STUDY OF FRACTALITY AND CHAOTICITY IN CENTRAL 4.5A GeV/c C-CU COLLISIONS

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ABSTRACT

Multifractality of charged particle pseudorapidity distributions is analyzed in central collisions of carbon and copper nuclei at 4.5 GeV/c per nucleon. Within the method of normalized factorial moments, modified to remove the bias of infinite statistics in the normalization, intermittency-like behavior (scaling) is observed up to eight-particle fluctuations. The study indicates a possible non-thermal phase transition inside cascading and two different regimes presented in multiparticle production. Dynamics of fractal structure formation is further studied using the chaoticity approach recently proposed. The distributions of the horizontal factorial moments are considered and a scaling behavior, referred to as erraticity, of the normalized moments of the distributions is obtained. The corresponding entropy indices are calculated indicating chaotic nature of multiparticle production with a specific self-similar structure.

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STUDY OF FRACTALITY AND CHAOTICITY 
IN CENTRAL 4.5\textit{A} GeV/c C-CU COLLISIONS

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Multifractality of charged particle pseudorapidity distributions is analyzed in central collisions of carbon and copper nuclei at 4.5 GeV/c per nucleon. Within the method of normalized factorial moments, modified to remove the bias of infinite statistics in the normalization, intermittency-like behavior (scaling) is observed up to eight-particle fluctuations. The study indicates a possible non-thermal phase transition inside cascading and two different regimes presented in multiparticle production. Dynamics of fractal structure formation is further studied using the chaoticity approach recently proposed. The distributions of the horizontal factorial moments are considered and a scaling behavior, referred to as erraticity, of the normalized moments of the distributions is obtained. The corresponding entropy indices are calculated indicating chaotic nature of multiparticle production with a specific self-similar structure.

Study of geometrical structure of dynamical fluctuations produced in high-energy collisions gives an exceptional opportunity to investigate hadroproduction process. The intermittency/fractal patterns of such density fluctuations observed in all types of collisions, one connects with two possible scenarios. A geometrical monofractal structure is associated with a (most expected) thermal phase transition e.g., from a quark-gluon plasma to nuclear matter, while multifractality is assigned to a self-similar cascading from partonic state to final hadrons with a possible “non-thermal” (non-equilibrium) phase transition during the cascade. On the other hand, the quantities under the study are usually calculated through averaged moments of the distributions and changes of the density fluctuations from event to event are not taken into account. These lead to the loss of information about more structure, namely, about chaoticity nature of multiparticle production. Recently, erraticity approach has been proposed to study chaotisity in high-energy physics, in particular, to calculate entropy indexes, large value of which can be considered as a signature of chaotic dynamics in self-similar multiparticle production (e.g. in branching processes in QCD).

In this report we perform an investigation of dynamics of fractality and chaoticity in multihadron production in central collisions of relativistic nuclei. Such studies have a specific interest due to the above mentioned expectation of quark-gluon plasma formation in high-energy nuclear reactions and, on the other hand, in a sense of a possible hadronic nature of intermittency, being also stronger with energy decrease. Note, the study of fractality continues our previous investigations, where different regimes in multiparticle productions has been mentioned.

The contribution presented deals with the data came from interactions of the JINR Synchrophasotron (Dubna) 4.5 \textit{A} GeV/c $^{12}\text{C}$ beam with a copper target inside the 2m Streamer Chamber SKM-200\textsuperscript{4}. A central collision trigger was used: absence of charged particles with momenta $p > 3$ GeV/c in a forward cone of 2.4° was required.

The scanning and the handling of the film data were carried out on special scanning tables of the Lebedev Physical Institute (Moscow).\textsuperscript{5} The average measurement error in the momentum $\langle \varepsilon_p/p \rangle$ was about 12%, and that in the polar angle measurements was $\langle \varepsilon_\vartheta \rangle \approx 2^\circ$.\textsuperscript{6} In total, 663 events with charged particles in the pseudorapidity window $\Delta \eta = 0.2 - 3.0$ are considered ($\eta = -\ln\tan(\vartheta/2)$). The accuracy $\langle \varepsilon_\eta \rangle$ does not exceed 0.1. In addition, particles with $p_T > 1$ GeV/c are excluded from the investigation as far as no negative charged particles were observed with such a transverse momentum. Under the assumption of an equal number of positive and negative pions, this cut was applied to eliminate the

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contribution of protons. The average multiplicity is $23.8 \pm 0.4$.

The density fluctuations are considered in the $\eta$-phase-space. To avoid the problem connected with a non-flat shape of the distribution $\rho(\eta)$, we use the “cumulative” variable

$$\tilde{\eta} = \frac{\int_{\eta_{\min}}^{\eta} \rho(\eta')d\eta' / \int_{\eta_{\min}}^{\eta_{\max}} \rho(\eta')d\eta'}{\int_{\eta_{\min}}^{\eta_{\max}} \rho(\eta')d\eta'},$$

with the uniform spectrum $\rho(\tilde{\eta})$ within the interval $[0,1]$.

