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Graded circular Bragg reflectors: a semi-analytical retrieval of approximate pitch profiles from Mueller-matrix data

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Abstract
Graded pitch profiles are found in structurally chiral materials like cholesteric liquid crystals (CLC) and in the cuticle of some scarab beetles. In most cases, the pitch profile is determined from electron microscopy techniques. Recently, it was shown that approximate pitch profiles in the cuticle of scarab beetles can be retrieved through an analysis of the spectral dependence of maxima and minima in normalized Mueller-matrix data. The analysis relies on basic concepts of interference in thin films, properties of optical modes in chiral systems, and the condition for circular Bragg reflection. In this work, the consistency of the procedure is demonstrated by analysis of normalized Mueller matrices of circular Bragg reflectors calculated for three predefined pitch profiles with (1) a stepwise decrease, (2) a stepwise increase and, (3) an exponential increase. The procedure does not require knowledge of the full Mueller matrix and can be used for non-destructive analysis of pitch in CLC, beetle cuticle and similar structures.

Supplementary material for this article is available online

Keywords: circular Bragg reflection, Mueller matrix, chiral structures, cholesteric liquid crystals

(Some figures may appear in colour only in the online journal)

1. Introduction
Cholesteric liquid crystals (CLC) and the cuticle of some scarab beetles are structural and optical analogs [1]. Rod-like molecules in the former and chitin-protein fibrils in the latter form a helicoidal arrangement producing the circular Bragg phenomenon [2]. Thus, for normal incident electromagnetic radiation the co-handed circular-polarization mode of the helicoidal structure is reflected. This occurs in a spectral band centered at wavelength \( \lambda_B = n_{av}\Lambda \) with bandwidth \( \Delta \lambda = \Delta n\Lambda \), where \( n_{av} \) is the in-plane average refractive index, \( \Delta n \) the in-plane birefringence and \( \Lambda \) the full turn pitch of the helicoidal structure. The basic approach for electromagnetic modeling considers the rotation of the dielectric axes along the helicoidal structure [3, 4]. Based on this model, there exist several reports on the optical characterization of CLC [5, 6] and the cuticle of beetles [7–11].

Since the bandwidth of circular Bragg reflection of CLC is limited by their birefringence, the broadening of selective reflection has been achieved by different methods producing a varying pitch across the thickness of the sample [12–22]. In these cases, the graded pitch profile has been quantified from cross-section electron microscopy images. Electron microscopy has also been applied to determine pitch variation across the cuticle of beetles [7–9]. Recently, we have shown...
that spectral analysis of maxima in minima in Mueller-matrix data can be used to determine approximate pitch variation across the cuticle of narrow- and broad-band circular Bragg reflectors [23–26]. A refined pitch profile was then determined by electromagnetic modeling and regression analysis of Mueller-matrix data which in addition provides optical functions of the constituents of these natural chiral systems [25, 26].

In this work, we investigate the consistency of the procedure to determine approximate pitch profiles of circular Bragg reflectors from Mueller-matrix data. The theoretical background of circular Bragg reflectors and basics of the Mueller-matrix approach are presented in section 2. The applicability of the procedure by using maxima and minima in reflectance spectra is discussed in section 3.1 for decreasing and increasing stepwise variation of pitch across the thickness. For the same pitch variations, the consistency of the procedure is tested for Mueller-matrix data in section 3.2. Motivated by data in the literature, an exponential increase of pitch with depth in liquid crystals [13] is also studied in section 3.3. Comments on implementation of the procedure appear in section 3.4. In the last section, conclusions of the work are summarized.

