QRF: Implicit Neural Representations with Quantum Radiance Fields

Yuan-Fu Yang  
National Tsing Hua University  
yfyangd@gmail.com

Min Sun  
National Tsing Hua University  
summin@ee.nthu.edu.tw

Abstract

Photorealistic rendering of real-world scenes is a tremendous challenge with a wide range of applications, including mixed reality (MR), and virtual reality (VR). Neural networks, which have long been investigated in the context of solving differential equations, have previously been introduced as implicit representations for photorealistic rendering. However, realistic rendering using classic computing is challenging because it requires time-consuming optical ray marching, and suffer computational bottlenecks due to the curse of dimensionality. In this paper, we propose Quantum Radiance Fields (QRF), which integrate the quantum circuit, quantum activation function, and quantum volume rendering for implicit scene representation. The results indicate that QRF not only exploits the advantage of quantum computing, such as high speed, fast convergence, and high parallelism, but also ensure high quality of volume rendering.

1. Introduction

Neural Scene Representations. Traditional 3D computer vision pipelines use multi-view stereo algorithms to estimate sparse point clouds, camera poses, and texture meshes from 2D input views. However, re-rendering these scene representations does not achieve photorealistic image quality. In contrast to these explicit scene representations, implicit scene representations produce significantly higher quality renderings and can be supervised directly with 3D data by using neural networks. Nevertheless, current neural networks consisting of MLPs are incapable of modeling signals with fine detail, and cannot accurately model high-frequency information and higher-order derivatives even with dense supervision. In addition, realistic rendering of real-world scenes using classic computer graphics techniques is challenging because it requires the difficult step of capturing detailed appearance and geometric models. Existing methods in practice often show blurry renderings due to limited network capacity. Synthesizing high-resolution imagery from these representations often requires time-consuming optical ray marching, and suffer computational bottlenecks due to the curse of dimensionality.

Quantum Neural Networks. The field of artificial neural networks has benefited greatly from recent developments in quantum computers in recent years. In particular, quantum neural networks (QNNs), a class of quantum algorithms which exploit qubits for creating trainable neural networks. Quantum computing can provide potentially exponential speedups due to their ability to perform massively parallel computations on the superposition of quantum states. The quantum neural networks for implicit representations that are capable of overcoming the computational challenges faced by conventional techniques performed on classical computers.

In this study, the neural rendering is performed with quantum circuits (as shown in Figure 2). The parameters calculation run on quantum computers, whose computation speeds are many times higher than supercomputers. In addition, we proposed a Quantum Volume Rendering based on quantum integration, which has been implemented on real quantum hardware. Our proposed Quantum Volume Rendering performs the fastest among various rendering tasks and offers speed-related advantages over implementing conventional integration on digital computers, such as Monte Carlo integration. To summarize, the contributions of our work include:
(1) The first QNNs-based system capable of rendering photorealistic novel views, hundreds of times faster than conventional neural networks.
(2) We present Quantum Radiance Fields (QRF) that consists of a set of quantum implicit fields, where for each quantum circuit, encoding circuit are learned to encode local properties for high-quality rendering.
(3) We leverage the quantum activation function, which can better represent details in signals than classical...
activation for implicit neural representations.
(4) We propose the Quantum Volume Rendering, a quantum algorithm for numerical integration, which is fundamentally better than classical Monte Carlo integration in volume rendering. The remainder of this paper is organized as follows. In Section 2, we introduce the neural scene representations, neural radiance fields, and quantum neural networks. In Section 3, the implicit neural representations with quantum radiance fields are introduced. In Section 4, we present results on 2D image regression and 3D scene reconstruction. For each task, we demonstrate the benefits of using circuit based QRF. Section 5 summarizes this work and briefly discusses possible future extensions.

2. Related Works

Neural Scene Representation. To model objects in a scene, many different scene geometry representations have been proposed. They can be divided into explicit and implicit representations. Explicit scene representations describe scenes as a collection of geometric primitives, and the output it produce can be classified into voxel-based [1][2], point-based [3][4], and mesh-based representations [5][6]. While explicit scene representations enable rapid generation of novel views, they are fundamentally limited by the internal resolution of their representations, which can lead to blurry outputs for high-frequency content.

To circumvent the above problem, many works have explored the potential of implicit neural scene representations directly infer outputs from a continuous input space. In contrast to explicit neural scene representations, implicit neural scene representations promise 3D structure-aware, continuous, memory-efficient representations of shape parts, objects, or scenes [7][8][9]. These representations use neural network to implicitly define objects or scenes, and can be supervised directly using 3D data, such as point clouds, or with 2D multi-view images [10][11][12].

Neural Radiance Fields. Recent work on Neural Radiation Fields (NeRF) has shown how neural network can be used to learn an implicit volumetric representation of the scene and encode complex 3D environments that can be rendered realistically from novel viewpoints [13]. However, NeRF needs to sample a large number of points along the ray for color accumulation to achieve high quality rendering.

Numerous works were developed with the purpose of speeding up NeRF. Neural Sparse Voxel Fields (NSVF) [15] speed up NeRF’s rendering using classical techniques like empty space skipping and early ray termination. MetaNeRF [16] proposed to apply standard meta-learning algorithms to learn the initial weight parameters of the MLPs. A meta-learned weight initialization leads to faster convergence and allows better reconstruction quality from fewer supervised views during test-time optimization. AutoInt [17] introduced an automatic integration framework that learns closed-form integral solutions that reduce the number of evaluations along the ray when raymarching through a NeRF. DONeRF [18] speed up inference by reducing the number of required samples along the ray. FastNeRF [19] proposed a graphics-inspired factorization that can be compactly cached and subsequently queried to compute pixel values in rendered images. KiloNeRF [20] represents a scene by thousands of small MLPs instead of a single large-capacity MLP representing the entire scene, so smaller and faster evaluation MLPs can be used. DS-NeRF [21] leveraged depth as an additional source of supervision to regularize the geometry learned by NeRF and improve the training of NeRF. Cheng Sun and Min Sun et al. [22] proposed a super-fast convergence approach to reconstructing the per-scene radiance field from a set of images that capture the scene with known poses, reducing training time from hours to 15 minutes with comparable quality to NeRF.

Quantum Neural Networks. QNNs is a relatively new field that blends the computational advantages brought by quantum computing and advances beyond classical computation [23]. QNNs not only bring more efficient algorithm performance, but are also able to find the global minimum in the sought solution with higher probability.
The main principles of quantum computing are those inherited from quantum physics, such as superposition, entanglement, and interference. A qubit system can hold multiple bits of information simultaneously, and thus also enables massive parallelism [25].

In this paper, we present Quantum Radiance Fields (QRF), which integrates the quantum circuit and quantum activation function with implicit representation. Then, a quantum computing assisted generative training process followed by supervised discriminative training is used to train the QRF model.

3. Method

To achieve real-time movie-quality rendering of compact neural representations for generated content, we employ the QNNs-based neural raymarching scheme of NeRF [13]. In addition, we are inspired by DONeRF [18] and represent the scene using a compact local sampling strategy. This strategy enables the raymarching-based neural representation to consider only important samples around the surface region, further reducing the usage of Qubit on simulated environments and quantum computers.

We review the baseline model of NeRF in Section 3.1, describe our Quantum Radiation Fields in Section 3.2, introduce Quantum Activation Function in Section 3.3 and Quantum Volume Rendering in Section 3.4.

