Chan-Paton Soliton Gauge States of Compatified Open String

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Abstract

We study the mechanism of enhanced gauge symmetry of bosonic open string compatified on torus by analyzing the zero-norm soliton (nonzero winding of wilson line) gauge states in the spectrum. Unlike the closed string case, we find that the soliton gauge state exists only at massive levels. These soliton gauge states correspond to the existence of enhanced massive gauge symmetries with transformation parameters containing both Einstein and Yang-Mills indices. In the T-dual picture, these symmetries exist only at some discrete values of compatified radii when N D-branes are coincident.
I. INTRODUCTION

The discovery of D-brane as R-R charge carrier and its applications to various string dualities has made it clear that open string is essential in the study of string theory. Historically, Yang-Mills gauge symmetry was incorporated into string theory through different mechanisms for closed and open strings. For the closed string, it was built into the theory through compatification of string coordinates or, more generally, by adding an internal Kac-Moody conformal field theory with the appropriate central charge. For the open string instead, Yang-Mills degree of freedom was built into the theory through Chan-Paton effect by adding charges at the end points of string.

In the previous paper, we related the closed string Kac-Moody gauge symmetry to the existence of massless zero-norm soliton gauge states (SGS) in the spectrum of torus compatification. The program was then extended to the massive states. The existence of massive SGS thus implies that there is an infinite enhanced gauge symmetry of compatified closed string theory. In this paper, we will study the SGS of compatified open string theory. Unlike the closed string case, we find that the SGS exists only at massive levels. These SGS correspond to the enhanced massive symmetries with transformation parameters containing both Einstein and Yang-Mills indices. This is reminiscent of the symmetry of closed Heterotic string massive modes discovered previously. In the T-dual picture, these SGS implies the existence of enhanced massive gauge symmetry at some discrete values of compatified radii when N D-branes are coincident.

This paper is organized as following. In section II we discuss the uncompatified open string. We first derive both massless and massive Chan-Paton zero-norm gauge states. The corresponding gauge symmetries and Ward identities are then derived. In the massive case, we get mixed Einstein-Yang-Mills-type symmetry, which is similar to the one we derived in the closed Heterotic string theory. Section III is devoted to the compatified open string case. Massive SGS, which is responsible for the enhancement of massive gauge symmetry, are shown to exist at any higher massive levels of the spectrum. A brief discussion is given
II. CHAN-PATON GAUGE STATES

In this section, we discuss (zero-norm) gauge state of uncompatified open string with Chan-Paton factor and its implication on on-shell symmetry and Ward identity. For simplicity, we consider the oriented $U(N)$ case. The vertex operators of massless gauge state is

$$\theta^a \lambda^a_{ij} k \cdot \partial x e^{ikx}$$

(1)

where $\lambda \in U(N), i \in N, j \in \tilde{N}$ and $a \in$ adjoint representation of $U(N)$. The on-shell conformal deformation and $U(N)$ gauge symmetry to lowest order in the weak background field approximation are ($\Box \theta^a = 0, \Box \equiv \partial_\mu \partial^\mu$)

$$\delta T = \lambda^a_{ij} \partial_\mu \theta^a \partial x^\mu$$

(2)

and

$$\delta A^a_\mu = \partial_\mu \theta^a$$

(3)

with $T$ the energy momentum tensor and $A^a_\mu$ the massless gauge field.

One can verify the corresponding Ward identity by calculating e.g., 1-vector and 3-tachyons four point correlators. The amplitude is calculated to be

$$T^{abcd}_\mu = \int \prod_{i=1}^4 dx_i \langle e^{ik_1 x_1} \partial x_\mu e^{ik_2 x_2} e^{ik_3 x_3} e^{ik_4 x_4} \rangle T_r (\lambda^a \lambda^b \lambda^c \lambda^d)$$

(4)

$$= \frac{\Gamma \left( -\frac{s}{2} - 1 \right) \Gamma \left( -\frac{t}{2} - 1 \right)}{\Gamma \left( \frac{u}{2} + 1 \right)} \left[ k_{3\mu} \left( \frac{s}{2} + 1 \right) - k_{1\mu} \left( \frac{t}{2} + 1 \right) \right] \times T_r (\lambda^a \lambda^b \lambda^c \lambda^d)$$

In equation (2.4), $s, t$ and $u$ are the usual Mandelstam variables. One can then verify the Ward identity

$$\theta^b k^\mu_2 T^{abcd}_\mu = 0$$

(5)
We now discuss the massive gauge states. The vertex operator of type I massive vector
gauge state is

$$\theta^a_\mu \lambda^a_{ij} [k \cdot \partial x \partial x^\mu + \partial^2 x^\mu] e^{ikx}$$  \hspace{1cm} (6)

