EXPLORING THE HELIUM CORE OF THE δ SCUTI STAR COROT 102749568 WITH ASTEROSEISMOLOGY

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ABSTRACT

Based on regularities in rotational splitting, we seek possible multiplets for the observed frequencies of CoRoT 102749568. There are 21 sets of multiplets identified, including four sets of multiplets with $l = 1$, nine sets of multiplets with $l = 2$, and eight sets of multiplets with $l = 3$. In particular, there are three complete triplets $(f_{10}, f_{12}, f_{14})$, $(f_{31}, f_{34}, f_{36})$, and $(f_{41}, f_{43}, f_{44})$. The rotational period of CoRoT 102749568 is estimated to be $1.34_{-0.05}^{+0.04}$ days. When doing model fittings, three $l = 1$ modes $(f_{12}, f_{34}$, and $f_{43})$ and the radial first overtone $f_{13}$ are used. Our results shows that the three nonradial modes $(f_{12}, f_{34}$, and $f_{43})$ are mixed modes, which mainly provide constraints on the helium core. The radial first overtone $f_{13}$ mainly provides constraint on the stellar envelope. Hence the size of the helium core of CoRoT 102749568 is determined to be $M_{He} = 0.148 \pm 0.003 \, M_\odot$ and $R_{He} = 0.0581 \pm 0.0007 \, R_\odot$. The fundamental parameters of CoRoT 102749568 are determined to be $M = 1.54 \pm 0.03 \, M_\odot$, $Z = 0.006$, $f_{"o} = 0.004 \pm 0.002$, $\log g = 3.696 \pm 0.003$, $T_{\text{eff}} = 6886 \pm 70 \, K$, $R = 2.916 \pm 0.039 \, R_\odot$, and $L = 17.12 \pm 1.13 \, L_\odot$.

Key words: stars: individual (CoRoT 102749568) – stars: rotation – stars: variables: delta Scuti

1. INTRODUCTION

Thanks to the space missions MOST (Walker et al. 2003), CoRoT (Baglin et al. 2006), and Kepler (Borucki et al. 2010), many δ Scuti stars are observed precisely (e.g., HD 144277 (Zwintz et al. 2011), HD 50844 (Poretti et al. 2009), and KIC 9700322 (Breger et al. 2011)). In particular, a large number of pulsation frequencies are detected in the light curves of some δ Scuti stars, such as HD 174936 (García Hernández et al. 2009), HD 50870 (Mantegazza et al. 2012), and HD 174966 (García Hernández et al. 2013). Due to the complexity of the frequency content, it is very difficult to disentangle the whole spectra of δ Scuti stars. Recently, Paparó et al. (2016) developed a sequence search method, and found a large number of series of quasi-equitably spaced frequencies in 77 δ Scuti stars. Besides, Chen et al. (2016) attempted to interpret the frequency spectra of the δ Scuti star HD 50844 using the rotational splitting.

CoRoT 102749568 was observed from 2007 October 24 to 2008 March 3 ($\Delta T = 131$ days) by CoRoT during the first long run in the anti-center direction (LRA01). Guenther et al. (2012) classified the δ Scuti star CoRoT 102749568 as an F1 IV star on the basis of the low-resolution $R = 1300$ spectra, which were observed in 2009 January with the AAOmega multi-object spectrograph mounted on the Anglo-Australian 3.9 m Telescope.

Paparó et al. (2013) converted the spectral type F1 IV of CoRoT 102749568 into effective temperature $T_{\text{eff}}$ and gravitational acceleration log$g$ using the calibrated values from Straizys & Kurilienne (1981), and then obtained $T_{\text{eff}} = 7000 \pm 200 \, K$ and log$g = 3.75 \pm 0.25$ by means of fitting AAOmega spectra with stellar atmosphere models of Kurucz (1979). Moreover, Paparó et al. (2013) obtained $v \sin i = 115 \pm 20 \, km \, s^{-1}$ from the high-resolution $R = 85,000$ spectra, which were observed with the Mercator Echelle Spectrograph mounted on the 1.2 m Mercator Telescope of Roque de los Muchachos Observatory. Furthermore, Paparó et al. (2013) extracted a total of 52 independent pulsation frequencies from the CoRoT timeseries. These frequencies are listed in Table 1. They identified the oscillation frequency $9.702 \, d^{-1}$ with the largest amplitude as the radial first overtone with the method of multi-color photometry. Moreover, Paparó et al. (2013) identified 11 other frequencies based on the regularities in frequency spacing.

Mode identification is very important for the asteroseismic study of pulsation stars. For a rotating star, the regularities due to rotational splitting in observed frequencies help us much to identify their spherical harmonic degree $l$ and azimuthal number $m$. Based on the rotational splitting law of $g$ modes, we successfully disentangled the frequency spectra of the δ Scuti star HD 50844 (Chen et al. 2016). That motivated us to analyze another δ Scuti star CoRoT 102749568 with the same method. In Section 2, we propose our mode identification by means of rotational splitting. In Section 3, we describe the details of input physics and model calculations: input physics are described in Section 3.1, model grids are elaborated in Section 3.2, and the optimal model is analyzed in Section 3.3. We discuss our results in Section 4, and summarize them in Section 5.

