A Closer Study of the Framed Standard Model Yielding Testable New Physics plus a Hidden Sector with Dark Matter Candidates

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Abstract

This closer study of the FSM I retains the earlier results of [1] in offering explanation for the existence of three fermion generations, as well as the hierarchical mass and mixing patterns of leptons and quarks;

II predicts a vector boson \( G \) with mass of order TeV which mixes with \( \gamma \) and \( Z \) of the standard model. The subsequent deviations from the standard mixing scheme are calculable in terms of the \( G \) mass. While these deviations for (i) \( m_Z - m_W \), (ii) \( \Gamma(Z \to \ell^+ \ell^-) \), and (iii) \( \Gamma(Z \to \text{hadrons}) \) are all within present experimental errors so long as \( m_G > 1 \) TeV, they should soon be detectable if the \( G \) mass is not too much bigger;

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III suggests that in parallel to the standard sector familiar to us, there is another where the roles of flavour and colour are interchanged. Though quite as copiously populated and as vibrant in self-interactions as our own, it communicates but little with the standard sector except via mixing through a couple of known portals, one of which is the $\gamma - Z - G$ complex noted in [II] above, and the other is a scalar complex which includes the standard model Higgs. As a result, the new sector appears hidden to us as we appear hidden to them, and so its lowest members with masses of order 10 MeV, being electrically neutral and seemingly stable, but abundant, may make eligible candidates as constituents of dark matter.

A more detailed summary of these results together with some remarks on the model’s special theoretical features can be found in the last section of the text.
1 Introduction

The framed standard model (FSM) \cite{1} is constructed from the standard model (SM) by adding to the usual gauge boson and matter fermion fields the frame vectors in internal space as dynamical variables (framons), thus making the particle theory more similar in spirit to the vierbein formulation \cite{2} of gravitation. In particle physics itself, the following then immediately result:

- (i) The standard Higgs boson, as framon in the electroweak sector.
- (ii) A global $\tilde{su}(3)$ symmetry, “dual” to the local colour symmetry, to act as fermion generations.
- (iii) A fermion mass matrix of the form:

$$m = m_T \alpha \alpha^\dagger,$$

(with $\alpha$ universal and only $m_T$ depending on the fermion species). At tree level, $\alpha$ is constant so that only the top generation has a mass and there is no mixing, but under renormalization by framon loops, the vector $\alpha$ rotates with changing scale $\mu$, leading to a hierarchical fermion mass spectrum and mixing between up and down fermion states, namely the CKM matrix for quarks and neutrino oscillations for leptons. Indeed, a fit with the renormalization group equation (RGE) so derived to 1-loop level \cite{1}, with 7 real parameters, gives already the close agreement with experiment shown in Table \cite{1} effectively replacing, to this accuracy, 17 independent parameters of the standard model by just 7.

The FSM seems thus to have given a geometric meaning to the Higgs field, and offered a solution to the fermion generation problem as well as an explanation for the bewildering mass and mixing pattern of quarks and leptons. It gives in addition a new solution to the strong CP problem translating it via the rotation of $\alpha$ to the CP-violating phase in the CKM matrix. Even if taken only as just a parametrization of the data, there is in the literature, as far as we know, no other fitting the same wide range of data to comparable accuracy with so few adjustable parameters.
|                  | Expt (June 2014) | FSM Calc | Agree to | Control Calc |
|------------------|------------------|----------|----------|--------------|
| **INPUT**        |                  |          |          |              |
| $m_c$            | 1.275 ± 0.025 GeV| 1.275 GeV| < 1σ     | 1.2755 GeV   |
| $m_\mu$          | 0.10566 GeV      | 0.1054 GeV| 0.2%     | 0.1056 GeV   |
| $m_e$            | 0.511 MeV        | 0.513 MeV| 0.4%     | 0.518 MeV    |
| $|V_{us}|$        | 0.22534 ± 0.00065| 0.22493  | < 1σ     | 0.22468      |
| $|V_{ub}|$        | 0.00351 +0.00015 -0.00014 | 0.00346 | < 1σ     | 0.00346      |
| $\sin^2 2\theta_{13}$ | 0.095 ± 0.010 | 0.101 | < 1σ     | 0.102 |
| **OUTPUT**       |                  |          |          |              |
| $m_s$            | 0.095 ± 0.005 GeV| 0.169 GeV| QCD running | 0.170 GeV |
| (at 2 GeV)       | (at $m_s$)       |          |          |              |
| $m_u/m_d$        | 0.38—0.58        | 0.56     | < 1σ     | 0.56         |
| $|V_{ud}|$        | 0.97427 ± 0.00015| 0.97437  | < 1σ     | 0.97443      |
| $|V_{cs}|$        | 0.97344 ± 0.00016| 0.97350  | < 1σ     | 0.97356      |
| $|V_{tb}|$        | 0.999146 +0.00021 -0.000046 | 0.99907 | 1.65σ     | 0.999075    |
| $|V_{cd}|$        | 0.22520 ± 0.00065| 0.22462  | < 1σ     | 0.22437      |
| $|V_{cb}|$        | 0.0412 +0.0011 -0.0005 | 0.0429 | 1.55σ     | 0.0429       |
| $|V_{ts}|$        | 0.0404 +0.0011 -0.0004 | 0.0413 | < 1σ     | 0.0412       |
| $|V_{td}|$        | 0.00867 +0.00029 -0.00031 | 0.01223 | 41%       | 0.01221      |
| $|J|$            | $(2.96^{+0.20}_{-0.16}) \times 10^{-5}$ | $2.35 \times 10^{-5}$ | 20%       | $2.34 \times 10^{-5}$ |
| $\sin^2 2\theta_{12}$ | 0.857 ± 0.024 | 0.841 | < 1σ     | 0.840 |
| $\sin^2 2\theta_{23}$ | > 0.95 | 0.89  | > 6%     | 0.89         |

Table 1: Calculated fermion masses and mixing parameters compared with experiment, reproduced from [1]
Despite these attractive features, however, the FSM begs a question which needs an urgent answer. The framons represent altogether 11 complex degrees of freedom, of which two correspond to the standard electroweak Higgs, which is already seen in experiment. But this still leaves 9, corresponding to the framons in the colour sector. Then:

- **Q** Why have we not been aware of colour framons in experiment?

The question **Q**, however, is not immediately answerable because these framons are colour triplets, and since colour is confined, they cannot propagate as particles in free space, but can manifest themselves, at best, as confined constituents of colourless bound states. A colour framon can combine with its conjugate in the s-wave to form scalar bosons (which we shall label generically as \( H \)), or in the p-wave (hence involving the gluon via the covariant derivative) to form vector bosons (which we shall call generically \( G \)), or else it can combine with a coloured fermion to form fermionic bound states (which we shall call generically \( F \)). The question **Q** can thus be rephrased as:

- **Q’** Why have we not seen these \( H, G \) and \( F \) in experiment?

to answer which, we shall need first to know something about the properties of these particles.

The particles \( H, G \), and \( F \) are in the colour theory the exact parallels of respectively the Higgs boson \( h_W \), the vector bosons \( W^\pm, \gamma - Z^0 \), and the leptons and quarks in the flavour theory. To see this, recall first an illuminating paper of 't Hooft [3], which pointed out that the standard electroweak theory, which is usually interpreted as having its local (flavour) \( su(2) \) symmetry spontaneously broken, has a “mathematically equivalent” interpretation as a confining theory where the local \( su(2) \) symmetry remains exact (see also [4]). What is broken then is only a global (often called the “accidental”) symmetry, say \( \widetilde{su}(2) \), hidden in the theory, and this global symmetry is here broken explicitly by electromagnetism (not spontaneously by weak hypercharge). In this alternative interpretation of the electroweak theory (which we shall refer to in this paper as the “confinement picture” (of 't Hooft)), fields carrying local flavour cannot propagate in free space but can appear only as constituents of flavour singlet bound states confined by flavour \( su(2) \). Thus \( h_W \) is the \( su(2) \) singlet bound state confined by flavour of the fundamental scalar field \( \phi \) with its own conjugate \( \phi^\dagger \) in s-wave, \( W^\pm, \gamma - Z^0 \) are
bound states of the same two constituents in \( p \)-wave, and leptons and quarks are bound states of the fundamental scalar to the fundamental fermion fields, making them in the flavour theory the respective parallels of the \( H \), \( G \) and \( F \) of the colour theory as claimed above.

We note in passing, when the confinement picture for the flavour theory of \( 't \) Hooft is adopted, the close parallel in the FSM between the two nonabelian sectors, flavour \( su(2) \) and colour \( su(3) \). Both theories are now confining and both are framed, for the flavour sector by (i) above, and for the colour sector by construction. In both theories, the vacuum is degenerate, leading in each to the breaking of a global symmetry “dual” to the local gauge one, giving in the flavour sector two (up-down) flavours and in the colour sector three fermion generations. And in each case, the global symmetry is broken explicitly by electromagnetism, in the flavour sector as already noted above, and in the colour sector as we shall see later in Section 4. And yet, despite this striking parallel in formulation, the physics that emerges in the two sectors will be very different, as we shall see, because of some inherent differences between the two nonabelian symmetries, flavour \( su(2) \) and colour \( su(3) \). These differences will be highlighted as points of interest as we move along.

Let us return now to the question of detecting the \( H \), \( G \) and \( F \). To save repeating long phrases in future, it will be convenient to introduce the following new terms:

- **B-ons** to denote the flavour neutral bound states of flavoured constituents held together by flavour \( su(2) \) confinement (\( B_\mu \) being in our notation the flavour \( su(2) \) gauge field).

- **C-ons** to denote colour neutral bound states of coloured constituents held together by colour \( su(3) \) confinement (\( C_\mu \) being in our notation the colour \( su(3) \) gauge field).

In this terminology, the Higgs boson \( h_W \), the vector bosons\(^2\) \( W, \gamma - Z \), and the quarks and leptons are all framonic \( B \)-ons, framonic in the sense that they are all obtained by binding a framon with other flavoured constituents via flavour confinement. On the other hand, \( H \), \( G \), and \( F \) are framonic \( C \)-ons and their respective analogues, only with the roles of flavour and colour interchanged. The framonic \( B \)-ons make up our world, that is, the world as

\(^2\)By \( \gamma - Z \), we mean the component of the \( \gamma \) and the \( Z \) in flavour \( su(2) \), i.e. \( W^3 \).
we have known it so far, which we shall call here the standard sector. They are the building blocks from which, for example, baryons are obtained as higher level constructs via colour confinement. Their $C$-on analogues, $H$, $G$, and $F$, on the other hand, have so far been hidden from us, for some reason yet unknown which is now our wish to find out.

To do so, we shall need first to envisage what properties these framonic $C$-ons are likely to possess, perhaps initially by drawing on their analogy in structure to particles already known to us. For this, however, we are placed immediately in a dilemma. On the one hand, as already noted, the framonic $C$-ons $H$, $G$, and $F$ are analogous to the framonic $B$-ons which appear to us as point-like objects, interacting via only "hard" interactions prescribed by and derivable perturbatively from the action, the two types of bound states being both framonic, but differing in the confining symmetry. On the other hand, the $H$, $G$, and $F$ are analogous also to hadrons which, in contrast, are bulky objects and have soft interactions, the two types being now both bound states via colour confinement, but differing in that the $H$, $G$, and $F$ each contains at least one colour framon as constituent, while hadrons have only quarks or antiquarks as constituents. The question then is whether the $H$, $G$, and $F$ are likely to resemble more the hadrons or the framonic $B$-ons.

To try to answer this, let us first examine the question why the framonic $B$-ons $h_{W}, W, \gamma - Z$, quarks and leptons, should be point-like while hadrons are fat, when both these types of particles are singlet compound states held together by gauge symmetry confinement. One can, of course, ascribe the difference to the different confining symmetries, one being flavour $su(2)$, the other being colour $su(3)$, deferring to answer the question because of our present incomplete knowledge on how confinement comes about. However, there is also between the two their structural difference, namely one type being framonic while the other not, which might give us a hint for their so very different properties. Framons have imaginary mass, meaning in the flavour case that the $\phi^{2}$ terms in the scalar potential has a negative coefficient. This is familiar, and is needed in the electroweak theory to make the vacuum degenerate and hence to break the flavour symmetry (local in the symmetry-breaking, global in the confinement picture). In contrast, the quark constituents in hadrons are all possessed with a real positive mass. Could this difference then hold the key to the question why hadrons have soft interactions while the framonic $B$-ons seem to have none?

Let us first recollect what little we know about soft interactions. Soft interactions, being supposedly nonperturbative effects, cannot be deduced
Consider then, in the language of quark diagrams, as a typical example of soft interactions, the decay: $\rho \to 2\pi$ which is represented by (a) of Figure 1. The quark and antiquark constituents in the $\rho$ recombine respectively with an antiquark and a quark of a pair emerging (say by quantum fluctuation) from the sea to form the 2 pions in the decay product. This decay occurs at a very high rate which cannot be obtained in perturbative QCD calculations. At first sight, barring any at present unknown difference in dynamics between the flavour and colour theory, it would appear that a very similar strong decay could occur to the Higgs boson $h_W$ as indicated in diagram (b) of Figure 1 giving a lepton-antilepton or quark-antiquark pair. Now, although the decay of $h_W$ into fermion-antifermion pairs does occur in experiment, it happens only at the standard perturbative rate obtained from the fundamental action, not at any unusually strong rate that the above analogy would lead us to expect. Why?

The Figure 1(b) differs from the Figure 1(a) in that for the $h_W$ in the former to decay softly, the flavour framon has to be separated from its partner so as to recombine with another to form the final state particles, whereas in the latter, it is only a quark-antiquark pair that has to be separated. The framon differs from a quark in having an imaginary mass. Now an imaginary

\[ \text{Figure 1: (a) quark diagram for } \rho\text{-decay, (b) quark diagram for } h_W\text{-decay?} \]
mass translates to a finite life time for the framon while the quark life time (inside hadronic matter) will be infinite. The possibility then arises that the framon, in contrast to a quark with infinite life time, may be too short-lived to have time finding and recombining with an alternative partner from the sea to form a new (flavour) singlet and emerge as a particle. A naive intuitive estimate of the framon life time made in Section 9 indeed suggests that the framon would have practically no chance to find and recombine with a new partner, so that the $h_W$ would have perforce almost to eschew soft decays altogether.

Generalizing the above argument to other soft interactions which all seem to involve recombinations of a composite’s constituents with particles emerging from the sea, one would be led to suggest that they will not occur in any of the electroweak particles, namely $h_W, W^\pm, \gamma - Z^0$, quarks and leptons, these being all framonic composites in 't Hooft’s confinement picture. Hence, these particles will all remain point-like and interact only weakly in contrast to hadrons which, being nonframonic composites, can have soft interactions and are therefore bulky and strong. One thus seems to have found here an answer to the question posed above, without assuming a basic difference in dynamics between flavour and colour.

If this conclusion is at all acceptable, then it would seem to apply to the framonic $C$-ons $H, G$ and $F$ as well. The vacuum in the colour sector has to be degenerate also, so as to break the generation symmetry, which means that the coefficient of the $\Phi^\dagger \Phi$ term in the framon potential (Section 3) is negative, and that the colour framon too will have finite (short) life time. The same arguments as above for the framonic $B$-ons will then lead to the conclusion that framonic $C$-ons too will have negligible soft interactions and remain point-like.

If this is true, then only hard interactions remain for the $H, G$ and $F$, and for these interactions one can deduce a fair amount of information for the following fortunate reasons:

- The terms in the action involving framons are strongly constrained in form by its necessary double invariance under both the local gauge symmetry $u(1) \times su(2) \times su(3)$ and its global dual $\tilde{u}(1) \times \tilde{su}(2) \times \tilde{su}(3)$, since physics should be independent of the choice of both the local and the global reference frames.

- From the action, using a method developed by 't Hooft originally for the
electroweak theory \[3\], one can deduce the mass matrices and couplings at tree level of the $H$, $G$, and $F$.

- Some of the freedom left over from the two preceding items can be tied down further by the fit to quark and lepton data cited in \[1\].

This programme will be carried out in some detail and reported in later sections. From these, higher order corrections can then be calculated perturbatively.

One result of immediate interest from this programme is that it gives many couplings among the framonic $C$-ons themselves but no direct coupling of these particles to leptons and quarks. Indeed, the framonic $C$-ons are found to couple to the light quarks and leptons which make up the present-day world effectively only via the exchange of $\gamma$ and $Z$, and of the latter only by virtue of some small admixture it has of framonic $C$-on. Hence, in the absence of soft interactions for them as suggested above, it would seem that framonic $C$-ons will have difficulty communicating with our world, and will not be easy to produce or detect with our experiments done with matter made up mostly of light quarks and leptons. If that is true, it would help to answer the question $Q'$ or $Q$ posed at the beginning.

But there is a surprise bonus. This check for internal consistency of the FSM, constructed initially just to explain the mass and mixing patterns of quarks and leptons, has uncovered, in parallel to ours, a strange new world populated by framonic $C$-ons. Though seemingly quite as complex and active within itself as our own, this new world may be hidden from us because framonic $C$-ons have difficulty communicating with us and we with them. Nevertheless, the two worlds came from the same roots and share the same vacuum. We recall from \[1\] that it is the renormalization of the vacuum by colour framon loops which leads to the rotation of the quark and lepton mass matrices, and hence to the mixing of their up and down states and to their mass hierarchy. Conversely, as will be seen later, the fit in \[1\] to quark and lepton properties gives useful information, via the same RGE, on the mass spectrum and couplings of the framonic $C$-ons.

The strange new world exists in theory, but does it form part of our universe? For this, we shall have to let our imagination loose. Presumably, like other coloured and flavoured constituents, framons would be present in abundance in the primordial soup. As the universe cooled down and expanded, these coloured and flavoured objects would each struggle to find partners to
neutralize their colour or flavour so as to survive as colour or flavour neutral particles in the confined phase. In those primordial circumstances, the density would be very high so that, one might suppose, even framons with their short life times would have no difficulty finding partners to emerge as flavour singlets (quarks, leptons, and so on) and as colour singlets ($H$, $G$ and $F$). And these, being framonic, would be tenacious and hold on to their partners for good. In the meantime, of course, quarks in parallel would be meeting partners and combining into baryons. One might even argue that being binary composites, the framonic states, in particular the framonic $C$-ons of interest to us, would be formed in greater abundance than baryons which are trinary composites, since it would presumably be easier statistically for a framon to find one partner than a quark to find two partners. The two worlds, ours of framonic $B$-ons and the new one of framonic $C$-ons, would then evolve separately, largely ignoring each other in the process. The heavier states in our world have decayed into the lowest stable states, that is, the baryons and leptons. In the other, framonic $C$-on, world, the lowest states we have found so far are certain $H$, $G$ and $F$ with masses of order 10 MeV (which are a special result of the fit in Table 1), with some $F$ possibly lower still because of some see-saw mechanism [8]. The lightest among them seem stable, with zero charge and little interaction with our sector. Thus, if these conclusions are maintained under further investigation, then those particles would be candidate constituents for dark matter.

However, these are early days yet, for the FSM gives a lot of information both in our standard sector and in the new hidden sector, which has to be checked through to ascertain, first, that it does not violate data already known and, secondly, whether some, and if so which pieces of it, can be tested against experiment. Some of these points will be dealt with in this paper as they occur, and some in separate papers, but there are still many which will need further careful scrutiny, not only by us but by the community. It is only if and when the FSM can manage to pass these tests will the scenario suggested above gain credibility, but this is a matter for the future.

Procedurally in this paper, one is dictated on by circumstances. In venturing into the hidden sector, where few empirical facts are known, one will have to rely mainly on theoretical arguments. Now in the applications we have made so far of the FSM, there were some gaps in the formulation of the scheme which did not figure, and were therefore left open. To proceed further, however, these gaps will now have to be filled. The first few sections of this paper will thus be devoted to going over some old grounds with a
finer comb, such as the specifications of the framon fields and the structure of the three terms in the action in which the framon occurs, namely the framon potential, the framon kinetic energy term, and the Yukawa coupling. Moreover, we shall find this reappraisal highly rewarding, providing us, as it does, with both a deeper understanding of the facts and a clarification of the concepts than we have had before.

In particular, while answering a question not posed earlier, because not needed then, on the electric charges carried by the various components of the colour framon, we came upon some new facts which have opened up for study a whole new phenomenology for the FSM. It is within the standard sector, but in an area outside that treated in for example [1] for which the FSM was originally constructed. This comes about as follows. We have already noted the close parallel between the flavour and colour sectors in FSM, and also the analogy between the $W$-bosons in the one and the $G$ in the other. Now, in the flavour theory, $W^3$ mixes with the $u(1)$ boson to form the $\gamma$ and the $Z$. So it is no surprise that in the colour theory, $G_8$ mixes with the $u(1)$ boson too. In the flavour sector, the charge chosen for the Higgs (framon) field matters for keeping the photon massless. So, it answers to reason that in the colour theory also, some particular choice of charges carried by the three components of the colour framon might do the same, as we shall show to be indeed the case in Section 4. Having then fixed the charges of the colour framon in this way, one is left with little freedom in the mixing problem. Instead of mixing just in the $\gamma - Z$ complex as in the electroweak theory, we have now in FSM the mixed $\gamma - Z - G$ complex, where the matrix relating these eigenstates of mass to the original gauge basis is given in terms of the gauge couplings and of the vacuum expectation values of flavour and colour framon fields. The couplings and the vacuum expectation value of the Higgs field are known, leaving the problem then with only one parameter, which we may take instead as the $G$ boson mass $m_G$. Now such a conclusion is phenomenologically very significant, given that the standard mixing scheme in the electroweak theory has been tested already to very high accuracy. Deviations from it are easily ruled out, and any which survive would qualify as interesting new physics to be sought by experiment and be tested. Some of the resulting phenomenology on this has already been done and reported in [9] which finds that the FSM has so far survived the tests to which it has been subjected. A summary of this can be found in Section 7.3. We consider this new phenomenology an important bonus, since it provides us with the means to test the model in a direction oblique to that for which it was
originally constructed.

Going next then into the hidden sector, one obtains from the framonic action the mass matrices and couplings of the $H$, $G$, and $F$. Further, from the scale-dependence of these quantities deduced from the RGEs via the fit of [1], one ends up with a fair amount of information on the mass spectra and interactions of the $H$, $G$, and $F$, which is what leads then, with a bit of guesswork, to the picture of the hidden sector outlined above.

In the end section (Section 11), we have listed some points of interest resulting from the present investigation, both experimental-phenomenological and theoretical-conceptual, at which the reader might wish to take a glance before getting involved with the details.

2 The framon fields

Being frame vectors to start with, framons carry, as do vierbeins in gravity, an index for the local gauge frame $r$ or $a$, as well as an index $\tilde{r}$ or $\tilde{a}$ for the global reference frame, and transform as representations under both local gauge transformations and global changes of the reference frame. Thus, if we denote the local gauge symmetry of SM as $G = u(1) \times su(2) \times su(3)$, and its corresponding (dual) global symmetry as; $\tilde{G} = \tilde{u}(1) \times \tilde{su}(2) \times \tilde{su}(3)$, then the framons should form together a representation of $G \times \tilde{G}$. The symmetries $G$ and $\tilde{G}$ being themselves product symmetries, one could choose for representations either the sum or the product for each pair. In FSM, we choose [10, 11] for the framon the representation $1 \times (2 + 3)$ of $G$, this being the “minimal representation” in the sense that it requires the introduction of the smallest number of independent scalar fields into the theory, since counting dimensions, $1 \times 2 < 1 + 2, 1 \times 3 < 1 + 3$, but $2 + 3 < 2 \times 3$. But we choose for the framon the representation $1 \times 2 \times 3$ of $\tilde{G}$, to avoid the flavour and colour sectors becoming completely disjoint. With this choice of representations, the framon for the whole FSM breaks into two sets:

* (FF) the flavour (“weak”) framon

\[
\alpha \Phi = (\alpha^{\tilde{a}} \phi^r_r) ; \quad r = 1, 2; \quad \tilde{r} = \tilde{1}, \tilde{2}; \quad \tilde{a} = \tilde{1}, \tilde{2}, \tilde{3}, \quad y = \pm \frac{1}{2}, \quad \tilde{y} = -\frac{1}{3} \mp \frac{1}{2}
\]  

(2)

where the columns of $\Phi$ transform as doublets of $SU(2)$ while its rows transform as anti-doublets of $\tilde{SU}(2)$, and:
• (CF) the colour ("strong") framon:

\[ \beta \Phi = (\beta^a \phi^a_\tilde a) ; \quad a = 1, 2, 3; \quad \tilde a = \tilde 1, \tilde 2, \tilde 3; \quad \tilde r = \tilde 1, \tilde 2, \tilde 3 \]

where the columns of \( \Phi \) transform as triplets of \( SU(3) \) while its rows transform as anti-triplets of \( \tilde SU(3) \). Each is multiplied by a spacetime independent factor \( \alpha \) and \( \beta \), which can be taken as real unit vectors without loss of generality. Here we denote by \( y \) and \( \tilde y \) the chosen \( u(1) \) and \( \tilde u(1) \) charges respectively, to be discussed a little further on.

For the flavour framon, the number of independent scalar fields can be reduced further by requiring the elements of \( \Phi \) to satisfy the following \([10, 11]\):

• (ME) "minimal embedding" condition:

\[ \phi^2_r = -\epsilon_{rs}(\phi^1_s)^* . \]  \hspace{1cm} (4)

We call this "minimal embedding" (ME) since its implementation is akin to embedding\(^3\) the group \( SU(2) \) in \( \mathbb{R}^4 \). A similar condition, however, cannot be imposed on the colour framons without changing the physical dimension of some components with respect to the others. The condition (4) allows us to eliminate one of the two columns of \( \Phi \), say \( \phi^2 \), in terms of the other \( \phi^1 \) (which we can call simply \( \phi \)), giving then:

• (FF') the flavour ("weak") framon:

\[ \alpha \phi = (\alpha^\tilde a \phi^a_r) ; \quad r = 1, 2; \quad \tilde a = \tilde 1, \tilde 2, \tilde 3, \]  \hspace{1cm} (5)

and leaving only one independent doublet \( \phi \) to be identified with the Higgs field in the standard electroweak theory.

