CP Violation in Supersymmetric Theories: $\tilde{t}_2 \rightarrow \tilde{t}_1 \tau^- \tau^+$

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Abstract

Supersymmetric (SUSY) theories include many new parameters, some of which are CP-violating. Assuming that SUSY is found at a future high-energy collider, we examine the decay $\tilde{t}_2 \rightarrow \tilde{t}_1 \tau^- \tau^+$ with an eye to obtaining information about the new CP phases. We show that there are two CP-violating asymmetries which can be large in some regions of the SUSY parameter space. These involve measuring one or both of the $\tau$ spins. These asymmetries are particularly sensitive to $\phi_{A_t}$, the phase of the trilinear coupling $A_t$.  

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Supersymmetry (SUSY) is usually invoked to solve the hierarchy problem of the Standard Model (SM). Because such models are so theoretically compelling, many believe that SUSY must be found in nature, and much work has gone into elucidating various aspects of such theories.

In SUSY theories each ordinary fermion and gauge boson has a superpartner, respectively of spin 0 and spin $\frac{1}{2}$. These models contain a large number of unknown parameters. Many of these parameters can be complex, so that, in general, SUSY theories predict large CP-violating effects due to the presence of SUSY particles. The purpose of this paper is to begin a systematic high-energy exploration of CP violation in SUSY couplings. That is, throughout this paper we assume that the CP-conserving parameters are known, but that those which violate CP remain to be measured. Of course, most measurements will not probe single CP-violating SUSY phases. However, if one makes many measurements, then the combined effect will be to measure or constrain all SUSY phases.

The low-energy CP-violating properties of the couplings of superparticles have been explored. For example, constraints due to electric dipole moments (EDMs) have been studied in detail [1–7]. Contributions to meson mixing have also been investigated [8], and the effect of SUSY theories on CP violation in the $B$-meson system has been examined [9].

The EDM experiments are particularly constraining and have led to the so-called SUSY CP problem – if sfermion masses are of order the weak scale and complex SUSY parameters have phases of order unity, calculations of EDMs generically yield values in excess of the experimental limits. Furthermore, the experimental limits are expected to be improved substantially over the next few years, which will lead to further constraints on the SUSY parameter space [7]. Theoretical calculations of EDMs are highly model-dependent, and there are several scenarios in which the problem is ameliorated [10]. For example, one simple way to avoid the SUSY CP problem is to assume that sfermion masses are of order the weak scale, but that all relevant CP-odd phases are highly suppressed. Another is to adopt the opposite stance, taking the phases to be of order unity, but sfermion masses to be very large (of order several TeV). Scenarios have also been studied in which only the first- and second-generation sfermions are very heavy, but the third-generation sfermions are near the electroweak scale. In other cases, authors have made assumptions concerning universality or alignment of various soft-SUSY-breaking terms. Finally, in principle there could be substantial cancellations between different diagrams, leading to a reduction in the EDMs.

Assuming that SUSY particles are found, it will be necessary to measure the CP-violating properties of their couplings in order to get a complete picture of the physical SUSY model. Several studies have already been performed, showing how high-energy observables could be used to measure or constrain various SUSY
phases \([11, 12]\). In the present work we consider CP-odd asymmetries related to a particular decay mode of the heavier top squark in SUSY.

The observables that we consider are sensitive primarily to parameters associated with the third-generation squarks. EDMs can in principle constrain quantities associated with the third generation. (For example, the two-loop Bar-Zee-type diagrams considered in Ref. \([3]\) can directly constrain the CP-violating parameters related to the third generation squarks.) We will assume that the SUSY parameters that we use, as well as those not directly involved in our calculation (i.e., those corresponding to the first two generations), are such that the various EDM constraints are not violated. Our point of view is that our observables may be used to provide an independent measurement of relevant CP-odd SUSY parameters. Such measurements would be complementary to the constraints coming from low-energy observables such as EDMs or meson mixing. If indeed the EDM constraints end up predicting particular values for the parameters we consider, then a high-energy measurement should verify such predictions.

In SUSY theories, there are two Higgs doublets. As a result, there are three neutral Higgs bosons – two scalars and one pseudoscalar. If CP is violated, when one evolves from the gauge basis to the mass basis, these neutral particles can mix, so that the couplings of the physical Higgs bosons are generally mixtures of scalar and pseudoscalar. Recently, it was shown that this mixing could be probed in the process \(H^0 \to t\bar{t}\) \([12]\). The measurement of such a mixing would be one signal of CP violation in SUSY theories. The authors of Ref. \([12]\) note that this signal could also be seen in \(H^0 \to \tau\bar{\tau}\) if the \(H^0\) is too light to decay to \(t\bar{t}\). As such, this method, and ours described below, are applicable to high-energy colliders such as the Large Hadron Collider or Next Linear Collider.

SUSY theories also contain two scalar superpartners of the top quark, one for each \(t\)-quark helicity, called \(\tilde{t}_L\) and \(\tilde{t}_R\). These two "stops" can mix, resulting in two mass eigenstates \(\tilde{t}_1\) and \(\tilde{t}_2\) whose masses can be very different. Here we adopt the standard notation \(m_{\tilde{t}_2} > m_{\tilde{t}_1}\). In this paper we examine the decay process \(\tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+\). This comes about principally through the exchange of an intermediate \(Z\) boson or any of the Higgs bosons. The measurement of CP violation in this process will therefore probe the CP phases in the stop couplings, as well as scalar-pseudoscalar mixing in the Higgs sector.

There are a variety of CP-violating asymmetries in this process, some of which depend on strong (CP-conserving) phases. However, since the \(\tau\)'s are leptons, they cannot emit gluons, and the process \(t_2 \to \tilde{t}_1 \tau^- \tau^+\) does not have QCD-induced strong phases (any gluons exchanged between the two stops only serve to renormalize the stop couplings). In order to generate strong phases, we therefore include the widths of the mediating particles.

In order to calculate the rate and the various CP-violating asymmetries for \(\tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+\), various SUSY parameters must be specified: the masses and couplings of \(m_{\tilde{t}_1}\) and \(m_{\tilde{t}_2}\), and the masses, mixings and widths of the Higgs bosons. However, these
parameters are not all independent – they can be computed from the underlying SUSY parameters. To be specific, there are 7 fundamental SUSY parameters which strongly affect this decay process (see Sec. IV for an explicit list).

However, since the aim here is to measure CP violation, we are mainly interested in the underlying CP-violating SUSY parameters. We note that all CP-violating asymmetries in \( \tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+ \) depend only on the stop couplings, the mixings of the Higgses, and their widths. These quantities in turn depend on several CP-violating (i.e., complex) parameters in the underlying theory – the various trilinear couplings, the gaugino masses and \( \mu \). Thus, by specifying the phases of the various complex parameters, one can predict all CP-violating effects in \( \tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+ \) (recall that we have assumed that all CP-conserving quantities are known). We shall assume throughout that \( \mu \) is real and positive. This may be arranged by rephasing the various complex parameters \([5]\). With this phase convention, the asymmetries in \( \tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+ \) have a strong dependence on \( \phi_A_t \) (the phase of the trilinear coupling \( A_t \)) and only a much weaker dependence on the other phases. (See Sec. IV for a more detailed discussion of this point.) Thus, the measurement of CP-violating effects in \( \tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+ \) will allow us to measure/constrain \( \phi_A_t \).

A computer program – “\( \text{CPsuperH} \)” – has been written by others to generate the physical SUSY parameters (including the masses, mixings and widths of the Higgs bosons, the masses of the stops and the couplings of the Higgs bosons to the (s)fermions) from the underlying SUSY parameters \([13]\). Thus, with this program in hand, it is necessary only to specify the values of the fundamental SUSY parameters. We use \( \text{CPsuperH} \) extensively in our numerical work below and have adopted their notation for the coupling constants.

In Sec. II, we compute the amplitude for \( \tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+ \), as well as its CP-conjugate. The various CP-violating signals are examined in Sec. III. We show that there are two CP asymmetries which can be large in some regions of SUSY parameter space. These involve the measurement of one or both of the \( \tau \) spins. In Sec. IV, we present the predictions for these asymmetries. The measurement of these asymmetries will provide information about the SUSY CP phases. We conclude in Sec. V. The Appendix contains additional information about the various CP asymmetries.

