CLUSTER VARIATION APPROACH TO THE RANDOM–ANISOTROPY BLUME–EMERY–GRIFFITHS MODEL

Carla Buzano\textsuperscript{a}, Amos Maritan\textsuperscript{b} and Alessandro Pelizzola\textsuperscript{a}

\textsuperscript{a}Dipartimento di Fisica and Unità INFM, Politecnico di Torino, I-10129 Torino, Italy
\textsuperscript{b}Dipartimento di Fisica, Università di Padova, and
INFN Sez. di Padova, I-35131 Padova, Italy

ABSTRACT

The random–anisotropy Blume–Emery–Griffiths model, which has been proposed to describe the critical behavior of $^3$He–$^4$He mixtures in a porous medium, is studied in the pair approximation of the cluster variation method extended to disordered systems. Several new features, with respect to mean field theory, are found, including a rich ground state, a nonzero percolation threshold, a reentrant coexistence curve and a miscibility gap on the high $^3$He concentration side down to zero temperature. Furthermore, nearest neighbor correlations are introduced in the random distribution of the anisotropy, which are shown to be responsible for the raising of the critical temperature with respect to the pure and uncorrelated random cases and contribute to the detachment of the coexistence curve from the $\lambda$ line.
1. Introduction

The study of the effects of randomness on phase transitions has a long story and only recently there has been considerable experimental and theoretical efforts to deeply understand them. Here we will be concerned with a simple model for the phase separation of $^3$He–$^4$He mixtures in aerogel[1].

Aerogel is an extremely porous medium, the silica glass, made via the sol–gel process[2]. Its porosity may be as high as 99.8%[3] corresponding to a density of 4mg/cm$^3$. There is experimental evidence[2,4] that the aerogel microstructure is rather ramified, composed of silica strands with a thickness of order 10Å and with an area of order 700m$^2$/gr[5].

In order to describe in a simple way the effects of the porous medium on the phase separation diagram of $^3$He–$^4$He we model the aerogel as a random external field which selects which of the two types of Helium to have nearby. Thus it is quite interesting, among various other things, to understand the role of the two types of impurities, i.e. the annealed $^3$He and the quenched randomness of the external field, on the superfluid transition of $^4$He.

Using mean field theory, transfer matrix and real space renormalization group calculations it was possible to predict a variety of physically acceptable scenarios for the phase separation diagram as the density of the quenched impurities is varied[1]. The main features are the disappearance of the tricritical point present only in the pure $^3$He–$^4$He system[6] and superfluidity in two co-existing phases, one rich in $^3$He and the other in $^4$He. Recent experiments[7] have confirmed the above qualitative picture.

The goal of this paper is to give a detailed study of the model proposed in [1] using the pair approximation of the cluster variation method (CVM)[8], generalized to random systems[9,10]. This approximation improves the standard mean field by taking into account effects of correlation among spins at different sites. Most of the qualitative features of the mean field phase diagram are preserved.

Our approach allows us to incorporate effects of nearest neighbors (n.n.) correlations in the quenched impurities, which are essential to a full understanding of the role of aerogel in the $^3$He–$^4$He phase diagram. Unlike the
coexistence curve, the $\lambda$ line separating the normal fluid from the superfluid, is rather sensitive to correlations of the quenched impurities at high $^3$He concentration. Interesting new features are found in the $T = 0$ phase diagram and the role of the percolation threshold is elucidated.

In the next section the random–anisotropy Blume–Emery–Griffiths model (RABEG) will be introduced. The CVM generalized to random systems will be discussed in detail in connection with the RABEG model. Sec. 3 is devoted to the discussion of the phase diagram obtained in the present approximation, also in connection with experimental results, and in Sec. 4 some conclusions are drawn.

