Modelling diameter distributions of *Quercus suber* L. stands in “Los Alcornocales” Natural Park (Cádiz-Málaga, Spain) by using the two parameter Weibull functions

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Abstract

**Aim of study:** The aim of this work was to model diameter distributions of *Quercus suber* stands. The ultimate goal was to construct models enabling the development of more affordable forest inventory methods. This is the first study of this type on cork oak forests in the area.

**Area of study:** The area of study is “Los Alcornocales” Natural Park (Cádiz-Málaga, Spain).

**Material and methods:** The diameter distributions of 100 permanent plots were modelled with the two-parameter Weibull function. Distribution parameters were fitted with the non-linear regression, maximum likelihood, moment and percentile-based methods. Goodness of fit with the different methods was compared in terms of number of plots rejected by the Kolmogorov-Smirnov test, bias, mean square error and mean absolute error. The scale and shape parameters in the Weibull function were related to the stand variables by using the parameter prediction model.

**Main results:** The best fitting was obtained with the non-linear regression approach, using as initial values those obtained by maximum likelihood method, the percentage of rejections by the Kolmogorov-Smirnov test was 2% of the total number of cases. The scale parameter “b” was successfully modelled in terms of the quadratic mean diameter under cork ($R^2_{adj} = 0.99$). The shape parameter “c” was modelled by using maximum diameter, minimum diameter and plot elevation ($R^2_{adj} = 0.40$).

**Research highlights:** The proposed model diameter distribution can be a highly useful tool for the inventorying and management of cork oak forests.

**Key words:** maximum likelihood method; moment method; non linear regression approach; parameter prediction model; percentile method; scale parameter; shape parameter.

Introduction

Cork oak (*Quercus suber* L.) is one of the major forest species in Spain, where it occupies an overall 475,000 ha (24% of the surface it covers worldwide). Spain is the second world producer of cork (the main product of *Q. suber*), with 24% of the world output (Pereira, 2007). Cork oak stands additionally possess ecological, social and economic significance by virtue of their many uses, which include grazing, masting, browsing, hunting, erosion control, climate regulation, biodiversity maintenance, wildlife habitation, carbon storage, landscape enrichment and recreation.

Reconciling the large variety of uses of *Q. suber* stands makes management of these forests rather complex. Growth models are especially useful for educated decision-making in this context. A number of models have been developed to estimate cork weight for individual trees or stands in Spain (Vázquez-Piqué and Pereira, 2008). However, none of these models has so far analyzed diameter distributions in this species.

Most *Q. suber* woodlands are natural stands that have received no systematic, sustained silvicultural treatment. As a result, only some specific stands exhibit well-defined stand structure (even-aged stand, uneven-aged stand or two-aged stand) (Montero and López, 2008). Cork oak woodlands in the Aljibe massif are very heterogeneous, with large differences in tree spatial distribution, species, density, diameter distribution and stand vertical structure (Torres and Montero, 2000). The silvicultural practices of cork oak forests in Cortes de la Frontera (Málaga, Spain) have recently been revised in depth. As a result, a growing trend to developing carefully designed management practices for increasingly small masses has emerged. In this context forest stand...
management was chosen as a suitable method for these areas (De Benito, 2008).

The associated inventories should provide detailed information about the dasometric characteristics of each stand, determination of which requires heavy and expensive sampling. Palahí et al. (2006a) developed an effective method to reduce the complexity and cost of sampling for major species in Catalonia — *Q. suber* included—. The method involves measuring stand variables and using diameter distribution models in subsequent calculations. Other authors have also noted the usefulness of diameter distribution models for forest inventories (Kilkki et al., 1989; Maltamo, 1997; Liu et al., 2002; Palahí et al., 2006a).

Broadly speaking, an accurate knowledge of diameter distributions is highly useful to describing and analysing the structure of forest stands; also, it can be useful for estimating age distribution and assessing stand stability in addition to calculating the number of trees in each diameter class with a view to planning silvicultural treatments.

