Numerical Simulations for Signals from MgB$_2$ Superconducting Detector

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Abstract. An MgB$_2$ neutron detector uses an energy released by a nuclear reaction between neutron and $^{10}$B. An instantaneous deterioration of superconductivity in a restricted regime can be observed as a pulsed signal under current-biased conditions. It would be useful to analyze local heat dynamics for designing an optimized MgB$_2$ neutron detector. We perform numerical simulations by means of the heat transport equation and the finite element method (FEM) to obtain a pulsed signal from a detector model.

1. Introduction

Cold neutrons of long wave length are very useful in wide fields of researches on bioscience, nanoscience and new materials. There are strong demands for higher intensity of neutrons for efficient measurements. As a matter of fact, several competitive high-intensity proton accelerators are under construction in the world. For example, Japan Proton Accelerator Research Complex (J-PARC) will be in full operation in a few years. The J-PARC converts from high-intensity pulsed protons to high-intensity pulsed neutrons by bombarding protons into mercury target. This is the so-called spalation neutron source. Unfortunately, conventional neutron detectors are too slow to count such high-intensity neutrons. Required innovations for neutron detectors are to pursue a high sensitivity, a fast response, a high energy resolution and a high spatial resolution.
A sharp resistive superconducting transition can be used to detect various sorts of radiations through a calorimetric change in the resistive superconducting transition. For example, an X-ray quantum on the order of keV can be measured by a transition edge sensor (TES). A TES type detector can operate at very low temperatures achieved by a dilution refrigerator or an adiabatic demagnetizing refrigerator.

In the present work, we apply a similar principle to an MgB$_2$ TES detector. The benefits of using MgB$_2$ detector are listed as follows. (1) The detector can operate at very high temperatures since the critical temperature of MgB$_2$ can reach 39 K. This means that an MgB$_2$ detector is much easier to achieve an operation temperature since an inexpensive cryogen-free refrigerator is enough to cool the detector down to operating temperatures. (2) An operation at higher temperatures is also helpful to pursue a higher speed since a thermal conductivity is primarily proportional to the specific heat. (3) When we measure a neutron by the MgB$_2$ detector, we are able to utilize huge energy on the order of MeV released by a nuclear reaction between neutron and $^{10}$B in MgB$_{10}$. This may overcome the increase of thermal noise at higher temperatures. (4) Since the ranges of alpha and lithium particles are on the order of $\mu$m, a hot spot size in the MgB$_2$ detector is also on the order of $\mu$m. This means that one has to use microfabrication techniques to build detectors, and is able to achieve a higher spatial resolution. (5) We intend to detect cold neutrons. Cold neutrons have higher cross section of nuclear reaction between $^{10}$B, and neutron. Hence, an MgB$_2$ thin film detector is able to use to detect a single cold neutron.

It is very helpful to prepare an efficient numerical simulator for designing an optimized MgB$_2$ device. We plan to solve the heat transport equation of the MgB$_2$ detector coded by FORTRAN on a desktop workstation. The finite element method (FEM) is employed to take into account various different shapes and structures in designing MgB$_2$ detectors. Thus, we obtain a spatial distribution of temperature variation of the detector as a function of sequential time. To reproduce a voltage pulse from an MgB$_2$ neutron detector, we use a superconducting transition curve of our actual MgB$_2$ detector as a function of temperature.

2. Numerical Method

The heat transport equation and the finite element method are employed in order to simulate composite structure consisting of several different materials. A three-dimensional heat transport equation is expressed by

$$C \frac{\partial \theta}{\partial t} = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} + \beta,$$

where $\theta$ is temperature of material, $X$, $Y$ and $Z$ are heat-flows along the x, y and z directions, respectively, $C$ is specific heat and $\beta$ is heat source per unit time. By introducing volume coordinate $N_i$ in FEM, a temperature of each element $\theta^i$ can be represented by

$$\theta^i = \sum_{i=1}^{4} N_i^i \theta^i = N_1 \theta_1 + N_2 \theta_2 + N_3 \theta_3 + N_4 \theta_4.$$

