KAON 99 — Summary and Perspective

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Abstract
An overview of KAON 99 with commentary is presented. Emphasis is placed on the state of CKM mixing and CP violation. The Jarlskog invariant, $J_{CP}$, is shown to provide a useful quantitative comparison of $K$ and $B$ phenomenology. The potential of future rare and “forbidden” decay experiments to probe $\mathcal{O}(3000 \text{ TeV})$ “New Physics” is also described.

1 Conference Overview and Commentary

For more than 50 years, Kaon physics has played a leading role in unveiling Nature’s fundamental intricacies and challenging our creative imaginations [1, 2]. The concept of hadronic “flavor” has its roots in the associated production of kaons and introduction of “strangeness” as a nearly conserved quantum number. SU(3)$_F$, current algebra, and the quark model all stemmed, to a large extent, from extensive follow-up studies of that discovery.

The $\theta$-$\tau$ puzzle in $K \rightarrow 2\pi$ and $3\pi$ (final states with different parities) provided the stimulus for Lee and Yang’s parity violation conjecture. Today, we easily accommodate parity violation via the chiral nature of the Standard Model’s SU(2)$_L \times$U(1)$_Y$ local gauge symmetry. However, that left-right asymmetry remains a deep fundamental mystery with potentially profound implications about the short distance properties of space-time and origin of mass.

Early null results in rare $K$ decay searches also led to important physics insights. The observed suppression of flavor changing neutral currents (FCNC) in $K_L \rightarrow \mu^+\mu^-$, $K \rightarrow \pi\nu\bar{\nu}$ etc. motivated the G.I.M. (Glashow-Iliopolous-Maiani) mechanism and introduction of charm. Today, medium rare $\mathcal{O}(10^{-8})$, branching ratios such as $K_L \rightarrow \mu^+\mu^-$ are routinely measured...
with high precision and used to search for or constrain potential “New Physics” effects.

The special (unique) $\Delta S = 2$ mixing features of the $K^0$-$\bar{K}^0$ system allowed CP violation to be unveiled in $K_L \to 2\pi$ decays. To explain that enigmatic effect, Kobayashi and Maskawa (KM) boldly proposed [3] the now discovered third generation of quarks, $t$ and $b$. Their parametrization of CP violation via angles and phases in a unitary $3 \times 3$ quark mixing matrix provided a simple but elegant solution to that outstanding puzzle. It also suggested many interesting predictions for FCNC and direct CP violation effects [4]. The recent measurement of $\epsilon'/\epsilon$ in $K \to 2\pi$ and initial studies of $B \to J/\psi K_s$ lend strong support to their hypothesis.

In a sense, the KM model of CP violation trivialized that previously mysterious phenomenon. It suggested that a mere non-vanishing weak interaction phase and quark mixing, rather than some new superweak interaction was responsible for CP violation. That beautiful solution now seems almost obvious. Also, if additional new interactions are eventually uncovered, it seems likely that they will similarly have relative phases which would provide additional sources of CP violation. That would be a welcome discovery, since electroweak baryogenesis [5] seems to require additional CP violation beyond the Standard Model.

Given its already rich and glorious history, what more can we hope to learn from $K$ decays? Are kaon studies passe, or competitive with $B$ physics and other ways to investigate CP violation?

This conference is proof of the excitement $K$ physics continues to generate. Its copious production cross-section and relative long lifetime make the kaon very special and experimentally popular worldwide. Indeed, there are many ongoing diverse experimental programs at labs around the world along with exciting ideas and proposals for new initiatives. I give in Table 1 a list of the kaon programs discussed at this meeting. Experiments at those facilities measure CKM (Cabibbo, Kobayashi, Maskawa) matrix elements, probe CP and possible CPT violation, search for very rare or forbidden decays, etc. In addition, they thoroughly study medium rare decays and other properties of kaons, thus providing an arena for refining theoretical skills such as chiral perturbation theory, lattice techniques, large $N_c$ approaches and perturbative QCD.

