Nonlinear conductivity of diffusive normal-metal contacts

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Metal microbridges with a high impurity content and shorter than the energy relaxation length are considered. Their conductance is calculated with allowance made for the Coulomb electron-electron interaction. It is shown that nonequilibrium electrons in the microbridges give rise to a nonlinear current-voltage characteristic.

It is common knowledge that tunnel junctions between disordered metals show nonlinear conductance at low temperatures (the so-called zero-bias anomalies), which results from the electron-electron interaction. On the other hand, short microbridges possess many properties of tunnel junctions. Therefore, it is interesting to investigate their nonlinear properties at low temperatures.

Nonlinear conductivity of weakly disordered metals at low temperatures was investigated both experimentally and theoretically in a number of papers. In these papers, the nonlinear behavior resulted from the heating of the electron gas by the current and from the temperature-dependent correction to the conductivity arising either from weak localization or electron-electron interaction effects. The resulting current-voltage characteristics were influenced, however, by several parameters describing the heat transfer from the electron gas in a massive sample, e.g., by the electron-phonon coupling constant and the acoustic transparency of the sample boundary. Therefore it was difficult to compare the theoretical results with theoretical predictions. For metal microbridges, the situation is different because the heat transfer is determined by the diffusion of hot electrons into the banks and no additional parameters are required.

In this paper we investigate the influence of Coulomb exchange electron-electron interactions on the current-voltage characteristics of disordered-metal microbridges shorter than the energy relaxation length. The characteristic energy scale in this case is the largest of the quantities \(eV\) and \(kT\), where \(V\) is the bias voltage and \(T\) is the temperature. Therefore the electron distribution function is essentially nonequilibrium at low temperatures \(kT \ll eV\), and one may expect a nonlinear current-voltage dependence.

Consider a metal microbridge connecting two massive banks with the \(x\) axis directed along the contact. The microbridge length \(L\) is assumed to be larger than the electron elastic mean free path and the microbridge width. Yet, \(L\) is assumed to be shorter than the energy relaxation length.

In studying the current-voltage characteristics of the contact under nonequilibrium conditions, the Keldysh diagrammatic technique may be conveniently used. In this technique, three types of Green functions are used,

\[
G^R(1, 2) = i\theta(t_2 - t_1)(\psi(1)\psi^+(2) + \psi^+(2)\psi(1)),
\]

\[
G^A(1, 2) = -i\theta(t_1 - t_2)(\psi(1)\psi^+(2) + \psi^+(2)\psi(1)),
\]

\[
G^K(1, 2) = -i(\psi(1)\psi^+(2) - \psi^+(2)\psi(1)).
\]

In the absence of the electron-electron interaction, the impurity-averaged retarded and advanced Green functions are given by the well-known expressions

\[
G^{RA}(\varepsilon, \mathbf{r}, b') = \int \frac{d^3p}{(2\pi)^3} \frac{\exp[ip(\mathbf{r} - b')]}{\varepsilon - p^2/2m \pm i/2\tau},
\]

where \(\tau\) is the elastic relaxation time. The Green function \(G^K\) obeys the integral equation

\[
G^K(\varepsilon, \mathbf{r}, \mathbf{r}') = \frac{1}{2\pi N\tau} \int d^3r_1 G^R(\varepsilon, \mathbf{r}, \mathbf{r}_1)
\times G^K(\varepsilon, \mathbf{r}_1, \mathbf{r}_1)G^A(\varepsilon, \mathbf{r}_1, \mathbf{r}'),
\]

where \(N\) is the electron density of states at the Fermi level. By setting \(\mathbf{r} = \mathbf{r}'\) in (2), one may obtain a diffusion equation for the Green function \(G^K(\varepsilon, \mathbf{r}, \mathbf{r}')\) (see Ref.1)

\[
D \nabla^2 G^K(\varepsilon, \mathbf{r}, \mathbf{r}) = 0, \quad D = \frac{1}{3}e^2/\tau.
\]

