Universal dynamics and deterministic switching of dissipative Kerr solitons in optical microresonators

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Temporal dissipative Kerr solitons in optical microresonators enable the generation of ultrashort pulses and low-noise frequency combs at microwave repetition rates. They have been demonstrated in a growing number of microresonator platforms, enabling chip-scale frequency combs, optical synthesis of low-noise microwaves and multichannel coherent communications. In all these applications, accessing and maintaining a single-soliton state is a key requirement—one that remains an outstanding challenge. Here, we study the dynamics of multiple-soliton states and report the discovery of a simple mechanism that deterministically switches the soliton state by reducing the number of solitons one by one. We demonstrate this control in SiN and MgF2 resonators and, moreover, we observe a secondary peak to emerge in the response of the system to a pump modulation, an effect uniquely associated with the soliton regime. Exploiting this feature, we map the multi-stability diagram of a microresonator experimentally. Our measurements show the physical mechanism of the soliton switching and provide insight into soliton dynamics in microresonators. The technique provides a method to sequentially reduce, monitor and stabilize an arbitrary state with solitons, in particular allowing for feedback stabilization of single-soliton states, which is necessary for practical applications.

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The discovery of microresonator frequency combs (Kerr combs) has opened a research area at the intersection of micro/nano-photonics and frequency metrology. Kerr combs are generated in high-Q millimetre- or micrometre-scale resonators via parametric processes driven by a continuous wave (CW) laser, and have attracted significant attention due to their unprecedented compactness, octave-spanning operation, repetition rates in the microwave domain (>10 GHz), and the ability to operate in low-noise regimes. So far, Kerr combs have been used in a number of proof-of-concept applications, such as atomic clocks, terabit coherent communications, or optical arbitrary waveform generation, and their spectral coverage extended towards the visible and mid-infrared. This technology could enable a new class of chip-scale optical frequency combs, that could make optical frequency metrology ubiquitous, and thus widely accessible beyond specialized metrology laboratories. A pivotal development in this regard has been the recent observation that Kerr combs can be operated in the regime of temporal dissipative Kerr solitons (DKS), which represent stable and self-reinforcing intracavity light patterns resulting from double balance between the parametric gain and cavity loss, as well as chromatic dispersion and the Kerr nonlinearity of the resonator. A key feature of dissipative solitons is the requirement for an external energy supply to compensate for the system's dissipative nature. Prior work has studied dissipative solitons and their nonlinear dynamics in mode-locked lasers, and rich nonlinear optical dynamics such as switching between soliton states has been observed. In optical microresonators, DKS are solutions of the Lugiato–Lefever equation, where the cavity loss is balanced by a CW laser and its associated parametric gain.

Originally demonstrated to spontaneously form in crystalline MgF2 resonators (and for the first time externally induced in optical fibre cavities), DKS have been demonstrated in a variety of high-Q resonator platforms, ranging from silica wedge resonators to SiN photonic chips and compact crystalline resonators pumped via distributed feedback lasers. Soliton-based Kerr combs allow for fully coherent optical frequency combs that can be sufficiently broadband for self-referencing via soliton-induced Cherenkov radiation, and provide access to stable ultrashort pulses of tunable duration at microwave repetition rates. Of particular interest are single-soliton states, which exhibit a spectrally smooth sech envelope. Such comb sources can promote the applications of Kerr combs and more generally be used in applications where short pulse duration at microwave repetition rates is desirable, such as phononic-based radar. Indeed, soliton Kerr combs have already been successfully used for terabit coherent communication, dual-comb spectroscopy, soliton-based low-noise microwave generation, and for realizing a phase coherent RF-to-optical link both with external broadening of the soliton pulses, as well as without external broadening using soliton Cherenkov radiation.

