On the Exotic Phases of M-theory

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Abstract

We study aspects of the new phases of M-theory recently conjectured using generalised dualities such as timelike T-duality. Our focus is on brane solutions. We derive the intersection rules in a general framework and then specialise to the new phases of M-theory. We discuss under which conditions a configuration with several branes leads to a regular extremal black hole under compactification. We point out that the entropy seems not to be constant when the radius of the physical timelike direction is varied. This could be interpreted as a non-conservation of the entropy (and the mass) under at least some of the new dualities.

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1 Introduction

The study of dualities [1, 2, 3, 4, 5] relating the consistent superstring theories led to the picture that these theories are descriptions of different phases, different regimes of a unique theory called M-theory. M-theory can take very different forms according to the limit one is considering. These phases differ by the presence or not of a gauge group, by the amount of supersymmetry and even by the dimensionality of the target space-time. The typical example is the opening up of a new dimension in the strong coupling limit of ten dimensional IIA theory which corresponds to 11 dimensional supergravity [2].

All these theories are formulated in a space-time with Lorentzian signature with one time coordinate, and the “orthodox” dualities do not affect the time direction which is supposed to be non-compact. Recently, timelike compactifications of M-theory and type II superstring theories on Lorentzian tori $T^n$ with $n$ spatial circles and one timelike circle have been considered [6, 7, 8]. The limits in which various cycles degenerate were studied. It was shown [8] that the type IIA (resp. IIB) theory on a timelike circle gives, in the limit in which the circle shrinks to zero size, a T-dual theory in 9+1 dimensions called IIB$^*$ (resp. IIA$^*$) characterised by the fact that the kinetic terms of the RR fields have the opposite (i.e. wrong) sign$^1$. The “exotic” dualities in which timelike directions are involved have been further analysed in [10]. For example, the S-dual theory of IIB$^*$ is a 9+1 theory called IIB$'$ characterised by a NSNS three form with a kinetic term of the wrong sign, implying the existence of an Euclidean fundamental string. Using the Buscher approach [11], it was furthermore shown in [11] that performing a T-duality in a string theory characterised by an Euclidean world-sheet maps the theory compactified on a spacelike (resp. timelike) circle $R$ onto a theory compactified on a timelike (resp. spacelike) circle of radius $1/R$ changing thus the space time signature of the theory.

Also the strong coupling of type IIA$^*$ appears to be a 9+2 supergravity called M$^*$ and characterised by the kinetic term of the four form field strength having the wrong sign [10]. Starting with, say, the usual IIA theory and performing a chain of T-dualities in various space or timelike directions and S dualities, one generates new “exotic” phases of M-theory corresponding to type IIA-like supergravities in 10+0, 9+1, 8+2, 6+4, 5+5, 4+6, 2+8, 0+10 dimensions, type IIB-like supergravities in 9+1, 7+3, 5+5, 3+7, 1+9 dimensions and eleven dimensional supergravities with signature 10+1, 9+2, 6+5, 5+6, 2+9 and 1+10. The action of all these theories differs from the usual supergravity actions by the fact that the signs of some of the kinetic terms are reversed [10]. All of these ‘new’ theories present however pathologies which are absent in the usual phases of M-theory, most notably ghost fields, and a better understanding of these phases and their relevance is certainly worthwhile. In a recent work [12], the study of solitons of these theories was initiated and a classification of all the $p$-brane type solutions preserving 1/2 of the supersymmetry was given. Note that some of these solitons had already been considered in the past [13] from the point of view of their supersymmetric world-volume action.

In this letter we propose to investigate further these new phases of M-theory, focusing on their brane solutions. We first derive the intersection rules [14, 15, 16] in arbitrary

$^1$Aspects of timelike T-duality have been previously discussed in a slightly different context in [8].
space-time dimensions, arbitrary signature and arbitrary signs of the kinetic terms of the field strengths (see also [17]), in the spirit of [18]. We then discuss compactifications of such configurations which lead to black holes living in an effective Lorentzian space-time (i.e. with one time coordinate), with particular attention on extremal black holes with non-vanishing entropy. We show that black holes sitting at the two opposite extremes of compactification of the physical time have seemingly different characteristics, namely one is non-singular and has non-vanishing entropy, and the other is singular and has zero entropy. This situation is in contrast with what happens for spacelike compactifications, and leads us to a critical assessment of this enlarged duality group and the new phases it uncovers.

