Cosmological implications of the full shape of anisotropic clustering measurements in BOSS and eBOSS

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ABSTRACT

We present the analysis of the full shape of anisotropic clustering measurement from the extended Baryon Oscillation Spectroscopic Survey (eBOSS) quasar sample together with the combined galaxy sample from the Baryon Oscillation Spectroscopic Survey (BOSS), re-analysed using an updated recipe for the non-linear matter power spectrum and the non-local bias parameters. We obtain constraints for flat ΛCDM cosmologies, focusing on the cosmological parameters that are independent of the Hubble parameter $h$. Our recovered value for the RMS linear perturbation theory variance as measured on the scale of 12 Mpc is $\sigma_8 = 0.805 \pm 0.049$, while using the traditional reference scale of 8 $h^{-1}$Mpc gives $\sigma_8 = 0.815 \pm 0.044$. We quantify the agreement between our measurements and the latest CMB data from Planck using the suspiciousness metric, and find them to be consistent within 0.64 $\pm$ 0.03. Combining our clustering constraints with the 3 $\times$ 2pt data sample from the Dark Energy Survey (DES) Year 1 release slightly degrades this agreement to the level of 1.54 $\pm$ 0.08, while still showing an overall consistency with Planck. We furthermore study the effect of imposing a Planck-like prior on the parameters that define the shape of the linear matter power spectrum, and find significantly tighter constraints on the parameters that control the evolution of density fluctuations. In particular, the combination of low-redshift data sets prefers a value of the physical dark energy density $\omega_{DE} = 0.335 \pm 0.011$, which is 1.7$\sigma$ higher than the one preferred by Planck.

Key words: large-scale structure of Universe – cosmological parameters

1 INTRODUCTION

The rapid progress of observational cosmology in recent years has been fuelled by an abundance of accurate observations (Riess et al. 1998; Perlmutter et al. 1999; Eisenstein et al. 2005; Cole et al. 2005; Hinshaw et al. 2013; Anderson et al. 2012; Alam et al. 2017; Planck Collaboration et al. 2020; Alam et al. 2021). The ΛCDM model has emerged as the new cosmological paradigm, being able to simultaneously describe all state-of-the-art observations. However, the two components making up the majority of the total energy budget of the Universe today in this model, dark energy and dark matter, remain poorly understood.

As it provides the most precise parameter constraints to date, the best-fitting ΛCDM model to the cosmic microwave background (CMB) observations by the Planck satellite (Planck Collaboration et al. 2020) has become the synonym to ‘standard cosmological model’. The comparison of predictions for the expansion history of the Universe $H(z)$, and the rate at which cosmic structures form at later times $f(z)$ with observations at lower redshifts serves as a powerful test of the ΛCDM paradigm. While a broad range of observations are in agreement with the CMB predictions, the increasingly precise measurements from the cosmic distance ladder (Riess et al. 2018; Riess et al. 2019), as well as the ever-increasing weak gravita-
tional lensing data sets (Hildebrandt et al. 2016; Abbott et al. 2018; Hikage et al. 2019), display hints of tension with Planck ΛCDM. In particular, local direct probes seem to prefer an up to 5σ (Riess et al. 2021) greater expansion rate of the Universe (the 'H₀ tension') while, less significantly (up to 3σ, Heymans et al. 2021), weak gravitational lensing measurements exhibit a lower amplitude (the 'σ₈ tension') than predicted by Planck. There is still no conclusive evidence on what drives these differences in the recovered values, and it is not uncommon for proposed solutions to the H₀ tension to make the σ₈ tension worse (see, for example, Hill et al. 2020).

With the lack of consensus on where (and whether) the inconsistencies with Planck ΛCDM arise, the cosmological community has put in an increased effort in assessing the internal consistency between different data sets within the ΛCDM scenario. Galaxy clustering allows to probe and distinguish how the underlying cosmology affects the background expansion of the Universe and its effects on the structure growth through baryon acoustic oscillations (BAO) and redshift-space distortions (RSD). Both of these effects set important features of the two-point correlation function, which can be fit to obtain summary statistics that carry compressed cosmological information. BAO set the angular scale of the acoustic peak, allowing to probe the distance – redshift relation. On the other hand, RSD provide information about structure growth through galaxy peculiar velocities, whose effect on the amplitude of the power spectrum is commonly characterised by the product of the logarithmic growth rate f and the RMS linear perturbation theory variance σ₈ (although see Sánchez 2020, for the problems caused by this approach). While analyses based solely on RSD and BAO summary statistics allow excellent internal consistency tests and may help constrain beyond - ΛCDM scenarios, it has been shown that they do not preserve all the information of the full measurement, in particular, losing the additional constraining power available from its shape (Brieden et al. 2021).

Other analyses, therefore, make use of the information recovered from fitting the full shape of two-point clustering measurements, either in Fourier or configuration space, directly comparing models against data (Tröster et al. 2020; d'Amico et al. 2020; Ivanov et al. 2020; Chen et al. 2021). These analyses tend to lose some of the immediate interpretability of the summary statistics but instead allow to directly obtain constraints of cosmological parameters independently of external data sets. This type of analyses have, therefore, recently received attention as a way to test the consistency between large-scale structure (LSS) and CMB measurements. Tröster et al. (2020) showed that the full shape analysis of galaxy clustering produces cosmological constraints that are comparable to those of other low-redshift probes. This work followed the analysis of correlation function wedges of Baryon Oscillation Spectroscopic Survey (BOSS) galaxies by Sánchez et al. (2017) in order to derive constraints on flat ΛCDM cosmologies from galaxy clustering alone (i.e. without combining it with CMB measurements, as was done in the BOSS Data Release 12 consensus analysis, Alam et al. 2017). Furthermore, the work also presented joint low-redshift constraints by combining galaxy clustering with weak lensing measurements from the Kilo-Degree Survey (KV450).

The σ₈ value recovered from the full shape analysis of the correlation function wedges by Tröster et al. (2020) is 2.1σ low compared to Planck’s prediction, with the difference increasing to 3.4σ when weak lensing measurements from KV450, are added, indicating that there may be some consistent discrepancy between CMB predictions and low redshift observations. This is also consistent with the more recent analysis by Heymans et al. (2021) where BOSS galaxies are used as lenses in the so called ‘3 x 2pt’ analysis (a set of three correlation functions consisting of autocorrelation of the lens galaxy positions, source galaxy shapes and the cross-correlation of the two), which finds a ~ 3σ discrepancy with Planck’s value of $S₈ = σ₈ \sqrt{Ωₘ/0.3}$, that combines σ₈ with the matter density Ωₘ in a way that minimises correlation between the two parameters. This result is consistent with the findings from other major weak lensing surveys (Dark Energy Survey (Collaboration et al. 2021) and Hyper Suprime-Cam (Hikage et al. 2019)), even though, most recently, DES reported consistency between their 3 x 2pt ΛCDM constraints and those of Planck when the full parameter space is considered. Furthermore, σ₈ may not be an entirely appropriate parameter for assessing consistency among the different surveys, as Sánchez (2020) have shown that it is affected by the different posterior distributions of Hubble parameter h recovered by different analyses. Alternatively, one may define $σ_{12}$ - the variance as measured on a fixed scale of 12 Mpc. In this work we adopt this notation and use $σ_{12}$ to both accurately characterise the amplitude of the power spectrum today as well as assess the consistency among the probes considered.