2. Definition of the model and the CVM pair approximation

The bulk phase diagram of $^3$He–$^4$He is well described by the Blume–Emery–Griffiths (BEG) model[6] which is a spin–1 system with nearest neighbor interactions and a uniform anisotropy external field distinguishing only between the $\pm 1$ and 0 values of the spin. Its generalization to the $^3$He–$^4$He mixture in a porous medium is described by the hamiltonian[1]

$$H = -J \sum_{<ij>} S_i S_j - K \sum_{<ij>} S_i^2 S_j^2 + \sum_i \Delta_i S_i^2,$$  \hspace{1cm} (1)

where the spin variables $S_i = 0, \pm 1$ are defined on sites of a lattice $\Lambda$. The first and the second sums are over n.n. pairs. When dealing with $^3$He–$^4$He mixtures[6] $S_i = \pm 1$ and $S_i = 0$ represents $^4$He and $^3$He respectively, and the superfluid transition is associated to the spontaneous symmetry breaking between the $\pm 1$ states. The exchange interaction with strength $J > 0$ is responsible for the superfluid ordering, while the random anisotropy $\Delta_i$ is related to the difference of chemical potentials $\mu_3 - \mu_4$ at site $i$, and $K = K_{33} + K_{44} - 2K_{34}$, where $K_{\alpha\beta}$ is the interaction energy between $^\alpha$He–$^\beta$He atoms. $K_{\alpha\beta}$ does not depend significantly on $\alpha$ and $\beta$ and then it is generally assumed $K = 0$.

Since we are going to study a model for $^3$He–$^4$He mixtures in aerogel, we will let each $\Delta_i$ take the value $\Delta_0$ or $\Delta_1$ with the same meaning as in [1], i.e.
\( \Delta_0 < 0 \) is the value of the anisotropy at the pore–grain interface of aerogel (where \(^4\)He prefers to stay), while \( \Delta_1 \) can be thought of as a bulk field which controls the \(^3\)He concentration \( x \).

Two order parameters can be defined: \( m = \langle \langle S_i \rangle \rangle_\Delta \) and \( q = \langle \langle S_i^2 \rangle \rangle_\Delta \), which are the quenched averages of the thermal equilibrium values \( \langle S_i \rangle \) and \( \langle S_i^2 \rangle \). \( m \) is the superfluid order parameter, while \( q = 1 - x \) is the \(^4\)He concentration.

As we mentioned in the introduction we take into account n.n. correlations in the random field distribution assuming the following joint probability density (with \( i \) and \( j \) nearest neighbors) for the \( \Delta_i \)’s:

\[
P(\Delta_i, \Delta_j) = \sum_{n_1, n_2 = 0, 1} p_{n_1 n_2} \delta(\Delta_i - \Delta_{n_1}) \delta(\Delta_j - \Delta_{n_2}),
\]

\[
p_{n_1 n_2} = p_{n_1} p_{n_2} + (2\delta_{n_1 n_2} - 1) \epsilon p^2,
\]

where \( p_0 = p \) is the fraction of sites at the aerogel interface (the grain space is neglected), \( p_1 = 1 - p \) and \( \epsilon \) is a measure of the interface correlation. Integrating \( \Delta_j \) one obtains

\[
P(\Delta_i) = p \delta(\Delta_i - \Delta_0) + (1 - p) \delta(\Delta_i - \Delta_1),
\]

which was used as a starting point in [1].

We are now ready to build the pair CVM free energy following the procedure outlined by Morita[9] for a general random system, which we briefly review.

Consider a random system on a finite lattice, for which the configuration of random fields and interactions is specified by a unique random variable \( h \) with distribution \( P(h) \), and define, according to the usual rules of statistical mechanics, a density matrix \( \rho_c(\sigma|h) \) for each configuration \( h \) (\( \sigma \) stands for the set of statistical degrees of freedom of the system, e.g. spin variables). In terms of \( \rho_c \), the free energy associated to the configuration \( h \) will be given by

\[
F(h) = \sum_{\sigma} \rho_c(\sigma|h) \left[ H(h, \sigma) + k_B T \ln \rho_c(\sigma|h) \right],
\]
where $H(h, \sigma)$ is the Hamiltonian, $k_B$ Boltzmann’s constant, and $T$ absolute temperature. The quenched free energy $F$ will then be given by

$$F = \sum_h P(h) F(h).$$

(6)

