Taking into Account the Centre-of-Mass Correlations in the Cross Sections for Elastic Scattering of Intermediate Energy Protons on the Exotic Nuclei $^6$He and $^8$He

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Abstract — We calculate the differential cross sections for proton elastic scattering on the exotic halo nuclei $^6$He and $^8$He at energies around $\sim 0.7 \text{ GeV}$ at the momentum transfers squared up to $0.30 \text{ (GeV/c)}^2$ and investigate the influence of the nucleon centre-of-mass correlations on the calculated cross sections. In particular, we show that the approximate account of the centre-of-mass correlations used previously considerably overestimates the cross sections at high values of the momentum transfer.

Comments: 12 pages, 2 figures. Submitted to Physics of Atomic Nuclei

Subjects: Nuclear Theory(nucl-th); High-Energy Physics-Theory(hep-th)

1. INTRODUCTION

Proton-nucleus elastic scattering at intermediate energy is an efficient means of studying the nuclear spatial structure. Several experiments were performed [1–3] in which the cross sections for proton elastic scattering on light exotic nuclei were measured in inverse kinematics at GSI-Darmstadt at an energy of $\sim 700 \text{ MeV/u}$ at small momentum transfers ($\sim 0.002 \leq |t| \leq \sim 0.05 \text{ (GeV/c)}^2$, where $t$ is the four momentum transfer squared). The cross sections were measured with the help of the hydrogen-filled ionization chamber IKAR [4], which served simultaneously as a gas target and a detector of the recoil protons. In particular, the $p^6$He and $p^8$He cross sections were measured [1].

The measured cross sections were analysed [5] in the framework of the Glauber multiple-scattering theory, and the root-mean-square (rms) radii of the nuclear total
matter $R_m$, the nuclear core $R_c$ and the neutron halo $R_h$ were deduced for $^6$He and $^8$He. Later, the $p^6$He and $p^8$He cross sections for elastic scattering were measured [6] practically at the same energy at higher momentum transfers (up to $|t| = 0.225$ (GeV/c)$^2$) using an experimental set-up with a liquid hydrogen target. The new experimental data at higher values of $|t|$ in combination with the cross sections [1] measured at low $|t|$-values allow, in principle, to determine the $^6$He and $^8$He nuclear radii $R_c$, $R_h$ and $R_m$ with better accuracy [7]. The many-body density distributions of $^6$He and $^8$He used in the calculations [7] of the cross sections were represented as products of one-body densities. The effect of the centre-of-mass (CM) correlations was taken into account by an approximate approach (to be considered below).

As is known, the effect of the CM correlations in the calculated cross sections is rather sizeable in the case of proton scattering on light nuclei at large momentum transfers. In the present paper, in order to find out the effect of the CM correlations on the $p^6$He and $p^8$He cross sections, we have calculated these cross sections neglecting the CM correlations and taking them into account by approximate and exact methods.

2. BASIC FORMULAS

According to the Glauber theory [8], the proton-nucleus elastic scattering amplitude $F(q)$ can be calculated as

$$F(q) = \frac{ik}{2\pi} \int d^2b \exp(iqb) \rho_A(r_1, ..., r_A) d^3r_1 d^3r_2 ... d^3r_A \times$$

$$\times \left\{ 1 - \prod_{j=1}^{A} [1 - \gamma(b - s_j)] \right\}. \quad (1)$$

Here, $k$ is the value of the wave vector of the incident proton, $q$ is the momentum transfer, $b$ is the impact-parameter vector, $\rho_A(r_1, ..., r_A)$ is the many-body density distribution of the nucleus, $r_1, ..., r_A$ and $s_1, ..., s_A$ stand for the radius vectors of the nucleons in the nucleus and their transverse coordinates, $A$ is the total number of nucleons in the nucleus, and $\gamma(b)$ is the profile function of the nucleon–nucleon
interaction. We employ a spin-independent amplitude of the free proton–nucleon (pN) scattering with the traditional high-energy parametrization of this amplitude and the corresponding profile-function

\[ \gamma(b) = \frac{\sigma_{pN}(1 - i\epsilon_{pN})}{2} \frac{1}{2\pi\beta_{pN}} \exp\left(-\frac{b^2}{2\beta_{pN}}\right), \]  

