Students creative thinking skills in solving two dimensional arithmetic series through research-based learning

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Abstract: Arithmetics is one of the topics in Mathematics, which deals with logic and detailed process upon generalizing formula. Creativity and flexibility are needed in generalizing formula of arithmetics series. This research aimed at analyzing students creative thinking skills in generalizing arithmetic series. The triangulation method and research-based learning was used in this research. The subjects were students of the Master Program of Mathematics Education in Faculty of Teacher Training and Education at Jember University. The data was collected by giving assignments to the students. The data collection was done by giving open problem-solving task and documentation study to the students to arrange generalization pattern based on the dependent function formula \( i \) and the function depend on \( i \) and \( j \). Then, the students finished the next problem-solving task to construct arithmetic generalization patterns based on the function formula which depends on \( i \) and \( i + n \) and the sum formula of functions dependent on \( i \) and \( j \) of the arithmetic compiled. The data analysis techniques operative in this study was Miles and Huberman analysis model. Based on the result of data analysis on task 1, the levels of students creative thinking skill were classified as follows; 22.22\% of the students categorized as "not creative"; 38.89\% of the students categorized as "less creative" category; 22.22\% of the students categorized as "sufficiently creative"; and 16.67\% of the students categorized as "creative". By contrast, the results of data analysis on task 2 found that the levels of students creative thinking skills were classified as follows; 22.22\% of the students categorized as "sufficiently creative", 44.44\% of the students categorized as "creative"; and 33.33\% of the students categorized as "very creative". This analysis result can set the basis for teaching references and actualizing a better teaching model in order to increase students creative thinking skills.

1. Introduction
Mathematics education at higher education tends to direct learners to understand mathematical formula by applying it in a lesson involving problem solving. This is the case particularly when the material contains certain formulas, such as arithmetic sequence and series. In fact, the ability to think in identifying and constructing formulas in Mathematics is required to foster students understanding on the material and accrue meaningful learning. One of thinking abilities essential for the students to experience more meaningful learning and improve their thinking skill in solving everyday problems is creative thinking.
Munandar suggests that creative thinking can be defined as the ability to reflect the aspects of fluency, flexibility, originality, and elaboration [1]. In creative thinking, learner will go a series of stages, inter alia, synthesizing ideas, building ideas, planning the implementation of ideas, and applying these ideas to produce something [2].

Based on the aforementioned discussion, it is imperative to develop an examination focusing on the graduate students creative thinking skills concern with combinatorics course. Knowing the graduate students creative thinking skills will shed light on what is essential to increase their mastery level, especially in the course of Combinatorics.

Learning method is also very important in measuring students creativity. As stated in the 21st century skills framework, creativity is one of the important things for students. For achieving and increasing their creativity, a student-centered learning method needs to be applied. One of student-centered learning methods is research-based learning, a method recently applied by several universities in several countries. The learning method has one of the ambitions of many universities, that is to bring the two core of educational endeavour and research closer together and to integrate research into teaching more than what has been done so far [3].

2. Research Methods

The research method applied in this research was triangulation with qualitative and quantitative approach. Qualitative research adheres to the natural background in which a study takes place (it views the context as a whole). In this regard, human serves as an instrument. Qualitative method requires inductive data analysis and theory development, which are all based on the data, which is descriptive in nature and more concerned with the process under investigation. The research is limited by its focus, the specific criteria for ensuring data validity, and its temporary design. Also, qualitative data are generated through joint decision [4]. However, this qualitative research was more concerned with results, rather than process, and applied by analyzing the students performance on task 1 and task 2 in order to gain details regarding their creative thinking skills and metacognition levels. Afterwards, the effectiveness of learning method operationalized can be proved by the result of data analysis.

The research subjects were 18 students of the Master Program of Mathematics Education in Faculty of Teacher Training and Education Jember University in the 2016/2017 academic year. The students were expected to give a comprehensive illustration of the level of their creative thinking skills.

