The lifespan method as a tool to study criticality in absorbing-state phase transitions

Angélica S. Mata,1,2 Marian Boguñá,3 Claudio Castellano,4,5 and Romualdo Pastor-Satorras2
1Departamento de Física, Universidade Federal de Viçosa, 36570-000, Viçosa - MG, Brazil
2Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Campus Nord B4, 08034 Barcelona, Spain
3Departament de Física Fonamental, Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain
4Istituto dei Sistemi Complessi (ISC-CNR), via dei Taurini 19, I-00185 Roma, Italy
5Dipartimento di Fisica, “Sapienza” Università di Roma, P.le A. Moro 2, I-00185 Roma, Italy
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In a recent work, a new numerical method (the lifespan method) has been introduced to study the critical properties of epidemic processes on complex networks [Phys. Rev. Lett. 111, 068701 (2013)]. Here, we present a detailed analysis of the viability of this method for the study of the critical properties of generic absorbing-state phase transitions in lattices. Focusing on the well understood case of the contact process, we develop a finite-size scaling theory to measure the critical point and its associated critical exponents. We show the validity of the method by studying numerically the contact process on a one-dimensional lattice and comparing the findings of the lifespan method with the standard quasi-stationary method. We find that the lifespan method gives results that are perfectly compatible with those of quasi-stationary simulations and with analytical results. Our observations confirm that the lifespan method is a fully legitimate tool for the study of the critical properties of absorbing phase-transitions in general substrates, either random or regular.

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I. INTRODUCTION

A key class of dynamical non-equilibrium systems are those with absorbing states, i.e., states from which the dynamics cannot escape once it falls on them. Classical examples of such systems are epidemic spreading processes [1]; obviously, a fully healthy state is absorbing in the above sense, provided we do not allow for immigration of infected individuals. A very relevant feature of many systems with absorbing states is their ability to exhibit absorbing-state phase transitions [2, 3], that is, non-equilibrium phase transitions between an active state, characterized by everlasting activity in the thermodynamic limit, and an absorbing state, where activity is absent.

The contact process (CP) represents the simplest theoretical model with an absorbing-state phase transition [4]. The CP is defined as follows: Sites in a lattice are characterized by a binary variable $\sigma_i$ that can take values $\sigma_i = 1$ (occupied by a particle) or $\sigma_i = 0$ (empty). Each occupied vertex can spontaneously become empty at a rate which, without loss of generality, is set equal to 1, thus fixing the time scale. On the other hand, at a rate $\lambda z$, where $z$ is the coordination number of the lattice, an occupied site creates offspring particles on its empty nearest neighbors. The creation of particles is a catalytic process occurring exclusively in pairs of empty-occupied sites, implying that the state devoid of particles is a fixed point of the dynamics (i.e. an absorbing state). In an infinite system, the CP displays an absorbing-state phase transition between the absorbing (empty) phase, for small values of the control parameter $\lambda$, and an active phase, with a finite average number of particles, for $\lambda$ larger than a critical value $\lambda_c$.

The theoretical characterization of absorbing-state phase transitions is based on mean-field approaches and field theory renormalization procedures [5]. While the former is only valid above the upper critical dimension, application of the latter in physical dimensions is hindered by considerable technical difficulties. For this reason, most of our knowledge about the properties of absorbing-state phase transitions is based on computer simulation of different representative models. The numerical analysis of this computer data represents a different sort of challenge, which is mainly hampered by finite size effects. In finite systems, any realization of the dynamics is bound to reach sooner or later the absorbing state, even above the critical point, due to dynamic fluctuations. This difficulty can be overcome by applying the finite-size scaling technique [6], based on the size dependence of physical observables that are averaged only over surviving runs, i.e., realizations which have not yet fallen into the absorbing state [3]. The critical point and various critical exponents can then be determined by studying the decay of the average of different observables over surviving runs as a function of the system size. Averaging over surviving runs is, however, computationally highly inefficient. A much more effective alternative is provided by the quasi-stationary (QS) method [7,9], in which every time the system tries to visit an absorbing state, it jumps instead to an active configuration previously stored during the simulation.

