A unified cosmic evolution: Inflation to late time acceleration

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The present work deals with a cosmological model having particle creation mechanism in the framework of irreversible thermodynamics. In the second order non-equilibrium thermodynamical prescription, the particle creation rate is treated as the dissipative effect. The non-equilibrium thermodynamical process is assumed to be isentropic, and, as a consequence, the entropy per particle is constant, and, hence, the dissipative pressure can be expressed linearly in terms of the particle creation rate in the background of the homogeneous and isotropic flat FLRW model. By proper choice of the particle creation rate as a function of the Hubble parameter, the model shows the evolution of the universe starting from the inflationary scenario to the present accelerating phase, considering the cosmic matter as normal perfect fluid with barotropic equation of state.

Keywords: Cosmic evolution, Particle creation, Bulk viscosity, Isentropic process, Cosmography

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I. INTRODUCTION

In cosmology, usually homogeneous and isotropic flat Friedmann-Lemaître-Robertson-Walker (FLRW) model is considered due to its agreement with inflation and the cosmic microwave background (CMB) observations. Here, the only dissipative phenomenon is in the form of bulk viscous pressure which may occur either due to coupling of different components of the cosmic substratum [1–5], or, due to non-conservation of (quantum) particle number [6–8]. In the present work, second option is only considered and for simplicity of calculations, isentropic (i.e., adiabatic) particle production [9, 10] of perfect fluid particles is considered, and, consequently, there is a simple linear relationship between particle production rate, and, the viscous pressure. However, it is to be noted that still there is entropy production due to enlargement of the phase space (due to increase in the number of fluid particles, and, also, due to the expansion of the universe in the present model) of the system.

Further, in the context of non-equilibrium thermodynamical prescription, second order deviations from equilibrium (following Israel and Stewart [11, 12]) are considered in order to eliminate the drawbacks of the 1st order Eckart theory related to causality and stability. As a result, entropy flow will be related to the cosmological particle production of isentropic nature, and, the bulk viscous pressure will become a dynamical degree of freedom having causal evolution equation. Furthermore, it is possible to eliminate bulk viscous inflation without particle production (as used by many authors), and, thus, one may have a model universe starting from a de Sitter phase which gradually evolves to standard FLRW model. The present work will be an attempt to incorporate the present accelerating phase within this causal second order theory.

Usually, to explain the recent accelerated expansion of the Universe as predicted by Supernovae Ia and complementary observations [13, 14], there are two common approaches – one within the framework of Einstein gravity by introducing an unknown type of matter component with a large negative pressure (dark energy), and, secondly, by modifying Einstein gravity theory, and interpreting the extra geometric terms as hypothetical matter component to explain the present accelerating phase. However, both the approaches are concentrated only to explain the recent observational predictions – there is no concern about the past or future evolution of the universe. In this context, the present work is an attempt not only to explain the recent observations, but also the past evolution of the universe without following any one of the above mentioned conventional approaches.

The paper is organized as follows: Section II deals with a brief review of the non-equilibrium thermodynamics in cosmology, section III describes the explicit solutions of the cosmological parameters corresponding to a unique particle creation rate. In section IV we have analyzed the cosmographic
parameters associated with our model. Section \[V\] we have described the particle productions in the language of field theory. In section \[VI\] we present an associated Hawking like radiation. Finally, we finish our discussions in section \[VII\]

II. PARTICLE CREATION MECHANISM AND NON-EQUILIBRIUM THERMODYNAMICS: BASIC EQUATIONS

In a closed thermodynamical system, suppose there are \(N\) particles having internal energy \(E\). Then the first law of thermodynamics which is essentially the conservation of internal energy reads

\[
dE = dQ - pdV, \tag{1}\]

where \(dQ\) is the amount of heat received by the system in time \(dt\), and as usual \(p\) is the thermodynamic pressure, \(V\) is any co-moving volume. The above energy conservation relation can be rewritten as Gibb’s equation

\[
T ds = d\tilde{q} = d\left(\frac{\rho}{n}\right) + pd\left(\frac{1}{n}\right), \tag{2}\]

where ‘\(s\)’ is the entropy per particle (specific entropy), \(T\) is the temperature of the fluid, \(n = N/V\) is the particle number density, and \(d\tilde{q} = dQ/N\) is the heat per unit particle. It should be noted that the above Gibb’s equation also holds for open thermodynamical system, i.e., when the particle number is not conserved [9, 10, 15].