2. Theoretical framework

2.1. Basics of Mueller-matrix calculations

In Stokes–Mueller formalism, light beams are represented by Stokes vectors \( \mathbf{S} = [I, Q, U, V]^T \) where \( I = I_p + I_s \) accounts for the total irradiance, \( Q = I_p - I_s \), and \( U = I_{+45^\circ} - I_{-45^\circ} \) are irradiances describing linear polarization. Here \( p \) is parallel to and \( s \) is perpendicular to the plane of incidence and \( +45^\circ \) and \( -45^\circ \) are measured from the plane of incidence. The Stokes parameter \( V = I_R - I_L \) accounts for circular polarization where \( R \) and \( L \) stand for irradiances of right- and left-handed (LH) light, respectively. The incident (\( \mathbf{S}_i \)) and reflected (\( \mathbf{S}_r \)) light beams are related by the \( 4 \times 4 \) Mueller matrix \( \mathbf{M} \) of the sample according to \( \mathbf{S}_r = \mathbf{M} \mathbf{S}_i \), where \( \mathbf{M} = \{ M_{ij} \} \). In this work, we focus on the total reflectance defined as \( R = M_{11} \) and the remaining 15 elements normalized according to \( m_{ij} = M_{ij}/M_{11} \).

The calculations of Mueller matrices were performed with the CompleteEASE software (J A Woollam Co., Inc.) in the spectral range 245–1200 nm with resolution 1 nm. The angle of incidence (\( \theta \)) is specified in each case. Briefly, the \( 2 \times 2 \) Jones matrix \( \mathbf{J} = \{ r_{ij} \} \) of the entire multilayer system is calculated first [27]

\[
\begin{bmatrix}
E_{tp} \\
E_{ts}
\end{bmatrix} =
\begin{bmatrix}
r_{pp} & r_{ps} \\
r_{sp} & r_{ss}
\end{bmatrix}
\begin{bmatrix}
E_{ip} \\
E_{is}
\end{bmatrix},
\]

where \( E_{ip,ts} \) and \( E_{ip,ts} \) are, respectively, the electric fields of the incident and reflected electromagnetic waves. Then, the Mueller matrix of the non-depolarizing optical system represented by the Jones matrix in equation (1) can be calculated with the standard procedure \( \mathbf{M} = \mathbf{T}(\mathbf{J} \otimes \mathbf{J}^*)\mathbf{T}^{-1} \) where \( \otimes \) denotes the Kronecker product, the asterisk means complex conjugation, and \( \mathbf{T} \) is the matrix relating the Stokes and coherency vectors [27]. The components of the coherency vector are the elements of the coherency matrix in the Jones representation.

Since experimental Mueller matrices also carry information about sample-induced depolarization of polarized incident light [24–26], deviations from an ideal model must be included to consider this effect. In this work, non-uniformity in the heliocoidal layer thickness \( d \) is assumed to be the source of depolarization. In practice, Mueller matrices are calculated for nine thicknesses in the interval \( d - \Delta d \) and \( d + \Delta d \), and the Gaussian weighted average of these Mueller matrices then represents the incoherent superposition of light reflected causing depolarization. In this work, \( \Delta d/d = 2\% \) was chosen which is a typical value found in real samples [25, 26].

2.2. Circular Bragg reflector with uniform pitch

Figure 1(a) shows a schematic representation of a circular Bragg reflector consisting of anisotropic twisted slices with principal refractive indices \( (n_1, n_2, n_3) \). The twist is parameterized by the variable azimuth angle \( \phi \) (in degrees) corresponding to the orientation of \( n_1 \) with respect to the plane of incidence [11]

\[
\phi(u) = \phi_0 + 360Tu/d,
\]

where \( u \) is the distance from the bottom of the chiral structure, \( d \) the thickness of the structure, \( T \) the number of turns, and \( \phi_0 \)
the azimuth offset of the \( n_1 \) direction. The sign of \( T \) defines the handedness of the chiral structure with \( T > 0 \) for LH and \( T < 0 \) for right-handed (RH). The cumulated number of periods is defined as

\[
N_p(u) = (\phi(u) - \phi_0)/360, \quad (3)
\]

and the full turn pitch is obtained from

\[
\Lambda = d/T. \quad (4)
\]

