A SEARCH FOR CONCENTRIC CIRCLES IN THE 7-YEAR WMAP TEMPERATURE SKY MAPS

I. K. WEHUS\textsuperscript{1} and H. K. ERIKSEN\textsuperscript{2,3}

\textbf{ABSTRACT}

In a recent analysis of the 7-year WMAP temperature sky maps, Gurzadyan and Penrose claim to find evidence for violent pre-Big Bang activity in the form of concentric low-variance circles at high statistical significance. In this paper, we perform an independent search for such concentric low-variance circles, employing both $\chi^2$ statistics and matched filters, and compare the results obtained from the 7-year WMAP temperature sky maps with those obtained from ΛCDM simulations. Our main findings are the following: We do reproduce the claimed ring structures observed in the WMAP data as presented by Gurzadyan and Penrose, thereby verifying their computational procedures. However, the results from our simulations do not agree with those presented by Gurzadyan and Penrose. On the contrary we obtain a substantially larger variance in our simulations, to the extent that the observed WMAP sky maps are fully consistent with the ΛCDM model as measured by these statistics.

\textbf{Subject headings:} cosmic microwave background — cosmology: observations — methods: statistical

1. INTRODUCTION

One of the main accomplishments in cosmology during the last decade is the establishment of the ΛCDM inflationary concordance model. According to this model, the universe consists of 5% baryonic matter, 22% dark matter and 73% dark energy (Jarosik et al. 2010), and is filled with random and Gaussian fluctuations. These fluctuations were generated during a short period of exponential expansion called inflation (e.g. Liddle & Lyth 2000), and during which the universe expanded by a factor of $\sim 10^{26}$ in $\sim 10^{-34}$ s. With only a handful of free parameters, this model is able to successfully fit thousands of observational data points.

Nevertheless, the ΛCDM model must at the current stage be considered an effective model rather than a fundamental model. First, it relies on several quantities that have never been directly observed except through their gravitational impact, such as both dark matter and dark energy. Second, it postulates the existence of an unknown scalar field, the inflaton. It is therefore important to put the inflationary framework to stringent tests, probing its range of validity in different ways. One approach to do so is to construct alternative cosmological theories, making different observational predictions than ΛCDM, and then compare the two models using high-precision data.

One example of such work has been demonstrated by the development of the “Conformal Cyclic Cosmology” (CCC) model by Penrose (2008, 2009, 2010). In this picture, the history of the universe is described in terms of a series of “aeons”, each of which is defined as a finite period between two Big Bang events. We will not consider further details of the CCC model in this paper, except for one interesting feature: According to Gurzadyan & Penrose (2010), the model postulates that super-massive black holes may collide in earlier aeons, releasing tremendous amounts of energy in the form of gravitational waves. These collisions may be observable in our current aeon in the form of concentric circles of low variance in the cosmic microwave background (CMB).

Following up on this prediction, Gurzadyan & Penrose (2010) analyzed the 7-year WMAP temperature sky maps, searching for concentric circles of low variance. And quite surprisingly, they claim to find evidence for such rings at the 6$\sigma$ confidence level, by comparing the results obtained from WMAP with ΛCDM simulations. This claim is sufficiently spectacular (involving both pre-Big Bang phenomena and super-massive black holes) to catch the interest of the general media, with numerous news stories and live media appearances following in the weeks after the release of the paper. Given these widespread reactions, the claims of Gurzadyan and Penrose deserve closer scrutiny through independent analysis; this paper presents one such independent analysis.

2. DATA AND SIMULATIONS

The pre-Big Bang black hole collisions discussed by Gurzadyan & Penrose (2010) observationally manifest themselves in the form of one or more rings of low variance centered on a central position, $\hat{p}$, in the CMB anisotropy field, $\Delta T(\hat{\mathbf{n}})$, observed today. In this paper, we therefore consider the best currently available full-sky maps of the CMB, namely the 7-year WMAP temperature sky maps. These data are provided in the form of pixelized HEALPix sky maps with a pixel resolution of 7'.

For simplicity we consider only the foreground-reduced WMAP W-band (94 GHz) data in this paper, as this channel has the highest resolution of all the WMAP bands, with a beam of 13' FWHM. (Note that Gurzadyan & Penrose (2010) additionally analyses the WMAP V-band data and the BOOMERanG98 data; however, we agree that instrumental effects are irrelevant to this analysis, and therefore consider only the W-band in the following.) We apply the WMAP KQ85 sky cut to this map, removing 22% of the sky, and

\textsuperscript{1}Department of Physics, University of Oslo, P.O. Box 1048 Blindern, N-0316 Oslo, Norway
\textsuperscript{2}Institute of Theoretical Astrophysics, University of Oslo, P.O. Box 1029 Blindern, N-0315 Oslo, Norway
\textsuperscript{3}Centre of Mathematics for Applications, University of Oslo, P.O. Box 1053 Blindern, N-0316 Oslo, Norway
taking into account both the galactic plane and high-latitude point sources (Gold et al. 2010). (Note that Gurzadyan & Penrose 2010 adopts a straight $|b| < 20^\circ$ sky cut; the difference is not significant.)

