The Enhançon, Black Holes, and the Second Law

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Abstract

We revisit the physics of five–dimensional black holes constructed from D5– and D1–branes and momentum modes in type IIB string theory compactified on K3. Since these black holes incorporate D5–branes wrapped on K3, an enhançon locus appears in the spacetime geometry. With a “small” number of D1–branes, the entropy of a black hole is maximised by including precisely half as many D5–branes as there are D1–branes in the black hole. Any attempts to introduce more D5–branes, and so reduce the entropy, are thwarted by the appearance of the enhançon locus above the horizon, which then prevents their approach. The enhançon mechanism thereby acts to uphold the Second Law of Thermodynamics. This result generalises: For each type of bound state object which can be made of both types of brane, we show that a new type of enhançon exists at successively smaller radii in the geometry, again acting to prevent any reduction of the entropy just when needed. We briefly explore the appearance of the enhançon locus in the black hole interior.

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1 Introduction

The idea behind the microscopic evaluation of the entropy of a class of charged black holes using D–branes is a simple one: First, one constructs an arrangement of D–branes with the same asymptotic macroscopic charges as the black hole. At weak coupling, this is a relatively benign situation, and we can evaluate the degeneracy associated to these macroscopic charges by counting the number of microscopic arrangements of D–branes which give rise to the configuration. Finally, we trust that since we have counted BPS microstates at weak coupling, the journey to strong coupling will not prevent us from associating the result with the entropy of the black hole which is supposed to form in the limit.

This worked very well for the prototype case, presented in ref. [1], of five–dimensional black holes in type IIB string theory. One wraps \( Q_5 \) D5–branes on a four–manifold \( \mathcal{M} \), which we shall take to lie in the \((x^6, x^7, x^8, x^9)\) directions, leaving a string in the \( x^5 \) direction. This string is combined with \( Q_1 \) D1–branes, also lying in \( x^5 \). Finally the \( x^5 \) direction is compactified and \( Q_P \) units of momentum are added along this circle direction. Thus the final configuration appears as a pointlike object, in the five uncompactified directions \((x^0, \ldots, x^4)\), which has three macroscopic charges \((Q_1, Q_5, Q_P)\) associated to it.

At strong coupling, (or large charges) there is a non trivial back–reaction on the geometry and the resulting spacetime solution is a five–dimensional extremal black hole, having a horizon of area \( A = 8\pi G \sqrt{Q_1Q_5Q_P} \) (where \( G \) is Newton’s constant). The microscopic count of the degeneracy of D–brane states giving rise to the microscopic charges gives exactly the result \( S = A/(4G) \) for the Bekenstein–Hawking entropy [2].

In the case where \( \mathcal{M} \) is K3, there are complications which have not been considered in the present black hole context. It is known [3, 4] that wrapping a D\( p \)–brane on K3 induces a negative unit of D\((p–4)\)–brane charge in the unwrapped part of the worldvolume. As this is a BPS object, this gives a corresponding negative contribution to the tension of the wrapped object [3, 4]. At strong coupling (or large charges), where the D–branes have a non–trivial back–reaction on the geometry, for generic combinations of parameters it was shown in ref. [7] that this situation can give rise to regions of the naive spacetime solution where the tension of the wrapped brane is unphysical. This is simply because the volume of the K3 upon which the brane is wrapped can vary as a function of position transverse to the branes. There are positions where \( V_{K3} \) drops below the stringy value of \( V_* \equiv (2\pi \ell_s)^4 \). Within such regions there are naked time–like singularities, resulting from the K3 volume shrinking all the way to zero.

This is particularly problematic when branes which supposedly generate the geometry have negative tension in the \( V_{K3} < V_* \) region. The proposal of ref. [7] was that the locus of points where the tension drops to zero \((i.e., \ V_{K3} = V_*\) — called the “enhançon” — marks the end of the validity of the naive spacetime geometry produced by those constituent branes, and the region within must be excised and replaced by a different geometry appropriate to the situation in hand: For the simplest case of just a single constituent type of D–brane, the
spacetime interior to the enhançon is proposed\[7\] to be flat. We shall see here that we will need more interesting geometries in the interior, since we have momentum, and both unwrapped and wrapped branes present, of which only the latter care about the enhançon. In fact, as we shall see, excision is not always necessary.

An issue which arises as an immediate consequence of considering the K3 case of constructing these black holes is the fact that one can choose parameters such that at strong coupling the enhançon appears outside of the black hole horizon. Naively, this may complicate some of the entropy counting story, and so we carefully reconsider this case. In fact, while the microscopic entropy story remains essentially unchanged, we will demonstrate that the enhançon does play a crucial role in the physics of the black holes. We find that its existence is essential to the correct operation of second law of thermodynamics! This is a quite satisfying additional facet of the connection, forged by D–branes, between microscopic and macroscopic physics of black holes.

2 The Consistency of Excision

We start by considering the strongly coupled limit, where we have non–trivial spacetime geometry. Using the conventions adopted for example in refs.[9, 10] the Einstein frame metric is:

$$ds^2 = f_1^{-3/4} f_5^{-1/4} \left( -dt^2 + dz^2 + k \left( dt - dz \right)^2 \right) + f_1^{1/4} f_5^{-1/4} ds_{K3}^2 + f_1^{1/4} f_5^{3/4} \left( dr^2 + r^2 d\Omega_3^2 \right) \tag{1}$$

where $ds_{K3}^2$ is the metric on a K3 manifold with a fixed volume $V$, and

$$d\Omega_3^2 = d\theta^2 + \sin^2 \theta (d\phi^2 + \sin^2 \phi d\chi^2) \tag{2}$$

is the metric on a round three sphere: $(t, r, \theta, \phi, \chi)$ constitute polar coordinates in the directions $(x^0, x^1, x^2, x^3, x^4)$, and the K3 is in the $(x^5, x^7, x^8, x^9)$ directions. The $x^5 \equiv z$ direction is compact with period $2\pi R_z$. The dilaton and Ramond–Ramond (R–R) fields are given by:

$$e^{2\phi} = \frac{f_1}{f_5} , \quad F^{(3)}_{rtz} = \partial_r f_1^{-1} , \quad F^{(3)}_{\phi\chi} = 2 r_5^2 \sin^2 \theta \sin \phi \ . \tag{3}$$

The harmonic functions are given by

$$f_1 = 1 + \frac{r_1^2}{r^2} , \quad f_5 = 1 + \frac{r_5^2}{r^2} , \quad k = \frac{r_P^2}{r^2} \tag{4}$$

where the various scales are set by

$$r_5^2 = g_s \ell_s^2 Q_5 , \quad r_1^2 = g_s \ell_s^2 \frac{V^*}{V} Q_1 , \quad r_P^2 = g_s \ell_s^2 \frac{V^*}{V} \frac{\ell_s^2}{R_z^2} Q_P \ , \tag{5}$$

\(^1\text{For a series of studies of the consistency of this type of construction and how it fits with enhançon and brane physics, see ref.\[3\].}\)
where \( V^* = (2\pi \ell_s)^4 \) is the magic duality volume of the K3. Newton’s constant is given by 
\[
16\pi G = (2\pi)^7 g_s^2 \ell_s^8.
\]

The apparent singularity at \( r = 0 \) is only a coordinate singularity. It is actually only an event horizon with vanishing surface gravity and area \( A_H = 4\pi^3 V R_2 r_1 r_3 r_p \), measured in the Einstein frame. These properties translate into a vanishing Hawking temperature and a Bekenstein–Hawking entropy of \( S = 2\pi \sqrt{Q_1 Q_5 Q_P} \).