Diameter distributions have so far relied preferentially on the Weibull function on account of its flexibility, the ability to describe a wide range of unimodal distributions and the relatively easy estimation of its parameters (Rubin et al., 2006). We chose to use the two-parameter Weibull function on the grounds of the good modelling results previously obtained for diameter distributions in various species (e.g. Maltamo et al., 1995; Álvarez González, 1997; Maltamo, 1997; Nanos and Montero, 2002; Bullock and Burkhart, 2005; Palahí et al., 2006a, 2006b; Gorgoso et al., 2007).

The aim of this work was thus to model diameter distributions of *Quercus suber* stands in the Aljibe massif by using the two-parameter Weibull function, the parameters “*b*” and “*c*” were related to ecological and stand variables. Four different fitting methods were compared: percentiles, moments, maximum likelihood and non-linear regression. The ultimate goal was to construct models enabling the development of more affordable forest inventory methods and, as an aside, to improve existing models for cork production.

### Data acquisition

Data were obtained by measuring the 100 circular plots between 2008 and 2009. Plot size ranged from 378 to 5,418 m² depending on the particular stand density (in order to achieve a minimum of 30 trees per plot). A total of 3,226 trees were measured. The study area was stratified in accordance with Torres and Montero (2000), who split “Los Alcornocales” into two large formations: sub-sclerophyllous cork oak forests and sclerophyllous cork oak forests. The number of plots in each stratum was calculated in proportion to its surface area. Also, the plots in each formation were chosen in such a way as to ensure that their mean tree diameters would be evenly distributed throughout their potential range and representative of the site qualities and densities of the study area. Table 1 lists the number of plots sampled per stratum and mean diameter range.

The trees in each plot were all subjected to the following measurements: perimeter at breast height over cork, total height and cork thickness in two normal diameters. The local relief is hilly, with steep slopes, 23.5% of the park area having slopes exceeding 34.5% (Junta de Andalucía, 2004). The area has a Mediterranean climate comprising two phytoclimatic subtypes: IV₂ and IV₄ (Allué, 1990). The average annual temperature is 14-18°C and annual precipitation ranges from 700 to 1,200 mm. The most abundant substrate consists of Aljibe sandstones, which form soils of the Cambisol and Luvisol types mainly (FAO), and are accompanied by clays, marls and limestones in Vertisols. Cork oak stands constitute the most extensive plant formation in the area. *Quercus suber* occurs either in pure stands or mixed with *Q. canariensis* in shady zones and valley bottoms, as well as interspersed with trees of *Olea europaea* var. *sylvestris* in drier, transition zones.

### Material and methods

#### Study area

“Los Alcornocales” is a natural park spanning 167,767 ha in one of the largest cork producing areas in Spain.

#### Table 1. Number of plots sampled per stratum and mean diameter range

| *D*_₀₀ | Sub-sclerophyllous cork oak forest | Sclerophyllous cor oak forest |
|--------|-----------------------------------|-----------------------------|
| < 25   | 11                                | 4                           |
| 25–30  | 17                                | 5                           |
| 30–35  | 19                                | 4                           |
| 35–40  | 20                                | 2                           |
| > 40   | 17                                | 1                           |

*D*_₀₀: quadratic mean diameter under cork (cm).
rections. The geographic coordinates for the centre of each plot were measured with a GPS receiver and the position of each tree was georeferenced by measuring its direction and distance to the plot centre.

**Preliminary calculations**

Prior to analysis, data were subjected to the Grübbbs test in order to identify potential outliers (Dagnelie, 2006). Only those trees having an abnormal diameter and lying outside the main stand (i.e. those residual trees belonging to a pre-existing stand) were excluded (22 trees from 18 different plots in total). Torres (1995) previously developed cork production models for cork oak forests in the Aljibe massif where he also excluded residual trees and considered the main stand only.

