Abstract

Time is one of the most significant characteristics of time-series, yet has received insufficient attention. Prior time-series forecasting research has mainly focused on mapping a past subseries (lookback window) to a future series (forecast window), and time of series often just play an auxiliary role even completely ignored in most cases. Due to the point-wise processing within these windows, extrapolating series to longer-term future is tough in the pattern. To overcome this barrier, we propose a brand-new time-series forecasting framework named Dateformer who turns attention to modeling time instead of following the above practice. Specifically, time-series are first split into patches by day to supervise the learning of dynamic date-representations with Date Encoder Representations from Transformers (DERT). These representations are then fed into a simple decoder to produce a coarser (or global) prediction, and used to help the model seek valuable information from the lookback window to learn a refined (or local) prediction. Dateformer obtains the final result by summing the above two parts. Our empirical studies on seven benchmarks show that the time-modeling method is more efficient for long-term series forecasting compared with sequence-modeling methods. Dateformer yields state-of-the-art accuracy with a 40% remarkable relative improvement, and broadens the maximum credible forecasting range to a half-yearly level.

1 Introduction And Related Work

Time-series forecasting is a critical demand across many domains, such as energy consumption, economics planning, traffic and weather forecasting. Numerous works have been promoted. ARIMA [3] and the filtering [6, 13] are representative methods of Statistics: the former tackles time-series forecasting transforming the non-stationary process to stationary through differencing [4], and the latter utilizes expert knowledge of Probability to predict. With the rise of deep learning, RNN-based and CNN-based methods are applied to time-series forecasting tasks [30, 23, 25, 2, 18, 26], in view of their great success in NLP and CV respectively. To a somewhat, these solutions meet the expectations. Recently, several studies have concentrated on long-term series forecasting, which involves longer-range forecast windows than regular time-series forecasting. RNN-based structures, that were once considered workhorses, are no longer qualified for the task due to their intrinsic flaws: difficulty capturing long-range dependency, slow reasoning and accumulating error. Fortunately, Transformer [29] appeared. As a variable-length-sequence-to-sequence inference structure that can calculate in parallel via the self-attention mechanism, it’s free of the first defect and has the potential to mitigate the late two. Nevertheless, directly applying Transformer to long-term series forecasting is computationally expensive because of its $L$-quadratic complexity on $L$-length sequence among both memory and time. Various sparse versions of self-attention emerge one after another to resolve the issue [13][12][16]. They’ve alleviated the problem, but also reached the bottlenecks: (i) they’ll...
sacrifice the information utilization because of the sparse point-wise connections. (ii) in terms of the series length that models can handle, the improvement brought by the sparse strategies is limited. (iii) they fail to exploit all merits of Transformer to accelerate reasoning and retard error accumulating. Informer further proposed a series-wise connection to address the first point, but still was confined by the second, and it’s hard to state that its architecture is similar to the vanilla Transformer. Instead, we contend that the bottlenecks are blamed for the rigid point-wise processing pattern, so any sparse strategy is merely a band-aid remedy. In this paper, we are no longer keen to propose any sparse version of self-attention, but tackle the time-series forecasting problem from another perspective, and explore the vanilla Transformer’s potential in this field. Inspired by ViT, we split time-series into patches, which can considerably reduce the amount of tokens and enhance efficiency. Unlike the image, for which we must carefully select patch size, day is a natural and perfect patch for time-series, and it has practical significance. We notice that many time-series exhibit seasonality regarding day as basic unit, and leveraging seasonality is crucial to a successful of numerous works.

Decomposition is a common tool for extracting the seasonality of time-series that are decomposed into trend, seasonality and remainder part via traditional techniques. The optimal prediction draw from the trend and seasonality, since the remainder is unpredictable. Following three issues plague the decomposition pattern. (a) Smoothing time-series is a chore that must be handled with caution. (b) Subject to the narrow lookback window, models are unable to capture long-range seasonality, and instead treats them as trend, which lead to suboptimal results (see Figure 1). (c) Above all, the static assumption on seasonality is coupled in the pattern. They extrapolate the seasonality captured in the look-back window to the forecast window unquestioningly, despite the fact that time-series’ seasonality is dynamic. Specifically, the seasonality of time-series is measured based on a variety of scales, including day, week, month, year, etc. A given time’s seasonality entail a mixture of seasonality of several time scales in a specified proportion. Then the dynamics can be summarized as follows: (i) to begin with, the proportion of seasonality at different scales fluctuates dynamically; for example, at ordinary times, the fraction of day and week is quite large, whereas approaching important festivals, the ratio of year is relatively high (see Figure 1). (ii) moreover, although the size of these scales is unlikely to change greatly, slight alterations are always conceivable; the number of days in a month is always changing, and in some lunar calendars, the days in a year can sometimes even vary up to 30 days. As the two points are intertwined and influence each other, inaccurate seasonality may be extrapolated to the forecast window under the decomposition pattern.

We believe that the fluctuation in this proportion is mostly owing to time’s polysemous, i.e., people’s attention to various time features could adjust depending on the context of time. The polysemous of date occurs when we use date to index time. Few people care that January 2, 2012 is a Monday, because the day before it is New Year’s Day. This reminds us that we should pay attention to the time of series, several works have been conducted: Time2Vec contributes a universal time representation, Informer provides a timestamp embedding. Models consume these representations by adding or concatenating them to their input series, just like positional encoding. In this sequence-modeling style, however, time representations act as only an auxiliary and hence fail to try their best. Not to mention that these representations are static, so can’t help models get rid of the aforementioned troubles at all. Some event prediction studies gave time more duty and achieved success. Learn from and exceed them, we decide to explore an unconventional time-series forecasting methodology by replacing sequences with time features as the primary modeling object: the time-modeling strategy.

