Chiral magnetic effect in the presence of an external axial-vector field

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Abstract

We study the excitation of the electric current of chiral fermions along the external magnetic field, known as the chiral magnetic effect, in the presence of the background axial-vector field. The calculation of the current is based on the exact solution of the Dirac equation for these fermions accounting for the external fields. First, this solution was obtained for massive particles and, then, we consider the chiral limit, which is used in the anomalous current computation. We obtain that, in this situation, the anomalous current does not contain the direct contribution of the axial-vector field. This result is compared with findings of other authors.

1 Introduction

The evolution of chiral charged particles in external fields reveals multiple quantum phenomena. First, we mention the Adler-Bell-Jackiw anomaly [1], which consists in the non-conservation of the axial current in the presence of an external electromagnetic field. This anomaly was shown in Ref. [2] to be closely related to the chiral magnetic effect (the CME), which is the excitation of the electric current of chiral particles $J_{\text{CME}} = \alpha_{\text{em}}(\mu_R - \mu_L)B/\pi$ along the external magnetic field $B$. Here $\alpha_{\text{em}} \approx 1/137$ is the fine structure constant and $\mu_{R,L}$ are the chemical potentials of right and left chiral fermions. One can also mention the chiral vortical effect [3], which is the generation of the anomalous current in a rotating matter. There are active searches for manifestations of the CME in astrophysics and cosmology [4], as well as in accelerator physics [5].

There is an open question on the influence of the external axial-vector field $V_5^{\mu}$ on the magnitude of the anomalous current in the CME. If a homogeneous and isotropic $V_5^{\mu}$ is

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present, the Lagrangian of the interaction of the fermion field $\psi$ with $V_5^\mu$ can be represented as $\mathcal{L}_5 \sim \bar{\psi} \gamma^\mu \gamma^5 \psi V_5^\mu \rightarrow \bar{\psi} \gamma^\mu \psi V_5$, which shows that the chiral imbalance $\mu_5 = (\mu_R - \mu_L)/2$ could be shifted by $V_5$. Here $\gamma^\mu = (\gamma^0, \gamma)$ and $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$ are the Dirac matrices. Thus the CME could contain the contribution of $V_5 \equiv V_5^0$. This idea was recently implemented in Ref. [6]. However the regularization used in Ref. [6] to compute the divergent integrals was ambiguous.

Another possibility for $V_5^\mu$ to affect the CME is the consideration of the polarization operator of a photon in the fermion plasma under the influence of $V_5^\mu$. In this case, the polarization operator could acquire the additional term $\Pi_{\mu\nu} \sim i\varepsilon_{\mu\nu\lambda\rho} V_5^\lambda \rho$, where $k^\rho$ is the photon momentum and $\varepsilon_{\mu\nu\lambda\rho}$ is the antisymmetric tensor. The appearance of this term is equivalent to the excitation of the current $J \sim V_5^\mu B_5$, which would be a direct contribution of $V_5^\mu$ to the CME.

The calculation of the antisymmetric contribution to $\Pi_{\mu\nu}$ was made in Refs. [7, 8] revealing nonzero results. However, a model with $\mathcal{L}_5 \neq 0$ is nonrenormalizable and the result of the calculation of $\Pi_{\mu\nu}$ was shown in Ref. [9] to depends on the regularization scheme applied. Basing on the topology arguments, one can conclude that any corrections to the CME from the photon polarization tensor are forbidden [10]. Nevertheless lattice calculations, performed in Ref. [11], show that the CME can get a contribution from an interfermion interaction.

The aim of this work is to find out whether there is an influence of the axial-vector field on the CME. We shall consider a particular example of this axial-vector field in the form of the electroweak interaction with background matter. We start in Sec. 2 with the motivation for this study. Then, in Sec. 3 we calculate the anomalous current along the external magnetic field basing on the exact solution of the Dirac equation in the external fields. Our results are discussed in Sec. 4.

## 2 Motivation

In this section, we compare the results of different methods for the calculation of $J_{\text{CME}} || B$ in the presence of $V_5^\mu$, which is taken in the form of an electroweak background matter.

