Hidden Sector Dirac Dark Matter, Stueckelberg $Z'$ Model and the CDMS and XENON Experiments

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Abstract

For the milli-charged Dirac dark matter in the Stueckelberg $Z'$ model, we discuss the contributions from the vector couplings of the dark matter with the neutral gauge bosons to the spin-independent scattering cross section in the direct detections. We also compute the effective coupling between the fermionic dark matter particle and the standard model Higgs boson generated through a triangular loop of $Z$ and/or $Z'$ bosons which may contribute to the spin-independent scattering cross section at the quantum level. We show that the latter contribution is consistent with the most recent experimental limits from CDMS II and XENON100. In the case that the dark matter particle carries a milli-charge of order $O(10^{-3}e)$, we found that it would lose all its kinetic energy by colliding with nucleons in the atmosphere before reaching the detector. Even though we use the Stueckelberg $Z'$ model for illustration, the results we obtain are rather general and applicable to other $Z$-$Z'$ portal-type hidden-sector models as well.
I. INTRODUCTION

The presence of cold dark matter (CDM) in our Universe is now well established by the very precise measurement of the cosmic microwave background radiation in the Wilkinson Microwave Anisotropy Probe (WMAP) experiment [1]. One of the most appealing and natural CDM particle candidates is provided by supersymmetric models with $R$-parity conservation [2]. This $R$-parity conservation ensures the stability of the lightest supersymmetric particle (LSP) so that the LSP can be CDM. The LSP is in general the lightest neutralino, a linear combination of neutral electroweak (EW) gauginos and Higgsinos. Since the nature of LSP depends on its precise compositions, its detection can vary a lot.

One of the proposed methods of detecting the dark matter is through direct search experiments. The dark matter particles move at a velocity relative to the detecting materials. It will recoil against the nucleons, and create either (1) phonon, (2) scintillation or (3) ionization signal, which can be amplified by electronics. The CDMS experiment is of the first type, while the XENON experiment is combination of types (2) and (3). Recently, the CDMS II finalized their search in Ref. [3]. When they opened the black box in their blind analysis, they found two candidate events, which are consistent with background fluctuation at a probability level of about 23%. Nevertheless, the signal is not conclusive. The CDMS then improves upon the upper limit on the spin-independent cross section $\sigma_{\chi N}^{SI}$ to $3.8 \times 10^{-44}$ cm$^2$ for $m_\chi \approx 70$ GeV. Using this new limit we have put a new bound on the Higgs-dark-matter coupling [4], which can be implied to an upper limit on the Higgs boson invisible width$^1$. More recently, XENON100 [6] has also reported no signals in their detectors and put a similar upper limit on the spin-independent cross section $\sigma_{\chi N}^{SI}$ to $3.4 \times 10^{-44}$ cm$^2$ for $m_\chi \approx 55$ GeV.

There is a new class of models for dark matter candidates, motivated by hidden-sector models. It could be a fermion or boson inside the hidden sector. The interactions with the standard model (SM) particles are only possible via $Z-Z'$ mixing, Stueckelberg-type mixing, Higgs-portal type models. We focus on the former two possibilities in the following. Two of us [7] and Nath et al. [8] had proposed the Dirac fermion, denoted by $\chi$, in the hidden sector as the dark matter candidate. Correct relic density can be obtained by adjusting

$^1$ There have been a few works [4, 5] on the Higgs invisible decay in light of the CDMS II result.
the mass of the dark matter, the mass of the $Z'$ boson, and the coupling strength $\mu$. In this work, we study the effective coupling between the dark matter and the nucleon in the Stueckelberg $Z'$ model. There are two possibilities for the effective interactions: (I) The general mixings among the photon, $Z$ and $Z'$ make it possible for the hidden sector dark matter couples to the SM quarks at the tree level; (II) It is obvious that the fermion $\chi$ has no direct coupling to the Higgs boson at the tree level. However, we point out that an effective coupling $g_{h\chi\chi}$ can be generated between the Dirac fermion $\chi$ and the Higgs boson through a triangular loop of $Z$ and/or $Z'$ bosons. It therefore gives rise to spin-independent scattering cross section that may be measurable with direct detection experiments. We show that the effective couplings in case (I) will provide useful constraints in the model, while case (II) will give rise to spin-independent cross sections, which are consistent with the most recent CDMS II and XENON100 results.

