Quantum chromodynamics (QCD)-like phase diagram with Efimov trimers and Cooper pairs in resonantly interacting SU(3) Fermi gases

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Abstract
We investigate color superfluidity and trimer formation in resonantly interacting SU(3) Fermi gases with a finite interaction range. The finite range is crucial to avoid the Thomas collapse and treat the Efimov effect occurring in this system. Using the Skorniakov–Ter-Martirosian equation with medium effects, we show the effects of the atomic Fermi distribution on the Efimov trimer energy at finite temperature. We show the critical temperature of color superfluidity within the many-body T-matrix approximation. In this way, we can provide a first insight into the phase diagram as a function of the temperature $T$ and the chemical potential $\mu$. This phase diagram consists of trimer, normal, and color-superfluid phases, and is similar to that of quantum chromodynamics at finite density and temperature.

1. Introduction

The concept of quantum simulation has opened new possibilities for exploring the properties of novel materials as well as exotic matter in extreme conditions [1–3]. Recently, ultracold atoms have been used as quantum simulators of strongly correlated systems thanks to the tunability of their physical parameters such as interparticle interaction [4–7]. For example, ultracold atomic Fermi gases loaded into an optical lattice can realize the Hubbard model, which is relevant to high-$T_c$ cuprate superconductors [3, 8–11]. The antiferromagnetic behavior which plays an important role for the superconducting mechanism of high-$T_c$ cuprates has been observed in this atomic system [3, 11]. As another example, strongly interacting two-component Fermi gases can be used as quantum simulators of dilute neutron matter [12–14]. Indeed, the observed thermodynamic quantities of homogeneous Fermi gases near the unitarity limit [15–18] quantitatively reproduce the equation of state of neutron matter obtained by numerical simulations [19] which is crucial for understanding the interior of a neutron star [20, 21].

The analog simulation of quantum chromodynamics (QCD) [22, 23], where quarks with three colors strongly interact with each other, can be regarded as one of the next important challenges for cold-atom physics [24–26]. While numerous efforts have been made to explore the phase diagram of finite-density QCD where various phenomena such as color superconductivity [27] have been theoretically proposed, they could not be confirmed by high-energy experiments due to the extreme densities, nor by exact numerical simulations due to the sign problem. On the other hand, three-component fermion systems have been experimentally realized by using mixtures of fermionic atoms in three different internal states [26, 28–35]. These systems reach the quantum degenerate regime around $T \approx 0.3T_F$ [28] where $T_F$ is the Fermi degenerate temperature of non-interacting atoms. In this regime, the existence of color superfluidity similar to color superconductivity in QCD has been theoretically examined in three-component Fermi gases [36–44]. Although there are several differences between these atomic systems and QCD, they constitute a good starting point to study the strong-coupling effects in three-component fermion systems.

In most of the previous works on color superfluidity [36–39, 41–44], zero-range interactions have been used to describe the attraction between the three components. However, such zero-range interactions are known to
lead to a collapse of the system [45, 46] in connection with the existence of Efimov trimer [30, 33, 35, 47–51]. A finite range of interactions is therefore necessary to prevent the collapse and properly treat Efimov trimers. The effect of finite range and the Efimov trimers on the phase diagram of three-component fermionic system was studied in [52]. However, this study was limited to zero temperature.

In this paper, we investigate in-medium Efimov trimers and color superfluidity in resonantly interacting SU(3) Fermi gases with finite interaction range and finite temperature. By including medium effects in the Skorniakov–Ter-Martirosian (STM) equation [53] which exactly describes Efimov physics in a three-body problem [34], we discuss how the trimer energy is changed at finite temperature. To explore many-body physics related to color superfluidity, we employ the non-selfconsistent T-matrix approximation (TMA) [55–65], which can correctly describe the Bardeen–Cooper–Schrieffer to Bose–Einstein-condensation (BCS–BEC) crossover [66–71] in strongly interacting two-component 40K [72] and 4Li [73] Fermi gases. Combining the results of these two approaches, we obtain the phase diagram with respect to the temperature T and the chemical potential μ.

This paper is organized as follows. In section 2, we explain the Hamiltonian for resonant SU(3) Fermi gases and our framework of in-medium STM equation and non-selfconsistent TMA. In section 3, we show our numerical results of the in-medium Efimov trimer energy and discuss the finite temperature behavior of them, as well as the color-superfluid instability. Finally, we summarize this paper in section 4. For simplicity, we take ℏ = k_B = 1 and the system volume V is taken to be unity.

