Center vortex model for the infrared sector of Yang-Mills theory

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A model for the infrared sector of $SU(2)$ Yang-Mills theory, based on magnetic vortices represented by (closed) random surfaces, is presented. The model quantitatively describes both confinement (including the finite-temperature transition to a deconfined phase) and the topological susceptibility of the Yang-Mills ensemble. A first (quenched) study of the spectrum of the Dirac operator furthermore yields a behavior for the chiral condensate which is compatible with results obtained in lattice gauge theory.

Diverse nonperturbative effects characterize strong interaction physics. Color charge is confined, chiral symmetry is spontaneously broken, and the axial $U(1)$ part of the flavor symmetry exhibits an anomaly. Various model explanations for these phenomena have been advanced; to name but two widely accepted ones, the dual superconductor mechanism of confinement, and instanton models, which describe the $U_A(1)$ anomaly and spontaneous chiral symmetry breaking. However, no clear picture has emerged which comprehensively describes infrared strong interaction physics within one common framework. The vortex model presented here aims to bridge this gap. On the basis of a simple effective dynamics, it simultaneously reproduces the confinement properties of $SU(2)$ Yang-Mills theory (including the finite-temperature deconfinement transition), as well as the topological susceptibility, which encodes the $U_A(1)$ anomaly. Furthermore, a first (quenched) study of the spectrum of the Dirac operator yields a behavior for the chiral condensate which is compatible with results obtained in lattice gauge theory; this indicates that the model also correctly reproduces the spontaneous breaking of chiral symmetry.

Center vortices are closed chromomagnetic flux lines in three-dimensional space; thus, they are described by closed two-dimensional world-surfaces in four-dimensional space-time. In the $SU(2)$ case, their magnetic flux is quantized such that they modify any Wilson loop by a phase factor $(-1)$ when they pierce an area spanned by the loop. To arrive at a tractable vortex model, it is useful to compose the vortex world-surfaces out of plaquettes on a hypercubic lattice. The spacing of this lattice is a fixed physical quantity (related to a thickness of the vortex fluxes), and represents the ultraviolet cutoff inherent in any infrared effective framework. The model vortex surfaces are regarded as random surfaces, and an ensemble of them is generated using Monte Carlo methods. The corresponding weight function penalizes curvature by associating an action increment $c$ with every instance of two plaquettes which are part of a vortex surface, but which do not lie in the same plane, sharing a link (note that several such pairs of plaquettes can occur for any given link).

Via the definition given above, Wilson loops (and, in complete analogy, Polyakov loop correlators) can be evaluated in the vortex ensemble, and string tensions extracted. For sufficiently small curvature coefficient $c$, one finds a confined phase (non-zero string tension) at low temperatures, and a transition to a high-temperature de-
confined phase. For $c = 0.24$, the $SU(2)$ Yang-Mills relation between the deconfinement temperature and the zero-temperature string tension, $T_C/\sqrt{\sigma_0} = 0.69$, is reproduced. When furthermore setting $\sigma_0 = (440 \text{ MeV})^2$ to fix the scale, measurement of $\sigma_0 a^2$ yields the lattice spacing $a = 0.39 \text{ fm}$. The full temperature dependence of the string tensions is displayed in Fig. 1.

![Figure 1](image1)

Figure 1. Confinement properties as a function of temperature, with $c = 0.24$. Crosses: string tension between static quarks; circles: spatial string tension $\sigma_s$. Whereas the former has largely been fitted using the freedom in the choice of $c$ (see text), the latter is predicted. In the deconfined regime, it begins to rise with temperature; the value $\sigma_s(T = 1.67 T_C) = 1.39 \sigma_0$ reproduces the one obtained in full $SU(2)$ Yang-Mills theory.

Note that the confined and deconfined phases can alternatively be characterized by certain percolation properties of the vortex clusters.

