The collective excitation of nuclear matter in a bosonized formula of Landau Fermi liquid theory

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Abstract

The collective excitation of the nuclear matter is analyzed in a bosonized formula of Landau Fermi liquid theory. When the nonlinear self-interacting terms of the scalar meson are taken into account in the Walecka model, the collective excitation energies of the nuclear matter can be obtained self-consistently. It shows that the calculation results are consistent with the corresponding experimental data of the nucleus $^{208}\text{Pb}$ when the quantum number of the orientation of the orbital angular momentum $m$ is zero. Moreover, the cases with the nonzero $m$ values are also studied, it manifests that the collective excitation energy with the fixed nonzero $m$ value are almost invariant when the quantum number of the orbital angular momentum $l$ changes. However, the collective excitation energy of the nuclear matter decreases with the absolute value of $m$ increasing. The direct interaction between two nucleons near the Fermi surface only changes the Fermi velocity, while the exchange interaction causes the collective excitation of the nuclear matter. At this point, it is different from the traditional views in nuclear physics, which announce that the random phase approximation is essential to generate the nuclear collective excitation states.

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I. INTRODUCTION

The collective excitation of nuclei had been studied in the framework of the macroscopic and microscopic model [1]. Until now, it is still an important topic in the nuclear physics. The central energy and strength distribution are calculated in the framework of random phase approximation, and then the results are compared with the experimental data [2–4].

Landau Fermi liquid theory has made a great success in the traditional condensed physics, and some people tried to solve the problems in the nuclear physics with this model. By using Landau Fermi liquid theory, the compression modulus of the nuclear matter is calculated [5]. Moreover, this theory is extended to study the collective excitation of nuclei [6–11]. In the recent years, it is still an interesting topic to study the nuclear structure in the framework of Landau Fermi liquid theory [12–18].

Actually, Landau Fermi liquid theory is a kind of quasi-particle method, which implies that the particles discussed in this theory are not real particles. In Landau Fermi liquid theory, the ground state of the Fermi system is treated as a vacuum. When a fermion is excited and jumps above the Fermi surface, a hole is left in the Fermi sea, as is called particle-hole excitation. In the particle-hole excitation, these excited particles are named as quasi-particles. If a great number of quasi-particles appear above the Fermi surface, the whole Fermi system lies in a collective excitation state, and the collective excitation energy of the Fermi system can be obtained with Landau Fermi liquid theory. Ever since it is established, this theory is successfully used to study the collective excitation of the many-electron system in metals widely in the condensed state physics.

The Boltzmann equation of the Fermi liquid in the two-dimensional space has been discussed in Ref. [19]. If the interaction from the external force is neglected, and supposing the quasi-particle lifetime is long enough, the bosonized equation of motion of the Fermi system can be obtained by integrating the momentum of the quasi-particle. Therefore, the collective excitation of the Fermi system becomes a wave motion of lots of quasi-particles in the momentum space.

The bosonization method of Landau Fermi liquid theory is exceeded to study the collective excitation of the nuclear matter, where the spin of the nucleons is taken into account [20, 21]. However, the Fermi liquid function, which is related to the interaction between nucleons near the Fermi surface, was constructed in the framework of the original Walecka model, where
the nonlinear self-interacting terms of the isoscalar meson are not included in the Lagrangian density, so the effective nucleon mass has to be regarded as a parameter in order to obtain the reasonable collective excitation energies, and the whole calculation is not self-consistent.

In this work, the calculation is performed in the Walecka model, where the nonlinear self-interaction terms of the isoscalar meson are considered, and the correct energy levels of the collective excitation of the nuclear matter are obtained self-consistently by using the parameters in the relativistic mean-field approximation. Moreover, the nuclear collective excitations related to the orientation of the orbital angular momentum are also discussed.

This manuscript is organized as follows: The theoretical framework of bosonized Landau Fermi liquid theory is evaluated in detail in Sect. II, and the collective excitation of the nuclear matter is discussed in Sect. III. Finally, the conclusion is summarized in Sect. IV.

The Fermi liquid function indicates the interaction of the quasi-nucleon near the Fermi surface, which is given in the Appendix part.

II. EQUATION OF MOTION OF THE COLLECTIVE EXCITATION OF THE NUCLEAR MATTER

If \( n_{0,\vec{k}\alpha} \) represents the nucleon number with momentum \( \vec{k} \) and spin orientation \( \alpha \) in the ground state of the nuclear matter, and \( n_{\vec{k}\alpha}(\vec{x},t) \) stands for the corresponding nucleon number in the excited state of the nuclear matter, the quasi-nucleon number with momentum \( \vec{k} \) and spin orientation \( \alpha \) is defined as

\[
\delta n_{\vec{k}\alpha}(\vec{x},t) = n_{\vec{k}\alpha}(\vec{x},t) - n_{0,\vec{k}\alpha},
\]

which is a function of the position \( \vec{x} \) and \( t \).

The quasi-nucleon energy in the nuclear matter can be written as

\[
\tilde{\varepsilon}_{\vec{k}\alpha}(\vec{x},t) = \xi_{\vec{k}\alpha}^* + \frac{1}{V} \sum_{\vec{k}',\beta} f(\vec{k},\alpha;\vec{k}',\beta) \delta n_{\vec{k}',\beta}(\vec{x},t),
\]

where \( \xi_{\vec{k}\alpha}^* \) is the energy of the quasi-nucleon with momentum \( \vec{k} \) and spin orientation \( \alpha \), and the Fermi liquid function \( f(\vec{k},\alpha;\vec{k}',\beta) \) stands for the interaction between two quasi-nucleon with momentum \( \vec{k}(\vec{k}') \) and spin orientation \( \alpha(\beta) \) respectively.

