Measuring the Gravitomagnetic Distortion from Rotating Halos. I. Methods

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Abstract

Source galaxy images are distorted not only by a static gravitational potential, but also by frame-dragging induced by massive rotating objects like clusters of galaxies. Such an effect is well understood theoretically; it is therefore of great interest to estimate its detectability for future surveys. In this work, we analyze the lensing convergence $\kappa$ around rotating dark matter halos. The rotation of the massive objects generates a gravitomagnetic potential giving rise to an anisotropic contribution to the lensing potential. We construct an estimator $\hat{\kappa}_d$ to describe the difference between the symmetric enhancement and reduction of $\kappa$ around the halo rotation axis, finding that it is well approximated by a function proportional to the halo velocity dispersion squared times a dimensionless angular momentum parameter. Using simulation mocks with realistic noise level for a survey like the Legacy Survey of Space and Time (LSST), we test our estimator, and show that the signal from frame-dragging of stacked rotating lenses is consistent with zero within $1\sigma$. However, we find that the most massive cluster in SDSS DR7 spectroscopic selected group catalog has a line-of-sight rotation velocity of $195.0 \, \text{km s}^{-1}$ and velocity dispersion of $667.8 \, \text{km s}^{-1}$, which is at $1.2 \times 10^{-8}$ odds according to the angular momentum probability distribution inferred from $N$-body simulations. By studying SDSS DR7 spectroscopic selected group catalog, we show how rotating clusters can be identified, and, finding that fast rotating clusters might be more abundant than in estimates based on simulations, a detection of gravitomagnetic distortion may be at reach in future surveys.

Unified Astronomy Thesaurus concepts: Galaxy clusters (584); General relativity (641)

1. Introduction

Gravitational lensing is the phenomenon that light rays from distant galaxies are bent by foreground potentials. According to the strength of the distortion, it can be classified as strong gravitational lensing or weak gravitational lensing.

In the strong lensing regime, it is represented by characteristic features of multiple images, or giant arcs. This was first observed in 1979 (Walsh et al. 1979) where a quasar (QSO) lensed into two images (0957+561 A, B, with the help of a 2.1 m telescope from Kit Peak National Observatory. Since then, many strong lensing cases have been observed, such as the famous Einstein cross QSO 2237+0305 (Huchra et al. 1985), the “smiling face” giant arc SDSS J1038+4431, among 37 systems from the Sloan Digital Sky Survey (SDSS; York et al. 2000; Sharon et al. 2020), and SDP 81 from ALAM observation (Hazeveh et al. 2016), among others. By modeling the position and number of images from strong lensing systems, one can infer the underlined dark matter distribution (see, e.g., Tamura et al. 2015; Keeton 2001; Caminha et al. 2019). Time delay measurements from strongly lensed images can be used to put constraints on the Hubble constant $H_0$, Suyu et al. (2017) and Wong et al. (2020) found $H_0$ constrained from time delay to be in $4.4\sigma$ tension with measurements from the cosmic microwave background (Planck Collaboration et al. 2020; see also Birrer et al. 2020), which shows that the tension is alleviated when combining TDCOSMO and the SLAC strong lensing catalog as well as taking the mass-sheet degeneracy in the modeling.

In the weak gravitational lensing regime, the distortion is much smaller, about a few percent of the intrinsic shapes of galaxies. However, weak lensing is ubiquitous as long as massive objects are present between distant light sources and the observer. By stacking multiple images, weak lensing signal can be extracted from spectroscopic surveys. A commonly used statistic is galaxy–galaxy lensing, which is a powerful tool to study dark matter halos traced by galaxies, or clusters of galaxies (see, e.g., Sheldon et al. 2004; Mandelbaum et al. 2006; Luo et al. 2017, 2018). The high-order weak lensing statistics, i.e., cosmic shear, can also be obtained from wide-field imaging surveys, such as KiDS (Asgari et al. 2021), DES (Troxel et al. 2018), and HSC-SSP (Hikage et al. 2019). Both galaxy–galaxy lensing and cosmic shear, being especially sensitive to the amplitude of density perturbations and the fraction of matter in the universe, can be used to place tight constraints on the cosmological models (Troxel et al. 2018; Hikage et al. 2019). In particular, recent studies have shown that weak lensing is useful to constrain various gravity theories, namely, Chen et al. (2020) confronted weak lensing observations to a $f(T)$ model from Cai et al. (2020) (see Cai et al. 2016 for a review) to test general relativity at galactic scales, while Luo et al. (2020) found emerging gravity (Verlinde 2017) to be inconsistent with galaxy–galaxy lensing signals from SDSS DR7 data (Abazajian et al. 2009). Given the wealth of information available in the data collected routinely in spectroscopic surveys, it is crucial to scrutinize all potential systematics in order to extract unbiased measurements from gravitational lensing.
On the observational side, much effort is made to mitigate instrumental systematics from, e.g., the inaccuracy of the point-spread function reconstruction (Mandelbaum et al. 2005; Lu et al. 2018), photo-z bias, or selection function (Mandelbaum et al. 2018).