With regard to determining the CKM quark mixing matrix, $V_{\text{CKM}}$, $K$ and $B$ measurements both play special key roles. Their importance is well illustrated by the Wolfenstein parametrization [3]

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \frac{\lambda}{2} & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

(1)
Table 1: Ongoing and future kaon physics programs reported on at this meeting

| Program                        | Talks by                      |
|--------------------------------|-------------------------------|
| KEK 12 GeV PS → 50 GeV PS      | T. Inagaki, G.-Y. Lim         |
| BNL 30 GeV AGS + Booster       | L. Littenberg, W. Molzon, M. Zeller |
| FNAL KTEV → KAMI               | P. Cooper, J. Whitmore        |
| CERN SPS                       | G. Kalmus, L. Koepke          |
| CP LEAR (Completed)            | P. Bloch                      |
| Frascati-DAφNE                 | P. Franzini, S. Di Falco      |
| Novosibirsk                    | L. Landsberg, N. Ryskulov     |

One would like to measure $\lambda$, $A$, $\rho$, and $\eta$ as precisely and with as much redundancy as possible. In that way, the unitarity conditions

$$\sum_i V_{ij}V_{ik}^* = \sum_i V_{ji}^*V_{ki} = \delta_{jk}$$

(2)

can be tested. A deviation from expectations in any mode would signal “New Physics”. Let me discuss some important experiments.

The theoretically cleanest direct measurement of the cornerstone CKM parameter, $\lambda$, comes from $K_{e3}$ decays ($K \rightarrow \pi e \nu$) [7]

$$\lambda = 0.2196 \pm 0.0023 \quad (K_{e3})$$

(3)

where the theoretical and experimental uncertainties (added in quadrature) are comparable. That value is to be compared with results from Hyperon and nuclear beta decays

$$\lambda = \begin{cases} 0.226 \pm 0.003 & \text{(Hyperons)} \\ 0.2265 \pm 0.0026 & \text{(}\beta \text{- decay)} \end{cases}$$

(4)

(5)

There is some inconsistency. Hopefully, ongoing efforts at BNL, FNAL, and Novosibirsk to remeasure the $K_{e3}$ decay rates for both $K^+$ and $K_L$ will help clarify the situation.

The parameter $A$ is obtained from $V_{cb}$ as measured in semi-leptonic $B$ decays (the counterpart of $K_{e3}$). Currently, one finds [8]

$$A = 0.83 \pm 0.05$$

(6)

In Ligeti’s talk it was suggested that ongoing and future studies of $B \rightarrow D^* e \nu$ decays may lead to a reduction in the $A$ uncertainty by a factor of
2 or 3 during the next 3–5 years. Such improvement would be a welcome advancement.

The $\rho$ and $\eta$ parameters are constrained by a combination of $K$ and $B$ measurements. For example, within the Standard Model, the CP violating mixing parameter $|\epsilon| = 2.28(1) \times 10^{-3}$ provides a determination of the combination

$$A^4\lambda^{10}\eta(1 - \rho + 0.44) \simeq 5.6(1.1) \times 10^{-8}$$

where the error is primarily due to the $K^0-\bar{K}^0$ matrix element uncertainty. For a given $A$ and $\lambda$, that constraint leads to a hyperbola in the $\rho$, $\eta$ plane. However, the $\pm 20\%$ uncertainty in (7) is amplified by the current $\pm 29\%$ uncertainty in $A^4\lambda^{10}$. So, the $\rho$, $\eta$ plane is not very favorable for displaying constraints from $K$ decays. In contrast, it reduces the uncertainties in constraints from $B$ decays, thus presenting them in a very favorable light. $K$ decay presentations should resist the lure of the $\rho$, $\eta$ plane.

$B$ physics already provides some powerful constraints on $\rho$ and $\eta$. Most useful is the ratio $\Gamma(b \to u)/\Gamma(b \to c)$ which implies the relatively narrow band

$$(\rho^2 + \eta^2)^{1/2} = 0.363 \pm 0.073$$

Taken together with the $B^0_s-\bar{B}^0_s$ mixing constraint $|1 - \rho - i\eta| = 1.01 \pm 0.22$ and Eq. (7), it suggests (roughly)

$$\rho \simeq 0.13 \ , \ \eta \simeq 0.34$$

Other constraints from $B^0_s-\bar{B}^0_s$ mixing and $B \to J/\psi K_s$ are consistent with values in that general region. Overall, there is good support for CKM mixing and unitarity.

Given the success of the CKM model, what more can we learn from CP violation, further CKM studies, and $K$ decays? There are compelling reasons to push those efforts further. Precision studies of CKM elements can not only further confirm the standard model, but can help explain the origin of electroweak mass and perhaps help uncover “New Physics”. Indeed, as I will later demonstrate, CP violation and FCNC measurements are sensitive to effects originating from scales as high as 3000 TeV!

