We calculate the QED corrections to deep inelastic scattering with tagged photons at HERA in the leading logarithmic approximation. Due to the special experimental setup, two large scales appear in the calculation that lead to two large logarithms of comparable size. The relation of our formalism to the conventional structure function formalism is outlined. We present some numerical results and compare with previous calculations.

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I. INTRODUCTION

One of the major tasks of the experiments at the HERA collider is the determination of the structure functions of the proton, $F_2(x, Q^2)$ and $F_L(x, Q^2)$, over a broad range of the kinematical variables. The extension of these measurements to the range of small Bjorken $x < 10^{-4}$ and $Q^2$ of a few GeV$^2$ is of particular interest, because it will help improve our understanding of the details of the dynamics of quarks and gluons inside the nucleon [1].

In order to separate $F_L(x, Q^2)$ from $F_2(x, Q^2)$, it is necessary to measure the cross section for $ep \rightarrow eX$ with different center-of-mass energies. However, instead of running the collider at reduced beam energies, one can employ a method suggested by Krasny et al. [2] that utilizes radiative events. This method takes advantage of a photon detector (PD) in the very forward direction, as seen from the incoming electron beam. Such a device is part of the luminosity monitors of the H1 and ZEUS experiments.

The idea of the method is that emission of photons in a direction close to the incoming electron can be interpreted as a reduction of the effective beam energy. The effective beam energy for each radiative event is determined from the energy of the hard photon that is observed in the PD. In fact, radiative events were already used in a measurement of the structure function $F_2$ down to $Q^2 \gtrsim 1.5$ GeV$^2$ [3].

The possibility to use radiative events for structure function analyses was already discussed by Jadach et al. [4]. However, these authors used radiative events with tagged photons in order to reduce the radiative corrections and to simplify the $F_2$ analysis. They did not consider higher order corrections explicitly, and it is not clear whether their method allows an extraction of $F_L$ without running at lower beam energies. On the other hand, the feasibility of a quantitative measurement of $F_L$ at HERA using the method of [2], for $Q^2$ below 5 GeV$^2$ and $x$ around $10^{-4}$, has been found in [5] and considered possible.

A more general treatment of the Born cross section for $ep \rightarrow e\gamma X$ that is also valid for non-collinear photon emission and in the range of high $Q^2$ can be found in the paper by Bardin et al. [6].

For an accurate description of the corresponding cross section one has to consider the radiative corrections. In this letter we calculate the QED radiative corrections to deep inelastic scattering with one tagged photon to leading logarithmic accuracy using the structure function formalism [7]. We will in particular discuss the appearance of two large logarithms corresponding to two large scales, which are due to the experimental setup peculiar to the HERA experiments. Finally, we will show some numerical results and compare our findings with the calculation by Bardin et al. [6].

II. KINEMATICS AND LOWEST ORDER CROSS SECTION

To begin with, let us start with a brief review of the kinematics adapted to the case of deep inelastic scattering with one exclusive hard photon,

$$e(p_e) + p(P) \rightarrow e(p'_e) + \gamma(k) + X(P'),$$ (1)

where the polar angle $\vartheta_\gamma$ of the photon (measured with respect to the incident electron beam) is assumed to be very small, $\vartheta_\gamma \lesssim \vartheta_0$, with $\vartheta_0$ being about 0.45 mrad in the case of H1.

A convenient set of invariants that takes into account the energy loss from the collinearly radiated photon is given by [6]:

$$Q^2 = -(p_e - p'_e - k)^2,$$
\[ x = \frac{Q^2}{2P \cdot (p_e - p'_e - k)}, \]
\[ y = \frac{P \cdot (p_e - p'_e - k)}{P \cdot (p_e - k)}. \tag{2} \]

Since we restrict ourselves to collinear photons, it is suitable to parameterize the energy of the radiated photon, \( E_\gamma \), with the help of
\[ z = \frac{E_e - E_\gamma}{E_e} = \frac{Q^2}{x y S}, \quad \text{with} \quad S = 2p_e \cdot P. \tag{3} \]