II. AMPLITUDE FOR \( \tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+ \)

We now present the details of the \( \tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+ \) calculation. There are box-diagram contributions to this process, but they are expected to be small. Thus, this decay is mediated principally by the \( Z \) boson and by any of the three neutral Higgs bosons. That is, we have \( \tilde{t}_2 \to \tilde{t}_1 (Z, H_i) \) followed by \( (Z, H_i) \to \tau^- \tau^+ \). In the following we always take the \( \tau^- \tau^+ \) invariant mass to be above the \( Z \) pole, and so we assume that only neutral Higgs-exchange diagrams are important. While interferences between the Higgs diagrams and the off-shell \( Z \) could in principle give contributions to the CP asymmetries we consider, these contributions are expected
to be small, unless the lightest Higgs boson has a mass very close to that of the Z. To simplify our discussion, we ignore any contributions from the Z.$^1$

As mentioned in the introduction, the scalar and pseudoscalar Higgs particles in SUSY can mix if CP is violated, leading to three physical neutral Higgs bosons that do not have well-defined CP transformation properties. In general, the scalar-pseudoscalar mixing need not be small. The squarks also mix, and in general the up-type squark mass-squared matrix is $6 \times 6$. However, it is usually assumed that the up-type squarks are block-diagonalized by the same unitary transformations that diagonalize the up-type quarks. In this case, one obtains a simplified form for the mixing matrix: one only needs to diagonalize a set of $2 \times 2$ matrices. We adopt this simplifying assumption for the stops and consider only the mixing between $\tilde{t}_L$ and $\tilde{t}_R$. The CPsuperH program calculates the masses and mixings of both the stops and the Higgs bosons.

The couplings between the stops and Higgs bosons may be described by the following Lagrangian:

$$
\mathcal{L}_{H\tilde{t}} = v \sum_{i,j,k} g_{H_i \tilde{t}_j \tilde{t}_k} H_i \tilde{t}_j^* \tilde{t}_k, \tag{1}
$$

where $v$ is related to the vacuum expectation values of the two Higgs doublets, $v = \sqrt{v_1^2 + v_2^2}$, and where $i = 1, 2, 3$ and $j, k = 1, 2$. The coupling constants $g_{H_i \tilde{t}_j \tilde{t}_k}$ are defined as follows [13],

$$
v g_{H_i \tilde{t}_j \tilde{t}_k} = \left(\Gamma^{\alpha \tilde{t}_j \tilde{t}_k}\right)_{\beta \gamma} O_{\alpha \beta} \tilde{U}^{\tilde{t}_j \tilde{t}_k}_{\gamma}, \tag{2}
$$

where $O$ and $U^{\tilde{t}}$ are the Higgs and stop mixing matrices, respectively. The three $2 \times 2$ matrices $\Gamma^{\alpha \tilde{t}_j \tilde{t}_k}$ depend on the SUSY parameters $A_t$, $\mu$, $\cos \beta$ and $\sin \beta$ (where $\tan \beta \equiv v_2/v_1$). Expressions for the matrices $\Gamma^{\alpha \tilde{t}_j \tilde{t}_k}$ may be found in Appendix B of Ref. [13]. In the CP-invariant limit, the couplings involving scalar Higgs bosons are real and those involving the pseudoscalar Higgs are purely imaginary. If CP is broken, these couplings are in general complex.

We also require the couplings between the tau leptons and the various Higgs bosons. The relevant piece of the Lagrangian is

$$
\mathcal{L}_{H\tau} = -g_\tau \sum_i H_i \tau \left(\gamma^5 g_{H_i \tau}^S + i g_{H_i \tau}^P \gamma^5 \right) \tau, \tag{3}
$$

where $i = 1, 2, 3$. The constant $g_\tau$ and the scalar and pseudoscalar coupling constants, $g_{H_i \tau}^S$ and $g_{H_i \tau}^P$, respectively, are real. At tree-level the couplings are given

$^1$ In order that this approximation be reasonable, we will always consider cases in which the lightest Higgs is relatively well-separated in mass from the Z. If Z contributions end up being required, they could be incorporated in a straightforward manner.
by the expressions [13]

\[ g_{\tau}^{\text{tree}} = \frac{g m_{\tau}}{2m_W}, \quad g_{H,\tau}^{s,\text{tree}} = \frac{O_{H}}{\cos \beta}, \quad g_{H,\tau}^{P,\text{tree}} = -O_{H} \tan \beta. \]  

(4)

The program CPsuperH produces quantum-corrected versions of the above expressions. In the limit that CP is conserved, a given Higgs boson \( H_i \) has either a scalar or a pseudoscalar coupling, but not both.

The final ingredient in our calculation is the Higgs-boson propagator. In the decay \( \tilde{t}_2 \to \tilde{t}_1 \tau^-\tau^+ \), we allow one or more of the Higgs bosons to go on-shell. The reason for this is two-fold. First, this will dramatically increase the rate, giving a possibility that experimentalists will in fact be able to measure the asymmetries that we construct. Second, as noted above, the strong phases in this process are obtained mathematically by including the widths, i.e. absorptive pieces in the propagator. These absorptive parts have a much larger effect near resonance.

There is one complication here. It is true that CPsuperH computes all the physical properties of the Higgs bosons. In particular, the width of the \( H_i \) boson is found by calculating the 1-loop contributions of the light particles (taken here to be principally \( b \) and \( \tau \)), and applying Cutkowsky rules. However, in our calculation the “off-diagonal widths” come into play. That is, this 1-loop diagram can also be used to relate a \( H_i \) boson to a \( H_j \) boson, and the absorptive parts of such diagrams can be important for those CP asymmetries in our decay process which rely on strong phases. CPsuperH does not give these off-diagonal widths, but they have been computed in Ref. [14].

A good approximation for the Higgs propagator (divided by ‘i’) is

\[ D(M^2) = \left( \begin{array}{ccc}
M^2 - m_{H_i}^2 + i \text{Im}\tilde{\Pi}_{11} & i \text{Im}\tilde{\Pi}_{12} & i \text{Im}\tilde{\Pi}_{13} \\
i \text{Im}\tilde{\Pi}_{21} & M^2 - m_{H_2}^2 + i \text{Im}\tilde{\Pi}_{22} & i \text{Im}\tilde{\Pi}_{23} \\
i \text{Im}\tilde{\Pi}_{31} & i \text{Im}\tilde{\Pi}_{32} & M^2 - m_{H_3}^2 + i \text{Im}\tilde{\Pi}_{33}
\end{array} \right)^{-1}, \tag{5} \]

where \( M^2 = (p_1 + p_2)^2 \), with \( p_1 \) and \( p_2 \) being the four-momenta of the \( \tau^- \) and \( \tau^+ \), respectively. Explicit expressions for the absorptive parts of the Higgs boson self-energies, \( \text{Im}\tilde{\Pi}_{ij}(M^2) \), may be found in Ref. [14]. For our purposes, it is sufficient to consider only the contributions to \( \text{Im}\tilde{\Pi}_{ij}(M^2) \) arising from the \( b\bar{b} \) and \( \tau\tau^\pm \) loops, so

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2 CPsuperH does include off-diagonal contributions to the Higgs self-energy in its calculation of the Higgs boson masses [15].

