Heavy Resonance Production in Ultrarelativistic Nuclear Collisions

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ABSTRACT

Are heavy quarks produced thermally or only by hard parton collisions? What is the probability that a produced heavy quark or antiquark is observed in a given resonance?

1. Introduction

In this talk, I discuss the problems of heavy quark production and of heavy resonance production, i.e., are heavy quarks produced thermally or only by hard parton collisions, and what is the probability that a produced heavy quark or antiquark is observed in a given resonance? My collaborators are Tanguy Alther at CERN, who has since died tragically in a climbing accident, and George Fai at Kent State University.

We use very simple ultrarelativistic nuclear scenarios, basically as proposed long ago by Bjorken. We assume that the collisions can be divided into the following periods:

1. hard parton collisions, in which essentially all of the final-state entropy is produced, for which the typical time scale is about 0.1 fm (we use the standard high energy conventions that $\hbar = c = k_B = 1$);
2. thermal equilibration, which ends after $0.2-0.3$ fm, and possibly even sooner;
3. expansion of thermally equilibrated gluon plasma (GP), or possibly quark gluon plasma (QGP);
4. phase transition, when the temperature $T \approx 1$ fm$^{-1}$;
5. resonance gas (RG) expansion; and
6. freezeout, after which the particles leave the hot matter and flow conveniently to the detectors.

Of course, it is possible that the process of thermal equilibration will coincide with some of the later equilibrium processes, but we believe that this will produce only small changes to most of our results. It is also possible that there is no phase transition.
but instead some fast but smooth change to the entropy and energy densities as a function of $T$, but again that should not produce large changes in our results.

We make the following approximations to model the dynamics. First, we assume that the hot matter is cylindrically symmetric, so

$$f(p_x, p_y, p_z) = f(p_T = \sqrt{p_x^2 + p_y^2}, p_z),$$

where $f$ is any distribution function. Second, we assume approximate boost-invariance as observed in proton collisions,

$$f(\tau, p_T, y) = f(\tau, p_T).$$

Finally, we assume that the hot matter does not expand transversely during the collision, so that the volume at proper time $\tau$ is

$$\frac{dV(\tau)}{dy} = \pi R^2 \tau,$$

where $R$ is the nuclear radius.

We also occasionally assume approximate entropy conservation,

$$\pi R^2 \tau s \simeq 3.6 \frac{dN}{dy},$$

where $s$ is the entropy density and $dN/dy$ is the number of produced particles per unit rapidity. In that case, we treat the hot matter as an ideal gas with zero chemical potential, so

$$s(T) = \sum_i \frac{g_i}{(2\pi)^3 T} \int d^3 p \frac{E_i(p) + k^2/3 E_i(p)}{e^{E_i(p)/T} \pm 1}.$$  

Combining eqs. 4 and 5, we then estimate $T$ as a function of $\tau$ for given $dN/dy$.

2. Heavy quark production

In this section I review the theory of heavy quark production. There have been three types of calculations of heavy quark production in ultrarelativistic heavy ion collisions. The initial production cross sections have been estimated from perturbative quantum chromodynamics (QCD), either by simply taking cross sections for proton collisions and scaling by $A^2$, where $A$ is the number of nucleons in the nucleus, or in a more sophisticated manner by using the nuclear (instead of the bare nucleon) parton structure functions. The results obtained with these two methods do not differ very much.

Thermal production cross sections have been calculated by taking the production matrix elements, convoluting with the four-volume element and the thermal distribution functions, and integrating over assumed collision histories. These thermal calculations have grown in sophistication with time, but the results have not
changed very much. Our major addition to these calculations has been the inclusion of heavy quark production through thermal gluon decay, which has been previously neglected but is the dominant term in the weak coupling limit, due to the anomalously large thermal gluon width. For typical values of the strong coupling constant probed in nuclear collisions, this new production term is comparable to those previously calculated, although it does not dominate.