2. Formalism

2.1. Hamiltonian

We consider a symmetric three-component fermionic system. We model the interaction between two different components by a two-channel Feshbach resonance model [7, 52] consisting of an open channel and a closed channel described by a single diatomic molecular state. We neglect the interaction within the open channel which corresponds to the limit of closed-channel dominated resonances. Furthermore, the open channel between two fermions and molecular states is taken as a contact-type coupling. The three-component fermionic system is therefore described by the following Hamiltonian

\[ H = \sum_{j=1,2,3} \sum_{p} \xi_{j}^{p} c_{p,j}^{†} c_{p,j} + \sum_{i<j} \sum_{q} \xi_{q}^{i,j} c_{q,i}^{†} c_{q,j} + g \sum_{i<j} \sum_{p,q} (b_{q,i}^{†} c_{p,q+i+j} c_{p,q+i+j}^{†} + \text{h.c.}), \]

where \( \xi_{j}^{p} = \frac{p^2}{2m} - \mu \) and \( \xi_{q}^{i,j} = \frac{q^2}{4m} + \nu - 2\mu \) are the kinetic energies of a fermion with mass m and the diatomic molecule, respectively, and \( \mu \) is the fermionic chemical potential. \( c_{p,j} \) and \( b_{q,i} \) are annihilation operators of a Fermi atom with the internal state \( j = 1, 2, 3 \) and a diatomic molecule made of \( i-j \) atomic pair. The energy of diatomic molecules \( \nu \) and the atom-dimer Feshbach coupling g are related to the scattering length a and the range parameter \( R_{e} \) as follows,

\[ \frac{m}{4\pi\alpha} = -\frac{\nu}{g} + \sum_{p} \frac{m}{p^2}, \]

\[ R_{e} = \frac{4\pi}{m^{2}g^{2}}. \]

We note that the effective range \( r_{e} \) of this two-channel interaction is here negative and is associated with \( R_{e} = -\frac{1}{2}r_{e} \). In this paper, we focus on the unitarity limit \( 1/\alpha = 0 \).

2.2. Non-selfconsistent TMA

We first employ the non-selfconsistent TMA within the Matsubara formalism [55–65] to determine thermodynamic properties such as critical temperature \( T_{c} \) of color superfluidity. In this framework, the thermal Green’s function of dressed atoms is given by

\[ G(p) = \frac{1}{i\omega_n - \xi_{p}^{F} - \Sigma(p)}, \]

where we used the four-momentum notation \( p = (p, i\omega_{n}) \) and \( \omega_{n} = (2n + 1)\pi T \) is the fermionic Matsubara frequency. \( \Sigma(p) \) is the fermionic self-energy. In the case of just two fermions in vacuum, the self-energy is given by the diagram shown in figure 1(a). Retaining only this diagram, we obtain [65]

\[ \Sigma(p) = 2g^{2}T \sum_{Q} D(Q) G_{0}(Q - p), \]

where \( Q = (Q, i\nu_{d}) \) and \( \nu_{d} = 2\pi n'T \) is the bosonic Matsubara frequency. \( G_{0}(p) = 1/(i\omega_{n} - \xi_{p}^{F}) \) and \( D(Q) \) are the in-medium Green’s functions of a non-interacting fermion and a dressed molecule, respectively. We note that although this equation (5) has the same form as that in vacuum, here \( G_{0} \) and \( D \) contain the medium...
effects. We also note that the factor 2 in equation (5) comes from the degree of freedom with respect to internal states.

Similarly, the thermal Green’s function of dressed molecules with the ultraviolet renormalization is given by

\[ D(Q) = \frac{1}{\nu \omega' - \xi^D_Q - \Xi(Q)}, \]

where \( \Xi(Q) \) is the bosonic self-energy diagrammatically shown in figure 1(b). Here again, we take the vacuum form

\[ \Xi(Q) = -g^2 T \sum_p G_0(p + Q)G_0(-p) = g^2 \sum_p \frac{1 - f(\xi^D_Q + t)}{\nu \omega' - \xi^D_Q + t}, \]

where \( f(\xi) = 1/(e^{\xi/T} + 1) \) is the Fermi–Dirac distribution function. The chemical potential \( \mu/\xi_F \) with fixed number density \( n \) (where \( \xi_F = (2\pi^2 n)^{1/3}/(2m) \) is the Fermi energy of an ideal Fermi gas at \( T = 0 \)) is obtained by solving the number equation,

\[ n = 3T \sum_Q G(p) - 6T \sum_Q D(Q). \]