2 Generalised Intersection Rules

In this section we determine how the extremal $p$-branes of the new phases of M-theory [12] intersect to form configurations with zero binding energy. The intersection rules will be obtained in a general framework for arbitrary space-time dimensions, arbitrary signature and arbitrary signs of the field strengths along the lines of [18]. The material contained in this section is rather technical, and it is aimed to satisfy the reader interested in the details of the derivation of the brane content of the exotic theories and their (generalised) intersection rules. Other readers need not understand thoroughly these details, and can go directly at the end of the section where the main results are recollected.

The starting point is the following action

$$I = \int d^D x \sqrt{|g|} \left\{ R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} \sum_I \frac{\theta_I}{n_I!} e^{a_I \phi} F_{n_I}^2 \right\}. \quad (1)$$

The overall sign (and factor) is not relevant. We allow for an arbitrary signature which we denote by $(S, T)$, with $S + T = D$, $S$ being the number of spacelike dimensions and $T$ the number of timelike dimensions [10, 12]. The $n$-forms’ kinetic terms also have an arbitrary sign given by $\theta_I = \pm 1$. The equations of motion (EOM) and Bianchi identities (BI) are:

$$R_{\mu}^\nu = \frac{1}{2} \partial^\mu \phi \partial_\nu \phi + \frac{1}{2} \sum_I \frac{\theta_I}{n_I!} e^{a_I \phi} \left[ n_I F^\mu_{\nu \lambda_2 \ldots \lambda_{n_I}} F_{\nu \lambda_2 \ldots \lambda_{n_I}} - \frac{n_I}{D - 2} \delta_{\mu}^{\nu} F_{n_I}^2 \right], \quad (2)$$

$$\Box \phi = \frac{1}{2} \sum_I \frac{\theta_I}{n_I!} a_I e^{a_I \phi} F_{n_I}^2, \quad (3)$$

$$\partial_\mu \left( \sqrt{|g|} e^{a_I \phi} F_{\mu \lambda_2 \ldots \lambda_{n_I}} \right) = 0, \quad \partial_\mu F_{\mu \lambda_2 \ldots \mu_{n_I + 1}}. \quad (4)$$

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2 We did not take into account in this general approach the various Chern-Simons-type terms which occur in the different theories. The solutions we will present below can be shown to be also consistent solutions of the full equations of motion including such terms.
Under Hodge duality which interchanges Maxwell EOM and BI one has:

\[ F_n \rightarrow \tilde{F}_{D-n}, \quad n \rightarrow \tilde{n} = D - n, \quad a \rightarrow \tilde{a} = -a, \quad \theta \rightarrow \tilde{\theta} = (-1)^{T+1}\theta. \quad (5) \]

Thus for even \( T \) the dual field strength has a kinetic term with opposite sign.

Let us now substitute the ansätze. For the metric we take:

\[ ds^2 = - t \sum_{a=1}^{D} B_{\alpha}^2 dt_{\alpha}^2 + \sum_{i=1}^{s} C_{i}^2 dy_{i}^2 + G_{ab}^2 \eta_{ab} dx^a dx^b, \quad (6) \]

with \( \eta_{ab} \) characterizing the overall transverse space-time of signature \((S-s,T-t)\) on which all the functions depend, and \( \delta_{a}^A = d = S - s + T - t. \)

The electric ansatz for a \((s_A,t_A)\)-brane is imposed on a \((s_A + t_A + 1)\)-form field strength (each brane is labelled by the index \( A \)):

\[ F_{\alpha_1 \ldots \alpha_{tA} i_1 \ldots i_{sA} a} = \epsilon_{\alpha_1 \ldots \alpha_{tA}} \epsilon_{i_1 \ldots i_{sA}} \partial_a E_A. \quad (7) \]

It satisfies trivially the BI. We actually do not need any magnetic ansatz, since we simply have to associate a ‘magnetic’ brane to a dual field strength and then translate the results according to the Hodge duality rules \((5)\).