The differential cross section for the process \( ep \rightarrow e\gamma X \), integrated over the photon emission angle \( 0 \leq \vartheta_\gamma \leq \vartheta_0 \), reads \( \bib{[2]}{[2]} \)
\[ \frac{d^3 \sigma_{\text{Born}}}{dx dQ^2 dz} = \sigma_0(x, Q^2; z) \cdot \frac{\alpha}{2\pi} P(z, L_0), \tag{4} \]
where
\[ \sigma_0(x, Q^2; z) = \frac{2\pi \alpha}{x Q^2} \left[ 2(1 - y) + \frac{y^2}{1 + R} \right] F_2(x, Q^2), \tag{5} \]
with
\[ y = \frac{Q^2}{xyz}, \tag{6} \]
and
\[ P(z, L_0) = \frac{1 + z^2}{1 - z} L_0 - \frac{2z}{1 - z}, \tag{7} \]
with
\[ L_0 \equiv \ln \frac{E_e}{\vartheta_0}, \quad \zeta_0 = \frac{E_e^2 \vartheta_0^2}{m_e^2}. \tag{8} \]

Here we have neglected terms of order \( O(\vartheta_0^2) \) as well as terms of order \( O(e^{-1}) \), and we have neglected the contributions from \( Z^0 \) exchange since we are mainly interested in the region of small \( Q^2 \) (see also \bib{[3]}{[3]}). \( R \) is defined as the ratio of cross sections for longitudinal to transverse photons,
\[ R(x, Q^2) = \frac{\sigma_L}{\sigma_T} = \frac{F_L}{F_2 - F_L}. \tag{9} \]

The allowed range for \( z \) follows from \bib{[3]}{[3]} and the restriction \( 0 \leq y \leq 1 \),
\[ \frac{Q^2}{xS} \leq z \leq 1. \tag{10} \]

III. RADIATIVE CORRECTIONS

There are two large sources of contributions to the radiative corrections from higher orders that contribute terms of the order of \( \alpha/\pi \cdot \ln Q^2/m_f^2 \), with \( m_f \) being the mass of a light fermion. One type of contributions are the corrections to the propagator of the exchanged boson between the electron and the hadronic system, while the others are due to radiation of photons off the incoming electron line, which we will treat in the structure function formalism.

We shall assume a calorimetric experimental setup, where a hard photon radiated collinearly to the outgoing electron line cannot be distinguished from a bare outgoing electron, so that final state radiation can be neglected to the desired leading logarithmic accuracy in accordance with the Kinoshita-Lee-Nauenberg theorem \bib{[4]}{[4]}. We also assume some minimum experimental cut on the transverse momentum of the outgoing hadronic system, in order to suppress the contribution from QED Compton events \bib{[1]}{[1]} for a leptonic measurement of the kinematical variables.

A. Vacuum polarization

The first important type of contribution to the radiative corrections comes from the vacuum polarization, which amounts to replacing the coupling constant \( \alpha \) in the hard cross section \( \sigma_0 \) (eq. \bib{[3]}{[3]}) by the QED running coupling \( \alpha(-Q^2) \). For the contribution from lepton loops we use the well-known one-loop perturbative result. Since we are interested in the region of rather low \( Q^2 \), we use a parameterization by Burkhardt and Pietrzyk \bib{[1]}{[1]} for the hadronic contribution.

B. Photonic corrections

In order to illustrate the calculation of radiative corrections to the cross section \bib{[5]}{[5]} due to photon emission in the framework of the structure function formalism \bib{[6]}{[6]}, let us first note that this cross section is already a QED correction: it is that part of the inclusive first order correction to deep inelastic scattering (\( ep \rightarrow eX \)), which is selected by requiring an exclusive hard photon seen in the PD. This may be exhibited by writing down the radiatively corrected inclusive cross section to DIS as a convolution of the electron non-singlet structure function \( D_{NS}(z, Q^2) \) with the hard cross section \bib{[5]}{[5]}
\[ \frac{d^3 \sigma_{\text{RC}}}{dx dQ^2} = \int dz D_{NS}(z, Q^2) \sigma_0(x, Q^2; z). \tag{11} \]

The electron non-singlet structure function,
\[ D_{NS}(z, Q^2) = \delta(1-z) + \frac{\alpha}{2\pi} L P^{(1)}(z) \]
\[ + \left( \frac{\alpha}{2\pi} \right)^2 \frac{L^2}{2!} P^{(2)}(z) + O((\alpha L)^3), \] (12)
depends on the large scale \( Q^2 \) only via the large logarithm \( L = \ln Q^2/m^2_\gamma \). It is known to properly sum the leading contributions \((\alpha L)^n\) to all orders in perturbation theory.

