Identification of mixed-symmetry states in an odd-mass nearly spherical nucleus

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The low-spin structure of $^{93}$Nb has been studied using the $(n,n'\gamma)$ reaction at neutron energies ranging from 1.5 to 3 MeV and the $(\pi n,p_{2n})$ $^{93}$Nb reaction at bombarding energies from 11.5 to 19 MeV. States at 1779.7 and 1840.6 keV, respectively, are proposed as mixed-symmetry states associated with the $\pi 2p_{1/2}^{-1} \otimes (2_{1,MS}^+)$ coupling. These assignments are derived from the observed $M1$ and $E2$ transition strengths to the $2p_{1/2}^{-1}$ symmetric one-phonon states.

Whereas $MS$ states have been studied extensively in even-even nuclei, little is known about $MS$ states in odd-mass nuclei, as only the scissors mode has been identified in a few deformed nuclei through strong dipole transitions to the ground-state. The nearly-spherical $N=52$ isotones form a bountiful region for $MS$ findings, where $2_{1,MS}^+$ states (of particular interest to this work) and $[2_{1,MS}^+ \otimes 2_{1,MS}^+]$ two-phonon $MS$ states have been identified. The characterization of such states indicates that the $2_{1,MS}^+$ state acts as a building block of nuclear excitations. In $^{42}$Zr, the proton subshell closure at $Z=40$ results in a much stronger individual particle strength than in the isotope $^{44}$Mo. However, a somewhat collective $MS$ state was identified as the $2_{1}^+$ level at 1.847 MeV. This $2_{1}^+$ state exhibits a large $B(M1; 2_{1,MS}^+\rightarrow 2_{1}^+)=0.37(4)$ $\mu_N^2$ and a weakly collective $B(E2; 2_{1,MS}^+\rightarrow 0_{1}^+) = 3.4(4)$ W.u. The nucleus $^{44}$Mo exhibits more collective features and the $2_{1,MS}^+$ state was clearly identified as the $2_{1}^+$ state at 2.067 MeV. It displays a stronger $M1$ transition, $B(M1; 2_{1,MS}^+\rightarrow 2_{1}^+)=0.56(5)$ $\mu_N^2$ and a weakly collective $E2$ transition to the ground-state, $B(E2; 2_{1,MS}^+\rightarrow 0_{1}^+) = 2.2(2)$ W.u. In a weak-coupling scenario, $MS$ states in the odd-$Z=40$ isotopes, $^{93}$Nb, might be expected at similar energies ($\sim$2 MeV) as their even-$Z$, $N=52$ isotope neighbors. If found in an odd-mass nucleus, these $MS$ states would give evidence for weak-coupling of the fermion to the collective excitations and affirm the effectiveness of IBM-2 in separating proton and neutron representations.

Excited states in $^{93}$Nb can be regarded as resulting from the coupling of a $1g_{9/2}$ proton to a $^{42}$Zr core, and a $2p_{1/2}^{-1}$ proton-hole to a $^{44}$Mo core. These couplings result in two independent and unmixed phonon structures of opposite parity. A quintet of positive-parity states built on the $J^\pi=9/2^+$ ground-state results from the $\pi 1g_{9/2}^{-1} \otimes (2_{1,MS}^+)$ particle-core coupling. This quintet has been identified in agreement with expectations of the center-of-gravity theorem, and the assignment is supported by Coulomb excitation results.

A simpler structure, a doublet of negative-parity states built on the $J^\pi=1/2^-$ first excited state at 31 keV (with a half-life of 16 years), is also observed and corresponds to the $\pi 2p_{1/2}^{-1} \otimes (2_{1,MS}^+)$ configuration. We have identified $MS$ states in the negative-parity structure of the nearly spherical odd-$Z$ nucleus $^{93}$Nb, which correspond to the $2_{1,MS}^+$ states found in neighboring even-even nuclei. Identifications are based on $M1$ and $E2$ strengths, energy systematics, and spin-parity assignments and from the comparison with shell model calculations using the low-momentum nucleon-nucleon interaction, $V_{low-k}$.

