Break-up mechanisms in heavy ion collisions at low energies

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We investigate reaction mechanisms occurring in heavy ion collisions at low energy (around 20 MeV/u). In particular, we focus on the competition between fusion and break-up processes (Deep-Inelastic and fragmentation) in semi-peripheral collisions, where the formation of excited systems in various conditions of shape and angular momentum is observed. Adopting a Langevin treatment for the dynamical evolution of the system configuration, described in terms of shape observables such as quadrupole and octupole moments, we derive fusion/fission probabilities, from which one can finally evaluate the corresponding fusion and break-up cross sections. The dependence of the results on shape, angular momentum and excitation energy is discussed.

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I. INTRODUCTION

Nuclear reactions between medium-mass nuclei at low energies (around 20 MeV/u) offer the possibility to investigate several aspects of dissipative mean-field dynamics and to probe nuclear matter under extreme conditions with respect to shape, spin and excitation energy. In this energy domain, well above the Coulomb barrier but below the Fermi energies, one essentially observes two types of reaction mechanisms: Fusion dominates in the case of central and semi-peripheral collisions, while binary reseparation processes, associated with deep-inelastic or fast fission mechanisms, essentially involve the remaining range of (semi-)peripheral reactions. However, along the transition from fusion to binary processes, composite systems with rather elongated shape and large intrinsic angular momentum can be formed, corresponding to metastable (or even unstable) conditions, where mean-field fluctuations may play a decisive role in determining the final outcome. The presence of large event by event variances related to the onset of new instabilities have been already noted in experiments, from the anomalous distribution of primary fragment properties in binary events. The observed variances (in mass, charge, excitation energy, angular distribution) appeared much larger than the ones predicted by mean-field nucleon exchange models. Similar conclusions were reached in theory simulations based on stochastic transport models.

Interaction times are quite long and a large coupling among various mean-field modes is expected, leading to a co-existence of the different reaction mechanisms in semi-central collisions. The study of the competition between fusionlike and binarylike processes and, more generally, of the fate of the hot nuclear residues created in these reactions is a longstanding problem, from which one can learn a lot about mean-field dynamics and fundamental properties of nuclear forces. This issue has recently found a renewed interest, due to the possibility to perform new analyses involving neutron-rich or even exotic systems. In these conditions the reaction mechanism characterizing dissipative collisions is expected to be sensitive to the density dependence of the isovector part of the nuclear interaction, a matter that is largely debated nowadays.

In reactions involving medium-heavy nuclei, as a result of the complex neck dynamics, one can also observe, in sufficiently inelastic collisions, new modes of reseparation of the colliding system, such as dynamical ternary breaking, with massive fragments nearly aligned along a common separation axis. Experimental evidences of this mechanism have been recently reported in the case of \(^{197}\text{Au} + ^{197}\text{Au}\) collisions at 15 MeV/u, where also aligned quaternary breaking has been observed. These effects could still be explained in terms of the persistence of the excitation of shape and rotational modes in the projectile-like(PLF) and/or target like(TLF) fragments that are formed in binarylike events, that would lead to further reseparation along a preferential axis, similarly to what happens in fast-fission processes of PLF or TLF. It is worth mentioning that, at higher beam energy (around 40 MeV/u), where apart from mean-field effects two-body correlations are important, ternary breakings become the dominant process and new features are observed, corresponding to the emission of small fragments coming directly from the strongly interacting neck region. Actually one may think in terms of a smooth transition between the different decay modes of PLF and/or TLF, from fast fission, characterized by the splitting into fragments with similar size and small relative velocity, to neck emission, where small fragments are emitted with larger relative velocity with respect to PLF and TLF.

From the above discussion, it is clear that the understanding of the competition between reaction mechanisms in dissipative collisions, as well as of the nature of new exotic reseparation modes, requires a thorough analysis of the underlying mean-field dynamics and associated shape fluctuations and rotational effects. In this paper, we attempt to improve the dynamical description of low energy collisions by coupling a microscopic transport approach based on mean-field concepts, suitable to follow the early stage of the collision up to the formation...
of composite excited sources, to a more refined treatment of the dynamics of shape observables, including the associated fluctuations within the Langevin scheme [11], for the following evolution up to the definition of the final outcome. In particular, we will discuss the dynamics of excited sources characterized by given values of quadrupole and octupole moments and intrinsic angular momentum. This allows one to investigate the competition between fusionlike and binarylike reaction mechanisms and to evaluate fusion cross sections, as well as the probability and the features of fast-fission processes of PLF (or TLF). The paper is organized as it follows: In Section 2 we present the hybrid transport treatment employed to follow the dynamical evolution of the system. Results concerning the competition between fusion and binary processes are discussed in Section 3. Finally conclusions and perspectives are drawn in Section 4.