To date, the formation process and dynamics of DKS in microresonators remain largely unexplored, in contrast to dissipative solitons in mode-locked fibre lasers, as well as solitons in fibre cavities. While solitons have been recently...
Figure 1 | Forward and backward tuning of the pump.  

a. The principle of microresonator frequency comb generation and the formation of dissipative Kerr solitons (DKS).  
b. Scanning electron microscope image of the Si$_3$N$_4$ on-chip microresonator with a free spectral range (FSR) of 14 GHz.  
c. Picture of the MgF$_2$ crystalline resonator with a FSR of 100 GHz.  
d. Scheme of the laser tuning method for the soliton generation in optical microresonators. The pump laser is tuned over the resonance from short to long wavelengths (forward tuning). Hatched region indicates the pump detuning range of multiple solitons (MS).  
e. Histogram plot of 200 overlaid experimental traces of the output comb light in the pump forward tuning over the resonance with the same pump power and tuning speed, which reveals the formation of a predominant multiple-soliton state with $N = 6$.  
f. Scheme of the laser backward tuning. To initiate the sequence, the forward tuning is first applied, and the pump is stopped in a multiple-soliton state (which can be stabilized by a suitable choice of the laser tuning speed). In the second stage, the pump is tuned back towards shorter wavelengths, which leads to successive soliton switching. $N \rightarrow N - 1 \rightarrow \ldots \rightarrow 1$. The area marked indicates the detuning range for multiple-soliton states, which is much larger compared to the forward tuning method. The single-soliton state (SS) is also accessible.  
g. Experimental trace in the forward tuning (yellow curve) followed by one trace in the backward tuning (white curve) with successive switching of multiple-soliton states from $N = 7$ to $N = 0$ (no solitons).  
h-j. Frequency comb spectra in soliton states with $N = 1, 2, 3$, measured during the backward tuning in a 100 GHz Si$_3$N$_4$ microresonator. The relative positions of the solitons result from the forward excitation path and are mostly random. They are retrieved via inverse Fourier transform of the optical spectrum.

reported in a number of high-Q microresonator platforms, the soliton generation procedures are inherently stochastic. CW laser tuning and 'power kicking' schemes were proposed for soliton generation, but at present these techniques do not allow the number of solitons formed in the resonator to be controlled or the deterministic manipulation of states with multiple solitons. Even though states with various numbers of solitons could be generated in optical microresonators, the switching between them takes place stochastically via, for example, pairwise interactions of solitons when the pump is tuned, and cannot be predicted so far. Due to these effects, deterministic generation of the single-soliton state still represents an outstanding challenge. A further challenge is to monitor and stabilize soliton states. The soliton regime is inherently stable but not immune to significant thermal drifts and other external perturbations. Reported passive lifetimes of DKS lasting several hours have been achieved in a stable laboratory environment. However, no technique has been developed to enable feedback-stabilized control of the soliton state and its underlying parameters (soliton power, pulse duration or laser detuning).
Figure 2 | Dynamical probing of temporal DKS in microresonators. a, Set-up scheme used for soliton generation, non-destructive soliton probing and deterministic soliton switching. An external-cavity diode laser (CW pump) is used as a pump source. AFG, arbitrary function generator; EDFA, erbium-doped fibre amplifier; EOM, electro-optical phase modulator; FBG, fibre Bragg grating; FPC, fibre polarization controller; OSA, optical spectrum analyser; OSC, oscilloscope; PD, photodiode; VNA, vector network analyser; WM, wavelength meter. b, (Left, top) Diagram of the double-resonance cavity transfer function in the state of DKS, the observed switching phenomenon, in two microresonator platforms that feature thermal effects, and provide a rich toolbox for the physical understanding of switching behaviour of DKS, highlight the influence of thermal effects, and provide a rich toolbox for the

In this paper we report the discovery of a phenomenon that allows the number of DKS in microresonators to be reduced (that is, from $N$ to $N - 1$), and thereby to reliably reach the single-soliton state. We present a detailed analysis of the observed phenomenon, in two microresonator platforms that feature thermal locking, and thereby demonstrate its universal nature. We also identify a unique system’s transfer function in the state of DKS, which allows both the control and stabilization of the soliton Kerr comb, preventing it from decay. The observed switching behaviour is shown to originate from the thermal effect of the microresonator, which leads to a power-dependent resonance frequency shift, in contrast to the Kerr nonlinear frequency shift, which is intensity dependent. This effect is not captured by standard modelling using the Lugiato–Lefever equation (LLE) or equivalently coupled-mode equations that only include the Kerr nonlinearity. The presented results contribute to the physical understanding of switching behaviour of DKS, highlight the influence of thermal effects, and provide a rich toolbox for the
study of the soliton dynamics. From an application perspective, the results present a route towards making reliable, feedback-stabilized pulse sources and frequency combs based on DKS in optical microresonators.

