Ultrarelativistic electron beam polarization in single-shot interaction with an ultraintense laser pulse

Yan-Fei Li, Rashid Shaisultanov, Karen Z. Hatsagortsyan, Feng Wan, Christoph H. Keitel, and Jian-Xing Li

1School of Science, Xi’an Jiaotong University, Xi’an 710049, China
2Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany
(Dated: December 19, 2018)

Spin-polarization of an ultrarelativistic electron beam head-on colliding with an ultraintense laser pulse is investigated in the quantum radiation-reaction regime. We develop a Monte-Carlo method to model electron radiative spin effects in arbitrary electromagnetic fields by employing spin-resolved radiation probabilities in the local constant field approximation. Due to spin-dependent radiation reaction, the applied elliptically polarized laser pulse polarizes the initially unpolarized electron beam and splits it along the propagation direction into two oppositely transversely polarized parts with a splitting angle of about tens of milliradians. Thus, a dense electron beam with above 70% polarization can be generated in tens of femtoseconds. The proposed method demonstrates a way for relativistic electron beam polarization with currently achievable laser facilities.

Introduction. Spin-polarized electron beams have been extensively employed to investigate matter properties, atomic and molecular structures [1–3]. In high-energy physics, relativistic polarized electron beams can be used to probe the nuclear structure [4, 5], generate polarized photons [6, 7] and positrons [8], study parity violation in Möller scattering [9] and new physics beyond the Standard Model [10]. There are many methods to generate polarized electron beams at low energies [11]. However, for relativistic electron beams, there are mainly two methods [11]. In the first method mostly used in the Stanford Linear Accelerator, the polarized electrons are first extracted from a photocathode (illuminated by a circularly polarized light) [12, 13] and then, accelerated by the linear accelerator (alternatively one may use polarized electrons from spin filters [14] or beam splitters [15], with subsequent laser wakefield acceleration [16]). The second method is a direct way of polarization of a relativistic electron beam in a storage ring via radiative polarization (Sokolov-Ternov effect) [17–24]. The polarization time of the latter due to the synchrotron radiation is rather slow (typically from minutes to hours), since the magnetic fields of a synchrotron are too weak (in the order of 1 Tesla). The electrons are polarized transversely due to Sokolov-Ternov effect. As mostly longitudinal polarization is interesting in high-energy physics, spin rotation systems are applied [25]. Moreover, for creating polarized positron beams (also applicable for electrons) Compton scattering or Bremsstrahlung of circularly polarized lasers and successive pair creation are commonly used [26–30]. The polarization of relativistic electrons can be detected by Compton scattering [31], Möller scattering [32], or other methods.

Strongest fields in a laboratory are provided by lasers, and the state-of-the-art ultraintense laser technology can reach a laser peak intensity in the scale of $10^{22}$ W/cm$^2$ (magnetic field strength $\sim 4 \cdot 10^5$ Tesla) [33–36]. Can such strong fields be employed to polarize electrons, similar to the Sokolov-Ternov effect? Unfortunately, previous investigations proved that electrons cannot be polarized via asymmetric spin-flip in nonlinear Compton scattering off a strong monochromatic plane laser wave [37–39]. In a plane-wave laser pulse the electron polar-
splitting along the propagation direction into two parts, which have opposite transverse polarizations, see the interaction scenario in Fig. 1. The splitting of the electron beam is due to spin-dependent radiation-reaction effect. After the interaction time of tens of femtoseconds the splitted electrons are highly polarized transversely, and the polarization rate can reach above 70% under currently achievable experimental conditions. It is interesting to note that the considered effect is damped in the circularly polarized (CP) and linearly polarized (LP) laser fields, but is significant in the elliptically polarized (EP) one.