II. SIMULATION OF THE COLLISIONAL DYNAMICS

A. Dynamical description of nuclear reactions

The evolution of systems governed by a complex phase space can be described by a transport equation, of the Boltzmann-Nordheim-Vlasov (BNV) type, with a fluctuating term, the so-called Boltzmann-Langevin equation (BLE) [12, 13]:

\[
\frac{df}{dt} = \frac{\partial f}{\partial t} + \{f, H\} = I_{\text{coll}}[f] + \delta I[f],
\]

where \(f(r,p,t)\) is the one-body distribution function, or Wigner transform of the one-body density, \(H(r,p,t)\) the mean field Hamiltonian, \(I_{\text{coll}}\) the two-body collision term (that accounts for the residual interaction) incorporating the Fermi statistics of the particles, and \(\delta I[f]\) the fluctuating part of the collision integral. The nuclear EOS, directly linked to the mean-field Hamiltonian \(H\), can be written as:

\[
E/A(\rho, I) = E_s/A(\rho) + C_{\text{sym}}(\rho) I^2 + O(I^4),
\]

where \(I = (N-Z)/A\) is the asymmetry parameter. We adopt a soft isoscalar EOS, \(E_s/A(\rho)\), with compressibility modulus \(K = 200\) MeV, which is favored e.g. from flow studies [14]. For the density \(\rho\) dependence of the symmetry energy, \(C_{\text{sym}}(\rho)\), we consider a linear increase of the potential part of the symmetry energy with density (asystiff):

\[
C_{\text{sym}}(\rho) = a \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3} + b \cdot (\rho/\rho_0),
\]

where \(\rho_0\) is the saturation density, \(a = 13.4\) MeV and \(b = 18\) MeV. From the expression of the energy density, Eq.(2), the mean-field potential is directly derived. The free energy- and angle-dependent nucleon-nucleon cross section is used in the collision integral [15].

Within such approach, the system is described in terms of the one-body distribution function \(f\), but this function may experience a stochastic evolution in response to the action of the fluctuating term \(\delta I(f)\).

However, the numerical resolution of the full BL equation is not available yet in 3D. Approximate treatments to the BLE have been introduced so far, see Refs. [15, 16], such as the Stochastic Mean Field (SMF) model, that consists in the implementation of stochastic density fluctuations only in coordinate space and can be solved numerically using the test particle method [13]. The latter approach has shown to be particularly appropriate for the description of the evolution of the dilute unstable sources that develop in dissipative collisions at Fermi energies (30-100 MeV/u [17]). However, here we are essentially interested in semi-central reactions at lower energies where, most likely, the formation of elongated (rather than dilute) systems is observed, and phenomena associated with surface (rather than volume) metastability and/or instability may take place. To improve the treatment of fluctuations suitable to describe the latter scenario, we will adopt a hybrid description of the dynamics: We follow the microscopic SMF evolution until the time instant when local thermal equilibrium is established and one observes the formation of quasi-stationary elongated systems, with density close to the normal value. Then, to deal with the following evolution of the system, we move to a more macroscopic model description, where the system is characterized in terms of global observables, for which the full treatment of fluctuations in phase space is numerically affordable, as explained below.

B. Dynamical evolution of shape observables

This Section is devoted to the description of the dynamical evolution of excited systems whose leading degrees of freedom are shape observables, while the density keeps always close to the normal value, \(\rho_0 = 4.10^{-26}\), being \(r_0\) the nuclear radius constant \((r_0 = 1.2\) fm\). The configuration of the system under study, having given charge \(Z\) and mass \(A\), is described by three global observables (and associated velocities): the quadrupole moment \(\beta_2\), the octupole moment \(\beta_3\) and the rotation angle \(\omega\). For situations far from the spherical shape the thermal agitation can induce fluctuations that may eventually lead to break-up channels. Hence the correct treatment of shape fluctuations is crucial for the characterization of the reaction mechanism. To this purpose, we consider the stochastic extension of the Rayleigh-Lagrange equations of motion [18] (the Langevin equation):

\[
\frac{d}{dt} \frac{\partial L}{\partial q_i} + \frac{\partial F}{\partial q_i} = \frac{\partial L}{\partial \dot{q}_i} + F_{\text{fluc}}(t),
\]

where \(q_i(i = 1, 2, 3) = (\omega, \beta_2, \beta_3)\), \(L(q_i, \dot{q}_i) = E_{\text{kin}}(q_i, \dot{q}_i) + E_{\text{rot}}(q_i, \dot{q}_i) - E_{\text{pot}}(q_i)\) denotes the La-
is the Rayleigh dissipation function. \( E_{\text{kin}}, E_{\text{rot}} \) and \( E_{\text{pot}} \) indicate the kinetic, rotational and potential energy of the system, respectively, and the quantity \( R_{ij} \) is the dissipation tensor. The difference with respect to the standard Rayleigh-Lagrange equations is the fluctuation term \( F_{\text{fluc}} \), that can be interpreted as a rapidly fluctuating stochastic force, in the same spirit of the Brownian motion, similar to the fluctuating term of the BLE, Eq. (1).