Data were used to calculate the following stand variables: number of trees per hectare (N), mean diameter over and under cork ($D_{mo}$ and $D_{mu}$), quadratic mean diameter over and under cork ($D_{go}$ and $D_{gu}$), basal area ($G$), mean height ($H_m$) and dominant height ($H_o$) was calculated from the percentage of the 100 thickest trees per ha (Assmann, 1970). Calculations also included the following percentiles for the frequency distribution of diameters under cork: 1% ($P_{1}$), 5% ($P_{5}$), 10% ($P_{10}$), 16.731% ($P_{17}$), 25% ($P_{25}$), 50% ($P_{50}$), 75% ($P_{75}$), 90% ($P_{90}$), 95% ($P_{95}$), 97.366% ($P_{97}$) and 99% ($P_{99}$). Table 2 shows the mean values for the main stand variables in the sampled plots.

**Estimation of the target parameters**

The Weibull cumulative distribution function is obtained by integrating its probability density function. For a random variable $x$, the function is expressed as follows:

$$F(x) = \int_{0}^{\frac{x-a}{b}} \left(1-e^{-\left(\frac{x-a}{b}\right)^{c}}\right) dx = 1-e^{-\left(\frac{x-a}{b}\right)^{c}}$$

with $x \geq a$, $a \geq 0$, $b \geq 0$ and $c \geq 0$. In this equation, $F(x)$ denotes the cumulative relative frequency of trees with a diameter equal to or smaller than the random variable $x$; “$a$” is the location parameter that is assumed to be zero in the two-parameter version of the equation; “$b$” is the scale parameter; and “$c$” is the shape parameter.

Diameter distributions were fitted for diameters under cork. This variable was preferred to the diameter over cork as it is more representative of tree size. Cork age can vary between sample plots, so diameter over cork may vary widely depending on cork thickness.

Parameters were estimated and compared by using four different methods based on percentiles, moments, maximum likelihood and non-linear regression.

**Percentile method**

The method of percentiles was previously employed for a similar purpose by several authors (Zarnoch and Dell, 1985; Shiver, 1988; Nanang, 1998; García Güemes et al., 2002; Zhang et al., 2003; Gorgoso et al., 2007). The parameters in the Weibull function were calculated from the following equations:

$$\hat{a} = \frac{\ln(x_r) - \ln(-\ln(1-r))}{\ln(-\ln(1-t)) - \ln(-\ln(1-r))}$$

$$\hat{b} = \frac{\ln(-\ln(1-t))}{\ln(-\ln(1-t))}$$

$$\hat{c} = \frac{\ln\left(\frac{1}{x_r}\right)}{\ln\left(\frac{x_t}{x_r}\right)}$$

where $x_r$ and $x_t$ are the sample percentiles with $0 < r < 1$ and $0 < t < 1$. Following Dubey (1967), Bailey and Dell (1973) proposed using two different pairs of percentiles to estimate the parameters, namely: $P_{17}$ and $P_{97}$ or $P_{40}$ and $P_{82}$. Later on, Shiver (1988) proposed using the 16.731 and 97.366 percentiles.

**Table 2. Summary of main stand variables measured in the sampled plots**

| Variable               | Mean   | Maximum | Minimum | Standard deviation |
|------------------------|--------|---------|---------|--------------------|
| Density (trees/ha)     | 269.9  | 858.9   | 55.4    | 146.3              |
| Basal area (m²/ha)     | 24.5   | 50.7    | 5.6     | 7.7                |
| Quadratic mean diameter (cm) | 32.7   | 50.4    | 18.1    | 7.4                |
| Mean height (m)        | 9.5    | 15.6    | 5.6     | 2.1                |
| Dominant height (m)    | 10.7   | 16.8    | 6.8     | 2.1                |
**Moment method**

