A COMPARISON OF ALTERNATIVE CRITERIA FOR DEFINING FUZZY BOUNDARIES ON FUZZY CATEGORICAL MAPS

ZHANG Jingxiong
Roger P. Kirby

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ABSTRACT This paper provides a brief introduction to the methods for generating fuzzy categorical maps from remotely sensed images (in graphical and digital forms). This is followed by a description of the slicing process for deriving fuzzy boundaries from fuzzy categorical maps, which can be based on the maximum fuzzy membership values, confusion index, or measure of entropy. Results from an empirical test performed in an Edinburgh suburb show that fuzzy boundaries of land cover can be derived from aerial photographs and satellite images by using the three criteria with small differences, and that slicing based on the maximum fuzzy membership values is the easiest and most straightforward solution. This, in turn, implies the suitability of maintaining both a crisp classification and its underlying certainty map for deriving fuzzy boundaries at different thresholds, which is a flexible and compact management of categorical map data and their uncertainty.

1 Introduction

Categorical maps represent an important type of data incorporated in GISs, which depict spatial distributions in form of exhaustive, non-overlapping areal units separated by boundary lines. An assumption underlying conventional categorical mapping is the object-based view of reality, that is, the real world can be regarded as occupied by a set of discrete point, line and areal objects (Goodchild, 1989). Clearly, an object-based model is suitable for spatial entities whose boundaries are well defined and for which attributes are exactly valued (NCD-CDS, 1988). Land parcel data provide a good example, where attributes for individual parcels such as ownership, land price and tax liability can be exactly evaluated, and boundary lines can be measured with high precision.

However, pure points, lines and areas do not exist in the real world. For instance, adjacent polygons on categorical maps are rarely separated by sharply-defined boundaries of zero width as might be interpreted in geometric terms of lines (Mark and Csillag, 1989). This means that boundaries separating adjacent areal units on categorical maps should be conceived as transition zones and as such, positional errors in boundaries need to be accommodated by using epsilon error band models (Chrisman, 1982; Guptill and Morrison, 1996; Kiiiveri, 1997). Epsilon error band models were proposed by Perkal (1956) and further tested by many authors including Chrisman (1982), Blakemore (1984), Drummond (1987), Dunn et al. (1990) and recently by Aspinall and Pearson (1994). Estimation of epsilon error band widths is mainly based on checking the test positional data against an independent set of reference data (Goodchild and Hunter, 1997).
There are many phenomena which are poorly defined and spatially varying at all scales, and therefore fuzzy (Altman, 1994; Burrough, 1996), for example, a variety of detail and complexity in many geographical phenomena such as land cover and soil results in many mixed pixels on remotely sensed images of coarse spatial resolutions (Campbell, 1987). Aerial photographs of fine quality do not necessarily resolve all the detail required, and even as the scales increase, spatial heterogeneity in the real world will still exist. Suppose a class of grassland is found for two patches A and B. It is not unusual to find that the growth of grass at these two patches is distinctively different. Even within each patch, the condition is not quite the same, no matter how fine the level of detail is applied. Therefore, a model based on discrete objects hardly works well for fuzzy phenomena, a more general mechanism for depicting the fuzziness inherent in many phenomena is through fuzzy set theory as opposed to crisp set theory (Kaufmann, 1975). For this reason, a more sensible representation scheme of categorical maps is via fuzzy categorical maps where each location possesses multiple and partial memberships, thus fuzzy membership values, to all the candidate classes under consideration.

The usefulness of fuzzy categorical maps had been promoted by researchers such as Fisher and Pathirana (1989), Berry (1993) and Lowell (1994). Widely recognised superiority of fuzzy over crisp categorical maps includes their representational and analytical capability, though there is an increased requirement for storing, in the case of c classes, c layers of raster data as opposed to l layer of vector data (Zhang, 1996). While this extra cost for expanded storage may well be justified by the need for information on errors in spatial databases, it is useful also to seek compactness in storing categorical maps by employing discrete objects while maintaining as much fuzziness as possible. Thus, an interesting question arises as to how fuzzy boundaries may be derived from fuzzy categorical maps. Fuzzy boundaries are referred here to as boundaries of nonzero widths on defuzzified, i.e., classified fuzzy categorical maps. Research had been carried out for modelling fuzzy boundaries of different natures (Edwards and Lowell, 1996; Wang and Hall, 1996). Burrough (1996) and Lagacherie et al. (1996) represented two of the few examples of modelling fuzzy boundaries on the basis of a more theoretical framework. They advocated the use of continuous fields as opposed to discrete objects in an attempt to model uncertainties, which had been addressed by Goodchild (1989).