We solve numerically the set \( \mathbf{4} \) of coupled equations.

For given values of the quadrupole and octupole moments, the shape of the system is parametrized, in terms of the polar angle \( \theta \), as it follows:

\[
R(\theta) = R_0(\beta_2, \beta_3)\{1 + \beta_1(\beta_2, \beta_3)Y_{10}(\theta) + \beta_2Y_{20}(\theta) + \beta_3Y_{30}(\theta)\},
\]

where the functions \( Y_{ij}(\theta) \) are spherical harmonics. The parameters \( \beta_1 \) and \( R_0 \) are introduced to conserve the position of the center of mass and the total volume \( V \) of the system and can be determined from the equations:

\[
\int dV = \frac{2\pi}{4} \int_0^\pi R^4(\theta) \sin \theta \cos \theta \, d\theta = 0,
\]

\[
\int dV = \frac{2\pi}{3} \int_0^\pi R^3(\theta) \sin \theta \, d\theta = \frac{4}{3} \pi r_0^3 A,
\]

where \( z \) denotes the coordinate along the system maximum elongation axis (or symmetry axis). In the following we discuss in detail the derivation of the different terms of the Lagrangian \( L \).

1. Rotational energy

The rotational energy is simply equal to:

\[
E_{\text{rot}} = \frac{1}{2} I(\beta_2, \beta_3) \dot{\omega}^2,
\]

where

\[
I(\beta_2, \beta_3) = \frac{\pi m r_0^2}{5} \int_0^\pi R^5(\theta) \{1 + \cos^2\theta\} \sin \theta \, d\theta
\]

is the moment of inertia for the whole system, being \( m \) the nucleon mass.

2. Kinetic energy

The kinetic energy can be expressed as it follows:

\[
E_{\text{kin}} = \frac{1}{2} \sum_{i,j=2}^3 M_{ij}(\beta_2, \beta_3) \dot{q}_i \dot{q}_j
\]

To calculate the mass tensor \( M_{ij} \), we adopt the prescriptions of Ref. [19]:

\[
M_{ij} = \frac{1}{2}(M'_{ij} + M'_{ji})
\]

with

\[
M'_{ij} = 2\pi m r_0 \int_0^\pi \sum_{l=1}^L b_{ij} R^{l+2}(\theta) P_l(\cos \theta) \left( \frac{\partial R_0}{\partial \beta_j} S + R_0 \left( \frac{\partial \beta_1}{\partial \beta_j} Y_{10} + \frac{\partial \beta_2}{\partial \beta_j} Y_{20} \right) \right) \sin \theta \, d\theta.
\]

Here \( P_l \) are Legendre polynomials and \( S(\theta) = \frac{R(\theta)}{R(0)} \). In our calculations we have \( L = 5 \). The coefficients \( b_{2l} \) and \( b_{3l} \) are obtained solving the system of equations:

\[
\sum_{l=1}^L A_{kl} b_{ml} = C_{mk} \quad k = 1 \ldots L, \quad m = 2, 3
\]

with

\[
A_{kl} = \int_0^\pi R^{l-1}(\theta) \left\{ P_l(\cos \theta) - \frac{1}{R(\theta)} \frac{\partial R(\theta)}{\partial \theta} \frac{\partial P_l(\cos \theta)}{\partial \theta} \right\} \sin \theta \, d\theta,
\]

\[
C_{mk} = \int_0^\pi R^{k-1}(\theta) \left\{ k P_k(\cos \theta) - \frac{1}{R(\theta)} \frac{\partial R(\theta)}{\partial \theta} \frac{\partial P_k(\cos \theta)}{\partial \theta} \right\}
\]

\[
\cdot \left\{ \frac{\partial R_0}{\partial \beta_m} S + R_0 \left( \frac{\partial \beta_1}{\partial \beta_m} Y_{10} + \frac{\partial \beta_2}{\partial \beta_m} Y_{20} \right) \right\} \sin \theta \, d\theta.
\]

