Search of $^1P_1$ charmonium in $B$ decay

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There is no doubt that the $^1P_1$ charmonium $h_c$ exists in the mass range between $J/\psi$ and $\psi'$. While experiment produced a candidate in the past, it still requires a confirmation. Given the recent progress in the exclusive $B$ decay into charmonia, we now have an opportunity to detect $h_c$ by measuring the final state $\gamma\eta_c$ of the cascade decay $B \to h_c K/K^* \to \gamma\eta_c K/K^*$. Confirmation of $h_c$ may turn out to be much easier in the $B$ decay than at charm factories although one may have to work a little harder to attain a high precision in the mass determination.

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I. INTRODUCTION

A few measurements suggested the $^1P_1$ charmonium around mass 3526 MeV [1,2]. In particular, the E760 Collaboration [2] studied the resonant production of $h_c$ in $p\overline{p}$ annihilation and quoted the $h_c$ mass at 3526.2 MeV. This value is almost exactly equal to the center of gravity (3525.17 MeV) of the $3P_J$ charmonia $\chi_{cJ}$ ($J = 0, 1, 2$). However, the result has yet to be confirmed by the E835 Collaboration [3]. No evidence has so far been seen for $h_c$ in $e^+e^-$ annihilation. From the theoretical viewpoint, there is no reason to expect that the $h_c$ mass should be so close to the center of gravity of the $3P_J$ masses, since such a relation based on the $\mathbf{L} \cdot \mathbf{S}$ coupling and the tensor force of one-gluon exchange would break down when general spin-dependent interactions are included. Experimentally, the $\chi_{cJ}$ mass splitting gives

$$R \equiv \frac{m_{\chi_{c2}} - m_{\chi_{c1}}}{m_{\chi_{c1}} - m_{\chi_{c0}}} \approx 0.476.$$  \hspace{1cm} (1)

The right-hand side would be equal to 2 for the $\mathbf{L} \cdot \mathbf{S}$ coupling alone, 0.8 with all spin-dependent forces of one-gluon exchange, and $0.8 \leq R \leq 1.4$ after including the more general spin-spin interaction arising from the confining potential [3]. Since our knowledge of the spin-dependent charmonium potential is incomplete, there is no accurate theoretical prediction of $m_{h_c}$ relative to $m_{\chi_{cJ}}$ even within the potential model. Furthermore, the E1 transition matrix elements for $\chi_{cJ} \to \gamma J/\psi$ deviate largely from the nonrelativistic values. When relativistic corrections are large for the motion of $c$ and $\overline{c}$, we should be cautious about accuracy of the potential model approach.

Review of Particle Physics [4] has not yet listed $h_c$ among the confirmed particles. Undoubtedly, much effort will be devoted to pursuit of $h_c$ at upcoming charm factories overcoming the odds against it. Meanwhile, the recent progress in $B$ physics suggests a new opportunity to search for $h_c$. The purpose of this short note is to point out that we may be able to observe $h_c$ more easily at the $B$-factories than at future charm factories and in hadron reactions.

Recently the Belle Collaboration discovered that the factorization-forbidden decay $B \to \chi_0 K$ occurs as vigorously as the factorization-allowed decays to other charmonia [5]. On the basis of this finding, we expect that another factorization-forbidden decay $B \to h_c K$ may also occur just as abundantly as $B \to \chi_0 K$. Since $h_c \to \gamma \eta_c$ is one of the two main decay modes of $h_c$, the decay $B \to h_c K$ cascades down to the final state $\gamma \eta_c K$ about half of the time. The only background for this process at the $B$-factories will be the process $B \to \psi' K \to \gamma \eta_c K$. Since the branching fraction for $\psi' \to \gamma \eta_c$ is minuscule, however, this background is two orders of magnitude smaller than the signal. If one can reconstruct $\eta_c$ from $K\overline{K}\pi$ or by $\eta \pi \pi$ with 50% efficiency, for instance, 10 million $B$’s translate to roughly 100 events of the signal. Therefore we have a very good chance to observe $h_c$ through $B \to \gamma \eta_c K$.

