Uplink and Downlink Performance Bounds for Full Duplex Cellular Networks

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Abstract—With Full Duplex (FD), wireless terminal is capable of transmitting and receiving data simultaneously in the same frequency resources, however, it introduces self interference and co-channel interference. Even though various signal processing techniques are emerged to cancel the self interference, the bottleneck for FD performance in cellular systems is the co-channel interference from the other uplink and downlink signals. In this work we have studied both the uplink and downlink performances of a FD cellular network, where users employ fractional power control in uplink. We use Matern Cluster Process to model the network, which provides a tractable and realistic model to characterize the user-base station distances which are needed for uplink power control. Based on the obtained coverage probabilities, rates and their robust approximations, we show that while FD improves downlink performance, it severely hurts the uplink performance. Also, we provide a trade-off between uplink and downlink performances. Our study suggests dense deployment of low power base stations can improve the performance of FD system.

Index Terms—Full duplex communications, Stochastic geometry, cluster process, uplink power control

I. INTRODUCTION

The full duplex communication is believed to be a potential candidate for the next generation cellular communication system. The FD systems allocate the same resources to both the uplink and downlink channels and provide bidirectional communications on the same temporal and spectral resources; unlike traditional Half Duplex (HD) systems, which segregate available resources among uplink and downlink channels. Thus FD systems can potentially lead to double the spectrum usage, however this spectrum utilization is at the cost of increased co-channel interference.

Recent studies of FD in the context of cellular communication systems show the performance improvements of FD over HD, [1]–[3]. The performance of FD system in heterogeneous networks is studied in [4], [5]. However, these works model BSs and UE locations as independent Poisson point processes, without considering the spatial dependencies of the users and the BSs. Recently, models based on cluster processes are proposed for cellular system analysis to incorporate this dependency, [6], [7]. In [8], the uplink and downlink performance of a full duplex cellular system is analyzed by using a Matern cluster process (MCP). They have imposed a minimum distance between BSs using the same frequency which is similar to fractional frequency reuse, which is against the fundamental principle of FD, i.e., the maximum reuse of spectral resources. So, we are interested to characterize the fundamental limits of FD cellular communications.

In this paper, we analyse a FD system both in uplink and downlink using MCP based model. We provide closed form expressions of coverage probabilities, average inverse SINR, rate distributions. The paper is organized as follows: Section II presents the system model Section III presents coverage probability analysis, Section IV presents uplink and downlink trade-offs; the paper is concluded in Section V.

II. SYSTEM MODEL

We now provide a mathematical model of the cellular system that will be used in the subsequent analysis. We begin with the spatial distribution of the base stations.

Network Model: The base station (BS) locations are distributed according to a homogeneous Poisson Point Process (PPP), Φb, with density λb. The locations of the users (UE)s are assumed to form a Matern Cluster Process (MCP) centred at the BS with a cell radius, Rc. Fig 1 shows a snapshot of the system model. Each BS schedules and allocates time-frequency resource elements to each of its UEs. In each cell, we assume a downlink UE and an uplink UE sharing the same resources (as has been allocated by the serving BS), which leads to cross-talk among the uplink and downlink paths. The limitation of this model is the overlapping of clusters of adjacent points of PPP in dense network with large clusters. The probability of this can be reduced by choosing a radius of cluster less than or equal to half of the average inter BS distance, 1/(2√λ).

Channel and path loss model: The channel between any two nodes are assumed to have Rayleigh fading with AWGN noise of variance σn^2. The pathloss is assumed to be exponential with parameter, α = 4, i.e., the power received at a distance x from a unit power transmitter will be l(x) = x^{-α}.

Received signal and interference: We consider the downlink UE, u0, of the serving BS, o. Let a_{x,d} be the signal transmitted by BS x in downlink and a_{x,u} be the signal transmitted by uplink UE of BS x ∈ Φb. Also, let h_{x,d} be the downlink channel seen by u0 from the BS x ∈ Φb and h_{x,u} be the uplink channel seen by u0 from the second user of the same BS. In Fig 1 various channels and distances are shown, readers may note that h_{x,d} and h_{x,u} are channels of different links and
not between the same links. Then, the received signal at $u_0$ is given by

$$y_d = h_{u,d} \frac{a_{0,d}}{\sqrt{r_{u,d}}} + h_{a,u} \frac{a_{0,u}}{\sqrt{r_{u,u}}} + \sum_{x \in \phi_{b}} h_{x,d} \frac{a_{x,d}}{\sqrt{r_{x,d}}} + h_{x,u} \frac{a_{x,u}}{\sqrt{r_{x,u}}}$$

(1)

where $r_{x,d}$ is the distance of $u_0$ from the BS $x$ and $r_{x,u}$ is the distance of $u_0$ from the uplink UE of BS $x$. The first term in the RHS of (1) is the desired signal and the second term is the same cell uplink interference. The interference from other cells includes both uplink and downlink interference.