In the three-dimensional FEM, a tetrahedron is chosen as a fundamental element. As shown in figure 1, $\theta^i$ is a temperature inside the element, $\theta_i$ is a temperature of each node of the tetrahedron and volume coordinate $N_i$ divides a single tetrahedron pyramid into the four tetrahedra.
We obtain an approximate total potential energy satisfying equation (1) as well as boundary conditions by

$$\sum_j (a_{ij} \dot{\theta}_j + C_{ij} \theta_j) = Q_i,$$  \hspace{1cm} (3)

where $a_{ij}, C_{ij}, Q_i$ represent heat capacity, thermal conductivity and heat source, respectively. We solve this equation by a one-step-$\theta$-method to obtain a temperature $\theta_n$ of each node at each time by

$$\theta_n = \left\{ A + \tau(1-\eta)C \right\}^{-1} \left[ \left( A - \eta \tau C \right) \theta_{n-1} + \tau \left\{ \eta f_{n-1} + (1-\eta) f_n \right\} \right],$$  \hspace{1cm} (4)

where $A = \{a_{ij}\}, C = \{C_{ij}\}, f = \{Q_i\}, \eta = 0.1$ is a tuning parameter to optimize a numerical approximation, and a total simulation time is $T (T=\eta \tau)$. Equation (4) is an actual form used to analyze hot spot dynamics of various different MgB$_2$ neutron detectors.

3. Numerical Simulation for different structures

3.1 Non membrane structure

Our MgB$_2$ detector is composed of 200-nm-thick MgB$_2$ thin film meander line on sapphire or silicon substrate. Critical temperature of our MgB$_2$ detector is approximately 27 K. In figure 2, an MgB$_2$ strip line (200 nm in thickness and 5 $\mu$m in length) deposited on an 800-nm thick sapphire substrate is supposed to be as a part of an elongated MgB$_2$ meander line. We choose input parameters of an MgB$_2$ detector at operating temperatures from the literature as follows, i.e., the specific heat of MgB$_2$ is $1.51 \times 10^3$ J/m$^3$K$^2$ [4], the specific heat of sapphire is $9.70 \times 10^3$ J/m$^3$K$^2$ [5], the thermal conductivity of MgB$_2$ is 8 W/mK [6] and the thermal conductivity of sapphire is 4000 W/m K [7]. We assume as an initial condition that an energy released by the nuclear reaction spreads uniformly over a 1-$\mu$m radius-hemisphere with the energy of $3.76 \times 10^{13}$ J (= the nuclear reaction energy 2.35 MeV between neutron and $^{10}$B). This is because the range of charged particles is on the order of micrometers (the range of alpha particle is 3.76 $\mu$m while the range of Li particle is 1.78 $\mu$m).
With the aid of equation (4) and the simulation program, we are able to simulate a time evolution of local temperature in the MgB$_2$ detector. Figure 3 shows a sequence of snapshots of thermal diffusion process in MgB$_2$ neutron detector in the 0.1-ns interval. A cross-sectional view is given (a) in the x-z plane or (b) in the y-z plane. First, a hot spot appears in the MgB$_2$ strip line. Then, it spreads over the sapphire substrate. Finally, the whole system becomes an equilibrium state as a sequence of time.

By using the actual superconducting transition curve of the MgB$_2$ stripline, we are able to produce the superconducting resistive response of the whole MgB$_2$ detector from the time-evolution of local temperatures. In our preceding publication [8], we simply employ a model transition curve expressed by a hyperbolic function of temperature. It was not appropriate to discuss the systematic variation of a pulse signal as functions of input power and bias current. This is due to an exponential tail of the model transition curve. In the present work, we modify our calculation by using a transition curve of our MgB$_2$ thin film detector determined experimentally (see figure 4). A temperature of each node
changes as a function of time. The detector total resistance was evaluated by summing up serial resistors of FEM elements along the x direction as well as parallel resistors along the y and z directions. In figure 5, a total resistance obtained by numerical simulations is shown as a function of time. Under constant current bias conditions, we suppose that a voltage signal is proportional to a produced resistance.

![Figure 4. Experimental superconducting transition curve of the MgB₂ detector is very sharp as a function of temperature. The vertical scale is presented in the logarithmic scale. The residual resistance at low temperatures is due to two-terminal measurements of the resistance of the detector. The MgB₂ detector is very homogeneous, but its lower critical temperature than bulk is due to the effect of off-stoichiometry in film deposition.](image)

![Figure 5. Reproduced pulsed signal by simulations in the scale of resistance. The transition curve of figure 4 is used to convert a local temperature to a local resistivity. The relaxation time is very short of approximately 0.4 ns.](image)

