Photoabsorption sum rules and quark structure parameters of hadrons

S.B. Gerasimov†

Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research, 141980 Dubna (Moscow region), Russia
† E-mail: gerash@thsun1.jinr.ru

Abstract
Following the idea of the quark-hadron duality we present, within the constituent quark model approach, the relations between different bremsstrahlung-weighted integrals of the nucleon resonance photoexcitation cross sections and correlation functions of the quark dipole moments in the nucleon ground state. These functions are of interest for checking detailed quark-configuration structure of the nucleon state vector. Some applications of this approach in meson sector are made and the role of meson degrees of freedom in the electromagnetic baryon observables is briefly discussed.

1 Introduction

In this paper, we make use the specific form of notion of ”quark-hadron duality” suggested by the correspondence principle with sum rule technique in nonrelativistic theory of interaction of radiation with matter and continue earlier started discussion[1, 2] about utility of a number of sum rules that cannot be derived via the dispersion-theoretic approach and require more specific assumptions on validity of asymptotic behaviour of the scattering amplitudes under consideration.

Sum rules equating integrals over photoabsorption cross-sections to static parameters of the ground state have long be in the use in nonrelativistic atomic and nuclear physics (e.g.[3]). In relativistic domain, one should meet problem of the convergence of corresponding integrals and the necessity to treat all radiative transitions relativistically, i.e. to include into consideration all higher multipoles in contrast with atomic and nuclear photoeffect where one can confine oneself by inclusion the electric dipole amplitudes in the reasonably large energy interval.

Here, our main idea is to merge the duality concept of the Regge-resonance type and the concept of the parton-hadron duality within the definite class of sum rules for the photoabsorption cross sections. These sum rules, at the same time, will qualitatively demonstrate justifiable common base with the known and much more simple sum rules in the nonrelativistic domain.

2 From Regge-duality to quark-hadron duality: The case of bremsstrahlung-weighted sum rules

In this section, we propose and discuss sum rules giving a connection between the valence quark part of the Dirac (charge) radii of nucleons (mesons) \(< r^2 >\) and the photoexcitation
cross sections of the nucleon (or, correspondingly, meson) resonances. We assume that both the ground and excited states of hadrons are the bound states of three (or two) constituent quarks and, tentatively, that the electroweak coupling constants of "constituent" and "current" quarks are the same (i.e. obtained via the minimal coupling principle).

It follows then, that the usual current algebra relations are valid for the currents constructed of the constituent quark field operators and, at the same time, one can to replace the sum over complete set of the hadron states in the dispersion integrals by the sum over all resonance states constructed, for example, of three constituent quarks and having the needed quantum numbers.

The first problem to deal with is the convergence of integrals, containing the moments of considered cross-sections. We rely here on the known idea of semi-local duality between the description of a given amplitude behaviour at high energies via the leading non-vacuum Regge trajectory exchanges in the t-channel and the sum of the s-channel resonances[4]. The vacuum Pomeron exchanges should then be associated with the non-resonance s-channel amplitudes[5].

Accepting that kind of duality between the resonance photoexcitation cross-sections and contributions of the positive charge-parity Regge exchanges, the f- and a2-exchange trajectories, we conclude that the bremsstrahlung-weighted integrals of the total photoproduction cross-section should be convergent ones.

Now, we consider first the nucleon case. Our basic idea consists in relating the electric dipole moment correlator and the mean-squared radii operators sandwiched by the nucleon state vectors in the "infinite - momentum frame"

\[
2 < \hat{D}^2 > _N - < \hat{D}^2 > _P + 8 < \hat{D}_S^2 > _{P(N)} = 2 < \hat{r}_1^2 > _N + < \hat{r}_1^2 > _P
\]  

(1)

where

\[
\hat{D} = \int \vec{x} \hat{\rho} \vec{x} d^3x = \sum_{j=1}^{3} Q_q(j) \bar{\xi}_j
\]  

(2)

\[
\hat{r}_1^2 = \int \vec{x}^2 \hat{\rho} \vec{x} d^3x = \sum_{j=1}^{3} Q_q(j) \bar{\xi}_j^2
\]  

