Light-curve modelling of superluminous supernova 2006gy: collision between supernova ejecta and a dense circumstellar medium

Takashi J. Moriya,1,2,3∗ Sergei I. Blinnikov,1,4,5 Nozomu Tominaga,1,6 Naoki Yoshida,1,7 Masaomi Tanaka,8 Keiichi Maeda1 and Ken’ichi Nomoto1

1Kavli Institute for the Physics and Mathematics of the Universe, Todai Institutes for Advanced Study, University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8583, Japan
2Department of Astronomy, Graduate School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan
3Research Center for the Early Universe, Graduate School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan
4Institute for Theoretical and Experimental Physics, Bolshaya Cheremushkinskaya 25, 117218 Moscow, Russia
5Sternberg Astronomical Institute, Moscow University, Universitetski pr. 13, 119992 Moscow, Russia
6Department of Physics, Faculty of Science and Engineering, Konan University, 8-9-1 Okamoto, Kobe, Hyogo 658-8501, Japan
7Department of Physics, Graduate School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan
8National Astronomical Observatory of Japan, 2-21-1 Ohsawa, Mitaka, Tokyo 181-8588, Japan

Accepted 2012 September 22. Received 2012 August 5; in original form 2012 April 26

ABSTRACT
We show model light curves of superluminous supernova 2006gy on the assumption that the supernova is powered by the collision of supernova ejecta and a dense circumstellar medium. The initial conditions are constructed based on the shock breakout condition, assuming that the circumstellar medium is dense enough to cause a shock breakout within it. We perform a set of numerical light-curve calculations using the one-dimensional multigroup radiation hydrodynamics code STELLA. We succeed in reproducing the overall features of the early light curve of SN 2006gy with a circumstellar medium of mass about $15 M_\odot$ (the average mass-loss rate is $\sim 0.1 M_\odot \text{yr}^{-1}$). Thus, the progenitor of SN 2006gy is likely a very massive star. The density profile of the circumstellar medium is not well constrained by light-curve modelling alone, but our modelling disfavours a circumstellar medium formed by steady mass loss. The ejecta mass is estimated to be comparable to or less than $15 M_\odot$ and the explosion energy is expected to be more than $4 \times 10^{51}$ erg. No $^{56}$Ni is required to explain the early light curve. We find that multidimensional effects, e.g. Rayleigh–Taylor instability, which is expected to take place in the cool dense shell between the supernova ejecta and the dense circumstellar medium, are important in understanding supernovae powered by shock interaction. We also show the evolution of optical and near-infrared model light curves of high-redshift superluminous supernovae. They can potentially be used to identify SN 2006gy-like superluminous supernovae in future optical and near-infrared transient surveys.

Key words: circumstellar matter – stars: mass-loss – supernovae: individual: SN 2006gy – early Universe.

1 INTRODUCTION
Recent untargeted transient surveys such as the Texas Supernova Search (Quimby 2006a), Palomar Transient Factory (Law et al. 2009; Rau et al. 2009), Catarina Real-time Transient Survey (Drake et al. 2009) and the Panoramic Survey Telescope & Rapid Response System (Hodapp et al. 2004) discovered new kinds of supernovae (SNe). Among the most spectacular are superluminous SNe (SLSNe), which typically become brighter than $\sim -21$ mag in the optical (e.g. Quimby et al. 2007, 2011; Smith et al. 2007; Barbary et al. 2009; Gal-Yam et al. 2009; Gezari et al. 2009; Miller et al. 2009; Drake et al. 2010, 2011; Pastorello et al. 2010; Chatzopoulos et al. 2011; Chomiuk et al. 2011; Rest et al. 2011; Berger et al. 2012; Leloudas et al. 2012).

SN 2006gy is the first example of a SLSN with detailed photometric and spectroscopic observations. The luminosity of SN 2006gy stayed brighter than $-21$ mag in the $R$ band for more than 50 d (see Section 2 for a brief summary of observations). It was the most luminous SN ever reported at that time. Many theoretical

∗E-mail: takashi.moriya@ipmu.jp
models have been proposed to explain the huge luminosity of SN 2006gy. SN 2006gy was, at first, linked to a Type Ia SN exploding inside a H-rich circumstellar medium (CSM) (e.g. SN 2002ic; Hamuy et al. 2003; Deng et al. 2004; Ofek et al. 2007). However, the total observed radiation energy exceeded the available energy from a Type Ia explosion and thus the notion was abandoned.

Currently, SN 2006gy is thought to have been caused by the death of a massive star. Several models have been proposed to explain the observed features as follows. All of them posit a massive progenitor star.

(i) Large production of $^{56}\text{Ni}$. The energy released by the radioactive decay of $^{56}\text{Ni}$ is a common luminosity source for SNe. However, more than 10 $M_\odot$ of $^{56}\text{Ni}$ would be required to account for the peak luminosity of SN 2006gy (e.g. Nomoto et al. 2007). The large amount of $^{56}\text{Ni}$ is not easily produced even by very high-energy core-collapse SNe (Umeda & Nomoto 2008). Pair-instability SNe (PISNe, e.g. Barkat, Rakavy & Sack 1967; Rakavy & Shaviv 1967) can produce the large $^{56}\text{Ni}$ required, but the rise time of SN 2006gy is found to be much shorter than that expected from a PISN (e.g. Nomoto et al. 2007; Moriya et al. 2010; Kasen, Woosley & Heger 2011). The weakness of Fe lines in spectra at the late phase also argues against the $^{56}\text{Ni}$ heating scenario (Kawabata et al. 2009).

(ii) Interaction between SN ejecta and a dense CSM. The interaction between SN ejecta and the dense CSM created by the progenitor before the explosion can make the luminosity of a SN very large. Strong shocks convert the kinetic energy of the SN ejecta to thermal energy, which is eventually released as radiation energy. Smith & McCray (2007) suggest that the declining phase of the light curve (LC) of SN 2006gy can be explained by emission from a shocked $\sim 10 M_\odot$ CSM shell. Agnoletto et al. (2009) combined $^{56}\text{Ni}$ and shock interaction to explain the early LC of SN 2006gy. They argue that the rising part of the LC is powered by $^{56}\text{Ni}$ decay and that shock interaction comes into play from around the LC peak. Then the required $^{56}\text{Ni}$ mass is reduced to a few $M_\odot$.

(iii) Pulsational pair-instability. Woosley, Blinnnikov & Heger (2007b) relate SN 2006gy to pulsational pair-instability. A very massive star with mass $\approx 80-130 M_\odot$ at the zero-age main sequence undergoes strong pulsations (see also Umeda & Nomoto 2008; Ohkubo et al. 2009), which induce intermittent extensive mass loss. If the mass ejected by such an eruption is caught up by the mass released by the next eruption, the two massive shells collide and the star can eventually be as luminous as SN 2006gy. In this case, the luminosity source is the kinetic energy of the materials ejected secondly. Hence, the radiation mechanism is essentially the same as that of (i), interaction between SN ejecta and a dense CSM. The ejecta from inside are released by pulsational pair-instability rather than by a SN explosion in this case. These kind of luminous transients are called pulsational pair-instability SNe.

(iv) Spin-down of newly born magnetars. If a magnetar born at the time of the core collapse of a massive star has suitable magnetic field and spin, it can release its rotational energy efficiently just after the SN explosion. If the radiation energy released at the magnetar can be converted to thermal energy, the SN ejecta surrounding the magnetar is heated up and the SN can become very bright (Kasen & Bildsten 2010; Woosley 2010; see also Maeda et al. 2007).

(v) Transition of a neutron star to a quark star. Neutrons in a neutron star formed at core collapse can be further decomposed to quarks and they can form a quark star. When a neutron transforms to quarks, latent heat due to the phase transition can heat up the SN ejecta surrounding the quark star and make a SLSN (e.g. Leahy & Ouyed 2008; Kostka et al. 2012; Ouyed et al. 2012; see also Benvenuto & Lugones 1999).

Among the possible heating sources, the CSM interaction (ii and iii) is plausible because the spectra of SN 2006gy show narrow P Cygni profiles that indicate the existence of a CSM outflow of $\sim 100 \text{km s}^{-1}$. Although the observed low X-ray flux of SN 2006gy (see Section 2) might appear to contradict the interaction scenario, the CSM could be optically thick enough to absorb X-rays from the shock wave or the shock wave itself could be cool (e.g. Blinnnikov 2008; Chevalier & Irwin 2012; Svirski, Nakar & Sari 2012).

Although scenario (ii), i.e. shock interaction between the SN ejecta and dense CSM, is suggested to be able to explain the early LC of SN 2006gy, detailed LC models have not been explored yet. Chevalier & Irwin (2011), Moriya & Tominaga (2012) and Svirski et al. (2012) investigate the possibility of explaining SN 2006gy by the interaction of SN ejecta and a dense CSM alone. They assume that the CSM is optically thick enough to cause a shock breakout within it (e.g. Weaver 1976; Nakar & Sari 2010, see also Ofek et al. 2010; Balberg & Loeb 2011 with regard to shock breakout in a dense CSM). Chatzopoulos, Wheeler & Vinko (2012) show the bolometric LC from the interaction scenario based on their semi-analytic LC model (see Section 6.4). Previous works regarding the collision of SN ejecta and a dense CSM to date have analysed the LC of SN 2006gy based on analytic approaches with several simplifications. The detailed numerical modelling of the LC of SN 2006gy has not been performed yet and is clearly required (see the very recent attempt by Ginzburg & Balberg 2012).