Another source of potential systematics can arise from the astrophysical properties of the lenses. In general relativity, a rotating massive object exerts an extra potential through the Einstein–Thirring–Lense effect (Bardeen & Petterson 1975). This “frame-dragging” can be seen as a gravitomagnetic distortion in the weak-field approximation. This effect is rather subtle for the perihelion of Mercury, contributing to the precession of –0°002 per century (Clemence 1947), which is orders of magnitude smaller than the other sources of precession. However, it can be important in the vicinity of very massive objects, e.g., in galaxy clusters (Miller et al. 2005; Oguri et al. 2018), or galaxy groups (Yang et al. 2007). At cluster scales, where dark matter halos can reach masses $10^{15}$ times heavier than the Sun and fast rotating speed, gravitomagnetic distortion may become significant enough to be measured. Combined with kinematic Sunyaev–Zel’dovich (SZ) effect (Chluba & Mannheim 2002), this effect can be used to further constrain halo rotation properties, thanks to high-resolution SZ spectral imaging (Mroczkowski et al. 2019). In weak lensing, rotation of the foreground object induces additional contribution to the shear (see, e.g., Ciufolini & Ricci 2003; Sereno 2003, 2005, 2007). In most analyses, the lenses are assumed to be static, such that the effect caused by the kinematic movement and rotation of foreground objects is neglected. There are also studies of relativistic correction based on Newtonian $N$-body simulations by applying a nonlinear post-Friedmann framework (Bruni et al. 2014a, 2014b; Milillo et al. 2015; Thomas et al. 2015b; Adamek et al. 2016) or simulations based on $f(R)$ gravity (Thomas et al. 2015a). Bonvin et al. (2018) and Gressel et al. (2019) extend such studies to possible observational effects such as redshift distortion and weak gravitational lensing. Barrera-Hinogoso et al. (2021) and Barrera-Hinogoso & Sapone (2020) further probe the vector modes and other relativistic effects based on relativistic simulations using GRAMSES code (Barrera-Hinogoso & Li 2020a, 2020b).

With the advent of large-imaging data from wide-field surveys, such as the Legacy Survey of Space and Time (LSST; LSST Science Collaboration et al. 2009), EUCLID (Laureijs et al. 2011), or the Wide Field Survey Telescope (WFST; Lou et al. 2016), it is worthwhile to reassess the detectability of shear distortion produced by gravitomagnetic effect from rotating dark matter halos. For instance, it has been argued that rotating masses lead to negligible errors in the measurements of the Hubble constant $H_0$ using gravitational time delay (Sereno 2005). However, Shajib et al. (2018) forecast that $H_0$ could be constraint to <1% with 40 time delay measurements, while it is expected that several hundreds will be detected by LSST (Liao 2019). The impact on the determination of $H_0$ from gravitomagnetic effect will be discussed elsewhere (C. Tang et al. 2021, in preparation). In this paper, we focus on weak lensing. We investigate to which extent the rotation of halos is relevant in weak lensing measurements from ongoing and future spectroscopic surveys. We construct a simple estimator, $\delta \kappa$, that measures the anisotropy induced by halo rotation on the lensing convergence field $\kappa$ of stacked clusters, and use simulations to quantify its significance.

The structure of this paper is organized as follows. In Section 2, we derive the gravitomagnetic distortion induced by rotating lenses with a singular isothermal sphere (SIS) density profile, and construct an estimator $\delta \kappa$ to quantify the resulting anisotropic contribution to the convergence field $\kappa$. Simple approximations for $\delta \kappa$ are given as a function of the velocity dispersion, angular momentum, and rotation axis orientation of the lens. In Section 3, using GalSim (Rowe et al. 2015) and astropy4.0.1, we generate a set of simulation mocks for LSST-like surveys of the lensing anisotropic signal from rotating halos, with various choices for the model parameters. In Section 4, we analyze the signal-to-noise ratio (S/N) of $\delta \kappa$ given the various configurations of the simulations, and discuss using the SDSS DR7 group catalog (Yang et al. 2007) and the characteristics of rotating galaxy clusters and their identification. We conclude in Section 5.

The code developed to measure gravitomagnetic effects in $\kappa$ maps is made publicly available via Zenodo (Luo 2021).

2. Gravitomagnetic Effect

In this section, we derive the gravitomagnetic distortion induced by rotating halos with an SIS density profile.

2.1. $\kappa$ Field from Gravitomagnetic Effect

We work within the parameterized post-Newtonian (PPN) approximation and consider dark matter halo with an SIS density profile. For static halos, the lensing convergence $\kappa(\xi)$ reads (Bartelmann & Schneider 2001)

$$\kappa(\xi) = \frac{\Sigma(\xi)}{\Sigma_{\text{crit}}(z_1, z_2)} = \frac{\sigma_z^2}{2G \Sigma_{\text{crit}}(z_1, z_2)} \xi^2,$$

where $\Sigma_{\text{crit}}(z_1, z_2) = \frac{c^2}{4\pi G} D_l D_s$ is the geometry factor given a lens at redshift $z_l$ and a source at $z_s$, $D_l$ and $D_s$ are the angular diameter distances between the observer and the lens, the lens and the source, and the lens and the source, respectively; $\sigma_z^2$ is the velocity dispersion of matter in the halo; and the vector $\xi$ denotes the two-dimensional position with respect to the gravitational potential center of the dark matter halo.

