CCD PHOTOMETRY OF THE CLASSIC SECOND-PARAMETER GLOBULAR CLUSTERS M3 AND M13

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Abstract

We present high-precision $V$, $B-V$ color-magnitude diagrams (CMDs) for the classic second-parameter globular clusters M3 and M13 from wide-field, deep CCD photometry. The data for the two clusters were obtained during the same photometric nights with the same instrument, allowing us to determine accurate relative ages. Based on a differential comparison of the CMDs using the $A(B-V)$ method, an age difference of $1.7 \pm 0.7$ Gyr is obtained between these two clusters. We compare this result with our updated horizontal-branch (HB) population models, which confirm that the observed age difference can produce the difference in HB morphology between the clusters. This provides further evidence that age is the dominant second parameter that influences HB morphology.

Key words: color-magnitude diagrams — globular clusters: individual (M3, M13) — stars: evolution — stars: horizontal-branch

On-line material: machine-readable tables

1. INTRODUCTION

Since the pioneering work by Sandage & Wallerstein (1960), van den Bergh (1967), and Sandage & Wildey (1967), it has been known that, in addition to [Fe/H] (the “first parameter”), there must be a second parameter controlling the morphology of the horizontal-branch (HB) in Galactic globular clusters (GGCs). Because of its important implications for the formation chronology of the Galaxy (Searle & Zinn 1978; Lee, Demarque, & Zinn 1994, hereafter LDZ94), determining the nature of this second parameter has been one of the key questions during the last 40 years. The high-precision CCD photometry of GCs in recent years (Bolte 1989; Buonanno et al. 1990, 1994; Green & Norris 1990; Chaboyer, Demarque, & Sarajedini 1996; Sarajedini, Chaboyer, & Demarque 1997; Sarajedini 1997; Stetson et al. 1999; Lee & Carney 1999) and the recent advances in HB modeling (LDZ94) have suggested that age is most likely the cause of the observed variations in HB morphology among clusters of the same [Fe/H] and as a function of Galactocentric distance. In particular, LDZ94 have concluded that age is the most natural candidate for the global second parameter because other candidates, such as helium abundance, CNO abundance, and core rotation, can be ruled out from the observational evidence, while supporting evidence does exist for the age hypothesis.

While the above view is generally accepted, some critics have argued that the relative age differences inferred from the main-sequence turnoffs (MSTOs) are sometimes too small to explain the differences in HB morphology between second-parameter clusters (VandenBerg, Bolte, & Stetson 1990, hereafter VBS90; Stetson, VandenBerg, & Bolte 1996; Catelan & de Freitas Pacheco 1995; Johnson & Bolte 1998, hereafter JB98). Among them, the most problematic case is M3 and M13, which is one of the most famous second-parameter pairs, along with NGC 288 and 362, where the age difference is well established. These studies argue that the age difference between M3 and M13 is appreciably smaller than the one suggested by HB population models and therefore is not sufficient to explain the observed difference in HB morphologies. The difference in HB morphology between M3 and M13, however, is not as dramatic as that of NGC 362 and 288 because M3, the cluster with the redder HB, possesses an intermediate HB type with both blue and red HB stars. Therefore, the predicted age difference is smaller in the M3/M13 pair (see Lee et al. 1999 and below) as compared with NGC 288/NGC 362 and is sometimes compatible with the observational uncertainties (1–2 Gyr) of the available CCD photometry.

