Opportunistic Multiuser Two-Way Amplify-and-Forward Relaying with a Multi Antenna Relay

Duckdong Hwang*, Bruno Clerckx**, Sung Sik Nam*** and Tae-Jin Lee†

Abstract—We consider the opportunistic multiuser diversity in the multiuser two-way amplify-and-forward (AF) relay channel. The relay, equipped with multiple antennas and a simple zero-forcing beam-forming scheme, selects a set of two way relaying user pairs to enhance the degree of freedom (DoF) and consequently the sum throughput of the system. The proposed channel aligned pair scheduling (CAPS) algorithm reduces the inter-pair interference and keeps the signal to interference plus noise power ratio (SINR) of user pairs relatively interference free in average sense when the number of user pairs become very large. For ideal situations, where the number of user pairs grows faster than the system signal to noise ratio (SNR), the DoF of $M$ pair channel use can be achieved when $M$ is the relay antenna size. With a limited number of pairs, the system is overloaded and the sum rates saturate at high signal to noise ratio (SNR) though modifications of CAPS can improve the performance to a certain amount. The performance of CAPS can be further enhanced by semi-orthogonal channel aligned pair scheduling (SCAPS) algorithm, which not only aligns the pair channels but also forms semi-orthogonal inter-pair channels. Simulation results show that we provide a set of approaches based on (S)CAPS and modified (S)CAPS, which provides system performance benefit depending on the SNR and the number of user pairs in the network.

Index Terms—Two-way relaying, opportunistic user pair selection, amplify-and-forward, degree of freedom.

I. INTRODUCTION

Two-way amplify-and-forward (AF) relaying [1], [2], [3], [4], [5] is an attractive technique to enhance the spectral efficiency of AF relaying system, where a pair of users exchanges bidirectional messages in two phases. The two transmissions from the users overlap in the first phase and the AF relay simply broadcasts the received signal toward the two users in the second phase. Each user subtracts out the reflected self-interference and can decode the message signal from the other user.

Space division multiple access (SDMA) techniques at the multiple antenna AF relay enable a set of user pairs to exchange the two-way traffics using the same spectral resource [6], [7], [8], [9], [10], [11]. Since the reflected self-interference of a user can be subtracted out, the handling of inter-pair interference is the key challenge when multiple two-way traffics share the same spectral resource. A base station and a set of users form a two-way traffic through a multi antenna relay in [9], [12], [13], [14]. Here, the inter-pair interference is jointly handled by the base station and the relay and the achievable degree of freedom (DoF) is the minimum of the numbers of the base station antennas and the relay antennas. Multiple pairs of two-way users through a multi antenna relay are considered in [15], [6], [16], [2], [10]. When users have a single antenna, the relay only handles the inter-pair interference [15], [6], [7], [17] and a DoF up to the integer floor of $\frac{M+1}{2}$ is achievable. Alternatively, the relay and the users collaborate to suppress the inter-pair interference [10], [16] with multi-antenna relay and users. In the latter schemes, $M$ pairs of users can be supported ($M$ DoF is achieved) when $M$ is the number of relay antennas, $N$ is the number of user antennas and they satisfy $N \geq (M + 1)/2$.

When there are multiple users as in the cellular network, the independent fading of user channels can be exploited to provide the system with various performance gains [18], [19], [20], [21]. This opportunistic multiuser diversity is utilized to enhance the figures of merit of scheduling algorithms [18] or is utilized to schedule semi-orthogonal user channels in the conventional multiuser multiple input multiple output (MU-MIMO) zero forcing beam-forming system [19]. Also, it can be used for interference alignment in the cellular networks [20] or in the interference channels [21]. For the two-way relay channel with single antenna users and an AF relay with $M$ antennas, we propose an opportunistic scheduling schemes in this paper, the DoF of which is guaranteed to be $M$ ($M$ pairs of two-way traffic can be served in two phases) if the number of pair users ($K$) scales proportionally to the signal to noise ratio (SNR). The DoF is improved compared to those of [15], [6], [7] but remains the same as the one in [10], [16]. However, the requirement for multiple antenna users is replaced by the opportunistic multiuser diversity from a large number of user pairs compared to the schemes in [10], [16]. For realistic values of $K$, the system sum rate performance saturates at high SNR and improvement is observed only in the low-to-mid SNR regime.

The contributions of this paper are summarized as follows

- A large number of users ($K \gg M$) in the system provide chances to find two-way pairs, the channel vectors of which are almost aligned as in [10], [16]. Therefore, we can schedule user pairs such that the inter-pair interference is minimized. The proposed channel aligned pair scheduling (CAPS) algorithm mitigates the impact of the inter-pair interference and leads the signal
to interference plus noise power ratio (SINR) of \( M \) pair users to overcome the sum rate saturation at high SNR in average sense. The zero-forcing beamformer (ZFBB) at the relay suppresses the inter-pair interference among the scheduled two-user pairs.