2. MODE IDENTIFICATION BASED ON ROTATIONAL SPLITTING

A pulsation mode is characterized by three indices: the radial order $k$, the spherical harmonic degree $l$, and the azimuthal number $m$ (Christensen-Dalsgaard 2003). The azimuthal number $m$ is degenerate for a spherically symmetric star. Namely, modes with the same $k$ and $l$ but different $m$ have the same frequency. Stellar rotation will break the structure of spherical symmetry and result in frequency splitting, i.e., one nonradial pulsation frequency will split into $2l + 1$ different frequencies. According to the theory of stellar oscillation, a general formula for rotational splitting is described as (Aerts
et al. 2010)

\[ \nu_{k,l,m} = \nu_{k,l} + \beta_{k,l,m} \frac{m}{P_{\text{rot}}} \]  

(1)

In Equation (1), \( \beta_{k,l} \) is the rotational parameter measuring the size of rotational splitting and \( P_{\text{rot}} \) the rotational period. For high-degree or high-order \( p \) modes, \( \beta_{k,l} \approx 1 \). Values of rotational splitting for pulsation modes with different spherical harmonic degree \( l \) are the same. For high-order \( g \) modes, \( \beta_{k,l} \approx 1 - \frac{1}{l(l+1)} \) (Brickhill 1975). The rotational splitting derived from \( l = 1 \) modes and those from \( l = 2 \) modes and \( l = 3 \) modes conform to the relation \( \delta \nu_{k,l=1} = \delta \nu_{k,l=2} = \delta \nu_{k,l=3} = 0.6:1:1.1 \) (Wiget et al. 1991). Based on these regularities in rotational splitting, we analyze the frequency spectra of CoRoT 102749568 and list possible multiplets in Table 2.

It can be noticed in Table 2 that we find 21 sets of multiplets, including three different types of rotational splitting. The averaged frequency splitting \( \delta \nu_{1} \) is 4.451 \( \mu \)Hz for multiplets 1, 2, 3, and 4. The averaged frequency splitting \( \delta \nu_{2} \) is 7.453 \( \mu \)Hz for multiplets 5, 6, 7, 8, 9, 10, 11, 12, and 13, and the averaged frequency splitting \( \delta \nu_{3} \) is 8.176 \( \mu \)Hz for multiplets 14, 15, 16, 17, 18, 19, 20, and 21. For these frequency differences in Table 2, we find that some of them approximate to the corresponding averaged value \( \delta \nu_{1}, \delta \nu_{2}, \) or \( \delta \nu_{3} \) (e.g., multiplets 1, 2, 3, and 5), and some of them are several times of the corresponding average value (e.g., multiplets 4, 11, 12, and 13). Moreover, we find that the ratio of \( \delta \nu_{1}/\delta \nu_{2}/\delta \nu_{3} \) is 0.597:1:1.097, which agrees well with the property of \( g \) modes. As shown in Figure 1, the \( \delta \) Scuti star CoRoT 102749568 is in the post-main-sequence evolution stage with a contracting helium core and an expanding envelope. Such stellar structure may reproduce these behaviors of rotational splitting.

Based on the property of rotational splitting for \( g \) modes, we identify frequencies in multiplets 1, 2, 3, and 4 as \( l = 1 \) modes, frequencies in multiplets 5, 6, 7, 8, 9, 10, 11, 12, and 13 as \( l = 2 \) modes, and frequencies in multiplets 14, 15, 16, 17, 18, 19, 20, and 21 as \( l = 3 \) modes. Furthermore, we find that the azimuthal number \( m \) of pulsation modes in multiplets 1, 2, 3, 4, 6, and 13 can be uniquely identified, and the azimuthal number \( m \) of pulsation modes in other multiplets allow of several possibilities (e.g., three possibilities for pulsation modes in multiplet 5).

Finally, there are three unidentified frequencies \( f_{1}, f_{48}, \) and \( f_{52}, \) which do not show frequency splitting. Frequencies \( f_{1} \) and \( f_{48} \) have a difference of 29.789 \( \mu \)Hz, about four times that of \( \delta \nu_{1,2} \). However, \( f_{10} \) has been regarded as one component of multiplet 1. Multiplet 1 consists of three components, being a complete triplet. Frequencies \( f_{10} \) and \( f_{12} \) have a difference of 4.485 \( \mu \)Hz, which agree well with the difference 4.399 \( \mu \)Hz between \( f_{12} \) and \( f_{14} \). Besides, modes with lower degree \( l \) are easier to observe because of the effect of geometrical cancellation. Frequencies \( f_{27} \) and \( f_{40} \) have a difference of 48.847 \( \mu \)Hz, about six times that of \( \delta \nu_{1,3} \). Similarly, the frequency \( f_{27} \) has been identified as one component of multiplet 14. Frequencies \( f_{48} \) and \( f_{51} \) have a difference of 29.712 \( \mu \)Hz, about four times that of \( \delta \nu_{1,3} \). Frequencies \( f_{48} \) and \( f_{52} \) have a difference of 40.597 \( \mu \)Hz, about five times that of \( \delta \nu_{1,3} \). The spherical harmonic degree of \( f_{48} \) allows two possibilities: \( l = 2 \) or \( l = 3 \). For the former case, the azimuthal numbers \( m \) of \( f_{48} \) and \( f_{51} \) are determined to be \( m = (−2, +2) \). This case is listed in Table 2. The azimuthal number of the latter case allows two possibilities: \( m = (−3, +2) \) and \( (−2, +3) \).

