Exploring lattice quantum chromodynamics by cooling

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Abstract

The effect of cooling on a number of observables is calculated in SU(2) lattice gauge theory. The static quark-antiquark potential and spin-dependent interactions are studied, and the topological charge is monitored. The chiral symmetry breaking order parameter $\langle \chi \rangle$ and meson correlators are calculated using staggered fermions. Interactions on the distance scale of a few lattice spacings are found to be essentially eliminated by cooling, including the spin-dependent potentials. $\langle \chi \rangle$ and meson correlators up to time separations of several lattice spacings relax very quickly to their free-field values. At larger times, there is evidence of a difference between the pseudoscalar and vector channels. A fit to the pseudoscalar correlation function yields “mass” values about $2/3$ (in lattice units) of the uncooled masses. These results raise the question of how to reconcile the large-time behavior of the hadron correlators with the fact that the spin-dependent potentials and $\langle \chi \rangle$ essentially disappear (in lattice units) after only a small amount of cooling.
I. INTRODUCTION

Cooling in lattice field theory is a technique for exposing the topological features of field configurations \([1,2]\). Recently, using this method, evidence has been presented \([3,4]\) that instantons play a dominant role in determining hadron properties in quantum chromodynamics (QCD). This conclusion was based on the calculation of long-distance properties of a variety of hadronic correlation functions. It has been argued, however, that the persistence of long-distance effects (particularly, confinement \([5]\)) is an inevitable consequence of the local nature of the cooling procedure and is not indicative of the underlying dynamics \([6]\).

To gain some more insight into what cooling is doing we have extended our work \([7]\) on three-dimensional QED to four-dimensional QCD with SU(2) color. We first examine the behavior of the static quark-antiquark potential (calculated from Wilson loops) and of the spin-dependent interaction (calculated from chromo-magnetic field correlations) under cooling. As in QED\(_3\), there is a good indication that the confining behavior of the static potential persists even after a significant amount of cooling although the overall scale of the potential is greatly reduced. On the other hand, the spin-dependent potentials (which are short-ranged) disappear rapidly upon cooling. In the spin-spin interaction, a residual effect, which one may associate with instantons, of only a few percent of the original potential was observed.

Next chiral symmetry breaking was calculated using staggered fermions in quenched approximation. The chiral symmetry order parameter \(\langle \chi \chi \rangle\) was calculated at a few nonzero quark mass values and compared to the result obtained for free fermions. Our first observation is that \(\langle \chi \chi \rangle\) relaxes to its free-field value even after a small amount of cooling. This is an indication that dynamical mass generation in the cooled configurations is small compared to that found in the uncooled vacuum. The extrapolation to zero quark mass is problematic since, in principle, it requires an extrapolation to infinite volume first. However, we can conservatively estimate that in the region of intermediate cooling (20 to 50 cooling steps) the chiral condensate has been reduced by at least an order of magnitude.

Finally, meson correlators for the spin-0 and spin-1 channels were calculated. The behavior of these time-correlation functions upon cooling is in qualitative agreement with what would be anticipated given the behavior of the potentials and the chiral order parameter. The meson correlators do not relax uniformly. The relaxation rate is most rapid at small times and it also decreases as quark mass decreases. At short time separations the spin-0 and spin-1 meson correlators reflect very clearly the effect of cooling on the spin-dependent potential (degeneracy of pseudoscalar and vector states) and on chiral symmetry breaking (parity doubling in the spin-1 channel). At larger times at fit to the pseudoscalar correlation functions yields “mass” values roughly 2/3 the size (in lattice units) of the uncooled masses.

Section II contains a description of the calculational methods used in this paper. The results are presented in Sec. III.

II. METHOD

The usual plaquette action

\[
S = \beta \sum_{x, \mu > \nu} \left\{ 1 - \frac{1}{2} \text{Tr} U_{\mu \nu}(x) \right\}
\]

(1)
is used. Periodic boundary conditions were imposed on the gauge field links in all directions. Field configurations were generated using a heatbath Monte Carlo algorithm.

For cooling, links were updated “vectorially” following the same checkerboard sequence as was used in the Monte Carlo algorithm. Each link in turn was replaced by a link proportional to the inverse of the sum of the “staples” of the plaquettes containing the link being updated so as to minimize the local contribution to the action. Some test runs were done with adiabatic cooling [8]. After 10 to 20 cooling steps the results become qualitatively the same as with full local minimization.

