We present results from magnetic monopoles in SU(2) lattice gauge theory at finite temperature. The lattices are $16^3 \times N_t$, for $N_t = 4, 6, 8, 12$, at $\beta = 2.5115$. Quantities discussed are: the spacial string tension, Polyakov loops, and the screening of timelike and spacelike magnetic currents.
1 Introduction

Magnetic monopoles found after gauge-fixing into the maximum Abelian gauge have been successful in explaining the fundamental string tension at $T = 0$ in $SU(2)$ lattice gauge theory \cite{1}. However, there has been an apparent serious problem with the spacial string tension at finite temperature \cite{2}. This problem has recently been resolved, and the monopole results are now in good agreement with the full $SU(2)$ answers. This is discussed in more detail in a recent paper \cite{3}, so here I will only mention that the difficulty was with the method of calculation rather than monopoles themselves. A contribution from Dirac sheets, derived first by Smit and van der Sijs \cite{4}, had been omitted. Evidently this is justified at $T = 0$, but not for temperatures above the deconfining temperature, which corresponds to $N_t = 8$ for the present calculations at $\beta = 5115$ \cite{5}.

In the remainder of this report, I will discuss the contributions of monopoles to correlators of Polyakov loops, and the closely related question of how the magnetic current screens itself.

2 Polyakov Loops

We denote by $C(R)$ the correlation function of Polyakov loops, $< P(R), P(0) >$, where the arguments refer to the spacial locations of the two Polyakov loops. For temperature $T < T_c$, or $N_t > 8$, $C(R)$ can be used to measure a $T$-dependent physical string tension, which is expected to approach 0 smoothly for $SU(2)$ lattice gauge theory as $T \rightarrow T_c$. For $T > T_c$, or $N_t < 8$, we are in the deconfined phase and $C(R)$ determines electric screening masses.

The above remarks are for $C(R)$ in full $SU(2)$. Here we are concerned with how much of this physics can be obtained from monopoles. The monopole contribution to $C(R)$ is readily calculated using the same method as for Wilson loops. The naive method \cite{1} can be used, since numerically the Dirac sheet contributions are negligible for Polyakov loop correlators. In the following we denote the correlator of Polyakov loops calculated from monopoles by $C_{mon}(R)$.

In Fig. (1) we show $-\ln(C_{mon})/N_t$ vs. $R$ for the four temperatures $T/T_c = 0.66, 1.00, 1.33, \text{ and } 2.00$ respectively, corresponding to $N_t = 12, 8, 6, \text{ and } 4$. The data points were generated by averaging over 500 widely spaced configurations at each $N_t$. The algorithm used and details of the gathering of configurations are given in \cite{3}.

First taking the case of $N_t = 12, \text{ or } T/T_c = 0.66$, the data points are well fit by a simple linear function. The slope of the straight line determines the physical string tension, $\sigma_p(T)$. The fit gives $\sigma_p(2/3T_c) = 0.029(2)$. We may translate this into a zero temperature value...
using the formula
\[
\sigma_p(0) = \sigma_p(T) + \frac{\pi}{3N_t^2}
\]
which gives \(\sigma_p(0) = 0.036(2)\), within errors of high precision numbers for the zero temperature string tension at \(\beta = 2.5115\). So for finite temperatures \(T < T_c\), monopoles appear to explain the physical string tension just as at zero temperature. We have no other runs at finite temperature below \(T_c\), but the data for \(T = T_c\) shows curvature at all values of \(R\), implying that the physical string tension has vanished.

Turning to \(T/T_c = 1.33, 2.00\), or \(N_t = 6, 4\), Fig.(1) shows a very different behavior for \(-\ln(C_{mon})/N_t\). The expected behavior for \(-\ln(C)/N_t\) in full SU(2) is of the general form, \(A - B/R^\gamma \exp(-\mu(T)R)\), where \(\mu(T)\) is an electric screening mass. At very high temperature, in the perturbative regime, \(\gamma = 2\), and \(\mu(T)\) can be calculated analytically, starting at one loop. At lower temperatures, in the non-perturbative regime, \(\gamma = 1\), and and \(\mu(T)\) is determined by the lightest glueball mass in three-dimensional SU(2) gauge theory. Our data is not extensive enough to determine \(\gamma\) and \(\mu(T)\) independently in fits to \(-\ln(C_{mon})/N_t\). Instead we explored fits where a value of \(\gamma\) was chosen, and then a minimum \(\chi^2\) was searched for by varying \(\mu(T)\). For the choice \(\gamma = 2\), we obtain \(\mu(T) = 0.0 \pm 0.1\), whereas for \(\gamma = 1\), the range of allowed screening masses is slightly larger, \(\mu(T) = 0.0 \pm 0.2\). In short, although qualitatively \(-\ln(C_{mon})/N_t\) is of the expected general shape, quantitatively we have negligible evidence for an electric screening mass arising from monopoles at temperatures \(T > T_c\). At ultra-high temperatures, this is not surprising, since the electric screening mass is calculable from gluons in perturbation theory, and monopoles although present, are not needed to explain it. At lower temperatures still satisfying \(T > T_c\), the electric screening mass can no longer be calculated perturbatively. Nevertheless, our results suggest that also here, its origin is not to be explained by monopoles.

It is important to emphasize that Polyakov correlators get contributions only from magnetic currents in purely spacial directions. Meanwhile a spacial Wilson loop, which determines the spacial string tension, receives contributions from magnetic currents in both space and time directions. (It is the directions dual to the plane of the generalized Wilson loop being computed that count.) In the next section, we will show that the time and space components of the magnetic current behave very differently.