Based on the above analyses, the detection of triplets, quintuplets, and septuplets helps us to identify four sets of multiplets with \( l = 1 \), nine sets of multiplets with \( l = 2 \), and eight sets of multiplets with \( l = 3 \). Owing to the deviations from the asymptotic expression, we find in Table 2 that slight differences of the rotational splitting exist in different multiplets (e.g., in multiplets 8 and 9). Besides, slight differences also exist in the same multiplet (e.g., in multiplet 1). Furthermore, we find in Table 2 that there are only two components in multiplets 4, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, and 21. Different physical origins are also possible, such as the large separation led by the so-called island modes (Lignières et al. 2006; García Hernández et al. 2013) and the phenomenon of avoided crossings (Aizenman et al. 1977). The \( \delta \) Scuti star CoRoT 102749568 is a slightly evolved star, the occurrence of avoided crossings will make the frequency spectra more complex. From the observed frequency spectra of CoRoT 102749568, it is difficult to find signs of avoided crossings. Hence the phenomenon of avoided crossings is not considered in our work.
Table 2

Possible Multiplets Due to Stellar Rotation

| Multiplet ID | Freq. (μHz) | δν (μHz) | l | m | Multiplet ID | Freq. (μHz) | δν (μHz) | l | m |
|--------------|-------------|----------|---|---|--------------|-------------|----------|---|---|
| f_{10}       | 106.152     | 4.485    | 1 | -1| f_{18}      | 122.559     | 22.375   | 2 | (-2, -1) |
| f_{12}       | 110.637     | 4.399    | 1 | 0 | f_{28}      | 144.934     | 2        | (+1, +2) |
| f_{14}       | 115.036     | 1        | 1 | +1| f_{42}      | 194.179     | 2        | (-2, -1) |
| f_{16}       | 162.625     | 4.382    | 1 | -1| f_{47}      | 216.758     | 2        | (+1, +2) |
| f_{31}       | 171.485     | 4.478    | 1 | +1| f_{48}      | 222.367     | 2        | -2    |
| f_{34}       | 197.503     | 8.157    | 3 | -1| f_{51}      | 252.079     | 2        | +2    |
| f_{41}       | 192.909     | 4.594    | 1 | -1| f_{52}      | 125.296     | 3        | (-3, -2, -1, 0, +1) |
| f_{33}       | 201.898     | 8.312    | 1 | +1| f_{54}      | 133.453     | 3        | (-2, -1, 0, +1, +2) |
| f_{44}       | 209.708     | 14.176   | 2 | (+1, +2, +3) |
| f_{5}        | 87.275      | 8.874    | 1 | -1| f_{15}      | 115.706     | 3        | (-3, -2, -1, 0, +1, +2) |
| f_{6}        | 96.149      | 8.106    | 1 | +1| f_{20}      | 123.812     | 3        | (-2, -1, 0, +1, +2, +3) |
| f_{8}        | 108.372     | 16.405   | 2 | (+1, +2, +3) |
| f_{11}       | 115.872     | 7.500    | 2 | (+1, +2) |
| f_{14}       | 202.072     | 22.499   | 2 | -2| f_{17}      | 117.666     | 3        | (-3, -2, -1, 0, +1) |
| f_{16}       | 124.571     | 7.457    | 2 | +1| f_{50}      | 249.725     | 3        | (-1, 0, +1, +2, +3) |
| f_{19}       | 132.028     | 134.762  | 2 | +2| f_{26}      | 158.977     | 3        | (-3, -2, -1, 0) |
| f_{22}       | 134.855     | 7.388    | 2 | (+1, +2, +3) |
| f_{23}       | 172.243     | 176.285  | 2 | (+1, +2, +3) |
| f_{25}       | 189.056     | 14.596   | 2 | (+1, +2, +3) |
| f_{28}       | 197.503     | 4.399    | 1 | 0 | f_{54}      | 133.453     | 3        | (-2, -1, 0, +1, +2) |
| f_{46}       | 209.708     | 33.423   | 2 | (+1, +2, +3) |
| f_{48}       | 222.367     | 29.712   | 2 | (+1, +2, +3) |
| f_{49}       | 233.083     | 16.405   | 2 | (+1, +2, +3) |
| f_{50}       | 249.725     | 16.642   | 2 | (+1, +2, +3) |
| f_{52}       | 252.079     | 8.106    | 2 | (+1, +2, +3) |
| f_{54}       | 255.380     | 32.002   | 3 | (+1, +2, +3) |

Note. δν—frequency difference in μHz.
3. INPUT PHYSICS AND MODEL CALCULATIONS

3.1. Input Physics

All of our theoretical models are computed with the Modules for Experiments in Stellar Astrophysics (MESA), which is developed by Paxton et al. (2011, 2013). We use the so-called module pulse from version 6596 to calculate stellar evolutionary models and their corresponding pulsation frequencies (Christensen-Dalsgaard 2008; Paxton et al. 2011, 2013).

In our calculations, the OPAL opacity table GS98 (Grevesse & Sauval 1998) series is adopted. We use the $T^{-\tau}$ relation of Eddington gray atmosphere in the atmosphere integration, and choose the mixing-length theory (MLT) of Böhm-Vitense (1958) to treat convection. Based on numerical calculations, we find that theoretical evolutionary models are not sensitive to the mixing-length parameter. However, the values of $\beta_{\text{MLT}}$ of theoretical models with slightly higher $\alpha_{\text{MLT}}$ agree better with asymptotic values of $g$ modes, hence $\alpha_{\text{MLT}} = 2.2$ is adopted in our work. Moreover, we find that theoretical models without convective core overshooting cannot reproduce those observed multiplets. Hence we introduce convective core overshooting in our calculations. For the overshooting mixing of the convective core, we adopt an exponentially decaying prescription. Following Freytag et al. (1996) and Herwig (2000), we introduce an overshoot mixing diffusion coefficient

$$D_{\text{ov}} = D_0 \exp \left( \frac{-2z}{f_{\text{ov}} H_p} \right).$$

In Equation (2), $D_0$ is the convective mixing coefficient, $z$ the distance into radiative zone away from the boundary of convective core, $H_p$ the pressure scale height, and $f_{\text{ov}}$ an adjustable parameter describing the efficiency of the overshooting mixing. In our calculations, we set the lower limit of the diffusion coefficient $D_{\text{ov}}^{\text{limit}} = 1 \times 10^{-2}$ cm$^2$ s$^{-1}$, below which no element mixing is allowed. In addition, effects of rotation and element diffusion are not considered in our work.