The physics of cooled configurations has been interpreted in terms of instantons. We also monitor the topological charge as a function of cooling and, following Chu et al. [4], the simple transcription of $F_{\mu\nu}\tilde{F}_{\mu\nu}$ to the lattice [9] is used for the topological charge density

$$q_t(x) = \frac{1}{32\pi^2} \sum_{\mu\rho,\nu\sigma} \epsilon_{\mu\rho\nu\sigma} \text{Tr} \{ U_{\mu\nu}(x) U_{\rho\sigma}(x) \},$$

where $U_{\mu\nu}, U_{\rho\sigma}$ are plaquettes of gauge field links. The total topological charge is

$$Q_t = \sum_x q_t(x).$$

As is well known this simple definition of topological charge does not accurately represent the topological charge of the uncooled lattice configurations [10]. However, for sufficiently smooth (cooled) configurations one can see instantons with the expected continuum action. The Creutz ratio

$$C(R, T) = -\ln \frac{W(R, T) W(R - 1, T - 1)}{W(R - 1, T) W(R, T - 1)},$$

where $W(R, T)$ denotes the $R$ by $T$ rectangular Wilson loop, can be used to determine the string tension. For large loops, which obey the area law, $C(R, R)$ gives the string tension. The potential $V(R)$ between static quarks was also calculated directly by extrapolating Wilson loops to large $T$

$$V(R) = -\lim_{T \to \infty} \frac{1}{T} \ln W(R, T).$$

Variance reduction methods [11] were used in the computation of the Wilson loops in the uncooled theory.

In addition to the confining central potential, spin-dependent interactions can also be calculated [12]. These are related to chromo-magnetic field correlations (see Ref. [13] for a simple derivation) which are computed by making magnetic field insertions on $R \times T$ Wilson loops. The spin-spin and tensor interactions are then given by

$$a^3g^2V_{SS}(R) = \frac{1}{T - 1} \sum_{t_1 = T/2, T/2 \pm 1} \sum_{t_2 = 1}^{T - 1} \langle B_L(0, t_1) B_L(R, t_2) \rangle + 2\langle B_{\perp}(0, t_1) B_{\perp}(R, t_2) \rangle \frac{1}{W(R, T)},$$

and
\[ a^3 g^2 V_T(R) = \frac{1}{T - 1} \sum_{t_1 = T/2, T/2 \pm 1}^{T-1} \sum_{t_2 = 1}^{T-1} \frac{\langle B_L(0, t_1) B_L(R, t_2) \rangle - \langle B_L(0, t_1) B_L(R, t_2) \rangle}{W(R, T)} \]  

(7)

where \( \langle B_L(0, t_1) B_L(R, t_2) \rangle \) and \( \langle B_L(0, t_1) B_L(R, t_2) \rangle \) are expectation values of Wilson loops with plaquette insertions (at \( t_1 \) and \( t_2 \)) corresponding to magnetic fields parallel and transverse to the spatial direction of the loop. In practice, the magnetic field insertion \( B \) that was used was the average over the eight spatial plaquettes whose corners lie on the Wilson loop \( W(R, T) \). This corresponds to operator II of Ref. [12].

In addition to observables constructed purely from gauge field variables we are also interested in how quarks behave in the cooled vacuum. In QCD a basic property of the vacuum is chiral symmetry breaking which can be studied most easily if staggered fermions are used. The action for staggered fermions is

\[ S_f = \frac{1}{2} \sum_{x, \mu} \eta_\mu(x) \left[ \bar{\chi}(x) U_\mu(x) \chi(x) \right] - \bar{\chi}(x) U_{\mu}^\dagger(x) \chi(x) + \sum_x m \bar{\chi}(x) \chi(x), \]

\[ \equiv \chi \mathcal{M}(\{U\}) \chi, \]  

(8)

where \( \chi, \bar{\chi} \) are single-component fermion fields, \( \eta_\mu(x) \) is the staggered fermion phase \[14], \( m \) is the mass in lattice units and the \( U \)’s are gauge field links. Antiperiodic boundary conditions were used for the fermion fields in all directions.

