PLANET TRAPS AND FIRST Planets: THE CRITICAL METALICITY FOR GAS GIANT FORMATION

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Received 2014 February 24; accepted 2014 April 19; published 2014 May 21

ABSTRACT

The ubiquity of planets poses an interesting question: when are first planets formed in galaxies? We investigate this by adopting a theoretical model where planet traps are combined with the standard core accretion scenario in which the efficiency of forming planetary cores directly relates to the metallicity ([Fe/H]) in disks. Three characteristic exoplanetary populations are examined: hot Jupiters, exo-Jupiters around 1 AU, and low-mass planets in tight orbits, such as super-Earths. We statistically compute planet formation frequencies (PFFs), as well as the orbital radius (⟨R_{rapid}⟩) within which gas accretion becomes efficient enough to form Jovian planets, as a function of metallicity (−2 ≲ [Fe/H] ≲ −0.6). We show that the total PFFs for these three populations increase steadily with metallicity. This is the direct outcome of the core accretion picture. For the metallicity range considered here, the population of low-mass planets dominates Jovian planets. The Jovian planets contribute to the PFFs above [Fe/H] ≃ −1. We find that the hot Jupiters form more efficiently than the exo-Jupiters at [Fe/H] ≲ −0.7. This arises from the slower growth of planetary cores and their more efficient radial inward transport by the host traps in lower metallicity disks. We show that the critical metallicity for forming Jovian planets is [Fe/H] ≃ −1.2 by comparing ⟨R_{rapid}⟩ of hot Jupiters and low-mass planets. The comparison intrinsically links to the different gas accretion efficiency between these two types of planets. Therefore, this study implies that important physical processes in planet formation may be tested by exoplanet observations around metal-poor stars.

Key words: accretion, accretion disks – methods: analytical – planet–disk interactions – planets and satellites: formation – protoplanetary disks – turbulence

Online-only material: color figures

1. INTRODUCTION

The unprecedented success in exoplanet observations has inferred that planet formation occurs in various environments (e.g., Udry & Santos 2007; Mayor et al. 2011; Batalha et al. 2013). The metallicity of host stars provides one of the major factors for discovering exoplanets (e.g., Santos et al. 2004; Schlaufman & Laughlin 2011; Buchhave et al. 2012). The observations suggest that massive planets such as our Jupiters are observed preferentially around metal-rich stars, whereas the detectability of low-mass planets, such as super-Earths, apparently does not correlate with stellar metallicity. The metallicity dependence is often referred to as the “planet–metallicity relation.” Currently, the presence of exoplanets is confirmed around stars with a wide range of metallicities ([Fe/H] ≲ 0.6). The lowest metallicity at which massive exoplanets are observed is so far [Fe/H] ≃ −2 (Setriawan et al. 2010; Setiawan et al. 2012), although these observations are currently a matter of debate. In fact, other observations show that the presence of planets for the same targets is very likely to be ruled out (Desidera et al. 2013; Müller et al. 2013; Jones & Jenkins 2014). Most observed massive and low-mass planets are confined well within [Fe/H] ≥ −0.6.

A successful theoretical framework to understand the exoplanet observations is the so-called core accretion scenario (e.g., Pollack et al. 1996; Ida & Lin 2004a). In this scenario, gas giants undergo sequential accretion of dust and gas: cores of gas giants form first via oligarchic growth by planetesimal collisions (e.g., Kokubo & Ida 1998; Thommes et al. 2003), and then the cores accrete surrounding gas and form massive envelopes around them (e.g., Mizuno 1980; Ikoma et al. 2000; Lissauer et al. 2009; Movshovitz et al. 2010). Population synthesis calculations developed by this picture confirmed that massive planets within the orbital radius r = 10 AU can be built within the disk lifetime (∼10^{6} yr; e.g., Ida & Lin 2004a; Mordasini et al. 2009; Hasegawa & Pudritz 2013). Furthermore, the calculations succeeded well in reproducing the planet–metallicity relation. This arises because the efficiency of forming planetary cores is directly linked to the number density of dust in disks in the model, and the cores accrete surrounding gas and form massive envelopes around them (e.g., Mizuno 1980; Ikoma et al. 2000; Lissauer et al. 2009; Movshovitz et al. 2010). Population synthesis calculations developed by this picture confirmed that massive planets within the orbital radius r = 10 AU can be built within the disk lifetime (∼10^{6} yr; e.g., Ida & Lin 2004a; Mordasini et al. 2009; Hasegawa & Pudritz 2013). Furthermore, the calculations succeeded well in reproducing the planet–metallicity relation. This arises because the efficiency of forming planetary cores is directly linked to the number density of dust in disks in the model (e.g., Ida & Lin 2004a; Mordasini et al. 2012; Hasegawa & Pudritz 2014). One of the intriguing questions that remain to be addressed is whether or not we can extrapolate these results to metal-poor stars ([Fe/H] < −0.6). What is the critical metallicity for forming gas giants in the standard core accretion picture?