3 We restrict our consideration to cases in which the neutral Higgs bosons can only decay into (non-t) quark and lepton pairs. In principle, \( M^2 \) could still be large enough that other terms should be included in the calculation of \( \text{Im}\tilde{\Pi}_{ij}(M^2) \). In practice, however, we are only interested in the effect of \( \text{Im}\tilde{\Pi}_{ij}(M^2) \) near resonances, i.e., when \( M^2 \sim m_{H_i}^2 \). In such cases it is sufficient to restrict our attention to the \( b \) and \( \tau \) contributions.
FIG. 1: Feynman diagram for the decay $\tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+$. Note that the Higgs propagator has off-diagonal terms, so transitions $H_i \to H_j$ are allowed. The shaded circle at the $H_j$-$\tau$-$\tau$ vertex denotes the fact that the effective coupling calculated by CPsuperH includes some loop corrections.

that

$$\text{Im}\hat{\Pi}_{ij}(M^2) \simeq \text{Im}\hat{\Pi}_{ij}^{bb}(M^2) + \text{Im}\hat{\Pi}_{ij}^{\tau\tau}(M^2),$$  \hspace{1cm} (6)$$

where

$$\text{Im}\hat{\Pi}_{ij}^{bb}(M^2) = \frac{3(M^2 g_b^2)}{8\pi} \left(1 + 5.67 \frac{\alpha_s(M^2)}{\pi}\right) \times \left[(1 - 4\kappa_b) g_{H_i H_b}^{S} g_{H_j H_b}^{S} + g_{H_i H_b}^{P} g_{H_j H_b}^{P}\right] \lambda^{1/2}(1, \kappa, \kappa_b),$$  \hspace{1cm} (7)$$

$$\text{Im}\hat{\Pi}_{ij}^{\tau\tau}(M^2) = \frac{(M^2 g_{\tau}^2)}{8\pi} \left[(1 - 4\kappa_{\tau}) g_{\tau \tau}^{S} g_{\tau \tau}^{S} + g_{\tau \tau}^{P} g_{\tau \tau}^{P}\right] \lambda^{1/2}(1, \kappa_{\tau}, \kappa_{\tau}),$$  \hspace{1cm} (8)$$

with $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2(xy + xz + yz)$ and $\kappa_{b,\tau} \equiv m_{b,\tau}^2/M^2$. The constants $g_b$ and $g_{H_i H_b}^{S,P}$ are associated with the $H_i H_b$ couplings and are defined in analogy with Eq. (3). (The tree-level expressions for the coupling constants are identical to those given in Eq. (4), but with the substitution $m_{\tau} \to m_{b,\tau}$.) Note that the diagonal elements are related to the widths of the Higgs bosons through the optical theorem, $\text{Im}\hat{\Pi}_{ii}(m_{H_i}^2) \simeq \Gamma(H_i) m_{H_i}$, giving rise to the usual Breit-Wigner form of the propagator if one takes $M^2 = m_{H_i}^2$ in the self-energies and ignores the off-diagonal terms. We include the off-diagonal terms in our calculation, since in some cases they have a non-negligible effect on the asymmetries that we consider.

Considering only the contributions due to the Higgs diagrams (see Fig. 1), the amplitude for the decay $\tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+$ is given by

$$A(s_1, s_2) = B u(p_1, s_1) v(p_2, s_2) + C \bar{u}(p_1, s_1) \gamma^5 v(p_2, s_2),$$  \hspace{1cm} (9)$$

where the indices ‘1’ and ‘2’ refer to $\tau^-$ and $\tau^+$, respectively. The complex coefficients $B$ and $C$ are given by

$$B = v g_{\tau} \sum_{i,j} g_{H_i H_j}^{S} D_{ij}(M^2) g_{\tau \tau}^{S},$$  \hspace{1cm} (10)$$
\[ C = ivg_r \sum_{i,j} g_{H,i}^\ast i_1 D_{ij}(M^2)g_{H,\tau_r}^P. \]  

(11)

Calculation of the absolute value squared of the amplitude, for specific spin states \( s_1 \) and \( s_2 \), yields the expression,

\[
|A(s_1, s_2)|^2 = \frac{1}{2} |B|^2 \left( (M^2 - 4m_\tau^2)(1 - s_1 \cdot s_2) + 2p_1 \cdot s_2 p_2 \cdot s_1 \right) \\
+ \frac{1}{2} |C|^2 \left[ M^2 (1 + s_1 \cdot s_2) - 2p_1 \cdot s_2 p_2 \cdot s_1 \right] \\
- m_\tau (B^* + B^C)(p_1 \cdot s_2 + p_2 \cdot s_1) \\
+ i (B^* - B^C) \epsilon_{0123} p_1^\beta s_1^\lambda p_2^\mu s_2^\mu,
\]

(12)

in which we have adopted the convention \( \epsilon_{0123} = +1 \).

In order to calculate CP-violating asymmetries we must determine the amplitude for the CP-conjugate process. This is

\[
\overline{A} = \overline{B}\bar{u}(p_1, s_1)v(p_2, s_2) + \overline{C}\bar{u}(p_1, s_1)\gamma^5 v(p_2, s_2),
\]

(13)

where

\[
\overline{B} = v g_r \sum_{i,j} g_{H,i}^\ast i_1 D_{ij}(M^2)g_{H,\tau_r}^S,
\]

(14)

\[
\overline{C} = -ivg_r \sum_{i,j} g_{H,i}^\ast i_1 D_{ij}(M^2)g_{H,\tau_r}^P
\]

(15)

(recall that \( g_r \) and \( g_{H,\tau_r}^{S,P} \) are real). As one might expect, the CP-conjugate amplitude is obtained from the original amplitude by complex-conjugating the weak phases and leaving the strong phases unchanged (the strong phases appear in the propagator matrix)\(^4\). The expression for \(|\overline{A}|^2\) is obtained from Eq. (12) by the replacement \((B, C) \rightarrow (\overline{B}, \overline{C})\). (Note that the explicit ‘i’ appearing in Eq. (12) does not change sign under CP conjugation.)

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\(^4\) Actually the situation is somewhat subtle. We have derived the expression for \(\overline{A}\) by determining the CP-conjugates of the initial and final states and then calculating the corresponding amplitude. Alternatively, one could reverse the signs on all weak phases \( \phi_i \) \( (\phi_{A_1}, \phi_{A_2}, \text{etc.}) \) and recalculate \( A \). Under the transformation \( \phi_i \rightarrow -\phi_i \), one finds that \( i\text{Im}\tilde{\Pi}_{13} \rightarrow -i\text{Im}\tilde{\Pi}_{13} \) and \( i\text{Im}\tilde{\Pi}_{23} \rightarrow -i\text{Im}\tilde{\Pi}_{23} \); i.e., some of the strong phases change sign. As a result, the 1-3 and 2-3 elements of the Higgs propagator also change sign. A careful consideration of sign changes that occur simultaneously in some of the coupling constants \( g_{H,i}^\ast i_1 \rightarrow -g_{H,i}^\ast i_1 \), \( g_{H,\tau_r}^S \rightarrow -g_{H,\tau_r}^S \), \( g_{H,\tau_r}^P \rightarrow -g_{H,\tau_r}^P \), and \( g_{H,\tau_r}^P \rightarrow -g_{H,\tau_r}^P \) shows that the expressions given for \( \overline{B} \) and \( \overline{C} \) are precisely correct.
III. CP ASYMMETRIES

We can form three CP-violating asymmetries from the expressions for $|A|^2$ and $|\bar{A}|^2$, using various combinations of $\tau^\pm$ polarizations to extract different terms. The first asymmetry is the usual rate asymmetry, for which it is assumed that spins are not measured. The second asymmetry is a “single-spin” asymmetry. It requires the measurement of a single $\tau$ spin in the decay and is sensitive to the real part of $BC^*$. The third asymmetry is a triple-product asymmetry and requires the measurement of both spins. It is sensitive to the imaginary part of $BC^*$.

In general, the construction of the CP asymmetries proceeds in two steps. In the first step, we sum over the $\tau^\pm$ spins (possibly in an asymmetric manner) for the process $\tilde{t}_2^- \rightarrow \tilde{t}_1^- \tau^- \tau^+$. We denote the resulting expression for the amplitude squared by “$|A|^2|_{(a)}$,” where ‘(a)’ refers to the specific prescription employed when summing over the spins. In the second step we construct $|\bar{A}|^2|_{(a)}$, the analogous quantity for the CP-conjugate of the process, and subtract it from $|A|^2|_{(a)}$. The resulting expression is odd under CP. We also integrate over the invariant mass $M$ of the $\tau^\pm$ pair, starting at some point above the $Z$ resonance but including one or more Higgs resonances. The CP asymmetries can in principle be increased by a judicious choice for the range of this integration. Assuming $M > M_{\text{min}}$, where $M_{\text{min}} \geq 2m_\tau$, we generically define the width associated with the $a^{th}$ prescription for summing over spins as

$$\Gamma_{M_{\text{min}}}^{(a)} \equiv \frac{1}{128\pi^3 m_{\tilde{t}_2}^3} \int_{M_{\text{min}}}^{m_{\tilde{t}_2}-m_{\tilde{t}_1}} \frac{dM}{M} \left(|A|^2|_{(a)}\right) \times \left[\lambda\left(m_{\tilde{t}_2}^2, m_{\tilde{t}_1}^2, M^2\right) \lambda\left(M^2, m_\tau^2, m_\tau^2\right)\right]^{1/2}.$$  (16)

The width for the CP-conjugate process is obtained by the replacement $(B, C) \rightarrow (\bar{B}, \bar{C})$ and is denoted $\Gamma_{M_{\text{min}}}^{(a)}$.