Let us now rewrite the EOM, enforcing the above ansätze \((6), (7)\) together with the ‘extremality’ ansatz \([18]\), namely:

\[ B_1 \ldots B_t C_1 \ldots C_s G^{d-2} = 1, \quad (8) \]

which all at once overwhelmingly simplifies the Ricci tensor and corresponds to singling out extremal configurations. The EOM \((2)-(4)\) then become:

\[ \partial^2 \ln B_{\alpha} = \frac{1}{2} \sum_{A} \gamma_A \frac{\delta_{a}^A}{D - 2} S_{A}^{2}(\partial E_A)^2, \quad (9) \]

\[ \partial^2 \ln C_{i} = \frac{1}{2} \sum_{A} \gamma_A \frac{\delta_{a}^A}{D - 2} S_{A}^{2}(\partial E_A)^2, \quad (10) \]

\[ \sum_{\alpha} \partial^a \ln B_{\alpha} \partial_b \ln B_{\alpha} + \sum_{i} \partial^a \ln C_{i} \partial_b \ln C_{i} + (d - 2) \partial^a \ln G \partial_b \ln G + \delta_{a}^b \partial^2 \ln G = \]

\[ = - \frac{1}{2} \partial^a \phi \partial_b \phi + \frac{1}{2} \sum_{A} \gamma_{A}s_{A}^{2} \left[ \partial^a E_A \partial_b E_A - \frac{s_{A}^2 + t_{A}}{D - 2} \delta_{a}^b (\partial E_A)^2 \right], \quad (11) \]

\[ \partial^2 \phi = - \frac{1}{2} \sum_{A} \gamma_{A}a_{A}s_{A}^{2}(\partial E_A)^2, \quad (12) \]

\[ \partial^a \left( S_{A}^{2} \partial_a E_A \right) = 0, \quad (13) \]
where the conventions are: $v^2 = \eta^{ab} v_a v_b$;

$$S_A^2 = V_A^{-2} e^{\alpha A \phi}, \quad V_A = \prod_{\alpha \parallel A} B_\alpha \prod_{i \parallel A} C_i. \quad (14)$$

The notation $\mu \parallel (\perp) A$ is meant to indicate that the direction labelled by $\mu$ is longitudinal (perpendicular) to the brane labelled by $A$. In the r.h.s. of the equations (9) we have defined:

$$\delta^\alpha_A = \begin{cases} 2 - s_A - t_A & \text{for } \alpha \parallel A \\ -(s_A + t_A) & \text{for } \alpha \perp A \end{cases} \quad (15)$$

and similarly for $\delta^i_A$ in eq. (10). Most importantly we have finally:

$$\gamma_A = \theta_A(-1)^{t_A+1}, \quad (16)$$

instead of having simply $\gamma_A = 1$ as in the usual case.

We now enforce the ‘independence’ ansatz (18), namely we take that:

$$E_A = l_A H_A^{-1}, \quad S_A = H_A, \quad (17)$$

and that all other functions are products of powers of the $H_A$’s. This amounts to enforcing the no-force condition between the intersecting branes. The Maxwell EOM (13) gives that $\partial^2 H_A = 0$, i.e. $H_A$ is harmonic in $d$-spacetime. The remaining EOM (together with the first ansatz) give:

$$\begin{align*}
\ln B_\alpha &= -\sum_A \alpha_A \delta^\alpha_A \ln H_A, \\
\ln C_i &= -\sum_A \alpha_A \delta^i_A \ln H_A, \\
\ln G &= \sum_A \alpha_A s_A t_A \ln H_A, \\
\phi &= \sum_A \alpha_A a_A \ln H_A,
\end{align*} \quad (18)$$

where, crucially, we have:

$$\alpha_A = \frac{1}{2} \gamma_A l_A^2. \quad (19)$$

This means that $\alpha_A$ is not necessarily positive from this definition; rather, it has the same sign of $\gamma_A$, which depends on $t_A$, $\theta_A$ and the electric or magnetic nature of the brane if $T$ is even.