In this paper, we explore the problem and quantify at what value of metallicity the formation of gas giants can proceed. To this end, we carry out a statistical analysis for planetary populations by using a semi-analytical model developed in a series of earlier papers (Hasegawa & Pudritz 2011, 2012, 2013, 2014, hereafter, HP11, HP12, HP13, HP14). By computing planet formation and migration in evolving gaseous disks, we estimate the planet formation frequency (PFF), as well as the averaged value of R_{rapid} (defined as ⟨R_{rapid}⟩) within which gas accretion onto the cores becomes rapid (∼10^{5} yr). The analysis is applied to three different planetary populations that are prominent in the exoplanet observations (see Table 1): hot Jupiters...
(0.01 < \( r_p/\text{AU} < 0.5 \), 30 < \( M_p/M_\oplus < 10^4 \), where \( r_p \) and \( M_p \) are the semi-major axis and the mass of planets, respectively), exo-Jupiters (0.5 < \( r_p/\text{AU} < 10 \), 30 < \( M_p/M_\oplus < 10^4 \)), and low-mass planets with short orbital periods, such as super-Earths (0.01 < \( r_p/\text{AU} < 0.5 \), 1 < \( M_p/M_\oplus < 30 \)).

As shown below, we find that the total PFFs for all the three populations are correlated positively with metallicity. This arises from the nature of the core accretion picture. The PFFs of the low-mass planets, which are formed as failed cores of gas giants and/or mini-gas giants in our model, correspond to those of the total at up to [Fe/H] \( \simeq -1 \) above which the population of the Jovian planets becomes non-negligible. Our results show that lower metallicity disks tend to create the hot Jupiters more easily than the exo-Jupiters, which originates from the combined effects of planet formation with planetary migration. We also demonstrate that the exo-Jupiters become dominant over the hot Jupiter population at [Fe/H] \( \gtrsim -0.7 \). Finally, we examine \( (R_{\text{rapid}}) \) for the hot Jupiters as well as the low-mass planets, which essentially traces the difference in the efficiency of gas accretion between them, and find that the critical metallicity for gas giant formation is [Fe/H] \( \simeq -1.2 \). Our results therefore imply that the behavior of the PFFs for different planetary populations and the resultant characteristic metallicities may link deeply to important physical processes involved with planet formation. Thus, exoplanet observations around metal-poor stars may contain an invaluable potential to examine these processes.

The plan of this paper is as follows. In Section 2, we briefly describe the semi-analytical model that we have employed. In Section 3, we present our results and derive the critical metallicity above which gas giant formation can take place. In Section 4, we discuss potential issues that may arise when planet formation around metal-poor stars is considered. We also examine how valid our results are by comparing the exoplanet observations that are currently available. Finally, we present our conclusions in Section 5.

2. SEMI-ANALYTICAL MODELS

We briefly summarize the model adopted in this paper. We heavily rely on a formulation developed in a series of papers (HP11; HP12; HP13; HP14) and refer the readers to these papers for details. Table 2 summarizes important quantities involved with this work. Also, all the parameters adopted in this paper are exactly the same as HP14, except for the value of \( \eta_{\text{bg}} \) or [Fe/H].

2.1. Basic Model

The model is designed to compute planet formation and migration in viscously evolving disks with photoevaporation of gas (HP12). In order to visualize a series of these processes, a set of theoretical evolutionary tracks of growing planets are constructed in the mass–semi-major axis diagram (see Figure 1). Since the full description of tracks can be found elsewhere (e.g., HP12 or HP14), we focus on the key ingredients in our model.

The essence of the model is to incorporate planet traps that are considered as one of the promising solutions to the well-known problem of rapid type I migration for planetary cores (Masset et al. 2006; Matsumura et al. 2007; Ida & Lin 2008b; Hasegawa & Pudritz 2010; Lyra et al. 2010; HP11). Planetary migration arises from resonant, tidal interactions between protoplanets and the surrounding gaseous disks out of which the protoplanets are born (e.g., Kley & Nelson 2012). It is well recognized that type
mass planets is very rapid (\( \sim 10^5 \text{ yr} \)), and its direction depends strongly on the disk properties such as the surface density and the temperature of disks. As a result, it is important to consider the disk properties in detail to examine the effects of type I migration. Note that type I migration is distinguished from type II migration, which occurs when protoplanets are massive enough to open up a gap in their disks.

The presence of planet traps in protoplanetary disks is a natural consequence of disk structures that deviate locally from simple power-law approximation (HP11). Due to the high sensitivity of rapid type I migration to disk structures (e.g., Paardekooper et al. 2010), such local variations can act as barriers for the migration. In addition, the location of the traps varies following disk evolution, since the disk properties evolve with time. As shown analytically by HP11 and numerically by Matsumura et al. (2009), planet traps move inward on a timescale similar to the disk lifetime (\( \sim 10^7 \text{ yr} \)). This slow, inward movement of traps is of fundamental importance both to prevent planetary cores from plunging into the central stars within the disk lifetime and to form gas giants at \( r \gtrsim 1 \text{ AU} \) efficiently (HP12). Planet traps are effective until trapped protoplanets become massive enough to switch to type II migration. We consider three disk properties (dead zones, ice lines, and heat transitions) and associated traps (HP11).

The significance of planet traps can be well recognized when evolutionary tracks of planets forming at planet traps are constructed. As demonstrated by HP12, the combination of the core accretion picture with planet traps leads to a planetary population that is consistent with the radial velocity observations of exoplanets in a sense that the end points of tracks line up with the observed planets.

