A thermodynamic theory for thermal-gradient-driven domain wall motion

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Spin waves (or magnons) interact with magnetic domain walls (DWs) in a complicated way that a DW can propagate either along or against magnon flow. However, thermally activated magnons always drive a DW to the hotter region of a nanowire of magnetic insulators under a temperature gradient. We theoretically illustrate why it is surely so by showing that DW entropy is always larger than that of a domain as long as material parameters do not depend on spin textures. Equivalently, the DW free energy is smaller than that of a domain. The larger DW entropy is related to the enhancement of magnon density of states at low energy originated from the phase shift of a spin wave passing through a DW.

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Manipulation of a magnetic domain wall (DW) in a nanostructure has attracted much attention due to its application prospects in logical operations [1] and data storage [2]. Moving DWs in a controlled manner is an important issue in those applications. Magnetic fields via energy dissipation [3,4] and electric current via angular momentum transfer [5–8] are well-known control parameters for DW motion. To overcome the Joule heating problem [5] in current driven magnetization reversal, heat itself has recently been proposed [9] as an efficient control parameter for spin manipulation. A temperature gradient can generate spin current [10–12] due to electron and/or magnon flow. This thermoelectric phenomenon of spin current generation is called spin Seebeck effect that has been experimentally observed through the inverse spin Hall effect [10]. The spin Seebeck effect has also been suggested [13,14] as a control parameter for DW manipulation. As spin 1 carriers, magnons can mediate a spin transfer torque (STT) [15] on a magnetic texture like a DW in a similar way as the electrons do. It was predicted [13,14] that a thermal-magnon-driven DW can propagate along a wire at a high speed, and this prediction was confirmed in a recent experiment [16].

There is little doubt that magnonic STT can drive a DW to move. In terms of DW propagation direction, the pure magnonic STT predicts [15] a DW moving against magnon propagation direction. However, subsequent studies show [17–19] that a DW may also propagate along magnon flow direction. This is very similar to electric-current-driven DW motion: A DW propagates along or against electron flow direction [20,22], depending on detail spin-orbit interactions and DW types [20,22]. It is not clear whether magnon-driven “wrong” DW propagation direction share a similar physics origin as its electron counterpart. In principle, angular momentum does not dictate DW motion since its governing dynamics, Landau-Lifshitz-Gilbert (LLG) equation, does not conserve the total angular momentum when the spin-lattice and spin-orbital interactions are involved. Nevertheless, thermally generated magnons seems drive a DW to a well-defined direction. All numerical simulations and experiments [13,14,16] indicate that a DW propagates to the hotter part of a wire under a temperature gradient. Although this result is consistent with the STT prediction, magnonic STT cannot be the sole physics behind. It is thus interesting to ask whether there is a general thermodynamic principle for thermal-gradient-driven DW motion. Previous theories [13,14,23] are based on magnon kinetics, multiscale micromagnetic framework as well as spin model simulations. In this paper, the underneath thermodynamic principle of thermal-gradient-driven DW motion is revealed. Due to the non-uniform phase shift of spin waves passing through a DW, the total number of magnon states below any energy in a DW is not smaller than that in a domain, resulting in a larger DW entropy and a lower DW free energy at any temperature. Thus, a DW must propagate to the hotter part of a wire under a thermal gradient in order to lower the wire free energy by taking the advantage of the larger DW entropy. Furthermore, our results explain also decrease of domain size by heating [24,26].

Magnetic domains and magnetic domain walls are stable and metastable states of a magnet. At thermal equilibrium, both domains and DWs fluctuate around their local energy minimums at a finite temperature $T$, creating magnons and microscopic states which contribute to the entropies and free energies of the systems. To calculate the entropy and free energy of a domain and a DW in a magnetic nanowire, we consider a head-to-head DW of a bi-axial wire of length $2L$ with $N$ spins along the $z$ direction, as shown in Fig. 1. The wire has an easy axis in $z$ direction and a hard axis along $x$ direction. The static DW structure is assumed to depends on $z$ only, not on $x$ and $y$. The DW is placed at the wire center and the temperature $T$ is far below the Curie temperature $T_c$. The magnetization is then governed by the dimensionless LLG equation [27],

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \mathbf{h}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t},$$

(1)
 FIG. 1: (color online) Schematic diagram of a nanowire with a head-to-head DW at its center \((z = 0)\). Pink arrows illustrate magnetization \(m\) and \(e_x, e_y, e_\phi\) are unit vectors of a spherical coordinates defined in terms of the \(z\)-axis and \(m\). The length of the wire is \(2L\) with \(N \gg 1\) spins, and the lattice constant is \(a = \frac{2\pi}{K}\). The cross section area is \(s\) whose length scale is much larger than \(\alpha\). \(\Delta\) is the DW width. The nanowire is under a temperature gradient \(\nabla T\). Blue color indicates the cold region and the red color for the hot one.

