Suppression of Bekenstein-Hawking radiation in $f(T)$-gravity

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We discuss semiclassical Nariai black holes in the framework of $f(T)$-gravity. For a diagonal choice of tetrades, stable Nariai metrics can be found, emitting Bekenstein-Hawking radiation in semiclassical limit. However, for a non-diagonal choice of tetrades, evaporation and antievaporation instabilities are turned on. In turn, this causes a back-reaction effect suppressing the Bekenstein-Hawking radiation. In particular, evaporation instabilities produce a new radiation – different by Bekenstein-Hawking emission – non-violating unitarity in particle physics sector.

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1. INTRODUCTION

In extended theories of gravity, extremal Schwarzschild-de Sitter black holes, also called Nariai solution, can be unstable. This phenomena was firstly discovered by Bousso, Hawking, Nojiri and Odintsov in context of quantum dilaton-gravity [1–4] and re-discovered in various extensions of General Relativity [5–17] later on. In this paper, we will consider quantum semiclassical Black holes in $f(T)$-gravity. We will discuss all possible classes of Narai black holes for diagonal or non-diagonal tetrads. In the non-diagonal tetrads case, evaporation and antievaporation instabilities of the Nariai solution appear out. In such a gauge choice, we will show that evaporating and antievaporating solutions turn off Bekenstein-Hawking radiation. Such a surprising result is based on the trapped surface analysis developed by Ellis and Firouzjaee in Refs. [27–29]. The pair creation process in a dynamical space-time is suppressed by the energy conservation and the causality condition. Particularly interesting is the evaporation phenomena in the non-diagonal choice. Contrary to Bekenstein-Hawking emission, the new evaporation effect does not lead to any information paradox or firewalls. Classical evaporation will push out both Bekenstein-Hawking pair before the effective tunneling time, i.e. there will be not any entanglement of the interior and the external regions of the black hole. More specifically, our argument is based on the analysis of the Raychaudhuri equation in $f(T)$-gravity [33]. In the antievaporation case, the torsion contribution introduces a gravitational focusing term into the Raychaduri equation. As a consequence, an emitting marginally trapped surface will transit from a time-like to a space-like manifold. In evaporation, on the contrary, the torsion term induces an anti-focusing contribution in the Raychauduri equation.

The paper is organized as follows: in Section 2, we briefly review basis of $f(T)$-gravity, Section 3-4 are devoted to a review of Nariai solutions in $f(T)$-gravity. In Section 5, our main argument is discussed. In Section 6, conclusions and further comments are shown.

2. $f(T)$-GRAVITY

The $f(T)$-action reads as follows:

$$ I = \frac{1}{16\pi} \int d^4 x \sqrt{-g} f(T) + S_m, \quad (1) $$

in units $G = c = 1$, where we project the metric and coordinates in representation of the tetrad matrices

$$ ds^2 = g_{\mu
u} dx^\mu dx^\nu = \eta_{ij} \theta^i \theta^j, \quad (2) $$

$$ dx^\mu = e^\mu_\nu \theta^\nu, \quad \theta^i = e^i_\mu dx^\mu, \quad (3) $$

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1 Other further analysis based on Ellis and Firouzjaee were discussed in Refs. [18, 30–32].

2 See Refs. [34–39] for recent progresses on $f(T)$-gravity.
where \( e^\mu_i e^i_\nu = \delta^\mu_\nu \), \( \eta_{ij} = \text{diag}(-1,1,1,1) \), \( \sqrt{-g} = e = \text{det}[e^i_j] \), \( i, j = 1, 2, 3, 4 \), \( \mu, \nu = 0, 1, 2, 3 \). We have introduced above the torsion tensor as follows:

\[
T^\alpha_{\mu
u} = \Gamma^\alpha_{\mu
u} - \Gamma^\alpha_{\nu\mu} = e^\rho_j (\partial_\mu e^i_\rho - \partial_\nu e^i_\mu)
\]  

(4)

The Euler-Lagrange equations can be obtained by varying the action with respect to the tetrad field \( e^i_\mu \):

\[
S_{\mu\nu}^\rho \partial_\rho T^{df}T_{df}^2 + [e^{-1} e^i_\mu \partial_\nu (e S_{\mu\alpha}^\rho e^\rho_i) + T_{\mu\alpha}^\sigma S_{\sigma\alpha}^\iota] \frac{dT}{dT} + \frac{1}{2} \delta_{\mu\nu} f = 4\pi T^{(m)}_{\mu\nu},
\]  

