Modeling, Identification, and Compensation Control of Friction for a Novel Dual-Drive Hydrostatic Lead Screw Micro-Feed System

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Abstract: This paper investigates the transmission performance of a novel dual-drive hydrostatic lead screw micro-nano feed system (DDHLS) that can obtain extremely low speed. Firstly, the oil film liquid friction of hydrostatic transmission is modelled, and the calculation model of oil film dynamic friction is proposed based on the variable viscosity theory. Secondly, on this basis, combined with the LuGre friction model, a novel all-components refinement friction identification method (ACRFIM) for DDHLS was developed. The friction parameters of the feed drive components such as LM guide and hydrostatic lead screw can be identified independently using the proposed method, ensuring precise friction force modelling in all components. Then, an all-component adaptive friction compensation control algorithm (AACA) was designed by introducing the temperature and disturbance influence factors into the friction model and considering the influence of the dynamic friction of liquid. The experiments illustrate that the calculation accuracy of the oil film friction model based on the variable viscosity theory is substantially improved. DDHLS can effectively suppress the adverse effects of nonlinear friction, and the proposed AACA has an obvious compensation effect for the friction of the time-varying system.

Keywords: hydrostatic lead screw; low-speed feed; nonlinear friction; adaptive control

1. Introduction

Ultra-high precision large stroke displacement is necessary to achieve ultra-precision machining and inspection [1]. The current high-precision displacement is mainly realized based on intelligent materials’ electrical, magnetic, thermal, optical, and acoustic effects. These have obvious shortcomings, such as tiny stroke, low rigidity, nonlinearity, and hysteresis. Although the ball screw and linear motion (LM) guide can realize large-stroke linear feed, the low-speed creeping phenomenon caused by the inherent properties of its electromechanical system makes it difficult to achieve precise and uniform micro-feeds [2]. Feng et al. proposed a two-axis differential micro-feed system (TDMS) based on a nut-driven ball screw pair and differential recombination principle [3]. On the premise of ensuring the micro-feed, the screw speed and nut speed can be increased at the same time. The low-speed nonlinear crawling zone of conventional electromechanical servo systems is avoided. Although TDMS has obtained lower transmission speed and micro-feed than traditional ball screws, it is limited by the high-frequency tremor caused by the machining errors of the balls and raceways and the influence of the backlash of the ball screw. The accuracy of TDMS cannot be further improved. In addition, the high rigidity, high moving accuracy, and almost frictionless transmission characteristic of hydrostatic transmission
components can make up for the deficiencies of rolling transmission components. However, they still cannot avoid the low-speed crawling area of the motors [4].

In order to realize the linear feed with large stroke, low speed, and ultra-high precision, Feng et al. proposed a dual-drive hydrostatic screw micro-nano feed system (DDHLS) based on the nut-driven hydrostatic lead screw [5]. Since the screw and the nut are supported by a lubricating oil film, DDHLS can significantly reduce the vibration and noise of the TDMS to achieve ultra-precision linear feed. However, in addition to the nonlinear friction of the LM guide, the friction force existing in the nut-driven hydrostatic nut and the shear force of the oil film in the hydrostatic components still interfere with the displacement accuracy of the DDHLS.

For decades, scholars have conducted extensive research on friction modeling and compensation control technology [6]. Various friction models have appeared successively, such as the Coulomb friction + viscous friction model, Stribeck friction model, Dahl friction model, and LuGre friction model [7,8]. Among them, the LuGre friction model can capture the complex static and dynamic characteristics in the friction process, including the Stribeck effect, hysteresis, sticking spring characteristics, and different separation forces. Therefore, the LuGre friction model was selected in this paper. Note that the effect of ultra-low friction is not considered, since the extremely low feed rate of the table in this study exceeds 100 nm/s [9]. Based on considering torque transmission, Li et al. proposed a dynamic model and parameter identification method to reduce the mechanical tracking error of the ball screw and proposed a feedforward compensation method [10]. To improve the model estimation performance of the feed system, Lee et al. proposed a fast identification algorithm for feed transmission mass and sliding friction coefficient based on the recursive least squares (RLS) method [11]. Yang et al. proposed a two-level friction model related to speed and established a tracking error pre-compensation model for CNC machine tools based on the feedforward friction compensation [12]. Thenozhi et al. identified the parameters of the continuous friction model through a two-step offline identification method and designed a servo-mechanism tracking controller by the backstepping method [13]. Dumanli et al. improved the tracking accuracy of the machine tool feed through a data-based closed-loop adjustment scheme. The positioning errors caused by servo dynamics and frictional interference were pre-compensated by modifying the reference trajectory [14]. In order to achieve high-precision tracking control of parallel manipulators, Sancak et al. used a dynamic LuGre model to model joint friction. They also designed the Luenberger-like observer and extended state observer [15]. Feng et al. described the nonlinear friction of the excavator electro-hydraulic system with the improved Strubeck model and designed a dynamic friction feedforward compensation method based on the structural invariance principle [16]. In the above study, the movement speed of the actuator can be obtained by multiplying the drive system by the transmission ratio, so it is considered that the drive end and the actuator end have the same friction characteristics.

However, due to the unique transmission structure of DDHLS, different macro-movement speeds can be selected for combination while ensuring the low-speed feed, and the traditional identification methods cannot be used. It should be noted that the friction characteristics of rolling contact components and hydrostatic transmission components are significantly different. The friction torque generated by the liquid friction of DDHLS at different macro speeds also complicates the feed system [17]. Although Feng et al. realized the separate identification of the ball screw and the LM guide and designed an observer-based PD friction compensator, it no longer applies to the more complex friction conditions of DDHLS [3]. In addition, the friction torque in the DDHLS is affected by various disturbance factors such as temperature change and mechanical vibration/shock. In order to achieve accurate compensation of friction torque, the friction model of each rolling contact friction component (such as the LM guide and rolling bearing) needs to consider the influence of the external environment [18]. At the same time, the lubricant viscosity of each hydrostatic component is affected by the heat generation of oil film friction.
at different speeds. Conventional PID control is challenging to deal with parameter uncertainty, modeling uncertainty, and nonlinearity for high-performance motion control [19]. Sliding mode control has strong robustness to system parameter perturbations and external disturbances, but the state trajectory is difficult to slide strictly along the sliding surface, which is prone to the chattering phenomenon [20]. A targeted design of friction compensation controller is required to obtain high control accuracy.