Consequently, in order to test the age hypothesis of the second-parameter effect between M3 and M13, high-quality CCD data should be taken that is good enough to discriminate a small age difference. For relative age dating of GGCs, it is essential to obtain color-magnitude diagrams (CMDs) that are reliable, at least, in a differential sense. The inhomogeneity of data sets and analysis methods in the various studies has been a major limitation of relative age dating of GCs. Until recently, many studies combined CMDs obtained from different instruments with different calibrations. These results have often been significantly hampered by this inhomogeneity and thus cannot be considered to be conclusive (see review in Stetson et al. 1996; Rosenberg et al. 1999). Indeed, even for the same GC, the fiducials that have been derived in different investigations...
and six short (20 and well-defined MS in each cluster. Six long (200
southwest of the cluster center in order to produce a deep
pixel scale of We observed two partially overlapping0
Filters. The field of view of this CCD is roughly with a9
available CCD CMDs for M3 and M13 are provided by
JB98. Their V1 photometry was obtained with the same
telescope during the same observing run, and the resulting
CMDs for the two clusters narrowly define the cluster
sequences from the red giant branch (RGB) to ~3 mag below
the MSTD. Using the differential age dating
described by VBS90, they concluded that the clusters are
unlikely to differ in age by the amount required to explain
the difference in their HB morphology purely as an age
effect. They proposed that the observations could be
explained better with a difference in the main-sequence
(MS) helium abundance (with M13 having the larger helium
abundance), while this contradicts the observational
evidence of similar helium abundances between M3 and M13
to within the errors (Y = 0.204±0.011 vs. 0.180±0.023; Sand-
quist 2000). On the other hand, Stetson (1998) obtained
homogeneous BI photometry for M3 and M13 using the
Canada-France-Hawaii Telescope and proposed that M3
appears to be about 12% older than M3, although he
argued that age is not the only appropriate difference
between M3 and M13, due to the bluer color of MSTD of M13
than that of M3 and different slopes of subgiant
branches (SGBs) in these two clusters. From his preliminary
instrumental B, B − R CMD for M3 and M13, Sarajedini
(1999) also presented a result that M3 appears to be older
than M3 by ~2 Gyr, and this age difference is consistent
with that implied by HB models. Consequently, although
there are at least three independent studies on the second-
parameter problem for M3 and M13 based on homoge-
neous data sets, the situation is still rather controversial.

The purpose of this paper is to present our new high-
quality homogeneous BV CCD photometry of M3 and
M13. Our new photometry was obtained on the same
photometric nights with the same instrument producing a
deeper and more extensive data set as compared with that
of JB98. This allows us to carry out a relative age dating for
these clusters that is more precise than previous studies.
These data are then compared with our new updated HB
models to test the age hypothesis of the second-parameter
effect between M3 and M13.

2. OBSERVATIONS AND DATA REDUCTIONS

All the observations were made using the Michigan-
Dartmouth-MIT (MDM) Observatory 2.4 m telescope
during two nights of an observing run in 2000 March.
Images were obtained with the thinned, back-illuminated,
SITe 2048 “Echelle” CCD and the standard Johnson B and
V filters. The field of view of this CCD is roughly 9.5 with a
pixel scale of 0.28. We observed two partially overlapping
fields in M3 and M13, respectively. One field was about 12'
west of the cluster center in order to produce a deep
and well-defined MS in each cluster. Six long (200–800 s)
and six short (20–60 s) exposures were obtained at this
location. In order to ensure a large sample of SGB and
RGB stars, a second field off the cluster center by 9’ west
was observed with three medium (80–240 s) and three short
(20–60 s) exposures. The nights were dark and of photo-
metric quality, and the seeing was also good—in the range
1’0–1’2. Figures 1 and 2 show the observed cluster fields for
M3 and M13, respectively.

The raw data frames were calibrated using twilight and
dawn sky flats and zero-level exposures. Calibration frames
were made by combining several individual exposures. All
exposure times were sufficiently long that the center-to-
corner shutter timing error was negligible. These pro-
cedures produced object frames with the sky flat to better
than 1% in all filters.

Photometry of the M3 and M13 stars was accomplished
using DAOPHOT II and ALLSTAR (Stetson 1987, 1995).
For each frame, a Moffat point-spread function (PSF),
varying quadratically with radial position, was constructed
using 50–100 bright, isolated, and unsaturated stars. The
PSF was improved iteratively by subtracting faint nearby
companions of the PSF stars. We calculated aperture cor-
rections using the program DAOGROW (Stetson 1990).
Using the aperture photometry data, growth curves were
constructed for each frame, in order to extrapolate from the
flux measurements over a circular area of finite radius to the
total flux observable for the star. The final aperture correc-
tion was made by adjusting the ALLSTAR magnitude of
each star by the weighted mean of the difference between
the total aperture magnitude returned by DAOGROW and
the profile-fitting ALLSTAR magnitude for selected stars
(e.g., PSF stars). The typical rms deviation for the aperture
correction for all frames corresponds to 0.013 mag, which
introduces a modest uncertainty to the zero point of the
calibration equation. After the aperture correction, DAO-
MATCH and DAOMASTER (Stetson 1992) were then
used to match stars of all frames covering the same field and
to derive the average instrumental magnitude on the same
photometric scale. For each frame, the magnitude offset
with respect to each master frame in B and V was calcu-
lated, and photometry for all frames of the same field was
transformed to a common instrumental system. In this way,
robust, intensity-weighted mean instrumental magnitudes
and rms scatters of magnitudes were obtained for all
matched stars. Last, the mean instrumental B and V magni-
tudes were matched to form B − V color.