In the spherical coordinate space, Eq. (2) takes the form of

\[
\tilde{\varepsilon}_{\vec{k}\alpha}(\vec{x},t) = \xi_{\vec{k}\alpha}^* + \frac{1}{(2\pi)^3} \int k'^2 dk' \int d\theta' d\varphi' \int d\phi' f(k,\theta,\phi;\alpha;k',\theta',\phi',\beta) \delta n_{\vec{k}',\beta}(\vec{x},t).
\]
Therefore,
\[
\frac{\partial}{\partial \vec{x}} \bar{\varepsilon}_{\vec{k}\alpha}(\vec{x}, t) = \frac{1}{(2\pi)^3} \sum_{\beta} \int k'^2 \, dk' \int \sin \theta' \, d\theta' \int d\phi' f(k, \theta, \phi, \alpha; k', \theta', \phi', \beta) \frac{\partial}{\partial \vec{x}} \delta n_{\vec{k}', \beta}(\vec{x}, t).
\]  
(4)

At the Fermi surface,
\[
\frac{\partial \bar{\varepsilon}_{\vec{k}\alpha}}{\partial \vec{k}} = v_F \hat{\vec{k}},
\]
\[
\frac{\partial n_{\vec{k}\alpha}}{\partial \vec{k}} = \frac{\partial n_{0\vec{k}\alpha}}{\partial \vec{k}} + \frac{\partial \delta n_{\vec{k}\alpha}}{\partial \vec{k}} \approx \frac{\partial n_{0\vec{k}\alpha}}{\partial \vec{k}} = -\hat{\vec{k}} \delta(|\vec{k}| - k_F),
\]  
(5)

with \(v_F\) Fermi velocity, and \(\hat{\vec{k}} = \vec{k}/|\vec{k}|\) the unit vector on the direction of momentum \(\vec{k}\).

According to the Hamilton principle, we obtain
\[
\frac{\partial}{\partial \vec{x}} = \frac{\partial}{\partial \vec{k}} \bar{\varepsilon}_{\vec{k}\alpha}(\vec{x}, t),
\]  
(6)

and
\[
\frac{\partial \vec{k}}{\partial t} = -\frac{\partial}{\partial \vec{x}} \bar{\varepsilon}_{\vec{k}\alpha}(\vec{x}, t).
\]  
(7)

Thus the Boltzmann equation of nucleons can be written as
\[
\frac{dn}{dt} = \frac{\partial n}{\partial t} + \frac{\partial n}{\partial \vec{x}} \cdot \frac{\partial \bar{\varepsilon}_{\vec{k}\alpha}}{\partial \vec{x}}(\vec{x}, t) + \frac{\partial n}{\partial \vec{k}} \cdot \frac{\partial \bar{\varepsilon}_{\vec{k}\alpha}}{\partial \vec{k}}(\vec{x}, t)
\]
\[
= \frac{\partial n}{\partial t} + \frac{\partial n}{\partial \vec{x}} \cdot \frac{\partial \bar{\varepsilon}_{\vec{k}\alpha}}{\partial \vec{x}}(\vec{x}, t) - \frac{\partial n}{\partial \vec{k}} \cdot \frac{\partial \bar{\varepsilon}_{\vec{k}\alpha}}{\partial \vec{k}}(\vec{x}, t)
\]
\[
= I[n],
\]  
(8)

with \(I[n]\) the collision term.

If the force term is taken into account, Boltzmann equation in Eq. (8) becomes
\[
\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \vec{x}} \cdot \frac{\partial \bar{\varepsilon}_{\vec{k}\alpha}}{\partial \vec{x}}(\vec{x}, t) + \frac{\partial n}{\partial \vec{k}} \cdot \left( \vec{F} - \frac{\partial}{\partial \vec{x}} \bar{\varepsilon}_{\vec{k}\alpha}(\vec{x}, t) \right) = I[n].
\]  
(9)

In the relaxation-time approximation, the collision term \(I[n] \approx -\tau^{-1} \delta n\), with \(\tau\) the lifetime of the quasi-nucleon. If \(\tau \to \infty\), the quasi-nucleon will not decay to other states. Thus the collision term \(I[n] \approx 0\). Moreover, if only the quasi-nucleon near the Fermi surface is considered in the nuclear matter, the collision term can be neglected since \(I[n] \sim |k_F - k|^2 \delta n\).

According to Eq. (1), \(n_{\vec{k}\alpha}(\vec{x}, t) = n_{0\vec{k}\alpha} + \delta n_{\vec{k}\alpha}(\vec{x}, t)\), we obtain \(\frac{\partial n_{\vec{k}\alpha}}{\partial t} = \frac{\partial}{\partial t} \delta n_{\vec{k}\alpha}\) and \(\frac{\partial n_{\vec{k}\alpha}}{\partial \vec{x}} = \frac{\partial}{\partial \vec{x}} \delta n_{\vec{k}\alpha}\), thus Eq. (9) becomes
\[
\frac{\partial \delta n_{\vec{k}\alpha}}{\partial t} + \frac{\partial \delta n_{\vec{k}\alpha}}{\partial \vec{x}} \cdot \frac{\partial \bar{\varepsilon}_{\vec{k}\alpha}}{\partial \vec{x}}(\vec{x}, t) + \frac{\partial n_{\vec{k}\alpha}}{\partial \vec{k}} \cdot \left( \vec{F} - \frac{\partial}{\partial \vec{x}} \bar{\varepsilon}_{\vec{k}\alpha}(\vec{x}, t) \right) = 0.
\]  
(10)
Supposing \( \vec{F} = 0 \), Eq. (10) takes the form of
\[
\frac{\partial}{\partial t} \delta n_{\vec{k}\alpha}(\vec{x}, t) + \frac{\partial \delta n_{\vec{k}\alpha}(\vec{x}, t)}{\partial \vec{x}} \cdot v^* \hat{\vec{k}} + \frac{\hat{\vec{k}} \delta(|\vec{k}| - k_F)}{(2\pi)^3} \sum_{\beta} \int k'^2 dk' \int \sin \theta' d\theta' \int d\phi' f(k, \theta, \phi, \alpha; k', \theta', \phi', \beta) \frac{\partial}{\partial \vec{x}} \delta n_{\vec{k}',\beta}(\vec{x}, t) = 0.
\]