As in (18) the last off-diagonal part of the equation (11) can be rewritten as:

$$\sum_{A,B} \partial^\alpha \ln H_A \partial^\beta \ln H_B [M_{AB}\alpha_A - \delta_{AB}] \alpha_B = 0, \quad (20)$$

with

$$M_{AB} = \sum_{\alpha} \frac{\delta^\alpha_A \delta^\beta_B}{(D-2)^2} + \sum_{i} \frac{\delta^i_A \delta^i_B}{(D-2)^2} + (d-2) \frac{(s_A + t_A)(s_B + t_B)}{(D-2)^2} + \frac{1}{2} a_A a_B. \quad (21)$$

We thus get:

$$\alpha_A = (M_{AA})^{-1} = \frac{D-2}{\Delta_A}, \quad (22)$$
with
\[ \Delta_A = (s_A + t_A)(D - 2 - s_A - t_A) + \frac{1}{2}a_A^2(D - 2). \]
(23)

Note that \( \Delta_A > 0 \) and thus \( a_A > 0 \). If we want to avoid imaginary \( l_A \)'s (and thus imaginary field strengths), then all branes for which \( \gamma_A = -1 \) are forbidden \[12\]. The powers of the harmonic functions \( H_A \)'s in the solutions are nevertheless exactly the same as for ordinary \((p, 1)\)-branes of single-time spacetime.

The last condition, which leads to the intersection rules, is given by \( M_{AB} = 0 \) for \( A \neq B \). It thus leads to:
\[ \bar{s} + \bar{t} = \frac{(s_A + t_A)(s_B + t_B)}{D - 2} - \frac{1}{2}a_Aa_B, \]
(24)
where \( \bar{s} \) and \( \bar{t} \) are respectively the number of common spacelike and timelike directions of the two branes involved.

To summarize the results of this section, we have found that a solution representing intersecting branes of one of the exotic phases of M-theory can be simply built taking into account that: a brane with a definite signature \((s_A, t_A)\) exists only if \( \gamma_A \equiv \theta_A(-1)^{t_A+1} = 1 \); the intersection rules are given by (24) and are not restrictive on \( \bar{s} \) and \( \bar{t} \) independently; the solution is actually given by the ‘harmonic superposition’ \[14\] of the single brane solutions, which involve exactly the same powers of the harmonic function as for the branes of the ‘orthodox’ theories.

## 3 Black holes and exotic dualities

In this section we will discuss under which conditions “exotic” configurations lead upon compactification to non-singular extremal black holes. We will also consider the action of the generalised dualities on configurations of the new phases of M-theory.

We first briefly recall the situation in the usual phases of M-theory namely type IIA and IIB in 9+1 dimensions and supergravity in 10+1 dimensions. The corresponding intersection rules are given by (24) in the special case \( T = t = \bar{t} = 1 \) and \( t_A = 1 \) for every \( A \). From the general solution corresponding to a configuration of intersecting branes, it is easy to derive the condition under which this configuration after compactification gives an extremal black hole with non-zero entropy \[18\]. The condition to have a non-zero area at \( r = 0 \) is:
\[ N = 2\frac{\bar{D} - 2}{\bar{D} - 3}, \]
(25)
where \( N \) is the total number of branes in the configuration and \( \bar{D} \) is the space-time dimensions in which the black hole is living (i.e. the overall transverse space directions and the time direction). This leads to the well-known result that there are only two cases giving regular extremal black holes namely \( N = 3, \bar{D} = 5 \) and \( N = 4, \bar{D} = 4 \). A key property of these configurations is that the dilaton and the other moduli approach a (finite) constant at the horizon \( r = 0 \), and this is precisely the reason

3Note that the \( a_A \)'s change sign according to the electric or magnetic nature of the brane.
why the geometry is also regular there, the Bekenstein-Hawking entropy is finite and moreover it can be matched with a microscopical derivation (which needs extrapolation from weak coupling). All the above $\bar{D} = 5$ and $\bar{D} = 4$ configurations are related by the transformations of the U-duality group of, say, type II theories on $T^5$ and $T^6$ respectively. The mass and the entropy of such black holes can actually be shown to be U-duality invariant. The microscopic counting of the entropy of these black holes is nevertheless performed in a particular realisation generically involving D-branes and momentum.

