Research Article
Computational Behavior of Second Law Poiseuille Flow of Micropolar Fluids in a Channel: Analytical Treatment

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1. Introduction

The knowledge of micropolar fluid has great applicability in many industrial areas and the field of biological research. Recent studies cross over these phenomena on the micropolar fluid by several researchers. The augmentation of both the influences of rotation and inertial between the microelements has been initiated by Eringen [1]. Furthermore, Eringen [2–4] used parallel plates through which the polar fluid past to develop the second law analysis. Eringen proposed a theory for polar fluids which exhibit the rotation of particles in conjunction with the vorticity vector independently and also the action of couple stress. However, such suspended particles into a viscous medium are made of hard particles. In particular, the blood flows in the human body. Ferro fluids are the best examples of micropolar fluids. Moreover, as described earlier in the field of biological research studies, i.e., the blood flows, colloidal elucidation and their movement, and solutions using the suspension, their application is vital. More precisely, the suspended dumbbell-shaped stiff cylindrical components are found in the micropolar fluid, and the governing equations for the fluid flow are based upon the conservation laws of mass, momentum, and constitutive relations. The investigation for the features of several components that characterize the flow using a semiinfinite plate is obtained by Ahmadi [5]. However, they have employed numerical solutions to get the results from the nonlinear ODEs. Recent development carries out the interest for the significant application for the enhancement of heat transfer properties in the polar fluid. The study of certain polymer solutions and the suspension of colloidal particles is vital for chemical engineering.
researchers. Moreover, in the biomedical sciences, complex biological structures are obtained using micropolar fluids. In comparison to conventional fluids, the properties of heat transfer are enhanced by using nanofluids, a new class of heat transfer fluids. The nanoparticles of size 1–100 nm are suspended into the base fluid to form such nanofluids. Due to the higher thermal conductivity of metallic nanoparticles, the conductivity of the nanofluid is also enhanced. The influence of various thermophysical parameters in the permeable medium for the mixed convection in a nanofluid has been conducted by Ellahi et al. [6]. The characteristics of Brownian motion for the heat transfer properties in a nanofluid are governed by Dogonchi and Ganji [7]. In a conclusive remark, they got that the fluid temperature rises due to an increase in the heat source, and the impact is reversed for thermal radiation. Jena and Mathur [8] worked on similarity solutions for the laminar-free convection flow of the micropolar fluid. They used the shooting method to establish the relationship between various parameters. Murthy and Srinivas [9] discussed entropy generation in steady Poiseuille flow of two immiscible micropolar fluids between two horizontal parallel plates of a channel with constant wall temperatures. The behaviors of several parameters, i.e., micropolarity, couple stress on the velocity, microrotation, and temperature, are discussed. In recent times, many researchers gave attention to micropolar fields [10–14].

Micropolar fluid flow and heat transfer over a non-linearly stretching plate with viscous dissipation was discussed by Ahmad and Ishak [15]. It is noteworthy that for polymer processing, the role of viscous dissipation is important. The fact is, it behaves like an energy source for the preparation of heat which in turn delays the process of solidification, which is treated as a coolant for the final product. The phenomena of heat and mass transfer are based on the involvement of thermal radiation and chemical reaction. Authors [16] evaluated in their study the interaction of Newtonian heat and mass processes in Walters-B fluid bounded by a moving surface. In past investigations, only the Newtonian heating of heat transfer has been used to examine the features of different fluid models under various aspects and flow geometries. Mathur and Mishra [17] calculated the heat and mass transfer of MHD-free convection through two infinite plates embedded with porous materials. Authors have studied the thermodiffusion effect which is not considered in the previous research. A numerical study for the two-dimensional steady incompressible mixed convective flow of an electrically conductive micro-nanofluid in a stretchable channel was reported by Rauf et al. [18]. They used the Runge–Kutta–Fehlberg fourth-fifth order (RKF45) method to solve the algebraic system of equations with boundary conditions. Mathur and Mishra [19] discussed the problem of MHD boundary layer flow in the presence of radiation and magnetic field over an exponentially stretching sheet. A semianalytical approach was applied by the authors [20] to study the Williamson nanofluid flow through a porous medium in the presence of melting heat transfer boundary conditions [21]. Sundar et al. have given a review on hybrid nanofluids preparation, thermal properties, heat transfer, and friction. To understand the behavior of melts and many polymer solutions, the Carreau–Yasuda model was established successfully. Hussain et al. [22] performed a numerical study on the Carreau–Yasuda nanofluid model over a convective heated surface near a stagnation point. Many researchers [23, 24] work on entropy generation of nanofluids in a different scenario. The numerical and graphical justification was given to strengthen their work. Bioconvection was imposed to study the rheology of MHD bioconvective nanofluid containing motile microorganisms by Muhammad Awais et al. [25]. Homogeneous and heterogeneous reactions of the 3D flow of Cu-water and Al2O3-water nanofluid and entropy generation estimation along stretching cylinder were investigated by Siddiqui and his fellow researchers [26, 27].

