TESTS FOR THE STATISTICS OF PAIR-PRODUCED NEW PARTICLES

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Abstract

Due to selection rules, new particles are sometimes discovered/predicted to be produced in pairs. In the current search for SUSY particles this will occur if R-parity is conserved. In local relativistic field theory, there can be identical particles which are neither bosons nor fermions which are associated with higher-dimensional representations of the permutation group. Such particles will generally be pair-produced and so empirical tests are required to exclude them. A parameter-free statistical model is used to study the unusual multiplicity signatures in coherent paraboson production versus the case of ordinary bosons.

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1 Comparison of Selection Rules in SUSY (from R-Parity) and Parastatistics

The general motivation is the question “Where are the fundamental particles corresponding to the other representations of the permutation group?” So far, fermions go in totally anti-symmetric representations and bosons go in totally symmetric representations. But there are mixed representations. Particles associated with such representations obey parastatistics [1]. The order of the paraparticles is denoted by “$p$” such that “$p = 1$” corresponds to the ordinary bosons and fermions. In general, “$p$” is the maximum number of parafermions (parabosons) that can occur in a totally symmetric (anti-symmetric) state.

1.1 Selection Rules

From R-parity, all supersymmetric particles are assigned a new conserved quantum number

$$R = (-)^{3B+L+2J}$$

(1)

so $R = \pm 1$ respectively for particles and superparticles. Consequently, superparticles are pairwise-produced and the lightest superpartner (LSP) must be stable.

From locality, the selection rules [1, 2] for the production of paraparticles are similar: Transitions with only one external $p > 1$ particle are forbidden, so the lightest such particle is stable. The number of parafermions is conserved modulo 2. For a $p$-even family, the number of external lines is even, so an even number of $p > 1$ must be produced. For a $p$-odd family, the number of external lines cannot be an odd number less than $p$. For instance, if $p = 5$, then there cannot be 3 paraparticles produced. However, in the case of $p = 3$ there is the important exception that while a single paraparticle cannot be produced, the production of 3 such particles is allowed.

1.2 Cross-Section Tests

In the case of SUSY, once the masses are known, the cross-sections are fixed. For instance, sleptons and electroweak gauginos interact with strength $\alpha$. Similarly, squarks and gluinos interact with strength $\alpha_{QCD}$ . In leading order in the coupling, the corresponding cross-section in the pair-production of two analogous order $p$ paraparticles is a factor of $p$ larger. In comparison with SUSY, such a cross-section test can be used to exclude the production of paraparticles. However, if a cross-section were found to be a factor of $p$ greater than expected, this would be the same prediction as in the case of the production of $p$-fold degenerate fermions/bosons. In particular, there is the issue “How to distinguish $p = 2$ parabosons from two-fold degenerate ordinary bosons?” In the case when SUSY is not involved in the pair-production processes, the magnitude of the cross-sections might not be known apriori, but this same question arises.
2 Coherent Production of Parabosons of Order 2

The multiplicity signatures for \( p = 2 \) parabosons and ordinary bosons are indeed different. To show this, we use a parameter-free statistical model \([3]\) to study multiplicity signatures for coherent production of charged-pairs of parabosons of order \( p = 2 \) in comparison with those arising in the case of ordinary bosons, \( p = 1 \). This model gives 3D plots of the pair probability \( P_m(q) \equiv \text{"the probability of } m \text{ paraboson charged-pairs } + q \text{ positive parabosons"} \) versus \( < n > \) and \( < n^2 > \). As shown in the figure from Ref. [3], the \( p = 1 \) curve is found to lie on the relatively narrow 2D \( p = 2 \) surface. Such signatures distinguish between \( p = 2 \) parabosons and two-fold degenerate ordinary bosons, \( p = 1 \).

Circa 1970, for ordinary bosons Horn and Silver \([4]\) constructed the analogous parameter-free statistical model, using conserved-charge coherent states, and showed that it well described inelastic \( \pi^+ \pi^- \) pair production from fixed targets with laboratory kinetic-energies up to 27 GeV. The model agreed with the universal trends of an experimental regularity reported by Wang. Horn and Silver argued for a statistical treatment of the gross features because (i) momentum conservation should be a weak constraint since the emitted pions occupy a small part of the available phase space, (ii) total isospin conservation on the distribution of charged pions should also be weak since neutral pions are summed over, and so (iii) charge conservation remains as the important constraint.

