Probing Composite Gravity in Colliders

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We explore scenarios in which the graviton is not a fundamental degree of freedom at short distances but merely emerges as an effective degree of freedom at long distances. In general, the scale of such graviton ‘compositeness’, $\Lambda_g$, can only be probed by measuring gravitational forces at short distances, which becomes increasingly difficult and eventually impossible as the distance is reduced. Here, however, we point out that if supersymmetry is an underlying symmetry, the gravitino can be used as an alternative probe to place a limit on $\Lambda_g$ in a collider environment, by demonstrating that there is a model-independent relation, $\Lambda_g \gtrsim m_{3/2}$. In other words, the gravitino knows that gravity is standard at least down to its Compton wavelength, so this can also be viewed as a test of general relativity possible at very short distances. If composite gravity is found first at some $\Lambda_g$, this would imply a model-independent upper bound on $m_{3/2}$.

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I. INTRODUCTION

Gravity at short distances is a vastly unexplored experimental frontier. It is possible that a deviation or even a drastic departure from the standard gravitational law may be found in future experiments. On the theoretical side, we have string theory which replaces general relativity (GR) at distances shorter than the string scale $M_s^{-1}$. However, since string theory not only modifies gravity but also governs the matter sector, the fact that we have not observed any stringy phenomena in particle physics experiments requires $M_s$ to be higher than at least a few TeV.

In contrast, for a theory which modifies only gravity, the bound on the scale of such new short-distance gravitational physics is significantly lowered to $\mathcal{O}(100\,\mu \text{m})^{-1} \approx \mathcal{O}(10^{-3})\,\text{eV}$ (also see the review [4]), which is 15 orders of magnitude larger than TeV$^{-1} \approx 10^{-17}$ cm! Therefore, there is huge room for a theory of this kind. This situation is quite intriguing, and this is the window that we will explore in this paper.

The striking fact about this range between 100 $\mu$ m and 10$^{-17}$ cm is that we know that matter is described by the standard local relativistic quantum field theory there. The standard model (SM) has been tested including nontrivial loop corrections with great precision [5]. This point cannot be emphasized too much. It means that a modification of gravity in this range cannot be as radical as, for example, abandoning the notion of a continuum spacetime; when we say the Bohr radius is 0.509 Å, we know perfectly what we are talking about! So, while we will boldly speak of modifying gravity in this paper, we will not mess around with matter; we take it for granted that the matter sector is completely normal, i.e., perfectly described by a local relativistic quantum field theory.

It should be also mentioned that, in general, changing the laws of gravity does not necessarily mean modifying or abandoning GR. For example, if we add $n$ extra spatial dimensions with the size $L$ in which only gravitons may propagate, then the Newton’s law changes from $1/r^2$ to $1/r^{2+n}$ for $r \ll L$ [6]. But gravity in this example is perfectly governed by the conventional GR; it is just living in more dimensions than four.

In this paper, however, we will explore the possibility that GR is abandoned at short distances in the sense that the graviton is not a fundamental propagating degree of freedom (d.o.f.) in whatever underlying theory, but is merely an effective d.o.f. appropriate at long distances. The scale, which we call $\Lambda_g^{-1}$, corresponding to the boundary between ‘short-distances’ and ‘long-distances’ could be anywhere shorter than $\mathcal{O}(100)\,\mu \text{m}$, but as we stated above, we will focus on the range $10^{-17}\,\text{cm} \lesssim \Lambda_g^{-1} \lesssim 100\,\mu \text{m}$ (or $10^{-3}\,\text{eV} \lesssim \Lambda_g \lesssim \text{TeV}$), so that we can exploit the fact that the matter sector is ‘normal’.

This includes various possibilities—the graviton may be a bound or solitonic state of the fundamental d.o.f. [7], or an extended state in some intrinsically nonlocal theory [8], or a sort of hydrodynamic state as in the scenarios often dubbed ‘emergent relativity’ [9]. We will not distinguish these varieties but just focus on their common feature that the graviton is not an elementary propagating d.o.f. in the fundamental theory but just appears as an effective d.o.f. in the long-distance description for $d > \Lambda_g^{-1}$. Admittedly not an optimal name, we call it a composite graviton, where by ‘composite’ we simply mean ‘not elementary’.

One may think such a composite graviton is excluded by the theorem by Weinberg and Witten [10]. Actually, what the Weinberg-Witten (WW) theorem excludes is not just a composite graviton but any massless spin-1 or -2 particle, composite or not! Therefore, we must be careful about the assumptions of the theorem; we all know QED and QCD which have a massless spin-1 particle, and GR which has a massless spin-2 particle. Note that the WW theorem states that if a theory allows the existence of a Lorentz-covariant conserved vector (or symmetric 2nd-rank tensor) current, then the theory cannot contain any massless spin-1 (or spin-2) particle charged
under this current. QED evades the spin-1 part of the theorem because the photon is not charged under the current. QCD evades the spin-1 part of the theorem because the current is not Lorentz covariant due to its dependence on the gluon field which is a 4-vector only up to a gauge transformation. Similarly, GR evades the spin-2 part of the theorem because the gravitational part of the energy-momentum ‘tensor’ is not really a tensor in GR.

Indeed, there is an explicit example of composite gauge bosons. Consider an $SU(N)$ supersymmetric QCD with $F$ flavors where $N + 1 < F < 3N/2$, with no superpotentials. In the far infrared (IR), this theory is described by an IR-free, weakly coupled $SU(F - N)$ gauge theory. However, these IR gauge bosons are not a subset of the original ultraviolet (UV) d.o.f.; rather, they are new effective d.o.f. appearing only in the IR description, which microscopically can be interpreted as solitonic states of the fundamental UV d.o.f. This is indeed a concrete example of composite massless gauge bosons, where the $SU(F - N)$ gauge symmetry emerges at low energies, making it consistent with the WW theorem.

Clearly, it is desirable to have a similar example for gravity. To this goal, Gherghetta, Peloso and Poppitz recently presented a theory in a 5-dimensional Anti-de-Sitter (AdS) space which is dual to a 4-dimensional conformal field theory in which the conformal symmetry is dynamically broken in the IR yielding a spectrum containing a massless spin-2 resonance. To complete their picture, analyses beyond the quadratic order in the gravitational constant problem (CCP) may instead be $O(10)$ MeV. Therefore, it is quite interesting to ask if composite gravity can solve the CCP by identifying $\Lambda_g$ with, say, $10^{-2}$ eV. However, it is not so hard to see the answer entirely depends on the nature of whatever underlying theory of composite gravity.