In this paper, the fluid friction model of each hydrostatic component is established, and a calculation method of oil film dynamic friction is proposed based on the variable viscosity theory. Afterward, an all-components refinement friction identification method (ACRFIM) was proposed to model DDHLS precisely. In addition, considering the influence of external disturbance factors, two factors of temperature change and disturbance were introduced to improve the friction model of the rolling contact component. Moreover, considering the influence of oil film dynamic friction, an all-component adaptive friction compensation control algorithm (AACA) is proposed. Finally, the dynamic characteristics and tracking performance of DDHLS at low-speed feed are verified by experiments.

2. Description and Modeling of System

2.1. System Description

The block diagram of the DDHLS is shown in Figure 1. The feed table is equipped with a nut-driven hydrostatic lead screw and a set of LM guides. Both ends of the hydrostatic lead screw are supported by the hydrostatic bearing, and the screw is driven by a servo motor. The self-driving nut is equipped with a double row angular contact ball bearing and glyd-rings and is driven by a hollow servo motor. The screw drive servo motor has the same parameters as the hollow servo motor. Depending on the requirements, either the conventional hydrostatic lead screw feed system (CHLS) or the DDHLS can be selected. In CHLS, the nut motor remains stationary, and the lead screw motor is driven. In DDHLS, the nut drive motor maintains a constant speed to avoid the low-speed crawling range. The screw motor and the nut drive motor rotate in the same direction and at approximately equal speed. The micro-feed speed and feed direction can be adjusted in real-time by adjusting the screw speed.

Figure 1. Block diagram of the DDHLS.

2.2. Dynamic Modeling of the DDHLS

Figure 2 shows the simplified dynamic model of the DDHLS. $K_s$, $T_{fs}$, $J_{ms}$, and $\theta_{ms}$ represent the torque coefficient, friction torque, equivalent rotational inertia, and rotation angle of the screw motor. $J_{ms}$ includes the rotational inertia of the screw motor shaft, coupling, and screw. $T_{fs}$ is the torque generated by the oil film friction in the hydrostatic bearings and the hydrostatic lead screw. $K_n$ is the torque coefficient of the hollow motor. $T_{fn}$ is the friction torque equivalent to the hollow motor shaft, including nut-bearing and glyd-rings. $F_d$ is the driving force acting on the table. $F_f$ is the friction force acting on the
LM guide. $J_{Me}$ is the equivalent rotational inertia on the hollow motor shaft, including the rotational inertia of the hollow motor shaft, connecting flange, and nut. $\theta_{Ms}$ is the rotation angle of the hollow motor shaft. $X_t$ is the displacement of the table.

\begin{equation}
\begin{aligned}
J_{Ms} \dot{\theta}_{Ms} &= K_{Ms} \omega_{Ms} - T_{Fs} - T_{Fs} - T_{ds} \\
J_{Mt} \dot{\theta}_{Mt} &= K_{Mt} \omega_{Mt} - T_{Fs} + T_{ds}
\end{aligned}
\end{equation}

where $T_{di}$ ($i = s$ or $n$) is the output torque generated by the interaction between the screw and the nut. When the screw speed is greater than the nut speed, it is the resistance torque for the screw motor and the incremental torque for the hollow motor. The $i_{ps}$ and $i_{pn}$ are the control outputs of the screw shaft and the nut shaft, respectively.

The motion analysis of the workbench is as follows [2]:

\begin{equation}
\begin{aligned}
F_{ds} &= K_{s} R_{s} (\theta_{Ms} - \theta_{Mt}) - X_t \\
F_{ds} - F_{f} &= M_{s} \ddot{X}_t \\
T_{di} &= F_{ds} R_{s} \\
R_{s} &= \frac{p}{2\pi}
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
F_{ds} &= \begin{cases} 
F_{f} + M_{s} \ddot{X}_t & |\theta_{Ms}| > |\theta_{Mt}| \\
0 & |\dot{\theta}_{Ms}| \leq |\dot{\theta}_{Ms}|
\end{cases} \\
F_{ds} &= \begin{cases} 
0 & |\dot{\theta}_{Ms}| > |\dot{\theta}_{Ms}|
\end{cases}
\end{aligned}
\end{equation}
where $K_{eq}$ is the equivalent stiffness of the mechanical system [21], which can be obtained by:

$$
K_{eq} = \left(\frac{1}{K_a} + \frac{R^2}{\eta \ K_{rl}}\right)^{-1}
$$

$$
K_{rl} = \left(\frac{1}{K_{cl}} + \frac{1}{K_{sl}}\right)^{-1}
$$

$$
K_a = \left(\frac{2}{K_{Sba}} + \frac{1}{K_{Sa}} + \frac{1}{K_{Na}} + \frac{1}{K_{Nba}}\right)^{-1}
$$

$$
K_{sr} = \frac{\pi d^2 EL}{4a(L-a)}
$$

where $K_{cl}$ and $K_{sl}$ are the torsional stiffness of the coupling and the lead screw, respectively. $K_{Sba}$, $K_{Sa}$, $K_{Na}$, and $K_{Nba}$ are the axial stiffness of bearing, lead screw, nut, and nut-bearing, respectively. $E$ is the elastic modulus of the screw shaft. $d$ is the diameter of the screw shaft. $L$ is the span between the screw supports. $a$ is the distance from the action point to the support.

By pre-designing the bearing bushing and the hollow motor bushing, the equivalent rotational inertia of the screw drive shaft and the nut drive shaft is guaranteed equal ($I = J_s = J_n$). The integrated stiffness of the worktable in different driving modes is equal ($K_{eq} = K_{eq1} = K_{eq2}$). The relevant parameters of the feeding system are shown in Table 1.

### Table 1. Parameters of the feed system.

| Parameters                                | Value          |
|-------------------------------------------|----------------|
| Torsional stiffness of the coupling ($K_{cl}$) | $4.1 \times 10^6$ N·m/rad |
| Torsional stiffness of the screw ($K_a$)    | $8.42 \times 10^6$ N·m/rad |
| Axial stiffness of the bearing ($K_{Sba}$)  | $8 \times 10^8$ N/m |
| Axial stiffness of the screw ($K_{Sa}$)     | $1.73 \times 10^8$ N/m |
| Axial stiffness of the nut ($K_{Na}$)       | $5.95 \times 10^8$ N/m |
| Axial stiffness of the nut-bearing ($K_{Nba}$) | $12 \times 10^8$ N/m |
| Moment of inertia equivalent to the motor ($J_s, J_n$) | $5.92 \times 10^{-3}$ Kg·m² |
| Oil supply pressure ($P_s$)                 | 3.2 MPa        |
| Density of hydraulic oil ($\rho$)           | 876 Kg/m³      |
| Specific heat capacity of hydraulic oil ($J$) | 1760 Kg·K      |

### 2.3. Modeling of Friction in DDHLS

For DDHLS, friction is mainly divided into four parts: the friction torque $T_{frm}$ of the screw motor, the friction torque $T_{fs}$ generated by the oil film, the equivalent friction torque $T_{fn}$ of the hollow motor shaft, and the friction force $F_f$ at the LM guide. The friction torque $T_{fs}$ includes the friction torque $T_{frm}$ generated by the hydrostatic lead screw oil film, the friction torque $T_{feb}$ generated by the front hydrostatic bearing oil film, and the friction torque $T_{fbr}$ generated by the rear hydrostatic bearing oil film. The friction torque $T_{fn}$ includes the friction torque of the nut-bearing and the glyd-ring.

