KIC 10736223: An Algol-type Eclipsing Binary That Has Just Undergone the Rapid Mass-transfer Stage

Xinghao Chen\textsuperscript{1,2}, Xiaobin Zhang\textsuperscript{3}, Yan Li\textsuperscript{1,2,4,5}, Hailling Chen\textsuperscript{1,2}, Changqing Luo\textsuperscript{3}, Jie Su\textsuperscript{1,2}, Xuefei Chen\textsuperscript{1,2}, and Zhanwen Han\textsuperscript{1,2,4,5}

\textsuperscript{1}Yunnan Observatories, Chinese Academy of Sciences, P.O. Box 110, Kunming 650216, People’s Republic of China; chenxinghao@ynao.ac.cn
\textsuperscript{2}Key Laboratory for Structure and Evolution of Celestial Objects, Chinese Academy of Sciences, P.O. Box 110, Kunming 650216, People’s Republic of China
\textsuperscript{3}Key Laboratory of Optical Astronomy, National Astronomical Observatories, Chinese Academy of Sciences, Beijing, 100012, People’s Republic of China
\textsuperscript{4}University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
\textsuperscript{5}Center for Astronomical Mega-Science, Chinese Academy of Sciences, 20 A Datun Road, Chaoyang District, Beijing, 100012, People’s Republic of China

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Abstract

This paper reports the discovery of an Algol system KIC 10736223 that just passed the rapid mass transfer stage. From the light-curve and radial-velocity modeling we find KIC 10736223 to be a detached Algol system with the less-massive secondary nearly filling its Roche lobe. Based on the short-cadence Kepler data, we analyzed intrinsic oscillations of the pulsator and identified six secured independent \( \delta \) Scuti-type pulsation modes \((f_1, f_3, f_6, f_{10}, f_{32}, \text{and } f_{38})\). We compute two grids of theoretical models to reproduce the \( \delta \) Scuti frequencies, and find that fitting results of mass-accreting models agree well with those of single-star evolutionary models. The fundamental parameters of the primary star yielded with asteroseismology are \( M = 1.57^{+0.05}_{-0.09} \, M_\odot, \) \( Z = 0.009 \pm 0.001, \) \( R = 1.484^{+0.016}_{-0.028} \, R_\odot, \) \( \log g = 4.291^{+0.004}_{-0.009}, \) \( T_{\text{eff}} = 7748^{+230}_{-738} \, K, \) and \( L = 7.136^{+1.014}_{-1.519} \, L_\odot. \) The asteroseismic parameters match well with the dynamical parameters derived from the binary model. Moreover, our asteroseismic results show that the pulsator is an almost unevolved star with an age between 9.46 and 11.65 Myr for single-star evolutionary models and 2.67–3.14 Myr for mass-accreting models. Therefore, KIC 10736223 may be an Algol system that has just undergone the rapid mass-transfer process.

Unified Astronomy Thesaurus concepts: Asteroseismology (73); Stellar oscillations (1617); Delta Scuti variable stars (370); Eclipsing binary stars (444)

1. Introduction

The pulsating stars in eclipsing binaries are particularly attractive objects since the synergy allows us to directly determine accurate stellar fundamental parameters and offers us significant insight into the interior of the star (Aerts & Harmanec 2004; Mkrtichian et al. 2005, 2007). There have been a handful of efforts to investigate the interior physics of stars in detail through simultaneous binary and asteroseismic modeling, such as for solar-like pulsating binaries (White et al. 2018; Keen et al. 2018), and g-mode pulsating binaries (Hambleton et al. 2013; Keen et al. 2015; Guo et al. 2016; Schmid & Aerts 2015; Johnston et al. 2019). Beck et al. (2018) presented an asteroseismic study of the eccentric binary system KIC 9163796, which includes two oscillating red giant stars, and found the two component stars to be in the early and late phases of the first dredge-up event on the red giant branch. Schmid & Aerts (2016) combined binary and asteroseismic modeling for the \( \delta \) Scuti-\( \gamma \) Doradus hybrid binary KIC 10080943 and found that the amount of core overshooting and diffusive mixing can be well constrained under the assumption of equal age for the two stars. Pulsating stars in a post-mass transfer binary also deserve more attention as they carry evidence of binary interaction, offering us the opportunity to refine the theory of stellar structure and evolution. Guo et al. (2017) analyzed a post-mass transfer hybrid pulsator in eclipsing binary KIC 9592855 and found that rotation rates of both the core and envelope are similar to that of the orbital motion. Bowman et al. (2019) analyzed the TESS light curve of the oEA system U Gru and discovered that tidally perturbed oscillations also occur in the p-mode region. At present, about 200 eclipsing binaries have been found to contain \( \delta \) Scuti-type pulsating components (Kahraman Alicavus et al. 2017; Liakos & Niarchos 2017; Gaulme & Guzik 2019). However, due to the low radial orders, mode identifications are very difficult for \( \delta \) Scuti pulsators; thus, studies of these systems mostly concentrate on binary properties (da Silva et al. 2014; Zhang et al. 2015; Wang et al. 2018; Lee et al. 2019). Comprehensive asteroseismic modeling of \( \delta \) Scuti pulsators in eclipsing binaries is still lacking.

