Non-Decoupling Effects of the Heavy $T$

in the $B_{d,s}^0 - \bar{B}_{d,s}^0$ Mixing and

Rare $K$ and $B$ Decays

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Abstract

We point out that in the case of a heavy top quark $T$, present in the Littlest Higgs model (LH), and the $t$-$T$ mixing parameter $x_L \geq 0.9$ the contribution to $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing from box diagrams with two $T$ exchanges cannot be neglected. Although formally $O(v^4/f^4)$ with $v = 246$ GeV and $f > 1$ TeV, this contribution increases linearly with $x_T = m_T^2/M_W^2$ and with $x_T = O(f^2/v^2)$ constitutes effectively an $O(v^2/f^2)$ correction. For $x_L \approx 1$, this contribution turns out to be more important than the genuine $O(v^2/f^2)$ corrections. In particular it is larger than the recently calculated $O(v^2/f^2)$ contribution of box diagrams with a single $T$ exchange that increases only logarithmically with $x_T$. For $x_L = 0.95$ and $f/v = 5, 10, 15$, the short distance function $S$ governing the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing mass differences $\Delta M_{d,s}$ receives 56%, 15% and 7% enhancements relative to its Standard Model (SM) value, implying a suppression of the CKM element $|V_{td}|$ and an enhancement of $\Delta M_s$. The short distance functions $X$ and $Y$, relevant for rare $K$ and $B$ decays, increase only logarithmically with $x_T$. With the suppressed $|V_{td}|$, $K \rightarrow \pi \nu \bar{\nu}$ and $B_d \rightarrow \mu^+ \mu^-$ decays are only insignificantly modified with respect to the SM, while the branching ratio $Br(B_s \rightarrow \mu^+ \mu^-)$ receives 66%, 19% and 9% enhancements for $x_L = 0.95$ and $f/v = 5, 10, 15$, respectively. Similar enhancement is found for $Br(B_s \rightarrow \mu \bar{\nu})/Br(B_d \rightarrow \mu \bar{\nu})$. 
1 Introduction

It is well known that in the Standard Model (SM), flavour changing neutral current processes (FCNC) such as $B^0_{d,s} - \bar{B}^0_{d,s}$ mixing, CP violation in $K \rightarrow \pi\pi$ and rare $K$ and $B$ decays are dominated by the contributions of top quark exchanges in box and penguin diagrams $[1, 2]$. This dominance originates in the large mass $m_t$ of the top quark and in its non-decoupling from low energy observables due to the corresponding Yukawa coupling that is proportional to $m_t$. In the evaluation of box and penguin diagrams in the Feynman-t’Hooft gauge this decoupling is realized through the diagrams with internal fictitious Goldstone boson and top quark exchanges. The couplings of Goldstone bosons to the top quark, being proportional to $m_t$, remove the suppression of the diagrams in question due to top quark propagators so that at the end the box and penguin diagrams increase with increasing $m_t$. In the unitary gauge, in which fictitious Goldstone bosons are absent, this behaviour originates from the longitudinal ($k_{\mu}k_{\mu}/M^2_W$) component of the $W^\pm$-propagators.

In particular, in the case of $B^0_{d,s} - \bar{B}^0_{d,s}$ mixing in the SM the relevant $m_t$ dependent Inami-Lim function $S(x_t) \equiv S_0(x_t)$ $[1, 2]$ has the following large $m_t$ behaviour $[2]$

$$S(x_t) \rightarrow \frac{x_t}{4}, \quad x_t = \frac{m_t^2}{M^2_W}.$$  \hfill (1.1)

Similarly the functions $X(x_t)$ and $Y(x_t)$, relevant for instance for $K \rightarrow \pi\nu\bar{\nu}$ and $B_{d,s} \rightarrow \mu^+\mu^-$, respectively, have the following large $m_t$ behaviour

$$X(x_t) \rightarrow \frac{x_t}{8}, \quad Y(x_t) \rightarrow \frac{x_t}{8}.$$  \hfill (1.2)

Yet, with $x_t \approx 4.4$, these formulae are very poor approximations of the true values $S = 2.42$, $X = 1.54$ and $Y = 0.99$. We will see below that in the case of the Littlest Higgs (LH) model, the value of the corresponding variable $x_T$ is at least 400 and the asymptotic formulae presented below are excellent approximations of the exact expressions.