We introduce Date Encoder Representations from Transformers, a supervised learning version BERT with time-series patches as labels, and take it as core to design a long-term forecasting

| Time   | Load  |
|--------|-------|
| 03:00  | 3000  |
| 06:00  | 4000  |
| 09:00  | 5000  |
| 12:00  | 6000  |
| 15:00  | 7000  |
| 18:00  | 8000  |
| 21:00  | 9000  |
| 24:00  | 10000 |

Figure 1: (a) depicts two-year power load of an area. (b) illustrates the area’s full day load on January 2, 2012, a week ago, a week later and a year ago. It can be seen that electricity consumption is mixed with different scales of seasonality. On January 2, 2012, the seasonality of week declined but it of year rose.
framework based on the canonical Transformer: **Dateformer.** It regards day as the atomic unit of time-series, considerably improving information input and utilization, reducing error accumulating and accelerating forecasting, particularly for fine-grained time-series (such as minute, quarter and half-hour levels). Above these, with the help of the dynamic date-representation given by DERT, a decomposition pattern that doesn’t involve any smoothing process is presented in Dateformer: time-series are decomposed into time-mapped (global) and lookback-window-mapped (local) part. The details are shown in section 7 and our contributions can be summarized as follows:

• We discover the polysemy of date and introduce DERT into the time-series field to address it. DERT is a pretrained structure that empowers the model to leverage information from entire visible dataset and even other datasets rather than merely from the lookback window.

• We design the Dateformer, a brand-new series forecasting framework of modeling time primarily, to demonstrate that the canonical Transformer is sufficient for long-term forecasting. To our best knowledge, it’s the first time-modeling model for time-series forecasting.

• Combining the advantages of the generative style and Autoregression, we develop a comprehensive day-wise inference strategy, which strikes a good balance between speed and correctness of reasoning: retards error accumulating while accelerating reasoning. Once trained, it can predict any days. That is, train once, run anyscale.

• A novel decomposition pattern is presented in Dateformer. Different from the classic seasonality-trend-level decomposition, the pattern takes a unique angle to time-series analysis, so as to obviate any smoothing process during decomposition.

• A new training methodology is tailor-made for Dateformer. Compared with common training strategies, it takes less time, is more conducive to the model extrapolation, and can be configured flexibly according to datasets and forecasting requirements.

2 Preliminary

Let’s redefine the time-series forecasting problem from the time-modeling perspective. Given a series of dates $D = \{D^1, D^2, ..., D^T\}$ and corresponding $N$ time series $X = \{X^1, X^2, ..., X^T \mid X^t \in \mathbb{R}^{G \times N}\}$ where $G$ is the numbers of time-series points in a day (in this paper, we denote the variable by granularity or $G$ for brevity), the target is to predict next $H$ days’ $n \leq N$ series values in the rolling forecasting fashion. And a large $H$ is encouraged in long-term series forecasting setting, that being sad, the long-term is a concept of time: the amount of time-series points in the next $H$ days varies depending on the series’ granularity. Although contrary to sequence-modeling methods that do well in fixed-step forecasting, we believe it’s more reasonable and in line with human thinking inertia.

3 DERT

Before getting the dynamic date-representation, the static date-embedding must be prepared in advance. Referring to the enlightening results of Word-embedding, we hope that each dimension of the date-embedding can reflect a time feature. Luckily, these features are artificially deduced and have strong regularity, thus can be generated fast by programs. We obtain the date-embeddings through a set of simple algorithms that integrate various time information of date: week, month, year, holiday, solar term, etc (see Appendix C for details). However, they can’t be immediately fed into DERT to produce the date-representations yet, the context of dates needs to be determined. The contextual boundaries of dates are open, which is distinct from words that are usually located in sentences and hence are in comparatively closed context. Intuitively, distant dates exert a weak influence on today, so it makes sense to select nearby dates as context. To achieve the idea, DERT slides the Transformer encoder with a fixed window on the date axis, analogous to convolution. It, as well as the static date-embedding, introduce inductive bias into the model, which helps generalization. A linear layer attached to the representation serves as implements of two tasks we design for pretraining.

Mean prediction It’s easy to think of predicting the total amount of series values in the day from the date-representation. Taking the day’s time-series patch as a label, we require DERT to predict the means of multivariate sequences on this day. For fixed time interval series, it’s equivalent to the total amount prediction.
Auto-correlation prediction. We observed that many time-series display varied intraday trends on different dates. Like more steady in holidays, or on the contrary. To leverage the characteristic, we utilize the circular auto-correlation in stochastic process theory [5, 21] to assess sequences’ stability. According to Wiener-Khinchin theorem [31], series auto-correlation can be calculated by Fast Fourier Transforms. Thus, we score the stability of a sequence with $L$-length by following equations:

$$S_{XX}(f) = \mathcal{F}(X_t)\mathcal{F}^*(X_t) = \int_{-\infty}^{\infty} X_te^{-i2\pi ft} dt \int_{-\infty}^{\infty} X_te^{-i2\pi ft} dt$$

$$R_{XX}(\tau) = \frac{F^{-1}(S_{XX}(f))}{\ell_2 Norm} = \frac{\int_{-\infty}^{\infty} S_{XX}(f)e^{i2\pi f\tau} df}{\sqrt{\lambda_1^2 + \lambda_2^2 + \cdots + \lambda_L^2}}$$

$$Score = \frac{1}{L} \sum_{\tau=0}^{L-1} R_{XX}(\tau)$$

where $\mathcal{F}$ denotes Fast Fourier Transforms, * and $F^{-1}$ denote the conjugate and inverse Transforms respectively. To balance pretrained losses between the two tasks, we average the normalized auto-correlation coefficients, and ask DERT to predict the means only relying on the date-representation through a auto-correlation prediction head. This task is proposed to extract some intraday information from time-series patches, so that plays the complementary part with the former task.

