The Location and Temperature of Event Horizon for General Black Hole via the Method of Damour-Ruffini-Zhao

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Abstract

In this paper, we investigate the general case of black hole at horizon by the method of Damour-Ruffini-Zhao. The proof of identification of the location of horizons determined both by Damour-Ruffini-Zhao’s (D-R-Z) method and by equation of null super-surface is given. The formula of temperature on the horizon for general black holes is obtained and is successfully checked by a variety of models of black holes.

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1 Introduction: The Schemes of Damour-Ruffini-Zhao

In 1976, Damour and Ruffini introduced an method to investigate the Hawking radiation[1]. This method prove the thermal radiation of black hole by the relativistic quantum mechanics on the background of curved space-time instead of using the quantization of field. The proof neither require the thermal balance between the black hole and the outside nor require the collapse of black hole. Therefore the method is valid to all the event horizon. The method was improved by Sannan[2] in

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1988. Zhao et al. introduce the $r_H = \xi$ and $\kappa$ as unknown parameters to generalized tortoise coordinates. The requirement that the Klein-Gordon equation in such tortoise coordinate has the form of standard wave equation does determined the event horizon $r_H$ and the temperature $T = \frac{\kappa}{2\pi r_H}$ simultaneously. Zhao et al. First use it to stationary black holes, later they generalized the method to dynamic black holes and successfully achieve plenty of results.

In this section Schwarzschild black hole will be used as a simple example to show the skeleton of Damour-Ruffini-Zhao’s schemes.

The Schwarzschild metric is

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{2M}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$ (1)

$$= (1 - \frac{2M}{r})(-dt^2 + dr_*^2) + r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$ (2)

where the tortoise coordinate $r_*$ is defined as

$$r_* = r + 2M\ln\left(\frac{r - 2M}{2M}\right) \quad (3)$$

The Klein-Gordon equation

$$(\Box - \mu^2)\Phi = 0, \quad \Box = \nabla^\mu \nabla_\mu = g^\mu\nu \partial_\mu \partial_\nu$$

or

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) - \mu^2 \Phi = 0.$$ (4)

$$i.e.$$

$$g^{\mu\nu} \partial_\mu \partial_\nu \Phi + \frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} g^{\mu\nu} \partial_\mu \Phi) - \mu^2 \Phi = 0.$$ (5)

Since the Klein-Gordon equation is invariant under coordinate transformation, in the tortoise coordinate system, multiplying Eq.(5) by $(g^{1+1*})^{-1}$ whose inverse is defined in Eq.(2), ones obtain

$$\frac{g^{00}}{g^{1+1*}} \frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial^2 \Phi}{\partial r_*^2} + \frac{1}{g^{1+1*}}\left\{g^{22} \frac{\partial^2 \Phi}{\partial \theta^2} + g^{33} \frac{\partial^2 \Phi}{\partial \varphi^2} + \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) - \mu^2 \Phi\right\} = 0.$$ (6)

From (2) it is clear to see $\frac{g^{00}}{g^{1+1*}} = -1$ and $g^{1+1*} \rightarrow \infty$, $g^{22}$, $g^{33}$ and $\partial_\mu (\sqrt{-g} g^{\mu\nu})$ do not approach $\infty$ when $r \rightarrow 2M$, So the the Klein-Gordon equation near the horizon is obtained as the wave equation

$$-\frac{\partial^2 \Phi}{\partial t^2} + \frac{\partial^2 \Phi}{\partial r_*^2} = 0.$$ (7)

This equation determines the radial solution $R(r, t)$ of $\Phi$. 

