Succinct Oblivious RAM

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Abstract

Reducing the database space overhead is critical in big-data processing. In this paper, we revisit oblivious RAM (ORAM) using big-data standard for the database space overhead.

ORAM is a cryptographic primitive that enables users to perform arbitrary database accesses without revealing the access pattern to the server. It is particularly important today since cloud services become increasingly common making it necessary to protect users’ private information from database access pattern analyses. Previous ORAM studies focused mostly on reducing the access overhead. Consequently, the access overhead of the state-of-the-art ORAM constructions is almost at practical levels in certain application scenarios such as secure processors. On the other hand, most existing ORAM constructions require \((1 + \Theta(1))n\) (say, \(10n\)) bits of server space where \(n\) is the database size. Though such space complexity is often considered to be “optimal”, overhead such as \(10x\) is prohibitive for big-data applications in practice.

We propose ORAM constructions that take only \((1+o(1))n\) bits of server space while maintaining state-of-the-art performance in terms of the access overhead and the user space. We also give non-asymptotic analyses and simulation results which indicate that the proposed ORAM constructions are practically effective.

1 Introduction

Oblivious RAM (ORAM) is a cryptographic primitive that enables users to access a database on a server without revealing the access pattern to the server. Although originally introduced in the context of software protection [14], ORAM is directly relevant to the present cloud computing scenarios.

In the previous studies on ORAM, researchers focused mainly on reducing the access bandwidth cost, a performance measure used as a proxy of the access time. This is because even the current most state-of-the-art ORAM constructions have two or three orders of magnitude larger bandwidth cost than the ordinary (non-secure) accesses. However, in certain settings, the ORAM access is already rather efficient. For example, Maas et al. proposed PHANTOM [23], an ORAM-based secure processor, and reported that if PHANTOM is deployed on the server, SQLite queries can be performed without revealing the access pattern at the cost of 1.2–6\(\times\) slowdown compared to non-secure SQLite queries. In such cases, it is reasonable to pay more attention to performance measures other than the access speed.

In particular, the server space usage is a very important performance measure for big-data applications. First, there are applications where the amount of data is virtually unbounded, and thus the limit of the available space defines the limit of the analyses. Second, due to the cache effect, small memory usage often leads to faster computation. Third, space costs money, especially in a cloud computing server. The second and the third points are especially relevant if the data is meant to be stored in the main memory (by default), which is exactly the case in ORAM application scenarios such as PHANTOM.

In most modern ORAM constructions, if the size of the original database is \(n\) bits, the amount of the space required by the server is \(n + \Theta(n)\) bits. In this paper, we investigate the possibility of ORAM constructions that need only \(n + o(n)\) bits of server space. We call such ORAM constructions succinct. This space efficiency formalization is widely used in the field of succinct data structures and has proved to be useful to design practically relevant space-efficient data structures in theoretically clean ways.

The main difficulty to achieve succinctness is that most existing ORAM construction approaches rely on the use of linear amount of “dummy” data. The situation is similar to conventional hash tables,
which need extra space linear to the stored keys size. Although it seems possible to reduce the constant factor of the extra space to some extent, it is not at all trivial if one can achieve sublinear extra space maintaining the state-of-the-art performance in other aspects such as access bandwidth and user space usage.

\textbf{Results.} Table\[\text{I}\] shows the performance comparison of the proposed methods and the existing methods. Our first construction takes \(n(1 + \Theta(\log n + \frac{g(n)}{f(\log n)}))\)-bit server space where \(n\) is the database size, \(f_1(\cdot)\) is an arbitrary function such that \(f_1(n) = \omega(\log n)\) and \(O(\log^2 n)\), \(g(\cdot)\) is an arbitrary function such that \(g(n) = \omega(1)\) and \(o(\sqrt{f_1(n)/\log n})\), and \(B\) is the size of a block, the unit of communication between the user and the server. The bandwidth blowup is \(O(\log^2 n)\) and the user space is \(O(f_1(n))\) blocks. Our second construction achieves \(n(1 + \Theta(\frac{\log n}{f_2(n)} + \frac{\log \log n}{f_2(n)}))\)-bit server space, \(O(\log^2 n)\)-bandwidth blowup and \(O(f_2(n) + R(n))\)-user space where \(f_2(\cdot)\) is an arbitrary function such that \(f_2(n) = \omega(\log \log n)\) and \(O(\log^2 n)\), \(R(\cdot)\) is an arbitrary function such that \(R(n) = \omega(\log n)\).

For example, suppose \(B = \log^2 n\), \(R = \log n \log \log n\), \(f_1(n) = f_2(n) = \log n \log \log n\) and \(g(n) = \log \log \log n\). Then, the user space of each of our first and the second constructions is \(O(\log n \log \log n)\) and the server space is \(n(1 + \Theta(\frac{\log \log n}{\log n}))\) (resp. \(n(1 + \Theta(\frac{1}{\log n}))\)) bits in the first (resp. second) construction.

The second construction has better theoretical performance than the first one. However, in practice, with some parameter settings, the first construction also works comparably well as the second construction depending on which performance measure one cares (See Section\[\text{5}\]). The first construction is also the basis of the second construction.

If \(B = \omega(\log n)\), Goldreich’s construction\[\text{14}\] and our constructions are succinct. Each of these methods works as long as \(B \geq c \log n\) for \(c\) around 3. The assumption \(B = \omega(\log n)\) is justified as follows. Stefanov et al.\[\text{40}\] mentioned that the typical block size is 64–256 KB (resp. from 128B to 4KB) in cloud computing scenario (resp. software protection scenario). Even \(B \geq \log^{1.5} n\) holds if \(n \leq 2^{501}\) (resp. \(n \leq 2^{97}\)) in cloud computing (resp. software protection) scenario with moderate block size of 64KB (resp. 128B).

We achieved exponentially smaller bandwidth blowup compared to Goldreich’s construction\[\text{14}\], which is the only preceding non-trivial succinct ORAM construction.

The bandwidth blowup of our constructions is smaller or equal to other non-succinct constructions except the construction of Kushilevitz et al.\[\text{22}\], the Onion ORAM\[\text{7}\] and the so-called SSS construction\[\text{39}\]. The construction of Kushilevitz et al.\[\text{22}\] and every other construction that is listed above it in Table\[\text{I}\] is based on a very expensive procedure called oblivious sorting and the constant factor hidden in the asymptotic notation of the bandwidth cost is prohibitively large. The Onion ORAM achieves \(O(1)\)-bandwidth blowup but it requires several assumptions. First, the Onion ORAM requires the server to perform some computation, e.g., homomorphic encryption evaluation. (In every other construction in Table\[\text{I}\] the server suffices to respond to read/write requests.) It also requires a computational assumption (decisional composite residuosity assumption or learning with errors assumption), and larger block size \((B = \tilde{\omega}(\log^2 n))\) to \(\tilde{\omega}(\log^6 n)\) depending on the exact construction, where \(\tilde{\omega}(\cdot)\) hides a poly-log-log factor. The SSS construction takes \(cn\)-bit user space where \(c \ll 1\). This method is effective for ordinary cloud computing setting but the user space is too large for secure processor setting — the PHANTOM-like applications where server space efficiency is more important.

\textbf{Possible applications.} There are several ORAM application scenarios with different requirements. Our methods are particularly relevant to secure processor scenario. In this scenario, it is assumed that a special processor under the control of the user is available in a remote server and the adversary cannot observe the activities inside the processor. The cloud service user sends a piece of code to the trusted processor, which, in turn, executes the code on the server. The communication between the cloud service user and the secure processor is protected by private key encryption. ORAM is implemented inside of the trusted processor using FPGA and it hides the processor’s access pattern to the main memory on the server. After executing the code, the secure processor may return the (encrypted) output to the cloud service user. One of the main advantages of this approach over the conventional ORAM application, in which the cloud service user locally executes ORAM, is that ORAM bandwidth blowup applies to the relatively cheap processor–memory communication rather than the costly over-network communication. Note that, with the ORAM user-server terminology, the secure processor (resp. the main memory) is the user (resp. the server).