Let us now turn to rotating halos. The halo angular momentum $J$ can be quantified by a dimensionless parameter $\lambda$, the ratio between the actual angular velocity and the theoretical one (Padmanabhan 2002):

$$J = \lambda \frac{GM^{5/2}}{|E|^2},$$

where $M$ and $E$ are the total mass and the total energy of the halo. The parameter $\lambda$ is almost independent of halo mass and of the large-scale structure. Its distribution is approximated by a log-normal function (Vitvitska et al. 2002):

$$p(\lambda) d\lambda = \frac{1}{\sqrt{2\pi} \sigma_\lambda} \exp \left[ -\frac{\ln^2(\lambda/\bar{\lambda})}{2\sigma_\lambda^2} \right] d\lambda,$$

where the mean value $\bar{\lambda} \approx 0.05$ and the scatter $\sigma_\lambda$ is around 0.5. For halos with an SIS profile, the angular momentum and energy are related to the total mass $M_{\text{SIS}} = \frac{2\pi \bar{\lambda}}{G} R_{\text{SIS}}$ as
(Bartelmann & Schneider 2001)

\[ E_{\text{SIS}} = -M_{\text{SIS}} \sigma_v^2, \]  
\[ J_{\text{SIS}} = \frac{4 \sigma_v^2 R_{\text{SIS}}^2}{G}, \]

where \( R_{\text{SIS}} \gg |\xi| \) is the truncation radius. Following Sereno (2005), we take the truncated radius such that the mean density within the radius is \( \sim 200 \) times larger than the critical density, yielding

\[ R_{\text{SIS}} = \frac{2 \sigma_v}{\sqrt{n} H(z)}, \]

where \( n \sim 200 \) characterizes the density ratio between the halo region and the mean density of the universe, and \( H(z) \) is the Hubble parameter at redshift \( z \). Although the halo velocity depends on multiple physical processes such as merging, we assume for simplicity that the rotation pattern is stable within the observational time. Under these assumptions, the halo rotation adds an extra term to the lensing potential via the Einstein–Thirring–Lense effect on top of the SIS potential (Sereno 2005):

\[ \phi = \phi_0 + \phi_{\text{GRM}}, \]

where the spherical SIS halo lensing potential reads

\[ \phi_0(\xi) = \frac{4G}{c^2} \int_{\mathcal{R}} d^2\xi' \Sigma(\xi') \ln|\xi - \xi'| = \frac{D_{\text{ls}}}{D_{\text{ls}} - D_{\text{ds}}} \frac{4\pi \sigma_v^2}{c^2} |\xi|. \]

The extra potential term introduced by gravitomagnetic effect \( \phi_{\text{GRM}} \) by an SIS density profile with velocity \( v \) is given by (Sereno 2005)

\[ \phi_0(\xi) \approx \frac{4G}{c^2} \int_{\mathcal{R}} d^2\xi' \Sigma(\xi') \langle v \cdot e_i \rangle \ln|\xi - \xi'|, \]

where \( \langle v \cdot e_i \rangle \) is the average velocity along the line of sight weighted by the projected density:

\[ \langle v \cdot e_i \rangle(\xi) = \frac{\int dD[v(\xi', l) \cdot e_i]|(\xi, l)|}{\Sigma(\xi)}. \]

The gravitomagnetic deflection angle is then given by the derivative of \( \phi_{\text{GRM}} \), yielding

\[ \hat{\alpha}_{\text{GRM}}(\xi) = -\frac{8G}{c^2} \int_{\mathcal{R}} d^2\xi' \Sigma(\xi') \frac{\langle v \cdot e_i \rangle(\xi')}{c} \frac{\xi - \xi'}{||\xi - \xi'||^2}. \]

Let us now consider a spherically symmetric lens that rotates about an arbitrary axis \( \hat{\ell} \), passing through its center (i.e., a main axis of inertia). To specify the orientation of the rotation axis, we need two Euler angles: \( \theta \), the angle between \( \hat{\ell} \) and the \( \xi_1 \) axis, and \( \beta \), the angle between the line of sight \( l \) and the line of nodes defined at the intersection of the \( \xi_1 \) plane and the equatorial plane (i.e., the plane orthogonal to the rotation axis and containing the lens center). The sketch map is shown in Figures 1 and 2. The former illustrates the geometry of the lensing system at large scales, while the latter, zoomed in, describes the relations among the line of sight, rotation axis, etc.

![Figure 1](image1.png)

**Figure 1.** The diagrammatic sketch of a gravitational lens system. The light ray propagates from the source \( S \) from the optic axis to the observer \( I \). The lens plane is orthogonal to the line of sight, which is shown as the \( \xi_1 - \xi_2 \) plane. The distances between the observer and the source, the observer and the lens, and the lens and the source are \( D_a, D_b, \) and \( D_{ds} \), respectively.

![Figure 2](image2.png)

**Figure 2.** Geometric relationships among the line of sight \( l \), the rotation axis \( \hat{\ell} \), the lens planes \( \xi_1 \) and \( \xi_2 \), and the two Euler angles \( \theta \) and \( \beta \).