The organization of the paper is as follows. In the next section, we briefly describe the spin-independent cross sections in terms of the effective couplings between the dark matter and the nucleon for both vector and scalar contributions in a model independent manner. In Sec. III, we summarize some major features of the Stueckelberg $Z'$ model and the generic $Z-Z'$ mixing model. In Sec. IV, we calculate the effective couplings for both vector and scalar contributions in the Stueckelberg $Z'$ model. Explicit results for the scalar contributions are presented for the Stueckelberg $Z'$ model as well as the generic $Z-Z'$ mixing model. Comparisons with the experimental limits are made. Finally, we summarize in Sec. V.

II. DIRECT DETECTION

Direct detection of the fermionic dark matter depends on the assumption how dark matter interacts with the nuclei or with the quarks from the more fundamental viewpoint. Contributions to the spin-independent cross section depend on the underlying mechanism is either dominated by vector gauge boson exchange or Higgs boson exchange or combination of both, depending on the specific model. We will first discuss these contributions in a model independent way.

Suppose the effective interactions between the dark matter particle (denoted by $\chi$ in the
and the quarks are given by

$$\mathcal{L}_{\text{eff}} = \sum_q \left\{ \alpha_q^S \bar{\chi} q q + \alpha_q^V \bar{\chi} \gamma^\mu \chi q q \gamma^\mu \right\},$$  \hspace{1cm} (1)$$

with the model dependent scalar and vector couplings specified by $\alpha_q^S$ and $\alpha_q^V$ respectively, then the spin-independent cross section between $\chi$ and each of the nucleon (taking the average between proton and neutron) is given by, (assuming a Dirac type fermion)

$$\sigma_{\chi N}^{\text{SI}} = \frac{\mu_{\chi N}^2}{\pi} \left( |G_N^s|^2 + \frac{|b_N|^2}{256} \right),$$  \hspace{1cm} (2)$$

where $\mu_{\chi N} = m_\chi m_N / (m_\chi + m_N)$ is the reduced mass between the dark matter particle and the nucleon $N = (p, n)$, and

$$G_N^s = \sum_q \langle N | \bar{q} q | N \rangle \alpha_q^S = \sum_q \alpha_q^S f_{Tq}^N \frac{m_N}{m_q},$$  \hspace{1cm} (3)$$

$$= \sum_{q=u,d,s} \alpha_q^S f_{Tq}^N \frac{m_N}{m_q} \frac{2}{27} \sum_{q=c,b,t} \alpha_q^S \frac{m_N}{m_q},$$  \hspace{1cm} (4)$$

where the relation $\langle N | \bar{q} q | N \rangle = f_{Tq}^N m_N / m_q$ has been used for the the nucleon matrix element $\langle N | \bar{q} q | N \rangle$.

While the expression for the vector contribution $b_N$ of a whole nucleus $(A, Z)$ is $b_N \equiv \alpha_u^V (A + Z) + \alpha_d^V (2A - Z)$, we take the average between proton and neutron (assume number of protons is about the same as neutrons in the nuclei) and thus obtain the expression for a single nucleon

$$b_N = \frac{3}{2} \left( \alpha_u^V + \alpha_d^V \right).$$  \hspace{1cm} (5)$$

This is useful for direct comparison with the results given by experiments, where usually the dark-matter-nucleon cross sections are reported.