Of course, the integers \( Q_1, Q_5 \) and \( Q_P \) measure the asymptotic charges associated with the electric and magnetic R–R fluxes and the internal \( z \)–momentum, respectively. We also introduce integers \( N_1 \) and \( N_5 \) to denote the number of D1–branes and D5–branes, respectively, in the system. Of course, we have \( N_5 = Q_5 \). However, as discussed above, wrapping the D5–branes on K3 induces a negative D1–brane charge. Hence we have \( N_1 = Q_1 + Q_5 \) or alternatively \( Q_1 = N_1 - N_5 \).

Now the volume of the K3 manifold (measured by the string frame metric \( \tilde{G}_{\mu\nu} = e^{\Phi/2} G_{\mu\nu} \)) is
\[
V(r) = \frac{f_1}{f_5} V, \tag{6}
\]
where \( V \) is the asymptotic volume of the K3. Now at the horizon,
\[
V_H \equiv V(r = 0) = \frac{r_1^2}{r_5^2} V = \frac{Q_1}{Q_5} V^* = \frac{N_1 - N_5}{N_5} V^*, \tag{7}
\]
and so if \( r_1 < r_5 \), then \( V_H < V \). So we see that as long as \( r_1 < r_5 \), that the volume of the K3 is shrinking as we move in from asymptotic infinity. Note that in the case of interest, \( r_1 < r_5 \), that the local string coupling, \( i.e., g_s e^\Phi = g_s (f_1/f_5)^{1/2} \), is also decreasing. When we reach \( V(r) = V^* \) at some radius, new physics will come into play, and this is the “enhançon” locus of ref.\[7\]. This radius may be computed easily to be:
\[
r_e^2 = g_s \ell_s^2 \frac{V^*}{(V - V^*)} (2N_5 - N_1), \quad \begin{cases} > 0 & \text{for } 2N_5 > N_1 \\ < 0 & \text{for } 2N_5 < N_1 \end{cases}, \tag{8}
\]
where \( r_e^2 < 0 \) simply indicates that the K3 volume reaches \( V^* \) inside the event horizon. Therefore we see that we can have the enhançon appearing either above or below the horizon, depending upon our choices of parameters. As we shall see, this will lead to very interesting physics.

### 2.1 Matching for \( r_e^2 > 0 \)

Now when the K3 volume reaches \( V^* \), at the enhançon radius, \( r_e \), the wrapped D5–branes will be unable to proceed supersymmetrically into smaller radius\[4\], due to the fact that their effective tensions are going through zero there. They are therefore forced to form an enhançon sphere at radius \( r_e \). Note however that there is nothing to prevent the D1–branes and momentum modes from moving inside of \( r = r_e \). They are not wrapped on K3 and therefore do not care that it is approaching a special radius there\[11\]. (We will illustrate these statements fully with a probe computation later in section 2.2.) Hence while eqns. (4)\[4\] provide a good supergravity solution
for \( r > r_e \), it seems that we need a different solution to describe the interior (although it will become apparent in the sequel that this need not be the case).

Naively, one may think that the interior solution should carry no D5–brane charge, \textit{i.e.}, the magnetic component of the R–R three–form should vanish. However, later we will show that the D5–branes can enter the region with \( V(r) < V_* \) and the black hole, if they are appropriately “dressed”. Hence we will consider a more general extension of the supergravity solution given above in eqns. (1–4). We introduce a shell at some arbitrary radius \( r = r_i \), which carries a fraction of the D–branes, \textit{i.e.}, \( \delta N_1' \) D1–branes and \( \delta N_5' \) D5–branes are uniformly distributed over the three–sphere at \( r = r_i \). Thus the black hole which remains in the interior contains \( N_1' = N_1 - \delta N_1 \) D1–branes and \( N_5' = N_5 - \delta N_5 \) D5–branes. Hence this black hole is characterised by the charges: \( Q_1' = N_1' - N_5' \), \( Q_5' = N_5' \) and \( Q_P \). The interior supergravity solution then takes essentially the same form as above

\[
\begin{align*}
\mathcal{L}^2 &= h_{1}^{-3/4} h_{5}^{-1/4} \left( -dt^2 + dz^2 + k \left( dt - dz \right)^2 \right) + h_{1}^{1/4} h_{5}^{-1/4} d\Omega_{K3}^2 + h_{1}^{1/4} h_{5}^{3/4} \left( dr^2 + r^2 d\Omega_{3}^2 \right) \\
\mathcal{E}^{2\phi} &= h_{1}/h_{5} , \quad F^{(3)}_{rtz} = \partial_t h_{1}^{-1} , \quad F^{(3)}_{\theta\phi\chi} = 2 r_{5}^2 \sin^2 \theta \sin \phi.
\end{align*}
\]

The two new harmonic functions introduced here are

\[
\begin{align*}
h_{1} &= 1 + \frac{r_{1}^2 - r_{5}^2}{r_{1}^2} + \frac{\tilde{r}_{1}^2}{r^2} , \quad h_{5} &= 1 + \frac{r_{5}^2 - r_{1}^2}{r_{1}^2} + \frac{\tilde{r}_{5}^2}{r^2}
\end{align*}
\]

while \( k \) remains as in eqn. (4), since we do not leave any momentum on the shell branes. The new D–brane scales are set by

\[
\begin{align*}
\tilde{r}_{5}^2 &= g_{s} \ell_{s}^2 Q_{5}' , \quad \tilde{r}_{1}^2 &= g_{s} \ell_{s}^2 \frac{V_{*}}{V} Q_{1}'.
\end{align*}
\]

Recall that these charges are chosen so that \( Q_5 - Q_5' = \delta N_5 \) and \( Q_1 - Q_1' = \delta N_1 - \delta N_5 \). Now the K3 volume at the horizon is given by

\[
V_{H} = \frac{Q_{1}'}{Q_{5}'} V_{*} = \frac{N_{1}' - N_{5}'}{N_{5}'} V_{*}.
\]

Note that this volume may be bigger than \( V_* \), and that in particular if \( \tilde{r}_{1} > \tilde{r}_{5} \), \( V(r) \) grows as we move to smaller radii inside the shell.

