On the $Q^2$ dependence of Nuclear Structure Functions

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Abstract

The recent high statistics NMC data on the Tin to Carbon structure function ratio seems to indicate, for the first time, a significant $Q^2$ dependence, especially at small values of Bjorken $x$, $x < 0.05$, and $Q^2 > 1$ GeV$^2$. Pure leading twist, perturbative QCD–based predictions, which are consistent with the free nucleon data, yield a fairly flat ratio with little or no $Q^2$ dependence. In view of this seeming contradiction, we re-examine the applicability of such a perturbative model to nuclear structure functions in such a kinematical regime. We find that the model is consistent with all data, within experimental errors, without any need for introducing additional higher twist contributions. The model correctly reproduces the $Q^2$ dependence of the Carbon structure function as well. We also critically examine the $Q^2$ dependence of the corresponding spin dependent structure functions.

1 Introduction

The structure function, $F_A^2(x, Q^2)$, for various nuclei of mass number, $A$, such as $^4$He, $^6$Li, $^{12}$C, $^{40}$Ca, and $^{208}$Pb, has been precisely measured in several lepton–nucleon deep inelastic scattering (DIS) experiments [1, 2], over a large range of the Bjorken variable, $x$, and momentum transfer, $Q^2$. In all cases, the ratio of the structure function for the bound nucleon in a nucleus, $A$, to that for the free nucleon, $D$ (taken to be an “average” nucleon in Deuteron), $R_A = F_A^2/F_D^2$, was found to be less than unity at small and large values of $x$, with antishadowing $(R > 1)$ at intermediate $x$ values around $x = 0.1$. No significant $Q^2$ dependence of these ratios has been observed so far [1, 2], within the error bars of these measurements. That is, the nuclear (bound nucleon) and free nucleon structure functions seem to exhibit the same $Q^2$ dependence.

A recent measurement [3] by the NMC presents, for the first time, a high-statistics study of the $Q^2$ behaviour of the Tin to Carbon structure function ratio, $F_{Sn}^2/F_C^2$; consequently, the statistical errors are much smaller than in the earlier measurements. The data appear to show significant $Q^2$ dependences, unlike what has been observed before, for other nuclei. Several theoretical models of nuclear structure functions are available [1, 5, 6], that can account for such a $Q^2$ dependence, especially at small values of $x$ and $Q^2$. Many of these use a hadronic approach to the problem, for example, vector meson dominance (VMD), which is valid down to nearly the photoproduction limit, and Regge phenomenology; this
results in effectively a higher twist (or a $1/Q^2$ type) contribution, that can reproduce the observed large, positive slope of the structure function ratio at small values of $x$, $x < 0.05$. As has been pointed out [4], a perturbative approach to the problem, addressed, similar to the free nucleon case, via the partonic content of the bound nucleon in the leading twist (LT) approximation, would result in small or essentially no $Q^2$ dependence of structure function ratios; such a partonic approach should also be more applicable at larger $Q^2$. A perturbative approach, in principle, also includes a higher twist (HT) contribution, that dies away as $1/Q^2$. Such a contribution would therefore not be significant for $Q^2$ much larger than, say, 10 GeV$^2$. There is no compulsive evidence for such HT contributions even down to 1 GeV$^2$ in the free nucleon case [7]. However, it is not known, at present, whether the $Q^2$ dependence of free and bound nucleon structure functions are the same. The current Sn/C data at small $x$, which show a significant $Q^2$ dependence, are precisely in the region of $1 < Q^2 < 10$ GeV$^2$ where it is not clearly established that HT effects are subleading. It is pertinent, therefore, to ask what is the smallest $Q^2$ down to which a pure LT contribution to bound nucleon structure functions is still consistent with the new Sn/C measurement. The answer, surprisingly enough, is that a pure DGLAP approach is still consistent with all available data, within the experimental errors. We now present the details of the calculation, and substantiate our claim.