In the momentum space, \( \frac{1}{i} \frac{\partial}{\partial t} \delta n_{\vec{k}\alpha}(\vec{q}, t) \rightarrow \hat{\vec{q}} \), we obtain
\[
i \frac{\partial}{\partial t} \delta n_{\vec{k}\alpha}(\vec{q}, t) = \left( \hat{\vec{k}} \cdot \hat{\vec{q}} \right) (v^* \delta n_{\vec{k}\alpha}(\vec{q}, t)
+ \delta(|\vec{k}| - k_F) \frac{1}{(2\pi)^3} \sum_{\beta} \int k'^2 dk' \int \sin \theta' d\theta' \int d\phi' f(k, \theta, \phi, \alpha; k', \theta', \phi', \beta) \delta n_{\vec{k}',\beta}(\vec{q}, t).\right)
\]

Since the quasi-nucleon number \( \delta n_{\vec{k}\alpha}(\vec{x}, t) \) will be a over complete set of variables when the spatial dependence is included, it turns out that the Fermi surface displacement with its spatial dependence is proper to describe the collective fluctuations of the Fermi liquid.

The quasi-nucleon density in the nuclear matter is defined as
\[
\bar{\rho}_\alpha(\theta, \phi) = \int \frac{k'^2 dk'}{(2\pi)^3} \delta n_{\vec{k}\alpha},
\]
with
\[
\delta n_{\vec{k}\alpha} = \begin{cases} 
1, \\
0, \\
-1.
\end{cases}
\]

The reduced Boltzmann equation is obtained by performing the integration \( \int \frac{k'^2 dk'}{(2\pi)^3} \) on both sides of Eq. (12),
\[
i \frac{\partial}{\partial t} \bar{\rho}_\alpha(\theta, \phi; \vec{q}, t) = \left( \hat{\vec{k}} \cdot \hat{\vec{q}} \right) \left( v^* \bar{\rho}_\alpha(\theta, \phi; \vec{q}, t)
+ \frac{k^2_F}{(2\pi)^3} \sum_{\beta} \int k'^2 dk' \int \sin \theta' d\theta' \int d\phi' f(k_F, \theta, \phi, \alpha; k', \theta', \phi', \beta) \delta n_{\vec{k}',\beta}(\vec{q}, t).\right)
\]

If only the nucleon near the Fermi surface on the collective excitation of the nuclear matter is taken into account, the nucleon momentum \( k' \) in the Fermi liquid function \( f(k_F, \theta, \phi, \alpha; k', \theta', \phi', \beta) \) takes the value of the Fermi momentum \( k_F \). Thus Eq. (15) becomes
\[
i \frac{\partial}{\partial t} \bar{\rho}_\alpha(\theta, \phi; \vec{q}, t) = \left( \hat{\vec{k}} \cdot \hat{\vec{q}} \right) \left( v^* \bar{\rho}_\alpha(\theta, \phi; \vec{q}, t)
+ \frac{k^2_F}{(2\pi)^3} \sum_{\beta} \int \sin \theta' d\theta' \int d\phi' f(k_F, \theta, \phi, \alpha; k_F, \theta', \phi', \beta) \bar{\rho}_\beta(\theta', \phi'; \vec{q}, t).\right).
\]
Supposing \((\theta_q, \phi_q)\) denotes the direction of \(\vec{q}\) in the spherical coordinate of the momentum space, we can obtain
\[
\hat{k} \cdot \vec{q} = q \left[ \sin \theta \sin \theta_q \cos (\phi - \phi_q) \right] + \cos \theta \cos \theta_q,
\]
so Eq. (16) can be written as
\[
i \frac{\partial}{\partial t} \tilde{\rho}_\alpha (\theta, \phi, \vec{q}, t) = q \sum_\beta \int d\Omega' \int d\Omega'' K(\theta, \phi; \theta', \phi') M(\theta', \phi', \alpha; \theta'', \phi'', \beta) \tilde{\rho}_\beta (\theta'', \phi'', \vec{q}, t),
\]
with
\[K(\theta, \phi; \theta', \phi') = \left[ \sin \theta \sin \theta_q \cos (\phi - \phi_q) + \cos \theta \cos \theta_q \right] \frac{1}{\sin \theta' \delta(\theta - \theta') \delta(\phi - \phi')},\]
and
\[M(\theta, \phi, \alpha; \theta', \phi', \beta) = v_F^* \frac{1}{\sin \theta} \delta_{\alpha \beta} \delta(\theta - \theta') \delta(\phi - \phi') + \frac{k_F^2}{(2\pi)^3} f(k_F, \theta, \phi, \alpha; k_F, \theta', \phi', \beta).\]

