Efficient polarization qubit transmission assisted by frequency degree of freedom

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Abstract

We present an efficient arbitrary polarization qubit transmission scheme against channel noise by utilizing frequency degree of freedom, which is more stable in transmission surroundings. The information of a quantum state is encoded in a frequency state during the transmission and transferred to a polarization state later. Both the fidelity of the quantum state transmitted and the success probability of this scheme are 100% in principle.

The transmission of quantum states between participants is an essential step in each quantum communication protocol. However, the fidelity of a quantum state rapidly degrades because of the photon loss and decoherence during the transmission, which will impact the efficiency and the security of quantum communication consequently. The coupling between the quantum system and the environment, which is called noise, is a serious obstacle to a perfect communication. The polarization degree of freedom (DOF) of photons is usually chosen as information carriers because of its maneuverability. However, this DOF is incident to be influenced by thermal fluctuation, vibration and the imperfection of fibres, i.e. the channel noise. Various methods have been proposed for rejecting or correcting the errors caused by noise, one of which can be called quantum error rejection [1–4].

In quantum error rejection schemes, there is an important assumption that the variation of noise is slow. If several qubits transmit through the channel simultaneously or close to each other, the impact of noise is identical. With this hypothesis, the arbitrary quantum state can be transmitted probabilistically by making two parts suffer from the same noise to interact and then reject errors by postselection. In 2005, Kalamidas proposed a single-photon error rejection protocol [1], which encodes the quantum states in two time bins and an uncorrupted state can be obtained at a definite time of arrival. However, a fast polarization modulator is employed in this scheme, whose synchronization makes it difficult to implement. Later, we presented a faithful qubit transmission scheme with passive linear optics [2]. Yamamoto et al proposed a qubit distribution scheme with ancillary qubit in 2005 [3]. However, the success probability is a bit low. All of these three protocols utilize the temporal DOF to encode the state where losses will be introduced inevitably during the postselection to rebuild the original state.

Recently, some other DOFs besides polarization have attracted considerable attention, one of which is the frequency DOF. Due to its stability, frequency DOF has been used in a series of quantum information schemes [4–11]. In this paper, we present a new method for transmitting a single-photon polarization state against channel noise with frequency DOF. The sender encodes the information of the quantum state into the frequency DOF and the receiver converts the information back to the polarization DOF and obtain the uncorrupted state deterministically.

Let us suppose that an arbitrary single-photon pure state to be transmitted is written as

\[ |\psi\rangle = \alpha |H\rangle + \beta |V\rangle, \quad (|\alpha|^2 + |\beta|^2 = 1). \]

Here $|H\rangle$ and $|V\rangle$ represent the horizontal and the vertical polarization states of a single photon, respectively. And $\alpha$ and $\beta$ are two parameters which carry the information of the state to be transmitted. The frequency of this photon is prepared in $\omega_2$. Considering the two DOFs, the whole quantum state to be transmitted can be written as

\[ |\psi\rangle = \alpha |H, \omega_2\rangle + \beta |V, \omega_2\rangle. \]

Before the transmission, the sender encodes the state with the encoder composed of a polarizing beam splitter (PBS),
The initial state evolves in the encoder as follows:

\[ \psi = \alpha |H, \omega_2\rangle + \beta |V, \omega_2\rangle \]

PBS \[\rightarrow \alpha |H, \omega_2\rangle_1 + \beta |V, \omega_2\rangle_2 \]

FS \[\rightarrow \alpha |H, \omega_1\rangle + \beta |H, \omega_2\rangle_2 \]

HWP \[\rightarrow \frac{1}{\sqrt{2}}(\alpha|H, \omega_1\rangle + i\beta|H, \omega_2\rangle)_{a} + (i\alpha|H, \omega_1\rangle + \beta|H, \omega_2\rangle)_{b}. \] (3)

The subscripts 1 and 2 represent the two paths of the interferometer, and \(a\) and \(b\) represent the two output ports of a BS and two independent noisy channels. The coefficient \(i\) comes from the phase shift aroused by the reflection of a BS. From the last two lines, we find that the information of the initial state is encoded into a frequency superposition state, and these two states in a different noise channel have similar form; we just analyze the situation of channel \(a\) in detail below.

Both the polarization state and the frequency state will be influenced by the channel noise during the transmission. However, the frequency DOF is much more stable than the polarization one [6–10]. For example, Naik et al demonstrated the Ekert protocol over only a few metres [12, 13]. And in the experiment of six-state protocol, the quantum bit error rate (QBER) increases to 33% [14, 15]. But for frequency coding, a quantum key distribution scheme over 20 km was implemented, in which approximately 4% QBER was recorded [16]. Therefore, we only consider the impact on the polarization DOF here. The effects of channel noise can be expressed with a unitary transformation

\[ |H\rangle \rightarrow \delta|H\rangle + \eta|V\rangle, \quad (|\delta|^2 + |\eta|^2 = 1) \] (4)

where \(\delta\) and \(\eta\) are the parameters of the noise. The quantum state the receiver obtains from channel \(a\) can be written as

\[ \frac{1}{\sqrt{2}}(\alpha|H, \omega_1\rangle + i\beta|H, \omega_2\rangle)_{a}^{\text{noise}} \rightarrow |\psi\rangle_a \]

\[ = \frac{1}{\sqrt{2}}(\alpha \delta|H, \omega_1\rangle + \alpha \eta|V, \omega_1\rangle + i\beta \delta|H, \omega_2\rangle + i\beta \eta|V, \omega_2\rangle). \] (5)

