countBF: A General-purpose High Accuracy and Space Efficient Counting Bloom Filter

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Abstract—Bloom Filter is a probabilistic data structure for the membership query, and it has been intensely experimented in various fields to reduce memory consumption and enhance a system’s performance. Bloom Filter is classified into two key categories: counting Bloom Filter (CBF), and non-counting Bloom Filter. CBF has a higher false positive probability than standard Bloom Filter (SBF), i.e., CBF uses a higher memory footprint than SBF. But CBF can address the issue of the false negative probability. Notably, SBF is also false negative free, but it cannot support delete operations like CBF. To address these issues, we present a novel counting Bloom Filter based on SBF and 2D Bloom Filter, called countBF. countBF uses a modified murmur hash function to enhance its various requirements, which is experimentally evaluated. Our experimental results show that countBF uses 1.96× and 7.85× less memory than SBF and CBF respectively, while preserving lower false positive probability and execution time than both SBF and CBF. The overall accuracy of countBF is 99.999921, and it proves the superiority of countBF over SBF and CBF. Also, we compare with other state-of-the-art counting Bloom Filters.

Index Terms—Bloom Filter, Counting Bloom Filter, Membership Filter, Frequency Count, Count-Min Sketch, Data Structures.

I. INTRODUCTION

Bloom Filter [1] is an extensively experimented data structure. It has met a vast application domains, namely, Big Data [2], IoT [3], Computer Networking [4], Network Security and Privacy [5], [6], Biometrics [7], and Bioinformatics [8]. Counting Bloom Filter is useful in Computer Network, Network Security and Privacy [9]. Therefore, there are diverse Bloom Filters available to address the various problems of diverse domains [10]–[14]. Bloom Filter is categorized into two key categories, namely, counting and non-counting Bloom Filter. Non-counting Bloom Filter (conventional) is faster than counting Bloom Filter. However, the non-counting Bloom Filter does not support delete operation due to a false negative issue. Delete operation introduces a false positive issue. On the contrary, counting Bloom Filter supports delete operation and can solve false negative issue [15]. However, the false positive probability counting Bloom Filter is higher than the conventional Bloom Filter [16]. [17]. Alternatively, Bloom Filter occupies more memory to achieve the desired false positive probability than conventional Bloom Filter.

Delete operation is crucial for Bloom Filter. Suppose a database management system integrates Bloom Filter to avoid unnecessary disk accesses and enhance its performance significantly with a tiny amount of memory [18]. Thus, the database management system requires to insert, query, and delete operation for which conventional Bloom Filter is not suitable. Therefore, counting Bloom Filter is used in such kind of requirements. Another example is a network router. Older packets are removed from the router databases. Therefore, the router requires counting Bloom Filter. Moreover, counting Bloom Filter is an effective approximation tool for frequency count. Therefore, counting Bloom Filter is adapted in diverse applications. Moreover, there are numerous variants of counting Bloom Filters available. Rottenstreich et al. [19] develops an excellent counting Bloom Filter based on $B_h$ sequences, called $B_h$-CBF and variable-increment counting Bloom Filter (VI-CBF). Unlike conventional counting Bloom Filter, insertion in $B_h$-CBF is incremented based on $B_h$ sequences. $B_h$-CBF able to reduce false positives, however, the space consumption is very high. In addition, Pontarelli et al. [20] develops a new counting Bloom Filter based on fingerprint, called FP-CBF. FP-CBF uses fingerprints to improve the counting Bloom Filter, and therefore, it requires $(C + F) \times m$, where $C$ is the number of bits, $F$ is the fingerprint size, and $m$ is the total number of positions. FP-CBF uses $(k + 1)$ hash functions to insert, delete and lookup operations. Also, FP-CBF outperforms VI-CBF in terms of false positive and space consumption. Moreover, Ternary Bloom Filter (TBF) [21] is a counting Bloom Filter to address the false positive and false negative.

As we know that counting Bloom Filter has a high false positive probability for which it requires a higher memory footprint than conventional Bloom Filter. CBF can eradicate the false negative issues, but it has a high false positive probability, and high memory footprint. To lower the false positive probability, counting Bloom Filter sacrifices memory footprint. Therefore, we propose a novel counting Bloom Filter to address the above-raised issues, called countBF. The countBF can reduce the false positive probability significantly while preserving a low memory footprint. Our experimental results show that countBF outperforms standard Bloom Filter (SBF) [16] and counting Bloom Filter [15] in every aspect.
Key objectives of our proposed system is to reduce memory footprint, to lower false positive probability and to increase its accuracy without compromising the insertion/query performance.

