We present some recently discovered infinite dimensional Lie algebras that can be understood as extensions of the algebra Map(M, g) of maps from a compact $p$-dimensional manifold $M$ to some finite dimensional Lie algebra $g$. In the first part of the paper, we describe the physical motivations for the study of these algebras. In the second part, we discuss their realization in terms of pseudo-differential operators and comment on their possible representation theory.

§ Talk given at the Gürsey Memorial Conference I on Strings and Symmetries, Istanbul, Turkey, June 6-10 1994.
Why study $\text{Map}(M, g)$ and its extensions?

Let us start with a situation that is familiar to everybody and consider the one dimensional case, $M = S^1$, with $g$ some finite dimensional Lie algebra. There, $\text{Map}(S^1, g)$ has a unique central extension $\widehat{\text{Map}}(S^1, g)$ given by the cocycle

$$c(\lambda, \xi) = \frac{i}{2\pi} \int_0^{2\pi} \text{tr} (\lambda(x)\xi'(x)) dx,$$

which is known as the (untwisted) affine Kac-Moody algebra [Ba-71] [Ka-67] [Ka-85b] [Mo-67] [Mi-89] [Wi-84]. An element of the algebra can be written as a pair $(\lambda; z)$ where $\lambda$ is a map from the circle into $g$ and $z$ is a complex number. The Lie product on these pairs is

$$[(\lambda; z), (\xi; w)] = ((\lambda, \xi); c(\lambda, \xi)).$$

Consider now the normal ordered currents $J_a(x) = (i/2) : \Psi^T(x) M^a \Psi(x) :$ where the $M^a$ are real antisymmetric generators of $g$ normalized as $\text{tr} M^a M^b = -2k \delta^{ab}$ and $\Psi(x)$ are Majorana-Weyl fermionic fields on the light-cone. The currents $J^a$ satisfy the celebrated current algebra

$$[J^a(x), J^b(y)] = i f^{abc} J^c(x) \delta(x - y) + \frac{i k}{2\pi} \delta^{ab} \delta'(x - y),$$

where the second term on the r.h.s. is the Schwinger term arising from normal ordering.

Noticing that the currents in (3) behave as operator valued scalar densities of weight one, we can then take the product $J(\lambda) = \int dx J^a(x) \lambda^a(x)$ to obtain a representation of the affine algebra (2) in the fermionic Fock space:

$$[J(\lambda) + k z, J(\xi) + k w] = J([\lambda, \xi]) + k c(\lambda, \xi).$$

Therefore, the study of representations of the algebra (2) is equivalent to the study of $(1 + 1)$-dimensional current algebras (3). It is then clear that current algebras in higher dimensions must be related to some extensions of the algebra $\text{Map}(M, g)$.

What are the “interesting” extensions of $\text{Map}(M, g)$ when $p > 1$?

When considering higher dimensional manifolds we first encounter the fact that, contrary to the one dimensional case, $\text{Map}(M, g)$ has an infinite number of central extensions [Pr-86]. Roughly speaking, for any loop one can draw on the manifold (even contractable loops) and for any two elements $\lambda, \xi \in \text{Map}(M, g)$, there is a two cocycle analogous to (1) given by evaluating the pull-back of the form $\text{tr} (\lambda d\xi)$ on such loop. These are essentially all the possible central extensions
one can have. However, none of these extensions are interesting in relation to the study of higher dimensional current algebras precisely because of their “one dimensional” nature.

We must therefore look for more general kinds of extensions (abelian or even non abelian) and this always means adding extra fields to the system. Also, if we are aiming at an algebra that does not depend on the detailed geometry of the manifold $M$, (e.g. its metric), it is a natural guess to consider adding differential forms. In order to keep the discussion contained, we will now present the algebra in an axiomatic way and, after that, argue about its relevance to physical systems of interest.

Our proposal of extension of $\text{Map}(M, g)$.

The algebra we are proposing is made of pairs $(X, z)$, where $X$ represents the free sum $X = X^{(0)} + (X^{(2)} + X^{(4)} + \cdots X^{([p])})$, $X^{(0)} \in \text{Map}(M, g)$ and $X^{(2)}, X^{(4)}, \cdots$ are $g$ valued differential forms on $M$ of even degree $2, 4, \cdots$. The symbol $[p]$ denotes the largest even integer less or equal than $p = \text{dim}(M)$ and $z$ is a complex number. In the following, $g$ is taken to be either $\mathfrak{gl}(n)$ or $\mathfrak{u}(n)$.