The data collection technique was the most important step in the research as it determined the success of gaining accurate data, which were then analyzed the students answers on the problem given. The data were collected by giving the assignment. The task was given to gather data pertinent to the level of students creative thinking skills in Combinatorics. The data collected were reduced, served, concluded, and verified. Data validity was ensured by triangulation, peer checking, and extended observation.

The researchers applied research-based learning when implementing the teaching and learning on Arithmetic series. Based on research-based learning conducted by Mr. Dafik (the lecturer), which adhered to Tremp’s model and focused on Syntax, seven steps were applied systematically [5]. The steps of research-based learning operative in the study included formulating a general question, overview of research-literature, defining the question, planning research activities, undertaking investigation, interpretation and consideration of results, and report as well as presentation.

The researchers implemented the research-based learning as a part of steps for getting the data related to the creative thinking analysis. Escalating students creative thinking skills can be done in numerous ways, one of which is by analyzing and predicting it. According to Siswono, the distribution of creative thinking levels is useful in predicting students ability to think creatively, especially in the field of Mathematics [2].

The data presentation in qualitative research can be made in a brief description and relations between categories. In Miles and Huberman’s model, as quoted by Sugiyono, the most common form of the data presentation in qualitative research, to date, has been narrative text [6]. Commonly, the data presentation in qualitative research is narrative text, but in this research, data presentation included the
classification and identification of the data, followed by the collection of data organized and categorized for drawing conclusion.

3. Research Finding
The data collection was commenced by providing learning material first and then distributing the task to the students, which focused on arithmetic progression. The arithmetic progression formula was $U_n = a + (n - 1)b$, where $U_n = f(x_i)$ and a series of to-function $i, a$ is the first term of the series $i$, $n$ is the number of rows $i$, $b$ is different from the series to-$i$, and $i$ own a tribe to-$n$. While, formula of arithmetic sequence is $S_n = \frac{n}{2}(a + U_n)$.

3.1 Formula of Arithmetic Sequence Function $f(x_i)$ Based on Term $i$
In this step, the students were required to determine the formula of arithmetic sequence. The formula might consist of 1 or more formulas, based on the sequence pattern. For example, if there are two kinds of formula on the sequence pattern, so the formula should be written as the following:

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|-----|---|---|---|---|---|---|
| $f(x)$ | 4 | 1 | 5 | 2 | 6 | 3 |

$\Rightarrow f(x_i) = \begin{cases} \frac{i + n + 2}{2}, & i \text{ odd} \\ \frac{i}{2}, & i \text{ even} \end{cases}$

The material above was discussed by the lecturer and students. Most of the student came up with the following results.

![Figure 1. Many Arithmetic sequences found by the students](image)

3.2 Formula of Functions Which Depends on $i$ and $j$ or $P_{m,d}(i, j)$
In the next step, arithmetic sequence had to be generalized. The formula could consist of many formulas based on the sequence pattern at each $i$-row or $j$-column. There were many examples about generalized arithmetic sequence, depending on variables $i$ and $j$:

| $i$ | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|---|
| $j$ | 1 | 1 | 2 | 3 | 4 | 5 |

| $i$ | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|---|
| $j$ | 1 | 1 | 4 | 7 | 10 | 13 |

| $i$ | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|---|
| $j$ | 1 | 2 | 5 | 8 | 11 |

| $i$ | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|---|
| $j$ | 1 | 3 | 6 | 9 | 12 |

| $i$ | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|---|
| $j$ | 1 | 3 | 6 | 9 | 12 |

Based on the example, students tried to find the new arithmetic sequence and they came up with the following results:
Figure 2. Many Arithmetic sequences depending on $i$ and $j$ found by the students