(3)

\(Q_q(j)\) and \(\bar{\xi}_j\) are the electric charges and configuration variables of presumably point-like constituent quarks in the infinite-momentum frame while the electric charge density operator \(\hat{\rho} = \hat{\rho}^S + \hat{\rho}^V\) is a sum of the isoscalar and isovector parts. The relation (13) comes from the following parametrization of the matrix elements

\[
< \hat{r}_1^2 > _P = \frac{4}{9} \alpha - \frac{1}{3} \beta
\]  

(4)

\[
< \hat{r}_1^2 > _N = -\frac{2}{3} \alpha + \frac{2}{3} \beta
\]  

(5)

\[
< \hat{D}^2 > _P = \frac{8}{9} \alpha + \frac{1}{9} \beta + \frac{8}{9} \gamma - \frac{8}{9} \delta
\]  

(6)

\[
< \hat{D}^2 > _N = \frac{2}{9} \alpha + \frac{4}{9} \beta + \frac{2}{9} \gamma - \frac{8}{9} \delta
\]  

(7)

\[
< \hat{D}_S^2 > _{P,N} = \frac{1}{36} (2\alpha + \beta + 2\gamma + 4\delta)
\]  

(8)
where \( \langle \xi_1^2 \rangle = \alpha, \langle \xi_3^2 \rangle = \beta, \langle \xi_1 \cdot \xi_2 \rangle = \gamma, \langle \xi_1 \cdot \xi_3 \rangle = \langle \xi_2 \cdot \xi_2 \rangle = \delta \) the indices "1" and "2" refer to the like quarks (i.e. to the u(d)-quarks inside proton (neutron)), and the "3" - to the odd quark. The matrix elements over the proton and neutron wave function are only assumed to obey to the charge symmetry relations.

The subsequent procedure is a standard technique of sum rule derivation within the framework of the dipole moment algebra at the "\( p_z \to \infty \)" frame[6, 7].

However, we attribute a new meaning to all appearing quantities, namely, all the cross sections are understood as the nucleon resonance excitation cross sections, all radii \( \langle r_1^2 \rangle_{p,n} \) as the constituent quark distribution radii \( \langle r_1^2 \rangle_{b,p,n} \), not including the sea quark and/or meson current effects. Further, we will replace the intermediate one-nucleon contributions proportional to the anomalous magnetic moments of nucleons, by the corresponding integrals entering in the anomalous magnetic sum rules. One can then to get

\[
\frac{4}{3} \pi^2 \alpha \langle \hat{D}^2 \rangle_{P(N)} = J_p^{P(N)} = J_p^{P(N)}(\frac{1}{2}) + J_p^{P(N)}(\frac{3}{2}) \tag{10}
\]

\[
\frac{4}{3} \pi^2 \alpha \langle \hat{r}_1^2 \rangle_{N} = J_a^P(\frac{3}{2}) - J_p^P(\frac{1}{2}) + 4J_p^S(\frac{1}{2}) \tag{11}
\]

where

\[
J^{V(S)}_{p,a}(I) = \int_{\omega_{thr}}^{\infty} \frac{d\omega}{\omega} \sigma_{p,a}(\gamma^{V(S)} N \to N^*(I)) \tag{12}
\]

the \( \sigma_{p,a} \) refers to the interaction cross section of the polarized "isovector" ("isoscalar") \( (\gamma^{V,S}) \) photons and polarized nucleons with parallel (or antiparallel) spins.

The relativistic dipole-moment fluctuation sum rule, Eq.(10), has been checked[2] in a number of the field-theoretical models, while the limiting case of Eq.(11) with the assumption of fully symmetrical quark distributions in nucleons, i.e. when \( \alpha = \beta \) and \( \gamma = \delta \), giving \( \langle r_1^2 \rangle_{b,n} = 0 \), has been discussed in[4].