We assume a fixed downlink power from all BS, i.e., $\mathbb{E}[|a_{x,d}|^2] = P_d$. In uplink, we consider transmission with fractional power control.

**Distance to the serving BS:** Since the UEs are distributed uniformly in each cell around the BS, the probability density function (PDF) of the distance to serving BS is given by [8],

$$f_R(r) = \frac{2r}{R_c^2}, \quad 0 \leq r \leq R_c.$$

(2)

### III. Coverage Probability Analysis

In this section, we provide the analysis of coverage probability for downlink and uplink users. Throughout our discussion we assume complete cancellation of self interference for each link and provide optimistic performance bounds. The traditional half duplex (HD) coverage probabilities can be found out as special cases from the FD coverage.

#### A. FD Downlink with uplink Power control

For a downlink UE, the interfering signals are from the uplink user of the same cell and the downlink and uplink signals of other cells as in equation (1). We consider a FD network with UEs employing the distance dependent fractional uplink power control as in [10], [11]. In this scheme the transmit power of user in uplink is set according to,

$$P_u^c(PL) = \min (P_{max}^u, P_0 + \epsilon PL) \text{ dBm}, \quad (3)$$

where $\epsilon \in [0, 1]$ and $P_0$ are the power control factors, designed to achieve a target SINR, $P_{max}^u$ is ceiling for uplink power, and $PL$ is the estimated pathloss. The nominal values of $P_0$, $-127 < P_0 \leq -64$ dBm, [11]. This transmission scheme compensates the pathloss and try to maintain the SINR needed to establish connection with the serving BS. So a UE near to the BS will be sending at a lower power and a user far from BS will be transmitting at higher power. This makes the spatial distribution of interference to be very different from that of transmission without power control. The various power control methods proposed in wireless communication standards can be seen in [10]–[12]. We assume perfect pathloss estimation at the user, i.e., $PL = r_u^\alpha$.

Neglecting the uplink signals of UEs of other cells, an upper bound for downlink coverage probability can be derived. Since the uplink powers of UEs are small compared to the downlink signal powers of BSs, this bound will be a tight approximation.

**Theorem 1.** Coverage probability for a downlink user in full duplex network with fractional uplink power control is given by

$$P_{c,DL}(T) \approx \mathbb{E}_{r_d,r_u} \left[ -e^{-\frac{\alpha}{\sqrt{\zeta} P_{d} T}} \frac{1}{\pi \alpha} \sqrt{\frac{\pi}{\zeta}} \tan^{-1}\left( \frac{\sqrt{\frac{\pi}{\zeta}}}{\frac{\pi}{\zeta} r_d^{\frac{\alpha - 1}{2}} \frac{\alpha - 2}{\pi} \sqrt{\frac{\pi}{\zeta}} \alpha \sigma^2} \right) \right],$$

(4)

where $\zeta$ is given in [13], the expectation is over the random variables $r_d$ and $r_u$, and their pdf is given in [8].

**Proof:** See Appendix A

The traditional HD coverage probability can be obtained as special case, by substituting $P_u = 0$ in (10) and given in the following corollary.

**Corollary 1.** When the network is interference limited, $(\sigma^2 = 0)$, the coverage probability of downlink user in half duplex network is given by

$$P_{c,DL}(T) \approx \mathbb{E}_r \left[ \exp \left( -2\pi T c \frac{\eta \alpha^{\frac{\alpha - 2}{\alpha}}}{\alpha} \right) \right].$$

This corollary is useful for the comparison of FD network with HD network.
We have verified our analytical results using extensive Monte-Carlo simulations in a network grid reproducing a real scenario in a 10 x 10 km² area. In simulations, we have considered all the interference including the uplink interference from other cells. The downlink coverage probabilities are plotted in Fig. 2. We can observe that all the bound are close to their respective simulations. As expected, the coverage of FD is slightly below HD due to the additional interference of uplink user. The coverage probability with power control will be closely matching for $0 < \epsilon \leq 0.8$, this is because of the approximation in (12). It can be observed that the coverage probability is not as severely affected in the downlink as compared to uplink, which will be discussed in detail in the next section.

