Prediction of a $Z_c(4000)\ D^*\bar{D}^*$ state and relationship to the claimed $Z_c(4025)$.

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Abstract

After discussing the OZI suppression of one light meson exchange in the interaction of $D^*\bar{D}^*$ with isospin $I=1$, we study the contribution of two pion exchange to the interaction and the exchange of a heavy vector $J/\psi$. We find this latter mechanism weak but enough to barely bind the system in $J=2$ with a mass around 4000 MeV, while the effect of the two pion exchange is a net attraction but weaker than that from $J/\psi$ exchange. We discuss this state and try to relate it to the $Z_c(4025)$ state, above the $D^*\bar{D}^*$ threshold, claimed in an experiment at BES from an enhancement of the $D^*\bar{D}^*$ distribution close to threshold. Together with the results from a recent reanalysis of the BES experiment showing that it is compatible with a $J=2$ state below threshold around 3990 MeV, we conclude that the BES experiment could be showing the existence of the state that we find in our approach.

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I. INTRODUCTION

The charmonium spectrum of $c\bar{c}$ states has been enriched with a plethora of new states called X,Y,Z states, which do not fit in the expected spectrum of ordinary $c\bar{c}$ quark states [1–4]. The theory has followed trend with also a rich offer of possible interpretations, like tetraquarks, or molecular states, and more exotic states [5]. One of the last surprises has been the finding of Z states with isospin $I=1$. In the hidden charm sector a state around 4020 MeV, called $Z_c(4020)$ and a width of about 8 MeV has been reported in [6], in the $e^+e^-\rightarrow \pi^+\pi^-h_c$ reaction, looking at the invariant mass of $\pi^\pm h_c$. Another BES experiment has found a peak in the $(D^*\bar{D}^*)^\pm$ spectrum close to threshold, which was interpreted in terms of a new resonance with mass around 4025 MeV and width about 25 MeV [7]. It is unclear whether these two states can be the same, and the quantum numbers are in any case not well determined. The peak seen in the $(D^*\bar{D}^*)^\pm$ spectrum is appealing since in [8] the study of the $D^*\bar{D}^*$ interaction gave rise to a state with $I=1$ in spin $J=2$. The state appeared around 3920 MeV, with uncertainties. Actually, we will claim here that it should be much less bound, but that most probably it is related to the peak seen in the $(D^*\bar{D}^*)^\pm$ in [7]. The threshold for $(D^*\bar{D}^*)^\pm$ is 4017 MeV, so a bound state of $D^*\bar{D}^*$ should have a smaller energy, while the energy of the state is claimed at 4025 in [7]. Yet, the interpretation of peaks around threshold is always problematic and a source of confusion. Indeed, most often an enhancement of the invariant mass at threshold is an indication of a bound state or resonance below threshold. There are multiple examples of it. In a similar reaction, $e^+e^-\rightarrow \psi D\bar{D}$ [9], a bump close to the threshold in the $D\bar{D}$ invariant mass distribution was reported by the Belle collaboration, which was tentatively interpreted as a new resonance. This peak was, however, interpreted in Ref. [10] in terms of a bound $D\bar{D}$ molecular state, called X(3700), which had been predicted in Ref. [11] and later on has also been reported in other works [12–17]. In a similar way, in Ref. [18], a peak seen in the $\phi\omega$ threshold in the $J/\psi \rightarrow \gamma\phi\omega$ reaction [19] was better interpreted as a manifestation of the $f_0(1710)$ resonance, below the $\phi\omega$ threshold, which couples strongly to $\phi\omega$ [20]. More recently a bump close to threshold in the $K^0\bar{K}^0$ invariant mass distribution, seen in the $J/\psi \rightarrow \eta K^0\bar{K}^0$ decay in Ref. [21], is interpreted in [22] as a signal of the formation of an $h_1$ resonance, predicted in Ref. [20], which couples mostly to the $K^*\bar{K}^*$. In the same direction as in the previous works, in [23] the experiment of [7] was reanalyzed and the enhancement in the $D^*\bar{D}^*$ invariant mass distribution was found compatible with a state with $J=2$, mass around 3990 MeV and width around 160 MeV, although fits with other solutions were also found acceptable. Yet, resonances with mass bigger than the $D^*\bar{D}^*$ mass were discouraged based on the difficulty to have single channel resonances with energy above threshold. Indeed, it was shown in [24] that an energy independent potential, smooth in momentum space, could not generate a resonance above the mass of the interacting particles. In this sense, any energy below threshold is preferred, and the $J=2$ solution with mass around 3990 was proposed as a good candidate to explain the experimental peak. Another reason in favor of this interpretation was that if the state were a $J^P=1^+$ produced in $S$-wave, as assumed in the experimental work [7], it can easily decay into $\pi J/\psi$. This decay channel is the same of the $Z_c(3900)$ [25]. However, while a peak is clearly visible in the $\pi J/\psi$ invariant mass distribution for the $Z_c(3900)$, no peak is seen around 4025 MeV (see Fig. 4 of Ref. [24]). In the present work we go back to [8] and perform some corrections to update the results of the local hidden gauge to the results of the heavy quark spin symmetry [26]. On the other hand we also show how for $I=1$, the exchange of light $q\bar{q}$ states is OZI forbidden,
which makes the exchange of light vectors and pseudoscalars cancel when equal masses are taken for them separately, in view of which we explore the exchange of two pions, both with or without interaction. The exchange of vector mesons is reduced to the exchange of $J/\psi$, which makes the potential small, in spite of which we still find it bigger than that of the two pion exchange. Altogether we find a state of the $D^*\bar{D}^*$ in $I=1$, $J=2$, close to threshold, which, together with the findings of [23], offers a natural interpretation for the peak observed in [7]. This molecular interpretation would also be supported by QCD sum rules calculations, [27], [28], [29], [30], although the uncertainty of $\pm 250$ MeV and $\pm 105$ MeV, and $\pm 280$ MeV respectively in the binding of these works offers only a weak support to our more precise determination of the mass. At the same time, QCD sum rules disfavor the interpretation of the $Z_c(4025)$ as a possible diquark-antidiquark type vector tetraquark states [31]. A molecular interpretation for this state is also assumed in [32], where the coupling to the $D^*\bar{D}^*$ components is evaluated by means of the Weinberg compositeness condition [33] and this picture is used to evaluate various strong decays widths of the resonance. The growing information around the claimed $Z_c(4025)$, together with the present work, comes to support a $D^*\bar{D}^*$ molecular state below threshold as an interpretation of the $(D^*\bar{D}^*)^\pm$ peak seen in [7].