The normalisation of the constants is chosen in eqn. (11) to ensure that the metric is continuous at \( r = r_i \). There is, however, a discontinuity in the extrinsic curvature which can be interpreted in terms of a \( \delta \)–function source of stress–energy at \( r = r_i \), using the standard Israel junction conditions \cite{12} — see also ref. \cite{13}. A more complete discussion of this analysis in the context of enhançon physics is given in ref. \cite{8}, so here we only sketch the calculations. The extrinsic curvature of the \( r = r_i \) surface is

\[
K_{AB}^\pm = \frac{1}{2} n_\pm^C \partial_c G_{AB} = \pm \frac{1}{2} \partial_r G_{AB}
\]
where \( n_\pm = \pm \sigma \partial_r \) is the outward directed unit normal vector, with \( \sigma = G_{rr}^{-1/2} \). Then defining the discontinuity in the extrinsic curvature across the gluing surface, \( \gamma_{AB} = K^+_A - K^-_B \), the surface stress–tensor becomes

\[
S_{AB} = \frac{1}{8\pi G} \left( \gamma_{AB} - G_{AB} \gamma^C_C \right). \tag{15}
\]

Note that as with all energy calculations in string theory, the above analysis is performed using the Einstein frame metric. Now a rather lengthy calculation leads to the following result:

\[
\begin{align*}
S_{\mu\nu} &= \frac{\sigma}{16\pi G} \left( \frac{f'_1}{f_1} + \frac{f'_5}{f_5} - \frac{h'_1}{h_1} - \frac{h'_5}{h_5} \right) G_{\mu\nu} \\
S_{ab} &= \frac{\sigma}{16\pi G} \left( \frac{f'_5}{f_5} - \frac{h'_5}{h_5} \right) G_{ab} \\
S_{ij} &= 0 \tag{16}
\end{align*}
\]

where \( \mu, \nu \) denote the \( t \) and \( z \) directions, \( a, b \) denote the K3 directions, and \( i, j \) denote the angular directions along the \( S^3 \) at the incision.

A few comments are in order here:

- The surface tension along the angular directions vanishes. This had to result since we are describing a BPS configuration and there should be no stresses required to support a shell of D1– and D5–branes at any radius \( r = r_i \).
- The tension in the K3 directions only depends on the D5–brane harmonic functions. This is, of course, the expected result since only the D5–branes wrap these directions.
- The momentum appears nowhere in \( S_{AB} \), again because this is a BPS configuration and none of the momentum is supported by the shell.
- Finally the surface stress–energy in the \( t \) and \( z \) directions is determined by a single “tension”,

\[
T_{\text{eff}} = \frac{\sigma}{16\pi G} \left( \frac{h'_1}{h_1} + \frac{h'_5}{h_5} - \frac{f'_1}{f_1} - \frac{f'_5}{f_5} \right). \tag{17}
\]

This tension should be that of the effective strings formed by the wrapped D5–branes and D1–branes in the shell. Note the units are that of \((\text{length})^{-9}\). This is as it should be. This is still a five–brane energy/(\text{length})^5 with a further average over the area, \( A_3 \), of the \( S^3 \) at \( r = r_i \). Taking \( r_i \) large, one finds exactly as expected,

\[
T_{\text{eff}} = \delta N_5 \frac{\tau_5}{A_3} \left( 1 - \frac{V_s}{V} \right) + \delta N_1 \frac{\tau_1}{A_3 V} \tag{18}
\]

where \( \tau_p = ((2\pi)^p \ell_s^{p+1} g_s)^{-1} \) is the standard tension of a single Dp–brane [14]. Hence we see two contributions: the first is that of the wrapped D5–branes and the second coming
from the D1–branes in the shell. Similarly at large $r_i$, the tension in the K3 directions becomes $T_{K3-\text{eff}} = \delta N_5 \tau_5 / A_3$, without the extra correction from the wrapping on K3. Further note that this simplified large $r_i$ calculation can be extended to any radius, where one finds that the shell acts as a source for the metric, dilaton and R–R fields precisely as expected from the probe brane action, as illustrated in ref. [8, 17].

Now recall the expressions for the harmonic functions in eqns. (4) and (11). One finds that for general $r_i$, up to a positive coefficient

$$T_{\text{eff}} \propto r_i^2 \left( \delta N_5 - \frac{V^*}{V} (\delta N_5 - \delta N_1) \right) - g_s \ell_s^2 \frac{V^*}{V} ((2N_5 - N_1)\delta N_5 - N_5\delta N_1).$$

Hence we have a positive tension as long as $r_i^2 > \tilde{r}_e^2 \equiv g_s \ell_s^2 V^* (2N_5 - N_1)\delta N_5 - N_5\delta N_1 / (V - V^*) \delta N_5 + V^* \delta N_1$. The expressions simplify somewhat if we set $\delta N_1 = 0$, i.e., if the shell contains no D1–branes. In this case,

$$\tilde{r}_e^2 = g_s \ell_s^2 \frac{V^*}{(V - V^*)} (2N_5 - N_1) = r_e^2.$$

which, as we have indicated, corresponds precisely to the usual enhançon radius. On the other hand, it is straightforward to see from eqn. (20) that for $\delta N_1 > 0$, $\tilde{r}_e^2 < r_e^2$ and hence the volume of K3 when the shell tension vanishes is smaller than the self–dual value: $V(\tilde{r}_e) < V_*$.  

### 2.2 Probing the Black Hole

In this section we shall ask D1– and D5–brane probes about their view of the geometry we have studied in the previous sections. Both of these types of probe are natural in this situation, since they preserve the same supersymmetries. However, we will consider a slightly more general calculation involving a composite probe brane consisting of $n_5$ D5–branes and $n_1$ D1–branes. It is important for the physics of the following that this composite probe is in the D5–branes’ Higgs phase. That is, this composite probe is not simply a collection of individual D5–branes and D1–branes moving together, but rather the D1–branes have been absorbed as instanton strings lying along the $z$–direction in the D5–brane world-volume. We regard these instantons to be maximally smeared over the K3 directions and that we have chosen the orientation of the vevs of the hypermultiplets arising from 1–5 strings such that the instantons are of maximal rank in the $U(n_5)$ gauge theory. In this phase, the composite probe brane is then a true bound state, i.e., the fields describing the relative separation of the branes in the Coulomb phase, are all massive.
The effective action for the composite brane probe regarded as an effective string becomes

\[ S = -\int_{\Sigma} d^2 \xi \ e^{-\Phi(r)} (n_5 \tau_5 V(r) + (n_1 - n_5) \tau_1) (-\det g_{ab})^{1/2} \]

\[ + n_5 \tau_5 \int_{\Sigma K3} C^{(6)} + (n_1 - n_5) \tau_1 \int_{\Sigma} C^{(2)}. \]  

(22)

where \( \Sigma \) is the unwrapped part of the brane’s world–volume, with coordinates \( \xi^{0,1} \). We note in the above action that the wrapping of the D5–branes on the K3 introduces negative contributions to both the tension and two–form R–R charge terms. Above \( g_{ab} \) is the pull–back of the string–frame spacetime metric:

\[ g_{ab} = e^{\Phi/2} G_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b}. \]

(23)

The background fields in which the probe moves are those of the black hole solution given in eqn. (3). The corresponding R–R potentials may be written as

\[ C^{(6)} = f_5^{-1} dx^0 \wedge dx^5 \wedge \varepsilon_{K3}, \quad C^{(2)} = f_1^{-1} dx^0 \wedge dx^5, \]

(24)

where \( \varepsilon_{K3} \) denotes the volume four–form on the K3 space with (fixed) volume \( V \). Note that these R–R potentials do not vanish asymptotically. However, this gauge choice is convenient because it eliminates a constant contribution to the energy which would otherwise appear in the following calculation. We should also mention that we adopt the conventions convenient for working with supergravity solutions, as described in ref. [13], so that the coefficients of the Wess–Zumino terms in eqn. (22) are \( \tau_p \) including a factor of \( 1/g_s \).

