Anisotropic parton escape is the dominant source of azimuthal anisotropy from A Multi-Phase Transport

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We trace the development of azimuthal anisotropy \( v_n \) in A Multi-Phase Transport (AMPT) model using parton-parton collision history. The parton \( v_n \) is studied as a function of the number of collisions of each parton in Au+Au and d+Au collisions at \( \sqrt{s_{NN}}=200 \) GeV. It is found that the majority of \( v_n \) comes from the anisotropic escape probability of partons, with no fundamental difference at low and high transverse momenta. The contribution to \( v_n \) from the parton collective flow appears small; however, it is this small anisotropy from the collective flow, not that from the anisotropic escape probability, that is most relevant for medium properties in heavy ion collisions.

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Relativistic heavy ion collisions aim to create the quark-gluon plasma (QGP) and study quantum chromodynamics (QCD) at the extreme conditions of high temperature and energy density [1]. The system created in these collisions is described well by hydrodynamics [2]. The high pressure buildup drives the system to expand at relativistic speed. Experimental data fit with hydrodynamics inspired models suggest that particles are locally thermalized and possess a common radial flow velocity [3]. Of particular interest are non-central collisions where the overlap volume of the colliding nuclei is anisotropic in the transverse plane (perpendicular to beam). The anisotropic pressure gradient generates anisotropic expansion and final-state particle azimuthal distribution [4]. The azimuthal anisotropy is often referred to as anisotropic flow. Large anisotropies have been measured, as large as hydrodynamics predict given the initial energy density and overlap geometry in heavy ion collisions [2]. This suggests that the collision system is strongly interacting and nearly thermalized [5].

Parton transport can also describe the measured large anisotropic flow, but requires an unusually large parton-parton interaction cross-sections [6]. It approaches the limiting case of ideal hydrodynamics with infinite cross-sections and may be an effective description of the strongly interacting QGP (sQGP). A modern transport model, A Multi-Phase Transport (AMPT) [7, 8], can describe the large anisotropic flow using normal parton-parton cross-section from QCD with the string melting mechanism [9]. String melting liberates strings into a larger number of quarks and antiquarks, effectively increasing the magnitude of early interactions in the system and generates an over-populated partonic matter [10].

It is generally perceived that the large anisotropic flow is generated by the anisotropic pressure gradient in large systems of heavy ion collisions, as in hydrodynamic calculations. Recent particle correlation data, however, hint at similar anisotropic flow in small systems of high multiplicity \( p+p \) and \( p+Pb \) collisions at the LHC [11] and \( d+Au \) collisions at RHIC [12]. Hydrodynamic calculations have been applied to these systems and seem to describe the experimental data well [13]. The AMPT model also seems to be able to describe the measured correlations [14]. This suggests that these small-system collisions might create a sQGP as well, in contrast to the general expectation that these small systems are not large enough to create a QGP medium.

The purpose of this Letter is to study the development of anisotropic flow to shed light on its connection to the properties of the sQGP and the thermalization mechanism. We employ the string melting version of the AMPT model [8], which consists of the initial condition, parton scatterings, quark coalescence for hadronization, and hadronic scatterings. In particular, parton scatterings are treated with the ZPC elastic parton cascade [15]. We analyze the entire history of parton-parton interactions in AMPT form the initial encounter of the colliding nuclei to the final-state partons. In this study only partons are analyzed to avoid complications from hadronization and hadronic scatterings.

We used AMPT version 2.26t1d. The parton screening mass was taken as \( \mu=2.265/\text{fm} \), and the strong coupling constant was set to \( \alpha_s=0.33 \). For this study, about 55k Au+Au (fixed impact parameter \( b=7.3 \) fm) and 700k \( d+Au \) \( (b=0 \) fm) collisions at nucleon-nucleon center-of-mass energy \( \sqrt{s_{NN}}=200 \) GeV were generated. The Lund string fragmentation parameters were set to 0.55 and 0.15/GeV [2]. Together with an upper limit of 0.40 on the relative production of strange to nonstrange quarks in string fragmentation, this parameter set has been shown to approximately reproduce the multiplicity density, transverse momentum \( (p_T) \) spectra and elliptic flow \( (v_2) \) of pions and kaons below \( p_T \) of 2 GeV/c in semi-central and central Au+Au collisions at 200 GeV [10].