The dependences of spectral location (\( \lambda_0 \)), strength (\( R_{max} \)) and bandwidth (\( \Delta \lambda \)) of a selective circular Bragg reflection on the parameters \( n_{av}, \Delta n, \Lambda, d, \) and \( \theta \) are illustrated first. Specifically, to illustrate the dependence on ‘low’ and ‘high’ birefringence \( \Delta n \), real data for \((n_1, n_2, n_3)\) were considered. They correspond to those determined for the scarab beetles Cotinis mutabilis and Chrysina chrysargyrea, respectively [25, 26] (see supplementary material available online at stacks.iop.org/JOPT/21/125401/mmedia). As an example of the magnitude of low and high \( \Delta n \) they are, respectively, 0.026 and 0.138 at wavelength 550 nm. The chiral layer is assumed as LH and supported on a substrate with refractive index \( n_s = 1.5 \). However, the value of \( n_s \) only affects the baseline and amplitude of the oscillations outside the band of the selective Bragg reflection. Once the optical functions are defined, the Jones matrix in equation (1) is calculated for the multilayer model of anisotropic twisted slices with azimuth given by equation (2), and then the corresponding Mueller matrix is obtained from \( M_J = T(J \otimes J^\tau)T^{-1} \). The reflectance is given by \( R = M_{11} \) as was mentioned before. Figures 2(b) and (c) show the calculated reflectance spectra at \( \theta = 20^\circ \) as a function of \( d \) expressed as multiples of \( \Lambda = 310 \) nm. Broader and weaker reflectance spectra are produced by thinner helicoidal layers, whereas higher birefringence strengthens and broadens the reflectance spectra. Normalized Mueller matrices were calculated for low and high birefringence with \( d = 62 \Lambda \) and \( d = 15 \Lambda \), respectively (see supplementary material).

The effects of the thickness (in terms of the number of turns) on \( R_{max} \) and \( \Delta \lambda \) are summarized in figures 2(a) and (b). About ten turns \( (d = 3.1 \mu \text{m}) \) are required to establish a selective Bragg reflection with a high birefringence. In structurally chiral systems with low birefringence, \( R_{max} \) and \( \Delta \lambda \) are clearly limited by the thickness. On the other hand, for increasing angle of incidence, the reflectance spectrum shifts to shorter wavelengths as is shown in figure 2(c) for the high birefringence case. It is also seen that the bandwidth decreases from 46 to 39 nm when \( \theta \) increases from 0° to 60°. However, in photon energy units the band broadens from 0.231 to 0.263 eV. As shown in figure 2(d), the central wavelength (\( \lambda_0 \)) of selective Bragg reflection follows the relationship

\[
\lambda_0 = n_{av}\Lambda \cos \theta_i, \quad (5)
\]

where \( \theta_i \) is determined from Snell’s law \( n_{air} \sin \theta = n_{av} \sin \theta_i \).

2.3. Optical modes in circular Bragg reflectors

The dispersion relation for light propagation in perfect CLC at normal incidence has an analytical solution [29], which is not the case for oblique incidence. However, numerical and approximate methods were developed by other authors some decades ago [30–32]. Fortunately, a so-called two-wave approximation is enough to explain the main features near the band of selective Bragg reflection [32]. Within this approximation, the optical modes for the component of the wave vector parallel to the helicoid axis (\( K_|| \)) can be obtained by solving a quartic equation. The details can be found elsewhere [24, 32]. As is known, at normal incidence the polarization eigenstates are circular but as the angle of incidence increases, elliptical to near linear (\( p \)- and \( s \)-polarized) polarized light is reflected [31].

Figures 3(a) and (b) show the calculated dispersion relation for \( K_|| \) in the forward direction for the two sets of refractive indices \((n_1, n_2, n_3)\) of low and high birefringence considered in this work. Photon energy units are used because the wave vectors scale with the frequency of electromagnetic waves. The LH mode is complex-valued (\( \text{Im}\{K_||\} \approx 0 \)) in the
The band of selective Bragg reflection and \( \text{Re}\{K||\} = 2\pi/\Lambda \) whereas the RH is real-valued in the whole spectral range. As expected, a higher birefringence broadens and strengthens (larger \( \text{Im}\{K||\} \)) the band of selective Bragg reflection. The attenuation length of the electric field of the LH mode is defined as \( \eta = \text{Im}\{K||\}^{-1} \). Furthermore, the LH mode propagates without attenuation outside the band of selective Bragg reflection at photon energies far enough from the resonance, both LH and RH modes become like those in an anisotropic material [31].