In this paper, we study model LCs for SN 2006gy by using the numerical radiation hydrodynamics code STELLA. Our purpose in modelling is to show numerically that the simple analytic shock breakout model actually works well to explain the overall features of the early LC of SLSN 2006gy. We focus on SN 2006gy because it is the best-observed SLSN currently reported. We do not aim at a perfect fitting of the LC, because it does not necessarily lead us to the correct physical parameters of SN 2006gy. This is because STELLA makes several simplifications to treat the radiation hydrodynamics numerically. Instead, we concentrate on the overall features of the LC of SN 2006gy and show that the dense CSM configurations predicted by the shock breakout model actually work well to reconstruct the LC of SN 2006gy.

The rest of the paper is organized as follows. Section 2 is a short summary of the observations of SN 2006gy. In Section 3, STELLA, which is used for our LC modelling, is summarized briefly. An overview of the ways multidimensional effects can be included in the one-dimensional code STELLA is given in Section 3. Our models are introduced in Section 4 and the LC calculations based on the models are shown in Section 5. A discussion is given in Section 6. We close the paper with conclusions in Section 7. We use the standard cosmology with $\Omega_0 = 0.7$.

2 BRIEF SUMMARY OF OBSERVATIONS

We briefly summarize the observational properties of SN 2006gy. SN 2006gy was discovered by the Texas Supernova Search on 2006 September 18 (UT) near the nucleus of an early-type galaxy NGC 1260 (Quimby 2006b). There are several suggested values for the extinction by the host galaxy (Ofek et al. 2007; Smith et al. 2007; Agnoletto et al. 2009). We adopt $E(B-V)_{\text{host}} = 0.40\text{mag}$.
following Agnoletto et al. (2009), with the Milky Way extinction $E(B - V)_{MW} = 0.16$ mag (Schlegel, Finkbeiner & Davis 1998). Thus, the total extinction is $E(B - V) = 0.56$ mag or $A_V = 1.3$ mag with $R_V = 3.1$ (Cardelli, Clayton & Mathis 1989). The distance modulus $\mu$ of the host galaxy is also taken from Agnoletto et al. (2009) ($\mu = 34.53$ mag).

Follow-up spectral observations classified SN 2006gy as Type IIn because of the narrow H emission lines presented in the spectra (see Schlegel (1990) and Filippenko (1997) for details of Type IIn). The luminosity of SN 2006gy kept rising until 2006 October 25 (UT) and the peak R-band luminosity was close to $\pm 22$ mag. The rise time is estimated as about 70 d (hereafter, days are in the rest frame). After reaching peak luminosity, the LC declined slowly ($\pm 0.02$ mag day$^{-1}$) for $\pm 120$ d and then the LC stayed almost constant for $\pm 20$ d until SN 2006gy was hid behind the Sun. Near-infrared (NIR) LCs in these early epochs are consistent with the blackbody temperature obtained from the optical spectra and no significant excess was detected (Miller et al. 2010). No X-rays were detected when the LC was rising (Ofek et al. 2007) but weak X-rays may have been detected during the declining phase (Smith et al. 2007). No radio emission is detected at any observed epochs (Argo et al. 2007; Bietenholz & Bartel 2007, 2008a,b; Chandra, Chakraborti & Ray 2007; Ofek et al. 2007).

Optical spectra of the early epochs have also been taken intensively (e.g. Agnoletto et al. 2009; Smith et al. 2010). Spectra taken before the LC peak are characterized by Lorentzian Hα emission lines (Smith et al. 2010). The origin of the Lorentzian profile is related to the existence of an optically thick CSM (e.g. Chugai 2001; Dessart et al. 2009). Except for the narrow H emission lines, the spectra are featureless and characterized by a blackbody with depletion in the blue (Agnoletto et al. 2009; Smith et al. 2010). The lack of features may arise partly because the spectra before the LC peak were taken only with low resolutions. After the LC peak, overall Hα line profiles can be fitted by two Gaussian components with full widths at half-maximum (FWHMs) of 1800 and 5200 km s$^{-1}$ and these are presumed to come from the interacting region between the ejecta and a dense CSM (e.g. Smith et al. 2010). Some broad absorption also exists in the blue part of the Hα profile, which is suggested to originate from the ejecta inside. In addition, the spectra of SN 2006gy show narrow P Cygni profiles from several elements (e.g. H, Fe) with an outflowing velocity $\sim 100$ km s$^{-1}$. As the velocity is too slow to attribute it to the ejecta inside, those narrow lines are presumed to originate from the unshocked CSM. The strengths of these narrow lines decline with time and they are barely seen in the spectrum taken at $\pm 140$ d after the LC peak (Smith et al. 2010).

About 100 d later, SN 2006gy emerged from behind the Sun and was observed again (Agnoletto et al. 2009; Kawabata et al. 2009; Miller et al. 2010). The optical luminosity of SN 2006gy declined dramatically (about 2 mag in the $R$ band) and was almost constant for $\pm 20$ d before the SN went behind the Sun. The luminosity declined very slowly ($\pm 0.002$ mag day$^{-1}$) after re-appearing from behind the Sun for more than 400 d until the last reported observation on 2008 November 22 (UT) (Kawabata et al. 2009; Miller et al. 2010). The decline rate is much slower than that of $^{56}$Co decay (0.01 mag day$^{-1}$) and so the main source of luminosity cannot be $^{56}$Co decay. Because of the high NIR luminosities, Smith et al. (2008) and Miller et al. (2010) suggest that the late-time luminosity is due to light echoes. The optical spectra of those epochs are dominated by intermediate-width emission lines ($\pm 2000$ km s$^{-1}$, Kawabata et al. 2009). H emission lines were weaker than those observed in previous epochs and suggest that the interaction is weak in those epochs and is no longer a main source of radiation (Agnoletto et al. 2009; Kawabata et al. 2009). The weakness of Fe lines in those epochs seems inconsistent with the large $^{56}$Ni production (Kawabata et al. 2009).

3 NUMERICAL METHOD

We briefly summarize the basics of the STELLA code, which we use for our LC modelling. We then describe in detail a key numerical method of STELLA that treats smearing due to multidimensional effects.

3.1 STELLA

STELLA is a one-dimensional multigroup radiation hydrodynamics code (Blinnikov & Bartunov 1993; Blinnikov et al. 1998, 2006; Blinnikov & Tolstov 2011) and calculates the spectral energy distributions (SEDs) at each time step. Multicolour LCs can be obtained by convolving filter functions with the SEDs. All the calculations are performed by adopting 100 frequency bins from 1 Å to $5 \times 10^4$ Å on a log scale. STELLA implicitly treats time-dependent equations of the angular moments of intensity averaged over a frequency bin. Local thermodynamic equilibrium is assumed to determine the ionization levels of materials. STELLA has been intensively used for modelling SN LCs powered by shock interaction (e.g. Chugai et al. 2004; Woosley et al. 2007b; Blinnikov & Sorokina 2010; Moriya et al. 2011) as well as other types of SNe (e.g. Baklanov, Blinnikov & Pavlyuk 2005; Folatelli et al. 2006; Tsvetkov et al. 2009; Tominaga et al. 2011). Comparisons of STELLA with other numerical codes are provided in Blinnikov et al. (1998, 2000, 2003) and Woosley et al. (2007a), for example, and analytical models are also compared with the numerical results of STELLA (e.g. Rabinak & Waxman 2011).

3.2 Smearing

As is shown in the previous works (e.g. Chevalier & Fransson 1994; Chugai 2001; Chugai et al. 2004) and as will be shown in the following sections, the interaction of SN ejecta and a dense CSM results in a dense cool shell between the SN ejecta and the dense CSM. This is because of radiative cooling of the shocked region. The shocked region becomes very dense and cools down efficiently by radiation. The cooling prevents the pressure from growing sufficiently to sustain the shell. Hence, the shell becomes thinner and denser and the cooling becomes more efficient. Thus, this cooling process is catastrophic. However, in reality such a shell is unstable, because of several instabilities like the Rayleigh–Taylor instability that require multidimensional calculations to treat (e.g. Chevalier & Blondin 1995). The multidimensional effect smears the shell and less kinetic energy is converted to radiation energy. In other words, cooling by radiation is less efficient in three dimensions than in one dimension. In the STELLA code, we take such multidimensional effects into account by introducing a smearing term in the equation of motion so that the conversion efficiency from kinetic energy to radiation energy can be reduced (Blinnikov et al. 1998). This term is similar to artificial viscosity, although the smearing term has completely the opposite effect.