Actually, Fermi liquid function denotes the interaction between two quasi-nucleons near the fermi surface, which can be obtained with the Lagrangian density of the Walecka model. The detailed evaluation can be found in the appendix part of this manuscript. According to Eq. (21), the Fermi liquid function can be written as
\[f(\vec{k}_1, \alpha; \vec{k}_2, \beta) = V_{\text{eff}}(0) - V_{\text{eff}}(\vec{k}_1 - \vec{k}_2) \delta_{\alpha \beta} - \left( \frac{-g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2} \right) + \left( \frac{-g_\sigma^2}{(\vec{k}_1 - \vec{k}_2)^2 + m_\sigma^2} + \frac{g_\omega^2}{(\vec{k}_1 - \vec{k}_2)^2 + m_\omega^2} \right) \delta_{\alpha \beta},\]
where
\[\vec{k}_1 = (k_F, \theta, \phi), \quad \vec{k}_2 = (k_F, \theta', \phi'),\]
and
\[ (\vec{k}_1 - \vec{k}_2)^2 = 2k_F^2 \left\{ 1 - \left[ \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\phi - \phi') \right] \right\} \]
\[= 2k_F^2 \left( 1 - \hat{k}_1 \cdot \hat{k}_2 \right).\]

In the Fermi liquid function in Eq. (21), the direct interaction between nucleons only gives a contribution to the ground energy of the nuclear matter, and is not relevant to the excitation of nucleon-hole pairs directly. Therefore, in the relativistic mean-field approximation, the Fermi energy of nucleons takes the form of
\[\varepsilon_F^* \simeq M_N + \frac{k_F^2}{2M_N^*} + (\frac{-g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2}) \sum_\gamma \frac{k_F^3}{6\pi^2},\]
where the summation runs over the spin orientation and isospin orientation of the nucleon in the nuclear matter. Correspondingly, the Fermi velocity of the nucleon \( v_F^* \) can be written as

\[
v_F^* = \frac{\partial \varepsilon_F^*}{\partial k_F} = \frac{k_F}{M_N^*} + \left( -\frac{g_2^2}{m_\pi^2} + \frac{g_\omega^2}{m_\omega^2} \right) \frac{k_F^2}{2\pi^2},\]

(25)

where the second term comes from the direct interaction part of the Fermi liquid function in Eq. (21). Therefore, only the exchanging interaction between nucleons is taken into account in the following calculation.

The quasi-nucleon density in Eq. (18) can be expanded in spherical harmonics, i.e.,

\[
\tilde{\rho}_\alpha(\theta, \phi, \vec{q}, t) = \sum_{l,m} \tilde{\rho}_\alpha(l, m, \vec{q}, t) Y_{l,m}^*(\theta, \phi). \tag{26}
\]

Similarly, \( K(\theta, \phi; \theta', \phi') \) and \( M(\theta', \phi', \alpha; \theta'', \phi'', \beta) \) can be expanded as

\[
K(\theta, \phi; \theta', \phi') = \sum_{l,m,l',m'} K(l, m; l', m') Y_{l,m}^*(\theta, \phi) Y_{l',m'}(\theta', \phi'), \tag{27}
\]

and

\[
M(\theta', \phi', \alpha; \theta'', \phi'', \beta) = \sum_{l_1,m_1,l_2,m_2} M(l_1, m_1, \alpha; l_2, m_2, \beta) Y_{l_1,m_1}^*(\theta', \phi') Y_{l_2,m_2}(\theta'', \phi''), \tag{28}
\]

respectively. Therefore, the equation of motion of the Fermi liquid takes the form of

\[
i \frac{\partial}{\partial t} \tilde{\rho}_\alpha(l, m, \vec{q}, t) = q \sum_{\beta} \sum_{l',m'} \sum_{l'',m''} K(l, m; l', m') M(l', m', \alpha; l'', m'', \beta) \tilde{\rho}_\beta(l'', m'', \vec{q}, t). \tag{29}
\]

Since the collective excitation of the nuclear matter is independent on the direction of the \( \vec{q} \), we can choose \( \theta_q = 0 \) and \( \phi_q = 0 \), and the function \( K(\theta, \phi; \theta', \phi') \) can be written as

\[
K(\theta, \phi; \theta', \phi') = \frac{\cos \theta}{\sin \theta'} \delta(\theta - \theta') \delta(\phi - \phi'). \tag{30}
\]

In the spherical harmonics of the momentum space,

\[
K(l, m; l', m') = \int \frac{\cos \theta}{\sin \theta'} \delta(\theta - \theta') \delta(\phi - \phi') Y_{l,m}(\theta, \phi) Y_{l',m'}^*(\theta', \phi') \sin \theta' d\theta' d\phi' \sin \theta d\theta d\phi
\]

\[
= (a_{lm} \delta_{l+1,l'} + a_{l-1,m} \delta_{l-1,l'}) \delta_{m,m'}, \tag{31}
\]

with

\[
a_{l,m} = \sqrt{\frac{(l + 1)^2 - m^2}{(2l + 1)(2l + 3)}}. \tag{32}
\]
In order to obtain Eq. (31), \( \cos \theta Y_{l,m}(\theta, \phi) = a_{l,m} Y_{l+1,m}(\theta, \phi) + a_{l-1,m} Y_{l-1,m}(\theta, \phi) \) is used. Moreover,

\[
M(l_1, m_1, \alpha; l_2, m_2, \beta) = \int \left[ v_F^* \frac{1}{\sin \theta'} \delta_{\alpha \beta} \delta(\theta - \theta') \delta(\phi - \phi') + \frac{k_F^2}{(2\pi)^3} f(k_F, \theta, \phi, \alpha, k_F, \theta', \phi', \beta) \right] Y_{l_1, m_1}(\theta, \phi) Y_{l_2, m_2}^*(\theta', \phi') d\Omega d\Omega' \\
= v_F^* \delta_{\alpha \beta} \delta_{l_1 l_2} \delta_{m_1 m_2} - \frac{k_F^2}{(2\pi)^3} f_F(l_1, m_1; l_2, m_2) \delta_{\alpha \beta},
\]

where the Fermi liquid function \( f_F(l_1, m_1; l_2, m_2) \) is only related to the exchange interaction between nucleons.