We note that the gauge state polarization contains both Einstein and Yang-Mills indices. This is very similar to the 10d closed Heterotic string case [5]. The only difference is that in
the Heterotic string, one could have more than one Yang-Mills index. The on-shell conformal
deformation and the mixed Einstein-Yang-Mills-type symmetry to lowest order weak field
approximation are \((\Box - 2) \theta^a_\mu = \partial \cdot \theta^a = 0\)

$$\delta T = \lambda^a_{ij} \partial_\mu \theta^a_\nu \partial x^\mu \partial x^\nu + \lambda^a_{ij} \theta^a_\mu \partial^2 x^\mu$$  \hspace{1cm} (7)

and

$$\delta M^a_\mu_\nu = \partial_\mu \theta^a_\nu + \partial_\nu \theta^a_\mu$$  \hspace{1cm} (8)

One can also derive the corresponding massive Ward identity by calculating the decay
rate of one massive state to three tachyons. The most general amplitude is calculated to be

$$A^{abcd} = \varepsilon^a \varepsilon^c \varepsilon^d (\varepsilon^b_\mu T^{\mu \nu} + \varepsilon^b_\mu T^\mu) \text{Tr} (\lambda^a \lambda^b \lambda^c \lambda^d)$$  \hspace{1cm} (9)

where

$$T^{\mu \nu} = \frac{\Gamma (-\frac{s}{2} - 1) \Gamma (-\frac{t}{2} - 1) \Gamma (\frac{n}{2} + 2)}{\Gamma (\frac{n}{2} + 1)} \left\{ \frac{s}{2} \left(\frac{s}{2} + 1\right) k^\mu_3 k^\nu_3 + \frac{t}{2} \left(\frac{t}{2} + 1\right) k^\mu_1 k^\nu_1 - 2 \left(\frac{s}{2} + 1\right) \left(\frac{t}{2} + 1\right) k^\mu_1 k^\nu_3 \right\}$$  \hspace{1cm} (10)

and

$$T^\mu = \frac{\Gamma (-\frac{s}{2} - 1) \Gamma (-\frac{t}{2} - 1) \Gamma (\frac{n}{2} + 2)}{\Gamma (\frac{n}{2} + 1)} \left\{ -k^\mu_3 \frac{s}{2} \left(\frac{s}{2} + 1\right) - k^\mu_1 \frac{t}{2} \left(\frac{t}{2} + 1\right) \right\}$$  \hspace{1cm} (11)

In equation (2.9) \(\varepsilon^a\) etc. are polarization of tachyons and \((\varepsilon^b_\mu, \varepsilon^b_\mu)\) is polarization of the
massive state. The above amplitude satisfies the following ward identity

$$k^\mu_\mu \theta^a_\mu T^{\mu \nu} + \theta^a_\mu T^\mu = 0$$  \hspace{1cm} (12)
Similar consideration can be applied to the following type II massive scalar gauge state

\[
\left[ \frac{1}{2} \alpha_{-1} \cdot \alpha_{-1} + \frac{5}{2} k \cdot \alpha_{-2} + \frac{3}{2} (k \cdot \alpha_{-1})^2 \right] |k, l = 0, i, j \rangle
\]  

(13)

which corresponds to a massive $U(N)$ symmetry.

III. CHAN-PATON SOLITON GAUGE STATE ON $R^{25} \otimes T^1$

In this section, we discuss soliton gauge states on torus compactification of bosonic open string. As is well known, the massless $U(N)$ gauge symmetry will be broken in general after compatification unless $N$ D-branes, in the T-dual picture, are coincident. We will see that when D-branes are coincident, one has enhancement of (unwinding) zero-norm gauge states and the massless $U(N)$ symmetry will be recovered. These zero-norm gauge states can be considered as charges or symmetry parameters of $U(N)$ group.

In the discussion of open string compatification, one needs to turn on the wilson line or nonzero background gauge field in the compact direction. This will effect the momentum in the compact direction, and the virasoro operators become

\[
L_0 = \frac{1}{2} \left( \frac{2\pi l - \theta_j + \theta_i}{2\pi R} \right)^2 + \frac{1}{2} (k^\mu)^2 + \sum_{n=1}^{\infty} \left( \alpha_-^\mu \alpha_n^\mu + \alpha_+^25 \alpha_n^25 \right) 
\]

(14)

\[
L_m = \frac{1}{2} \sum_{n=\infty}^{\infty} \vec{\alpha}_{m-n} \cdot \vec{\alpha}_n
\]

(15)

Note that in equation (3.2), $\alpha_0^{25} \equiv p^{25}$ which also appears in the first term in equation (3.1). $k$ is the 25d momentum. $\theta_i, R$ are the gauge and space-time moduli respectively and $l$ is the winding number in the compact direction. The spectrums of type I and type II zero-norm gauge states become

\[
M^2 = \left( \frac{2\pi l - \theta_j + \theta_i}{2\pi R} \right)^2 + 2I
\]

(16)

and

\[
M^2 = \left( \frac{2\pi l - \theta_j + \theta_i}{2\pi R} \right)^2 + 2 (I + 1)
\]

(17)
where \( I = \sum_{n=1}^{\infty} (\alpha_{-n}^{\mu} \alpha_{n}^{\mu} + \alpha_{-n}^{25} \alpha_{n}^{25}) \).

