A hybrid simulation code is developed that is suited for fast one-dimensional simulations of shower profiles, including fluctuations. It combines the Monte Carlo simulation of high energy interactions with a fast numerical solution of cascade equations for the resulting distributions of secondary particles. Results obtained with this new code, called CONEX, are presented and compared to CORSIKA predictions.

1. INTRODUCTION

Ultra-high energy cosmic rays (UHECR) are investigated by measuring the characteristics of the secondary particle cascades that they induce in the Earth’s atmosphere. The information obtained from these extensive air showers (EAS) is used to infer the properties of the primary particle, relying on a proper theoretical description of the cascade processes.

The most natural way to get detailed information on the atmospheric particle cascading seems to be a direct Monte Carlo (MC) simulation of the EAS development, like it is done, for example, in the CORSIKA program \cite{1}. Nevertheless, for primary particles of very high energies, this is not a viable option because of unreasonably large calculation time required. The situation can be improved by applying some weighted sampling algorithms, like the so-called “thinning” method \cite{2}. Although this approach allows the reduction of EAS calculation times to practically affordable values, it has limitations. The summation of particle contributions with very large weights creates significant artificial fluctuations for EAS characteristics of interest \cite{3}.

A possible alternative procedure is to describe EAS development numerically, based on the solution of the corresponding cascade equations \cite{4,5,6}. Combining this with an explicit MC simulation of the most energetic part of an EAS allows one to obtain accurate results both for average EAS characteristics and for their fluctuations \cite{7}. In this article we report on the development of an extensive air shower calculation program of such a type: CONEX. The MC treatment of above-threshold particle cascading is treated in the standard way and does not differ significantly from e.g. CORSIKA procedure \cite{1}. On the other hand, the numerical description of lower energy sub-cascades is based on the solution of hadronic cascade equations, using an updated algorithm of Ref. \cite{6} and a newly developed procedure for solving electro-magnetic (e/m) cascade equations.

The outline of the paper is as follows. Section \ref{sec:scheme} describes the calculation scheme and its basic procedures. In Section \ref{sec:results} we present examples of EAS characteristics obtained with CONEX and investigate the accuracy of the predictions comparing the hybrid approach with pure MC or

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numerical procedures, and with the results of the CORSIKA code. Finally, Section 4 contains a summary of our work and discusses both potential applications of the program and the prospects for its further development.

2. SCHEME

A hybrid air-shower calculation scheme consists of two main stages: an explicit MC simulation of particle cascade at energies above some chosen threshold $E_{\text{thr}}$ (typically a factor of 100 smaller than the energy of the primary particle) and a solution of nuclear-electro-magnetic cascade equations for sub-cascades of smaller energies. Both MC and numerical parts model the same physics, but the numerical solution to the cascade equations is calculated only along the direction of the shower axis.

In the hadronic cascade one follows the propagation, interactions and decays (if relevant) of (anti-)nucleons, charged pions, charged and neutral kaons. All other types of hadrons produced in the interactions and decays are assumed to decay at the place. Particle interactions in the MC part are treated within a chosen high energy hadronic interaction model (optionally NEXUS [8] or QGSJET [9]) and decays are simulated using the corresponding routines of the NEXUS model. The same models are applied to pre-calculate average secondary particle spectra for later use by the numerical scheme. Optionally, below some energy $E_{\text{had}}^{\text{low}} \sim 100$ GeV one employs the GHEISHA model [10]. Ionization energy loss of charged hadrons is approximated by the Bethe-Bloch equation.

The MC treatment of the e/m cascade is realized by means of the EGS4 code [11], supplemented by an account for the Landau-Pomeranchuk-Migdal (LPM) effect for very energetic electrons (positrons) and photons. The numerical part is based on the same interaction processes as the MC one, using Bethe-Heitler cross sections for bremsstrahlung and pair production with corrections at low energy according to Storm and Israel, the Klein-Nishina formula for the Compton process, accounting for Möller and Bhabha processes as well as for positron-electron annihilation (see, for example [12,13]). Both LPM-effect and photo-effect are neglected in the numerical part as it is not supposed to be used in the energy ranges where the corresponding processes give an essential contribution. Ionization losses of electrons and positrons are described by the Bethe-Bloch formula with corrections due to the density effect.