In this work we are, therefore, interested in building upon Tröster et al. (2020) and exploring, whether the discrepancy between the low-redshift probes and Planck within the ΛCDM model holds when extending the redshift range probed by the clustering measurements with the addition of eBOSS quasar clustering. We provide the joint constraints from the full shape analysis of BOSS galaxy and eBOSS quasar clustering on their own, as well as in combination with weak lensing information. For our weak lensing data set we use the 3 x 2pt measurements from the Dark Energy Survey Year 1 (DES Y1, Abbott et al. 2018) release, which both cover a larger area than KV450 and include galaxy clustering and galaxy-galaxy lensing as well as the shear-only measurements. If the tension seen between the low-redshift probes and Planck is purely statistical, adding more data should not only tighten the posterior contours but be able to bring the constraints to a better agreement. The results from an equivalent analysis with KiDS-450 shear measurements are available in the appendix A.

In addition to expanding our data sets we also aim to re-define the parameter space following Sanchez et al. (2021), who distinguish ‘shape’ and ‘evolution’ cosmological parameters. This classification is introduced to describe the degenerate way in which evolution parameters affect the linear matter power spectrum when expressed in Mpc units. In such parameter space, $σ₈$ is replaced by $σ_{12}$, as discussed above, and the relative matter and dark energy densities ($Ω_m, Ω_{DE}$) are replaced by their physical counterparts ($ω_m, ω_{DE}$). While in Sanchez et al. (2021) the h-independent parameter space is presented to create a framework which allows them to reduce the number of parameters required to model the cosmology dependence of the matter power spectrum, the advantage of such parameter choice for this work is two-fold. First, the derived constraints do not depend on the posterior of h of the particular analysis and can, therefore, be directly compared with constraints from other data sets and, second, the effect that each of the parameters has on the power spectrum is clear, with evolution parameters affecting its amplitude and shape parameters determining the shape.

We provide a more detailed description of the parameter space we use (including the prior choices) in Section 2, together with a summary of our data and models, and illustrate how it compares with its h-dependent equivalent in Section 3.1, where we also present our cosmological constraints from BOSS and eBOSS. The results obtained when adding DES are further presented in Section 3.3 We finish with a discussion of our results in Section 4 and present our conclusions in Section 5.
2 METHODOLOGY

This work is an extension of Tröster et al. (2020) and largely follows the same structure and methods - we assume flat ΛCDM cosmology and obtain the joint low-redshift parameter constraints by combining the likelihoods for each data set considered independently. Our model for anisotropic galaxy and quasar clustering measurements follows that described in Sánchez et al. (2017) for the so-called ‘full shape’ analysis (with the exception of the matter power spectrum model) whereas for the ‘3x2pt’ analysis (galaxy shear, galaxy-galaxy lensing, and galaxy clustering) we use the model described in Abbott et al. (2018). In this section we summarise the data and models used with a more detailed description available in the references above. The measurements here are as used in the respective original analyses and, therefore, had been tested against various systematics and include the appropriate corrections.

2.1 Galaxy and QSO clustering measurements

The Sloan Digital Sky Survey (SDSS) has mapped the large-scale structure of the Universe thanks to the accurate measurements by the double-armed spectrographs (Smee et al. 2013) at the Sloan Foundation Telescope at Apache Point Observatory (Gunn et al. 2006). Throughout its different stages (York et al. 2000; Eisenstein et al. 2011; Blanton et al. 2017) the SDSS has provided redshift information on millions of galaxies and quasars.

We consider clustering measurements in configuration space from two data sets: the galaxy samples of BOSS (Dawson et al. 2013), corresponding to SDSS DR12 (Alam et al. 2015; Reid et al. 2016), and the QSO catalogue (Lyke et al. 2020) from eBOSS (Dawson et al. 2016), contained in SDSS DR16 (Ahumada et al. 2020; Ross et al. 2020). In each case, the information from the full anisotropic correlation function $\xi(s, \mu)$, where $s$ denotes the comoving pair separation and $\mu$ represents the cosine of the angle between the separation vector and the line of sight, was compressed into different but closely related statistics.

We analyse the clustering properties of the combined BOSS galaxy sample using the measurements of Sánchez et al. (2017), who employs the clustering wedges statistic (Kazin et al. 2012), $\xi_{\Delta \mu}(s)$, which corresponds to the average of $\xi(s, \mu)$, over the interval $\Delta \mu = \mu_2 - \mu_1$, that is

$$\xi_{\Delta \mu}(s) = \frac{1}{\Delta \mu} \int_{\mu_1}^{\mu_2} \xi(s, \mu) \, d\mu.$$  \hspace{1cm} (1)

Sánchez et al. (2017) measured three wedges by splitting the $\mu$ range from 0 to 1 into three equal-width intervals. We consider their measurements in two redshift bins, with $0.2 < z < 0.5$ (the LOWZ sample) and $0.5 < z < 0.75$ (CMASS), corresponding to the effective redshifts $z_{\text{eff}} = 0.38$ and 0.61, respectively. The covariance matrices, $C$, of these data were estimated using the set of 2045 MDPATCHY mock catalogues described in Kitaura et al. (2016). These measurements were also used in the analysis of Tröster et al. (2020) and the recent studies of the cosmological implications of the KiDS 1000 data set (Heymans et al. 2021; Tröster et al. 2021).

For the eBOSS QSO catalogue we use the measurements of Hou et al. (2021), who considered the Legendre multipoles given by

$$\xi_\ell(s) = \frac{2\ell + 1}{2} \int \xi(s, \mu) L_\ell(\mu) \, d\mu,$$  \hspace{1cm} (2)

where $L_\ell(\mu)$ denotes the $\ell$-th order Legendre polynomial. We consider the multipoles $\ell = 0, 2, 4$ obtained using the redshift range $0.8 < z < 2.2$, with an effective redshift $z_{\text{eff}} = 1.48$. The covariance matrix of these measurements were obtained using the set of 1000 mock catalogues described in Zhao et al. (2021). Besides the QSO sample used here, the full eBOSS data set contains two additional tracers, the luminous red galaxies (LRG) and emission line galaxies (ELG) samples (for the corresponding BAO and RSD analyses, see Bautista et al. 2021; Gil-Marín et al. 2020; de Mattia et al. 2021; Tambone et al. 2020). These samples overlap in redshift among them and with the galaxies from BOSS. We therefore restrict our analysis of eBOSS data to the QSO sample to, in combination with BOSS, cover the maximum possible redshift range while ensuring that the clustering measurements can be treated as independent in our likelihood analysis.

We treat the measurements from BOSS and eBOSS as in the original analyses of Sánchez et al. (2017) and Hou et al. (2021). We restrict our analysis to the parameter $\mu$ separations within the range $20 \, h^{-1} \text{Mpc} < s < 160 \, h^{-1} \text{Mpc}$. We assume a Gaussian likelihood for each set of measurements, in which the covariance matrices are kept fixed. We account for the impact of the finite number of mock catalogues used to derive $C$ (Kaufman 1967; Hartlap et al. 2007; Percival et al. 2014). The large number of mock catalogues used ensures that the effect of the noise in $C$ on the obtained cosmological constraints corresponds to a modest correction factor of less than 2 per cent.

2.2 Modelling anisotropic clustering measurements

Our modelling of the full shape of the Legendre multipoles and clustering wedges of the anisotropic two-point correlation function largely follows the treatment of Sánchez et al. (2017), with some important differences.

