Reliability of the two-point measurement of the spatial correlation length from Gaussian-shaped fluctuating signals in fusion-grade plasmas

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Abstract
A statistical method for the estimation of the spatial correlation lengths of Gaussian-shaped fluctuating signals with two measurement points is examined to quantitatively evaluate its reliability (variance) and accuracy (bias error). The standard deviation of the correlation value is analytically derived for randomly distributed Gaussian shaped fluctuations satisfying stationarity and homogeneity, allowing us to evaluate, as a function of fluctuation-to-noise ratios, the sizes of averaging time windows and the ratios of the distance between the two measurement points to the true correlation length, and the goodness of the two-point measurement for estimating the spatial correlation length. Analytic results are confirmed with numerically generated synthetic data and real experimental data obtained with the KSTAR beam emission spectroscopy diagnostic. Our results can be applied to Gaussian-shaped fluctuating signals where a correlation length must be measured with only two measurement points.

Keywords: turbulence, two-points correlation measurement, KSTAR, beam emission spectroscopy

(Some figures may appear in colour only in the online journal)
are not aligned in a straight line (or to a background magnetic field line).

As we do not find any systematic studies on the reliability of the obtained two-point correlation function, i.e. variance of the correlation function, which is equally as important as the correlation function itself, we investigate how accurately and reliably one can measure the correlation function with only two measurement points. This is then used to calculate the accuracy and reliability of the estimated spatial correlation length.

We focus on the two-point measurement because it is the smallest necessary number to get the spatial structure, unless Taylor’s hypothesis [6] can be validly applied where temporal information at one spatial position contains upstream spatial information. We focus on the spatial correlation length because it is one of the basic properties of turbulence. The decorrelation rate in time and fluctuation levels are also important characteristics; however these suffer less from system hardware due to the availability of many time points. Furthermore, turbulence is believed to be critically balanced [7–10] as reported in observations of solar winds [11–13] and a gyro-kinetic simulation of ion-temperature-gradient driven turbulence [14] in a fusion relevant geometry. However, in fusion experiments, the parallel correlation length has never been measured in the core of the plasmas; consequently only an experimental ‘signature’ of critically balanced turbulence in MAST (Mega Amp Spherical Tokamak) is reported [15].

The parallel correlation length of the fluctuations are typically measured using the probes only at the edge of the fusion-grade plasma [16–18]. As the diagnostic systems are being developed, there is a possibility of measuring parallel correlation lengths in the core of KSTAR (Korea Superconducting Tokamak Advanced Research) plasmas with beam emission spectroscopy (BES) [19] and microwave imaging reflectometry [20]. Both systems are 2D (poloidal and radial) but installed at different toroidal locations measuring the same physical quantity—density fluctuations. Again, this motivates us to study the reliability and accuracy of the correlation lengths obtained with the two measurement points.

This paper is organized as follows: in section 2, the correlation function and its variance of randomly distributed moving Gaussian-shaped (both in time and space) fluctuations are analytically derived with a finite averaging time window and noise. This expression is, then, compared with the numerically generated synthetic data where we find good agreement between the analytic and numerical results. This correlation function is used to obtain the accuracy (bias error) and reliability (variance) of the estimated correlation length in section 3, both analytically and numerically, whose results are also confirmed quantitatively with the experimental data obtained from the KSTAR BES system. Our conclusions are provided in section 4.

\[ S(t) = \sum_{n=1}^{N} S_n(t), \] (1)

where \( N \) is the total number of eddies and \( S_n(t) \) is the signal due to \( n \)th ‘eddy’ at the location \( x = x_n \). As has been done previously [21–23] motivated by the experimental observations [24, 25] of ion-scale turbulence in fusion-grade plasmas, we use a Gaussian-shaped structure in both time and space for each eddy:

\[ S_n(t) = A_n \exp -\frac{(t - t_n)^2}{2\tau_{life}^2} - \frac{(x_n - v(t - t_n) - x_i)^2}{2\lambda_i^2} \] (2)

where the coherent properties of each eddy in time and space are parameterized by the characteristic temporal scale \( \tau_{life} \) and the spatial scale \( \lambda_i \). Furthermore, we allow the eddy to move with the speed \( v \) to mimic observed eddy motions due to background plasma flows [22] or wave-like propagations.

We let the \( n \)th eddy have the maximum amplitude \( A_n \) at \( x = x_i \) and \( t = t_i \) where random number \( A_n \) is selected from a normal distribution with zero mean and variance of \( \lambda_i^2 \), \( x_i \) and \( t_i \) are selected from uniformly distributed random numbers within the finite domain of \( [x_i - \Delta x, x_i + \Delta x] \) and \( [t_i - \Delta t, t_i + \Delta t] \), respectively.

This means that our result is strictly valid within a flux-surface where the radial gradients of various equilibrium quantities correlated with the turbulence [26] are constant.

A spatio-temporal filling factor \( F \) is defined as [22]

\[ F \equiv N \left( \frac{\lambda_i}{\Delta L} \right) \left( \frac{\tau_{life}}{\Delta T} \right) \] (3)

and we control the total number of eddies \( N \) such that \( F \propto C(1) \) do not have too frequent or too rare eddies. Later on, we show that \( F \propto C(1) \) ensures that the generated signal has the square of the fluctuation level of the order of \( A_i^2 \) as specified. More detailed descriptions on the model of the fluctuating signal can be found elsewhere [21, 23].

For the readers who question the validity of the Gaussian-shaped eddies, as the Lorentzian eddies are also observed in the scrape-off-layer region of the magnetically confined plasmas [27–29], we provide the discussions with the Lorentzian eddies in appendix D. This section is recommended to be read after reading section 3.

2.1. Correlation function and its variance

In this section, we analytically derive the correlation value between the two spatial positions at \( x = x_a \) and \( x = x_p \) at the correlation time delay \( \tau = 0 \) (as we are interested in obtaining the correlation length of the fluctuating signals) and its associated variance.