(5)

where \( T^{(m)}_{\mu\nu} \) is the energy-momentum tensor and \( S_{\mu\nu}^\rho \)

\[
S_{\mu\alpha}^\rho = \frac{1}{2} (\delta^\rho_\alpha T^{\nu\beta} - \delta^\mu_\beta T^{\nu\alpha} + K^\rho_{\alpha\nu}),
\]  

(6)

where \( K^\rho_{\alpha\nu} \) is the cotorsion and the scalar torsion is

\[
T = T_{\mu\alpha}^\sigma S_{\sigma\alpha}^\mu.
\]  

(7)

General relativity with a cosmological constant is recovered in the limit \( \frac{dT}{dT} \to 0 \), i.e. \( f(T) = a + bT \)

3. NARIAI BLACK HOLE IN DIAGONAL TETRADS

The Naraii space-time has the following form:

\[
ds^2 = \frac{1}{\Lambda} \left[-\frac{1}{\cos^2 \tau} (dx^2 - dr^2) + d\Omega^2 \right],
\]  

(8)

where \( \Lambda \) is the cosmological constant, and the \( d\Omega^2 \) is the solid angle on a 2-sphere \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \), \( 0 < \tau < \pi/2 \), \( 0 < t < \infty \), \( \cosh t = 1/\cos \tau \). The Ricci scalar of Naraii space-time is a constant being \( R = 4\Lambda \). The Naraii space-time is a solution of Eq. (6) in the diagonal tetrad ansatz

\[
ds^2 = e^{2\rho(x,t)} (-dx^2 + dt^2) + e^{-2\phi(x,t)} d\Omega^2,
\]  

(9)

\[
e^a_\mu = [e^\rho, e^\phi, e^{-\phi}, e^{-\phi} \sin \theta].
\]  

(10)

Dynamical aspects of Naraii solutions can be studied with perturbation theory methods:

\[
\rho = \ln[\sqrt{\Lambda} \cos \tau] + \delta \rho(\tau, x)
\]  

(11)

\[
\phi = \ln \sqrt{\Lambda} + \delta \phi(\tau, x)
\]  

(12)

and

\[
\delta T = -2\Lambda \sin(2\tau) \dot{\delta \phi}
\]  

(13)

is found. Inserting Eq. (12,13) in Eq. (6) one finds

\[
\delta \phi(x, \tau) = k_1 \sin(x - \bar{x}) \sec \tau + k_2
\]  

(14)

where \( \bar{x} \) is the fixed initial condition, \( k_{1,2} \) are two integration constants. Now, the horizon is defined through the condition

\[
\left( \frac{\partial \phi}{\partial \tau} \right)^2 = \left( \frac{\partial \phi}{\partial x} \right)^2.
\]  

(15)

From this, we obtain

\[
x_h = \bar{x} - \tau + m \pi - \frac{\pi}{2},
\]  

(16)

where \( m = 0, 1, \ldots \), corresponding to

\[
\delta \phi(\tau, x_h) = k_1 (-1)^{n+1} + k_2,
\]  

(17)

\[
r_h(\tau) = 1 + \delta \phi(\tau, x_h).
\]  

(18)

This means that the Black hole radius is fixed, i.e. no evaporation or antievaporation instabilities. In this case, thermodynamic proprieties were studied in Ref. [19].
4. CLASSICAL EVAPORATION AND ANTI-EVAPORATION IN NON-DIAGONAL TETRADS

Let us consider a non-diagonal tetrad base as follows:

\[ e_0^0 = e_3, \quad e_3^1 = e_0^{1,2,3} = e_0^0 = 0, \]  
\[ e_1^1 = \cos \psi \sin \theta e_3, \quad e_1^2 = \cos \psi \cos \theta e_3 - \phi, \quad e_1^3 = -\sin \psi \sin \theta e_3 - \phi, \]  
\[ e_2^1 = \sin \psi \sin \theta e_3, \quad e_2^2 = \cos \theta e_3, \quad e_2^3 = \sin \psi \cos \theta e_3 - \phi, \]  
\[ e_3^1 = \cos \psi \sin \theta e_3, \quad e_3^2 = -\sin \theta e_3. \]  
\[ (19) \]
\[ (20) \]
\[ (21) \]
\[ (22) \]