A. Rate Asymmetry

The first CP asymmetry is the usual rate asymmetry and is obtained by summing symmetrically over the spin states $s_1$ and $s_2$ to yield

$$|A|^2_{\text{rate}} \equiv \sum_{\text{spins}} |A(s_1, s_2)|^2 = 2 |B|^2 \left(M^2 - 4m_\tau^2\right) + 2 |C|^2 M^2.$$  (17)

\footnote{In practice, we always assume that $M_{\text{min}}$ is above the $Z$ resonance so that we can ignore $Z$ contributions.}
This expression may be inserted in Eq. (16) to obtain \( \Gamma_{M_{\text{min}}} \), and similarly for \( \Gamma_{M_{\text{min}}} \) (we drop the ‘rate’ superscripts in this case). The resulting CP asymmetry is then

\[
A_{\text{CP}}^{\text{rate}} \equiv \frac{\Gamma_{M_{\text{min}}} - \Gamma_{M_{\text{min}}}}{\Gamma_{M_{\text{min}}} + \Gamma_{M_{\text{min}}}}.
\]  

(18)

A non-zero value for this asymmetry requires both a weak-phase difference and a strong-phase difference between the diagrams contributing to the decay. Since we obtain our strong phases from absorptive parts of the Higgs boson propagator, simultaneously non-negligible values for the asymmetry and the rate itself require the interference of two or three nearby Higgs boson resonances.

We have found numerically that the rate asymmetry tends to be very small for \( \tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+ \), with values typically at the sub-one percent level. There are a few factors that conspire to make the rate asymmetries small; these are described in the Appendix. At this point we simply note that this particular observable will not be a very useful tool for the study of SUSY CP violation in this decay process. It turns out, however, that the measurement of the spin of one or both final-state leptons can lead to a dramatic increase in the sensitivity of \( \tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+ \) to CP phases. The two asymmetries described below use lepton spins to probe the cross-terms containing \( B \) and \( C \) in Eq. (12). These asymmetries can be quite large numerically.

**B. Single-spin Asymmetry**

We first construct a CP asymmetry that uses the polarization of only one of the leptons. This single-spin asymmetry extracts the piece in \( |A|^2 \) that is proportional to the real part of \( BC^* \). In constructing this asymmetry, it is useful to calculate \( |A(s_1, s_2)|^2 \) explicitly for the different spin combinations of the \( \tau^\pm \). The spin four-vectors \( s_1 \) and \( s_2 \) are determined in the usual manner by Lorentz-transforming unit three-vectors from the rest frames of the particle and antiparticle. Let the unit three-vector \( \hat{s} \) denote a specific spin state in the rest frame of the \( \tau^+ \) or \( \tau^- \). Lorentz-transforming \( \hat{s} \) to a reference frame in which the \( \tau^+ \) or \( \tau^- \) has four-momentum \( p^\mu = (E, \vec{p}) \) yields

\[
s^\mu = \left( \frac{\vec{p} \cdot \hat{s}}{m}, \hat{s} + \frac{(\vec{p} \cdot \hat{s}) \vec{p}}{m(E + m)} \right).
\]  

(19)

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6 We present our results as asymmetries depending on the spins of the \( \tau^- \) and \( \tau^+ \). In practice, experimentalists would use the decay products of the taus to probe their spins. In that case, one could construct CP asymmetries depending on the momenta of the decay products. See Ref. [16] for an explicit comparison of ‘theoretical’ triple-product asymmetries involving the \( D^* \) polarization states in \( B \to D^* \ell\overline{\nu} \) with more ‘experimental’ asymmetries that use only the momenta in the four-body final state of the decay \( B \to (D^* \to D\pi)\ell\overline{\nu} \).
For a given set of momenta in the process $\tilde{t}_2 \rightarrow \tilde{t}_1 \tau^- \tau^+$ there are four spin combinations that one could consider, corresponding to the four combinations of $\pm \hat{s}_1$ and $\pm \hat{s}_2$. In constructing the single-spin asymmetry it is convenient to work in the helicity basis, taking the unit three-vectors to be aligned or anti-aligned with the lepton three-momenta in the rest frame of the lepton pair. This choice also maximizes the single-spin asymmetry. (In the following subsection we will construct triple products using the spins $s_1$ and $s_2$. In that case it will be convenient to employ a basis in which the unit three-vectors are perpendicular to the lepton three-momenta.)

Since $|A|^2$ is Lorentz-invariant, we may choose any convenient frame of reference. We work in the rest frame of the $\tau^\pm$ pair, so that $p_{\tau^\pm} = (E, \pm \vec{p})$, with $\vec{p}$ denoting the three-momentum of the $\tau^-$ in that frame. Defining $\hat{s}_1 = \vec{p}/|\vec{p}|$ and Lorentz-transforming $\pm \hat{s}_1$ from the $\tau^-$ rest frame to the Higgs rest frame, we have $s_1^{\pm \mu} = \pm (|\vec{p}|/m_\tau, E\vec{p}/(m_\tau|\vec{p}|))$. Similarly, for $\tau^+$ spins that are in the $\pm \vec{p}/|\vec{p}|$ direction in the $\tau^+$ rest frame, we have $s_2^{\pm \mu} = \pm (-|\vec{p}|/m_\tau, E\vec{p}/(m_\tau|\vec{p}|))$ in the Higgs rest frame.

Explicit calculations for the four spin combinations yield

$$|A_{(\pm, \pm)}|^2 = 0,$$  

$$|A_{(\pm, \mp)}|^2 = |B|^2 (M^2 - 4m_\tau^2) + |C|^2 M^2 \mp (BC^* + B^*C) M \sqrt{M^2 - 4m_\tau^2}.$$  

Summing all four spin combinations reproduces the result in Eq. (17), as expected.

Taking the difference of the two non-zero spin combinations yields

$$|A|^2 \mid_{\text{spin}} \equiv |A_{(+, -)}|^2 - |A_{(-, +)}|^2 = -4 \, \text{Re} \,(BC^*) \, M \sqrt{M^2 - 4m_\tau^2}.$$  

The resulting CP asymmetry is given by

$$A_{\text{CP}}^{\text{spin}} \equiv \frac{\Gamma_{M_{\min}}^{\text{spin}} - \Gamma_{M_{\min}}^{\text{spin}}}{\Gamma_{M_{\min}} + \Gamma_{M_{\min}}} ,$$  

in which we have normalized the asymmetry using the total widths $\Gamma_{M_{\min}}^{\text{spin}}$ and $\Gamma_{M_{\min}}^{\text{spin}}$. Note that this asymmetry only requires the measurement of a single spin, since the $++$ and $--$ combinations are zero. As was the case for the rate asymmetry, this single-spin asymmetry requires the interference of two amplitudes with a non-zero relative strong phase.

In contrast to the rate asymmetry, the single-spin asymmetry can be fairly large numerically since it does not suffer from the same cancellations as those of the rate asymmetry.

\[ ^7 \text{In the remainder of this work we will refer to the rest frame of the } \tau^\pm \text{ pair as the rest frame of the decaying (virtual) Higgs.} \]
FIG. 2: Definitions of angular variables and unit vectors used in the calculation of the triple-product CP asymmetry. The unit vector \( \hat{z} \) specifies the direction of the virtual Higgs as seen in the \( \tilde{t}_2 \) rest frame. The vector \( \bar{p} = p \hat{n} \) specifies the \( \tau^- \) momentum in the Higgs rest frame.