2 Brief description of the model

Specifically, we choose, for the purposes of effecting this comparison, a model [8] that uses standard leading order DGLAP [9] (or leading twist) evolution equations to obtain the nuclear structure functions at various $Q^2$ values, starting from a low input scale, $Q^2 = \mu^2 = 0.23$ GeV$^2$. The Glück-Reya-Vogt (GRV) [7] parametrisation is used for the free nucleon input, which is well-known to provide a good fit to the available proton data; the corresponding bound nucleon input densities are obtained by calculating distortions of the free ones due to the effects of nuclear binding and swelling. Binding, in particular, plays a special rôle at low-$x$, where shadowing is observed. Details of the model can be found in Ref. [8]. In summary, the increased radius of the bound nucleon, due to nucleon swelling [10], results in a redistribution of the partons inside the nucleon [11]. The increase in the radius, $\delta_A$, of a bound nucleon, relative to a free one, is parametrised as,

$$\delta_A = \delta_{\text{vol}}(1 - 1.1A^{-1/3}) ;$$

the free parameter, $\delta_{\text{vol}}$, was chosen to be 15%. Eq. (1), therefore, fixes the swelling for all $A$. Conservation of parton number and energy–momentum, along with use of Heisenberg’s uncertainty principle, then specifies the input bound nucleon parton densities, starting from the free nucleon ones, using only one free parameter that parametrises the extent of swelling, and with fixed $A$ dependence, as indicated in eq. (1). Nuclear binding is introduced by assigning the consequent energy loss, as computed using the Weizsäcker mass formula, to the sea quark sector; the sea quark density is thus depleted in comparison to the free nucleon case, by an amount dependent on its mass number, $A$. Such a depletion also occurs at the time of scattering, for partons with small $x$, $x < 0.1$ [8]. The model predicts structure functions with very few free parameters, and with well-defined $A$ dependences. Results have
been obtained for various structure function ratios at average values of $Q^2$, corresponding to the measured values; good agreement with data has been found in the region of validity of the model. (The model is not valid at large $x$, $x \gtrsim 0.5$, since Fermi motion effects have not been included).

### 2.1 Comparison with data

In view of the new Sn/C data, we now examine in detail the $Q^2$ dependence of the $^{118}$Sn and $^{12}$C nuclear structure functions and their ratios within this dynamical model. We emphasise that, once the model has been fitted to data from nuclei such as Helium or Carbon, as has already been done, the resulting Tin structure function is essentially fixed, with no more free parameters. Furthermore, the magnitude and the $Q^2$ dependence in the base case, viz., for the structure function of proton or deuteron (which is considered to be the prototype “average nucleon”) is well reproduced by this model. (It is in fact tailored to do so). Since the bound nucleon densities are computed in a manner similar to that of the free nucleon ones, that is, by the DGLAP evolution equations, the $Q^2$ dependence of the bound densities is similar to that of the free ones. The only difference lies in the input (starting) densities, which are computed according to standard nuclear inputs. Hence our model exhibits only a weak $Q^2$ dependence for all structure function ratios \([12]\). This can be seen in Fig. 1, where the model prediction for the structure function ratio, $R = F_2^{Sn}/F_2^{C}$, is plotted for various $x$ values in the range between about 0.01 and 0.4 (the central value of the corresponding experimental $x$ bin was used) as a function of $Q^2$, for $1 < Q^2 < 100 \text{ GeV}^2$. Note that all the data lie in the accepted perturbative regime of $Q^2 > 1 \text{ GeV}^2$.

Although the central values of the data at low-$x$ are strongly increasing with $Q^2$, it is seen that the model is in good agreement with the available data, within the errors of the experiment. The $\chi^2$ per degree of freedom of the fits, as shown in Table 1, is seen to be around unity or less. The only exception is the last $x$-bin, $x = 0.55$; however, here our model is not expected to hold, since we have not included Fermi motion effects. (This also seems to indicate that these effects are severely $A$-dependent and so do not cancel in the ratio of structure functions). The average $\chi^2$/d.o.f. = 138/142 for the complete data set, indicating that the model is consistent with the data. This further improves to 109/136 if we drop the last data point, $x = 0.55$.