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The incoming and outgoing wave solutions of (8) are respectively

\[ R_{in} = e^{-i\omega(t+r_*)}, \]
\[ R_{out} = e^{-i\omega(t-r_*)}. \]

The Eddington-Finkelstein coordinates is

\[ v = t + r_*, \]

with which the Eqs.(9)(10) are rewritten as

\[ R_{in} = e^{-i\omega v}, \]
\[ R_{out} = e^{2i\omega r_*}e^{-i\omega v}. \]

It is clear that on the horizon \( r = 2M \), \( R_{in} \) is analytical and \( R_{out} \) is logarithmically singular. Following Damour and Ruffi[1] and Sannan[2], ones obtain the spectral distribution of the outgoing wave

\[ N_\omega = \Gamma_\omega e^{\omega/K_B T} \pm 1, \]
\[ T = \frac{\kappa}{2\pi K_B}, \quad \kappa = \frac{1}{4M}, \]

where the upper sign + is for fermions and the low sign − is for bosons. \( \Gamma_\omega \) is the transformation coefficient caused by the potential barrier in the exterior gravitational field( \( 2M < r < \infty \)). \( K_B \) is the Boltzmann constant. It is evidently thermal radiation whose temperature is shown by Eq. (15).

In Eddington coordinates the metric has the form

\[ ds^2 = -(1 - \frac{2M}{r})dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2). \]

The wave equation on the horizon has the standard form

\[ \frac{\partial^2 \Phi}{\partial r_*^2} + 2\frac{\partial^2 \Phi}{\partial v \partial r_*} = 0. \]

The outgoing solution is singular on the horizon. Via analytic continuation similar with that in the general coordinates, the same result of thermal radiation temperature is obtained.

Recently Zhao et al. brought forward a simple method to determine the event horizon and the temperature quickly and precisely. The main idea is as follows. Generalize the tortoise coordinates transformation to be

\[ r_* = r + \frac{1}{2\kappa}ln(r - \xi), \]
where the parameters $\xi$ and $\kappa$ will be determined in the next step. The key step is that on the horizon $\xi$ in terms of tortoise coordinates the dynamic equation is required to have the standard form of the wave equation (8) or (17). This requirement can quickly determine both the horizon and the temperature simultaneously. This method is first used to investigate the stationary black holes, then Zhao et al. generalized to some models of dynamic black holes. Their results is consistent with the known ones. Whether or not for general black holes the horizon determined by this method is identified with that determined by the null surface equation $n_\mu n^\mu = 0$ naturally rises. The answer is formative and a proof will be completed in section two. In section three the formula of temperature for general black holes is given by Zhao’s method. The last section is some remarks.

2 the Equivalence of the Horizons Determined by Damour-Ruffini-Zhao’s Method and Equation of Null Supersurface

The Klein-Gordon equation in curved space is

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} g^{\mu\nu} \frac{\partial \Phi}{\partial x^\nu} \right) - \mu^2 \Phi = 0. \quad (19)$$

whose 2-order derivative (our interest) term is $g^{\mu\nu} \partial_\mu \partial_\nu \Phi$. Suppose the horizon sites at $x^1 = \xi(x^0, x^2, x^3)$. Suppose $g^{00} \propto \frac{1}{(x^1 - \xi)^m}$. Tortoise coordinates transformation is,

$$x^1_* = x^1 + \frac{n}{2\kappa} \ln(x^1 - \xi) \quad (20)$$

with other components invariant. So $\Phi(x^\mu) \mapsto \tilde{\Phi}(x^\mu_*)$, and

$$\partial_\mu = \frac{\partial x^\nu_*}{\partial x^\mu} \frac{\partial}{\partial x^\nu_*} := A^\nu_\mu \partial_{\nu_*} \quad (21)$$

from which and (20) ones obtain

$$A^1_\mu = \delta_{\mu1} \left( 1 + \frac{n}{\epsilon} \right) - \frac{n \xi_\mu'}{\epsilon}, \quad (22)$$

in which $\epsilon = 2\kappa(x^1 - \xi)$ and $\xi_\mu' = \frac{\partial \xi_\mu}{\partial x^\nu}$. The 2-order derivative term $g^{\mu\nu} \partial_\mu \partial_\nu \Phi$ becomes

$$g^{\mu\nu} A^\rho_* \partial_{\rho_*} (A^\lambda_* \partial_{\lambda_*} \Phi). \quad (23)$$

Then the 2-order derivative term in $x^\nu_*$ is
\[ g^{\mu
u} A_\mu^\lambda A_\nu^\kappa \partial_\rho \partial_\lambda \Phi. \]  
(24)