We will now choose static gauge, aligning the coordinates of the effective probe string with the \( x^5 \) direction and letting it move in the directions transverse to K3 while freezing and smearing the degrees of freedom on the K3:

\[ \xi^0 = x^0 \equiv t \]

\[ \xi^1 = x^5 \equiv z \]

\[ x^i = x^i(t, z) , \quad i = 1, 2, 3, 4, \]

(25)

After a brief computation, the result can be written as the effective Lagrangian \( L \) for a string moving in the \( (x^1, x^2, x^3, x^4) \) directions:

\[ L = \frac{1}{2} (n_5 \tau_5 V f_1 + (n_1 - n_5) \tau_1 f_5) \left[ \dot{r}^2 - r'^2 + k(\dot{r} - r')^2 + r^2 \left( \Omega_3^2 - \Omega_3'^2 + k(\dot{\Omega}_4 - \Omega_4')^2 \right) \right], \]

(26)

where dot and prime are used to denote \( \partial/\partial t \) and \( \partial/\partial z \), respectively. The notation for the angular contributions is such that the derivatives of the angular positions are contracted with the standard metric on the unit \( S^3 \), e.g., \( \Omega_3^2 = \dot{\theta}^2 + \sin^2 \theta (\dot{\phi}^2 + \sin^2 \phi \dot{\chi}^2) \). Notice that there is no non–trivial potential, since supersymmetry cancelled the mass against the R–R charge.
The effective tension of the probe is given by the prefactor in eqn. (26). We can already see that there is the possibility that the tension will go negative when \( n_5 > n_1 \).

Putting in the definitions of the harmonic functions given in eqn. (4), we get that the tension remains positive as long as

\[
(n_5 \tau_5 V f_1 + (n_1 - n_5) \tau_1 f_5) > 0
\]

\( \text{i.e., } r^2 > g_s \ell_s^2 V_* (\frac{2N_5 - N_1}{V - V_*}) n_5 - N_5 n_1 \),

where the lower bound is actually the same as \( \tilde{r}_e^2 \) in eqn. (20) with the substitutions: \( \delta N_1 \rightarrow n_1 \), \( \delta N_5 \rightarrow n_5 \). It is satisfying that this fits perfectly with the consistency condition we derived from the supergravity solution with the composite shell in the previous section.

Let us consider some special cases of this result. If we remove all of the D5–branes, the result for pure D1–brane probes is quite simple, as setting \( n_5 \) to zero in the above result gives:

\[
L_{D1} = \frac{1}{2} n_1 \tau_1 f_5 \left[ \dot{r}^2 - r'^2 + k(\dot{r} - r')^2 \right]
\]

This is a natural result: The D1–brane is not wrapped on the K3 and so its tension is positive everywhere. It simply floats past the enhançon radius on its way to the origin without seeing anything particularly interesting there [11].

Note that the result (28) is the same as would have been obtained in the case of probing for a \( T^4 \) compactification, assuming that we consider only motion in the directions transverse to the torus. Similarly in the case that \( n_1 = n_5 \), we get:

\[
L = \frac{1}{2} n_5 \tau_5 f_5 \left[ \dot{r}^2 - r'^2 + k(\dot{r} - r')^2 \right]
\]

which is the same as the result for pure D5–brane probes in the case where they are wrapped on \( T^4 \). The cancellation of the induced tensions from K3 wrapping and non–trivial instanton number in constructing the bound state probe provided a simple result: the wrapped D5–branes, when appropriately dressed with instantons, can indeed pass through the enhançon shell.

If we instead remove all of the D1–branes, we just get the familiar result of ref. [7] that the probe, made of pure D5–branes, hangs up at the enhançon radius \( r_e \). Now we discover that ref. [7]’s result is just a special case of a more general result: whenever there are more D5–branes than D1–branes making up the probe (i.e., \( n_5 > n_1 \)), there is a generalisation of the enhançon radius, \( \tilde{r}_e^2 \), where the composite probe will become tensionless and must stop. Notice that this happens in a “substringy” regime where \( V_{K3} < V_* \).
3 The Physics of the Black Holes

3.1 Constructing a Black Hole

The probe results of the previous section now can be seen to highlight the relevance of the supergravity solution we studied in section 2, where we placed some of the D5–branes inside the black hole along with the D1–branes, despite the fact that sometimes the enhançon radius appears outside the horizon. To orient the discussion, we consider making the black hole by beginning with a “large” enhançon shell containing $N_5$ D5–branes and (adiabatically) bringing in $N_1$ D1–branes from infinity, as well as $Q_P$ momentum modes. As the latter play no role relevant to the enhançon, we will simply assume that they are carried in to the origin along with the first few D1–branes. As emphasised in the probe discussion, since the D1–branes are not wrapped on the K3, their tension remains positive everywhere. Hence the D1–branes can simply pass through the enhançon shell on their way to the origin[11].

With a small number of D1–branes (i.e., $N_1 < 2N_5$), the enhançon radius (8) remains well away from the origin. Hence naively, one might think that no black hole is formed, rather the D5–branes must remain fixed in the enhançon shell. However, let us examine the interior solution (9–11) when we choose $r_i = r_e$ so that the K3 volume $V(r)$ starts at $V^*$ at the incision radius. If $N'_1 = N_1$ and $N'_5 = 0$ (i.e., all of the D1–branes at the origin and all of the D5–branes at the enhançon radius), then it is easy to show that $V(r)$ grows as the radius decreases inside the shell. Since $V(r) > V_*$ in this region, there is no obstacle to moving some of the D5–branes from the enhançon shell to the origin and forming a black hole. As D5–branes are moved to the origin, the growth of the K3 volume is suppressed and stops when $N'_5 = N_1/2$ as can be seen from eqn. (13). At this point, the volume at the horizon and throughout the interior region is a fixed constant, i.e., $V(r < r_e) = V_*$. While this solution (with $N'_1 = N_1$, $N'_5 = N_1/2$) may seem to be a limiting configuration, the black hole can absorb more D5–branes using the bound states considered in the previous section. For example, illustrated in eqn. (29), a bound state with $n_1 = n_5$ has no problem moving in a region where $V(r) < V_*$. Hence when the D1–branes move in from infinity, rather than making a passive transit through enhançon shell, some of these D1–branes can bind to D5–branes as instanton strings and the resulting D1/D5 bound states can move to the origin. In this way, a black hole can be constructed which contains $N'_5 \leq N_1$ D5–branes. Note that from eqn. (13), we see $V_H < V_*$ for $N_1/2 < N'_5 < N_1$. That is, we are able to construct black holes surrounded by a region where the K3 volume is less than $V_*$. From the above discussion, we conclude there are several different regimes: For $0 < N_1 < N_5$, the black hole can only absorb a fraction of the total number of D5–branes (up to $N'_5 = N_1$) and so the black hole is naturally dressed by an enhançon shell. For $N_5 < N_1 < 2N_5$, the black

\[ V(r) = \frac{h_1}{h_5} V \]
hole can absorb all of the D5–branes but there is still a region where \( V(r) < V_* \) outside of the horizon. Finally for \( N_1 > 2N_5 \), the black hole can again absorb all of the D5–branes and since \( r_e^2 < 0 \) the K3 volume reaches \( V_* \) inside the event horizon.