II. $B \to \text{CHARMONIUM} + K$

The Belle Collaboration reported for the decay $B \to \chi_0 K$ [6]

$$B(B^+ \to \chi_0 K^+) = (8.0^{+2.7}_{-2.4} \pm 1.0 \pm 1.1) \times 10^{-4}. \hspace{1cm} (2)$$

This number should be compared with the recent measurement by the BaBar Collaboration on the $B$ decay into other charmonia [7]:

$$B(B^+ \to J/\psi K^+) = (10.1 \pm 0.3 \pm 0.5) \times 10^{-4},$$
$$B(B^+ \to \chi_{c1} K^+) = (7.5 \pm 0.8 \pm 0.8) \times 10^{-4},$$
$$B(B^+ \to \psi' K^+) = (6.4 \pm 0.5 \pm 0.8) \times 10^{-4}. \hspace{1cm} (3)$$

Added to these is an earlier measurement on the branching fraction for $B \to \eta_c K$ by CLEO [8]:

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1 Although it is an odd naming, I call the $^1P_1$ charmonium as $h_c$ following the Particle Data tabulation [9].
B(B^+ \rightarrow \eta_c K^+) = (6.9^{+2.6}_{-2.1} \pm 0.8 \pm 2.0) \times 10^{-4}. \quad (4)

Most recently, however, BaBar gave a preliminary result for this decay as \[10\]

B(B^+ \rightarrow \eta_c K^+) = (15.0 \pm 1.9 \pm 1.5 \pm 4.6) \times 10^{-4}. \quad (5)

We should notice here that the decay $B \rightarrow \chi_{c0} K$ is forbidden by the factorization while $B \rightarrow J/\psi(\psi') K$, $B \rightarrow \eta_c K$, and $B \rightarrow \chi_{c1} K$ are all allowed. Nonetheless the branching fraction to $\chi_{c0} K^+$ is just as large as those into $J/\psi(\psi') K^+$, $\eta_c K^+$, and $\chi_{c1} K^+$. Since no effective decay operators allows $B \rightarrow \chi_{c1} K$ in the factorization limit, its decay amplitude must arise from the loop corrections of the energy scale below $m_b$ to the tree-decay operators $\mathcal{O}_{1,2}$. The relevant $t\bar{c}$ operator for production of $\chi_{c0}$ is generated when the bilocal operator $\mathcal{T}_x(x) c(y)$ due to the loop correction is expanded in the series of local operators; $\mathcal{T}_x(x) c(y) \rightarrow \mathcal{T}_x(x) c(x) + \mathcal{T}_x(x) (y-x) \mu \bar{\nu} c(x) + \cdots$. If the relevant part of the loop energy is between $m_b$ and $m_c$, then $|y-x| \approx 1/m_b \sim 1/m_c$, so that one may keep only the leading term of the expansion. In this case the QCD coupling $\alpha_s/\pi$ would suppress the $B \rightarrow \chi_{c0} K^+$ decay branching by $(\alpha_s/\pi)^2$ though the suppression relative to the factorization-allowed processes may be somewhat moderated by the color structure. The experimental fact that $B(B^+ \rightarrow \chi_{c0} K^+)$ is comparable with $B(B^+ \rightarrow \chi_{c1} K^+)$ indicates that the factorization, even improved with perturbative QCD corrections, is in serious doubt for the $B$ decay into charmonia. In terms of the local expansion of $\mathcal{T}_x(x) c(y)$, the magnitude of the relevant $|y-x|$ is as large as $1/\Lambda_{QCD}$ or, in the case of charmonia, could be the charmonium radius $1/\alpha_s m_c$. If so, we must include not only all terms of the local expansion but also all orders of $\alpha_s$ in computation of decay amplitudes. Then a quantitative calculation based on perturbative QCD is intractable.

The decay $B \rightarrow h_c K$ is also forbidden by the factorization and has the same chiral structure ($\mathcal{T}_{1L} R \pm \mathcal{T}_{3L} L$) for charmonium as $B \rightarrow \chi_{c0} K$. The local operator of the lowest dimension leading to the decay $B \rightarrow h_c K$ is $\mathcal{T}_{1L} \bar{c} \gamma^\mu c$. When the factorization and perturbative QCD fail as proven by the $B \rightarrow \chi_{c0} K$ decay rate, it is very likely that the decay $B \rightarrow h_c K$ occurs as abundantly as $B \rightarrow \chi_{c0} K$ and the factorization-allowed $B$ decays into charmonia.