II. FORMALISM

We want to study states of $I = 1$. The first consideration is that one light meson exchange is OZI forbidden. To realize that, we look at the $D^{*+}\bar{D}^{*0}$ interaction diagram in Fig. 1 and we see that a $d\bar{d}$ state exchange is forced to be converted into a $u\bar{u}$ state. In terms of physical mesons, that would mean that the $\rho$, $\omega$ cancel if we take equal masses (as it is indeed the case in the hidden gauge approach) and $\pi$, $\eta$, $\eta'$ also cancel if equal masses are taken or for large momenta bigger than the mass of the mesons.

![Diagram](attachment://fig1.png)

FIG. 1.

In view of this, we want to evaluate the contribution of two pion exchange in the next section, where the OZI restriction no longer holds.

A. The $D^*\bar{D}^*$ interaction by means of $\sigma$ exchange

The nucleon-nucleon interaction calls for an intermediate range attraction which was traditionally taken into account by means of “$\sigma$” exchange. With ups and downs the $\sigma$ resonance appears now in the PDG [34] as the $f_0(500)$. This resonance appears unavoidably in a study of the $\pi\pi$ interaction with a unitary approach using as input the kernel from
the chiral Lagrangians [35–37]. The analysis of $\pi\pi$ data with Roy equations allows one to establish the mass and width of this resonance with some precision [38, 39], compatible with the prediction of the chiral unitary approach, with mass around 460 MeV and half width around 280 MeV. From this point of view it was interesting to provide a microscopic picture for $\sigma$ exchange, based on the nature of the $\sigma$ resonance stemming from the interaction of two pions. This job was done in [40] considering the exchange of two correlated (interacting) pions for the NN interaction. In this section we extend these ideas to the interaction of $D^*\bar{D}^*$.

We have four diagrams contributing to this process and they are shown in Fig. 2. Each one of them contains four PPV vertices involving a $D^*$ ($\bar{D}^*$) vector meson and the two pseudoscalar, the pion and the $D$ ($\bar{D}$) meson. These vertices are of four different types, which are depicted in Fig. 3. Their evaluation is easily done with the local hidden gauge Lagrangians [41–45], which are very useful when dealing with vector mesons. The crossing of the pion lines indicates that we have there the $\pi\pi$ scattering amplitude.

![Fig. 2. Lowest order $\pi\pi$ interaction in the $I = 1$ channel for $D^*\bar{D}^* \rightarrow D^*\bar{D}^*$.](image)

In particular, the Lagrangian we need in order to evaluate the amplitudes of the diagrams in Fig. 2 is given by

$$\mathcal{L}_{PPV} = -ig \langle V^\mu [P, \partial_\mu P] \rangle,$$

(1)

where the symbol $\langle \rangle$ stands for the trace in $SU(4)$ and the constant $g$ is the strong coupling of the $D^*$ meson to $D\pi$. We use for $g$ the theoretical value obtained within the heavy quark spin symmetry approach, which is given by the $SU(3)$ value $g = \frac{m_V}{m_{D^*}}$ with $m_V \approx m_\rho$ and $f = 93$ MeV the pion decay constant, multiplied by $m_{D^*}/m_{K^*}$ [40], as we discuss below. This gives the effective value of $g = 9.40$. With this value, we obtain 71 keV for the width of $D^{*+} \rightarrow D^0\pi^+$ compared to the experimental value ($65 \pm 15$) keV of [47].

The P matrix contains the 15-plet of the pseudoscalar mesons written in the physical
basis in which $\eta, \eta'$ mixing is considered \[48\],

\[
P = \begin{pmatrix}
\frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \frac{\pi^+}{\sqrt{3}} + \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^+ & \bar{D}^0 \\
\frac{\pi^-}{\sqrt{3}} - \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 & D^- & \frac{-\eta}{\sqrt{3}} + \sqrt{\frac{2}{3} \eta' \bar{D}^-} \\
K^- & \bar{K}^0 & D^+ & \bar{s} \\
D^0 & D^+ & \bar{s} J/\psi & \eta_c
\end{pmatrix},
\]

while the $V$ matrix contains the 15-plet of vector mesons,

\[
V_{\mu} = \begin{pmatrix}
\frac{\omega}{\sqrt{2}} + \frac{\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\
\rho^- & \frac{\omega}{\sqrt{2}} - \frac{\rho^0}{\sqrt{2}} & K^{*0} & D^{*-} \\
K^{*-} & \frac{K^{*0}}{\sqrt{2}} & \phi & D^{*-}_s \\
D^{*0} & \bar{D}^{*+} & D^{*+}_s & J/\psi
\end{pmatrix}_{\mu}.
\]

Using Eq. (1) we can write the amplitudes of the four vertices of Fig. 3 as

\[-it_{PPV} = -iq C(p_D + p_\pi)_\mu \epsilon_{\mu},\]

where $p_D$ and $p_\pi$ are the four-momenta of the $D$ meson and of the pion, respectively, and $\epsilon_V$ is the polarization vector of the $D^*$ meson in the vertex. The coefficients $C$, for each one of the four vertices, are listed in Tab. I.