To analytically derive the correlation function following Kim et al [23], \( \Delta L \) and \( \Delta T \) are assumed to be infinitely large compared to \( \lambda_i \) and \( \tau_{life} \). A correlation value \( C_{a,b} \) between \( x = x_a \) and \( x = x_j \) is calculated as \( E[S_a(t)S_b(t)] \), where \( E[\cdot] \) is the time averaging operator over the ‘sub-time window’ whose size is set by \( \Delta T_{sub} \). Then, the ensemble averaged correlation value \( \langle C_{a,b} \rangle \) is [23]
\[ \langle C_{a,b} \rangle \approx \pi \lambda^2 F \left( \exp \left[ \frac{-(x_a - x_b)^2}{2(2\lambda)^2} \right] - 2\sqrt{\frac{\tau_{ac}}{\Delta T_{ub}}} \exp \left[ \frac{-(x_a - x_b)^2}{4(\lambda^2 + \tau_{life}^2)} \right] \right) \]

\[ \approx \pi \lambda^2 F \left( \exp \left[ \frac{-(x_a - x_b)^2}{2(2\lambda)^2} \right] \right) \quad (4) \]

where the approximation in the last line is valid for \( \tau_{ac}/\Delta T_{ub} \ll 1 \). \( \Delta T_{ub} \) is much smaller than \( \Delta T \) allowing many of the \( \Delta T_{ub} \) to exist within \( \Delta T \) (for an ensemble average [23]) but is still much larger than \( \tau_{ac} \), the usual auto-correlation time of the fluctuating signal in the lab frame defined as [30]

\[ \tau_{ac} = \frac{\lambda \tau_{life}}{\sqrt{\lambda^2 + \tau_{life}^2}}. \quad (5) \]

Note that the square of the fluctuation level \( \langle C_{a,b} \rangle \) is \( \sim O(A^2) \) for the spatio-temporal filling factor \( F \sim O(1) \).

The correlation values, i.e. \( C_{a,b} \), have a certain distribution resulting from the fact that the amplitude \( (A) \) and the spatio-temporal location \( (x_i \text{ and } t_i) \) of the eddies are randomly distributed. This distribution gives us randomness in \( C_{a,b} \), i.e. the variance \( \sigma_{C_{a,b}}^2 \) (see appendix A), which is

\[ \sigma_{C_{a,b}}^2 = \frac{\sqrt{\pi}}{2} \lambda_{\text{life}} (1 + \exp \left[ \frac{-(x_a - x_b)^2}{2\lambda^2} \right]) + \frac{3}{\sqrt{2}} \pi^{3/2} A^4 F \frac{\tau_{ac}}{\Delta T_{ub}} \exp \left[ \frac{-(x_a - x_b)^2}{2\lambda^2} \right] \quad (6) \]

where the approximation is valid for \( \Delta L \gg \lambda_x \), \( \Delta T \gg \Delta T_{ub} \gg \tau_{life} \) (or \( \tau_{ac} \)), and \( N \gg 1 \) with the condition of the spatio-temporal filling factor \( F \sim O(1) \) consistent with the assumptions we made to obtain \( \langle C_{a,b} \rangle \) in equation (4). Note that large \( N \) is, in fact, a consequence of having \( F \sim O(1) \) with large \( \Delta L \) and \( \Delta T \).

2.2. Normalized correlation function and its variance with and without noise

We now calculate the normalized correlation functions with and without noise and their associated variances. Again, we are interested in the correlation value between the two spatial points \( x_a \) and \( x_b \) at the correlation time delay \( \tau = 0 \). Although the correlation length can be estimated with the ensemble averaged correlation value derived above, we derive the normalized correlation value because the normalized one estimates the correlation length more accurately (smaller bias error) and reliably (smaller variance) compared to the unnormalized one, which we show later in section 3.2.

The ensemble averaged normalized correlation value without noise \( \langle \bar{C}_{a,b} \rangle \) is

\[ \langle \bar{C}_{a,b} \rangle = \frac{C_{a,b}}{\delta_a \delta_b} \approx \frac{C_{a,b}}{\delta_a^2} \approx \frac{C_{a,b}}{\delta_b^2} = \exp \left[ \frac{-(x_a - x_b)^2}{2(2\lambda)^2} \right] \quad (7) \]

where \( \delta_a = \sqrt{C_{a,b}} \) and \( \delta_b = \sqrt{C_{b,b}} \) are the fluctuation levels at the locations \( x_a \) and \( x_b \), respectively. We have denoted this fluctuation level as \( \delta_f \) and assumed \( \delta_a \approx \delta_f \) and \( \delta_b \approx \delta_f \) based on the homogeneity of the data. Note that \( \langle \delta_f^2 \rangle = \pi A^2 F \) from equation (4). The validity of the approximation in the second line of equation (7) is provided in appendix B.

As for the case of \( C_{a,b} \), \( \bar{C}_{a,b} \) also has a distribution resulting in a finite variance solely due to the randomness of the eddies. This variance denoted as \( \sigma_{\bar{C}_{a,b}}^2 \) is

\[ \sigma_{\bar{C}_{a,b}}^2 \approx \frac{\pi \lambda_{\text{life}}}{\sqrt{\lambda^2 + \tau_{life}^2}} \left( 1 - \exp \left[ \frac{-(x_a - x_b)^2}{2\lambda^2} \right] \right)^2 \quad (8) \]

The detailed derivation of \( \sigma_{\bar{C}_{a,b}}^2 \) and assumptions we made are provided in appendix C.1.

