ACCURACY VERIFICATION OF DISCHARGE COMPUTED BY THE HK-CURVE AND ESTIMATION OF MISSING DATA BY USING THE CURVE

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 Compared with the conventional HQ-curve method, the HK-curve method computes river discharge $Q$ more accurately by considering hydrological data at water level $H$ and water-surface slope $i$. In the analysis, the actual observed hydrological data during the flood is used, and the results indicated that the error of the computed $Q$ by the HK-curve method for the observed value was less than almost 5%, while the error of the computed $Q$ by the HQ-curve method occasionally exceeded 10%. Additionally, the results suggested that the HK-curve was almost identical to the one that considered the whole observed data from the river rising period to the river falling period via peak time during the flood although only the data in the period of drawdown were used. Thus, even when the data were observed only in the drawdown period of flood, the HK-curve method estimated highly accurate missing $Q$ at the river rising period.

Key Words: HK-curve, water-surface slope, relative error rate, uncertainty, estimation of missing data

1. INTRODUCTION

Water discharge constitutes important information for river planning and river management. Therefore, it is necessary to know river water discharge for every hour throughout the year although it is impractical to continuously observe hourly water discharge by using a float or a current meter, which is widely accepted for the observation of river water discharge in Japan. Subsequently, the method of converting water stage into water discharge by using the Stage-Discharge rating curve (hereinafter referred to as the HQ curve), is widely accepted. Based on the Technical Criteria for River Works: Practical Guide for Searching¹ edited by the Japanese Ministry of Land, Infrastructure, Transport and Tourism, it is necessary to initially conduct low water discharge observations at least 36 times a year (approximately three times a month) and high water discharge observations for as many floods as possible including middle-sized ones in a discharge observation station. Subsequently, a second-order approximate curve expression should be established by the least square method with respect to the plot of the observed water stage $H$ on the vertical axis and the observed water discharge $Q$ on the horizontal axis. The second-order approximate curve is the HQ curve that expresses the relation between $H$ and the corresponding $Q$. In practice, multiple HQ curves are often established corresponding to the water stage and period²). Each hour water discharge is calculated by substituting the water stage observed by an automatic water gauge installed at the discharge observation station into the HQ curves established in this manner. In the method, it is assumed that a one-to-one relationship exists between $H$ and $Q$. However, in actual rivers, there may be differences in the water discharge between the river rising period and the river falling period during a flood even at the same water stage due to the influence of the water-surface slope when the longitudinal river-bed slope is gentle. This is well known as a feature in which the plots of observed $H$ and $Q$ as mentioned above exhibit “a counterclockwise loop.”³) Motonaga et al.⁴) indicated that a non-negligible difference can
occur between the discharge converted from the stage measured by an automatic water gauge by using a HQ curve (hereafter, the discharge is termed as “HQ-discharge”) and the actual observed discharge (hereafter, the discharge is termed as “Obs.-discharge”) at the rising period and falling period of the flood due to the feature of the high water observation data at the I-Observatory in the I-river, which is a first-class river in Japan, and the HQ curve actually established by the observed data from the observatory. Conversely, they also indicated that the HQ-discharge and Obs.-discharge are nearly equal at the flood peak.

