Spin effects in forward high energy proton scattering

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Abstract. The relation between elastic amplitudes for polarized proton collisions and spin-dependent scattering observables is studied near the forward direction with a view to providing information on particular amplitudes at low values of momentum transfer. Such information has been singularly successful, for instance, in evaluating the amount of beam polarization achieved by the RHIC facility for its spin programme. A more extensive deployment of polarized facilities at a number of accelerators would permit a much deeper analysis of spin physics in a range of kinematic domains.

1. Introduction

Ever since the discovery of the nucleon spin puzzle at CERN in polarized muon deep inelastic scattering off polarized nucleons [1] polarized protons have been important probes in exploring nucleon spin structure. The RHIC programme was extended to include colliding polarized proton beams [2] for which precise knowledge of the beam polarization was required.

A number of methods of polarimetry were tested using the proton and antiproton beams at Fermilab [3] and the most appropriate method appeared to be the measurement of scattering near the forward direction, where hadronic and photon exchange processes interfere. Furthermore, proton proton elastic scattering also provided the possibility of a self-calibrating polarimeter system based on a gaseous atomic hydrogen polarized jet target [4].

2. Forward spin dependence

Hadronic spin dependence in the elastic collisions of protons and ions can be substantially enhanced by studying their effects in the peripheral region [5]. Spin averaged electromagnetic and hadronic elastic scattering amplitudes are of comparable magnitude at the interference invariant momentum transfer \( t_c = -8 \pi \alpha / \sigma_{\text{tot}} \) with \( \alpha \) being the fine structure constant [6].

Higher order photon contributions may be represented by the following spin independent Coulomb phase whose imaginary exponent multiplies each electromagnetic amplitude

\[
\delta = -\alpha \ln \left| Bt/2s + 4t/\Lambda^2 \right| - \alpha \gamma, \quad G_E(t) \approx G_M(t)/\mu \approx (1 - t/\Lambda^2)^{-2} \quad (1)
\]

where \( B \) relates to the exponential \( t \)-behaviour of the differential cross section, Euler’s constant is \( \gamma = 0.5772 \ldots \), and \( \Lambda^2 = 0.71 \text{GeV}^2 \) reflects the momentum transfer dependence of the electromagnetic form factors with \( \mu = 2.793 \) as magnetic moment and \( m \) as mass of the proton.
3. Single spin asymmetry

The single spin asymmetry $A_N$ [7] measured normal to the scattering plane [8] may be expressed in terms of hadronic amplitude ratios $r_5$ for single helicity flip [9] as in eqn (4) and $r_2$ for double helicity flip, the denominator being the spin averaged imaginary part of the hadronic amplitude

$$\frac{m A_N}{\sqrt{-t}} \frac{16 \pi}{\sigma_{\text{tot}}} \frac{d \sigma}{dt} e^{-Bt} = \left[ \kappa (1 + \text{Im} r_2) - \kappa \delta (\rho + \text{Re} r_2) - 2 (\text{Im} r_5 - \delta \text{Re} r_5) \right] \frac{t_c}{t} - 2 (1 + \text{Im} r_2) \text{Re} r_5 + 2 (\rho + \text{Re} r_2) \text{Im} r_5$$

(2)

though in the case of a spin zero carbon target the double helicity flip $r_2$ plays no rôle. Even for the proton case, evidence suggests that $|r_2|$ is small [10] at high energy, indicating that at very low $|t|$ < $|t_c|$ ≈ 0.002 (GeV/c)$^2$ the singular term in $t$ has $\kappa - 2 \text{Im} r_5$ as a major element of its coefficient, where $\kappa = 1.793$ is the anomalous moment of the proton. At larger values of $|t|$ beyond interference, the non-singular second term $2\rho \text{Im} r_5 - 2 \text{Re} r_5$ contributes appreciably. The differential cross section for elastic scattering in the interference region has an expansion

$$\frac{16 \pi}{\sigma_{\text{tot}}} \frac{d \sigma}{dt} e^{-Bt} = \frac{t_c^2}{t^2} - 2 (\rho + \delta + \epsilon) \frac{t_c}{t} - \frac{2t}{m^2} \left[ \left( \frac{\kappa t_c}{2t} - \text{Re} r_5 \right)^2 + \left( \text{Im} r_5 \right)^2 \right] + 1 + \rho^2$$

(3)

that includes singular terms in $t$ and small electromagnetic and hadronic form factor effects [11], though transverse and longitudinal double helicity flip contributions have been neglected. The asymmetry $A_N$ typically reaches a peak of about 4% at momentum transfers in the region of $\sqrt{3} t_c$ and measurements can then provide values of the single helicity flip quantity $r_5$. The Coulomb phase $\delta$ is about 2% for proton proton collisions but it is substantially larger at around 11% for proton carbon elastic scattering due to the greater electric charge. The extra interference term $\epsilon$ includes the hadronic slope in $t$ parameter $B(s)$ and electromagnetic form factor effects

$$r_5 = \frac{2m \phi_5}{\sqrt{-t} \text{Im} (\phi_1 + \phi_3)}, \quad \epsilon = \left( \frac{B}{2} - \frac{4}{\Lambda^2} + \frac{\mu - 1}{2m^2} \right) t_c = \frac{B - 9.23}{2} t_c.$$  