Our proposed counting Bloom Filter is similar to the conventional Bloom Filters. countBF has counters, while SBF does not have counters. Also, countBF is implemented in the platform of a 2D Bloom Filter (2DBF) [22]. 2DBF uses a 2D integer array instead of relying on the bitmap array. This 2D integer array is used as a bitmap where each integer represents a block of bits. countBF enhances its performance by tuning the murmur hash function [23]. Murmur hash function is the best non-cryptographic string hash functions [24]. There are also cryptographic string hash functions, however, it does not enhances the performance and the false positive probability [25]. Therefore, we compare countBF with SBF and CBF to evaluate the characteristics using various test cases. In our experimental work, we have compared countBF with SBF because counting Bloom Filter is unable outperform SBF, and thus, it is justified to compare with SBF and CBF. Also, we compare with the other filters with countBF.

II. countBF: The Proposed System

![Architecture of countBF with 8 bit counters](image)

We present a novel counting Bloom Filter, called countBF, by deploying 2-Dimensional Bloom Filter [22]. countBF uses a few arithmetic operations to increase its performance. Let, \( \mathbb{B}_{x,y} \) be the two-dimensional integer array to implement counting Bloom Filter where \( x \) and \( y \) are the dimensions of the filter. The \( x \neq y \) and these are prime numbers. A cell of the \( \mathbb{B}_{x,y} \) is constituted by \( \eta \) counters, as shown in Figure 1. Each counter contains \( \alpha \) bits, which is defined by the user, and it can be 1 to 64 bits depending on the user’s requirement. The data type of a cell is \textbf{unsigned long int} or \textbf{unsigned int}. Each cell of \( \mathbb{B}_{x,y} \) occupies \( \beta \) bits. Therefore, the total number of counters is calculated as \( \eta = \frac{\beta}{\alpha} \). Particularly, \( \eta = 8 \) counters, if each cell occupies \( \beta = 64 \) bits and each counter contains \( \alpha = 8 \) bits. Our key objective is to reduce memory footprint, lower the false positive probability, increase the accuracy and enhance the query performance of the counting Bloom Filter. countBF uses two masks, namely, extract mask and reset mask. An extract mask is used to extract the bit information of a counter from a cell. Similarly, a reset mask is used to reset the bit information of a counter in a cell. These two masks are used because of countBF relies on the integer array instead of the bitmap array. The extract mask is dependent on the number of counters \( \eta \). Therefore, there are \( \eta \) extract masks which are stored in an array. Extract mask is defined as \( M^e = \{M_1^e, M_2^e, M_3^e, \ldots, M_\eta^e\} \). For instance, \( M_2^e = \ldots00001111111100000000 \) for 8 bits counters. The third extract mask is \( 0x0000000000000000FC000 \) and it is the correct representation of the extract mask in 64 bits measures for 7 bits counter. Extract masks are used to extract certain counter’s value using bit operations. For instance, \textbf{unsigned long int} occupies 64 bits and there are 8 counters with 8 bits each. To extract \( l^{th} \) counter information, we perform \( C_l = (C_l >> (\eta * l)) \). This \( C_l \) gives the counter information.

Similar to extract mask, there are also \( \eta \) masks for reset a counter to zero. Reset mask is defined as \( M^r = \{M_1^r, M_2^r, M_3^r, \ldots, M_\eta^r\} \). For instance, \( M_2^r = \ldots11111000000011111111 \) for a 8 bit counter. The third reset mask is \( 0x00000000000000000E03FFF \) and it is the correct representation of the reset mask in 64 bits measures for 7 bits counter. Extract masks are used to extract certain counter’s value. Reset masks are used to reset the counter’s value to zero. To reset \( l^{th} \) counter’s value to zero, we need to perform \( B_{i,j} = (B_{i,j} \land M^r_l) \).