We compute the \( n^{th} \) harmonic plane of each event from...
The momentum anisotropy is characterized by Fourier coefficients [17]

\[ \langle r^2 \sin n\phi_r, \langle r^2 \cos n\phi_r \rangle \rangle + \pi \frac{n}{N} \]  

Here \( r \) and \( \phi_r \) are the polar coordinate of each initial parton (after its formation time) in the transverse plane, and \( \langle \rangle \) denotes the per-event average. The \( \psi^{(r)}_n \) is the azimuth of the short axis direction of the corresponding \( n \)th harmonic component of the overlap geometry.

We follow the parton two-body elastic scattering history in AMPT. The momentum anisotropy in the initial state, final state, and any intermediate state in-between. The momentum anisotropy is characterized by Fourier coefficients [17]

\[ \psi_n^{(r)} = \frac{1}{n} \frac{1}{N} \langle r^2 \sin n\phi_r, \langle r^2 \cos n\phi_r \rangle \rangle + \pi \frac{n}{N} \]  

where \( \phi \) is the azimuthal angle of the parton momentum.

Results. We trace the history of AMPT parton cascading by the number of collisions \( (N_{\text{coll}}) \) a parton suffers with other partons. Figure 1(a) shows the probability distributions of partons freezing out after \( N_{\text{coll}} \) collisions in \( \text{Au+Au (b=7.3 fm)} \) and \( d+Au \) \( (b=0 \text{ fm}) \) collisions. Partons with high final \( p_\perp \) have fewer collisions and freeze out earlier. In contrast, low \( p_\perp \) partons freeze out later, and they suffer relatively more collisions. Integrated over \( p_\perp \), partons suffer on average \( (N_{\text{coll}})=4.5 \) and 1.1 collisions before freezeout in \( \text{Au+Au} \) and \( d+\text{Au} \) collisions, respectively.

As shown in Fig. 1(a), some partons do not interact at all and thus instantly freeze out at \( N_{\text{coll}}=0 \). These partons tend to reside in the outer region of the overlap volume, as shown in Fig. 1(b) for \( \text{Au+Au} \) where the transverse radius \( (r_\perp) \) distribution of freezeout partons is shown by the thick solid curve. Those continuing to interact tend to have smaller \( r_\perp \) as shown by the thick dashed curve. This feature is qualitatively similar for all \( N_{\text{coll}} \) values (e.g. see the thin curves in Fig. 1(b) for \( N_{\text{coll}}=5 \)). This is consistent with the general expectation—the energy density is smaller in the outer shell thus the probability for further interactions is smaller. It is interesting to note that the freezeout “surface” moves inward, indicating an outside-to-inside freezeout scenario.

We track the development of \( v_n \) by studying parton \( v_n \) as a function of \( N_{\text{coll}} \). The \( v_2 \) results are shown in Fig. 2 for \( \text{Au+Au} \) collisions. The \( v_3 \) results are qualitatively similar. Qualitatively similar results are also found for \( d+\text{Au} \) collisions. In Fig. 2 the solid curve is the \( v_2 \) of all active partons after \( N_{\text{coll}} \) collisions; they are defined as partons that have suffered at least \( N_{\text{coll}} \) collisions, including those that freeze out after \( N_{\text{coll}} \) collisions but excluding partons that have frozen out with smaller \( N_{\text{coll}} \) values. The dashed curve is the \( v_2 \) of the partons that freeze out after suffering exactly \( N_{\text{coll}} \) collisions (i.e. without further interactions). At \( N_{\text{coll}}=0 \), the active parton \( v_n \) is strictly zero in the AMPT model. Some partons do not interact at all and instantly freeze out with \( N_{\text{coll}}=0 \). Because there is a larger probability for the partons to escape along the short axis of the overlap volume, those freeze-out partons have positive \( v_n \). Similar behavior has been seen before in another parton transport model [18]. In the low density limit, the anisotropy may be analytically derived [19]. In fact, this escape mechanism is rather general as it happens throughout the entire evolution of the collision system. After \( N_{\text{coll}} \) collisions, the \( v_n \) of all active partons is still roughly zero. Some of these partons freeze out; they have positive \( v_n \) due to the preferential escape along the short axis. The rest non-freezeout partons, having a negative \( v_n \) (shown as the dotted curve in Fig. 2), continue to interact. With one more collision, the azimuthal distribution of these non-freezeout partons, initially with negative \( v_n \), becomes roughly isotropic again, with approximately zero \( v_n \) (solid curve). This process then repeats itself.

