Heavy quarkonium production in ultraperipheral nuclear collisions

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Abstract

Heavy quarkonium production in ultraperipheral nuclear collisions is described within the QCD dipole formalism. A realistic quarkonium wave functions in the rest frame are calculated solving the Schrödinger equation with a subsequent Lorentz boost to high energy. We rely on several selected $Q\bar{Q}$ potentials, which provide the best description of quarkonium spectra and decay widths, as well as data on diffractive electroproduction of quarkonia on protons. Nuclear effects are calculated with the phenomenological dipole cross sections fitted to DIS data. Higher twist effect related to the lowest $Q\bar{Q}$ Fock component of the photon, as well as the leading twist effects, related to higher components containing gluons, are included. The results for coherent and incoherent photoproduction of charmonia and bottomonia on nuclei are in a good accord with available data from the recent UPC measurements at the LHC. They can also be verified in future experiments at the planned electron-ion colliders.

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I. INTRODUCTION

In comparison to hadro-production of heavy quarkonia states ($V = J/\psi(1S), \psi'(2S), \Upsilon(1S), \Upsilon'(2S), \Upsilon''(3S), \ldots$), where the production mechanism is not well interpreted up to now, the main motivation for their investigation in heavy-ion ultra-peripheral collisions (UPC) at Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) arises from the following reasons:

i) Here the mechanism of almost real photoproduction of heavy quarkonia is well phenomenologically described [1–5] and corresponding theoretical uncertainties are rather well understood [6–8] (a comprehensive review on quarkonium phenomenology can be found in e.g. Refs. [9–11]);

ii) Relatively small size of heavy quarkonia should lead to minimization of uncertainties inherent from the nonperturbative region what, consequently, allows to probe and verify mainly a (semi)perturbative QCD (pQCD) origin of calculated nuclear production amplitudes;

iii) Besides the first data on charmonium production in UPC at RHIC reported by the PHENIX collaboration [12], during last few years a lot of new data have been presented at the LHC by the CMS [13], ALICE [14–16] collaborations at c.m. collision energy $\sqrt{s_{NN}} = 2.76$ TeV, as well as by the LHCb [17] and ALICE [18] collaborations at even higher energy $\sqrt{s_{NN}} = 5.02$ TeV. This can provide us with an additional information about the dynamics of the corresponding photoproduction processes and gives a further possibility to verify our ideas about the onset of various nuclear effects occurring in interactions with nuclei in a wider energy range.

The UPC are associated with sufficiently large relative impact parameter of colliding nuclei exceeding the sum of their radii. Then the electromagnetic field generated by large nuclear charges induces an interaction of quasireal photons (where the photon virtuality $Q^2 \rightarrow 0$) with nuclei followed by a subsequent production of vector mesons (quarkonia). Such a quasi-real diffractive electroproduction is accompanied by two main phenomena affecting the production rate.

The first one, representing the final state absorption of produced quarkonia, is related to the phenomenon known as the color transparency (CT). Within the popular light-front (LF) color dipole approach [1, 2, 4, 19–26], frequently used in the literature, such a diffractive process can be described as a production of the $Q\bar{Q}$ pair by the real or virtual photon. Treating the UPC, the transverse separation of a $Q\bar{Q}$ fluctuation is proportional to $1/m_Q$, where $m_Q$ is the quark mass. In photoproduction of heavy quarkonia off nuclei the large quark mass related to the small sized $Q\bar{Q}$ pair should leads to a small absorption during its propagation through the nucleus. This represents the basic idea of CT. Here the large-sized photon configurations with larger absorption cross sections are filtered out by the medium what is known as the color filtering phenomenon. The estimation of the corresponding length scale controlling the evolution of the $Q\bar{Q}$ wave packet can be obtained in the rest frame of the nucleus from the condition that the relative phase shift between the two lowest levels, $V$ and $V'$ becomes of the order of unity [1, 21]

$$l_f = \frac{2k}{M_{V'}^2 - M_V^2},$$

(1.1)

where $k$ is the photon energy and $M_V$ and $M_{V'}$ are the quarkonium masses in 1$S$ and 2$S$ states, respectively. This scale $l_f$ in Eq. (1.1) is usually called formation length. In the
present paper we assume a very slow expansion of the $Q\bar{Q}$ transverse size relying on large photon energies $k$, in particular in the LHC kinematics. A more detailed description based on the light-front Green function formalism will be presented elsewhere.

Another important source of nuclear suppression is shadowing, which is caused by a phase shift between amplitudes of photoproduction of $V$ on different bound nucleons. When this phase shift reaches unity the interference become destructive, i.e. the amplitudes tend to cancel each other. The corresponding distance is called coherence length (CL), \[ l_c = \frac{2k}{M_V^2} = \frac{s - M^2}{M M_V^2}, \] where $M$ is the nucleon mass.

The CL phenomenon in charmonium production off nuclei has been included via corrections for the finite $l_c$ as is described in Sect. II C. The corresponding factors have been calculated up to now within a simplified Glauber approximation \[27–29\] whose applicability to quarkonium production (especially for $2S$-radially excited states) is questionable. For this reason in the present paper the values of the finite-$l_c$ factors have been obtained for the first time within the rigorous Green function formalism, which includes naturally the CL effects.

Another initial state interaction phenomenon known to cause the nuclear suppression in production of quarkonia is the gluon shadowing (GS). In the present paper, similarly as for quarks discussed above, the GS is treated in terms of the LF QCD dipole approach (see also Sect. II D). It is related to higher Fock components, $|Q\bar{Q} nG\rangle$, of the photon containing gluons \[30\]. In comparison with the lowest $|Q\bar{Q}\rangle$ Fock state the corresponding lifetime of multi-gluon states is shorter \[31\] due to their larger effective masses. This requires a higher photon energy to reach a sufficient onset of the corresponding shadowing corrections. The higher is the Fock state the higher must be the photon energy to manifest its relevance for a shadowing.

Following analyses in Refs. \[6–8\], in the present paper we treat a simple non-photon-like $V \to Q\bar{Q}$ structure in order to avoid $D$-wave admixture excluding so the additional non-specified contribution to the $S$-wave quarkonium electroproduction cross sections.

The advantage of the $S$-wave heavy quarkonia, studied in the present paper, is based on a simple factorization of radial and spin-dependent components of their wave functions. Here the former component is well defined in the $Q\bar{Q}$ rest frame and can be obtained by solving the Schrödinger equation for realistic interaction potentials between $Q$ and $\bar{Q}$ proposed in the literature. In our calculations we choose two of them, power-like potential (POW) \[32, 33\] and Buchmüller-Tye potential (BT) \[34\], which provide the best description of available data on charmonium electroproduction off a nucleon target as was analyzed in Ref. \[7\]. The same reason is related to our choice of the dipole cross section, which represents the important ingredient inherent from the LF color dipole formalism. Here we adopt two popular parametrizations, denoted as KST from Ref. \[35\] and GBW from Refs. \[36, 37\].

Performing the boosting to the LF frame we rely on the widely used procedure \[38\] for the generation of the corresponding LF wave functions. In the spin-dependent component, two-dimensional spinors describing the heavy $Q$ and $\bar{Q}$ in the LF frame are related to those in the rest frame by the transformation known as the Melosh spin rotation \[6–8, 24, 39\]. The onset of spin rotation effects for different LF wave functions, corresponding to various realistic potential models for the $Q$-$\bar{Q}$ interaction in the rest frame, have been analyzed in electroproduction of heavy quarkonia on a nucleon target in Refs. \[6, 7\]. However, in
this work the study of UPC requires the further extended investigation of spin effects in quarkonium production off nuclei.

In this paper we extend our previous studies [27–29] performing a comprehensive analysis of nuclear effects occurring in heavy quarkonium production in UPC. Here we present for the first time how the onset of GS and CL effects is manifested at different rapidities and c.m. collision energies corresponding to RHIC and the LHC experiments. Besides, treating a simple non-photon-like structure of the quarkonium vertex without $D$-wave admixture, we derive new expressions for the nucleon electroproduction cross sections including the Melosh spin transformation and containing an explicit dependence only on the dipole cross section. This allows a simple straightforward generalization of spin effects to nuclear targets, as well as to perform predictions for the cross sections of coherent and incoherent photoproduction of various heavy quarkonium states generated by different $Q - \bar{Q}$ interaction potentials in the $Q\bar{Q}$ rest frame.

