Radio frequency spectrum of fermions near a narrow Feshbach resonance

Junjun Xu,1,2 Qiang Gu,1 and Erich J. Mueller2

1Department of Physics, University of Science and Technology Beijing, Beijing 100083, China
2Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853, USA

(Dated: May 23, 2013)

We calculate the radio frequency (RF) spectrum of fermions near a narrow Feshbach resonance, explaining observations made in ultracold samples of 6Li [Hazlett, et al., PRL 108, 045304 (2012)]. Using a two channel resonance model, we show that the RF spectrum contains two peaks. In the wide resonance limit, nearly all spectral weight lies in one of these peaks. We find strong temperature dependence which can be traced to the energy dependence of the two-particle scattering. In addition to microscopic calculations, we use sum rule arguments to find generic features of the spectrum which are model independent.

PACS numbers: 03.75.Ss, 67.85.Lm, 37.10.Pq, 34.50.Cx

I. INTRODUCTION

Magnetic field induced scattering resonances give cold atom experiments the ability to tune the strength of inter-atomic interactions [1–3]. For example, for fields in the range $824 – 844$G, the scattering length in a gas of fermionic $^6$Li changes by several orders of magnitude. Studies of superfluidity at these fields have revolutionized our understanding of the connections between BCS-pairing of fermions and Bose-Einstein condensation of composite bosons. Recent attention has turned to “narrow resonances, where the characteristic field over which the scattering length changes are $\sim 0.1$G [4–8]. As will be described below, the scattering properties near narrow resonances are more complicated, featuring energy dependences which are not captured by the scattering length. Here we study how this energy dependence is manifest in radio frequency (RF) spectra.

When the de Broglie wavelength of an atomic gas is large compared to the range of interactions, one is in the cold-collision limit, and all scattering properties are encoded in the s-wave scattering amplitude $\text{Re}(f_0(k)) = -a_s - r_{\text{eff}}u_s^2 k^2 + O(k^3)$. The scattering cross section between particles with relative momentum $k$ is proportional to $|f_0(k)|^2$. Low energy scattering is typically parameterized by the s-wave scattering length $a_s = -f_0(k = 0)$. The scattering length is a function of magnetic field, diverging at the Feshbach resonance field $B_0$, with the functional form

$$a_s(B) = a_{\text{bg}}(1 - \frac{\Delta B}{B - B_0}).$$

Here, $\Delta B$ is the width of the resonance, and $a_{\text{bg}}$ is the background scattering length, describing the scattering far from resonance. These resonances are generally associated with a crossing between a “closed channel” molecular state and the open-channel continuum. The characteristic scale over which $f_0$ changes is given by $r_{\text{eff}} = h^2/\mu a_{\text{bg}} \Delta B$ [9], where $\mu$ $\approx \mu_\mu$ is the difference in the magnetic moments of the two channels. If $r_{\text{eff}} k \gg 1$ for a typical collision, then the scattering length is insufficient to describe the physics.

Recently, Ho has pointed out that for a narrow resonance, because of the energy dependent of the phase shift, the interaction energy is highly asymmetric and strong interactions persist even for $B - B_0 \gg \Delta B$ on the BCS side [7]. O’Hara’s experimental group has studied a narrow resonance in $^6$Li, finding that the interaction energy and three-body recombination rate are both strongly energy dependent [8]. This energy dependence can lead to novel many-body physics, such as breached-pair superfluidity [9].

In this paper, we will calculate the RF spectrum of $^6$Li atoms near the narrow resonance around 543G. As in the experiment of Hazlett, et al., we consider the system initially in the lowest and third lowest hyperfine state (defined as 1 and 3). The Feshbach resonance does not couple these atoms, and the system is readily modeled as non-interacting. RF waves will induce a transition between 3 and the second lowest hyperfine state (defined as 2). The shape of the absorption line will be modified by the interactions between atoms in state 1 and 2. Consequently the absorption spectrum will have strong dependence on the magnetic field. One hopes to use details of the RF lineshape to learn about the underlying many-body physics [10–20].

To calculate the lineshape we sum an infinite set of diagrams, restricting ourselves to intermediate states without particle-hole excitations. Similarly we do not include the inelastic decay of the excited Feshbach molecules. These latter processes should slightly broaden the spectrum. This approach yields relatively simple results, and obeys all of the appropriate sum rules. Including more complicated intermediate states will quantitatively change the detailed lineshape, leaving gross features (such as its first few moments) unchanged.