### 3 Magnetic Screening

We begin by reviewing the reasons for believing the magnetic current screens itself. One comes from the classic work of Polyakov for d=3 \(U(1)\) theory with monopoles \[\Box\]. This is relevant here, since at high temperature a \(d = 4\) theory effectively looks three dimensional. Polyakov’s analysis ties an area law for Wilson loops to plasma-like behavior for the monopole gas, with strong Debye screening. Another argument for screening comes from the Abelian Higgs model when it is equivalent to a type II superconductor \[\Box\]. Taking the “dual”, the tube of electric flux connecting a pair of opposite sign external charges is accompanied by strong screening of the (magnetic) supercurrent in directions perpendicular to the flux tube. In both cases, confinement is accompanied by strong screening of the magnetic charges (or currents) which cause confinement.

Returning to our finite temperature calculations, for a Wilson loop in xy,xz, or yz planes, the magnetic current which contributes is in tz, ty, or tx planes. For such purely spacial loops we do see an area law (spacial string ten-
The only component of current common in all these cases is the time component. This suggests strong screening of the time component of the magnetic current in spatial directions.

For a Polyakov correlator, the generalized Wilson loop is in $t_z$, $t_x$, or $t_y$ planes and the corresponding magnetic current which contributes is in $xy$, $yz$, or $xz$ planes. For Polyakov loop correlators we do not have an area law. Further, the time component of the magnetic current never contributes. We may then have a consistent situation if strong screening of the time component of magnetic current is accompanied by weak, partial screening of the spatial components.

To test these ideas, we study screening using a formalism explained in more detail in [9]. An infinitesimal external “static” monopole-antimonopole pair is inserted into the system of magnetic current, then the potential between this pair in momentum space is calculated.

The magnitude of the external charges is $\kappa g$, where $\kappa \ll 1$, and $g = 2\pi/e$ is the unit of magnetic charge. For the screening of the time-component of the current, we obtain for the screened potential $V_\kappa(k)$,

$$V_\kappa(k) = -g^2V(k)(1 - g^2m_{44}(k)V(k)),$$

where $V(k)$ is the $d = 3$ Fourier Transform (FT) of the lattice Coulomb potential, and $m_{44}(k)$ is the FT of the static magnetic charge correlation function. The effect of magnetic vacuum polarization on $V_\kappa(k)$ is contained in $f(k) = m_{44}(k)V(k)$.

In Fig.(2), we show $f(k)$ vs $k^2$ for all four temperatures. The screening is stronger ($f(k)$ larger) for $T = 2T_c$, and rather similar for the other temperatures, consistent with the much larger spacial string tension for $T = 2T_c$.

More quantitatively, if $V_\kappa$ is exponentially screened in position space, then $1/f(k)$ should be linear in $k^2$ for small $k^2$. In Fig.(3) we show $1/f(k)$ vs. $k^2$ for the time-like current. The slope defines the square of a magnetic screening mass $M^m_{sc}$. Fits to the data give $M^m_{sc} = 0.6(1)/a$ for $T/T_c = 2$, and $0.5(1)/a$ for $T/T_c = 1.33$, 1.0, 0.66. An intriguing question for the future is the relation between this screening mass found from the time-like magnetic current and the magnetic mass found from the gluon propagator [10].

If $V_\kappa$ has no long range part, $1/f(0)$ must equal $g^2$. Using straight line fits to $1/f(k)$, the fitted value of $1/f(0) = 24.3 \pm 1.2$. This compares well with

$$g^2 = \frac{4\pi^2}{e^2} = \beta \pi^2 = 24.789$$

Turning now to the screening of the spacial current, we change our viewpoint and regard a spacial direction as the “static” one. The resulting screened potential $V_\kappa$ is given by a formula identical to the one above except now...
the screening function $f(k) = m_{xx}(k)V(k)$, if the static direction was chosen as the $x$-direction, etc for $y, z$. In Fig. (4), we show the results averaged over the three choices $x, y, z$. Comparing Figs. (2) and (4), we see that the screening of the spacial current, in contrast to the timelike, decreases with temperature. To take the most extreme case, for $T = 2T_c$, the time component of the magnetic current, the screening is strongest ($f(k)$ largest), while for the spacial component of the current, the screening is weakest ($f(k)$ smallest).

To summarize, we have a consistent picture of monopole dynamics emerging from study of spacial Wilson loops, Polyakov loop correlators, and magnetic current screening. The Coulombic behavior (zero screening mass) of Polyakov loop correlators reflects the weak screening of the spacial magnetic current.

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References

[1] J. D. Stack, S. D. Neiman, and R. J. Wensley, Phys. Rev. D50 (1994) 3399
[2] J. D. Stack, Nucl. Phys. B(Proc. Suppl) 42 (1995) 554.
[3] J. D. Stack, S. D. Neiman, and R. J. Wensley, hep-lat 9605045, to be published in Physics Letters B.
[4] J. Smit and A. J. van der Sijs, Nucl. Phys. B355 (1991) 603.
[5] J. Fingberg, U. Heller, and F. Karsch, *Nucl. Phys.* B392 (1993) 493.

[6] G. Bali, J. Fingberg, U. Heller, F. Karsch, and K. Schilling, *Phys. Rev. Letters* 71, 3059 (1993).

[7] A. M. Polyakov, *Nucl. Phys.* B120 (1977) 429.

[8] See H. W. Wyld and R. T. Cutler, *Phys. Rev.* D14 (1976),1648, and references therein.

[9] J. D. Stack and R. J. Wensley, *Nucl. Phys.* B371 (1992) 597.

[10] U. M. Heller, F. Karsch, and J. Rank *Phys. Lett* B 355 (1995)511.