3.1. Baseline Model

In the neural radiation field, the scene is represented by a neural network \( f_\theta \) with parameter \( \theta_n \) to capture the volume 3D representation. NeRF’s neural network \( f_\theta: (p,d) \rightarrow (c,\sigma) \) maps 3D position \( p \in \mathbb{R}^3 \) and light direction \( d \in \mathbb{R}^2 \) to color value \( c \) and transparency \( \sigma \). The architecture of \( f_\theta \) is chosen such that only the color \( c \) depends on the viewing direction \( d \). This encourages the learning of consistent geometry. In the deterministic preprocessing step, \( x \) and \( d \) are transformed via a positional encoding \( y \), which promotes the learning of high frequency details. To render a single image pixel, a ray is cast from the center of the camera, through that pixel and into the scene. We denote the direction of this ray as \( d \).

Multiple 3D positions \( (p_1, \ldots, p_k) \) are then sampled along the ray between the near and far boundaries defined by the camera parameters. The neural network \( f_\theta \) is evaluated at each position \( p_i \) and ray direction \( d \) to produce color \( c_i \) and transparency \( \sigma_i \). These intermediate outputs are then integrated as follows to produce the final pixel color \( \hat{c} \):

\[
\hat{c} = \sum_{i=1}^{K} T_i (1 - e^{-\sigma_i \delta_i}) c_i
\]

where \( T_i = e^{-\sum_{j=1}^{i-1} \sigma_j} \) is the transmittance and \( \delta_i = (p_i+1 - p_i) \) is the distance between samples. Since \( f_\theta \) is dependent on ray direction, NeRF can model viewpoint-dependent effects.

3.2. Quantum Radiance Fields

The quantum radiance fields is implemented by various quantum circuits built into the continuous-variable architecture. It consists of three consecutive parts (as shown in Figure 3). An encoding circuit encodes the classical data into the states of the qubits, followed by a parameterized quantum circuits (PQCs), which is used to transform these states to their optimal location on the Hilbert space. Finally, quantum activation is used to make nonlinear mapping to the input and add some nonlinear factors to neural networks so that neural networks can better represent the color and transparency of each position along the ray.

3.2.1 Encoding Circuit

The encoding circuit is used to encode the classical data into the physical states of Hilbert space for quantum computing [26], which is critical to the success of quantum neural networks. Quantum encoding can be thought of as loading a data point \( x \in X \) from memory into a quantum state so that it can be processed by the QNNs. The loading is accomplished by encoding from the set \( X \) to the \( n \)-qubit quantum state \( D_n \). Many QNNs papers [27][28][29] proposed wavefunction encoding with \( n = \log_2 2N \). This provides an exponential space savings at the cost of an
exponential increasing in time. That is, a quantum state of \( \log_2 2N \) qubits can represent a data point with \( N \) features, but such a quantum state takes time \( O(2^n) \) to prepare. Some recent authors [30][31][32] have considered an angle encoding which can efficiently encode classical data into quantum state.

**Angle Encoding.** Angle encoding makes use of rotation gates to encode classical information \( x_k \in \mathbb{R}^N \) without any normalization condition. Angle encoding can be constructed using a single rotation with angle \( \theta_k \) (normalized to be in \([-\pi, \pi]\)) for each qubit, and can therefore encode \( N \) features with \( N \) qubits. Angle encoding consists in the following transformation:

\[
S_k |0\rangle = \otimes_{k=0}^{N} \cos(\theta_k) |0\rangle + \sin(\theta_k) |1\rangle
\]

where the circuit starts with the \( |0\rangle \) state, encodes a data point \( x_k \) using a circuit \( S_k \). \( N \) is the number of qubits, which used for encoding is equal to the dimension of vector \( x_k \). Angle encoding can be very easily constructed and has a depth of only 1. The main advantage of angle encoding is that it is very efficient in terms of operations. Only a constant number of parallel operations are needed regardless of how many data values need to be encoded. This is not optimal from a qubit point of view since each input vector component requires one qubit [52].

**Dense Angle Encoding.** Angle encoding can be slightly generalized to encode two features per qubit by exploiting the relative phase degree of freedom [53]. We refer to this as the dense angle encoding and include a definition below:

\[
|X\rangle = \otimes_{k=1}^{N/2} \cos(\theta_{2k}) |0\rangle + e^{i\pi \theta_{2k}} \sin(\pi \theta_{2k}) |1\rangle
\]

where \( \theta \) is a feature vector \( x = [x_1, \ldots, x_N]^T \in \mathbb{R}^N \), the dense angle encoding maps \( x \rightarrow E(x) \). Dense encoding is derived by extending the above formula into two features using relative phase degrees of freedom. It exploits the additional property of relative phase qubits to encode \( N \) data points using only \( N/2 \) qubits.

**3.2.2 Parametrized Quantum Circuits.**

Parametrized Quantum Circuits is composed of a set of parameterized single and controlled single qubit gates. The parameters are iteratively optimized by a classical optimizer to attain a desired input-output relationship. A block-diagonal approximation to the Fubini-Study metric tensor of the PQC's can be evaluated on quantum hardware. In general, an \( n \) qubits PQC can be written as:

\[
u(\theta)\varphi_0 = \left( \prod_{\ell=1}^{k} W_\ell u_\ell(\theta_\ell) \right)\varphi_0
\]

where \( \varphi_0 \) is the initial quantum state, \( m \) is the maximum circuit depth, \( W_\ell \) is the non-parameterized quantum gate at \( \ell \)-th layer, \( u_\ell(\theta_\ell) \) is the parametrized quantum gate with parameters \( \{ \theta_0, \theta_1, \ldots, \theta_k \} \) at \( \ell \)-th layer, which is a sequence consisting of parameterized qubit gates. Herein, the form of \( u_\ell(\theta_\ell) \) is variable and agrees with any physical constraint such as highly limited connectivity between physical qubits.

To achieve better entanglement of the qubits before appending nonlinear operations, the \( n \) qubits PQC's has \( N \) repeated layers in our model. In order to provide computational speedup by orchestrating constructive and destructive interference of the amplitudes in quantum computing, we constructed \( m \) rotation gates on the \( n \) qubits PQC's as our basic quantum circuit, which can be written as:

\[
\left( \prod_{\ell=1}^{N} \hat{CNOT}_{i,i+1} R(\theta_{i+nj}) \right)\varphi_0
\]

where \( CNOT_{i,i+1} \) represents \( CNOT \) gate at the control qubit. \( R(\theta_{i+nj}) \) represents the rotation gate along each of the X, Y, and Z-axis. \( \theta_{i+nj} \) is adjustable parameter of rotation gates \( R \). With the Pauli matrix, we can define single-qubit rotation along each of the \( X, Y, \) and \( Z \)-axis as:

\[
R_x(\theta) = e^{-i \theta \sigma_x}, R_y(\theta) = e^{-i \theta \sigma_y}, R_z(\theta) = e^{-i \theta \sigma_z}
\]

where \( \{ \sigma_x, \sigma_y, \sigma_z \} \) is Pauli matrices. The operation of \( R(\theta) \) can be modified by changing parameters \( \theta \). Thus, the output state can be optimized to approximate the wanted state. By optimizing the parameters, the general PQC's tries to approximate arbitrary states so that it can be used for different specific molecules. The goal of PQC's is to solve an optimization problem encoded into a cost function:

\[
\theta^* = \arg \min_{\theta \in \mathbb{C}} \langle \psi(\theta)|H|\psi(\theta)\rangle
\]

where \( H \) is the Hamiltonian with the ground energy to seek. As parameters \( \theta \) are continuous, many gradient-based optimization algorithms can be used to find the optimal ones. Figure 3 shows an example of Parametrized Quantum Circuits with \( n=6 \) and \( m=4 \). Four qubits use the rotation gate \( R(\theta) \) by the angle \( \theta \) around \( z \)-axis on the Hilbert space, and \( CNOT \) gate is used for 2 specific qubits.