Recently, in the context of epidemic modeling on complex networks [10], Boguñá et al. [11] proposed to consider the lifespan of spreading simulations starting from a single infected site as a tool to determine the position of the critical point. Inspired by the satisfactory performance of the lifespan method (LS) on epidemic models in networks [11], in this paper we consider its extension and application to general models with absorbing states.
on regular Euclidean lattices. Focusing on the classic CP, we present a detailed investigation of the application of this new approach to determine critical properties of absorbing state phase transitions. We present a finite size scaling theory of the method and apply it to the analysis of numerical results in the well-known controlled case of the CP in a one dimensional lattice, for which theoretical and high-quality numerical results are already available. A close comparison with the results of quasi-stationary simulations is also performed. Our results indicate that the lifetime method is a perfectly viable alternative to investigate critical properties of general absorbing phase-transitions not only on networks, but also in regular, finite dimensional lattices.

We have organized our paper as follows: In Sec. II we present the numerical implementation and main properties of the CP. Section III reviews briefly the QS method and the finite-size scaling form of the properties computed from it. Sections IV and V present the LS method and discuss its finite-size scaling theory, respectively. In Sec. VI we present numerical results comparing the predictions of both QS and LS methods for the CP in a \( d = 1 \) lattice. Conclusions and perspectives are finally discussed in Sec. VII.

II. THE CONTACT PROCESS

On a lattice or network with \( N \) nodes, the CP is numerically simulated as follows \([3]\): An occupied site \( j \) is randomly selected. With probability \( p = 1/(1 + \lambda) \) the selected site becomes empty. With complementary probability \( 1 - p \) one of the neighbors of \( j \) is randomly chosen and, if empty, it becomes occupied. Time is incremented by \( \Delta t = 1/[(1 + \lambda)n(t)] \), where \( n(t) \) is the number of occupied sites at time \( t \).

In an infinite system, the CP displays an absorbing-state phase transition at a critical point \( \lambda_c \), between an absorbing phase for \( \lambda \leq \lambda_c \), and an active one for \( \lambda > \lambda_c \). The order parameter of the transition is the stationary density of occupied sites \( \rho_{\text{st}}(\lambda) \equiv \lim_{t \to \infty} \lim_{N \to \infty} n(t)/N \), which is zero below the threshold \( \lambda_c \) and larger than zero above it. Near the critical point \( \rho_{\text{st}}(\lambda) \) vanishes as a power law

\[
\rho_{\text{st}}(\lambda) \propto (\lambda - \lambda_c)^\beta, \tag{1}
\]
characterized by the critical exponent \( \beta \). The onset of critical fluctuations at the transition is ruled by a diverging correlation length \( \xi \), given by

\[
\xi \propto |\lambda - \lambda_c|^{-\nu}, \tag{2}
\]
where \( \nu \) is the finite size scaling exponent.

III. THE QUASI-STATIONARY METHOD

The standard numerical procedure to investigate the finite-size scaling at absorbing phase transitions—by measuring the average of the order parameter restricted only to surviving runs—is extremely inefficient, since surviving configurations are very rare at long times. The quasi-stationary method represents an alternative strategy which consists in constraining the system to be in a quasi-stationary state. In practice, this is implemented by replacing the absorbing state, every time the system tries to visit it, with an active configuration randomly taken from the history of the simulation \([7]\). For this task, a list of \( M \) active configurations is stored and constantly updated. An update consists in randomly choosing a configuration in the list and replacing it by the present active configuration with a small probability \( p_r \Delta t \ll 1 \). The parameter \( p_r \) is typically chosen to be equal to 0.02. In any case, in the simulations presented here, no significant dependence on this parameter was detected for a wide range of variations in simulations.