In secure processor scenario,

- the user space is very limited, e.g., 6MB;
Table 1: Comparison of theoretical performances. Bandwidth blowup is the number of blocks required to be communicated for accessing one block of data. User space includes the temporary space needed during access procedures. \( n \) is the database size in bits and \( B \) is the block size in bits. \( B \) must satisfy \( B \geq c_1 \lg n \) and \( B = O(n^{c_2}) \) for constants \( c_1 > 1, 0 < c_2 < 1 \). Typically, \( c_1 \) is around 3. \( f_1(\cdot) \) is an arbitrary function such that \( f_1(n) = \omega(\log n) \) and \( O(\log^m n) \). \( f_2(\cdot) \) is an arbitrary function such that \( f_2(n) = \omega(\log^2 n) \) and \( O(\log^m n) \). \( R(\cdot) \) is an arbitrary function such that \( R(n) = \omega(\log n) \). \( g(\cdot) \) is an arbitrary function such that \( g(n) = \omega(1) \) and \( o(\sqrt{f_1(n)}/\log n) \). Bounds with \( \dagger \) are amortized. The method in [7] requires additional assumptions. The user space bound of the method in [39] has a constant factor \( \ll 1 \).

|                      | Server space (#bits)                                      | Bandwidth blowup                  | User space (#block)         |
|----------------------|-----------------------------------------------------------|-----------------------------------|-----------------------------|
| Goldreich [14]       | \( n \left( 1 + \Theta \left( \frac{\log n}{B} + \frac{1}{\sqrt{n}} \right) \right) \) | \( O(\sqrt{n} \log n) \dagger \) | \( O(1) \)                |
| Ostrovsky [29]       | \( O(n \log n) \)                                      | \( O(\log^3 n) \dagger \)        | \( O(1) \)                |
| Ostrovsky, Shoup [30] | \( n(1 + \Theta(1)) \)                                | \( O(\sqrt{n} \log n) \)        | \( O(1) \)                |
| Our result (Theorem 1)| \( n(1 + \Theta(1)) \)                                | \( O(\log^2 n) \)               | \( O(1) \)                |
| Our result (Theorem 2)| \( n(1 + \Theta(1)) \)                                | \( O(\log^2 n) \)               | \( O(R(n)) \)             |

- The server usually does not perform complex computation;
- Simple ORAM algorithms are desirable for hardware implementation;
- The server space is much larger than the user space but there is some noticeable limit. The server can use disks if needed but it greatly slows down accesses.

In most existing secure processor systems, the Path ORAM [40] or its close variants are used [11, 12, 23, 33]. Indeed, the Path ORAM satisfies the first three requirements above. However, it does not capture the last point. For example, suppose 128GB database is stored in the Path ORAM. If the block size is 128B, it takes about 10G blocks, i.e., 1.28TB (to ensure rigorous security). Then, each ORAM access procedure takes about 31\( \mu \)s assuming each memory access takes 100ns. If half of the 10G blocks are stored in the main memory and the other half is stored in the disk, due to the randomized access pattern of the Path ORAM, almost every ORAM access procedure ends up a disk seek, which takes milliseconds order time. In such cases, it is reasonable to use another ORAM construction that takes, say, half the space of the Path ORAM even though it requires twice as many memory accesses.

Tree-based ORAM. Our ORAM constructions are tree-based. In a typical tree-based ORAM construction, \( N \) blocks are stored in a complete binary tree with \( N \) leaves on the server. Each node of the tree can store up to \( Z \) blocks where \( Z \) is a constant. Each block is assigned a position label, which is an integer chosen uniformly at random from \([N]\). A block with position label \( i \) must be stored at some node on the path from the root to the \( i \)-th leaf. This framework was introduced by Shi et al. [38] and used in many subsequent studies [5, 7, 13, 35, 40].

Consider a particular block \( b \). As the user continuously issues access requests, \( b \) moves around the tree in roughly the following manner. First, when the user issues an access request to \( b \), \( b \) is picked out of the tree and given a new uniformly random position label. Then, \( b \) is inserted into the tree from the root. If the user issues an access request to another block, then, with some probability, \( b \) will move down the path to the leaf indicated by its position label. If the next node on the path is full, \( b \) must wait for the blocks “ahead” to move down. If the pace at which the blocks move down the tree cannot keep up with the pace at which blocks are picked out and reinserted from the root, then, some blocks will not be able to reenter the tree. If such “congestion” occurs, the user must maintain the overflown blocks locally.

Note that most space in the tree is wasted: there are \( 2N - 1 \) nodes in the tree, each with capacity \( Z \), whereas there are only \( N \) blocks. Thus, to save server space, it is desirable to make the tree more
compact, for example, by reducing $Z$. However, to maintain a low probability of “congestion”, it is desirable to make the tree larger, for example, by increasing $Z$. To construct a succinct tree-based ORAM, we need to satisfy these conflicting demands simultaneously.

**Our ideas.** One of our key ideas is the following two-stage tree layout. We first change the tree to a complete binary tree with $N/\lg^{1.4} N$ leaves (assume this is a power of 2). In addition, we set the capacity of each leaf node to $\lg^{1.4} N + \lg^{1.3} N$ while keeping the capacity of each internal node at $Z$. The total size of the leaf nodes is then $N + N/\lg^{0.4} N$, and the total size of all tree nodes except the leaves is $\Theta(N/\lg^{1.4} N)$. Thus, the total size of the entire tree is $N + o(N)$. We choose each position label from $[N/\lg^{1.4} N]$.

To see why blocks can flow around in this tree without much congestion, suppose that the user inserts each block directly into the leaf node pointed to by the block’s position label. Clearly, the loads of leaves in this hypothetical setting dominates the loads of leaves in the real setting. Then, the situation would exactly be the same as the “balls-into-bins” game [24] with $N$ balls and $N/\lg^{1.4} N$ bins. In particular, the number of blocks stored in each leaf node is $\log^{1.2} N + \Theta(\log^{0.6} N)$ with high probability. Thus, every leaf node has sufficient capacity to store all of its assigned blocks.

Furthermore, the blocks in the internal nodes flow as smoothly as in the original non-succinct ORAM construction since we did not modify that part. Therefore, the blocks flow without much congestion throughout the tree. This is the idea behind the first construction (Theorem 1).

Another key idea follows naturally from the above argument, specifically from the connection to the balls-into-bins game. A remarkable phenomenon known as “the power of two choices” states that, in the balls-into-bins game, if one chooses two bins uniformly and independently for each ball, and throws the ball into the least loaded bin, the bin loads will be distributed much more tightly around the mean than they are in the one-choice game [1] [3] [24]. The maximum bin load corresponds to the leaf node size in tree-based ORAM constructions. Thus, the size of the tree can be further decreased by using the two-choice strategy to assign the position labels. This is the idea behind the second construction (Theorem 2).

We note that the current paper is the first to apply the power of two choices to tree-based ORAM. (Some non-tree-based constructions [16] [22] [32] use the two choices idea in the form of cuckoo hashing [31].) Moreover, the resulting algorithms keep the simplicity of the Path ORAM [40], which is a highly valuable asset in the relevant application scenario as mentioned above. As for the analysis, the existing stash size analyses [35] [40] do not seem to work with parameter regimes required for succinctness. We will give a different proof route (though it still heavily borrows from [35] [40]).

**Our contributions.** Our contributions in the current paper are as follows:

- We introduce the notion of succinct oblivious RAM. This is a promising first step to systematically design ORAM constructions with small server space usage;
- We propose two succinct ORAM constructions. Not only being succinct, these constructions exhibit state-of-the-art performance in terms of the bandwidth blowup. The methods are simple and easy to implement;
- We also give non-asymptotic bounds and simulation results which indicate that the proposed methods are practically effective.

### 1.1 Related Work

In the field of succinct data structures [15] [20], the goal is to represent an object such as a string [9] [10] [15] [17] [18] [21] [26] [27] [36] [37] or a tree [2] [6] [8] [25] [28] [33] in such a way that a) only $OPT + o(OPT)$ bits are required, and b) relevant queries such as random access or substring search are efficiently supported. Here, $OPT$ is the information theoretic optimum, i.e., the minimum number of bits needed to represent the object.

The current study is related to succinct data structures in the following way. Suppose a remote server hosts a database that is implemented by a succinct data structure, and a user wishes to access the database without revealing the access pattern to the server. The user, of course, can apply any existing ORAM constructions. However, if ORAM increases the database size by some constant factor, it destroys the $OPT + o(OPT)$ bound guaranteed by the succinct data structure. One can apply the succinct ORAM constructions proposed in this paper to hide succinct data structure access pattern on a remote storage device without harming the theoretical guarantee on the data structure size.
1.2 Organization of the Paper

In Section 2, we introduce basic notions that will be used in later sections. We describe our first succinct ORAM construction (encapsulated in Theorem 1) in Section 3 and the second construction (encapsulated in Theorem 2) in Section 4. Then, we present non-asymptotic analyses and simulation results in Section 5. We conclude the paper in Section 6.