On each night, a number of standard stars from the list of
Landolt (1992) were observed. Sixty-three standard stars
were observed in B and V, covering a color range of −0.3 to
2.2 for B − V and an air-mass range of 1.16–1.46. All
standard-star exposure times were long (greater than 7 s)
enough so that the systematic error resulting from shutter
timing (approximately a few tens of milliseconds) is insig-
nificant (less than 1%). DAOGROW was also performed to
measure the total aperture magnitude of the standard stars.
The aperture magnitudes and the known standard system
magnitudes were then used to derive coefficients of the
transformation equations. The atmospheric extinction coef-
ficients in each color have been determined by the same
standard stars at different air masses. The final transfor-
mation equations were obtained by a linear least-squares fit.
They are

\[ B - V = 1.130(b - v) + 0.280 , \]
\[ V - v = -0.037(B - V) - 1.118 , \]
Fig. 1.—Observed cluster regions in the (a) western and (b) southwestern field of M3. North is to the left, and east is down.
Fig. 2.—Same as Fig. 1, but for M13
where $B-V$ and $V$ are the color indices and visual magnitude in the standard $BV$ system, and $(b-v)_o$ and $v_o$ refer to instrumental ones corrected for extinction. No other trends in the residuals were noticeable, and therefore no additional terms in the transformation equations appear to be necessary. The calibration equations relate observed to standard values for $V$ and $B-V$ with rms residuals of 0.006 and 0.005 mag, respectively, as shown in Figure 3.

From comparison of the stars belonging to overlapping area of two adjacent fields, we confirmed that there are no systematic differences in either color or magnitude between the cluster fields. The mean offsets of the photometric zero point are never greater than 0.01 mag. We compared our CCD photometry with data taken from other studies for the stars in common. Figures 4a and 5a show comparison of our $V$ magnitudes with those of JB98 for M3 and M13 stars, respectively. The mean difference in the sense our measurements minus others is $0.002 \pm 0.028$ and $0.001 \pm 0.024$ for M3 and M13, respectively, where the uncertainty is the standard deviation of the mean. Figure 4b shows comparisons with photometry of Stetson & Harris (1988) for M3 (their secondary standard field). The mean differences are $0.026 \pm 0.019$ and $0.010 \pm 0.022$ in $V$ and $B-V$, respectively. We have also made a comparison with the photometry of Richer & Fahlman (1986) for M13, as shown in
Figure 5b. The mean differences are $0.019 \pm 0.019$ and $0.009 \pm 0.014$ in $V$ and $B-V$, respectively. We conclude that there are no significant systematic zero-point differences between our photometry and others.

3. COLOR-MAGNITUDE DIAGRAMS

For the final photometric list, we selected stars with a detection on at least three frames in each band, error in the $B$ and $V$ magnitudes of less than $0.05$ mag, and a mean value of $\chi$ (quality of the PSF fit to the stellar image returned by DAOPHOT II) less than 2. Figure 6 shows our $V$, $B-V$ CMDs for the M3 and M13, respectively, representing stars used in our analysis. All the long-exposure data for the southwestern field of M3 and M13 were included except saturated, bright RGB stars that were recovered from the short-exposure data. However, in the case of the western field data, the following selection criteria were adopted to have the best definitions of the CMD branches, since crowding worsens the quality of the photometry in the inner cluster region. We included only stars with $V < 18$ mag from the medium-exposure data of M3. For the sparsely populated bright RGB and SGB of M13, we added medium- and short-exposure photometry for the relatively bright stars ($V < 18.7, B-V > 0.3$) in the outer part of the CCD frame ($Y > 700$ pixel) to help define the sequences more accurately. In order to represent the HB blue tail of M13, which extends to about the MSTO magnitude, we include all medium-exposure data with $B-V < 0.3$. Note that the CMD of M13 is being used here to understand the morphology of the CMD and does not accurately represent the population ratios of stars in different evolutionary stages. The color-magnitude data for M3 and M13 are tabulated in Tables 1 and 2, respectively.