Finally from (22) and (24) one gets the coefficient \( c_{11} \) of the term \( \partial_1 \partial_1 \Phi \)

\[ g^{\mu
u} A_\mu^1 A_\nu^1 = g^{\mu
u} \left[ \delta_{\mu 1} (1 + \frac{n}{\epsilon}) - \frac{n \xi'_\nu}{\epsilon} \right] \left[ \delta_{\nu 1} (1 + \frac{n}{\epsilon}) - \frac{n \xi'_\mu}{\epsilon} \right] 
= \frac{n}{\epsilon} \left[ g^{11} - 2g^{1\nu} \xi'_\nu + g^{\mu\nu} \xi'_\mu \xi'_\nu \right] + 2g^{11} - 2g^{1\nu} \xi'_\nu + \frac{\epsilon}{n} g^{11} \]  
(25)

Similarly the coefficient \( c_{00} \) of the \( \partial_0 \partial_0 \Phi \) is \((-g^{00})\). Therefore the Klein-Gordon equation has the form

\[ c_{11} \partial_1 \partial_1 \Phi - (-g^{00}) \partial_0 \partial_0 \Phi + (other \ terms) = 0 \]  
(27)

Multiplies (27) by \( \frac{1}{(-g^{00})} \), one obtains

\[ \frac{c_{11}}{(-g^{00})} \partial_0 \partial_0 \Phi - \partial_0 \partial_0 \Phi + (other \ terms) = 0, \]  
(28)

in which \( c_{11} \) is defined in (25).

On the horizon \( x^1 \mapsto \xi, \epsilon \mapsto 0 \), therefore in order that in the Eq. (28) the coefficient of \( \partial_0 \partial_0 \Phi \) be well defined when \( x^1 \mapsto \xi \), one must has

\[ g^{11} - 2g^{1\nu} \xi'_\nu + g^{\mu\nu} \xi'_\mu \xi'_\nu = 0, \]  
(29)

or

\[ \frac{1}{-g^{00}} \propto \epsilon^2 \mapsto 0. \]  
(30)

We now prove that when \( x^1 \mapsto \xi \) Eq. (29) is just the equation of zero-supersurface. Because the horizon is at \( x^1 = \xi, \frac{\partial F}{\partial x^1} \neq 0 \). Multiplying both sides of (29) with \( \frac{\partial F}{\partial x^1} \frac{\partial F}{\partial x^1} \),

\[ g^{11} \frac{\partial F}{\partial x^1} \frac{\partial F}{\partial x^1} - 2g^{1\nu} \xi'_\nu \frac{\partial F}{\partial x^1} \frac{\partial F}{\partial x^1} + g^{\mu\nu} \xi'_\mu \xi'_\nu \frac{\partial F}{\partial x^1} \frac{\partial F}{\partial x^1} = 0 \]  
(31)

is obtained.

The zero surface is

\[ F(x^0, x^1, x^2, x^3) = 0 \]  
(32)

whose solution can be written formally

\[ x^1 = x^1(x^0, x^2, x^3) \]  
(33)
The $x^1$ in the above expression is just the location of horizon $\xi$. Substituting Eq.(33) into Eq.(32), one obtains

$$F(x^0, \xi(x^0, x^2, x^3), x^2, x^3) = 0,$$  

(34)

from which ones derive

$$\frac{\partial F}{\partial x^\mu} + \frac{\partial F}{\partial x^1} \frac{\partial \xi}{\partial x^\mu} = 0.$$  

(35)

Using (35) ones obtain

$$g_{\mu\nu} \frac{\partial F}{\partial x^\mu} \frac{\partial F}{\partial x^\nu} = 0,$$  

(36)

which is just the equation $n_\mu n^\mu = 0$ of zero-supersurface.[8]

This result perhaps indicates that the D-R-Z’s scheme contains something essential.