In the case of the dominance by Higgs boson exchange, we write $\alpha_q^S$ as

$$\alpha_q^S = - \frac{g_{h\chi\chi} g_{hqq}}{m_h^2},$$

and $G_N^s$ takes the form

$$G_N^s = \sum_{q=u,d,s} f_{Tq}^N \frac{m_N}{m_q} \left( - \frac{g_{h\chi\chi} g_{hqq}}{m_h^2} \right) + \frac{2}{27} \sum_{q=c,b,t} \frac{m_N}{m_q} \left( - \frac{g_{h\chi\chi} g_{hqq}}{m_h^2} \right),$$  \hspace{1cm} (6)$$

where $g_{h\chi\chi}$ is the effective coupling between the dark matter particle $\chi$ and the Higgs boson, $g_{hqq}$ is the Yukawa coupling of the quark $q$. Note that the $m_q$ dependence in the Yukawa
coupling $g_{hqq}$ will be cancelled by the $1/m_q$ dependence coming from the matrix element $\langle N|\bar{q}q|N\rangle$. In the scenario where the Higgs boson exchange is dominant, the experimental upper limit on the spin-independent cross section can imply an upper limit for the dark matter-Higgs coupling, which is more or less model independent.

Default values of the parameters used, e.g. in DarkSUSY [9] are

$$f_{Tu}^p = 0.023, \quad f_{Td}^p = 0.034, \quad f_{Ts}^p = 0.14, \quad f_{Tu}^g = 1 - f_{Tu}^p - f_{Td}^p - f_{Ts}^p = 0.803, \quad f_{Tu}^n = 0.019, \quad f_{Td}^n = 0.041, \quad f_{Ts}^n = 0.14, \quad f_{Tu}^g = 1 - f_{Tu}^n - f_{Td}^n - f_{Ts}^n = 0.8. \quad (7)$$

We take the average between proton and neutron for $\sigma_{SI}^{\chi N}$. For $m_{\chi} \sim O(100) \text{ GeV}$, $\mu_{\chi N} \approx m_N$ and the spin-independent cross section was estimated to be [4]

$$\sigma_{SI}^{\chi N} \approx \frac{g_{h\chi\chi}^2 m_{\chi}^2}{4\pi m_W^2} \frac{1}{m_h^2} (0.3766)^2. \quad (8)$$

Using the new CDMS II limit of $\sigma_{SI}^{\chi N} < 3.8 \times 10^{-44} \text{ cm}^2$ and taking $m_h = 120 \text{ GeV}$, we can obtain an upper limit on the Higgs-dark-matter coupling [4]

$$g_{h\chi\chi}^2 \lesssim 0.04. \quad (9)$$

Similar constraint on $g_{h\chi\chi}^2$ can be deduced by using the XENON100 limit.

III. MODELS

A. Stueckelberg $Z'$ model

The details of the Stueckelberg $Z'$ model can be found in Ref. [10], and more specifically the couplings used in this study can be found in Ref. [11]. Here we give a brief account. The Stueckelberg extension of the SM (StSM) [10] is obtained by adding [7, 8] a hidden sector associated with an extra $U(1)_C$ interaction, under which the SM particles are neutral.

Assuming there is no kinetic mixing between the two $U(1)$’s, the Lagrangian describing the system is given by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{StSM}}$$

with

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + i \bar{f} \gamma^\mu D_\mu f + D_\mu \Phi^\dagger D^\mu \Phi - V(\Phi^\dagger \Phi), \quad (10)$$

$$\mathcal{L}_{\text{StSM}} = -\frac{1}{4} C_{\mu\nu} C^{\mu\nu} + \frac{1}{2} (\partial_\mu \sigma + M_1 C_\mu + M_2 B_\mu)^2 + \bar{\chi} (i\gamma^\mu D_\mu - M_\chi) \chi, \quad (11)$$
where $W^a_{\mu\nu}(a = 1, 2, 3)$, $B_{\mu\nu}$ and $C_{\mu\nu}$ are the field strength tensors of the gauge fields $W^a_{\mu}$, $B_{\mu}$, and $C_{\mu}$, respectively; $f$ denotes a SM fermion, while $\chi$ is a Dirac fermion in the hidden sector which may play a role as milli-charged dark matter in the Universe \cite{7, 8} and $M_\chi$ is its mass; $\Phi$ is the SM Higgs doublet; and $\sigma$ is the Stueckelberg axion scalar. The covariant derivatives $D_\mu = (\partial_\mu + ig_2 \vec{T} \cdot \vec{W}_\mu + ig_Y \frac{Y}{2} B_\mu)$ and $D^X_\mu = (\partial_\mu + ig_X Q_X C_\mu)$.