![Fig. 2: Coverage probability of a typical downlink user in HD and FD for various different power control factor $\epsilon$ with fixed $P_0 = -64$ dBm, $P_d = 40$ dBm and Inter-BS distance 400m. Simulations are marked by ⊕.](image)

**B. FD Uplink with power control**

In this section, we analyze the uplink coverage probability with power control. Conditioned that, the user is at a distance $r_u$ from the parent BS, the SINR is given by

$$\text{SINR}|_{r_u} = \frac{P_0 r_u^{\alpha \epsilon} |h_u|^2 r_u^{-\alpha}}{\sigma^2 + \sum_{x \in \Phi_c^u} P_d |h_{x,d}|^2 r_{x,d}^{-\alpha} + P_c(r_{x,u}) |h_{x,u}|^2 r_{x,u}^{-\alpha}},$$

where $OBI$ and $OUI$ are other BS downlink interference and uplink UE interferences. Since, the uplink signal powers are small compared to that of downlink, the users of other cells are far from the user of interest, we neglect the other cell uplink interference.

**Theorem 2.** Coverage probability for an uplink user in full duplex network with fractional uplink power control and constant downlink power is given by

$$P_{c,u/L}(T) \approx E_{r_u}\left[ e^{-\sigma^2 2^\alpha \frac{(1-\epsilon)}{\epsilon} r_u - 2^\alpha \frac{2\lambda}{\pi} \left( \frac{P_0 r_u^{\alpha \epsilon}}{P_d} \right)^{2\alpha}} \right]$$

where the expectation is over the random variable $r_u$, which is uniformly distributed and the pdf is given in (2).

**Proof:** Follows the similar procedure of theorem (1). □

The uplink coverage probabilities for various configurations are plotted in Fig. 3. It can be seen that the uplink coverage probability is severely affected by the same channel downlink interference. By reducing the downlink transmission power, we can improve the uplink coverage without affecting downlink coverage. Also, we can see that by increasing $\epsilon$ close to unity, a slight improvement in uplink coverage, but this may hurt the downlink users which are in close proximity of the uplink user. Increasing $P_0$ (impractical as the limited power capability of UEs) increases the uplink coverage, but this will cause higher interference and reduce the downlink coverage, especially to the cell edge users. In next section, we will discuss, in detail, interactions of various network parameters and different trade-off between uplink and downlink rate.

![Fig. 3: Coverage probability of a typical uplink user for various configuration $\epsilon$ and $P_d(\epsilon, P_d)$ with $P_0 = -64$ dBm and Inter-BS distance 400 m. Simulations are marked by ⊕.](image)

**IV. UPLINK AND DOWNLINK RATE TRADE OFFS**

In this section we analyze the dependencies of average rate and coverage on various system parameters both in uplink and downlink.

**A. Average inverse SINR**

In order to establish the interdependencies of system parameters and obtain a simplified closed-form theoretical expression, we evaluate the average inverse SINR. The uplink SINR
is given in [5]; a simple average of inverse of this quantity can be deduced as following,

$$\mathbb{E}(1/{\text{SINR}_{UL}}) \approx \mathbb{E} \left( \frac{\sigma^2}{P_0^{\alpha + \epsilon} |h_u|^2 r_u^{-\alpha}} + \sum_{x \in S_k} \frac{P_d |h_x|^2 r_x^{-\alpha}}{P_0^{\alpha + \epsilon} |h_u|^2 r_u^{-\alpha}} \right) = \frac{1}{P_0} \frac{2R^\alpha(1-\epsilon)}{\alpha(1-\epsilon) + 2} \left( \sigma^2 + \frac{2\pi \lambda P_d}{\alpha^2 - 3\alpha + 2} \right).$$  \hspace{1cm} (7)