In addition, we obtain the critical temperature \( T_c \) from the Thouless criterion \( D^{-1}(Q = 0, \nu \omega' = 0) = 0 \) \([74]\), which gives \( 1/a = 0 \)

\[ \frac{m^2 \mu R_s}{2\pi} + \sum_p \left[ \frac{1}{2\xi^F_p} \tanh \left( \frac{\xi^F_p}{2T_c} \right) \right] - \frac{m^2}{p^2} = 0. \]

2.3. In-medium STM equation

To determine the trimer energy \( E_3^M \) in the medium, we consider the three-body \( T \)-matrix equation \([75, 76]\) which is diagrammatically shown in figure 2. In the vacuum case, it gives the so-called STM equation which exactly describes Efimov physics in three-body problems. Using the thermal Green’s functions within the Matsubara formalism, we obtain the in-medium three-body \( T \)-matrix

\[ T_3^M(p, p'; P) = -g^2 G_0(p - p' - q) D(q) G_0(q) D(p - q) T_3^M(q, p'; P). \]

We note that \( p, p', \) and \( P \) are the four-momenta of incoming fermion, outgoing fermion, and the center-of-the-mass, respectively (we suppress the index of internal states for simplicity). The factor 2 in the second term of rhs of equation (10) comes from the degree of freedom with respect to internal states. \( G_0(q) \) with \( q = (q, i\Omega_n) \) indicates intermediate states of fermions which are integrated. By including Pauli-blocking effects on \( G_0(p) \) (see appendix), we obtain the in-medium STM equation in the unitarity limit \( (1/a \to 0) \).
where $\kappa(q)^2 = \frac{3}{2}q^2 - mE^M_3L(p)$ is the function defined by equation (A.9) in appendix, and

$$F(p, q) = 1 - f(\xi^{\mathcal{E}}_{p+q/2}) - f(\xi^{\mathcal{E}}_{p-q/2})$$

is the statistical factor associated with the Fermi–Dirac distribution of atoms given by $f(\xi^{\mathcal{E}}_p) = 1/(e^{\xi^{\mathcal{E}}_p/T} + 1)$.

One can find that the ordinary STM equation in a three-body problem is recovered by setting $F(p, q) = 1$. A similar equation to equation (11) was used in [77, 78]. We note however that these works are restricted to $T = 0$ and moreover, the form of $F(p, q)$ in these works corresponds to

$$F(p, q) = \theta(\xi^{\mathcal{E}}_{p+q/2})\theta(\xi^{\mathcal{E}}_{p-q/2}),$$

instead of equation (12). Here $\theta(x)$ denotes the Heaviside step function. Equation (13) corresponds to the absence of propagator for holes below the Fermi sea (see equation (A.12)). While there is no physical reason to neglect this propagator, interestingly, this choice allows the possibility of positive-energy solutions, called Cooper triples. However, if we solve equation (11) with equation (13) extended to finite temperature, namely,

$$F(p, q) = [1 - f(\xi^{\mathcal{E}}_{p+q/2})][1 - f(\xi^{\mathcal{E}}_{p-q/2})],$$

such positive-energy solutions cannot be calculated due to the appearance of divergences coming from the denominators in equation (11). Our equation (12) leads to the same problem at any temperature, and therefore does not allow positive-energy solutions, even at $T = 0$. We suppose that this is a technical issue due to the incompleteness of the theories, which should be addressed in the future. Nevertheless, in [79], it was found that the difference between the choices of equations (12) and (14) are not significant at finite temperature and negative energy. Therefore, we focus on the region where $E^M_3 \leq 0$. We briefly note that similar in-medium three-body equations were employed in nuclear physics [80–87].

