Theory, Solution Method and Applications of Kinematic Wave

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This study first summarizes the Kinematic Wave Theory which has been used to describe vehicle motions and the Variational Theory which is an efficient solution method to estimate vehicle trajectories based on the Kinematic Wave Theory. Then, several demonstrative applications on a signalized arterial and on an intercity motorway are presented by a data fusion technique using probe vehicle data and conventional traffic detector data.

KEYWORDS: kinematic wave, vehicle trajectory, variational theory, traffic flow

1. Introduction

Kinematic Wave Theory was first proposed by Lighthill and Whitham in 1955 and almost same time by Richards in 1956 and therefore the theory has been called as the LWR theory. Since then, the theory has been commonly used to analyze vehicle motions on a highway. The theory is basically based on the flow conservation in a differential equation form combined with the fundamental diagram specified by highway characteristics. Vehicle motions can be described on a time-space diagram by solving the differential equation. Some later time in 1960s, Newell proposed a three-dimensional representation of traffic flow that combined a time-space diagram with a cumulative curve introducing three dimensional axes: time, space and cumulative trips. The 3-D representation is a very convenient way to describe traffic encompassing queueing phenomena and vehicle motions. Then, recently in 2005, Dagnzo proposed an efficient solution method to find out vehicle trajectories based on the Kinematic Wave Theory on the 3-D surface. Once vehicle trajectories are estimated, they can be used for several purposes including travel time estimation and prediction, signal coordination and emission monitoring. In this study, the method is applied to real world data and its robustness is confirmed through a couple of sensitivity tests.

This manuscript first briefly summarizes these historical studies and demonstrates several applications of the theory to a signalized arterial and an intercity motorway. A unique feature of the applications is the data fusion of different sensing data such as probe vehicle data, traffic detector data and signal timing data. Probe vehicles provide spatial traffic information and direct measurements of travel time. However nowadays, their uplink frequency is restricted and there is sometimes GPS reception errors included in the data. On the other hand fixed sensors (ultrasonic and loop detectors) record traffic data continuously but only at fixed locations. Considering different characteristics of these traffic data from various sources, data fusion techniques are applied to estimate trajectories of all vehicles running on a road section with several signalized intersections and on an intercity motorway.

2. Fundamental Theory

2.1 Time-space diagram and cumulative trips

Vehicle Trajectories can be shown on a time-space diagram which is a two-dimensional space of time \( t \) and distance \( x \) of travel. Adding one more dimension of the cumulative trips \( N \) as a vertical axis as shown in Fig. 1, vehicle trajectories are piled up so that the cumulative number of trips \( N(x, t) \) appears at any time \( t \) and distance \( x \). This is the 3-D representation proposed by Newell. If you look at the \( t-x \) plane, you will see vehicle trajectories, but if you look at the \( N-t \) plane, you will also understand the cumulative curves at any location \( x \).

2.2 Kinematic wave

The key feature of the LWR theory is that there is some functional relation between the flow \( q \) and the density \( k \) which might vary with location \( x \) and time \( t \).
Equation (1) is sometimes called ‘fundamental diagram’ and the conservation equation is written as below with no entering or exiting traffic.

\[
\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \frac{\partial k(q(x,t),x)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0
\]

(2)

Newell (1993a, 1993b, 1993c) combined the concept of cumulative curves with the LWR theory and extended it to the three-dimensional kinematic wave theory shown in Fig. 1. Given \(N(x,t)\) as cumulative number of vehicles at location \(x\) by time \(t\), Newell suggested evaluating \(N(x,t)\) rather than \(q(x,t)\). By definition, relationships among flow \(q\), density \(k\) and cumulative number \(N\) is described as below.

\[
\frac{\partial N(x,t)}{\partial x} = -k(x,t) \quad \frac{\partial N(x,t)}{\partial t} = q(x,t)
\]

Assuming a one-dimensional traffic movement where first in-first out (FIFO) condition holds, variations in the cumulative number of vehicles over space can be evaluated through (3).

\[
\frac{dN(x,t)}{dx} = \frac{\partial N}{\partial x} + \frac{\partial N}{\partial t} \frac{dt}{dx} = -k(x,t) + q(x,t) \frac{dt}{dx}
\]

(3)

This means that \(dN(x,t)/dx\) depends on \(dt/dx\), which is the inverse of a velocity on the two-dimensional space \(x-t\) specifying how much time \(t\) shifts when location \(x\) changes. Suppose that we evaluate \(dN(x,t)/dx\) along the characteristic curve; that is, \(dx/dt = \partial q(x,t)/\partial k(x,t)\), Eq. (3) is rewritten as follows.

\[
\frac{dN(x,t)}{dx} = -k(x,t) + q(x,t) \frac{dt}{dx} = -k(x,t) + q(x,t) \frac{\partial k}{\partial q} = -k(x,t) + q(x,t)/w(x,t)
\]

(4)

where \(w(x,t)\) = wave speed = \(\partial q(x,t)/\partial k(x,t)\)

This result is graphically shown that the value of \(-dN(x,t)/dx\) is found at an intersection of the wave speed and density axis on Fig. 2.