The counting Bloom Filter is comprised of a set of counters to counts the input items. Conventional Bloom Filter uses bitmap array to manipulate the bits to store information of input items. However, countBF does not use bitmap arrays. Instead, it uses a 2D integer array where each cell occupies some memory depending on the data type. For instance, \textbf{unsigned long int} occupies 64 bits in modern computers, and therefore, each cell occupies 64 bits memory, and it is initialized by zero. Let \( \eta \) be the total number of counters, \( \alpha \) be the bits per counters, and \( \mu \) be the bits per cell in a 2D array. Therefore, the total number of counters in each cell of countBF is \( \eta = \frac{\mu}{\alpha} \), and the remainder is not used. Thus, the total number of masks varies depending on the bits used per counter \( \alpha \).

countBF is a counting Bloom Filter that comprises many counters. The counters are incremented upon insertion of an item. Let, \( B_{i,j} \) be a 2D Bloom Filter (2DBF) and \( C_l \) be the \( l^{th} \) counter in a cell of a 2DBF. Let, \( H() \) be a hash function. We use the murmur hash function. Difference seed values create a different hash value for the same key. For insertion of a single item, countBF calls \( k \) hash functions, and the item is inserted into the \( k \) counters. Algorithm 1 demonstrates insertion of an item \( K \) using \( k \) hash functions. It requires increment the
Algorithm 1 Insertion of an item $K$ into countBF using $k$ hash functions.
1: procedure INSERTION($B_{x,y}$, $K$)
2: for $i = 1$ to $k$
3: $h = H_i(K, Seed_i)$
4: INCREMENT($B_{x,y}$, $h$)
5: end for
6: end procedure

Algorithm 2 Increment a single counter while inserting an item $K$ into countBF using a single hash function.
1: procedure INCREMENT($B_{x,y}$, $h$)
2: $i = h \% x$, $j = h \% y$, $l = h \% \eta$
3: $C_i = B_{i,j} \land M^i_l$
4: $C_i = C_i \rightarrow (\alpha \times l)$
5: $C_i = C_i + 1$
6: if $C_i$ = MAX then
7: Counter Overflow.
8: return
9: end if
10: $C_i = C_i <\!\!< (\alpha \times l)$
11: $B_{i,j} = B_{i,j} \land M^i_l$
12: $B_{i,j} = B_{i,j} \lor C_i$
13: end procedure

counters’ value. Algorithm 2 shows the incrementing process of a counter.

Algorithm 3 Lookup an item $K$ in countBF using $k$ hash functions.
1: procedure LOOKUP($B_{x,y}$, $K$)
2: for $i = 1$ to $k$
3: $h = H_i(K, Seed_i)$
4: $flag = flag \land TEST(B_{x,y}, h)$
5: end for
6: return $flag$
7: end procedure

Lookup or query operation is similar to insertion operation except the increment steps. Querying an item $K$ requires $k$ hash functions with $k$ seed values. The seed values and hash functions of lookup procedure cannot be different from the seed values and hash functions of insertion and delete operations. Algorithm 3 calls murmur hash function $k$ times and TEST() function $k$ times. The TEST() function is demonstrated in Algorithm 4 that returns either true or false. The variable $flag$ holds the final result of all test functions and returns the variable $flag$. The $flag$ is a Boolean variable that can hold either true or false. All TEST() function results are ANDed which produce Boolean value of true or false and assigned to $flag$.

Conventional Bloom Filter does not support delete operation due to false negatives. The counting Bloom Filter was introduced to address the issue of false negatives [15]. The delete operation creates the issue of false negatives in conventional Bloom Filter. Therefore, counting Bloom Filter is used in many domains. Similar to conventional counting Bloom Filter, countBF also supports a delete operation without any false negative issue. To delete an item $K$, countBF requires $k$ hash functions call and $k$ DECREMENT() functions calls as demonstrated in Algorithm 5. The insertion and delete operations are required the same steps except for the decrement of the counters in delete operation, as shown in Algorithm 5.

III. EXPERIMENTAL RESULTS

Our proposed algorithm is evaluated in 8GB RAM, Intel® Core™ i7-7700 CPU @ 3.60GHz × 8, Ubuntu 18.04.5 LTS and GCC version 7.5.0. We created four different datasets, namely, Same Set, Mixed Set, Disjoint Set, and Random Set. Let $S = \{x_1, x_2, x_3, \ldots, x_n\}$ be an inserted set into the

Algorithm 4 Lookup in a single counter while querying an item $K$ in countBF using a single hash function.
1: procedure TEST($B_{x,y}$, Hashvalue $h$)
2: $i = h \% x$, $j = h \% y$, $l = h \% \eta$
3: $C_i = B_{i,j} \land M^i_l$
4: $C_i = C_i \rightarrow (\alpha \times l)$
5: if $C_i \geq 1$ then
6: return true
7: else
8: return false
9: end if
10: end procedure

