Constraining deceleration, jerk and transition redshift using cosmic chronometers, Type Ia supernovae and ISW effect.

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Abstract

In this study we present constraints on the deceleration (q) and jerk (j) parameters using the late time integrated Sachs-Wolfe effect, type Ia supernovae, and H(z) data. We first directly measure the deceleration and jerk parameters using the cosmic chronometers data with the Taylor series expression of H(z). However, due to the unusual variations in the deceleration parameter with slight changes in other parameters like snap (s) and lerk (l), we found that direct measurements using the series expansion of the H(z) is not a suitable method for non-Lambda-CDM models and so we will need to derive the deceleration parameter after constraining density parameters and dark energy equation of state parameters. Then we present derived values of the deceleration parameter from Lambda CDM, WCDM and CPL models. We also discuss the transition redshift (zt) in relation with the deceleration parameter. Our best fit values for the deceleration parameter, after combining results from H(z), Union 2.1 and NVSS-ISW are obtained as -0.5808±0.025 for Lambda CDM, -0.61±0.15 for both WCDM and CPL model. Our best fit for the combined jerk parameter for Lambda-CDM model is 1±3.97e-07, for WCDM model is 1.054±0.141 and for CPL model is 1.0654±0.1345. Also, the combined transition redshift is obtained as 0.724±0.047 for Lambda-CDM model.

1 Introduction

Accelerated expansion of the universe and related concept of dark energy are among the most important concepts of modern cosmology. Cosmological studies before the discovery of the accelerated expansion of our universe were mainly focused at constraining the expansion rate of our universe via Hubble Constant and the

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deceleration parameter \( q \) [1] as then the matter dominated universe was assumed to be expanding with a decelerating rate. In this study we are constraining the deceleration parameter to see how this evolution evolves over the expansion history of universe. In an expanding universe, the deceleration parameter gives negative values which is observed by various observational results. While going back in time we can observe the transition of our universe from current accelerating phase to the deceleration phase in past at some redshift point known as transition redshift. We will also measure transition redshift by deriving it using matter and dark energy density parameters in \( \Lambda \)-CDM model and then compare it with transition redshift observed via the deceleration parameter under \( \Lambda \)-CDM, WCDM and CPL model assumptions.

In an isotropic and homogeneous universe, we can start by defining the distance element as [1] [2] [3]:

\[
\text{d}s^2 = -\text{d}t^2 + a(t)^2 \left( \frac{\text{d}r^2}{1-kr^2} + r^2 d\Omega^2 \right) \tag{1}
\]

Here \( a(t) \) is the scaling factor and \( k \) represent the spatial curvature. The ratio of the rate of change of the scale factor to the current value of scale factor is represented by the Hubble parameter:

\[
H(t) = \frac{\text{d}a}{\text{d}t} \tag{2}
\]

The deceleration parameter in relation with the Hubble parameter can be written as [4] [1] [5]:

\[
q(t) = -1 - \frac{H'}{H} \tag{3}
\]

Here \( H' \) is the time derivative of the Hubble parameter. We can write the deceleration parameter as a function of redshift in a flat universe as:

\[
q(z) = \Omega_r(z) + \frac{\Omega_m(z)}{2} + \Omega_\Lambda(z) \frac{1+w(z)}{2} \tag{4}
\]

Here \( \Omega_r \) is radiation density parameter, \( \Omega_m \) is the matter density parameter, \( \Omega_\Lambda \) is the dark energy density parameter and \( w(z) \) is the dark energy equation of state parameter which in \( \Lambda \)-CDM model is taken as a constant value of \( w=-1 \).