The paper is organized as follows. In the next Section we present expressions for calculation of differential cross sections $d\sigma/dy$ corresponding to coherent (elastic) and incoherent (quasi-elastic) heavy quarkonium production in UPC as is described in Sects. II A and II B respectively. Consequently, in Sects. II C and II D we discuss two main effects modifying nuclear suppression in UPC, the corrections for a finite coherence length and the gluon shadowing. The former is calculated for the first time within the LF dipole approach based on the Green function formalism giving results that are substantially different from values based on the vector dominance model (VDM). The next Sect. III is devoted to comparison of model predictions with available data and to analysis of particular nuclear effects in coherent and incoherent quarkonium production in UPC. Finally, the last Sect. IV contains a summary with the main concluding remarks about manifestations and the onset rate of various nuclear effects occurring in UPC.

II. QUARKONIUM PRODUCTION CROSS SECTION IN ULTRAPERIPHERAL COLLISIONS

The large charge $Z$ of heavy nuclei gives rise to strong electromagnetic fields: in a heavy-ion UPC the photon field of one nucleus can produce a photo-nuclear reaction in the other. Then the cross section for the photoproduction of a vector meson $V$ by the Weizsäcker-Williams photons can be written in the rest frame of the target nucleus $A$ as follows:

$$k \frac{d\sigma}{dk} = \int d^2\tau \int d^2b \ n(k, \vec{b} - \vec{\tau}) \frac{d^2\sigma_A(b, s)}{d^2b}.$$  \hspace{1cm} (2.1)

This formula is derived in the one-photon-exchange approximation. Here the variable $\vec{\tau}$ is the relative impact parameter of a nuclear collision and $\vec{b}$ is the impact parameter of the photon-nucleon collision relative to the center of one of the nuclei. Treating the collision of identical nuclei with nuclear radius $R_A$ in UPC, the nuclei interact with each other through their electromagnetic fields, what leads to a condition that the impact parameter $\tau$ should be larger than the sum of the corresponding nuclear radii, i.e. $\tau > 2R_A$.

The variable $n(k, \vec{b})$ in Eq. (2.1) represents the photons flux induced by the projectile nucleus with Lorenz factor $\gamma$ and has the following form,

$$n(k, \vec{b}) = \frac{\alpha_{em} Z^2 k^2}{\pi^2 \gamma^2} \left\{ K_1^2 \left( \frac{bk}{\gamma} \right) + \frac{1}{\gamma^2} K_0^2 \left( \frac{bk}{\gamma} \right) \right\},$$  \hspace{1cm} (2.2)
where $Z$ is the ion charge, $\alpha_{em} = 1/137.036$ is the fine-structure constant, $K_{0,1}$ are the modified Bessel functions and the Lorentz factor $\gamma = 2\gamma_{col}^2$ with $\gamma_{col} = \sqrt{s_N}/2M$. Here $\sqrt{s_N}$ is c.m. collision energy. The first and the second term in Eq. (2.2) corresponds to the flux of photons transversely and longitudinally polarized to the ion direction, respectively. The former photons flux dominates in ultra-relativistic collisions with $\gamma \gg 1$. Consequently, in heavy-ion UPC at RHIC and the LHC one can safely neglect the second term in Eq. (2.2) treating the photons as almost real due to a very small virtuality, $-q^2 = Q^2 < 1/R_A^2$.

A. Coherent production

To calculate the cross sections for coherent (coh) quarkonium production ($\gamma A \rightarrow VA$) we use the light-front dipole approach [1], which has been applied to describe $J/\psi$ photoproduction off nucleons [6, 7, 24] and nuclei [5, 27]. In this approach, assuming sufficiently large photon energies, corresponding to the most of kinematic regions studied in the present paper, when the CL $l_c (1.2)$ exceeds substantially the nuclear radius $R_A$, $l_c \gg R_A$, the corresponding nuclear cross sections take a simple asymptotic form,

$$
\frac{d^2\sigma_{coh}^A(s,b)}{d^2b} \bigg|_{l_c \gg R_A} = \left| \int d^2r \int_0^1 d\alpha \Psi_V^*(\vec{r},\alpha) \left( 1 - \exp \left[ -\frac{1}{2} \sigma_{q\bar{q}}(r,s) T_A(b) \right] \right) \Psi_\gamma(\vec{r},\alpha) \right|^2.
$$

We rely on the optical approximation, which is rather accurate for heavy nuclei. Here $T_A(b) = \int_{-\infty}^{\infty} dz \rho_A(b,z)$ is the nuclear thickness function normalized as $\int d^2b T_A(b) = A$, where $\rho_A(b,z)$ is the nuclear density function of realistic Wood-Saxon form with parameters taken from [40]; $\Psi_V(r,\alpha)$ is the LF wave function for heavy quarkonium, and $\Psi_\gamma(r,\alpha)$ is the LF distribution or the wave function of the $Q\bar{Q}$ Fock component of the quasi-real (transversely polarized) photon, where the $Q\bar{Q}$ fluctuation (dipole) has the transverse size $\vec{r}$ and the variable $\alpha = p_{Q\bar{Q}}^+/p_+^+$ is the boost-invariant fraction of the photon momentum carried by a heavy quark (or antiquark).

The universal dipole-nucleon total cross section $\sigma_{q\bar{q}}(r,s)$ depends on transverse dipole separation $r$ and c.m. energy squared $s = M_V \sqrt{s_N} \exp[y]$. Energy dependence of the dipole cross section can be alternatively included also via variable $x = M_V^2/W^2 = M_V \exp[-y]/\sqrt{s_N}$, where $y$ is the rapidity variable.

Notice that the coherent cross section, Eq. (2.3), is different from the usual Glauber expression [41] due to presence of the dipole cross section [42]. It effectively includes the Gribov inelastic shadowing corrections [43, 44] in all orders for the $Q\bar{Q}$ Fock component of the photon.

B. Incoherent production

Besides "elastic" coherent photoproduction $\gamma A \rightarrow VA$, where the nucleus remains intact, the vector meson can be produced in a quasi-elastic process $\gamma A \rightarrow VA^*$, where the nucleus is excited and decays to fragments. Important is that additional meson production is excluded. In this case one can sum over different products of nuclear excitation and employ
the conditions of completeness. Of course one channel of elastic photoproduction must be subtracted. It is instructive to see the result within the Glauber approximation \cite{45},

\[
\frac{d^2\sigma_A^{inc}(s, b)}{d^2b} \bigg|_{t_c > R_A} \propto \exp \left[ -\sigma_{V} T_A(b) \right] - \exp \left[ -\sigma_{tot} T_A(b) \right] = \exp \left[ -\sigma_{tot} T_A(b) \right] \left\{ \exp \left[ -\sigma_{el} T_A(b) \right] - 1 \right\} . \tag{2.4}
\]

Here the inelastic V-N cross section \(\sigma_{V} = \sigma_{tot} - \sigma_{el}^{V}\), where the elastic cross section

\[
\sigma_{el} \approx \frac{(\sigma_{tot})^2}{16\pi B^V} \tag{2.5}
\]

and \(B^V\) is the slope of the differential elastic \(V-N\) cross section.