3.3 Finding Formulas of Total Function $\sum_{j=1}^{m} P_{i,j}^{m,n}(i,j)$

If the formula $P_{i,j}^{m,n}(i,j)$ can be determined correctly, the formula amount $\sum_{j=1}^{m} P_{i,j}^{m,n}(i,j)$ exactly can be determined as well. Therefore, the formula of $\sum_{j=1}^{m} P_{i,j}^{m,n}(i,j)$ can be generalized properly. The following example portrays how the students found the function formula:

|   | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 |
| 2 | 5 | 7 | 9 | 11 | 13 |
| 3 | 10 | 13 | 16 | 19 | 22 |
| 4 | 15 | 18 | 21 | 24 | 27 |
| 5 | 20 | 23 | 26 | 29 | 32 |
| 6 | 25 | 28 | 31 | 34 | 37 |
| 7 | 30 | 33 | 36 | 39 | 42 |
| 8 | 35 | 38 | 41 | 44 | 47 |
| 9 | 40 | 43 | 46 | 49 | 52 |
| 10 | 45 | 48 | 51 | 54 | 57 |

Thus, the formula $P_{i,j}^{m,n} = (j-1)n + i$

Then formula Number $\sum_{j=1}^{m} P_{i,j}^{m,n}(i,j)$

$$\sum_{j=1}^{m} P_{i,j}^{m,n}(i,j) = \sum_{j=1}^{m} (j-1)n + i = \sum_{j=1}^{m} jn - n + i = \sum_{j=1}^{m} jn + \sum_{j=1}^{m} (-n + i) = (1 + 2 + 3 + ... + m)n + (-n + i)m$$

$$= \frac{m}{2}(1 + m)n - mn + mi = \frac{mn}{2}(m - 1) + mi$$

3.4 Achievement of Test Result Based on Student Creative Thinking Skill’s Indicators

The data concerning the students creative thinking skills classified them into high, medium, and low group, the complete of which details are presented in Figure 3.
Figure 3 shows that the students performance on task 1 and task 2 results were significantly different at all levels. The low-level students improved their performance from level 0 to level 2 or 3. Similar increase was also evident among medium-level students, who successfully increased their performance from level 1 to level 3. The other improvement, among high-level students, was evinced by the improvement from level 2 or 3 to level 4. The improvement in all indicators of students creative thinking skills could not be separated from the guidance of research-based learning process. This finding was in line with the results of research conducted by Bani, the results of which indicated that guided discovery methods can improve students mathematical reasoning [8]. This teaching method not only encourages the students to be more active, but also guides them to be more active in learning. Hudojo states that if students are actively involved in finding concepts, then they will understand the concept better, and their memory will retain the learnt concept longer, allowing it to be used in other contexts [9].

Based on the research results, the researchers classified the students critical thinking skills into the following categories, as stipulated by the indicators of critical thinking skills.

Table 1. The Students Creative Thinking Skills on Task 1 and Task 2

| Task 1 | Category | Task 2 |
|--------|----------|--------|
| Sum    | Percentage (%) | Sum    | Percentage (%) |
| 4      | 22,22 (Not Creative) | -      | - |
| 7      | 38,89 (Less Creative) | -      | - |
| 4      | 22,22 (Sufficiently Creative) | 4      | 22,22 |
| 3      | 16,67 (Creative) | 8      | 44,44 |
Based on the data presented in Table 1, it is obvious that majority of the students creative thinking skill levels, based on task 1, are in the "less creative" category, and the level of students creative thinking skill, upon accomplishing task 2, is in "creative" category. Therefore, the levels of students creative thinking skill were significantly different between the two tasks. However, based on the results of the interview, there were some interesting obstacles experienced by students. These findings revealed (1) that the students arranged non-arithmetic sequence; (2) that they tried to arrange a multilevel arithmetic sequence; (3) that when constructing the formula, many students assumed the parameters as constants, upon operating them; (4) that the students generally did not think that the formula obtained had to be applied in a general fashion to any extent; (5) that when finding the function of formula from the existing data, they found it difficult modifying the first term of sequence, this was the case because the first term always moved when generalized, be the first term start from the front or back or even the middle in arithmetic sequence; and (6) that when generalizing arithmetic pattern, there was a new arithmetic sequence with more n-terms and the initial value lied on the same position as previous one. Thus, research-based learning was proven effective in aiding the students in improving their creative thinking skills, although there were some obstacles experienced by the students.

4. Discussion
The following figure is an example of students work when finding the arithmetic sequence, along with its analysis result.
The data was more creative, because the number 1 was already in the 3\textsuperscript{rd} term.