We now give the relevant coefficients of the power series expansion of \( D_{NS} \). Introducing a small auxiliary parameter \( \Delta \) that serves as an infrared (IR) regulator to separate virtual+soft and hard photon contributions, the first two coefficients of the expansion of \( D_{NS} \) are

\[ P^{(1,2)}(z) = P^{(1,2)}_\delta \cdot \delta(1-z) + P^{(1,2)}_{\Theta}(z) \cdot \Theta(1-\Delta-z), \]
with
\[ P^{(1)}_{\Theta}(z) = 1 + \frac{z^2}{1-z}, \quad P^{(1)}_\delta = 2 \ln \Delta + \frac{3}{2}, \] (13)
and similarly (see e.g. [3]),

\[ P^{(2)}_{\Theta}(z) = \int^{1-\Delta}_{z/(1-\Delta)} \frac{dt}{t} P^{(1)}_{\Theta}(t) P^{(1)}_{\Theta}(\frac{z}{t}) + 2 P^{(1)}_{\Theta} P^{(1)}_{\Theta}(z) \]
\[ = 2 \left[ 1 + \frac{z^2}{1-z} \left( 2 \ln(1-z) - \ln z + \frac{3}{2} \right) \right] + \frac{1+z}{2} \ln(z-1+z). \] (14)

Inspecting (14), it is obvious that the logarithmic piece of \( P(z) \) is contained in the first order correction to the inclusive cross section; the difference in the logarithms \((L-L_0)\) is accounted for by the remaining phase space due to photons emitted with angle larger than \( \theta_0 \).

Let us now turn to the contributions from the second order. Since the maximum emission angle \( \theta_0 \) is about 0.45 mrad for the HERA detectors, the “large logarithm” \( L_0 \) appearing in (14) turns out to be moderate,

\[ \zeta_0 \approx 600, \quad L_0 \approx 6.4 \quad \text{at} \quad E_e = 27.5 \text{ GeV}, \]
so that the complement

\[ L_1 \equiv L - L_0 \approx 6.5 \ldots 15.7 \]
(for e.g., \( Q^2 = 0.1 \ldots 1000 \text{ GeV}^2 \))
is of similar magnitude or even larger than \( L_0 \). For this reason we have two large logarithms \( L_0, L_1 \) entering the game.

We shall separate the contributions into virtual+collinear, soft+collinear, two collinear photons, and one collinear plus one semicollinear photon.

The sum of the contributions of virtual+soft correction to collinear photon emission, with the emission angle of the soft photon being integrated over the full solid angle, but the hard photon only over the angular range of the PD, can be obtained from the expression for the one-loop Compton tensor. It reads [13]

\[ \left( \frac{\alpha}{2\pi} \right)^2 L_0 \left[ (L_0 + L_1) P^{(1)}_\Theta - (L_0 + 2L_1) \ln z \right] \]
\[ \times P^{(1)}_{\Theta}(z) \cdot \sigma_0(x, Q^2; z). \] (15)

The contribution from two hard photons in the PD, with the energy fraction of each being larger than \( \Delta \), is found to be [13]

\[ \left( \frac{\alpha}{2\pi} \right)^2 \frac{L_0^2}{2} \left[ P^{(2)}_{\Theta}(z) - 2P^{(1)}_{\Theta}(z) \left( P^{(1)}_\Theta - \ln z \right) \right] \]
\[ \times \sigma_0(x, Q^2; z), \] (16)

with \( z = 1 - (\sum \gamma_e)/E_e \), where \( \sum \gamma_e \) is the total photon energy registered in the PD.

Finally, we consider the contribution from one collinear photon that hits the PD, while the other one is emitted at an angle larger than \( \theta_0 \).

Let us investigate the regions of phase space where the large logarithmic contributions originate. For the hard photon that hits the PD, the major contribution comes from the region of polar angles \( \theta_0^{(1)} \approx m_e/E_e \ll \theta_0 \). Similarly, for the other photon the biggest contribution comes from polar angles close to the lower limit, \( \theta_0^{(2)} \gtrsim \theta_0 \). Thus, the leading contributions come essentially from the region where \( \theta_0^{(1,2)} \approx \zeta_0 \gg 1 \). One can easily see that one obtains the double logarithms entirely from the contribution where the photon hitting the PD is emitted first, with the “lost” photon being emitted second, while the reversed case is suppressed if the following condition is satisfied:

\[ x_2 \cdot \zeta_0 \gg 1, \quad \text{where} \quad x_2 = \frac{E_\gamma^{(2)}}{E_e}. \] (17)

Thus we shall assume in the following \( \Delta \ll 1 \), but \( \Delta \cdot \zeta_0 \gg 1 \).

