Multi-Objective Optimization of Moving-magnet Linear Oscillatory Motor Using Response Surface Methodology with Quantum-Behaved PSO Operator

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Abstract. To reduce the difficulty of manufacturing and increase the magnetic thrust density, a moving-magnet linear oscillatory motor (MMLOM) without inner-stators was proposed. To get the optimal design of maximum electromagnetic thrust with minimal permanent magnetic material, firstly, the 3D finite element analysis (FEA) model of the MMLOM was built and verified by comparison with prototype experiment result. Then the influence of design parameters of permanent magnet (PM) on the electromagnetic thrust was systematically analyzed by the 3D FEA to get the design parameters. Secondly, response surface methodology (RSM) was employed to build the response surface model of the new MMLOM, which can obtain an analytical model of the PM volume and thrust. Then a multi-objective optimization methods for design parameters of PM, using response surface methodology (RSM) with a quantum-behaved PSO (QPSO) operator, was proposed. Then the way to choose the best design parameters of PM among the multi-objective optimization solution sets was proposed. Then the 3D FEA of the optimal design candidates was compared. The comparison results showed that the proposed method can obtain the best combination of the geometric parameters of reducing the PM volume and increasing the thrust.

1. Introduction
As an electromagnetic driving device, linear oscillatory motor (LOM) is able to achieve high reciprocating linear motion without intermediate conversion mechanism, which changes rotary motion into linear motion, such as screw, chain, and gear. With the advantages of simple structure, low noise, fast response and high efficiency [1-2]. LOM has been widely used to drive refrigeration compressors, pumps, artificial hearts and shock absorbers.

The MMLOM has becoming more and more important in LOM areas due to its light mover mass, high resonant frequency and high thrust density [3-5]. The LG company has already used MMLOM for driving linear compressors of commercial refrigerators. Compared with the compressor drive system composed of rotatory motor and crank shaft [6], the energy-saving effect of the LG linear compressor is remarkable. However, the key issues which restrict the traditional MM LOM are complicated structure, difficulty in inner-stators stacking and high production cost. A novel MMLOM...
without inner-stators was proposed [7], which has the novel topology of two symmetry “C” type external divided stators with separation distance, and the mover is set between two “C” type teeth. The new topology has the characteristics of compact configuration, easy manufacture, little flux leakage, high thrust density and good thrust controllability.

The thrust performance and PM volume are regularly used to evaluate the performance of MMLOM, which is significantly influenced by the geometric parameters. Therefore, techniques of reducing the PM volume and increasing the thrust are very important in the novel MMLOM design. To find the most suitable combination of design parameters to achieve the design goal, the RSM with a QPSO operator was employed. Meanwhile, the 3D FEA is employed as the analyzing tool for the motor thrust and performance.

PM thickness, PM width, stators space, and PM axial length define the search space for the optimization problem. RSM is well adapted to obtain an analytical model of the PM volume and thrust. The RSM enables objective functions to be easily created and a great computational time to be saved. The 3D FEA is used as numerical experiments on geometrical design variables to provide the response. The QPSO algorithm is used as a searching tool for design optimization. With the proposed QPSO technique, the PM volume of initially designed MMLOM can be reduced and its thrust can be increased.

2. Basic Structure and Principle

Fig. 1 shows the structure of the proposed MMLOM, which has two divided “C” type stators and one mover, with two concentrated stator windings connected in series. In particular, the PM mover is set between the two teeth of “C” iron core, so it has no inner-stator compared to the conventional MMLOM. When the winding is fed in single-phase alternating current from external driving circuit, the alternating magnetic field is produced interacts with the constant PM exciting field, then the alternating thrust force is produced. Together with two springs, it could drive the mover to produce fast linear oscillatory motion.
3. D FEA of the Thrust

3.1. 3D FEA Model
The finite element method (FEM) is an effective method for numerical calculation of electromagnetic field distribution. The commercial ANSOFT software is used for this purpose. The nonlinear material properties and end magnetic flux leakage are taken into consideration. The specifications of this MMLOM are listed in Table 1.

Table 1. Specification of MMLOM

| Parameters          | Value   | Parameters                              | Value  |
|---------------------|---------|-----------------------------------------|--------|
| Gap /mm             | 1       | Coil turns /mm                           | 300    |
| PM material         | N35H    | Coil diameter /mm                       | 0.8    |
| PM Axial length /mm | 40      | Stator stack thickness /mm               | 50     |
| PM thickness /mm    | 3       | Stator tooth width /mm                   | 15     |
| Silicon material    | 50W470  | Stators space /mm                       | 10     |

To verify the results of 3D FEA model, the thrust of the prototype, as shown in Fig. 2, was measured. Fig. 3 shows a comparison of thrust results between the 3D FEA model and the measurement, and the error is less than 5%. Therefore, the 3D FEA model can work as an accurate numerical method of analysis for the motor optimization.

3.2. Analysis on Geometric Parameters of PM
The PM provides the excitation flux for the MMLOM, which not only affects the cost of the MMLOM, but also determines the thrust characteristics. The PM geometric parameters of this MMLOM are thickness, width and axial length. Therefore, it is necessary to analyze the effect of PM geometric parameters on thrust by 3D FEA.