After a relaxation time, the QS quantities are determined during a given averaging time window. Following this approach, it is possible to evaluate the full probability distribution of the number of occupied vertices in the quasi-stationary state and use it to calculate all quantities of interest. The transition point is then determined by considering the modified susceptibility \([12]\)

\[
\chi = \frac{L^d \left( \left\langle \rho^2 \right\rangle - \left\langle \rho \right\rangle^2 \right)}{\left\langle \rho \right\rangle}. \tag{3}
\]
Close to the critical point, the susceptibility diverges as \( \chi \sim (\lambda_c - \lambda)^{-(\gamma + \beta)} \). As we see, the critical exponent of this susceptibility is larger than the standard one \( (\gamma) \), which simplifies its numerical evaluation while preserving all the scaling properties. In a finite lattice of side \( L \), \( \chi \) shows a diverging peak at \( \lambda = \lambda_p^{\text{QS}}(L) \), providing a finite size approximation of the critical point. In the thermodynamic limit, \( \lambda_p^{\text{QS}}(L) \) approaches the true critical point with the scaling form \([13]\)

\[
\lambda_p^{\text{QS}}(L) = \lambda_c + A_{\text{QS}} L^{-1/\nu} \tag{4}
\]
In finite but large systems, the density of occupied sites and the susceptibility can be written near the critical point with the finite-size scaling form \([3]\)

\[
\rho_{\text{st}}(\lambda, L) \propto L^{-\beta/\nu} \int \left( \lambda - \lambda_c \right) L^{1/\nu} \left[ \left( \lambda - \lambda_c \right) L^{1/\nu} \right], \tag{5}
\]
and

\[
\chi(\lambda, L) \propto L^{(\gamma + \beta)/\nu} g \left[ \left( \lambda - \lambda_c \right) L^{1/\nu} \right], \tag{6}
\]
where \( f(x) \) and \( g(x) \) are scaling functions that satisfy \( f(x) \propto x^\alpha \) for \( x \gg 1 \), \( f(x) \propto |x|^{-\alpha + \beta} \) for \( -x \gg 1 \), and \( f(x) = \text{const.} \) for \( |x| \ll 1 \), and \( g(x) \propto |x|^{-(\gamma + \beta)} \) for \( |x| \gg 1 \), \( g(x) = \text{const.} \) for \( |x| \ll 1 \). Equations \([5]\) and \([6]\) imply that, at the critical point, the QS observables depend on \( L \) as

\[
\rho_{\text{st}}(\lambda_c, L) \propto L^{-\beta/\nu} \quad \text{and} \quad \chi(\lambda_c, L) \propto L^{(\gamma + \beta)/\nu}. \tag{7}
\]
IV. THE LIFESPAN METHOD

The LS method proposed by Boguñá et al. [11] considers spreading simulations starting from a single occupied site. Each realization of the dynamical process is characterized by its lifespan \( \tau \) and its coverage \( C \), where the latter is defined as the number of distinct sites which have been occupied at least once during the realization. In the thermodynamic limit, realizations can be either finite or endemic. Endemic realizations have an infinite lifetime and their coverage is equal to the system size; such realizations are only possible above the critical point. Finite realizations, on the other hand, have finite lifetime and coverage. Finite realizations can be found both below and above the critical point, although the probability to find a finite realization decreases when \( \lambda \) is increased.