The method of the relativistic quantum mechanics was used for the first time to describe the CME in Ref. [12]. The idea of this method is the following. First, one obtains the exact solution of the Dirac equation for a massless particle in an external magnetic field. Then the electric current is computed as

$$J_\chi = q \langle \bar{\psi}_\chi \gamma \psi_\chi \rangle, \quad (2.1)$$

where $\psi_\chi$ is the wave function, obtained in the solution of the Dirac equation and $q$ is the particle charge. The averaging $\langle \ldots \rangle$ in Eq. (2.1) is made over the statistical ensemble. The contribution of any chirality $\chi = L, R$ is accounted for in Eq. (2.1). This method allows one to take into account the contribution of the external field nonperturbatively. Nevertheless, it is restricted to the constant and homogeneous magnetic field.
The application of the relativistic quantum mechanics method for the study of the CME in the presence of the electroweak parity violating interaction was made in Refs. [13,14]. Let us consider a massless electron, electroweakly interacting with nonmoving and unpolarized background matter under the influence of an external magnetic field along the $z$-axis, $B = Be_z$. The Lagrangian for such an electron, described by the bispinor $\psi_e$, has the form,

$$L = \bar{\psi}_e \left[ \gamma_\mu (i\partial^\mu + eA^\mu) - \gamma_0 (V_L P_L + V_R P_R) \right] \psi_e,$$  \hspace{1cm} (2.2)

where $A^\mu = (0, 0, Bx, 0)$ is the vector potential corresponding to the constant and homogeneous magnetic field, $e > 0$ is the elementary charge, $P_L, R = (1 \pm \gamma^5)/2$ are the chiral projectors, and $V_{L,R}$ are the effective potentials of the electroweak interaction of the electron chiral projections with background matter. The explicit form of $V_{L,R}$ is given in Ref. [13] for the case of background matter consisting of neutrons and protons.

The energy spectra of the chiral projections of the electron were found in Refs. [13,14] as

$$E_{L,R} = V_{L,R} + E_0, \quad E_0 = \sqrt{p_z^2 + 2enB},$$  \hspace{1cm} (2.3)

where $p_z$ is the longitudinal momentum along the magnetic field, and $n = 0, 1, \ldots$ is the discrete quantum number. If $n = 0$ in Eq. (2.3), we have obtained in Refs. [13,14] for left electrons,

$$E_{L}^{(n=0)} = V_L + p_z, \quad 0 < p_z < +\infty,$$ \hspace{1cm} (2.4)

and for right particles,

$$E_{R}^{(n=0)} = V_R - p_z, \quad -\infty < p_z < 0.$$  \hspace{1cm} (2.5)

In Eqs. (2.4) and (2.5) we assumed, in the analogy with the situation when $V_{L,R} = 0$, that left and right electrons move in a certain direction along the magnetic field.

Using the spectra in Eqs. (2.3)-(2.5) and the wave functions, found in Refs. [13,14], as well as applying the general expression for the current in Eq. (2.1), one finds that only lowest energy level with $n = 0$ contributes to the current, giving one its nonzero component along the magnetic field as

$$J = \frac{2\alpha_{em}}{\pi} (\mu_5 + V_5) B,$$ \hspace{1cm} (2.6)

where $\alpha_{em} = e^2/4\pi$ is the fine structure constant and $V_5 = (V_L - V_R)/2$ is the contribution of the electroweak interaction.

The result in Eq. (2.6) was criticized in Ref. [15], where it was found that

$$J = \frac{2\alpha_{em}}{\pi} \mu_5 B \equiv J_{\text{CME}},$$ \hspace{1cm} (2.7)

even in the presence of the background electroweak matter, i.e. when $V_{L,R} \neq 0$. The authors of Ref. [15] used the alternative derivation of the CME based on the energy balance in the motion of a massless charged particle in parallel electric and magnetic fields, previously proposed in Ref. [16]. Note that, even if one uses the method of Ref. [16] to derive the CME
in the presence the electroweakly interacting matter and accounts for the dispersion relation in Eqs. (2.4) and (2.5), the expression for the current in Eq. (2.6) can be reproduced \[17\].

