Formalization of dynamic scheduling for an educational center

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Abstract. The model of scheduling with the opportunity of daily adjustment, which contains four groups of constrains, is made on the basis of the analysis of the provided services features and legal requirements. Necessary scalar and matrix variables are entered. For the decision of similar tasks the methods based on combinations of genetic programming with greedy algorithms and procedures of local search, traditional for similar tasks are well proved. In view of inaccuracy of the initial sending, inaccuracy of the regulatory base, existence of inevitable noises and many other unaccounted factors, in real tasks of design (where it is required to define expiration date of the project or its cost) special accuracy is not required. Besides, the found option of distribution of resources can always be used as first approximation for more exact calculation, and specification can be made by means of much less labor-consuming methods of the perturbation theory.

1. Introduction
The competent scheduling is necessary for children developing centers, which means the maximum load of classroom fund, considering balanced load of teachers and clients. Therefore, even if there are many analogs in educational sphere, creating a new scheduling informational system is still relevant topic. The research presented in the article is devoted to formalization of this task. The necessity of schedule daily adjustment, which is important because of constant addition of the held events, has been considered.

Many Russian and foreign researchers were engaged and still working on a problem of automatization and formalization of the process of scheduling. It is possible to allocate problems of creating the admissible schedule and more difficult task of creating an optimum schedule [1]. Professor V.V. Kovalyov is one of the founders of Russian school of schedules optimization. In his research [2] scheduling task is about structuring staff working hours in the most convenient way for all employees and clients. That is completely new class of problems, which belongs to the category of large combinatorial optimization problems, many of which are NP-complete and traditionally considered as intricate problems [3, 4].

Generally, the problem of scheduling with limited resources is formulated so that considering a set of events and a set of resources and capacity measurement, the best way of allocating resources can be chosen to maximize capacity. As a result, the majority of researches has been devoted to simplification of scheduling problem to a moment where some algorithms can find solutions, or to development of effective heuristics to find good solutions [5].
2. Problem statement

The initial problem of the schedules theory consists in allocation of limited resources possessed by the organization, to certain works in time that will allow solving an optimizing problem for this distribution. Therefore, it is possible to find an optimum algorithm to order known properties of task and resources and present restrictions. The schedule length and task time in the studied system are usually studied as main measures of efficiency. At the same time, models of these tasks will be determined within the fact that the existing information, upon which ordering decisions are made, is known in advance [6].

The offered formalization is based upon the methods described in the article "Formalization of a problem of scheduling in a higher educational institution" [7] by Popov G.A. In this work all the restrictions are divided into four groups: A, H, M, and L.

The group A has an absolute priority; these restrictions are always considered and can not be broken. The group H has a high priority. These restrictions can be broken in isolated case of emergency. The group M has a medium priority, meaning that breaking of these restrictions is highly undesirable, but it is possible. The group L has a low priority. It is desirable to observe restrictions of this group, but their violation doesn’t involve any serious consequences.

In the case of educational center, the restrictions are following.

A1. A class for one group shouldn’t last more than 3 hours.
A2. The duration of one class can be different; it depends on a type of given classes: art garden, master class, class in a course, birthday, and tenancy. For example, class in the course "Experimental Painting" will be considered as 2 hours = 1 class hour + 1 preparatory hour (to prepare the room and to clean it up after the class).
A3. If two rooms are adjacent and an active class takes place in one of them, the other must be empty. If two rooms are separate, it is possible to put there any classes, as well as putting an active class in the first room, and passive in the other.
A4. Art garden must work only in the morning.
H1. Classes for children from 1,5 to 5 years have to finish no later than 7 p.m.
H2. Classes for children from 6 to 9 years have to finish no later than 8 p.m.
H3. Classes for children from 10 to 15 years have to finish no later than 9 p.m.
H4. For group of children of the same age the schedule have to be without gaps.
H5. The teacher has not to give lessons more than 6 hours per day.
M1. If during one day, a teacher has several groups to give lessons to, whenever it is possible, the schedule should be created without gaps.
L1. It is better to put active classes after the passive or even the last for the person.

Variables, necessary for scheduling formalization, are specified in table 1.

Restriction A1 satisfies the SanRaN 2.4.1.-3049-13 “Sanitary and epidemiologic requirements to providing, keeping and organizing of the preschool educational organizations’ working hours” part “I. General provisions and scope”, point 1.3. Restriction A2 for children's fitness satisfies SanRaN 2.4.1.-3049-13 part “XI. The requirements for enrolment of children in the preschool educational organizations, to a day regimen and the organization of educational process” point 11.4 and restrictions H1-H3 satisfy the point 12.5 of the same part [8].

The description of variables used to formalize a scheduling problem for creative space for children and parents.

All groups are sequentially numbered – an index \( g \). There is only one group for one class (further, in the text \( g \) can be both group and class). Other group is formed for other class. In this case classes are divided into an art garden, a specific master class, a course, birthday and tenancy. It is necessary to consider that duration of one class can be different from 30 minutes to 3 hours without preparation of the room for class, UV sterilizing and airing of the room.