In the case where the HQ curves do not represent the actual river discharge, Motonaga et al.⁴ proposed the HK curve method in which the HK curve is used as opposed to the HQ curve. The HK curve represents the relation between water stage $H$ and conveyance $K$. Conveyance $K$ is a parameter that is not affected by the water-surface slope. In the method, water discharge is converted from stage and water-surface slope by using the HK curve. The water-surface slope is observed by two automatic water gauges set apart in the longitudinal direction of the river. It is observed that the relationship between $H$ and $K$ exhibits one-to-one correspondence irrespective of whether it is at the rising period and the falling period in contrast to the relation between $H$ and $Q$. The water discharge computed by the HK curve method (hereafter termed as the discharge “HK-discharge”) represents the actual discharge accurately as opposed to the HQ-discharge because the HK-discharge is computed by using $K$ that is determined uniquely for an arbitrary stage and the observed water-surface slope corresponding to it. Motonaga et al.⁵ conducted a 2D unsteady flow simulation on the longitudinal section from 53.6 KP to 55.6 KP from the river mouth of the A-river, which is a first-class river in Japan, to demonstrate the usefulness of the method. It is considered that the usefulness of the HK curve method is theoretically demonstrated by these existing studies. Conversely, it is considered that the result of the 2D unsteady flow simulation on the A-river did not include a small-scale turbulent component because the scale of the mesh used in the simulation corresponded to dozens m scale. Thus, scattering was not observed in the plots of water stage $H$ and conveyance $K$ calculated in the 2D unsteady flow simulation, and these data plot groups almost completely matched a HK curve. Conversely, the uncertainty⁶ must be included in any actual observed data irrespective of the extent to which the float method was performed adequately because the flood flow in the actual rivers contains the turbulent component that is not considered in the numerical simulation mentioned above. For example, fluctuations in the float path due to the turbulent flow component of the river flow are considered with respect to the water discharge. Additionally, the error caused by visually reading the scale of the staff gauge to measure the fluctuating height of the water surface is considered as a reason for uncertainty with respect to the water stage. It is impossible to repeat the observation for the same river flow, and thus the result of an observation appropriately performed based on the prescribed manual⁷ is considered as the “true value” except for cases in which it mostly deviates from the overall results. Therefore, when the observation results of the water stage $H$ and conveyance $K$ are plotted, it is expected that the observation data group slightly differs from the finally obtained HK curve. Extant studies do not confirm the accuracy of the reproduction of the actual observed discharge by the HK curve that is obtained by using the observation data including such uncertainty. The study involved an attempt to verify the accuracy of the HK curve method by using actual high water discharge observation data.

In addition to improving the accuracy of the computed discharge, examples of the utilization of the HK curve method are shown in the study. A common problem in the process of implementing the HQ curve method involves cases in which an HQ curve must be established only by the data of river falling period because the flood peak arrives extremely quickly such that observation preparation is not performed in time, and the data of the river rising period are omitted. The study estimates the missing data of the river rising period with high accuracy by considering the feature of one-to-one correspondence between $H$ and $K$.

2. METHODS

(1) HK curve method

Conveyance $K$ is obtained by using the following equation from observed water discharge $Q$ and water-surface slope $i$ as follows:

$$Q=K\cdot i^{1/2}$$  (1)

Based on Manning’s equation, the water discharge is expressed by the following equation:

$$Q=A\cdot R^{2/3}\cdot i^{1/2}$$  (2)

where $A$: discharge section area, $n$: Manning’s roughness coefficient, and $R$: hydraulic radius. The following equation is obtained by comparing Eq.(1) and Eq.(2) as follows:

$$K=\frac{A}{n}R^{2/3}$$  (3)

As shown in Eq.(3), conveyance $K$ is a parameter that contains the variation in the roughness coefficient during the flood, and thus Motonaga et al.⁴ recommended using the parameter given the possibility of
future use in river channel management. In Eq. (1), water-surface slope \( i \) is obtained from the difference between the reading value of the staff gauge at the observatory and water stage by the automatic water gauges installed separately in the longitudinal direction of the river. Water stage \( H \) is plotted on the vertical axis and conveyance \( K \) is plotted on the horizontal axis. Conveyance \( K \) is calculated from the observed water discharge and water-surface slope by using Eq. (1). The HK curve is an approximate expression of a quadratic curve with the same shape as the HQ curve obtained by the least squares method for these data plot groups. In the HK curve method, \( K \) is converted from the stage by automatic water gauge by using the HK curve, and subsequently, water discharge is computed from Eq. (1) from the \( K \) and the water-surface slope obtained by the two gauges. A gauge is set at the water discharge observatory, and another gauge is set away from it in the longitudinal direction of the river.

(2) Verification of the accuracy of the HK curve method by actual data

The accuracy of the discharge computed by HK curve method was verified by using the actual observed high water data (water stage and discharge) for I-Observatory that is located at the 44.5 km point from the river mouth in the I-river, which is a Japanese first-class river. The water-surface slope \( i \) used to calculate \( K \) was obtained from the water stage data at the I-Observatory (reading data of the staff gauge) and data of the automatic water gauge at the T-Observatory (58.0 km point from the river-mouth) that is the nearest observatory of the I-Observatory station. The automatic water gauge data at the T-Observatory recorded hourly data, and there was a case in which a time lag occurred between the observation time of the automatic water gauge at T-Observatory and the observation execution time at I-Observatory. In this case, successive hourly data were linearly interpolated at the T-Observatory, and the T-Observatory water stage at the time of execution of an observation at I-Observatory was obtained. The author of the study confirmed that the water stage data by the automatic water gauge were almost equal to the visual reading data of the staff gauge at T-Observatory, and the automatic water gauge was properly managed during the flood of the I-river that corresponded to the subject of the study although the detail was omitted due to space limitations. As mentioned above, the water stage observation by visually reading the water staff gauge can cause an error due to the fluctuations in the water surface. However, the results indicated that the water stage observation accuracy was maintained if the water surface fluctuation was not extreme because the reading value of the staff gauge was the averaged value of the maximum and the minimum values of the fluctuating water surface level.

