Probing the Bose glass–superfluid transition using quantum quenches of disorder

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The disordered Bose–Hubbard model—a paradigm for strongly correlated and disordered bosonic systems—is central to our understanding of quantum phase transitions. Despite extensive theoretical work on the disordered Bose–Hubbard model, little is known about the impact of temperature, the dynamical behaviour of quantum phases, and how equilibrium is affected during quantum phase transitions. These issues are critically important to applications such as quantum annealing and electronics based on quantum phase transitions. Here, we use a quantum quench of disorder in an ultracold lattice gas to dynamically probe the superfluid–Bose glass quantum phase transition at non-zero temperature (Fig. 1). By measuring excitations generated during the quench, we provide evidence for superfluid puddles in the Bose glass phase and produce a superfluid–Bose glass phase diagram consistent with completely constrained, finite temperature, and equilibrium quantum Monte Carlo simulations. The residual energy from the quench, which is an efficacy measure for optimization through quantum annealing, is unchanged for quench times spanning nearly a hundred tunneling times.

To make these measurements, we create an atomic realization of the three-dimensional (3D) disordered Bose–Hubbard model (DBHM) using ultracold 87Rb atoms trapped in a disordered optical lattice. The DBHM has been used to describe systems as diverse as 4He in disordered media, Josephson-junction arrays, and long-wavelength properties of superconducting electron pairs. In the DBHM, a strongly interacting superfluid (SF) undergoes a quantum phase transition (QPT) into a Bose glass (BG) when subjected to disorder. The critical exponents of this transition fall in the percolation class. The BG phase exhibits the peculiar property of lacking long-range order while possessing infinite superfluid susceptibility, and it is therefore viewed as a gapless insulator with finite compressibility that arises from the presence of quasi-condensates, or SF puddles, embedded in an insulating background. Disordered ultracold atom gases have been used to indirectly measure the SF–BG transition by means of transport and coherence measurements in 1D (ref. 12) and 3D (ref. 13) disordered lattices and in 1D quasi-periodic lattices.

In our experiment, we prepare a gas consisting of \( (27 \pm 2) \times 10^4 \) atoms cooled below the critical temperature \( T_c \) for Bose–Einstein condensation in a parabolic trap such that the condensate fraction is greater than 90%. A disordered cubic optical lattice formed from pairs of counter-propagating \( \lambda = 812 \) nm laser beams and a 532 nm optical speckle field is superimposed on the gas (see Supplementary Methods). The atoms experience a potential energy shift proportional to the speckle intensity, which varies randomly in space, leading to disorder in the Hubbard parameters. The DBHM we realize is characterized by the Hamiltonian

\[
H = - \sum_{\langle ij \rangle} \left( t_i b_i^\dagger b_j + \text{h.c.} \right) + \sum_i (\epsilon_i - \mu) n_i + \frac{1}{2} \sum_i U_i n_i (n_i - 1) + \frac{1}{2} \sum_i m \omega_i r_i^2 n_i
\]

where \( i \) and \( j \) index the lattice sites, \( \langle ij \rangle \) indicates that tunnelling occurs only between adjacent sites, and \( t_i, U_i, \) and \( \epsilon_i \) are the Hubbard energies. In equation (1), \( n_i \) is the number of particles on site \( i \), \( b_i \) (\( b_i^\dagger \) ) removes (adds) a particle from site \( i \), \( m \) is the atomic mass, \( \omega_i \) is the geometric mean of the trap frequencies, \( r_i \) is the distance to the centre of the trap, and \( \mu \) is the chemical potential. We measure all energies in terms of the recoil energy \( E_R = \hbar^2 / 2m \lambda^2 \approx k_B \times 170 \) nK. The distribution of the Hubbard parameters, which are broadened around the values for the uniform system, are precisely known. The strength of the disorder is characterized by the average potential energy \( \Delta \) associated with the speckle, which is approximately equal to the standard deviation of the \( \epsilon_i \) distribution. The lattice potential depth \( s \) (which controls \( U_i / \lambda \) and \( \Delta \) are independently adjusted by tuning the power of the lattice laser and 532 nm light. For the values of \( s \) sampled in this work, the gas is a strongly correlated, quantum-depleted SF when \( \Delta = 0 \). We do not explore sufficiently high \( s \) to generate a Mott insulator phase in the gas.