A comment is in order for another factorization-forbidden decay $B \rightarrow \chi_{c2} K$. The decay $B \rightarrow \chi_{c2} K$ occurs with $\mathcal{T}_{1L} \bar{c} \gamma^\mu \gamma^\nu c$. The Belle Collaboration did not see a signal of $B \rightarrow \chi_{c2} K$ with a statistical significance \[12\]. However, since they searched $\chi_{c0, c2} \rightarrow \pi^+\pi^-$ and $K^+K^-$ decay modes, their failure to see a clear signal for $B \rightarrow \chi_{c2} K$ may be due to the smaller branching fractions for $\chi_{c2} \rightarrow \pi^+\pi^-$ and $K^+K^-$ as compared with $\chi_{c0} \rightarrow \pi^+\pi^-$ and $K^+K^-$.

On the other hand the CLEO Collaboration identified $\chi_{c2}$ by $\chi_{c2} \rightarrow J/\psi \gamma$ and concluded that $B(B \rightarrow \chi_{c2} X)$ is significantly less than $B(B \rightarrow \chi_{c1} X)$. But they focused on the inclusive decays and the uncertainties were large for the exclusive decays: $0.04 < B(B \rightarrow \chi_{c2} K/K^+)/B(B \rightarrow \chi_{c1} K/K^*) < 0.58$ with the 95% confidence level. (See the Sample B of Ref. \[12\].) Very recently, however, the Belle Collaboration reported the branching fraction for inclusive $\chi_{c2}$ production \[13\],

\[B(B \rightarrow \chi_{c2} X) = (15.3^{+2.3}_{-2.8} \pm 2.6) \times 10^{-4}, \quad (6)\]

where the $\chi_{c2}$’s fed by $\psi' \rightarrow \gamma \chi_{c2}$ have been subtracted out. This number is twice as large as the corresponding one of CLEO \[12\]. In view of this latest Belle measurement, it is possible that $B(B \rightarrow \chi_{c2} K)$ will eventually turn out to be comparable with $B(B \rightarrow \chi_{c0} K)$.

With these observations in theory and experiment, we proceed for the moment with the assumption,

\[B(B \rightarrow h_c K) \approx B(B \rightarrow \chi_{c0} K) \quad (7)\]

to explore the opportunity to detect $h_c$. Once we assume Eq.\[7\], we are assuming the same relation with $K$ replaced with $K^*$ or a higher strange meson. We emphasize that Eq.\[7\] is an assumption at present. However, the measurement we are discussing will test its validity, as we discuss below, and determine or set an upper bound on $B(B \rightarrow h_c K)$ with a good accuracy.

III. DECAY OF $^1P_1$

Numerous calculations were performed for the properties of charmonia in potential models \[14\]. The decay property of $\chi_{c1}$ and $h_c$ was specifically studied by Bodwin et al \[15\]. Production of $h_c$ through $\psi' \rightarrow h_c \pi^0$ in $e^+e^-$ annihilation was also studied \[16\]. However, all that we need for our purpose here can be obtained directly from the experimental numbers for other charmonia if we make the approximation to use a common orbital wave function for the spin singlet and triplet of the same principal quantum number. This approximation is justified for the $c$ and $\overline{c}$ in nonrelativistic motion, and the results are independent of specific bound-state wave functions. Although the nonrelativistic treatment of charmonia is often limited in precision, we do not need much more than that for our discussion below.

The main decay modes of $h_c$ are $h_c \rightarrow gg$ and $h_c \rightarrow \gamma \eta_c$. The former is given by perturbative QCD to the leading logarithm of the $h_c$ size \[13\]. By equating the $h_c$ bound-state wave function at the origin to that of $\chi_{c1}$, we obtain with the experimental value $\Gamma(\chi_{c1} \rightarrow gg) = \Gamma(\chi_{c1} \rightarrow hadrons) = 640 \pm 100$ keV,

\[\Gamma(h_c \rightarrow gg) = \frac{5}{6} \times \Gamma(\chi_{c1} \rightarrow gg), \quad = 530 \pm 80 \text{ keV}. \quad (8)\]