Through out this paper we restrict ourselves to a uniform gas whose density corresponds to the average density of the experimental harmonically trapped system. A more sophisticated treatment would include inhomogeneous broadening from the trap [21].

Our paper is organized as follows: We first introduce the two channel resonance model which describes the system. Then we give a simple sum rule argument to extract
generic features of the RF lineshape as one changes the
resonance width. Next we calculate the RF spectrum
from a $T = 0$ variational ansatz. Next we generalize
our calculation to finite temperature using Matsubara
Green’s function techniques. Finally, we compare our
results with experiments.

II. MODEL

To describe the 3-component fermions near a narrow
Feshbach resonance, we use the following two channel
resonance model

$$
H = \sum_{k,\sigma} \epsilon_{k,\sigma} a_{k,\sigma}^\dagger a_{k,\sigma} + \sum_k (\epsilon_{k,\sigma} + \delta) b_k^\dagger b_k + \frac{\lambda}{\sqrt{\Omega}} \sum_{p,q} \left( b_{p+q} a_{p,1} a_{q,2} + \text{h.c.} \right),
$$

where the first term in the Hamiltonian corresponds to
the energy of isolated atoms: $a_{k,\sigma}$ annihilates an atom
with momentum $k$ and spin $\sigma = 1,2,3$, whose energy is
$\epsilon_{k,\sigma} = \hbar^2 k^2/2m - \mu_\sigma$, where $\mu_\sigma$ is the chemical
potential. The second term corresponds to the energy
of isolated molecules, $\epsilon_{k,b} = \hbar^2 k^2/4m - \mu_1 - \mu_2$ and
$\delta = 2\mu_B (B - B_\infty)$ is the detuning between the open
and closed channel, where $2\mu_B$ is the magnetic moment
difference between open and closed channel in $^6$Li with $\mu_B$
the Bohr magneton. The last term in the Hamiltonian
$\lambda/\sqrt{\Omega} \sum_{p,q} \left( b_{p+q} a_{p,1} a_{q,2} + \text{h.c.} \right)$
parametrizes the coupling between open and closed channels via
a single coefficient $\lambda$. $\Omega$ is the volume of the system.
To second order in $\lambda$, the two-body T-matrix describing
scattering between states 1 and 2 is $T^{2B}(k) = m^2 \lambda^2 \delta(k - \epsilon_k - \epsilon_b)$, where $\epsilon_k = \hbar^2 k^2/2m$ is the energy of one particle before scattering. Thus the s-wave scattering length of the system is

$$
a_s = \frac{m\Omega}{4\pi\hbar^2} T^{2B}(0) = -\frac{m\lambda^2}{4\pi\hbar^2} \delta,
$$

which can be compared with the empirical magnetic field
dependence $a_s = a_{bg}(1 - \Delta B/B - B_\infty) \approx -a_{bg} \Delta B/B - B_\infty$. Hence $\lambda$ is related to the experimental observables via

$$
\lambda = \sqrt{\frac{8\pi\hbar^2 a_{bg} \Delta B \mu_B}{m}}.
$$

As introduced in section I, the spin states $\sigma = 1,2,3$
model the three lowest energy hyperfine states of $^6$Li near
the narrow Feshbach resonance at $B = 543$G. Inserting
known experimental parameters for $^6$Li with $a_{bg} \approx 62a_0$
and $a_0$ the Bohr radius, we find $\lambda \approx 2.9 \times 10^{-10}$J/m$^2$.
The effective range of the model is $r_{\text{eff}} = 4\pi\hbar^2/m^2\lambda^2 \approx
3.5 \times 10^4a_0$. For a uniform gas of $^6$Li gas with density
$n = 10^{13}$cm$^{-3}$, the Fermi wave vector $k_F = (6\pi^2n)^{1/3} \approx
8.4 \times 10^3$cm$^{-1}$. In this case, we have $r_{\text{eff}}k_F \approx 15.4$, corre-
sponding to a narrow resonance.