**3.3. Quantum Activation Function**

Recent implicit neural representations are built on ReLU-based multilayer perceptron. These architectures lack the capacity to represent the fine details in the underlying signal, and they often do not represent the derivative of the target signal well. This is partly due to the fact that ReLU networks are piecewise linear, and their second derivatives are zero everywhere, so they cannot model the information contained in the higher-order derivatives of natural signals. To address these limitations, we leverage quantum activation functions, which can better represent details in signals than ReLU-MLPs for
implicit neural representations.

We apply a multi-step quantum approach by selecting the ReLU’s solution for positive values $R(z)_{ReLU}$, and the LReLU’s solution for negative values $R(z)_{LReLU}$ [54]. By applying the quantum principle of entanglement, the tensor product of the two candidate Hilbert state spaces from $H_{ReLU}$ and $H_{LReLU}$ was performed as:

$$H_{ReLU} \otimes H_{LReLU}$$

(8)

The quantum entanglement in Eq. 8 allows to overcome the limitation of the ReLU being dying for negative inputs. The resulting state in the blended system is described by:

$$|\varphi_{ReLU}\rangle \otimes |\varphi_{LReLU}\rangle$$

(9)

In an entangled or inseparable state, the formulation of product states of Quantum ReLU (QReLU) can be generalized as:

$$|\varphi_{QReLU}\rangle = \sum |0\rangle_{ReLU} \otimes |0\rangle_{LReLU}$$

(10)

where keeping output for positive values in the QReLU, and with the added novelty of the entangled solution for negative values. This fits complicated signals, such as natural images and 3D shapes, and their derivatives robustly.

### 3.4. Quantum Volume Rendering

Conventional implicit neural representations represent a scene as an implicit function $F_{\theta}(p, v) \rightarrow (c, \omega)$, where $\theta$ are parameters of an underlying neural network [13]. It evaluate a volume rendering integral to compute the color of camera ray $p(z) = p_0 + z \cdot v$ as:

$$c(p_0, v) = \int_0^\infty \omega(p(z)) \cdot c(p(z), v) dz$$

(11)

where $\int_0^\infty \omega(p(z)) dz = 1$, $c$ is the scene color, $w$ is the probability density at spatial location $p$ and ray direction $v$. $c(p_0, v)$ describes the scene color $c$ and its probability density $\omega$ at spatial location $p$ and ray direction $v$.

Volume rendering methods estimate the integral $c(p_0, v)$ by densely sampling points on each camera ray and accumulating the colors and densities of the sampled points into a 2D image as:

$$c(p_0, v) \approx \sum_{i=1}^N \left( \prod_{j=1}^k \alpha(z_i, \Delta_i) \right) \cdot (1 - \alpha(z_i, \Delta_i)) \cdot c(p(z_i), v)$$

(12)

where $\alpha(z_i, \Delta_i) = \exp(-\sigma(p(z_i), v))$, and $(\sigma(p(z_i)))_{i=1}^N$ are the colors and the volume densities of the sampled points. Although volume rendering offer unprecedented image quality, they are also extremely slow and memory inefficient. This is because volume rendering methods need to sample a large number of points along the rays for
color accumulation to achieve high quality rendering.

Volume rendering is essentially a numerical integration problem in each pixel, which is commonly done by Monte Carlo integration on classical computers. In this paper, we propose the quantum ray tracing, a quantum algorithm for numerical integration that is fundamentally superior to classical Monte Carlo integration. Furthermore, we apply Grover’s search [33] to design clever algorithms to take full advantage of quantum parallelism. Given a ray tracing oracle that implements the following transformations:

$$O_f(p_{\text{pixel}}, \text{channel}) : \sum_{j=0}^{N-1} x_j |j\rangle \rightarrow \sum_{j=0}^{N-1} x_j |f(j)\rangle$$

(13)

where $\text{pixel}$ and $\text{channel}$ are classical parameters, $j$ plays the role of ray identity, and $f(j)$ is a real number that stands for the ray energy. In the rest of this paper $f$ is specified as the function that maps ray $j$ to ray energy. The oracle can trace $N = 2^n$ paths simultaneously, and the final color we hope to write to the corresponding pixel and channel is the average of those energies $\frac{1}{N} \sum_{j=0}^{N-1} f(j)$. Suppose those real numbers $f(j)$ are stored in a fixed-point format with integer bit length $b_0$ and total bit length $b$, we can transfer the estimation problem of Eq. 13 into quantum counting by constructing a Boolean function:

$$g(j, k) = \begin{cases} 1, f(j) \geq 2^{b_0-b-k} \\ 0, f(j) < 2^{b_0-b-k} \end{cases}$$

(14)

where $k = (0,1,\ldots,2^b-1)$. The phase oracle $O_g$ for $g$ in Grover’s search [55] as:

$$O_g : \sum_{j,k} |j,k\rangle \rightarrow \sum_{j,k} (-1)^{g(j,k)} |j,k\rangle$$

(15)

To construct $O_g$, we need a comparison gate that performs the comparison operation $COMP$ on the two integers $2^{b-b_0}f(j)$ and $k$.

$$COMP : \sum_{j,k} |f(j),k\rangle \rightarrow \sum_{j,k} |f(j),k|g(j,k)\rangle$$

(16)

The $O_g$ gate can be constructed as Figure 4. The quantity $\sum_{j,k} g(j,k)$ can be estimated by quantum counting algorithm. In the paper we assume one call to $O_g$ quantum ray tracing takes the same samples as tracing one path in classical numerical integration. We evaluate the cost of
classical path tracing by the number of ray paths $N_c$, as the noise comes mostly from the Monte Carlo integration. And in quantum ray tracing, the time cost is evaluated by the number of queries $N_q$ to the ray tracing oracle $O_f$. The quantum integration has a convergence rate of $O(1/N_q)$, hence has a quadratic speedup over classical Monte Carlo integration with convergence rate of $O(1/\sqrt{N_c})$.

4. Results

4.1. Task

2D Image Regression. We train a QNNs to regress from 2D input pixel coordinates to the corresponding RGB values of an image (as shown in Figure 5). We consider two different distributions $\mathcal{H}$: face images (CelebA [34]) and natural images (Div2K [35]). Given a sampled image $h \sim \mathcal{H}$, we resize all images to $256 \times 256$ as observations for network weights $\theta$ in the optimization inner loop. At each inner loop step, the entire image is reconstructed and used to compute the loss. We then compare the classical MLPs and QNNs over these two distributions.