Algorithm 5 Delete an item $K$ in countBF using $k$ hash functions.
1: procedure DELETE($B_{x,y}$, Keys $K$)
2: for $i = 1$ to $k$
3: $h = H_i(K, Seed_i)$
4: DECREMENT($B_{x,y}$, $h$)
5: end for
6: end procedure

Algorithm 6 Decrement a single counter while deleting an item $K$ from countBF using a single hash function.
1: procedure DECREMENT($B_{x,y}$, Hashvalue $h$)
2: $i = h \% x$, $j = h \% y$, $l = h \% \eta$
3: $C_i = B_{i,j} \land M^i_l$
4: $C_i = C_i \rightarrow (\alpha \times l)$
5: $C_i = C_i - 1$
6: if $C_i < 1$ then
7: No deletion.
8: return
9: end if
10: $C_i = C_i <\!\!< (\alpha \times l)$
11: $B_{i,j} = B_{i,j} \land M^i_l$
12: $B_{i,j} = B_{i,j} \lor C_i$
13: end procedure
Bloom Filter, \( Q \) be the query set. The Same Set defines \( S = Q \) whereas Disjoint Set is defined as \( S \cap Q = \phi \). The definition of the Mixed Set follows any one condition, either \( q_1 \in S \) and \( q_2 \notin S \) or \( q_1 \notin S \) and \( q_2 \in S \) where \( q_1 \subset Q \) and \( q_2 \subset Q \). However, the Random Set is randomly generated dataset. These test cases are able to unearth the strengths and weaknesses of a Bloom Filter. We have assessed our proposed countBF for various counter’s sizes for the fair judgement.

countBF is compared with standard Bloom Filter (SBF) [16] and conventional counting Bloom Filter (CBF) [15]. The countBF is evaluated by setting the counter’s bits by 3 bits, 4 bits, 5 bits, 6 bits, 7 bits, and 8 bits. A counter’s bit can be a maximum of 64-bits. In our experimental evaluation, each cell occupies 64 bits, and hence, each cell has different counters. The total number of counters for 3 bits, 4 bits, 5 bits, 6 bits, 7 bits, and 8 bits counters are 21, 16, 12, 10, 9, and 8 counters in each cell of countBF. In this configuration, we conduct the experiments, and Figure 2 demonstrates the insertion time taken by countBF, SBF, and CBF. On an average, countBF is faster than SBF and CBF in the insertion of items. countBF with 4 bits counter is slowest among countBF with 3 bits, 4 bits, 5 bits, 6 bits, 7 bits, and 8 bits counters and countBF with 7 bits counter is fastest among countBF with 3 bits, 4 bits, 5 bits, 6 bits, 7 bits, and 8 bits counters in the insertion of items. Individually, countBF with 4 bits, 8 bits, 7 bits, 7 bits, and 7 bits counters are the fastest in the insertion of 10M, 20M, 30M, 40M, and 50M dataset, respectively. Overall, countBF with 7 bits is the fastest counter in the insertion operation.

Figures 3, 4, 5, 6, and 7 demonstrates the lookup operations of 10M, 20M, 30M, 40M, and 50M items respectively by countBF, SBF, and CBF. countBF is faster than SBF and CBF in all sized query with all test cases. On an average, countBF with 3 bits, 7 bits, 7 bits, and 3 bits counters are the fastest in 10M, 20M, 30M, 40M, and 50M items lookup, respectively. It is observed that countBF with 8 bits counter exhibits the worst performance overall. For any test case, countBF with 4 bits, 8 bits, 8 bits, 7 bits, and 6 bits are the fastest in the insertion of 10M, 20M, 30M, 40M, and 50M dataset, respectively. Overall, countBF with 7 bits is the fastest filter. countBF outperforms SBF and CBF by 50.73% and 55.68%, 45.33% and 54.67%, 44.84% and 53.82%, 43.99% and 57.14%, and 44.46% and 52.01% in 10M, 20M, 30M, 40M, and 50M items’ lookup respectively. Overall, our proposed system has a higher lookup performance rate than SBF and CBF.