As is shown in Blinnikov et al. (1998), the smearing term is defined such that the total energy is manifestly conserved. Only the neighbouring zones are affected by the smearing term. The overall normalization factor $R_{\text{cut}}(\tau)$ of the smearing is expressed as

$$R_{\text{cut}}(\tau) = B_0 f(\tau).$$

See Blinnikov et al. (1998) for the definitions of $R_{\text{cut}}(\tau)$ and the smearing term. $f(\tau)$ is introduced so that the artificial smearing is
4 INITIAL CONDITIONS

In this section, we show how the initial conditions for our LC calculations are constructed. Two components exist in the initial conditions: SN ejecta inside and CSM outside. We assume that there is a gap between the progenitor and the dense CSM. Then, it takes some time for the SN ejecta to reach the dense CSM and start to collide. We assume that the SN ejecta expand freely in the gap before the collision. We follow the LCs after the collision numerically. The initial conditions for the two components, SN ejecta and dense CSM, are constructed in the way explained in this section. These initial conditions will then be confirmed by our numerical LC calculations, as we show in later sections. The initial density structures of two representative models are shown in Fig. 1 as examples. Both SN ejecta and CSM are assumed to have solar metallicity and no $^{56}$Ni is included in our calculations unless otherwise mentioned. The summary of the models is given in Table 1.

4.1 Supernova ejecta

The SN ejecta before the collision are assumed to be freely expanding with a homologous velocity profile. The analytic approximation for the density structure of SN ejecta provided by e.g. Chevalier & Soker (1989) is adopted:

$$\rho(r, t) = \begin{cases} 
\frac{M_{ej}}{v_i r^3} \left( \frac{r}{v_i t} \right)^{-3} & (v < v_i), \\
\frac{M_{ej}}{v_i r^3} \left( \frac{r}{v_i t} \right)^{-n} & (v > v_i), 
\end{cases}$$

where $M_{ej}$ is the SN ejecta mass, $t$ is the time since the explosion, $v_i = \zeta_i (E_{ej}/M_{ej})^{1/2}$ and $E_{ej}$ is the kinetic energy of the SN ejecta. The constants $\zeta_\rho$ and $\zeta_v$ are constrained by the condition that the sum of the density and the kinetic energy should be $M_{ej}$ and $E_{ej}$:

$$\zeta_v = \frac{2(5 - \delta)(n - 5)}{(3 - \delta)(n - 3)}^{1/2},$$

$$\zeta_\rho = \frac{1}{4\pi} \frac{(n - 3)(3 - \delta)}{n - \delta}.$$
In this paper we adopt $\delta = 1$ and $n = 7$, as used by Chevalier & Irwin (2011). We have also tried $n = 6$ and $n = 8$ for some models but results showed little difference compared with those for $n = 7$. The maximum velocity of the SN ejecta before the interaction is chosen to be high enough that most of the assumed $E_{32}$ is contained in the SN ejecta. It is around 20 000–50 000 km s$^{-1}$.

$M_{ij}$ is difficult to constrain only by observations of the LC of SN 2006gy, because the LC is mainly affected by the CSM as is shown in the following sections. In most of the models, we adopt $M_{ij} = 20 M_\odot$ because the progenitors of Type IIn SNe are presumed to originate from relatively massive stars. The effect of $M_{ij}$ on LCs is discussed in Sections 5.2.3 and 6.1.

### 4.2 Circumstellar medium

In the calculations a dense CSM is assumed to exist from $R_i$ to $R_o$, from the centre, with a density structure $\rho \propto r^{-w}$. The outflowing velocity of the CSM is 100 km s$^{-1}$. It is estimated from the narrow P Cygni profile of H$\alpha$ visible in the spectra of SN 2006gy (e.g. Smith et al. 2010). The CSM is assumed to be optically thick enough to cause a shock breakout within it. We estimate the physical conditions of the CSM from observations by using the shock breakout condition described in Moriya & Tominaga (2012). Our main purpose of this paper is to see how well the properties of the dense CSM predicted by this simple shock breakout model can explain the overall LC features of SN 2006gy. We use the following three values, which can be estimated from the observations to derive the CSM properties: the photon diffusion time $t_d$ in the CSM, the propagation time $t_i$ of the forward shock through the CSM and the forward shock velocity $v_s$. $t_d$ corresponds to the rise time of the LC and $t_i$ corresponds to the time at which the narrow P Cygni H$\alpha$ profiles disappear from the CSM. We adopt $t_d = 70$ d and $t_i = 194$ d (Moriya & Tominaga 2012). $v_i$ can be estimated from the spectral evolution. With $t_d$, $t_i$ and $v_s$, we can estimate the outer radius $R_o$ of the CSM and the radius $xR_o$ at which the shock breakout occurs ($R_i/R_o < x < 1$) for a given $w$ based on the shock breakout model. The shock breakout condition predicts the following relations for the three values (Moriya & Tominaga 2012):

\[
\begin{align*}
    t_d & \simeq \left( \frac{R_o}{v_s} \right) \left[ \frac{c/v_s + x^{-1} - u}{c/v_s + 1} \right]^{1/(1-u)} - x \quad (w \neq 1), \\
    t_d & \simeq \left( \frac{R_o}{v_s} \right) \left( x^{1/(1+c/w)} - x \right) \quad (w = 1), \\
    t_i & \simeq R_o - xR_o \quad \frac{v_s}{v_i}.
\end{align*}
\]

We try three values for $w$: $w = 0, 2, 5$. The models with $w = 2$ correspond to the case of steady mass loss and they are naturally expected structures for the CSM. Note that X-ray observations of Type IIn SNe suggest that the CSM around Type IIn SN progenitors often does not have $w = 2$ density structures and most of the CSM is likely from non-steady mass loss (e.g. Dwarkadas & Gruszo 2011). A steep CSM density gradient with $w = 5$ is suggested for SN 2006gy in Moriya & Tominaga (2012). We also show $w = 0$ models, which are difficult to exclude only through LC modelling.

It turns out in the later sections that it is difficult to estimate $v_s$ from observational values self-consistently. This is partly because $v_s$ is not an independent parameter and, in principle, can be derived for a given CSM structure if we specify $E_{32}$ and $M_{ij}$. However, $v_s$ is also strongly affected by the conversion efficiency from kinetic energy to radiation through the interaction and it is unknown at first. Thus, it is difficult to estimate $v_s$ from first principles. Hence, we set $v_s$ as a free parameter in this paper. At first, we try to estimate it from observations. As the blackbody radius of SN 2006gy expands linearly with velocity 5200 km s$^{-1}$, one may estimate that $v_s = 5200$ km s$^{-1}$. However, the required $E_{32}$ for the $v_s = 5200$ km s$^{-1}$ models to explain the peak luminosity of SN 2006gy is found to be very high and it becomes inconsistent with the relatively low $v_i$. In other words, the SLSN models obtained by setting $v_s = 5200$ km s$^{-1}$ are not self-consistent. Thus, we also try models with higher $v_s$, namely $v_s = 10000$ km s$^{-1}$. The $v_s = 10000$ km s$^{-1}$ models are found to work self-consistently well, as is shown in the following sections. In addition, the linear evolution of the blackbody radius with 5200 km s$^{-1}$ can be explained by the $v_s = 10000$ km s$^{-1}$ models. With $t_d = 70$ d, $t_i = 194$ d and the given $v_s$, we can derive $R_o$ and $xR_o$ from equations (5) and (6) for a specified $w$. In the remainder of this section, we show the details of the two $v_s$ models.

#### 4.2.1 $v_s = 5200$ km s$^{-1}$ model

This model corresponds to the SN 2006gy model in Moriya & Tominaga (2012). The shock velocity $v_s = 5200$ km s$^{-1}$ is estimated from the observed evolution of the blackbody radius of SN 2006gy (Smith et al. 2010). For $w = 5$, we obtain $R_o = 1.1 \times 10^{15}$ cm and $xR_o = 1.8 \times 10^{15}$ cm from equations (5) and (6). The inner radius $R_i$ of the CSM cannot be constrained by the above observables and is a free parameter. Two $R_i$ are tried: $10^{15}$ cm (A1) and $1.5 \times 10^{15}$ cm (A2). With $w = 2$ (steady mass loss), we obtain $R_o = 8.8 \times 10^{15}$ cm and $xR_o = 9.0 \times 10^{15}$ cm (B1). We set $R_i = xR_o$ in the $w = 2$ models. The results do not depend so much on $R_i$ in this case because most of the mass in the $w = 2$ CSM is distributed in the outer part of the CSM. We also calculate LCs from model (B2), in which the CSM mass is artificially increased 30 times from model B1.