Finally, there is the parton cascade model, which attempts to include both of the previous calculations by using a perturbative QCD cascade, in which interactions are more or less arbitrarily cut off at some lower momentum so that cross sections remain finite. This model seems to work reasonably well for strange (s) quark production, which at least seems to be correctly predicted for proton collisions and scales with projectile and target in a reasonable manner. Results for charm (c) and bottom (b) production are probably not reliable, since proton collision results are not reproduced and the cross sections scale as $A^{5/3}$ in proton-nucleus collisions, instead of $A$ as observed by experimenters.

In my opinion, the best density estimates and dominant quark production mechanisms for an Au+Au collision at RHIC energy ($\sqrt{s} = 200$ GeV/nucleon) are:

\begin{itemize}
  \item \textbf{s}: Pure thermal production gives $dN_s/dy \simeq 100$, the parton cascade model gives $dN_s/dy \simeq 50$, so I expect that $dN_s/dy \simeq 100$, mostly from thermal collisions.
  \item \textbf{c}: Thermal production gives $dN_c/dy \simeq 1$, perturbative QCD gives $dN_c/dy \simeq 1$, so I expect that $dN_c/dy \simeq 1$ with roughly equal production from hard parton collisions and thermal collisions.
  \item \textbf{b}: Thermal production gives $dN_b/dy < 10^{-3}$, perturbative QCD gives $dN_b/dy \simeq 0.02$, so I expect $dN_b/dy \simeq 0.02$ mostly from hard parton collisions.
\end{itemize}

3. Freezeout conditions

Once we have our heavy quarks, the next question is, “When do they freeze out?” We attempted to solve this problem with a simple model of heavy quark production and dynamics in an ultrarelativistic nuclear collision. We first assume that all heavy quark-antiquark pairs are produced at time $\tau = 0$; this is reasonable for $c$ and $b$ quarks even if they are produced thermally, since thermal production is also concentrated at early times, although it is not such a good assumption for $s$ quarks. We then assume that these quarks quickly thermalize, and that their subsequent trajectory is a random walk in a thermal bath, with collision frequencies taken from Pisarski, the bath temperature is estimated by assuming entropy conservation. Finally, we say that the quarks have frozen out when either (i) their mean free path is larger than the mean distance to leave the hot matter, or (ii) their random walk has taken them out of the hot matter.

This is of course complicated by the fact that we expect a phase transition to RG during the evolution of the hot matter. We take this into account by assuming that
the interaction cross section per unit entropy is approximately the same in the two phases, and that in the RG phase the quarks are contained in mesons which we model as $q\bar{q}$ pairs to estimate the mesonic mean free paths. We thus obtain different results by varying the transition temperature, $T_c$, and the ratio of the number of degrees of freedom in the two phases, $\nu$.

I show results in Table 1 for S+S collisions at SPS energy ($\sqrt{s} = 20$ GeV/nucleon), and for Au+Au collisions at SPS, RHIC, and LHC ($\sqrt{s} = 7$ TeV/nucleon) energies, averaged over $T_c = 150$ and 200 MeV, and over $\nu = 5$ and 10. Here $T_f$ is the freezeout temperature, $\tau_f$ is the proper time, $r_Q$ is the distance the quark has moved from its point of creation, and $n_f$ is the mean number of collisions before freezeout. This last is the most important number – when $n_f \simeq 1$ (S+S collisions at SPS), statistical recombination models should not be expected to work very well, while when $n_f \gg 1$ (Au+Au collisions at all energies considered) we expect that statistical models should describe heavy resonance data very well.