KIC 10736223 is an Algol-type eclipsing binary with an orbital period of 1.1051 days. It was discovered by Dahlmark (2000) in Cygnus. In the Kepler Input Catalog, the effective temperature \( T_{\text{eff}} \) and the metallicity of [Fe/H] for the star were given to be 7797 K and \(-0.193\) (Pinsonneault et al. 2012), respectively. The low-resolution spectroscopy observed by the LAMOST project (Luo et al. 2012) yields an effective temperature of 7554 K and metallicity of [Fe/H] = \(-0.511\) for the star. Matson et al. (2017) obtained a set of radial-velocity measurements for KIC 10736223 and determined the masses of the two component stars to be \(1.6 \pm 0.1 \, M_\odot\) and \(0.35 \pm 0.03 \, M_\odot\), respectively. Moreover, Debosscher et al. (2011) surveyed the variability of KIC 10736223 and found three main frequencies at 1.810503, 0.905251, and 4.523270 day\(^{-1}\). These frequencies are 2, 1, and 5 times the orbital frequencies, respectively. The intrinsic pulsations of KIC 10736223 are still poorly studied.

In this work, we carry out a comprehensive analysis of the eclipsing binary system KIC 10736223, including eclipse
analysis and asteroseismic modeling. In Section 2, we describe how physical parameters of KIC 10736223 are determined through the analysis of light and radial-velocity curves. In Section 3, we analyze pulsation characteristics of the primary star. In Section 4, we elaborate on details of input physics and our asteroseismic modeling. Finally, we summarize and discuss the results in Section 5.

2. The Eclipsing Binary System

KIC 10736223 was observed in both the long-cadence mode and the short-cadence mode by the Kepler satellite. For the purpose of this work, we used only the short-cadence data, which were obtained during Kepler’s observing quarter 16, from 2013 March 7 to April 8 with a time span of $\Delta T = 31.8$ days. We downloaded the data from the MAST data archive center (http://archive.stsci.edu/kepler/). Following the method described in Slawson et al. (2011), the simple aperture photometry light curve extracted from the data file was detrended and normalized, and outliers were removed. The radial-velocity data of the binary system are adopted from Matson et al. (2017) and contain six measurements, three for the primary and three for the secondary component. Using the ephemerides given in the Kepler eclipsing binary catalog (Kirk et al. 2016), phases were computed and the light and radial-velocity curves were folded as displayed in Figure 1.

In order to obtain reliable parameters for this eclipsing binary system, we model the light and radial-velocity curves simultaneously following Zhang et al. (2018) by using the Wilson–Devinney method (Wilson & Devinney 1971; Wilson 1979, hereafter W-D). The 2013 version of the W-D code was employed for light-curve analysis. In doing that, the effective temperature of the primary star was fixed at $T_{eff,1} = 7554$ K based on the LAMOST spectroscopy. The gravity-darkening exponent and the bolometric albedo of the primary star were both set to 1.0 according to Lucy (1967) and Rucinski (1969), considering that it could have a radiative envelope. The parameters of the secondary component were taken as 0.32 and 0.5, respectively, according to the effective temperature that resulted from the iteration. The initial bolometric limb-darkening coefficients in logarithmic form ($X_{1,2}$, $Y_{1,2}$) were taken from van Hamme (1993), and the monochromatic ones ($x_{1,2}$, $y_{1,2}$) in the Kepler band were taken from Claret & Bloemen (2011). Since the orbit of the binary is tight and the light curve is symmetric with two eclipses that are separated by exactly 0.5 and have equal eclipsing durations in phase, KIC 10736223 ought to be a circularized and synchronized system. A synchronous rotation for both
Table 1
Physical Parameters of the Binary KIC 10736223

| Parameter   | Without Spot | With Spot |
|-------------|--------------|-----------|
| $p_{\text{rot}}$(days) | 1.1050943 | 1.1050943 |
| $i$(deg)   | 89.657 ± 0.007 | 89.411 ± 0.006 |
| $q = M_2/M_1$ | 0.211 ± 0.001 | 0.1998 ± 0.0003 |
| $T_1$(K)   | 7554 ± 200$^a$ | 7554 ± 200$^a$ |
| $T_2$(K)   | 5045 ± 135 | 5006 ± 135 |
| $\Omega$  | 4.1529 ± 0.0005 | 4.1431 ± 0.0005 |
| $\Omega_2$ | 2.3167 ± 0.0013 | 2.2713 ± 0.0011 |
| $L_1/(L_1 + L_2)$($\nu_p$) | 0.878 ± 0.001 | 0.878 ± 0.001 |
| $r_1$(pole) | 0.2533 ± 0.0001 | 0.2532 ± 0.0001 |
| $r_1$(point) | 0.2575 ± 0.0001 | 0.2573 ± 0.0001 |
| $r_1$(side) | 0.2558 ± 0.0001 | 0.2557 ± 0.0001 |
| $r_1$(back) | 0.2570 ± 0.0001 | 0.2569 ± 0.0001 |
| $r_2$(pole) | 0.2230 ± 0.0007 | 0.2230 ± 0.0006 |
| $r_2$(point) | 0.2720 ± 0.0020 | 0.2781 ± 0.0021 |
| $r_2$(side) | 0.2305 ± 0.0008 | 0.2308 ± 0.0007 |
| $r_2$(back) | 0.2537 ± 0.0012 | 0.2560 ± 0.0011 |

Spot Parameters:

| Parameter   | Value          |
|-------------|----------------|
| Colatitude (deg) | ...            |
| Longitude (deg) | ...            |
| Radius (deg)   | ...            |
| $T_{\text{spot}}/T_{\text{local}}$ | ...            |
| $\sum(O - C)^2$ ($10^{-4}$) | 0.8876          | 0.8736          |