It appears, therefore, that \( v_n \) comes mostly from an iterative escape mechanism. The traditional wisdom of low \( p_\perp \) particles accumulating \( v_n \) after multiple collisions plays only a minor role. In fact, partons at all stages of the system evolution are approximately isotropic (see the solid curves in Fig. 2 where only very small \( v_n \) is accumulated); whether they freeze out or not is determined by the anisotropic escape probability at that point of evolution. This includes initial partons before any scatterings, where some of them freeze out with a positive \( v_n \) due to the anisotropic escape probability. The high-\( p_\perp \) anisotropy is generally believed to result mostly from the escape mechanism [20, 21], which we have also verified with our results. Our finding here is that both high-\( p_\perp \) and low-\( p_\perp \) anisotropies result mostly from the escape mechanism, and there does not seem to be a qualitative
The freezeout partons (thick dashed curve) are for freezeout partons. The solid curves (thick solid and dashed curves) are for all(active) partons after suffering $N_{\text{coll}}$ collisions, the dashed curve freezeout partons, and the dotted curve non-freezeout partons.

This is a space-momentum correlation with a non-zero local velocity due to the anisotropic escape mechanism, but different from a collectivity that represents a common collective flow velocity achieved only via interactions. As Fig. 3 shows, there are space-momentum correlations for the active partons at any given $N_{\text{coll}}>0$; these correlations are good indicators of collective flow. Some of the active partons freeze out at a given $N_{\text{coll}}$; the additional $\beta_1$ for these freezeout partons relative to the active partons is the effect of the anisotropic escape mechanism.

The question is then whether the collective flow of the active partons is important for the final $v_n$. Thus we did test calculations with no collective flow by randomizing the parton azimuthal direction after each parton-parton scattering. The system continues to evolve in AMPT, but the evolution is different from the original one. The $\beta_1$ variable from this modified evolution is shown in Fig. 3 for Au+Au. The results for $d$+Au are again similar. The active parton $\beta_1$ is now zero because of the randomization. The freezeout parton $\beta_1$ is non-zero purely due to the anisotropic escape mechanism.

We show in Fig. 4 the $v_2$ of active partons and freezeout partons from this azimuth-randomized AMPT run by the thin solid and dashed curves, respectively, for Au+Au (left panel) and $d$+Au collisions (right panel). In the randomized case, the parton azimuthal angles are randomized and hence the active parton $v_2$ is zero; thus the final-state freezeout anisotropy is entirely due to the anisotropic escape mechanism. For comparison, the $v_2$ results from the normal AMPT run for Au+Au (already shown in Fig. 2) are superimposed in Fig. 4 as the thick solid and dashed curves, where the active parton $v_2$ is slightly positive and the freezeout parton $v_2$ is much higher. The gain in $v_2$ from the active partons to the freezeout partons is due to the escape mechanism. The gain in the normal AMPT results is slightly different from that in the azimuth-randomized results. This is not surprising because the anisotropies in the escape probability differ in these two cases: in the former case the parton $\hat{p}_1$’s are correlated with their $\hat{r}_1$’s while in the latter case the parton $\hat{r}_1$’s are random. The integrated $v_2$ of all final partons in Au+Au ($b=7.3$ fm) collisions is 3.9% from the normal AMPT results and 2.7% from the azimuth-randomized results, indicating that the collective flow of active partons plays only a minor role for the final $v_n$. The corresponding values for $d$+Au ($b=0$ fm) collisions are 2.7% and 2.5%.

We have shown mostly results in 200 GeV Au+Au collisions at impact parameter $b=7.3$ fm. There seems to be no qualitative difference between the behaviors in Au+Au and $d$+Au collisions, as shown in Fig. 4. The results for other impact parameters are also similar. Although we discussed only $v_2$, the same conclusions hold for $v_3$ as well, suggesting that the development mechanism of $v_n$ in AMPT is universal. We note that a fixed $N_{\text{coll}}$ value does not correspond to partons at identical

\[ \beta_1 \equiv \langle \hat{r}_1 \cdot \hat{p}_1 \rangle \]

where $\hat{r}_1$ and $\hat{p}_1$ are the transverse radial position and momentum unit vectors, as a function of $N_{\text{coll}}$. The $\beta_1$ parameter represents approximately the collective transverse radial velocity. The plot is for Au+Au collisions, but the result is similar for $d$+Au collisions. The $\beta_1$ of active partons (thick solid curve) at $N_{\text{coll}}=0$ is not exactly zero because partons can form only after a finite formation time over which a parton’s displacement depends on its momentum. The freezeout partons (thick dashed curve) at $N_{\text{coll}}=0$ have a strong space-momentum correlation.