Here, we assume the effect of fading on the rate to be completely mitigated by channel inversion power control; consequently we obtain a converging limit. This assumption is meaningful as in practice channel inversion is also done as part of power control [13]. Also, we have approximated $r^{-\alpha} \approx (1+r)^{-\alpha}$ to avoid the singularity in the pathloss model. In practice, $r \gg 1$ and hence this assumption works well. A similar average can be derived for downlink users also,

$$\mathbb{E}(1/{\text{SINR}_{DL}}) \approx \frac{2R^\alpha}{\alpha + 2} \left( \frac{\sigma^2}{P_d} + \frac{2\pi \lambda}{\alpha^2 - 3\alpha + 2} \right) + \frac{P_0 R^{\alpha + 2}(2\alpha - 1)}{P_d(\alpha - 1)(\alpha + 2)}. \hspace{1cm} (8)$$

The bounds derived in (8) and (7) are plotted in Fig. 4, along with $\mathbb{E}[^{\text{SINR}}]$ for validation (see their inverse behavior). From the bounds, we can observe that in uplink increasing the cell radius increases the average inverse SINR, which in turn reduces the coverage probability. Also, from (7), we can see that increasing $P_u$ will increase the average inverse SINR$_{UL}$, hence reduce coverage, which is severe since $P_0 \ll P_d$. On the other hand, in downlink increasing $P_d$ increases desired as well as interference power and hence downlink coverage will not be affected. These observations, reduced cell radius and low power, recommend small cells for FD.

In Fig. 4, increasing power control factor $\epsilon$ increases UL coverage. This is not true in practice for the cell edge users, since a cell edge user may have a closer interfering user of the other cell. So, our bound is an optimistic result in a sense, i.e., for a large percentage of users our bounds will be applicable and higher $\epsilon \approx 1$ is suggested. It can also be seen that increasing $P_0$ improves the uplink SINR and reduces downlink SINR, this is not studied since transmission powers of mobile UEIs are limited.

### B. Rate Profile

The average rate $\mathbb{E}(R = \log_2(1 + \text{SINR}))$ can be deduced from the SINR distribution using the property of positive random variables, $\mathbb{E}(X) = \int_0^{\infty} P(X > t) dt$. The cumulative distribution function (CDF) of rate, $F_C(c) = \mathbb{P}(C \leq c)$, can also be obtained from the coverage probability. For HD systems,

$$F_C^H(c) = \mathbb{P}[\log_2(1 + \text{SINR}) < c] = 1 - P_c^H[2^c - 1]$$

where $c$ is the rate in bits/sec/Hz. Similarly for FD systems,

$$F_C^F(c) = \mathbb{P}[2\log_2(1 + \text{SINR}) < c] = 1 - P_c^F[2^{2c} - 1]$$

The rate CDF for uplink and downlink for various configurations are plotted in Fig. 5. A rate profile is given in Table I. Both of them shows the improvement of average and cell edge rate (5% point) in downlink of FD system. When $P_d = 40$ dBm and $R = 200$, the mean rate in HD downlink is 2.02 bps/Hz, and the same is 4.04 for FD with $\epsilon = 0.2$ and 4.00 for $\epsilon = 0.8$, i.e., in downlink FD almost doubles the average rate. Similar observations can be seen in literature [3].

Even though FD promises notable improvement in the achievable rate for the downlink users, without diminishing their coverage probabilities, the uplink users are severely affected by the strong same channel downlink interfering signals. This can be seen in rate cdf (Fig. 5) and in rate profile, Table I. The same observation can be seen in [14], but without substantial mathematics to support the result. Our results are in line with their observations and provide a mathematical framework to analyze the FD networks with uplink power control.
TABLE I: Mean and Cell edge rate for different configurations of \((\epsilon, P_d)\) at \(P_0 = -64\) dBm and inter-BS distance 400 m.

| \((\epsilon, P_d)\) | Uplink | Downlink |
|---------------------|--------|----------|
|                     | Mean Rate | Mean Rate | Mean Rate | Mean Rate |
| 0.2, 40             | 2.8 × 10^{-11} | 2.8 × 10^{-10} | 0.0125 | 4.04 |
| 0.8, 40             | 4.1 × 10^{-10} | 3.6 × 10^{-9} | 0.0120 | 4.90 |
| 0.2, 23             | 1.4 × 10^{-9} | 3.3 × 10^{-8} | 0.0125 | 4.04 |
| 0.8, 23             | 2.0 × 10^{-8} | 1.5 × 10^{-7} | 0.0099 | 3.82 |
| HD                 | -        | -         | 0.0062 | 2.02 |

