An evidence-based assumption that helps to reduce the discrepancy between the observed and predicted $^7$Be abundances in novae

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ABSTRACT

Recent spectroscopic measurements of the equivalent widths of the resonant Be II doublet and Ca II K lines and their ratios in expanding nova ejecta indicate surprisingly high abundances of $^7$Be with a typical mass fraction $X_{\text{obs}}(^7\text{Be}) = 10^{-4}$. This is an order of magnitude larger than theoretically predicted values of $X_{\text{theor}}(^7\text{Be}) \sim 10^{-5}$ for novae. We propose how this discrepancy can be reduced. We use an analytical solution of the $^7$Be production equations to demonstrate that $X_{\text{theor}}(^7\text{Be})$ is proportional to the $^4$He mass fraction $Y$ in the nova accreted envelope and then we perform computations of 1D hydrostatic evolution of the 1.15 $M_\odot$ CO nova model that confirm our conclusion based on the analytical solution. Our assumption of enhanced $^4$He abundances that helps to reduce the discrepancy between $X_{\text{obs}}(^7\text{Be})$ and $X_{\text{theor}}(^7\text{Be})$ is supported by UV, optical and IR spectroscopy data that reveal unusually high values of $Y$ in nova ejecta. We also show that a significantly increased abundance of $^3$He in nova accreted envelopes does not lead to higher values of $X_{\text{theor}}(^7\text{Be})$ because this assumption affects the evolution of nova models resulting in a decrease of both their peak temperatures and accreted masses and, as a consequence, in a reduced production of $^7$Be.

Key words: nuclear reactions, nucleosynthesis, abundances, stars: abundances, stars: novae, cataclysmic variables

1 INTRODUCTION

It had been suggested a long time ago that novae could produce significant amounts of $^7$Li (Starrfield et al. 1978). This hypothesis has recently been supported by direct spectroscopic detection and measurement of high abundances of $^7$Be in expanding ejecta of seven novae, taking into account that $^7$Li is a decay product (100% electron capture) of $^7$Be whose terrestrial half-life is 53.2 days (Tajitsu et al. 2015, 2016; Molaro et al. 2016; Selvelli et al. 2018; Izzo et al. 2018; Molaro et al. 2020). However, a problem arises when we compare the observed $^7$Be mass fractions in the novae with their theoretically predicted values and find that, on average, the former, $X_{\text{obs}}(^7\text{Be}) \approx 10^{-4}$ (Molaro et al. 2020), exceed the latter, $X_{\text{theor}}(^7\text{Be}) \sim 10^{-5}$ (e.g., Hernanz et al. 1996; José & Hernanz 1998; Starrfield et al. 2020), by an order of magnitude.

The discrepancy between the observed and predicted $^7$Be abundances in novae can obviously be reduced either by casting doubt on the spectroscopically derived $^7$Be abundances, showing that they were overestimated, or by tuning up nova models under reasonable assumptions, so that they can predict higher $^7$Be abundances. Chugai & Kudryashov (2020) have explored the first possibility criticizing the assumption made in the cited observational works that the ionic number density ratio $N(\text{Be II})/N(\text{Ca II})$, obtained by comparing equivalent widths of the resonant Be II doublet and Ca II K lines to derive $X_{\text{obs}}(^7\text{Be})$, is equal to the total number ratio $N(\text{Be})/N(\text{Ca})$. Chugai & Kudryashov (2020) have used observational data of Molaro et al. (2016) for the nova V5668 Sgr to construct a simple model of its expanding photosphere and atmosphere with which they have demonstrated that it was a wrong assumption. According to
their model, the ionization fraction $N$(Be II)/$N$(Be) is $\sim$10 to $\sim$ 100 times larger than $N$(Ca II)/$N$(Ca), which should decrease the abundance of $^7$Be in this nova, $X_{\text{obs}}$(Be) $\approx$ 7.3 $\times$ 10$^{-5}$, by the corresponding factors, bringing it into an agreement with theoretical predictions.