As mentioned earlier, we consider spatially flat FLRW model of the universe as an open thermodynamical system, and, non-equilibrium nature comes into picture due to the particle creation mechanism. Thus, the Einstein’s field equations take the form

\[
3H^2 = \kappa \rho, \quad \text{and,} \quad 2\dot{H} = -\kappa(\rho + p + \Pi), \tag{3}\]

where \(\kappa = 8\pi G\) is the Einstein’s gravitational constant. For a relativistic fluid with dissipation in the form of bulk viscosity has the energy-momentum tensor

\[
T_{\mu\nu} = (\rho + p + \Pi)u_\mu u_\nu + (p + \Pi)g_{\mu\nu}, \tag{4}\]

and the conservation equation \(T^{\mu\nu}_{\quad;\nu} = 0\) (Bianchi’s identity) reads
\[ \dot{\rho} + \theta (\rho + p + \Pi) = 0, \]  \hspace{1cm} (5)

where the bulk viscous pressure \( \Pi \) is related to the entropy production.

Further, in an open thermodynamical system, the non-conservation of fluid particles is reflected by the equation

\[ N^\mu_{,\mu} \equiv \dot{n} + \theta n = n \Gamma, \]  \hspace{1cm} (6)

where \( \Gamma \) denotes the rate of change of the particle number \((N = na^3)\) in a co-moving volume \(a^3\). \(N^\mu = nu^\mu\) is the particle flow vector, \(n\) is the particle number density, \(u^\mu\) is the unit time-like vector (4−velocity), \(\theta = u^\mu_{,\mu}\) is the expansion of the congruence of time-like geodesics and notationally \(\dot{n} = na \dot{a}\). So, creation and annihilation of particles is characterized by the sign of \(\Gamma\) (creation: \(\Gamma > 0\) and annihilation: \(\Gamma < 0\)). Further, a non-zero \(\Gamma\) is dynamically equivalent to an effective bulk pressure \([8, 16–21]\) of the fluid, and, hence, non-equilibrium thermodynamics comes into consideration. However, Lima et al. \([22]\) showed that such scalar processes (bulk viscosity and matter creation) are not equivalent from a thermodynamic viewpoint — only the dynamic behavior can simply be demonstrated in the case of “adiabatic” particle creation as follows:

Using the above conservation equations (5) and (6) into the Gibb’s relation (i.e., Eq. (2)), one obtains the entropy variation per particle as

\[ \dot{s} = -\frac{\theta}{nT} \left[ \Pi + \frac{\Gamma}{\theta} (\rho + p) \right]. \]  \hspace{1cm} (7)

For simplicity, if the thermal process is assumed to be isentropic (i.e., adiabatic) then the entropy per particle remains constant (in contrast to dissipative process), i.e., \(\dot{s} = 0\) and we have

\[ \Pi = -\frac{\Gamma}{\theta} (\rho + p). \]  \hspace{1cm} (8)

The above relation shows that a non-vanishing \(\Gamma\) will produce an effective bulk pressure on the thermodynamic fluid and non-equilibrium thermodynamics comes into the picture. In other words, a dissipative fluid is equivalent to a perfect fluid having a non-conserved particle number. It should be noted that there is entropy production only due to the enlargement of the phase space of the system. Further, one may note that, this effective bulk pressure does not correspond to conventional
non-equilibrium phase, rather a state having equilibrium properties as well (but not the equilibrium era with \( \Gamma = 0 \)).

In the framework of second order non-equilibrium thermodynamics due to Israel and Stewart \cite{11}, the entropy flow vector \( (S^a) \) has the expression

\[
S^a = s N^a - \frac{\tau \Pi^2}{2\zeta T} u^a,
\]

(9)

where \( \zeta \) is the coefficient of bulk viscosity and \( \tau \) represents the time of relaxation. Now, using the conservation equations (5) and (6), one obtains the entropy production density from the above Eq. (9) as

\[
T S^a ; a = -n \mu \Gamma - \Pi \left[ \theta + \frac{\tau \Pi}{\zeta} + \frac{1}{2} \Pi T \left( \frac{\tau}{\zeta T} u^a \right) ; a \right],
\]