The cross section of incoherent photoproduction has the form, analogous to (2.4), but with additional integrations over the dipole size (see derivation in Sect. VII of Ref. \cite{46}),

\[
\frac{d^2\sigma_A^{inc}(s, b)}{d^2b} \bigg|_{t_c > R_A} = \int d^2r_1 \int_0^1 d\alpha_1 \Psi_V^*(\vec{r}_1, \alpha_1) \Psi_\gamma(\vec{r}_1, \alpha_1) \exp \left[ -\frac{1}{2} \sigma_{qq}(r_1, s) T_A(b) \right] \times \int d^2r_2 \int_0^1 d\alpha_2 \Psi_V^*(\vec{r}_2, \alpha_2) \Psi_\gamma(\vec{r}_2, \alpha_2) \exp \left[ -\frac{1}{2} \sigma_{qq}(r_2, s) T_A(b) \right] \times \left\{ \exp \left[ \frac{\sigma_{qq}(r_1) \sigma_{qq}(r_2)}{16\pi B(s)} \right] - 1 \right\} . \tag{2.6}
\]

The elastic cross section of a heavy quarkonium on a nucleon is rather small and the exponential in the last row of Eq. (2.6) can be expanded. Then we arrive at a simple result \cite{27},

\[
\frac{d^2\sigma_A^{inc}(s, b)}{d^2b} \bigg|_{t_c > R_A} \approx \frac{T_A(b)}{16\pi B(s)} \int d^2r \int_0^1 d\alpha \Psi_V^*(\vec{r}, \alpha) \Psi_\gamma(\vec{r}, \alpha) \sigma_{qq}(r, s) \times \exp \left[ -\frac{1}{2} \sigma_{qq}(r, s) T_A(b) \right] \bigg|^2 = \frac{T_A(b)}{16\pi B(s)} \int d^2r \int_0^1 d\alpha \Psi_V^*(\vec{r}, \alpha) \Sigma_A^{inc}(r, s, b) \Psi_\gamma(\vec{r}, \alpha) \bigg|^2 . \tag{2.7}
\]

The real part of the \(\gamma N \rightarrow VN\) amplitude in Eqs. (2.3) and (2.7) is included via the following replacement \cite{44,47,48},

\[
\sigma_{qq}(s, r) \Rightarrow \sigma_{qq}(s, r) \left( 1 - i \frac{\pi}{2} \frac{\partial}{\partial \ln s} \ln \sigma_{qq}(s, r) \right) . \tag{2.8}
\]

Treating the non-photon-like structure of the \(V \rightarrow QQ\) vertex \cite{6,7,27,29} the Melosh spin transformation is incorporated performing the following substitutions in Eqs. (2.3) and (2.7),

\[
\Sigma_A^{coh}(r, s, b) \Rightarrow \Sigma_A^{coh}(r, s, b) \left[ \Sigma^{(1)}(r, \alpha) + \Sigma^{(2)}(r, \alpha) \right] \]

\[
\Sigma_A^{inc}(r, s, b) \Rightarrow \Sigma_A^{inc}(r, s, b) \left[ \Sigma^{(1)}(r, \alpha) + \Sigma^{(2)}(r, \alpha) \right] . \tag{2.9}
\]

\[
\text{\[8\]}\]
as a result of the straightforward generalization from the nucleon amplitude $\gamma N \rightarrow VN$ implementing replacements $\sigma_{q\bar{q}} \Rightarrow \Sigma_{coh}^{inc}$ and $\sigma_{q\bar{q}} \Rightarrow \Sigma_{inc}^{coh}$, where

$$\Sigma^{(1)}(r, \alpha) = N K_0(m_Q r) \int_0^\infty dp_T p_T J_0(p_T r) \Psi_V(\alpha, p_T) \left[ \frac{m_Q^2 (m_L + m_T) + m_L m_T^2}{m_T (m_L + m_T)} \right]$$

and

$$\Sigma^{(2)}(r, \alpha) = \frac{N}{2} K_1(m_Q r) \int_0^\infty dp_T p_T^2 J_1(p_T r) \Psi_V(\alpha, p_T) \left[ \frac{m_Q^2 (m_L + 2m_T) - m_T m_L^2}{m_Q m_T (m_L + m_T)} \right].$$

Here $N = Z_Q \sqrt{2 N_c \alpha_{em}/2 \pi}$, where the factor $N_c = 3$ represents the number of colors in QCD, $Z_Q$ is the electric charge of the heavy quark ($Z_c = 2/3$ and $Z_b = 1/3$ for the production of charmonia and bottomonia, respectively), $J_{0,1}$ and $K_{0,1}$ are the Bessel functions of the first kind and the modified Bessel functions of the second kind, respectively. The variables $m_{T,L}$ in above formulas have the following form,

$$m_T = \sqrt{m_Q^2 + p_T^2}, \quad m_L = 2 m_Q \sqrt{\alpha(1 - \alpha)}.$$  \hspace{1cm} (2.12)

In comparison with the results from Ref. [27], the new form of Eqs. (2.9)-(2.11) does not require to perform the so called “resummation procedure” proposed in [27] in order to include properly the spin rotation effects in nuclear photoproduction cross sections (2.3) and (2.7). However, it is unclear whether these effects have been incorporated properly in Ref. [49] since the nucleon amplitude presented there depends also on a derivative of $\sigma_{q\bar{q}}$ and the corresponding explanation of adaptability of spin effects to nuclear targets is missing.

In our numerical calculations, following the results from Refs. [6, 7, 27–29], we take the charm and the bottom quark masses corresponding to the values from realistic phenomenological models for the $Q\bar{Q}$ interaction potential, such as POW and BT, used in our analysis. Similarly, the LF quarkonium wave functions $\Psi_V(\alpha, r)$, after Lorentz boosting procedure from Ref. [38], are the counterparts of their nonrelativistic form generated by these potentials in the $Q\bar{Q}$ rest frame.

C. Corrections for a finite coherence length

As was already mentioned above, the Green function approach allows to include directly the effects of quantum coherence without any restrictions for the coherence length, Eq. (1.2). However, as an alternative and a more simple way, instead of such a complicated method one can use expressions for nuclear cross sections in the limit of long CL, $l_c \gg R_A$ (see Eqs. (2.3) and (2.7)), and then provide additional corrections for a finite CL when $l_c \lesssim R_A$. Such an incorporation of finite-CL effects via the effective correction factors (form factors) $F(s, l_c)$ has been suggested in Ref. [50] and employed in Ref. [27] for calculations of charmonium photoproduction off nuclei. The corresponding form of factors $F(s, l_c)$ for the coherent and
incoherent scattering is the following:

\[ F^{\text{coh}}(s, l_c) = \int d^2 b \int_{-\infty}^{\infty} dz \rho_A(b, z) F_1(s, b, z) e^{iz/l_c} \mid \left( \ldots \right) \mid_{l_c \to \infty}, \]  
\[ F^{\text{inc}}(s, l_c) = \int d^2 b \int_{-\infty}^{\infty} dz \rho_A(b, z) \left| F_1(s, b, z) - F_2(s, b, z, l_c) \right|^2 / \left( \ldots \right) \mid_{l_c \to \infty}, \]  
\[ F_1(s, b, z) = \exp \left( -\frac{1}{2} \sigma_{VN}(s) \int dz' \rho_A(b, z') \right), \]  
\[ F_2(s, b, z, l_c) = \frac{1}{2} \sigma_{VN}(s) \int_{-\infty}^{z} dz' \rho_A(b, z') F_1(s, b, z') e^{-i(z-z')/l_c}, \]  

where the variable \( \sigma_{VN}(s) \equiv \sigma_{VN}^{\text{tot}}(s) \) represents the quarkonium-nucleon total cross section introduced in the previous Sect. II B. Then the correction to the finite coherence length is usually incorporated by multiplying the nuclear cross sections in the limit of infinite coherence length, Eqs. (2.13) and (2.14), by the corresponding \( l_c \)-correction factors (form factors) [27],

\[ d^2 \sigma_A^{\text{coh}}(s, b) = d^2 \sigma_A^{\text{coh}}(s, b) \mid_{l_c \to \infty}, \]  
\[ d^2 \sigma_A^{\text{inc}}(s, b) = d^2 \sigma_A^{\text{inc}}(s, b) \mid_{l_c \to \infty}, \]

The quarkonium-nucleon cross sections \( \sigma_{VN}(s) \) in Eqs. (2.15) and (2.16) have been calculated in Ref. [24] for the 1S and 2S charmonium states \( J/\psi \) and \( \psi' \) including also the Melosh spin effects. However, here we present the alternative and new expressions for \( \sigma_{VN}(s) \) based on Eqs. (2.10) and (2.11),

\[ \sigma_{VN}(s) = \frac{1}{d\alpha} \int d^2 r \sigma_{qq}(r, s) \left[ \Xi^{(1)}(r, \alpha) + \Xi^{(2)}(r, \alpha) \right]^2, \]  

where

\[ \Xi^{(1)}(r, \alpha) = \int_{0}^{\infty} dp_T p_T J_0(p_T r) \Psi_V(\alpha, p_T) \left[ m_Q^2 (m_L + m_T) + m_T p_T^2 \right] \left[ m_Q m_T (m_L + m_T) \right], \]  
\[ \Xi^{(2)}(r, \alpha) = \int_{0}^{\infty} dp_T p_T J_1(p_T r) \Psi_V(\alpha, p_T) \left[ \frac{p_T}{2} m_Q^2 (m_L + 2m_T) - m_T m_L^2 \right] \left[ m_Q m_T (m_L + m_T) \right]. \]
As an example, using quarkonium wave functions generated by the BT potential and adopting KST and GBW models for the dipole cross section, we plot in Fig. 1 by the solid lines the quarkonium-nucleon total cross sections for various quarkonium states. Our calculations are based on Eq. (2.19), which includes the spin rotation effects. Here the onset of such effects is tested by comparing with results without any spin rotation depicted by the dashed lines. One can see that the Melosh spin transformation manifests itself only for the \( J/\psi \) production, enhancing the corresponding charmonium-nucleon cross section by about 10%. For other quarkonium states the onset of spin effects is very weak and can be neglected.