Under this ansatz, we obtain

\[ \delta \phi = A \sec \tau \cos(x - \bar{x}) + B (\tan \tau)^{3/2} e^{1 + 2 \cos^2 \tau} e^{4 \cos^4 \tau}, \]  
\[ (23) \]

where \( A, B \) are integration constants; corresponding to an horizon

\[ x_h = \bar{x} - \tau + \arcsin \left( \frac{\cos^2 \tau}{A} \frac{d}{d\tau} \frac{d \phi(\tau)}{d\tau} \right), \]  
\[ (24) \]

Eq. (23) has a divergence in \( \tau \to \pi / 2 \) -in the extreme time-like angle not included in the range of the Nariai solution. Depending on the integration constants, Eq. (24) solution is increasing or decreasing in time. The first class of instabilities corresponds to the classical antievaporation, while the second class to the classical evaporation.

5. NO BEKENSTEIN-HAWKING IN NON-DIAGONAL ANTI-EVAPORATING NARAI SOLUTION

Let us consider the Raychaudhuri equation in \( f(T) \)-gravity [33]:

\[ \dot{\theta} = -\frac{1}{3} \hat{\theta}^2 - \sigma_{\mu\nu} \hat{\sigma}^{\mu\nu} + \omega_{\mu\nu} \hat{\omega}^{\mu\nu} - R_{\mu\nu} U^\mu U^\nu - \tilde{\nabla}^\rho \hat{\alpha}^\rho - 2 U^\nu T^\sigma_{\mu\nu} \left( \frac{1}{3} h^\mu_{\sigma} \hat{\theta} + \hat{\sigma}^\mu + \hat{\omega}^\mu - U^\sigma \hat{a}^\mu \right), \]  
\[ (26) \]

\( \dot{\theta}, \sigma, \omega, \alpha \) are the expansion, shear, vorticity and acceleration in \( f(T) \)-gravity and \( R_{\mu\nu} \) is the Ricci tensor corrected by contributions from the torsion:

\[ R_{\mu\nu} = R_{\mu\nu} - 2 \nabla_\mu T_\rho + \nabla_\nu K_{\sigma}^{\mu\nu} K^{\lambda}_{\mu\rho}, \]  
\[ (27) \]

\[ R_{\sigma\mu\nu} = \partial_\mu \Gamma^\sigma_{\nu\rho} - \partial_\nu \Gamma^\sigma_{\mu\rho} + \Gamma^\sigma_{\mu\lambda} \Gamma^\lambda_{\nu\rho} - \Gamma^\sigma_{\nu\lambda} \Gamma^\lambda_{\mu\rho}. \]  
\[ (28) \]

In general, \( \dot{\theta}, \sigma, \omega, \alpha \) include the correction from the torsion as follows:

\[ \dot{\theta} = \dot{\theta}(GR) - 2 T^\rho U_\rho, \]  
\[ (29) \]

\[ \sigma_{\mu\nu} = \sigma(GR)_{\mu\nu} + 2 h^\rho_{\mu} h^\sigma_{\nu} K^\lambda_{\rho\sigma} U^\lambda, \]  
\[ (30) \]

\[ \omega_{\mu\nu} = \omega(GR)_{\mu\nu} + 2 h^\rho_{\mu} h^\sigma_{\nu} K^\lambda_{\rho\sigma} U^\lambda, \]  
\[ (31) \]

\[ \alpha_\rho = a_\rho(GR) + U^\nu K^\sigma_{\mu\rho} U_\sigma, \]  
\[ (32) \]
\[ \nabla_\mu U_\nu = \dot{\sigma}_{\mu\nu} + \frac{1}{3} h_{\mu\nu} \dot{\theta} + \omega_{\mu\nu} - U_\mu \dot{\omega}_\nu , \]

where \( \dot{\theta} = \frac{\partial}{\partial \lambda} \hat{\theta} \), where \( \lambda \) is the affine parameter. In the optical null case, \( U^a \equiv k^a \) with \( k^a = \frac{d\sigma^a}{d\lambda} \), with \( k^2 = 0 \) and

\[ \dot{\theta} = k^a_a = 2 \frac{1}{\Sigma} \frac{d\Sigma}{d\lambda} . \]

We can define an emitting marginally outer 2-surface \( \Sigma_{\text{time-like}} \) and the non-emitting inner 2-surface \( \Sigma_{\text{space-like}} \).

