Neutrinos, Axions and Conformal Symmetry

Krzysztof A. Meissner and Hermann Nicolai

1 Institute of Theoretical Physics, University of Warsaw
Hoża 69, 00-681 Warsaw, Poland
2 Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut)
Mühlengraben 1, D-14476 Potsdam, Germany

We demonstrate that radiative breaking of conformal symmetry (and simultaneously electroweak symmetry) in the Standard Model with right-chiral neutrinos and a minimally enlarged scalar sector induces spontaneous breaking of lepton number symmetry, which naturally gives rise to an axion-like particle with some unusual features. The couplings of this ‘axion’ to Standard Model particles, in particular photons and gluons, are entirely determined (and computable) via the conformal anomaly, and their smallness turns out to be directly related to the smallness of the masses of light neutrinos.

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1. Introduction. It has been known for some time that the assumption of classically unbroken conformal symmetry may provide an alternative mechanism towards explaining the stability of the weak scale w.r.t to the Planck scale. In such a scheme all observed mass scales must arise via the quantum mechanical breaking of conformal invariance induced by the Coleman-Weinberg effective potential [1]). In [2, 3] (see also [4, 5] for an earlier, but different proposal along these lines), we have recently shown that a minimal extension of the Standard Model (SM) with right-chiral neutrinos and one extra scalar (‘Majoron’) can realize this possibility, provided

- there are no intermediate mass scales between the weak scale and the Planck scale \( M_P \); and
- the RG evolved couplings exhibit neither Landau poles nor instabilities over this range of energies.

The second assumption is motivated by the expectation that any extension of the SM (with or without supersymmetry) that stays within the framework of quantum field theory will eventually fail, and that the main task is therefore to delay the onset of this breakdown to the Planck scale, where a proper theory of quantum gravity is expected to replace quantum field theory. This requirement leads to important restrictions on the SM parameters, which can in principle be tested at LHC.

In this Letter, we study an extension of the model [2] with the usual Higgs doublet \( \Phi(x) \), and one extra (weak singlet) scalar field \( \phi(x) \), but with the main difference that this extra scalar field is now taken complex. Writing

\[
\phi(x) = \varphi(x) \exp \left( \frac{ia(x)}{\sqrt{2}\mu} \right) \tag{1}
\]

with real fields \( \varphi(x) \) and \( a(x) \), we will show that the ‘Majoron’ \( \varphi \) acquires a vacuum expectation value \( \langle \varphi \rangle \neq 0 \) via radiative corrections. The field \( a(x) \) then gives rise to a (pseudo-)Goldstone particle associated with the spontaneous breaking of a new global \( U(1)_L \) (modified lepton number) symmetry. This boson shares many properties with the standard axion \( f \), but also exhibits some very unusual features (in particular, it cannot be assigned a definite parity, unlike the standard axion). We here explain and explore some of these features, especially concerning the effective couplings of this ‘axion’ to photons and gluons, as well as its potential suitability as a dark matter candidate and for solving the strong CP problem. Further details will be given in a forthcoming article [6].

Apart from its compatibility with the known SM phenomenology, and its implications for the hierarchy problem, the main virtue of the present proposal is that it provides a single source of explanation for axion couplings and neutrino masses via the conformal anomaly, thereby tying together in a most economical manner features of the SM previously thought to be unrelated.

2. Lagrangian. We refer to [6, 7] for basic properties of the SM, and here only quote the relevant interaction terms in the (classically conformal) lagrangian, viz.

\[
\mathcal{L}_{\text{int}} = \left( \overline{\epsilon} \Phi Y^{E \nu}_{ij} E^j + \overline{Q} \Phi Y^{D \nu}_{ij} D^j + \overline{Q} \Phi Y^{U \nu}_{ij} U^j \\
+ \overline{L} \Phi Y^{\nu 
}_{ij} N^j + \phi N^{ij} \varepsilon^{-1} Y^{M \nu}_{ij} N^j + \text{h.c.} \right) \\
- \frac{1}{4} \left( \Phi^i \Phi \right)^2 - \frac{1}{2} \left( \Phi^i \Phi \right) \left( \phi^i \phi \right) - \frac{1}{4} \left( \phi^i \phi \right)^2 \tag{2}
\]