Upon introducing the generalized density matrix

$$\rho(h, \sigma) = P(h) \rho_c(\sigma|h)$$

(7)

one can easily show that

$$F = \sum_{h,\sigma} \rho(h, \sigma) [H(h, \sigma) + k_B T \ln \rho(h, \sigma)] + T S_c,$$

(8)

where $S_c$ depends only on $P(h)$ and is given by

$$S_c = -k_B \sum_h P(h) \ln P(h).$$

(9)

As in the pure case, we have a variational principle: $\rho(h, \sigma)$ can be determined as the matrix which minimizes $F$, as given by (8), with the constraint

$$\sum_\sigma \rho(h, \sigma) = P(h).$$

(10)

In such a scheme it can be shown that the quenched average of the expectation value $\langle A(h, \sigma) \rangle$ of an operator $A(h, \sigma)$ is given by

$$\langle\langle A(h, \sigma)\rangle\rangle_h = \sum_h P(h) \langle A(h, \sigma) \rangle = \sum_{h,\sigma} \rho(h, \sigma) A(h, \sigma).$$

(11)

The cluster variation method can then be obtained by taking the thermodynamic limit and truncating the cumulant expansion for the entropy $S = -k_B T \sum_{h,\sigma} \rho(h, \sigma) \ln \rho(h, \sigma)$ to a set of ”maximal preserved clusters” $\Gamma_i$, $i = 1, 2, \ldots r$ (and all their translates). The variational principle will then be applied to the reduced density matrices $\rho_{\Gamma_i}(h, \sigma)$ associated to the maximal preserved clusters.

In the following, our maximal preserved cluster will be the n.n. pair, and thus we will introduce a density matrix $\rho_p$ with elements $\rho_p(h, \sigma) \equiv$
\[ \rho_p^{(n_1, n_2)}(S_1, S_2) \ (n_1, n_2 = 0, 1, S_1, S_2 = \pm 1, 0), \text{ where } h = (\Delta n_1, \Delta n_2) \text{ is the random field configuration on the cluster and } \sigma = (S_1, S_2) \text{ denotes the set of spin variables.} \]

\[ \rho_p \text{ (which is diagonal, since we are dealing only with the } z \text{-component of the spins) is subject to the constraint } \sum_{\sigma} \rho_p(h, \sigma) = P(h), \]

that is

\[ \sum_{S_1 S_2} \rho_p^{(n_1, n_2)}(S_1, S_2) = p_{n_1 n_2}. \]  

(12)

Writing \( \rho_p \) as a direct sum of density matrices, one for each possible configuration of the random fields on the cluster:

\[ \rho_p = \bigoplus_{n_1, n_2 = 0, 1} p_{n_1 n_2} \tilde{\rho}_p^{(n_1, n_2)} \]  

(13)

the constraint (12) becomes

\[ \text{Tr} \tilde{\rho}_p^{(n_1, n_2)} = 1, \quad n_1, n_2 = 0, 1. \]  

(14)

Furthermore, \( \rho_p \) has the obvious symmetry property

\[ \rho_p^{(n_1, n_2)}(S_1, S_2) = \rho_p^{(n_2, n_1)}(S_2, S_1) \geq 0. \]  

(15)

As usual, a (reduced) site density matrix can be obtained from \( \rho_p \) by a partial trace:

\[ \rho_s = \text{Tr}_{\rho \setminus s} \rho_p, \]  

(16)

or, more explicitly,

\[ \rho_s^{(n_1)}(S_1) = \sum_{n_2 S_2} \rho_p^{(n_1, n_2)}(S_1, S_2). \]  

(17)

The pair CVM free energy is then given by

\[ F = U + k_B T \left[ (1 - \nu) \text{Tr}(\rho_s \ln \rho_s) + \frac{\nu}{2} \text{Tr}(\rho_p \ln \rho_p) \right], \]  