3.2. Model Grids

The internal structure and the evolutionary track of a star depend on the initial mass $M$, the initial chemical composition $(X, Y, Z)$, and the overshooting parameters $f_{\text{ov}}$. A grid of theoretical models is computed with MESA, $M$ varying from 1.5 $M_\odot$ to 2.2 $M_\odot$ with a step of 0.01 $M_\odot$, $Z$ varying from 0.005 to 0.030 with a step of 0.001, and $f_{\text{ov}}$ varying from 0 to 0.016 with a step of 0.001. In our calculations, we choose the initial helium fraction $Y = 0.245 + 1.54Z$ (e.g., Dotter et al. 2008; Thompson et al. 2014; Tian et al. 2015) as a function of mass fraction of heavy elements $Z$.

Theoretical models for each star are computed from the zero-age main sequence to post-main-sequence stage. The error box in Figure 1 corresponds to the observed stellar parameters, i.e., the effective temperature $6800 \, K < T_{\text{eff}} < 7200 \, K$ and the gravitational acceleration $3.50 < \log g < 4.00$. We calculate frequencies of oscillation modes with $l = 0, 1, 2, 3$ and $0$ for every stellar model which falls inside the error box along the evolutionary track.

3.3. Optimal Models

We try to use theoretical oscillation frequencies derived from a grid of evolutionary models to fit those of identified pulsation modes. According to the analyses in Section 2, mode identifications are unique only in multiplets 1, 2, and 3, and their $m = 0$ components are observed. When doing model fittings, we hence use four identified pulsation modes, i.e., three $l = 1$ modes ($f_{12}, f_{34},$ and $f_{53}$) and the radial first overtone $f_{13}$. Paparó et al. (2013) identify the frequency $f_{13}$ with the largest amplitude as the radial first overtone with the method of multicolor photometry. In our calculations, we use the identification of $f_{13}$ as the radial first overtone. When doing model fittings, we use the following criterion:

$$\chi^2 = \frac{1}{n} \sum \left( \nu_i^{\text{obs}} - \nu_i^{\text{theo}} \right)^2,$$

where $\nu_i^{\text{obs}}$ is the observed frequency, $\nu_i^{\text{theo}}$ the theoretically calculated frequency, and $n$ the amount of observed frequencies.

Figure 2 shows a plot of $1/\chi^2$ to the effective temperature $T_{\text{eff}}$ for all theoretical models. Each curve in Figure 2 corresponds to one theoretical evolutionary track. In Figure 2,
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Table 3
Candidate Models with $\chi^2 < 0.05$ of Four Observed Frequencies for the $\delta$ Scuti Star CoRoT 102749568

| Model | $Z$ | $M$ ($M_\odot$) | $f_{\text{ov}}$ | $T_{\text{eff}}$ (K) | $\log g$ (dex) | $R$ ($R_\odot$) | $L$ ($L_\odot$) | $n_0$ (hr) | $\Pi_0$ (s) | $\chi^2$ |
|-------|----|----------------|-------------|------------------|-------------|-------------|-------------|----------|----------|--------|
| 1     | 0.010 | 1.74 | 0 | 7111 | 3.706 | 3.065 | 21.52 | 4.21 | 431.1 | 0.030 |
| 2     | 0.010 | 1.75 | 0 | 7132 | 3.704 | 3.080 | 21.99 | 4.23 | 430.7 | 0.020 |
| 3     | 0.006 | 1.57 | 0.001 | 6951 | 3.693 | 2.953 | 18.24 | 4.14 | 330.0 | 0.028 |
| 4     | 0.006 | 1.54 | 0.004 | 6886 | 3.696 | 2.916 | 17.12 | 4.08 | 331.8 | 0.016 |
| 5     | 0.006 | 1.52 | 0.005 | 6837 | 3.698 | 2.889 | 16.34 | 4.03 | 331.5 | 0.041 |
| 6     | 0.006 | 1.51 | 0.006 | 6816 | 3.699 | 2.877 | 15.99 | 4.01 | 331.8 | 0.043 |
| 7     | 0.007 | 1.59 | 0.013 | 6993 | 3.695 | 2.965 | 18.84 | 4.16 | 427.7 | 0.023 |

Table 4
Theoretical Rotational Parameters of the Three Nonradial Pulsation Modes ($f_{12}$, $f_{34}$, and $f_{43}$) for the Candidate Models in Table 3

| Model | $f_{12}(\xi_r^i, \xi_h^i)$ ($\mu$Hz) | $f_{34}(\xi_r^i, \xi_h^i)$ ($\mu$Hz) | $f_{43}(\xi_r^i, \xi_h^i)$ ($\mu$Hz) |
|-------|----------------|----------------|----------------|
| obs   | 110.637(0.5) | 167.007(0.5) | 197.503(0.5) |
| 1     | 110.618(0.519) | 166.977(0.528) | 197.704(0.541) |
| 2     | 110.622(0.528) | 166.843(0.535) | 197.606(0.553) |
| 3     | 110.411(0.538) | 167.118(0.514) | 197.421(0.537) |
| 4     | 110.476(0.517) | 166.873(0.524) | 197.617(0.511) |
| 5     | 110.332(0.511) | 167.215(0.513) | 197.401(0.508) |
| 6     | 110.440(0.523) | 166.798(0.516) | 197.773(0.507) |
| 7     | 110.753(0.573) | 167.170(0.570) | 197.499(0.522) |

Table 5
Fundamental Parameters of the $\delta$ Scuti Star CoRoT 102749568

| Parameter | Values |
|-----------|--------|
| $M/M_\odot$ | 1.54 ± 0.03 |
| $Z$ | 0.006 |
| $f_{\text{ov}}$ | 0.004 ± 0.002 |
| $T_{\text{eff}}$ (K) | 6886 ± 70 |
| $\log g$ | 3.696 ± 0.003 |
| $R/R_\odot$ | 2.916 ± 0.039 |
| $L/L_\odot$ | 17.12 ± 1.13 |
| $M_{\text{eff}}/M_\odot$ | 0.148 ± 0.003 |
| $R_{\text{eff}}/R_\odot$ | 0.0581 ± 0.0007 |

Note. The rotational parameters $\beta_{kl}$ of observed frequencies inside brackets are the asymptotic value of $g$ modes according to Equation (1).