3. Potential energy: Nuclear term

Concerning the nuclear part of the potential energy, \( E_n \), we discuss essentially the surface contribution, since our system keeps a volume constant in time. We adopt a double volume integral of the Yukawa-plus-exponential folding function [20]:

\[
E_n = -\frac{a_s(1 - k_s T^2)}{8\pi^2 r_0^3 a^3} \int_V \int_{\sigma = 2} \left( \frac{\sigma}{a} - 2 \right) e^{-\sigma/a} d^3r \, d^3r',
\]

where \( a_s \) is the surface-energy constant, \( k_s \) is the surface-asymmetry constant and \( a \) is the range of the Yukawa-plus-exponential potential. \( \sigma \) denotes the modulus of the relative distance \( \sigma = |r - r'| \). Parameters have been fitted to the ground-state energies and fission barrier heights [21, 22]. In order to reduce the numerical efforts, the integral of Eq. (17) can be transformed into a double surface integral, by using the twofold Gauss divergence theorem. For axially symmetric shapes, one of the azimuthal
integrations can be performed trivially \[20, 23\] and the resulting threefold integral is:
\[
E_n = \frac{a_s(1-k_s f^2)}{4\pi r_0^2} \int \int \int \left\{ 2 - \left[ \left( \frac{\sigma}{a} \right)^2 + 2 \frac{\sigma}{a} + 2 \right] e^{-\sigma/a} \right\}
\]
\[
\times \frac{P(\theta, \theta', \phi)P(\theta', \theta, -\phi)}{\sigma^4} d\theta d\theta' d\phi,
\]
where the distance \(\sigma\) can be expressed as:
\[
\sigma = \left[ R^2(\theta) + R^2(\theta') - 2R(\theta)R(\theta') \right]^{1/2}
\]
\[
P(\theta, \theta', \phi) = R(\theta) \sin \theta \left\{ R^2(\theta) - R(\theta)R(\theta') \right\} \cos \theta \cos \theta' +
\]
\[
+ \sin \theta \sin \theta' \cos \phi - R(\theta') \frac{\partial R(\theta)}{\partial \theta} \left\{ \sin \theta \cos \theta' - \cos \theta \sin \theta' \cos \phi \right\}
\]
and
\[
\frac{\partial R(\theta)}{\partial \theta} = \frac{\sigma}{2} \left( \frac{\sigma}{a_C} - 5 + \left[ \frac{1}{2} \left( \frac{\sigma}{a_C} \right)^2 + \frac{3 \sigma}{a_C} + 5 \right] \right)
\]
\[
\times e^{-\sigma/a_C} \frac{P(\theta, \theta', \phi)P(\theta', \theta, -\phi)}{\sigma^4} d\theta d\theta' d\phi,
\]
where \(a_C\) is the range parameter of the Yukawa function generating the diffuse charge distribution \[23, 25\].

5. Dissipation function

The one-body dissipation mechanism is evaluated as it follows (see Ref. \[18\] for details):
\[
\frac{dE}{dt} = m\rho v \int \hat{n}^2 dS
\]
where the integration is performed over the whole surface of the system, \(\hat{n} = \frac{\nabla R}{|\nabla R|}\) is the average nucleon velocity and
\[
\hat{n}^2 = \frac{\left| \frac{\partial R}{\partial \theta} \right|^2}{|\nabla R|^2}, \quad R = r - R(\theta)
\]
Hence we get the following expressions for the dissipation tensor \(R_{ij}\):
\[
R_{ij} = 2[j-i] \pi m\rho_0 \bar{v}
\]
\[
\times \int_0^\pi \left\{ \frac{\partial R}{\partial \beta_j}Y_{10} + Y_{10} \right\} \left\{ \frac{\partial R}{\partial \beta_j}Y_0 + \frac{\partial R_0}{\partial \beta_j}S \right\} \left(2\theta\right) \sin \theta d\theta
\]

