon}\right) + (-1)^{d+1} \log\left(\frac{1}{q\tau}\right) \propto \left\{ \frac{\log(\varepsilon)}{\log\left(\frac{1}{q^{2+\tau\varepsilon}}\right)} \right\}$$

for even and odd cases. So, in the even case the instability is weaker than in the odd. It is interesting that the above formula is different from (34) when $q\tau \ll 1$. This is not surprising, since the equivalence of the anti-Poincare and the global case works only for two point functions with the points inside the light cone. The above condition violates this constraint.

The above considerations refer to the lowest non-trivial order of perturbation theory. It is not clear at this point if they survive in the higher orders.

8 The Ansatz for the Green functions.

In order to sum the leading logarithms we need an ansatz for the Green functions. This problem has not been solved yet. In this section we will indicate a plausible approach to its solution.

Let us first remember how the secular terms in the Green functions are summed up in the case of the ordinary Boltzmann equation. In this case we have two different time scales, one given by the characteristic energies of the particles and another, much longer time scale - that of the free flight. As a result, in the leading approximation, the Green functions retain their non-interacting form but the occupation numbers, defining these functions are slowly changing. The evolution of these numbers is governed by the collision integral and as noticed in [3] can be viewed as an IR analogue of the renormalization group. So, the key point concerning kinetics is that the weak enough interaction modifies the Green functions only through the slow variation of the occupation numbers. Another point is that the kinetic equation is just the Dyson equation in which the lowest order self-energy
part is expressed in terms of the dressed Green functions, while the vertex corrections can be neglected.

Something very similar is also true in our case. The formula (20) shows that the interaction leaves the Green function free, except that we have a slowly varying factors \( N(q, \tau) \) and \( \alpha(q, \tau) \), the two soft modes. These factors can be interpreted as the occupation number and the infinitesimal Bogolyubov rotation. Therefore we shall conjecture the following structure of the Green functions in the global dS space

\[
G(q, \tau) = (1 + N(l)) f^*(q, \tau_>) f(q, \tau_<) + N(l) f^*(q, \tau_<) f(q, \tau_>) 
\]

(45)

where \( l = \log \frac{\tau}{\varepsilon} \) and

\[
f(q, \tau) = \alpha(l) h(q\tau) + \beta(l) h^*(q\tau) \quad (46)
\]

\[
|\alpha|^2 - |\beta|^2 = 1 \quad (47)
\]

This ansatz assumes that beyond the first approximation we get a finite Bogolyubov transformation. When we substitute this expression into the Schwinger- Keldysh self energy part, \( \Sigma \sim \int GG \) we will get the collision integral. It contains many terms involving products of \( N, \alpha, \beta \). The overlap integrals \( A, B, C \) (14 - 16 ) now contain the functions \( f \) instead of \( h \). The effect of the non-zero occupation numbers, say, in the \( A \) amplitude is seen from the replacement of (32 ) by

\[
N(q, \tau) \sim \int d^d k |A(q, k)|^2 \{ (1 + N(q)(1 + N(k))(1 + N(q-k))) - N(q)N(k)N(q-k) \} + ... 
\]

(48)

The calculations in the preceding sections indicate that these quantities (with the logarithmic accuracy) depend on its arguments in a specific way. Namely, if we introduce the physical momentum \( p = q\tau \), the occupation numbers at the fixed \( p \) depend on \( \tau \) only and not on \( p \sim m \). More precisely, there are extra logarithmic terms if \( p \ll m \), but, at least in the leading order this domain is negligible. After going to the scaling limit the above term becomes

\[
N(\tau) = \int_\varepsilon^{\tau} \frac{d\tau_1}{\tau_1} |A(\tau_1)|^2 \{ (1 + N(\tau_1))^2(1 + N(\tau)) - N^2(\tau_1)N(\tau) \} + ... 
\]

(49)

\[\text{A similar ansatz was considered in [11] and by D. Krotov (unpublished) in the Poincare patch.}\]
here the \( A \) depends on \( \tau \) through the Bogolyubov coefficients; the dots stand for the contribution of the \( B \) and \( C \) processes described above. We need another equation for these coefficients. It follows from the matching of the terms in the Green function proportional to \( \hbar^2(q\tau) \) and its complex conjugate. This is a straightforward procedure, but it results in long clumsy equations. Perhaps going to the geometrical variables of \( \text{SL}(2,\mathbb{C}) \), describing the Bogolyubov transformation, will bring some simplification.

There is, however, another problem with our ansatz. The \( C \)-type amplitudes, according to (14) contain the integrals involving the combinations \( hh^* \) which don’t oscillate at very large \( k \). This can cause a divergence which will invalidate our scaling limit. The problem has not been fully resolved. Let us notice that it is related to the bremsstrahlung radiation of the soft quanta, moreover, the domain of large \( k \) corresponds to the flat space limit. The emission of such quanta produces the well-known divergences in perturbation theory, which, however, disappear under the appropriate treatment. We hope that the same cancellation occurs in our case and the above divergences can be renormalized away. Formally, it is easy to give a prescription which eliminates the power-like divergences. It is sufficient to define the overlap integrals as an analytic continuation in \( d \) from the region \( d < 2 \) where they converge.

\section{Conclusions}

The main conclusion of this paper is that the de Sitter spaces of various kinds are unstable on the quantum level. In this paper we discussed only the interacting massive scalar fields, which is technically the easiest case. It is of course very important to include massless gravitons in our picture. Formally the massless case corresponds to taking \( \mu = i\frac{d}{2} \). The overlap integrals become superficially divergent. However, this leading divergence must cancel due to gauge invariance. We will be left with the subleading divergence, similar to the ones considered above. A lot of work remains to be done, mostly related to the proper treatment of gauge degrees of freedom.

The summation of the leading logarithms is another unfinished business. It is likely that it will lead to the Landau poles in various observables and will require new methods. Whatever the answer will be, this is a well posed fascinating problem.

Another interesting problem is to consider instead the global dS space the
“centaur” - a Coleman - de Lucia bubble appeared through tunneling. It is described by the same Feynman diagrams, but, as usual, the time variable is imaginary under the barrier. It seems likely (but not proved) that dS symmetry is unstable also in this case.

It may be too early to try to build models based on the above mechanisms (see, however, [16]). Still it might be relevant in various cosmological scenarios. Indeed, the standard inflation is based on the assumption of slow roll towards zero cosmological constant. Even if we assume that the cosmological constant problem is somehow solved, the question remains, why the theory was originally in a highly excited state and not simply in the Minkowsky vacuum. In the above picture the presence of the original cosmological constant will make the steady state impossible.

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