A flow model for optimization of highway toll stations

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Abstract. This paper is aimed at developing a highway toll stations route optimization model (HTSROM) for cost-effective and sustainable highway infrastructure planning and design. There are many factors for the locations of highway toll stations, including potential ridership, costs of land, construction and operation, and travel times. We developed HTSROM algorithm which used a genetic model to optimize and integrate a geographic information system for transfer of land-use, environmental, and topographic data during the optimal search process. The experimental results are discussed on the application, and more extensions of the HTSROM model remain to be studied in the future work.

1 Introduction

As road transportation brings great changes of our lives, it also brings safety and environmental problems [1-3]. The road constructions consumed uncountable resources. New transportation technologies such as connected vehicle and travel demand control, are urgently in need of reducing the negative environmental and safety impacts from transportation [2]. Barrier tolls are used as an instrument to finance new highway infrastructure. A barrier toll is a series of tollbooths which are placed across the highway, perpendicular to the traffic flow. Drivers pay their tolls when vehicles pass through the tollbooths. There are usually more tollbooths than lanes of the highway. Congestion arises when vehicles converge on the exit of the toll plaza.

A traditional mode of toll plaza on one side is illustrated in Fig.1, and the depth of color shows the willingness of drivers to go through the booth. The darker the block is, the lower that drivers are likely to pass it. The red block marks the area where congestion is mostly like to arise. Most traffic delays when an evacuation occur at intersections, so lane-based routing is important for reducing the delays [3,4]. Typically all changing lanes makes the traffic much slower, because three lines of traffic smoothly move from the left to the right, as shown in Fig.2. Suddenly a vehicle marked by a indigo dot changes its lane from the upper to the middle lane, causing a series of vehicles slowing down, marked in deep gray dots. Therefore several vehicles, marked in purple dots, decide to change lane, resulting in more deceleration, finally result in a congestion by the butterfly effect.
This paper presents a highway toll stations route optimization model (HTSROM) for cost-effective and sustainable highway infrastructure planning and design. The aim of the algorithm is that *Average Lane-Changing Times (ALT)* of all the vehicles involved in the system may serve as a key factor to decide the performance of the toll plaza (the lower, the better). Thereafter, another method to increase the *Occupancy Rate of Lanes (ORL)* is developed with a method to decrease the ALT. We review the previous traditional work in prescriptive routing criteria for routing plan. The next section presents the model and formulation. Manual analysis and simulation are used to evaluate the efficiency of the HTSROM model. In the end, the paper concludes with the discussion of results and further works.

2 The proposed algorithm

As illustrated in Fig.3, if one lane has more empty places than the other lane beside, there will be a tendency for vehicles on the other lane to change lane. That is because vehicles of sparser traffic flow enjoy higher speed limit, due to safety consideration. Therefore by increasing ORL, less lane-changing will happen in the traffic system, increasing the handling capacity of the toll plaza, and meanwhile ensuring a safer traffic environment.

Figure 3. The basic assumption in the HTSROM mode.

Then, we choose the case when $B = 8$ and $L = 4$ as a general representation of different tolls. To simplify our discussion, we assume that the highway before and after the toll plaza contain the same number of lanes, which is always less than the number of toll booths. The table below shows some
variables that we cover later in this paper.

| Variables | Definitions |
|-----------|-------------|
| B         | the number of tollbooths |
| L         | the number of lanes in the highway before and after the toll plaza |
| plaza     | the plaza matrix |
| $d_i$     | the distance between two vehicles |
| p         | the probability of random deceleration of vehicles |
| P         | the probability that a driver choose a certain booth |
| t         | token of the booths, varies from one to eight |
| $v_{max}$ | maximal vehicle speed |
| influx    | the number of vehicles that reach the toll plaza in an hour (veh/hr) |
| J         | the number of vehicles per hour passing the area where the end of the plaza joins the L outgoing traffic lanes (veh/hr) |
| $J_{max}$ | the limit of handling capacity of the toll station |

2.1 CA Model
The first key of HTSROM is cellular automation model (CA Model), which is used for the simulation of specified toll plaza conditions. In the CA model, we set a main matrix to represent the toll plaza on highway. Each element of the matrix represents a cell which has four different states. The specific definition of each state is shown in Table.2. The main part of matrix plaza and the simulation diagram of CA model are shown in Fig.4. It is shown that the yellow cells are occupied by a toll barrier. Then, we mark the toll booths (yellow cells in Fig.4) with a list of numbers, shown in Fig.5.