C. Triple-product Asymmetry

For the triple-product CP asymmetry we must use a different basis for the spins \( s_{1,2} \). We work in the rest frame of the virtual Higgs (i.e., in the rest frame of the \( \tau^{\pm} \) pair), defining the \(+z\) direction in that frame to be the direction of the Higgs as seen in the \( \tilde{t}_2 \) rest frame. We let the unit vector \( \hat{z} \) denote this direction (see Fig. 2). We let the unit vector \( \hat{n} \) denote the direction of the \( \tau^- \) in the Higgs rest frame. To specify the spin four-vectors of the \( \tau^{\pm} \), we start by defining the corresponding unit three-vectors in the individual rest frames of the \( \tau^- \) and the \( \tau^+ \). These are then Lorentz-transformed to the Higgs rest frame. In the individual \( \tau^- \) and \( \tau^+ \) rest frames we define the ‘+1’ and ‘−1’ spin states as

\[
\hat{s}^+_1 = \pm \frac{\hat{z} - \hat{n} \cos \theta}{|\hat{z} - \hat{n} \cos \theta|} = \pm \frac{\hat{z} - \hat{n} \cos \theta}{\sin \theta},
\]

\[
\hat{s}^+_2 = \pm \frac{\hat{z} \times \hat{n}}{|\hat{z} \times \hat{n}|} = \pm \frac{\hat{z} \times \hat{n}}{\sin \theta},
\]

where \( \theta \) is defined in Fig. 2. Then the vectors \( \hat{s}^+_1, \hat{s}^+_2 \) and \( \hat{n} \) are mutually perpendicular. The ‘\( \epsilon \)’ term may then be extracted from Eq. (12) by the following prescription:

\[
|A|_{TP}^2 \equiv -\frac{1}{4\pi} \int \left[ |A(+, +)|^2 + |A(-, -)|^2 - |A(+, -)|^2 - |A(-, +)|^2 \right] d\Omega = 4 \text{ Im} (BC^*) M \sqrt{M^2 - 4m_\tau^2},
\]
where the angular integration is performed in the Higgs rest frame, and where the ± spins are defined as in Eqs. (24) and (25). (This basis is different than that employed in Eqs. (20) and (21).) Inserting Eq. (26) in Eq. (16), and similarly for the CP-conjugate process, we form the CP-odd triple product asymmetry as follows:

\[ A_{\text{CP}}^{\text{TP}} \equiv \frac{\Gamma_{M_{\text{min}}}^{\text{TP}} - \Gamma_{M_{\text{min}}}^{\text{TP}}}{\Gamma_{M_{\text{min}}}^{\text{TP}} + \Gamma_{M_{\text{min}}}^{\text{TP}}} \]  

(27)

Note that the prescription used to define \( |A|^2 \) is equivalent, in this spin basis, to the definition

\[ \Gamma_{M_{\text{min}}}^{\text{TP}} = \Gamma_{M_{\text{min}}} (\epsilon(p_1, s_1, p_2, s_2) > 0) - \Gamma_{M_{\text{min}}} (\epsilon(p_1, s_1, p_2, s_2) < 0) . \]  

(28)

The triple-product asymmetry does not require a relative strong phase between interfering amplitudes. In fact, the triple-product asymmetry does not even necessarily require the interference of two different Higgs resonances – one resonance suffices as long as there is scalar-pseudoscalar mixing among the Higgs bosons.

IV. RESULTS

As has been noted above and discussed in the Appendix, the rate asymmetry in \( t\bar{t}_2 \rightarrow t_1 \tau^- \tau^+ \) will in general be very small, so we do not consider it further here. The single-spin CP asymmetry can be large, but requires the interference of two or three nearby Higgs resonances. If the resonances are spread far apart, the product of any two interfering amplitudes will be suppressed, since one amplitude will be much smaller than the other. The strong-phase difference in this CP asymmetry is due to the width difference between the contributing Higgs bosons as well as the relative values of their masses (see Eq. (A.7)). The triple-product CP asymmetry does not require a strong-phase difference and can receive large contributions from single resonances (due to scalar-pseudoscalar mixing) and from the interference between two resonances. The Appendix contains further discussions of these two distinct types of contributions.

In SUSY it is somewhat natural to have nearby Higgs resonances, so that it is not unreasonable to hope for large single-spin and triple-product asymmetries. At tree level there is a well-known relation between the masses of the heavier scalar Higgs (\( H \)) and the pseudoscalar Higgs (\( A \)),

\[ m_H^2 - m_A^2 \approx m_Z^2 \sin^2 2\beta , \]  

(29)

a relation which is valid in the limit \( m_A^2 \gg m_Z^2 \) [17]. For large \( \tan \beta \), \( \sin^2 2\beta \approx 0 \) and thus \( m_H^2 \approx m_A^2 \). If CP is not conserved, loop corrections lead to the mixing of the scalar and pseudoscalar Higgs bosons [18]. This mixing can lead to the breaking
of the near degeneracy in masses, as has been explored in some detail in Ref. [19]. Nevertheless, for some combinations of parameters (in particular, it appears, if $|\mu|$ is not too large) there remains a near degeneracy of the two heavier Higgs bosons.

In the following we choose a few points in the SUSY parameter space and use these to illustrate some of the features of the single-spin and triple-product asymmetries. Our treatment is certainly not exhaustive, but is meant simply to point to the usefulness of using these asymmetries as a tool for exploring SUSY CP violation.

We choose values for the charged Higgs mass and $\tan \beta$ that are consistent with the recent bound from Belle, $\tan \beta/m_{H^\pm} < \sim 0.146$ GeV$^{-1}$ [20].

The asymmetries $A_{\text{spin}}^{\text{CP}}$ and $A_{\text{TP}}^{\text{CP}}$ defined in Eqs. (23) and (27) have been integrated over $M$, where $M = \sqrt{(p_1 + p_2)^2}$ is the invariant mass of the $\tau^\pm$ pair (see Eq. (16)). Since the $\tau$'s are produced by the s-channel decays of Higgs bosons (see Fig. 1), the differential widths (in $M$) for the decays $\tilde{t}^\pm_2 \rightarrow \tilde{t}^\pm_1 \tau^- \tau^+$ are expected to exhibit resonant peaks in the vicinities of the neutral Higgs masses; i.e., when $M \approx m_{H_i}$, with $i = 1, 2, 3$. The CP asymmetries that we have constructed have a sensitive dependence on the resonance structure of the decay. For example, as mentioned above, the single-spin asymmetry requires the interference of different resonances, and will be suppressed if the resonances are too far apart. The triple-product asymmetry can receive contributions from single resonances, but in some cases the contributions due to different resonances could be of opposite signs, leading to a suppression of the (integrated) quantity $A_{\text{TP}}^{\text{CP}}$. Experimentalists may wish to enforce various cuts on the integration over $M$ in order to optimize both the CP asymmetry and its statistical significance.