The present model is developed for the study on the plane Poiseuille flow of micropolar fluid within a channel formed to be the two horizontal parallel plates. Analytical treatment is carried out for the flow phenomena to get the result using the symbolic routine code by MAPLE. The behavior of no-slip and hyperstick conditions plays a vital role in the velocity and the angular momentum profiles. The computation for the several pertinent parameters is obtained and presented. The numerical results of the volume flow rate, shear stress, and couple stress coefficients are presented via the tabular form.

2. Problem Formalism

The plane Poiseuille flow of a micropolar fluid between the two horizontal parallel plates within a channel is considered. Both the plates are extended along the x-direction with a fixed distance between them as 2h. Here, the x-axis is treated as the axial, and the y-axis is represented as transverse direction with a center of the channel mentioned as the origin (Figure 1). The lower half of the channel is equipped with the region $-h < y < 0$ called zone I and the upper half region is $0 < y < h$ is known as zone II. It is assumed that the density of the fluid in zone I is heavier than the fluid present in zone II. The governing equations for the incompressible micropolar fluid in both the zones are described following Eringen [2, 4].

Conservation of mass is given as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0.$$  \hspace{1cm} (1)

Conservation of momentum is given as

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \rho \mathbf{f} - \nabla \mathbf{P} - \kappa \nabla \times \mathbf{v}$$

$$- (\mu + k) \nabla \times \nabla \times \mathbf{v} + (\lambda + 2\mu + k) \nabla \nabla \cdot \mathbf{v}.$$  \hspace{1cm} (2)

Conservation of angular momentum is given as

$$\rho \frac{\partial \mathbf{g}}{\partial t} = \rho \mathbf{f} - \nabla \mathbf{P} + \kappa \nabla \times \mathbf{q} + \gamma \nabla \times \nabla \times \mathbf{v} + (\alpha + \beta + \gamma) \nabla \nabla \cdot \mathbf{v}.$$  \hspace{1cm} (3)

Here, the fluid density $\rho$ and the gyration coefficient $\mathbf{f}$ are treated as constants, and $\mathbf{P}$ is the fluid pressure at any point.
Also, $\lambda, \mu, \kappa$ are the material constants known as viscosity coefficients, and $\alpha, \beta, \gamma$ are the gyroviscosity coefficients. From equation (3), it is very clear for vanishing $\kappa, \alpha, \beta, \gamma$ and along with $l$ and $f$, the microrotation vanishes. Furthermore, for vanishing $\kappa$, equation (2) for the velocity profiles becomes Newtonian. The stress tensor and the couple stress tensor are given by

$$
T_{ij} = (-P + \lambda \nabla \cdot q) \delta_{ij} + (2\mu + \kappa) \delta_{ij} + \kappa \varepsilon_{ijk}(\omega_m - \omega_i),
$$

$$
\tau_{ij} = \alpha \varepsilon_{ijk}\omega_j + \beta \varepsilon_{ijk}\omega_j + \gamma \varepsilon_{ijk}\omega_j,
$$

where $\tau_{ij}$ is the microrotating vector, $\omega_i$ is the vorticity vector, $\delta_{ij}$ is the rate of shear strain component, and $\varepsilon_{ijk}$ is the Kronecker symbol.