Fig.4 shows the thrust characteristics with PM geometric parameters, which showed that the thrust increases with PM thickness, but the increment is lowered due to magnetic saturation. When the PM width is less than the stator laminated thickness (50mm), the thrust increases nearly linearly with the PM width. Due to the end flux leakage, the thrust increment begins to decrease sharply when the PM width is larger than 53mm. Although the PM axial length does not affect the magnitude of the thrust, it can affect the stability range of the thrust. When the PM axial length is approximately equal to the sum of the stator tooth width and the stators space, a smoother thrust can be obtained, then the effective stroke of the MMLOM is maximum. Therefore, it is necessary to analyze the effect of stators space on thrust. As shown in Fig.5, thrust increases little when the stator space increases by 3 mm.
4. Optimization Design

4.1. Analysis on Geometric Parameters of PM

RSM seeks to find the relationship between design variables and responses through the statistical fitting method [8]. This model can avoid iteration numerical calculation of electromagnetic fields when solving the traditional inverse problems. The response can be obtained from real experiments or computer simulation. Therefore, in this paper, the 3D FEA is used as numerical experiments to provide the response. The maximum thrust force and the minimum PM volume of this MMLOM are the responses. They are changed by the PM geometric parameters. The RSM assumes that the true functional relationship can be written as Eqn. 1.

\[ f(X) = \sum_{j=1}^{N} c_j H(\|X - X_j\|) \]  

(1)

\[ [c_j] = [X_{ij}]^{-1} [f_i] \]  

(2)

\[
\begin{cases}
  H(r) = r, \\
  H(r) = (r^2 + h)^\beta & (0 < \beta < 1), \\
  H(r) = \frac{1}{(r^2 + h)^\alpha} & (\alpha > 0)
\end{cases}
\]  

(3)

\( \forall f \in R \), where \( H(r) \) is the RBF, \( N \) is the total sampling points, \( X_j \) is the sampling point coordinates, \( c_j \) is the undetermined coefficient which can be written as Eqn. 2, where \( f_i = f(X_i) \), \( X_{ij} (i,j = 1,2,...,N) \) depends on the selected basis function. The common RBF may be written as Eqn. 3, where \( r = \|X - X_j\| \) is the Euclidean Norm.
In Eqn. 4, the first and second basis function are relatively simple, and suitable for optimization application with little optimizing variable dimension. The third and fourth basis function are more complex, and the function reconstruction need to be adjusted for different goals. In this paper, the third basis function was chosen to estimate the interactions of the design variables and curvature properties of the response surface.

4.2. QPSO

In traditional PSO, status and trajectory of particles is determined by both the position and velocity, according to the equations as Eqn. 4 [9], where \( w \) is inertia weight, \( c_1 \) and \( c_2 \) are acceleration constants, \( r_1 \) and \( r_2 \) are random values.

\[
\begin{align*}
  v_i(t + 1) &= w \cdot v_i(t) + c_1 r_1 \left[ p_i(t) - x_i(t) \right] + c_2 r_2 \left[ g_i(t) - x_i(t) \right] \\
  x_i(t + 1) &= x_i(t) + v_i(t + 1)
\end{align*}
\]

(4)

The main disadvantage of the traditional PSO is that the global convergence can’t be guaranteed. Many researchers have devoted efforts to improve PSO performance in various ways. QPSO is an improvement on the standard PSO algorithm from a quantum mechanics angle [10]. In QPSO, the state of a particle is depicted by the wave function, instead of position and velocity. Hence, the quantum system possesses more states and each particle can appear in an arbitrary position to seek space according to a certain probability. Using the Monte Carlo method to collapse the wave function into the desired region, particles move according to the following iterative equation as Eqn. 5 and Eqn. 6, where \( \phi \in [0, 1] \) is a random number, \( \beta \in [0.5, 1] \) is the contraction-expansion coefficient, which can be tuned to control the convergence speed of the algorithms. \( G_j(t) \) is the average optimal position defined as the mean of all the best positions of the population, \( p_j(t) \) is the equilibrium position, which varies during iteration and acts as a mutation.

\[
\begin{align*}
  x_j(t + 1) &= \beta \left[ G_j(t) - x_j(t) \right] \cdot \ln(1 / \mu) + p_j(t) \quad (if \; k > 0.5) \\
  x_j(t + 1) &= -\beta \left[ G_j(t) - x_j(t) \right] \cdot \ln(1 / \mu) + p_j(t) \quad (if \; k < 0.5)
\end{align*}
\]

(5)

\[
\begin{align*}
  x_{i_{\text{min}}} \leq x_i(t + 1) \leq x_{i_{\text{max}}} \\
  p_j(t) &= \phi \cdot p_j(t) + (1 - \phi) \cdot p_{g_j}(t) \\
  G_j(t) &= \frac{1}{M} \sum_{i=1}^{M} p_j(t)
\end{align*}
\]

(6)

4.3. Optimization Procedures, an optimization design of the MMLOM with lower PM volume and lager thrust, using RSM with a QPSO operator for linear compressor application, has been presented in this paper. The design procedure according to the total design strategy is as follows:

4.3.1. Geometric Parameters selection and response surface model build, the selection of design variables is a very important step in the optimization procedure. In this paper, four geometric parameters are selected as the design variables, \( x_1 \) is the thickness, \( x_2 \) is the width, \( x_3 \) is the stators space, and \( x_4 \) is the axial length.
In order to determine the RBF of the response surface, several experimental designs have been developed to establish the approximate equation using the 3D FEA results. 200 sample points are obtained via 3D FEA, including 150 sample points for training RBF neural network, to build the thrust and the PM volume of response surface models. The remaining 50 sample points as the test samples, the error between RSM predictions and test samples is shown in Fig. 6. Response surface model constructed with radial basis neural network is accurately sufficiently close to 3D FEA model, so it can be used to further multi-objective optimization.

![Thrust prediction error](a) PM volume prediction error](b)

Figure 6. Relative error of RSM objective function prediction

4.3.2. QPSO Approach, the main objective of this paper is to search the best combination of geometric parameters of the proposed MMLOM for simultaneously reducing the PM volume and increasing the thrust. The objective functions $F_1(x_1, x_2, x_3, x_4) = \min[f_1(x_1, x_2, x_3, x_4)]$ and $F_2(x_1, x_2, x_3, x_4) = \max[f_2(x_1, x_2, x_3, x_4)]$, where $f_1$ is the PM volume and $f_2$ is the thrust, the constraints of the optimization problem is $x_1 \in [2, 30]$, $x_2 \in [2, 30]$, $x_3 \in [0, 10]$ and $x_4 = x_3 + 15$.

To solve the optimization problem, a QPSO algorithm was presented. The initial number of group is set to be 20, and the maximum number of generations is set to be 500, the particle is composed of four coded variables. The near best solution is obtained through the calculation of the proposed OPSQ method as follows:

1. Set $t = 0$, initialize the position vectors of the initial particle group.
2. Objective function value calculation according to the two objective functions.
3. Calculation of averaged global optimal position of the particle group.
4. Update of random points of every particle.
5. Recalculation of new position vector of every particle.
6. Set $t = t + 1$, repeat step (2) to step (6), until the termination conditions satisfied.

4.3.3. Optimization results and verification, Fig.7 shows the Pareto curve which is obtained through optimal solution set of the objective functions.

The objective constraints of the multi-objective optimization approach for the proposed MMLOM are $F_1(x_1, x_2, x_3, x_4) < 3000 \text{mm}^3$ and $F_2(x_1, x_2, x_3, x_4) > 20 \text{N}$.
Table 2 shows the optimal solution sets. Then the optimum parameter range is: PM thickness range in [2.75, 3] mm, PM width range in [53, 57.25] mm, stators space in [2.75, 3] mm, and PM axial length range in [17.75, 18] mm. Considering actual PM geometrical size and difficulty in prototype motor stacking, we select the optimum parameter set in which the PM thickness is 3 mm, PM width is 53 mm, and the PM axial length is 18 mm.

4.3.4. Verification of Multi-Objective Optimization Result. Table. 3 shows the comparison of the 3D FEA result before optimization, optimization result via RSM and optimization result verified by 3D FEA. By comparison, we can find that the optimization result via RSM is found in good coincidence with the FEA result, where the error is below 0.3%; the thrust increases by 4.6% from 19.6N to 20.5N after optimization; the PM volume decreases by 23.7% from 3750 mm$^3$ to 2862 mm$^3$ after optimization.

Table 2. Optimal sets in pareto curve

| ID | PM thickness (m) | PM width (mm) | Stators space (mm) | Thrust (N) | PM Volume (mm$^3$) |
|----|-----------------|---------------|--------------------|------------|--------------------|
| 1  | 3.00            | 53.00         | 3.00               | 20.45      | 2865               |
| 2  | 2.75            | 57.25         | 3.00               | 20.30      | 2827               |
| 3  | 3.00            | 53.50         | 2.75               | 20.55      | 2849               |
| 4  | 2.75            | 57.50         | 2.75               | 20.40      | 2806               |

Table 3. Comparison of 3D FEA result

| Para. FEA result | PM thickness (mm) | PM width (mm) | Stators space (mm) | Thrust (N) | PM volume (mm$^3$) |
|-----------------|-------------------|---------------|--------------------|------------|--------------------|
| Before opt.     | 3.00              | 50.00         | 10.00              | 19.60      | 3750               |
| After opt.      | 3.00              | 53.00         | 3.00               | 20.50      | 2862               |

5. Conclusion

In this paper, a rigorous design of MMLOM with both larger thrust and smaller PM volume, using RSM with a QPSO operator. Four candidate design parameters of PM thickness, PM width, stators space and PM axial length are selected by the proposed method. The FEA based on ANSOFT has been used as numerical experiments to provide the response for RSM. The QPSO algorithm is used as a searching tool for design optimization. The optimized 3D FEA results showed that the thrust of MMLOM is increased by 4.6% and the PM volume is reduced by 23.7% compared with the initial model.

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