(18)

with \( U = \langle \langle H \rangle \rangle_\Delta \) and \( \nu \) coordination number of the lattice, and has to be minimized with respect to \( \rho_p \), with the constraints (14) and (15). Taking derivatives of \( F \) with respect to \( \rho_p \) leads to three systems of equations, which
are linear in the elements of \( \ln \tilde{\rho}_p^{(00)} \), \( \ln \tilde{\rho}_p^{(01)} \) and \( \ln \tilde{\rho}_p^{(11)} \) respectively, and where the elements of \( \ln \rho_s \) can be treated as parameters. Exponentiating the solution and making use of (14) yields

\[
\rho_p^{(n_1,n_2)}(S_1, S_2) = p_{n_1n_2} \sum_{S_1, S_2} e^{\frac{1}{\nu} S_1 S_2 V_s^{(n_1)}} V_s^{(n_2)},
\]

with \( \beta = \nu J / k_B T \) and

\[
V^{(n)}_\pm = e^{-\frac{\beta}{\nu} d_n} \left[ \frac{q_n \pm m_n}{2(1 - q_n)} \right]^\alpha, \quad V^{(n)}_0 = 1,
\]

where \( \alpha = (\nu - 1) / \nu \), \( d_n = \Delta_n / (\nu J) \). The parameters \( m_n \) and \( q_n \), which can be thought of as “local order parameters” at the aerogel interface \( (n = 0) \) and in the interior \( (n = 1) \), are given by

\[
m_n = \tilde{\rho}_s^{(n)}(+) - \tilde{\rho}_s^{(n)}(-), \quad q_n = \tilde{\rho}_s^{(n)}(+) + \tilde{\rho}_s^{(n)}(-).
\]

It can be easily shown that the parameters \( m_n \) and \( q_n \) are solutions of the following equations

\[
\begin{align*}
p_n m_n &= \sum_{n_2, S_2} \left( \rho_p^{(n,n_2)}(+, S_2) - \rho_p^{(n,n_2)}(-, S_2) \right) \\
p_n q_n &= \sum_{n_2, S_2} \left( \rho_p^{(n,n_2)}(+, S_2) + \rho_p^{(n,n_2)}(-, S_2) \right)
\end{align*}
\]

which, together with (19), are the basic results of this section, and the starting point for the analysis of the phase diagram of our model at finite temperature.

Finally, the order parameters of the model are related to our “local order parameters” by

\[
m = p m_0 + (1 - p) m_1 \\
q = p q_0 + (1 - p) q_1
\]

(22)
3. The phase diagram

At zero temperature, several different phases can be found, which will be identified by the set \((m_{00} = q_{00}, m_{01} = q_{01}, m_{10} = q_{10}, m_{11} = q_{11})\), where

\[
\begin{align*}
    m_{n_1 n_2} &= \sum_S \left( \rho_p^{(n_1, n_2)}(+, S) - \rho_p^{(n_1, n_2)}(-, S) \right), \\
    q_{n_1 n_2} &= \sum_S \left( \rho_p^{(n_1, n_2)}(+, S) + \rho_p^{(n_1, n_2)}(-, S) \right).
\end{align*}
\]

These are related to the expectation values of \(S_i\) and \(S_i^2\) respectively when the anisotropy field is \(\Delta_{n_1}\) at site \(i\) and \(\Delta_{n_2}\) in one of its neighbours.

The phase diagram is easily determined by comparing the energies \(U\) of the different phases, and is reported in Fig. 1. Two comments are in order: i) as opposite to the mean field result[1], the boundaries between the different ground states do no longer depend on the probability distribution of the random variables and ii) increasing the level of the approximation we have obtained a more complicated phase diagram, with a larger number of possible ground states. This is reminiscent of what happens in the exact solution at \(T = 0\) of the random field Ising model on a Bethe lattice[11]. Notice that our model (1) reduces to a random field Ising model for \(J = 0\) and \(K \neq 0\) since all spin variables appear squared and thus assuming only the two values 0, 1.