2 Preliminaries

2.1 Notations

We denote the set \{0, 1, \ldots, n - 1\} as \([n]\) for a non-negative integer \(n\). We write \(\lg x\) to denote the base-2 logarithm of \(x\) and \(\ln x\) to denote the natural logarithm of \(x\). We write \(\log x\) to denote the logarithm of \(x\) in the context where the base can be any positive constant. We write \(\text{poly}(n)\) to denote \(n^c\) for some constant \(c > 0\). A negligible function of \(n\) is defined to be a function that is asymptotically smaller than \(1/n^c\) for any constant \(c > 0\).

2.2 Oblivious RAM

**Definition.** Oblivious RAM is defined through the interaction between three parties the user, the server and the oblivious RAM (ORAM) simulator. The user wishes to perform random access to the database on the server without revealing the “access pattern” to the server. Roughly speaking, the ORAM simulator works as a mediator between the user and the server. It takes access requests to the database from the user and translates them to “appropriate” access requests to the server. The database on the server can be maintained as some “data structure” instead of the raw form on which the user intends to perform random access and thus, the access requests to the server need not be (and are not) the same as the access requests to the database. The ORAM simulator then, performs random accesses to the server on behalf of the user (using translated requests), thereby making it impossible for the server to infer the access patterns to the database even though the accesses to the server is visible from the server.

Formally, let each of \(B\) and \(n\) be a positive integer and \(N := \lceil n/B \rceil\). The value \(B\) models the unit of communication and \(n\) models the database size. We call a chunk of \(B\) bits a block. For brevity, we assume \(n\) is a multiple of \(B\) in the rest of the paper. A logical (resp. physical) access request is a triplet \((\text{op}, \text{addr}, \text{val})\), where \(\text{op} \in \{\text{read}, \text{write}\}\), \(\text{addr} \in [N]\) (resp. \(\text{addr} \in \mathbb{N}\)), \(\text{val} \in \{0, 1\}^B\). The user sends logical access requests to the ORAM simulator and receives a block for each request. The server receives physical access requests from the ORAM simulator and returns a block for each request in the following way: for \((\text{read}, i, v)\), the server returns \(v\) of the most recent request \((\text{write}, i, v)\). The ORAM simulator takes a sequence of logical access requests from the user and for each logical access request, it makes a sequence of physical access requests to the server receiving a returned block for each of them, and returns a block to the user. The ORAM simulator is possibly stateful and probabilistic. It must respond to logical access requests online and must satisfy the following conditions:

**Correctness** The ORAM simulator is correct if and only if, for a logical access request with \(\text{addr} = i\), it returns \(v\) of the previous and most recent logical access request \((\text{write}, i, v)\).\(^2\)

**Security** The ORAM simulator is computationally (resp. information theoretically) secure if and only if, for any logical access request sequences of the same length, the distributions of the addr values of the resulting physical access requests are computationally (resp. information theoretically) indistinguishable.

An ORAM construction is an ORAM simulator implementation. We have distinguished the user from the ORAM simulator for exposition but in practice, an ORAM simulator is a program run by the user. Thus, we do not distinguish them in the rest of the paper.

**Encryption.** In the ORAM constructions considered in this paper, the user holds a symmetric cipher key and every block is encrypted when it is stored on the server. Encryption can increase the database size. Theoretically, we can bound the space overhead due to encryption to \(o(1)\)-factor. For example, one can encrypt a block \(m\) as \((r, m \oplus F(r))\) where \(r\) is a random bits of size \(\omega(\log n)\) and \(o(B)\), \(F\) is encrypted.

\(^2\)We use the convention that not only read but also write requests have return values.
a pseudorandom function (key is omitted) and \( \oplus \) denotes bitwise XOR. Or, in practice, one can use “counter mode” of block cipher, i.e., encrypting a block \( m \) as \((i, m \oplus F(z \| i))\) where \( F \) is AES, \( i \) is the number of blocks encrypted so far and \( z \) is a nonce. Assuming that we allocate 128 bits to \( i \) and the typical block sizes mentioned in Section 1, the additional space is 1/4096–1/16384 (resp. 1/8–1/256) factor of the original database size in cloud computing (resp. software protection) scenario. Since the space overhead due to encryption is rather small, we ignore it in the rest of the paper.

Performance measures. The most popular ORAM performance measures include the amount of the space required by the user/server and the amount of time required for each logical access.

In most ORAM constructions, the user needs to maintain a small amount of information locally. In addition to this, in some constructions, the user temporarily needs to store more information during the access procedure. We refer the amount of the space the user temporarily needs during access procedure as temporary space usage and the amount of the space the user needs even if no access is made as permanent space usage.

In this paper, we pay special attention to the server space usage. In particular, we use the following notion of succinctness as a criterion for ORAM server-space efficiency:

**Definition 1.** If the server space usage of an ORAM construction representing an \( n \)-bit database is \( n + o(n) \) bits, the ORAM construction is said to be succinct.

As for the access efficiency, following the previous studies, we use the amount of communication between the user and the server as a proxy for the access time. We define the bandwidth blowup of an ORAM construction to be the number of blocks that needs to be communicated between the user and the server per logical access. In other words, the bandwidth blowup is the ratio of communication amount needed for secure access to communication amount needed for ordinary (insecure) access.

Asymptotic behavior of parameters. Among the ORAM-related parameters, the original database size \( n \) and block size \( B \) are outside of the user’s control. Other parameters, e.g., the metadata size, can be chosen by the user. We assume that \( B \) is a function of \( n \) satisfying \( B = \omega(\log n) \). (See Section 1 for the justification.) Thus, after all, \( n \) is the only free parameter on which the other parameters depend. In all asymptotic statements in this paper, the limit is taken as \( n \rightarrow \infty \).

2.3 Sub-ORAM

We use an ORAM construction encapsulated into the following proposition as a blackbox. Concretely, the Path ORAM \(^{[40]}\) suffices.

**Proposition 1.** Let \( n \) be the database size and \( B \) be the block size, both in bits. If \( B \geq 3 \log n \) and \( B = O(n^{c}) \) for some \( 0 < c < 1 \), there exists an information theoretically secure ORAM construction such that i) the server’s space usage is \( n \left(10 + \Theta \left(\frac{\log n}{B}\right)\right) \) bits; \(^{[3]}\)

ii) the worst-case bandwidth blowup is \( O(\log^{2} n) \); iii) the user’s temporary space usage is \( O(\log n) \) blocks; and iv) for any \( R = \omega(\log n) \), the probability that the user’s permanent space usage becomes larger than \( R \) blocks during \( \text{poly}(n) \) logical accesses is negligible.

3 Succinct ORAM Construction

In this section, we prove the following theorem.

**Theorem 1.** Let \( n \) be the database size and \( B \) be the block size, both in bits. If \( B \geq 3 \log n \) and \( B = O(n^{c}) \) for some constant \( 0 < c < 1 \), then for any \( f : \mathbb{N} \rightarrow \mathbb{R} \) such that \( f(n) = \omega(\log n) \) and \( f(n) = O(\log^{2} n) \) and any \( g : \mathbb{N} \rightarrow \mathbb{R} \) such that \( g(n) = \omega(1) \) and \( g(n) = o(\sqrt{f(n)/\log n}) \), there exists an information theoretically secure ORAM construction such that i) the server’s space usage is bounded by \( n \left(1 + \Theta \left(\frac{\log n}{B} + \frac{g(n)}{\sqrt{f(n)/\log n}}\right)\right) \) bits;

\(^{[3]}\) The bandwidth blowup is a ratio and does not have a unit.

\(^{[4]}\) The description of the original paper depends on the assumption that \( N \) is a power of two. If this assumption is not true and we pad the database to make \( N \) a power of two, the factor 10 in the server space bound becomes 20.
Corollary 1. If, in addition to the conditions of Theorem 1, \( R \) blocks during internal buckets the internal nodes as a container that can accommodate a certain number of blocks. We call the buckets corresponding to labels of all real blocks. Below, we explain each of them more in detail.

Position table contains the real blocks that are not in the data tree, and stash at each point of time, it contains most real blocks with high probability. The user maintains metadata tree containing metadata blocks, which we call \( N \) where

\[
\frac{n}{B} \leq N = n/B.
\]

We assume, for brevity, that each of \( \log n \) and \( \sqrt{NL} \) is an integer.