In the $w = 0$ models, $t_i$ becomes similar to $t_d$ ($t_i \approx t_d$) (Moriya & Tominaga 2012). Thus, the entire CSM is shocked with $t_d$ and no unshocked wind remains after the LC peak. This is in contradiction to the observations of SN 2006gy because narrow P Cygni profiles are observed after the LC peak. However, this is only true when we only consider a single $w = 0$ CSM component. If there is another CSM component outside the main CSM, which is not dense enough to affect the LC but does affect the spectra, SN 2006gy-like SNe can appear. Thus, $w = 0$ models are difficult to exclude based only on the LC. From the rise time of the LC, we can presume $y_iR_o - xR_o \approx R_o - xR_o \approx v_s t_i = 3.1 \times 10^{15}$ cm. $y_iR_o$ is the radius at which the optical depth from the surface of the CSM becomes 1 and $y_iR_o \approx R_o$ when $w = 0$ (see Moriya & Tominaga 2012 for details). We set the last scattering surface of the $w = 5$ model ($y_iR_o = 4.9 \times 10^{15}$ cm) as $R_o$, so that we can compare the results with those of the $w = 5$ models. Thus, $xR_o = 1.8 \times 10^{15}$ cm is also the same as the $w = 5$ model and we adopt $R_i = 10^{15}$ cm (C1).

#### 4.2.2 $v_s = 10000$ km s$^{-1}$ model

$v_s = 10000$ km s$^{-1}$ models are constructed by following the same approach as for the $v_s = 5200$ km s$^{-1}$ models. With $t_d = 70$ d, $t_i = 194$ d and $v_s = 10000$ km s$^{-1}$, $R_o = 2.1 \times 10^{16}$ cm and $xR_o = 4.6 \times 10^{15}$ cm are obtained for $w = 5$. We try two $R_i$: $4.5 \times 10^{15}$ cm (D1) and $5 \times 10^{15}$ cm (D2–D7). Although $R_i = 5 \times 10^{15}$ cm is slightly larger than the shock breakout radius $xR_o = 4.6 \times 10^{15}$ cm, it turns out that the model approaches the SN 2006gy LC more closely. Given the approximate form of our estimations, the difference is
The rise times of the models are broadly consistent with the observational properties of SN 2006gy.

5.1 $v_e = 5200 \text{ km s}^{-1}$ model

Fig. 2 shows the LCs from the $v_e = 5200 \text{ km s}^{-1}$ models with the observed R-band LC of Smith et al. (2007). $E_{ej}$ is chosen so that the peak luminosities of the model LCs can be as high as that of SN 2006gy. However, $E_{ej}$ should be very high ($\gtrsim 5 \times 10^{52} \text{ erg}$) for the $v_e = 5200 \text{ km s}^{-1}$ models to be as luminous as SN 2006gy, and assuming the relatively low $v_e = 5200 \text{ km s}^{-1}$ is not consistent with the high kinetic energy. This inconsistency can also be seen from the rise times of the models. Although the models are constructed so that the rise times of the LCs become $t_d = 70 \text{ d}$, the rise times of the numerical results are much shorter than 70 d. If we set smaller $E_{ej}$, the rise times can be the same as that of SN 2006gy but then the luminosities become much smaller than that of SN 2006gy. In brief, the models derived by assuming $v_e = 5200 \text{ km s}^{-1}$ are not compatible with the large luminosity of SN 2006gy. Given these results, we adopt models with a higher $v_e$, $v_e = 10000 \text{ km s}^{-1}$, and they are able to explain the LC of SN 2006gy self-consistently (Section 5.2).

One important question of the interaction model is whether the CSM from the steady mass-loss case ($v_e = 2$) can explain the properties of SLSNe, and we look into the $w = 2$ models more carefully. If it can, a mechanism to achieve such huge steady mass loss may exist. If not, it is indicated that explosive non-steady mass loss must take place in their progenitors and there should exist some mechanism to cause such mass loss just before their explosions.

5.2 $v_e = 10000 \text{ km s}^{-1}$ model

As $v_e = 5200 \text{ km s}^{-1}$ models are not able to explain SN 2006gy, we investigate models with a higher $v_e$, $v_e = 10000 \text{ km s}^{-1}$. $v_e = 5200 \text{ km s}^{-1}$ is estimated from the evolution of the blackbody radius but it is shown that the evolution of the blackbody radius in the $v_e = 10000 \text{ km s}^{-1}$ models is consistent with that of SN 2006gy.

5.2.1 Light curve

The R-band LCs from the $v_e = 10000 \text{ km s}^{-1}$ models are shown in Fig. 3. Multicolour LCs of models D2 ($w = 5$) and F1 ($w = 0$) are shown in Fig. 4 and the bolometric LCs of the two models are shown in Fig. 5. The colour evolution of models D2 and F1 is shown in Fig. 6.

The rising parts and the peak luminosities of the LCs of the $w = 0$, $5$ models are consistent with SN 2006gy. Thus, the $v_e = 10000 \text{ km s}^{-1}$ models are self-consistent with the assumed $E_{ej}$ and $M_{ej}$. The $v_e = 10000 \text{ km s}^{-1}$ models only require $E_{ej} = 10^{52} \text{ erg}$ to achieve the peak luminosity of SN 2006gy, instead of $E_{ej} \simeq 5 \times 10^{52} \text{ erg}$ required for the $v_e = 5200 \text{ km s}^{-1}$ models. This is
because the blackbody radius in the CSM can be larger in the $v_s = 10000 \text{ km s}^{-1}$ models and less energy is required to achieve the same luminosity. The steady mass-loss models ($w = 2$) are, however, still not consistent with SN 2006gy.

The model LCs with $w = 0, 5$ after the peak start to deviate from the observed LCs, although the deviations stay less than 1 mag before the plateau in the observed LC at around 200 d. Our model $R$-band LCs take some time after the LCs have reached the peak until they start to decline, contrary to the observed $R$-band LC of SN 2006gy. This is because unshocked optically thick CSM remains even after the LC peak in our numerical models and the photosphere remains there for a while. The analytic model of Moriya & Tominaga (2012), which we use for the estimate of the initial condition, assumes a constant $v_s$. However, $v_s$ actually reduces as the interaction goes on and the optically thick part of the CSM is not shocked away entirely at time $t_d$ when the optically thick CSM is assumed to be swept up by the forward shock in the model of Moriya & Tominaga (2012). This effect is more significant in $w = 0$ models because $w = 0$ models suffer more on account of deceleration than $w = 5$ models. A severe failure of our models is that all of them fail to reproduce the plateau in the LC of SN 2006gy at around 200 d. We discuss this separately in Section 6.2.

Looking at the multicolour LCs (Fig. 4), the $w = 0$ model (F1) is closer to the observed LC, especially the $B$-band LC of the rising epochs. This is presumed to be because the initial density jump between SN ejecta and CSM is smaller in the $w = 0$ model (Fig. 1) and the temperature becomes lower in the $w = 0$ model. However, the LCs in the $U$ and $B$ bands can be affected by many weak absorption lines of Fe-group elements, which are not taken into account in our opacity. Those weak absorptions may reduce the luminosity of the $U$- and $B$-band LCs and we cannot judge

---

**Figure 4.** Multicolour LCs of models D2 ($w = 5$, left) and F1 ($w = 0$, right). The observational data are from Smith et al. (2007), Agnoletto et al. (2009) and Kawabata et al. (2009). The origin of the time axis in the left (right) panel is 5 (0) days since the collision.

**Figure 5.** Bolometric LCs of models D2 ($w = 5$) and F1 ($w = 0$). The origin of the time axis in the D2 (F1) model is 5 (0) days since the collision.

**Figure 6.** Colour evolution of models D2 ($w = 5$, left) and F1 ($w = 0$, right). Observational points are from Agnoletto et al. (2009).
which model is better based just on the blue part of the LCs. In
addition, the difference is ≃ 0.5 mag and it is not significant. The
colour evolution of the two models (Fig. 6) roughly follows the ob-
served evolution, although some deviations exist, especially in $R - I$
and $u - R$.

Although it is possible to continue LC modelling to obtain much
better fits to the SN 2006gy LC, it does not necessarily lead us to
a better understanding of the properties of the SN ejecta and the
dense CSM involved in the progenitor system of SN 2006gy. This
is mainly because of the simplified physics adopted in STELLA. Es-
specially, STELLA is a one-dimensional code and multidimensional
effects are approximately incorporated by adopting the smearing
parameter. As the uncertainties involved in the parameter are large
(see Section 5.2.4), making a perfect fit to the observed LC does
not necessarily provide us with the best parameter. In addition, the
differences in the LCs in the declining phases are less than 1 mag
(or a factor ≃ 2) and the differences in the rising phases are much
less. Thus, the properties of the SN ejecta and the dense CSM in
the D2 and F1 models are presumed not to be very different from
the ‘actual’ values. We therefore conclude that the CSM param-
ters predicted by the shock breakout model can explain the overall
properties of SN 2006gy. We also note that a systematic study of
the effect of CSM properties on LCs powered by the interaction
between SN ejecta and dense CSM is summarized in Moriya et al.
(2011). The durations of the LCs of models D2 and F1 are slightly
longer than that of SN 2006gy. To reduce the durations of the LCs
while keeping their peak luminosities, we can, for example, change
the radii of the CSM.