\[ \langle \hat{r}_1 \cdot \hat{p}_1 \rangle = 0 \]
freezeout partons

\[(a) \text{Au+Au} \quad (b) \text{d+Au} \]

all(active) partons

escape mechanism is at work at both high and low \(p\) the final anisotropic flow. Our study indicates that the
This escape mechanism contributes to the majority of
partons possess positive \(v_n\) and therefore a fundamental difference in the anisotropic escape probability those escaped (freezeout) partons escape from the overlap volume, and due to the N after suffering (thick curves) and the azimuth-randomized (thin curves) AMPT results are shown. The solid curves are for all(active) partons after suffering \(N_{\text{coll}}\) collisions, and the dashed curves are for freezeout partons.

FIG. 4: Parton \(v_2\) as a function of the number of collisions \(N_{\text{coll}}\) in (a) \text{Au+Au} and (b) \text{d+Au} collisions. Both the normal (thick curves) and the azimuth-randomized (thin curves) AMPT results are shown. The solid curves are for all(active) partons after suffering \(N_{\text{coll}}\) collisions, and the dashed curves are for freezeout partons.

times but rather a convolution over different times. We have also studied results as a function of time instead of \(N_{\text{coll}}\), and our qualitative conclusions remain unchanged.

Discussions. The unique finding of our study is that partons escape from the overlap volume, and due to the anisotropic escape probability those escaped (freezeout) partons possess positive \(v_n\), even without interactions. This escape mechanism contributes to the majority of the final anisotropic flow. Our study indicates that the escape mechanism is at work at both high and low \(p_\perp\); there appears to be no fundamental difference in the \(v_n\) development of high- and low-\(p_\perp\) partons.

The correlations between space and momentum of freezeout partons are largely due to the escape mechanism. This mimics “surface emission,” [21] which selects partons to freeze out from a pool of active partons depending on the parton’s momentum and position. It thus creates a space-momentum correlation which is not a result of collective flow. It is important here to distinguish between space-momentum correlation and collective flow, where the latter means a collective motion that is generated by interactions so that particles acquire additional energy stored in their common motion (e.g., particles in nearly local thermal equilibrium moving on top of a common velocity field).

Figure 4 shows that the traditional picture of low \(p_\perp\) particles accumulating \(v_n\) after multiple collisions plays only a minor role for the final \(v_n\). There is indeed a finite collective flow in AMPT, as shown by the thick solid curve in Fig. 3. This radial flow is presumably generated by hydrodynamic-type interactions and pressure gradients. The azimuthal modulation of this radial flow is the anisotropic flow \(v_n\) of active partons, as shown in the solid thick curves in Figs. 2 and 4. It is the \(v_n\) of these active partons that is the most relevant for the standard hydrodynamic flow description, or the collective properties of the sQGP.

Hydrodynamics have been used successfully to describe heavy ion collisions [2]. Hydrodynamic evolution and expansion are pressure driven, and anisotropy stems from this expansion. Hydrodynamic evolution is typically stopped in a calculation when the local energy density reaches a given temperature. Particle production is then modeled by the Cooper-Frye formalism. Thus the escape mechanism shown in this transport study is not obviously present in hydrodynamics. On the other hand, a parton transport with large enough cross sections should approach hydrodynamic behavior. Therefore it is worthwhile to examine the possible role of the escape mechanism in the hydrodynamics framework. Previous studies have shown that continuous particle emission instead of sudden freezeout in hydrodynamics can have important implications for pion interferometry [22].

Since AMPT describes the bulk experimental data well [10], it is also worthwhile to investigate how to identify the possible escape mechanism in the experimental data. One discriminator would be radial flow: collectivity generates extra \(p_\perp\) but a simple space-momentum correlation from the escape mechanism does not.

