On Quantum Interference:
No Decoherence From an Actual Movable Mirror

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Abstract

There exists a commonly accepted viewpoint that a movable mirror in an interferometer should cause interference breakdown due to a quantum jump to one of the two components of a photon mode. That effect goes back to Dirac. We argue that the conventional reasoning is inadequate: First, it would be more circumspect to interpret interference breakdown as being due to the entanglement of the photon with the mirror, not referring to quantum jumps. Second—and crucial—even in such an interpretation, the reasoning does not take into account the uncertainty of the mirror momentum. The effect of the entanglement and interference breakdown would take place if uncertainty were much less than the recoil momentum, which is of the order of the photon momentum. However, an examination leads to the conclusion that for an actual mirror the opposite situation occurs. Thus there should be no such effect.

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Introduction

There exists a commonly accepted point of view that a movable mirror in an interferometer should cause interference breakdown due to decoherence, i.e., a quantum jump to one of the two components of a photon mode. That effect goes back to Dirac. It was described in the first edition of The Principles of Quantum Mechanics [1] and examined in later editions including the last one [2]. The decoherence effect forms the basis for the Elitzur-Vaidman bomb-testing problem [3]. A detailed analysis of the effect in the context of the conventional approach is given by Penrose [4].

The underlying idea is this [1,2]: A movable mirror is an apparatus for a photon energy measurement, and the result of the measurement is either the whole photon or nothing, which means a quantum jump.

But the conventional reasoning provokes a question and suffers from an essential shortcoming. The question is this: Where is the demarcation between unmovable (mass $M = \infty$) and movable ($M < \infty$) mirrors? The shortcoming consists in that the reasoning does not involve any analysis of the measurement per se. First, it would be more circumspect to interpret interference breakdown as being due to the entanglement of the state of the photon and the mirror as a quantum object, not referring to quantum jumps. Second—and crucial—even in such an interpretation, the reasoning does not take into account the uncertainty $\Delta p$ of the mirror momentum. In order that the effect of interference breakdown take place, the initial and changed states of the mirror must be almost orthogonal, which implies the inequality $\Delta p \ll \delta p$ where $\delta p$ is the recoil momentum, and the latter is of order of the photon momentum $k$. Thus the inequality $\Delta p \ll k$ must hold. However for any actual mirror, the reverse inequality holds: $\Delta p \gg k$. Therefore there should be no interference breakdown.

In this paper, we examine the effect of the quantum nature of the mirror in detail. The central point is this: It is the mirror momentum uncertainty—rather than the mirror movableness and the recoil—that plays an essential role.

1 A conventional approach and its shortcoming

1.1 A conventional formulation and solution of the problem

The problem of decoherence in quantum interferometry was first examined by Dirac. There is no better way to describe the problem than quoting Dirac himself [2]:

"If we are given a beam of roughly monochromatic light, then we know something about the location and momentum of the associated photons. We know that each of them is located somewhere in the region of space through which the beam is passing and has a momentum in the direction of the beam of magnitude in terms of the frequency of the beam... When we have such information about the location and momentum of a photon we shall say that it is in a definite translational state.

We shall discuss the description which quantum mechanics provides of the interference of photons... Suppose we have a beam... which is passed through some kind of interferometer, so that it gets split up into two components and the two components are subsequently made to interfere. We may... take an incident beam consisting of only a single photon and inquire what
will happen to it as it goes through the apparatus. This will present to us the difficulty of the conflict between the wave and corpuscular theories of light in an acute form.

...we must now describe the photon as going partly into each of the two components into which the incident beam is split. The photon is then, as we may say, in a translational state given by the superposition of the two translational states associated with the two components. We are thus led to a generalization of the term ‘translational state’ applied to a photon. For a photon to be in a definite translational state it need not be associated with one single beam..., but may be associated with two or more beams... which are the components into which one original beam has been split... In the accurate mathematical theory each translational function may describe either a single beam or two or more beams into which one original beam has been split. Translational states are thus superposable in a similar way to wave functions.

