Investigation of heavy ions diffusion under the influence of current-driven mechanism and compositional waves in plasma

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Abstract

We consider diffusion caused by a combined influence of the Hall effect and electric currents, and argue that such diffusion forms chemical inhomogeneities in plasma. The considered mechanism can be responsible for the formation of element spots in laboratory and astrophysical plasmas. Such current-driven diffusion can be accompanied by the propagation of a particular type of waves which have not been considered earlier. In these waves, the impurity number density oscillates alone and their frequency is determined by the electric currents and sort of impurity ions. These compositional waves exist if the magnetic pressure in plasma is much greater than the gas pressure. Such waves lead to local variations of chemical composition in plasma and, hence, can manifest themself by variations of the emission in spectral lines.
INTRODUCTION

Often laboratory and astrophysical plasmas are multicomponent, and diffusion of elements plays an important role in many phenomena. For instance, diffusion can be responsible for the formation of chemical inhomogeneities which influence emission, heat transport, conductivity, etc (1–3). In thermonuclear fusion experiments, the source of impurities is usually the chamber walls, and diffusion determines the penetration depth of these impurities and their distribution in plasma (4–6). Even a small admixture of heavy ions increases drastically radiative losses of plasma and changes its thermal properties. In astrophysical conditions, chemical inhomogeneities have been detected in many stars which have relatively quiescent surface layers. It is widely accepted opinion that these inhomogeneities are determined by diffusion processes (7) but, however, the mechanisms resulting in formation of chemical spots is rather uncertain. Usually, diffusion in astrophysical bodies is influenced by a number of factors (gravity, radiative force, magnetic field, temperature gradient, etc. (8)) and, therefore, chemically peculiar stars are excellent laboratories to study diffusion processes in plasma.

Diffusion in plasma can differ qualitatively from that in neutral gases because of the presence of electrons and electric currents. This particularly concerns hydrogen plasma where the rate of momentum exchange between electrons and protons is comparable to the rate of the momentum redistribution among protons (9). In such plasma, the influence of electrons on diffusion of heavy ions is especially pronounced. Chemical inhomogeneities can appear in plasma because of a number of reasons, for instance, because of a non-uniform temperature. Also, it is often thought that chemical spots occur due to the presence of the magnetic field. The magnetic field can magnetize electrons in plasma that, generally, leads to anisotropic transport and can produce an inhomogeneous distribution of heavy ions. Anisotropy of diffusion is characterized by the Hall parameter, $x_e = \omega_{Be} \tau_e$, where $\omega_{Be} = eB/m_ec$ is the gyrofrequency of electrons and $\tau_e$ is their relaxation time; $B$ is the magnetic field. In hydrogen plasma, $\tau_e = 3\sqrt{m_e(k_bT)^{3/2}}/4\sqrt{2\pi}e^4n\Lambda$ (8) where $n$ and $T$ are the number density of electrons and their temperature, respectively, $\Lambda$ is the Coulomb logarithm. At $x_e \geq 1$, the rates of diffusion along and across the magnetic field become different and, in general, diffusion can produce the inhomogeneous distribution of elements.

In this paper, we consider one more diffusion process that leads to formation of chemical
inhomogeneities. This process is caused by a combined influence of the Hall effect and electric currents. Only fully ionized plasma is considered consisting electrons $e$, protons $p$, and small admixture of heavy ions $i$. Generally, similar processes can occur in any system of charged particles but they will be considered elsewhere. Using a simple model, we show that interaction of the electric current with impurities leads to their diffusion in the direction perpendicular to both the electric current and magnetic field. This type of diffusion can alter the distribution of chemical elements and contribute to the formation of chemical spots even if the magnetic field is relatively weak and does not magnetize electrons ($x_e \ll 1$). We also argue that the current-driven diffusion in combination with the Hall effect can be the reason of the particular type of waves in which the impurity number density oscillates alone.

