Primordial Lepton Oscillations and Baryogenesis

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Abstract

The baryon asymmetry of the Universe should have been produced after the inflation era. We consider the possibility that the asymmetry is generated by the flavor oscillations in the reheating process after inflation, so that the baryon asymmetry is realized already at the beginning of the radiation dominated era. In the seesaw model, we show that the propagators of the left-handed leptons generically have flavor mixings in the thermal background, that can generate flavor-dependent lepton asymmetry through the \( CP \) violation in the oscillation phenomena. The flavor dependent rates for the wash-out process can leave the net asymmetry today.
1 Introduction

The $CP$ violation in the neutrino oscillation can produce flavor dependent particle-antiparticle asymmetry. Although it has not been established experimentally, the discovery of such phenomena will be a quite important hint for the mystery of the baryon asymmetry of the Universe. The mechanism for the baryon asymmetry before the electroweak phase transition requires generation of primordial $B-L$ asymmetry, rather than $B$ asymmetry, due to the $B+L$ breaking electroweak sphaleron process [1]. Within the field content of the Standard Model, the Majorana neutrino mass term, $l l H H$, is the lowest dimensional operator which breaks $B-L$ explicitly. Therefore, it is quite reasonable that the $CP$ violation in the neutrino interactions is responsible for baryogenesis.

The standard leptogenesis scenario [2] assumes that the $CP$ violation in the decays of right-handed neutrinos produces the lepton asymmetry, and thus it requires the production of the right-handed neutrinos after the inflation and their decays when the $L$-violating interactions gets sufficiently ineffective. It has been extensively studied such possibilities and viable parameter regions are investigated. It has been found that the mechanism works for high enough reheating temperatures. For the review, see Ref. [3] for example.

There is another interesting period of time where the out-of-equilibrium condition is satisfied automatically. After the inflation, the decays of the inflaton can reheat the Universe, producing the radiation energy which eventually dominates the Universe. In the course of reheating, high-energy particles are produced by the decay process, and each particles lose their energy by scattering with thermal plasma. This energy-loss process is obviously a one-way process, and thus provides us with perfect environment for baryogenesis. In Ref. [5], such a possibility has been studied where the scattering of the leptons during the reheating process produces the baryon asymmetry of the Universe.

In this paper, we consider the oscillation phenomena of the left-handed leptons (neutrinos and charged leptons) in the thermal background and the possibility to produce lepton asymmetry via its $CP$ violation. The inflaton decays into leptons in a particular combination of the flavor eigenstates. Since the lepton propagators in the thermal background are not diagonal in the flavor basis, the leptons can change the flavor during the propagation. The $CP$ violation in the oscillation, i.e., the indirect $CP$ violation in the inflaton decays, can produce the flavor dependent lepton asymmetry although no net lepton asymmetry is generated. The

\* See Ref. [4] for baryogenesis from the oscillations of left-handed leptons which are originated from the decay of right-handed neutrinos.
flavor dependent lepton asymmetry, in turn, can be converted into the net asymmetry by the
flavor dependent wash-out process due to, for example, the \( lllHH \) interactions. We find that
the baryon asymmetry of the Universe can be explained by this mechanism, even in the case
where the reheating temperature is much lower than the masses of the right-handed neutrinos
and there is no decays of the inflatons into right-handed neutrinos, which, therefore, have
never shown up in the history of the Universe.

This paper is organized as follows. In Sec. 2, we discuss the lepton oscillation phenomena
in the context of the seesaw model. We show that observed baryon asymmetry of the Universe
can be explained by the primordial lepton oscillation in Sec. 3. Sec. 4 is devoted for summary
and conclusion.

2 Lepton propagators in the early Universe

In this section, we first calculate the propagators of the left-handed leptons in the thermal
background, and show that the thermal correction to the dispersion relation causes the flavor
oscillations as in the case of the neutrino oscillation in the vacuum by the mass differences.
As a concrete scenario, we consider the seesaw model \([6]\) in which the left-handed leptons
have flavor off-diagonal interactions.