4 Dateformer

Dateformer in the narrow sense is not a Transformer-like model. It’s more like a scheduler who obtains predicting outcomes by utilizing two Transformer-based components: Dert and Longlongformer. Dert maps time features to the forecast window, while Longlongformer extrapolates the lookback window to future. Dateformer schedules them with an algorithm and then fuses their outputs to produce the final forecast. In this section, we go over them in as much detail as possible.

Model input. The inputs of Dateformer include two parts: the static date-embeddings and the lookback series. To get the dynamic representations of days situated in the lookback window and forecast window, we should input their static date-embeddings and some days before and after them as padding. In addition, a crucial distinction between Dateformer and sequence-modeling methods is that the lookback window in Dateformer plays merely the role of providing short-term (local) information. Dateformer is not interested in learning a mapping from the lookback window to the forecast window. The local information is just leveraged to improve the prediction more precise.

4.1 Dert

Dert holds the classic encoder-decoder architecture. It employs a DERT encoder to encode the static date-embedding, and the output representation is duplicated into $G$ copies. In order to subdivide them
to the finer time scales, we add the canonical positional encoding [29] to these copies. A MLP acts as the decoder of Dert to map these time-representations to predictive values. One may ask: why don’t we use Dert to pretrain DERT? Actually, Dert is a fairly good pretrained task for most datasets. It can achieve the same effects as the previously mentioned two tasks on these datasets. On some datasets, however, we found that Dert could converge to a slightly better result if we pretrain the encoder by the two pretrained tasks. In addition, Dert-pretrained DERT is not convenient to transfer between different datasets. As a result, we didn’t include it in the list of pretrained tasks.

4.2 Longlongformer

Longlongformer is a Transformer with a few tweaks to match our demand of it: extrapolating the lookback window to future. Although the setting is the major competition arena for sequence-modeling methods, Longlongformer would prove that the time-modeling method can also be effective.

The encoder of Longlongformer takes time-series patches in the lookback window as input and embeds them by a linear layer. For multivariate time-series, flattening should be conducted in advance. Outputs of the encoder will be fed into the decoder as cross information to help it refine the date-representations. Longlongformer always predicts only one day next to its lookback window. Its decoder eats the dynamic representations of the lookback window and the next day. After decoding, it first picks up the refined representation of the day that needs to be predicted, duplicates it into \( G \) copies, and plus positional encoding. Subsequently, they are mapped to a preliminary prediction through a MLP, just like we did in Dert, and the MLP usually shares parameters with the Dert’s decoder. Having that done, a position-wise feed-forward network is attached to the decoder for the purpose of converting its output tokens with dimension \( d_{\text{model}} \) to the logits with dimension \( d_{\text{variate}} \). Then, we get a distribution through a non-position-wise softmax activation on the tokens dimension. The preliminary prediction and all time-series patches in the lookback window will be summed using this distribution as weights. Finally, Longlongformer outputs the weight-sum result as its forecast.

4.3 Scheduling Algorithm

Dateformer decomposes time-series into time-mapped and lookback-window-mapped part, schedules Dert and Longlongformer to extrapolate them respectively. It begins by sliding the Dert encoder on input static embeddings to get the dynamic representations of all dates in the lookback window and forecast window. The Dert decoder will then construct a coarse forecast from these representations, which is also called the global forecast because Dert makes the prediction based on the information from entire visible series condensed in it. To get a more refined forecast, we leverage the adjacent time-series to provide recent information: we subtract the global forecast from the input lookback window series, and the series residuals will be sent into Longlongformer encoder later. The just
generated representations are reused as the input of Longlongformer decoder. Since it’s extrapolated from the local lookback time-series, the refined forecast output by Longlongformer is referred to as the local forecast. It’s worth noting that Longlongformer can only predict for one day at a time. Therefore, Dateformer recursively calls it until the local forecast is aligned with the global forecast. Autoregression is employed in the recursion: a day prediction appended to the residuals’ tail, a day ground-truth residual is inserted into the head of the inputs at the same time to delay the error accumulating. This also means that, at first, Dateformer will not feed all of the series residuals patches into Longlongformer, but will instead maintain a gradually expanding lookback window for it. Finally, as its output, Dateformer adds the global and local forecast together.

5 Training Methodology

The common training methodologies fail to tap the full capacity of Dateformer. In addition, it’s difficult for the model to achieve the best effects in complex settings depending on only one training configuration. Thus, we devise a three-steps training technique for Dateformer, which can be customized to deal with a variety of forecasting setups on different datasets.

Pretraining In section 3, we introduce two tasks to pretrain DERT, and the first step is to pretrain the DERT encoder. Time-series are split into patches by day to supervise the learning of DERT. Each task contributes half of the pretrained loss. The pretraining stage concentrates information from the whole visible dataset and even other datasets into DERT encoder, which breaks the information bottlenecks, and hence improves the model’s performance.

Warm-up The second step is to train the Dert separately, we call this warm-up phase. During warm-up, the pretrained DERT encoder is loaded into Dert, then it’s asked to predict time-series patches in training set. We insert this stage to force Dert to remember the global characteristics of time-series, so as to help Dateformer produce more robust long-range predictions. Furthermore, it also serves as the adapter when transferring a DERT that is pretrained on other datasets. This phase is optional, and we observed that a better short-term forecast is usually provided by the Dateformer skipping warm-up. More credible long-term predictions, however, always draw from the preheated.
Training  Dateformer loads pretrained DERT or Dert then start training. We would use a small learning rate to fine-tune the pretrained parameters. With enough memory, Dateformer can extrapolate to any number of days in the future, once trained.