#### 2.3.1. Fluid Friction Modeling

The Hydrostatic Lead Screw

The structure of the hydrostatic lead screw and the schematic diagram of the oil film are shown in Figure 3. In the hydrostatic lead screw, the screw and the nut are separated by a lubricant film, and the fluid friction originates from the oil film shearing effect. As shown in Equation (5), the internal friction generated by the sheared oil film follows Newton’s fluid shear law.
where $\eta$ is the viscosity coefficient of the lubricant, and $h$ is the oil film thickness in the gap.

\[
\tau = \eta \frac{v}{h}
\]  

The existence of the oil film makes the torsional stiffness of the screw-nut pair extremely small, which can be ignored in the following calculation. When the lead screw and nut rotate at rotational speeds $\omega_l$ and $\omega_n$, respectively, the DDHLS drives the table feed through the differential rotational speed $\omega_d = \omega_l - \omega_n$ (assuming $\omega_l > \omega_n$). In order to simplify the calculation, the lateral taper of the single-turn helical oil film is divided into several circular units using the micro-element method. The friction torque $T_{Fsn}$ generated by the sheared oil film in the DDHSL can be obtained by:

\[
T_{Fsn} = 6 \int_{h}^{R} r dF,
\]

\[
dF = (\omega_l - \omega_n) R \frac{\mu_s}{h} dA
\]

\[
dA = \frac{2\pi r \cos \alpha}{\mu_s} dr
\]

where $r$ is the radius of the unit, $dA$ is the area of the unit, $dF$ is the liquid friction force, and $\mu_s$ is the kinematic viscosity of the hydrostatic lead screw lubricant.

Substituting the parameters in Table 2 into Equation (6), the oil film friction energy dissipation and generated torque are as follows:

\[
\begin{align*}
\mu_s &= 5.3515 \times 10^{-3} \mu_s (\omega_l - \omega_n)^2 \\
T_{Fsn} &= 5.11 \times 10^{-3} \mu_s (\omega_l - \omega_n)
\end{align*}
\]

**Table 2.** Parameters of the hydrostatic lead screw.

| Parameters                  | Value  |
|-----------------------------|--------|
| Inner radius of the nut ($R_1$) | 14 mm  |
| Outer radius of the lead screw ($R_2$) | 20 mm  |
| Half angle of thread ($\alpha$)    | 10°    |
| Pitch ($P$)                   | 8 mm   |
Number of nut threads with helical recesses \( (N) \) 6
Designed axial clearance for the hydrostatic lead screw \( (h_0) \) 0.025 mm

The Front Hydrostatic Bearing

For tapered bearing in the front hydrostatic bearing (Figure 4a), the oil velocity and frictional resistance vary with radius, which also is solved using the micro-element method. The friction energy dissipation and the friction torque generated by the conical oil film can be calculated by:

\[
\begin{align*}
P_{psc} &= \frac{\mu \pi \sin^2 \alpha_{bc} \omega^2}{900 h_b} \left( \frac{\theta_s/2}{2} \sin \alpha_{bc} \left( r_a^4 - r_b^4 + \frac{r_a^4 - r_b^4}{1 + h_b / h_r} \right) + \frac{c \left( r_d^4 - r_e^4 \right)}{3 \left( 1 + h_b / h_r \right)} \right) \\
T_{psc} &= \frac{\mu \pi \sin^2 \alpha_{bc} \omega}{30 h_b} \left( \frac{\theta_s/2}{2} \sin \alpha_{bc} \left( r_a^4 - r_b^4 + \frac{r_a^4 - r_b^4}{1 + h_b / h_r} \right) + \frac{c \left( r_d^4 - r_e^4 \right)}{3 \left( 1 + h_b / h_r \right)} \right)
\end{align*}
\]  
(8)

where \( \mu \) is the kinematic viscosity of the front hydrostatic bearing hydraulic oil, \( h_b \) is the thickness of the oil film, and \( h_r \) is the depth of the oil groove.

\[\begin{align*}
&\text{(a)} \quad \text{Figure 4. The front hydrostatic bearing. (a) the tapered bearing. (b) the thrust bearing.}
\end{align*}\]

For the thrust bearing in the front hydrostatic bearing (Figure 4b), the friction energy dissipation and the generated torque of the thrust bearing can be obtained by substituting...
the relevant parameters of the thrust bearing into Equation (8) and making \( \alpha_{bc} = 90^\circ \) and \( \theta_s = 360^\circ \):

\[
\begin{align*}
P_{f_{bt}} &= 1.7273 \times 10^{-4} \mu_s \omega_s^3 \\
T_{f_{bt}} &= 1.6 \times 10^{-3} \mu_s \omega_s
\end{align*}
\] (10)

According to Equations (9) and (10), the total friction energy dissipation and generated torque of the front hydrostatic bearing are as follows:

\[
\begin{align*}
P_{f_{br}} &= 4.7201 \times 10^{-4} \mu_s \omega_s^3 \\
T_{f_{br}} &= 4.5 \times 10^{-3} \mu_s \omega_s
\end{align*}
\] (11)

The Rear Hydrostatic Bearing

According to the product description of the rear hydrostatic bearing (Figure 5), \( r_{br} = 12.5 \text{ mm} \), \( H_{br} = 70 \text{ mm} \), oil film thickness \( h_{br} = 0.02 \text{ mm} \). The rear hydrostatic bearing needs to arrange the oil chamber, and the mating surface of the screw and the hydrostatic bearing is not an entire circumferential surface. The oil film support area \( A_{brf} = 2146.75 \text{ mm}^2 \) is known from the data provided by the product.