The moment method, which was previously used to estimate the parameters in the Weibull function by some authors (Shifley and Lentz, 1985; Shiver, 1988; Lindsay et al., 1996; Nanang, 1998; Rio and Montero, 2001; Zhang et al., 2003; Gove, 2003; Merganic and Sterba, 2006; Gorgoso et al., 2007), relies on first- and second-order sample moments for calculation. The expressions relating parameters “b” and “c” in the Weibull function to the sample mean and variance are as follows:

\[
\hat{b} = \frac{\bar{d}}{\Gamma\left(\frac{1}{c}\right)}
\]

\[
\sigma^2 = \frac{\bar{d}^2}{\Gamma^2\left(\frac{1}{c}\right) \Gamma\left(\frac{2}{c}\right)} - \left[\frac{\Gamma\left(\frac{1}{c}\right)}{\Gamma\left(\frac{1}{c}\right)}\right]^2
\]

where \(\bar{d}\) is the mean diameter of the distribution, \(\sigma^2\) the variance and \(\Gamma\) the gamma function. Substituting the calculated mean and variance into the previous expressions provides two equations in two unknowns that allow “b” and “c” to be readily estimated.

**Maximum likelihood method**

The maximum likelihood method has also been widely used to estimate the parameters in the Weibull distribution function (Maltamo et al., 1995; Maltamo, 1997; Álvarez González and Ruiz González, 1998; Maltamo et al., 2000; Nanos and Montero, 2002; Gove, 2003; Cao, 2004; Newton et al., 2005; Palahí et al., 2006a, 2006b; Gorgoso et al., 2007). This method allows parameters “b” and “c” to be calculated from the following equations:

\[
\hat{b} = \frac{1}{\bar{d}^2} \sum_{i=1}^{n} \left[ \ln(x_i) \right]^2 - \frac{1}{\hat{c}} = \frac{1}{\bar{d}^2} \sum_{i=1}^{n} \ln(x_i)
\]

\[
\hat{c} = \left[ \frac{1}{n} \sum_{i=1}^{n} \left( \ln(x_i) \right)^2 \right] \hat{b}
\]

where \(n\) is the number of observations and \(x_i\) the diameter for each individual tree in the sample.

**Non-linear regression method**

This method, which has been successfully used by authors such as Álvarez González (1997), García Güemes et al. (2002) and Gorgoso et al. (2007), requires appropriate initial values as input data. Such values can be obtained by using the maximum likelihood method (Álvarez González, 1997) or the percentile method (García Güemes et al., 2002; Gorgoso et al., 2007). In this work, parameter convergence was obtained with the Marquardt iterative algorithm (Marquardt, 1963) in SPSS, using the maximum likelihood values as input data.

**Goodness of fit**

Goodness of fit was assessed via the Kolmogorov-Smirnov test at the 20% significance level. Mean absolute error (MAE), mean square error (MSE) and bias were calculated, which were obtained from the following equations:

\[
MAE = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n}
\]

\[
MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}
\]

\[
Bias = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)}{n}
\]

where \(\hat{y}_i\) is the cumulative frequency obtained by applying the model to each plot datum and \(y_i\) is the cumulative frequency for each observation in each plot.

**Modelling distribution parameters in terms of stand variables**

Prior to modelling, the presence or absence of significant differences in Weibull parameters between the two strata (sclerophyllous an sub-sclerophyllous cork oak forest) was confirmed with an analysis of variance. The Pearson correlation coefficient between the Weibull function parameters obtained with the non-linear regression method and the following variables was calculated and analysed: plot elevation (\(Z\)); number of trees per hectare (\(N\)); basal area (\(G\)); mean diameter over and under cork (\(D_{mo}\) and \(D_{mu}\)), quadratic mean diameter over and under cork (\(D_{go}\) and \(D_{gu}\)); mean height (\(H_m\)); dominant height (\(H_o\)); the percentiles \(P_{10}\), \(P_{25}\), \(P_{50}\), \(P_{75}\), \(P_{90}\) and \(P_{95}\); maximum and minimum diameter (\(D_{max}\) and \(D_{min}\)); the ratios of \(P_{25}\) to the quadratic mean diameters over and under cork (\(P_{25}/D_{go}\) and \(P_{25}/D_{gu}\)), and their respective natural logarithms \([L(P_{25}/D_{go})]\) and \([L(P_{25}/D_{gu})]\).
**Parameter prediction method**

This method related the parameters in the Weibull function to stand variables via straightforward linear models. Parameter “$b$” was modelled by stepwise regression, using the input variables mentioned in the preceding paragraph. The shape parameter, “$c$”, was modelled in two different ways; one (Method 1) involved stepwise regression with the same input variables as with parameter “$b$” and the other (Method 2) relating “$c$” to the ratio of $P_{25}$ to the quadratic mean diameter, a procedure which provided good results in previous studies (Álvarez González, 1997; Gorgoso et al., 2007). In order to facilitate application of the resulting equation, a second model predicting $P_{25}$ in terms of stand variables was also fitted.