If a true deviation from the Standard Model in $K$ decays or other rare reactions is uncovered, there will certainly not be a lack of interesting explanations. SUSY [10, 11], Dynamical Symmetry Breaking, Large Extra Dimensions, etc. can potentially provide new significant sources of CP violation and FCNC effects.
2 New CP Violation Results

The most exciting kaon physics announcement of 1999 was the measurement of $\text{Re} \epsilon'/\epsilon$ by $K\text{TeV}$ [12] and NA48 [13]. Taken together with earlier studies, those new results

\[
\frac{\text{Re} \epsilon'}{\epsilon} = \begin{cases} 
23.0 \pm 6.5 \times 10^{-4} & \text{NA31} \\
7.4 \pm 5.9 \times 10^{-4} & \text{FNAL} \\
28.0 \pm 4.1 \times 10^{-4} & K\text{TeV} \\
18.5 \pm 7.3 \times 10^{-4} & \text{NA48}
\end{cases}
\]

(10)

given an average (with PDG expanded error) [13]

\[
(\frac{\text{Re} \epsilon'}{\epsilon})_{\text{Ave}} = 21.2 \pm 4.6 \times 10^{-4}
\]

(11)

That rather solid observation of direct CP violation rules out (old) Superweak Models. Is it consistent with CKM expectations? Pre 1999, the main theory predictions were (labeled by their home cities) [14, 15]

\[
(\frac{\text{Re} \epsilon'}{\epsilon})_{\text{Theory}} = \begin{cases} 
4.6 \pm 3.0 \pm 0.4 \times 10^{-4} & \text{(Rome)} \\
3.6 \pm 3.4 \times 10^{-4} & \text{Munich} \\
17_{-10}^{+14} \times 10^{-4} & \text{(Trieste)}
\end{cases}
\]

(12)

The broad range of those estimates does not allow for a definitive conclusion. The experimental result does, however, appear to be somewhat high.

Let me comment on the utility of $\epsilon'/\epsilon$ to probe sources of CP violation beyond the standard model. It is quite conceivable that some part of the experimentally observed $\epsilon'/\epsilon$ comes from “New Physics”. However, the current theoretical uncertainty of at least $\pm 100\%$ (probably more) makes such an interpretation very premature. Nevertheless, as discussed in Isidori’s talk [16], even with that large a theory error one can still obtain interesting constraints on, for example, potentially large new CP violating $Z_{\mu}d_L\gamma^\mu s_L$ interactions induced by SUSY loops in some models [17].

The ongoing experiments ($K\text{TeV}$ and NA48) were, however, designed to reach a $\Delta \epsilon'/\epsilon$ of $\pm 1-2 \times 10^{-4}$, i.e. a $\pm 5-10\%$ determination of that important quantity. In addition, the KLOE experiment [18] at Frascati will provide independent confirmation with very different systematic uncertainties. It would be a shame if such elegant measurements could not be fully utilized because of theoretical shortcomings.
To significantly reduce the theoretical uncertainty in $\epsilon'/\epsilon$ requires a systematic first principles calculation of the $K \to 2\pi$ amplitudes in, for example, a lattice gauge theory approach. With today’s powerful QCD teraflop computers and new theoretical methods such as domain wall fermions, much more precise calculations may, in fact, be possible. Indeed, T. Blum [15] described just such an ongoing effort at the RIKEN BNL Research Center. That collaboration aims for about $\pm 20\%$ theoretical uncertainty. Of course, before any new method is accepted, it must undergo close theoretical scrutiny and pass various consistency checks. For example, it should quantitatively explain the $\Delta I = 1/2$ amplitude enhancement relative to $\Delta I = 3/2$ amplitude in $K$ decays (a factor of 22). Also, it should demonstrate control of isospin violating effects which can feed $\Delta I = 1/2$ enhancements into the $\Delta I = 3/2$ amplitudes of $\epsilon'/\epsilon$. Perhaps, most important, as emphasized by Martinelli [17], the lattice approach should be self contained. Rather than patch together pieces of calculations from other prescriptions, it should be as complete as possible.

If a $\pm 20\%$ theoretical calculation of $\epsilon'/\epsilon$ is achieved, it will provide a very interesting confrontation with experiment. It would either allow for a powerful precise determination of the Standard Model CP violation parameter or point to “New Physics”. Either case justifies the effort.