The calculations have shown that the Dirac charge radius of the neutron is indeed a small quantity, especially if the model amplitudes are used taking into account the relativistic corrections and the effects of the mixing of the basis \( SU(6) \otimes O(3) \)-configurations in the ground and excited wave functions[8]. However, if the use is made of the phenomenological amplitudes (especially enhanced, as compared with the quark model calculations, of the \( \Delta(1232) \) excitation amplitude), the right-hand side of Eq.(11) acquire larger positive value

\[
\frac{4}{3} \pi^2 \alpha \langle \hat{r}_1^2 \rangle_{N} \approx 0; \ (34 \mu b) \tag{13}
\]

where the first value corresponds to the model[8] and the second one - to phenomenological \( \gamma N \to N^*(I = 1/2, 3/2) \)-transition amplitudes, respectively[10].

The most precise determination of the neutron charge radius follows from the neutron-electron scattering lengths measured in the thermal neutron scattering off the inert gases (Ne,Ar,Kr,Xe), or metal (W,Pb,Bi) atoms. The dominant contribution to the (Sachs) charge radius of the neutron

\[
\langle \hat{r}_{eh}^2 \rangle_{N} = \langle \hat{r}_1^2 \rangle_{N} + \frac{3\kappa_N}{2m_N} \tag{14}
\]
is due to the second (Foldy) term, proportional to $\kappa_N = -1.913$ n.m. The experimental uncertainties ascribed to a measured values of $< r_{ch}^2 >_N$ enable to extract a very small contribution of the Dirac (charge) radius $< r_{ch}^2 >_N$ from Eq.(14).

The sum rules for mesons have been written down as well

\[ \frac{4}{3} \pi^2 \alpha < r^2 > = J_{tot}(\gamma\pi^+) + \frac{4}{5} J_{tot}(\gamma\pi^o), \]

\[ \frac{4}{3} \pi^2 \alpha < r^2 > = J_{tot}(\gammaK^+) + 2J_{tot}(\gammaK^o). \]

and have been checked analytically in the lowest perturbation order in the meson-quark coupling constant. Their phenomenological consequences have been discussed in more detail in [9].

In what follows we rather turn to sum rules for meson resonances in photon-photon collisions. Their evident advantage is in that the partonic "wave function" of the photon follows from the lowest order perturbation calculation in the fine coupling constant and standard local coupling in the photon-charged-parton vertices. Furthermore, it can be shown that choosing different polarizations of colliding photons, one can obtain the linear combination of certain $\gamma\gamma \rightarrow q\bar{q}$ cross-sections that will dominantly collect the $q\bar{q}$-states with definite spin-parity, hence, by the adopted quark-hadron duality, the meson resonances with the same quantum numbers. In particular, the combinations of the integrals over the bremsstrahlung-weighted and polarized $\gamma\gamma \rightarrow q\bar{q}$ cross-sections, $I_{\perp} - (1/2)I_{\parallel}$, will be related with low mass meson resonances having spatial quantum numbers ($J^{PC} = 0^{-+}, 0^{++}, 2^{++}$), correspondingly. The $\gamma\gamma$-cross-sections $\sigma_{\perp(\parallel)}^{\gamma\gamma}$ (and the integrals thereof) refer to plane-polarized photons with the perpendicular (parallel) polarizations, and the $\sigma_{p}^{\gamma\gamma}$ corresponds to circularly polarized photons with parallel spins.

Evaluating cross-sections and elementary integrals for the experimentally best known case of pseudoscalar mesons we get the sum rule for radiative widths of the $\pi^o, \eta$ and $\eta'$ mesons

\[ \frac{1}{16\pi^2} \int_{thr}^{\infty} \frac{ds}{s} (\sigma_{\perp}^{\gamma\gamma}(s) - \frac{1}{2} \sigma_{p}^{\gamma\gamma}(s)) = \frac{\alpha^2}{24\pi^2 m_q^2} = \sum_{i=\pi,\eta,\eta'} \frac{\Gamma(i \rightarrow \gamma\gamma)}{m_i^3} \]

\[ (12.3 \cdot 10^{-6}\text{GeV}^{-2}) \simeq (10.9 \cdot 10^{-6}\text{GeV}^{-2}) \]

The value $m_q \approx 240$ MeV used in this calculation have been taken from [2] (in Eq.(17), we have taken approximately $m_{u,d} \approx m_s$ which is within accuracy of our calculation). Close numerical values of the integral and sum of known radiative widths [11] leave little place for contribution of higher mass pseudoscalar radial excitations and give good evidence for the relevance of quark-hadron duality in the considered context.

