How fast is a quantum jump?

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Abstract

A proposal is put forward for an estimate of the duration of a transition between atomic states. The proposal rests on the consideration that a resonance of the atomic electron with modes of the zero-point radiation field of Compton’s frequency is at the core of the phenomenon. The theoretical result, given essentially by the expression $(\alpha \omega_C)^{-1}$, where $\alpha$ is the fine structure constant and $\omega_C$ the Compton angular frequency for the electron, lies well within the range of the recently experimentally estimated values of the order of attoseconds ($10^{-18}$ s).

Keywords: zitterbewegung; atomic transition; jumping time; zero-point field; stochastic electrodynamics

1 Introduction

It is in a way puzzling that the physics involved in atomic quantum jumps (or single atomic transitions) has remained in almost complete darkness, the more so considering the crucial role spectroscopy has played for more than a century, and the impressive advances in both theoretical and experimental quantum physics. Attention to this intriguing subject has presumably been hindered for a long time by the masterful dogma of the instantaneous character of atomic transitions postulated by Bohr [1]—and bitterly opposed by Schrödinger [2, 3]. One can still come across articles negating quantum jumps—and any other kind of discontinuities, for that matter (e.g. [4])—or taking them as a sudden increase of our knowledge of the system (e. g., [5, 6]) rather than a physical phenomenon.

For atomic and molecular spectroscopists it is clear that quantum jumps exist; this is part of their daily bread. Most spectroscopists are also aware that the time involved in a transition is finite but very short; so short indeed that the Franck-Condon principle applies, which sets an upper limit to their duration, on the order of femtoseconds ($10^{-15}$ s). Because atomic transitions are so fast, up to recently they were considered “instantaneous”, this term being taken by

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some in the rigorous sense, and by others as meaning “in an unmeasurably (or
unobservably) short time”. This picture, however, is changing thanks to recent
calculational and experimental work, notably using attosecond spectroscopy ap-
p lied to photoionization [7, 8]. Photoionization experiments in bulk materials
are known to involve electronic correlations, which makes it difficult to ascertain
the time it takes for one atom to lose one electron. With this caveat, chronos-
cope measurement of the times involved in the photoelectric effect assigns to
the primary photoexcitation process a duration on the order of $10^{-17}$ s [9]. Fur-
ther, although not directly comparable to (natural) atomic transitions, recent
experimental work with an artificial atom (a superconducting circuit consisting
of two hybridized qubits on a chip) in which a quantum jump is intercepted and
reverted by means of an electric pulse, seems to confirm Schrödinger’s intuition
that the evolution of the jump itself is continuous and needs a finite time to take
place [10]. This is in line with Schulman’s definition of “jump time” as the time
scale such that perturbations occurring at intervals of this duration affect the
transition [11]. Based on his definition, Schulman’s own estimate made in terms
of the “Zeno time” (related to the second moment of the Hamiltonian) and the
natural lifetime, results however in a much shorter time than the experimental
estimates, as short as $10^{-20}$ s for atomic transitions.

The various computational and experimental estimates have contributed to
establish the existence of (finite-time) quantum jumps, and have apparently set
tighter bounds on their duration. The basic physics behind the process, however,
has not been clarified, so the question remains: what is it that determines the
duration of a transition?

In the present work we attempt to throw light on this question via a the-
etorical analysis that does not rely on specific experimental settings. We do
so by invoking the existence of the electromagnetic zero-point radiation field
(ZPF) and applying the conventional approach followed in stochastic electrodyn-
amics (SED) to the specific problem of the dynamics of the electron during a
transition. We start by recalling Schrödinger’s work on the zitterbewegung as
a rapid oscillation of the Dirac electron, and appeal to SED to identify it as a
result of its resonance with the Compton-frequency components of the ZPF. The
stationary solution of the equation of motion for the rapidly oscillating electron
corresponds to the zitterbewegung; the transient solution, in its turn, describes
the dynamics of the transition between states. The decay time associated with
the transient solution, which we propose to take as an approximate measure of
the transition time, is expressed in terms of universal constants and its value is
of the order of $10^{-18}$ s.

2 The guiding premise

To put the discussion on track we start by recalling the source of the zitterbe-
wegung as disclosed by Schrödinger [12] in his revision of the properties of the
free particle in Dirac’s theory of the electron. This will signal the importance
of the Compton-frequency modes of the ZPF for the dynamics of the electron,
and pave the way for their consideration as a central element in the transition.

A well-known result in Dirac’s theory of the free electron is that the velocity operator is \( c \hat{\alpha} \), where \( \hat{\alpha} \) is one of the matrices in the Dirac theory and \( c \) stands for the velocity of light in vacuum; this is expressed in the equation \( \dot{x} = c \hat{\alpha} \).

