Mathematical-model-based Simulation of an Automotive Alternator Considering the Operation of a Rectifying Circuit

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This paper presents a method for the mathematical-model-based simulation of automotive alternators. In the proposed method, a rectifier and motor are expressed by an electrical circuit constant that includes resistance and inductance. Moreover, a power correction factor is introduced for considering the operation of the rectifier. Owing to this, the calculation accuracy can be improved over that of a conventional mathematical simulation method. To validate the effectiveness, calculation results are compared with the experimental results.

Keywords: Alternator, mathematical-model-based simulation, rectifier circuit, claw-pole motor.

1. Introduction

Model-based simulation techniques have received significant attention recently (1-2). The purpose of those simulations is to reduce the development period by considering systematic behaviors in the designing process. Hence, each electrical device for automotive use must be expressed as a simplified electric constant (1-2). The simulation accuracy of the model-based simulation depends on the electric constant. Therefore, highly accurate models of electrical components, such as motors and inverters, must be constructed.

Automotive alternators, which are generators for charging a car battery, comprises a motor and a rectifier. An automobile alternator is shown in Fig. 1(a). For downsizing, electromechanical integral structures with high reliability and insulation performance are typically selected, as shown in Fig. 1(b) (3-4). To construct the simulation model of an alternator, as shown in Fig. 1(c), a simplified motor and rectifier model must be established (3-4). Meanwhile, motor inductance has a nonlinear characteristic owing to magnetic saturation. Therefore, electromagnetic field analysis must be performed to express motor inductance (5-6).

Fig. 2(a) illustrates the rotor of a claw-pole motor. To calculate the inductance characteristics, three-dimensional magnetic field analysis must be performed to express the complicated magnetic structure of the claw-shaped rotors (5-6). An analysis model of a claw-pole motor is shown in Fig. 1(b). Circuit and electromagnetic coupled analysis is an effective method for achieving a simulation with high calculation accuracy. However, coupled analysis simulations are not suitable for model-based simulations as they require significant calculation costs.

Herein, a mathematical-model-based method for simulating an automotive alternator is proposed. In the proposed method, motor inductance is expressed by a behavior model (7-8). In the behavior model, the relationship between current and interlinkage flux is formed to table data. Using the data, d/q-axis terminal voltages are calculated based on a d/q-axis voltage equation. Moreover, a mathematical-based model is constructed to express the rectifier circuit. In the circuit model, power loss is expressed by a resistance network. In addition, a rectifying operation is simulated using an impedance equation. Hence, the accuracy calculation of the motor output power can be improved via a mathematical-model-based simulation.

In this study, the proposed method was applied to an alternator. Moreover, calculations and experimental results were compared to validate its effectiveness.
2. Computational method

The flow chart of the proposed method is shown in Fig. 3. A flux map was generated using electromagnetic field analysis. In this process, a three-dimensional magnetostatic equation, which can be expressed as shown in the equation below, was solved.

\[ \nabla \times (\nabla \times A) = J_f \]  
\[ \text{(1)} \]

In Eqn. (1), \( \nabla \), \( A \), and \( J_f \) denote the magnetic reluctivity, vector potential, and coil current density, respectively. A sample of a d/q-axis flux map is shown in Fig. 4, in which the nonlinearity of the inductance of claw-pole motors can be observed. Hence, the inductance characteristics in terms of current width and phase angle must be expressed in tabular form to realize highly accurate calculations. 

In the convergence process (as shown in the flow chart of Fig. 3), the equations below were imposed to select the \( I_d \), \( I_f \), and \( I_q \) values.

Since the impedance component in the circuit load of the alternator can be assumed as small, it can be assumed that the difference between the phase of the motor-induced voltage and that of the motor-phase current is small. The assumption above cannot be considered in the conventional method. Therefore, we considered the operation of the rectifier circuit. Finally, the volume of the field current is selected for Eqn. (2-c). In Eqn. (2-c), \( I_f^{\text{max}} \) denotes the maximum value of the field coil, and it is selected by heat limitation.

A simplified equivalent circuit of alternators is shown in Fig. 5. The DC current \( I_f \) is supplied to the current field coil from a car battery. The rotor pole is magnetized by the DC current, and the rotor is rotated using the driving force of the engine. Consequently, an induced voltage is accrued at the terminal of the motor. When the motor voltage exceeds the bus voltage, the generated AC current flows to the rectifier from the motor. The AC current is converted from AC to DC, and the battery is charged. In this regard, the car battery can be charged only in the section in which the motor voltage is higher than the bus voltage. Meanwhile, the operating condition above cannot be considered in the conventional mathematical model. Therefore, we considered the operation of the rectifier using a mathematical model. The details of the model will be described later.