| $x$   | $\chi^2$/d.o.f. | d.o.f. | $x$   | $\chi^2$/d.o.f. | d.o.f. | $x$   | $\chi^2$/d.o.f. | d.o.f. |
|-------|-----------------|-------|-------|-----------------|-------|-------|-----------------|-------|
| 0.0125| 0.59            | 8     | 0.055 | 0.78            | 11    | 0.25  | 0.53            | 11    |
| 0.0175| 1.18            | 9     | 0.07  | 0.93            | 12    | 0.35  | 1.10            | 9     |
| 0.025 | 0.65            | 9     | 0.09  | 0.27            | 12    | 0.45  | 1.16            | 7     |
| 0.035 | 0.96            | 11    | 0.125 | 0.90            | 14    | 0.55  | 4.77            | 6     |
| 0.045 | 0.55            | 11    | 0.175 | 0.99            | 12    |       |                 |       |

Table 1: The $\chi^2$ fits of the model with the NMC data on the $Q^2$ dependence of the Tin to Carbon structure function ratio \([8]\), for various $x$ bins, the central values of which are shown.

Hence, we see that a purely DGLAP-type perturbative evolution of an input set of densi-
ties predicts a very small $Q^2$ dependence of the Sn/C structure function ratio [4, 5]. Moreover, the prediction is consistent with the available NMC data.

The $Q^2$ dependence of the structure function ratios, $R^A = F^A_2/F^D_2$, for $A = \text{Li, C, Ca, and Pb}$, also show very little $Q^2$ dependence in the shadowing regime, $x \leq 0.1$. The resulting fit to a linear $Q^2$ dependence, of the form,

$$\frac{F^A_2}{F^D_2} = b \ln Q^2 + c,$$

(2)

yields a slope, $b$, that, though positive at small-$x$, and negative at large $x$ ($x > 0.1$), is almost vanishing throughout, and compatible with the corresponding measurements available from NMC and E665 [1, 2] in the region of overlap (we do not include the points which have $x > 0.5$ and $Q^2 \leq 1 \text{ GeV}^2$ since we are using a perturbative evolution equation without Fermi motion effects). We therefore believe that the model predictions are compatible with the available data.

This implies that the $Q^2$ dependence of the bound and free structure functions are essentially the same. Note that the corresponding free nucleon data in the same $x$ region can be fitted with a DGLAP type $Q^2$ behaviour and do not seem to need inclusion of HT effects in the region $1 < Q^2 < 10 \text{ GeV}^2$. Hence the $Q^2$ dependence of free and bound nucleon structure functions seems to show the same origin, viz., parton splitting. We test this by comparing the $Q^2$ dependence of the Carbon structure function data alone, not just structure function ratios, against the preliminary high statistics data available from the NMC [3]. This is shown in Fig. 2, for the same ranges of $x$, and for the same $Q^2$ bins as the Sn/C data. Note that the error bars shown here correspond to statistical errors alone. We see that there is reasonable agreement with the data for all $x$ ranges. However, the $\chi^2$/d.o.f. here is worse, being only 122/87; this is because of worse fits for $0.04 \leq x \leq 0.1$ as can be seen from Table 3. This can be greatly improved by an overall normalisation change and does not require a change in the slope. For instance, a decrease in the overall normalisation by just 3% decreases the $\chi^2$/d.o.f. to 74/87 (or to 68/83, if we drop the last data point), which is very acceptable. However, we do not do this here, and merely keep in mind that there would also be an improved agreement on using a next-to-leading (NLO) order fit rather than just a leading order one.

### 2.2 The polarised case

Finally, we consider the $Q^2$ dependence of the corresponding polarised structure function, $g_1^A(x, Q^2)$. In particular, the structure function of interest [4] is that of $^3\text{He}$, since it was the target used in determining the neutron spin dependent structure function [5]. We use an extension of the same dynamical model to the polarised case [8, 16] to study the $Q^2$ dependence, where the free nucleon input now corresponds to the fits from the Glück-Rey-

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1 This can be accomplished by a small change in the extent of swelling, which remains a free parameter in this model. This would cause a corresponding change in the structure functions of all other nuclei; however, this should not worsen the agreement of the model with these other measured structure functions.
Table 2: The $\chi^2$ fits of the model with the NMC data on the $Q^2$ dependence of the Carbon structure function, $F^C_2(x,Q^2)$ \[13\], for various available $x$ bins, the central values of which are shown.