As shown by Schrödinger [12], the Heisenberg equation of motion gives for \( \dot{x} \) in terms of the canonical momentum \( p \) acquired by the particle with energy \( E \) at time \( t \), the expression (for simplicity we use one-dimensional notation)

\[
\frac{E}{c^2} \dot{x} = p - \left( p - \frac{E}{c^2} \dot{x}(0) \right) e^{-i2Et/\hbar}.
\]

To arrive at this result, Schrödinger considered both the energy and the momentum of the free particle as having a constant value; the coefficient \( E/c^2 \) stands for the mass of the particle.

Let us now compare this result with the conventional definition of the canonical momentum \( p \) for a particle in the presence of an electromagnetic field \( A \), namely

\[
m \ddot{x} = p - \frac{e}{c} A.
\]

The comparison may seem unwarranted, since from the viewpoint usually adopted to read the quantum-mechanical formalism, no electromagnetic field exists other than any expressly recognized external field, whence for the free particle one should take \( A = 0 \). Here, by contrast, we propose to explore the possibility of taking the comparison at face value. This means that the quantity \( (p - \frac{E}{c^2} \dot{x}(0)) e^{-i2Et/\hbar} \) in Eq. (1) represents the effect of an acting electromagnetic field of very high frequency, essentially twice Compton’s frequency \( \omega_C = mc^2/\hbar \), and an amplitude of the order of Compton’s wavelength \( \lambda_C = \hbar/mc \), as follows from an integration of Eq. (1). This extra oscillation of the (free) particle predicted by the Dirac equation exhibits the zitterbewegung as a real helicoidal motion with velocity \( c \) around the particle trajectory [12]. Incidentally, notice that Schrödinger had real particle trajectories in mind.

The above identification acquires full sense within the framework of SED, which is based on the premise that the electron is permanently embedded in the ZPF (for different reviews of SED see, e.g., [13][14][15][16]). The (nonrelativistic) SED equation of motion for a particle of charge \( e \) and mass \( m \), subject to an external force \( f(x) \), is the corresponding Abraham-Lorentz equation of classical electrodynamics, extended to include the ZPF. It reads (here the dynamical variables are c-numbers)

\[
m \dddot{x} = f(x) + m \tau \ddot{x} + e \left[ E(x, t) + \frac{\dot{x}}{c} \times B(x, t) \right].
\]

The term \( m \tau \ddot{x} \) stands for the (nonrelativistic) expression for the force due to radiation damping, with \( \tau = 2e^2/3mc^3 \sim 10^{-23} \) s for the electron. The term within brackets is the Lorentz force due to the radiation field. In consonance with the nonrelativistic treatment, the field is normally taken in the dipole approximation, whence (we resume one-dimensional notation, for simplicity),

\[
m \dddot{x} = f(x) + m \tau \ddot{x} + eE_x(t).
\]
For the treatment of the majority of atomic problems in quantum mechanics
this approximation has proven legitimate and sufficient; even the radiative life-
times and (nonrelativistic) radiative corrections are correctly obtained under
this approximation [14, 15].

Using canonical variables, for which \( \dot{p} = f(x) \), and writing in the Coulomb
gauge \( E_x = -\frac{1}{c} \partial A_x / \partial t \), integration of Eq. (4) leads to
\[
mx' = p + m\tau \ddot{x} - \frac{e}{c} A_x(t) = p - \frac{e}{c} A_T(t).
\] (5)

In the second equality, \( A_T(t) \) represents the total radiation field in the \( x \)
direction, \( A_T = A_x - (2e/3c^2)\dot{x} \). In the absence of external radiation fields, this
reduces to the ZPF plus particle radiation (a more detailed discussion can be
seen in chapter 6 of Ref. [14]).

Now the comparison of Eqs. (1) and (5) is immediate, (5) being the non-
relativistic (sed) counterpart of (1). This comparison suggests that—as is fre-
quently the case—the relativistic treatment of the quantum problem automatic-
ically includes (some of) the effects of the ZPF on the motion of the particle, even
if this field is not expressly introduced. In other words, the quantum description
already contains information about the presence of the ZPF.

According to this discussion, within the framework of sed the oscillations
manifested as zitterbewegung are the result of a resonant interaction of the
particle with the components of the ZPF having a frequency of the order of
Compton’s frequency. This suggests a prominent role for \( \omega_C \) in the dynamical
behaviour of the atomic electron, and gives a clue for understanding other
dynamical effects, even in the nonrelativistic scenario, as will be shown in the
following section in relation with atomic transitions.