**BASIC EQUATIONS**

Consider a cylindrical plasma configuration with the magnetic field parallel to the axis $z$, $\vec{B} = B(s)\vec{e}_z$; $(s, \varphi, z)$ and $(\vec{e}_s, \vec{e}_\varphi, \vec{e}_z)$ are cylindrical coordinates and the corresponding unit vectors. Then, the electric current is

$$j_\varphi = -(c/4\pi)(dB/ds). \quad (1)$$

We suppose that $j_\varphi \to 0$ at large $s$ and, hence, $B \to B_0=\text{const}$ at $s \to \infty$. Note that $B(s)$ cannot be an arbitrary function of $s$ because, generally, the magnetic configurations are unstable for some dependences $B(s)$ ([12–14]). The characteristic timescale of this instability is usually much shorter than the diffusion timescale and, therefore, a formation of chemical structures in unstable magnetic configurations is impossible. Often, the magnetic field has a more complex topology than our simple model. However, this model describes correctly the main qualitative features of current-driven diffusion. In some cases, this model can even mimic the magnetic field in certain regions. For example, the field near the magnetic pole has a topology very close to our model [10].

We assume that plasma is fully ionized and consists of electrons $e$, protons $p$, and small admixture of heavy ions $i$. The number density of species $i$ is small and it does not influence the dynamics of plasma. Therefore, these ions can be treated as test particles interacting only with a background hydrogen plasma.

The partial momentum equations in fully ionized plasma have been considered by a
number of authors ([9, 11]). The paper [11] deals mainly with the hydrogen-helium plasma. However, the derived equations can be applied for hydrogen plasma with a small admixture of any other ions if their number density is small. If the mean hydrodynamic velocity is zero and only small diffusive velocities are non-vanishing, the partial momentum equation for the species $i$ reads

$$-
abla p_i + Z_i e n_i \left( \vec{E} + \frac{\vec{V}_i}{c} \times \vec{B} \right) + \vec{R}_{ie} + \vec{R}_{ip} + \vec{F}_i = 0$$

(1), where $Z_i$ is the charge number of the species $i$, $p_i$ and $n_i$ are the partial pressure and number density, respectively, $\vec{V}_i$ is the velocity, and $\vec{E}$ is the electric field. The force $\vec{F}_i$ is the external force on species $i$; in astrophysical conditions, $\vec{F}_i$ is usually the sum of the gravitational and radiation forces. For the sake of simplicity, we neglect external forces in our simplified model. The forces $\vec{R}_{ie}$ and $\vec{R}_{ip}$ are caused by the interaction of ions $i$ with electrons and protons, respectively. The forces $\vec{R}_{ie}$ and $\vec{R}_{ip}$ are internal and their sum over all plasma components is zero in accordance with Newton’s third law. Since diffusive velocities are small, we neglect the term proportional $(\vec{V}_i \cdot \nabla) \vec{V}_1$ in Eq. (6).

If $n_i$ is small compared to the number density of protons, $\vec{R}_{ie}$ is given by

$$\vec{R}_{ie} = -(Z_i^2 n_i/n) \vec{R}_e$$

(3), where $\vec{R}_e$ is the force acting on the electron gas [11]. Since $n_i \ll n$, $\vec{R}_e$ is determined mainly by scattering of electrons on protons but scattering on ions $i$ gives a small contribution to $\vec{R}_e$. Therefore, we can use for $\vec{R}_e$ the expression for one component hydrogen plasma calculated by Braginskii [9]. In our model of a cylindrical isothermal plasma, this expression reads

$$\vec{R}_e = -\alpha_\perp \vec{u} + \alpha_\land \vec{b} \times \vec{u},$$

(4) where $\vec{u} = -\vec{j}/en$ is the current velocity of electrons; $\vec{b} = \vec{B}/B$; $\alpha_\perp$ and $\alpha_\land$ are coefficients calculated by [9]. The force (4) describes the standard friction caused by the relative motion of electrons and protons. Taking into account Eq.(1), we have

$$\vec{u} = (c/4\pi en)(dB/ds)\vec{e}_\varphi.$$
coefficients $\alpha_\perp$ and $\alpha_\wedge$ from [9] with the accuracy in linear terms in $x_e$, we obtain

$$R_{ie\varphi} = Z_i^2 n_i \left( 0.51 \frac{m_e}{\tau_e} u \right), \quad R_{i-es} = Z_i^2 n_i \left( 0.21 \frac{m_e}{\tau_e} u \right). \quad (6)$$

If $T = \text{const}$, the friction force $\vec{R}_{ip}$ is proportional to the relative velocity of ions $i$ and protons, $\vec{R}_{ip} \propto (\vec{V}_p - \vec{V}_i)$. This force can be easily calculated in the case $A_i = m_i/m_p \gg 1$.

