Research on Optimization of Phase Design for an Isolated Intersection

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Abstract. Based on the phase conflict matrix and safety principle, a phase design optimization model is proposed to minimize the sum of flow-capacity ratios concerning critical phases. Both dynamic programming method and improved elite multi-population genetic algorithm are used to automatically generate the optimal phase design scheme. Finally, the model and its algorithm are verified by a numerical example, which shows this optimization model can reduce the total flow-capacity ratio of critical phases by 6.5%, compared with the current phase design.

1. Introduction

Signal optimization for the isolated intersection mainly focuses on optimizing cycle length, green signal ratio and phase sequence, but pays less attention to phase design when traffic states vary over time. The unreasonable phase design lowers effectiveness of the subsequent signal timing optimization. Furthermore, signal control of an isolated intersection lays a solid foundation for coordinated signal control for arterial intersections. Therefore, it is crucial to optimize the phase design of an isolated intersection.

Ambili proposed the vertex coloring and binary integer linear programming (BILP) method to find the minimum number of stages in a signal cycle at an isolated intersection, but did not consider designing overlapping phases [1]. Shen developed the improved K-medoids algorithm to optimize the phase and signal timing achieved the saturation balance of each phase at the intersection. However, the object of this study is not an arbitrary intersection [2]. Considering the mutual coupling of phase and sequence, some scholars adopted two-stage method to optimize phase combination and phase sequence [3]. The first stage optimizes the phase combination with the minimum critical flow-capacity ratio and the second stage optimizes the phase sequence for the shortest green light interval. However, the optimization of the phase sequence is achieved by enumerating the phase sequence combination plans that have been obtained in the first stage. Therefore, the calculation efficiency of this method is low, and it is only suitable for typical intersections, not for arbitrary intersections. Similarly, for the purpose of minimizing the average delay of vehicles at the intersection, the authors determined the phase design and green time of each phase by enumerating the limited phase design schemes in the complete signal group [4]. On the other hand, different from the enumeration method, Liang adopted heuristic methods to optimize generic signal phase and timing plans at the signalized intersection based on connected vehicle technology [5]. The results showed that heuristic algorithms provide very similar operational performance as the complete enumeration approach but with significantly reduced computational costs (up to 98% savings). Through the comparison of various control strategies, Cai started with the dual-ring structure in NEMA TS2 (U.S.), redefined signal phases, formulated unified nesting rules, and
finally established the on-line phase transition structure with two patterns [6]. However, that rules did not apply to irregular intersections. Lu developed an intersection signal phase design and timing synchronization optimization model based on the basic phase time allocation model [7], which can handle complex phase design conditions, such as overlapping phases, and the model could optimize phase time, but the phase design schemes were known, and phase optimization was only a selection process.

It is noted that the previous studies rely too much on experience, and pay less attention to overlapping phase design. Accordingly, considering overlapping phases, this paper proposes an intersection phase design optimization model regardless of the right-turn traffic flows, and automatically generates the optimal phase design scheme through the dynamic programming algorithm and improved elite multi-population genetic algorithm.

The paper is organized as follows: Section II establishes the phase design optimization model. Section III designs its optimization algorithm. In section IV, experimental results are provided to verify the effectiveness of our proposed model. Section V concludes this paper.

2. Model Establishment
This section proposes the model to optimize phase design of an isolated intersection, based on phase conflict matrix, phase flow, saturation flow rate and configuration design. The model consists of two parts: its constraints and the associated objective function.

2.1. Model constraints
Intersection phase conflict matrix \( W = (w_{j,v})_{n \times n} \), where \( w_{j,v} \) is the phase conflict element, \( n \) is the number of phases, \( j \) and \( v \) are intersection phase respectively, \( w_{j,v} \) is defined as:

\[
0, 1, \quad j, v \in J, \quad (1)
\]

where \( w_{j,v} = 1 \) indicates phase \( j \) and \( v \) are conflict, otherwise, phase \( j \) and \( v \) are compatible, \( J \) is the set of phase \( j \).