The expression for the current in Eq. (2.7) was also derived in Ref. \[18\], where the CME in the presence of the axial-vector field was studied using the chiral hydrodynamics approach, which was developed earlier in Ref. \[19\]. No explicit contribution of the electroweak interaction to the current, like in Eq. (2.6), was found in Ref. \[18\].

The apparent contradiction between the relativistic quantum mechanics method in Refs. \[13,14\] and other approaches \[15,18\] for the description of the CME in the presence of the axial-vector external field requires a special analysis.

### 3 Anomalous current in the presence of the electroweak interaction with matter

To analyze the contribution of the parity violating interaction to the CME, using the relativistic quantum mechanics approach, we shall start with the consideration of massive particles interacting with the axial-vector and magnetic fields and then discuss the chiral limit. As in Sec. 2, we shall discuss the electroweak interaction of an electron with background matter. Since the relativistic quantum mechanics method deals with an exact solution of the Dirac equation an external field, the corresponding solution should be utilized. For the first time the Dirac equation for a massive electron, electroweakly interacting with background matter under the influence of an external magnetic field, was solved in Ref. \[20\]. Then this solution was used in Ref. \[21\] to compute the induced current along the magnetic field.

Thus, instead of the Lagrangian in Eq. (2.2), we shall discuss the following Lagrangian:

\[
\mathcal{L} = \bar{\psi}_e \left[ \gamma_\mu (i \partial^\mu + e A^\mu) - m - \gamma_0 (V_L P_L + V_R P_R) \right] \psi_e, 
\]

where \( m \) is the electron mass. The remaining parameters have the same meaning as in Sec. 2.

Let us look for the solution of the Dirac equation, which results from Eq. (3.1), in the form,

\[
\psi_e = \exp (-i Et + ip_y y + ip_z z) \psi_x ,
\]

where \( \psi_x = \psi(x) \) is the bispinor which depends on \( x \) and \( p_{y,z} \) are the momentum projections along the \( y \)- and \( z \)-axes. We shall choose the chiral representation of the Dirac matrices \[22\],

\[
\gamma^\mu = \begin{pmatrix} 0 & -\sigma^\mu \\ -\bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \sigma^\mu = (\sigma_0, -\sigma), \quad \bar{\sigma}^\mu = (\sigma_0, \sigma),
\]

where \( \sigma_0 \) is the unit \( 2 \times 2 \) matrix and \( \sigma \) are the Pauli matrices. Using Eq. (3.3), we can represent \( \psi_x \) in the form \[21\],

\[
\psi_x^T = (C_1 u_{n-1}, iC_2 u_n, C_3 u_{n-1}, iC_4 u_n),
\]
where $C_i$, $i = 1, \ldots, 4$, are the spin coefficients,
\[ u_n(\eta) = \left( \frac{eB}{\pi} \right)^{1/4} \exp \left( -\frac{\eta^2}{2} \right) \frac{H_n(\eta)}{\sqrt{2^n n!}}, \quad n = 0, 1, \ldots, \] (3.5)
are the Hermite functions, $H_n(\eta)$ are the Hermite polynomials, and $\eta = \sqrt{eBx + p_y}/\sqrt{eB}$.

The energy spectrum for $n > 0$ reads [20, 21]
\[ E = \bar{V} + \lambda \mathcal{E}, \quad \mathcal{E} = \sqrt{(\mathcal{E}_0 + sV_5)^2 + m^2}, \] (3.6)
where $s = \pm 1$ is the discrete quantum number dealing with the spin operator [20], $\mathcal{E}_0$ is defined in Eq. (2.3), $\bar{V} = (V_L + V_R)/2$, and $\lambda = \pm 1$ is the sign of the energy, i.e. the electron energy reads $E_e = E(\lambda = +1) = \mathcal{E} + \bar{V}$, and the positron energy has the form, $E_{\bar{e}} = -E(\lambda = -1) = \mathcal{E} - \bar{V}$. For $n = 0$, one has [21]
\[ E = \bar{V} + \lambda \mathcal{E}, \quad \mathcal{E} = \sqrt{(p_z + V_5)^2 + m^2}. \] (3.7)

Note that, at $n = 0$, there is only one spin state of the electron.