The filled circles correspond to seven candidate models of CoRoT 102749568 in Table 3.

Christensen-Dalsgaard (2003) defines the general expression of the rotational parameter $\beta_{kl}$ of a pulsation mode for a rigid body as

$$
\beta_{kl} = \frac{\int_0^R \left( \xi_r^2 + L_2 \xi_h^2 - 2 \xi_r \xi_h - \xi_r^2 \right) r^2 \rho dr}{\int_0^R \left( \xi_r^2 + L_2 \xi_h^2 \right) r^2 \rho dr},
$$

where the subscripts “$r$” and “$h$” correspond to the radial displacement and the horizontal displacement, $\rho$ denotes the local density, and $L^2 = l(l + 1)$. Based on the asymptotic behavior of the eigenfunctions of high-order $g$ modes, $\beta_{kl}$ can be simplified as the asymptotic value $1 - \frac{1}{L^2}$. According to asymptotic value of $g$ modes, $\beta_{k,l=1} = 0.5$, $\beta_{k,l=2} = 0.833$, and $\beta_{k,l=3} = 0.917$.

For the three identified $l = 1$ modes $f_{12}$, $f_{34}$, and $f_{43}$, their corresponding $\beta_{kl}$ of these candidate modes are listed in Table 4. In Table 4, rotational parameters $\beta_{kl}$ of observed frequencies are asymptotic values of $g$ modes based on Equation (1). It can be found in Table 4 that theoretical values of $\beta_{kl}$ for models 1, 2, 3, and 7 significantly deviate from those asymptotic values of $g$ modes. We therefore exclude these four models from our considerations. The physical parameters of CoRoT 102749568 are obtained based on models 4, 5, and 6. These parameters are listed in Table 5. In our work, we select the theoretical model (model 4) with minimum value of $\chi^2 = 0.016$ as the optimal model. Its theoretical evolutionary track corresponds to the curve in Figure 1.

Theoretical pulsation frequencies of the optimal model are listed in Table 6, in which $n_\rho$ is the amount of radial nodes in the propagation of $p$ modes and $n_g$ the amount of radial nodes in the propagation of $g$ modes. We notice in Table 6 that most of the pulsation modes are gravity and mixed modes. Figure 3 shows a plot of $\beta_{kl}$ to theoretical pulsation frequencies for the optimal model. We can find in Figure 3 that most of $\beta_{kl}$ are in good agreement with the asymptotic values of $g$ modes. These pulsation modes possess more pronounced $g$-mode features. Besides, there are several pulsation modes whose $\beta_{kl}$ obviously deviate from the asymptotic values of $g$ modes. They have more pronounced $p$-mode features.

Comparisons of results of pulsation frequencies in Table 2 are listed in Table 7. The $m = 0$ pulsation frequencies in columns denoted with $\nu^{\text{liner}}$ are derived from $m = 0$ modes according to Equation (1). The filled circles in Figure 3 correspond to $m = 0$ components of the multiplets in Table 7. It can be noticed in Table 7 that $m = 0$ components in multiplets 1, 2, 3, 5, 9, 10, 14, and 18 are observed, while $m = 0$ components in multiplets 4, 6, 7, 8, 11, 12, 13, 15, 16, 17, 19, 20, and 21 are absent. In Figure 3, we notice that $\beta_{kl}$, of corresponding $m = 0$ components in multiplet 1, 2, 3, 5, 9, 10, 14, and 18 agree well with the asymptotic value of $g$ modes. These $m = 0$ components in multiplet 13 are slightly larger than the asymptotic value from Equation (1). Moreover, we find that $\beta_{kl}$ of corresponding $m = 0$ components in multiplet 13 are slightly larger than the asymptotic value from Equation (1). These results also prove that our approach of mode identification based on the rotational splitting of $g$ modes is self-consistent.
Table 6
Theoretically Calculated Frequencies of the Optimal Model

| \( \nu^{th}(l, \eta_p, n) \) (MHz) | \( \Delta \nu \) | \( \nu^{th}(l, \eta_p, n) \) (MHz) | \( \Delta \nu \) | \( \nu^{th}(l, \eta_p, n) \) (MHz) | \( \Delta \nu \) | \( \nu^{th}(l, \eta_p, n) \) (MHz) | \( \Delta \nu \) | \( \nu^{th}(l, \eta_p, n) \) (MHz) | \( \Delta \nu \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 86.4460 (1, 0, -9) | 0.833 | 121.149 (1, 1, -58) | 0.844 | 68.292 (3, 0, -150) | 0.917 | 112.635 (1, 1, 89) | 0.918 |
| 118.212 (1, 0, -9) | 0.833 | 122.819 (2, 0, -58) | 0.857 | 68.772 (3, 0, -149) | 0.917 | 113.962 (1, 1, 88) | 0.919 |
| 140.555 (0, 2, -8) | 0.833 | 124.380 (2, 0, -57) | 0.844 | 69.238 (3, 0, -148) | 0.917 | 115.230 (1, 1, 87) | 0.922 |
| 170.701 (0, 3, -6) | 0.833 | 126.576 (2, 0, -56) | 0.837 | 69.665 (3, 0, -147) | 0.917 | 116.273 (1, 1, 86) | 0.925 |
| 201.090 (0, 4, -6) | 0.833 | 128.974 (2, 0, -55) | 0.836 | 70.089 (3, 0, -146) | 0.917 | 117.415 (1, 1, 85) | 0.921 |
| 232.369 (0, 5, 0) | 0.833 | 131.076 (2, 0, -54) | 0.841 | 70.565 (3, 0, -145) | 0.917 | 118.825 (1, 1, 84) | 0.919 |
| 263.224 (0, 6, 0) | 0.833 | 132.641 (2, 0, -53) | 0.840 | 71.076 (3, 0, -144) | 0.917 | 120.332 (1, 1, 83) | 0.919 |