2 Measuring the deceleration parameter using \( H(z) \), ISW effect and type Ia supernovae data

2.1 Cosmic Chronometers

In order to derive the results using equation [4] we will first fit energy densities and dark energy equation of state (EoS) parameters using some available datasets. For this purpose we use direct \( H(z) \) measurement data which uses the cosmic chronometers approach to study the expansion rate of the Universe as a function of redshift. As discussed in Moresco et al. 2018 [7] [6], the expansion history of the Universe via the Hubble parameter as a function of redshift \( (z) \) can be constrained by
measuring the differential age evolution of cosmic chronometers without assuming any particular cosmology. This cosmology independent way of measurement of \( H(z) \) evolution itself provides an interesting opportunity to test various models of cosmology quickly and more directly than some other methods. Theoretically, for flat Λ-CDM model and extensions \( H(z) \) can be written as:

\[
H(z) = H_0 \sqrt{\Omega_{\Lambda} I(z) + \Omega_r (1 + z)^4 + \Omega_m (1 + z)^3}
\]

(5)

Where, \( I(z) \) is defined as:

\[
I(z) = \exp \left( 3 \int_0^z \left( \frac{1 + w_{de}(z')}{1 + z'} \right) dz' \right)
\]

(6)

\( I(z) \) depends on the parametrization of the dark energy equation of state (EoS) and for standard Λ-CDM model with EoS as \( w_{de}(z) = -1 \) (constant), the multiplier \( I(z) \) becomes ‘1’.

Using the cosmic chronometer data, we can also measure the current deceleration parameter value at \( z=0 \) \( (q_0) \) by using the Taylor expansion for \( H(y)=H_0 E(y) \).

For this we can define a value \( y \) as [8] [9]:

\[
y = \frac{z}{z + 1}
\]

Now, we can define \( E(y) \) as:

\[
E(y) = 1 + k_1 y + (k_2 y^2)/2 + (k_3 y^3)/6 + (k_4 y^4)/24
\]

(7)

With,

\[
\begin{align*}
k_1 &= 1 + q_0 \\
k_2 &= 2 - (q_0^2) + 2q_0 + j_0 \\
k_3 &= 6 + 3(q_0^3) - 3(q_0^2) + 6q_0 + 4q_0j_0 + 3j_0 - s_0 \\
k_4 &= -15(q_0^4) + 12(q_0^3) + 25(q_0^2)j_0 + 7q_0s_0 + 4j_0 + 12(q_0^2) + l_0 - 4s_0 + 12j_0 + 24q_0 + 24
\end{align*}
\]

For Λ-CDM case, we use \( j_0 = 1, s_0 = 0 \) and \( l_0 = 0 \) [11]. For non-Λ CDM cases, we use three configurations. In the first we one we fix \( s_0 = 0 \) and \( l_0 \) as zero and keep \( H_0, q_0 \) and \( j_0 \) free. In the second non-Λ CDM case, we fix \( H_0 \) to Λ-CDM result, and \( s_0 = 0 \) and \( j_0 = 0 \). In the third case, we only fix \( H_0 \) and use other parameters as free. Table 1 presents the results for different models. For our study we use boundary conditions \( 65 \leq H_0 \leq 75, -5 \leq q_0 \leq 5, -10 \leq j_0 \leq 10, -500 \leq s_0 \leq 500 \) and \(-2000 \leq l_0 \leq 2000 \).

We use \( H(z) \) data set provided by Moresco et al., 2016 [6] [10] for our cosmic chronometer related analysis. We can see in table 1 that Taylor series expansion results show extremely large uncertainties if we use snap and jerk as free parameters. Also, they don’t give us much idea about the underlying cosmology. Therefore, we will use the derived deceleration and jerk parameters for our further analysis. Table 2 presents the results for the derived deceleration and jerk parameters using the cosmic chronometers or \( H(z) \) data.
Table 1: Measurement of $q_0$, $j_0$, $l_0$ and $s_0$ from Taylor series expression by using Moresco et al., 2016 data [6] with $H(y=\frac{z}{\sqrt{1+z}}$ from Capozziello et al., 2011 [8] and Rezaei et al. 2020 [9].

| Model          | Parameters | $H_0$  | $q_0$       | $j_0$         | $s_0$         | $l_0$         |
|----------------|------------|--------|-------------|---------------|---------------|---------------|
| Λ-CDM          | 68.4±3.1   | -0.34±0.24 | 1(fixed)     | 0(fixed)     | 0(fixed)     |
| Non-Lambda-CDM-I | 70.1±3.2   | -0.82±0.26 | 2.46±0.63   | 0(fixed)     | 0(fixed)     |
| Non-Lambda-CDM-II | 68.4(fixed) | -0.72±0.19 | 2.32±0.64   | 0(fixed)     | 0(fixed)     |
| Non-Lambda-CDM-III | 68.4(fixed) | -0.45±0.48 | -0.8±6.6    | -9±90        | 210±740      |