At finite temperature, and for \(\Delta_0 < 0\), it has been shown in the mean field analysis[1] that the phase diagram has a quite rich structure, with a second order transition separating the ordered phases from the disordered one, a first order transition, terminating in a critical point, between the ordered phases above, and several multicritical points. Our approximation yields an even richer structure: indeed we have three different ground states (see dotted line in Fig. 1), \((1111)\) for \(d_1 < 1/2\) corresponding to \(^4\)He present everywhere, \((1110)\) for \(1/2 < d_1 < 1\), which should correspond to \(^4\)He present in the aerogel interface and diffusing near it, and \((1100)\) for \(d_1 > 1\), i.e. \(^4\)He only at the aerogel interface. Again, ordered phases are separated by the disordered (high temperature) one by a second order transition line, for which an equation can be obtained by expanding (23) around \(m_0 = m_1 = 0\). Up to
the first order, the equations for $m_n$ yield

$$\begin{align*}
m_0 &= a_{00}m_0 + a_{01}m_1, \\
m_1 &= a_{10}m_1 + a_{11}m_1,
\end{align*}$$

where

$$a_{\gamma\lambda} = \alpha \left[ \delta_{\gamma\lambda} + 4 \frac{p_{\gamma\lambda} V^{(\gamma)} V^{(\lambda)} }{p_\gamma q_\lambda Z_{\gamma\lambda}} \sinh \frac{\beta}{\nu} \right].$$

The equation for the critical temperature turns out to be

$$(a_{00} - 1)(a_{11} - 1) = a_{01}a_{10},$$

meaning that a non trivial solution of eq. (24) exists, where $q_0$ and $q_1$ must be determined from the corresponding equations with $m_0 = m_1 = 0$. A solution for the critical temperature is obtained for any $d_1$ only if $p$ is greater than the percolation threshold $p_c$.

The percolation threshold can be defined as follows: in the limit $\Delta_1 \to +\infty$, $S_i$ will be 0 for all sites with anisotropy $\Delta_1$ and our model will be equivalent to a random-diluted BEG model with anisotropy $\Delta_0$ and concentration $p$. Such a model will undergo a second order transition at $T_c > 0$ for any large enough $p$, and the percolation threshold is just that value of $p$ for which $T_c$ becomes zero. It can be obtained by taking the limits $\Delta_1 \to +\infty$ and $\beta \to +\infty$ in the equation for the critical temperature, and the result is

$$p_c = \frac{1}{(\nu - 1)(1 + \epsilon)},$$

representing a remarkable improvement with respect to the mean field theory, which gives $p_c = 0$. Notice that for uncorrelated disorder, i.e. $\epsilon = 0$, the exact result for the Bethe lattice with coordination $\nu$ is recovered[12].

For $p > p_c$ the phase diagram has the structure shown in Figs. 2(a) and 2(b) (heavy lines): solid lines are used for the second order transition, while dashed lines stand for the first order transition separating (1111) and (1110) phases (if in the $(d_1, \tau = \beta^{-1})$ plane) or for the corresponding coexistence region (if in the $(x, \tau)$ plane, where $x$ is the $^3$He concentration). $C$ is a critical point. No first order line separating (1110) and (1100) phases seems to start
from the point $F(d_1 = 1, \tau = 0)$, where a first order transition occurs at zero temperature. Again this is reminescent of the behaviour of the random field Ising model on the Bethe lattice[11]. The phase diagram at $T > 0$ is qualitatively the same as that given by mean field theory, which is reported for comparison (light lines). In all considered cases ($\Delta_0 < 0$) $x < 1 - p$.