\[
f_F(l_1, m_1; l_2, m_2) = \int V_{\text{eff}}(\vec{k}_1 - \vec{k}_2) Y_{l_1, m_1}(\theta, \phi) Y_{l_2, m_2}^*(\theta', \phi') \sin \theta d\theta d\phi \sin \theta' d\theta' d\phi' \\
= \int \left( -\frac{g_\sigma^2}{(k_1 - k_2)^2 + m_\sigma^2} + \frac{g_\omega^2}{(k_1 - k_2)^2 + m_\omega^2} \right) Y_{l_1, m_1}(\theta, \phi) Y_{l_2, m_2}^*(\theta', \phi') \sin \theta d\theta d\phi \sin \theta' d\theta' d\phi' \\
= f_F(l_1, l_2) \delta_{l_1 l_2} \delta_{m_1 m_2}.
\]

Apparently, the exchange interaction between nucleons plays an important role on the collective excitation of the nuclear matter when \( l_1 = l_2 \) and \( m_1 = m_2 \).

According to Eqs. (31), (33) and (34), the equation of motion of the nuclear matter in the Landau Fermi liquid theory can be written as

\[
i \frac{\partial}{\partial t} \tilde{\rho}_\alpha(l, m, \vec{q}, t) = q \sum_{l'} (a_{l m} \delta_{l+1, l'} + a_{l-1, m} \delta_{l-1, l'}) \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l', l') \right) \tilde{\rho}_\alpha(l', m, \vec{q}, t).
\]

(35)

The equation of motion of the nuclear matter in Eq. (35) can be rewritten in the matrix form, i.e.,

\[
i \frac{\partial}{\partial t} \tilde{\rho}_\alpha(l, m, \vec{q}, t) = q \tilde{K} \tilde{M} \tilde{\rho}_\alpha(l, m, \vec{q}, t),
\]

(36)

with

\[
\tilde{K}_{l, l'} = (a_{l m} \delta_{l+1, l'} + a_{l-1, m} \delta_{l-1, l'}),
\]

(37)

and

\[
\tilde{M}_{l', l} = \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l', l') \right) \delta_{l, l} \delta_{m, m'}.
\]

(38)
The Fermi liquid is unstable if one of the $f_F(l', l')$ is larger than $(2\pi)^3 v_F^* k_F^2$. Supposing $\tilde{M} = WW^T$ and $u_\alpha = W^T \tilde{\rho}_\alpha$, Eq. (36) becomes

$$i \frac{\partial}{\partial t} u_\alpha(l, m, \vec{q}, t) = q W^T \tilde{K} W u_\alpha(l, m, \vec{q}, t) = H u_\alpha(l, m, \vec{q}, t),$$

(39)

with

$$H_{l,l'}(m) = q(W^T \tilde{K} W)_{l,l'}$$

(40)

$$= q (a_{lm} \delta_{l+1,l} + a_{l-1,m} \delta_{l-1,l'}) \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l, l) \right)^{1/2} \left( v_F^* - \frac{k_F^2}{(2\pi)^3} f_F(l', l') \right)^{1/2} .$$

It is apparent that the Hamiltonian in Eq. (40) is hermitian, $H = H^\dagger$. The collective excitation energies of the nucleon with momentum $q$ corresponds to the eigenvalues of the Hamiltonian in Eq. (40), respectively.

FIG. 1: The energy levels of the Fermi liquid. (a) represents the case of $f_F(l, l) = 0$, while (b) stands for the the case of $f_F(l, l) > 0$.

Assuming the Fermi liquid function is zero, i.e., $f_F(l, l) = 0$, the continuous eigenenergy levels of the Hamiltonian in Eq. (40) are produced, which correspond to particle-hole excitations in the nuclear matter. However, if the value of the Fermi liquid function is large enough, and $f_F(l, l) > 0$, it is more possible to produce two discrete energy levels besides the continuous ones. The positive discrete energy represents the creation of the collective excitation mode of the nuclear matter, while the negative one stands for the annihilation of the collective excitation mode of the nuclear matter, as depicted in Fig. 1.
III. THE COLLECTIVE EXCITATION OF THE NUCLEAR MATTER

In the relativistic mean-field approximation, the parameters are fitted according to the saturation properties of the nuclear matter. However, only if the nonlinear self-interacting terms of the scalar meson are included in the Lagrangian, the compression modulus of the nuclear matter will be in a reasonable range, and the equation of state will not be too stiff. In this work, the parameter set NL3 is adopted in the calculation, i.e., \( g_\sigma = 10.217 \), \( g_\omega = 12.868 \), \( g_3 = -10.431 \, fm^{-1} \), \( g_3 = -28.885 \), \( m_\sigma = 508.194 \, MeV \), \( m_\omega = 782.501 \, MeV \) and \( M_N = 939 \, MeV \).

According to Eq. (25), the Fermi velocity is related to the nucleon effective mass \( M_N^* \), which takes form of \( M_N^* = M_N + g_\sigma \sigma_0 \) in the relativistic mean-field approximation, where \( \sigma_0 \) is the expectation of the scalar meson field in the nuclear matter. Apparently, the self-consistency must be conserved when the calculation is performed. Since the nucleon near the Fermi surface is easier to be excited than the others, we assume \( q \equiv k_F \) in Eq. (40), and the Fermi momentum \( k_F = 1.36 \, fm^{-1} \) for the saturation nuclear matter.

