Neutrino masses, muon g-2, dark matter, lithium problem, and leptogenesis at TeV-scale

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Observational evidences of nonzero neutrino masses and the existence of dark matter request physics beyond standard model. A model with extra scalars and leptonic vector-like fermions is introduced. By imposing a $Z_2$ symmetry, the neutrino masses as well as anomalous muon magnetic moment can be generated via one-loop effects at TeV-scale. An effort of explaining dark matter, Lithium problem, and leptogenesis is presented. This scenario can be tested at LHC and/or future experiments.

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I. INTRODUCTION

There are some solid evidences for the physics beyond the standard model (SM) of particle physics. One is the observations of neutrino oscillations which has established that neutrinos have very small masses. The low energy accelerator experiment of the muon anomalous magnetic moment also gives another hint for the physics beyond the SM. Besides that there are also evidences from early Universe cosmology and astronomy: existence of dark matter and matter-antimatter asymmetry of the Universe\cite{1}. The observed baryon asymmetry of the Universe can not be explained within the SM with one CP violating phase.

Standard big-bang nucleosynthesis (SBBN) is one of the most reliable and farthest reaching probes of early Universe cosmology. One can calculate the relative abundances of light elements to H at the end of the "first three minutes" after the big bang. Despite the great success of SBBN, it has been noted that the prediction for the ratio of $^7\text{Li}/H$ and the isotopic ratio $^6\text{Li}/^7\text{Li}$ do not agree with current observations, called the "lithium problems"\cite{2}. The SBBN model predicts primordial $^6\text{Li}$ abundance about three orders of magnitude smaller than the observed abundance level and $^7\text{Li}$ abundance a factor of two to three larger when one adopts a value of the energy density of baryon inferred from the WMAP data. They do not have an astrophysical solution in a complete manner at present. One of the plausible solution is the existence of primordial late-decaying charged particles in the early Universe.

In this paper, we consider a novel model which can address all these issues within a single framework. Our model is an extension of the radiative seesaw mechanism of neutrino mass with extra scalars and leptonic vector-like fermions. By imposing a $Z_2$ symmetry, the neutrino masses as well as anomalous muon magnetic moment can be generated via one-loop effects at TeV-scale.

II. THE MODEL

Besides the inert doublet model, we introduced a set of new fermionic lepton doublet $L_i$ in our model. The new fermions are assumed to be vector-like to make sure that the theory is anomaly free as for self consistency. A discrete symmetry is imposed such that all the new particles are odd and the SM particles are even under this $Z_2$ projection. The content of the model is as following, scalar sectors

$$\phi_{i=1,2} \quad \text{and} \quad S^+$$

and an extra fermionic part

$$L_i = \begin{pmatrix} N_i^0 \\ E_i^- \end{pmatrix},$$

where $\phi_1$ corresponds to the SM Higgs which is even under $Z_2$. So we have the new Yukawa couplings

$$L_Y = f_{\alpha i} \bar{L}_i S^+ + y_{\alpha i} L_i \phi_2 + h.c.$$

$$= \left[ f_{\alpha i} \left( \bar{N}_i^0 E_i^- + l_{\tau i} \tilde{N}_\tau^0 \right) \right] S^+$$

$$+ y_{\alpha i} \left( l_{Ra}^+ \bar{E}_i^+ \phi_0^0 + l_{Ra}^+ \tilde{N}_{1i} \phi_2^0 \right) + h.c.,$$

where $\alpha$ runs for $e, \mu$, and $\tau$, while $i$ stands for the number of new fermionic doublet, we need at least two of them in order to achieve successful leptogenesis.