As an extension to the work by Kirby, Zhang and Du (1997), this paper seeks to define possible ways, by which fuzzy boundaries may be derived from fuzzy categorical maps. The utility of the proposed approaches is illustrated by examples in the context of suburban land cover mapping, in which real data sets of both raster and vector data formats over an area of continuum of land cover types (some are well-defined, others are less so) were incorporated. The next section will describe the concepts underlying fuzzy categorical maps and fuzzy boundaries. Emphasis is placed on how fuzzy boundaries can be generated from the slicing process by using three criteria; the maximum fuzzy membership values, confusion index, and measure of entropy. This is followed by a section detailing on the test carried out, discussing the process of data acquisition, derivation of fuzzy categorical maps and fuzzy boundaries. Discussion of the results obtained is given. And finally the conclusion is presented.

2 Concepts and methods

2.1 Fuzzy categorical maps

The basis for fuzzy categorical maps is the concept of fields. A field-based model conceives the real world as a set of single-valued functions defined at each location. Both numerical and categorical variables are relevant. For the former we have elevation and rainfall as examples, and the latter exemplified by land cover and soil type (Goodchild, 1989). For categorical variables, every point of a field is concerned with a discrete outcome such as a nominal or an ordinal label in a classification system. Suppose c classes are possible for a categorical variable. It is suitable to view this variable as multi-categorical field $p_i(x)$, where $p_i(x)$ represents the probability of point $x$ belonging to a candidate class $i (i =$
1, 2, ..., c). It is required that the probabilities range from 0.0 to 1.0, and sum to 1.0 across all the classes for a particular case, i.e., a point. As fuzzy meaning may be more realistic and sensible, the values of \( p_i(x) \) are often known as fuzzy membership values (Lowell, 1994).

Shown in Fig. 1 is a comparative illustration of an object-based variable versus a multi-categorical field-based representation of a categorical variable with four possible classes: A, B, C and D.

![Object Model](image1)

![Multi-Categorical Field Model](image2)

Fig. 1 Two distinctive views of categorical mapping

(a) object model

(b) multi-categorical field model

The key to the derivation of fuzzy categorical maps is the process of fuzzy classification, which, in turn, relies on defining appropriate fuzzy membership functions (Klir and Yuan, 1995). For categorical mapping such as land cover mapping, conventionally used methods include semiautomatic computerised classifications of digital images and visual interpretation of graphical images. Various methods exist for deriving the fuzzy membership values for each type of classification. When using digital images, a typical method for fuzzy classification is the so-called fuzzy c-means clustering (Bezdek et al., 1984), which seeks to optimise the partition of observations (pixels) among target classes by minimising a certain distance measure adopted. Such a process assigns fuzzy membership values for each pixel belonging to all the candidate classes in an iterative way. It is briefly illustrated in Fig. 2 below, while a detailed discussion can be found in Bezdek et al. (1984).

![Interpolation Process](image3)

Fig. 2 An example of fuzzy 3-means clustering in a two-dimensional spectral space with dash lines representing contours of fuzzy membership values

For graphical images such as aerial photographs in graphical (not digital) form, the derivation of fuzzy classification takes place in spatial rather than spectral domain. The basis for deriving fuzzy membership values lies on the recognition that spatial heterogeneity is central to many naturally occurring phenomena; in land cover mapping, for example, homogeneous locations can be identified more accurately than heterogeneous, transitional zones. This fact implies the plausibility of spatial interpolation methods for deriving fuzzy membership values at heterogeneous locations, when a set of classified samples at homogeneous locations given.
signed by a membership value of 1.0 to their respective classes. At \( x \) (Fig. 3(a) and Fig. 3(b)), the membership will reflect the component probability of each adjacent polygon. Fuzzy membership value will decrease when moving from the centre towards and beyond the boundaries until it reaches 0.0 in the centre of each adjacent polygon. Polygon boundaries are seen somewhere within the transitional zones indicated by dashed lines in Fig. 3(a). The changing pattern of class probabilities along a transect may be modelled by some function; for example, fuzzy membership functions for finding 1 or 2 along the transect of \( C_1 \) to \( C_2 \) are shown in Fig. 3(b), where \( p(d_1/1) \) stands for class 1 at a distance of \( d_1 \) away from the centre \( C_1 \), while \( p(d_2/2) \) stands for the fuzzy membership value of class 2 at a distance of \( d_2 \) away from the centre \( C_2 \). A theoretically more sound approach is via indicator kriging, which is described in Bierkens and Burrough (1993a and 1993b) and applied in Zhang and Kirby (1997), but not covered further here.

