On the Stability of the CMB Autocorrelation Function on Cosmological Parameters

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Abstract. The properties of the Cosmic Microwave Background Radiation angular autocorrelation function in Friedman Universe with negative curvature ($k = -1$) are studied. The dependence of the spectral index of autocorrelation function on the density parameter of the Universe is studied numerically, taking into account the effect of geodesic mixing, occurring in $k = -1$ curvature Universes.

1. Introduction

The outstanding importance of the knowledge of the power spectrum of the CMB is connected with the fact that it is expected to contain direct information on the spectrum of initial perturbations responsible for the formation of the present large scale matter distribution of the Universe (Peebles 1993, Peacock and Dodds 1994). Usually it is assumed that the CMB power spectrum which is observed now besides being influenced by Sachs-Wolfe effect, basically has to be the same as at the last scattering epoch. If the Universe has negative curvature, due to the effect of geodesic mixing as shown by (Gurzadyan & Kocharian 1997, Gurzadyan & Torres 1997), the present CMB spectrum has to be modified (flattened) after the time of the last scattering. The information on the spectrum is obtained via the angular autocorrelation function. The COBE data indicate the spectrum $n = 1.0 \pm 0.1$ (Smoot et al 1992, Hancock et al 1997, Nordberg & Smoot 1998).

Among the important goals of the study of anisotropy of CMB is to check the inflationary models (Efstathiou 1996). It is well known, that inflationary scenario has been suggested to explain several essential peculiarities of the Universe, including flatness ($\Omega = 1$). Among those predictions the temperature autocorrelation function $C(\theta, \beta)$ will correspond to Harrison-Zeldovich scale-invariant spectrum of initial fluctuations.

Below we discuss of the interpretation of the properties of $C(\theta, \beta)$, where $\theta$ is the modulation angle and $\beta$ the observing beam angle in view of the peculiarities of motion of CMB photon beams in Friedmann-Robertson-Walker Universe with negative curvature. It was proved (Gurzadyan & Kocharian 1997) that in Universe with negative curvature any spectrum of perturbations at last scattering epoch tends to a scale-invariant one with time. Therefore our aim should be to reconstruct the CMB autocorrelation function at the last scattering epoch from the present form of that function.

The CMB temperature anisotropy amplitude $\Delta T/T$ can be expanded in a series of spherical harmonics (Efstathiou 1996):
\[ \Delta T/T = \Sigma l,m a_{l,m} Y_{l,m}(\theta, \phi). \]

To be valid this decomposition functions have to satisfy the following conditions:

- The set of \( \{ Y_{l,m} \} \) functions has to be complete;
- \( \{ Y_{l,m} \} \) functions have to be orthogonal.

However, these conditions certainly valid in flat (k=0) Riemannian space, but we do not know the basis of functions in the case of hyperbolic (k=-1) Universe. So, in this case it is impossible to investigate the temperature anisotropy by common method of harmonic analysis.

This difficulty can be avoided if one applies the methods of theory of dynamical systems: the photon beam motion is reduced to the problem of the behaviour of time correlation function of geodesic flow on 3-manifold with negative constant curvature. The problem is how the properties of geodesics flow depend on the geometrical and topological properties of the (k=-1) Universe (Gurzadyan & Kocharian 1994). The study of the exponential decay of time autocorrelation function leads to the decrease of CMB anisotropy after last scattering epoch depending on the value of the density parameter \( \Omega \).

This effect was first calculated in (Gurzadyan & Kocharian 1993).

Note, that the mixing concept is a result of a ”roughness” of the system in the phase space, i.e. the possibility of the investigation of its small, but finite regions (Lichtenberg & Lieberman 1984).

This effect occurs only in the Universe with negative curvature and disappears in cases k=0 and k=+1.

2. Autocorrelation function

The consideration of the problem of exponential decay of correlators of geodesic flow have rather interesting observational consequences concerning the properties of temperature autocorrelation function \( C(\theta, \beta) \). Anosov (Anosov 1962, 1967) and others had obtained fundamental results concerning the behaviour of geodesic flow on 3-manifold with negative curvature. In particular, it was proved that a geodesic flow on a closed (compact, without boundary) manifold with constant negative curvature is an Anosov system, possessing the strongest chaotic properties (mixing of all degrees) and have positive Kolmogorov-Sinai entropy (KSE). Note that systems of this type are structurally stable, i.e. at the influence of a small perturbation is ignored and system will remain an Anosov system This is an important point, since FRW Universe is not absolutely homogeneous and isotropic, but is perturbed.

Consider the following correlator:

\[ b(\lambda) = \int_{SM} A_1 \circ f^\lambda A_2 d\mu - \int_{SM} A_1 d\mu \int_{SM} A_2 d\mu, \]

where \( f^\lambda \) is a geodesic flow on a space with measure \( \mu \).
The correlation function of geodesic flow $f^\lambda$ on $SM$ with negative curvature is decreasing by exponential low, i.e.

For $\exists c > 0$ (Pollicott 1992)

$$|b(\lambda)| \leq c |b(0)| e^{-h \lambda},$$

where $h$ is the KS-entropy of the geodesic flow $f^\lambda$.

The problem is to describe the free motion of photon on 4D space. This problem is solved in (Gurzadyan & Kocharian 1993) and for matter-dominated Universe the following formula was derived:

$$|C_\lambda(\theta, \beta) - 1| \leq c |C_0(\theta, \beta) - 1| \frac{1}{(1 + z)^2} \left[ \frac{\sqrt{1 + z \Omega} + \sqrt{1 - \Omega}}{1 + \sqrt{a - \Omega}} \right]^4,$$  \hfill (1)

where $z$ is the redshift of the last scattering epoch, $c$ is constant.

### 3. Results

Thus for any direction in sky the "measured" temperature tends to the constant mean temperature. Since this effect is model independent and appears only in the case of isotropic and homogeneous Universe with negative curvature. It provides possibility to getting some information on the curvature of the Universe.

Therefore we used (1) to reconstruct the autocorrelation function at the last scattering epoch. Calculations were performed for several values of the spectral
The results are shown in Figures 1-3. The dependence on density parameter $\Omega$ is clearly visible.

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References

Anosov, D.V. 1962, DAN SSSR, 707, 145
Anosov, D.V. 1967, Comm. Steklov Math. Inst., 90
Efstathiou, G. 1996, in Proc. XXXI Moriond Meeting, Ed. Frontieres
Gurzadyan, V.G., Kocharian, A.A. 1993, Int.J.Mod.Phys.D., 97, 2
Gurzadyan, V.G., Kocharian, A.A. 1994, Paradigms of the Large-Scale Universe, Gordon & Breach
Gurzadyan, V.G., Kocharian, A.A. 1997 in Quantum Gravity VI. World Sci.
Hancock, S et al 1997, MNRAS, 289, 505
Lichtenberg, A.J. & Lieberman, M.A. 1984, Regular and Stochastic Motion, Wiley
Nordberg, H.P., Smoot, G.F. astro-ph/9805112
Peacock, J.A., Dodds, S.J. 1994, MNRAS, 267, 1020
Peebles, P.J.E. 1993, Principles of Physical Cosmology, Princeton Univ. Press
Pollicott, M. 1992, Journ.Stat.Phys., 667, 67
Smoot, G. et al 1992, ApJ, L1, 396