We now turn to the new phases of M-theory and first state clearly the setting of the problem. We will be considering only the compactifications of these ‘exotic’ theories which lead to an effective (reduced) space-time of Lorentzian signature, i.e. with only one timelike direction. This means that although the underlying theories may have unconventional signature, we ask that the effective physics be conventional, in order to compare with the standard results.

Our focus is thus on configurations of intersecting branes that satisfy the following two requirements:

- We restrict ourselves to static configurations, i.e. the harmonic functions do not depend on any time. All the time directions are generically longitudinal to at least one brane of the configuration, namely $T = t$.

- We will compactify on all the timelike directions but one. The non-compact timelike direction will correspond to the physical time and the expected black hole is living in a spacetime with $S - s$ spacelike directions and one timelike direction.

For a given configuration with $\mathcal{N}$ branes, one has to distinguish between the branes which are longitudinal to the non-compact time which is chosen as being the physical one and the branes which are transverse to this time.

The branes transverse to the physical time appear to have quite unusual features. Indeed, they can be considered from the point of view of the effective spacetime as some kind of instantons, whose localisation in the time coordinate is however ‘smeared’ out as far as the present solutions are concerned. Because of all their longitudinal directions being transverse to the physical time, their ADM mass turns out to vanish identically. As a consistency check, one can see that the space-time transverse to a single such ‘instantonic’ brane reduces indeed to flat Minkowski space-time (in the effective Einstein frame). Thus, in accordance with the harmonic superposition principle, only the branes longitudinal to the physical time will contribute to the total mass of the configuration.

Focusing now on the entropy, if one assume that the configuration considered contains $n$ branes longitudinal to the physical time and $m$ branes perpendicular to it, one has obviously the total number of branes which is given by:

$$\mathcal{N} = n + m,$$

and one finds that the condition to have a non-zero area is given by:

$$n = 2\frac{\bar{D} - 2}{\bar{D} - 3},$$
namely the condition depends only on the number \( n \) of branes which “share” the physical time. The reason for this is that the branes perpendicular to the ‘physical’ time contribute a factor of 1 to the classical entropy formula, and thus the total entropy of the compound does not ‘feel’ them.

The independence with respect to the ‘instantonic’ branes of the mass and the entropy has physical implications. In some of the new phases of M-theory it is possible to build configurations which are singular even if the values of \( N \) and \( \bar{D} \) would have led to regular configurations in the “orthodox” phases.

To illustrate this fact, we consider an example in the framework of the type II\( A_{5+5} \) phase. Type II\( A_{5+5} \) is a theory with signature (5,5) characterised by the same lagrangian as the usual IIA theory, namely all the kinetic terms of the field strengths have the “correct” sign [10]. A configuration of II\( A_{5+5} \) with \( N = 4 \) and \( \bar{D} = 4 \) with \( n = 2 \) and \( m = 2 \) is given by (each brane is characterised by \((s_A, t_A)\) the number of spacelike and timelike directions of its worldvolume):

\[
\begin{array}{cccccc|cccc}
  t_1 & t_2 & t_3 & t_4 & t_5 & y_1 & y_2 & x_1 & x_2 & x_3 \\
 F1(1,1) & X & - & - & - & | & X & - & - & - \\
 F1(1,1) & - & X & - & - & | & - & X & - & - \\
 D4(0,5) & X & X & X & X & | & - & - & - & - \\
 D4(2,3) & - & - & X & X & | & X & X & - & - \\
\end{array}
\]  

(28)

For every choice of non-compact physical time this configuration corresponds to \( N = 4 \) and \( \bar{D} = 4 \) with \( n = 2 \) and \( m = 2 \). It is thus a singular configuration, with vanishing Bekenstein-Hawking entropy and with the mass being the sum of two charges, the nature of which depends on which time is considered as the physical one (hereafter we will pick \( t_1 \)). One can also easily check that the dilaton, as well as all the other moduli, is singular at \( r = 0 \).

