Analytical calculation method of free static magnetic field point in magnetic signal measurement

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Abstract. In the application of magnetic signal measurement such as non-invasive temperature of ferromagnetic resonance, magnetic nanoparticle fluid hyperthermia, magnetic nanoparticle concentration imaging, etc., it is necessary to generate a free static magnetic field point (FFP), so as to the signal is extracted and the corresponding magnetic signal is obtained by calculation. FFP is the point at which the static magnetic field produces a magnetic field strength that is approximately zero. FFP is established by a set of coils that generate static magnetic field with different circular currents. Based on Biot-Savar's law, using the geometric relationship of the coils, an analytical method is proposed for calculating the current of each coil when an arbitrary point on the XOY plane as FFP. The analytical method not only overcomes the shortcomings of the traditional iterative method, such as slow convergence and uncertain results, but also has a theoretical basis, fast calculation speed and high precision. The simulation results show that the position error of FFP is within 3 mm, a gradient magnetic field is formed around the point, which satisfies the needs of magnetic signal measurement in the above applications. This is helpful for the improvement of the application of magnetic signal measurement.

1. Introduction

As a new means of tumor Hyperthermia (MFH), Magnetic Fluid Hyperthermia (MFH) has attracted the attention and research of domestic and foreign scholars [1-4]. In this application, magnetic nanoparticle temperature measurement [5-6], magnetic nanoparticle fluid heating control [7-8], magnetic nanoparticle concentration imaging [9-10] and other technologies are included. All these techniques require measurement of magnetic signal. In order to measure the magnetic signal at a certain point, a static gradient magnetic field should be constructed so that the point is a free static magnetic field point (FFP). The magnitude of the static magnetic field at the point is zero, and the magnetic field around the point presents a gradient distribution [11]. The signal at the non-free static magnetic field point will not affect the magnetic signal measurement of the free static magnetic field point due to the saturation of static magnetic field [12]. The collected signal is extracted by using the Fourier transform method, and the corresponding magnetic signal is obtained by calculation. The static gradient magnetic field is mainly composed of multiple coils, which are formed into a circle in space [13-14]. The model constructed in this paper consists of six coils. Through the size design, the coil of the static gradient magnetic field model is more compact.
Currently, swarm intelligence algorithm, such as particle swarm optimization algorithm, is traditionally used to calculate the coil current for zero static magnetic field\textsuperscript{[15-16]}. However, the swarm intelligence algorithm has the disadvantages of easily falling into local optimum, low convergence accuracy, difficult convergence and slow calculation speed. In this paper, based on the Biot-Savar’s law and the geometry of the coil, the required coil current is deduced theoretically, so that the specific set point meets the requirement of free static magnetic field. The theoretical analysis method avoids the disadvantages of the traditional algorithm, and its calculation speed is fast, the result is reliable and stable. This makes high-precision real-time computing possible, which will facilitate the development of application technologies based on magnetic signal measurement.

2. Introduction to theory and method

2.1. The magnetic field excited by a coil

In magnetostatics, Biot-Savar's law describes the magnetic field excited by a current element at any point P in space. It is suitable for calculating the magnetic field generated by a steady current. Adopting the International System of Units, the law is expressed by Equation (1).

$$d\vec{B} = \frac{\mu_0 \vec{I} \times \vec{e}}{r^2}$$

Where, $I$ is the current, $dl$ is the tiny line element of the current, $\vec{e}$ is the unit vector of the current element pointing to the field point, and $\mu_0$ is the vacuum magnetic permeability.

