We assume that dark matter and dark energy satisfy the unified equation of state:

\[ p = B(z) \rho, \]

with \( p = p_{dE}, \rho = \rho_{dm} + \rho_{dE} \), where the pressure of dark matter \( p_{dm} = 0 \) has been taken into account. A special function \( B = -A(1+z)^{-\alpha} \) is presented, which can well describe the evolution of the universe. In this model, the universe will end up with a Big Rip. By further simple analysis, we know other choices of the function \( B \) can also describe the universe but lead to a different doomsday.

Keywords: the unified equation of state; dark matter; dark energy

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1. Introduction

The astronomical observations indicate that the expansion of our universe accelerated. This accelerated expansion is ascribed to a mysterious component with the negative pressure called dark energy that comprises about \( \frac{2}{3} \) of the energy density of the universe. The simplest model for the dark energy is a cosmological constant. There are many more complex models proposed attempting to explain dark energy either as a dynamic substance (for example Quintessence, Phantom) or via some forms of modified gravitational theory perhaps related to extra dimensions and string physics. Now the unified models for dark energy and dark matter have been also discussed, and characteristics of some kinds of models are summarized in Alam’s works.

The research team of A. G. Riess et al. have pinpointed the transition epoch from matter domination to dark energy domination when cosmic expansion began to accelerate and also indicated the transition between the two epochs is constrained to be at \( z_T = 0.46 \pm 0.13 \). It is therefore pressing that an independent model of dark energy can be employed to elaborate the present phenomena and trace back to the past and even forecast the future evolution. To attain this aim, in this paper we present the simplest linear equation of state \( p = B(z) \rho, \) with \( p = p_{dE}, \rho = \rho_{dm} + \rho_{dE} \), where the pressure of dark matter \( p_{dm} = 0 \) has been taken into account. This model can predict a smooth transition from nonrelativistic matter (including both baryons
and dark matter) domination phase to dark energy domination one in a natural way. Directed by this objective, the paper is organized as follows. The central part of the paper is Sec. 2 in which the two constraints in choosing the function $B$ are discussed and a suitable function $B = -\frac{A}{(1+z)^\alpha}$ is proposed. From the function some parameters of the universe are derived. In Sec. 3 the conclusion is put up that only by choosing a suitable function $B$ can the unified equation of state proposed describe the evolution of the universe.

2. Analysis and deduction

It is known that our universe begins with the big bang. The radiation should have been the dominant contribution to the total density of the universe when the redshift $z$ was more than $10^4$ in the early universe. The density of radiation decreased and quickly approached 0 with the evolution of the universe. However, the density of nonrelativistic matter (including both baryons and dark matter) increased and became the dominant contribution to the universe. Subsequently, the density of nonrelativistic matter decreased and that of dark energy increased and at the same time cosmic expansion transited from deceleration to acceleration, the process of which is discussed in this paper.

We assume dark matter and dark energy satisfy the unified equation of state $p = B(z)\rho$ with $p = p_{dm} + p_{dE}; \rho = \rho_{dm} + \rho_{dE}$, where the pressure of dark matter and that of dark energy satisfy respectively: $p_{dm} = 0, p_{dE} = \omega_{dE}\rho_{dE}(\omega_{dE} < 0)$. Because of the above equations and the evolution of our universe, there are some constraints for function $B$ proposed:

(i) Function $B$ should maintain negative value forever and vary in the range $(\omega_{dE}, 0)$.

(ii) For $z > 0$, function $B$ should have been approaching 0 but not equaling to 0 for a long time in the high redshift and be smaller and smaller as redshift becomes low.

In terms of the two constraints proposed above, we give a concrete form of function $B$ as $B = -\frac{A}{(1+z)^\alpha}$, and we can obtain the solutions of some parameters including fractional components, deceleration-acceleration transition, the equation of state of dark energy, which are coincident with the evolution of the universe and consistent with the present astrophysical observations.

For a spatially flat FRW universe, Friedmann equation can be written as:

$$H^2 = \frac{8\pi G}{3}\rho_{tot}. \quad (1)$$

Also the continuity equation:

$$\dot{\rho}_{tot} = -3H(\rho_{tot} + \rho_{tot}), \quad (2)$$

has to be taken into account, where $\rho_{tot}$ is the total energy density of our universe $\rho_{tot} = \rho_b + \rho_{dm} + \rho_{dE} = \rho_b + \rho$, where subscript “$b$” denotes baryons. So $\rho_b$ is the
density of baryons and Eq. (3) is separated into two parts:

\[ \dot{\rho}_b = -3H(p_b + \rho_b), \]  

(3)

\[ \dot{\rho} = -3H(p + \rho). \]  

(4)

If the function \( B(z) = -\frac{A}{(1+z)^\alpha} \), where \( A, \alpha \) are the positive real constants, substituting \( p = B(z)\rho \) to Eq. (4) then

\[ \dot{\rho} = -3H\left( -\frac{A}{(1+z)^\alpha}\rho + \rho \right). \]  