where \(m\) is the unit direction of magnetization with a saturation value \(M_s\). \(t\) is in the units of \((\gamma M_s)^{-1}\) where \(\gamma\) is gyromagnetic ratio. \(\alpha\) is the dimensionless Gilbert damping constant which is negligibly small \((10^{-4})\) for a magnetic insulator like YIG \([28]\). \(h_{\text{eff}} = A\nabla^2 m + K_x m_x \hat{x} - K_z m_z \hat{z}\) is the effective field in units of \(M_s\) where \(A, K_x, K_z\) are exchange constant, the anisotropy constants of the easy- and hard-axis, respectively. The energy density is in units of in unit of \(\mu_0 M_s^2\) so that \(K_s\) is dimensionless and \(\Lambda\) has dimension of length square. All parameters are assumed to be independent of spin texture and \(T\). The spin wave equation is obtained by linearize Eq. \((1)\) for the small fluctuation of the magnetization. \(\psi\) is 

\[
\frac{d}{dq} = \frac{dn}{dq_z} = \frac{L}{\pi} - \frac{\Delta}{\pi[1 + (\Delta q_z^2)^2]}.
\]

\(|q|^2 \psi(x,y,z) = (-\Delta^2 \nabla^2 + 2\text{sech}^2) \psi(x,y,z), \tag{2}\]

with \(\sqrt{q_z^2} = 1 + (\omega c)/K_z\). The equation has a bound state of \(\psi = \rho \text{sech}(z/\Delta)\) for \(\omega = 0\) and propagating (spin wave) states

\[
\psi_q = \sqrt{1 + (\Delta q_z^2)} (-i \Delta q_z + \text{tanh}\frac{z}{\Delta}) e^{i q_x x}, \tag{3}\]

where \(\rho\) is the amplitude of spin wave of wave vector \(q\).

The spin wave spectrum in a DW as well as in a domain is

\[
\omega(q) = \sqrt{(A|q|^2 + K_z)(A|q|^2 + K_z + K_x)}. \tag{4}\]

The bound state is the Goldstone mode of the translational symmetry \([30]\) which describes the fact that a DW center can locate at any position along the wire. It is not relevant here since we consider a DW fixed in the wire. The propagating spin wave are assumed to be plane waves in the transverse directions which gives a trivial factor of \(s/(2\pi)^2\) to the magnon density of states (DOS), where \(s\) is the cross section area. The following discussion will be on \(q_z\) that is determined by the anti-boundary condition of \(\psi_q(x,y,-L) = -\psi_q(x,y,L)\). From Eq. \((3)\), one has \(e^{i q_z L - \eta} = e^{-i q_z L + \eta}\) with \(\eta = \arctan(\Delta_q z)\) for \(L \gg \Delta\). Thus allowed \(q_z s\) are

\[
q_z L - \eta = -q_z L + (2n + 1)\pi + \eta, \tag{5}\]

where \(\eta\) is \(\pi\) for \(q_x L\) (like its counterpart of electrons, DOS does not depend on the types of boundary conditions) for a DW is

\[
g_{\text{DW}}(q_z) = \frac{dn}{dq_z} = \frac{L}{\pi} - \frac{\Delta}{\pi[1 + (\Delta q_z^2)^2]}.
\]

This differs from the domain magnon DOS, \(g_{\text{D}} = L/\pi\). It might be interesting to point out that the number of spin waves \(N\) should not depend on whether we are considering a DW or a domain. Similar to the Debye cut-off wave number of phonons, there is a maximum wave number \(q_c\) satisfying \(\int_{-q_c}^{q_c} g(q_z) dq_z = N\) for both DW and domain. Thus, \(q_{c,\text{D}} = \pi/a\), the boundary of the Brillouin zone, is the cut-off wave number for a domain since \(\eta = 2L\). From Eq. \((5)\), \(q_{c,\text{DW}}\) for a DW is given by

\[
q_{c,\text{DW}} = \frac{\pi}{a} + \frac{\arctan(\pi \Delta/a)}{L - \Delta[1 + (\pi \Delta/a)^2]}.
\]