The spin coefficients obey the system [21],
\[ \begin{align*}
(\mathcal{E} \pm p_z \pm V_5) C_{1,3} &+ \sqrt{2} eBnC_{2,4} + mC_{3,1} = 0, \\
(\mathcal{E} \pm p_z \pm V_5) C_{2,4} &+ \sqrt{2} eBnC_{1,3} + mC_{4,2} = 0,
\end{align*} \] (3.8)
where we consider the particle (electron) degrees of freedom, $\lambda = 1$. Since we are mainly interested in the dynamics of electrons at the lowest energy level, we should set $n = 0$ in Eq. (3.8). It results from Eq. (3.4) that, in the situation, $C_1 = C_3 = 0$ to avoid the appearance of Hermite functions with negative indexes.

If, besides setting $n = 0$ in Eq. (3.8), we approach to the limit $m \to 0$ there, one gets
\[ \begin{align*}
(\mathcal{E} + p_z + V_5) C_2 &= 0, \quad \text{or} \quad \begin{cases} 
\mathcal{E} = -p_z - V_5, \\
C_2 \neq 0, \quad \text{and} \quad C_4 = 0,
\end{cases} \quad \text{(3.9)} \\
(\mathcal{E} - p_z - V_5) C_4 &= 0, \quad \text{or} \quad \begin{cases} 
\mathcal{E} = p_z + V_5, \\
C_4 \neq 0, \quad \text{and} \quad C_2 = 0.
\end{cases} \quad \text{(3.10)}
\end{align*} \]

We can see that Eq. (3.9) corresponds to a right electron and Eq. (3.10) to a left one.

The energy spectrum in Eq. (3.7) in the limit $m \to 0$ reads
\[ \mathcal{E} = |p_z + V_5|. \] (3.11)

Comparing Eq. (3.11) with Eqs. (3.9) and (3.10), we obtain that for a right electron
\[ |p_z + V_5| = -p_z - V_5, \quad \text{or} \quad p_z < -V_5, \] (3.12)
and
\[ |p_z + V_5| = p_z + V_5, \quad \text{or} \quad p_z > -V_5, \] (3.13)
for a left particle.

Therefore the total energy of a left electron at the lowest energy level has the form,

\[ E_{eL}^{(n=0)} = V_L + p_z, \quad -V_5 < p_z < +\infty, \quad (3.14) \]

and

\[ E_{eR}^{(n=0)} = V_R - p_z, \quad -\infty < p_z < -V_5, \quad (3.15) \]

of a right particle. Comparing Eqs. (3.14) and (3.15) with Eqs. (2.4) and (2.5), we can see that the form of the spectrum at \( n = 0 \), obtained here, formally coincides with that used in Refs. [13,14]. However, the range of the \( p_z \) variation is different.

To complete the solution of the Dirac equation at \( n = 0 \) and \( m \to 0 \) we should fix the remaining spin coefficients. One gets that

\[ C_2^{(R)} = C_4^{(L)} = \frac{1}{2\pi}, \quad C_2^{(L)} = C_4^{(R)} = 0, \quad (3.16) \]

which results from the normalization condition

\[ \int d^3x \psi_{\bar{e}p_{y,n}p_{z,n'}}^\dagger \psi_{\bar{e}p_{y,n}p_{z,n'}} = \delta (p_y - p'_y) \delta (p_z - p'_z) \delta_{nn'}, \quad (3.17) \]

of the total wave function.

The wave function of a positron can be obtained from Eqs. (3.2) and (3.4) by applying the charge conjugation \( \psi_{\bar{e}} = i\gamma^2 \psi_e^* \) and setting \( \lambda = -1 \) in Eq. (3.6). Finally one has

\[ \psi_{\bar{e}}^T = \exp(-iE_{\bar{e}}t - ip_y y - ip_z z) \times (-iC_4u_n, -C_3u_{n-1}, iC_2u_n, C_1u_{n-1})^T, \quad (3.18) \]

where the coefficients \( C_i \) obey the system in Eq. (3.8).