In the LS method, the role of the order parameter is played by the probability that a run is endemic, \( P_{\text{end}}(\lambda) \). This probability is zero below the critical point and grows monotonously for \( \lambda > \lambda_c \), approaching 1 in the limit \( \lambda \to \infty \). The role of the susceptibility is played by the average duration of finite realizations, \( \langle \tau \rangle \). For small values of \( \lambda \) all realizations are finite and have a very short duration. As \( \lambda \) grows the average duration of finite realizations increases, diverging at the critical point. Above the critical point, the probability of a realization to be endemic increases and those realizations that remain finite have necessarily a short lifespan. This is so because once a realization has been alive for a very long time, the probability that it becomes finally endemic increases. As a result, \( \langle \tau \rangle \) diverges when approaching the critical point from the right and decreases as \( \lambda \) is increased. In a finite system with \( N \) nodes, \( \langle \tau \rangle \) exhibits a peak for a value \( \lambda^{LS}_p(N) \) that converges to \( \lambda_c \) in the thermodynamic limit.

In finite systems, the program described above has to be implemented with care. Indeed, in a finite system any realization is bound to end, reaching the absorbing state, even though this might occur over astronomically long temporal scales. Therefore, the distinction between finite and endemic realizations is, a priori, not clear-cut. In practice, we declare a realization as endemic whenever its coverage reaches a predefined threshold value \( C_{th} = c_{th}N \), with \( c_{th} \) a constant value between zero and one. Realizations ending before the value \( C = C_{th} \) is reached are considered to be finite. In the thermodynamic limit, reaching \( C_{th} \) means that an infinite number of nodes have been reached by the outbreak. If so, the probability that such realization is eventually trapped in the absorbing state is zero, meaning that the realization is endemic with probability 1.

V. FINITE-SIZE SCALING OF THE LIFESPAN METHOD

In this section, we present a finite-size scaling theory of the LS method, which enables the detailed analysis of numerical simulations. In general, the theory can be applied to any type of discrete structure. For this reason, hereafter we use the number of sites \( N \) as the measure of the size of the system. The case of a lattice of side \( L \) in \( d \) dimensions can be easily recovered by replacing \( N = L^d \). Let \( \Psi(\tau, C; \lambda) \) be the joint probability of a realization of the CP process to have, in an infinite size system, a (finite) lifespan \( \tau \) and coverage \( C \). This joint probability can be written as

\[
\Psi(\tau, C; \lambda) = \psi(\tau; \lambda)\Theta(C|\tau; \lambda),
\]

where \( \psi(\tau; \lambda) \) is the probability density of the lifespan \( \tau \) and \( \Theta(C|\tau; \lambda) \) is the probability that the coverage is \( C \), given that the lifespan is \( \tau \). The usual scaling assumption for \( \psi(\tau; \lambda) \), near the critical point, is [3]

\[
\psi(\tau; \lambda) = \tau^{-1-\delta} f[(\lambda_c - \lambda)\tau^{\beta}],
\]

for \( \tau > \tau_{\text{min}} \), some minimum time scale. The scaling function \( f(x) \) is non-symmetric, continuous at \( x = 0 \), constant when \( |x| \ll 1 \), and decays faster than a power law when \( |x| \gg 1 \). The scaling hypothesis Eq. (9) can be used to derive a scaling relation between the exponent \( \beta \) and the exponents \( \delta \) and \( \sigma \). Below the critical point, \( \lambda < \lambda_c \), all realizations are finite and, thus, \( \int \psi(\tau; \lambda) d\tau = 1 \). Above this point, there is a finite probability that a realization is endemic and, therefore, \( \int \psi(\tau; \lambda) d\tau = 1 - P_{\text{end}}(\lambda) \). Combining these two results and Eq. (9) leads to

\[
P_{\text{end}}(\lambda) \sim \frac{1}{\sigma}(\lambda - \lambda_c)^{\delta} \int_{0}^{\infty} x^{-1-\delta} \left[ f(x) - f(-x) \right] dx,
\]

which provides the relation \( \beta = \delta/\sigma \).

The scaling assumption Eq. (9) tells us that the lifespan is power law distributed up to the cut-off value \( \tau_c \equiv |\lambda_c - \lambda|^{-1/\sigma} \).

\[
\tau_c \sim |\lambda_c - \lambda|^{-1/\sigma},
\]

depending on the deviation from the critical point [11]. In turn, this implies that, close to the critical point, the moments \( \langle \tau^n \rangle \) behave as

\[
\langle \tau^n \rangle \sim |\lambda_c - \lambda|^{-\frac{x^n}{\nu}}.
\]

This result is similar to the behavior of the size of finite clusters in regular percolation [13].