![Figure 5. The rear hydrostatic bearing.](image)

The rear hydrostatic bearing friction energy dissipation and generated torque can be calculated by:

\[
\begin{align*}
P_{r_{br}} &= \frac{\mu \pi r_{br}^2}{900 h_{br}} A_{brf} r_{br}^2 \\
T_{f_{br}} &= \frac{\mu \pi \omega_s}{30 h_{br}} A_{brf} r_{br}^2
\end{align*}
\] (12)

By substituting the relevant parameters of the rear hydrostatic bearing into Equation (12), the following Equation can be obtained:

\[
\begin{align*}
P_{r_{br}} &= 5.8544 \times 10^{-4} \mu_s \omega_s^3 \\
T_{f_{br}} &= 5.591 \times 10^{-4} \mu_s \omega_s
\end{align*}
\] (13)

According to Equations (7), (11), and (13), the torque generated by the oil film friction of the screw drive shaft is as follows:
\[
T_{Pr} = T_{Fna} + T_{Fbat} + T_{Fbr}
\]
\[
= 5.11 \times 10^{-3} \mu_m (\omega_f - \omega_l) + 4.5 \times 10^{-3} \mu_m \omega_l + 5.591 \times 10^{-4} \mu \omega_l
\]

2.3.2. Fluid Friction Calculation Model Based on Variable Viscosity Theory

Most existing temperature rise calculations for oil films are based on the adiabatic iso-viscosity assumption [17,22]. However, the fluid friction of the hydrostatic bearing and the hydrostatic screw varies with speed. At the same time, the fluid friction energy is mainly dissipated in the form of heat, and the dissipated heat energy reacts on the oil film to change its dynamic viscosity. Therefore, the operation of hydrostatic components is a dynamic process.

Equation (15) is the Vogel viscosity–temperature equation. Numerous scholars have confirmed its higher accuracy in the commonly used hydraulic oil viscosity–temperature equation by analyzing and comparing experimental curves, regression analysis, and fitting equations [23].

\[
\mu(T) = a e^{b/Tc}
\]

In this study, the maximum speed of the hydrostatic lead screw does not exceed 1000 rpm, and the oil temperature does not exceed 40 °C. Therefore, the dynamic viscosity of hydraulic oil at 10 °C, 20 °C, and 30 °C is selected and brought into the Equation (15) [24]. The viscosity–temperature equation of 46# hydraulic oil is obtained by solving three sets of equations:

\[
\mu(T) = 4.3795 \times 10^{-5} e^{1025/T-167.485}
\]

\(T\) is the thermodynamic temperature, the unit is K.

In the theoretical calculation of temperature rise, the factors affecting the transfer and conversion of heat energy are complex, resulting in the diversification of solution boundary conditions. In order to simplify the calculation, it is assumed that all the heat generated by fluid friction energy loss is taken away by the lubricant. Then, the lubricant temperature rise can be calculated as follows:

\[
\Delta T\ (P_r) = \frac{P_r}{\rho c q}
\]

where \(P_r\) is the friction power, \(\rho\) is the lubricating oil density, \(c\) is the specific heat capacity of the lubricating oil, and \(q\) is the flow rate.

The temperature rise of the hydrostatic lead screw, the front hydrostatic bearing, and the rear hydrostatic bearing can be obtained by bringing the parameters in Table 4 into Equation (17):

\[
\begin{align*}
\Delta T_{\text{sn}} &= P_{\text{sn}} / 19.272 \\
\Delta T_{\text{bat}} &= P_{\text{bat}} / 10.2784 \\
\Delta T_{\text{br}} &= P_{\text{br}} / 6.424
\end{align*}
\]
Table 4. Parameters of oil flow.

| Parameters                                      | Value  |
|------------------------------------------------|--------|
| Oil flow of hydrostatic lead screw \((q_{sn})\) | 0.75 L/min |
| Oil flow of front hydrostatic bearing \((q_{bat})\) | 0.4 L/min |
| Oil flow of rear hydrostatic bearing \((q_{br})\) | 0.25 L/min |

For the DDHLS, the oil film temperature rise of the same hydrostatic component is different at different speeds, as is the temperature rise of different hydrostatic components at the same speed. In order to accurately calculate the temperature rise of each hydrostatic component at different rotational speeds, a dynamic friction calculation model based on variable viscosity theory is proposed, as shown in Figure 6. The initial oil temperature is 19°C, and the initial viscosity can be obtained according to the viscosity–temperature in Equation (16). The target rotational speed is divided into \(n\) speed intervals, and the hydraulic oil temperature rise of the hydrostatic component in each speed interval \(\Delta \omega\) can be obtained by associating Equations (7), (11), (13), and (18) until the target rotational speed is reached. Finally, the energy consumption and torque generated by the fluid friction of the hydrostatic component at different rotational speeds can be accurately calculated.

![Fluid friction calculation flowchart based on variable viscosity theory.](image)

2.3.3. Boundary Friction Modeling

In recent years, the theory and application of the dynamic friction model have become more and more mature [25,26]. Among them, the LuGre friction model is widely used due to its comprehensive and accurate description of pre-sliding and sliding friction [27]. This paper uses the LuGre model to model the boundary friction separately.

The friction of the screw drive motor shaft and the nut drive shaft is modeled as follows:
The friction model of the LM guide is as follows:

\[
T_f = \sigma_{0g} z + \sigma_{1g} \dot{z} + \sigma_{2g} \ddot{z},
\]

\[
\dot{z} = \dot{\theta} - \frac{\dot{\theta}}{g (\dot{\theta})} z,
\]

\[
\sigma_{0g} \dot{\theta} = T_C + (T_f - F_s) e^{-(\theta/\theta_0)}.
\]

(19)

where \( i = m \) and \( n \) represent the screw motor shaft and the nut drive shaft. \( T_f \) is the friction torque. \( F_f \) is the LM guide friction force. \( T_C \) is the maximum static friction torque. \( F_C \) is the maximum coulomb friction force. \( \sigma_0 \) is the stiffness coefficient. \( \sigma_1 \) is the damping coefficient. \( \sigma_2 \) is the viscous friction coefficient.

3. Friction Identification in DDHLS

The static and dynamic friction model parameters in the LuGre friction model are identified separately by the genetic algorithm. The flow of ACRFIM is shown in Figure 7. Among them, the identification involving hydrostatic components is explicitly described in Section 5.2.