3. Results

Figure 3 shows the ground-state trimer energy $E^M_3$ with the medium effects, which can be obtained numerically from equation (11) as a dimensionless function

$$E^M_3 = \frac{1}{mR^2_s}X(R_s/\lambda_T, \mu/T),$$

where $\lambda_T = \frac{\hbar}{\sqrt{2mT}}$ is the thermal de Broglie wavelength. Physically, the range parameter $R_s$ gives the typical size of the Efimov trimer [88]. In this regard, the ratio between $R_s$ and $\lambda_T$ represents how trimer states are affected by finite temperature effects. In the case of $R_s \ll \lambda_T$ and $\mu/T \lesssim 0$, $E^M_3$ approaches the vacuum limit given by $E^V_3 = -0.013\,85/(mR^2_s)$ which is close to that of a universal trimer [89], since the trimer size is small enough compared to the typical thermal length scale.
In addition, the ratio $\mu/T$ is associated with the fugacity $z = e^{\mu/\epsilon_F}$ and represents Pauli-blocking effects due to the atomic Fermi distribution, which plays a significant role when $\mu > 0$. The absolute value of $E_3^M$ is greatly reduced in the Fermi degenerate region. Finally, $E_3^M$ disappears in the region where $\mu/T$ and $R_*/\lambda_T$ are relatively large. The physical interpretation of these effects is that Fermi atoms from the medium weaken the $E_3^M$ attraction between three atoms forming a trimer state. This phenomenon is somewhat similar to the Gor’kov–Melik-Barkhudarov (GMB) correction in weak-coupling superconductors for which the size of Cooper pairs is large [90] and the pairing interaction is screened by the medium [91–95].

To see how these effects would appear in actual experiments at given temperatures and densities, we plot in figure 4(a) the typical temperature dependence of trimer energy $E_3^M$ at different range parameters according to the density equation of state obtained from the non-selfconsistent T-matrix approximation (TMA) above $T_c$. $k_F$ and $\epsilon_F$ are the Fermi degenerate temperature, the Fermi momentum, and the Fermi energy of ideal Fermi gases, respectively.

Figure 4. (a) Trimer energy $E_3^M$ with medium effects as a function of $T/T_F$ obtained from the chemical potential $\mu/\epsilon_F$ calculated within the non-selfconsistent T-matrix approximation (TMA) above $T_c$. $k_F$ and $\epsilon_F$ are the Fermi degenerate temperature, the Fermi momenta, and the Fermi energy of ideal Fermi gases, respectively.

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From these results, we get a first insight into the phase diagram with respect to the chemical potential $\mu$ and temperature $T$, as shown in figure 5. We expect a trimer phase (TP) where all of the atoms are bound into
ground-state Efimov trimers, a color superfluid phase (CSF) where all three kinds of pairs are condensed, and a normal phase (NP) where the atoms form neither trimers nor condensed pairs. The boundary between TP and NP is estimated by the curve where \( E_0^M = 0 \). This curve does not represent any phase transition between trimer and NPs but it should be a good indication of how the trimer character disappears in the high-density region of this system. CSF is defined by the region below \( T_c \). We note that \( T_c \) approaches zero in the low-density (zero-range) limit at \( mR^* \). We also note that the high-density limit of the critical temperature \( T_{Tc} \) is simply obtained from the BEC temperature of diatomic molecules in the presence of thermally excited fermions and \( \mu \) approaches zero in this limit. This indicates that the system undergoes a crossover from unitary Cooper pairs to BEC of closed channel molecules. This behavior is specific to the narrow-resonance two-channel model used in this work. Interestingly, figure 6 is similar to the phenomenological phase diagram of QCD consisting of the hadron phase (analogue of TP), the color superconducting phase (analogue of CSF), and the deconfined quark phase (analogue of NP) [23]. However, while the phase transition at \( T = T_c \) between CSF and NP is of the second order, that of color superconductivity is of the first order due to the gauge coupling [27, 96]. Moreover, the conjectured BEC–BCS crossover [97, 98] in QCD with increasing the chemical potential is opposite to the BCS–BEC crossover found in this model at \( 1/a = 0 \). We stress again this is a particularity of the narrow-resonance two-channel model.
Although our in-medium STM equation cannot be justified near \( T = T_c \) where the Bose–Einstein distribution of diatomic molecules increases in the low-energy region, one can find a quantum-phase-transition-like behavior around \( \mu mR_k^d = 0 \). The inset of figure 5 shows the magnification around \( \mu mR_k^d = 0 \), where the two curves we calculated cross each other. In reality, the region near this point is expected to be dominated by strong multi-body correlation due to the competition between trimer formation and color superfluidity \([82] \), which cannot be captured by our treatment.

Finally, we look at pairing fluctuations above \( T_c \). Indeed, it is known that pairing fluctuations become strong near \( T_c \) in two-component Fermi gases near the unitarity limit.