This numerical value does not depend on the magnitude of the fuzzy cutoff variable in the leading logarithmic term nor on specific binding potentials.
The radiative decay $h_c \rightarrow \gamma \eta_c$ is an allowed E1 transition similar to $\chi_{cJ} \rightarrow \gamma J/\psi$. We can eliminate the E1 transition matrix element $(f|\mathbf{P}|i)$ between the 1P and the 1S state by relating $h_c \rightarrow \gamma \eta_c$ to $\chi_{c1} \rightarrow \gamma J/\psi$:

$$\Gamma(h_c \rightarrow \gamma \eta_c) = \left(\frac{|f|}{|i|}\right)^3 \Gamma(\chi_{c1} \rightarrow \gamma J/\psi),$$

$$= 520 \pm 90 \text{ keV}. \quad (9)$$

The central value of Eq. (9) is about 15\% higher than the value computed by Bodwin et al. [15], while the value 530±80 keV of Eq. (8) coincides with theirs. The rates for other modes such as $h_c \rightarrow J/\psi \pi^0$ and $\gamma \chi_{c0}$ are of $O(1)$ keV. Therefore we obtain from the $ggg$ and $\gamma \eta_c$ decay modes the $h_c \rightarrow \gamma \eta_c$ branching fraction:

$$B(h_c \rightarrow \gamma \eta_c) = 0.50 \pm 0.11. \quad (10)$$

In this estimate the uncertainty is entirely due to those of the measured values for $\Gamma_{\chi_{c1}}(\chi_{c1})$ and $B(\chi_{c1} \rightarrow \gamma J/\psi)$. The value of Eq. (10) is a firm number up to relativistic corrections and higher-order QCD corrections though the former corrections may turn out to be larger than we imagine.

Combining $B(h_c \rightarrow \gamma \eta_c)$ of Eq. (10) with Eqs. (9) and (8), we obtain the cascade branching fraction for $B \rightarrow h_c \rightarrow \gamma \eta_c K$:

$$B(B^+ \rightarrow h_c K^+ \rightarrow \gamma \eta_c K^+) = \left(4.0^{+1.5}_{-1.2} \pm 0.5 \pm 0.6\right) \times 10^{-4}. \quad (11)$$

It goes without saying that the number on the right-hand side is subject to the uncertainty of the assumed equality in Eq. (9). If $\eta_c$ is searched by $K\overline{K}\pi$ or $\eta\pi\pi$ (the branching fraction $\simeq 5\%$ each), the cascade branching fraction is

$$B(B^+ \rightarrow h_c K^+ \rightarrow \gamma \eta_c K^+ \rightarrow \gamma (K\overline{K}\pi)K^+) \simeq 2 \times 10^{-5}. \quad (12)$$

When 10 millions of $B$ mesons are accumulated, there will be about 100 events of the $\gamma \eta_c K^+$ signal just from $K\overline{K}\pi$ or from $\eta\pi\pi$ alone in the case that the reconstruction efficiency of $h_c$ is 50\% for these decay modes. One can increase statistics by including $B^\pm \rightarrow h_c K^{*\pm}$ and by combining $B^0 \overline{B}^0$ with $B^\pm$. There will be a sufficient number of the cascade $B \rightarrow h_c X \rightarrow \gamma \eta_c X$ events to search for $h_c$.

Let us compare Eq. (12) with the corresponding number in the $h_c$ search through $\psi' \rightarrow \gamma h_c$ at charm factories. According to the calculation by Yan et al. [16] and more recently by Kuang [20], who included S-D mixing of $\psi'$, the branching fraction for $\psi' \rightarrow \gamma h_c \pi^0$ is at the level of $1 \times 10^{-3}$ at most, for $m_{h_c} = 3526.2$ MeV. Taking account of the low reconstruction efficiency of the soft $\pi^0 \rightarrow \gamma \gamma$, Kuang estimates that detection of $h_c$ through $\psi' \rightarrow h_c \pi^0$ requires 30 million $\psi'$s at charm factories. While $h_c$ can be produced only through $\psi' \rightarrow h_c \pi^0$ at charm factories, the $h_c$ production occurs in the $B$ decay in conjunction with $K^*$ or a strange meson as well as with $K$. Furthermore, the production in conjunction with $K^*$ tends to be stronger than that with $K$ in the $B \rightarrow \text{charm factory}$ decay. By and large, the search of $h_c$ will be quite competitive with the search at charm factories, if not superior to it.