We compute model predictions of the non-linear matter power spectrum, $P_{nm}(k)$, using the Rapid and Efficient SPerceptron calculation based on RESPonSe functiOn approach (RESPRESSO, Nishimichi et al. 2017). The key ingredient of RESPRESSO is the response function, $K(k, q)$, which quantifies the variation of the non-linear matter power spectrum at scale $k$ induced by a change of the linear power at scale $q$ as

$$K(k, q) \equiv \frac{\partial P_{nm}(k)}{\partial P_L(q)}.$$  \hspace{1cm} (3)

Nishimichi et al. (2016) presented a phenomenological model for $K(k, q)$ based on renormalised perturbation theory (Taruya et al. 2012), which gives a good agreement with simulation results over a wide range of scales for $k$ and $q$. The response function allows to obtain $P_{nm}(k)$ for arbitrary cosmological parameters $\theta$ based on a measurement from N-body simulations of a fiducial cosmology $\theta_{\text{fid}}$ as

$$P_{nm}(k|\theta) = P_{nm}(k|\theta_{\text{fid}}) \int \ln q \, K(k, q)$$ 

$$\times \left[ P_L(q|\theta) - P_L(q|\theta_{\text{fid}}) \right].$$  \hspace{1cm} (4)

The choice of $\theta_{\text{fid}}$ in RESPRESSO corresponds to the best-fitting ΛCDM model to the Planck 2015 data (Planck Collaboration et al. 2016). Equation (4) is most accurate for cosmologies that are close to $\theta_{\text{fid}}$. For cosmologies further away from the fiducial, its accuracy can be improved by performing a multi-step reconstruction. Eggemeier et al. (2020) showed that RESPRESSO outperforms other perturbation theory based models in terms of the range of validity and accurate recovery of mean posterior values.

Following the notation of Eggemeier et al. (2019), we describe the relation between the galaxy density fluctuations, $\delta$, and the matter density fluctuations, $\delta_m$, at one loop in terms of the four-parameter
where the first two terms represent contributions from linear and quadratic local bias, while the remaining ones correspond to non-local terms. Here, \( G_2 \) is the Galileon operator of the normalized velocity potential \( \Phi_v \), and \( \varphi_1 \) is the linear Lagrangian perturbation potential with \( \varphi_2 \) as a second-order potential that accounts for the non-locality of the gravitational evolution.

\[
G_2(\Phi_v) = (\nabla_i \nabla_j \Phi_v)^2 - (\nabla^2 \Phi_v)^2, \\
G_2(\varphi_1, \varphi_2) = \nabla_i \varphi_1 \nabla_j \varphi_1 - \nabla^2 \varphi_2 \nabla^2 \varphi_1. 
\]

(6) \hspace{1cm} (7)

Two-point statistics alone do not constrain \( \gamma_2 \) well, because \( \gamma_2 \) enters at higher order and is degenerate with \( \gamma_{21} \). Therefore, we set the value of this parameter in terms of the linear bias \( b_1 \) using the quadratic relation

\[
\gamma_2(b_1) = 0.524 - 0.547b_1 + 0.046b_1^2, 
\]

(8)

which describes the results of Sheth, Chan & Scoccimarro (2013) using excursion set theory. Eggemeier et al. (2020) showed that this relation is more accurate for tracers with \( b_1 \geq 1.3 \) than the one obtained under the assumption of local bias in Lagrangian space used in Sánchez et al. (2017).

The value of \( \gamma_{21} \) can also be derived in terms of \( b_1 \) under the assumption of the conserved evolution of galaxies (hereafter co-evolution) after their formation as (Fry 1996; Catelan et al. 1998, 2000; Chan et al. 2012)

\[
\gamma_{21} = -\frac{2}{21} (b_1 - 1) + \frac{6}{7} \gamma_2. 
\]

(9)

This relation was thoroughly tested against constraints derived from a combination of power spectrum and bispectrum data in Eggemeier et al. (2021), and found to be in excellent agreement for BOSS galaxies. In addition to this, in Sec. 2.5, we confirm that the use of this relation gives an accurate description of the results of N-body simulations and we therefore implement it in our analysis of the BOSS eBOSS data. In this way, the only required free bias parameters in our recipe are \( b_1 \) and \( b_2 \), while the non-local bias terms can be fully expressed in terms of the linear bias through equations (8) and (9).

Our description of the effects of RSD matches that of Sánchez et al. (2017), Following Scoccimarro (2004) and Taruya et al. (2010), we write the two dimensional redshift-space power spectrum as

\[
P(k, \mu) = W_{\text{tot}}(i f k, \mu) P_{\text{nov}}(k, \mu), 
\]

(10)

where the ‘no-virial’ power spectrum, \( P_{\text{nov}}(k, \mu) \), is computed using the one-loop approximation and includes three terms, one representing a non-linear version of the Kaiser formula (Kaiser 1987) and two higher-order terms that include the contributions of the cross-spectrum and bispectrum between densities and velocities. Besides the non-linear matter power spectrum, \( P_{\text{nov}}(k, \mu) \) requires also the the velocity-velocity and matter-velocity power spectra, which we compute using the empirical relations measured from N-body simulations of (Bel et al. 2019). The function \( W_{\text{tot}}(\lambda = i f k \mu) \) represents the large-scale limit of the generating function of the pairwise velocity distribution, which accounts for non-linear corrections due to fingers-of-God (FOG) or virial motions and can be parametrised as (Sánchez et al. 2017)

\[
W_{\text{tot}}(\lambda) = \frac{1}{\sqrt{1 - \lambda^2 \sigma^2_{\text{vir}}}} \exp \left( \frac{\lambda^2 \sigma^2_{\text{vir}}}{1 - \lambda^2 \sigma^2_{\text{vir}}} \right), 
\]

(11)

where \( \alpha_{\text{vir}} \) is a free parameter characterizing the kurtosis of the small-scale velocity distribution, and \( \sigma_{\text{vir}} \) is the one-dimensional linear velocity dispersion defined in terms of the linear matter power spectrum as

\[
\sigma_v^2 = \frac{1}{6\pi^2} \int dk P_L(k). 
\]

(12)

The QSO sample is known to be affected by non-negligible redshift errors which also affect the clustering measurements (Zarrouk et al. 2018). We account for this following Hou et al. (2018), who showed that this effect can be correctly described by including an additional damping factor to the power spectrum of equation (10) of the form \( \exp(-k^2 \sigma_v^2) \), where \( \sigma_v \) is treated as an additional free parameter.

Finally, the Alcock-Paczyński distortions (Alcock & Paczyński 1979) due to the difference between the true and fiducial cosmologies are accounted for by introducing the geometric distortion factors

\[
q_\perp = D_M(z_{\text{eff}})/D_M(z_{\text{eff}}), \\
q_\parallel = H'(z_{\text{eff}})/H(z_{\text{eff}}). 
\]

(13) \hspace{1cm} (14)

Here, \( D_M(z) \) is the comoving angular diameter distance and \( H(z) \) is the Hubble parameter, with primed quantities corresponding to the fiducial cosmology used to convert redshifts to distances. The distortion factors are then applied to rescale the separations \( s \) of galaxy pairs and the angles between the separation vector and the line of sight \( \mu \) such that

\[
s = s' \left( q_\parallel^2 \mu^2 + q_\perp^2 (1 - \mu^2) \right), \\
\mu = \mu' \sqrt{q_\parallel^2 \mu^2 + q_\perp^2 (1 - \mu^2)}. 
\]

(15) \hspace{1cm} (16)

In summary, our model of the clustering wedges from BOSS requires three free parameters, \( b_1, b_2, \) and \( \alpha_{\text{vir}} \), with the values of \( \gamma_2 \) and \( \gamma_{21} \) given in terms of \( b_1 \) using equations (8) and (9). This is one less free parameter than in the original analysis of Sánchez et al. (2017). The Legendre multipoles of the eBOSS QSO require the addition of \( \sigma_v \), leading to a total of four free parameters.