Here \( Q^i \) and \( L^i \) are the left-chiral quark and lepton doublets, \( U^i \) and \( D^i \) the right-chiral up- and down-like quarks, while \( E^i \) are the right-chiral electron-like leptons, and \( N^i \equiv \nu_R^i \) the right-chiral neutrinos. As in [2] we suppress all indices except the family indices. One can use global redefinitions of the fermion fields to transform the Yukawa matrices \( Y^E_{ij}, Y^U_{ij}, Y^D_{ij} \) and \( Y^{\nu M}_{ij} \) to real diagonal matrices. Furthermore, \( Y^{M \nu}_{ij} = (K_D M_D)_{ij} \) where \( K_D \) is a CKM matrix and \( M_D \) is real diagonal. Using the remaining freedom we can then set \( Y^{\nu} = K_{\nu} M_{\nu} U_{\nu} \), where \( K_{\nu} \) is a CKM-like matrix (i.e. with three angles and one phase), \( M_{\nu} \) is real diagonal and \( U_{\nu} \) is a unitary matrix with \( \det U_{\nu} = 1 \).

Besides the standard (local) \( SU(3)_c \times SU(2)_w \times U(1)_Y \) symmetries, the lagrangian [2] admits two global \( U(1) \)
symmetries. The first is baryon number symmetry
\[ Q^i \rightarrow e^{i\beta} Q^i, \quad U^i \rightarrow e^{i\beta} U^i, \quad D^i \rightarrow e^{i\beta} D^i \] (3)
The other is the modified lepton number symmetry
\[ L^i \rightarrow e^{i\alpha} L^i, \quad E^i \rightarrow e^{i\alpha} E^i, \quad N^i \rightarrow e^{i\alpha} N^i, \quad \phi \rightarrow e^{-2i\alpha} \phi \] (4)
with associated Noether current
\[ J^\mu_L = \overline{L} \gamma^\mu L + \overline{E} \gamma^\mu E + \overline{N} \gamma^\mu N - 2i\phi \overline{\theta} \]
(5)
Notice that the first three terms add up to a purely vectorlike current \[ e^\pm \gamma^\mu \bar{e}^\pm + \rho^\pm \gamma^\mu \nu^\pm . \] An unusual feature is that the scalar field \( \phi \) also carries lepton charge, thus modifying the standard lepton number current by a ‘leakage term’.

3. Minimization of effective potential. While the classical potential (2) does not induce spontaneous symmetry breaking, the effective one-loop potential computed from (2) can develop non-vanishing vacuum expectation values for the scalar fields (see also \[ 10 \] and references therein for more recent work on the Coleman Weinberg mechanism). This potential is given by formula (3) of \[ 2 \] (with \( N = 4 \) and \( M = 2 \)), to which we also add the contribution from the \( SU(2)_w \) gauge fields which was not taken into account in \[ 2 \]. We then perform the numerical minimization subject to the requirements stated in the Introduction. As in \[ 2 \], the numerical search shows that there exists a (small) subset of parameter space compatible with our requirements, and in particular allowing for the following set of values:
\[ \lambda_1 = 3.77, \quad \lambda_2 = 3.72, \quad \lambda_3 = 3.73, \quad g_t = 1, \quad Y_\lambda^2 = 0.4 \] (6)
(the approximate \( O(6) \) symmetry of the scalar self-couplings is accidental, and not preserved by quantum corrections, cf. \[ 11 \] below). For these values, the minimum is located at \( \langle H \rangle = 2.74 \times 10^{-5} \, v, \quad \langle \varphi \rangle = 1.51 \times 10^{-4} \, v \). Here \( v \) is the mass parameter introduced by dimensional regularization, whose choice sets the scale for all dimensionful quantities in the model. Accordingly, we impose \( \langle H \rangle = 174 \) GeV, where \( H^2 \equiv \Phi^\dag \Phi \). Hence,
\[ \langle H \rangle = 174 \text{ GeV}, \quad \langle \varphi \rangle = 958 \text{ GeV}, \quad v = 3.65 \times 10^4 \langle H \rangle \] (7)
After symmetry breaking three degrees of freedom of \( \Phi \) are converted into longitudinal components of \( W^\pm \) and \( Z^0 \), so we are left with one real scalar field \( H \) and the complex field \[ 1 \]. Defining (at the potential minimum
\[ H' = H \cos \beta + \varphi \sin \beta, \quad \varphi' = -H \sin \beta + \varphi \cos \beta \] (8)
the numerical analysis yields the following values
\[ m_{H'} = 207 \text{ GeV}, \quad m_{\varphi'} = 477 \text{ GeV}, \quad \sin \beta = 0.179 \] (9)
while the Goldstone field \( \alpha(x) \) stays massless. Although these numbers do not constitute a definitive prediction of our model, it turns out that the ‘window’ left open by