| Table 2. Four States of Cell |
|-------------------------------|
| 1 the cell is occupied by a vehicle |
| -1 the cell is forbidden to pass(including the road boundary) |
| 0 the cell is unoccupied |
| -3 the cell is occupied by a toll barrier |

Figure 4. Matrix and simulation diagram of CA model.
2.2 NS Rules
Another key of HTSROM is **NS rules**, which is employed to initialize our CA model with NS rules that were first suggested by Nagel and Schreckenberg [4,6]. NS rules definite three states of cars-acceleration, deceleration and random moderation. And the maximum speed of cars is larger than 1. We set this CA model on the basis of NS rules as discrete model of which parameters like time, speed and place are all discrete. The set of rules is shown in Table 3.

**Table 3. Rules of Vehicles Moving Forward Based on NS Rules**

| States | CA code |
|--------|---------|
| initial state | $v_1 = 2, v_2 = 1, v_3 = 1, v_4 = 0$ |
| acceleration rules | $v_i(t + 1) = \min(v_i(t) + 1, v_{max})$ |
| accident prevention | $v_i = \min(v_i, d_i - 1)$ |
| deceleration rules | $v_i(t + 1) = \min(v_i(t + 1), gap_i(t))$ |
| random deceleration rules | $v_i(t + 1) = \max(v_i(t + 1) - 1, 0)$ if $rand < p$ |
| update place rules | $x_i(t + 1) = x_i(t) + v_i(t + 1)$ |

2.3 Experimental Assumption
Besides, we assume that the number of vehicles reaching the toll plaza per time period obeys the **poisson** distribution. By real life experience we know that drivers tend to make a short cut and are less likely to choose lanes on the side. Therefore in the following table we define the probability for drivers to choose a certain booth.

**Table 4. The probability for drivers to choose a booth.**

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----|---|---|---|---|---|---|---|---|
| $P(t)$ | 24 | 24 | 24 | 24 | 8 | 4 | 2 | 1 |
|     | 111 | 111 | 111 | 111 | 111 | 111 | 111 | 111 |

It can be seen that the probabilities of choosing the **No.1,2,3,4** booths are the same, that is because these booths directly face the high way. The probability significantly decreases when it comes to **No.5** booth, and smoothly goes down on **No.6,7,8** booths.

In the following discussion we will not encounter with large change of the space of toll plaza,
therefore we take no consideration to the cost of land occupied by the construction. We introduce **Average Lane-Changing Times (ALT)** to decide the safety performance of the system. We use the handling capacity per hour $J$ to decide the capacity performance of the system. Fig. 6 shows that the handling capacity increases as the influx improves, and the limit handling capacity $J_{\text{max}}$ of the original system is around 2290 veh/hr. In Fig. 7, the fluctuation of ALT when the influx varies. As illustrated in Fig. 6 and Fig. 7, when the influx comes to the limit, vehicles change lanes for more than 2.5 times on average, which contributes to the congestion.

![Figure 6. The handling capacity increases as the influx improves.](image6)

![Figure 7. The fluctuation of ALT when the influx varies.](image7)

3 Optimization and analysis

3.1 Arrangement of ETC

ETC (Electronic Toll Collection) is nowadays the most advanced technique of collecting tolls. It allows drivers to pay their tolls without stopping or even slowing down. Apparently this technique improve the efficiency of the barrier toll, yet it to some extent guides the choice of lanes of the drivers as well. The Fig. 8 illustrates how the HTSROM is realized in program. We mark the ETC with light blue. Vehicles do not decelerate when passing through the ETC. To make our research simple, we first assume that one of eight booths is replaced by ETC booth, then which booth should we change?
Figure 8. The simulation diagram with ETC and digital number tokens.

From above perspective views, we illustrate the fluctuation of $J$ and $ALT$ when the position of ETC varies in Fig. 9, respectively. Then we can see that we should replace the No.1,2,3,4 booths by the ETC. It is because these four lanes directly face the highway and are the closest route for the drivers. It seems that the No.8 booth is the worst choice to be replaced. However, in the discussion above we fail to take the influence of the drivers’ choice of booths into account. The fact is that ETC will attract drivers to pass through because it simply saves time. Now let’s reset the distribution that drivers choose the booths and see what will happen.