The scalar potential is given by

$$V(\phi_1, \phi_2, S^-) = -\mu_1^2 |\phi_1|^2 + \lambda_1 |\phi_1|^4 + m_2^2 |\phi_2|^2 + \lambda_2 |\phi_2|^4$$
where we already set neutrino masses can be approximated as two charged scalars as shown in Fig. 1. Note that there is a mixing between the two new charged scalar, it is the important parameter associated with neutrino mass matrix. The mixing matrix between \( S^\pm \) and \( \phi_2^\pm \) is

\[
\begin{pmatrix}
\phi_2^+ \\ S^+
\end{pmatrix}
= \begin{pmatrix}
m^2_2 + \frac{\lambda_2}{\sqrt{2}} \mu S^2_2 \\ \mu
\end{pmatrix}
\begin{pmatrix}
\phi_2^- \\ S^-
\end{pmatrix}
\]

(5)

A. Neutrino mass generation

The neutrino masses can be generated at one-loop level as shown in Fig. 1. Note that there is a mixing between two charged scalars \( S^\pm \) and \( \phi_2^\pm \) in the loop which associated with a GIM cancellation that make the corrections finite.

\[\mu < \phi_1^0 > \]

FIG. 1: 1-loop diagram for neutrino mass.

The generated neutrino mass matrix is

\[
(m_\nu)_{\alpha\beta} = -i f_{\alpha\beta} f_{\bar{\beta}} M_{E_i} \mu^2 < \phi_1^0 >^2 \\
\times \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - M_{\phi_2}^2)^2} \frac{1}{(q^2 - M_{\phi_1}^2)} \frac{1}{(q^2 - M_{E_i}^2)}
\]

\[
= \frac{f_{\alpha\beta} f_{\bar{\beta}} \mu^2 \sigma^2}{32 \pi^2 (M_{E_i}^2 - M_{\phi_2}^2)^2} \left[F(M_{E_i}^2) - F(M_{\phi_2}^2)\right]
\]

(6)

where \( F(M^2) = \frac{1}{(M^2 - M_{\phi_2}^2)^2} + \frac{M_{E_i}^2}{M_{E_i}^2 - M_{\phi_2}^2} \ln \frac{M_{E_i}^2}{M_{\phi_2}^2} \).

Under the assumption that \( M_{E_i} \sim M_{\phi_2} \sim M_{\phi_1} \), the neutrino masses can be approximated as

\[
(m_\nu)_{\alpha\beta} \approx \frac{f_{\alpha\beta} f_{\bar{\beta}} \mu^2 \sigma^2}{64 \pi^2 M_{E_i} M_{\phi_2}^2} \left(\frac{\mu^2}{M_{E_i}} - \frac{\mu^2}{M_{\phi_2}^2}\right) v
\]

\[\sim 10^{-3} \times f^2 \mu^2 M_{E_i} \]

(7)

This mass matrix contains both a loop suppression factor and a mass suppression factor hence it has similar structure as the radiative seesaw models. Thus we obtain the results that if \( \mu \) is around \( O(1) \sim O(100) \) GeV, \( f \sim 10^{-2} - 10^{-4} \) to have the neutrino masses as 0.1 eV, where we already set \( M_{\phi_2} \sim 500 \) GeV which is from the constraint of dark matter relic abundance.

B. Muon magnetic moment

The current limit of the muon magnetic moment is

\[
\Delta a_\mu = (290 \pm 90) \times 10^{-11},
\]

(8)

which is 3.2\( \sigma \) deviation between theory and experiment. The contributions to muon \( g - 2 \) from our model are shown in Fig. 2 and Fig. 3. In Fig. 2 one can see that the neutrino masses and the muon \( g - 2 \) are generated by similar mechanism. And we can get the enhancement by the chirality flip in the internal fermion line, the contributions are

\[
\Delta a_{\mu(N_e)}^{NP} = -\frac{\sin \delta \cos \delta}{16\pi^2} \sum_k (f_{\mu k} y_{\mu k}) \frac{m_\mu}{M_k} [F(x_{P_1}) - F(x_{P_2})],
\]

where \( x_{P_i} = m_{P_i}^2 / M_k^2 \) and \( P_i \) are the mass eigenstates of the charged scalars. And the function \( F(x) = \frac{1}{x^{3/2}} [1 - x^2 + 2x \ln x] \). The mixing angle satisfies the relation,

\[
\sin \delta \cos \delta = \frac{\mu \nu}{\sqrt{2}(m_{P_1}^2 - m_{P_2}^2)},
\]