Absolute Parameters:

| Parameter   | Value          |
|-------------|----------------|
| $a$ ($R_\odot$) | 5.68 ± 0.11    | 5.69 ± 0.11    |
| $\gamma$ (km s$^{-1}$) | -5.56 ± 1.73  | -6.40 ± 1.74   |
| $M_1$($M_\odot$) | 1.66 ± 0.09    | 1.69 ± 0.09    |
| $M_2$($M_\odot$) | 0.35 ± 0.02    | 0.34 ± 0.02    |
| $R_1$($R_\odot$) | 1.45 ± 0.03    | 1.45 ± 0.03    |
| $R_2$($R_\odot$) | 1.34 ± 0.03    | 1.35 ± 0.03    |
| $L_1$($L_\odot$) | 6.08 ± 0.72    | 6.14 ± 0.72    |
| $L_2$($L_\odot$) | 1.04 ± 0.12    | 1.14 ± 0.12    |
| log $g_1$ | 4.34 ± 0.03    | 4.34 ± 0.03    |
| log $g_2$ | 3.71 ± 0.03    | 3.71 ± 0.03    |

Note.

* Assumed values.

components and a circular orbit were therefore adopted for the binary model.

We started the differential-correction (DC) program of the W-D code from Mod 2 (with detached configuration). The free parameters are the mass ratio ($q = M_2/M_1$), the orbital inclination ($i$), the phase shift, the effective temperature of star 2 ($T_2$), the dimensionless surface potential ($\Omega$), and the luminosity of the two stars. These parameters were alternately adjusted until a converged solution was reached. The results are listed in the second column of Table 1. Based on the solution, the theoretical light curve was calculated as presented by the dashed curve in the top panel of Figure 1. The $O - C$ light residuals of the solution are plotted in the second panel of the figure, wherein it can be seen that there is a structure remaining in the residuals that is coherently phased with the orbit. The unsotted binary model does not describe the observed light curve perfectly around phases 0.2 and 0.5.

The initial binary solution reveals that KIC 10736223 is very likely a detached system consisting of a K-type secondary component. The light-curve discrepancy may result from the probable magnetic activity of the cool secondary. We thus tested this possibility by placing a cool spot on the secondary star to solve the light discrepancy. The spot parameters are the spot temperature given as a fraction of the surrounding photospheric temperature, the spot radius, and its colatitude and longitude. The preliminary spot longitude could be found approximately from the phases of the light distortion. The other three parameters were calculated by adjusting the theoretical light curve to approximately fit the observed light curve. The spot parameters were then adjusted along with the free parameters. Finally, we obtained the best-fitting model. The best solution is given in the third column of Table 1 and is described by a solid curve in the top panel of Figure 1. The parameter errors given in Table 1 are produced by the DC code computed from the covariance matrix using the standard method. The bottom panel of Figure 1 presents the $O - C$ residuals computed from the spotted binary model. It shows that the cool spot on the secondary component could almost entirely explain the light variation. We calculate the sum of the squared residuals for the models with the spot and without the spot, respectively. As shown in Table 1, the $\sum(O - C)^2$ value of the model with the spot is smaller than that of the model without the spot.

The synthesis to the radial-velocity measurements based on the best solution is illustrated in the lower two panels of Figure 1. The synthesis yields a semiaxis of 5.69 ± 0.11 $R_\odot$ for the binary system. Combining the spectroscopic solution with the results of the light-curve modeling, the physical parameters including mass, radius, luminosity, and surface gravity of the two components were determined as given in Table 1. The physical parameters derived for the primary component suggest that it is an unevolved main-sequence star. The less-massive secondary is much more evolved. With a short orbital period and a small mass ratio, KIC 10736223 is a typical classical Algol system that is formed through mass exchange and mass-ratio reversal. With a filling factor of $R_2/R_{\text{crit}} = 0.97$, the secondary star is almost filling its Roche lobe, probably indicating that the binary system just passed the rapid mass-transfer stage.

3. Frequency Analysis

Based on the derived photometric analysis, we compute the time-resolved theoretical light curves with the binary model, and then obtain the pulsational light variations by subtracting it from the short-cadence data. Figure 2 presents light residuals in plots of magnitude versus BJD. All of the residuals are shown in the upper panel and the close-up view of a portion of the residuals is shown in the lower panel. In order to investigate the pulsation features in detail, we carry out a multiple frequency analysis of the light residuals with the Period04 program (Lenz & Breger 2005). No signals are detected in the high frequency region ($f > 100$ day$^{-1}$). We then perform further frequency extraction in the frequency range of 0–100 day$^{-1}$.

At each step of the iteration, we select the frequency with the highest amplitude and perform a multiperiod least-square fit to the data using all frequencies already detected. The data are then prewhitened and the residuals are used for further analysis until arriving at the empirical threshold of the signal-to-noise ratio ($S/N$) = 5.4 (Baran et al. 2015). Finally, a total of 61 frequencies with $S/N > 5.4$ were detected, which are listed in Table 2. The noises are calculated in the range of 2 day$^{-1}$ around each frequency, and uncertainties of frequencies and amplitudes are calculated based on the treatment proposed by
Figure 3 presents Fourier amplitude spectra of the light residuals; the original spectrum is shown in the upper panel and the residual spectrum of the 61 frequencies after prewhitening is shown in the lower panel.

