Superior Exploration-Exploitation Balance with Quantum-Inspired Hadamard Walks

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ABSTRACT
This paper extends the analogies employed in the development of quantum-inspired evolutionary algorithms by proposing quantum-inspired Hadamard walks, called QHW. A novel quantum-inspired evolutionary algorithm, called HQEA, for solving combinatorial optimization problems, is also proposed. The novelty of HQEA lies in its incorporation of QHW Remote Search and QHW Local Search - the quantum equivalents of classical mutation and local search, that this paper defines. The intuitive reasoning behind this approach, and the exploration-exploitation balance thus occurring is explained. From the results of the experiments carried out on the 0,1-knapsack problem, HQEA performs significantly better than a conventional genetic algorithm, CGA, and two quantum-inspired evolutionary algorithms - QEA and NQEA, in terms of convergence speed and accuracy.

Categories and Subject Descriptors
I.2.8 [Artificial Intelligence]: Problem Solving, Control Methods, and Search

General Terms
Algorithms, Experimentation, Theory

Keywords
Evolutionary Algorithms, Quantum Computing, Random Walks, Combinatorial Optimization, Knapsack Problem, Late Breaking Abstract

1. INTRODUCTION
In recent years, several quantum-inspired evolutionary algorithms have been proposed, drawing upon quantum computing concepts such as quantum bits and the superposition of states. Q-bit representations for individuals and quantum gate-based evolution operators have been proposed. It was experimentally observed that the performance of quantum-inspired class of evolutionary algorithms was superior to that of conventional genetic algorithms.\[1,3\]

Quantum walks have been known for showing properties quite unlike classical random walks.\[2\] A frequently used balanced unitary coin, which lends additional degrees of freedom to the quantum walk over its classical counterpart, is the Hadamard coin \(H\),

\[H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\]
Algorithm 1 QHW

Initialization: Hadamard Walk centered at $\Delta \theta = 0$ with $2n + 1$ states from $[-n, n]$
Enter $\pi$ and $\pi$ in the quantum state as $\psi_0 = \begin{pmatrix} 1 \cr 0 \end{pmatrix}$, $\psi_1 = \begin{pmatrix} 1 \cr 0 \end{pmatrix}$
Do the Hadamard walk by flipping the Hadamard coin $n$ times
Probability of $\Delta \theta_k$ (state $k$) is given by $|\psi_{0,k}|^2 + |\psi_{1,k}|^2$
Determine $\Delta \theta$ given the weighted probability distribution of $\Delta \theta_k$
return $\Delta \theta$

Superposition concedes interesting properties to the quantum walk. The quantum walk propagates relatively faster along the line: it’s variance grows quadratically with the number of steps $n$, $\sigma^2 \propto n^2$, compared to $\sigma^2 \propto n$ for the classical random walk. QHW takes advantage of this rapid convergence as shown in Figure 1-2, and Algorithm 1.

3. QHW LOCAL AND REMOTE SEARCH

The novelty of QHW lies in that it can be used for both remote searches as well as local searches, by suitably altering the parameter $n$ relative to $n_{\text{max}}$. By operating with a high $n$ value on the worst solutions, exploration is achieved. By using a smaller $n$ value on the good solutions, exploitation is achieved.

The algorithm for QHW Local and Remote Search is summarized below as Algorithm 2.

Algorithm 2 QHW Local and Remote Search

Select best (for local) and worst (for remote) $n\%$ of population by fitness values
for each of the selected best/worst $n\%$ individuals do
while $i < m$ do
$\theta' = \theta + \Delta \theta$ where $\Delta \theta = \text{QHW}[n]$
Generate new $\alpha'_i, \beta'_i$ by using $\theta'$
if (Fitness[NewIndividual] > Fitness[OriginalIndividual]) then
Replace OriginalIndividual with NewIndividual
end if
end while
end for
return Updated $n\%$ individuals

4. HQEA

Algorithm 3 describes a novel quantum-inspired evolutionary algorithm that utilizes QHW Remote Search for exploration and QHW Local Search for exploitation.

5. EXPERIMENTS

The performance of HQEA to that of a conventional genetic algorithm, CGA, and two quantum-inspired evolutionary algorithms, QEA and NQEA, on the 0-1 knapsack problem is compared.

Table 1 reports the average fitness of the best solutions found by the respective algorithms over 10 runs on two instances of the 0-1 knapsack problem with 200 and 500 items, rounded to the nearest integer. In the experiments of HQEA, the parameters are set to $n = 10$ for QHW Local Search and $n = 100$ for QHW Remote Search, with $n_{\text{max}} = 100$ for both. All other parameters for HQEA and those of QEA and NQEA are set according to those proposed in [1] and [3], with a population of 10 individuals and $\Delta \theta = 0.01\pi$. In CGA, a uniform crossover, and roulette wheel selection are used.

6. CONCLUSIONS

In this paper, the convergence and accuracy in HQEA is noted to be significantly quicker compared to CGA, QEA and NQEA. Note that HQEA does not build upon NQEA - rather, it is a new and innovative look at quantum-inspired evolutionary algorithms.

7. REFERENCES

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