our requirements is fairly small for \( m_{H'} \), while \( m_{\varphi'} \) can vary over a larger range of values \( > 400 \text{ GeV} \). Note also that only the components along \( H \) of the mass eigenstates couple to the usual SM particles. This leads to a rather striking (and testable) prediction of the present model: the decay rates of \( H' \) and \( \varphi' \) should have the same branching ratios in those channels which are kinematically accessible to both particles \[ 2 \].

The effective coupling constants are calculated numerically as the respective fourth-order derivatives of the effective potential at the minimum:
\[ \lambda_1^\text{eff} = 1.447, \quad \lambda_2^\text{eff} = 0.648, \quad \lambda_3^\text{eff} = 0.871 \] (10)
Checking the consistency of our basic assumptions now amounts to evolving all couplings according to their RG equations, with \[ 10 \] and the known SM couplings at the weak scale as the initial values. Performing the steps described in \[ 2 \], and also taking into account the weak \( SU(2)_w \) coupling \( g_2 \), we verify that there are indeed no Landau poles or instabilities up to the Planck scale.

4. Neutrino propagators. The new effects reported in this Letter all depend crucially on the mixing of the neutrino degrees of freedom within each neutrino species. For the computation of loop diagrams we temporarily switch to \( SL(2, C) \) spinor notation, see e.g. \[ 13 \] for details. With \( \nu_L \equiv \nu \left( 1 - \gamma^5 \right) \nu \equiv \bar{\nu}^\alpha \) and \( \nu_R \equiv \frac{1}{2} \left( 1 + \gamma^5 \right) \nu \equiv N_\alpha \), the relevant (free) part of the Lagrangian reads
\[ \mathcal{L} = \frac{i}{2} \left( \nu^\alpha \partial_\mu \phi^{\alpha \beta} + N^\alpha \phi_{\alpha \beta} \phi^{\beta} \right) + \text{c.c.} \] (11)
\[ + m \left( \bar{\nu}^\alpha N_\alpha + \bar{N}_\alpha N^\alpha \right) + \frac{M}{2} \left( N^\alpha N_\alpha + \bar{N}_\alpha \bar{N}^\alpha \right) \]
with the Dirac and Majorana mass parameters \( m = Y^\nu \langle H \rangle \) and \( M = Y^M \langle \varphi \rangle \), respectively. For simplicity, we here consider only one neutrino generation; in the general case the formulas below will contain additional factors of \( Y^{+Y^{-1}} Y^{+} \), or traces over family indices (which may alter our estimates below). Rather than diagonalize the fields w.r.t to these mass terms, we prefer to work with non-diagonal propagators, leaving the fields as they are in the interaction vertices. The poles of the propagators are obtained via the standard seesaw formula \[ 14 \] \[ \mu^2 = m^2 + \frac{1}{2} M^2 \pm \sqrt{1 + \frac{4 m^2}{M^2}} \] (12)
whence \( \mu_+ \approx M \) and \( \mu_- \approx m^2 / M \approx m_\nu \) for \( m_\nu < 1 \text{ eV} \). Defining \( D(p) := \left( p^2 - m^2 \right) \left( p^2 - m_\nu^2 \right) \) we obtain the propagators (in momentum space)
\[ \langle \nu_\alpha \nu_\beta \rangle = i m^2 M D(p) \varepsilon_{\alpha \beta} \]
\[ \langle \bar{\nu}_\alpha \bar{\nu}_\beta \rangle = i (p^2 - M^2) D(p) \varepsilon_{\alpha \beta} \]
\[ \langle N_\alpha N_\beta \rangle = \frac{1}{2} (p^2 - m^2) D(p) \varepsilon_{\alpha \beta} \]
\[ \langle N_\alpha \bar{N}_\beta \rangle = -i M D(p) \varepsilon_{\alpha \beta} \]
\[ \langle \nu_\alpha \bar{N}_\beta \rangle = -i m M D(p) \varepsilon_{\alpha \beta} \] (13)
together with their complex conjugate components. It will be essential for the UV finiteness of the diagrams to be computed below, that some of these propagators fall off like $\sim p^{-3}$, unlike the standard Dirac propagator.