(10)

where \( \mu = \left( \frac{\mu + \mu_n}{n} \right) - Ts \) is the chemical potential. Now, for the validity of the second law of thermodynamics (i.e., \( S^a ; a \geq 0 \)) one can choose the ansatz for bulk viscous pressure as

\[
\Pi = -\zeta \left[ \theta + \frac{\tau \Pi}{\zeta} + \frac{1}{2} \Pi T \left( \frac{\tau}{\zeta T} u^a \right) ; a + \frac{\mu n \Gamma}{\zeta \Pi} \right].
\]

(11)

As a result, \( \Pi \) becomes a dynamical variable with an inhomogeneous evolution equation

\[
\Pi^2 + \tau \Pi \dot{\Pi} + \frac{1}{2} \zeta \Pi^2 T \left( \frac{\tau}{\zeta T} u^a \right) ; a + \zeta \Pi \theta = -\zeta \mu n \Gamma.
\]

(12)

Here, the chemical potential \( \mu \) may act as an effective symmetry-breaking parameter in relativistic field theories. Also, the evolution equation becomes linear first order in nature in absence of chemical potential.

In this context, it is relevant to mention that the basic physical difference between the noncausal and the causal theory is the introduction of the time of relaxation (in the later one). As a result, in causal theory \( \Pi \) decays to zero after \( \Gamma \) has been switched off (assuming non-vanishing \( \Gamma \) produces the effective viscous pressure). Moreover, if the above second order theory is isentropic in nature then the entropy production density simplifies to

\[
S^a ; a = -\Pi \left[ \frac{\Pi}{2} \left( \frac{\tau}{\zeta T} u^a \right) ; a + \frac{\tau}{\zeta T} \ddot{\Pi} - \frac{ns \Gamma}{\Pi} \right].
\]

(13)
Further, due to the isentropic condition (8), the evolution of the relevant thermodynamical variables are

\[ \dot{\rho} = - (\theta - \Gamma)(\rho + p), \quad \dot{p} = -c_s^2(\theta - \Gamma)(\rho + p), \quad \dot{n} = - (\theta - \Gamma), \quad \frac{T}{\dot{\rho}} = - (\theta - \Gamma) \frac{\partial p}{\partial \rho}, \]  

(14)

where \( c_s^2 = (\frac{\partial p}{\partial \rho})_{ad.} \) is the square of the adiabatic sound velocity (23).

III. PHENOMENOLOGICAL CHOICE OF PARTICLE CREATION RATE AND COSMIC EVOLUTION

To describe the cosmic evolution (for a given particle creation rate as a function of the Hubble parameter), one can eliminate the effective bulk pressure \( \Pi \) from the Einstein field equations (3) using the isentropic condition (8) to obtain

\[ \frac{\Gamma}{3H} = 1 + \frac{2}{3\gamma} \left( \frac{\dot{H}}{H^2} \right). \]  

(15)

On the other hand, this equation can also be considered as the determining equation for the particle creation rate \( \Gamma \) from the given cosmic evolution. Also, the deceleration parameter \( q \) takes the form

\[ q = -1 + \frac{3\gamma}{2} \left( 1 - \frac{\Gamma}{3H} \right). \]  

(16)

So, in the present context, the cosmic history is characterized by the fundamental physical quantities, namely, the expansion rate \( H \) and the energy density \( \rho \), and, as a result, the gravitational creation rate \( \Gamma \) can be defined in a natural way.

In earlier studies (22-27), the particle creation rate \( \Gamma \) has been chosen for different phases of the evolution from thermodynamical viewpoint. As \( \Gamma \) should be greater than \( H \) in the very early universe so that the created radiation may be considered as a thermalized heat bath. Hence, at the very early Universe, \( \Gamma \) is chosen to be proportional to \( H^2 \) (i.e., \( \Gamma \propto \rho \)) and the corresponding cosmological solution (22, 23, 26, 27) shows a smooth transition from the inflationary scenario to the radiation era. Also for this adiabatic production of relativistic particles, the energy density scales as \( \rho_r \propto T^4 \), i.e., black body radiation (26, 27).