In particular with a piece-wise linear fundamental diagram as shown in Fig. 3, \(dN\) takes only two values: 0 for the forward wave and \(-k_{jam}dx\) for the backward wave in which \(k_{jam}\) is the jam density.

\[
dN(x,t) = \begin{cases} 
0, & \text{if } (x,t) \text{ lies in free flow region} \\
-k_{jam}dx, & \text{if } (x,t) \text{ lies in congested region}
\end{cases}
\]

(5)

2.3 Variational theory

Variational theory (Daganzo, 2005a, b) provides an efficient solution method to estimate \(N\) at any point \((x,t)\) in time-space given appropriate boundary conditions at \((x_0,t_0)\). Let us start the discussion from the relative capacity observed by a moving observer.
Relative Capacity

A highway has the fundamental diagram \( q(k, t, x) = q(k, x, t) \) in which \( q(k, x, t) \) is flow as a function of density \( k(x, t) \) and location \( x \). If you stand at the roadside at location \( x \), you observe flow \( q(k, x, t) \) at time \( t \). However, if you also move at speed \( u \), your observed flow would be \( q - ku \) [vehicles/unit time]. This is the moving observer.

Considering a moving observer, let us estimate the maximum observed flow called Relative Capacity; that is, the maximum flow possibly observed by a moving observer. Apparently, if a moving observer does not move, the Relative Capacity is the same as \( q^{\text{max}} \). Similarly, if a moving observer travels at speed of \( u \), the Relative Capacity is evaluated as

\[
\text{Relative Capacity} = \sup_k \{q(k, t, x) - k(x, t) \cdot u(x, t)\} \quad \text{with respect to } k
\]  

As shown in Fig. 4, \( q(k, x, t) - ku \) takes the maximum when the wave speed is equal to the speed of the moving observer.
(2) Maximum Number of Passing and Cumulative Trips

A moving observer starts moving from point B to P where the cumulative trips are NB and NP as shown in Fig. 5. If the moving observer actually passed by ΔN vehicles from B to P, NP must be NB + ΔN. Using Relative Capacity evaluated above, the maximum number of passing denoted as ΔBP by the moving observer from B to P is

\[ \Delta BP = \int_{t_B}^{t_P} \sup_{k} (q(k, x, t) - k(x, t) \cdot u(x, t)) \, dt \]

This is the max number of passing, when you travel at speed u, which may change overtime. Since ΔBP is the maximum number of passing,

\[ NP \leq NB + \Delta BP. \]

If we know the cumulative trips at every point B on the boundary, NP must be smaller or equal to NB + ΔBP for all B on the boundary. Therefore,

\[ NP \leq \inf_B \{NB + \Delta BP\}. \tag{7} \]

On the other hand, suppose that you can trace back from P to the boundary along the wave, W. Provided that you successfully reach point B' on the boundary, NP must be written as

\[ NP = NB' + \Delta W. \]

Since, from Eq. (6), \( q(k, x, t) - ku \) takes the maximum when the wave speed is equal to the speed of the moving observer, \( \Delta W \) must be equal to the relative capacity: \( \Delta W = \Delta BP \) and we can write NP as

\[ NP = NB' + \Delta W = NB' + \Delta BP \]

Since B' exists on the boundary,

\[ NP = NB' + \Delta W = NB' + \Delta BP \geq \inf_B \{NB + \Delta BP\} \tag{8} \]

Therefore, from (7) and (8),

\[ \inf_B \{NB + \Delta BP\} \leq NP \leq \inf_B \{NB + \Delta BP\} \rightarrow NP = \inf_B \{NB + \Delta BP\} \tag{9} \]

As a whole, NP is evaluated by taking the minimum value of NB + ΔBP from all boundary points, provided that the boundary can be successfully traced back from point P at the wave speed. Note that the trajectory of W from P to B' may not be a straight line, since the fundamental diagram, \( q(k, x, t) \) could be different at different locations. Only when \( q(k, x, t) \) is the same at all locations, W must be a straight line because flow does not change along a wave.