The marginally outer trapped 2-surface \( \Sigma_{\text{MOT}} \) has a topology of space-like 2-sphere with the condition

\[ \hat{\theta} (\Sigma_{\text{MOT}}) = 0 , \]

where \( \hat{\theta} \) in a \( S^2 \)-surface is the divergence of the outgoing null geodesics. The \( \hat{\theta} \) decreases with the increasing of the gravitational field – for example \( \hat{\theta}_T > 0 \) for \( r > 2M \) in the Schwarzschild case.

The radius of the \( S^2 \)-sphere \( \Sigma_{\text{MOT}} \) coincides with the Schwarzschild radius. \( S^2 \)-spheres with smaller radii than \( r_S = 2M \) will be trapped surfaces (TS), i.e. \( \theta (\Sigma_{\text{TS}}) < 0 \). Such a topological definition can be generalized for 3d surfaces. The dynamical horizon is a marginally outer trapped 3d surface. It is foliated by marginally trapped 2d surfaces. In particular, a dynamical horizon can be foliated by a chosen family of \( S^2 \) with \( \theta(a) \) of a null normal vector \( n_a \) vanishing while \( \theta_n \neq m < 0 \), for each \( S^2 \). In particular, one can distinguish among an emitting marginally outer trapped 3d surface \( \Sigma_{\text{time-like}} \) and a non-emitting one \( \Sigma_{\text{space-like}} \) by their derivative of \( \theta_m \) with respect to an ingoing null tangent vector \( n_a \):

\[ \hat{\theta}_m (\Sigma_{\text{time-like}}) = 0, \quad \frac{\partial \theta_m (\Sigma_{\text{time-like}})}{\partial n^a} > 0 \]

and the non-emitting one is defined as

\[ \hat{\theta}_m (\Sigma_{\text{space-like}}) = 0, \quad \frac{\partial \theta_m (\Sigma_{\text{space-like}})}{\partial n^a} < 0 . \]

Now, adopting these definitions, we demonstrate that the antievaporation will transmute the emitting marginally trapped 3d surface to a non-emitting space-like 3d surface. We can consider the Raychaudhuri equation associated to our problem. Let us suppose an initial condition \( \theta (\bar{\lambda}) > 0 \) with \( \bar{\lambda} \) an initial value of the affine parameter \( \lambda \). In the antievaporation phenomena, the null Raychauduri equation is bounded as

\[ \frac{d\hat{\theta}}{d\lambda} < -R_{ab}k^a k^b , \]

where \( R_{ab}k^a k^b \) is the effective contraction of the Ricci tensor \( R_{ab} \) with null 4-vectors, corrected by torsion contributions, in turn governed by the EoMs \( \mathbf{29} \).

Let us consider the antievaporation case: for \( \lambda > \bar{\lambda} \), it is \( R_{ab}k^a k^b > K > 0 \), where \( K \) is the 0-th leading order of the scalar function \( R_{ab}k^a k^b(t) \). Such a case coincide with the antievaporating solutions by definitions. So that

\[ \hat{\theta}(\lambda) < \hat{\theta}(\bar{\lambda}) - K(\lambda - \bar{\lambda}) - O((\lambda - \bar{\lambda})^2) \]

Neglecting higher order terms, this leads to \( \hat{\theta}(\lambda) < 0 \) for \( \lambda > \bar{\lambda} + \hat{\theta}_0 / K \), where \( \hat{\theta}_0 \equiv \hat{\theta}(\bar{\lambda}) \) are defined at a characteristic time \( \hat{t} \). For a small \( \delta t \), a constant \( 0 \)th contribution sourced by the torsion will cause an effective focusing term in the Raychauduri equation. So that, an emitting marginally trapped 3d surface will exponentially evolve to a non-emitting marginally one.

