Vector Meson Production in Ultraperipheral Heavy Ion Collisions

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The ultraperipheral heavy ion collisions (UPC’s) are an important alternative to study the QCD dynamics until the next generation of $e^+e^−/ep/eA$ colliders become reality. Due to the coherent action of all the protons in the nucleus, the electromagnetic field is very strong and the resulting flux of equivalent photons is large, which allows to study two-photon as well as photonuclear interactions at high energies. In this paper we present a brief review of the vector meson production in UPC’s at high energies using the QCD color dipole approach to describe their photonuclear production and the perturbative QCD Pomeron (BFKL dynamics) to describe the double meson production in photon-photon process. Predictions for rates and integrated cross sections are presented for energies of RHIC and LHC.

Keywords: Quantum Chromodynamics; Vector Meson Production; Ultraperipheral Heavy Ion Collisions.

PACS Nos.: 12.38.-t; 24.85.+p 25.75.-q; 14.40.-n

1. Introduction

In ultraperipheral relativistic heavy-ion collisions (UPC’s) the ions do not interact directly with each other and move essentially undisturbed along the beam direction. The only possible interaction is due to the long range electromagnetic interaction and diffractive processes (For a review see, e.g. Refs. [12]). Due to the coherent action of all the protons in the nucleus, the electromagnetic field is very strong and the resulting flux of equivalent photons is large. A photon stemming from the electromagnetic field of one of the two colliding nuclei can penetrate into the other nucleus and interact with one or more of its hadrons, giving rise to photon-nucleus collisions to an energy region hitherto unexplored experimentally. For example, the interaction of quasi-real photons with protons has been studied extensively at the electron-proton collider at HERA, with $W_{\gamma p} \leq 200$ GeV. Due to the larger number of photons coming from one of the colliding nuclei in heavy ion collisions, a similar and more detailed study will be possible in these collisions, with $W_{\gamma N}$
reaching 950 GeV for the Large Hadron Collider (LHC) operating in its heavy ion mode. Similarly, one can investigates QCD effects in the context of coherent two-photon scattering at UPC's. The important advantage of investigating two-photon interactions in nuclear collisions is the flux of virtual photons from the electromagnetic field of the nuclei scaling as $Z^2$, giving a two-photon luminosity scaling as $Z^4$. The maximum $\gamma\gamma$ collision energy $W_{\gamma\gamma}$ is $2\gamma/R_A$, about 6 GeV at RHIC and 200 GeV at LHC. In particular, the LHC will have a significant energy and luminosity reaching beyond LEP2, and could be a bridge to $\gamma\gamma$ collisions at a future $e^+e^-$ linear collider.

Over the past few years a comprehensive analysis of the heavy quark and vector meson production in UPC's has been made considering different theoretical approaches. For a recent review on heavy quark production in UPC's, see Ref. 13. In particular, much effort has been devoted to obtain signatures of the QCD Pomeron in such processes, which can be used to constrain the QCD dynamics at high energies. On the other hand, recently the STAR Collaboration released the first data on the cross section of the coherent $\rho$ production in gold-gold UPC's at $\sqrt{s} = 130$ GeV, which provides the first opportunity to check the basic features and main approximations of the distinct approaches describing nuclear vector meson photoproduction.

Here, we analyze the possibility of using UPC's as a photon-photon/nucleus collider, and study the production of vector mesons considering different QCD dynamics. Probably, only the next generation of $e^+e^-/ep/eA$ colliders will allow to discriminate between the distinct approaches for the QCD dynamics at high energies. However, until these colliders become reality we need to consider alternative searches in the current and/or scheduled accelerators which allow us to constraint the QCD dynamics. The first analysis, summarized in Sec. 2 concerns to the photonuclear meson production, where we consider the QCD color dipole approach to describe its production in photon-nucleus process. That approach is suitable at high energies and allows take into account the corrections of partons saturation phenomenon (For recent reviews see, e.g. Ref. 15) and nuclear effects in a simple and intuitive way. The basic quantity is the dipole cross section, which encodes all information on the interaction of the color dipoles with the nucleus target. The scattering of dipoles off nuclei is modeled through the Glauber-Gribov formalism. The conversion of dipoles on mesons is accounted by the meson wave function. Predictions are given for the ion modes and energies of RHIC and LHC. A comparison with the recent STAR data on coherent $\rho$ production is also presented. The second analysis, summarized in Sec. 3 treats the double meson production in two-photon processes, in particular double $J/\Psi$, $\rho - J/\Psi$ and double $\rho$ production. In the case of heavy mesons, we consider the underlying dynamics at high energies as given by the perturbative QCD Pomeron, with the evolution described by the BFKL equation. Then we investigate the energy behavior in LO and NLO accuracy. The hard scale for the process is guarantee by the large mass of the heavy meson. For the
mixed light-heavy meson case, we use the QCD double logarithmic approximation, which depends on the gluon content of the light meson. The sensitivity to different gluon distributions is addressed. As a byproduct, we obtain the double $\rho$ cross section using the previous results through the Pomeron-exchange factorization theorem. In the last section we discuss the main features of the theoretical estimates and also address the background processes.