In particular, the Levi–Civita symbol $\varepsilon_{ijk}$ is defined as

$$
\varepsilon_{ijk} = \begin{cases} 
-1, & \text{if } i, j, m \text{ are cyclic}, \\
0, & \text{if any two of } i, j, m \text{ are equal}, \\
1, & \text{if } i, j, m \text{ are acyclic}.
\end{cases}
$$

Also, comma denotes covariant differentiation.

The present model is designed assuming steady, one-dimensional incompressible micropolar fluid with a negligible gravity effect.

The velocity and microrotation vector field components are assumed to be $q = (U(Y, 0), 0, 0)$ and $\vec{\tau} = (0, 0, C(Y))$, respectively.

The following nondimensional quantities are introduced to transform the governing equations into a dimensionless form:

$$
x = \frac{X}{h},
$$

$$
y = \frac{X}{h},
$$

$$
u = \frac{U}{U_0},
$$

$$
p = \frac{P}{\rho_1 U_0^2},
$$

$$
C = \frac{CU_0}{h}
$$

where within the channel, the maximum fluid velocity is $U_0$.

Employing the aforesaid nondimensional quantities, equation (1) satisfies automatically, and equations (2) and (3) are expressed for the different zones in the following form.

### 3. Governing Equations

The velocity profiles are

$$
u(y) = \begin{cases} 
u_1(y), & -1 \leq y \leq 0 \text{ (zone } I), \\
u_2(y), & 0 \leq y \leq 1 \text{ (zone } II),
\end{cases}
$$

and the microrotation profile is

$$\omega(y) = \begin{cases} \omega_1(y), & -1 \leq y \leq 0 \text{ (zone } I), \\
\omega_2(y), & 0 \leq y \leq 1 \text{ (zone } II).
\end{cases}
$$

#### 3.1. Zone I.

In zone I, the transformed equations are

$$
\frac{d^2\nu_1}{dy^2} + \left(\frac{\delta_1}{1 + \delta_1}\right) \frac{d\omega_1}{dy} = \left(\frac{1}{1 + \delta_2}\right) Re \frac{dp}{dx},
$$

$$
\frac{d^2\omega_1}{dy^2} - s_1 \frac{d\nu_1}{dy} - 2s_2 \omega_1 = 0.
$$

#### 3.2. Zone II.

Similarly, in zone II, the transformed equations are

$$
\frac{d^2\nu_2}{dy^2} + \left(\frac{\delta_2}{1 + \delta_2}\right) \frac{d\omega_2}{dy} = \left(\frac{1}{1 + \delta_2}\right) \frac{n_p}{n_\mu} - Re \frac{dp}{dx},
$$

$$
\frac{d^2\omega_2}{dy^2} - s_2 \frac{d\nu_2}{dy} - 2s_2 \omega_2 = 0,
$$

where $Re = \rho_1 U_0 h/\mu_1, \delta_i = \kappa_i/\mu_1, s_i = \kappa_i h^2 / \gamma_1, i = 1, 2$ and $n_\mu = \mu_2/\mu_1$.

It is clear to understand from the transformed equations that the velocity and microrotation profiles in both the zones are coupled in nature. Since we have considered the Poiseuille flow, both the plates are fixed and a constant pressure gradient is an act through which the flow is maintained. Here, $dp/dx = B$ is a constant. Due to no-slip and hyperstick conditions, the boundary conditions are assumed as...
$u_1 = 0, \quad \omega_1 = 0,$