The electron polarization. Probabilities in Eqs. (1) and (2) are summed up by photon emission probability depending on the emitted photon frequency, \( \omega \), the emitted photon frequency, \( \epsilon \), the electron energy before radiation, \( \eta = k \cdot r \) the laser phase, \( p_z \) and \( k \) and \( r \) are 4-vectors of the electron momentum before radiation, laser wave-vector, and coordinate, respectively, \( \beta = v/c \), \( \mathbf{a} = u/|u| \) is the acceleration, \( \mathbf{S}_i \) and \( \mathbf{S}_f \) denote the electron spin polarization vector before and after radiation, respectively, \( \mathbf{S}_i = S_i \mathbf{S}_i \), and \( \mathbf{S}_f = S_f \mathbf{S}_f \). Summing over \( \mathbf{S}_f \), the radiation probability depending on the initial spin is obtained:

\[
\frac{dW_{fi}}{d\eta} = 8W_R \left\{ - (1 + u)^2 \left[ \text{IntK}_1(u') - 2\text{K}_1(u') \right] + u^2 \left[ \text{IntK}_1(u') + 2\text{K}_1(u') \right] - u^2 \left[ \text{IntK}_1(u') - 2\text{K}_1(u') \right] \left( \mathbf{S}_f \cdot \beta \right) \left( \mathbf{S}_f \cdot \beta \right) \right\}.
\]  

Averaging by the electron initial spin, the widely used radiation probability is obtained [52-55]. Note that the radiation probabilities in Eqs. (1) and (2) are summed up by photon polarization.

The spin dynamics due to photon emissions are described in the spirit of the quantum jump approach [55,57], applicable when the photon formation time is much smaller than the typical time of the regular quantum dynamics. After a photon emission, the electron spin state is collapsed into one of its basis states defined with respect to the instantaneous spin quantization axis (SQA), which is chosen along the magnetic field \( \mathbf{B} \) in the rest frame of electron, i.e., along \( \beta \times \mathbf{a} \). We consider the stochastic spin flip at photon emission using three random numbers \( N_r, N_\nu \), and \( N''_\nu \) in [0, 1], as follows. First, at each emission length, as the spin-dependent radiation probability in Eq. (2) is damped in the strong field processes [56,57], applicability of the quantum jump approach [58-59,60,61].
We employ a tightly-focused EP laser pulse with a Gaussian temporal profile. And, the spatial distribution of the electromagnetic fields takes into account up to $(w_0/z_L)^3$-order of the nonparaxial solution \[62\]–\[64\], where $w_0$ is the laser beam waist, and $z_L$ the Rayleigh length.

**Results.** The considered effect of polarization of an electron beam is illustrated in Fig. 2. The laser peak intensity $I_0 \approx 1.37 \times 10^{22}$ W/cm$^2$ ($\xi = 100$), wavelength $\lambda_0 = 1$ µm, the laser pulse duration $\tau = 5T_0$, with the laser period $T_0$, the laser focal radius $w_0 = 5$ µm, and the ellipticity $\epsilon = |E_x|/|E_y| = 0.05$. An electron bunch of a cylindrical form collides head-on with the laser pulse at the polar angle $\theta_0 = 180^\circ$ and the azimuthal angle $\phi_0 = 0^\circ$ with an angular divergence of 0.3 mrad. The electron initial kinetic energy $e_0 = 4$ GeV ($\gamma \approx 7827.8$) with an energy spread $\Delta e_0/e_0 = 0.06$, $\chi_{\text{max}} \approx 1.5$ (the pair production is estimated to be negligible for present parameters), the electron bunch radius $w_e = \lambda_0$, the length $L_e = 5\lambda_0$, and the density $n_e \approx 2.6 \times 10^{17}$ cm$^{-3}$ with a transversely Gaussian and longitudinally uniform distribution. This kind of electron bunch can be obtained by current laser wakefield accelerators \[65\]–\[66\].

The simulation results presented in Fig. 2 show that an initially unpolarized electron bunch is polarized and splitted into two beams polarizing parallel and anti-parallel to the minor axis of elliptical polarization (+y axis), respectively, with a splitting angle of about 20 mrad, see Fig. 2(a), which is much larger than the angular divergence of the electron beams \[62\]. The corresponding electron density mainly concentrates in the beam center, since the transverse ponderomotive force is relatively small, see Fig. 2(b). Figure 2(c) represents the average spin $\bar{S}_y$ (magenta-solid curve) and the electron density distribution (black-dashed curve) integrated over $\theta_x$. Near $\theta_x = 0$, the electron density is rather high, but $\bar{S}_y$ is very low.