To display some of the above features more clearly, let us define several differential quantities. First, we define $d\Gamma_{\text{sum}}/dM$ to be the sum of the total (i.e., summed over spins) differential widths for the process and the anti-process. That is,

$$
\frac{d\Gamma_{\text{sum}}}{dM} = \frac{1}{128\pi^3 m_{\tilde{t}_2}^2} \frac{1}{M} \left( |\mathcal{A}|^2 \mid_{\text{rate}} + |\overline{\mathcal{A}}|^2 \mid_{\text{rate}} \right) \\
\times \left[ \lambda \left( m_{\tilde{t}_2}^2, m_{\tilde{t}_1}^2, M^2 \right) \lambda \left( M^2, m_{\tau}^2, m_{\tau}^2 \right) \right]^{1/2}.
$$

A plot of $d\Gamma_{\text{sum}}/dM$ as a function of $M$ will show resonance structure, with peaks appearing at the Higgs masses. Furthermore, integrating $d\Gamma_{\text{sum}}/dM$ over $M$ gives the sum of the widths for the process and the anti-process,

$$
\Gamma_{\text{sum}} \equiv \int_{M_{\text{min}}}^{m_{\tilde{t}_2} - m_{\tilde{t}_1}} \frac{d\Gamma_{\text{sum}}}{dM} dM = \Gamma_{m_{\text{min}}} + \Gamma_{M_{\text{min}}}.
$$

$\Gamma_{\text{sum}}$ is the quantity that appears in the denominator of the CP asymmetries defined in Eqs. (23) and (27). Recall as well that it has an implicit dependence on the lower limit, $M_{\text{min}}$, of the integration in the above expression. We also define two CP-
FIG. 3: Plot of $\frac{d\Gamma_{\text{sum}}}{dM}$ (solid line), $\frac{d\Gamma_{\text{TP}}}{dM}$ (long dashed line) and $\frac{d\Gamma_{\text{spin}}}{dM}$ (short dashed line) for a case in which the two heaviest Higgs resonances (near 126.1 GeV and 128.5 GeV) have a significant overlap.

asymmetric differential widths,

$$
\frac{d\Gamma^{(a)}_{\text{CP}}}{dM} = \frac{1}{128\pi^3 m_{t_2}^2} \frac{1}{M} \left( |A|^2 \big|_{(a)} - |\overline{A}|^2 \big|_{(a)} \right) \\
\times \left[ \lambda \left( m_{t_2}^2, m_{t_1}^2, M^2 \right) \lambda \left( M^2, m_{\tau}^2, m_{\tau}^2 \right) \right]^{1/2},
$$

(32)

where ‘a’ corresponds to ‘spin’ or ‘TP’. When integrated over $M$, these differential widths form the numerators in the expressions for the CP asymmetries in Eqs. (23) and (27), so that

$$
A^{(a)}_{\text{CP}} = \frac{1}{\Gamma_{\text{sum}}} \int_{m_{t_2}-m_{t_1}}^{m_{t_2}} \frac{d\Gamma^{(a)}_{\text{CP}}}{dM} dM.
$$

(33)

In the following we will show plots of $d\Gamma^{\text{spin}}_{\text{CP}}/dM$, $d\Gamma^{\text{TP}}_{\text{CP}}/dM$ and $d\Gamma_{\text{sum}}/dM$. The relation between these plots and the integrated CP asymmetries $A^{\text{spin}}_{\text{CP}}$ and $A^{\text{TP}}_{\text{CP}}$ follows from Eq. (33): The spin (TP) asymmetry is the ratio of the area under the curve $d\Gamma^{(\text{spin/TP})}_{\text{CP}}/dM$ to the area under the curve $d\Gamma_{\text{sum}}/dM$. 

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Figure 3 shows plots of the differential widths $d\Gamma_{\text{CP}}^{\text{spin}}/dM$, $d\Gamma_{\text{CP}}^{\text{TP}}/dM$ and $d\Gamma_{\text{sum}}/dM$ for a particular set of input parameters. To generate these plots we have used \texttt{CPsuperH} to determine the masses and couplings of the Higgs bosons as well as the masses of the stops. Some of the input parameters are as follows: $m_{H^\pm} = 150$ GeV, $\mu = +200$ GeV, $|A_t| = 600$ GeV, $\phi_{A_t} = 90^\circ$, $\tan \beta = 20$, $m_{\tilde{Q}_3} = 350$ GeV and $m_{\tilde{U}_3} = 400$ GeV, where we have followed the notation of \texttt{CPsuperH}. (The decay process strongly depends on all of these parameters. Other parameters have been chosen such that the neutral Higgs bosons cannot decay into SUSY particles.) The only non-zero phase is that of the trilinear coupling $A_t$. The masses of the stops are determined by \texttt{CPsuperH} to be $m_{\tilde{t}_1} \simeq 259$ GeV and $m_{\tilde{t}_2} \simeq 511$ GeV. Similarly, the neutral Higgs masses (widths) are $m_{H_1} \simeq 114.38$ (0.15) GeV, $m_{H_2} \simeq 126.12$ (1.09) GeV and $m_{H_3} \simeq 128.45$ (0.95) GeV.

As noted above, to obtain the integrated CP asymmetries, we determine the areas under the curves in Fig. 3 and calculate the appropriate ratios. A comparison of the area under the dashed curve ($d\Gamma_{\text{CP}}^{\text{TP}}/dM$) to that under the solid curve ($d\Gamma_{\text{sum}}/dM$) indicates that the CP asymmetry will be relatively large in this case. A similar analysis for the single-spin asymmetry leads one to the conclusion that that asymmetry will be somewhat smaller. Our numerical results confirm these qualitative observations. Integrating over all three Higgs resonances we have $A_{\text{CP}}^{\text{spin}} \simeq -14\%$ and $A_{\text{CP}}^{\text{TP}} \simeq -31\%$.

Consider also a few other technical details associated with this example:

1. One might wonder why the resonance evident in $d\Gamma_{\text{sum}}/dM$ near the heaviest Higgs at 128 GeV is larger than that near 126 GeV. It turns out that the difference in the strengths of these resonances is due primarily to differences in the Higgs-stop-stop couplings.

2. In this example the two heaviest Higgs resonances have a relatively significant overlap, allowing for some amount of interference between the resonances. The single-spin asymmetry requires such interference, while the triple-product asymmetry can also receive contributions from single resonances.

3. Including only the heavier two resonances in the integration over $M$ increases $A_{\text{CP}}^{\text{spin}}$ (in terms of its magnitude) to about $-19\%$, but decreases $A_{\text{CP}}^{\text{TP}}$ (again in terms of its magnitude) to about $-24\%$. The increase in the single-spin asymmetry is not difficult to understand: The numerator in Eq. (33) is nearly unchanged (because the $H_1$ resonance is so far separated from the other two and does interfere very much), but the denominator decreases from $\Gamma_{\text{sum}} \simeq 0.13$ GeV to $\Gamma_{\text{sum}} \simeq 0.098$ GeV.

4. Observation of a triple-product asymmetry in the vicinity of a single, widely-separated resonance (such as $H_1$ in this case) would be evidence of scalar-pseudoscalar mixing in the Higgs sector.
A second example is shown in Fig. 4. The input parameters for this example are the same as those of the previous example, except that $\mu$ and $A_t$ have been increased in magnitude ($\mu = +500$ GeV and $|A_t| = 700$ GeV in this case). As a result, $m_{H_2}$ and $m_{H_3}$ have moved farther away from each other and $m_{H_1}$ has increased. (The neutral Higgs masses (widths), as determined by \textit{CPsuperH}, are $m_{H_1} \simeq 116.09$ (0.16) GeV, $m_{H_2} \simeq 124.73$ (0.83) GeV and $m_{H_3} \simeq 130.11$ (0.74) GeV, in this case. The stop masses are $m_{\tilde{t}_1} \simeq 228$ GeV and $m_{\tilde{t}_2} \simeq 526$ GeV.) The CP asymmetries and integrated width in this example, when integrated over all three (only the heavier two) resonances, are $A^\text{spin}_{\text{CP}} \simeq -5\%$ ($-6\%$), $A^\text{TP}_{\text{CP}} \simeq -36\%$ ($-30\%$) and $\Gamma_{\text{sum}} \simeq 0.93$ (0.83) GeV. The single-spin asymmetry has been reduced somewhat, compared to the previous example, due in part to the fact that the resonances are more widely separated in this case. Also note that the triple-product asymmetry suffers from some amount of cancellation between the second and third resonances, since the area under the curve near these two resonances is of opposite sign. In such a scenario it might be experimentally preferable when measuring the triple-product asymmetry to restrict the integration over $M$ to smaller regions or to weight the region near the middle resonance with a relative negative sign (to avoid the cancellation).
Figure 5 shows the variation of the integrated CP asymmetries as functions of the phase of $A_t$, $\phi_{A_t}$, for the same set of input parameters used in Fig. 3 (except, of course, for $\phi_{A_t}$). Note that the masses of the Higgs bosons and stops, as well as the widths of the Higgs bosons, are themselves functions of $\phi_{A_t}$, so their values change over the course of the plot. The four curves correspond to $A_{CP}^{TP}$ and $A_{CP}^{spin}$, with two different choices for $M_{min}$, one of which includes all three resonances and one of which includes only those corresponding to $H_2$ and $H_3$. In this example, including the first resonance tends to decrease the magnitude of $A_{CP}^{spin}$, while it sometimes increases the magnitude of $A_{CP}^{TP}$ and sometimes decreases it. This is evident from the plot. Values as large as about 64% and 49% are observed for the triple-product and single-spin asymmetries, respectively. It is possible to obtain even larger triple-product asymmetries than those shown in Fig. 5. For example, using the same parameters as those used for Fig. 4, but taking $\phi_{A_t} = 135^\circ$, one obtains $A_{CP}^{TP} \simeq -92\%$ and $\Gamma_{sum} \simeq 0.73$ GeV when integrating over all three resonances.