We probe the BG–SF transition by measuring the amount of excitation produced by quenching \( \Delta \) at fixed \( s \). The disorder strength is linearly ramped from an initial value of \( \Delta_0 \) to zero in 30 ms (Fig. 1b), which is slow enough to avoid creating excitations solely through the time variation of the spatially inhomogeneous disorder potential (see Supplementary Methods). On the basis of general arguments regarding the phase diagram in untrapped systems, the BG phase will appear in the low-density edge of the gas for sufficiently high \( \Delta_0 \) (ref. 1). For stronger disorder, the BG–SF boundary moves inwards, encompassing more of the atoms. Excitations produced by the quench are measured using time-of-flight (TOF) imaging. By imaging after a long (50 ms) period of free expansion, vortices and any other excitations involving velocity fields (including phonons) are transformed into modulations of the density profile and the measured optical depth (OD). These excitations are visible in the characteristic images shown in Figs 1c and 2b. For low \( \Delta_0 \), the density profile after the quench and TOF is smooth, whereas for high \( \Delta_0 \), features consistent with vortices are present.
In the KZ scenario, an adiabatic transition from a phase-disordered description by the quantum KZ effect in the context of QPTs (Ref. 19). The process and the threshold behaviour evident in Fig. 2a is generally spatially distinct SF puddles combine, the random phases associated behaviour is observed for all a `clean' lattice through a quench between Mott insulator (MI) and SF states in Kibble-Zurek (KZ) effect by measuring excitations generated (Methods). This method was previously used to observe the quantum disorder and quench. The gas is quenched from the BG to the SF regime by DBHM and quench. The lattice potential depth and disorder strength are shown using red and green configurations and the disordered lattice potential (false colour) are shown at three values of $\Delta$. For sufficiently high $\Delta$, BG (blue) and SF (light grey) phases coexist in the trap. b, Time sequence for the measurement. The lattice potential depth and disorder strength are shown using red and green lines. c, Equilibrium is disrupted during the quench and excitations are produced, which are measured in TOF images (greyscale). Images are shown for $\Delta_0 \approx 0.5E_F$ (i) and $\Delta_0 = 0$ (ii) at $s = 12E_F$. The white ellipse indicates the fitted Thomas-Fermi radius, and the fitted profile is subtracted in the lower images. For sufficiently high disorder, excitations such as vortices are apparent (red arrow) after the quench, whereas smooth profiles are obtained at low $\Delta_0$. Clear images of vortices are rare, because vortices generated by the quench are randomly oriented relative to the imaging direction.

To quantitatively characterize the amount of excitation present after the quench, we measure

$$\tilde{\chi}^2 = \sum_{ij} \left( \frac{O_{ij} - f_{ij}}{f_{ij}} \right)^2 \sum_{ij} O_{ij}$$

(2)

where $O_{ij}$ is the measured OD at the pixel indexed by $i$ and $j$, and $f_{ij}$ is a smooth fitting function (see Supplementary Methods). This method was previously used to observe the quantum Kibble-Zurek (KZ) effect by measuring excitations generated through a quench between Mott insulator (MI) and SF states in a `clean' lattice. Data for $s = 11E_F$ and $\Delta_0 \approx 0$–$1E_F$ are shown in Fig. 2a. It is apparent that excitations are not generated by the quench until a threshold disorder strength is crossed, above which $\tilde{\chi}^2$ increases approximately linearly with $\Delta_0$. Similar threshold behaviour is observed for all $s$ we sample in this work.