For the more practical use and also for simplicity, let us consider a homogeneous highway section with a piecewise linear fundamental diagram as shown in Fig. 6. When a moving observer travelling at speed u from point B to P, the observer will count the number of passing vehicles based on the relative capacity. For a homogeneous highway, the number of passing is independent of the path from B to P as shown below.

Maximum Number of Passings = \[ \int_{t_B}^{t_P} \left( 1 - \frac{u}{v} \right) \cdot q_{\text{max}} \, dt \]

= \[ q_{\text{max}} (t_P - t_B) - \frac{q_{\text{max}}}{v} (x_P - x_B) \]

= \[ \begin{cases} 0, & \text{if } u = v \\ \frac{q_{\text{max}}}{u} (x_P - x_B) - \frac{q_{\text{max}}}{v} (x_P - x_B) = k_{\text{max}} (x_B - x_P), & \text{if } u = -w \end{cases} \]

2.4 Solution domain

In Variational Theory the time-space is modeled by a dense discrete network consisting of nodes and short straight valid paths as links with the following two properties: (i) the slopes of links branching from each node which represent feasible wave speeds; and (ii) link costs which represent the allowable change in the cumulative number of vehicles along valid links.

Considering a piece-wise linear fundamental diagram with feasible wave speeds of u (forward wave speed) and \(-w\) (backward wave speed), the solution domain in time-space can be modeled with a lopsided-network in which the mesh resembles triangular fundamental diagrams with identical steps in time-space. The nodes are on a rectangular lattice with space separation \( \Delta x \) and time separation \( \Delta t \). There are sets of links pointing to any node with slopes \( u \) and \(-w\). The coordinate system \( F^c \) is defined with \( i \)-coordinate aligned with the backward wave, and \( j \)-coordinate aligned with the forward wave. The cumulative number of the vehicles at node \((i, j)\), is presented by \( N(i, j) \). From (5), considering the feasible wave speeds \( [u, -w] \), the link cost along the forward wave is zero, which means there is no change in the cumulative number of the vehicles along the forward wave. On the other hand, the link cost along the backward wave is
estimated by multiplying the jam density $k_{jam}$ by $\Delta x$. Given the piecewise linear fundamental diagram and time separation $\Delta t$, the space separation between network nodes $\Delta x$ is estimated from (10).

$$\Delta x = \frac{q_{max} \Delta t}{k_{jam}} = \frac{\Delta t}{\left(\frac{1}{u} + \frac{1}{w}\right)}$$  \hspace{1cm} (10)

2.5 Shortest path algorithm from boundary nodes

In order to estimate the value of $N$ at each node, appropriate boundary conditions should be set in the solution network. As shown in Fig. 7, the solution domain is bounded within upstream (CD) and downstream (AB) boundaries.
Considering the passing times of the vehicles, the cumulative traffic counts at the upstream and the downstream are assigned to the relevant nodes along the upstream and the downstream. Vehicle passing times can be retrieved from loop detector data or video surveys. Additional boundary conditions can be set by using probe trajectory data. Any probe trajectory can be approximated in time-space plane by using the nearest nodes to its path. Considering the FIFO discipline, \( N \) along a probe trajectory path on the solution network is a constant value. Therefore, a constant height should be assigned to those nodes which represent a probe trajectory in solution domain. Starting from the boundary nodes, the value of \( N \) at each node in solution domain \( N(i,j) \) can be estimated using the dynamic programming algorithm presented in (11).

\[
N(i,j) = \min\{N(i, j - 1), N(i - 1, j) + k_{jam} \cdot \Delta x\}
\]

The boundary conditions can be reset as soon as newer probe trajectory data become available. Given the boundary conditions at upstream, downstream and the reference probe trajectory in Fig. 7, the basic data fusion framework is only suitable for reconstructing vehicle trajectories from the passage of the reference probe trajectory up to the present time (ABCD area). Application of the basic data fusion model to reconstruct vehicle trajectories in offline condition is demonstrated in Mehran et al. (2012). In the following sections, a methodology is proposed which extends the applicability of the basic data fusion framework to real-time traffic prediction. The proposed extension enables short-term prediction of vehicle trajectories beyond present time. However, before proceeding further, description of the study area and available traffic data for this study are presented.