An alternative approach which does not need the use of SU(4) is provided in [46]. It relates the $D^*D\pi$ coupling to the $K^*K\pi$ one assuming that the $s$ or $c$ quarks are spectators in both processes. There is need to renormalize the coupling due to the different normalization of the meson fields ($1/\sqrt{2E}$ factor) and the results obtained reproduce the couplings provided by the heavy quark spin symmetry [26] and provide a width for $D^* \to D\pi$ compatible with experiment, as we have mentioned above.

The weight of a single diagram in the process can be obtained looking at the numbers written besides each vertex of the diagrams in Fig. 2 and multiplying the different coefficients.
| Diagram | C  |
|---------|----|
| 1) $D^+\pi^0D^{*+}$ | $1/\sqrt{2}$ |
| 2) $D^+\pi^-D^{*0}$ | $-1$ |
| 3) $\bar{D}^0\pi^0\bar{D}^{*0}$ | $1/\sqrt{2}$ |
| 4) $\bar{D}^0\pi^-D^{*-}$ | $1$ |

**TABLE I.** Coefficients $C$ for the different vertices in Fig. 3.

$C$, shown in Tab. 1, corresponding to the vertices involved. This allows us to write the amplitude of the process as

$$-it_\sigma = -i V^2 (\frac{1}{4}t_{\pi^0\pi^0\to\pi^0\pi^0} + \frac{1}{2}t_{\pi^0\pi^0\to\pi^+\pi^-} + \frac{1}{2}t_{\pi^+\pi^-\to\pi^0\pi^0} + t_{\pi^+\pi^-\to\pi^+\pi^-}) ,$$

(5)

where the factor $V$ is the contribution to the diagram of the triangular loops, which we shall evaluate later. Note that, in order to write the amplitude, we must assume two initial pions and two final pions all pointing to the right in the diagrams of Fig. 2, hence providing the amplitude of Eq. (5).

Considering the unitary normalization of the $\pi\pi$ states [35],

$$|\pi\pi, I = 0\rangle = \frac{1}{\sqrt{6}} |\pi^0\pi^0 + \pi^+\pi^- + \pi^-\pi^+\rangle ,$$

(6)

and writing explicitly the isoscalar amplitude

$$t_{\pi^0\pi^0\to\pi^0\pi^0}^{I=0} = \frac{1}{6} (t_{\pi^0\pi^0\to\pi^0\pi^0} + 2t_{\pi^0\pi^0\to\pi^+\pi^-} + 2t_{\pi^+\pi^-\to\pi^0\pi^0} + 4t_{\pi^+\pi^-\to\pi^+\pi^-}) ,$$

(7)

we can rewrite Eq. (5) as

$$-it_\sigma = -i V^2 \frac{3}{2} t_{\pi^0\to\pi\pi}^{I=0} .$$

(8)

Since the pions in the diagrams in Fig. 2 are off-shell, we need to use the off shell $t$-matrix obtained from the lowest order meson-meson Lagrangian [35]

$$t_{\pi\pi\to\pi\pi}^{I=0} = -\frac{1}{9f^2} \left(9s + \frac{15m_\pi^2}{2} - 3 \sum_i p_i^2 \right) ,$$

(9)

with $s$ the Mandelstam variable and $m_\pi$ and $p_i$ the mass and momenta of the pions, respectively. As done in Ref. [40], we can obtain the on-shell amplitude simply putting $p_i^2 = m_\pi^2$ and this allow us to rewrite Eq. (9) as

$$t_{\pi\pi\to\pi\pi}^{I=0} = t_{\pi\pi\to\pi\pi}^{I=0, OS} + \frac{1}{3f^2} \sum_i (p_i^2 - m_\pi^2) ,$$

(10)

where

$$t_{\pi\pi\to\pi\pi}^{I=0, OS} = -\frac{1}{f^2} (s - m_\pi^2) .$$

(11)
Following the approach of Ref. [40], it can be shown that the off-shell part cancels exactly with other diagrams at the same order in the chiral counting. Thus, at lowest order, we can write
\[ t_\sigma = V^2 \frac{3}{2} \frac{1}{f^2} \left( s - \frac{m^2}{2} \right). \] (12)

In order to apply the unitary Bethe-Salpeter approach to the scalar mesons amplitude, we need to sum the set of diagrams in Fig. 4. This is easily done substituting the on-shell meson-meson amplitude of Eq. (11) by
\[ t_{\pi\pi \rightarrow \pi\pi}^{I=0} = -\frac{1}{f^2} \frac{s - \frac{m^2}{2}}{1 + \frac{1}{f^2}(s - \frac{m^2}{2})G(s)}, \] (13)
where \( G(s) \) is the two pions loop function, conveniently regularized [40],
\[ G(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m^2_\pi + i\epsilon} \frac{1}{(P - q)^2 - m^2_\pi + i\epsilon}, \] (14)
with \( P \) the total momentum of the two pion sistem and \( P^2 = s \).

FIG. 4. \( \bar{D}D^* \) interaction when the \( \pi\pi \) scattering matrix is summed up to all orders in the unitary approach.

We need, now, to evaluate the factor \( V \) that appears in Eq. (12), related, as already mentioned, to the triangular loop, which is shown in Fig. 5. For simplicity, we use the Breit reference frame. This means that
\[ p_1 \equiv (p^0_1, \vec{q}/2), \]
\[ p'_1 \equiv (p^0'_1, -\vec{q}/2), \]
\[ p \equiv (p^0, p'), \] (15)
where \( \vec{q} \) is the three-momentum transferred in the process. Since there is no energy exchange, \( s = -\vec{q}^2 \). It is also useful to define the variable \( q \equiv (0, \vec{q}) \).

Thus, by means of Eq. (4) and keeping in mind that we already factorized outside \( V \) the coefficients \( C \), we can write the expression of \( V \) as
\[ V = i g^2 \int \frac{d^4p}{(2\pi)^4} \epsilon_\mu (2p - p_1)^\nu \epsilon'_\nu (2p - p'_1)^\mu \frac{1}{p^2 - m^2_D + i\epsilon} \frac{1}{(p - p_1)^2 - m^2_\pi + i\epsilon} \frac{1}{(p - p'_1)^2 - m^2_\pi + i\epsilon}, \] (16)
with $m_D$ the mass of the $D$ meson.