(2)

where \( \sigma_{pN} \) is the total cross section for the pN interaction, \( \beta_{pN} \) is the pN amplitude slope parameter, and \( \epsilon_{pN} \) is the ratio of the real to imaginary parts of the pN scattering amplitude. The differential cross section \( d\sigma/dt \) for proton-nucleus elastic scattering is connected with the amplitude \( F(q) \) as

\[ d\sigma/dt = (\pi/k^2)|F(q)|^2. \]  

(3)

If we neglect all nucleon correlations in the nuclear many-body density distribution \( \rho_A(r_1, ..., r_A) \), then it can be represented as a product of similar one-body densities \( \rho_0(r_j) \):

\[ \rho_A(r_1, ..., r_A) = \prod_{j=1}^{A} \rho_0(r_j). \]  

(4)

Let \( F_0(q) \) be the proton-nucleus elastic scattering amplitude calculated by Eq. (1) with many-body density (4). Then in the case where the one-body density \( \rho_0(r_j) \) is a Gaussian distribution

\[ \rho_0(r_j) = (2\pi a_0^2)^{-3/2} \exp(-r_j^2/2a_0^2), \]  

(5)

the CM correlations can be taken into account in the calculated amplitude \( F_1(q) \) as it is known [9] by multiplying the amplitude \( F_0(q) \) with a CM correction factor \( H_0(q) \):

\[ F_1(q) = H_0(q)F_0(q), \]  

(6)

where \( H_0(q) = \exp[q^2R_m^2/6(A-1)] \)

(7)

and \( R_m \equiv \langle r^2 \rangle^{1/2} = [3(1 - 1/A)]^{1/2}a_0 \) is the nuclear rms matter radius calculated in the nucleus CM system.

Equation (6) with correction factor (7) takes into account the CM correlations exactly in the case of a Gaussian one-body nuclear matter density distribution.
(Eq. (5)). For non-Gaussian distributions such an approach is approximate and it is justified for the cases when the distributions of the total nuclear density do not differ significantly from a Gaussian one and for proton scattering on middle-weight and heavy nuclei at relatively small momentum transfers where the effect of the CM correlations is small. However, such a correction factor was used in [2, 3, 5, 7] for proton elastic scattering on light exotic halo nuclei, the total matter density distributions in which differ significantly from a Gaussian one.

In the following, in order to see how big is the effect of the CM correlations in the cross sections and how accurate is the approximate account of the correlations, we perform calculations of the cross sections for proton elastic scattering on the exotic nuclei $^6\text{He}$ and $^8\text{He}$ at an energy of $\sim 700$ MeV neglecting the CM correlations, taking them into account exactly, and using an approximate approach with the correction factor given by Eq. (7).

If we neglect the CM correlations in the nuclear many-body density $\rho_A(r_1, ..., r_A)$, then for exotic nuclei $^6\text{He}$ and $^8\text{He}$ it can be represented as

$$\rho_A(r_1, ..., r_A) = \prod_{j=1}^{A_c} \rho_c(r_j) \times \prod_{j=A_c+1}^{A_c+A_h} \rho_h(r_j).$$

Here $\rho_c(r_j)$ and $\rho_h(r_j)$ are the one-body density distributions of the core and halo of the exotic nucleus correspondingly; $A_c$ and $A_h$ are the numbers of the core and halo nucleons ($A_c + A_h = A$; $A_c = 4$ in $^6\text{He}$ and $^8\text{He}$; $A_h = 2$ in $^6\text{He}$ and $A_h = 4$ in $^8\text{He}$; the core consists of 2 protons and 2 neutrons, the halo consists of neutrons). We describe the matter density distributions in the core and in the halo by Gaussian functions

$$\rho_c(r_j) = (2\pi a_c^2)^{-3/2} \exp(-r_j^2/2a_c^2),$$
$$\rho_h(r_j) = (2\pi a_h^2)^{-3/2} \exp(-r_j^2/2a_h^2),$$

where $a_c = R_c/\sqrt{3}$, $a_h = R_h/\sqrt{3}$, $R_c$ and $R_h$ are the rms radii of the core and halo matter density distributions (we assume $R_h > R_c$). As for the rms radius $R_m$ of the total matter density distribution, it is connected with the core and halo radii $R_c$ and $R_h$ as