In previous works, we have claimed that a spectral analysis of maxima and minima in the normalized Mueller matrix elements \( m_{ij} \) of data from the cuticle of beetles, carry information about the dispersion relation [23, 24]. Here, the consistency of the procedure is demonstrated by analyzing the element \( m_{21} \) from calculated data of a high birefringence chiral layer with \( d = 15\AA \) (for the full normalized Mueller matrix see supplemental material). Recall that for an isotropic non-absorbing film of refractive index \( n_{av} \) and thickness \( d \), the wave vector component is \( K_\theta = 2\pi n_{av}^{-1} n_{av} \cos \theta_1 \) and maxima and minima appear in the optical spectra at wavelengths \( \lambda_n \) (or photon energies \( E_m = 1240/\lambda_m \), for \( E_m \) in eV and \( \lambda_m \) in nm) where the phase factor \( \beta \) is

\[
\beta = 2K_\theta d = 4\pi n_{av}^{-1} d n_{av} \cos \theta_1 = m\pi.
\]

where \( m \) is an integer number. Since the true order of interference \( m \) is not known, a temporary index \( m \) is used to label the minima and maxima as is shown in figure 3(c). Then, \( m(E_m) \) is plotted and a linear fit on the nearly linear longwave limit is performed; more details are given in [23, 24]. The linear fit provides the offset necessary to obtain the correct order \( m \) and then \( K_\parallel \) is determined according to equation (6) and the known values \( d = 15\AA \) and \( n_{av} = (n_1 + n_2)/2 \). The results are plotted in figure 3(d) together with the values calculated within the two-wave approximation. The evident agreement demonstrates the consistency of our claim. In practice, both \( d \) and \( n_{av} \) might be unknown, but using a reasonable value of \( n_{av} \), an approximate value of the thickness can be obtained from the slope of the linear fitting.

### 2.4. Circular Bragg reflectors with graded pitch

In the case of a graded pitch, equation (4) must be generalized and the pitch profile is determined by the derivative of the cumulated number of periods as function of position [25, 26]

\[
\Lambda(u) = (dN_p/du)^{-1},
\]

from which the direction of the refractive index \( n_1 \) as a function of position is determined

\[
\phi(u) = \phi_0 + 360\int \frac{du}{\Lambda(u)}.
\]

In the case of graded transitions in pitch, it has been shown that an adequate representation is [25, 26]

\[
\phi(u) = \phi_0 + 360T \left( \frac{u}{d} + \sum_{j} a_j \ln \left| 1 + \exp \left( \frac{u - u_j}{b_j} \right) \right| \right),
\]

where the parameters \( a_j \), \( u_j \), and \( b_j \) are, respectively, the strength, position, and broadening of the \( j \)th transition in pitch. If \( a_j > 0 \) (<0), the pitch increases (decreases) with depth. Another case of interest in this work is an exponential variation in pitch with depth represented by

\[
\phi(u) = \phi_0 \pm \frac{360T_{\exp}}{\sigma} \exp \left[ \pm \sigma \left( \frac{u}{d} - 1 \right) \right],
\]

where \( T_{\exp} = d/\Lambda_0 \), \( \Lambda_0 \) the value of pitch at the surface, the plus (minus) sign is for decreasing (increasing) pitch with depth, and \( \sigma \) is the dimensionless growth (decay) parameter.
3. Results and discussion

3.1. Circular Bragg reflectors with graded transitions in pitch: reflectance analysis

Let us consider a circular Bragg reflector with \( d = 10 \mu m \) with a single jump in pitch between 300 and 500 nm which either can be decreasing or increasing as shown in figure 4(a). Gradual change in the pitch across the thickness has been produced in CLC and found in the cuticle of some beetles. The graded profiles were calculated with equations (3), (7), and (9), in the latter the parameters were \( T = 34, a = -0.02, u_0 = 5 \mu m, \) and \( b = 0.05 \) for decreasing pitch, whereas for the gradual increase \( T = 20.5, a = 0.033, u_0 = 5 \mu m, \) and \( b = 0.05 \) were used.