Let us now consider the signal with uncorrelated noise. Our model signal with noise \( S_n(t) \) is then,

\[ S_n^\text{noise}(t) = S_f(t) + n_d(t), \quad (9) \]

where \( n_d(t) \) is the noise as a function of time at \( x = x_a \). A normalized correlation value with noise \( \bar{C}_{a,b}^\text{noise} \) between the signals from two spatial positions \( x_a \) and \( x_b \) is

\[ \bar{C}_{a,b}^\text{noise} = \frac{E [S_f(t) S_f(t)] + E [n_d(t) n_d(t)]}{\sqrt{E [S_f^2(t)] + E [n_d^2(t)]}}. \quad (10) \]

Note that \( E [n_d(t) n_d(t)] = 0 \) unless \( x_a = x_b \) due to the uncorrelated noise assumption. The normalized correlation value at \( x_a = x_b \) is by definition one whether or not noise exists, i.e. \( \bar{C}_{a,a} = 1 \) and \( C_{a,a} = 1 \). It is obvious that the effect of \( E [n_d(t) n_d(t)] \) on \( \bar{C}_{a,b}^\text{noise} \) for \( x_a = x_b \) can be removed for our model signal, but it is also possible to do so with experimental data by considering the auto-correlation function as a function of the correlation time delay \( \tau \) [31]. The idea is, basically, interpolating the auto-correlation signal around \( \tau = 0 \) to eliminate the effect of \( E [n_d(t) n_d(t)] \) present only around a very short interval of \( \tau = 0 \).

Once we remove the noise effect at \( x_a = x_b \) both for our model data and experimental data, we can define \( C_{a,b}^\text{noise} \) as

\[ C_{a,b}^\text{noise} = \frac{E [S_f(t) S_f(t)]}{\sqrt{E [S_f^2(t)] + E [n_d^2(t)]}} \]

\[ \approx \frac{\delta_f^2}{\delta_f^2 + \delta_n^2} \delta_f, \quad (11) \]

where we assumed that the noise level \( \delta_n \) is homogenous, as is the case for \( \delta_f \). Then, the ensemble averaged normalized correlation value with noise is
The first line equality is owing to the fact that the normalized correlation value $C_{a,b}^{\delta_f}$ is not correlated with a function of fluctuation levels, i.e. $\delta_f^2/\langle \delta_f^2 \rangle$. The second line approximation is based on the same rationale for $\langle C_{a,b}^{\delta_f} \rangle$ in equation (7) (see appendix B). Notice that the effective role of the noise is to decrease the normalized correlation value, and it is not even unity at $x_a = x_b$, rather it is a function of fluctuation($\delta_f$)-to-noise($\delta_n$) ratio (FNR).

\[
\langle C_{a,b}^{\text{noise}} \rangle = \left\langle \frac{\delta_f^2}{\delta_f^2 + \delta_n^2} \right\rangle \langle C_{a,b} \rangle \\
\approx \frac{\langle \delta_f^2 \rangle}{\langle \delta_f^2 \rangle + \langle \delta_n^2 \rangle} \langle C_{a,b} \rangle \\
= \frac{\langle \delta_f^2 \rangle}{\langle \delta_f^2 \rangle + \langle \delta_n^2 \rangle} \exp \left[ \frac{(x_a - x_b)^2}{2(\sqrt{2} \lambda_f)^2} \right]. \tag{12}
\]

The variance $\sigma_{C_{a,b}}^2$ of the ensemble averaged normalized correlation value with noise calculated in appendix C.2 is

\[
\sigma_{C_{a,b}}^2 \approx \langle C_{a,b}^{\text{noise}} \rangle^2 \left[ \left( \frac{\sigma_{\delta_f}}{\delta_f} - \frac{\sigma_{\delta_n}}{\delta_n} \right)^2 + \frac{\sigma_{C_{a,b}}^2}{\langle C_{a,b} \rangle^2} \right]^2. \tag{13}
\]

where $\sigma_{\delta_f}$ and $\sigma_{\delta_n}$ are the variances of $\delta_f$ and $\delta_n$, respectively. Since $\delta_f^2 = C_{a,a}$, we have $\sigma_{\delta_f}^2 = \sigma_{\delta_n}^2$, which can be calculated using equation (6).

Note that equations (12) and (13) which are the ensemble averaged normalized correlation value and its variance with noise, respectively, become equations (7) and (8) for $\delta_n = 0$, and none of these equations contain unknowns for our model fluctuation data.

2.3. Comparisons between analytic expressions and numerical results

To confirm that the analytically derived unnormalized and normalized correlation values and their associated variances are indeed correct, we generate 50 sets of 1D synthetic data

![Figure 1](image_url)

(a) Correlation values and (b) their standard deviations as a function of $\Delta x = |x_a - x_b|$ estimated from the synthetic data, $\langle C_{a,b} \rangle$ and $\sigma_{C_{a,b}}^2$ (blue lines), and calculated analytically, $\langle C_{a,b} \rangle$ and $\sigma_{C_{a,b}}$ (red lines). The error bars in (a) represent $\sigma_{C_{a,b}}$, and those in (b) show 95% confidence interval.

The variance $\sigma_{C_{a,b}}^2$ of the ensemble averaged normalized correlation value with noise calculated in appendix C.2 is

\[
\sigma_{C_{a,b}}^2 \approx \langle C_{a,b}^{\text{noise}} \rangle^2 \left[ \left( \frac{\sigma_{\delta_f}}{\delta_f} - \frac{\sigma_{\delta_n}}{\delta_n} \right)^2 + \frac{\sigma_{C_{a,b}}^2}{\langle C_{a,b} \rangle^2} \right]^2. \tag{13}
\]
based on equations (1) and (2). These sets of synthetic fluctuation data are generated such that they are guaranteed to be homogeneous and stationary [23].

The basic properties of each set of the synthetic data are: the characteristic spatial scale $\lambda = 0.3 \, \text{m}$, the characteristic temporal scale $\tau = 15 \, \text{s}$, the velocity of the eddies $v = 5000 \, \text{m s}^{-1}$, the total time window for the data $\Delta T = 15000 \, \text{s}$ with the sub-time window $\Delta T_{\text{sub}} = 585 \, \text{s}$ (giving 25 sub-time windows) and the spatial domain size $\Delta L = 2.7 \, \text{m}$. To make sure that the synthetic data are homogeneous, $\sigma^2_f$ is kept constant in space as well as satisfying the criteria on the domain size according to reference [23].

