The Numerical Analysis of Soliton Propagation with Split-Step Fourier Transform Method

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Abstract. It is very important for practicability and system designing to study on the optical soliton transform and evolution with numerical simulation. In this paper, the split-step Fourier transform method (SSFM) is particularly introduced and the simulation is also given. The Matlab language is used to simulate the soliton propagation in the optical fiber. The influence of optical fiber waste is discussed and it also pointed that if two optical solitons is very near, they will strongly infect each other, it can make the soliton shape aberrant.

1. Introduction

In mathematics, the nonlinear Schrödinger equation (NLSE) is used to describe the soliton propagation in the fiber. It is a high order nonlinear partial differential equation, and very difficult to obtain the analytical solution. So the numerical methods are introduced to simulate the propagation of the solitons. There are frequently-used methods like perturbation method, differential method, split-step Fourier method (SSFM) et al. The principle of the SSFM is simpler among them, it is easy to write program, and employed by many scholars.

In this paper, the SSFM is particularly introduced, and the friendly interface Matlab software is used to numerically simulate the corresponding problems.

2. Another section of your paper

The NLSE can be obtained from Maxwell equation group to describe the soliton propagation in the optical fiber.

\[ \frac{\partial A}{\partial z} + \frac{\alpha}{2} A + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial T^3} = i \gamma |A|^2 A \]  

(1)

Where \( A(z,T) \) is the complex envelope of the pulse, \( T = t - \beta y z \) represents the normalized time; \( \beta_1 = 1/\nu \), \( \nu \) is the group velocity; \( \beta_2, \beta_3 \) are respective the quadratic and cubic dispersion coefficients; \( \gamma \) is the nonlinear parameter; \( \alpha \) is the fiber loss coefficient.

\( U \) is defined as \( U = A(z,T) / \sqrt{P_0} \), and \( P_0 \) is the peak value power of the initial pulse. Then (1) can be changed into (2).

\[ \frac{\partial U}{\partial z} + \frac{\alpha}{2} U + i \frac{\beta_2}{2} \frac{\partial^2 U}{\partial T^2} - \frac{1}{6} \beta_3 \frac{\partial^3 U}{\partial T^3} = i \gamma |U|^2 U \]  

(2)

The SSFM is used to obtain the numerical solution of (2). The SSFM consists of two major steps to calculate when the optical pulse transmits a fetch in the optic fiber, namely, let the nonlinear and the
dispersion separately works. When the distance is very small, this analysis has enough high precision. Equation (2) can be changed into:

\[
\frac{\partial U}{\partial z} = (\hat{D} + \hat{N})U
\]  

(3)

Where \( \hat{D} \) and \( \hat{N} \) are difference operators, the former represents dispersion part and the rear represents nonlinear part. When only the nonlinear part or the dispersion part works, we can expediently obtain:

\[
U(I(z,T)) = U(0,T) \exp[i\gamma P_0 |U(0,T)|^2 z]
\]

(4)

\[
U(z,T) = \text{IFFT}\left[ \left( \frac{i}{2} \beta_0 \omega^2 - \frac{i}{6} \beta_2 \omega^4 - \frac{\alpha}{2} \right) \text{FFT}\left[ U(1(z,T)) \right] \right]
\]

(5)

Where \( \omega \) is the Fourier frequency.

In the Matlab language, there are particular commands to implement the Fourier transform and inverse Fourier transform. They are respective fft() and ifft() functions, the zero frequency components is needed to shift to the center when the independent variable is a vector, and it needs to rearrange the outputs of the fft() and ifft(). So we must pay attention to use the fftshift() and ifftshift() functions when the program is written.

3. A subsection

The dispersive length and nonlinear length of the optical soliton are respectively defined as

\[
L_D = \frac{T_0^2}{|\beta_2|} \text{ and } L_{NL} = \frac{1}{\gamma P_0} 
\]

\( T_0 \) is related to the full-width at half-maximum(FWHM) intensity of the input pulse. And \( z_0 = \frac{\pi}{2} L_D \), \( N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \), \( N \) is the solution order. In the simulation, the parameters are selected as \( T_0=20ps \), \( \lambda=1.55\mu m \), \( \beta_2=-20(ps^2/km) \), \( \gamma=2.43(1/kmW) \), \( \beta_3=0.1(ps^3/km) \).