In finite systems, an additional temporal cutoff competes with \( \tau_c \) in Eq. (11), namely, the temporal cutoff \( \tau_c(N) \) arising from the finiteness of the system size. To define this temporal scale, we consider the behavior near the critical point of the average coverage \( \bar{C}(\tau; \lambda_c) \equiv \sum_{C} C\Theta(C|\tau; \lambda_c) \). By assuming that the coverage \( C \) is a self-averaging quantity, so that its fluctuations around \( \bar{C}(\tau; \lambda_c) \) can be neglected, we can write

\[
\bar{C}(\tau; \lambda_c) \sim \tau^\mu.
\]

1 Note that the pre-factor in Eq. (11) can be different when approaching the critical point from below or from above.
However, since \( \bar{C}(x; \lambda_c) \) cannot become larger than \( N \), Eq. (13) can only hold up to a cut-off value \( \tau_c(N) \sim N^{1/\nu} \). The interplay between the two cut-offs present in the system, \( \tau_c \sim |\lambda_c - \lambda|^{-1/\sigma} \) (due to the distance from the critical point) and \( \tau_c(N) \) (due to the finite size) determines the scaling of the moments \( \langle \tau^n \rangle \). When \( \tau_c(N) \gg \tau_c \), the system does not notice its finiteness and, therefore, all moments are given by Eq. (12). Instead, when \( \tau_c(N) \ll \tau_c \), the distribution is cut-off by \( \tau_c(N) \) and, thus all moments behave as \( \langle \tau^n \rangle \sim [\tau_c(N)]^{n-\delta} \). To sum up:

\[
\langle \tau^n \rangle \sim \begin{cases} 
|\lambda_c - \lambda|^{\frac{n-\delta}{\sigma}} & \text{if } |\lambda_c - \lambda|N^{\sigma/\mu} \gg 1 \\
N^{-\frac{n-\delta}{\nu}} & \text{if } |\lambda_c - \lambda|N^{\sigma/\mu} \ll 1
\end{cases} \quad (14)
\]

Defining the exponents \( \gamma_n \equiv (n - \delta)/\sigma \) and \( \nu_\perp \equiv \mu/\sigma \), the behavior of Eq. (14) can be captured by the following finite size scaling form

\[
\langle \tau^n \rangle(N) = N^{\gamma_n/\nu_\perp} G_n \left[ (\lambda_c - \lambda)N^{1/\nu_\perp} \right] , \quad (15)
\]

where the scaling function \( G_n(x) \) is constant if \( |x| \ll 1 \) and goes as \( |x|^{-\gamma_1} \) when \( |x| \gg 1 \). As usual, we expect to find a maximum of \( \langle \tau^n \rangle \) around a value \( \lambda_p^{LS}(N) \), which depends on the system size as

\[
\lambda_p^{LS}(N) = \lambda_c + A_{LS} N^{-1/\nu_\perp} . \quad (16)
\]

We can then use, in general, the average lifespan to determine numerically the critical point and some of the critical exponents. There is, however, a pathological case if

The critical properties of the CP on a one-dimensional lattice and the corresponding finite-size scaling theory for the transition are very well known, and accurate theoretical and numerical values are readily available for comparison. This makes the CP on a one-dimensional lattice the ideal testbed for numerical methods. In this section, we present results of numerical simulations of the CP on a \( d = 1 \) lattice, applying both the QS and LS methods. Hereafter, we use \( N = L \).
A. Quasi-stationary simulations