It is beyond the scope of our paper to discuss if the conclusion about a significant overestimate of the $^7$Be abundance in V5668 Sgr and possibly in other novae made by Chugai \& Kudryashov (2020) is true or not. We can only add that the assumption that most of the $^7$Be and Ca atoms are in the singly ionized state is supported, according to Tajitsu et al. (2016), by the fact that they failed to detect doubly ionized states of Fe-poor elements with second ionization potentials intermediate to those of Be II (18.21 eV) and Ca II (11.87 eV) in the spectra of novae V5668 Sgr and V2944 Oph. Also, Selvelli et al. (2018) obtained very close values that the assumption that most of the $^7$Be abundance in the nova V838 Her using four different methods, one of which was based on the assumption that $N$(Be II)/$N$(Mg II) = $N$(Be)/$N$(Mg) and used an equivalent width of Mg II whose ionization potential 15.03 eV is closer to that of Be II. We leave a further discussion of the veracity of the reported anomalously high abundances of $^7$Be in novae to stellar spectroscopists and proceed to a brief summary of previously proposed tuning-ups of nova models that result in an increase of their predicted $^7$Be abundances.

There are three types of models that follow in detail thermonuclear runaway (TNR) and convective nucleosynthesis in novae: 1D hydrodynamic models, 1D hydrostatic stellar evolution models, and one- or two-zone parametric models, the last ones being only used for simple estimates. For the frequently modelled CO nova with the white-dwarf (WD) mass 1.15 $M_\odot$, the largest value of $X_{\text{theor}}$(Be) = 1.9 $\times$ 10$^{-5}$ has been obtained by Starrfield et al. (2020) in their 1D hydrodynamic simulation in which the solar composition of accreted envelope was switched to a mixture of 25% of the WD and 75% of the solar compositions immediately after a start of the TNR, when convective mixing had begun, to mimic the dredge-up of the WD material by convection, as revealed by 3D hydro simulations of Casanova et al. (2016). Our nova evolutionary model with 50% pre-mixed WD composition in the accreted envelope computed using the MESA code implemented in the Nova Framework (Denissenkov et al. 2014) predicts the same value of $X_{\text{theor}}$(Be) = 1.2 $\times$ 10$^{-5}$ as the one obtained by Starrfield et al. (2020) for their second considered case of a mixture of equal amounts of the WD and solar compositions. This is larger than the value of $X_{\text{theor}}$(Be) = 8.1 $\times$ 10$^{-6}$ reported by José \& Hernanz (1998) for their 1D hydrodynamic nova model with the similar parameters. The two-zone parametric model of Chugai \& Kudryashov (2020) has an upper limit of $X_{\text{theor}}$(Be) = 5 $\times$ 10$^{-5}$.

Because $^7$Be is produced in the reaction $^3$He($^3$He,$^7$Be) $^7$Be, it has been suggested many times, but, as far as we know, has not been verified in nova simulations yet, that a higher $^3$He mass fraction in accreted matter could help to reduce the discrepancy between the observed and predicted $^7$Be abundances in novae. In this Letter, we will show that, instead of $^3$He, it is rather an assumed enhanced abundance of $^4$He, which is indeed observed in nova envelopes (Gehrz et al. 1998; Downen et al. 2013), that can help to push the predicted abundance of $^7$Be in novae closer to its observed values, that will be represented here by the value of $X_{\text{obs}}$(Be) = 10$^{-4}$ considered as a typical $^7$Be yield for novae by Molaro et al. (2020), thus reducing the discrepancy between observations and theory.

### 2 EQUATIONS OF THE $^7$BE PRODUCTION AND THEIR ANALYTICAL SOLUTION

The rates of the reactions that affect the production of $^7$Be in novae have negligible uncertainties, which leaves us with a few parameters whose variations within their reasonable limits may lead to a significant increase of this production. The surprisingly good agreement between the $^7$Be yields provided for the similar nova models by different simulations means that it is probably not details of the physics of nova explosion that mainly determine the predicted $^7$Be abundance in its ejecta. Therefore, we have decided to vary the initial abundance of $^3$He, as was previously proposed, and also that of $^4$He, because these parameters can still be considered as relatively free (their possible variation ranges will be discussed later) and because their values are expected to directly affect the $^7$Be production in the reaction $^3$He($^3$He,$^7$Be)$^7$Be. The fruitful consumption of $^3$He by this reaction is accompanied by its waste in the competing reaction $^3$He($^3$He,2$p$)$^4$He whose cross section is five orders of magnitude larger. At given temperature $T$, density $\rho$, and constant $^4$He mass fraction $Y$, this competition is described by the following $^7$Be production equations:

$$
\frac{dX(^7\text{Be})}{dt} = -\frac{1}{3} \lambda_1 \rho [X(^7\text{He})]^2 - \frac{1}{4} \lambda_2 \rho X(^3\text{He})Y,$$

$$
\frac{dX(^3\text{He})}{dt} = \frac{7}{12} \lambda_2 \rho X(^3\text{He})Y,
$$

where $\lambda_1$ and $\lambda_2$ are $T$-dependent rates ($\lambda_i \equiv \langle \sigma v \rangle_i N_A$ is the product of the Maxwellian-averaged cross section and Avogadro number) of the reactions $^3$He($^3$He,2$p$)$^4$He and $^3$He($^3$He,$^7$Be)$^7$Be, respectively. The analytical solution of these equations is

$$
X(^7\text{Be}) = \frac{7}{4} Y \frac{\lambda_2}{\lambda_1} \left[ 1 + \frac{4}{3} \frac{X_0(^3\text{He}) \lambda_1}{Y \lambda_2} \left( 1 - e^{-t/\tau} \right) \right],
$$

where $\tau = 4/(\lambda_2 \rho Y)$, and $X_0(^3\text{He})$ is the initial mass fraction of $^3$He.

Taking a half-solar value of $Y = 0.14$, for the density $\rho = 261$ g cm$^{-3}$ and temperature $T$ = $10^{39}$ K = 0.232 that are achieved near the maxima of energy generation rates in both the CNO cycle and the pp chains at the bottom of the accreted envelope in our 1.15 $M_\odot$ CO nova model (Section 3), we find the timescale $\tau \approx 40$ s that is comparable to the timescale of very fast changes of the nova $\rho$ and $T$ profiles during a time interval of $\sim$ 100 s around its peak temperature. By the end of this period, $X(^7\text{Be})$ has already attained its maximum value. Therefore, when estimating the final values of $X(^7\text{Be})$ predicted by our analytical solution, we ignore the exponent in Equation (1), assuming that $t/\tau \gg 1$. These values are plotted in Figure 1 for a range of $T$ and for different values of the parameters $X_0(^3\text{He})$ and $Y$, keeping one of them fixed at its half-solar value, i.e. $X_0(^3\text{He}) = 2.96 \times 10^{-5}$ and $Y = 0.14$ for the 50% WD pre-mixed accreted matter, and assuming that all $^2$H was transformed into $^3$He in the donor star, when changing the other.
From Figure 1 we see that $X(\textsuperscript{7}\text{Be})$ increases with $T$, which agrees with its increase with the WD mass found by Starrfield et al. (2020). The range of the $\textsuperscript{7}\text{Be}$ yield predicted for novae by our analytical solution for the half-solar values of both $X_0(\text{He})$ and $Y$ encompasses almost all the values of $X(\textsuperscript{7}\text{Be})$ predicted by the corresponding 1D nova models. It is clear from Equation (1) that the persistent prediction of $X(\textsuperscript{7}\text{Be}) \approx 10^{-5}$ by nova models is simply caused by the fact that $\lambda_2/\lambda_1 \approx 10^{-5}$ and this ratio weakly depends on $T$.

Given that the predicted value of $X(\textsuperscript{7}\text{Be}) \gg X_0(\text{Li})$, Equation (1) can also be written as

$$\frac{X(\textsuperscript{7}\text{Li})}{X_0(\text{Li})} = 1 + C \log_{10} \frac{X_0(\text{He})}{X_0(\text{He})},$$

where $C = \left\{ \log_{10} \left[ \frac{4 X_0(\text{He})}{Y} \lambda_1 \lambda_2 \right] \right\}^{-1}$, and $X_0(\text{Li})$ is the abundance of $\text{Li}$ for the case of $X_0(\text{He}) = X_0(\text{He})$. This equation assumes that $X(\text{Li}) = X(\text{Be})$ and again that $t/\tau \gg 1$. It has been derived by Boffin et al. (1993) with the value of $C = 1.5$ constrained by results of their parametric one-zone nova model calculations. For their preferred value of $T_B = 0.3$, our analytical solution provides $C = 0.75$. The logarithmic dependence of $X(\textsuperscript{7}\text{Be})$ on $X_0(\text{He})$ is clearly seen in Figure 1.