The Lagrangian is given by

\[
\mathcal{L}_{\text{int}} = -y_{ij}^{\nu} \bar{N}_i P_L (l_j \cdot \tilde{H}) + \text{h.c.} \\
+ \frac{M_i}{2} \bar{N}_i^c N_i + \text{h.c.}
+ y_{\ell}^{\nu} \bar{N}_i^c P_L (l_i \cdot H) + \text{h.c.}
\]

(1)

We have diagonalized the charged lepton Yukawa coupling \( y_{\ell} \), that defines the flavor eigen-
state. The right-handed neutrinos, \( N_i \), are introduced and \( M_i \) are their Majorana masses.

In a high temperature medium where the Higgs particles are in the thermal bath, the
Yukawa interactions in Eq. (1) affect the propagators of the \( l_i \) fields. Following Ref. [7], the
propagator of a fermion field in the momentum space is parametrized as

\[
S(K) = \left[ (1 + a) \frac{K}{u} + b\hat{p}\right]^{-1},
\]

(2)

where \( K^\mu = (\omega, k) \) is the four momentum and \( u^\mu = (1, 0, 0, 0) \). The coefficients, \( a \) and \( b \),
are functions of \( \omega \) and \( k = |k| \). By looking at the pole of the propagator, the dispersion relation
is modified to

\[ \omega = k - \text{Re} \left[ \frac{b}{1 + a} \right]. \tag{3} \]

The non-vanishing functions, \( a \) and \( b \), are the effects of the interactions. At the leading order in the perturbation theory, one can ignore \( a \). The real part of the function \( b \) is calculated to be

\[ b_{\mu'\nu} \simeq -\frac{T^2}{16k}(y_e^2)^2\delta_{\mu'\nu} + \frac{\pi^2T^4}{9M_k^2}g_{\nu}^{kl*}g_{\mu'k}^{kl'}, \tag{4} \]

for \( \omega \sim k \gg T \), where \( T \) is the temperature. We have assumed here that \( Tk \ll M_i^2 \). For \( Tk \gtrsim M_i^2 \), one obtains the term similar to the first term which stems from the charged lepton Yukawa interactions. The second term provides flavor non-diagonal entries, responsible for the oscillation. The second term is not quite the “thermal mass,” since the dispersion relation is still \( \omega = 0 \) in the \( k \to 0 \) limit. However, it does modify the dispersion relation as in Eq. (3), and causes the flavor oscillation phenomena as we discuss below. The contribution from the gauge interactions are flavor universal, and thus can be ignored for our purpose.

The second term is suppressed by \( M_i^4 \), which can be understood by the operator analyses. For \( M_i \gg T \), one can integrate out the \( N_i \) fields, and effective interaction terms with dimension five or higher are generated. The odd-dimensional operators, such as \( l\bar{l}HH \) and \( llH\partial^2H \), break the \( L \) number as well as the Higgs number, and thus would not cause the forward elastic scattering to modify the propagators. The dimension-six operators, \( \bar{l}\gamma^\mu l(iH^\dagger\partial_\mu H + \text{h.c.}) \), would not contribute as long as there is no chemical potential for the Higgs fields. Therefore, the first contribution appears from the dimension-eight operators which are suppressed by \( M_i^4 \). Nevertheless, for a large enough \( k \),

\[ k^2 \geq \frac{9}{16\pi^2} \frac{(y_e^2)^2}{y_{\nu}^{kl*}y_{\nu'k}^{kl'}} \frac{M_k^2}{T^2}M_i^2, \tag{5} \]

the second term in Eq. (4) dominates over the first one.