\( K \) – set of age groups. \( k \) defines the age of the person \( b \). \( k_1 \) - children at the age from 1,5 up to 5 years, \( k_2 \) - children at the age from 6 up to 9 years, \( k_3 \) - children at the age from 10 up to 14 years, \( k_4 \) - children more senior than 14 years and adults.
Z is set of all classes’ duration within one day. Each $z_i$ increases by 30 minutes because after every class the room should be UV sterilized for 20 minutes and aired for 10 minutes, according to norms [8]. The maximum duration of all classes is designated as $t_g$, $t_i$ - time of the beginning of a class. The total hours $T_g$ is set for each class.

**Table 1. Variables, necessary for scheduling formalization**

| Variable | Notation | Data type |
|----------|----------|-----------|
| $g$      | all groups are sequentially numbered | string |
| $q$      | total number of group’s classes | integer |
| $B(b_1,..b_n)$ | set of all people | string |
| $b_n$    | a person who may belong to different groups $g$ | string |
| $k$      | set of age groups | string |
| $Z$      | the set of all classes’ duration within one day | string |
| $z_i$    | increases by 30 minutes because after every class the room should be UV sterilized for 20 minutes and be aired for 10 minutes | integer |
| $t_g$    | the maximum duration of all classes for specific group | integer |
| $t$      | the time from the beginning of a 24-hours day | integer |
| $T_g$    | the total hours is set for each class | integer |
| $t^g_t$  | time of the beginning of a class for group $g$ | integer |
| $A$      | all rooms are numbered | string |
| $V_a$    | for each room capacity is specified | integer |
| $a_{ij}$ | room type | integer |
| $l$      | all teachers are numbered | string |
| $d$      | duration of the program of carried out birthday | string |
| $r_g$    | class type for group $g$ | boolean |
| $x_{tg} = 1$ | if there is a class in class $t$ in room $a$ for group $g$ | boolean |
| $x_{tg} = 1$ | if a person $b$ belongs to group $g$ | boolean |
| $x_{tg} = 1$ | if the teacher $l$ gives classes in group $g$ | boolean |
| $x_{tg} = 1$ | if person $b$ belongs to age group $k$ | boolean |
| $x_{tg} = 1$ | if equipment of the room $a$ suits to specific class for group $g$ | boolean |

All rooms are numbered – an index $a$. For each room capacity $V_a$ is specified; it depends on room area and norms [8]: for one child aged up to 3 years it has to be given a place not less than $2.5 \, m^2$, for children over 3 years not less than $2.0 \, m^2$, and type of the room $a_{ij}$ (side or enclosed).

### 3. Theory

The restrictions following from the general reasons concerning the class scheduling:

R1. One person can not be in two classes at the same time, it means $b_1 \neq b_2$, for all $t, g, b_1, b_2, b_1 \neq b_2$:

$$x_{q_1 g} \cdot x_{q_2 g} = 0$$ (1)

R2. Classes can not be given at the same time in two different rooms; it means $a_1 \neq a_2$, for all $t, g, a_1, a_2, a_1 \neq a_2$,

$$x_{q_1 g} \cdot x_{q_2 g} = 0$$ (2)

R3. At the same time two different disciplines can’t be given in the same group it means if $g_1 \neq g_2$:

$$x_{t_1 g_1} \cdot x_{t_2 g_2} = 0$$ (3)

R5. The total hours of each class for each group has to be equal to the set size recorded in the course curriculum:
\begin{equation}
\sum_{q} x_{qt} = T_q
\end{equation}

R7. Restriction of the group enrolment. The group starts if there belong not less than three people and no more than the capacity of the room:
\begin{equation}
3 \leq \sum_{b=1}^{n} x_{bg_i} \leq V_i x_{bg_i}.
\end{equation}

A1. One class for one group should not last more than 180 minutes for all g:
\begin{equation}
\sum_{t} z_{tg} x_{tg} \leq t \cdot
g.
\end{equation}

A3. If two rooms are adjacent and an active class takes place in one of them, the other must be empty. If two rooms are separate, it is possible to put there any classes, as well as putting an active class in the first room, and a passive in the other:
\begin{equation}
\left(1-a_{ij} x_{g_1 g_2} r_{g_1} x_{g_2} r_{g_2}\right) \left(1-a_{ij} x_{g_1} r_{g_1} x_{g_2} r_{g_2}\right) \left(1-a_{ij} x_{g_1} r_{g_1} x_{g_2} r_{g_2}\right) \leq 0.
\end{equation}

A4. Art garden must work only in the morning. \(t_i\) - time of the beginning of an art garden class which has to begin no later than 10 a.m. (the 600th minute). Let \(g_1\) be designated as a group of an art garden:
\begin{equation}
t_{gi}^{g_1} \leq 600.
\end{equation}

H1. Classes for children from 1.5 to 5 years have to finish no later than 7 p.m. (1140 minutes). \(t_i^{g}\) is the time of the first class for all groups g:
\begin{equation}
t_i^g + \sum_{t} x_{tg} x_{rbk_1} z_{tg} \leq 1140.\end{equation}

Thus, for the considered task it is enough to receive a certain "good" decision. For the decision of similar tasks well proved the methods based on combinations of genetic programming with greedy algorithms and procedures of local search, traditional for similar tasks [9-11].

