Small $x$ phenomenology: summary and status

The Small x Collaboration

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Abstract. The aim of this paper is to summarize the general status of our understanding of small-$x$ physics. It is based on presentations and discussions at an informal meeting on this topic held in Lund, Sweden, in March 2001. This document also marks the founding of an informal collaboration between experimentalists and theorists with a special interest in small-$x$ physics.

This paper is dedicated to the memory of Bo Andersson, who died unexpectedly from a heart attack on March 4th, 2002.

1 Introduction

In this paper we present a summary of the workshop on small-$x$ parton dynamics held in Lund in the beginning of March 2001. During two days we went through a number of theoretical and phenomenological aspects of small-$x$ physics in short talks and long discussions. Here we will present the main points of these discussions and try to summarize the general status of the work in this field.

For almost thirty years, QCD has been the theory of strong interactions. Although it has been very successful, there are still a number of problems which have not been solved. Most of these have to do with the transition between the perturbative and non-perturbative description of the theory. Although perturbative techniques work surprisingly well down to very small scales where the running coupling starts to become large, in the end what is observed are hadrons, the transition to which is still not on firm theoretical grounds. At very high energies another problem arises. Even at high scales where the running coupling is small the phase space for additional emissions increases rapidly and makes the perturbative expansion ill-behaved. The solution to this problem is to resum the leading logarithmic behavior of the cross section to all orders, thus rearranging the perturbative expansion into a more rapidly converging series.

The DGLAP [1–4] evolution is the most familiar resummation strategy. Given that a cross section involving incoming hadrons is dominated by diagrams where successive emissions are strongly ordered in virtuality, the resulting large logarithms of ratios of subsequent virtualities can be resummed. The cross section can then be rewritten in terms of a process-dependent hard matrix element convoluted with universal parton density functions, the scaling violations of which are described by the DGLAP evolution. This is called collinear factorization. Because of the strong ordering of virtualities, the virtuality of the parton entering the hard scattering matrix element can be neglected (treated collinear with the incoming hadron) compared to the large scale $Q^2$. This approach has been very successful in describing the bulk of experimental measurements at lepton–hadron and hadron–hadron colliders.

With HERA, a new kinematic regime has opened up where the very small $x$ parts of the proton parton dis-
Table 1. Summary of the ability of the collinear and $k_{\perp}$-factorization approaches to reproduce the current measurements of some observables: OK means a satisfactory description; 1/2 means a not perfect but also a not too bad description, or in part of the phase space an acceptable description; OK? means satisfactory description if a heavy quark excitation component is added in leading order; NO means that the description is bad; and? means that no thorough comparison has been made.

| HERA observables                  | collinear factorization | $k_{\perp}$-factorization |
|-----------------------------------|-------------------------|---------------------------|
| high $Q^2$ $D^*$ production       | OK [8,9]                | OK [9,10]                 |
| low $Q^2$ $D^*$ production        | OK [8,9]                | OK [9,10]                 |
| direct photoproduction of $D^*$   | 1/2 [11]                | OK [10,12–15]             |
| resolved photoproduction of $D^*$ | NO [11]                 | 1/2 [12–15]               |
| high $Q^2$ B production           | NO [16]                 | ?                         |
| low $Q^2$ B production            | NO [16]                 | ?                         |
| direct photoproduction of B       | OK? [17], NO [18]       | OK [19–21]                |
| resolved photoproduction of B     | OK? [17]                | OK [19–21]                |
| high $Q^2$ di-jets                | OK [22,23]              | ?                         |
| low $Q^2$ di-jets                 | NO [22–25]              | ?                         |
| direct photoproduction of di-jets | 1/2 [22,24,25]          | ?                         |
| resolved photoproduction of di-jets| NO [22,24,25]          | ?                         |

| HERA small-$x$ observables        |                          |                           |
|-----------------------------------|-------------------------|---------------------------|
| forward jet production            | NO [26]                 | OK [14]                   |
| forward $\pi$ production          | NO [26]                 | 1/2 [27]                  |
| particle spectra                  | NO [28]                 | OK [14]                   |
| energy flow                       | NO [28]                 | ?                         |
| photoproduction of $J/\Psi$       | NO [29]                 | 1/2 [30,31]               |
| $J/\Psi$ production in DIS        | NO                       | ?                         |

| TEVATRON observables             |                          |                           |
|----------------------------------|-------------------------|---------------------------|
| high-$p_{\perp}$ $D^*$ production| ?                       | ?                         |
| low-$p_{\perp}$ $D^*$ production | ?                       | ?                         |
| high-$p_{\perp}$ B production    | OK? [32]                | OK [20,21,33,34]          |
| low-$p_{\perp}$ B production     | OK? [32]                | OK [20,21,33,34]          |
| $J/\Psi$ production              | NO                      | ?                         |
| high-$p_{\perp}$ jets at large rapidity differences | NO | ? |

distributions are being probed. The hard scale, $Q^2$, is not very high in such events and it was expected that the DGLAP evolution should break down. To some surprise, the DGLAP evolution has been quite successful in describing the strong rise of the cross section with decreasing $x$. For some non-inclusive observables there are, however, clear discrepancies as summarized in Table 1.

At asymptotically large energies, it is believed that the theoretically correct description is given by the BFKL [5–7] evolution. Here, each emitted gluon is assumed to take a large fraction of the energy of the propagating gluon, $(1-z)$ for $z \to 0$, and large logarithms of $1/z$ are summed up to all orders. Although the rise of $F_2$ with decreasing $x$ as measured at HERA can be described with the DGLAP evolution, a strong power-like rise was predicted by BFKL. Just as for DGLAP, it is possible to factorize an observable into a convolution of process-dependent hard matrix elements with universal parton distributions. But as the virtuality and transverse momentum of the propagating gluon are no longer ordered, the matrix elements have to be taken off-shell and the convolution is also over transverse momentum with unintegrated parton distributions. We therefore talk about $k_{\perp}$-factorization [35,36] or the semihard approach [37,38].

Recently, the next-to-leading logarithmic (NLL) corrections to the BFKL equation were calculated and found