The nucleus $^{93}$Nb was studied using the $(n,n'\gamma)$ reaction at the University of Kentucky and the

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94Zr(p,2nγ)93Nb reaction at the University of Cologne. Excitation functions, lifetimes and branching ratios were measured using the 93Nb(n,n′γ) reaction at neutron energies ranging from 1.5 to 2.6 MeV. Gamma-gamma coincidences were also measured with the (n,n′γ) reaction at a neutron energy of 3 MeV. Excitation functions, together with the analysis of background subtracted coincidence spectra allowed the construction of a comprehensive level scheme up to 2.1 MeV. The coincidence methods following neutron inelastic scattering are described by McGrath et al. 16. Lifetimes were measured through the Doppler-shift attenuation method following the (n,n′γ) reaction 17. From an angular correlation experiment with the 94Zr(p,2nγ)93Nb reaction, branching ratios were measured and multipolarities and spin assignments determined. We used seven angular correlation groups defined by three angles: \( \theta_1 \) and \( \theta_2 \) are the angles of the two detector axes relative to the beam axis, and the angle \( \phi \) between the planes defined by the beam axis and emission directions of the γ-rays. When fitting angular correlation data, we fixed the Gaussian width, \( \sigma \), of the m-substate distribution from fits to previously known transitions in 93Nb, and treated the multipole mixing ratio, \( \delta \) 18, of the unknown transition as a free parameter following the formalism developed by Krane, Steffen and Wheeler 19. The current results for the strongest populated transitions are in good agreement with those identified in previous work 14,11. Table I gives the properties of selected negative-parity levels in 93Nb. The proposed \( MS \) states are the 1779.7 keV and 1840.6 keV levels (as shown in Figure 1) arising from the \( 2p_{1/2}^{-1} \otimes (2^+_{1,MS}, 94\text{Mo}) \) configuration.

FIG. 1: 93Nb partial level scheme showing the \( 2p_{1/2}^{-1} \) single-particle, symmetric one-phonon and proposed \( MS \) states of importance to this discussion.

1779.7 keV state — The previously proposed \( (5/2^-) \) level at 1779.7 keV yields a new 1092 keV branch to the first \( 3/2^- \) excited state that has been revealed from the excitation function and coincidence data. The level has been assigned as \( J^\pi=5/2^- \) by the analysis of the angular correlation data (see Table I and top panel of Fig. 2), and as shown on the top panel of Fig. 2, a mean life of \( 105\pm25 \) fs has been measured. The 969 and 1092 keV transitions depopulating this state to the \( 2p_{1/2}^{-1} \otimes 2^+ \) symmetric one-phonon states provide branching ratios of 100(5) and 8(5), respectively, and mixing ratios, \( \delta \), of

FIG. 2: (Color online) Angular correlation data for a) the 1092 keV transition gated by the 656 keV γ-ray and b) the 1153 keV transition gated by the 656 keV γ-ray. Spin and parity assignments of \( J^\pi = 3/2^- \) and \( 5/2^- \) are equally possible for the 1840.6 keV level with fits to the data of \( \delta = 0.14(4) \) and 0.26(6), respectively. The Gaussian width of the magnetic substate distribution, \( \sigma = 2.5 \), was fixed from fits to previously known transitions in 93Nb. The \( \chi^2 \) values given in our angular correlation fits are reduced, and are given by the total \( \chi^2 \) divided by the “number of correlation groups – 2”.

0.04(6) and 0.05(9), respectively. Hence, the 969 keV transition to the \( 5/2^- \) state has a large \( B(M1) \) value of \( 0.55^{+0.24}_{-0.18} \mu_N^2 \) and a small \( B(E2) \) value of 0.5(2) W.u., while the 1092 keV transition to the \( 3/2^- \) state exhibits a much weaker \( B(M1) \) strength of \( 0.03^{+0.04}_{-0.02} \mu_N^2 \) and a \( B(E2) \) value of 0.04(4) W.u.