Under the axial symmetry about the rotation axis, we get

\[ \langle v \cdot e_i \rangle(\xi_1, \xi_2, l) = -\omega(R)[\xi_1 \cos(\theta) + \xi_2 \sin(\theta)\cos(\beta)] = -\omega_1(R)\xi_1 + \omega_2(R)\xi_2, \]

where \( \omega(R) \) is the modulus of the angular velocity at a distance \( R = \sqrt{R_1^2 + R_2^2} \) from the rotation axis. In the case of a rigid body, there is no dependence on \( R \) for \( \omega \), and Equation (12) simplifies to \( v \cdot e_i(\xi_1, \xi_2, l) = -\omega_1(R)\xi_1 + \omega_2(R)\xi_2 \). Thus, for spherically symmetric rigid halos, the gravitomagnetic deflection angle, Equation (11), becomes

\[ \hat{\alpha}_{\text{GRM}}(\xi_1) = \frac{2 \kappa}{3c} \left[ \frac{\omega_2(2\xi_1^2 + \xi_2^2) - \omega_1\xi_1\xi_2}{|\xi|} - \frac{3\omega_2 R_{\text{SIS}}}{2} \right], \]

\[ \hat{\alpha}_{\text{GRM}}(\xi_2) = \frac{2 \kappa}{3c} \left[ \frac{\omega_1(2\xi_2^2 + \xi_1^2) - \omega_2\xi_1\xi_2}{|\xi|} - \frac{3\omega_1 R_{\text{SIS}}}{2} \right], \]

where \( \xi = \sqrt{\xi_1^2 + \xi_2^2} \).
By taking the derivative of Equation (13), we obtain the contribution from gravitomagnetic effect to the $\kappa$ field:

$$\kappa(\xi)_{\text{GRM}} = \frac{D_d}{2} (\nabla_\xi \cdot \hat{\alpha}) = \frac{\kappa(\xi)(\omega_2 \xi_1 - \omega_1 \xi_2)}{c}.$$  

(14)

2.2. $\delta\kappa$ Estimator

Gravitomagnetic effect induced by rotating lens yields an anisotropic contribution to the lensing convergence field $\kappa$ in the form of a dipole, as given by Equation (14). This is illustrated in Figure 3. The first panel shows the $\kappa$ field for a static SIS halo, with a velocity dispersion of 1000 km s$^{-1}$. The second panel depicts the contribution by gravitomagnetic effect with $\lambda = 0.2$. It shows an enhancement from one side of the rotation axis and reduction from the other side. It is clear from Figure 3 that the dipole induced by the rotation of the lens is much smaller than the total signal. We can however be a bit more quantitative in order to investigate if such gravitomagnetic distortion can be detected.

One can construct an estimator to quantify the anisotropy of the signal by taking the difference of the mean $\kappa$ divided by the rotation axis:

$$\delta\kappa = \langle \kappa_{\text{enhance}} \rangle - \langle \kappa_{\text{reduce}} \rangle.$$  

(15)

The mean value of $\kappa$ from both sides of the rotation axis is measured inside the virial radius as in Equation (6) of a halo to minimize the effect from large-scale structure. The virial radius of a $10^{14}$ solar mass is about 2 Mpc $h^{-1}$ according to this calculation, which is similar to that of a Navarro–Frenk–White (NFW) profile (Navarro et al. 1997), based on numerical simulations.

In the upper right corner of the right panel of Figure 3, the two regions can be clearly seen in two different colors. In Figure 4, we evaluate the dependence of $\delta\kappa$ on the halo velocity dispersion $\sigma_v$ and the rotation parameter $\lambda$. To do so, we generate $\kappa$ maps induced by gravitomagnetic effects given by Equation (14) and measure $\delta\kappa$ as described above. We see that $\delta\kappa$ gets bigger as $\sigma_v$ or $\lambda$ get bigger. For a halo with velocity dispersion of 1300 km s$^{-1}$ and rotation parameter of 0.8, $\delta\kappa$ can be comparable in size to $\kappa$ at the edge of the halo.

We now look for a simple functional form for $\delta\kappa$. As for an SIS profile, there is no rotation if there is no velocity dispersion, $\delta\kappa$ depends at leading order on $\sigma_v^2$. We measure $\delta\kappa$ on a grid of 500 $\times$ 500 points of $(\lambda, \sigma_v)$ and fit with the following ansatz:

$$\delta\kappa(\lambda, \sigma_v) = \mu \lambda \left( \frac{\sigma_v}{1000 \text{ km s}^{-1}} \right)^2,$$  

(16)

where we get $\mu \approx 5.69 \times 10^{-4}$, an overall normalization that can be considered as the typical magnitude of gravitomagnetic effects from rotating halos. Equation (16) shows that there is a strong degeneracy between $\sigma_v^2$ and $\lambda$: they are thus strongly dependent on observations.