5. **Neutrino triangles and ‘axion’ vertices.** With the above propagators we can now proceed to compute various couplings involving the ‘axion’ $a$, which are mediated by neutrino mixing via two or three-loop diagrams. In order to extract the new vertices from (2) (and to have canonically normalized kinetic terms for these scalar fields), we set $\mu = \langle \varphi \rangle$ in (1) and expand

$$\phi(x) = \langle \varphi \rangle + \frac{1}{\sqrt{2}}(\varphi(x) + ia(x)) + \ldots \quad (14)$$

We here only present results for the axion-photon coupling $aF\tilde{F}$, whose lowest order contribution is given by the two-loop diagram depicted in Fig.1. Because parity is (in fact, maximally) broken, there are also ‘dilatonic’ couplings of type $aFF$, whose determination will require in addition the consideration of diagrams with photons emanating from the internal $W$-line (there will be similar couplings involving the ‘Majoron’ $\varphi$). To simplify the calculation we first compute the ‘blob’ in Fig. 1, which is proportional to the integral (in Euclidean signature)

$$\int \frac{d^4l}{(2\pi)^4} \frac{f_{\alpha \beta}}{(l^2 + M_f^2)(l + q)^2 + M_f^2(l + p_1)^2 + m_W^2} + (p_1 \leftrightarrow p_2) \quad (15)$$

where $m_W$ is the $W$-boson mass, and where we set $m = m_\nu = 0$ in order to simplify the integrand (a valid approximation because $m, m_\nu \ll m_W, M$). Here we are interested in the result for small axion momentum $q^\mu = p_1^\mu + p_2^\mu$. Putting in all factors and reverting back to 4-spinor notation, the ‘form factor’ for the electron-axion vertex for large $p_1, p_2$ is well approximated by (modulo terms of order $O(M^2/p_1^2)$ and $O(m_W^2/p_2^2)$)

$$F(q, p_1) = \frac{m_\nu Y_M \alpha_2}{8\pi} \frac{1 + \gamma^5}{2} \frac{q}{p_1^2 + m_W^2} \quad (16)$$

where $\alpha_2 \equiv q^2/(4\pi)$ and $\varphi \equiv \gamma^5 q_\mu$. This formula makes obvious one of our main assertions, namely that the effective axionic couplings are proportional to the light neutrino masses, and vanish in the limit $m_\nu \to 0$ (or for vanishing Yukawa coupling $Y_M$). The complete expression for the effective axion-electron vertex at small $q^\mu$ will be given in (2). Inter alia the above vertex would also determine the rate of energy loss through radiation of ‘axions’ from charged plasma in the Sun’s core.

In order to arrive at an estimate of the axion-photon coupling, we next substitute the result (16) into the expression for the electron triangle shown in Fig. 1. Though superficially similar to the diagram giving rise to the well known $\gamma^5$-anomaly in QED, the integral here is convergent because of the damping of the integrand due to the ‘form factor’ (16). After some computation we obtain

$$\Omega_{\text{eff}}^\alpha \gamma = \frac{1}{4f_a} F^{\mu \nu} F_{\mu \nu}, \quad f_a = \frac{2\pi^2 m_W^2}{\alpha \omega_\text{em} m_\nu Y_M} \quad (17)$$

Substituting numbers we find $f_a = O(10^{15} \text{GeV})$ which is outside the range of existing or planned experiments \[\text{[10]}\]. However, apart from the simplifications introduced above which may still affect this estimate, one must keep in mind that the simultaneous presence of $aF$ couplings may substantially alter the analysis with regard to observable effects.

