Neutrino masses, mixing and leptogenesis in a two Higgs doublet model “for the third generation”

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We examine neutrino oscillations in a two Higgs doublet model (2HDM) in which the second doublet couples only to the third generation right-handed up-fermions, i.e., to \(t_R\) and to \(N_3\) which is the heaviest right-handed Majorana neutrino. The inherently large tan \(\beta\) of this model can naturally account for the large top mass and, based on a quark-lepton similarity ansatz, when embedded into a seesaw mechanism it can also account for the observed neutrino masses and mixing angles giving a very small \(\theta_{13}\): \(-0.017 \lesssim \theta_{13} \lesssim 0.021\) at 99% CL, and a very restrictive prediction for the atmospheric mixing angle: \(42.9^\circ \lesssim \theta_{\text{atm}} \lesssim 45.2^\circ\) at 99% CL. The large value of tan \(\beta\) also sets the mass scale of the heaviest right-handed Majorana neutrino \(N_3\) and triggers successful leptogenesis through a CP-asymmetry in the decays of the \(N_1\) (lightest right-handed Majorana) which is \(\tan^2 \beta\) enhanced compared to the CP-asymmetry obtained in models for leptogenesis with one Higgs doublet or in the MSSM. This enhancement allows us to relax the lower bound on \(M_{N_1}\) and consequently also the lower bound on the reheating temperature of the early universe.

A monumental discovery of the past decade is that neutrinos have mass! This discovery is bound to have a significant impact on our understanding of the universe. It also presents us with a pressing challenge: unraveling of the mixing matrix in the lepton sector. When we recapture the analogous quark case we can easily grasp the importance and the difficulties of this new task. Indeed, an obvious but nonetheless crucial question is the relation of the mixing matrix in the lepton sector to that in the quark sector. Intensive experimental effort is under way and much more is being planned, round the globe, to address these issues of vital importance.

In this work we suggest that these impressive findings in the lepton sector are closely related to another remarkable finding of the 90’s in the quark sector, namely that the top quark is enormously heavy compared to all the other quarks. Specifically, we will propose a simple, grounds-up approach which treats the 3rd generation neutrino in a completely analogous manner to that in the quark sector. This reasoning leads us to a concrete framework for neutrino masses and mixings with considerable predictive power. For example, we find that the crucial mixing angle \(\theta_{13}\), which is an important target of many neutrino experiments, is quite constrained, \(-0.017 \lesssim \theta_{13} \lesssim 0.021\) at 99% CL, in our picture.

We note that these observations of neutrino oscillations\(^1\), confirming the existence of massive neutrinos, implies that the Standard Model (SM) can no longer be regarded as a minimal theory that explains all observed phenomena in Particle Physics. Bearing this in mind, our objective here is to present a simple extension of the SM, which will serve as a low energy effective theory which captures the dominant features of phenomenology and models some underlying complicated dynamics of electroweak (EW) symmetry breaking at distances much shorter than the EW scale.

Recall the allowed 3\(\sigma\) ranges on the atmospheric and solar neutrino square mass differences and mixing angles (including the CHOOZ \(\bar{\nu}_e\) disappearance experiment)\(^2\):

\[
7.3 \leq \Delta m^2_{\text{sol}} \cdot 10^3 \text{ eV}^2 \leq 9.3 \quad , \quad 0.28 \leq \tan^2 \theta_{\text{sol}} \leq 0.6 \quad ,
\]

\[
1.6 \leq \Delta m^2_{\text{atm}} \cdot 10^3 \text{ eV}^2 \leq 3.6 \quad , \quad 0.5 \leq \tan^2 \theta_{\text{atm}} \leq 2.1 \quad ,
\]

\[
\sin^2 \theta_{\text{cp}} \leq 0.041 \quad ,
\]

where in the normal hierarchical scheme, \(m_1 << m_2 << m_3\), we have \(\Delta m^2_{\text{sol}} = \Delta m^2_{21}\), \(\Delta m^2_{\text{atm}} = \Delta m^2_{23}\) and \(\theta_{\text{sol}} = \theta_{23}\), \(\theta_{\text{atm}} = \theta_{21}\), \(\sin^2 \theta_{\text{cp}} = \theta_{13}\).