Taking into account that the velocity of the background plasma is zero in our model, $\vec{R}_{ip}$ can be represented as [11]

$$\vec{R}_{ip} = (0.42 m_i n_i Z_i^2 / \tau_i) (-\vec{V}_i), \quad (7)$$

where $\tau_i = 3 \sqrt{m_i (k_B T)^{3/2} / 4 \sqrt{2 \pi e^4 n \Lambda}}$; $\tau_i / Z_i^2$ is the timescale of ion-proton scattering; we assume that $\Lambda$ is the same for all types of scattering.

The momentum equation for the species $i$ (Eq.(2)) depends on cylindrical components of the electric field, $E_s$ and $E_\varphi$. These components can be determined from the momentum equations for electrons and protons

$$- \nabla (nk_B T) - en \left( \vec{E} + \frac{\vec{u}}{c} \times \vec{B} \right) + \vec{R}_e + \vec{F}_e = 0, \quad (8)$$

$$- \nabla (nk_B T) + en \vec{E} - \vec{R}_e + \vec{F}_p = 0 \quad (9)$$

([9]). Taking into account the friction force $\vec{R}_e$ (Eq. (3)), we obtain with accuracy in linear terms in $x_e$

$$E_s = \frac{u B}{2c} - \frac{1}{e} \left( 0.21 \frac{m_e u}{\tau_e} x \right), \quad E_\varphi = \frac{1}{e} \left( 0.51 \frac{m_e u}{\tau_e} \right). \quad (10)$$

Substituting Eqs.(6), (7), and (10) into the $s$- and $\varphi$-components of Eq.(2), we arrive to the expression for a diffusion velocity, $\vec{V}_i$:

$$\vec{V}_i = V_{is} \vec{e}_s + V_{i\varphi} \vec{e}_\varphi, \quad V_{is} = V_{ni} + V_B, \quad (11)$$

where

$$V_{ni} = -D \frac{d \ln n_i}{ds}, \quad V_B = D_B \frac{d \ln B}{ds}, \quad V_{i\varphi} = D_{B\varphi} \frac{dB}{ds}; \quad (12)$$

$V_{ni}$ is the velocities of ordinary diffusion and $V_B$ is the diffusion velocity caused by the electric current. The corresponding diffusion coefficients are

$$D = \frac{2.4 c_i^2 \tau_i}{Z_i^2}, \quad D_B = \frac{2.4 c_i^2 \tau_i}{Z_i A_i} (0.21 Z_i - 0.71), \quad (13)$$

$$D_{B\varphi} = 1.22 \sqrt{\frac{m_e c (Z_i - 1)}{m_i 4 \pi e n Z_i}}. \quad (14)$$
where \(c_i^2 = k_B T/m_i\) and \(c_A^2 = B^2/(4\pi n m_p)\). Eqs. (11)-(12) describe the drift of ions \(i\) under the combined influence of \(\nabla n_i\) and \(\vec{j}\). The azimuthal drift velocity, \(V_{i\phi}\), is relevant to the current motion of electrons because heavy ions are carried away by electrons in the direction of their motion. The radial velocity is caused mainly by the Hall effect. In the presence of electric currents, this effect produces a force that is perpendicular to both the electric current (azimuthal) and magnetic field (vertical).