### 2.3 Additional data sets

We complement the information from our clustering measurements with the \( 3 \times 2 pt \) measurements from DES Y1 (Abbott et al. 2018). We also use the shear measurements from the Kilo-Degree Survey (KiDS-450, Hildebrandt et al. 2016) and present the results in Appendix A.

The source galaxy samples from DES are split into four redshift bins, spanning the redshift range of \( 0.2 < z < 1.3 \). In addition to shear measurements from the source galaxies, the DES Y1 data set also includes galaxy clustering and galaxy-galaxy lensing two-point correlation function measurements, as well as the lens redshift distributions for five redshift bins in the range of \( 0.15 < z < 0.9 \). Our scale cuts for these measurements match those of Abbott et al. (2018).

For our \( 3 \times 2 pt \) analysis, we use the DES likelihood as implemented in CosmoMC (Lewis & Bridle 2002), which corresponds to the model described in Abbott et al. (2018). The likelihood includes models for the two-point correlation functions describing galaxy-galaxy lensing, galaxy clustering, and cosmic shear. The correlation functions are modelled making use of Limber and flat-sky approximations (Limber 1954; Kaiser 1992; LoVerde & Afshordi 2008; Kilbinger et al. 2017) with the non-linear power spectrum obtained using HMCODE (Mead et al. 2015) as implemented in camb (Lewis...
et al. 2000). The smallest angular separations considered correspond to a comoving scale of $8\ h^{-1}\text{Mpc}$. The intrinsic alignment is modelled using a ‘non-linear linear’ alignment recipe (Hirata & Seljak 2004; Bridle & King 2007). The model also includes a treatment for multiplicative shear bias and photometric redshift uncertainty. The former is accounted for by introducing multiplicative bias terms of the form $(1 + m^i)$ for each bin $i$ for shear and galaxy-galaxy lensing. The latter is modelled by the shift parameters $\delta^s$ assigned to each bin for both source and lens galaxies. Finally, baryonic effects are not included as they are expected to be below the measurement errors for the range of scales considered in the analysis. For all the weak lensing nuisance parameters we impose the same priors as the ones listed in Abbott et al. (2018).

Additionally, we test the consistency of the low-redshift LSS measurements with the latest CMB temperature and polarization power spectra from the Planck satellite (Planck Collaboration et al. 2020), to which we refer simply as ‘Planck’. We do not include CMB lensing information. We use the public nuisance parameter-marginalised likelihood plik lite TTTEEE+lowl+lowlE for all Planck constraints (Planck Collaboration et al. 2020).

### 2.4 Parameter spaces and prior ranges

Our goal is to obtain constraints on the parameters of the standard $\Lambda$CDM model, which corresponds to a flat universe, where dark energy is characterized by a constant equation of state parameter $\omega_{\Lambda} = -1$. Following Sanchez et al. (2021), we focus on cosmological parameters that can be classified as either “shape” or “evolution”. The former are parameters that control the shape of the linear-theory power spectrum expressed in Mpc units. The latter only affect the amplitude of $P_l(k)$ at any given redshift. Assuming a fixed total neutrino mass of $\sum m_\nu = 0.06\ eV$, the $\Lambda$CDM model can be described by the parameters

$$\theta = (\omega_b, \omega_c, \omega_{\text{DE}}, A_s, n_s).$$

These are the present-day physical energy densities of baryons, cold dark matter, and dark energy, and the amplitude and spectral index of the primordial power spectrum of scalar perturbations at the pivot wavenumber of $k_0 = 0.05\ \text{Mpc}^{-1}$.

Additional parameters can be derived from the set of equation (17). The dimensionless Hubble parameter, $H$, is defined by the sum of all energy contributions. For a $\Lambda$CDM model, this is

$$H^2 = \omega_b + \omega_c + \omega_\nu + \omega_{\text{DE}}.$$

| Parameter | Prior |
|-----------|-------|
| $\Omega_b h^2$ | $U(0.019, 0.026)$ |
| $\Omega_c h^2$ | $U(0.01, 0.2)$ |
| $\Omega_{\text{DE}} h^2$ | $U(0.5, 10.0)$ |
| $\tau$ | $U(0.01, 0.8)$ |
| $\ln(10^{10} \Lambda_0)$ | $U(1.5, 4.0)$ |
| $n_s$ | $U(0.5, 1.5)$ |
| $b_1$ | $U(0.3, 0.9)$ |
| $b_2$ | $U(-0.0, 0.3)$ |
| $\omega_{\text{vir}}$ | $U(0.0, 12.0)$ |
| $\sigma_{\text{eff}}$ (eBOSS only) | $U(0.01, 6.0)$ |

It is also common to express the contributions of the various energy components in terms of the density parameters

$$\Omega_i = \omega_i / H^2,$$

which represent the fraction of the total energy density of the Universe corresponding to a given component $i$. The overall amplitude of matter density fluctuations is often characterized in terms of $\sigma_8$, the linear-theory RMS mass fluctuations in spheres of radius $R = 8\ h^{-1}\text{Mpc}$. A common property of these parameters is their dependence on the value of $h$. The issues associated with this dependence are discussed in detail by Sánchez (2020) and can be summarised as follows.

The main consequence of using quantities that depend on $h$ in cosmological analyses is that this complicates the comparison of constraints derived from probes that lead to different posterior distributions on $h$. This can be illustrated the most straightforwardly when considering $\sigma_8$, which is defined in terms of a scale in $h^{-1}\text{Mpc}$ units. As done by Sánchez (2020), the one-dimensional marginalised posterior distribution for $h$ can be used to obtain the corresponding posterior for $(8/h)\ Mpc$ to explore what physical distances this radius corresponds to. Fig. 1 repeats this simple exercise for the data sets considered in this work - as expected, the range of scales recovered in each case heavily depends on the type of probe considered (Planck displaying an extremely narrow posterior at the physical scale of approximately 12 Mpc, while the remaining probes cover varying ranges), especially in the case where the posterior of $h$ is simply limited by the prior imposed, as is the case for weak lensing data sets.

The solid line in Fig. 2 shows the density field variance $\sigma_R$ as a
the amplitude of a weighted average of \( \sigma_R \), the value measured is, in fact, a weighted average of \( \sigma_R \) over a range of \( R \).

function of the scale \( R \) in a Planck ΛCDM Universe. The shaded areas indicate the range of physical scales covered by the the posterior distributions of \( (8/h) \) Mpc for DES, BOSS, and Planck shown in Fig. 1. The issue with \( \sigma_8 \) is then that its marginalized value corresponds to a weighted average of \( \sigma_R \) on a range of scales that is different for each data set. A further complication is that the value of \( h \) also has an impact on the amplitude of \( \sigma_R \). As discussed in Sánchez (2020), these issues can be avoided by considering the variance of the density field on a reference scale in Mpc such as \( \sigma_{12} \), which is equivalent to \( \sigma_8 \) but is defined on a physical scale of 12 Mpc. We, therefore, opt to focus on \( \sigma_{12} \) and quantities that carry no explicit dependence on the Hubble constant \( h \) in order to enable us to appropriately compare and the constraints from our data sets.