Both our M3 and M13 CMDs extend to about $3.5$ mag fainter than the MSTO. All the cluster sequences that we are particularly interested in (the lower RGB, SGB, MS, and MSTO) are significantly well defined, allowing us to derive the cluster parameters accurately. Our CMDs show the similar quality and overall morphology with those of previous investigators (e.g., JB98) but include more stars in all the cluster sequences. Although there is a paucity of stars in the brightest RGB and asymptotic giant branch regions

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**TABLE 1**

| ID  | $x$    | $y$    | $V$      | $\sigma_V$ | $B-V$  | $\sigma_{B-V}$ |
|-----|--------|--------|----------|------------|--------|----------------|
| 1   | 1467.63| 1106.58| 11.406   | 0.005      | 0.702  | 0.007          |
| 2   | 644.02 | 26.79  | 12.853   | 0.004      | 1.426  | 0.006          |
| 3   | 241.15 | 354.39 | 14.047   | 0.003      | 0.664  | 0.005          |
| 4   | 285.23 | 1191.12| 14.165   | 0.003      | 1.024  | 0.005          |
| 5   | 1559.35| 1816.76| 14.840   | 0.003      | 0.804  | 0.005          |
| 6   | 400.20 | 1052.79| 15.180   | 0.003      | 1.119  | 0.005          |
| 7   | 286.05 | 484.68 | 15.349   | 0.003      | 0.851  | 0.005          |
| 8   | 313.97 | 470.24 | 15.392   | 0.003      | 0.389  | 0.008          |
| 9   | 99.29  | 227.80 | 15.415   | 0.005      | 0.550  | 0.007          |
| 10  | 835.15 | 1745.64| 15.540   | 0.006      | 0.820  | 0.008          |

Note.—Table 1 is available in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.

**TABLE 2**

| ID  | $x$    | $y$    | $V$      | $\sigma_V$ | $B-V$  | $\sigma_{B-V}$ |
|-----|--------|--------|----------|------------|--------|----------------|
| 1   | 466.76 | 880.00 | 14.242   | 0.005      | 0.673  | 0.007          |
| 2   | 322.68 | 1512.17| 14.308   | 0.004      | 0.889  | 0.006          |
| 3   | 144.05 | 115.67 | 14.503   | 0.004      | 0.844  | 0.006          |
| 4   | 1373.81| 235.60 | 14.721   | 0.004      | 0.597  | 0.006          |
| 5   | 54.15  | 183.29 | 14.736   | 0.004      | 0.824  | 0.006          |
| 6   | 453.66 | 688.43 | 14.889   | 0.004      | 0.581  | 0.006          |
| 7   | 1697.89| 520.95 | 15.030   | 0.004      | 1.379  | 0.006          |
| 8   | 400.83 | 97.12  | 15.045   | 0.004      | 0.788  | 0.006          |
| 9   | 893.09 | 604.51 | 15.063   | 0.004      | 0.073  | 0.006          |
| 10  | 98.33  | 687.16 | 15.076   | 0.004      | 0.076  | 0.006          |

Note.—Table 2 is available in its entirety in the electronic edition of the Astronomical Journal. A portion is shown here for guidance regarding its form and content.
in both cluster CMDs, this does not hamper our differential age analysis, which mainly uses the lower RGB stars.