Figures 8, 9, 10, 11, and 12 demonstrates the false positive probability of countBF, SBF and CBF in log scale. Counting Bloom Filters are designed to deal with the false negative issue. Therefore, conventional counting Bloom Filter has an issue of trade-off between false positive probability and
memory consumption. Normally, counting Bloom Filters have a higher false positive probability than other variants of the membership filter. However, countBF outperforms in the false positive probability in Mixed Set, Disjoint Set, and Random Set queries. There is no false positive probability for the Same Set query. countBF exhibits an excellent false positive probability in Disjoint Set query, and it is zero in Disjoint set for the lookup of 10M dataset, which is demonstrated in Figure 9. However, the highest false positive probability of countBF is recorded as 0.000035. countBF with 7 bits
counter exhibits the highest false positive probability in 20M dataset lookup. On an average, the lowest and highest false positive probability of countBF is 0.000006 and 0.000022, respectively. Overall, the false positive probability of countBF is 0.000013. On an average, the false positive probability of SBF and CBF are 0.001004533333333 and 0.000999133333333, where CBF exhibits a lower false positive probability. It is possible due to the higher memory footprint of CBF than SBF. Thus, our proposed system exhibits the lowest false positive probability, whereas SBF and CBF exhibit equivalent false positive probability. Both SBF and CBF are configured to the desired false positive probability as 0.001, and therefore, their false positive probability is equivalent.

Figures 13, 14, 15, 16, and 17 show the accuracy of countBF, SBF and CBF. The accuracy of SBF and CBF does not differ more, and both are equivalent due to the desired false positive setting to 0.001. However, the accuracy of countBF outperforms SBF and CBF in all sized dataset with all test cases. The highest accuracy is 100% in Disjoint Set dataset of 20M lookup, and it is countBF with 3 bits counter, which is demonstrated in Figure 14. On an average, the lowest and highest accuracy of countBF are 99.9999% and 99.99999% respectively. Overall, countBF accuracy is 99.999921%. The overall accuracy of SBF and CBF are 99.8997% and 99.90%, respectively.

Figures 13, 14, 15, 16 and 17 show the accuracy of countBF, SBF and CBF. The accuracy of SBF and CBF does not differ more, and both are equivalent due to the desired false positive setting to 0.001. However, the accuracy of countBF outperforms SBF and CBF in all sized dataset with all test cases. The highest accuracy is 100% in Disjoint Set dataset of 20M lookup, and it is countBF with 3 bits counter, which is demonstrated in Figure 14. On an average, the lowest and highest accuracy of countBF are 99.999897%, and 99.99995% respectively. Overall, countBF accuracy is 99.999921%. The overall accuracy of SBF and CBF are 99.9997% and 99.90%, respectively.

As we know that the CBF has a higher false positive probability than SBF. On the contrary, the experimental results show that SBF and CBF similar false positive probability. We have indeed configured the desired false positive probability to 0.001, and therefore, the false positive probability of SBF and
CBF are equivalent. However, the memory requirements are different. CBF uses higher memory than SBF. It means CBF has a higher false positive probability than SBF. We adjust the memory allocation to achieve the desired false positive probability. Therefore, CBF has allocated more memory to achieve the desired false positive probability. Thus, counting Bloom Filter occupies more memory to achieve the desired false positive probability. Notably, counting Bloom Filter consume more memory than other variants of Bloom Filter or membership filter to achieve certain false positive probability. On the contrary, SBF uses 1.96× more memory than countBF, and CBF uses 7.85× more memory than countBF on an average. The lower memory footprint of countBF shows the highest accuracy and the lowest false positive probability. On an average, countBF, SBF and CBF consume memory of 26.11 MB, 51.42 MB, and 205.67 MB for all datasets, respectively. The memory, false positive probability, and accuracy are the key decisive factor of the Bloom Filter. However, countBF also faster than SBF and CBF. But there are much faster membership filters available, however, countBF is more accurate than any other counting variant of Bloom Filters.

Figure 19 demonstrates the bits per item of the countBF, SBF and CBF. Now, countBF uses the lowest bits per item, and CBF uses the highest bits per item. countBF, SBF, and CBF use 7.32 bits, 14.38 bits, and 57.51 bits per item on average, respectively. Our proposed counting Bloom Filter uses the lowest bits per item and provides a lower false positive probability.

