A divided Universe: red and blue galaxies and their preferred environments

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ABSTRACT

Making use of scaling relations between the central and the total galaxy luminosity of a dark matter halo as a function of the halo mass, and the scatter in these relations, we present an empirical model to describe the luminosity function (LF) of galaxies. We extend this model to describe relative statistics of early-type (or red) and late-type (or blue) galaxies, with the fraction of early-type galaxies at halo centres, relative to the total sample, determined only by the halo mass; the same fraction in the case of satellites is taken to be dependent on both the halo mass and the satellite galaxy luminosity. This simple model describes the conditional LF (the LF of galaxies as a function of the halo mass) measured with the 2dF galaxy group catalogue from cluster to group mass scales. Given the observational measurements of the LF as a function of the environment using the 2dF, with environment defined by the galaxy overdensity measured over a given volume, we extend our model to describe environmental LFs. Using 2dF measurements, we extract information related to the conditional mass function for haloes from extreme voids to dense regions in terms of the galaxy overdensity. We also calculate the probability distribution function of halo mass, as a function of the galaxy overdensity, and use these probabilities to address preferred environments of red and blue galaxies. Our model also allows us to make predictions, e.g. galaxy bias as a function of the galaxy type and luminosity, the void mass function, and the average galaxy luminosity as a function of the density environment. The extension of the halo model to construct conditional and environmental LFs of galaxies is a powerful approach in the era of wide-field large-scale structure surveys, given the ability to extract information beyond the average LF.

Key words: galaxies: formation – galaxies: fundamental parameters – galaxies: haloes – cosmology: observations – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

An important aspect of understanding underlying astrophysical reasons for galaxy formation and evolution involves studying the relative distribution of red (early-type) and blue (late-type) galaxies, as a function of the galaxy environment. Observational measurements of this so-called density–morphology relation suggest evidence that early-type galaxies are predominantly found in dense environments such as galaxy groups and clusters (Dressler 1980; Goto et al. 2003). Evidence also suggests that the formation of early-type galaxies predates the formation of galaxy clusters (Dressler et al. 1997). In the era of wide-field galaxy surveys such as the 2dF Galaxy Redshift Survey (2dFGRS; Colless et al. 2001) or the Sloan Digital Sky Survey (SDSS; York et al. 2000), detailed statistics on galaxy types and their environments allow us to now construct detailed models related statistics of galaxy types and where these galaxies are located. These models may then aid in explaining the underlying reasons for the occurrence of galaxy types and their preferred environments with initial conditions given by the primordial density fluctuations and cosmological parameters that determine the expansion.

Following these lines, numerical and semi-analytical models of galaxy formation are generally pursued to model and understand galaxy statistics including luminosity functions (LFs), occurrence of galaxy types, their spatial distribution, and clustering properties. While initial conditions and background cosmology are known adequately, these techniques are yet to describe statistical measurements of the galaxy distribution with reasonable accuracy. The main unknown here comes from our limited understanding of astrophysics involving how baryons cool to form stars and how the formation of stars and their evolution, including the stellar end products, may lead to various feedback processes that affect subsequent star formation. The standard text book description of galaxy formation involves

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gas first heating to virial temperature during the formation of dark matter haloes and subsequently cooling at halo centres to form stars (Rees & Ostriker 1977; White & Rees 1978). A characteristic scale in galaxy formation is then related to the amount of gas, basically within the ‘cooling radius’, which can cool within the Hubble time given a mass scale for the halo. Semi-analytical models of galaxy formation (e.g. Benson et al. 2001) and direct hydrodynamical simulations of the galaxy distribution (e.g. Kay et al. 2002) generally overpredict the number of galaxies both at the low end and the high end of the galaxy LF. The bright end of the LF is always associated with the ‘overcooling problem’ in numerical simulations (Balogh et al. 2001), where hot gas cools rapidly to form luminous galaxies at halo centres. The faint-end problem, involving a lack of faint galaxies in the observed LF, is generally explained as due to a feedback process during the era of reionization (Barkana & Loeb 2001) when primordial galaxies started to form. Models are generally evoked to expel gas from haloes, such as through heating associated with reionization or a first generation of supernovae (e.g. Bullock, Kravtsov & Weinberg 2000; Benson et al. 2002), but the gas expelled from small haloes settles eventually in more massive haloes and cools to form bright central galaxies with luminosities exceeding those observed (e.g. Benson et al. 2003).

Ignoring galaxy growth through continuous cooling of hot gas, in Cooray & Milosavljević (2005a), the central galaxy luminosity growth, as a function of the halo mass was explained based on a simple description for galaxy merging to the halo centre with an efficiency determined by the dynamical friction alone. These models also lead to a characteristic scale in the galaxy formation reflected in terms of a flattening of the relation between central galaxy luminosity and halo mass, or \( L_\star(M) \) relation, as the halo mass is increased. This characteristic luminosity is associated with a critical mass scale when the dynamical friction time-scale becomes close to or exceeds the Hubble time. In Cooray & Milosavljević (2005b), a model for the LF of galaxies that relied primarily on this relation, and to a lesser extent on the relation between total galaxy luminosity in a given halo and its halo mass, was used to show that this characteristic luminosity is the same as \( L_\star \) in the LF, when the LF is described with the Schechter (1976) form of \( \Phi(L) \propto (L/L_\star)^\alpha \exp(-L/L_\star) \). The success in describing the LF of galaxies using the \( L_\star(M) \) relation and its scatter led to the conclusion that the \( L_\star \) in the LF is not a reflection of efficiency associated with gas cooling, as has been argued in the past based on traditional models of galaxy formation dominated by hot gas cooling in dark matter haloes (e.g. Dekel 2005). The characteristic luminosity is rather due to decreasing efficiency of dissipationless merging of galaxies to a central galaxy as hierarchical structure formation builds up massive parent haloes.

Other evidence for a departure from traditional ideas of galaxy formation and evolution comes from numerical simulations, where some simulations now suggest that gas, as dark matter haloes virialize, never heats to the ‘virial’ temperature completely, but rather shock heating during virialization forms a bimodal temperature distribution (Dekel & Birnboim 2005; Keres et al. 2005). Binney (2004) suggested that only the colder component cools to form a galaxy, while the hotter component remains at the same temperature. There is a lower characteristic scale in galaxy formation associated with the mass scale where most gas is never shock heated and remains at the virial temperature, but rapidly cools at the centre to form a galaxy. One-dimensional numerical simulation suggests this lower mass scale is \((1-6) \times 10^{11} M_\odot \) (Dekel 2005). In this description for galaxy formation, galaxies in more massive haloes can grow in luminosity only through mergers with other galaxies. The dynamical friction process involved with merging produces a consistent \( L_\star(M) \) relation that agrees with observations (Cooray & Milosavljević 2005a). The same relation, when combined with the mass function, leads to the LF, and can be fitted with the Schechter (1976) form. The exponential drop-off of the LF at the bright end is reflection of the scatter in the \( L_\star(M) \) relation (Cooray & Milosavljević 2005b).

Here, we extend the model of Cooray & Milosavljević (2005b) to describe galaxy statistics measured by the 2dFGRS. The main advantage of using 2dFGRS data is the availability of the LF measured as a function of the galaxy type, and the environment, measured in terms of the galaxy overdensity over the volume determined by size scale of \( 8 h^{-1} \text{Mpc} \) (Croton et al. 2005). The 2dFGRS data also allow measurements of the conditional luminosity function (CLF; Yang, Mo & van den Bosch 2003b), the LF of galaxies as a function of the halo mass (Yang et al. 2005). Our models on the CLF can be directly compared to these measurements and interesting information on the relative distribution of galaxy types can be extracted from the data.