Furthermore, it has been recently shown (28, 29) that, \( \Gamma \propto H \) and \( \Gamma \propto 1/H \) describe respectively the intermediate matter dominated era (starting from radiation) and the transition from matter dominated era to late time acceleration. The scale factor, the Hubble parameter and the thermodynamical
parameters are shown to be continuous across the transition points (i.e., the epochs from the inflationary era to the radiation era and from the matter dominated era to the late time acceleration). Also, $\Gamma = \Gamma_0$ has been shown \cite{30} to correspond the emergent scenario. Subsequently, in another work \cite{31}, a linear combinations of all these choices, i.e., $\Gamma = \Gamma_0 + lH^2 + mH + n/H$ has been examined to describe the whole evolution of the Universe. Although no exact analytic solution is possible for this choice of $\Gamma$ (from Eq. \cite{15}), still the graphical representation of the deceleration parameter has shown the whole evolution of the universe starting from an early inflationary epoch to the present accelerating phase, and, the model predicts a possible transition from present accelerating stage to decelerating phase again in the future.
With this background in mind, the present work is a partial modification of the above general choice with an aim to have an exact analytic solution so that more sophisticated cosmic study, namely, the cosmographic analysis can be done. By choosing the coefficients appropriately (which will be clear subsequently) the form of $\Gamma$ is taken as

$$\Gamma = -\mu^2 + 3H + \frac{\alpha^2}{H}$$  \hspace{1cm} (17)

with $\mu$ and $\alpha$ as real constants. Now substituting this $\Gamma$ into the evolution equation (15), the explicit solution reads as

$$a = a_0 e^{\left(\frac{\alpha^2}{\mu^2}t\right)\exp\left[-\frac{2}{\gamma\mu^4} \exp\left\{-\frac{\mu^2\gamma}{2}(t - t_0)\right\}\right]}$$ \hspace{1cm} (18)

$$H = \frac{\alpha^2}{\mu^2} + \frac{1}{\mu^2} \exp\left[-\frac{\mu^2\gamma}{2}(t - t_0)\right]$$ \hspace{1cm} (19)

where $a_0$ and $t_0$ are the constants of integration. The graphical representation of the cosmic evolution, namely, the scale factor $a$, the Hubble parameter $H$, and the deceleration parameter $q$ are presented in FIGs. (1)–(3) respectively for various choices of $\gamma$, the equation of state parameter for the cosmic fluid. The diagramatic representation of $q$ shows two transitions of $q$ (from accelerating phase to deceleration, and, then again acceleration) which indicates that the present model of the Universe describes the evolution from the inflationary scenario to the present late time acceleration through the decelerated matter dominated era. Thus, we have a complete cosmic history after the big bang till today. It is interesting to note that as the cosmic time becomes very large, we have the $\Lambda$CDM model:

$$a \simeq a_0 e^{H_0 t}, \quad H \simeq H_0 = \frac{\alpha^2}{\mu^2}, \quad q \simeq -1, \quad \rho \simeq 3H_0^2 = \Lambda = -p,$$  \hspace{1cm} (20)

and it agrees with the recent Planck data set [32]. Now, corresponding to the cosmological solution (i.e., Eqns. (17) and (18)) of the present model, the relevant thermodynamical parameters evolve as (see Eq. (14))

$$T = T_0 \left[\frac{\alpha^2}{\mu^2} + \frac{1}{\mu^2} \exp\left\{-\frac{\mu^2\gamma}{2}(t - t_0)\right\}\right]^{\left(\frac{2}{\gamma}\right)}$$

$$n = n_0 \left[\frac{\alpha^2}{\mu^2} + \frac{1}{\mu^2} \exp\left\{-\frac{\mu^2\gamma}{2}(t - t_0)\right\}\right]^{\left(\frac{1}{\gamma}\right)}$$

$$S = S_0 e^{\left(\frac{\alpha^2}{\mu^2}t\right)\exp\left[-\frac{2}{\gamma\mu^4} \exp\left\{-\frac{\mu^2\gamma}{2}(t - t_0)\right\}\right]} \left[\frac{\alpha^2}{\mu^2} + \frac{1}{\mu^2} \exp\left\{-\frac{\mu^2\gamma}{2}(t - t_0)\right\}\right]^{\left(\frac{4}{\gamma}\right)}$$ \hspace{1cm} (21)
FIG. 4. This is the behavior of the temperature over the cosmic time for three different choices of $\gamma$.