In the proposed method, the behavior of motor part was expressed by the d/q-axis voltage equation. Moreover, rectifier loss was calculated using a concentrated resistance constant. Although the parasitic inductance in the converter was high, its effect was not considered in this study because the power factor of the alternator was almost 1.0. Moreover, the internal resistance of the battery and various harnesses was defined by resistance constants, i.e., the same as for rectification loss.
2.1 d/q axis voltage equation

The d/q-axis voltage equation can be expressed as follows:

\[
\begin{bmatrix}
V_d(t) \\
V_q(t)
\end{bmatrix} = \begin{bmatrix}
(R_o + R_w) & L_q(t) \\
L_d(t) & (R_o + R_w)
\end{bmatrix} \begin{bmatrix}
I_d(t) \\
I_q(t)
\end{bmatrix} + \eta \begin{bmatrix}
0 \\
\phi_d(t)
\end{bmatrix} + \omega \begin{bmatrix}
0 \\
\phi_q(t)
\end{bmatrix}.
\]

In Eqn. (3), the stator and AC harness resistances must be calculated subject to temperature conversion. Using the equation above, the motor output power can be obtained as follows:

\[
P_e(t) = V_u(t)I_d(t) + R_wI_d^2(t) - 3R_oI_q^2(t) - 3(R_o + R_w)I_dI_q(t)
\]

\[\text{......................................................... (3)}\]

In Eqn. (4), the diode chip and bar resistances are considered for the rectification loss. Moreover, the DC power is expressed as follows:

\[
P_u(t) = V_{bat}I_d(t) + (R_w + R_a)I_d^2(t)
\]

\[\text{......................................................... (4)}\]

Using Eqn. (5), the generated DC current can be calculated as follows:

\[
I_d(t) = \frac{-V_{bat} - \sqrt{V_{bat}^2 + 4.0(R_w + R_a)P_u(t)}}{2.0(R_w + R_a)}
\]

\[\text{......................................................... (5)}\]

In the conventional mathematical model, the power generated by the alternator is computed as described above \(^{(9)}\). In this process, it is assumed that the induced voltage and three-phase current contribute to the generated power in all periods of one electric cycle. Meanwhile, the generating current can be obtained only in the section in which the motor voltage is higher than the bus voltage. Hence, the generation characteristics may be overestimated in the conventional method. Moreover, the power estimation process above is unclear in the conventional model-based simulation \(^{(10)}\).

2.2 Rectifier operation

We assume that the U-phase-induced voltage can be written as

\[
E_{u}(t) \geq E_{v}(t) \wedge E_{w}(t) \geq E_{u}(t)
\]

\[\text{......................................................... (7)}\]

Therefore, the condition in which the U-phase-generated current can be obtained can be written as

\[
\max\{E_u(t), E_v(t), E_w(t)\} - \min\{E_u(t), E_v(t), E_w(t)\} \geq V_{bat}
\]

\[\text{......................................................... (8)}\]

Furthermore, several other cases can be assumed. First, we assume a situation expressed as follows:

\[
E_u(t) \geq E_v(t) \wedge E_w(t) \geq E_u(t)
\]

\[\text{......................................................... (9)}\]

Relationships involving the U-, V-, and W-phase voltages and neutral points can be written as follows:

\[
V_{u}(t) = -\frac{V_{bat} + E_v(t) + E_w(t)}{3}
\]

\[\text{......................................................... (10)}\]

\[
V_{u}(t) = \frac{V_{bat} + E_v(t) + E_w(t)}{3} + I_u(t)Z
\]

\[\text{......................................................... (10)}\]

\[
V_{u}(t) = \frac{V_{bat} + E_v(t) + E_w(t)}{3} - I_u(t)Z
\]

\[\text{......................................................... (10)}\]

In Eqn. (10), Z denotes the motor impedance. In this study, we assumed that the motor impedance did not change in response to the exposure time and rotor position. Moreover, the impedance values were identical in the U-, V-, and W-phases. Since \(V_{u} + E_{u} \leq 0\) or \(V_{u} + E_{u} \geq V_{bat}\) must be satisfied to obtain the generated U-phase current, the condition in which the U-phase voltage can be observed from the bus line can be written as follows:

\[
V_{u}(t) + E_u(t) \leq 0
\]

\[\text{......................................................... (11)}\]

\[
V_{u}(t) + E_u(t) \geq V_{bat}
\]

\[\text{......................................................... (11)}\]

Subsequently, Eqn. (11) can be rewritten as

\[
E_u(t) \geq -\frac{V_{bat}}{3}
\]

\[\text{......................................................... (12)}\]

\[
E_u(t) \leq \frac{V_{bat}}{3}
\]

\[\text{......................................................... (12)}\]
In the proposed method, the ratio for satisfying Eqn. (12) in one electrical angle period was calculated using the induced voltage. Subsequently, we define situation expressed as follows:

\[
E_x(t) \geq E_y(t) \cap E_z(t) \geq E_x(t) \\
E_x(t) \leq E_y(t) \cap E_z(t) \leq E_x(t)
\] .......................... (13)