The matrix \( b \) can be diagonalized by a unitary transformation:

\[ U^\dagger bU = b^{\text{diag}}. \tag{6} \]

The unitary matrix \( U \) is different from the PMNS matrix \([8]\), and the actual form cannot be determined only from the low energy data. The neutrino oscillation can be understood in the standard quantum mechanical considerations. In the basis of diagonal \( b \), the neutrino wave function is given by \([9]\)

\[ |t = \Delta t\rangle = e^{-i\omega|\Delta t}e^{ik\cdot\Delta x}|t = 0\rangle, \tag{7} \]
and the differences of the dispersion relations appear in the phase of the interference terms in the transition rates of $l_l \to l_{l'}$:

$$\delta \phi_{jk} = (\omega_j - \omega_k)\Delta t - (k_j - k_k)\Delta x$$

$$= (k_j - k_k)(\Delta t - \Delta x) - \Delta b_{jk}\Delta t$$

$$\simeq -\Delta b_{jk}\Delta t,$$

(8)

where $\Delta b_{jk} = b^\text{diag}_j - b^\text{diag}_k$, and $\Delta t$ and $\Delta x$ are time and length of the travel, respectively. The time $\Delta t$ should be taken as the mean free time of the lepton propagation, $\Delta t \simeq \Gamma^{-1}$, after which the leptons lose their momenta or pair annihilate with the leptons in the thermal bath. The dominant scattering process is through the $SU(2)$ gauge interactions which give a flavor universal $\Gamma$:

$$\Gamma \sim \frac{g^2}{4\pi}T.$$

(9)

Essentially the same result can be derived from field theory [10]. In the basis of diagonal $b$, the denominator of the propagators is

$$\frac{1}{(\omega + b^\text{diag}_j^k)^2 - k^2 + i\omega\Gamma},$$

(10)

where $\Gamma$ is given in Eq. (9). The field theoretic computation of the probability involves the phase space integration over the propagators:

$$\int_0^\infty d\omega \frac{1}{(\omega + b^\text{diag}_j^k)^2 - k^2 + i\omega\Gamma} \simeq \frac{\pi}{k^2 \Gamma} \left[ \frac{1}{1 + 2i\Delta b_{jk}/\Gamma} \right].$$

(11)

We see the oscillation effect $1/(1 + 2i\Delta b_{jk}/\Gamma)$, which is similar to the quantum mechanical consideration, $\exp(-i\Delta b_{jk}/\Gamma)$, for $\Delta b_{jk}/\Gamma \ll 1$.

3 Baryon asymmetry

Now we consider the possible scenario for realizing the baryon asymmetry through the primordial oscillation phenomena.

We assume that the high energy leptons are generated by the decays of the inflaton $\phi$:

$$\phi \to l_i + X$$

(12)

and its $CP$ conjugate process. The part of the final state $X$ is arbitrary, and the decay mode above is not even necessary to be the dominant one. For example, $X = H + e^c_j$ makes $\phi$ gauge
singlet. We also assume that the reheating temperature of the Universe, $T_R$, is lower than $10^{12}$ GeV so that the Yukawa interaction of the tau leptons is in the thermal equilibrium \[11\]. In this circumstance, one can distinguish lepton asymmetries in $l_\tau$ and $l_{e,\mu}$ as in the case of flavored leptogenesis \[12\], where indices are defined in the basis where the charged lepton Yukawa matrix is diagonal.

As we have seen in the previous section, the neutrino Yukawa interaction is not diagonal in this basis. The leptons obtain the off-diagonal components in their propagator in the thermal plasma as in Eq. (4). Therefore, the leptons generated through the inflaton decays undergo the flavor oscillations until the first scattering where the leptons lose their energy.

In the basis where the propagator is diagonalized, the flavor eigenstate is expressed as

$$|l(e, \mu, \tau)\rangle = \sum_{j=1,2,3} U_{jl}^\dagger |j\rangle,$$

and the inflaton decays provide lepton states which are a linear combination of the eigenstates of the Hamiltonian:

$$|l_\phi\rangle = \sum_{j=1,2,3} V_j^* |j\rangle.$$

Here, $U$ is a unitary matrix same as that in Eq. (6), whereas $V$ is a normalized vector. By combining Eqs. (8), (13), and (14), the $CP$ asymmetry is given by the oscillation formula \[9, 13\]:

$$P_l - \bar{P}_l = 4 \sum_{j>k} \text{Im} \left( U_{lj} V_j^* V_k^* U_{lk}^\dagger \right) \int dk f(k) \sin \frac{\Delta b_{jk}(k)}{\Gamma}.$$