(4)

4. Amplitudes from FNAL E-704 results

The first high energy measurements of the single spin asymmetry $A_N$ at $\sqrt{s} = 20$ GeV (200 GeV/c beam momentum) in the Coulomb Nuclear Interference (CNI) region of momentum transfer $-t < 0.1$ (GeV/c)$^2$ [8], provided estimates of the spin flip hadronic amplitude $\phi_5$ [9] in the form of the ratio $r_5$

$$\text{Re} r_5 = -0.025 \pm 0.039, \quad \text{Im} r_5 = 0.145 \pm 0.311, \quad (200 \text{ GeV/c}),$$

(5)

and a combination of the CNI data at 200 GeV/c with previous high energy (45–300 GeV/c) $A_N$ results, measured over a more extensive momentum transfer interval 0.1 < $-t$ < 0.6 (GeV/c)$^2$, yielded an improved determination in two domains of laboratory momentum [9]

$$\text{Re} r_5 = -0.037 \pm 0.022, \quad \text{Im} r_5 = 0.078 \pm 0.182, \quad (45 - 205 \text{ GeV/c}),$$

$$\text{Re} r_5 = -0.010 \pm 0.004, \quad \text{Im} r_5 = 0.082 \pm 0.138, \quad (150 - 300 \text{ GeV/c}).$$

(6)

The energy dependence of the ratio $r_5$ does not appear to be significant at these momenta [12] and the above estimates are compatible with the value $\text{Im} r_5 = (6.9 \pm 3.5\%)$ calculated from the proton proton spin rotation parameter $R_{pp}$ measured at 45 GeV/c in the range 0.2 < $-t$ < 0.5 GeV$^2$ [13] assuming that one may neglect [14] the small quantity $\rho \text{Re} r_5$

$$R_{pp} \approx \left( \frac{\sqrt{-t}}{m} \right) \left( 2 \rho \text{Re} r_5 + 2 \text{Im} r_5 - 1/2 \right).$$

(7)
5. Amplitudes from RHIC measurements

Many values of the asymmetry $A_N$ have also been obtained at Brookhaven National Laboratory. Proton proton collisions at small $t$ using Roman Pots in the RHIC rings provided measurements at $\sqrt{s} = 200$ GeV [15]. Further results on $A_N$ were obtained from the polarized hydrogen jet fixed target being used as a polarimeter at RHIC [16]. The amplitude $\phi_5$ at small $-t$ was extracted at each energy

$$\Re r_5 = -0.006 \pm 0.038, \quad \Im r_5 = -0.108 \pm 0.038, \quad (\sqrt{s} = 6.8 \text{ GeV}),$$

$$\Re r_5 = -0.008 \pm 0.009, \quad \Im r_5 = -0.015 \pm 0.029, \quad (\sqrt{s} = 13.7 \text{ GeV}), \quad (8)$$

$$\Re r_5 = -0.033 \pm 0.035, \quad \Im r_5 = -0.43 \pm 0.56, \quad (\sqrt{s} = 200 \text{ GeV}).$$

The jet target experiment also measured the double spin asymmetry $A_{NN}$ and the total cross section $\sigma_{tot} = 38.4 \pm 0.05 \text{ mb}$ for both energies. In addition, the proton carbon asymmetry was obtained at RHIC with a carbon filament intercepting the beams, measuring elastic $pC$ scattering as a fast relative polarimeter [17]. These measurements, as well as those taken with the jet target have high statistics and therefore they allow the extraction of the spin flip amplitude with good precision, once the systematic uncertainties have been fully understood. Using these $A_N$ measurements, the single helicity flip hadronic amplitude divided by the forward imaginary spin averaged amplitude $r_5$, can be limited in absolute value:

$$|r_5| < 0.11 \pm 0.08 \quad (6.8 \text{ GeV}), \quad |r_5| < 0.02 \pm 0.03 \quad (13.7 \text{ GeV}),$$

$$|r_5| < 0.15 \pm 0.26 \quad (19.4 \text{ GeV}), \quad |r_5| < 0.44 \pm 0.42 \quad (200 \text{ GeV}). \quad (9)$$

6. Discussion of the proton proton results

The data from RHIC tend to display a negative value of $\Im \phi_5$ while the E704 analyses prefer positive values for $\Im \phi_5$, essentially the Pomeron flip amplitude [18]. Within experimental accuracy there is no sizeable discrepancy between the data, which are all consistent with $|r_5| \approx 0$. Recent preliminary data from $pp2pp$ [19] suggest that there is little hadronic spin flip at high energy.