2. Photonuclear vector meson production at UPC’s

In heavy ion collisions the large number of photons coming from one of the colliding nuclei will allow to study photoproduction, with the photonuclear cross sections given by the convolution between the photon flux from one of the nuclei and the cross section for the scattering photon-nuclei. The final expression for the production of vector mesons in UPC’s is then given by,

$$
\sigma_{AA\to AV} \left( \sqrt{S_{NN}} \right) = \int_{\omega_{\text{min}}}^{\infty} d\omega \frac{dN(\omega)}{d\omega} \sigma_{\gamma A \to V A} \left( W_{\gamma A}^2 = 2\omega \sqrt{S_{NN}} \right)
$$

where $\omega$ is the photon energy ($\omega_{\text{min}} = m_V^2/4\gamma_L m_p$), $m_V$ is the meson mass and $\sqrt{S_{NN}}$ is the ion-ion c.m.s. energy. For instance, the Lorentz factor for LHC is $\gamma_L = 2930$, giving the maximum c.m.s. $\gamma N$ energy $W_{\gamma A} \lesssim 950$ GeV. In this process we have that the nuclei are not disrupted and the final state consists solely of the two nuclei and the vector meson decay products. Consequently, we have that the final state is characterized by a small number of centrally produced particles, with rapidity gaps separating the central final state from both beams. Moreover, due to the coherence requirement, the transverse momentum is limited to be smaller than $p_T = \sqrt{2}/R_A$, where $R_A$ is the nuclear radius. Therefore, these reactions can be studied experimentally by selecting events with low multiplicity and small total $p_T$.

The photon flux is given by the Weizsacker-Williams method\(^1\). The flux from a charge $Z$ nucleus a distance $b$ away is

$$
d^3N(\omega, b^2) = \frac{Z^2 \alpha_{\text{em}} \eta^2}{\pi^2 \omega^2 b^2} \left[ K_0^2(\eta) + \frac{\eta}{\gamma_L} K_0^2(\eta) \right]
$$

where $\gamma_L$ is the Lorentz boost of a single beam and $\eta = \omega b/\gamma_L$; $K_{0,1}(\eta)$ are the modified Bessel functions. The requirement that photoproduction is not accompanied by hadronic interaction (ultraperipheral collision) can be done by restricting the impact parameter $b$ to be larger than twice the nuclear radius, $R_A = 1.2 A^{1/3}$ fm. Therefore, the total photon flux interacting with the target nucleus is given by Eq. (2) integrated over the transverse area of the target for all impact parameters subject to the constraint that the two nuclei do not interact hadronically. An analytic approximation for $AA$ collisions can be obtained using as integration limit $b > 2 R_A$, producing

$$
dN(\omega) = \frac{2 Z^2 \alpha_{\text{em}}}{\pi \omega} \left[ \bar{\eta} K_0(\bar{\eta}) K_1(\bar{\eta}) + \frac{\bar{\eta}^2}{2} \left( K_1^2(\bar{\eta}) - K_0^2(\bar{\eta}) \right) \right]
$$
where $\bar{\eta} = 2\omega R_A/\gamma_L$.

Analyzing Eq. (1) we obtain that the main input in the calculations of the vector mesons production cross sections in UPC’s is the photonuclear cross section $\sigma (\gamma A \rightarrow VA)$ which is given by

$$\sigma (\gamma A \rightarrow VA) = \frac{|\text{Im} A_{\text{nuc}}(s, t = 0)|^2}{16\pi} (1 + \beta^2) \int_{t_{\text{min}}}^{\infty} dt |F(t)|^2,$$

(4)

with $t_{\text{min}} = (m_V^2/2\omega)^2$. Here, the scattering amplitude for meson production on photon-nuclei collisions at zero momentum transfer is labeled by $A_{\text{nuc}}$. The quantity $\beta$ is the ratio between the imaginary and real part of the amplitude. Using the analytical approximation of the Woods-Saxon distribution as a hard sphere, with radius $R_A$, convoluted with a Yukawa potential with range $a = 0.7$ fm, we obtain that the nuclear form factor reads as

$$F(q = \sqrt{|t|}) = \frac{4\pi\rho_0}{A q^3} \left[ \sin(qR_A) - qR_A \cos(qR_A) \right] \left[ \frac{1}{1 + a^2q^2} \right],$$

(5)

where $\rho_0 = 0.16$ fm$^{-3}$.

Currently, there are in literature different models for the photonuclear vector meson production cross section which differ basically in the treatment for the photon and for its interaction with the target. The main aspect is that real photons have a complicated nature. In a first approximation, the photon is a point-like particle, although in field theory it may fluctuate also into a fermion pair (See discussions in Refs. [1,11]). In the case where there is a photon transition in a colorless antiquark-quark pair, the propagation of this colorless hadronic wave packet in a nuclear medium can be treated either in the hadronic basis as a result of Gribov’s inelastic corrections or in QCD in terms of the partonic basis, which are complementary. Let’s briefly discuss these two representations (For a detailed discussion see Ref. [15]). The time scale characterizing the evolution of a $q\bar{q}$ wave packet can be estimated based on the uncertainty principle and Lorentz dilation. The lifetime of the photon fluctuation is given by $t_c = \nu/(Q^2 + m_V^2)$, where $\nu$ is the photon energy, $m_V$ is the mass of the fluctuation and $Q^2$ is the photon virtuality. It is usually called coherence time. Using light-cone kinematics we can define the coherence length, which is given by $l_c = t_c$. Moreover, one cannot decide whether a ground state $V$ is produced or the next-excited state $V'$, unless the process lasts longer than the inverse mass difference between these states. In the rest frame of the nucleus, this formation time is Lorentz dilated and is given by $t_f = 2\nu/(m_{V'}^2 - m_V^2)$. Similarly, we can define a formation length given by $l_f = t_f$. In the hadronic basis, the same process looks quite different. The incident photon may produce different states on a bound nucleon, the $V$ meson ground state or an excited state. Those states propagate through the nucleus experiencing multiple-diagonal and off-diagonal diffractive interactions, and eventually the ground state is detected. According the quark-hadron duality, we expect that these two descriptions to be equivalent. However, as these two approaches have been used assuming different approximations, their comparison may provide a scale
for the theoretical uncertainty involved. Furthermore, it is important to emphasize that at high photon energy $\nu$, both $l_c$ and $l_f$ greatly exceed the nuclear radius $R_A$, which implies in the partonic basis that the transverse size of the $q\bar{q}$ pair do not change during the interaction with the target. This enables one to introduce the QCD dipole picture \cite{19}, where the process is factorized into the photon fluctuation in a $q\bar{q}$ pair and the dipole cross section. These aspects become the interpretation in the partonic basis more intuitive and straightforward than in the hadronic basis.