![Figure 1](image.png)

**TABLE I:** Values of parameters \( \sigma_0 \) and \( \Delta \) for parametrizations Eq. (2.22) of the quarkonium-nucleon cross sections calculated according to Eqs. (2.19)-(2.21) using quarkonium wave functions determined from the BT potential, as well as the KST and GBW parametrizations for the dipole cross section.

| Quarkonium State | \( \sigma_0 \) (mb): KST | \( \Delta \): KST | \( \sigma_0 \) (mb): GBW | \( \Delta \): GBW |
|-----------------|----------------|-------------|----------------|-------------|
| \( J/\psi(1S) \) | 6.01 | 0.201 | 5.44 | 0.179 |
| \( \psi'(2S) \) | 16.35 | 0.116 | 11.99 | 0.095 |
| \( \Upsilon(1S) \) | 1.92 | 0.251 | 1.10 | 0.238 |
| \( \Upsilon'(2S) \) | 7.77 | 0.187 | 4.68 | 0.164 |

The quarkonium-nucleon total cross sections \( \sigma_{VN}(s) \) calculated from Eq. (2.19) can be also parametrized as suggested in Ref. [24]. In Tab. I we present the values of parameters \( \sigma_0 \) and \( \Delta \) inherent from the following parametrization of \( \sigma_{VN}(s) \),

\[
\sigma_{VN}(s) = \sigma_0 \cdot \left( \frac{s}{s_0} \right)^\Delta, \quad s_0 = 1000 \text{ GeV}^2, \quad (2.22)
\]
where the parameters are averaged over the energy interval $10^2 \text{ GeV}^2 < s < 10^5 \text{ GeV}^2$ and $10^3 \text{ GeV}^2 < s < 10^6 \text{ GeV}^2$ for charmonium and bottomonium states, respectively. Differences between values of these parameters related to KST and GBW dipole models can be treated as a measure of the theoretical uncertainty in determination of their magnitudes.

From known values of $\sigma_{VN}(s)$ one can calculate the $l_c$-correction factors $F(s)$ using Eqs. (2.13)-(2.16). The corresponding values of $F(s)$ for coherent and incoherent $J/\psi$ (thick lines) and $\psi'$ (thin lines) photoproduction on the gold target are depicted by dashed lines on left panels of Fig. 2 as function of the square of c.m. energy $s$. Here the top and bottom panel corresponds to calculations using KST and GBW parametrization for the dipole cross section, respectively. The analogous results for $F(s)$ but for photoproduction of $\Upsilon$ and $\Upsilon'$ are depicted on right panels of the same Fig. 2.

Here we would like to emphasize that the above simple estimation of corrections for a finite coherence length, based on VDM, can be applied with a reasonable accuracy only for photoproduction of $\rho$ mesons off nuclei (for the review of photo- and electroproduction of vector mesons and VDM see Ref. [41], for example). For the heavy quarkonium production (especially for production of $2S$-radially excited states), however, it can not provide reliable results (see estimations in Ref. [51]). For this reason, it is worth switching to a more rigorous quantum-mechanical description of factors $F^{coh}$ and $F^{inc}$ within the sophisticated
Green function formalism. Here the corresponding functions \( F_1 \) and \( F_2 \) in Eqs. (2.13) and (2.14) have the following form,

\[
F_1(s, b, z) = \int_0^1 d\alpha \int d^2r_1 d^2r_2 \Psi^*_V(\vec{r}_2, \alpha) G_{qq}(z', \vec{r}_2; z, \vec{r}_1) \sigma_{qq}(r_1, s) \Psi_V(\vec{r}_1, \alpha) \bigg|_{z' \to \infty} \tag{2.23}
\]

\[
F_2(s, b, z) = \frac{1}{2} \int_{-\infty}^z dz_1 \rho_A(b, z_1) \int_0^1 d\alpha \int d^2r_1 d^2r_2 d^2r \Psi^*_V(\vec{r}_2, \alpha)
\]

\[
\times G_{qq}(z' \to \infty, \vec{r}_2; z, \vec{r}) \sigma_{qq}(\vec{r}, s) G_{qq}(z, \vec{r}; z_1, \vec{r}_1) \sigma_{qq}(\vec{r}_1, s) \Psi_V(\vec{r}_1, \alpha), \tag{2.24}
\]

where \( \Psi_v(\vec{r}, \alpha) = K_0(qr) \) and the Green function \( G_{qq}(z', \vec{r}_2; z, \vec{r}_1) \) describes the propagation of an interacting \( QQ \) pair in a nuclear medium between points with longitudinal coordinates \( z \) and \( z' \) and with initial and final separations \( \vec{r}_1 \) and \( \vec{r}_2 \). Here, for simplicity, we took the quadratic form for the dipole cross section \( \sigma_{qq}(r, s) = C(s) r^2 \) and for the LF \( QQ \) interaction potential in the evolution equation for the Green function (see Ref. [21], for example), as well as the constant nuclear density \( \rho_A(b, z) = \rho_0 \Theta(R_A^2 - b^2 - z^2) \). For the LF quarkonium wave functions we used the following Gaussian shape [21] for the \( 1S \) and \( 2S \) states,

\[
\Psi_V(r, \alpha) = C_V a^2(\alpha) f(\alpha) \exp\left[-\frac{1}{2} a^2(\alpha) r^2\right] \tag{2.25}
\]

\[
\Psi_{V'}(r, \alpha) = C_{V'} a^2(\alpha) f(\alpha) \exp\left[-\frac{1}{2} a^2(\alpha) r^2\right] \left\{1 + 4h(\alpha) - \frac{\beta}{2} 2a^2(\alpha) r^2\right\}, \tag{2.26}
\]

where

\[
f(\alpha) = \exp\left[-h(\alpha)\right] = \exp\left[-\frac{m_Q^2}{2a^2(\alpha)} + \frac{4\alpha(1-\alpha)m_Q^2}{2a^2(\alpha)}\right], \tag{2.27}
\]

and the parameter \( \beta \), controlling the position of the node, has been determined from the orthogonality condition \( \int d^3r \, d\alpha \Psi_V(r, \alpha) \, \Psi_{V'}(r, \alpha) = \delta_{VV'} \). We have found \( \beta = 0.908 \) and 0.963 for production of \( \psi' \) and \( \Upsilon' \), respectively. The function \( a^2(\alpha) = 2\alpha(1-\alpha) m_Q \omega \), where the oscillatory frequency \( \omega = (M_{V'}^2 - M_{V}^2)/2 \approx 0.3 \text{ GeV} \). This allowed to obtain an explicit analytical harmonic oscillatory form for the Green function [52], what substantially simplified the calculations of the \( l_c \)-correction factors \( F^{\text{coh}} \) and \( F^{\text{inc}} \).

The Fig. 2 clearly shows that more accurate calculations of factors \( F^{\text{coh}} \) and \( F^{\text{inc}} \) within a rigorous Green function formalism (solid lines) are different from the simplified standard estimations based on the VDM (dashed line). For the \( 1S \)-quarkonium states the variation between both results is not very large. Not so for the \( 2S \)-radially excited quarkonia where the latter approximation cannot be applied anymore for calculation of the \( l_c \)-correction factors. The more detailed comparison of both approaches in heavy quarkonium production will be presented elsewhere [53].