6  Experiment

Datasets  We extensively perform experiments on seven real-world benchmarks, including energy, traffic, economics and weather: (1) \(ETT^3\) dataset collects 2 years (July 2016 to July 2018) electricity data from 2 transformers located in China, including oil temperature and load recorded every 15 minutes or hourly; (2) \(ECL^2\) dataset collects the hourly electricity consumption data of 321 Portugal clients from 2012 to 2015; (3) \(PL^4\) dataset contains power load series of 2 areas in China from 2009 to 2015. It’s recorded every 15 minutes and carries incomplete weather data. We eliminate the climate information and stack the 2 series into a multivariate time-series; (4) \(Traffic^3\) contains hourly record value of road occupancy rate in San Francisco from 2015 to 2016; (5) \(Weather^4\) is a collection of 21 meteorological indicators series recorded every 10 minutes by a German station for the 2020 whole year; (6) \(ER^5\) collects the daily exchange rate of 8 countries from March 2001 to March 2022. We split all datasets into training, validation and test set by the ratio of 6:2:2 for the \(ETT\) and \(PL\) datasets and 7:1:2 for others, just following the standard protocol.

Baselines  We select 6 baselines for multivariate forecasting comparisons, including 4 Transformer-based, 1 RNN-based and 1 CNN-based models: Autoformer \[32\], Informer \[33\], Pyraformer \[16\], Reformer \[12\], LSTM+T2V \[11\] and TCN \[1\]. Among them, Autoformer is the latest state-of-the-art.

Implementation details  Our proposed models are trained with L2 loss, and using AdamW \[17\] optimizer with weight decay of \(7e^{-4}\). We adjust the learning rate by the OneCycleLR \[27\] which use three phase scheduling with the percentage of the cycle spent increasing the learning rate is 0.4, and the max learning rate is \(1e^{-3}\). Batch size for pretraining is 8192, and 32 for others. The total epochs are 100, but the model normally super converges very quickly. All experiments are repeated three times, implemented by PyTorch\[22\], and trained on a NVIDIA RTX3090 24GB GPUs. The numbers of the preday and postday padding are 7 and 14 respectively. There are 1 or 4 layers in the Dert encoder, and Longlongformer contains 4 layers both in the encoder and decoder.

6.1 Main Results

We exhibit the performance of Dateformer in this section. As the first time-modeling work, we reset the forecasting setups from the time-modeling angle under the rolling forecasting setting. We evaluate models with a wide range of prediction days: 1d, 3d, 7d, 30d, 90d and 180d covering short-, medium- and long-term forecasting demand. Any length of input is allowed as long as the models can handle it. For other models, if existing, we’ll choose the input length recommended in their paper. Otherwise, an estimated input length will be given by us. Our Dateformer selects 7d, 9d, 12d, 26d, 46d and 60d as corresponding lookback lengths, and we just train Dateformer once on the 7d-predict-1d task to test all setups for each dataset. For fairness, all these sequence-modeling baselines are trained time-step-wise but tested day-wise. Due to the space limitation, only the multivariate forecasting comparison is shown here, see Appendix A for full experiments and ablation studies.

Multivariate results  Our proposed Dateformer achieves the consistent state-of-the-art performance under all setups of all benchmarks. The longer forecasting range, the more significant improvement. In the long-term forecasting setting (at least 90 days), Dateformer gives MSE reduction: 88% (1.393 → 0.151, 1.337 → 0.185) in \(PL\), 42% (1.181 → 0.690) in \(ETTm1\), 38% (0.697 → 0.431) in \(ETTh2\), 88% (1.921 → 0.240) in \(ECL\), 79% (2.489 → 0.527) in \(Traffic\) and 36% in \(ER\). Overall, Dateformer yields a 40% averaged accuracy improvement among all setups. And compared with other models, its errors rise most steadily as the forecasting range grows. It means that Dateformer

1\[https://archive.ics.uci.edu/ml/datasets/ElectricityLoadDiagrams20112014\]
2\[This dataset is provided by the 9th China Society of Electrical Engineering cup competition.\]
3\[http://pems.dot.ca.gov/\]
4\[https://www.bgc-jena.mpg.de/wetter/\]
5\[https://fred.stlouisfed.org/categories/158\]
Table 1: Multivariate results. OOM: Out Of Memory. - means failing to train because the validation set is too small to provide even a sample. Number in parentheses denotes the dataset’s granularity.