$u_1 = u_2$ and $\omega_1 = \omega_2,$

$$\left[ \frac{\partial u_1}{\partial y} + 2 \left( \frac{\delta_1}{1 + \delta_1} \right) \omega_1 \right]_{y=0} = \left[ \frac{\partial u_2}{\partial y} + 2 \left( \frac{\delta_2}{1 + \delta_2} \right) \omega_2 \right]_{y=0},$$

at $y = -1$

$$\frac{d\omega_1}{dy} \bigg|_{y=0} = \frac{d\omega_2}{dy} \bigg|_{y=0},$$

$u_2 = 0, \quad \omega_2 = 0,$

$$\frac{d^4 u_1}{dy^4} + \left( \frac{2 + \delta_1}{1 + \delta_1} \right) \frac{d^3 u_1}{dy^3} = - \left( \frac{2 \delta_1}{1 + \delta_1} \right) \frac{n_p}{n_\mu} \text{Re} B,$$ (18)

$$\omega_1 = \frac{1}{2} \frac{d u_1}{dy} - \frac{1 + \delta_1}{2 s_1 \delta_1} \frac{d^3 u_1}{dy^3}.$$ (19)

3.3. Engineering Coefficients. The shear stress coefficients at the fluid interfaces are expressed as

$$\tau_{xy} = \left[ \frac{\partial u_i}{\partial y} + 2 \left( \frac{\delta_i}{1 + \delta_i} \right) \omega_i \right]_{y=0}, \quad i = 1, 2.$$ (14)

However, the shear stresses near the lower and upper plates are

$$\tau_{xy} = \left[ \frac{\partial u_1}{\partial y} \right]_{y=-1},$$ (15)

$$\tau_{xy} = \left[ \frac{\partial u_2}{\partial y} \right]_{y=1}.$$ (16)

Similarly, the couple stress coefficients are expressed as

$$m_{xy\mid \text{zone-1}} = \frac{d\omega_1}{dy} \bigg|_{y=0}, \quad \text{and} \quad m_{xy\mid \text{zone-II}} = n_p \frac{d\omega_2}{dy} \bigg|_{y=0}.$$ (17)

3.4. Volumetric Flow Rate. The volumetric flow rate is computed as

$$q = q_1 + q_2 = \int_{-1}^{0} u_1 (y) dy + \int_{0}^{1} u_2 (y) dy.$$ (18)

4. Solution of the Problem

Eliminating $\omega_1$ from equations (9) and (10) of zone I, the transformed equations are of the form

$$\omega_1 = C_1 B_7 \exp (B_2 y) + C_2 B_8 \exp (-B_2 y) - 0.5 C_3 - B_3 y,$$ (24)

$$\omega_2 = C_5 B_9 \exp (B_2 y) + C_6 B_{10} \exp (-B_2 y) - 0.5 C_7 - B_6 y.$$ (25)
where the eight unknowns \( C_i, i = 1, \ldots, 8 \) are to be determined.

Employing the boundary conditions (13) in equations (22)–(25), we get the following matrix to get the unknowns.

\[
\begin{pmatrix}
B_1 e^{-B_1} & B_1 e^{B_1} & -1 & 1 & 0 & 0 & 0 & 0 \\
B_2 e^{-B_2} & B_2 e^{B_2} & -0.5 & 0 & 0 & 0 & 0 & 0 \\
B_1 & B_1 & 0 & 1 & -B_4 & -B_4 & 0 & -1 \\
B_7 & B_7 & -0.5 & 0 & -B_9 & -B_{10} & 0.5 & 0 \\
B_{13} & B_{14} & -0.5 B_{13} & 0 & -B_{15} & -B_{16} & 0.5B_{12} & 0 \\
B_{2B} & B_{2B} & -1 & 0 & -B_3B_9 & B_3B_{10} & 0 & 0 \\
0 & 0 & 0 & 0 & B_4 e^{B_4} & B_4 e^{-B_4} & 1 & 1 \\
0 & 0 & 0 & 0 & B_5 e^{B_5} & B_5 e^{-B_5} & -0.5 & 0 \\
\end{pmatrix}
\begin{pmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
C_5 \\
C_6 \\
C_7 \\
C_8 \\
\end{pmatrix}
= \begin{pmatrix}
-B_3 \\
-B_4 \\
0 \\
0 \\
0 \\
-B_6 \\
C_6 \\
-B_6 \\
\end{pmatrix}
\]

(26)

where the known values \( B_i, i = 1, \ldots, 16 \) are presented in the appendix.