Next, we build an ensemble of 1000 simulations with a spectrum given by the best-fit WMAP 7-year $\Lambda$CDM model (Komatsu et al. 2010). These realizations are convolved with the W-band beam and HEALPix pixel window, and projected onto a HEALPix $N_{\text{side}} = 512$ grid. Finally, uncorrelated Gaussian noise with RMS equal to $\sigma_0/\sqrt{N_{\text{obs}}(p)}$ are added to pixel $p$, where $\sigma_0$ is the W-band noise RMS per observation and $N_{\text{obs}}(p)$ is the number of observations in that pixel. The KQ85 sky cut is also applied to the simulations.

3. METHOD

The central concept in the following search is the radial standard deviation profile

\[ \sigma_r(b) \equiv \sqrt{\frac{1}{N-b} \sum_{\theta \in b} \Delta T(n)^2}, \]  

where $p$ denotes a fixed reference pixel on the sky, $n$ is a variable pixel index, and $\theta = \cos(b \cdot \hat{n})$ is the angular distance between those two positions; $b$ denotes an angular bin, $[\theta_{b-1}, \theta_b)$. Thus, this function measures the standard deviation of the CMB field as a function of radial distance from $p$, and a sharp negative spike in this function corresponds to the postulated signature of the black hole collisions. To reduce correlations between different bins, we subtract the mean from each function $\sigma_p(b)$ separately before further processing.

We measure $\sigma_r(b)$ for all positions over a HEALPix $N_{\text{side}} = 32$ grid, for a total of 12,288 points. Each function is evaluated from 0 to $20^\circ$ in 40 bins, corresponding to a bin size of 0.5$^\circ$. If more than 40% of the pixels within $20^\circ$ of a given pixel $p$ are masked out, that pixel is removed from further analysis. Finally, we also remove any pixels that are immediate neighbors to the galactic cut.

Having computed these functions for both data and simulations, the next step is to identify potential low-variance ring candidates, and quantify their significance. To this end, we introduce two different statistics. First, we consider matched filters designed to highlight negative spikes in $\sigma_p(b)$,

\[ \hat{\sigma}_r(b) = \sum_{b'} \sigma_p(b')f(b' - b). \]  

Here $f(b)$ defines a discrete filter, and we consider three different cases in this paper, namely $f_1 = [0.5, -1, 0.5]$, $f_2 = [0.25, 0.25, -0.5, -0.5, 0.25, 0.25]$ and $f_3 = [0.5, -0.125, -0.75, -0.125, 0.5]$; for $|b - b'|$ larger than the length of the filter, $f'$ is zero. These filters correspond to 1) a negative top-hat filter of width 1; 2) a negative top-hat filter of width 2; and 3) a negative wedge filter, each sensitive to typical interesting candidates. For each case, we adopt the maximum of $\hat{\sigma}_r(b)$ as our statistic, considering only angular distances larger than $2^\circ$, as the intrinsic estimator variance is very large on the smallest scales. We then make both sky maps and histograms of the maximum values of $\hat{\sigma}_r(b)$, and compare these between the observed data and the simulations.

Our second statistic is based on the standard $\chi^2$ estimator, which is sensitive to the overall fluctuation level of $\sigma_p(b)$, rather than individual spikes. Specifically, we compute

\[ \chi^2 = \sum_{bb'} (\sigma_p(b) - \mu_b)C_{bb'}^{-1}(\sigma_p(b') - \mu_{b'}) \]

for each pixel and data set, where $\mu_b$ is the mean of $\sigma_p(b)$ computed from the simulation set, and $C_{bb'}$ is the covariance matrix. Again, we make sky maps of these values for the observed data, and compare the observed data with the simulations in terms of histograms.

4. RESULTS

First, we show in the left panel of Figure 1 the standard deviation profile computed from the WMAP data centered on Galactic coordinates $(l, b) = (105.04^\circ, 37^\circ)$. (The mean is not subtracted in this plot.) This is the same profile as shown in Figure 2 of...
The agreement between the two results is excellent, despite the different masks used by the two analyses. This validates the routines used to compute the radial profiles for both analyses.