From the synthetic data we estimate the correlation value $\langle C_{a,b} \rangle$ (we use a superscript asterisk to denote that it is estimated with the synthetic data) and its standard deviation, i.e. square root of the variance, $\sigma^*_{C_{a,b}}$ as

$$\langle C_{a,b} \rangle^* = \frac{1}{50} \sum_{i=1}^{50} \left[ \frac{1}{25} \sum_{j=1}^{25} C_{a,b}^{ij} \right],$$

$$\sigma^*_{C_{a,b}} = \frac{1}{50} \sum_{i=1}^{50} \left[ \frac{1}{25} \sum_{j=1}^{25} \left( C_{a,b}^{ij} - \langle C_{a,b} \rangle^* \right)^2 \right],$$

(14)

where the superscripts $i$ and $j$ denote the $i$th set and the $j$th sub-time window within a set of the synthetic data, respectively. Here, $C_{a,b}^{ij}$ is estimated as

$$C_{a,b}^{ij} = \frac{1}{\Delta T_{\text{sub}}} \int_{\Delta T_{\text{sub}}}^{} S_a^j(t) S_b^i(t) \, dt.$$
Figure 1(a) shows analytically calculated $C_{ab}$ (red line) using equation (4) and $\ast C_{ab}$ (blue line) using equation (14) on the synthetic data as a function of $\Delta x = |x_a - x_b|$. They show a good agreement. The error bars on $\ast C_{ab}$ are $\sigma \ast C_{ab}$. Furthermore, figure 1(b) illustrates a good agreement within the 95% confidence interval between the analytically calculated $\sigma C_{ab}$ (red line) using equation (6) and estimated $\ast \sigma C_{ab}$ from the synthetic data using equation (14) as a function of $\Delta x$.

Similar comparisons are shown in figure 2 for the normalized correlation values with various FNRs (FNR = $\delta/\delta_f$): FNR = 1.0 (blue), 2.0 (red) and 4.0 (yellow). Normalized correlation values and their standard deviations with noise, $C_{a,a}^{\text{noise}}$ and $\sigma C_{a,a}^{\text{noise}}$, are calculated using equations (12) and (13); while they are also obtained from the synthetic data, $C_{a,a}^{\ast \text{noise}}$ and $\sigma C_{a,a}^{\ast \text{noise}}$ using equation (14) with normalized $\ast C_{a,a}^{\ast \text{noise}}$. Again, we find good agreements between the analytical and numerical results.

Note that the value of $C_{a,a}^{\text{noise}}$ with $x_a = x_b$, i.e. $\Delta x = 0$ is $\left< \xi_a^2 \right> \left< \xi_b^2 \right> + \left< \xi_a^2 \right> \left< \xi_b^2 \right>$ according to equation (12). Thus, we expect this value to be 0.50, 0.80 and 0.94 for FNR = 1.0, 2.0 and 4.0, respectively. These values coincide with the results from the synthetic data which are obtained by removing the noise in the signal by using the interpolation technique around the correlation time delay $\tau = 0$ of the auto-correlation function [31] as mentioned earlier.

As vindicated by the numerical results, we can use the analytically obtained normalized correlation values with and without noise and their associated variances to examine the accuracy (bias error) and reliability (variance) of the two-point measurement of the spatial correlation length.

3. Accuracy and reliability of the correlation length measurement with two spatial points

With experimentally obtained fluctuation data one may estimate the normalized auto- and cross-correlation values at the correlation time delay $\tau = 0$ denoted as $C_1$ and $C_2$, respectively; while the true normalized correlation values are $C_{a,a}^{\text{true}}$ and $C_{a,b}^{\text{true}}$ as shown in figure 3. Here, $C_1$ is the normalized auto-correlation value after removing the noise by using the technique explained earlier [31]. As we are restricted to use only two spatial points, one may fit any arbitrary functions to the data. We have chosen to fit a Gaussian function to the experimentally obtained values $C_1$ and $C_2$ motivated by the experimentally observed ion-scale turbulence in tokamaks [24, 25]:

$$C_2 = C_1 \exp \left( -\frac{(x_a - x_b)^2}{2\ell_x^2} \right),$$

where we have only one unknown $\ell_x$ which is the correlation length that we wish to estimate. The correlation length $\ell_x$ is...
$\sqrt{2}$ times larger than the characteristic spatial scale $\lambda_x$ (see equations (7) and (12)). The measured correlation length $\ell_x$ is, then, estimated to be

$$\ell_x = \begin{cases} \infty & C_1 \leq C_2 \\ \frac{|x_a - x_b|}{\sqrt{2}} \sqrt{\frac{1}{\ln C_1 - \ln C_2}} & C_1 > C_2 > 0 \\ 0 & C_2 \leq 0. \end{cases}$$

(17)

The expected value $\mu_{\ell_x}$ and the variance $\sigma_{\ell_x}^2$ of the measured correlation length $\ell_x$ are

$$\mu_{\ell_x} = \int \int \ell_x p(C_1)p(C_2) dC_1 dC_2$$
$$\sigma_{\ell_x}^2 = \int \int (\ell_x)^2 p(C_1)p(C_2) dC_1 dC_2 - \mu_{\ell_x}^2,$$

(18)

where $p(C_1)$ and $p(C_2)$ are the probability density functions of obtaining $C_1$ and $C_2$, respectively.

### 3.1. Correlation length measurement with two spatial points from the analytic expression and synthetic data

To be able to calculate $\mu_{\ell_x}$ and $\sigma_{\ell_x}^2$, we need to know the probability density function of $\{C_{\text{noise}}^{a,b}\}$, i.e. $p(C_1)$ and $p(C_2)$ in equation (18). The variance of the ensemble averaged correlation value is $1/M$ times the variance of the correlation value where $M$ is the sample number, i.e. the number of sub-time windows used to measure the ensemble average of the correlation value. Owing to the central limit theorem, this probability density function, $p(C_1)$ or $p(C_2)$, is a normal distribution function with the mean of $\{C_{\text{noise}}^{a,b}\}$ and the standard deviation of $\sigma_{\text{noise}}/\sqrt{M}$ which can be estimated either analytically or numerically with the synthetic data.