| $x$   | $\chi^2$/d.o.f. | d.o.f. | $x$   | $\chi^2$/d.o.f. | d.o.f. | $x$   | $\chi^2$/d.o.f. | d.o.f. |
|-------|-----------------|--------|-------|-----------------|--------|-------|-----------------|--------|
| 0.0125| -               | -      | 0.055 | 2.95            | 7      | 0.25  | 0.79            | 9      |
| 0.0175| 0.64            | 4      | 0.07  | 2.70            | 8      | 0.35  | 0.43            | 7      |
| 0.025 | 0.95            | 5      | 0.09  | 1.91            | 8      | 0.45  | 0.09            | 5      |
| 0.035 | 1.26            | 6      | 0.125 | 1.19            | 9      | 0.55  | 1.35            | 4      |
| 0.045 | 1.78            | 6      | 0.175 | 1.35            | 9      |        |                 |        |

Vogelsang (GRVs) \[17\] parametrisation. We see from Fig. 3 that the polarised ratio,

$$\tilde{R} = \frac{g_{\text{He/n}}}{g_{\text{n}}},$$  \hspace{1cm} (3)

is more sensitive to $Q^2$ than the corresponding unpolarised ratio,

$$R^n = \frac{F_{2\text{He/n}}}{F_{2\text{n}}},$$  \hspace{1cm} (4)

where the superscript, $n$, on the unpolarised ratio indicates that the “average neutron” structure function in He was computed, in order to effect a meaningful comparison with $\tilde{R}$. (See Table 3 for a comparison with data.) At small $x$, where the sea densities dominate, the $Q^2$ behaviour of $\tilde{R}$ and $R$ are different, owing to the different $Q^2$ dependence of the corresponding unpolarised and polarised splitting functions, $\Delta P_{ij}$ and $\Delta P_{ij}$. At larger $x$, from $x \gtrsim 0.1$, since $P_{qq} = \Delta P_{qq}$, i.e., the nonsinglet unpolarised and polarised splitting functions are the same, $\tilde{R}$ and $R$ show the same $Q^2$ behaviour.

Such a $Q^2$ dependence \[17\] implies that the corresponding spin asymmetry,

$$A_{1/n} \equiv \frac{g_{1/n}}{F_{1/n}},$$  \hspace{1cm} (5)

which is what is usually measured in DIS with polarised targets, will show more sensitivity to $Q^2$. This can be seen in Fig. 4 where both the free ($A_{1/n}$) and bound ($A_{1\text{He/n}}$) nucleon asymmetries are shown (using $F_{2} = 2xF_{1}$) for various $x$ values accessible by current experiments. The extent of the $Q^2$ dependence, as has been exhibited by a comparison with the available data in Table 3, gives hope that, when more data is available, this large $Q^2$ dependence can actually be observed. For instance, the asymmetry changes by about 15–20% for $x$ from 0.025–0.2, in going from $Q^2 = 1$ to $Q^2 = 10$ GeV$^2$. As has already been pointed out \[10\], the nuclear dependence effectively cancels in the ratio, eq. (5), so that the free and bound nucleon asymmetries are nearly equal; hence the E142 measurement (which measures the bound nucleon neutron asymmetry in Helium) still provides a good measure of the free neutron asymmetry, and hence no information at all about nuclear effects. It
Table 3: The model prediction for the polarisation asymmetry in Helium compared with data
[15] at the same central values of \((x, Q^2)\) as the data; errors shown are statistical and systematic,
combined. The \(Q^2\) dependence of the computed asymmetry is indicated by showing the asymmetry
at \(Q^2 = 1, 10\) GeV\(^2\) in the last column.