After the electroweak symmetry breaking of $\langle \Phi \rangle = v/\sqrt{2}$ with a vacuum expectation value $v \simeq 246$ GeV, the mass term for $V \equiv (C_\mu, B_\mu, W^3_\mu)^T$ is given by

$$-\frac{1}{2} V^T M_{Stu}^2 V \equiv -\frac{1}{2} \begin{pmatrix} C_\mu & B_\mu & W^3_\mu \end{pmatrix} \begin{pmatrix} M_1^2 & M_1 M_2 & 0 \\ M_1 M_2 & M_2^2 + \frac{1}{4} g_Y^2 v^2 - \frac{1}{4} g_2 g_Y v^2 & \frac{1}{4} g_2 g_Y^2 v^2 \\ 0 & \frac{1}{4} g_2 g_Y^2 v^2 & \frac{1}{4} g_2^2 v^2 \end{pmatrix} \begin{pmatrix} C_\mu \\ B_\mu \\ W^3_\mu \end{pmatrix}.$$  \hspace{1cm} (12)

One can easily show that the determinant of $M_{Stu}^2$ is zero, indicating the existence of at least one zero eigenvalue to be identified as the photon mass. A similarity transformation $O$ can bring the mass matrix $M_{Stu}^2$ into a diagonal form

$$\begin{pmatrix} C_\mu \\ B_\mu \\ W^3_\mu \end{pmatrix} = O \begin{pmatrix} Z'_\mu \\ Z_\mu \\ A_\mu \end{pmatrix}, \quad O^T M_{Stu}^2 O = \text{Diag} \begin{pmatrix} M_{Z'}^2, M_Z^2, 0 \end{pmatrix}. \hspace{1cm} (13)$$

Explicit formulas for the matrix elements of $O$ in terms of the fundamental parameters in the Lagrangian can be found in Refs. \cite{7, 8, 10, 11}. The couplings between the neutral gauge bosons and the Higgs are given by

$$L_{\text{Higgs}}^{Z-Z'} = \frac{1}{8} \left( H^2 + 2vH \right) \left[ (g_2 O_{32} - g_Y O_{22})^2 Z_\mu Z^\mu + (g_2 O_{31} - g_Y O_{21})^2 Z'_\mu Z''^\mu \right. $$
$$+ \left. 2 (g_2 O_{31} - g_Y O_{21}) (g_2 O_{32} - g_Y O_{22}) Z_\mu Z''^\mu \right]. \hspace{1cm} (14)$$

The neutral current interactions are given by

$$-L_{\text{int}}^{NC} = \bar{f} \gamma^\mu \left[ \left( \epsilon Z^\mu_P L + \epsilon Z^\mu_R R \right) Z'_\mu + \left( \epsilon Z^\mu_P L + \epsilon Z^\mu_R R \right) Z_\mu + e Q_{em} A_\mu \right] f $$
$$+ \bar{\chi} \gamma^\mu \left[ \epsilon Z^\mu Z'_\mu + \epsilon Z^\mu Z_\mu + \epsilon A_\mu \right] \chi, \hspace{1cm} (15)$$
with

\[
\begin{align*}
\epsilon_{\mathcal{L}}^f_L &= \frac{g_2}{\cos \theta} \cos \psi \left[ (1 - \epsilon \sin \theta \tan \psi) T_f^3 - \sin^2 \theta (1 - \epsilon \csc \theta \tan \psi) Q_f \right] , \\
\epsilon_{\mathcal{L}}^f_R &= -\frac{g_2}{\cos \theta} \cos \psi \sin^2 \theta (1 - \epsilon \csc \theta \tan \psi) Q_f , \\
\epsilon_{\mathcal{L}}^{f'}_L &= -\frac{g_2}{\cos \theta} \cos \psi \left[ (\tan \psi + \epsilon \sin \theta) T_f^3 - \sin^2 \theta (\epsilon \csc \theta + \tan \psi) Q_f \right] , \\
\epsilon_{\mathcal{L}}^{f'}_R &= \frac{g_2}{\cos \theta} \cos \psi \sin^2 \theta (\epsilon \csc \theta + \tan \psi) Q_f , \\
\epsilon^{\chi}_{V_i} &= g_X Q_X^\chi O_{1i} ,
\end{align*}
\]  