3D Scene Representation. The goal of 3D scene representation is to generate a novel view of the scene from a set of reference images. We validate our QRFS through an extensive series of ablation studies and comparisons to recent techniques for accelerating NeRF. We evaluate our method on three inward-facing datasets:

1. Synthetic-NeRF [13] consists of 360-degree views of complex objects in 8 scenes, where each scene has a central object with 100 inward facing cameras distributed randomly on the upper hemisphere. The images are 800×800 with provided ground truth camera poses.
2. Synthetic-NSVF [15] contains 8 objects synthesized by NSVF. Strictly following the settings of NSVF, we set the image resolution to 800 × 800 pixels and let each scene have 100 views for training and 200 views for testing.
3. Tanks & Temples [36] is a real-world dataset containing 5 scenes of real objects captured by an inward-facing camera surrounding the scene. Each scene contains between 152-384 images of size 1920 × 1080.

4.2. Implementation

The principal baseline for our experiments is NeRF [13]. We report the results of the original NeRF implementation, as well as the reimplementations in Jax (JaxNeRF) [14]. We also compare two older methods, Scene Representation Network (SRN) [7] and Neural Volume [8], as well as five recent papers introducing NeRF accelerations, Neural Sparse Voxel Field (NSVF) [15], AutoInt [17], FastNeRF [19], KioNeRF [20] and Depth-supervised NeRF (DS-NeRF) [21]. To evaluate QRF, we focus on two competing requirements of scene representations: quality of the generated images, and efficiency of the image generation. To quantify the rendering quality, we rely on three metrics:

1. Peak Signal to Noise Ratio (PSNR): A classic metric to measure the corruption of a signal.
2. Structural Similarity Index Measure (SSIM) [37]: A perceptual image quality assessment based on the degradation of structural information.
3. Learned Perceptual Image Patch Similarity (LPIPS) [38]: A perceptual metric based on the deep features of a trained network that is more consistent with human judgement.

where higher PSNR, SSIM and lower LPIPS is most desirable. We train both the NeRF baseline and QRF for a high number of iterations to find the limits of the representation capabilities of the respective architectures. We train each model for 350k iterations. For the classical scene representation, we train it in 48 hours using 2 Tesla V100 GPUs. For our 2D quantum representations and QRF model, we train it with Borealis, a photonic quantum processor from Xanadu that can be programmed and entangled. The inference time performance is measured on a Tesla V100 for classical NeRF and a Borealis for QRF.

4.3. Ablation Study

Activation Function. We first compare the performance of the classical activation function and the quantum activation function in the 2D image regression and 3D scene representation task. We use Rectified Linear Unit (ReLU), Exponential Linear Unit (ELU), Smooth ReLU (SoftPlus), SIREN [9] as the classical activation function, and QReLU as quantum activation function. For 2D image regression, we train an MLP with 4 layers/256 channels and apply sigmoid activation to the output for each task. For 3D scene representation, we train baseline NeRF with 6 layers/256 channels and apply positional

![Figure 5: Quantum Implicit Neural Representations on 2D image regression with encoding circuit and PQC (n=3, m=3).](image-url)
encoding to the input coordinates. Table 1 shows that when the QReLU function serves as the activation function for each task, it outperforms the state-of-the-art activation function, which indicates that the quantum activation function has better convergence.

**Encoding Circuit and Quantum Circuit.** We validate our QRF on our quantum system using various encoding circuits and quantum circuits. We use circuit-A/B/C by Paulo Sousa et al. and circuit-D by Maria Schuld et al. as baselines for quantum circuits (as shown in Figure 6). In addition, we adopt general qubit gates, waveform encoding, angle encoding, and dense angle encoding as our encoding circuit strategy. More details about our quantum circuit and encoding circuit can be found in the supplemental. The ablation results yield many significant findings (as shown in Table 2). First, the model with Dense Angle Encoding has higher test PSNR regardless of the quantum circuit used. Then, circuit-B/C/D has higher test PSNR than circuit-A, because the static CNOTs of circuit-A are replaced with parametric two-qubit gates to construct the circuits. Next, circuit-B and D, which employed two-qubit gates in a nearest-neighbor fashion, were more expressible than circuit C that employed two-qubit gates in a ring topology. Finally, circuit-D has the highest PSNR.

### Table 1: Ablation Study of activation function from the Div2K [35] and Synthetic-NeRF [13] dataset.

| Task       | 2D Image Regression | 3D Scene Representation |
|------------|---------------------|-------------------------|
| Datasets   | Div2K [35]          | Synthetic-NeRF [13]     |
| Evaluate Metrics | PSNR | SSIM | LPIPS | PSNR | SSIM | LPIPS |
| Classical AF | ReLU   | 32.89 | 0.961 | 0.044 | 24.85 | 0.812 | 0.208 |
|            | ELU     | 23.12 | 0.380 | 0.258 | -     | -     | -     |
|            | Softplus | 19.37 | 0.273 | 0.482 | -     | -     | -     |
| Quantum AF  | QReLU   | 38.43 | 0.971 | 0.016 | 27.12 | 0.919 | 0.168 |

This is because circuit-D uses all-to-all configuration of two-qubit gates, which led to both favorable expressibility and entangling capability.

### 4.4. Experiment Results

**2D Image Regression.** We first compare the performance between classical MLPs and QNNs model in the task of 2D image regression. According to the ablation study result, we use QReLU as the activation function, dense angle encoding as the encoding circuit, and two-qubit gates as our parameterized quantum circuits (as Figure 5). Table 3 shows that our proposed quantum model has significant advantages in each evaluate metric.

**3D Scene Representation.** We employ QReLU, Dense Angle Encoding, and circuit-D to construct our QRF model

### Table 2: Ablation Study of encoding circuit and quantum circuit from the Synthetic-NeRF dataset [13].

| Evaluate Metrics | Encoding Circuit | Quantum Circuit |
|------------------|------------------|-----------------|
|                  | A               | B               | C               | D               |
| PSNR             | General Qubit Encoding | 24.82 | 26.33 | 25.52 | 26.65 |
| Wavefunction Encoding | 24.88 | 26.39 | 25.58 | 26.65 |
| Angle Encoding   | 25.20 | 26.80 | 25.99 | 26.98 |
| Dense Angle Encoding | 25.64 | 27.15 | 26.33 | 27.33 |
| General Qubit Encoding | 0.809 | 0.875 | 0.833 | 0.893 |
| Wavefunction Encoding | 0.820 | 0.885 | 0.838 | 0.880 |
| Angle Encoding   | 0.832 | 0.900 | 0.860 | 0.911 |
| Dense Angle Encoding | 0.838 | 0.915 | 0.873 | 0.921 |
| LPIPS            | General Qubit Encoding | 0.211 | 0.183 | 0.195 | 0.183 |
| Wavefunction Encoding | 0.208 | 0.189 | 0.195 | 0.182 |
| Angle Encoding   | 0.206 | 0.176 | 0.188 | 0.175 |
| Dense Angle Encoding | 0.199 | 0.176 | 0.183 | 0.169 |

### Table 3: Quantitative comparisons for 2D image regression. Compared with classical MLPs, our proposed QNNs model outperforms in the rendering quality.

| Datasets | CelebA [33] | Div2K Dataset [34] |
|----------|-------------|---------------------|
| Evaluate Metrics | PSNR | SSIM | LPIPS | PSNR | SSIM | LPIPS |
| Classical MLPs | 30.67 | 0.945 | 0.112 | 32.89 | 0.961 | 0.044 |
| QNNs       | 33.71 | 0.963 | 0.038 | 36.36 | 0.969 | 0.027 |

Figure 6: Quantum Circuit by Paulo Sousa, et al. [39] and Maria Schuld et al. [40].
based on the ablation study. As shown in Table 4, we find that QRF inference is over 2000 times faster than NeRF, and faster than other methods except FastNeRF. It is because that FastNeRF can compactly cached and subsequently queried to compute the pixel values in the rendered image by factorization approach. Furthermore, QRF performed the best among all image quality metrics. Notably, our method does not address the training speed issue. We propose to improve the training speed of NeRF models by finding initialization through Meta Learning [16], or reconstructing the per-scene radiance field by Direct Voxel Grid Optimization NeRF [22].