Note: \( \nu^{th} \) denotes calculated frequency in MHz, \( \eta_p \) is the amount of radial nodes in the propagation cavities of p modes, \( n_q \) is the amount of radial nodes in the propagation cavities of g modes. \( \Delta \nu \) is one rotational parameter measuring the size of rotational splitting.

Finally, we try to conduct mode identification for the three isolated pulsation frequencies based on the optimal model, and list in Table 8. We notice in Table 8 that there are two possible model counterparts for \( f_g \), i.e., (1, 0, -51, -1) 76.394 MHz and (2, 0, -117, 2) 76.303 MHz. For \( f_{\delta \theta} \), (2, 3, -35, +2) 190.803 MHz may be a possible model counterpart. According
to the analyses in Section 2, the spherical harmonic degree of $f_{48}$ allows two possibilities: $l = 2$ and $l = 3$. When identifying $f_{48}$ and $f_{51}$ as being two $l = 2$ modes, their possible model counterparts are listed in Table 7. If $f_{48}$ and $f_{52}$ are identified as being two $l = 3$ modes, there are no suitable model counterparts for them.

4. DISCUSSIONS

When conducting model fittings, we use four identified pulsation modes including the radial first overtone $f_{13}$ and three $l = 1$ modes ($f_{12}$, $f_{34}$, and $f_{43}$). Figure 4 shows a propagation diagram of the optimal model. Based on the default parameters, we adopt the position where the hydrogen fraction $X_{He} = 0.01$ as the boundary of the helium core. The outer zone is the stellar envelope, and the inner zone is the helium core. The vertical curves in Figures 4 and 5 indicate the boundary of the helium core. Figure 5 shows the scaled eigenfunctions of the radial first overtone and the three $l = 1$ nonradial pulsation modes. It can be seen clearly in Figure 5 that the radial first overtone mainly propagates in the stellar envelope, and therefore mainly provides constraints on the stellar envelope. For the three nonradial pulsation modes, Figure 5 shows that they have g-mode features in the helium core and p-mode features in the stellar envelope. Then the three nonradial pulsation modes mainly provide constraints on the helium core.

Following Chen et al. (2016), we introduce two asteroseismic parameters: the acoustic radius $\tau_0$ and the period separation $\Pi_0$. The acoustic radius $\tau_0$ is a significant physical parameter in asteroseismic study. The acoustic radius $\tau_0$ carries information on the stellar envelope (e.g., Ballot et al. 2004; Miglio et al. 2010; Chen et al. 2016). The acoustic radius $\tau_0$ is defined as (Aerts et al. 2010)

$$\tau_0 = \int_0^R \frac{dr}{c_s},$$

(5)

in which $R$ is the stellar radius and $c_s$ the adiabatic sound speed. According to Equation (5), the value of acoustic radius $\tau_0$ is mainly dominated by the profile of $c_s$ inside the stellar envelope.

According to the theory of stellar oscillations, g-mode oscillations are gravity waves. They mainly propagate inside the helium core. Their properties can be characterized by $\Pi_0$, which is defined as

$$\Pi_0 = 2\pi^2 \left( \int_0^R N \frac{N}{r} dr \right)^{-1},$$

(6)

(Unno et al. 1979; Tassoul 1980; Aerts et al. 2010), where $N$ is the Brunt–Väisälä frequency. According to Equation (6), $\Pi_0$ is mainly dominated by the profile of Brunt–Väisälä frequency $N$ inside the helium core.

To fit the four pulsation modes ($f_{12}$, $f_{13}$, $f_{34}$, and $f_{43}$), both the helium core and the stellar envelope of the theoretical model need to be matched to the actual structure of CoRoT 102749568. It can be found in Table 3 that $\tau_0$ and $\Pi_0$ of the three preferred models (models 4, 5, and 6) are very close. This is because they are nearly alike in structure. Thus the size of the helium core of CoRoT 102749568 is determined to be $M_{He} = 0.148 \pm 0.003 \ M_\odot$ and $R_{He} = 0.0581 \pm 0.0007 \ R_\odot$. The errors are estimated on the basis of the deviations of the helium cores of models 5 and 6 from that of model 4.

According to Equation (1), the rotational period $P_{rot}$ of the $\delta$ Scuti star CoRoT 102749568 is determined to be $P_{rot} = 1.34 \pm 0.04$ days. Meanwhile, we find in Table 5 that the theoretical radius $R$ of CoRoT 102749568 is $2.916 \pm 0.039 \ R_\odot$. 