Finally, we look into steady mass-loss models to see whether
they actually fail to reproduce the LC of SN 2006gy. The steady
mass-loss model ($w = 2$, E1) is, again, too faint to explain SN
2006gy with $E_{\text{ej}} = 3 \times 10^{52}$ erg. The rise time is already too short
and reaching the peak luminosity of SN 2006gy by increasing $E_{\text{ej}}$
do not work, as is discussed in Section 5.1. This is because of the
too small $M_{\text{CSM}}$ and matters can be improved if $M_{\text{CSM}}$ is increased
(see models B1 and B2). Another possible way to make $w = 2$
models work is to increase the conversion efficiency from $E_{\text{ej}}$ to
radiation energy, so that $E_{\text{ej}}$ can be reduced (see Section 6.1 for a
discussion of the conversion efficiency). In model E2, $M_{\text{ej}}$ is set to
be comparable to $M_{\text{CSM}}$ so that the conversion efficiency becomes
higher (Section 6.1). A similar peak luminosity to that of model E1
is reached with a lower $E_{\text{ej}} (10^{52}$ erg) in model E2. However, as the
diffusion time of the CSM is not affected very much by this, the
LCs become similar to each other and increasing the efficiency does
not revive the $w = 2$ models. To summarize, it is still difficult to
use the dense CSM resulting from steady mass loss to explain the
LC of SN 2006gy with the shock breakout model.

5.2.2 Dynamical evolution

Fig. 7 shows the dynamical structures of model D2 ($w = 5$) at around
the LC peak. The left panel shows the structure in the physical
coordinate (radius) whereas the right panel shows the structure in
the mass coordinate. A cool dense shell is created between SN ejecta
and CSM, in which about 10 M$_{\odot}$ of shocked CSM and 3 M$_{\odot}$ of
shocked SN ejecta are contained.

Fig. 8 shows the evolution of the colour temperature ($T_{\text{col}}$) and
the effective temperature ($T_{\text{eff}}$) of model D2. The colour tem-
perature is derived by fitting the spectra obtained by numerical cal-
culations with the blackbody spectral distribution, whereas the
effective temperature is obtained by using the bolometric lumin-
osity ($L_{\text{bol}}$) and the radius ($R_{\text{eff}}$) of the photosphere, which
is defined as the radius where the Rosseland mean optical depth
$\tau_R$ from the surface becomes $2/3$ in STELLA and is expressed as
$T_{\text{eff}} = (L_{\text{bol}}/(4\pi\sigma R_{\text{eff}}^2\tau_R^{2/3}))^{1/4}$. Here, $\sigma$ is the Stefan–Boltzmann
constant. As radiation comes mainly from the shell and Thomson
scattering is the dominant opacity source in the CSM above the

Figure 7. Physical structures of model D2 in radius (left) and mass coordinates (right) at around the LC peak (74 d since the collision). Black lines show the density structure (left y-axis). Dotted lines (blue in the online article) are the velocity scaled by 10$^6$ cm s$^{-1}$ (right y-axis), shaded solid lines are the logarithm of the absolute value of luminosity scaled by 10$^{54}$ erg s$^{-1}$ (purple in the online article, right y-axis), the logarithm of the temperature in K (green in the online article, right y-axis) and the Rosseland optical depth measured from the outside (red in the online article, right y-axis).
shell, $T_{\text{eff}}$ roughly traces the temperature of the shell. The photosphere ($R_{\text{BB}}(t=2/3)$) is well above the shell and $T_{\text{eff}}$ becomes very low because of the large $R_{\text{BB}}(t=2/3)$ (see also Fig. 9). At the time when $T_{\text{col}}$ starts to increase for the second time (from $\approx 180$ days), the photosphere is in the SN ejecta, the density structure and composition of which are expressed in an approximated way, and the results around these epochs and later should not be taken seriously.

Fig. 9 shows the evolution of the blackbody radius $R_{\text{BB}} = \sqrt{L_{\text{bol}}/4\pi\sigma T_{\text{eff}}^4}$ and the photosphere $R_{\text{BB}}(t=2/3)$. The constant-velocity line with 5200 km s$^{-1}$ is the evolution of the blackbody radius obtained by Smith et al. (2010) along which $R_{\text{BB}}$ follows until around 125 days. The observational blackbody radius follows the line until around 125 days and starts to decline (see Smith et al. 2010 for details).

The $R_{\text{BB}}$ obtained by our calculations tends to be smaller than the shell radius from where the radiation comes. For example, $R_{\text{BB}}$ of model D2 shown in Fig. 9 stays lower than the radius at which the interaction starts ($R_i = 5 \times 10^{13}$ cm). The reason is presumed to be similar to that for the discrepancy in $R_{\text{BB}}$ values obtained from observations and numerical calculations. $T_{\text{eff}}$ is obtained from spectral fitting, but the actual spectra suffer from line depletion, especially in the blue. Since $T_{\text{eff}}$ is reduced from the temperature at the photon production site, $L_{\text{bol}}$ is affected by such depletion and is less than the value expected from a blackbody with $T_{\text{eff}}$. Thus, with the smaller $L_{\text{bol}}$, $R_{\text{BB}} = \sqrt{L_{\text{bol}}/4\pi\sigma T_{\text{eff}}^4}$ becomes smaller than the actual emitting region.

As most hydrogen in the CSM remains to be ionized, $R_{\text{BB}}(t=2/3)$ continues to be the radius where the Rosseland mean opacity from the surface of the CSM is 2/3 and remains constant until the shock wave comes close to the radius. Then, $R_{\text{BB}}(t=2/3)$ evolves roughly following the forward shock.

### 5.2.3 $M_{\text{ej}}$ and $E_{\text{ej}}$

The properties of SN ejecta ($M_{\text{ej}}$ and $E_{\text{ej}}$) determine many aspects of SNe powered by shock interaction (e.g. luminosities), because $E_{\text{ej}}$ determines the available energy and $M_{\text{ej}}$ affects the efficiency of conversion of available kinetic energy to radiation energy. We discuss the effect of $M_{\text{ej}}$ and $E_{\text{ej}}$ in Section 6.1, including the results of the $v_i = 5200$ km s$^{-1}$ models, and here we simply show the results of LC calculations with different $M_{\text{ej}}$ and $E_{\text{ej}}$ (Fig. 10). LCs are similar to each other and we can see that it is difficult to constrain $M_{\text{ej}}$ and $E_{\text{ej}}$ only by the LC. This can also be seen by comparing models E1 and E2 in Fig. 3. The two models have different $M_{\text{ej}}$ and $E_{\text{ej}}$ with the same CSM but the resulting LCs are similar (see the discussion in Section 6.1).

In the best LC model of the pulsational pair-instability model presented by Woosley et al. (2007b), ejecta with 5.1 M$_\odot$ and 2.9 $\times$ 10$^{52}$ erg collide with a CSM with 24.5 M$_\odot$. Our canonical models (D2 and F1) have much higher $E_{\text{ej}}$ ($10^{52}$ erg). One of the reasons is presumed to be the smaller photospheric radius ($y_i R_i = 1.1 \times 10^{16}$ cm) in our models. The dense CSM in the pulsational pair-instability model extends to about 3 $\times$ 10$^{16}$ cm and the photosphere can be larger than in our models. This effect of the location of the photosphere can also be seen in comparisons with $v_i = 5200$ km s$^{-1}$
models. The photospheric radii of these models are only \( \gamma R_e \simeq 5 \times 10^{15} \text{ cm} \) at most and the required energy to achieve the maximum luminosity of SN 2006gy is \( 5 \times 10^{52} \text{ erg} \), which is even larger than the value of \( 10^{52} \text{ erg} \) required for the \( v = 10000 \text{ km s}^{-1} \) models \((\gamma R_e = 1.1 \times 10^{16} \text{ cm})\). This shows the difficulties in constraining \( E_{\text{tot}} \) only through the LC.

In addition, the efficiency of conversion of kinetic energy to radiation is mainly determined by the relative mass of the ejecta and the colliding CSM. It does not depend strongly on the ejecta mass if the CSM mass is much larger than the ejecta mass. To get high conversion efficiencies of kinetic energy to radiation energy, \( M_0 \) should be comparable to or less than \( M_{\text{CSM}} \), and we can at least obtain some constraint on \( M_0 \) from the LC based on the viewpoint of conversion efficiency (Section 6.3).

### 5.2.4 Smearing

The dense shell that appears between SN ejecta and the CSM is unstable in the multidimensional case, as is discussed in Section 3.2. As a result of the instabilities, we expect less kinetic energy to be converted to radiation because there will be extra multidimensional motions caused by the instabilities. To take such multidimensional effects into account in the one-dimensional code STELLA, we include a smearing term in the equation of motion (the parameter \( B_q \), Section 3.2).

Fig. 11 shows LCs with different values of the smearing parameter \( B_q \). With larger \( B_q \), the effect of the smearing becomes larger and less kinetic energy is converted to radiation. In other words, radiative cooling becomes less efficient. The model D2 is calculated with our standard \( B_q = 1 \). Model D6 has \( B_q = 0.33 \) and model D7 has \( B_q = 3 \). The shape of the LC is different even if we only change \( B_q \) by a factor of 3. We also show the effect of \( B_q \) on the LCs obtained from the pulsational pair-instability SN models of Woosley et al. (2007b) in Appendix A and show that this effect is not unique to our models. We discuss the efficiency in detail in Section 6.1.