While not as complicated as semi-analytical models of galaxy formation, the empirical modelling approach utilized here has the advantage that one is able to understand the main ingredients that shape the CLFs easily. The approach builds upon attempts by Yang et al. (2003b) to describe the LF with CLFs, but assuming Schechter forms a priori for the CLF, and approaches that are built upon the halo model for the galaxy distribution (Cooray & Sheth 2002), but now extended to discuss conditional functions (Zheng et al. 2005 with the stellar mass function and Zehavi et al. 2005 in the case of CLFs). While the halo model has been successfully used to describe statistics of the dark matter field – such as clustering (Seljak 2000; Peacock & Smith 2000) or weak lensing (Cooray, Hu & Miralda-Escudé 2000; Cooray & Hu 2001) – and basic properties of galaxy clustering (e.g. Scoccimarro et al. 2001), it is useful to consider more applications of this technique which can provide information on underlying physics related to the galaxy distribution.

Here, we will separate our discussion into central and satellite galaxies and make as few assumptions as possible through simple model descriptions. The motivations for the separation of galaxies into these two divisions are numerous: from the theoretical side, a better description of the galaxy occupation statistics is obtained when one separates into central and satellite galaxies (Kravtsov et al. 2004), while from observations, central and satellites galaxies are known to show different properties, such as colour and luminosity (e.g. Berlind et al. 2004). Our goal here is to consider an analytical description for the LF with built-in model ingredients that recover the observations. We then argue that instead of attempting to understand mass-averaged statistics such as the LF, it may be best to reproduce the main ingredients that shape CLFs with numerical and semi-analytical models of galaxy formation in order to understand the underlying physics. In the case of CLFs, we will argue that the main ingredient is the \( L_\star(M) \) relation, and in the case of galaxy types, a model on the fraction of early-type and late-type galaxies as a function of the halo mass. If these relations and their observed scatter can be explained with simple physics, then it is guaranteed that the LF would be recovered.

To model the environmental LF (Croton et al. 2005), we need information on the mass function of dark matter haloes that corresponds to the environment of interest, whether it is a void or a dense region. Because this information is not directly available under a simple model (see, for example, Mo et al. 2004 and the approach there that utilized numerical simulations), here we make use of the observed measurements to extract information on
these conditional mass functions. These mass functions, as well as related probabilities, then provide us with general information on how blue and red galaxies are distributed in the Universe.

The paper is organized as follows. In the next section, we outline the basic ingredients in the empirical model for CLFs and present a comparison to measurements using 2dF data (from Yang et al. 2005), both in terms of the average number of galaxies and in terms of galaxy types. In Section 3, we describe the LF and in Section 4 the environmental LF based on the conditional mass function. Using data from Croton et al. (2005), we extract information on the conditional mass function and various statistical measurements related to the relative distribution of early- and late-type galaxies. We also study the void mass function and compare with predictions in the literature. We conclude with a brief discussion of our main results in Section 5. Throughout the paper, we assume the concordance cosmological parameters consistent with Wilkinson Microwave Anisotropy Probe (WMAP) data (Spergel et al. 2003). Throughout this paper, to be consistent with observations, we take the scaled Hubble constant to be $h = 1$, in units of 100 km s$^{-1}$ Mpc$^{-1}$.

2 CONDITIONAL LUMINOSITY FUNCTION: EMPIRICAL MODEL

In order to construct the LF, we follow Cooray & Milosavljević (2005b). The CLF, denoted by $\Phi(L | M)$, is the average number of galaxies with luminosities between $L$ and $L + dL$ that reside in haloes of mass $M$ (Yang et al. 2003b). First, we separate the CLF into terms associated with central and satellite galaxies, such that

$$\Phi(L | M) = \Phi_c(L | M) + \Phi_s(L | M)$$

$$\Phi_c(L | M) = \frac{f_c(M)}{\sqrt{2\pi} \ln(10) L} \exp \left\{ - \frac{\ln[L/L_c(M)]^2}{2\Sigma^2} \right\}$$

$$\Phi_s(L | M) = \Lambda(M) f_s(L).$$

(1)

Here, $L_c(M)$ is the relation between the central galaxy luminosity of a given dark matter halo and its halo mass, while $\ln(10)\Sigma$ is the dispersion in this relation. The central galaxy CLF takes a lognormal form, while the satellite galaxy CLF takes a power-law form in luminosity. Such a separation describes the LF best, with an overall better fit to the data in the $K$ band as explored by Cooray & Milosavljević (2005b). While a previous attempt to describe CLFs, as appropriate for 2dFGRS, involved a priori assumed Schechter (1976) forms, we believe the description here is more appropriate. Our motivation for lognormal distributions also comes from measured conditional LFs, such as galaxy cluster LFs including bright galaxies, where data do require an additional lognormal component in addition to the Schechter (1976) form (Trentham & Tully 2002). Similarly, the stellar mass function, as a function of haloes mass in semi-analytical models, is best described with a lognormal component for the central galaxies (Zheng et al. 2005).

In the next few subsections, we describe in detail other parameters associated with the CLF and how numerical values for these parameters are obtained. First, we discuss the CLF of central galaxies and then move on to discuss satellites. We end this section with a comparison to CLFs measured in the 2dFGRS by Yang et al. (2005).

2.1 Central galaxies

In our description for CLFs (equation 1), central galaxies have a lognormal distribution in luminosity with a mean determined by the $L_c(M)$ relation. The scatter in the $L_c(M)$ relation is captured through the dispersion $\Sigma$ of the lognormal distribution. In Cooray & Milosavljević (2005b), we found $\Sigma \sim 0.23$ to describe the field-galaxy LF in the $K$ band (Huang et al. 2003). Here, to describe 2dFGRS data in the $b_1$ band, we found a lower dispersion, with $\Sigma \sim 0.17$, with a 95 per cent confidence level between 0.162 and 0.176 based on the 2dF galaxy LF. While the exact reason for differences between the dispersion at two wavelengths is not understood, because $\Sigma$ reflects the scatter in the $L_c(M)$ relation, we expect this relation for luminosities measured in the $b_1$ band to have less scatter than in the $K$ band. A comparison of the Yang et al. (2005) data shown in Fig. 1 and the Lin, Mohr & Stanford (2004) data shown in fig. 1(a) of Cooray & Milosavljević (2005b) suggests this may be the case, but the reason for the difference in scatter has yet to be understood. Incidentally, a value for the dispersion of $\sim 0.17 \pm 0.1$ is in good agreement with the value of 0.168 found for the dispersion of central galaxy luminosities by Yang et al. (2003b), where these authors used a completely different parametrization for the CLF than that described here.

In equation (1), the normalization factor $f_c(M)$ in the central galaxy CLF captures the efficiency for galaxy formation as a function of the halo. In Cooray & Milosavljević (2005b), this was set by requiring $\int \Phi(L | M)LdL$ equals the average total luminosity of galaxies $L_{\text{tot}}(M)$ in a halo of mass $M$. This condition does not include the fact that at low-mass haloes, galaxy formation is inefficient and not all dark matter haloes host a galaxy. This is equivalent to modifying the halo mass function at the low-mass end to select only

![Figure 1](https://academic.oup.com/mnras/article-abstract/363/1/337/1302621/337)
haloes that host a galaxy. Motivated by the halo occupation number models for central galaxies (e.g. Kravtsov et al. 2004), where not all low-mass haloes occupy galaxies, to fit the low-end luminosity data of the 2dFGRS galaxy LF, we allow a description of the form

\[ f_c(M) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log(M) - \log(M_{\text{min}})}{\sigma} \right) \right], \]

with parameters \( M_{\text{min}} = 5 \times 10^{10} \, M_\odot \) and \( \sigma = 0.75 \). These parameters were determined by comparing the 2dFGRS LF at the low end as discussed in Section 3. This efficiency function is such that it is 0.1 when \( M \sim 10^{10} \, M_\odot \), but is unity when \( M > \) few times \( 10^{11} \, M_\odot \). When describing the environmental LFs, we will continue to use this form.