3 Formula of Temperature of General Black Hole

3.1 In General Coordinate

Followed the method presented by D-R and developed by Zhao et al., to determine the temperature, the coefficient of (28) is required equal to 1

$$n = \frac{n}{-g^{00} \epsilon} \{ n [g^{11} - 2g^{1\nu} \xi'_\nu + g^{\mu\nu} \xi'_\mu \xi'_\nu] + 2g^{11} - 2g^{1\nu} \xi'_\nu + \frac{\epsilon}{n} g^{11} \} = 1.$$  

(37)

For $n = 1$, $g^{00} \epsilon \propto 1$, the term $\frac{1}{\epsilon}[g^{11} - 2g^{1\nu} \xi'_\nu + g^{\mu\nu} \xi'_\mu \xi'_\nu]$ in Eq.(37) is of type $\epsilon_0$, which equals to

$$\frac{1}{2\kappa} \frac{\partial g^{11}}{\partial x^1} - 2 \frac{\partial g^{1\nu}}{\partial x^1} \xi'_\nu + \frac{g^{\mu\nu}}{\partial x^1} \xi'_\mu \xi'_\nu := \frac{D}{2\kappa}.$$  

(38)

Using Eq.(37) and noticing $\epsilon = 2\kappa(x^1 - \xi)$, ones obtain the resolution of $\kappa$

$$\kappa = \frac{D}{-2g^{11} + 2g^{1\nu} \xi'_\nu + \sqrt{(2g^{11} - 2g^{1\nu} \xi'_\nu)^2 - 4g^{00}(x^1 - \xi)D}},$$  

(39)

where $D$ is defined in (38). Making use of the type $\epsilon_0$

$$-g^{00}(x^1 - \xi) = \left(\frac{g^{00}}{\partial x^1}\right)^2.$$  

(40)
Then the result of $\kappa$ is obtained as

$$\kappa = \frac{D}{-(2g^{11} - 2g^{1\nu}\xi'_\nu) + \sqrt{(2g^{11} - 2g^{1\nu}\xi'_\nu)^2 + 4\frac{(g^{0\nu})^2}{\partial x^\nu} D}} \quad (41)$$

For $n = 2$, $g^{00} \propto (\epsilon)^{-1}$, Eq. (37) simlyfies to

$$\frac{n^2}{-g^{00}}[g^{11} - 2g^{1\nu}\xi'_\nu + g^{\mu\nu}\xi'_\mu \xi'_\nu] = 1,$$  \quad (42)

from which the solution of $\kappa$ is obtained as

$$\kappa = \frac{\partial g^{00}}{2(-g^{00})^{\frac{1}{2}}} \sqrt{g^{11} - 2g^{1\nu}\xi'_\nu + g^{\mu\nu}\xi'_\mu \xi'_\nu}. \quad (43)$$

We pointed out that when the metric is stationary, Eq.(41) and Eq.(43) both reduce to the result

$$\kappa = \frac{n}{2} \frac{\partial \sqrt{g^{11}(g^{00})^{-1}}}{\partial x^1} \bigg|_{x^1, \xi}.$$  \quad (44)

Similarly, analytical continuation on the horizon gives the temperature as

$$T = \frac{\kappa}{2\pi K_B}$$

with $\kappa$ defined in Eq.(44).

### 3.2 Eddington-Finkelstein coordinate

Since most complex black hole models are given in Eddington-Finkelstein coordinates, we now investigate $\kappa$ in Eddington-Finkelstein coordinates. Since Klein-Gordon equation is covariant in different coordinates, it keep the form as (13). The tortoise coordinate transformation is

$$x^*_1 = x^1 + \frac{1}{2\kappa} \ln(x^1 - \xi). \quad (45)$$

From (21)(24) the coefficient of $\frac{\partial}{\partial x^0} \frac{\partial}{\partial x^*_1}$ is obtained as

$$g^{\mu\nu} \frac{\partial x^0}{\partial x^\mu} \frac{\partial x^1}{\partial x^\nu} + g^{\mu\nu} \frac{\partial x^1}{\partial x^\mu} \frac{\partial x^0}{\partial x^\nu} = 2g^{0\nu} \left( \frac{\delta_{1\nu} - \xi'_\nu}{\epsilon} + \delta_{1\nu} \right). \quad (46)$$

Considering $n = 1$ in (23) the coefficient of $\frac{\partial}{\partial x^*_1} \frac{\partial}{\partial x^*_1}$ is

$$\frac{1}{\epsilon} \left[ \frac{1}{\epsilon} [g^{11} - 2g^{1\nu}\xi'_\nu + g^{\mu\nu}\xi'_\mu \xi'_\nu] + 2g^{11} - 2g^{1\nu}\xi'_\nu + \epsilon g^{11} \right]. \quad (47)$$