1840.6 keV state — The 1840.6 keV level has been placed from our measurements. The angular correlation analysis of the competing branches depopulating this state (see Table I and bottom panel of Fig. 2) leads equally to either \( J^\pi = 3/2^- \) or \( 5/2^- \) assignments. A mean life of \( 103\pm25 \) fs has been measured for this state.

FIG. 3: (Color online) Doppler-shift attenuation data for γ-rays de-exciting a) the 1779.7 keV level and b) the 1840.6 keV level.
### Table I: Results obtained from the $^{93}$Nb(n,n$'$γ) and $^{94}$Zr(p,2n)$^{93}$Nb measurements.

Negative-parity levels, lifetimes, initial and final spin of the states, γ-ray energies, branching and mixing ratios, and experimental $B$(M1) and $B$(E2) transition rates are listed. Shell model $B$(M1) and $B$(E2) predictions for relevant transitions together with isoscalar (IS) and isovector (IV) components of the $E2$ operator are shown in the last four columns. An asterisk labels newly identified levels and γ-ray transitions.

| $E_L$ (keV) | $\tau$ (fs) | $J_f^\pi \rightarrow J_i^\pi$ | $E_\gamma$ (keV) | $I_\gamma$ | $\delta$ | $B$(M1) | $B$(E2) | $B$(M1) | $B$(E2) | IS | IV |
|-------------|-------------|------------------|-----------------|------------|--------|--------|--------|--------|--------|----|----|
| 1395.8(2)  | $> 792$     | $\frac{7}{2}^-$  | $\frac{7}{2}^-$ | 708.6      | 9(4)    | $E2$   | $< 18$ | -       | 8.50   | 28.61 | -13.85 |
| 1500.0(1)  | 1164(301)   | $\frac{9}{2}^-$  | $\frac{9}{2}^-$ | 585.1      | 100(4)  | $-0.10(2)$ | $< 0.31$ | -       | 5.2    | 0.0003 | 0.879 | 9.17 | -4.39 |
| 1572.1(2)  | $281^{+213}_{-94}$ | $\frac{9}{2}^-$  | $\frac{9}{2}^-$ | 885.1      | 37(5)   | $-1.60(14)$ | 0.02(1) | 36$^{+27}_{-19}$ | 0.002 | 14.46 | 25.71 | -8.85 |
| 1588.4(2)  | $> 1260$    | $\frac{7}{2}^-$  | $\frac{7}{2}^-$ | 901.2      | 100(8)  | $-0.53(6)$ | $< 0.04$ | $< 8$ | 0.0012 | 4.36  | 17.18 | -5.26 |
| 1779.7(2)  | $105^{+43}_{-28}$ | $\frac{7}{2}^-$  | $\frac{7}{2}^-$ | 1092.4     | 8(5)    | 0.05(9) | 0.03$^{+0.04}_{-0.02}$ | 0.04$^{+0.04}_{-0.03}$ | 0.037 | 0.306 | -0.74 | 2.35 |
| 1840.6(2)  | $103^{+35}_{-24}$ | $\frac{7}{2}^-$  | $\frac{7}{2}^-$ | 1153.4     | 100(4)  | 0.14(4), 0.26(6), 0.29(8), 0.28(9), 2.5(7), 8(2) | 0.462 | 0.306 | -4.36 | 4.90 |
| 1948.1(2)  | $230^{+133}_{-69}$ | $\frac{7}{2}^-$  | $\frac{7}{2}^-$ | 1137.4     | 100     | 0.05(4) | 0.17(7) | 0.2(1) | -      | 5.99 | 13.03 | 14.55 |
| 1997.6(2)  | $92^{+22}_{-17}$ | $\frac{7}{2}^-$  | $\frac{7}{2}^-$ | 1310.2     | 12(5)   | $-0.29(12)$ | 0.03(2) | 0.8$^{+0.06}_{-0.04}$ | - | 0.046 | 0.078 | -1.91 | 2.63 |

The $B$(M1) values from the 1779.7 keV state to the $\frac{5}{2}^1$ level and from the proposed 1840.6 keV state to the $\frac{3}{2}^1$ level are greater than from any other negative-parity states feeding the symmetric one-phonon states. These observations, together with the appearance of these states in the expected energy range (~ 2 MeV), support their assignment as the primary MS states.