We check the extracted frequencies and search for possible orbital harmonics and linear combination frequencies in the form of $f_k = f_i \pm m f_{orb}$ or $f_k = m f + n f_i$ (Pápics 2012; Kurtz et al. 2015), where $m$ and $n$ are integers, $f_i$ and $f_j$ are the parent frequencies, $f_k$ is the combination frequency, and $f_{orb} = 0.9049 \text{ day}^{-1}$. A peak is accepted as a combination if the amplitudes of both parent frequencies are larger than that of the presumed combination term, and the difference between the observed frequency and the predicted frequency is smaller than the frequency resolution $1.5/\Delta T = 0.047 \text{ day}^{-1}$ (Loumos & Deeming 1978; Lee et al. 2019). As shown in Table 2, we identify 46 such frequencies. When two frequencies are closer than $1.5/\Delta T = 0.047 \text{ day}^{-1}$, the lower amplitude frequency is identified as the possible side lobe. Given that the Kepler satellite rotates 90° every 93 days, the corresponding frequency is $f_{sat} = 0.011 \text{ day}^{-1}$ (Hass et al. 2010; Van Cleve & Caldwell 2016; Yang et al. 2018). We also discard the peak $f_{48}$, which is two times the frequency $f_{sat}$. Finally, six confident independent frequencies $f_1, f_3, f_4, f_{19}, f_{42}$, and $f_{48}$ are retained and marked in bold in Table 2.

The six independent frequencies range from 23.1127 to 50.5292 days$^{-1}$. Connecting with physical parameters of the components in Table 1, we identify the primary star as a δ Scuti pulsator with multiperiodic pulsations.

4. Stellar Models
4.1. Input Physics

The one-dimensional stellar evolution code Modules for Experiments in Stellar Astrophysics (MESA, version 10398; Paxton et al. 2011, 2013, 2015, 2018) is used to generate theoretical models. In particular, the submodule called “pulse_a-dipls” was adopted to compute evolutionary models of stars and calculate adiabatic frequencies of their corresponding radial oscillations and nonradial oscillations (Christensen-Dalsgaard 2008; Paxton et al. 2011, 2013, 2015, 2018).

In the calculations, the 2005 update of the OPAL equation of state tables (Rogers & Nayfonov 2002) is used. The OPAL opacity tables of Iglesias & Rogers (1996) are used for the high temperature region, and the tables of Ferguson et al. (2005) are used for the low temperature region. We adopt “simple_photosphere” for the atmosphere boundary condition, and use the solar metal composition AGSS09 (Asplund et al. 2009) as the initial ingredient in metal. In the convective region, we use the classical mixing length theory of Böhm-Vitense (1958) with the solar value of $\alpha = 1.9$ (Paxton et al. 2013) to treat convection. In addition, effects of the stellar rotation, the convective overshooting, and the element diffusion, as well as
magnetic fields on the structure and evolution of the star are not included in this work (for more details, refer to the Appendix).

4.2. Single-star Evolutionary Models

A grid of single-star evolutionary models is computed with the MESA code. The mass fraction of helium $Y$ in the grid is set to $Y = 0.249 + 1.33Z$ (Li et al. 2018), as a function of the mass fraction of heavy elements $Z$. Thus the evolutionary track and interior structure of a star are completely determined by $M$ and $Z$. We consider the stellar mass $M$ between 1.45 $M_\odot$ and 2.00 $M_\odot$ in a mass interval of 0.01 $M_\odot$ and metallicities $Z$ between 0.003 and 0.030 in an interval of 0.001.

Each star in the grid is computed starting from the pre-main-sequence stage and ending when the central hydrogen of the star is exhausted ($X_c < 1 \times 10^{-5}$). Based on the binary model, we adopt $7000 < T_{\text{eff}} < 8000$ K, $4.26 < \log g < 4.40$, and $1.40 R_\odot < R < 1.60 R_\odot$ as the observational constraints for potential models. When a star evolves along its evolutionary track into this region, adiabatic frequencies of the radial oscillations ($\ell = 0$) and nonradial oscillations with $\ell = 1$ and 2 are calculated for the structure model at each evolutionary step. Due to the cancellation effects of the surface geometry, oscillation modes with a higher degree are hardly visible, we then do not include oscillation modes with $\ell \geq 3$.

The component stars in binaries always rotate along with their orbital motion. We introduce the rotation period $P_{\text{rot}}$ of the star as another adjustable parameter (Chen et al. 2019) and consider $P_{\text{rot}}$ between 0 and 3 days with a step of 0.02 days. Effects of rotation will result in each nonradial oscillation mode with the spherical harmonic degree $\ell$ splitting into 2$\ell + 1$ different frequencies. The general expression of the first-order effect of rotation on pulsation was derived as

$$v_{\ell,m} = v_{\ell,n} + m \delta v_{\ell,n} = v_{\ell,n} + \beta_{\ell,n} \frac{m}{P_{\text{rot}}}$$  \hspace{1cm} (1)

(Aerts et al. 2010), where $\delta v_{\ell,n}$ is the splitting frequency, and the radial orders $n$, the spherical harmonic degree $\ell$, and the azimuthal number $m$ are three indices characterizing oscillation modes. As shown in Equation (1), the effect of the rotation on pulsation is completely determined by the constant $\beta_{\ell,n}$, which
is deduced to be

\[ \beta_{\ell,m} = \frac{\int_0^R (\xi_r^2 + L^2 \xi_h^2 - 2 \xi_r \xi_h - \xi_h^2) r^2 \rho \, dr}{\int_0^R (\xi_r^2 + L^2 \xi_h^2) r^2 \rho \, dr} \]  

(Aerts et al. 2010). In Equation (2), \( \xi_r \) represents the radial displacement, \( \xi_h \) represents the horizontal displacement, \( \rho \) represents the local density of the star, and \( L^2 = \ell (\ell + 1) \). According to Equation (2), each dipole mode splits into three different components, corresponding to modes with \( m = -1, 0, \) and \( +1 \), respectively. Each quadrupole mode splits into five different components, corresponding to modes with \( m = -2, -1, 0, +1, \) and \( +2 \), respectively.