We perform now a series of dualities to map II\( A_{5+5} \) onto the ordinary IIA theory in order to see onto which configuration (28) is mapped. The series of transformation is the following (see fig.5 of [10]):

II\( A_{5+5} \) \( \Downarrow T_{t_5} \) \( \Uparrow T_{t_3} \) \( \Downarrow S \) \( \Uparrow S \) \( \Downarrow T \) \( \Rightarrow \) II\( A_{6+4} \) \( \Rightarrow \) II\( B_{7+3} \) \( \Rightarrow \) II\( A_{8+2} \) \( \Rightarrow \) II\( B_{9+1} \)

where \( S \) corresponds to a S-duality, \( T_{t_i} \) stands for a T-duality in the direction \( t_i \) which does not affect the signature of spacetime, and \( T_{t_i \rightarrow y_j} \) corresponds to a T-duality which changes the signature of the spacetime, namely mapping a theory compactified on a
timelike circle of radius $R$ in the $t_i$ direction onto a theory compactified on a spacelike circle of radius $1/R$, the new spacelike direction being labelled by $y_j$.

Note that the last duality is the trickiest one. While for all the other ones we can imagine that the direction on which we are performing the T-duality is compact to begin with and we stick to the same point in the moduli space, in the case of $t_1$ we have to deal with the fact we take it as being the physical time, and thus we would like to consider it as non-compact both at the departure and at the arrival. This entails going, in the variables pertaining to (28), from an infinite radius of compactification $L$ to a vanishingly small one, $L = 0$, which is dual to an infinite radius after the last T-duality.

The configuration (28) becomes after having applied these transformations the following one:

$$
t_1 | y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 \ x_1 \ x_2 \ x_3
NS5(5,1) \ X \ | \ X - X X X X - - -
D4(4,1) \ X \ | \ - - X X X X - - -
W(1,1) \ X \ | \ - - - X - - - - -
D4(4,1) \ X \ | \ X X X - - X - - -
$$

This configuration is perfectly regular and leads upon compactification to the usual $\mathcal{N} = 4$, $\mathcal{D} = 4$ extremal black hole with non-zero entropy.

We can take as a second example one of the usual configurations that lead to ‘countable’ black holes, namely those made essentially by D-branes in type II theories. To be more precise, one can consider either a 5-dimensional black hole made of two kinds of D-branes plus F-strings (as in the first reference of [19]), or a 4-dimensional black hole composed uniquely of (four different kinds of) D-branes, like in [21].

In both of the cases above, all the moduli have a constant value at the horizon $r = 0$, thus allowing for a finite value of the entropy. If we focus on the dilaton, then we can easily see that each $(s, t)$ D-brane contributes in the following way:

$$e^{2\phi} = H^{4+s+t} \times \ldots,$$

where $H$ is the harmonic function related to the D-brane. We now perform a timelike T-duality and thus go to type II* theories. The (only) timelike direction is common to all the D-branes in the compound, thus the world-volume of each D-brane will loose one dimension. This entails that every D-brane will contribute a half-power more of its harmonic function to $e^{2\phi}$, thus making now the latter function explode at the horizon $r = 0$. A consequence of the explosion of this modulus (and possibly of others) is that the Bekenstein-Hawking entropy is no longer finite, but rather it vanishes, as can be checked independently.

In the particularly striking case of the four-dimensional black hole made out only of D-branes, one gets after the timelike T-duality a configuration which has also a vanishing mass, and actually the four-dimensional Einstein frame metric is simply the four-dimensional flat Minkowski one.