In particular, it appears that the underlying theory should not be a local field theory if one wishes to suppress loop corrections to the cosmological constant by abandoning elementary gravitons. The argument goes as follows. Consider three diagrams in FIG. 1. The diagram (a) is a correction to the vacuum energy, (b) is a correction to the gravitational mass, and (c) is a correction to the inertial mass. In a field theory, the loop integral in (a) can be suppressed only if the vertex has a form factor that depends on the loop momentum. Now, the problem is, once (a) is suppressed by such a form factor, the correction (b) also gets suppressed because it has the same form factor, while the correction (c) does not get suppressed because there is no such form factor. This violates the equivalence principle, and we need fine-tuning to restore it. However, for a composite graviton which is not from a local field theory, there does not have to be tension like this, and suppressing loop corrections to the cosmological constant may be consistent with the equivalence principle. But even supposing we did find such a nonlocal underlying theory, it would still be halfway to solving the cosmological constant problem, since there are also tree-level or classical contributions to the vacuum energy from phase transitions which must be somehow suppressed. The door is not shut yet, and Ref. discusses a toy model for such a nonlocal theory without problems with the equivalence principle or the classical contributions. In the rest of the paper, we will not concern ourselves with the cosmological constant problem any further, and just focus on the physics of composite gravity.

Is there any reason or motivation to consider such drastic modification of gravity in this range? Just near the edge of the range, there is a cosmologically interesting scale $\Lambda_g \approx (20 \mu m)^{-1} \approx 10^{-2}$ eV $= (16\pi^2\rho_{\text{vac}})^{1/4}$, where $\rho_{\text{vac}}$ is the vacuum energy density corresponding to the observed acceleration of the expansion of the universe. Kaplan and Sundrum also recently pointed out that the interesting scale in the context of composite gravity, the scale $\Lambda_g$ is the only quantity we can discuss. So far, the lower bound on $\Lambda_g$ has been placed by measuring gravitational forces between test masses, which has reached the scale of $O(100) \mu m$. But it is
clear that such direct measurement will be increasingly difficult and eventually impossible as the distance gets reduced. Soon, some other methods must replace it to probe the scale of composite gravity.

Such an alternative can arise if there is something that is related to the graviton but is more accessible than the graviton at short distances. In general, there is nothing that is related to gravity except the graviton itself. However, if nature possesses (spontaneously broken) supersymmetry (SUSY), the gravitino precisely satisfies the criteria—it is related to the graviton and may be accessible even in colliders! The introduction of SUSY allows us to extract some informations relating the graviton and the gravitino without knowing what the underlying theory is. In fact, we will show that if a gravitino exists, it can indeed be used to probe gravity at very short distances where direct measurement of gravitational forces is impossible.

To keep our discussions as model independent as possible, we would like to have an effective field theory and ask questions that can be answered by it. This effective theory must have the following features:

- It must contain a physical scale $\Lambda_g$ above which the graviton is no longer an elementary degree of freedom. The scale $\Lambda_g$ is not a scale chosen for convenience but corresponds to a physical boundary between two completely different phases of the theory, just like $\Lambda_{QCD}$ separates two different descriptions with totally different degrees of freedom (i.e. partons versus hadrons).

(Recall that $\Lambda_g$ is a parameter anywhere from $\mathcal{O}(10^{-3})$ eV to $\mathcal{O}$(TeV) or whatever cutoff for the matter sector.)

- Nevertheless, to reproduce all the known gravitational physics, it must include all the matter particles, even the ones heavier than $\Lambda_g$! And, as emphasized already, we know that the matter sector is perfectly described by a local relativistic quantum field theory with a cutoff higher than TeV $> \Lambda_g$.

Because of the second feature, we cannot use the usual effective field theory formalism in which all the particles heavier than $\Lambda_g$ are simply integrated out; that would fail to capture all the known long-distance gravitational physics such as the $1/r^2$ law, the perihelion precession, the bending of light, etc.

Therefore, the first important question is whether or not there exists a sensible effective theory that can deal with this highly asymmetric situation in which gravity has a low cutoff and matter has a high cutoff. This question was answered by R. Sundrum, who developed a formalism, soft graviton effective theory (SGET) which assures that we can consistently analyze this asymmetric situation without referring to the underlying theory of composite gravity. We will review the essential ideas of SGET in Sec. III to keep our discussions self-contained.

Given that there is a consistent effective field theory to describe the low-cutoff gravity with the high-cutoff heavy matter, there seems nothing wrong to have a gravitino heavier than $\Lambda_g$, since we should be able to treat it just as one of heavy matter particles. After all, $\Lambda_g$ is the scale of graviton’s compositeness which does not have to be equal to that of gravitino’s once supersymmetry is broken. Also, there is nothing wrong a priori for a composite particle to be heavier than the scale of its compositeness, like the B-mesons, the hydrogen atom, etc.

Nevertheless, as we will show in Sec. III there is a non-trivial remnant of the underlying supersymmetry which gives rise to the relation

$$m_{3/2} \lesssim \Lambda_g.$$  \hspace{1cm} (1)

Therefore, in fact a gravitino—if it exists—knows that gravity should be just GR (i.e. the graviton is an elementary d.o.f.) at least down to its Compton wavelength! In other words, the discovery of a gravitino and the measurement of its mass offers a short-distance test of GR and places a model-independent lower-bound on $\Lambda_g$! In particular, depending on the value of $m_{3/2}$, we may be able to completely exclude the possibility of composite gravity as a solution to the CCP.

On the other hand, if we first discover composite gravity somehow and measure $\Lambda_g$ before discovering a gravitino, then this inequality predicts that, once we see a gravitino, we will find its mass be lighter than $\Lambda_g$.

In Sec. IV A and IV B, we will continue the discussions to gain a further understanding of the inequality, followed by a brief comment in Sec. IV C on the possibility of independent theoretical tests of the inequality.

In order for our prediction to be useful, it is clearly crucial to experimentally convince ourselves that what we are observing is really a gravitino, not a random spin-3/2 resonance which may just happen to be there. This issue will be discussed in Sec. V We will then conclude in Sec. VI.

II. SOFT GRAVITON EFFECTIVE THEORY

As we have already mentioned, we need to describe all experimentally known gravitational physics occurring among heavy ($\gg \Lambda_g$) matter particles, without extrapolating our knowledge of gravity beyond $\Lambda_g$. Soft graviton effective theory (SGET) is designed precisely for this purpose.\footnote{Strictly speaking, to describe the typical observed gravitational phenomena involving gravitational bound states, we should switch to yet another effective field theory to have a transparent power-counting scheme appropriate for that purpose. The interested reader should read Ref. \cite{15} which develops such an effective theory, dubbed “nonrelativistic general relativity” (NRGR).} Here, we will review its central concepts to keep the discussions self-contained.
To start, let us consider gravity only. In this case, the theory takes the form of a familiar effective field theory with the cutoff $\Lambda_g$ imposed on the graviton field $h_{\mu \nu}$ defined via

$$g_{\mu \nu} = \eta_{\mu \nu} + \frac{h_{\mu \nu}}{M_{Pl}}. \tag{2}$$

Namely, the lagrangian is just the usual Ricci scalar term plus a whole series of higher-dimensional operators suppressed by powers of $\Lambda_g$:

$$\mathcal{L}_{\text{grav}} \sim M_{Pl}^2 \left( R + \frac{R^2}{\Lambda_g^2} + \frac{R_{\mu \nu} R^{\mu \nu}}{\Lambda_g^2} + \cdots \right), \tag{3}$$

where dimensionless $O(1)$ coefficients are suppressed.\(^2\) As we mentioned earlier, $\Lambda_g$ is a physical scale above which $h_{\mu \nu}$ is no longer an elementary degree of freedom.

