Programming of the Average Windward Area of Finite Element Fragments Based on MCSPS

Xingyu Liu\textsuperscript{1,3,*}, Renjun Zhan\textsuperscript{1} and Jiaying Wang\textsuperscript{2}

\textsuperscript{1} College of Equipment Management and Support, Armed Police Engineering University, Xi'an 710086, Shaanxi Province
\textsuperscript{2} Basic Department, Armed Police Engineering University, Xi'an 710086, Shaanxi Province
\textsuperscript{3} Laboratory of the Ministry of Marine Intelligent Equipment and Systems Education, Shanghai Jiaotong University, Xuhui District, Shanghai 200030

*Corresponding author email: lzclsrlxy@163.com

Abstract. In order to obtain the average windward area of the natural fragments in finite element method, a simulation model of the average windward area is established based on the Monte Carlo Subdivision Projection Method. The projection boundary of the fragments is obtained. Based on the cross multiplication, the simulation program of the windward area of the FE natural fragment is compiled. According to the concave and convex polygon vertices of the different quadrants, the average windward area of the finite element fragments is solved. The results show that the average windward area simulation program based on cross multiplication is applicable to the area of concave and convex polygons; the results of different quadrants of the same polygon are exactly the same; the program is applicable to the average windward area of simulation fragments with arbitrary irregular shape.

1. Introduction

In terms of the average windward area calculation of warhead prefabricated fragments, a large number of theoretical and experimental studies have been carried out. Based on the experimental results, the empirical formula of the windward area of regular shaped fragments such as cylinder, sphere, cuboid and diamond has been established \cite{1,2}. At present, the average windward area of natural fragments is mainly calculated by the test method of uniform orientation theory \cite{3-7}. However, the test method is not suitable for the finite element (FE for short) natural fragments. Simulation calculation model of the average windward area of the FE natural fragments has not been reported. The most commonly method for solving the fragment projection area is the Triangulation method, which divides the polygonal area into several triangles, and then calculates the area of each triangle \cite{8}. Compared with Helen's formula and Pick's theorem, the cross multiplication is more accurate and efficient. Therefore, based on the FE results of the 37mm stun grenade, the number, coordinate, velocity and volume of elements in different fragments are obtained by the same fragment search algorithm. Based on the calculation model of the FE fragments average windward area established by the Monte Carlo subdivision projection simulation (MCSPS for short), the projection boundary of the fragment nodes is obtained. Then, based on the cross multiplication, the simulation program of FE fragment average windward area is compiled. According to the concave and convex polygon vertices of the different quadrants, the program applicability is tested. Through the program calculation, the number of
fragments, the coordinates of the centroid, the velocity of the centroid, the volume, the number of elements and the average windward area are obtained.

2. FE Fragments and the Same Fragment Search Algorithm

2.1. FE fragments and Data Integration
In this paper, the shell breaking model of 37mm stun grenade is solved [9], which is established by the regular hexahedron elements of the 1/4 symmetrical model. After LS-DYNA solution, the shell breaking situation of natural fragments at different times is obtained, as shown in Figure 1. At 30us, the package shell has been completely broken.

Figure 1. The shell breaking situation of natural fragments at different times

Export the data of elements and nodes at this time. Data files such as element ID, element coordinate-volume and node velocity are established respectively. According to MATLAB, element ID and coordinate-volume are imported and consolidated, which is defined as database $E$; node velocity file is also imported, which is defined as database $N$. In this process, the file pointer is used to read the data, so as to remove the keyword format and solution information of LS-DYNA output file.

Database $N$ contains all node numbers and corresponding velocity components ($v_{n1}$, $v_{n2}$, $v_{n3}$). Database $E$ contains all element numbers, eight node numbers of the regular hexahedron element, element coordinates ($x$, $y$, $z$) and element volume. Due to the different stress-strain transport, the velocities of the eight nodes are not the same, which needs to be averaged. By traversing the node number column of database $N$, eight nodes velocities of elements in $E$ are searched and obtained. The velocity matrix of elements ($v_{e1}$, $v_{e2}$, $v_{e3}$) is calculated by the arithmetic average. The expression is as follows:

$$v_{ei} = \frac{1}{8} \sum_{j=1}^{8} v_{ji}$$  \hspace{1cm} (1)

Where, $i$ is free index of tensor; $v_{ji}$ is the node velocity component; $v_{ei}$ is the element velocity component, which is put into the new column of Database $E$.

