Two-body nucleon-nucleon correlations in Glauber models of relativistic heavy-ion collisions

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We investigate the influence of the central two-body nucleon-nucleon correlations on several quantities observed in relativistic heavy-ion collisions. It is demonstrated with explicit Monte Carlo simulations, that the basic correlation measures observed in relativistic heavy-ion collisions, such as the fluctuations of participant eccentricity, initial size fluctuations, or the fluctuations of the number of sources producing particles, are all sensitive to the inclusion of the two-body correlations. The effect is at the level of about 10-20%. Moreover, the realistic (Gaussian) correlation function gives indistinguishable results from the hard-core repulsion, with the expulsion distance set to 0.9 fm. Thus, we verify that for investigations of the considered correlation measures, it is sufficient to use the Monte Carlo generators accounting for the hard-core repulsion.

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I. INTRODUCTION

The atomic nucleus is closer to a self-bound saturated liquid than to a Fermi gas of non-interacting particles, as is for simplicity frequently assumed in studies of relativistic heavy-ion collisions. Thus the inclusion of correlations in the initial configuration of nucleons in the colliding nuclei is a priori very important. Recently Alvioli, Drescher, and Strikman [1,2] generated distributions of nucleons in nuclei which account for the central two-body nucleon-nucleon (NN) correlations. The procedure, based on the Metropolis search for configurations satisfying constraints imposed by the NN correlations, reproduces the one-body Woods-Saxon distributions, as well as central NN correlations, taken in the Gaussian form. This calculation is a very important step in the investigations using the Glauber approach [3,4] to relativistic heavy-ion collisions, as it is well known [5,6] that correlations induce event-by-event fluctuations of the measured quantities.

The Glauber Monte Carlo codes [7,11] (for a discussion of physics issues see Ref. [8] and the review [11]) which model the early phase of the collision, have not been incorporating, for practical reasons, realistic NN correlations. Instead, the hard-core expulsion, easy to implement, is used. In that method, centers of nucleons, whose positions are randomly generated according to the Woods-Saxon one-body distribution, are not allowed to be placed closer to one-another than the expulsion distance $d \sim 1$ fm, which simulates the hard-core NN repulsion. It is not a priori clear that the results obtained with the realistic (Gaussian) and the hard-core correlations should be the same for various correlation measures used in the heavy-ion studies. Moreover, it is not obvious what precise value of $d$ should be taken to make the simulations most realistic.

The purpose of this paper is to investigate, with the help of explicit Glauber Monte-Carlo simulations by GLISSANDO [9], the role of the central two-body NN correlations for several popular observables in relativistic heavy-ion collisions. In particular, we look at the following fluctuation measures: the participant eccentricity fluctuations related to the fluctuations of the elliptic flow [12-24], the multiplicity fluctuations as analyzed in the set up of the CERN NA49 experiment [25], and the recently investigated initial size fluctuations [26], which influence the transverse-momentum fluctuations [27-42]. We find that all these measures are sensitive to the inclusion of the two-body correlations at a level of about 10-20%. However, the realistic (Gaussian) correlation function gives virtually indistinguishable results from the calculations with the hard-core repulsion, with the expulsion distance tuned to $d = 0.9$ fm. Thus, we will argue that for all practical terms of modeling the Glauber initial phase of the collision, it is sufficient to use the Monte Carlo generators with the hard-core repulsion.

Certainly, the method of Ref. [1] is more general, as it allows to include correlations from attractive forces, as well as introduce the isospin dependence. These were recently considered in Ref. [43], and when these distributions are published, they can be implemented in Glauber generators and tested in a similar way as in the present work.
II. NUCLEAR CORRELATIONS

The method of Ref. [1] imposes a given form of one- and two-body nucleon distributions. The one-body density is parametrized with the standard Woods-Saxon form

$$\rho^{(1)}(r) = \frac{A}{1 + e^{r - R}}. \quad (1)$$

Our fit to the distributions for $^{208}\text{Pb}$ from [2] yields the optimum parameters

$$R = 6.59(1) \text{ fm}, \quad a = 0.549(2) \text{ fm}, \quad (2)$$

where the uncertainties follow from the regression analysis on the available sample [2] of $10^5$ configurations. The result of our numerical simulation is displayed in Fig. 1.