A coil is in the rectangular coordinate system with Z axis as the center axis and center as the origin, as shown in figure 1(a). The magnetic induction intensity excited at any point $P(x, y, z)$ in space is $\vec{B} = (B_x, B_y, B_z)$. Formula (2) is the calculation formula of the three components\textsuperscript{[17]}.

\[
\begin{align*}
B_x &= \frac{\mu_0 I R^2}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{\cos \theta d\theta}{\left[(x-R\cos \theta)^2 + (y-R\sin \theta)^2 + z^2\right]^{3/2}} \\
B_y &= \frac{\mu_0 I R^2}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{\sin \theta d\theta}{\left[(x-R\cos \theta)^2 + (y-R\sin \theta)^2 + z^2\right]^{3/2}} \\
B_z &= \frac{\mu_0 I R^2}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{R(R-x\cos \theta-y\sin \theta) d\theta}{\left[(x-R\cos \theta)^2 + (y-R\sin \theta)^2 + z^2\right]^{3/2}}
\end{align*}
\]

In the formula, I is the current passing through the coil, $\mu_0$ is the vacuum permeability, R is the coil radius, x, y and z are the coordinates of P, and $B_x$, $B_y$ and $B_z$ are the components of magnetic induction intensity on the three coordinate axes respectively. The Simpson's numerical integration is used to solve the numerical value.

To facilitate the calculation of the established model, a coordinate system is established as shown in figure 1(b). Place the coil in the rectangular coordinate system with the y axis as the center axis and the center as (0,-L,0). The free magnetic field point $P(x, y, 0)$ is located in the XOY plane, and the free magnetic field point is abbreviated as $P(x, y)$.
In this coordinate system, the magnetic induction intensity generated by the coil 1 at any point \(P(x,y,z)\) can be calculated by the coordinate transformation of Equation 2, which is Equation (3). The magnetic induction intensity generated by the coil 1 is a vector \(B_1 = (B_{1x}, B_{1y}, B_{1z})\).

\[
B_{1x} = \frac{\mu_0 I_1}{4\pi} \int_0^{2\pi} \left( \frac{(y+L)\sin \theta}{(z-R\cos \theta)^2 + (x-R\sin \theta)^2 + (y+L)^2} \right) \, d\theta
\]

\[
B_{1y} = \frac{\mu_0 I_1}{4\pi} \int_0^{2\pi} \left( \frac{R(z-R\cos \theta - x\sin \theta)}{(z-R\cos \theta)^2 + (x-R\sin \theta)^2 + (y+L)^2} \right) \, d\theta
\]

\[
B_{1z} = \frac{\mu_0 I_1}{4\pi} \int_0^{2\pi} \left( \frac{(z-R\cos \theta)^2 + (x-R\sin \theta)^2 + (y+L)^2}{(y+L)\cos \theta \, d\theta} \right)
\]

Since the free magnetic field point \(P(x,y)\) lies in the XOY plane, according to the symmetry, \(B_{1z} = 0\). The magnetic induction intensity at the free magnetic field point \(P\) is \(B_1 = (B_{1x}, B_{1y}, 0)\), abbreviated as \(B_1 = (B_{1x}, B_{1y})\).

2.2. Methods

In this paper, three coils are used as a group to calculate the coil current. Using Biot-Savar's law, we obtain the magnetic field excited by coil 1 to any point \(P\) in the coordinate system established by the model. Coil 2 and coil 3 can be viewed as the coils that coil 1 rotating counterclockwise about the Z axis at 120 degrees and 240 degrees respectively. According to the knowledge of geometry and coordinate transformation, after the point P coordinates are rotated 120 degrees and 240 degrees clockwise around the Z axis respectively, magnetic field excited by coil 2 and coil 3 to point P and the magnetic field calculated by the calculation method of the coil 1 are the same in size, 240 degrees and 120 degrees phase difference in the direction, respectively. After rotating the magnetic field in the direction of 120 degrees and 240 degrees counterclockwise around the Z axis respectively, it is concluded that the magnetic field excited by coil 2 and coil 3 in this coordinate system for P is obtained. In order to make P meet the requirements of free static magnetic field, the combined magnetic field generated by the set of coils to P is free, that is, the three magnetic field vectors generated by the coils 1-3 to P are added to zero.