(16)

and \( \epsilon \equiv \tan \phi \). Therefore, the couplings among the Higgs boson, \( Z \) and \( Z' \) are

\[
\begin{align*}
C_{Z'^'Z} &= (g_2 O_{31} - g_Y O_{21})^2 O_{11} , \\
C_{ZZ} &= (g_2 O_{32} - g_Y O_{22})^2 O_{12} , \\
C_{Z'Z} &= C_{Z'^'Z} = \sqrt{C_{Z'^'Z} C_{ZZ}} , \\
&= (g_2 O_{31} - g_Y O_{21}) (g_2 O_{32} - g_Y O_{22}) O_{11} O_{12} .
\end{align*}
\]  

(17) - (19)

With all these couplings and inputs we are ready to compute the effective coupling \( g_{h\chi\chi} \) and thus the spin-independent cross section \( \sigma^\text{SI}_{\chi N} \).

\section*{B. \( Z-Z' \) mixing models}

Before mixing the neutral current interactions are

\[
- \mathcal{L}_{\text{NC}} = \frac{g_2}{\cos \theta_w} \sum_f \bar{f} \gamma^\mu (g_v - g_a \gamma_5) f \ Z_{1\mu} + g_X Q_X^\chi \bar{\chi} \gamma^\mu \chi \ Z_{2\mu}
\]  

(20)

where \( Z_1 \) and \( Z_2 \) are the unmixed states, which are then rotated into mass eigenstates \( Z, Z' \) via a mixing

\[
\begin{pmatrix}
Z_1 \\
Z_2
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
Z \\
Z'
\end{pmatrix} .
\]  

(21)

The neutral current gauge interactions of the hidden fermion \( \chi \) in this model are then

\[
- \mathcal{L}_{\text{NC}}^\chi = g_X Q_X^\chi (\cos \theta Z_{\mu}' + \sin \theta Z_{\mu}) \bar{\chi} \gamma^\mu \chi .
\]  

(22)

The interactions among the Higgs boson and the \( Z, Z' \) are

\[
- \mathcal{L}_{hZZ} = \frac{1}{8 \cos^2 \theta_w} (H^2 + 2vH) \left( \cos \theta Z_{\mu} - \sin \theta Z_{\mu}' \right)^2
\]  

(23)
where $\theta_w$ is the Weinberg’s angle. Therefore, the coefficients $C$’s in this model are given by

$$C_{Z'Z'} = \frac{g_2^2}{\cos^2 \theta_w} \sin^2 \theta \cos^2 \theta , \quad (24)$$

$$C_{ZZ} = \frac{g_2^2}{\cos^2 \theta_w} \cos^2 \theta \sin^2 \theta , \quad (25)$$

$$C_{Z'Z} = C_{Z'Z}' = -\frac{g_2^2}{\cos^2 \theta_w} \sin^2 \theta \cos^2 \theta . \quad (26)$$

**IV. EFFECTIVE COUPLINGS IN STUECKELBERG $Z'$ MODEL**

**A. Tree-level mixing**

We note that the dark matter $\chi$ in the present Stueckelberg $Z'$ model is assumed to be a Dirac particle. Thus it has vector couplings to all the neutral gauge bosons $\gamma$, $Z$ and $Z'$. In particular, it carries milli-charged while couples to photon $[7, 8]$. There are also axial couplings among the dark matter with $Z$ and $Z'$, but they only contribute to the spin-dependent cross sections. For multi-component dark matter model of Stueckelberg type with both Dirac and Majorana hidden fermions, see the recent work of Ref.[12].