For $0 < p < p_c$ a new feature arises, that cannot be described by mean field theory: the second order line has a limit point $F$ at zero temperature and $d_1 = 1$, independent of $p$, $\epsilon$ and $d_0$, and the critical temperature is zero for any $d_1 > 1$, meaning that, even if the ground state is ordered, aerogel concentration is too low for the pore–grain interface to sustain an ordered phase at finite (i.e. nonzero) temperature. The corresponding phase diagrams are given in Figs. 3(a) and 3(b) and in Figs. 4(a) and 4(b), for typical values of the parameters. In the latter case the second order line is reentrant and intersects the first order line in the critical end points $A$ and $B$. It is easily realized that there exists a value $p^* < p_c$ of $p$ for which the points $A$ and $B$ merge into one, giving rise to a new multicritical point where the second and the first order lines are tangent with respect to each other. It is also interesting to remark that the first order transition has always a reentrant behavior, a feature that does not exist in the mean field treatment[1].

In our scheme the behavior of order parameters and n.n. correlation functions can be obtained in a natural way: in Fig. 5 we report the behavior of the local order parameters $m_n$ and $q_n$ vs. temperature for the case $1/2 < d_1 < 1$ (dotted line in Fig. 3(a)), where $m_0 = q_0 = 1$ and $m_1 = q_1 = p_{10}/(1 - p)$ in the ground state. It is interesting that $m_0$ and $m_1$ exhibit a quite unusual oscillation. Furthermore, in Fig. 6, we report the global order parameter $m$ vs. temperature at fixed concentration, corresponding to the dotted line in Fig. 2(b).

Finally, let us discuss the effect of a correlation $\epsilon > 0$ in the random distribution on the phase diagram, and especially on the second order transition. In the mean field analysis[1] it was shown that the second order transition occurs at $\tau = 1 - x = q$, independent of $p$, while our analysis shows that the second order critical temperature at fixed concentration is indeed weakly
dependent on $p$ (Fig. 7). Introducing a correlation $\epsilon$ of order unity in the random distribution of the $\Delta_i$'s causes the critical temperature to raise significantly, especially in the low-$q$ (high-$x$) region (Fig. 8), and we believe that this effect should account for the experimentally observed[7] increase in the critical temperature with respect to the pure case. The behavior of $T_c$ as a function of $\epsilon$ is shown in Fig. 9.

4. Conclusions

A model for the critical behavior of $^3$He–$^4$He mixtures in a porous medium (aerogel), namely the random–anisotropy Blume–Emery–Griffiths model[1], has been investigated in the pair approximation of the cluster variation method. This improves the mean field analysis given in [1] in several directions. A rather rich phase diagram is found already at zero temperature, with various ground states corresponding to ($\Delta_0 < 0$): $^4$He present everywhere, (1111) in Fig. 1, at the aerogel interface and in a neighbour of it (1110), and only at the aerogel interface (1100). The symmetric situations, with $^3$He preferentially near aerogel are of course possible within the context of a model (Fig. 1 for $d_0 > 1/2$).

A nonzero percolation threshold can be determined, above which the phase diagram is mean field like, and below which it changes qualitatively, with the appearance of a zero temperature limit point for the second order transition line. A completely open and extremely interesting question concerns the universality class of the whole line of continuous transitions: is there a zero temperature or $d_1 = +\infty$ fixed point attracting a finite part of the critical line and an unstable fixed point at finite temperature separating two different critical behaviors (e.g. $d_1 < 0$ and $d_1 \approx 1$)? A real space renormalization group analysis might be suitable to address this problem.

In all considered cases, the first order transition line (in the temperature–anisotropy phase diagram) and the coexistence region (in the temperature–concentration phase diagram) exhibit reentrant behavior. This reentrance is also partially responsible for the existence of a miscibility gap on the high $^3$He concentration side going down to zero temperature.
The experimental phase diagrams are in remarkable agreement with the ones presented in figs. 2(b) and 3(b). Quantitative comparisons would of course require more adequate treatments of the superfluid transition.

Particular attention has been devoted to the analysis of the effects of a nearest neighbor correlation in the random distribution of the anisotropy, which should not be neglected due to the high correlation characterizing the aerogel interface. The main result is the increase in the critical temperature with respect to the pure case, contributing to the detachment of the $\lambda$ line from the coexistence curve and which has been experimentally observed [7] and cannot be explained in terms of the uncorrelated randomness alone. Correlations in the quenched impurities make the question of the universality class of the critical line more involved even at low disorder concentration.