The Lie product in such an algebra can be written concisely as

$$[[X; z], [Y; w]] = ([X, Y] + [dX, dY]) \frac{i}{(2\pi)^p} \int_M \text{tr} (X dY),$$

where the following conventions have been made:

- a. The wedge product is understood in all the commutators of forms on the r.h.s.
- b. Forms of degree larger that $p$, arising from the wedge products, all vanish.
- c. The integral on the r.h.s. vanishes on forms of degree not equal to $p$. (In particular it always vanishes when $p$ is even.)

We will now argue that algebras of this kind are of interest in higher dimensional physics by studying a few examples in various dimensions.

$p = 1$ In this case the algebra (5) obviously reduces to the affine Kac-Moody algebra (2) presented in the introduction.

$p = 2$ This is the current algebra of planar QCD in the parity conserving phase [Fe-92] [Fu-94] (i.e., with an even number of flavors $N_f$ and a large number of colors $N_c$). Explicitly, in terms of the flavor currents $J^a(x)$ and the Goldstone field $\Phi^a(x)$, it takes the form (up to some rescaling)

$$[J^a(x), J^b(y)] = i f^{abc} J^c(x) \delta(x - y) + \frac{iN_c}{4\pi} d^{abc} e^{\mu\nu} \partial_\mu \Phi^c(x) \partial_\nu \delta(x - y)$$

$$[J^a(x), \Phi^b(y)] = i f^{abc} \Phi^c(x) \delta(x - y)$$

$$[\Phi^a(x), \Phi^b(y)] = 0.$$

\[6\]
This is probably the most interesting case, because, in three dimensions, (5) describes the gauge algebras arising from the canonical formulations of anomalous gauge theories with chiral fermions [Mi-83] [Fa-84a] [Fa-84b] [Mi-85] [Pe-88] [Fl-89] [Jo-85]. These algebras are known in the literature under the name of Mickelsson-Faddeev-Shatsvili (MFS) algebras. In their most explicit form, as linear algebras in the gauge generators $G^a(x)$ and the gauge fields $A^a_\mu(x)$ they can be written as [Fa-84a]:

\[
\begin{align*}
[G^a(x), G^b(y)] &= if^{abc}G^c(x)\delta(x - y) + \frac{1}{12i\pi^2}d^{abc}\epsilon^{\mu\nu\rho}\partial_\mu A^c_\nu(x)\partial_\rho\delta(x - y) \\
[G^a(x), A^b_\mu(y)] &= if^{abc}A^c_\mu(x)\delta(x - y) + \delta^{ab}\partial_\mu\delta(x - y) \\
[A^a_\mu(x), A^b_\nu(y)] &= 0.
\end{align*}
\]

(7)

The gauge field $A^a_\mu$ is associated with the two form $X^{(2)}$ and the scalar density $G^a$ with the scalar $X^{(0)}$. There is also a connection with the BRST algebra of three dimensional extended objects (3-branes) [Di-92].

$p = 4$ This is the lowest dimension in which (5) becomes a non-abelian extension of $\text{Map}(M, g)$ (the wedge product of two 2-forms is a 4-form). More interesting is the case:

$p = 5$ This algebra has been studied in connection with the bosonic sector of the 5-brane [Ce-94], believed to be of interest in the study of non-perturbative effects in string theory [Se-94]. It is an alternative and inequivalent formulation of the problem studied in [Di-93]. It should also be stressed that for odd dimensions higher than three, the algebras obtained from (5) are not equivalent to the higher dimensional versions of the MFS algebras nor can the two algebras be related by contraction. In this particular case, (5) yields:

\[
\begin{align*}
[T^a(x), T^b(y)] &= if^{abc}T^c(x)\delta(x - y) + d^{abc}\partial_i T^{ij}(x)\partial_j\delta(x - y) \\
[T^a(x), T^{b mn}(y)] &= if^{abc}T^{c mn}(x)\delta(x - y) - d^{abc}\epsilon^{ijk mn}\partial_i T_j^c(x)\partial_k\delta(x - y) \\
[T^a(x), T^b_i(y)] &= if^{abc}T^c_i(x)\delta(x - y) + k\delta^{ab}\partial_i\delta(x - y) \\
[T^a kl(x), T^{b mn}(y)] &= if^{abc}\epsilon^{ikl mn}T^c(x)\delta(x - y) + k\delta^{ab}\epsilon^{ikl mn}\partial_i\delta(x - y) \\
[T^{a mn}(x), T^b_i(y)] &= 0 \\
[T^a_i(x), T^b_j(y)] &= 0,
\end{align*}
\]

(8)

in terms of a set of generators $T^a(x)$, $T^{a ij}(x)$, $T^a_i(x)$ and $k$. (See [Ce-94] for details.)