4. Experimental results
The authors assume that there are 7 types of classes and 10 actions, values classes-teacher is put in direct correspondence. In general, classes can be given in any room, but in practice, passive classes are given in the room \(a_1\), active classes - in the room \(a_2\), and action is taking place in two classes at once, then the variable \(a_3\) is used. It is conceivable that the room on a defined class is assigned to the teacher. If to look at a surname of the teacher, the employee understands what class will be and in what room.

In this regard the data can be presented in the form of a two-dimensional matrix where groups which indexes are described in group of restrictions A2 down register. It is known that \(g_i\) is engaged during \(t_i\) at the teacher of \(l_i\) in the \(a_1\) room. Across there is time of the class beginning is specified. And one two-dimensional matrix \((g_i, t_i)\) shows the schedule on one day of the week, next day there will be a counting again with \(t_i\) which is formed anew.

The authors suppose that classes begin from 9 a.m., time of the beginning of class, it means \(t_i=540\) minute of the present day. The first group of day, for example, \(g_1\) consists of 10 children \(b_1, b_2, ..., b_{10} \in g_1\) the teacher of \(l_1\) conducts a class. The duration of a class of \(z_1 = 180\) minutes, class lasts for three hours, the following begins from 750th minute according to restriction of A2.

Then the Monday schedule for the program will be presented in the matrix form the table 2.

**Table 2. Monday schedule**
For an employee the schedule will take the form presented in the table 3.

| t1 | t2 | t3 | t4 | t5 |
|----|----|----|----|----|
| g1 | 1  | 0  | 0  | 0  |
| g2 | 0  | 1  | 0  | 0  |
| g3 | 0  | 0  | 1  | 0  |
| g4 | 0  | 0  | 0  | 1  |
| g17| 0  | 0  | 0  | 1  |

Table 3. Monday schedule for an employee

| Time  | Activity | Teacher | Children | Classes |
|-------|----------|---------|----------|---------|
| 9:00  | Art-garden morning | Muhina | 10 | 1,2 |
| 12:30 | Art-garden day | Muhina | 7 | 1,2 |
| 16:00 | Clay modeling | Rodinova | 5 | 1 |
| 17:30 | Forwardness | Muhina | 6 | 2 |
| 19:00 | Action | Rosin | 15 | 1,2 |

The calculation was carried out using the Mathcad program, the schedule is adequate, but in objective function (10) minimization of idle time of the class and a gap between classes of one teacher is put.

\[ F = \alpha_1 \sum_t \sum_a \left| x_{1a} - x_{t+1,a} \right| + \alpha_2 \sum_t \sum_a \left| y_{1a} - y_{t+1,a} \right| \rightarrow \text{min}, \]  

(10)

Where \( s \) is the schedule, \( t \) is time from the beginning of 24-hours day, \( l \) is teachers, \( a \) is rooms, \( \alpha_1 \) - weight coefficient the teacher's priority, \( \alpha_2 \) - weight coefficient a room priority. Let in a priority there will be a minimization of gap in the teachers schedule, then will take \( \alpha_1 = 0.6 \), then \( \alpha_2 = 0.4 \).

As a result it turned out the number of rooms idle time in minutes in a week on average if to consider that the organization works from 9 a.m. to 9 p.m. (720 minutes per day, 5040 minutes per week), has made 1260 minutes per week (180 minutes per day). The quantity of gaps in the teacher schedule on average takes 210 minutes per day, 1470 minutes per week.

5. Results and discussion

The solving of optimum schedules compilation problem is probed by methods of integer linear programming in many operations, for example, [9-11]. This approach is classical. This task can be also linearized [12, 13]. The formalization of similar tasks [14-16] in the form of tasks of the linear programming and integer linear programming generates models with very large number of variables – up to million [17]. The attempts of using exact methods to solve such problems, only formally are successful as require either multi-day computation [14], or use of powerful and expensive computing resources. In view of inaccuracy of the initial sending, inaccuracy of the regulatory base, the existence of inevitable noises and many other unaccounted factors, in real tasks of design (where it is required to define expiration date of the project or its cost) special accuracy is not required. Besides, the found option of distribution of resources can be always used as first approximation for more exact calculation,
and specification can be made by means of much less labor-consuming methods of the perturbation theory [18].

6. Conclusion
Thus, in the article the problem of scheduling formalization is solved, mathematical restrictions for arrangement of classes in the schedule are applied; it means that the mathematical model of a problem has been created. The provided mathematical model satisfies all requirements and restrictions, allows passing directly to automation of this process.

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