We can also express Equation (16) as a function of the halo mass:

$$\delta\kappa(\lambda, \sigma_v) = \mu \lambda \frac{GM}{2R_{\text{SIS}}},$$  

(17)

where $R_{\text{SIS}}$ is the truncation radius defined in Equation (6). Taking into account the distribution of $\lambda$ as given in
Equation (3), Equation (16) becomes

$$\langle \delta \kappa (\sigma_i) \rangle = \int P(\lambda)(\mu \sigma_i^2) \lambda d\lambda.$$  

So far we have considered that the real (halo) rotation axis is perfectly aligned with the observed (tracer) rotation axis. If there is a misalignment $\delta \theta$ between those two, $\delta \kappa$ will be reduced by a factor of $\cos(\delta \theta)$, such that the misaligned $\delta \kappa_m$ is related to the aligned $\delta \kappa_o$ by

$$\delta \kappa_m = \cos(\delta \theta) \delta \kappa_o.$$  

For a given distribution $P(\delta \theta)$, Equation (19) becomes

$$\langle \delta \kappa \rangle = \int d\delta \theta P(\delta \theta) \cos(\delta \theta) \delta \kappa.$$  

Combining Equation (16), Equation (19), and Equation (20), we obtain a general expression for $\delta \kappa$ taking into account the scatters of both the angular momentum and the misalignment of the observed-to-real rotation axis:

$$\langle \delta \kappa \rangle = \int d\delta \theta P(\delta \theta) \cos(\delta \theta) \int P(\lambda) \mu \lambda \left( \frac{\sigma_i}{1000 \text{ km s}^{-1}} \right)^2 d\lambda.$$  

To summarize, Equation (19) describes gravitomagnetic distortion around a single cluster, while Equation (21) provides an estimator for stacked clusters. We will discuss next this later case, asking ourselves if stacking multiple rotating lenses can achieve sufficient S/N for measurements of gravitomagnetic effect. Meanwhile, we finish this section by discussing related issues on the measurements of gravitomagnetic distortion in the single cluster case.

We show the dependence of $\delta \kappa$ on the azimuthal angle, Equation (19), in Figure 5. The angular dependence (angle between the true rotation axis and an arbitrary axis) of $\delta \kappa$ is clear. However, this is difficult to estimate by stacking without any knowledge of the rotation axis. We will show later in Section 4 that we can infer the rotation axis by observing the same pattern of angular dependence as in Figure 5. Such specific dependence can be used to distinguish gravitomagnetic distortions from other sub-leading lensing contributions, such as from halo asymmetry, biased tracers, and so on.

In particular, one should keep in mind that irregularities in the halo shapes can lead to a very different signal than the one we derived here, assuming perfect spherical symmetry and an SIS density profile. In Figure 6, we show for illustration the measured $\kappa$ map and values of $\delta \kappa$ as a function of $\delta \theta$ on a typical halo identified in IllustrisTNG300-300 (Nelson et al. 2018). Note that we did not add shape noise here, since for a single cluster, the shape noise will be overwhelmingly dominant. The signal is comparable in size to a gravitomagnetic distortion with $\sigma_i = 1000 \text{ km s}^{-1}$ and $\lambda = 0.2$, but we can see that the halo morphology leads to a very different profile for $\delta \kappa(\delta \theta)$. Furthermore, the effect of biased tracers has not been taken into account here (this will be investigated in an upcoming work). However, at this stage, we expect that biased tracers will present a similar signal as the one for halos for gravitomagnetic effect, at least for isolated clusters. Thus, to hope to detect Einstein–Thirring–Lense effect in weak lensing surveys, one should consider relatively isolated clusters, i.e., with cylindrical selection (e.g., Mandelbaum et al. 2006), or isolated galaxies (e.g., Luo et al. 2020).

![Figure 5](image-url)  

In the following section, we perform a suite of simulations to test the model above, with realistic shape noise of galaxies and galaxy number density to evaluate the detectability in LSST-like surveys.

### 3. Simulation

We simulate $\kappa$ maps as described by Equation (14) assuming a shape noise of 0.5 and galaxy number density of $50 \text{ arcmin}^{-2}$ following the characteristics of LSST (LSST Science Collaboration et al. 2009). In real observations, $\kappa$ maps are reconstructed from the shear distortion field $\gamma_{i, j}, i = 1, 2$. All the simulated rotating halos are assuming SIS profiles in order to be consistent with our formulations. Thus, the error propagation of the shape noise to $\kappa$ can be estimated in Fourier space as (Starck et al. 2006)

$$\bar{\kappa} = \bar{P}_1 \bar{\gamma}_1 + \bar{P}_2 \bar{\gamma}_2,$$

where $\bar{P}_1$ and $\bar{P}_2$ are defined as:

$$\bar{P}_1 = \frac{k_1^2 - k_2^2}{k_1^2 + k_2^2}, \quad \bar{P}_2 = \frac{2k_1^2k_2^2}{k_1^2 + k_2^2}.$$  

$\bar{P}_1 = 0$ when $k_2^2 = k_2^2$ and $\bar{P}_2 = 0$ when $k_i = 0, (i = 1, 2)$. Propagating the shape noise of both components of $\gamma_i$ and $N_\kappa$
there are 26 clusters with halo masses ~\log 15 at redshift \( z = 0.2 \) and source redshift \( z = 0.4 \). Lower: \( \theta_n \) measured around this halo as a function of \( \delta \theta \). The \( \delta \theta \) dependence is very different than the one shown in Figure 5.

\[
\kappa = \bar{P}_1(\gamma_1 + \bar{N}_1) + \bar{P}_2(\gamma_2 + \bar{N}_2),
\]

such that the error on \( \kappa \) is

\[
\delta \kappa = \bar{P}_1 \bar{N}_1 + \bar{P}_2 \bar{N}_2.
\]

Here, we simply have \( \bar{N}_1 = N_i = 0.3 \), where \( N_i \) is the shape noise, given that \( N_1 = N_2 = 0.3 \) and \( k_1 = k_2 \). In this work we neglect the error and bias associated with the reconstruction algorithm. The final error that we use for our simulations is then \( \sigma = \sqrt{\frac{0.3}{N_i}} \).