Figure 3. Plot of $\beta_{kl}$ vs. theoretically calculated frequency $\nu$ of the optimal model. The filled circles correspond to $m = 0$ components of the multiplets in Table 7.
| Multiplet ID | $f_{\text{obs}}$ (Hz) | $f_{\text{theo}}$ (Hz) | $\Delta f$ (Hz) | Multiplet ID | $f_{\text{obs}}$ (Hz) | $f_{\text{theo}}$ (Hz) | $\Delta f$ (Hz) |
|-------------|----------------------|----------------------|----------------|-------------|----------------------|----------------------|----------------|
| $f_1$       | 106.152              | 106.024 (1, −1)      | 0.128          | 11          | 122.559              | 123.228 (2, −2)      | 0.669          |
| $f_2$       | 110.637              | 110.476 (1, 0)       | 0.161          | $f_{18}$    | 144.934              | 144.904 (2, +1)      | 0.030          |
| $f_{14}$    | 115.036              | 114.928 (1, +1)      | 0.108          | $f_{28}$    | 194.179              | 194.638 (2, −2)      | 0.459          |
| $f_{31}$    | 162.625              | 162.360 (1, −1)      | 0.265          | $f_{12}$    | 216.758              | 216.547 (2, +1)      | 0.211          |
| $f_{34}$    | 167.007              | 166.873 (1, 0)       | 0.134          | $f_{35}$    | 222.367              | 222.005 (2, −2)      | 0.362          |
| $f_{41}$    | 192.909              | 193.216 (1, −1)      | 0.307          | $f_{13}$    | 252.079              | 251.871 (2, +2)      | 0.208          |
| $f_{43}$    | 197.503              | 197.617 (1, 0)       | 0.114          | $f_{44}$    | 125.296              | 125.696 (3, −1)      | 0.400          |
| $f_{44}$    | 201.898              | 202.018 (1, +1)      | 0.120          | $f_{14}$    | 133.453              | 133.602 (3, 0)       | 0.149          |
| $f_{55}$    | 171.485              | 171.386 (1, +1)      | 0.099          | $f_{45}$    | 141.765              | 141.508 (3, +1)      | 0.257          |
| $f_{5}$     | 87.275               | 86.767 (1, −1)       | 0.508          | $f_{15}$    | 115.706              | 116.152 (3, −2)      | 0.446          |
| $f_6$       | 96.149               | 95.707 (1, +1)       | 0.442          | $f_{20}$    | 123.812              | 124.093 (3, −1)      | 0.281          |
| $f_8$       | 100.779              | 100.560 (2, −2)      | 0.219          | $f_{17}$    | 117.666              | 118.118 (3, −3)      | 0.452          |
| $f_{11}$    | 108.372              | 107.932 (2, −1)      | 0.440          | $f_{25}$    | 134.071              | 133.964 (3, −1)      | 0.107          |
| $f_{16}$    | 115.872              | 115.304 (2, 0)       | 0.568          | $f_{40}$    | 233.083              | 233.576 (3, −1)      | 0.493          |
| $f_9$       | 102.072              | 102.510 (2, −2)      | 0.438          | $f_{50}$    | 249.725              | 249.422 (3, +1)      | 0.303          |
| $f_{21}$    | 124.571              | 124.264 (2, +1)      | 0.307          | $f_{26}$    | 134.762              | 135.456 (3, 0)       | 0.694          |
| $f_{23}$    | 132.028              | 131.516 (2, +2)      | 0.512          | $f_{50}$    | 158.977              | 159.148 (3, +3)      | 0.171          |
| $f_5$       | 65.541               | 65.633 (2, −2)       | 0.092          | $f_1$       | 64.936               | 65.120 (3, −1)       | 0.184          |
| $f_3$       | 72.978               | 72.807 (2, −1)       | 0.171          | $f_3$       | 96.938               | 96.675 (3, +3)       | 0.263          |
| $f_{32}$    | 164.262              | 164.281 (2, −2)      | 0.019          | $f_{19}$    | 122.769              | 122.827 (3, −1)      | 0.058          |
| $f_{36}$    | 171.638              | 171.635 (2, −1)      | 0.003          | $f_{20}$    | 155.380              | 154.588 (3, +3)      | 0.792          |
| $f_{33}$    | 164.855              | 164.831 (2, 0)       | 0.024          | $f_{58}$    | 176.285              | 177.108 (3, −2)      | 0.823          |
| $f_{37}$    | 172.243              | 172.143 (2, +1)      | 0.100          | $f_{46}$    | 209.708              | 208.766 (3, +2)      | 0.942          |
| $f_{50}$    | 189.056              | 188.534 (2, 0)       | 0.522          | $f_{45}$    | 203.652              | 203.106 (2, +2)      | 0.546          |

Note. $f_{\text{obs}}$ denotes the observed frequencies in Hz, $f_{\text{theo}}$ denotes the calculated frequencies in Hz. $\Delta f = |f_{\text{obs}} - f_{\text{theo}}|$. 

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According to $v_{\text{rot}} = 2\pi R/P_{\text{rot}}$, the rotational velocity at the equator is then deduced to be $v_{\text{rot}} = 109.8 \pm 4.6$ km s$^{-1}$, which is in agreement with the value of $v \sin i = 115 \pm 20$ km s$^{-1}$ (Paparó et al. 2013).

It should be noticed that Equation (1) only contains the first-order effect of rotation $C_1 m/P_{\text{rot}}$, in which $C_1 = 1 - \frac{1}{2}$. The second-order effect of rotation is derived by Dziembowski & Goode (1992) as being $\frac{m C_2}{P_{\text{rot}}^{1/3}}$. The coefficient $C_2 = \frac{4\pi^2(2l^2 - 3) - 9}{2\pi^2(4l^2 - 3)}$. The ratio of the second-order effect and the first-order effect is then deduced to be $\phi_l = \frac{C_2 m}{C_1 P_{\text{rot}}^{1/3}}$.