The chiral symmetry order parameter is calculated from the inverse of the fermion matrix \( \mathcal{M} \) of Eq. (8)

\[ \langle \overline{\chi} \chi \rangle = \frac{1}{V} \langle \text{Tr} \mathcal{M}^{-1}(\{U\}) \rangle, \]  

(9)

where \( V \) is the lattice volume and the angle brackets denote the gauge field configuration average. A random source method \[15,16] was used to calculate \( \text{Tr} \mathcal{M}^{-1}(\{U\}) \). Sixteen Gaussian random sources were used for each gauge field configuration.

Meson correlation functions can be constructed from local bilinears of the single-component \( \chi \) fields. We consider two such correlators which after integration over the fermion fields, can be expressed in terms of the inverse of the fermion matrix as

\[ g_0(t) = \sum_x \text{Tr} \left\{ \mathcal{M}^{-1}(\vec{x}, t; 0) \left[ \mathcal{M}^{-1}(\vec{x}, t; 0) \right]^\dagger \right\}, \]

(10)

and

\[ g_1(t) = \sum_x \left[ (-1)^{x_1} + (-1)^{x_2} + (-1)^{x_3} \right] \text{Tr} \left\{ \mathcal{M}^{-1}(\vec{x}, t; 0) \left[ \mathcal{M}^{-1}(\vec{x}, t; 0) \right]^\dagger \right\}. \]

(11)

These two functions describe the propagation of zero-momentum meson states of spin 0 and 1 respectively. As is well known, with local staggered fermion operators mixing between states of different parity can in principle occur. In practice, the spin-0 channel is essentially pure pseudoscalar and describes the pseudo-goldstone boson. The spin-1 correlator is dominated by the vector meson (at least in the uncooled theory) with some admixture of axial-vector meson states.
III. RESULTS

Most of the calculations were done on a $12^4$ lattice at $\beta = 2.4$. This value was chosen as it is well into the scaling region for SU(2) color. For comparison some calculations were done at $\beta = 2.2$ on a $12^4$ lattice and at $\beta = 2.4$ on a $16^4$ lattice. However, not all results will be shown here since they are qualitatively the same in all cases.

It is useful to consider first the topological properties of the gauge field configurations. A sample of 300 configurations, separated by 100 heat-bath Monte-Carlo sweeps after 4000 sweeps of thermalization, was analyzed. In the uncooled configurations our value for the (lattice) topological susceptibility $\langle Q_t^2/L^4 \rangle$ of $(3.7 \pm 0.3) \times 10^{-5}$ agrees well with the high statistics result of $(3.5 \pm 0.1) \times 10^{-5}$ obtained by Campostrini et al. [8]. The simple transcription of the continuum topological charge operator is not a true topological quantity on the lattice [10]. It need not take integer values as can be seen in Fig. 1 which shows a histogram of the number of configurations versus $Q_t$. However in cooled configurations which are sufficiently smooth, the operator $Q_t$ does cluster around integer values. This is shown in Fig. 2 constructed from our sample of 300 configurations at 25, 50, 75 and 100 cooling steps.

It is also useful to examine the behavior of the action under cooling. Histograms of the average plaquette are plotted in Fig. 3 for different amounts of cooling. Fig. 4 shows a scatter plot of $Q_t$ versus average plaquette. At 100 cooling steps there is a fairly obvious instanton interpretation. The vertical dashed lines in Fig. 3d correspond to values of the total action of $8\pi^2 n/g^2$ for $n=1, 2, 3$ and 4. After about 100 cooling steps the configurations are dominated by a single classical instanton. 75 cooling steps seems to be in a transition region [17]. Single instanton peaks are seen in the action but there are many configurations which have a more complicated structure. The region of 25 to 50 cooling has been interpreted as being dominated by multi-instanton–anti-instanton fluctuations. However one has to be aware that, as can be inferred from Fig. 3 and 4, these configurations are not simply superpositions of isolated (noninteracting) classical instantons.

The Creutz ratio and static potential can be calculated from Wilson loops as in Eqs. (4) and (5). A $12^4$ lattice is too small to see the true asymptotic confining behavior but we can infer the general trend. The Creutz ratio $C(R, R)$ calculated from our sample of 300 uncooled configurations is shown in Fig. 5(a). On the same graph we also show the Creutz ratio after 25 cooling steps. Further cooling results in the Creutz ratios of Fig. 5(b).