Results

Observation of switching between dissipative Kerr soliton states by laser backward tuning. The principles of microresonator frequency comb generation and the formation of dissipative Kerr solitons (DKS) are shown in Fig. 1a. In this work we study two microresonator platforms: Si$_3$N$_4$ on-chip ring microresonators$^{21,30,33}$ (Fig. 1b) and MgF$_2$ crystalline resonators$^{15,25,39}$ (Fig. 1c). The laser tuning technique was developed as an effective method for the formation of DKS$^{15}$, in which the CW pump laser is tuned over the cavity resonance from the effectively blue-detuned to the red-detuned side$^{12-23}$, referred to as the ‘forward tuning’. Initially, the frequency of the CW pump is in the blue-detuned regime and tuned into the resonance. Due to the slow thermal and fast Kerr nonlinearity of the microresonator, the cavity resonance is shifted and locked to the pump laser$^{15,24}$. In this regime the Kerr comb formation can be observed provided that the optical power built up in the cavity reaches the gain threshold of modulation instability (MI). The mechanism results in a triangular trace in the generated comb light, over the pump frequency detuning. When the pump is further tuned forwards, it enters the effectively red-detuned regime where multiple intracavity solitons (that is, a multiple-soliton state) can be formed. The soliton state is accompanied by a step-like power trace in the generated comb light, where the step height corresponds to the number of solitons (N) inside the resonator. Switching to states with a lower number of solitons may also occur and the power trace will exhibit characteristic steps. Eventually, by stopping the pump laser tuning at a step while ensuring thermal equilibrium in the resonator, stable multiple-soliton and even single-soliton states can be accessed (Fig. 1d). This forward tuning method was applied in MgF$_2$, Si$_3$N$_4$ and silica resonators for single dissipative Kerr soliton generation$^{15,22-24}$.

However, the thermal nonlinearity can significantly impact the soliton step pattern, such that single-soliton states become rarely accessible with the forward tuning method. Figure 1e shows 200 overlaid experimental power traces of the generated comb light obtained in a Si$_3$N$_4$ microresonator via the forward tuning, in which only multiple-soliton states are stochastically accessed having N = 6 (predominantly), 7, 8 or 9 solitons. Careful studies further reveal several common features in Si$_3$N$_4$ microresonators, irrespective of the employed pump power (see Supplementary Information for more details) and the laser tuning speed: the distributions mostly consist of traces with one step corresponding to a high-N multiple-soliton state; the accessible step length decreases with decreasing N; and the number of generated solitons increases with increasing pump power. All of these imply that the single-soliton state is not readily accessible with the forward tuning technique.

Remarkably, we observed that an additional laser tuning towards shorter wavelengths—the ‘backward tuning’—provides a way to reliably access the single-soliton state starting from an arbitrary multiple-soliton state. The result of this backward tuning sequence, shown in Fig. 1f, allows for successive extinction of intracavity solitons (soliton switching) down to the single-soliton state (N → N − 1 → ⋮ → 1). Figure 1g shows one trace of the generated light from a Si$_3$N$_4$ microresonator, where switching from seven solitons to a single soliton is observed. Strikingly, the generated comb light reveals a regular staircase pattern with equal stair length and height. The exact soliton number in each step can be precisely inferred from the step height. The pattern is almost identical over multiple experimental runs (using the same tuning speed and pump power) regardless of the initial soliton number N. Each switching between multiple-soliton states occurs with the extinction of one soliton at a time, which is confirmed by the relative positions of the intracavity solitons that are retrieved from the optical spectrum (insets in Fig. 1h–j).