\[
\begin{align*}
\text{(a)} & \\
\text{(b)} &
\end{align*}
\]

Fig. 3 The process of interpolating on fuzzy maps

2.2 Fuzzy boundaries

Suppose that fuzzy categorical maps are derived by using any of the methods for fuzzy classification described in the previous section. Then, denote a vector

\[
P(x) = (p_1(x), p_2(x), \ldots, p_c(x))
\]

where \( p_i(x) (i = 1, 2, \ldots, c) \) are the fuzzy membership values of location \( x \) belonging to class \( i \), and hence comprise a set of fuzzy maps with a total \( c \) classes. Though provision of complete fuzzy membership values for individual locations offers analytical advantage in the handling of fuzziness in categorical mapping, it is useful also to seek compactness in storing categorical maps while maintaining as much fuzziness as possible. This can be done by deriving fuzzy boundaries for a crisp classification represented by discrete area objects.

To produce a crisp classification, an analogy with the classification of raster-based remotely sensed images using the maximum likelihood classifier is helpful: the maximum likelihood classifier assigns pixels to classes to which they have the maximum probability of belonging, measured by specific class membership functions. Similarly, for fuzzy categorical maps with readily available fuzzy membership values for individual grid cells, to generate a conventional maximum likelihood classification, vector \( P(x) \) is subjected to a maximisation process, by which cell \( x \) is labelled as the class having the maximum values. For example, cell \( x \) is to be classified into class \( j \) on the condition as expressed in Eq. (2):

\[
p_j(x) = \max(p_1(x), p_2(x), \ldots, p_c(x)), \quad j = 1, 2, \ldots, c
\]

where class labels \( j \)'s form the classified data layer.

During a conventional crisp classification, information contained in a fuzzy vector \( P(x) \) is filtered out, leaving only the class labels having the maximum fuzzy membership values for individual locations. Boundaries in the resulting categorical map are defined where classes are separated, as shown, for example, in Fig. 3. In order to acknowledge the spatial heterogeneity of class membership in the classified map, information contained in fuzzy membership values should be explored.

Firstly, the maximum fuzzy membership values of individual locations can be maintained to assist in defining fuzzy boundaries. Denote the maximum fuzzy membership values by \( p_{\max} \). The process defining fuzzy boundaries can be done via a slicing
process, by which $p_{\text{max}}$ is examined with reference to a prescribed threshold $r$ (Zhang, 1996). Specifically, this processing is so performed that a location $x$ is selected if the value of $p_{\text{max}}$ is less than value $r$. This is illustrated in Fig. 4(a) where a two-class example is developed for the profile along $C_1$ and $C_2$ in Fig. 3 (Fig. 4(b) and 4(c) will be discussed below).

Secondly, there is the criterion known as confusion index for defining fuzzy boundaries, which involves two fuzzy membership values for each location, thus it becomes more complex than the previous criterion. Burrough (1996) reported a rare example of deriving fuzzy boundaries from a simulated set of fuzzy maps. He employed the concept of confusion index, which is evaluated by $1.0$ minus the difference between the fuzzy membership values of location $x$ belonging to the first most likely and the second most likely classes. The assumption underlying such an index is that the greater the confusion index, the smaller the difference in fuzzy membership values between the first and the second most likely classes, the fuzzier location $x$, and thus the more likely that location $x$ defines a fuzzy boundary. Usually, a threshold $t$ is applied so that location $x$ defines a fuzzy boundary if its confusion index is greater than a pre-defined threshold $r$, as shown in Fig. 4(b).

Finally, there is the third, also the most complex criterion, i.e., the measure of entropy, for defining fuzzy boundaries, which uses the complete fuzzy membership values for each location. Foody (1995) described the use of entropy for evaluating the degree of fuzziness for fuzzy maps. Measures of entropy express the way in which the probability of class membership is partitioned between the classes. It is based on the assumption that in an accurate classification each location will have a high probability of membership in only one class. Large values indicate low accuracy in classification, while small values indicate high accuracy in classification. Then, it is logical to assert that boundaries usually occur where locations have high degrees of fuzziness, that is, big values of entropy. Entropy $H(p(x))$ is measured using Eq. (3)

$$H(p(x)) = -\sum_{i=1}^{c} p_i(x) \log^2 p_i(x)$$

where $p_i(x)$ is the fuzzy membership value of grid cell $x$ belonging to class $i$, where the index $i$ ranges from 1 to $c$ (the total number of classes). Again, a threshold $t$ is applied so that location $x$ defines a fuzzy boundary if its measure of entropy is greater than a pre-determined threshold $r$, as shown in Fig. 4(c).