(9)

this factor appears implicitly in neutrino masses due to the GIM mechanism. We found \( \sin \delta \cos \delta \times f_{\mu k} \sim (10^{-3} - 10^{-4}) \), \( m_\mu / (16\pi^2 M_k) \sim 10^{-5} \), and by setting \( y_{\mu k} \sim O(10^{-1} - 10^{-3}) \) will give us magnetic moment of order \( 10^{-9} \sim 10^{-10} \) which could give enough contribution for the observed deviation between the SM prediction and experiment.

For the contributions from the heavy charged lepton as shown in Fig. 3, we see the enhanced FCNC type leading contribution in the parameter regime where \( m_{E_i} \ll M_{E_i} \sim M_{\phi_2} \) and \( M_{E_i} \sim M_{\phi_2} \) is

\[
\Delta a_{\mu(E_i)}^{NP} \approx \frac{y_{\mu k} m_\mu}{48\pi^2 M_{\phi_2}^3} \approx y_{\mu k}^2 \times 10^{-11}.
\]

(10)
It will not give us sufficient muon anomalous magnetic moment unless the couplings \( y_{uk} \) is of order of \( O(10) \). We will not consider this case in this paper.

C. Dark matter, lithium problem and leptogenesis

**Dark matter.** The neutral component of inert doublet (ID) can be the dark matter candidate has been investigated in [4]. The mass difference between the scalar (\( \phi^0_{dR} \)) and pseudoscalar (\( \phi^0_{sR} \)) of the neutral component is determined by the quartic coupling constant \( \lambda_3 \) in the potential. To realize the dark matter relic abundance, (co)annihilations of ID into gauge bosons or Higgs should be carefully treated. The quartic couplings \( \lambda's \) in the potential are bounded by considering (co)annihilations into/through Higgs. We should point out that besides the quartic couplings \( \lambda's \), the terms associated with \( \mu \) in our model will also contribute to the coannihilations between \( S^- \) and \( \phi^-_2 \), \( \mu \) and \( \lambda's \) can be related as \( \mu \sim \lambda v \), thus as similar to the discussions in [4], \( M_s - M_{\phi^-_2} \) can be constrained in a few GeV and \( \mu \) is around \( O(100) \) GeV.

**Lithium problem.** The lithium problem arises from the significant discrepancy between the primordial \( ^7Li \) abundance as predicted by Standard Big Bang Nucleosynthesis (SBBN) and the WMAP baryon density, and the pre-Galactic lithium abundance inferred from observations of metal-poor stars[2, 8].

One of the solution to this is the so-called Catalytic Big Bang Nucleosynthesis (CBBN) [7] which states if a long-lived negatively-charged particle exists, it would form an exotic atom and work as a catalyzer. The bound state will induce reactions which can produce suitable primordial abundance of \( ^6Li \) and \( ^7Li \). In our model the scalar particle \( S^- \) will form the bound state with \( ^4He \) and this bound state will play the role as the catalyzer. The catalyptic path to \( ^6Li \) and \( ^9Be \) is

\[
S^- \rightarrow (^4HeS^-) \rightarrow ^6Li \quad \text{and} \quad S^- \rightarrow (^4HeS^-) \rightarrow (^8BeS^-) \rightarrow ^9Be. \tag{11}
\]

And the key for the nuclear catalysis is an enormous enhancement of the reaction rates in the photonless recoil reactions mediated by \( S^- \):

\[
(^4HeS^-) + D \rightarrow ^6Li + S^- \quad \text{and} \quad (^8BeS^-) + n \rightarrow ^9Be + S^-.
\tag{12}
\]

**FIG. 4:** three body decays of \( S^- \).

The rates of these catalyzed reactions depend sensitively on the abundance of \( S^- \) at the relevant times. The observations impose strong constraints on the lifetime of the negative charged particle to be \( \sim 10^8 \text{sec} \) to live long enough to form the exotic atom and catalyze the reactions. In our model a long-lived \( S^- \) can be achieved through the three body decays into the lepton sectors and dark matter in the final states as showed in Fig. 4.