Let us consider now what happens when we determine the energy in one of the components. The result of such a determination must be either the whole photon or nothing at all. Thus the photon must change suddenly from being partly in one beam and partly in the other to being entirely in one of the beams. This sudden change is due to the disturbance in the translational state of the photon which the observation necessarily makes...

One could carry out the energy measurement without destroying the component beam by, for example, reflecting the beam from a movable mirror and observing the recoil. Our description of the photon allows us to infer that, after such an energy measurement, it would not be possible to bring about any interference effects between the two components. So long as the photon is partly in one beam and partly in the other, interference can occur when the two beams are superposed, but this possibility disappears when the photon is forced entirely into one of the beams by an observation. The other beam then no longer enters into the description of the photon, so that it counts as being entirely in the one beam in the ordinary way for any experiment that may subsequently be performed on it”.

In the modern terminology, wave functions of ordinary wave optics are called mode functions [5]. They are by no means quantum wave functions of a photon: those do not exist. In this terminology, the sudden change of the mode function discussed above is called decoherence.

1.2 An essential shortcoming: No analysis of the measurement

The conventional reasoning given above lacks precision: It contains no analysis of the measuring process. In particular, even if an apparatus is considered as a classical object, its quantum wave function is involved in the description of the measurement. We quote Landau and Lifshitz [6]:

“...consider a system consisting of two parts: a classical apparatus and an electron... The states of the apparatus are described by quasiclassical wavefunctions $\Phi_n$, where the suffix $n$ corresponds to the ‘reading’ $g_n$ of the apparatus...

...Let $\Phi_0$ be the wavefunction of the initial state of the apparatus... and $\Psi$ of the electron... the initial wavefunction of the whole system is $\Psi\Phi_0$. After the measuring process we obtain a sum of the form $\sum_n A_n\Phi_n$...

...the classical nature of the apparatus means that, at any instant, the quantity $g$... has some definite value. This enables us to say that the state of the system apparatus+electron after the measurement will in actual fact be described, not by the entire sum, but by only the one term which corresponds to the ‘reading’ $g_n$ of the apparatus $A_n\Phi_n$. It follows from this that $A_n$ is proportional to the wavefunction of the electron after the measurement...”
It is essential that from $g_n' \neq g_n$ follows
\[(\Phi_n', \Phi_n) = 0 \quad (1.2.1)\]
Thus the conventional reasoning implies tacitly that the initial, $\Phi_{in}$, and changed, $\Phi_{ch}$, states of the mirror are orthogonal or, at least, that
\[|\langle \Phi_{ch}, \Phi_{in} \rangle| \ll 1 \quad (1.2.2)\]
So we have to revise the decoherence problem taking into account the quantum nature of the mirror. Although principal results may be achieved elementarily, it seems instructive to carry out a comprehensive examination.

2 A mirror as a quantum object
and the mode based picture

2.1 A mirror as a quantum object
A movable mirror is one with a finite mass, $M < \infty$. So any actual mirror is movable. In the simplest case, its movement is translational. Thus the quantum mechanical description of the mirror is given by the wave function $\Phi$ of the center of mass.
Had a photon been described by a wave function, the problem would have been represented by the transition
\[\Psi \Phi_0 \rightarrow \sum_n c_n \Psi_n \Phi_n \quad (2.1.1)\]
without the preconditions (1.2.1) or (1.2.2). However, such is not the case. In actual fact a photon is described by a mode function, $f$, rather then a wave function, $\Psi$. Therefore, for the sake of unification, we will describe the mirror by a mode function, $\varphi$, too, i.e., in terms of quantum field theory.