Conveyance \( K \) of each observation time was calculated from the observed discharge at the I-Observatory and the water-surface slope between the I-Observatory and T-Observatory by using Eq. (1). The HK curve was established by the least square method for the plot of the measured data of the water stage \( H \) and conveyance \( K \). The data used to obtain HQ curve and HK curve corresponded to data observed simultaneously. The accuracy of the HQ-discharge and HK-discharge at I-Observatory was verified. Verifying the accuracy of the water discharge was done using the “relative error rate” as defined by Eq. (4). The relative error rate is the ratio of the difference between the Obs.-discharge and the HQ-discharge or HK-discharge to the Obs.-discharge. It exhibits the deviation of each computed discharge from the actual observed discharge. The expression is as follows:

\[
\text{The relative error rate (\%)} = \frac{\text{HQ discharge or HK discharge} - \text{Obs.-discharge}}{\text{Obs.-discharge}} \times 100
\]

(3) Investigation of the distance for the water-surface slope observation

In the study, in order to investigate the HK curve method, the water-surface slope was obtained from the water stage difference between the I-Observatory that is subject to consideration and the T-Observatory located 13.5 km upstream from the I-Observatory. The automatic water gauges set apart in the longitudinal direction for observing the water-surface slope averaged the water stage observed in each second and outputted the averaged value over a certain time, and thus the error due to the water surface fluctuation smoothed and decreased originally. While obtaining the water-surface slope, the water stage difference increases when the distance between the observatories increases, and the influence of the above-mentioned water surface fluctuation decreases relatively. Conversely, when the distance between observatories increases, there are possibilities including bends in river channels, merging of tributaries, and distributaries. They may affect the water-surface slope. In addition to the T-Observatory, other water stage observatories as shown in Table 1 are installed upstream of the I-Observatory. In the study, the water-surface slope was observed by using the data of these observatories. In the same manner as when calculating the water-surface slope between I-Observatory and T-Observatory, the visual reading data of the staff gauge at I-Observa-
Table 1 Observatories located upstream of I-Observatory.

| Observatory | Distance from I-Observatory[km] | Note |
|-------------|--------------------------------|------|
| T           | 13.5                           |      |
| N           | 32.5                           |      |
| H           | 49.4                           | Big branches joins from the left bank between N-H. |

and the observed data by automatic water gauges at the upstream observatories (N-Observatory, H-Observatory) were simultaneously used to obtain water-surface slope. Significant meandering does not exist in the longitudinal section of the I-river that was considered in the study.

(4) Estimation of missing discharge data by using the HK curve method

In Japanese rivers, the flood peak is reached extremely fast such that there are several cases in which the observation preparation is not constructed in time and subsequently the observation before the flood peak is not performed, and the data during the river rising period are missing. Therefore, it is relatively frequently observed that only the observation data of the river falling period are present. Given parameters including the decrease in manpower and the reduction in financial resources for future hydrological observations, the tendency may be more significant henceforth. When the relation between water stage and corresponding discharge during flood exhibits “a counterclockwise loop,” the gradient of the obtained HQ curve becomes steep and the calculated discharge may be excessively low if high water observation is conducted only during the river falling period of the flood. Conversely, as mentioned above, even when a relationship between the water stage \( H \) and the discharge \( Q \) indicates a loop, a feature exists wherein the water stage \( H \) and the conveyance \( K \) exhibit a one-to-one relationship. From the result, even if the observation before the flood peak is not performed and subsequently observation data exist only at the river falling period, it is conceivable that the HK curve in this case may be equal to the HK curve in the case in which the observation is performed from the river rising period to the river falling period via the flood peak. Therefore, it is assumed that the data were missing at the river rising period, and thus the data at this period were initially deleted from the observation data used to verify the accuracy of the HK discharge in section (2) of the chapter. Subsequently, the accuracy verification by using Eq.(4) was performed on the water discharge computed by the HQ curve and the HK curve that were established only by the remaining data at the river falling period. Based on the results, an attempt focused on estimating the missing discharge data from the river rising period to the peak of the flood by using the HK curve method for the case in which only data of the river falling period after the flood peak existed.

