Modeling of the Dynacard of the Rocking Machine Taking into Account the Characteristics of the Drive Unit

V B Sadov

Ph.D., Associate Professor, Department of Automated Control Systems, South Ural State University, 76, Prospect Lenina, Chelyabinsk 454080, Russian Federation

E-mail: sv_2005@inbox.ru

Abstract. The issue of synthesis of models of the "well-rod deep-well pump" system is considered taking into account the characteristics of the mechanical part of the installation with the rod depth pump and the characteristics of the drive. This model allows to obtain dynacards and wattmeterograms, which is important when using it to develop diagnostic and control algorithms. An example of using the mechanics of a traditional rocking machine is considered. The necessary formulas for modeling the oil production process are presented. The obtained model can serve as a basis for constructing virtual and physical stands designed for testing the algorithms for controlling and diagnosing installations with a rod deep pump and testing the efficiency of their control systems. Conclusions are made about the applicability of this approach to the synthesis of models and for other types of actuators of installations with sucker-rod deep pumps.

1. Introduction
To simulate the "well-rod deep-well pump" system, a partial-wave equation is solved that describes the behavior of both the rod columns and the deep pump itself. In this area, the works of Sam Gibbs [1, 2] are the most famous, in Russia the articles of V.M. Kasyanov [3] are considered classical. Currently, this topic in Russia is most actively engaged in the Ufa State Oil Technical University and in the group of enterprises "Grant" [4, 5]. In these papers, the authors consider various models of the "well-pump" system in order to obtain dynacard data with faulty and faulty submersible equipment. The obtained models allow to shorten the processing time of dynacard processing algorithms and to check the algorithms of automatic well control. In this case, the drive is often not taken into account because of its rather low contribution to the dynamics of the system, and it is taken into account only in the law of motion of the polished rod. But in modern conditions it becomes important to take into account both the mechanical characteristics of the units and the characteristics of the drive in the models for the purpose of constructing drive control systems with no sensors on the moving part of the plants and the synthesis of control algorithms for such systems [6, 7].

2. Building a Model
The load on the polished rod, its speed and movement is expressed through the load, speed and movement on the rod of the sucker rod as [8]
\[ P(t) = P_p (t - \delta) + Q + [V(t) - V_p (t - \delta)] \cdot \frac{E \cdot f}{a}, \]

\[ V(t) = 0.5 \cdot [V_p (t + \delta) + V_p (t - \delta)] + 0.5 \cdot [P_p (t + \delta) - P_p (t - \delta)] \cdot \frac{d}{E \cdot f}, \]

\[ U(t) = 0.5 \cdot [U_p (t + \delta) + U_p (t - \delta)] + 0.5 \cdot \left[ \int_0^t P_p (t + \delta) \cdot dt - \int_0^t P_p (t - \delta) \cdot dt \right] \cdot \frac{a}{E \cdot f}, \]

where \( t \) – time variable;
\( \delta = \frac{H}{a} \) – time of propagation of the displacement wave from one end of the rod to the other (\( a \) – bar speed);
\( f \) – bar cross-sectional area;
\( E \) – modulus of elasticity of rod material;
\( Q \) – rod weight in liquid (in oil mixture).

The scheme of the traditional rocking machine is shown in Figure 1 [9]. Here \( l_1 \) and \( l_2 \) – lengths of the arms of the rocker, \( M_1 \) and \( M_2 \) – counterweights, \( L \) – conrod length, \( l \) – crank length. The output shaft of the reducer is taken as the origin of the coordinate reference (\( p = 0 \)). The diagram shows the forces and their projections. The forces acting on the crank from the deep rod pump (force \( P \) in Figure 1) and the weights of the balancing weight \( M_1 \) (force \( P_3 \) in Figure 1), are shown in Figure 2.

Figure 1. Scheme of the rocking machine

Figure 2. The diagram of forces on the crank

The equations for the forces participating in the process of motion of the mechanism:
\[ P_0' = P \cdot \cos \Delta, \]  

where \( P \) – force on polished rod.
\[ P_0 = P_0' \cdot \frac{l_1}{l_2}. \]  

The force acting along the connecting rod can be found as
\[ P_3 = \frac{P_0}{\cos(90^\circ - \beta)}. \]  

Hence the component of this force, perpendicular to the balance, is calculated as
With the known dimensions of the components of the rocking machine $l_1, l_2, L, l, R, H$ and the angle of rotation of the output shaft of the reducer $\alpha$, it is possible to determine the positions of the points A, B, C, D and, respectively, the angles of rotation of the links of the mechanism.