In order to explain the above neutrino mass differences without introducing extremely small and unnatural neutrino Dirac Yukawa couplings, the general practice is to add superheavy right-handed Majorana neutrino fields, \(N_j\), and rely on the seesaw mechanism for generating sub-eV light neutrinos:

\[
\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_Y(N, L, \phi) + M^\nu_N N_i N_j/2 \quad ,
\]

where, in the minimal setup, \(\mathcal{L}_Y(N, L, \phi) = Y^\nu_{ij} L_i H N_j\), with \(L_i = (\nu_{iL}, l_{iL})\) and \(H\) is the only Higgs doublet that couples to neutrinos. Then, the light neutrino mass matrix is given by the see-saw mechanism as:

\[
m_\nu = -m_D M^{-1}_N m^\nu_D \quad ,
\]

with \(m_D = v Y^\nu\) and \(v = < H >\).

In this work we wish to propose a 2HDM, which we will name the “2HDM for the 3rd generation” (3g2HDM).
The model relates the neutrino sector to the up-quark sector, and by giving the third generation right-handed neutrino similar status as the top quark, it provides a natural framework for accommodating the observed pattern of neutrino masses and mixing angles. In addition, our 3g2HDM assumes a specific structure in the Yukawa sector which should be viewed as an effective low-energy parametrization of the underlying short distance theory. This concrete phenomenological framework for the Yukawa sector allows us to successfully address both the heaviness of the top-quark and the apparent hierarchical structure in the neutrino sector.

In particular, we extend the idea of the so called “2HDM for the top-quark” (t2HDM) \cite{3} to the leptonic sector. In the t2HDM one assumes that $\phi_2$ [the second Higgs doublet with a much larger vacuum expectation value (VEV)] couples only to the top-quark, while the other Higgs doublet $\phi_1$ (with a much smaller VEV) couples to all the other fermions. The large mass hierarchy between the top-quark and the other quarks is then viewed as a consequence of $v_2/v_1 \equiv \tan \beta >> O(1)$, which, therefore, becomes the “working assumption” of the t2HDM.

The t2HDM has several other notable features, such as enhanced $H^+b\ell$ and $H^0c\bar{c}$ couplings, new flavor changing $H^0tc$ and $H^0tu$ interactions and new CP phases. Thus, the t2HDM can influence CP-violation in $B$ physics, flavor changing $Z$-decays \cite{4} and the production of a Higgs in association with a $c$-jet or a $b$-jet in hadron colliders \cite{5}.

Following the idea of the t2HDM, we propose the following Yukawa interactions in the 3g2HDM:

$$
\mathcal{L}_Y = - Y^e \bar{L}_L \phi_1 \ell_R - Y^d \bar{Q}_L \phi_1 d_R
- Y^u \bar{Q}_L \phi_2 u_R - Y^e \bar{Q}_L \phi_2 e_R
- Y^e \bar{L}_L \phi_1 N - Y^u \bar{L}_L \phi_2 N + h.c.,
$$

where $N$ are right-handed Majorana neutrinos, $Q$ and $L$ are the usual quark and lepton doublets and

$$
Y_{1}^{\alpha,\nu} = \begin{pmatrix}
0 & b_{\nu}^{1,\alpha} & 0 \\
0 & a_{\alpha,\nu} & 0 \\
0 & 0 & 0 \\
\end{pmatrix},
Y_{2}^{\alpha,\nu} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix},
$$

such that, in both the quark and lepton sectors, $\phi_2$ couples only to the third generation right-handed up-fermions. The Yukawa texture in \cite{6} yields the following Dirac neutrino mass matrix (dropping the superscript $\nu$):

$$
m_D \sim \frac{v_1}{\sqrt{2}} \begin{pmatrix}
a & b & 0 \\
a & b & ct_\beta \\
0 & 0 & ct_\beta \\
\end{pmatrix},
$$

where $t_\beta \equiv \tan \beta = v_2/v_1$.