Let us consider the scattering process $\gamma A \rightarrow VA$ in the QCD dipole approach, where $V$ stands for both light and heavy mesons. The scattering process can be seen in the target rest frame as a succession in time of three factorizable subprocesses: i) the photon fluctuates in a quark-antiquark pair (the dipole), ii) this color dipole interacts with the target, and iii) the pair converts into vector meson final state. Using as kinematic variables the $\gamma^* A$ c.m.s. energy squared $s = W_{\gamma^* A}^2 = (p + q)^2$, where $p$ and $q$ are the target and the photon momenta, respectively, the photon virtuality squared $Q^2 = -q^2$ and the scaling variable $\tilde{x} = (Q^2 + 4m_f^2)/(W_{\gamma A}^2 + Q^2)$, the corresponding imaginary part of the amplitude at zero momentum transfer reads as \cite{20,21,18},

$$Im A (\gamma A \rightarrow VA) = \sum_{h,\bar{h}} \int dz d^2r \Psi^\gamma_{h,\bar{h}}(z, r, Q^2) \sigma_{target}^{\text{dip}}(\tilde{x}, r) \Psi^V_{h,\bar{h}}(z, r),$$

where $\Psi^\gamma_{h,\bar{h}}(z, r)$ and $\Psi^V_{h,\bar{h}}(z, r)$ are the light-cone wavefunctions of the photon and vector meson, respectively. The quark and antiquark helicities are labeled by $h$ and $\bar{h}$ and reference to the meson and photon helicities are implicitly understood. The variable $r$ defines the relative transverse separation of the pair (dipole) and $z$ $(1 - z)$ is the longitudinal momentum fractions of the quark (antiquark). The basic blocks are the photon wavefunction, $\Psi^\gamma$, the meson wavefunction, $\Psi^V_{T, L}$, and the dipole-target cross section, $\sigma_{target}^{\text{dip}}$.

In the dipole formalism, the light-cone wavefunctions $\Psi_{h,\bar{h}}(z, r)$ in the mixed representation $(z, r)$ are obtained through two dimensional Fourier transform of the momentum space light-cone wavefunctions, $\Psi_{h,\bar{h}}(z, k)$, which can be completely determined using light cone perturbation theory. On the other hand, for vector mesons, the light-cone wavefunctions are not known in a systematic way and they are thus obtained through models (For a recent detailed discussion see Ref. \cite{18}). Here, we follows the analytically simple DGKP approach \cite{22}, which assumes that the dependencies on $r$ and $z$ of the wavefunction are factorised, with a Gaussian dependence on $r$ (For a detailed discussion see Refs. \cite{23,24}). The main shortcoming of this approach is that it breaks the rotational invariance between transverse and longitudinally polarized vector mesons \cite{18}. However, as it describes reasonably the HERA data for vector meson production, as pointed out in Ref. \cite{23}, we will use it in our phenomenological analysis. Finally, the imaginary part of the forward amplitude can be obtained by putting the expressions for photon and vector meson (DGKP) wavefunctions into Eq. (6). Moreover, summation over the quark/antiquark helicities and an average over the transverse polarization states of the photon should
be taken into account. The transverse component (the longitudinal one does not contribute for photoproduction) is then written as

\[
\text{Im} \mathcal{A}_T(s, t = 0) = \int d^2r \int_0^1 dz \alpha_{em}^{1/2} f_V f_T(z) \exp \left[ -\frac{\omega_T^2 r^2}{2} \right] \times \left\{ \frac{\omega_T^2 \varepsilon r}{m_V} \left[ z^2 + (1 - z)^2 \right] K_1(\varepsilon r) + \frac{m_f^2}{m_V} K_0(\varepsilon r) \right\} \sigma_{\text{dip}}^{\text{target}}(\tilde{x}, r) \tag{7}
\]

with \( \sigma_{\text{dip}}^{\text{target}} \) being the dipole-nucleus cross section. In the photoproduction case, \( \varepsilon = m_f \), where \( m_f \) is the quark mass of flavour \( f \). The corresponding parameters for the vector mesons wavefunctions \( (m_V, \omega_T, f_V, \text{etc}) \) are presented in Table 1 of Ref. 24. Following Ref. 21 we have estimated contribution from real part for the photoproduction of vector mesons. This correction is about 3% for light mesons and it reaches 13% for \( J/\Psi \) at high energies 24. Additionally for heavy mesons we have taken into account the skewness effects, associated to off-forward features of the process (different transverse momenta of the exchanged gluons in the \( t \)-channel), which are increasingly important in this case.