Note the scales and kinematic configurations to which this $\mathcal{L}_{\text{grav}}$ is applicable. It is appropriate only for processes where all momentum transfers among the gravitons are less than $O(\Lambda_g)$. For example, it can not be used to calculate the cross-section for two highly energetic ($E \gg \Lambda_g$) gravitons scattering with a large angle. In fact, we do not even know if such a scattering occurs at all—maybe they would end up with ‘jets’, like in hadron-hadron collisions—who knows? No experiments so far have told us what would happen to such processes, and performing theoretical calculations requires specifying the full theory valid at distances shorter than $\Lambda_g^{-1}$. The moral here is that a large momentum transfer should not be delivered to a graviton within our effective theory.

To also understand that a graviton should not be exchanged to mediate a large momentum transfer, imagine a theory with a fermion $\psi$ and a scalar $\phi$, and suppose we have verified that a Yukawa coupling $\bar{\psi} \psi \phi$ perfectly describes the $\psi \psi \rightarrow \bar{\psi} \bar{\psi}$ scattering when both $\psi$’s get only very low recoils, i.e. the momentum transfer mediated by $\phi$ is very small. But it may be completely wrong to use this Yukawa theory to describe the scattering of two very energetic $\psi$’s by a large angle, corresponding to a large momentum transfer mediated by $\phi$. For instance, suppose that the $\phi$ is actually a strongly bound state of two new fermions $\Psi$ interacting with $\psi$ via a 4-fermion coupling $\bar{\psi} \psi \bar{\Psi} \Psi$. Then, when the $\psi$’s get recoils much larger than $\Lambda_{QCD}$, of this new strong interaction, we must use the 4-fermion theory with $\Psi$ rather than the Yukawa theory with $\phi$. Here, $\phi$ is meant to be the analog of the graviton, and therefore, within our effective theory, a graviton should not be exchanged to mediate a large momentum transfer ($\gg \Lambda_g$).

Now, let us move on and include matter fields. First, note that for a given value of $\Lambda_g$, some elementary particles in the standard model (SM) are too short-lived ($\tau \ll \Lambda_g^{-1}$) to be included in SGET. On the other hand, some composite particles in the SM live long enough ($\tau \gg \Lambda_g^{-1}$) and also are too small in size ($\ll \Lambda_g^{-1}$) for a soft graviton to recognize that they are composite. For example, if $\Lambda_g$ is, say, $10^{-2}$ eV, then the proton, the hydrogen atoms in 1S and 2P states would be all elementary fields (fermion, scalar, and vector, respectively) in SGET.

Secondly, there are many ‘hard processes’ among those matter particles involving momentum transfers much larger than $\Lambda_g$. For example, if $\Lambda_g$ is, say, 1 eV, then the pair annihilation, $e^+ e^- \rightarrow \gamma \gamma$, would be a hard process. Since a soft graviton in this case cannot resolve the $t$-channel electron propagator there, we should shrink it to a point and express the entire process by a single local operator. Also, since soft gravitons in this case cannot pair-produce an electron and a positron, they are completely unrelated particles from soft gravitons’ viewpoint. Whereas if $\Lambda_g$ is, say, 1 GeV, then there are soft gravitons who can see the $t$-channel propagator in $e^+ e^- \rightarrow \gamma \gamma$, and electrons and positrons must be described by a single field operator.

The general matching procedure for a SGET may be best explained by comparing it with the construction of a usual effective field theory in which heavy particles are simply integrated out. In the derivation of a usual effective theory, we consider one-light-particle-irreducible (1LPI) diagrams; in a 1LPI diagram, all external lines represent light particles to be kept in the effective theory, and the diagram would not split in two if any one of internal light-particle propagators were cut. We then obtain effective vertices in the effective theory by shrinking every heavy propagator to a point.

Similarly, for a SGET, we consider one-nearly-on-shell-particle-irreducible (1NOSPI) diagrams; in a 1NOSPI diagram, all external lines are nearly on-shell, i.e., its deviation from the mass shell is less than $O(\Lambda_g)$, and the diagram would not split in two if any one of internal nearly-on-shell propagators were cut. We then obtain effective vertices by shrinking every far-off-shell (i.e. not nearly on-shell) propagator to a point. For the technical detail of ‘shrinking’ or matching procedure, see Ref. [18].

Having matched all hard SM processes onto effective operators, we are now ready to couple to it the soft graviton described by $\mathcal{L}_{\text{grav}}$. This step is straightforward—we just use general covariance as a guide, just as we do for the conventional general relativity.

By construction, SGET respects all fundamental requirements such as the equivalence principle, Lorentz invariance, and unitarity, as long as we stay within its applicability we have discussed above [18]. In particular, unitarity holds because all propagators that can be on-shell are included in SGET, so it correctly reproduces the imaginary part of any amplitude.

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\(^2\) The operator $R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma}$ can be omitted in perturbation theory since it can be expressed as a linear combination of the two operators explicitly written in (3) plus a total derivative. Furthermore, in the absence of matter, even those two operators could be removed by field redefinition, but we have kept them in (3) because we are interested in including matter. See Refs. [20] for more discussions on these operators.
III. THE COMPOSITE GRAVITINO

Now, let us consider putting a gravitino in the story, with the hope that a gravitino may be more experimentally accessible than gravitons at short distances so we can learn something about gravity. The new ingredient in this section is supersymmetry (SUSY) as a spontaneously broken exact underlying symmetry, not only in the matter sector but also in the gravity sector. As mentioned in Sec. II, the introduction of SUSY is a necessary and minimal additional ingredient if we wish to have an alternative probe for $\Lambda_g$ which is as model-independent as possible, because without SUSY there is nothing that is necessarily related to gravity except the graviton itself.

Since the graviton is not a fundamental degree of freedom at short distances, neither is the gravitino.\(^3\) Let $\Lambda_g$ be the scale above which the gravitino ceases to be an elementary degree of freedom. Because supersymmetry is broken, $\Lambda_g$ does not have to be equal to $\Lambda_g$. There is another scale in the theory, the gravitino mass $m_{3/2}$. *A priori, these three scales may come in any order.* SGET assures that there is a consistent framework to describe particles which are much heavier than $\Lambda_g$, so $m_{3/2}$ may be higher or lower than $\Lambda_g$. While $\Lambda_g$ is roughly the ‘size’ of the gravitino, there is nothing wrong for a composite particle to be heavier than the inverse of its size, or the compositeness scale. In fact, heavy quark effective theory (HQET)\(^2\), which describes a single $B$-meson system, takes advantage of the fact that the $B$-meson’s compositeness scale $\Lambda_{QCD}$ is much less than its mass $m_B \approx m_b$.