(5)

So the density of dark matter and dark energy evolves with the red shift as

\[ \rho = C(1 + z)^3e^{\frac{3A}{\alpha(1+z)^\alpha}}, \]  

(6)

where \( C \) is an integral constant. Using the constraint condition of \( z = 0 \) makes it satisfy:

\[ \rho_0 = Ce^{\frac{3A}{\alpha}}, \]  

(7)

where \( \rho_0 \) is the present density of dark matter and dark energy and today’s evaluated quantities will hereafter denoted by the label “0”, and

\[ \rho_{c0} = \rho_{tot0} = \rho_0 + \rho_{b0} = \frac{3H_0^2}{8\pi G}, \]  

(8)

is how the present critical energy density is defined. Replacing Eq. (8) in Eq. (7), we obtain:

\[ C = (\rho_{c0} - \rho_{b0})e^{-\frac{3A}{\alpha}}. \]  

(9)

The density of baryons and that of dark matter are respectively recognized as:

\[ \rho_b = \rho_{b0}(1 + z)^3, \]  

(10)

\[ \rho_{dm} = \rho_{dm0}(1 + z)^3. \]  

(11)

So the total density takes the form:

\[ \rho_{tot} = (\rho_{c0} - \rho_{b0})(1 + z)^3e^{\frac{3A}{\alpha(1+z)^\alpha}} - \frac{3A}{\alpha} + \rho_{b0}(1 + z)^3, \]  

(12)

and the density of dark energy can be obtained by Eqs. (10), (11) and (12) as:

\[ \rho_{dE} = (\rho_{c0} - \rho_{b0})(1 + z)^3e^{\frac{3A}{\alpha(1+z)^\alpha}} - \frac{3A}{\alpha} - \rho_{dm0}(1 + z)^3. \]  

(13)

One can express the fractional energy densities \( \Omega_b, \Omega_{dm}, \Omega_{dE} \) as

\[ \Omega_b = \frac{\Omega_{b0}}{(1 - \Omega_{b0})e^{\frac{3A}{\alpha(1+z)^\alpha}} - \frac{3A}{\alpha} + \Omega_{b0}}, \]  

(14)

\[ \Omega_{dm} = \frac{\Omega_{dm0}}{(1 - \Omega_{b0})e^{\frac{3A}{\alpha(1+z)^\alpha}} - \frac{3A}{\alpha} + \Omega_{b0}}, \]  

(15)

\[ \Omega_{dE} = \frac{(1 - \Omega_{b0})e^{\frac{3A}{\alpha(1+z)^\alpha}} - \frac{3A}{\alpha} - \Omega_{dm0}}{(1 - \Omega_{b0})e^{\frac{3A}{\alpha(1+z)^\alpha}} - \frac{3A}{\alpha} + \Omega_{b0}}. \]  

(16)
where $\Omega_{b0} = \frac{\rho_{b0}}{\rho_{c0}}$, $\Omega_{dm0} = \frac{\rho_{dm0}}{\rho_{c0}}$.

Assuming $A = 0.7$ and $\alpha = 1.81$, the fractional energy densities are only the functions of redshift with the prior of $\Omega_{b0} = 0.04$, $\Omega_{dm0} = 0.3$. The assumption is still used in the following calculation.

The results of the three fractional energy densities at the present time are compatible with observations using the priors of $\Omega_{b0} = 0.04$, $\Omega_{dm0} = 0.3$. Meanwhile, according to FIG. 1A we know that the past evolution of baryons, dark matter and dark energy is consistent with what is recognized. The cosmology will end up with a Big Rip in that the universe will be filled with dark energy in future. The conclusion is shared in many literatures (e.g. [3]).

![Figure 1](image_url)

**Fig. 1.** A: The fractional energy densities as the functions of redshift. B: The equation of state of dark energy $\omega_{dE}$ as the function of redshift. The priors $\Omega_{b0} = 0.04$; $\Omega_{dm0} = 0.3$ have been used.

It follows that the equation of state parameter of dark energy can be obtained as:

$$\omega_{dE} = \frac{p_{dE}}{\rho_{dE}} = \frac{A}{(1+z)^\alpha} \cdot \frac{(1 - \Omega_{b0})e^{\frac{3A}{1 + \alpha + z}} - \frac{3A}{1 + \alpha + z} - \Omega_{dm0}}{(1 - \Omega_{b0})e^{\frac{3A}{1 + \alpha + z}} - \frac{3A}{1 + \alpha + z} - \Omega_{dm0}}. \quad (17)$$

Using Eq. (17) we can plot $\omega_{dE}$ as the function of redshift (see FIG. 1B). The equation of state of dark energy is $\omega_{dE0} \approx -1.019$ at present. Tracing back to the past, the larger the redshift is, the more the density of nonrelativistic matter (including both baryons and dark matter) is and the more adjacent to 0 the equation of state of dark energy $\omega_{dE}$ is, i.e., we realize an evolving dark energy with the equation of state being below $-1$ around the present epoch evolved from $\omega_{dE} > -1$ in the past [9]. This is another consistency between our theory and the evolution.

