Investigation of distributed-porosity fields for urban flood modelling using single-porosity models

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Abstract. Porosity-based models rely on the assumption that an urban area can be compared to a porous medium, where the porosity is defined as the ratio between the actual area available to the flow, i.e. not occupied by buildings, and the total area of the considered urban environment. In classical single-porosity models, the resulting value of the porosity parameter is considered as constant and accounts essentially for the reduced water storage capacity and reduced space available for the flow. As a consequence, the source term involving the porosity parameter only accounts for a local head loss at the entrance and at the exit of the urban area. Therefore, the head losses occurring inside the urban area are accounted for using drag-type source terms. In the present work, we tested different definitions of the porosity parameter, showing the benefits of accounting for areas with distributed porosity based on the actual layout of buildings and streets. This formulation is still compatible with the basic idea of porosity-based model, i.e allowing for the use of coarse computational meshes instead of refined meshing of the urban area.

1. Introduction

An urban area located along a river can be flooded when the discharge in the river exceeds a given threshold. In that case, the districts built in the floodplains of the river are progressively filled with water. The flooding water constitutes essentially an additional water storage area while, due to generally small velocities, dynamic effects are limited. However, in other circumstances, such as dike breaching or flash floods following heavy rainfall, the city might be flooded by flowing water, with important dynamic effects in addition to storage effect. In such a situation, the resulting flow features complex phenomena such as momentum exchanges and head-losses at cross-roads, or dispersion due to the presence of buildings. For such situations, performing detailed numerical simulations is often not possible, as it would require a detailed representation of all streets and buildings, leading to prohibitive data acquisition and computational costs. Porosity models offer an alternative as these can be used with coarse grids, larger than the street width. Such
models consider the urban area as a continuum and assume that the urban fabric can be considered as a porous medium, the buildings being the “grains” and the streets the “pores”.

In such models, the porosity is defined as the ratio between the area offered to the flow and the total area. The first porosity models [1-3] assumed a single constant value of the porosity parameter in the urban area (SP – single porosity models), and were complemented by head loss source terms to represent the effects of the momentum losses and exchanges inside the urban area [4, 5], or on the development of dual porosity models (DIP) to differentiate between a so-called “storage” and “conveyance” porosity [6].

In the present study, the effect of a variable-porosity field is investigated, in the framework of single-porosity models. In this variable-porosity model, the value of the porosity parameter assigned to the urban area is no more constant, but is adjusted to the local layout of the urban fabric, i.e. a higher porosity in the regions occupied by the streets and a lower value in the regions occupied by the buildings. With such a model, we show that parts of the dispersion effects induced by the buildings are accounted for directly by the porosity variations. This model is complemented by a drag-type source term, written as a drag matrix, allowing for the representation of directional effects, as proposed in [7].

The paper is structured as follows. First, the governing equations and the drag tensor are presented. Then, using laboratory test cases of steady flows in idealised urban environments [7], the results of a variable-porosity field are illustrated, as well as the capacity of the drag tensor to reproduce directional effects. The model is calibrated on a test case involving an isotropic city layout, and then validated using the measurements obtained for other layouts of the idealised city. Finally, conclusions are drawn and future possible improvements are discussed.

2. Governing equations

Considering the porosity parameter \( \phi \), the governing equations of the SP model [7] read

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S}
\]

(1a)

\[
\mathbf{U} = \begin{pmatrix} \phi h \\ \phi u h \\ \phi v h \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \phi u h \\ \phi u^2 h + \phi \frac{gh^2}{2} \\ \phi u v h \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \phi v h \\ \phi u v h \\ \phi v^2 h + \phi \frac{gh^2}{2} \end{pmatrix}
\]

(1b)

\[
\mathbf{S} = \begin{pmatrix} 0 \\ \phi gh (S_{0,x} - S_{f,x} - S_{l,x}) + \frac{gh^2}{2} \frac{\partial \phi}{\partial x} \\ \phi gh (S_{0,y} - S_{f,y} - S_{l,y}) + \frac{gh^2}{2} \frac{\partial \phi}{\partial y} \end{pmatrix}
\]

(1c)

where \( S_{0,x} \) and \( S_{0,y} \) are the bed slope terms in the \( x \) and \( y \) directions, \( S_{f,x} \) and \( S_{f,y} \) are the friction source terms calculated by the Manning formula, \( S_{l,x} \) and \( S_{l,y} \) are the drag source terms calculated as

\[
\mathbf{S}_l = \begin{pmatrix} S_{l,x} \\ S_{l,y} \end{pmatrix} = \left\| \mathbf{V} \right\| \left( \mathbf{C}_D \cdot \mathbf{V} \right)
\]

(2)

where \( \mathbf{V} \) is the velocity vector and \( \mathbf{C}_D \) the drag matrix with the directional drag coefficients...
3. Porosity distribution and drag tensor

From the governing equations, where the only terms not directly proportional to \( \phi \) are the terms containing the derivatives of \( \phi \), it can be observed that for a constant porosity field, the spatial derivatives involving the porosity parameter \( \phi \) can only have a very local influence at the transitions between the urban area with a \( \phi < 1 \) and the open flow area with \( \phi = 1 \).

Considering the experiments conducted at the Hydraulics Laboratory of the Université Catholique de Louvain, Belgium, in a channel with an idealised urban area constituted by different arrangements of square wooden blocks featuring the buildings (see Fig. 1 for the experimental set-up), as described in [7], a constant porosity parameter would result in the porosity distribution illustrated in Figure 2a. The porosity of this area was calculated as \( \phi = 0.4375 \). In this study, we propose to use a variable porosity field for the same building layout, as illustrated in Figure 2b. The average value of the porosity in the area is the same, i.e. \( \phi = 0.4375 \), but it is distributed according to the buildings layout, with higher values in the streets and lower values centred on each building.