In the case of HQET, the effective theory breaks down if a gluon delivers a momentum transfer larger than $m_b$ to the $b$ quark. But in general effective theories, the breakdown may happen at an energy much lower than any obvious mass scale in the theory. For example, consider the effective field theory of a hydrogen atom in the ground state interacting with soft photons ($E_\gamma \ll \mathcal{O}(\text{eV})$). This effective theory contains an elementary scalar field (the hydrogen atom in the $1S$ state) and the electromagnetic field, and it correctly accounts for the Rayleigh scattering, explaining why the sky is blue.\(^4\) But this effective theory clearly goes wrong if a photon delivers an energy of $\mathcal{O}(\text{eV})$ or higher, where we should take into account the fact that the scalar is actually not elementary. But this breakdown scale is much less than the scalar mass, $\mathcal{O}(\text{GeV})$.\(^5\)\(^6\)

\(^3\) Of course, there is also a possibility that a gravitino just does not exist. Here, we assume that a gravitino exists and its lifetime is long enough ($\gg \Lambda_g^{-1}$) to be in the effective theory. We will come back to this caveat in Secs. IV A and IV B.

\(^4\) The reader not familiar with this cute application of effective field theory may like to read Ref. 22.

\(^5\) We get a different breakdown scale if we are interested in capturing a different physics, such as the pair-annihilation of a hydrogen and an anti-hydrogen.

\(^6\) Interestingly, even if we take into account the internal structure, the breakdown scale $\mathcal{O}(\text{eV})$ is still much smaller than the lightest mass in the theory $m_{3/2} \approx 0.5 \text{ MeV}$. See Ref. 23 for an illuminating formalism making this breakdown scale manifest.

Therefore, *a priori* there seems no restriction on possible values for $m_{3/2}$. (For further discussions shedding different light on this matter, see Sec. IV A.) Nevertheless, we will show below that $m_{3/2}$ should be bounded from above by $\Lambda_g$, which is a nontrivial constraint arising from the underlying supersymmetry.

First, we must be clear about what we mean by ‘gravitino’. For instance, say, we have found a new spin-3/2 fermion which has no $SU(3) \times SU(2) \times U(1)$ interactions with the rest of the standard model. Does it mean we have seen a gravitino? Not necessarily. In order for some spin-3/2 fermion to be a candidate for a gravitino, at least it must have—possibly among other things—a coupling to the supersymmetry current of the matter sector; in other words, it should be able to convert a matter particle to its superpartner. Without this feature, it would be no different from a random spin-3/2 resonance.

So, we begin by supposing that we have seen a spin-3/2 fermion $X$ emitted in a process of the type $\tilde{Y} \to X + Y$, where $Y$ is the superpartner of a particle $Y$. For $m_{3/2} \ll \Lambda_g$, it is clearly consistent to add the gravitino in the pure gravity effective lagrangian\(^3\), treating it just like the graviton. In other words, we can first forget about the graviton and gravitino, construct the nearly-on-shell effective lagrangian for matter, then couple the graviton and the gravitino using general covariance and local supersymmetry as a guide, where the effects of $m_{3/2}$ can be systematically included as perturbation.

For $m_{3/2} \gg \Lambda_g$, we clearly cannot include the gravitino in\(^3\) together with the soft graviton, because whenever such a heavy gravitino is produced or exchanged, it is a hard process ($\gg \Lambda_g$) by definition. But this simply suggests that we should treat it just as one of heavy matter fields instead. The only difference seems that unlike all the other matter particles, we do not have a fundamental theory for the composite gravitino, so we cannot calculate the coefficients in SGET lagrangian—that is fine, we just leave them as parameters.

However, we have to be careful, because this splitting of the graviton and gravitino into the soft and hard sectors may be incompatible with the underlying SUSY, which pairs them.

Let us build a gauge-theory analog of our problem. First, recall our *global* symmetry structure: the underlying symmetry is the super-Poincaré group, which is spontaneously broken to its subgroup, the Poincaré group. So, consider a global $SU(2)$ symmetry which spontaneously breaks down to a $U(1)$ by a triplet scalar $\phi = \phi^a \sigma^a$ getting a VEV $\langle \phi^3 \rangle = 0$ and $\langle \phi^3 \rangle = v$. (Here, $\phi$ is treated just as a spurion.) The $SU(2)$ is the analog of the underlying supersymmetry, while the unbroken $U(1)$ is the
analog of the unbroken Poincaré symmetry.

Now, at long distances, the Poincaré group is gauged by the existence of the soft graviton which, however, is not a fundamental degree of freedom at short distances. So, correspondingly, we gauge the $U(1)$ at long distances by introducing a soft massless vector field $W_\mu^3$, which we call ‘toy soft graviton’. And just like the graviton, $W_\mu^3$ is not a propagating degree of freedom at short distances. Finally, we also need a ‘toy gravitino’, i.e., a massive vector $W_\mu^3 \equiv (W_\mu^1 - iW_\mu^2)/\sqrt{2}$.

Let us assume $m_\chi \gg \Lambda_g$ which is the case of our interest. We want to write down ‘toy SGET’ for the toy gravitino. The only property of $W_\mu^3$ which possibly makes it different from other heavy particles is that it is the $SU(2)$-partner of the toy graviton $W_\mu^3$. So, the question is whether there is any constraint on the structure of the toy SGET from the underlying $SU(2)$, or the toy SUSY.

Let us forget $\Lambda_g$ for a moment, and recall how a spontaneously broken symmetry leaves its trace in low-energy physics. To be concrete, consider couplings of $W_\mu^3$ and $W_\mu^3$ to a heavy Dirac fermion doublet $\psi$. (\psi is of course the analog of the pair of a SM particle and its superpartner.) If we limit to only renormalizable operators, all three $W_\mu^3$ must couple to the three currents $J^{a\mu} = \bar{\psi} \sigma^a \gamma^\mu \psi$ with a single common coupling constant $g$. This equality is a consequence of the underlying $SU(2)$, even though it is broken.

However, once we take into account higher-dimensional operators, the coupling of $W^{1,2}_\mu$ to $J^{1,2}_\mu$ does not have to be equal to that of $W_\mu^3$ to $J^{3}_\mu$, because there are higher dimensional operators that reduce to these couplings after picking up the VEV. Among such, the one with the lowest dimension is the dimension-5 operator $\bar{\psi} \phi D\phi \psi$. We could go on and analyze this operator, but it turns out that we can learn the same lesson with much less arithmetic from the following dimension-6 operator:

$$L_6 = -\frac{16\pi^2 v^2 c}{M^2} \bar{\psi} \phi iD\phi \psi,$$

where we take $c \sim 1$ so that $M$ corresponds to the scale obtained via ‘naive dimensional analysis’ (NDA), i.e., the scale at which this operator would lead to strong coupling if the theory is not replaced with a more fundamental theory by then \cite{24}. After substituting the VEV for $\phi$ and canonically normalizing the fields, we find that the coupling of $W_\mu^3$ stays equal to $g$ as expected from the unbroken $U(1)$ gauge invariance, but the coupling of $W_\mu^3$ does get modified as

$$g \rightarrow g_+ = \frac{1 + a}{1 - a} g,$$

where

$$a \equiv \frac{16\pi^2 v^2 c}{M^2} \sim \left(\frac{4\pi v}{M}\right)^2.$$

Therefore, the equality of the $W_\mu^3$ and $W_\mu^3$ couplings no longer holds. Especially, if $v$ is $O(M/4\pi)$, then $g_+/g$ could be anywhere between zero and infinity, and there would be no remnants of the underlying $SU(2)$ symmetry.