2.2 The mode based picture:
Tensor product of mode function spaces
To analyze the system photon+mirror in terms of modes, i.e., mode functions, we introduce the tensor product of mode function spaces. Let
\[F^K := \{f^K_{\alpha}\}, \quad K = I, \Pi \quad (2.2.1)\]
be a space of mode functions. We introduce a compound mode with a compound mode function
\[f = \sum_n c_n f^I_n \otimes f^\Pi_n \quad (2.2.2)\]
and the space of such functions
\[F = F^I \otimes F^\Pi = \{f^I_{\beta}\} \otimes \{f^\Pi_{\gamma}\} \quad (2.2.3)\]
Such a description may be called mode based picture.

For the system photon+mirror, we write

\[ X := \{\chi_\alpha\} = \{f_\beta\} \otimes \{\varphi_\gamma\} \]  
\[ \chi = \sum_n c_n f_n \otimes \varphi_n \]  

(2.2.4)

(2.2.5)

where the modes \( f \) and \( \varphi \) relate to the photon and center of mass of the mirror, respectively.

2.3 In and out modes

Now the problem reduces to that of scattering theory. Introducing in and out modes we have

\[ \chi^\text{in} = f^\text{in} \otimes \varphi^\text{in} \rightarrow \chi^\text{out} = \sum_n c_n f_n^\text{out} \otimes \varphi_n^\text{out} \]  

(2.3.1)

The problem is rather simple due to three properties: one of the two particles is a mirror; the latter is a nonrelativistic object with a mass much greater than the photon energy; photon modes are translational.

2.4 A nonrelativistic mirror

Let us consider the normal incidence of a photon on a fully reflecting nonrelativistic mirror. We have

\[ f^\text{in} = f_k, \quad k = |k| \]  
\[ \varphi^\text{in} = \int dp b(p)\varphi_p \]  
\[ \Delta p, \quad |k| \ll M \]  

(2.4.1)

(2.4.2)

(2.4.3)

So

\[ \chi^\text{in} = \int dp b(p)f_k \otimes \varphi_p \]  

(2.4.4)

Now

\[ f_k \otimes \varphi_p \rightarrow f_{k'} \otimes \varphi_{p'} \]  

(2.4.5)

where

\[ k' + p' = k + p, \quad |k'| + \frac{p'^2}{2M} = |k| + \frac{p^2}{2M} \]  

(2.4.6)

\((\hbar = 1, \ c = 1)\), and

\[ \chi^\text{out} = \int dp b(p)f_{k'} \otimes \varphi_{p'} \]  

(2.4.7)

From (2.4.6) follows

\[ |k| - |k'| = \frac{1}{2M} (|k| - k')(|k| - k') + 2p \]  

(2.4.8)

Putting

\[ k' = |k'| \neq |k| \]  

(2.4.9)
would result in
\[ 2M + |k'| = |k| + 2p \ll M \] (2.4.10)
so that
\[ k' = -|k'| \] (2.4.11)
We find
\[ |k'| = -|k| - (M + p) + [(M + p)^2 + 4M|k|]\frac{1}{2} \approx |k| - \frac{2|k|p}{M} \approx |k| \] (2.4.12)
Thus
\[ k' \approx -k, \quad p' \approx p + 2k \] (2.4.13)
so that
\[ \chi^{\text{out}} \approx f^{\text{out}} \otimes \varphi^{\text{out}} \] (2.4.14)
\[ f^{\text{out}} = f_{-k} \] (2.4.15)
\[ \varphi^{\text{out}} = \int dp b(p) \varphi_{p+2k} = \int dp b(p - 2k) \varphi_p \] (2.4.16)
Reducing (2.4.7) to (2.4.14) simplifies significantly the sum in (2.3.1).

3 Semitransparent and fully reflecting mirrors

Let us consider two cases of interest for interferometry: a semitransparent mirror and a fully reflecting one.