With the new heavy leptonic doublet in the intermediate states plus the small Yukawa couplings and the phase space suppression of the mass differences between \( S^- \) and \( \phi^-_2 \), the long-lived \( S^- \) can be easily realized. The decay rate of \( S^- \) is

\[
\Gamma_{s|\alpha\beta(N_i)} \approx \frac{(f_{e1}y_{u\beta})^2}{30\pi^3 M^4_{N_i}} \times (\delta m)^5 (1 - \frac{5m^2}{\delta m^2}) \approx f_{e1}^2y_{u\beta}^2 \times 10^{-15}(\frac{\delta m}{1\text{GeV}})^5 \text{GeV}, \tag{13}
\]

where \( \delta m = M_s - M_{\phi^-_2} \). The lifetime will be around

\[
\tau_{\alpha\beta} \approx 6.6 \times f_{e1}^2y_{u\beta}^2 \times (\frac{\delta m}{1\text{GeV}})^{-5} \times 10^{-10} \text{sec}. \tag{14}
\]

Combining the parameters from neutrino mass, muon magnetic moment, and dark matter, i.e. \( f \approx O(10^{-4} \sim 10^{-5}) \) if \( \mu \approx O(100)\text{GeV}, y \approx (10^{-1} \sim 10^{-2}) \), and \( \delta m \approx O(1\text{GeV}) \), one naturally obtains the lifetime \( \tau_{\alpha\beta} \) to be within the required range to solve the lithium problem. Note that since the \( M_s - M_{\phi^-_2} \approx O(1\text{GeV}) \) from dark matter relic abundance, the three-body decay of \( S^- \) with a final \( \tau \) lepton is kinematically suppressed.

**Leptogenesis.** There are two sources of CP asymmetry from each of the new Yukawa interactions as drawn in Fig. 5. The decay rates of \( N_i \) are the sum of

\[
\Gamma_{N_i} = \sum_{\alpha}(y_{u\alpha})^2 M_{N_i} \quad \text{and} \quad \Gamma_{N_i} = \frac{(f_{e1})^2}{8\pi} M_{N_i}, \tag{15}
\]

which correspond to right-handed (left-handed) sector leptogenesis. Let’s consider the right-handed leptogenesis first [8, 9], we obtain the CP asymmetry

\[
\epsilon_1 = \frac{\Gamma(N_{i} \rightarrow \nu_{j} \phi_{2}^{+}) - \Gamma(N_{i} \rightarrow \nu_{j} \phi_{0}^{+})}{\Gamma(N_{i} \rightarrow \nu_{j} \phi_{2}^{+}) + \Gamma(N_{i} \rightarrow \nu_{j} \phi_{0}^{+})} = \frac{1}{8\pi} \sum_{m \neq 1} \frac{\text{Im}(y_{u}y_{u}^{\dagger})}{\sum_{\alpha}(y_{u}y_{u}^{\dagger})} \left( f_{e1} \frac{M_{m}}{M_{1}} + f_{e2} \frac{M_{m}}{M_{2}} \right), \tag{16}
\]

with the same notation as in previous equation.
where we have assumed the hierarchical masses of heavy neutrinos. In this channel, the usual constraints from neutrino masses disapparuch such that a hierarchical Yukawa couplings $y/s$ can easily be satisfied to realize the amount of the baryon asymmetry through leptogenesis, $\frac{1}{s} = -0.9 \times 10^{-3} \epsilon_1 \eta = 9 \times 10^{-11}$. For maximal efficiency, $\eta = 1$, we have the relation \[ \frac{y^{(1)}}{y^{(2)}} < 0.28 \times \sqrt{\frac{M_{N_1}}{M_{N_2}}} \frac{M_{N_1}}{M_{N_2}} 10^6 \text{GeV}. \] (17)