This lesson can be generalized. In general $g_+$ differs from $g$ as

$$g_+ = \alpha g,$$

where the factor $\alpha$ includes contributions from all the operators that can mix with $W_\mu^3 J^{a\mu}$. The relation $\alpha \simeq 1$ holds as long as $v \ll M/4\pi$, but for $v \sim M/4\pi$, all those operators would contribute to $\alpha$ equally in magnitude, and consequently $\alpha$ could be anywhere between zero and infinity.

Now, let us go back to the case of our interest and take $\Lambda_g$ into account. Let us write the doublet $\psi$ as

$$\psi = \begin{pmatrix} \tilde{\chi} \\ \chi \end{pmatrix},$$

and, for definiteness, take $\tilde{\chi}$ to be heavier than $\chi$ with the mass difference larger than $m_\chi$ so that $\tilde{\chi}$ can decay into $\chi$ and $W^+$. Clearly, this is the analog of a sparticle decaying into its SM partner and a gravitino.

Once we have seen a toy gravitino produced via this decay, $g_+$ must be nonzero. In the rest frame of the decaying $\tilde{\chi}$, this decay is caused by the operator

$$\mathcal{H}_{\text{int}} \supset g_+ W^+_{-\bm{p}} \chi_{\bm{p}} \tilde{\chi}_{0},$$

where the irrelevant indices, bars and daggers are suppressed, while the important quantity here is $|\bm{p}| = \sqrt{E^2_{\chi} - m_{\chi}^2}$ where $E_{\chi}$ is the energy of the outgoing $\chi$ given by

$$E_{\chi} = \frac{m_{\chi}^2 + m_{\chi}^2 - m_w^2}{2m_{\chi}}.$$

Now, if $g$ is also nonzero, there would also be a term

$$\mathcal{H}_{\text{int}} \supset g W^3_{-\bm{p}} \tilde{\chi}_{\bm{p}} \tilde{\chi}_{0},$$

with the same $\bm{p}$. The problem is that, at the second order in perturbation theory, this operator could cause the process $\tilde{\chi}_{\bm{p}} \chi_{0} \rightarrow \chi_{0} \tilde{\chi}_{\bm{p}}$ via the $t$-channel $W^3_{\mu}$ exchange.
(FIG 2). Note that the momentum transfer \( Q^2 \) mediated by the \( W_\mu^3 \) is given by

\[
Q^2 = -\left( \sqrt{p^2 + m_\tilde{\chi}^2} - m_\chi \right)^2 + p^2 \\
= 2m_\tilde{\chi}^2 \left( 1 + \frac{p^2}{m_\tilde{\chi}^2} - 1 \right). \tag{12}
\]

From \( |p| = \sqrt{E_\chi^2 - m_\chi^2} \) and \( \mu \), we see that for generic \( m_\chi \) and \( m_W \), we have \( |Q| \sim m_\chi > m_W \gg \Lambda_g \), which is a hard momentum transfer. However, as discussed in Sec. 11, a graviton cannot be exchanged to mediate such a large momentum transfer within SGET. Therefore, the operator 11 should not be present in the effective theory.

Therefore, to decouple the operator 11, we must take the limit \( g \to 0 \) while keeping \( g_+ \) fixed to a finite value. Then the relation 4 requires \( \alpha \to \infty \), which, however, is possible only if \( v \sim M/4\pi \), as noted before. In this limit, all the higher-dimensional operators that can contribute to \( g_+ \) do contribute equally in magnitude, while all the other interactions have the full NDA strength. Furthermore, having taken this limit, we have decoupled the soft \( W_\mu^3 \) as well, so we have to couple it back to the theory. This can be easily done by using the \( U(1) \) invariance, but now the coupling of this \( U(1) \)—let us call it \( g_{\text{soft}} \)—is completely arbitrary, with no relation to \( g_+ \)!

Therefore, although we cannot perform any quantitatively reliable analysis beyond estimates due to \( v \sim M/4\pi \), this is good enough to give us the following qualitative understanding of what \( W_\mu^3 \) is like. First, its coupling to the \( SU(2) \) current, \( g_+ \), is not related at all to the coupling of the soft \( W_\mu^3 \) to the \( U(1) \) current, \( g_{\text{soft}} \). Second, it has all kinds of additional interactions, all with the full NDA strength. Because of these two features, \( W_\mu^3 \) should be viewed just as a random spin-1 resonance, rather than the ‘\( SU(2) \)-partner’ of \( W_\mu^3 \).

Recalling the dictionary of our analogy, translating this gauge-theory lesson back to gravity is straightforward. (The only slight mismatch in the dictionary, which is not at all essential for us, appears in the \( 4\pi \) counting for broken SUSY, where the relation \( v \sim M/4\pi \) should be translated as \( F \sim M^2/4\pi \) where \( F \) is the decay constant of the goldstino, or the square of the SUSY breaking scale \( \Lambda_g \).) Therefore, we have found

- If \( m_{3/2} \ll \Lambda_g \), it is consistent for the gravitino to be just ‘canonical’, with all the properties we expect from the standard supergravity, except for the fact that the gravitino—like the graviton—is not an elementary degree of freedom at short distances. In other words, as long as we avoid processes where a gravitino receives or mediates a large momentum transfer, the gravitino can behave normally.

- If \( m_{3/2} \gg \Lambda_g \), this ‘gravitino’ is not really a gravitino, because the coupling of this ‘gravitino’ to a SM particle and its superpartner can have any value, with no relation to the ‘canonical’ strength, and we also expect this ‘gravitino’ to have a whole series of other couplings, all equally important with the full NDA strength. In short, it behaves just like a random spin-3/2 resonance with no relation to the gravity sector.

Hereafter, to distinguish these cases, we will use the term gravitino only to refer to the first case, while we will call the second case pseudo-gravitino.

We postpone the issue of experimentally distinguishing a gravitino from a pseudo-gravitino until Sec. 17. At this point, let us just assume that the distinction can be made. Then, we have found the model-independent relation between the gravitino mass and the composite gravity scale:

\[
m_{3/2} \lesssim \Lambda_g. \tag{13}
\]

By definition, gravity is described by GR at distances longer than \( \Lambda_g^{-1} \), because GR is the only consistent theory once we have a graviton coupled to matter described by a local relativistic quantum field theory. (Note that we could not have said this if we had not restricted \( \Lambda_g \) below TeV which assures the matter sector is ‘normal’.) Therefore, the relation 13 means that the existence of a gravitino guarantees that GR is correct at least down to its Compton wavelength! Hence, this is a short-distance test of GR, which in turn places a lower bound on \( \Lambda_g \). On the other hand, the relation 13 implies that if we find composite gravity first at some \( \Lambda_g \), then we will not discover a gravitino above the scale \( \Lambda_g \)—at best we may just see a pseudo-gravitino which is nothing but a random spin-3/2 state.