M13 has an exclusively blue HB with a long blue tail extending well below the level of the MSTO. Moreover, the HB appears to have two gaps at extending well below the level of the MSTO. Moreover, the age analysis, which mainly uses the lower RGB stars, in both cluster CMDs, this does not hamper our differential age dating techniques, was sampled with a smaller magnitude bin of 0.1 mag. Because of the very small number of potential members brighter than $V \sim 14.5$ and $V \sim 13.5$ for M3 and M13, respectively, the fiducial line of bright RGB sequence is determined by eye. Our unsmoothed M3 and M13 fiducial sequences are listed in Tables 3 and 4, respectively. We found no significant difference between RGB fiducials from the western and southwestern fields. There is some disagreement of our fiducial for M13 with that of Paltrinieri et al. (1998). However, we found that our fiducial has good agreement with those obtained from other studies, such as Richer & Fahamian (1986) and Yim et al. (2000). While these two fiducials agree well with ours in the region of MS and SGB, Paltrinieri et al.'s fiducial shows large deviation in this region.

4. RELATIVE AGE DATING: $\Delta(B-V)$ METHOD

In contrast to absolute age dating techniques, relative age determinations are less affected by uncertainties in stellar evolution theory because most of the effects of theoretical uncertainties are removed in a relative comparison. Furthermore, differential comparisons between GCs can reduce the effects of observational uncertainties involved in the absolute age determination (see Stetson et al. 1996; Sarajedini et al. 1997). For example, VBS90 and Sarajedini & Demarque (1990) independently showed that, for clusters having similar chemical compositions, the color difference between the MSTO and the lower giant branch (the SGB) is an excellent age diagnostic, in the sense that a larger color difference denotes a younger age; one major advantage of this diagnostic is that it is independent of distance, reddening, and photometric zero-point/color calibrations.

The technique outlined by VBS90 involves normalizing cluster principal sequences by the TO color, $(B-V)_{TO}$, and the visual magnitude, $V_{+0.05}$, of the MS at the point 0.05 mag redder than the TO color. Once this shifting has been accomplished, older clusters will have bluer giant branches relative to younger clusters and vice versa.

The $(B-V)_{TO}$ and $V_{+0.05}$ were determined by fitting a parabola to the stars in the region near the MSTO and upper MS point 0.05 mag redder than the TO, respectively. We derived $(B-V)_{TO} = 0.440 \pm 0.005$ and $0.435 \pm 0.006$ for M3 and M13, respectively, where the uncertainty is the observed scatter of the stars in the TO region. The $V_{+0.05}$ was also obtained to be $19.91 \pm 0.06$ and $19.49 \pm 0.06$ for M3 and M13, respectively, where the error is also the observed scatter of the stars within the parabola-sided boxes including $V_{+0.05}$.

In Figure 7, we show our color-magnitude data and fiducial sequences for M3 and M13 using the registration prescription specified by VBS90. In Figure 7b, also shown are Yale-Yale (Y2) isochrones (Yi et al. 2001) having $[\text{Fe/H}] = -1.66^3$ (Zinn & West 1984) and $[\alpha/\text{Fe}] = 0.3$ and ages ranging from 10 to 14 Gyr. The M3 and M13 sequences and the isochrones have been shifted horizontally to match at $(B-V)_{TO}$ and vertically to agree with one another at $V_{+0.05}$. As shown in the figures, the RGB sequences of M3 and M13 are well separated from each other, indicating a real age difference.

It is important to note that in Figure 7 the separation in the SGB region between M3 and M13 appears to be inconsistent with the original registration scheme of VBS90, where the SGBs of the two clusters should be coincident.

3 Our models reveal that the $\Delta age/\Delta(B-V)$ of RGBs will not be affected by choice of $[\text{Fe/H}]$ from $-1.9$ to $-1.4$. 