5. Results and Discussion

A computational study on the flow of second law Poiseuille flow for a micropolar fluid within a channel is carried out in the current problem. The non-Newtonian fluid passed through two horizontal plates that were partitioned into two zones separated by the central region. The pressure gradient is assumed to be the constant. The crux of the investigation is the assumption of no-slip and hyperstick boundary conditions. The transformed governing equations for the velocity and the microrotation profiles in both the zones are coupled in nature, and a symbolic routine is handled by the MAPLE to solve the differential equations. The computed results for different characterizing parameters on the flow profiles are presented via graphs, and the numerical computations for the volume flow rate are obtained and displayed through a table. Throughout the computation, the following values are considered to be fixed, except the variation of the particular parameter displayed in the corresponding graphs. The velocity distributions for several parameters are deployed in Figures 2–7, and the microrotation profiles are presented through Figures 8–13. In all the figures, the dotted line indicates the variation of the parameters in the zone I and the bold line represents the variation in zone II.

5.1. Velocity Profiles. Figure 2 elaborates the behavior of the cross-viscosity parameter \( \delta_1 \) on the velocity profiles. From the governing equations, it is clear to see that for large \( \delta_1 \), the fluid particles rotate about themselves with high angular velocity. Therefore, the velocity profile retards both the regions resulting in the channel thickness increase. It is interesting to observe that the profile picks near the lower plate up to the central region, and thereafter, the fall in the profile is marked from the central region to the upper plate region. The profile behavior validates with the work of Umavati et al. [28]. The trend of the graph in both the regions is a very similar pattern to that of the earlier study. Finally, it is concluded that increasing cross-viscosity or the micropolarity suitably reduces the velocity profiles in both zones. The influence of the couple stress parameter \( s_1 \) on the velocity profiles for fixed values of other pertinent parameters is displayed in Figure 3. The distributions in both zones are displayed significantly. It is clearly visible that increasing couple stress parameter, the profile increases, showing the channel with decreases at both the plates. The large value of couple stress characterizes the Newtonian case. The couple stress tensor occurs due to the rotation of the particles. Figures 4 and 5, respectively, present the variation of cross-viscosity \( \delta_2 \) and the couple stress parameter \( s_2 \) on the velocity distributions. As described earlier, for the increasing micropolarity parameter, the velocity profile retards significantly in both the zones, whereas the reverse impact is rendered for the increasing couple stress. However, the increasing couple stresses the increase in the velocity distribution is insignificant. Reynolds number characterizes the relation between the inertial and the viscous force. From the mathematical form of the Reynolds number, it is observed that as inertial force increases, the Reynolds number increases. The role of Reynolds number is an important aspect of the velocity distribution that is presented in Figure 6. The higher the Reynolds number, the rate increases significantly. The profile is tilted near the central region with a pick starting from the lower plate region to the central region in zone I and retards from the central region to the upper plate in zone II. The physical behavior of the Reynolds number shows as a controlling parameter for the flow phenomena. Dual characteristics of the constant pressure gradient are marked on the velocity distribution in both the zones that are reflected in Figure 7. Flow separation occurs due to a change in pressure gradient. Adverse pressure gradient presents, if the pressure is acted along the direction of flow. In zone I, for an increase in negative pressure gradient, the rise in velocity profile is marked, whereas the backflow occurs with increasing pressure gradient. A similar observation is reflected in zone II for various values of a pressure gradient.

5.2. Angular Velocity Profiles. The characteristics of pertinent physical parameters on the angular velocity are
Figure 2: Velocity profiles for various $\delta_1$.

Figure 3: Velocity profiles for various $s_1$.

Figure 4: Velocity profiles for various $\delta_2$. 
Figure 5: Velocity profiles for various $s_2$.  

Figure 6: Velocity profiles for various Re.  

Figure 7: Velocity profiles for various $dp/dx$.  

