Analysis of new charmless strange $B$ decay data leaves high $B \to K\eta'$ and $B \to K\eta'X$ still unexplained

Harry J. Lipkin$^{b,c,*}$

$b$ School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv, Israel

c Department of Particle Physics
Weizmann Institute of Science, Rehovot 76100, Israel
and
High Energy Physics Division, Argonne National Laboratory
Argonne, IL 60439-4815, USA

Abstract

The question of whether the anomalously large $B \to K\eta'$ and $B \to K\eta'X$ (inclusive) branching ratios are consistent with the standard model or a sign of new physics is still open and in the experimental court. The nature of the extra hadron $X$ in $B \to K\eta'X$ is completely unknown. Directions for investigation and analysis of inclusive data are suggested as well as for comparable inclusive data on $B \to K\eta X$.

New data confirming the prediction $BR(K^+\rho^o) = BR(K^+\omega)$ support flavor-topology $B \to VP$ sum rules based on the OZI rule and emphasize the sharp contrast with the failure of the analogous $B \to PP$ sum rule due to the anomalously large $BR(B \to K\eta')$. Confirmation of OZI validity in $B$-decay analyses for VP final states suggests improving data on the analogous neutral decay difference $BR(B^o \to K^o\rho^o) - BR(B^o \to K^o\omega)$ which measures tree-penguin interference and possible direct CP violation.

A successful approximate isospin sum rule is rearranged and reinterpreted to pinpoint tree-penguin interference in $B \to K\pi$ and $B \to K\rho$ transitions. The magnitude of the interference is shown to still be at the statistical noise level with present data.

*e-mail: ftlipkin@weizmann.ac.il
I. TWO INTERESTING SUM RULES AND THEIR IMPLICATIONS

A. An interesting flavor-topology sum rule

The large branching ratio \( BR(B \to K\eta') \approx 70 \cdot 10^{-6} \) still remains a problem [1], together with the large inclusive branching ratio for \( B \to K\eta'X \) (inclusive).

The real problem here is that \( BR(B \to K\eta') \gg BR(B \to K\pi) \). In the standard description of the \( \eta \) and \( \eta' \), their SU(3) octet components belong to the same pseudoscalar octet as the pions. Thus this large difference suggests a contribution to \( BR(B \to K\eta') \) via the SU(3) singlet component of the \( \eta \) and \( \eta' \).

The necessity for this extra contribution is seen in the gross violation of the sum rule [2] which was derived specifically to test for such contributions.

\[
BR(B^\pm \to K^{\pm}\eta') + BR(B^\pm \to K^{\pm}\eta) \leq BR(B^\pm \to K^{\pm}\pi^0) + BR(B^\pm \to \bar{K}\pi^0) \quad (1.1)
\]

where \( \bar{K}\pi^0 \) denotes \( K\pi^0 \) for the \( B^+ \) decay and \( \bar{K}\pi^0 \) for the \( B^- \) decay. The experimental values [3] in units of \( 10^{-6} \) are

\[
BR(K^\pm\eta')(70.8 \pm 3.4) + BR(K^\pm\eta)(2.6 \pm 0.5) \leq BR(K^\pm\pi^0)(12.1 \pm 0.8) + BR(K^\pm\pi^0)(24.1 \pm 1.3) \quad (1.2)
\]

We review here the derivation which exploits known [4] flavor-topology [5] characteristics of charmless strange \( B^\pm \) decays. Common calculations of weak decays are subject to uncertainties arising from unknown contributions of final state interactions. The flavor-topology approach automatically includes the contributions to all orders from all final state interactions described by quark-gluon interactions which conserve flavor SU(3).

The final states considered for \( B^- \) decay all have the quark composition \( s\bar{u}q\bar{q} \) where \( q\bar{q} \) denotes a pair of the same flavor which can be \( u\bar{u}, \ d\bar{d} \) or \( s\bar{s} \). Charm admixture in the final state is not considered.

We first review the flavor-topology properties of all the diagrams that can lead to final states \( KM \), where \( \tilde{K} \) denotes a \( K^- \) or \( \bar{K}\pi^0 \) or any analogous pair of \( K^* \) resonances and \( M \) denotes the members of any meson nonet, labeled \( M_1 \) for the unitary singlet state, \( M_u, M_d \) and \( M_s \) respectively for the \( u\bar{u}, d\bar{d} \) and \( s\bar{s} \) states and \( M^- \) for the \( d\bar{u} \) state.