In the Little Higgs models $[4]-[8]$, that offer an attractive and a rather simple solution to the gauge hierarchy problem, there is a new very heavy top quark $T$. In the LH model $[7]$ its mass is given by

$$m_T = \frac{f}{\lambda_1^2 + \lambda_2^2}, \quad x_T = \frac{x_L}{\lambda_1^2 + \lambda_2^2}. \hfill (1.3)$$

Here $\lambda_i$ parametrize the Yukawa interactions of the top quark and $v = 246$ GeV is the vacuum expectation value of the SM Higgs doublet. The parameter $x_L$ enters the sine of
the $t$-$T$ mixing which is simply given by $x_L v/f$. The new scale $f > 1$ TeV is related to
\[ \Lambda \sim 4\pi f \sim \mathcal{O}(10 \text{ TeV}) \]
at which the gauge group of the LH model $[SU(2)_1 \otimes U(1)_1] \otimes [SU(2)_2 \otimes U(1)_2]$ is broken down to the SM gauge group. The SM results for various
observables of interest receive $\mathcal{O}(v^2/f^2)$ corrections that originate in new heavy gauge
boson and scalar exchanges and in particular in the diagrams with the heavy $T$. The
constraints from various processes, in particular from electroweak precision observables
and direct new particles searches, have been extensively analyzed in [9]-[16].

As already discussed in [9, 17], the parameter $x_L$ describes together with $v/f$ the size
of the violation of the three generation CKM unitarity and is also crucial for the gauge
interactions of the heavy $T$ quark with the ordinary down quarks. $x_L$ can in principle
vary in the range $0 < x_L < 1$. For $x_L \approx 1$, the mass $m_T$ becomes large and its coupling
to the ordinary $W^\pm_L$ bosons and the down quarks, $W^\pm_L \bar{T}d_j$, being $\mathcal{O}(x_L v/f)$, is only
suppressed by $v/f$. In Fig. 1 we show the dependence of $m_T$ on $x_L$ for three values of
$f/v$.

We are aware of the fact that with increasing $m_T$ also one-loop corrections to the
SM Higgs mass increase. Typically for $m_T \geq 6$ TeV a fine-tuning of at least 1\% has to be made in order to keep $m_H$ below 200GeV [11, 18]. As roughly $f/v \geq 8$ is required
by electroweak precision studies [9]-[16], the non-decoupling effects of $T$ considered here can be significant and simultaneously consistent with these constraints only in a narrow range of $f/v$. But these bounds are clearly model dependent and we will consider the range $5 \leq f/v \leq 15$ and $x_L \leq 0.95$ for completeness.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The mass of $T$ as a function of $x_L$ for different values of $f/v$.}
\end{figure}

As with $f/v \geq 5$ the $T$ quark is by an order of magnitude heavier than the ordinary
top quark, it is of interest to ask what are its effects in the FCNC processes. In the
present letter we summarize the main results of an analysis of $T$ contributions to the
function $S$ for $x_L$ close to unity, pointing out that the existing $\mathcal{O}(v^2/f^2)$ calculations

of the function $S$ in the LH model \[17,19\] are inadequate for the description of this parameter region. The $O(v^4/f^4)$ contributions involving the $T$ quark have to be taken into account as well and in fact for $x_L > 0.85$ these $O(v^4/f^4)$ corrections turn out to be as important as the genuine $O(v^2/f^2)$ corrections. For higher values of $x_L$ they become even dominant. The details of these investigations will appear in an update of \[17\].

On the other hand our analysis of the functions $X$ and $Y$ \[20\] shows that they increase with $x_T$ only as log $x_T$ and that an $O(v^2/f^2)$ analysis is sufficient for the study of the large $x_T$ behaviour. Below we will summarize the main results of this analysis. The details will be presented in \[20\].

2 The Function $S$ and $\Delta M_{d,s}$ in the LH Model

In \[17\] we have calculated the $O(v^2/f^2)$ contributions to the function $S$ in the LH model. Our results have been recently confirmed in \[19\]. The $O(v^2/f^2)$ correction $\Delta S$ can be written as follows

$$\Delta S = (\Delta S)_T + (\Delta S)_{W^\pm} + (\Delta S)_{\text{Rest}}$$

with the three contributions representing the $T$, $W^\pm$ (new heavy charged gauge bosons) and the remaining contributions that result from $O(v^2/f^2)$ corrections to the vertices involving only SM particles.