$$R_m = \left[(A_c R_c^2 + A_h R_h^2)/A\right]^{1/2}.$$
Amplitude (1) calculated with many-body density distribution (8) does not include the CM correlations. If this amplitude is multiplied by correction factor (7), then the CM correlations are taken into account approximately. Note that in the latter case the parameters \(a_c\) and \(a_h\) are connected with the core and halo radii \(R_c\) and \(R_h\) as

\[
a_c^2 = \frac{[R_c^2 + R_m^2/(A - 1)]}{3}, \quad (12)
\]

\[
a_h^2 = \frac{[R_h^2 + R_m^2/(A - 1)]}{3}. \quad (13)
\]

In the case when the core and halo distributions are Gaussian ones, the CM correlations can be taken into account exactly in a simple way as described below. We assume that the core of \(^6\)He and \(^8\)He consists of a four-nucleon cluster with the size \(R_c^*\) the same as that of the \(^4\)He nucleus \((R_c^* \approx 1.46 \text{ fm}[10])\). This cluster experiences some motion around the CM of the nucleus so that the effective core size \(R_c\) is larger than \(R_c^*\). Similarly, the effective halo size \(R_h\) is larger than the size \(R_h^*\) of the halo cluster (2 neutrons in \(^6\)He and 4 neutrons in \(^8\)He) in its CM system. It is easy to show (see, for example, [7]) that

\[
R_h^* = [R_h^2 - (A_c/A_h)2(R_c^2 - R_c^{*2})]^{1/2}. \quad (14)
\]

We start the calculation of the proton-nucleus scattering amplitude \(F_A(q)\) by calculations of the amplitudes \(F_c^*(q)\) and \(F_h^*(q)\) of proton scattering on the core and halo clusters using the Glauber formula (1). The many-body density distributions of the core and halo clusters are represented as products of one-body density distributions \(\rho_c^*(r_j) \sim \exp(-r_j^2/2a_c^*)\) and \(\rho_h^*(r_j) \sim \exp(-r_j^2/2a_h^*)\) where the parameters \(a_c^*\) and \(a_h^*\) are connected with the rms matter radii \(R_c^*\) and \(R_h^*\) as

\[
a_c^* = R_c^*/[3(1 - 1/A_c)]^{1/2}, \quad (15)
\]

\[
a_h^* = R_h^*/[3(1 - 1/A_h)]^{1/2}. \quad (16)
\]

In order to take into account the CM nucleon correlations in the core and halo clusters, the calculated amplitudes are multiplied by the correction factors \(H_c^*(q)\) and \(H_h^*(q)\), where

\[
H_c^*(q) = \exp[q^2 R_c^{*2}/6(A_c - 1)], \quad (17)
\]
\[ H^*_h(q) = \exp[q^2 R_{h}^* / (2(A_h - 1))]. \]  

Then we prescribe some artificial independent motion of these clusters in the laboratory system in accordance with the distributions \( \rho_c(r_c) \) and \( \rho_h(r_h) \):

\[
\rho_c(r_c) = (2\pi a_{c,h}^2 / A_c)^{-3/2} \exp\left(-A_c r_c^2 / 2a_{c,h}^2\right),
\]

\[
\rho_h(r_h) = (2\pi a_{c,h}^2 / A_h)^{-3/2} \exp\left(-A_h r_h^2 / 2a_{c,h}^2\right),
\]

where \( r_c = \sum_{j=1}^{A_c} r_j / A_c \), \( r_h = \sum_{j=A_c+1}^{A_c+A_h} r_j / A_h \), and \( a_{c,h} \) is a radial parameter to be defined later. Correspondingly, the two-body density distribution \( \rho_{c,h}(r_c, r_h) \) describing the core and halo clusters motion in the laboratory system is

\[
\rho_{c,h}(r_c, r_h) = \rho_c(r_c) \cdot \rho_h(r_h) \sim \exp[-(A_c r_c^2 + A_h r_h^2) / 2a_{c,h}^2].
\]