A charged-paraboson pair in order \( p = 2 \) consists of one A quantum of charge ‘+1’ and one B quantum of charge ‘-1’. The Hermitian charge operator is defined by \( Q = N_a - N_b \) where \( N_{a,b} \) are the parabose number operators. \( Q \) does not commute with \( a \) or \( b \), and the paraboson pair operators \( ab \neq ba \). Nevertheless, since \([Q, ab] = 0, [Q, ba] = 0, [ab, ba] = 0\), the \( p = 2 \) conserved-charge coherent state can be defined as a simultaneous eigenstate of \( Q, ab, \) and \( ba \):

\[
Q|q, z, z' > = q|q, z, z' >, \quad ab|q, z, z' > = z|q, z, z' >, \quad ba|q, z, z' > = z'|q, z, z' > \tag{2}
\]

Unlike in the \( p = 1 \) case \([4]\) where only one parameter arises, here two complex numbers \( z \) and \( z' \) arise because \( ab \) and \( ba \) are fundamentally distinct operators. Consequently, in the following multiplicity considerations, two non-negative parameters occur which are the moduli of these two complex numbers, \( u \equiv |z| \) and \( v \equiv |z'| \). These parameters can be interpreted as the intensity strengths of the “\( ab \)” and “\( ba \)” sources. The explicit expressions for the conserved-charge coherent states \( |q, z, z' > \) are given in Ref. [5]. For \( q \) fixed, the percentage of events with \( m \) such pairs, \( P_m(q) \), is the square of the moduli of the expansion coefficients of \(|q, z, z' > \) in terms of the two-mode parabose number Fock states.

The figure shows the pair-probability \( P_1(1) \) for the production of two \( A^+ \) and one \( B^- \). The formulas for the \( p = 1 \) curve are for the source-intensity-strength \( x \) non-negative

\[
< n > = \frac{xI_2(2x)}{I_1(2x)}, \quad < n^2 > = < n > + \frac{x^2I_3(2x)}{I_1(2x)} \tag{3}
\]

\[
P_1^{(1)}(1) = \frac{x^3}{2I_1(2x)} \tag{4}
\]

and for the \( p = 2 \) ribbon are for the intensity strengths \( u, v \) non-negative

\[
< n > = \frac{1}{2} \left( \frac{uI_1(u)}{I_0(u)} + \frac{vI_2(v)}{I_1(v)} \right) \tag{5}
\]
\[<n^2> = <n> + \frac{1}{4} \left( \frac{u^2 I_2(u)}{I_0(u)} + \frac{2uv I_1(u) I_2(v)}{I_0(u) I_1(v)} + \frac{v^2 I_3(v)}{I_1(v)} \right) \] (6)

\[P_1^{(2)}(1) = \frac{u^2 v + v^3}{8I_0(u)I_1(v)} \] (7)

where the \(I_\nu\)'s are modified Bessel functions.

In the figure, the solid line is the \(p = 1\) curve. This \(p = 1\) curve is also the \(\{0, v\}\) line on the \(p = 2\) ribbon. Near the peak, there is a “fold” at the bottom edge of the ribbon. As shown, the \(p = 2\) ribbon consists of open-circles for the non-folded \(u \geq v\) region, and of solid-circles for the folded \(u \leq v\) region. The upper edge of the ribbon is the \(\{u, 0\}\) set of points. Slightly to the right of the peak, one can see from the solid-circles that each line of dots travelling leftward down the page, bends under (or “over”, whichever as the viewer prefers) the fold to reach the \(\{0, v\}\) line on the ribbon.

For the cases of \(q = 0\) and \(q = 1\), the \(p = 1\) curve always lies on the \(p = 2\) two dimensional surface. For \(q\) odd, as shown in the figure, the ribbon is \(u \leftrightarrow v\) asymmetric and the ribbon is only partly folded over. For \(q\) even, there is a complete fold \(u = v\) of the ribbon because then the \(u > v\) and \(v > u\) surfaces are identical.

The “line of dots travelling leftward down the page” in the peak region of the figure are a set of \(\{u, v\}\) values from a unit-negative-slope diagonal in the \(\{u, v\}\) domain. In independent-particle-emission models, the total energy in the emitted particles is monotonically related to the intensity strength of the source. This suggests that with the sum \(E_{total_A} + E_{total_B}\) fixed, there will be a significant \(E_{total_A} - E_{total_B}\) energy dependence in coherent paraboson pair-production. \(E_{total_A, B}\) are respectively the total emitted \(Q = 1\) and \(Q = -1\) quanta’s energies.

There is overall \(A^+ \leftrightarrow B^-\) symmetry (\(U(1)\) charge symmetry): The “Probability for \((m + |q|) A^+\)'s and \((m) B^-\)'s ” equals “Probability for \((m) A^+\)'s and \((m + |q|) B^-\)'s”, because \(P_m(|q|) = P_{m+|q|}(-|q|)\) and \(<n_b^M >_{-|q|} = < n_a^M >_{|q|}\) for \(M = \text{integer}\). Nevertheless, for \(q\) odd, physical observables such as \(P_m(q)\) are not symmetric under the \(u \leftrightarrow v\) exchange of the “\(ab\)” and “\(ba\)” intensity strengths. See Eqs. (5-7).

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Figure Caption

Figure 1: The pair-probability $P_1(1)$ for the production of two $A^+$ parabosons and one $B^-$ paraboson versus the mean number of charged-pairs $<n>$ and $<n^2>$. 