3.1. Influence of the fiber loss to the soliton propagation

The initial pulse is assumed to be \( U(0,t)=\text{sech}(t) \), \( t=\frac{T}{T_0} \). \( P_0 \) is suitable selected in order to let the soliton order be \( N=1 \). The optical pulse can transmit a long distance without fiber loss, its pulse shape is shown in Figure. 1. If the fiber has loss, for example, the loss coefficient \( \gamma \) equals 0.14/km, its pulse shape is shown in figure 2. The soliton amplitude will change due to the loss when the soliton propagates in fiber, it can de distinctly shown in the two dimension diagram of figure 3 and figure 4. It can be obtained from figure 4 that the power of the first order soliton will reduce almost to zero when it transmits two soliton periods due to the fiber loss.

**Figure 1.** The first order soliton propagating in the fiber without loss.

**Figure 2.** The first order soliton propagating in the fiber with loss.
3.2. Interaction of two first order solitons
The amplitudes of two solitons are chosen to be the same strength and equal 1. The two solitons are also assumed to be symmetrized, so the initial pulse can be written as \( U(0,t) = \text{sech}(t-t_0) + \text{sech}(t+t_0) \), let \( N=1 \), \( 2t_0 \) is the initial separation between the neighboring two solitons. Their propagations without loss are shown in figure 5 and figure 6. And the propagations with loss are shown in figure 7 and figure 8. The loss coefficient \( \alpha \) is also selected to be 0.14/km. In figure 5 and figure 7, the half initial separation between two solitons \( t_0=3 \). In figure 7 and figure 8, \( t_0=1.5 \).

Two solitons will almost parallelly propagate for a long distance when the initial separation is 6. It can be obtained from figure 5. And when the initial separation reduces to 3, first the two solitons attract each other and combine to be one at the point of two soliton periods. Later, they repel each other and re-split to two first order solitons. In figure 7 and figure 8, the energy of two solitons reduced to be zero before the serious interaction occurred due to the higher fiber loss. If the soliton energy is not compensated, the soliton will not transmit for a long distance, and the soliton communication will have not significance. Along with the development of the erbium doped optical amplifier(EDFA) technology, its energy can be compensated periodically and the fiber loss is not taken into account at the later analysis.
3.3. Interaction of two second order solitons

Let $N=2$, the initial pulse is also assumed to be $U(0,t) = \text{sech}(t-t_0) + \text{sech}(t+t_0)$, its evolutions in the fiber are shown in figure 9 and figure 11. The spectrums at the end of the transmission are shown in the figure 10 and figure 12. In figure 9 and figure 10, $t_0=3$, and in figure 11 and figure 12, $t_0=1.5$. We can know from figure 9, the interaction of two solitons is less when the initial separation is 6, and it shows attraction character. It has arose the third soliton at the end of the transmission. Its spectrum has less blue shift and red shift, it is shown in figure 10. When the initial separation decreases to 3, the two solitons transmit no longer than half period, each begin to compress and it arises the third soliton. Due to the strong attraction, the two solitons compress into one at about 2 periods. At 2.5 periods, it re-splits into three solitons, and the energy of the middle one is bigger than both sides two symmetrized solitons, it also shows periodical fluctuation. Its energy has decreasing direction along with the transmission distance increasing, it belongs to the consuming a part of energy of the dispersion. At the end of the transmission, the blue shift and red shift of the spectrum increased, and the frequency components centralized relatively.
4. Conclusion
In this paper, the SSFM is particularly introduced and the attentive questions in the simulation are indicated. The Matlab language is used to simulate the soliton propagation in the optical fiber. The influence of optical fiber waste is discussed and it also pointed that if two optical solitons is very near, they will strongly infect each other, it can make the soliton shape aberrant.

References
[1] Zhang S M, Xu W C and Luo A P 2001 An improved method to compress femtosecond optical solitons in fibers Acta Photonica Sinica 30 280-283
[2] Xie X P, Zhao S X and Wang X H 2002 The high-order factor analysis of femtosecond soliton pulse propagation in fiber Acta Photonica Sinica 31 429-431
[3] Wang T and Huang T L 2005 Numerical research of interaction between femtosecond solitons in optical fibers Journal of Xi’an Comjunication Institute 4 16-18
[4] Yin D J 2001 Analysis and control of second-order optical soliton split with tird-order dispersion Journal of Optoelectronics.Laser. 12 1305-09
[5] Agrawal D P 1989 Nonlinear Fiber Optics (Bsoton: Academic Press)