**DISTRIBUTION OF IONS IN THE PRESENCE OF ELECTRIC CURRENTS**

In our model, the condition of hydrostatic equilibrium is given by

\[-\nabla p + \vec{j} \times \vec{B}/c = 0\]  
(15)

where \(p\) and \(\rho\) are the pressure and density, respectively. Since the background plasma is hydrogen, \(p \approx 2n k_B T\) where \(k_B\) is the Boltzmann constant. Integrating the \(s\)-component of Eq. (15) and assuming that the temperature is constant, we obtain

\[n = n_0 \left(1 + \beta_0^{-1} - \beta^{-1}\right),\]  
(16)

where \(\beta = 8\pi p_0/B^2\); \((p_0, n_0, T_0, \beta_0)\) are the values of \((p, n, T, \beta)\) at \(s \to \infty\).

Consider the equilibrium distribution of elements. In equilibrium, we have \(V_{is} = 0\) and Eq.(11) yields

\[
\frac{d\ln n_i}{ds} = \frac{D_B}{D} \frac{d\ln B}{ds}.
\]  
(17)

The term on the r.h.s. describes the effect of electric currents on the distribution of impurities. Note that this type of diffusion is driven namely by the electric current rather than an inhomogeneity of the magnetic field. Occasionally, the conditions \(dB/ds \neq 0\) and \(j \neq 0\) are equivalent in our simplified model. One has from Eq. (15)

\[
\frac{d}{ds}(nk_B T) = -\frac{B}{8\pi} \frac{dB}{ds}.
\]  
(18)

Substituting Eq. (18) into Eq.(17) and integrating, we obtain

\[
\frac{n_i}{n_{i0}} = \left(\frac{n}{n_0}\right)^\mu,
\]  
(19)

where

\[
\mu = -2Z_i(0.21Z_i - 0.71)
\]  
(20)
and \( n_{i0} \) is the value of \( n_i \) at \( s \to \infty \). Denoting the local abundance of the element \( i \) as 
\[ \gamma_i = \frac{n_i}{n} \]
and taking into account Eq. (16), we have 
\[ \frac{\gamma_i}{\gamma_{i0}} = \left( \frac{n}{n_0} \right)^{\mu - 1} = \left( 1 + \frac{1}{\beta_0} - \frac{1}{\beta} \right)^{\mu - 1}, \tag{21} \]
where \( \gamma_{i0} = \frac{n_{i0}}{n_0} \). Local abundances turn out to be flexible to the field strength and, particularly, this concerns the ions with large charge numbers. If other mechanisms of diffusion are negligibl

e and the distribution of elements is basically current-driven, then the exponent \((\mu - 1)\) can reach large negative values for elements with large \( Z_i \) and, hence, produce strong abundance anomalies. For instance, \((\mu - 1)\) is equal 1.16, -0.52, and -2.04 for \( Z_i =2, 3, \) and 4, respectively. Note that \((\mu - 1)\) changes its sign as \( Z_i \) increases: \((\mu - 1) > 0\) if \( Z_i =2 \) but \((\mu - 1) < 0\) for \( Z_i \geq 3 \). Therefore, elements with \( Z_i \geq 3 \) are in deficit \((\gamma_i < \gamma_{i0})\) in the region with a weak magnetic field \((B < B_0)\) but, on the contrary, these elements should be overabundant in the region where the magnetic field is stronger than \( B_0 \).

Eq. (21) describes the distribution of impurities in diffusive equilibrium. The characteristic timescale to reach this equilibrium, \( t_B \), can be estimated as
\[ t_B \sim L/V_B \sim L^2/D_B. \tag{22} \]
where \( L \) is the magnetic lengthscale, \( L = |d \ln B/ds|^{-1} \). The characteristic timescale of baro-diffusion is given by the well-known expression
\[ t_n \sim L/V_n \sim L^2/D. \tag{23} \]
Hence, the current-driven diffusion operates on a shorter timescale if \( D_B > D \) or
\[ \frac{c_A^2}{c_s^2} > Z_i^{-1}(0.21Z_i - 0.71)^{-1}, \tag{24} \]
where \( c_s \) is the sound speed, \( c_s^2 = k_B T/m_p \). Therefore, the current-driven diffusion can be more efficient if the magnetic pressure is greater than the gas pressure.