Further confirmation of CKM mixing and CP violation is also starting to come from $B$ decays. (Of course $B$ studies offer tremendous potential for future studies.) CDF [19] has been able to observe an asymmetry in $B \to J/\psi K_S$. Using a time integrated sample of 400 events, they have determined (see J. Kroll’s talk) $\beta$ of the unitarity triangle

$$\sin 2\beta = 0.79^{+0.41}_{-0.44}. \quad (13)$$

That result is in good accord with Standard Model expectations (see M. Gronau’s talk [20]). Although currently only a 2$\sigma$ effect, CDF expects to reduce the error in Eq. (13) by a factor of 5 to $\pm 0.084$. In the longer term, $B$ factories (now up and running), $B$TeV, and LHC-B hope to achieve $\pm 0.02$ precision. The CDF result indicates that CDF and $D\emptyset$ with their significant upgrades can be expected to play major roles in future $b$ physics.

Other probes of CP violation discussed at this meeting include: 1) Measurement of $T$ odd asymmetries in $K_L \to \pi^+\pi^-e^+e^-$ and $p\bar{p} \to K^\pm\pi^\mp (\gamma)_0$, 2) Search for transverse muon polarization in $K_{\mu3}$ decay, 3) Hyperon decay asymmetries, and 4) Electric dipole moments.

The measured experimental 13.6% $T$ odd asymmetry between the $\pi^+\pi^-$ and $e^+e^-$ planes in $K_L \to \pi^+\pi^-e^+e^-$ observed at Fermilab (see Ladovsky’s talk [21]) is in good accord with the 13–14% expectation due to $\epsilon$ in the Standard Model (see talks by Sehgal [22] and Savage [23]). That result was
based on 1811 events at KTeV. Such a large asymmetry in $K$ decays is quite spectacular and was to most people very surprising. Future efforts at KAMI could yield $10^5$ decays in that channel and perhaps provide another probe of direct CP violation.

We were also reminded here of an earlier $T$ odd study from CP LEAR

$$A = \frac{R(K^0 \rightarrow \pi^- e^+ \nu) - R(K^0 \rightarrow \pi^+ e^- \bar{\nu})}{R(K^0 \rightarrow \pi^- e^+ \nu) + R(K^0 \rightarrow \pi^+ e^- \bar{\nu})} = 6.6 \pm 1.3 \pm 1.6 \times 10^{-3} \quad (14)$$

That Kabir test is in good agreement with the Standard Model prediction $A = 6.4 \times 10^{-3}$, again due to $\epsilon$.

G.-Y. Lim reported a recent KEK result for the muon transverse polarization in $K_{\mu 3}(K^+ \rightarrow \pi^0 \mu^+ \nu\mu)$ decay

$$p^{T}_\mu = \hat{s}_\mu \cdot (\hat{p}_\mu \times \hat{p}_\pi) \quad (15)$$

They have reached

$$p^{T}_\mu = -0.0042 \pm 0.0049 \pm 0.0009 \quad (16)$$

and aim for $10^{-3}$ sensitivity. The Standard Model predicts $P^{T}_\mu \sim 0$; so, a non-zero experimental result would directly point to a new source of CP violation. The leading candidate would be a charged Higgs exchange amplitude with a relatively large CP violating phase. Such direct searches for completely new sources of CP violation are extremely important and must be pushed as far as possible. An approved BNL experiment would reach $10^{-4}$ sensitivity, but unfortunately, it may never get to take data because of uncertainties in future AGS running for fixed target experiments.

Larger than expected CP violating asymmetries in Hyperon decays (see talks by Pakvasa, White, and Solomey) could also point to “New Physics”. An extensive Hyperon decay program is being proposed at Fermilab.

Perhaps the most promising way to uncover new sources of CP violation is the study of electric dipole moments. Such effects are predicted to be non-zero, but unobservably small in the CKM framework. However, “New Physics” of the type needed in some Baryogenesis scenarios, for example, could provide much larger edm signals, near the current experimental bounds. In the talk by M. Romalis we heard of ambitious efforts to push the sensitivity for the neutron and electron edm’s from $6.3 \times 10^{-26} \rightarrow 10^{-28} e$-cm and $4 \times 10^{-27} \rightarrow 10^{-31} e$-cm respectively. A proposal by the $g_{\mu}-2$ collaboration at BNL would also greatly extend the search for a muon edm from $10^{-18} \rightarrow 10^{-24} e$-cm. All such advances should
be strongly encouraged, since a positive finding would be revolutionary and may, in fact, be just waiting to be unveiled.