As with correlations, only the data for the plots not rejected by the Kolmogorov-Smirnov test were considered in the analysis.

**Validation of the model**

The ensuing model was verified by cross-validation. The fitting procedure was applied as many times as plots were studied, with omission of the data for one plot in each run. Calculations included the Kolmogorov-Smirnov test, MSE, MAE and bias between the observations for each plot and the predicted values provided by the model with exclusion of the plot concerned.

**Results**

**Estimation of the target parameters**

Table 3 shows the errors, bias and number of fits rejected by the Kolmogorov-Smirnov test in the fitting of “$b$” and “$c$” in the Weibull function with the percentile, moment, maximum likelihood and non-linear regression methods. Table 4 gives the means and standard deviations for the shape and scale parameters.

Based on the results of the Kolmogorov-Smirnov test, the non-linear regression method is the most suitable for fitting diameter distributions of *Q. suber* stands; in fact, only 2% of the fits were rejected. The moment and maximum likelihood methods provided worse results, which, however, were similar to each other. Finally, the percentile method was the worst performer, with 34% of rejections.

The non-linear regression method provided the lowest mean square error (MSE), mean absolute error (MAE) and bias; by contrast, the percentile method had the poorest results in these respects. The moment and maximum likelihood methods performed similarly except for a slightly higher bias with the former. Based on the foregoing, the non-linear regression method provided the best fits.

**Modelling distribution parameters in terms of stand variables**

An analysis of variance of the two parameters in the Weibull function revealed the absence of significant differences ($p > 0.05$) between the two strata (sclerophyllous and sub-sclerophyllous cork oak forest).

Table 5 shows the correlations found between the Weibull parameters as calculated with the non-linear regression method and stand variables. As can be seen, the highest correlation for parameter “$c$” was that with the ratio of $P_{25}$ to the quadratic mean diameter under

| Table 3. Mean values of bias, mean absolute error, mean square error (both in number of trees per one) and number of plots rejected by the Kolmogorov-Smirnov test ($\alpha = 0.20$) for the four fitting methods applied to the Weibull distribution function |
|---------------------------------|-------|-------|-------|-------|
| Fitting method                  | Bias  | MAE   | MSE   | KS    |
|---------------------------------|-------|-------|-------|-------|
| Percentile                      | 0.1187| 0.0635| 0.0075| 34    |
| Moment                          | 0.0308| 0.0481| 0.0039| 9     |
| Maximum likelihood              | 0.0256| 0.0469| 0.0036| 9     |
| Non-linear regression           | -0.0031| 0.0325| 0.0017| 2     |

MAE: mean absolute error. MSE: mean square error. KS: number of plots rejected by the Kolmogorov-Smirnov test.

| Table 4. Mean and standard deviation of the scale and shape parameters in the Weibull function |
|-------------------------------------------------------|------|------|
| Parameter     | Mean | Standard deviation |
|----------------|------|-------------------|
| $b$ (scale)    | 33.3 | 7.3               |
| $c$ (shape)    | 5.9  | 1.5               |
cork: in addition, “c” exhibited moderately high correlation ($p < 0.01$) with the diameter distribution percentiles from $P_1$ to $P_{50}$, and also with the minimum diameter. There was also significant correlation between “c” and plot elevation. On the other hand, the highest correlation for “b” was that with the quadratic mean diameter under cork. This parameter was additionally correlated, with a high significance, with all other variables except the basal area.