5. Conclusion

In this paper, we presented Quantum Radiance Fields, a novel extension to NeRF that enables the rendering of photorealistic images using quantum computing. As a result, our method not only renders much faster, but can also deliver higher quality images. Moreover, the presented acceleration quantum strategy might also apply more broadly to other methods. We hope this paper can be a demonstration that quantum computing has the potential to provide satisfactory solutions for scene representation and volume rendering.

Table 4: Quantitative results on each scene from the Synthetic-NeRF [13], Synthetic-NSVF [15], and Tanks and Temples [36]. We highlight the top 3 results in each column are color coded as Top 1, Top 2 and Top 3.

| Dataset          | Evaluate Metrics | Synthetic-NeRF [13] | Synthetic-NSVF [15] | Tanks and Temples [36] |
|------------------|------------------|---------------------|---------------------|------------------------|
|                  | PSNR  | SSIM  | LPIPS | FPS | PSNR  | SSIM  | LPIPS | FPS | PSNR  | SSIM  | LPIPS | FPS |
| SRN [7]          | 22.26 | 0.846 | 0.170 | 0.909 | 24.33 | 0.882 | 0.141 | 1.304 | 24.10 | 0.847 | 0.251 | 0.250 |
| Neural Volumes [8] | 26.05 | 0.893 | 0.160 | 3.330 | 25.83 | 0.892 | 0.124 | 4.778 | 23.70 | 0.834 | 0.260 | 1.000 |
| NeRF [13]        | 31.01 | 0.947 | 0.081 | 0.023 | 30.81 | 0.952 | 0.043 | 0.033 | 25.78 | 0.864 | 0.198 | 0.007 |
| JaxNeRF [14]     | 31.69 | 0.953 | 0.049 | 0.045 | 31.49 | 0.958 | 0.026 | 0.065 | 27.94 | 0.904 | 0.168 | 0.013 |
| NSVF [15]        | 31.75 | 0.953 | 0.047 | 0.815 | 35.18 | 0.979 | 0.015 | 0.095 | 28.42 | 0.907 | 0.153 | 0.163 |
| AutoINT [17]     | 25.55 | 0.911 | 0.170 | 0.380 | 26.63 | 0.916 | 0.090 | 0.545 | 22.28 | 0.766 | 0.278 | 0.116 |
| DoNeRF [18]      | 32.50 | 0.957 | 0.037 | 5.635 | 32.29 | 0.962 | 0.027 | 8.085 | 27.02 | 0.805 | 0.174 | 1.715 |
| FastNeRF [19]    | 29.97 | 0.941 | 0.053 | 47.26 | 29.78 | 0.946 | 0.083 | 172.42 | 24.92 | 0.792 | 0.213 | 47.67 |
| KioNeRF [20]     | 31.02 | 0.950 | 0.051 | 38.46 | 33.37 | 0.970 | 0.020 | 55.07 | 28.41 | 0.910 | 0.091 | 11.68 |
| QRF              | 32.65 | 0.960 | 0.029 | 47.26 | 35.44 | 0.980 | 0.014 | 67.70 | 29.65 | 0.820 | 0.085 | 14.36 |

Figure 7: Qualitative comparisons on Synthetic-NeRF [13]. We compare classical implicit representation, NeRF, NeRF accelerations, and our proposed method. On this dataset, we find that our method better recovers the fine details in the scene. The results are similar in other datasets, please refer to our supplementary for more details.
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Supplemental Material

QRF: Implicit Neural Representations with Quantum Radiance Fields

We provide additional implementation details, discussions, and experimental results in the supplementary material to provide more detailed results and algorithms. In Section 1, we discuss the limitations and societal impact of our paper. In Section 2, we present the derivation details, including encoding circuit, quantum circuit, Grover algorithm, and optimization. Finally, we show the detailed quantitative and qualitative results on each scene in the additional experiments.

1. Discussion

1.1. Limitations

In deep learning, it is very useful to transform data into a higher-dimensional feature space. In the same way, there are two strategies for using quantum circuits to generate higher dimensional features: entangling more and more qubits. However, quantum computers are now in their infancy, the available qubits are limited. In this paper, we use photonic quantum computer with 216 squeezed-state qubits to construct our QRF. While inference is fast, the training time is longer than on a classical computer in the case of qubit limitation.

In addition, QRF and NeRFs are unable to sample the volume to infinity and instead assume that the scene lies between a near and far bound due to practical constraints. To ensure efficient execution of volume rendering, we treat the furthest radiance as an opaque wall for boundary limitation.

1.2. Societal Impact

QRF is a technique which empowers NeRFs to operate on a variety of scenes. While novel view synthesis is not a synthetic media, it can open the door to abuse when generating face image through the scene. There could also be privacy concerns, as using QRF technology to better render the sharper details of a face could capture personally identifiable information.

Significant drawbacks with the technology of QRF and NeRFs are concerns surrounding privacy. For example, the specific face rendering and recognition technology used, how it’s deployed, and what computing method it relies on play a part in determining how much of a privacy risk it is. These worries about privacy aren’t unreasonable. People don’t like to feel they’re being watched or that personal information could be leaked. These anxieties are reflected by institutions worldwide that have implemented strict regulations to protect people’s biometric data. Europe’s General Data Protection Regulation (GDPR) [1] is an example of this. Europe’s GDPR details that citizens are entitled to their privacy and that any breach of privacy will be met with consequences.

Whether we are delivering the NeRFs app or user experience directly to consumers or any company, we require appropriate notification. This means that consumers are very aware of the personally identifiable data we will be collecting.

2. Derivation Details

We present the encoding circuit, quantum logic gate, and our proposed detail quantum circuits, then we introduce the Grover algorithm for calculating numerical integrals in this paper.

2.1. Encoding Circuit

The encoding circuit is used to encode the classical data into the physical states of Hilbert space for quantum computing. In this section, we introduce another 3 encoding methods [2] applied in the paper, including General Qubit Encoding, Wavefunction Encoding, Angle Encoding.

**General Qubit Encoding.** In this encoding, the data has to be in form of a binary string to get encoding. Approximating a scalar value to its binary form and then transforming it to a quantum state. For example, if we have a classical dataset containing two examples $x_k = 01$ and $x_{k+1} = 11$, the corresponding quantum state after basis encoding is $|x_k⟩ = |01⟩$ and $|x_{k+1}⟩ = |11⟩$. In short, Basis encoding encodes an n-bit binary string $x_k$ to an n-qubit quantum state as:

$$|x_k⟩ = |i_k⟩$$

where $|i_k⟩$ is a computational basis state, and every binary string has a unique integer representation $i_k = \sum_{k=0}^{n-1} 2^k x_k$.