3.1 A semitransparent mirror

In the case of a semitransparent mirror,
\[ f^{\text{in}} = f^{\text{in}}_{k}, \quad \chi^{\text{in}} = f^{\text{in}}_{k} \otimes \varphi^{\text{in}} \] (3.1.1)
\[ \chi^{\text{out}} = c f^{\text{out}}_{k} \otimes \varphi^{\text{out}} + c' f^{\text{out}}_{k'} \otimes \varphi^{\text{out}}' \] (3.1.2)
\[ \varphi^{\text{out}} \approx \varphi^{\text{in}} \] (3.1.3)
Here \( f^{\text{in}}_{k}, f^{\text{out}}_{k}, \) and \( f^{\text{out}}_{k'} \) relate to the incident, transmitted, and reflected modes, respectively.

3.2 A fully reflecting mirror

In the case of a fully reflecting mirror in an interferometer,
\[ f^{\text{in}} = c_1 f^{\text{in}}_{k_1} + c_2 f^{\text{in}}_{k_2}, \quad \chi^{\text{in}} = c_1 f^{\text{in}}_{k_1} \otimes \varphi^{\text{in}} + c_2 f^{\text{in}}_{k_2} \otimes \varphi^{\text{in}} \] (3.2.1)
\[ \chi^{\text{out}} = c_1 f^{\text{out}}_{k_1} \otimes \varphi^{\text{out}}_1 + c_2 f^{\text{out}}_{k_2} \otimes \varphi^{\text{out}}_2 \] (3.2.2)
\[ \varphi^{\text{out}}_1 \approx \varphi^{\text{in}} \] (3.2.3)
Here the second component of the photon mode reflects from the mirror.
3.3 Unification

In the decoherence problem, it is the compound mode function $\chi^{\text{out}}$ that is subject to analysis. The two above cases may be unified:

$$\chi^{\text{out}} = \chi = c_1 f_1 \otimes \varphi_1 + c_2 f_2 \otimes \varphi_2, \quad f_j := \tilde{f}_{k_j} \quad (3.3.1)$$

$$(f_{j'}, f_j) = \delta_{j'j} \quad (3.3.2)$$

$$\varphi_1 \approx \varphi^{\text{in}} \quad (3.3.3)$$

4 The essence of the problem

4.1 Mode state operator

In the mode based picture, the states of the photon and mirror are described by mode state operators:

$$\varrho_{\text{ph}} = \text{Tr}_m \varrho, \quad \varrho_{\text{m}} = \text{Tr}_{\text{ph}} \varrho \quad (4.1.1)$$

where the subscripts ph and m stand for photon and mirror, respectively, and

$$\varrho = \chi \chi^\dagger \quad (4.1.2)$$

is a mode state operator for the system photon+mirror.

4.2 An ideal mirror and the essence of the problem

The compound mode function $\chi$ is given by (3.3.1). Let us introduce an imaginary object—an ideal mirror for which

$$\varphi_2 = \varphi_1 = \varphi^{\text{in}} \quad (4.2.1)$$

so that

$$\chi_{\text{ideal}} = f_{\text{ideal}} \otimes \varphi^{\text{in}} \quad (4.2.2)$$

and

$$\varrho_{\text{ph ideal}} = f_{\text{ideal}} f_{\text{ideal}}^\dagger \quad (4.2.3)$$

where

$$f_{\text{ideal}} = c_1 f_1 + c_2 f_2 \quad (4.2.4)$$

Now the essence of the problem amounts to comparing

$$\varrho_{\text{ph}} = \text{Tr}_m \chi \chi^\dagger \quad (4.2.5)$$

with (4.2.3).
5 The solution: Mathematical aspect

5.1 The normal form of the compound mode function

To find the \( \varphi_{ph} (4.2.5) \) it is expedient to represent the compound mode function (3.3.1) in the normal form [7]:

\[
\chi = \alpha_1 \bar{f}_1 \otimes \bar{\varphi}_1 + \alpha_2 \bar{f}_2 \otimes \bar{\varphi}_2, \quad \alpha_j > 0, \quad \alpha_j^2 + \alpha_j^2 = 1 \tag{5.1.1}
\]

\[
(\varphi_{j'}, \varphi_j) = \delta_{j'j} \tag{5.1.2}
\]

\[
(\bar{f}_{j'}, \bar{f}_j) = \delta_{j'j} \tag{5.1.3}
\]