We simulate five sets of stacked rotating clusters with a fixed velocity dispersion of 1000 km s\(^{-1}\). Table 1 shows the specification chosen for each simulation. The first three (Simulations 1, 2, and 3) are with rotation parameters \( \lambda > 0.1, 0.2, \) and 0.25, These values are motivated by the following. We choose from Miller et al. (2005) the C4 identification algorithm as an informative reference for cluster characteristics: a cluster with a richness of 36 has been identified with a velocity dispersion of 2182 km s\(^{-1}\). However, we restrict ourselves to a more conservative upper limit for the velocity dispersion of 1000 km s\(^{-1}\) from the observation of Abell 2255 identified by the C4 identification algorithm. This corresponds to a halo mass of log\((M_{\odot}/h^{-1} M_{\odot})\) ~ 15, where \( M_{\odot} \) is a solar mass, according to the scaling relation of Zahid et al. (2016). According to Vitvitska et al. (2002), the rotation parameter is about \( \lambda \approx 0.05 \) on average; however, faster rotating halos can be expected from a distribution such as in Equation (3). If we consider SDSS DR7 north cap spectroscopic survey volume (~7500 deg\(^2\)) the redshift range [0.01, 0.2], there are 26 clusters with halo mass > 10\(^{15}\)h\(^{-1}\)M\(_{\odot}\). If we consider an LSST-like survey with ~20000 deg\(^2\), the redshift range from 0.01 to 1, the volume is roughly 184 times the SDSS DR7 volume group catalog. This estimation is based on a recent group finder algorithm by Yang et al. (2021), who modify the halo-based group finder designed for SDSS survey to adopt the photometric only data for DESI Legacy Survey (Dey et al. 2019). The group catalog has photometric redshift ranging from 0.0 to 1.0. In that case, roughly about 4785 clusters with log\((M_{\odot}/h^{-1} M_{\odot})\) ~ 15 can be detected. For \( \lambda > 0.1 \), only ~400 clusters can be detected, with mean \( \lambda = 0.128 \). The number of detectable clusters further reduces to 13 for \( \lambda > 0.2 \) and to 3 for \( \lambda > 0.25 \).

We also simulate six unrealistic cases with \( \lambda = 0.3-0.8 \) and a velocity dispersion of 1000 km s\(^{-1}\) and 400 stacked clusters with zero scatter between the rotation axis indicator and the real rotation axis (Simulation 4). Figure 7 shows one of the

![Figure 6.](image1.png)

**Figure 6.** Upper: \( \kappa \) map of an illustrisTNG300-300 halo, with log\((M_{\odot}/h^{-1} M_{\odot})\) ~ 15 at redshift \( z = 0.2 \) and source redshift \( z = 0.4 \). Lower: \( \theta_n \) measured around this halo as a function of \( \delta \theta \). The \( \delta \theta \) dependence is very different than the one shown in Figure 5.

**Table 1**

| Simulation | \( \sigma_v \) (km s\(^{-1}\)) | \( \lambda \) | \( \sigma_\theta \) (deg) | Number |
|------------|-----------------|-----------------|-----------------|-------|
| Sim 1      | 1000            | >0.1            | 0.0             | 412   |
| Sim 2      | 1000            | >0.2            | 0.0             | 13    |
| Sim 3      | 1000            | >0.25           | 0.0             | 3     |
| Sim 4      | 1000            | 0.3-0.8         | 0.0             | 400   |
| Sim 5      | 1000            | 0.8             | [1, 5, 10, 20, 30] | 400   |

![Figure 7.](image2.png)

**Figure 7.** The angular dependence of \( \delta \theta_n \) from 400 stacked clusters with \( \sigma_v = 1000 \) km s\(^{-1}\) and \( \lambda = 0.8 \). The black dots are the mocked data. The theoretical input is shown by the red line.
simulations for the fourth set with $\lambda = 0.8$ assuming zero scatter on the rotation axis. The last simulation (Simulation 5) is set to estimate the signal reduction from the misalignment between the true rotation axis and the tracer axis from observations.

4. Results

4.1. Simulation Results

We now investigate the detectability of gravitomagnetic distortion in weak lensing surveys by measuring $\delta\kappa$ on the simulations designed in previous section. The results are shown in Figure 8.

Let us first focus on Simulations 1, 2, and 3, designed to mock LSST-like surveys: for $\lambda > 0.1$, we have 412 cluster halos of mass $> 10^{15}h^{-1}M_\odot$, for an average of $\lambda$ around 0.128. Only 13 clusters are found for $\lambda > 0.2$, and 3 for $\lambda > 0.25$. This is summarized in Table 1. As expected, we find that $\delta\kappa$ is smaller for Simulation 1 than for Simulations 2 and 3, as on average the rotation of halos is smaller. Nevertheless, as the number of stacked halos in Simulation 1 is bigger, the error bars are much smaller. For the other two, which only have 13 and 3 clusters, the shape noise overweights the signal by a factor of $\sim 10$. In all these “realistic” cases, we find null detection of gravitomagnetic effect, given that $\delta\kappa$ is compatible with zero within $1\sigma$.