2.2 Type Ia supernovae

Type Ia supernovae have been a useful tool in constraining cosmological parameters especially in context of the accelerated expansion of our universe. We can write the distance modulus for as a difference of apparent magnitude ($m$) and absolute magnitude ($M$) of the type Ia supernovae as [12] [13] [14]:

$$\mu = m - M$$

Also, luminosity distance and distance modulus are related as:

$$\mu(z) = 5\log [DL(z)] + 25$$

We can get apparent magnitude in the form:

$$m(z) = 5\log (DL(z)) + M + 25$$

With the inclusion of observational factors like color ($k$), shape($s$) and the probability that the supernova belongs in the low-host-mass category ($P$), equation [9] becomes:

$$m(z) = 5\log (DL(z)) + M - \alpha s - \beta k - \delta P + 25$$

We use Union 2.1 distance modulus dataset which is publicly shared by Supernova Cosmology Project (SCP). The dataset is comprised of 580 type Ia supernovae which passed the usability cuts. The dataset is comprised of redshift range 0.015 ≤ $z$ ≤ 1.414 with median redshift at $z \approx 0.294$. Due to the degeneracy issue between Hubble Constant ($H_0$) and the absolute magnitude of type Ia supernovae ($M$), we can separate contribution of $H_0$ and uncertainty in the absolute magnitude 'M' from equations [8] and [9] as [12] :

$$M' = 25 + 5\log (\frac{H_0}{100}) + \sigma_M$$

We can then marginalize over this part to constrain densities and EoS parameters. We use Union 2.1 data set for our type Ia supernovae related analysis [13].
2.3 ISW Effect

The late time integrated Sachs-Wolfe effect (ISW) deals with the blue-shifting and red-shifting of the CMB photons due to the presence of large scale structures and super-voids respectively (please read: [15] [16] [17] [18] [19] [20] [21] [22]). The cross-correlation angular power spectrum coefficient ’Cl’ can be calculated as:

\[ C_l = 4 \pi \int \frac{dk}{k} \Delta^2(k) W_l^g(k) W_l^t(k) \]  

(12)

Here, \( W_l^g(k) \) and \( W_l^t(k) \) are galaxy and temperature window functions respectively and \( \Delta^2(k) \) is the logarithmic matter power spectrum. Galaxy window function depends on redshift distribution (dN/dz) and galaxy bias (b). We use redshift distribution as in dataset for NVSS-ISW provided by Stölzner et al. 2018 [24], Brinckmann & Lesgourgues 2019 [10] and Audren et al. 2013 [23] which provide CMB-galaxy cross-correlation angular power spectrum for Planck 2015 [25] [26] and NVSS [27] [28]. We also use redshift dependent galaxy bias for NVSS [26] [25].

\[ b(z) = 0.90[1 + 0.54(1 + z)^2] \]  

(13)

Another factor which affects the galaxy window function and so the ISW effect is the magnification bias due to gravitational lensing. Magnification bias is dependent on the slope ‘\( \alpha \)’ of the integral count, \( N(> S) = CS^{-\alpha} \). However as discussed in [29] [30], ‘\( \alpha \)’ only plays its part when it is greater or less than ‘1’. In NVSS integral count’s case, ‘\( \alpha \)’ is almost equal to ‘1’ and so did not significantly affect our theoretical calculations.

For calculations related to the minimum \( \chi^2 \) and mean likelihood, we use diagonal of the covariance matrix provided by Stölzner et al. 2018 [24], Brinckmann & Lesgourgues 2019 [10] and Audren et al. 2013 [23]. We use cross-correlation Cl values from multipole (l)=10 to 100 as for higher multipole ranges CMB lensing and Sunyaev–Zeldovich effect play a significant part in the overall CMB anisotropy power spectrum [31] [32]. Apart from cosmology parameters, we also fit a parameter AISW which quantifies ISW amplitude as in Stölzner et al. 2018 [24] and also to deal with possible effect on ‘Cl’ values due to error in modeling the galaxy bias, b(z). AISW is not equal to 1 can either mean disagreement with \( \Lambda \)-CDM or modeling issues with the galaxy bias. To calculate AISW, we use:

\[ \chi^2 = \sum ((Cl_{th} - AISW \cdot Cl_{obs})/(\sigma^2)) \]  