| Models | Dateformer | Autoformer | Informer | Pyraformer | Refformer | LSTM+T2V | TCM |
|--------|------------|------------|----------|------------|-----------|---------|-----|
|        | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE |
| PL (90) | 1 | 0.043 | 0.142 | 0.103 | 0.205 | 0.059 | 0.152 | 0.064 | 0.164 | 0.119 | 0.242 | 1.564 | 0.882 | 0.138 | 0.249 |
|        | 2 | 0.085 | 0.194 | 0.269 | 0.365 | 0.310 | 0.352 | 0.148 | 0.251 | 0.307 | 0.419 | 1.775 | 1.030 | 0.284 | 0.384 |
|        | 3 | 0.098 | 0.217 | 0.386 | 0.453 | 0.390 | 0.440 | 0.228 | 0.336 | 0.410 | 0.510 | 8.946 | 2.594 | 0.209 | 0.302 |
|        | 4 | 0.119 | 0.253 | 0.833 | 0.710 | 0.461 | 0.499 | OOM | OOM | OOM | OOM | 5.302 | 1.846 | 0.613 | 0.565 |
|        | 5 | 0.151 | 0.297 | OOM | OOM | OOM | OOM | OOM | OOM | OOM | 4.061 | 1.593 | 1.393 | 0.912 |
|        | 6 | 0.185 | 0.329 | OOM | OOM | OOM | OOM | OOM | OOM | OOM | 4.688 | 1.654 | 1.337 | 0.883 |
|        | 7 | 0.322 | 0.355 | 0.519 | 0.478 | 0.609 | 0.551 | 0.515 | 0.504 | 0.758 | 0.619 | 1.006 | 0.688 | 0.754 | 0.678 |
|        | 8 | 0.368 | 0.381 | 0.657 | 0.544 | 1.007 | 0.772 | 0.837 | 0.701 | 0.974 | 0.723 | 1.175 | 0.757 | 0.832 | 0.714 |
|        | 9 | 0.417 | 0.410 | 0.661 | 0.546 | 1.119 | 0.794 | 1.087 | 0.819 | 1.121 | 0.789 | 1.785 | 1.002 | 0.868 | 0.731 |
|        | 10 | 0.438 | 0.446 | 0.659 | 0.572 | 1.137 | 0.829 | OOM | OOM | OOM | 1.196 | 0.773 | 1.177 | 0.843 |
|        | 11 | 0.690 | 0.595 | OOM | OOM | OOM | OOM | OOM | 2.719 | 1.193 | 1.181 | 0.984 |
| ETL (90) | 1 | 0.234 | 0.306 | 0.315 | 0.381 | 0.501 | 0.547 | 0.356 | 0.453 | 0.764 | 0.702 | 2.794 | 1.196 | 0.951 | 0.732 |
|        | 2 | 0.311 | 0.363 | 0.355 | 0.404 | 1.665 | 1.025 | 1.063 | 0.799 | 1.311 | 0.921 | 4.967 | 1.563 | 1.449 | 0.968 |
|        | 3 | 0.383 | 0.413 | 0.475 | 0.405 | 4.128 | 1.732 | 5.600 | 1.848 | 2.522 | 1.272 | 4.714 | 1.695 | 5.205 | 2.000 |
|        | 4 | 0.437 | 0.472 | 0.502 | 0.506 | 3.773 | 1.635 | 4.293 | 1.762 | 2.823 | 1.302 | 6.203 | 1.874 | 4.213 | 1.750 |
|        | 5 | 0.431 | 0.486 | 0.697 | 0.621 | 2.711 | 1.255 | 3.270 | 1.457 | 3.398 | 1.417 | 7.585 | 2.063 | 2.759 | 1.332 |
|        | 6 | 0.113 | 0.218 | 0.158 | 0.281 | 0.290 | 0.387 | 0.261 | 0.365 | 0.290 | 0.393 | 0.498 | 0.494 | 0.351 | 0.428 |
|        | 7 | 0.148 | 0.251 | 0.184 | 0.307 | 0.334 | 0.414 | 0.264 | 0.367 | 0.273 | 0.371 | 1.023 | 0.791 | 0.411 | 0.458 |
|        | 8 | 0.163 | 0.266 | 0.206 | 0.323 | 0.355 | 0.439 | 0.278 | 0.386 | 0.322 | 0.407 | 1.226 | 0.834 | 0.351 | 0.428 |
|        | 9 | 0.187 | 0.291 | 0.328 | 0.380 | 0.315 | 0.395 | 0.280 | 0.389 | 0.288 | 0.397 | 1.738 | 1.032 | 0.465 | 0.491 |
|        | 10 | 0.220 | 0.322 | 0.361 | 0.436 | 0.578 | 0.572 | OOM | OOM | OOM | 1.815 | 1.062 | 0.436 | 0.473 |
|        | 11 | 0.240 | 0.339 | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

7 Discussion

![Figure 5: Auto-correlation of four time-series from ETT oil temperature series: (a) Ground-truth; (b) Time-Mapped; (c) Exclude Time-Mapped; (d) Remainder](image)

Figure 5: Auto-correlation of four time-series from ETT oil temperature series: (a) Ground-truth; (b) The forecast of Dert; (c) Ground-truth — Dert’s forecast; (d) Ground-truth — final forecast.

Decomposition Analysis  Auto-correlation is a common tool for analyzing series in stochastic process theory. Auto-correlation series exhibit the same seasonality as their source series. To acquire an intuitive idea of the decomposition pattern in Dateformer, we use formula 2 to perform a preliminary analysis, where τ denotes the lag time-step. For a more visual display, we intercept a period of auto-correlation and normalize it, as shown in figure 5. Combining figures 5a and 5b, gives the most credible and robust long-range forecast. What’s more, Dateformer is the only one model that is able to provide credible forecast for all demands in here.
we found that the ETT oil temperature series has complex seasonality and Dert captures it almost perfectly. Nevertheless, Dert fails to predict the series mean completely accurately: eliminating the time-mapped part, a sharp descent appears at the left end of figure 5c but didn’t immediately drop to near 0. This, we believe, is due to the fact that the trend change is not only dependent on time. Some sudden factors would affect it, and we suppose that these effects have a certain inertia, hence can be approximated from the lookback window. Although not done instantly, the series in figure 5c still converges to 0 within a short lag. This also proves our hypothesis: the impact of sudden factors will not be long-lasting. After subtracting the final forecast, we get a auto-correlation series of random oscillations around 0 in figure 5d which is as unpredictable as white noise series.

\[ R_{XX}(\tau) = \int_{-\infty}^{\infty} X_t X_{t+\tau} \, dt \]  (2)

**Forecasting Pattern** All the time, most studies on time-series forecasting are high on learning a mapping from a past subseries to a nearby future series. An important hypothesis supporting the forecasting pattern is that the adjacent subseries can provide enough information to predict. For short-term series forecasting, on most occasions, the premise is intuitively acceptable. In some cases, however, we argue that it’s no longer reliable: we can’t expect that an accurate prediction mapped from an anomalous subseries. Under the long-term series forecasting setting, the assumption is utterly incorrect. Information contained in the adjacent subseries is insufficient for long-term series forecasting. To obtain credible long-range prediction, we should exploit as much data as possible, even whole visible time-series. Previous sequence-modeling methods can’t achieve the idea, thus we design the Dert to remember the global information of the entire series. The most immediate reason why Dateformer works, we believe, is that more information is leveraged. We hope to reform the forecasting pattern to utilize all available time-series, and the time-modeling method is merely an approximate implementation, we look forward to implementing it in the sequence-modeling fashion.