One can see from Fig. 2 that the effects of a finite CL are effective for the energy region \( s \lesssim 10^3 \text{ GeV}^2 \) and \( s \lesssim 10^4 \text{ GeV}^2 \) in production of charmonia and bottomonia, respectively. This is a direct consequence of the CL dependence on a quarkonium mass as given by Eq. (1.2). The Fig. 2 also demonstrates that a contraction of the CL toward smaller values of \( s \) leads to a significant reduction of \( J/\psi \) resp. \( \Upsilon \) coherent photoproduction cross sections. However, the corresponding cross sections for the incoherent process are enhanced only slightly, by \( \sim 20 \div 30\% \) resp. \( \sim 7 \div 12\% \).
For the 2S-radially excited quarkonia the onset of the finite-$l_c$ effects, generated by the Green function formalism, is significantly different from VDM, especially in the incoherent case, as a direct consequence of the nodal structure of their wave functions. Such the node effect is stronger for $\psi'(2S)$ compared to $\Upsilon'(2S)$ state leading thus to much complicated non-monotonic behaviour of the factor $F_{\text{inc}}(s)$ towards small values of $s$. On the other hand, a stronger energy dependence from the region of small $Q\bar{Q}$ transverse separations below the node position compared to large $Q\bar{Q}$ dipole sizes above the node position causes a weakening of the node effect with the energy resulting thus in a gradual convergence of factors $F_{\text{inc}}^{V'}(s)$ to values of $F_{\text{inc}}^{V}(s)$ towards large $s$.

Note that authors in the recent paper [49] used the above presented Glauber approximation for calculations of $F^{\text{coh}}$ and $F^{\text{inc}}$ repeating only our previous studies [27–29, 50]. As we have already mentioned above, such an approximation cannot be applied properly for the photo- and electro-production of heavy quarkonia off nuclei. Moreover, these form factors have been calculated for values of the CL which are not acquired in the target rest frame (see Eqs. (3.1) and (3.4) in Ref. [49]). This cannot lead to a correct and proper inclusion of the finite-CL effects in calculations and, consequently, cannot provide a correct predictions for nuclear production cross sections in regions of large positive resp. negative rapidities. Besides, the corresponding analysis itself of the onset of such effects as function of rapidity and collision energy is missing. Especially, their strong onset is expected in the RHIC kinematic region, which is also excluded in Ref. [49] from a theoretical study of quarkonium production in UPC. Here, apart from the question about a validity of the Glauber approximation for estimation of factors $F^{\text{coh}}$ and $F^{\text{inc}}$, the corresponding analysis of finite-CL effects is based on the lack of calculational details for the total quarkonium-nucleon cross section (without explaining how effects of the Melosh spin rotation have been included), which is an important ingredient of $l_c$-correction form factors (see Eqs. (2.15) and (2.16)).

D. Gluon shadowing

Leading twist gluon shadowing (GS) was introduced within the dipole representation in [21] and applied to photoproduction of vector mesons on nuclei in Refs. [5, 22]. Such effects are generated by higher Fock components of the photon containing besides the $Q\bar{Q}$ pair also gluons, i.e. $|Q\bar{Q}G\rangle$, $|Q\bar{Q}2G\rangle$ ... $|Q\bar{Q}nG\rangle$. Since the dipole cross section $\sigma_{q\bar{q}}(r,s)$ depends at small dipole sizes $r$ on the gluon distribution in the target, nuclear shadowing of the gluon distribution is expected to reduce $\sigma_{q\bar{q}}(r,s)$ in a nuclear reaction relative to the one on the nucleon,

$$\sigma_{q\bar{q}}(r,s) \Rightarrow \sigma_{q\bar{q}}(r,s) \cdot R_G(s, \mu^2, b).$$  (2.28)

Here the gluon shadowing factor $R_G$, defined as the nuclear-to-nucleon ratio of gluon densities, $R_G(s, \mu^2, b) = G_A(s, \mu^2, b)/(A G_A(s, \mu^2))$, is probed at the factorisation scale $\mu^2 \sim \tau M_V^2 = B/r_s^2$, where $\tau = B/Y^2$. The large scale factor $B \approx 10$ has been estimated in Ref. [54] and the factor $Y$ is related to the scanning radius $r_S = 2$ as $Y = r_s M_V$. The factors $Y$ have been estimated in Ref. [3] with corresponding values as presented in Tab. II for various quarkonium states using wave functions generated by the BT and POW potential.

Within the LF dipole formalism based on the path integral technique we calculate the gluon shadowing correction corresponding to the lowest Fock component $|q\bar{q}G\rangle$ containing only one gluon [51, 21, 27, 35, 55]. Consequently, eikonalization of the $R_G$ factor [56] by
Table II: Values of the factor $Y$ for various quarkonium states with wave functions generated by the BT and POW potential.

|       | $J/\psi (1S)$ | $\psi' (2S)$ | $\Upsilon (1S)$ | $\Upsilon' (2S)$ |
|-------|---------------|---------------|-----------------|------------------|
| $Y: BT$ | 5.3           | 4.7           | 5.7             | 5.3              |
| $Y: POW$ | 5.8           | 5.0           | 6.0             | 5.7              |

Eq. (2.28) includes effectively the effect of higher multi-gluon Fock states of the photon simulating thus the interference effects in a nucleus (for alternative estimation of gluon shadowing effects in charmonium production in UPC see [57, 58], for example).

As an example, using the KST dipole model for $\sigma_{q\bar{q}}$, we plot in Fig. 3 the magnitude of the gluon shadowing factor $R_G$ for $J/\psi$ and $\Upsilon$ photoproduction off lead target. Here we show that the GS rises with c.m. energy $s$ at several fixed values of the nuclear path length $L = 1, 3, 5, 10$ and $14$ fm.

In comparison to $J/\psi$ photoproduction, the onset of GS effects is much weaker for $\Upsilon$ due to much larger factorization scale related to the bottomonium mass.

### III. MODEL PREDICTIONS VS AVAILABLE DATA

In order to calculate the nuclear cross sections for the incoherent (quasi-elastic) process one should know the slope parameter for the elastic process $\gamma N \rightarrow VN$. Here we rely on the standard Regge form,

$$B_{J/\psi}(s) = B_0 + 2 \alpha'(0) \ln \left( \frac{s}{s_0} \right), \quad (3.1)$$

where the parameters $\alpha' = 0.171 \text{GeV}^{-2}$, the slope of the Pomeron trajectory, and $B_0 = 1.54 \text{ GeV}^{-2}$ were fitted in [7] to data on $J/\psi$ photoproduction with $s_0 = 1 \text{ GeV}^2$. 

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The slope for 1S-bottomonium photoproduction was fitted to data in [7] and found to have a smaller value than for $J/\psi$, $B_T(s) \approx B_{J/\psi}(s) - 1 \text{GeV}^{-2}$.

For production of 2S-radially excited bottomonium states, the node effect is negligibly small and one can safely use the same magnitudes of the slope parameter for both 1S and 2S states, i.e. $B_T(s) \sim B_{J/\psi}(s)$. Not so for production of radially excited charmonia where the difference of diffraction slopes $\Delta_B = B_{J/\psi}(s) - B_{\psi'}(s)$ cannot be neglected. Here we adopt a parametrization of the factor $\Delta_B(s)$ as function of c.m. energy $s$ from Ref. [7] (see also Ref. [59]).

We calculated the coherent and incoherent heavy quarkonium photoproduction according to Eq. (2.1). Here we included additionally also a possibility that the photo-nuclear reaction can be also initiated by the photon formed from the second nucleus of the colliding nuclei, performing the replacement in Eq. (2.1) $y \Rightarrow -y$. The corresponding nuclear cross sections are calculated in the limit of large photon energy (when the CL (1.2) exceeds substantially the nuclear radius $R_A$) using Eqs. (2.3) and (2.7).

For the dipole cross section inherent in nuclear cross sections we adopt two phenomenological parametrizations of a saturated form: KST from Ref. [35] and GBW from Refs. [36, 37]. Here, at small c.m. energies squared $s$ these cross sections can be scanned at large values of $x = M^2_{Q\bar{Q}}/s$, where the dipole model was modified including quark distributions in the limit $x \to 1$. Similarly as was done in Ref. [60] the large-$x$ quark distributions have been taken into account multiplying the structure functions by the factor $(1 - x)^7$, which follows from the see quark counting rules.