Since \(q\) has to be limited within the Brillouin zone, the states between \(q_z = \pi/a\) and \(q_{c,\text{DW}}\) should be folded back to the Brillouin zone center as shown in Fig. 2(a) (shadowed area). Thus, the spin wave (magnon) DOS of a DW is

\[
g_{\text{DW}}(q_z) = \begin{cases} 
\frac{2L}{\pi} - \frac{\arctan(\pi \Delta/a)}{L - \Delta[1 + (\pi \Delta/a)^2]}, q_z < q'_c \\
\frac{\Delta / \pi}{L - \Delta[1 + (\pi \Delta/a)^2]}, q_z > q'_c \end{cases}
\]

where \(q'_c = \frac{\arctan(\pi \Delta/a)}{L - \Delta[1 + (\pi \Delta/a)^2]}\). As a result of the phase shift and the Brillouin zone folding, the magnon DOS is enhanced at small \(q_z\) in comparison with that of a domain. This phase shift induced DOS modification has the similar physics of the Friedel Sum Rule electronic system. Magnon DOS for both a domain (blue line) and a DW (thick black curve) are plotted schematically in Fig. 2(a). In the limit of \(L \gg \Delta \gg a, q'_c \approx \frac{\pi}{2L}\) and as \(L \to \infty\), \(g_{\text{DW}} - g_{\text{D}}\) becomes \(\delta(q_z) - \frac{\Delta / \pi}{L - \Delta[1 + (\pi \Delta/a)^2]}\) according


\[ \beta = \text{dependence of } \delta F \text{ function} \]

Let \( U_{\text{DW}} = 4s\sqrt{AK_z} \) in our model be the static DW free energy and/or the domain free energy, instead of DW free energy only. \( \delta F \) is over all possible magnon configurations \( \{n(q)\} \). The grand partition function \( Z = \sum_{\{n(q)\}} e^{-\beta E} \), where the summation is over all possible magnon configurations \( \{n(q)\} \), and \( \beta = \frac{1}{k_B T} \) with \( k_B \) the Boltzmann constant. Since we assume the static configurations are uniform in \( xy \) plane for both a DW and a domain, it’s convenient to consider the free energy density per unit cross section area which can be evaluated in a straightforward fashion. By defining the function

\[ \Theta(q) = \ln(1 - e^{-\beta \omega(q)}) \]

where \( \omega \) is given by Eq. 4, the results is

\[ F_{\text{DW}} = -\frac{1}{s} k_B T \ln Z_{\text{DW}} = 4\sqrt{AK_z} + \frac{k_B T}{(2\pi)^2} \int \Theta(q) g_{\text{DW}}(q_z) d^3q. \]  

The integration is over Brillouin zone and the factor \((2\pi)^{-2}\) comes from the DOS of transverse wave components \(s/\pi^2\). Similarly, domain free energy density with the same spin wave spectrum, Eq. 4, is

\[ F_D = -\frac{1}{s} k_B T \ln Z_D = \frac{k_B T}{(2\pi)^2} \int \Theta(q) g_D d^3q. \]

The free energy density difference between a DW and a domain is then

\[ \delta F(T) \equiv F_{\text{DW}} - F_D = 4\sqrt{AK_z} + \frac{k_B T}{(2\pi)^2} \int \frac{\pi}{\Delta} \int \Theta(q_x, q_y, 0) dq_x dq_y \]

\[ - \int \frac{\Delta}{\pi[1 + (q_z/\Delta)^2]} \Theta(q) d^3q, \]

where the second integration is over the Brillouin Zone. The conversion from a sum over \( q \) to an integral over \( q \) is justifiable in the case of \( L \gg \Delta \gg a \). Notice that, as expected, \( \delta F \) is independent of \( L \). Physically, this is because the free energy difference between a DW and a domain should only relate to the properties of a DW and a domain, characterized by \( U_{\text{DW}} \) and \( \Delta \) or \( K_z \), and \( A \).

The black line in Fig. 2(b) is the temperature dependence of \( \delta F \) with typical YIG parameters: \( a = 1.2\text{nm}, A = 3 \times 10^{-16}\text{m}^2, M_s = 1.75 \times 10^5\text{A/m}, \gamma = 3.4 \times 10^4\text{Hz}\cdot\text{m/A}, K_z = 0.032 \) and a shape anisotropy for a strip \( K_x = 1 \). One can see that \( \delta F \) always decreases with the increase of the temperature, in a linear form as shown in the figure. This behavior is directly related to the enhancement of magnon DOS near \( q_z = 0 \): Larger DOS at \( q_z = 0 \) means that the number of thermally excited magnons is larger, this leads to a larger entropy. The blue line in Fig. 2(b) is the temperature dependence of the density difference of the DW entropy and the domain entropy, \( \delta S = -\frac{\partial \delta F}{\partial T} \). At OK, only the lowest energy states (static DW or uniform domain) are allowed for DWs or domains without any magnons. Thus the entropy difference is zero and the free energy difference equals \( U_{\text{DW}} \). As the temperature increases, the entropy of a DW is always larger than that for a domain, and the entropy difference becomes almost a constant at a higher temperature of order of 50K with YIG parameters. Therefore, the free energy difference decreases with \( T \) in a linear form as shown by the black line in Fig. 2(b). Thus the total free energy can be lowered by moving the DW to the hotter part of the wire. The basic thermodynamics principles require a system to evolve in a way that lowers its free energy. So as long as the spins interact with heat baths this thermodynamic force should always drive the DW to a well-defined direction towards the hotter part of the wire. In order to estimate the DW speed driven by a temperature gradient, one can use \( \delta F \) to find the equivalent magnetic field. Then DW speed can be estimated by the Walker formula. Consider two points \( A \) and \( B \), which are \( l \) apart from each other on the wire, and assume \( T_A \) and \( T_B \) be the temperature at \( A \) and \( B \), respectively, one assumes \( T_B > T_A \). A DW moves from \( A \) to \( B \) if a DW is initially centered at \( A \). The free energy density of the wire is then lowered by \( \delta F(T_A) - \delta F(T_B) \). Equating the decrease of this free energy with the Zeeman energy by an equivalent magnetic field \( H_{\text{eq}} \), one has