The integral in Eq. (16) is logarithmically divergent. As in Ref. [40], the regularization is accomplished by means of a cutoff in the space of intermediate states ($p_{\text{max}} = 2$ GeV) and a form factor. In order to keep the integration in $p^0$ simple, we use the product of static form factors

$$F = F_1(p^2 + \frac{q^2}{2}) F_2(p^2 - \frac{q^2}{2}) = \frac{\Lambda^2}{\Lambda^2 + (\frac{p + q}{2})^2} \frac{\Lambda^2}{\Lambda^2 + (\frac{p - q}{2})^2},$$

with $\Lambda = 1$ GeV.

Since $\epsilon_\mu p_1^\mu = 0$ and $\epsilon'_\nu p_1'^\nu = 0$, Eq. (16) can be rewritten as

$$V = 4g^2 \int \frac{d^4p}{(2\pi)^4} \epsilon_\mu p_1^\mu \epsilon'_\nu p_1'^\nu \frac{1}{p^2 - m_D^2 + i\epsilon} \frac{1}{(p - p_1)^2 - m_N^2 + i\epsilon} \frac{F}{(p - p'_1)^2 - m_N^2 + i\epsilon}. \quad (18)$$

The integral in Eq. (18) is symmetric with respect to $p_1$ and $p_1'$ and this allows us to derive the structure of the result of the integration, which will be of the type

$$V = \epsilon_\mu \epsilon'_\nu (ag_{\mu\nu} + b(p_1^\mu p_1'^\nu + p_1'^\mu p_1^\nu) + c(p_1^\mu p_1'^\nu + p_1'^\mu p_1^\nu)). \quad (19)$$

In the last expression, due to the Lorentz condition, only the terms $ag_{\mu\nu}$ and $cp_1^\mu p_1'^\nu$ survive but we need the entire structure to evaluate them. This is done taking the trace of Eq. (18) and multiplying the equation by $(p_1\mu p_1^\nu + p_1'^\mu p_1'^\nu)$ and $(p_1\mu p_1^\nu + p_1'^\mu p_1'^\nu)$, in order to obtain a system of three equations. Solving the system, we find the expressions of the three coefficients in Eq. (19) but, as we already said, we are only interested in

$$a = \frac{-Y m_D^2 + Z(p_1 p_1') + X(m_D^2 - (p_1 p_1')^2)}{2(m_D^2 - (p_1 p_1')^2)},$$

$$c = \frac{-3Y m_D^2 (p_1 p_1') + X(m_D^4 - (p_1 p_1')^2) + Z(m_D^4 + 2(p_1 p_1')^2)}{2(m_D^4 - (p_1 p_1')^2)}.$$ 

FIG. 5. Two pion exchange triangle vertex.
where

\[ X = 4g^2I_1 + 4g^2m_D^2I_2 , \]
\[ Y = 8g^2p_1^{02}I_1 + 8g^2I_3 , \]
\[ Z = 8g^2p_1^{02}I_1 + 8g^2I_4 . \]  

The four integrals in the equations above, \( I_1, I_2, I_3 \) and \( I_4 \), have the following expressions:

\[
I_1 = \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p-p_1)^2 - m^2_\pi + i\epsilon} \frac{1}{(p-p'_1)^2 - m^2_\pi + i\epsilon} F, \\
I_2 = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2_D + i\epsilon} \frac{1}{(p-p_1)^2 - m^2_\pi + i\epsilon} \frac{1}{(p-p'_1)^2 - m^2_\pi + i\epsilon} F, \\
I_3 = \int \frac{d^4p}{(2\pi)^4} \frac{1}{(\bar{p}^2 + m^2_D)p^0_1 + (\bar{p}'^2)^2} \frac{1}{(p-p_1)^2 - m^2_\pi + i\epsilon} \frac{1}{(p-p'_1)^2 - m^2_\pi + i\epsilon} F, \\
I_4 = \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2_D + i\epsilon} \frac{1}{(\bar{p}^2 + m^2_D)p^0_1 - (\bar{p}'^2)^2} \frac{1}{(p-p_1)^2 - m^2_\pi + i\epsilon} \frac{1}{(p-p'_1)^2 - m^2_\pi + i\epsilon} F. 
\]

After performing the integration in \( dp^0 \), which can be done analytically using Cauchy’s theorem, we obtain

\[
I_1 = \int \frac{d^3p}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1\omega_2 - \bar{q}^2 - (\omega_1 + \omega_2)^2} F, \\
I_2 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_D} \frac{1}{2\omega_1\omega_2} \frac{\omega_1 + \omega_2 + E_D - m_D - i\epsilon}{E_D + \omega_2 - m_D - i\epsilon} \frac{1}{E_D + \omega_2 - m_D - i\epsilon} F, \\
I_3 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_D} \frac{1}{2\omega_1\omega_2} \frac{\omega_1 + \omega_2 + E_D - m_D - i\epsilon}{E_D + \omega_1 - m_D - i\epsilon} \frac{1}{E_D + \omega_1 - m_D - i\epsilon} \frac{(\bar{p}^2 + m^2_D)p^0_1 + (\bar{p}'^2)^2}{F}, \\
I_4 = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_D} \frac{1}{2\omega_1\omega_2} \frac{\omega_1 + \omega_2 + E_D - m_D - i\epsilon}{E_D + \omega_1 - m_D - i\epsilon} \frac{1}{E_D + \omega_1 - m_D - i\epsilon} \frac{(\bar{p}^2 + m^2_D)p^0_1 - (\bar{p}'^2)^2}{F}, 
\]

where \( \omega_1 = \sqrt{(\bar{p} + \bar{q}/2)^2 + m^2_\pi} \), \( \omega_2 = \sqrt{(\bar{p} - \bar{q}/2)^2 + m^2_\pi} \), and \( E_D = \sqrt{\bar{p}^2 + m^2_D} \) are the energies of the two pions and of the \( D \) meson involved in the loop, respectively, and \( m_D \) is the mass of the \( \bar{D}^* \) meson. Since the mass of the \( D \) meson is so large, we have taken the positive energy component of the propagator \([ (p^0 - E_D)2E_D ]^{-1} \), which simplifies the integration.