- As the authors in [19] utilized the multiuser diversity to find a semi-orthogonal user set for scheduling, we can modify the CAPS algorithm so that the performance of CAPS is further enhanced. Thus, we also provide semi-orthogonal channel aligned scheduling (SCAPS) algorithm where the semi-orthogonal user scheduling (SUS) algorithm in [19] is embedded into CAPS.
- With a fixed \( K \), CAPS (SCAPS) overloads the system with user pairs beyond the relay’s capability to handle the inter-pair interference. Therefore, the sum rates of CAPS and SCAPS saturate at high SNR. Only when \( K \) grows proportionally with the SNR, the high SNR ceiling can be removed. With a fixed \( K \), we can apply a modified scheduling strategy. Suppose CAPS (or SCAPS) schedules only \( M (\leq M) \) two way pairs. Then, a mixed set of pairs, one part (\( M \) pairs) by CAPS (or SCAPS) and the other part (arbitrarily chosen the integer floor of \( J = (M - M)/2 \) pairs) can be scheduled together in the modified CAPS (or modified SCAPS) strategy. Together with this modification, we provide a set of alternative approaches to improve the system performance depending on SNR and the number of user pairs in the system. It is shown that modified SCAPS performs the best at low to mid SNR with a mitigated high SNR performance. Further, it is shown that we can adapt the loaded pairs in the system according to different modifications of SCAPS and provide more gain.

The paper is organized as follows. The system model appears in Section II. The presentation of CAPS algorithm, its modification and numerical results follow in Section III. The SCAPS algorithm, adaptation of SCAPS and its numerical results are presented in Section IV. Section V concludes the paper.

**Notations:** The bold lower case letter represents a vector and the bold upper case letter represents a matrix. \( E[a] \) denotes the average of a random variable \( a \). The notations \( A^T, A^H, A^\dagger \) and \( T_r[A] \) are the transpose, the Hermitian transpose, the pseudo inverse and the trace of a matrix \( A \), respectively. \( A^{-1} \) and \( \|a\| \) denote the projection onto the space orthogonal to the columns of \( A \) and the norm of a vector \( a \), respectively. \( |A| \) denotes the cardinality of a set \( A \). \( I_k \) denotes the identity matrix with \( k \times k \) dimensions. \( CN(0, C) \) denotes the complex white Gaussian random vector with zero mean vector \( 0 \) and the covariance matrix \( C \). The integer floor function \( \lfloor a \rfloor \) returns the largest integer less than or equal to \( a \).

II. SYSTEM MODEL

In Fig. 1 the multiuser two-way relay channel is depicted, where the half-duplex AF relay has \( M (M \geq 2) \) antennas and the \( 2K \) single antenna user terminals make \( K \) two way pairs, where two users in a pair exchange bidirectional information through the relay. The relay can be a relay station or an access point of a macro/small cell as long as it serves the direct transmissions among the user terminals inside the coverage area. The \( M \times 1 \) channel vector between the user \( i \) and the relay is denoted by \( h_i \). The elements of these channel vectors are independent identically distributed (i.i.d.) \( CN(0, I_M) \). We assume that the \( i \)-th user \( (i = 1, \ldots, M) \) is paired with the \( (i+K) \)-th user without loss of generality. Each user sends a pilot signal so that the relay can learn the channels for all users \( (h_i, i = 1, \ldots, 2K) \), based on which \( M \) two way pairs are selected by the relay.

The transmission of two-way relaying is composed of two phases. The \( 2M \) users in the selected pairs transmit their messages toward the relay in the first phase and the relay broadcasts the beam-formed and physical layer network coded messages toward the \( 2M \) users in the second phase. All the channel vectors do not change during the two transmission phases. Suppose user pairs \( i = 1, \ldots, M \) are selected. The \( i \)-th user sends the message symbol \( x_i \), \( (E[|x_i|^2] = P_s) \) through the antenna in the first phase. The received signal at the relay is given as

\[
y_r = \sum_{i=1}^{M} (h_{ix_i} + h_{i+Kx_{i+K}}) + n_r, \tag{1}
\]

where the \( M \times 1 \) vector \( n_r \) is \( CN(0, I_M) \). To maximize the SINR at the final destinations, the relay applies a \( M \times M \) beam-former \( \mathbf{W}_r \) to the received signal \( \mathbf{y}_r \) and transmits the product vector \( \mathbf{W}_r \mathbf{y}_r \) in the second phase. Then the signal received at the \( j \)-th user in the second phase is given as

\[
y_j = \mathbf{h}_j^T \mathbf{W}_r \mathbf{y}_r + n_j = \mathbf{h}_j^T \mathbf{W}_r \sum_{i=1}^{M} (h_{ix_i} + h_{i+Kx_{i+K}}) + n_r] + n_j, \tag{2}
\]

where \( n_j \) is \( CN(0, 1) \) again.

If \( \mathbf{h}_j^T \mathbf{W}_r \mathbf{h}_j \) is known to the \( j \)-th user, it can subtract out the self-interference term \( \mathbf{h}_j^T \mathbf{W}_r \mathbf{h}_j x_j \). Then, \( (2) \) becomes

\[
y_j = h_{j+Kx_{i+K}} + \sum_{i \neq j} (h_{ix_i} + h_{i+Kx_{i+K}}) + n_r] + n_j = \mathbf{h}_j^T \mathbf{W}_r \mathbf{h}_{i+Kx_{i+K}} + \mathbb{I}_j + n_r,j + n_j, \tag{3}
\]
where \( n_{r,j} = h_j^H W_r n_r \) and the inter-pair interference term \( I_j = h_j^H W_r \sum_{i \neq j} (h_i x_i + h_{i+K} x_{i+K}) \). The \( j \)-th user can decode the message \( x_{j+K} \) from the other user in the same pair. The signal-to-interference plus noise power ratio (SINR) of the \( j \)-th user is given as

\[
SINR_j = \frac{P_j |h_j^H W_r h_{j+K}|^2}{|I_j|^2 + ||W_r^H h_j||^2 + 1}.
\]

All the other users in the \( M \) pairs have similar SINR expressions.