As was described in section 2.1, to calculate the Mueller matrix of the graded circular reflectors, the Jones matrix for the multilayer model of anisotropic twisted slices with azimuth given by equation (9) is calculated first. Then the Mueller matrix is obtained from \( \mathbf{M} = \mathbf{T} \left( \mathbf{J} \otimes \mathbf{J}^* \right) \mathbf{T}^{-1} \) where the reflectance is given by \( R = M_{11} \). The calculated \( R \) spectra at \( \theta = 20^\circ \) for low and high birefringence are shown in figure 4(b). The two maxima in the spectra are located at photon energies 1.65 and 2.65 eV. These values correspond to wavelengths 782 and 469 nm where selective Bragg reflection is expected for \( \Lambda = 300 \) and 500 nm, respectively, according to equation (5). Furthermore, the values of \( R \) agree with those expected according to figure 2(a). However, for the two cases in figure 4(b) it is not possible to distinguish whether the gradual change in pitch is increasing or decreasing.
To investigate the applicability of the procedure outlined in section 2.3, minima and maxima in the $R$ spectra were labeled and the spectral dependence of index $m$ is shown in figures 4(c) and (d) for low and high birefringence, respectively. In both panels, the data resemble the dispersion relation in figure 3(d). However, in figure 4(c) for the low birefringence case, the data are nearly identical for decreasing and increasing pitch profiles. In the case of chiral structures with higher birefringence, the values of $m$ of the increasing pitch case deviates from those of the decreasing pitch case at photon energies around the band of selective Bragg reflection. This behavior results from additional maxima and/or minima in the $R$ spectra. However, not substantial changes of slope can be noticed. Earlier we have shown (see also section 2.4) that for chiral structures with graded pitch, the derivative of $m$ is the relevant quantity [26]. In summary, it is not possible to distinguish between decreasing or increasing pitch profiles from an $R$ spectrum.

### 3.2. Circular Bragg reflectors with graded transitions in pitch: Mueller-matrix analysis

In this section we investigate the consistency of the procedure employed in our previous work to retrieve the pitch profile in the cuticle of the scarab beetle *C. Chrysargyrea* [26]. The normalized Mueller matrices of LH chiral structures with the graded pitch profiles shown in figure 4(a) calculated for low and high birefringence show a very rich oscillatory behavior (see supplementary material). It is known that in chiral systems, only nine of the fifteen elements of the normalized Mueller matrix are independent [25, 26]. A careful inspection of the nine independent elements shows that the phase of the oscillations is not independent. For example, the spectral location of maxima in $m_{21}$, $m_{41}$, and $m_{22}$ coincides with minima in $m_{33}$, $m_{24}$, and $m_{44}$ as well as with maximum slope in $m_{31}$, $m_{23}$, and $m_{43}$ (see supplementary material). Furthermore, this relation of phase is maintained along spectral regions of selective Bragg reflection or free propagation of LH and RH modes. From this result, it is concluded that all elements carry the same information and we choose $m_{21}$ for the analysis.

Inspection of $m_{21}$ for the low birefringence case, shown in figures 5(a) and (b), reveals that slower spectral variations occurs at different photon energies, about 1.65 and 2.65 eV for decreasing and increasing pitch, respectively. Thus, it is possible to identify whether the pitch is large or small near the surface or the bottom. Indeed, this reasoning was applied to identify the pitch variation in the cuticle of the scarab beetle *C. mutabilis* [24]. As can be noticed in figures 5(c) and (d), a similar behavior is found for higher birefringence but with the slow variation covering a wider spectral range.

To retrieve an approximate pitch profile from data in figure 5, first, maxima and minima were labeled with the index $m$. Figures 6(a) and (b) show the spectral dependence $m(E_m)$ for the low and high birefringence cases, respectively. Below 1.5 eV and above 2.75 eV the same slope is noticed regardless of decreasing or increasing pitch structures and in some cases the values of $m$ are even indistinguishable as can be seen in figure 6(a). That is because outside the selective Bragg reflection region the structures behave as anisotropic
materials and both LH and RH modes propagate without attenuation.