Relationships involving the U-, V-, and W-phase voltages and neutral points can be written as follows:

\[
V_c(t) = \frac{V_x(t) + V_y(t) + V_z(t)}{3} .......................... (14)
\]

\[
V_c(t) = V_x(t) + E_z(t) - I_z(t)Z  \\
V_c(t) = V_x(t) + E_y(t) - I_y(t)Z  \\
V_c(t) = V_x(t) + E_z(t) - I_z(t)Z \rightarrow V_{bat}
\]

Since \( V_c + E_z \leq 0 \) or \( V_c + E_\alpha \approx V_{bat} \) must be satisfied to obtain the generated U-phase current, the condition in which the U-phase voltage can be observed from the bus line can be expressed as

\[
V_c(t) + E_z(t) \leq 0  \\
V_c(t) + E_\alpha \geq V_{bat}
\] .......................... (15)

Subsequently, Eqn. (15) can be rewritten as follows:

\[
E_z(t) \leq V_c(t) - \frac{V_{bat}}{3}  \\
E_\alpha(t) \geq V_{bat} - V_c(t) = \frac{V_{bat}}{3}
\] .......................... (16)

In the proposed method, the ratio for satisfying Eqn. (16) in one electrical angle period was calculated using the induced voltage. In the proposed method, the ratio \( \alpha \) for satisfying Eqs. (8), (12), and (16) in one electrical angle period was calculated. The procedures above were conducted for the V and W phases. Subsequently, the claw-pole output AC power was calculated using the equation below:

\[
P_{\alpha}(t) = \alpha^2(V_x(t), I_x(t), V_y(t), I_y(t), V_z(t), I_z(t)) - R_u I_u^2(t) - 3R_a I_a^2(t) - 3(R_\alpha + R_w) I_w^2(t)
\] .......................... (17)

From Eqs. (7)–(17), the condition in which the U-phase-generated current can be obtained can be summarized as follows:

\[
(i). \quad [E_x(t)] \geq [E_y(t)] \cap [E_z(t)] \geq [E_x(t)] \\
\max (E_x(t), E_y(t), E_z(t)) - \min (E_x(t), E_y(t), E_z(t)) \geq V_{bat} \land \|E_i(t)\| \geq \frac{V_{bat}}{3}
\] .......................... (18-a)

\[
(ii). \quad \|E_x(t)\| \geq \|E_y(t)\| \cap \|E_z(t)\| \geq |E_x(t)| \\
\max (E_x(t), E_y(t), E_z(t)) - \min (E_x(t), E_y(t), E_z(t)) \geq V_{bat}
\] .......................... (18-b)

Finally, the condition pertaining to the modification ratio, which is multiplied by square to AC output power, can be expressed as follows:

\[
\alpha = \frac{1}{T_B} \int_0^T \frac{V_x(t)^2}{V_{\alpha}} dt \\
V_x(t) = \left\{ \begin{array}{ll}
V_x(t) & \text{subject to Eqn.}(18) \\
0.0 & \text{etc}
\end{array} \right.
\]

Using Eqn. (19), the calculation accuracy of the proposed method can be improved.

3. Numerical and Experimental results

3.1 Condition

The conditions for the calculation are summarized in Table II. Since the number of poles was set to 16, the flux map was created using a 1/16 model, as shown in Fig. 2(b). To consider thermal effects, ordinary and hot conditions were selected. To set the motor temperature for case B, an experiment was conducted after placing the alternator in the thermostatic chamber for a significant amount of time. Moreover, the field current condition in case C differed from those of cases A and B. By comparing the calculation results with those of the conventional method in these three cases, the thermal effects and output power can be analyzed. In the conventional method, the correction factor \( \alpha \) was set to 1.0. In other words, the calculated AC power was not corrected using Eqn. (17).

| Table 2 Motor Specification and calculation conditions |
|-------------------------------------------------------|
| Case A | Case B | Case C |
| Number of pole Pairs | 16 | ← | ← |
| Thermal condition (°C) | 25 | ← | ← |
| Bus voltage (V) | 13.5 | ← | ← |
| Stator | Slot number | 96 | ← | ← |
| | Coil turn | 3T-Y | ← | ← |
| Rotor | Coil turn | 99.5 | ← | ← |
| | Field coil current (A) | 28.0 | ← | ← |
| | | 21.5 | ← | ← |

3.2 Results

The calculation results obtained using the conventional and the proposed methods are shown in Fig. 6. The value of the generated current shown in Fig. 6 was normalized by multiplying the maximum value for each condition. As shown, the generated current was revised from the conventional method in the proposed method. This was due to the revision by multiplying correction factor \( \alpha \). Hence, it was clear that the calculation accuracy can be improved using the proposed method. Moreover, this tendency was evident in all conditions summarized in Table II. The results confirm that the proposed method is valid for evaluating the generation characteristics of claw-pole alternators.