**Interacting Boson Fermion Model** — According to the IBM-2, even-even nuclei in the vibrational U(5) limit exhibit $M1$ transition strengths from the $2^+_\text{MS}$ state to the symmetric one-phonon $2^+_1$ state given by:

$$B(M1; 2^+_\text{MS} \rightarrow 2^+_1) = \frac{3}{4\pi} (g_\pi - g_\nu)^2 \frac{N_\pi - N_\nu}{N^2} \mu_N^2$$  \hspace{1cm} (1)$$

where $N_\pi$ and $N_\nu$ are the number of proton and neutron pairs, respectively and $N=N_\pi+N_\nu$. The standard boson $g$-factors, $g_\pi$ and $g_\nu$, are $g_\pi=1$ for proton bosons and $g_\nu=0$ for neutron bosons. Considering $^{88}$Sr$_{50}$ as the inert core for the lowest MS state in $^{94}$Mo, the proton and neutron boson numbers are $N_\pi=2$ and $N_\nu=1$, giving $B(M1; 2^+_\text{MS} \rightarrow 2^+_1) = 0.32 \mu_N^2$. In the weak-coupling limit, the Interacting Boson Fermion Model (IBFM) predicts that the strength of $B(M1; 2^+_\text{MS} \rightarrow 2^+_1)$ in the IBM-2 should equal the strength of the sum of the $B$(M1) values for states arising from the coupling of the unpaired particle, $p$, with the $MS$ state in $^{94}$Mo. Hence,

$$\sum B(M1; [p \otimes 2^+_\text{MS}]J \rightarrow [p \otimes 2^+_1]3/2-) = 0.32^{+0.12}_{-0.10} \mu_N^2$$

$$\sum B(M1; [p \otimes 2^+_\text{MS}]J \rightarrow [p \otimes 2^+_1]5/2-) = 0.63^{+0.27}_{-0.21} \mu_N^2$$

Whereas the former value of $0.32^{+0.12}_{-0.10} \mu_N^2$ is consistent within the weak coupling limit, the latter value exceeds the schematic $U(5)$ estimate from above. This strong $B(M1)$ might be partly due to the spin contribution of the unpaired proton to the M1 strength which is absent in the IBM-2.

**Shell Model Calculations** — To gain further insight into the nature and structure of the proposed MS states, we have examined $^{93}$Nb in the framework of the nuclear shell model. While a more complete description will be given in a subsequent publication, our focus here will be on providing a theoretical confirmation for the MS interpretations proposed above and quantifying the spin contribution to the M1 transition. The starting point for these calculations is the low-momentum nucleon-nucleon interaction $V_{\text{low-k}}$ [14], an energy independent interaction whose only free parameter, the
momentum cutoff $\Lambda$, is fixed at 2.1 fm$^{-1}$. All calculations were carried out with the Oxbash shell model code \[22\], which has been used with $V_{low-k}$ to reproduce the mixed-symmetry structures of both $^{92}$Zr and $^{94}$Mo \[21\]. As in \[4\] we have chosen $^{88}$Sr as the inert core and used the following model space orbits for protons and neutrons: $\pi[2p_{1/2}, 1g_{9/2}, 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}]$ and $\nu[1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2}]$, where the single-particle energies were optimized to fit the experimental spectra of $^{90}$Zr and $^{90}$Sr. To make our current calculations fully predictive, we fixed the effective quadrupole charges, $e_p = 1.85e$ and $e_n = 1.3e$, as the averages of the values used for $^{92}$Zr and $^{94}$Mo and quenched the spin $g$-factors by 0.7.