We now discuss the core accretion scenario that is adopted as the formulation of planetary growth in the model. This scenario consists of two distinct processes: the formation of cores, followed by gas accretion onto the cores (HP12; HP14; see also Ida & Lin 2004a). For the core formation, the timescale of the oligarchic growth is used for computing the mass evolution of planetary cores (Kokubo & Ida 2002). Since the timescale decreases with increasing the dust density or \([\text{Fe/H}]\) in disks (see Equation (1)), the efficiency of planet formation, especially massive planets, becomes an increasing function of \([\text{Fe/H}]\). The feature of the core accretion picture is already shown in previous studies (Ida & Lin 2004b; Mordasini et al. 2012, HP14) and is regarded as clear evidence that the core accretion picture is preferred for understanding the exoplanet observations.

For gas accretion, we adopt the conservative approach that is used in one of the popular population synthesis calculations (Ida & Lin 2004a, 2008a; Ida et al. 2013). Practically, gas accretion starts after the mass of planetary cores exceeds the critical value \( (M_{\text{crit}})\) beyond which hydrostatic envelopes around the cores cannot be maintained (Mizuno 1980; Bodenheimer & Pollack 1986; Ikoma et al. 2000; Hori & Ikoma 2010). The mass increase via the gas accretion is then determined by the Kelvin–Helmholtz timescale \( (\tau_{KH})\) that characterizes the gravitational contraction of the envelopes (e.g., Ikoma et al. 2000; Ida & Lin 2004a). The actual formulae for \( M_{\text{crit}}\) and \( \tau_{KH}\) are derived from detailed numerical simulations (Ikoma et al. 2000). Note that these two formulae contain three fundamental parameters (see Table 2). \( M_{\text{crit}}\) is the key parameter to regulate the value of \( M_{\text{crit}}\); whereas \( \tau_{KH}\) is determined entirely by a set of two parameters, \( c \) and \( d \). As shown in HP14, these three parameters can be constrained by the exoplanet observations in which the number of exoplanets that are detected by the radial velocity methods is examined as a function of stellar metallicity; the best set is \( M_{\text{crit}} = 5 \ M_\odot\), \( c = 9\), and \( d = 3\). We adopt these three values for the following calculations.

Finally, the metallicity \( ([\text{Fe/H}])\) is treated as a parameter in our model and is handled by (Ida & Lin 2004b, HP14, see also Tables 2 and 3 for a definition of quantities):

\[
[\text{Fe/H}] \simeq [m/\text{H}] \equiv \log_{10}\left(\frac{\dot{\Sigma}_{\text{dust}}}{\dot{\Sigma}_{\text{gas},\odot}}\right) = \log_{10}(\eta_{\text{dust}}),
\]

where \( m \) represents a mixture of metals. Note that \( \eta_{\text{dust}}\) regulates the surface density of dust in disks that is related to the surface density of gas \( (\Sigma_g)\) as \( \Sigma_g \propto \eta_{\text{dust}}\Sigma_g\). As pointed out above, the variation of \( \eta_{\text{dust}}\) thus affects the efficiency of forming cores of gas giants (Ida & Lin 2004b; Mordasini et al. 2012; HP14).

### 2.2. The Condition for Rapid Gas Accretion

In this paper, the timescale of gas accretion \( (\tau_{KH})\) plays an important role. This is because the fate of growing planets, either the Jovian planets or the low-mass planets, is determined by the efficiency of gas accretion. As an example, when the formation of a core completes in the early stage of disk evolution and the core mass is large enough, the subsequent gas accretion proceeds efficiently, and the core can grow to a gas giant

### Table 3

**Input Parameters for the Statistical Analysis**

| Symbol | Meaning | Values |
|--------|---------|--------|
|         |         |        |
| \( M_* \) | Stellar mass | 1 \( M_\odot \) |
| \( R_* \) | Stellar radius | 1 \( R_\odot \) |
| \( T_* \) | Stellar effective temperature | 5780 K |
| \( \eta_{\text{acc}} \) | A dimensionless factor for varying the disk accretion rate \( (\dot{M}) \) | \( 0.1 \leq \eta_{\text{acc}} \leq 10 \) |
| \( w_{\text{mass}}(\eta_{\text{acc}}) \) | Weight function for \( \eta_{\text{acc}} \) modeled by the Gaussian function |        |
| \( \eta_{\text{dust}} \) | A parameter for varying the metallicity \( ([\text{Fe/H}], \text{see Equation (1)}) \) | \( 0.01 \leq \eta_{\text{dust}} \leq 0.25 \) |

**Disk lifetime parameters**

| \( \eta_{\text{dep}} \) | A dimensionless factor for varying \( \tau_{\text{dep}} \) (see Table 2) | \( 0.1 \leq \eta_{\text{dep}} \leq 10 \) |

**Note.** These quantities are derived from the observations of protoplanetary disks.
within the disk lifetime. For instance, \( \tau_{KH} < 10^6 \) yr for \( M_p > 10 \, M_{\oplus} \), which is comparable to or shorter than the typical disk lifetime \( (\sim 10^6 \text{–} 10^7 \) yr). The opposite situation can be attained if a core needs a long time to initiate gas accretion and its mass is small. When \( M_p < 1 \, M_{\oplus} \), \( \tau_{KH} > 10^6 \) yr, which is beyond the disk lifetime. This argument therefore indicates that the examination of \( \tau_{KH} \) can allow differentiation of the Jovian planets from the low-mass planets that are formed as failed cores of gas giants and/or mini-gas giants. Note that some studies, including this work, show that a large fraction of low-mass planets in tight orbits such as super-Earths can be generated via the same mechanism as forming gas giants, but with the different efficiency of gas accretion (see one of the dotted lines without a circle in Figure 1).