The phase design of the intersection adopts 0-1 planning and decision variable \( z_{ij} \) is defined as follows:

\[
z_{ij} = 0, 1, \quad i \in I, \quad j \in J, \quad (2)
\]

where \( i \) is signal stage, \( I \) is the set of stage \( i \), \( z_{ij} = 1 \) indicates that phase \( j \) belongs to stage \( i \), otherwise, phase \( j \) does not belongs to stage \( i \).

Signal stages at the intersection will easily increase the total loss time when the number of stages is excessive, and few signal stages are not conducive to release all phases Therefore, this model defines at least two and at most \( n - 1 \) signal stages in one cycle:

\[
2 \leq m \leq n - 1, \quad (3)
\]

where \( m \) is the number of stages in the phase design scheme, and \( n \) is the number of phases.

Each signal stage releases at lowest one phase to ensure that stages will not be idle:

\[
\sum_{j=1}^{n} z_{ij} \geq 1, \quad i \in I. \quad (4)
\]

No phase could span the entire cycle, otherwise, green light interval will be non-existent. Without loss of generality, the model requires that each phase must be released once, but only twice at most, as shown in formula (5):

\[
1 \leq \sum_{i=1}^{m} z_{ij} \leq 2, \quad j \in J. \quad (5)
\]

To prevent the entire cycle from being combined by overlapping phases, it is assumed that at least one non-overlapping phase is included in stage one:

\[
z_{i,j}(1 - z_{i,j}) = 1, \quad \forall j \in J. \quad (6)
\]
The overlapping phase could effectively reduce the waste of the green light and increase utilization. If the overlapping phase is set, it must be continuous:
\[
z_u z_v = 0, \text{ if } |u - v| \geq 2, \forall i, u \in I, \forall j \in J,
\]
where \( u \) represents the signal stage.

For safety, it is prohibited to release conflicting phases at the same stage:
\[
w_{ij} w_{vj} (z_u + z_v) = 1, \text{ if } w_{ij} = w_{vj} = 1, \forall i \in I, \forall j, v \in J.
\]

Taking into account the efficiency of vehicle pass through the intersection, non-conflict phases should be released in the same stage as far as possible:
\[
(1 - w_{ij})(1 - w_{vj}) z_u z_v = 1, \text{ if } w_{ij} = w_{vj} = 0, \\
\forall i \in I, \forall j, v \in J.
\]

The variable \( e_p \) is defined to calculate the cycle total flow-capacity ratio for convenience:
\[
e_p = \begin{cases} 
1, & \text{if } j \in p, \forall p \in P \\
0, & \text{otherwise}
\end{cases}
\]
where \( p \) represents a path from stage one to stage \( m \), \( P \) is the set of \( p \), \( e_p = 1 \) indicates that phase \( j \) belongs to the path \( p \), otherwise, phase \( j \) does not belongs to the path \( p \).

### 2.2. Objective function

The objective function of the phase design optimization model at an isolation intersection is to minimize the total flow-capacity ratio of the critical phases:

\[
\min Y = \max \left( \sum_{j \in P} q_j e_p \right) \\
\text{s.t. Eq. (2)—Eq. (11)}
\]

Where \( Y \) is the sum of flow-capacity ratios of critical phases, \( q_j \) is the traffic flow of phase \( j \), and \( s_j \) is saturation flow rate.

In summary, the phase design constraints of formulas (2) to (10) and the objective function of equation (11) are established.

### 3. Model Solution

#### 3.1. Problem difficulty analysis

The phase design problem of an isolated intersection can be abstracted as a permutation and combination problem. In order to facilitate the analysis of the complexity, the problem is simplified. Assuming that the isolated intersection have \( n \) phases and \( m \) signal stages, regardless of overlapping phases, each phase is only released once and the stage is allowed to be empty, hence, there are \( n^m \) phase design schemes from stage 1 to stage \( m \). This is an NP problem, so the original problem is at least an NP problem. For the solution of NP problems, with the increase of the phase number, brute force search is no longer applicable, because it requires extremely large time and space resources, but genetic algorithms can solve it well [5].