Due to the $t$-channel pole, one might expect the photon exchange diagram will be the dominant contribution for the spin-independent dark matter-nucleon cross section and thus the CDMS II result can be used to eliminate the milli-charged dark matter model. However, for the milli-charged dark matter to get detected inside the underground detectors, it has to traverse our whole atmosphere and penetrate through the surface rock. The milli-charged dark matter interacting with the ordinary matter through the long range Coulombic force might lose all its kinetic energy before it reaches the detector. Following the analysis in [13], one can estimate the stopping distance to be

$$L \approx \frac{m_A^2 m_{\chi} v_{\chi}^4}{8\pi \rho (\tilde{e}_\gamma^\chi \alpha Z)^2 \log (E_R^{\text{max}} / E_R^{\text{min}})}$$

$$\approx 0.27 \left( \frac{10^{-3}}{\tilde{e}_\gamma^\chi} \right)^2 \left( \frac{m_A}{32 \text{ GeV}} \right)^2 \left( \frac{16}{Z} \right)^2 \left( \frac{m_{\chi}}{100 \text{ GeV}} \right) \left( \frac{5 \text{ g/cm}^3}{\rho} \right) \left( \frac{v_{\chi}}{300 \text{ km/s}} \right)^4 \text{ [m]} \quad (27)$$

Here, $m_A$ is the mass of the ordinary matter with atomic number $Z$ and density $\rho$, $\tilde{e}_\gamma^\chi = e_{\gamma}^\chi / e$ is the milli-charge of the dark matter in unit of $e$, and $E_R^{\text{min,max}}$ are the minimum and maximum recoil energies of the matter, respectively. Following [13], we set $\log (E_R^{\text{max}} / E_R^{\text{min}}) \sim 10$ to obtain the above estimation. This small stopping distance suggests milli-charged dark
matter will not be able to reach the underground detectors unless $\tilde{\epsilon}_\gamma$ is very small of the order of $10^{-8}$. Such kind of dark matter can arise in models with kinetic mixing. In the present context of Stueckelberg $Z'$ model, $\tilde{\epsilon}_\gamma$ can be considerably larger, typically of the size of $10^{-3}$ \cite{7, 8}.

For completeness, we also present the contributions from the $Z$ and $Z'$ diagrams. The spin-independent dark matter-nucleon cross section from their vector couplings is given by the second term of Eq. (2),

$$\sigma_{\chi N}^{\text{SI}} = \frac{\mu_{\chi N}^2 |b_N|^2}{256\pi},$$

(28)

where $b_N = \frac{3}{2}(\alpha_u^V + \alpha_d^V)$ assuming the number of protons and the number of neutrons are about the same and taking the average between proton and neutron. For the $Z$ and $Z'$ couplings in the Stueckelberg $Z'$ model described above, we have

$$\alpha_f^V = \frac{\epsilon_f}{2m_Z^2} \left( \epsilon_{fL}^Z + \epsilon_{fR}^Z \right) + \frac{\epsilon_f}{2m_{Z'}^2} \left( \epsilon_{fL}^{Z'} + \epsilon_{fR}^{Z'} \right),$$

(29)

for $f = u$ or $d$-quark. For sufficiently heavy $Z'$, its contribution can be ignored. With $\tan \phi = 0.01$ and $m_{Z'} = 1$ TeV, we obtain an estimate for

$$\sigma_{\chi N}^{\text{SI}} \approx 0.44 \times 10^{-45} \text{ [cm}^2]\right].$$

(30)

This is roughly two orders of magnitude below the current CDMS II and XENON100 limits. Thus, one can not put stringent constraint on the vector couplings of the dark matter $\chi$ with the $Z$ and $Z'$ bosons in the Stueckelberg model yet. However, the long range Coulomb interaction between the milli-charged dark matter and the nuclei of the detectors may be too severe for the dark matter to reach the detector in this scenario. In this case the above formula (28) for the spin-independent cross section derived from the effective 4-fermion local operator is no longer applicable. On the other hand, the CDMS II and XENON100 limits can be used to place useful constraint on the mixing angle in generic $Z$-$Z'$ mixing model presented in previous section. Furthermore, XENON100 is expected to improve its upper limit by one to two order of magnitudes in the future. The parameter space of the Stueckelberg $Z'$ model will be probed more effectively by this future improvement.