If \( n = 0 \), we obtain on the basis of Eqs. (3.18) and (3.7) that

\[ \psi_{\bar{e}R}^{(n=0)} = \exp(-iE_{\bar{e}R}t - ip_y y - ip_z z) \times \frac{i\mu_0}{2\pi} (-1, 0, 0, 0)^T, \quad (3.19) \]

where

\[ E_{\bar{e}R}^{(n=0)} = p_z - V_R, \quad -V_5 < p_z < +\infty, \quad (3.20) \]

is the energy of right positrons at the lowest energy level. For left positrons one has

\[ \psi_{\bar{e}L}^{(n=0)} = \exp(-iE_{\bar{e}L}t - ip_y y - ip_z z) \times \frac{i\mu_0}{2\pi} (0, 0, 1, 0)^T, \quad (3.21) \]

where

\[ E_{\bar{e}L}^{(n=0)} = -p_z - V_L, \quad -\infty < p_z < -V_5, \quad (3.22) \]

is the energy of left positrons at the lowest energy level. The positron wave functions in Eqs. (3.19) and (3.21) satisfy the normalization condition in Eq. (3.17).
Figure 1: The energy spectrum of massless left and right electrons/positrons at the lowest energy level with \( n = 0 \) electroweakly interacting with background matter. This plot corresponds to Eqs. (3.14), (3.15), (3.20), and (3.22).

The energy spectrum for electrons and positrons at the lowest energy level is shown in Fig. 1. One can see that there is no gap between the dispersion relations of left and right electrons/positrons, predicted in Refs. [13,14,17], i.e. the assumption that \( E_{\text{Lmin}}^{(n=0)} \neq E_{\text{Rmin}}^{(n=0)} \) at \( p_z = 0 \) (see Eqs. (2.4) and (2.5)) is incorrect. In the presence of the electroweak matter, the spectrum of massless electrons/positrons with \( n = 0 \) is parallel transported to the point \( (p_z = -V_5, E = \bar{V}) \) for electrons and to \( (p_z = -V_5, E = -\bar{V}) \) for positrons from the point \( (p_z = 0, E = 0) \) corresponding to the vacuum case.

According to Eq. (2.1), the contributions of left and right electrons at the lowest energy level to the current are

\[
J_{\text{eL,R}}^{(n=0)} = -e \int dp_y dp_z \bar{\psi}_{\text{eL,R}} \gamma \psi_{\text{eL,R}} f(E_{\text{eL,R}}^{(n=0)} - \mu_{\text{L,R}}),
\]

where \( f(E) = [\exp(\beta E) + 1]^{-1} \) is the Fermi-Dirac distribution function, \( \beta = 1/T \) is the reciprocal temperature, \( \mu_{\text{L,R}} \) are the chemical potentials of left and right particles. First we notice that the components of the current, transverse with respect to \( \mathbf{B} \), are vanishing. Performing the integration over \( -\infty < p_y < +\infty \) and accounting for Eqs. (3.14)-(3.16), on the basis of Eq. (3.23) we obtain the expression for the total current of electrons \( J_{\text{e}}^{(n=0)} = J_{\text{eL}}^{(n=0)} + J_{\text{eR}}^{(n=0)} \) at \( n = 0 \),

\[
J_{\text{e}}^{(n=0)} = \frac{e^2 B}{(2\pi)^2} \left[ \int_{-V_5}^{+\infty} dp_z f(p_z + V_L - \mu_L) - \int_{-\infty}^{-V_5} dp_z f(-p_z + V_R - \mu_R) \right] = \frac{e^2 B}{(2\pi)^2} \int_0^{+\infty} dp \left[ f(p + \bar{V} - \mu_L) - f(p + \bar{V} - \mu_R) \right].
\]

Analogously to Refs. [13,14] one can show that higher energy levels with \( n > 0 \) do not contribute to the current. Thus we shall omit the superscript in Eq. (3.24) for brevity.
The positron contribution to the current $J_{\bar{e}}$ can be obtained analogously to Eq. (3.23) as
\[ J^{(n=0)}_{\bar{e}L,R} = e \int dp_y dp_z \bar{\psi}_{\bar{e}L,R} \gamma \psi_{\bar{e}L,R} f(E_{\bar{e}L,R}^{(n=0)} + \mu_{L,R}). \] (3.25)