![Comparison of bits per item of countBF, SBF and CBF](image)

**Fig. 19.** Comparison of bits per item of countBF, SBF and CBF. Lower is better.

Figure 20 depicts each cell's memory wastage in bits per memory cell of countBF with 3 bits, 4 bits, 5 bits, 6 bits, 7 bits, and 8 bits counters. Assuming, these countBF occupies 64-bits memory in each cell, and therefore, it shows the total number of unused bits (allocated but not used). Lower is better.

![Comparison of memory wastage in bits per memory cell of countBF with other filters](image)

**Fig. 20.** Comparison of memory wastage in bits per memory cell of countBF with 3 bits, 4 bits, 5 bits, 6 bits, 7 bits, and 8 bits counters. Assuming, these countBF occupies 64-bits memory in each cell, and therefore, it shows the total number of unused bits (allocated but not used). Lower is better.

IV. Analysis

countBF is derived from 2DBF and conventional Bloom Filter. Therefore, we derive the memory requirements from conventional Bloom Filter. Let, \( m \) be the bit size of Bloom Filter, \( n \) be the total number of input items, and \( k \) be the total number of hash functions. Therefore, the probability of a particular bit is not set to 1 is \( (1 - \frac{1}{m}) \). The probability of that particular bit is not set to 1 by \( k \) hash functions is \( (1 - \frac{1}{m})^k \). There are \( n \) input items and the probability of that particular bit is not set to 1 is \( \epsilon = (1 - e^{-kn/m})^k \). The probability of the particular bit to be 1 is \( (1 - e^{-kn/m}) \). The probability of all bits are set to 1 is \( \frac{1}{\epsilon} \).

The memory requirements of countBF depends on dimensions of 2DBF. Therefore, countBF uses \( \frac{k}{2} \) hash functions. The memory requirement is calculated as follows- we divide the total memory requirement of \( m \) using \( b = \frac{m}{2\times\beta} \) where \( \beta \) is the memory bits occupied by each cell in countBF. Now, \( q = \sqrt{b} \) to select the \( q \)th prime number. Let, \( \mathcal{PN}_\psi \) be the array of \( \psi \) prime numbers. Let, \( i = \text{selectPrimeNumber}(q) \) select a prime number from \( \mathcal{PN}_\psi \) indexed at \( q \) and assigned the returned value to \( i \). The dimension of countBF is calculated as \( X = \mathcal{PN}_{i+3} \) and \( Y = \mathcal{PN}_{i-3} \), because the dimension of countBF cannot be same, i.e., \( X \neq Y \) and these are prime numbers. Therefore, the total memory requirement is \( X \times Y \times \beta \) bits or \( \frac{X \times Y \times \beta}{MB} \) MB where \( MB = 8 \times 1024 \times 1024 \) is a megabyte.

We exploit the property of the hashing technique with prime numbers. Prime number reduces collision in hashing. Therefore, our proposed counting Bloom Filter uses two prime numbers to perform modulus operation and define the dimension of the 2D filter. Thus, two modulus operations using prime numbers significantly reduce the false positive probability. Therefore, countBF is able to increase its accuracy using a lower memory footprint. Moreover, we are able to reduce the number of hash functions to half.

A. Comparison of countBF with other filters

There are numerous faster membership filters available than countBF. For instance, counting Quotient Filter (CBF) [27], Morton Filter (MF) [28], XOR Filter (XF) [29]. Morton and XOR filters are the fastest filters. However, there is a trade-off among speed, memory and false positive probability. Also, there is a compressed Bloom Filter, which compromises
performance and accuracy with memory \[31\]. Various membership filters are available and a few membership filters are compared in Table \[1\]. Many of the membership filters do not provide high accuracy and low false positive probability with a tiny amount of memory, while \textit{countBF} can do the same. \textit{CBF} has the highest false positive probability and consumes the highest memory \[15\]. The lookup and query performance are same for \textit{Cuckoo Filter (CF)} \[26\] and \textit{countBF}. Cuckoo Filter exhibits poor in the false positive probability.