For the massless case \( I = l = 0 \), one gets \( N^2 \) massless solution from equation (3.3)

\[
k_{\mu} \alpha_{-1}^{\mu} |k, l = 0, i, j\rangle
\]  

(18)

if all \( \theta_i \) are equal, or in the T-dual picture when \( N \) D-branes are coincident. These \( N^2 \) massless gauge states correspond to the charges of massless \( U(N) \) gauge symmetry. There is no type II massless solution in equation (3.4).

We are now ready to discuss the interesting massive case. For \( M^2 = 2 \) and general moduli \( (R, \theta_i) \),

1. \( I = 1, l = 0 \), one gets two gauge states solutions from equation (3.3):

\[
[(\varepsilon \cdot \alpha_{-1}) (k \cdot \alpha_{-1}) + \varepsilon \cdot \alpha_{-2}] |k, l = 0, i, i\rangle, \; \varepsilon \cdot k = 0
\] 

(19)

and

\[
(k \cdot \alpha_{-1} \alpha_{-1}^{25} + \alpha_{-2}^{25}) |k, l = 0, i, i\rangle
\] 

(20)

If all \( \theta_i \) are equal, the \((i, i)\) is enhanced to \((i, j)\). Equation (3.7) implies a massive \( U(N) \) symmetry with transformation parameter \( \theta^a \). Equation (3.6) implies a massive Einstein-Yang-Mills-type symmetry with transformation parameter \( \theta^a_{\mu} \)

2. \( I = 0, \frac{2\pi l - \theta_j + \theta_i}{2\pi R} = \pm \sqrt{2} \), one gets solution from equation (3.3)

\[
(k \cdot \alpha_{-1} \pm \sqrt{2} \alpha_{-1}^{25}) |k, l, i, j\rangle
\] 

(21)

Now since \( |\theta_i - \theta_j| < 2\pi \), for any given \( R \), there is at most one solution of \((|l|, |\theta_i - \theta_j|)\). One is tempted to consider the case

\[
(k \cdot \alpha_{-1} \pm \sqrt{2} \alpha_{-1}^{25}) |k, l = \pm \sqrt{2} R, i, i\rangle
\] 

(22)

That means in the moduli \((R = \sqrt{2} n, \theta_i)\) with \( n \in Z^+ \), one has soliton gauge states which imply a massive \( U(1)^N \) symmetry. If all \( \theta_i \) are equal, the \((i, i)\) is enhanced to \((i, j)\). Equation (3.9) implies a massive \( U(N) \) symmetry at the discrete values of moduli point \( R = \sqrt{2} n \).
For example, in the T-dual picture, for $R = \sqrt{2}, l = \pm 2$, and if all D-branes are coincident, we have an enhanced massive $U(N)$ symmetry. This phenomenon is very different from the massless case, where one gets enhanced $U(N)$ symmetry at any radius $R$ when $N$ D-branes are coincident.

We would like to point out that similar Einstein-Yang-Mills-type symmetry was discovered before in the closed Heterotic string theory. There, however, one could have more than one Yang-Mills indices on the transformation parameters.

For the type II states with $M^2 = 2$ in equation (3.4), $I = l = 0$. One gets one more $U(N)$ gauge states

$$
\left[ \frac{1}{2} \alpha_{-1} \cdot \alpha_{-1} + \frac{1}{2} \alpha_{-2} \alpha_{-2} + \frac{5}{2} k \cdot \alpha_{-2} + \frac{3}{2} (k \cdot \alpha_{-1})^2 \right] |k; l = 0, i, j \rangle
$$

if all $\theta_i$ are equal.

For the general mass level, choosing $I = 0$ and $i, j$ in equation (3.2), we have $l/R = \pm M$. For say $R = \sqrt{2}$ and $l = \pm \sqrt{2}M$, which implies

$$
M^2 = 2n^2, \; n = 0, 1, 2, \ldots
$$

So we have Chan-Paton soliton gauge states at any higher massive level of the spectrum. Similar result was found in the closed string case.

IV. CONCLUSION

Zero-norm gauge state solution in the old covariant quantization of string theory is closely related to the BRST cohomology of the theory. Physically, they correspond to charges of the symmetries. It is believed that all space-time symmetry of string theory, including closed or open and compatified or uncompatified ones, are due to the existence of (soliton) gauge state in the spectrum. Similar consideration can be applied to the R-R charges and D-branes. Presumably, there is no R-R zero-norm gauge states as charges of R-R gauge fields in the type II string spectrum. How D-branes carry the zero-norm gauge state charges to emit R-R fields is an interesting question to study.
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