The paper introduces several concepts for super Young tableaux which already exist for classical Young tableaux, and proves fundamental properties about them.

Super Young tableaux are the super analogue of Young tableaux. Here, instead of filling the tableau with integers, one fills the tableau with entries from a signed alphabet. A signed alphabet $S$ is a finite or countably infinite totally ordered set which is the disjoint union of two subsets $S_0$ and $S_1$. A super Young tableaux is a filling of a Young diagram with a signed alphabet $S$ which is weakly increasing with respect to rows and columns. If one denotes the $(i, j)$-th entry of a super Young tableaux $T$ by $T(i, j)$, then we must have $T(i, j) < T(i, j + 1)$ unless $T(i, j) \in S_0$, in which case we may have $T(i, j) = T(i, j + 1)$. Similarly, we must have $T(i, j) < T(i + 1, j)$ unless $T(i, j) \in S_1$, in which case we may have $T(i, j) = T(i + 1, j)$.

Many features of the theory of Young tableaux carry over to the super case. One can define insertion algorithms for inserting entries into super tableaux. Two words in the signed alphabet produce the same tableau via repeated insertion starting from the empty tableau if and only if they are related by “super Knuth-like relations”. One can therefore introduce a super plactic monoid as the quotient of the free monoid on the signed alphabet by the super Knuth-like relations.

As the title of the paper suggests, the primary object introduced by the paper is the super jeu de taquin, a procedure for turning super skew tableaux into super tableaux using forward sliding moves. Key properties of the super jeu de taquin are shown. Firstly, it is shown that the super jeu de taquin is compatible with the super plactic congruence: that is, if one super tableaux can be obtained from another via forward sliding moves, then their row-reading words are super plactic equivalent. Consequently, it is shown that the super jeu de taquin is confluent: all choices of forwarding sliding moves will eventually produce the same super tableau. It is also shown that the super tableaux obtained from concatenating the words of two super skew tableaux are the same as the tableaux obtained from concatenating the two super skew tableaux and applying the super jeu de taquin. A super evacuation procedure is defined, which is a procedure for producing an opposite tableau from a given tableau.

In the final section, a super Littlewood-Richardson rule is obtained, and the super jeu de taquin is described using the growth diagrams introduced by S. Fomin [“Appendix 1: Knuth equivalence, jeu de taquin, and the Littlewood-Richardson rule”, in: R. P. Stanley, Enumerative combinatorics. Volume 2. Cambridge: Cambridge University Press (1999; Zbl 0928.05001)].

Reviewer: Nicholas Williams (Lancaster)

MSC:

05E10 Combinatorial aspects of representation theory
20M05 Free semigroups, generators and relations, word problems
14N15 Classical problems, Schubert calculus

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