With Eq.(16) and Eq.(17), one have the Klein-Gordon equation on the horizon $x^1 = \xi$

$$A \frac{\partial^2}{\partial (x^*_1)^2} \Phi + 2 \frac{\partial}{\partial x^*_1} \frac{\partial}{\partial x^0} \Phi + \text{(other terms)} = 0, \quad (48)$$
where
\[
A = \frac{1}{\epsilon g^{00}(\delta_{\nu} - \epsilon) + \delta_{\nu}} \left\{ \frac{1}{\epsilon} \left[ g^{11} - 2g^{1\nu} \xi'_\nu + g^{\mu\nu} \xi'_\mu \xi'_\nu + 2g^{11} - 2g^{1\nu} \xi'_\nu + \epsilon g^{11} \right] \right\}
\]
\[
= \frac{1}{\gamma^{0\mu}g_{\nu\rho} \xi'_\rho} \left\{ \frac{1}{\epsilon} \left[ g^{11} - 2g^{1\nu} \xi'_\nu + g^{\mu\nu} \xi'_\mu \xi'_\nu + 2g^{11} - 2g^{1\nu} \xi'_\nu + \epsilon g^{11} \right] \right\}_{x^1 \to \xi}
\]

Obviously the well-definition of coefficient of $\frac{\partial^2}{\partial x^1^2}$ again requires the existence of condition Eq.(29) which determine the event horizon. Via the technique suggested by Zhao et al., the above expression being 1 brings $\kappa$ an unique value

\[
\kappa = \frac{1}{2} \left[ \frac{\partial g^{11}}{\partial x_1^1} - 2 \frac{\partial g^{1\nu}}{\partial x_1^1} \xi'_\nu + \frac{g^{\mu\nu}}{\partial x_1^1} \xi'_\mu \xi'_\nu \right].
\]

Eq.(50) is the expression of $\kappa$ for general metric in Eddington-Finkelstein coordinate. Since $g^{\mu\nu} \to -g^{\mu\nu}$ does not affect $\kappa$ in (50), it does not matter to choose $+++$ or $+--$ for metrics.

The analytical continuation gives temperature similar as before.

In the following of this section, we will use the result (50) to investigate various models of black holes.

example 1: vaidya black hole

The coordinate is $v, r, \theta, \varphi$

the metric is

\[
g_{\mu\nu} = \begin{bmatrix}
-(1 - \frac{2m(v)}{r}) & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & r^2 & 0 \\
0 & 0 & 0 & r^2 \sin^2 \theta
\end{bmatrix}
\]

and its inverse is

\[
g^{\mu\nu} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 1 - \frac{2m(v)}{r} & 0 & 0 \\
0 & 0 & r^{-2} & 0 \\
0 & 0 & 0 & r^{-2} \sin^{-2} \theta
\end{bmatrix}
\]

Putting (52) into (29),

\[
\xi = r_H = \frac{2m(v)}{1 - 2\xi'_0}.
\]

Putting (52) into (50),
\[ 2\kappa = \frac{\frac{\partial g^{11}}{\partial x^1}}{g^{10} - 2g^{11} + 2g^{10}\xi'_0} = \frac{1}{\xi} \]  

(54)

is obtained. We have used (53) to obtain (54).

**example 2: spherically charged black hole**

The metric is

\[
g_{\mu\nu} = \begin{bmatrix}
-(1 - \frac{2m}{r} + \frac{Q^2}{r^2}) & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & r^2 & 0 \\
0 & 0 & 0 & r^2 \sin^2 \theta
\end{bmatrix}
\]  

(55)

and its inverse is

\[
g^{\mu\nu} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 1 - \frac{2m}{r} + \frac{Q^2}{r^2} & 0 & 0 \\
0 & 0 & r^{-2} & 0 \\
0 & 0 & 0 & r^{-2} \sin^{-2} \theta
\end{bmatrix}
\]  

(56)

By similar calculations as that in the Vaidya BH, one obtains

\[
\xi = \frac{m \pm \sqrt{m^2 - Q^2(1 - 2\xi'_0)}}{1 - 2\xi'_0}.
\]  