However, these minimality arguments for the choice of framon fields, whether of the representation or of the embedding, may be fortuitous, since they have been made with foreknowledge of what is needed for the standard model, and no physical reason is as yet known why the minimal choices are

\(^{3}\text{We can get a simple and clear picture of this embedding by identifying the elements of } SU(2) \text{ as unit quarternions, so that the whole group just sits inside } \mathbb{R}^4 \text{ as its unit sphere. In fact, this is a special property of the group } SU(2) \text{ not shared by the other unitary groups.} \)
to be preferred. Nevertheless, it seems interesting that such arguments do exist.

We note that the condition (ME) does not break the $\tilde{su}(2)$ symmetry. Indeed, even after the elimination of $\phi^2$ in terms of $\phi$, as in the traditional formulation, the theory still has this hidden $\tilde{su}(2)$ symmetry, which is sometimes called an “accidental” symmetry (see equation (7)). The only difference here is that this symmetry has been built into the theory as part of the framon concept, thus explaining in this context its origin. To exhibit, in what follows, this underlying $\tilde{su}(2)$ symmetry, it is often useful to write the flavour framon field in the form (2), leaving the elimination by (4) of one column in terms of the other understood to be performed later.

To complete the specification of framons as representations of $G \times \tilde{G}$, we have still to assign them $u(1)$ and $\tilde{u}(1)$ charges $y$ and $\tilde{y}$. For this, we shall need to specify not just the gauge algebra but also the gauge group $U(1) \times SU(2) \times SU(3)$. Analyses of the particle spectrum in the standard model give then the local gauge group as what we called in [10] $U(3, 2, 1)$ (now renamed $U(1, 2, 3)$) which is obtained from the group $U(1) \times SU(2) \times SU(3)$ by identifying in it certain sextets of elements. Symbolically: $U(1, 2, 3) \sim (U(1) \times SU(2) \times SU(3)) / \mathbb{Z}_6$, for which the $u(1)$ charges $y$ take on the following values:

\begin{align*}
(1, 1); & \quad y = 0 + k, \\
(2, 1); & \quad y = \frac{1}{2} + l, \\
(1, 3); & \quad y = -\frac{1}{3} + m, \\
(2, 3); & \quad y = \frac{1}{6} + n,
\end{align*}

where the first number inside the brackets denotes the dimension of the $SU(2)$ representation, the second number that of $SU(3)$, and $k, l, m, n$ can be any integer, positive or negative.

As in [10, 11], we choose to assign to the flavour framon the $U(1)$ charges $y = \pm \frac{1}{2}$ with the smallest absolute values. However, because of “minimal

\footnote{Here we would like to clarify our notation. For many purposes, such as at the level of the Lagrangian, one can neglect, or in fact cannot know of, any discrete identification of the group elements, meaning that we need, or are able, just to specify the corresponding Lie algebra. However, if we want to study the charges, then we have to take into account discrete identifications of the group elements, meaning that we need to specify the Lie group itself. To underline this point, in the first case we use lower case symbols, usually reserved for the Lie algebras, and in the second case we use upper case symbols, as is usual for the Lie groups.}
embedding” (ME), it follows that if \( \phi^1 \) is assigned the charge \( -\frac{1}{2} \), then \( \phi^2 \) must have charge \( +\frac{1}{2} \). In other words, the global flavour \( \tilde{su}(2) \) symmetry is now broken along the direction \( \tilde{1} \). The breaking need not always be made along \( \tilde{1} \), which is after all only a co-ordinate choice, but can be along any direction specified by a vector, say \( \gamma \), as we shall have occasion to prefer. In that case, introducing a further vector \( \gamma^\perp \) orthogonal to \( \gamma \) in \( \tilde{su}(2) \) space, we rewrite (ME) as:

\[
\phi^\perp_r = -\epsilon_{rs}(\phi_s)^*,
\]

with \( \phi = \Phi \gamma \), \( \phi^\perp = \Phi \gamma^\perp \), which version is often useful for exhibiting more clearly the underlying symmetry. We recall that in the confinement picture of ’t Hooft, \( u(1) \) represents electromagnetism and \( y \) the electric charge. We can thus ascribe to the vector \( \gamma \) the function of specifying the direction in which electromagnetism breaks the global symmetry \( \tilde{su}(2) \).

The assignment of \( y \) to the colour framons (CF) is less straightforward. In [11], we have tentatively assigned the common charge \( -\frac{1}{3} \) to all colour framons, this being the smallest in absolute value allowed them by (6), and we not knowing then any reason for doing otherwise. But that was a mistake. It will be seen later in Section 4 that one of the three colour framons must be assigned a charge \( +\frac{2}{3} \) to keep the photon massless. Correction of this error does not change our earlier result in for example [1] which did not make use of the wrong charge assignments, but will lead to interesting physics in other areas, as will be spelt out in Section 7.3.

Lastly, we come to the \( \tilde{u}(1) \) charges \( \tilde{y} \). For the factors \( \Phi \) and \( \Phi^\perp \) in respectively (FF) and (CF), just as their representations in the nonabelian parts of \( \tilde{G} \) and \( \tilde{G} \) are conjugates, so should their representations of \( \tilde{u}(1) \) be the conjugate of their representations in \( u(1) \), meaning that \( \tilde{y} = -y \). For \( \Phi \) in (FF), this gives then \( \tilde{y} = +\frac{1}{2} \) for \( \phi^1 \) but \( \tilde{y} = -\frac{1}{2} \) for \( \phi^2 \). Similarly, when we have specified in Section 4 separate \( y \) for the 3 components of \( \Phi \) in (CF), each component will have to be assigned separately the value \( \tilde{y} = -y \). There remains, however, a question on whether the global factors \( \alpha \) in (FF) and \( \beta \) in (CF) should be assigned a \( \tilde{y} \) too, and if so, which value.

To answer this question, it is again necessary to specify the group corresponding to the global symmetry \( \tilde{G} = \tilde{u}(1) \times \tilde{su}(2) \times \tilde{su}(3) \). A reasonable choice would seem to be again \( U(1,2,3) \). One might even argue that it is necessary that the local and global groups be the same, as follows. Any infinitesimal change of the global reference frame under the symmetry \( \tilde{G} \) can be counteracted by the opposite change in the local frame under \( G \). The sug-
gestion above that the global group is the same as the local group is basically just the extension of the said requirement even to finite changes. If so, then the list in (6) implies that the \( \tilde{su}(3) \) triplet \( \alpha \) should be assigned a \( \tilde{y} = -\frac{1}{3} \), and the \( \tilde{su}(2) \) doublet \( \beta \) a \( \tilde{y} = \pm \frac{1}{2} \). These assignments are then what give the values of \( \tilde{y} \) listed in \([2]\) and \([3]\).

Just as \( u(1) \) invariance leads to charge conservation, so does \( \tilde{u}(1) \) invariance lead to the conservation of the \( \tilde{y} \) charge, which is in FSM connected to baryon number and lepton number conservation. The details of how they are connected will be postponed to Section 7.2 after Yukawa couplings have been considered.

These assignments of the \( y \) and \( \tilde{y} \) charges complete then the specification of the framon fields. We note that the framons are frame vectors only in the internal symmetry space, and transform under internal symmetry operations as indicated. They are, on the other hand, invariant under proper Lorentz transformations, and are therefore spacetime scalar fields.

3 The framon self-interaction potential

The self-interaction potential of framons is required to be invariant under \( G \times \tilde{G} \). Including up to quartic terms for renormalizability, and contracting indices in all possible ways, one can construct \([10]\) an invariant potential thus:

\[
V = -\mu_W \text{tr}[\Phi^\dagger \Phi] + \lambda_W (\text{tr}[\Phi^\dagger \Phi])^2 + \kappa_W \text{tr}[\Phi^\dagger \Phi \Phi^\dagger \Phi] \\
-\mu_S \text{Tr}[\Phi^\dagger \Phi] + \lambda_S (\text{Tr}[\Phi^\dagger \Phi])^2 + \kappa_S \text{Tr}[\Phi^\dagger \Phi \Phi^\dagger \Phi] \\
+\nu_1' \text{tr}[\Phi^\dagger \Phi] \text{Tr}[\Phi^\dagger \Phi] - \nu_2' [\Phi^\dagger \beta \cdot (\Phi \beta)] [\Phi^\dagger \alpha \cdot (\Phi \alpha)].
\]

This is seen to be invariant under \( su(2) \times su(3) \times \tilde{su}(2) \times \tilde{su}(3) \). However, to show that this is in fact invariant under \( u(1) \times \tilde{u}(1) \) as well, we shall have to leave until Section 4, after we have specified the assignment of the corresponding charges to the various components of the colour framon.

We note in particular the \( \nu_1', \nu_2' \) terms linking the flavour and colour framons, especially the \( \nu_2' \) term, which breaks the global flavour symmetry \( \tilde{su}(2) \) explicitly via the vector \( \beta \) coming from the colour framon \([3]\), and also the global colour symmetry \( \tilde{su}(3) \) explicitly via the vector \( \alpha \) from the flavour framon \([2]\).

Eliminating one of the columns of \( \Phi \) in terms of the other using \([4]\) according to the minimal embedding \((\text{ME})\) of \( SU(2) \), we have:

\[
V = -\mu_W |\phi|^2 + \lambda_W (|\phi|^2)^2
\]
\[-\mu_S \sum_a |\phi^a|^2 + \lambda_S \left( \sum_a |\phi^a|^2 \right)^2 + \kappa_S \sum_{\tilde{a}\tilde{b}} |\phi^\tilde{a} \cdot \phi^\tilde{b}|^2 \]
\[+ \nu_1 |\phi|^2 \sum_a |\phi^a|^2 - \nu_2 |\phi|^2 \left( \sum_a \alpha^\tilde{a} \phi^{\tilde{a}} \right) \cdot \left( \sum_a \alpha^\tilde{a} \phi^{\tilde{a}} \right), \quad (9)\]

with the new unprimed parameters simply given in terms of the primed parameters in (8). The \(\kappa_W\) term is absorbed into the \(\lambda_W\) term leaving just the usual Mexican hat potential for the electroweak sector. We note in particular, that:

- (ME') the dependence of the \(\nu_2\) term on \(\beta\) drops out,

while its dependence on \(\alpha\) remains, which fact will be seen to be important.

For this reason, it is worthwhile making explicit how this comes about. The term in (8) under consideration is of the form:

\[\left[ \beta^1 \phi^1 + \beta^2 \phi^2 \right]^\dagger \cdot \left[ \beta^1 \phi^1 + \beta^2 \phi^2 \right], \quad (10)\]

where \(\phi\) is here a vector in \(su(2)\) space and the dot denotes the inner product between such vectors. Now the minimal embedding condition (ME) in (4), implies:

\[\phi^{\tilde{a}} \cdot \phi^\dagger = 0, \quad |\phi^\dagger| = |\phi^\tilde{a}| = |\phi|^2, \quad (11)\]

so the expression (10) becomes:

\[|\beta^1|^2 |\phi^1|^2 + |\beta^2|^2 |\phi^2|^2 = |\beta^1|^2 + |\beta^2|^2 |\phi|^2 = |\phi|^2. \quad (12)\]

Thus one sees that because of minimal embedding, the vector \(\beta\) drops out of the \(\nu_2\) term, and indeed out of the framon potential altogether. Hence the vacuum obtained from the potential will not depend on \(\beta\).

We recall that in (8), it was \(\beta\) which broke explicitly the \(\tilde{su}(2)\) symmetry. Now, after minimal embedding, \(\beta\) has dropped out, but the symmetry remains broken, this time in the direction \(\phi^\dagger\), or more generally along the vector \(\gamma\). We can say that it is the vector \(\gamma\) which prescribes the direction of the \(u(1)\) of electromagnetism that breaks the \(\tilde{su}(2)\) of the framon potential. However, the \(\tilde{su}(3)\) is still broken by the vector \(\alpha\), the colour equivalent of \(\beta\) in the flavour sector. As we shall see, this particular lack of parallel between
the flavour and colour sectors, due directly to the special minimal embedding property of SU(2), is the source of much of the difference in physics to emerge from the FSM in the two sectors.

The vacuum is found by minimizing $V$, where the coefficients of the 7 terms are by choice all positive. In particular $\mu_W, \mu_S$ being positive means that the minimum is degenerate in both the flavour and colour sectors. In the flavour sector, this degeneracy is the same as in the standard electroweak theory. For the colour sector, it is found that any vacuum within the degenerate set can be cast by an appropriate choice of gauges (both local and global) into the following diagonal form:

$$\Phi_{\text{VAC}} \rightarrow \zeta S V_0 = \zeta_S \begin{pmatrix} Q & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & P \end{pmatrix},$$

(13)

with:

$$P = \sqrt{\frac{1}{3}(1 + 2R)},$$

(14)

$$Q = \sqrt{\frac{1}{3}(1 - R)},$$

(15)

$$R = \frac{\nu_2^2 \kappa^2 W_2}{2 \kappa_S \zeta_S^2},$$

(16)

where $\zeta_W$ and $\zeta_S$ are the vacuum expectation values of the flavour and colour framons respectively, and $\alpha$, which is coupled to the vacuum, takes the form:

$$\alpha \rightarrow \alpha_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

(17)

In other words, it is the vector $\alpha$ coming from the flavour framon which, we recall, broke “explicitly” the global colour symmetry of the $\nu_2$ term in $V$, that now gives a special direction $\tilde{3}$ to the vacuum in the chosen gauge, which, however, still remains degenerate in the other two (the $\tilde{1}$ and $\tilde{2}$) directions.

Expanding the potential $V$ in fluctuations about the vacuum, thus: $\Phi \rightarrow \Phi_{\text{VAC}} + \delta \Phi$, with $\delta \Phi$ hermitian, one obtains in this gauge the mass

---

5When $\delta \Phi$ is antihermitean, it corresponds to only a gauge transformation which is used to fix the gauge II, or, in more colourful language, it represents a degree of freedom which is eaten up by a colour gauge boson to acquire a mass.
squared matrix, and the couplings to one another, of quanta we call gener-
cically \( H \), these being, as mentioned in the introduction, the analogues in 
the colour sector of the Higgs boson \( h_W \). In ’t Hooft’s confinement pic-
ture, these \( H \) appear as bound states of a framon-antiframon pair: 
\( \Phi^\dagger \Phi \sim \Phi^\dagger_{\text{vac}} (\Phi_{\text{vac}} + \delta \Phi) = \Phi^\dagger_{\text{vac}} \Phi_{\text{vac}} + \Phi^\dagger_{\text{vac}} \delta \Phi \), meaning therefore that an \( H \) is 
to be labelled by \( \zeta_{S}^{-1} \Phi^\dagger_{\text{vac}} \) times a hermitian matrix. As in [1], we adopt as 
basis the following matrices:

\[
\begin{align*}
V_1 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
V_2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
V_3 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
V_4 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
V_5 &= \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
V_6 &= \frac{1}{\sqrt{(P^2 + Q^2)}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Q \\ 0 & P & 0 \end{pmatrix} \\
V_7 &= \frac{i}{\sqrt{(P^2 + Q^2)}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -Q \\ 0 & P & 0 \end{pmatrix} \\
V_8 &= \frac{1}{\sqrt{(P^2 + Q^2)}} \begin{pmatrix} 0 & 0 & Q \\ 0 & 0 & 0 \\ P & 0 & 0 \end{pmatrix} \\
V_9 &= \frac{i}{\sqrt{(P^2 + Q^2)}} \begin{pmatrix} 0 & 0 & -Q \\ 0 & 0 & 0 \\ P & 0 & 0 \end{pmatrix}
\end{align*}
\]
giving a tree-level mass (squared) matrix which is almost diagonal:

\[
M_H = \begin{pmatrix}
4\lambda_W \xi_W & 2\xi_W (\nu_1 - \nu_2) \sqrt{\frac{1+2R}{3}} & 2\sqrt{2} \xi_W \nu_1 \sqrt{\frac{1-R}{3}} & 0 \\
* & 4(\kappa_S + \lambda_S) \xi_S^2 \left( \frac{1+2R}{3} \right) & 4\sqrt{2} \lambda_S \xi_S^2 \sqrt{(1+2R)(1-R)} & 0 \\
* & * & 4(\kappa_S + 2\lambda_S) \xi_S^2 \left( \frac{1-R}{3} \right) & 0 \\
0 & 0 & 0 & D
\end{pmatrix}
\]

where

\[
D = \kappa_S \xi_S^2 \begin{pmatrix}
4(\frac{1-R}{3}) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4(\frac{1-R}{3}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 4(\frac{1-R}{3}) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2(\frac{2+R}{3}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2(\frac{2+R}{3}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2(\frac{2+R}{3}) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2(\frac{2+R}{3})
\end{pmatrix}
\]

In (19), the rows and columns are labelled, for later convenience, by respectively: \( h_W, H_3, H_+ = \sqrt{\frac{1}{2}}(H_1 + H_2), H_- = \sqrt{\frac{1}{2}}(H_1 - H_2), H_4, H_5, H_6, H_7, H_8, H_9 \), in that order.

Expanding further the potential \( V \) in fluctuations about the vacuum, we get the tree-level couplings of these states with one another given in Appendix A.

### 4 The framon kinetic energy term

Recalling that our framon field is chosen by minimality arguments to be in the representation \( 1 \times (2 + 3) \) of \( u(1) \times su(2) \times su(3) \), we can write its kinetic energy as a sum of two terms. For the flavour framon we have:

\[
\mathcal{A}_{KE}^F = \text{tr}[(D_\mu \Phi)^\dagger D_\mu \Phi],
\]

with

\[
D_\mu = \partial_\mu - ig_1 \Gamma A_\mu - \frac{1}{2}ig_2 B_\mu,
\]

and for the colour framon:

\[
\mathcal{A}_{KE}^C = \text{Tr}[(D_\mu \Phi)^\dagger D_\mu \Phi],
\]
\[ D_\mu = \partial_\mu - ig_1 A_\mu - \frac{i}{2} g_3 C_\mu. \] (24)

In both cases, \( \Gamma \) is a charge operator which specifies the \( u(1) \) charge of the various components of the framon. This can depend only on the global indices \( \tilde{r} \) or \( \tilde{a} \), not on the local indices \( r \) or \( a \), since both local flavour and colour are confined and have to remain exact. Thus \( \Gamma \) can be taken as a matrix in \( \tilde{su}(2) \) for the flavour framon or in \( \tilde{su}(3) \) for the colour framon.\(^6\)

Let us first examine the expression (21) for the flavour framon, recalling that \( \phi^{\tilde{1}} \) has charge \(-\frac{1}{2}\), and \( \phi^{\tilde{2}} \) charge \(+\frac{1}{2}\). Eliminating then by (ME) \( \phi^{\tilde{2}} \) in (21) in terms of \( \phi = \phi^{\tilde{1}} \), one has:

\[ A_{KE}^F = 2[(D_\mu \phi)^\dagger D_\mu] \phi, \] (25)

with

\[ D_\mu = \partial_\mu + \frac{i}{2} g_1 A_\mu - \frac{i}{2} g_2 B_\mu, \] (26)

differing from the standard electroweak theory only by a harmless factor 2 which we shall henceforth neglect.

Proceeding from (25) to derive the masses of the vector bosons \( \gamma, Z^0, W^\pm \) in the usual symmetry-breaking picture is familiar. Let us repeat the derivation, however, in the confinement picture of ’t Hooft, which we wish later to apply to the colour sector.

In the \( su(2) \) theory, there are three local gauge degrees of freedom, which we can use to fix the gauge by rotating, with an \( SU(2) \) transformation \( \Omega(x) \), the doublet scalar field \( \phi \) (containing 4 parameters) to point, at every space-time point \( x \), in the first direction and to be real, thus:

\[ \phi = \Omega \left( \begin{array}{c} \rho \\ 0 \end{array} \right) = \Omega \phi_{GF}, \] (27)

with \( \rho \) real. For a theory with the usual Mexican hat potential in this sector, such as (9) above, we can write:

\[ \rho = \zeta_W + h_W, \] (28)

where \( \zeta_W \) is the vacuum expectation value of \( \phi \), and \( h_W \), as its fluctuation about the vacuum value, is the Higgs boson field.

\(^6\)Hence, when applied as in (21) on \( \Phi \) or in (23) on \( \Phi \), the rows of which are labelled by local indices flavour \( r \) or colour \( a \) and columns are labelled by dual flavour \( \tilde{r} \) or dual colour \( \tilde{a} \), \( \Gamma \) as a matrix has to operate from the right.
Since to zeroth order $\rho = \zeta_W$, we can rewrite to this order the KE term of $\phi$ in (25) as:

$$[D_\mu \phi]^\dagger [D_\mu \phi] = \phi_{\Phi}^\dagger (D_\mu \Omega)^\dagger \Omega^\dagger D_\mu \Omega \phi_{\Phi},$$

and, by introducing

$$\frac{1}{2} \tilde{B}_\mu = \frac{i}{g_2} \Omega^\dagger (\partial_\mu - \frac{1}{2} ig_2 B_\mu) \Omega$$

as:

$$[D_\mu \phi]^\dagger [D_\mu \phi] = \phi_{\Phi}^\dagger [+\frac{1}{2} ig_1 A_\mu - \frac{1}{2} ig_2 \tilde{B}_\mu]^\dagger [+\frac{1}{2} ig_1 A_\mu - \frac{1}{2} ig_2 \tilde{B}_\mu] \phi_{\Phi},$$

To leading order then, this gives for the mass term, $\tilde{B}_\mu$ being hermitian:

$$(\zeta_W, 0) \frac{1}{4} [g_1^2 A_\mu^2 + g_2^2 \tilde{B}_\mu^2 - 2g_1 g_2 A_\mu \tilde{B}_\mu] \left(\begin{array}{c}
\zeta_W \\
0
\end{array}\right),$$

that is, $\frac{1}{4} \zeta_W^2$ times the 11 element of the quantity inside the square brackets.

Next, we note that $\tilde{B}_\mu$ as defined in (30) is just:

$$\tilde{B}_\mu = \Omega^\dagger B_\mu \Omega + \frac{2i}{g_2} \Omega^\dagger \partial_\mu \Omega,$$

namely the gauge transform of $B$ under $\Omega$. Then, since $\text{tr}[G^{\mu\nu} G_{\mu\nu}]$ is a gauge invariant quantity, it follows that:

$$\text{tr}[G^{\mu\nu} G_{\mu\nu}] = \text{tr}[\tilde{G}^{\mu\nu} \tilde{G}_{\mu\nu}],$$

where

$$\tilde{G}_{\mu\nu} = \partial_\mu \tilde{B}_\nu - \partial_\nu \tilde{B}_\mu + ig_2 [\tilde{B}_\mu, \tilde{B}_\nu].$$

Hence, the action in terms of $\tilde{B}$ is exactly the same as the action obtained by the symmetry-breaking picture in terms of $B$, with a mass squared matrix worked out from (32) as again

$$\frac{1}{4} \zeta_W^2 \left(\begin{array}{cccc}
g_2^2 & 0 & 0 & 0 \\
0 & g_2^2 & 0 & 0 \\
0 & 0 & g_2^2 & -g_1 g_2 \\
0 & 0 & -g_1 g_2 & g_1^2
\end{array}\right),$$

21
with only the $\tilde{B}^3$ component mixing with the photon giving:

$$\gamma_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} [g_2 A_\mu + g_1 \tilde{B}^3_\mu], \quad (37)$$

$$Z_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} [-g_1 A_\mu + g_2 \tilde{B}^3_\mu], \quad (38)$$

while

$$W^{\pm}_\mu = \tilde{B}^{\pm}_\mu. \quad (39)$$

Although the result is the same as in the symmetry-breaking picture and even most of the algebra used in its derivation looks familiar, one gains a very different physical interpretation to the phenomenon. We note first that the massive vector bosons $W^{\pm}, Z$ are no longer given by the triplet of gauge fields $B_\mu$ of the local su(2) symmetry, but by the fields $\tilde{B}_\mu$ in (30). These are su(2) singlets, with all the local su(2) indices originally in $D_\mu$ saturated by those in the transformation matrices $\Omega$ and $\Omega^{-1}$ between which it is sandwiched. Now $\Omega$ is a matrix which transforms the local su(2) frame to a fixed global frame. Hence, the fields $\tilde{B}_\mu$ are su(2) scalars, only triplets in a new global symmetry which we may call $\tilde{\text{su}}(2)$. Indeed, $\Omega$ as the transformation matrix between the local and global frames can also be taken as the vacuum expectation value of the framon matrix $\Phi$ (See Section 2). For this reason, the $\tilde{B}_\mu$ in (30) are interpreted by ’t Hooft as a bound state of a $\Phi$-$\Phi^\dagger$ pair in "p-wave" (because of the $D_\mu$) formed by local su(2) confinement. In other words, in our adopted language here, they are (framonic) $B$-ons. And they have acquired masses, as bound states usually do. Now $\tilde{B}^3_\mu$ mixes with $A_\mu$ to form the $Z^0$, but this mixing is just the usual mixing between quantum states, and involves no mixing of the local gauge symmetries $u(1)$ and $su(2)$ as it is said to do in the symmetry-breaking picture. It breaks the original degeneracy between $\tilde{B}^3$ and $\tilde{B}^{\pm}$, but this breaks only the global symmetry $\tilde{\text{su}}(2)$, while the local $su(2)$, being confining, remains exact.