The coordinates of point B are defined as
\[
\begin{align*}
x_B &= -R, \\
y_B &= H.
\end{align*}
\]

The coordinates of point D:
\[
\begin{align*}
x_D &= -l \cdot \cos \alpha, \\
y_D &= -l \cdot \sin \alpha.
\end{align*}
\]

The coordinates of point A
\[
\begin{align*}
(x_A - x_B)^2 + (y_A - y_B)^2 &= l_2^2, \\
(x_A - x_D)^2 + (y_A - y_D)^2 &= L^2.
\end{align*}
\]

Hence
\[
\begin{align*}
x_A^2 + y_A^2 - 2 \cdot x_A \cdot x_B - 2 \cdot y_A \cdot y_B + x_B^2 + y_B^2 &= l_2^2, \\
x_A^2 + y_A^2 - 2 \cdot x_A \cdot x_D - 2 \cdot y_A \cdot y_D + x_D^2 + y_D^2 &= L^2.
\end{align*}
\]

Subtracting the second equation from the first equation of this system, we have:
\[
-2 \cdot x_A \cdot (x_B - x_D) - 2 \cdot y_A \cdot (y_B - y_D) + x_B^2 + y_B^2 - x_D^2 - y_D^2 - l_2^2 + L^2 = 0
\]

or
\[
x_A = -y_A \cdot \frac{x_B - x_D}{x_B - x_D} + \frac{x_B^2 + y_B^2 - x_D^2 - y_D^2 - l_2^2 + L^2}{2 \cdot (x_B - x_D)}.
\]

The last expression can be represented as
\[
x_A = k_1 \cdot y_A + k_2,
\]

where
\[
k_1 = -\frac{y_B - y_D}{x_B - x_D} ; k_2 = \frac{x_B^2 + y_B^2 - x_D^2 - y_D^2 - l_2^2 + L^2}{2 \cdot (x_B - x_D)}.
\]

Substituting (12) into the first equation of system (8), we have:
\[
(k_1 \cdot y_A + k_2 - x_B)^2 + (y_A - y_B)^2 = l_2^2,
\]

hence
\[
y_A^2 \cdot (k_1^2 + 1) + y_A \cdot [2 \cdot k_1 \cdot (k_2 - x_B) - 2 \cdot y_B] + (k_2 - x_B)^2 + y_B^2 - l_2^2 = 0.
\]

Solving this quadratic equation, we obtain the value $y_A$, and substituting the value obtained in (12), we obtain the value $x_A$. 

\[
P_2 = P_1 \cdot \sin \gamma.
\]
Having the coordinates of points A, B and D, you can get the value of the angles. For an angle $\Delta$ it is:

$$
\Delta = \arcsin\left(\frac{y_A - H}{l_2}\right).
$$

(16)

When calculating the angle $\beta$ it should be noted that it is always possible to form a triangle from points A, B and D, therefore, to calculate this angle, one can use the known formulas for calculating angles in a triangle:

$$
k_{BA} = \frac{y_A - y_B}{x_A - x_B},
k_{AD} = \frac{y_D - y_A}{x_D - x_A},
$$

(17)

$$
tg\beta = \frac{k_{AD} - k_{BA}}{1 + k_{AD} \cdot k_{BA}}.
$$

When calculating the angle $\gamma$ it must be taken into account that this angle can take practically any value, therefore it is possible to determine the angle between the origin of OX and the crank and calculate the angle $\gamma$ taking into account this angle and the angle of rotation of the crank $\alpha$.

The coordinates of point C after determining the angles between the links of the rocking machine can be obtained as

$$
\begin{align*}
x_C &= -R - l_1 \cdot \cos \Delta, \\
y_C &= H - l_1 \cdot \sin \Delta.
\end{align*}
$$

(18)

Here the component $(-l_1 \cdot \sin \Delta)$ will determine in us the movement of the polished rod (the real geometry of the cable traverse assembly is not taken into account).

The torque acting output shaft of the gearbox can be obtained as

$$
M_{up} = P_2 \cdot l.
$$

(19)

When considering the movement of the rocking machine, it should be noted that the electric motor is mechanically connected via a gear (with a gear ratio $k_n$), and the output shaft of the reducer corresponds to the point 0 in Figure 1. Therefore, the load torque that occurs on the motor shaft due to the force on the polished rod can be obtained using

$$
M_u = M_{up} \cdot k_n.
$$

(20)

It is possible to write the equation of torques on the motor shaft as

$$
M = M_u - M_y - M_x,
$$

(21)

where $M_y$ – torque due to inertia of counterbalancing weights (counterweights), $M_x$ – torque from the gravity of the counterbalancing cargo located on the crank.