\[ H_{\text{eq}} = \frac{\delta F(T_A) - \delta F(T_B)}{2\mu_0 M_s l} = \frac{\delta S \nabla T}{2\mu_0 M_s}. \]

According to Eq. 11, the equivalent field is independent of sample size, \( L \) and \( s \), as expected. Then according to the well-known Walker formula below the Walker breakdown field \( \alpha K_z M_s \), the DW speed \( v \) under a field \( H_{\text{eq}} \) is \( v = \gamma \frac{H_{\text{eq}} \Delta}{\alpha} \text{[3, 4]} \). Together with Eq. 12 we can see the propagating speed \( v \) is proportional to the temperature gradient \( \nabla T \). To compare with the recent experiment 16, we use YIG parameters with \( K_x = 1 \), experimental temperature gradient of \( \nabla T = 2.25 \times 10^4\text{K/m} \), and \( \alpha = 0.0075 \). Then the equivalent field is \( H_{\text{eq}} \approx 0.003\text{A/m} \). The speed is then \( v \approx 1.3\text{mm/s} \) which is about 6 times larger than the experimental value of \( \approx 200\text{µm/s} \). This is not a small value since modern laser technology can create a temperature gradient as large as \( 10^5\text{K/m} \). The discrepancy is
FIG. 2: (color online) (a) Schematic diagram of magnon density of states inside a DW (black thick curve) and inside a domain (blue line). Shadowed area between $q_z = \pi/a$ and $q_c$ illustrates the folded states to the Brillouin zone center. The color indicates relative contribution of $q_z$ state to the free energy (or entropy) in fixed $q_x, q_y$. The heavier a color is, the more important of the state is. (b) The temperature dependence of the free energy density difference $\delta F$ (black curve) and the entropy density difference $\delta S$ (blue curve) of a DW and a domain.

not surprising, considering complications involved in an experiment.

There are fundamental differences in the STT interpretations of magnon-driven DW motion and the thermodynamic viewpoint. Magnonic STT can only predict DW motion correctly to a system where the angular momentum dominates the DW dynamics. However, the thermodynamic theory present here is general and applicable to any wire with all possible microscopic interactions as long as material parameters do not depend on the spin textures. In case that material parameters depend on the spin textures, one should expect very interesting and very rich physics. Of course, all parameter changes should obey thermodynamic principles [32]. Of course, our thermodynamic theory is phenomenological in nature. It provides no microscopic description of how spins interact with other degrees of freedoms to generate a global DW propagation. It should be pointed out that our theory considers only magnon effects without electron contribution, important for a metallic wire. It is known that electronic STT and magnonic STT have the opposite sign under a temperature gradient. Thus, it is better to use magnetic insulating wires if one wants to test the current theory so that electron effects can be totally neglected. In a magnetic film, magnetic domains form strips whose width decreases as the temperature increase [24-26]. It is interesting to note that our theory can also provide a natural explanation to this well-known fact: Because DW entropy is larger than that of a domain, thus it is favorable to increase the number of DWs, or decrease strip width in order to decrease the total free energy of the magnetic film. Of course, generation of DWs shall cost system energy, and the equilibrium value of strip width is the compromise between the entropy gain and DW energy cost.

In conclusion, we compute magnonic contribution to the free energies and entropies of a DW and a domain. It is analytically found, with a clear physics picture, that a DW always has a larger entropy. Thus, the driving force behind DW propagation under a temperature gradient is the entropy. A DW propagates to the hotter region of a wire in order to lower the wire free energy. This result is robust and general. It does not depend on the microscopic details of a wire or a DW as long as the material parameters such as the exchange coefficient do not depend on the spin texture. The DW propagating speed is proportional to temperature gradient and can be as large as tens of m/s in reasonable parameters. The free energy and/or entropy results can also explain deduction of domain size at a higher temperature.

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