Regularization of the Algebra

\[\text{§}\] In general, these algebras do not arise as Noether symmetries of the $p$-brane action [Pe-93] but they can still be used to construct a BRST operator for the $p$-brane functional.
Having presented the algebra, we will now examine its explicit realization as algebra of operators in some Fock space. For definitiveness, let us focus on the three dimensional case, where one can write the Lie product of $\hat{\text{Map}}(M, g)$ as
\[
[(X^{(0)} + X^{(2)}; z), (Y^{(0)} + Y^{(2)}; w)] = ([X^{(0)}, Y^{(0)}] + [X^{(0)}, Y^{(2)}] + [X^{(2)}, Y^{(0)}] + \frac{i}{(2\pi)^3} \int_M \text{tr} (X^{(0)} dY^{(2)} + X^{(2)} dY^{(0)})).
\] (9)

The simplest and most obvious choice for the Fock space is a fermionic Fock space $\mathcal{F}$ constructed by considering Weyl spinor fields $\Psi(x)$ at equal time $t = 0$. (Contrary to $1 + 1$ dimensions, there are no Majorana-Weyl spinors in $3 + 1$ dimensional space time.) Using these fields one can easily construct the charge densities (i.e. time components of the currents):
\[
J^a(x) = \frac{i}{2} : \Psi^\dagger(x) M^a \Psi(x) : ,
\] (10)

Where $M^a$ are now antihermitian generators of $\mathfrak{g}$. If, in analogy with the one dimensional case, we tried to set $J(X^{(0)}) = \int_M J^a(x) X^{(0)\dagger a}(x) d^3x$, we would immediately run into the following problem: Contrary to the one dimensional case, the operator $J(X^{(0)})$ creates states of infinite norm out of the ordinary vacuum §. A few years ago, Mickelsson and Rajeev proposed to enlarge the Fock space to “make room” for these extra states [Mi-88] [Fu-90] [La-91] [La-94]. Unfortunately, it was later realized by Pickerell that there is a no-go theorem that prevents one from finding unitary representations this way [Pi-87] [Pi-89].

More recently, Mickelsson has proposed a different approach [Mi-93a] [Mi-93b] [Mi-94]. The basic idea is to regularize the argument of $J$ in a way that, on one hand, preserves the algebra and, on the other hand, makes the norms converge. Originally the idea was applied to the MFS algebra with explicit dependence of the regulators on the gauge fields. This creates some problems in the implementation, and one has to content oneself with finding a projective representation. We follow a simpler approach, where all generators are regularized by the action of a linear operator and there is no explicit dependence on the gauge fields.

Let then $A : \mathcal{H} \to \mathcal{H}$ be an arbitrary operator in the first quantized Hilbert space of Weyl spinorial wave functions $\psi \in \mathcal{H}$. (E.g. when $A = X^{(0)}$ we think of it as a multiplicative operator.) Also, let $h$ be the helicity operator, which coincides with $(-1)^x$ the sign of the energy. It is a well known calculation that shows that
\[
||J(A)||_0 ^2 = \text{Tr}_{\mathcal{H}} ([h, A]^\dagger [h, A]).
\] (11)

§ Even in one dimension, there are cases where normal ordering is not enough [Ev-94] but they do not arise in this context.
This calculation follows straightforwardly by writing the l.h.s. in terms of the fermionic oscillators and expanding the normal ordered bilinears. Expressed in more mathematical language, what (11) says is that $J(A)|0>$ has finite norm iff $[h, A]$ is an Hilbert-Schmidt operator, i.e. the trace of its square is finite.

To be more explicit, let $A$ be a pseudo-differential operator (PSDO) [Co-85] [Co-88] [Gu-85] [La-89] [Va-92] defined through its symbol $a(x, p)$, function on the phase space:

$$\text{Sym } (A) = a(x, p) \approx \sum_{k=0}^{\infty} a_{-k}(x, p) \quad \text{where} \quad a_{-k}(x, p) \approx \frac{1}{|p|^k}, \quad a_{-k} = O(-k). \quad (12)$$