| $V$ | $B-V$ | $V$ | $B-V$ |
|-----|-------|-----|-------|
| 13.75 | 0.943 | 18.68 | 0.436 |
| 14.25 | 0.876 | 18.90 | 0.444 |
| 14.76 | 0.821 | 19.10 | 0.457 |
| 15.25 | 0.775 | 19.31 | 0.469 |
| 15.76 | 0.734 | 19.51 | 0.485 |
| 16.24 | 0.701 | 19.69 | 0.508 |
| 16.77 | 0.668 | 19.90 | 0.531 |
| 17.13 | 0.654 | 20.10 | 0.564 |
| 17.29 | 0.639 | 20.31 | 0.603 |
| 17.49 | 0.631 | 20.50 | 0.637 |
| 17.70 | 0.613 | 20.69 | 0.678 |
| 17.90 | 0.532 | 20.90 | 0.712 |
| 18.11 | 0.464 | 21.10 | 0.763 |
| 18.29 | 0.443 | 21.30 | 0.800 |
| 18.51 | 0.436 | 21.49 | 0.837 |
As shown in the isochrones of Figure 7b, this separation is partly explained as a real feature if the absolute ages of the two clusters are as young as 10–12 Gyr. This feature was also found in the $VI$ CMDs reported by JB98. However, their CMDs show a larger separation between the SGBs of the two clusters than ours, relative to the separation of the RGBs caused by an age difference (see Figs. 8 and 9 of JB98). VandenBerg (2000) found that this is due to the failure of JB98 to match their data in the TO region and explained that matching the CMDs at $V_{+0.05}$ will not produce a superposition of the cluster TOs if the MS loci have different slopes.

In Figure 7, we also see that the TO of M13 is slightly brighter than that of M3 after the cluster fiducials have been registered following the VBS90 method. Following the recommended procedures of VandenBerg (2000, see his footnote 3), we therefore shifted our M13 fiducial vertically until the differences between the two clusters were minimized at the two points that were 0.05 mag redder than the TO (i.e., both above and below the TO). The M13 sequence has been shifted fainter and bluer by 0.06 mag and 0.001 mag, respectively. In Figure 8, we present our M3 and M13 fiducial sequences and individual color-magnitude data, after the TOs of both clusters are made to coincide with each other, along with the same isochrones shown in Figure 7b. It should be noted that there is still a sufficiently large
separation between the M3 and M13 RGBs, implying an age difference.

The relative age is measured by comparing the colors of the RGB, where the color separation is largely independent of magnitude. In the range of $-4 < V - V_{PG} < -2$, we fitted a parabola to the M3 RGB sequence and calculated the mean offset between this parabola and the shifted RGB stars of M13. In order to check the reliability of this estimate, we repeated this procedure for the case of the M13 RGB sequence and the M3 stars. We find the mean value of the color difference between the RGBs, in the sense of M3 minus M13, to be $\delta(B - V)_{RGB} = 0.029 \pm 0.008$. The error is the standard error of the mean color difference. We estimated $\sigma(B - V) = 0.008$ in the separation of the RGBs due to the error of the MTO color. Considering the additional error (0.08 mag) from the magnitude of the MTO, which corresponds to the $\sigma(B - V) = 0.004$ in the RGBs, we estimated $\sigma(B - V) = 0.009$ because of the combined error in the magnitude and color of MTO. When this error is combined with that of the mean color difference between the RGBs, the total uncertainty of the color difference between the RGBs of M3 and M13 corresponds to 0.012 mag.

We calculate the offset in RGB color associated with an offset in presumed age by noting that $\Delta \log t_o = -2.19 \log(B - V)$. This corresponds to about 0.59 Gyr per 0.01 mag, if we adopt an age of 12 Gyr for M13. It should be noted that the color difference between the RGBs is not uniform, but depends on the absolute age (see the spacing of isochrones presented in Figs. 7 and 8), in the sense that using younger isochrones would reduce the inferred age differences. Adopting the absolute age of M13 to be about 12 Gyr (Chaboyer et al. 1998; Yi et al. 2001), the color difference between the two clusters corresponds to a relative age difference of $1.7 \pm 0.7$ Gyr, in the sense that M13 is older.  