With the increase of $|\theta_y|$, the electron density exponentially declines, however, $\bar{S}_y$ remarkably ascends until about 80%. Separating the part of the electron beam within $\theta_x > 0$ (or $\theta_x < 0$), one will obtain an electron beam with positive (or negative) transverse polarization. When splitting the beams exactly at $\theta_y = 0$, one obtains $\bar{S}_y \approx 34.21\%$ for both of splitted beams. However, we can increase the polarization of beams if we exclude the electrons near $\theta_x = 0$. For instance, as is shown by blue and red boxes in Fig. 2(b), the corresponding average spin $\bar{S}_y$ and electron number ratio $N_p/N_e$ are approximately (-65%, 10%) and (71%, 5%), respectively, see Fig. 2(d). The
corresponding splitting angle is of about 3 mrad, which is much larger than the angular resolution (less than 0.1 mrad) with current technique of electron detectors \[66-69\].

Moreover, for experimental convenience, we consider the cases of larger energy spread \(\Delta \theta_0/\theta_0 = 0.1\), larger angular divergence of 1 mrad and different collision angles \(\theta_0 = 179^{\circ}\) and \(\phi_0 = 90^{\circ}\), and all show stable and uniform results \[62\].

The reason for the electron beam polarization and splitting is analyzed in Fig. 3. The spin effect in the radiation probability is due to the third term in Eq. (2), and its contribution is rather significant (about 30\%) for high-energy photon emission, see Figs. 3(a), (b) at \(\hbar \omega_p/E_0 \approx 0.5 \sim 0.6\), which is negative (positive) for S, parallel (anti-parallel) to SQA. For simplicity, we analyze the electron radiative dynamics in plane wave cases, see Figs. 3(c), (d). Let us assume that the relativistic electrons initially move along \(-z\) direction, have no transverse momentum, and the final polarization along \(y\) axis is detected. When in the laser field the electron emits a photon (mostly at large \(\chi\) with a transverse momentum, finally it will appear with an opposite one due to the momentum conservation. The ultrarelativistic electron is assumed to emit a photon along its momentum direction, since the emission angle \(\sim 1/\gamma\) is rather small. Therefore, the electron final transverse momentum will be opposite by sign to its momentum at the photon emission point. In the laser field the transverse momentum \(p_x = eA(\eta)\), with the vector potential \(A(\eta)\), is delayed by \(\pi/2\) with respect to the field \(E(\eta)\). The SQA is along \(\beta \times \alpha = e\beta \times E + e\gamma \times (\beta \times B) \sim e(1 - \beta_\gamma)\beta \times E\), and note that \(\beta\) is negative.

For the LP plane wave polarized along \(x\) axis, see Figs. 3(c1), (c2), \(\chi \approx \xi \gamma y\) oscillates with \(|E_x|\), and the SQA is along \(y\) axis, with a sign following \(E_x\). According to Eq. (2) and Figs. 3(a), (b), at points of \(\gamma_1\) and \(\gamma_2\), the photon emission is more probable for spin-up (with respect to \(+y\) direction) electrons, because the corresponding \(E_x\) (green curve), and consequently, SQA are both negative. At points of \(\gamma_1\) and \(\gamma_2\), spin-down electrons mostly radiate. The final transverse momenta of electrons emitting photons at \(\gamma_1\) and \(\gamma_2\) are positive and at \(\gamma_2\) and \(\gamma_3\) negative. Consequently, spin-up and spin-down electrons move symmetrically with respect to the \(x\) axis and mix together, as indicated in Figs. 3(f), (g).

For the CP plane wave, see Figs. 3(d1)-(d3), \(\chi\) is constant, and the SQA rotates along the propagation \(z\) axis. In Fig. 3(d2), at points of \(\gamma_1\) and \(\gamma_2\), spin-down (with respect to \(+y\) direction) electrons more probably radiate (since the corresponding \(E_x\) and \(y\) component of SQA are both positive), and final \(p_x < 0\) for \(\gamma_1\) and \(p_x > 0\) for \(\gamma_2\). The similar analysis applies for other points, e.g., for \(\gamma_3\) the final \(p_x < 0\) (spin-up) and for \(\gamma_4\), \(p_x > 0\) (spin-down). Thus, spin-up and spin-down electrons mix together with respect to \(x\) axis. Similar electron spin dynamics exist for \(p_x\) in Fig. 3(d3) as well. Finally, spin-up and spin-down electrons mix together in \(x-y\) plane, as indicated in Figs. 3(d), (e).