(14)

3 Likelihood

We calculated likelihood (L) as:

\[ L = \exp(-\frac{\Delta \chi^2}{2}) \]  

(15)

Here, \( \Delta \chi^2 \) is the chi-square minus the minimum chi-square value for the parameter sets being tested by using the theoretical models and data\(^1\). For parameter fitting, [Sample code for likelihood calculations using H(z) data: https://github.com/faisalrahman36/Workshop_labs/tree/master/Workshop_2_Cosmology_examples/Hz_Data]
we use mean likelihood instead of maximum likelihood in order to minimize the effect of parameter boundary cuts [12].

For our study we use boundary conditions \( 65 \leq H_0 \leq 75, \quad 0.6 \leq \Omega_\Lambda \leq 0.8, \quad -1.5 \leq w_0 \leq -1/3 \) and \(-2 \leq w_a \leq 2\). We are assuming flatness with all the models for our analysis.

We use mean likelihood analysis to constrain parameters with \( \Lambda \)-CDM, WCDM and CPL models [33] [34]. Figures [125] show deceleration parameter evolution over different redshifts from \( H(z) \), NVSS-ISW, and Union 2.1 for \( \Lambda \)-CDM, WCDM and CPL models and their best fit values can be seen in table 2.

The results are given in 2. We can see that for \( \Lambda \)-CDM model, results for the deceleration parameter \( q_0 \) derived from densities and EoS parameters using \( H(z) \), ISW and Union 2.1 type Ia supernovae are not only consistent with each other within one standard deviation but are also in agreement with table 1 within one standard deviation.

However, results for WCDM and CPL parameters, derived from \( H(z) \), ISW and Union 2.1 type Ia supernovae, are quite different from non \( \Lambda \)-CDM \( q_0 \) results from table 1. This is mainly because of the limitations with Taylor series luminosity distance form used for measurements in table 1. It is also worth noting that all \( w_0 \) results from WCDM and CPL using \( H(z) \), ISW and Union 2.1 type Ia supernovae are not too different from our Lambda-CDM assumption of a constant equation of EoS parameter, \( w=-1 \). They all agree with \( w=-1 \), within one standard deviation despite being different in nature and redshift ranges. Even though we kept phantom energy region \( (w<-1) \) in our boundary conditions, but our results still agree with a likely constant \( w=-1 \). Phantom energy is theoretical concept associated with the acceleration of our universe where \( w<-1 \) can lead to the possibility of a "Big Rip" fate of our universe, according to which all matter from large scale structures to subatomic particles getting torn apart [35] [36]. Vagnozzi et al. 2018 [37] suggested that quintessence dark energy models can be ruled out in coming years, independently of cosmological observations if long-baseline neutrino experiments measure the neutrino mass ordering to be inverted. However, this will require confirmation from cosmological studies using other signatures as well.

Another interesting thing are the values of Hubble constant \( H_0 \) in \( H(z) \) and ISW from NVSS, are in agreement with each other. The mean values are closer to 70 as observed by the gravitational waves and \( H_0 \) based on a calibration of the Tip of the Red Giant Branch (TRGB) applied to type Ia supernovae (SNeIa) [38] [39] [40] [41] and standard deviation margins bring them in the range of the extreme values observed by Planck’s CMB measurements, Dark Energy Survey (DES), H0LiCOW’s combined results [42] [43] [44]. There are still other challenges associated with the observational signatures from which the datasets are taken, apart from disagreements over the expansion rate of our universe. There are issues like the cosmic cold spot or the CMB cold spot [45] [46] which isn’t agreeing with the ISW effect results based on current observations. This can be due to issues related to galaxy bias, magnification bias, galaxy classification, redshift distribution and other issues involved in understanding the galaxy window function and galaxy-CMB maps cross-correlations [21] [47] [48]. There are also challenges associated with the type Ia supernovae observations which is attracting some attention mainly due to the expansion rate debate.