5. COMPARISON WITH SYNTHETIC HB MODELS

There are several recent developments that affect the relative age dating from HB morphology. We have included them in our most updated version of the HB population models (see Lee et al. 1999; Lee & Yoon 2001). First of all, there is now reason to believe that the absolute ages of the oldest GGCs is reduced to about 12 Gyr, as suggested by the Hipparcos distance calibration and other improvements in stellar models (Reid 1998; Gratton et al. 1997; Chaboyer et al. 1998; Grundahl, VandenBerg, & Andersen 1998; Yi et al. 2001). This has a strong impact on the relative age estimation from HB morphology because the variation of the HB mass is more sensitive to age at younger ages because of the nonlinear relationship between RGB tip mass ($M_{TRG}$) and age (see LDZ94). Second, in the new population models, we have used new HB tracks with improved input physics (Yi, Demarque, & Kim 1997) and corresponding $Y^2$ isochrones (Yi et al. 2001). Third, it is now well established that the $\alpha$-elements are enhanced in halo populations. Specifically, we adopted $[\alpha/Fe] = 0.3$ for clusters with $[Fe/H] < -1.0$, and thereafter we assume that it steadily declines to 0.0 at solar metallicity (see, e.g., Wheeler, Sneden, & Truran 1989). In practice, the treatment suggested by Salaris, Chieffi, & Straniero (1993) was used to simulate the effect of $\alpha$-element enhancement. Finally, empirical mass-loss law of Reimers (1975) suggests more mass loss at larger ages. The result of this effect was also presented in LDZ94, but unfortunately their most widely used diagram (their Fig. 7) is the one based on fixed mass loss.

We found that all the above effects make the HB morphology more sensitive to age. Figure 9 illustrates the HB morphology versus $[Fe/H]$ relations for the GGCs, along with the theoretical HB models that were produced by our synthetic HB models. As shown in Figure 9b, now the required age difference is significantly reduced as compared with LDZ94. Only 1.1 Gyr of age difference, rather than 2 Gyr, is enough to explain the systematic shift of the HB morphology between the inner and outer halo clusters. To within the observational uncertainties, age differences of about 1–2 Gyr are now enough to explain the observed differences in HB morphology between M3 and M13 (or M2) and between the outer halo clusters (Pal 3, Pal 4, Pal 14, and Eridanus) and M3. These values are consistent with the observations presented in this paper and also with the recent relative age datings both from HST and high-quality, ground-based data (Stetson et al. 1999; Lee & Carney 1999).

In Figure 10, the observed CMDs of M3 and M13 are compared with our new population models, which include the scatter expected from the random errors in magnitude and in color as estimated by our photometry. For the two clusters, we adopted the same metallicity ($[Fe/H] = -1.66$) on the Zinn & West (1984) scale, mass dispersion ($\sigma_M = 0.025 M_\odot$) on the HB, MS helium abundance ($Y_{MS} = 0.23$), and $\alpha$-element enhancement ($[\alpha/Fe] = 0.3$), but applied an age difference of 1.7 Gyr between the two clusters as estimated from our relative age dating (see § 4). As shown in the figure, there is a reasonable match between the synthetic HB models and the observations. However, the observations indicate that M13 has a long blue tail on the HB, while the standard HB model (with $\sigma_M = 0.02–0.03 M_\odot$) fails to reproduce this detail.

The wide color range of the M13 HB would be reproduced by using a larger value for $\sigma_M$, roughly twice as large as M3, together with a slightly larger age difference ($\Delta t \sim 2.4$ Gyr) between the two clusters (see Lee & Yoon 2001). Certainly, this uncertainty has some impact on the age difference that one infers from HB morphology. However, the magnitude of this age uncertainty ($\sim 0.7$ Gyr) is still compatible with the errors ($\sim 0.7$ Gyr) of the relative age difference among the clusters.
Fig. 9.—HB morphology vs. [Fe/H] relations for GGCs with theoretical isochrones that were produced by synthetic HB models (cluster data from Table 1 of LDZ94). Our new HB models with the effects of recent developments \( (b) \) are more sensitive to age compared with our earlier models \( (a) \). Here \( \Delta t = 0 \) corresponds to the mean age of the inner halo \( (R_{\text{GC}} < 8 \text{kpc}) \) clusters, and the relative ages are in gigayears. Note that with our new HB models age differences of about 1–2 Gyr are now enough to explain the observed differences in HB morphology between M3 and M13.