The quarkonium wave functions in the $Q\bar{Q}$ rest frame are obtained as a solution of the Schrödinger equation for two distinct realistic $Q - \bar{Q}$ interaction potentials, power-like (POW) [32, 33] and Buchmüller-Tye (BT) [34], which lead to the best description of data on charmonium electroproduction off nucleons as was investigated in Ref. [7]. This requires to perform the Lorentz boost of their radial parts to the LF frame. Here we used the standard prescription from Ref. [35]. The spin-dependent part is a subject to the Melosh spin transformation as presented in Refs. [6, 7, 24], which contains also details about the photon and quarkonium wave functions. Such spin transformation has been incorporated in calculations of nuclear cross section via Eqs. (2.9)-(2.11).

We include in our predictions two main phenomena affecting the nuclear cross sections: the gluon shadowing and the finite-CL corrections. Whereas the former are described in Sect. II D and dominates at large photon energies, the latter prefer smaller energies (when the CL of the short-lived $Q\bar{Q}$ fluctuations is comparable with the nuclear radius) and are included via $l_c$-correction factors calculated within a rigorous Green function formalism as described in Sect. II C. Our predictions for the differential cross sections $d\sigma/dy$ of quarkonium production in UPC as function of the rapidity $y$ have been performed for one RHIC $\sqrt{s_N} = 200$ GeV and two different LHC c.m. collision energies $\sqrt{s_N} = 2.76$ and 5.02 TeV.

In Fig. 4 we present our results for the rapidity $y$ distributions of coherent (left panels) and incoherent (right panels) charmonium photoproduction in UPC obtained for $\sqrt{s_N} = 200$ GeV (top panels), $\sqrt{s_N} = 2.76$ TeV (middle panels) and $\sqrt{s_N} = 5.02$ TeV (bottom panels). Calculations have been performed for charmonium wave functions generated by two distinct $Q\bar{Q}$ potentials, POW (dashed lines) and BT (solid lines). For the dipole cross sections $\sigma_{q\bar{q}}$ we adopted two different parameterizations, KST (thick lines) and GBW (thin lines). Here the model predictions are tested by the LHC data from the CMS [13] and ALICE [14,16] collaborations at c.m. collision energy $\sqrt{s_N} = 2.76$ TeV, as well as by the LHCb [17] and ALICE [18] data at $\sqrt{s_N} = 5.02$ TeV.
FIG. 4: Rapidity distributions of coherent (left panels) and incoherent (right panels) charmonium photoproduction in UPC at RHIC collision energy $\sqrt{s_N} = 200$ GeV (top panels) and at LHC energies $\sqrt{s_N} = 2.76$ TeV (middle panels) and $\sqrt{s_N} = 5.02$ TeV (bottom panels). The nuclear cross sections are calculated with charmonium wave functions generated by the POW (dashed lines) and BT (solid lines) potentials and with KST (thick lines) and GBW (thin lines) models for the dipole cross section. The data are taken from CMS [13], ALICE [14–16, 18] and LHCb [17] collaborations.
One can see from Fig. 4 that values of $d\sigma/dy$ are strongly correlated with the shape of quarkonium wave functions determined from various $Q - \bar{Q}$ interaction potentials. Whereas in charmonium production in UPC the POW (dashed lines) and BT (solid lines) models lead to rather different predictions for $d\sigma/dy$, in the bottomonium case the both models give very similar results (see Fig. 5), what is in correspondence with our previous studies of quarkonium electroproduction off protons. However, it is in contrast with results from Ref. [49], where a rather weak resp. strong sensitivity of $d\sigma/dy$ to the shape of charmonium resp. bottomonium wave functions has been predicted. Such results have been achieved only at the cost of universal heavy quark masses $m_c = 1.4$ GeV and $m_b = 4.75$ GeV for all $Q - \bar{Q}$ interaction potentials used in the analysis. Because of a strong sensitivity of quarkonium cross sections $\sigma(\gamma N \rightarrow VN)$ to the values of the quark mass as analysed in Ref. [7], the above mentioned artificial and universal values for $m_c$ and $m_b$, that are different from those extracted from various distinct potential models, cannot provides us with proper and consistent predictions for nuclear phenomena occurring in quarkonium production in UPC.

The experimental data on production of radially excited heavy quarkonia in UPC are very scanty. The ALICE collaboration [16] has been measured $d\sigma/dy$ for coherent production of $\psi'(2S)$ at $y = 0$ as is depicted in the left panel of Fig. 5 together with our predictions. One can see a reasonable agreement for the both $Q - \bar{Q}$ interaction potentials, as well as for the both models of the dipole cross section. For completeness, the right panel of Fig. 5 shows predicted values of $d\sigma/dy$ for the incoherent production of $\psi'(2S)$ in UPC.

The next Fig. 6 represents analogous predictions as Fig. 4 but for production of bottomonia in UPC. Here the KST and GBW parametrizations give very similar results in a correspondence with analysis of the process $\gamma N \rightarrow \Upsilon N$ in Ref. [7]. Differences in predictions using the KST and GBW models for $\sigma_{q\bar{q}}(r)$ can be treated as a measure of the underlined theoretical uncertainty.

The next Fig. 7 demonstrates how the onset of particular nuclear effects is manifested in distributions $d\sigma/dy$ at RHIC (top panels) and the LHC (bottom panels) energies in coherent production of charmonia (left panels) and bottomonia (right panels) in UPC, respectively. Here the dotted lines represent our calculations in the standard high energy limit, including only the maximal shadowing generated by the lowest long-lived $Q\bar{Q}$ Fock component of the

![Figure 5: The same as Fig. 4 but for the $\psi'(2S)$ production in UPC. The experimental value is taken from the ALICE [16] collaboration.](image-url)
FIG. 6: The same as Fig. 4 but for the bottomonium production in UPC.

photon, without any other corrections and effects occurring in interactions with nuclei. The dashed and solid lines incorporate additionally and subsequently the spin rotation effects and corrections to a finite CL with the gluon shadowing, respectively. Here our predictions correspond to the KST model for the dipole cross section.

Since in the RHIC kinematic region the gluon shadowing effects can be neglected, the
FIG. 7: Manifestation of particular nuclear effects in coherent charmonium (left panels) and bottomonium (right panels) photoproduction in UPC at RHIC collision energy \( \sqrt{s_N} = 200 \) GeV (top panels) and at LHC energy \( \sqrt{s_N} = 5.02 \) TeV (bottom panels). The nuclear cross sections are calculated with charmonium wave functions generated by the BT potential adopting the KST model for the dipole cross section. The dotted lined represents predictions in the high energy limit, Eqs. (2.3) and (2.7), containing only the maximal shadowing from the long-lived lowest \( \bar{Q}Q \) Fock states of the photon. The dashed and solid lines additionally include the spin rotation effects and corrections to a finite CL with the gluon shadowing, respectively. The data are taken from PHENIX [12], CMS [13], ALICE [14–16, 18] and LHCb [17] collaborations.

The differences between the solid and dashed lines represent the onset of the finite-\( l_c \) corrections which is more pronounced in coherent production of bottomonia compared to \( J/\psi \). Whereas at the LHC collision energy \( \sqrt{s_N} = 5.02 \) TeV such differences exhibit the dominance of the former effects mainly in the midrapidity \( (y = 0) \) region, the latter effects manifest themselves in regions of large negative and positive rapidities. The both effects substantially reduce the nuclear cross sections \( d\sigma/dy \). In comparison to charmonium production at the LHC, a much weaker onset of gluon shadowing effects in production of bottomonia is caused by a larger corresponding scale \( \propto M_{\Upsilon}^2 \gg M_{J/\psi}^2 \) (see also Fig. 3).

Analogously, the Fig. 8 shows contributions of above nuclear effects to a magnitude of rapidity distributions \( d\sigma/dy \) for coherent production of 2S-radially excited quarkonia.
FIG. 8: The same as Fig. 7 but for coherent production of 2S-radially excited quarkonia in UPC.

Here in contrast to our predictions for coherent J/ψ production in Fig. 7, where the spin rotation effects considerably enhance nuclear cross sections, the nodal structure of ψ′(2S) wave function leads to a counter-intuitive reduction of dσ/dy as one can see in the left panels of Fig. 8. In comparison to the RHIC energy range, such a reduction is smaller at the larger LHC energy due to a weakening of the node effect with energy. In coherent production of Υ′(2S) a very weak node effect leads to a standard enhancement of dσ/dy as is expected for 1S quarkonia (see right panels of Fig. 8).