We perform further checks with Simulations 4 and 5. Simulation 4 is set to understand the relation between the measured $\delta\kappa$ and $\lambda$. The values of $\lambda$ range from 0.3 to 0.8, for fixed number of 400 of stacked clusters. The theoretical prediction, Equation (16), agrees with Simulation 4 data points.

Simulation 5 is designed to study the effect of a misalignment between the true rotation axis of clusters and the ones selected using observational tracers, such as the major axis of central galaxies (Okumura et al. 2009), the distribution of satellite galaxies inside clusters (Baxter et al. 2016), or spin axis of spiral galaxies (Zhang et al. 2015). In general, the tracers are misaligned with their dark counterpart. One famous case is the bullet cluster (Clowe et al. 2006), in which the baryonic distribution significantly differs from the dark matter distribution inferred from weak lensing. To probe such a discrepancy, we choose randomly for each cluster in Simulation 5 the misalignment angle between the tracer axis and the true rotation axis among a normal distribution centered on 0 with scatter: $\sigma_\theta = 1^\circ, 5^\circ, 10^\circ, 20^\circ, \text{ and } 30^\circ$. We find that a $5^\circ$ scatter leads to a damping of $\sim 10\%$, which remains consistent with the true input value denoted by the solid black line in the lower panel of Figure 8. For larger scatters, the signal becomes too low to be detected, where even with $10^\circ$ scatter the signal is already consistent with zero. This highlights the importance of selecting precisely the tracer axis in order to extract the gravitomagnetic distortion signal.

4.2. Observational Indications

We now proceed with a rough quantification of the rotation axis and speed from the groups of galaxies of SDSS DR7 using redshift selection following Yang et al. (2007). The same method to measure $\delta\kappa(\delta\theta)$ can be used to measure the mean redshift of member galaxies separated by an arbitrary axis that has an angle of $\delta\theta$ with respect to the true axis in two-dimensional projected plane. Here, we use $\phi$ to denote the angle between the true rotation axis and the arbitrary axis from observations in order to distinguish it from $\delta\theta$ used for simulations. The difference of the mean redshift from the two sides divided by the 2D rotation axis is given by $\Delta z = \langle z_1 \rangle - \langle z_2 \rangle$, where 1 and 2 indicate the two sides of the axis. Following Sofue (2013) in which the rotation curve of our own galaxy is found to be well fitted by a sine function, we fit $\Delta z$ with the parameterization:

$$\Delta z(\phi) = z_{\text{off}} \pm z_{\text{amp}} \times \sin(\phi - \phi_0).$$

The first term on the right-hand side of Equation (26) is the offset of the group along the line of sight, which should be zero since we choose the central galaxies as the reference, one such that the peculiar velocity of the cluster becomes zero (Hwang & Lee 2007). However, we caution that there might be uncertainties in the choice of the brightest central galaxies (BCGs). For our purpose, we consider that a nonzero offset will be much smaller than the error and thus can be neglected. The amplitude $z_{\text{amp}}$ represents the rotation speed along the line of sight, and $\phi_0$ is the rotation axis angle with respect to the east to north. The relation between $\Delta z$ and velocity dispersion along the line of sight is related by (Danese et al. 1980)

$$v_{\text{los}} = \frac{\Delta z}{1 + z_{\text{bcg}}},$$

where $z_{\text{bcg}}$ is the redshift of the BCG. With this procedure outlined above, we are thus able to extract all information to evaluate $\delta\kappa$.

As an illustration, we select the richest group in the Yang et al. (2007) catalog with 623 members with velocity dispersion of about 667.8 km s$^{-1}$ estimated from the scatter of $\Delta z = z_{\text{member}} - z_{\text{bcg}}$ as shown in Figure 9, and with a line-of-sight rotation speed of about 195.0 km s$^{-1}$ based on Equations (26) and (27). The upper right panel of Figure 9 illustrates that the rotation axis alignment can be measured by fitting the rotation curve using a sine function as in Equation (26). We emphasize that, however, one also has to account for the projection effects in order to obtain a reliable
measurement of the rotation axis. This will be discussed in a follow-up study based on the combination of observational group catalog and simulations. This leads to \( \lambda = 0.292 \), which is extremely unlikely given the probability distribution \( P(\lambda) \) given in Equation (3) that we used for our simulations: the fraction of clusters with halo mass larger than \( 10^{15} h^{-1} M_\odot \) in SDSS DR7 is about 0.00006 and the probability of a halo with \( \lambda \geq 0.3 \) is about 0.0002, yielding a joint probability of such a cluster to appear in this catalog of only \( 1.2 \times 10^{-8} \), according to the probability distribution inferred from simulations (Vitvitska et al. 2002). Yet we note that this is still more likely than the bullet cluster. The probability of forming a bullet cluster-like object with such high colliding speed in the Lambda cold dark matter (ΛCDM) framework is about \( 3.3 \times 10^{-11} \) (Lee & Komatsu 2010). The reason for such an occurrence may be due to the fact that the velocity dispersion of this cluster is underestimated because SDSS spectroscopic objects need to be brighter than 17.77 mag in the \( r \) band, which means that fainter members are not included. Even if we set the velocity dispersion to be 1000 km s\(^{-1}\), \( \lambda \) is still about 0.2. It thus seems that highly rotating clusters are not as rare as predicted by Vitvitska et al. (2002).