![Fig. 4](image_url) Three criteria applied in slicing process for deriving fuzzy boundaries

3 An empirical test

3.1 The test site and the data sources

In the test, land cover data, as a classic example of categorical data, were used to evaluate the performance of different criteria in defining fuzzy boundaries from fuzzy categorical maps, and to investigate whether these approaches provide similar, if not identical, results. The chosen test site is an area of about 2 square kilometres, located within the city of Edinburgh, around Blackford Hill, it is shown in
Fig. 5. There are a wooded valley, residential, commercial and academic buildings, road networks and footpaths, recreational areas, a small lake, agricultural fields and worked allotments, hills and flat ground, as shown in Fig. 5. The residential districts are built densely together: the roads, the pavements, the roofs, the walls and the hedges exist in complex spatial arrangements, creating difficulties for the interpreter of suburban photography. Further difficulties arise from the indistinct nature of boundaries between land cover types. For example, on Blackford Hills, the dispersed individual trees or groups of trees blend into adjacent land cover types. It appears that the study area chosen provides a good environment with significant fuzziness to test the alternative criteria for defining fuzzy boundaries.

Ground control points (GCPs) consist of field surveyed control points, densified points using photogrammetric block adjustment based on 1:5 000 scale aerial photographs, and those digitised from Ordnance Survey large scale plans (Zhang, 1996). This set of GCPs is sufficient for photogrammetric digitising and remote sensing image rectification.

3.2 Deriving fuzzy maps of land cover

In order to provide a layer of reference data, photogrammetric plotting was performed on the basis of reconstituted stereo photographic pair. The USGS land use and land cover classification system for use with remote sensing data was used in the following classes appropriate to the scene (Anderson et al., 1976):

1. grass (park and grass land);
2. built-up (built-up and barren land);
3. wood (wooded land, no distinction made between deciduous and coniferous woodland);
4. shrub (shrub land, including open wooded land), and
5. water (water bodies and water works).

For both SPOT HRV and Landsat TM data, a fuzzy clustering algorithm based on Bezdek et al. (1984), programmed in FORTRAN 77 on VAX/VMS, was used to produce fuzzy membership vectors across the 5 target classes pixel by pixel within the study area (Zhang 1996). To generate fuzzy maps from aerial photographs, indicator kriging is a recommendable approach to estimate the probabilities of all candidate land cover types occurring at other uncertain locations (Zhang and Kirby 1997). Indicator kriging is supported in geostatistical package GSLIB, which was employed to generate fuzzy maps based on a set of representative and classified samples taken from aerial photographs (Deutsch and Journel, 1992). The outputs from both fuzzy c-means clustering and indicator kriging were transformed to ASC format files via some written FORTRAN programs, which could be loaded to ARC/INFO GRID data files in order to facilitate data management and analysis.

3.3 Deriving fuzzy boundaries

After the fuzzy maps are produced, it is possible to apply the three criteria described in Section 2.2 to derive fuzzy boundaries. As one of the main purposes of this paper is to check if the three criteria produce similar results with respect to the fuzzy boundaries defined, it is necessary to require that the numbers of classified grid cells remain the same when using different criteria for slicing.

Towards this aim, layers of the maximum fuzzy membership values, confusion index and measure of entropy for each set of fuzzy maps derived in Section 3.2 are sliced into 30 intervals of equal areas, and are labelled with the serial number of intervals from 1 to 30 accordingly. The number 30 was chosen for statistical reason. This process results in 3 labelled maps for each set of fuzzy maps based on the 1:24 000 scale aerial photographs, SPOT HRV...
data and Landsat TM data respectively. These labelled maps can be easily used to create sliced maps by applying integer thresholds ranging from 1 to 30. The sliced maps with the same threshold for each set of fuzzy maps are then overlaid with each other, creating 30 overlaid maps for each combination. Their average agreements are reported in Table 1.

Table 1: Average agreements for fuzzy boundaries defined by using different criteria

| FMVs = fuzzy membership values | the maximum FMVs | the maximum FMVs | confusion index versus entropy | versus entropy |
|--------------------------------|------------------|------------------|--------------------------------|---------------|
|                                | 99.6             | 92.1             | 92.1                           |
|                                | 96.5             | 91.8             | 93.9                           |
|                                | 95.2             | 90.4             | 91.5                           |

Note: Fuzzy maps based on Aerial photographs: 1:24 000 scale, SPOT HRV data, Landsat TM data.