It is thus rather easy to find configurations in which the number of branes longitudinal to the physical time vary when the generalised dualities are performed. Note however
that since in the ‘orthodox’ theories all the branes have to be longitudinal to the only
time, the entropy is finite whenever the condition (25) is met. To put it in other words,
it is possible to go from an ‘ordinary’ configuration which leads to a black hole with
non-vanishing entropy to an exotic one related to a black hole with vanishing entropy,
but not from an ordinary configuration with vanishing entropy to an exotic one with
non-zero entropy (always keeping the dimensionality of the black hole fixed).

As a last remark, one could be worried about the possibility that, starting with a
configuration which already counts the right number of branes longitudinal to the phys-
tical time in order to have a finite entropy, one could still add more branes perpendicular
to the physical time. It turns out however that the generalised intersection rules (24)
prevent such a possibility.

4 Discussion

We have analysed in this letter the exotic phases of M-theory from the point of view
of the branes that they contain. The first result is that the intersection rules of these
branes (24) display no surprises with respect to the intersection rules of the ordinary
type II and M-theories [18]. Most notably, the same conclusions can be drawn with
respect to the microscopic interpretation of some of them, namely as branes ending on
other branes or as branes within other branes (associated respectively to world-volume
gauge theory monopoles and instantons).

The second result, which might lead to a critical assessment of the new exotic theories,
is that the new dualities, involving T-dualities along timelike directions, seem to connect
smoothly black holes with different characteristics, namely ones which are non-singular
and have a finite entropy to others which are singular and have a vanishing entropy.

A heuristic explanation of the mechanism which leads to the non-invariance of the
entropy is the following. Let us begin by noting that quantities such as the mass and the
entropy of black holes are usually defined in a Hamiltonian context, for which a physical
time coordinate has to be inequivocally defined and plays a special rôle. It is thus a
legitimate possibility that the procedure of compactifying the physical timelike direction,
then performing the T-duality, and eventually decompactifying the same direction can
affect these Hamiltonian quantities. The precise effect of this procedure on the D-branes
is that they become ‘instantonic’ branes, as it can be seen by a simple string theory
argument. This is not straightforward to see in our classical set-up, since the solutions
we consider are static, but nevertheless we argued in the preceding section that these
branes give a vanishing contribution to the ADM mass, a fact which is consistent with
the picture of these branes being events in the transverse effective space-time. On these
same grounds we do not expect them to contribute to the entropy of the configuration.

Our precise computation is performed at two points of the moduli sp-
cification of the physical time, namely where the original radius and the dual one are
respectively infinite. This is in order to be on safe grounds to perform a Hamiltonian
analysis and derive the black hole thermodynamics. What happens in the middle, i.e.
at finite radius of the physical time, has to be considered cautiously.
Given that it makes sense to consider a theory in which the physical time is compact (and thus a moduli space of timelike compactifications exists), we are thus left with three options: the entropy does depend on the radius $L$ of compactification of the physical time, while it is independent of all the other moduli; the entropy is independent of $L$, but the entropy and the mass in one theory are mapped to other quantities in the dual theory, and vice-versa—thus the theories might be dual but we do not know how to translate their variables; the entropy is independent of $L$, and the mismatch on the two sides means that the transformation is not a duality.

In all cases, the above discussion indicates that timelike dualities (or, even before, theories with a compact physical time) lead to rather exotic results. The last option above implies that the new theories presented in [10] are actually disconnected from the usual phases of M-theory. This would mean that the presence of non-unitary fields prevents these theories to play a rôle whatsoever. The other options yield a milder assessment, but nevertheless single out the timelike T-dualities as very particular, seemingly singular, ones. Note however that acting with T-duality in a timelike direction which is not taken as the physically relevant one is much closer to a traditional duality.

Of course we have considered here only solutions with the harmonic functions depending only on spacelike coordinates (i.e. static solutions with respect to any time). Moreover, we based our discussion on solutions for which we required that the effective space has only one time. This means that although we consider exotic theories with possibly several times, we nevertheless stick to a traditional Lorentzian signature as far as the effective physics is concerned. A more radical attitude would be to consider on an equal footing also all the ‘effective’ phases, i.e. all possible compactifications, but the problems discussed above could actually multiply.

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