3. RESULTS AND DISCUSSION

(1) Result of the accuracy verification for the HK curve method

Figure 1 shows an overlay diagram (hereinafter referred to as the \( H-Q \) diagram) of the plots of the high water observation data at the I-Observatory in the I-river and the HQ curves that were actually established by using these data. The two floods shown in Figure 1 occurred in the same year, and it was confirmed that the relationship between the stage and the corresponding discharge exhibited a counterclockwise loop in each case. Figure 1 also indicates that there were non-negligible differences between the HQ-discharge and the Obs.-discharge at the river rising period and the river falling period. For example, at the stage of 6 m with respect to the high water observation data I as shown in the figure, the HQ-discharge was approximately 400 m\(^3\)/s lower than the Obs.-discharge at the river rising period. Conversely, the HQ-discharge was approximately 250 m\(^3\)/s higher than the Obs.-discharge at the river falling period. Subsequently, conveyance \( K \) was calculated based on the water-surface slope between the I-Observatory and T-Observatory. Figure 2 shows an overlay diagram (hereinafter referred to as the \( H-K \) diagram) of the observed data of the water stage \( H \) and conveyance \( K \) and the HK curves established by using the
data. The data used to establish the HQ curve and the HK curve were data that were simultaneously observed. In Fig.2, when the actual observation data were used, although the relationship between \( H \) and \( K \) indicated variations due to the uncertainty of the data as mentioned above, the variability was not extremely high such that the relation between \( H \) and \( K \) indicated almost a one-to-one relationship. Figure 3 shows the time-series diagram of the HQ-discharge, HK-discharge, and Obs.-discharge. Numbers in square brackets in the figure denote the observation number attached to each observation time. Figure 4 shows the relative error rates of the HQ-discharge and HK-discharge of each observation time as shown in Fig.3. Figure 4 shows that the relative error rate of the HK-discharge was lower than that of the HQ-discharge over the entire flood time from the river rising period to the river falling period via the flood peak. With the exception of the peak time, the HQ-discharge exhibited a relative error rate exceeding 10%, and specifically it could exceed 15% at the river rising period while the HK-discharge displayed a relative error rate that was generally less than 5% and the improvement in the accuracy of the discharge computed by the HK curve method was significant. Both the HQ-discharge and HK-discharge tended to exhibit a higher relative error rate at the river rising period than those at the river falling period. The reason for this tendency is assumed to be that the disturbance component of the flow, as widely known, usually predominates at the rising period and subsequently the variation in observation data was relatively higher at the rising period than at the falling period due to the influence of the uncertainty caused by it. Sago\(^7\) indicated that the error of the discharge computed by HQ curve method was approximately 13%. However, the error referred to by Sago was strictly different from the relative error rate expressed in Eq. (4) since he defined “the average value calculated from many data” as the “true value” and defined the difference from it as an error. However, it was considered that the observation at the I-Observatory was performed adequately because any problems were not reported during the observation, and in this case, as previously mentioned, the observation result was considered as the “true value.” Therefore, both errors were considered identical. Even when compared with the error indicated by Sago, the HK-discharge in the study exhibited high accuracy.

(2) Result of the investigation of the distance for the water-surface slope measurement

Figure 5 shows the time-series of the stage and various water-surface slope corresponding to the high water data I at the I-Observatory as shown in Fig.1. The time-series of water-surface slope due to the stage difference between the I-Observatory and T-Observatory used in the HK curve method appeared to indicate a reasonable result because it changed smoothly and its peak arrived before the stage peak time. The water-surface slope due to the stage difference between the I-Observatory and N-Observatory and between the I-Observatory and H-Observatory also ex-
hibited a smooth change although the tendency of the fluctuation was different from the water-surface slope due to the stage difference between the I-Observatory and T-Observatory. It is not currently clear whether the difference was due to the difference in distance between observatories or the influence of a branch that joins from the left bank between the N-Observatory and H-Observatory. (In the tributary, the peak discharge reached approximately 30% of the peak discharge at I-Observatory during the flood corresponding to high water data I as shown in Fig.1, and thus it is considered to be a relatively large branch.) In any case, it is necessary to focus on the various influences if the distance between the water gauges increases.