Here, we focus on the characteristic orbital radius \( R_{\text{rapid}} \) rather than directly examining \( \tau_{KH} \), where \( R_{\text{rapid}} \) is defined as the orbital radius within which the gas accretion becomes efficient enough so that protoplanets can grow to be gas giants. This is because examination of \( R_{\text{rapid}} \) for different planetary populations can be readily achieved in computing evolutionary tracks in the mass–semi-major axis diagram (see Figure 1). Practically, we assume that the evolutionary tracks arrive at \( R_{\text{rapid}} \) if the following condition is satisfied.

\[
\tau_{KH} / \tau_{\text{rapid}} < 1, \quad (2)
\]

where \( \tau_{\text{rapid}} = 10^5 \) yr. We confirmed that the specific choice of \( \tau_{\text{rapid}} \) does not affect our conclusions if \( \tau_{\text{rapid}} \) is sufficiently shorter than the typical disk lifetime \( (\sim 10^6 \text{–} 10^7 \) yr).

Figure 1, as an example, shows how \( R_{\text{rapid}} \) is estimated along the tracks (see the circles). Note that one of the tracks that end up in the zone of low-mass planets does not experience the rapid gas accretion and hence no circle on the track (see one of the dotted lines).

It is important that these examples show that gas giants end up with a larger value of \( R_{\text{rapid}} \) than low-mass planets. Such a difference in \( R_{\text{rapid}} \) for different planetary populations can be understood as follows. In our model, planet formation is intimately coupled with inward planetary migration, either via the inward movement of planet traps or via the inward type II migration. As a result, the orbital radius of planets shrinks with time as they grow. Since it is preferred to undergo the rapid gas accretion in an earlier stage of disk evolution in order to form gas giants efficiently, such planets tend to have a larger value of \( R_{\text{rapid}} \) than planets that eventually fill out the regime of the low-mass planets. Note that, as pointed out above, the low-mass planets are formed as failed cores of gas giants and/or mini-gas giants in our model. This in turn indicates that planets ending up in the zone of the low-mass planets do not necessarily have \( R_{\text{rapid}} = 0 \). For the case that \( R_{\text{rapid}} > 0 \), the low-mass planets undergo rapid gas accretion, predominantly in the final disk evolution stage, where the disk dissipation proceeds very rapidly, and hence the cores cannot build massive envelopes within the disk lifetime. The finally formed planets for this case therefore are regarded as mini-gas giants (see the circle on one of the dotted lines in Figure 1). For the case \( R_{\text{rapid}} = 0 \), on the contrary, the rapid gas accretion never occurs. As a result, the low-mass planets are considered as failed cores of gas giants with low-mass or negligible envelopes (see one of the dotted lines without a circle in Figure 1).

Thus, it is expected that the statistical analysis of \( R_{\text{rapid}} \) for different planetary populations can be a useful tool for distinguishing these populations, and eventually allow one to quantify the critical metallicity for gas giant formation (see below).

### 2.3. Statistical Approach

The quantitative comparison with observations is crucial for deriving invaluable constraints on theories of planet formation. For this purpose, a new statistical approach was developed in HP13 (see also HP14). The input parameters for the statistical analysis are summarized in Table 3.

This approach starts off from dividing the mass–semi-major axis diagram into zones (see Figure 1; see also Table 1). Such partition is well motivated by the exoplanet observations which show that most currently observed exoplanets distribute well in these zones (Chiang & Laughlin 2013, HP13).

Then, a large number \( (N_{\text{int}} = 300) \) of tracks are computed by focusing on two important disk parameters: the disk accretion rate \( (\eta_{\text{acc}}) \) and disk lifetime \( (\eta_{\text{dep}}) \); see Table 3). The former essentially scales the disk mass, whereas the latter involves the efficiency of photoevaporation that dissipates gas disks completely at the final stage of disk evolution. In addition to the stellar parameters, population synthesis calculations confirm that a set of these three parameters are crucial for regulating planet formation in protoplanetary disks (e.g., Ida & Lin 2004a; Mordasini et al. 2009).

Counting the number of final points of tracks that end up in a certain zone, Zone \( i \), PFFs are evaluated as (HP13)

\[
PFFs(\text{Zone } i) \equiv \sum_{\eta_{\text{acc}}} \sum_{\eta_{\text{dep}}} w_{\text{mass}}(\eta_{\text{acc}}) w_{\text{lifetime}}(\eta_{\text{dep}}) \times \frac{N(\text{Zone } i, \eta_{\text{acc}}, \eta_{\text{dep}})}{N_{\text{int}}}, \quad (3)
\]

where \( N(\text{Zone } i, \eta_{\text{acc}}, \eta_{\text{dep}}) \) is the total number of tracks that eventually distribute in Zone \( i \) after gas disks dissipate, \( N_{\text{int}} = 300 \) is the total number of tracks considered in single calculations, and \( w_{\text{mass}} \) and \( w_{\text{lifetime}} \) are both weight functions for \( \eta_{\text{acc}} \) and \( \eta_{\text{dep}} \), respectively.