A general theoretical framework for discussing CPT and Lorentz invariance violation was given by A. Kostelecky [29]. $K$ physics studies currently provide the most sensitive tests of CPT [30]. Measurements of $m_{K^0} - m_{\bar{K}^0}$ at $K_{\text{TeV}}$, NA48, and CP LEAR have reached the incredible $10^{-18}$ GeV level and are beginning to approach the interesting $m_K^2/m_{\text{planck}}$ sensitivity. Future measurements at Frascati (see S. DiFalco’s talk on KLOE) will further advance the cause.

3 Medium Rare, Rare, and Forbidden Decays

In recent years, great progress has been made in the study of flavor changing neutral current (FCNC) decays of $K$ mesons. Experimental studies have been accompanied by an expansion in our theoretical arsenal of tools which now includes: chiral perturbation theory, large $N_c$, lattice gauge theories etc. Together, they have allowed us to test the Standard Model as well as to probe for and constrain possible “New Physics”. Here, I divide rare decays into three categories: 1) Medium Rare which includes roughly $10^{-5}$- $10^{-9}$ branching ratios, 2) Rare decays with branching ratios $\lesssim 10^{-9}$ and 3) Forbidden decays which do not occur in the Standard Model. For the last of those, I will discuss only muon-number non-conservation, because it provides such a sensitive probe of “New Physics”.

At this meeting, we heard about many measurements of medium rare decays [31]. In table 2, I list some of the results that were discussed. Most impressive to me is the fact that some measurements of historical importance such as $K_L \rightarrow \mu^+\mu^-$ have gone from a handful of events to precision measurements based on 5–10 thousand events [32]. Indeed, they now confront the standard model at its quantum loop level so as to constrain “New Physics” such as SUSY or Technicolor inspired models. In addition, the abundance of events in $K_L \rightarrow \mu^+\mu^-$, $\pi^+\pi^- e^+e^-$, $K^+ \rightarrow \pi^+\mu^+\mu^-$ etc. suggest that they may be further used to study CP violation effects in the future. Note, also that those measurements have been very useful in fine tuning the parameters of chiral perturbation theory and advancing its techniques (see talk by J. Bijnens [33]).

Rare $K$ decay experiments have made spectacular progress in measuring incredibly small branching ratios or pushing bounds. D. Ambrose [34] reported the smallest branching ratio ever measured in a decay process

$$B(K_L \rightarrow e^+e^-) = 8.7^{+3.7}_{-4.1} \times 10^{-12}$$ (17)
Table 2: Examples of Medium Rare K Decay Branching Ratios

| Decay Mode | Branching Ratio | Comments |
|------------|-----------------|----------|
| \(K^+ \rightarrow \pi^+ e^+ e^-\) | \(2.82 \pm 0.04 \pm 0.07 \times 10^{-7}\) | BNL E865 - Preliminary |
| \(K^+ \rightarrow \pi^+ \mu^+ \mu^-\) | \(9.23 \pm 0.6 \pm 0.6 \times 10^{-8}\) | " " |
| \(K^+ \rightarrow \pi^+ e^+ \nu_e\) | \(\sim 3.9 \times 10^{-5}\) | (300,000 events) |
| \(K^+ \rightarrow \pi^+ \pi^0\gamma\) | \(4.72 \pm 0.77 \times 10^{-6}\) | BNL E787 |
| \(K_L \rightarrow \pi^+ \pi^- e^+ e^-\) | \(4.4 \pm 1.3 \pm 0.5 \times 10^{-7}\) | KEK E162 |
| \(K_L \rightarrow e^+ e^-\gamma\) | \(1.06 \pm 0.02 \pm 0.02 \pm 0.04 \times 10^{-5}\) | CERN NA48 |
| \(K_L \rightarrow \mu^+ \mu^-\) | \(7.18 \pm 0.17 \times 10^{-9}\) | BNL E871 |

That result is in good agreement with the Standard Model prediction of \(9 \times 10^{-12}\). It indicates that even such rare decays can be cleanly observed and measured with precision. That bodes well for other more interesting rare decays for which only bounds currently exist \[35\]

\[
B(K_L \rightarrow \pi^0 e^+ e^-) < 5.6 \times 10^{-10} \tag{a}
\]
\[
B(K_L \rightarrow \pi^0 \mu^+ \mu^-) < 3.4 \times 10^{-10} \tag{b}
\]
\[
B(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 5.9 \times 10^{-7} \tag{c}
\]

but are expected to occur at about \(5 \times 10^{-12}\), \(1 \times 10^{-12}\) and \(3 \times 10^{-11}\) respectively. Each of those decays provides a nice test of direct CP violation, if a real measurement can be achieved. The golden mode \[36\] \(K_L \rightarrow \pi^0 \nu \bar{\nu}\) is particularly attractive because it is theoretically pristine (with only about \(\pm 1-2\%\) theoretical uncertainty). In fact, as I will subsequently describe, it has the unique potential of determining the extremely important Jarlskog \[37\] CP violating parameter \(J_{CP}\) at about the \(\pm 5\%\) level. Such a measurement is so compelling, that it must be carried out, if experimentally feasible (more commentary and discussion of experimental goals later).