Block usage. The ORAM is supposed to provide the user with an interface to access the database as if it is stored in array \( A \) of \( B \)-bit blocks (Subsection 2.2). We use blocks as follows:

- Each block is either a data block or a metadata block;
- Each data block is either a real block or a dummy block. A real block contains an entry of \( A \). A dummy block does not contain any information on the database contents and is used only to hide the access pattern;
- Each real block is given a position label, a value in \( [2^L] \);
- A metadata block contains the metadata of several data blocks. For each data block, its metadata consists of
  - type: A flag indicating whether the block is real or dummy;
  - addr: If the block is real and represents \( A[i] \), the value of addr is \( i \). If the block is a dummy, the value is arbitrary;
  - pos: If the block is real with position label \( i \), the value of pos is \( i \). If the block is a dummy, the value is arbitrary.

Data layout. The server maintains a tree containing data blocks, which we call data tree, and another tree containing metadata blocks, which we call metadata tree. The data tree is used in such a way that at each point of time, it contains most real blocks with high probability. The user maintains stash, which contains the real blocks that are not in the data tree, and position table, which contains the position labels of all real blocks. Below, we explain each of them more in detail.

The data tree is a complete binary tree with \( 2^L \) leaves. Each node of the tree is a bucket, which is a container that can accommodate a certain number of blocks. We call the buckets corresponding to the internal nodes as internal buckets and the buckets corresponding to the leaf nodes as leaf buckets. The size of each internal bucket is \( Z \) (blocks) while the size of each leaf bucket is \( M \) (blocks). We will determine \( Z \) to be 3 in Subsection 5.5 but for now, we consider it as an arbitrary constant. The data tree is represented as the bitstring derived by concatenating all buckets in breadth first order. As is well-known, with this representation, given an index of a node, the index of the parent or left/right child can be derived by simple arithmetic. The total space usage of the data tree is equal to the sum of the bucket sizes.

The metadata tree is also a complete binary tree with \( 2^L \) leaves. Each node of the tree is the metadata of the data blocks in the corresponding bucket of the data tree. The metadata tree is represented similarly to the data tree but there is a subtlety. If the metadata of the blocks in a bucket has a size smaller than \( B \), it is wasteful to allocate one full block for them. To avoid this waste, we represent metadata tree as the bitstring derived by concatenating the metadata of all data blocks in the data tree in breadth first order. The space usage of the metadata tree is equal to the sum of all metadata of all data blocks.
Each real block in the stash is maintained with its addr and pos. The stash can be any linear-space data structure that efficiently supports insertion, deletion and range query by pos, e.g., a self balancing binary search tree.

The position table stores the position label of the real block storing $A[i]$ in the $i$-th entry.

**Access procedure.** Access requests are processed in such a way that the following invariant conditions are always satisfied:

- Each real block is stored either in the data tree or in the stash;
- If a real block with position label $\ell$ is stored in the data tree, it is in the bucket on the path from the root to the $\ell$-th leaf.

The main routine of the access procedure is described in Algorithm 1 and the subroutines for Access are described in Algorithm 2. The notations used in the access procedure are summarized in Table 2.

| Notation   | Description                                                                 |
|------------|-----------------------------------------------------------------------------|
| Pos        | the position table                                                          |
| $P(\ell)$  | the path from the root to the $\ell$-th leaf of the data tree               |
| $P(\ell, i)$ | the depth $i$ bucket on $P(\ell)$ (the root is at depth 0)                |
| $P(\ell, i, j)$ | the $j$-th block in $P(\ell, i)$ (counted from one)              |
| meta[$P(\ell, i)$] | the metadata of the blocks in $P(\ell, i)$   |
| $|P(\ell, i)|$ | the number of blocks in $P(\ell, i)$ ($|P(\ell, i)| = Z$ for $i < L$ and $|P(\ell, L)| = M$) |
| md[i]      | the $i$-th metadata in md (if md = md[$P(\ell, i)$], the metadata of $P(\ell, i, j)$) |
| RANDOM($b$) | returns a uniformly random $b$-bit integer                                 |
| BitReversal($\ell$) | returns the $L$-bit integer derived by reversing the bits of $L$-bit integer $\ell$ |
| $G$        | a persistent/global variable storing the number of Access called so far   |

![Table 2: The notations for access procedure](image)

$^5$We note that the pseudocode and notations borrow much from existing work [33] [40].
Algorithm 1 Main routine

1: function Access(a, op, v')
2: \[\ell' \leftarrow \text{Random}(L)\]
3: \[\ell \leftarrow \text{Pos}[a]\]
4: \[\text{Pos}[a] \leftarrow \ell'\]
5: \[v \leftarrow \text{ReadPath}(\ell, a)\]
6: if \(v = \bot\) then
7: \[\text{find } (a, \ell, v'') \in \text{stash} > \text{there exists } (a, \ell, v'') \in \text{stash}\]
8: \[v \leftarrow v''\]
9: \[\text{stash} \leftarrow \text{stash} \setminus (a, \ell, v'')\]
10: \[\text{ret} \leftarrow v\]
11: if \(\text{op} = \text{write}\) then
12: \[v \leftarrow v'\]
13: \[\text{stash} \leftarrow \text{stash} \cup (a, \ell', v)\]
14: \[\text{EvictPath}()\]
15: return \text{ret}

Outsourcing position table. In the construction described so far, the user space usage is much larger than the bound claimed in Theorem 1 since the user needs to maintain the position table locally. To obtain Theorem 1 we modify the construction so that the position table is stored on the server using the sub-ORAM in Proposition 1, e.g., the Path ORAM \[40\]. Access procedure is the same except that the line 3–4 of \(\text{Access}(\ell \leftarrow \text{Pos}[a] \text{ and Pos}[a] \leftarrow \ell)\) is replaced by a sub-ORAM write access.

3.2 Security

Fix \(t > 0\). Let \(a\) be a length \(t > 0\) sequence of logical addresses to be accessed and \(a'\) be the corresponding sequence of physical addresses (indices of the server memory) to be accessed. The sequence \(a'\) is determined by \(a\) and the randomness used by the ORAM simulator. To prove the information theoretic security, it suffices to show that \(a'\) really does not depend on \(a\). The sequence \(a'\) consists of \(a'_1\), the physical addresses accessed in step 3–4 of \(\text{Access}\) and \(a'_2\), those accessed in the rest parts of \(\text{Access}\). The addresses \(a'_1\) is determined by the sub-ORAM access procedure and is independent of \(a\) due to the information theoretic security of the sub-ORAM. The addresses \(a'_2\) consists of addresses accessed by \(\text{ReadPath}(\ell, a)\) and \(\text{EvictPath}()\). \(\text{ReadPath}(\ell, a)\) accesses the path \(P(\ell)\), which is determined by \(\ell\), the position label of the accessed block. Since the position labels are chosen independently and uniformly at random, the \(\text{ReadPath}\) accesses are independent of \(a\). \(\text{EvictPath}\) accesses \(P(\text{BitReversal}(G))\), which is determined by \(G\), the number of times \(\text{Access}\) was called (modulo \(2^L\)). Thus, the accesses of \(\text{EvictPath}\) is also independent of \(a\). Therefore, \(a'\) is independent of \(a\).

3.3 Server Space

First, it is helpful to observe the followings:

\[
\log N = \Theta(\log n), \quad L = \Theta(\log n), \quad M = \Theta(f(n)). \tag{1}
\]

Remember that the server holds the data tree, the metadata tree and the position table.

The total size of the internal (resp. leaf) buckets is \(Z(2^L - 1)\) (resp. \(M2^L\)) blocks. Since

\[
Z(2^L - 1) < Z2^L = ZN/f(n), \quad M2^L = N + g(n)\sqrt{NL2^L} = N \left(1 + \Theta\left(\frac{g(n)}{\sqrt{f(n)/\log n}}\right)\right),
\]

the number of the blocks in the data tree is bounded by

\[
ZN/f(n) + N \left(1 + \Theta\left(\frac{g(n)}{\sqrt{f(n)/\log n}}\right)\right) = N \left(1 + \Theta\left(\frac{1}{f(n)} + \frac{g(n)}{\sqrt{f(n)/\log n}}\right)\right).
\]
Algorithm 2 Subroutines for \textsc{Access}

1: \textbf{function} \textsc{ReadPath}(\ell,a) \\
2: \hspace{1em} v \leftarrow \perp \\
3: \hspace{1em} \textbf{for} i \leftarrow 0 \text{ to } L \textbf{ do} \\
4: \hspace{2em} md \leftarrow \text{meta}[P(\ell,i)] \\
5: \hspace{2em} \textbf{for} j \leftarrow 1 \text{ to } |P(\ell,i)| \textbf{ do} \\
6: \hspace{3em} v' \leftarrow P(\ell,i,j) \\
7: \hspace{3em} \textbf{if} md[j] = (\text{real},a,\ell) \text{ then} \\
8: \hspace{4em} v \leftarrow v' \\
9: \hspace{4em} md[j] \leftarrow (\text{dummy},\cdot,\cdot) \\
10: \hspace{2em} \text{meta}[P(\ell,i)] \leftarrow md \\
11: \hspace{1em} \textbf{return} v \\