It is shown in Table 1 that the average agreements between fuzzy boundaries defined by use of the three different criteria all are very great, more than 90%, for fuzzy maps based on the 1:24 000 scale aerial photographs, SPOT HRV data and Landsat TM data. This suggests that the three criteria for defining fuzzy maps are quite similar in terms of the position and spatial extent of resulting fuzzy boundaries. At this stage, a specific example is interesting. Suppose that half of the grid cells are classified. The thresholds used in different methods are listed in Table 2 below.

Table 2: Thresholds for deriving fuzzy boundaries

| FMVs = fuzzy membership values | Thresholds             | confusion index | measure of entropy |
|--------------------------------|------------------------|-----------------|-------------------|
|                                | maximum FMVs           | 0.78            | 0.22              | 0.80              |
|                                | 0.51                   | 0.57            | 1.67              |
|                                | 0.40                   | 0.67            | 1.67              |

Note: Fuzzy maps based on Aerial photographs: 1:24 000 scale, SPOT HRV data, Landsat TM data.

The slicing process creates a kind of categorical maps where classified locations belong to their named classes with, at least, levels of certainty implied in the thresholds applied, and unclassified locations comprise the fuzzy boundaries which should be excluded from evaluating classification accuracy. Take the fuzzy maps created from the 1:24 000 scale aerial photographs as an example. The sliced maps using thresholds indicated in Table 1 (confusion index: 0.22, measure of entropy: 0.80, and the maximum fuzzy membership values: 0.78) are shown in Fig. 6(a), Fig. 6(b) and Fig. 6(c), respectively, while Fig. 6(d) shows the overlaid map of the three sliced maps, where differences among the three sliced maps are indicated by black dots.

A classification accuracy test is usually based on an error matrix, which is constructed by comparisons between the test data and the reference data. From the error matrix, it is possible to derive several useful classification accuracy parameters such as the overall classification accuracy and the Kappa coefficient of agreement (Congalton, 1991). In this case study, classified maps are checked against the assumed reference map constructed via photogrammetric digitising. This processing results in overall classification accuracies listed in Table 3 below for classified fuzzy maps based on the 1:24 000 scale aerial photographs, SPOT HRV data and Landsat TM data respectively.

As can be seen from Table 3, the three criteria for defining fuzzy boundaries yield very similar overall classification accuracies, except for the case of the Landsat TM data, where fluctuation deserving attention is observed: its result is in favour of the maximum fuzzy membership values method. It is thus confirmed that the criterion based on the maximum fuzzy membership values is preferred in defining fuzzy boundaries, when its apparent simplicity given.

Table 3: Accuracy assessment for classified maps excluding fuzzy boundaries (FMVs = fuzzy membership values)

| Overall classification accuracies | the maximum FMVs | confusion index | measure of entropy |
|-----------------------------------|------------------|-----------------|-------------------|
|                                   | 83.7             | 83.7            | 83.6              |
|                                   | 46.7             | 47.9            | 47.8              |
|                                   | 36.6             | 34.6            | 31.1              |

Note: Fuzzy maps based on Aerial photographs: 1:24 000 scale, SPOT HRV data, Landsat TM data.

4 Conclusion

It has been shown that fuzzy boundaries can be
derived, in a quantitative way, using different criteria on fuzzy categorical maps with very similar performances, and the whole process followed permits a theoretically sound and data-driven solution to estimate errors in attributes and boundaries of categorical maps. Interpretation is consistent with both automatic and interpretative classification methods, commonly used in natural resources survey such as land cover mapping. Among all the three techniques assessed, the direct extension of the maximum likelihood classification is the easiest and most straightforward solution.

(a) Confusion index (b) Entropy

(c) Maximum fuzzy membership values

(d) Showing the differences of the three maps

For (a), (b) and (c)

\[\text{grassland} \]
\[\text{builtup land} \]
\[\text{woodland} \]
\[\text{shrubland} \]
\[\text{water bodies} \]
\[\text{unclassified} \]

For (d)

\[\text{difference} \]

Fig. 6 Classified maps of land cover incorporating fuzzy boundaries indicated by unclassified areas, using alternative methods of thresholding

Conventionally used methods for categorical mapping such as land cover mapping include semiautomatic computerised classifications of digital images and visual interpretation of graphical images, as has been explained in this case study. Usually, discrete area objects, i.e., polygons, are employed in categorical maps to represent the two-dimensional distributions under study. In order to extend object-based data models into the domain of fuzzy categorical maps, fuzzy boundaries need to be defined properly. The results obtained in this test implies the suitability of maintaining both a crisp classification and its underlying certainty map for deriving fuzzy boundaries at different thresholds, which offers both flexibility and compactness for managing categorical maps and information on their uncertainties.

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