4. Statistical recombination

Our statistical recombination model is very similar to those used by recent authors. The main difference is that, in addition to ensuring that the quark and antiquark densities, respectively $\rho_Q$ and $\rho_{\bar{Q}}$, are conserved at freezeout, we ensure that the local quark-antiquark density, $\rho_{Q\bar{Q}}^{(2)}(x, x)$, is also conserved. This is not trivial, as typically $\rho_{Q\bar{Q}}^{(2)}(x, x) \gg \rho_Q\rho_{\bar{Q}}$. We thus include three chemical potentials:

| $A$  | $dN/dy$ | $Q$ | $T_f/T_c$ | $\tau_f$ (fm/c) | $r_Q$ (fm) | $n_f$ |
|------|---------|-----|-----------|-----------------|------------|-------|
| 32   | 85      | $s$ | 1.0c      | 2.0             | 2.5        | 1     |
|      |         | $c$ | 1.0c      | 3.6             | 2.5        | 1     |
|      |         | $b$ | 0.8c      | 8.3             | 2.7        | 1     |
| 197  | 1000    | $s$ | 0.9b      | 15              | 8.8        | 4     |
|      |         | $c$ | 0.9b      | 26              | 8.7        | 5     |
|      |         | $b$ | 0.7b      | 56              | 8.6        | 5     |
| 197  | 2000    | $s$ | 1.0w      | 21              | 9.3        | 7     |
|      |         | $c$ | 0.9w      | 37              | 9.3        | 8     |
|      |         | $b$ | 0.7w      | 78              | 9.3        | 10    |
| 197  | 3500    | $s$ | 1.0w      | 26              | 9.3        | 10    |
|      |         | $c$ | 1.0w      | 45              | 9.3        | 11    |
|      |         | $b$ | 0.8w      | 97              | 9.3        | 15    |

Table 1: Freezeout parameters, adapted from Ref. 2.
1. quark, $\mu_Q$;
2. antiquark, $\mu_{\bar{Q}} \neq -\mu_Q$, since the heavy quarks are not in chemical equilibrium;
3. pair, $\mu_{Q\bar{Q}}$.

The chemical potential for resonance $i$ is

$$\mu_i = \sum_Q \left( k_i^{(Q)} \mu_Q + k_i^{(\bar{Q})} \mu_{\bar{Q}} + k_i^{(Q\bar{Q})} \mu_{Q\bar{Q}} \right). \quad (6)$$

Here $k_i^{(Q)}$ ($k_i^{(\bar{Q})}$) is the number of quarks (antiquarks) of flavor $Q$, and $k_i^{(Q\bar{Q})}$ is the number of pairs (the smaller of $k_i^{(Q)}$ and $k_i^{(\bar{Q})}$). We include all confirmed meson and baryon resonances in our statistical recombination model.

We estimate the two-particle density (and thus $\mu_{Q\bar{Q}}$) from the freezeout conditions obtained from our simulation.

$$\frac{\rho_{Q\bar{Q}}^{(2)}}{\rho_Q} = \rho_{Q\bar{Q}} + \frac{3}{4\pi} \left( \frac{3}{5} \right)^{3/2} \left( \frac{3}{5} \right)^{3/2}. \quad (7)$$

Results are shown in Table 2, again averaged over $T_c$ and $\nu$. Although the corrections to $s$ resonance production from the pair chemical potential are less than order unity, the corrections to $c$ and $b$ resonance production are huge, giving orders of magnitude changes in fugacities. Thus, this pair chemical potential (or its equivalent) must be included to calculate $c$ and $b$ resonance production.

Finally, this recombination model can be used to gain insight into predicted phenomena such as suppression of $J/\psi$ resonance production in ultrarelativistic nuclear collision. Predictions for the fractions of $c\bar{c}$ and $b\bar{b}$ pairs that freeze out as $c\bar{c}$ and $b\bar{b}$ mesons are given in Tables 3 and 4. These quantities serve as surrogates for the $J/\psi$ and $\Upsilon$ suppression, as these resonances are the most common sources for observed $J/\psi$ and $\Upsilon$ resonances. It is worth noting that the heavy mesons are all predicted to freeze out in the RG in ultrarelativistic nuclear collisions, so their suppression probably depends most strongly on (and thus carries the most information about) their interactions with the RG, and not with the QGP; thus, their dynamics is only weakly dependent on the presence or absence of QGP. Otherwise, it is obvious from the strong dependences on $T_c$ and $\nu$ that the theoretical uncertainty in these calculations is huge, so much work remains to be done on the problem of how produced heavy quarks emerge as observed heavy resonances.