**Wavefunction Encoding.** The wavefunction encoding is also known as amplitude embedding. The amplitude is the height of a wave. In this encoding, the data points are transformed into amplitudes of the quantum state. A normalized classical $N$-dimensional $x_k$ is represented by the amplitudes of a $n$-qubit quantum state $|x_k⟩$ as:

$$|x_k⟩ = \sum_{k=0}^{n} x_k |i_k⟩$$

where N is the length of vector $x_k$ into amplitudes of an n-qubit quantum state with $n = \log_2(N)$. $\{|i_k⟩\}$ is the
computational basis for the Hilbert space. Since the classical information forms the amplitudes of a quantum state, the input needs to satisfy the normalization condition: \(|x|^2 = 1\).

**Angle Encoding.** Angle encoding makes use of rotation gates to encode classical information \(x_k \in \mathbb{R}^N\) without any normalization condition. The classical information determines angles of rotation gates:

\[
|x_k\rangle = \bigotimes_{k=0}^{N} R(x_k)|0^N\rangle
\]  

(3)

Where \(N\) is the number of qubits, \(R\) can be one of \(R_x, R_y, R_z\). Usually, \(N\) used for encoding is equal to the dimension of vector \(x_k\). Since it is \(2\pi\)-periodic one may want to limit \(\mathbb{R}^N\) to the hypercube \([0, 2\pi]^\otimes n\). The \(i\)-th feature \(x_k\) is encoded into the \(k\)-th qubit via a Pauli-X rotation. Figure 1 show the angle encoding with a rotation angle of \(\theta\) around z-axis on the Hilbert space.

After activation function \(\text{tanh}(x)\), the output \(x_k \in [-1, 1]\) from the end of classical layer. Then, the rotation angle is mapped between \([0, \pi]\) due to the periodicity of the cosine function. This is relevant since the expectation value is taken with respect to the \(\sigma_z\) operator at the end of the circuit execution.

### 2.2. Quantum Logic Gate

Quantum logical gate is used in quantum computing, which is essentially a transformation of one or more qubits. We have given a description of the quantum gate types in Figure 2.

**CNOT Gate** means the quantum logic gate which acts on a pair of qubits by the linear map which is given in this way on the canonical linear basis-elements of a pair of qubits, where the first qubit is usually referred to as the control qubit and the second qubit as the target qubit. Expressed in basis states, the CNOT gate:

1. leaves the control qubit unchanged and performs a Pauli-X gate on the target qubit when the control qubit is in state \([1]\).
2. leaves the target qubit unchanged when the control qubit is in state \([0]\).

**Rotation Gate** give rise to the rotation operators, which rotate the Bloch vector about the x, y and z axes, by a given angle \(\theta\):

\[
R_x(\theta) = e^{-i\frac{\theta}{2} \gamma_x}, R_y(\theta) = e^{-i\frac{\theta}{2} \gamma_y}, R_z(\theta) = e^{-i\frac{\theta}{2} \gamma_z}
\]  

(4)

Now, if an operator \(A\) satisfies \(A^2 = I\), it can be shown that:

\[
e^{i\theta A} = \cos(\theta)I + i \sin(\theta)A
\]  

(5)

And since all the Pauli matrices satisfy \(X^2 + Y^2 + Z^2 = I\), the rotation operator can be expanded as:

\[
R_x(\theta) = e^{-i\frac{\theta}{2} \gamma_x} = \cos\left(\frac{\theta}{2}\right)I - i \sin\left(\frac{\theta}{2}\right)\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}
\]  

(6)

\[
= \begin{bmatrix} \cos(\theta/2) -i\sin(\theta/2) \\ -i\sin(\theta/2) \cos(\theta/2) \end{bmatrix}
\]

**Figure 2: Quantum Logic Gates**
Figure 3: Quantum Circuit from prototype model (3-1, 3-3, 3-5, 3-7) [3][4] and its’ extend model of Quantum Radiance Fields (3-2, 3-4, 3-6, 3-8). Similar to the architecture of NeRF [5], QRF enforces that the predicted $\sigma$ is independent of view direction.
\[ R_y(\theta) = e^{-i \frac{\theta}{2} y} = \cos \left( \frac{\theta}{2} \right) I - i \sin \left( \frac{\theta}{2} \right) |0\rangle \langle 0| - i \]

\[
= \begin{bmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}
\] (7)

\[ R_x(\theta) = e^{-i \frac{\theta}{2} x} = \cos \left( \frac{\theta}{2} \right) I - i \sin \left( \frac{\theta}{2} \right) |1\rangle \langle 1| - i \]

\[
= \begin{bmatrix} \cos(\theta/2) -i\sin(\theta/2) & 0 \\ 0 & \cos(\theta/2) +i\sin(\theta/2) \end{bmatrix}
\] (8)

2.3. Quantum Circuit

Quantum circuit is a system comprising multiple qubits. A quantum computer stores its quantum data in one or more quantum circuits. We use circuit-A/B/C by Paulo Sousa et al. [3] and circuit-D by Maria Schuld et al. [4] as baselines for quantum circuits (as shown in Figure 3).

Figure 3-1, 3-3, 3-5, 3-7 is the prototype model by Paulo Sousa et al. and circuit-D by Maria Schuld et al., and Figure 3-2, 3-4, 3-6, 3-7 is the extend of Quantum Radiance Fields. CNOT gates, rotation gates are used to construct our quantum circuit. For input, 3D position \((x, y, z)\) and viewing direction \((\theta, \phi)\) are encoded by dense angle encoding, and it output colors \((r, g, b)\) and transparency values \((\sigma)\). Similar to the NeRF [5], QRF enforces that the predicted \(\sigma\) is independent of view direction. Therefore, we do not perform any cross quantum operation on 4th qubit \((q_3)\) and 5th/6th qubit \((q_4, q_5)\). Finally, we perform the measurement on \(q_0/q_1/q_2/q_3\), mapping a quantum state to a classical vector, to get the final result (colors and transparency values).

2.4. Grover Search

Rendering on conventional computers is capable of generating realistic imagery, but the computational complexity of these light transport algorithms is a limiting problem that needs to be addressed is supersampling of the image, which can be seen as a mean estimation per pixel. Grover algorithm [6] was the first to introduce a quantum algorithm for estimating a mean. The idea is to combine Amplitude Amplification (AA) with Quantum Fourier Transformation (QFT). Johnston [8] first implemented Grover algorithm to conduct numerical integrations in the context of rendering. Volume rendering is essentially a numerical integration problem in each pixel, which is commonly done by Monte Carlo integration on classical computers. In this paper, we apply Grover algorithm to design a ray tracing oracle for volume rendering.