The above quantity can be regarded as the PFFs for the following reasons. First, the ratio \( N(\text{Zone } i, \eta_{\text{acc}}, \eta_{\text{dep}})/N_{\text{int}} \) can be considered as the “efficiency” of forming planets in Zone \( i \) for a specific set of \( \eta_{\text{acc}} \) and \( \eta_{\text{dep}} \). Second, the ratio is then integrated over a wide range of both \( \eta_{\text{acc}} \) and \( \eta_{\text{dep}} \) with the weight functions \( w_{\text{mass}} \) and \( w_{\text{lifetime}} \). These two weight functions are represented by the Gaussian function and are formulated by the observations of protoplanetary disks (HP13). Specifically, \( w_{\text{mass}} \) and \( w_{\text{lifetime}} \) are essentially analytical modeling of the observed distribution of the disk accretion rate and the disk lifetime, respectively. Consequently, the PFFs can be compared with the observations directly without the standard population synthesis calculations being performed.

One of the strong advantages in this approach is that it allows one to examine theories of planet formation quantitatively. As discussed above, HP14 have recently applied this approach to the observational planet–metallicity relation and derived the best set of three fundamental parameters \( (M_{c, \text{crit}}, c, \text{ and } d) \) in the model of planet formation (see Table 2). One of the major findings of HP14 is that the canonical value of \( M_{c, \text{crit}} (\sim 10 \, M_{\oplus}) \) that is widely adopted in the literature is likely to be too large to reproduce the observations.

As already discussed in Section 2.2, it is important to estimate the value of \( R_{\text{rapid}} \) within which the gas accretion becomes efficient. We calculate \( R_{\text{rapid}}(\text{Zone } i) \), a statistically averaged
We first discuss the results of the PFFs for the total, as well as the low-mass planets, and then examine those for both hot and exo-Jupiters.

Figure 2 (top panel) shows that the total PFFs steadily increase with metallicity (see the thick line). For the low-mass planets (see the dotted line), the PFFs behave similarly to those of the total until the metallicity goes up to [Fe/H] ≃ −1.2. Beyond such metallicities, the rapid increase in the PFFs ceases, and their value becomes almost constant toward [Fe/H] ≃ −0.6.

The behavior of the total PFFs can be understood as a direct reflection of the core accretion scenario. As discussed above, the efficiency of planet formation in this picture is a direct reflection of the core accretion scenario. As discussed above, the efficiency of planet formation in this picture is correlated positively with the amount of the dust in disks (see Equation (1)). Thus, it is evident in our model that more planets are formed for higher metallicity disks.

The deviation of the low-mass planets from the total PFFs originates from the rapid increase in the PFFs for the Jovian planets. As shown in the middle panel, both the Jovian planets achieve higher values of PFFs at [Fe/H] ≥ −0.7. Consequently, the contribution of gas giants becomes non-negligible around such metallicities. In addition, we recall that low-mass planets are formed via the same mechanism as gas giants in our model. Specifically, they are regarded as failed cores of gas giants and/or mini-gas giants. Our results therefore imply that the dominant end product starts switching from low-mass planets to massive ones at [Fe/H] ≥ −0.7 (see the following discussion).

Figure 2 (middle panel) shows the resultant PFFs for both the hot and exo-Jupiters (see the dashed and solid lines, respectively). We find that the PFFs for the hot Jupiters are higher than those for the exo-Jupiters at lower metallicity ([Fe/H] ≲ −0.7). The results also show that the PFFs for the exo-Jupiters increase with metallicity more rapidly than those for the hot Jupiters. The crossover occurs at [Fe/H] ≃ −0.7 (see the vertical solid line).

We discuss a physical reason of why the formation of the hot Jupiters proceeds more efficiently in lower metallicity disks than that of the exo-Jupiters. This can be understood as follows. First, the lower metallicity environments result in the slower rate of planetary growth. This is obvious from the core accretion picture (e.g., Kokubo & Ida 2002; see also HP14). Second, the effects of

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Blow-up version of the bottom panel of Figure 2. The value of \( \langle R_{\text{rapid}} \rangle \) for the hot Jupiters sharply exceeds that of the low-mass planets at [Fe/H] ≃ −1.2. (A color version of this figure is available in the online journal.)

### 3.1. The PFFs

We present our results in Figures 2 and 3. We show the resultant PFFs for three different planetary populations and their total, as a function of metallicity, and discuss the critical metallicity for forming Jovian planets. Based on our preliminary results, we confirmed that it is sufficient to consider a certain range of metallicities (−2 ≤ [Fe/H] ≤ −0.6).
planet traps then become more prominent. As described briefly in Section 2.1 and substantially in HP12, planet traps can play a crucial role in capturing and moving planetary cores inward. Such trapped cores drop out from their host traps when they obtain the gap-opening mass that is an increasing function of the distance from the central stars. As a result, when planetary cores take a long time to become massive enough to achieve the gap-opening mass, they experience radial drift by their traps over a long distance. This ends up with the distribution of finally formed gas giants that are more likely to fill out the zone of the hot Jupiters than that of the exo-Jupiters in the lower metallicity environment.