8 Conclusions

In this paper, we presented Dateformer, the first time-modeling framework for time-series forecasting, to prove that the canonical Transformer is sufficient to deal with the long-term series forecasting problem. Benefitting from the time-modeling style, a novel decomposition pattern is presented in Dateformer. And more information intake allows it make robust and credible long-range forecast. Dateformer yields consistent state-of-the-art performance in extensive real-world time-series datasets.

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A. Full Experiment

A.1 Multivariate Forecast

| Models | Dateformer | Autoformer | Informer | Pyraformer | Reformer | LSTM+T2V | TCN |
|--------|------------|------------|----------|------------|----------|----------|-----|
| Metric | MSE MAE   | MSE MAE   | MSE MAE  | MSE MAE    | MSE MAE  | MSE MAE  | MSE MAE |
| ETTm1(24) | 0.324 0.357 | 0.389 0.427 | 0.587 0.545 | 0.575 0.532 | 0.531 0.555 | 0.858 0.655 | 0.724 0.661 |
| ETTm2(96) | 0.369 0.383 | 0.431 0.441 | 0.781 0.669 | 0.606 0.572 | 0.787 0.636 | 1.192 0.772 | 0.863 0.732 |
| ETTm3(384) | 0.418 0.411 | 0.509 0.492 | 0.913 0.729 | 0.787 0.673 | 0.864 0.699 | 1.760 0.965 | 0.908 0.745 |
| ETTm4(960) | 0.434 0.443 | 0.535 0.534 | 1.171 0.847 | 0.985 0.791 | 1.171 0.827 | 2.151 1.031 | 1.099 0.835 |
| ETTm5(3840) | 0.691 0.599 | 0.908 0.741 | 1.294 0.895 | 1.094 0.846 | 1.224 0.832 | 1.508 0.987 | 1.088 0.829 |

Multivariate results We list multivariate forecast results on full ETT dataset in here. It can be seen that Dateformer achieves the consistent state-of-the-art performance for all setups. In the long-term forecasting setting, Dateformer gives MSE reduction: 24% (0.908 → 0.691) in ETTm1, 42% (1.181 → 0.690) in ETTm2, 38% (0.697 → 0.431) in ETTm3, 91% (0.512 → 0.580) in ETTm4.

A.2 Univariate Forecast

Baselines We also select 6 competitive baselines for univariate forecasting comparisons, covering deep learning and statistical methods: Autoformer [32], Informer [33], N-BEATS [20], LSTM+T2V [11], DeepAR [25] and ARIMA [3].

Table 3: Univariate results

| Models | Dateformer | Autoformer | Informer | N-BEATS | LSTM+T2V | ARIMA |
|--------|------------|------------|----------|---------|----------|-------|
| Metric | MSE MAE | MSE MAE | MSE MAE | MSE MAE | MSE MAE | MSE MAE |
| Pl(40) | 0.040 0.133 | 0.133 0.231 | 0.058 0.148 | 0.076 0.164 | 1.186 0.712 | 0.645 0.518 | 0.996 0.801 |
| Traffic(24) | 0.087 0.190 | 0.366 0.427 | 0.232 0.298 | 0.202 0.272 | 1.663 0.912 | 1.163 0.770 | 1.163 0.878 |
| Weather(164) | 0.103 0.212 | 0.830 0.672 | 0.353 0.390 | 0.226 0.304 | 1.601 0.908 | 1.789 1.018 | 1.325 0.926 |
| EBR(116) | 0.113 0.239 | 0.955 0.753 | 0.395 0.452 | 0.483 0.685 | 5.200 1.847 | 3.026 1.375 | 5.994 1.292 |

Univariate results In the univariate forecasting setting, Dateformer still achieves the consistent state-of-the-art performance under all setups. In the long-term forecasting setting, Dateformer gives 24% (0.908 → 0.691) in ETTm1, 42% (1.181 → 0.690) in ETTm2, 38% (0.697 → 0.431) in ETTm3, 91% (0.512 → 0.580) in ETTm4.
In this section, we compare the results of several training methods, and verify the effectiveness of the dynamic date-representations produced by DERT. Meanwhile, we modify a Dateformer to adopt the generative style proposed by Informer, to compare our reasoning strategy. In Table 4, we list Dert and 6 Dateformers as objects to compare: Dateformer\(^1\) goes through all training stages in section 5; Dateformer\(^0\) is free of the first two phases; Structurally, we replace the canonical self-attention \(^{29}\) with the probSparse self-attention \(^{33}\), so as to examine compatibility of Dateformer; As another modification version, Dateformer-rush employs a one-step generative style decoder to output forecast. * denotes the result is selected to compare with other models in the previous text. For simplicity, Dateformer-prob and

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### Table 4: Ablation of multivariate forecast results