A. Relay Pre-coder Design

To limit the transmit power at the relay, the relay beam-former \( W_r \) should meet

\[
Tr[H^H W_r^H W_r H + I_{2M}] = P_r,
\]

where the columns of \( H \) are \( 2M \) channel vectors \( (h_j) \) of the selected user pairs and \( P_r \) is the relay power constraint. Let \( \beta \) be the power control parameter so that Eq. (5) is to be satisfied, then we have \( W_r = \beta W_r^H W_r \), where the \( j \)-th row of the \( M \times M \) matrix \( W_r \) is denoted as a \( 1 \times M \) vector \( w_j \), \( ||w_j|| = 1 \) and has the property that the angular distance toward the user channels of the other pairs are bounded. Also, let the selected user pair set \( S = \{1, \ldots, M, K + 1, \ldots, K + M \} \). Mathematically, we can write the property of the \( j \)-th row as

\[
\frac{|w_j h_k|}{||h_k||} \leq \delta, \quad k \in S_j,
\]

where \( S_j = S \setminus \{j, j + K\} \). When \( \delta \) can be made to zero, the inter-pair interference can be forced to zero by the relay beam-former \( W_r \). However, in practice, we can only maintain \( \delta \) small enough through the opportunistic multiuser diversity and control the inter-pair interference within a certain level.

With the aid of the property in (6), the inter-pair interference power can be bounded as in the following Lemma 1.

**Lemma 1**: The inter-pair interference power is bounded as \( \frac{|I_j|^2}{P_r} \leq \beta^2 \delta^2 (||h_j||^2 + (M-1)||h_j|| + (M-1)^2 \delta^2) \sum_{k \in S_j} ||h_k||^2 \).

**Proof**: Let \( H_j \) be the \( M \times (M-2) \) matrix, where the columns corresponding to the \( j \)-th user pair are struck out from the matrix \( H \). Also, let \( \Pi \) and \( \Pi_j \) be the \( M \times (M-1) \) and \( M \times M \) matrices, respectively, with all one entries and \( \Lambda \) be the \( 2(M-1) \times 2(M-1) \) diagonal matrix with elements \( ||h_k||^2 \), \( k \in S_j \) on its diagonal. Then,

\[
\frac{|I_j|^2}{P_r} = \beta^2 ||h_j^H \tilde{W}_r^H \tilde{W}_r h_j||^2
\]

\[
= \beta^2 Tr[\tilde{W}_r H_j H_j^H \tilde{W}_r^H \tilde{W}_r h_j^H \tilde{W}_r^H]
\]

\[
\leq \beta^2 \delta^2 Tr[\Pi \Lambda \Pi \tilde{W}_r h_j^H \tilde{W}_r^H]
\]

\[
= \beta^2 \delta^2 \sum_{k \in S_j} ||h_k||^2 Tr[\tilde{H} \tilde{W}_r h_j^H \tilde{W}_r^H].
\]

The property in (6) is used in the inequality. Define \( a \) as the \( M \times 1 \) vector with \( ||h_j|| \) on its \( j \)-th position and \( \delta \) on all other positions. Then, using (6) again, the righthand side of (7) is further bounded as

\[
\leq \beta^2 \delta^2 \sum_{k \in S_j} ||h_k||^2 Tr[\tilde{H} \tilde{W}_r h_j^H \tilde{W}_r^H]
\]

\[
= \beta^2 \delta^2 \sum_{k \in S_j} ||h_k||^2 \frac{P_j}{Tr[\tilde{H} \tilde{W}_r h_j^H \tilde{W}_r^H]}
\]

Similarly, we can show that the relay noise power term delivered to the \( j \)-th user receiver \( ||W_r^H h_j^2|| \) is bounded as \( ||W_r^H h_j^2|| \leq \beta^2 (||h_j||^2 + (M-1)^2 \delta^2) \). Once the \( j \)-th user pair beam-formers \( (w_j) \) satisfy the property in (6), the power of the two hop channel \( h_j^H W_r h_{j+K} \) is lower bounded as \( ||h_j^H W_r h_{j+K}||^2 \geq \beta^2 ||h_j^H W_j^H w_j h_{j+K}||^2 \). Therefore, a reasonable choice of \( w_j \) is to maximize \( ||h_j^H W_r h_{j+K}|| \) within the constraint given in (6).

III. CHANNEL ALIGNED PAIR SCHEDULING

A. The Algorithm

The opportunistic multiuser diversity acquired by clever scheduling strategies has been studied for many years in cellular networks [18, 19, 20, 22]. In this section, we utilize the large number of user channels to align the channels of users in a pair so that the well known beam-formers like ZFBF; applied at the relay, can handle the inter-pair interference easily. The following channel aligned pair scheduling (CAPS) algorithm picks up \( M \) user pairs whose pair channels are mostly aligned.