Because of (3.3.2) we have

\[
g_m = w_1 \varphi_1 \varphi_1^\dagger + w_2 \varphi_2 \varphi_2^\dagger, \quad w_j = |c_j|^2 \tag{5.1.4}
\]

The \( \varphi_j \) are determined by the equation

\[
\rho_m \bar{\varphi} = \bar{\varphi} \tag{5.1.5}
\]

with the conditions (5.1.2), whence

\[
g_m = \bar{w}_1 \varphi_1 \varphi_1^\dagger + \bar{w}_2 \varphi_2 \varphi_2^\dagger \tag{5.1.6}
\]

which, in turn, implies (5.1.3) and

\[
|\alpha_j|^2 = \bar{w}_j \tag{5.1.7}
\]

We may choose

\[
\alpha_j = \sqrt{\bar{w}_j} \tag{5.1.8}
\]

Put

\[
\bar{\varphi}_j = b_j \varphi_i := \sum_{i=1,2} b_j^i \varphi_i, \quad \varphi_i = (b^{-1})_i^j \bar{\varphi}_j \tag{5.1.9}
\]

Then

\[
\chi = \sum_j c_j f_j \otimes (b^{-1})_j^i \varphi_i = \bar{f}^i \otimes \varphi_i \tag{5.1.10}
\]

\[
\bar{f}^i = \sum_j c_j (b^{-1})_j^i f_j \tag{5.1.11}
\]

Introduce

\[
\alpha_i = \|\bar{f}^i\|, \quad \bar{f}_i = \frac{1}{\alpha_i} \bar{f}^i \tag{5.1.12}
\]

so that

\[
\|\bar{f}_i\| = 1 \tag{5.1.13}
\]

Thus we obtain (5.1.1).

From (5.1.2) and (5.1.9) follows

\[
|b_1|^2 + |b_2|^2 + 2\text{Re}\{b_1^* b_2 (\varphi_1, \varphi_2)\} = 1 \tag{5.1.14}
\]
Equation (5.1.5) results in

\[
\begin{align*}
[\bar{w} - w_2(1 - r^2)]b^1 + re^{i\beta}\bar{w}b^2 &= 0 \\
re^{-i\beta}\bar{w}b^1 + [\bar{w} - w_1(1 - r^2)]b^2 &= 0
\end{align*}
\] (5.1.15)

where

\[
\bar{w} = 1 - \bar{w}, \quad re^{i\beta} = (\varphi_1, \varphi_2)
\] (5.1.16)

We obtain

\[
\bar{w} = \bar{w}_\pm = \frac{1}{2}[1 \pm \sqrt{1 - 4w_1w_2\bar{r}^2}], \quad \bar{r}^2 = 1 - r^2
\] (5.1.17)

From (5.1.15) follows

\[
b^2 = -\frac{\bar{w} - w_2\bar{r}^2}{r\bar{w}}e^{-i\beta}b^1
\] (5.1.18)

Substituting (5.1.18) into (5.1.14) gives

\[
|b^1|^2 \frac{\bar{r}^2}{1 - \bar{r}^2} \left[1 - 2 \frac{w_2\bar{r}^2}{\bar{w}} + \left(\frac{w_2}{w}\right)^2 \bar{r}^2\right] = 1
\] (5.1.19)

5.2 Two opposite cases

Consider two opposite cases:

I \quad \varphi_2 \approx \varphi_1, \ i.e., \ |1 - (\varphi_1, \varphi_2)| \ll 1, \ |1 - re^{i\beta}| \ll 1, \ \bar{r} \ll 1, \ |\beta| \ll 1 (5.2.1)