As demonstrated in \[17\] the three contributions in question are given to a very good approximation as follows ($W^\pm_L \equiv W^\pm$)

$$(\Delta S)_T = \left[\frac{1}{2} \frac{v^2}{f^2} x_t^2\right] x_t (\log x_T - 1.57), \quad x_T = \frac{m_t^2}{M_{W^\pm}^2},$$

$$(\Delta S)_{W^\mp} = 2 \frac{c_s^2}{s^2} m_t^2 M_{W^\pm}^2 = 2 \frac{v^2}{f^2} c_t^4 x_t,$$

$$(\Delta S)_{\text{Rest}} = -2 \frac{v^2}{f^2} c_t^4 S_{\text{SM}}(x_t),$$

where $x_t$ has been defined in \[14\], $S(x_t)_{\text{SM}} = S_0(x_t)$ in \[17\] and the numerical factor in (2.2) corresponds to $m_t(m_t) = 168.1$ GeV. The mixing parameters $s$ and $c$ \[9\] are related through $s = \sqrt{1 - c^2}$ with $0 < s < 1$.

For $x_L \leq 0.8$ considered in \[17,19\] and $s < 0.4$ the most important contribution turns out to be $(\Delta S)_{W^\pm}$. However, for higher values of $s$ and $x_L > 0.7$, the contribution $(\Delta S)_T$ becomes more important.

We observe that all three contributions have a characteristic linear behaviour in $x_t$ that signals the non-decoupling of the ordinary top quark. However, the corresponding non-decoupling of $T$ is only logarithmic. This is related to the fact that with the $W^\pm_L T d_j$
coupling being $\mathcal{O}(v/f)$ only box diagrams with a single $T$ exchange (see Fig. 2) contribute at $\mathcal{O}(v^2/f^2)$. Similarly to the SM box diagrams with a single $t$ exchange, that increase as $\log x_t$, the $T$ contribution in the LH model increases only as $\log x_T$.

Figure 2: Single and double heavy Top contributions to the function $S$ in the LH model at $\mathcal{O}(v^2/f^2)$ and $\mathcal{O}(v^4/f^4)$, respectively.

Here we would like to point out that for $x_L \geq 0.85$, the term $(\Delta S)_T$ in (2.2) does not give a proper description of the non-decoupling of $T$. Indeed in this case also the box diagram with two $T$ exchanges given in Fig. 2 has to be considered. Although formally $\mathcal{O}(v^4/f^4)$, this contribution increases linearly with $x_T$ and with $x_T = \mathcal{O}(f^2/v^2)$ constitutes effectively an $\mathcal{O}(v^2/f^2)$ contribution.

Calculating the diagram with two $T$ exchanges in Fig. 2 and adding those $\mathcal{O}(v^4/f^4)$ corrections from box diagrams with $t$, $T$ and $u$ quark exchanges [17], that have to be taken into account in order to remove the divergences characteristic for a unitary gauge calculation and for the GIM mechanism [21] to become effective, we find

$$(\Delta S)_{TT} \approx \frac{v^4 x_L^4 x_T}{4} \frac{x_t^3}{f^2 (1 - x_L)} \frac{x_t}{4}.$$  (2.5)

Formula (2.5) represents for $x_L > 0.85$ and $f/v \geq 5$ the exact expression given in [17] to within 3% and becomes rather accurate for $x_L > 0.90$ and $f/v \geq 10$.

In fact the result in (2.5) can easily be understood. $(\Delta S)_{TT}$ has a GIM structure [17]

$$(\Delta S)_{TT} = \frac{v^4}{f^4 x_L^4} [F(x_T, x_T; W_L) + F(x_t, x_t; W_L) - 2F(x_t, x_T; W_L)]$$  (2.6)

with the function $F(x_i, x_j; W_L)$ resulting up to an overall factor from box diagram with two $W_L^\pm$ and two quarks $(i, j)$ exchanges. This GIM structure is identical to the one of $S$ in the SM that depends on $x_t$ and $x_u$ and is given by

$$S(x_t) = F(x_t, x_t; W_L) + F(x_u, x_u; W_L) - 2F(x_u, x_t; W_L).$$  (2.7)
For large $x_T$ it turns out to be a good approximation to evaluate $(\Delta S)_{TT}$ with $x_t = 0$. In this case (2.6) reduces to $S(x_t)$ in (2.7) with $x_t$ replaced by $x_T$ and $x_u$ by $x_t$. The factor $x_t/4$ in (1.1) is then replaced by $x_T/4$ as seen in (2.5).