In either of the last two cases, the supergravity solution is given in eqns. (1–4), taken for \( 0 < r < \infty \). We have a black hole with horizon area set by the product \( Q_1Q_5Q_P \). In the first case \( (0 < N_1 < N_5) \), there is an enhançon shell and so we must introduce the interior solution (9–11) to describe the region \( r < r_e \). Then we have an interior black hole with horizon area set by the product \( Q'_1Q'_5Q_P \).

We should emphasise that all of the black holes as well as the intermediate configurations involved in their construction are supersymmetric. Hence we can choose, if we wish, to leave extra D1–branes and D5–branes outside the horizon and not contributing to making the black hole. Further, in the regime \( 0 < N_1 < N_5 \), we may choose to put the excess D5–branes in an enhançon shell around the black hole, or we may place them in some distant region thereby essentially removing them from the problem. We leave it to the reader to verify that the area of the black hole is still given by precisely \( A = 8\pi G \sqrt{Q_1Q_5Q_P} \) in either case.

From the discussion here, we conclude that enhançon physics does play a role in the black holes when the number of D1–branes is small. We can sharpen our understanding of the precise nature of this role by carefully examining the formulae for the horizon area (or black hole entropy).

### 3.2 The Second Law of Thermodynamics

The entropy and area of the black holes which we construct are given by the familiar formula

\[
S = \frac{A}{4G} = 2\pi \sqrt{Q_1Q_5Q_P} = 2\pi \sqrt{(N_1 - N_5)N_5Q_P} .
\]  

(31)

For fixed \( N_1 \) and \( Q_P \), considering the dependence of the entropy on the number of five–branes, we see that it gives a semi–ellipse, as depicted in figure 1 on the left. Now while it is clear that black holes form for any number of D5–branes, the maximal entropy black holes that we can make are those for which \( N_5 = N_1/2 \), or in other words \( Q_1 = Q_5 \). This is the apside of the ellipse on the left in figure 1.

Hence if we wish to consider the maximum entropy that can be achieved for a given set of parameters, \( N_1, N_5 \) and \( Q_P \), we see that the behaviour of this entropy changes at precisely \( N_1 = 2N_5 \). The curve on the right of figure 1 shows the (square of the) maximal entropy as a function of \( Q_1 \) for fixed \( N_5 \) and \( Q_P \). For a “large” number of D1–branes \( (N_1 > 2N_5) \), the maximal area squared is simply proportional to \( Q_1 \), as expected from eqn. (31). However, for a “small” number of D1–branes \( (N_1 < 2N_5) \), the entropy is maximised if only \( N'_5 = N_1/2 \) of the available D5–branes participate in the formation of the black hole. In this regime, we have

\[
A^2_{\text{max}} \propto N_1^2 = (Q_1 + Q_5)^2
\]  

(32)
Figure 1: On the left is the area as a function of \(N_5\), for fixed \(Q_P\) and \(N_1\), which forms half of an ellipse. As the number of five–branes increases past \(N_1/2\), the area decreases. On the right is the square of the maximal horizon area as a function of \(Q_1\), for fixed \(Q_P\) (=1) and \(Q_5\) (=2). For \(N_1 > 2N_5\), the \((\text{area})^2\) increases linearly. For \(N_1 < 2N_5\), to maximise the area, one must use only \(N_1/2\) of the available D5–branes (see left graph), and therefore the dependence on \(N_1\) is quadratic.

and so the curve becomes a parabola which only reaches zero at \(Q_1 = -Q_5\). Note that in this regime, the maximum entropy is greater than one would calculate from eqn. \([31]\). Assuming the excess D5–branes have accumulated in an enhançon shell around the black hole, the maximum entropy configuration corresponds to precisely that where the K3 volume is frozen at \(V_*\) throughout the interior region.

Let us return to the curve on the left of figure 1. Imagine that we begin with a black hole with a “large” number of D1–branes. It lies on the left hand side of the ellipse in the figure. We may now consider increasing the number of D5–branes in the system by adding more one at a time. As we do so, the black hole moves up the ellipse to the extremum at \(N_5 = N_1/2\). At this point, however, if we were to add one more D5–brane, we we see that we will in fact decrease the horizon area, and hence the entropy of the resulting system. We can in principle bring this D5–brane up to the black hole horizon as slowly as we like. We seem, therefore, to have found a way of reducing the entropy of the hole by an adiabatic process. This is a violation of the second law of thermodynamics, which appears to be a previously unconsidered flaw in the microphysics of black hole thermodynamics, as represented by D–branes.

Happily, there is a very satisfying resolution of this problem. It is precisely for this class of black holes that the enhançon appears above the horizon. So an attempt to bring our extra D5–brane into the hole is thwarted by the fact that it will be forced to stop at the enhançon radius \(r_e\) just above the horizon\([4]\).

We could bind the extra D5–brane with an extra D1–brane to bring it in, but in this case

\(^2\)We are grateful to Joe Polchinski for a conversation in which this possibility arose.
Q_1 \) remains fixed while \( Q_5 \) increases. Thus dropping in the D1/D5 bound state increases black hole entropy.

If we begin with a black hole on the right half of the ellipse \((N_1/2 < N_5 < N_1)\), the enhançon again ensures that we cannot move further to the right decreasing the horizon area by dropping D5–branes into the black hole. These were configurations where the black hole is already surrounded by a region where \( V(r) < V_* \) and hence the extra D5–branes are restrained from reaching the horizon by the enhançon mechanism.

However, we have seen in section 2.2 that D1/D5 bound states can move through such regions where \( V(r) < V_* \) and so we must still investigate if we are able to decrease the entropy by sending in a bound D1/D5 probe brane. Adopting the previous notation, let the probe consist of a bound state with \( n_1 \) D1–branes and \( n_5 \) D5–branes. Assuming that the black hole already contains many more of each type of brane, \( i.e., n_1, n_5 << N_1, N_5 \), dropping in such a probe would cause an infinitesimal shift in the entropy (squared) given by

\[
\delta S^2 = 4\pi^2 Q_P (N_5 n_1 + (N_1 - 2N_5)n_5) .
\]

Note that implicitly we are assuming \( N_1, N_5, n_1, n_5 > 0 \). Even so the expression in parentheses has the potential to be negative which would signal a decrease in the black hole entropy. However, we found that this expression also appears in the numerator of eqn. (27) for the radius of vanishing probe tension, but with the opposite sign! Hence the probe–brane finds no obstacle to dropping inside the horizon only in those situations where the entropy increases. Precisely in those cases where second law would be violated, the enhançon locus stands guard outside of the event horizon and the composite branes are restrained from entering the black hole. Thus the enhançon provides string theory with precisely the mechanism needed to maintain consistency with the second law of black hole thermodynamics.

### 3.3 Evaluating the Entropy in Gauge Theory

Let us review briefly what the crucial elements of the entropy counting argument are\footnote{See ref.\[16\] for a review.}. We assume that the scale of the K3 is much smaller than that of the circle, so that we have an effective 1+1 dimensional gauge theory on the effective D1–brane formed by wrapping the D5–branes and binding it with D1–branes. At strong coupling the theory will flow to a conformal field theory in the infra–red. The important feature of the conformal field theory is its central charge, which can be computed from the gauge theory as proportional to \( n_H - n_V \), the difference between the numbers of hypermultiplets and the number of vector multiplets. Counting the bosonic parts, the D1–branes contribute \( N_1^2 \) vectors and \( N_1^2 \) hypers, the latter coming from \((x^6, x^7, x^8, x^9)\) fluctuations. The D5–branes contribute \( N_5^2 \) vectors, but there are no massless modes coming from oscillator excitations in the \((x^6, x^7, x^8, x^9)\) (K3) directions. There are, in addition, 1–5 strings which give \( N_1 N_5 \) hypermultiplets. Evaluating the difference gives:
$N_1 N_5 - N_5^2 = Q_1 Q_5$ hypermultiplets. Hence in total, there are $4Q_1 Q_5$ bosonic excitations and an equal number of fermions, since a hypermultiplet contains four scalars and their superpartners.