In the experiments, the backward tuning process must be adiabatic to induce the successive reduction of the soliton number: the thermal equilibrium is required at each multiple-soliton state. This prerequisite is satisfied by choosing a tuning speed much slower than the thermal relaxation rate, which depends on the effective mode volume and the thermal diffusivity of a microresonator$^{22}$. For Si$_3$N$_4$ microresonator used in these experiments the backward tuning speed is chosen to be ~40 MHz s$^{-1}$, while the forward tuning speed is ~100 GHz s$^{-1}$. In this way all soliton states (N ≤ 7) are deterministically accessible. In contrast to the robust backward tuning that enables successive extinction of intracavity solitons, the forward tuning in Si$_3$N$_4$ microresonators typically leads to collective extinction of solitons when in a multiple-soliton state.

The backward tuning was also studied in MgF$_2$ crystalline microresonators, where the deterministic soliton switching to the single-soliton state is equally observed. In contrast to the Si$_3$N$_4$ platform, the single-soliton state can directly be accessed via the forward tuning in MgF$_2$ microresonators$^{15}$. However, this requires suitable adjustments to the cavity mode coupling, the pump power and the tuning speed. By contrast, it was observed that the backward tuning is significantly more robust and facilitates the generation of single-soliton states in crystalline resonators.

The soliton switching in both Si$_3$N$_4$ and crystalline MgF$_2$ resonators reveals that the backward tuning represents a universal approach to the generation of a single-soliton state in microresonators, provided that the thermal locking can be achieved.

Dynamical probing of the soliton response. In the state of DKS, the key parameter is the effective laser-resonance frequency detuning that determines both the intensity and the duration of soliton pulses$^{15,21}$. This detuning is defined as 2πδω = ωp − ω0, where ωp indicates the frequency of the pumped cavity resonance and ω0 is the pump laser frequency. In experiments, the pump frequency is precisely controlled, but the resonance frequency is thermally shifted from the initial cold cavity resonance frequency ω0 due to the amount of optical power dissipated in the cavity, making it a priori not possible to evaluate the effective detuning. On the other hand, the absolute detuning 2πδ = ωp − ω0 can be readily measured. It has been shown that solitons are supported within a certain range of the effective detuning$^{1,2,7}$, when the pump is effectively red-detuned (ωp < ω0) with a constant power, which we refer to as the soliton existence range.

We developed a non-destructive probing scheme that allows the tracking of the effective detuning and the determination of the soliton number N. It is based on measuring the conversion (that is, transfer function) of a phase-modulated pump to amplitude modulation on the comb power, using a network analyser (the practical aspects of this measurement are described in the Methods and in Fig. 2a). Similar schemes were applied in mode-locked lasers$^{4,5}$.

This probing method enables the identification of the different stages of Kerr comb generation, including the soliton formation, as shown in Fig. 2b. First, when the pump is in the blue-detuned regime (ωp > ω0), the system’s transfer function exhibits a Lorentzian-like resonance profile that originates from the cavity resonance. Second, when (forward) tuning the pump frequency into the cavity resonance, to obtain a frequency comb in the MI regime, the transfer function shows an asymmetric profile with fixed peak position, indicating the thermal and Kerr locking of the cavity resonance to the pump frequency. Third, when the pump laser is tuned into the soliton existence range (that is, in the effective red-detuned regime), the transfer function unexpectedly shows a
Figure 3 | Deterministic switching of the soliton states. a, Evolution of the generated comb light obtained from the 100 GHz Si$_3$N$_4$ microresonator when the pump is tuned backwards from a multiple-soliton state with $N=6$ (effectively red-detuned) to the effectively blue-detuned regime. b, Set of 500 concatenated VNA traces acquired during the backward tuning shown in a. The orange arrow indicates the switching from a single-soliton state to a no-soliton state, while the pump is still red-detuned with respect to the cavity resonance. The yellow arrow indicates the transition from the red-detuned operating regime to the blue-detuned regime. c, Evolution of the transfer function during the backward tuning process in the effectively red-detuned regime, with no soliton present ($N=0$). d, Evolution of the transfer function in the multiple-soliton state with $N=6$. e, Evolution of the generated comb light obtained from 14 GHz MgF$_2$ crystalline resonator when the pump is tuned backwards from a multiple-soliton state with $N=6$ (effectively red-detuned) to the effectively blue-detuned regime. f, Set of ~1,700 concatenated VNA traces acquired during the backward tuning shown in e. g, Evolution of the transfer function during the backward tuning in the state with no soliton present. h, Evolution of modulation response in the multiple-soliton state with $N=6$.

double-resonance feature. Finally, when the pump frequency is tuned out of the soliton existence range where no comb is observed, the transfer function shows again a single, Lorentzian-like resonance similar to the first stage.