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displayed in Figures 8–13. The behavior of cross-viscosity \( \delta_1 \) is displayed in Figure 8. In zone I, the profile behaves in the opposite phenomenon from the point of inflection. The increase in micropolarity increases the profile in the first half, whereas the reverse impact is observed in the second half. However, the backflow occurs in zone I. Due to the rotation of the particles, in zone II, the increase in cross-viscosity retards the angular velocity significantly. Near the central region, the pick in the profile is marked, and with a successive fall, the profile leads towards the upper plate. Figure 9 exhibits the variation of couple stress \( s_1 \) on the angular velocity profiles. Initiation with a backflow, the profile enhances irrespective the values of \( s_1 \) towards the central region, but for the increasing \( s_1 \), the profiles of angular velocity retards. Moreover, in zone II, the profile behaves with a similar trend in the positive domain. The variation of the cross-viscosity \( \delta_2 \) and the couple stress \( s_2 \) on the angular velocity is displayed in Figures 10 and 11, respectively. The profile behaves oppositely in different zones. Exhibiting a backflow in zone I, the angular velocity enhances with increasing \( \delta_2 \), and retardation is marked throughout the positive domain in zone II. For the variation \( s_2 \) in zone I, the change in angular velocity is very negligible. In a close remark, it is seen that with a backflow, the dual behavior is highlighted. However, in zone II, the attenuate in angular velocity is shown with enhancing couple stress parameter. Figure 12 portrays the contribution of the Reynolds number on the profiles for the fixed values of other parameters. As described earlier about the physical significance of the Reynolds number, the increasing value retards the angular momentum drastically in zone I. However, in zone II, the profile enhances in similar magnitude in the positive domain. The characteristics of the pressure gradient are exhibited on the angular velocity that is presented in Figure 13. The profiles of angular momentum seem to be symmetric in nature about the central region that is shown in both the zones. It is clear to see that for the increase in pressure gradient from negative to positive, the profile enhances significantly in zone I and in zone II, and the impact is reversed. Therefore, the symmetricity in the profiles is rendered.

5.3. Engineering Coefficients. Finally, the shear stress coefficients, as well as the couple stress coefficients at the fluid interface, are computed for various parameters and are displayed in Table 1. It is seen that the increase in cross-viscosity/micropolarity parameters retards the rate of shear stress and couple stress near the fluid interface. Whereas, reverse impact is observed with the increase in couple stress parameters \( s_1 \), i.e., the rate coefficients are enhanced. It is also pointed that the rate of shear stress coefficient increases, but the rate of couple stress coefficient decreases for increasing couple stress parameter \( s_2 \). Also, the Reynolds number parameter enhances both the rate coefficients significantly. Table 2 displays the rate of shear stress coefficients at the lower as well as the upper plate for different contributing parameters. The rate of shear stress at the lower plate attenuates in magnitude with the increasing cross-viscosity parameter, whereas the retardation is marked at the
Figure 10: Microrotation profiles for various $\delta_2$.

Figure 11: Microrotation profiles for various $s_2$.

Figure 12: Microrotation profiles for various Re.
Figure 13: Microrotation profiles for various $dp/dx$.

Table 1: Rate of shear stress coefficients and the couple stress coefficients at the fluid interfaces.

| $\delta_1$ | $s_1$ | $\delta_2$ | $s_2$ | Re | $(\partial u_1/\partial y) + 2(\delta_1/(1 + \delta_1))\omega_1$ | $(\partial u_2/\partial y) + 2(\delta_2/(1 + \delta_2))\omega_2$ | $\partial \omega_1/\partial y$ | $n_p\partial \omega_2/\partial y$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 1 | 5 | 1.5 | $-0.209728562$ | $-0.209728562$ | $-0.042152147$ | $-0.033721718$ |
| 2 | $-0.170874742$ | $-0.181053192$ | $-0.024428281$ | $0.034092599$ |
| 3 | $-0.150570191$ | $-0.161806471$ | $-0.071689941$ | $0.076368414$ |
| 4 | $-0.138921138$ | $-0.148996849$ | $-0.024428281$ | $0.033424504$ |

