Recently, the BES Collaboration in Beijing observed a near threshold enhancement in the proton-antiproton (pp) mass spectrum from $J/\psi \rightarrow \gamma pp$ radiative decay \[.\] Fitting the enhancement with an $S$-wave Breit-Wigner resonance result, results in peak mass at $M = 1859^{+3}_{-10} (\text{stat})^{+5}_{-2} (\text{sys})$ with a total width $\Gamma < 30 \text{ MeV}/c^2$. With a $P$-wave fit, the peak mass is very close to the threshold, $M = 1876.4 \pm 0.9 \text{ MeV}$ and the total width is very narrow, $\Gamma = 4.6 \pm 1.8 \text{ MeV}$. This discovery is subsequently confirmed by the Belle Collaboration in different reactions of the decays $B^+ \rightarrow K^+ pp$ \[ and $B^0 \rightarrow D^0 pp$ \], showing enhancements in the $pp$ invariant mass distribution near $2m_p$. Such a mass and width does not match that of any known particle \[.\]

Theoretical existence of possible proton-antiproton bound state has long been speculated in quark model and conventional nucleon potentials \[.\] \[.\] \[.\] \[.\] However, it was only in a very recent study, made by Yan et al us using the Skyrme model \[], that predicted mass and the width close with the experiment. Experimentally, the quantum numbers of $pp(1857)$ are not well determined yet. The photon polar angle distribution was found consistent with $1 + \cos^2 \theta$, suggesting the angular momentum is very likely to be $J = 0$. Making full use of general symmetry requirement and available experimental information the corresponding spin and parity are $J^{PC} = 0^+ \[.\] \[.\] In this letter we report first quenched lattice QCD calculation capable of studying the $pp(1859)$.

In contrast with conventional baryons and mesons, it is difficult to deal with $qq^3$ couplings even in the absence of unquenched effect. The two-point function, in general, couples not only to the single hadron but also to the two-hadron states. We seek operators that have a little overlap with the hadronic two-body states in order to identify the signal of our hexaquark state in lattice QCD. For this purpose, we first consider a non two-body-type field for $I(J^{PC}) = 0(0^{-+})$ and construct our interpolating fields based on the diquark description.

In Jaffe model \[], each pair of $[ud]$ form a diquark which transforms like a spin singlet ($1_s$), colour anti-triplet ($3_c$), and the flavour anti-triplet ($3_f$). Therefore, for a diquark operator, one has \[.\]

$$Q^{\alpha}_T(x) = \frac{1}{2} \epsilon_{ijk} \epsilon_{abc} q^T_{j,b}(x) CT q_{a,c}(x),$$

where $\epsilon_{abc}$ is completely antisymmetric tensor, and $(a,b,c)$ and $(i,j,k)$ denote the colour and flavour indices, respectively. The superscript $T$ denotes the transpose of the Dirac spinor and $C$ is the charge conjugation matrix. The Dirac structure of the operator is represented by the matrices $\Gamma$, satisfying $\Gamma_{\alpha\beta} = -\Gamma_{\beta\alpha}$ ($\alpha$ and $\beta$ are Dirac indices) such that the diquark operator transforms like a scalar or pseudoscalar. The colour and flavour antisymmetries restrict the possible $\Gamma$’s within $1$, $\gamma_5$ and $\gamma_5\gamma_\mu$. The hexaquark hadron $pp([ud][ud][wu])$ emerges as a member with $S = 0$ and $I = 0$ in $\left(Q^3 \otimes Q^3\right) \otimes Q^f = \left([15]_c \otimes [15]_c\right) \otimes [21]_c$ in SU(6) colour-spin representation and a flavour singlet in $(\bar{3}_f \otimes \bar{3}_f) \otimes 6_f$. With this picture the interpolating operator for $0^{-+}$ is obtained as

$$\chi_{df}(x) = \epsilon_{abc} Q^{\alpha}_T(x) Q^{\beta}_T(x) Q^{\gamma}_T(x)$$

where $Q' = u^T C^{-1} \gamma_5 \Gamma'' u$. This identification looks familiar if we represent one of the quarks by charge conjugate field: $q_a q_b \rightarrow q_c a q_b$, where $q_c = -i \gamma^T \sigma^{\alpha} \gamma_5$. Then the classification of diquark bilinears is analogous to that of $qq$ bilinears. We choose $\Gamma' = 1$ and $\Gamma'' = \gamma_5$. There are many possibilities of constructing the operator even in the $I = 0$ channel. In principle testing various other interpolating operators for the best overlap with $0^{-+}$ state should provide information on the wave function of the particle. Our second operator, which we will refer to as the $NN$-interpolating field, is generalised to obtain an isospin $I = 0$ colour-singlet $NN$-type hexaquark interpolating field:

$$\chi_{NN} = \frac{1}{\sqrt{2}} \left[ O_N O_{\bar{N}} - O_{\bar{N}} O_N \right],$$
where
\[ O_N(x) = \sum_x \epsilon_{abc} \left[ (u_a^T(x)C\gamma_5d_b(x))u_c(x) - (u \leftrightarrow d)u_c(x) \right] \]
and
\[ O_{\bar{N}}(x) = \sum_x \epsilon_{abc} \left[ (\bar{u}_a(x)C\gamma_5\bar{d}_b^T(x))\bar{u}_c(x) - (\bar{u} \leftrightarrow \bar{d})\bar{u}_c(x) \right] \]
are the standard interpolating fields for the upper two Dirac components of the nucleon and antinucleon operators, respectively. These definitions take account of the symmetries for isospin and nonrelativistic spin representations. In absence of quark annihilation diagrams, correlation function is expressed in terms of basis determined by direct (36 members) and cross (324 members) contributions of quark contractions. From the dispersion of the two-point function, the contribution from direct and the cross amplitudes can be easily calculated. We have taken single quark-antiquark exchange and a double quark-antiquark exchange to be equivalent because the \( N \) and \( \bar{N} \) operators are summed over all spatial sites for \( s \)-wave state. A more accurate procedure will be to measure the \( 2 \times 2 \) correlation matrix of two different interpolating operators in Eqs. (2) and (3) and extract the effective mass from the eigenvalues using variational techniques [13]. In this study, however, we do not intend to pursue this issue further.

To examine the possible proton-antiproton ground state in lattice QCD, we generate quenched configurations on a \( 20^3 \times 60 \) lattice (with periodic boundary conditions in all directions) with tadpole-improved anisotropic gluon action [14] in the coupling range of \( 3.0 - 4.0 \). Gauge configurations are generated by a 5-hit pseudo heat bath update supplemented by four over-relaxation steps. These configurations are then fixed to the Coulomb gauge at every 500 sweeps. After discarding the initial sweeps, a total of 500 configurations for each coupling are accumulated for measurements. Using the tadpole-improved clover quark action on the anisotropic lattice [13] over the lattice spacing range of \( 0.26 - 0.39 \) fm, we compute the light-quark propagators at six values of the hopping parameter \( \kappa_t \) which cover the quark mass region of \( m_u < m_q < 2m_u \).

To obtain a better overlap with the ground state we used iterative smearing of gauge links and the application of fuzzing technique for the fermion fields [16]. The application of fuzzing for two of the six quarks inside the hexaquark state flattens the curvature of the effective mass. The largest plateau in the region with small errors is obtained with fuzzed \( u \)- and \( d \)-quarks. We used this variant to calculate our correlation functions.

From the correlation functions we extract the mass (energy) by standard \( \chi^2 \) fitting with multi-hyperbolic cosine ansatz
\[ C(t) = \sum_{i=1}^{n} A_i \cosh(tm_i). \]
However, to ensure the validity of our results, we compared them with those obtained as the solution to the equation
\[ \frac{C(t+1)}{C(t)} = \frac{\cosh([t+1-N/2]a_m_{eff})}{\cosh([t-N/2]a_m_{eff})}, \]
for a given \( C(t+1)/C(t) \) at fixed \( t \).

The fitting range \( [t_{min}, t_{max}] \) for the final analysis is determined by fixing \( t_{max} \) and finding a range of \( t_{min} \) where the ground state mass is stable against \( t_{min} \). We choose one “best fit” which is insensitive to the fit range, has high confidence level and reasonable statistical errors. Typical example of the effective mass plot is shown in Fig. 1. The effective mass decreases monotonically, which implies that excited spectral contributions are greatly reduced. For the \( \chi_{N\bar{N}} \)-type field, an impressive plateau with reasonable statistical errors is observed in the interval \( 35 \leq t \leq 50 \), where we expect to achieve single state dominance. For the \( \chi_{d\bar{q}} \)-type field, the signal is noisy at earlier times, and hence we fit in the interval \( 45 \leq t \leq 55 \). The best-fit curve to the \( M_{N\bar{N}} \) data has \( \chi^2/N_{DF} = 1.01 \) and for \( M_{d\bar{q}} \), we find \( \chi^2/N_{DF} = 0.89 \). Statistical errors of masses are estimated by a single elimination jackknife method. We kept statistical errors under control by ensuring that analyzed configuration are uncorrelated, which is made possible by separating them by as many as 500 sweeps. The statistical uncertainties on our hadron masses are typically on the few percent level. The masses of the non-strange mesons \( \pi, \rho \) as well as the nucleon were computed for scale setting and analyzing the stability of hexa-quark state, respectively.