According to the knowledge of geometry, three vectors add up to zero, that is, three vectors can form the head and tail connected triangle. According to Biot-Savar's law, the sine of the Angle between the three magnetic field vectors generated by the coil 1-3 to P is unchanged. According to the relation between the three sides and the corresponding Angle in the sine theorem, the ratio relation of the current of the three coils can be obtained. Under the assumption that coil 3 current size is 1, the current size of coil 1-2 is determined according to the ratio relation. The direction of the three currents is determined by judging whether the three magnetic field vectors add up to zero.

3. Calculation of current for a set of coils

3.1. Calculation of magnetic induction intensity of a set of coils

The coil 1 is rotated counterclockwise by 120 degrees around the z-axis to become the coil 2. As shown in figure 2(a).

![Figure 2. coil 1 turns 120 and 240 degrees counterclockwise around the z axis.](image)
When calculating the magnetic induction intensity of coil 2 to the point P, it can be seen that coil 2 is rotated 120 degrees clockwise around the z axis to the position of coil 1, that is, position \( P(x, y) \) is rotated 120 degrees clockwise around the z axis to \( P_1(x_1, y_1) \). The rotation formula is formula (4). Among them, \( \delta = \frac{2\pi}{3} \).

\[
(x, y) \begin{bmatrix}
\cos\delta & \sin\delta \\
-\sin\delta & \cos\delta
\end{bmatrix} = (x_1, y_1)
\]

(4)

The calculation method of coil 1 was used to calculate \( p_1 \), and the magnetic induction intensity \( B_{p1} = (B_{p1x}, B_{p1y}) \) of coil 1 to \( p_1 \) was obtained, and then the calculation result of coil 1 to \( p_1 \) was rotated 120 degrees counterclockwise around the z axis, namely the magnetic induction intensity of coil 2 to \( p \), denoted as vector \( B_2 = (B_{2x}, B_{2y}) \). The rotation formula is formula (5). Among them, \( \delta = \frac{2\pi}{3} \).

\[
(B_{p1x}, B_{p1y}) \begin{bmatrix}
\cos\delta & -\sin\delta \\
\sin\delta & \cos\delta
\end{bmatrix} = (B_{2x}, B_{2y})
\]

(5)

The coil 1 is rotated counterclockwise by 240 degrees around the z-axis to become the coil 3. As shown in figure 2(b).

When calculating the magnetic induction intensity of coil 3 to the point P, it can be seen that coil 3 is rotated 240 degrees clockwise around the z axis to the position of coil 1, that is, the position \( P(x, y) \) is rotated 240 degrees clockwise around the z axis to \( P_2(x_2, y_2) \). The rotation formula is formula (6). Among them, \( \delta = \frac{4\pi}{3} \).

\[
(x, y) \begin{bmatrix}
\cos\delta & \sin\delta \\
-\sin\delta & \cos\delta
\end{bmatrix} = (x_2, y_2)
\]

(6)

The calculation method of coil 1 was used to calculate \( p_2 \), and the magnetic induction intensity \( B_{p2} = (B_{p2x}, B_{p2y}) \) of coil 1 to \( p_2 \) was obtained. Then, the calculation result of coil 1 to \( p_2 \) was rotated 240 degree counterclockwise around the z axis, namely the magnetic induction intensity of coil 3 to \( p \), denoted as a vector \( B_3 = (B_{3x}, B_{3y}) \). The rotation formula is formula (7). Among them, \( \delta = \frac{4\pi}{3} \).

\[
(B_{p2x}, B_{p2y}) \begin{bmatrix}
\cos\delta & -\sin\delta \\
\sin\delta & \cos\delta
\end{bmatrix} = (B_{3x}, B_{3y})
\]

(7)

3.2. Calculation of current size of a set of coils

Through the calculation of 3.1, the magnetic induction intensity of the three coils at point \( p \), \( B_1 \), \( B_2 \) and \( B_3 \), respectively, can be obtained. Since \( p \) is specified to be in the XOY plane, the component of magnetic induction intensity in the z-axis direction is 0. \( B_1 \), \( B_2 \), and \( B_3 \) can be considered as two-dimensional vectors.