5. Conclusions

We have calculated heavy quark production; $s$ quarks are mostly produced thermally, $b$ quarks are mostly produced by hard parton collisions, while $c$ quarks are produced by both mechanisms. We have attempted to estimate when freezeout will occur for the various heavy quark species, and whether there are enough collisions
$$A \frac{dN}{dy} Q \frac{dN_Q}{dy} \rho_Q (\text{fm}^{-3}) \frac{\rho_Q^{(2)}}{\rho_Q}$$

| $A$ | $dN/dy$ | $Q$ | $dN_Q/dy$ | $\rho_Q$ (fm$^{-3}$) | $\rho_Q^{(2)}/\rho_Q$ |
|-----|---------|-----|-----------|---------------------|---------------------|
| 32  | 85      | $s$ | 5         | 0.06                | 1                   |
|     |         |     |           |                     |                     |
|     | c       |     | 0.03      | $2 \times 10^{-4}$  | 40                  |
|     | b       |     | 0.0003    | $9 \times 10^{-7}$  | 7000                |
| 197 | 1000    | $s$ | 25        | 0.01                | 1                   |
|     |         |     |           |                     |                     |
|     | c       |     | 0.3       | $8 \times 10^{-5}$  | 3                   |
|     | b       |     | 0.003     | $4 \times 10^{-7}$  | 500                 |
| 197 | 2000    | $s$ | 50        | 0.016               | 1                   |
|     |         |     |           |                     |                     |
|     | c       |     | 1         | $2 \times 10^{-4}$  | 2                   |
|     | b       |     | 0.02      | $2 \times 10^{-6}$  | 80                  |
| 197 | 3500    | $s$ | 250       | 0.06                | 1                   |
|     |         |     |           |                     |                     |
|     | c       |     | 5         | $7 \times 10^{-4}$  | 1                   |
|     | b       |     | 0.4       | $3 \times 10^{-5}$  | 6                   |

Table 2: Freezeout densities, adapted from Ref. 2.

$$T_c (\text{MeV}): \quad 150 \quad 200$$

| $A$ | $dN/dy$ | $\nu$: 5 | 10 | 5 | 10 |
|-----|---------|-----------|----|---|----|
| 32  | 85      | 0.066     | 0.075 | 0.029 | 0.021 |
| 197 | 1000    | 0.021     | 0.002 | 0.008 | 0.001 |
| 197 | 2000    | 0.009     | 0.004 | 0.004 | 0.001 |
| 197 | 3500    | 0.009     | 0.010 | 0.005 | 0.002 |

Table 3: $\pi$ meson fractions, from Ref. 2.

$$T_c (\text{MeV}): \quad 150 \quad 200$$

| $A$ | $dN/dy$ | $\nu$: 5 | 10 | 5 | 10 |
|-----|---------|-----------|---|---|---|
| 32  | 85      | 0.62      | 0.16 | 0.36 | 0.072 |
| 197 | 1000    | 0.50      | 0.053 | 0.20 | 0.016 |
| 197 | 2000    | 0.20      | 0.013 | 0.070 | 0.005 |
| 197 | 3500    | 0.075     | 0.008 | 0.027 | 0.002 |

Table 4: $b\bar{b}$ meson fractions, from Ref. 2.
that statistical recombination models should be reliable guides to heavy resonance production; the latter seems to be true for Au+Au collisions at SPS energy and above, but not necessarily for S+S collisions at SPS. We found that heavy quarks typically freeze out from the RG, and not from the QGP, so the relative numbers of the various heavy resonances are only weakly dependent on the presence or absence of QGP. Finally, we found that it is necessary to include a pair chemical potential in statistical recombination models, or to otherwise allow for the fact that heavy quarks and antiquarks are always produced in coincidence.

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6. References

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