- for \( k = 1 \) to \( K \), calculate \( \nu_k = \frac{h_k^H h_{k+K}}{||h_k|| ||h_{k+K}||} \)
- pick up \( M \) user pairs with the largest channel correlation values \( \nu_k \)
- Order the selected pairs for notational convenience. Let \( i = \varphi(k) \) denote this reordering, where \( i \) runs from 1 to \( M \)
- Take the mean direction between the channel vectors of user \( j \) and user \( j + K \) as \( h_{j-1}(j) = h_{j+1}(j) ||h_{j+1}(j)|| + h_{j+1}(j+K) ||h_{j+1}(j+K)|| \)
- Set \( G = \{h_{j-1}(1), \ldots, h_{j+1}(M)\} \) and find \( \tilde{W}_r = \rho G \) to meet the property in (6). Here, \( \rho \) is set to make \( Tr[\tilde{W}_r^H \tilde{W}_r] = M \)
- Set \( W_r = \beta \tilde{W}_r^H \tilde{W}_r \) and find \( \beta \) using equation (5)

From the SINR expression in (3), it is easy to see that the inter-pair interference power \( |I_j|^2 \) becomes the bottleneck to the \( j \)-th user throughout in high SNR. Therefore, it is important to see that the simple CAPS algorithm in this section actually works toward reducing this interference power through the opportunistic multiuser diversity. Lemma 2 presents the probability density function (pdf) of \( \nu_{j-1}(M) \) selected by CAPS algorithm.

**Lemma 2**: The pdf of \( \mu = \nu_{j-1}(M) \) is given as

\[
f_\mu(\mu) = M(M-1) \left( \frac{K}{M} \right) \sum_{n=0}^{K-M} \left( \frac{K-M}{n} \right) \left( 1 - \mu \right)^{(M-1)(K-n)-1} \left( -1 \right)^{(K-M-n)}.
\]

**Proof**: The cumulative density function (cdf) of the angular distance (\( \nu \)) of two complex random vectors is derived
in [23] through a reinterpretation of Theorem 1 of [24]. It is given as
\[ F_{\nu}(\nu) = 1 - (1 - \nu)^{M-1}, \]
onumber
over the set \( \nu \in [0,1] \). Using the property of order statistics [25], the pdf of \( \mu \) can be found as
\[
\begin{align*}
    f_{\mu}(\mu) &= M \left( \frac{K}{M} \right) f_{\nu}(\mu)^{(K-M)} \left( 1 - F_{\nu}(\mu) \right)^{(M-1)} \\
    &= M \left( \frac{K}{M} \right) (M-1)(1 - \mu)^{M-2} \\
    &\quad \cdot (1 - (1 - \mu)^{(K-M)}(1 - \mu)^{(M-1)^2}) \\
    &= M(M-1) \left( \frac{K}{M} \right) \sum_{n=0}^{K-M} \left( \frac{K-M}{n} \right) \\
    &\quad \cdot (1 - \mu)^{(M-1)(K-n)-1}(1)^{(K-M-n)}. \\
\end{align*}
\]

(10)

From (9), it is straightforward to see that the average \( E[\mu] \) is
\[
E[\mu] = M(M-1) \left( \frac{K}{M} \right) \sum_{n=0}^{K-M} \left( \frac{K-M}{n} \right) \\
\cdot (-1)^{(K-M-n)} (K-M-n+1). \\
\]

(11)

Let us define \( \theta_k = \cos^{-1} \nu_k \), the angle between the two channel vectors of the \( k \)-th pair. Since the relay beamformer \( \hat{\mathbf{w}}_k \) of CAPS is constructed to zero force the mean channel vectors of other pairs, the \( \delta \) in (6) is determined by \( \delta^2 = \sin^2 \theta_k / 2 \). Using the trigonometric identity, we can define \( \rho(\hat{\mathbf{K}},M) = E[\delta^2] = 1 - E[\mu] \). Though it is hard to see how \( \rho(\hat{\mathbf{K}},M) \) varies as a function of \( K \) value from Eq. (11), the plots in Fig. 2 show that this value diminishes quickly when \( K \) value grows. In the large \( K \) region, we can notice that the curve \( \rho(\hat{\mathbf{K}},2) \) upper bounds the curves of other values of \( M \) (\( M \geq 3 \)). In Appendix A we provide a large \( K \) approximation of \( \rho(\hat{\mathbf{K}},M) \) and show that it is upper bounded by \( \rho(\hat{\mathbf{K}},2) = \frac{1}{1+\frac{1}{K+1}} \). Lemma 3 presents an upper bound of the inter-pair interference power \( |I_j|^2 \) when CAPS algorithm is applied.

**Lemma 3:** With the CAPS algorithm, the inter-pair interference power \( |I_j|^2 \) can be upper bounded, on average, as
\[
|I_j|^2 \leq \beta^2 P_s \frac{1}{K+1} \left[ (M-1) \| \mathbf{h}_j \| + \left( \frac{1}{K+1} \right) \sum_{k \in S_j} \| \mathbf{h}_k \|^2 \right] \\
+ (M-1) \left( \frac{1}{K+1} \right) \sum_{k \in S_j} \| \mathbf{h}_k \|^2. \\
\]

(12)

**Proof:** Combining the results of Lemma 1 and Lemma 2 and the upper bound of \( \rho(\hat{\mathbf{K}},M) \), we can arrive at the upper bound in (12).