Note that we have assumed the texture zeros $(Y_{1}^{\alpha,\nu})_{13} = 0$ and $(Y_{2}^{\alpha,\nu})_{31} = 0$, which gives a Dirac neutrino mass matrix in which the upper right and lower left entries vanish. This is required in order to simplify our discussion below and to avoid fine tuning in the neutrino Yukawa sector when confronted with the experimental limits on neutrino masses and mixings (in fact, it is sufficient to set only $(Y_{2}^{\alpha,\nu})_{13} \to 0$ in order to avoid fine tuning). The Dirac mass texture in \cite{7} is similar to the one which arises in the minimal approach suggested in \cite{7, 8}, that is, from the requirement of the most economic addition to the SM that can accommodate both neutrino oscillation data and the observed baryon asymmetry in the universe. Clearly, for such texture zeros to be natural, they have to be protected by underlying symmetries of the short distance theory. For example, SO(10) GUT theories dictate a symmetry between the up-quark and the Dirac neutrino mass matrices which, along with the empirically known properties of the up-quark mass matrix, yields the texture zeros in \cite{8}, see e.g., \cite{9}. In fact, as was shown in \cite{10}, there always exists a scalar sector such that any texture zero in the fermion mass matrices can be enforced by means of an abelian symmetry once the scalar sector is enlarged to the extent required. Another alternative and attractive framework for the zero entries in $Y_1$ and $Y_2$ was suggested in \cite{11}: such texture zeros may easily result from a vanishingly small overlap between the wave functions of $N$ and $L$ with the corresponding indexes in extra-dimensional models. The above several possible ways to accommodate the desired texture zeros [in \cite{12}] do not give us any further insight regarding the underlying reason for the structure of the Yukawa couplings in our model. Nonetheless, they do motivate us to study the implications and phenomenology of the 3g2HDM with the texture zeros assumed in \cite{6}.

We note also that, without loss of generality, further simplifications were taken in \cite{6} that allow us to give a more transparent analytical derivation of the neutrino oscillations pattern in our 3g2HDM. In particular, we have chosen similar Yukawa couplings within each generation: $(Y_{1}^{\alpha,\nu})_{11}(i = 1, 2) \sim a_{\alpha,\nu}$, $(Y_{1}^{\alpha,\nu})_{12}(i = 1, 2, 3) \sim b_{\nu}^{1,\alpha}$ and $(Y_{2}^{\alpha,\nu})_{33}(i = 2, 3) \sim c_{\nu}^{\alpha}$. In addition, a real parameter of $O(1)$ (the parameter $\delta$) was added to the $(Y_{1})_{23}$ entry in \cite{6}. We emphasize that these simplifying assumptions have no effect on our main results below, aside from simplifying the analytical formulæ given below.

In the basis where $M_N$ is diagonal (one can always choose a basis for the fields $N$ such that $M_N$ is diagonal), $M_N = M \cdot \text{diag}(\epsilon_1, \epsilon_2, \epsilon_3)$, we then obtain from the seesaw mechanism in \cite{3}:

$$
m_{\nu} = m_{\nu}^{0} \begin{pmatrix}
\epsilon & \epsilon & \delta \bar{c} + \omega \\
\epsilon + \omega & \delta \bar{c} + \omega \\
\delta \bar{c} + \omega \\
\end{pmatrix},
$$

where

$$
\epsilon \equiv \frac{a_2^2}{\epsilon_{M_1}} + \frac{b^2}{\epsilon_{M_2}},
\bar{c} \equiv \epsilon - \frac{a_2^2}{\epsilon_{M_1}},
\omega \equiv \frac{c_2^2 \bar{c}^2}{\epsilon_{M_3}},
$$

and

$$
m_{\nu}^{0} = (v_1)^2/2M.
$$
In the limit $\omega \gg \epsilon, \bar{\epsilon}, \delta$, the physical neutrino masses and mixing angles are then given by (11):

$$
\tan 2\theta_{23} \sim \frac{2r\omega}{\epsilon(\delta^2 - r)}, \quad \tan 2\theta_{12} \sim \frac{2\bar{\epsilon}}{f}, \quad \theta_{13} \sim \frac{\epsilon(\delta + r)}{2\sqrt{2}r\omega},
$$