Complementarily, also the topological properties of the Yang-Mills ensemble encode important nonperturbative effects. The topological charge $Q$ of a vortex surface configuration is carried by its singular points, i.e. points at which the set of tangent vectors to the surface configuration spans all four space-time directions (a simple example are surface self-intersection points). Since a vortex surface carries a field strength characterized by a nonvanishing tensor component associated with the two space-time directions locally orthogonal to the surface, these singular points are precisely the points at which the topological charge density $\epsilon_{\mu \nu \lambda \tau} \text{Tr} F_{\mu \nu} F_{\lambda \tau}$ is non-vanishing. Note that, in order to carry nontrivial global topological charge, surfaces must be non-oriented; lines along which the orientation flips can be associated with Abelian magnetic monopoles. Generic vortex surfaces in the ensemble studied here are indeed non-orientable. In practice, identifying all singular points of the hypercubic lattice surfaces used in the present model involves resolving ambiguities reminiscent of those contained in lattice Yang-Mills link configurations. The resulting topological susceptibility $\chi = \langle Q^2 \rangle / V$, where $V$ denotes the space-time volume under consideration, is exhibited in Fig. 2.

![Figure 2](image2)

Figure 2. (Fourth root of) the topological susceptibility as a function of temperature, with $c = 0.24$. Also this result is quantitatively compatible with the one in full Yang-Mills theory.

A comprehensive description of the nonperturbative phenomena which determine strong interaction physics must furthermore include the coupling of the vortices to quark degrees of freedom and the associated spontaneous breaking of chiral symmetry. The latter can be quantified via the chiral condensate $\langle \bar{\psi} \psi \rangle$, which is related to the spectral density $\rho(\lambda)$ of the Dirac operator in a vortex background via the Casher-Banks formula $\langle \bar{\psi} \psi \rangle = \pi \rho(0)$. One can locally associate the chromomagnetic flux represented by an arbitrary vortex surface with a continuum gauge field; this allows to construct the Dirac operator directly in the continuum, preserving exact chiral symmetry. Globally, one must allow for the gauge field to be...
defined on different space-time patches, in order to accommodate the non-orientability of the vortex surfaces. While the gauge fields on each patch can be chosen Abelian (i.e. in 3-direction in color space), the transition functions in the overlap regions in general have to be non-Abelian. Since the spectrum of the Dirac operator here is obtained using the finite element method, it is natural to use the different finite elements as the space-time patches. A sample Dirac spectrum generated using this method is displayed in Fig. 3.

![Figure 3. Dirac spectral density ρ(λ) in a universe of volume (1.17 fm)^4, with c = 0.24.](image)

Note the anomalous enhancement of ρ(λ) near λ = 0, in accordance with the divergence expected for a Dirac operator with good chiral properties [3]. Also the linear extrapolation of the bulk of the spectrum to λ = 0, i.e. the spectral density obtained by truncating the λ = 0 divergence, quantitatively lies in the range obtained in full SU(2) Yang-Mills theory [5]. Of course, the chiral condensate by itself does not directly represent a renormalization group invariant, physical quantity, but can only interpreted in conjunction with a definition of the current quark masses. These can only be reliably fixed in the present model after more detailed (ultimately unquenched) measurements of hadronic properties.

It is also instructive to consider a change in the construction of the gauge field which trivializes the topology. This can be done by replacing the monopole loops on the vortex surfaces by double vortex sheets spanning areas bounded by the loops. Then, the surfaces can be oriented, the global topological charge vanishes, and presumably also many local clusters of nontrivial topology are trivialized (although the topological density as such does not necessarily vanish).

In the Dirac spectrum, this leads to a truncation of the λ = 0 divergence, and instead the finite-volume gap near λ = 0 familiar e.g. from chiral random matrix theory opens, without the bulk of the spectrum being qualitatively changed. While the λ = 0 divergence thus seems intimately related to topological properties [8], chiral symmetry breaking as such merely appears to require randomness of the gauge fields in a very general sense, without specific reference to topology.

Several improvements of the present model are envisaged. For one, the treatment must be generalized from SU(2) to SU(3) color. Also the Dirac operator can still be developed further to explicitly take into account a finite thickness of the vortices; the above construction, while embodying the correct topology, still deals with magnetic flux localized on infinitely thin world-surfaces. Eventually, it is hoped that this model will become a useful tool for phenomenological considerations.

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