Physically, in qualitative terms, the double-resonance feature originates from the fundamental cavity bistability in the presence of the Kerr nonlinearity. In the soliton state, the intracavity field consists simultaneously of a weak CW background and intense soliton pulses. The CW background represents the `lower branch' of the cavity field solution, which is effectively red-detuned to the cavity resonance. On the other hand, the soliton pulse with its high peak intensity will induce an additional shift of the cavity resonance, due to the intensity-dependent Kerr phase shift. This process in return allows the pump laser to be coupled into the resonance, such that the soliton experiences gain. Therefore, there exist two resonance states, each having a different detuning from the pump frequency, and thereby each inducing different quadrature rotations on the incoming probing sidebands. The presence of this response causes the input phase modulation of the pump to be converted to amplitude modulation of the generated comb power. The net result appears as a double-resonance feature—the soliton-induced ‘$S$-resonance’ and ‘$C$-resonance’ related to the CW, as schematically shown in Fig. 2b.

It is the soliton-induced nonlinear phase shift that produces the $S$-resonance in the transfer function, making the double-resonance feature a unique identification of the soliton state in Kerr combs. In the absence of comb formation where a weak CW component is also coupled into the cavity, only the $C$-resonance is present. In fact, transfer function measurements of this type were also applied in mode-locked laser systems, but the soliton feature was not captured$^{15-45}$. In the present work, both $C$- and $S$-resonances can be described theoretically (see Supplementary Information for details). Rich information from the microresonator system can be determined from the transfer function. First, the $C$-resonance peak (frequency) indicates the effective detuning between the pump and the cavity resonance ($\Omega_c \approx 2\pi\delta_w$). Second, the $S$-resonance frequency is less sensitive to the detuning (Supplementary Equation 20). Third, the amplitude of the $S$-resonance is related to the number ($N$) of solitons, as the response signal is enhanced by the higher comb power associated with a larger number of solitons.

We applied the modulation probing scheme to both Si$_3$N$_4$ and MgF$_2$ microresonators. The double-resonance transfer function is
observed in both platforms when a soliton state is obtained. Transfer functions for different soliton numbers and at various pump detunings are shown in Fig. 2c–f. We noticed that it is qualitatively similar for both platforms. The C-resonance position varies with the detuning (Fig. 2c,e), while the S-resonance frequency is practically fixed, as predicted theoretically. The peak amplitude of the S-resonance depends linearly on the soliton number N (Fig. 2d,f). A theoretical simulation of the transfer function in the soliton state is shown in Fig. 2g,h, which confirms the double-resonance feature and its properties.

We note that the cavity transfer function between a weak pump modulation and the modulation on the comb power in the soliton state was earlier numerically investigated in ref. 46. While two peaks of the transfer function were also numerically observed in this work, they were attributed conceptually to Feshbach and relaxation oscillations in the presence of third-order dispersion. The present work reveals a different underlying physical origin of the two resonances, which do not result from higher-order dispersion.

Importantly, the probing technique allows for tracking of the effective detuning δ_{\text{eff}}, which is a key parameter in soliton generation experiments, as it determines, for example, the pulse duration. In a soliton state, thermal drifts of the cavity resonance originating from various external sources may cause variations of δ_{\text{eff}}. Using the measured transfer function, the effective detuning can be monitored and adjusted (for example, by tuning the pump frequency) to maintain the effective detuning within the soliton existence range. In practice, feedback-locking of δ_{\text{eff}} is also possible, which allows for long-term operation of soliton states in a microresonator.

**Deterministic switching of soliton states.** We next investigate the switching of soliton states via the backward tuning scheme by applying the modulation probing in Si$_3$N$_4$ microresonators. We first employ forward tuning to generate a multiple-soliton state with N = 6, and subsequently perform a slow backward laser tuning. The generated comb light in the microresonator again shows the staircase pattern in the backward tuning, which corresponds to successive soliton switching from N = 6 to the single-soliton state (Fig. 3a). The system's transfer function traces are simultaneously recorded and continuously stacked to monitor the evolution of the transfer function during the process (Fig. 3b).