Let \( \rho_j = \frac{\| \mathbf{h}_j \|^2 + (M-1)\| \mathbf{h}_j \| + (M-1)^2 \frac{1}{K+1} \sum_{k \in S_j} \| \mathbf{h}_k \|^2}{\| \mathbf{h}_j \|^2 + (M-1)\| \mathbf{h}_j \| + (M-1)^2 \frac{1}{K+1} \sum_{k \in S_j} \| \mathbf{h}_k \|^2} \). Then, the SINR expression in (4), on average when CAPS is applied, can be lower bounded (invoking Jensen’s Inequality) as
\[
\begin{align*}
\text{SINR}_j &\geq \frac{\beta^2 P_s \| \mathbf{h}_j \|^2 \| \mathbf{w}_j \|^2 \| \mathbf{h}_{j+k} \|^2}{\frac{2\beta^2 P_s}{K+1} \rho_j + \beta^2 (\| \mathbf{h}_j \|^2 + \frac{2(M-1)}{K+1}) + 1} \\
\text{SINR}_{j+k} &\geq \frac{\beta^2 P_s \| \mathbf{h}_j \|^2 \| \mathbf{w}_j \|^2 \| \mathbf{h}_{j+k} \|^2}{\frac{2\beta^2 P_s}{K+1} \rho_j + \beta^2 (\| \mathbf{h}_{j+k} \|^2 + \frac{2(M-1)}{K+1}) + 1}. \\
\end{align*}
\]

(13)

After some manipulations, \( E[\beta^2] = (P_r - M) / E\left[ T \left[ \mathbf{H}^T \mathbf{W} \right] \mathbf{W} \right] \approx (P_r - M)/(2M\| \mathbf{w}_j \|^2) \). Substituting this into (13) and assuming that \( K \) grows much faster than \( P_s P_r \), we can conclude that CAPS can improve user SINR and consequently the user throughput in average sense as long as the relay beam-former keeps the term \( \| \mathbf{h}_j \|^2 \| \mathbf{w}_j \|^2 \| \mathbf{h}_{j+k} \|^2 \) non-zero. Therefore, we can ignore the inter-pair interference terms for such case and have average SINR expressions as
\[
\begin{align*}
\text{SINR}_j &\approx \frac{\beta^2 P_s \| \mathbf{h}_j \|^2 \| \mathbf{w}_j \|^2 \| \mathbf{h}_{j+k} \|^2}{\beta^2 (\| \mathbf{h}_j \|^2 + 1)} \\
\text{SINR}_{j+k} &\approx \frac{\beta^2 P_s \| \mathbf{h}_j \|^2 \| \mathbf{w}_j \|^2 \| \mathbf{h}_{j+k} \|^2}{\beta^2 (\| \mathbf{h}_{j+k} \|^2 + 1)}. \\
\end{align*}
\]

(14)

Since (14) is satisfied for all \( M \) pairs, we can achieve 2M DoFs in two phases (DoF of \( M \) achieved). This does not mean that DoF of \( M \) can be achieved for practical values of \( K \). Though future cellular scenarios conceive massive numbers of wireless devices assisting a human user [26], [27], it is hardly imaginable that an access point manages the traffic of more than hundreds of user pairs. Therefore, the assumption of large \( K \) has only limited implications in practice.

**B. Modified CAPS**

We can modify the CAPS algorithm such that only \( \hat{M}(< M) \) two way pairs are selected by CAPS and arbitrary \( J = [(M - \hat{M})/2] \) two way pairs are additionally supported at the same time. In this way the total number of channels in the system is given as \( 2(M + J) \). If we consider the pair...
channel vectors selected by CAPS to take up only \(\tilde{M}\) spatial dimension, then the size of spatial dimensions taken by the total channel vectors at the relay is \(\tilde{M} + 2J (\leq M)\). The relay beam-former can handle the inter-pair interference. The relay beam-former is constructed as follows to suppress the inter-pair interference.

\[
G = [h_1, \ldots, h_J, h_{K+1}, \ldots, h_{K+J}, \hat{h}_{\varphi^{-1}(1)}, \ldots, \hat{h}_{\varphi^{-1}(\tilde{M})}] \\
\tilde{W}_r = \rho G^\dagger \\
W_r = \beta \tilde{W}_r H \tilde{W}_r. \tag{15}
\]

Here, \(h_j, h_{K+j}\) denote the channel vectors of an additionally supporting pair.

Note that the value \(\tilde{M}\) gives a set of alternatives for implementing CAPS. Also, this modification is applicable to SCAPS introduced in section IV. Thus, we are given a set of combination strategies to choose depending on SNR and the number of user pairs in the network. The numerical results will illustrate how the combinations affect the sum rate performance.