**Acknowledgements**

We are indebted to Jayanth Banavar, Moses Chan and Flavio Toigo for enlightening discussions. We also thank Moses Chan for sending us the results of ref. [7] before publication.
References

[1] A. Maritan, M. Cieplak, M.R. Swift, F. Toigo and J.R. Banavar, Phys. Rev. Lett. 69 (1992) 221.

[2] J. Fricke, Aerogel in Scientific American 258 (1988) 92.

[3] T.M. Tillotson, L.W. Hrubesh and I.M. Thomas, Partially Hydrolized Alkoxy silanes as Precursors of Silicon Aerogels, in Better Ceramic through Chemistry III, C. Brinker, D. Clark and D. Ulrich ed., Mat. Res. Soc. Proc. 121 (1988) 685.

[4] D.W. Schaefer, C.J. Brinker, D. Richter, B. Farago and B. Frick, Phys. Rev. Lett. 64 (1990) 2316.

[5] A. Wong, Liquid–Vapor Critical Point of Fluids in Porous Glasses, Ph.D. thesis, Penn State University (1991).

[6] M. Blume, V.J. Emery and R.B. Griffiths, Phys. Rev. A 4 (1971) 1071.

[7] J. Ma, S. Kim and M.H.W. Chan, in preparation.

[8] R. Kikuchi, Phys. Rev. 81 (1951) 988; G. An, J. Stat. Phys. 52 (1988) 727; T. Morita, J. Stat. Phys. 59 (1990) 819.

[9] T. Morita, Progr. Theor. Phys. Suppl. 80 (1984) 103; see also H. Falk, J. Phys. C 9 (1976) L213.

[10] Ph. de Smedt, J.O. Indekeu and L. Zhang, Physica A 140 (1987) 450.

[11] R. Bruinsma, Phys. Rev. B 30 (1984) 289.

[12] M.E. Fisher and J.W. Essam, J. Math. Phys. 2 (1961) 609.
Figure captions

Fig. 1 : Phase diagram at $T = 0$.

Fig. 2(a) : Phase diagram in the $(d_1, \tau)$ plane for $\nu = 6, \epsilon = 0, p = 0.21 > p_c,$ $d_0 = -0.5$. Heavy lines are our results and light lines are results from mean field approximation.

Fig. 2(b) : Phase diagram in the $(x, \tau)$ plane for the case of Fig. 2(a). Symbols as in fig. 2(a) (for the dotted line see Fig. 6).

Fig. 3(a) : The same as Fig. 2(a) with $p = 0.19 < p_c$ (for the dotted line see Fig. 5).

Fig. 3(b) : Phase diagram in the $(x, \tau)$ plane for the case of Fig. 3(a).

Fig. 4(a) : The same as Fig. 2(a) with $p = 0.1 < p_c$.

Fig. 4(b) : Phase diagram in the $(x, \tau)$ plane for the case of Fig. 4(a).

Fig. 5 : $m_n$ and $q_n$ vs. $T$ along the dotted line in Fig. 3(a) ($d_1 = 0.65$).

Fig. 6 : $m$ vs. $T$ along the dotted line in Fig. 2(b) ($x = 0.69$).

Fig. 7 : The critical temperature in the $(x, \tau)$ plane for $\nu = 6, \epsilon = 0, d_0 = -0.5$ and $p = 0$ (solid line), 0.1 (dashed line) 0.2 (dotted line).

Fig. 8 : The critical temperature in the $(x, \tau)$ plane for $\nu = 6, d_0 = -0.5$, $p = 0.1$ and $\epsilon = 0$ (solid line), 0.5 (dotted line), 1.0 (dot–dashed line) and 2.0 (dashed line).

Fig. 9 : The critical temperature vs. $\epsilon$ for the case of Fig. 7 and with $x = 0.6$. 