1: \textbf{function} \textsc{EvictPath}( ) \\
2: \hspace{1em} \ell \leftarrow G \mod 2^L \hspace{1em} \triangleright G \text{ is global/persistent, and initially zero} \\
3: \hspace{1em} G \leftarrow G + 1 \\
4: \hspace{1em} \ell' \leftarrow \text{BitReversal}(\ell) \\
5: \hspace{1em} \textbf{for} i \leftarrow 0 \text{ to } L \textbf{ do} \\
6: \hspace{2em} \text{stash} \leftarrow \text{stash} \cup \text{ReadBucket}(P(\ell',i)) \\
7: \hspace{1em} \textbf{for} i \leftarrow L \text{ to } 0 \textbf{ do} \\
8: \hspace{2em} \text{WriteBucket}(P(\ell',i),\text{stash}) \\

1: \textbf{function} \textsc{ReadBucket}(P(\ell,i)) \\
2: \hspace{1em} S \leftarrow \emptyset \\
3: \hspace{1em} md \leftarrow \text{meta}[P(\ell,i)] \\
4: \hspace{1em} \textbf{for} j \leftarrow 1 \text{ to } |P(\ell,i)| \textbf{ do} \\
5: \hspace{2em} v \leftarrow P(\ell,i,j) \\
6: \hspace{2em} \textbf{if} md[j] = (\text{real},a,\ell') \text{ for some } a \text{ and } \ell' \text{ then} \\
7: \hspace{3em} S \leftarrow S \cup (a,\ell',v) \\
8: \hspace{3em} md[j] \leftarrow (\text{dummy},\cdot,\cdot) \\
9: \hspace{1em} \text{meta}[P(\ell,i)] \leftarrow md \\
10: \hspace{1em} \textbf{return} S \\

1: \textbf{function} \textsc{WriteBucket}(P(\ell,i),\text{stash}) \\
2: \hspace{1em} S \leftarrow \text{blocks in the stash whose labels have the same length } i \text{ prefix as } \ell \\
3: \hspace{1em} \textbf{for} j \leftarrow 1 \text{ to } |P(\ell,i)| \textbf{ do} \\
4: \hspace{2em} \textbf{if} S \neq \emptyset \text{ then} \\
5: \hspace{3em} \text{pick arbitrary } (a,\ell,v) \in S \\
6: \hspace{3em} P(\ell,i,j) \leftarrow v \\
7: \hspace{3em} md[j] \leftarrow (\text{real},a,\ell) \\
8: \hspace{3em} S \leftarrow S \setminus (a,\ell,v) \\
9: \hspace{2em} \textbf{else} \\
10: \hspace{3em} P(\ell,i,j) \leftarrow \text{garbage} \\
11: \hspace{3em} md[j] \leftarrow (\text{dummy},\cdot,\cdot) \\
12: \hspace{1em} \text{md}[P(\ell,i)] \leftarrow md
The metadata for each data block takes 1 bit for type, $\lceil \log N \rceil$ bits for $addr$ and $L$ bits for $pos$. The total is $\Theta(\log n)$ bits, which is $\Theta\left(\frac{\log n}{B}\right)$ blocks. Thus, the number of bits in the data tree and the metadata tree combined is

$$BN\left(1 + \Theta\left(\frac{1}{f(n)} + \frac{g(n)}{\sqrt{f(n)/\log n}}\right)\right)\left(1 + \Theta\left(\frac{\log n}{B} + \frac{g(n)}{\sqrt{f(n)/\log n}}\right)\right) = n\left(1 + \Theta\left(\frac{\log n}{B} + \frac{g(n)}{\sqrt{f(n)/\log n}}\right)\right).$$

The position labels take $\Theta(n\log B)$ bits. By Proposition 1, the sub-ORAM containing the position table takes $\Theta(n\log B)$ bits. Thus, the server space is $n(1 + \Theta(\frac{\log n}{B} + \frac{g(n)}{\sqrt{f(n)/\log n}}))$ bits.

### 3.4 Bandwidth Blowup

The bandwidth cost of each of ReadPath and EvictPath is proportional to the sum of the numbers of the blocks in a root–leaf path in the data tree and the metadata tree. The number for the data tree is $ZL + M = O(\log n) + O(f(n)) = O(f(n))$. The number for the metadata tree is around $\frac{2\log N + 1}{B} = o(1)$ factor of that for the data tree. The bandwidth cost for accessing the position table is $O(\log^2 n)$ by Proposition 1. Therefore, the bandwidth blowup of Access is $O(\log^2 n)$.

### 3.5 User Space

The temporary user space usage is proportional to the sum of the numbers of the blocks in a root–leaf path in the data tree and the metadata tree. As is shown in the bandwidth analysis, the latter is bounded by $O(f(n))$.

In the rest of this subsection, we bound the permanent user space usage, i.e., the stash size. First, we import some concepts and tools from [10] and [34]. Fix a sequence of input logical access requests. Later, we will specify a concrete request sequence that we use for the analysis. Let ORAM$_Z$ be the real ORAM construction that we are analyzing and ORAM$_{\infty}$ be the hypothetical ORAM construction derived by modifying the size of each bucket in ORAM$_Z$ to $\infty$. Let $S_Z$ (resp. $S_{\infty}$) be the ORAM$_Z$ (resp. ORAM$_{\infty}$) after processing the access requests. Note that ORAM$_Z$ (resp. ORAM$_{\infty}$) is an ORAM construction while $S_Z$ (resp. $S_{\infty}$) is the state of the construction at a particular point of time. We write $G$ to denote a post-processing algorithm that takes $S_Z$ and $S_{\infty}$ and modify $S_{\infty}$ in the following way. The algorithm $G$ enumerates the buckets in ORAM$_{\infty}$ in reverse breadth first order. We define $b_{2i}^{Z}$ to be the root bucket of ORAM$_Z$ and $b_{2i+1}^{Z}$ (resp. $b_{2i+1}^Z$) to be the left (resp. right) child of $b_{2i}^{Z}$. We define $b_{0}^{Z}$ to be the stash of ORAM$_Z$. For $i \in [2^{\log + 1}], b_{2i}^{\infty}$ is defined similarly. For each $i$ from $2^{\log + 1}$ to 1, $G$ processes each block $v \in b_{2i}^{\infty}$ as follows: i) if $v \in b_{2i}^{Z}$, $v$ is left as it is; ii) if in $S_Z$, $v$ is stored in some proper ancestor of $b_{2i}^{Z}$, $G$ moves $v$ to $b_{2i/2}^{\infty}$, i.e., $b_{2i}^{\infty}$’s parent. If the number of blocks left in $b_{2i}^{\infty}$ after such transportation is less than $Z$, $G$ outputs an error; iii) if, in $S_Z$, $v$ is not stored in any ancestor of $b_{2i}^{Z}$, $G$ outputs an error. We denote the output of $G$ with input $S_Z$ and $S_{\infty}$ as $G_{S_Z}(S_{\infty})$. For $S \in \{S_Z,G_{S_Z}(S_{\infty})\}$, we define $st(S)$ to be the number of blocks in the stash of $S$. Due to the following lemma, $st(S_Z)$ and $st(G_{S_Z}(S_{\infty}))$ are equivalent as random variables.

**Lemma 1 (34 Lemma 1).** *If the randomness used in ORAM$_Z$ is the same as ORAM$_{\infty}$, i.e., the position labels assigned to the accessed blocks are the same in the two ORAM constructions, then $G$ does not output an error and $S_Z = G_{S_Z}(S_{\infty})$.***

Let a subtree be a connected subgraph of the complete binary tree with $2^L$ leaves that contains the root. For a subtree $T$, let $C(T)$ be the number of blocks that can be stored in the corresponding buckets of ORAM$_Z$, and $X(T)$ be the number of blocks that are stored in the corresponding buckets of $S_{\infty}$. Note that $C(T)$ is a constant while $X(T)$ is a random variable. Also, let $n(T)$ denote the number of nodes in $T$.

**Lemma 2 (34 Lemma 2).** *For any integer $R > 0$, $st(G_{S_Z}(S_{\infty})) > R$ iff there exists a subtree $T$ such that $X(T) > C(T) + R$.*

We call those subtrees that contain only internal nodes (of the enclosing complete binary tree) as internal subtrees.