The maximum 2 dex enhancement of the initial $\text{He}$ abundance relative to its solar value, assumed in Figure 1 for the accreted matter, could potentially come from the so-called $\text{He}$ bump inside the donor star, formed as a result of incomplete H burning in the pp I branch, like in all solar-type stars. Then, given that the WD companion is most likely to be tidally locked, its rapid rotation and tidal deformation should drive meridional circulation that may reach the bump and bring the abundant $\text{He}$ to the surface from where it will be donated to the WD. It is also possible that the companion has lost enough mass to expose the $\text{He}$ bump at its surface (Shen & Bildsten 2009).

Equation (1) and Figure 1 also show that the amount of $\text{Be}$ produced in novae is proportional to $Y$, hence its increase by the factors of 2 and 3 should result in the enhancements of $X(\text{Be})$ comparable to those obtained for $X_0(\text{He})/X_0(\text{He}) = 10$ and 100. Table 2 of Gehrz et al. (1998) and Table 1 of Downen et al. (2013) summarize UV, optical and IR spectroscopy data on mass fractions of H, He and heavy elements in nova ejecta. Most of the $Y$ values presented in this table exceed the half-solar value of $Y \approx 0.14$ used in our and other similar nova models with the 50% pre-mixed WD composition, nearly 30% of them being larger than 0.3 with six stars having $Y > 0.4$. Therefore, the assumption of $Y > 0.14$ in the nova envelope that we make in this work is supported by observations.

Because $\text{He}$ burning starts at the bottom of the accreted envelope before the TNR is triggered by the reaction $^{12}\text{C}(p,\gamma)\text{N}$ (e.g., Shen & Bildsten 2009, and references therein), the impact of a significant increase of its initial abundance on the $\text{Be}$ production in novae can only be studied using full nova models, since neither simple one- or two-zone parametric models nor even multi-zone post-processing nucleosynthesis models, like the one of the Nova Framework that uses the NuGrid mppp code (Denissenkov et al. 2014), take into account a feedback of this assumption on nova properties. Therefore, we have employed the MESA code setup of the Nova Framework to perform 1D hydrostatic evolutionary computations of our $1.15 M_\odot$ CO nova model with increased abundances of $\text{He}$ and $\text{He}$ in its accreted envelope, results of which are presented in the next section.

### 3 RESULTS OF 1.15 $M_\odot$ CO NOVA EVOLUTIONARY COMPUTATIONS WITH INCREASED ABUNDANCES OF $\text{He}$ AND $\text{He}$ IN THE ACCRETED ENVELOPE

Our Nova Framework has been using the revision 5329 of the MESA stellar evolution code (Paxton et al. 2011, 2013) with a content of its inlist file set up to model CO and ONe nova evolution (Denissenkov et al. 2014). In this work, we have chosen the largest of the nuclear reaction networks available in the Nova Framework nova.net with 77 species from $^1\text{H}$ to $^{48}\text{Ca}$ coupled by 442 reactions for which we have used the rates from the JINA Reaclib v1.1 database (Cyburt et al. 2010). This network includes most of the reactions affecting the $\text{Be}$ production in novae discussed by Boffin et al. (1993) and Hernanz et al. (1996), except $^{7}\text{Be}(\alpha,\gamma)^{13}\text{N}$ and the chain $^{8}\text{B}(p,\gamma)^{9}\text{C}(e^+,\nu)^{12}\text{He}$ because these are much slower than their competing reactions $^{7}\text{Be}(p,\gamma)^{12}\text{C}$ and $^{8}\text{B}(e^+,\nu)^{12}\text{He}$.