where $S_0$ is some constant, and, $T_0$, $n_0$ are integration constants. We have plotted these thermodynamical parameters in FIGs. (4)-(6) for various choices of $\gamma$. It should be mentioned that if $\gamma = 2$, i.e., $p = \rho$ (relativistic fluid), then $T \propto \rho^{\frac{4}{3}}$, which represents the usual black body radiation. Note that, in the $\Lambda$CDM limit the above thermodynamical parameters become

$$T \rightarrow T_0 \left(\frac{\alpha^2}{\mu^2}\right)^{\left(\frac{2}{3}\gamma - 1\right)}, \quad n \rightarrow n_0 \left(\frac{\alpha^2}{\mu^2}\right)^{\left(\frac{1}{3}\right)}, \quad S \rightarrow S_0 e^{H_0 t} \left(\frac{\alpha^2}{\mu^2}\right)^{\left(\frac{1}{3}\right)},$$

(22)

which shows that although the temperature and the number density become constant, but the entropy in a co-moving volume evolves as the scale factor. Lastly, we note that, in the above limit, the particle creation rate becomes constant: $\Gamma \rightarrow \Gamma_0 = 3H_0$.

IV. COSMOGRAPHIC ANALYSIS AND OBSERVATIONAL DATA

In this section, we make a comparative study of the present model of the universe with the presently available observed data set. At first, we discuss a geometric view of the DE models. Sahni et al. [33] first proposed this idea with two dimensionless and model independent geometric parameters $\{r, s\}$ defined as

$$r = \frac{1}{aH^3} \frac{d^3a}{dt^3}, \quad \text{and}, \quad s = \frac{r - 1}{3 \left(q - \frac{1}{2}\right)}.$$
These two parameters in Eq. (23) are used to filter the observationally supported DE models from other phenomenological DE models existing in the literature. Subsequently, these geometric investigation was further extended by considering the Taylor series expansion of the scale factor about the present time as

$$a(t) = a(t_p) + H_p (t - t_p) + \frac{1}{2!} q_p H_p^2 (t - t_p)^2 + \frac{1}{3!} j_p H_p^3 (t - t_p)^3 + \frac{1}{4!} s_p H_p^4 (t - t_p)^4 + \mathcal{O}[(t - t_p)^5], \quad (24)$$

where the model independent parameters $j, s, l, m$ are known as cosmographic parameters [34, 35], and, are defined as

$$j = \frac{1}{aH^3} \frac{d^3 a}{dt^3}, \quad s = \frac{1}{aH^4} \frac{d^4 a}{dt^4}, \quad l = \frac{1}{aH^5} \frac{d^5 a}{dt^5}, \quad \text{and,} \quad m = \frac{1}{aH^6} \frac{d^6 a}{dt^6}. \quad \quad (25)$$

Here, the suffix ‘p’ stands for the value of the corresponding variable at the present epoch $(t_p)$. It should be noted that, the cosmographic parameters are individually named as jerk ($j$) (this ‘j’ is same as ‘r’ defined by Sahni et al. [33]), snap $(s)$ (this ‘s’ is different from one defined by Sahni et al. [33]), jerk, and $m$ parameter [34, 35].

The above cosmographic parameters (CP) can be expressed in terms of the deceleration parameter $(q)$, and, its higher derivatives in the following way:

$$j = -\frac{1}{H} \frac{dq}{dt} + q(1 + 2q), \quad (26)$$
\[ s = \frac{1}{H} \frac{dj}{dt} + j - 3(1 + q)j, \] (27)

\[ l = \frac{1}{H} \frac{ds}{dt} + s - 4(1 + q)s, \] (28)

\[ m = \frac{1}{H} \frac{dl}{dt} + l - 5(1 + q)l \] (29)

**FIG. 7.** The figure shows a comparative study of the deceleration parameter of our model with the 4 different latest observational data sets.