2.2 Satellites

For satellites, the normalization \( A(M) \) of the satellite CLF can be obtained by defining \( L_c(M) \equiv L_{\text{sat}}(M) - L_c(M) \) and requiring that \( L_c(M) = \int_{L_{\text{max}}}^{L_c} \Phi_s(L) \, dL \) with \( f_s(L) = 1 \), where the minimum luminosity of a satellite is \( L_{\text{sat}} \). In the luminosity ranges of interest, our CLFs are mostly independent of the exact value assumed for \( L_{\text{min}} \) as long as it lies in the range \( (10^9 - 10^9) \, L_\odot \). For the maximum luminosity of satellites, following the result found in Cooray & Milosavljević (2005b), by comparing predictions to the K-band cluster LF of Lin & Mohr (2004), we set \( L_{\text{max}} = L_c/2 \). A comparison to 2dFGRS CLFs as measured by Yang et al. (2005), however, suggested that such a sharp cut-off is inconsistent and that to account for scatter in the total galaxy luminosity, as a function of the halo mass, one must allow for a distribution in \( L_{\text{max}} \). Instead of additional numerical integrals, we allow for a luminosity dependence with the introduction of \( f_s(L) \) centred around the maximum luminosity of satellites such that \( \Phi_s(L) \) does not go to zero rapidly at \( L_{\text{max}} \). With a comparison to the data, we again found a lognormal description with

\[ f_s(L) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log(L_s/2) - \log(L)}{\sigma_s} \right) \right], \]

where \( \sigma_s = 0.3 \). The description here is such that \( f_s = 1 \) when \( L < L_{\text{max}} = L_c/2 \), but falls to zero at a luminosity beyond \( L_c/2 \) avoiding the sharp drop-off at \( L_c/2 \) with \( f_s(L) = 1 \). When model fitting the LF, or the LF as a function of the environment, this description is unimportant as the central galaxies dominate galaxy statistics. This is due to the fact that, as discussed in Cooray & Milosavljević (2005b), the LF is dominated by central galaxies instead of satellites, which has also been noted by Zheng et al. (2005) when describing the LFs in the present paper, but when describing the CLFs of Yang et al. (2005), we allow for a mass dependence for \( \gamma \) (see Fig. 3).

For comparison, the halo occupation numbers are given by

\[ N_i(M) = \int N_i(L) \, dL \]

where the subscript \( i \) refers to central and satellite galaxies. By design, when \( f_s(L) \rightarrow 1 \), \( N_c(M) = 1 \) as there is always a galaxy in the halo centre. When \( L_{\text{sat}}(M) \gg L_c(M) \), \( N_s \) scales with halo mass in the same way total luminosity scales with the halo mass.

2.3 Galaxy types

Here, in addition to the 2dFGRS LF, we also model the LF of galaxy types, broadly divided into two classes involving red, or early-type, and blue, or late-type, galaxies. A previous modelling of galaxy types using CLFs is described in van den Bosch, Yang & Mo (2003); our approach differs because of the overall division of the sample to central and satellite galaxies. Thus, the division to two types applies to these two components, separately, in a given halo. As it is clear, the CLFs we have developed facilitate this easy separation.

As we find later by comparing to 2dFGRS data, central galaxies tend to be early type when found in massive haloes, corresponding to groups and clusters, but late type when in low-mass haloes with a few or no satellites. Thus, the division of central galaxies to the two types can simply be described as a function of mass. While we have extracted this division in a function form here, we have not investigated the underlying reasons how galaxy types in the centre of haloes change from primarily late to early type, as the halo mass function increases. The form is given and it remains to be seen if this analytical description is reproduced in numerical or semi-analytical models of galaxy formation or not.

In the case of satellites, a comparison to the 2dFGRS CLFs measured by Yang et al. (2005) shows that the division to early- and late-type galaxies is not simply a function of mass alone, but rather a function of both mass and luminosity of the galaxy. For example, in low-mass haloes corresponding to galaxy groups, the low luminous satellites with luminosities less than \( 10^9 \, L_\odot \) tend to be mostly late-type or blue galaxies, while the bright satellites, with luminosities around \( 10^{10} \, L_\odot \) are dominated by early-type or red galaxies.

To describe these general behaviours, we consider the division of central and satellite CLFs in to early and late types separately, and write

\[
\begin{align*}
\Phi_{\text{early}-\text{cen}}(L | M) &= \Phi_c(L | M) f_{\text{early}-\text{cen}}(M) \\
\Phi_{\text{late}-\text{sat}}(L | M) &= \Phi_s(L | M) f_{\text{late}-\text{sat}}(M, L),
\end{align*}
\]

where the two functions that divide between early and late types are taken to be functions of mass, in the case of central galaxies, and both mass and luminosity in the case of satellites. Note that these fractions are defined with respect to the total galaxy number of a halo. Because the early- and late-type fractions should sum to unity, late-type fractions are simply \( 1 - f_{\text{early}-\text{cen}}(M) \) and \( 1 - f_{\text{early}-\text{sat}}(M, L) \) for central and satellite galaxies, respectively.
Given this simple dependence, we do not write late-type fractions separately.

In the case of central galaxies, we assume that there is a smooth transition between a dominant fraction of late type to a dominant early-type fraction, as a function of the halo mass, as the halo mass is increased. A description that fits the 2dFGRS CLFs of Yang et al. (2005) is

\[
 f_{\text{early-cen}}(M) = \frac{1}{2} \left\{ 1 + \text{erf} \left[ \frac{\log(M) - \log(M_{\text{cen}})}{\sigma_{\text{early-cen}}} \right] \right\},
\]

with \(M_{\text{cen}} = 5 \times 10^{12} \text{M}_\odot\) and \(\sigma_{\text{early-cen}} = 2.0\). As the halo mass increases, a large width for the transition from predominantly late-type to early-type galaxies in the centres of massive haloes results in an early-type fraction that never falls to unity or zero at either of the two ends in the mass ranges of interest.

For satellites, at high-mass haloes, we found the early-type fraction to be roughly two-thirds while at the low-mass end, this fraction decreases to around one-third. At the low end of halo masses where satellites are found, however, this fraction is luminosity-dependent. The model that describes this behaviour is

\[
 f_{\text{early-sat}}(M, L) = \frac{1}{6} g(M) + \frac{1}{6} h(L) + \frac{1}{3},
\]

where

\[
g(M) = \frac{1}{2} \left\{ 1 + \text{erf} \left[ \frac{\log(M) - \log(M_{\text{sat}})}{\sigma_{\text{sat}}} \right] \right\},
\]

\[
h(L) = \frac{1}{2} \left\{ 1 + \text{erf} \left[ \frac{\log(L) - \log(L_{\text{sat}})}{\sigma_{\text{sat}}} \right] \right\},
\]

with \(M_{\text{sat}} = 3 \times 10^{13} \text{M}_\odot\), \(L_{\text{sat}} = 3 \times 10^{9} \text{L}_{\odot}\), and \(\sigma_{\text{sat}} = 1\). This function varies between 1/3 and 2/3 when low-mass, low luminosities to high-mass, high luminosities. Note that satellites are subhaloes that have merged with a central halo. Thus, before these galaxies became satellites, they were in fact central galaxies. Thus, the fact that the fraction of early-to-late satellite galaxies is both mass- and luminosity-dependent should not be considered a drawback in this description or the halo approach to galaxy statistics in general. In fact, using a model where redshift dependences are also included, including a merging hierarchy, one may be able to start with a simple description for the early- to late-type galaxy fraction that is mass-dependent alone, and understand how the merging of these galaxies results in the fractional dependence of early galaxies, say, as satellites in massive dark matter haloes. Clearly, such work follows underlying motivations of semi-analytical models of galaxy formation. Here, we provide the relations that need to be explored in such an approach.