Figure 9. The fluctuation of $J$ and $ALT$ when the position of ETC varies, when influx is set by 4000 veh/hr. Left: fluctuation of $J$, Right: fluctuation of $ALT$.

The update of distribution is not remarkable if we replace the No.2,3,4 booths. However the distribution significantly changes when we replace the No.5,6,7,8 booths. Note that since the No.1 booth has already been replaced by ETC, the probability of choosing the other booths can not exceed the probability of choosing the No.1 booth. And when a new ETC is set on the No.8 booth, an example of the update of distribution is shown in Table 5. Note that the No.1 booth has already been replaced. Then, the new fluctuation of $J$ and $ALT$ when the position of ETC varies in Fig. 10, respectively. From above we can see that the $ALT$ experiences a remarkable decrease, and the number of $J$ increases. This inspires us the best choice of the booths to be replaced, the No.1,8 booths. In common cases, we should replace the booths on the side.

Table 5. The update of distribution.

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----|---|---|---|---|---|---|---|---|
| $P(t)$ | $\frac{27}{111}$ | $\frac{19}{111}$ | $\frac{19}{111}$ | $\frac{19}{111}$ | $\frac{6}{111}$ | $\frac{2}{111}$ | $\frac{1}{111}$ | $\frac{18}{111}$ |
Figure 10. The fluctuation of $J$ and $ALT$ when the position of ETC varies. Note that the $No.1$ booth has already been replaced by $ETC$. The $influx$ is set by 1200, 2400, 3600 veh/hr. Left: fluctuation of $J$, Right: fluctuation of $ALT$.

3.2 Rearrangement of Toll Booths

After the former discussion we have noticed that the traffic condition can be improved if there are vehicles guided to the lanes on the side. Since we know that drivers hardly see things behind the toll booths, let us analyze the new situation if we artificially “pull” the booths on the side closer to the drivers. The rearrangement of toll booths is shown in Fig. 11. By changing the distribution of toll booths we substantially change the probability of drivers choosing a certain lane, as shown in Table 6. Since we have no data on the influence to the choice of drivers, we could simply assume that the change of arrangement do bring the influence we except, by real life experience. We can later rearrange the booths again if we gain the data of the influence after actual operation, and then reach the most optimized rearrangement. From above, we can see that the rearrangement of toll booths successfully brings a great improve to the system.
3.3 Truck Lanes
Drivers do meet such cases when they see a better lane beside yet blocked by large trucks. Trucks significantly affect traffic efficiency due to their volume and poor steering ability. One way to handle the trouble caused by trucks is to set truck lanes. Apparently truck lanes should be set right on the side of the road. We will try to figure out whether they should be set in the No.1 booth or the No.8 booth. We assume that trucks are two times longer than small vehicles, which means they occupy two blocks in the CA model. We also assume that the limiting speed of trucks is 70% of small vehicles. Notice that most of the vehicles on road are small cars, therefore we assume that the proportion of trucks is no larger than 50%. What’s more, trucks should only move on truck lanes, yet the route of small vehicles is not limited.

Fig. 12 (a) shows the different arrangement of truck lanes after rearranging toll booths. Truck lanes are marked in light blue. Fig. 12(b) illustrates the fluctuation of $J$ when there is no truck lanes, when the truck lane is set before toll station No.1, and when the truck lane is set before toll station No.8. The proportion of trucks goes from 10% to 50%. The influx is set by 4000veh/hr. The handling capacity significantly improves after the set of truck lanes, when there is not too much trucks on the road. However, with the increase of the proportion of trucks, truck lanes loses its competitiveness. That is because too much vehicles are gathered in one certain lane, causing a great waste of space on the other lanes.
Figure 12(a): the different arrangement of truck lanes after rearranging toll booths

Figure 12(b): the fluctuation of $J$

4 Conclusion
Toll stations are always the weak spots of high way. Through the discussion above we come up with different ways of alleviating congestion. The set of ETC increases the speed of passing, and together with the rearrangement of toll booths, optimizes the utilization of road spaces. The set of Truck lanes resolves the contradiction between trucks and small vehicles. Hopefully the future technique of intelligent autonomous vehicles will get the congestion solved once and forever.

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