One way to account for such behaviour is by excitations created through SF puddles merging as the BG–SF boundary is crossed. As spatially distinct SF puddles combine, the random phases associated with each island naturally lead to vortices and other excitations. This process and the threshold behaviour evident in Fig. 2a is generally described by the quantum KZ effect in the context of QPTs (Ref. 19).

In the KZ scenario, an adiabatic transition from a phase-disordered (for example, BG) to an ordered (for example, SF) state is impossible because of diverging characteristic scales of length and time in the vicinity of the transition. Dynamically traversing a continuous QPT by tuning (or quenching) a Hamiltonian parameter leads to the formation of excitations that persist even after the transition is crossed. The quantum KZ effect has been used to describe this phenomenon, and it has been observed for the MI–SF transition in a `clean' optical lattice, where sensitivity to the equilibrium phase boundary and power-law scaling were observed, and the scaling of the coherence length after the quench with the quench rate was measured. Our knowledge of how KZ physics is modified by disorder and glassy phases is limited to simulations and theory of certain one-dimensional spin models. The prime example of a quantum quench in a disordered system before this work was in an Ising magnet, where a smaller residual energy for quantum annealing compared with thermal annealing was observed by means of magnetic susceptibility measurements.

To connect the observed threshold disorder with the SF–BG transition, we carry out exact quantum Monte Carlo (QMC) simulations of the equilibrium system using the same trap and lattice parameters, atom number, and speckle disorder as in the experiment (see Supplementary Methods). For trap-free geometries in the thermodynamic limit, the BG is characterized by a vanishing superfluid order parameter and non-zero compressibility. In contrast, the trapped system we consider exhibits domains corresponding to SF and BG phases that we distinguish using the spatial extent of the condensate. The condensate is identified as the macroscopic.
occupation of a single-particle eigenstate that we can obtain from the single-particle density matrix $\rho_i = \sum_k \langle \hat{b}_k \rangle^2$ (refs 25,26).

For clean systems (that is, $\Delta = 0$) with $U/t < 29.34 \pm 0.02$ that are below $T_c$, a single condensate extends throughout the system that coincides with the local superfluid density order parameter (see Supplementary Methods). As $\Delta$ is increased, this behaviour changes, and the extent of the macroscopic condensate shrinks, leaving behind regions devoid of coherence. Because the SF–BG transition is of the continuous type, phase coexistence is forbidden, and we identify these regions as BG phase. To illustrate how the BG phase emerges in the gas, we show the two highest occupation eigenfunctions of $\rho_i$ for $s = 11E_0$ and $\Delta = 0.05E_0$ and $1E_0$ in Fig. 2c. At low $\Delta$, all single-particle states are spatially overlapped with the SF domain, and the second highest occupied state results from interaction-induced quantum depletion. For sufficiently high $\Delta$, however, this extended state is replaced by a spatially localized mode that corresponds to a non-macroscopic and locally coherent superfluid puddle, characteristic of the BG phase.

To compare with the measurements, we compute the BG fraction $N_{\text{BG}}/N$ as the fraction of atoms in regions without a macroscopic condensate present. This estimate is an upper bound at non-zero temperature because of thermal excitations, which we find is small in the regime we study (see Supplementary Methods). As shown in Fig. 2d, the BG as defined by this criterion emerges at the edge of the gas and grows in extent and number as $\Delta$ is increased. The typical behaviour for $N_{\text{BG}}/N$ at $s = 11E_0$ as $\Delta$ is varied (shown in Fig. 2c) mirrors the amount of excitation created by the quench in the experiment: $N_{\text{BG}}/N$ is only non-zero above a threshold disorder, above which it increases approximately linearly with $\Delta$.