| \(x\)  | \((Q^2)\) | \(A_1^{He}(data)\) | \(A_1^{He}(model)\) | \(A_1^{He, model}(Q^2)\) |
|-------|----------|---------------------|---------------------|---------------------|
| 0.0055| \(-\)     | \(-\)               | \(-\)               | \(-0.022 : -0.020\) |
| 0.0085| \(-\)     | \(-\)               | \(-\)               | \(-0.030 : -0.027\) |
| 0.0125| \(-\)     | \(-\)               | \(-\)               | \(-0.039 : -0.034\) |
| 0.025 | 0.96      | 0.066 \pm 0.111     | \(-0.063\)          | \(-0.063 : -0.054\) |
| 0.035 | 1.10      | \(-0.058 \pm 0.060\)| \(-0.078\)          | \(-0.078 : -0.066\) |
| 0.05  | 1.30      | \(-0.095 \pm 0.045\)| \(-0.096\)          | \(-0.098 : -0.080\) |
| 0.078 | 1.60      | \(-0.062 \pm 0.044\)| \(-0.116\)          | \(-0.123 : -0.098\) |
| 0.124 | 2.30      | \(-0.137 \pm 0.048\)| \(-0.127\)          | \(-0.142 : -0.110\) |
| 0.175 | 2.70      | \(-0.087 \pm 0.055\)| \(-0.122\)          | \(-0.141 : -0.108\) |
| 0.248 | 3.10      | \(-0.020 \pm 0.072\)| \(-0.098\)          | \(-0.117 : -0.086\) |
| 0.344 | 3.40      | 0.029 \pm 0.114     | \(-0.046\)          | \(-0.064 : -0.037\) |
| 0.466 | 5.20      | 0.030 \pm 0.241     | 0.021               | 0.002 : 0.028        |

seems as if information on spin dependent nuclear structure functions can only be obtained by measuring ratios similar to that shown in Eq. (3), or plotted in Fig. 3:

\[
\bar{R}^{AB}(x, Q^2) = \frac{g_A^A(x, Q^2)}{g_B^B(x, Q^2)};
\]

rather than from measurement of the conventional polarisation asymmetry, \(A_1^{A/n}\).

3 Discussion and Conclusion

We conclude that a perturbative approach to nuclear structure functions with the use of the leading twist DGLAP evolution equations predicts, not just the ratios, but also the magnitudes of various nuclear structure functions, and their \(Q^2\) behaviour as well, that are consistent with all data so far available. Such an approach typically yields a very flat \(Q^2\) behaviour for structure function ratios at small \(x\). A similar \(Q^2\) behaviour of bound and free nucleon spin independent structure functions is therefore indicated. Another perturbative approach, using parton recombination and \(Q^2\) rescaling arguments, has been able to obtain a non-trivial \(Q^2\) dependence [3], similar in nature to that obtained by non-partonic approaches [1, 4]. The \(Q^2\) dependence in this model, though, is very sensitive to the input parameters [5]. However, a persistence of the significant \(Q^2\) dependence in the Sn/C ratio at small \(x\) (for \(Q^2 \geq 1\) GeV\(^2\)), accompanied by a shrinking of the corresponding error bars by the acquisition of more data, could well signify the presence of higher twist effects, which have not so far been observed for the free nucleon case at such \(Q^2\) values. This would significantly affect
the extraction of spin dependent structure functions from polarised deep inelastic scattering experiments off polarised nuclear targets as well. Hence, a detailed experimental study of the $Q^2$ dependence of the spin polarisation asymmetry is also urged. In short, it would be interesting to obtain more data for various nuclei in the small-$x$ region, both in polarised and unpolarised experiments, in order to verify if a universal $Q^2$ behaviour of free and bound nucleon structure functions exists. This is bound to throw more light on the nature and origin of the modifications of parton densities in nuclei.

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References

[1] P. Amaudruz et al., The NMC, Nucl. Phys. B441 (1995) 3; M. Arneodo et al., The NMC, Nucl. Phys. B441 (1995) 12.

[2] M.R. Adams, et al., The E665 Collaboration, Z. Phys. C67 (1995) 403.

[3] Michal Szleper, private communication.

[4] W. Melnitchouk and A.W. Thomas, Phys. Lett. B346 (1995) 165; Phys. Rev. C52 (1996) 3373; G. Piller, W. Ratzka, and W. Weise, Z. Phys. A352 (1995) 427.

[5] L.L. Frankfurt, M.I. Strikman, S. Liuti, Phys. Rev. Lett. 65 (1990) 1725; B. Kopeliovich, B. Povh, preprint, MPIH-V12-1995, to appear in Phys. Lett.; see also, [13] below.