We have performed a limited study of the effects of allowing other SUSY pa-
rameters to have non-zero phases. The phase in the trilinear coupling $A_b$ could in principle affect our asymmetries through the mixing of the Higgs bosons. In practice, we have found that this phase only seems to have a non-negligible effect when $|A_b|$ is quite large – of order 5-10 TeV in the cases studied – due to the difference in the top and bottom Yukawa couplings. The phases of the gaugino masses could also in principle give contributions due to their effects on Higgs-fermion-fermion effective vertices. Taking the magnitudes of $M_1$, $M_2$ and $M_3$ to be approximately 200, 400 and 1000 GeV, we have found that in the cases considered these quantities can make changes to the triple product asymmetry that are of order $\pm 0.1$. The effect on the single-spin asymmetry tends to be smaller in magnitude.

We have also considered scenarios involving different values for $\tan \beta$ or $A_t$. The triple-product and single-spin asymmetries are strongly dependent on the particular values chosen for these quantities. In the few examples we studied, decreasing $\tan \beta$ from 20 (the value used in Figs. 3-5) tended to decrease the asymmetries. We also increased $\tan \beta$ in one case (and increased $m_{H^\pm}$ to respect the experimental bound on $\tan \beta/m_{H^\pm}$) and found that the triple-product asymmetry could increase in that case. Varying the magnitude of $A_t$ (while holding its phase constant) can also affect the asymmetries. In the cases studied, the asymmetries tended to be somewhat constant as $|A_t|$ was varied between 400 GeV and 700 GeV. There were more significant changes in the range 700 GeV to 1 TeV.

We should also make a comment regarding the invariance of our results under rephasing of the complex SUSY parameters. As is well-known, it is possible to rotate away two phases associated with the various complex parameters in supersymmetry (see, for example, Ref. [5]). CPsuperH appears to use one of these degrees of freedom to force the SUSY parameter $m_{12}^2$ to be real [18, 21]. The remaining complex quantities within CPsuperH are the gaugino masses ($M_i$, with $i = 1, 2, 3$), the third-generation $A$-terms ($A_\alpha$, with $\alpha = b, t, \tau$) and $\mu$. Six physical CP phases may be constructed from these seven complex parameters. These may be taken to be $\text{Arg} (A_t^* M_i)$ and $\text{Arg} (\mu A_\alpha)$ [5]. (Although our scenario is not identical to that in Ref. [5], we may borrow the results from that reference because for our calculation we may consider the first two generations to be decoupled from the third. The more general case is considered in Ref. [22].) In this work we have used the remaining available phase rotation to set $\text{Arg} (\mu) = 0$, in which case the phases of the $A$-terms are physical. Nevertheless, an important check of our asymmetries would be to study their behavior under a rephasing transformation. For example, if one wished to make $A_t$ real and positive through the transformation $A_t \rightarrow A_t e^{-i\phi_{A_t}}$, then the phase $\phi_{A_t}$ would appear in the other complex parameters, since one would also need to take $\mu \rightarrow \mu e^{i\phi_{A_t}}$, $A_{b,\tau} \rightarrow A_{b,\tau} e^{-i\phi_{A_t}}$ and $M_i \rightarrow M_i e^{-i\phi_{A_t}}$. The asymmetries should be unchanged under such transformations. We have performed a numerical check of the rephasing invariance of our asymmetries in a few particular examples and have found them to be invariant to within the expected degree of numerical precision.

To summarize the numerical work, we note that while the regular rate asymmetry
is expected to be very small for $\tilde{t}^+_2 \rightarrow \tilde{t}^+_1 \tau^- \tau^+$, we have seen that the single-spin and triple-product asymmetries can be quite large. In this work we have seen examples where the former can be of order 50% and the latter of order 90%. Such large values indicate that these asymmetries provide a promising avenue for the measurement of CP violation within SUSY. In particular, the observables studied here are very sensitive to the phase of the trilinear coupling, $A_t$, as is evidenced in Fig. 5. We have also seen that the differential CP asymmetries can have an interesting functional dependence on $M$, the invariant mass of the $\tau^\pm$ pair. Depending on the particular set of circumstances, experimentalists may wish to consider certain resonances separately, or to find other ways to optimize the CP-odd signals without sacrificing their statistical significance.

V. CONCLUSIONS

Many physicists believe that supersymmetry (SUSY) will be discovered at future high-energy colliders. If so, first measurements will involve CP-conserving quantities such as the masses of SUSY particles, etc. However, SUSY also contains a number of CP-violating parameters. It is only through their measurement that one will be able to identify the type of SUSY theory which is found in nature.

In this paper we have studied CP violation in the decay $\tilde{t}_2 \rightarrow \tilde{t}_1 \tau^- \tau^+$. We have shown that two CP asymmetries can be quite large in some regions of the SUSY parameter space (up to 90% for the parameters we have chosen). They are the single-spin and triple-product asymmetries, and involve the measurement of one or both of the $\tau$ spins. Both of these asymmetries depend almost entirely on a single CP-violating parameter, the phase of $A_t$, $\phi_{A_t}$. Thus, the measurement of the CP asymmetries in this decay will allow one to extract or constrain $\phi_{A_t}$. (Future work will involve other channels, sensitive to other SUSY phases.)

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APPENDIX: SOME APPROXIMATE EXPRESSIONS

In this appendix we provide some approximate expressions that help to clarify the discussions of the three asymmetries.
1. Rate Asymmetry

In the text it is noted that the rate asymmetry is suppressed for $\tilde{t}_2 \to \tilde{t}_1 \tau^- \tau^+$. To see why this is the case, consider the difference in the amplitudes-squared (which appears in the numerator of the expression for the rate asymmetry, Eq. (18)),

$$|A|^2 \Big|_{\text{rate}} - |\bar{A}|^2 \Big|_{\text{rate}} = -\frac{64\pi v^2}{\lambda^{1/2}(1, \kappa_\tau, \kappa_\tau)} \sum_{k,i} |g_{H_i\tilde{t}_2,\tilde{t}_1}| \sin (\alpha_i - \alpha_k) \times \sin (\alpha_i - \alpha_k) \text{Im} \left[ \left( D\text{Im}\hat{\Pi}_{\tau\tau}^\tau D^* \right)_{ik} \right].$$

(A.1)

To derive this expression we have made use of the definition of Im$\hat{\Pi}_{\tau\tau}^\tau (M^2)$ in Eq. (8); we have also set

$$\alpha_i \equiv \arg (g_{H_i\tilde{t}_2,\tilde{t}_1}).$$

(A.2)

The above expression may be simplified by assuming that the off-diagonal terms in the propagator are small (although not negligible) compared to the diagonal elements. Expanding Eq. (A.1) under this assumption, we find

$$|A|^2 \Big|_{\text{rate}} - |\bar{A}|^2 \Big|_{\text{rate}} \simeq -\frac{64\pi v^2}{\lambda^{1/2}(1, \kappa_\tau, \kappa_\tau)} \sum_{i,k} |g_{H_i\tilde{t}_2,\tilde{t}_1}| \sin (\alpha_i - \alpha_k) \times \frac{1}{|X_{ik}|^2} \times \left( M^2 - m_{H_i}^2 \right) \left( \text{Im} \hat{\Pi}_{ii}^\tau \text{Im} \hat{\Pi}_{\tau\tau}^\tau - \text{Im} \hat{\Pi}_{ii}^\tau \text{Im} \hat{\Pi}_{\tau\tau}^\tau \right)$$

(A.3)

$$= -\frac{64\pi v^2}{\lambda^{1/2}(1, \kappa_\tau, \kappa_\tau)} \sum_{i,k} |g_{H_i\tilde{t}_2,\tilde{t}_1}| \sin (\alpha_i - \alpha_k) \times \frac{1}{|X_{ik}|^2} \times \left( M^2 - m_{H_k}^2 \right) \left( \text{Im} \hat{\Pi}_{ii}^{\tau\tau} \text{Im} \hat{\Pi}_{\tau\tau}^\tau - \text{Im} \hat{\Pi}_{ii}^{\tau\tau} \text{Im} \hat{\Pi}_{\tau\tau}^\tau \right).$$

(A.4)

where

$$X_{ik} \equiv \left( M^2 - m_{H_i}^2 + i \text{Im} \hat{\Pi}_{ii} \right) \left( M^2 - m_{H_k}^2 + i \text{Im} \hat{\Pi}_{kk} \right).$$

(A.5)

In writing down these expressions, we have omitted a class of terms that could be important if all three resonances are close together.