In Fig. 2, as a summary, we show, as a function of the halo mass, various fractions encountered in our model. Note that the central galaxy fraction falls below unity at halo masses below \(10^{11} \text{M}_\odot\), which is due to the efficiency factor we included to account for the fact that not all haloes at the low end may host a galaxy. Setting \(f_c(M) = 1\) results in an overprediction for the abundance of galaxies at that low-luminosity end. We discuss this in the context of modelling the 2dFGRS galaxy LF (Section 3).

### 2.4 Central and total luminosity relation

The main ingredient in the modelling the LF using this empirical approach is the \(L_0(M)\) relation. As discussed in Cooray & Milosavljević (2005b), the shape of the \(L_0(M)\) relation determines the shape of the LF. The slope of this relation is directly reflected in the faint-end slope of the LF, while the scatter of this relation determines the exponential-like drop-off of the LF at the bright end.

For the \(L_0(M)\) relation, here we make use of the suggested relation in Vale & Ostriker (2004). These authors established this relation by inverting the 2dFGRS LF given a analytical description for the subhalo mass function of the Universe (e.g. De Lucia et al. 2004; Oguri & Lee 2004). The relation is described with a general fitting formula given by

\[
 L(M) = L_0 \left( \frac{M}{M_0} \right)^{a} \left( b + \frac{M}{M_0} \right)^{c} / d.
\]

For central galaxy luminosities, the parameters are \(L_0 = 5.7 \times 10^9 \text{L}_{\odot}\), \(M_0 = 10^{11} \text{M}_{\odot}\), \(a = 4.0\), \(b = 0.57\), \(c = 3.72\) and \(d = 0.23\) (Vale & Ostriker 2004). These values are different from Cooray & Milosavljević (2005b) because we use \(K\)-band luminosities there, while the relation given in this paper is expected to describe the 2dFGRS data adequately, as it is extracted from 2dFGRS LF in the \(b_3\) band of Norberg et al. (2002a).

For the total galaxy luminosity, as a function of the halo mass, we also use the fitting formula in equation (8), but with \(c = 3.57\), which we picked based on model fits to the 2dFGRS CLFs of Yang et al. (2005). At the massive end, the total luminosity can alternatively be described by a power law or a double power law with the break around \(\sim 3.5 \times 10^{13} \text{M}_{\odot}\) (Vale & Ostriker 2004). The constructed LF does not change if power-law behaviour is enforced at the high end of the halo masses. This is because the average LF is dominated by central galaxies on any scale. The overall shape of the LF is strongly sensitive to the shape of the \(L_0(M)\) relation, and its scatter, and less on details related to the \(L_{\text{sat}}(M)\) relation.

In Fig. 1, we show the two relations used for the central galaxy and total galaxy luminosity of a given halo, on average, as a function of the halo mass. Incidentally, we found the relation extracted by Vale & Ostriker (2004) to be in good agreement with direct measurements of central galaxy luminosities in the 2dF galaxy group catalogue by Yang et al. (2005). Similarly, the \(L_{\text{sat}}(M)\) relation based on Vale & Ostriker (2004) agrees with total luminosity measurements.
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independently obtained by a different technique, involving clustering relation, in van den Bosch et al. (2005). In fact, these agreements are remarkable and suggest that we have a good starting point to build up a model for the galaxy LF both as a function of the environment and galaxy type.

2.5 Conditional luminosity functions: a comparison to 2dF

Now that we have an analytical description for the CLF with parameters determined either by results already in the literature, such as $L_c(M)$ from Vale & Ostriker (2004), or based on model fitting the data, we can discuss how well our model fits the 2dF CLFs of Yang et al. (2005). These CLFs were previously modelled with a priori assumed Schechter (1976) functions following the models in Yang et al. (2003b), but here we make use of the lognormal description for central galaxies and power laws for satellite galaxies to build the CLF.

In Fig. 3, we show a comparison of our CLFs to those extracted from the 2dF galaxy group catalogue by Yang et al. (2005). We show our models in the mass ranges in which they measured the CLFs. These CLF model fits can in fact be compared with figs 9 and 11 in Yang et al. (2005). The model fits generally support our lognormal description for central galaxies and the separate description of power law for satellite galaxies. The model fits require that the slope of the power law be $\gamma \sim -1$ in cluster scales, as found by Cooray & Milosavljević (2005b) by comparing to the cluster LF of Lin et al. (2004) in the $K$ band, but flattens to $\gamma \sim 0$ at galaxy group scales. The transition from late to early type is also adequately modelled with our simple description, except that we find our models to underpredict the number of bright, and potentially central, galaxies at poor galaxy group mass scales. This difference may be modelled by updating $L_c(M)$ at these mass scales. However, given the overall adequate description, and the fact that we wanted to build this model with the least number of parameter variations as possible but by using...
existing results from the literature (such as from Vale & Ostriker 2004), we have not pursued such a possibility here. It is also not clear how well masses have been estimated by Yang et al. (2005) for low-mass galaxy groups where few galaxies are found in each group.

Based on a simple $\chi^2$ analysis, the combination of central and satellite galaxies considered here and the Schechter (1976) function approach considered in Yang et al. (2005) provide equally appropriate fits to the data. The separation to central and satellite galaxies, however, provides a better physical motivation and is more consistent with a large number of studies related to the halo occupation distribution where statistics are generally considered in terms of central and satellite galaxies (e.g. Kravtsov et al. 2004; Zheng et al. 2005). With improving quality of the data, it may be possible to identify which approach, Schechter (1976) functions or the sum of lognormal and power-law models, provides a better description.

As discussed in Yang et al. (2005) for 2dF $b$-band data, and in Cooray & Milosavljević (2005b) for the $K$-band data, the CLF represents galaxy statistics better than the LF when wide-field data sets are available in which redshifts are measured for tens of thousands of galaxies. While Yang et al. (2003b) described CLFs using the a priori assumption that $\Phi(L | M)$ is given by the Schechter (1976) function with no separation to central and satellite galaxies, as is clear from Fig. 3, our description involving lognormal distribution for central galaxies and a power law for satellite galaxies may provide a better description. However, the peak of galaxies at the bright end may be due to a problem associated with mass assignment in the 2dFGRS galaxy group catalogue by Yang et al. (2005) (van den Bosch, private communication). The same reasons could also explain the apparent increase in bright galaxies at low-mass haloes, such as poor groups, when compared to our model predictions. Even if improvements are minor, when comparing CLFs based on the Schechter form used in Yang et al. (2005), the method suggested here, based on a division of the galaxy sample to central and satellite galaxies, is more physical. As described, the model considered also provides a more useful approach to divide galaxies into galaxy types, which is advantageous because we are trying to gain an understanding of how galaxy types are distributed in varying dark matter halo masses.

### 3 Luminosity Functions

Given our model for the CLFs, we can now construct the LF, which is an average of CLFs in mass with the halo mass distribution given by the mass function. Here, we use the Sheth & Tormen (1999, hereafter ST) mass function $dn/dM$ for dark matter haloes. This mass function is in better agreement with numerical simulations (Jenkins et al. 2001), when compared to the more familiar Press & Schechter (1974, hereafter PS) mass function.