Taking this ordering of photon emissions into account, the contribution from one tagged plus one undetected photon can be calculated by means of the quasireal electron method [12]

\[ \left( \frac{\alpha}{2\pi} \right)^2 L_0 L_1 \cdot P^{(1)}_{\Theta}(z) \int_{\Delta}^{x_{\gamma_{\max}}^{(2)}} \frac{dx_2}{z} P^{(1)}_{\Theta} \left( 1 - \frac{x_2}{z} \right) \]
\[ \times \tilde{\sigma}_0(x, Q^2; z-x_2), \] (18)
where \( \tilde{\sigma}_0 \) is understood to be expressed by the kinematical variables \( \hat{x}, \hat{Q}^2 \), of the hard subprocess,

\[ \tilde{\sigma}_0(x, Q^2; z-x_2) \equiv \sigma_0(\hat{x}, \hat{Q}^2, z-x_2) \cdot J(x, Q^2; x_2). \] (19)

Note that this contribution explicitly depends on the experimental determination of the kinematical variables \( x \) and \( Q^2 \), since the almost collinear emission of the second photon shifts the “true” kinematical variables \( (\hat{x}, \hat{Q}^2) \)
with respect to the measured ones \((x, Q^2)\). The Jacobian \(J\) accounts for scaling properties of the chosen kinematical variables under radiation of the second photon. The upper limit of the \(x_2\)-integration in (13) is given by either some experimental cut on the maximal energy of the second photon, or by the kinematical limit, which also depends on the choice of the experimental determination of the kinematical variables, as we will discuss later.

After change of variables \(x_2 = zu, u_0 = x_2^{\text{max}}/z\), the integral in (13) may be conveniently decomposed into IR divergent (\(\Delta\) dependent) and IR convergent contributions as (suppressing the arguments \(x, Q^2\))

\[
\int_{\Delta}^{x_2^{\text{max}}} \frac{dx_2}{z} P^{(1)}_{\Theta}(1 - \frac{x_2}{z}) \bar{\sigma}_0(z - x_2) = \\
= \int_{\Delta/z}^{u_0} du \quad P^{(1)}_{\Theta}(1 - u) [\bar{\sigma}_0(z(1 - u)) - \bar{\sigma}_0(z)] + \bar{\sigma}_0(z) \cdot \left[ \int_0^{u_0} du \quad P^{(1)}_{\Theta}(1 - u) \left( \frac{\bar{\sigma}_0(z(1 - u))}{\bar{\sigma}_0(z)} - 1 \right) + 2 \ln z - P^{(1)}_\delta - \int_{u_0}^1 du \quad P^{(1)}_{\Theta}(1 - u) \right],
\]

where in the last step we have extended the \(u\)-integration of the IR convergent piece to 0 due to the smallness of \(\Delta\), and we have used the property

\[
\int_0^1 du \quad P^{(1)}(u) = 0
\]
in the simplification of the IR divergent piece.

If we sum the contributions (13), (16), and (18), we see that the dependence on the auxiliary parameter \(\Delta\) cancels, as it should.

C. The radiatively corrected cross section

We may now write down our final result for the radiative corrections. Since we restricted ourselves to the leading logarithmic corrections, it follows that the radiatively corrected cross section may be written in factorized form,

\[
\frac{d^3\sigma_{RC}}{dx dQ^2 dz} = \frac{d^3\sigma_{\text{Born}}}{dx dQ^2 dz} \cdot (1 + \delta_{ho}(x, Q^2, z)) \cdot (1 + \delta_{ho}(x, Q^2, z)).
\]

where we retain in the correction factor \((1 + \delta_{ho})\) only the logarithmic terms \(L_0, L_1\) from (13), (16), and (18), and the contribution from vacuum polarization,

\[
1 + \delta_{ho} = \left( \frac{\alpha(-Q^2)}{\alpha(0)} \right)^2 \left\{ 1 + \frac{\alpha L_0}{2\pi} \left[ \int_0^{u_0} du \quad P^{(1)}_{\Theta}(1 - u) \left( \frac{\bar{\sigma}_0(z(1 - u))}{\bar{\sigma}_0(z)} - 1 \right) - \int_{u_0}^1 du \quad P^{(1)}_{\Theta}(1 - u) \right] \right\}.
\]

Note that the contribution from the undetected hard photon depends on the choice of kinematical variables and on the upper limit \(u_0\).

