SW parameter in magnetic multilayers with rough interface

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The magnetization (M) and spin-wave (SW) parameter of Fe (110)/Ag (111) multilayers has been calculated in the presence of the random magnetic field (RMF) and interlayer exchange coupling (IEC), using simple SW theory. We introduce roughness as RMF caused by fluctuations in the spacer thickness (ST). The results show that the M and SW parameter oscillate as a function of ST and they are affected by roughness strongly.

Keywords: Spin waves; Interface structure and roughness; magnetic multilayer
PACS: 75.30.Ds; 68.35.At; 75.70.-i

4. Introduction
Since the discovery of IEC, many studies have been implemented on the properties of ultrathin magnetic structures which their magnetic properties have been studied theoretically [1]. Oscillatory coupling has been found for a number of FM/non-magnetic metal systems [2, 3] at the origin of the discovery of GMR effect. One common approach is to study M and SW parameter (B) of FM thin layers. The observed results show that Curie temperature and B oscillate with ST [5, 6], which is investigated theoretically [7, 8]. A SW theory has been used to explain the ST-dependence of M and B. In real films, we expect interfacial roughness which affects IEC [8]. By employing the RKKY theory [9], we can relate IEC to RMF. A calculation was performed using a model described in Ref [10]. The Fe (110)/Ag (111) multilayer is considered as multiplane with RMFs concerned to IEC [8]. In this paper, the cases of FM and anti-FM interlayer coupling are solved for the ST-dependence of M and B, in the presence of RMF at low T. In section 2 the theoretical method is described. We present results and discussion in section 3 and finally the major points are summarized in section 4.

5. Theoretical method
The Heisenberg Hamiltonian (H) with the nearest neighbour (NN) inter (J₁) and intra (J₀) plane exchange coupling (EC) is used to describe the system. We approximate each thin FM layer by 2D square lattice of spin ½ parallel to the x-z plane. The x, y, and z axes are corresponding to the [101], [110], and [T10] directions. Suppose the RMF is applied along the z direction. This model is only valid for a super-lattice with the FM film thin enough to behave 2D. Thus SW spectrum and M are derived.

5.1. FM coupling
The Heisenberg-type H in the presence of a small RMF is given by
\[ H = -J_0 \sum \sum S(l_i, l_j) S(l_i + \delta l, l_j) - J_1 \sum \sum [S(l_i, l_j) S(l_i, l_j - 1) + S(l_i, l_j) S(l_i, l_j + 1)] \]
\[ - \sum g \mu_B H_s S^z l. \]  
(1)

S(l) is spin at lth site. \( \delta \) and \( l_1 \) are NN vector and the component of I, in x-z plane respectively. \( H_s \) is RMF acting on lth site. In equation (1) the first and second terms are intra and inter plane EC energy and the third one is the RMF energy. Lattice parameter, in the film plane is a and along y is b. We can apply Holestein-Primakoff transformation at low T. H is diagonal in the k space, thus SW spectrum in the limit of \( J_0 >> k_B T \), which only SWs with long wave length can be excited in the x-z plane, is [9]

\[ \omega(k) = S J_0 \left[ (k_x a)^2 + \left( \sqrt{2} k_z a \right)^2 \right] + 4 S J_1 [1 - \cos(k_y b)] + h_l \approx S J_0 \rho^2 + 4 S J_1 [1 - \cos(k_y b)] + h_l. \]  
(2)

The thermal average of magnetization, \( \langle S_z \rangle \), is given by [9]

\[ \frac{\langle S_z \rangle}{S} = 1 - \frac{1}{S} \frac{1}{(2\pi)^3} \int d^3k \frac{1}{e^{\omega(k)/k_B} - 1} = 1 + \frac{k_B T}{16 \pi^2 S^2 J_0} \int \frac{d(k_y b)}{\sqrt{2} \pi \Delta_{-\pi}^\pi} \ln(1 - e^{-\frac{1}{k_B T} [4 S J_1 (1 - \cos(k_y b)) + h_l]}). \]  
(3)

If \( \Delta \to 0 \), for strong interlayer coupling \( (J_1 >> k_B T) \), we get the Bloch law \( (T^{3/2}) \), but for weak coupling \( (J_1 << k_B T) \), we get a quasilinear T-dependence magnetization [9].

### 2.2. Anti-ferromagnetic (AF) coupling

At \( T = 0 \) K and for \( \Delta = 0 \), due to anti-ferromagnetic IEC we can divide the superlattice into two sublattice A and B with the property of tending to align all spins on sublattice A in the +z direction and all spins on sublattice B in the -z direction. Thus \( H \) in the presence of a RMF is given by

\[ H = -J_0 \sum \sum S(l_i, l_j) S(l_i + \delta l, l_j) + J_1 \sum \sum [S(l_i, l_j) S(l_i, l_j - 1) + S(l_i, l_j) S(l_i, l_j + 1)] \]
\[ - \sum g \mu_B H_s S^z l. \]  
(4)