3 How fast is a quantum jump?

We turn now to our task of estimating an order of magnitude for the time it
takes the atomic electron to make a transition between states, guided by the
above considerations. The gist of our argument is, as stated above, the acknowl-
edgement that the electron resonates with the modes of the ZPF of Compton’s
frequency, in addition to the (slow) motion impressed upon it by the external
forces and the low frequency components of the ZPF. We shall take the simplest
nonrelativistic approach to tackle the problem. Therefore, we apply Eq. (4)
to the actual position variable—which we denote now as \( x'(t) \) instead of \( x(t) \),
with \( x' = x + z \)—and separate the terms corresponding to the slow motion,
represented by \( x(t) \), from those associated with the (normally) small but rapid
motion, represented by \( z(t) \).

\[1\] The relativistic frequency is \( 2mc^2/\hbar \) due to the simultaneous consideration of both the
positive and negative energies. In the nonrelativistic case the Compton frequency \( mc^2/\hbar \) is a
more natural limit for the descriptive capacity of the theory. More detailed discussions about
the electron resonance at Compton’s frequency can be seen in Refs. [17, 18].
A Taylor series expansion up to first order in $z$, of $x'(t)$ around $x(t)$, gives for the equation describing the slow motion

$$m\ddot{x} = f(x) + m\tau \dot{x} + eE'(t),$$

(6)

where $E'(t)$ represents the zpf except for its high-frequency modes. This is equivalent to the usual SED equation of motion in the long-wavelength approximation, and, as said above, it serves in general to solve nonrelativistic atomic problems. In particular, the correct radiative lifetimes are obtained both for spontaneous and induced transitions, as a result of the resonant response of the atomic electron to the (long-wavelength) modes of the radiation field, the zpf included [15, 19].

To study the dynamics of the transition itself we need the equation of motion for the fast variable $z(t)$, which is obtained by collecting the remaining terms not contained in Eq. (6) and including a force term $-m\omega_C^2 z$ to account for the resonance of the electron at the Compton frequency,

$$m\ddot{z} = zf'(x) - m\omega_C^2 z + eE_C(t) + m\tau \dot{z},$$

(7)

where $E_C(t)$ stands for the high-frequency modes of the zpf. The term $zf'(x)$ is small compared with the remaining force terms and can be neglected; we are thus left with

$$m\ddot{z} = -m\omega_C^2 z + m\tau \dot{z} + eE_C(t).$$

(8)

The stationary (forced) solution of this inhomogenous equation represents the zitterbewegung, which takes place during the entire life of the electron, thanks to the permanent action of the high-frequency zpf modes represented by $E_C$, as discussed in Sect. 2. In addition, the homogeneous part of the equation admits a transient solution $z_{tr}(t)$. Writing to first order in $\tau$

$$\omega_C \sqrt{1 + i\tau \omega_C} \simeq \omega_C + \frac{1}{2} i\tau \omega_C^2,$$

(9)

we obtain

$$z_{tr}(t) = z_0 \exp(i\omega_C \sqrt{1 + i\tau \omega_C} t) + c.c. \simeq e^{-\frac{\tau \omega_C^2 t}{2}} \left( z_0 e^{i\omega_C t} + z_0^* e^{-i\omega_C t} \right),$$

(10)

where the constants of integration $z_0$ and $z_0^*$ are to be determined by the initial value $z_{tr}(0)$.

We propose to identify the irreversible change in the state of motion described by Eq. (10) with a transition between (atomic) states, $z_{tr}(0)$ giving an idea of the distance traveled during the transition. The change of state implies a well-defined increase or decrease in the energy of the system, normally accompanied by an absorption or emission of radiation. It does not, however, entail in principle any discontinuity in the trajectory—as the image implicit in the notion of a “quantum jump” seems to suggest.

The characteristic time $T_{tr}$ for the decay, which according to this proposal can be taken as a measure of the duration of the transition between states, is
(using $\tau = 2e^2/3mc^3$ and $\omega_C = mc^2/\hbar$)

$$T_{tr} \simeq \frac{2}{\tau \omega_C^2} = \frac{3\hbar^2}{e^2mc},$$

(11)

It is remarkable that $T_{tr}$ becomes expressed in terms of the four fundamental constants $e, m, c, \hbar$, meaning that its order of magnitude can be evaluated from simple dimensional considerations. Noting that $\tau \omega_C = 2\alpha/3$, where $\alpha = e^2/\hbar c$ is the fine-structure constant, we get for $T_{tr}$ the alternative expressions

$$T_{tr} = \frac{3}{\alpha \omega_C} = \frac{3T_C}{2\pi\alpha},$$

(12)

which shows that the transition time is larger by two orders of magnitude than the Compton time $T_C$. With $\lambda_C = 2.43 \times 10^{-10} \text{ cm}$, we have $T_C = 8.1 \times 10^{-21} \text{ s}$, which gives

$$T_{tr} \simeq 65.4 \ T_C = 0.53 \times 10^{-18} \text{ s}.$$ (13)

The theoretical value thus obtained for the jumping (transition) time lies well within the range of recent empirical evaluations. A more elaborate, relativistic treatment would most certainly produce more precise results.

Finally, it is interesting to note that the times corresponding to the (inverse of the) frequencies of atomic spectral lines lie approximately between $0.3 \times 10^{-18} \text{ s}$ and $0.3 \times 10^{-17} \text{ s}$, which would indicate that the (emitted or absorbed) radiation field performs about one oscillation during the transition process.

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