It is interesting that the dominant Jovian population switches from the hot Jupiters to the exo-Jupiters at $[\text{Fe/H}] \simeq -0.7$. This occurs because once the formation of gas giants becomes efficient enough, the zone of exo-Jupiters is a more preferred place for gas giants to end up. This trend is already shown in HP13, which demonstrates that the resultant PFFs are in good agreement with the exoplanet observations, wherein most observed gas giants at $r \lesssim 10$ AU are densely populated around 1 AU with fewer hot Jupiters (see also HP14).

### 3.2. The Critical Metallicity

We discuss various characteristic metallicities for forming planets, utilizing the above results. Our results show that the minimum metallicity for low-mass planets is $[\text{Fe/H}] \simeq -1.8$ (see the top panel of Figure 2). Nonetheless, we focus the following argument only on gas giant formation because low-mass planet formation can also proceed in planetesimal disks that emerge after gas disks totally dissipate, which is not modeled in this study. As a result, the above estimate would provide only an upper limit for the low-mass planets.

Based on the above discussion, the critical metallicity for forming gas giants can be estimated intuitively as $[\text{Fe/H}] \simeq -1$ (see the middle panel of Figure 2). In the following, we specify the value more sharply, examining the value of $\langle R_{\text{rapid}} \rangle$ (see Sections 2.2 and 2.3 for the definition).

Our results suggest that the critical metallicity for gas giant formation will be derived from the hot Jupiters, rather than the exo-Jupiters. The main reason for this is that the population of the hot Jupiters becomes dominant over the exo-Jupiters at $[\text{Fe/H}] \lesssim -0.7$. As already pointed out, this occurs due to the lower growth rate of cores in disks with lower metallicities and the resultant enhancement of the inward transport of the cores by their traps. We refer to the metallicity above which the PFFs for the exo-Jupiters exceed those for the hot Jupiters as the “crossover metallicity” (see Figure 2).

We now estimate the critical metallicity for forming gas giants, focusing on the behavior of $\langle R_{\text{rapid}} \rangle$ for different planetary populations. As discussed in Section 2.2, the Jovian planets tend to have a larger value of $R_{\text{rapid}}$ than the low-mass planets. In addition, we recall that the radial extent of the zones of the hot Jupiters and the low-mass planets corresponds with each other (see Figure 1). We can therefore point out that the difference in $\langle R_{\text{rapid}} \rangle$ for these two different planetary populations originates from the gas accretion efficiency.

Figure 2 (bottom panel) shows the resultant $\langle R_{\text{rapid}} \rangle$ for both the hot Jupiters, as well as the low-mass planets (see the dashed and dotted lines, respectively; see also Figure 3 for a clear demonstration). We find that $\langle R_{\text{rapid}} \rangle$ for both populations is zero up to $[\text{Fe/H}] \simeq -1.4$. As the metallicity increases, the low-mass planets first achieve a non-zero value of $\langle R_{\text{rapid}} \rangle$ at $[\text{Fe/H}] \simeq -1.3$, and then the hot Jupiters do at $[\text{Fe/H}] \simeq -1.2$. Once the value of $\langle R_{\text{rapid}} \rangle$ for the hot Jupiters becomes larger than zero, it increases with metallicity more rapidly than for the low-mass planets. The sharp rise occurs at $[\text{Fe/H}] \simeq -1.2$.

Our results show that when $[\text{Fe/H}] > -1.2$, at which the PFFs for the hot Jupiters are non-negligible, $\langle R_{\text{rapid}} \rangle$ for the hot Jupiters obtains a larger value than that for the low-mass planets. For the regime of $[\text{Fe/H}] \lesssim -1.2$, on the contrary, the opposite trend is established, where almost no hot Jupiters are formed (see Figure 3). Thus, our results indicate that the critical metallicity for gas giant formation is $[\text{Fe/H}] \simeq -1.2$. This is the major finding of this paper and is of quantitative significance in putting a constraint on the core accretion picture for forming massive planets around metal-poor stars.

The resultant behavior of $\langle R_{\text{rapid}} \rangle$ also provides a suggestion for possible members of the low-mass planets. When $[\text{Fe/H}] \lesssim -1.4$, all of the formed planets are failed cores of gas giants so that they are solid cores with low-mass or almost negligible atmospheres. As the metallicity goes up, the members of the low-mass planets become the combination of failed cores as well as mini-gas giants that contain more masses in their envelopes.

In summary, we conclude that various kinds of characteristic metallicities derived here are very likely to depend on physical processes in planet formation. Specifically, the crossover metallicity can calibrate how planet formation and migration are coupled with each other. The examination of $\langle R_{\text{rapid}} \rangle$ for different planetary populations, which acts as a crucial agent for quantifying the critical metallicity for gas giant formation, can trace the different gas accretion histories between them. We also note that these metallicities are derived from the model that can well reproduce the current observational data of exoplanets.

### 4. DISCUSSION

As discussed above, we have derived various characteristic metallicities for planet formation (see Figure 2). These values may provide a good reference for exoplanet observations around metal-poor stars. This is because we have adopted the best choice of the weight functions ($w_{\text{mass}}$ and $w_{\text{lifetime}}$) that can fit well to the observations of protoplanetary disks, as well as the best set of parameters for planetary growth ($M_{\text{crit}}$, $\epsilon$, $v_{\text{esc}}$, and $d$; see Table 2) that can reproduce well the statistical properties of currently observed exoplanets ($-0.6 \leq [\text{Fe/H}] \leq 0.6$). Nonetheless, there may be a number of potential issues that may affect the above values. We summarize them and discuss how important they are for deriving the critical metallicity. Also, we discuss how consistent our results are with the current observations of exoplanets around metal-poor stars.