(We do not allow for positive powers of $p$ in the asymptotic expansion of the symbol.) The relation between the operator $A$ and its symbol $a$ is given by defining the action of $A$ on $H$ as:

$$A\psi(x) = \frac{1}{(2\pi)^{3/2}} \int e^{ip \cdot x} a(x, p) \tilde{\psi}(p) d^3p, \quad (13)$$

where $\tilde{\psi}$ is the ordinary Fourier transform of $\psi$. For example, the symbol of $X^{(0)}$ as a multiplicative operator is $X^{(0)}$ itself as a function of $x$, whereas the symbol of $h$ is $\text{Sym } (h) \equiv \epsilon = (p_\mu \sigma_\mu)/|p|$, where $\sigma_\mu$ are the ordinary Pauli matrices. The algebra of PSDO’s of this kind is defined through the star product $*$

$$a \ast b = \text{Sym } (AB) = a(x, p) \exp(-i \frac{\epsilon}{p_\mu} \cdot \frac{\partial}{\partial x_\mu}) b(x, p). \quad (14)$$

One can then restate the Hilbert-Schmidt condition in terms of the ultra-violet asymptotic behavior of the symbols and say that $J(A)|0>$ has finite norm iff $[\epsilon, a]_* \approx p^{-2} \equiv O(-2)$ (square integrable in $p \in \mathbb{R}^3$.) We shall denote the space of these operators as $W$.

The multiplicative operator $X^{(0)}$ does not have this property, being the $*$-commutator with $\epsilon$ only of degree $O(-1)$. We must therefore add a counterterm of degree $O(-1)$ and try to cancel the offending term. Define:

$$\theta(X^{(0)}) = X^{(0)} + S_\mu \frac{\partial X^{(0)}}{\partial x_\mu} + O(-2). \quad (15)$$

then, by using the asymptotic behavior of the symbols,

$$[\epsilon, \theta(X^{(0)})]_* = O(-2) \quad \text{iff} \quad [\epsilon, S_\mu \frac{\partial X^{(0)}}{\partial x_\mu}] = i \frac{\partial \epsilon}{\partial p_\mu} \frac{\partial X^{(0)}}{\partial x_\mu}, \quad (16)$$

where the commutator in the r.h.s. of (16) is an ordinary matrix commutator. Equation (16) has a solution that can be written as, up to unimportant terms that commute with $\epsilon$ [Mi-93a] [Mi-94],

$$S_\mu = \frac{\epsilon_{\mu \nu \rho} p_{\nu} \sigma_\rho}{2p^2} \quad (17)$$
where $\epsilon_{\mu\nu\rho}$ denotes the totally antisymmetric tensor. This choice makes $\theta(X^{(0)})$ into an operator with a good second quantized picture (i.e., an element of $\mathcal{W}$) and preserves the original algebra to degree $O(-1)$. At this point one may wonder how far one should go and how to incorporate the term $X^{(2)}$ into the picture. We answer both questions by considering the central term of the algebra.

The twisted Radul cocycle

In the space of PSDO’s there is essentially only one trace we can define. This is given by the so-called Wodzicki residue [Wo-85], a higher dimensional version of the Adler-Manin residue [Ad-79] [Ma-79] that, on a three-dimensional manifold, can be written as

$$\text{Res}\,\langle a \rangle = \frac{1}{(2\pi)^3} \int_{|p| = \Lambda} \text{tr} \, a_{-3}(x,p) \eta \wedge d\eta \wedge d\eta,$$

(18)

where $\eta = p_{\mu} dx^\mu$ is the canonical one-form on the cotangent bundle of $M$. We assume that the symbol has compact support in $x$ to avoid considering global properties. Using this trace, one can write cochains in the space of PSDO’s. In particular, we need a two-cocycle that is a version of the so-called Radul cocycle [Ra-91a] [Ra-91b] twisted by the presence of a factor $\epsilon$ [Mi-94]. The Radul cocycle is, again, a higher dimensional generalization of the one-dimensional Kravchenko-Khesin cocycle [Kr-91] that we define as follows:

$$c_R(a,b) = \frac{3i}{\pi} \text{Res} \, (\epsilon \ast [\log |p|, a]_{\ast} \ast b).$$

(19)

Looking at (18) we see that the trace depends on the terms of degree $O(-3)$ in the asymptotic expansion. However, because of the presence of $\log |p|$ in (19), it is easy to see that the cohomology only depends on terms up to degree $O(-2)$. The result of this analysis is that, in order to have a non-trivial central term, which we expect to be generated at the quantum level by rearranging the normal ordered fermions, one has to go one more step and compute the terms of degree $O(-2)$.