In Fig. 1 we show the results obtained by performing QS simulations of the CP on a one-dimensional lattice of length $L$. The susceptibility $\chi$, Fig. 1(a), shows a well defined peak, which becomes narrower and taller as the system size $L$ grows. The plot of the quasi-stationary density $n_{st}(L)$, Fig. 1(b), also displays a transition becoming narrower and sharper as $L$ grows. From the position of the susceptibility peak $\lambda_p^Q(N)$ it is possible to obtain asymptotically an estimate of the transition point $\lambda_c$ by applying the relation in Eq. 1. We have used this expression to perform a nonlinear regression to determine the critical point $\lambda_c$ and the exponent $\nu_\perp$, see Fig. 2 and Table I. The values obtained by this procedure are in very good agreement with the best estimates accepted in the literature.\[\]

Right at the critical point, the average density of particles and the susceptibility should scale with the system size as given by Eq. 7. From this analysis, see Fig. 1(c) and (d), we can compute the exponents $\beta/\nu_\perp$ and $(\gamma + \beta)/\nu_\perp$, which again reproduce with good accuracy the known values of the CP, see Table I.

B. The lifespan method

As discussed in Sec. V, in the LS method for finite systems, the role of the order parameter is played by the probability $P_{\text{end}}(\lambda, L)$ that a run reaches the predefined coverage $C_{th}$ (i.e. it is effectively endemic), while the analogue of the susceptibility is given by the average duration $\langle \tau \rangle$ of finite realizations.

In Fig. 3(a), we plot the average lifespan $\langle \tau \rangle$ as a function of $\lambda$, for different system sizes, computed for a fixed coverage threshold $c_{th} = 0.5$; the effect of varying the coverage threshold is discussed in Sec. VI C. From this figure, we can observe that the lifespan $\langle \tau \rangle$ has a well-defined peak at a value $\lambda_p^{LS}(L)$, signaling the presence of a phase transition. The dependence of the peak position as a function of the system size $L$ is reported in Fig. 2. A non-linear fitting of the data according to Eq. 15 provides numerical estimates for the critical point $\lambda_c$ and the exponent $\nu_\perp$, see Table II which are compatible with the exact results derived analytically. Hence, we conclude that both the QS and the LS method recover compatible results for the position of the critical point and the exponent $\nu_\perp$.

The peak value $\langle \tau \rangle_p$ of the average lifespan grows as a

![Fig. 3](image-url)  

**Fig. 3.** (color online) (a) Lifetime $\langle \tau \rangle$ against creation rate for the CP on a one dimensional lattice. Curves are for system size (bottom to top) $L = 1000, 2000, 5000, 10000, 20000$ and $50000$. (b) Size dependence of the height of the peak of the average lifespan $\tau_p(L)$. The dashed line represents a power law regression with slope $1 - \delta/\mu = 1.32(1)$. We perform $5 \times 10^5$ individual realizations for each size. Error bars are smaller than symbols. (c) The scaling plot of the lifetime according to Eq. 15 for the same data of panel (a).

![Fig. 4](image-url)  

**Fig. 4.** (color online) Average coverage as a function of $\tau$ evaluated at $\lambda = \lambda_p^{LS}(N)$. The dashed line has a slope 0.64 and serves as a guide to the eyes.

| $\lambda_c$ | Theoretical | QS | LS |
|------------|-------------|----|----|
| $\nu_\perp$ | 1.09684(6)  | 1.098(5) | 1.100(5) |
| $\beta/\nu_\perp$ | 0.25208(4) | 0.253(5) | 0.255(5) |
| $(\gamma + \beta)/\nu_\perp$ | 0.7479(1) | 0.736(3) | — |
| $\mu (= \nu_\perp/\nu_\parallel)$ | 0.63261(4) | — | 0.64(1) |

**Table I.** Critical point and exponents of the CP in a $d = 1$ lattice obtained using the QS and LS methods. For comparison, we quote also the best estimates of those, from Refs. 3, 15.
power-law as a function of $L$, see Fig. 3(b). According to the scaling theory presented in Sec. IV, the exponent of this growth is equal to $(1-\delta)/\mu$, for which we obtain a value 1.32(1). The value of $\delta$ is well-known in the literature, namely $\delta = 0.15947(3)$ \cite{ref}. From here, we obtain the exponent $\mu = 0.64(1)$. We can also determine this exponent directly from the scaling of the average coverage near the critical point, $\bar{C}(\tau_\lambda; \lambda_c) \sim \tau^\mu$, see Eq. (13).