2.2. The Same Fragment Search-sort Algorithm
Because the element numbers in $E$ are arranged in ascending order, the quantity, volume, velocity and coordinate of the same fragment cannot be obtained directly. Database $E$ need to be searched and sorted according to the same fragment elements. By observing the hexahedral elements, it is not difficult to find the following criteria: any adjacent element must belong to the same fragment; at least one of the eight node numbers of the adjacent element are equal. According to the above criteria, the search can be started from the first element in $E$, and the elements satisfying the conditions are taken out of $E$ and put into a new database $P_{e}$. Through the program loop, one element will be extended to the neighbouring elements, and the neighbouring elements will be further extended to all elements of the same fragment. The flow chart of the same fragment search-sort algorithm is as shown in Figure 2. After searching and sorting, database $P_{e}$ arranged by the same fragment number can be obtained. The data result is shown in Figure 3. The fragment quantity after shell broken is obtained by the maximum value of fragment number. The volume of each fragment can be obtained by summing the volume of elements in the same fragment. The centroid coordinate and velocity of each fragment can be obtained by averaging the coordinate component and velocity component. In addition, the average windward area of each fragment can be solved by MCSPS method.
3. The Simulation Program of Average Windward Area Based on MCSPS Method

3.1. The Overview of MCSPS Method

The calculation model of the average windward area of the FE fragments is established by the Monte Carlo Subdivision Projection Simulation (MCSPS). With $N$ times translation, random rotation, projection and triangulation of the node coordinates of FE fragments, the mean value of $N$ times projection area $A_{ji}$ can be solved and obtained. When the $N$ value tends to infinity, the mean value can be approximated as the average windward area of the fragment. The expression can be defined as follows:

$$\bar{A}_j = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} A_{ji}$$  \hspace{1cm} (2)$$

Where, $\bar{A}_j$ is the average windward area of the natural fragment $j$; $A_{ji}$ is the windward area of the fragment $j$ of the $i$th MCSPS; $N$ is the total number of MCSPS.

Algorithm flow chart of average windward area based on MCSPS is shown in Figure 4. $P_e$ is the database of fragment elements; $P$ is the fragment database; $i_2$ and $i$ are row search mark of $P_e$; $q$ is the fragment number; $mc$ is the variable of Monte Carlo number; $N$ is the total Monte Carlo number, which is set to 50000 times; $T$ is the projection points matrix; $dt$ is the structure variable used to store the triangulation data; and $area_{mc}$ is used to store the area matrix of each Monte Carlo projection.
3.2. Cross Multiplication for Polygon Area

According to the simulation program, the polygon boundary index \( pb \) arranged clockwise or anticlockwise can be obtained. Assume the coordinates of the boundary points are arranged in clockwise direction, which are \( P_1 (x_1, y_1), P_2 (x_2, y_2), \ldots, P_n (x_n, y_n) \). It can be seen from Figure 5 that the polygon area \( A \) is equal to the sum of the clockwise triangle areas minus the sum of the anticlockwise. The expression is as follows:

\[
A = \sum_{i=1}^{n-1} \frac{1}{2} \left| \begin{array}{ccc} x_1 & y_1 & 1 \\ x_{i+1} & y_{i+1} & 1 \\ x_1 & y_1 & 1 \end{array} \right| - \sum_{i=1}^{n-1} \frac{1}{2} \left| \begin{array}{ccc} x_{i+1} & y_{i+1} & 1 \\ x_{i+2} & y_{i+2} & 1 \\ x_1 & y_1 & 1 \end{array} \right|
\]

For the different quadrants vertex, the result of \( A \) could be negative. The absolute value of \( A \) should be taken. The triangle area is equal to half of the modulo of two vector cross multiplication. which is the directed area. Therefore, the calculation formula of concave or convex polygon area based on cross multiplication can be deduced as shown in formula (4), and the MATLAB code of cross multiplication solving polygon area is shown in Figure 6.

\[
A = \frac{1}{2} \sum_{i=1}^{n-1} \left( x_i y_{i+1} - x_{i+1} y_i \right) + \left( x_{i+1} y_n - x_n y_{i+1} \right)
\]