In Fig. 3 we show the four contributions $(\Delta S)_i$ as functions of $x_L$ for $f/v = 5$, $f/v = 10$ and $s = \sqrt{1-c^2} = 0.2$. We observe that for $x_L < 0.85$ the new contribution is smaller than $(\Delta S)_T$ but for $x_L > 0.90$ it becomes a significant new effect in $S$.

A general upper bound on the function $S$ in models with minimal flavour violation (MFV) can be obtained from the usual analysis of the unitarity triangle [22]. It is valid also in the LH model considered here. A recent update [23] of this bound gives

$$S \leq 3.3, \quad (95\% \; \text{C.L.}) \quad (2.8)$$

to be compared with $S_{SM} = 2.42$ in the SM [2]. In Fig. 4 we plot

$$S_{LH} = S_{SM}(x_t) + (\Delta S)_{TT} + (\Delta S)_T + (\Delta S)_{W^\pm} + (\Delta S)_{\text{Rest}} \quad (2.9)$$
as a function of $x_L$ for different values of $f/v$ and $s = 0.2$. We also show there the SM value and the upper bound in (2.8). For a comparison we recall that from the studies of the $\rho$ parameter an upper bound on $x_L$ of 0.95, almost independently of $f/v$, has been obtained [13]. We observe that for $f/v = 5$ a bound $x_L \leq 0.90$ at 95% C.L. can be obtained. Weaker bounds are found for $5 < f/v < 10$ and in order to find a bound stronger than in [13] for $f/v > 10$, $S_{\text{max}}$ should be within 10% of the SM value. This would require significant reduction of the theoretical uncertainties in the analysis of the unitarity triangle.

With the precisely measured $\Delta M_d$ and $|V_{ts}| \approx |V_{cb}|$, the observed enhancement of the
function $S$ implies
\[
\frac{(|V_{td}|)_{\text{LH}}}{(|V_{td}|)_{\text{SM}}} = \sqrt{\frac{S_{\text{SM}}}{S_{\text{LH}}}}, \quad \frac{(\Delta M_s)_{\text{LH}}}{(\Delta M_s)_{\text{SM}}} = \frac{S_{\text{LH}}}{S_{\text{SM}}},
\]
that is the suppression of $|V_{td}|$ and the enhancement of $\Delta M_s$. As we will see below in the context of $K \to \pi \nu \bar{\nu}$ and $B_d \to \mu^+ \mu^-$ decays, the suppression of $|V_{td}|$ compensates to a large extent the enhancements of the functions $X$ and $Y$ in the relevant branching ratios. On the other hand, $\Delta M_s$ involving $|V_{ts}|$ and being proportional to $S_{\text{LH}}$, is for $x_L = 0.95$ enhanced by 56%, 15% and 7% for $f/v = 5, 10, 15$, respectively.

3 Rare $K$ and $B$ Decays in the LH Model

The analysis of the functions $X$ and $Y$ is much more involved due to many box and penguin diagrams with $T$ and new heavy gauge boson $Z_H^0$, $A_H^0$ and $W_H^\pm$ exchanges. The details of this analysis in the full space of the parameters involved will be presented in [20]. Here we present only the results for $x_L > 0.7$, where the diagrams with the heavy $T$ and ordinary SM particles shown in Fig. 5 become dominant.

Due to a different topology of penguin diagrams and the fact that in the relevant box diagrams with $\nu \bar{\nu}$ and $\mu \bar{\mu}$ in the final state only a single $T$ can be exchanged, the $O(v^2/f^2)$ diagrams in Fig. 5 give an adequate description of the $x_L > 0.7$ region. The diamonds in this figure indicate $O(x_T^2 v^2/f^2)$ corrections to the SM vertices that are explicitly given in [9, 17]. Many of the diagrams in this figure give in the unitary gauge contributions to $X$ and $Y$ that grow as $x_T$ and $x_T \log x_T$ but due to the GIM mechanism all these contributions cancel each other in the sum. In particular the penguin diagram involving two $T$ propagators, being proportional to $\sin^2 \theta_W$, is canceled by other diagrams.
involving $\sin^2 \theta_w$ so that $\sin^2 \theta_w$ does not appear in the final result for $X$ and $Y$ as it should be.

$$\sin^2 \theta_w$$

Figure 5: Top and heavy top quark contributions to the function $X$ in the LH model at $\mathcal{O}(v^2/f^2)$ which are proportional to $x_L^2$.