In Fig. 4, we analyze this coverage, obtaining numerically an exponent $\mu = 0.64(1)$, in perfect agreement with the value found from the scaling of the peak of the average lifespan. In Fig. 3(c), we finally check the full finite-size scaling form of the lifespan $\langle \tau \rangle$ as given by Eq. (17). We perform a data collapse analysis by plotting $L^{1/\nu_\perp} P_{\text{end}}(\lambda, L)$ as a function of $L^{1/\nu_\perp}(\lambda - \lambda_c)$. The perfect collapse of the plots shown in Fig. 3(c) confirms the validity of the finite-size scaling proposed in Eq. (5).

Concerning the order parameter, in Fig. 5(a) we plot $P_{\text{end}}(\lambda, L)$ evaluated with threshold coverage $c_{\text{th}} = 0.5$ as a function of $\lambda$ and different values of $L$. As we can see, it displays a sharp phase transition at the critical point when the size of the system increases. Close to criticality, and for large $L$, this probability exhibits a power law form with system size given by Eq. (18).

By analyzing $P_{\text{end}}(\lambda_c, L)$ as a function of $L$, we can obtain the exponent $\beta/\nu_\perp$, see Table II again in very good agreement with QS estimates. Finally, in Fig. 5(b), we check the full finite size scaling form Eq. (17) by plotting $L^{1/\nu_\perp} P_{\text{end}}(\lambda, L)$ as a function of $L^{1/\nu_\perp}(\lambda - \lambda_c)$, using the numerical exponents found. The perfect data collapse found demonstrates, once again, the correctness of the finite-size scaling form for the order parameter of the LS method.

C. Robustness with respect to the coverage threshold $c_{\text{th}}$

In the results presented above, we have used a fixed value of the coverage threshold $c_{\text{th}} = 0.5$. As we have discussed in Sec. IV, our results are however independent of the precise value of $c_{\text{th}}$. To check such a claim, we perform additional simulations for threshold values $c_{\text{th}} = 0.75$ and $c_{\text{th}} = 0.90$. In Fig. 6(a), we plot the average lifespan as a function of $\lambda$ for a fixed system size, $L = 5000$, and different values of the coverage threshold $c_{\text{th}}$. As we can see, increasing the coverage threshold slightly shifts both the position of the peak as well as the height of the maximum lifespan. Nevertheless, as we show in
problem in non-equilibrium statistical mechanics. Indeed, while powerful analytical strategies, such as field theoretic methods and their renormalization group analysis, are available, these methods are technically complex and ensuing loop expansions lead to approximate values for critical exponents, sometimes of uncontrolled validity in physical dimensions. For this reason, good numerical tools are of invaluable help. Here, we have reported a new numerical technique, the lifespan method, which is able to determine with great accuracy the critical properties of absorbing-state phase transitions. To this end, we have developed the corresponding finite-size scaling theory, which allows us to determine precisely both the the critical point and the critical exponents by looking at the size dependence of the associated susceptibility and order parameter. Results of the application of the lifespan method to the contact process in a $d = 1$ lattice are compared with results from the quasi-stationary method and other numerical and analytical results, showing that the new approach is fully reliable.

The lifespan method is thus an alternative way to numerically studying systems with absorbing states, which complements more traditional techniques, such as the quasi-stationary method, and that will represent in the future a useful addition to the numerical toolset of the statistical physics practitioner.

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