Self-similar (fractal) dynamics is revealed by a scaling-law,

$$F_q \propto M^{\varphi_q}, \quad 0 < \varphi_q \leq q - 1, \quad (q \geq 2),$$

of the $q$th order normalized (“vertical”) factorial moments

$$F_q = \frac{N^q}{M} \sum_{m=1}^{M} \frac{\langle n[m] \rangle}{N[m]},$$

over $M$ number of the bins into which the phase space of produced particles is divided. Here, $n_m$ is the number of particles in the $m$-th bin per event, $N_m$ is the similar number but calculated for all $N$ events, and $\langle \cdots \rangle$ denotes averaging over the events. $n[q]$ is the $q$th power factorial multinomial, $n(n-1)\cdots(n-q+1)$. Note, we apply the modified moments proposed to remove the biased estimator of the normalization (especially in small bins), that along with the use of the “transformed” variable allow to study higher-order moments.

![Fig. 1: ln $F_q$ vs. ln $M$ for $q = 3$ and 5.](image1.png)

![Fig. 2: $\lambda_q$ vs. $q$.](image2.png)

The exponents $\varphi_q$ show the strength of an intermittent structure of the distribution. Fractality is characterized by the codimensions, $d_q = \varphi_q/(q-1)$, indicating monofractal objects if $d_q = \text{const.}(q)$, and multifractal ones if the hierarchy, $d_q > d_p, q > p$, is satisfied. Then, a corresponding connection with a phase transition can be found as discussed in the beginning.

As a signal of the transition, the existence of a minimum of the function

$$\lambda_q = (\varphi_q + 1)/q$$

at a certain “critical” value of $q = q_c$ is suggested. The minimum of eq. (4) may also be a manifestation of coexistence of many small (liquid-type) fluctuations and a few high-density ones.
Fig. 1 presents dependence of $\ln F_q$ on $\ln M$, depicted for $q = 3$ and 5. The different increase of the moments with increasing $M$ for the different $M$-regions seen on the plots continues up to $q = 8$ (not shown), confirming our earlier results of the existence of distinguished regimes of particle production at various bin-averaging scales.

As in the preceding analysis, multifractality in hadroproduction has been observed, i.e. $d_q > d_p$ at $q > p$ (not shown), supporting a cascading scenario of particle production. Different increase of the $d_q$ with $q$ was found, indicating possible change of the regime of particle creation.

Fig. 2 shows the $\lambda_q$ function (4), confirming that at least two regimes of particle production exist: one with the phase transition at $q_c$ between 4 and 5, and another one for which no critical behavior is reached. The $q_c$-value and the $M$-intervals, which exhibit the minimum of $\lambda_q$, $11 \leq M \leq 17$, $11 \leq M \leq 24$, are found to be about the same as in our previous studies of the most “critical” $M$-region, $11 \leq M \leq 17$, as well as in recent similar investigations of heavy-ion collisions at ultra-high energies.

Taking into account multifractality, the critical $q_c$ indicates a “non-thermal” phase transition rather during the cascade than within one phase. Although the interpretation may be a matter of debate, it must be noted that the minimum was found earlier also in hadronic interactions at small $p_T$ and has been indicated in high-energy nuclear interactions.

Intermittency has not fully exhausted the characterics of dynamical fluctuations. To study change of such fluctuations from event to event and then, to find out the chaotiicity characteristics of the observed self-similar process it was proposed to consider the normalized moments $C_{p,q}$, defined by

$$C_{p,q} = \langle f_q^p \rangle / \langle F_q^H \rangle^p ,$$

of the distributions of the normalized “horizontal” factorial moments $F_q^H = \langle f_q \rangle$. Here,

$$f_q = \frac{N^q}{M} \sum_{m=1}^{M} \frac{n_m^{[q]}}{(N/M)^{[q]}}$$

and $N$ is a total number of particles. Note, the order $p$ of the $C_{p,q}$-moments can be non-integer, negative or zero, but are restricted with zero $f_q$’s (“empty-bin effect”).

The degree of erratic fluctuations of the self-similar dynamics, indicated by $\lambda_q$, is suggested to have a scaling behavior,

$$C_{p,q} \propto M^{\psi_{p,q}} .$$
The exponents $\psi_{p,q}$ are referred to as erraticity indices and show a strength of “spatial” fluctuations.

The power-law (7) of the $C_{p,q}$-moments vs. $M$ is shown in Figs. 3 and 4. The first one illustrates the $M$-dependence of the moments for different $p = 0.4, 1.2, 1.6$, and fixed $q = 3$, while Fig. 4 presents the case, when $p$ is fixed, $p = 2$, and $q$ changes, $q = 2, 3, 5$. As seen, the scaling behavior is satisfied up to small bins, however for $q > 2$ contributions of zero $f$’s (6) are sensitive making the moments to saturate. This also is a reason why we do not consider higher $q$-orders ($q \geq 6$) as well as $p < 0$.