IV. RESULTS FOR HERA

In the presence of an undetected (lost) photon, the determination of the kinematical variables \(x, Q^2\) becomes ambiguous, it will depend on the method chosen, and in turn the corrections (22) will depend on this choice.

For HERA, several methods are being used to determine the “true” kinematical variables \(\hat{x}, \hat{Q}^2\), in order to reduce systematic errors or to control the influence of initial state radiation (see e.g., [3] for an illuminating discussion of the latter). For the sake of brevity we will discuss only the so-called electron method (E) and the Jacquet-Blondel method (JB).

In the case of the electron method, the kinematical variables (3) are determined via

\[
Q^2 = 4E^\text{eff}_e E^\text{eff}_e \cos^2 \vartheta_e /2, \\
y_e = 1 - \frac{E^\text{eff}_e}{E_e} \sin^2 \vartheta_e /2, \\
x_e = \frac{\hat{Q}^2}{y_e s^\text{eff}}.
\]

with

\[
s^\text{eff} = 4E^\text{eff}_e E^\text{eff}_p, \\
E^\text{eff}_e = E_e - E_\gamma = zE_e,
\]

and \(E_\gamma\) is the energy deposited in the PD. Radiating an additional almost collinear photon with energy \(E^\text{coll}_\gamma\) leads to a shift of the “true” variables \(\hat{x}, \hat{Q}^2\), of the hard subprocess with respect to the measured ones, \(x_e, Q^2_e\),

\[
\hat{Q}^2 = Q^2 \cdot z', \\
\hat{x} = \frac{y_e z' - 1}{y_e + z' - 1}, \\
z' \geq \frac{1 - y_e}{1 - x_e y_e}.
\]

The Jacobian (13) for this choice of kinematical variables is

\[
J(x_e, Q^2_e) = \left( \frac{y_e z'}{y_e + z' - 1} \right)^2.
\]

In the case of the Jacquet-Blondel method, the kinematical variables are determined using
\[ y_{JB} = \frac{1}{2E_{\text{eff}}^h} \sum_h (E_h - p_{z,h}), \]
\[ Q_{JB}^2 = \frac{\sum_h p_{h}^1}{1 - y_{JB}} \cdot x_{JB} = \frac{Q_{JB}^2}{y_{JB,\text{eff}}}, \]

where \( E_h, p_{z,h}, p_{h}^1 \) are the energy, longitudinal and transverse momentum components of the final state hadrons.

The relation between the measured and “true” variables in presence of an undetected photon are now given by

\[ \hat{Q}^2 = Q_{JB}^2 \frac{1 - y_{JB}}{1 - y_{JB}/z'}, \quad \hat{y} = \frac{y_{JB}}{z'}, \quad \hat{x} = x_{JB} \frac{1 - y_{JB}}{1 - y_{JB}/z'}, \]

with the Jacobian

\[ J(x_{JB}, Q_{JB}^2) = \left( \frac{1 - y_{JB}}{1 - y_{JB}/z'} \right)^2, \]

and with the kinematical limit being

\[ z' \geq \frac{y_{JB}}{1 - x_{JB}(1 - y_{JB})}. \]

Comparing (24) and (28), one finds that one can in principle determine the energy of the undetected photon (assuming that it is emitted almost collinearly) via

\[ z' = 1 + y_{JB} - y_e. \]

This may be used to impose an experimental cut to reduce the size of the radiative corrections.