Genetic algorithm is inspired by the biological evolution driven by natural selection and mutation. It is suitable for dealing with complex optimization problems that are difficult to be solved by traditional search algorithms. The algorithm starts from a randomly generated initial solution, and iteratively generates new solutions through selection, crossover and mutation operations. Each individual in the population represents a solution to the problem, called a chromosome, whose quality is measured by the fitness function. According to the fitness level, a certain number of outstanding individuals are selected from the previous generation and passed on to the next generation to form a new population. As a positive feedback mechanism and enhanced learning system, it has stronger robustness and parallel
search ability than enumeration method, which can significantly reduce the calculation time [5], because the model established in this paper has a certain search space, so it can be solved by genetic algorithm.

Due to the particularity of phase combination in phase design, the traditional genetic algorithm is easy to destroy the optimal individual in the process of crossover and mutation, resulting in the algorithm cannot converge to the global optimal solution. For purpose of the optimal individual is not destroyed, the improved elite multi-population genetic algorithm is proposed. Elite solutions are selected at each generation and used for reproduction, which are expected to “evolve” toward a near-optimal solution over successive generations.

3.2. Improved elite multi-population genetic algorithm

Fig. 1 shows the solution process of the improved elite multi-population genetic algorithm, where the population diversity operator \( P_v \) is guided by probability selection. If the random number is less than \( P_v \), the population will be reinitialized, and the elite retention strategy will be followed to reproduce the best individuals in the history to the new population, while ensuring that the elite individuals do not perform any crossover and mutation operations in the next iteration. Otherwise, the population is reproduced according to roulette and the elite retention strategy. The improved elite multi-population genetic algorithm inherits the information of the optimal individual in historical populations, retains the characteristics of biological evolution, expands the population diversity and avoids the characteristics of decreasing diversity in the process of traditional population renewal.

3.3. Algorithm design

Step 1: Initialize. Initialize the population size, number of iterations, crossover probability, mutation probability, population diversity operator, individual fitness and the isolated intersection related parameters.

Step 2: Real number coding. The combinations of non-conflicting phases are regarded as genes on the chromosome.

Step 3: Crossover. Exchange the parental chromosomes information, supplement the phase of the intersected individuals, and delete the individuals that do not satisfy with formulas (3)-(5).

Step 4: Mutation. Exchange the genes of chromosomes and delete the individuals that do not satisfy constraints formulas (6)-(7).

Step 5: Calculate the individual’s total flow-capacity ratio through dynamic programming algorithm.

Step 6: Calculate the fitness value and screen the individual with the highest fitness.

Step 7: Update populations according to the \( P_v \) operator.

Step 8: Repeat step 3-step 7 until the iteration terminates.
Step 9: Output the optimal phase design.

4. Numerical experiment and results analysis

4.1. Intersection status

We consider here a signalized intersection with four unique approaches in Shaoxing to verify the effect of the model and algorithm. The intersection configuration is shown in Fig. 2, and the traffic parameters are shown in the TABLE1.

![Figure 2: Layout of the intersection and signal phases](image)

| Parameters       | East approach |  | West approach |  |  |
|------------------|---------------|---|---------------|---|---|
|                  | Straight      | Left | Right | Straight | Left | Right |
| $q/($pcu·h$^{-1}$) | 984           | 120  | 168   | 780     | 120  | 84    |
| $s/($pcu·h$^{-1}$) | 4 842         | 1 450 | —     | 4 773   | 1 520 | —     |

![Figure 3: Current phase design scheme of the intersection](image)

Fig. 2 shows the current phase design scheme of the intersection. In the first stage, the straight phases of the north-south approach are released, then the left-turn phases are released in the second stage, in the third stage, the straight phases of the east-west approach are released and finally release the left-turn phases.