![FIG. 1: Effective mass of the \( J^{PC} = 0^{++} \) colour-singlet state as a function of \( t \). The data correspond to interpolators \( \chi_{d\bar{q}} \) (open circles) and \( \chi_{N\bar{N}} \) (open triangles) at \( \beta = 3.5 \) and \( \kappa_t = 0.2380 \).](image-url)
The chiral extrapolation to the physical limit is the next important issue. From the view point of chiral perturbation theory, data points with smallest $m_s^2$ should be used to capture the chiral log behaviour. Leinweber et al [17] demonstrated that the chiral extrapolation method based upon finite-range regulator leads to extremely accurate value for the mass of the physical nucleon with systematic errors of less than one percent. Unfortunately there does not seem to exist such a chiral extrapolation technique for multi-quark hadrons beyond a naive one. Since quenched spectroscopy is quite reliable for mass ratio of stable particles, it is physically even more motivated to extrapolate mass ratio instead of mass. This allows for the cancellation of systematic errors since the hadron states are generated from the same gauge field configurations and hence systematic errors are strongly correlated. Fig. 2 collects and displays the resulting par-ticle mass ratios, $M_{N_Ndq}/M_p$, extrapolated to the physical mass values using linear and quadratic fits in $m_s^2$.

$$m_h = a + bm_s^2, \quad m_h = a + bm_s^2 + cm_s^4. \quad (6)$$

For the nucleon we have employed the procedure adopted in Ref. [17]. The difference between these two extrapolations gives some information about systematic uncertainties in the extrapolated quantities. The data for the mass ratios $M_{N_Ndq}/M_p$ behave almost linearly in $m_s^2$ and both the linear and quadratic fits, in Eq. (6), essentially gave the identical results. The mass ratios extracted from the fields $\chi_{dq}$ and $\chi_{NN}$ are very close and agree within the errors. It can be seen that mass ratios show a weaker dependence on the quark mass. We expect the contributions from the uncertainties due to chiral logarithms in the physical limit significantly less dominant at present statistics.

The question whether the lowest-lying $J^{PC} = 0^{-+}$ state extracted is scattering state or a resonance is better resolved by analysing the ratio of the mass differences $\Delta M = M_{N_Ndq} - 2M_P$ (between the candidate low-lying hexa-quark state and the free two-particle state) and the nucleon mass. The fitted data, illustrated in Fig. 3, show that both the field operators give the consistent estimates of the ratio and the masses derived from both the operators are consistently higher than the lowest-mass two-particle state. The ratios are clearly positive and increase in magnitude as we approach the physical regime. At zero quark mass the mass difference for both the field operators is $\sim 55(2)$ MeV with $\chi^2/ND_F = 1.05$. In other words, binding becomes weaker near the physical regime with a general trend of positive binding as the zero quark mass limit is approached. The chiral uncertainties in the physical limit are less than $2\%$.

A similar behaviour observed for the mass differences between the hexaquark and the two-particle states at smaller lattice spacing $a_s = 0.26$ fm is illustrated in Fig. 4. The ratio obtained from the $\chi_{dq}$ operator for the five smallest quark masses is slightly higher than that obtained from the $\chi_{NN}$ operator. For $N\bar{N}$-type field the mass difference is $\sim 56(1)$ MeV and $\sim 59(2)$ MeV for diquark-type interpolator. The positive mass splitting in both these channels could be interpreted as the evidence of repulsion. The chiral uncertainties are again estimated to be less than a percent. Similarly, positive mass differences that approximately remained constant with $m_s^2$ and consistent with those obtained at our regular lattices was observed on a larger lattice volume of $6.24^3$ fm$^3$. Again the signature of repulsion at quark masses near the physical regime would imply no evidence of the resonance.

\[ \text{FIG. 2: Chiral extrapolation of hadron mass ratios, } M_{N_Ndq}/M_p, \text{ extracted with the diquark-type (open triangles) and } NN \text{ (open circles) interpolating fields at } a_s = 0.32 \text{ fm. The solid triangles correspond to the data from proton interpolating field. The dashed lines are chiral extrapolation (linear in } m_s^2 \text{) to the mass ratios using Eq. (6).} \]

\[ \text{FIG. 3: Chiral extrapolation of the ratios, } \Delta M/M_p = (M_{N_Ndq} - 2M_P)/M_P, \text{ extracted from diquark-type (open circles) and } NN \text{ (open triangles) fields at } a_s = 0.29 \text{ fm. The solid and the dashed curves are chiral extrapolation (linear in } m_s^2 \text{) using smallest five masses.} \]
in the $J^{PC} = 0^{-+}$ channel.