### C. Numerical Results

In Fig. 3 and Fig. 4 we plot the system sum rates of the opportunistic two-way multiuser channel with full CAPS (\(M = \tilde{M}\)) when \(M = 6\) and different values of fixed \(K\). The \(m\) in legends denotes the number of scheduled two way pairs (\(m = \tilde{M} + J\)). Here, the system sum rate is defined as

\[
R = \sum_{k=1}^{M} \frac{1}{2} \left[ \log_2 (1 + SINR_k) + \log_2 (1 + SINR_{k+K}) \right].
\]

The user terminals are assumed to be the same distance apart from the relay and use the same power (\(P_s\)) while the relay power is \(P_r\). For fixed \(K\), the curves saturate as SNR increases though the ceiling can be lifted as the \(K\) value grows. However, \(K\) can only take a finite value in reality and thus the system sum-rate ultimately saturates.

Plotted in Fig. 4 and Fig. 5 are system sum rates comparisons of full CAPS scheduling (\(M = \tilde{M} = 4\) in Fig. 4 and \(M = \tilde{M} = 6\) in Fig. 5) with those of modified CAPS scheduling cases when \(M = 2, J = 1\) (denoted as \(m = 3\) in the legend of Fig. 4) and the case when \(M = 2, J = 2\) (denoted as \(m = 4\) in the legend of Fig. 5), respectively. Also plotted are the no CAPS applied cases \(M = 0, J = 2\) (denoted as \(m = 2\) in the legend of Fig. 4) and the cases \(M = 0, J = 3\) (denoted as \(m = 3\) in the legend of Fig. 5), respectively.

Hardly seen from the figures, there exist regions in very low SNR, where CAPS outperforms modified CAPS slightly. Overall, the modified CAPS (\(m = 3\) in Fig. 4 and \(m = 4\) in Fig. 5) improves the performance of CAPS though the curves saturate at high SNRs as well. On the other hand, the curves of \(m = 2\) and \(m = 3\) without CAPS do not suffer from saturation.
and the sum rate performances exhibit \( M = 4 \) and \( M = 6 \) DoF, respectively. This is reasonable since allocating less channel aligned pairs avoids system overloading and improves the performance by reducing the ceiling effect. Therefore, it is tempting to conclude that CAPS is worthless with realistic \( K \) values though \( m = 4 \) cases in Fig. 5 show some improvement over the case \( m = 3 \) in the low SNR region. The SCAPS algorithm in section IV enhances CAPS to widen the system performance advantage in the low to mid SNR region.

IV. SEMI-ORTHOGONAL CHANNEL ALIGNED PAIR SCHEDULING

A. The Algorithm

The multiuser diversity in the cellular networks has been used to schedule users with semi-orthogonal channel vectors to enhance the system throughput [19]. With CAPS algorithm, we can align the pair channels so that the degree of freedom of the multiuser two-way relay network is enhanced. The semi-orthogonal channel aligned pair scheduling (SCAPS), introduced in this section, provides additional system throughput gain over the CAPS algorithm. It first selects a set of user pairs whose pair channel alignments are greater than a threshold. Then, it sequentially chooses one pair by one pair, the minimum magnitude of the pair channel vectors after the orthogonal projection onto the space formed by the set already selected pair channels is the strongest. The SCAPS algorithm works as follows.

- **Step 0**: Take a small values \( \epsilon \) \((0 < \epsilon \ll 1)\) and take an empty set \( S = \phi \).
- **Step 1**: for \( k = 1 \) to \( K \), calculate \( \nu_k = \frac{\| h_k \| \| h_{k+K} \|}{\| h_k \| \| h_{k+K} \|} \).
- **Step 2**: Let \( S_0 \) be the set of user pairs with \( \nu_k \) greater than \( 1 - \epsilon \).
- **Step 3**: Set \( t = 1 \). Among the user pairs in \( S_0 \), pick up a pair \( k^* = \arg \max_{k \in S_0} \min(\| h_k \|, \| h_{k+K} \|) \). Set \( S = S \cup \{k^*\} \), \( S_0 = S_0 \setminus \{k^*\} \), \( \varphi(k^*) = t \) and \( H_S = [h_k, h_{k+K}] \). Set \( t = t + 1 \).
- **Step 4**: Among the user pairs in \( S_0 \), pick up a pair \( k^* = \arg \max_{k \in S_0} \min(\| h_k^o \|, \| h_{k+K}^o \|) \). Set \( S = S \cup \{k^*\} \), \( S_0 = S_0 \setminus \{k^*\} \), \( \varphi(k^*) = t \) and append \( h_k \) and \( h_{k+K} \) to the last two columns of \( H_S \) \((2t-1)\)-th and \(2t\)-th columns of \( H_S \)). Set \( t = t + 1 \) and repeat Step 4 while \( t \leq M \).
- **Step 5**: Set \( G = [\hat{h}_{k^*-1(1)}, \ldots, \hat{h}_{k^*-1(M)}] \) and find \( \hat{W}_r = \rho G^T \). Here, \( \rho \) is set to make \( \text{Tr}[\hat{W}_r^H \hat{W}_r] = M \).
- **Step 6**: Set \( W_r = \beta \hat{W}_r^H \hat{W}_r \) and find \( \beta \) using equation (5).