Table \[1\] demonstrates the overall comparison among SBF, CBF, CF, CQF, MF, XF, VI-CBF, TCBF and \textit{countBF}. Cuckoo Filter \[29\] exhibits the worst in false positive probability. SBF, CBF, CF, CQF, MF, XF, VI-CBF, TCBF and \textit{countBF} using rating points from 1 to 10. Rating 1 represents the worst, and rating 10 is the best. SBF is the standard, and the overall rating is 6.5. MF and XF are the fastest, and their ratings are 10 for both. The rating of \textit{countBF} and CF is the same in execution time while \textit{countBF} outperforms in other features. Moreover, SBF, CF, MF, and XF are the same in false positive probability and accuracy. SBF, CF, MF, XF, VI-CBF, and TCBF have an almost similar memory footprint. \textit{countBF} outperforms all other filters in false positive probability, accuracy, and memory footprint except execution times.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Filter  & False positives & Accuracy & Memory Consumption & Query Speed \\
\hline
SBF \[16\] & 7 & 7 & 6 & 6 \\
CBF \[15\] & 4 & 4 & 7 & 5 \\
Cuckoo Filter \[26\] & 4 & 4 & 5 & 9 \\
CQF \[27\] & 6 & 6 & 7 & 9 \\
Morton Filter \[28\] & 6 & 6 & 7 & 10 \\
XOR Filter \[29\] & 6 & 6 & 7 & 10 \\
VI-CBF \[19\] & 6 & 6 & 5 & 9 \\
TCBF \[30\] & 7 & 7 & 7 & 9 \\
\textit{countBF} & 10 & 10 & 10 & 8 \\
\hline
\end{tabular}
\caption{Comparison among a few membership filter. Rating scale from 1-10. Rating 1 is the worst and 10 is the best.}
\end{table}

\textbf{B. Frequency count}

Count-Min Sketch (CMS) is a probabilistic data structure for frequency count of input stream \[32\]. \textit{countBF} is converted into CMS. Insertion of an item causes an increment of \(k\) counters by \(k\) hash functions. Counter value can be extracted by extract masks in \textit{countBF} and let these counters be \(C_1, C_2, C_3, \ldots C_k\). These counters are placed in different locations of \textit{countBF} depending on the hash function. Therefore, the frequency of an item \(K\) in \textit{countBF} can be found in Equation \[1\].

\[
\text{Count}(K) = \min_{i=1}^{k} C_i 
\]

The minimum count among all counters will be the frequency of \(K\). This frequency count is an approximation counting. CMS requires a large number of bits in each counters and high accuracy. These capabilities have already been demonstrated experimentally and evaluated using different parameters.

\textbf{V. Related works and Discussion}

Counting Bloom Filter (CBF) is introduced on 2000 by Li et al. \[15\]. Counting Bloom Filter become popular due to false negative free. Deletion introduces false negatives in conventional Bloom Filter \[25\], but deletion is the most important operation in many applications. Therefore, diverse variants of counting Bloom Filter have been introduced, and Luo et al. \[33\] reported 15 counting Bloom Filters till 24 December 2018. Recently, a few counting Bloom Filters are developed, tandem counting Bloom Filter (TCBF) \[30\], Einziger and Friedman \[34\], and mergeCBF \[35\]. Current literature shows that even if there are diverse counting Bloom Filters available, but are unable to reduce memory consumption per element. Even the comparatively memory consumption per item of state-of-the-art CBFs are higher than SBF. False positive and memory have a strong relationship. Higher memory is required to lower the false positive probability. Thus, we can conclude that counting Bloom Filter has a higher false positive probability than SBF \[16\]. The key challenge is to decrease false positive probability using less memory without compromising the insertion/query/deletion operations’ performance. We have already demonstrated experimentally that our proposed \textit{countBF} uses less memory footprint than any state-of-the-art counting Bloom Filters. Also, \textit{countBF} can increase its accuracy by reducing false positive probability using a low memory footprint.

\textbf{VI. Conclusion}

This article has demonstrated our proposed counting Bloom Filter, called \textit{countBF}, significantly improved over SBF and CBF. \textit{countBF} can insert, query, and delete an item in \(O(k)\) time complexity while preserving high accuracy, low false positive probability, low memory footprint, and fast execution time. Moreover, the properties of \textit{countBF} make more room for incoming items. We have evaluated the false positive probability using various test cases experimentally. The false positive probability is lower than the SBF and CBF. Alternatively, the accuracy of \textit{countBF} is higher than SBF and CBF. Therefore, \textit{countBF} is able to outperform the existing Bloom Filter in terms of false positives, accuracy, memory footprint, and performance. Also, we have compared with various state-of-the-art counting Bloom Filters and it shows that our proposed counting Bloom Filter is an ideal solution. Moreover, we have demonstrated how to adapt \textit{countBF} in frequency count, similar to CMS. It can also be applied in diverse domains where delete operation is a crucial part of the system.

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