Note that $y^{(1)} = \sqrt{\sum_i |\langle y_{ii} \rangle|^2} < 3 \times 10^{-4} (\frac{M_{N_1}}{10^6 \text{GeV}})^{1/2}$ is bounded by the out-of-equilibrium condition, and $y^{(2)} = (\frac{\langle M_{N_1} \rangle}{\sum_i |\langle y_{ii} \rangle|^2})^{1/2} \geq 1.05 \times 10^{-3} (\frac{M_{N_1}}{M_{N_2}})^{1/2}$ is the amount of CP asymmetry we need. From these conditions, we can find the TeV solution of leptogenesis, for example, if $M_{N_1} = 1 T e V$, $M_{N_2} = 5 T e V$, $y^{(2)} \simeq 2.3 \times 10^{-3}$, and $y^{(1)} \simeq 3 \times 10^{-7}$. The effect of CP asymmetry from the left-handed sector leptogenesis is constrained by the scale of the neutrino mass and we find that the right-handed one gives the dominate contribution.

Note that there will be extra washout effects due to gauge interactions because of the non-trivial quantum numbers carried by $N_i$. A similar washout effects can be found in type II seesaw with decaying scalar triplet [10] and type III seesaw with decaying fermionic triplet [11]. It was shown in [10] that due to the Boltzmann suppression factor at temperatures below the gauge boson masses, they can not wash-out the lepton asymmetry in an efficient way. We should emphasis here that the DM $\phi^\mu$ is formed in the decays of $S^-$ (in the process of nucleosynthesis) and $N_i$ (in the process of leptogenesis).

D. Direct detection and Collider phenomenology

Direct detection of DM can be measured through the elastic scattering of a DM particle with a nuclei inside the detector. The Z boson exchange channel constrains the lower bound of the mass splitting $(M_{\phi^\mu} - M_{\phi^\tau})$ of order a few 100 keV [12]. The cross-section of the processes through exchange a Higgs scalar $h$ at tree level and gauge bosons at one-loop level are $\sigma h \approx f^2 R \lambda^2 \phi^\mu / 4\pi \times (m^2 / m_{DM} m^2)$ and $\sigma_{1-loop} = \frac{\eta_F^2 \pi N_{\text{Dirac}}}{64 M_W^2} \left( \frac{1}{M_W^2} + \frac{1}{m_N^2} \right)^2$. The late contribution is very interesting because it is independent of DM mass which sets a lower bound around $10^{-10} \text{ pb}$ [4]. Although it is beyond the current experimental sensitivity but it is reachable in next-generation experiments.

The new particles in our model are reachable at LHC. The productions are dominated by the processes $q + \bar{q} \rightarrow Z' \rightarrow X + X$, where $X$ represents $S^-$, $\phi^\tau$, and $L_i$, and $q + q' \rightarrow W^* \rightarrow h \phi^\tau (E^-) + \phi^\mu (N_i)$. Note the new particles must be produced in pairs according to $Z_2$ symmetry. The novel signatures will be the missing energy while producing the DM $\phi^\mu$ , and the decays of $\phi^\mu$ and $E^-_1$ are mainly into $\phi^\mu \pi^-$ and $N_i \pi^-$ respectively. The charged particles in our model will leave charged tracks in detectors and provide clean signatures due to their long lifetimes. One should note that $S^-$ and $\phi^\mu$ can only be produced through $h$ which is proportional to $\mu$.

III. SUMMARY AND DISCUSSIONS

In summary, we propose a model which can give neutrino mass with radiative seesaw mechanism and can account for the muon anomalous magnetic moment. In these two cases the GIM mechanism associated with the scale parameter $\mu$ plays an important role. By setting $\mu \sim O(100) \text{GeV}$ the relic abundance of dark matter from inert doublet scalar can be realized, and a long-lived negative charged particle which can fulfill the CBBN scheme for the Lithium problem. The model also contains the low scale leptogenesis which can help to solve the problem of baryon asymmetry in the Universe. The nearly degenerate spectrum of the particle content will give interesting collider phenomena and can be tested in the near future.

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