The uncertainty in the smearing parameter adds another difficulty to our estimations of physical parameters of the progenitor system. This is one reason why we think that making the perfect fitting now will not lead us to the exact parameters of the progenitor system. Calibrations for the smearing parameter should be performed, at least. However, the rise time and the peak luminosity are not so sensitive to the smearing parameter and the parameters of SN ejecta and dense CSM obtained with the current uncertainty are presumed to be close to the real ones.

### 5.2.5 Effect of \( ^{56}\text{Ni} \)

We have also examined the effect of \( ^{56}\text{Ni} \) decay on the LCs. Fig. 12 shows the results. We include \( ^{56}\text{Ni} \) at the centre of model D2. If we include \( ^{56}\text{Ni} \), the length of the peak is extended due to the extra heat source. A significant effect can only be seen when we include \( >10 \text{ M}_\odot \) of \( ^{56}\text{Ni} \). However, the amount of \( ^{56}\text{Ni} \) is observationally constrained to be less than \( 2.5 \text{ M}_\odot \) (Miller et al. 2010) and hence the effect of \( ^{56}\text{Ni} \) is negligible.

### 6 DISCUSSION

#### 6.1 Conversion efficiency

The source of radiation in our LC models is the kinetic energy of SN ejecta. The amount of energy converted from kinetic energy to radiation can be estimated by the conservation laws of energy and momentum. If we assume that the radiation pressure does not change the dynamics of the materials very much, the conservation of momentum requires

\[
M_{\text{colej}} v_{\text{colej}} = (M_{\text{colej}} + M_{\text{CSM}}) v_{\text{shell}}, \tag{7}
\]

where \( M_{\text{colej}} \) is the mass of the collided SN ejecta, \( v_{\text{colej}} \) is the mean velocity of the collided SN ejecta, \( M_{\text{CSM}} \) is the mass of the shocked CSM and \( v_{\text{shell}} \) is the velocity of the dense shell between SN ejecta and CSM. The radiation energy \( E_{\text{rad}} \) emitted as a result of the interaction can be derived from the conservation of energy:

\[
E_{\text{rad}} = \alpha \left[ \frac{1}{2} M_{\text{colej}} v_{\text{colej}}^2 - \frac{1}{2} (M_{\text{colej}} + M_{\text{CSM}}) v_{\text{shell}}^2 \right], \tag{8}
\]

where \( \alpha \) is the fraction of kinetic energy converted to radiation. From equations (7) and (8),

\[
\frac{1}{2} M_{\text{colej}} v_{\text{colej}}^2 = \frac{\alpha M_{\text{CSM}}}{M_{\text{colej}} + M_{\text{CSM}}}. \tag{9}
\]
If most of the SN ejecta and CSM is shocked, i.e. \( M_{\text{ej}} \approx M_j \) and \( M_{\text{CSM}} \approx M_{\text{CSM}} \), we obtain a rough estimate for the radiation energy emitted:

\[
E_{\text{rad}} \approx \frac{\alpha M_{\text{CSM}}}{M_j + M_{\text{CSM}}} E_{\text{ej}}. \tag{10}
\]

Here, \( \alpha \) is expected to be close to 1 without the smearing parameter \( B_q \) because most thermal energy gained by the shock is eventually emitted as radiation. Since the parameter \( B_q \) adds additional acceleration to reduce the amount of energy converted to thermal energy, \( \alpha \) is expected to become lower as \( B_q \) becomes larger. The remaining energy is mostly in the form of kinetic energy. We may also express the effect as a reduction of the radiative cooling efficiency because less radiation energy is emitted with the smearing term.

Table 2 is the list of radiation energy obtained by adding up the bolometric luminosity from the time of collision to around 300 d after the collision. Model D3 is excluded because we do not have the entire numerical LC. We also show the parameter \( \alpha \), which is derived by using equation (10). The efficiency \( E_{\text{rad}}/E_{\text{ej}} \) to convert SN kinetic energy to radiation is plotted in Fig. 13 as a function of \( M_{\text{CSM}}/(M_j + M_{\text{CSM}}) \) with the results obtained by van Marle et al. (2010).

In the high \( M_{\text{CSM}}/(M_j + M_{\text{CSM}}) \) region, our standard \( B_q = 1 \) results follow the line of \( \alpha = 0.5 \). This means that the efficiency of conversion of kinetic energy to radiation is reduced by 50 per cent. On the other hand, the results of van Marle et al. (2010) follow the \( \alpha = 1 \) line and the effect of multidimensional instabilities is not significant. Although van Marle et al. (2010) use a three-dimensional code and multidimensional instabilities are included in principle, their approximate way of treating the radiation and limited spatial resolution may have prevented multidimensional instabilities from growing.

As \( M_{\text{CSM}}/(M_j + M_{\text{CSM}}) \) becomes lower, the results start to deviate from the constant \( \alpha \) line. This is because \( M_{\text{CSM}} \) gets very small and the bulk of the ejecta is not affected by the interaction, i.e. the assumption \( M_{\text{ej}} = M_j \) is no longer valid and equation (10) should not be used. We should use equation (8) instead. As \( M_{\text{ej}} \ll M_j \) in this regime, the efficiencies tend to be higher than the values obtained from equation (10).

The combinations of \( E_{\text{ej}} \) and \( M_j \) that give a similar \( [M_{\text{CSM}}/(M_j + M_{\text{CSM}})]^{1/2} M_{\text{ej}} v_{\text{ej}}^2 \) are expected to result in similar LCs and are degenerate. Thus, it is difficult to constrain the exact values for \( M_j \) and \( E_{\text{ej}} \) from LCs. This is clearly seen in models E1 and E2 in Fig. 3. Both models have similar LCs. The CSM of the two models is exactly the same but \( M_j \) and \( E_{\text{ej}} \) are different. Although \( M_{\text{ej}} \approx M_j \) in model E1 and \( M_{\text{ej}} \approx M_j \) in model E2, \( M_{\text{ej}} \) and \( [M_{\text{CSM}}/(M_j + M_{\text{CSM}})]^{1/2} M_{\text{ej}} v_{\text{ej}}^2 \) happen to be similar in the two models, with similar \( M_{\text{CSM}} \approx M_{\text{CSM}} \). Thus, the two models have similar \( [M_{\text{CSM}}/(M_j + M_{\text{CSM}})]^{1/2} M_{\text{ej}} v_{\text{ej}}^2 \) and result in similar LCs.

### 6.2 Origin of the plateau phase

There exists a plateau in the LC of SN 2006gy at around 200 d. None of our models succeeds in producing the plateau. This is because the remaining CSM at these epochs is too thin to affect the LC. Note that the LC observations at later epochs reject the possibility of explaining this plateau by \(^{56}\text{Ni}\) heating (see also Section 5.2.5). There are several other possible ways to explain the plateau. One possibility is recombination in the SN ejecta. Because we use simplified SN ejecta structures, our results of LC calculations after the photosphere gets inside the SN ejecta are beyond the applicability of our simple models. Increasing the SN ejecta mass may also help, because a plateau phase can be longer with larger hydrogen mass, although the conversion efficiency from kinetic energy to radiation is also affected at the same time (Section 6.1). By putting more realistic SN ejecta with realistic hydrogen-rich envelopes or more massive SN ejecta, the recombination wave may stay in the envelope for a while and may end up in the plateau phase, as is the case in Type IIP SNe (see e.g. Kasen & Woosley 2009). We note that the blackbody temperatures of these epochs are \( \approx 6000 \) K and they are consistent with this scenario. Recombination may also occur in the shocked CSM or dense cool shell.

Light echoes from the remaining CSM may also play a role. The LC after this plateau phase remains almost constant for more than 200 d, although the luminosity is about 10 times smaller (Miller et al. 2010). Thus, it is possible that there existed another CSM component that caused the echoes at around 200 d and shocked away when SN 2006gy was behind the Sun.

Smearing may also be relevant to the plateau. In the models in Appendix A, we can see that the plateau can appear or disappear depending on the degree of smearing. Less smearing makes the cool...
within \( gC_m \) CSM (e.g. Smith & Owocki 2006). Their \( W \) is similar to or less than \( M \)⊙, the zero-

ise required to be \( +0, \sim \) erg. Thus, the SN explosion inside \( \sim 1. \) is expected to be \( M \)⊙ or less. This

profiles in SN 2006gy (e.g. \( M \)⊙ erg, the SN ejecta

× \( n \) × \( 50 \) per cent at most (Section 6.1). As the radiation

\[ E \ll M \], and \( \delta \) = \( 1.5 \times 10^{-13} \) g cm\(^{-2}\). Following the result of Chatzopoulos et al. (2012), we put 2 \( M \)⊙ of \( ^{56} \)Ni at the centre of the ejecta but the value is too small to affect the main part of the LC (Section 5.2.5). Except for the central region, the composition is set as the solar metallicity. We use our standard \( B_q = 1. \)

Fig. 14 shows the result of the numerical calculation. Overall, the parameters suggested by Chatzopoulos et al. (2012) do not result in a similar LC to that of SN 2006gy. The peak luminosity of the bolometric LC is close to that of SN 2006gy but the duration is much shorter that that of SN 2006gy. In addition, the \( R \)-band peak luminosity is much smaller than that of SN 2006gy because the

6.4 Comparison with a semi-analytic model

Recently, Chatzopoulos et al. (2012) proposed a semi-analytic model of LCs powered by shock interaction. The model involves several simplifications but the overall features predicted by the model are shown to match some numerical results. They show a LC model of SN 2006gy and we perform LC calculations with the same parameters as obtained by them. The parameters are \( \delta = 0, n = 12, w = 0, E_{\text{ej}} = 4.4 \times 10^{51} \) erg, \( M_{\text{ej}} = 40 \) M⊙, \( R_i = 5 \times 10^{14} \) cm and \( R_e = 2.5 \times 10^{15} \) cm (corresponds to \( M_{\text{CSM}} = 5 \) M⊙ with a constant CSM density \( 1.5 \times 10^{-13} \) g cm\(^{-2}\)).