| Models | Dateformer\(^{1}\) | Dateformer\(^{0}\) | Dateformer\(^{0}\) | Dateformer\(^{0}\) | Dateformer-prob | Dateformer-rush |
|--------|------------------|------------------|------------------|------------------|-----------------|----------------|
|        | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE |
| ECL(1) | 1   | 0.043* | 0.142* | 0.032 | 0.119 | 0.044 | 0.142 | 0.027 | 0.112 | 0.145 | 0.272 | 0.044 | 0.147 | 0.197 | 0.227 |
|        | 2   | 0.083* | 0.194* | 0.107 | 0.220 | 0.078 | 0.192 | 0.077 | 0.176 | 0.145 | 0.273 | 0.085 | 0.202 | 0.284 | 0.396 |
|        | 3   | 0.098* | 0.217* | 0.147 | 0.271 | 0.097 | 0.219 | 0.122 | 0.227 | 0.146 | 0.274 | 0.101 | 0.223 | 0.329 | 0.266 |
|        | 5   | 0.119* | 0.253* | 0.275 | 0.375 | 0.133 | 0.269 | 0.324 | 0.384 | 0.149 | 0.277 | 0.119 | 0.254 | 0.545 | 0.485 |
|        | 7   | 0.151* | 0.297* | 0.600 | 0.587 | 0.166 | 0.311 | 0.798 | 0.679 | 0.165 | 0.295 | 0.147 | 0.294 | 0.826 | 0.692 |
|        | 90  | 0.185* | 0.329* | 0.911 | 0.733 | 0.218 | 0.358 | 1.238 | 0.843 | 0.176 | 0.308 | 0.179 | 0.323 | 0.767 | 0.613 |
|        | 180 | 0.336  | 0.364  | 0.327 | 0.357 | 0.342 | 0.371 | 0.328* | 0.357 | 0.123 | 0.876 | 0.325 | 0.363 | 0.704 | 0.647 |
|        | 200 | 0.376  | 0.390  | 0.367 | 0.383 | 0.392 | 0.400 | 0.360* | 0.383 | 0.120 | 0.874 | 0.367 | 0.389 | 0.753 | 0.702 |
|        | 300 | 0.432  | 0.426  | 0.425 | 0.416 | 0.459 | 0.440 | 0.448* | 0.411* | 0.123 | 0.873 | 0.418 | 0.419 | 0.808 | 0.739 |
|        | 400 | 0.507  | 0.492  | 0.444 | 0.447 | 0.642 | 0.560 | 0.430* | 0.443* | 0.125 | 0.869 | 0.452 | 0.462 | 1.023 | 0.825 |
|        | 500 | 0.790  | 0.640  | 0.681 | 0.589 | 1.055 | 0.741 | 0.691* | 0.599* | 0.125 | 0.865 | 0.700 | 0.610 | 1.176 | 0.848 |

In this section, we compare the results of several training methods, and verify the effectiveness of the dynamic date-representations produced by DERT. Meanwhile, we modify a Dateformer to adopt the generative style proposed by Informer, to compare our reasoning strategy. In Table 4, we list Dert and 6 Dateformers as objects to compare: Dateformer\(^{1}\) goes through all training stages in section 5; Dateformer\(^0\) is free of the first two phases; Structurally, we replace the canonical self-attention \(^{29}\) with the probSparse self-attention \(^{33}\), so as to examine compatibility of Dateformer; As another modification version, Dateformer-rush employs a one-step generative style decoder to output forecast. * denotes the result is selected to compare with other models in the previous text. For simplicity, Dateformer-prob and Dateformer-rush will be trained with the same steps as them. **Training methods comparison** Comparing various training methods, we concluded there is no versatile training strategy that does the best for all forecast tasks on every dataset yet. For most datasets here, pretraining can bring a slight improvement, at least not a significant decrease, so it's
recommended by us. On the other hand, a better short-term forecast is usually provided by the Dateformer without warm-up, and the majority of best long-term forecasts are produced by the preheated one. We suggest that the training stage should be flexibly configured according to the different forecast requirements. It’s desirable to train a Dateformer individually for each setup if conditions permit. It should be noted that we use only one Dateformer to compare with other models, for each dataset. A greater improvement will be reported if we follow the above procedure. To keep the scalability of Dateformer, we didn’t do so. In addition, with the increase of forecast length, in some datasets like PL, the predictive accuracy of Dateformer that skips warm-up plummets, even sometimes worse than a separate Dert. We call this degradation: the Dateformer predicts too much depending on the local information (lookback series), which hurts its long-term forecasting ability. It clearly demonstrates why, for the long-term forecasting demand on these time-series, Dert should be forced to retain the series’ global characteristics before formal training.

Self-attention compatibility Dateformer is compatible with sparse versions of self-attention. Replacing the canonical self-attention with the probSparse version, there is no obvious performance damage that occurred in Dateformer, even a little better in some datasets. Despite this, Dateformer is unable to further extend its forecast range using the sparse technique: the factor restricting the model to conduct a longer-term forecast is that so much time-series data can’t be stored in limited memory at the same time. This trouble can be easily solved through simple lazy loading and instant output strategy, so as to broaden the maximum forecast length to theoretical infinity at the expense of a little reasoning speed. That is an engineering problem, thus we won’t discuss it here.

Inference style The poor performance of Dateformer-rush can prove the superiority of our comprehensive reasoning style. The model is good for nothing but inference speed. Despite the shortcomings of slow reasoning and cumulative error, Autoregression is still an appropriate reasoning strategy. It’s feasible to develop strengths and fix weaknesses of Autoregression. Combining the day-wise reasoning style, we take advantage of the variable-length processing capability of Transformer, delaying the error accumulating, accelerating inference and giving Dateformer with scalability.

| Metric | 1  | 3  | 7  | 30 | 90 | 180 |
|--------|----|----|----|----|----|-----|
| Dert   | 0.145 | 0.145 | 0.146 | 0.149 | 0.165 | 0.176 |
| FFN    | 0.189 | 0.190 | 0.191 | 0.192 | 0.207 | 0.201 |

Dynamic date-representations To verify the polysemy of date and effectiveness of dynamic representations, we replace the self-attention layers in Dert encoder with a position-wise feed-forward network that has roughly the same amount of parameters, and test them on the PL time-series. In Table 5, Dert consistently outperformed the FFN with same input static date-embeddings. It’s enough to prove the effectiveness of the dynamic date-representation. In order to show the polysemy of date more intuitively, we visualized the distribution of the Dert encoder’s attention, see Appendix E.

A.4 Few Sample Learning

We are amazed to find that only a small amount of data is needed to train Dateformer, especially when a DERT pretrained from other datasets is prepared. Take the ETTm1 dataset as example, we train Dateformer on time-series ranging full 12 months (353 samples), 8 months (233 samples), 4 months (113 samples) and 8 days (1 sample), to test all setups on the same test set. Among these series, the full 12 months time-series is the standard setting. A Dateformer that employs a DERT encoder pretrained from PL acts as the comparison to check if the model can benefit from other bigger datasets. The two Dateformers both skip the warm-up phase during training.