The radial two-body correlation function $C(r)$ is defined as [1]

$$C(r) = 1 - \frac{\rho^{(2)}_C(r)}{\rho^{(2)}_U(r)}, \quad (3)$$

where $\rho^{(2)}_C(r)$ and $\rho^{(2)}_U(r)$ are the correlated and uncorrelated radial two-body densities,

$$\rho^{(2)}_i(r) = \int d^2\Omega \int d^3R \rho^{(2)}_i(R + r/2, R - r/2). \quad (4)$$

Here $\rho^{(2)}_i(r_1, r_2)$, $i = C, U$, denotes the appropriate two-nucleon density, $r$ is the relative coordinate, $r = |r|$, and $\Omega$ corresponds to the two angles associated with $r$, over which the density is integrated. The correlated density is read off from the distributions [2] with the help of GLISSANDO by histogramming the relative distances between the centers of nucleons in the same nucleus, while the uncorrelated density is found by taking the pairs of nucleons from different nuclei (this corresponds to the well-known mixing technique, which gets rid of correlations). The result of our procedure is shown in Fig. 2.

We recover the Gaussian central NN correlation, implemented in the procedure of Ref. [1],

$$C(r) = e^{-\frac{r^2}{2b^2}}, \quad (5)$$

with

$$b = 0.561(1). \quad (6)$$

The uncertainty comes from the finite sample of $10^5$ configurations from [2]. Thus indeed the distributions of [1, 2] properly implement the one-body density and the Gaussian central two-body correlations. The purpose of the above study was to read off the one-body parameters [2], which in the following sections will be input in the generation of the uncorrelated distributions by the Glauber simulations with GLISSANDO [9]. Results from the uncorrelated distributions will be compared to the correlated case, where the correlated distributions of Ref. [1] will be fed directly into our simulations.

III. GLAUBER MODELS

The prototype Glauber model used in the heavy-ion phenomenology is the wounded-nucleon model [14]. A wounded nucleon has collided inelastically at least once in the collision process. Variants of the approach [9, 15–47] admix a certain fraction of binary collisions to the wounded nucleons, which leads to a better overall description of multiplicities of the produced particles. In this mixed model, investigated in this work, the number of the produced particles is proportional to the number of sources

$$N_s = (1 - \alpha)N_w/2 + \alpha N_{\text{bin}}, \quad (7)$$

where $N_w$ is the number of the wounded nucleons, and $N_{\text{bin}}$ the number of binary NN collisions. The fits to
particle multiplicities of Ref. [47] give \( \alpha = 0.145 \) for collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \), and \( \alpha = 0.12 \) for \( \sqrt{s_{NN}} = 19.6 \text{ GeV} \). Extrapolation to the LHC energies yields \( \alpha \approx 0.2 \).

More sophisticated approaches [48–50] discriminate between the nucleons which have collided only once (corona) and more than once (core), which leads to an appealing physical picture. Also, the wounded-quark model [51–56] yields a successful phenomenology, All in all, the Glauber picture of the initial stage of the relativistic heavy-ion collision is a key element of many phenomenological analyses of the particle production mechanism.

In this paper we apply the mixed model for the \(^{208}\text{Pb}\)-\(^{208}\text{Pb} \) collisions, with \( \alpha = 0.12 \), corresponding to the highest SPS energy. We term the locations of centers of the wounded nucleons or the binary collisions as “sources”, with the weight of the wounded nucleon \( w_i = (1 - \alpha)/2 \), and the weight of the binary collision \( w = \alpha \). A source emits particles, according to a superposed distribution [9].

While for the one-body measures, such as the particle multiplicities or spectra, only the one-body distributions matter and correlations are irrelevant, the fluctuations measures are expected to be sensitive to the NN correlations in the nucleon distributions. These are examined in detail in the next section.

### IV. RESULTS OF SIMULATIONS

In this section we compare the results of the Glauber calculation initialized with the distributions of Ref. [1, 2] (solid lines in the figures), with uncorrelated distributions (dashed lines), and with the distributions accounting for the hard-core repulsion with the expulsion radius \( d = 0.9 \text{ fm} \) (dotted lines). The simulations are performed with GLISSANDO [8].

We note that in the case with no correlations we simply use the Woods-Saxon parameters [2], while in the case with the hard-core repulsion we need to start with a somewhat more compact distribution, as the expulsion leads to swelling, as explained in Ref. [9]. We find that starting the Monte Carlo generation with \( R = 6.44 \text{ fm} \) and \( \alpha = 0.549 \text{ fm} \), leads, with \( d = 0.9 \text{ fm} \), to the one-body distribution with parameter values [2]. This construction, with shrunken “bare” one-body distributions, is important, as that way all calculations presented in the figures correspond to identical one-body distribution, and the differences in results are caused entirely by the two-body correlations.