The \( q\bar{q} \) pair observed in the final two-meson state may come from a very complicated diagram involving many quarks and gluons. Flavor topology avoids these complications by focusing on the vertex in the diagram which creates this \( q\bar{q} \) pair. There are only two possible vertices describing this pair creation, one where the pair is created from a gluon and one in which it is created from a \( W \) boson. The diagrams illustrated in figs. 1-7 show all possible diagrams in which a \( b\bar{u} \) initial state enters a black box and emerges as a state of a quark-antiquark pair and a boson, \( W \) or gluon, that hadronizes into the final state by QCD interactions including gluon exchanges that do not change quark flavor quantum numbers and conserve flavor SU(3). The black box includes all the possible standard model diagrams with quark-gluon interactions and flavor exchanges that conserve flavor SU(3).

The two relevant pair creation vertices are:

1. The pair is created by gluons and must therefore be a flavor singlet denoted by \( (q\bar{q})_1 \).

The four possible diagrams of this type are illustrated in figs. 1, 2, 3 and 4. The
black boxes in these diagrams represent the sum of all possible diagrams which can lead from a $b\bar{u}$ initial state to the $s\bar{u}G$ state. These diagrams include not only the gluonic penguin diagram and the annihilation diagram but also all tree diagrams in which a $q\bar{q}$ pair is annihilated somewhere in the black box and another pair is created by interactions that conserve flavor-SU(3) symmetry.

The three contributions to the decay amplitude from the diagrams shown in figs. 2, 3 and 4 are equal by SU(3) symmetry.

2. There is no pair creation by gluons in the diagram and the strange quark must come from the initial weak vertex as an $s\bar{u}$ pair as illustrated in figs. 5 and 6. The black boxes in these diagrams represent the sum of all possible diagrams which can lead from a $b\bar{u}$ initial state to the $u\bar{u}W$ state. These diagrams include the tree diagrams along with all other diagrams containing a tree and all strong rescatterings which do not change the flavor of any quark. The line from the initial spectator $\bar{u}$ antiquark then passes through the black box with gluon exchanges but no flavor change and combines either with either of the two final quarks to make either a $K^-$ or an $M_u$.

These two contributions to the decay amplitude add coherently and are commonly called color favored and color suppressed tree diagrams.

Figs. 1-7 show all the flavor topology diagrams considered here. The treatment is exact in the standard model except for the effects of flavor-SU(3) symmetry breaking and the contributions from other diagrams like the electroweak penguin diagram.

We further assume the A...Z [6,7] or OZI [8–10] rule that the quark and antiquark in the flavor singlet pair created by gluons cannot both appear in the same hadron; i.e. that the gluonic hairpin diagram illustrated in fig. 1 is forbidden. Since the remaining diagrams satisfy flavor-SU(3) symmetry there are only two independent amplitudes for describing the creation of the $q\bar{q}$ pair in the final two-meson state.

The decays are therefore described by three parameters:

1. The flavor singlet $s\bar{u}(q\bar{q})_1$ amplitude summing the contributions from diagrams illustrated in figs. 2, 3 and 4, in which the quark and antiquark appear in different final mesons.

2. A $K^-M_u$ amplitude summing the contributions from diagrams illustrated in figs. 5 and 6.

3. A relative phase.

For decays into two pseudoscalar mesons, the one relation obtainable between the decays to four final states is the sum rule:

$$\tilde{\Gamma}(B^\pm \to K^\mp\eta') + \tilde{\Gamma}(B^\pm \to K^\pm\eta) \leq \tilde{\Gamma}(B^\pm \to K^\mp\pi^o) + \tilde{\Gamma}(B^\pm \to K^o\pi^\pm)$$

(1.3)

where $\tilde{\Gamma}$ denotes the partial width when phase space corrections due to mass differences are neglected. $K^o$ denotes $K^o$ for the $B^+$ decay and $K^o$ for the $B^-$ decay.

The equality holds in the flavor-SU(3) limit. The direction of the inequality follows from the assumption that SU(3) symmetry breaking will suppress the $s\bar{s}$ contribution to the
singlet \((q\bar{q})_1\). The left hand side of the sum rule (1.3) is seen to be invariant under the \(\eta - \eta'\) mixing transformation. Thus the result holds for the pseudoscalar nonet with any \(\eta - \eta'\) mixing angle.