**COMPOSITIONAL WAVES**

The continuity equation for ions \( i \) reads in our model \[ [11] \]
\[ \frac{\partial n_i}{\partial t} + \frac{1}{s} \frac{\partial}{\partial s} \left( s n_i V_{is} \right) + \frac{1}{s} \frac{\partial}{\partial \varphi} \left( n_i V_{i\varphi} \right) = 0. \tag{25} \]
Consider the behaviour of small disturbances in the impurity number density, \( n_i \), by making use of a linear analysis of Eq. (25). Assume that plasma is in equilibrium in the unperturbed state. Since the number density of impurity is small, its influence on parameters of the basic state is negligible. We consider disturbances that do no depend on \( z \). Denoting the disturbances of \( n_i \) by \( \delta n_i \) and linearizing Eq.(25), we obtain
\[
\frac{\partial \delta n_i}{\partial t} - \frac{1}{s} \frac{\partial}{\partial s} \left( sD \frac{\partial \delta n_i}{\partial s} - s \delta n_i \frac{D_B dB}{ds} \right) + \frac{1}{s} \frac{\partial}{\partial \varphi} \left( \delta n_i D_{B\varphi} \frac{dB}{ds} \right) = 0.
\]  
(26)

We consider disturbances with the wavelength shorter than the lengthscale of \( B \). In this case, we can use the so called local approximation and assume that disturbances are \( \propto \exp(-iks - M\varphi) \) where \( k \) is the wavevector, \( ks \gg 1 \), and \( M \) is the azimuthal wavenumber. Since the basic state does not depend on \( t \), \( \delta n_i \) can be represented as \( \delta n_i \propto \exp(i\omega t - ifs - iM\varphi) \) where \( \omega \) should be calculated from the dispersion equation. We consider two particular cases of the compositional waves, \( M = 0 \) and \( M \gg ks \).

*Cylindrical waves with \( M = 0 \).* Substituting \( \delta n_i \) into Eq. (26), we obtain the dispersion equation for \( M = 0 
\]
\[
i\omega = -\omega_R + i\omega_B, \ \omega_R = Dk^2, \ \omega_B = kD_B(d \ln B/ds).
\]  
(27)

This dispersion equation describes cylindrical waves in which only the number density of impurity oscillates. The quantity \( \omega_R \) describes decay of waves with the characteristic timescale \( \sim (Dk^2)^{-1} \) typical for a standard diffusion. The frequency \( \omega_B \) describes oscillations of impurities caused by the combined action of electric current and the Hall effect. The frequency is non-vanishing only in the presence of electric currents since \( dB/ds = -(4\pi/cB)j_\varphi \). Therefore, such waves cannot exist in multicomponent current-free plasma even if the magnetic field is sufficiently strong to magnetize plasma. Note that the frequency can be of any sign but \( \omega_R \) is always positive. The compositional waves are aperiodic if \( \omega_R > |\omega_B| \) and oscillatory if \( |\omega_B| > \omega_R \). This condition is equivalent to
\[
c^2_A/c^2_s > Z_i^{-1}|0.21Z_i - 0.71|^{-1}kL,
\]  
(28)

where \( c_s \) is the sound speed, \( c^2_s = k_BT/m_p \). The compositional waves become oscillatory if the field is strong and the magnetic pressure is substantially greater than the gas pressure.
The frequency is higher in the region where the magnetic field has a strong gradient. The order of magnitude estimate of $\omega_I$ is

$$\omega_I \sim k c_A (1/Z_i A_i) (c_A/c_i) (l_i/L),$$

(29)

where $l_i = c_i \tau_i$ is the mean free-path of ions $i$. Note that different impurities oscillate with different frequencies.