Given the mass function, the galaxy LF is

$$
\Phi_i(L) = \int_0^\infty \Phi(L | M) \frac{dn}{dM} dM,
$$

where $i$ is an index for early- and late-type galaxies. The CLF for each type involves the sum of central and satellite galaxies. To understand our model for the LF, we will plot these two divisions, as well as the sum, separately in each of the LF figures shown in this paper.

Fig. 4 presents a general fit to the 2dF LF, where in Fig. 4(a) we assume $f_c(M) = 1$. While the model describes data adequately at magnitudes below $-19$, the fit at the low end of the luminosity is poor. Our model suggests that the low-end slope of the LF should be around $-1.33$ to $-1.25$, with the latter coming from our assumption that $L_c(M) \sim M^2$ at low-mass haloes (Cooray & Milosavljević 2005b). The flattening of the slope at the low end of the luminosity, as measured in the 2dF LF, may be a reflection of either (i) the galaxy selection function such that faint galaxies are missed, or (ii) a real effect in the Universe such that low-mass haloes do not host a large number of central galaxies. The difference from the model in Fig. 4(a), when compared to data, can be reduced with the inclusion of a mass-dependent $f_c(M)$ function. In Fig. 4(b), we show the case with equation (2). Note that the underlying reasons for this mass function can either be a selection effect or a real effect. We cannot distinguish between the two possibilities; however, it is likely that most selection biases are already accounted when constructing the LF and the effect we are seeing mostly is due to the fact that not all low-mass dark matter haloes host a galaxy.

---

**Figure 4.** The galaxy LF in the 2dFGRS. The measured data points are from Croton et al. (2005). In addition to the total LF, we also show the early-type (dotted lines; red online) and late-type (dashed lines; blue) LFs. The dotted lines are the central galaxies while the dashed lines are for satellites. In (a), we set $f_c(M) = 1$. This results in a faint-end slope of $\sim -1.3$, which is steeper than the measured value of $\sim -1.05$. In order to account for the flattening, in (b), we allow for $f_c(M)$ to be mass-dependent with the form given in equation (2).
To understand the mass dependence of the LF, in Fig. 5, we plot the LF separated into mass bins between $10^9$ and $10^{16} M_\odot$. Galaxies in low-mass haloes dominate the statistics of the LF at the faint end; in fact, the 2dF LF at magnitudes fainter than $-19$ is associated with galaxies in haloes with masses below $10^{12} M_\odot$. On the other hand, the exponential decrease in the Schechter form for the LF at the bright end is associated with galaxies in haloes of mass $10^{13} M_\odot$ and above. As shown in Fig. 1, the $L_\star(\mathcal{M})$ relation begins to flatten at halo masses above $10^{12} M_\odot$. The characteristic luminosity, $L_\star$, can be identified with the luminosity of galaxies in haloes of this mass scale. The exponential drop-off, instead of a sharp cut-off, is a reflection of the scatter in the mass scale. The exponential decrease in the Schechter form for the LF at the bright end is associated with galaxies in haloes of mass $10^{13} M_\odot$ and above. As shown in Fig. 1, the $L_\star(\mathcal{M})$ relation begins to flatten at halo masses above $10^{12} M_\odot$. The characteristic luminosity, $L_\star$, can be identified with the luminosity of galaxies in haloes of this mass scale. The exponential drop-off, instead of a sharp cut-off, is a reflection of the scatter in the mass scale. This, however, does not imply that at each luminosity satellites dominate, but rather, as Fig. 4 shows, central galaxies dominate.

As shown in Fig. 7, the LF is dominated by galaxies that occupy dark matter haloes in the mass ranges between $10^{10}$ and $10^{11} M_\odot$. While central galaxies dominate statistics in this mass range, satellite galaxies become the dominant contributor to the LF from each mass scale. This, however, does not imply that at each luminosity satellites dominate, but rather, as Fig. 4 shows, central galaxies dominate.

In Fig. 6, we show these conditional probability distribution functions, as a function of the halo mass, when $L = 10^9-10^{11} h^{-2} L_\odot$. These probabilities show a peak at low masses, associated with central galaxies, and a tail towards higher masses, associated with satellites of the same luminosity. As the mass scale is increased, the peak related to central galaxies broadens because the $L_\star(\mathcal{M})$ relation increases slowly with the increase in mass such that one encounters a fractionally higher mass range with an increase in luminosity. Fainter late-type galaxies are in low-mass haloes, while brighter early-type galaxies are in galaxy groups and clusters. By integrating the conditional probability distribution functions over luminosities, we can calculate the probability distribution of mass associated with the 2dF LF (van den Bosch et al. 2003):

$$P(\mathcal{M}) d\mathcal{M} = \frac{\int \Phi(L|\mathcal{M}) dL \frac{dn}{d\mathcal{M}}} {\int \Phi(L) dL} d\mathcal{M}.$$  

(11)

As shown in Fig. 7, the LF is dominated by galaxies that occupy dark matter haloes in the mass ranges between $10^{10}$ and $10^{11} M_\odot$. While central galaxies dominate statistics in this mass range, satellite galaxies become the dominant contributor to the LF from each mass scale. This, however, does not imply that at each luminosity satellites dominate, but rather, as Fig. 4 shows, central galaxies dominate.

To understand the relative distribution of mass given the galaxy luminosity, we also calculate the conditional probability distribution $P(\mathcal{M} | L)$ that a galaxy of given luminosity $L$ is in a halo of mass $\mathcal{M}$ as (Yang et al. 2003b):

$$P(\mathcal{M} | L) d\mathcal{M} = \frac{\Phi(L | \mathcal{M})}{\Phi(L)} \frac{dn}{d\mathcal{M}} d\mathcal{M}.$$  

(10)
Figure 6. The conditional probability distribution function of halo mass $P(M \mid L)$ to host a galaxy of the given luminosity, as a function of the halo mass. The black lines are the total galaxy sample, while red and blue lines show the sample divided to early- and late-type galaxies. These are the same models that were used to describe the LFs of Croton et al. (2005), as shown in Fig. 4. From (a) to (d), we show these probabilities for $L = 10^8, 10^9, 10^{10}$ and $10^{11} \ h^{-2} \ L_\odot$, respectively. The low-mass end peak, which tends to be narrow for lower luminosities, are the central galaxies, while the tail to high masses is associated with satellite galaxies. The width of the central galaxy luminosity peak increases because the $L_c(M)$ relation increases while flattening such that one encounters a fractionally higher mass range with an increase in luminosity. Low-luminous late-type galaxies are found in low-mass haloes, while luminous red galaxies are in haloes with masses corresponding to galaxy groups and clusters.

Note that the behaviour of central galaxies is similar to the average, but on the other hand, early-type or red galaxies are primarily in haloes with masses above $10^{11} \ M_\odot$.

3.1 Galaxy bias

Another useful quantity to compare with observed data is the galaxy bias, as a function of the luminosity. Using the conditional LFs, we can calculate these as

$$b(L) = \int b_{halo}(M) \frac{\Phi_i(L \mid M)}{\Phi_i(L)} \frac{dn}{dM} \ dM,$$

where $b_{halo}(M)$ is the halo bias with respect to the linear density field (Sheth, Mo & Tormen 2001; also Efstathiou et al. 1988; Cole & Kaiser 1989; Mo, Jing & White 1997) and $i$ denotes the galaxy type.

In Fig. 8, we show the galaxy bias as a function of the luminosity. We also divide the sample into galaxy types.