**B. Effective Higgs-Dark Matter Coupling**

The effective coupling between the SM Higgs and the hidden milli-charged dark matter $\chi$ can be induced at one-loop level. The calculation is similar to the one loop electroweak
correction to the $H\bar{b}b$ coupling except there are no $W^\pm$ and unphysical Higgs bosons $G^{\pm,0}$ running inside the loops. Thus to simplify our calculation we will proceed using the ’t Hooft-Feynman gauge.

We can write down the following amplitude for the effective coupling of Higgs-$\chi-\chi$

$$\overline{\chi}(p') \left[ F(q^2) + i\gamma_5 G(q^2) \right] \chi(p) H$$

(31)

with $F(q^2)$ and $G(q^2)$ being the scalar and pseudoscalar form factors, and $q = p - p'$. For on-shell $\chi$ in the initial and final states, we find that $G(q^2) = 0$ and

$$F(q^2) = -\frac{(g_X Q_X^\chi)^2 m_W m_\chi}{8\pi^2} g_2 \sum_{(i,j)=\{(Z',Z'),(Z,Z),\ldots\}} C_{ij} \int_0^1 dx \int_0^x dy \frac{(1 + y)}{\Delta_{ij}(x,y)}$$

(32)

where $g_X Q_X^\chi$ is the gauge coupling of the fermion $\chi$ to the vector gauge boson in the hidden sector,

$$\Delta_{ij}(x,y) = y^2 m_\chi^2 + (x-y)m_i^2 + (1-x)m_j^2 - (1-x)(x-y)q^2 - i0^+$$

(33)

and $C_{Z_iZ_j}$ contains the coupling of $HZ_iZ_j$ and the mixing angles of $Z_i$ and $Z_j$ with the hidden-sector dark matter. We will give these couplings explicitly in the Stueckelberg $Z'$ model and the generic $Z-Z'$ mixing model in the subsections. The effective Higgs-dark-matter coupling $g_{h\chi\chi}$ relevant for spin-independent cross section is then given by

$$g_{h\chi\chi} = F(q^2 = 0)$$

(34)

in which the elastic scattering of the $\chi$ is at $q^2 \approx 0$.

If kinematically allowed, the SM Higgs can decay into the invisible $\overline{\chi}\chi$ mode and its width is given by

$$\Gamma (h \rightarrow \overline{\chi}\chi) = \frac{m_h}{8\pi} |F (m_h^2)|^2 \left( 1 - \frac{4m_\chi^2}{m_h^2} \right)^{\frac{3}{2}}.$$  \hspace{1cm} (35)

With $\tan \phi = M_2/M_1$ and $m_{Z'}$, fixed, all parameters of the Stueckelberg $Z'$ model are fixed. In Fig. [1] we show the $\sigma^{\text{SI}}_{\chi\chi}$ versus the mass of $Z'$ with the inputs: $g_X Q_X^\chi = g_2$, $m_\chi = 100$ GeV, $m_h = 120$ GeV and three different values of $\tan \phi = 0.01$, 0.03 and 0.05. It is evident from this plot that for a wide range of parameters in this model, the predictions of the spin-independent cross sections are consistent with the latest CDMS II experiment.

\[\text{The spin-independent cross section can get a significant boost in the large } \tan \phi \text{ scenario discussed in [11]. However, the results are still several orders below the current CDMS II limit.}\]
FIG. 1: The spin-independent cross section $\sigma_{\chi N}^{SI}$ versus the mass of the $Z'$ boson in the Stueckelberg model. Inputs are $g_X Q_X^\chi = g_2$, $m_\chi = 100$ GeV and $m_h = 120$ GeV.