Let us now present some numerical results for the radiative corrections (22). As input we used

\[ E_e = 27.5 \text{ GeV}, \quad E_p = 820 \text{ GeV}, \quad \vartheta_0 = 0.45 \text{ mrad}, \]

the GRV94-LO structure functions from PDFLIB for \( x > 10^{-3} \), the BK parameterization for GRV94 for \( x < 10^{-3} \), and for simplicity a fixed value \( R = 0.3 \) (see e.g., [3] for a table of measured \( R \) values). Figure 1 shows the corrections \( \delta_{ho} \) for an energy of 5 GeV deposited in the PD.

\[ \text{FIG. 1. Radiative corrections } \delta_{ho} \text{ in leptonic variables for different values of } x \text{ and a tagged photon energy of 5 GeV.} \]

At small \( y_e \), the corrections are negative, since the contribution from virtual+soft corrections dominates, because the kinematical limit on the energy of the undetected photon tends to zero as \( y_e \to 0 \),

\[ u_0 = 1 - z'_{\min} = \frac{y_e(1 - x_e)}{1 - x_e y_e}. \]

For large \( y_e \), the phase space for photon emission increases, increasing also the shifts between “true” and measured variables (24), which leads to large positive corrections as \( y_e \to 1 \).

\[ \text{FIG. 2. Radiative corrections } \delta_{ho} \text{ in Jacquet-Blondel variables for different values of } x \text{ and a tagged photon energy of 5 GeV.} \]

\[ {\text{[3]} } \]

We have verified that the results for the corrections do not change significantly when we use the ALLM parameterization for \( x \lesssim 10^{-3} \), although the difference in the predictions for the low-\( x \), low-\( Q^2 \) range affects the corrections already for \( x \) around \( 10^{-3} \).
Figure 2 shows the corrections $\delta_{\text{JB}}$ for Jacquet-Blondel variables, again for a tagged photon energy of 5 GeV. In this case the corrections are negative for $y_{\text{JB}} \to 1$, because in this limit the phase space for the undetected photon tends to zero,

$$u_0 = 1 - x'_{\min} = \frac{(1 - x_{\text{JB}})(1 - y_{\text{JB}})}{1 - x_{\text{JB}}(1 - y_{\text{JB}})},$$

(34)

whereas for small $y_{\text{JB}}$ the corrections remain moderate. Since the Jacquet-Blondel variables correspond to a “more inclusive” measurement than the leptonic variables, the corrections due to radiation of an additional photon are generally smaller.

V. DISCUSSION AND CONCLUSIONS

In this letter, we have studied the radiative corrections to deep inelastic scattering with tagged photons at HERA taking into account only the leading logarithms. The relevant expansion parameter in the present case is not simply $\alpha/\pi \cdot L$, but we have two large logarithms $L_0, L_1 = L - L_0$ which happen to be of similar order of magnitude due to the particular choice of the geometrical acceptance of the photon detector. The corrections do depend on the choice of the experimental determination of the kinematical variables, but they turn out to be of the order of 20–40% in most regions of phase space for leptonic variables, and below 10% for Jacquet-Blondel variables. Large negative corrections occur in those regions of phase space that are dominated by soft photon emission. These large corrections are however well under control once one takes into account the resummation of multiple photon emission.

We have not considered the contributions from next-to-leading order, $(\alpha/\pi)^2 \times L$, which may be sizable since the individual logarithms $L_0, L_1$ are not very large, especially for the experimentally interesting region of low $Q^2$. However, the calculation of those terms is more involved and will be presented elsewhere.

We have compared our results with the results given in [8]. At the Born level, we find very good numerical agreement between [4] and the program HECTOR [21] if $Z$-exchange is neglected, which is a good approximation since the experimental analysis is restricted to the region of not too large $Q^2$. However, at the level of radiative corrections, there are major differences. First, we note that our derivation of the calculation of the corrections that is based on the structure function formalism [3] disagrees with formula (4.1) given in their paper. In particular, the coefficient of the leading logarithmic term derived from (4.1) equals two times our coefficient. The naive usage of the quasireal electron approximation in their formula leads to a loss of interference of amplitudes, as was claimed by the authors. However, the interference actually does contribute leading logarithms when both photons hit the PD, contrary to the statement given in [8]. More details on this interference between photon emissions can be found in [3]. Furthermore, it seems that the authors omitted the statistical factor $1/2!$, while summing the contributions (iib, iiic) of the semicollinear kinematics.

Some care is needed in the comparison of numerical results since Bardin et al. choose kinematical variables different from ours. Taking into account the abovementioned factor of 2 and the difference due to our treatment of the ordering condition for the photon emission, we can qualitatively reproduce their results as presented in figure 6 of their paper in the range of smaller $y$ values (thus confirming the lack of the symmetry factor).

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