Refer to Table 5, we found the Dateformers trained with few samples also can produce acceptable forecasts. In the extreme case of only one training sample provided, Dateformer still achieves the state-of-the-art performance compared with other models. With the increase of samples, the performance of the model is better and better until it reaches saturation: 233 samples are enough for Dateformer to convergence on the ETTm1 dataset. In addition, the Dateformer whose Dert encoder is pretrained on the PL dataset always delivers a little superior output, particularly under the 1 sample
learning setting. This may prove that Dateformer can transfer the knowledge learned from other datasets to current work. The full details are not yet clear, we will improve it in the future work.

**B Hyper Parameter Sensitivity**

In this section, we check Dateformer’s robustness with respect to the hyper-parameters: preday padding and postday padding. We select 6 groups of paddings: (1, 1), (3, 3), (7, 7), (7, 14), (14, 14) and (30, 30). And test them on the 7d-predict-1d or 47d-predict-96d (only for the ER dataset) multivariate forecasting tasks. All these Dateformers are trained from scratch and skip the warm-up, the results are shown in Table 7. It can be seen that the size of paddings doesn’t significantly affect the performance of Dateformer, but we still recommend a moderate size for the paddings.

**C Date-embedding**

Observing the movement of celestial planets, changes in temperature, or the rise and fall of plants, our forefathers distilled a set of laws, that is the calendar. A wealth of wisdom is encapsulated in the calendar. People’s activities are guided by the calendar and we believe that’s the primary cause of seasonality in many time-series. We try to introduce the wisdom in our models as a priori knowledge. We use the Gregorian calendar as the solar calendar and the traditional Chinese calendar as the lunar calendar to deduce our date-embedding. Besides, the vacation and weekday information is also taken into count. In our code, we provide the date-embeddings from 2001 to 2023 for 12 countries or regions: Australia, British, Canada, China, Germany, Japan, New Zealand, Portugal, Singapore, Switzerland, USA and San Francisco, California.

Our static date-embedding contains the following time features: abs_day, year, day (month_day), year_day, weekofyear, lunar_year, lunar_month, lunar_day, lunar_year_day, dayofyear, dayofmonth, monthofyear, dayofweek, dayoflunaryear, dayoflunarmonth, monthoflunaryear, jieqiofyear, jieqi_day, dayoffieqi, holidays, workdays, residual_holiday, residual_workday. As an example, we embed today by following equations:

\[
\text{abs}_\text{day} = \frac{\text{days that have passed}^1 \text{ from December 31, 2000}}{365.25 \times 5}
\]

\[
\text{year} = \frac{\text{this year} - 1998.5}{25}
\]

\(^1\)All similar expressions in this section include today.
day = \text{days that have passed in this month}\n\frac{31}{31}
\text{year}_\text{day} = \text{days that have passed in this year}\n\frac{366}{366}
\text{weekofyear} = \text{weeks that have passed in this year}\n\frac{54}{54}
\text{lunar}_\text{year} = \text{this lunar year} - 1998.5
\frac{25}{25}
\text{lunar}_\text{month} = \text{this lunar month}\n\frac{12}{12}
\text{lunar}_\text{day} = \text{days that have passed in this lunar month}\n\frac{30}{30}
\text{lunar}_\text{year}_\text{day} = \text{days that have passed in this lunar year}\n\frac{384}{384}
\text{dayofyear} = \text{days that have passed in this year} - 1 \frac{\text{total days in this year} - 1}{366} - 0.5
\text{dayofmonth} = \text{days that have passed in this month} - 1 \frac{\text{total days in this month} - 1}{31} - 0.5
\text{monthofyear} = \text{months that have passed in this year} - 1 \frac{11}{366} - 0.5
\text{dayofweek} = \text{days that have passed in this week} - 1 \frac{6}{6} - 0.5
\text{dayoflunaryear} = \text{days that have passed in this lunar year} - 1 \frac{\text{total days in this lunaryear} - 1}{384} - 0.5
\text{dayoflunarmonth} = \text{days that have passed in this lunar month} - 1 \frac{\text{total days in this lunar month} - 1}{29} - 0.5
\text{monthoflunaryear} = \text{lunar months that have passed in this lunaryear} - 1 \frac{11}{384} - 0.5
\text{jieqiofyear} = \text{solar terms that have passed in this year} - 1 \frac{23}{23} - 0.5
\text{jiei}_\text{day} = \text{days that have passed in this solar term}\n\frac{15}{15}
\text{dayofjieqi} = \text{days that have passed in this solar term} - 1 \frac{\text{total days in this solar term} - 1}{23} - 0.5

We are sorry to say that the calculation of the last four features is far too complex for us to explain using formulas or simple words. Please email the first author if you are interested in it.
D Broader Impact

This work focuses on pure scientific research, so there is no potential ethical risk.

E Visualization of Date’s Polysemy

Figure 6: DeRt encoder’s attention weight distribution of PL dataset during 2012. (a) is encoding Jan 2, Jan 1 is New Year’s day; (b) is encoding Jan 24, in the traditional Chinese lunar calendar, Jan 22 is New Year’s Eve and Jan 23 is the Spring Festival; (c) is encoding Apr 5, Apr 4 is the Qingming Festival. (d) is encoding Oct 2, Sep 30 is the Mid-Autumn Festival in Chinese lunar calendar, and Oct 1 is National Day in the Gregorian calendar.

F Showcases

Here, we visualize some of the forecast results and compare them with other models.

Figure 7: 3 days forecast case from the PL dataset
Figure 8: 7 days forecast case from the PL dataset

Figure 9: 3 days forecast case from the Traffic dataset

Figure 10: 7 days forecast case from the Traffic dataset

Figure 11: Mid-long-term forecast cases of Dateformer from the PL dataset