#### 4.1. Metallicity Effects on Disk Evolution

Disk evolution can be affected by the metallicity. This has recently been inferred from the observations that show that the disk lifetime in the low metallicity environment is likely to be much shorter than that in the solar metallicity region with $\sim 10^{(0.1[\text{Fe/H}])}$ dependence (Yasui et al. 2010). This dependence suggests that the disk lifetime at $[\text{Fe/H}] = -1$ shortens by a factor of 10, compared with disks with solar metallicity. It is obvious that such short disk lifetimes will make a considerable impact on planet formation.

In our model, photoevaporation, which plays a dominant role in the final stage of disk evolution (e.g., Armitage 2011), defines the disk lifetime ($t_{\text{dissip}}$; see Section 2.3). The above observed disk lifetime—metallicity correlation can be explained marginally by the effects of photoevaporation (e.g., Gorti &
One may wonder then how gas accretion onto the cores is affected by the metallicity of surrounding materials that are eventually accreted by the cores. If it could be assumed that the opacity in the planetary envelope is directly related to the metallicity of the disk, then the disk metallicity would provide an important effect for gas accretion. This occurs because the critical core mass ($M_{\text{crit}}$) above which gas accretion initiates depends on the opacity in the envelope (see Table 2).

In our model, $M_{\text{crit}}$ includes a parameter $M_{\text{crit0}}$ that is written as (Ikoma et al. 2000; Ida & Lin 2004a)

$$M_{\text{crit0}} = 10 M_\oplus (\kappa/1 \, \text{cm}^2 \, \text{g}^{-1})^{0.2-0.3}, \tag{5}$$

where $\kappa$ is the grain opacity of the envelope surrounding the planetary core. We have adopted $M_{\text{crit0}} = 5 M_\oplus$, equivalently $\kappa \equiv 0.1 \, \text{cm}^2 \, \text{g}^{-1}$ following HP14, where the value of $M_{\text{crit0}}$ is determined by comparing the currently available exoplanet observations that are well confined in a certain range of metallicities ($-0.6 \leq \text{[Fe/H]} \leq 0.6$). How can we justify the usage of this fixed value for planet formation in disks around more metal-poor stars ($-2 \leq \text{[Fe/H]} \leq -0.6$)?

Recently, Movshovitz & Podolak (2008) have shown through detailed numerical simulation of planetary envelopes that the resultant opacity in the envelope is very unlikely to be sensitive to the accreted materials. Namely, the metallicity of disks is very unlikely to determine the efficiency of the gas accretion directly (see Equation (5)). Instead, the growth of dust grains and their subsequent settling in the envelope play a more important role in regulating the gas accretion (Movshovitz et al. 2010). If this is the case, we can use the same value of $M_{\text{crit0}}$ as above, for examining the formation of first gas giants. In other words, the resultant opacity, $\kappa \equiv 0.1 \, \text{cm}^2 \, \text{g}^{-1}$, which is about one order of magnitude lower than the canonical value, is considered as a consequence of grain growth and subsequent settling in the envelope surrounding planetary cores, rather than the reflection of the accreted materials with low metallicities.

Thus, it is very unlikely that gas accretion adopted in our model significantly affects our results.  

### 4.3. Relation between $\eta_{\text{dust}}$ and $\text{[Fe/H]}$

In the above calculations, we have assumed a simple relationship between the dust-to-gas ratio ($\eta_{\text{dust}}$) and the metallicity ($\text{[Fe/H]}$; see Equation (1)). On the contrary, there is currently accumulating evidence that $\eta_{\text{dust}}$ does not scale simply with $\text{[Fe/H]}$. In fact, both theoretical and observational studies suggest that $\eta_{\text{dust}}$ is very likely to drop more rapidly than $\text{[Fe/H]}$ at low metallicity (e.g., Lisenfeld & Ferrara 1998; Hirota 1999; Inoue 2003; Asano et al. 2013; Rémy-Ruyer et al. 2014; Fisher et al. 2014). Note that this nonlinear effect may be valid predominantly in low gas density regimes. In dense gas regions, such as protoplanetary disks, gas-phase metals may be accreted onto dust grains (Chakraborty et al. 2013), which tend to weaken the effect. Nonetheless, it is interesting to examine the effect of the nonlinear $\eta_{\text{dust}}$–$\text{[Fe/H]}$ relation on the critical metallicity of planet formation.

In order to proceed, we will utilize the results of Asano et al. (2013), where dust formation history in a galaxy is investigated.

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3 We recall that our model adopts the core accretion scenario so that the above argument can be applied. When a gravitational instability scenario, in which planets are formed directly from gravitationally unstable disks, such as stars, would be more important to account for planet formation around metal-poor stars, the disk metallicity may play a more significant role (e.g., Boss 2002; Cai et al. 2006; Meru & Bate 2010).
It is obviously important to perform a more comprehensive observational survey for detecting exoplanets around metal-poor stars and to examine our results.

