Landau-Ginsberg Theory of Quark Confinement

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We describe the SU(3) deconfinement transition using Landau-Ginsberg theory. Drawing on perturbation theory and symmetry principles, we construct the free energy as a function of temperature and the Polyakov loop. Once the two adjustable parameters of the model are fixed, the pressure $p$, energy $\epsilon$ and Polyakov loop expectation value $P_F$ are calculable functions of temperature. An excellent fit to the continuum extrapolation of lattice thermodynamics data can be achieved. In an extended form of the model, the glueball potential is responsible for breaking scale invariance at low temperatures. Three parameters are required, but the glueball mass and the gluon condensate are calculable functions of temperature, along with $p$, $\epsilon$ and $P_F$.

1. Theory

We take the free energy density $f$ of the gluon plasma to be a function of the temperature $T$ and the fundamental representation Polyakov loop $P$. The theory also depends on a renormalization group invariant scale-setting parameter $\Lambda$. Perturbation theory gives a free energy $f$ of the form $T^4 f_4(P,g(T/\Lambda))$. Perturbation theory does not describe $f$ near the deconfining transition, but is probably adequate for $T$ much greater than the deconfinement temperature $T_d$. Subleading terms, of the form $T^{4-\Lambda'} f_4(P,g(T/\Lambda))$, are likely needed to describe the deconfining transition. Such terms are inherently non-perturbative, due to the appearance of the factor $\Lambda'$. It is easy to show that $\Delta \equiv \epsilon - 3p$ is given by

$$\Delta = [4 - T \partial_T] f$$

and therefore contains information about the sub-leading terms. Note that $\Delta$ is also directly related to the finite temperature contribution to the stress-energy tensor anomaly, which depends in a non-trivial way on the Polyakov loop.

Given the close connection between $\Delta$ and the subleading terms in $f$ which drive the deconfinement transition, it is natural to examine the behavior of $\Delta(T)$ near $T_d$ as measured in simulations. Using the data of Boyd et al for SU(3) lattice gauge theory, we find that $\Delta(T) \propto T^2$ over a large range of temperatures above $T_d$. This suggests that a term in $f$ proportional to $T^2$ plays an important role in the deconfinement transition. It is also necessary to have a term proportional to $T^0$ (independent of $T$), so that there is a non-zero free energy density difference between confined and deconfined phases at very low temperatures. Thus we conjecture the simple form for the free energy

$$f(T, P) = T^4 f_4(P) + T^2 \Lambda^2 f_2(P) + \Lambda^4 f_0(P)$$

where $f_0$ must favor the confined phase to yield confinement at arbitrarily low temperatures.

We look at the one loop perturbative result for guidance on the possible forms for $f_r$. We define the eigenvalues $q_j$ by diagonalizing the fundamental representation Polyakov loop $P$: $P_{j,k} = \exp(i\pi q_j) \delta_{jk}$. The free energy for gluons in a constant $A_0$ background is:

$$f_g(q) = \frac{2}{\beta} T r A \int \frac{d^3 k}{(2\pi)^3} \ln \left[ 1 - e^{-\beta \omega_k} P \right]$$

$$= -\frac{2}{\beta} T r A \int \frac{d^3 k}{(2\pi)^3} \sum_{n=1}^{\infty} \frac{1}{n} e^{-n\beta \omega_k} P^n$$

$$= \frac{2\pi^2 T^4}{3} \sum_{j,k=1}^{N} (1 - \frac{1}{N} \delta_{jk}) B_4 \left( \frac{\Delta q_{jk}}{2} \right)$$

where $|\Delta q_{jk}|_2 \equiv (q_j - q_k) \mod(2)$ and $B_4$ is the fourth Bernoulli polynomial, given by $B_4(x) = \frac{x^4}{4}$.
$x^4 - 2x^3 + x^2 - \frac{1}{3}$. The free energy is a sum of terms, each of which represents field configurations in which a net number of $n$ gluons go around space-time in the Euclidean time direction.

3. Results

The above potential has built in the correct low and high temperature behavior, and has two free parameters, $b$ and $c$. We can use one of these to set the overall scale by fixing the deconfinement temperature. To determine the remaining parameter, we fit the lattice data for $\Delta$ at $N_t = 8$, which is well measured and a good approximation to the continuum limit. With $T_d = 0.272 \text{GeV}$, we obtain $b^{1/4} = 0.356 \text{GeV}$ and $c^{1/2} = 0.313 \text{GeV}$. The results of our fitting procedure are shown in figures 1, 2 and 3 for $\Delta$, $p$ and $\varepsilon$. The agreement is good throughout the range $T_d - 4$, with a discrepancy in the high-temperature behavior of $p$ and $\varepsilon$ is probably accounted for by HTL-improved perturbation theory.

4. Extended Model

The physical origin of the parameters $b$ and $c$ above is obscure. Since the trace of the stress-energy tensor $\theta_{\mu}^\mu$ couples to the scalar glueball, we introduce a scalar glueball field $\phi$ as the source of scale symmetry breaking in an extended model. For $SU(3)$, our extended model is

$$ f = aT^4 \left( \psi^4 - \frac{2}{3} \psi^3 + \psi^2 \right) + (bT^2) \psi^2 + \lambda \phi^4 \log \left( \frac{\phi^2}{e^{1/2} \mu^2} \right). $$

Spontaneous symmetry breaking of $\phi$ via a Coleman-Weinberg potential introduces the scale.
µ. If we make the identification $\phi^4 \propto Tr \left( F_{\mu\nu}^2 \right)$, the $T = 0$ potential for $\phi$ can be derived in a variety of ways: 1) renormalization group [7]; 2) explicit calculation for constant fields [8]; 3) stress-energy tensor anomaly [9]; 4) stress-energy sum rules [10]. The values of $\lambda$ and $\mu$ can be determined from the values of the gluon condensate and the glueball mass. The $T = 0$ condensate from the Coleman-Weinberg potential is given by $-2\lambda\phi^4 \rightarrow -2\lambda\mu^4$ and the glueball mass is given by $M_g^2 = 8\lambda\mu^2$. A similar glueball potential has been used to model the chiral transition [11]. A coupling between $\phi$ and $P^2$ can be inferred from perturbation theory [3] [12]; similar couplings to the chiral order parameter exist [3] [13] [14] [15].

\[ \text{Figure 3. } \varepsilon/T^4 \text{ versus } T. \]

We have found values for the parameters $\alpha, \beta, \lambda$ and $\mu$ which mimic the behavior of our simpler model near $T_d$. Our extended model has a potentially fatal problem associated with the restoration of scale symmetry. For plausible values of the gluon condensate and glueball mass, restoration of scale symmetry at $T_d$ leads to a single abrupt phase transition incompatible with lattice data. The alternative, with unrealistic values, is restoration above $T_d$ via a first order transition, which would be observable in lattice data. This argues against any simple role of the glueball in the thermodynamics of the gluon plasma.

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