3. Treatment of In/Out Vehicles in a Study Section

The basic assumption in the kinematic wave theory as well as in the Variational Theory (Daganzo, 2005a; Daganzo, 2005b) is that there are no vehicles entering or leaving the study area from midblock intersections. However, incoming/outgoing vehicles from the midblock sections as well as those joining or leaving the study section from the shops and minor roads directly affect the movement of other vehicles and shape of the estimated trajectories. Mehran and Kuwahara (2011) improved the theory so as to consider incoming/outgoing vehicles.

3.1 Influencing area by an incoming/outgoing vehicle

To proceed further, coordinate system \( F^* \) is implemented in Fig. 8 with the origin set at A which divides the time-space plane into four distinct regions. Considering the time-space plane in Fig. 8, let us discuss the influential area on the cumulative heights by an incoming vehicle trajectory at location A in the midblock of the study section. Suppose that cumulative heights \( N(i,j) \) for all \( i \) and \( j \) have been calculated without considering the incoming/outgoing vehicles in the midblock section. Considering an incoming vehicle at A, since the wave speed is bounded within the range of \([-w, u]\), the followings are true:

Fig. 7. Solution domain and network configuration in time-space.
The cumulative height at A influences only the cumulative heights in Region I.

From the above, it is concluded that the cumulative height at A due to an incoming vehicle influences the cumulative heights only in Region I; that is, \( N(i,j) \) already calculated are not necessarily changed for \( (i,j) \) in Regions II, III and IV.

### 3.2 Treatment to an incoming/outgoing vehicle

Therefore, given the cumulative heights in Regions II, III and IV with a vehicle added at A as in Fig. 8, the heights along \( i \)-coordinate, \( j \)-coordinate and the thick solid and dashed lines at the upstream and the downstream boundaries can be determined. These heights should be employed as the new initial condition to estimate the cumulative heights in Region I. If a vehicle is added at A, the cumulative heights along \( i \)-coordinate and D–E must be increased by 1. The reason is that vehicles passing \( i \)-coordinate and D–E must be those running after the vehicle was added and hence the cumulative heights along \( i \)-coordinate and D–E must be increased by 1. When an outgoing vehicle leaves the study section at A, the cumulative heights along \( i \)-coordinate and D–E must be decreased by 1 instead.

\[
N(i,j) = N(i,j) + \delta \quad \text{for} \quad (i,j) \quad \text{along} \quad i \text{-coordinate and D–E in Fig. 8}
\]

\[
\delta = \begin{cases} 
+1, & \text{if a vehicle is added} \\
-1, & \text{if a vehicle is subtracted} 
\end{cases}
\]

On the other hand, given an incoming vehicle at A, the heights along \( j \)-coordinate and B–C would remain the same. Since the slope along \( j \)-coordinate is the free-flow speed, all the vehicles passing \( j \)-coordinate must be the vehicles running ahead of the vehicle coming in at A. Because an incoming vehicle at A increases the cumulative heights of the vehicles only after itself, the cumulative heights along \( j \)-coordinate would not change. For boundary B–C, since the vehicle counts at the downstream end must already include the incoming vehicles, the cumulative heights along B–C do not have to be changed, too. When an outgoing vehicle leaves the study section at A, based on above discussion, the cumulative heights along \( j \)-coordinate and B–C remain unchanged as well. Given the new initial conditions at the boundaries, the cumulative heights for each node in Region I is found by calculating the shortest paths.

The above procedure is sequentially repeating the setting of the new initial conditions at the boundaries and the calculation of the cumulative heights until the next incoming/outgoing vehicle. However, by changing some of the link costs in advance, we can estimate the cumulative heights at once. If we know there is an incoming vehicle at A, instead of defining new initial conditions at the boundaries, the costs of the links connected with \( i \)-coordinate are increased by 1 and the cumulative heights along D–E should be increased by 1 in advance as shown in Fig. 8.

For an outgoing vehicle at A, the link costs along \( i \)-coordinate and D–E are decreased by 1. If we know the locations of all incoming/outgoing vehicles over time-space plane, these modifications of the link costs as well as the heights of the boundary nodes can be done in advance before start calculating the shortest paths.

Therefore, with these treatments, we could estimate the cumulative heights of all nodes over the study area at once.

