HIGHER SPIN HADRONS AS RELATIVISTIC FIELDS *

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I discuss the problem of consistent interactions of higher-spin fields and its relevance to resonance physics.

1. Introduction

The complexity of QCD does not yet allow us to describe low-energy hadronic reactions in terms of the underlying quark-gluon dynamics. A simpler, albeit more phenomenological, approach is to seek for a description in terms of hadronic degrees of freedom. This approach is usually based on some form of effective Lagrangian (or Hamiltonian) written in terms of hadronic fields, corresponding to pions, nucleons, $\rho$-mesons, $\Delta$-isobars, etc.

One of the basic problems here arises in the treatment of hadrons with spin one and higher. This is a very old problem of consistent interaction of higher-spin fields. It first had been addressed in the works of Dirac$^1$, Fierz and Pauli$^2$, Johnson and Sudarshan$^3$, Velo and Zwanziger$^4$, who discovered that by far not any interacting theory of higher-spin ($s \geq 1$) fields is consistent.

The problem has to do with the unphysical spin degrees of freedom (DOF) which are necessarily introduced to achieve a relativistic formulation of a higher-spin field theory. These unphysical DOF are introduced in addition to the physical $2s + 1$ (or, 2) spin DOF which describe the polarizations of a massive (or, massless) particle with spin $s$. It turns out a theory is consistent only if the unphysical DOF decouple, i.e., do not influence the observables. Here I will formulate a consistency condition which insures the decoupling of unphysical DOF and advocate its importance in formulating the higher-spin $N^*$ couplings.

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2. DOF counting and higher-spin gauge symmetries

For the sake of manifest Lorentz-invariance we must operate in terms of tensor and spinor fields. One of the most common formulations is based on symmetric tensors. A boson with spin $s$ is represented by a rank-$s$ tensor field $h_{\mu_1 \cdots \mu_s}(x)$, while a fermion is described by a rank $j = s - 1/2$ tensor-spinor $\psi^{(\alpha)\mu_1 \cdots \mu_j}(x)$, symmetrized in indices $\mu$; index $\alpha$ is a spinor index. The question is how to reconcile the number of independent components of these fields with the number of spin DOF of the corresponding particle. For $s \geq 1$ additional constraints must be imposed on the fields to reduce the number of independent components to 2 for massless and $2s + 1$ for massive situations. For the massless fields this is done by demanding invariance of the action under local (gauge) variations of the fields:

$$\delta h^{\mu_1 \cdots \mu_s}(x) = \partial^{(\mu_1} \phi^{\mu_2 \cdots \mu_s)}(x),$$
$$\delta \psi^{(\alpha)\mu_1 \cdots \mu_j}(x) = \partial^{(\mu_1} \epsilon^{(\alpha)\mu_2 \cdots \mu_j)}(x).$$

Local symmetries generate constraints and thus it is possible to formulate free actions with only 2 spin DOF. The mass term is then introduced such as to (partially) break the symmetry to raise the number of DOF to $2s + 1$.

Rather than going into the DOF counting for arbitrary $s$ (which can be found in, e.g., Ref. 5) let us consider two simplest examples. A massless spin-1 particle is described by a vector field $h^\mu$ and the well-known Lagrangian

$$L_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad F^{\mu\nu} = \partial^{[\mu} h^{\nu]},$$

Thus, even though the field has 4 components to begin with, the symmetry of the action under $\delta h^\mu = \partial^\mu \phi$ leaves only 2 independent components. The mass term: $L_m = -\frac{1}{2}m^2 h_\mu h^\mu$, raises the spin DOF number to 3 as is appropriate for a massive spin-1 particle.

A massless spin-3/2 particle is described by a 16-component $\psi_\mu$ and the Rarita-Schwinger Lagrangian (spinor indices omitted):

$$L_0 = i \bar{\psi}_\mu \gamma^{\mu\nu\lambda} \partial_\lambda \psi_\nu,$$

with $\gamma^{\mu\nu\lambda} = \frac{1}{2}(\gamma^\mu \gamma^\nu \gamma^\lambda - \gamma^\lambda \gamma^\nu \gamma^\mu)$. Here again, due to the symmetry under $\delta \psi_\mu = \partial_\mu \epsilon$, (where $\epsilon$ is a spinor) only 2 components are independent. The mass term needs to be introduced such that this number is raised to 4. The following form is known to be uniquely appropriate: $L_m = -m \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu$, where $\gamma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$.