At time $t = 0$, we imagine the system is prepared with
an equal number of particles in states 1 and 3, $N_1 =
N_3 = N$, and no particles in state 2, $N_2 = 0$. Within
our model, interactions vanish for this initial state. To
investigate the narrow resonance between states 1 and 2,
we introduce a radio frequency probe which drives atoms
from state 3 into state 2. This probe can be modeled by a
perturbation

$$
V = \sum_k \left( a_{k,3}^\dagger a_{k,3} e^{-i\omega t} + a_{k,3}^\dagger a_{k,2} e^{i\omega t} \right),
$$

where $w' = w - (\mu_2 - \mu_3)$. The physical radio waves
have frequency $\nu = \nu_0 + w/\hbar$ where $\nu_0$ is the free-space
resonance frequency for the transition from state 3 to 2.
For simplicity, we use units where $\hbar = k_B = 1$ and denote
$\mu_1 = \mu_3 = \mu$, $\mu_2 = 0$. Thus in our model $w' = w + \mu$.

A. Sum rules

At zero temperature, the ground state of our system (in
the absence of the probe) is a Fermi sea of equal numbers
of 1 and 3 particles $|GS\rangle = |F\rangle = \Pi_{k < k_F} a_{k,1}^\dagger a_{k,3}^\dagger |0\rangle$. The probe in Eq. (5) generates transitions from state 3 to
state 2 at a rate

$$
I(w) = 2\pi \sum_f |\langle GS|V|f\rangle|^2 \delta (w + \mu - E_f + E_0)
\times \text{Im} \langle GS|V|\frac{1}{w-H}V^\dagger|GS\rangle,
$$

where $H = H_0 - \mu$. The energy of the ground state is
$E_0$, and the sum is over all final states $|f\rangle$ with energy $E_f$.
In this subsection, we calculate moments of $I(w)$. Our
results will be exact. We will then use these moments
to describe qualitative features of the spectrum.

First, the total spectral weight is simply given by the
number of atoms initially in state 3,

$$
S_0 = \int \frac{dw}{2\pi} I(w) = \sum_f |\langle GS|V|f\rangle|^2 = \langle GS|VV^\dagger|GS\rangle = N.
$$

Second, the first moment vanishes

$$
S_1 = \int \frac{dw}{2\pi} wI(w) = \langle GS|V\hat{H}V^\dagger|GS\rangle = 0,
$$

implying that the spectrum should extend over both the
negative and positive RF frequencies with a centroid at
$w = 0$. Third, the second moment is

$$
S_2 = \int \frac{dw}{2\pi} w^2 I(w) = \langle GS|V\hat{H}\hat{V}\dagger|GS\rangle = \langle GS|[V,\hat{H}]^2|GS\rangle
= \frac{\lambda^2}{2\Omega} N^2.
$$
Finally, detuning dependence is encoded in the third order sum rule

$$S_3 = \int \frac{dw}{2\pi} \omega^3 I(w) = \frac{\lambda^2}{\Omega} \langle GS | V \Lambda H_0 \Lambda V | GS \rangle = \frac{\lambda^2}{\Omega} N^2 \delta_{\text{shift}},$$

(10)

where $H_0 = H - \lambda \Lambda / \sqrt{\Omega}$. $\delta_{\text{shift}} = \delta - 3E_F/5$ with $E_F = k_F^2/2m$ the Fermi energy and $k_F = (6\pi^2 N/\Omega)^{1/3}$.

To get a qualitative picture of the spectrum, we imagine a bimodal distribution made up from two $\delta$-function peaks,

$$I(w) \approx A_+ \delta(w - w_+) + A_- \delta(w - w_-).$$

(11)

From the sum rules Eq. (7)-(10) we find

$$A_\pm = \pi N \left( 1 \mp \frac{\delta_{\text{shift}}}{\sqrt{\delta_{\text{shift}}^2 + 4N\lambda^2/\Omega}} \right),$$

(12)

$$w_\pm = \frac{1}{2} \left( \delta_{\text{shift}} \pm \sqrt{\delta_{\text{shift}}^2 + 4N\lambda^2/\Omega} \right).$$

(13)

Fig. 1. Illustration of the evolution of the RF peaks from wide to narrow resonance. Vertical axis shows the frequency of one of the two peaks in the RF spectrum, as estimated by our sum rule argument. Different lines are labeled by their value of $z = k_F a'$. A wide resonance corresponds to $z \to 0$.