5. CONCLUSIONS

We have quantitatively investigated various characteristic metallicities for planet formation that were motivated by the recent success of exoplanet observations. To achieve such a goal, we have adopted a formalism developed in a series of earlier papers (HP11; HP12; HP13; HP14). The main features of the model are planet traps at which rapid type I migration for planetary cores is halted. Three types of planet traps (dead zone, ice line, and heat transition) have been considered. For planetary growth, we have relied on the standard core accretion scenario, where the formation of planetary cores is sensitive to the dust density in disks or the metallicity ([Fe/H]). Coupling the scenario with the planet traps in viscously evolving disks with photoevaporation has enabled one to compute the evolution of planets in the mass–semi-major axis diagram (see Figure 1).

We have utilized a set of theoretical evolutionary tracks of planets for deriving the PFF as well as the statistically averaged orbital radius at which growing protoplanets undergo rapid gas accretion ([R_{\text{rapid}}]; see Figure 1). Specification of \( R_{\text{rapid}} \) is crucial in this study because it allows one to estimate the critical metallicity for gas giant formation. This was achieved by dividing the mass–semi-major axis diagram into zones, as well as counting the end points of the tracks for each zone. Zoning the diagram is in fact inferred from the accumulation of observed exoplanets. We have considered three different planetary populations: hot Jupiters, exo-Jupiters, and low-mass planets (see Table 1).

We have obtained the PFFs for three types of planets as well as the total as a function of metallicity (see Figure 2). We have shown that the total PFF is a steady increasing function of metallicity, which is straightforward to understand from the core accretion scenario. The PFFs of the low-mass planets follow those of the total until the population of the Jovian planets provides a significant contribution, which occurs at \([\text{Fe/H]} \gtrsim -1\). We have also computed the PFFs for both types of Jovian planets, which show that the hot Jupiters can form at lower metallicities than the exo-Jupiters. Switching of the dominant population between these two types is a consequence of the intimate coupling of planet formation and migration and takes place at \([\text{Fe/H]} \simeq -0.7\), which is referred to as the crossover metallicity in this paper.

We have also estimated various characteristic metallicities for planet formation (see Figure 2; see also Table 5). The minimum metallicity for low-mass planets is \([\text{Fe/H]} \simeq -1.8\), although this probably provides only an upper limit (Section 3.2). We have quantified the critical metallicity for gas giant formation. This was derived from the behavior of \((R_{\text{rapid}})\), which essentially links to the different efficiencies of gas accretion between the hot Jupiters and the low-mass planets. We have found that \([\text{Fe/H]} \simeq -1.2\) is the critical metallicity for forming gas giants around metal-poor stars. This is the most important finding of this study. We have also pointed out that these values of metallicity depend on the processes of planet formation and migration. Therefore, our results infer that the observations of exoplanets around metal-poor stars may be used as a probe for calibration of these processes, and hence such observations are very important for examining the current understanding of planet formation in protoplanetary disks.
We have discussed a number of potential issues that may affect our results (Section 4). Specifically, we have examined the effects of metallicity on disk evolution as well as planet formation. The recent observations suggest that the reduction in metallicity shortens the disk lifetime significantly. It is obvious that this effect prevents planet formation. Nonetheless, we have neglected it because more intensive observations are required to make a reliable model. Planet formation is also affected by lowering the metallicity. We have discussed the effects on gas accretion. It is important that the resultant opacity in planetary envelopes is unlikely to depend strongly on the metallicity of materials accreted by planetary cores. Thus, the setup adopted in this paper may be sufficient.

We have also examined the nonlinear effect on the relationship between the dust-to-gas ratio ($\eta_{\text{d/g}}$) and the metallicity. This was motivated by the recent observational progress on the relationship in the lower metallicity environment, where $\eta_{\text{d/g}}$ is very likely to decrease more rapidly than [Fe/H]. We have calibrated the effect, making use of one of the most recent results in which how the relationship evolves with time was investigated. We have found that the corresponding metallicities that are derived from the theoretical nonlinear $\eta_{\text{d/g}}$-[Fe/H] relation depend sensitively on the star formation timescale (see Table 5). This is because the formation and evolution of stars play an important role in enriching the metallicity. Therefore, our results may be useful for bridging the observations of exoplanets around metal-poor stars with the estimate of the star formation history through galaxy evolution.

Finally, we have provided some implications for exoplanets around metal-poor stars that are currently observed so far. There is no example which violates our prediction at the moment. It is, however, obvious that more observational data are required to examine our results.

In a subsequent paper, we will undertake a more comprehensive study in which the effect of stellar masses will be examined. Also, we will consider galaxy evolution simultaneously, which triggers the change of metallicity with time. Thus, the study will allow a more complete discussion about when the first planets form in galaxies.

The authors thank Ralph Pudritz for stimulating discussion, Ryosuke Asano for providing us with the data of dust enrichment in galaxies, and an anonymous referee for useful comments on our manuscript. Y.H. is supported by EACOA Fellowship, which is supported by East Asia Core Observatories Association, which consists of the Academia Sinica Institute of Astronomy and Astrophysics, the National Astronomical Observatory of Japan, the National Astronomical Observatory of China, and the Korea Astronomy and Space Science Institute. H.H. is supported by NSC grant NSC102-2119-M-001-006-MY3.

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