This turns out to be important also for another reason. In order to have the regularized operators satisfying the original algebra (9) to degree $O(-2)$, one must include a two-form realized as a PSDO of degree $O(-2)$; it is simply impossible to obtain a regulator that works to this degree by using the scalars $X^{(0)}$ alone. This is very pleasing because it gives a reason for the forms of higher degree to be included in the algebra. Finally, we would like to point out that this way of proceeding breaks down for space dimensions strictly higher than three (i.e., quantum fields in space-time dimensions strictly higher than $3+1$), indicating some sort of “upper critical dimension” for this regularization procedure. Without giving all the details of the calculations (to be presented...
in a forthcoming paper), we shall present the final answer for the regulated algebra:

$$\theta(X(0) + X(2)) = X(0) + S_\mu \frac{\partial X(0)}{\partial x^\mu} + \frac{1}{2} S_\mu S_\nu \frac{\partial^2 X(0)}{\partial x^\mu \partial x^\nu} + \frac{1}{2} A_{\mu\nu} X(2)_{\mu\nu},$$  \tag{20}$$

where $S_\mu$ was given in (17) and the other two terms can be expressed as functions of $S_\mu$ as

$$S_{\mu\nu} = \frac{1}{2} (S_\mu S_\nu + S_\nu S_\mu) - \frac{i}{2} \left( \frac{\partial S_\mu}{\partial p^\nu} + \frac{\partial S_\nu}{\partial p^\mu} \right)$$

$$A_{\mu\nu} = \frac{1}{2} (S_\mu S_\nu - S_\nu S_\mu) - \frac{i}{2} \left( \frac{\partial S_\mu}{\partial p^\nu} - \frac{\partial S_\nu}{\partial p^\mu} \right).$$  \tag{21}$$

It can then be checked by an explicit calculation that eq. (20) satisfies all the requirements for our regulated algebra. In particular, not only is the $\ast$-commutator of $\theta(X(0) + X(2))$ with $\epsilon$ defining an Hilbert-Schmidt operator, but also the Lie product (9) is preserved to degree $O(-2)$

$$[\theta(X(0) + X(2)), \theta(Y(0) + Y(2))]_\ast = \theta([X(0), Y(0)] + [X(2), Y(0)] + [X(0), Y(2)] + [dX(0), dY(0)]),$$  \tag{22}$$

and the twisted Radul cocycle (19) generates the correct central term after integrating out the momentum variables

$$c_R(\theta(X(0) + X(2)), \theta(Y(0) + Y(2))) = \frac{i}{(2\pi)^3} \int_M \text{tr} \left( X(0) dY(2) + X(2) dY(0) \right).$$  \tag{23}$$

Since the $\ast$-product of two PSDO’s always generates terms of arbitrary low degree, if one wants to interpret $\theta$ as a Lie algebra homomorphism, one has to quotient out an ideal from the space of PSDO’s under consideration. In other words, the homomorphism $\theta$ is defined only on the equivalence classes of operators that differ by an (uninteresting) PSDO of degree $O(-3)$ or less. We denote this homomorphism by

$$\theta : \widehat{\text{Map}}(M, g) \to \widehat{\mathcal{W}} / \mathcal{I}_{-3},$$  \tag{24}$$

where $\widehat{\mathcal{W}}$ is the extension, via the cocycle (19), of the algebra $\mathcal{W}$ defined above and $\mathcal{I}_{-3}$ is the ideal of $\mathcal{W}$ consisting of PSDO’s of degree $O(-3)$ or less. Notice that the quotient $\widehat{\mathcal{W}} / \mathcal{I}_{-3}$ is well defined because the cocycle (19) vanishes if one of its arguments is in the ideal $\mathcal{I}_{-3}$.

**Conclusions and further projects**

In conclusion, we have presented some extensions of the algebra $\text{Map}(M, g)$ that have applications in various fields of physics (current algebras, gauge anomalies, $p$-branes etc...). For the three dimensional case, we have found a way of regularizing these algebras that allows one to have well defined second quantized operators in a fermionic Fock space, and “looks promising” as far as the representation theory is concerned.
Obviously, much work still needs to be done. We would like to emphasize two points that deserve further investigations: The first is the need to gain a better understanding of how the central term can arise from second quantization. We know that the cocycle (19) must somehow be related to the universal cocycle of $\mathfrak{gl}_1$ [Ka-85a] [Pr-86] [Lu-76] but we would like to have an explicit derivation in more physical terms. The second point is to better understand the representation theory of these algebras [Ki-76], in particular, to understand how, and if, it is possible to quotient the representations of $\hat{\mathcal{W}}$ by its ideal $\mathcal{I}_{-3}$ and how to use these results in the study of some specific quantum field theories.

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