[6] M. Miyama, S. Kumano, Phys. Rev. C50 (1994) 1247; Phys. Rev. C48 (1993) 2016; S. Kumano, M. Miyama, Phys. Lett. B378 (1996) 267; R. Kobayashi, S. Kumano, and M. Miyama, Phys. Lett. B354 (1995) 465.

[7] M. Glück, E. Reya, and A. Vogt, Z. Phys. C84 (1990) 1725; Z. Phys. C67 (1995) 433.

[8] D. Indumathi, W. Zhu, Dortmund University preprint, DO-TH-95/03, 1995, to appear in Z. für Physik C.

[9] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438; ibid, 675; Yu.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641; G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.

[10] R. L. Jaffe, Phys. Rev. Lett. 50 (1983) 228; F. E. Close, R. G. Roberts, and G. G. Ross, Phys. Lett. B129 (1983) 346.

[11] W. Zhu and J. G. Shen, Phys. Lett. B219 (1989) 107; W. Zhu and L. Qian, Phys. Rev. C45 (1992) 1397.

[12] D. Indumathi, W. Zhu, Dortmund University preprint DO-TH-96/12, to appear in the Proceedings of the Workshop, “Future Physics at HERA”, DESY, Hamburg, 1996.
[13] A. Mücklich, Ph. D. Thesis, Ruprecht-Karls Universität, Heidelberg, 1995.

[14] W. Melnitchouk, G. Piller, and A.W. Thomas, Phys. Lett. B346 (1995) 165; L.D. Kaptari et al., Phys. Lett. B321 (1994) 271; H. Khan and P. Hoodbhoy, Phys. Lett. B298 (1993) 181; M.V. Tokarev, Phys. Lett. B318 (1993) 559; B. Badelek and J. Kwiecinski, Nucl. Phys. B370 (1992) 278; L.L. Frankfurt and M.I. Strikman, Nucl. Phys. B405 (1983) 557; L.L. Frankfurt, V. Guzey, and M.I. Strikman, Tel Aviv University, Israel, preprint 1996, hep-ph9602301; R.M. Woloshyn, Nucl. Phys. A496 (1989) 749; C. Ciofi degli Atti, E. Pace, G. Salme, Phys. Rev. C46 (1992) R1591; C. Ciofi degli Atti, S. Scopetta, E. Pace, G. Salme, Phys. Rev. C48 (1993) R968.

[15] P.L. Anthony, et al., SLAC-E142 Collaboration, Phys. Rev. Lett. 71 (1993) 959; data from C.C. Young, “Measurements of the Neutron Polarised structure function at SLAC”, Talk presented at the International workshop on DIS and related subjects, Eilat, Israel, Feb 6-11, 1994; See also J. Ashman et al., EMC, Nucl. Phys. B238 (1989) 1; D. Adams et al., SMC, Phys. Lett. B329 (1994) 399; Erratum B339 (1994) 332; K. Abe et al., SLAC-E143 Collaboration, Phys. Rev. Lett. 74 (1995) 346, and preprints SLAC-PUB-94-6508 and SLAC-PUB-95-6734.

[16] D. Indumathi, Phys. Lett. B374 (1996) 193.

[17] M. Glück, E. Reya, and W. Vogelsang, Phys. Lett. B359 (1995) 201.
Figure 1: The model prediction for the $Q^2$ dependence of the structure function ratio for Sn/C, in comparison with data from the NMC [3], with statistical and systematic errors added in quadrature. Average (central bin) values of $x$ are shown.
Figure 2: The model prediction for the $Q^2$ dependence of the bound nucleon structure function in Carbon for various values of $x$. Data shown correspond to preliminary results from the NMC [13], with only statistical errors shown.
Figure 3: The model prediction for the $Q^2$ dependence of the spin independent ($R$) and spin dependent ($\tilde{R}$) structure function ratios, for He/n, are shown as solid and dashed lines respectively for various $x$ values.

Figure 4: The model prediction for the $Q^2$ dependence of the polarisation asymmetry, $A_{1}^{He}$ (dashed lines) and the free neutron asymmetry, $A_{1}^{n}$ (solid lines), for various values of $x$ from 0.0055 to 0.5. A comparison with available data [15] can be found in Table 3.