**Lemma 3 (34 Lemma 3).** *For any internal subtree $T$, $E[X(T)] \leq n(T)/2$.*

Let the working set of a sequence of access requests $\{op_i, addr_i, val_i\}$, be the set $\{addr_i\}_i$. 11
Lemma 4 (\cite{ORAM} Lemma 3). Among all access request sequences of working set size \(t\), the probability \(\Pr[st(S_Z) > R]\) is maximized by the sequence that contains exactly one access to each of the \(t\) different addresses.

Because of Lemma 4, we fix the input access request sequence to \((op_i, i, val_i)_{i \in [N]}\) without loss of generality. (\(op_i\) and \(val_i\) are arbitrary since they do not affect the stash size.)

Now we prove

\[
\Pr[st(S_Z) > R] = n^{-\omega(1)}
\]

for \(R = \omega(\log n)\). Remember that (2) is a bound on the stash size at a particular point of time. Given (2), the probability that the stash size becomes larger than \(R\) at any point in \(\text{poly}(n)\) logical accesses is also bounded as \(n^{-\omega(1)}\) by union bound.

Let \(G\) be the event that no leaf bucket of \(S_\infty\) contains more than \(M\) blocks. Let \(B\) be the complement of \(G\).

Lemma 5. \(\Pr[B] = n^{-\omega(1)}\).

Proof. Consider ORAM\(_\infty\) just before post-processing. For \(i \in [2^L]\), let \(load_i\) be the number of real blocks in the \(i\)-th leaf bucket and \(ctr_i\) be the number of real blocks with position label \(i\). Since a real block can be stored in the \(i\)-th leaf bucket only if it has position label \(i\), \(load_i \leq ctr_i\). For \(i \in [2^L]\) and \(j \in [N]\) let \(ctr_{i,j}\) be the indicator random variable of the event that the \(j\)-th accessed real block is assigned position label \(i\).

Clearly, \(ctr_i = \sum_j ctr_{i,j}\) for each \(i \in [2^L]\). and \(E[ctr_{i,j}] = \Pr[ctr_{i,j} = 1] = 1/2^L\) for each \(i \in [2^L]\) and \(j \in [N]\) Thus, \(E[ctr_i] = E[\sum_j ctr_{i,j}] = \sum_j E[ctr_{i,j}] = N/2^L\) for each \(i \in [2^L]\). For each \(i \in [2^L]\), \(\{ctr_{i,j}\}_{j \in [N]}\) are mutually independent. The lemma follows as

\[
\Pr[B] = \Pr[\bigcup_{i \in [2^L]} load_i > M]
\leq \Pr[\bigcup_{i \in [2^L]} ctr_i > M]
\leq \sum_{i \in [2^L]} \Pr[ctr_i > M]
= \sum_{i \in [2^L]} \Pr[ctr_i > N/2^L]
\leq 2^L \exp\left(-\frac{1}{3}g(n)^2 L\right)
\leq n \exp(-\omega(1) \ln n)
= n^{-\omega(1)}.
\]

We used Chernoff bound in the fifth step.

Let \(T\) be the set of all subtrees and \(T’\) be the set of all internal subtrees. Then,

\[
\Pr[st(S_Z) > R] = \Pr[st(G_{S_Z}(S_\infty)) > R]
= \Pr[\bigcup_{T \in T} X(T) > C(T) + R]
\leq \Pr[\bigcup_{T \in T} X(T) > C(T) + R|G] + \Pr[B]
= \Pr[\bigcup_{T \in T'} X(T') > C(T) + R|G] + \Pr[B]
\leq \sum_{m \geq 1} \frac{4^m}{n(T)^m} \max_{n(T) = m} \Pr[X(T) > C(T) + R|G] + \Pr[B].
\]

In the last step, we used the fact that the number of ordered binary trees with \(m\) nodes is bounded by \(4^m\).

Lemma 6. For any internal subtree \(T\) with \(n(T) = m\),

\[
\Pr[X(T) > C(T) + R|G] \leq (2Z)^{-R} \exp\left(-m(Z \ln 2Z + 1/2 - Z)\right) \Pr[G]^{-1}.
\]

Proof. For any \(t > 0\),

\[
\Pr[X(T) > C(T) + R|G] = \Pr[e^{tX(T)} > e^{t(C(T)+R)}|G]
\leq E[e^{tX(T)}|G]e^{-t(C(T)+R)}
\leq E[e^{tX(T)}] \Pr[G]^{-1} e^{-t(C(T)+R)}.
\]
For \( j \in [N] \), let \( X_j(T) \) be the indicator random variable of the event that, in \( S_\infty \), the \( j \)-th accessed real block is in \( T \) and let \( p_j := \Pr[X_j(T) = 1] \). Clearly, \( \sum_j X_j(T) = X(T) \) and \( E[X(T)] = E[\sum_j X_j(T)] = \sum_j E[X_j(T)] = \sum_j p_j \). The random variable \( X_j(T) \) depends only on \( j \) and the position label of the \( j \)-th accessed real block. Thus, \( \{X_j(T)\}_{j \in [N]} \) are mutually independent. Then,

\[
E[e^{tX(T)}] = E[e^{t\sum_{j \in [N]} X_j(T)}] \\
= E[\prod_{j \in [N]} e^{tX_j(T)}] \\
= \prod_{j \in [N]} E[e^{tX_j(T)}] \\
= \prod_{j \in [N]} (p_j(e^t - 1) + 1) \\
\leq \prod_{j \in [N]} (\exp(p_j(e^t - 1)) \\
= \exp((e^t - 1)\sum_{j \in [N]} p_j) \\
= \exp((e^t - 1)E[X(T)]).
\]

(5)

We used the independence of \( \{X_j(T)\}_{j \in [N]} \) in the third step. Let \( m := n(T) \). From bounds (4), (5) and Lemma 3 \( \Pr[X(T) > C(T) + R|G] \) is bounded by

\[
\exp((e^t - 1)m/2)e^{-(mZ+R)}\Pr[G]^{-1} = \exp(-tR)\exp(-m(tZ - (1/2)(e^t - 1)))\Pr[G]^{-1}.
\]

The lemma follows by setting \( t = \ln 2Z \).

If \( Z = 3 \), \( q := Z \ln 2 + 1/2 - Z - \ln 4 = 1.4889 \cdots > 0 \).

By (3) and Lemma 6 \( \Pr[st(S_Z) > R] \) is bounded by

\[
\sum_{m \geq 1} 4^m 6^{-R} \exp(-m(q + \ln 4))\Pr[G]^{-1} + \Pr[B] \leq \frac{(1/6)^R}{1 - e^{-q}}\Pr[G]^{-1} + \Pr[B].
\]

By Lemma 3 the bound above is \( n^{-\omega(1)} \) if \( R = \omega(\log n) \).

4 Succincter ORAM Construction

In this section, we prove the following theorem.

**Theorem 2.** Let \( n \) be the database size and \( B \) be the block size, both in bits. If \( B \geq 3 \log n \) and \( B = O(n^c) \) for some \( 0 < c < 1 \), then for any \( f : \mathbb{N} \rightarrow \mathbb{R} \) such that \( f(n) = \omega(\log \log n) \) and \( f(n) = O(\log^2 n) \), there exists an information theoretically secure ORAM construction for which i) the server’s space usage is bounded by

\[
n \left( 1 + \Theta \left( \frac{\log n}{B} + \frac{\log \log n}{f(n)} \right) \right)
\]

bits;

ii) the worst case bandwidth blowup is \( O(\log^2 n) \); iii) the user’s temporary space usage is \( O(\log n + f(n)) \) blocks; and iv) for any \( R = \omega(\log n) \), the probability that the user’s permanent space usage becomes larger than \( R \) blocks during \( \poly(n) \) logical accesses is \( n^{-\omega(1)} \).

**Corollary 2.** If, in addition to the conditions of Theorem 2 \( B = \omega(\log n) \), then, the ORAM construction of Theorem 2 is succinct.

Theorem 2 is stronger than Theorem 1. For example, if \( B = \Omega(\log^2 n) \) and \( f(n) = \Theta(\log n \log \log n) \), the server space bound of Theorem 2 implies that the extra server space is \( \Theta(n/\log n) \) and the user temporary space usage is \( \Theta(\log n \log \log n) \). In contrast, the extra server space bound of Theorem 1 is \( \omega(n/\sqrt{n \log n}) \) even if we allow the user’s temporary space to become \( \Theta(\log^2 n) \).

In the rest of this section, \( n, B, f(\cdot) \) are as described in the statement of Theorem 2.