The last Fig. 9 illustrates a manifestation of particular nuclear effects at the LHC collision energy \( \sqrt{s_N} = 5.02 \text{ TeV} \) in incoherent production of 1S-ground state (top panels) and 2S-radially excited (bottom panels) quarkonia, respectively. Since we expect very small corrections for a finite CL and weak gluon shadowing effects at small energies (see Figs. 2 and 3), here we do not present our predictions for the RHIC kinematic range. Differences between the solid and dashed lines represent the onset of net gluon shadowing effects at the LHC which is maximal at \( y = 0 \).

Finally, for completeness, we present in Tab. III the values of cross sections \( \sigma_{coh}(V) \) integrated over rapidity \( y \) for coherent AA → VAA production of various quarkonium states (\( V = J/ψ, ψ′, Υ, Υ′ \)) in UPC using wave functions generated by the BT and POW potentials and adopting two different models KST and GBW for the dipole cross section.
The corresponding values of $\sigma_{\text{coh}}$ clearly demonstrates that the onset of the node effect in coherent production of $\psi'(2S)$ and $\Upsilon'(2S)$ is weaker at larger collision energies, what is demonstrated as a rise of ratios $\sigma_{\text{coh}}(\psi')/\sigma_{\text{coh}}(J/\psi)$ and $\sigma_{\text{coh}}(\Upsilon')/\sigma_{\text{coh}}(\Upsilon)$ with $\sqrt{s_N}$.

IV. CONCLUSIONS

In this paper we treat the heavy quarkonium ($J/\psi(1S), \psi'(2S), \Upsilon(1S), \Upsilon'(2S),...$) production in heavy-ion UPC in the energy range accessible by experiments at RHIC and the LHC. Here the main observations are the following:

- Calculations of the corresponding photoproduction cross sections for the coherent, as well as incoherent process have been performed within the LF color dipole approach. Here we used the standard formulas for nuclear cross sections in the high energy limit where the CL is much longer than the nuclear radius, $l_c \gg R_A$. However, we included also the corrections for a finite coherence length via form factors, which have been treated for the first time within a rigorous Green function formalism. The standard Glauber approximation based on the VDM is relatively rather vague for 1S quarkonia,
but cannot be applied anymore for production of $2S$-radially excited quarkonia (see differences between VDM and Green function results in Fig. 2). Such corrections for a more proper inclusion of the quark shadowing effects are absent in the most of recent papers devoted to electroproduction of heavy quarkonia off nuclei.

- We have found that the finite-$l_c$ corrections are important especially in the RHIC energy range modifying significantly rapidity distributions $d\sigma/dy$ at small photon energies when $l_c \lesssim R_A$, i.e. in regions of large negative and positive rapidities. At the LHC such a modification of $d\sigma/dy$ is rather small. Whereas in coherent production of quarkonia they substantially reduce nuclear cross sections, they lead to a weak enhancement of $d\sigma/dy$ in the incoherent process (see Fig. 2). In comparison to $1S$ states we predict much weaker onset of CL effects in incoherent production of $2S$ radially excited heavy quarkonia due to nodal structure of their wave functions. For production of $\Upsilon'(2S)$ these effects are practically invisible and can be neglected (see right panels of Fig. 2).

- At larger photon energies we included also the gluon shadowing correction related to higher multi-gluon Fock components of the photon. Here we did not adopt any phenomenological parametrization of the gluon shadowing factor $R_G$ from the literature but calculated this factor directly within the LF color dipole formalism based on the Green function technique. This allows to obtain the magnitude of GS as function of the nuclear impact parameter, as well as to include naturally the effects of coherence length related to such higher Fock fluctuations of the photon. Here we relied on a dominant contribution to GS from the lowest $|Q\bar{Q}G\rangle$ Fock state with a subsequent omission of higher multi-gluon contributions to GS due to their very large effective masses and, consequently, very weak onset of quantum coherence effects. Note that an alternative calculations of GS effects based on a very popular Balitsky-Kovchegov equation [61, 62], which sums up the all but only long-lived multi-gluon fluctuations,

| A u A u ($\sqrt{s_N} = 200$ GeV) | P b P b ($\sqrt{s_N} = 2.76$ GeV) | P b P b ($\sqrt{s_N} = 5.5$ TeV) |
|-----------------|-----------------|-----------------|
| $J/\psi$: GBW, POW | 237.0 $\mu$b | 20.9 mb | 36.0 mb |
| $J/\psi$: GBW, BT | 162.8 $\mu$b | 15.4 mb | 26.3 mb |
| $\psi'$: GBW, BT | 10.12 $\mu$b | 2.61 mb | 4.91 mb |
| $J/\psi$: KST, POW | 235.1 $\mu$b | 19.7 mb | 33.8 mb |
| $J/\psi$: KST, BT | 159.9 $\mu$b | 13.7 mb | 24.4 mb |
| $\psi'$: KST, BT | 15.9 $\mu$b | 2.62 mb | 5.01 mb |
| $\Upsilon$: GBW, POW | 0.97 nb | 8.18 $\mu$b | 18.3 $\mu$b |
| $\Upsilon$: GBW, BT | 0.99 nb | 8.39 $\mu$b | 19.3 $\mu$b |
| $\Upsilon'$: GBW, BT | 0.17 nb | 2.24 $\mu$b | 5.44 $\mu$b |
| $\Upsilon$: KST, POW | 2.83 nb | 21.4 $\mu$b | 50.6 $\mu$b |
| $\Upsilon$: KST, BT | 2.88 nb | 22.1 $\mu$b | 52.5 $\mu$b |
| $\Upsilon'$: KST, BT | 0.53 nb | 6.74 $\mu$b | 16.1 $\mu$b |

**TABLE III:** Total (integrated over rapidity $y$) cross sections $\sigma_{coh}(V)$ for coherent $AA \rightarrow VAA$ vector meson ($V = J/\psi, \psi', \Upsilon, \Upsilon'$) production in UPC using quarkonium wave functions obtained from the POW and BT potentials and adopting GBW and KST models for the dipole cross sections.
cannot provide us with reliable results and can be applied only when transverse sizes
of all Fock states are “frozen” during propagation through the nucleus.

- In the LHC kinematic region we have found a significant onset of GS effects, reducing
the nuclear cross sections for charmonium coherent and incoherent photoproduction
by a factor of $1.1 \div 1.2$. They are gradually diminishing toward smaller energies and
can be neglected in kinematic regions accessible by RHIC experiments (see Fig. 3).

- The quarkonium wave functions are well defined in the $Q\bar{Q}$ rest frame. They have
been included in our calculations and obtained by solving the Schrödinger equation
for two distinct realistic $Q - \bar{Q}$ interaction potentials, POW and BT, which give the
best agreement with available data on charmonium real and virtual photoproduction
off a nucleon target. Consequently, the corresponding LF wave functions have been
generated performing the boosting to the LF frame using the so called Terent’ev
prescription from Ref. [38], whose validity has been verify in Ref. [63] for symmetric $c\bar{c}$ photon fluctuations with $\alpha \approx 0.5$.

- Spin dependent part of $S$-wave quarkonium wave functions can be safely factorized
from the radial component. In order to avoid the $D$-wave component in the $Q\bar{Q}$ rest
frame, here we treat a simple non-photon-like structure of the $V \rightarrow Q\bar{Q}$ transition.
Consequently, we perform explicitly the transformation of 2-dim. heavy (anti)quark
spinors from the rest to the LF frame known as the Melosh spin rotation. We derived
the new formulas for coherent and incoherent nuclear cross sections incorporating such
a transformation. We have found that spin effects have a significant impact on the
magnitude of nuclear cross sections. Standardly, they cause a substantial enhance-
ment of the nuclear production rates. Such a scenario of enlargement corresponds
to charmonium photoproduction results on the proton target [6, 7, 24]. However, in
production of $2S$-radially excited charmonia the Melosh spin transformation leads to
a counter-intuitive reduction of nuclear rapidity distributions $d\sigma/dy$ as a consequence
of the nodal structure of $\psi'(2S)$ radial wave functions. In production of $\Upsilon'(2S)$ state,
the node effect becomes much weaker and leads again to the standard scenario of the
$\frac{d\sigma}{dy}$ enhancement.