Using Eqs. (3.19)-(3.22), we obtain on the basis of Eq. (3.25) the total contribution of positrons at the lowest energy level to the current in the form,
\[ J^{(n=0)}_{\bar{e}} = e^2 B \left( \frac{2}{\pi} \right)^2 \int_{-V_5}^{+\infty} dp \left[ f(p + \bar{V} \pm \mu_L) - f(p - \bar{V} \pm \mu_L) \right] \]
\[ = e^2 B \left( \frac{2}{\pi} \right)^2 \int_{-\infty}^{+\infty} dp \left[ f(p - \bar{V} \pm \mu_R) - f(p + \bar{V} \pm \mu_R) \right]. \] (3.26)

It should be noted that higher energy levels do not contribute to the current.

Using Eqs. (3.24) and (3.26), we obtain that the total current $J = J_e + J_{\bar{e}}$ reads
\[ J = \frac{e^2 B}{(2\pi)^2} \int_{-\infty}^{+\infty} dp \left[ f(p + \bar{V} - \mu_L) - f(p - \bar{V} + \mu_L) - f(p + \bar{V} + \mu_R) + f(p - \bar{V} + \mu_R) \right] \]
\[ = \frac{2\alpha_{em}}{\pi} \mu_5 B, \] (3.27)
which is in agreement with Eq. (2.7).

4 Discussion

In the present work, we have elaborated the improved derivation of the anomalous current of massless charged fermions, interacting with an axial-vector field under the influence of the external magnetic field, induced along the magnetic field. We have chosen a particular example of the axial-vector field as the electroweak interaction of an electron with non-moving and unpolarized background matter. Unlike Refs. [15,18], here we have used the method of the relativistic quantum mechanics, originally proposed in Ref. [12] to describe the CME.

Using the exact solution of the Dirac equation, found in Refs. [20,21], we have shown in Sec. 3 that the axial-vector field does not contribute to the current $J \parallel B$; cf. Eq. (3.27). The value of the current coincides with the prediction of the CME in Eq. (2.7) even in the case when chiral fermions electroweakly interact with background matter, confirming the findings of Refs. [15,18].

To obtain this result in frames of the relativistic quantum mechanics one has to consider the solution of the Dirac equation for a massive electron in the external fields and then approach to the limit $m \to 0$. If one sets $m = 0$ in the Dirac equation from the very beginning, i.e. if one considers the chiral Lagrangian in Eq. (2.2), one obtains the current in Eq. (2.6) as in Refs. [13,14], which is inconsistent with the results of Refs. [15,18]. Thus we conclude that the system of chiral fermions, where the external axial-vector field is present, can be prepared in two non-equivalent ways. This fact is reflected in Fig. 2.
It is also interesting to notice that, at $n = 0$, particles of a certain chirality, say left-handed, are indirectly affected by the parameters corresponding to the opposite (i.e., right-handed) chirality. It can be seen in Eqs. (3.14) and (3.15). Indeed, if one adiabatically changes $V_L$, not only $E_L^{(n=0)}$ but also $E_R^{(n=0)}$ will be modified since the range of the $p_z$ variation in $E_R^{(n=0)}$ depends on $V_5$. One would expect that left- and right-handed electrons behave totally independent for purely massless particles.

The effect of the change of the particle momentum in the presence of the electroweak interaction (see Eq. (3.11)) was known previously. In case when a particle moves in a background electroweak matter, both particle energy and its momentum acquire the contributions $\sim G_F$, where $G_F$ is the Fermi constant. It results, e.g., in the appearance of the ponderomotive force in the situation of the anisotropic matter with inhomogeneous density [23].

Finally, we mention that one does not need involve the concept of the the Cern-Simons current [24], as suggested in Ref. [15], to reconcile the results for the derivation of the CME in the presence of the axial-vector field based on the relativistic quantum mechanics [12] and the energy balance arguments [16]. We can obtain the coinciding results just by using the correct energy spectrum of massless electrons at the lowest energy level; cf. Eqs. (3.14) and (3.15).

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