While the bias factors have similar shapes, early-type galaxies are biased higher relative to the late-type galaxies; the difference in the bias factor between early- and late-type galaxies is at the level of $\sim$5 per cent. At low luminosities, the total sample bias is close to that of late-type galaxies, while at the bright end the average bias for the whole sample is close to that of early-type galaxies. We also note an important difference in the bias when comparing satellite galaxies and central galaxies; satellite galaxies, on average, have a higher bias factor at all luminosities when compared to central galaxies. This is due to the fact that satellite galaxies are preferentially in higher-mass haloes that are, on average, biased higher with respect to the linear density field. The average bias factor, for the whole sample, however, is dominated by central galaxies, for the same reasons the LF is also dominated by central galaxies. Bias measurements as a function of galaxy type exist in the form of clustering information such as the correlation length as a function of luminosity and type (Norberg et al. 2002b). While we have not attempted to convert this clustering information to obtain bias $b(L)$ as a function of galaxy type here, because it involves an additional step of modelling, our next improvement in this approach is to compute clustering statistics, in which case a direct comparison could easily be made. This clustering information has been used in van den Bosch et al. (2003) when modelling the CLFs appropriate for the 2dFGRS.
ENVIRONMENTAL LUMINOSITY FUNCTIONS

Having discussed average statistics, we now focus on the LFs measured by Croton et al. (2005) as a function of the galaxy overdensity, $\delta_{gal}$. These overdensities correspond to a volume of radius $8 \, h^{-1}$ Mpc, and are measured following $\delta_{gal} = (N_g - \bar{N}_g)/\bar{N}_g$ such that $-1 > \delta_{gal}$.

To calculate the environmental LFs given by $\Phi(L | \delta_{gal})$, the LF given $\delta_{gal}$, following Mo et al. (2004), we make one important assumption. We assume that the CLFs, $\Phi(L | M)$, are independent of the environment. The motivation for such an assumption is inherent in the halo model (Cooray & Sheth 2002), where one makes the assumption that galaxy distribution can be associated with the halo mass rather than the environment. Similarly, observational studies based on the Sloan survey indicate that galaxy colour may also be independent of the environment, implying that the CLFs for galaxy types are not dependent on $\delta_{gal}$ (Blanton et al. 2004). Thus, given that $\Phi(L | M)$ is $\delta_{gal}$ independent, all variations in $\Phi(L | \delta_{gal})$ must come from variations in the halo mass function, as a function of $\delta_{gal}$. The halo mass function, in fact, is $\delta_{dm}$ dependent; the peak background split (Sheth & Tormen 1999, 2002) makes use of this dependence to extract information on, for example, relative biasing of the haloes, as a function of mass, to calculate halo bias factors.

While we make use of the assumption that CLFs can be considered to be independent of $\delta_{gal}$, the real situation may be more complicated. For example, as found by Kodama et al. (2001) and Goto et al. (2005), galaxy types start to change in small groups around a massive cluster and this behaviour is different when the same mass groups are found in less dense environments. This result would imply that the CLF depends on two parameters given by the halo mass and the galaxy overdensity, or one needs to consider $\Phi(L | M, \delta_{gal})$ instead of $\Phi(L | M)$. Here, we do not pursue such a complicated description of the CLF given limited knowledge on how $\delta_{gal}$ may be related to the luminosity distribution of galaxies. One way to include a dependence on overdensity would be to consider dependence on formation time-scale; haloes that virialized earlier could be found predominantly in denser environments when compared to haloes that formed later. Thus, because galaxy properties evolve, haloes in denser environments, and formed earlier, may show different properties than haloes of the same mass, but formed later, in less dense environments. Thus, a more appropriate approach would be to consider a CLF of the form $\Phi(L | M, \delta_{gal})$, where $t_i$ is the formation time, measured with respect to some epoch, and then to understand the statistical relation between $t_i$ and $\delta_{gal}$. While we hope to pursue such a possibility in a future paper, we make use of the first-step approach here where CLFs are assumed to be dependent only on the halo mass.

Thus, to calculate the LF as a function of the environment, as defined by the galaxy overdensity, we define

$$\Phi_i(L | \delta_{gal}) = \int_{0}^{\infty} \Phi_i(L | M) \frac{dn(M | \delta_{gal})}{dM} dM,$$

where $dn(M | \delta_{gal})/dM$ is the conditional halo mass function. Unfortunately, analytical techniques are not adequate enough to reliably calculate the conditional mass function, $dn/dM(M | \delta_{gal})$, given the galaxy overdensity, $\delta_{gal}$ (see, for example, Mo et al. 2004 and discussion below). Here, making use of the assumption that CLFs are independent of $\delta_{gal}$, we use observed measurements of $\Phi(L | \delta_{gal})$ to extract information on $dn(M | \delta_{gal})/dM$, the conditional mass function of dark matter haloes as a function of the galaxy overdensity. In this approach, $dn(M | \delta_{gal})/dM$, necessary to reproduce Croton et al. (2005) data, can potentially be compared with either improved model of the conditional mass function. While this comparison is beyond the scope of this paper, as it involves understanding certain aspects of the mass function, we plan to return to this topic later. Later in this section, however, we comment on the void mass function or the conditional mass function when $\delta_{gal} \sim -1$. Because Croton et al. (2005) measurements include the LF of galaxies in
Figure 9. The environmental LFs, or $\Phi(L | \delta_{\text{gal}})$, where $\delta_{\text{gal}}$ is the galaxy overdensity measured over a volume of $8 \, h^{-1} \, \text{Mpc}$. The measured data are from Croton et al. (2005). From (a) to (f), the overdensity varies from $\sim -0.95$ to greater than 6. In each of the panels, we show the ‘best-fitting’ models for the conditional mass function, $dn(M | \delta_{\text{gal}})/dM$, based on parameters $A(\delta_{\text{gal}})$ and $\alpha(\delta_{\text{gal}})$ following equation (14). The line styles are the same as in Fig. 4. For comparison, we also show the average LF in Fig. 4(b), with dot-dashed lines.

voids, we can directly establish the void mass function using our modelling technique.

To extract information on $dn(M | \delta_{\text{gal}})/dM$, motivated by numerical simulation based measurements in Mo et al. (2004) of the conditional mass function, we assume one can model this as

$$dn(M | \delta_{\text{gal}})/dM = A(\delta) \left( 1 + \frac{M}{10^{13}} \right)^{-\alpha(\delta)} \frac{dn}{dM} \quad (14)$$

where $A(\delta)$ and $\alpha(\delta)$ are two parameters we will extract from the data given the average mass function $dn/dM$ following the ST mass function.

These model fits to the Croton et al. (2005) data are shown in Fig. 9. For comparison, we also show the average LF (shown in Fig. 4). Our model fits generally describe the data, except in overdense regions, such as when $\delta_{\text{gal}} > 6$, when we underpredict the...
abundance of early-type galaxies and overpredict the abundance of late-type galaxies.