Assuming $v_{l,1,0} = 100$ $\mu$Hz, the absolute value of $\phi_l$ is estimated to be 0.0043, $\phi_{l=2}$ to be 0.0141, and $\phi_{l=3}$ to be 0.0073. For pulsation modes with $l = 1$, the second-order effect is 0.43% of that of the first-order effect. For pulsation modes with $l = 2$ and $l = 3$, the ratios $\phi_{l=2}$ and $\phi_{l=3}$ are in direct proportion to the azimuthal number $m$. The second-order effect is 2.82% that of the first-order for modes with $l = 2$ and $|m| = 2$. The second-order effect is 2.19% that of the first-order for modes with $l = 3$ and $|m| = 3$. In brief, the impact of the second-order effect on stellar rotation is much less than that of the first-order effect. Hence the second-order effect of rotation is not considered in our work.

Finally, our model fitting results show that a slight increase in the convective core size is essential to explain these multiplets. There are two different ways to increase the convective core size: convective core overshooting (Herwig 2000; Li & Yang 2007; Zhang 2013) and rotation (Eggenberger et al. 2010; Girardi et al. 2011). Maeder & Meynet (2000) and Yang et al. (2013) found that the effects of rotation on stellar structure and evolution depend on the masses of stellar models. Moreover, Yang et al. (2013) noticed that 2.05 $M_\odot$ is a critical mass. Rotation results in an increase in the convective core size for stars with $M > 2.05 M_\odot$. The effect is similar to that of convective core overshooting. However for stars with $M < 2.05 M_\odot$, rotation leads to a decrease in the convective core size. The optimal model in our work corresponds to a star with $M = 1.54 M_\odot$, $Z = 0.006$, $f_{\text{ov}} = 0.004$. According to the analyses of Yang et al. (2013), rotation will result in a slight decrease in the convective core size. If the effects of rotation are included in theoretical evolutionary models, a larger convective core overshooting may be indispensable.

5. SUMMARY

In this work, we carry out asteroseismic analyses and numerical calculations for the δ Scuti star CoRoT 102749568. The main results are as follows:

1. We identify 21 sets of multiplets using the regularities in rotational splitting, including four sets of multiplets with $l = 1$, nine sets of multiplets with $l = 2$, and eight sets of multiplets with $l = 3$. In particular, there are three complete triplets, i.e., $(f_{10}, f_{12}, f_{14}, f_{31}, f_{33}, f_{35})$, and $(f_{41}, f_{43}, f_{45})$. The rotational period $P_{\text{rot}}$ is estimated to be $1.34^{+0.04}_{-0.05}$ days according to the frequency differences in these multiplets.

2. Based on our model calculations, the δ Scuti star CoRoT 102749568 is in the post-main-sequence evolution stage. The stellar parameters of the δ Scuti star CoRoT 102749568 are determined to be $M = 1.54 \pm 0.03$ $M_\odot$, $Z = 0.006$, $f_{\text{ov}} = 0.004 \pm 0.002$, $\log g = 3.696 \pm 0.003$, $T_{\text{eff}} = 6886 \pm 70$ K, $R = 2.916 \pm 0.039$ $R_\odot$, and $L = 17.12 \pm 1.13$ $L_\odot$.

3. Based on our optimal model, we notice that most of the oscillation frequencies are mixed modes. The radial first overtone $f_{11}$ mainly provides constraints on the stellar envelope. The three nonradial pulsation modes $f_{12}$, $f_{33}$, and $f_{35}$ possess more pronounced g-mode features, which mainly provide constraints on the helium core. The property of the stellar envelope is characterized by the acoustic radius $\bar{R}$, and the property of the helium core is characterized by the period separation $\Pi_0$. Finally, the size of the helium core of CoRoT 102749568 is determined to be $M_{\text{He}} = 0.148 \pm 0.003$ $M_\odot$, and $R_{\text{He}} = 0.0581 \pm 0.0007$ $R_\odot$.

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\begin{table}
\centering
\begin{tabular}{|c|c|c|c|}
\hline
ID & $\nu^{\text{obs}}$ ($\mu$Hz) & $\nu^{\text{obs}}(l, n_p, n_q, n_m)$ ($\mu$Hz) & $\Delta \nu$ ($\mu$Hz) \\
\hline
$f_{13}$ & 112.291 & 112.212(0, 1, 0) & 0.079 \\
$f_4$ & 76.363 & 76.394(1, 0, −51, −1) & 0.031 \\
 & & 76.303(2, 0, −117, +2) & 0.060 \\
$f_{30}$ & 190.612 & 190.803(2, 3, −39, +2) & 0.191 \\
$f_{52}$ & 262.964 & 262.382(1, 6, −14, −1) & 0.582 \\
\hline
\end{tabular}
\caption{Possible Mode Identifications for the Unidentified Observed Frequencies Based on the Optimal Model}
\end{table}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{N is Brunt–Väisälä frequency and $L_l$ (l = 1, 2, 3) are Lamb frequencies. $M_k$ is the stellar mass. The vertical line indicates the boundary of the helium core.}
\end{figure}
Figure 5. Scaled eigenfunctions of the radial first overtone $f_{112}$ and the three nonradial modes $f_{123}, f_{345},$ and $f_{436}$. $X_q = \sqrt{q(1 - q)}$ and $q = M/M_*$. Panel (a) is for the radial first overtone $112.212 \mu\text{Hz}$ ($l = 0, n_p = 1, n_q = 0$). Panel (b) is for the mode $110.476 \mu\text{Hz}$ ($l = 1, n_p = 1, n_q = -37$). Panel (c) is for the mode $166.873 \mu\text{Hz}$ ($l = 1, n_p = 4, n_q = -24$). The vertical line indicates the boundary of the helium core.