Abstract: The livestock and agriculture is like lifeline of Indian villagers and economy. India is ranked first in livestock population and second in fish production. Fish is major part of eating in south and most of eastern part of India and use aquaculture for fish production India also a major producer of fish through aquaculture, and ranks second in the world. In such condition forecasting of livestock is important in making policies and marketing, planning of products related to livestock and fishes. Fuzzy time series (FTS) is of great importance for such forecasting. But the problem with FTS forecast lies with the accuracy. There are many methods for linguistic values as variables for data of time series, but limitations starts with the error in forecasted and actual value. The present work studies the forecasting of fish production in India rainfall forecasting by FTS using two interval differences is proposed. The presented method is tested on the official data of deptt. of Animal Husbandry, Dairying and Fisheries, Government of India and compared with Chen’s models used for university enrollment. The forecasted values shows better result compared to Chen model.

Keywords: Fish production, Fuzzy time series (FTS), Fuzzy logical relations (FLR), Nested difference interval.

I. INTRODUCTION

When Zadeh [5] introduced the FS theory to deal with having uncertainty and impression, this becomes an amazing tool for many real life problems involving vagueness and impression in the information. Song and Chissom [7], [8], [9] successfully use this idea of having non mathematical variables for FTS model and tested it for University of Alabama’s enrollment data. The models given by Song and Chissom [7], [8], [9] became one of the most sought after model for researchers. Chen [10] proposed a model by defining arithmetic operations instead of complicated max-min operators and obtained better results than [7], [9]. Huaung [3], [4] contributed with two models, in one he proposed in heuristic increasing and decreasing relation model which improved the results of forecasting university enrollment and tested on Taiwan Futures Exchange forecasting. In other model [3], [4] presented to choose the effective length of interval which resulting in improving the accuracy of the forecasting values. Chen [11] used high order FTS for his model and tested this model on university enrollment forecasting. Singh [12], [13] proposed a third order model with a computational method for FTS forecasting and tested this model on real historical data of university enrollment with a counter implementation on crop production forecasting. Financial forecasting is another thrilling field for researchers, Bose and Mali [6] presented a data partitioning and rule selection techniques for FTS. Rana [1] studied on the rice production FTS Forecasting model. Rana [2] worked on time invariant models and presented a comparative study for forecasting crop production using time invariant FTS models and models are compared on the basis of statistical errors. Nested interval differences are differences within one interval. We used nested interval differences in this prediction.

II. MATERIALS AND METHODS

Definition 1. FS $A_i$ is defined as a collection of class of objects with their grade of membership. Let $U$ represents the Universe of discourse with $U = \{u_1, u_2, u_3, \ldots, u_n\}$, where $u_i$ are linguistic objects of $U$ then a fuzzy set $A_i$ of $U$ is defined by

$$A_i = \frac{\mu_{A_1}(u_1)}{u_1} + \frac{\mu_{A_2}(u_2)}{u_2} + \frac{\mu_{A_3}(u_2)}{u_2} + \cdots + \frac{\mu_{A_n}(u_n)}{u_n}$$

Where $U = \{u_i\; i = 1, 2, \ldots\}$ is universe and $\mu_{A_i}$ is membership function and $u_i$ are word variables.

Definition 2. FS $f_i(t) ; i = 1, 2, \ldots,$ defined on $Y(t)$ and $F(t) = \{f_i\}$ then $F(t)$ is called FTS on $Y(t)$.

Definition 3. The equation: $F(t) = F(t - 1) OR(t, t - 1)$ where ‘$\circ$ ‘ is Max–Min composition operator. Relation $R$ is called 1st order model of $F(t)$ if $R(t_1, t_2) = R(t_2, t_1)$, for $t_1 \neq t_2$ then $F(t)$ is called a time invariant.

III. PROPOSED METHODOLOGY

Step I. Define $U$ as $U = [u_{min} - U_1, U_{max} - U_2]$ where $U_1$ and $U_2$ are two proper positive numbers.

Step II. Construct equal length sub intervals $u_{1}, u_{2}, \ldots, u_{m}$ from $U$.

Step III. Make FS $A_i$ according to step II subintervals.

Step IV. Get FLR after fuzzifying data using “if $A_c \oplus A_n$ is current and next year fuzzify production then the FLR is $A_c \rightarrow A_n$ where $A_c$ is called current state and $A_n$ is next state of fuzzified data.

Step V. Using FLR, obtain the fuzzified output.

Step VI. Defuzzify values of step V.