However, a few differences were observed between the calculation and experiment results, particularly in the low-rotational-speed condition. This tendency was observed in all conditions. Moreover, the impedance characteristics between the low- and high-speed conditions differed. In more detail, the resistance component was dominant in the low-speed condition. By constant, the inductance component was dominant in the high-speed condition. Hence, it can be assumed that the phase-current comparison for the low- and high-speed conditions is valid to evaluate the error in those results.

In the next section, we discuss the calculation error of the proposed method based on a finite element analysis of the U-phase
3.3 Discussion In Fig. 7, the relationship between the rotational speed and each of the line voltage and correction factor is shown. As shown, the latter two components increased with the rotational speed. Since the correction factor was calculated using Eqn. (18), its value increased with the line voltage. Hence, it can be assumed that the tendency of the actual motor can be expressed using the proposed method.

The U-phase current obtained using the finite element method in the generation mode is shown in Fig. 8. In this simulation, Eqn. (1) and Eqn. (20) below were coupled in the analysis.

\[
\frac{d\phi_u}{dt} - (R_s + R_d) I_u(t) = R_e I_u(t)
\]

(20)

In Eqn. (20), \(R_e\) is the equivalent load resistance per phase \(^{(1)}\). The results indicate that a difference existed in the current wave between the low- and high-speed condition. Specifically, a larger harmonic current component existed in the low-speed condition than in the high-speed condition. In the generation mode of the alternators, the period when the U-phase current flowed was limited to the period when the U-phase-induced voltage satisfied Eqn. (17). Therefore, the duration in which the motor output terminals were in the released state was longer in the low-speed condition than in the high-speed condition. Therefore, the U-phase current was 0 A for a certain duration. Moreover, the resistance component of the motor impedance dominated because of the low electrical frequency in the low-speed condition. Since the resistance component is mainly due to the winding resistance, its space and time harmonics are lower than those of the inductance component. In addition, the interlinkage flux appearing in the stator windings included spatial harmonics, which depended on the rotor position, determined by the rotor magnetomotive force harmonics. In the high-speed condition, these harmonics were weakened by the stator weakening current, which was proportional to the generated current. Hence, more harmonic currents were included in the low-speed condition than in the high-speed condition. As described in Section II, a sine wave was imposed in creating the step of the flux map. Therefore, the harmonic iron loss and power factor were not considered in the present method. To improve the calculation accuracy, the fifth and seventh harmonic currents must be introduced for creating the flux map.

In addition, the BH curve of the magnetic property must be reconsidered. Fig. 9 shows the magnetic contours obtained using the finite element method. As shown, the magnetic flux density of the two conditions differed significantly. Because the current generated in the low- and high-speed conditions differed, the flux-weakening volumes differed in both conditions. Hence, the magnetic material properties, particularly the saturation magnetic flux density, must be expressed appropriately. However, it is difficult to express the effect of punching degradation in magnetic materials.

It is clear that the harmonic current wave and magnetic material properties must be reconstructed to improve the calculation accuracy. To consider the fifth and seventh harmonic currents, exorbitant computational costs will be incurred owing to the increase in the number of analysis cases. Moreover, the size of the flux map table data would be large. Therefore, the proposed method is suitable as a model simplification method, e.g., for model-based simulations. Because only mathematical simulations were involved, the proposed method offered low computational costs instead of calculation accuracy compared with circuit and magnetic coupled finite element simulations. Therefore, the use of the present method to express the magnetic saturation characteristics of magnetic materials should be investigated in future studies.
4. Conclusion

A novel mathematical-model-based simulation method for automobile alternators was presented herein. In the proposed method, motor inductance is expressed by a behavior model. Using relevant data, d/q-axis terminal voltages were calculated based on a d/q-axis voltage equation. Moreover, a mathematical-based model was constructed to express a rectifier circuit. In the circuit model, power loss was expressed based on a resistance network. In addition, the rectifying operation was simulated using an impedance equation. The simulation and experimental results indicated that the analysis accuracy of the alternator improved compared with conventional mathematical simulations.

For future work, the magnetic saturation characteristics of the magnetic material will be investigated while considering degradation caused by mechanical stress. Moreover, engine and electrical loads will be introduced in the proposed method. Based on simulation, the relationship between the generation efficiency of an alternator and CO₂ emission amount will be discussed.

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