Further, the deceleration parameter can be expressed in terms of the redshift parameter \( z = 1/a - 1 \) as

\[ q(z) = q_p + (-q_p - 2q_p^2 + j_p)z + \frac{1}{2} \left( 2q_p + 8q_p^2 + 8q_p^3 - 7q_p j_p - 4j_p - s_p \right) z^2 + O(z^3). \] (30)

For the observed data sets, we choose the following
(i) 192 Sne Ia and 69 GRBs with CPL parametrization (data 1) [36]
(ii) 192 Sne Ia and 69 GRBs with linear parametrization (data 2) [36]
(iii) Supernovae Union 2+ BAO+ OHD+ GRBs data (data 3) [37]
(iv) Supernovae Union 2+ BAO+ GRBs data (data 4) [37]

In FIG. 7, we have made a comparative study of the deceleration parameter for the present model with those for the above four data sets. From the graph we see that the behavior of the deceleration parameter in our model almost matches with the 4 latest observed data sets.
FIG. 8. The figures show the variation of the “jerk” \( j \) throughout the evolution of the universe.

Also, we have shown the graphical representations of the above four CP parameters for different choices of \( \gamma \).

In fact, FIG. 8 shows the complete evolution of the jerk parameter \( j \), while FIG. 9 shows explicitly the variation of \( j \) in the early phase of the universe. There is a transition of \( j \) from \(-\) ve values to \(+\) ve values. The variation of the snap parameter \( s \) over the entire evolution of the universe has been shown in FIG. 10, while FIGs. 11 and 12 represent the detailed variation of \( s \) in the early and late phases respectively. There are three transitions of \( s \) during the whole evolution. FIG. 13 shows the variation of the lerk parameter \( l \) over the entire cosmic time, and its variation at early phase is presented in FIG. 14. The figures show two transitions of the lerk parameter. Finally, the \( m \) parameter is graphically represented in FIGs. 15–17, which also has the two transitions over the entire evolution.

V. FIELD THEORETIC DESCRIPTION OF COSMIC HISTORY

In this section, we shall describe the cosmic evolution from the field theoretic point of view by describing the whole dynamical process as the evolution of a scalar field \( \phi \) having self interacting potential \( V(\phi) \), or, equivalently, the evolution of the present effective imperfect fluid can be described by a minimally coupled scalar field. Thus, the energy density and the thermodynamic pressure of the cosmic fluid are given by
FIG. 10. The figures show the variation of the “snap” \( s \) parameter throughout the entire evolution of the universe.

FIG. 11. The figures are just to show that during the early phase of the universe, \( s \) started from \(-ve\) values.

FIG. 12. This is just to show that when \( t \rightarrow \infty \), \( s \) becomes \(+ve\).

\[
\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad \text{and} \quad p_{\text{eff}} = p + \Pi = \frac{1}{2} \dot{\phi}^2 - V(\phi). \tag{31}
\]

Thus, for the present isentropic thermodynamical system, we have

\[
\dot{\phi}^2 = \frac{\gamma}{\kappa} (\mu^2 H - \alpha^2), \tag{32}
\]

\[
V(\phi) = \frac{1}{2\kappa} [6H^2 - \gamma(\mu^2 H - \alpha^2)], \tag{33}
\]
where $\Pi$ is eliminated by the isentropic condition equation (8), particle creation rate $\Gamma$ is obtained from Eq. (17), and the first Friedmann equation in (3) has been used. Thus, integrating the above equation (32), the explicit form of $\phi$ is

$$\phi = \phi_0 - \frac{4}{\mu^2 \sqrt{\kappa \gamma}} \exp \left( \frac{-\mu^2 \gamma}{4} (t - t_0) \right),$$

and the potential can be expressed explicitly in $\phi$ as

$$V(\phi) = V_0 + V_1 (\phi - \phi_0)^2 + V_2 (\phi - \phi_0)^4,$$

where $\phi_0$ is the constant of integration, and, $V_0, V_1, V_2$ are the constants depending on $\mu, \alpha, \gamma$. Note that, the scalar field has always a value less than $\phi_0$. Further, the particle creation rate $\Gamma$ can be expressed as a function of $\phi$ as

$$\Gamma = \Gamma_0 + \Gamma_1 (\phi - \phi_0)^2 + \frac{\mu^2}{1 + \Gamma_2 (\phi - \phi_0)^2},$$

with $\Gamma_0, \Gamma_1, \Gamma_2$ as constants.

It is worthwhile to mention that, in the asymptotic limit (i.e., $\Lambda$CDM era), $\phi$ becomes a constant ($= \phi_0$), and the potential behaves as the cosmological constant $\Lambda$. 