Table 2: The rate of shear stress coefficients at the lower and upper plates, respectively.

| $\delta_1$ | $s_1$ | $\delta_2$ | $s_2$ | Re | $\partial u_1/\partial y$ | $\partial u_2/\partial y$ |
|---|---|---|---|---|---|---|
| 1 | 5 | 1 | 5 | 1.5 | $-2.22E-16$ | $5.55E-17$ |
| 2 | $8.33E-17$ | 0 | 0 | $-5.55E-17$ | $-5.55E-17$ |
| 3 | $6.66E-16$ | $6.66E-16$ | $5.55E-17$ | $5.55E-17$ |

Table 2: The rate of shear stress coefficients at the lower and upper plates, respectively.
upper plate. A distinct characteristic is marked in the rate of shear stress for a couple of stress parameters. An increase in $s_1$ creases the shear stress at both the plates, whereas increasing $s_2$ retardation is marked. With the increasing Reynolds number, the rate increases in magnitude at the lower plate, and the effect is opposite at the upper plate. Moreover, Table 3 escalates the volumetric flow rate for various contributing parameters. BT_the flow rate decreases with the increase in micropolarity parameters as well as the couple stress parameter $s_1$, and increasing $s_2$, the flow rate increases significantly.

### Appendix

$$B_1 = \frac{1 + \delta_1}{s_1(2 + \delta_1)},$$

$$B_2 = \sqrt{\frac{s_2(2 + \delta_1)}{1 + \delta_1}},$$

$$B_3 = \frac{B}{(2 + \delta_1)},$$

$$B_4 = \frac{1 + \delta_2}{s_2(2 + \delta_2)},$$

$$B_5 = \sqrt{\frac{s_2(2 + \delta_2)}{1 + \delta_2}},$$

$$B_6 = \frac{n_p B}{n_g (2 + \delta_2)}.$$

$$B_7 = -\frac{1}{2} B_1 B_2 + \frac{1 + \delta_1}{2 \delta_1 s_1} B_1 B_2^3,$$

$$B_8 = \frac{1}{2} B_1 B_2 + \frac{1 + \delta_1}{2 \delta_1 s_1} B_1 B_2^3,$$

$$B_9 = -\frac{1}{2} B_4 B_5 - \frac{1 + \delta_2}{2 \delta_2 s_2} B_4 B_5^3,$$

$$B_{10} = \frac{1}{2} B_4 B_5 + \frac{1 + \delta_2}{2 \delta_2 s_2} B_4 B_5^3,$$

$$B_{11} = \frac{2 \delta_1}{(1 + \delta_1)},$$

$$B_{12} = \frac{2 \delta_2}{1 + \delta_2}.$$
Nomenclature

Be: Bejan number
Br: Brinkman number
\( \overline{q} \): Velocity vector
\( a_f \): Microinertial parameter
Bi: Biot number
\( Br/f_k \): Viscous dissipation parameter
D: Deformation tensor
Cp: Specific heat
Gr: Grashof number
h: Channel width
\( k_f \): Thermal conductivity
\( m^2 \): Micropolar parameter
N: Coupling number
\( Nh \): Entropy generation (heat transfer)
Ns: Entropy generation number
Nv: Entropy generation (viscous dissipation)
Re: Reynolds number
T: Dimensional temperature
\( \eta_f \): Ratio of couple stress viscosity coefficients
\( \eta_k \): Ratio of thermal conductivities
\( \eta_v \): Ratio of viscosities
\( T_1 \): Ambient temperature
\( T_2 \): Fluid temperature
u: Dimensional axial velocity
X: Flow direction
Y: Normal to the flow direction.

Greek Symbols
\( \alpha \): Inclined angle
\( \alpha_f \): Slip parameter
\( \beta, \gamma \): Gyration viscosity coefficients
\( \kappa \): Vortex viscosity
\( \rho \): Density of the fluid
\( \theta \): Dimensionless temperature
\( \mu \): Viscosity of the fluid
\( \sigma \): Microinertial component
f: Velocity profile.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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