A. The case of \( m = 0 \)

In this subsection, the quantum number \( m \) in the coefficient \( a_{l,m} \) in Eq. (32) is assumed to be zero, and then the collective excitation mode with \( m = 0 \) will be studied firstly.

The collective excitation energies of the nuclear matter \( E_l \) with different \( l \) values as functions of the Fermi momentum \( k_F \) are shown in Fig. 2, where the dotted line represents the case of \( l = 0 \), the dashed line denotes the case of \( l = 1 \), and the solid line stands for the case of \( l = 2 \). It indicates that the collective excitation energies of the nuclear matter \( E_l \) increase with the Fermi momentum \( k_F \).

The collective excitation energies of the nuclear matter at the saturation density for different values of \( l \) are summarized in Table I. Moreover, the corresponding central energies of the isoscalar giant monopole resonance, the isoscalar giant dipole resonance and the isoscalar giant quadrupole resonance of the nucleus \(^{208}Pb\) are also listed in Table I. Apparently, when the self-interacting terms of the scalar meson are taken into account, by solving the equation of motion of the Fermi liquid in Eq. (32), the reasonable collective excitation energies of the saturation nuclear matter can be obtained, which are consistent
with the corresponding experimental values of the nucleus $^{208}Pb$, respectively.

If the self-interacting terms of the scalar meson are not included in the calculation of the relativistic mean-field approximation, the compression modulus of the nuclear matter at the saturation density is about 500MeV, which is too large and the nuclear matter becomes stiff. When the equation of motion in Eq. (39) is solved, the obtained collective excitation energies are far larger than the corresponding experimental values, just as done in Ref. [20, 21].

| $l$ | $E_l$ (MeV) | $E_{\text{exp}}$ (MeV) |
|-----|-------------|------------------------|
| 0   | 15.20       | 14.17 ± 0.28           |
| 1   | 13.88       | 13.5 ± 0.2             |
| 2   | 10.58       | 10.9 ± 0.1             |

TABLE I: The collective excitation energies of the saturation nuclear matter $E_l$ for $l = 0, 1, 2$ and the corresponding experimental central energies $E_{\text{exp}}$ of the isoscalar giant resonances of the nucleus $^{208}Pb$.

FIG. 2: The collective excitation energies of the nuclear matter for different values of $l$ vs. the Fermi momentum. The dotted line represents the case of $l = 0$, the dashed line denotes the case of $l = 1$, and the solid line stands for the case of $l = 2$. 
B. The case of \( m \neq 0 \)

The Hamiltonian in Eq. (40) is not only relevant to the quantum number of the orbital angular momentum \( l \), but to the quantum number of the orientation of the orbital angular momentum \( m \). In this subsection, the case of \( m \neq 0 \) will be discussed.

| \( E_{l,m} \) (MeV) | \( m = -3 \) | \( m = -2 \) | \( m = -1 \) | \( m = 0 \) | \( m = 1 \) | \( m = 2 \) | \( m = 3 \) |
|---------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| \( l = 0 \)         |             |             |             |             |             |             | 15.20       |
| \( l = 1 \)         |             |             |             | 10.12       |             |             |             |
| \( l = 2 \)         | 10.09       | 10.11       | 10.58       | 10.11       | 10.09       |             |             |
| \( l = 3 \)         | 10.06       | 10.09       | 10.11       | 10.15       | 10.11       | 10.09       | 10.06       |

TABLE II: The collective excitation energies of the saturation nuclear matter for different values of \( l \) and \( m \).

The collective excitation energies of the saturation nuclear matter with Fermi momentum \( k_F = 1.36 \text{fm}^{-1} \) for different \( l \) and \( m \) are listed in Table II. For the same value of \( l \), the quantum number of the orientation of the orbital angular momentum \( m \) changes from \(-l\) to \( l \), and the collective excitation energy with \( m \) is the same as the \(-m\) case. When the \( l \) value is conserved, the more larger the absolute value of \( m \) is, the more lower the collective excitation energy of the saturation nuclear matter is. Especially, it is surprised that the collective excitation energies of the saturation nuclear matter with different \( l \) but the same nonzero \( m \) almost take same values, as shown in Table II.

IV. CONCLUSIONS

Landau Fermi liquid theory is bosonized and used to describe the collective fluctuation of the two-dimensional Fermi system in Ref. [19]. In this work, this method is generalized to the situation of the three-dimension Fermi system, and the spin of the fermion is also taken into account. Therefore, we tried to study the collective excitation of the nuclear matter in the framework of the bosonized Landau Fermi liquid theory. When the nonlinear self-interacting terms of the scalar meson is included in the Lagrangian, which is necessary to produce a correct compression modulus of the nuclear mater in the relativistic mean-field
approximation, the collective excitation energies for the different orbital angular momentum $l$ and the different orientation of the orbital angular momentum $m$ can be obtained self-consistently. Moreover, the results with $m = 0$ are consistent with the corresponding central energies of the giant resonances of the nucleus $^{208}$Pb, respectively.

For the case of $m \neq 0$, the collective excitation energy with fixed $(l, m)$ takes the same value as that of $(l, -m)$. It is found that the collective excitation energy of the nuclear matter with the fixed nonzero $m$ value is almost invariant when the $l$ value changes. Moreover, these energy values decrease with the absolute value of $m$ increasing when the $l$ value is fixed.