We obtain the posterior distribution of all these parameters by performing Monte Carlo Markov chain (MCMC) sampling with CosmoMC (Lewis & Bridle 2002), which uses CAMB to calculate the linear-theory matter power spectra (Lewis et al. 2000), adapted to compute the theoretical model of our anisotropic clustering measurements described in Sec. 2.2. CosmoMC uses as basis parameters the set

\[
\theta_{\text{base}} = (\omega_b, \omega_c, \theta_{\text{MC}}, A_s, n_s) ,
\]

where \( \theta_{\text{MC}} \) is defined by a factor 100 times the approximate angular size of the sound horizon at recombination. With the exception of the physical baryon density, we assign flat uninformative priors to all the parameters of equation (20) as was done in Tröster et al. (2020). Our prior for \( \omega_c \) has to be restrictive, as our clustering measurements cannot constrain this parameter by themselves. Nevertheless, it is chosen to be approximately 25 times wider than the constraints on this parameter derived from Planck data alone (Planck Collaboration et al. 2020). Even though we do not sample the Hubble parameter \( h \), we still need to specify the values allowed - our chosen range 0.5 < \( h \) < 0.9 is wider than that of the KiDS-450 analysis of Hildebrandt et al. (2016) and comparable to the one used in the DES-yr1 fiducial analysis of Abbott et al. (2018). Joudaki et al. (2016) showed that the prior on \( h \) has no impact on the significance of the \( \sigma_8 \) tension. We list all the priors used in this analysis in Table 1.

As discussed by Sanchez et al. (2021), the effect of all evolution parameters on the linear matter power spectrum is degenerate: for a given set of shape parameters, the linear power spectra of all possible combinations of evolution parameters that lead to the same value of \( \sigma_{12}(z) \) are identical. This behaviour is inherited by the non-linear matter power spectrum predicted by RESPRESSO, which depends exclusively on \( P_L(k) \). However, the full model of \( P(k, \mu) \) does not follow this simple degeneracy due to the effect of bias, RSD and AP distortions. Of the parameters listed in equation (17), \( \omega_b \), \( \omega_c \), and \( n_s \) are shape parameters, while \( \omega_{\text{DM}} \) and \( A_s \) are purely evolution parameters. Other quantities such as \( h \) or \( \Omega_k \) represent a mixture of both shape and evolution parameters.

Present-day CMB measurements can constrain the values of most shape parameters with high accuracy, with posterior distributions that are well described by a multivariate Gaussian, independently of the evolution parameters being explored. On the other hand, clustering measurements on their own provide only weak constraints on the values of the shape parameters. However, if the shape parameters are fixed, clustering data can provide precise measurements of the evolution parameters. To test the impact of the additional information on the shape of the linear power spectrum, along with the priors described above, we use another set of priors to explore the constraints on the evolution parameters. For these runs, we impose Gaussian priors on the cosmological parameters that control the shape of the linear power spectrum - \( \omega_b \), \( \omega_c \), and \( n_s \). We derived the covariance matrix and mean values for these priors from our Planck-only posterior distributions. We refer to these constraints as the ‘Planck shape’ case.

### 2.5 Model validation

As we are using an updated prescription for the modelling of both the non-linear matter power spectrum and galaxy bias compared to the previous work of Tröster et al. (2020), we want to assess if it can recover unbiased cosmological parameter estimates, using mock data based on numerical simulations as a testing ground. We do so by applying our model to the mocks that were used for model validation in the original analyses: the Minerva simulations (Grieb et al. 2016; Lippich et al. 2019) for a BOSS-like sample and outerRim (Heitmann et al. 2019) for an eBOSS-like data set.

Minerva mocks are produced from a set of 300 N-body simulations with \( 1000^3 \) particles and a box size of \( L = 1.5 h^{-1}\text{Gpc} \). The snapshots at \( z = 0.31 \) and \( z = 0.57 \) were used to create halo catalogues with a minimum halo mass of \( M_{\text{min}} = 2.67 \times 10^{12}\text{h}^{-1}\text{M}_\odot \), which were populated with synthetic galaxies using the halo occupation distribution (HOD) model by Zheng et al. (2007) with parameters designed to reproduce the clustering properties of the LOWZ and CMASS galaxy samples from BOSS.

The outerRim (Heitmann et al. 2019) simulation uses \( 10 \times 240^3 \) dark matter particles to trace the dark matter density field in a \( L = 3 h^{-1}\text{Gpc} \) size box. We use a set of 100 mock catalogues constructed from the snapshot at \( z = 1.433 \), which was populated using an HOD model matching the clustering of the eBOSS QSO sample and tested extensively in the mock challenge (labeled as HOD0 in Smith et al. 2020). These realizations include catastrophic redshift failures at a rate of 1.5%, which corresponds to that of the eBOSS quasars.

We measured the mean clustering wedges of the samples from Minerva and the Legendre multipoles from outerRim with the same binning and range of scales as those of the real data from
BOSS and eBOSS and computed their corresponding theoretical covariance matrices using the Gaussian recipe of Grieb et al. (2016). We analysed these measurements using identical nuisance and cosmological parameter priors as for our final results and tested the validity of the model described in Sec. 2.2 with and without the assumption of the co-evolution relation for $\gamma_2$ of equation (9). We performed a joint fit of the two BOSS-like samples from MINERVA while the OuterRim measurements, which correspond to a different cosmology, were analysed separately. Fig. 3 shows the posterior distributions recovered from these measurements, which are in excellent agreement with the true input cosmology for all cases (shown by the dashed lines). Nevertheless, we find that setting the value of $\gamma_2$ according to equation (9) recovers the true parameter values more accurately for both samples and in tighter constraints than when it is freely varied. We therefore adopt this approach in the analysis of the clustering measurements from BOSS and eBOSS.

### 3 RESULTS

Our main results come from the combination of the full shape analyses of the BOSS galaxy clustering wedges and eBOSS QSO Legendre multiples described in Sec. 2.1. We also present combined late-Universe constraints obtained from the joint analysis of these clustering measurements with the $3 \times 2pt$ data set from DES Y1. For comparison, in Appendix A we present the constraints obtained using instead the cosmic shear measurements from KiDS-450, which lead to similar results. As we find a good agreement between BOSS + eBOSS + DES and Planck, we also present the parameter constraints obtained from the combination of all four data sets. These constraints are summarized in Table 2 and are discussed in Sects. 3.1 – 3.3.

![Figures](image-url)
3.1 Clustering constraints

Here we present the main result of our work - the combined flat CDM constraints from the anisotropic clustering measurements from BOSS and eBOSS. Fig. 4 shows the posterior distributions for BOSS and eBOSS separately (light blue and orange contours, respectively) as well as their combined constraints (green contours) for two sub-sets of cosmological parameters. For comparison, we also show the Planck-only constraints in dark blue. The panels on the left show the results on the more traditional parameter set of \( \sigma_8, \Omega_m, \) and \( H_0 \) whereas the ones on the right correspond to the alternative basis discussed in Sec. 2.4 of this paper.