Note, shown by linear fit the most “critical” $M$-interval in Figs. 3 and 4 exhibits the same rise character as in Fig. 1. Corresponding erraticity indexes are shown in Fig. 5a, compared to the “non-critical” fit for $2 \leq M \leq 17$. The fourth order polynomials are used to approximate the $\psi_{p,q}$-slopes as functions of $p$. The general properties of the $\psi$-functions are seen: $\psi_{p,q} < 0$ for $0 < p < 1$, $\psi_{p,q} > 0$ for $p > 1$ (see also Fig. 3). Nevertheless, it is clear, that the fits deviates from zero for $p = 0$ that is connected with the empty-bin contributions.

![Fig. 5: Erraticity indexes $\psi_{p,q}$ (7), $q = 3, 4$, as functions of $p$ for different $M$-intervals: (a) $11 \leq M \leq 17$; (b) $2 \leq M \leq 17$.](image)

Of particular interest is an index $\mu_q$ defined as

$$\mu_q = \frac{d}{dp}\psi_{p,q}\bigg|_{p=1}, \quad (8)$$

shown to be related to the entropy in event space and be a possible measure of chaoticity. The larger $\mu_q$ is, the more chaotic the system is. In the case of a phase transition high values of $\mu_q$ are expected.

Table 1 presents the $\mu_q$ calculated for different $M$-intervals and $q = 2...5$. The large values found for each interval indicate very chaotic dynamics of particle production, confirming its cascading nature. It is worthwhile to note increase of the entropy index with approaching to the “critical” $M$-intervals that lends support for a phase transition. However, we must emphasize contribution of empty bins at high $q$’s.

![Table 1](image)
Table 1. The entropy indexes $\mu_q$.

| $M$-interval | $q = 2$ | $q = 3$ | $q = 4$ | $q = 5$ |
|--------------|--------|--------|--------|--------|
| 11-17        | 0.033 ± 0.014 | 0.251 ± 0.081 | 0.926 ± 0.218 | 2.070 ± 0.355 |
| 11-24        | 0.032 ± 0.012 | 0.222 ± 0.023 | 0.743 ± 0.082 | 1.581 ± 0.170 |
| 2-17         | 0.012 ± 0.002 | 0.095 ± 0.010 | 0.309 ± 0.034 | 0.631 ± 0.065 |
| 2-43         | 0.024 ± 0.001 | 0.175 ± 0.005 | 0.516 ± 0.014 | 0.967 ± 0.025 |

In summary, the dynamics of fractality and chaoticity in multiparticle production is studied with the pseudorapidity spectra of charged particles produced in central C-Cu collisions at 4.5 GeV/c/nucleon. The scaled factorial moments are calculated in the transformed variables and corrected to take into account the bias of infinite statistics. Multifractal structure of pseudorapidity density fluctuations is observed to be of dynamical nature, indicating a possible non-thermal phase transition. The existence of two different regimes of particle production during the hadronization cascade is mentioned. These results based on the increased statistics confirm conclusions of our previous analysis.

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References

1. E.A. De Wolf, I. Dremin, W. Kittel, Phys. Reports 270, 1 (1996).
2. A. Białas, Nucl. Phys. A 525, 345c (1991); ibid. A 545, 285c (1992).
3. R. Peschanski, Int. J. Mod. Phys. A 6, 3681 (1991).
4. R.C. Hwa, Acta Phys. Pol. B 27, 1789 (1996).
5. Z. Cao and R.C. Hwa, Phys. Rev. Lett. 75, 1268 (1995); Preprint OITS-610 (Eugene, 1997).
6. Z. Cao and R.C. Hwa, Phys. Rev. D 53, 6608 (1996).
7. E.K. Sarkisyan et al., Phys. Lett. B 347, 439 (1995).
8. E.K. Sarkisyan et al., 7th Int. Workshop on Multiparticle Production, “Correlations and Fluctuations” (Nijmegen, 1996), eds. R.C. Hwa et al. (World Scientific, 1997), p. 271.
9. A. Abdurakhimov et al., Prib. Tekhn. Eksp. 5, 53 (1978).
10. G.G. Taran et al., FIAN preprint No.20 (Moscow, 1987).
11. A. Białas and M. Gazdzicki, Phys. Lett. B 252, 483 (1990); W. Ochs, Z. Phys. C 50, 339 (1991).
12. K. Kadija and P. Seyboth, Z. Phys. C 61, 465 (1994).
13. M. Blażeck, Int. J. Mod. Phys. A 12, 839 (1997), and 7th Int. Workshop on Multiparticle Production, “Correlations and Fluctuations” (Nijmegen, 1996), eds. R.C. Hwa et al. (World Scientific, 1997), p. 229.
14. D. Ghosh et al., Z. Phys. C 71, 243 (1996); ibid. C 73, 269 (1997).
15. P.L. Jain and G. Singh, Phys. Rev. C 44, 854 (1991).
16. R.C. Hwa, 7th Int. Workshop on Multiparticle Production, “Correlations and Fluctuations” (Nijmegen, 1996), eds. R.C. Hwa et al. (World Scientific, 1997), p. 303.