However, for the EP plane wave with a rather small ellipticity \((E_\perp \ll E_\parallel\)), the radiation probability and the SQA both mainly relies on \(E_x\), and the SQA is along \(y\) axis. In Fig. 3(c3), \(p_x\) has a \(\pi\) delay with respect to \(E_x\); at points of \(\gamma_1\) and \(\gamma_2\), \(E_x\) and SQA are both negative, thus, spin-up (with respect to \(+y\) direction) electrons more probably radiate and finally acquire negative \(p_y\). And, at points of \(\gamma_3\) and \(\gamma_4\), spin-down electrons more probably radiate and finally have positive \(p_y\). Consequently, electrons split up with respect to the \(+y\) axis, see Figs. 1(b), (c) in which, since \(p_x\) is negative, spin-up (spin-down) electrons move at positive (negative) \(\theta_0 = \arctan(p_y/p_x)\). The trajectories of sample electrons in Fig. 3(e) illustrate those behaviors.

We underline that the considered effect of the spin-dependent splitting of the beam relies on the spin-dependent radiation reaction, rather than on the asymmetric spin flip. Moreover, multiple flips of spin will smear out the considered effect and we judiciously have chosen parameters to reduce the flip effect via limiting the number of emitted photons: \(N_{ph} \sim \xi_\gamma \tau_0/T_0 \approx 3.65 \times 10^{10}\). along with rather small spin flip probability, see Fig. 3(f).

Furthermore, impacts of the laser and electron beam parameters on the polarization are analyzed in Fig. 4. First, the ellipticity \(\epsilon\) is a very crucial parameter. If \(\epsilon\) is too small, the splitting angle \(\theta_0 \sim p_\parallel/\epsilon E_0\) is very small as well, and the polarized electrons partially overlap near \(p_\parallel = 0\) (e.g., the ultimate case of the LP laser), which reduces the degree of polarization. Oppositely, largely increasing ellipticity can increase the splitting angle, but unfortunately also the SQA rotation (cf., the ultimate case of the CP laser). As a result the average polarization decreases, see Fig. 4(a). The optimal ellipticity is of order of \(10^{-2}\) to \(10^{-1}\). The trade off exists also for the laser intensity, pulse duration, and the electron energy. From one side, the effect relies on the radiation reaction and requires large \(\chi \approx 10^{-6}\xi \gamma \gg 1\) and many photon emission. From another side, the spin flips smear out the considered effect which imposes restriction on the photon emissions. For this reason, with increasing \(\xi\) and the electron kinetic energy \(E_0\), the polarization is first enhanced due to the increase of \(\chi\), and then saturates, see Fig. 4(b), (d). The mentioned trade off yields nonuniform dependence on the laser pulse duration. The polarization is weak at too short or too long pulses, and the optimum is \(\tau = 5\tau_0\) for the given parameters, see Fig. 4(c).

For a simple estimation of radiative polarization effects, we also develop a semi-classical analytical method based on

![FIG. 4. Impacts of (a) ellipticity \(\epsilon\), (b) laser intensity \(\xi\), (c) laser pulse duration \(\tau\), and (d) initial kinetic energy of electrons \(E_0\) on the polarization. Other parameters are the same as in Fig. 2](image-url)
the modified Landau-Lifshitz equation with a radiation-reaction force accounting for quantum-recoil and spin effects. This model further confirms above obtained results qualitatively.

In conclusion, we have developed a Monte-Carlo method for simulating radiative spin effects. We show that adding a proper small ellipticity to the strong laser pulse allows to directly polarize and split a counterpropagating relativistic electron beam into highly polarized parts with current achievable experimental techniques, which can be used in high-energy physics.

Acknowledgement: We are grateful to A. Di Piazza and M. Tamburini for helpful discussions. This work is supported by the Science Challenge Project of China (No. TZ2016099), the National Key Research and Development Program of China (Grant No. 2018YFA0404801), and the National Natural Science Foundation of China (Grants Nos. 11874295, 11804269).