At $\delta_{\text{shift}} = 0$, the two peaks have equal weight, and it is natural to define an effective scattering length $a'_s = -m\lambda^2/4\pi\delta_{\text{shift}}$. In the limit where the Fermi energy is small compared to the detuning this corresponds to the standard definition in Eq. (3). As will be more precisely described below, for a wide resonance, one almost always has $E_F \ll \delta$, so $a'_s \approx a_s$.

To illustrate the structure of Eq. (13), we rewrite it in terms of the dimensionless variables $x = 1/k_F a'_s$ and $z = k_F r_{\text{eff}}$,

$$\frac{w_\pm}{E_F} = g_\pm (x, z),$$

(14)

where $g_\pm (x, z) = -(x \pm \sqrt{x^2 + 8z/3\pi})/z$. The variable $x$ is a measure of the interaction strength, while $z$ is a measure of the resonance width. Fig. 1 shows $w$ as a function of $x$ for several values of $z$. We only include the positive frequency $w_+$ in Fig. 1. The equivalent picture for $w_-$ is generated by noting that $g_- (x, z) = -g_+ (-x, z)$. As $x \to 0$, the peak moves to $w_+ = \sqrt{8/(3\pi)}E_F$. In the wide resonance limit, $z \to 0$, the frequency shift diverges at $x = 0$. Additionally, for $x \gg \sqrt{z}$, the coefficients simplify $A_+ \to 1$ and $A_- \to 0$, and there is effectively only a single peak, with $w_+ \approx 4\pi n a_s/m$. On the other hand, in the narrow resonance limit, $z \gg 1$, the peaks have nearly equal weight and disperse slowly as a function of the scattering length. The curves in Fig. 1 become flatter as the resonance width decreases.

In summary, the sum rules suggest the following:

(1) In the limit of a wide resonance, the spectrum is dominated by a single peak at frequency $w \approx 4\pi n a_s/m$, and $w \to \infty$ as $a_s \to \infty$.

(2) For a finite width resonance this divergence is cut off.

(3) Generically, the spectrum will be bimodal near resonance.

(4) The location of the resonance, defined by when equal spectral weight lies in each peak, is shifted from its free-space value.

Beyond these generalities, the sum rule arguments do not tell us about the detailed lineshapes. In the following subsections we present more sophisticated arguments to access these details. We will find that near resonance the peaks become quite broad, with spectral width growing as the resonance increases.

B. Zero temperature

In the following subsections, we give a quantitative description of the RF spectrum. First we consider zero temperature, approximating the sum in Eq. (6) by projecting $\hat{H}$ into a restricted subspace. In subsection II. C, we show that this projection is equivalent to summing a certain set of Feynman diagrams.

We consider intermediate states of the form

$$|n\rangle = \begin{cases} |q\rangle = a_{q,2}^\dagger a_{q,3}|F\rangle & (q < k_F) \\ |p, q\rangle = b_{p+q,2}^\dagger a_{p,1} a_{q,3}|F\rangle & (p, q < k_F) \end{cases},$$

(15)