This result can be expressed as an inequality in terms of branching ratios since the difference in the phase space factors between the left and right due to the low pion mass depresses the branching ratios to the \(K\eta\) and and \(K\eta'\) more than those for the \(K\pi\) final states. This gives the inequality sum rule (1.1).

The experimental violation of the pseudoscalar sum rule (1.1) indicates a violation of one of the basic assumptions leading to the derivation. Since SU(3) symmetry breaking which keeps the pseudoscalar nonet intact is not likely to produce such a result, the most likely explanations are a breakdown of the pseudoscalar nonet picture or a violation of the OZI rule. There have been suggestions of an additional contribution [11] outside the nonet like a glueball, charm admixture or radial excitation [12–14] in the \(\eta'\) wave function or an A...Z or OZI-violating hairpin diagram illustrated by fig. 1.

It is therefore of interest to look for tests of the nonet picture. New direct experimental tests of this standard mixing picture in \(B\) decays to \(\eta\) and \(\eta'\) together with charmonium have been suggested [15]. The suggestion that the OZI rule is violated in these decays leads to expectations for similar violations or additional contributions to decays to final states containing the \(\eta'\) to occur elsewhere.

However the present new data [3] show no such OZI violation in any of the vector pseudoscalar decays and no additional \(\eta'\) enhancement in other related decays; e.g. \(B^\pm \to K^\pm \eta, B^\pm \to K^{\pm}\eta',\) or \(B^\pm \to K^{\pm}\eta.\) Further tests are obtainable by looking for \(\eta'\) enhancement in \(B^\pm \to \pi^{\pm}\eta', B^\pm \to \pi^{\pm}\eta, B^\pm \to \rho^{\pm}\eta'\) and \(B^\pm \to \rho^{\pm}\eta\) decays, where a charm admixture can give direct CP violation asymmetry.

A serious combined analysis of all decays involving \(\eta\) and \(\eta'\) final states might show which decays conform to the conventional picture where the \(\eta\) and \(\eta'\) behave as normal members of the pseudoscalar nonet and where there is evidence of anomalous enhancement and a possible violation of the standard mixing picture. Here the open question of the nature of the enhancement in the inclusive \(B \to K\eta'X\) deserves a serious experimental investigation along with the analogous inclusive decays \(B \to K\eta X.\)

The paradox of the absence of strong anomalous enhancements elsewhere is sharpened by the agreement of new data [3] with the similar flavor-topology relations holding for the vector-pseudoscalar final states. There are two cases depending upon whether the strange or the nonstrange meson is a vector. In the nonstrange vector case, the ideal \(\omega - \phi\) mixing separates the sum rule into two equalities [2],

\[
BR(K^\pm \rho^o) = BR(K^\pm \omega) \tag{1.4}
\]

\[
BR(K^\pm \phi) \leq BR(K^o \rho^\pm) \tag{1.5}
\]

where the \(\rho - \omega\) approximate degeneracy preserves the approximate equalities of the branching ratios in the relation (1.4). The inequality in the relation (1.5) arises from the \(\rho - \phi\) mass difference.

The new data [3] show agreement with experiment for the equality (1.4).

\[
BR(K^\pm \rho^o)(5.15 \pm 0.9) = BR(K^\pm \omega)(5.1 \pm 0.7) \tag{1.6}
\]
Better statistics are needed for a significant test of the equality (1.5).

\[ BR(K^\pm \phi)(9.7 \pm 1.5) \leq BR(K^o \rho^\pm)(\leq 48) \]  

(1.7)

The other vector-pseudoscalar sum rule with the strange vector is

\[ \tilde{\Gamma}(B^\pm \to K^{*\pm} \eta') + \tilde{\Gamma}(B^\pm \to K^{*\pm} \eta) \leq \tilde{\Gamma}(B^{*\pm} \to K^{*\pm} \pi^o) + \tilde{\Gamma}(B^\pm \to K^o \pi^\pm) \]  

(1.8)

The experimental values from the new data [3] also show no evidence for a strong violation indicating a large contribution [12–14] of the type needed to explain the large violation of the sum rule (1.1).

\[ BR(K^{*\pm} \eta')(\leq 14) + BR(K^{*\pm} \eta)(24.3 \pm 3.0) \leq BR(K^{*\pm} \pi^o)(6.9 \pm 2.3) + BR(K^{*o} \pi^\pm)(9.7 \pm 1.2) \]  

(1.9)

The success of the equality (1.4) and the absence of any strong violation in the other relations raise the question of why the additional contribution needed to explain the violation required for the pseudoscalar sum rule (1.1) does not appear elsewhere.