IV. DISCUSSIONS

A. Should a Gravitino Exist?

The quick answer is, we don’t know. There is no strong argument indicating whether it should or shouldn’t. We will present below several arguments, not to answer this question but to shed different light and gain more insights on the result of Sec. 11.

Imagine a huge hierarchy between the SUSY breaking scale \( \sqrt{F} \) and \( \Lambda_g \), as \( \sqrt{F} \gg \Lambda_g \). Above \( \Lambda_g \), the gravity sector is described by some exotic degrees of freedom—which may not even be field-theoretic—with
no gravitons. Here, there is no point of asking what the superpartner of the graviton is, because the graviton is not even in the theory. When we go below $\Lambda_g$, the graviton emerges, but we do not expect that a gravitino appears there, because from the usual effective-field-theoretic viewpoint, the dynamics at $\Lambda_g$ that generates the graviton should not ‘know’ about SUSY which is broken away above $\Lambda_g$.

This argument is too naive, however. As we will argue below, not only is it possible that a pseudo-gravitino may exist, but also even an honest gravitino with all the (approximately) canonical properties may exist! Consider a supersymmetric $SU(3)$ gauge theory with two flavors with no superpotentials, and suppose that SUSY is broken with the soft masses much larger than $\Lambda_{QCD}$. For simplicity and definiteness, also assume that the squark masses are all degenerate, respecting the flavor symmetry, and that the gluino is much heavier than the squarks. This theory possesses an $R$ parity under which all the quarks and gluons are even while all the squarks and gluino are odd. Hence, the squarks are stable, and there are stable fermionic meson-like bound states (‘mesinos’) with one quark and one anti-squark.\(^9\)

So, apparently, the mesons have superpartners, the mesinos. But look at other particles; for example, there is no ‘sproton’ or ‘sneutron’, because they would decay too quickly to form a bound state. In fact, most particles lack their superpartner, so the interactions between the meson-mesino sector and the rest are completely non-supersymmetric. Therefore, if these non-supersymmetric couplings are significant, there is no sense in which the mesinos are the superpartners of the mesons, except for the quantum numbers. In other words, the mesinos in this case are just analogous to our pseudo-gravitino.

However, there is also a logical possibility that the couplings between the meson-mesino sector and the rest are sufficiently small for some reason. Then, it is at least consistent for the mesinos to retain the properties expected from supersymmetry.\(^10\) A similar situation could happen to a gravitino. For example, if $\Lambda_g$ is, say, $10^{-2}$ eV, then it could be perfectly consistent for a gravitino with, say, $m_{3/2} = 10^{-3}$ eV to carry all the (approximately) canonical couplings we expect from supergravity—as long as the gravitino does not receive or mediate a momentum transfer larger than $\Lambda_g$—even though $\sqrt{F}$ here would be $\sim$ TeV which is way above $\Lambda_g$. To sum up, from the standard effective-field-theoretic view, there seems no preference among ‘nothing’, ‘a pseudo-gravitino’, and ‘a (real) gravitino’.

To gain more insight, let us consider the limit in the opposite order. This time we start with a finite $\Lambda_g$ but no SUSY breaking ($F = 0$).\(^11\) So we start with a degenerate pair of massless graviton and gravitino. This gravitino is of course exactly what we expect from supergravity, as long as we avoid momentum transfers larger than $\Lambda_g$. As we raise $F$, the gravitino mass goes up according to the usual relation $m_{3/2} \sim F/M_{Pl}$, as long as $m_{3/2} \ll \Lambda_g$. If we keep raising $F$, $m_{3/2}$ eventually hits $\Lambda_g$, beyond which the gravitino may start looking strange. (The result of Sec. [III] says it will start looking strange, but here let us pretend that we did not know Sec. [III]. Then, in particular we no longer know how $m_{3/2}$ should vary as a function of $F$. (We will come back to this issue in detail in Sec. [IV].) Here, let us suppose that it still keeps going up, although not necessarily obeying the usual linear relation. Will this ‘gravitino’ eventually disappear? Note that it will disappear from SGET if its lifetime becomes shorter than $\Lambda_g$\(^1\). Naively, we expect that the lifetime should be quite long because the coupling $1/M_{Pl}$ is extremely weak, so it would stay in the effective theory even if $m_{3/2}$ is as high as the weak scale. But this ‘gravitino’ may have unusual interactions, and there are probably many new states around $E \sim \Lambda_g$ into which the ‘gravitino’ could decay. So the lifetime may or may not be quick enough for the ‘gravitino’ to disappear from SGET. We need the underlying theory to see which way it goes.

Finally, it is also conceivable that $m_{3/2}$ ‘saturates’ at $\Lambda_g$ as we raise $F$. We would expect this if there is an exotic state at $E \sim \Lambda_g$ which can mix with the gravitino. Then, by the ‘no-level-crossing’ theorem, $m_{3/2}$ cannot go up any further, and the ‘gravitino’ becomes a mixture of the original gravitino and this exotic state. Therefore, in this case, we expect a pseudo-gravitino with $m_{3/2} \sim \Lambda_g$.

To summarize, qualitative arguments seem completely inconclusive about the nature and fate of a gravitino. The result of Sec. [III] is therefore quite nontrivial.

B. Relation of $m_{3/2}$ to SUSY Breaking Scale

Here, we comment on the validity of the famous relation between the gravitino mass and the SUSY breaking scale:

$$m_{3/2} = \frac{F}{\sqrt{3}M_{Pl}}. \quad (14)$$

In the pseudo-gravitino case ($m_{3/2} \gg \Lambda_g$), this usual relation has no reason to be true. Clearly, we cannot use

\(^9\) We are assuming that these mesinos are the lightest among the hadrons containing superpartners.

\(^10\) Note that this is exactly what is happening in typical weak-scale SUSY models in which the visible-sector interactions at the weak scale are taken to be (approximately) supersymmetric, even though the actual SUSY breaking scale is often as high as $10^{11}$ GeV. This is consistent because the interaction that transmits SUSY breaking to the visible sector is assumed to be sufficiently feeble.