### 4. Application to a Signalized Arterial

#### 4.1 Study section

As shown in Fig. 9, a 1.15 km stretch of Komazawa Street in Tokyo, Japan, was considered for the purpose of this...
study. Selected section is a single lane facility and includes six signalized intersections. Analysis period was chosen during the morning peak period from 8:15 to 9:15 on 1 September 2006. A pair of AVI cameras was installed at the entrance and the exit points of the study area to record the number plates of vehicles entering and/or leaving the section. Number plates were matched afterwards in order to estimate travel time during the analysis period. In addition to travel time, AVI data provide passing times of individual vehicles at the upstream and the downstream of the study area. Probe data were collected using GPS equipped probe taxes on the study area. Probe data included the time and the travelled distance at 1 second intervals. Required AVI and probe data were provided from the COSE (Consortium for Software Engineering) project. Traffic signal data including phasing, cycle length and signal timings were provided by Tokyo Metropolitan Police Department for all signalized intersections except for the third signalized intersection, where such data were not available. Locations of the intersections, signal timing patterns and probe trajectories are also shown in Fig. 9.

At this stage it is assumed that there are no vehicles entering or leaving the study area from midblock intersections. Required parameters to estimate vehicle trajectories are summarize in Table 1. The forward wave speed \( u \) was estimated from probe data during the free-flow conditions. The saturation flow rate was estimated using AVI data during the green intervals and an assumed value was considered for the jam density on the study section. The time separation between the network nodes \( \Delta t \) is set to 1 second.

For the treatment of signal red intervals, the cumulative height along a red interval must be the same because no vehicles can leave from the intersection. Therefore, a shortcut link with zero link cost (no increment of cumulative height) is created along the red interval.

| Parameter                      | Value |
|-------------------------------|-------|
| Forward wave speed (km/h)     | 32.4  |
| Saturation flow rate (veh/h)  | 1750  |
| Jam density (Veh/km)          | 165   |

4.2 Result

Figure 10 shows the estimated trajectories when the 10th probe trajectory is used as a reference. Even though the signal timings of the 3rd intersection is not considered, the accordance between the estimated trajectories and the corresponding probe trajectories looks quite good.

The agreement between the estimated and observed trajectories is evaluated by comparing the accordance between the trajectory paths for each set of the probe and its corresponding estimated trajectory. Beginning from the first probe.
trajectory, pairs of adjacent probe trajectories is considered. For each pair, the first probe trajectory is used as a reference and the trajectories are estimated up to the second probe trajectory. Then the second probe trajectory is compared with its corresponding estimated trajectory. The agreement is evaluated as a function of the difference between the entry times of the reference probe trajectory and the estimated trajectory corresponding to the subsequent probe trajectory. Suppose the solution network is consisting of \( n \) rows in time-space. For each pair of the probe and its estimated trajectory, \( T_x(y) \) represents the measured time at space \( y \) for a probe vehicle leaving the upstream boundary at time \( x \) while \( t_x(y) \) refers to the estimated time at space \( y \) for a vehicle leaving the upstream boundary at the same time \( x \). For each set of probe and its corresponding trajectory which leave the upstream at time \( x \), Eq. (13) is used to estimate the Mean Absolute Error (MAE). Afterwards, the relative error (RE) for each set of the probe and the estimated trajectories is calculated from (14).

For each set of probe and its corresponding trajectory which leave the upstream at time \( x \), the Mean Absolute Error (MAE) is estimated as:

\[
\text{MAE}_x = \frac{1}{n} \sum_{y=1}^{n} |T_x(y) - t_x(y)|
\]

and the relative error (RE) is:

\[
\text{RE}_x = \frac{\text{MAE}_x}{T_{T_x}} \times 100
\]

where \( T_{T_x} \) represents the total travel time from the upstream to the downstream measured from the probe trajectory leaving the upstream boundary at time \( x \).

According to Fig. 11, generally when the difference between the entry time of the reference probe trajectory and the estimated trajectory increase, the relative error increases. The results imply that the estimated trajectories are more accurate over shorter intervals. Those trajectories which are closer to a reference probe trajectory are strongly influenced by the shape of the reference trajectory. However, the shape of the farther trajectories is mostly governed by the solution network characteristics. Since proposed methodology is deterministic, stochastic aspects of the traffic flow and the driving behavior factors are not considered. In addition, impacts of incoming/outgoing vehicles from midblock intersections are not considered which result in some discrepancy between the probes and corresponding estimated trajectories.

In this application, since the number of incoming and outgoing vehicles were not observed, the consideration of those vehicles cannot be incorporated. We will however examine the treatment of incoming/outgoing vehicles in an application to an intercity motorway in the later section.