(10)

where we have defined the parameters:

$$
r = \frac{\epsilon}{\bar{\epsilon}}, \quad g = \frac{|r - \delta|}{\sqrt{2r}}, \quad f = \frac{\delta^2 - 2\delta - r}{2r}.
$$

(11)

In the following, we will assume normal hierarchy for the light neutrinos: $m_3 >> m_2 >> m_1$, therefore leading to $m_3 \sim \sqrt{\Delta m_{atm}^2}$ and $m_2 \sim \sqrt{\Delta m_{sol}^2}$. Thus, setting for example $2m_0^2 \sim \sqrt{\Delta m_{sol}^2}$, it follows from the expression for $m_3$ in (10) that $\omega \sim \Delta m_{atm}^2/\Delta m_{sol}^2$.

Note that, using the definitions for $\omega$ in (3), for $m_0^2$ in (9) and the fact that $m_1 \sim c_\beta \times (v_1/\sqrt{2})$, our model yields the following seesaw triple-relation between the heaviest light neutrino, the heaviest right-handed Majorana neutrino and the top quark:

$$
m_3 \sim \frac{2m_2^2}{M_{\chi}},
$$

(12)

Moreover, from (9) with $2m_0^2 \sim \sqrt{\Delta m_{sol}^2}$ we obtain the typical mass scale ($M$) of the heavy right handed neutrinos:

$$
M \sim \frac{v^2}{\sqrt{\Delta m_{sol}^2}} \sim \frac{2m_2^2}{t_\beta \sqrt{\Delta m_{sol}^2}} \sim 10^{13} \text{ GeV},
$$

(13)

where, in the last equality, we have taken $m_1 \sim v_2/\sqrt{2}$ and $t_\beta \sim O(1)$, which are the “working assumption” values within the 3g2HDM.

In the following numerical analysis, we will set $\omega \sim \sqrt{\Delta m_{atm}^2/\Delta m_{sol}^2} = 5.18$ and $m_0^2 \sim \sqrt{\Delta m_{sol}^2}/2 = 4.53 \cdot 10^{-3} \text{ eV}$, which correspond to the best fitted values (2): $\Delta m_{atm}^2 = 2.2 \cdot 10^{-3} \text{ [eV]^2}$ and $\Delta m_{sol}^2 = 8.2 \cdot 10^{-5} \text{ [eV]^2}$. We note, though, that the best fitted set of input parameters (obtained by performing a minimum $\chi^2$ fit of our model to the experimentally measured values of atmospheric and solar neutrino masses and mixing angles (2)) is: (20) $\omega \sim 5.34, \epsilon \sim 0.57, \delta \sim 1.28$ and $r \sim 1$.

In Fig. 1 we give a scatter plot of the allowed ranges in the $\chi_{13} - \chi_{23}$ and $\chi_{13} - \chi_{12}$ planes, i.e., subject to the $3\sigma$ limits listed in (11). This is done by randomly varying the three input parameters $\epsilon$, $r$ and $\delta$ (requiring $\omega \gg \epsilon, \bar{\epsilon}, \delta$) with a sample of $3 \cdot 10^6$ points. We see that the 3g2HDM predicts $\theta_{13}$ to lie within (in radians) $-0.05 \lesssim \theta_{13} \lesssim 0.035$ and the atmospheric mixing angle $\theta_{23}$ to be at most a few degrees away from maximal. In fact, a minimum $\chi^2$ analysis with respect to $\theta_{13}$ and $\theta_{23}$ yields:

$$
-0.017 \lesssim \theta_{13} \lesssim 0.021 \quad 99\% \text{ CL}, \quad 42.9^0 \lesssim \theta_{23} \lesssim 45.2^0 \quad 99\% \text{ CL},
$$

(14)

with the best fitted value at $\theta_{13} \sim -0.011$ and $\theta_{23} \sim 43.9^0$. Notice that this is a rather restrictive prediction for the atmospheric mixing angle since the experimentally allowed $3\sigma$ range for $\theta_{23}$ spans over about $20^0$, i.e., $35^0 \lesssim \theta_{23}^{exp} \lesssim 55^0$ [see (1)].