The main idea of Grover algorithm is to exploit the existence of a periodic cycle when we keep applying amplitude amplification on \(|\psi\rangle\). Applying QFT on the history of rotated \(|\psi\rangle\), we can extract the frequency of this periodic cycle, which then allows us to calculate the corresponding \(|\psi\rangle\). Given a function \(F(a) : a \rightarrow [0, 1]\) and a quantum oracle operator:

\[ \tilde{Q}_F = |0\rangle \langle i| \rightarrow (\sqrt{1 - F(i)}|0\rangle + \sqrt{F(i)}|1\rangle) \otimes |i\rangle \] (9)

where the objective is to get the average \(f\) of \(F(a)\) with \(N\) samples: \(\frac{1}{N} \sum_{j=0}^{N-1} F(j)\). We use the oracle \(\tilde{Q}_F\) and make a superposition state \(|\psi_0\rangle\) from the initial state whose all qubits are \(|0\rangle\), where the numbers of qubits for each are \(\log_2 P, 1, \log_2 N\). We thus write the initial state as

\[ |0 \ldots 0\rangle \otimes |0\rangle \otimes |0 \ldots 0\rangle = |0\rangle \otimes |0\rangle \otimes |0\rangle \] (10)

We generate a superposition state \(|\psi_0\rangle\) as

\[ |\psi_0\rangle = \tilde{Q}_F(R \otimes I \otimes R)|0\rangle \otimes |0\rangle \otimes |0\rangle \]

\[ = \frac{1}{\sqrt{PN}} \sum_{m=0}^{P-1} \sum_{i=0}^{N-1} |m\rangle \otimes \tilde{Q}_F(|0\rangle \otimes |i\rangle) \]

\[ = \frac{1}{\sqrt{PN}} \sum_{m=0}^{P-1} \sum_{i=0}^{N-1} |m\rangle \otimes (\sqrt{1 - F(i)}|0\rangle + \sqrt{F(i)}|1\rangle) \otimes |i\rangle \] (11)

We can define \(\cos(\theta)\) and \(\sin(\theta)\) as \(\sqrt{1 - F(i)}\) and \(\sqrt{F(i)}\), and \(|\psi_0\rangle\) is:

\[ |\psi_0\rangle = \frac{1}{\sqrt{P}} \sum_{m=0}^{P-1} |m\rangle \otimes (\cos \theta |0\rangle + \sin \theta |1\rangle) \otimes |i\rangle \] (12)

We then apply Amplitude Amplification to the \(\cos \theta |0\rangle + \sin \theta |1\rangle\) state for \(P\) times:

\[ |\psi_1\rangle = \frac{1}{\sqrt{P}} \sum_{m=0}^{P-1} |m\rangle \otimes (\cos(2m + 1) |0\rangle 

+ \sin(2m + 1) |1\rangle) \otimes |i\rangle \] (13)

We then measure the target qubits and assume that the state is converged to \(|1\rangle\):

\[ |\psi_2\rangle = \frac{1}{C} \sum_{m=0}^{P-1} \sin(2m + 1) \theta |m\rangle |1\rangle \] (14)

Finally, we perform QFT on \(|\psi_2\rangle\). With a sufficiently large probability, the result of measurement will be:

\[ t \approx \frac{P \theta}{\pi}, \frac{P(\pi - \theta)}{\pi} \] (15)
If the measured and converged state is $|0\rangle$, we get the same result. Therefore, we can deduce the estimated average $f$ by:

$$f' = \sin^2\left(\frac{t\pi}{P}\right)$$  \hfill (16)

Since $\frac{e}{p}$ can be determined by the precision $O(1/P)$ in this process, $f'$ also has the precision of $O(1/P)$. The estimation error $|f - f'|$ is inversely proportional to the number of Amplitude Amplification operations $P$. Since Amplitude Amplification uses two queries per operation, we perform $O(N)$ queries to achieve $O(1/N)$ error. Note that this convergence rate is faster than $O(1/\sqrt{N})$ of Monte Carlo integration.

2.5. Optimization

In the Quantum Radiance Fields, the scene is represented by a Parametrized Quantum Circuits $PQCG_\theta$ with parameter $\theta_n$ to capture the volume 3D representation. QRF's model $PQCG_\theta: (p, d) \rightarrow (c, \sigma)$ maps 3D position $p \in \mathbb{R}^3$ and light direction $d \in \mathbb{R}^2$ to color value $c$ and transparency $\sigma$ (as shown in Figure 4). The architecture of $PQCG_\theta$ is chosen such that only the color $c$ depends on the viewing direction $d$. This encourages the learning of consistent geometry. In the deterministic preprocessing step, $x$ and $d$ are transformed via a positional encoding $\gamma$, which promotes the learning of high frequency details. To render a single image pixel, a ray is cast from the center of the camera, through that pixel and into the scene. We denote the direction of this ray as $d$. Multiple 3D positions ($p_1, \ldots, p_k$) are then sampled along the ray between the near and far boundaries defined by the camera parameters. The $PQCG_\theta$ is evaluated at each position $p_i$ and ray direction $d$ to produce color $c_i$ and transparency $\sigma_i$. These intermediate outputs are then integrated as follows to produce the final pixel color $\hat{c}$:

$$\hat{c} = \sum_{i=1}^{K} T_i (1 - e^{-\sigma_i \delta_i}) c_i$$  \hfill (17)

where $T_i = e^{-\sum_{j=1}^{i-1} \sigma_j}$ is the transmittance and $\delta_i = (p_{i+1} - p_i)$ is the distance between samples.

QRF is trained by minimizing a photometric loss between rendered images and training images. More specifically, for each parameter update step, we randomly sample a training image and $N$ pixels $p_i \in (c_1, \ldots, c_N)$ inside that image. Subsequently, the corresponding pixels $\hat{p}_i \in (\hat{c}_1, \ldots, \hat{c}_N)$ are rendered according to Eq. 17. As shown in Figure 4, the model parameters $\theta$ are optimized by minimizing an $L2$ reconstruction loss between the rendered pixels and their ground truth:

$$Loss = \sum_{i=1}^{N} \|\hat{p}_i - p_i\|^2$$  \hfill (17)

The quantum processor implements unitary transformations on input states. We have a set of basic unitaries $\{U_\theta(\delta)\}$. The input state $|\psi, 1\rangle$ is prepared and then transformed via a sequence of few qubit unitaries $U_\theta(\delta)$ that depend on parameters $\theta$. These $(\theta_i)$ get adjusted during learning such that the measurement on the readout qubit tends to produce the desired label for $|\psi\rangle$. Our goal is to find parameters $\hat{\theta}$ so that the predicted render color $\hat{c}_i$ is near the true label. We will address the question of whether such parameters even exist as well as the question of whether such optimal parameters can then be efficiently found. For a given circuit, a set of parameters $\hat{\theta}$, and an input string $(p, d)$, consider the sample loss:

$$Loss = 1 - \langle (p, d), 1 | U'(\hat{\theta}) (c, \sigma) U(\hat{\theta}) | (p, d), 1 \rangle$$  \hfill (18)

Consider the derivative of the sample loss (as Eq. 18) with respect to $\theta_k$ which is associated with the unitary $U_\theta(\delta_k)$ which has the generalized Pauli operator $\Sigma k$. Now:

$$\frac{dLoss(\hat{\theta}, z)}{d\theta_k} = 2 \text{Im} \langle z, 1 | U'(\hat{\theta}) Y_{n+1} U(\hat{\theta}) | z, 1 \rangle$$  \hfill (19)

where $z = (p, d)$ is the input of PQCs, $Y_{n+1} = (c, \sigma)$ is the prediction after measurement. Given an accurate estimate
of the gradient we need a strategy for how to update $\hat{\theta}$. Let $\hat{g}$ be the gradient of $\text{loss}(\hat{\theta}, z)$ with respect to $\hat{\theta}$. Now we want to change $\hat{\theta}$ in the direction of $\hat{g}$. To lowest order in $\gamma$ we have that:

$$\text{Loss}(\hat{\theta} + \gamma \hat{g}, z) = \text{Loss}(\hat{\theta}, z) + \gamma \hat{g}^2 + O(\gamma^2) \quad (20)$$

We want to move the loss to its minimum at 0 so the first thought is to make $\gamma = -\frac{\text{Loss}(\hat{\theta}, z)}{\hat{g}^2}$, then introduce a learning rate $\alpha$ and apply the basic gradient descent rule:

$$\hat{\theta} = \hat{\theta} - \alpha \frac{\text{Loss}(\hat{\theta}, z)}{\hat{g}^2} \hat{g} \quad (21)$$

We use Adam optimizer with a learning rate parameter $\alpha$ to keep track of gradient moments over time to redirect the optimization trajectory.