Then we perform a \( S^2 \) minimization by comparing frequencies between model and observations according to

\[ S^2 = \frac{1}{k} \sum (\nu_{\text{mod},i} - \nu_{\text{obs},i})^2, \]

where \( k \) is the number of the observed frequencies, and \( \nu_{\text{mod},i} \) and \( \nu_{\text{obs},i} \) are a pair of matched model-observed frequencies. Due to there being no preconceived idea of identifications of the six \( \delta \) Scuti frequencies \( f_1, f_3, f_6, f_{19}, f_{42}, \) and \( f_{48} \), the random fitting algorithm is adopted. The model frequency nearest to the observed frequency is treated as the most likely matched model counterpart.

Figures 4 and 5 depict changes of fitting results \( S^2_m \) versus various physical parameters. Similar to the work of Chen et al. (2016), the solution is found to be limited to a small parameter space for a given evolutionary track, we thus choose a single model with the minimum value of \( S^2 \) per evolutionary track and denote the minimum value with \( S^2_m \). The horizontal lines in the figures mark the position of \( S^2_m = 0.13 \), which corresponds to the square of \( 1/\Delta T \). The circles above the horizontal line correspond to 34 candidate models in Table 3. Model A24 has a minimum value of \( S^2_m = 0.023 \), we thus consider Model A24 as the best-fitting model and mark it with the filled circles in Figures 4 and 5.

Figures 4(a)–(c) present changes of \( S^2_m \) as a function of the metallicity \( Z \), the stellar mass \( M \), and the rotation period \( P_{\text{rot}} \), respectively. In the figures, we find that values of \( Z \) and \( P_{\text{rot}} \) show excellent convergence, i.e., \( Z = 0.009 \pm 0.001 \) and \( P_{\text{rot}} = 1.00^{+0.02}_{-0.06} \) days. However, values of \( M \) are found to cover a wide range between 1.48 \( M_\odot \) and 1.62 \( M_\odot \).

Figure 4(d) presents changes of \( S^2_m \) as a function of the age of the star. It can be clearly seen in the figure that ages of candidate models converge well to \( 10.27^{+1.38}_{-0.81} \) Myr. The pulsating primary appears to be an almost unevolved star on the zero-age main sequence, probably indicating that the binary system KIC 10736223 has just undergone a rapid mass-transfer stage.

Figures 5(a)–(d) present changes of \( S^2_m \) as a function of various fundamental stellar parameters: the gravitational
Figure 4. Visualization of fitting results $S_m^2$ vs. physical parameters: the metallicity $Z$, the stellar mass $M$, the rotation period $P_{rot}$, and the ages of stars, respectively. The horizontal line in orange marks the position of $S_m^2 = 0.13$. The filled circle denotes the best-fitting model.

Figure 5. Visualization of fitting results $S_m^2$ vs. stellar fundamental parameters: the gravitational acceleration $\log g$, the stellar radius $R$, the effective temperature $T_{\text{eff}}$, and the stellar luminosity $L$, respectively. The horizontal line marks the position of $S_m^2 = 0.13$. The filled circle denotes the best-fitting model.
acceleration log g, the stellar radius R, the effective temperature $T_{\text{eff}}$, and the stellar luminosity L, respectively. As illustrated in the figures, values of log g and R converge well to 4.291$^{+0.009}_{-0.001}$ and 1.484$^{+0.016}_{-0.025}$ $R_\odot$, respectively. However, the convergence of $T_{\text{eff}}$ and L are relatively worse, i.e., $T_{\text{eff}} = 7748^{+240}_{-230}$ K and $L = 7.136^{+1.014}_{-1.319} L_\odot$.

Based on the above analyses, stellar parameters of the primary star obtained by asteroseismology are listed in the second column of Table 4. These parameters are in agreement with those derived from the binary model. Table 5 lists model frequencies of the best-fitting model. Table 6 lists comparisons between model frequencies derived from the best-fitting model and observations. According to the comparisons, $f_0$ and $f_{19}$ are identified as two dipole modes, and $f_1, f_3, f_{42}$, and $f_{48}$ as four quadrupole modes.