Similar to \(K_L \rightarrow \pi^0 \nu \bar{\nu}\) is the rare decay \(K^+ \rightarrow \pi^+ \nu \bar{\nu}\) being pursued by the E787 collaboration at BNL. That group saw a single event in its 1995 run. Further analysis, as described by G. Redlinger \[38\], did not uncover additional candidates. The collaboration has not updated their 1 event branching ratio, but one expects that it now corresponds to about \(1.5 \times 10^{-10}\) with fairly large errors. About 2 times as much data remains to be analyzed, but already the experiment appears to be consistent with the Standard Model expectation \(B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \simeq 0.9 \times 10^{-10}\).

The theoretical error \[39\] on \(B(K^+ \rightarrow \pi^+ \nu \bar{\nu})\) due to charm mass and QCD uncertainties is only about \(\pm 7\%\). So, it would be extremely useful to measure that branching ratio with a similar \(\pm 10\%\) experimental error,
both as a means of determining CKM mixing parameters and constraining “New Physics”. E787 could wind up with several events when the analysis is complete. Its approved follow-up E949 at BNL has a goal of $0.8 \times 10^{-11}$ sensitivity, or about 10 Standard Model events (about a $\pm 30\%$ determination of $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$). In the longer term, the CKM proposal at Fermilab’s KAMI facility would aim for 100 events or $\pm 10\%$. As I will describe later, a $\pm 10\%$ measurement of that important branching ratio will allow “New Physics” to be probed beyond the 1000 TeV (PeV) level!

Muon-number violating (forbidden) decays have also been searched for with impressive sensitivities. Kaon and muon decays have achieved the bounds [32] given in Table 3. If no events appear in the ongoing E865 analysis at BNL, the bound on $K^+ \rightarrow \pi^+ \mu e$ is expected to reach $8 \times 10^{-12}$. Searches for those forbidden $K$ decays could probably be pushed by about another order of magnitude at future high intensity kaon facilities. However, currently, most planning activity involves forbidden muon decays such as $\mu^+ \rightarrow e^+ \gamma$ and $\mu^- N \rightarrow e^- N$ (coherent muon conversion in muonic atoms) because ideas for extending the current experimental sensitivity by 3 or 4 orders of magnitude exist. (New forbidden decay searches should generally strive for at least 2 orders of magnitude improvement.)

Table 3: Current bounds on muon-number violating decays and future potential.

| Decay Mode                | Current Bound     | Future Potential         |
|---------------------------|-------------------|--------------------------|
| $B(K_L \rightarrow \mu e)$ | $< 4.7 \times 10^{-12}$ BNL E781 |                        |
| $B(K^+ \rightarrow \pi^+ \mu e)$ | $< 4.8 \times 10^{-11}$ BNL E865 | Probably could be pushed to a few $\times 10^{-13}$ |
| $B(K_L \rightarrow \pi^0 \mu e)$ | $< 3.2 \times 10^{-9}$ FNAL – KTeV |                        |
| $B(\mu^+ \rightarrow e^+ \gamma)$ | $< 1.2 \times 10^{-11}$ MEGA | $10^{-14}$ PSI Proposals |
| $B(\mu^+ \rightarrow e^+ e^- e^+)$ | $< 1 \times 10^{-12}$ | —                       |
| $B(\mu^- N \rightarrow e^- N)$ | $< 6 \times 10^{-13}$ SINDRUM II | $5 \times 10^{-17}$ MECO at BNL |

Coherent muon-electron conversion, $\mu^- N \rightarrow e^- N$, is a particularly powerful probe of “New Physics”. Its discovery potential is very robust, including SUSY loops, heavy neutrino mixing, $Z'$ bosons, Multi-Higgs models, compositeness etc. To demonstrate its reach, consider the muon number non-conserving four fermion interaction

$$\mathcal{L} = \frac{4\pi}{\Lambda^2} \eta_q \bar{e} \gamma_\alpha \mu_q \bar{\gamma}^\alpha q \quad q = u, d$$ (19)

where $\Lambda$ is a generic scale of “New Physics” and $\eta_q$ represents a model dependent combination of couplings, mixing parameters, etc. At a sensitivity
of $5 \times 10^{-17}$, the goal of the proposed MECO experiment at BNL, one is probing (approximately)

$$\Lambda \gtrsim 3000 \text{ TeV} \sqrt{\eta_q}$$  \hspace{1cm} (20)

Few experiments are capable of exploring such short-distance scales. Of course, a discovery would be revolutionary. Given its potential, experiments such as MECO must be pushed as far and as soon as possible.