3. Additional Experiments

Quantitative Results. We present the per-scene quantitative comparisons in Table 1 for three dataset: Synthetic-NeRF [9], Synthetic-NSVF [10], Tanks and Temples [11]. We present the per-scene quantitative comparisons in Table 1 for three dataset: Synthetic-NeRF [9], Synthetic-NSVF [10], Tanks and Temples [11]. We adopt PSNR as the evaluation metric. QRF perform the best result to most of the recent methods, except the JaxNeRF [14] and NSVF [10]. Moreover, all the methods after NeRF on the tables take quite a few minutes to render. In Figure 6, we show our QRF qualitative results on 8 scenes of Tanks and Temples [11]. In Figure 7, we show our QRF qualitative results on 4 scenes of Tanks and Temples [11].

Qualitative Results. We provide more qualitative results in Figure 5 for Synthetic-NeRF [9], Figure 6 for Synthetic-NSVF [10], and Figure 7 for Tanks and Temples [11]. In Figure 5, we compare our results with the original NeRF implementation [9], Neural Sparse Voxel Field (NSVF) [10], AutoInt [15], DoNeRF [16], FastNeRF [17], and KioNeRF [17] on Synthetic-NeRF [9] dataset. In Figure 6, we show our QRF qualitative results on 8 scenes of Synthetic-NSVF [10]. In Figure 7, we show our QRF qualitative results on 4 scenes of Tanks and Temples [11].

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| Scenes                  | Avg. | Chair | Ship | Hotdog | Drums | Ficus | Materials | Lego | Mic |
|-------------------------|------|-------|------|--------|-------|-------|-----------|------|-----|
| SRN [12]                | 22.26| 26.82 | 20.67| 26.78  | 17.14 | 20.85 | 18.03     | 20.96| 26.79|
| Neural Volumes [13]     | 26.05| 28.21 | 23.79| 30.58  | 22.53 | 25.14 | 24.33     | 26.10| 27.73|
| NeRF [9]                | 31.01| 33.05 | 28.60| 35.16  | 25.34 | 30.37 | 29.67     | 32.17| 33.73|
| JaxNeRF [14]            | 31.69| **35.49** | 29.55| 34.98  | 25.37 | **30.84** | 29.90  | 33.69| 33.72|
| NSVF [10]               | 31.75| 33.21 | 27.91| 37.16  | 25.16 | 31.25 | **32.66** | 32.31| 34.35|
| AutoInt [15]            | 25.55| 25.14 | 22.12| 30.14  | 20.57 | 22.50 | 25.38     | 31.14| 27.38|
| DoNeRF [16]             | **32.50** | 35.17 | 30.44| 37.73  | 25.65 | 30.15 | 31.29     | 35.17| 34.42|
| FastNeRF [17]           | 29.97| 32.43 | 27.88| 34.79  | 23.74 | 27.80 | 28.94     | 32.41| 31.76|
| KioNeRF [18]            | 31.00| 28.85 | 35.99| 24.56  | 28.76 | 29.94 | 33.52     | 32.85|     |
| QRF                     | **32.65** | 35.34 | 30.35| 37.96  | 25.87 | 30.29 | 31.51     | 35.30| 34.57|

| Scenes                  | Avg. | Bikes | Wineholder | Spaceship | Streamtrain | Palace | Lifestyle | Toad | Robot |
|-------------------------|------|-------|------------|-----------|-------------|--------|-----------|------|-------|
| SRN [12]                | 24.33| 23.79 | 20.71      | 27.95     | 25.53       | 24.46  | 24.57     | 25.35| 22.28|
| Neural Volumes [13]     | 25.83| 26.64 | 21.33      | 29.92     | 25.30       | 26.37  | 27.69     | 24.62| 24.75|
| NeRF [9]                | 30.81| 31.76 | 28.24      | 34.69     | 30.82       | 31.75  | 31.09     | 29.44| 28.70|
| JaxNeRF [14]            | 31.49| 32.46 | 28.86      | 35.46     | 31.50       | 32.45  | 31.28     | 30.09| 29.33|
| NSVF [10]               | **35.18** | 37.56 | 32.45      | 39.12     | 33.11       | **34.41** | **35.92** | 33.41| 35.48|
| AutoInt [15]            | 26.63| 28.43 | 24.34      | 29.61     | 26.58       | 25.82  | 26.28     | 25.21| 26.78|
| DoNeRF [16]             | 32.29| 34.47 | 29.51      | 35.91     | 32.23       | 31.31  | 31.87     | 30.57| 32.47|
| FastNeRF [17]           | 29.78| 31.79 | 27.22      | 33.12     | 29.72       | 28.87  | 29.39     | 28.20| 29.95|
| KioNeRF [18]            | 33.37| 35.63 | 30.50      | 37.11     | 33.30       | 32.36  | 32.93     | 31.60| 33.56|
| QRF                     | **35.44** | 37.84 | 32.39      | **39.41** | **35.37**   | **34.36** | **34.98** | **35.36** | **35.64** |

| Scenes                  | Avg. | Family | Barn | Truck | Caterpillar | Ignatius | - | - | - |
|-------------------------|------|--------|------|-------|-------------|----------|---|---|---|
| SRN [12]                | 24.10| 27.55  | 22.42| 22.65  | 21.15       | 26.71    | - | - | - |
| Neural Volumes [13]     | 23.70| 27.73  | 21.69| 21.39  | 21.57       | 26.14    | - | - | - |
| NeRF [9]                | 25.78| 30.25  | 24.11| 24.58  | 23.66       | 26.30    | - | - | - |
| JaxNeRF [14]            | 27.94| 32.48  | 27.38| 26.65  | 25.21       | 27.96    | - | - | - |
| NSVF [10]               | 28.42| 33.58  | 27.17| 26.95  | 26.46       | **27.92** | - | - | - |
| AutoInt [15]            | 22.28| 26.33  | 21.30| 21.13  | 20.74       | 21.89    | - | - | - |
| DoNeRF [16]             | 27.02| 31.93  | 25.83| 25.62  | 25.16       | 26.54    | - | - | - |
| FastNeRF [17]           | 24.92| 29.44  | 23.82| 23.63  | 23.20       | 24.48    | - | - | - |
| KioNeRF [18]            | 28.41| 33.57  | 27.16| 26.94  | 26.45       | 27.91    | - | - | - |
| QRF                     | 29.65| **35.03** | 28.35| 28.12  | **27.61**   | 29.13    | - | - | - |

Table 1: Quantitative results on each scene from the Synthetic-NeRF [9], Synthetic-NSVF [10], and Tanks and Temples [11]. We highlight the top 3 results in each column are color coded as Top 1, Top 2, and Top 3.
Figure 5: Qualitative comparisons on Synthetic-NeRF [9].
Figure 6: Qualitative Result of QRF on Synthetic-NSVF [10].
Figure 7: Qualitative Result of QRF on Tanks and Temples [11].