The corresponding results for the potentials are shown in Fig. 6. The potentials are steadily reduced by cooling but the concave upward curvature is consistent with the idea put forward by Teper [6] that the string tension survives cooling, albeit, at increasingly large distances.

Spin-dependent potentials are calculated from chromo-magnetic field correlations. The spin-spin and tensor potentials are plotted in Fig. 7 and 8 respectively. The uncooled potentials are qualitatively consistent with the $\beta = 2.575$ operator II results of Michael and Rakow [12]. The spin-dependent potentials are short ranged and decrease rapidly upon cooling. In the region of 25 to 50 cooling steps the residual effect in these potentials is only at the level of a few percent of the uncooled values.

Of course it has to be remembered that the comparison of cooled and uncooled results is being done here in terms of lattice (not “physical”) units. It is clear that if one attempted
to keep the magnitude of the potentials quantitatively similar in physical units while cooling a very large change in the lattice spacing would be required.

Spontaneous chiral symmetry breaking is a basic property of QCD. A signature for this phenomenon is the persistence of a nonzero value of the “quark condensate” in the limit of zero quark mass. To study chiral symmetry breaking we calculate the expectation value of the local staggered fermion operator $\langle \chi \chi \rangle$. For our $12^4$ calculation, 40 configurations (separated by 200 Monte Carlo sweeps) at $\beta = 2.4$ and staggered fermion masses $m a = 0.3, 0.2$ and 0.1 were used. Fig. 9 shows $\langle \chi \chi \rangle$ versus quark mass (in lattice units). The results without cooling are consistent with those of Billoire et al. \cite{18} at the same value of $\beta$. The quantity $\langle \chi \chi \rangle$ was also calculated using 20 configurations on a $16^4$ lattice and quark masses down to $ma = 0.05$. The results are shown in Fig. 10. At the masses that are common to both calculations, the $12^4$ and $16^4$ lattice results are consistent. The results for $\langle \chi \chi \rangle$ after 10 cooling steps and for massive free staggered fermions are also plotted in Figs. 9 and 10. After 10 cooling steps the values of $\langle \chi \chi \rangle$ are already quite close to the free field results at the same value of $ma$. Therefore, dynamical mass generation seems to be quite small in the cooled vacuum.

In principle the extrapolation of $\langle \chi \chi \rangle$ to zero quark mass to extract the genuine chiral condensate requires that the infinite volume limit be taken first. This would allow calculations at arbitrarily small quark masses. In practise we can only do a limited number of calculations so the procedure of Billoire et al. \cite{18} is adopted. Three values of $\langle \chi \chi \rangle$ at the lowest nonzero mass are used to determine the coefficients of the expansion

$$\langle \chi \chi \rangle(m) = \langle \chi \chi \rangle_0 + \langle \chi \chi \rangle_1 m + \langle \chi \chi \rangle_2 m^2.$$  \hspace{1cm} (12)

The extrapolated values of $\langle \chi \chi \rangle_0$ from this procedure are plotted in Fig. 11 versus cooling up to 50 cooling steps. Unfortunately the values from the $12^4$ and $16^4$ lattice calculations do not agree indicating that perhaps the correct mass window is not being used for the extrapolation or that the extrapolation function is not adequate. Therefore only a qualitative conclusion is possible, namely, that the chiral condensate (in lattice units) is greatly reduced in cooled configurations. Of course, one could keep the condensate fixed in physical units but this would require a decrease of the lattice spacing by a factor of about 2.5 to 3.

Finally we examine the meson correlation functions. First we consider the qualitative effects of cooling by comparing the meson correlators in the vector and pseudoscalar channels. A sample of results at the lightest quark mass values for both $12^4$ and $16^4$ lattice are presented.

Fig. 12 shows the pseudoscalar and vector zero-momentum correlators on the $12^4$ lattice for $ma = 0.1$. After only a small amount of cooling the correlation functions for small time separations display essentially the same behavior as would be obtained for free fields. The pronounced oscillation of the vector correlator indicates near degeneracy of even and odd parity spin-1 states (parity doubling). It is clear that, after cooling, pseudoscalar and vector states are also nearly degenerate. In other words, after 20 to 50 cooling steps, quark propagation at $ma = 0.1$, at least in the limited time region available on the $12^4$ lattice, shows very little evidence of spin-dependent forces or chiral symmetry breaking.