6. The Langevin term

The stochastic force \(F_{fluc}(t)\) will determine fluctuations in momentum space, according to the value of the diffusion coefficient \(D\). We assume that
\[
\langle F_{fluc}(t)F_{fluc}(t+s) \rangle = D\delta(s)
\]
The action of the stochastic force \(F_{fluc}\) may be simulated numerically by repeatedly producing a random kick \(\delta P\) in the collective velocity associated with the quadrupole and octupole moments. The value of \(\delta P\) is chosen randomly from a Gaussian distribution with mean value and variance given by:
\[
\langle \delta P \rangle = 0
\]
\[
\langle \delta P \rangle^2 = D\delta t
\]
where \(\delta t\) is the small time step between two kicks. The diffusion coefficient \(D\) can be found using the Einstein relation:
\[
D = 2T \gamma,
\]
where \(\gamma\) is the dissipation coefficient and \(T\) is the temperature of the system \[20\]. Hence the fluctuations that we are considering are induced essentially by the thermal agitation. We notice that our dissipation tensor \(R_{ij}\), introduced above, has also nondiagonal terms. Hence, to correctly extract the dissipation coefficients, we diagonalize the dissipation tensor \(R_{ij} \rightarrow \gamma_{ij}\). The tensor \(\gamma_{ij}\)
will have only diagonal elements: \( \gamma_2 \) and \( \gamma_3 \). Now we can find \( D_2 \) and \( D_3 \) in the new coordinate system and evaluate \( \delta P_2 \) and \( \delta P_3 \), the random kicks for the new coordinates. Finally it is possible to go back to the general coordinates \( \beta_2 \) and \( \beta_3 \), by the inverse transformation, and obtain \( \delta P_{\beta_2} \) and \( \delta P_{\beta_3} \).

### III. RESULTS

We will exploit the Langevin treatment outlined above to investigate the competition between (incomplete) fusion and binary break-up mechanisms in low energy reactions. We consider the system \(^{36}\text{Ar} + ^{96}\text{Zr}\) at two beam energies, 9 and 16 MeV/u, in the following range of impact parameters: \( b = 5-7 \) fm and \( b = 4-6 \) fm at 9 and 16 MeV/u, respectively. Within this selection, according to the SMF dynamical evolution, one observes the formation of rather elongated configurations for which fluctuations are expected to be crucial in determining the following evolution. For lower impact parameters, the conditions of the reactions are such that one always obtains incomplete fusion, while for larger impact parameters binary break-up is observed. Contour plots of the density in the reaction plane, as obtained in the SMF calculations, are displayed in Figs.1-2, for the two reactions.

The description of the system in terms of the global observables \( \beta_2, \beta_3 \) and \( \omega \) begins at the moment when, according to the full SMF evolution, the composite system reaches a quasi-stationary shape, having dissipated almost completely the radial part of the kinetic energy deposited into the system, while the angular part is converted into intrinsic spin. This time instant is estimated to be around \( t_{freeze-out} \approx 200 \) fm/c. During the earlier dynamical evolution, pre-equilibrium nucleon emission takes place. As a consequence, mass and charge of the system are smaller than the total mass and charge numbers, respectively. We get \( A \approx 122, Z \approx 53 \). As one can see from Figs.1-2, the system configuration can be suitably parametrized in terms of quadrupole and octupole moments. From this point of view, the Langevin treatment introduced above appears appropriate to describe the following evolution, though the dynamical description is devolved to few leading degrees of freedom. The initial conditions of the Langevin equation have been determined running 10 SMF trajectories. The corresponding parameters are listed in Tables I-II, for a couple of events, for each considered case.

Then, within the Langevin treatment, 200 stochastic events were considered for each SMF trajectory. Fluctuations are injected each 3 fm/c.

According to the values listed in Tables I-II, we test essentially the behavior of composite systems with a variety of conditions of angular momentum, ranging from 50 \( \hbar \) to 100 \( \hbar \) and quadrupole moment \( \beta_2 \), from 0.2 to 1. The excitation energy is about 250 MeV, corresponding to temperatures of the order of 4 MeV. Apart from the situation observed in the case of \( b = 7 \) fm, \( E/A = 9 \) MeV/u, the octupole moment, \( \beta_3 \), always takes rather small values, of both signs, indicating that the memory of the entrance channel mass asymmetry is lost. Also the quadrupole and octupole collective velocities are rather small and may take values of both signs, suggesting that collective motions, apart from the rotation associated with the intrinsic spin, are damped. These conditions correspond closely to quasi-stationary, metastable situations, i.e. the system is stable against small shape fluctuations. From one side, it may evolve radiating its excitation energy and spin and relaxing slowly towards the spherical configuration. On the other hand, if the amplitude of the kicks of the associated collective velocities is large enough, the system may overcome the fission barrier and reach configurations corresponding to surface instabilities, from which it rapidly separates in two pieces. However, one should also consider that the latter possibility is in competition with nucleon emission, that reduces the excitation energy (and the associated amplitude of thermal fluctuations), while the shape of the system is evolving. The nucleon emission rate can be evaluated according to the standard Weisskopf formalism [27]. For the situations under study, the excitation energy reduces, due to nucleon emission, approximately by 2.5 MeV each 30 fm/c. We follow the trajectory of the system until the available excitation energy is fully dissipated.