We find then that to a very good approximation the corrections to $X$ and $Y$ in the LH model are given in the $x_L > 0.7$ region as follows

$$\Delta X = \left[ \frac{v^2}{f^2} x_L^2 \right] \left[ (x_t \frac{3}{8} + \frac{3}{8}) \log x_T - 3.32 \right],$$

(3.1)

$$\Delta Y = \left[ \frac{v^2}{f^2} x_L^2 \right] \left[ (x_t \frac{3}{8} + \frac{3}{8}) \log x_T - 3.53 \right],$$

(3.2)

with the numerical factors corresponding to $m_t(m_t) = 168.1$ GeV. Exact formulae will be presented in [20]. A characteristic $x_t$-decoupling is observed with a logarithmic dependence on $x_T$. The combined dependence on $x_t$ and $x_T$ in this leading contribution
makes it clear from which diagrams in Fig. 5 this leading contribution comes from. This is the $Z^0$-penguin diagram with both $T$ and $t$ exchanges and the corresponding diagram with $t$ and $T$.

In Fig. 6 we show

$$X_{\text{LH}} = X_{\text{SM}} + \Delta X, \quad Y_{\text{LH}} = Y_{\text{SM}} + \Delta Y$$

as functions of $x_L$ for three values of $f/v$. We observe that due to the fact that $\Delta X \approx \Delta Y$ but $X_{\text{SM}} \approx 1.6 \, Y_{\text{SM}}$, the relative corrections to $Y$ are larger.

The branching ratios for $K_L \to \pi^0 \nu \bar{\nu}$ and $B_{s,d} \to \mu^+ \mu^-$ are proportional to $X_{\text{LH}}^2$ and $Y_{\text{LH}}^2$, respectively. The branching ratio for $K^+ \to \pi^+ \nu \bar{\nu}$ grows slower with $X_{\text{LH}}$ due to an additive charm contribution that is essentially unaffected by the LH contributions. Now, $K_L \to \pi^0 \nu \bar{\nu}$, $K^+ \to \pi^+ \nu \bar{\nu}$ and $B_d \to \mu^+ \mu^-$ involve the CKM element $V_{td}$, whose value is decreased in the LH model as discussed in the previous section. Consequently the enhancement of $X$ and $Y$ in the LH model is compensated by the suppression of $V_{td}$ at the level of the branching ratios. The situation is quite different in $B_s \to \mu^+ \mu^-$, where the relevant CKM element $|V_{ts}|$ is approximately equal to $|V_{cb}|$ and the enhancement of $Y$ is fully visible in the branching ratio. Explicitly we have

$$\frac{Br(K_L \to \pi^0 \nu \bar{\nu})_{\text{LH}}}{Br(K_L \to \pi^0 \nu \bar{\nu})_{\text{SM}}} = \frac{S_{\text{SM}}}{S_{\text{LH}}} \frac{X_{\text{LH}}^2}{X_{\text{SM}}^2},$$

$$Br(B_d \to \mu^+ \mu^-)_{\text{LH}} = \frac{S_{\text{SM}}}{S_{\text{LH}}} \frac{Y_{\text{LH}}^2}{Y_{\text{SM}}^2},$$

$$Br(B_s \to \mu^+ \mu^-)_{\text{LH}} = \frac{Y_{\text{LH}}^2}{Y_{\text{SM}}^2}.$$
This pattern is clearly seen in Table 1. The values given there have been obtained by using the MFV formulae for branching ratios given in [24] with $F_{B_d} = 203$ MeV, $F_{B_s} = 238$ MeV and setting the three CKM parameters $|V_{us}|$, $|V_{cb}|$ and the angle $\beta$ in the unitarity triangle to

$$|V_{us}| = 0.224, \quad |V_{cb}| = 0.0415, \quad \beta = 23.3^\circ. \quad (3.7)$$

The fourth parameter, the UT side $R_t$ is then calculated according to

$$(R_t)_{\text{LH}} = (R_t)_{\text{SM}} \sqrt{\frac{S_{\text{SM}}}{S_{\text{LH}}}}, \quad (R_t)_{\text{SM}} = 0.89 \quad (3.8)$$

with $(R_t)_{\text{SM}} = 0.89$ being the central value of the UT fit in [25].