It is better to make \( \epsilon \) small to keep the channels of a pair well aligned, which forces the cardinality of the set \( S_0 \) to be small as well. On the other hand, \( |S_0| \) needs to be large enough to reap the benefit of semi-orthogonal channel scheduling. The expected value of \( |S_0| \) is given as \( K \varphi_r(\epsilon) \). Note that the modification of SCAPS similar to subsection III-B is possible.

B. Numerical Results

We notice in subsection III-C that the CAPS in networks of realistic user pair size provides only marginally enhanced performance in the low to mid SNR regimes. The SCAPS algorithm in this section is introduced to further enhance the system sum rate performance in the low to mid SNR region. In Fig. 6 and Fig. 7 the system sum rates of SCAPS are compared with those of CAPS for \( M = 4 \) and \( M = 6 \) respectively. In simulations, we control \( \epsilon \) so that \( 2M \) user pairs are selected for Step 2 of SCAPS algorithm. For both CAPS and SCAPS, we use the modified scheduling algorithms, where \( M = 2 \), \( J = 1 \) and \( M = 2 \), \( J = 2 \) pairs are scheduled, respectively. Hence, they are modified CAPS and modified SCAPS. It is shown that the modified SCAPS outperforms modified CAPS in the low to mid SNR regimes albeit it incurs a performance loss compared to modified CAPS in high SNRs.

In summary, noticeable amount of system sum rate gains over no CAPS case and modified CAPS case are achieved with sophisticated modified SCAPS even when the number of user
pairs $K$ is not large enough. In general for practical $K$, the modified SCAPS gives the best alternatives for the low to mid SNR region while no CAPS at all ($M = 0$) is beneficial at high SNR since it enjoys perfect removal of inter-pair interference. In practical systems, the scheduler can search over a set of different modifications of SCAPS (different combinations of $(M, J)$) until it meets the combination which results in the best sum-rate performance. We call this scheduling approach as adaptive SCAPS. Figure 8 shows the comparison of adaptive SCAPS ($m = 4$) and modified SCAPS when $M = 4$ and $K = 100$. We notice that the adaptive SCAPS exhibits an additional scheduling gain throughout the entire SNR region.

V. CONCLUSION

We show that the opportunistic multiuser diversity can be utilized to enhance the sum rate performance of the multiuser two-way AF relay channel. Simple zero-forcing based beamforming and efficient scheduling algorithm implemented at the multi antenna relay enhances the degree of freedom and the sum throughput of the system. To keep the SINR of user pairs relatively interference free in average sense when the number of user pairs becomes very large, we propose CAPS algorithm. When the number of user pairs grows proportionally to the system SNR, the DoF of $M$ per channel use can be achieved. The SCAPS algorithm, which not only aligns the pair channels but also forms the inter-pair channels semi-orthogonal, enhances the performance of CAPS in the low and mid SNR regime. In practice where the number of pairs is limited, CAPS and SCAPS overload the system with a large number of user pairs and suffers from saturation at high SNR. In the modified scheduling methods, we can schedule a mixed set of pairs, where a part of pairs are selected by (S)CAPS and the other part of pairs are chosen arbitrarily. Simulation results show that we provide a set of alternative approaches of (S)CAPS and modified (S)CAPS CAPS depending on the SNR and the number of user pairs in the network. Adaptation among the modifications of SCAPS provides an additional scheduling gain throughout the entire SNR region.

APPENDIX

APPENDIX A

PROOF THAT $\varphi(K, 2) \geq \varphi(K, M)$, $M \geq 3$

We show that $\varphi(K, 2) \geq \varphi(K, M)$, for $M \geq 3$ by deriving a different form of $E(\mu)$ and making an approximation of the term in a large $K$. Starting from (10), we have

$$E(\mu) = \int_{0}^{1} \mu f_{\mu}(\mu) d\mu = M(M - 1) \left(\frac{K}{M}\right) \times \int_{0}^{1} \mu(1 - \mu)^{M-2 + (M-1)^2} \left(1 - (1 - \mu)^{M-1}\right)^{K-M} d\mu. \quad (16)$$

In (16), let $x = (1 - \mu)^{M-1}$, then $\mu = 1 - x^{1/M}$ and $d\mu = (1/M)(1 - x^{1/M}) dx$. As a result, we can re-write the integral expression in (16) as

$$\int_{0}^{1} \mu(1 - \mu)^{M-2 + (M-1)^2} \left(1 - (1 - \mu)^{M-1}\right)^{K-M} d\mu = \frac{1}{(M - 1)} \int_{0}^{1} (1 - x^{1/M}) x^{M-1}(1 - x)^{K-M} d\mu. \quad (17)$$

Now, (17) can be re-written as the following two simple integral expressions

$$\frac{1}{(M - 1)} \left[ \int_{0}^{1} x^{M-1}(1 - x)^{K-M} d\mu - \int_{0}^{1} x^{M-1 + 1/M - 1}(1 - x)^{K-M} d\mu \right]. \quad (18)$$

With the help of [23] (3.191.11), the first and second inner integrals in (18) can be shown to be equal to the following closed-form expressions, respectively

$$\int_{0}^{1} x^{M-1}(1 - x)^{K-M} d\mu = B(K - M + 1, M) \quad (19)$$

for $K - M + 1 > 0$,

and

$$\int_{0}^{1} x^{M-1 + 1/M - 1}(1 - x)^{K-M} d\mu = B\left(K - M + 1 + \frac{1}{M - 1}, M\right) \quad (20)$$

for $K - M + \frac{1}{M - 1} + 1 > 0$,

where $B(\cdot, \cdot)$ is the beta function [22] (6.2).