It is generally accepted that anisotropic expansion would quickly diminish the anisotropy in the overlap geometry and hence in the pressure gradient. One would thus expect that anisotropic flow is sensitive mainly to the early stage of the collision and thermalization must happen early and quickly [4]. Our AMPT study indicates that anisotropic flow of active partons continues to build up and the anisotropy of freezeout partons does not decrease much with \(N_{\text{coll}}\), presumably due to the smallness of the anisotropic flow (of active partons) that is not sufficient to quickly diminish the spatial anisotropy. Therefore anisotropic flow is generated throughout the entire system evolution and fast thermalization may not be needed.

In summary, we have studied the development of
anisotropic flow $v_n$ in AMPT as a function of the number of collisions $N_{\text{coll}}$ that a parton suffers in Au+Au and $d+Au$ collisions at $\sqrt{s_{\text{NN}}}=200$ GeV. It is found that the majority of $v_n$ comes from the anisotropic escape probability of partons, and this picture applies similarly to partons at both high $p_\perp$ and low $p_\perp$. The anisotropic flow of active partons as a result of parton-parton interactions or hydrodynamic-type pressure gradient seems to be small; however, it is the collectivity and anisotropic flow of these active partons that are most relevant for the physics of the sQGP.

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[1] I. Arsene et al. (BRAHMS Collaboration), Nucl.Phys. A757, 1 (2005); B. Back et al. (PHOBOS Collaboration), ibid., 28; J. Adams et al. (STAR Collaboration), ibid., 102; K. Adcox et al. (PHENIX Collaboration), ibid., 184; B. Muller, J. Schukraft, and B. Wyslouch, Ann.Rev.Nucl.Part.Sci. 62, 361 (2012).
[2] U. Heinz and R. Snellings, Ann.Rev.Nucl.Part.Sci. 63, 123 (2013); C. Gale, S. Jeon, and B. Schenke, Int.J.Mod.Phys. A28, 1340011 (2013).
[3] B. Abelev et al. (STAR Collaboration), Phys.Rev. C79, 034909 (2009).
[4] J.-Y. Ollitrault, Phys.Rev. D46, 229 (1992).
[5] M. Gyulassy and L. McLerran, Nucl.Phys. A750, 30 (2005).
[6] D. Molnar and M. Gyulassy, Nucl.Phys. A697, 495 (2002).
[7] B. Zhang, C.M. Ko, B.-A. Li, and Z.-W. Lin, Phys.Rev. C61, 067901 (2000).
[8] Z.-W. Lin, C. M. Ko, B.-A. Li, B. Zhang, and S. Pal, Phys.Rev. C72, 064901 (2005).
[9] Z.-W. Lin and C.M. Ko, Phys.Rev. C65, 034904 (2002).
[10] Z.-W. Lin, Phys.Rev. C90, 014904 (2014).
[11] V. Khachatryan et al. (CMS Collaboration), JHEP 1009, 091 (2010); S. Chatrchyan et al. (CMS Collaboration), Phys.Lett. B718, 795 (2013); B. Abelev et al. (ALICE Collaboration), ibid. B719, 29 (2013); G. And et al. (ATLAS Collaboration), Phys.Rev.Lett. 110, 182302 (2013).
[12] A. Adare et al. (PHENIX Collaboration) (2014), 1404.7461.
[13] P. Bozek, Eur.Phys.J. C71, 1530 (2011); P. Bozek and W. Broniowski, Phys.Lett. B718, 1557 (2013).
[14] A. Bzdak and G.-L. Ma, Phys.Rev.Lett. 113, 252301 (2014).
[15] B. Zhang, Comput.Phys.Commun. 109, 193 (1998).
[16] J.-Y. Ollitrault, Phys.Rev. D48, 1132 (1993).
[17] S. Voloshin and Y. Zhang, Z.Phys. C70, 665 (1996).
[18] D. Molnar (2005), nucl-th/0503051.
[19] H. Heiselberg and A.-M. Levy, Phys.Rev. C59, 2716 (1999); S. Voloshin and A. M. Poskanzer, Phys.Lett. B474, 27 (2000).
[20] X.-N. Wang and M. Gyulassy, Phys.Rev.Lett. 68, 1480, (1992).
[21] E.V. Shuryak, Phys.Rev. C66, 027902 (2002).
[22] F. Grassi, Y. Hama, and T. Kodama, Phys.Lett. B355, 9 (1995); O. Socolowski, F. Grassi, Y. Hama, and T. Kodama, Phys.Rev.Lett. 93, 182301 (2004).