Free field actions for arbitrary $s$ based on symmetric tensors were successfully formulated by Singh and Hagen 6 for massive and by Fronsdal 7 for
massless situation. Interacting theories, on the other hand, appeared to be much more formidable to formulate consistently. Just giving a charge to higher-spin particle by a minimal coupling to electromagnetic field would already lead to serious pathologies, e.g., negative norm states\(^3\), solutions to the field equations propagating faster than light\(^4\). For the spin-3/2 case, all these pathologies can be related to the fact that the coupling changes the number constraints leading thus to a theory with wrong DOF content\(^8\).

This just emphasizes an obvious consistency condition on the interactions of higher-spin fields: they must preserve the DOF counting of the free theory. The following statement is less obvious: interactions will be consistent (preserve the DOF counting) only if they are invariant under the gauge transformations similar\(^a\) to transformations (1). In other words, interactions must support the gauge symmetries of the free massless theory, so that only the mass terms break the symmetries.

Indeed, since only the breaking of the symmetry changes the DOF content, gauge-invariant couplings leave the DOF counting of the free theory intact, for both the massless and massive situations. This argument proves at least the sufficiency of the gauge-invariance requirement. Proof of necessity exists but is more involved and will not be presented here.

The requirement of gauge-invariance of higher-spin couplings is thus crucial for consistency for both the massless and massive fields. It has however been totally ignored in formulating the couplings of the \(N^*\) fields. The aim of our investigation is to correct this situation and implement the consistent couplings in the \(N^*\) phenomenology.

3. Decoupling of the lower-spin DOF

We will require the \(N^*\) couplings to be invariant under the free massless field transformations\(^b\), Eq. (1). Corresponding vertex will satisfy the following transversality condition:

\[ p_{[\mu_1} \Gamma^{\mu_1 \cdots \mu_j]} = 0, \]  

\(^a\)Transformations are similar if they have the same number of transformation parameters and the same order of the differential operator acting on them. These are the two parameters which determine the number of constraints imposed by the gauge symmetry.

\(^b\)In principle, symmetry under more general (similar) transformations should be allowed. However, a nontrivial modification of the transformations, e.g., making them dependent on other fields, may lead to appearance of new conserved charges associated with the \(N^*\) fields. As long as no such charges are observed in nature one is restricted to transformations (1).
where \( p \) is the momentum and \( \mu_1 \ldots j \) are the tensor indices corresponding to the higher-spin particle; \( j = s \) for bosons and \( j = s - 1/2 \) for fermions.

Using this property one can show that there is no coupling to the lower-spin sector of the propagator. Indeed the lower-spin sectors always enter with at least one factor of \( p^\mu / m \) so when matrix elements such as \( \Gamma^{\mu_1 \ldots \mu_j} S_{\mu_1 \ldots \mu_j \nu_1 \ldots \nu_j} \Gamma^{\nu_1 \ldots \nu_j} \) are computed the net contribution of the lower-spin sector vanishes.

To see this effect more explicitly it is helpful to write out the propagator in terms of the spin projection operators. For instance, in the spin-1 case the projector on pure spin-1 and spin-0 states are:

\[
P_{(1)}^{\mu \nu} (p) = g_{\mu \nu} - \frac{p_\mu p_\nu}{p^2}, \quad P_{(0)}^{\mu \nu} (p) = \frac{p_\mu p_\nu}{p^2},
\]

and the massive spin-1 propagator is written therefore as

\[
S_{\mu \nu} (p) = \frac{1}{p^2 - m^2} P_{(1)}^{\mu \nu} (p) - \frac{1}{m^2} P_{(0)}^{\mu \nu} (p).
\]

Obviously, as long as the vertices involving the spin-1 particle obey transversality, the spin-0 term drops out of the matrix elements.

In the spin-3/2 case the spin-projection operators are:

\[
P_{(1/2)}^{\mu \nu} (p) = g_{\mu \nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3p^2} (p_\mu p_\nu + p_\mu \gamma_\nu \not{p}), \quad P_{(0)}^{(1/2)} = \frac{p_\mu p_\nu \gamma_\mu \gamma_\nu}{(\sqrt{3} p^2)}, \quad P_{(1/2)}^{(1/2)} = \frac{p_\mu p_\nu \gamma_\mu \gamma_\nu}{(\sqrt{3} p^2)}, \quad P_{(21/2)} = \frac{p_\mu p_\nu \gamma_\mu \gamma_\nu}{(\sqrt{3} p^2)}.
\]

and the propagator reads:

\[
S_{\mu \nu} (p) = \frac{1}{p - m} P_{(3/2)}^{\mu \nu} - \frac{2}{3m^2} (\not{p} + m) P_{(1/2)}^{(1/2)} + \frac{1}{\sqrt{3} m} \left( P_{(1/2)}^{(1/2)} + P_{(21/2)}^{(1/2)} \right).
\]

Again, if \( p \cdot \Gamma = 0 \) then \( \Gamma \cdot P^{(1/2)} \cdot \Gamma = 0 \), and thus the spin-1/2 sector decouples from the matrix elements.