Once we have the probability density functions $p(C_1) = P\left(C_{\text{noise}}^{a}\right)$ and $p(C_2) = P\left(C_{a,b}^{\text{noise}}\right)$ either from the analytic results or numerical synthetic data, a Monte-Carlo method by generating 10000 samples from these probability density functions is used to generate a histogram of the correlation length normalized to the true correlation length $\ell_x^\text{true}$ as a function of the separation distance between the two measured positions $\Delta x$ normalized to the true correlation length $\ell_x^\text{true}$ as well.

This histogram is, then, used to estimate the mean $\mu_{\ell_x}$ and the variance $\sigma_{\ell_x}^2$ of the two-point correlation length measurements. Note that we ignore the case of $C_1 \leq C_2$ which corresponds to $\ell_x = \infty$. Nevertheless, if $C_1 \leq C_2$
occurs frequently, indicating that the correlation length $\ell_x$ is much larger than the separation distance of the two measurement positions, such a signature will be shown as a large variance according to equations (17) and (18). Here, we examine two major factors that affect the quality of the correlation length measurements: (1) size of the total time window and (2) noise level.

To quantify how the size of the total time window affects the measured correlation length, we generate three sets of synthetic data with $\lambda = 0.3$ m, $\tau_{\text{life}} = 15$ $\mu$s, $v = 5000$ m s$^{-1}$ and $\Delta T_{\text{lab}} = 585$ $\mu$s at the fixed FNR of 0.5, whereas $\Delta T$ are set to be 10000 $\mu$s, 30000 $\mu$s and 50000 $\mu$s. Figure 4 shows the histograms of the obtained correlation length using the analytic results (left panels) and synthetic data (right panels). Based on these histograms, we finally obtain the average and standard deviation of the correlation length (normalized to $\ell_x^{\text{true}}$) as a function of $\Delta x/\ell_x^{\text{true}}$ shown in figure 5. Although figure 5 is more succinct than figure 4, it is better to consider figure 4 because the first ($\mu_1$) and second ($\sigma_1^2$) moments become less representative as the histogram deviates from a normal distribution, especially for small and large $\Delta x/\ell_x^{\text{true}}$.

A similar trend between the analytic and numerical results shows that increasing the size of the total window decreases the standard deviation of the measurements, and the two-point correlation length measurement becomes less reliable if the separation distance between the two measured points are either smaller than or more than two times larger than the actual correlation length of the fluctuation data.

Next, we examine the effect of noise level, i.e. the FNR, on the two-point correlation length measurements. Three sets of synthetic data with $\lambda = 0.3$ m, $\tau_{\text{life}} = 15$ $\mu$s, $v = 5000$ m s$^{-1}$, $\Delta T = 15000$ $\mu$s and $\Delta T_{\text{lab}} = 585$ $\mu$s are generated with different values of FNR: 1.0, 2.0 and 4.0. Figure 6 shows the histograms of the obtained correlation length using the analytic results (left panel) and synthetic data (right panel), whereas the average and standard deviation of the correlation length is shown in figure 7. We find that the lower limit on the $\Delta x/\ell_x^{\text{true}}$ is dependent on the FNR; while the upper limit is not. This observation can be explained based on figure 2. We see that the standard deviation levels are larger at small $\Delta x$ for the smaller FNR. This results in a larger value on the lower limit of $\Delta x/\ell_x^{\text{true}}$ with the smaller FNR. In addition, larger standard deviation at the small $\Delta x/\ell_x^{\text{true}}$ is more likely to have $C_1 \leq C_2$ in equation (17). Figure 2 also shows that although the standard deviation levels are larger with the larger FNR at large $\Delta x$, they are smaller at $\Delta x = 0$. Therefore, we observe a similar upper limit on $\Delta x/\ell_x^{\text{true}}$ for the investigated FNR cases.

3.2. Unnormalized versus normalized correlation function for the correlation length measurement

The variance of the correlation length $\sigma_1^2$ is a function of $p(C_1)$ and $p(C_2)$ as shown in equation (18), i.e. the narrower the $p(C_1)$ and $p(C_2)$, the smaller the variance. Considering an extreme case of absolutely no noise, we see that $p(C_1)$ is a delta function, i.e. no variance, for the normalized correlation function.
(see equation (8)) while it always has a non-zero width, i.e. non-zero variance, for the unnormalized correlation function (see equation (6)). Figure 8 shows the unnormalized (blue) and normalized correlation functions with no noise (red) and FNR = 4.0 (yellow) as a function of $\Delta x / t_{\text{true}}$ calculated with the analytic results. As the $\Delta x / t_{\text{true}}$ approaches zero, error bars become smaller for the normalized correlation functions than the unnormalized ones. Here, the error bars represent the standard deviation of the mean, i.e. $1/\sqrt{M}$ times the standard deviation of the correlation value.

**Figure 7.** Same as figure 5 for a fixed $\Delta T = 15000 \mu s$ with the different values of FNR: 1.0 (blue), 2.0 (red) and 4.0 (yellow).

**Figure 8.** Unnormalized (blue) and normalized correlation functions with no noise (red) and noise level of FNR = 4.0 (yellow) as a function of $\Delta x / t_{\text{true}}$ calculated with the analytic results. Here, the error bars represent the standard deviation of the mean, i.e. $1/\sqrt{M}$ times the standard deviation of the correlation value.
deviation of the mean, i.e. $1/\sqrt{M}$ times the standard deviation of the correlation value; thus, they provide approximate widths of $p(C_1)$ and $p(C_2)$. Direct comparison of the widths of $p(C_2)$ between unnormalized and normalized correlation functions can be tricky as they are functions of $\Delta x$, but the very small width of the $p(C_1)$ at $\Delta x = 0$ for the normalized correlation function, in general, provides better correlation length measurements compared to the unnormalized correlation function.