ALGORITHM to get forecasted value $[S_n']$ is corresponding interval $u_n$ for which $A_n$ has member $= 1$

$\text{IF} S_n' \text{ length of } u_n$

$\text{Mid value of } u_n$
Fish Production Forecasting in India using Nested Interval Based Fuzzy Time Series Model

For FLR $A_c \rightarrow A_n$

$E_c$ real data of $n^{th}$ year
$E_{c-1}$ real data of $(n-1)^{th}$ year
$E_{c-2}$ real data of $(n-2)^{th}$ year
$F_n$ forecasted value $(n + 1)^{th}$ year

Applying Algorithm for forecasting production of fish in India is as -

Fish production forecasting for $(n + 1)^{th}$ year, $c = 3$ to ... 
FLR for $k^{th}$ year to $(k + 1)^{th}$ year is $A_c \rightarrow A_n$

Now calculating the nested differences $\alpha_c$, $\beta_c$ and $\gamma_c$ using $\frac{D_c}{4}$ and $D_c$ as

$D_c = |E_c - E_{c-1}| - |E_{c-1} - E_{c-2}|$

$\alpha_c = E_c + \frac{D_c}{4}$, $\alpha'_c = E_c - \frac{D_c}{4}$

$\beta_c = E_c + \frac{3D_c}{4}$, $\beta'_c = E_c - \frac{3D_c}{4}$

For using “If Then rule” for nested interval forecasting value $F_i$, $i = 1$ to $6$

If $L[S_n^*] \leq \alpha_c \leq U[S_n^*]$ then $F_1 = \alpha_c$, $n_1 = 1$. Else

$F_1 = 0$, $n_1 = 0$

Next $l$

If $L[S_n^*] \leq \alpha'_c \leq U[S_n^*]$ then $F_2 = \alpha'_c$, $n_2 = 1$. Else

$F_2 = 0$, $n_2 = 0$

Next $l$

If $L[S_n^*] \leq \beta_c \leq U[S_n^*]$ then $F_3 = \beta_c$, $n_3 = 1$. Else

$F_3 = 0$, $n_3 = 0$

Next $l$

If $L[S_n^*] \leq \beta'_c \leq U[S_n^*]$ then $F_4 = \beta'_c$, $n_4 = 1$. Else

$F_4 = 0$, $n_4 = 0$

Next $l$

If $L[S_n^*] \leq \gamma_c \leq U[S_n^*]$ then $F_5 = \gamma_c$, $n_5 = 1$. Else

$F_5 = 0$, $n_5 = 0$

Next $l$

If $L[S_n^*] \leq \gamma'_c \leq U[S_n^*]$ then $F_6 = \gamma'_c$, $n_6 = 1$. Else

$F_6 = 0$, $n_6 = 0$

Now $F = \sum_{i=1}^{6} F_i$

If $F = 0$ then $F_n = M[S_n^*]$

Else $F_n = \frac{(F + M[S_n^*])}{(n_1 + n_2 + n_3 + n_4 + n_5 + n_6)}$

Step $k$

Application of the proposed method to the fish production data in tones from 1994-95 to 2016-17

Step 1. $U = [4777, 8095, 11413]$

Step 2. Devide $U$ in equal length of subintervals

$u_1 = [4777, 5330, 5883]$ $u_2 = [5883, 6436, 6989]$

$u_3 = [6989, 7542, 8095]$ $u_4 = [8095, 8598, 9101]$ $u_5 = [9101, 9654, 10207]$

$u_6 = [10207, 10760, 11313]$

Step 3. Defining FS $A_i$ as

$A_1$ poor production
$A_2$ average production
$A_3$ ab. average production
$A_4$ good production
$A_5$ excellent production
$A_6$ bumper production

and the membership grades to these fuzzy sets of linguistic values are defined as

$A_1 = \frac{1}{u_1} + \frac{1}{u_2} + \frac{1}{u_3} + \frac{1}{u_4} + \frac{1}{u_5} + \frac{1}{u_6}$

$A_2 = \frac{0}{u_1} + \frac{1}{u_2} + \frac{1}{u_3} + \frac{1}{u_4} + \frac{1}{u_5} + \frac{1}{u_6}$

$A_3 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{1}{u_3} + \frac{1}{u_4} + \frac{1}{u_5} + \frac{1}{u_6}$

$A_4 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{1}{u_4} + \frac{1}{u_5} + \frac{1}{u_6}$

$A_5 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{1}{u_5} + \frac{1}{u_6}$

$A_6 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{1}{u_6}$

Step 4. FLRs are obtained and are in table 1.