FIG. 1: The allowed ranges in the $\chi_{13} - \chi_{23}$ (left) and $\chi_{13} - \chi_{12}$ (right) planes, for $\omega \sim 5.18$ and $m_0^2 \sim 4.53 \cdot 10^{-3} \text{ eV}$. See also text.

To better understand what is enforcing maximal atmospheric mixing and a very small $\theta_{13}$ (as is evident from Fig. 1), we note that from (10) we have:

$$
\theta_{13} \sim \frac{\epsilon(\delta + r)}{\sqrt{2}r\omega} \times \bar{\omega}^{-1}, \quad \theta_{23} \sim \frac{1}{2} \tan^{-1} \left[ \frac{\bar{\omega}}{\epsilon(\delta^2 - r)} \right], (15)
$$

where

$$
\bar{\omega} \equiv 2r\omega \sim 2r \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}} > 1,
$$

(16)

since by definition $r \geq 1$. Therefore, for values of $\epsilon, \delta$ and $r$ of $O(1)$, we have:

$$
|\theta_{13}| \sim \frac{1}{2} \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}}, \quad (17)
$$

as is typical to grand unified (or quark-lepton unified) theories (13). Furthermore, from (15) we have (taking $\delta < 1$, see discussion below and Fig. 2):

$$
\theta_{23} - \frac{\pi}{4} \sim - \frac{1}{2} \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}} \sim - |\theta_{13}|, \quad (18)
$$

as can be seen in Fig. 1.

In Fig. 2 we give a scatter plot of the allowed regions in the $\delta - \epsilon$ plane, in two limiting cases (12):

- case I: $r \sim 1$ ($\sim \bar{\epsilon}$), corresponding to $\frac{\delta^2}{\epsilon m_1} << \frac{\epsilon^2}{\epsilon m_2}$;
- case II: $r \sim 2$ ($\sim 2\bar{\epsilon}$), corresponding to $\frac{\delta^2}{\epsilon m_1} \sim \frac{\epsilon^2}{\epsilon m_2}$.

We see that case I (corresponding to the best fitted value for $r$ (12)) is compatible with neutrino oscillation data for $0.4 \lesssim \epsilon \lesssim 0.7$ with $-1.7 \lesssim \delta \lesssim -1$, while the allowed
FIG. 2: The allowed ranges in the $\epsilon - \delta$ plane, for cases I (left) and II (right). See also caption to Fig. 4.

TABLE I: Mixing angles and neutrino masses in cases I and II with $\epsilon = 0.5$ and $\epsilon = 0.75$, respectively. For both cases $\delta = -1.5$, $\omega \sim 5.18$ and $m_2^0 \sim 4.53 \cdot 10^{-3}$ eV.

| case (c) | $\theta_{23}$ | $\theta_{13}$ | $\theta_{12}$ | $m_1$ [eV] | $m_2$ [eV] | $m_3$ [eV] |
|----------|----------------|----------------|--------------|------------|------------|------------|
| I (0.5)  | $43^\circ$     | $29.5^\circ$   | $-11.4^\circ$| 0          | 0.0093     | 0.047      |
| II (0.75)| $44.7^\circ$   | $35.9^\circ$   | $0.69^\circ$ | 3.5 - 10^{-4} | 0.0092     | 0.048      |

range for case II is $0.5 \lesssim \epsilon \lesssim 0.8$ with $-2.4 \lesssim \delta \lesssim -1.4$. As an example, in Table I we give the neutrino mixing angles and masses in cases I and II, for some specific values of the allowed parameter space.

Let us now examine the consequences of our model on the mass spectrum of the heavy Majorana neutrinos, taking into account the above constraints coming from neutrino oscillation. For definiteness, we will consider case I (i.e., $r \sim 1$, which we find as the best fitted value with respect to oscillation data), and based on quark-lepton similarity (perhaps motivated by GUT scenarios, see e.g., [9]), we will take the following Ansatz for the neutrino Dirac Yukawa couplings:

$$ a \sim O(10^{-3}), \quad b \sim O(10^{-1}), \quad c \sim O(1) .$$

Indeed, this choice follows from the assumption that the Dirac mass matrices of the neutrinos and up-quarks exhibit a similar hierarchical structure. This Ansatz is, therefore, natural in the 3g2HDM under consideration, since this model is constructed in order to simultaneously explain the large top mass and the hierarchical structure of the neutrino masses, by relating the quark sector to the leptonic sector. That is, taking $v_1 \sim O(10)$ and $\ell_{\beta} \sim O(10)$, this Ansatz follows from the up-quark sector: $a^u v_1 \sim O(m_u)$, $b^u v_1 \sim O(m_c)$ and $m_l \sim O(c^u v_1 \tan \beta)$, if one assumes the quark-lepton similarity: $a^u \sim a^\nu \sim a$, $b^u \sim b^\nu \sim b$ and $c^u \sim c^\nu \sim c$. It is then easy to extract the mass spectrum of the heavy Majorana neutrinos in our Ansatz, subject to the constraints given in [14]. In particular, from [12] we have:

$$ M_{N_1} \sim 2m_1^2/\sqrt{\Delta m_{atm}} \sim 10^{15} \text{ GeV} .$$

Furthermore, we find that our Ansatz [19] yields the following masses for $N_2$ and $N_1$ for $r \sim 1$ and for $c \sim O(1)$ in accord with the constrains coming from neutrino oscillations, see Fig. 1:

$$ M_{N_2} \sim 10^{-2} M_\odot, \quad M_{N_1} \gg 10^{-6} M_\odot ,$$

which, for $M \sim 10^{13} \text{ GeV}$ [see [13]], gives $M_{N_2} \sim 10^{11}$ GeV and $M_{N_1} \gg 10^{7} \text{ GeV}$. Let us now examine how the 3g2HDM with our quark-lepton similarity Ansatz in [19] and with $r \sim 1$ and $c \sim O(1)$ fits into the mechanism of leptogenesis [16]. Leptogenesis is an additional attractive feature, associated with the seesaw mechanism, since it allows the possibility of generating the observed baryon excess in the universe from a lepton asymmetry, driven by decays of the heavy Majorana neutrinos [14, 15, 16]. In particular, a CP-asymmetry, $\epsilon_{N_1}$, in the decay $N_1 \rightarrow \ell\phi_j$ can generate the lepton asymmetry (see e.g., [14, 15]),

$$ n_L/\epsilon = \epsilon_{N_1} Y_{N_1} (T >> M_{N_1}) \eta ,$$

where $Y_{N_1} = n_{N_1}/s$, $n_{N_1}$ being the number density of $N_1$ and $s$ the entropy, with $Y_{N_1} (T >> M_{N_1}) = 135(3)/(4\pi^2 g_*)$ and $g_*$ being the effective number of spin-degrees of freedom in thermal equilibrium. Also, $\eta$ is the “washout” parameter (efficiency factor) that measures the amount of deviation from the out-of-equilibrium condition at the time of the $N_1$ decay.

The lepton asymmetry $n_L/\epsilon$ can then be converted into a baryon asymmetry through nonperturbative sphaleron processes. The conversion factor is [15]:

$$ n_B/\epsilon = -(8N_G + 4N_H)/(22N_G + 13N_H) \times n_L/\epsilon ,$$

where $N_G$ and $N_H$ are the number of generations and Higgs doublets, respectively. Thus, in our case, i.e., $N_G = 3$, $N_H = 2$ and $g_* \gtrsim 100$, we obtain:

$$ n_B/\epsilon \sim 1.4 \times 10^{-3}\epsilon_{N_1} \eta .$$

As seen from (20) and (21), our 3g2HDM can lead to a hierarchical mass spectrum for the heavy Majorana neutrinos, $M_{N_1} << M_{N_2} << M_{N_3}$, within a large portion of the allowed parameter space. In particular, imposing $M_{N_1} << M_{N_2}$ from (21) and with $M \sim 10^{13} \text{ GeV}$, we get: $M_{N_1} \sim 10^{8}$ to $10^{10} \text{ GeV}$. With such an hierarchical mass spectrum, only the CP-asymmetry produced by the decay of $N_1$ survives.