where $|F\rangle$ is the filled Fermi sea of atoms in states 1 and 3. The state $|q\rangle$ represents the situation where the atom in spin-state 3 with momentum $q$, has been transferred into spin-state 2. This atom can bind with an atom in spin-state 1 with momentum $p$, forming a molecule with momentum $p+q$, described by state $|p, q\rangle$. We neglect possible intermediate states where this molecule then breaks up into a pair of atoms with momentum $p'$ and $q'$. These latter states look similar to $|q\rangle$, but have extra particle-hole excitations. In the limit $\lambda \to 0$, such processes are suppressed relative to the terms we keep.
Our approximation satisfies the sum rules in subsection II. A. In this truncated space, the coupling interaction
\[ \Lambda = \sum_{p,q} \left( b_{p+q}^\dagger a_{p,1} a_{q,2} + h.c. \right) \]
relates these two states. The relationship is shown diagrammatically in Fig. 2. The Fermi distribution function is given by
\[ f(x) = \frac{1}{e^{x/\beta} + 1} \]
and
\[ \frac{1}{\beta} = \frac{1}{w_n} \]
where \( w_n = 2\pi n / \beta \) and \( w_m = (2\pi + 1)m / \beta \) with \( m, n \in \mathbb{N} \). The relationship is shown diagrammatically in Fig. 2. Since the Hamiltonian contains no interactions involving particles in state 3, \( G_3 = \frac{1}{2} (i w_3 - \epsilon_{k,3}) \) is a bare propagator. Using the standard techniques of many-body perturbation theory, \( G_2 \) can be expressed as an infinite sum of diagrams. The natural extension of the approximation in subsection II. B. involves truncating this sum to only include terms without particle-hole pairs. The resulting series is expressed as a Dyson sum in Fig. 3. It corresponds to writing the propagator as a geometric series
\[ G_2(k,iw_2) = \frac{1}{G_2^0(k,iw_2) - \lambda^2 \Sigma(k,iw_2)} \]
where
\[ G_{\sigma=1,2,3}(k,iw) = \frac{1}{(iw_\sigma - \epsilon_{k,\sigma})} \]
is the bare atomic propagator. \( \Sigma(k,iw_2) \) is the self energy of particle 2, which we take to be
\[ \Sigma(k,iw_2) = \frac{1}{\beta} \sum_{p,w_1} G_b^0(p + k, i(w_1 + w_2)) G_f^0(p, iw_1) \]
and
\[ \int d^3p \frac{f(\epsilon_{p,1})}{i w_2 + \epsilon_{p,1} - \epsilon_{k+p,2} - \delta} \]
where \( G_b^0(k,iw_n) = 1/(iw_n - \epsilon_{k,b}) \) is the bare molecular propagator. The Fermi distribution function is \( f(x) = 1/(e^{x/\beta} + 1) \). Thus we have
\[ R(k,iw_n) = \frac{1}{\beta} \frac{f(\epsilon_{k,3})}{i w_n + \epsilon_{k,3} - \epsilon_{k,2} - \Gamma(k,iw_n)} \]
with $\Gamma(k, iw_n) = \lambda^2 \Sigma(k, iw_n + \epsilon_{k,3})$. The RF spectrum is recovered as

$$I(w) \propto \text{Im} \frac{1}{V} \sum_k R(k, iw_n \rightarrow w + \mu + i0)$$

$$= \text{Im} \frac{1}{\beta} \int d^3k \frac{f(\epsilon_{k,3})}{w + \mu + \epsilon_{k,3} - \epsilon_{k,2} - \Gamma(k, w + \mu)},$$

where in the last expression we implicitly assumed that $w$ has a small positive imaginary part.

At zero temperature, Eq. (28) reduces to our previous approximation in Eq. (19). In terms of dimensionless variables we write

$$I(\tilde{w}) \propto \text{Im} \int_0^\infty d\tilde{k} \frac{\tilde{k}^2 \tilde{f}(\tilde{k}^2 - \tilde{\mu})}{\tilde{w} - \tilde{\lambda}^2 \tilde{\Sigma}}$$

with

$$\tilde{\Sigma} = \int_0^\infty d\tilde{p} \tilde{f}(\tilde{p}^2 - \tilde{\mu}) \tilde{p} \log \left( \frac{\tilde{w} - \tilde{\delta} + (\tilde{k} + \tilde{\mu})^2/2}{\tilde{w} - \tilde{\delta} + (\tilde{k} - \tilde{\mu})^2/2} \right),$$

where $\tilde{w} = w/\epsilon_F$, $\tilde{k} = k/k_F$, $\tilde{\mu} = \mu/\epsilon_F$, $\tilde{\delta} = \delta/\epsilon_F$, and $\tilde{f}(x) = 1/(e^{\beta x} + 1)$, where $\epsilon_F$ is an arbitrary scale (typically taken to be the chemical potential), and $k_F^2/2m = \epsilon_F$.

### D. Results

In Fig. 4 we show the evolution of the RF spectrum as a function of detuning and frequency for temperatures $T = 0$, $\mu$, and $2\mu$. The chemical potential is fixed as $\tilde{\mu} = 1$ and the coupling strength $\tilde{\lambda} = 1$. Many of the features in Fig. 4 were anticipated by our sum rule calculation in section II. A. There are two peaks, which disperse in opposite directions, with spectral weight continuously shifting from one to another. A new feature, particularly apparent at higher temperatures, is the peaks become quite broad near resonance. There is also a marked asymmetry, where the negative energy peak is broader. One can attribute this broadening to the energy dependence of the scattering.