Results of our computations of the evolution of the $1.15 M_\odot$ CO nova model with the WD central temperature $12 \times 10^6$ K and accretion rate $2 \times 10^{-10} M_\odot$ yr$^{-1}$ are presented in Figure 2 for the 50% WD pre-mixed composition of the accreted envelope, including the $\text{He}$ and $\text{He}$ mass fractions, i.e. for $Y = 0.14$ and $X_0(\text{He}) = 2.96 \times 10^{-5}$. As already mentioned, we see that the $\text{Be}$ abundance attains its maximum value of $X_{\text{max}}(\text{Be}) = 1.24 \times 10^{-5}$ (panel c) around the peak temperature $T_B = 0.232$ (panel b), when...
Figure 2. The track on the Hertzsprung-Russell diagram (panel a), temperature (b), $^7$Be mass fraction (c) and density (d) mass-coordinate profiles inside the envelope of our 1.15 $M_\odot$ CO nova model shown for different evolutionary phases indicated by model numbers. The $^7$Be abundance attains its maximum value between models 360 and 400, at the knee of the evolutionary track (between the light-gray and brown circles in panel a), when $T$ reaches its peak value at the bottom of the accreted envelope (panel b). This computation used the 50% WD pre-mixed chemical composition for the envelope.

the star approaches the knee of its evolutionary track (between the models 360 and 400 in panel a). By this moment, the density at the bottom of the convective envelope has already dropped to $\rho = 261$ g cm$^{-3}$ from its maximum value of $8 \times 10^3$ g cm$^{-3}$. The model has accreted $M_{\text{acc}} = 2.53 \times 10^{-5} M_\odot$ by the beginning of its TNR, and its evolution was followed for 71 minutes after the peak temperature.

After the $^4$He mass fraction in the accreted envelope was increased to the values of $Y = 0.28$ and $Y = 0.42$, that are within the limits of $Y$ measured in nova ejecta (Gehrz et al. 1998; Downen et al. 2013), at the expense of the $^{16}$O abundance to keep the sum of all mass fractions equal to one, the maximum $^7$Be abundances in our nova model have reached the values of $X_{\text{max}}(^7$Be) = $1.97 \times 10^{-5}$ and $X_{\text{max}}(^7$Be) = $2.54 \times 10^{-5}$, respectively. The other nova properties have essentially remained unchanged. All the three values of $X_{\text{max}}(^7$Be) predicted by the nova model are displayed in Figure 1 as star symbols of the same colours that we used to plot the analytical solution curves for the corresponding values of $Y$. Although resulting in slightly different values, unsurprisingly because we compare the simple one-zone and detailed multi-zone evolutionary nova models, the two sets of computations show the same trend of the predicted $^7$Be abundance increasing with $Y$. It is interesting that our value of $X_{\text{max}}(^7$Be) obtained for $Y = 0.28$ agrees very well with the value of $X(^7$Be) = $1.9 \times 10^{-5}$ reported by Starrfield et al. (2020) for their 1D hydrodynamic 1.15 $M_\odot$ CO nova model with a mixture of 25% WD and 75% solar compositions in the envelope that has a close value of $Y = 0.23$.