The dominance of the dark energy leads to the acceleration expansion of our universe. The increasing density of dark energy is the reason why expansion of
our universe transited from deceleration to acceleration. Our theory describes the transition. The deduction is as follows:

\[ q = -\frac{\ddot{a}}{a^2} = -\frac{\ddot{a}}{aH^2}, \]  

(18)

where \( q \) is the deceleration parameter. Using Eqs. (1), (6) and (9), the Friedmann equation may be rewritten as

\[ H^2 = \frac{8\pi G}{3}(\rho_c - \rho_b)(1 + z)^3 e^{3A(1+z)^{\alpha}} - \frac{3A}{\alpha} + \frac{8\pi G}{3} \rho_b(1 + z)^3. \]  

(19)

Differentiating Eq. (19) with respect to time and changing variable from the cosmic time to the redshift, we finally obtain:

\[ \frac{\ddot{a}}{a} = -\frac{1}{2}H^2 + \frac{3A(\rho_c - \rho_b)}{2(1 + z)^{\alpha - 3}} e^{\frac{3A}{\alpha}(1+z)^{\alpha}} - \frac{3A}{\alpha}. \]  

(20)

Combining the above two equations, we find that \( q \) satisfies

\[ q = \frac{1}{2} - \frac{3A}{2(1 + z)^{\alpha}} \frac{(1 - \Omega_{b0}) e^{\frac{3A}{\alpha}(1+z)^{\alpha}} - \frac{3A}{\alpha}}{(1 - \Omega_{b0}) e^{\frac{3A}{\alpha}(1+z)^{\alpha}} - \frac{3A}{\alpha} + \Omega_{b0}}. \]  

(21)

From this expression, we get the present deceleration parameter \( q_0 = -0.508 \) according to \( z = 0 \) and the transition redshift \( z_T \approx 0.449 \) according to \( q = 0 \). The deceleration parameter as the function of redshift is shown in FIG. 2.

Fig. 2. The deceleration parameter \( q \) as the function of redshift. The priors \( \Omega_{b0} = 0.04, \Omega_{dm0} = 0.3 \) have been used.

From above deduction, we know the acceleration of our cosmic expansion will be faster and faster when \( B = -\frac{A}{(1+z)^{\alpha}} \) is chosen. As a result of smaller and smaller \( B \), \( q \) and \( \omega_{dE} \) will also be smaller and smaller and even will approach negative infinity from FIG. 3. It is inevitable that the universe will end up with a Big Rip. On the
condition that choosing another function based on the above two constraints and using another symbol: \( B_2 = -0.7 \exp(-z^2) \) and \( (z > -1) \) in order to distinguish the two functions, we can still obtain the solutions coincident with the past and the present observational data because of the semblable curves of the two functions in \( z > 0 \). But the fate of the universe will be different. New function \( B_2 \) is axial symmetry. Smaller \( z \) will lead to larger \( B_2 \) in future which also reflects the tendency of the equation of state of dark energy. Both of the densities of baryons, dark matter will approach small quantities, so we think that the total energy density approximately equal to the density of dark energy. Because of

\[
q = -\frac{\ddot{a}}{a^2} = -\frac{\ddot{a}}{aH^2} = \frac{1}{2} + \frac{3}{2} \frac{p_{\text{tot}}}{\rho_{\text{tot}}},
\]

one obtains

\[
q \approx \frac{1}{2} + \frac{3}{2} B_2.
\]

When \( B_2 > -\frac{1}{3} \), \( q > 0 \), the expansion of the universe will come back to deceleration from acceleration. We don’t discuss the detailed deduction owing to the difficulty of the integral of Gaussian function.

As shown in the above discussion, the results can be obtained which are in agreement with the observational data of the past time and the present time as long as we choose the function \( B \) in terms of the above two analyzed constraints. At the same time the tendency of \( B \) in the range \(-1 < z < 0\) determines the fate of the universe. So it is possible to find a suitable function to describe the evolution of the universe from the early time to the future.
3. conclusion

In summary, this paper considers that the utilization of a unified linear equation of state can describe the history of the universe and its evolution of the future. It is worth highlighting that we don’t simulate function $B$ without foundation but based on some remarkable constraints. Analyzing the constraints of the choice of the function $B$ in terms of the evolution of our universe and choosing the appropriate function $B$ according to the constraints, the evolution of our universe can be obtained in consistence with what is recognized. Both dark matter and dark energy are considered the essential but missing pieces in the cosmic jigsaw puzzle. But the nature of either dark matter or dark energy is currently unknown. We propose this model-independent unified equation of state which can be utilized to forecast the nature of dark matter and dark energy possibly.

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