After substituting (19) and (20) into (18) and some manipulations, the desired closed-form expression of (16) can be obtained as

$$\varphi(K, M) = \frac{1}{2} \left[ 1 - \frac{B\left(K - M + 1 + \frac{1}{M - 1}, M\right)}{B(K - M + 1, M)} \right]. \quad (21)$$
In [21], Stirling’s approximation gives the asymptotic formula $B(x, y) \sim \Gamma(y) x^{-y}$ when $x$ is very large and $y$ is fixed. Therefore, for a large $K$, we have

$$1 - \frac{B(K - M + 1 + \frac{1}{M-1}, M)}{B(K - M + 1, M)} \approx 1 - \left(\frac{K - M + 1}{K - M + 1 + \frac{1}{M-1}}\right)^M. \tag{22}$$

Now, for $K \gg M$, we also have the following

$$1 - \left(\frac{K - M + 1}{K - M + 1 + \frac{1}{M-1}}\right)^M \approx 1 - \left(\frac{K}{K + \frac{1}{M-1}}\right)^M. \tag{23}$$

Note that for a large $K$, the approximation expression, (23), decreases as $M (M \geq 2)$ increases. As a result, for a large value of $K (K \gg M)$, (21) has the upper bound when $M$ has the smallest value, $M = 2$, as

$$\varphi(K, M) \leq \varphi(K, 2) = \frac{1}{K + 1}. \tag{24}$$

### References

[1] B. Rankov and A. Wittenben, “Spectrally efficient protocols for half-duplex fading relay channels,” IEEE Select. Areas Commun., vol. 25, no. 2, pp. 379–389, Feb. 2007.

[2] T. Oechtering and H. Boche, “Bidirectional regenerative half-duplex relaying using relay selection,” IEEE Trans. Wireless Commun., vol. 7, no. 5, pp. 1819–1821, May 2008.

[3] R. Zhang, Y. Liang, C. Chai, and S. Cui, “Optimal beamforming for two-way multi-antenna relay channel with analogue network coding,” IEEE Select. Areas Commun., vol. 27, no. 5, pp. 699–712, Jun. 2009.

[4] Z. Zhao, Z. Ding, M. Peng, W. Wang, and K. Leung, “On the design of network coding for multiple two-way relaying channels,” IEEE Trans. Wireless Commun., vol. 10, no. 6, pp. 1820–1832, Jun. 2011.

[5] C. Esli and A. Wittenben, “One and two-way decode-and-forward relaying for wireless multiuser MIMO networks,” in IEEE Globecom, Dec. 2008, pp. 1–6.

[6] Z. Zhao, Z. Ding, M. Peng, W. Wang, and K. Leung, “A special case of multi-way relay channel: When beamforming is not applicable,” IEEE Trans. Wireless Commun., vol. 10, no. 7, pp. 2046–2051, Jul. 2011.

[7] C. Y. Leow, Z. Ding, K. K. Leung, and D. L. Goeckel, “On the study of analogue network coding for multi-pair bidirectional relay channels,” IEEE Trans. Wireless Commun., vol. 10, no. 2, pp. 670–681, Feb. 2011.

[8] M. Chen and A. Yener, “Multi-user two-way relaying: detection and interference management strategies,” IEEE Trans. Wireless Commun., vol. 8, no. 8, pp. 4296–4305, Aug. 2009.

[9] Z. Ding, I. Krikidis, J. Thompson, and K. Leung, “Physical layer network coding and precoding for the two-way relay channel in cellular systems,” IEEE Trans. on Signal Processing, vol. 59, no. 2, pp. 696–712, Jan. 2011.

[10] D. Hwang, S. Kim, and C. Park, “Channel aligned beamforming in two-way multi-pair decode-and-forward relay down-link channels,” IEEE Wireless Commun. Lett., vol. 1, no. 5, pp. 464–467, Oct. 2012.

[11] C. Esli and A. Wittenben, “Multis user MIMO two-way relaying for cellular communications,” in IEEE PI MRC, 2008, pp. 1–6.

[12] H. Yang, B. Jung, and J. Chun, “Zero-forcing based two-phase relaying with multiple mobile stations,” in Asilomar Conf., 2008, pp. 351–355.

[13] S. Toh and D. Slock, “A linear beamforming scheme for multi-user MIMO AF two-phase two-way relaying,” in IEEE PI MRC, 2009, pp. 1003–1007.

[14] E. Chiu and V. K. Lau, “Cellular multiuser two-way MIMO AF relaying via signal space alignment: Minimum weighted SINR maximization,” IEEE Trans. on Signal Processing, vol. 60, no. 9, pp. 4864–4873, Sep. 2012.

[15] J. Joung and A. H. Sayed, “Multi-user two-way amplify-and-forward relay processing and power control methods for beamforming systems,” IEEE Trans. on Signal Processing, vol. 58, no. 3, pp. 1833–1846, Mar. 2010.