We note as an aside that, just as for the framon potential, the $\tilde{s}u(2)$ can be broken in the $\tilde{1}$ direction as above, or along a vector $\gamma$. What really breaks the symmetry is again the $u(1)$ of electromagnetism.

We turn next to the kinetic energy term for the colour framons. Since colour is by general consensus confining and the kinetic energy term very similar to that in the flavour case, the above treatment in ’t Hooft’s confinement picture would seem to be tailor-made for it. As in the flavour case, we
wish first to gauge-fix to a convenient frame, but now with only 8 degrees of freedom in \( su(3) \) but many more parameters in the colour framon field \( \Phi \), we can at best only make \( \Phi \) triangular or hermitian. In either case, we shall call it \( \Phi_{GF} \), and write:

\[
\Phi = \Omega \Phi_{GF}.
\]

(40)

Since to zeroth order \( \Phi_{GF} = \Phi_{VAC} \), we can proceed with the same manipulations as in the flavour sector. Thus the kinetic energy is given to this order by

\[
\text{Tr}[\Phi_{GF}^\dagger (D_\mu \Omega)\Omega^\dagger D_\mu \Omega \Phi_{GF}] = \text{Tr}[\Phi_{GF}^\dagger (-ig_1 \Gamma A_\mu - \frac{1}{2}ig_3 \tilde{C}_\mu)\Omega^\dagger (-ig_1 \Gamma A_\mu - \frac{1}{2}ig_3 \tilde{C}_\mu) \Phi_{GF}],
\]

with:

\[
\frac{1}{2} \tilde{C}_\mu = \frac{i}{g_3} \Omega^\dagger (\partial_\mu - \frac{1}{2}ig_3 C_\mu) \Omega.
\]

(41)

(42)

In deriving the result in (41), we have used the fact that \( \Gamma \), being a matrix in \( \tilde{su}(3) \) space with both rows and columns labelled by \( \tilde{a} \) indices, is not affected by sandwiching \( A_\mu \Gamma \) between the (local) gauge fixing transformation \( \Omega^\dagger \cdots \Omega \). And since \( A_\mu \) itself is proportional to the identity in colour, one has just:

\[
\Omega^\dagger D_\mu \Omega = -ig_1 \Gamma A_\mu - \frac{1}{2}ig_3 \tilde{C}_\mu.
\]

(43)

Expanding next \( \tilde{C}_\mu \) in terms of the Gell-Mann matrices, we have:

\[
\tilde{C}_\mu = \sum_K \tilde{C}_K^\mu \lambda_K.
\]

(44)

These represent then our framonic vector \( C \)-ons, which we shall call \( G \) generically.

To find the mass matrix of the \( G \) from (41), we substitute as usual for \( \Phi_{GF} \) its vacuum expectation value, in the global \( \tilde{su}(3) \) gauge where it is diagonal:

\[
\Phi_{GF} \rightarrow \frac{\xi_s}{\sqrt{3}} \begin{pmatrix}
\sqrt{1-R} & 0 & 0 \\
0 & \sqrt{1-R} & 0 \\
0 & 0 & \sqrt{1+2R}
\end{pmatrix} = \Phi_{VAC},
\]

(45)

and obtain the mass term as:

\[
\text{Tr} \left( \Phi_{VAC}^2 (-ig_1 \Gamma A_\mu - \frac{1}{2}ig_3 \tilde{C}_\mu)^\dagger (-ig_1 \Gamma A_\mu - \frac{1}{2}ig_3 \tilde{C}_\mu) \right).
\]

(46)
We now have to specify $\Gamma$ or, in other words, the charges of the framons labelled by the column index $\tilde{1}, \tilde{2}, \tilde{3}$ of the matrix $\Phi$. We recall that the framon charges are constrained to be those listed in (6) for representations of $U(1,2,3)$, with preference for the lowest values as being more fundamental. Beyond this, in contrast to the flavour case where the condition (ME) of (4) gives a unique choice for the framon charges, the choice here for colour is for the moment still open. Now in [11], it was suggested tentatively for simplicity that one gives to all 3 colour framon components the same minimal value $-\frac{1}{3}$ allowed by (6), but it will be shown later that this leads to a mass matrix with no zero mode, giving thus the photon a mass, which would be physically unacceptable.

However, we recall from (6) that what the gauge group $U(1,2,3)$ implies is only that the strong framon (colour triplet, weak $su(2)$ singlet) should have $u(1)$ (electric) charge $-\frac{1}{3} + n$, $n$ being any integer, positive or negative. There is no need as far as the group representation is concerned for all colour framons to have the same charge. In fact, in the flavour sector the two framons have opposite charges and this seems to have played a role in giving the mass matrix a zero mode, hence keeping the photon massless. The question is thus whether we can devise a similar arrangement here in the colour sector.

We suggest the following, namely that $\phi^{\tilde{1},\tilde{2}}$ have charge $-\frac{1}{3}$ but that $\phi^{\tilde{3}}$ has charge $+\frac{2}{3}$, both allowable as $U(1,2,3)$ representations. We have kept the charges the same for $\phi^{\tilde{1}}$ and $\phi^{\tilde{2}}$ because of the $\tilde{su}(2)$ symmetry residual in our theory, and arranged the total charge of all three framons to be zero, in parallel to the opposite charges of the two framons in the electroweak theory.

Suppose we do that, what will happen? In (41), the charge operator $\Gamma$ now takes the form:

$$\Gamma = \begin{pmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & +\frac{2}{3} \end{pmatrix},$$

(48)

Evaluating now the mass matrix as per (47), we have, first, from the $\tilde{C}^\mu \tilde{C}_\mu$ term, no nonzero crossed terms, leaving thus only the diagonal $\tilde{C}^K \tilde{C}_K$ term...
tributions as:
\[ \frac{1}{4} g_3^2 \text{Tr}[\Phi^2_{\text{vac}} \lambda^2_K] = \frac{1}{12} g_3^2 \zeta^2_S (1 - R) \quad \text{for } K = 1, 2, 3, \]  
\[ \frac{1}{4} g_3^2 \text{Tr}[\Phi^2_{\text{vac}} \lambda^2_K] = \frac{1}{24} g_3^2 \zeta^2_S (2 + R) \quad \text{for } K = 4, 5, 6, 7, \]  
and for \( K = 8 \)
\[ \frac{1}{4} g_3^2 \text{Tr}[\Phi^2_{\text{vac}} \lambda^2_8] = \frac{1}{6} g_3^2 \zeta^2_S (1 + R). \]

Second, from the \( A^\mu \bar{C}_\mu \) term, only the \( K = 8 \) term gives nonzero contribution:
\[ \frac{1}{2} g_1 g_3 \text{Tr}[\Phi^2_{\text{vac}} (\Gamma^\dagger \lambda_8 + \lambda_8 \Gamma)] = -\frac{2}{3 \sqrt{3}} g_1 g_3 \zeta^2_S (1 + R). \]  

Third, from the \( A_\mu A_\mu \) term, we obtain:
\[ g_1^2 \text{Tr}[\Phi^2_{\text{vac}} \Gamma^\dagger \Gamma] = \frac{2}{9} g_1^2 \zeta^2_S (1 + R). \]  

The mass matrix so obtained is almost diagonal except for a mixing between the photon and \( G_8 \), with a mass submatrix of the form:
\[ \frac{1}{6} (1 + R) \zeta^2_S \begin{pmatrix} \frac{4}{3} g_1^2 & -\frac{2}{\sqrt{3}} g_1 g_3 \\ -\frac{2}{\sqrt{3}} g_1 g_3 & g_3^2 \end{pmatrix}. \]  

This matrix has a zero mode, meaning that \( G_8 \), though mixing with \( A_\mu \), will leave the photon massless, as we want.

We note the seemingly crucial fact that the two matrices \( \Gamma \) and \( \lambda_8 \) are proportional:
\[ \Gamma = -\frac{1}{\sqrt{3}} \lambda_8, \]  
for one to arrive at the above result. Given the structure of the vacuum in (45), we believe that the choice of \( \Gamma \) in (48) is essentially unique for the photon to remain massless although we have as yet no formal proof that this is so. For example, had we taken \( \Gamma \) as \(-\frac{1}{3}\) times the identity as we did in \[III\], we would have found a mass matrix which is again diagonal except for the mixing between the photon and \( K = 8 \) state as follows:
\[ \frac{1}{3} \zeta^2_S \begin{pmatrix} \frac{1}{3} g_1^2 & -\frac{R}{\sqrt{3}} g_1 g_3 \\ -\frac{R}{\sqrt{3}} g_1 g_3 & \frac{1}{2} (1 + R) g_3^2 \end{pmatrix}. \]  

This has no zero mode giving thus, as noted before, an unacceptable mass to the photon.
At this stage, it is interesting to compare the flavour and colour sectors and note their similarities and differences. In the electroweak theory if we had used for the kinetic energy the form (21) without (ME) instead of (25), then the charge matrix in (22) would be

\[ \Gamma = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}. \]  

(57)

The net charge is zero as for (48) above. In the electroweak theory, only \( \tilde{B}_\mu^3 \) mixes with \( A_\mu \), and \( \Gamma \) is proportional to \( \tau_3 \), just as \( \Gamma \) is to \( \lambda_8 \) in the colour sector. In both cases, it is this property of \( \Gamma \) which guarantees that the mass-mixing matrix has a zero mode leaving thus the photon massless.

Despite these similarities, however, there is a notable difference between the electroweak and colour sectors which, as we shall see, is responsible for some marked divergences in the physics emerging for the two sectors. First, as noted, it was the vector \( \alpha \) coming from the weak framon which breaks the \( \tilde{\text{su}}(3) \) symmetry. Given this vector, the strong vacuum in, say, the hermitian gauge, will automatically align itself in such a way as to have its long axis (for \( R > 0 \)) pointing in the direction of \( \alpha \), leaving the two shorter axes orthogonal to it and a residual symmetry about it. Now we find that the direction in which the \( u(1) \) of electromagnetism is embedded in \( \tilde{\text{su}}(3) \) space is also given by \( \alpha \), with the \( \phi \) pointing in the direction of \( \alpha \) (again in the hermitian gauge) given the charge \( +\frac{2}{3} \) while the two \( \phi \) orthogonal to \( \alpha \) are given the charges \( -\frac{1}{3} \). Now, this is quite different from what happens to the \( \text{su}(2) \) symmetry in the flavour sector which, though also broken by the \( u(1) \) of electromagnetism via the vector \( \gamma \), this last, as far as is known, need not have any relation to the vector \( \beta \) coming from the strong framon, since the dependence of the vacuum on \( \beta \) has been eliminated by “minimal embedding”, which is applicable to \( \text{su}(2) \) but not to \( \text{su}(3) \).

Combining the above result for the flavour and colour sectors in treating the two kinetic energy terms together, one finds a mass matrix which is diagonal for all the vector states except for \( A_\mu, \tilde{B}_\mu^3, \tilde{C}_\mu^8 \) which mix together via the submatrix:

\[
\begin{pmatrix}
\frac{1}{4}\zeta^2_W g_1^2 + \frac{2}{5}(1 + R)\zeta^2_S g_1^2 & -\frac{1}{4}\zeta^2_W g_1 g_2 & -\frac{1}{3\sqrt{3}}(1 + R)\zeta^2_S g_1 g_3 \\
-\frac{1}{4}\zeta^2_W g_2 g_3 & \frac{1}{4}\zeta^2_W g_2^2 & 0 \\
-\frac{1}{3\sqrt{3}}(1 + R)\zeta^2_S g_1 g_3 & 0 & \frac{1}{6}(1 + R)\zeta^2_S g_3^2
\end{pmatrix}.
\]  

(58)
This has an eigenvector with zero eigenvalue giving the massless photon as:

$$\gamma = \left( \frac{e}{g_1} A_\mu + \frac{e}{g_2} \tilde{B}_\mu + \frac{2}{\sqrt{3} g_3} e \tilde{C}_8 \right),$$  \hspace{1cm} (59)

with the normalization given by the electromagnetic coupling

$$\frac{1}{e^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2} + \frac{1}{3 g_3^2}. \hspace{1cm} (60)$$

Though leaving the photon massless, as is indispensable for the theory to be viable, the result differs from the standard electroweak theory which has already been tested to great accuracy by experiment. It is thus important to check whether the deviations still remain within the limits of the present experimental errors and, if so, whether they can be detected as new physics by future experiments. An answer to these questions will require consideration too lengthy to be treated in this paper and so has to be delegated to another \cite{9}, only a brief summary of which will be given in Section 7.3.

Having now assigned $u(1)$ charges to the colour framon so as to leave the photon massless, we are ready to turn back to answer the question left open before on the $u(1)$ and $\tilde{u}(1)$ invariance of the framon potential \cite{9}. First, we note that as for the flavour framon, $\tilde{y} = -y$ for each column of $\Phi$. It is then immediately clear that all terms in \cite{9} are invariant under $u(1)$ and $\tilde{u}(1)$, except perhaps for a hesitation over the last term with coefficient $\nu_2$. However, even this last term is seen to be invariant when we recall that $\Phi \alpha$ is just that component of $\Phi$ which carries a $y$ of $+\frac{2}{3}$ and a $\tilde{y}$ of $-\frac{2}{3}$, so that the change in phase under either a $u(1)$ or $\tilde{u}(1)$ transformation of $\Phi \alpha$ will cancel the opposite change in phase of the factor $(\Phi \alpha)^\dagger$ also present. This is independent of what $\tilde{y}$ charges are assigned to $\alpha$ and $\beta$ since these factors always occur in \cite{9} in conjugate pairs. Hence, we conclude that $V$ in \cite{9} is indeed fully invariant under $G \times \tilde{G}$ as required.

We have already dealt with the tree-level mass matrix of the $G$. Expanding further the expression \cite{41} gives the tree-level couplings of the $H$ to the $G$ detailed in Appendix B.

5 Yukawa couplings of framon to fermions

At the present stage of our understanding, the construction of Yukawa couplings is conceptually different from that of the framon potential and kinetic
energy term treated above. These latter terms involve only framons and
gauge potentials, each of which has been assigned a geometrical significance
and each has a specific function to discharge. Their construction requires
thus merely an insistence that the basic invariance principles be satisfied, al-
though at times this might have required some deftness to achieve. Yukawa
couplings, on the other hand, involve matter fermions, for which no geo-
metrical significance has yet been discovered. One does not therefore know
a priori which fermion fields should figure in these couplings. Input from
experiment or other conditions is thus needed.

Let us take first as example the flavour coupling of the standard model
in its usual formulation. To accommodate the quarks and leptons seen in
experiment, it is suggested that we take as fundamental fermion fields the
following:

\[ \psi_L\left(\frac{1}{6}, 2, 3\right), \psi_L\left(-\frac{1}{2}, 2, 1\right), \psi_R\left(\frac{2}{3}, 1, 3\right), \psi_R\left(-\frac{1}{3}, 1, 3\right); \psi_R(-1, 1, 1), \psi_R(0, 1, 1), \]  

(61)

(where the first argument inside the brackets denotes the \(u(1)\) charge, the
second the dimension of the \(su(2)\) representation and the third that of the
\(su(3)\) representation). As far as the group representations are concerned,
this choice seems reasonable, these representations being indeed the simplest
for the gauge group \(U(1, 2, 3)\). However, it is a bit of a mystery that

- [CH] The flavour doublet fields are to be left-handed while the flavour
  singlets are to be right-handed,

which we shall refer to as the “chirality puzzle”, since the origin of this
requirement imposed on us by experiment is theoretically unknown.

From (61), Yukawa couplings are constructed as:

\[ Y_\psi \bar{\psi}_L \phi_r \psi_R + Y_\phi \bar{\phi}_L \phi_R \psi_R. \]  

(62)

Or, to exhibit its underlying symmetry under global \(\tilde{su}(2)\), this can be rewrit-
ten as:

\[ Y_\psi \bar{\psi}_L (\phi_r \cdot \gamma) \psi_R + Y_\phi \bar{\phi}_L (\phi_R \cdot \gamma^\dagger) \psi_R. \]  

(63)

in terms of the vector \(\gamma\) introduced in the previous section.

Further, to accommodate the three generations of quarks and leptons seen
in experiment, it is postulated in the standard model that:

- [GE] There are to be three copies each of the fields in (61)
which we shall call the “generation puzzle”, this being in the standard model also theoretically unexplained.

Then, from these fermion fields, one constructs 6 Yukawa terms, each of the form (62), and each with its own set of \((Y_-, Y_+)_i\), that is, 12 independent empirical parameters altogether. Besides, these \((Y_-, Y_+)_i\), being proportional to the quark and lepton masses, are required to take on a hierarchical array of values, in which can be discerned:

- \([h1]\) a hierarchy among generations of the same species, e.g. \(m_t \gg m_c \gg m_u\),
- \([h2]\) a hierarchy between species of different charges, e.g. \(m_t \gg m_b, m_\tau \gg m_\nu_3\),
- \([h3]\) a hierarchy between quarks and leptons, e.g. \(m_t \gg m_\tau, m_b \gg m_\nu_3\).

And all these properties are built into the standard model as empirical input.

In the FSM, things perhaps look somewhat better. First, the flavour framon \((FF)\) in (2) carries with it a global colour vector \(\alpha\), so that in constructing a Yukawa term with the fermion fields in (61) along the lines of (62), one would need to introduce three copies of each to maintain \(\tilde{su}(3)\) invariance, as is demanded by the framon hypothesis. One obtains then from \(\psi_L\left(\frac{1}{2}, 2, 1\right)\) and \(\psi_R\left(-1, 1, 1\right), \psi_R(0, 1, 1), \psi_R(0, 1, 1)\), the Yukawa terms for leptons [111 11]:

\[
A_{\text{YF}} = \sum_{[\tilde{a}] \neq [\tilde{b}]} Y_{[\tilde{a}] \neq [\tilde{b}]} \bar{\psi}_{[\tilde{a}]} \tilde{\alpha} (\Phi \gamma) \frac{1}{2} (1 + \gamma_5) \psi^{[\tilde{b}]} - \sum_{[\tilde{a}] \neq [\tilde{b}]} Y_{[\tilde{a}] \neq [\tilde{b}]} \bar{\psi}_{[\tilde{a}]} \tilde{\alpha} (\Phi \gamma^\perp) \frac{1}{2} (1 + \gamma_5) \psi^{[\tilde{b}]} + \text{h.c.,} \tag{64}
\]

and from \(\psi_L(\frac{1}{6}, 2, 3)\) and \(\psi_R(\frac{2}{3}, 1, 3), \psi_R(-\frac{1}{3}, 1, 3)\) similar terms for quarks.

Hence all three generations now appear as a natural requirement of invariance and are incorporated into a single Yukawa term. Secondly, these Yukawa couplings give, at tree level, a rank-one mass matrix [1] which is already putatively hierarchical, and if one believes further the result cited above from [1], then realistic hierarchical patterns of masses for generations will automatically emerge as a result of (64), through the rotation of the mass matrix [1] with scale under renormalization by framon loops and need no longer be taken as input from experiment. In this sense then, the FSM appears to have solved the puzzle [GE] why there should be three generations
of quarks and leptons, and also the puzzle \[h1\] why their masses should be hierarchical. In the following Section 6, a solution along similar lines will be suggested for the two other hierarchy puzzles \[h2\] and \[h3\]. However, the chirality puzzle remains largely unresolved.

For the flavour Yukawa coupling in the FSM, there is one further point to check for consistency. The terms \((64)\) is by construction invariant under the local gauge symmetry \(u(1) \times su(2) \times su(3)\) and under the global symmetry \(\tilde{su}(2) \times \tilde{su}(3)\). But is it also invariant under \(\tilde{u}(1)\), as it is required to be? We recall from our starting premises that of the fundamental fields only the framon field \(\alpha\Phi\) has any reason to carry a global quantum number \(\tilde{y}\). Thus, all the fundamental fermion fields \(\psi\) appearing in \((64)\), whether left-handed or right-handed, should have \(\tilde{y} = 0\). This statement agrees with the assertion in Section 2 that the group for the global symmetry \(\tilde{G}\) is \(U(1, 2, 3)\) so that by \((6)\), these fermion fields, being singlets in both global flavour and colour, should indeed have \(\tilde{y} = 0\). What then in \((64)\) will cancel the \(\tilde{y}\) value \(-\frac{1}{3} \mp \frac{1}{2}\) given in \((2)\) for flavour framon \(\alpha\Phi\) so as to leave the Yukawa term invariant? We notice first that the vectors \(\gamma, \gamma^\perp\), though not field variables, are \(\tilde{su}(2)\) doublets, and so are nevertheless required by \((6)\) to have \(\tilde{y} = \pm \frac{1}{2}\), hence cancelling part of this deficit, namely \(\mp \frac{1}{2}\), from \(\alpha\Phi\). Secondly, we notice that appearing in \((64)\) above is not \(\alpha\Phi\) but are \(\alpha^a\), the components of \(\alpha\) in the directions \(\tilde{a}\), which, being scalars in global colour \(\tilde{su}(3)\), should, again by \((6)\) carry \(\tilde{y} = 0\), meaning that the other part of the \(\tilde{y}\) charge, namely \(\frac{1}{3}\), supposed to be carried by \(\alpha\Phi\) as cited above, should not have appeared. If we insist on writing \((64)\) in terms of \(\alpha\Phi\), then we should have written \(\alpha^a\) as \((e^{[a]}_\dagger \cdot \alpha)\) where \(e^{[a]}\) are three orthonormal vectors associated with the three fundamental left-handed fermions fields \(\psi^{[a]}\). Then these \(e^\dagger\), being anti-triplets, will carry each an appropriate \(\tilde{y} = \frac{1}{3}\) to cancel off the \(-\frac{1}{3}\) from \(\alpha\), leaving then both the components \(\alpha^a\) and the whole Yukawa term \((64)\) invariant under \(\tilde{u}(1)\) as required. Though seemingly pedantic, these considerations for checking the invariance of Yukawa terms under \(\tilde{u}(1)\) are to our minds worth going through once in detail, so as to clarify later (Section 7.2) the relationship between \(\tilde{y}\) and the lepton and baryon numbers.

Next, we turn to the construction of the Yukawa terms for the colour framon \((CF)\), using \((64)\) as template. Instead of \(\tilde{su}(2)\) we now have \(\tilde{su}(3)\), which is to be broken explicitly again by electromagnetism. According to the preceding section, however, the global nonabelian symmetry is here broken not by an “external” vector like \(\gamma\) in the flavour case, but by the vector \(\alpha\).
which comes from the flavour framon (FF) in [2]. The reason is that, as already noted, the vacuum has itself a broken symmetry depending on the direction of \( \alpha \), which forces us to have the breaking by electromagnetism occurring only in that direction if we want to keep the photon massless. In place of \( \gamma^\perp \) orthogonal to \( \gamma \) in the flavour case, let us introduce thus two corresponding (3-d) unit vectors \( \delta \) and \( \delta' \) both orthogonal to \( \alpha \) and also mutually orthogonal [7]. The colour framon in the direction \( \alpha \) carries the charge \(+\frac{2}{3}\), but the framons in the directions \( \delta, \delta' \) carry the charge \(-\frac{1}{3}\).

With these observations and new notations, we propose then to write the Yukawa term for the colour framon in the following generic form, provided we have at our disposal the appropriate left- and right-handed fermion fields:

\[
A_{YC} = Z_\delta \bar{\psi}_L(\Phi \delta) \psi^+_R + Z_{\delta'} \bar{\psi}_L(\Phi \delta') \psi'^+_R + Z_\alpha \bar{\psi}_L(\Phi \alpha) \psi^+_R.
\] (65)

This is by construction invariant under the double symmetry \( G \times \tilde{G} \), including the \( \tilde{u}(1) \) factor, as can be seen following similar arguments for the flavour term (64) above.

However, this formula hides an important difference from the flavour case. In parallel to the vector \( \alpha \) in (64) coming from the flavour framon [2], which played such an important role there in explaining the intricacies of fermion generations [1], there ought to appear in (65) also the vector \( \beta \) coming from the colour framon [3]. In strict parallel to (64), we ought thus to have written for (65) something like:

\[
A_{YC} = \sum_{[\tilde{r}][\tilde{s}]} Z_{\tilde{r}\tilde{s}} \bar{\psi}_{\tilde{r}L}(\Phi \delta) \psi^+_{\tilde{s}R} + \sum_{[\tilde{r}][\tilde{s}]} Z_{\tilde{r}\tilde{s}} \bar{\psi}_{\tilde{r}L}(\Phi \delta') \psi'^+_{\tilde{s}R}.
\] (65)

This introduction in the colour Yukawa terms of three (extra) vectors \( \alpha, \delta, \delta' \), in parallel with \( \gamma, \gamma^\perp \) in the flavour case, may seem somewhat arbitrary and perhaps unfounded. The same may be said of the three (3d) vectors \( e^{[a]} \) introduced earlier. However, this comes about from a result in (multi)linear algebra about tensor products, which says that there is a natural isomorphism (but bases-dependent)

\[
\mathbb{C}^3 \otimes \mathbb{C} H \cong H \oplus H \oplus H
\]

where the right hand side represents three scalar fields in the Hilbert space \( H \). So we are saying that it makes good sense (as proposed above) to attach to each of these scalar fields a basis vector from the representation space of \( \tilde{su}(3) \) with its inherent linear structure, and thus make the invariance manifest and give a clearer mathematical formulation of the procedure.
\[ + \sum_{\{\tilde{r}\}^{[s]} \bar{\psi}_{[\tilde{r}]L} \beta^\tilde{r} (\Phi \alpha) \psi^R_{[\tilde{r}]} \]

\[ + \text{h.c.} \]

(66)

We recall, however, that in contrast to \( \alpha \), which is coupled to the vacuum and therefore rotates with changing scale leading to all the intricacies of fermion generations, the vector \( \beta \) (ultimately again because of the “minimal embedding” of \( \tilde{su}(2) \) special to \( su(2) \)) is decoupled from the vacuum and scale-independent. Thus, for example, the expression:

\[ \sum_{\{\tilde{r}\}} \bar{\psi}_{[\tilde{r}]L} \beta^\tilde{r}, \]

being a constant (scale-independent) linear combination of the \( \bar{\psi}_{[\tilde{r}]L} \), represents really only one single field, which we could therefore just as well denote by \( \bar{\psi}_L \) as in (65). In other words, we could just as well remain with (65) above. This means that there is in the colour Yukawa coupling no parallel to generations, a significant feature in the flavour case.