The experiments reveal a relationship between the evolution of the transfer function and the soliton switching. Within each soliton state, the C-resonance shifts towards the S-resonance due to the decrease of the effective detuning when the laser is tuned backwards. When the two resonances overlap and form a single-peak profile, the amplitude of the S-resonance is significantly enhanced (Fig. 3d). The phenomenon is also confirmed by the theory (see Supplementary Information). Accompanying this feature, soliton switching occurs, which results in a drop in the comb power as one soliton is lost (N → N − 1). After the switching, the C-resonance abruptly separates from the S-resonance. Simultaneously, while still being fixed, the S-resonance amplitude is reduced to a lower level than the previous state, since the number of solitons is reduced by one. In the absence of solitons (N = 0), the S-resonance is absent in the transfer function, but the C-resonance is still present (Fig. 2c).

The same measurement was carried out in MgF$_2$ resonators, see Fig. 3e–h. While similar switching dynamics are observed, there are several details which differ between Si$_3$N$_4$ and MgF$_2$ platforms. First, the optical quality factor Q of MgF$_2$ crystalline resonators (∼10$^4$) is three orders of magnitude higher than for Si$_3$N$_4$ micro-rings (∼10$^3$). The C- and S-resonances in the transfer function of crystalline resonator are therefore better resolved as a result of the narrower resonance linewidth. The soliton existence range in Si$_3$N$_4$ microresonators is typically $\mathcal{O}$1(1 GHz), while that in MgF$_2$ resonators is $\mathcal{O}$1(1 MHz). Second, after each soliton switching, the MgF$_2$ resonator shows slower recol of the C-resonance than the Si$_3$N$_4$ microresonator. This is attributed to the distinct thermal relaxation of the two platforms. The MgF$_2$ resonator has a larger effective mode volume and physical size than the chip-scale Si$_3$N$_4$ micro-ring resonators, such that the thermal relaxation time is longer. In the evolution of the transfer function of the MgF$_2$ resonator (Fig. 3f), the recoil of the C-resonance leaves a curved trajectory while it is very abrupt in the Si$_3$N$_4$ microresonator (Fig. 3b).

The soliton probing scheme using the phase modulation, combined with the backward tuning, allows an understanding of the soliton switching dynamics in microresonators. The system's transfer function clearly predicts the switching, and therefore provides a convenient tool to control the soliton states and induce switching on demand. In experiments, one can perform deterministic switching by tuning the pump frequency while monitoring the effective laser frequency detuning.

**Thermally enabled switching of soliton states.** We attribute the successive soliton switching in the backward tuning to the thermal nonlinearity of optical microresonators. Due to material absorption, the intracavity optical field thermally shifts the cavity resonance as a result of thermal expansion and thermal change of the refractive index. The pumped cavity resonance is therefore $\omega_a = \omega_0 - \Delta_1$, where $\Delta_1$ is the thermally induced resonance shift, which, in the soliton state with thermal equilibrium, is approximately proportional to the energy of the intracavity field:

$$\Delta_1(N) \propto E_C + N E_S \quad (1)$$

where $E_C$ is the energy of the CW component, $E_S$ is the energy of one soliton and N the number of solitons. Thus, the effective detuning can be expressed as $2\pi \delta_{\text{eff}} = \omega_a - \omega_0 - \Delta_1 = 2\pi \delta - \Delta_1$, where $\delta$ indicates the absolute detuning $2\pi \delta = \omega_a - \omega_0$. It is known that the microresonator system has a soliton existence range that, based on the LLE, is degenerate with respect to the number of solitons (N)$^{15}$. The lower boundary of this range is represented by the position of the S-resonance. Therefore, when the C-resonance is overlapped with the S-resonance via backward tuning, soliton switching occurs. Each time a soliton is lost, the thermal shift $\Delta_1$ is reduced and thus the effective detuning is increased, remaining in the soliton existence range. Therefore, this opens the possibility of tuning the laser further backwards (reducing $\delta$), inducing switching and deterministically reaching the single-soliton state.