“Observed discharge table” contained in the detailed rules of hydrological observation work regulations is a form to record the results for low water observation and high water observation for each time period. The form includes an entry column of the “water-surface slope.” This is a value that is calculated from visual readings of the staff gauges attached to the first and second sighting cross-section at an observatory. The distance between the first and the second sighting cross-section is approximately 100 m generally, and it is expected that the accuracy of the water stage difference observed by using visual reading of stage at the distance is not high. Figure 5 also shows the water-surface slope between these two sighting sections, and it is obvious that the fluctuations in the water-surface slope are high and not suitable as the water-surface slope used for the HK curve method.

(3) Result of the estimation of the missing discharge data by using the HK curve method

Figure 6 is a case study for the discharge time-series assuming that the discharge observation preparation was not constructed in time and the data at the river rising period were missing during the flood corresponding to high water observation data I shown in Fig.1. In Fig.6, the plotted points marked with “x” are data assumed missing. Such data that are observed only after the peak time actually appear frequently. Figure 7 and Fig.8 show the H-Q diagram and H-K diagram, respectively, which are created by assuming that the discharge observation data at the river rising period were missing during the flood corresponding to high water observation data I shown in Fig.1. In a manner similar to Fig.6, the plotted data marked with “x” are data assumed missing. Both the figures show HQ curves and HK curves established by using observation data given the assumption that data were missing at the river rising period (hereinafter referred to as HQ curve [1] and HK curve [1], respectively). Both the figures also show the HQ curves and the HK curves established by using all high water observation data shown in Fig.1 and Fig.2 (hereinafter referred to as HQ curve [2] and HK curve [2], respectively). As shown in Fig.7, the HQ curve [1] accurately reproduces the actual observation data at the river falling period although it indicates a separation from the observed data higher than the HQ curve [2] at the river rising period. Conversely, as shown in Fig.8, the HK curve [1] and the HK curve [2] are generally in agreement. From this fact, a possibility exists that the discharge is computed with high accuracy when the HK curve method is used even if the observation preparation is not constructed in time and the observation data at the river rising period are omitted. Figure 9 shows the relative error rate calculated by Eq.(4) for the discharge computed by using HQ curves [1], [2] (HQ-discharge[1], [2]), respectively, and the discharge computed by using HK curves [1], [2] (HK-discharge[1],[2]), respectively. The relative error rates are referred to as HQ relative error rate[1], [2] and HK relative error rate[1], [2], respectively. In the figure, Numbers in square brackets correspond to Fig.3. This indicates that the
HQ relative error rate [1] exceeds the HQ relative error rate [2] and the maximum exceeds 20% at the river rising period of the flood. Conversely, the HQ relative error rate [1] is lower than the HQ relative error rate [2] at the river falling period after the flood peak. This is because HQ curve [1] is established only with the data at the river falling period, and thus it is possible to accurately reproduce the Obs.-discharge with respect to the river falling period. Conversely, a significant difference does not exist between the HK relative error rate [1] and HK relative error rate [2] and both error rates are generally lower than the HQ relative error rate [1]. Figure 9 also shows that HK relative error rates may exceed the HQ relative error rates around the peak time, although a particular problem does not exist because the original error is low in the vicinity of peak time. This reveals that HK curves with high accuracy are established even if observation data are present only at the river falling period. Moreover, the result indicates that even if the observation data at the river rising period are missing, it is possible to estimate the value of the missing discharge with relatively high accuracy.

4. CONCLUSION

The conclusions obtained in this study are summarized as follows:
(1) The actual observed data indicate that the HK curve method computes the discharge more accurately when compared with the HQ curve method, which is widely used conventionally. The error due to the HQ curve method may exceed 10%, whereas the error while using the HK curve method falls within approximately 5%.
(2) The results indicate that an increase in the distance between the two water gauges to calculate the water-surface slope increases influences such as the joining of the tributaries. It is also shown that the water-surface slope (which is recorded in the observed discharge table) calculated from the difference in water stage between the first and the second sighting cross-section has extremely severe fluctuations, such that it does not constitute appropriate data for use in the HK curve method.
(3) The situation involving missing data at the river rising period prior to the flood peak in the high water discharge observation is observed relatively frequently. Even in this situation, in contrast to the HQ curve, the HK curve that is established only by the data at the river falling period is approximately equal to the HK curve established by using regular observation data (continuous data from the river rising period to the river
falling period via the flood peak).
(4) A method to estimate missing data accurately by using HK curve is proposed by using the above results.

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