A possible reason for the sign difference may derive from the fact that fits to RHIC asymmetry data cover a limited forward $t$ region, while previous E704 analyses have included larger $|t|$ measurements. In this case the $A_N$ asymmetry data in the medium $-t$ region can be interpreted from the expression $\Re \phi_+ \Im \phi_5 - \Im \phi_+ \Re \phi_5$. At small $-t$ values outside the CNI region, where the second term should prevail due to a dominating quantity $\Im \phi_+$, the asymmetry $A_N$ is positive and therefore $\Re \phi_5$ should be negative. The real part $\Re \phi_+$, close to zero at $t = 0$ around $\sqrt{s} = 20 \text{ GeV}$, is expected to become negative at larger $-t$ values on the basis of dispersion relations [20] and eventually, approaching the dip, the first term would take over as $\Im \phi_+ \approx 0$. In this case, $\Im \phi_5$ should be positive to give a negative asymmetry $A_N$.

7. Amplitudes at larger momentum transfers

Spin-dependent differential cross sections at other small angles have been studied [21]. A global fit, including both the above BNL and FNAL data at very small $-t$ and previous high energy ($45$–$300$ GeV/$c$) results on the asymmetry $A_N$ over an interval $0.1 < -t < 0.6$ (GeV/$c$)$^2$, allows one to estimate the ratio of amplitudes $r_5$ over the intermediate $-t$ region. Using eqn (2) and eqn (3) with a zero-crossing term [9] to reflect the behaviour of the asymmetry $A_N$ at larger values of $-t$ for energies above $45$ GeV gives

$$\Re r_5 = -0.0042 \pm 0.0020, \quad \Im r_5 = 0.059 \pm 0.0080.$$  

The trend of $r_5$ may be better described by exhibiting its modulus and phase $\theta_5$. For instance, the above data provide the estimate shown in Fig. 1 for the ratio $r_5$ given in eqn 4.
8. Proton carbon scattering
The analysis for a carbon nucleus with \( Z = 6 \) parallels that of the proton case with the simplification that, for a spin zero nucleus, the double helicity flip amplitudes \( \phi_2 \) and \( \phi_4 \) and the difference amplitude \( \phi_1 - \phi_3 \) are absent. Gluon and meson exchange also contribute in different ways to high energy proton proton and proton carbon scattering because of the C-parity of the exchanges [22]. A comparison of the two processes and their asymmetries would lead to additional understanding of the high energy dynamics involved.

Interference between hadronic and electromagnetic helicity nonflip amplitudes is prominent in the collisions of protons on nuclei of atomic number \( Z \) for momentum transfers close to \( t_c = -8\pi Z\alpha/\sigma_{\text{tot}}^\text{pC} \approx -0.0013 \) (GeV/c)\(^2\). The maximum asymmetry for the pC case results from a simple scaling of the expression for \( A_N(pp) \), taking \( t_c(pC) = Z t_c(pp)/A^\gamma \), where \( \gamma \approx (2/3) \) [14], [17].

The analyzing power for proton-carbon elastic scattering in the Coulomb-nuclear interference region of momentum transfer, \( 0.009 < -t < 0.041 \) (GeV/c)\(^2\), has been measured at 21.7 GeV/c [23] and it would be most interesting to learn of the energy dependence of the hadronic spin-flip amplitude in proton carbon elastic scattering by studying the systematic errors of asymmetry measurements taken at other energies.

9. Achievements and Future Progress
Polarized protons scattering on ions and protons probe the dependence of the hadronic interaction on spin not only for the soft processes discussed here but also for the large transverse momenta reactions that yield information on the helicity parton distribution functions [24]. Measurement of the level of proton beam polarization itself [25] relies upon an understanding

\[ r_5 \text{ (eqn 4)} \]
of the helicity structure of the interactions of hydrogen and carbon nuclei at low momentum transfers, particularly the spin dependent couplings of the Pomeron [26]. Analyticity in the form of dispersion relations relates the low and high energy helicity amplitudes for particle and antiparticle forward scattering, emphasising the importance of the spin programmes at the Fermilab [8], BNL [27], RHIC [16], PAX [28], FAIR and CERN [29] facilities.

10. Conclusions
To first order, the single spin asymmetry \( A_{10} \) in the CNI region of momentum transfer can be satisfactorily described in terms of a dominant hadronic amplitude \( \text{Im} \phi_5(h) \) and a singular, largely real, electromagnetic helicity flip amplitude \( \phi_5(\text{em}) \). The main correction, associated with \( \text{Im} \phi_5(h) \), has been identified, giving an experimental evaluation of the magnitude and phase of the forward helicity flip amplitude \( \phi_5(h) \) which turns out to be small, except at lower energies. With improving data quality and range over other spin observables, the contribution of the hadronic double helicity flip amplitudes \( \phi_3(h) \) and \( \phi_4(h) \) may also be investigated in addition to the effect of the difference \( \phi_1(h) - \phi_3(h) \) of the hadronic helicity nonflip amplitudes.

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