The rotation speed is the most important quantity to estimate gravitomagnetic distortion. If we assume that the rotation speed is 195.0 km s\(^{-1}\), for a cluster halo mass of \( \log(M_h/h^{-1} M_\odot) \sim 15 \), this leads to a value of \( \delta \kappa = 0.0002 \). But for such a cluster, the shape noise will be of order 0.003, which is more than 10 times larger than the signal. Manolopoulou & Plionis (2017) have already started this research on the angular dependence of the line-of-sight rotation velocity and rotation axis using Monte Carlo mocks of rotating clusters. They found that using their algorithm up to 28% clusters can be identified as rotating.
The error bars from Figure 9 are estimated by the bootstrap resampling method: we create 600 samples out of 623 member galaxies and take the distribution of mean value of each sample to estimate the velocity and error. The lower panel of Figure 9 shows a 72°62 ± 1°42 offset between the major axis of satellite distribution (red solid line) and rotation axis (blue dashed line), which is simply $\pi/2 + \phi_0$. This means that the major axis from satellites cannot be used as the rotation axis and tends to be antialigned with the major axis in this cluster. Note that we mention the major axis of clusters, as it is of more interest than the minor axis in the sense that it is more likely aligned with large-scale filament structure. The alignment between the two axes is another interesting study, which we will explore in the future.

In this figure we have all the information needed to extract $\delta\kappa$ from observational data, i.e., the velocity dispersion, $\lambda = v_{los}/\sigma_v$, and the rotation axis. We will further explore if this can be achieved using the group catalog from Yang et al. (2007) and the SDSS DR7 shape catalog of Luo et al. (2017) (C. Tang et al. 2021, in preparation).

5. Conclusion

We have analyzed the Einstein–Thirring–Lense effect in lensing convergence maps around rotating halos. Making the bridge between observations and previous theoretical works (Ciufolini & Ricci 2003; Sereno 2005, 2007), we have constructed an estimator to measure the anistropic signal from gravitomagnetic effects induced by the rotation of foregrounds in weak lensing surveys. We find that this signal is two or three orders of magnitude smaller than the distortion from the static halo potential, in accordance with Cuesta-Lazaro et al. (2018).

Assuming lenses with a spherical SIS density profile, we have run a suite of simulations calibrated for an LSST-like survey, for which we estimate that only about 400 clusters with relatively high rotation parameters $\lambda > 0.1$ can be found. Applying our estimator on the simulations, even by stacking ~400 rotating clusters with $\lambda > 0.1$ and velocity dispersion within the clusters of $\sigma_v > 1000 \text{ km s}^{-1}$, assumes perfect knowledge of the halo rotation axis. We also simulate data to test observational effects, such as the impact on the signal amplitude of a misalignment between the observational tracer and the halo rotation axis, finding that the signal is strongly reduced for a misalignment scatter of more than 5°. However, we observe that the chosen characteristics for our mocks, in particular the dependence on the rotation parameter $\lambda$, is strongly motivated based on inputs from $N$-body simulations. In real observations, fast rotating clusters may not be as hard to find as in simulations. Indeed, we have further discussed methods to identify rotating clusters in maps from spectroscopic surveys. By selecting the most massive cluster from Yang et al. (2007), we have found that this cluster has an extremely big angular momentum with $\lambda = 0.295$, which is very unlikely given the distribution we assumed for $\lambda$ based on simulations. This serves as a preliminary indication that the story might be different in real observations: rotating halos may be more abundant and at higher speed in our universe than what simulations show, motivating us to investigate further gravitomagnetic effects in the context of weak lensing. This will be explored in an upcoming work.

We finish by highlighting a number of interesting open questions related to the current work.

1. We found that the major axis of satellite position distribution in clusters cannot be used to probe gravitomagnetic effect, as it is completely misaligned with the cluster rotation axis. However, the alignment between the major axis and the cluster rotation axis can be an interesting topic to study, as it can give hints on the direction of the real rotation axis.
2. We have sketched a procedure how to select rotating clusters in a group catalog. This can be used to measure the distribution of $\lambda$, to see whether it is consistent with the one predicted from first principles or with the use of simulations.
3. Modified gravity may lead to different lensing signals, in particular through the gravitomagnetic potential. Although gravitomagnetic distortions are difficult to detect, it may be interesting to study the impact of modification to gravity on frame-draving effects, as the resulting signal may be different in shape and size.
4. The elongation of the dark matter halo along the major axis does not introduce any signal to $\delta\kappa$ due to the subtractive nature of the estimator. As long as the density profile is symmetric with respect to the rotation axis, this elongation will not cause any extra $\delta\kappa$.

We leave these issues to future studies.

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