Stimulated Excitation by Seeding with Cherenkov Radiation in an Optical Cavity

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Abstract. By seeding with narrow-band Cherenkov radiation from a dielectric loaded waveguide (DLW), stimulated excitation in an optical cavity is presented. The evolution and energy loss of the field oscillating in the optical cavity is analyzed by theoretical and numerical calculation. The results show that the high order TM modes of the Cherenkov radiation can be better preserved after a large number of roundtrips in the optical cavity and this scheme offers a potential method of realizing high power Terahertz radiation source in a compact facility.

1. Introduction
In the last decades, Terahertz (THz) radiation has played a more and more important role in scientific progress and a lot of achievements have been gained with its help in various fields\cite{1, 2}. With the rapid development of THz application in science and technology, the demand for high power radiation sources becomes urgent. One of the most attractive solution is to excite the THz radiation by the relativistic electron bunch in magnetic or slow wave structures\cite{3, 4, 5}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Radiation scheme.}
\end{figure}
Generally, the equipments which are used for generating the relativistic electron bunch are large and expensive. Therefore, a lot of efforts have been made for minimization of accelerator-based THz radiation sources, and one is to apply optical cavity in radiation structures. When the electrons excite the radiation in a cavity at resonance, the electromagnetic field in the cavity is stacked and obtains energy from the electron. This mechanism is called stimulated radiation[6].

Free-electron laser (FEL) oscillators which consists of a resonator with undulator have been widely used at wavelengths across the electromagnetic spectrum from the THz through the ultraviolet[4]. The stimulated transition radiation excited by femtosecond electron bunches in the cavity was observed[7]. Recently, the stimulated excitation of a cavity by trains of electron bunches via coherent diffraction radiation process was tested[6].

As shown in Fig. 1, we propose a novel radiation scheme: the direct current from electron gun is accelerated to relativistic velocity in RF cavity and repetition rate of the micro electron bunch is equal to the frequency of RF field. Cherenkov radiation, which is excited in a dielectric loaded waveguide(DLW) by electron bunch, enter the optical cavity and meet the next micro bunch after being reflected by the mirror. In this scheme, Cherenkov radiation excited in a DLW is the narrow-band electromagnetic wave with abundant harmonics. By properly designing the optical cavity parameters, resonance with harmonics in optical cavity occurs within the THz frequency range. Therefore, there is no need to confine the DLW and electron bunch to tiny size to get fundamental mode with high frequency. In this paper, the loss of radiation with each mode in optical cavity is analyzed and the distribution of the field is shown after a number of roundtrips in the optical cavity. It shows that the stimulated radiation in an optical cavity will be realized and proves that this scheme will be an attractive candidate for the compact THz radiation sources.

2. Theory and Numerical Calculation

2.1. Cherenkov Radiation

![Figure 2](https://example.com/fig2.png)

**Figure 2.** The structure of the dielectric loaded waveguide and the modes of Cherenkov radiation.

Dielectric loaded waveguide (DLW) is a dielectric tube (relative permittivity is $\varepsilon_r$) which is coated with a layer of metal on the outer face as shown in Fig. 2. As the velocity of electromagnetic wave in dielectric material is lower than the velocity of electron $\beta c$ ($c$ is the speed of light and $\beta$ is the relative velocity of electrons), the Cherenkov radiation will be excited when the electron goes through the DLW. Because of the existence of the metal boundary, the
Table 1. Parameters of electron source

| Electron Parameters | Value  |
|---------------------|--------|
| Energy              | 5 MeV  |
| Micro Bunch Repetition($f_b$) | 2.856 GHz |
| Micro Bunch length(RMS) | 3 ps    |
| Micro Bunch Charge  | 0.2 nC |

radiation is confined to a discrete of modes. The dispersion relation of DLW can be expressed as[5]:

$$I_1(k_1a) = \frac{\varepsilon r k_1 J_0(k_2b) Y_1(k_2a) - Y_0(k_2b) J_1(k_2a)}{k_2 J_0(k_2b) Y_0(k_2a) - Y_0(k_2b) J_0(k_2a)},$$  \hspace{1cm} (1)$$

where $k_1 = \frac{\omega}{c} \sqrt{1 - \beta^2}$, $k_2 = \frac{\omega}{c} \sqrt{\varepsilon - \frac{1}{\varepsilon}}$, $\omega$ is the angular frequency of the field in DLW, and $J_m$, $Y_m$, $I_m$ are the Bessel functions with order m. $a$ and $b$ are the inner and outer radius of the dielectric tube. Here, a DLW with $a = 2.9$ mm, $b = 3.95$ mm, $\varepsilon_r \approx 3.8$ (fused silica) is considered. When a train of electron bunches with parameters in Table 1 go through the DLW, narrow-band Cherenkov radiation will be excited with a series of modes. As shown in Fig. 2, the frequencies of the first three modes with high amplitude are 0.034 THz, 0.1028 THz, 0.18136 THz, individually.