In the case of massive banks, the electron distribution at the ends of the microbridge is equilibrium and therefore the boundary conditions for Eq. (3) are

\[
G^K(\varepsilon, \mathbf{r}, \mathbf{r})|_{x=-L/2} = -2\pi N[1 - 2n(\varepsilon - eV/2)],
\]

\[
G^K(\varepsilon, \mathbf{r}, \mathbf{r})|_{x=L/2} = -2\pi N[1 - 2n(\varepsilon + eV/2)],
\]

where \(n(\varepsilon)\) is the Fermi distribution function and \(V\) is the voltage drop across the contact. The current flowing through the contact may be expressed in terms of the Green function \(G^K\) via the formula

\[
j_x = -\frac{e}{2m} \int d\varepsilon \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) G^K(\varepsilon, \mathbf{r}, \mathbf{r}')|_{\mathbf{r} = \mathbf{r}'}. \quad (5)
\]
The corrections to the current from the electron-electron interaction are represented by the three diagrams shown in Fig. 1 and their complex conjugates. In these diagrams, solid lines denote the advanced and retarded electron Green functions, the black circles denote the Green functions $G^K(r, r)$. The single dashed lines represent the impurity-potential correlator $(2\pi N\tau)^{-1}\delta(r-r')$. The shaded rectangles denote the impurity-averaged two-particle Green functions

\[ P(\omega, r, r') = \langle G^A(\varepsilon + \omega, r, r')G^R(\varepsilon, r', r) \rangle_{\text{imp}}, \]

which obey the equation

\[ (i\omega + D\nabla^2)P(\omega, r, r') = -2\pi N\delta(r-r'), \]

\[ P(\omega, r, r')|_{x=\pm L/2} = 0. \]

The shaded semicircles denote the impurity-renormalized electron vertex $(2\pi N\tau)^{-1}P(\omega, r, r')$. The electron-electron interaction is represented by the wavy lines. Taking into account the Debye screening, one obtains the following equation for the retarded potential of the electron-electron interaction,

\[ D\nabla^2 V^R(\omega, r, r') = N^{-1}(-i\omega + D\nabla^2)\delta(r-r'), \]

\[ V^R(\omega, r, r')|_{x=\pm L/2} = 0. \]

The advanced potential $V^A$ is the complex conjugate of $V^R$. These diagrams result in a contribution to the current density in the form

\[ j_1(r) = 2(2\pi)^{-5}eDN^{-3} \]

\[ \times \text{Im} \int d\varepsilon \int d\omega \int d^3r_1 \int d^3r_2 G^K(\varepsilon, r, r) \]

\[ \times P(-\omega, r, r_1)G^K(\varepsilon - \omega, r_1, r_1)\nabla_r P(-\omega, r_2, r). \]

To take into account the current-conservation law, this current density should be averaged over the contact length.

Substitute the solutions of (3), (6), and (8) into (4). In the low-temperature limit $kT \ll eV$, the Fermi distribution function is step-like and the integration over $\varepsilon$ may be easily performed. The integration with respect to $\omega$ in (4) may be conveniently replaced by the integration with respect to a dimensionless quantity $y$, which is related to $\omega$ by $y^2 = (L^2/2D)\omega$. Therefore, the correction to the current flowing through the contact may be presented in the form

\[ I_1 = -\frac{16eD}{\pi L^2} \left( s^2 \int_0^\infty dy Q(y) + \int_0^s dy Q(y) \right), \]

where $s = L(eV/2hD)^{1/2}$ and

\[ Q(y) = \{ (1/4)y[\sinh(2y) - \sin(2y)] - y^{-1}[\sinh(2y) + \sin(2y)] + 2y^{-1}[\sinh(y)\cos(y) + \sin(y)\cosh(y)] \}

\times [\cosh(2y) - \cos(2y)]^{-1}. \]

The corresponding differential resistivity versus voltage curve is shown in Fig. 2. At high voltages $eV \gg hD/L^2$, $I_1 \approx -(8/\pi)(DeV/2hL^2)^{1/2}$. Therefore, the voltage dependence of resistivity in this voltage range has the shape characteristic of the temperature dependence of resistivity in one-dimensional conductors. This is quite natural, because the width of the drop in the electron distribution function is now determined by the applied voltage and not by the temperature. Therefore, the characteristic momentum transfer in (3) is of the order of $(eV/h)^{1/2}$. In the opposite case of small voltages $eV \ll hD/L^2$, the momentum transfer is limited by $h/L$. Therefore, the conductivity tends to a constant value, i.e., $I_1 = -0.17(e^2/h)V$. Hence, increasing voltage gives rise to a cross-over from a zero-dimensional case to a quasi one-dimensional case.
Note that the effect of the electric field is not reduced to a simple heating of the electron gas to some effective temperature. Instead, the electron distribution function consists of two subsequent steps positioned at $\varepsilon_F \pm eV/2$ (see the inset in Fig. 2) and therefore has an essentially non-Fermian shape. In principle, the nonequilibrium electron distribution may also affect the phase-breaking time and therefore the weak localization contribution to the conductance. However, weak localization corrections may be suppressed by a sufficiently strong magnetic field, and the universal conduction fluctuations may be eliminated by averaging the results over a number of samples.

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