V. CONCLUSION

In this work, we have analyzed the downlink and uplink performance limits of FD network. We modeled the network using Matern Cluster Process, where users are distributed around the parent PPP of BSs. This makes the user-base station interactions not completely independent, while majority of literature uses independent point processes for BSs and UEs. We derived approximations for the coverage probabilities both in uplink and downlink. We have provided closed form expression for average inverse SINR, which provides many insights about the trade of between different system parameters. Also, we provided mean rate and rate CDFs for uplink and downlink users. The results show that, the uplink user coverage is severely affected by the larger interference from the downlink, while the downlink performance is not affected by the co-channel uplink signals. The analysis recommends small cells with low power BSs for FD cellular system. This model can be extended to analyze effectiveness of FD in cloud radio and MIMO.

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Fig. 6: Illustration of distances between UEs and BS \(\phi\). The downlink UE, \(u_0\) is at a distance \(r_d\) from \(\phi\) and the uplink interfering UE, \(u_i\) is \(r_i\) from \(\phi\).

The power control given in equation (3) can be restated in linear scale as, \(P_u^{\text{lin}}(r_u) = \min (P_{max}, P_{0}r_u^{\alpha})\). Assume the
user adjust its uplink power based on the long term average RSRP, then

\[ P_c^u(r_u) = \begin{cases} P_{0}^{\text{up}} & 0 \leq r_u \leq \eta \\ P_{\text{max}}^u & \eta < r_u \leq R_c \end{cases}, \]  

where \( \eta = \left( \frac{P_{\text{max}}^u}{P_0} \right)^{1/\alpha} \). The user power is upper limited by \( P_{\text{max}}^u \) when distance \( r_u > \eta \). The expectation in (10) has to be taken along \( r_d, P_u, \gamma \) in which only the term \( T_0 \) depends on \( r_u \) and \( \gamma \). By substituting (11) in \( T_0 \), we have

\[ \mathbb{E}[T_0] = \mathbb{E}[\gamma] \int_{r_u=0}^{R_c} \frac{f_R(r_u) r_u}{1 + P_0 \rho_{\text{up}} T r_d^\alpha / P_d d^\alpha} \prod_{r_u=r_d}^{R^\alpha} + \int_{r_u=\eta}^{R_c} \frac{f_R(r_u) r_u}{1 + P_{\text{max}}^u T r_d^\alpha / P_d d^\alpha} \prod_{r_u=r_d}^{R^\alpha}, \]  

where (a) is by noting that for moderate and dense networks, the power saturating distance, \( \eta > R_c \), and hence second term is neglected.

Now substituting (12) in (10) and taking expectation over \( \gamma \), we have,

\[ P_c^u(T) = \mathbb{E}[\gamma] \mathbb{E}[\text{SINR}] \left\{ r_d, r_u, \gamma > T \right\}, \]

\[ = \exp \left( -\frac{T r_d^\alpha}{P_d} \left( 2\pi^2 \lambda T \frac{c}{T} \frac{r_d^\alpha}{\alpha} \left( \frac{r_d^\alpha}{\alpha} \right)^{\alpha-2} \right) -\frac{1}{1 + P_0 r_{\text{up}}^\alpha T r_d^\alpha / P_d (r_d^2 + r_u^2 - 2 r_d r_u \cos \gamma)^\alpha} \right), \]

\[ \approx -\exp \left( -\frac{T r_d^\alpha}{P_d} \left( 2\pi^2 \lambda T \frac{c}{T} \frac{r_d^\alpha}{\alpha} \left( \frac{r_d^\alpha}{\alpha} \right)^{\alpha-2} \right) \right) \times \left( \frac{\sqrt{2} P_d r_d \tan^{-1} \left( \frac{\sqrt{2} \pi \alpha P_0 T}{r_d} \left( \frac{r_d}{r_u} \right)^{\alpha-2} \right)}{\pi \alpha P_0 T} \right)^\alpha, \]

where (a) follows by approximating \( P_{0}^{\text{up}} T r_d^\alpha / P_d (r_d^2 + r_u^2 - 2 r_d r_u \cos \gamma)^\alpha \) by power series expansion up to to second order and

\[ \zeta = -\frac{\alpha P_0 T r_d r_u}{(r_d - r_u)^2 \left( P_d \frac{(r_d - r_u)^2 r_d}{r_d} \right)^{\alpha/2} + P_0 T} . \]  

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