Figure 14. Observed \( R \)-band LC and the bolometric and \( R \)-band LCs calculated with the parameter shown in Chatzopoulos et al. (2012). The observational points are shifted arbitrarily to match the numerical results in the figure.

6.3 Progenitor of SN 2006gy

As the properties of the progenitor system that can be obtained from the LC modelling strongly depend on the CSM, it is difficult to obtain information on the progenitor from the LC of SN 2006gy. The CSM properties and SN ejecta properties are degenerate. However, we can gain some indications regarding it. As the origin of the luminosity is the kinetic energy of SN ejecta, the kinetic energy should be converted to radiation efficiently. If \( M_{\text{CSM}} \ll M_{\text{ej}} \), the conversion efficiency \( a M_{\text{CSM}}/(M_{\text{ej}} + M_{\text{CSM}}) \) is so small that the kinetic energy cannot be converted efficiently enough to explain the LC of SN 2006gy. Thus, the mass of the CSM should be close to or larger than \( M_{\text{ej}} \).

According to our modelling, \( M_{\text{CSM}} \) is required to be \( \sim 10 \) M⊙ and this means that \( M_{\text{ej}} \) is expected to be \( \sim 10 \) M⊙ or less. This indicates that the total mass of the system well exceeds \( 10 \) M⊙ and the progenitor of SN 2006gy should be a very massive star. In addition, the progenitor should lose \( M_{\text{CSM}} \) within \( \sim 10 \) yr before the explosion. Our models for SN 2006gy have \( M_{\text{CSM}} \simeq 18 \) M⊙ and it may be difficult for red supergiants (RSGs) to have such mass loss for the following reason: to have a CSM with \( 18 \) M⊙, the zero-age main-sequence mass of RSGs should be very large, but such massive stars suffer more from the radiation-driven wind during their main-sequence phase because of their large luminosities. Thus, losing most of their mass only just before their explosions might be difficult. However, extensive mass loss of RSGs is suggested by many authors (e.g. van Loon et al. 2005; Vanbeveren, Van Bever & Belkus 2007; Smith, Hinkle & Ryde 2009; Boyer et al. 2010; Yoon & Cantiiolo 2010; Moriya et al. 2011; Georgy 2012) and it is still possible that very massive RSGs lose \( \sim 10 \) M⊙ just before their explosions due to pulsations (e.g. Li & Gong 1994; Heger et al. 1997; Yoon & Cantiiolo 2010), dust (e.g. van Loon et al. 2005) or g-mode oscillations (Quataert & Shiode 2012, see also Arnett & Meakin 2011).

Another possible progenitor of SN 2006gy is a very massive star in the luminous blue variable (LBV; see e.g. Humphreys & Davidson 1994) phase, as suggested by e.g. Smith et al. (2007). LBVs experience extensive mass loss and some of them, e.g. \( \eta \) Carinae, have \( \sim 10 \) M⊙ CSM (e.g. Smith & Owocki 2006). Their typical wind velocities are also consistent with the wind velocities estimated from the narrow P Cygni H\( \alpha \) profiles in SN 2006gy (e.g. Smith et al. 2010). Although LBVs are theoretically considered to be on the way to Wolf–Rayet stars and hence would not explode (e.g. Crowther 2007; Vink 2009), the progenitor of Type IIn SN 2005gl was found to be an LBV (Gal-Yam et al. 2007; Gal-Yam & Leonard 2009). Several other Type IIn SNe are also suggested to have evidence for LBV progenitors (e.g. Kotak & Vink 2006; Trundle et al. 2008; Smith et al. 2011; Kiewe et al. 2012).

A shell created due to the interaction between the RSG wind and the Wolf–Rayet wind is another possible way to have a massive CSM (e.g. Dwarkadas 2011). Alternatively, a shell created by pulsational instability can be followed up by the SN ejecta, instead of the ejecta of the next pulse as suggested by Woosley et al. (2007b). Some binary interaction may cause extensive mass loss (e.g. Hachisu, Kato & Nomoto 2008) but binary interaction has not been considered deeply as a possible progenitor of SLSNe yet (see Chevalier 2012; Soker 2013). The collision of massive stars in a dense stellar cluster can make a massive star surrounded by a massive CSM and it may also result in SN 2006gy-like SLSNe (Portegies Zwart & van den Heuvel 2007, see also Pan, Loeb & Kasen 2012b).

With the condition that \( M_{\text{ej}} \) is similar to or less than \( M_{\text{CSM}} \), the conversion efficiency of kinetic energy to radiation (equation 9) is expected to be \( \sim 50 \) per cent at most (Section 6.1). As the radiation energy emitted by SN 2006gy exceeds \( 2 \times 10^{51} \) erg, the SN ejecta should have more than \( \sim 4 \times 10^{51} \) erg. Thus, the SN explosion inside should be very energetic. As the energy of our models is comparable to that of energetic broad-line Type Ic SNe, the progenitors of which are suggested to be very massive (e.g. Nomoto et al. 2011), the estimated high energy may also indicate that the progenitor mass is rather close to those of LBVs. Note, however, that the host galaxy of SN 2006gy is not metal-poor (e.g. Ofek et al. 2007), while broad-line Type Ic SNe appear more preferentially in low-metallicity environments (e.g. Arcavi et al. 2010; Modjaz et al. 2011; Sanders et al. 2012). In addition, the late-time spectra of SN 2006gy are not similar to those of broad-line Type Ic SNe (Kawabata et al. 2009), although the late-time spectra of Type Ic SLSN 2010gx show such features (Pastorello et al. 2010).
photospheric temperature is much higher than that of SN 2006gy around the peak and the model has much bluer spectra. The duration can be longer if we use a smaller $B_q$, but our results shown in Fig. 11 imply that it is difficult to make the duration twice as long as the $B_q = 1$ LC to match the LC of SN 2006gy simply by making $B_q$ small. What is more, changing $B_q$ does not improve the colour of the LC and the $R$-band LC is expected to remain much fainter than the observed $R$-band LC. The plateau phase after the drop in the LC is due to recombination in the 40-M$_\odot$ SN ejecta inside. The duration of the plateau phase should actually be much shorter than that in Fig. 14, as the entire SN ejecta is composed of material with solar metallicity in the model (except for the central $56^{\text{Ni}}$) and we use an approximate density structure for the SN ejecta.

There are several possible reasons for the discrepancy. The semi-analytic model assumes that the thermal energy gained by the forward shock is always released at the centre of the CSM because this assumption is required to treat the transport equations analytically. However, this assumption keeps the diffusion time of the photons from the forward shock constant and the diffusion time is fixed at the initial value. In reality, the forward shock travels outward and the diffusion time decreases with time as the remaining unshocked wind decreases. This effect leads to overestimation of the duration of the LCs in the semi-analytic model. This is presumed to be the main reason why the duration of the LC obtained by the numerical calculation is much shorter than that obtained by the analytical model. This indicates that the semi-analytic model should not be applied to a system with a CSM having a large initial diffusion time, in which the kinetic energy of shock waves is the main source of radiation.

Another possible reason is that the energy released by the reverse shock is overestimated in the semi-analytic model. In the semi-analytic model, the self-similar solution of Chevalier (1982) and Nadezhin (1985) is used as the evolution of the hydrodynamical structure. However, in reality the effect of cooling, which is not taken into account in the adiabatic self-similar solution, is so strong in the case of SLSNe powered by shock interaction that a thin cool dense shell is created between the SN ejecta and the dense CSM. Thus, the reverse shock cannot travel as fast as expected from the adiabatic self-similar solution and instead stays close to the forward shock.

In summary, many important effects that are essential in modelling a LC powered by shock interaction between SN ejecta and a dense CSM with a large photon diffusion time are lacking in the semi-analytic model, so it may not be appropriate to use it for the modelling of SLSNe powered by such a strong interaction.

### 6.5 High-redshift Type IIn superluminous supernovae

SLSNe powered by shock interaction are not only brighter but also bluer than other SNe. Thus, SLSNe are expected to be a good probe with which to study the high-redshift Universe, especially as their progenitors are expected to be very massive (Section 6.3): SLSNe can thus provide us with information regarding very massive stars in the early Universe. Some high-redshift Type IIn SNe have already been detected up to $z = 2.36$ (Cooke et al. 2009, see also Barton & Cooke 2010).