There are also possibilities of such issues arising due to our lack of understanding of things like curvature [49] [50] [51] [52] or nature of inflation in different areas [53] or some other exotic phenomenon [46].
4 Transition redshift

A useful quantity to study the accelerated expansion of our universe is to measure the transition redshift \((z_t)\). The transition redshift can be defined as the redshift at which the universe enters into the accelerating phase from an earlier decelerating phase. In flat Lambda-CDM models, we can write \(z_t\) as \([6]\):

\[
z_t = \frac{2}{3} \Omega_\Lambda^{1/3} - 1
\]

(16)

Table 3 gives the results we get from \(H(z)\), NVSS-ISW, and Union 2.1 for flat Lambda-CDM model. The results are consistent within one standard deviation, with the results obtained from previous studies using CMB anisotropies, BAO, type Ia supernovae and cosmic chronometers for \(\Lambda\)-CDM model \([6, 54, 55, 56, 57]\). However, our best fit transition redshift results are slightly larger than the measurements from cosmological model independent approach discussed in Moresco et al. 2016 \([6]\). Figures (123) show transition redshifts from \(H(z)\), NVSS-ISW, and Union 2.1 for flat \(\Lambda\)-CDM model and their best fit values can be seen in table 3.

5 Jerk parameter

Another quantity which is helpful in understanding the acceleration of the universe especially its deviations from the \(\Lambda\)-CDM is the jerk parameter. Jerk parameter can be helpful in understanding the transitions between phases of different cosmic accelerations. The jerk parameter, \(j\) \([1, 58, 59]\), is a dimensionless quantity obtained by taking the third derivative of the scale factor ‘\(a(t)\)’ w.r.t cosmic time \([58]\). We can calculate the jerk parameter as a function of redshift, using the
Figure 2: Deceleration parameter and transition redshift from Union 2.1 type Ia supernovae data.

Figure 3: Deceleration parameter and transition redshift from NVSS-ISW data.
Figure 4: Jerk parameter and transition redshift measurements from H(z) data.

We can see that they agree with the Λ-CDM assumption of $j_0=1$ for all models and data. In the figures 4, 5, 6, we plot both $j(z)=1$ assumption and $j(z)$ evolution from the data for Λ-CDM along with WCDM and CPL models. Similar can be seen in tables 2 and 4 for the current $j_0$ values using H(z), Union 2.1 and ISW-NVSS datasets with Λ-CDM, WCDM and CPL models.

6 Combined constraints

In order to obtain combined constraints for the deceleration parameter ($q_0$), jerk parameter ($j_0$) and transition redshift ($z_t$), we use inverse-variance weighted average mean. We first combine lower redshift signatures from Union 2.1 and H(z), then we combine both higher redshift ISW signatures and then we combine results from all datasets in our study. We can see in table 4 that results agree with each other within one standard deviation. However in case of both ($q_0$) and ($j_0$), deviations for WCDM and CPL are relatively higher especially for high redshift ISW studies. However, standard deviations improved when all datasets are combined together. For Λ-CDM case, results from tables 1, 2 and 4 agree with each other within one standard deviation but for non-Λ CDM cases, we can see some disagreement in the jerk parameter which is likely due to the greater uncertainties involved with the H(z)’s Taylor series expression approximations when we free jerk, snap and lerk parameters.

We can also see that results for transition redshift in table 5 giving a value $z_t \approx 0.724$, giving slightly lesser standard deviations than table 3. The combined transition redshift is closed to previous Λ-CDM based estimates [6] [54] [55] [56]
Figure 5: Jerk parameter and transition redshift from Union 2.1 type Ia supernovae data.

Figure 6: Jerk parameter and transition redshift from NVSS-ISW data.
but higher than the cosmological model independent method results presented in Moresco et al. 2016 [6].

7 Conclusion

In this study, we started with an introduction to the deceleration parameter ($q_0$) and then estimated it directly using the cosmic chronometer or $H(z)$ data, and in the later parts of the chapter, we estimated $q_0$ by deriving it from the cosmological parameter results obtained using the cosmic chronometers data, type Ia supernovae data and then ISW data. We obtained the combined $q_0=-0.5808\pm0.025$ for Lambda-CDM model, and $q_0=-0.61\pm0.15$ for both WCDM and CPL models. We also measured the jerk parameter ($j_0$). The best fit measurement we got for the combined jerk parameter for Lambda-CDM model is $1+3.971e-07$, for WCDM model is $1.054\pm0.141$ and for CPL model is $1.0654\pm0.1345$. We also estimated the transition redshift. We measured the combined $zt=0.724\pm0.047$ from the Lambda-CDM model parameters.