In our model $N_1$ can only decay to the “light” doublet $\phi_1$, as can be seen from [14] and [15]. Therefore, the CP-asymmetry $\epsilon_{N_1}$ is similar to the one obtained in single-Higgs models in the limit $M_{N_1} << M_{N_2}, M_{N_3}$ (see e.g., [15]):

$$ \epsilon_{N_1} = -\frac{3}{16\pi} \sum_{i=2,3} \frac{\text{Im} \left[ \left(Y_{N_1}^{\nu}\right)^2 \right]}{\left(Y_{N_1}^{\nu}\right)^2} \frac{M_{N_{1i}}}{M_{N_{1i}}} ,$$

where $Y_{N_1}^{\nu}$ is the Yukawa coupling of $N_1$ to $\phi_1$ given in [15]. Note that, since $N_3$ does not couple to $\phi_1$ in our
model, it does not contribute to the sum in (21) above. Therefore, $\epsilon_{N_1}$ is generated only from the $i = 2$ term in (24), giving:

$$\epsilon_{N_1} = -\frac{3b_2^2}{8\pi} \frac{M_{N_1}}{M_{N_2}} \sin 2(\theta_h - \theta_a),$$  \hspace{1cm} (25)$$

where the CP-phases arise from the possible complex entries in $Y_1^i$: $a = |a|e^{i\theta_a}$ and $b = |b|e^{i\theta_b}$. Thus, using (13) and $\epsilon \sim b^2/\epsilon_{M_2}$ (as implied by a fit of the 3g2HDM to neutrino oscillation data, i.e., $r \sim 1$), we obtain:

$$\epsilon_{N_1} \sim \frac{3}{16\pi} \frac{b_2^2}{m_i^2} \frac{\Delta m_{sol}^2}{m_i^2} \epsilon M_{N_i} \sin 2(\theta_h - \theta_a).$$  \hspace{1cm} (26)$$

It is interesting to note that the above CP-asymmetry is a factor of $\sim b_2^2$ larger than the CP-asymmetry obtained in models with one Higgs doublet or for that matter in SUSY-like models where the condensate of a single Higgs is responsible for generating the Dirac neutrino mass term. Thus, in our 3g2HDM the enhancement factor is of $\mathcal{O}(100)$ since $t_3 \sim \mathcal{O}(10)$. To see that we can rewrite $\epsilon_{N_1}$ in (26) as follows:

$$\frac{\epsilon_{N_1}}{\sin 2(\theta_h - \theta_a)} \sim -\epsilon_{\text{max}} \frac{2\epsilon_1^2}{\omega} \frac{m_i^2}{\Delta m_{sol}^2} \epsilon M_{N_i} \sin 2(\theta_h - \theta_a),$$  \hspace{1cm} (27)$$

where in our model $\epsilon \sim 0.5$ and $\omega \sim 5$ are fixed by oscillation data and $\epsilon_{\text{max}}$ is the maximum of the CP-asymmetry in models with one Higgs doublet (see e.g., [15]):

$$\epsilon_{\text{max}} \sim \frac{3}{16\pi} \frac{M_{N_i} m_{atm}}{v^2},$$  \hspace{1cm} (28)$$

with the atmospheric neutrino mass $m_{atm} = m_3$ given in [10] and $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV. As mentioned above, this enhancement is possible due to the presence of the second Higgs doublet, since in this case the light neutrino mass spectrum is determined by $v_1$ (the VEV of the lighter Higgs field) and not by the EW-scale VEV $v$ (as in the usual scenario with one Higgs doublet), and the hierarchy $m_{atm} \gg m_{sol}$ is dictated by the large tan $\beta$. In this way, the CP-asymmetry no longer suffers from a direct relation to the Dirac neutrino mass term and is, therefore, not directly constrained by oscillation data. This observation was also made by Fukuyama and Okada in [13] who have investigated leptogenesis within a generic two Higgs doublet model. As will be shown below, this enhancement of the CP-asymmetry allows us to reproduce the observed baryon asymmetry in the universe with $M_{N_1}$ as low as $10^9$ GeV, which in turn relaxes the lower bound on the reheating temperature in the early universe to $T_{RH} \gtrsim 2 \times 10^8$ GeV. [13]