With upcoming optical deep surveys with e.g. Subaru/Hyper Suprime-Cam (HSC: Miyazaki et al. 2006), SLSNe powered by interaction can be detected up to $z \sim 5$ (e.g. Cooke 2008; Tanaka et al. 2012) whereas PISNe can be detected up to $z \sim 2$ (e.g. Pan et al. 2012b). Upcoming NIR surveys by e.g. the James Webb Space Telescope (JWST), Euclid$^3$ or the Wide-field Imaging Surveyor for High-redshift (WISH)$^3$ can approach much higher redshifts (e.g. Scannapieco et al. 2005; Hummel et al. 2011; Pan, Kasen & Loeb 2012a; Tanaka et al. 2012). Here, we show how the LC of SN 2006gy is observed if it appears at high redshifts. This can be used for the identification of candidate transients obtained in upcoming optical and NIR surveys.

Fig. 15 shows the $i$-band LCs of models D2 ($w = 5$) and F1 ($w = 0$) for several redshifts. As model F1 has a similar colour to SN 2006gy at around the LC peak, the F1 model provides a better estimate of high-redshift SLSN LCs. Fig. 16 shows the LCs as seen with the F277w filter$^4$, which is one of the wide filters planned for the JWST. The central wavelength of the F277w filter is at 2.77 $\mu$m. The JWST can reach $\sim 30$ mag and is able to detect SLSNe beyond $z = 10$. See Tanaka et al. (2012) for details of the observational strategies and estimated number of detections of SLSNe with upcoming optical and NIR surveys.

---

1. http://www.jwst.nasa.gov/
2. http://sci.esa.int/euclid
3. http://www.wishmission.org/en/index.html
4. http://www.stsci.edu/jwst/instruments/nircam/instrumentdesign/filters/
7 SUMMARY AND CONCLUSIONS

We have shown that interaction between SN ejecta and a dense CSM is a viable mechanism to power SLSNe such as SN 2006gy. The interaction in the dense CSM accounts for the huge luminosity and long duration of the SN 2006gy LC. Shock breakout within the dense CSM is key to our understanding of interaction-powered SLSNe. Our canonical models have $M_{ej} = 20 M_{\odot}$, $E_{ej} = 10^{52}$ erg and $M_{CSM} = 18 M_{\odot}$ ($w = 5$) or $15 M_{\odot}$ ($w = 0$), where the CSM is assumed to have a density profile of $\rho \propto r^{-w}$. The corresponding average mass-loss rate of the progenitor is about $0.4 M_{\odot}$ yr$^{-1}$ if we assume that the dense CSM originates from a 100 km s$^{-1}$ wind. Our steady mass-loss models ($w = 2$) fail to explain the SN 2006gy LC. No $^{56}$Ni is required to explain the early LC of SN 2006gy.

It is difficult to break the degeneracy between $M_{ej}$, $E_{ej}$ and $M_{CSM}$. One can obtain constraints on the progenitor of SN 2006gy based on the efficiency, as the conversion efficiency of SN kinetic energy to radiation becomes high when $M_{ej}$ is comparable to or less than $M_{CSM}$. The progenitor of SN 2006gy should be a very massive star because $M_{CSM} = 18 M_{\odot}$ or $15 M_{\odot}$. As the conversion efficiency is $\lesssim 50$ per cent at most and the radiation energy emitted by SN 2006gy is more than $2 \times 10^{51}$ erg, $E_{ej}$ should be larger than $4 \times 10^{50}$ erg.

We have also examined the effect of multidimensional instabilities in the dense cool shell on the model LCs. Such instabilities are expected to reduce the amount of kinetic energy converted to radiation. Our LC modelling is based on a one-dimensional radiation hydrodynamics code in which multidimensional instabilities are implemented only in an approximate way. We have thus explored the effect qualitatively. Further studies on multidimensional effect(s), perhaps using three-dimensional radiation hydrodynamics simulations, are needed for better understanding of SNe powered by interaction.

Finally, we have provided predictions from our model for high-redshift SLSNe. We have calculated the optical and NIR LC evolution in the observer frame. The results can be used for the identification of SLSNe detected at high redshifts in future transient surveys.

The existence of a very massive CSM close to the progenitor, which is required to explain the LC of SN 2006gy, challenges our current understanding of stellar mass loss. A better understanding of SNe powered by interaction will lead to a better understanding of the mass-loss mechanisms of massive stars. This is a critical key with which to reveal the evolution and fates of massive stars, which are a fundamental component of the Universe.

ACKNOWLEDGMENTS

We thank the anonymous referee for useful comments. TJM is supported by the Japan Society for the Promotion of Science Research Fellowship for Young Scientists (23.5929). SIB is partly supported by the grants of the Government of the Russian Federation (No 11.G34.31.0047), RFBR 10-02-00249, 10-02-01398, by RF Sci. Schools 3458.2010.2 and 3899.2010.2 and by a grant IZ73Z0-128180/1 of the Swiss National Science Foundation (SCOPES). All the numerical calculations were carried out on the general-purpose PC farm at Center for Computational Astrophysics, CICA, of National Astronomical Observatory of Japan. NY acknowledges support by the Grants-in-Aid for Young Scientists (S: 20674003) by the Japan Society for the Promotion of Science. This research is supported in part by a grant from the Hayakawa Satio Fund awarded by the Astronomical Society of Japan. This research is also supported by World Premier International Research Center Initiative, MEXT, Japan.

REFERENCES

Agnoletto I. et al., 2009, ApJ, 691, 1348
Arcavi I. et al., 2010, ApJ, 721, 777
Argo M. K., Beswick R. J., Muxlow T. W. B., Pedlar A., 2007, Astron. Telegram, 1084,
Arnett W. D., Meakin C., 2011, ApJ, 741, 33
 Arkalanov P. V., Blinnikov S. I., Pavlyuk N. N., 2005, Astron. Lett., 31, 429
Bar-El A., Loeb A., 2011, MNRAS, 414, 1715
Barbary K. et al., 2009, ApJ, 690, 1358
Barth A., Rakavy G., Sack N., 1967, Phys. Rev. Lett., 18, 379
Barton E. J., Cooke J., 2010, in Whalen D. J., Bromm V., Yoshida N., eds, AIP Conf. Ser. Vol. 1294, The First Stars and Galaxies: Challenges for the Next Decade. AIP, New York, p. 196
Benedevuto O. G., Lugones G., 1999, MNRAS, 304, L25
Berger E. et al., 2012, ApJ, 755, L29
Bietenholz M., Bartel N., 2007, Astron. Telegram, 1254, 1
Bietenholz M., Bartel N., 2008a, Astron. Telegram, 1525, 1
Bietenholz M., Bartel N., 2008b, Astron. Telegram, 1657, 1
Blinnikov S. I., 2008, in Suda T., Nozawa T., Ohnishi A., Kato K., Fujimoto M.Y., Kajino T., Kubono S., eds, The 10th Meeting on the Next Decade. AIP, New York, p. 196
Blinnikov S. I., Bartunov O. S., 1993, A&A, 273, 106
Blinnikov S. I., Sorokina E. I., 2010, preprint (arXiv:1009.4353)
Blinnikov S. I., Tolstov A. G., 2011, Astron. Lett., 37, 194
APPENDIX A: EFFECT OF THE SMEARING TERM ON LIGHT CURVES OF PULSATIONAL PAIR-INSTABILITY SUPERNOVAE

We show the results of our additional investigations regarding the dependence of the smearing term $B_q$ on the results of LC calculations. We show the results obtained from the pulsational-pair instability SN models presented by Woosley et al. (2007b) and show that the effect of $B_q$ on LCs is similar to that in our LC models. This means that the effect of $B_q$ seen in our models is not unique to our models.

The models presented here are based on the same progenitor model ($M_{ZAMS} = 110 \, M_\odot$) shown in Woosley et al. (2007b). The shell ejected by the first pulsation ($24.5 \, M_\odot$) is caught up by the materials ejected by the next pulsation ($5.1 \, M_\odot$). Woosley et al. (2007b) have artificially increased the kinetic energy of the materials ejected by the second pulsation to obtain a better fit to the LC of SN 2006gy and their best LC model has four times as much kinetic energy as that of the original model. The models shown here have more kinetic energy. The kinetic energy of the second pulse is increased nine times and has a value of $6.5 \times 10^{51}$ erg. The rest of the model is the same as those discussed in Woosley et al. (2007b).

Figs A1, A2 and A3 show the results of LC calculations with different $B_q$. Comparing the results with those shown in Fig. 11, we can see that the dependence of the results on $B_q$ is basically the same. This is because the source of the luminosity in both models is the kinetic energy of material coming from inside and $B_q$ directly affects the conversion efficiency of kinetic energy to radiation energy. For accurate modelling of the LCs resulting from the conversion of kinetic energy to radiation, we need more investigations of multidimensional effects during the interaction.

This paper has been typeset from a TeX/LaTeX file prepared by the author.