**Figure 7.** Flowchart of the ACRFIM.

3.1. Friction Identification of the Screw Drive Motor

**Step A:** Firstly, the LuGre model parameters of the servo motor are identified by referring to the method of Wang et al. [28]. The friction parameter identification process of the screw drive motor is as follows: The torque and steady-state speed are used for static parameter identification. The torque, rotation angle, speed, and acceleration are used for dynamic parameter identification.
3.2. Identification for the Friction Model of LM Guides

**Step B:** According to the structural characteristics of DDHLS, when the screw drive motor runs at low speed and the nut remains stationary, the shear force on the oil film is almost zero. Therefore, the friction torque generated by the hydrostatic components can be ignored in the analysis. The friction torque of the feed system only originates from the friction of the LM guide. The following equation can be obtained when the feed rate of the worktable remains stable:

\[ T_m = T_{fm} + R_f \cdot F_f \]  

(21)

where \( T_m \) is the output torque of the screw drive motor in the steady state.

The average deformation of the bristle of the LM guides at the steady state can be described by:

\[ z_e = g(\dot{X}_i) \operatorname{sgn}(\dot{X}_i) \]  

(22)

when the worktable moves at a constant speed, ignoring the system disturbance, and the frictional force of the LM guide can be expressed as:

\[
F_f = \frac{1}{R_f} \left[ T_{fm} - \sigma_{m}\dot{z}_m - \sigma_{m}\left( \dot{\theta}_m - \frac{\sigma_{m}}{T_{cm} + (T_{cm} - T_{en})e^{-|\theta_m/\theta_{en}|}} \right) - \sigma_{2m}\dot{\theta}_m \right] \\
= \left[ F_{cg} + (F_0 - F_{cg})e^{-|\dot{X}_i/\dot{\theta}_{en}|} \right] \operatorname{sgn}(\dot{X}_i) + \sigma_{2g}\dot{X}_i
\]  

(23)

The LM guide friction model consists of four static friction parameters \((F_{cg}, F_0, v_{cg}, \sigma_{2g})\) and two dynamic friction parameters \((\sigma_0, \sigma_1)\). The static friction parameters are identified by the experimentally obtained Stribeck curve, which describes the sliding friction behavior of LM guides. The steady-state identification error and optimization objective function of the workbench are defined as:

\[
J_g = \sum_{i=1}^{N_g} \left\{ F_{gs} \left( X_{gs}, \dot{X}_i \right) - \dot{F}_{gs} \left( X_{gs}, \dot{X}_i \right) \right\}^2
\]

\[
J_g = \sum_{i=1}^{N_g} \left\{ F_{gs} \left( X_{gs}, \dot{X}_i \right) - \dot{F}_{gs} \left( X_{gs}, \dot{X}_i \right) \right\}
\]  

(24)

where \( X_{gs} \) is given by \( \left[ F_{cg}, F_{0g}, v_{cg}, \sigma_{2g} \right] \). \( F_{gs} \) is the experimentally measured friction force during steady-state operation, and \( \dot{F}_{gs} \) is the estimated value calculated by the friction model. \( N_{gs} \) is the number of points contained in the Stribeck curve.

Parameter identification is achieved by solving the minimum value of \( J_g \). The Stribeck curve drawn by the static friction identification experiment of LM guides is shown in Figure 8a. The static friction parameters identified by the genetic algorithm are shown in Table 5.
Figure 8. Friction characteristics of LM guides and identified feed drive model. (a) Stribeck curves. (b) Hysteresis curves.

Table 5. Friction parameters identified in DDHLS.

| Parameters | Values | Parameters |
|------------|--------|------------|
| Serve Motor LM guide | The Nut Shaft |
| $T_s$ (N·m) | 0.0608 | 17.721 |
| $F_s$ (N) | $-0.10594$ | $-18.223$ |
| $\sigma_1$ (N·m/r) | 0.0006 | 41.34 |
| $\sigma_0$ (N·m/r) | 0.22 | 1.87 x 10^3 |
| $\sigma_1$ (N·m/s/r) | 0.22 | 1.73 x 10^3 |

The identified static friction parameters are used to identify dynamic friction parameters. The dynamic friction parameters are identified through experimentally obtained hysteresis curves representing pre-slip behavior. The frictional force in the pre-sliding stage of the LM guide can be expressed as:

$$
F_f = \sigma_g z_g + \sigma_z \dot{z}_g + \sigma_z \dot{X}_g
$$

The identification error and optimization objective function at time $t$ are as follows:

$$
E(X_{gd}, t) = F_{gd}(t) - \hat{F}_{gd}(X_{gd}, t)
$$

$$
J_{gd} = \sum_{i=1}^{N_{gd}} E^2(X_{gd}, t_i) + \max \left\{ E(X_{gd}, t) \right\}
$$

where $X_{gd}$ is given by $[\sigma_0, \sigma_1]$. $F_{gd}$ is the experimentally measured friction force during pre-slip operation, and $\hat{F}_{gd}$ is the estimated value calculated by the friction model. $N_{gd}$ is the number of samples contained in the hysteresis curve.

The hysteresis curve drawn by the dynamic friction identification experiment of the LM guide is shown in Figure 8b. The dynamic friction parameters of the LM guide obtained from the identification are shown in Table 5.

3.3. Identification for the Friction Model of Nut Shaft

**Step C:** By adjusting the rotational speed of the screw drive motor and the nut drive motor simultaneously, the nut shaft parameters and the hydrostatic bearings friction
model can be identified. Due to the unique transmission structure of DDHLS, when the screw and the nut rotate at the same speed and in the opposite direction, in theory, the worktable remains stationary, and the shear force of the hydrostatic lead screw oil film is zero. The friction of the LM guide can be negligible. The static and dynamic friction parameter identification of the nut shaft adopts the same identification process as the LM guide. The identification results of the nut shaft are shown in Table 5. In order to compare, the friction parameter units, except the screw drive motor shaft, were converted to linear motion mode.

4. Controller Design

In the proposed DDHLS, the motor and the nut drive shaft are highly susceptible to nonlinear friction at low rotational speeds. At the same time, the torque generated by the oil film friction of hydrostatic transmission components varies at different rotational speeds. In addition, the friction model parameters of the rolling contact components are inevitably affected by temperature, wear, lubrication, and position during operation. It is necessary to design the friction compensation controller to achieve high-precision feed control.