![Experimental setup and flume dimensions](image)

**Fig. 1.** Experimental setup and flume dimensions in (m): (a) plane view and (b) cross-section.

The porosity distribution illustrated in Fig. 2 corresponds to the aligned isotropic layout of [7], i.e. the same street width in the \( x \) and \( y \) directions. Other city layouts were also considered: (i) with narrower streets in the \( x \) direction, (ii) with wider streets in the \( x \) direction, (ii) rotated layouts, where the city features an angle of \( 22.5^\circ \) with respect to the \( x \) direction. For all simulations, the mesh comprised 10160 computational cells, while simulations run with a fine mesh in [7] comprised 46682 elements.
3.1. Effect of the distributed porosity field

The effect of such a variable porosity field is illustrated in Figure 3 for the case of an isotropic aligned city layout, for a steady flow with $Q = 75$ l/s. It can be observed that the variable porosity field improves the results compared to the constant porosity value. Similar results are obtained for the other city layouts of [7]. However, the headlosses at the entrance of the city are still not reproduced, which confirms the need for additional headloss formulations, as detailed in the next section.

3.2. Effect of the drag formulation

The coefficients of the drag matrix (Eq. 3) were calibrated in [8] on the isotropic aligned city layout, yielding the following values for the drag coefficients: $C_{D,xx} = 10$, $C_{D,yy} = 10$, $C_{D,xy} = 30$. The results obtained with the variable porosity field and these drag coefficients are illustrated in Fig. 4 for the isotropic layout. It can be observed that the computed results much better predict the total head loss, as the upstream water level is now well reproduced.
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Then, for the anisotropic layouts, the drag coefficients are modified according to the ratio between the street width in the main longitudinal direction $w_l$ and the street width in the main transversal direction $w_t$, while the non-diagonal terms in the drag matrix remain unchanged

$$\left(C_{D,xx} \right)_{rot} = C_{D,xx} \frac{w_l}{w_l} \quad (4a)$$

$$\left(C_{D,xy} \right)_{rot} = C_{D,xy} \frac{w_l}{w_l} \quad (4b)$$

For the rotated city layouts, the drag matrix $C_D$ is multiplied by the rotation matrix $A$ as indicated by Eqs. (5) and (6).

$$C'_D = A^{T} \begin{pmatrix} C_{D,xx} & C_{D,xy} \\ C_{D,xy} & C_{D,yy} \end{pmatrix} A \quad (5)$$

$$A = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \quad (6)$$

where $\beta$ is the angle of rotation of the city. Then, in order to account for the additional headlosses induced by the orientation of the city and thus of the velocity vectors, the resulting source term is multiplied by amplification factors $a_x$ and $a_y$ as follows:

$$S_i = \begin{pmatrix} a_x S_{i,x} \\ a_y S_{i,y} \end{pmatrix} \quad (7)$$

$$a_x = (5 \left| \sin 2\theta \right| + 1) \left(1 + \left(\frac{\theta}{45} \right)^2 \right)^m \quad (8a)$$

$$a_y = (5 \left| \sin 2\theta \right| + 1) \left(1 - \left(\frac{\theta}{45} \right)^2 \right)^m \quad (a)$$

where

$$\theta = \beta - \tan^{-1} \left(\frac{v}{u} \right) \quad (9)$$

The amplification factors were calibrated by Linkens and Snaps in [8] who proposed $m = 4$. The amplification factor for the head losses is maximum when the deviation between the city alignment and the actual velocity vectors is maximum.
4. Model validation

The model is then validated by comparing the numerical results with the experimental measurements on the rotated city cases. The comparisons are shown in Figure 5 for the cases of the rotated isotropic city (left) and the rotated anisotropic city with wide streets (right), for a discharge $Q = 75 \text{ l/s}$.

It can be observed that the proposed drag formulation and amplification factors allow for a good reproduction of the experimental measurements, especially as regards the upstream water level which is the result of the different head losses in the city. However, in the streets 3 and 4, an important decrease of the water depth occurs before the exit of the city, yielding a too low water level in the city and a less realistic water profile.

Fig. 5. Validation of the model, $Q = 75 \text{ l/s}$, variable porosity field. Left: isotropic rotated layout. Right: anisotropic wide rotated layout. Experiments and distributed porosity model with drag.
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Fig. 5. Validation of the model, $Q = 75$ l/s, variable porosity field. Left: isotropic rotated layout. Right: anisotropic wide rotated layout. Experiments and distributed porosity model with drag.

5. Conclusions

We have investigated different adaptations to the single porosity model in order to better represent the head losses occurring when water flows through an urban district. These adaptations consisted in adopting a distributed porosity field, with higher values in the streets and lower values centred on the buildings, and in proposing a drag formulation with amplification factors accounting for the eventual difference in the orientation of the main city streets and of the velocity vectors.

The proposed model was calibrated and validated using experimental data by [7]. First, it was shown that adopting a variable porosity coefficient allowed for a better representation of the water level variations in the city, and provided additional head losses when compared to the case of constant porosity field. Then, the simplest case of an isotropic city aligned with the flow direction was used to calibrate the drag coefficients. Finally, applying the proposed model, drag coefficients for the anisotropic and rotated cases were calculated. These coefficients were validated by comparing the simulations results to the measurements for different city layouts.

From the comparisons, it can be concluded that the proposed porosity distribution and drag model result in a better agreement between simulations and measurements. The proposed technique should now be further validated using more complex city layouts. Especially, for real urban areas, the definition of the angle $\beta$ between the main city streets and the main flow direction would certainly not be straightforward. However, the model could allow for different values of $\beta$ in different parts of the urban area, which would allow circumventing this difficulty and going towards more realistic cases.

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