![Figure 3: Current phase design scheme of the intersection](image)

4.2. Optimization results and discussion

In view of the traffic safety, it can be seen from Fig. 2 that the phase conflict matrix $W$ is given as follows:

![Table 1: Traffic parameters of the intersection](image)
On the basis of the optimization model and algorithm, the improved phase design scheme processed by Matlab program is shown in Fig. 4.

Figure 4 Improved phase design scheme of the intersection

Compared with the current phase design scheme, as shown in Fig. 4, Phase 8 overlaps Phases 4 and 7 respectively. The main reason for designing overlapping phase is the unbalanced distribution of traffic flows at the north-south approach. The traffic flows of Phases 7 and 8 were much more than that of Phases 3 and 4. If symmetrical phase design is adopted, the green time of Phases 7 and 8 will be wasted.

### TABLE 2. Critical phases and total flow-capacity ratio

| Critical phases | Total flow-capacity ratio |
|-----------------|---------------------------|
| Status          | 8, 7, 1, 2                | 0.728 |
| Optimization    | 1, 2, 4, 7, 3             | 0.681 |
| Optimization percentage | —                | 6.5    |

After the phase design optimization, the original phase 8 is no longer the critical phase, which is replaced by Phases 3 and 4, and the total flow-capacity ratio reduces from 0.728 to 0.681, namely a decrease of about 6.5 percentage points as shown in TABLE 2. Therefore, our proposed model has improved the current phase design.

5. Conclusions

On basis of the phase conflict matrix at an isolated intersection, a phase design optimization model is established for minimizing the total flow-capacity ratio of critical phases, and then it is solved by dynamic programming algorithm and improved elite multi-population genetic algorithm. Specifically, the dynamic programming algorithm was designed to calculate the total flow-capacity ratio and identify critical phases, and the improved genetic algorithm solves the optimization problem of total flow-capacity ratio.

It should be noted that, the proposed model and algorithm are only focus on an isolated intersection. An important yet challenging task is to extend the proposed methodology to deal with arterials and networks in the future, which is worth studying.

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References

[1] T. A. Ambili, S. D. Kumaravel, M. S. Thilagavathy, R. Ayyagari, “Design of optimal phase plans for isolated intersections using vertex coloring and binary integer linear programming,” 2018 21st International Conference on Intelligent Transportation Systems (ITSC). IEEE, 2018: 3591-3595.

[2] G. J. Shen, X. Y. Zhu, W. Xu, L. F. Tang, X. J. Kong, “Research on phase combination and signal
timing based on improved K-Medoids algorithm for intersection signal control,” Wireless Communications and Mobile Computing, 2020.

[3] L. Nie, W. J. Ma, “Novel model for generation and optimization of signal phase and phase sequence at isolated intersection,” Journal of Jilin University (Engineering and Technology Edition), 2019:1-9.

[4] C. J. Zhai, K. L. Li, J. M. Xu, “Adaptive control of an isolated intersection based on the complete phase group,” 2017 Chinese Automation Congress (CAC). IEEE, 2017: 2808-2813.

[5] X. J. Liang, S. I. Guler, V. V. Gayah, “A heuristic method to optimize generic signal phasing and timing plans at signalized intersections using connected vehicle technology,” Transportation Research Part C: Emerging Technologies, 2020, 111: 156-170.

[6] Y. Cai, X. G. Yang, H. Wang, “A flexible online traffic signal phase switching structure,” Urban Transport of China, 2009, 7(03): 80-85.

[7] K. Lu, X. Tian, G. R. Lin, X. D. Deng, “Simultaneous optimization model of signal phase design and timing at intersection,” Journal of Zhejiang University (Engineering Science), 2020, 43(01): 106-110.