Based on the research of Jiang et al. [18], two influence factors are introduced into the LuGre friction model according to the temperature and disturbance of each rolling contact friction component. The AACA was proposed based on the all-component friction model, as shown in Figure 9. After that, it was combined with Lyapunov stability theory for stability analysis. When the screw speed is greater than the nut speed, the nut drive motor only needs to offset the friction torque of the nut shaft, and the friction torque is caused by the LM guide acts on the screw shaft. Substituting Equations (14), (19), and (20) into (1) and making \( F_L = M_n \dot{X}_n \), the following equation can be obtained:

\[
\begin{aligned}
J_{\alpha d} \dot{\theta}_{\alpha b} &= K_n l_y - \left[ \alpha_{n} \left( \sigma_{bn} z_{n} + \sigma_{mn} \dot{z}_{n} \right) + \beta_{n} \sigma_{bn} \dot{\theta}_{n} \right] - \left( T_{m} + T_{h} + T_{l} \right) \\
&- R_{\alpha} \left[ \left( \sigma_{n} z_{n} + \sigma_{l} \dot{z}_{n} \right) + \beta_{n} \sigma_{n} \dot{X}_{n} + F_{L} \right] \\
J_{\alpha d} \dot{\theta}_{\beta b} &= K_n l_{q} - \left[ \alpha_{n} \left( \sigma_{bn} z_{n} + \sigma_{mn} \dot{z}_{n} \right) + \beta_{n} \sigma_{bn} \dot{\theta}_{n} \right]
\end{aligned}
\]  

\( (27) \)

Where \( \alpha_{i} > 0 \) is the influence factor of the load force on the bristle offset, and \( \beta_{i} > 0 \) is the change in viscous friction moment due to temperature change. \( l = m, g, \) and \( n \) represent the screw motor shaft, LM guide, and nut shaft.

![Figure 9. Block diagram of the AACA controller.](image-url)
tracking error of the screw shaft servo system and the dynamic change of the tracking error are defined as:

\[
\begin{align*}
\dot{e}_{s1} &= \theta^*_{Ms} - \theta_{Ms} \\
\ddot{e}_{s1} &= \ddot{\theta}^*_{Ms} - \ddot{\theta}_{Ms}
\end{align*}
\]  

(28)

where \( \theta^*_{Ms} \) is the desired rotation angle.

Select the first Lyapunov function and derive it:

\[
\begin{align*}
V_{s1} &= \frac{1}{2} e_{s1}^2 \\
\dot{V}_{s1} &= e_{s1} \dot{e}_{s1} = e_{s1} (\dot{\theta}^*_{Ms} - \dot{\theta}_{Ms})
\end{align*}
\]  

(29)

The speed control signal is designed as follows:

\[\omega = \dot{\theta}^*_{Ms} + k_1 e_{s1} + k_2 \int_0^t e_{s1} dt\]  

(30)

The integral term \( \int_0^t e_{s1} dt \) is used to ensure that the tracking error of the system approaches zero under the condition of model or load uncertainty. \( k_1 > 0 \) and \( k_2 > 0 \) are design parameters.

The second error variable of the screw shaft servo system is designed as follows:

\[
\begin{align*}
\dot{e}_{s2} &= \omega - \dot{\theta}_{Ms} = \dot{\theta}^*_{Ms} + k_1 e_{s1} + k_2 \int_0^t e_{s1} dt - \dot{\theta}_{Ms} \\
\ddot{e}_{s2} &= \ddot{\theta}^*_{Ms} + k_1 \dot{e}_{s1} + k_2 e_{s1} - \ddot{\theta}_{Ms}
\end{align*}
\]  

(31)

From Equations (19), (20), (27), and (31), the following equation can be obtained:

\[
J_{Ms} \ddot{e}_{s2} = (T_{mef} + T_{mef} + T_{mef}) - K_{m} e_{s2} + J_{Ms} \ddot{\theta}_{Ms} + J_{Ms} k_1 \dot{e}_{s1} + J_{Ms} k_2 e_{s1}
\]

\[
+ \alpha_n z_n \left[ \sigma_{n} - \sigma_{n} \frac{\theta_{n}}{g(\theta_{n})} \right] + \alpha_e \left[ \theta_{e} \sigma_{e} \ddot{\theta}_{e} + \beta_e \sigma_{e} \dot{\theta}_{e} \ddot{\theta}_{e} + F_{L} \right]
\]

(32)

where \( \alpha_n, \beta_n, \alpha_e, \beta_e, \) and \( F_L \) denote the uncertain parameters present in the system, and their real values are represented by the estimated values \( \hat{\alpha}_n, \hat{\beta}_n, \hat{\alpha}_e, \hat{\beta}_e, \) and \( \hat{F}_L \).

The error between the estimated and actual values can be expressed as:

\[
\begin{align*}
\hat{\alpha}_n &= \alpha_n - \hat{\alpha}_n \\
\hat{\beta}_n &= \beta_n - \hat{\beta}_n \\
\hat{\alpha}_e &= \alpha_e - \hat{\alpha}_e \\
\hat{\beta}_e &= \beta_e - \hat{\beta}_e \\
\hat{F}_L &= F_L - \hat{F}_L
\end{align*}
\]  

(33)

The average bristle offset \( z \) in the LuGre friction model is unknown and cannot be measured. Here, four nonlinear observers are designed to estimate the \( z \) value. The nonlinear state observer equations are as follows:
\[
\begin{cases}
\dot{\hat{z}}_{am} = \hat{\theta}_{am} - \frac{\partial \hat{z}_{am}}{\partial \hat{\theta}_{am}} \hat{z}_{am} + \tau_{am} \\
\dot{\hat{z}}_{pm} = \hat{\theta}_{pm} - \frac{\partial \hat{z}_{pm}}{\partial \hat{\theta}_{pm}} \hat{z}_{pm} \\
\dot{\hat{z}}_{ag} = \hat{\theta}_{ag} - \frac{\partial \hat{z}_{ag}}{\partial \hat{\theta}_{ag}} \hat{z}_{ag} + \tau_{ag} \\
\dot{\hat{z}}_{pg} = \hat{\theta}_{pg} - \frac{\partial \hat{z}_{pg}}{\partial \hat{\theta}_{pg}} \hat{z}_{pg}
\end{cases}
\tag{34}
\]

where \( \hat{z}_{am}, \hat{z}_{pm}, \hat{z}_{ag}, \hat{z}_{pg} \) are the estimated values of \( z \). \( \tau_{am} \) and \( \tau_{ag} \) are the observer compensation items.