On the $16^4$ lattice the correlations functions can be explored over a larger time range and at smaller masses. The results for $ma = 0.1$ and $ma = 0.05$ are plotted in Figs. 13 and 14 respectively. At large time separations the correlation functions differ from those obtained
for free fields. For example, comparing Figs. 12 and 13 one sees at the largest time separation some evidence of a difference between the pseudoscalar and vector channels. Similarly, if the mass is decreased (compare Figs. 13 and 14) the difference between pseudoscalar and vector correlators after cooling is enhanced.

Although our lattices are too small to do very accurate mass determinations we were able to get reasonable fits to the pseudoscalar correlator on the $16^4$ lattice in a restricted time window with the function

$$g_0(t) = C \left[ e^{-M_P t} + e^{-M_P (L-t)} \right],$$

where $L$ is the size of the lattice. The meson decay constant $f_P$ can be calculated \cite{19} from the coefficient $C$ according to $f_P = 2m\sqrt{C}/M_P^{3/2}$. The results for the mass and decay constant are given in Table 1. The pattern is consistent with the findings of Chu et al. \cite{4}. The masses (in lattice units) are reduced to about 2/3 of the uncooled values and do not change very much in the range of intermediate cooling (20 to 50 cooling steps). The meson decay constant, which reflects the short-distance behavior of the wave function, is reduced to about 1/2 of the uncooled value.

How is one to interpret the behavior of the meson correlators under cooling? One way is that the correlators reflect the true dynamics of the cooled vacuum. At sufficiently large times and sufficiently small masses the correlation functions are not affected by cooling very much so hadron properties (and the uncooled lattice scale) survive essentially intact. This is the approach of Chu et al. \cite{4}. An obvious question with this interpretation is how to reconcile it with the finding of direct calculation that the spin-dependent potentials and chiral symmetry breaking essentially disappear (in lattice units) after only a small amount of cooling.

Another interpretation is that the effect identified by Teper \cite{6} in the string tension is also present in the meson correlators. Namely, correlations functions do not relax uniformly under cooling. At sufficiently large time separations it is inevitable that the behavior of the original uncooled configurations persists and this does not reflect the dynamics of the cooled vacuum.

IV. SUMMARY

In this work the effect of cooling on a number of observables was calculated in SU(2) lattice gauge theory. These include the central and spin-dependent potentials, the chiral symmetry breaking order parameter and meson correlators.

Even after 100 cooling steps a remnant of the confining static potential is seen at large distance. However interactions on the distance scale of a few lattice spacings are essentially eliminated. This includes the spin-dependent interactions induced by chromo-magnetic field correlations.

The quantity $\langle \overline{\chi}\chi \rangle$ was found to approach its free-field value very quickly with cooling. This may indicate that on the lattice, with staggered fermions, chiral symmetry breaking is driven more by local fluctuations in the topological charge density rather than by global topological properties. As cooling smooths out local fluctuations, chiral symmetry breaking decreases rapidly, even though instantons remain. This is in line with the study of Hands.
and Teper [20] who suggest that the instanton-induced zero modes for staggered fermions do not become delocalized as would be required for chiral symmetry breaking.

Meson correlators up to time separations of several lattice spacings relax quickly to free-field values reflecting the behavior of the potentials. At larger times, differences from free-field behavior persist. A fit to the pseudoscalar correlation function yields “mass” values about $2/3$ of the uncooled masses. To this extent our results confirm the calculation of Chu et al. [4].

On the other hand, we cannot conclude that our results provide evidence for the dominant role of instantons. There are properties of QCD that are changed by cooling. The lack of a direct signature for spin-dependent forces and chiral symmetry breaking has to be reconciled with the large-time behavior of the hadron correlators. Until this can be done the interpretation of the hadron properties after cooling remains imprecise.

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FIGURES

FIG. 1. Histogram of the number of configurations versus the absolute value of the topological charge in configurations with no cooling.

FIG. 2. Histogram of the number of configurations versus the absolute value of the topological charge after (a) 25 cooling steps, (b) 50 cooling steps, (c) 75 cooling steps, and (d) 100 cooling steps.