The efficiency factor, $\eta_1$, which measures the amount of lepton asymmetry left from the CP-asymmetry $\epsilon_{N_1}$, is calculated by solving the appropriate Boltzmann equations. The key parameter entering the Boltzmann equations is the “decay parameter”, defined as [13]:

$$K \equiv \frac{\Gamma_{N_1}/H(T \sim M_{N_1})}{\Gamma_{N_1} } ,$$

where $\Gamma_{N_1} = (Y_{\nu}^r Y_{\nu}^\dagger)_{ii} M_{N_1}/8\pi$ is the total decay width of $N_1$ and $H(T) = \sqrt{4\pi^3 g_*/45 T^2/M_{\text{Planck}}}$ is the Hubble expansion rate. In particular, using (4), the decay parameter $K$ is given by (for $g_* \sim 100$):

$$K \sim 4.8 \times 10^{-3} a^2 \epsilon M_{N_1}^2 / M_{\text{Planck}},$$  \hspace{1cm} (29)$$

which, for $a \sim 10^{-3}$ [i.e., Ansatz (19)], gives $6 \lesssim K \lesssim 60$ if $M_{N_1} \sim 10^9 - 10^{10}$ GeV. In this range of values for the decay parameter (corresponding to the “mildly strong wash out” regime), the relation $\eta \sim 0.5 K^{1.2}$ constitutes a good fit to the numerical solution of the Boltzmann equations, see e.g., P. Di Bari in [13]. Thus, using the above fit for $\eta$ and the CP-asymmetry in (26), the baryon asymmetry in (28) becomes:

$$\frac{n_B}{s} \sim 10^{-17} \frac{t_3^2 \sqrt{\Delta m_{sol}^2}}{2m_i^2} \epsilon M_{N_1} \left( \frac{M_{N_1}}{\text{GeV}} \right)^{1.2} \sin 2(\theta_h - \theta_a) .$$  \hspace{1cm} (30)$$

This has to be compared with the observed baryon to photon number ratio $n_B/s \sim 6 \times 10^{-10}$ [19], implying $n_B/s \sim 8.5 \times 10^{-11}$. For example, taking $\epsilon \sim 0.5$ (see Fig. 2) and $\Delta m_{sol}^2 \sim 8.2 \cdot 10^{-5}$ eV$^2$, along with $t_3 \sim 10$ and $m_i \sim 170$ GeV, (30) reproduces the observed baryon asymmetry for e.g. $M_{N_1} \sim 10^{10}$ GeV and $\sin 2(\theta_h - \theta_a) \sim 0.1$, or for $M_{N_1} \sim 10^9$ if CP is maximally violated in the sense that $\sin 2(\theta_h - \theta_a) \sim 1$.

Summarizing, we have presented here a simple extension to the Standard Model, in a grounds-up approach, which leads us to a framework for neutrino masses and mixings with considerable predictable power. This can clearly have significant impact on on-going as well as planned experiments. To briefly recapitulate, we have constructed a 2HDM in which the second doublet, with a much larger VEV, couples only to the third generation right-handed up-fermions (the top-quark and the 3rd generation right-handed neutrino) and the other doublet couples to all other fermions. Thus, this model is a possible effective low energy parametrization of an underlying short distance theory which envisions a close relation between quark dynamics and neutrino physics. The key parameter of this 2HDM is tan $\beta$ which is assumed to be of $\mathcal{O}(10)$ in order to naturally accommodate a large mass for the top quark. We have shown that the large value of tan $\beta$ in this model is directly responsible for successfully reproducing the observed neutrino oscillation data, predicting a very small $\theta_{13}$: $0.017 \lesssim \theta_{13} \lesssim 0.021$ at 99% CL, a very restricted allowed range for the atmospheric mixing angle: $43^\circ \lesssim \theta_{23} \lesssim 45^\circ$ at 99% CL, as well as successfully reproducing the observed baryon asymmetry of the universe through leptogenesis. In particular, in our model, the CP-asymmetry in the N1 decays that drives leptogenesis is larger by a factor of $\mathcal{O}(\tan^2 \beta) \sim \mathcal{O}(100)$ than the CP-asymmetry obtained in models for leptogenesis with one Higgs doublet or in the MSSM. This enhancement allows to relax the lower bound on $M_{N_1}$.
and accordingly also the lower bound on the reheating temperature of the early universe.

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