The torque due to the inertia of the counterbalancing loads can be considered as the sum of two components: the torque of inertia of the counterweight on the crank and the torque of inertia of the counterweight on the balance bar. At the same time, the torques arising in the rest of the mechanics of the rocking machine are neglected because of their relative smallness. If you bring all the torques of inertia to the crank, you can write:
\[ M_y = -J_y \cdot \frac{\partial \omega}{\partial t} = -J_y \cdot k_y \cdot \frac{\partial n}{\partial t}, \]  

(22)

where \( J_y \) – torque of inertia of the counterweight, reduced to the motor shaft \( J_y = J_y \cdot k_y \) (\( J \) – torque of inertia of the balancing weight on the crank), \( \omega \) – angular speed of rotation of the counterweight relative to its axis of rotation (the speed of rotation of the output shaft of the reducer), \( n \) – current engine speed per minute, \( k_y \) – transfer factor, defined as

\[ k_y = \frac{2 \cdot \pi}{60}. \]  

(23)

The torque of the gravity of the balancer weight located on the crank can be found as

\[ M_c = P_3 \cdot l \cdot k_p \cdot \cos \alpha, \]  

(24)

where \( P_3 \) is determined by the weight of the balancing cargo on the crank, reduced to the connection point with the connecting rod, and is calculated by the formula

\[ P_3 = \frac{J}{l^2} \cdot g. \]  

(25)

To determine the torque on the engine \( M \) consider the mechanical characteristic of an asynchronous AC motor [10], shown in Figure 3.

![Figure 3. Mechanical characteristic of an AC induction motor.](image)

We will consider only the stable part of this characteristic, i.e. section 1-3, where point 2 roughly corresponds to the average value of the rotation speed of the engine output shaft \( n_0 \) (we take from the engine’s passport the speed of rotation at the rated load). Since the drive of the pumping machine mainly uses motors with increased slip, the slope of the characteristic of the section 1-3 can be significant. From this we can write the linearized equation of the dependence of the angular torque \( M \) on the motor shaft from the speed of rotation of the output shaft of the reducer \( n \):

\[ M = \frac{n_0 - n}{k_{nx}} - M_0, \]  

(26)

where \( n_0 \) – nominal motor rotation speed (at point 2 in Figure 3), \( M_0 \) – rated torque, \( k_{nx} > 0 \) – coefficient of decline in mechanical characteristics.

It is possible to write an expression for the law of motion of a polished rod:

\[ U(t) = l_1 \cdot \sin \Delta. \]  

(27)

All values will be calculated through the angle of rotation of the output shaft of the reducer (crank rotation), which can be defined as
\[ \alpha = \frac{2 \cdot \pi}{60} \int_0^t n \cdot k_p \cdot \frac{dt}{n dt}. \] (28)

It is also possible to determine the instantaneous power for the motor to realize the motion of the polished rod as

\[ W_{\text{ INST}} = M \cdot \omega_{\text{ INST}} = \frac{M \cdot 2 \cdot \pi \cdot n}{60}, \] (29)

where \( \omega_{\text{ INST}} \) – engine speed.

This model was implemented in the form of a computer program that, by specifying the nominal speed of rotation of the output shaft of the engine with a rod-type deep-well pump, obtains all points of movement of the polished rod, the forces on it, as well as torques in the mechanism and the current drive power.

3. Conclusion

The synthesized model of the "well-rod deep-well pump" system, taking into account the characteristics of the mechanical part of the installation with the rod depth pump and the characteristics of the drive, allows to take into account the real, non-sinusoidal motion of the polished rod during oil production, the unevenness of this movement. The model also makes it possible to obtain dynacards and watt-meters, which is important when using it to develop diagnostic and control algorithms [11]. The obtained model can serve as a basis for constructing virtual and physical stands designed for testing the algorithms for controlling and diagnosing installations with a rod deep pump and testing the efficiency of their control systems. Such stands allow to sharply reduce the material costs associated with carrying out field trials of the developed control systems. This approach to the synthesis of models can easily be adapted to chain and linear drives [12], which is often used in installations with a rod depth pump.

4. References

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Acknowledgement
The work was supported by Act 211 of the Government of the Russian Federation, contract № 02.A03.21.0011.