This decoupling property offers tremendous simplifications in the treatment of higher-spin hadron fields. When gauge-invariant couplings are used the lower-spin can be dropped from the full relativistic propagators and only the highest-spin term must be kept\(^c\). Certain relativistic hadron-exchange amplitudes for any \( s \) can easily be found, see, e.g., Appendix of Ref.\(^9\), where the \( \pi N \) amplitudes are discussed.

\(^c\)This cannot be done for a coupling that involves the lower-spin sector. Removing the lower-spin sector by hand in that case violates locality because of the \( 1/p^2 \) singularity of the projection operators.
4. Consistent versus conventional couplings

Most of the commonly used $N^*$ couplings are inconsistent from the DOF-counting point of view presented here, simply because they do not have the higher-spin gauge symmetry. However, it can be shown that in perturbation theory the difference between certain consistent and inconsistent couplings can be accommodated by specific contact terms\textsuperscript{10}. In other words, when the lower-spin sectors are involved due to bad couplings, their contribution takes form of contact terms. One manages to control, to some extent, the strength of these contact terms by introducing the so-called off-shell couplings and thus additional “off-shell parameters”. The best known are the off-shell parameters associated with the $\pi N\Delta$ and $\gamma N\Delta$ couplings, where $\Delta$ is the spin-3/2 Delta(1232) isobar.

Often it is claimed that these off-shell couplings play an important role and are necessary to describe experimental data. In my view this merely indicates that some physics is missing but can be reasonably well mimicked by the contact terms introduced via the off-shell couplings of the higher-spin $N^*$. If these contact terms are of unnaturally large importance than an effort needs to be made to find the explicit mechanisms that it mimics.

A nice example of this situation is provided by two different calculations of the Compton scattering on the nucleon in the Delta(1232) resonance region. First calculation, done by Olaf Scholten and myself\textsuperscript{11}, includes the Delta-excitation using the conventional $\gamma N\Delta$ couplings $G_1$, $G_2$, and two off-shell couplings $z_1$, $z_2$. We found that the off-shell couplings play a crucial role in reproducing the data even in relatively low-energy region, around the pion-production threshold. Second calculation, by Daniel Phillips and myself\textsuperscript{12}, includes the Delta using consistent $\gamma N\Delta$ couplings $g_{M}$ and $g_{E}$, as well as chiral one-loop diagrams. In this case there are no off-shell couplings but the pion loops are included instead. Nevertheless, the description of the data are of better quality than in the first calculation. And this is despite the fact the second calculation has two less free parameters, since the pion loops in this case bring only the well-established $m_\pi$, $f_\pi$, and $g_A$.

Certainly without the loops, the calculation with consistent couplings does much worse phenomenologically than the conventional one with off-shell couplings. So when comparing the performance of consistent versus conventional couplings one should keep in mind that conventional couplings have more free parameters which adjust the contact term produced by the lower-spin components. It is in many ways better to separate the discussion of the genuine $N^*$ contribution and the shorter range effects, which
is achieved by using the consistent couplings and including any necessary contact terms separately.

5. Conclusions and outlook

Relativistic field-theoretic treatment of $N^*$ resonances is needed in approaches based on hadronic degrees of freedom, such as relativistic potential models, $K$-matrix approach, chiral perturbation theory. The requirement of physical spin DOF counting (i.e., $2s+1$ polarization for a massive particle) constrains the allowed form of the couplings of particles with spin higher than one. Such couplings must support the gauge symmetries of the free massless field. At the level of Feynman rules, corresponding vertices will satisfy the transversality condition, Eq. (4). It is then easy to see that such gauge-invariant couplings do not couple to the unphysical lower-spin sector of the field. These couplings thus allow for a consistent and straightforward treatment of the higher-spin $N^*$. It is therefore appears to be promising to implement these couplings in the $N^*$ phenomenology. The work in this direction is underway. One of the most challenging aspects of this program — the problems of minimal electromagnetic and chiral couplings of the higher-spin field — has not been discussed here. We plan to report on a solution of this problem in an effective-field-theoretic framework in a nearest future.

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