FIG. 13. The figures show the variation of the “lerk” ($l$) parameter throughout the entire evolution of the universe.

FIG. 14. The figures are just to show that during the early phase of the universe, ‘$l$’ was +ve.
FIG. 15. The figures show the variation of the \( \m \) parameter throughout the entire evolution of the universe.

FIG. 16. The figures are just to show that during the early phase of the universe, \( \m \) started from + ve.

FIG. 17. This is just to show that when \( t \to \infty \), \( \m \) becomes + ve.

Furthermore, in view of the slow roll approximation during the inflationary phase, the density fluctuations are of the form \( \delta_H \sim H^2/\dot{\phi}^2 \sim 10^{-5} \) \( [21, 27] \). So, for the present model, we see

\[
\delta_H \sim \frac{\kappa H}{\gamma \mu^2 \left(1 - \frac{H}{H_0}\right)} \quad \text{Initial epoch: } H = H_I, \quad \text{i.e.,} \quad (37)
\]

\[
H_I/H_0 \sim \delta_H \left(\frac{\gamma H_0^4}{\kappa \alpha^2}\right) - 1 \approx \delta_H \left(\frac{\gamma H_0^4}{\kappa \alpha^2}\right). \quad (38)
\]
As \( \kappa = 8\pi G = 8\pi l_{pl}^2 \) (in units \( \hbar = c = 1 \)), so, the ratio of the Hubble parameter at the two accelerating phases (initial and present) is proportional to \( l_{pl}^{-2} \). At the Planck size of the universe, i.e., \( l_{pl} = 10^{-35} \text{ m} \), the Hubble parameter is \( H_I \sim 10^{45} \text{ sec}^{-1} \), so, \( H_0 \sim 10^{-23} \text{ sec}^{-1} \). Thus, the cosmological parameter \( \Lambda \) is \( \sim 10^{-47} \), the present observed value.

VI. HAWKING LIKE RADIATION IN THE CONTEXT OF PARTICLE CREATION MECHANISM

We have made attempts in this section to interpret the mechanism of particle creation as the phenomenon of Hawking like radiation from the homogeneous and isotropic FLRW space-time model. Normally, in the particle creation mechanism, the dissipative term behaves as an effective bulk viscous pressure, and, as a result, there is a negative pressure term in the Einstein’s field equations. Also, it is found that, by proper choice of the particle creation rate, there is an accelerated expansion of the universe both at the early stage (inflation), and at late-time, i.e., a complete description of the cosmic history. So, it is very natural to enquire whether there is any similarity between this mechanism with Hawking radiation as the inflationary stage can be described by the Hawking radiation in the FLRW universe.

In Hawking radiation, initially due to the enormous size of the black hole, the evaporation process was very slow, and the process gradually became faster and faster with the diminishment of the size of the black hole. As a result, the temperature of the black hole also increases with the process. At the end, when the black hole becomes of Planck dimension, quantum gravity should come into picture. On the other hand, the evolution of the universe is in the reverse direction of the black hole evaporation. At the beginning, the quantum gravity effects are important due to Planck size of the universe. But, with the evolution of the universe, Hawking radiation comes into picture, and the temperature gradually decreases.

Moreover, there is another basic difference between these two evolution processes. In black hole evaporation, the created particles escape outside the event horizon, and, move to asymptotic infinity, while for evolution of the FLRW, the situation is just reversed — the particles created near the (apparent) horizon will move inside. As a result, due to black hole evaporation, there is a loss of energy, but the universe gains energy due to particle creations. Furthermore, due to isotropic nature of the FLRW model, the radiation should be uniform in all direction, and, we have from the Stefan–Boltzmann radiation law (SBRL)

\[
P = \frac{dQ}{dt} = \sigma A_H T^4, \tag{39}
\]
where $\sigma = \pi^2 k_B^2 / 60 h^3 c^2$ is the Stefan-Boltzmann constant, $T$ is the radiation temperature, $Q$ is the heat radiated by the black body, $P$ is the net radiated power, and $A_H$ is the radiating area.