We construct a SF–BG phase diagram (Fig. 3) by estimating the threshold disorder $\Delta_0$ for excitations to appear in the experiment and for BG to appear in QMC simulations using a piecewise-linear fit to data such as those shown in Fig. 2. The fitting function assumes constant behaviour for disorder strengths less than $\Delta_0$, and linearly increasing behaviour characterized by the free parameters $\Delta_0$ and a slope for disorder strengths greater than $\Delta_0$. Several important features of the phase diagram are evident. The threshold disorder $\Delta_0$ is weakly dependent on $U/t$, and the QMC and experimental results agree within the 25% systematic uncertainty in $\Delta_0$; there are additional systematic and statistical uncertainties arising from finite temperature and disorder averaging (see Supplementary Methods). This agreement demonstrates that the quench dynamics and production of excitations in this strongly disordered system are sensitive to the ground-state, equilibrium phases (which are not significantly distorted by non-zero temperature), and that the QMC simulations accurately describe the physical experiment. Moreover, it implies that the SF puddles present in the QMC simulations (Fig. 2c) exist in the experiment. The observed threshold behaviour cannot be explained by mean field theory, which predicts that a BG appears for infinitesimal disorder$^{27}$.

The decrease in $\Delta_0$ at higher $s$ (that is, larger $U/t$), which cannot be accounted for by general classical percolation mechanisms, indicates that stronger interactions weaken the SF against localization in this regime. Finally, the observed $\Delta_0$ is lower than that required to convert the entire gas to a localized BG phase, as determined through transport measurements$^{13}$. This behaviour is expected, because the entire gas must be converted to a BG phase in order to halt transport.

We explore the dynamical timescale of the SF–BG transition by varying the quench time $\tau_Q$ at fixed $s$ and $\Delta_0$. In the KZ scenario for clean systems, the amount of excitation and heat produced during a quantum quench typically exhibit power-law dependences on the quench time$^{19}$. The knowledge of how this changes in disordered systems is restricted to theory and simulations of one-dimensional spin chains, which possess an inverse logarithmic dependence of the residual energy (that is, the energy generated by the quench) on the quench time$^{20,21}$. Data for nearly two orders of magnitude in $\tau_Q/(\hbar/t)$, where $\hbar/t$ is the tunnelling time, are shown in Fig. 4 at $s = 10E_0$ for the BG regime at $\Delta_0 = 1E_0$. To avoid complications from decay of excitations during the quench, we measure the temperature of the gas after allowing rethermalization in the trap for 150 ms (see Supplementary Methods). We show the residual energy as the change in the thermal energy $\delta \epsilon = \epsilon - \epsilon_0$, where $\epsilon$ and $\epsilon_0$ are the thermal energy per particle inferred from the measured temperature with and without the quench.

The extremely weak dependence of $\delta \epsilon$ on $\tau_Q$ is characteristic of the BG regime for all $s$ sampled in this work. The residual energy
is nearly fixed for quenches spanning approximately six to sixty times the tunneling time. We do not observe significant changes in this behaviour with different $s$. The range of $\Delta_0$ we can sample is limited to approximately $1E_h$ at $s = 10E_h$—and less at higher $s$—because this measurement requires the heat generated by the quench to be greater than that from the lattice light, and for the gas after the quench to remain condensed. Constraining the scaling with $t_0$ is challenging given the observed weak dependence and our inability to explore longer quench times because scattering of the lattice light heats the gas above $T_e$. The fits to an inverse logarithm proportional to $1/\log(t_0/\hbar)^{0.4\pm0.6}$ and a power law $(t_0/\hbar)^{-3\pm0.1}$ shown in Fig. 4 are equally consistent with the data.

Theoretical predictions for how the residual energy changes with quench time in this system are unavailable, as exact numerical simulation of a quench is intractable for experimentally relevant numbers of particles in two and three dimensions. Approximate simulations of other dynamics have been carried out, including quantum systems.

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Author contributions

B.D., C.M., U.R. and D.M.C. conceived the research. C.M. and U.R. contributed equally to this work: C.M. conducted and analysed the measurements, and U.R. performed and analysed the numerical simulations. P.R. and D.C. contributed to the measurements and data analysis. B.D. and D.M.C. supervised the experimental and theoretical work, respectively. B.D., U.R. and C.M. wrote the manuscript, which was discussed by and commented on by all authors.

Additional information

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Competing financial interests

The authors declare no competing financial interests.