II \quad |(\varphi_1, \varphi_2)| \ll 1, \ r \ll 1 (5.2.2)

Case I. We find

\[
\bar{w}_1 := \bar{w}_- \approx w_1w_2\bar{r}^2, \quad \bar{w}_1 = 1 - w_1w_2\bar{r}^2 \approx 1, \quad \bar{w}_2 = w_1w_2\bar{r}^2 \ll 1
\] (5.2.3)

Now (5.1.19) for \(i = 1\) gives

\[
|b^1|^2 \frac{1}{w_1^2} \approx 1
\] (5.2.4)

Put

\[
b_1^1 = w_1
\] (5.2.5)

Then from (5.1.18)

\[
b_1^2 = w_2
\] (5.2.6)

For \(i = 2\), (5.1.19) gives

\[
|b^2_1| \approx \frac{1}{r}
\] (5.2.7)

Put

\[
b_2^1 = -\frac{1}{r}
\] (5.2.8)

then

\[
b_2^2 = \frac{1}{r}
\] (5.2.9)
Thus
\[(b_i^j) = \begin{pmatrix} w_1 & w_2 \\ -1/\tilde{r} & 1/\tilde{r} \end{pmatrix}, \quad ((b^{-1})_i^j) = \begin{pmatrix} 1 & -\tilde{r}w_2 \\ 1 & \tilde{r}w_1 \end{pmatrix}\] (5.2.10)

We obtain
\[
\bar{f}_1 = c_1f_1 + c_2f_2 = f_{\text{ideal}}, \quad \bar{f}_2 = \frac{1}{\sqrt{w_1w_2}}(-c_1w_2f_1 + c_2w_1f_2) \] (5.2.11)
\[
\bar{\varphi}_1 = w_1\varphi_1 + w_2\varphi_2, \quad \bar{\varphi}_2 = \frac{1}{\tilde{r}}(-\varphi_1 + \varphi_2) \] (5.2.12)
\[
\chi = \sqrt{1 - w_1w_2\tilde{r}^2}f_{\text{ideal}} \otimes \bar{\varphi}_1 + \sqrt{w_1w_2\tilde{r}^2}\bar{f}_2 \otimes \bar{\varphi}_2 \] (5.2.13)

So
\[
\rho_{\text{ph}} = (1 - w_1w_2\tilde{r}^2)\rho_{\text{ph ideal}} + w_1w_2\tilde{r}^2\bar{f}_2\bar{f}_2^\dagger \approx \rho_{\text{ph ideal}} \] (5.2.14)

There is practically no decoherence and no interference breakdown.

Case II. In the zeroth approximation,
\[r = 0, \quad (\varphi_1, \varphi_2) = 0 \] (5.2.15)

and (5.1.1) results in
\[
\chi = \sqrt{w_1}c_1f_1 \otimes \varphi_1 + \sqrt{w_2}c_2f_2 \otimes \varphi_2 = c_1f_1 \otimes \varphi_1 + c_2f_2 \otimes \varphi_2 \] (5.2.16)

So
\[
\rho_{\text{ph}} = w_1f_1f_1^\dagger + w_2f_2f_2^\dagger \] (5.2.17)

There is entanglement of the photon with the mirror and interference breakdown. This corresponds to the result of the conventional reasoning.

Note that in actual fact, the measurement problem per se has nothing to do with the question of interference breakdown. It is only the entanglement that is essential.