The corresponding state estimation errors and their changes can be expressed as follows:

\[
\begin{cases}
\dot{\hat{z}}_{am} = z_{am} - \hat{z}_{am} \\
\dot{\hat{z}}_{pm} = z_{pm} - \hat{z}_{pm} \\
\dot{\hat{z}}_{ag} = z_{ag} - \hat{z}_{ag} \\
\dot{\hat{z}}_{pg} = z_{pg} - \hat{z}_{pg}
\end{cases}
\tag{35}
\]

The Lyapunov function is further defined as follows:

\[
V_\omega = V_s + \frac{1}{2} k_3 \left( \int_0^t \dot{e}_d \, dt \right)^2 + \frac{1}{2} k_1 J_{el} \hat{e}_d^2 + \frac{1}{2} \alpha \hat{\beta}_a^2 + \frac{1}{2} \beta \hat{\alpha}_a^2 + \frac{1}{2} \beta \hat{\alpha}_g^2 + \frac{1}{2} \beta \hat{\alpha}_g^2
\tag{37}
\]

where \( k_3 > 0 \) is the design parameter, \( r_0 > 0, r_1 > 0, r_2 > 0, r_3 > 0, \) and \( r_4 > 0 \) are the adaptive gains. Since \( k_3, r_0, r_1, r_2, r_3, \) and \( r_4 \) are positive real numbers, it can be determined that \( V_\omega \) is positive definite.

The differentiation of the Lyapunov function with respect to time is calculated by Equations (32) and (37), as follows:
\[
\begin{align*}
\dot{v}_{s2} &= -k_4 e_1^2 + e_4 e_2 \\
&= \begin{bmatrix}
-K_n i_{up} + J_m \dot{\theta}_m^* + J_m k_e \dot{e}_s + J_m k_e e_s \\
+\alpha_e z_m \left( \sigma_{0e} - \sigma_{m} \frac{\dot{\theta}_m}{g(\theta_m)} \right) + \alpha_e \sigma_m \dot{\theta}_m + \beta_e \sigma_{m} \dot{\theta}_m \\
+ e_2 k_i \\
+ R_f \left[ \alpha_g z_g \left( \sigma_{0g} - \sigma_{1g} \frac{\dot{X}_i}{g(X_i)} \right) + F_i + \alpha_g \sigma_{1g} \dot{X}_i + \beta_g \sigma_{1g} \dot{X}_i \\
+ \left( T_{nef} + T_{nc} \right) \right] \\
+ \alpha_n \dot{\alpha}_n + \frac{1}{r_1} \beta_n \dot{\beta}_n + \frac{1}{r_2} \alpha_g \dot{\alpha}_g + \frac{1}{r_3} \beta_g \dot{\beta}_g + \frac{1}{r_4} \dot{F}_i \end{bmatrix}
\end{align*}
\]

Combining Equations (31) and (32), the control law of the screw axis servo system is designed as follows:

\[
i_{np} = \frac{1}{K_n} \begin{bmatrix}
J_m \dot{\theta}_m^* + J_m k_e \dot{e}_s + J_m k_e e_s + \frac{e_1}{k_3} + k_4 e_2 \\
\dot{\alpha}_n \sigma_{0m} - \dot{\alpha}_n \sigma_{m} \frac{\dot{\theta}_m}{g(\theta_m)} \\
+ \left( \dot{\sigma}_m \right) \\
+ \left( T_{nef} + T_{nc} \right) \dot{\theta}_m + \left( T_{nec} + T_{nc} \right) \\
+ R_f \left[ \dot{\alpha}_g \sigma_{0g} - \dot{\alpha}_g \sigma_{1g} \frac{\dot{X}_i}{g(X_i)} \right] \\
+ \dot{F}_i + \left( \dot{\alpha}_g \sigma_{1g} + \dot{\beta}_g \sigma_{1g} \right) \dot{X}_i
\end{bmatrix}
\]

Substitute Equation (39) into Equation (38) and simplify it:
\[ \dot{v}_{j2} = -k_i e_{i2} - k_i e_{i2}^2 + \tilde{F}_L \left( e_{i2} k_i R_\tau - \frac{\dot{\hat{F}}}{r_i} \right) - \alpha_n \left( \frac{\theta_{an}}{g(\theta_{an})} \right) z_{an}^2 \]

\[-\beta_n \left( \frac{\theta_{an}}{g(\theta_{an})} \right) z_{an}^2 - \alpha_n \left( \frac{\theta_{an}}{g(\theta_{an})} \right) z_{an}^2 \]

\[+ \alpha_m \tilde{z}_{an} \left( \sigma_{an} e_{i2} k_3 - \sigma_{an} e_{i2} k_3 \right) \left( \frac{|\dot{\theta}_m|}{g(\theta_{m})} - \tau_{an} \right) \]

\[+ \alpha \tilde{z}_{ag} \left( R_n \sigma_{ag} e_{i2} k_3 - R_n \sigma_{ag} e_{i2} k_3 \right) \left( \frac{|\dot{\theta}_g|}{g(\theta_{g})} - \tau_{ag} \right) \]

\[-\beta_n \tilde{z}_{fn} \cdot \tau_{fn} - \beta_n \tilde{z}_{fg} \cdot \tau_{fg} \]

\[+ \tilde{a}_m \left( e_{i2} k_3 \left( -\tilde{z}_{an} \sigma_{am} + \tilde{z}_{an} \sigma_{am} - \frac{|\dot{\theta}_m|}{g(\theta_{m})} + \sigma_{am} \theta_{m} \right) - \tilde{a}_m \right) \]

\[+ \tilde{a}_g \left( R_n \sigma_{ag} e_{i2} k_3 \left( -\tilde{z}_{ag} \sigma_{ag} + \tilde{z}_{ag} \sigma_{ag} - \frac{|\dot{\theta}_g|}{g(\theta_{g})} + \sigma_{ag} \dot{X}_i \right) - \tilde{a}_g \right) \]

\[+ \tilde{b}_g \left( R_n \sigma_{ag} \dot{X}_i e_{i2} k_3 - \tilde{b}_g \right) \]

For the screw shaft servo system, the adaptive laws and the observer error compensation terms for each unknown parameter are selected as shown in Equation (41):

\[ \dot{\alpha}_m = r_n e_{i2} k_3 \left( -\tilde{z}_{an} \sigma_{an} + \tilde{z}_{an} \sigma_{an} - \frac{|\dot{\theta}_m|}{g(\theta_{m})} + \sigma_{am} \theta_{m} \right) \]

\[ \dot{\beta}_m = r_n \sigma_{an} e_{i2} k_3 \theta_{m} \]

\[ \dot{\alpha}_g = r_n R_n e_{i2} k_3 \left( -\tilde{z}_{ag} \sigma_{ag} + \tilde{z}_{ag} \sigma_{ag} - \frac{|\dot{\theta}_g|}{g(\theta_{g})} + \sigma_{ag} \dot{X}_i \right) \]

\[ \dot{\beta}_g = r_n R_n \sigma_{ag} \dot{X}_i e_{i2} k_3 \]

\[ \dot{\hat{F}}_L = r_n e_{i2} k_3 \dot{X}_i \]

\[ \tau_{an} = \sigma_{an} e_{i2} k_3 - \sigma_{an} e_{i2} k_3 \left( \frac{|\dot{\theta}_m|}{g(\theta_{m})} \right) \]

\[ \tau_{ag} = R_n \sigma_{ag} e_{i2} k_3 - R_n \sigma_{ag} e_{i2} k_3 \left( \frac{|\dot{\theta}_g|}{g(\theta_{g})} \right) \]