Especially, it should be emphasized that the exchange interaction between nucleons near the Fermi surface plays a critical role in the collective excitation of the nuclear matter. At this point, it is different from the traditional ideals in the nuclear physics, where it is believed that the random phase approximation results in the collective excitation of nuclei.[26]

In a conclusion, the collective excitation of the nuclear matter is analyzed dynamically in the framework of the bosonized Landau Fermi liquid theory, and an intuitive description on these phenomena has been accomplished in this work.

**Appendix: Fermi liquid function**

If the effective potential between two nucleons is $V_{\text{eff}}(\vec{r} - \vec{r}')$, the two-body interaction operator can be written as

$$
\hat{V} = \sum_{\alpha, \alpha', \beta, \beta'} \int d^3 r \int d^3 r' \psi^\dagger_{\alpha}(\vec{r}) \psi^\dagger_{\alpha'}(\vec{r}') V_{\text{eff}}^{\alpha \alpha', \beta \beta'}(\vec{r} - \vec{r}') \psi_{\beta'}(\vec{r}') \psi_{\beta}(\vec{r}),
$$

(41)

where $V_{\text{eff}}^{\alpha \alpha', \beta \beta'}(\vec{r} - \vec{r}) = V_{\text{eff}}(\vec{r} - \vec{r})$,

$$
\psi^\dagger_{\lambda}(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3 p \hat{b}^\dagger(p, \lambda) \hat{U}(p, \lambda) \gamma_0 \exp(i p \cdot r),
$$

(42)

and

$$
\psi_{\lambda}(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3 p \hat{b}(p, \lambda) \hat{U}(p, \lambda) \exp(-i p \cdot r),
$$

(43)

with $p$ the nucleon momentum in Minkowski space. Since there are not anti-nucleons in the ground state of the nuclear matter, the terms related to the creation and annihilation operators of antinucleons are eliminated in Eqs. (42) and (43).
The scattering matrix element between two nucleons takes the form of

\[ S_{fi}^{(2)} = \langle f | S^{(2)} | i \rangle = \langle p', \lambda', k'; \delta' \rangle \langle S^{(2)} | p, \lambda, k; \delta \rangle, \] (44)

with

\[ S^{(2)} = -i \int dt \hat{V}, \] (45)

The anti-symmetric wave function of the two-nucleon system can be written as

\[ |p, \lambda, k; \delta\rangle = \frac{1}{\sqrt{2}} \left[ |p, \lambda\rangle_1 |k; \delta\rangle_2 - |k; \delta\rangle_1 |p, \lambda\rangle_2 \right], \] (46)

and

\[ \langle p', \lambda', k'; \delta' | = \frac{1}{\sqrt{2}} \left[ 2 \langle k'; \delta' | |p', \lambda' \rangle_1 \langle p', \lambda' | |k'; \delta' \rangle_1 - 2 \langle k'; \delta' | |p'; \lambda' \rangle_1 \langle k'; \delta' | |p'; \lambda' \rangle_1 \right], \] (47)

where the labels 1 and 2 represent the first and second nucleons, respectively. In what follows, these two labels will be neglected.

In the non-relativistic approximation, Assuming the interaction between two nucleons is realized instantaneously, we can make an approximation \( r_0' = r_0 \rightarrow t \) in Eq. (44). Moreover, the nucleon wave function is independent on the momentum, and only relevant to the nucleon spin, so we can obtain

\[ \bar{U}(k', \delta') U(k, \delta) = \delta_{\delta', \delta}, \] and

\[ \bar{U}(k', \delta') \gamma_{\mu} U(k, \delta) = \begin{cases} \delta_{\delta', \delta}, & \mu = 0, \\ 0, & \mu = 1, 2, 3. \end{cases} \] (48)

If the effective potential \( V_{eff} (\vec{q}) \) is only relevant to the exchanged momentum squared \( \vec{q}^2 \), and independent on the nucleon spin, the scattering matrix element \( S_{fi}^{(2)} \) can be written as

\[ S_{fi}^{(2)} = -i \frac{1}{(2\pi)^2} \delta^{(4)}(k + p - k' - p') \left[ V_{eff} (\vec{k} - \vec{k}') \delta_{\delta', \delta} \delta_{\lambda', \lambda} - V_{eff} (\vec{p} - \vec{k}') \delta_{\delta', \lambda} \delta_{\lambda', \delta} \right]. \] (49)

The Lagrangian density of the nuclear matter in Walecka model can be written as

\[ \mathcal{L} = \bar{\psi} (i \gamma_{\mu} \partial^\mu - M_N) \psi + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^{\mu} \omega_{\mu} - g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma_{\mu} \omega^{\mu} \psi, \] (50)

where the tensor of the vector meson is \( \omega_{\mu\nu} = \partial_{\mu} \omega_{\nu} - \partial_{\nu} \omega_{\mu} \), and \( M_N, m_\sigma \) and \( m_\omega \) represent masses of the nucleon, \( \sigma \) and \( \omega \) mesons, \( g_2 \) and \( g_3 \) are coefficients related to the nonlinear
FIG. 3: Nucleon-nucleon interaction, where (a) represents the direct interaction, and (b) stands for the exchange interaction.

self-interacting terms of the scalar meson, $g_\sigma$ and $g_\omega$ denote the coupling constants of the nucleon to $\sigma$ and $\omega$ mesons, respectively.

According to the Lagrangian density in Eq. (50), the interacting Hamiltonian can be written as

$$ H_I(x) = g_\sigma \bar{\psi} \sigma \psi + g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi. $$

Therefore, the second-order scattering matrix element can be written as

$$ \hat{S}_2 = \left( -i \right)^2 \frac{2!}{2!} \int d^4 x_1 \int d^4 x_2 T \left[ H_I(x_1) H_I(x_2) \right]. $$

In order to obtain the amplitude of nucleons, the Feynmann diagrams in Fig. 3 must be calculated.