Regardless of the parameter space considered, we find all of our data sets to be in good agreement with each other. The largest deviation between the joint BOSS + eBOSS constraints and those recovered from Planck can be observed in the matter density \( \Omega_m \), which displays a difference at the 1.7\( \sigma \) level. Nevertheless, this deviation does not indicate a similarly significant disagreement in the physical matter density preferred by these probes, as the value of \( \omega_m \) recovered by our clustering constraints matches that of Planck within 0.8\( \sigma \). This suggests that the differences seen in \( \Omega_m \) are related to the posterior distributions on \( h \) recovered from these data sets. Indeed, looking at our \( h \)-independent parameter space, we see that the marginalised constraint of the physical dark energy density also differs from the value preferred by Planck by 0.8\( \sigma \), with clustering measurements preferring slightly higher \( \omega_{\text{DE}} \), which translates into a higher value for \( H_0 \) and a lower \( \Omega_m \).

Tröster et al. 2020 found that the clustering measurements from BOSS wedges prefer a 2.1\( \sigma \) lower value of \( \sigma_8 \) as compared to Planck. Here we confirm the low preference, albeit with much lower significance due to the differences in the modelling of the power spectrum, for both \( \sigma_8 \) and \( \sigma_{12} \) (consistent with Planck at the 1.1\( \sigma \) and 1.3\( \sigma \) level, respectively). The increased consistency between these results is mainly due to the tighter constraints enabled by the use of the co-evolution relation of equation (9), which restricts the allowed region of the parameter space to higher values of \( \sigma_8 \) and \( \sigma_{12} \), as can be seen in Fig. 3. The constraints on \( \sigma_8 \) and \( \sigma_{12} \) recovered from eBOSS are at similar levels of agreement with Planck, however, the values recovered are 1.3\( \sigma \) and 1.2\( \sigma \) higher than the CMB results. This is also consistent with the most recent analysis by Hou et al. (2021) and Neveux et al. (2020), who found the inferred growth rate \( f \sigma_8 \) to be \( \sim 2 \sigma \) higher than the CDM model with the best-fitting Planck parameters. The combination of the clustering measurements from BOSS and eBOSS is, therefore, in an overall excellent agreement with Planck - with differences at the level of 0.05\( \sigma \) for \( \sigma_8 \) and 0.04\( \sigma \) for \( \sigma_{12} \).

As discussed in Sec. 2.4, the shape parameters \( \omega_0, \omega_c, \) and \( n_s \) are all tightly constrained by Planck with posterior distributions that are in complete agreement with those inferred from the other cosmological probes considered here. We can, therefore, study the improvement in the constraints on the evolution parameters \( \omega_{\text{DE}} \) and \( A_s \) that are obtained from the LSS probes when the shape of the power spectrum is constrained to match that of Planck’s cosmology. As described in Sec. 2.4, we achieve this by adding an informative Gaussian prior on the shape parameters based on our Planck runs and repeating our analysis with an otherwise identical set up.

The results of this exercise are shown in Fig. 5. As the two data sets were already in a good agreement across the parameter space, including the shape parameters, imposing additional priors simply adds constraining power on the degenerate evolution parameters, most notably \( \omega_{\text{DE}} \) (degenerate with \( \omega_m \)), which is recovered to be slightly higher than the Planck value to compensate the slight shifts in \( \sigma_{12} \) and \( \ln(10^{10}A_s) \) to lower values.

3.2 Consistency with Planck

When looking at marginalised posteriors we are limited by our selection of the parameter space as well as the associated projection effects and, while we can use the standard deviation to quantify agreement...
First, we want to establish agreement over the whole parameter space considered. In order to do this, we use the suspiciousness tension metric, $S$, introduced by Handley & Lemos 2019. The main advantages of using suspiciousness include the fact that it measures the agreement between two data sets across the entire parameter space, similarly to the Bayes factor $R$. However, unlike $R$, the suspiciousness is by construction insensitive to prior widths, as long as the posterior is not prior-limited. Given two data sets, $A$ and $B$, the suspiciousness quantifies the mismatch between them by comparing the relative gain in confidence in data set $A$ when data set $B$ is added (as measured by $R$) with the unlikeliness of the two data sets ever matching as measured by the information ratio $I$, that is

$$\ln I = \ln R - \ln S.$$  

(21)

Following the method described in Heymans et al. (2021), we redefine $\ln R$ and $\ln I$ in terms of the expectation values of the log-likelihoods ($\ln L$) and evidences $Z$. The evidences, however, cancel out and we are able to calculate $S$ from the expectation values only:

$$\ln S = \langle \ln L_{A+B} \rangle_{p_{A+B}} - \langle \ln L_A \rangle_{p_A} - \langle \ln L_B \rangle_{p_B}.$$  

(22)

The value of $S$ can then be interpreted using the fact, that for Gaussian posteriors, the difference $d - 2 \ln S$, where $d$ is the Bayesian model dimensionality, is $\chi^2_d$ distributed. We calculate $d$ for each of the data sets separately, $d_A$ and $d_B$, and their combination, $d_{A+B}$, as described in Handley & Lemos (2019) and combine the results as $d = d_A + d_B - d_{A+B}$.

Applying this procedure to eBOSS + BOSS and Planck, we find $\ln S = 0.41 \pm 0.07$ with a Bayesian dimensionality of $d=4.5 \pm 0.4$, which correctly indicates that there are approximately 5 cosmological parameters shared between the two data sets. This can then be related to a p-value of $p = 0.52 \pm 0.02$ or a tension of $0.64 \pm 0.03 \sigma$, which is consistent with the $0.76 \pm 0.05 \sigma$ tension between Planck and BOSS alone found by Tröster et al. (2020) and indicates a good agreement between these data sets.

In addition to the suspiciousness, we want to use a tension metric that allows for a greater control to focus only on a selected subset of parameters. For this purpose, we use the update difference-in-mean statistic, $Q_{UDM}$, as described in Raveri & Hu (2019) and implemented in Tensimeter$^1$ (Lemos et al. 2021). This statistic extends the simple difference in means, where the difference in mean parameter values $\bar{\theta}$ measured by two data sets is weighted by their covariance $C$. The ‘update’ in UDM refers to the fact that instead of comparing data set $A$ with data set $B$, we consider the updated information in the combination $A + B$ with respect to $A$ by means of

$$Q_{UDM} = \left( \hat{\theta}_{A+B} - \hat{\theta}_A \right)^T \left( C_A - C_{A+B} \right)^{-1} \left( \hat{\theta}_{A+B} - \hat{\theta}_A \right).$$  

(23)

This has the advantage of the posterior of $A + B$ being more Gaussian than that of $B$ alone. For Gaussian distributed parameters, $Q_{UDM}$ is chi-square distributed with a number of degrees of freedom given by the rank of $\left( C_A - C_{A+B} \right)$. The calculation of $Q_{UDM}$ may be performed by finding the Karhunen–Loève (KL) modes of the covariances and re-expressing the cosmological parameters in this basis. This transformation allows us to reduce the sampling noise by imposing a limit to the eigenvalues of the modes that are considered

and in this way cutting out those that are dominated by noise (which represent the directions in which adding $B$ does not improve the constraints with respect to $A$). The number of remaining modes correspond to the degrees of freedom with which $Q_{UDM}$ is distributed. For our tension calculations we, therefore, only select the modes $\alpha$ whose eigenvalues $\lambda$ satisfy:

$$0.05 < \lambda_\alpha - 1 < 100.$$  

(24)

This corresponds to requiring that a mode of the base data set is updated by at least 5 per-cent. We subsequently find that there are 2 modes being constrained when Planck is updated by both probe combinations considered in this work (BOSS + eBOSS and BOSS + eBOSS + DES).