The adaptive friction compensation algorithm of the nut drive shaft adopts the same design method as the screw drive shaft, and the control law of the nut shaft servo system is designed as follows:
For the nut shaft servo system, the adaptive laws of the unknown parameters and the observer error compensation term are shown in Equation (43):

\[
i_{in} = \frac{1}{K_m} \left[ J_m \dot{\theta}^2 + J_m k \dot{\theta} + J_m k \ddot{\theta} + k_3 e + k_4 e_2 + \dot{\alpha}_n \right] + \left[ -\dot{\alpha}_n \ddot{\theta} \right] \frac{\ddot{\theta}}{\dot{\theta}^2} + \left[ (\dot{\alpha}_n + \dot{\beta}_n) \right] \theta_n \tag{42}
\]

\[
\begin{aligned}
\dot{\alpha}_n &= r_0 e^2 k_3 \left( -2 \sigma \sigma_0 + 2 \sigma_1 \sigma_0 \right) \frac{\ddot{\theta}_n}{\dot{\theta}_n} + \sigma \theta_n \\
\dot{\beta}_n &= r_1 \sigma_2 e_2 k \dot{\theta}_n \\
\tau_{in} &= \sigma_0 e_2 k_3 - \sigma_1 e_2 k_3 \frac{\ddot{\theta}_n}{\dot{\theta}_n}
\end{aligned}
\tag{43}
\]

5. Experiments and Results

5.1. Experimental Setup

The DDHLS experimental device is shown in Figure 10. The laser coaxial displacement measuring instrument (CL-3000, KEYENCE) is used as the table displacement sensor to generate real-time position signals, and the sampling period was set to 0.001 s. In addition, the Matlab/Simulink and Rapid Control Prototyping system (MicroLabBox, dSPACE) were used to deploy the feedback control algorithm implementation.

![Figure 10. Experimental setup of DDHLS.](image)

5.2. Fluid Friction Model Estimation

As shown in Figure 10, the platinum resistance temperature sensors PT100 were used to measure the real-time oil temperature of the hydrostatic components. A, B, and C are the oil temperature measurement points of the front hydrostatic bearing, the hydrostatic
lead screw, and the rear hydrostatic bearing, respectively. Figures 11 and 12 show the experimentally measured fluid friction torque and oil heating of the hydrostatic components compared with their estimated values using the fluid friction calculation model. As shown in Table 6, the fluid friction calculation model based on the variable viscosity theory has superior fitting property compared with conventional constant viscosity model, which confirms the correctness of the proposed variable viscosity theory.

Figure 11. Comparison of the experimental friction torque and oil heating of hydrostatic bearings with their estimation.

Figure 12. Comparison of the experimental friction torque and oil heating of hydrostatic lead screw with their estimation.
Table 6. Comparisons of estimation errors of the fluid friction calculation model.

| Viscosity Types | Estimation Error (N·m) | Estimation Error (℃) |
|-----------------|------------------------|----------------------|
|                 | Max        | Mean     | Max        | Mean      | Max        | Mean      | Max        | Mean      |
| Constant        | 0.289      | 0.127    | 0.115      | 0.045     | 5.461      | 2.611     | 1.582      | 0.674     |
| Variable        | 0.032      | 0.016    | 0.029      | 0.009     | 0.421      | 0.138     | 0.367      | 0.120     |

5.3. Friction Compensation

In order to verify the effectiveness of the proposed control method, a comparative analysis of the tracking performance between the PID control algorithm and the ACCA control algorithm is performed. Figures 13 and 14 show the experimental results of the periodic sinusoidal trajectory tracking when CHLS and DDHLS adopt traditional PID control and AACA control, respectively. The table position tracking command used in the experiment is \( y = \sin(0.4\pi t) \). In order to avoid the nonlinear creep region for both drive axes, the position commands of the screw drive shaft and the nut drive shaft are set as \( y = \sin(0.4\pi t) + 7t \) and \( y = 7t \), respectively. Figures 13a and 14a show the tracking responses of the workbench to the periodic sinusoidal reference trajectories in CHLS and DDHLS, respectively. The position tracking error of the table is shown in Figures 13b and 14b. When CHLS passes through the speed zero point at low speed, the trajectory tracking is affected by the nonlinear friction of the servo system itself, resulting in a large tracking error (the maximum error is 7.93 \( \mu \)m). However, the differential drive structure in DDHLS allows both the screw drive motor and the nut drive motor to rotate at high speed. The nonlinear frictional interference of the servo system is reduced significantly (the maximum error is 4.64 \( \mu \)m). Thus, the tracking error of the table at ultra-low speed is reduced.

It can be seen from Figures 13b and 14b that, for both CHLS and DDHLS, the AACA control algorithm can more effectively compensate the effect of low-speed nonlinear friction on the servo feed system compared with the PID control algorithm. The tracking errors in CHLS and DDHLS are reduced by 40.6% and 62.9%, respectively. The tracking accuracy of the feeding system is effectively improved, and the friction disturbance during the operation of the servo system is better suppressed.

Figure 13. Experimental results of position and tracking errors before and after friction compensation by CHLS. (a) Tracking responses. (b) Tracking errors.
Figure 14. Experimental results of position and tracking errors before and after friction compensation by DDHLS. (a) Tracking responses. (b) Tracking errors.

6. Conclusions

This paper presents a comprehensive investigation on the modeling, identification, and compensation control of friction for a novel DDHLS. A fluid friction calculation model based on variable viscosity theory is proposed. Experimental results reveal that it has greater accuracy than the fluid friction calculation model based on the constant viscosity hypothesis. The proposed ACRFIM can identify independently the friction model of all the rolling contact components and the hydrostatic components. This characteristic assured the precision control of the DDHLS. The AACA control strategy was designed by introducing the temperature and the disturbance influence factors into the friction model of the rolling contact component and considering the influence of the dynamic fluid friction of the hydrostatic component. The experimental results show that, compared with CHLS, the nonlinear friction disturbance of DDHLS is significantly reduced in low-speed feed. In addition, the tracking accuracy of the ACCA control strategy is improved compared with the PID control strategy.

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