3 The pionic "dressing" of hadrons: examples and problems

In this section we discuss briefly the role of nonvalence degrees of freedom (the nucleon sea partons and/or peripheral meson currents) in parametrization and description of
hadron magnetic moments, both diagonal and non-diagonal, including the $N\Delta\gamma$-transition moment. Earlier we have considered\[12\] a number of consequences of sum rules for the static, electroweak characteristics of baryons following from the theory of broken internal symmetries and common features of the quark models including relativistic effects and corrections due to nonvalence degrees of freedom – the sea partons and/or the meson clouds at the periphery of baryons.

Now, we list some consequences of the obtained sum rules. The numerical relevance of adopted parametrization is seen from results enabling even to estimate from one of obtained sum rules, namely,

$$
(S^+ - S^-)(S^+ + S^- - 6\Lambda + 2\Xi^0 + 2\Xi^-) 
- (\Xi^0 - \Xi^-)(S^+ + S^- + 6\Lambda - 4\Xi^0 - 4\Xi^-) = 0.
$$

The necessary effect of the isospin-violating $\Sigma^0\Lambda$-mixing. By definition, the $\Lambda$-value entering into Eq.(18) should be ”refined” from the electromagnetic $\Lambda\Sigma^0$–mixing affecting $\mu(\Lambda)_{exp}$. Hence, the numerical value of $\Lambda$, extracted from Eq.(18), can be used to determine the $\Lambda\Sigma^0$–mixing angle through the relation

$$
\sin \theta_{\Lambda\Sigma} \simeq \frac{\Lambda - \Lambda_{exp}}{2 \mu(\Lambda\Sigma)} = (1.43 \pm 0.31) 10^{-2}
$$

in accord with the independent estimate of $\theta_{\Lambda\Sigma}$ from the electromagnetic mass-splitting sum rule \[13\]. Owing to interaction of the $u$– and $d$– quarks with charged pions the ”magnetic anomaly” is developing, i.e. $u/d = -1.80 \pm 0.02 \neq Q_u/Q_d = -2$. Evaluation of the lowest order quark–pion loop diagrams gives \[12\]: $u/d = (Q_u + \kappa_u)/(Q_d + \kappa_d) = -1.77$, where $\kappa_q$ is the quark anomalous magnetic moment in natural units.

Of course, this approach is free of a problem raised by Lipkin\[14\] and concerning the ratio $R_{\Sigma/\Lambda}$ of magnetic moments of $\Sigma$- and $\Lambda$- hyperons. With the parameters $u/d = -1.80$ and $\alpha_D = (D/(F+D))_{mag} = 0.58$, defined without including in fit the $\Lambda$-hyperon magnetic moment, we obtain

$$
R_{\Sigma/\Lambda} = \frac{\Sigma^+ + 2\Sigma^-}{\Lambda} = -.27 \quad (vs \quad - .23 \quad [10])
$$

while in the standard nonrelativistic quark model without inclusion of non-valence d.o.f. this ratio would equals $-1$.

The meson–baryon universality of the quark characteristics, suggested long ago \[15\], is confirmed by the calculation of the ratio of $K^*$ radiative widths

$$
\frac{\Gamma(K^{*+} \to K^+\gamma)}{\Gamma(K^{*+} \to K^+\gamma)} = \left(\frac{u/d + s/d}{1 + s/d}\right)^2 = 0.42 \pm 0.03 \quad (vs \quad 0.44 \pm 0.06[10]).
$$