Here $P_{l(\bar{l})}$ represents the oscillation probability of $l_\phi \rightarrow l_\ell$ ($\bar{l}_\phi \rightarrow \bar{l}_\ell$), $f(k)$ is the momentum distribution of neutrinos from inflaton decay with $\int dk f(k) = 1$, and we have evaluated asymmetry at the stage of the first scattering, that is, $\Delta t \simeq \Gamma^{-1}$. In the following, we take $V_1 = 0$ for simplicity. Then, the $CP$ asymmetry is

$$P_l - \bar{P}_l = 4 \text{Im} \left( U_{l3} V_3^* V_2^* U_{l2}^\dagger \right) \int dk f(k) \sin \frac{\Delta b_{32}(k)}{\Gamma} \simeq 4 \text{Im} \left( U_{l3} V_3^* V_2^* U_{l2}^\dagger \right) \frac{\Delta b_{32}(m_\phi)}{\Gamma}.$$

In the second line, we have expanded the sine function, and used the relation that the expectation value of momentum, $\int dk f(k)$, is the order of $m_\phi$ which is the mass of the inflaton. Eq. (16) is generally non-vanishing and the $CP$-asymmetry factor, $A_{CP} = \text{Im} (U_{l3} V_3^* V_2^* U_{l2}^\dagger)$, can easily be of order unity. Since the asymmetry vanishes when we take the sum over the $l$
index, there is no net lepton asymmetry generated. The tau asymmetry is compensated by the $e$ and $\mu$ asymmetry.

At the stage of the first scattering, however, the flavor eigenstate is not a meaningful concept since the time scale of the first scattering is much faster than the scattering through the Yukawa interaction for the charged tau lepton. This means that the density matrix of the lepton number is not diagonal in the flavor basis. The flavor dependent asymmetry becomes physical later at the time scale of the scattering through the tau Yukawa interaction, where the off-diagonal components of the density matrix dump to zero while the diagonal components, which we calculated, are left time-independent\textsuperscript{12}. Note that, if the interaction rate of neutrino Yukawa couplings is larger than that of tau Yukawa, it could generate the off-diagonal components of the density matrix, which threatens our baryogenesis mechanism. As long as right-handed neutrinos are heavy, it is safe because we know that the dominant interaction rate through the $llHH$ operator is smaller than Hubble rate as we discuss later.

Since we assume that the temperature is low enough that the tau Yukawa interaction is in the thermal equilibrium, one can identify the tau asymmetry, and thus the generated flavor dependent asymmetry can be separately treated. In particular, the $\Delta L = 2$ process through the $llHH$ interaction can wash out the lepton asymmetry in a flavor dependent manner, that results in the generation of the net lepton asymmetry. If we assume the hierarchical neutrino masses in normal hierarchy, one can consider the situation where only the $l_3l_3HH$ interaction is effective. The interaction rate for the $\Delta L = 2$ process is approximately given by

$$\Gamma_{l_{\ell}}^{w.o.} \sim \frac{|U_{\alpha l}|^2 m_{\nu l}^2 T_R^3}{\pi^2 v^4},$$

where $U_{PMNS}$ is the PMNS matrix, $m_{\nu l}$ is the neutrino mass, and $v \simeq 246$GeV. Compared with the expansion rate of the Universe, we obtain

$$\frac{\Gamma_{l_{\ell}}^{w.o.}}{H} = \frac{T}{10^{12} \text{ GeV}} \times \begin{cases} 0.03, & l = \tau \\ 0.02, & l = \mu + e \end{cases}$$

(18)

Here we take the observed values of mixing angles and $m_{\nu 3} \simeq 0.05$ eV\textsuperscript{9}. Note that this quantity does not depend on the Dirac or Majorana phases.