We can now go back to the \( D^* \bar{D}^* \) potential in momentum space, whose final expression, according to Eqs. (8) and (13), is given by

\[
t_\sigma(\bar{q}) = V^2 \frac{3}{2} \frac{1}{f^2} \frac{1}{1 - G(-\bar{q}^2)} \frac{\bar{q}^2 + m^2_\pi}{\bar{q}^2 + \frac{m^2_\pi}{2}} , \tag{24}
\]

with

\[
V = \epsilon_\mu\epsilon^\nu (g^{\mu\nu} + cp_1^\mu p_1^\nu) \tag{25}
\]

and \( a \) and \( c \) derived using Eqs. (20), (21) and (23).
Since we assume small initial momenta $\vec{p}_1$ and $\vec{p}_1'$ of the vectors compared to the vector mass, we can take $\epsilon^0 \equiv 0$ and only the $ae\epsilon^l$ combination remains. The other vertex will provide a similar structure. Hence, we have the combination

$$\epsilon^{(1)}_i \epsilon^{(2)}_j \epsilon^{(3)}_i \epsilon^{(4)}_j,$$

with $1 + 2 \to 3 + 4$. By recalling the form of spin projector operators [49],

$$P^{(0)} = \frac{1}{3} \epsilon_i \epsilon_i \epsilon_j \epsilon_j,$$

$$P^{(1)} = \frac{1}{2} \left( \epsilon_i \epsilon_j \epsilon_i \epsilon_j - \epsilon_i \epsilon_j \epsilon_j \epsilon_i \right),$$

$$P^{(2)} = \frac{1}{2} \left( \epsilon_i \epsilon_j \epsilon_i \epsilon_j + \epsilon_i \epsilon_j \epsilon_j \epsilon_i \right) - \frac{1}{3} \epsilon_i \epsilon_i \epsilon_j \epsilon_j,$$

we see that

$$\epsilon^{(1)}_i \epsilon^{(2)}_j \epsilon^{(3)}_i \epsilon^{(4)}_j \equiv P^{(0)} + P^{(1)} + P^{(2)}.$$

The strength of $t_{\sigma}(q)$, removing $g^{\mu\nu} \epsilon_\mu \epsilon_\nu$, gives already the strength of the two pion exchange potential in $J = 2$. The potential $t_{\sigma}$ as a function of the transferred momentum $q$ is plotted in Fig. 6.

FIG. 6. Potential $t_{\sigma}$ as a function of the momentum transferred in the process.

B. Uncorrelated crossed two pion exchange

Now we want to study the $D^* \bar{D}^*$ interaction when the pions exchanged are not interacting. In this case, only the diagrams $a)$ and $d)$ of Fig. 2 contribute to the process. This means that the isospin factor, given by the different vertices involved (see Tab. I), will be $\frac{5}{2}$, and we do not have the $\pi\pi$ amplitude (see Eq. (9)). Note that we take only the crossed diagrams. The iterated one $\pi$ exchange requires the exchange of $q\bar{q}$ which we saw was OZI suppressed and we do not consider it.
Recalling the expression of the vertices given in Eq. (4), and choosing the momenta assignment as shown in Fig. 7, we can directly write the amplitude of the process as

\[ t = \frac{5}{4} ig^4 \int \frac{d^4 p}{(2\pi)^4} \epsilon_{\mu}(2p - p_1)\epsilon_{\nu}(2p - p'_1)\epsilon_\alpha(2p - 2p'_1 + p_2)\epsilon_\beta(2p - p'_1 - p_1 + p_2) \]

\[ \times F^2 \left( \frac{1}{p^2 - m_D^2 + i\epsilon} \frac{1}{(p - p'_1 + p_2)^2 - m_D^2 + i\epsilon} \frac{1}{(p - p_1)^2 - m_\pi^2 + i\epsilon} \right) \]

\[ \times \left( \frac{1}{(p - p'_1)^2 - m_\pi^2 + i\epsilon} \right). \tag{29} \]

**FIG. 7.** Momenta assignment in the two pion exchange in $D\bar{D}^* \rightarrow D\bar{D}^*$.

Since the momenta of the particles in the loop are small, we can use the non-relativistic approximation. The consequence is that only the spatial components of the polarization vectors survive and we can rewrite the amplitude of Eq. (29) as

\[ t = \frac{5}{4} ig^4 \int \frac{d^4 p}{(2\pi)^4} \epsilon_i(2p - p_1)\epsilon_j(2p - p'_1)\epsilon_l(2p - 2p'_1 + p_2)\epsilon_m(2p - p'_1 - p_1 + p_2) \]

\[ \times F^2 \left( \frac{1}{p^2 - m_D^2 + i\epsilon} \frac{1}{(p - p'_1 + p_2)^2 - m_D^2 + i\epsilon} \frac{1}{(p - p_1)^2 - m_\pi^2 + i\epsilon} \right) \]

\[ \times \left( \frac{1}{(p - p'_1)^2 - m_\pi^2 + i\epsilon} \right). \tag{30} \]

We also assume that $4p^2 \gg q^2/4$, such that the dominant term in Eq. (30) is the one with the form $p_ip_jp_lp_m$. This means that the amplitude in Eq. (30) will have the following structure:

\[ a (\delta_{ij}\delta_{lm} + \delta_{il}\delta_{jm} + \delta_{im}\delta_{jl}) , \tag{31} \]

where $a$ is a coefficient that can be determined putting, for example, $i = j$ and $l = m$ in both Eqs. (30) and (31). Doing this, we obtain that $a = 1/15$. 