However, in the spectral range 1.5–2.75 eV corresponding to the selective Bragg reflection where $m_{41}$ is different between decreasing and increasing pitch. Since in this spectral range the propagation of the LH mode is limited up to a certain effective attenuation length $\langle \eta \rangle$, equation (6) can be generalized by associating $d \to \langle \eta \rangle$ (in units of nm) and taking the numerical photon energy derivative of $m(E_m)$. After some elementary algebra it is found

$$
\langle \eta \rangle = \frac{310 \cos \theta_i}{n_{av} \cos^2 \theta_i + E_m \frac{dn_{av}}{dE_m} \frac{dE_m}{dE}} \frac{dm}{dE_m}. \tag{11}
$$

Figures 6(c) and (d) show $\langle \eta \rangle$ as a function of $\lambda_m$ as calculated with equation (11) with $n_{av} = (n_1 + n_2)/2$. As expected, in both cases $\langle \eta \rangle \approx d = 10 \mu m$ for $\lambda_m < 450 \mu m$ and $\lambda_m > 800 \mu m$. On the other hand, abrupt changes in $\langle \eta \rangle$ are observed at wavelengths where the pitch near the surface produce selective Bragg reflection, that is, 800 nm for the decreasing and 450 nm for the increasing profile. At other wavelengths rather smooth variations are noticed.

The fact that $\langle \eta \rangle$ reaches the value of $d$ outside the band of selective Bragg reflection, makes it plausible to consider $\langle \eta \rangle$ to be a measure of penetration depth, that is $\langle \eta \rangle \rightarrow d - u$. Furthermore, the $\lambda_m$ axis can be transformed according to equation (5), that is, $\Delta = \lambda_m/n_{av} \cos \theta_i$, which is valid for wavelengths of selective Bragg reflection, i.e. $\lambda_m$ between 450 and 800 nm. Figure 7 was obtained by making these transforms to the data in figures 6(c) and (d). By cutting the data corresponding to the sample thickness, the approximate variation of pitch as a function of depth is obtained. The very good agreement between original and retrieved graded profiles demonstrates the consistency of the procedure, regardless of the value of birefringence. Near the surface the pitch profile is not resolved because the LH mode propagates a finite length inside the sample to attain the values of reflectance.

![Figure 7](image-url) Comparison between approximate and original pitch profiles for: low birefringence with decreasing (a) and increasing (b) graded pitch; high birefringence with decreasing (c) and increasing (d) graded pitch.

![Figure 8](image-url) (a) Assumed exponential pitch variation for samples of different thickness calculated with equation (10) and parameters in table 1 to simulate data from [13]. (b) Calculated reflectance spectra at normal incidence.

Table 1. Values of the parameters in equation (10) to generate the graded pitch profiles in figure 8(a): $\Lambda_0 = 235 nm$ and $\phi_0 = 0$. Growth parameter $\alpha$ from [13].

| d (μm) | $T_{exp}$ | $\sigma$ | $\alpha(\mu m^{-1})$ |
|-------|-----------|----------|----------------------|
| 5     | 21.55     | 0.74     | 0.148                |
| 10    | 42.55     | 0.92     | 0.092                |
| 20    | 85.11     | 1.02     | 0.051                |

$\lambda_m$ growth parameter $\alpha$ from [13].
shown in figure 4(b). This is qualitatively explained with the data in figure 2(a) although they are for \( \Lambda = 310 \text{ nm} \).

### 3.3. Circular Bragg reflectors with exponential variation in pitch

In the previous section the pitch is nearly constant at the boundaries of the chiral structure. In this section a continuous pitch variation across the whole structure is analyzed. An exponential variation of pitch with depth \( \zeta \) according to \( \Lambda(\zeta) = \Lambda_0 \exp(\alpha \zeta) \), where \( \alpha \) is the growth parameter. This type of structure has been reported \cite{13} and the broadband reflection \( R \) spectra at normal incidence of three RH CLC with thicknesses 5, 10, and 20 \( \mu \text{m} \) were analyzed by calculations using the Berreman \( 4 \times 4 \) method. The CLC samples were assumed as embedded in media of refractive index 1.62, with \( n_{av} = 1.62, \Delta n = 0.16 \), and \( \alpha \) as shown in table 1 \cite{13}.