FIG. 3. Histogram of the number of configurations versus the average plaquette after (a) 25 cooling steps, (b) 50 cooling steps, (c) 75 cooling steps, and (d) 100 cooling steps. The vertical dashed lines in (d) indicate the value of the average plaquette corresponding to an instanton action with winding number 1, 2, 3, and 4.

FIG. 4. Scatterplot of the topological charge versus the average plaquette after (a) 25 cooling steps, (b) 50 cooling steps, (c) 75 cooling steps, and (d) 100 cooling steps.

FIG. 5. The Creutz $C(R, R)$ as a function of loop size $R$ for (a) no cooling (△) and 25 cooling steps (●), (b) 25 cooling steps (●), 50 cooling steps (■) and 100 cooling steps (filled triangles).

FIG. 6. The static potential $V(R)$ versus $R$ for (a) no cooling (△) and 25 cooling steps (●), (b) 25 cooling steps (●), 50 cooling steps (■) and 100 cooling steps (filled triangles).

FIG. 7. The spin-spin potential $V_{SS}(R)$ versus $R$ for (a) no cooling (△) and 25 cooling steps (●), (b) 25 cooling steps (●), 50 cooling steps (■) and 100 cooling steps (filled triangles).

FIG. 8. The tensor potential $V_{T}(R)$ versus $R$ for (a) no cooling (△) and 25 cooling steps (●), (b) 25 cooling steps (●), 50 cooling steps (■) and 100 cooling steps (filled triangles).

FIG. 9. The chiral order parameter $\langle \bar{\chi} \chi \rangle$ versus fermion mass $m_a$ calculated on a $12^4$ lattice for no cooling (△) and 10 cooling steps (□). The lattice free-field values are also shown (◦).

FIG. 10. The chiral order parameter $\langle \bar{\chi} \chi \rangle$ versus fermion mass $m_a$ calculated on a $16^4$ lattice for no cooling (△) and 10 cooling steps (□). The lattice free-field values are also shown (◦).

FIG. 11. The chiral order parameter $\langle \bar{\chi} \chi \rangle$ extrapolated to zero fermion mass as a function of cooling step on the $12^4$ lattice (△) and the $16^4$ lattice (◦).
FIG. 12. Comparison of pseudoscalar ($\triangle$) and vector ($\circ$) meson correlators calculated on a $12^4$ lattice at $ma = 0.1$ for (a) no cooling, (b) 10 cooling steps, (c) 20 cooling steps, and (d) 50 cooling steps.

FIG. 13. Comparison of pseudoscalar ($\triangle$) and vector ($\circ$) meson correlators calculated on a $16^4$ lattice at $ma = 0.1$ for (a) no cooling, (b) 10 cooling steps, (c) 20 cooling steps, and (d) 50 cooling steps.

FIG. 14. Comparison of pseudoscalar ($\triangle$) and vector ($\circ$) meson correlators calculated on a $16^4$ lattice at $ma = 0.05$ for (a) no cooling, (b) 10 cooling steps, (c) 20 cooling steps, and (d) 50 cooling steps.
TABLES

TABLE I. Results of fits to pseudoscalar meson mass and decay constant. A rough estimate of the systematic error due to the fitting procedure is about 5% in the values for $M_P$, and about 10% in the values for $f_P$.

| $ma$ | cooling step | $M_{Pa}$ | $f_{Pa}$ | $ma$ | cooling step | $M_{Pa}$ | $f_{Pa}$ |
|------|--------------|----------|----------|------|--------------|----------|----------|
| 0.05 | 0            | 0.61     | 0.13     | 0.10 | 0            | 0.83     | 0.17     |
| 10   | 0.45         | 0.077    |          | 10   | 0.56         | 0.089    |          |
| 20   | 0.38         | 0.075    |          | 20   | 0.58         | 0.085    |          |
| 50   | 0.34         | 0.070    |          | 50   | 0.51         | 0.075    |          |
| 0.20 | 0            | 1.11     | 0.22     | 0.30 | 0            | 1.33     | 0.26     |
| 10   | 0.74         | 0.11     |          | 10   | 0.98         | 0.13     |          |
| 20   | 0.74         | 0.10     |          | 20   | 0.94         | 0.12     |          |
| 50   | 0.74         | 0.10     |          | 50   | 0.92         | 0.11     |          |
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