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Thus, we can write
\[
    t = \frac{5}{4} g^4 \frac{1}{15} \int \frac{d^4 p}{(2\pi)^4} \left( \frac{\vec{q}^2}{4} - \frac{\vec{q}^2}{4} \right)^2 \left( \epsilon_i \epsilon_i \epsilon_i \epsilon_i + \epsilon_i \epsilon_i \epsilon_i \epsilon_i + \epsilon_i \epsilon_i \epsilon_i \epsilon_i \right) \frac{F^2}{p^2 - m_D^2 + i\epsilon} \frac{1}{E^2_D} \frac{1}{\omega_1 + \omega_2} \frac{1}{2\omega_1 \omega_2} \frac{1}{p_1^0 - \omega_1 - E_D + i\epsilon} \frac{1}{p_1^0 - \omega_2 - E_D + i\epsilon},
\]
that, performing the analytical integration in \( dp^0 \), becomes
\[
    t = \frac{5}{4} g^4 \frac{1}{15} \int \frac{d^3 p}{(2\pi)^3} \left( \frac{\vec{q}^2}{4} - \frac{\vec{q}^2}{4} \right)^2 \left( \epsilon_i \epsilon_i \epsilon_i \epsilon_i + \epsilon_i \epsilon_i \epsilon_i \epsilon_i + \epsilon_i \epsilon_i \epsilon_i \epsilon_i \right) \frac{F^2}{p^2 - m_D^2 + i\epsilon} \frac{1}{E^2_D} \frac{1}{\omega_1 + \omega_2} \frac{1}{2\omega_1 \omega_2} \frac{1}{p_1^0 - \omega_1 - E_D + i\epsilon} \frac{1}{p_1^0 - \omega_2 - E_D + i\epsilon}.
\]

The combination of polarization vectors appearing in Eq. (33) can be rewritten in terms of the spin projector operators [19] as
\[
    \epsilon_i \epsilon_i \epsilon_i \epsilon_i + \epsilon_i \epsilon_i \epsilon_i \epsilon_i + \epsilon_i \epsilon_i \epsilon_i \epsilon_i = 5p^{(0)} + 2p^{(2)}.
\]
Thus, the final expression of the amplitude reads
\[
    t = \frac{5}{4} g^4 \frac{A}{15} \int \frac{d^3 p}{(2\pi)^3} \left( \frac{\vec{q}^2}{4} - \frac{\vec{q}^2}{4} \right)^2 \frac{F^2}{\omega_1 + \omega_2} \frac{1}{2\omega_1 \omega_2} \frac{1}{4E^2_D} \frac{1}{p_1^0 - \omega_1 - E_D + i\epsilon} \frac{1}{p_1^0 - \omega_2 - E_D + i\epsilon} \times \frac{1}{p_1^0 - \omega_2 - E_D + i\epsilon} \left( 1 + \frac{E_D + \omega_1 + \omega_2 - p_1^0}{p_1^0 - \omega_1 - E_D + i\epsilon} + \frac{E_D + \omega_1 + \omega_2 - p_1^0}{p_1^0 - \omega_2 - E_D + i\epsilon} \right),
\]
where \( A = 5 \) for the \( J = 0 \) case and \( A = 2 \) for the \( J = 2 \) case. The amplitude \( t \) in the two cases is shown in Fig. 8.

### C. Vector exchange

We follow the approach of Ref. [8], a study of the vector-vector interaction in the framework of hidden gauge formalism for the channels with quantum numbers \( C = 0 \) and \( S = 0 \).
In this paper possible vector-vector states are investigated using a unitary approach in coupled channels.

The starting point is once again a Lagrangian coming from the hidden gauge formalism, this time describing the interaction of vector mesons among themselves,

\[ \mathcal{L} = -\frac{1}{4} \langle V_{\mu\nu} V^{\mu\nu} \rangle, \tag{36} \]

where \( V_{\mu\nu} \) is defined as

\[ V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu], \tag{37} \]

\( V_\mu \) is given by Eq. (3) and the coupling constant \( g \) in \( SU(3) \) is the same as used in Eq. (1).

We shall comment on its extrapolation to the heavy sector later on.

From the Lagrangian in Eq. (36), two different types of interaction can be derived: a contact interaction, coming from the \([V_\mu, V_\nu]\) term,

\[ \mathcal{L}^{(c)} = \frac{g^2}{2} \langle V_\mu V_\nu V^{\mu\nu} - V_\nu V_\mu V^{\mu\nu} \rangle, \tag{38} \]

and the three-vector vertex

\[ \mathcal{L}^{(3V)} = ig \langle (\partial_\mu V_\nu - \partial_\nu V_\mu) V^{\mu\nu} \rangle. \tag{39} \]

The Lagrangian \( \mathcal{L}^{(3V)} \) produces the \( VV \rightarrow VV \) interaction by means of the exchange of one vector meson.

The channels we are interested in are the ones with quantum numbers \( I = 1, \) charm \( C = 0 \) and strangeness \( S = 0, \) which are \( D^* \bar{D}^*, K^* \bar{K}^*, \) \( \rho \rho, \rho \omega, \rho J/\psi, \rho \phi. \) From the Lagrangians in Eqs. (38) and (39), the amplitudes that will be used as the kernel to solve the Bethe-Salpeter equation can be evaluated. The reader can find all the details of the calculation in Ref. [8].

Here, we will only consider the case with \( J = 2 \) since this is the only spin channel where the interaction gives an attractive potential for \( D^* \bar{D}^* \rightarrow D^* \bar{D}^*. \) In [8], in addition to the \( J/\psi \rho \) channel, which is the most important after the \( D^* \bar{D}^* \), the \( \rho \rho, \rho \omega, \rho \phi \) light vector channels were also considered. However, the thresholds of these channels are situated at energies much smaller than the mass of the state we are looking for, such that the results would be only slightly affected by their inclusion.