In order to obtain the dipole-nucleus cross section we will assume the validity of the Glauber-Gribov picture 25 which allows to write

\[
\sigma_{\text{nucleus}}(\tilde{x}, r^2; A) = 2 \int d^2b \left\{ 1 - \exp \left[ -\frac{1}{2} T_A(b) \sigma_{\text{dip}}^{\text{proton}}(\tilde{x}, r^2) \right] \right\}, \tag{8}
\]

where \( b \) is the impact parameter of the center of the dipole relative to the center of the nucleus and the integrand gives the total dipole-nucleus cross section for a fixed impact parameter. The nuclear profile function is labeled by \( T_A(b) \), which will be obtained from a 3-parameter Fermi distribution for the nuclear density. The above equation sums up all the multiple elastic rescattering diagrams of the \( q\bar{q} \) pair and is justified for large coherence length, where the transverse separation \( r \) of partons in the multiparton Fock state of the photon becomes as good a conserved quantity as the angular momentum, i.e. the size of the pair \( r \) becomes eigenvalue of the scattering matrix. For the dipole-proton cross section we follow the quite successful saturation model 29 which interpolates between the small and large dipole configurations, providing color transparency behavior, \( \sigma_{\text{dip}}^{\text{proton}} \sim r^2 \), as \( r \to 0 \) and constant behavior, \( \sigma_{\text{dip}}^{\text{proton}} \sim \sigma_0 \), at large dipole separations. The parameters of the model have been obtained from an adjustment to small \( x \) HERA data. The parameterization for the dipole cross section takes the eikonal-like form 29,

\[
\sigma_{\text{dip}}^{\text{proton}}(\tilde{x}, r^2) = \sigma_0 \left[ 1 - \exp \left( -\frac{Q^2_{\text{sat}}(\tilde{x}) r^2}{4} \right) \right], \quad Q^2_{\text{sat}}(\tilde{x}) = \left( \frac{x_0}{\tilde{x}} \right)^{\lambda} \text{GeV}^2 \tag{9}
\]

where the saturation scale \( Q^2_{\text{sat}} \) defines the onset of the saturation phenomenon, which depends on energy. An additional parameter is the effective light quark mass, \( m_f = 0.14 \text{ GeV} \), which plays the role of a regulator for the photoproduction \( (Q^2 = 0) \) cross section. The charm quark mass is considered to be \( m_c = 1.5 \text{ GeV} \).
Let’s present the estimates from the QCD saturation model for vector meson photoproduction in the kinematical range of the colliders RHIC and LHC. It is noticeable that the energy dependence of the dipole-nucleus cross section \[ Q^2_{\text{sat}} \] is strongly connected with the saturation scale \( Q^2_{\text{sat}}(W_{\gamma A}) \propto A^{1/3} Q^2_{\text{sat}} \). Namely, the saturation effects are larger whether the momentum scale is of order or larger than the correspondent size of the vector meson and the energy growth of the cross section is then slowed down. In Table 1 one shows the corresponding integrated cross section. For LHC energy, one considers lead (Pb) and calcium (Ca), whereas for RHIC one takes gold (Au) and silicon (Si). The integrated cross sections can be contrasted with the theoretical estimations using GVDM plus Glauber-Gribov approach of Refs. 11 as well as the estimation of Ref. 10, which considers VDM plus a classical mechanical calculation for nuclear scattering and uses as input for the \( \gamma p \rightarrow V p \) reaction an extrapolation of the experimental DESY-HERA fits for meson photoproduction. Initially, let’s consider the latter approach (see Tab. III in Ref. 10). At RHIC energy and Si nucleus, our results are about 20 % lower for \( \rho \) and \( \omega \), whereas gives a larger \( \phi \) cross section and almost the same \( J/\Psi \) cross section. However, the situation changes for gold nucleus, where the present results are about 50 % larger than the estimates in Ref. 10. At LHC energy and for Ca nucleus, our results gives higher cross section by a factor of order 10 %, whereas for lead nucleus the factor reaches a factor 2 for light mesons and almost similar for the \( J/\Psi \) meson. Basically, the values are quite similar for light nuclei. For heaviest nuclei, the results overestimate those ones in Ref. 10 when one considers light mesons and become similar for the \( J/\Psi \) case. These features can be understood through the theoretical procedure when considering the nuclear scattering.

Now we compare our results with those ones in Ref. 11, where the main focus is on the \( \rho \) and \( J/\Psi \) production. For \( \rho \) the predictions are computed only for RHIC energy \( \sqrt{s_{NN}} = 130 \text{ GeV} \) and we will consider it later on. We can anticipate that their results are closer to ours since a Glauber-Gribov approach is used in describing the scattering on nuclei. For \( J/\Psi \) the theoretical approach for the photonnuclear production was the collinear QCD double logarithmic approximation, where the \( \gamma A \rightarrow J/\Psi A \) cross section is directly proportional to the squared nuclear gluon density distribution. There, it was considered an impulse approximation (no nuclear shadowing) and a leading twist shadowing version. The impulse approximation gives a larger cross section at central rapidity (about a factor 4 higher for Ca and
Fig. 1. Energy dependence of coherent $\rho$ meson production in gold-gold in UPC’s at RHIC ($\sqrt{s_{NN}} = 130$ GeV). Experimental data from STAR Collaboration.

Our results are closer to their impulse approximation, which suggests nuclear shadowing could be weak for $J/\Psi$ production. This feature can be easily tested in the first experimental measurements of coherent $J/\Psi$ production on UPC’s at LHC. Concerning the integrated cross section, they found 0.6 mb for Ca nucleus and 70 mb for Pb at LHC. Our results are 0.44 mb and 41.5 mb, respectively. Thus, our results are about 15% lower for Ca and also 40% lower for Pb. The difference between the predictions comes from mostly from the distinct QCD approaches considered used and the different photon flux in the UPC calculation.