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| Dataset/Model | AISW | H0       | Omega_Lambda | w0       | wa       | q0 (derived) | j0 (derived) |
|---------------|------|----------|--------------|----------|----------|-------------|--------------|
| **H(z)**      |      |          |              |          |          |             |              |
| Lambda-CDM    | N/A  | 68.8±2.4 | 0.688±0.046  | -0.532±0.069 | 1±3.4786e-06 |
| WCDM          | N/A  | 69.5±3   | 0.688±0.048  | -1.1±0.26 | -0.6352±0.28 | 1.1±0.2684  |
| CPL           | N/A  | 69.7±3   | 0.689±0.053  | -1.11±0.26 | -0.647±0.282 | 1.1137±0.269 |
| **NVSS-ISW**  |      |          |              |          |          |             |              |
| Lambda-CDM    | 2.92±0.79 | 70.2±3.4 | 0.742±0.039  | -0.613±0.0585 | 1±4.633e-06 |
| WCDM          | 2.93±0.78 | 70.2 (fixed from LCDM) | 0.743±0.039 | -0.86±0.37 | -0.458±0.415 | 0.844±0.41245 |
| CPL           | 2.93 (fixed from WCDM) | 70.2 (fixed from LCDM) | 0.746±0.036 | -0.9±0.39 | -0.507±0.439 | 0.888±0.4364 |
| **Union 2.1** |      |          |              |          |          |             |              |
| Lambda-CDM    | N/A  | Marginalized | 0.721±0.02  | -0.5815±0.03 | 1±4.012e-07 |
| WCDM          | N/A  | Marginalized | 0.705±0.055 | -1.07±0.17 | -0.6315±0.2 | 1.074±0.18  |
| CPL           | N/A  | Marginalized | 0.692±0.059 | -1.07±0.16 | -0.4±1   | -0.61066±0.19118 | 1.07266±0.1662 |
Table 3: Transition redshift ($z_t$) from $H(z)$, NVSS-ISW and Union 2.1 for flat Lambda-CDM model.

| Dataset      | Transition Redshift ($z_t$) |
|--------------|-----------------------------|
| $H(z)$       | $0.639915 \pm 0.11714$      |
| NVSS-ISW     | $0.792 \pm 0.122$          |
| Union 2.1    | $0.728968 \pm 0.0573$      |

Table 4: Combined deceleration and jerk parameters from $H(z)$, NVSS-ISW and Union 2.1

| Model        | Datasets                  | $q_0$        | $j_0$            |
|--------------|---------------------------|--------------|------------------|
| Lambda-CDM   | $H(z)$+Union 2.1           | $-0.574 \pm 0.0275$ | $1 \pm 3.986e^{-07}$ |
|              | $H(z)$+Union 2.1+NVSS-ISW | $-0.5808 \pm 0.025$  | $1 \pm 3.971e^{-07}$ |
| WCDM         | $H(z)$+Union 2.1           | $-0.633 \pm 0.163$  | $1.082 \pm 0.1495$ |
|              | $H(z)$+Union 2.1+NVSS-ISW | $-0.61 \pm 0.15$    | $1.054 \pm 0.141$  |
| CPL          | $H(z)$+Union 2.1           | $-0.622 \pm 0.158$  | $1.084 \pm 0.1414$ |
|              | $H(z)$+Union 2.1+NVSS-ISW | $-0.61 \pm 0.15$    | $1.0654 \pm 0.1345$ |

Table 5: Combined transition redshift ($z_t$) from $H(z)$, NVSS-ISW and Union 2.1 for flat Lambda-CDM model

| Datasets                  | Transition Redshift ($z_t$) |
|---------------------------|-----------------------------|
| $H(z)$+Union 2.1           | $0.712 \pm 0.051$          |
| $H(z)$+Union 2.1+NVSS-ISW | $0.724 \pm 0.047$          |
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