### IV. POSSIBLE BACKGROUND EVENTS

The only decay mode that feeds $\gamma \eta_c K$ with the $\gamma \eta_c$ invariant mass close to $m_{h_c}$ is the cascade decay $B \rightarrow \psi'K \rightarrow \gamma \eta_c K$. Since $\psi' \rightarrow \gamma \eta_c$ is a hindered M1 transition with the branching fraction $\simeq 5\%$ each), the cascade branching fraction is tiny;

$$B(B \rightarrow \psi'K^+ \rightarrow \gamma \eta_c K^+) = (1.8 \pm 0.4 \pm 0.2) \times 10^{-6}. \quad (13)$$

It is more than two orders of magnitude smaller than the signal of Eq. (11). We can therefore choose a wide bin for $(p_y + p_n)^2$ in reconstruction of $h_c$ without concern about the $\psi'$ contamination in $\gamma \eta_c$. This is fortunate from the viewpoint of raising the precision in mass determination. Since there is no competing decay process, we may fix the invariant mass of $K\overline{K}\pi$ or $\eta\pi\pi$ to $m_{h_c}$ once we find a cluster of candidate events. Although we certainly do not expect to determine the $h_c$ mass to the accuracy anywhere close to its width ($\Gamma_{h_c} \simeq 1$ MeV), it will be easy to notice if the $h_c$ mass is located substantially off the center of gravity of $\chi_{cJ}$.

It will be challenging to identify $h_c$ directly by its hadronic decay modes. Since $h_c$ is G-parity odd, the simplest decay mode is $h_c \rightarrow \pi\pi\pi$, then $K\overline{K}\pi$. The branching fractions to $\pi\pi\pi$ and $K\overline{K}\pi$ are no larger than at the level of 1\% if we make a guess by rescaling the corresponding decays for $\psi'$. Then the cascade branching fraction is most likely of the order,

$$B(B \rightarrow h_c K \rightarrow \pi^+\pi^-\pi^0 K) = O(1) \times 10^{-5}. \quad (14)$$

After multiplying it with the reconstruction efficiency of $\pi^0 \rightarrow \gamma \gamma$, it does not appear competitive with $B \rightarrow h_c K \rightarrow \gamma \eta_c K$. Although one can distinguish $h_c$ from $\chi_{c1}$ by G-parity of the decay products, one can separate $h_c$ from $\psi'$ only by the mass resolution when one searches $h_c$ by hadron decays. There are clear advantages for studying the cascade decay $B \rightarrow h_c K \rightarrow \gamma \eta_c K$.

### V. SUMMARY

Nobody disputes the presence of $h_c$. Our real interest is in the values of its parameters. For this purpose the cascade decay process $B \rightarrow h_c K/K^* \rightarrow \gamma \eta_c K/K^*$ deserves a careful study at the B-factories. The search of $h_c$ through the $B$ decay is very competitive with the
search at charm factories and presumably superior to it. It will either confirm the controversial $P_c$ charmonium at the center of mass gravity of $\chi_{cJ}$ or discover it off the value suggested by $p\bar{p}$ annihilation. We should keep in mind that theory does not require that the $h_c$ mass should be so close to the center of gravity of $\chi_{cJ}$.

We shall obtain the product of the branching fractions, $B(B \to h_c K) \times B(h_c \to \gamma \eta_c K)$ from the proposed $B$ decay measurement. Since the value of $B(h_c \to \eta_c K)$ given in Eq. (13) is a fairly firm number, measurement or nil measurement of the process $B \to h_c K$ will provide us with a meaningful number or a tight upper bound for $B(B \to \chi_c K)$. If we end up with a nil result for $B \to h_c K \to \gamma \eta_c K$, it would mean that $B(B \to h_c K)$ is for some reason much smaller than $B(B \to \chi_c K)$. Whatever the experimental outcome will be, such information will provide us with an opportunity to examine all of $B \to \text{charmonium}$ decays together and will advance our understanding of how or if the factorization plays a role in the $B$ decay into charmonia.

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