Finally, we performed a continuum extrapolation for the chirally extrapolated quantities in Fig. 5. Expecting that dominant part of scaling violation errors is largely eliminated by tadpole improvement, we adopt an $a_s$-linear extrapolation for the continuum limit. We also perform an $a_s$-linear extrapolation to estimate systematic errors. Performing such extrapolations, we adopt the choice which shows the smoothest scaling behavior for the final values, and use others to estimate the systematic errors. As can be seen from Fig. 5, the mass ratio again shows a weak dependence on the lattice spacing and varies only slightly over the fitting range. The four non-zero lattice spacing values of the ratio are within $0.01 - 0.02$ standard deviations of the extrapolated zero lattice spacing result. This will make for unambiguous and accurate continuum extrapolation. Using the physical nucleon mass $m_p = 938$ MeV, we obtain a continuum mass estimate of 1935(19)(60) and 1940(26)(72) MeV for the ground state hexa quark state from $\chi_{N\bar{N}}$ and $\chi_{dq}$ interpolators, respectively. The results from two extrapolations seem to be in good agreement, within errors. Given the fact that the ratio does not show any scaling violations, we could also quote the value of this quantity on our finest lattice, which has the smallest error. Nevertheless, order 4% errors on the finally quoted values are mostly due to the chiral and the continuum extrapolations. The quenching errors might be the largest source of uncertainty. Note however, that in the case of stable hadrons, this is not expected to be very important. It has been shown [18] that with an appropriate definition of scale, the mass ratios of stable hadrons are described correctly by the quenched approximation on the $1 - 2\%$ level. To this end we also calculated the pseudoscalar to vector meson ratio $R_{SP}$ and pseudoscalar to nucleon mass ratio $R_{SN}$ and found that in the continuum these ratios differ about 3% from their corresponding experimental values. So we quote our quenching errors to be less than five percent. This implies that dynamical quarks might play a less important role on the spectrum in question.

Since the mass difference between the reported experimental $p\bar{p}$ mass and the physical $2m_p$ continuum is $\sim 30$ MeV, naively one may be tempted to interpret the results in Figs. 3 and 4 as a signature of the proton-antiproton bound state on the lattice. But the positive mass difference observed in the range of pion mass differs from what is seen for the $p\bar{p}$ state studied on the lattice. The continuum results imply that the mass difference does not move into the continuum with an attractive interaction between $p$ and $\bar{p}$. This suggests that the observed signal is too heavy to be identified with the empirical $p\bar{p}(1859)$ and unlikely to be a signature of a possible proton-antiproton system as a six quark state. However the interaction between a baryon and an antibaryon can be as strong as that of baryon-baryon systems. Therefore we do not discard the interesting possibility that it could be a mixed molecular state of baryon and antibaryon bound by some long-range force of strong or electromagnetic nature or both.

We have presented the results of the first lattice investigation on the $p\bar{p}$ state employing improved gauge and fermion anisotropic actions, relatively light quark masses as well as smearing techniques to enhance the overlap with the ground state of the particle. Our analysis takes into account all possible uncertainties, such as statistical, finite-size, and quenching errors when performing the chiral and continuum extrapolations. On the basis of our lattice calculation we speculate that the state is not to be identified as a bound state of six quarks. However a thorough examination of this question would require the implementation of flavour $SU(3)$ violation. The $I = 0, p\bar{p}$ state couples to $4\pi\eta$ through the $s\bar{s}$ component of the $\eta$ in the quenched approximation. By giving the strange

![FIG. 4: As in Fig. 3 but at $a_s = 0.26$ fm.](image)

![FIG. 5: Continuum extrapolation of the mass ratios extracted from diquark-type (solid circles) and $N\bar{N}$ (solid triangles) fields. The solid and the dashed curves are linear extrapolation to the continuum limit.](image)
quark a larger mass would alter threshold which in turn would affect the manifestation of the bound state. As differences between meson masses in full and quenched QCD of order 60 MeV [19] and 100 MeV or more for baryons have been observed [20], one might wonder what effect the dynamics of full QCD could have on this state.

Whereas the quenched approximation is found to give quite useful results at zero temperature, the fermions cannot be neglected if precise contact with nature is needed. One cannot yet rule out the possible existence of a $p\bar{p}$ bound state in full QCD.

I would like to thank R. Jaffe, C. Michael, C. Detar, S. Sasaki, C. Hamer and D. Leinweber for valuable suggestions. It is a pleasure to acknowledge Shan Jin for his hospitality at CCAST and stimulating discussions. We are grateful for the access to the computing facility at the Shenzhen University on 128 nodes of Deepsuper-21C. The computations were done using a modified version of the publicly available MILC code (see www.physics.indiana.edu/sg/milc.html). This work was supported by the Guangdong Provincial Ministry of Education.

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