Recently, the STAR Collaboration published the first measurement for coherent $\rho$ production in gold-gold UPC’s at $\sqrt{s} = 130$ GeV [14], providing the first opportunity to test the distinct approaches describing nuclear vector meson photoproduction. Our predictions are presented in Fig. 1. As the parameters in our approach were constrained in the study of the photoproduction of vector mesons in $\gamma p(A)$ interactions [23,24], our predictions are parameter free. In the range of rapidities $|y| \leq 3$, at energy $\sqrt{s_{NN}} = 130$ GeV, we have found $\sigma_{\text{sat}}(-3 \leq y \leq 3) = 410$ mb in good agreement with the STAR measurement $\sigma_{\text{STAR}}(-3 \leq y \leq 3) = 370 \pm 170 \pm 80$ mb. For the cut $|y| \leq 1$, we have obtained $\sigma_{\text{sat}}(-1 \leq y \leq 1) = 221$ mb, whereas the STAR result is $\sigma_{\text{STAR}}(-1 \leq y \leq 1) = 140 \pm 60 \pm 30$ mb. The values presented here are somewhat similar to the ones obtained in Ref [11], which uses the generalized vector dominance model (GVDM) and the Glauber-Gribov approach, including in addition the finite coherence length effects.
3. Vector meson production in two-photon processes at UPC’s

Relativistic heavy-ion collisions are a potentially prolific source of $\gamma\gamma$ collisions at high energy colliders. The advantage of using heavy ions is that the cross sections vary as $Z^4\alpha^4$ rather just as $\alpha^4$. Moreover, the maximum $\gamma\gamma$ collision energy $W_{\gamma\gamma}$ is $2\gamma/R_A$, about 6 GeV at RHIC and 200 GeV at LHC, where $R_A$ is the nuclear radius and $\gamma$ is the center-of-mass system Lorentz factor of each ion. In particular, the LHC will have a significant energy and luminosity reach beyond LEP2, and could be a bridge to $\gamma\gamma$ collisions at a future $e^+e^-$ linear collider. For two-photon collisions, the cross section for the reaction $AA \rightarrow AAV_1V_2$ will be given by

$$\sigma_{AA\rightarrow AA} = \int \frac{d\omega_1}{\omega_1} n_1(\omega_1) \int \frac{d\omega_2}{\omega_2} n_2(\omega_2) \sigma_{\gamma\gamma\rightarrow V_1V_2}(W = \sqrt{4\omega_1\omega_2})$$

where the photon energy distribution $n(\omega)$ is calculated within the Weizsäcker-Williams approximation. In general, the total cross section $AA \rightarrow AA \gamma\gamma \rightarrow AX$, where $X$ is the system produced within the rapidity gap, factorizes into the photon-photon luminosity $dL_{\gamma\gamma}d\tau$ and the cross section of the $\gamma\gamma$ interaction,

$$\sigma_{AA\rightarrow AA} = \int d\tau \frac{dL_{\gamma\gamma}}{d\tau} \hat{\sigma}_{\gamma\gamma\rightarrow V_1V_2}(\hat{s})$$

where $\tau = \hat{s}/s$, $\hat{s} = W^2$ is the square of the center of mass (c.m.s.) system energy of the two photons and $s$ of the ion-ion system. The $\gamma\gamma$ luminosity is given by the convolution of the photon fluxes from two ultrarelativistic nuclei:

$$\frac{dL_{\gamma\gamma}}{d\tau} = \int_1^\tau \frac{dx}{x} f(x) f(\tau/x)$$

where the photon distribution function $f(x)$ is related to the equivalent photon number $n(\omega)$ via $f(x) = (E/\omega) n(xE)$, with $x = \omega/E$ and $E$ is the total energy of the initial particle in a given reference frame. The remaining quantity to be determined in order to proceed is the quantity $f(x)$, which has been investigated by several groups (for more details, see e.g. Ref. 31). Here, we consider the photon distribution of Ref. 31, providing a photon distribution which is not factorizable. The authors of Ref. 31 produced practical parametric expressions for the differential luminosity by adjusting the theoretical results. The comparison with the complete form is consistent within a few percents. The approach given above excludes possible final state interactions of the produced particles with the colliding particles, allowing reliable calculations of UPC’s. Therefore, in order to estimate the double vector meson production cross section in UPC’s it is only necessary to consider a suitable QCD model for this process in $\gamma\gamma$ collisions (For a recent analysis see Ref. 33).

Let’s start our discussion considering the double $J/\Psi$ production. In Ref. 32 the double $J/\Psi$ production in $\gamma\gamma$ collisions has been proposed as a probe of the hard QCD Pomeron. At the photon level we have that the calculation of the cross section can be made perturbatively, due to the presence of the charm mass. The non-perturbative content is provided only by the $J/\Psi$ light-cone wave function, which is well constrained through the experimental measurement of its leptonic...
width $\Gamma_{J/\Psi \rightarrow t^+t^-}$. In the following we use the high energy factorization and the BFKL dynamics in order to perform estimates for the referred reaction. Assuming a small $t$ approximation, the total cross section for this process can be written as,

$$
\sigma_{\text{tot}}(\gamma \gamma \rightarrow J/\Psi J/\Psi) = \frac{1}{B_{J/\Psi J/\Psi}} \left. \frac{d\sigma(\gamma \gamma \rightarrow J/\Psi J/\Psi)}{dt} \right|_{t=0}, \tag{13}
$$

with

$$
\frac{d\sigma(\gamma \gamma \rightarrow J/\Psi J/\Psi)}{dt} = \frac{|A(W^2, t = 0)|^2}{16\pi} \exp(-B_{J/\Psi J/\Psi} \cdot |t|), \tag{14}
$$

where $B_{J/\Psi J/\Psi}$ is the corresponding slope parameter, which we assume as being $B_{J/\Psi J/\Psi} = m_c^{-2}$ (See discussion in Ref. [3]). In the general case, the imaginary part of the scattering amplitude is given as follows [32],