### 4.3. Mass-accreting Models

KIC 10736223 is a classical Algol binary system formed through mass exchange and mass-ratio reversal. Ideally, binary evolution models with mass transfer should be adopted. However,
Table 5: Theoretical Frequencies Derived from the Optimal Single-star Evolutionary Model (Model A24)

| $\nu_{\text{mod}}(\ell, n)$ | $\beta_{\ell,n}$ | $\nu_{\text{obs}}(\ell, n)$ | $\beta_{\ell,n}$ | $\nu_{\text{mod}}(\ell, n)$ | $\beta_{\ell,n}$ |
|--------------------------|-----------------|------------------|-----------------|------------------|-----------------|
| 248.342(0.0)            | 133.496(1.0)    | 0.515            | 200.425(2.0)    | 0.800            |
| 318.170(0.1)            | 255.109(1.1)    | 0.989            | 256.119(2.0)    | 0.991            |
| 385.157(0.2)            | 332.967(1.2)    | 0.991            | 308.923(2.1)    | 0.830            |
| 455.871(0.3)            | 412.580(1.3)    | 0.988            | 368.927(2.2)    | 0.856            |
| 532.781(0.4)            | 492.669(1.4)    | 0.984            | 444.292(2.3)    | 0.923            |
| 613.349(0.5)            | 573.554(1.5)    | 0.980            | 524.448(2.4)    | 0.951            |
| 694.314(0.6)            | 654.779(1.6)    | 0.979            | 606.248(2.5)    | 0.964            |
| 774.716(0.7)            | 735.115(1.7)    | 0.979            | 687.592(2.6)    | 0.972            |

Note. $\nu_{\text{mod}}$ denotes the model frequency, $\ell$ and $n$ are its spherical harmonic degree and radial order, respectively. $\beta_{\ell,n}$ is the rotational parameters defined as Equation (2).

Table 6: Comparisons between Model Frequencies of the Optimal Single-star Evolutionary Model (Model A24) and Observations

| ID  | $\nu_{\text{obs}}$ (Hz) | $\nu_{\text{mod}}$ (Hz) | $(\ell, n, m)$ | $\nu_{\text{obs}} - \nu_{\text{mod}}$ (Hz) |
|-----|-------------------------|-------------------------|----------------|---------------------------------|
| $f_1$ | 289.867                | 289.710                 | (2, 1, −2)    | 0.157                          |
| $f_2$ | 267.508                | 267.589                 | (2, 0, +1)    | 0.081                          |
| $f_3$ | 584.829                | 584.897                 | (1, 5, +1)    | 0.068                          |
| $f_4$ | 344.619                | 344.427                 | (1, 2, +1)    | 0.192                          |
| $f_5$ | 348.891                | 349.112                 | (2, 2, −2)    | 0.221                          |
| $f_6$ | 546.582                | 546.462                 | (2, 4, +2)    | 0.120                          |

Note. $\nu_{\text{obs}}$ is the observed frequency, $\nu_{\text{mod}}$ is the model frequency, $\nu_{\text{obs}} - \nu_{\text{mod}}$ denotes the difference between the observed frequency and its model counterpart.

Figure 7(d) shows changes of $S_2^0$ as a function of the age of the star. Here, the age corresponds to the evolutionary time since the mass accretion ended. It can be clearly seen in the figure that ages of candidate models converge well to $2.68^{+0.04}_{-0.01}$ Myr, also suggesting that the binary system just passed the rapid mass transfer stage.

Table 8 lists comparisons between model frequencies of the optimal mass-accreting model and observations. Table 9 shows that identifications of $f_1$, $f_2$, $f_3$, $f_4$, $f_5$, and $f_6$ are the same as those of single-star evolution. For $f_5$, mass-accreting models suggest it to be a dipole mode.

5. Summary and Discussion

We have carried out a detailed analysis of the eclipsing binary KIC 10736223, through binary properties and asteroseismology. The results of light-curve modeling reveal that the light curve of KIC 10736223 can be almost entirely explained by including a cool spot on the secondary star. Stellar parameters of the two component stars derived from the binary model are $M_1 = 1.69 \pm 0.09$ $M_\odot$, $R_1 = 1.45 \pm 0.03$ $R_\odot$, log $g_1 = 4.34 \pm 0.03$, and $M_2 = 0.34 \pm 0.02$ $M_\odot$, $R_2 = 1.35 \pm 0.05$ $R_\odot$, log $g_2 = 3.71 \pm 0.03$, respectively. The simultaneous light-curve and radial-velocity modeling reveals a detached configuration for the binary system with the less-massive secondary nearly filling its Roche lobe.

By subtracting the binary model from the original Kepler data, we obtain the light variations due to intrinsic stellar pulsations. Through a multiple frequency analysis for the light residuals, we identify six confident independent frequencies ($f_1$, $f_2$, $f_3$, $f_4$, $f_5$, and $f_6$). These frequencies range from 23.1127 to 50.5292 day$^{-1}$, we then identify KIC 10736223 as a member of eclipsing binaries consisting of a $\delta$ Scuti pulsator.

To reproduce the six $\delta$ Scuti frequencies, we compute two grids of theoretical models, including a grid of single-star evolutionary models and a grid of mass-accreting models. Due to there being no preconceived idea of mode identifications for the observed frequencies, we adopt a random fitting algorithm. Fitting results of mass-accreting models are in good agreement with those of single-star evolutionary models.

Fundamental parameters of the primary star are determined to be $M_1 = 1.57^{+0.05}_{-0.05}$ $M_\odot$, $Z = 0.009 \pm 0.001$, $R = 1.48^{+0.01}_{-0.02}$ $R_\odot$, log $g = 4.291^{+0.006}_{-0.021}$, $T_{\text{eff}} = 7748^{+230}_{-75}$ K, and $L = 7.136^{+1.014}_{-1.519}$ $L_\odot$. According to the theory of stellar oscillations, properties of $p$ modes can be roughly characterized by the acoustic radius $\tau_0$, which is the sound travel time from the surface of the star to the core. Aerts et al. (2010) defined the acoustic radius $\tau_0$ as

$$\tau_0 = \int_0^R \frac{dr}{c_s}$$

in which $c_s$ is the adiabatic sound speed and $R$ is the stellar radius. As an important asteroseismic parameter, $\tau_0$ is usually used to characterize features of the stellar envelope (e.g., Ballot et al. 2004; Miglio et al. 2010; Chen et al. 2016). Comparing the two kinds of models, we find that their acoustic radii $\tau_0$ match each other well, i.e., between 1.50 and 1.69 hr for single-star evolutionary models and 1.61–1.69 hr for mass-accreting models.
Figure 6. Visualization of fitting results $S_m^2$ vs. adjustable parameters of mass-accreting models: the metallicity $Z$, initial stellar mass $M_1$, final stellar mass $M_2$, and rotation period $P_{\text{rot}}$, respectively. The horizontal line marks the position of $S_m^2 = 0.075$.