For BOSS + eBOSS we get $Q_{UDM} = 2.0$ for the full parameter space, resulting in a ‘tension’ with Planck of $0.90 \sigma$ - only slightly higher than what $S$ suggests.

### 3.3 Joint analysis with DES data

Following Tröster et al. (2020), we want to further investigate the constraints from multiple low-redshift probes together by adding a weak lensing data set - in this case, the $3 \times 2pt$ measurements from DES Y1. Tröster et al. (2020) used the suspiciousness statistic and showed that the combination of BOSS clustering and KiDS-450 shear measurements are in $\sim 2\sigma$ tension with Planck. The most recent KiDS-1000 $3 \times 2pt$ analysis (Heymans et al. 2021), where the BOSS galaxy sample was used for galaxy clustering and galaxy-galaxy lensing measurements, also found a similar level of tension when the entire parameter space is considered. As DES Y1 measurements have no overlap with either BOSS or eBOSS, we can treat these data sets as independent and easily combine them to test whether we also find a similar trend.
The resulting constraints are presented in Fig. 6. We confirm that DES is in good agreement with eBOSS + BOSS (with ln S = −1.08 ± 0.05, which corresponds to a 1.3 ± 0.08σ tension) and it is, therefore, safe to combine them. The addition of DES data to the analysis provides only slightly tighter constraints with respect to eBOSS + BOSS, with the greatest improvement in σ₁₂, and an overall good agreement with Planck.

Nevertheless, it is worth noting that, when considering the two-dimensional posterior projections, there are two parameter combinations in particular for which the 1σ contours of DES + BOSS + eBOSS and Planck do not overlap. The slight discrepancy we observe in the Ωₘ − σ₁₂ plane is reminiscent of the ‘c₈’ tension seen in Ωₘ − σ₈ and is larger than the discrepancy displayed by either of the probes individually. In addition to that, we also see a similarly slight disagreement in the ln(10¹⁰A₈) − σ₁₂ plane. The projection of A₈ with σ₁₂ (as opposed to σ₈) allows us to recover the tight degeneracy between the two parameters which exposes how, for a given present-day clustering amplitude, low-redshift probes prefer a higher initial power spectrum amplitude.

We find that adding the DES Y1 3 × 2pt measurements worsens the agreement with Planck with respect to the results obtained from the combination of BOSS and eBOSS alone. We obtain a suspiciousness of ln S = −1.86 ± 0.04, corresponding to a tension of 1.54 ± 0.08σ. When considering the UDM statistic across the entire shared parameter space, we find Q_{UDM} = 6.3 distributed with 2 degrees of freedom, which translates into a tension at the 1.9σ level. As for the case of the clustering-only constraints, Q_{UDM} indicates a greater level of tension than S.

Lemos et al. (2021) found that the DES Y1 3 × 2pt measurements alone are in a 2.3σ tension with Planck, as measured by Q_{UDM}. This increases to 2.4 ± 0.02σ when using the suspiciousness statistic. These levels of tension are comparable with what we find from the full combination of low-redshift probes. The tension between Planck and weak lensing data sets is usually interpreted as a reflection of tension in the parameter combination S₈ = σ₈(Ωₘ/0.3)^0.5, that is taken to describe the ‘lensing strength’. The S₈ value that we recover from the joint low redshift probes is also about 2σ lower than the Planck constraint (see Table 2). Nevertheless, as we see in Fig. 6, there is a comparable discrepancy in ln(10¹⁰A₈) − σ₁₂ plane. We can use Q_{UDM} in order to quantify and compare the level of tension present in these two-dimensional projections by calculating it for a subset of shared parameter space. We find that the amount of tension in both Ωₘ − σ₈ and its h-independent equivalent is ~ 2.0σ, whereas ln(10¹⁰A₈) − σ₁₂ displays a slightly higher tension of 2.5σ.

We also repeated our fitting procedure with an additional Gaussian prior on the parameters controlling the shape of the power spectrum to be consistent with Planck, as described in Sec. 2.4. The resulting posteriors are shown in Fig. 7. We observe the same general trends as from the analysis of our clustering data alone discussed in Sec. 3.1. However, the prior on the shape parameters leads to larger shifts in the evolution parameters. This is expected, as DES data on their own cannot constrain the shape parameters well. Adding the informative priors breaks the degeneracies between shape and evolution parameters and increases the constraining power significantly. This, in turn, exposes any discrepancies in the evolution parameters. The values of σ₁₂ and ln(10¹⁰A₈) preferred by our low-redshift probes when an informative prior is imposed are, respectively, 1.89σ and 1.22σ lower than the corresponding Planck values. Meanwhile, the recovered value for ωₐDE is 1.73σ higher.
4 DISCUSSION

The flat $\Lambda$CDM constraints from the low-redshift probes presented in Sec. 3 show a consistent picture. Updating the power spectrum model and supplementing the clustering measurements with eBOSS data brings the joint BOSS + eBOSS constraints to a better agreement with Planck than the BOSS-only results from Tröster et al. (2020). These constraints are not significantly modified when these data are combined with DES, resulting in a good overall consistency with Planck across the entire parameter space, as indicated by both $S$ and $Q_{\text{UDM}}$.

Nevertheless, when considering specific two-dimensional projections we still see intriguing differences, mainly driven by the lensing data. Although the constraints in the $\sigma_{12} - \omega_m$ plane obtained using BOSS + eBOSS and DES data separately do not show the discrepancy with Planck that characterizes the results in their $h$-dependent counterparts of $\sigma_8$ and $\Omega_m$, the full combination of low-redshift data tightens the degeneracy between these parameters and leads to constraints that are just outside the region of the parameter space preferred by Planck.

We also see differences in the $\log(10^{10} A_s) - \sigma_{12}$ plane between DES and Planck, which are inherited by the full combination of low-redshift data sets. The tight relation between these parameters, which is not seen when using $\sigma_8$, illustrates the closer link between $\sigma_{12}$ and the overall amplitude of density fluctuations obtained by eliminating the ambiguity caused by the dependency on $h$. For a given value of $\sigma_{12}$, Planck measurements prefer a lower initial amplitude of density fluctuations than DES, suggesting a discrepancy in the total growth of structures predicted by these two data sets.

Within the context of a $\Lambda$CDM model, the key parameter controlling the growth of structure at low redshift is the physical dark energy density. Indeed, as can be seen in Fig. 6 the posterior distribution of $\omega_{\text{DE}}$ recovered from DES extends to significantly higher values than the one obtained using Planck CMB measurements. The tendency of the low-redshift data to prefer a higher value of $\omega_{\text{DE}}$ than that of Planck can be seen more clearly in the results obtained after imposing a prior on the shape parameters shown in Fig. 7. In this case, we find $\omega_{\text{DE}} = 0.335 \pm 0.011$ using BOSS + eBOSS + DES while Planck data lead to $\omega_{\text{DE}} = 0.3093 \pm 0.0093$. A higher value of $\omega_{\text{DE}}$ corresponds also to a higher value of $h$. Therefore, this difference is also interesting in the context of the Hubble tension, as many of the proposed solutions to this issue focus on modifying the dark energy component.