We can calculate the amplitude \( F_{c,h}(q) \) of proton scattering on the system of these clusters by the basic Glauber formula using the amplitudes \( F^*_c(q) \) and \( F^*_h(q) \), the two-body density distribution \( \rho_{c,h}(r_c, r_h) \) and performing integrations over the radius-vectors \( r_c \) and \( r_h \). We obtain

\[
F_{c,h}(q) = \frac{ik}{2\pi} \int d^2 b \exp(\imath q b) \left\{ 1 - \left[ 1 - \Gamma_c(b) \right] \left[ 1 - \Gamma_h(b) \right] \right\},
\]

where

\[
\Gamma_c(b) = \frac{1}{2\pi i k} \int d^2 q \exp(-q^2 a_{c,h}^2 / 2A_c) F^*_c(q) \exp(-\imath q b),
\]

\[
\Gamma_h(b) = \frac{1}{2\pi i k} \int d^2 q \exp(-q^2 a_{c,h}^2 / 2A_h) F^*_h(q) \exp(-\imath q b).
\]

Now we note that due to the following equality

\[
A_c r_c^2 + A_h r_h^2 = (A_c r_c^2 + A_h r_h^2) + A r_{CM}^2
\]

the two-body density distribution \( \rho_{c,h}(r_c, r_h) \) (given by Eq. (21)) can be represented as a product of the distribution \( \rho(r_{CM}) \) of the CM radius \( r_{CM} = (A_c r_c + A_h r_h) / A \) and the distribution \( \rho'(r_c', r_h') \) of the relative coordinates \( r_c' = r_c - r_{CM} \) and \( r_h' = r_h - r_{CM} \):

\[
\rho_{c,h}(r_c, r_h) = \rho(r_{CM}) \cdot \rho'(r_c', r_h'),
\]

where \( \rho(r_{CM}) \sim \exp(-A r_{CM}^2 / 2a_{c,h}^2) \)
and $\rho'(r'_c, r'_h) \sim \exp[-(A_cr'^2_c + A_h r'^2_h)/2a_{c,h}^2]$.  

(28)

Let us calculate the amplitude $F_{c,h}(q)$ using the Glauber formula and the distribution $\rho_{c,h}(r_c, r_h)$ given by Eq. (26). Then performing integrations over the coordinates $r_{CM}$ and $r'_c$ (or $r'_h$) we obtain:

$$F_{c,h}(q) = \exp(-q^2 a_{c,h}^2 / 2A) \cdot F_A(q),$$

(29)

where $F_A(q)$ is the amplitude of interest. Consequently, the amplitude of proton-nucleus elastic scattering can be calculated as

$$F_A(q) = \exp(q^2 a_{c,h}^2 / 2A) \cdot F_{c,h}(q),$$

(30)

where the amplitude $F_{c,h}(q)$ is given by Eqs. (22–24).

Now let us calculate the square of the effective core size $R_c$. Taking into account the value of the internal core size $R^*_c$ and equations (19) and (27) we obtain:

$$R_c^2 = R^*_c^2 + 3a_{c,h}^2 A_c - 3a_{c,h}^2 A_h = R^*_c^2 + 3a_{c,h}^2 A_h / AA_c.$$  

(31)

Therefore, the parameter $a_{c,h}$ used in the calculations of the amplitude $F_{c,h}(q)$ is determined through the nuclear rms radii $R_c$ and $R^*_c$ as

$$a_{c,h} = [\frac{AA_c}{3A_h}(R^*_c^2 - R_c^2)]^{1/2}.$$  

(32)

3. RESULTS of CALCULATIONS

We have calculated the cross sections for proton elastic scattering on the $^6$He and $^8$He nuclei correspondingly at the energies 717 MeV and 674 MeV in the momentum transfer range ~0 < \(|t| < 0.30 \ (\text{GeV}/c)^2\). The values of the rms radii $R_c$ and $R_h$ of the $^6$He and $^8$He nuclei and the input parameters $\sigma_{pN}$, $\beta_{pN}$, and $\epsilon_{pN}$ of the proton-nucleon scattering amplitudes were taken from [7]:

$R_c = 1.96 \ \text{fm}, \ R_h = 3.30 \ \text{fm}$ for $^6$He,

$R_c = 1.81 \ \text{fm}, \ R_h = 3.12 \ \text{fm}$ for $^8$He;
\[ \sigma_{pp} = 44.6 \text{ mb}, \quad \beta_{pp} = 0.20 \text{ fm}^2, \quad \epsilon_{pp} = 0.069, \]
\[ \sigma_{pn} = 37.7 \text{ mb}, \quad \beta_{pn} = 0.24 \text{ fm}^2, \quad \epsilon_{pn} = -0.307 \]
for the proton-proton (\(pp\)) and proton-neutron (\(pn\)) interaction in the case of \(p^6\)He scattering, and
\[ \sigma_{pp} = 41.9 \text{ mb}, \quad \beta_{pp} = 0.20 \text{ fm}^2, \quad \epsilon_{pp} = 0.129, \]
\[ \sigma_{pn} = 37.4 \text{ mb}, \quad \beta_{pn} = 0.24 \text{ fm}^2, \quad \epsilon_{pn} = -0.283 \]
for the \(pp\) and \(pn\) interaction in the case of \(p^8\)He scattering. The internal size of the core \(R_c^*\) in these nuclei was taken as \(R_c^* = 1.46 \text{ fm}\) [10]. The Coulomb interaction was taken into account as in [11].

The results of the calculations are presented in Figs. 1 and 2. The dotted, dashed and solid curves correspond respectively to the calculations where the CM correlations were neglected, included with approximate correction factor (7) and taken into account exactly as was described above. Note that in these three calculations the same nuclear one-body density distribution was used:

\[ \rho_A(r) = [A_c(3/2\pi R_c^2)^{3/2}\exp(-3r^2/2R_c^2) + A_h(3/2\pi R_h^2)^{3/2}\exp(-3r^2/2R_h^2)]/A. \quad (33) \]

So the difference between the results of these calculations is due to neglect or different account of the CM correlations. The experimental cross sections are also shown in the figures: the hollow circles – the data of [1], and the solid squares – the data of [6].

As it is seen in the figures, the effect of the CM correlations in the calculated cross sections at \(0 < |t| < 0.10 \text{ (GeV/c)}^2\) is rather small and in a first approximation can be neglected. At \(|t| > \sim 0.10 \text{ (GeV/c)}^2\), especially in the region of the first diffraction minimum and the second diffraction maximum, the effect of the CM correlations is rather sizeable. We also see that at high momentum transfers the approximate approach of taking the CM correlations into account results in a significant overestimation of the cross sections as compared with the exact calculations.

The calculated cross sections at \(|t| > \sim 0.13 \text{ (GeV/c)}^2\) are smaller than the experimental ones. The behaviour of the calculated cross sections at high values of \(|t|\) is governed mainly by the size of the nuclear core and its radial shape. So, varying the
core size and its shape, in principle, it is possible to fit the calculated cross sections to the data. In the present paper, however, we did not try to perform such a fit. We note that the experimental cross sections of [6] have rather large uncertainties at $|t| > 0.15$ (GeV/c)$^2$. In order to get more precise sizes of the cores of the studied nuclei and to get information on the radial shapes of the cores, new experimental data of better quality at high $|t|$-values and new theoretical analyses with accurate accounts of the CM nucleon correlations are needed.

4. CONCLUSION

We have shown that the effect of the CM correlations in the cross sections for intermediate-energy proton elastic scattering on light exotic nuclei at high momentum transfers is rather sizeable, and it is important to take it accurately into account. We have also shown that an approximate account of the CM correlations with correction factor (7) is not justified since it results in a significant overestimation of the calculated cross sections at high $|t|$-values.

We are grateful to O. A. Kiselev for sending us the data of [6] in the tabular form.
Fig. 1. Cross sections for proton elastic scattering on the $^6$He nuclei at an energy of 717 MeV. Hollow circles – data of [1], solid squares – data of [6]. Dotted, dashed and solid curves correspond respectively to the cross sections calculated not taking the CM correlations into account, with the CM correlations taken into account using approximate correction factor (7), and with the CM correlations taken into account exactly.
Fig. 2. The same as in Fig. 1 for the cross sections for proton elastic scattering on the \(^8\)He nuclei at an energy of 674 MeV.
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