Figure 4. Algorithm flow chart of average windward area based on MCSPS

Figure 5. Schematic diagram of polygon area by cross multiplication

Figure 6. MATLAB codes of cross multiplication solving concave or convex polygon area
4. Results and Discussion

4.1. Program Applicability Test of Concave and Convex Polygon

In order to test the applicability of cross multiplication for concave and convex polygons, the concave and convex quadrilaterals which are convenient for geometric calculation are selected for discussion, as shown in Figure 7. According to the cross-multiplication program calculation, the convex polygon area is 6, and the concave polygon area is 2.25. By geometric calculation, the convex polygon area is the sum of the left triangle area 3 and the right triangle area 3; the concave polygon area is the difference of the two triangles, which is completely consistent with the program results. It is shown that the cross multiplication is suitable for both concave and convex polygons.

![Figure 7. Concave and convex polygons with four vertices](image)

4.2. Uniqueness Verification of Different Quadrant Results

In order to verify uniqueness of different quadrant results, respectively translate the vertices in Figure 7 (b) to a single quadrant, two quadrants, three quadrants, and four quadrants for discussion. The results are shown in Figure 8. The program results in four situations are completely consistent with the geometric calculation results. The polygon area obtained by the cross-multiplication program is independent of the quadrant of the vertex.

![Figure 8. Different quadrant polygons](image)

4.3. Average Windward Area of FE Fragments

The result of the average windward area of the FE fragments in Figure 1 can be obtained by solving, as shown in Figure 9. There are 15 fragments in shell breaking model, whose shapes and numbers are shown in Figure 10. Fragment 4, 6, 8, 10, 14 and 15 are shell debris with less than 3 elements, which cannot be triangulated effectively.

![Figure 9. The results of database P by MCSPS](image)  
![Figure 10. Shapes and numbers of FE fragments](image)
4.4. Accuracy and Complexity of MCSPS Method

The calculation time and accuracy of MCSPS and uniform orientation method \(^{[5]}\) of fragment 5 are compared, and the results are shown in Table 1. Operating environment: Intel (R) core (TM) i7-8750h CPU @ 2.20GHz, 16GB and MATLAB r2017a. In the table, as the cycles times increases, the average windward area of MCSPS method gradually converges to 2.281cm\(^2\). The uniform orientation method is not converged, which is greatly affected by the initial projection surface. For the same cycle times, the algorithm complexity of MCSPS method and uniform orientation method is basically the same. In general, the uniform orientation method can get a low precision result only by 16 cycles times, while the MCSPS method can get a high accurate result by thousands of cycles.

| cycle times | MCSPS solution time/s | windward area/m\(^2\) | uniform orientation method solution time/s | windward area/m\(^2\) |
|-------------|-----------------------|------------------------|------------------------------------------|------------------------|
| 16          | 6.230E+0              | 2.247E-4               | 6.332E+0                                 | 2.302E-4               |
| 32          | 1.249E+1              | 2.247E-4               | 1.265E+1                                 | 2.370E-4               |
| 64          | 2.499E+1              | 2.286E-4               | 2.516E+1                                 | 2.476E-4               |
| 128         | 4.979E+1              | 2.227E-4               | 5.014E+1                                 | 2.138E-4               |
| 256         | 1.824E+2              | 2.245E-4               | 1.001E+2                                 | 2.542E-4               |
| 512         | 2.829E+2              | 2.238E-4               | 2.002E+2                                 | 2.146E-4               |
| 1024        | 4.828E+2              | 2.260E-4               | 4.013E+2                                 | 2.207E-4               |
| 2048        | 8.875E+2              | 2.264E-4               | 8.013E+2                                 | 2.327E-4               |
| 4096        | 1.689E+3              | 2.273E-4               | 1.597E+3                                 | 2.527E-4               |
| 8192        | 3.278E+3              | 2.281E-4               | 3.187E+3                                 | 2.505E-4               |
| 16384       | 6.457E+3              | 2.281E-4               | 6.372E+3                                 | 2.101E-4               |

5. Conclusion

The windward area of irregular FE fragments can be solved by the simulation program of MCSPS. The cross-multiplication program is applicable to the solution of concave and convex polygon area, and the polygon area is independent of the quadrant of the vertex. Compared with the uniform orientation method, MCSPS method is more accurate, but the solution time is longer.

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