In another language all that we have done is evaluated the dimension the Higgs branch of the D5–brane moduli space of vacua, where the $N_1$ D1–branes can become instanton strings of the $U(N_5)$ gauge theory on the world–volume of the D5–branes. The vacuum expectation values of the 1–5 strings is precisely what constitutes this branch. In this language, the absence of hypers coming from the 5–5 sector corresponds to the absence of Wilson lines on the K3 surface (since the latter has trivial first homotopy class).

Giving the D1–D5 bound string an overall momentum $P = Q_P/R_z$ in the $x^5$ direction can be achieved in a number of ways, because of these $4Q_1 Q_5$ microstates and their fermionic superpartners, and in fact the precise formula for this comes from the standard partition function:

$$
\left( \prod_{Q_P=0}^{\infty} \frac{1 + q^n}{1 - q^n} \right)^{4Q_1 Q_5} = \sum_{Q_P=1}^{\infty} \Omega(Q_P)q^{Q_P}, \tag{34}
$$

where $\Omega(Q_P)$ is the degeneracy at level $Q_P$. (Recall that for BPS excitations, the energy level (appearing in the partition function) and the left (or right) moving momentum are equal.) At large $Q_P$ there is the result $\Omega(Q_P) \sim \exp(2\pi \sqrt{Q_1 Q_5 Q_P})$. So the entropy $S \equiv \log(\Omega) = 2\pi \sqrt{Q_1 Q_5 Q_P}$, which precisely matches the strong coupling Bekenstein–Hawking result from the supergravity solution listed below eqn. (5).

Note that we have a mild paradox here. For $N_1 < 2N_5$, we know from the analysis of the previous section that, at any given value of the momentum, the entropy can be maximised by using only $N_1/2$ of the D5–branes in the problem. So, on the one hand, it would seem that it is favourable to Higgs the $U(N_5)$ gauge theory leaving massless only a $U(N_1/2)$ subgroup. On the other hand, the gauge theory cannot know this, since all of these supersymmetric vacua are degenerate. Therefore all black holes appear to be on the same footing from a field theory point of view, despite the fact that we can increase the entropy by not using all the five–branes.

Clearly this puzzle is simply an artifact of the thermodynamically peculiar situation that we are at zero temperature while having a finite entropy. In such a situation, the entropy strictly has a meaning as a degeneracy of ground states. Processes which maximise the entropy require dynamics, and hence must take the system (slightly) away from extremality. That is, we must slightly excite the system in order that it can explore the phase space, and find the configurations of maximal degeneracy or entropy as it settles back to the ground state energy. It would be interesting to study the effective couplings in the full 1+1 dimensional model to see if they are consistent with the system being able to “seek” the higher entropy configurations once fluctuations are included. Macroscopically, this must correspond to the system being able to expel D5–branes in order to increase its entropy, which is fascinating.
4 Beyond the horizon

Now we would like to consider the physics of the black hole interior, the metric for which may be obtained by analytically continuing the geometry of section 2. In particular, one might wish to consider the case where the volume of the K3 shrinks but does not reach $V_*$ until inside the event horizon, and so there is enhançon physics inside the black hole.

A simple choice of coordinates which cover the black hole interior\cite{18} is constructed as follows: As an intermediate step, change the radial coordinate with $R^2 = r_0^2 + r^2$, which positions the horizon at $R = r_0$. However, we can consider interior points with $R < r_0$. We do not present here the solution written in terms of $R$, rather we make a second coordinate change $R^2 = r_0^2 - r^2$ — note the sign — which puts the solution in a simple form. The interior metric becomes

$$ds^2 = F_1^{-3/4} F_5^{-1/4} (dt^2 - dz^2 + k (dt - dz)^2) + F_1^{1/4} F_5^{-1/4} ds_{K3}^2 + F_1^{1/4} F_5^{3/4} (dr^2 + r^2 d\Omega_3^2) ,$$

where now

$$F_1 = -1 + \frac{r_1^2}{r^2} , \quad F_5 = -1 + \frac{r_5^2}{r^2} ,$$

where $k$ and the scales $r_{1,5}$ are as before in eqns. (44). The dilaton and R–R fields for the interior are given by:

$$e^{2\Phi} = \frac{F_1}{F_5} , \quad F_{rtz}^{(3)} = -\partial_t F_1^{-1} , \quad F_{\theta\phi\chi}^{(3)} = 2 r_5^2 \sin^2 \theta \sin \phi .$$

These coordinates are convenient, since they yield a structural form for the interior solution similar to the one which we had outside of the event horizon. Consequently, it is easy to compare results. Note, however, that we have turned the radial coordinate “inside out.” The horizon is again positioned at $r = 0$, and moving to larger values of $r$ takes us further into the black hole interior. For example,

$$G_{\theta\theta} = (r_1^2 - r_2^2)^{1/4} (r_5^2 - r_2^2)^{3/4}$$

and so the three-sphere part of the geometry does indeed get smaller as we move to larger values of $r$. The above metric element also suggests that a curvature singularity may arise at $r = r_1$ or $r_5$. The regime of interest here is $r_1 < r_5$, in which case one does indeed encounter a time–like singularity at $r = r_1$. We are also assuming that $r_P > r_1$ so that there are no closed time–like curves in the interior geometry, as would result if $G_{zz} \propto r_P^2 / r^2 - 1$ became negative.

The volume of the K3 manifold (as measured by the string–frame metric) is now given by

$$V(r) = \frac{F_1}{F_5} V ,$$

and we see again that as long as $r_1 < r_5$, the K3 volume continues to shrink as we move from the horizon to larger values of $r$. Note that $V(r)$ reaches zero at the singularity $r = r_1$. 

14
The enhançon \( r_e \) in the new coordinates is given by the formula:

\[
\frac{r_e^2}{\ell_s^2} = g_s \frac{V_*}{(V - V_*)} (N_1 - 2N_5) , \quad \begin{cases} < 0 & \text{for } 2N_5 > N_1 , \\ > 0 & \text{for } 2N_5 < N_1 , \end{cases}
\]  

(40)

which is precisely the expected result compared to eqn. (3). That is, for \( 2N_5 < N_1 \) the enhançon radius appears in the black hole interior.