Figure 9 shows the average and standard deviation of the correlation length estimated with the unnormalized (blue) and normalized correlation functions without noise (red) and with the noise level of FNR = 4.0 (yellow) with the analytic results as in figure 8. We see that the normalized correlation function provides smaller standard deviation and bias error of the measured correlation length, especially for $\Delta x \leq \ell_{x}^{true}$.

3.3. Experimental data: two point correlation length measurement for real density fluctuations

Reliability and accuracy of the two-point correlation length measurement have been examined analytically and numerically. We now apply the two-point correlation length measurement using normalized correlation function (blue line) and unnormalized correlation function (red line) when the noise level is zero. Solid lines show the mean and dashed lines show the standard deviation.

Figure 10. An example of a Gaussian fitting (red) to the correlation values (blue) from poloidally aligned four-point detectors for KSTAR shot #9133. Correlation values for $\Delta x < 0$ are copies of the values from $\Delta x > 0$ to aid visualization. This Gaussian fitting provides the ‘true’ poloidal correlation length $\ell_{x}^{true}$.
measurement on the density fluctuation data experimentally obtained via the BES system [19] installed in KSTAR.

Since the ‘true’ correlation length of the fluctuation data is not available, we estimate the correlation length using the four points (poloidally aligned four-point detectors) denoted as \( \ell_{\theta(4\text{points})} \) and treat this value as the ‘true’ correlation length. Figure 10 shows an example of the four-point measurement of the correlation values (blue) at the time delay \( \tau = 0 \). Note that the correlation values for \( \Delta x < 0 \) are just copies of the values from \( \Delta x > 0 \) to aid visualization. Gaussian fitting (red) to the correlation values provides the correlation length \( \ell_{\theta(4\text{points})} \).

Then, we estimate the correlation length using only two measurement points denoted as \( \ell_{\theta(2\text{points})} \). Figure 11 shows \( \ell_{\theta(2\text{points})}/\ell_{\theta(4\text{points})} \) as a function of \( \Delta x/\ell_{\theta(4\text{points})} \) for different sizes of the total time window \( \Delta T = 10 \text{ ms} \) (blue), 30ms (red) and 50ms (yellow). The result is quantitatively similar to figure 4 (or figure 5), i.e. results obtained analytically and numerically: two-point correlation length measurements are not biased for \( 1 \leq \Delta x/\ell_{\theta(4\text{points})} \leq 2 \), and a larger size of the total time window provides smaller variance.

4. Conclusion

We have analytically derived the normalized and unnormalized correlation functions of Gaussian shaped moving fluctuation data as well as their associated variances. The analytic results are found to have good agreement with the numerical results. Based on the correlation functions, the reliability and accuracy of the two-point correlation length measurement have been examined, where we have found that the separation distance between the two measurements points needs to be within one and two times of the correlation length to obtain reliable and accurate true results. The lower limit is found to be dependent on the ratio of fluctuation to noise. This indicates that if an obtained correlation length from two-point measurements is much larger or smaller than the separation distance, the validity of the obtained correlation length must be questioned. Our results can also be used as design criteria when one builds a diagnostic system measuring fluctuations of any physical quantities with an aim of obtaining correlation lengths.

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Appendix A. Variance of the correlation value

The variance of the estimated correlation value \( \sigma_{\hat{C}_{a,b}}^2 \) by definition is

\[
\sigma_{\hat{C}_{a,b}}^2 = \langle \hat{C}_{a,b}^2 \rangle - \langle \hat{C}_{a,b} \rangle^2.
\] (A.1)
Appendix B. Ensemble average approximation of the normalized correlation function

Here, we explain the rationale and the assumptions we have made in $\left\langle \frac{\sigma_{ab}}{\sigma_f} \right\rangle \approx \left\langle \frac{C_{ab}}{\delta_f^2} \right\rangle$ which is the approximation in the second line of the right-hand-side of equation (7).

Suppose there are two random variables $x$ and $y$ as the realizations of $X$ and $Y$, respectively. For any $f(x, y)$, the bivariate first order Taylor expansion about $(x_0, y_0)$ is

$$f(x, y) = f(x_0, y_0) + \frac{\partial f(x, y)}{\partial x} \bigg|_{(x_0, y_0)} (x - x_0)$$

$$+ \frac{\partial f(x, y)}{\partial y} \bigg|_{(x_0, y_0)} (y - y_0) + \text{remainder.}$$

(B.1)

Applying equation (B.1) with $f(x, y) = x/y$ for $x = C_{ab}$ and $y = \delta_f^2$, the ensemble average of $f(x, y)$ about $x_0 = \langle X \rangle$ and $y_0 = \langle Y \rangle$ is

$$\left\langle \frac{C_{ab}}{\delta_f^2} \right\rangle = \left\langle f(x, y) \right\rangle$$

$$= \left\langle f(\langle X \rangle, \langle Y \rangle) \right\rangle + \frac{\partial f(x, y)}{\partial x} \bigg|_{(\langle X \rangle, \langle Y \rangle)} (\langle X \rangle - \langle X \rangle)$$

$$+ \frac{\partial f(x, y)}{\partial y} \bigg|_{(\langle X \rangle, \langle Y \rangle)} (\langle Y \rangle - \langle Y \rangle) + \text{remainder}$$

$$\approx \left\langle \frac{C_{ab}}{\delta_f^2} \right\rangle = \left\langle \frac{C_{ab}}{\delta_f^2} \right\rangle.$$

(B.2)

Ignoring the remainder, i.e. the higher order terms, the last line is valid when the standard deviation of a random variable ($\sigma_f$) is small compared to its mean (or expectation) value ($\langle Y \rangle$) [32]. Note that the higher order terms in $x$, i.e. $\partial^n/\partial x^n$ for $n \geq 2$, are zeros for $f(x, y) = x/y$. Thus, the ensemble average approximation $\left\langle \frac{C_{ab}}{\delta_f^2} \right\rangle \approx \left\langle \frac{C_{ab}}{\delta_f^2} \right\rangle$ is valid for $\sigma_{ab} = \sigma_f^2 \ll \langle \delta_f^2 \rangle = \langle C_{ab} \rangle$, which is satisfied for $\tau_{life} \ll \Delta T_{sub}$ according to equations (4) and (6).