#### A. Eccentricity

We start with a measure sensitive to the fluctuations, the so-called participant eccentricity. This measure appears in the studies of the event-by-event fluctuations of the initial shape, in particular of its elliptic component [14–24]. The effect is important, as the fluctuations lead to enhanced eccentricity of the initial system, and as a result of the subsequent hydrodynamic evolution, to enhanced elliptic flow. The participant eccentricity is defined in each event as

\[
\varepsilon^* = \frac{\sqrt{(\sigma_x^2 - \sigma_y^2)^2 + 4\sigma_{xy}^2}}{\sigma_x^2 + \sigma_y^2},
\]

where \( \sigma_x^2 \) and \( \sigma_y^2 \) are the variances of the two transverse coordinates, and \( \sigma_{xy} \) is the covariance. Specifically, in each event

\[
\langle x \rangle = \sum_i w_i x_i, \quad \sigma_x^2 = \sum_i w_i (x_i - \langle x \rangle)^2,
\]

and similarly for the \( y \) variable and the covariance. The index \( i \) runs over all generated sources, and \( w_i \) are the weights. The quantity \( \varepsilon^* \) has the interpretation of the eccentricity evaluated event-by-event in a variable reference frame [10], rotated in such a way that the eccentricity in a given event is maximized.

In the top panel of Fig. 3 we show the dependence of the event-by-event average, \( \langle \varepsilon^* \rangle \), on the number of wounded nucleons (determining the centrality of the event). We note that the three calculations are virtually indistinguishable, except for a tiny difference for the most central collisions, where the uncorrelated case is a few percent higher. The same conclusions were reached in the analogous study of eccentricity in Ref. [57].

The bottom panel of Fig. 3 shows the scaled standard deviation, \( \Delta \varepsilon^*/\langle \varepsilon^* \rangle \), obtained from our event-by-event analysis. We note a significant difference between the uncorrelated case, which has up to 10% larger fluctuations at intermediate centralities, and the cases with correlations. However, the calculations with the realistic NN correlations and the hard-core correlations give an indistinguishable result, with the two curves overlapping within the statistical noise.

The short horizontal line at the most central events corresponds to the theoretical value \( \sqrt{4/\pi - 1} \) of Ref. [19], following from the central limit theorem.

#### B. Multiplicity fluctuations

Next, we consider a quantity relevant for the multiplicity fluctuations as measured in the NA49 experimental setup [25], where the number of participants in the projectile is determined via the VETO calorimeter. Significant fluctuations of the number of sources may follow in this case from the fact that even at a fixed number of the wounded nucleons in the projectile, the number of wounded nucleons in the target fluctuates due to the statistical nature of the Glauber approach. The fluctuations of multiplicity in nucleus-nucleus collisions were also investigated experimentally in [58–61]. We recall [62–63]
that the simple superposition models with the effect of fluctuations of the target wounded nucleons are not able to explain the data of Ref. [25]. Nevertheless, for the present purpose of analyzing the importance of the NN correlations, the effect serves its purpose.

In Fig. 4 we show the scaled variance of the total number of sources, \( \omega \), plotted as a function of the wounded nucleons in the projectile, \( N_{w}^{\text{proj}} \). It was recently shown in Ref. [26] that the initial size fluctuations are carried over via hydrodynamics and statistical hadronization into the event-by-event transverse-momentum fluctuations [27–42], where they lead to a natural description of the RHIC data for the measure \( \sigma_{\text{dyn}}(p_T) \). In Fig. 5 we show the scaled standard deviation of the size variable \( r \) of Eq. (11), plotted as a function of the total number of wounded nucleons, \( N_{w} \).

**C. Size fluctuations**

Finally, we look at the event-by-event size fluctuations, namely the fluctuations of the variable

\[
r = \sum_{i} w_i \sqrt{(x_i - \langle x \rangle)^2 + (y_i - \langle y \rangle)^2},
\]

(11)

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V. CONCLUSIONS

We have checked by carrying out explicit Glauber Monte Carlo simulations with GLISSANDO [9], that the inclusion of the central NN correlations influences the fluctuation measures in relativistic heavy-ion collisions at a level of, say, 10-20%. Comparison of the realistic (Gaussian) correlations implemented in Ref. [1] and the hard-core correlations, typically used in the Glauber Monte Carlo codes, shows that they lead to the same results when the hard-core expulsion distance between the centers of nucleons is tuned to
\[ d = 0.9 \text{ fm.} \tag{12} \]

Thus the main message for the practitioners of the Glauber Monte Carlo models is that, at least for the investigated observables, the implementation of the hard-core repulsion with \( d \) given by Eq. (12), straightforward to implement in Monte Carlo generators, leads to realistic predictions. We note that the dependence of the results on the value of \( d \) is sensitive, as the excluded volume scales as \( d^3 \).

Certainly, the general method of Ref. [1] allows one to implement channel-dependent NN correlations, as well as the nuclear attraction, relevant at intermediate distances. The role of these effects for the fluctuation measures in relativistic heavy-ion collisions can be investigated in a similar manner as in this work. In essence, every effect which increases the “regularity” of the initial nucleon distributions of the colliding nuclei, such as the considered central NN correlations, will have the tendency of decreasing the event-by-event fluctuations in nuclear collisions generated by the Glauber models.
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