Next, we need to ask the question: for which fermions are we to construct such Yukawa terms? In our previous applications of the Yukawa term in for example \( [\text{1}] \), we did not have to answer this question, since for deriving the scale dependence of \( \alpha \) needed by the programme there, merely the generic form of the Yukawa term sufficed. But now, to study the spectrum of the \( F \) we shall have to do so. In contrast to the flavour case, where we know from experiment which quarks and leptons occur as the known bound states, the same question is not easily answerable for the colour case since the bound state fermions \( F \) are still unknown to us. Therefore, with little other information to guide us, any answer, it would seem, can only be tentative, standing to be rectified and/or supplemented if and when further empirical information becomes available or if and when it is understood what theoretical grounding or geometrical significance fermions have, in parallel to those of the gauge bosons or of the framons. Even so, we believe that a working model for the colour Yukawa terms would be useful to serve as a sort of base camp for exploration and we propose to construct one as follows.

We start with the list (61) of fermion fields which, from previous analyses, we know must be present in our theory so as to give us the quarks and leptons. We need, however, to introduce three copies of these, both in the SM by fiat to accommodate the three generations and in the FSM to account for \( \tilde{su}(3) \).
invariance and hence to deduce the existence of the three generations. For example, we need three copies of \( \psi_L(\frac{1}{6}, 2, 3) \) in the FSM to bind with the flavour framon so as to form the three generations of left-handed flavour-doublet quarks. We can understand this if we regard the fermions in the list (61) as fundamental quantized fields, and the three copies needed of them as just their quanta of excitation, since these quanta are automatically represented by wave functions all transforming under symmetry operations in the same way as the fundamental fields themselves, as we want our “copies” to do.

This is similar in spirit to what we may call the standard scenario where the quark field is regarded as fundamental, when we take a \( u \) quark and combine it with an anti-\( d \) quark to form a \( \pi^+ \), and another \( u \) quark and combine it with two \( d \) quarks to form a neutron. These two \( u \) quarks are but two different quanta of excitation (identical copies) of the same “fundamental” quark field, which combine with two separate entities (the \( \bar{d} \) and the \( dd \)) respectively to form the \( \pi^+ \) and the neutron. Indeed, there was a time when physicists would construct field theories with the compound states \( \pi^+ \) and neutron as second quantized fields, as we do now with quarks and leptons. In a sense then, by accepting ’t Hooft’s confinement picture one has gone a level deeper, by starting with the fermions (61) as “fundamental fields”, while the quarks and leptons themselves appear as compound states of the flavour framon with quanta of the fundamental fermions fields to give the three different generations.

Let us note that, in doing so, we make no pretence of dealing really with physics at a more fundamental level. For example, we cannot tell at this stage which bound states should exist between the framon and the quanta of the fundamental fermion fields. We are treating (61) merely as a list of allowed representations to draw copies from so as to form the bound states we want and then to write down effective actions for them as one used to do in the old days for pions and nucleons before the advent of chromodynamics. If this does really represent a deeper level of physics, it is for the future to explore.

If we accept this interpretation of (61) as fundamental fields, then there seems nothing in principle to stop their quanta, if these are coloured, from combining via colour confinement with the colour framons to form the \( F \). Now of the fields in (61), three carry colour, namely:

\[
\psi_L(\frac{1}{6}, 2, 3), \quad \psi_R(\frac{2}{3}, 1, 3), \quad \psi_R(-\frac{1}{3}, 1, 3). \quad (68)
\]
Combining these with the colour framon $\Phi$, what $F$-states will they give?

First, from $\psi_L(\frac{1}{6}, 2, 3)$ combined with $\Phi^\dagger$ we obtain the left-handed $F$ in the first column in the following list (69). These are colour singlet bound states but are flavour doublets. They are in fact the counterparts of the left-handed quarks which are flavour singlet bound states carrying colour, only with the roles of flavour and colour interchanged, and will be called co-quarks. In order to construct a Yukawa term for each of these left-handed $F$ so as to give it a (Dirac) mass, they have to be matched with fields of the opposite handedness, as listed in the second column, resulting in the mass eigenvalues in the third column, as will be shown later.

$$\Phi^\dagger \psi_L(\frac{1}{6}, 2, 3) = \begin{pmatrix}
\psi_L(\frac{1}{2}, 2, 1) \\
\psi_L(\frac{1}{2}, 2, 1) \\
\psi_L(-\frac{1}{2}, 2, 1)
\end{pmatrix} : \begin{pmatrix}
\psi_R(\frac{1}{2}, 2, 1) \\
\psi_R(\frac{1}{2}, 2, 1) \\
\psi_R(-\frac{1}{2}, 2, 1)
\end{pmatrix} : ZQ\sqrt{1-R \zeta S}$$

Then from the other two fundamental fields on the list (68), we have similarly:

$$\Phi^\dagger \psi_R(\frac{2}{3}, 1, 3) = \begin{pmatrix}
\psi_R(1, 1, 1) \\
\psi_R(1, 1, 1) \\
\psi_R(0, 1, 1)
\end{pmatrix} : \begin{pmatrix}
\psi_L(1, 1, 1) \\
\psi_L(1, 1, 1) \\
\psi_L(0, 1, 1)
\end{pmatrix} : Z_L\sqrt{1-R \zeta S}$$

and:

$$\Phi^\dagger \psi_R(-\frac{1}{3}, 1, 3) = \begin{pmatrix}
\psi_R(0, 1, 1) \\
\psi_R(0, 1, 1) \\
\psi_R(-1, 1, 1)
\end{pmatrix} : \begin{pmatrix}
\psi_L(0, 1, 1) \\
\psi_L(0, 1, 1) \\
\psi_L(-1, 1, 1)
\end{pmatrix} : Z_L\sqrt{1-R \zeta S}$$

which will be called co-leptons, with the electrically neutral members labelled also as co-neutrinos.

Some of the fields listed in the second columns, specifically those coupled via $\delta$ or $\delta'$, carry the same quantum numbers as the charge conjugates of the 3 colour neutral fields in (61), and can be interpreted as such. But of those three others coupled via $\alpha$, only $\psi_L(0, 1, 1)$ can be taken as $\psi_R(0, 1, 1)^C$ but
the other two are not contained in the list \( \{61\} \) and have to be added as new fields giving the full list as:

\[
\psi_L(\frac{1}{6}, 2, 3), \quad \psi_L(-\frac{1}{3}, 2, 1), \quad \psi_R(\frac{2}{3}, 1, 3), \\
\psi_R(-\frac{1}{3}, 1, 3), \quad \psi_R(-1, 1, 1), \quad \psi_R(0, 1, 1) \\
\psi_R(-\frac{1}{2}, 2, 1), \quad \psi_L(-1, 1, 1), \quad \psi_L(0, 1, 1),
\]

(72)

where the last item is repeated for convenience for later reference despite being the charge conjugate of \( \psi_R(0, 1, 1) \) already listed. This list seems to provide an interesting new take on the question of chirality in that the un-coloured fields in it are present initially in both handedness, but components of opposite handedness get pulled off in different directions, with half of them forming via flavour confinement the left-handed leptons, and the other half posing as the right-handed partners of some \( F \). However, this raises the question what if \( \psi_R(-\frac{1}{2}, 2, 1) \) should bind with a flavour framon via flavour confinement as \( \psi_L(-\frac{1}{2}, 2, 1) \) does. This will result in a state with the same quantum numbers as a right-handed lepton which is not wanted. But if it is not a right-handed lepton, then what is it? To this, we shall suggest an answer later in Section 8 \[g\]

With (72) as our list of fundamental fermion fields, we propose then to work with the colour Yukawa terms:

\[
\mathcal{A}_{YQ} = \begin{bmatrix}
Z_{Q8}\bar{\psi}_L(\frac{1}{6}, 2, 3)\Phi\delta\psi_R(\frac{1}{2}, 2, 1) \\
+Z_{Q8}\bar{\psi}_L(\frac{2}{3}, 1, 3)\Phi\delta'\psi'_R(\frac{1}{2}, 2, 1) \\
+Z_{Q\alpha}\bar{\psi}_L(\frac{1}{6}, 2, 3)\Phi\alpha\psi_R(-\frac{1}{2}, 2, 1)
\end{bmatrix} + \text{h.c.}
\]

(73)

for the co-quarks, plus

\[
\mathcal{A}_{YL1} = \begin{bmatrix}
Z_{L1\delta}\bar{\psi}_R(\frac{2}{3}, 1, 3)\Phi\delta\psi_L(1, 1, 1) \\
+Z_{L1\delta}\bar{\psi}_R(\frac{2}{3}, 1, 3)\Phi\delta'\psi'_L(1, 1, 1) \\
+Z_{L1\alpha}\bar{\psi}_R(\frac{2}{3}, 1, 3)\Phi\alpha\psi_L(0, 1, 1)
\end{bmatrix} + \text{h.c.}
\]

(74)

and

\[
\mathcal{A}_{YL2} = \begin{bmatrix}
Z_{L2\delta}\bar{\psi}_R(-\frac{1}{3}, 1, 3)\Phi\delta\psi_L(0, 1, 1) \\
+Z_{L2\delta}\bar{\psi}_R(-\frac{1}{3}, 1, 3)\Phi\delta'\psi'_L(0, 1, 1) \\
+Z_{L2\alpha}\bar{\psi}_R(-\frac{1}{3}, 1, 3)\Phi\alpha\psi_L(-1, 1, 1)
\end{bmatrix} + \text{h.c.}
\]

(75)

for the two sets of co-leptons.

We stress, however, that they are to be regarded as mere working hypotheses and do not have the same theoretical basis as the framon potential.
and kinetic energy term considered in the two preceding sections, nor the phenomenological justification as the flavour Yukawa couplings studied earlier in this section.

In these couplings, by substituting for the framon field $\Phi$ its vacuum expectation value, one obtains the tree-level mass matrices of the $F$, which in the canonical gauge (13) are already diagonal with elements listed in the last columns of (69), (70) and (71). Further, by expanding about the vacuum in fluctuations of the framon fields, one obtains in the same gauge the tree-level coupling matrices of the $H_K$ to the $F$ as:

$$\Gamma_K = V_K Z^{1/2} (1 + \gamma_5) + Z V_K^\dagger Z^{1/2} (1 - \gamma_5),$$

(76)

where $V_K$ are given in (18), and

$$Z_Q = \begin{pmatrix} Z_Q \delta & 0 & 0 \\ 0 & Z_Q \delta' & 0 \\ 0 & 0 & Z_Q \alpha \end{pmatrix},$$

(77)

with similar formulae for $Z_{L1}$ and $Z_{L2}$. These will be useful in the following section.

6 Scale-dependence from framon loops

Apart from the terms in the action involving the framon fields detailed in the last 3 sections, there are of course also the usual terms of the standard model which we may call the kinetic energy terms of the gauge bosons and the fermions, which have no explicit dependence on the framon fields. The only question then is how they translate in the confinement picture of 't Hooft into couplings of the $G$ and $F$ states. Now in Section 4, it is shown in the parallel flavour case that the kinetic energy term in (34) of the flavour gauge bosons $B_\mu$ is formally the same when translated into the $\tilde{B}_\mu$ states. The same arguments will show that the kinetic energy term for the colour boson $C_\mu$ will also be formally the same when translated into the $\tilde{C}_\mu$ states representing the $G$. This means that the couplings of the $G$ among themselves will be the same as the colour gauge bosons among themselves. Very similar arguments when applied to the kinetic energy term of the fermions will show that the couplings between the $G$ and $F$ are formally the same as those between the colour gauge bosons and the fundamental fermions fields. Though fairly straightforward, these arguments will be outlined for completeness in Appendix C.
Together then with the tree-level couplings of the $H$ to themselves and to the $G$ listed in Appendices A and B, plus those to the $F$ listed at the end of the preceding section, one can in principle proceed to evaluate the higher order loop corrections. However, at this early explorative stage, we are obviously not yet in a position to embark on a full investigation in this direction. We shall here restrict ourselves only to 1-framon loop correction of the fermion self-energy with only the limited aim of checking how it fits in with the noted hierarchies in the fermion spectrum, and with the result obtained before in [1] using a Yukawa coupling which is generically similar but differs in detail from that suggested in the preceding section.

With hindsight, the choice of framon loop effects on the fermion self-energy as a first example of higher order corrections is seen to be a particularly lucky one to study, given that the Yukawa couplings (76) are so much simpler than the other couplings listed in Appendix A or B, but that they already display that unique property of framons in carrying both local and global indices, so that framon loops lead, with changing scales, not only to changing strengths to quantities as gauge boson loops do, but also to changing orientations in the global symmetry space, which was what we call “rotation” in the Introduction, from which the results of Table [1] were derived.

Fermion masses derived from Yukawa couplings have a common feature in that at tree level the mass is given as a product of the coupling strength times the vacuum value of the scalar (framon) field, where the latter is a property of the vacuum and therefore independent of the fermion appearing in the coupling. This means that the fermion mass is proportional to the coupling strength which governs the size of the framon loop, and thus also of the renormalizing effects they give. Hence, the bigger the mass, the bigger also the renormalization effects.

Consider first as example the flavour Yukawa terms for quarks as given in (64), which for the present purpose can be replaced simply by (62) for $t$ and $b$ where we have suppressed the complexities due to generations given that the properties of the lower generations, according to [1], will emerge automatically as a consequence of the rotation of $\alpha$. Let us consider the scale dependence of the quark masses under renormalization by a framon loop, which in this case is due just to the single standard model Higgs boson $h_W$. It is clear that the masses will start to run with scale with a speed proportional to the couplings $Y^+, Y^-$. Given that $Y^+$ and $Y^-$ are proportional respectively to $m_b$ and $m_t$ at the scale we choose to measure these masses, it
follows that $Y_+ \ll Y_-$ at that scale and that $Y_+ / Y_-$ will decrease further as the scale increases further. Hence, if we choose to write:

$$(Y_+, Y_-) = Y\eta = Y(\sin \theta, \cos \theta)$$

then $\eta$ will have a fixed point $(0, 1)$, or $\theta = 0$ when $\mu \to \infty$. Using the language adopted in [1], though in a different context, we may consider $\eta$ as a vector with a high scale fixed point at $(0, 1)$ which rotates with scale.

Suppose we turn the question around and choose to start with the assertion that there is such a fixed point at $\mu = \infty$, then at a high finite scale we expect that $\theta$ will be small, or $Y_+ \ll Y_-$, or $m_b \ll m_t$, obtaining this last empirical fact (Section 5 [h2]) as a consequence of rotation, in much the same way as the “leakage mechanism” in [1] which gave the hierarchical mass spectrum for the generations, although the similarity is only formal, the physics being very different. In this case, however, the assertion is of only conceptual but no concrete value, since with no other information to fix the integration constant (that is, the “initial value”) for the implied rotation equation, one cannot derive the actual value for $m_b / m_t$ as one would like to.

Similar arguments can be applied of course to the ratio $m_\tau / m_t$ for a qualitative understanding of why leptons are light compared with quarks (Section 5 [h3]). Indeed, from such considerations, it would seem to follow that Yukawa couplings and fermion masses will in general be hierarchical if they are measured at scales close to the fixed point at infinity. The only question is the order of the hierarchy, for which, from the above examples, it seems that the following rule-of-thumb from ancient folklore:

- [RT] The more interactions, and hence the more self-energy it has, the heavier will a particle be compared with its peers.

They can be applied in principle also to the lepton mass ratio $m_{\nu_3} / m_\tau$ to complete our picture for the hierarchy among fermion species, but by $m_{\nu_3}$ here, one presumably means the Dirac mass of the heaviest neutrino, which is unknown experimentally, and not its measured physical mass which is thought to be affected by a see-saw mechanism. However, in the fit of [1] summarized in Table I, a value for $m_{\nu_3} \sim 29.5$ MeV emerged, the ratio of which to $m_\tau$ is remarkably close to $m_b / m_t$, namely $m_{\nu_3} / m_\tau \sim 0.166$ compared with $m_b / m_t \sim 0.24$. If the running with changing scale of $m_b$ due to its renormalization by gluon loops is taken into account, the agreement is even closer, giving $m_b / m_t \sim 0.165$ at the $t$ mass. It is amusing to note that this agreement will result if one assumes (i) that $\eta$ is, for some reason, the same (or similar) for both quarks and leptons, and (ii) that the rotation of $\eta$ induced by $h_W$ loop will essentially stop below the $h_W \sim m_t$ mass scale so that the value of $\theta$ is frozen at that scale.
still applies (apart, of course, from the famous exception that the proton is lighter than the neutron, or in its modern guise, $m_u < m_d$, for which an explanation was already offered in [1]). Thus, $m_t > m_b$ because $t$ has the bigger charge, and quarks are heavier than leptons because quarks have colour interactions while leptons do not.

Turning next to the colour Yukawa coupling, we recall that for the working model suggested above in Section 5, there are altogether 9 Yukawa terms: for each $F$-fermion type, namely whether co-quark, co-lepton 1 or co-lepton 2, there are three Yukawa terms corresponding to the three columns of the colour framon $\Phi$ and coupled respectively via $\delta, \delta'$ and $\alpha$. Let us take first just one $F$-fermion type, to be specific say the co-quark although, as it will be seen later, it does not really matter which. If we were to introduce, in analogy to $\eta$, some 3-vectors $\zeta$ defined as:

\[
(Z_{Q\delta}, Z_{Q\delta'}, Z_{Q\alpha}) = Z_Q \zeta_Q,
\]

with $\zeta$ again a unit vector and:

\[
Z_Q = \sqrt{Z_{Q\alpha}^2 + Z_{Q\delta}^2 + Z_{Q\delta'}^2}
\]

and assign to $\zeta$, in analogy to $\eta$, a high scale fixed point at (0, 0, 1), then at high finite scales we have

\[
Z_{Q\alpha} \gg Z_{Q\delta}, Z_{Q\delta'}
\]

or that the couplings of the co-quarks corresponding to the three components will be hierarchical, in close analogy to [h2] above for flavour.

There is a slight complication. Compared to the flavour Yukawa coupling, the colour coupling differs by replacing, in the RGE, $\zeta_W$ not just by $\zeta_S$ but by the matrix $\Phi_{\text{VAC}}$ in [13]. This means that the above conclusion for $Z_{Q\delta}, Z_{Q\delta'}, Z_{Q\alpha}$ has to be modified by some factors depending on $R$ and hence also on scale, which differ for the $\alpha$ and the $\delta, \delta'$ components. But this will not make much difference, especially at high scales where it matters most, and where, if account is taken of the fit in [1], $R \sim 0$ and the modifying factors become identical.

Very similar considerations to the above suggest that the coupling strengths for the $F$-fermion types would again be hierarchical, and thus by [RT]:

\[
Z_{\text{co-quark}} > Z_{\text{co-(charged)leptons}} > Z_{\text{co-neutrinos}},
\]
which is just the colour analogue of \([h3]\).

Account of these conclusions (81) and (82) will be taken when we later come to consider the mass spectrum of the \(F\).

Next we turn to the question of consistency between the working model of the preceding section and the scheme of \([1]\) and the result in Table \([1]\). The former has 9 Yukawa couplings each giving an RGE, but not all these are of independent relevance to the problem treated there. For instance, the three RGEs for the three \(F\)-fermion types which give the mass hierarchies (82) above are seen to be equivalent as far as \([1]\) is concerned. As noted before, the \(F\) mass obtained from a Yukawa coupling is a product, symbolically \(Z_{\alpha} \Phi_{\text{VAC}}\), and so on, and the hierarchy would affect only the couplings \(Z_{\alpha}\) but not \(\Phi_{\text{VAC}}\) which belongs to the vacuum and should be the same whatever the \(F\)-fermion type it is coupled to. What matters to the problem there is only the scale-dependence of \(\alpha\) induced by that of the vacuum. In other words, the RGE for the rotation of the vacuum on which that problem depends would be the same irrespective of which Yukawa term for the three fermion type from which it is derived.

Next, whichever \(F\) type we choose to focus on, there are still three Yukawa terms coupled respectively via \(\delta, \delta'\) and \(\alpha\). But only the last will be directly relevant for the problem in \([1]\), the results of which were all derived from the rotation of \(\alpha\). As explained there, the term coupled via \(\alpha\) gives the scale-dependence of the third row of the rotation matrix \(A\); so the terms coupled via \(\delta\) and \(\delta'\) will give the scale-dependence of the first and second row of \(A\). We recall however that \(A\) is a rotation matrix which depends on only three parameters which we may take to be the Euler angles of equation (40) in \([1]\). We have seen there that the equation from the term coupled via \(\alpha\) already determines the scale-dependence of two of the Euler angles called \(\theta_1\) and \(\theta_2\) leaving only \(\theta_3\) unconstrained. It is then the scale-dependence of \(\theta_3\) which will now result from the new equations implied by the terms in (75) coupled via \(\delta\) and \(\delta'\), but this is in principle irrelevant for the results derived in \([1]\) and cited in Table \([1]\). The only hesitation is in the very low scale region where one expects the heavy modes associated with \(\alpha\) to decouple but the vectors \(\delta\) and \(\delta'\) continue to run, and these might give indirect effects of which the RGE in \([1]\) has yet taken no account.

We turn our attention now on to the last remaining term coupled via \(\alpha\) which is similar in form to the coupling studied in \([1]\). Compared to equation (20) of \([1]\), this terms differs still in two respects: (i) by replacing \(\sum_{[b]} Z_{[b]} \psi^R_{[b]}\)
with $Z\psi_R$, and (ii) by replacing what is called $\alpha_Y$ there by $\alpha$ here.

• For (i), we notice that what we call $Z\psi_R$ here is in fact just a special case of $\sum_b Z[b]\psi_R[b]$ when we take the array $(Z[1], Z[2], Z[3])$ to be $(0, 0, Z)$. Since, in the derivation of the rotation equation in [1], $(Z[1], Z[2], Z[3])$ never figures except via its norm $\rho^2_S = \langle Z|Z\rangle$, the replacement above will have no effect except for replacing $\rho_S$ there by $Z$ here.

• For (ii), although $\alpha_Y$ and $\alpha$ both take the value $(0, 0, 1)$ when the reference vacuum is diagonal, the former is supposed to be a fixed vector while the latter will be seen eventually to depend on scale. However, being but a global quantity carrying no local indices, $\alpha$ does not emit or absorb framons, and do not thus get renormalized. It is thus not involved in the renormalization calculation of [1] which only affect the vacuum expectation value of $\Phi$. The vector $\alpha$ gets rotated under scale change only because it is coupled to the vacuum which changes direction under scale change, and so gets dragged along by the vacuum as the latter rotates. For this reason, the rotation equation derived there for $\Phi_{\text{vac}}$ remains still valid in form after the replacement.

We conclude therefore, that the analysis done in [1] would remain valid for the working model of Yukawa couplings suggested in the preceding section, despite the apparent difference in couplings. Indeed, from the above analysis, it would appear that almost any other model of Yukawa couplings constructed along the lines described in Section 5 would lead to the same result, since what is required from the Yukawa couplings there is merely the rotation of $\alpha$ which appears in the fermion mass matrix (1). The meaning of the fitted parameters may change depending on the choice of model, but the fitted result would not. And this is helpful since the present choice made in Section 5 is only a tentative one, as warned.

7 New physics in the standard sector

With the last section on scale-dependence, we have completed the present round of theoretical scrutiny on the basic structure of the FSM, which is needed for its extension into the hidden sector of framonic C-ons: $H$, $G$, and $F$. Besides providing some conceptual clarifications and new insights, this closer study made three material changes to our earlier formulation, namely, in order of appearance:
• **[R1]** New assignments of the $u(1)$ (electric) charge $y$ to the colour framon $(CF)$;

• **[R2]** New assignments of the the $\tilde{u}(1)$ charge $\tilde{y}$ to the colour framon $(CF)$;

• **[R3]** A working model for the Yukawa terms spelling out some details not needed before.

Hence, before going on to study the hidden sector, which is the stated main aim of this paper, we ought first to check whether, in the standard sector itself, these changes may (i) alter the results obtained earlier, or (ii) imply new physics which has to be tested against experiment. We shall deal with them in the reverse order, leaving **[R1]**, the most fundamental with the most important consequences—and therefore the main concern of this section—to the last, to be considered at some length.

### 7.1 Deviations from SM predictions for some rare decays

It was already shown in the preceding section that despite **[R3]** the result of the fit summarized in Table 1 should remain valid. We can thus also accept some other effects deduced earlier from the rotation of $\alpha$, such as the deviations from the standard model in some rare Higgs decays \[\text{[14, 15]}\]. These deviations come about as follows. The Yukawa coupling of the Higgs boson $h$ to quarks and leptons in FSM is subsumed in the coupling of the flavour framon $[2]$ which carries as a factor the vector $\alpha$, the rotation of which in the resulting mass matrix for quarks and leptons is what leads to their hierarchical mass patterns in the FSM. The same $\alpha$ will thus appear in the couplings of $h$ to quark-antiquark and lepton-antilepton pairs, and its rotation is what in the FSM governs the widths for the Higgs boson decaying into the various $qq$ and $\ell\ell$ modes. This prediction differs from the SM where the couplings are given by the fermion masses. For this reason, it was suggested in \[\text{[14]}\] that:

- the widths into lower generation fermions such as $h \to \bar{c}c$, $h \to \bar{s}s$ and $h \to \mu^+\mu^-$ are all much suppressed compared with what the standard model would expect.
The first two quark modes are apparently very hard to look for in LHC experiments because of background, but the $\mu^+\mu^-$ mode can and has been searched for but has not yet been seen. The latest bound from LHC is given as [16]:

\[
\frac{\Gamma(h \to \mu^+\mu^-)}{\text{SM prediction}} = 0.1 \pm 2.5, \tag{83}
\]

with no event seen. If the experimental error can be reduced further and still no such mode is seen, then it would be a result in favour of FSM. Predictions were made also of some flavour-violating modes such as $h \to \mu\tau$ at a low rate [14, 15], which might soon be experimentally accessible.