For the case \( 2N_5 < N_1 \), we might expect that some of the wrapped five–branes to hang at radius \( r_e \). Hence consider an excision construction where the above “exterior” solution is matched to an “interior” solution at some radius \( r = r_i \), as was done in section 2. Note that with the present coordinates inside the event horizon the new interior solution will be describing \( r > r_i \). As in section 2, the interior solution is characterised by the charge:

\[
Q'_{5} = N'_5 - N'_1 \quad \text{and} \quad Q'_{1} = N'_1 - N'_5 .
\]

This solution then takes essentially the same form as above

\[
\begin{align*}
\frac{ds^2}{H_{1}^{3/4}H_5^{-1/4}} &= \left( dt^2 - dz^2 + k (dt - dz)^2 \right) + H_1^{1/4}H_5^{-1/4} \left( ds_{K3}^2 + H_1^{1/4}H_5^{3/4} \left( dr^2 + r^2 d\Omega_3^2 \right) \right) , \\
\text{e}^{2\Phi} &= H_1/H_5 , \quad F^{(3)}_{rtz} = -\partial_r H_1^{-1} , \quad F^{(3)}_{\theta \phi \chi} = 2\tilde{r}_5^2 \sin^2 \theta \sin \phi .
\end{align*}
\]  

(41)

The two new harmonic functions introduced here are

\[
H_1 = -1 + \frac{r_i^2}{r_1^2} + \frac{\tilde{r}_1^2}{r^2} , \quad H_5 = -1 + \frac{r_5^2}{r_i^2} + \frac{\tilde{r}_5^2}{r^2}
\]  

(42)

while \( k \) still remains as in eqn. (3), since we do not consider the situation where the shell carries some of the momentum. The new D–brane scales are set by

\[
\tilde{r}_5^2 = g_s \ell_s^2 Q'_5 , \quad \tilde{r}_1^2 = g_s \ell_s^2 \frac{V_*}{V} Q'_1 .
\]

(43)

Hence the R–R charges would indicate that the shell contains \( \delta N_5 = Q_5 - Q'_5 \) D5–branes and \( \delta N_1 = Q_1 + Q_5 - Q'_1 - Q'_5 \) D1–branes. Notice that even if the shell contains no D1–branes, the location of the singularity would be altered. The latter is now at \( r = \tilde{r}_1 r_i / \sqrt{\tilde{r}_1^2 + \tilde{r}_5^2 - r_i^2} \).

Next, let us examine the stress tensor associated with the shell. This is given by:

\[
\begin{align*}
S_{\mu \nu} &= -\frac{\sigma}{16\pi G} \left( \frac{F'_{1}}{F_{1}} + \frac{F'_{5}}{F_{5}} - \frac{H'_{1}}{H_{1}} - \frac{H'_{5}}{H_{5}} \right) G_{\mu \nu} , \\
S_{ab} &= -\frac{\sigma}{16\pi G} \left( \frac{F'_{5}}{F_{5}} - \frac{H'_{5}}{H_{5}} \right) G_{ab} , \\
S_{ij} &= 0
\end{align*}
\]  

(44)

where as before: \( \mu, \nu \) denote the \( t \) and \( z \) directions; \( a, b \) denote the K3 directions; and \( i, j \) denote the angular directions along the \( S^3 \) at the incision. Note that with our inside-out coordinates, the normal vectors switch their signs compared to section 2. That is, \( n_\pm = \pm \sigma \partial_r \) compared to those introduced after eqn. (14).
The important observation about this result, however, is that the tensions appearing in this shell stress-energy are negative! For example, one finds that the tension characteristic of the K3 directions is now

\[ T_{\text{K3-eff}} \propto -\delta N_5 \tau_5/A_3. \] (45)

There is a similar minus sign in the effective string tension for the t and z directions, compared to eqn. (18). However, the effect of wrapping five–branes on the K3 space is still apparent, so while this tension is negative near the horizon (i.e., for small \( r_i \)), it vanishes at the radius:

\[ r_i^2 = r_e^2 = g_s l_s^2 V (N_1 - 2N_5)\delta N_5 + N_5 \delta N_1 \]

(46)

and becomes positive for smaller values of \( r \) — note that the K3 tension remains negative in this region. As a consequence, one cannot claim that the shell is constructed of D5–branes and D1–branes (alone).

These problematic results arise from the peculiar properties of the black hole singularity\[19\], and are, in fact, entirely consistent with probe calculations\[20\]. The region near the time–like singularity has a negative effective mass. Hence any positive mass probe with the same charge as the singularity is naturally pushed outward\[19\]. That is, the trajectory of such a probe heads towards the future Cauchy horizon at \( r = 0 \). This effect can be compensated for by also reversing the sign of the probe so that the R–R forces are attractive while the effective gravitational force is repulsive. Hence anti–branes become the natural probes of the interior geometry, in the sense that the potential for their motion vanishes\[20\]. This is consistent with the excision construction above in that the characteristic tensions appearing in eqn. (44) would be positive if the parameters \( \delta N_5 \) and \( \delta N_1 \) were negative. That is, the stress–energy would be well–behaved if the D5– and D1–brane charges of the solution inserted for \( r > r_i \) were larger for the original exterior solution, indicating that the shell was composed of anti–D5–branes and anti–D1–branes. Note that the effective string tension of the shell of anti–branes is positive for small values of \( r_i \) and vanishes precisely at the radius given in eqn. (46). This latter result remains unchanged when the signs of both \( \delta N_5 \) and \( \delta N_1 \) are flipped.

5 Discussion

An interesting and satisfying result of our investigation here is the discovery that the enhançon radius is just the outer shell of an onion–like structure arising when the supergravity solution is constructed of multiple species of brane. There is a series of concentric “generalised enhançon” shells where various D\((p + 4)\)–Dp bound states become tensionless, for successively smaller values of the K3 volume below the stringy value \( V_* \). It is immediately apparent that there are interesting cousins of these rich structures to be found in the various U–dual situations involving D–branes stretched between NS5–branes, fractional branes on a collapsed two–cycle of K3 (with varying flux), and heterotic string theory on \( T^4 \). For \( p \) even (type IIA) it is clear that there is enhanced gauge symmetry at these radii, while for \( p \) odd (type IIB), we get the
enhanced two–form gauge symmetry associated to a rich family of tensionless strings. These issues clearly deserve more exploration.

The central issue of our investigation has been the interplay of the physics of the enhançon with that of five–dimensional black holes. After some initial thought, the enhançon mechanism seems to play no essential role in the construction of the black holes. The only situation where an enhançon must necessarily dress the black hole exterior is in the regime $0 < N_1 < N_5$ (or $-Q_5 < Q_1 < 0$), i.e., when the wrapped D5–branes make the largest contribution to the asymptotic D1–charge.

Reconsidering the standard entropy formula (31), we noted that when the system contains a small number of D1–branes the entropy is not maximised when all of the D5–branes are included in the black hole. Rather, in the regime $0 < N_1 < 2N_5$ (or $-Q_5 < Q_1 < Q_5$), the entropy is maximised when the black hole only absorbs $N'_5 = N_1/2$ of the available wrapped D5–branes. This extended regime precisely matches that where the enhançon locus (8) appears outside of the event horizon, and so it is natural to keep some of the D5–branes at a distance away from the black hole. Recall that for $N_5 < N_1 < 2N_5$ (or $0 < Q_1 < Q_5$), in principle the black hole could still absorb all of the D5–branes using D1/D5 bound states.

Naïvely, this well–known entropy formula (31) allows us to decrease the entropy in certain cases by throwing more wrapped D5–branes into the black hole. Hence string theory might appear to allow us to violate the second law of thermodynamics. Here, however, the enhançon mechanism provides an elegant resolution of this paradox. In precisely those configurations where the addition of D5–branes would produce a violation of the second law, the enhançon locus sits outside of the event horizon to prevent their infall. So we discovered a fascinating interplay between the micro–physics of branes and the macro–physics of supergravity black holes.