In the following exposition, we often refer to Section 3 to avoid repetition. We recommend the readers to read Section 3 beforehand.
4.1 Description
As in Section 3, we first explain a simplified version with a large user space usage, and construct the full version that achieves the claimed bounds from the simplified version.

Let
\[ L := \lceil \log(N/f(n)) \rceil \quad \text{and} \quad M := \lceil N/2^L + (1 + \varepsilon) \log L \rceil \]
where \( N = n/B \) and \( \varepsilon > 0 \) is a constant. We assume, for brevity, that each of \( \log(N/f(n)) \) and \( N/2^L + (1 + \varepsilon) \log L \) is an integer.

Block usage. The block usage is the same as the ORAM construction described in Section 3 except that each real block is given two position labels instead of one. We call them the primary position label and the secondary position label. Only the primary position labels are stored in the metadata blocks.

Data layout. The data layout is basically the same in Section 3. We only explain the difference from Section 3.

First, the position table stores both the primary position labels and the secondary position labels.

Second, the user maintains an additional table called counter table. It is a size \( 2^L \) array whose \( i \)-th entry is the number of real blocks with primary position label \( i \).

Last, since the value of each of \( L \) and \( M \) is different from that in Section 3, the tree/bucket size is changed accordingly.

Access procedure. The same invariant conditions as Section 3 are maintained except that the “position label” in the second condition is replaced by “primary position label”.

The main routine is described in Algorithm 3. The array Pos and the subroutines Random, ReadPath and EvictPath are the same as in Section 3 while Ctr is the counter table. We let \( P(\ell) \) denote the path from the root to the \( \ell \)-th leaf in the data tree. For brevity, we assume that every block is already initialized, i.e., each real block is assigned a valid value with the metadata stored in the corresponding node of the metadata tree and the position table and counter table contain the correct values.

Let \( b_a \) be the accessed block. We first retrieve the two position labels \( \ell_1 \) and \( \ell_2 \) of \( b_a \) from the position table and update each of the two position table values to a number chosen independently and uniformly at random from \([L]\), which will become the new position labels of \( b_a \) (line 2–4). One of \( \ell_1 \) and \( \ell_2 \) is the primary position label and the other is the secondary position label but we do not know (and do not need to know) which is which. By the invariant conditions, \( b_a \) is either in the stash or in \( P(\ell_1) \) or \( P(\ell_2) \). We scan \( P(\ell_1) \) and \( P(\ell_2) \) and retrieve \( b_a \) from \( P(\ell_i) \) if the primary position label is \( \ell_i \) and \( b_a \) is in \( P(\ell_i) \) (line 5). If \( b_a \) is not found in the paths, it must be in the stash and we retrieve it from the stash (line 11–13). At this point, we know the primary position label \( \ell \) of \( b_a \) (since it is written in the pos entry of the block) and we decrement the \( \ell \)-th entry of the counter table, determine the new primary position label \( \ell' \) and increment the \( \ell' \)-th entry of the counter table (line 14–17). After, that, we update the block contents if it is a write request (line 19–20), insert \( b_a \) into the stash (line 21), call EvictPath (line 22) and returns the retrieved block content (line 23) all in the same way as Algorithm 1.

Outsourcing the position/counter table. In the full version of the construction, the position table and the counter table are stored on the server using the sub-ORAM in Proposition 1. Every access to each of these tables is done using the sub-ORAM access procedure.

4.2 Security
The security proof of the current ORAM construction is almost the same as in Subsection 3.2. The only difference in the situation is that now, the sequence of accessed addresses \( a'_2 \) depends on two position labels instead of one. Anyway, these position labels are distributed independently and uniformly at random and thus, are independent of \( a \).

4.3 Server Space
The bounds still hold.
Algorithm 3 Main routine (two choices)

1: function Access(a, op, v')
2:  \( \ell_1' \leftarrow \text{Random}(L) \)
3:  \( \ell_2' \leftarrow \text{Random}(L) \)
4:  \((\ell_1, \ell_2) \leftarrow \text{Pos}[a] \)
5:  Pos[a] \leftarrow (\ell_1', \ell_2')
6:  v_1 \leftarrow \text{ReadPath}(\ell_1, a), v_2 \leftarrow \text{ReadPath}(\ell_2, a)
7:  \text{if } v_1 \neq \bot \text{ then}
8:  (v, \ell) \leftarrow (v_1, \ell_1)
9:  \text{else if } v_2 \neq \bot \text{ then}
10: (v, \ell) \leftarrow (v_2, \ell_2)
11: \text{else}
12:  \text{Find } (a, \ell'', v'') \in \text{stash}
13:  (v, \ell) \leftarrow (v'', \ell'')
14:  \text{stash} \leftarrow \text{stash} \setminus (a, \ell'', v'')
15: \text{else if } op = \text{write} \text{ then}
16:  v \leftarrow v'
17:  \text{stash} \leftarrow \text{stash} \cup (a, \ell_i', v)
18: \text{EvictPath}()
19: \text{return } ret
The number of blocks in the leaf buckets is
\[ M2^L = N\left(1 + (1 + \varepsilon) \frac{\log L}{f(n)}\right) = N\left(1 + \Theta\left(\frac{\log \log n}{f(n)}\right)\right). \]

The number of blocks in the internal buckets is \( Z(2^L - 1) < ZN/f(n) \), which is \( O\left(\frac{\log \log n}{f(n)}\right) \). Thus, the data tree size is bounded by \( N(1 + \Theta\left(\frac{\log \log n}{f(n)}\right)) \) blocks. As in Section 3, the metadata size of each data block is \( \Theta\left(\frac{\log n}{B}\right) \) blocks. Thus, the number of blocks in the data tree and the metadata tree combined is at most \( 1 + \Theta\left(\frac{\log n}{B}\right) \) times larger than \( N(1 + \Theta\left(\frac{\log \log n}{f(n)}\right)) \), which is
\[ n\left(1 + \Theta\left(\frac{\log n}{B} + \frac{\log \log n}{f(n)}\right)\right) \] bits.

Position labels take \( 2NL = 2nL/B \leq 2n\log n \) bits while counter table values take \( 2^L [\log N] = N[\log N]/f(n) \leq N = n/B \) bits. By Proposition 1, the sub-ORAM containing the position table (resp. counter table) takes \( \Theta(n\log n) \) (resp. \( \Theta(n/B) \)) bits.

Therefore, the server space usage is bounded by \( n(1 + \Theta\left(\frac{\log n}{B} + \frac{\log \log n}{f(n)}\right)) \) bits.

### 4.4 Bandwidth Blowup

By the same argument as in the bandwidth analysis, the bandwidth cost of each of \( \text{READPATH} \) and \( \text{EVICTPATH} \) is proportional to \( ZL + M = O(\log n + f(n)) \) (in blocks). By Proposition 1, the bandwidth cost of access to each of the position table and the counter table is \( O(\log^2 n) \). Thus, the bandwidth blowup is \( O(\log^2 n) \).

### 4.5 User Space

By the same argument as in Subsection 3.5, the temporary user space is proportional to \( ZL + M = O(\log n + f(n)) \).

In the rest of the subsection, we bound the permanent user space, i.e., the stash size. Using the current ORAM construction, define \( \text{ORAM}_Z \), \( \text{ORAM}_{\infty} \), \( \text{S}_Z \) and \( \text{S}_{\infty} \) analogously to Subsection 3.5. Then, we prove \( \Pr[s(S_Z) > R] = n^{-\omega(1)} \) for \( R = \omega(\log n) \). Most arguments in Subsection 3.5 can be reused and we focus on the differences.

First, Lemma 5 still holds for the current construction but the proof is different from Subsection 3.5.

**Proof of Lemma 5 for the two-choice construction.** Define \( \text{load}_i \), \( \text{ctr}_i \) and \( \text{ctr}_{i,j} \) in the same way as we did in the proof of Lemma 5 except that the primary position labels are used instead of the position labels. By the same argument as the proof of Lemma 5, it suffices to prove \( \Pr[\bigcup_{i \in [2^L]} \text{ctr}_i > M] = n^{-\omega(1)} \).

We apply an existing bound for the heavily loaded case of the balls-into-bins game with two choices. In the balls-into-bins game with \( n \) balls and \( n \) bins (with one choice), each of the \( m \) balls is thrown into one of the \( n \) bins chosen uniformly and independently at random. In the balls-into-bins game with two choices, for each ball, two bins are chosen uniformly and independently at random. Then, the ball is thrown into the least loaded bin. The gap of a balls-into-bins game with \( m \) balls and \( n \) bins is defined to be the difference between the number of balls in the bin with the maximum load and the average number of balls in a bin, i.e., \( m/n \). Berenbrink et al. [3] proved the following proposition.