Figure 6 shows the single-particle spectral function

\[
A(p, \omega) = -\frac{1}{\pi} \text{Im} G(p, \omega_{\text{in}} \rightarrow \omega + i\delta)
\]

obtained from the analytic continuation of \( G(p) \) given by equation (4) (\( \delta \) is an infinitesimally small positive number), at \( \mu mR_k^d = 0.0298 \) and \( TmR_k^d = 0.0302 \) which is just above \( T_c \) indicated in the inset of figure 5. One can see that the atomic dispersion has a gap structure near \( \omega = 0 \) even in the absence of the superfluid gap. This excitation gap in the NP originates from strong pairing fluctuations (preformed Cooper pair) and is called pseudogap, which has been extensively discussed for various strongly correlated quantum systems such as high-\( T_c \) superconductors \([99,100]\), ultracold Fermi gases \([55–65,101–108]\), color superconductivity \([109,110]\), and nuclear matter \([111–114]\). This single-particle excitation property is accessible by photo-emission spectrum measurement in cold atom systems \([102–105]\). In principle, such experiments could also observe many-body effects associated with in-medium Efimov trimers. However, treating such many-body effects theoretically would require a self-consistent approach including both two-body and three-body correlations in the self-energy.

The pseudogap can also be seen in the single-particle density of states \( \rho(\omega) \), which is defined by

\[
\rho(\omega) = \sum_p A(p, \omega).
\]

It is shown in figure 7(a) at \( T = T_c \). This quantity clearly shows the pseudogap effect as a dip structure around \( \omega = 0 \). The pseudogap disappears away from \( \mu = T = 0 \), that is, in the high-density (large range parameter) limit as found in the case of two-component Fermi gases with negative effective range \([65]\). To characterize this many-body phenomenon, we introduce the pseudogap size \( \Delta_{\text{pg}} \) defined as the half width of the dip \([60]\) (see the inset of figure 7(b)). One can find that \( \Delta_{\text{pg}} \) grows when \( \mu mR_k^d \) approaches zero and reaches a maximum value \( \Delta_{\text{pg}} \approx 0.55 \) at \( \mu mR_k^d = 0 \). This enhancement of \( \Delta_{\text{pg}} \) indicates that many-body effects associated with pairing fluctuations are important in the region around \( \mu = T = 0 \) in the phase diagram of figure 5. This confirms the expected competition between formation of Cooper pairs and that of Efimov trimers. This competition would occur around \( E_{\text{fi}} = 0 \), which is shown as the vertical dashed line in figure 7(b).

4. Summary

In this paper, we have investigated some of the strong-coupling effects occurring in resonantly interacting SU(3) Fermi gases with a finite interaction range, namely, the in-medium Efimov trimer and the critical temperature \( T_c \) of color superfluidity.

The trimer formation is weakened by the medium effects, which consist of thermal agitation and Fermi pressure due to Pauli exclusion. The trimer is affected by thermal agitation when the thermal de Broglie wavelength is comparable to the trimer size given by the range of interaction. The Pauli-blocking effects are significant when the chemical potential \( \mu \) becomes large. As in the case of the GMB corrections in weakly coupled superconductors where the pairing interaction forming loosely bound Cooper pairs is screened by electrons from the medium, the medium effects in three-component Fermi gases become stronger when the trimer size or the atomic Fermi sphere becomes larger. We have shown how these effects would appear in actual experiments varying temperature at fixed number density.

Finally, we have investigated the phase diagram with respect to the chemical potential \( \mu \) and temperature \( T \). Our calculations indicate the existence of three phases: trimer, normal, and color superfluid phases. Interestingly, the obtained phase diagram is analogous to the phenomenological QCD phase diagram which consists of hadron, deconfined quark, and color superconducting phases. We emphasize that the finite interaction range plays an important role to obtain such a QCD-like phase diagram in this atomic system.

Near \( \mu = T = 0 \) in our phase diagram, the system is expected to be dominated by strong two-body and three-body correlations resulting from the competition between trimer formation and color superfluidity. This idea is supported by our calculation of the in-medium trimer energy and the single-particle spectral function which exhibits strong pairing fluctuations near the CSF transition. A self-consistent treatment of two-body and three-body correlations is required to understand this interesting regime, which is left as a future problem. Our
analysis does not exclude the possibility of trimer superfluidity due to a possible residual attraction between the trimers [115]. Such a state would be similar to the $p$-wave superfluidity in a Bose–Fermi mixture [116]. Although we consider a resonant interaction in this paper, the phase diagram would quantitatively change when tuning the scattering length. In particular, the CSF (or molecular BEC) would start at a lower chemical potential in the case of a finite scattering length [52].