In these \( B^\pm \) decays all amplitudes arising from the \( b \to u\bar{u}s \) transition depend only upon a single sum of the color-favored and color-suppressed tree contributions illustrated respectively in figs. 5 and 6. This simplification provides predictive power and allows crucial tests of the basic assumptions for charged \( B \) decays. This simplification does not arise in the neutral decays, where independent color-favored and color-suppressed tree contributions must be separately considered. Thus amplitudes derived here for charged decays are not simply related by isospin to amplitudes for neutral decays.

The possible implication of these results for the neutral decays is discussed below.

**B. An approximate isospin sum rule that agrees with experiment**

An approximate equality [16,17] has been expressed as the “Lipkin sum rule” [18].

\[ R_L \equiv \frac{\Gamma(B^+ \to K^+ \pi^o) + \Gamma(B^o \to K^o \pi^o)}{\Gamma(B^+ \to K^o \pi^+) + \Gamma(B^o \to K^+ \pi^-)} \approx 1 \]  

(1.10)

However, writing the relation in this form obscures the fact that it has real significance only if there is interference between the dominant penguin and another amplitude leading to an \( I=3/2 \) final state. It is trivially satisfied if the decays are described entirely by a pure penguin contribution. This physics can be seen explicitly by rearranging the sum rule (1.10) as the approximate equality,

\[ 2BR(B^+ \to K^+ \pi^o) - BR(B^+ \to K^o \pi^+) \approx BR(B^o \to K^+ \pi^-) - 2BR(B^o \to K^o \pi^o) \]  

(1.11)

where for simplicity we express the result in terms of branching ratios, which can be corrected for lifetime differences when there is sufficient precision.

For the case where the decays are described entirely by a pure penguin contribution, the final states are both isospin eigenstates with \( I=1/2 \) and both sides of the relation (1.11) vanish. In this case the relation reduces to the trivial \( 0=0 \). For the case where there is an
additional small contribution leading to an I=3/2 final state, the approximate equality (1.11) relates the contributions to the charged and neutral decays from the interference between this I=3/2 final state and the dominant penguin.

Substituting experimental values gives

\[
2 \cdot (12.1 \pm 0.8) - (24.1 \pm 1.3) = 0.1 \pm 2.1 \approx (18.2 \pm 0.8) - 2 \cdot (11.5 \pm 1.0) = -4.8 \pm 2.2
\]

(1.12)

The agreement here within two standard deviations also exhibits a finite small contribution of tree-penguin interference whose true value is unfortunately down in the noise of the statistics. However, its smallness justifies the approximation of treating the tree contribution only in first order.

The same approximate equalities can be examined for the vector-pseudoscalar final states.

\[
2 \text{BR}(B^+ \to K^{*+} \pi^0) - \text{BR}(B^+ \to K^{*0} \pi^+) \approx \text{BR}(B^o \to K^{*+} \pi^-) - 2 \text{BR}(B^o \to K^{*0} \pi^o)
\]

(1.13)

Substituting experimental values gives

\[
2 \cdot (6.9 \pm 2.3) - (9.2 \pm 1.2) = 4.6 \pm 4.8 \approx (12.1 \pm 1.8) - 2 \cdot (1.7 \pm 0.8) = 8.7 \pm 2.4
\]

(1.14)

Here again, the agreement is within statistics but the error is still too large to give a significant estimate of the tree-penguin interference.

C. Some implications for neutral decays and direct CP violation

The success of the equality (1.4) between the \(K^{*0} \rho^0\) and \(K^{*0} \omega\) decays of the charged \(B\)'s has interesting implications for these decays of neutral \(B\)'s. The equality (1.4) follows for charged \(B\) decays, where the initial state has a spectator \(u\) quark, because both the \(\rho^0\) and \(\omega\) are produced via their \(u\bar{u}\) component. There is no diagram producing the \(d\bar{d}\) component if the OZI rule holds. The success of the equality (1.4) thus indicates that the OZI rule holds and that it can be expected also to hold for the neutral decays.