4 Quantitative Tests of CKM Unitarity - CP Violation

The $3 \times 3$ CKM mixing matrix, $V_{CKM}$, must be unitary. A convenient parametrization

$$V_{CKM} = \begin{pmatrix} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \begin{align*} c_i &= \cos \theta_i \\ s_i &= \sin \theta_i \end{align*}$$  \hspace{1cm} (21)

exhibits the features that allow the orthonormal relationships in Eq. (2) to be satisfied.

One can test the Standard Model and search for “New Physics” by making clean precision measurements of $V_{CKM}$ elements and seeing if unitarity is satisfied. For example, 4 measurements determine $\theta_1$, $\theta_2$, $\theta_3$ and $\delta$ (or Wolfenstein’s $\lambda$, $A$, $\rho$, and $\eta$). A fifth measurement then tests unitarity. Alternatively, each of the individual relationships in Eq. (2) can be tested by 3 (or more) measurements. Unitarity can be tested within $K$ or $B$ decays alone or in comparison with one another. Of course, in all cases theoretical uncertainties should be minimized. Also, it is useful to have as many different consistency checks as possible, since that allows many potential “New Physics” effects to be explored.

CP violation and FCNC effects are particularly good probes of “New Physics”, because the Standard Model predictions are generally so small. Which system is more sensitive, $K$ or $B$ decays? How does one compare the potential of $K$ and $B$ studies in an unbiased manner? A nice answer is provided by Cecilia Jarlskog’s $J_{CP}$ parameter. Let me describe its utility.

The six orthogonal relations in Eq. (2) with $j \neq k$ give rise to so-called unitarity triangles. I will label the 6 distinct triangles by their $(j, k)$ indices. The $(1,3)$ or $(d,b)$ triangle

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$  \hspace{1cm} (22)
is best known because of its general use in illustrating $b$ physics studies. $B$ programs aim to measure the angles and sides of those triangles in as many ways as possible. A deviation from closure or single inconsistent measurement would signal "New Physics". In addition, if one factors out $V_{cd}V_{cb}$ from that relation, the remaining triangle is nicely illustrated in the $\rho, \eta$ plane.

In $K$ physics there is also a useful unitarity triangle, the $(1,2)$ or $(d, s)$ relation

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$

Both triangles are illustrated in fig. 1. The $(1,2)$ triangle has angles near 0 and $90^\circ$ which imply very small CP violating decay asymmetries (in contrast with $B$ decays). Does that make it uninteresting? No. As pointed out by C. Jarlskog, the most interesting feature of any unitarity triangle is its area and that quantity is the same for all 6 triangles.

"All CKM triangles are created equal in Area!"

In fact, she observed that a quantity $J_{CP} = 2 \times$ the triangle area was the unique real measure of CP violation in the Standard Model. Unitarity requires

$$J_{CP} = J_{12} = J_{13} = J_{23} = J_{21} = J_{31} = J_{32}$$

In terms of the parametrizations of Eq. (21) or Eq. (1)

$$J_{CP} = s_1 s_2 s_3 c_1 c_2 c_3^2 \sin \delta \simeq A^2 \lambda^6 \eta$$

Standard model CP violation is tested by measuring $J_{CP}$ as precisely and in as many distinct ways as possible. A deviation would signal "New Physics"

Currently, a global fit to all $K$ and $B$ studies indicates

$$J_{CP} = 2.7 \pm 1.1 \times 10^{-5},$$

i.e. it is determined to about $\pm 40\%$. How well can the next generation of $K$ and $B$ studies individually determine $J_{CP}$? In the case of $B$ physics, the long term prospects are that $J_{13}$ will be measured to about $\pm 15\%$. Pushing to $\pm 5\%$ is extremely difficult, but worth trying to achieve.