- We have studied also differences in our predictions employing two phenomenological
parametrizations for the dipole cross section, KST and GBW, in order to estimate a
corresponding measure of the theoretical uncertainty in the current analysis.

- Our calculations are in a rather good agreement with available data on coherent pro-
duction of $J/\psi(1S)$ and $\psi'(2S)$ in UPC at the LHC (see Figs. 4 and 5). They can be
tested not only by the future measurements of UPC at RHIC and the LHC but also
by future experiments at planned electron-ion colliders.

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[1] B.Z. Kopeliovich and B.G. Zakharov; Phys. Rev. D 44, 3466 (1991).
[2] B.Z. Kopeliovich, J. Nemchik, N.N. Nikolaev and B.G. Zakharov; Phys. Lett. B 324, 469 (1994).
[3] J. Nemchik, N. N. Nikolaev, E. Predazzi and B. Zakharov, Phys. Lett. B 374, 199-204 (1996).
[4] J. Nemchik, N. N. Nikolaev, E. Predazzi and B. Zakharov, Z. Phys. C 75, 71-87 (1997).
[5] J. Nemchik, Phys. Rev. C 66, 045204 (2002).
[6] M. Krelina, J. Nemchik, R. Pasechnik and J. Cepila, Eur. Phys. J. C 79, no.2, 154 (2019).
[7] J. Cepila, J. Nemchik, M. Krelina and R. Pasechnik, Eur. Phys. J. C 79, no.6, 495 (2019).
[8] M. Krelina, J. Nemchik and R. Pasechnik, Eur. Phys. J. C 80, no.2, 92 (2020).
[9] I.P. Ivanov, N.N. Nikolaev and A.A. Savin; Phys. Part. Nucl. 37, 1 (2006).
[10] N. Brambilla et al. [Quarkonium Working Group], arXiv:hep-ph/0412158.
[11] N. Brambilla et al.; Eur. Phys. J. C 71, 1534 (2011).
[12] S. Afanasiev et al. [PHENIX], Phys. Lett. B 679, 321-329 (2009).
[13] V. Khachatryan et al. [CMS], Phys. Lett. B 772, 489-511 (2017).
[14] B. Abelev et al. [ALICE], Phys. Lett. B 718, 1273-1283 (2013).
[15] E. Abbas et al. [ALICE], Eur. Phys. J. C 73, no.11, 2617 (2013).
[16] J. Adam et al. [ALICE], Phys. Lett. B 751, 358-370 (2015).
[17] A. Bursche [LHCb], Nucl. Phys. A 982, 247-250 (2019).
[18] S. Acharya et al. [ALICE], Phys. Lett. B 798, 134926 (2019).
[19] B. Kopeliovich, J. Nemchik, N. N. Nikolaev and B. Zakharov, Phys. Lett. B 309, 179 (1993).
[20] J. Nemchik, N. N. Nikolaev and B. Zakharov, Phys. Lett. B 341, 228 (1994).
[21] B. Kopeliovich, J. Nemchik, A. Schafer and A. Tarasov, Phys. Rev. C 65, 035201 (2002).
[22] B. Kopeliovich, J. Nemchik and I. Schmidt, Phys. Rev. C 76, 025210 (2007).
[23] J. Nemchik, N. N. Nikolaev and B. Zakharov, Phys. Lett. B 339, 194-200 (1994).
[24] J. Hufner, Y.P. Ivanov, B.Z. Kopeliovich and A.V. Tarasov; Phys. Rev. D 62, 094022 (2000).
[25] J. Nemchik; Phys. Rev. D 63, 074007 (2001).
[26] J. Nemchik; Eur. Phys. J. C 18, 711 (2001).
[27] Y. Ivanov, B. Kopeliovich, A. Tarasov and J. Hufner, Phys. Rev. C 66, 024903 (2002).
[28] Y. P. Ivanov, B. Kopeliovich, A. Tarasov and J. Hufner, AIP Conf. Proc. 660, 283 (2003).
[29] Y. Ivanov, B. Kopeliovich and I. Schmidt, arXiv:0706.1532 [hep-ph].
[30] A. H. Mueller, Nucl. Phys. B 335, 115 (1990).
[31] B.Z. Kopeliovich, J. Raufesien and A.V. Tarasov, Phys. Rev. C 62, 035204 (2000).
[32] A. Martin; Phys. Lett. B 93, 338 (1980).
[33] N. Barik and S.N. Jena; Phys. Lett. B 97, 265 (1980).
[34] W. Buchmuller and S.H.H. Tye; Phys. Rev. D 24, 132 (1981).
[35] B.Z. Kopeliovich, A. Schafer and A.V. Tarasov; Phys. Rev. D 62, 054022 (2000).
[36] K.J. Golec-Biernat and M. Wusthoff; Phys. Rev. D 59, 014017 (1998).
[37] K.J. Golec-Biernat and M. Wusthoff; Phys. Rev. D 60, 114023 (1999).
[38] M.V. Terentev; Sov. J. Nucl. Phys. 24, 106 (1976) [Yad. Fiz. 24, 207 (1976)].
[39] H.J. Melosh; Phys. Rev. D 9, 1095 (1974).
[40] H. De Vries, C. W. De Jager and C. De Vries, Atom. Data Nucl. Data Tabl. 36, 495 (1987).
[41] T. Bauer, R. Spital, D. Yennie and F. Pipkin, Rev. Mod. Phys. 50, 261 (1978).
[42] B. Z. Kopeliovich, L. I. Lapidus and A. B. Zamolodchikov, JETP Lett. 33, 595 (1981).
[43] V. N. Gribov, Sov. Phys. JETP 29, 483 (1969) [Zh. Eksp. Teor. Fiz. 56, 892 (1969)].
[44] B. Z. Kopeliovich, Int. J. Mod. Phys. A 31, no. 28n29, 1645021 (2016).
[45] J. Hufner, B. Kopeliovich and J. Nemchik, Phys. Lett. B 383, 362-366 (1996).
[46] B. Z. Kopeliovich, I. K. Potashnikova and I. Schmidt, Phys. Rev. C 73, 034901 (2006).
[47] J.B. Bronzan, G.L. Kane and U.P. Sukhatme; Phys. Lett. B 49, 272 (1974).
[48] J.R. Forshaw, R. Sandapen and G. Shaw; Phys. Rev. D 69, 094013 (2004).
[49] C. Henkels, E. G. de Oliveira, R. Pasechnik and H. Trebien, Phys. Rev. D 102, 014024 (2020).
[50] J. Hufner, B. Kopeliovich and A. B. Zamolodchikov, Z. Phys. A 357, 113 (1997).
[51] J. Hufner and B. Z. Kopeliovich, Phys. Lett. B 426, 154-160 (1998).
[52] R.P. Feynman and A.R. Gibbs, Quantum Mechanics and Path Integrals, McGraw-Hill Book Company, NY 1965.
[53] M. Krelina, B. Z. Kopeliovich and J. Nemchik, paper in preparation.
[54] N. N. Nikolaev and B. Zakharov, Phys. Lett. B 332, 184-190 (1994).
[55] B. Kopeliovich, J. Nemchik, I. Potashnikova and I. Schmidt, J. Phys. G 35, 115010 (2008).
[56] B. Kopeliovich, A. Tarasov and J. Hufner; Nucl. Phys. A 696, 669 (2001).
[57] V. Guzey and M. Zhalov, JHEP 10, 207 (2013).
[58] V. Guzey, E. Kryshen, M. Strikman and M. Zhalov, Phys. Lett. B 726, 290-295 (2013).
[59] J. Nemchik, N.N. Nikolaev, E. Predazzi, B.G. Zakharov and V.R. Zoller; J. Exp. Theor. Phys. 86, 1054 (1998).
[60] K. Kutak and J. Kwiecinski, Eur. Phys. J. C 29, 521 (2003).
[61] I. Balitsky, Nucl. Phys. B 463, 99 (1996).
[62] Y. V. Kovchegov, Phys. Rev. D 60, 034008 (1999).
[63] B.Z. Kopeliovich, E. Levin, I. Schmidt and M. Siddikov; Phys. Rev. D 92, 034023 (2015).