This physics underlying the equality (1.4) is exactly the same as that which motivated the originally surprising prediction [6] of equality for the strong interaction reactions

\[
\sigma(K^- p \to \Lambda \omega) = \sigma(K^- p \to \Lambda \rho^0)
\]

(1.15)

Here the \(\omega - \rho^0\) equality follows also because there is no diagram producing a \(d\bar{d}\) component.

For the neutral \(B_d\) decays, where the initial state has a spectator \(d\) quark, the penguin diagram produces both the \(\rho^0\) and \(\omega\) via their \(d\bar{d}\) component, as in fig. 3 with the spectator antiquark changed to a \(\bar{d}\). The color suppressed tree diagram still produces both via their \(u\bar{u}\) component as in fig. 5. Note that the color favored tree diagram produces only charged vector mesons in neutral \(B_d\) decays as in fig. 4 with the spectator antiquark changed to a \(d\). The tree-penguin interference thus has opposite phase for \(K^{*}\rho^0\) and \(K^{*}\omega\) decays of neutral \(B\)'s. A difference between \(BR(K^{*}\rho^0)\) and \(BR(K^{*}\omega)\) would indicate the presence of tree-penguin interference and therefore a possibility for observing direct CP violation in the charged as well as the neutral \(B \to K \rho\) and \(B \to K \omega\) decays.

It is therefore of interest to refine the data for the \(K \rho^0\) and \(K \omega\) decays of both charged and neutral \(B\)'s to look for evidence for tree-penguin interference.
II. DIRECTIONS FOR INVESTIGATING INCLUSIVE B → Kη'X

To gain a better understanding for why the additional contribution needed to explain the large violation of the sum rule (1.1) does not seem to appear elsewhere, we return to the large inclusive branching ratios which also are not yet sufficiently investigated.

A. Parity Selection Rules from Gluonic Penguin Diagrams for Final States Containing the η and η'

Consider the model where the gluonic penguin diagram produces the η and η' in charmless strange final states both via the uū (or dd) and ss components of these mesons. As shown [1] in figs. 2-4 and the equation under fig. 4, the amplitudes for the two components interfere constructively for the η' and destructively for the η in all final states of even parity and vice versa for states of odd parity.

This model predicts that the η should appear in states strongly dominated by ODD parity and the η' in states of EVEN parity. This prediction should be violated in most models which introduce some other mechanism for explaining the large η' enhancement found in the Kη' final state.

This leads to the large η'/η ratio for the Kη and Kη' final states and the reverse [1] for the K*(890)η and K*η'.

\[ BR(K^+\eta') \gg BR(K^0\eta); \quad \text{while} \quad BR(K^*\eta') \ll BR(K^{*\pm}\eta) \] (2.1)

B. Experimental consequences of the Parity Selection Rules

We now list some further experimental consequences of this parity rule [1] which can be checked possibly with already available data.

1. The Kπη and Kπη' states all have odd parity, even when the Kπ is not in a K*. Therefore the model predicts that the Kπη should be much stronger than Kπη' when summed over all final states and that this holds both for the charged and neutral B decays. Thus one can get better statistics for a comparison of the η to the η' by summing over all of them. If this gives a strong enhancement of the η over the η' this can be strong evidence against models that produce the η' via the SU(3) singlet component; e.g., gluons, anomaly or intrinsic charm, since it is hard to find reasons why this should hold only for a two-pseudoscalar state and not for a three-pseudoscalar state.

2. If B → Kη'X (inclusive) data are plotted against the mass of the state X recoiling against the η', the parity arguments suggest that the contribution to the Kη'X final state should be small; rather KηX should be strong. This can be tested by noting whether the X mass in the strong signal observed for Kη'X is a pion mass or more than 2mπ. If the single pion contribution is indeed small, large contributions are ruled out from states [3] in which the state X is the scalar resonance K_0(1430) → (93%−Kπ).
or the tensor $K_2(1430) \rightarrow (50\% - K\pi)$ and limit contributions from higher states like $K^*(1680) \rightarrow (39\% - K\pi)$. This test would allow ruling out these resonances without needing any complicated fits to mass plots and simplify the analysis of what is left as well as putting constraints on models which explain the $\eta'$ excess by some kind of singlet creation of the $\eta'$.