We note that a conventional neutron counter such as a BF₃ gas counter is known to work at the speed of 10 ms. An actual signal from our MgB₂ detector on a sapphire substrate is typically 3.5 ns after passing through a 200 MHz bandpass filter [9] and is much faster than any existing neutron detector. The intrinsic relaxation time is expected to be even shorter than the measured one by considering the fastness of the signal and the limited bandwidth of the measurement system. A relaxation time obtained by numerical simulations (0.4 ns) is also very fast, and we consider that our numerical simulations are successful for explaining the fastness of the experimental relaxation time. A part of the reasons for this discrepancy is due to the contribution from the Joule heating of the transport current, which is not taken into account in present simulations. This effect may act to prolong a relaxation time in simulations. Further improvements of numerical simulations are in progress by taking into account the Joule heating of the transport current in the MgB₂ stripline. We argue that our simulations are valid only in the limit of the weak bias current, where the Joule heating is negligibly small compared to the external energy dissipation in the detector.

Figure 6 shows a peak height of a voltage signal as a function of temperature, where an input heat is chosen as 2.35 MeV, 1.18 MeV or 0.59 MeV. A signal for 2.35 MeV and 1.18 MeV is saturated at
some lower temperatures near the critical temperature. We argue that an offset point of the superconducting transition is suitable to choose as a bias point for achieving good detection efficiency.

![Figure 6](image)

**Figure 6.** Temperature dependence of peak height of a signal in numerical simulations. The transition curve is also shown for comparison. We choose input heat power as 2.35 MeV, 1.18 MeV and 0.59 MeV.

3.2 Membrane type structure

Superconducting TES detectors use a membrane structure to reduce the heat capacity and to control the thermal conduction in the detector. Next, we examine a membrane type structure as well as a non-membrane type structure for an MgB$_2$ detector. By employing the membrane structure, one expects an enhanced sensitivity but a longer relaxation time due to a narrower heat conduction path. That is, there is a trade-off between the sensitivity and the response time of the detector. We use a silicon technology to produce a membrane structure. Therefore, it is very important to perform numerical simulations to optimize the detector design. In figure 7, we show a designed membrane structure for numerical simulations. In figure 7 (a), there is a sandwiched structure of an MgB$_2$ stripline, a SiN layer, and a silicon substrate. The Si substrate just beneath the MgB$_2$ stripline is removed by etching to produce a membrane structure of SiN. A poor thermal conductivity of the SiN layer is useful to control the heat conduction in an actual characteristic of the MgB$_2$ device. For numerical calculations, we use the Si specific heat of 4.6×10$^4$ J/m$^3$ K [10] and the Si thermal conductivity of 3000 W/mK [11].

![Figure 7](image)

**Figure 7.** Model structure of numerical simulations for (a) non-membrane type structure and (b) membrane type structure. Both structures have a SiN layer just beneath an MgB$_2$ stripline. In the membrane structure, the silicon substrate is supposed to be removed by etching.
We carried out numerical simulations for both non-membrane structure and membrane structure. According to snapshots, we found that a hot spot of the membrane detector expands over the wider area than that of the non-membrane structured detector. Therefore, it takes a longer time to quench the hot spot in membrane type structure compared to the non-membrane one.

Figure 8 is the reproduced pulse signals for both non-membrane type and membrane type structures, where an input power is set as 0.59 MeV. The bias temperature is chosen as 27 K. We find that a membrane type structure has a higher sensitivity and a longer response time compared to the non-membrane type structure. The relaxation time is prolonged to the order of 1 ns, but this is still much faster than any existing neutron detectors. We consider that our simulator for the MgB$_2$ detector is very useful to optimize the detector design. We also do not consider the nonlinearity of which the researches remain to be taken into account in simulations.

![Figure 8. Reproduced pulsed signals for non-membrane type structure (solid line) and for membrane type structure (symbols) at 27 K. We assume that a bias current through the detector is small enough so that we are able to neglect the contribution from the Joule heating.](image)

4. Conclusion

We developed a compact simulation code to analyze the thermal properties of MgB$_2$ neutron detectors. Numerical simulations can be conveniently carried out on the basis of the desktop workstation. By combining with experimental superconducting transition curve of the MgB$_2$ thin film and numerical simulations, we succeeded in reproducing a pulsed voltage signal from an MgB$_2$ neutron detector. A relaxation time of the MgB$_2$ detector with sapphire substrate explains the fastness of the MgB$_2$ detector observed experimentally. We also compare numerical simulations for a membrane structured MgB$_2$ detector with a non-membrane structured one. We believe that our numerical method is very useful in order to aid an optimized design of MgB$_2$ neutron detectors.

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