### III. COMPARISON WITH EXPERIMENT

Here we compare the spectra in Fig. 4 with the experiments in Ref [8]. Rather than modeling the harmonic trap, we will treat the gas as homogeneous, using the mean density quoted in Ref [8], $n = 10^{13} \text{cm}^{-3}$. Since the initial state is effectively non-interacting, we can extract the chemical potential at a given temperature by solving $n = \int d^3k f(\epsilon_k - \mu)$. Our results are shown in Fig. 5 (cf. Fig. 4 from Ref [8]). We see that on the scale of fields used in the experiment, the bimodal structure is not apparent, and it is reasonable to model the spectra by a single peak. At higher temperatures near resonance the peak is quite broad. The qualitative position of the peak tracks well with the observations in Ref [8], but deviates quantitatively. The discrepancies are likely attributable to inhomogeneities and uncertainty in the density.

### IV. SUMMARY

In summary, we have studied the RF spectrum of fermions near a narrow resonance. We presented a sum rule calculation, which shows how the spectrum evolves from a wide to narrow resonance. Wide resonances possess a divergence which is cut off by the effective range. We found bimodal behavior near resonance. This bimodality becomes less apparent at high temperature and can be masked by inhomogeneous broadening. At temperatures of order the Fermi temperature, both peaks broaden near resonance. The positive energy peak, however, is distinctly sharper. This asymmetry can be attributed to the energy dependence of the scattering.
FIG. 5. RF spectrum as function of magnetic field $B$ and RF frequency $\omega$ for different temperature $T = 0.86\mu K$, $3.8\mu K$, $10\mu K$, and $24\mu K$. Brighter color corresponding to higher intensity. Spectra and fields are given in physical units.

ACKNOWLEDGMENTS

We thank K. O’Hara for discussions of experimental details. This research is supported by the National Science Foundation (PHY-1068165), the National Key Basic Research Program of China (Grant No. 2013CB922000) and the National Natural Science Foundation of China (Grant No. 11074021). J. X. is also supported by China Scholarship Council.

[1] S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper-Kurn, and W. Ketterle, Nature 392, 151 (1998).
[2] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008).
[3] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. 82, 1225 (2010).
[4] K. E. Strecker, G. B. Partridge, and R. G. Hulet, Phys. Rev. Lett. 91, 080406 (2003).
[5] A. Schwenk, and C. J. Pethick, Phys. Rev. Lett. 95, 160401 (2005).
[6] D. S. Petrov, Phys. Rev. Lett. 93, 143201 (2004).
[7] T.-L. Ho, X. Cui, and W. Li, Phys. Rev. Lett. 108, 250401 (2012).
[8] E. L. Hazlett, Y. Zhang, R. W. Stites, and K. M. O’Hara, Phys. Rev. Lett. 108, 045304 (2012).
[9] M. M. Forbes, E. Gubankova, W. V. Liu, and F. Wilczek, Phys. Rev. Lett. 94, 017001 (2005).
[10] K. R. A. Hazzard and E. J. Mueller, Phys. Rev. A 81, 033404 (2010).
[11] S. Basu and E. J. Mueller, Phys. Rev. Lett. 101, 060405 (2008).
[12] J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010).
[13] Y. He, C. C. Chien, Q. Chen, and K. Levin, Phys. Rev. Lett. 102, 020402 (2009).
[14] P. Pieri, A. Perali, and G. C. Strinati, Nature Physics 5, 736 (2009).
[15] R. Haussmann, M. Punk, and W. Zwerger, Phys. Rev. A 80, 063612 (2009).
[16] M. Greiner, C. A. Regal, and D. S. Jin, Phys. Rev. Lett. 94, 070403 (2005).
[17] C. Chin, M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, J. H. Denschlag, and R. Grimm, Science 305, 1128 (2004).
[18] C. H. Schunck, Y. Shin, A. Schirotzek, M. W. Zwierlein, and W. Ketterle, Science 316, 867 (2007).
[19] L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Yefsah, W. S. Bakr, and M. W. Zwierlein, Phys. Rev. Lett. 109, 095302 (2012).
[20] P. Pieri, A. Perali, G. C. Strinati, S. Riedl, M. J. Wright, A. Altmeyer, C. Kohstall, E. R. Sanchez Guajardo, J. Hecker Denschlag, and R. Grimm, Phys. Rev. A 84, 011608(R) (2011).
[21] E. J. Mueller, Phys. Rev. A 78 045601 (2008).
[22] E. Timmermans, P. Tommasini, M. Hussein, A. Kerman, Phys. Rep. 315, 199 (1999).