3. The measurement of the TRANSVERSITY in the final states $\eta\rho K$, $\eta'\rho K$, $\eta\pi K^*(890)$ and $\eta'\pi K^*(890)$. This is the measurement of the polarization of the vector meson in its rest frame with respect to an axis normal to the VPP plane [19]. This gives an unambiguous signal for the PARITY of the final state (whether it is $0^+$ or $0^-$) independent of the quantum numbers of the $K\pi\pi$ state recoiling against the $\eta$ or $\eta'$.

4. An $\eta$ or $\eta'$ recoiling against a $K^*$ resonance with NATURAL parity (even P for even J and odd P for odd J) has odd parity and should give a final state favoring the $\eta$ over the $\eta'$. The opposite is true for a recoil against a state with UNNATURAL parity. The $K$ and $K^*(890)$ states are special cases of this prediction.

5. One should look for $K\eta$ and $K\eta'$ resonances in the states $K\eta X$ and $K\eta' X$. Here the even parity resonances should favor the $\eta'$ and the odd parity resonances favor the $\eta$.

ACKNOWLEDGEMENTS

This research was supported in part by the U.S. Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38. It is a pleasure to thank Michael Gronau, Yuval Grossman, Zoltan Ligeti, Yosef Nir, Jonathan Rosner, J.G. Smith, and Frank Wuerthwein for discussions and comments.

REFERENCES

[1] Harry J. Lipkin, Experimental Puzzles in Heavy Flavor Decays - hep-ph/0009241

[2] Harry J. Lipkin, hep-ph/9708253

[3] Heavy Flavor Averaging Group http://www.slac.stanford.edu/xorg/hfag/rare/

[4] Harry J. Lipkin, Phys. Rev. Lett. 46 (1981) 1307

[5] Frank E. Close and Harry J. Lipkin, Physics Letters B405 (1997) 157

[6] G. Alexander, H.J. Lipkin and F. Scheck, Phys. Rev. Lett. 17 (1966) 412

[7] Harry J. Lipkin, The Alexander-Zweig (OZI) in Multiparticle Production Are There Strange Quarks in Nucleons and Pions Physics Letters B225, (1989) 287

[8] S. Okubo, Phys. Lett 5 (1963) 1975 and Phys. Rev. D16 (1977) 2336

[9] G. Zweig, CERN Report No. 8419/TH412 unpublished, 1964;

[10] J. Iizuka, Prog. Theor. Phys. Suppl. 37-38 (1966) 21 and in Symmetries in Elementary Particle Physics (Academic Press, New York, 1965) p. 192

[11] Harry J. Lipkin, hep-ph/9710342, Physics Letters B415 (1997) 186

8
[12] H. Harari, Phys. Lett. 60B (1976) 172
[13] H. Fritzsch and J. D. Jackson, Phys. Lett. 66B (1977) 365
[14] D. Atwood and A. Soni, Phys. Lett. B405 (1997) 150
[15] Alakabaha Datta, Harry J. Lipkin and Patrick J. O’Donnell, hep-ph/0111336, Physics Letters B 529 (2002) 93
[16] Harry J. Lipkin, hep-ph/9810351, Physics Letters B445 (1999) 403
[17] Michael Gronau and Jonathan L. Rosner, hep-ph/9809384, Phys. Rev. D 59 (1999) 113002
[18] Ahmed Ali, Invited Plenary Talk at the 32nd International Conference on High Energy Physics (ICHEP’04), Beijing, China, Aug. 16-22, 2004, hep-ph/0412128
[19] H.J. Lipkin, Getting the Maximum Information from B-Decays into CP Eigenstates, in Proceedings of the SLAC Workshop on Physics and Detector Issues for a High-Luminosity Asymmetric B Factory, edited by David Hitlin, Published as SLAC, LBL and Caltech reports SLAC-373, LBL-30097 and CALT-68-1697, p. 49
FIG. 1.

“Gluonic hairpin” diagram. $G$ denotes any number of gluons.
\[ A[M_s(\vec{k})K^-(\vec{k})] = A[M_u(-\vec{k})K^-(\vec{k})] = P \cdot A[M_u(\vec{k})K^-(\vec{k})] \]
FIG. 5.
Color favored tree diagram.

FIG. 6.
Color suppressed tree diagram.