6 The solution: Physical aspect

6.1 A crucial role of the mirror momentum uncertainty

From (2.4.2) and (2.4.16) we infer that generally
\[
(\varphi_1, \varphi_2) = \int d\tilde{p}\varphi_1(\tilde{p})b(\tilde{p} - \delta\tilde{p}), \quad |\delta\tilde{p}| \sim k = |\vec{k}| \] (6.1.1)

where \(\delta\tilde{p}\) is the change of the mirror momentum and \(\vec{k}\) is the momentum of the photon mode (or mode component). Therefore it is the parameter
\[\varsigma := \frac{\Delta p}{k} \] (6.1.2)
that plays a crucial role in the interference breakdown problem. If
\[ \kappa \ll 1 \] (6.1.3)
then case II is realized: the entanglement and interference breakdown.
If
\[ \kappa \gg 1 \] (6.1.4)
then case I is realized: no entanglement and no interference breakdown.

6.2 The mirror movement
The mirror displacement is
\[ \delta x(t) = \frac{\delta p}{M} t, \quad \delta p = |\delta \vec{p}| \] (6.2.1)
On the other hand, the mirror position uncertainty is
\[ \Delta x(t) = \Delta x(0) + \frac{\Delta p}{M} t \] (6.2.2)
Thus the necessary condition for
\[ \delta x(t) \gg \Delta x(t) \] (6.2.3)
is
\[ \delta p \gg \Delta p \] (6.2.4)
i.e., (6.1.3).

6.3 The effect of mirror fuzziness
We have
\[ \kappa \gtrsim \frac{\lambda}{\Delta x(0)} \] (6.3.1)
where \( \lambda \) is the photon wavelength. Thus a small \( \kappa \) (6.1.3) implies
\[ \Delta x(0) \gg \lambda \] (6.3.2)
which means a fuzzy mirror. On the other hand,
\[ \Delta x(0) \ll \lambda \] (6.3.3)
i.e., a regular mirror implies a large \( \kappa \) (6.1.4). Thus interference breakdown implies fuzziness, and regularity implies no interference breakdown.
6.4 The effect of thermal fluctuations

Let

\[ \langle \vec{p} \rangle = 0 \quad (6.4.1) \]

then

\[ \frac{(\Delta T \ p)^2}{2M} = \langle E \rangle \sim T \quad (6.4.2) \]

Thus interference breakdown implies

\[ T \ll \frac{k^2}{M} \quad (6.4.3) \]

For an actual mirror, we have

\[ M \gg \left( \frac{10^8}{\text{cm}} \right)^2 \lambda^2 \times 10^{-24} \text{g} \times 10^{48} \text{sec}^{-1} \text{g} = \left( \frac{10^8}{\text{cm}} \right)^2 (5 \times 10^{-5} \text{cm})^2 \times 10^{24} \text{sec}^{-1} = 2.5 \times 10^{31} \text{sec}^{-1} \quad (6.4.4) \]

So for

\[ k = 3 \times 10^{15} \text{sec}^{-1} \quad (6.4.5) \]

we obtain

\[ T \ll \frac{9 \times 10^{30} \text{sec}^{-2}}{M} \ll \frac{9 \times 10^{30}}{2.5 \times 10^{31} \text{sec}^{-1}} \sim 1 \text{sec}^{-1} \quad (6.4.6) \]

i.e.,

\[ T \ll 10^{-11} \text{K} \quad (6.4.7) \]

Conclusion

The analysis conducted leads to the conclusion that there is no entanglement and no interference breakdown stemming from an actual movable mirror.

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References

[1] P.A.M. Dirac, The Principles of Quantum Mechanics (Oxford University Press, 1930).
[2] P.A.M. Dirac, The Principles of Quantum Mechanics, fourth ed. Revised (Oxford University Press, 1978).
[3] A.C. Elitzur, L. Vaidman, Quantum-machanical interaction-free measurements, Found. of Phys., 23, 987-97.
[4] Roger Penrose, Shadows Of The Mind (Vintage, 1995).
[5] Leslie E. Ballentine, Quantum Mechanics (World Scientific, 2001).

[6] L.D. Landau, E.M. Lifshitz, Quantum Mechanics (Pergamon, 1977).

[7] Josef M. Jauch, Foundations of Quantum Mechanics (Addison-Wesley Publishing Company, 1968).