However, reservations made in [14] on the tentative nature of these predictions persist since a systematic approach to calculating reaction amplitudes with a rotating mass matrix has not yet been developed.

### 7.2 $\tilde{y}$ conservation versus $B$ and $L$ conservation

It was shown in Section 5 that the Yukawa couplings constructed are invariant under $\tilde{u}(1)$ as required, which means that they conserve $\tilde{y}$ as newly defined in [R2]. These Yukawa terms conserve also baryon number and lepton number separately. It is then natural to ask, as we did, whether these conservation laws are connected, and if so in what way. The conservation of $\tilde{y}$ comes in FSM from a gauge principle, namely $\tilde{u}(1)$ invariance, and so long as this symmetry is unbroken, the conservation has to hold. On the other hand, the conservation of baryon and lepton numbers are, as far as is known, only empirical with no generally accepted theoretical basis [17]. Besides, lepton number conservation is violated by the Majorana mass term for right-handed neutrinos, which term is wanted for neutrinoless double beta-decay and for the see-saw mechanism for explaining the very small physical masses of neutrinos. However, even this term conserves $\tilde{y}$ since, according to the analysis given in Section 5, right-handed neutrinos have $\tilde{y} = 0$, though assigned lepton number 1 by convention. In other words, $\tilde{u}(1)$ invariance or $\tilde{y}$ conservation admits the Yukawa terms in Section 5 as well as the Majorana mass term for right-handed neutrinos, so that, so long as no other terms are found which say otherwise and none are known so far, the FSM conserves $B$, but it conserves $L$ only up to the Majorana term, and so it allows both
neutrinoless double beta-decay and the see-saw mechanism to operate.\footnote{But we were wrong before \cite{10} to identify $\tilde{y}$ with $B - L$ and to claim that we had found a gauge principle for $B - L$ conservation.}

### 7.3 New mixing scheme in the $\gamma - Z - G$ complex giving deviations from the standard model

We turn now to $[R1]$, namely the assignment in (3) of different electric charges to different components of the colour framon, as necessitated by the imperative that the photon should remain massless. This change is more fundamental since it implies a mixing scheme for the vector bosons different from that of the standard model, involving not only the photon and the $Z$ but also another vector boson we call $G$ (Section 4). This will thus lead to departures from the standard model already at the tree level in the electroweak sector in which the standard model has already been tested against experiment to great accuracy, and any sizeable departures from it would have already been ruled out. An examination of these departures as a test of the FSM is thus urgently due.

However, a thorough examination of this question is not yet possible at the present stage of the FSM’s development, as will be explained in the next paragraph. Besides, given the vast amount of data and the sophistication with which they have been analysed as regards consistency with the SM mixing scheme, a thorough re-examination with the FSM scheme to the same breadth and precision is beyond our immediate capability. We have therefore limited our analysis so far to only the following three very well measured quantities:

- (a) $m_Z - m_W$
- (b) $\Gamma(Z \to \ell\bar{\ell})$,
- (c) $\Gamma(Z \to q\bar{q})$.

The details of this analysis will be reported in a separate paper $[9]$, as they would occupy more space than can be allowed for in the present one. Here, we shall give as an example only an outline of the analysis for (a) $m_Z - m_W$, together with just a mention of the results on (b) and (c).

The comparison of the FSM to the standard model and to experiment can at present be done only at tree level because one is not yet in a position in the
FSM to investigate loop corrections in general (see Section 6). We propose therefore to adopt the following criterion. Assuming that the deviations of loop corrections in FSM from SM to be of higher order in smallness, and that the SM itself is in agreement with experiment, we compare the tree level results of the two models, and if the difference in tree-level predictions for a certain quantity is less than the present experimental error in that quantity, we consider that the FSM prediction is also within that experimental error.

Let us then look at the vector boson masses as example and see how the FSM differs from the standard model. At tree level, the description of the $W$ boson is the same in the two models; only the neutral bosons are mixed differently. Without mixing, $W$ and $Z$ would be degenerate in mass in either model; it is the mixing which gives the shift in mass $m_Z - m_W$, and which is what differs between the two models. It is thus the quantity $m_{\text{shift}} = m_Z - m_W$, for which the predicted values of the two models are to be compared. Let us then proceed as follows. From the experimental information on the mass and widths of the $W$ and the tree-level formula $m_W = \frac{1}{2} g_2 \zeta_W$, we determine the vacuum expectation value of the Higgs scalar field as $\zeta_W \sim 246$ GeV, and the coupling $g_2$ of the flavour gauge field as $g_2^2 \sim 0.4271$. These are the central values with the experimental errors yet to be folded in. Supplying further the accurately measured value of the electron charge $e$, and the coupling $g_3$ as independently determined in perturbative QCD and $Z$ hadronic decays, we calculate the shifts of the $Z$ mass from the $W$ mass in the mixing schemes of the SM and of the FSM. Then we compare the results and see whether the difference between the two predicted mass shifts remains within the bounds quoted by experiment, according to the criterion proposed in the preceding paragraph. This may not be the usual way that precision tests for the electroweak theory are phrased, but here it makes the logic clearer since for the mass shifts $m_Z - m_W$ to be compared, the experimental error is dominated by that of $m_W$ which should therefore be folded in, not just that of the more precisely measure $m_Z$.

For the standard model then, from the Weinberg mixing formula:

$$\frac{1}{e^2} = \frac{1}{(g_{1^{\text{SM}}}^2)^2 + \frac{1}{g_2^2}},$$

and the previously settled values of $\zeta_W$, $g_2$ and $e$, we can calculate the value of $g_{1^{\text{SM}}}$ and hence the tree-level prediction for the mass $m_{Z^{\text{SM}}}$ of $Z$ as:

$$m_{Z^{\text{SM}}} = \frac{1}{2} \zeta_W \sqrt{(g_{1^{\text{SM}}}^2)^2 + g_2^2}.$$
For the FSM, the tree-level mass of the $Z$, which we shall call $m_Z$, is to be obtained as the lower non-zero eigenvalue of the mass matrix (58) obtained in Section 4, or equivalently, after some algebra, as the lower eigenvalue of the matrix:

$$
\begin{pmatrix}
\ell(g_1^2 + g_2^2) & -\frac{1}{\sqrt{3}}\sqrt{k\ell}g_2^2 \\
-\frac{1}{\sqrt{3}}\sqrt{k\ell}g_2^2 & k(\frac{1}{3}g_1^2 + \frac{1}{4}g_3^2)
\end{pmatrix},
$$

(86)

with $\ell = \frac{1}{4}\zeta^2_W$ and $k = \frac{2}{3}(1 + R)\zeta^2_S$, and $g_1$ given by the relation:

$$
\frac{1}{e^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2} + \frac{4}{3}\frac{1}{g_3^2},
$$

(87)

obtained from FSM mixing [9].

The formulae for the two values, $m_Z^{SM}$ and $m_Z$ look very different. There is little freedom, all the couplings $e, g_2, g_3$ being known, with the only unknown parameter being $\zeta_S$, and even this, as argued in the next section, is loosely constrained by the fit in [1] to be of order TeV. The mass difference $m_Z - m_W$ is now given by the PDG [16] to an impressive accuracy of about 15 MeV and consistent with the SM predictions. Hence this looks like an extremely stringent test for FSM.

One is however saved by the following result:

- Provided that $\zeta_S \gg \zeta_W$, then to first order in $\zeta^2_W/\zeta^2_S$, the value of $m_Z$ given by the FSM at tree level is identical to $m_Z^{SM}$ predicted by the SM also at tree level, whatever the values of $e, g_2, g_3$ and $\zeta_W$.

This can be seen as follows. That $\zeta_S \gg \zeta_W$ means that in the matrix (86) the mixing off-diagonal elements:

$$
C = \frac{1}{\sqrt{3}}g_2^2\sqrt{k\ell}
$$

(88)

are small compared to the larger ($B$) of the two diagonal elements:

$$
A = \ell(g_1^2 + g_2^2), \quad B = k(\frac{1}{3}g_1^2 + \frac{1}{4}g_3^2).
$$

(89)

Usual perturbation method then gives the approximate eigenvalue as:

$$
m_Z^2 \sim \ell(g_1^2 + g_2^2) + \Delta,
$$

(90)

where

$$
\Delta = \frac{C^2}{A - B} \sim -\frac{1}{3}g_1^4\ell + \frac{1}{4}g_3^2.
$$

(91)
One notices in (90) that \( (m_Z')^2 \) is the sum of two terms. The first, coming from the diagonal element \( A \), differs from \( (m_{SM}^Z)^2 \) by having \( g_1^2 \) satisfying (87) instead of \( (g_{SM}^Z)^2 \) satisfying (84), while the \( \Delta \) represents the effect of mixing. Now, \( g_1^2 \) being larger than \( (g_{SM}^Z)^2 \), the first difference pushes \( m_Z' \) up compared with \( (m_{SM}^Z)^2 \), while the second term \( \Delta \) from the usual repulsion by mixing pushes \( m_Z' \) down. Hence, the two shifts of \( A \) and \( \Delta \) in opposite directions will tend to cancel and make \( m_Z' \) end up closer to \( m_{SM}^Z \) in any case. However, explicit calculation shows that they actually cancel exactly for all values of the coupling parameters, giving thus the above noted result as a pleasant surprise. Notice that for \( \zeta_S \) of order TeV, the ratio \( \zeta_W^2 / \zeta_5^2 \) which appears in the mixing matrix is of the order of a few percent, which would be many times larger then the experimental error on the \( Z \) and \( W \) mass difference, if it were not for the above noted cancellation. With the cancellation, however, the estimate for the deviation would be of order \( (\zeta_W^2 / \zeta_5^2)^2 \sim 10^{-4} \), and within the present experimental error. This is borne out by explicit calculations performed in \([9]\), which gives for \( \zeta_S \sim 2 \) TeV, or \( m_G \sim 1 \) TeV:

\[
(m_{SM}^Z - m_W) - (m_Z - m_W) = 10.4 \text{ MeV}, \tag{92}
\]
well within the quoted experimental error of 15 MeV.

The interesting point is that, as shown in \([9]\), a similar scenario obtains in the deviations of the FSM from SM for the decay widths (b) and (c). To zeroth order in the expansion in powers of \( \zeta_W^2 / \zeta_5^2 \), the decay widths calculated with the FSM mixing scheme are identical to those worked out from the standard model, independently of the values of the couplings. That this should happen is again due to the relation between \( g_1 \) and \( g_{SM}^1 \) as given by (87) and (84). To first order in the expansion, on the other hand, in contrast to the mass difference considered above, the decay widths as given by the FSM do differ from those given by the SM. However, the difference is only of order \( g_1^4 \zeta_W^2 / \zeta_5^2 \) which, given the known value of \( g_1 \), is numerically similar to \( (\zeta_W^2 / \zeta_5^2)^2 \) for \( \zeta_S \) of order TeV, and ensures that it remains within the present experimental bounds. For example, in \([9]\) we find that, using the same parameters and also at tree level, the difference in \( \Gamma(Z \rightarrow e^+e^-) \) between the two schemes is only 0.03 MeV, while the quoted experimental error is 0.12 MeV.

That the deviations of the FSM from the SM should vanish (effectively) to first order in \( \zeta_W^2 / \zeta_5^2 \) in both the \( Z-W \) mass difference and \( Z \) decays raises the suspicion that there may be some deeper reason for these cancellations which we have not yet understood.
Since the exact form of the mass mixing matrix for the FSM is given above in (86), it is straightforward to work out, as it is done in [9], a limit for $\zeta_S$ above which the predicted $m_Z - m_W$ will lie within the present experimental bounds. The same have been done also for the decay widths (b) and (c), and the conclusion is that so long as $\zeta_S \geq 2$ TeV, then the tree-level predictions of the FSM will differ from those of the SM for all the 3 listed items (a), (b), and (c) only by amounts less than the present experimental errors, hence surviving the test we posed above.

There are other hurdles yet to get over, of course, before one can claim the FSM to be consistent with the extensive and very accurate data now available in the electroweak sector, but the above examples are a good start, being seemingly the most stringent. It will be a massive programme to check consistency of the FSM mixing scheme with all the data available, which we are not yet in a position to undertake. In this paper, we shall adopt an optimistic view and tentatively assume that the programme will go through, so as to free ourselves for interesting explorations further afield.

Whatever the actual value of $\zeta_S$, however, there will be deviations of the FSM from the SM. Turning the argument around, therefore, one can regard these deviations as new physics to be searched for in future experiments. For instance, the tree-level FSM prediction, worked out in [9], for the mass shift $m_Z - m_W$ in the manner described above is actually smaller than that predicted by the standard model, namely

$$m_{Z}^{SM} - m_{W} < (m_Z - m_W).$$

In other words, had one started from the experimentally better measured value of the $Z$ mass and worked backwards to predict the $W$ mass, as is more usually done, then the FSM will give a $W$ mass somewhat larger than that predicted by the standard model. It is amusing to note that measurements [18] at LEP, Tevatron, and LHC so far actually all give central values for $m_W$ larger than that predicted by the standard model, though each by only 1-2 $\sigma$. If the experimental error can, with more data, be further reduced, then it is not excluded that we shall soon be asking whether the deviation suggested by the FSM can in fact be observed.

Similar deviations of the FSM from the SM have been worked out in [9] also for the partial widths of the decays (b) $Z \rightarrow \ell \bar{\ell}$ and (c) $Z \rightarrow q\bar{q}$, and these can again be regarded as new physics to be searched for in experiment. Indeed, there being only the one parameter $\zeta_S$, these three deviations are correlated, as they are connected also to the mass of the vector boson we
call $G$, which, as will be discussed later in Section 8 [b], can probably be observed as an $e^+e^-$ anomaly in the multi-TeV range. In short, a point that will be taken up again in the next section and is expanded further in [9], this complex of effects holds out a promise not only of new physics to be tested in the standard sector but also of an opening into the mysterious world of framonic $C$-ons so far hidden from us.

8 Mass spectra of $H$, $G$ and $F$

Having survived, for the moment it seems, the most immediate tests in the standard sector, let us now proceed boldly to explore the new hidden sector populated by the framonic $C$-ons $H$, $G$ and $F$. To navigate such uncharted waters, however, will require some audacity, supplementing theoretical results sometimes with intuition or even just imagination based merely on hints from various bits of physics. In case our readers should think that we do so on occasion to excess, to them we proffer now our apology.

Let us start with the physical mass spectra of the $H$, $G$, and $F$. In Sections 3, 4 and 5 the tree-level mass matrices for these states are derived already from the fundamental action. These matrices, however, are scale-dependent according to Section 6, and so one has to specify at what scales to diagonalize them so as to evaluate the physical masses of these physical particles. It is a commonly accepted prescription that:

- (PM) Physical masses should be evaluated at the mass-scales of the particles themselves,

a criterion we have used in the FSM, in deriving results such as that in Table 1. That seems to have worked. We aim to follow the same procedure now with the $H$, $G$ and $F$.

The criterion means that the physical mass $m_x$ of a state $x$ is to be a solution of an equation of the form:

$$m_x(\mu) = \mu$$  \hspace{1cm} (93)

where $m_x(\mu)$ represents the scale-dependent eigenvalue of the mass matrix corresponding to the state $x$. We can distinguish three cases in the solutions of this equation, all of which we shall meet in what follows:
• (C1) There is a real positive solution to (93) which is unique. In this case, the physical mass $m_x$ of the state $x$ is unambiguously defined as that solution.

• (C2) There are two real positive solutions to (93), in which case we define the physical mass $m_x$ of $x$ to be the lower of the two solutions since the higher solution will be unstable against decay into the lower one.

• (C3) There is no real positive solution to (93), in which case we shall interpret the state $x$ as an inherent but covert degree of freedom in our theory which does not materialize and manifest itself as a physical particle.

Let us take first the $H$ states, the tree-level mass squared matrix for which is given in (19). This is seen to depend on the parameter $R$, which in turn is shown in Section 6 and [1] to depend on the scale $\mu$. There may be further scale dependences coming from the other parameters on which the matrix also depends, but since these have not been studied, we can only ignore them for the moment and take account only of the scale dependence via $R$ that we know about. This is also the philosophy adopted in our earlier work in the standard sector, for example in [1], which seems to have worked, although in that case it is the dependence on $\mu$ via the rotating vector $\alpha$ that matters, an interesting point of difference that we shall return to later.

How then does $R$ depend on scale? In Section 6 and [1], we have obtained the RGEs via framon loops which are derivable from the given Yukawa terms. These depend on certain parameters, and when integrated will depend on some more integration constants, all of which are to be determined empirically by fitting with experiment. At first sight, this looks difficult since these $H$ are still unknown: how is one to do so? However, one interesting feature—one might even be tempted to say the beauty—of the FSM scheme is that the hidden and standard sectors share the same vacuum, so that when the vacuum moves with scale, it will affect simultaneously both sectors. This also means that information obtained in either sector can be used to determine how the vacuum moves and the conclusion will be valid for both. Indeed, we recall that it is actually from the Yukawa couplings of the $F$ in the hidden sector that the RGEs in Section 6 and [1] are derived, but they are applied to reproduce the mass and mixing patterns of quarks and leptons in the standard sector. Conversely then, now that we have determined the parameters
on which the RGEs depend by fitting data in the standard sector, we can
apply the result to the hidden sector to explore the mass spectra of the $H$,
$G$ and $F$. That being the case, the values of $R$ at any scale $\mu$ can just be
read off from Figure 2 of [1], reproduced here as Figure 2 for easy reference,
but of course only to the extent that we can rely on that result.

As it stands, the matrix (19) is already almost diagonal except for the
$3 \times 3$ upper left corner block. And apart from that labelled by $h_W$ (the
electroweak Higgs), the diagonal elements are of the following 3 types:

- (i) those with $Q = 0$ and values proportional to $1 + 2R$: $[H_{(3|\bar{3})}]$;

- (ii) those with $Q = \pm 1$ and values proportional to $\frac{1}{2}(2 + R)$:
  $[H_{(1|\bar{3})}, H_{(2|\bar{3})}, H_{(3|1)}, H_{(3|\bar{2})}]$;

- (iii) those with $Q = 0$ and values proportional to $1 - R$:
  $[H_{\text{odd}} = \frac{1}{\sqrt{2}}(H_{(1|\bar{1})} - H_{(2|\bar{2})}), H_{\text{even}} = \frac{1}{\sqrt{2}}(H_{(1|\bar{1})} + H_{(2|\bar{2})}), H_{(1|\bar{2})}, H_{(2|\bar{1})}]$.

Given that $R$ as seen in Figure 2 is about 0.02 at $\mu = m_Z$ and is smaller
still in the TeV region that, as will be seen, interests us, we can largely neglect
in that region the difference in $R$-dependent factors which distinguishes the
three types (i)—(iii). This means, first, that the already diagonal types (ii)
and (iii) are approximately degenerate. Secondly, when $R \sim 0$, the submatrix
for the states labelled $H_{\langle \bar{3}|\bar{3}\rangle}$ and $H_{\text{even}}$ in the list above simplifies and gives
as eigenstates:

$$H_{\text{high}} = \sqrt{\frac{1}{3}} H_{\langle \bar{3}|\bar{3}\rangle} + \sqrt{\frac{2}{3}} H_{\text{even}}; \quad M^2 = 4(\kappa_S + 3\lambda_S)\zeta_S^2; \quad (94)$$

$$H_{\text{low}} = -\sqrt{\frac{2}{3}} H_{\langle \bar{3}|\bar{3}\rangle} + \sqrt{\frac{1}{3}} H_{\text{even}}; \quad M^2 = 4\kappa_S\zeta_S^2; \quad (95)$$

where only $H_{\text{high}}$ now still mixes with $h_W$. The state $H_{\text{low}}$ is again degenerate
with the other $H$, but $H_{\text{high}}$ is singled out with a larger eigenvalue.

To obtain the mass of the $H$ we solve the equation (93). For this we
need an estimate of the vacuum expectation value $\zeta_S$ of the colour framon.
Interestingly, this same vacuum expectation value has already been given
a lower bound of about 2 TeV (at $\mu \sim m_Z$) (Section 7.3) when we were
considering $m_Z - m_W$ and $Z$-decay into quark and lepton pairs, a completely
different area of physics to the present one. On the other hand, Figure 2 gives
$R \sim 0.02$ at the $Z$ mass scale. If one then assumes that the dimensionless
couplings $\nu_2$ and $\kappa_S$ appearing in $R = \nu_2\zeta_W^2/2\kappa_S\zeta_S^2$ both have values of order
unity, one would obtain a value for $\zeta_S$ of order TeV, which is not that far from
the bound above, and converts that bound into a crude, order-of-magnitude
estimate for $\zeta_S$.

This means then that at $\mu \sim m_Z$ the eigenvalue $m_x(\mu)$ is of order TeV,
that is, larger than the scale itself for all the states listed in (i)—(iii) above.
As the scale increases further, the eigenvalue is expected to increase only
logarithmically with scale and will eventually be caught up by the scale it-
self to give a solution to the equation (93) in the multi-TeV region. For the
$H$ states of types (i) and (ii), this solution is unique since $m_x(\mu)$ decreases
only logarithmically with decreasing scale and cannot catch up again with $\mu$
to give another solution, so that, by (C1) above, we would obtain physical
masses $m_x$ of order TeV (or higher) for these states. However, for the re-
mainning states in (iii) with eigenvalues proportional to $\sqrt{1 - R}$, there will be
another solution to the equation (93) as follows. We see from Figure 2 that
$R$ approaches 1 rapidly at around 17 MeV so that shortly above this scale
(whatever may be the values of the parameters $\kappa_S$ and $\zeta_S$ at this scale), there
is bound to be another solution of the equation. Intriguingly, the accuracy
for the estimate of 17 MeV for this second solution is limited only by the
credibility of the fit in [1], there being no need to actually solve the equation
Particle | State | Mass
--- | --- | ---
$H^0$ | mixture of $H_{(3\bar{3})}$, $H_{even}$ and $h_W$ | $\gtrsim$ multi-TeV
$H^0_{1/2}$ | $H_{(1\bar{3})}$ $H_{(2\bar{3})}$ $H_{(3\bar{1})}$ $H_{\bar{3}(2)}$ mixture of $H_{even}$, $H_{(3\bar{3})}$ | $\gtrsim$ TeV
$H^0_{low}$ | $H_{(1\bar{2})}$ $H_{(2\bar{1})}$ $\frac{1}{\sqrt{2}}(H_{(1\bar{1})} - H_{(2\bar{2})})$ | $\sim 17$ MeV

Table 2: Suggested spectrum of the $H$ states

, since what is required is that $(1 - R) \sim [17\text{MeV}/\text{TeV}]^2$, which is close enough to zero for us not to bother about the difference, especially since the curve for $R$ in Figure 2 is very steep there. We conclude then by (C2) above that the physical masses of the states of type (iii) will be given by this second and lower solution.

These conclusions are summarized in Table 2, where we have simplified the notation and inserted the charges as superscripts for easy reference.

The analysis for the $G$ states is very similar. Apart from the mixing of $G_8$ with $Z$ and $\gamma$, the mass matrix is already diagonal in the Gell-Mann basis, and as seen in (50) the $G$ fall naturally into three groups, as did the $H$, with eigenvalues proportional to, respectively: $(1 + 2R), \frac{1}{2}(2 + R), (1 - R)$. Again for $\mu \sim \zeta_S$ of order TeV, $R \sim 0$ so that these values are all nearly degenerate, but for the last group with eigenvalue $\propto (1 - R)$ there is a second solution for the physical mass at $\sim 17$ MeV near $R = 1$. Unlike the $H$, however, the coupling $g_3$ here is known from QCD, so that the spectrum depends only on the one parameter $\zeta_S$. Thus once that value is known, say for example from the analysis in Section 7.3, then the spectrum can actually be calculated. These conclusions are summarized in Table 3, where the equal sign in the bound denotes the mass evaluated at tree level with the benchmark value $\zeta_S = 2$ TeV (Section 7.3).

We note in Tables 2 and 3 the following interesting points:

- [a]

Comparing the spectra of the $H$ and $G$ with that of the quarks and
leptons, we find that there is a strange sort of duality between the two. The RGE derived in [1] (see also Section 6) gives the variations with scale of the two quantities $R$ and $\alpha$. They are correlated, which is not surprising given that $R = \nu_2\zeta^2_w/2\kappa_s\zeta^2_s$ measures the relative strength of the $\tilde{u}(3)$ symmetry-breaking $\nu_2$ term in the framon potential against the $\tilde{u}(3)$ symmetry-maintaining $\kappa_s$ term, hence governing the amount of breaking, or in other words the direction of $\alpha$. The mass matrices of the quarks and leptons are independent of $R$ but depends on $\alpha$ whose rotation then gives the details of the quark and lepton spectra. In parallel, the mass matrices of the $H$ and $G$ are not affected by the rotation of $\alpha$ since the vacuum value of $\Phi$ rotates covariantly with it, but they depend explicitly on $R$ and this dependence is what prescribes the spectrum. However, despite the marked differences in the mechanisms by which the two spectra are generated, and despite the fact that one concerns fermions while the other bosons, their properties seem to echo one another. The $H$ and $G$ each fall naturally into three groups of decreasing masses (although the differences may not be big at high scales where $R \sim 0$), echoing the three generations of quarks and leptons. Of these, the lowest group has a particularly low mass because of the existence of a second solution to the mass condition (93), again echoing the lowest generation quarks and leptons which acquire their particularly low masses also by virtue of a second solution [1].