$$
\mathcal{I} \mathcal{m} A(W^2_{\gamma\gamma}, t) = \int \frac{d^2k}{\pi} \frac{\Phi_{\gamma J/\Psi}(k, q) \tilde{\Phi}_{\gamma J/\Psi}(W^2_{\gamma\gamma}, k, q)}{(k + q/2)^2 (k - q/2)^2}, \tag{15}
$$

where $W$ is the center of mass energy of the two photon system and the photon-meson impact factor is denoted by $\Phi_{\gamma J/\Psi}$. At the Born level $\Phi = \Phi$ and the reaction is described by the two-gluon exchange, which have transverse momenta $q/2 \pm k$ and where the momentum transfer is $t = -q^2$. When considering the complete gluon ladder contribution, the quantity $\tilde{\Phi}$ contains the impact factor and the gluon emission on the ladder, which is driven by the QCD dynamics. At the LLA level, the BFKL ladder contribution for the $t$-channel exchange provides the following expression for it,

$$
\tilde{\Phi}_{\gamma J/\Psi}(W^2_{\gamma\gamma}, k, q) = \int d^2k' \frac{k^2}{k'^2} K(W^2_{\gamma\gamma}, k, k', q) \Phi_{\gamma J/\Psi}(k', q), \tag{16}
$$

where

$$
\Phi_{\gamma J/\Psi}(k, q) = C \left[ \frac{1}{m_{\pi}^2 + q^2} - \frac{1}{m_{\Omega}^2 + k^2} \right], \tag{17}
$$

and $K(W^2_{\gamma\gamma}, k, k', q)$ is solution of the LLA BFKL equation at $t \neq 0$. The Eq. [17] defines the impact factor in the nonrelativistic approximation, where it is assumed that the quark and antiquark have the same longitudinal momentum. A comparison between the HERA data and the theoretical predictions for $J/\Psi$ photoproduction using this approximation has found good agreement [34]. We have considered the following parameters for the further calculations: $m_c = m_{J/\Psi}/2 = 1.55$ GeV, $C = \sqrt{\alpha_s} \alpha_s (\mu^2) e_c \frac{8}{3} \pi m_c f_{J/\Psi}$, with $e_c = 2/3$ and $f_{J/\Psi} = 0.38$ GeV.

In the Born two-gluon level we have that $\mathcal{K} = \delta^2(k - k')$ and the cross section is energy independent. On the other hand, in order to perform a LLA BFKL calculation, the following solution for the evolution equation in the forward case was considered [3]

$$
\mathcal{K}(W^2_{\gamma\gamma}, k, k') = \frac{1}{\sqrt{2} \pi^3 \alpha s k^2 k'^2} \frac{1}{\sqrt{\ln(W^2_{\gamma\gamma}/s)}} \left( \frac{W^2_{\gamma\gamma}}{s} \right)^{\omega_F} e^{-\frac{\ln^2 k^2 k'^2}{2 \pi m_c W^2_{\gamma\gamma}/s}}, \tag{18}
$$
where $\omega_{I\Delta} = \frac{3\alpha_s}{\pi} \ln 2$ and $a = \frac{32\alpha_s}{\pi} 28 \zeta(3)$. The Pomeron intercept is given by $1 + \omega_{I\Delta}$ in the leading logarithmic approximation, which depends on $\alpha_s$. We have taken $\hat{s} = 1$ GeV$^2$. The results are sensitive to the choice for the intercept, providing an enhancement of the total cross section by one/two orders of magnitude in relation to the Born level at the considered energy range. The behavior presented by the approximation above is considerably steep on energy. Consequently, it is important to evaluate the corrections which comes from the next-to-leading order corrections to the BFKL equation or those associated to unitarity effects. In order to simulate the NLO effects, we have used a value $\omega_{I\Delta} = 0.37$. The effective behavior obtained is given by $W_{\lambda\gamma\gamma}$, with $\lambda = 0.29$, which value is somewhat close to $\lambda \approx 0.23 - 0.29$ obtained in Ref. 32. A more detailed analysis of the double $J/\Psi$ production at the photon level can be found in Ref. 3.

Having determined the $\gamma\gamma$ cross section we can estimate the double $J/\Psi$ cross section in UPC’s. In Ref. 3 the energy dependence of the cross section was investigated, considering the Pb + Pb collisions and different approximations for the $\gamma\gamma$ subprocess. Shortly, the LLA BFKL solution predicts large values of cross section for LHC energies in comparison with the Born approximation 3. The ratio between the BFKL prediction and the two-gluon cross section is around 40 at $\sqrt{s} = 5500$ GeV. Taking into account non-leading corrections, this ratio is reduced to approximately 1.7. Therefore, these results indicate that if the NLO corrections to the BFKL approach are account in the approximation considered here, a future experimental analyzes of double $J/\Psi$ production in ultraperipheral heavy ion collisions could not easily constrain the QCD dynamics, since the BFKL(NLO) result is similar to the Born prediction. Only accurate measurements would allow discriminate between the two cases. Moreover, it is important to emphasize that the NLO corrections can be larger, implying a full suppression of energy enhancement associated with the iteration of gluons in the $t$-channel present in the QCD dynamics at high energies. However, if the energy dependence of the $\gamma\gamma$ cross section was driven for a large intercept, closer to the LO prediction, the analysis of UPC’s can be useful. It is worth to mention that our results are quite similar to those ones computed for the FELIX proposal at LHC 35. For PbPb collisions with energies of center of mass equal to $\sqrt{s} = 5500$ A GeV, luminosities of $L_{AA} = 4.2 \times 10^{26} cm^{-2} s^{-1}$ are considered. Consequently, during a standard $10^6 s/ month$ heavy ion run at the LHC, we predict approximately 30, 50 and 1100 events for Born, BFKL (MOD) and BFKL (LO), respectively 32. It is interesting to compare these predictions with the results for double $J/\Psi$ production at LEP2, since the $\gamma\gamma$ center of mass energies are similar. Considering an integrated luminosity of 500 pb$^{-1}$ in three years and $\sqrt{s} = 175$ GeV, almost 70 events are expected taking into account the non-leading corrections to the BFKL approach 32. Therefore, we predict a large number of events in ultraperipheral heavy ion collisions, allowing future experimental analyses, even if the acceptance for the $J/\Psi$ detection being low. It should be stressed that the LHC probably will operate in its heavy ion mode only four weeks per year.
Let’s now discuss the $\rho J/\Psi$ production in UPC’s. At the photon level, this process can be calculated similarly as made for the elastic $J/\Psi$ photoproduction off the proton. Basically, the differential cross section is given by