Firstly, the case that one vector meson exchanges between two nucleons is discussed. The scattering matrix element can be written as

$$ S_i^{(2)}(\omega) = -g_\omega^2 (2\pi)^4 \delta^4(p'+k'-p-k) \left( \frac{1}{(2\pi)^{3/2}} \right)^4 \left[ \bar{U}(k', \delta') \gamma_\mu U(k, \delta) \frac{-ig^{\mu\nu}}{(k'-k)^2 - m_\omega^2 + i\varepsilon} \bar{U}(p', \lambda') \gamma_\mu U(p, \lambda) 
- \bar{U}(k', \delta') \gamma_\mu U(p, \lambda) \frac{-ig^{\mu\nu}}{(k'-p)^2 - m_\omega^2 + i\varepsilon} \bar{U}(p', \lambda') \gamma_\mu U(k, \delta) \right]. $$

The three-momentum of the nucleon is far lower than the nucleon mass, $|\vec{k}| << M_N$, $|\vec{k}'| << M_N$, $|\vec{p}| << M_N$, $|\vec{p}'| << M_N$, which corresponds to the nucleon mass becomes
infinite in the non-relativistic approximation, i.e., \( M_N \rightarrow +\infty \). Thus the zero component of the momentum of the intermediate vector meson in the direct interaction of nucleons can be written as

\[
q_0 = k'_0 - k_0 = \sqrt{k'^2 + M_N^2} - \sqrt{k^2 + M_N^2} \\
\approx M_N \left( 1 + \frac{k'^2}{2M_N^2} \right) - M_N \left( 1 + \frac{k^2}{2M_N^2} \right) \\
= \frac{k'^2}{2M_N} - \frac{k^2}{2M_N} \\
\rightarrow 0. \tag{54}
\]

Similarly, it can be certified that the zero component of the vector meson momentum in the exchanging interaction of nucleons \( q'_0 = k'_0 - p_0 \) tends to zero in the non-relativistic approximation.

According to Eq. (48), the scattering matrix element in Eq. (53) can be simplified as

\[
S^{(2)}_{fi}(\omega) = -g^2 \omega (2\pi)^4 \delta^4(p' + k' - p - k) \left( \frac{1}{(2\pi)^{3/2}} \right)^4 \\
\left[ \delta_{\delta',\delta} \delta_{\mu,0} \left( \frac{-i g^{\mu\nu}}{(k' - k)^2 - m_\omega^2 + i\varepsilon} \delta_{\nu,\lambda} \delta_{\nu,0} \right) \\
- \left(\delta_{\delta',\delta} \delta_{\mu,0} \frac{-i g^{\mu\nu}}{(k' - p)^2 - m_\omega^2 + i\varepsilon} \delta_{\nu,\lambda} \delta_{\nu,0} \right) \right]. \\
= -g^2 \omega \frac{i}{(2\pi)^2} \delta^4(p' + k' - p - k) \\
\left[ \frac{1}{(k' - k)^2 + m_\omega^2 + i\varepsilon} \delta_{\delta',\delta} \delta_{\mu,\lambda} \\
- \frac{1}{(k' - p)^2 + m_\omega^2 + i\varepsilon} \delta_{\delta',\delta} \delta_{\mu,\lambda} \right]. \tag{55}
\]

Comparing Eqs. (49) and (55), the nucleon potential by exchanging a vector meson can be obtained as

\[
V_{\omega eff}(\vec{q}) = \frac{g^2_\omega}{\vec{q}^2 + m_\omega^2}. \tag{56}
\]

Similarly, the nucleon potential by exchanging a scalar meson is obtained as

\[
V_{\sigma eff}(\vec{q}) = \frac{-g^2_\sigma}{\vec{q}^2 + m_\sigma^2}. \tag{57}
\]
Therefore, the total potential between two nucleons can be written as

$$V_{\text{eff}}(\vec{q}) = \frac{-g_\sigma^2}{q^2 + m_\sigma^2} + \frac{g_\omega^2}{q^2 + m_\omega^2}. \quad (58)$$

In the coordinate representation, the nucleon potential in Eq. (58) takes the form of Yukawa potential,

$$\tilde{V}_{\text{eff}}(r) = \frac{1}{(2\pi)^3} \int d^3q V_{\text{eff}}(\vec{q}) \exp(i\vec{q} \cdot \vec{r})$$

$$= -\frac{g_\sigma^2}{4\pi} \frac{\exp(-m_\sigma r)}{r} + \frac{g_\omega^2}{4\pi} \frac{\exp(-m_\omega r)}{r}, \quad (59)$$

which is a Fourier transformation of the potential $V_{\text{eff}}(\vec{q})$ in Eq. (58).

If $\vec{p}(\vec{k})$ and $\lambda(\delta)$ represent the three-momentum and spin orientation of two incoming nucleons near the Fermi surface, respectively, while in the final state, the three-momentum and spin orientation of two outgoing nucleons are still $\vec{p}(\vec{k})$ and $\lambda(\delta)$, respectively, the Fermi liquid function is defined as the sum of the direct and exchanging potential, as depicted in Fig. 3. According to the nucleon potential in Eq. (58), the Fermi liquid function can be written as

$$f(\vec{p}, \lambda; \vec{k}, \delta) = V_{\text{eff}}(0) - V_{\text{eff}}(\vec{p} - \vec{k})\delta_{\lambda, \delta}. \quad (60)$$
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