Using the above SBRL in the first law of thermodynamics, i.e.,

$$P = \frac{dQ}{dt} = \frac{d}{dt}(\rho V) + p \frac{dV}{dt},$$

we obtain

$$\dot{\rho} + 3H\{(\rho + p) - \sigma T^4\} = 0.$$  \hspace{1cm} (41)

So, comparing with matter conservation equation (5), and considering the thermal process to be isentropic (i.e., using Eq. (8)), the particle creation rate is related to the temperature as

$$\Gamma = \frac{\sigma T^4}{\gamma H},$$

Now, choosing $\Gamma$ for the present model (in Eq. (17)), we get

$$\sigma T^4 = \gamma (-\mu^2 H + 3H^2 + \alpha^2).$$  \hspace{1cm} (43)

In the early phases of the evolution of the universe, $H$ is very large, so, the above equation can approximately be written as

$$\sigma T^4 \simeq 3\gamma H^2 = \kappa \gamma \rho,$$  \hspace{1cm} (44)

which is the usual black body radiation. On the other hand, at late-times, when the universe is very nearly to the equilibrium configuration, the Clausius relation becomes

$$T\dot{S} = \dot{Q} = \text{constant}.$$  \hspace{1cm} (45)

But, at late-time, the entropy is proportional to ‘$a$’ (see Eqns. (20) and (22)), and, hence, $\dot{S} \propto H_0 a$. Thus, from the above relation, we get
\[ T \propto 1/a, \]

which is nothing but the present CMB temperature. Lastly, it should be noted that, in the early phase of the universe, when \( \Gamma \simeq H \), \( T \propto H^{1/2} \), so, the temperature is not exactly Hawking temperature \( (T \propto H) \), rather, Hawking-type radiation. The same is also true for the late-time evolution (where CMB temperature is dominant over Hawking temperature). Finally, the model becomes very close to the \( \Lambda \)CDM era; \( H \) is found to be a constant, and, hence, the particle creation rate, as well as, the temperature are constant, and there is no analogy with Hawking type radiation.

VII. SUMMARY AND DISCUSSIONS

Relativistic cosmology with usual perfect fluid (having barotropic equation of state) as cosmic substratum is considered in the present work in the context of non-equilibrium thermodynamics. In the framework of particle creation mechanism, dissipative phenomenon is reflected as an effective bulk viscous pressure. In the second order formulation of non-equilibrium thermodynamics due to Israel and Stewart, the dissipative pressure acts as a dynamical variable whose evolution is characterized by non-linear inhomogeneous evolution, however, the entropy flow vector satisfies the second law of thermodynamics. Due to our inability in solving the non-linear evolution equation, the thermal process is assumed to be adiabatic (i.e., the entropy per particle is constant), and, as a result, the dissipative pressure is linearly related with the particle creation rate. In earlier works \[22-29\], the particle creation rate at different cosmic stages had been chosen phenomenologically (with some basis from thermodynamics), and the solutions for different physical and thermodynamical variables show a continuity across the transition epochs. Subsequently, a single particle creation rate \[31\] describes the whole evolution, but no analytic solution was found. The present work also considers the particle creation rate as a function of the Hubble parameter in a phenomenological way which not only describes the cosmic story from inflation to late-time acceleration, but also, the model has an analytic solution. Also, graphically, we have shown the evolution of the scale factor, Hubble parameter, deceleration parameter, the thermodynamic variables, namely, the temperature, entropy, and the number density. It is found that, at late-time, the model asymptotically approaches to \( \Lambda \)CDM, and it agrees with the latest Planck data set \[32\]. A cosmographic analysis has also been done for our model, and, the cosmographic parameters, namely, the jerk \( (j) \), snap \( (s) \), lerk \( (l) \), and the \( m \) parameter have been graphically shown against the cosmic time. From the figures, we conclude that the above parameters respectively have one, three, two, and two transitions through the cosmic evolution. Further, from the
field theoretic view point, it has been shown that, it is possible to have a minimally coupled scalar field as equivalent to the cosmic fluid for the description of the evolution of the universe. In the late-time asymptotic limit, the self similar potential behaves as cosmological constant. Finally, in resemblance with Hawking radiation, it is found that, at early epoch of the evolution, the temperature corresponds to black body radiation, while for very close to ΛCDM, the temperature is related to the CMBR — in both cases, the temperature is not the Hawking temperature, rather, Hawking type radiation.

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