However, there is serious diffraction effect when the THz wave is propagating. By applying the Kirchhoff diffraction formula, the far field from the DLW export can be obtained. The formula can be written as[8]:

$$U(R, \theta) = \int_S U_0 e^{-ikR} K(\theta, \lambda) dS,$$

(2)

where $K(\theta, \lambda) = \frac{i}{\lambda} (1 + \cos \theta)$, $U_0$ is the distribution of source field in face $S$, $\theta$ is the angle between the vector of the probe and normal vector of the export face as shown in Fig. 3(b), $R$ is the distance between the source and probe, $\lambda$ is the wavelength of the field. The far field of radiation with first three modes from the DLW export is shown in Fig. 3(a). It is clear that the higher order mode has a lower diffraction angle. For a reflector with limited size, the lower mode will lose more energy.

2.2. Diffraction in Open cavity

In this section, a confocal cavity (which is shown in Fig. 3(b) ) with open boundary is selected for its high stability. In this cavity, the radius of the spherical mirror is equal to the distance between the mirrors and the angle $\phi$ is 45°. In numerical calculation, the process where the propagating wave is reflected back and forth by two mirrors can be simplified as a process where the wave is going through an array of lens and diaphragms[9]. And this process can be expressed by rewriting Eq. 2 as:

$$U_{n+1}(R, \theta) = \int_S U_n \frac{e^{-ikR}}{R} K(\theta, \lambda) dS,$$

(3)

$U_n$ is the distribution of the field in former mirror, and $U_{n+1}$ is the later. To match the phase of the former and the later radiation pulse, the relation between the length of the optical cavity $L$ and field wavelength $\lambda$ satisfy:

$$L = \frac{q}{2} \lambda,$$

(4)
Figure 3. (a) The far field distribution of the Cherenkov radiation with the first three modes from the DLW export; (b) The scheme of the Kirchhoff diffraction and confocal cavity; (c) The variation of radiation relative intensity in three modes with wave propagating in cavity (just one radiation pulse enters the cavity).

here, \( q \) is an integer and \( L = \frac{c}{2f_b} \), so the electron bunch will meet with the reflected radiation pulse when it enters the cavity. In case of mode TM\(_{02}\), the cavity will resonate with a cavity length corresponding to \( q = 36 \). As a pulse of Cherenkov radiation enters the cavity, the variation of the radiation intensity in each roundtrip can be obtained by numerical calculation. In Fig. 3(c), it is clearly shown that the first mode will lose more energy in an open cavity for its severe diffraction effect. TM\(_{02}\) mode shows a similar loss rate with TM\(_{03}\) mode in the previous roundtrips, but the loss rate of radiation increase after 40 roundtrips. Although TM\(_{03}\) mode shows the less loss rate in the cavity, but its amplitude is lower than the TM\(_{02}\) and its frequency is not at resonance with optical cavity. Therefore, TM\(_{02}\) is the only mode which can be excited in the cavity.

3. Discussion
In this paper, the mode and the diffraction effect of Cherenkov radiation from DLW is introduced and the loss rate of the radiation in optical cavity is also considered. However, the mechanism of the radiation energy gain is hard to analyze. If we just consider the Cherenkov radiation, which enter the cavity along with the electron bunch every roundtrip, the distribution of the electric field after 50 roundtrips can be shown in Fig. 4. The energy of the radiation is stacked and longitudinal field is also built in the electron trajectory. However, in this case, the electron bunch will interact with the electron like the mechanism in DLW and the field will obtain the energy from the electron. This progress can be analyzed by the Particle-in-cell code (but it requires the very good computer performance). Meanwhile, when the electron bunch enters the cavity, the diffraction radiation will also induced, but it has the low amplitude in THz frequency compared with the Cherenkov radiation.
Figure 4. The distribution of the radial electric field $E_r$ (upper) and the longitudinal electric field $E_z$ (down) after 50 roundtrips (radiation pulses enter the cavity in each roundtrip).

4. CONCLUSION
This paper shows a novel scheme that excites the stimulated THz radiation in an optical cavity by seeding with narrow-band Cherenkov radiation from a dielectric loaded waveguide (DLW). By theoretical and numerical calculation, the loss of the radiation with each mode in optical cavity is analyzed. It shows that the stimulated radiation with TM$_{02}$ mode in optical cavity will be realized and this proves that the this scheme will be an attractive candidate for the compact THz radiation sources.

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