The number 10 was a parameter, so it was chosen to be 7 plus 3, where the number 7 was the n\textsuperscript{th} term formula for i = 3 \text{ mod } 3.

During the process of finding the function of the second step, it should be noted that term i on the first difference become the first term at the second step, and so on. So that the n\textsuperscript{th} reduced one. It should be noted also that the first term in the process of finding the n\textsuperscript{th} term is difference between the two first term in function f (x).

Figure 4. Students work upon finding the new arithmetic sequence and its formula.
The following interview result portrays the response of very-creative students, describing the process he went through upon finding new arithmetic sequence up until getting the generalization of formula.

Lecturer: Did you face a problem when finding the arithmetic sequence which could be generalized?

Student A: Yes, I did. I ran into a problem when finding it because I tried to find an unusual arithmetic sequence. It was the first time I tried it.

Lecturer: Can you explain more about your problems?

Student A: I think it was because I did not do it this way before. We should tried to find an unusual arithmetic sequence. In the other courses, there was no task like this. In this course, we should find the unique arithmetic sequence which could be generalized and then expand it very carefully. When expanding the sequence, I experienced many problems. For example, sometimes it could not be expanded for $m$ or $n$ even. When it happened, I had to find another arithmetic sequence or use it and then tried to generalize it with many cases of $m$ and $n$.

Lecturer: From all of the learning steps you went through, what was your most difficult problem when finishing it?

Student A: This learning method was new for me. I got many finishing steps clearly but the most difficult problem was when it came to predicting of arithmetic sequence which I already found. I should think deeply about the possibility of expanding the sequence and generalizing it. Otherwise, if I did not think about it deeply about it then I got stuck or got many risks. For example, I got an arithmetic sequence going through careful thoughts, and, if I could not expand it, then I had to repeat the whole process all the way from the beginning. Unluckily, sometimes, I got an arithmetic sequence and already managed to expand but I could not find the formula, then I started all steps over again.

Lecturer: How can you handle your problems in finishing the test?

Student A: I should understand the task very carefully, think deeply before going onto the next step and being more creative when finding unique sequence. Once, I tried to never give up.

Lecturer: What is your opinion about the learning method already applied in your class?

Student A: It was good for me. Even though, I got difficulties but I enjoyed the learning process. It was meaningful for me.

Based on figure 4 and interview result above, the study has concluded that the students work signified creative thinking ability at “very creative” category because the first term already started from in the third position and all of the numbers were not in ordered. According to Wulantina’s research, student who has high ability tries to get more detailed information and can indentify the problems given very well when choosing the information during problem solving steps [10]. On the other hand, the results of Saefudin’s research concludes that when applying ideas, students with high Mathematics ability does not make a mistake in solving problems, and they feel so much encouraged to solve problems in different ways using various answers [11]. The accuracy to apply a chosen strategy is based on student’s previous experiences. This is in line with the opinion Cropley and Urban, which corroborates that the incubation stage is the stage at which students put their relation completion of the ideas they have had previously [7]. Based on the above findings, the formulas of second step or $P_{n,m}(i,j)$ were created. At this stage, only 61.11% of graduate students managed to find a formula function properly.

Based on the analysis results of data obtained from the students, some students were found unable to relate the first function to another function at the next level. Therefore, the research shifted to the formula functions that depend on $i$ and $j$ or $P_{n,m}(i,j)$ formulas. The following description shown in Figure 5 provides more details on that finding. What follows is student B’s work upon generalizing formula, followed by the result of interview conducted to him.
The order of numbers in the table is not sorted.

The limit for the sequence \( j \) is \( 1 \leq j \leq m \).

The limit of \( j \) in this table is \( 1 \leq j \leq 4 \).

The generalizing formula requires a four-stage process of the invention depending on row position \( j \) and \( i \).

**Step 1 for \( j \) and \( i \) odd:**
The process of finding the formula of \( U_n \) and \( U_m \) functions in the series for \( j \) and \( i \) odd is done by considering the parameter \( b \) on \( U_n \) and parameter \( a \) on \( U_m \).

