Mutual inductance calculation for coils with misalignment in wireless power transfer

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Abstract: In wireless power transfer (WPT) systems, the energy transmission channel is established by the coupling relationship between transmitting coil and receiving coil, and the coupling strength is usually measured by coupling coefficient. Therefore, it is necessary to calculate the mutual inductance between transmitting coil and receiving coil when describing the coupling strength. Here, filamentary circular coils and cylindrical helix coils are taken as the object of study, and the formulas for mutual inductance of two coils with misalignment are derived based on Neumann’s formula. In addition, the influence of multiple factors, such as axial distance, lateral distance and angle, on mutual inductance is analysed, and the calculated results are verified by experiments. The research results of this paper can be used to estimate the mutual inductance between the couplers in WPT systems, especially when there is misalignment.

1 Introduction

Wireless power transfer (WPT), as a new power transmission mode, has been widely concerned by scholars in this field. It has been applied to electric vehicles, implantable medical devices, and consumer electronics [1–4], which has greatly facilitating people’s lives.

WPT systems allow electrical power to be transmitted between transmitting coil and receiving coil using high-frequency magnetic fields [5, 6]. During the transmission, the coupling relationship between the two coils is established, which is usually measured by coupling coefficient. In addition, there is often a misalignment between the couplers in the operation of WPT systems and this will affect energy transmission. Therefore, it is of great significance to calculate mutual inductance in the case of misalignment.

The mutual inductance calculation between coils with misalignment has been studied in some literatures, using the Grover’s formula and filament method [7–9], but only give the result of theoretical calculation. The problem of couplings misalignment in WPT systems has also been researched, which involves mutual inductance calculation [10–13]. The mutual inductance depends on the shapes and orientations of the two coils [5], and there are many different shapes of couplers in WPT systems. We take the filamentary circular coils and the cylindrical helix coils as the object of study here, and the formulas for mutual inductance of two coils with misalignment are derived based on Neumann’s formula. Meanwhile, the results of calculation are verified by experiments. The research results of this paper provide methods for the mutual inductance calculation when transmitting and receiving coils are not coaxial in WPT systems.

2 Mutual inductance model

2.1 Filamentary circular coils

Setting up a Cartesian coordinate system as shown in Fig. 1a, the centre of the filamentary circular coil $l_1$ is $O (0, 0, 0)$, which is origin of the Cartesian coordinate system. Two filamentary circular coils with radius $r_1$ and $r_2$ are coaxial, and the distance between them is $d$.

Using magnetic vector potential approach, the formula of calculating mutual inductance is as follows (Neumann’s formula):

$$M = \frac{N_1 N_2 \mu_0}{4\pi} \int \oint \frac{dl_1 \times dl_2}{R}$$

The line elements of an arbitrary point $P, Q$ are

$$\begin{align*}
dl_1 &= (-r_1 \sin \theta_1 + r_1 \cos \theta_1) d\theta \quad \text{(1)} \\
dl_2 &= (-r_2 \sin \varphi + r_2 \cos \varphi) d\varphi
\end{align*}$$

and then

$$dl_1 \times dl_2 = r_1 r_2 \cos (\theta - \varphi) d\theta d\varphi$$

The distance $R_{PQ}$ between two line elements is

$$R_{PQ} = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos (\theta - \varphi) + d^2}$$

Therefore, the mutual inductance between two filamentary circular coils can be expressed as:

$$M = \frac{N_1 N_2 \mu_0}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{r_1 r_2 \cos (\theta - \varphi) d\theta d\varphi}{\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos (\theta - \varphi) + d^2}}$$

Fig. 1 Relative position of two filamentary circular coils
(a) Two coaxial filamentary circular coils, (b) Two filamentary circular coils with lateral and angular misalignments
where $\mu_0$ is magnetic permeability of vacuum, $N_1$ and $N_2$ are the number of turns for filamentary circular coil $l_1$ and $l_2$, $r_1$ and $r_2$ are radius of filamentary circular coil $l_1$ and $l_2$.

When there are misalignments (lateral and angular misalignments) between two filamentary circular coils, the relative position of two coils in the Cartesian coordinate system is shown in Fig. 1b. Axis $z'$ and axis $z$ are in the same plane, and the angle of them is $\alpha$. Meanwhile, axis $x'$ is parallel to axis $x$. In other words, the coil $l_2$ in Figs. 1b is obtained by the rotation and translation of $l_2$ in Fig. 1a. The rotation matrix is as follows:

$$R_{\alpha}(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} \tag{6}$$

The centre of the filamentary circular coil $l_2$ is $O'$ $(0, c, d)$. $Q$ is an arbitrary point of the filamentary circular coil $l_2$, whose coordinates in the coordinate system $xyz$ are $Q (r \cos \theta, r \sin \theta \cos \phi, c - r \sin \theta \sin \phi)$, which is converted from the coordinate system $x'y'z'$ by the rotation matrix.

Following from (1), the mutual inductance between two filamentary circular coils with lateral and angular misalignments can be expressed as:

$$M = \frac{N_1 N_2 \mu_0}{4\pi} \int_0^{2\pi} \int_0^\infty \frac{r_1 r_2 \sin \theta \sin \phi + \cos \alpha \cos \theta \cos \phi}{(r_1 \cos \theta - r_2 \cos \phi)^2 + (r_1 \sin \theta - r_2 \sin \phi \cos \alpha - c)^2} r_1 r_2 \sin \theta \sin \phi + \cos \alpha \cos \theta \cos \phi \, d\theta \, d\phi$$

$$+ \frac{1}{(r_1^2 + r_2^2 - 2 r_1 r_2 \cos \theta - \phi)^2 + ((h/2\pi) (\theta - \phi) - d)^2} \tag{7}$$

2.2 Cylindrical helix coils

The cylindrical helix is divided into left-handed helix and right-handed helix, and its parameter equation is (8). Where $a$ is the radius of cylinder, $b$ is the pitch of cylindrical helix. In addition, right-handed helix takes plus sign and left-handed helix takes minus sign.

$$\begin{cases} x = a \cos \theta \\ y = a \sin \theta \\ z = \pm h \theta / (2\pi) \end{cases} \tag{8}$$

Assuming that both cylindrical helix coils are right-handed, the radii of two helix coils are $r_1$ and $r_2$ respectively, their turns are $n$ and $n$, and the pitches of them are $h$, two coaxial helix coils are shown in Fig. 2a.