The expression in Eq. (A.4) is small for a few reasons. First, a cancellation has occurred so that only self-energy loops with $b$ quarks contribute to the required strong phase difference (the $\tau$ contributions disappear in going from Eq. (A.3) to Eq. (A.4))\textsuperscript{8}. Second, the terms Im$\hat{\Pi}_{ii}^{\tau\tau}$, Im$\hat{\Pi}_{\tau\tau}^\tau$ and −Im$\hat{\Pi}_{ii}^{\tau\tau}$, Im$\hat{\Pi}_{\tau\tau}^\tau$ tend to cancel each other. Part of the reason for this is that the coupling constants $g_{H_i\tilde{t}_2,\tilde{t}_1}$ and $g_{H_i\tilde{t}_2,\tilde{t}_1}$ are equal at tree-level.

\textsuperscript{8} Note the importance of keeping the off-diagonal propagator terms in this case.
2. Single-spin Asymmetry

In our numerical work we keep the full $3 \times 3$ Higgs propagator, but to understand some of the dynamics for the single-spin asymmetry it is sufficient to keep only the diagonal terms. In that case we obtain

$$|\mathcal{A}|^2 \approx 8M \sqrt{M^2 - 4m^2 g^2},$$

$$|\bar{\mathcal{A}}|^2 \approx -8M \sqrt{M^2 - 4m^2 g^2},$$

where we have also made the approximation $\text{Im} \tilde{\Pi}_{jj}(M^2) \approx \Gamma_j m_H$, with $\Gamma_j \equiv \Gamma(H_j)$. If we define the strong phases $\delta_j(M^2)$ as follows

$$\cos \delta_j + i \sin \delta_j \equiv \frac{(M^2 - m^2_{H_j}) - i\Gamma_j m_H}{\sqrt{(M^2 - m^2_{H_j})^2 + \Gamma_j^2 m^2_{H_j}}},$$

we have

$$|\mathcal{A}|^2 - |\bar{\mathcal{A}}|^2 \approx -8M \sqrt{M^2 - 4m^2 g^2}$$

where

$$\sum_{j \neq k} \left[ g_{H_j \tilde{\tau} \tilde{\tau}} g_{H_k \tilde{\tau} \tilde{\tau}} \cos (\alpha_k - \alpha_j) \right] \left[ (M^2 - m^2_{H_j})^2 + \Gamma_j^2 m^2_{H_j} \right] \left[ (M^2 - m^2_{H_k})^2 + \Gamma_k^2 m^2_{H_k} \right].$$

In this form we can see clearly that the single-spin asymmetry depends on the sine of the relative strong phase between the interfering resonances. But it is perhaps somewhat counterintuitive that the single-spin asymmetry depends on the cosine of $\alpha_k - \alpha_j$, the weak phase difference.

To understand the presence of ‘$\cos(\alpha_k - \alpha_j)$’ in Eq. (A.8), consider first the CP-invariant limit. In this case there is no scalar-pseudoscalar mixing – two of the Higgs bosons are pure scalars and one is a pure pseudoscalar. Let the indices $j_S$ correspond to the two scalar Higgs bosons and let $j_P$ correspond to the pseudoscalar Higgs. Then $\alpha_{j_S} = 0$ or $\pi$ and $\alpha_{j_P} = \pm \pi/2$ (i.e., the stops have real (imaginary) couplings to the scalars (pseudoscalar) – see Eq. (A.2)). Furthermore, $g_{H_j \tilde{\tau} \tilde{\tau}} = 0$ (i.e., the Higgs bosons have pure scalar or pseudoscalar couplings to the taus). It is clear, then, that Eq. (A.8) is zero in the CP-invariant limit. Either the product of the scalar and pseudoscalar couplings is zero, or the weak-phase difference is $\pm \pi/2$, so that the cosine is zero.
Now suppose we allow CP to be broken by a small amount, so that there are two “mostly scalar” and one “mostly pseudoscalar” Higgs bosons. In this case there are two distinct types of contributions to the asymmetry. One contribution comes from the interference of a scalar and a pseudoscalar, denoted by the indices $k$ and $j$, respectively. In this case we can redefine $\tilde{\alpha}_j = \alpha_j \mp \pi/2$ for the (mostly) pseudoscalar Higgs, so that $\tilde{\alpha}_j$ measures the departure of $\alpha_j$ from the CP-invariant limit of $\pm \pi/2$. Then $\cos(\alpha_k - \alpha_j) = \pm \sin(\alpha_k - \tilde{\alpha}_j)$, and the expected sine of the weak phase difference appears. Alternatively, suppose $j$ and $k$ are both mostly scalars, but $g_{H_k \tau \tau}$ is not quite zero. In this case there can be a contribution to the asymmetry even if $\alpha_k = \alpha_j$.

To summarize, the single-spin CP asymmetry can receive contributions both from a weak phase difference in the interfering amplitudes, as well as from scalar-pseudoscalar mixing of the Higgs bosons. In general there will be some combination of these effects. Both types of contributions require a strong phase difference.

3. Triple-product Asymmetry

Making the same approximations for the Higgs propagator as was made in the previous subsection, we find the following expression in the case of the triple-product asymmetry,

$$
|A|^2_{TP} - |\overline{A}|^2_{TP} \simeq -8M\sqrt{M^2 - 4m_h^2v^2g_r^2} \\
\times \left[ \sum_k \left| g_{H_k \tilde{\tau} \tilde{i} i} \right|^2 \frac{g^S_{H_k \tau \tau} g^P_{H_k \tau \tau}}{\left( M^2 - m^2_{H_k} \right)^2 + \Gamma_k^2 m^2_{H_k}} \right] \\
+ \sum_{j \neq k} \left| g_{H_j \tilde{\tau} \tilde{j} i} g_{H_k \tilde{\tau} \tilde{i} i} \right| g^S_{H_k \tau \tau} g^P_{H_j \tau \tau} \cos (\alpha_k - \alpha_j) \cos (\delta_k - \delta_j) \right] \sqrt{\left[ \left( M^2 - m^2_{H_k} \right)^2 + \Gamma_k^2 m^2_{H_k} \right]} \right]
$$

(A.9)

This expression bears some similarity to that derived for the single-spin asymmetry (Eq. (A.8)), but with two important differences. In the first place, the cosine of the relative strong phase appears – this asymmetry does not require a strong phase difference. In the second place, there is a new term compared to the single-spin asymmetry – the term with a single sum over $k$. Since there is no need to have a relative strong phase for the triple-product asymmetry, it is possible to receive contributions from single resonances. Such a contribution contains an implicit dependence on CP-violating phases because it is only non-zero if $g^S_{H_k \tau \tau}$ and $g^P_{H_k \tau \tau}$ are both non-zero; i.e., if the Higgs boson in question has both scalar and pseudoscalar couplings. This can only happen if CP has been broken. Note that this first term is
somewhat analogous to the asymmetry considered by Valencia and Wang in Ref. [12].

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Note that there is also another allowed region with a slightly larger value for \( \tan \beta/m_{H^\pm} \).

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