The expressions of these potentials are reported in the following equations, including both the contact and the vector-exchange term:

\[ t_{D^* \bar{D}^* \rightarrow D^* \bar{D}^*} = -g_D^2 + g_D^2 \frac{2m_\omega^2 m_\rho^2 + m_{J/\psi}^2 (-m_\omega^2 + m_\rho^2)) (4m_D^2 - 3s)}{4m_{J/\psi}^2 m_\omega^2 m_\rho^2}, \tag{40} \]

\[ t_{D^* \bar{D}^* \rightarrow \rho J/\psi} = -2gg_D + g_D^2 \frac{2m_D^2 + m_{J/\psi}^2 + m_\rho^2 - 3s}{m_D^2}, \tag{41} \]

where \( m_\rho, m_\omega \) and \( m_{J/\psi} \) are the masses of the \( \rho, \omega \) and \( J/\psi \) mesons respectively. The constant \( g_D = m_D^*/(2f_D) \), which was used in [8], is analogous to the coupling \( g \) for light mesons, with \( f_D = 206/\sqrt{2} = 145.66 \) MeV. However, as we discuss below, we can use constrains of heavy quark spin symmetry to provide a more accurate coupling.

The exercise to relate the \( D^* D \pi \) vertex to the \( K^* K \pi \) in [36] is repeated in that work for the Weinberg-Tomozawa term that we are considering now, based on the exchange of
vector mesons. The $D^*\bar{D}^* \rightarrow D^*\bar{D}^*$ is now mediated by $J/\psi$ exchange ($c\bar{c}$) in analogy to the $\phi$ exchange in $K^*\bar{K}^* \rightarrow K^*\bar{K}^*$. There is a factor $\omega_{D^*}/\omega_K$ between the $D^* D^* J/\psi$ and the $K^* K^* \phi$ vertices and, since the $K^* K^* \phi$ vertex is proportional to $\omega_K$, the $D^* D^* J/\psi$ will have the same proportionality coefficient multiplied by $\omega_{D^*}$, which is what the straight application of $SU(4)$ provides in this case. Note that the vector exchange term in Eq. (40) at the $D^*\bar{D}^*$ threshold, for simplicity, gives, with $m_{\omega} = m_{\rho}$, $g_{D^*}^2 \frac{m_{\omega}^2}{m_{J/\psi}^2}$. There we see explicitly the energy of the $m_{D^*}$ from the two vertices and $m_{J/\psi}^2$ from the $J/\psi$ propagator. According with the previous argument this should be $g_{D^*}^2 \frac{m_{\omega}^2}{m_{J/\psi}^2}$. We, thus, use here the normal $g$ coupling which is in agreement with the heavy quark spin symmetry. For consistency, we also take $g^2$ in the contact term, which is smaller than the $J/\psi$ exchange one, and in the transition potential of Eq. (41). The use of the new coupling will have as a consequence the reduction of the binding of the $I = 1$ state with respect to the one found in [5].

The two potentials are plotted in Fig. 9 as functions of the centre of mass energy $\sqrt{s}$. The expressions of Eqs. (40) and (41) provide the potential $V$ that must be used to solve the Bethe-Salpeter equation in coupled channels

$$T = (1 - VG)^{-1}V ,$$

where $V$ is a $2 \times 2$ matrix with elements $V_{11} = t_{D^* D^* \rightarrow D^* D^*}$, $V_{12} = V_{21} = t_{D^* D^* \rightarrow \rho J/\psi}$, and $V_{22} = 0$. The matrix $G$ is the $2 \times 2$ diagonal loop function matrix whose elements are given by

$$G_i = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\epsilon} \left( \frac{1}{(q-P)^2 - m_2^2 + i\epsilon} \right) ,$$

with $m_1$ and $m_2$ the masses of the two mesons involved in the loop in the channel $i$ and $P$ the total four-momentum of the mesons.

After the integration in $dq^0$, Eq. (43) becomes

$$G_i = \int \frac{d^3q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\omega_1 \omega_2} \left( \frac{1}{(P^0)^2 - (\omega_1 + \omega_2)^2 + i\epsilon} \right) ,$$

which is regularized by means of a cutoff in the three-momentum $q_{max}$.
The function $G_i$ can be also written in dimensional regularization as

\[
G_i = \frac{1}{16\pi^2} \left( \alpha_i + \log \frac{m_1^2}{\mu^2} + \frac{m_2^2 - m_1^2 + s}{2s} \log \frac{m_2^2}{m_1^2} + \frac{p}{\sqrt{s}} \log \frac{s - m_2^2 + m_1^2 + 2p\sqrt{s}}{s + m_2^2 - m_1^2 + 2p\sqrt{s}} \right) \\
+ \left( \log \frac{s + m_2^2 - m_1^2 + 2p\sqrt{s}}{-s - m_2^2 + m_1^2 + 2p\sqrt{s}} \right),
\]

where $p$ is the three-momentum of the mesons in the centre of mass

\[
p = \frac{\sqrt{(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)}}{2\sqrt{s}}.
\]

### III. RESULTS

The first thing we shall do is to compare the strength of the $2\pi$ exchange with that of the local hidden gauge. A rough comparison can be done by comparing $\int d^3q V(q)$ in both cases. As we can see by looking at Figs. 6 and 8, the correlated and uncorrelated $2\pi$ exchange have opposite sign and cancel partially. We find for $\pi^\ast$ that the strength of the vector exchange potential for $D^\ast\bar{D}^\ast$ was seen in the $(D^\ast\bar{D})^\pm$ invariant mass spectrum close to the $(D^\ast\bar{D})^\pm$ threshold, which was interpreted in [7] as a signal of a $J = 0$ resonance at 4025 MeV. However in [23] it was found that the spectrum could be equally reproduced assuming a $J = 2$ resonance below threshold, with a mass around 3990 MeV and a width of 160 MeV. A fit with about 8 MeV less binding and smaller width is also acceptable by looking at the different options discussed in [23]. Our choice of the parameters is motivated to get a binding similar to that suggested in [23], but we discuss below our uncertainties. The finding of the present paper would give support to the interpretation of the results of [7] as a consequence of an $I = 1$ resonance coming from the $D^\ast\bar{D}^\ast$ interaction, with the option suggested in [23] of a bound $D^\ast\bar{D}^\ast$ state with relatively large width.