5. Application to an Intercity Motorway

5.1 Study section

As shown in Fig. 12, the length of study section is 64 km between Shiroishi IC and Motomiya IC on the inbound of Tohoku expressway which is running from Aomori to Tokyo. We examine if the proposed method can reproduce traffic congestion from 10:00 to 19:00 on the 14th of August, 2012 in the middle of the Buddhist festival.
As shown in Fig. 13, the available input data are 5-minute vehicle count data measured by five loop detectors located between the six interchanges. Although total 27 probe trajectories are available during the study period, 5 of those are selected for the reference probe trajectories shown in thick lines in the figure. Each of the probe trajectories are
measured every 6 seconds. Vehicle passing times at detector locations are made assuming vehicles pass uniformly during each of 5-minute period. Also, incoming and outgoing vehicles at each of four interchanges (1) to (4) are estimated from vehicles count differences at the adjacent detectors. Note that we cannot estimate the number of individual incoming and outgoing vehicle counts separately at each of the interchanges but only estimate ‘incoming vehicles–outgoing vehicles’ can be estimated.

5.2 Result

Figure 14 shows a result using only information of probe 1 as the reference without considering incoming and outgoing vehicles at interchanges. Figure 15 shows estimates using all of five selected probe information. Although the both results give fairly good estimates of the congested region where the estimated trajectories have slower speeds, estimated trajectories using all probe information as in Fig. 15 shows the better agreement with the probe trajectories measured especially at the end of the study period. In this application, even if a probe trajectory in about every two hours is used, quite reasonable estimates of congested region is obtained.

Furthermore, the method can predict traffic condition in near future. For example in Fig. 14, even based on input data (detector data and probe data) by time A, vehicle trajectories in region ABC can be predicted in which slope AC and BC are wave speed $u$ and $w$ respectively. Therefore, at current present time is A, from the estimated result in region ABC, we can predict time and location caught by the queue for a vehicle departing from the entry at time. This is the strong advantage in the information provision to users about to enter the motorway.

Figure 16 shows estimates using all probe information and also considering incoming and outgoing vehicles at interchanges (1) to (4). Although we expected that consideration of incoming and outgoing vehicles improves the reproducibility, the agreement with observed probe trajectories looks almost the same as one in Fig. 15. Also, we find a problem in estimated trajectories that they are discontinuous at referenced probe trajectories as seen at about 18:00 before and after the probe 4 trajectory. Basically, the same problem must occur when several probe data are used as

![Fig. 14. Estimated trajectories using probe 1 only without considering in/out vehicles.](image1)

![Fig. 15. Estimated trajectories using all probes without considering in/out vehicles.](image2)
references. However, from the visual inspection, this discontinuity problem is more serious in the case considering incoming and outgoing vehicles. Probably, the cause might be some counting errors of those vehicles in the midblock of the study section.

Figure 17 summarizes the above three result in terms of the root mean square error (RMS [sec]) of estimated trajectories compared to observed probe trajectory leaving the entrance at the same time. For the case using only probe 1 information, RMS is the largest and it gets increasing as time goes. Similar to the previous visual inspection, RMS is the minimum in the case shown in Fig. 15 that uses all probes but without considering incoming and outgoing vehicles.

An interesting finding is that trajectories may be reasonably estimated even without considering incoming and outgoing vehicles. Basically, probe trajectories are already reflecting incoming and outgoing vehicles. This may be the reason why the reproducibility without considering incoming and outgoing vehicles is quite reasonable in this case study.

**Summary**

This research proposes a method that estimates trajectories of all vehicles running along a road section taking incoming and outgoing vehicles in the midblock of the study section. The method is then applied to a signalized arterials and to an intercity motorway.
The agreements of the estimated trajectories with observed probe trajectories is in general quite good. Also, since the method can predict vehicle trajectories in near future, it provides drivers several useful forecasted information such as when a driver will be caught by a queue, how long the queue length, etc.

For the future study, we are considering the use of traffic simulator for the analysis. Currently, kinematic wave theory is used to describe vehicle motions. However, there are several restrictions and problems such as no overtaking, no lane changing, discontinuity in estimated trajectories, etc. Use of a traffic simulator could be one of the solution to solve these problems and restrictions. But, a traffic simulator must be modified so that (i) it can describe probe vehicle motion exactly as observed and (ii) it can keep the departure time constraint for every vehicle at the exit of a study section. In general, a standard traffic simulator only considers vehicle leaving times at the entrance. However, to incorporate an impact of a backward wave, a traffic simulator has to be modified so that every vehicle leaves the study section at the given timing.

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