The experimentally interesting quantities $\mu(\Delta^+P) = \mu(\Delta^0N)$ and $\mu(\Sigma^{*0}\Lambda)$ are affected by the exchange current contributions and for their estimation we need additional assumptions. We use the analogy with the one–pion–exchange current, well–known in nuclear physics, to assume for the exchange magnetic moment operator

$$
\hat{\mu}_{exch} = \sum_{i<j} [\vec{\sigma}_i \times \vec{\sigma}_j]_3 [\vec{\tau}_i \times \vec{\tau}_j]_3 f(r_{ij}),
$$
where \( f(r_{ij}) \) is an unspecified function of the interquark distances, \( \vec{\sigma}_i(\vec{\tau}_i) \) are spin (isospin) operators of quarks. Calculating the matrix elements of \( \hat{\mu}_{\text{exch}} \) between the baryon wave functions, belonging to the 56-plet of \( SU(6) \), one can find

\[
C(P) = \frac{1}{\sqrt{2}} C(\Delta^+ P) = \sqrt{3} C(\Lambda \Sigma), \tag{23}
\]

\[
\mu(\Delta^+ P)_{56} = \frac{1}{\sqrt{2}} \left( P - N + \frac{1}{3} (P + N) \frac{1 - u/d}{1 + u/d} \right). \tag{24}
\]

where Eq.(24) may serve as a generalization of the well-known \( SU(6) \)-relation [16].

We list below the limiting relations following from the neglect of the meson degrees of freedom

\[
\Sigma^+ [\Sigma^-] = P[-P - N] + (\Lambda - \frac{N}{2})(1 + \frac{2N}{P}), \tag{25}
\]

\[
\Xi^0 [\Xi^-] = N[-P - N] + 2(\Lambda - \frac{N}{2})(1 + \frac{N}{2P}), \tag{26}
\]

\[
\mu(\Lambda \Sigma) = -\frac{\sqrt{3}}{2} N. \tag{27}
\]

The numerical values of magnetic moments following from this assumption coincide almost identically with the results of the \( SU(6) \)-based NRQM taking account of the \( SU(3) \) breaking due to the quark–mass differences [15]. We stress, however, that no NR assumption or explicit \( SU(6) \)-wave function are used this time. The ratio \( \alpha_D = D/(F + D) = .61 \) in this case and it is definitely less than \( \alpha_D = .58 \), when non-valence degrees of freedom are included [12]. This is demonstrating a substantial influence of the nonvalence degrees of freedom on this important parameter.

### 4 Concluding remarks

The empirical spectrum of mesons and baryons suggests that hadrons are largely composed of the spin-1/2 constituent quarks confined to \( q\bar{q} \) and \( qqq \) systems. Naturally, one needs to understand these relevant degrees of freedom, their effective structure parameters and forces acting between them to reach the understanding of QCD in the confining regime. The constituent quark model is a useful descriptive tool to systematize the phenomenology of the resonance physics. The photon-hadron inelastic reactions in the resonance region have an additional attractiveness in that many phenomenologically successful ingredients of the description of static electromagnetic properties of hadrons, forming one of cornerstones of the constituent quark model itself, can be invoked and included in these processes. Despite many successes the fundamental question of the connection between the three (for baryons) ( or the \( \bar{q}q \)-pair, for mesons) ”spectroscopic” quarks, bearing the quantum numbers of a given hadron, and the infinite number of the ”current” (or fundamental) quarks required, e.g., by the deep-inelastic lepton-hadron scattering, is still a problem. One can hope that the soon available precise experimental data on the resonance photoexcitation amplitudes will provide desirable constraints on the ground state quark-configuration structure and the effective coupling constants of corresponding constituent quarks. Tentatively, one can conclude that, for mesons, the abovementioned application of the developed approach to derivation of the \( \gamma\gamma \)- sum rule looks very encouraging. As
far as it is the most clean test of our approach, its extension for radiative decays of higher spin meson resonances appears very desirable and interesting.

For the nucleon and nucleon resonances, the situation looks intriguing in that the $SU(6)$-value of the $\Delta N$-transition magnetic moment (remaining also in the $N_c \to \infty$-approach) turned out intact to the included pionic and relativistic corrections and, probably, may indicate to the presence of significant "live" meson component in the Fock-state vector of this resonance. In this respect, the ongoing and forthcoming detailed investigation of higher nucleon resonance may also bring interesting surprises of the same sort. The pertinent generalization of sum rules taking into account these additional degrees of freedom is still to be done.

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