• [b]
Both the $H$ and $G$ spectra are such that all charged states are heavy of order TeV while all the light states of order 17 MeV are neutral. This is an important property for the spectra to possess in order to remain realistically viable, for any charged particle of light mass is unlikely to have escaped detection by experiment.

• [c]
The heaviest states, $H^0$ in Table 2 and $G^0$ in Table 3 are both distinguished further by being mixed states each with a component in the standard sector. The mixing of the vector bosons which gives $G^0$ as eigenstate has already been detailed in Section 4. For the scalar states, the state $H_{\text{high}}$ in (94) mixes with the standard electroweak Higgs state $h_W$, to give diagonal states, say, $H$ and $h$, where the lower mixed state $h$, which is mostly $h_W$, is to be identified with the Higgs state already observed at 125 GeV, while the higher mixed state $H$, which is mostly $H_{\text{high}}$, would be a new state yet to be observed.

By virtue of their (small) components in the standard sector, both $H$ and $G$ can be produced by experiments which produce the $h$ and the $Z$, and decay also into final states into which the $h$ and $Z$ decay. Thus, $H$ can appear as a diphoton and the $G$ as a $\ell^+\ell^-$ bump at LHC. Indeed, at one stage, a diphoton enhancement [19, 20] was reported by ATLAS and CMS at a mass of around 750 GeV which could have suited $H$ although the mass is lower than expected, but this was in any case not confirmed by later data with higher statistics [21, 22]. However, looking further along these lines might well reveal the $H$ and $G$ in the multi-TeV range.

Of the two, $G$ looks the more promising. Besides the lower predicted mass, it has the virtue of depending on only the one parameter $\zeta_S$, since its couplings are governed by $g_3$ which is already known from perturative QCD. Thus, we know already its partial widths into lepton pairs and into hadrons [9], and with more work, there is a chance that even its production cross section at the LHC and its total width might be estimated. Whether it is so is at present under investigation.

From the FSM point of view, the search for $H$ and $G$ will be extremely worthwhile, not only for just their own sake as new particles. By virtue of their being mixed states partly in the standard sector but mostly in the hidden sector, they would serve us as valuable portals into the
hidden sector. Indeed, they are the only two such portals so far known to us. For example, once produced, they would decay mostly into particles in the hidden sector which will be all new to us, and give us a glimpse into that other world.

- [d]

Perhaps the most striking feature of Tables 2 and 3 is the appearance of states at as low a mass as 17 MeV in a spectrum the natural scale of which as given by $\zeta$ is of order TeV. We recall that these entries were made based on:

- (i) the $\mu$-dependence of $R$ obtained from the fit in [1] to the mass and mixing data of quarks and leptons.
- (ii) strict adherence to the criterion ($\text{PM}$) that physical masses of particles are to be obtained from the running mass at the scale equal to the mass itself.
- (iii) that when a second solution exists for the physical mass, we choose the lower as being more stable ($C2$).

Given that these are theoretically none too strong, the entries themselves are a rather bold assertion which can do with some phenomenological support.

As indirect support, we can cite the example of the quark and lepton spectra obtained in [1] which was based on the same premises (i)—(iii) except for the replacement of (i) by the parallel $\mu$-dependence of the rotation of $\alpha$. And those parallel arguments there have yielded explanations for the unusual properties of the lowest generation which we had previously found very puzzling: (i) $m_u$ of order MeV, from an input scale $m_t$ of order 100 GeV, (ii) $m_u \sim m_d \sim m_e$ in magnitude, despite $m_c \gg m_s \gg m_\mu$, (iii) $m_u < m_d$, despite the fact that for the higher generations, $m_u \gg m_b > m_\tau$ and $m_c > m_s > m_\mu$. It is this prior experience for the quarks and leptons in the standard sector which gives one now some confidence to suggest the same interpretation ($C2$) of the second solution for the $H$ and $G$ above.

Nevertheless, some direct phenomenological support from the hidden sector itself will be needed for the assertion to be really creditable. Such, of course, will be much harder to come by, or so we thought.
One is agreeably surprised, however, by a recent development in an unexpected area which might have a bearing on this subject. Recent observations of such phenomena as the $g-2$ anomaly have led to theoretical speculations of new low mass particles, which in turn prompted experimental searches in this region. Although most of these searches are negative, one experiment on excited beryllium decay [23], has reported a $7\sigma$ anomaly in the $e^+e^-$ effective mass plot in the final state, which the authors interpret as a possible new particle with a mass of coincidently 17 MeV, just the mass predicted for the low mass $H$ and $G$ states in Tables 2 and 3. The observed ratio

$$\frac{\Gamma(Be^8* \rightarrow Be^8 + X)}{\Gamma(Be^8* \rightarrow Be^8 + \gamma)}$$

is given as $5.8 \times 10^{-6}$. This, and the fact that it has not been observed in other different experiments scanning this mass region, imply that it must have rather unusual properties. Hence, despite its not having been independently confirmed by other experiments, the anomaly has sparked a fair amount of theoretical speculations [24, 25, 26] on what this new particle could be, including the possibility that it could be a $1^-$ state.

The interest for us is that the above considerations in FSM do predict new particles in the hidden sector at just that mass region. Of these particles, that labelled $G_3$ in Table 3 in particular, can decay with a small width into $e^+e^-$ via a framon loop (the framon being charged), and can thus be a candidate for the anomaly. We have not understood enough of FSM dynamics as yet to ascertain whether its candidacy is indeed viable, given the intricacy of the many bounds from other experiments which a candidate has to satisfy. However, if it is, and the anomaly itself should survive independent scrutiny, then this can be a big boost to the credibility of both, and to FSM as a whole. If confirmed, then this $G_3$, like $G$ in [c] above, will serve as a useful window into the hidden sector.

Next, we turn to the mass spectrum of the $F$. This depends on the Yukawa couplings (73), (74), and (75), which are, as said, but working models, and does not therefore deserve our confidence as much as the preceding spectra for the $H$ and $G$. 

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Nevertheless, to proceed with the analysis, we note first that in the lists \((69), (70)\) and \((71)\), those \(F\) of type (i) which are coupled via \(\delta\) and \(\delta'\) will have very different properties from those of type (ii) coupled via \(\alpha\). Those of type (i) have mass matrix elements proportional to \(\sqrt{1 - R}\) while those of type (ii) have mass matrix elements proportional to \(\sqrt{1 + 2R}\). Further, we recall (Section 6) that the vector \(\zeta\) is supposed to rotate with scale starting from a fixed point \((0, 0, 1)\) at infinite scale, so that at high but finite scales these coefficients will be hierarchical, meaning that \(\zeta_{\alpha} \gg \zeta_{\delta}, \zeta_{\delta'}\).

Let us consider first the three \(F\) of type (ii). They will have mass matrix elements of large finite values at high scales, decreasing logarithmically by virtue of renormalization with decreasing scales, giving thus case \((C1)\) solutions for their physical masses of order TeV in close parallel to the \(H\) and \(G\) studied above. The six \(F\) of type (i), on the other hand, will start at infinite scale with zero mass matrix elements because of the vanishing there of \(\zeta_{\delta}\) and \(\zeta_{\delta'}\). These elements may increase as the scale lowers and \(\zeta\) rotates, but only slowly, if at all, since the overall factor \(\zeta_{S}\) decreases. However, the mass matrix elements have to vanish again as the scale nears 17 MeV because of the factor \(\sqrt{1 - R}\) they carry. It is thus not obvious whether the matrix elements will ever match the scale and give solutions to the physical masses. In other words, it is unclear whether the \(F\) of type (i) will be the case \((C3)\) or the case \((C2)\) as above listed. For the charged co-quarks of \((69)\) and charged co-leptons of \((70)\), however, it had better be the case \((C3)\), or otherwise one would have light charged co-quarks and co-leptons with masses of order 17 MeV. This would be unacceptable because any charged particle at this mass would be unlikely to have escaped experimental detection. Thus, if this should happen, then we would have to give up the model of Yukawa couplings \((69)-(71)\) altogether and construct another. For the two co-neutrinos of type (i) in \((71)\), however, case \((C2)\) solutions are not ruled out.

Ignoring then, tentatively, the \(F\) of type (i), there remain only the three states of type (ii) to consider: the co-quark with charge \(-\frac{1}{2}\) of \((69)\), the co-lepton with charge \(-1\) of \((71)\), and the co-neutrino of \((70)\), with coefficients \(Z\) (and hence also masses) which, we recall, are likely to be hierarchical, presumably in that order according to \((RT)\). For the co-neutrinos, however, there is the possibility of the following added complication:

- [e]

These states, which have zero charges, can form Majorana-type mass terms (Section 7.2) as right-handed neutrinos do in the standard sector.
If so, then they may be subject to a see-saw mechanism, which may give them lower (perhaps very much lower) masses than the TeV scale originally suggested. (We recall also that by \[RT\], co-neutrinos are likely to start off in any case with a much lower value for Z.) Whether this is indeed the case, and what physical masses they will end up with, will be crucial to the question to be discussed in Section 10, namely whether \(H_{\text{low}}\) and \(G_{\text{low}}\) are stable and be part of the dark matter, or whether they will decay into co-neutrino-anti-co-neutrino pairs and thus be absent in the universe today.

Taken all together then, the above arguments suggest the spectrum for \(F\) listed in Table 4

- \([f]\) The charged co-quarks in Table 4 would give a step rise of size \(\frac{1}{4}\) in \(R = \sigma_{\text{total}}/\sigma_{\text{elastic}}\) in \(e^+e^-\) collision, while the charged leptons would give a step rise of 1, but both only at centre-of-mass energies of order TeV, which are, however, much beyond the reach of all planned colliders at present.

- \([g]\) There is, however, one question about the particle spectrum which we cannot answer at present, and which arises as follows. Among the fundamental fermion fields listed in (72) above, there is one which stands out, namely \(\psi_L(\frac{1}{6}, 2, 3)\), which, unlike any other, carries both flavour and colour. According to the above treatment, it combines with the flavour framon to form a quark, and with the colour framon to form a co-quark, neither of which can propagate in free space, the quark being still coloured and the co-quark, flavoured. A quark can combine with other quarks and antiquarks via colour confinement to form hadrons. In parallel, we expected that co-quarks would combine with other co-quarks and anti-co-quarks to form co-hadrons. But cannot a quark combine with a colour anti-framon via colour confinement, or equivalently, a co-quark combine with a flavour anti-framon via flavour confinement, to form a doubly framonic, simultaneous \(B\)-on and \(C\)-on? Let us say, symbolically \(\Phi^d\Phi^d\psi_L(\frac{1}{6}, 2, 3)\), which, for want of anything better at present, we may call tentatively a “lepto-\(F\)”. Conceptually, there does not seem to be any principle against “lepto-\(F\)”s, but we are at a loss to ascertain whether they ought to exist, or to speculate on their properties, since the tools we have been using for the other particles do not seem to extend easily to this class of objects. Intuitively, one
would guess that they are point-like, like framonic $B$-ons and $C$-ons, but perhaps even more so, being doubly framonic. They will probably be hard to form in the early universe, because, like baryons, they are trinary objects, but much smaller in size, and so they may be rather rare in nature. But if they exist, they may disturb the picture we have built above of two parallel worlds, one of framonic $B$-ons and one of framonic $C$-ons, for, being doubly framonic they would belong to, and communicate with, both sectors, hence breaking the dichotomy, and playing a possibly crucial role in the FSM framework.

Unable at present to determine whether “lepto-$F$” may or may not exist, we shall work on the hypothesis that even if they do, they will have such a high mass, or else interact with the rest so weakly, that the dichotomous picture we have been developing will still hold to a good approximation, and leave the problem to be sorted out, we hope, in the future.

If the “lepto-$F$” does exist, then its right-handed component, being a bound state of the flavour framon via flavour confinement with the right-handed $F$, $\psi_R(-\frac{1}{2}, 2, 1)$, is exactly the answer to the question posed in Section 5 at the end of the paragraph after equation (72). That being the case, the chirality puzzle $[\text{CH}]$ simplifies with (72), since we require now only the single field $\psi(\frac{1}{6}, 2, 3)$ (but no longer $\psi(-\frac{1}{2}, 2, 1)$) to be purely left-handed, which might, in the long run, be a little easier to explain.

### 9 Interactions of $H$, $G$ and $F$

The situation as regards the tree-level couplings of the $H$, $G$, and $F$ as derived from the FSM action is summarized at the beginning of Section 6. The details of all these couplings are rather complicated, as are seen...
especially in Appendices A and B, which would imply a lot of complexity in the interactions of framonic C-ons among themselves. They may be of interest in the future, but need not bother us at present. One point to note, however, is that despite all these myriad interactions, our derivations have not revealed any couplings linking the \(H, G,\) and \(F\) directly to the quarks and leptons with which we do our experiments. Indeed, the only couplings we found which link these framonic C-ons to the framonic B-ons which constitute our standard sector are the few in Appendix A proportional to \(\nu_1\) and \(\nu_2\) linking the \(H_K\) to the \(h_W\).

How then can the two sectors communicate? As answers to this question, we can identify only the following.

- **[bc1]** A framonic C-on can interact with a framonic B-on, if both are charged, by exchanging a photon, the photon being mostly the \(u(1)\) gauge boson \(A_\mu\) and this couples to both sides. But they cannot interact by exchanging either a flavour or a colour gauge boson since a framonic B-on is, by definition, neutral in flavour and a framonic C-on, neutral in colour. Furthermore, they cannot easily interact by exchanging one of their own members, given the paucity of direct couplings linking the two groups; unless the particle exchanged happens to be a mixed state which is partly a B-on and partly a C-on. Of such, we have seen above 4 examples, namely \(h\) and \(H\) which are mixtures of the usual (framonic B-on) Higgs \(h_W\) with the framonic C-on \(H_{\text{high}}\), plus \(Z\) and \(G\), which are mixtures of \(\gamma\), the framonic B-on \(Z\) (of SM) and the framonic C-on \(G_8\), as described in Sections 4 and 7. These are the only examples of mixed states we know, hence

- **[bc2]** A framonic C-on can interact with a framonic B-on by exchanging the mixed vector states \(Z, G\) or the mixed scalar states \(h, H\).

Further,

- **[bc3]** In the case where one of the interacting particles is \(h_W\) or an \(H_K\), then the interaction can also go by exchanging \(h_W\) or an \(H_K\) via the couplings proportional to \(\nu_1\) or \(\nu_2\) of Appendix A.

But these are all we have found, and it does not amount to very much. Taken at face value then, it would seem that, despite each sector having plenty of interactions within itself, the sector composed of framonic B-ons and the
sector composed of framonic C-ons have but little communcation with each other. As we shall see, this will go some way towards explaining why the framonic C-on sector containing the $H$, $G$ and $F$ should appear “hidden” to us as our sector containing quarks, leptons and us should appear “hidden” to them.

However, this is so only if we assume that the $H$, $G$ and $F$ have no soft interactions, as argued in the Introduction, for if there were such soft interactions with strength much superior to the hard interactions derived perturbatively from the fundamental action, then they would dominate over the latter, making the suggestion of the sector being “hidden” completely irrelevant.

The centre of our attention is thus shifted to the question whether framonic C-ons have soft interactions or not. In the Introduction, we have outlined already an argument suggesting that framonic C-ons have no, or at least very little, soft interactions. Here we shall just flesh out this argument some more, although we shall still be very far from clinching it.

In the Introduction, we started by asking the question why hadrons have soft interactions while the framonic B-ons (quarks, leptons and so on) seem to have none. We say that framonic B-ons have no soft interaction because they all appear point-like to the furthest extent that we have probed, unlike hadrons which we have already seen to be bulky, that is, about a fermi in size. But are we sure that the B-on interactions we do see, which we have ascribed to the hard couplings derived before are not partly soft interactions in disguise? Let us take the decay of the standard Higgs boson $h \rightarrow \ell^+\ell^-$ as an example. This can arise via the standard Yukawa coupling (“hard”) but it can also arise by the framon constituents of $h$ separating and recombining with a fundamental fermion-antifermion pair emerging from the sea as pictured in Figure 1(b) (“soft”). Can the two processes be distinguished from one another? In this case, the answer is “yes”, and quite easily. We recall that the Yukawa couplings of $h$ to fermions depend critically on the fermion mass. In the SM they are simply proportional to the mass, and in the FSM they come from the rotation of $\alpha$, but in either case, the branching ratios satisfy $\Gamma(h \rightarrow \tau^+\tau^-) \gg \Gamma(h \rightarrow \mu^+\mu^-) \gg \Gamma(h \rightarrow e^+e^-)$, recalling that the physical $h$ in SM is $h_W$, and in FSM it is mostly $h_W$. This condition is very well satisfied by experiment, where $h \rightarrow \tau^+\tau^-$ is relatively copious, but $h \rightarrow \mu^+\mu^-$ has not been seen, let alone the mode into $e^+e^-$. (See also Section 7.1 above.) On the other hand, if the decay had occurred via soft interactions as pictured in Figure 1(b), one would expect instead from soft
hadron phenomenological lore that the light decay products be favoured over
the heavy ones. For example, we have from the PDG tables [16] the following:

\[
\begin{align*}
  f_2(1270) & : \quad r = \frac{84.8}{4.6} = 18.4 \\
  \rho_3(1690) & : \quad r = \frac{23.6}{1.58} = 14.9 \\
  f_4(2050) & : \quad r = \frac{17.0}{0.68} = 25.0
\end{align*}
\]

for the ratio \( r = (BR : X \rightarrow \pi\pi)/(BR : X \rightarrow KK) \). In all three typical
soft decays, the light products (\( \pi\pi \)) are favoured over the heavy ones (\( KK \)),
that is, directly opposite to what was seen above for \( h \) decay. It would thus
seem that if there is any soft component at all in \( h \) leptonic decay, it has to
be very weak.

It has already been suggested in the Introduction that this difference
between hadrons being dominated by soft interactions and the framonic \( B \)-
ones having apparently none might be ascribed, not to possible differences in
the soft dynamics between colour \( su(3) \) and flavour \( su(2) \), but to the fact
that hadrons are not framonic while the framonic \( B \)-ons are. It was thus
argued that, for example, the framon and antiframon constituents of the \( h \)
are so short-lived that they will have no time to seek out and recombine each
with an alternative partner from the sea to effect a soft decay. If that is
the case, then the same argument when applied to the \( H, G \) and \( F \) would
suggest that they too will have little soft interactions and remain point-like,
just as framonic \( B \)-ons do.

But do we in fact know enough of the parameters of the model to make estimates of the framonic life time and the recombination time in soft decays so as to make the above assertion creditable? An estimate of the recombination time, that is, the time for a quark to find and recombine with an antiquark coming from the sea, can be estimated from the typical widths of
hadronic resonances, say of order 100 MeV, corresponding to a life time of
\( \sim 7 \times 10^{-24} \) s. The life time of the colour framon is supposedly given by
\( \sqrt{\mu_S} \), which parameter is as yet unknown but its flavour analogue \( \sqrt{\mu_W} \) is.
Using the long known value 246 GeV for the vacuum expectation value of
the Higgs scalar field, and the more recently known value 125 GeV of the
Higgs mass, one obtains \( \sqrt{\mu_W} \sim 88 \) Gev, corresponding to \( \sim 7.5 \times 10^{-27} \) s.
Thus, if \( \mu_S \) is anywhere near the same order as \( \mu_W \), one would conclude that
the chance of a coloured constituent from a framonic \( C \)-on combining with
a quark partner from the sea within the framonic life time so as to effect a
soft decay for the framonic \( C \)-on would indeed be very small.
So far we have only given examples in soft decays, but similar arguments would apply also to suppress other soft interactions for framonic C-ons. Let us take for example:

- **S1 No nuclear-like forces between framonic C-ons?** Nuclear forces between two nucleons, at least at long range, are generally thought to arise from one-pion exchange, which may be depicted as in Figure 3. This we interpret as a nucleon splitting off a quark which then combines with an antiquark from a quark-antiquark pair coming from the sea to form a pion, while the remaining diquark combines with the other quark and goes off. And then the pion gets exchanged to the other nucleon and do the reverse. The analogue for a nuclear-like force between two $H$s would be one depicted by Figure 4 with the two $H$s exchanging another $H$. This, one sees, will require the framon constituents in one $H$ to split and recombine with framons created from the sea, which we already claimed above to be very unlikely. The process can go, of course, via the hard couplings listed in Appendices A and B, but these latter will not have hadronic or nuclear strength.

- **S2 No framonic jets in hadron collisions?** A quark jet is supposedly produced in hadron collisions by hard collisions knocking off from the hadron a quark which then hadronizes, meaning that it combines with other quarks from the sea to form other hadrons which then emerge. In a similar way, a framon constituent (if any) in the original hadron can be knocked off by a hard collision, but our previous argument would suggest that it will not live long enough to find other partners from the

Figure 3: Quark diagram of one-pion exchange giving nuclear force.
Figure 4: Hypothetical quark-like diagram of one $H$-exchange giving “soft” interaction between two $H$s.

sea to form new hadrons and to emerge as a jet. Conversely, a quark knocked off by the hard collision will also find it hard to find a framonic partner from the sea to emerge as an $F$, the framon from the sea not being long-lived enough. Hence, we argue, there will be no framonic jets either way.

• **S3 No change of $R$ in $e^+e^-$ collision at framon-antiframon threshold?** The colour framon being charged, the $e^+e^-$ can create via the intermediate photon a framon-antiframon pair and so might seem to affect $R$. However, the framons, being coloured, have to combine with some other coloured object to form a colour singlet before it can emerge in the final state. Hence, by the same argument as in **S2** above against hadronization, we claim that it will not have time to do so. Besides, in the parallel flavour case, the coupling of the photon to a flavour framon-antiframon pair has also no effect on $R$. There, a change in $R$ occurs only at the thresholds of framonic $B$-on pairs such as $q\bar{q}$ or $\ell\bar{\ell}$. So also here, one would expect changes in $R$ only at the production thresholds of the framonic $C$-ons such as $Q\bar{Q}$ and $L\bar{L}$ as suggested in Section 8 [e].

If one accepts the above suggestion that framonic $B$-ons and $C$-ons have no or little soft interactions, one is led to two, at first sight, rather astonishing conclusions. First,

• **[a]**
A quite revolutionarily new answer to the old question why strong interactions are strong and weak interactions are weak. When we look at the actual values of the flavour and the colour gauge couplings $g_2$ and $g_3$, there seems to be not that much between them for size. Why then should one find such disparity between the strengths of the weak and strong interactions? It used to be that one can give as reason that flavour is spontaneously broken and colour confined, but if we accept 't Hooft’s confinement picture for the electroweak theory, this distinction is removed. The new answer instead would seem to be that the particles we know in the flavour sector, that is, again the Higgs, $W^\pm, \gamma - Z^0$, quarks and leptons, are all framonic, and those particles we know in the colour sector, that is, mesons and baryons, are all non-framonic, and in both sectors, non-framonic bound state can have soft interactions but framonic bound states have none, and soft interactions are strong but hard interactions are weak.

Conclusion [a] restores the parity between the flavour and colour sectors as far as the strengths of interactions are concerned, but begs, of course, the equally intriguing question why one has seen in nature so far only framonic $B$-ons and the nonframonic $C$-ons which are the hadrons. To answer this question, let us first correct an inaccuracy in it. When we said that hadrons are nonframonic as $C$-ons, we only meant that they contain no colour framons as constituent. Hadrons are made up of quarks and antiquarks but these, in the confinement picture, are themselves compound states of the flavour framon with fundamental fermion fields, confined by flavour. Hadrons are thus, in this language, second-level constructs made by colour confinement out of the more basic and point-like framonic $B$-ons, quarks and antiquarks. We note that they are not made out of just the fundamental fermions themselves, presumably because hadrons are relatively large objects, and at that size-scale, the fundamental fermions are already confined by flavour with flavour framons to form the point-like flavour singlet states quarks and antiquarks. In parallel, one expects framonic $C$-ons, if flavoured, to combine also via flavour confinement to form co-hadrons which are analogues of hadrons, only with the role of flavour and colour interchanged. And these, like hadrons, are in a sense only second-level constructs made out of framonic $C$-ons and are presumably also fairly bulky objects. In this language then the question asked above simplifies to merely why we have seen in experiments so far only framonic $B$-ons but not framonic $C$-ons. And this is answerable by the
second astonishing conclusion already mentioned, namely

- [b] That, with little interaction between framonic B-ons and framonic C-ons, the latter may well exist in abundance without us humans made out of the former being immediately aware of it. Whether that is indeed the case, or in other words whether we can answer the question (Q) or (Q’) posed at the beginning, has to be postponed until we have considered what framonic C-ons are likely to occur naturally in the world today. But if it is, then we would end up with the picture given in the Introduction of two worlds which are not communicating much with each other, although each is as complex and interactive within itself as the other. We choose to call ours (the framonic B-on world) the standard sector, but the other (the framonic C-on world) the hidden sector, only because we ourselves happen to be composed of framonic B-ons. This lack of communication between the two sectors is not absolute. There are the interactions [bc1]—[bc3] already listed which can be used by us to probe the hidden sector. Besides, the suppression of soft interactions in framonic B-ons and C-ons as argued is not due to any conservation law or selection rule, only to lack of probability. This means that these soft interactions may still occur though generally at a very low rate, that is, except in unusual circumstances as could exist in, for example, the early universe.