$$\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \rho J/\Psi) = C \frac{16\pi^3 g_\rho^2 [\alpha_s(\mu^2)]^2 \Gamma_{ee}^{J/\Psi}}{3 M_{J/\Psi}^3} \left[ xG^p(x, \mu^2) \right]^2 e^{-B_{\rho J/\Psi}|t|},$$

(19)

where $C$ denotes a product of corrections factors. The two-photon cms energy is denoted by $W_{\gamma\gamma}$, where $x = M_{J/\Psi}^2/W_{\gamma\gamma}^2$ and $M_{J/\Psi}$ is the heavy meson mass. The mass scale is given by $\mu^2 = M_{J/\Psi}^2/4$. In the small-$t$ approximation, the slope is estimated to be $B_{\rho J/\Psi} = 5.5 \pm 1.0$ GeV$^{-2}$. The light meson-photon coupling is denoted by $g_\rho = 0.454$ and the heavy meson decay width into a lepton pair is $\Gamma_{ee}$. Moreover, $xG^p(x, \mu^2)$ is the gluon content of the light meson, which can be constrained from experimental data for the cross section. Currently, there are different parameterizations for this distribution in the literature, associated to distinct QCD dynamics. In Ref. we investigate if an experimental analysis of the $\rho J/\Psi$ allows discriminate among distinct parameterizations for the gluon content of the light meson. In particular, we use both LO and NLO GRS parameterization, the SaS1D parameterization and a phenomenological Regge motivated ansatz. The latter is given by $xG^\rho_{\text{Regge}}(x, \mu^2) = x_0 G^\rho(x_0, \mu^2)$, with $x_0 = 0.1$ and the Pomeron intercept is considered as $\omega_P = 1.25$, in agreement with the effective power in the HERA data. For the nuclear case we allow for a higher intercept, in order to simulate a BFKL-like behavior, motivated by the studies in double $J/\Psi$ production. The normalization, $x_0 G^\rho(x_0, M_{J/\Psi}^2/4)$, is given by the NLO-GRS parameterization. The differences among the parameterizations are sizeable already at photon level as a consequence of distinct effective exponent $\lambda$. The GRS LO one presents the steeper behavior, followed by SaS1D ($\lambda = 0.3038$). The Regge motivated and GRS NLO parameterizations are more close since the intercept for the Regge ansatz is similar to the effective power of GRS NLO ($\lambda \simeq 0.228$). The results for the $\rho J/\Psi$ production cross sections at the photon level can be found in Ref.

For $\rho J/\Psi$ meson production in UPC’s at LHC energy range we have that an upper bound is obtained using the SaS1D parameterization and the lower one by the Regge ansatz with the lower Pomeron intercept $\omega_P = 1.25$. At the LHC energies $\sqrt{s} = 5500$ GeV, the cross section takes values between 680 nb and 2.6 mb, showing the process can be used to discriminate among parameterizations. It should be noted that the ultraperipheral cross section is dominated by not so high two-photons energies. Although the deviations among the models are quite visible at very high energies, in the nuclear cross section such deviations are less sizeable. This is directly related to the effective two-photon luminosity, which peaks at smaller $W_{\gamma\gamma}$. Therefore, correct estimates should include careful treatment of the low energy threshold effects. The nuclear cross section is enhanced in relation to the $e^+e^-$ case, which provides values of hundreds of pb’s in contrast with units of mb in the peripheral.
Table 2. The double $\rho$ production cross sections in UPC’s at LHC ($\sqrt{s} = 5500$ GeV) for PbPb.

| SCENARIO                        | $\sigma(\AA \rightarrow \AA \rho\rho)$ (ab) |
|---------------------------------|------------------------------------------|
| BFKL(LO) + GRS(LO)              | $25 \times 10^4$                        |
| BFKL(LO) + GRS(NLO)             | $4 \times 10^4$                         |
| BFKL(LO) + Regge ($\omega_P = 1.25$) | $810$                                |
| BFKL(MOD) + GRS(LO)             | $125 \times 10^4$                      |
| BFKL(MOD) + GRS(NLO)            | $20 \times 10^4$                       |
| BFKL(MOD) + Regge ($\omega_P = 1.25$) | $405 \times 10^2$                   |

nuclear case. Comparing our estimates with the FELIX parameterization, it was found the values predicted by FELIX collaboration are ever larger than obtained using the approach presented here, independently of the value of slope used. As a cross check, we also calculated the cross section for $\rho J/\Psi$ production in UPC’s using the same kinematical cut proposed in Ref. 35. In such a situation, the agreement was much better.