We can calculate the mutual inductance between two coaxial helix coils according to (1). The line elements of an arbitrary point on the helix coils are as follows:

$$\begin{align*}
 dl_1 &= \left[ -r_1 \sin \theta \, d\theta + r_1 \cos \theta \, d\phi + \frac{h}{2\pi} \, d\phi \right] \\
 dl_2 &= \left[ -r_2 \sin \phi \, d\phi + r_2 \cos \phi \, d\theta + \frac{h}{2\pi} \, d\theta \right]
\end{align*} \tag{9}$$

and then

$$dl_1 \cdot dl_2 = \left[ r_1 \cos \phi - r_2 \cos \theta + \frac{h}{2\pi} \right] \sin \theta \sin \phi \, d\theta \, d\phi \tag{10}$$

The distance $R$ between two line elements is

$$R = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos \theta - \phi} + \frac{h}{2\pi} (\theta - \phi - d)^2 \tag{11}$$

The mutual inductance between two coaxial helix coils can be obtained as follows:

$$M = \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_0^\infty \frac{r_1 r_2 \cos \theta \cos \phi}{(r_1 \cos \theta - r_2 \cos \phi)^2 + (r_1 \sin \theta - r_2 \sin \phi \cos \alpha - c)^2}$$

$$+ \frac{1}{(r_1^2 + r_2^2 - 2 r_1 r_2 \cos \theta - \phi)^2 + ((h/2\pi) (\theta - \phi) - d)^2} \tag{12}$$

We can set up a Cartesian coordinate system like Fig. 1b when there are misalignments (lateral and angular misalignments) between two cylindrical helix coils, as shown in Fig. 2b. The rotation matrix is the same as (6). The coordinates of an arbitrary point of the helix coil $l_2$ in the coordinate system $x'y'z'$ are $(r \cos \theta, r \sin \theta + h \phi / (2\pi), c - r \sin \phi \cos \alpha)$, which can be converted to $(r \cos \theta, r \sin \theta + h \phi / (2\pi), c - r \sin \phi \cos \alpha + h \cos \theta / (2\pi) + d)$ in the coordinate system $xyz$ by the rotation matrix.

Using (1), we can get the mutual inductance between two helix coils with lateral and angular misalignment, as follows: (see (13)).

3 Calculated results and experimental verification

In order to verify the theoretical analysis, we use both calculated analysis and experimental analysis. The experimental device is shown in Fig. 3. Both coils are made of copper wire and the main characteristic parameters of the experimental coils are listed in Table 1.

In the experiment, the shape of coils has been fixed, and we can study the factors that influence orientations, such as axial distance, lateral misalignment, and angular misalignment. We measured the influence of these factors on mutual inductance, and these measurements are compared with the calculated results of (7) and (13). The calculated results and experimental results are shown in Figs. 4 and 5. Several variables are involved in this process: $c$, $d$ and $\alpha$, representing lateral misalignment, axial distance, and angular misalignment, respectively. These variables correspond to the unknown quantities in (7) and (13).

Figs. 4 and 5a–c show the mutual inductance under the influence of a single factor and Fig. 5d shows the calculated results of mutual inductance under the influence of two factors (lateral and
We can see that the experimental results are not exactly the same as the calculated results. After all, the calculated results are obtained under the ideal condition and there are measurement errors. However, the trend of their change is consistent. The research results of this paper can be used to estimate the mutual inductance between the couplers in WPT systems, especially when there is misalignment.

4 Conclusion

Here, we take filamentary circular coils and cylindrical helix coils as the object of study, and the formulas for mutual inductance of two coils with misalignment are derived based on Neumann's formula. However, the axes of these two coils are limited to the same plane. We use experiments to verify the influence of multiple factors (axial distance, lateral and angular misalignment) on mutual inductance and find that the experimental results are almost consistent with the calculated results. The research results of this paper can be used to estimate the mutual inductance between the couplers in WPT systems, especially when there is misalignment.

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Table 1 Main characteristic parameters of the experimental coils

| Type of coil   | Radius of coil, mm | Number of turns | Wire diameter, mm | Pitch between turns |
|---------------|--------------------|-----------------|-------------------|--------------------|
| circular coil | 100                | 5               | 2.5               | —                  |
| helix coil    | 100                | 5               | 2.5               | 10                 |

**Fig. 3 Experimental device**

**Fig. 4 Mutual inductance for two filamentary circular coils**

(a) Axial distance \((c = 0, \alpha = 0)\), (b) Lateral misalignment \((d = 0.2 \text{ m}, \alpha = 0)\), (c) Angular misalignment \((c = 0, d = 0.2 \text{ m})\), (d) Lateral and angular misalignment \((d = 0.2 \text{ m})\)
Fig. 5 Mutual inductance for two cylindrical helix coils:
(a) Axial distance \((c = 0, \alpha = 0)\), (b) Lateral misalignment \((d = 0.2 \text{ m}, \alpha = 0)\), (c) Angular misalignment \((c = 0, d = 0.2 \text{ m})\), (d) Lateral and angular misalignment \((d = 0.2 \text{ m})\)

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