The gluonic couplings can be analyzed in a similar way (detailed results will be presented in \[\text{[3]}\]). For their determination we must evaluate the three-loop diagram shown in Fig. 2, as well as analogous diagrams not shown, with $Z$ boson exchange and a triangle consisting only of neutrino lines, and with gluons emanating from the quark line connecting two $W$ or $Z$ bosons. This calculation is complicated by the fact that the the ‘quark box’ is dominated by small momenta, where $\alpha_s$ is large (and could thus dynamically enhance the coupling). A very crude estimate can be derived by replacing $\alpha_{em}$ in (17) by $6\alpha_2\alpha_s/\pi$, which yields

$$\Omega_{\text{eff}}^\alpha \gamma = \frac{\alpha_s}{8\pi g_a} G^{\mu \nu} \tilde{G}_{\mu \nu}, \quad g_a \equiv \frac{\pi^2 m_W^2}{\alpha_0^\text{em} m_\nu Y_M} \quad (18)$$

This gives $g_a = O(10^{16} \text{GeV})$. Summing over quark flavors, as well as taking into account all relevant diagrams could bring this number down, and into the range that could make the axion a viable dark matter candidate according to standard reasoning (see e.g. \[\text{[17]}\], p. 280ff, but the actual numbers are subject to large uncertainties \[\text{[18]}\]). However, before starting the actual comparison, conventional lines of reasoning must be re-examined in view of the fact that parity violation now also allows for ‘dilatonic’ couplings $\propto aGG$, which may vitiate some of the accepted arguments (for instance, concerning periodicity properties of the ‘axion’ potential and the location of the minimum in this potential). Similar caveats may apply to the use of our ‘axion’ for solving the strong CP problem: although for this purpose the precise value of $g_a$ does not matter so much, it is not clear whether the present scenario would lead to $\langle a \rangle = 0$, as generally required for the solution of the strong CP problem.

6. **Conclusions and Outlook.** In \[\text{[2]}\] it was shown that all mass scales of the SM can be generated via radiative corrections and the conformal anomaly from a single scale $v$, which is set here by the choice of $\langle H \rangle$ in eq. (7). In particular, no large scales beyond the SM are needed to explain the smallness of neutrino masses, if one allows for the entries of the Yukawa coupling matrix $Y_\nu$ to assume values in the range $O(1) - O(10^{-5})$ (this is the main difference with more conventional seesaw-type scenarios, for which $m \approx O(100 \text{GeV})$ and $M$ must be assumed very large, resulting in very large masses for the heavy neutrinos, see e.g. \[\text{[19]}\]). In this Letter, we have extended these considerations to another sector of the SM, by showing that the replacement of the real ‘Majoron’ field $\varphi$ by the complex scalar $\phi$ makes it possible to also accommodate an axion-like degree of freedom in the model. Again, the relevant (large) mass scales emerge rather naturally by
radiative corrections, and without the need to introduce any new large mass scales by hand.

The present work also opens interesting new perspectives on the one remaining problem that must be solved within the framework of particle physics and quantum field theory (and not a future theory of quantum gravity), namely the issue of leptogenesis in the early universe (solving, through sphalerons, the baryon asymmetry problem). The current current $\mathcal{L}$ differs from the ordinary lepton number current by an extra scalar contribution; the ‘leakage’ of lepton number induced by the last term in $\mathcal{L}$ could thus lead to interesting new effects. In $\mathcal{L}$ it was pointed out that leptogenesis might be explained in terms of heavy neutrinos with an unconventionally small mass of $\sim O(10 \text{GeV})$; in our model this scenario would correspond to $Y_M \sim O(0.01)$ and $Y_\nu \sim O(10^{-6})$. Perhaps a combination of these ideas could lead to a satisfactory resolution of the problem.

Finally, we would like to emphasize that the present model offers an attractive and economical alternative to MSSM type models, because low energy supersymmetry may not be needed after all to stabilize the weak scale with regard to the Planck scale if the two conditions stated in the Introduction are met (see also [2] for a similar point of view).

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![Fig.1. Axion-photon-photon effective vertex](image1)

![Fig.2. Axion-gluon-gluon effective vertex](image2)