The analysis of the consistency between low- and high-redshift data has been focused on the comparison of constraints on $S_8$, which depends on the present-day value of $\sigma_8$. Fig. 8 shows the redshift evolution of $S_8(z)$ predicted by Planck and the combination of all low-redshift data sets. These curves are consistent at high redshift during matter domination and start to diverge at $z < 1$ to reach a difference at the $2\sigma$ level at $z = 0$. However, as this redshift is not probed by any LSS data set, the value of $S_8(z = 0)$ is an extrapolation based on the assumption of a $\Lambda$CDM background evolution. Extending this extrapolation to $a > 1$, the difference between the two cosmologies continues to increase and becomes even more significant. Therefore, quoting the statistical significance of any discrepancy in the recovered values of $S_8(z = 0)$ might not be the best characterization of the difference in the cosmological information content of these measurements.

As discussed before, DES and Planck data appear to prefer different evolutions for the growth of cosmic structure, which in a $\Lambda$CDM universe depends on $\Omega_m$ and $\omega_{\text{DE}}$. As the former is exquisitely constrained by Planck for general parameter spaces, the latter is perhaps the most interesting parameter to consider. As $\omega_{\text{DE}}$ is constant in redshift for a $\Lambda$CDM universe, the deviations in the value of this parameter recovered from different data sets could be used as an indication of their consistency within the standard cosmological model.

5 CONCLUSIONS

We obtained constraints on the parameters of the standard $\Lambda$CDM model from the joint analysis of anisotropic clustering measurements in configuration space from BOSS and eBOSS. In particular, we used the information of the full shape of the clustering wedges of the final BOSS galaxy samples obtained by Sánchez et al. (2017) and the legendre multipoles of the eBOSS DR16 QSO catalogue of Hou et al. (2021). We updated the recipes to describe the non-linear matter power spectrum and the non-local bias parameters with respect to those used in the BOSS-only analyses of Sánchez et al. (2017) and Tröster et al. (2020). We directly compared our theoretical predictions for different cosmologies against the BOSS and eBOSS clustering measurements, without the commonly used RSD and BAO summary statistics. We focus on cosmological parameters that can be classified either as shape or evolution parameters (Sánchez et al. 2021), such as the physical matter and dark energy densities, instead of other commonly used quantities such as $\Omega_m$ and $\omega_{\text{DE}}$, that depend on the value of $h$. Our constraints from the combination of BOSS + eBOSS represent improvements ranging from 20 to 25 per-cent with respect to those of Tröster et al. (2020) and are in excellent agreement with Planck, with the suspiciousness and updated difference in means tension metrics indicating agreement at the level of 0.64$\sigma$ and 0.90$\sigma$, respectively.

We combined the clustering data from BOSS and eBOSS with the $3 \times 2$pt correlation function measurements from DES Y1 to obtain joint low-redshift cosmological constraints that are also consistent with the $\Lambda$CDM Planck results, albeit with larger deviations (1.54$\sigma$...
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DATA AVAILABILITY

The clustering measurements from BOSS and eBOSS used in this analysis are publicly available via the SDSS Science Archive Server (https://sas.sdss.org/).

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2.00σ differences as inferred from S and Q_{UDM}, respectively). We do see interesting discrepancies in certain parameter combinations at the level of 2σ or more, such as the Ω_m − σ_S and ω_m − σ_{12} planes, and, more significantly, in the log(10^{10} A_S) − σ_{12} projection. For a given value of σ_{12}, low-redshift probes (mostly driven by DES) prefer a higher amplitude of primordial density fluctuations than Planck, indicating differences in the total growth of structure predicted by these data sets.

We further tested the impact of imposing a Gaussian prior on ω_h, ω_c, and n_s representing the constraints on these shape parameters recovered from Planck data. Such prior leads to a significant improvement in the constraints on the evolution parameters, such as ω_{DE} and A_s. In this case, we find that the full combination of low-redshift data sets prefers a value of the physical dark energy density ω_{DE} that is 1.7σ higher than that preferred by Planck. This discrepancy, which is also related to the amount of structure growth preferred by these data sets, offers an interesting link with the H_0 tension, as it points to a higher value of h being preferred by the low-redshift data.

The advent of new large, high-quality data sets such as the Dark Energy Spectroscopic Instrument (DESI, DESI Collaboration et al. 2016), the ESA space mission Euclid (Laureijs et al. 2011), and the Legacy Survey of Space and Time (LSST) at the Rubin Observatory (Ivezic et al. 2019), will allow us to combine multiple probes and significantly tighten our cosmological constraints. The discussion of the consistency between different data sets has so far been centred on the comparison of constraints on S_8(z = 0). As we move on to the analysis of Stage IV data sets, it would be beneficial to shift our focus from best constrained parameter combinations within a ΛCDM scenario to quantities that more closely represent the cosmological information content of these data, or that have a more direct physical interpretation.
Full-shape analysis of BOSS & eBOSS

APPENDIX A: JOINT ANALYSIS WITH KIDS-450 DATA

In this section we present the joint analysis of the anisotropic clustering measurements from BOSS and eBOSS together with cosmic shear measurements from KiDS-450, Hildebrandt et al. 2016.

We use cosmic shear measurements from the Kilo-Degree Survey, (Kuijken et al. 2015; Hildebrandt et al. 2016; Fenech Conti et al. 2017), hereafter referred to as KiDS. The KiDS data are processed by THELI (Erben et al. 2013) and Astro-WISE (Begeman et al. 2012; de Jong et al. 2017). Shears are measured using lensfit (Miller et al. 2013), and photometric redshifts are obtained from PSF-matched photometry and calibrated using external overlapping spectroscopic surveys (see Hildebrandt et al. (2016)).

The KiDS-450 weak lensing data set consists of tomographic shear measurements from four redshift bins spanning the total range of 0.1 < z ≤ 0.9 and the corresponding source redshift distributions estimated from the weighted direct calibration (‘DIR’) for each bin (Lima et al. 2008). We use the recommended scale cuts and use the angular bins with θ < 72 arcmin for ξ_r(θ) and θ > 6 arcmin for ξ_s(θ).

It is important to note that here we are using the same DES shear model as in the main analysis. This means that the treatment of the nuisance parameters (namely, the baryonic effects, the photometric redshift uncertainty and the additive and multiplicative bias parameters) differs from the original analysis of KiDS-450. We do not include baryonic effects and our priors for photometric redshift uncertainty match those of Hildebrandt, H. et al. 2020. We impose flat priors for multiplicative bias U(−0.1, 0.1) with the additive bias parameter taken to be zero, mimicking the DES set up. Finally, we also follow the DES intrinsic alignment model, imposing a flat prior on redshift evolution of intrinsic alignment parameter. We compare our final posteriors with the ones obtained from the publicly available KV450 chains (Hildebrandt, H. et al. 2020) and find a good agreement between the two, as shown in Fig. A1. Moreover, there is a weak tendency for our analysis to prefer lower RMS variance values, which leads to a more conservative assessment of any potential tensions with Planck.

Fig. A2 shows the combined constraints from the combination of KiDS-450 + BOSS + eBOSS (orange contours), which are in near-perfect agreement with the equivalent combination using DES (grey contours). The suspiciousness statistic shows an agreement between Planck and this set of low-redshift measurements of 1.5 ± 0.3σ, which is also consistent with our results from BOSS + eBOSS + DES.

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**Figure A1.** Comparison of the marginalised posterior distributions between the fiducial KV450 analysis and this work. For this comparison, we adapted the cosmological parameter priors to match those of the fiducial analysis.

**Figure A2.** Comparison of low-redshift constraints obtained by combining BOSS + eBOSS with either DES Y1 3 × 2pt data (grey contours) or the KiDS-450 shear measurements (orange contours).