Figure 7. Visualization of fitting results $S_m^2$ vs. stellar fundamental parameters of mass-accreting models: the effective temperature $T_{\text{eff}}$, the gravitational acceleration $\log g$, the stellar radius $R$, and the age of stars, respectively. The age represents the evolutionary time since the mass accretion ends. The horizontal line marks the position of $S_m^2 = 0.075$. 

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Table 7
Candidate Mass-accreting Models Where $S_n^2 \leq 0.13$ $P_{rot}$ Is the Rotation Period

| Model | $P_{rot}$ (day) | Z | $M_1$ ($M_\odot$) | $M_2$ ($M_\odot$) | $T_{eff}$ (K) | $\log g$ | $R$ ($R_\odot$) | $L$ ($L_\odot$) | $\tau_0$ (hr) | $X_e$ | Age (Myr) | $S_n^2$ |
|-------|----------------|---|-------------------|-------------------|-------------|----------|-------------|-------------|--------------|-------|----------|--------|
| B1    | 0.94           | 0.008 | 0.45             | 1.50              | 7673        | 4.283    | 1.464       | 6.678       | 1.564        | 0.7308 | 3.14     | 0.126  |
| B2    | 0.94           | 0.008 | 0.45             | 1.52              | 7777        | 4.285    | 1.471       | 7.095       | 1.564        | 0.7308 | 2.98     | 0.120  |
| B3    | 0.94           | 0.008 | 0.50             | 1.50              | 7677        | 4.283    | 1.465       | 6.694       | 1.561        | 0.7308 | 3.13     | 0.124  |
| B4    | 0.95           | 0.008 | 0.50             | 1.52              | 7777        | 4.285    | 1.472       | 7.118       | 1.564        | 0.7308 | 2.96     | 0.119  |
| B5    | 0.95           | 0.008 | 0.50             | 1.52              | 7777        | 4.285    | 1.471       | 7.093       | 1.564        | 0.7308 | 2.98     | 0.124  |
| B6    | 0.95           | 0.008 | 0.45             | 1.54              | 7878        | 4.287    | 1.478       | 7.564       | 1.566        | 0.7308 | 2.81     | 0.108  |
| B7    | 0.95           | 0.008 | 0.45             | 1.56              | 7963        | 4.289    | 1.484       | 7.955       | 1.569        | 0.7308 | 2.68     | 0.106  |
| B8    | 0.95           | 0.008 | 0.50             | 1.52              | 7777        | 4.285    | 1.471       | 7.117       | 1.564        | 0.7308 | 2.97     | 0.118  |
| B9    | 0.95           | 0.008 | 0.50             | 1.54              | 7862        | 4.287    | 1.477       | 7.491       | 1.566        | 0.7308 | 2.83     | 0.107  |
| B10   | 0.95           | 0.008 | 0.50             | 1.56              | 7972        | 4.289    | 1.484       | 7.997       | 1.569        | 0.7308 | 2.67     | 0.107  |
| B11   | 0.96           | 0.008 | 0.45             | 1.56              | 7963        | 4.289    | 1.484       | 7.955       | 1.569        | 0.7308 | 2.68     | 0.111  |
| B12   | 0.96           | 0.008 | 0.50             | 1.56              | 7962        | 4.289    | 1.484       | 7.994       | 1.569        | 0.7308 | 2.67     | 0.114  |

Note. $\tau_0$ is the acoustic radius. $X_e$ is the mass fraction of hydrogen in the center of the star. The age represents the evolutionary time since the mass accretion ended.

Table 8
Comparisons between Model Frequencies of the Optimal Mass-accreting Model (Model B7) and Observations

| ID | $\nu_{obs}$ ($\mu$Hz) | $\nu_{mod}$ ($\mu$Hz) | ($\ell, n, m$) | $|\nu_{obs} - \nu_{mod}|$ ($\mu$Hz) |
|----|-----------------------|------------------------|---------------|-----------------------------------|
| $f_1$ | 289.867 | 289.912 | (2, 1, 2) | 0.045 |
| $f_2$ | 267.508 | 267.098 | (1, 1, 1) | 0.410 |
| $f_3$ | 584.829 | 584.475 | (1, 5, +1) | 0.354 |
| $f_4$ | 344.619 | 344.826 | (2, 1, +1) | 0.207 |
| $f_5$ | 348.891 | 348.763 | (2, 2, -2) | 0.128 |
| $f_6$ | 546.582 | 547.114 | (2, 4, +2) | 0.532 |

Note. $\nu_{obs}$ is the observed frequency. $\nu_{mod}$ is the model frequency. $|\nu_{obs} - \nu_{mod}|$ denotes the difference between the observed frequency and its model counterpart.