In the case of $K$ decays, we are extremely fortunate. The decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ directly determines the height of the $(1,2)$ triangle and the base is already well known from $\beta$-decay and $K_{e3}$ decays. One finds

$$J_{12} = J_{CP} = 5.60[B(K_L \rightarrow \pi^0 \nu \bar{\nu})]^{1/2}$$
\[ B \text{ Physics: } V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \]

\[ K \text{ Physics: } V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \]

\[ J_{13}^{CP} = 2 \times \text{Area} \]

\[ J_{12}^{CP} = 2 \times \text{Area} = 5.60 \left[ \text{BR}(K_L \rightarrow \pi^0\nu\bar{\nu}) \right]^{1/2} \]

Figure 1: Unitarity triangles for \( B \) and \( K \) studies.

That result is extremely clean. Theoretical uncertainties are at the level of 1–2%. So, the only real limitation is how well \( B(K_L \rightarrow \pi^0\nu\bar{\nu}) \) can be measured. A proposed measurement at the ±25% level (about 16 events) would determine \( J_{CP} \) to about ±12 1/2%, which is better than long term \( B \) physics expectations. In the longer term, a 10% measurement of \( B(K_L \rightarrow \pi^0\nu\bar{\nu}) \) would give \( J_{CP} \) to ±5%. Of course, we need at least 2 measurements of similar precision \( J_{CP} \) for comparison; so, it would be nice if \( B \) efforts could remain competitive.

How might determinations and comparison of \( J_{12} \) and \( J_{13} \) with high precision be utilized? As a simple illustration, consider a strangeness changing interaction \[ \mathcal{L} = \frac{4\pi}{\Lambda^2} B\bar{d}_\alpha \gamma_\alpha s_L \bar{\nu}_i \gamma^\alpha \nu_i + \text{h.c.} \] 

\[ (28) \]
do to “New Physics” at scale \( \Lambda \). A ±10% measurement of \( B(K_L \rightarrow \pi^0\nu\bar{\nu}) \) would probe \( \Lambda \sim 3000 \text{ TeV} \) \((\text{Im}B)^{1/2}\). Note that ±10% precision in \( B(K^+ \rightarrow \pi^+\nu\bar{\nu}) \) provides similar probing power. Clearly, studies of \( K^+ \rightarrow \pi^+\nu\bar{\nu} \) and \( K_L \rightarrow \pi^0\nu\bar{\nu} \) must be pushed as far as possible.

5 Concluding Remarks (Future Outlook)

Direct CP violation in \( K \rightarrow 2\pi \) decays has finally been unambiguously observed. Ongoing experimental efforts should eventually determine \( \text{Re} \epsilon' / \epsilon \)
to ±5–10%. Theoretical calculations must strive to reach a similar level of precision.

$B$ physics has come of age. Studies at CLEO will soon share the spotlight and be challenged by asymmetric $B$ factories with CP violation as their primary goal. CDF and $D\emptyset$ will also be important players in the future along with LHCb, TeV etc.

$B$ studies open a new exciting frontier, but they do not close the door on $K$ or rare muon decays. The Kaon system is still the best place to look for CPT violation and the new $\phi$ factory at Frascati will be at the forefront of that effort. The rare decays $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$ are exceptionally clean theoretically. Besides testing CKM mixing with great precision, they are capable of probing “New Physics” up to about the 3000 TeV level. The muon number violating reaction $\mu^{-}N \rightarrow e^{-}N$, similarly probes 3000 TeV physics, but in a very different channel. Such outstanding experimental opportunities are extremely scarce. They must be seized and pushed as far as possible.

The decay $K_L \rightarrow \pi^0\nu\bar{\nu}$ is very special. It alone can determine the all important Jarlskog parameter $J_{CP}$ to about ±5% in the long term. It would then set the standard for comparing other manifestations of CP violation in $K$ and $B$ decays. It must be pursued with the same zeal and priority as $B$ physics.

Other rare $K$ decays, $K_L \rightarrow \pi^0 e^+ e^-$, $K_L \rightarrow \pi^0 \mu^+ \mu^-$, $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ etc. can also contribute to our understanding of CP violation and search for “New Physics”. Kinematic and polarization asymmetries may be particularly useful in those endeavors.

Kaon physics has had a glorious history. It continues to be exciting (e.g. $\epsilon'/\epsilon$, $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $K_L \rightarrow \pi^0\nu\bar{\nu}$ etc.) Are there any future big surprises or great discoveries waiting still to be uncovered in the kaon system? We will find out only if we continue to expand our efforts and follow our instinct to explore.

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