Appendix C. Variance of the normalized correlation value with and without noise

C.1. Without noise

To derive the variance of the normalized correlation value $\sigma_{C_{ab}}^2$, we invoke an approximated form of the variance of products [33] for $g = A/B$, namely

$$\sigma_g^2 \approx g^2 \left[ \left( \frac{\sigma_A}{A} \right)^2 + \left( \frac{\sigma_B}{B} \right)^2 - 2 \rho_{AB} \frac{\sigma_A \sigma_B}{AB} \right]$$

(C.1)
where $\sigma_g$, $\sigma_A$, and $\sigma_B$ are the standard deviations of $g$, $A$, and $B$, respectively. $\rho_{AB}$ is the normalized correlation between $A$ and $B$. For our case, $g = \langle C_{a,b} \rangle$, $A = \langle C_{a,b} \rangle$, and $B = \langle \delta_a \delta_b \rangle = \langle \delta_f^2 \rangle$. A difficulty to derive $\sigma_C^{2a}$ arises due to the correlation between $C_{a,b}$ and $\delta_f^2$, i.e., $\rho_{AB}$. For this reason, we take two extreme cases where $\rho_{AB} \approx 1$ and $\rho_{AB} \approx 0$, and connect them smoothly based on an educated guess. Note that we confirm numerically in section 2.3 that our result is indeed valid.

For $x_a = x_b$, we know that $C_{a,b} = C_{a,a} = \delta_f^2$, i.e. $A = B$, resulting in $\rho_{AB} = 1$. This simply gives $\sigma_g^2 = 0$, i.e. $\sigma_C^{2a} = 0$. This result is what we expect since we know that the normalized auto-correlation at the time delay $\tau = 0$ is unity by definition, and there is no variance associated with it if the signal contains no noise.

The opposite extreme case is obtained by considering two spatial positions where $|x_a - x_b| \gg \lambda$, such that there is absolutely no correlation between the signals at $x_a$ and $x_b$, i.e.
Thus, there is no correlation between $C_{a,b}$ and $\delta_f^2$, resulting in $\rho_{AB} \approx 0$. For this case, $\sigma_{C_{a,b}}^2$ becomes

$$
\sigma_{C_{a,b}}^2 \approx \left( \frac{\langle C_{a,b} \rangle}{\delta_f^2} \right)^2 \left[ \frac{\sigma_{C_{a,b}}}{\langle C_{a,b} \rangle} + \left( \frac{\sigma_{\delta_f^2}}{\langle \delta_f^2 \rangle} \right)^2 \right],
$$

$$
\approx \frac{\sigma_{C_{a,b}}^2}{\pi^2 A^2 F^2},
$$

(C.2)

where we have ignored the second term inside the square bracket for very small $\langle C_{a,b} \rangle$ to get the last line and used the fact that $\langle \delta_f^2 \rangle = \pi A^2 F$.

With these two results on $\sigma_{C_{a,b}}^2$ for $|x_a - x_b| = 0$ and $|x_a - x_b| \gg \lambda_e$, we connect them with a Gaussian shape motivated by the form of variance for the unnormalized correlation value $\sigma_{C_{a,b}}^2$ as in equation (6). Thus, we finally obtain the variance of the normalized correlation value as:

$$
\sigma_{\tilde{C}_{a,b}}^2 \approx \frac{\sigma_{C_{a,b}}^2}{\pi^2 A^2 F^2} \left( 1 - \exp \left[ -\frac{(x_a - x_b)^2}{2\lambda_e^2} \right] \right)^2,
$$

(C.3)

which is what we have in equation (8).

C.2. With noise

To obtain the variance of the normalized correlation value with noise $\sigma_{\tilde{C}_{a,b}}^2$, we again use an approximated form of the variance of products [33] for $g = AB$ which is

$$
\sigma_{\tilde{C}_{a,b}}^2 \approx g^2 \left[ \frac{\sigma_A^2}{A^2} + \frac{\sigma_B^2}{B^2} + 2\rho_{AB} \frac{\sigma_A \sigma_B}{AB} \right],
$$

(C.4)

where $g = \langle \tilde{C}_{a,b}^\text{noise} \rangle$, $A = \langle \delta_f^2 \rangle / \langle \delta_f^2 + \delta_m^2 \rangle$ and $B = \langle \tilde{C}_{a,b} \rangle$ with $\sigma_g^2 = \sigma_{\tilde{C}_{a,b}}^2$, $\sigma_A^2 = \sigma_{\delta_f^2}^2 \langle \delta_f^2 \rangle$ and $\sigma_B^2 = \sigma_{\delta_m^2}^2 \langle \delta_m^2 \rangle$ is the normalized correlation between $\delta_f^2(\delta_f^2 + \delta_m^2)$ and $\tilde{C}_{a,b}$ which is zero because the normalized correlation value $\tilde{C}_{a,b}$ is independent of the fluctuation level.