10 A speculative survey of the hidden sector

Let us try next to imagine what the world would look like if there were indeed such a “hidden” sector in the particle spectrum. This would be instructive, we think, even though, given the flimsy knowledge we have at present, any picture our imagination can produce is bound to be inaccurate and incomplete.

In the very early universe when it was still very hot and dense and when presumably both flavour and colour were deconfined, the fundamental particles including framons of both types would be swimming around free. When the universe began to expand and cool, however, these particles would be looking for appropriate partners to form flavour and colour neutral objects so as to survive into the next epoch when both flavour and colour would be
confined. The first objects to form, it seems, would be the tightly bound (in the sense of appearing point-like at scales familiar to us) framonic $B$-ons and $C$-ons, and later the more loosely bound hadrons and co-hadrons, these being bound states by respectively colour and flavour confinement of the coloured framonic $B$-ons (quarks) and flavoured framonic $C$-ons (co-quarks). It is interesting—perhaps even sobering—to note that baryons, which are destined to become the staple of our standard sector, would appear to be late-comers and of a rather rare occurrence in this formation, requiring as they would the concurrence of three objects of the appropriate combination of colours. It would be rare compared with almost all the other particles, namely the framonic $B$-ons (for example leptons), $C$-ons (for example $H$, $G$ and $F$) and even the co-baryons, which require the concurrence of only two objects of the appropriate flavour or colour. This observation may go towards an eventual understanding of why dark matter makes up the bulk of our material universe.

As soon as formed, some of the composite particles would start to decay. In the standard sector, the scenario is familiar. All the unstable particles would have decayed away by our epoch, leaving only the stable few: the photon, the neutrinos, the electron, the proton, and on some occasions the neutron too if it happens by luck to be bound up in a stable nucleus.

What would happen in the hidden sector? There, besides the already familiar $H$, $G$ and $F$, we should take account of the co-hadrons too. These are of two general types, co-mesons made up of a co-quark and an anti-co-quark, thus $Q_r\bar{Q}_r$, and co-baryons made up of two co-quarks $\epsilon_{rs}Q_rQ_s$. We note that unlike ordinary baryons, the co-baryons are bosonic, not fermionic. Like ordinary hadrons, however, co-hadrons are probably bulky objects, liable to soft interactions, and likely to exist in many excited (resonance) states. The excited states, being short-lived, would have decayed quickly via soft interactions into the ground state, so that by now we have only these latter to contend with. We do not know whether the $H$, $G$ and $F$ exist also in excited states. For their counterparts in the flavour sector, that is, the framonic $B$-ons, no excited states are known, but this may only mean that they are too high in mass to have been detected. In any case, even if excited states for the framonic $C$-ons exist, they can all easily decay into the ground states by emitting $H_{\text{light}}$ and $G_{\text{light}}$, namely those with masses $\sim 17$ MeV, and into $F_{\text{light}}\bar{F}_{\text{light}}$ pairs via many of the couplings listed in Appendices A, B and Section 5, leaving again only their ground states to be considered. Let us then go through the remaining ground state particles in the hidden sector
and see what one might expect to happen to them.

Let us start with the heaviest states $H^0$ and $G^0$ in Tables 2 and 3. Both of these can easily decay into $H_{\text{light}}$ and $G_{\text{light}}$ via the couplings listed in Appendices A and B, and also into ordinary particles via their mixing respectively with $h_W$ and $\gamma - Z$, so that by now no such particles would remain. Next, consider the charged $H$ and $G$ which, we recall, are still of order TeV in mass. Those of opposite charges would have attracted one another and annihilated into $H_{\text{light}}, G_{\text{light}}$, and even into ordinary light particles, but some might have survived. These survivors, however, could not decay further into $H_{\text{light}}$ or $G_{\text{light}}$ for these latter are neutral. Like their $B$-on counterparts, they are more likely, if the masses involved permit, to decay into fermion-antifermion pairs, for example into a light co-neutrino plus a heavy charged anti-co-lepton, or vice versa. If this decay is permitted, then again no heavy charged $H$ and $G$ would remain. Of the remaining light states in Table 2, $H_{\text{odd}}$, can decay further into $e^+e^-$ by mixing with $h_W$ via a framon loop and disappear, and so would $G_3$, one of the $G_{\text{light}}$, by coupling to $\gamma$ via a loop. The rest would be stable unless co-neutrinos can acquire a mass lower than 8 MeV via a see-saw mechanism, as envisaged in Section 8 [e], in which case none of the $H$ and $G$ would survive.

We recall that, partly by choice, all charged $F$ are heavy (that is, with masses of order TeV) while all light $F$ (that is, with masses $\leq 17$ MeV) are neutral, so that unless there are other couplings than those we have derived from the action above, there is no way for the heavy charged $F$ to decay. As pointed out in Section 5, co-quarks and co-leptons have no equivalent to the generations of quarks and leptons, so that there are no lighter charged states for them to decay into via normal hard couplings derivable from the action. However, given that the suppression of soft interactions suggested in Section 9 is not meant to be absolute, it is possible that some residue of soft interaction would allow the charged co-quark and co-lepton to decay into ordinary charged mesons (e.g. $\pi^\pm$) plus co-neutrinos. We think that is likely, although we have not been able to work it through. If not, then some heavy co-quarks and co-leptons would remain, forming co-protons and perhaps even co-hydrogen atoms. We do not know whether this latter scenario has any chance at all of being viable.

However, independently of what happens to the charged co-quark and/or co-lepton, the light $H$, light $G$ and light co-neutrino will be there in abundance, both from formation in the early universe, and as decay products from the heavier states as described. And these, being neutral, cannot inter-
act with our standard sector via the exchange of a photon \([bc1]\). They can, however, interact with particles in the standard sector via \([bc2]\) by exchanging a \(Z\) as neutrinos do, except that the interaction rate will be suppressed compared with that for neutrinos by the smallness of the component of \(Z\) in \(G^8\) which is what allows the \(Z\) to couple into the framonic \(C\)-on sector. Now this mixing element has already been calculated in \([9]\) and given there (in equations (28), (60)) a value of around 0.16, for \(\zeta_S\) at its smallest permitted value of about 2 TeV. This means that the light \(H\), light \(G\) and light co-neutrinos are all estimated to interact with ordinary matter via \(Z\)-exchange with cross sections of no more than a few percent of that for neutrinos. They can interact with the standard sector also by exchanging the other mixed vector state \(G\), but the cross section will be even smaller, because \(G\) has both a higher mass and a smaller component in \(Z\). Further, the light \(H\), light \(G\) and light co-neutrinos can interact with the standard sector via \([bc2]\) also by exchanging an \(h\) or an \(H\), of which the mixing is less known though similar. But these interactions can be safely neglected compared to \(Z\)-exchange, since they all involve the \(h_W\)-coupling to light quarks and leptons, which are all that we have naturally occurring at present in the standard sector. The same can be said as well for the interactions of the light \(H\) via \([bc3]\) by exchanging \(h_W\). Hence, having gone through the already limited list in Section 9 above and found no interactions with our sector of significance, we suggest that the light \(H\), the light \(G\) and the light co-neutrinos may all qualify as candidates for dark matter. They are light, of mass \(\leq 17\) MeV, but very numerous compared with baryons as already noted, and could thus make up an appreciable component of the missing dark matter, although without further investigation one cannot ascertain whether they could make up the bulk of it. But, being light, they would have escaped detection by current dark matter experiments which gives stringent bounds on heavy dark matter particles but almost no constraint on dark matter particles in the mass range of our concern \([27]\).

This scenario suggested by the spectra for \(H\), \(G\), and \(F\) listed in Tables \([2]–[4]\) would seem to give a tentative answer also to the question \(Q\) or \(Q'\) posed at the beginning. The heavy states are supposedly unstable, so that none would occur naturally. Their high masses and meagre interactions with the standard sector mean that they would not have been produced easily by our experiments to-date. On the other hand, the stable, low mass states which remain are neutral and barely interacting with us, and so the whole sector could so far have escaped our notice altogether (except, of course,
through their gravitational effect). However, if the estimate of $\zeta_S \sim \text{TeV}$ is anything to go by, one may be starting soon to catch glimpses of this new sector populated by framonic $C$-ons which has hitherto been hidden from our view.

11 Concluding summary

The material dealt with in this paper, being well beyond the original physics for which the FSM was first constructed, and still mostly at an exploratory stage, does not admit yet of a proper conclusion, in lieu of which, therefore, only a summary will be given of what seems so far to have emerged.

11.1 Points of experimental and phenomenological interest

Reproduction of standard model results

We start by listing what the FSM has done previously [1] in reproducing standard model results.

- [i] It gives the standard model Higgs boson as part of the flavour framon.
- [ii] It gives three generations (each) of quarks and leptons, with generations arising as the global dual of the local colour symmetry.
- [iii] It reproduces the hierarchical mass spectrum for the 3 generations of quarks and leptons.
- [iv] It reproduces the CKM mixing matrix for quarks including the Kobayashi-Maskawa CP-violating phase.
- [v] It reproduces the 3 mixing angles in neutrino oscillations, that is, the PMNS matrix for leptons except a possible CP-violating phase.

Of these, [iii]—[v] come about as consequences of the quark and lepton mass matrices rotating with scale, which, as shown in Section 6, is itself a consequence of renormalization under framon loops. As a result, the number of empirical parameters is much reduced compared with the standard model (17 SM parameters replaced by 7 for FSM).
Deviations from the SM in the standard sector

When probed deeper, the FSM reveals deviations (not always small) from the standard model, among which are (Section 7.1) the following,

- [vi] Departures from SM in rare decays, such as the suppresson of $h \rightarrow \mu^+\mu^-$ and the likely occurrence of some flavour-violating modes such as $h \rightarrow \tau\bar{\tau}$.

which were suggested some time ago [14, 15] but may soon be detectable. These deviations, like [iii]—[v] above, are consequences of the scale dependence derived in Section 6. But there are deviations from SM which are more fundamental (Section 7.3):

- [vii] Departures from SM in the electroweak mixing scheme: for example, $m_Z - m_W, \Gamma(Z \rightarrow \ell^+\ell^-), \Gamma(Z \rightarrow q^+q^-)$, all slightly different from that predicted by SM.

For the vacuum expectation value of the colour framon $\zeta_S \geq 2 \text{ TeV}$, these deviations are all within the present experimental errors [9] but they should show up when experimental accuracy is improved.

Glimpses of the hidden sector

Framonic C-ons in the hidden sector are supposed to communicate little with our standard sector except via the photon coupled to charges on either side, and via the mixing of their $G$ with our $Z$ and the photon (Section 4), and the mixing of their $H_{\text{high}}$ and $H_{\text{even}}$ with the standard Higgs $h_W$ (Sections 3 and 8). But through these few chinks, one can deduce the following:

- [viii] Possibly at LHC, a $\ell^+\ell^-$ bump at the invariant mass of $G$ and a diphoton bump at the invariant mass of the $H$, again both in the TeV range (Section 8 [b]).

- [ix] Atomki-like anomalies [23] at around 17 MeV (Section 8 [d]).

- [x] Step increases in $R(e^+e^-)$ at production thresholds of $Q\bar{Q}$ (co-quark, anti-co-quark), $LL$ (co-lepton, anti-co-lepton) (Section 8 [f]), and a peak in $R(e^+e^-)$ at the $G$ mass (Section 8 [c], [9]), all in the TeV range, unfortunately beyond that of any collider being planned.
Within the hidden sector itself

Most heavy framonic C-ons would have decayed away by our epoch leaving only the lowest as dark matter candidates.

- [xi] These dark matter candidates are light, of masses ~17 Mev, and can be both bosonic and fermionic if no see-saw mechanism occurs for co-neutrinos, but can be of masses < 8 MeV and all fermionic, if there is see-saw for co-neutrinos. They are estimated to have cross sections with ordinary matter of the order of a few percent of neutrino cross sections.

Though light, these dark candidates can occur in abundance and make up an appreciable portion of the missing dark matter, though not necessarily the bulk of it (Section 10).

11.2 Points of conceptual and theoretical interest

[a] Parallel between the flavour and colour sectors

Perhaps the most striking feature of the FSM as here formulated is the close parallel in structure between the flavour and colour sectors. One is used to thinking of the flavour sector as describing what are called weak interactions, and of the colour sector as describing what are called strong interactions. The physics one sees in weak and strong interactions are very different, and one would have expected the theories governing these two types of interactions to be very different too. This is true in the usual SM formulation where the the flavour $su(2)$ symmetry is spontaneously broken, while the colour $su(3)$ symmetry is confining and exact. However, the confinement picture of ’t Hooft for the electroweak theory adopted in this paper changes all that, for one can now regard the flavour theory as confining also, in parallel with the colour theory, which parallel is further accentuated in the FSM, where both theories are framed as well. It might therefore appear mysterious how so very different physics in the two sectors can emerge.

Some of these differences are traced to the difference in basic properties between the flavour $SU(2)$ and colour $SU(3)$ groups. As pointed out in Section 2, the number of independent framons in the flavour symmetry can be reduced by imposing the condition $[4]$ while a similar reduction is not available for the colour framon without changing the physical dimension of
some of its components. And this difference is traced directly to the special property of the $SU(2)$ group being embeddable in $\mathbb{R}^4$ (Footnote 3).

Now the difference just noted between flavour and colour is the source in the FSM of some great differences in the physics outcome for the two sectors. In Section 3, it is this reduction in the number of flavour framons which makes the factor $\beta$ coming from the colour framon $(CF)$ in (3) disappear from the framon potential (9), while the corresponding factor $\alpha$ coming from the flavour framon $(FF)$ in (2) remains. As a result, the FSM vacuum depends on $\alpha$ but not on $\beta$, and makes the vector $\alpha$ rotate with changing scales, while $\beta$ is unchanged. It is this rotation of $\alpha$ which leads in [1] to three generations of quarks and leptons with hierarchical masses, to CKM mixing for quarks, and to neutrino oscillations for leptons. But, because of the above, there is no equivalent to all these in the $F$ spectrum.

These observations, however, still do not explain the following:

[b] Why flavour interactions appear weak while colour interactions strong

By this, what we really mean is why hadrons which occur in the colour theory interact strongly while particles occurring in the flavour theory, such as the leptons, the vector and Higgs bosons do not. It is not that the hadrons do not have interactions similar to those of the “weak” particles. They do, and these their “hard interactions” are the subject of study of perturbative QCD. The real difference is that apart from these hard interactions, hadrons have in addition strong “soft interactions” which cannot be derived perturbatively from the fundamental Lagrangian. Instead of ascribing this difference to the possible difference in dynamics between the flavour and colour symmetries, as one may be tempted to, the FSM suggests that the difference stems instead from the difference in structure between the “weak” particles and hadrons, namely that the former are framonic while the latter are not, as set out in Section 9. And the framon, being short-lived—the argument goes—will find it hard to separate from its partners in framonic states to recombine with alternative partners from the sea, as is needed to effect a soft interaction. Admittedly, the arguments presented there are no better grounded theoretically than the previous assumption of different dynamics operating for the flavour and colour symmetries. But we think that the hypothesis is worth entertaining in that it restores the parallel observed above between the flavour and colour sectors, while opening a door to a possible hidden sector rich in structure for us to explore.
As to the question whether soft interactions for framonic composites, be they $B$-ons or $C$-ons, are suppressed or not, it is not easily settled definitively since it requires an understanding of nonperturbative effects which we do not have. The only tool one has at present for probing non-perturbative physics is by lattice calculations, and we very much hope that experts in this field might consider throwing some light on to the matter.

[c] **Strong and weak CP-violations identified**

The observation in Section 9[b] separates physics according to the FSM into what we call the standard and the hidden sectors. In the standard sector, FSM has reproduced the results listed in [i]—[v] in the preceding subsection. This does not end with just numbers but make conceptual changes as well. For example, geometrical significance has been given to the Higgs boson as a framon, and to generations as the dual to colour; the kaleidoscopic mass and mixing patterns of quarks and leptons are not just accidents of nature but some at least have a dynamical origin. But these have already been much discussed in our earlier work and need not be repeated. We recall only one point, namely that FSM offers a solution of the strong CP problem without axions but links it instead via the mass matrix rotation to the Kobayashi-Maskawa phase in the CKM matrix. This we think worth emphasizing again, now that the axion is proving elusive to experimental searches.

Traditionally, in the standard model, CP-violation for quarks can come from two sources:

- (i) the Kobayashi-Maskawa phase admissible in the $3 \times 3$ quark mixing matrix,

- (ii) the CP-violating theta-angle term in the strong Lagrangian allowed by gauge invariance [28].

That the phase (i) is admissible is just a property of a $3 \times 3$ unitary matrix, but the standard model gives no physical reason for its presence nor an estimate for its size. That the theta-angle term (ii) is allowed by gauge invariance is a bit of an embarrassment for the standard model, given that the natural choice of order unity for this theta-angle would give CP-violations for strong interactions many orders larger than acceptable to experiment. The FSM scheme interestingly identifies the two problems, exploiting the existence of zero eigenvalues in a rank-one quark mass matrix and using a well-known loop-hole in the problem in this case to transform away the theta-angle term,
avoiding thus gross CP-violation in strong interactions, while making use of rotation in the mass matrix to transmit the effect into the CKM matrix to predict a KM phase of the right magnitude \[29\]. It has, in a sense, killed two birds with a single stone, solving, on the one hand, the strong CP problem (without axions) and on the other the weak CP problem by supplying a raison d’etre and even an estimate for the KM phase.

\[\text{[d] New scale at order 10 MeV}\]

In the fit to data summarized in Table \footnote{1} one obtains a trajectory for \(\alpha\) which passes through \(\theta = 0\) at a scale of around 17 MeV, at which point its normal curvature changes sign, while the following ratio of parameters \(R = \nu_2\zeta_W^2/2\kappa_S\zeta_S^2\) goes to the value 1, and stays there when the scale lowers further. The actual value of \(\mu\) where this happens, given in \footnote{1} as 17 MeV, depends on details but the outcome that there is such a point does not seem to. From this, we recall, one has obtained the following quite significant consequences:

- that \(m_u < m_d\), on which crucial empirical fact depends the stability of the proton and hence our own existence,
- that there are possibly Atomki-like anomalies as listed in \(\text{x}\) of the preceding subsection,
- that there are likely to be dark particles at or below that mass, as listed in \(\text{xi}\) of the preceding subsection.

Superficially, \(R \to 1\) means that the vacuum value of the colour framom in its canonical gauge \footnote{45} vanishes in 2 directions. Or if, as suggested in equation (90) of \footnote{11}, in analogy to the vierbeins in gravity, this is interpreted as the “inverse square root” of the metric in \(\tilde{\text{su}}(3)\) space, thus:

\[
(g_{\tilde{a}\tilde{b}}) = 3\zeta_S^{-2} \left( \begin{array}{ccc}
1 - R & 0 & 0 \\
0 & (1 - R)^{-1} & 0 \\
0 & 0 & (1 + 2R)^{-1}
\end{array} \right)
\]

then it would mean that generation space has collapsed at that scale from 3 into only 2 dimensions. We have not yet succeeded in understanding what this really means, nor theoretically how and why the trajectory for \(\alpha\) should have such a behaviour, nor yet why there should be such an additional scale in the problem. But it is clearly something worth understanding in future.\footnote{10}
11.3 Remark

In the above summaries of Sections 11.1, 11.2, we might have been guilty of emphasizing the positives while ignoring possible negatives. If we have done so, however, it was due merely to the common human weakness of optimism with no intention to deceive. The fact is that, having embarked on an exploration of a domain both vast and unfamiliar, much more so than we at first suspected, we quickly found our understanding and knowledge under strain, and barely equal to the task. Guided only by some distant spots of light to direct our search, we have just reported what we found, but have not had yet the sagacity or the courage, nor even the time, to dig into dark corners for things we might have missed. We have no doubt that there are many mysteries in the FSM that we have not yet understood. In summarizing now the results we find interesting, when the investigation is still far from complete, we do so in the hope that abler minds than ours might be tempted to turn their power on to the problem to give it the illumination it needs.

Appendix A. Tree-level couplings of $H_K$ and $h_W$ among themselves

In the framon potential (9) we shall omit the purely electroweak part (the first two terms), since they are identical to the standard model, and shall consider the strong framon terms and the terms linking the strong and weak framons.

In order to obtain the tree-level interactions between the fields we expand the framon fields around their vacuum values:

\[
\phi = \zeta_W + h_W
\]

\[
\Phi = \Phi_{\text{VAC}} + \delta\Phi
\]

with $\alpha = (0, 0, 1)^T$ and $\delta\Phi = \sum_{K=\pm,3} V_K H_K$.

In the subsequent expansion linear terms in the fields vanish after considering the minimum condition of the potential and we shall omit the quadratic

---

which has adopted as one ingredient our scheme of the rotating mass matrix (though not the details of the FSM here) [30] Bjorken has suggested a new scale at around the same order (7 MeV for him vs 17 MeV for us, but close enough at this juncture) which he ascribes to the Zeldovitch effect in gravity. However, we have not understood enough as yet either to concur or to disagree with his suggestion.

\[ \text{For ease of writing, we put } H_+ = \frac{1}{\sqrt{2}}(H_1 + H_2), \quad H_- = \frac{1}{\sqrt{2}}(H_1 - H_2). \]
terms that give the mass matrix of the framon fields already worked out in
the text (Section 3). In the following we shall consider the tree-level interactions of the framons among themselves, that is, cubic and quadratic terms in
the fields. In a self-explanatory notation we distinguish the following terms:

\[ V^{\text{int.}} = V_S + V_{SW} \]

Each term in turn is decomposed according to their couplings into:

\[ V_S = \lambda S V_S^{(\lambda S)} + \kappa S V_S^{(\kappa S)} \]
\[ V_{SW} = \nu_1 V_{SW}^{(\nu_1)} - \nu_2 V_{SW}^{(\nu_2)} \]

where

\[ V_S^{(\lambda S)} = (\text{Tr}[\delta \Phi^\dagger \delta \Phi])^2 + 2 \text{Tr}[\delta \Phi^\dagger \Phi_{\text{VAC}} + \Phi_{\text{VAC}}^\dagger \delta \Phi] \text{Tr}[\delta \Phi^\dagger \delta \Phi] \]
\[ V_S^{(\kappa S)} = \text{Tr}[\delta \Phi^\dagger \delta \Phi \delta \Phi^\dagger \delta \Phi] + 2 \text{Tr}[(\delta \Phi^\dagger \Phi_{\text{VAC}} + \Phi_{\text{VAC}}^\dagger \delta \Phi)\delta \Phi^\dagger \delta \Phi] \]
\[ V_{SW}^{(\nu_1)} = 2\zeta_w h_w \text{Tr}[\delta \Phi^\dagger \delta \Phi] + h_w^2 \text{Tr}[\delta \Phi^\dagger \Phi_{\text{VAC}} + \Phi_{\text{VAC}}^\dagger \delta \Phi] \]
\[ V_{SW}^{(\nu_2)} = 2h_w \zeta_w [(\delta \Phi \alpha)^\dagger \cdot (\delta \Phi \alpha)] + h_w^2 [(\delta \Phi \alpha)^\dagger \cdot (\delta \Phi \alpha)] \]

After the expansion, the interaction terms in the potential are the following.