We have also plotted the mean of the variance profiles computed from all pixels in Figure 1 together with the corresponding 1 and 2σ confidence regions. Here we note several interesting features. First, the WMAP example profile is consistently low compared to the mean, suggesting strong correlations between bins. This is typical for all profiles for both WMAP and simulations; the local variance in a CMB map depends on the scanning strategy of WMAP, and the entire profile can shift up or down depending on whether the corresponding pixel is located in the ecliptic plane (high noise) or in the ecliptic poles (low noise). Since this effect is of little interest in the search for concentric rings, we subtract the mean before further analysis to reduce bin-to-bin correlations.

Second, the simulated distribution is very similar to that of WMAP, having consistent mean and standard deviation. We have also checked that this distribution is consistent from simulation to simulation.

Third, we see that the lowest point in the WMAP example corresponds to a ∼3.3σ outlier compared to the typical WMAP profile. This is the point that was claimed to be a 6σ outlier by Gurzadyan & Penrose (2010). With our simulations, this point appears considerably less anomalous. Further, it is important to note that this particular profile is by no means randomly chosen. On the contrary, it was picked out precisely due to its anomalous behaviour after searching through more than 10 000 points. The actual statistical significance of this particular 3.3σ outlier is therefore likely quite low.

Fourth, as illustrated by the example simulation profile in the right panel of Figure 1, it is not difficult to find individual simulated profiles with “anomalous” behaviour. Here one can also see “rings”, similar to those observed in WMAP, but created by chance alone.

To address these issues more rigorously, we are forced to adopt statistical techniques; visual inspection is not sufficient. We therefore employ the search algorithms described in Section 3 to identify peculiar candidates in both the data and simulations, and consider various statistics based on these results to interpret significances statistically.

In Figure 2 we show sky maps of the third matched filter, \( f_3 \), statistic, where each pixel indicates the maximum of \( \overline{\sigma}_p(b) \) over \( b \). Thus, pixels with large values in these plots indicate variance profiles with a notable negative wedge-like structure. Such maps are shown for both the WMAP data and a random simulation. At least visually, the two maps appear statistically consistent.

This statement is quantified more rigorously in Figure 3, where we show histograms for each of the four statistics computed from the WMAP data, and compared to the mean properties of the simulated ensemble. Again, the WMAP properties appear fully consistent with the simulations.

However, the histogram statistics shown above are sensitive only to the mean properties of the CMB field. They therefore only show that there is no evidence for a large number of concentric circles in the WMAP data, not that there cannot be a small number of highly significant cases. Finally, we therefore also consider the maximum value of each of the four statistics, which should be sensitive to single extreme cases. Figure 4 shows histograms of such maximum values, as computed from the simulated ensemble, with the corresponding WMAP value indicated by a vertical line. Again, we see that WMAP appears fully consistent with the ΛCDM simulations.

5. CONCLUDING REMARKS

In this paper we search for concentric low-variance rings in the 7-year WMAP temperature sky maps, seeking to reproduce the results recently presented by Gurzadyan & Penrose (2010). While our two analyses do agree in terms of specific variance profiles, they clearly disagree in terms of statistical interpretation and signifi-
Fig. 4.— Comparison of the full-sky maximum value for each statistic between the simulations (histograms) and the WMAP data (vertical dashed line).

Our analysis. Specifically, we find a substantially larger variance in our ΛCDM simulations than Gurzadyan & Penrose (2010) do in theirs. When taking into account this larger variance, and accounting for statistical selection effects, the evidence for concentric circles in the WMAP data appears minimal. Rather, the WMAP data appears fully consistent with our simulations.

The main difference between the two analyses must lie in the construction of the simulations. However, we have good reason to believe that our simulations are correct. First, we note that in our simulations the mean variance profile decreases towards small angular distances. This is bound to happen for two reasons: The CMB anisotropies constitute a correlated field with a characteristic scale of 1°, corresponding to the first peak in the CMB spectrum. Also, the instrumental beam of WMAP smooths out all small scale structure. It is therefore surprising that this effect is not seen in Figure 2 of Gurzadyan & Penrose (2010). Second, the fact that our simulations agree with WMAP provides further confidence; it is difficult to imagine an error in the simulation pipeline that would cause the simulations to become more similar to the observed data. Third, as noted by Gurzadyan & Penrose (2010), there are both low and high peaks in the reported variance profiles. While Gurzadyan and Penrose assign no significance to the high variance peaks (stating that “the peaks of high variance are of no importance, as these can result from numerous irrelevant effects”), they are clearly relevant in our interpretation, as they illustrate the substantial statistical variance present in these profiles. This large variance is observed both in the real data and the simulations.

Thus, we conclude that there is no evidence for the CCC model in the current WMAP data. Of course, Planck (Tauber et al. 2010) may provide new light on this issue, having higher sensitivity and resolution than WMAP, but one should probably not have too high expectations in this regard. Even if the CCC model should turn out to describe the real universe, it will quite likely be difficult to unambiguously identify such concentric circles due to the dominant background cosmic variance.

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