Hence, thanks to the introduction of fluctuations in

### TABLE I: Characteristics of the composite system, as obtained in the reaction \(^{36}\text{Ar} + ^{96}\text{Zr}\) at 9 MeV/u at the time \( t_{freeze-out} \): Excitation energy, intrinsic angular momentum, quadrupole moment, octupole moment and associated collective velocities. The time unit adopted to define the collective velocities is \( 10^{-22} s = 30 \) fm/c. Two events are displayed for each impact parameter. The fission probability (see text) is reported in the last column.

| \( b \) (fm) | \( E^* \) (MeV) | \( L \) (\( \hbar \)) | \( \beta_2 \) | \( \beta_3 \) | \( \frac{d\beta_2}{dt} \) | \( \frac{d\beta_3}{dt} \) | \( P \) |
|-------|----------|---------|-------|-------|---------|---------|-----|
| 7     | 225      | 100     | 1.14  | -0.73 | 0.099   | 0.024   | 0.990 |
| 7     | 242      | 95      | 1.00  | -0.76 | 0.143   | -0.129  | 0.990 |
| 6     | 240      | 77      | 0.83  | 0.47  | 0.062   | -0.010  | 0.645 |
| 6     | 224      | 84      | 1.01  | -0.52 | 0.113   | -0.063  | 0.880 |
| 5     | 216      | 64      | 0.58  | -0.32 | 0.125   | 0.938   | 0.375 |
| 5     | 227      | 58      | 0.56  | 0.36  | -0.004  | 0.005   | 0.145 |

### TABLE II: Same as in Table I, but for the reaction at 16 MeV/u.

| \( b \) (fm) | \( E^* \) (MeV) | \( L \) (\( \hbar \)) | \( \beta_2 \) | \( \beta_3 \) | \( \frac{d\beta_2}{dt} \) | \( \frac{d\beta_3}{dt} \) | \( P \) |
|-------|----------|---------|-------|-------|---------|---------|-----|
| 6     | 279      | 90      | 0.88  | 0.34  | 0.016   | 0.059   | 1.000 |
| 6     | 277      | 97      | 0.88  | 0.44  | -0.015  | -0.031  | 1.000 |
| 5     | 241      | 73      | 0.37  | 0.15  | -0.063  | -0.047  | 0.320 |
| 5     | 252      | 77      | 0.63  | 0.40  | 0.136   | -0.020  | 0.580 |
| 4     | 258      | 63      | 0.31  | 0.06  | 0.052   | 0.018   | 0.110 |
| 4     | 247      | 52      | 0.22  | 0.05  | -0.007  | 0.002   | 0.035 |
the dynamical evolution, for a given impact parameter one observes a bifurcation of trajectories, leading either to compact shapes (fusion) or to elongated shapes, with large values of quadrupole and/or octupole moments, that eventually cause the break-up of the system. Actually the two possible outcomes are associated with a kind of bimodal behavior of the shape observables, related to configurations corresponding to local minima of the total (surface + Coulomb) energy. In may be interesting to notice that bimodality has been recently observed also in the context of liquid-gas phase transitions, where volume instabilities are concerned and dilute systems may either recompact to normal density or split into a huge number of small fragments [28].

A. Fission rates

In the following, we will first discuss some illustrative results obtained in the case of the reaction at 16 MeV/u, \( b = 5-6 \) fm. In Fig. 3 we present one example of trajectories corresponding to the two possible exit channels (fusion or fission), in the \( \beta_2, \beta_3 \) plane. Due to the random kicks, starting from the same initial conditions, rather different paths are explored. It should be noticed that, also in the case of trajectories leading to fusion, the final configuration is not exactly spherical, but is associated with small (not vanishing) values of the quadrupole moment. This corresponds to the stationary configuration compatible with the amount of intrinsic angular momentum present in the system. On the other hand, break-up configurations are characterized by rather large values of

FIG. 1: Contour plots of the density projected on the reaction plane calculated with SMF for the reaction \(^{36}\text{Ar} + ^{96}\text{Zr}\) at 9 MeV/u, at several times (fm/c). The size of each box is 40 fm.

FIG. 2: The same as in Fig. 1 but at 16 MeV/u.
impact parameters, after an initially increasing trend, re-
in each of the cases considered. For the most peripheral
is displayed as a function of time for a set of 200 events
$= 6$ $b$

$\beta$
tions in the case of the reaction at 16 MeV/u, $b = 5$ fm.

octupole moments, as obtained for the break-up configura-
$\beta$

FIG. 3: One example of trajectories leading either to fusion or
to break-up, in the ($\beta_2,\beta_3$) plane, as obtained in the reaction
at 16 MeV/u, $b = 5$ fm.