Table 1: Actual and Fuzzified Yield

| Year    | Actual Yield ("000 Tonnes) | Fuzzified Yield |
|---------|-----------------------------|-----------------|
| 1994-95 | 4789                        | $A_1$           |
| 1995-96 | 4949                        | $A_1$           |
| 1996-97 | 5348                        | $A_1$           |
| 1997-98 | 5388                        | $A_1$           |
| 1998-99 | 5298                        | $A_1$           |
| 1999-00 | 5675                        | $A_1$           |
| 2000-01 | 5656                        | $A_1$           |
| 2001-02 | 5926                        | $A_2$           |
| 2002-03 | 6200                        | $A_2$           |
| 2003-04 | 6399                        | $A_2$           |
| 2004-05 | 6305                        | $A_2$           |
| 2005-06 | 6572                        | $A_2$           |
| 2006-07 | 6869                        | $A_2$           |
| 2007-08 | 7127                        | $A_2$           |
| 2008-09 | 7616                        | $A_2$           |
| 2009-10 | 7998                        | $A_3$           |
| 2010-11 | 8231                        | $A_4$           |
| 2011-12 | 8666                        | $A_4$           |
| 2012-13 | 9040                        | $A_4$           |
| 2013-14 | 9579                        | $A_5$           |
| 2014-15 | 10335                       | $A_6$           |
| 2015-16 | 10795                       | $A_6$           |
| 2016-17 | 11410                       | $A_6$           |

Step 5. The forecasted defuzzified fish production is calculated by using the proposed algorithms and put in the table 2.

Step 6. Calculate mean square error (MSE), forecasting error (FE), average FE are as

$MSE = \frac{1}{n} \sum (\text{actual val.} - \text{forecast val.})^2$

$FE = \frac{\text{absolute error}}{\text{sum of FE}} \times 100$

$AEF = \frac{\text{number of errors}}{\text{sum of FE}} \times 100$
### Table 2: Forecasted Fish Yield (‘000 Tonnes)

| Year     | Actual Yield | Forecasted Yield Present Model | Forecasted Yield Chen Model |
|----------|--------------|---------------------------------|----------------------------|
| 1994-95  | 4789         | –                               | –                          |
| 1995-96  | 4949         | –                               | 5330                       |
| 1996-97  | 5348         | –                               | 5330                       |
| 1997-98  | 5388         | 5345                            | 5330                       |
| 1998-99  | 5298         | 5380                            | 5330                       |
| 1999-00  | 5675         | 5303                            | 5330                       |
| 2000-01  | 5656         | 5489                            | 5330                       |
| 2001-02  | 5926         | 6125                            | 6436                       |
| 2002-03  | 6200         | 6179                            | 6436                       |
| 2003-04  | 6399         | 6234                            | 6436                       |
| 2004-05  | 6305         | 6404                            | 6436                       |
| 2005-06  | 6572         | 6324                            | 6436                       |
| 2006-07  | 6869         | 6509                            | 6436                       |
| 2007-08  | 7127         | 7265                            | 7542                       |
| 2008-09  | 7616         | 7205                            | 7542                       |
| 2009-10  | 7998         | 7606                            | 7542                       |
| 2010-11  | 8231         | 8352                            | 8598                       |
| 2011-12  | 8666         | 8317                            | 8598                       |
| 2012-13  | 9040         | 8656                            | 8598                       |
| 2013-14  | 9579         | 9378                            | 9654                       |
| 2014-15  | 10335        | 10760                           | 10760                      |
| 2015-16  | 10795        | 10497                           | 10760                      |
| 2016-17  | 11410        | 10790                           | 10760                      |
| MSE      | =            | 76642.39                        | 97642.35                   |
| % FE     | =            | 2.830331                        | 3.570238                   |
| AFE      | =            | 3.570238                        | 3.570238                   |

Figure 1 shows the year wise comparison between actual fish Yield and forecasted fish Yield by proposed and Chen’s models.

![Figure 1. Comparison of forecasted and actual fish production](image)

**IV. CONCLUSION**

In the proposed work forecast is done using nested differences in an interval which reduces the error in forecasting values significantly. This procedure also simplifies the computational work for the fish production forecasting. For the countries like India depending heavily on agriculture and livestock it will be of great importance to get the information in advance to give an idea regarding production which will improve the management and policies of future planning.

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