The relevant SM results are shown in Table I, where we include theoretical $B(M1)$ and $B(E2)$ values with the isoscalar and isovector components of the $E2$ transition from the $1/2^-$ level. We list only those SM states that have a clear manifestation in the experimental data, identifying the 1396 keV $7/2^-$ and 1500 keV $9/2^-$ states as the splitting of the $4^+_1$ two-phonon state and the 1572 keV $3/2^-$ and 1588 keV $5/2^-$ states as the splitting of the two-phonon $2^+_1$ state.

1779.7 keV $5/2^-$ state — We identify the $5/2^-_2$ SM state (at an energy of 1734 keV) with the experimentally observed 1779.7 keV state. In Table I we see the SM predicts $B(M1)$ values to the fully symmetric one-phonon $5/2^-$ and $3/2^-$ states that are in good agreement with the experimental results, noting in particular the strong $M1$ transition to the $5/2^-$ final state. By analyzing the dominant components of the calculated SM wavefunctions \[21\], we can verify that the $^{93}$Nb states result from the orthogonal bosonic core states of $^{94}$Mo coupling to a $p_{1/2}$ hole. Under this assumption, the ratio $B(M1 : 5/2^-_{MS} \rightarrow 5/2^-_1)/B(M1 : 5/2^-_1 \rightarrow 3/2^-_1)$ can be expressed as a ratio of Racah coefficients, from which we predict the $B(M1)$ value for the $J_i \neq J_f$ transition be suppressed by a factor of 14 with respect to the $J_i = J_f$ transition. This is clearly seen in both the calculations and experiment. A final theoretical prediction consistent with the $MS$ identification of this state is the weakly-collective $E2$ transition with significant isovector character to the $1/2^-$ first excited state.

We now examine the spin and orbital parts of the $B(M1)$ strength to see if the shell model predicts an enhanced spin contribution from the unpaired proton. Using vanishing orbital $g$-factors ($g^i_p = g^i_n = 0$), we find the spin $B(M1)$ value to be 0.155 $\mu_N^3$, and with vanishing spin $g$-factors ($g^i_p = g^i_n = 0$), we have the orbital $B(M1)$ value of 0.153 $\mu_N^3$. Since a similar finding was reported for $MS$ states in even-mass nuclei \[4\], it appears that there is little significant spin enhancement to the $B(M1)$ values.

1810.6 keV state — The shell model predicts a 1811 keV $3/2^-_{MS}$ spin-flip partner to the $5/2^-_{MS}$ state. An obvious candidate would be the 1840.6 keV experimental state. In Table I we see that, however, there are two possible $J$ assignments for this level. Theoretical considerations support the labeling of this state as $3/2^-_{MS}$. Primarily, its close proximity in energy to the 1779.7 keV $5/2^-$ state suggests its candidacy as the spin-flip partner. It also exhibits a strong $M1$ transition to the symmetric $3/2^-$ one-phonon state and a suppressed (now by a theoretically-predicted factor of 9) $M1$ to the symmetric $5/2^-$ one-phonon state, further confirming the $3/2^-_{MS}$ assignment. With a $3/2^-$ label for this state, the shell model predicts a weakly-collective $E2$ transition with significant isovector character to the $1/2^-$ first excited state, just as with the $5/2^-_{MS}$ state identified above. Similarly, we see no sign of an enhanced spin contribution to the $M1$ transition, where $B(M1)_s = 0.110$ $\mu_N^3$ and $B(M1)_l = 0.121$ $\mu_N^3$, and conclude that this state can reasonably be identified as $3/2^-_{MS}$. In conclusion, we propose, for the first time, $MS$ states in a nearly spherical odd-mass nucleus from both experimental and theoretical evidence.

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