In general, an individual shower is simulated as follows. One starts with the primary particle of given energy, direction, at a given initial position in the atmosphere. These define the shower axis along which the shower will be calculated using slant depth. For a primary hadron initiating a shower one simulates the hadronic cascade explicitly until all produced hadrons fall below the energy threshold $E_{\text{thr}}$. The characteristics of all sub-threshold hadrons and e/m particles (type, energy, and slant depth position) are written to corresponding stacks to form the “source terms” for the numerical solution of the cascade equations. The above-threshold e/m particles are transferred to the EGS4 program, where the simulation of the e/m particle cascade is performed in a similar way, with all sub-threshold e/m particles being added to the e/m source term. As the next step, the hadronic cascade at energies below $E_{\text{thr}}$ is described numerically based on the solution of corresponding cascade equations and the initial condition specified by the source term. The results are discretized energy spectra of hadrons of different types at various depth positions. All sub-threshold e/m particles produced at this stage are added to the e/m source term. Finally, sub-threshold e/m cascades are described numerically based on the solution of corresponding e/m cascade equations with the initial conditions set by the corresponding source term. In case of the primary particle being a photon or an electron the simulation process consists just from e/m MC cascading for above-threshold particles and numerical treatment of the process for sub-threshold particles. No feedback from the e/m to the hadronic part is considered.

3. RESULTS

Currently CONEX can be used for the simulation of showers initiated by any primary particle
and energy accepted by the high energy interaction model implemented (γ, proton, iron and more up to 10^{22} eV). The basic output consists of the longitudinal profile along the shower axis for all produced particles (e/m and hadronic) except for the muons (under development). Additionally, Gaisser-Hillas fit parameters, longitudinal energy deposit profiles and energy spectra for 3 different slant depth levels can be provided. The geometry of the model allows the simulation of showers at any angle up to 90° assuming US standard atmosphere, but can easily be adapted to any atmosphere and upward going showers. On the other hand, since numerical calculations are only done along the shower axis, no lateral distribution is available.

In Fig. 1 the average longitudinal profile of charged particles for 10^{18} eV proton-initiated vertical showers as calculated both in the hybrid scheme (E_{thr} = 10^7 GeV) and using pure MC (E_{thr} = 1 GeV) or numerical (E_{thr} = 10^9 GeV) approaches with GHEISHA and QGSJET is compared to CORSIKA simulations. All methods are fully compatible with CORSIKA results. Fig. 2 shows in more detail how well the different methods and the CORSIKA results agree in the description of the energy spectra of e/m particles around the maximum (X = 700 g/cm²) and at the ground level (X = 1000 g/cm²) for 10^{18} eV proton-initiated vertical showers.

The most important feature for the hybrid scheme is to reproduce the event-by-event fluctuations of the shower development as a function of the primary mass. In Fig. 3 fluctuations of the shower maximum depth X_{max} around the mean shower maximum depth \langle X_{max} \rangle for a primary energy of 10^{18} eV are shown. Proton and iron-initiated showers were simulated with CONEX and are compared with CORSIKA results. A perfect agreement is observed for both primary particles.

In Fig. 4 the average depth of maximum, \langle X_{max} \rangle, of proton and iron-induced air showers is shown as function of the energy for various models and data. The the predictions of the CONEX and CORSIKA simulation codes agree within the statistical uncertainties.

4. SUMMARY

CONEX is a new, reliable tool for fast one-dimensional air shower simulations. Realistic shower profiles, including fluctuations that lead, for example, to showers with two maxima, can be generated for particles of primary mass, energy and incident angle as relevant to UHECR studies.
With a calculation time of about 2 minutes per shower (depending weakly on the primary energy and mass), CONEX provides necessary information for applications such as fluorescence detector simulations or theoretical studies of shower profiles and $X_{\text{max}}$ distributions. Work is in progress to include muon calculations and new high and low-energy hadronic interaction models in CONEX. Furthermore it is planned to link CONEX to CORSIKA to provide full 3D information and low energy particle tracking for surface detector analysis or shower radio emission generation.

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