$\beta_2$ and/or $\beta_3$. Actually one sees an interesting correlation
between the two parameters, that is represented in Fig.4.
In fact, both a large quadrupole or octupole moments
are linked to break-up configurations, that correspond to
tangent spheroids. Fluctuations of the octupole moment
are rather large, though the majority of the events is lo-
cated near $\beta_3 = 0$, corresponding to symmetric fission.

In Figs.5-6 (left) the fission rate, $dN/dt$, as obtained for
$b = 6 - 7$ $fm$ at 9 $MeV/u$ and $b = 5 - 6$ $fm$ at 16 $MeV/u$,is displayed as a function of time for a set of 200 events
in each of the cases considered. For the most peripheral
impact parameters, after an initially increasing trend, re-
lated to the time interval needed to build and propagate
fluctuations, we observe an almost exponential decrease,
as expected in the case of constant break-up probability
$\gamma_{break}$. In this case one can write: $dN/dt = N_0(\gamma_{break})$, with $N_0 = N_{0e}^{-\gamma_{break}}$ and $N_0 = 200$ (the total num-er of events considered). This corresponds to situations
where the break up probability ($\gamma_{break} \approx 0.002$ $c/fm$)
is not much affected by the competing nucleon emission.
All events practically lead to fission over a time interval
that is shorter than the one needed to exhaust the avail-
able excitation energy by nucleon emission. In fact, the
maximum of the emission rate is observed at about 300
fm/c and the system needs, on average, roughly 500 fm/c
to reach the break-up configuration (this is actually the
half life time $\tau_{break} = 1/\gamma_{break}$). On the other hand, for
smaller impact parameters (corresponding to lower defor-
mation of the system and lower angular momentum), the
break-up probability $\gamma_{break}$ is quenched approximately
by a factor 4 (see the left panel of Figs.5-6) and de-
creases in the course of time because of nucleon emission,
that reduces the excitation energy and the correspond-
ing amount of thermal fluctuations. In most cases, the
excitation energy deposited into the system is dissipated
before the break up configuration may be reached. It is
interesting to notice that, even in the most favourable
case, the typical times of the process are rather large
(500 fm/c), as compared for instance to the time scales
associated with the development of volume instabilities
in multifragmentation processes at higher energies (about
150 fm/c). This can be explained in terms of the larger
amount of excitation energy deposited into the system in
the latter case (that induces fluctuations of higher ampli-
tude and collective radial expansion) and of the smaller
growth times associated with volume instabilities [17].

The corresponding fraction of events that undergo
break-up, $P_{break}$, is reported in Table I-II, at the two
energies and for all impact parameters considered. From
the estimated break-up probabilities it is possible to con-
struct the fusion cross-section, $\sigma_f(b) = (1 - P_{break}) \pi bdb$, that is displayed in Fig.7, for the two energies. We also
show, for comparison, the results obtained within the
SMF approach only, where due to the approximate treat-
ment of fluctuations, one gets distributions close to a
sharp cut-off (approximated by a sharp cut-off in the fig-
ure). It is interesting to notice that, especially in the case
of the reaction at 9 $MeV/u$, the fusion cross section is
reduced significantly by the introduction of fluctuations
that, in turn, help the system to overcome the fission
barrier and to break-up.

B. Features of fission fragments

The time $t_{break}$, needed to reach the break-up configu-
ration, is connected to other interesting features of the
reaction dynamics, depending on the various entrance
channel conditions. In fact, due to the intrinsic spin,
the system rotates while its shape evolves according to
FIG. 5: (Color online) Left panel: Distribution of the time $t_{\text{break}}$ (see text), as obtained for the reaction at 9 MeV/u and impact parameters $b = 7$ fm (full histogram) and $b = 6$ fm (shaded histogram). Right panel: Angular distribution of the break-up direction. Full line and circles refer to $b = 7$ fm; dashed line and open squares are for $b = 6$ fm.

FIG. 6: (Color online) Same as Fig.5, for the reaction at 16 MeV/u and $b = 5$-6 fm.

As a consequence, the direction along which the system separates into pieces is strictly connected to $t_{\text{break}}$. Hence the shape of the angular distribution of fission fragments can be used as a clock of the collision, from which one can extract information on the break-up probability and the underlying reaction mechanism. This is an appealing issue that can be investigated also experimentally by looking at the angular distribution of the emerging reaction products and at the possible existence of alignment effects [9, 10]. In the case of a fast break-up (fast fission) the angular distribution should exhibit a peak: due to the elongated shape of the system, the emission is not isotropic. Along the separation process, fragments acquire velocities essentially due to the Coulomb repulsion, according to the Viola systematics, like in standard fission, but with a preferential emission axis. The distribution of the angle, $\theta_{\text{break}}$, corresponding to the rotation (on the plane perpendicular to the direction of the intrinsic spin of the system) until the break-up configuration is reached is shown in Figs.5-6 (right panel), for the two energies and two impact parameters. Obviously, the shape of this distribution depends on the fission probability, but also on the system angular velocity (that in turn depends on the intrinsic spin). In fact, in absence of rotation (vanishing spin) the fragments would always be emitted along a fixed axis. In the case of the most peripheral events, a clear peak is observed in the distribution. On the other hand, for more central impact parameters, the half life time is much larger and one essentially gets a flat distribution for $\theta_{\text{break}}$, similarly to what is expected in the case of standard statistical fission.