Now, let us consider a Bekenstein-Hawking pair in an antievaporating solution. They are imagined to be created in the black hole horizon as virtual pair. The external gravitational field can promote them to be real particles. Then, a particle of this pair can tunnel outside the black hole horizon, with a certain characteristic time scale \( \tau_{bh} \). With an understood correction to the Black hole entropy formula, this process seems compatible with Nariai solutions in diagonal tetrad choice. Bekenstein-Hawking’s calculations are performed in the limit of a static horizon an a black hole in thermal equilibrium with the environment. This approximation cannot work for antievaporating black holes. In fact, the horizon is displacing outward the previous radius. Bekenstein-Hawking pair will be trapped in the black hole interior, foliated in space-like surfaces \( \Sigma_{\text{space-like}} \). But from a space-like surface, the tunneling effect of a particle is impossible: otherwise causality will be violated. The suppression effect is a subleading effect if and only if the
Bekenstein-Hawking radiation has a characteristic time $\tau_{bh} < \delta t$, where $\delta t$ is the minimal effective time scale in the external rest frame for a $\Sigma_{\text{time-like}} \rightarrow \Sigma_{\text{space-like}}$ transition. However, this cannot be possible for an arbitrary small $\delta t$. This leads to the conclusion that the Bekenstein-Hawking radiation is exponentially turned off in time. This conclusion is solid for every antievaporating solution in $f(T)$-gravity discussed in Section 4.

Now, let us comment what happen in the opposite case: evaporating solutions. In this case $f(T)$-gravity will source an extra anti-focalizing term in the the null Raychauduri equation. This will cause exactly the opposite transition: a null-like horizon is pushed out the black hole radius and it will become time-like. Defining $\delta t$ as the transition time $\Sigma_{\text{space-like}} \rightarrow \Sigma_{\text{time-like}}$, Bekenstein-Hawking effect will happen if $\tau_{bh} << \delta t$. However, with $\delta t < \tau_{bh}$, the Bekenstein-Hawking pair is pushed-off from the black hole horizon. In other words, they both will be emitted from the black hole. So that they can annihilate out-side the black hole producing radiation. Contrary to Bekenstein-Hawking radiation, unitarity is not violated in black hole formation during the gravitational collapse. In fact, firewall paradox is exactly coming by the entanglement of the two pairs combined by the fact the one is falling inside the interior while its twin tunnels out. In our case, both are emitted out. In Bekenstein-Hawking case, outgoing information is exactly copied with the interior information. In our case, there is not any entanglement among black hole interior and external environment. So that this radiation does not introduce any new information paradoxes.

6. CONCLUSIONS AND OUTLOOKS

In this paper, extremal Schwarzschild-de Sitter solutions of $f(T)$-gravity have been analyzed in quantum semi-classical regime. These solutions are known in literature as Nariai Black Holes. In $f(T)$-gravity, Nariai solutions are static in diagonal tetrad basis and their thermodynamic proprieties were studied in [19]: a Bekenstein-Hawking radiation is emitted in this case [20, 21]. However, the Nariai solution has antievaporation or evaporation instabilities in non-diagonal tetrad basis [8].

In these two cases, we have studied the the null Raychauduri equation. In the case of antievaporation, we have demonstrated that the torsion contribution provides a new term in Raychauduri equation trapping the emitting surfaces in the black hole space-like interior before the effective Bekenstein-Hawking emission time. Surprisingly, Bekenstein-Hawking radiation is suppressed. On the other hand, the evaporation instability, sourced by torsion contribution of $f(T)$-theory, leads to a new peculiar quantum mechanical effect. The external gravitational field will promote virtual pairs to a real particle-antiparticle in the null-like event horizon. However, before the Bekenstein-Hawking emission by tunneling, both particles are pushed out the black hole horizon. In fact the torsion contribution provides an anti-focalizing term in the Raychauduri equation, pushing out space-like surfaces into time-like regions. As a consequence, both the particle and the antiparticle are emitted by the black hole. This new radiation does not lead to paradoxical losing of unitarity in quantum mechanics. The so called firewall paradox is related to an entanglement of Bekenstein-Hawking pair, leading to an entanglement of the black hole interior and its external environment [23, 24].

In our case, the entangled particles are both emitted out.

The extension of the Einstein-Hilbert action as $f(R)$-gravity or $f(T)$-gravity can be also motivated as EFT of quantum gravity. An UV completion of $f(T)$-gravity is not still understood and in particular the quantization procedure in tetrad basis. This is a crucial aspect to clarify in future. Further other issues concern evaporation and antievaporation instabilities in virtual micro black holes pairs 3. In particular, it should be possible that such instabilities promote the spontaneous production of micro-black holes from virtuality. Finally, possible quantum chaos effects on the event horizon may be crucially relevant in context of unstable antievaporating black holes [42, 43]. These aspects certainly deserve further investigations beyond the purposes of this paper.

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3 The relevance of virtual black holes contribution in vacuum energy density and in quantum information processing was recently studied in Refs. [40, 41].
[gr-qc].