**Mapping of the soliton stability diagram in optical microresonator.** The pump backward tuning enables deterministic and successive soliton switching, opening access to soliton states $N, N - 1, \ldots , 1$. It is therefore possible to experimentally explore the soliton existence range in each state (thus forming multi-stability diagram of the microresonator system$^{15,21,47}$), which to the authors' best knowledge has never been directly measured for optical cavity solitons of any kind. In terms of the effective detuning, we express the soliton existence range as $\delta_1 < \delta_{\text{eff}} < \delta_{\text{max}}$. The lower boundary $\delta_1$ is identified in the backward tuning soliton switching: it corresponds to the frequency at which the C-resonance and the fixed S-resonance overlap. In the studied Si$_3$N$_4$ microresonator with the chosen pumping conditions, this quantity is measured to be $\delta_1 \sim 0.78$ GHz. The upper detuning boundary $\delta_{\text{max}}$ of the soliton existence range can be explored for each soliton state when the pump laser is tuned forwards until the soliton comb disappears. Based on the theory and standard LLE simulations, this detuning is expected to be identical for all states corresponding to different number of solitons$^{15}$ (see Fig. 4d and also Supplementary Information), defined as the energy balance boundary of DKS. Under the same pumping conditions the maximum effective detuning $\delta_{\text{max}}$ is found for all soliton states to be $\sim 2.0$ GHz, yet
We performed numerical simulations based on both LLE and coupled-mode equations with the additional thermal relaxation equation included (see Supplementary Information), which verify that the deterministic soliton switching is enabled by the thermal nonlinearity of the microresonator (Fig. 4c,d). By including the thermal effects into numerical simulations, we are able to reproduce the staircase feature, corresponding to the successive reduction of the soliton number in the backward pump tuning (see red curve in Fig. 4c). Analytical power traces of soliton steps (black dashed lines) indicate soliton existence ranges for multiple-soliton states with different \( N \). They reveal a displacement of the soliton existence range between different soliton states (qualitatively similar to the measurement in Fig. 4a) as a consequence of the thermal nonlinearity.

When thermal effects in the simulations are switched off, soliton steps are well aligned and the soliton existence range is again degenerate with respect to the soliton number \( N \), see Fig. 4d. No soliton switching is therefore observed in the backward tuning. Numerical simulation also revealed the soliton breather states that are considered as an intermediate state between the chaotic MI regime and the stable soliton state. In the breather state, the soliton pulse peak power and the pulse duration, as well as the average intracavity energy, will experience periodical oscillations. This induces thermal perturbations to the cavity resonance and initiates the soliton switching. The breather states were indeed experimentally observed and identified, and will be reported elsewhere.

Discussion

We experimentally, numerically and analytically demonstrated a successive reduction of the number of solitons in microresonators in the state of DKS \( (N \rightarrow N - 1 \rightarrow \cdots \rightarrow 1) \) and showed that the phenomenon is platform independent. We observed that the soliton existence range is not fully degenerate with respect to the number of solitons, and each stable soliton state can be accessed by simply
backward tuning the pump frequency. This provides a deterministic route towards soliton switching and the generation of a single soliton inside microresonators. It is the power-dependent thermal nonlinearity that lifts the degeneracy of the soliton existence range, and leads to a staircase trace in the generated comb light when tuning backwards. We also investigated the transfer function of the microresonator system, between phase modulation on the pump and amplitude modulation on the system’s transmitted power. We observed a unique double-resonance feature in the transfer function when the microresonator is in the state of DKS, which is induced by the nonlinear cavity bistability. This feature reveals the laser-resonance frequency detuning as well as the number of solitons, and provides insights into the dynamics of the soliton switching. We also developed a theory to explain the system’s transfer function and, in particular, the double-resonance feature. Moreover, we performed numerical simulations where the thermal nonlinearity is included. The analytical treatment as well as the simulation results confirm the experimental observations. Combining the soliton probing scheme with the laser backward tuning allows for the stabilization of the soliton-based microresonator frequency comb, with a desired number, which could be applied to a variety of Kerr nonlinear microresonators.