Since $\sigma_{\delta_f^2(\delta_f^2 + \delta_m^2)}^2$ is unknown, we calculate it using equation (C.1) where the variances of $\delta_f^2$ and $\delta_m^2$ are $\sigma_{\delta_f^2}$ and $\sigma_{\delta_m^2}$ respectively. Then, the variance of $\delta_f^2 + \delta_m^2$ denoted as $\sigma_{\delta_f^2 + \delta_m^2}^2$ is $\sigma_{\delta_f^2}^2 + \sigma_{\delta_m^2}^2$ because $\delta_f^2$ and $\delta_m^2$ are uncorrelated. Furthermore, the normalized correlation between $\langle \delta_f^2 \rangle$ and $\langle \delta_f^2 + \delta_m^2 \rangle$ are approximately unity since $\delta_m^2$ is a constant as long as the sub-time window size $\Delta T_{ab}$ is much larger than the auto-correlation time of the noise which is a sampling time. Thus, $\sigma_{\tilde{C}_{a,b}}^2$ is

$$
\sigma_{\tilde{C}_{a,b}}^2 \approx \sigma_{\delta_f^2(\delta_f^2 + \delta_m^2)}^2 \left[ \frac{\langle \delta_f^2 \rangle}{\langle \delta_f^2 \rangle + \langle \delta_m^2 \rangle} \right]^2 \times \left[ \frac{\sigma_{\delta_f^2}^2}{\langle \delta_f^2 \rangle} + \frac{\sigma_{\delta_m^2}^2}{\langle \delta_m^2 \rangle - 2 \frac{\sigma_{\delta_f^2} \sigma_{\delta_m^2}}{\langle \delta_f^2 \rangle \langle \delta_m^2 \rangle} \right] = \left( \frac{\langle \delta_f^2 \rangle}{\langle \delta_f^2 \rangle + \langle \delta_m^2 \rangle} \right)^2 \left[ \frac{\sigma_{\delta_f^2}^2}{\langle \delta_f^2 \rangle} + \frac{\sigma_{\delta_m^2}^2}{\langle \delta_m^2 \rangle} \right].
$$

We then obtain $\sigma_{\tilde{C}_{a,b}}^2$ as

$$
\sigma_{\tilde{C}_{a,b}}^2 \approx \left( \frac{\langle \tilde{C}_{a,b} \rangle}{\langle \delta_f^2 \rangle + \langle \delta_m^2 \rangle} \right)^2 \left[ \frac{\sigma_{\delta_f^2}^2}{\langle \delta_f^2 \rangle} + \frac{\sigma_{\delta_m^2}^2}{\langle \delta_m^2 \rangle} \right] = \left( \frac{\langle \tilde{C}_{a,b} \rangle}{\langle \delta_f^2 \rangle + \langle \delta_m^2 \rangle} \right)^2 \left[ \frac{\sigma_{\delta_f^2}^2}{\langle \delta_f^2 \rangle} + \frac{\sigma_{\delta_m^2}^2}{\langle \delta_m^2 \rangle} \right].
$$

(C.6)

where we assume that $\sigma_{\delta_f^2}^2 \gg \sigma_{\delta_m^2}^2$ in the last line approximation since $\delta_m^2$ is almost a constant. This is the equation (13). Figure 2 shows how our analytical result agrees with the numerically estimated variances using the synthetic data including the noise effects.
Appendix D. The two-point measurement of the correlation length for the Lorentzian-shaped eddies

As the Lorentzian-shaped eddies at the edge (SOL) of the magnetically confined plasmas are observed [27–29], we discuss the accuracy and reliability of the two-point measurement of the correlation length for the Lorentzian-shaped eddies in this section. Fluctuating signals are the sum of eddies as in equation (1) with moving Lorentzian eddies:

\[ S_n(t) = A_i \frac{1}{\left( t - t_i \right)^2 + 1} \frac{1}{\left( x_n - x_i \right)^2 + 1}, \]  

(D.1)

where \( A_i \) is selected from a normal distribution; while \( x_i \) and \( t_i \) are generated from uniform distributions.

Once we have the synthetic data with Lorentzian eddies, we follow similar steps as in section 3 with the Lorentzian fitting function:

\[ C_2 = C_1 \frac{1}{\left( x_n - x_i \right)^2 + 1}, \]  

(D.2)

and the measured correlation length (\( \ell_x \)) is estimated to be

\[ \ell_x = \begin{cases} \infty & C_1 \leq C_2 \\ \sqrt{\frac{C_2}{C_1}} & C_1 > C_2 > 0 \\ 0 & C_2 \leq 0. \end{cases} \]  

(D.3)

Note that the covariance function of the Lorentzian function shows that the correlation length (\( \ell_x \)) is twice as large as the characteristic length (\( \lambda_\lambda \)), i.e. \( \ell_x^{true} = 2 \lambda_\lambda \):

\[ \int_{-\infty}^{\infty} \frac{1}{\left( x/\lambda_\lambda \right)^2 + 1} \frac{1}{\left( x-x'\right)^2 + 1} \frac{1}{\left( x/\lambda_\lambda \right)^2 + 1} dx' = \frac{\pi \lambda_\lambda}{2} \left( \frac{x}{\lambda_\lambda} \right)^2 + 1. \]  

(D.4)

We obtain the expected value \( \mu_{\ell_x} \) and the variance \( \sigma_{\ell_x}^2 \) using equation (18). The probability density functions \( p(C_1) = P\left( C_{a,\text{true}} \right) \) and \( p(C_2) = P\left( C_{a,\text{true}} \right) \) are obtained from the synthetic data, and a Monte-Carlo method is also used to generate histograms similar to figures 4 and 6. Again, we investigate the accuracy and reliability of the two-point measurement for different sizes of the total time window and different values of FNR.

To investigate how the size of the total time window affects the measured correlation length, we generate three sets of synthetic data with \( \lambda_\lambda = 0.3 \) m, \( \tau_{\text{life}} = 15 \) µs, \( v = 5000 \) m s\(^{-1} \) and \( \Delta T_{\text{lab}} = 585 \) µs at the fixed FNR of 0.5, whereas \( \Delta T \) are set to be 10 000 µs, 30 000 µs and 50 000 µs. Note that these values are the same as those used to generate figure 4. Figure D1 shows the histogram of the obtained correlation length with the \( p(C_1) \) and \( p(C_2) \) from the synthetic data, and figure D2 shows the mean and the standard deviation estimated using equation (18).

For the cases of different values of FNR, we generate the synthetic data with FNR \( = 1.0, 2.0 \) and 4.0 with \( \Delta T = 15 \) 000 µs; whereas all other values are kept the same as before. The results are shown in figures D3 and D4. We find that the Lorentzian-shaped eddies are not very much different from the Gaussian-shaped eddies, see figures 4–7.

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