C. Fast-fission of PLF and TLF

Several shape, angular momentum and excitation energy conditions can be observed also, in the case of collisions between heavy systems, after the separation into PLF and TLF, for one (or both) of these products. Thus it is interesting to investigate fast fission processes of these objects, leading to ternary (or quaternary) breaking of the whole system. For instance, we display in Fig.8, density contour plots as obtained in SMF simulations of semi-peripheral collisions of Au + Au at 15 MeV/u, for which aligned ternary and quaternary breaking has been recently observed experimentally [9]. One can see that similar shape configurations, as the ones observed in the reactions investigated above, may appear for PLF/TLF fragments. However, these fragments have lower angu-

FIG. 7: (Color online) Fusion cross section, as a function of the impact parameter $b$, as obtained in the reactions at 9 MeV/u (black histogram) and 16 MeV/u (grey histogram) with the Langevin treatment, Eq.(4). The lines correspond to SMF simulations at 9 MeV/u (full) and 16 MeV/u (dashed).
lar momentum (about $20 - 40\hbar$) and excitation energy
(of the order of 100 MeV). The corresponding break-up
probability is of the order of 10% and emission times
are longer ($\approx 2000$ fm/c). The fast-fission mechanism
could explain qualitatively some of the features observed
experimentally, such as alignment effects and fragment
relative velocities and charge distributions. However, a
thorough analysis of the kinematical properties of the
reaction products \cite{9}, as well as the estimated rather
short break-up times, suggests the persistence of non-
equilibrium effects in momentum space, i.e. the presence
of collective velocities in $\beta_2$ and/or $\beta_3$, in addition to the
tangential velocity generated by the intrinsic angular mo-
momentum. Collective velocities, probably underestimated
in the SMF calculations, would speed up the fragmenta-
tion process since the system is pushed towards more
exotic shapes, from which it is easier to overcome the
fission barrier.

**IV. CONCLUSIONS**

In this article we have investigated the role of shape
fluctuations in the dynamical evolution of excited sys-
tems that can be formed in semi-peripheral reactions at
low energy (around 20 MeV/u). Quasi-stationary com-
posite systems, with quadrupole and/or octupole defor-
mation, are observed, for which shape fluctuations are
essential to overcome the fission barrier and eventually
break-up. This analysis is performed within a hybrid
treatment that couples the study of the early stage of the
dynamics, devolved to a microscopic stochastic transport
approach, up to the formation of primary excited sources,
to a full Langevin description of the leading degrees of
freedom of these objects: quadrupole, octupole moments
and angular velocity. For temperature, shape and an-
gular momentum conditions obtained in semi-peripheral
reactions, typical time scales of the break-up process are
of the order of 500 fm/c. The fission fragments are emit-
ted along a preferential direction, that corresponds to
the maximum elongation axis. Due to angular momentum
effects, this direction may rotate while the shape of the
system is evolving towards break-up configurations.
Hence a careful analysis of the angular distribution of the
reaction products may give relevant information about
fission probabilities and the involved time scales, that in
turn are closely linked to the mean-field dynamics and
the properties of the nuclear interaction (range, surface
energy, two-body correlations). From this study it is clear
that a good treatment of mean-field fluctuations is a cru-
usal point in the characterization of dissipative reactions.
The model employed here provides a suitable description
of surface modes, parametrized in terms of quadrupole
and octupole oscillations, but it could miss some non-
equilibrium effects that can help the system to break-
up. In fact, collective velocities related to shape ob-
servables are likely underestimated in the SMF approach
\cite{29} and the role of multipolarities higher than octupole
is neglected in the Langevin treatment. A fully micro-
scopic description of the whole process would be highly
desirable, though it is far from being trivial. Some at-
tempts are represented by improved quantum molecular
dynamics calculations (ImQMD) \cite{30}. Stochastic exten-
sions of Time-Dependent-Hartree-Fock (TDHF) calcula-
tions should also provide a valuable tool to characterize
reaction mechanisms in low energy collisions \cite{31}. Work
is in progress in this direction.

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