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Appendix. Derivation of the in-medium STM equation

Since we calculate the ground-state trimer energy $E^M_3$ with the medium effects, we set $P = (0, \ i\zeta)$ in the three-body $T$-matrix equation given by equation (10) [75, 76] (where $i\zeta = (2l + 1)\pi T$ is the fermionic Matsubara frequency). We obtain

$$T^M_3(p, p'; P) = -2g^2 T \sum_q G_0(p - q) G_0(q) D(p - q) T^M_3(q, p'; P), \quad (A.1)$$

Figure 7. (a) Single-particle density of states $\rho(\omega)$ and (b) pseudogap size $\Delta_{pg}$ [60] at $T = T_c$ where $\rho_0$ is that at a Fermi level in a non-interacting Fermi gas. The vertical dashed line shows $E^M_3 = 0$. $\Delta_{pg}$ is determined from the half width of the dip in $\rho(\omega)$ around $\omega = 0$ as shown in the inset of the panel (b), where the horizontal dotted lines exhibit the local maximum, the half depth, and the local minimum, respectively.
where we ignore the first term of rhs of equation (10) which is negligible near the pole of $T^M_3$. The summation over the Matsubara frequencies $\Omega_n$ (where $q = (\mathbf{q}, \Omega_n)$) in equation (A.1) can be replaced by the contour integral with respect to an anticlockwise path $C$ enclosing the pole of the Fermi–Dirac distribution function $f(x)$, namely, $x = i\Omega_n$ as

$$T \sum_{\Omega_n} = -\oint_C \frac{dx}{2\pi i} f(x), \quad \text{(A.2)}$$

since $\text{Res}(f(x = i\Omega_n)) = -T$. We note that $C$ can be deformed to a clockwise path $C'$ which encloses the poles of $G_0$, $D$, and $T^M_3$, which give medium effects associated with the momentum distributions of atoms, molecules, and trimers, respectively. For simplicity, we consider only the pole of $G_0$ to incorporate the effects of the atomic Fermi–Dirac distribution. This approximation is justified in the high temperature regime where the fugacity $\zeta = e^{\mu/T}$ is small. In this regime, the atomic Fermi distribution function is approximately given by

$$f(\xi) \approx e^{-\frac{\xi - \mu}{T}},$$

and trimers, respectively. For simplicity, we consider only the pole of $\text{Fermi}$

$$G_0 \text{and } T^M_3 \text{, which give medium effects associated with the momentum distributions of atoms, molecules, and trimers, respectively. For simplicity, we consider only the pole of } G_0 \text{ to incorporate the effects of the atomic Fermi–Dirac distribution. This approximation is justified in the high temperature regime where the fugacity } \zeta = e^{\mu/T} \text{ is small. In this regime, the atomic Fermi distribution function is approximately given by}$$

$$f(\xi) \approx e^{-\frac{\xi - \mu}{T}},$$

$$T^M_3(p, p'; P) = 2g^2 \sum_q \int_C \frac{dx}{2\pi i} f(x)D(P - q)T^M_3(q, p'; P)\]

$$= 2g^2 \sum_q \left[ \frac{\{1 - f(\xi^q_{p+q})\}D(P - q)T^M_3(q, p'; P)}{\zeta_q - i\omega_n - \xi^q_{p+q}} \right. \]

$$\left. - \frac{f(\xi^q_q)D(p - q)T^M_3(q, p'; P)}{\zeta_q - i\omega_n - \xi^q_q - \xi^q_{p+q}} \right]. \quad \text{(A.3)}$$

where $\zeta_q = (q, i\zeta_q - i\omega_n - \xi^q_{p+q})$ and $\zeta_{q'} = (q', \xi^q_{q'}p)$. To obtain the in-medium STM equation, we perform the analytic continuations $i\omega_n \rightarrow \xi^q_q$ and $i\zeta_q \rightarrow E^M_3 - 3\mu$ in equation (A.3). In this way, we obtain

$$T^M_3(p, p'; P) = 2g^2 \sum_q \left[ \frac{\{1 - f(\xi^q_{p+q})\}D(q, \xi^q_{p+q} + \xi^q_p)}{E^3 - 3\mu - \xi^q_p - \xi^q_{p+q}} \right. \]

$$\times T^M_3((q, E^3_3 - 3\mu - \xi^q_p - \xi^q_{p+q}), p'; P)\]

$$\left. - \frac{f(\xi^q_q)D(q, E^3_3 - 3\mu - \xi^q_q)T^M_3((q, \xi^q_q), p'; P)}{E^3 - 3\mu - \xi^q_q - \xi^q_{p+q}} \right]. \quad \text{(A.4)}$$