In order to achieve the magnetic induction intensity generated by the three coils at point \( p \) is 0, the vectors of \( B_1 \), \( B_2 \) and \( B_3 \) are required to add up to 0, that is, the three vectors need to satisfy the triangle that can form the head and tail connection, as shown in figure 3(a).

![Figure 3. three vectors form an end-to-end triangle.](image-url)
Formula (8) is derived from the sine theorem.

\[
\frac{|B_1|}{\sin \alpha} = \frac{|B_2|}{\sin \beta} = \frac{|B_3|}{\sin \gamma}
\]  

(8)

According to the Biot-Savar's law, it can be seen that the current element \(dl\) produces the magnetic field \(dB\) at \(p\). The current value only affects the size of the magnetic field \(dB\), not the direction of \(dB\). The direction of \(dB\) is determined by the cross product of \(dl\) and \(e_r\). It can be seen from formula 2 that if the magnetic induction intensity of coil 1 with current value of 1 against \(p\) is vector \(B\), then the magnetic induction intensity when current is \(I\) is vector \(IB\). Thus, when the current is positive, the current does not affect the direction of the magnetic induction intensity, and when the current is negative, the direction of the magnetic induction intensity is reversed. The current doesn't affect the sine of the angles between the three vectors \(\alpha\), \(\beta\) and \(\gamma\).

For the convenience of calculation, the magnetic induction intensity \(CoilB1\), \(CoilB2\) and \(CoilB3\) generated at \(p\) point are calculated respectively when the current of the three coils is 1, and then calculate the \(\alpha\), \(\beta\) and \(\gamma\) respectively.

In order to make the magnetic induction intensity at point \(p\) equal to 0, the magnetic induction intensity \(I_1*CoilB1\), \(I_2*CoilB2\) and \(I_3*CoilB3\) generated by the three coils can form a triangle connected head to tail, as shown in figure 3(b), which satisfies formula (9).

\[
\frac{|I_1*CoilB1|}{\sin \alpha} = \frac{|I_2*CoilB2|}{\sin \beta} = \frac{|I_3*CoilB3|}{\sin \gamma}
\]  

(9)

If \(I_1\) is set as 1, the ratio of the current size of the three coils can be obtained according to formula (9), as shown in formula (10).

\[
I_1 : I_2 : I_3 = \frac{\sin \alpha}{\sin \gamma} \frac{|CoilB3|}{|CoilB1|} \frac{\sin \beta}{\sin \gamma} \frac{|CoilB3|}{|CoilB2|}
\]  

(10)

According to the actual needs, the current of the three coils can meet the ratio relationship.

3.3. Calculation of current direction of a set of coils

Finally determine the direction of the current.

If \(-I_1*CoilB1 + I_2*CoilB2 + I_3*CoilB3 = 0\), the current direction of \(I_1\) is reversed.

If \(-I_1*CoilB1 - I_2*CoilB2 + I_3*CoilB3 = 0\), the current direction of \(I_2\) is reversed.

If \(-I_1*CoilB1 - I_2*CoilB2 - I_3*CoilB3 = 0\), the current direction of \(I_1\) and \(I_2\) are reversed.

4. Experimental simulation analysis

4.1. The experiment design

The gradient magnetic field model is composed of 6 coils, 3 coils are a group, and there are 2 groups, which are deployed in the peripheral symmetric space. The radius of the coil around the magnetic field is 0.15m, and the radius of the center circle is 0.3m. The center of the first set of three coils is uniformly distributed on the circle in the XOY plane at every 120 degrees, as shown in figure 4(a). The position of the second set of coils is obtained by rotating the first set of coils 180 degrees counterclockwise. After rotating the free magnetic field point clockwise for 180 degrees, the calculation procedure of the first set of coils is adopted to calculate the current size and direction of the second set of coils. See figure 4(b). The coil radius R is set to be 0.15m, and the central position of the first set of coil 1 is (0,-0.3,0), namely L=0.3m.
In order to verify the feasibility and accuracy of the analytic method in this paper, it is assumed that the free static magnetic field point on XOY plane is located at the origin (0,0,0), X-axis (0.1,0,0), Y-axis (0,0.1,0) and the first quadrant (0.1,0.1,0), respectively. The analytic method in this paper is used to calculate the current of each coil and bring it into comsol finite element simulation software for verification.