Note added in proof: During submission of this work, Yi et al. reported on the stabilization of a soliton state based on the comb power level18.

Methods
Methods, including statements of data availability and any associated accession codes and references, are available in the online version of this paper.

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Author contributions
M.K. designed and performed the experiments and analysed the data. H.G. conceived and initiated the numerical simulations with thermal effects. E.L. performed experiments in MgF$_2$ microresonators and analysed the data. A.K. fabricated the Si$_3$N$_4$ samples and M.H.P.P developed the fabrication method. V.B. assisted in experiments. G.L. and V.E.L. assisted in simulations. M.L.G. developed the theory and performed the simulations. M.K., H.G., E.L., M.L.G. and T.J.K. discussed all data in the manuscript. M.K. and H.G. wrote the manuscript, with input from E.L., M.L.G. and T.J.K. The project was supervised by T.J.K.

Additional information
Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to M.L.G. or T.J.K.

Competing financial interests
The authors declare no competing financial interests.
Methods

Fabrication. Si$_3$N$_4$ microresonators investigated in this work were fabricated using the Photonic Damascene process.$^{49}$ The microresonators have a FSR of 100 GHz. A single-mode ‘filtering’ section was added to the microresonators to suppress high-order modes.$^{50,51}$ The dispersion parameters of the microresonators were measured using the frequency-comb-assisted laser spectroscopy method.$^{52}$

$D_2 = 2 \Delta f_0 / D_1$ MHz, $D_3 = 2 \Delta f_0 / D_0$ kHz (where the resonance frequencies near $\omega_0$ are expressed in a series $\omega_n = \omega_0 + \sum_{i=1}^{\infty} D_i \mu^i / i!$, where $i \in \mathbb{N}$ and $\mu \in \mathbb{Z}$ is the mode number). The pumped resonance is at 1,553.4 nm, the tuning speed of the forward tuning for the soliton excitation is ~1 nm s$^{-1}$ and the pump power is ~2–3 W in the waveguide.

The MgF$_2$ crystalline resonator was fabricated by diamond turning a cylindrical blank and subsequent hand polishing to achieve a high $Q$ (linewidth $= 2 \Delta f_0 / D_1$ kHz). The diameter of 5 mm yields a FSR $D_1 = 14$ GHz. The dispersion parameters at the pump wavelength of 1,553 nm are: $D_2 = 1.9$ kHz, $D_3 = 0.1$ Hz. The pump laser (fibre laser, wavelength 1,553 nm; short-term linewidth 10 kHz) is amplified to 250 mW. The relative laser frequency is monitored by counting the heterodyne beat between the pump laser and a reference laser stabilized to an ultra-stable cavity. The light is evanescently coupled to a whispering gallery mode of the resonator with a tapered optical fibre.

Dynamical soliton probing. Regarding the dynamical soliton probing scheme, presented in the Fig. 2a, we employ an external-cavity diode laser as the pump, which is phase modulated using an electro-optical modulator (EOM, bandwidth 10 GHz). We employed a vector network analyser (VNA) to drive the EOM with a swept frequency from 5 kHz to 4.5 GHz. The Si$_3$N$_4$ resonator is pumped, with the power being amplified by an erbium-doped fibre amplifier (EDFA) to 3–5 W. We use lensed fibres that introduce a coupling loss of 2.5–3 dB per facet. An arbitrary function generator is used for tuning the frequency of the CW pump. The pump frequency during the backward tuning is measured by a wave meter with a resolution of ~50 MHz. The output signal from the chip is split into several branches: to measure the comb spectrum with the OSA and to track the generated light of the comb with an oscilloscope where the residual pump is filtered. A third portion is used to measure the system’s transfer function by recording the cavity-induced quadrature rotation from the phase to the amplitude quadrature with a photodiode, and subsequent demodulation via a RF-homodyne detection inside the VNA. The phase modulation imprints two out-of-phase sidebands symmetrically around the pump (we use a small modulation index of 0.01, so that only first-order sidebands are considered). Passing through the cavity, the sidebands pick up a different relative phase, which leads to an amplitude modulation of the measured signal. In other words, we measure the system’s transfer function between the phase modulation of the pump and amplitude modulation of the comb power.

Data availability. The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

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