The conditional mass functions that describe these environmental LFs are shown in Fig. 10, where we plot separately the mass functions and the ratio of mass functions to the average ST mass function. Underdense regions show a sharp decrease in the abundance of high-mass haloes, while the overdense regions are such that it is a scaled version of the ST mass function, but with an increasing abundance of high-mass haloes. In principle, these mass functions should be described by the conditional mass functions that are used to calculate merger trees of dark matter haloes or based on the peak-background split (Sheth & Tormen 1999, 2002). However, the required modification to the mass function with \( \delta \rightarrow \delta_c - \delta_{L,m} \) and \( \sigma^2(M) \rightarrow \sigma^2(M') \) did not produce shapes of the extracted mass functions. In this replacement, \( \delta_c \sim 1.686 \) is the standard overdensity for collapse, \( M_M = 4/3\pi R^3 \rho_m(1+\delta_m) \), \( \delta_m \) is the overdensity in mass that corresponds to the overdensity in galaxies, and \( \delta_{L,m} \) is the linearized overdensity corresponding to the mass overdensity. The main reason for the difficulty in obtaining \( d\int(n(\delta_{gal})) / d\int n \) is that we do not yet have an accurate model for the relation between \( \delta_{gal} \), which is non-linear, and \( \delta_m \) and \( \delta_{L,m} \). In fact, these relations capture the biasing of galaxies with respect to the linear density field. Mo et al. (2004) used simulations to estimate the environmental mass functions, as a function of the galaxy overdensity, which were then compared with same LFs directly measured in mock catalogues. Here, we used observed data to extract information on the conditional mass functions. We find reasonable agreement with the mass functions plotted in their fig. 2.1

To further understand the dependence of these LFs on halo masses, we calculate the conditional probability for haloes of mass \( M \) to host galaxies given the environmental overdensity, \( \delta_{gal} \). These probabilities are calculated from

\[
P(M | \delta_{gal}) = \int \frac{\Phi(L | M) dL}{\int \Phi(L | L) dL} \frac{dn(M | \delta_{gal})}{dM}.
\]

In Fig. 11, we plot \( P(\log M | \delta_{gal}) \). These probabilities show mass scales that are important for the environmental LF, as a function of the galaxy overdensity. In addition to the total distribution, we also show probabilities in terms of the galaxy type, and separated into both central and satellite galaxies. The haloes that contribute to the LF in underdense regions host a higher fraction of late-type galaxies relative to early-type galaxies. On the other hand, in overdense regions, one finds both late-type and early-type galaxies, with a large fraction of early-type galaxies coming from more massive haloes or galaxy groups and clusters, while the late-type fractional contribution is dominated by galaxies in low-mass haloes.

To compare with measurements in Croton et al. (2005), we calculate one more quantity involving the mean luminosity per galaxy, as a function of the galaxy density environment:

\[
\vec{\rho}_L = \frac{\int_{L_{min}} \Phi(L | \delta_{gal}) L dL}{\int_{L_{min}} \Phi(L | \delta_{gal}) dL}.
\]

Here, \( L_{min} \) was set at an absolute magnitude of \(-17\) following the measurements of Croton et al. (2005). The Croton et al. (2005) data and the same quantity based on model fits in Fig. 9 are plotted in Fig. 12. They show general agreement; except in high dense environments, the mean luminosity per galaxy, in the case of early-type galaxies, is somewhat higher in our models when compared to the data. This is due to the fact that our models underpredict the abundance of early-type galaxies in dense environments, making them brighter on average than observed.

4.1 Galaxies in voids

As a final use of the data in Croton et al. (2005), we use their environmental LF in extreme voids, with \(-1 \leq \delta_{gal} \leq -0.9\), to extract information on the conditional mass function of the same environment. Our results are summarized in Fig. 13. Note that the

\[\text{Figure 10.} \quad (a) \text{The predicted mass functions, as a function of the environment captured by the galaxy overdensity over a size scale of } 8 h^{-1} \text{ Mpc. The dot-dashed line is the standard ST mass function, while other curves show the mass function from voids with } \delta_{gal} < 0 \text{ to dense regions with } \delta > 1. \quad (b) \text{The ratio of environmental mass functions to the mean mass function. Note the sharp cut-off at high masses when } \delta < -0.5 \text{. In the case of dense regions when } \delta \gg 1 \text{, the mass function increases at all mass scales, relative to the mean, but with a greater increase at high masses.}\]
massive haloes, while late types dominate the fractional contribution from low-mass haloes. In the case of overdense regions, a variety of mass scales contribute to the LF of that region, with early types dominating the fraction from the mass function required by the 2dF extreme void LF. In fact, the description for the void mass function comes closest to describing generally produced. The Sheth & van de Weygaert (2004) analytical mass function, following the $L_N(M)$ relation, to restrict galaxy luminosities to be fainter than the observed cut-off requires that no haloes with masses greater than $10^{13}$ M⊙ be present in voids with a fractional abundance $M_{\text{dn}}/dM$ greater than $10^{-8}h^3$ Mpc$^{-3}$.

Given that the void mass function has received some attention in the literature, we can make a direct comparison with our estimate of the mass function with previous estimates to the extent analytical formulae are available. In Fig. 13, we show several comparisons drawn from published attempts to analytically model the void mass function (also, see Gottlöber et al. 2003). The methods by Goldberg et al. (2005) and Patiri et al. (2005) generally fail to describe the void mass function both at the low and high ends of the mass function, although the abundance at a halo mass of $10^{11}$ M⊙ is generally produced. The Sheth & van de Weygaert (2004) analytical description for the void mass function comes closest to describing the mass function required by the 2dF extreme void LF. In fact, the void LF shows a sharp decrease in the abundance of galaxies with absolute magnitudes brighter than $-19$. Because our conditional LFs are independent of $\delta_{\text{gal}}$, this puts a strong constraint on the void mass function at the high-mass end. Because luminosities grow with mass, following the $L_N(M)$ relation, to restrict galaxy luminosities to be fainter than the observed cut-off requires that no haloes with masses greater than $10^{13}$ M⊙ be present in voids with a fractional abundance $M_{\text{dn}}/dM$ greater than $10^{-8}h^3$ Mpc$^{-3}$.

5 DISCUSSION AND SUMMARY

In this paper, we have made use of the scaling relation between the central and the total galaxy luminosity of a dark matter halo as a function of the halo mass, to construct the CLF of galaxies. The

Figure 11. The probability distribution function of halo masses hosting 2dF galaxies as a function of the environment as measured by the galaxy overdensity in $8 h^{-1}$ Mpc volumes. Here, we plot $P(\log M | \delta_{\text{gal}})$. The black lines are the total galaxy sample, while red and blue lines show the sample divided to early- and late-type galaxies following measurements by Croton et al. (2005). From (a) to (d), we show four ranges in $\delta_{\text{gal}}$ corresponding to both underdense and overdense regions. In the case of underdense regions, haloes with mass $10^{11}$ M⊙ host most galaxies and the fraction of galaxies is dominated by the late types (blue galaxies). In the case of overdense regions, a variety of mass scales contribute to the LF of that region, with early types dominating the fraction from massive haloes, while late types dominate the fractional contribution from low-mass haloes.

Sheth & van de Weygaert (2004) description managed to capture the low-end turnover in the void mass function, which is simply associated with our efficiency function $f_c(M)$. This model, however, overproduces the abundance of haloes at the high-mass end with a prediction for the presence of $\sim 10^{14}$ M⊙ haloes with the same abundance as haloes of $\sim 10^{13}$ M⊙ required by the 2dF LF. The dashed lines in Fig. 13(a) show the predicted LF based on the Sheth & van de Weygaert (2004) void mass function. The bright end is overpopulated and the turn-off in the LF moves to a higher luminosity than seen in the data. Our suggestion for the void mass function, based on the Croton et al. (2005) extreme void LF, may provide useful guidance to analytically calculate the void mass function. As part of an attempt to understand the conditional mass functions, we hope to return to this issue again in the future.
Figure 12. The mean luminosity per galaxy as a function of the environment, as measured by \( \delta_{\text{gal}} \). The data plotted are direct measurements established by Croton et al. (2005) based on the measured environmental LFs, while the lines show the predictions based on our model fitted to describe the same environmental LFs (Fig. 9). The solid circles show the total galaxy sample, open (red online) squares show early-type galaxies, and open (blue online) triangles show late-type galaxies. We find a general agreement, except at the high dense regions, where the mean luminosity per galaxy for early-type galaxies is slightly higher in our models when compared to the data. This is due to the fact that in our models, we underpredict the abundance of early-type galaxies, relative to the observed LF.