References:

[1] J. Kessler, Polarized Electrons (Springer, Berlin, 1985).
[2] M. Getzlaf, Surface Magnetism (Springer, Berlin, 2010).
[3] T. Gay, Adv. At. Mol. Opt. Phys. 57, 157 (2009).
[4] K. Abe et al., Phys. Rev. Lett. 75, 25 (1995).
[5] V. Alexakhin et al., Phys. Lett. B 647, 8 (2007).
[6] H. Olsen and L. C. Maximon, Phys. Rev. 114, 887 (1959).
[7] R. Martin, G. Weber, R. Barday, Y. Fritzsche, U. Spillmann, W. Chen, R. D. DuBois, J. Enders, M. Hegewald, S. Hess, A. Surzhykov, D. B. Thorn, S. Trotsenko, M. Wagner, D. F. A. Winters, V. A. Yerokhin, and T. Stöhlker, Phys. Rev. Lett. 108, 264801 (2012).
[8] D. Abbott, P. Adderley, A. Adeyemi, P. Aguilera, M. Ali, H. Areti, M. Baylac, J. Benesch, G. Bosson, B. Cadé, A. Camsonne, L. S. Cardman, J. Clark, P. Cole, S. Covert, C. Cuevas, O. Dadoun, D. Dale, H. Dong, J. Dumas, E. Fanchini, T. Forest, E. Forman, A. Freyberger, E. Frieodifog, S. Golge, J. Grames, P. Guéye, J. Hansknecht, P. Harrell, J. Hoskins, C. Hyde, B. Josey, R. Kazimi, Y. Kim, D. Machie, K. Mahoney, R. Mamie, M. Marton, J. McCarter, M. McKeown, R. Michaels, M. Olson, K. D. Owen, A. Vacheret, D. Walz, T. Weber, J. Weisend, D. Whittum, M. Woods, and I. Younis (SLAC E158 Collaboration), Phys. Rev. Lett. 92, 181602 (2004).

[9] P. L. Anthony, R. G. Arnold, C. Arroyo, K. Baird, K. Bega, J. Biesiada, P. E. Bosted, M. Breuer, R. Carr, G. D. Cates, J.-P. Chen, E. Chudakov, M. Cooke, F. J. Decker, P. Decowski, A. Deur, W. Emami, R. Erickson, T. Fieguth, C. Field, J. Gao, K. Gustafsson, R. S. Hicks, R. Holmes, E. W. Hughes, T. B. Humensky, G. M. Jones, L. J. Kaufman, Y. G. Kolomensky, K. S. Kumar, D. Lhuillier, R. Lombard-Nelsen, P. Mastromarino, B. Mayer, R. D. McKeown, R. Michaels, M. Olson, K. D. Paschke, G. A. Peterson, R. Pitthan, K. Pope, D. Relyea, S. E. Rock, O. Saxton, G. Shapiro, J. Singh, P. A. Souder, Z. M. Szalata, W. A. Tobias, B. T. Tongue, J. Turner, B. Tweedie, A. Vacheret, D. Walz, T. Weber, J. Weisend, D. Whittum, M. Woods, and I. Younis (SLAC E158 Collaboration), Phys. Rev. Lett. 92, 181602 (2004).

[10] G. Moortgat-Pick, T. Abe, G. Alexander, B. Anantharayan, A. Babich, V. Bharadwaj, D. Barber, A. Bartl, A. Brachmann, S. Chen, J. Clarke, J. Clendenin, J. Dainton, K. Desh, M. Diehl, B. Dobos, T. Dorland, H. Dreiner, H. Eberl, J. Ellis, K. Fittemann, H. Fraas, F. Franco-Solovia, F. Franke, A. Freitas, J. Goodson, J. Gray, A. Han, S. Heinemeyer, S. Hesselbach, T. Hirose, K. Hohenwarter-Sodek, A. Juste, J. Kalinowski, T. Kernreiter, O. Kittel, S. Kraml, U. Langenfeld, W. Majerotto, A. Martinez, H.-U. Martyn, A. Mikhailichenko, C. Milstene, W. Menges, N. Meyners, K. Migg, K. Moffett, S. Moretti, O. Nachtmann, F. Nagel, T. Nakaniishi, U. Nauenberg, H. Nowak, T. Omori, P. Osland, A. Pankov, N. Paver, R. Pitthan, R. Pschl, W. Porod, J. Proulx, P. Richardson, S. Riemann, S. Rindani, T. Rizzo, A. Schlicke, P. Schler, C. Schwaneberger, D. Scott, J. Sheppard, R. Singh, A. Sopczak, H. Spiesberger, A. Stahl, H. Steiner, A. Wagner, A. Weber, G. Weiglein, G. Wilson, M. Woods, P. Zervas, J. Zhang, and F. Zomer, Phys. Rep. 460, 131 (2008).