**Step 2 for \( j \) and \( i \) even:**
The process of finding the formula of the function \( U_m \) is related to the parameters \( a \).

**Step 3 and 4:**
As for \( j \) odd and \( i \) even, and for \( j \) even and \( i \) odd, finding the function on these steps can be done by identifying the same parameters as the previous two processes.

The conclusion of the formula for \( P_{n,0}^+ = P_{4,0}^+ \) states that there are four different formulas based on the position lied on \( j \) and \( i \).

Figure 5. Student’s work of finding the new arithmetic sequence and it’s formula.

**Lecturer:** Did you face any problem when finding the arithmetic sequence which could be generalized?

**Student B:** Yes, I did. I faced many problems about it.
Lecturer: Can you explain more about your problems?
Student B: I got many problems. For example, I found difficulties finding a new arithmetic sequence at first, so I had to try it again and again using multiple ways. When generalizing formulas, sometimes, I forgot to assume the current number as parameters so, I got stuck in the last step.

Lecturer: From all of the steps of the learning which you went through, what was your most difficult problem when finishing it?
Student B: Finding the new arithmetic sequence at first was tough, and also formulation part was hard.

Lecturer: How did you handle your problems in finishing the test?
Student B: I had to find a new arithmetic sequence through many different ways and then tried to finish the formulation very carefully.

Lecturer: What was your opinion about the learning method already applied in your class?
Student B: I faced many problems as I explained before but I think that learning method was unusual and different from others. As long as I was capable of finishing the task on every step, I had to think seriously and very carefully.

Based on the students performance portrayed on figure 5 and interview to student B, the step of generalizing formula required a four-stage invention process, depending on the row position $j$ and $i$. There was also one part in the work done by student B that indicated “creative” level because the arrangement of the numbers was already scrambled but the formulas could not be found. The student possessing creative ability tried to get more details information yet he could not indentify the problems very well when choosing the right information during problems solving steps. This premise supported the research undertaken by Wulanta, claiming that students with fair ability to gain information can precisely analyse question being asked, but he is not so much consistent in selecting the information and the information needed to solve the problem [12]. Another research result by Defitriani contends that students are not so creative to solve problems at hand, although they try to think of a solution to the problems they face [10]. While based on a hypothetical theory developed by Siswono, the less creative students tend to end up with unchecked answers once they complete a task [12].

The students who have low-level creative thinking skills just answered the given problem without thinking deeply whether the answer was correct or false. It happened because students had not understood the main problems and some of them believed that their answer was correct, but they got stuck in the middle of finishing the steps. Referring to Defitriani, student categorized at not creative level tries to understand problems given but he misunderstands it [12]. Another research conducted by Siswono states that student categorized at not creative is convinced about his ideas for problem solution, but he makes mistakes when operationalizing the ideas [12].

5. Conclusion

Based on the abovementioned results and discussion, the study has drawn the following conclusions. Firstly, based on test results, the students who possess high, middle, and low creative thinking skills make a significant improvement between task 1 and task 2. All of the results are related to the learning process implemented, i.e. guided research-based learning. Then, based on the results of interviews to the students, several obstacles are present upon task accomplishment. These occur because the students are not familiar with guided research-based learning. There are only small number of students who can adapt to the learning quickly and well in this research-based learning. Secondly, the levels of students creative thinking skill based on their performance on task 1 are in the "less creative" category, and those based on their performance on task 2 are in "creative" category. Thirdly, based on the results of analysis and discussion, factors affecting the levels of students creative thinking skills in Combinatorics include the accuracy in expanding the existing data to find the function, the misunderstanding about the parameters when generalizing formulas, the inclination of graduate
students to rely on rote (learning), and motivation. Lastly, developing students creative thinking skills can be carried out by increasing their motivation in developing the concepts taught by lecturers, continuously doing exercises in problem solving, and understanding problems given more carefully. So and so, the guided research-based learning method, when integrated into Combinatorics, can successfully lead to improved level of students critical thinking skills.

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