**Proposition 2.** In the two-choice balls-into-bins game with \( m \) balls and \( n \) bins, for any \( c \), \( \Pr[\text{gap} > \lg \lg n + \gamma(c)] < 1/n^c \), where \( \gamma(c) \) is a constant that depends only on \( c \).

**Corollary 3.** In the two-choice balls-into-bins game with \( m \) balls and \( n \) bins, \( \Pr[\text{gap} > (1 + \varepsilon) \lg \lg n] = n^{-\omega(1)} \) for any \( \varepsilon > 0 \).

After processing the access requests, the \( 2^L \) values in the counter table are distributed in exactly the same way as the bin loads after the balls-into-bins game with two choices with \( N \) balls and \( 2^L \) bins.
where we used Corollary 3 in the third step.

Next, we modify Lemma 6.

Lemma 7. For any internal subtree $T$ with $n(T) = m$,

$$\Pr[X(T) > C(T) + R|G] \leq (Z)^{-R} \exp(-m(Z \ln Z + 1 - Z)) \Pr[G]^{-1}.$$  

Proof. The part where the arguments in Subsection 3.5 breaks down is (5). In Subsection 3.5, we used the mutual independence of $\{X_j\}_{j \in [N]}$ for the third step of (5) but here, $\{X_j\}_{j \in [N]}$ are not mutually independent. We fix this problem as follows.

Let ORAM$_{\infty}$ be another hypothetical ORAM construction derived by modifying ORAM$_{\infty}$ so that every time a just accessed real block $b$ is inserted into the stash, another block called $b$'s shadow is also inserted into the stash. If $b$ is given the primary position label $\ell_1$ and the secondary position label $\ell_2$ at the time $b$'s shadow $b'$ is inserted into the stash, $b'$ is given the primary position label $\ell_2$ and the secondary position label $\ell_1$. A shadow is evicted in the same way as a real block but it does not affect the counter table. Let $S'_\infty$ be ORAM$_{\infty}$ after processing the access requests. Since each of the $N$ real blocks is accessed exactly once, each real block in $S'_\infty$ has one shadow. For $j \in [N]$, let $Y_j(T)$ (resp. $Y'_j(T)$) be the indicator random variable of the event that, in $S'_\infty$, the $j$-th accessed real block (resp. the $j$-th accessed real block’s shadow) is in subtree $T$. Since shadows do not affect real blocks’ move, $X_j$ and $Y_j$ are equivalent random variables. Also, since the primary and the secondary position label of each real block is chosen independently and uniformly at random, $Y_j + Y'_j$ is distributed equally as $U_j + U'_j$ where $U_j$ and $U'_j$ are independent random variables, each distributed equally with the $X_j$ in the proof of Lemma 6. Thus, with $Y(T) := \sum_j Y_j(T)$,

$$E[e^{tX(T)}] \leq E[e^{t \sum_j (Y_j(T) + Y'_j(T))}] = E[e^{t \sum_j (U_j + U'_j)}] = E[e^{t \sum_j U_j}] E[e^{t \sum_j U'_j}] = E[e^{t \sum_j U_j}]^2 = \exp((e^t - 1)2E[X(T)])$$

where we used (5) in the last step.

Then, from (4), (6) and Lemma 3, $\Pr[X(T) > C(T) + R|G]$ is bounded by

$$\exp((e^t - 1)m - t(mZ + R)) \Pr[G]^{-1} = \exp(-tR) \exp(-m(tZ - (e^t - 1))) \Pr[G]^{-1}.$$  

The lemma follows by setting $t = \ln Z$.

If $Z = 4$, $q = Z \ln Z + 1 - Z - \ln 4 > 1.15888 \cdots > 0$. By (3) and Lemma 7, $\Pr[s(t(S_Z) > R)]$ is bounded by

$$\sum_{m \geq 1} 4^m 4^{-R} \exp(-m(q + \ln 4)) \Pr[G]^{-1} + \Pr[B] < \frac{(1/4)^R}{1 - e^{-q}} \Pr[G]^{-1} + \Pr[B].$$

By Lemma 8 the bound above is $n^{-\omega(1)}$ if $R = \omega(\log n)$.

---

$^a$We need to do this since $\{X_j(T)\}_{j \in [N]}$ (defined analogously in Lemma 6) is not mutually independent due to the two-choice strategy.
Table 3: Performance comparison with concrete parameters. The symbol † means the integration of Ring ORAM techniques. \( N = 2^{20}, B = 2^{10} \). \( A \) and \( S \) are parameters for the Ring ORAM. \( A \) specifies the infrequency of \text{EvictPath} and \( S \) is the space in each bucket reserved for dummy blocks.) The cost for recursive calls and metadata handling are relatively minor and not included. The stash overflow probability is \( < 2^{-80} \) for rigorous settings. Aggressive settings do not have security guarantees (stash size bounds) and, in particular, are not suitable for fair comparison.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Parameters} & \text{Extra server space} & \text{Bandwidth} & \text{Stash size} \\
\hline
\text{Rigorous} & 5,20,–,–,– & 9N & 210 & 114 \\
\text{Th. 1} & 5,19,–,4,6 & 10N & 109 & 63 \\
\text{Th. 1} & 3,15,112,– & 2.59N & 471 & 32 \\
\text{Th. 1} & 5,15,112,4,7 & 2.91N & 253 & 64 \\
\hline
\text{Aggressive} & 4,19,–,–,– & 3N & 160 & \\
\text{Th. 1} & 5,19,–,4,6 & 7N & 145 & \\
\text{Th. 1} & 4,15,36,– & .25N & 288 & \\
\text{Th. 1} & 5,15,36,4,6 & .46875N & 163 & \\
\text{Th. 2} & 3,16,14,– & .0625N & 248 & \\
\text{Th. 2} & 5,15,28,4,7 & .25N & 194 & \\
\hline
\end{array}
\]

5 Practicality of the Proposed Methods

Table 3 shows the performance of the proposed methods, the Path ORAM \([40]\) and the Ring ORAM \([34]\) with concrete parameters. The Ring ORAM has asymptotically the same performance as the Path ORAM but it achieves constant factor smaller bandwidth at the cost of larger server space. It is easy to integrate the main technique of the Ring ORAM to the internal nodes of the proposed methods and we also show the performance of these variants.

The table contains “rigorous” and “aggressive” parameter settings. Rigorous parameters were derived from theoretical analysis with additional care for constant factors. The aggressive parameters for existing methods were taken from the experiments in the original papers. We chose the aggressive parameters for the proposed methods by simulation: we simulated database scan (accessing addresses 1, 2, \ldots, \( N \)) for 100 times and found some parameters for which the stash size after every scan was zero. (Such usage of scan is standard in literature since Lemma 4 means scan maximizes the stash size.) We emphasize that constructions with aggressive parameters lack rigorous security and they are not suitable for fair comparison.

Unfortunately, we could not derive rigorous bounds for the second construction (Theorem 2) for reasonable size of \( N \) since the balls-into-bins analysis of Berenbrink et al. \([3]\), used in the stash size analysis, requires a very large number of bins. However, the simulation results indicate that the second construction works for reasonable size of \( N \).

6 Conclusion

ORAM is a multifaceted problem and recently, researchers have been recognizing the importance of rethinking the relevancy of multiple aspects of ORAM using modern standards \([4, 39]\). In this paper, we provided another point of view and insight for this exploration by introducing the notion of succinctness to ORAM and proposing succinct ORAM constructions. We think our methods are particularly suitable for secure processor setting. It is interesting to consider succinct constructions optimized for other settings.

As we already mentioned, we could not derive non-asymptotic bounds for Theorem 2 (for reasonable size of \( N \)) since the analysis of Berenbrink et al. \([3]\) for heavily loaded case of two-choice balls-into-bins game, on which we rely, requires a large number of bins. However, simulation results suggest that Theorem 2 does have a significant impact in practice even for reasonably small \( N \) and it is desirable to close this gap between theory and practice. One obvious approach is to fine tune the analysis of Berenbrink et al. to get a better (non-asymptotic) bound but this seems difficult because of the complexity of the

\[^7\text{Specifically, we modify the access procedure to access only one block per each bucket (instead of all blocks in the bucket) by permuting the blocks in each bucket. For this technique to work, we need to introduce, for each bucket, additional space dedicated only for dummy blocks and this is why we cannot apply this technique to the leaves maintaining succinctness.}\]
analysis. Talwar and Wieder gave an alternative simpler analysis [41] but the resulting bound is not as tight as that of Berenbrink et al.

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