The pulsating primary component of KIC 4544587 is found to be an almost unevolved star near the zero-age main sequence. Ages of the primary star converge to $10.27^{+1.38}_{-0.81}$ Myr for single-star evolutionary models and $2.68^{+0.46}_{-0.41}$ Myr for mass-accreting models, respectively. Given that KIC 10736223 is a classical Algol system formed through mass exchange and mass-ratio reversal, the system KIC 10736223 probably has just passed a rapid mass-transfer stage.

Comparisons between model frequencies and observations suggest that $f_6$ and $f_{10}$ are two dipole modes and $f_1, f_{12}$, and $f_{18}$ are three quadrupole modes. For $f_3$, there are two possible identifications, i.e., $\ell = 2$ by single-star evolutionary models and $\ell = 1$ by mass-accreting models. Moreover, we find that $f_9$ and $f_{19}$ are identified as two $m = +1$ dipole modes and $f_{13}, f_{14},$ and $f_{18}$ as three $|m| = 2$ quadrupole modes. This feature reveals that the primary star has a high rotational inclination angle according to Gizon & Solanki (2003), which meets with the orbital inclination angle of $89.411 \pm 0.006$ degrees.

Finally, our astroseismic results show that the rotation of the primary star is very close to the synchronous rotation, which matches the previous hypothesis. The rotation period $P_{rot}$ of the primary star is determined to be $1.00^{+0.02}_{-0.01}$ days for single-star evolutionary models and $0.95 \pm 0.01$ days for mass-accreting models. According to the work of Saio (1981), Dziembowski & Goode (1992), and Aerts et al. (2010), the first-order effect of rotation on pulsation is in proportion to $1/P_{rot}$, and that of the second-order is in proportion to $1/(P_{rot}^2 \nu_{e,n})$. Then their ratio can be estimated to be on the order of $1/(P_{rot} \nu_{e,n})$, where $\nu_{e,n}$ ranges from $23.1127$ to $50.5292$ days$^{-1}$. The second-order effect of rotation on pulsation is much less than that of the first-order one, thus the second-order effect of rotation on pulsation is not included in this work.

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Appendix
Inlist Files Used in This Work (Version 10398)

A.1. The Inlist File for Single-Star Evolution

```plaintext
! inlist pulse
&star_job
asterojust_call_myExtras_check_model = .true.
show_log_description_at_start = .false.
create_pre_main_sequence_model = .true.
change_lastPgas_flag = .true.
new_lastPgas_flag = .true.
change_initial_net = .true.
new_net_name = 'o18_and_ne22.net'
kappa_file_prefix = 'a09'
kappa_lowT_prefix = 'lowT_fa05_a09p'
initial_zfractions = 6
! end of star_job namelist
&controls
initial_mass = 1.57
```
A.2. The Inlist File for Mass Accretion from M₁ to M₂

```
! inlist_pulse
@star_job
astero.just.call.my.extras.check.model = .true.
show_log.description.at.start = .false.
create.pre.main.sequence.model = .true.
save.model.when.terminate = .true.
save.model.filename = 'final_mass.mod'
change.lnPgas_flag = .true.
new.lnPgas_flag = .true.
change.initial.net = .true.
new.net.name = 'o18_and_ne22.net'
kappa.file.prefix = 'a09'
kappa.lowT.prefix = 'lowT_fa05_a09p'
initial.zfracs = 6
! end of star_job namelist
@controls
initial_z = 0.008
initial_y = 0.25964
initial_mass = 0.45
mass_change = 1d-6 !if star.age > 2d9 years
accrete.same.as.surface = .true. !if star.age > 2d9 years
star_mass_max.limit = 1.56 !final stellar mass
MLT_option = 'MLI'
mixing_length.alpha = 1.90
calculate.Brunt.N2 = .true.
use.brunt.gradmuX.form = .true.
which.atm.option = 'simple.photosphere' !default
max_number_backups = 50
max_number_retries = 100
max_model_number = 80000
history_interval = 1
max_num_profile_models = 80000
xa.central_lower_limit_species(1) = 'h1'
xa.central_lower_limit(1) = 1d-5
use_other_mesh_functions = .true.
mesh_delta_coeff = 0.9
M.function.weight = 50
max.center_cell_dq = 1d-10
max.allowed.nz = 80000
varcontrol_target = 2d-5
max.years_for.timestep = 5d6 ! 5d4 if star.age > 2d9 years
! end of controls namelist
```

A.3. The Inlist File for Evolution of the Accreted Models

```
! inlist_pulse
@star_job
astero.just.call_my.extras_check.model = .true.
show_log.description_at_start = .false.
load_saved_model = .true.
saved_model_name = 'final_mass.mod'
change.lnPgas_flag = .true.
new.lnPgas_flag = .true.
change.initial.net = .true.
new.net.name = 'o18_and_ne22.net'
kappa.file.prefix = 'a09'
kappa.lowT.prefix = 'lowT.fa05.a09p'
initial.zfracs = 6
! end of star_job namelist
@controls
initial_mass = 1.56
initial_z = 0.008
initial.y = 0.25964
M.function.weight = 50
max.center_cell_dq = 1d-10
max.allowed.nz = 80000
varcontrol_target = 2d-5
max.years_for.timestep = 1d6!for main-sequence models
(1d3 for pre-main sequence models)
! end of controls namelist
```

ORCID iDs
Xiaobin Zhang @ https://orcid.org/0000-0002-5164-3773
Yan Li @ https://orcid.org/0000-0002-1424-3164
Jie Su @ https://orcid.org/0000-0001-7566-9436

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