4.2. Experimental results and analysis

Through the analytic method in this paper, the free static magnetic field point is located in the XOY plane, respectively at the origin (0,0,0), X-axis (0.1,0,0), Y-axis (0,0.1,0) and the first quadrant (0.1,0.1,0). The calculation results of coil current of each group are shown in table 1.

| Position | Group | Coil1 | Coil2 | Coil3 |
|----------|-------|-------|-------|-------|
| (0,0,0)  | 1     | 1     | 1     | 1     |
|          | 2     | 1     | 1     | 1     |
| (0.1,0,0)| 1     | 1.3141| 0.6334| 1     |
|          | 2     | 2.0748| 1.5789| 1     |
| (0,0.1,0)| 1     | 0.7813| 1     | 1     |
|          | 2     | 0.3897| 1     | 1     |
| (0.1,0.1,0)| 1  | 0.6223| 0.2219| 1     |
|          | 2     | 0.5074| -0.1161| 1     |

The calculation result in Table 1 is a proportional relationship required for each group of three coils in the case where the current of the coil 3 in each group of coils is assumed to be 1. According to the actual needs, the three coil currents in each group can be scaled up and down. The two sets of coil currents in Table 1 which can satisfy the four free static magnetic field points were taken into the Comsol simulation software respectively and simulated using the finite element method. The simulation results are shown in figure 5. The simulation position of the free static magnetic field point and its magnetic field are shown in table 2.

| set position | simulation position | Position error | magnetic field size |
|--------------|---------------------|---------------|--------------------|
| (0,0,0)      | (-5.63e-4,-1.23e-3,0)| (5.63e-4,-1.23e-3,0) | 4.03e-8          |
| (0.1,0,0)    | (9.81e-2,-1.66e-3,0) | (1.90e-3,-1.66e-3,0) | 1.93e-8          |
| (0.0,1,0)    | (-2.53e-3,9.96e-2,0) | (2.53e-3,4.00e-4,0) | 1.21e-8          |
| (0.1,0.1,0)  | (9.71e-2,9.73e-2,0)  | (2.90e-3,2.70e-3,0) | 6.68e-9          |

The deviation between the simulated free static magnetic field point position and the set position is within the range of 3mm. The source of the error may be Simpson's numerical integration and the subdivision degree of mesh during simulation. The magnitude of the magnetic field at the simulated position is close to zero.
Figure 5. FFP at four position.

As can be seen from figure 5, the magnetic field forms a gradient magnetic field centered at this point near the free static magnetic field point. When the free static magnetic field point is at the origin, the magnetic field gradient around it is large. At other locations, the gradient of the magnetic field is relatively small. Especially at $(0.1, 0.1, 0)$, the gradient of the magnetic field is small in one direction. However, by increasing the coil current according to the current ratio, the gradient of the magnetic field can be increased to satisfy the gradient requirement of the magnetic field at the free static magnetic field point.

From the experimental results, it can be concluded that the position error of the free static magnetic field point or the gradient at that point can meet the requirements of the magnetic signal measurement application.

5. Conclusion

In the application based on magnetic signal measurement, the traditional swarm intelligence algorithm has the disadvantages of unstable results, slow calculation speed and easy to fall into local solutions when calculating free static magnetic field points. Based on Biot-Savar's law and positional relationship between coils, this paper proposes an analytical method for free static magnetic field points based on coil grouping calculation. Compared with the traditional swarm intelligence algorithm, the analytical method has theoretical support, the result is unique, and the calculation speed is fast. The simulation results of the analytical method are carried out by the finite element method. The experimental results show that the simulated free static magnetic field point and the set free static magnetic field point error are less than 3mm, which meets the requirements of magnetic signal measurement applications. This analytical method facilitates real-time calculation of free static magnetic field points.
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