We observe that $Br(K^+ \to \pi^+ \nu \bar{\nu})$ and $Br(K_L \to \pi^0 \nu \bar{\nu})$ are very close to the SM value and with increasing $x_L$ they are even slightly suppressed. $Br(B_d \to \mu^+ \mu^-)$ is slightly enhanced with respect to the SM, while the enhancement of $Br(B_s \to \mu^+ \mu^-)$ for $f/v \leq 10$ is significant.

### Table 1: Branching ratios for rare decays in the LH model and the SM for $f/v = 5$. In the last two rows the results for $f/v = 10$ are given.

| $x_L$ | 0.80 | 0.90 | 0.95 | SM |
|-------|------|------|------|----|
| $Br(K^+ \to \pi^+ \nu \bar{\nu}) \times 10^{11}$ | 7.91 | 7.78 | 7.34 | 7.88 |
| $Br(K_L \to \pi^0 \nu \bar{\nu}) \times 10^{11}$ | 3.07 | 3.00 | 2.76 | 3.05 |
| $Br(B_d \to \mu^+ \mu^-) \times 10^{10}$ | 1.32 | 1.34 | 1.27 | 1.20 |
| $Br(B_s \to \mu^+ \mu^-) \times 10^9$ | 5.17 | 5.81 | 6.38 | 3.86 |
| $R_{sd}$ | 39.2 | 43.4 | 50.4 | 32.2 |
| $Br(B_s \to \mu^+ \mu^-) \times 10^9$ | 4.27 | 4.44 | 4.58 | 3.86 |
| $R_{sd}$ | 34.3 | 35.4 | 37.2 | 32.2 |

The branching ratios $Br(B_{s,d} \to \mu \bar{\mu})$ and $\Delta M_{s,d}$ are subject to uncertainties in $|V_{td}|$ and the meson decay constants $F_{B_{s,d}}$. On the other hand the ratio

$$R_{sd} = \frac{Br(B_s \to \mu \bar{\mu})}{Br(B_d \to \mu \bar{\mu})} \quad (3.9)$$

is theoretically cleaner. We show this ratio in Table 1.

### 4 Conclusions

We have calculated the dominant $\mathcal{O}(v^4/f^4)$ corrections to the function $S$ in the LH model. Due to a large value of $m_T$, this contribution can compete with the genuine
$O(v^2/f^2)$ corrections. For $x_L > 0.90$ it becomes a significant new contribution to the function $S$ in this model. As seen in (2.10), the enhancement of $S$ implies the suppression of the value of $|V_{td}|$ in the LH model and the enhancement of $\Delta M_s$ with the size of these effects depending sensitively on $x_L$ and $f/v$ (see Fig. 4). With the improved precision of the unitarity triangle analysis an upper bound on $x_L$, that is stronger than the one coming from the analysis of the $\rho$ parameter, could in principle be obtained.

The non-decoupling effects of $T$ in rare $K$ and $B$ decays discussed above are weaker, with the functions $X$ and $Y$ growing only logarithmically with $x_T$. We find that the branching ratios for $K \to \pi \nu \bar{\nu}$ and $B_d \to \mu^+\mu^-$ are only insignificantly modified by the LH effects because the enhancements of $X$ and $Y$ are compensated by the decrease of $|V_{td}|$. On the other hand $Br(B_s \to \mu^+\mu^-)$ can be enhanced up to 66%, 19% and 9% for $x_L = 0.95$ and $f/v = 5, 10, 15$, respectively. This pattern is insignificantly modified through the effects of contributions involving the new gauge bosons $Z_H^0, A_H^0$ and $W_H^\pm$.

The effects presented here are certainly of theoretical interest, although for $f/v \geq 8$ as required by other studies [9]-[16], the corrections to the SM results for $S$, $X$ and $Y$ with $x_L$, even as high as 0.95, are only at most 15% with the only relevant enhancements seen in $\Delta M_s$ and $Br(B_s \to \mu^+\mu^-)$. On the other hand the effects found here could be larger in other Little Higgs models and our analysis of the function $S$ shows that $O(v^4/f^4)$ contributions cannot be always neglected.

The non-decoupling of $T$ in the LH model has been already emphasized in [26] in the context of $K_L \to \pi^0 \nu \bar{\nu}$. However, our result for this decay differs significantly from the large enhancement found by these authors. We will make a comparison with that paper in [20], where also other contributions to the rare decays in the LH model will be presented.

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