The CLF provides a powerful technique to understand galaxy properties, especially on the spatial distribution of galaxies, in the era of wide-field large-scale structure surveys where large galaxy samples can easily be subdivided with adequate statistics (Yang et al. 2003b). The CLF is closely related to the halo occupation number, which is the average number of galaxies given the halo mass (Cooray & Sheth 2002). The halo occupation number captures galaxy statistics that simply treat each galaxy in the sample equally and is useful when attempting to understand statistics such as galaxy clustering statistics, on average. Higher-order statistics of the galaxy distribution require detailed statistics of halo occupation beyond the mean. On the other hand, galaxies vary in luminosity and colour. Therefore, a more useful quantity to consider is the conditional occupation number, i.e. the number of galaxies in a given halo mass with given luminosity. These conditional occupation numbers are in fact the conditional mass functions we have described here.

When compared to previous descriptions of the CLF in the literature (e.g. Yang et al. 2003b, 2005), we make several improvements by dividing the galaxy sample into galaxies that are in the centres of dark matter haloes (central galaxies), and in the subhaloes of a main halo (satellite galaxies). This division is central to our arguments on how the CLF is shaped, and assumes that different physics govern the evolution of these two components. In our underlying physical description, following Cooray & Milosavljević (2005a), central galaxies grow in luminosity through dissipationless merging of satellite galaxies. These satellites are both early and late types, with the fraction of early-type galaxies taken to be dependent on both the halo mass and the satellite galaxy luminosity. The underlying physical reason for this dependence is not clear, but could very well be associated with a tidal stripping effect that removes gas and the stellar content of late-type galaxies in more massive haloes and converts these galaxies to early types. It will be helpful to understand if numerical and semi-analytical models predict the fractions of late- to early-type galaxies, including the suggestion that the fraction in satellites changes at a luminosity around \( \sim 5 \times 10^9 L_\odot \) from more late types at lower luminosities than this value to more early types. Using Sloan data, Kauffmann et al. (2003) found that galaxy properties dramatically change at mass scales between \( 10^9 \) and \( 10^{10} M_\odot \). The underlying reasons for such a change could also be the same reason for changes in the galaxy fraction at \( \sim 5 \times 10^9 L_\odot \) as found here based on a modelling of the LF.

Our simple empirical model describes the CLFs of galaxy types, as well as the total sample, measured with the 2dF galaxy group catalogue from cluster to group mass scales by Yang et al. (2005). A comparison of our CLFs those in Yang et al. (2005, their figs 9 and 11) reveals that our model can account for the peak of the CLF at high luminosities; this peak is associated with central galaxies, which we have modelled with a lognormal distribution. Similar peaks are also observed in the conditional baryonic mass function of stars in semi-analytical models of galaxy formation (Zhang et al. 2005) and in the LFs of clusters where all galaxies are included (Trentham & Tully 2002). The slope of the satellite CLF changes from \( -1 \) at cluster mass scales to \( 0 \) at group mass scales. It is hard to understand such a change in the context of the merging evolution of satellite galaxies as hierarchical merging builds up bigger haloes. A simple extension of the analytical model of Cooray & Milosavljević (2005a), which involves dynamical friction, cannot explain the change in the slope as parent halo mass is increased. The trend would be to flatten the slope, not steepen. A combination of the 2dF CLFs, which probe the bright end of the luminosities, and the dwarf galaxy LFs of clusters and groups from Trentham & Tully (2002) shows that the satellite LFs are more complicated than assuming a simple power law. As discussed in Cooray & Cen (2005), the satellite LFs could show various evidences for feedback processes, both due to reionization and galaxy formation at early times.

Given the observational measurements of the LF as a function of the environment using 2dF data by Croton et al. (2005), as measured by the galaxy overdensity measured over volumes corresponding to a radius of \( 8 h^{-1} \) Mpc, we extended our CLF model to describe environmental LFs. While it would have been useful to have direct predictions that can compare with observations, we failed to do this due to lack of information on the conditional mass function, or the mass function of dark matter haloes given the galaxy overdensity. In Mo et al. (2004), predictions were made using the conditional mass function measured in numerical simulations. Here, we use Croton et al. (2005) measurements to establish information on the conditional mass functions, from environments such as galaxy voids to dense regions. With these mass functions, we also estimate statistical quantities such as the probability distribution function of halo mass, as a function of the galaxy overdensity. We find that the preferred environment of blue galaxies are underdense regions in low-mass haloes, while the early-type, red galaxies are mostly in dense environments and dominated by satellites of larger-mass haloes. The shapes of the mass functions we have extracted could eventually be compared to numerical simulations or analytical techniques.

Using the Croton et al. (2005) LF for galaxies in voids, we also establish the void mass function. We find that the void mass function is peaked at halo masses around \( \sim 10^{13} M_\odot \). Such a peak in the void mass function is predicted in the analytical calculation by Sheth & van de Weygaert (2004). We do not, however, find a tail to higher halo masses as suggested by the description in Sheth & van de Weygaert (2004). The void mass function must sharply turn over; if
not, one would predict brighter galaxies in void environments than suggested by the measurements in Croton et al. (2005).

To summarize our paper, the main results are as follows.

(i) The galaxy LF, which is an average of CLFs over the halo mass function, is primarily shaped by the $L_c(M)$ relation; the faint-end slope of the LF reflects the faint-end scaling of the $L_c(M)$ relation while the bright-end turn-off in the LF, generally described by the exponential cut-off in the Schechter (1976) fitting form, is determined by the scatter in the $L_c(M)$ relation. Understanding the $L_c(M)$ relation and its scatter is central to understanding the galaxy LF.

(ii) The fraction of galaxy types as a function of the halo mass (Fig. 2). The galaxy types are distributed such that one finds essentially all late types in low-mass haloes and mostly early types in high-mass haloes. In the case of early types, the fraction is dominated by satellite galaxies rather than central galaxies in halo centers. These fractions may capture interesting physics such as tidal stripping, which occur in dense environments and could also be responsible for the fractional change of galaxy types, from dominant late type to early type, as the luminosity of satellite galaxies is increased.

(iii) The mass-dependent slope for the satellite CLF, where $\gamma(M) \sim -1$ at galaxy cluster mass scales and $\gamma \sim 0$ at poor galaxy group mass scales. The slope, when combined with dwarf LFs of clusters that probe the faint end of the luminosity distribution, shows a more complicated behaviour than one would naively expect from the subhalo mass function of dark matter haloes. As discussed in Cooray & Cen (2005), the slope may be a reflection of feedback processes associated with reionization and the first generation of dwarf galaxy formation.

(iv) The conditional mass functions (Fig. 10), or the mass function of dark matter haloes given the galaxy overdensity measured over a volume corresponding to a radius of $8h^{-1}\text{Mpc}$. We will leave it as a challenge to improve analytical techniques to produce the required mass functions to compare with what we suggest is needed to explain the Croton et al. (2005) measurements. Any disagreements, if understood, may suggest that our central assumption that the CLF is independent of the galaxy overdensity is incorrect. If this is the case, galaxy formation and evolution involves additional parameters beyond the halo mass and could question the viability of the halo approach to describe galaxy statistics.

(v) Galaxy bias predictions as a function of luminosity given the galaxy type (Fig. 8). While our predictions agree with the bias predicted for total samples, it will be useful to understand clustering bias as a function of galaxy colour as well. In an upcoming paper, we will extend the CLFs developed here to describe clustering properties of galaxies and a direct comparison to measurements in Norberg et al. (2002b).

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A numerical code to model the LF is available from the author or can be downloaded from http://www.cooray.org/lumfunc.html.

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