Furthermore, the first integrand in equation (A.4) gives dominant contributions near its pole, corresponding to

$$E^3_3 - 3\mu - \xi^q_p - \xi^q_{p+q} = 0.$$ We can then approximate the arguments of $D$ and $T^M_3$ as

$$D(q, \xi^q_{p+q} + \xi^q_p) \approx D(q, E^3_3 - 3\mu - \xi^q_q), \quad \text{(A.5)}$$

and

$$T^M_3((q, E^3_3 - 3\mu - \xi^q_q), p'; P) \approx T^M_3((q, \xi^q_q), p'; P). \quad \text{(A.6)}$$

By substituting equations (A.5) and (A.6) into equation (A.4), we obtain

$$T^M_3(p, p'; P) = 2g^2 \sum_q \left[ \frac{1 - f(\xi^q_q) - f(\xi^q_{p+q})}{E^3 - 3\mu - \xi^q_p - \xi^q_{p+q}} \right. \]

$$\times D(q, E^3_3 - 3\mu - \xi^q_q)T^M_3((q, \xi^q_q), p'; P), \quad \text{(A.7)}$$

where

$$D(q, E^3_3 - 3\mu - \xi^q_q) \approx \frac{4\pi}{mg^2} \sum_k \frac{1}{\left\{ \frac{f(k, q)}{k^2 + s(q)^2} - \frac{1}{k^2} \right\} - R_\kappa(q)^2}. \quad \text{(A.8)}$$

is obtained from the analytic continuation of equation (6). We note that $\kappa(q)^2 = \frac{3}{2}q^2 - mE^3_3$ and $F(k, q)$ is the statistical factor defined by equation (12). We introduce
\[ L(q) = \frac{m g^2}{4 \pi} D(q, E_s^M) - 3 \mu - \frac{\xi^F}{q} T_5^M(q, \xi^F, p'; P). \] (A.9)

We note that although \( L(q) \) implicitly depends on \( p' \) and \( P \), they do not change the equation of \( E^M \) because we consider only the s-wave component. By using \( L(q) \), equation (A.7) can be rewritten by

\[
\left[ -R_{\alpha k}(p)^2 + 4\pi \sum_k \left\{ \frac{F(k, p)}{k^2 + \alpha^2(p)^2} - \frac{1}{k^2} \right\} \right] L(p) = 8\pi \sum_q \frac{1 - f(\xi^F_{p+q}) - f(\xi^F_q)}{m E^M_q - p^2 - q^2 - p \cdot q} L(q). \] (A.10)

Finally, we obtain the in-medium STM equation, that is, equation (11) by making the substitutions \( q \to q + p/2 \) and \( p \to -p \) in equation (A.10). We note that by taking the vacuum limit \( \mu \to -\infty \) where \( f(\xi^F_q) \to 0 \), equation (11) reproduces the ordinary STM equation of a three-body problem at \( 1/a = 0 \) [52] given by

\[
\left[ -R_{\alpha k}(p)^2 + 4\pi \sum_k \left\{ \frac{1}{k^2 + \alpha^2(p)^2} - \frac{1}{k^2} \right\} \right] L(p) = -8\pi \sum_q \frac{L(q + p/2)}{q^2 + \alpha^2(p)^2}, \] (A.11)

which gives the ground-state trimer energy \( E_s^T = -0.013 \ 85/(mR_s^2) \) in vacuum.

At \( T = 0 \), we can obtain the same equation by using the time-ordered Green’s function [117]

\[
G^T_{\alpha i}(p) = \frac{1 - f(\xi^F_p)}{\omega + i\delta - \xi^F_p} + \frac{f(\xi^F_p)}{\omega - i\delta - \xi^F_p}, \] (A.12)

which consists of the propagator of a particle above the Fermi sea and that of a hole below the Fermi sea. Here, \( \delta \) is an infinitesimally small positive number. If the hole propagator is neglected, the statistical factor of [77], equation (13) is obtained.

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