\(^11\) An extreme but trivial limit of this case is to take $\Lambda_g \gg M_{Pl}$, i.e., the limit of an elementary graviton. Note that for any $\sqrt{F} \lesssim M_{Pl}$, the gravitino is a normal gravitino, and the inequality is trivially satisfied since $m_{3/2} \sim F/M_{Pl} \lesssim M_{Pl} \lesssim \Lambda_g$.\(^1\)}
the supergravity formalism to derive it, because supergravity contains general relativity which is not applicable for $E \gg \Lambda_g$ in our scenario. But more fundamentally, recall that this relation is just a consequence of the equivalence between the goldstino and the longitudinal component of the gravitino at high energies ($E_{3/2} \gg m_{3/2}$). Usually, we derive the relation by demanding that the amplitude of exchanging a gravitino between two supersymmetry currents be equal to that of exchanging a goldstino, in the global SUSY limit ($M_{Pl} \to \infty$) for $E_{3/2} \gg m_{3/2}$. However, in the pseudo-gravitino case, it has a different coupling to the supersymmetry current as well as a host of additional interactions. Hence, the formula \cite{14} does not hold for a pseudo-gravitino. In other words, since the pseudo-gravitino does not eat the goldstino by exactly the right amount, the SUSY currents must exchange something else to match the goldstino-exchange amplitude. But this `something else' must be among the new exotic states in the full theory of gravity, which we have no idea about. (If we had the underlying theory, we could subtract the exotic contribution from the amplitude and figure out how the formula \cite{14} should get modified.)

On the other hand, for $m_{3/2} \ll \Lambda_g$, we can apply the derivation for $m_{3/2} \ll E_{3/2} \ll \Lambda_g$, and obtain the usual relation \cite{14}, assuming that the gravitino has the standard $1/M_{Pl}$ coupling to the SUSY current, which is at least a consistent thing to do as we discussed in Sec. \textbf{III}

C. Theoretical Tests

It is certainly desirable to confirm the result of Sec. \textbf{III} by a theoretical argument that has a firmer foundation. Recall the concrete example of composite gauge bosons mentioned in Sec. \textbf{I} the $SU(N)$ supersymmetric QCD with $F$ flavors, where $N + 1 < F < 3N/2$. Below the $\Lambda_{QCD}$ of the $SU(N)$, this theory is described in terms of an IR-free $SU(F-N)$ gauge theory whose gauge bosons are composites of the original degrees of freedom \cite{12}.

Now let us deform the theory such that the low-energy gauge group $SU(F-N)$ gets spontaneously broken down to $SU(M)$ where $M < F - N$. If we apply the argument of Sec. \textbf{III} to this theory, we predict that the massive $W$ bosons, with all the `normal' couplings retained, cannot be heavier than $\Lambda_{QCD}$. $W$ bosons heavier than $\Lambda_{QCD}$ may exist but they should behave like random spin-1 resonances, rather than as the `$SU(F-N)$-partners' of the $SU(M)$ gauge bosons. While it sounds plausible, the currently available theoretical wisdoms are not powerful enough to definitively confirm the statement.

This SUSY QCD example also illustrates how extremely nontrivial it is to have a composite graviton coupled to elementary matter particles. In the case of the SUSY QCD model, this corresponds to the composite $SU(F-N)$ gauge bosons coupled to elementary quarks that are point-like even far above $\Lambda_{QCD}$! This is clearly a very difficult, if possible, thing to do. In the AdS composite graviton model of Ref. \cite{13}, the graviton wavefunction is highly peaked toward the IR brane, but there is an exponentially suppressed tail overlapping the UV brane where the SM fields live, which can be thought of as an explanation for the weakness of gravity. Adding supersymmetry to their setup to study the gravitino properties is saved for future work.

V. PRECISION GRAVITINO STUDY AND PROBING $\Lambda_g$ IN COLLIDERS

Clearly, the most important quantity in any composite graviton scenario is the scale $\Lambda_g$. As we mentioned already in Sec. \textbf{III} in order to probe the scale $\Lambda_g$, it is crucial to experimentally distinguish a gravitino from a pseudo-gravitino.

Unfortunately, if the results of such `precision gravitino study' turn out that what we have seen is actually a pseudo-gravitino, this will not be a sufficient evidence that gravity is modified at short distances. For example, a pseudo-gravitino is also present in a scenario where supersymmetry is not a fundamental symmetry at high energies but merely an (approximate) accidental global symmetry of the matter sector at low energies \cite{25}. In this scenario, the gravity sector is just the conventional GR (with no supersymmetry). Therefore, for a pseudo-gravitino, we need the underlying theory to derive more specific predictions to be tested.

On the other hand, if we can convince ourselves that it is not a pseudo-gravitino, then we can put a model-independent lower bound on $\Lambda_g$, as $\Lambda_g \gtrsim m_{3/2}$! Interestingly, as we will see shortly, in precisely the regime that the direct gravity measurement between test masses is impossible, the measurement of $m_{3/2}$ becomes possible, so the precision gravitino study can potentially exclude composite graviton scenarios dramatically at very short distances.

Since it is impossible to see a gravitino $\psi_{3/2}$ directly, the only hope to learn something about it lies in the case where both $\tilde{X}$ and $X$ can be precisely studied in the decay $\tilde{X} \to X + \psi_{3/2}$. This means that the decay must be sufficiently slow and that $\tilde{X}$ and $X$ both must be visible. This will indeed be realized if the $\tilde{X}$ is the next-to-lightest supersymmetric particle (NLSP) (the lightest (LSP) being the gravitino) and is electrically charged and/or strongly-interacting. In such a case, due to the very weak coupling of $\tilde{X}$ to the gravitino, there will be a long, highly visible track of the NLSP inside a collider detector before it decays \cite{28}, unless $\psi_{3/2}$ is too light. It is even possible that the NLSP stops in the detector if it is strongly interacting or produced sufficiently slow. In such circumstances, the momenta and energies of the NLSP and its SM partner as well as the NLSP lifetime should be measurable, which in turn allows us to deduce the mass and the coupling of the gravitino to see whether it is a pseudo-gravitino or not.
This ‘gravitino LSP with charged NLSP’ scenario has already been a great interest in SUSY phenomenology, especially in the context of gauge-mediated SUSY models where the gravitino is the LSP and is often a scalar tau lepton [24, 31, 32]. Note that once X and \( \tilde{X} \) have been observed, the gravitino mass can be simply determined from rewriting (10):

\[
m_{3/2} = \left( m_X^2 + m_{\tilde{X}}^2 - 2m_{\tilde{X}} E_X \right)^{1/2},
\]

where \( E_X \) is the energy of the X measured in the rest frame of the \( \tilde{X} \). If \( \tilde{X} \) stops inside a detector, \( E_X \) can be directly measured. Even if it does not, since both the X and \( \tilde{X} \) are highly visible in the detector, the measurement of their energies and the relative angle (the ‘kink’ in the track) can determine \( E_X \).