The third and fourth terms are the RMF energy in the sublattices A and B. In this equation, \( l_x, l_y, l_z = nb \) \( \epsilon \) A with even n compose sublattice A; and \( l_x, l_y, l_z = nb \) \( \epsilon \) B with odd n is compose sublattice B. At low T, we can apply Holestein-Primakoff approximation within each sublattice A and B. The \( H \) transformed to the magnon variables, \( a(k) \) and \( b(k) \), that satisfy the boson communication relations is not diagonal in the k space:

\[ H = \sum_k [\omega_0(k) + h_l + 4 S J_1] [a^+(k) a(k) + b^+(k) b(k)] + 4 S J_1 \sum_k \cos(k_y b) [a(k) b(k) + a^+(k) b^+(k)] + c.c. \]  
(5)

By defining \( \omega(k) = \sqrt{[S J_0 (k_x a)^2 + (\sqrt{2} k_z a)^2] + 4 S J_1 + h_l}^2 - [4 S J_1 \cos(k_y b)]^2 \), at low \( T \), and appropriate transformations [9] in the presence of RMF, \( H \) will be diagonal in the k space. Therefore, the configuration average of magnetization, \( \langle S_z \rangle \), can be expressed as equation (3).
The results show that if $\Delta \to 0$, we get the $T^2$ dependence for strong IEC and a quasi-linear $T$-dependence for weak IEC [9].

6. results and discussions

We use RKKY function for IEC strength ($J_1$) [8]. The average of RMF, $\Delta$, was related to IEC by $\Delta^2 = 2\partial J/\partial R^2 \langle \delta R \rangle^2$ [7]. Thus, we can find $\Delta$ as a function of ST. Using equations (3) and (6), SW magnetization is evaluated in the presence of roughness, numerically (the points in the figures 1 and 2 show the calculated values). We fixed $J_0 = 710k_B$ [10], and the parameter $J_1 = 180k_B$ for ST = 4 monolayer (ML) was allowed to vary with Ag spacer thickness. Thus the magnetization is plotted versus ST for various $\Delta (\Delta R = 0.0, 0.25, 0.5$ and 1.0 ML) at $T = 10$ K, figure 1a. As a result of oscillatory behaviour of IEC versus ST, SW magnetization oscillates with the period of $J_1$ (in the absence of roughness). When $\partial J/\partial R$ has minimum value, we observe small dips in the presence of RMF, which occur alternatively with the same period as IEC, and their depth increases by roughness. Hence, introducing RMF result in the period of magnetization is two times of IEC periodicity. The magnetization will vanish when $J_1$ passes through zero ($\partial J/\partial R$ has maximum value) alternatively, figure 1a ($\Delta = 0$). But in the presence of roughness the heights of the valleys will reduce which is closer to real multilayers. The plot also show that SW magnetization increases in the presence of RMF which means that the deviation of $M(T \neq 0)$ from saturation value, $M(T=0)$, caused by existence of spin-waves in the system, decreases with RMF. The results also show that SW magnetization decreases with ST because of weakening IEC. The SW magnetization reduces with $T$ due to increase of the thermal excitations of SWs, figure 1b. It is obvious that the depth of dips increases by $T$.

We define $B(T) = (1/B_0 T^{-1/2}) \times [1 - \langle \langle S_z (h_0) \rangle \rangle / S_0]$, where $B_0 (= 5 \times 10^{-6} \text{K}^{-3/2})$ is SW parameter of bulk Fe. Figure 2 shows $B(T)$ versus ST in the presence of roughness for $T=10$ K. Due to IEC, $B(T)$ oscillates with the period of $J_1$. In general, $B(T)$ is dependent on the effective exchange seen by the Fe atoms. For thin ferromagnetic films in a general compare with thick ones [11], the reduced coordination results in larger $B(T)$ as is seen in the figure 2. $\Delta = 0$, in compare with figure 2b [11] for

![Figure 1](image-url)
$T^{3/2}$ prefactor in the Bloch law. In the presence of RMF, when $\partial J/\partial R$ has minimum value, we observe small peaks, which occur alternatively with the same period as $J_1$. Hence, the period of $B$ will be two times of $J_1$, the same as SW magnetization. These peaks which just occur in the presence of RMF can be important in explaining experimental results. Unfortunately, there isn't experimental result for thin Fe/Ag multilayers. In ref. [11], we have experimental data for thick ferromagnetic Fe films [11] which behave bulk like and their SW magnetization obeys the Bloch law. Our calculations are only valid for FM films thin enough to behave 2D. Hence, their magnetic behaviour is more complicated and shows dimensional crossover [9]. Thus, it isn't expected that our theoretical results is the same as experimental one [11] in details. But, as it was mentioned, introducing RMF result in the period of SW parameter is two times of IEC periodicity, which maybe explain some parts of the experimental results [11].

Figure 2. $B(T)/B_0$ versus spacer thickness with $\delta R = 0$, 0.25, 0.5 and 1 ML (monolayer) at $T=10$ K. The points show the calculated values

4. Conclusion
The SW magnetization and SW parameter of ultra-thin Fe/Ag multilayers has been investigated in the presence of RMF for various ST. The results show that the existence of the RMF can strongly influence the behaviour of the basic magnetic characteristics of ultra-thin magnetic multilayers included SW magnetization and SW parameter. Our results show both of them oscillates with ST, and the period of oscillations by considering RMF is two times of period of oscillations when $\Delta = 0$. The largest effect of the RMF on $M$ and $B$ is about where $J_1$ is passing through zero.

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Figure 1a

Figure 1b
Figure 2