(57)

\[
\kappa = \frac{m - \frac{Q^2}{\xi}}{2m\xi - Q^2}.
\]  

(58)

**example 3: Dynamically Kerr black hole**

the coordinates are \(v, r, \theta, \varphi\)

The metric is
\[
g_{\mu\nu} = \begin{bmatrix}
-(1 - \frac{2mr}{\rho^2}) & 1 & 0 & -2mra \sin^2 \theta \\
1 & 0 & 0 & -a \sin^2 \theta \\
0 & 0 & \rho^2 & 0 \\
-2mra \sin^2 \theta & -a \sin^2 \theta & 0 & [r^2 + a^2 + \frac{2mra^2 \sin^2 \theta}{\rho^2}] \sin^2 \theta
\end{bmatrix}
\]  
(59)

and its inverse is

\[
g^{\mu\nu} = \frac{1}{\rho^2} \begin{bmatrix}
a^2 \sin^2 \theta & r^2 + a^2 & 0 & a \\
r^2 + a^2 & \Delta & 0 & a \\
0 & 0 & 1 & 0 \\
a & a & 0 & \frac{1}{\sin^2 \theta}
\end{bmatrix},
\]  
(60)

where \( \rho^2 = r^2 + a^2 \cos^2 \theta \), \( \Delta = r^2 + a^2 - 2mr \).

Putting (60) into Eq. (29), the equation to determine horizon is obtained as

\[
r^2(1 - 2\xi'_0) - 2mr + a^2[1 - 2\xi'_0 + (\xi'_0)^2 \sin^2 \theta] + (\xi'_2)^2 = 0.
\]  
(61)

Using Eq. (54), ones get

\[
\kappa = \frac{1}{2} \frac{\partial g^{11}}{\partial r} - 2 \frac{\partial g^{10}}{\partial r} \xi'_0 + \frac{\partial g^{00}}{\partial r} \xi'_0 \xi'_0 + \frac{\partial g^{22}}{\partial r} \xi'_0 \xi'_0.
\]  
(62)

Putting Eq. (60) into Eq. (62),

\[
\kappa = \frac{(1 - 2\xi'_0)\xi - m}{2m\xi - (1 - \xi'_0)\xi'_0 a^2 \sin^2 \theta + \xi'_0 \xi'_2}.
\]  
(63)

is obtained.

example 4: Black Hole with variable linear acceleration

the metric is

\[
g_{\mu\nu} = \begin{bmatrix}
-(1 - 2ar \cos \theta - r^2 f - \frac{2m}{r}) & 1 & r^2 f & 0 \\
1 & 0 & 0 & 0 \\
r^2 f & 0 & r^2 & 0 \\
0 & 0 & 0 & r^2 \sin^2 \theta
\end{bmatrix}
\]  
(64)
and its inverse is
\[
g^{\mu\nu} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 1 - 2ar\cos\theta - \frac{2m}{r} & -f & 0 \\
0 & -f & r^{-2} & 0 \\
0 & 0 & 0 & r^{-2}\sin^{-2}\theta
\end{bmatrix}.
\] (65)

The equation that determine \(\xi\) is
\[
2\xi' - (1 - 2a\xi\cos\theta - \frac{2m}{\xi}) - 2f\xi' - \frac{\xi'\xi'\xi'}{\xi\xi} = 0
\] (66)
and
\[
\kappa = \frac{1}{2\xi}\left[\frac{m}{\xi} - a\cos\theta - \frac{\xi'\xi'}{\xi'}\right].
\] (67)

4 Remarks

There are two points that should be emphasized. One is the location of event horizon determined by Eq.(29) is only the local horizon, i.e, it is the necessary condition to determine the event horizon. The consistence of horizon with the null surface equation indicates that Zhao’s method grasps some essence of the issue. The other is the other terms in Eqs.(27)(28)(48) is not calculated in detail. In some cases these terms may change the properties of the wave equation near the event horizon. But If the equation can be simplified to the form of wave equation, the results in this paper is obtained necessarily. Therefore we use the necessary condition rather than sufficient condition. The general formula for Dirac particle is excluded, that will be investigated in another paper.

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