1. Cubic terms proportional to \( \lambda S \) coming from \( V_S^{(\lambda S)} \), containing the combination of the fields \( H_I H_J H_K \).

\[ V_S^{(\lambda S)} = 4\sqrt{2}\zeta S \left[ QH_+ + \frac{P}{\sqrt{2}}H_3 \right] \left( \sum_{K=\pm,3} H_K^2 \right) \]

2. Cubic terms proportional to \( \kappa S \) coming from \( V_S^{(\kappa S)} \), containing the combination of the fields \( H_I H_J H_K \).

\[ V_S^{(\kappa S)} = 2\sqrt{2}\zeta S \left\{ QH_+^3 + \sqrt{2}PH_3^3 + 3QH_+ (H_-^2 + H_4^2 + H_5^2) \right\} + Q \frac{2P^2 + Q^2}{P^2 + Q^2} \left[ H_+ (H_6^2 + H_7^2 + H_8^2 + H_9^2) + H_- (-H_6^2 - H_7^2 + H_8^2 + H_9^2) \right] + 2H_4 (H_6 H_8 + H_7 H_9) + 2H_5 (H_6 H_9 - H_7 H_8) \]
\[ + \sqrt{2}P \frac{2P^2 + Q^2}{P^2 + Q^2} H_3 (H_6^2 + H_7^2 + H_8^2 + H_9^2) \]
3. Cubic terms proportional to $\nu_1$ coming from $V_{SW}^{(\nu_1)}$, mixing strong and weak framon fields, containing the combination of the fields $h_W^2 H_K$ and $h_W H_K^2$.

\[ V_{SW}^{(\nu_1)} = \zeta_s h_W^2 2\sqrt{2} \left( Q H_+ + \frac{P}{\sqrt{2}} H_3 \right) + 2\zeta_s h_W \left( \sum_{K=\pm,3} H_K^2 \right) \]

4. Cubic terms proportional to $\nu_2$ coming from $V_{SW}^{(\nu_2)}$, mixing strong and weak framon fields:

\[ V_{SW}^{(\nu_2)} = 2\zeta_s P h_W^2 H_3 + 2\zeta_s h_W \left[ H_3^2 + \frac{Q^2}{P^2 + Q^2} \left( H_6^2 + H_7^2 + H_8^2 + H_9^2 \right) \right] \]

5. Quartic terms proportional to $\lambda_S$ coming from $V_S^{(\lambda_S)}$, containing the combination of the fields $H_1^2 H_2$.

\[ V_S^{(\lambda_S)} = \lambda_S \left( \sum_{K=\pm,3} H_K^2 \right)^2 \]

6. Quartic terms proportional to $\kappa_S$ coming from $V_S^{(\kappa_S)}$, containing the combination of the fields $H_1 H_2 H_K H_L$.

\[ V_{SW}^{(\kappa_S)} = \frac{1}{2} \left[ H_+^4 + H_-^4 + (H_4^4 + H_5^4)^2 \right] + H_4^4 \\
+ H_+^2 \left[ 3 (H_2^2 + H_4^2 + H_5^2) + H_6^2 + H_7^2 + H_8^2 + H_9^2 \right] \\
+ 2H_3^2 \left( H_6^2 + H_7^2 + H_8^2 + H_9^2 \right) \\
+ H_2^2 (H_4^2 + H_5^2 + H_6^2 + H_7^2 + H_8^2 + H_9^2) + 2H_3^2 (H_6^2 + H_7^2 + H_8^2 + H_9^2) \\
+ 2H_4 H_5 (H_6^2 + H_7^2 + H_8^2 + H_9^2) + (H_4^2 + H_5^2) \left( H_6^2 + H_7^2 + H_8^2 + H_9^2 \right) \\
+ 4H_+ \left( H_4 H_6 H_8 + H_4 H_7 H_9 + H_5 H_6 H_9 - H_5 H_7 H_8 \right) \\
+ \frac{2\sqrt{2} P Q}{P^2 + Q^2} \left[ H_+ H_3 (-H_6^2 - H_7^2 + H_8^2 + H_9^2) + H_+ H_3 \left( H_6^2 + H_7^2 \right) \right. \\
\left. + H_8^2 + H_9^2 \right) + 2H_3 \left( H_4 H_6 H_8 + H_4 H_7 H_9 + H_5 H_6 H_9 - H_5 H_7 H_8 \right) \right] \\
+ \frac{P^4 + Q^4}{(P^2 + Q^2)^2} \left( H_6^2 + H_7^2 + H_8^2 + H_9^2 \right)^2 \]

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7. Quartic terms proportional to $\nu_1$ coming from $V_{SW}^{(\nu_1)}$, containing the combination of the fields $h_{W}^{2}H_{K}^{2}$.

$$V_{SW}^{(\nu_1)} = h_{W}^{2} \left( \sum_{K=+3}^{8} H_{K}^{2} \right)$$

8. Quartic terms proportional to $\nu_2$ coming from $V_{SW}^{(\nu_2)}$, containing the combination of the fields $h_{W}^{2}H_{K}^{2}$.

$$V_{SW}^{(\nu_2)} = h_{W}^{2} \left\{ H_{3}^{2} + \frac{Q^{2}}{P^{2}+Q^{2}} \left( H_{6}^{2} + H_{7}^{2} + H_{8}^{2} + H_{9}^{2} \right) \right\}$$

Appendix B. Tree-level couplings of the $H$ with the $\bar{C}$ (or $G$)

The component of the kinetic energy corresponding to the colour framon is given in (23). After expanding the framon fields about their vacuum values as it has been done in Appendix A the result can be decomposed according to the field content in

$$A_{KE}^{C} = K^{(2)} + K^{(3)} + K^{(4)}$$

The first term ($K^{(2)}$) corresponds to the mass matrix of the $\bar{C}_i^\mu$ and $A_\mu$ fields that has been worked out in the text (Sections 4 and 8). The remaining terms give the tree-level interactions between the $H_K$ with the $\bar{C}_\alpha^\mu$ and $A_\mu$ and will be given in the following.

We start with the term cubic in the fields and write:

$$K^{(3)} = g_3^2 \zeta_S K_1^{(3)} + g_1^2 \zeta_S K_2^{(3)} + 2g_3g_1 \zeta_S K_3^{(3)}$$

with each term given by:

$$K_1^{(3)} = \frac{1}{\zeta_S} \sum_{\alpha,\beta} \bar{C}^\alpha \bar{C}^\beta \text{Tr} \left( \lambda_\alpha \lambda_\beta \left[ \Phi_{\text{vac}} \delta \Phi + \delta \Phi \Phi_{\text{vac}}^\dagger \right] \right)$$

$$K_2^{(3)} = \frac{1}{\zeta_S} A_\mu^2 \text{Tr} \left( \Gamma \left[ \Phi_{\text{vac}}^\dagger \delta \Phi + \delta \Phi \Phi_{\text{vac}} \right] \Gamma \right)$$

$$K_3^{(3)} = \frac{1}{2\zeta_S} A_\mu \sum_\alpha \bar{C}^\alpha \text{Tr} \left( \Gamma \delta \Phi \lambda_\alpha \Phi_{\text{vac}} + \Gamma \Phi_{\text{vac}}^\dagger \lambda_\alpha \delta \Phi \right)$$
Working out the traces of the matrix fields we get the following parts.

1. Terms proportional to $g_3^2 \zeta_S$ coming from $K_1^{(3)}$, containing the combinations of fields $\tilde{C}_\mu^\alpha \tilde{C}_\mu^\beta H_K$.

\[
K_1^{(3)} = 2 \left\{ \tilde{C}_\mu^4 \tilde{C}_\mu^4 + \tilde{C}_\mu^5 \tilde{C}_\mu^5 + \frac{1}{3} \tilde{C}_\mu^8 \tilde{C}_\mu^8 \right\} \left\{ PH_3 + \frac{Q}{\sqrt{2}} (H_+ + H_-) \right\} \\
+ 2 \left\{ \tilde{C}_\mu^6 \tilde{C}_\mu^6 + \tilde{C}_\mu^7 \tilde{C}_\mu^7 + \frac{1}{3} \tilde{C}_\mu^8 \tilde{C}_\mu^8 \right\} \left\{ PH_3 + \frac{Q}{\sqrt{2}} (H_+ - H_-) \right\} + \frac{4P}{3} \tilde{C}_\mu^8 \tilde{C}_\mu^8 \\
+ \frac{4PQ}{\sqrt{P^2 + Q^2}} \left\{ (\tilde{C}_\mu^1 \tilde{C}_\mu^4 + \tilde{C}_\mu^2 \tilde{C}_\mu^5 - \tilde{C}_\mu^3 \tilde{C}_\mu^6 - \frac{1}{\sqrt{3}} \tilde{C}_\mu^7 \tilde{C}_\mu^8) H_6 \\
+ (\tilde{C}_\mu^3 \tilde{C}_\mu^5 + \tilde{C}_\mu^4 \tilde{C}_\mu^6 - \tilde{C}_\mu^2 \tilde{C}_\mu^7 - \frac{1}{\sqrt{3}} \tilde{C}_\mu^4 \tilde{C}_\mu^8) H_7 \\
+ (\tilde{C}_\mu^5 \tilde{C}_\mu^3 + \tilde{C}_\mu^4 \tilde{C}_\mu^6 - \tilde{C}_\mu^7 \tilde{C}_\mu^8 - \frac{1}{\sqrt{3}} \tilde{C}_\mu^3 \tilde{C}_\mu^8) H_8 \\
+ (\tilde{C}_\mu^4 \tilde{C}_\mu^5 + \tilde{C}_\mu^4 \tilde{C}_\mu^6 - \tilde{C}_\mu^8 \tilde{C}_\mu^8 - \frac{1}{\sqrt{3}} \tilde{C}_\mu^4 \tilde{C}_\mu^8) H_9 \\
+ 2\sqrt{2}Q \left\{ (\tilde{C}_\mu^1 \tilde{C}_\mu^4 + \tilde{C}_\mu^2 \tilde{C}_\mu^5 + \tilde{C}_\mu^3 \tilde{C}_\mu^3) H_+ + \frac{2}{\sqrt{3}} \tilde{C}_\mu^3 \tilde{C}_\mu^8 H_- \\
+ (\tilde{C}_\mu^4 \tilde{C}_\mu^6 + \tilde{C}_\mu^5 \tilde{C}_\mu^7 + \frac{2}{\sqrt{3}} \tilde{C}_\mu^4 \tilde{C}_\mu^8) H_4 + (\tilde{C}_\mu^5 \tilde{C}_\mu^6 - \tilde{C}_\mu^4 \tilde{C}_\mu^7 + \frac{2}{\sqrt{3}} \tilde{C}_\mu^2 \tilde{C}_\mu^8) H_5 \right\} \right\}
\]

2. Terms proportional to $g_1^2 \zeta_S$ coming from $K_2^{(3)}$, containing the combinations of fields $A^2_\mu H_K$.

\[
K_2^{(3)} = \frac{2}{9} A^2_\mu \left( 4P H_3 + Q\sqrt{2} H_+ \right)
\]

3. Terms proportional to $g_1 g_3 \zeta_S$ coming from $K_3^{(3)}$, containing the combinations of fields $A_\mu \tilde{C}_\mu^\alpha H_K$.

\[
K_3^{(3)} = \frac{2}{3} A_\mu \left\{ \sqrt{2}Q \left\{ -\tilde{C}_\mu^1 H_4 - \tilde{C}_\mu^2 H_5 - \sqrt{2} \tilde{C}_\mu^3 H_- \right\} \\
+ \frac{PQ}{\sqrt{P^2 + Q^2}} \left( \tilde{C}_\mu^4 H_8 + \tilde{C}_\mu^5 H_9 + \tilde{C}_\mu^6 H_6 + \tilde{C}_\mu^7 H_7 \right) \right\}
\]
Next we write the quartic terms:

$$K^{(4)} = K^{(4)}_1 + 2g_3K^{(4)}_2 + 2g_1K^{(4)}_3 + g_3^2K^{(4)}_4 + g_1^2K^{(4)}_5 + 2g_3g_1K^{(4)}_6$$

where:

$$K^{(4)}_1 = \text{Tr} \left( [\partial_\mu \delta \Phi]^\dagger [\partial_\mu \delta \Phi] \right)$$

$$K^{(4)}_2 = -\frac{i}{2} \sum_{\alpha=1}^8 \tilde{C}_\mu^\alpha \text{Tr} \left( [\partial_\mu \delta \Phi]^\dagger \lambda_\alpha \delta \Phi - \delta \Phi^\dagger \lambda_\alpha [\partial_\mu \delta \Phi] \right)$$

$$K^{(4)}_3 = -\frac{i}{2} A_\mu \text{Tr} \left( [\partial_\mu \delta \Phi]^\dagger \delta \Phi \Gamma - \Gamma \delta \Phi \dagger [\partial_\mu \delta \Phi] \right)$$

$$K^{(4)}_4 = \sum_{\alpha,\beta=1}^8 \tilde{C}_\mu^\alpha \tilde{C}_\mu^\beta \text{Tr} \left( \delta \Phi^\dagger \lambda_\alpha \lambda_\beta \delta \Phi \right)$$

$$K^{(4)}_5 = A_\mu^2 \text{Tr} \left( \Gamma \delta \Phi^\dagger \delta \Phi \Gamma \right)$$

$$K^{(4)}_6 = \frac{1}{2} A_\mu \sum_{\alpha=1}^8 \tilde{C}_\mu^\alpha \text{Tr} \left( \delta \Phi^\dagger \lambda_\alpha \Gamma \delta \Phi + \Gamma \delta \Phi^\dagger \lambda_\alpha \delta \Phi \right)$$

Working out the traces of the matrix fields we get the following terms.

1. Terms from $K^{(4)}_1$, containing the combinations of fields $\partial_\mu H^2_K$.

$$K^{(4)}_1 = \partial_\mu H^2_+ + \partial_\mu H^2_- + \partial_\mu H^2_3 + \partial_\mu H^2_4 + \partial_\mu H^2_5 + \partial_\mu H^2_6 + \partial_\mu H^2_7 + \partial_\mu H^2_8 + \partial_\mu H^2_9$$

2. Terms proportional to $g_3$ coming from $K^{(4)}_2$, containing the combinations of fields $\tilde{C}_\mu^\alpha H_1 \partial_\mu H_J$.

$$K^{(4)}_2 = \frac{1}{2} \left\{ \tilde{C}_\mu^\alpha \left[ H_3 \partial_\mu H_7 - H_7 \partial_\mu H_3 + \frac{Q_2^2}{P_2^2 + Q_2^2} \left( H_6 \partial_\mu H_9 - H_9 \partial_\mu H_6 + H_8 \partial_\mu H_7 - H_7 \partial_\mu H_8 \right) \right] \right\}$$
+ 3\tilde{C}_\mu^2 \left\{ H_- \partial_\mu H_4 - H_4 \partial_\mu H_- + \frac{Q^2}{P^2 + Q^2} \left[ H_8 \partial_\mu H_6 - H_6 \partial_\mu H_8 + H_9 \partial_\mu H_7 - H_7 \partial_\mu H_9 \right] \right\} \\
+ \frac{1}{\sqrt{2(P^2 + Q^2)}} \tilde{C}_\mu^5 \left\{ P \left[ H_9 \partial_\mu H_+ - H_+ \partial_\mu H_9 + H_9 \partial_\mu H_- - H_- \partial_\mu H_9 + H_7 \partial_\mu H_4 \\
- H_4 \partial_\mu H_7 + H_6 \partial_\mu H_5 - H_5 \partial_\mu H_6 \right] + Q \left[ H_3 \partial_\mu H_9 - H_9 \partial_\mu H_3 \right] \right\} \\
+ \frac{1}{\sqrt{2(P^2 + Q^2)}} \tilde{C}_\mu^6 \left\{ \frac{P}{\sqrt{2}} \left[ H_7 \partial_\mu H_+ - H_+ \partial_\mu H_7 - H_7 \partial_\mu H_- - H_- \partial_\mu H_7 + H_9 \partial_\mu H_4 \\
- H_4 \partial_\mu H_9 - H_8 \partial_\mu H_5 + H_5 \partial_\mu H_8 \right] + Q \left[ - H_7 \partial_\mu H_3 + H_3 \partial_\mu H_7 \right] \right\} \\
+ \frac{1}{\sqrt{2(P^2 + Q^2)}} \tilde{C}_\mu^7 \left\{ \frac{P}{\sqrt{2}} \left[ H_8 \partial_\mu H_6 - H_6 \partial_\mu H_8 + H_6 \partial_\mu H_- - H_- \partial_\mu H_6 - H_8 \partial_\mu H_4 \\
+ H_4 \partial_\mu H_8 - H_9 \partial_\mu H_5 + H_5 \partial_\mu H_9 \right] + Q \left[ H_6 \partial_\mu H_3 - H_3 \partial_\mu H_6 \right] \right\} \\
+ \frac{2P^2 + Q^2}{\sqrt{3(P^2 + Q^2)}} \tilde{C}_\mu^8 \left\{ H_9 \partial_\mu H_7 - H_7 \partial_\mu H_9 - H_9 \partial_\mu H_7 + H_8 \partial_\mu H_9 \right\} \\

3. Terms proportional to \( g_1 \) coming from \( K_3^{(4)} \), containing the combinations of fields \( A_\mu H_I \partial_\mu H_J \).

\[ K_3^{(4)} = \frac{1}{3(P^2 + Q^2)} A_\mu \left\{ - H_7 \partial_\mu H_6 + H_6 \partial_\mu H_7 - H_9 \partial_\mu H_8 + H_8 \partial_\mu H_9 \right\} \]

4. Terms proportional to \( g_1^2 \) coming from \( K_4^{(4)} \), containing the combinations of fields \( \tilde{C}_\mu^a \tilde{C}_\mu^b H_1 H_J \).

\[ K_4^{(4)} = \left\{ \tilde{C}_\mu^1 \tilde{C}_\mu^1 + \tilde{C}_\mu^2 \tilde{C}_\mu^2 + \tilde{C}_\mu^3 \tilde{C}_\mu^3 \right\} \\\n\left\{ H_+^2 + H_+^2 + H_4^2 + H_5^2 + \frac{Q^2}{P^2 + Q^2} \left[ H_6^2 + H_7^2 + H_8^2 + H_9^2 \right] \right\} \]

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\[\begin{align*}
&+ \frac{2}{\sqrt{P^2 + Q^2}} \left\{ \bar{C}_\mu^1 \tilde{C}_\mu^4 + \bar{C}_\mu^2 \tilde{C}_\mu^5 - \bar{C}_\mu^3 \tilde{C}_\mu^6 - \frac{1}{\sqrt{3}} \bar{C}_\mu^6 \tilde{C}_\mu^8 \right\} \\
&\{ \frac{P}{\sqrt{2}} [H_+ H_6 - H_- H_6 + H_4 H_8 + H_5 H_9] + Q H_3 H_6 \} \\
&+ \frac{2}{\sqrt{P^2 + Q^2}} \left\{ \bar{C}_\mu^1 \tilde{C}_\mu^5 - \bar{C}_\mu^2 \tilde{C}_\mu^4 - \bar{C}_\mu^3 \tilde{C}_\mu^7 - \frac{1}{\sqrt{3}} \bar{C}_\mu^7 \tilde{C}_\mu^8 \right\} \\
&\{ \frac{P}{\sqrt{2}} [H_+ H_7 - H_- H_7 + H_4 H_9 - H_5 H_8] + Q H_3 H_7 \} \\
&+ \frac{2}{\sqrt{P^2 + Q^2}} \left\{ \bar{C}_\mu^1 \tilde{C}_\mu^6 - \bar{C}_\mu^2 \tilde{C}_\mu^4 - \bar{C}_\mu^3 \tilde{C}_\mu^5 - \frac{1}{\sqrt{3}} \bar{C}_\mu^5 \tilde{C}_\mu^8 \right\} \\
&\{ \frac{P}{\sqrt{2}} [H_+ H_8 + H_- H_8 + H_4 H_6 - H_5 H_7] + Q H_3 H_8 \} \\
&+ \frac{2}{\sqrt{P^2 + Q^2}} \left\{ \bar{C}_\mu^1 \tilde{C}_\mu^7 + \bar{C}_\mu^2 \tilde{C}_\mu^6 + \bar{C}_\mu^3 \tilde{C}_\mu^5 - \frac{1}{\sqrt{3}} \bar{C}_\mu^5 \tilde{C}_\mu^8 \right\} \\
&\{ \frac{P}{\sqrt{2}} [H_+ H_9 + H_- H_9 + H_4 H_7 + H_5 H_6] + Q H_3 H_9 \} \\
&+ \left\{ \frac{4}{\sqrt{3}} \bar{C}_\mu^1 \tilde{C}_\mu^8 + 2 \bar{C}_\mu^4 \tilde{C}_\mu^6 + 2 \bar{C}_\mu^5 \tilde{C}_\mu^7 \right\} \left\{ H_+ H_4 + \frac{Q^2}{P^2 + Q^2} [H_6 H_8 + H_7 H_9] \right\} \\
&+ \left\{ \frac{4}{\sqrt{3}} \bar{C}_\mu^2 \tilde{C}_\mu^8 - 2 \bar{C}_\mu^4 \tilde{C}_\mu^7 + 2 \bar{C}_\mu^5 \tilde{C}_\mu^6 \right\} \left\{ H_+ H_5 + \frac{Q^2}{P^2 + Q^2} [H_6 H_9 - H_7 H_8] \right\} \\
&+ \frac{2}{\sqrt{3}} \bar{C}_\mu^3 \tilde{C}_\mu^8 \left\{ 2 H_+ H_- + \frac{Q^2}{P^2 + Q^2} \left[ - H_6^2 - H_7^2 + H_8^2 + H_9^2 \right] \right\} \\
&+ \left\{ \bar{C}_\mu^4 \tilde{C}_\mu^8 + \bar{C}_\mu^5 \tilde{C}_\mu^6 \right\} \\
&\left\{ \frac{(H_+ + H_-)^2}{2} + H_3^2 + \frac{H_4^2}{2} + \frac{H_5^2}{2} + \frac{P^2}{P^2 + Q^2} [H_6^2 + H_7^2] + H_8^2 + H_9^2 \right\} \\
&+ \left\{ \bar{C}_\mu^6 \tilde{C}_\mu^8 + \bar{C}_\mu^7 \tilde{C}_\mu^7 \right\} \\
&\left\{ \frac{(H_+ - H_-)^2}{2} + H_3^2 + \frac{H_4^2}{2} + \frac{H_5^2}{2} + H_6^2 + H_7^2 + \frac{P^2}{P^2 + Q^2} [H_8^2 + H_9^2] \right\} \\
&+ \frac{1}{3} \bar{C}_\mu^8 \tilde{C}_\mu^8 \left\{ H_+^2 + H_-^2 + 4 H_3^2 + H_4^2 + H_5^2 + \frac{4 P^2 + Q^2}{P^2 + Q^2} [H_6^2 + H_7^2 + H_8^2 + H_9^2] \right\}
\end{align*}\]

5. Terms proportional to $g_1^2$ coming from $K_5^{(4)}$, containing the combinations of fields $A_\mu^2 H_1 H_f$. 

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\[ K_5^{(4)} = A_{\mu}^2 \frac{1}{9} \{ H_1^2 + H_2^2 + 4H_3^2 + H_4^2 + H_5^2 + \frac{P^2 + 4Q^2}{P^2 + Q^2} [H_6^2 + H_7^2 + H_8^2 + H_9^2] \} \]

6. Terms proportional to \( g_1 g_3 \) coming from \( K_6^{(4)} \), containing the combinations of fields \( A_{\mu} \tilde{C}_\mu H_I H_J \).

\[ K_6^{(4)} = A_{\mu} \tilde{C}_\mu \frac{2}{3} \left\{ -H_+ H_4 + \frac{2Q^2}{P^2 + Q^2} [H_6 H_8 + H_7 H_9] \right\} \]
\[ + A_{\mu} \tilde{C}_\mu \frac{2}{3} \left\{ -H_+ H_5 + \frac{2Q^2}{P^2 + Q^2} [H_6 H_9 - H_7 H_8] \right\} \]
\[ + A_{\mu} \tilde{C}_\mu \frac{2}{3} \left\{ -H_+ H_- + \frac{Q^2}{P^2 + Q^2} [H_8^2 + H_9^2 - H_6^2 - H_7^2] \right\} \]
\[ + A_{\mu} \tilde{C}_\mu \frac{2}{3} \left\{ -H_+ H_8 + \frac{P}{2} \left[ -H_+ H_8 + H_- H_8 - H_4 H_6 + H_5 H_7 \right] + 2QH_3 H_8 \right\} \]
\[ + A_{\mu} \tilde{C}_\mu \frac{2}{3} \left\{ -H_+ H_9 + \frac{P}{2} \left[ -H_+ H_9 - H_- H_9 - H_5 H_6 - H_4 H_7 \right] + 2QH_3 H_9 \right\} \]
\[ + A_{\mu} \tilde{C}_\mu \frac{2}{3} \left\{ -H_+ H_6 + \frac{P}{2} \left[ -H_+ H_8 + H_- H_6 - H_4 H_8 - H_5 H_7 \right] + 2QH_3 H_6 \right\} \]
\[ + A_{\mu} \tilde{C}_\mu \frac{2}{3} \left\{ -H_+ H_7 + \frac{P}{2} \left[ -H_+ H_7 + H_- H_7 + H_5 H_8 - H_4 H_9 \right] + 2QH_3 H_7 \right\} \]
\[ + A_{\mu} \tilde{C}_\mu \frac{1}{3} \left\{ -H_+^2 - H_-^2 - 4H_3^2 - H_4^2 - H_5^2 + 2H_6^2 + 2H_7^2 + 2H_8^2 + 2H_9^2 \right\} \]

Appendix C. Tree-level couplings of the \( F \) with the \( \tilde{C}_\mu \) (or \( G \))

The kinetic energy term of a fundamental fermion field \( \psi \), say a flavour doublet and colour triplet, can be written as:

\[ \bar{\psi} D_{\mu} \psi = \bar{\psi} (\partial_{\mu} - ig_1 \Gamma A_{\mu} - i g_2 B_{\mu} - i g_3 C_{\mu}) \psi, \]

where \( \Gamma \) is the charge operator operating on whatever follows. Using the operator \( \Omega \) introduced in (40) to fix the gauge so that the colour framion field \( \Phi \) becomes hermitian as in (45), we can rewrite the above as:

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\[
\bar{\psi} D_\mu \psi = \bar{\psi} \Omega^{-1} (\partial_\mu - ig_1 \Gamma A_\mu - i \frac{1}{2} g_2 B_\mu - i \frac{1}{2} g_3 C_\mu) \Omega^{-1} \psi,
\]
where \(\chi = \Omega^{-1} \psi\), being a bound state of the fundamental fermion field \(\psi\) with \(\Phi^1\) (approximated by \(\Omega^{-1}\) as explained in Section 4) represents in our present language an \(F\) field. We note also that since \(\Omega\) acts on only the colour sector, it leaves \(A_\mu\) and \(B_\mu\) in the preceding formula unchanged. Using then (42) of Section 4, the above formula can be rewritten again as:

\[
\bar{\psi} D_\mu \psi = \bar{\chi} (\partial_\mu - ig_1 \Gamma A_\mu - i \frac{1}{2} g_2 B_\mu - i \frac{1}{2} g_3 \tilde{C}_\mu) \chi,
\]
where \(\tilde{C}_\mu\) now represents our \(G\). This formula is the same in form as the first equation, except for the replacement of \(\psi\) by \(\chi\) and \(C_\mu\) by \(\tilde{C}_\mu\). From this we deduce that the \(F\) do couple to the \(G\) in the same way that the fundamental fields \(\psi\) couple to the colour gauge fields \(C_\mu\), as claimed in Sections 6 and 9.

We note that the above derivation is essentially just a paraphrase of that given by ’t Hooft [3] and Banks and Rabinovici [4] for the quarks and leptons in the confinement picture of the electroweak theory, only with flavour and colour interchanged. Indeed, had we taken \(\Omega\) for colour above as the \(\Omega\) for flavour in (27) of Section 4, we would reproduce their result, namely that quarks and leptons, as (in our language) framonic \(B\)-ons, have interactions with \(W^\pm, \gamma - Z\) (also as framonic \(B\)-ons) the same as the fundamental fermions \(\psi\) have with the original gauge fields.

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