As a byproduct, using the Pomeron-exchange factorization theorem, we can use the results for double $J/\Psi$ production and for $\rho J/\Psi$ production to obtain a rough estimate of the double $\rho$ production in ultraperipheral heavy ion collisions. In principle, this theorem cannot be applied for BFKL Pomeron, since it is not an isolated Regge pole. As pointed out in Ref. 41 (See also Ref. 42), only when the asymptotic freedom is incorporated into the BFKL equation is that the BFKL Pomeron can be expressed in terms of a series of isolated poles in the angular-momentum plane, with the contribution of each isolated pole satisfying the factorization theorem. An open question is how much the factorization is violated in different processes. In what follows we estimate the double $\rho$ cross section assuming the Pomeron-exchange factorization theorem. These predictions should be compared with those obtained in a QCD calculation. Preliminary results presented in Ref. 33 shown that the violation is not large. However, more detailed studies are necessary in order to quantify this violation. Given the assumption that single Pomeron exchange dominates, the following relation among the total cross sections can be posed

$$\sigma(\gamma\gamma \rightarrow \rho\rho) = \frac{(B_{\rho J/\Psi})^2}{B_{\rho\rho} \times B_{J/\Psi J/\Psi}} \times \frac{[\sigma(\gamma\gamma \rightarrow \rho J/\Psi)]^2}{\sigma(\gamma\gamma \rightarrow J/\Psi J/\Psi)}. \quad (20)$$

Using the factorization theorem for the slopes, we estimate $B_{\rho\rho} \approx 12.1 \text{ GeV}^{-2}$. In the Table 2 we present our predictions for the double $\rho$ production in ultraperipheral $AA$ collisions, considering different scenarios for the QCD dynamics. We can see that there is a large range of possible values for the cross section, which implies that future experimental data are essential to constraint the dynamics as well as the violations of the factorization theorem. It is important to emphasize that our predictions agree with those presented in Ref. 35.
4. Summary

Before presenting a summary of the main results, let’s discuss the background processes and experimental separation. In Ref. 9, a detailed analysis of the experimental separation between photoproduction and two-photon interactions was presented. There, the authors have estimated that the two-photon cross sections are at least 1000 smaller than the photoproduction cross section, which makes the experimental separation between the two interactions very hard. Our calculations indicate that the inclusion of the QCD Pomeron effects implies higher cross sections at two-photon level and, consequently, larger cross sections in ultraperipheral collisions. Therefore, the inclusion of these effects implies that, in general, the contribution of two-photon interactions is non-negligible. However, the experimental separation of two-photon process still remains a challenge. In principle, an analysis of the impact parameter dependence should allow to separate between the two classes of reactions, since two-photon interactions can occur at a significant distance from both nuclei, while a photonuclear interaction must occur inside or very near a nucleus.

Other possible background process is that associated to Pomeron-Pomeron interactions. It has been verified that such reactions would be non-negligible for light ions, while they are significantly suppressed for heavy ions 13. In the particular case of the double heavy meson production this contribution deserves more detailed studies, since the current treatments rely on the Regge formalism instead of a QCD approach. An additional contribution in two photon ultraperipheral collisions is the meson production accompanied by mutual Coulomb dissociation. It has been estimated that these reactions increase the total cross section for an amount of $\sim 10\%$ at RHIC/LHC energies 44. We disregarded this contribution in the present calculations, since that it would be smaller than the associated theoretical uncertainties.

Let’s consider the experimental feasibility of photonuclear production. Although the vector meson photoproduction at $AA$ collisions to be a small fraction in comparison to the total hadronic cross sections, the experimental separation of these reaction channels is possible. As photoproduction is an exclusive reaction, $A+A \rightarrow A+A+V$, the separation of the signal from hadronic background would be relatively clear. Namely, the characteristic features in photoproduction at UPC’s are low $p_T$ meson spectra and a double rapidity gap pattern. Moreover, the detection (Roman pots) of the scattered nuclei can be an additional useful feature. In hadroproduction, the spectra on transverse momentum of mesons are often peaked around meson mass, $p_T \approx m_V$. An experimental cut $p_T < 1$ GeV would eliminate most part of the hadronic background. Hence, the rapidity cut would enter as an auxiliary separation mechanism. This procedure is specially powerful, since there will be rapidity gaps on both sides of the produced meson.

As a summary, the cross sections for the $A+A \rightarrow A+A+V$ ($V = \rho, \omega, \phi, J/\Psi$) process were computed and theoretical estimates for scattering on both light and heavy nuclei are given for RHIC and LHC energies. The rates are very high, mostly for light mesons and at LHC energies. As an important result, we compare our
prediction for the coherent $\rho$ meson production with RHIC data at $\sqrt{s_{NN}} = 130$ GeV. The corresponding results are in good agreement with the experimental results when considered the cuts on momentum transfer and on rapidity. Moreover, we have computed the double vector production in UPC where one considers the perturbative QCD Pomeron, described by the BFKL equation. For the mixed light-heavy meson production, we use the DDLA pQCD approximation, which depends on the gluon content of the light meson and obtain the double $\rho$ cross section using the previous results through the Pomeron-exchange factorization theorem.

As a final remark, in a similar way the strong electromagnetic fields generated by high-energy protons allow us to study photon - nucleon processes in proton - proton interactions in a large kinematical range. These events can be experimentally studied by selecting events with low multiplicity and small total transverse momentum. We have studied this case for the photoproduction of vector mesons in proton-(anti)proton in Ref. 7 and the photoproduction of heavy quarks in Ref. 45

Acknowledgments

This work was partially financed by the Brazilian funding agencies CNPq and FAPERGS.

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