[11] M. L. Swartz, SLAC-PUB-4656 (1988).
[12] D. T. Pierce and F. Meier, Phys. Rev. B 13, 5484 (2008).
[13] D. T. Pierce, F. Meier, and P. Zürcher, Appl. Phys. Lett. 26, 670 (2008).
[14] H. Batelaan, A. S. Green, B. A. Hitt, and T. J. Gay, Phys. Rev. Lett. 82, 4216 (1999).
[15] M. M. Dellweg and C. Müller, Phys. Rev. Lett. 118, 070403 (2017).
[16] M. Wen, M. Tamburini, and C. H. Keitel, arXiv: 1809.10570 (2018).
[17] A. A. Sokolov and I. M. Ternov, Sov. Phys. Dokl. 8, 1203 (1964).
[18] A. A. Sokolov and I. M. Ternov, Synchrotron Radiation (Akademie, Germany, 1968).
[19] V. Baier and V. Katkov, Phys. Lett. A 24, 327 (1967).
[20] V. N. Baier, Sov. Phys. Usp. 14, 695 (1972).
[21] Y. Derbenev and A. M. Kondratenko, Zh. Eksp. Teoret. Fiz. 64, 1918 (1973).
[22] Y. N. Baier, V. M. Katkov, and V. M. Strakhovenko, Phys. Lett. B 70, 83 (1977).
[23] Y. S. Derbenev, A. M. Kondratenko, and A. N. Skrinsky, Part. Accel. 9, 247 (1979).
[24] S. R. Mane, Phys. Rev. A 36, 105 (1987).
[25] J. Buon and K. Steffen, Nucl. Instrum. Methods A 245, 248 (1986).
[26] T. Hirose, K. Dobashi, Y. Kurihara, T. Muto, T. Omori, T. Okugi, I. Sakai, J. Urakawa, and M. Washio, Nucl. Instrum. Methods A 455, 15 (2000).
[27] T. Omori, M. Fukuda, T. Hirose, Y. Kurihara, R. Kuroda, M. Nozawa, K. Ohashi, T. Okugi, S. Sakaue, T. Saito, J. Urakawa, M. Washio, and I. Yamazaki, Phys. Rev. Lett. 96, 114801 (2006).
[28] X. Artru, R. Chehab, M. Chevallier, V. Strakhovenko, A. Variola, and A. Vivoli, Nucl. Instrum. Methods B 266, 3868 (2008).
[29] T.-O. Müller and C. Müller, Phys. Lett. B 696, 201 (2011).
[30] A. Di Piazza, A. I. Milstein, and C. Müller, Phys. Rev. A 82, 062110 (2010).
[31] D. Barber, H.-D. Bremer, M. Böge, R. Brinkmann, W. Brückner, C. Büscher, M. Chapman, K. Coulter, P. Delheij, M. Düren, T. Elner, F. Gianfelice-Wendt, P. Green, H. Gaul, H. Gressmann, O. Häusser, R. Henderson, T. Janke, H. Kaiser, R. Kaiser, P. Kitching, R. Klanner, P. Levy, H.-C. Lewin, M. Lomperski, W. Lorenzoon, L. Losev, R. McKeown, N. Meyners, B. Michael, R. Milner, A. Mücklich, F. Neureither, W.-D. Nowak, P. Patel, K. Rith, C. Schulz, E. Steffen, M. Veltri, M. Vetterli, W. Vogel, W. Wander, D. Westphal, K. Zapfe, and F. Zetsche, Nucl. Instrum. Methods A 329, 79 (1993).
[32] P. S. Cooper, M. J. Alguard, R. D. Ehrlich, V. W. Hughes,