**Non-axisymmetric waves with $M \gg ks$.** In this case, the dispersion equation reads

$$i\omega = -\omega_R + i\omega_{B\varphi}, \quad \omega_{B\varphi} = (M/s) BD_{B\varphi} (d \ln B/ds).$$

(30)

Like cylindrical waves, the non-axisymmetric compositional waves exist only in the presence of electric currents. Non-axisymmetric waves rotate around the cylindric axis with the frequency $\omega_{B\varphi}$ and decay slowly on the diffusion timescale $\sim \omega_R^{-1}$. The frequency of such waves is typically higher than that of cylindrical waves. One can estimate the ratio of these frequencies as

$$\left( \frac{\omega_{B\varphi}}{\omega_B} \right) \sim \left( BD_{B\varphi}/D_B \right) \sim (1/A_i x_e)(M/ks).$$

(31)

Since we consider only weak magnetic fields ($x_e \ll 1$), the period of non-axisymmetric waves is shorter for waves with $M > A_i x_e (ks)$. The ratio of the diffusion timescale and period of non-axisymmetric waves is

$$\left( \frac{\omega_{B\varphi}}{\omega_R} \right) \sim (1/x_e) (c_A^2/c_*^2) (Z_i/A_i) (1/kL)$$

(32)

and it can be large. Hence, these waves can be oscillatory.

**DISCUSSION**

We have considered diffusion of heavy ions under the influence of the current-driven mechanism. This mechanism is well known in plasma physics (see [1] for review) and can be responsible for many phenomena in laboratory devices. Generally, the velocity of such diffusion can be comparable to or even greater than that caused by other diffusion mechanisms. It turns out that the electric currents lead to the formation of chemical inhomogeneities (spots) in multicomponent plasma. The chemical composition in such spots can differ substantially from the average one (see Eq. (21)). The efficiency of the considered diffusion mechanism depends on a strength of the magnetic field. This mechanism can operate even
if the magnetic field is relatively weak whereas other diffusion mechanisms require a substantially stronger magnetic field. For example, some impurities in astrophysical conditions can drift under the influence of a radiation force \[19, 20\]. This occurs in bright stars with a high surface temperature. However, such diffusion is influenced by the magnetic field only if it is sufficiently strong and can magnetize ions.

The current-driven diffusion is relevant to the Hall effect and, therefore, it leads to a drift of ions in the direction perpendicular to both the magnetic field and electric current. As a result, a distribution of chemical elements in plasma depends essentially on the geometry of the magnetic field and electric current. In our simple model of the magnetic field, chemical inhomogeneities have a cylindrical form but, generally, their distribution can be much more complicated. A cylindrical geometry is suitable for various experimental designs and often used for numerical modelling (see \[1\] for a review). Chemical inhomogeneities can manifest themself by emission in spectral lines and a non-uniform distribution of the temperature. Note that the considered mechanism of diffusion can be important not only in plasma but in some conductive fluids if the magnetic field is sufficiently strong.

Our study reveals that a particular type of waves may exist in multicomponent plasma in the presence of electric currents. These waves are slowly decaying and characterized by oscillations of the impurity number density alone. They exist only if the magnetic pressure is greater than the gas pressure. Such condition is fulfilled in many laboratory experiments. The frequency of compositional waves turns out to be different for different impurities. These waves should manifest itself by oscillations in spectrum. Compositional waves can occur in both astrophysical and laboratory plasmas but their frequency is essentially higher in laboratory conditions. For example, if \(B \sim 10^5 \text{ G}, n \sim 10^{15} \text{ cm}^{-3}, T \sim 10^6\text{K}, \) and \(L \sim \lambda \sim 10^2 \text{ cm},\) then the period of compositional waves is \(\sim 10^{-8} \text{ s}.\) Note that this is only the order of magnitude estimate but frequencies of various impurities may differ essentially since the period of compositional waves depend on the sort of heavy ions.

Usually, diffusion processes play an important role in plasma if hydrodynamic motions are very slow. In some cases, however, chemical spots can be formed even in flows with a relatively large velocity but with some particular topology (for example, a rotating flow). This can occur usually in laminar flows. Unfortunately, such flows often are unstable in magnetized plasma. This is particularily concerned by flows with a large Hall parameter
since hydrodynamic motions in such plasma typically are unstable even in the presence of a very small shear ([15–18]). As a result, a formation of the chemical spots is unlikely if there are hydrodynamic motions even with a relatively weak shear.

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