The results change drastically when we assume a significantly increased initial abundance of $^4$He in the accreted envelope. For $X_0(^4$He)/$X_0(^{16}$O) = 100 and $Y = 0.42$ our nova model yields $X_{\text{max}}(^7$Be) = $1.98 \times 10^{-5}$ which is 22% lower than for the nova model with the same value of $Y$ and the half-solar mass fraction of $^4$He (the filled and open star symbols of the same colour in Figure 1). The other nova properties also undergo considerable changes, e.g. the peak temperature and the accreted mass are reduced to $T_p = 0.168$ and $M_{\text{acc}} = 6.72 \times 10^{-6} M_\odot$. These changes are caused by the substantially increased heating of the bottom of the accreted envelope by the energy released in the reaction $^3$He($^3$He,2$p$)$^4$He whose $Q$ value 12.86 MeV is much larger than $Q = 1.95$ MeV for the TNR triggering reaction $^{12}$C($p$,r)$^{13}$N. As a result, the TNR starts earlier, with the lower peak $T$ and $M_{\text{acc}}$. Thus, taking into account the feedback of the increased value of $X_0(^4$He) on the nova properties leads to a decrease, rather than to the previously expected increase, of the maximum $^7$Be abundance predicted with the evolutionary nova model. Therefore, Equations (1) and (2) derived for the one-zone parametric model do not provide correct dependencies of the $^7$Be and $^7$Li nova yields on the initial abundance of $^4$He.
We have done 1D stellar evolution computations of the popular 1.15 $M_\odot$ CO nova model to demonstrate that the $^7\text{Be}$ production in novae increases proportionally to the $^4\text{He}$ mass fraction $Y$ in their accreted envelopes, as predicted by the analytical solution (1). Therefore, the assumption of a 2 to 3 times enhanced value of $Y$, as compared to the half-solar value of $Y = 0.14$ used in nova models accreting a mixture of equal amounts of the WD and solar compositions, helps to reduce the discrepancy between the observed and theoretically predicted $^7\text{Be}$ mass fractions in novae, $X_{\text{obs}}(^7\text{Be}) \approx 10^{-4}$ and $X_{\text{theor}}(^7\text{Be}) \sim 10^{-5}$. This assumption is supported by the surprisingly high values of $Y$, with the maximum value of $Y = 0.6$, reported in nova ejecta by Gehrz et al. (1998) and Downen et al. (2013) that have not been explained yet. In principle, the high $^4\text{He}$ abundances in nova ejecta could come from a $^4\text{He}$ buffer layer atop of the accreting white dwarf that was either left by its AGB progenitor (Iben et al. 1992; Starrfield et al. 1998; Denissenkov et al. 2013) or formed as a result of H afterburning during the supersoft X-ray phase following each nova explosion (Wolf et al. 2013).

We have also shown for the first time that the previously proposed hypothesis of a significantly enhanced, up to 2 dex, mass fraction of $^3\text{He}$ in the nova accreted envelope does not raise $X_{\text{theor}}(^7\text{Be})$ closer to $X_{\text{obs}}(^7\text{Be})$. Instead, it results in a decrease of $X_{\text{theor}}(^7\text{Be})$ because of the feedback of this assumption on nova properties — the increased release of heat in the highly energetic reaction $^3\text{He}(^3\text{He},2p)^4\text{He}$ at the bottom of the accreted envelope leads to an earlier TNR with lower both peak temperature and accreted mass.

If we extrapolate our result obtained for $Y = 0.42$ (the orange star symbol in Figure 1) to the peak temperature $T_B = 0.350$ of the 1.3 $M_\odot$ ONe nova model (e.g., see Table 1 of Denissenkov et al. 2014) we will get an estimate of $X_{\text{theor}}(^7\text{Be}) \approx 3 \times 10^{-5}$ that is only ~3 times smaller than the average mass fraction of $^7\text{Be}$ observed in novae, although a slight decrease of the latter is still required for a better match.

Of note for future work, the higher $^7\text{Be}$ mass fraction calculated here would lead to a higher yield of the characteristic 478 keV gamma ray via the $10\%^7\text{Be}(e^−,\nu)^7\text{Li}$, as calculated in e.g. Gaump-Gomar et al. (1998), in which the terrestrial value of the $^7\text{Be}$ EC lifetime is adopted under the assumption that full Be ionization is not substantial. Applying the formalism of Iben et al. (1967) to the conditions at the end of our nova trajectory (71 minutes) implies a very small free electron capture rate, as expected due to the low density. Furthermore, the occupation probabilities for K-shell electrons under these conditions are very small, implying that most of the $^7\text{Be}$ is fully-ionized at this stage and will not convert at the terrestrial rate to $^7\text{Li}$ until conditions are met such that bound state captures occur frequently again. Thus the effective lifetime, considering only the small K-shell capture rate, is on the order of $5 \times 10^6$ days. Due to the timescales involved in the nova trajectory, it is not clear that this variation in lifetime will affect the temporal evolution of the 478 keV line, and requires careful consideration of the conditions in the hot expanding ejecta.

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**DATA AVAILABILITY**

The data underlying this article will be shared on reasonable request to the corresponding author.

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