To reproduce the pitch variation reported in \cite{13}, equation (11) was used with values of the corresponding parameters shown in table 1 with resulting pitch as shown in figure 8(a). Figure 8(b) shows the \( R \) spectra at normal incidence calculated with the model of twisted anisotropic slices described in section 2.2. It is noticed a good agreement with those reported \cite{13}.

To test the consistency of the procedure described in section 3.2, normalized Mueller matrices were calculated at \( \theta = 20^\circ \) with the graded pitch profiles shown in figure 8(a) (see supplementary material). Oblique incidence was assumed because that is the current design for measurements. Figure 9(a) shows the calculated \( m_{21} \) element. As can be noticed, besides the increasing number of maxima and minima with increasing thickness, it is almost impossible to recognize any other special features in the spectra. However, the corresponding \( m \) indices show a smooth dependence on \( E_m \) as can be seen in figure 9(b). As expected, larger values of \( m \) and increasing curvature are found for thicker films. Further details are revealed when taking the derivative in equation (11). Figure 9(c) shows the spectral dependence of \( \langle \eta \rangle \) where it can be noticed that the corresponding values of the thickness in each case are asymptotically approached at long-wavelengths. At short wavelengths, \( \langle \eta \rangle \) slowly approaches 5 \( \mu \text{m} \) but for thicker samples the approaching is much more abrupt. The approximate pitch profiles in figure 9(d) were calculated with the transforms as described above, \( \Lambda = \lambda_m / n_{av} \cos \theta_l \) and \( \langle \eta \rangle \rightarrow d - u \). For clarity, data of \( \langle \eta \rangle \) approaching the value of the sample thickness were cut. A very good agreement between the approximate and original pitch profiles is noticed.

### 3.4. Comments on the implementation of the procedure

Depending on the experimental setup of ellipsometers, the number of accessible elements \( m_{ij} \) is different \cite{33}. In some cases, Mueller matrix elements outside the principal and secondary diagonal contain much richer information as in the case of single and stepwise pitch (see supplementary material). However, elements in the 2 \( \times \) 2 central block can be useful as well, e.g. for exponential variation of pitch in section 3.3 (see supplementary material). Fortunately, most ellipsometric systems can measure enough elements. Of course, a wide spectral range covering the band of selective Bragg reflection, high spectral resolution, and sample uniformity are preferred. In summary, the procedure to retrieve an approximate pitch profile is: (i) acquire accurate measurements of some \( m_{ij} \); (ii) label maxima and minima in one of the \( m_{ij} \) using the index \( m \); (iii) plot the spectral dependence \( m(E_m) \); (iv) calculate \( \langle \eta \rangle \) according to equation (11) assuming a value for \( n_{av} \); (v) plot \( \langle \eta \rangle \) as function of \( \lambda_m \) and identify the approximate value of sample thickness; (vi) transform the \( \lambda_m \) axis to a \( \Lambda \) axis using equation (5); (viii) plot \( \Lambda \) as function of \( \langle \eta \rangle \) renamed as depth. Some noise is expected in the data due
to the numerical derivative in equation (11). Finally, the approximate pitch profile can be determined by nonlinear regression analysis. To account for sample inhomogeneities, the use of a full normalized Mueller matrix is necessary to evaluate the depolarization index.

4. Conclusions

A semi-analytical procedure has been developed to retrieve the pitch profile in circular Bragg reflectors. The procedure employs elements of the normalized Mueller matrix and is based on a spectral analysis of maxima and minima. It is performed through transformations using fundamental concepts of interference in thin films, properties of optical modes in structurally chiral materials, and the condition for selective Bragg reflection. The consistency of the procedure was demonstrated by retrieving pitch profiles from data calculated using three profiles: stepwise increase, stepwise decrease, and exponential increase with depth. An estimated value of the thickness can also be determined.

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