No severe constraints can be deduced for this model from the current CDMS experiment. Similar conclusions can be obtained for the generic $Z-Z'$ mixing model since the mixing angle is generally constrained to be rather tiny ($\lesssim 10^{-3}$).

We have also checked that the invisible decay width of the Higgs decay $H \to \chi \chi$ given by Eq. (35) is very tiny and will not have any significant effects on the Higgs boson decay.

V. SUMMARY

In summary, we have calculated the nucleon-dark-matter scattering due to (i) tree-level mixing between the hidden sector gauge boson and the SM gauge bosons, and (ii) the one-loop induced coupling between the SM Higgs boson and the Dirac fermionic dark matter in the hidden sector. Such couplings can contribute to the spin-independent cross section for the dark matter direct detection. However, for a wide class of $Z-Z'$ models we found that these scalar contributions to the spin-independent cross sections are well below the current CDMS II and XENON100 experiments. The current experimental limits do not provide
useful constraints for the vector couplings of the dark matter with the neutral gauge bosons in the Stueckelberg $Z'$ model but may place more restrictive constraints in generic $Z-Z'$ mixing models. Furthermore, in the case that the dark matter particle has a milli-charge of order $O(10^{-3}e)$ (e.g. in the Stueckelberg-type mixing), it cannot traverse the atmosphere to the detector because it lost all its kinetic energy by colliding with nucleons. In such a case, the direct detection limits cannot apply. Projected upper limit on the spin-independent cross section for XENON100 is expected to have improvements by one to two orders of magnitudes and thus it can probe the parameter space of the model more effectively.

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[1] D. Larson et al., arXiv:1001.4635 [astro-ph.CO].
[2] G. Bertone, D. Hooper and J. Silk, Phys. Rept. 405, 279 (2005) arXiv:hep-ph/0404175.
[3] Z. Ahmed et al. [The CDMS-II Collaboration], Science 327, 1619 (2010) arXiv:0912.3592 [astro-ph.CO]].
[4] K. Cheung and T. C. Yuan, Phys. Lett. B 685, 182 (2010) arXiv:0912.4599 [hep-ph]].
[5] X. G. He, T. Li, X. Q. Li, J. Tandean and H. C. Tsai, Phys. Lett. B 688, 332 (2010) arXiv:0912.4722 [hep-ph];
J. Hisano, K. Nakayama and M. Yamanaka, Phys. Lett. B 684, 246 (2010) arXiv:0912.4701 [hep-ph];
M. Aoki, S. Kanemura and O. Seto, Phys. Lett. B 685, 313 (2010) arXiv:0912.5536 [hep-ph];
L. Wang and J. M. Yang, JHEP 1005, 024 (2010) arXiv:1003.4492 [hep-ph].
[6] E. Aprile et al. [XENON100 Collaboration], Phys. Rev. Lett. 105, 131302 (2010) arXiv:1005.0380 [astro-ph.CO].
[7] K. Cheung and T. C. Yuan, JHEP **0703**, 120 (2007) [arXiv:hep-ph/0701107].
[8] D. Feldman, Z. Liu and P. Nath, Phys. Rev. D **75**, 115001 (2007) [arXiv:hep-ph/0702123].
[9] P. Gondolo, J. Edsjo, P. Ullio, L. Bergstrom, M. Schelke and E. A. Baltz, JCAP **0407**, 008 (2004) [arXiv:astro-ph/0406204].
[10] D. Feldman, Z. Liu and P. Nath, JHEP **0611**, 007 (2006) [arXiv:hep-ph/0606294].
[11] K. Cheung, C. W. Chiang, Y. K. Hsiao and T. C. Yuan, Phys. Rev. D **81**, 053001 (2010) [arXiv:0911.0734 [hep-ph]].
[12] D. Feldman, Z. Liu, P. Nath and G. Peim, Phys. Rev. D **81**, 095017 (2010) [arXiv:1004.0649 [hep-ph]].
[13] R. Foot, Phys. Rev. D **69**, 036001 (2004) [arXiv:hep-ph/0308254].