One is able to probe regions where the K3 volume is less than $V_*$ with D1/D5 bound states in which D1–branes are absorbed as instantonic strings on wrapped D5–branes. Thus there is a further potential to violate the second law with these composite branes. However, we found that the previous result generalised: There is a family of generalised enhançon loci, inside the usual vanilla enhançon, where these composite branes become tensionless. They emerge in the black hole exterior precisely in the regime where the infall of such branes would reduce the horizon area. Hence the black hole solutions actually display a very delicate behaviour on macroscopic scales to ward off the probes through the details of the micro–physics of branes.

One may claim that supergravity shows a similar prescience of braney physics in the matching calculations for the shells of section 2. That is, the shell provides a source with precisely the correct R–R charges, dilaton charge and stress–energy to match those of a shell of D–branes, as was shown in great detail in ref.[8, 17]. We would argue, however, that this result is essentially a result of supersymmetry, i.e., that supergravity knows that supersymmetric sources with fixed R–R charges must have the characteristics we associate the D–branes. The results of the matching calculations become much more clouded for non–supersymmetric configurations[8].
On the other hand, we must allow that the original entropy matching calculations for these black holes\[1\] are highly suggestive that supergravity does have remarkable insight into at least certain aspects of the brane physics, and of course the AdS/CFT correspondence\[25\] confirms this. Specifically, how does supergravity know to construct an event horizon whose area matches the ground state degeneracy of the constituent branes? The enforcement of the second law via the enhançon mechanism is a further compelling demonstration that supergravity has a much deeper “knowledge” of the micro–physics of branes than meets the eye. Note that this new phenomenon lies outside the AdS/CFT correspondence, as the usual decoupling limit removes the enhançon.

Of course, the type IIB supergravity of interest here is embedded in a superstring theory, and much of what we have found here relies on the fact that supergravity retains a greater memory of the underlying string theory than we might have had the right to expect: The enhançon radius and its generalised cousins found here are supergravity’s mementos of the parent string theory. These radii are special, even in supergravity, as we have seen here and as is emphasised also in ref.\[8\]. One might argue that supersymmetry may play an essential role in producing our results, but the connection seems somewhat obscure since the second law seems somewhat removed from typical supersymmetry considerations.

Further insight might be gained by revisiting the supersymmetric attractor equations which govern the exterior geometry of these black holes\[21\]. It would interesting to see if the attractor flows “know” about the enhançon locus. Presumably the enhançon must appear for the flows with $0 < N_1 < N_5$ (or $-Q_5 < Q_1 < 0$) if they are to avoid a repulson–like singularity. Perhaps Denef’s careful analysis of ref.\[22\] can be extended to the present context to show that supersymmetry naturally applies an excision at the enhançon radius.

A related question would be to express our results in a U–duality invariant way. That is, the relation $N_1 = 2N_5$ is distinguished in the present analysis as a boundary between the regimes where the maximal entropy black hole does or does not incorporate all of the available D5–branes. Expressing this boundary using the U–duality invariants\[23\] in the type IIB theory compactified on K3 seems to be a nontrivial problem.

Just as the microscopic entropy counting can be extended to four dimensional black holes\[24\], one can also investigate the role of the enhançon for these black holes. It seems that the enhançon mechanism is again essential to enforcing the second law for these black holes as well\[17\]. Of course, the structure of the attractor flows and U–duality invariants would be even richer in this setting.

As we commented at the end of section 3, supersymmetric configurations provide a rather abstract framework for a discussion of thermodynamics. Considerations such as maximising the entropy only make sense in the context of dynamical processes which take the black holes at least slightly away from extremality. Then one can imagine that the system explores the full space of accessible states as it settles back to being a supersymmetric black hole. Of course, the non–extremal version of the black holes considered here is well–known — see, for example, refs.\[23, 26\]. In this context, the application of the excision technique of section 2
may produce ambiguous results. However, this is unlikely to be an obstacle to studying the important question of how the enhançon physics works to enforce the second law for these non–extremal black holes.

It would be interesting to consider near–extremal black holes in the regime \( N_5 < N_1 < 2N_5 \) (or \( 0 < Q_1 < Q_5 \)). Here, if the black hole begins with all of the D5–branes at the horizon, its entropy is not maximised. So we anticipate a new type of instability in this context, leading to the expulsion of some of the D5–charge from the black hole, possibly by the creation of D5/anti–D5 pairs near the horizon. It would be interesting to see if this is a classical instability or if it remains a quantum instability similar to those arising in the discharge of Reissner–Nordstrom black holes. It would be interesting to see if the local stability analysis suggested in ref.[28] might give some insight into this question. It would also be interesting to understand this new expulsion mechanism in the microscopic framework of the conformal field theory living on the D–branes.

Another interesting situation to reconsider for non–extremal black holes is the case where the enhançon radius occurs inside the horizon. Inside of a non–extremal horizon, the enhançon locus will be a time, not a place! Hence not only will an infalling D5–brane necessarily reach the enhançon radius, it must pass through this surface into the region where \( V(r) < V_* \). At first sight, this would seem a paradoxical situation, but we can propose a simple resolution. As the D5–brane approaches the enhançon radius, it can nucleate the creation of a D1/anti–D1 pair. If the D1–brane binds to the D5–brane, the bound state will have no problem in passing beyond the enhançon radius, and the anti–D1–brane sees no obstacle to passing this surface on its own. Note that as a result of the time–like nature of the radius in this situation, the non–extremal black hole interior is a effectively dynamical background and “energy” conservation does not represent an obstacle for such a pair creation process. Still it would be interesting to study the infall quantitatively to confirm whether this process and/or some other mechanisms come into play.

In the case of the interior of the extremal black holes considered briefly in section 4, we uncovered essentially no new insights. As a result of the peculiar structure of the spacetime geometry, the physics of D–branes in the black hole exterior seems to be replaced by the physics of anti–D–branes in the interior. It seems this even extends to the appearance of an enhançon locus. However, one of our initial goals had been to investigate whether there is a compelling role for the enhançon in resolving time–like singularity appearing inside the black hole. Unfortunately, it seems that the answer to this particular question is no. However, any possible disappointment which we might have felt has been eclipsed by our excitement about the myriad new avenues for interesting physics investigations uncovered here. We hope that we have infected the reader with similar feelings.
Acknowledgements

Research by RCM was supported by NSERC of Canada and Fonds FCAR du Québec. That of CVJ was supported in part by the University of Durham and the U.K. Particle Physics and Astronomy Research Council. We would like to thank the Aspen Center for Physics for hospitality during the initial stages of this project. CVJ would like to thank the ITP, UCSB for hospitality during part of this work, and the organisers of the “M–Theory” workshop for enabling him to participate. Research at the ITP was supported in part by the U.S. National Science Foundation under Grant No. PHY99–07949. We would like to thank Neil Constable, Frederik Denef, Don Marolf, Greg Moore, Amanda Peet, Simon Ross, Oyvind Tafjord and especially Joe Polchinski for useful conversations. We also thank Neil Constable for a careful reading of the manuscript.

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