We have also evaluated the uncertainties in the results due to the possible contribution of the two pion exchange in the interaction. As we already mentioned in the beginning of this section, this contribution is attractive. In order to take it into account, we increase the magnitude of the vector exchange potential for $D^\ast\bar{D}^\ast \to D^\ast\bar{D}^\ast$ of Eq. (40) and see how the position of the peak changes. We find that, with an increase in the magnitude of the
potential of 50%, the energy of the peak decreases of about 5 MeV. Then we did the same thing, but adjusting the cutoff used in Eq. (45) in order to maintain fixed at 3998 MeV the position of the peak. The results obtained are shown in Fig. 11. Increasing the magnitude of $t_{D^* D^* \to D^* D^*}$, the peak in $|T_{11}|^2$ is maintained in the same position using a lower cutoff. In the case of an increase of 20% (the thin, continuous line in Fig. 11), we need a cutoff of $q_{\text{max}} \simeq 940$ MeV, while in the case of 50% (the dashed line in Fig. 11), $q_{\text{max}} \simeq 930$ MeV. The shape of $|T_{11}|^2$ is slightly changed when going to higher magnitudes, giving a narrower peak and a higher strength.

We should comment on our use of a cutoff around 960 MeV, when in other calculations based on the same extension of the local hidden gauge theory smaller values of the cutoff are
used. Indeed, in the study of hidden charm baryon states in [50] and hidden beauty states in [51] values around 800 MeV are used. This is easily justified if one recalls that in the hidden charm or hidden beauty cases one is exchanging light vector mesons and the mass of the vector mesons is already giving the range in momenta of the interaction [52]. Here we are exchanging $J/\psi$, which barely restricts the momentum range. Hence the cutoff is expected to be larger. We have taken natural values for $\alpha_i$, or the cutoff, guided by the results of the analysis of [23]. Yet, it is interesting to see what happens if we reduce the cutoff. In Fig. 12 we show $|T_{11}|^2$ for different values of the cutoff. We can see that as $q_{\text{max}}$ decreases, the peak of $|T_{11}|^2$ is moving closer to the threshold and its strength decreases. At $q_{\text{max}} = 700$ MeV we already have a clear cusp and, for lower values of $q_{\text{max}}$, the cusp remains but the strength of $|T_{11}|^2$ at the peak is very weak and we would no longer be able to produce an enhancement of the $D^* \bar{D}^*$ invariant mass distribution as seen in the experiment of [7]. It is also interesting to see that even for values of $q_{\text{max}} \simeq 800$ MeV as in [50, 51], which we argued are too low, we still find a state bound by a few MeV. On the other hand bigger values of $q_{\text{max}}$ would produce a too large binding that would contradict the results of the analysis of [23]. Hence, considering uncertainties in our model, we can say that we are obtaining a bound $D^* \bar{D}^*$ state (decaying in $J/\psi \rho$) within 3990 – 4000 MeV, with a width of about 100 MeV.

As already mentioned in Section II C in [8] also the $\rho \rho$, $\rho \omega$, $\rho \phi$ light vector channels were considered and the $\rho \omega$ and $\rho \phi$ also give some contribution to the width. A slight increase in the value of $\Gamma \simeq 100$ MeV, would be in agreement with the analysis of [23] where $\Gamma = 160$ MeV.

![Graph showing $|T_{11}|^2$ as a function of $\sqrt{s}$](image)

FIG. 12. $|T_{11}|^2$ as a function of $\sqrt{s}$, for different values of the cutoff $q_{\text{max}}$. From up down, $q_{\text{max}} = 960, 900, 850, 800, 750, 700, 650, 600, 550, 500$ MeV.

### IV. CONCLUSIONS

We have studied the interaction of $D^* \bar{D}^*$ in $I = 1$ from the perspective of the local hidden gauge approach, extrapolating the model to account for the exchange of heavy vectors. This is necessary here once we prove that the exchange of light vectors is OZI forbidden in
$I = 1$. The interaction is then weaker than for $I = 0$, where the exchange of light vectors is allowed, but still strong enough to weakly bind the system. We have also taken into account the coupled channel $J/\psi \rho$, which is open for decay and is responsible for a width of the state of the order of 100 MeV. We also mention that the exchange of light $q\bar{q}$ is OZI forbidden, which implies that the sum of the exchange of light pseudoscalar mesons also vanishes if the masses of these mesons are taken degenerate. Because of that, we study the effect of two pion exchange, with or without interaction, but we find that this contribution is smaller than the exchange of heavy vectors. The study conducted here complements the one of [23] where the peak seen in the $D^*\bar{D}^*$ spectrum in the $e^+e^- \rightarrow (D^*\bar{D}^*)^{\pm}\pi^{\pm}$ reaction, that led the experimental team to claim a $J^P = 1^+ Z_c(4025)$, was reinterpreted as a possible $2^+$ bound state of $D^*\bar{D}^*$ with $I = 1$. Both the mass and width that we obtain are compatible with the results obtained in [23] from a fit to the experimental data, from where we would conclude that the state that we find in our approach provides a natural explanation of the experimental results of [7] and one can claim a resonance from this experiment but with a different energy, width ($M = 3990 - 4000$ MeV, $\Gamma \simeq 100$ MeV) and quantum numbers ($I^G = 1^-, J^{PC} = 2^{++}$).

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