The receiver uses a decoder to obtain the original polarization state, shown in figure 2. The frequency beam splitter (FBS) is polarization independent and used to guide photons to different spatial modes according to their frequencies. That is, the frequency state \(|\omega_1\rangle\) goes to path 1 (2) according to the figure. The FS in path 1 adjusts the frequency state from \(|\omega_1\rangle\) to \(|\omega_2\rangle\), so as to make the quantum states in these two paths have the same frequency. And the HWP in path 2 rotates the horizontal polarization \(|H\rangle\) to \(|V\rangle\). The phase modulator (PM) modulates the phase of the quantum state passing through path 2 as \(|\psi\rangle \rightarrow e^{i\theta}|\psi\rangle\), where \(\theta = \pi/2\). The effect of the decoder can be written as

\[ |\psi\rangle_a \rightarrow \frac{1}{\sqrt{2}}(\alpha \delta|H, \omega_1\rangle_1 + \alpha \eta|V, \omega_1\rangle_1 + i\beta \delta|H, \omega_2\rangle_2 + i\beta \eta|V, \omega_2\rangle_2) \]

\[ \rightarrow \frac{1}{\sqrt{2}}(\alpha \delta|H, \omega_2\rangle_1 + \alpha \eta|V, \omega_2\rangle_1 + \beta \delta|V, \omega_2\rangle_2 + \beta \eta|H, \omega_2\rangle_2) \]

\[ \rightarrow \frac{1}{\sqrt{2}}(\delta|H, \omega_2\rangle + \beta|V, \omega_2\rangle)_{d} + \eta(\alpha|H, \omega_2\rangle + \beta|V, \omega_2\rangle). \] (6)

The subscripts 1 and 2 represent two paths and \(c, d\) represent two output ports of the decoder. These two states in two different spatial modes have the same form with the initial one, retaining the information of the quantum state. With the decoder set in channel \(a\), the receiver can obtain the initial state with success probability 1/2. The situation of channel \(b\) is similar. Suppose the noise is \(|H\rangle \rightarrow \delta'|H\rangle + \eta'|V\rangle\) in channel \(b\), the state received with another decoder set in channel \(b\) is

\[ |\psi\rangle_b \rightarrow \frac{i}{\sqrt{2}}[\delta'(|H, \omega_2\rangle + \beta|V, \omega_2\rangle)_{d} + \eta'(|\alpha|H, \omega_2\rangle + \beta|V, \omega_2\rangle)]. \] (7)

which are also equal states with the original one. Therefore the total success probability to transmit an arbitrary polarization state is 1 in principle.

In the present scheme, the frequency DOF is introduced to defeat the channel noise. Before the transmission, the secret information encoded in the polarization state was transferred to the frequency DOF, which will be converted back to the original polarization state after transmission. Therefore the modulation of the frequency DOF is important in our
This scheme proposed a useful method to transmit polarization qubit against channel noise. Then it will have good applications in one-way quantum communication protocols such as the BB84 QKD protocol [22]. The two parties can choose the two nonorthogonal bases, \(| \pm x \rangle = \frac{1}{\sqrt{2}}((H) \pm |V\rangle)\) and \(| \pm y \rangle = \frac{1}{\sqrt{2}}((H) \pm i|V\rangle)\), to realize the sharing of secret keys. With our encoder and decoder, the efficiency of the quantum key distribution scheme is the same as the original one and the errors caused by the channel noise are eliminated.

In theory, the success probability is 100% based on an ideal condition. In practice, the success probability will be impacted by the efficiency of these elements in the encoder and decoder, among which the proportion of the FS is non-ignorable. The success probability decreases to \(\eta^2\) with practical conditions, where \(\eta\) is the efficiency of a FS. The FS can be realized by several means with the current technique, such as AOM [17, 18], SFG [19, 20] and so on. And sufficient high modulation efficiency is accessible. It is reported that the internal conversion efficiency of an SFG process is 99% and the overall efficiency can be 65% [19]. And in [20], the SFG efficiency is roughly 90%. The use of AOM is widespread and the efficiency is high. Therefore, compared with the scheme using temporal DOF [2], this scheme has higher success probability in practice.

We have discussed our qubit transmission scheme in the case that the frequency DOF is insensitive to the noise. Previous experiments showed that the frequency DOF is more stable than the polarization one. Therefore we use the frequency DOF to encode the information during the transmission. In practical transmission, the two parts used to rebuild the original state will catch the relative phase \(\Delta \varphi\) due to the difference in frequency. The state \(\frac{1}{\sqrt{2}}[|H, \omega_1 \rangle + i|V, \omega_1 \rangle + \beta|V, \omega_2 \rangle]_a\) changes to \(\frac{1}{\sqrt{2}}[|H, \omega_1 \rangle + i|V, \omega_1 \rangle + e^{i\Delta \varphi}(|H, \omega_2 \rangle + i\beta|V, \omega_2 \rangle)]_a\) after the transmission. And the final state will be \(\frac{1}{\sqrt{2}}|H, \omega_1 \rangle + e^{i\Delta \varphi}\beta|V, \omega_2 \rangle\). Here \(\Delta \varphi = L_a(\omega_2 - \omega_1)/v\), where \(L_a\) is the length of channel \(a\) and \(v\) is the velocity of the photon in the quantum channel [10]. As the channel length and frequencies chosen are fixed, the relative phase \(\Delta \varphi\) does not fluctuate with time in theory. With a phase compensation, the receiver can obtain the initial state.

In summary, we have proposed a single-photon error correction scheme against collective noise with frequency DOF. In this scheme, additional qubits are not required. The polarization state is encoded in the frequency DOF which is immune to noise and decrypted after transmission. The success probability is 100% in theory. As in our scheme two pulses travelling in different time slots are not interacted to obtain the uncorrupted state, the channel noise should not be restricted as a collective one. In addition, our scheme is not only suitable for pure states but also for mixed ones, which will have good applications in one-way quantum communication.

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