6. DISCUSSION

Although the presence of a blue tail on the HB has only a mild impact on the relative age dating from HB morphology, it is still important to understand the origin of this effect in order to use the HB as a more reliable age indicator. The blue tail phenomenon is widely considered to be a result of local effects, such as enhanced mass loss in high-density environments. Buonanno et al. (1997) examined the role of stellar density in the morphology of the HB and suggested that clusters with higher central densities are more likely to populate the bluest extremes of the HB (see also Fusi Pecci et al. 1993). However, the correlation between stellar density and HB morphology is rather weak with large scatter and/or limited to a small fraction of GCs within an intermediate metallicity range (LDZ94; Sarajedini et al. 1997). Note that the central densities (\( \log \rho = 3.51 \) vs. 3.33; Harris 1996) and concentration (\( c = \log r_c/r_t = 1.84 \) vs. 1.51; Harris 1996) parameters for M3 and M13 are very similar despite their apparent difference in \( \sigma_M \) on the HB. Furthermore, as suggested by LDZ94, there is also no clear evidence that the variation of HB morphology with Galactocentric distance \( (R_{\text{GC}}) \) is related to central densities of GCs.

It is also suggested that some noncanonical effects in the stellar interior, such as rapid rotation and deep mixing, would make a star both bluer and brighter on the HB and are thus related to the presence of blue tails (Mengel & Gross 1976; Kraft et al. 1993, 1997; Kraft 1994; Peterson, Rood, & Crocker 1995; Langer & Hoffmann 1995; Sweigart 1997a, 1997b; Cavallo & Nagar 2000). However, the predicted increase in the rotation velocity with effective temperature along the HB has not been confirmed from the recent high-resolution spectroscopy of blue HB stars of M13 (Behr et al. 2000). On the other hand, Behr et al. (1999) reported that in the hotter HB stars of M13 helium is underabundant, while iron and other metals are enhanced. It is suggested that these abundance anomalies are most likely due to the diffusion effects in the radiative atmospheres. Similarly, a number of interesting phenomena have recently been reported to occur in blue HB stars around a temperature of 11,000 K; these include a gap (i.e., G1 gap) in the HB distribution (Ferraro et al. 1998; Piotto et al. 1999; Caloi 1999), a jump in the Strömgren \( u \) magnitudes and the onset of radiative levitation (Grundahl et al. 1998, 1999), and a shift to lower surface gravities (Moehler et al. 1999, 2000). All these phenomena suggest that the blue tail feature may be related to the disappearance of surface convection and the formation of a radiative stellar atmosphere followed by radiative levitation of heavy elements and helium diffusion for stars hotter than about 11,000 K (Sweigart 2001; Moehler et al. 1999). From the theoretical point of view, it is probably possible that the levitation of heavy elements along with helium diffusion would push the blue HB stars to even hotter \( T_{\text{eff}} \) values on the HR diagram, creating the blue tail. If this is confirmed by detailed modeling, the blue tail phenomenon may not be considered as adding noise to the second-parameter effect, since it is then rather a general feature of extremely blue HB clusters. In this case, a more reasonable relative age would be estimated from the HB morphology by ignoring the blue tail, since radiative levitation and diffusion effects are not included in our standard HB models (i.e., Fig. 10).

7. SUMMARY

We present new high-quality \( V, B-V \) CMDs for the GGCs M3 and M13, constructed from wide-field, deep CCD photometry obtained during the same nights with the
same instrument. From our homogeneous data set, we draw the following conclusions: (1) Based on a careful differential comparison of the CMDs using the \(\Delta(B-V)\) method, we confirm a significant difference between these two clusters, indicating an age difference of \(1.7 \pm 0.7\) Gyr in the sense that M13 is older than M3. (2) We present updated HB models, which suggest that HB morphology is more sensitive to cluster age compared with our previous models. From a comparison of observations with the new HB models, we find that the observed age difference can reproduce the difference in HB morphology between the clusters. This provides further evidence that cluster age is the dominant second parameter that influences HB morphology, which in turn suggests that HB morphology is a reliable age indicator in most Population II stellar systems. (3) While the physical origin of the blue tail phenomenon is still uncertain, there is now a growing body of evidence that suggests this is an ubiquitous characteristic of clusters with extremely blue HB stars hotter than 11,000 K. If true, the presence of blue tail would have less impact on relative age dating based on HB morphology.

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Fig. 10.—Comparison of observational and synthetic CMDs for M3 and M13, assuming that M13 is older by 1.7 Gyr. Crosses in the model CMDs are RR Lyrae variables.
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