As a result, we obtain the net lepton asymmetry as

$$\frac{n_L}{s} \sim \frac{n_\phi}{s} \times B_{\phi \rightarrow l} \times A_{CP} \times \frac{\Delta b_{23}}{\Gamma} \times \left( \frac{\Gamma_{l_{\ell}}^{w.o.}}{H} - \frac{\Gamma_{\mu + e}^{w.o.}}{H} \right) \times 0.01 \times \left( \frac{T_R}{10^{12} \text{ GeV}} \right),$$

(19)\textsuperscript{1}

\textsuperscript{1}We thank Sacha Davidson for discussion on the flavor basis and the time scales of the various interactions.
where $B_{\phi \rightarrow l}$ is the branching ratio of the inflaton decays into leptons. The first factor is 
the inflaton abundance, and the factor of the difference of the $\Gamma^{w.o.}/H$ describes the flavor-
dependent wash-out effects. The number density of the inflaton is

$$\frac{n_\phi}{s} \sim \frac{T_R}{m_\phi},$$ \hspace{1cm} (20)

and the splitting of the dispersion relation is

$$\frac{\Delta b_{23}}{\Gamma} \sim \frac{4\pi^3 m_\phi T_R^3}{9g_2^2 M_k^4} |y_{\nu}^k|^2,$$ \hspace{1cm} (21)

by taking $k \sim m_\phi$. Therefore, putting altogether, we obtain the baryon asymmetry

$$\frac{n_B}{s} \sim \frac{n_L}{s} \sim 10^{-7} \times B_{\phi \rightarrow l} \times A_{CP} \times \left( \frac{T_R}{10^{12} \text{ GeV}} \right)^2 \left( \frac{10}{M/T_R} \right)^{-3},$$ \hspace{1cm} (22)

where $M$ is the right-handed neutrino mass which we take, for simplicity, to be common for all flavors. We see that the baryon asymmetry, $n_B/s \sim 10^{-10}$ \cite{14}, can be explained by this mechanism. It is interesting to note that the final formula is independent of the inflaton mass as long as $m_\phi \gg T_R$. We also note that Eq. (5),

$$k \geq \frac{3}{4\pi} \frac{y_{\nu}^k}{\sqrt{y_{\nu}^{kl} y_{\nu}^{kl}}} \frac{M}{T_R} \sim 10^{12} \text{ GeV} \left( \frac{y_{\nu}^k}{10^{-2}} \right) \left( \frac{M}{10^{15} \text{ GeV}} \right)^{3/2} \sqrt{\frac{0.05 \text{ eV}}{m_{\nu 3}}} \left( \frac{10^{12} \text{ GeV}}{T_R} \right),$$ \hspace{1cm} (23)

can be satisfied for large $m_\phi \sim k$.

The estimate above is only valid when $M^2 \gg T_R m_\phi$. For more general situations, the wash-out rates as well as the thermal correction to the propagators can be enhanced, and thus a larger asymmetry may be realized unless the neutrino Yukawa couplings are larger than tau Yukawa coupling. We leave such general analyses for future studies.

4 Conclusion

In the inflationary Universe, one of the natural possibilities to explain the baryon asymmetry 
is that the asymmetry is generated during the reheating era such that the radiation dominated 
Universe has started with non-vanishing asymmetry. Since the transition from the inflaton 
dominated era to the radiation dominated one is a one-way process, one can naturally satisfy 
the out-of-equilibrium condition for baryogenesis. Also, for high enough reheating temper-
atures, the $L$-violating interactions responsible for the neutrino masses and the electroweak 
sphaleron process is effective, that can provide the $B$-violating condition. The final condition 
is the $CP$ violation. In the reheating era, the natural location to look for the $CP$ violation is
the decays of the inflaton. We indeed have shown that $CP$ violation in the flavor oscillation of the leptons can happen in the inflaton decays and that can explain the baryon asymmetry of the Universe.

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