On the other hand, the measurement of the \( \tilde{X} \) lifetime gives us the gravitino’s coupling. If what we are seeing is not a pseudo-gravitino but is a real one, then the coupling should go as \( 1/M_{\text{Pl}} \) times the polarization factor \( E_{3/2}/m_{3/2} \) for the helicity-\( \pm 1/2 \) components, so the rate is given by

\[
\Gamma_{\tilde{X}} = \frac{m_{\tilde{X}}^5}{48\pi M_{\text{Pl}}^2 m_{3/2}^5/2}
\approx (20 \mu \text{m})^{-1} \left( \frac{\text{eV}}{m_{3/2}} \right)^2 \left( \frac{m_{\tilde{X}}}{100 \text{ GeV}} \right)^5
\approx (20 \text{ hours})^{-1} \left( \frac{\text{GeV}}{m_{3/2}} \right)^2 \left( \frac{m_{\tilde{X}}}{100 \text{ GeV}} \right)^5
\]

where we have dropped \( m_X \) and \( m_{3/2} \) for simplicity. (The helicity-\( \pm 3/2 \) components have no \( E_{3/2}/m_{3/2} \) enhancement and thus have been neglected.) The consistency of \( m_{3/2} \) determined from this formula with the value extracted from pure kinematics [15] will be an almost convincing evidence that the gravitino is not a pseudo, because it would be such a coincidence if the pseudo-gravitino coupling, which could be any size, just happened to be \( 1/M_{\text{Pl}} \). Ref. [31] proposes to go even further, to test the gravitino’s spin by using the angular distribution in the 3-body decay \( \tilde{\tau} \rightarrow \tau + \gamma + \psi_{3/2} \).

Now, it is probably extremely hard to directly measure the gravitational force between test masses for distances smaller than the micron scale which would correspond to \( \Lambda_g \lesssim 10^{-1} \text{ eV} \). Let us see whether the precision gravitino study can be used to place a bound on \( \Lambda_g \) beyond this limitation. Taking \( m_{\tilde{X}} = 100 \text{ GeV} \), the rate (10) tells us that for \( m_{3/2} = 10^{-1} \text{ eV} \), the NLSP will decay within \( \mathcal{O}(1) \mu \text{m} \), since the relativistic \( \gamma \) factor for the NLSP cannot be larger than \( \mathcal{O}(10) \) in a TeV-scale collider. This is unfortunately too short to be seen. Demanding that the NLSP must fly at least a few \( 100 \mu \text{m} \) to be clearly observed by a micro vertex detector, we need \( m_{3/2} \) to be at least a few eV. However, for such low values for \( m_{3/2} \), the formula (15) requires \( m_X, m_{\tilde{X}} \) and \( E_X \) to be measured with unrealistically high precision. The problem is, to determine a small \( m_{3/2} \) from (15), we have to nearly cancel two large terms and take the square-root. Therefore, the lowest possible value for \( \Lambda_g \) that can be probed is actually limited by the accuracy in measuring these parameters rather than the minimal NLSP flight length that a detector can resolve. For example, if we are anticipating \( m_{3/2} \) of order 1 GeV and if we are content with determining \( m_{3/2} \) only up to a factor of a few, then for \( m_{\tilde{X}} \approx 100 \text{ GeV} \) (neglecting \( m_X \) for simplicity), we would need to measure \( m_{\tilde{X}} \) and \( E_X \) with the accuracy of ±10 MeV. Therefore, measuring \( m_{3/2} \) of \( \mathcal{O}(1) \text{ GeV} \) event-by-event is unrealistic, so it must be done statistically. Taking the uncertainty in the individual \( E_X \) measurement to be \( \mathcal{O}(1) \text{ GeV} \), we need to observe \( \mathcal{O}(10^4) \) NLSP decays to have enough statistics for \( m_{\tilde{X}} \approx 100 \text{ GeV} \) and \( m_{3/2} \approx \text{GeV} \).

Also, note that for \( m_{3/2} \approx \text{GeV} \), the \( \tilde{X} \) lifetime is about a few hours to a week, so the NLSPs must be collected and stored to do the measurement. Such a possibility for \( \tilde{X} = \tilde{\tau} \) has been extensively studied in Refs. [31], and the bottom line is that collecting \( \mathcal{O}(10^4) \) or even \( \mathcal{O}(10^5) \) NLSPs and observing their decays should be possible in the LHC and/or the ILC, although the prospect depends on other SUSY parameters.

Those analyses also conclude that we may be able to go up to \( m_{3/2} \) of \( \mathcal{O}(100) \text{ GeV} \). Therefore, it is not too optimistic to expect that precision gravitino study may be able to probe the scale \( \Lambda_g \) between GeV and 100 GeV. While this is still quite challenging (and we also have to be lucky with the SUSY spectrum), note that this is a regime where direct measurement of gravitational forces is absolutely impossible, so precision gravitino study is the only available probe for composite gravity.

VI. CONCLUSIONS

In this paper, we have considered ‘composite gravity’, namely, the possibility that the graviton is not an elementary propagating degree of freedom at distances shorter than \( \Lambda_g^{-1} \). We pointed out that such a scenario is not necessarily forbidden by the Weinberg-Witten theorem. Another important assumption we made is that the matter sector is completely described by a local quantum field theory, which is true for \( \Lambda_g \) between the current experimental limit \( \sim 10^{-3} \text{ eV} \), and \( \sim \text{TeV} \) or whatever cutoff for the matter sector. To perform a model-independent, effective-field-theoretical analysis, it is necessary to reconcile ‘elementary matter with a high cutoff’ and ‘composite gravity with a low cutoff’, and for this purpose we have utilized soft graviton effective theory (SGET) by

\[ \text{Note that this agreement between the two measurements of } m_{3/2} \text{ is equivalent to checking if the gravitino has really eaten the goldstino as it should if it is not a pseudo.} \]
Sundrum.

In general, the only way to place a lower limit on the scale $\Lambda_g$ is by a null result in experiments seeking a deviation from the standard $1/r^2$ law between macroscopic test masses. This method becomes increasingly difficult as the distance gets reduced. Therefore, it is desirable to have an alternative probe. The problem is, however, that in general there is nothing related to gravity except the graviton itself, so there is no other way to probe $\Lambda_g$ without using gravity.

However, we noted that if there is an underlying supersymmetry, it may lead to the existence of a gravitino, which is related to gravity but easier to observe than the graviton. Applying the SGET framework to the gravitino, we have shown the relation, $\Lambda_g \gtrsim m_{3/2}$, i.e., the gravitino remains an elementary degree of freedom at least down to the graviton’s Compton wavelength. In other words, we can use a gravitino to test general relativity at short distances—once we see a gravitino, we know that GR is correct at least up to $m_{3/2}$, which in turn places a lower bound on $\Lambda_g$! This can have a significant impact on the possibility of composite graviton as a solution to the cosmological constant problem. For example, if we find $m_{3/2}$ to be, say, 1 GeV, the door will be completely shut.

On the other hand, if we first find gravitino compositeness and measure $\Lambda_g$, then our inequality says that we will not discover a gravitino above the scale $\Lambda_g$—at best we may just see some random spin-3/2 fermion with completely random couplings, nothing to do with gravity.

To utilize this inequality to place a limit on $\Lambda_g$, it is crucial to experimentally convince ourselves that what we are looking at is really a gravitino, rather than a random spin-3/2 fermion. In the future colliders such as the LHC and ILC, the prospect of being able to do so seems quite bright for the range $10^{-14}$ cm and $10^{-16}$ cm. Therefore, precision gravitino study can indeed be an alternative model-independent probe for $\Lambda_g$ or a test of general relativity, in a regime where direct measurement of gravitational force is absolutely impossible.

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