**Parameter prediction method**

Following fitting with the non-linear regression method, the parameters in the Weibull function were processed with the parameter prediction method. Stepwise regression of the data led to the following equation:

$$b = 1.02 + 0.257D_{gu} \left(R^2_{adj} = 0.99\right)$$

**Method 1**

Parameter “c” was modelled as a function of the maximum diameter, minimum diameter and plot elevation ($Z$). The ensuing equation was

$$c = 5.262 + 0.151D_{min} - 0.055D_{max} + 1.58 \times 10^{-6} Z^2 \left(R^2_{adj} = 0.40\right)$$

**Method 2**

The equation relating parameter “c” to the ratio of $P_{25}$ to the quadratic mean diameter was:

$$c = 10.463 \left(\frac{P_{25}}{D_{gu}}\right)^{1.074} \left(R^2_{adj} = 0.77\right)$$

Using this equation required fitting a second model capable of predicting $P_{25}$ in terms of stand variables:

$$P_{25} = 18.285 - 0.038N + 0.433G + 0.915H_m \left(R^2_{adj} = 0.789\right)$$

**Validation of the model**

Table 6 shows the validation results obtained with the two methods used. Method 1, which calculated “c” as a function of the maximum and minimum diameters, and plot elevation, provided better results in terms of bias, errors and number of rejections in the Kolmogorov-Smirnov test than did Method 2, which calculated “c” as a function of $P_{25}$ and the quadratic mean diameter. The parameter “b” was modelled as a function of the quadratic mean diameter under cork in Method 1 and 2. Fig. 1 shows the absolute histogram of observed

### Table 5.

| Variable | $c$   | $b$   |
|----------|-------|-------|
| Z        | 0.271*| 0.246*|
| N        | -0.086| -0.723**|
| G        | 0.118 | -0.023|
| $D_{mo}$ | 0.226*| 0.942**|
| $D_{mu}$ | 0.259*| 0.979**|
| $D_{go}$ | 0.210*| 0.970**|
| $D_{gu}$ | 0.235*| 0.996**|
| $H_m$    | 0.251*| 0.701**|
| $H_o$    | 0.247*| 0.627**|
| $P_1$    | 0.533**| 0.694**|
| $P_{10}$ | 0.542**| 0.808**|
| $P_{25}$ | 0.476**| 0.902**|
| $P_{50}$ | 0.436**| 0.968**|
| $P_{75}$ | 0.309**| 0.993**|
| $P_{95}$ | 0.156 | 0.993**|
| $P_{99}$ | 0.059 | 0.956**|
| $P_{90}$ | 0.016 | 0.922**|
| $P_{99}$ | 0.001 | 0.885**|
| $D_{max}$| 0.003 | 0.869**|
| $D_{min}$| 0.524**| 0.734**|
| $P_{25}/D_{gu}$| 0.843**| 0.734**|
| $P_{25}/D_{gu}$| 0.702**| 0.516**|
| $L(P_{25}/D_{gu})$| 0.823**| 0.203**|
| $L(P_{25}/D_{gu})$| 0.570**| 0.446**|

**Table 6.** Bias, mean absolute error, mean square error (both in number of trees per one) and number of plots rejected by the Kolmogorov-Smirnov test ($\alpha = 0.20$) for the two methods derived by cross-validation

| Method | Bias  | MAE   | MSE   | KS   |
|--------|-------|-------|-------|------|
| 1      | -0.0017 | 0.0469 | 0.0037 | 14   |
| 2      | -0.0116 | 0.0784 | 0.0126 | 33   |

MAE: mean absolute error. MSE: mean square error. KS: number of plots rejected by the Kolmogorov-Smirnov test.
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Discussion

Judging by the plot rejection results of the Kolmogorov-Smirnov test, errors and bias, the most accurate fitting method was non-linear regression using the values from the maximum likelihood method as input data. Non-linear regression was previously used to estimate the parameters in the Weibull distribution function and proved an advantageous choice over alternative methods (Álvarez González and Ruiz González, 1998; García Güemes et al., 2002; Gorgoso et al., 2007). Thus, the non-linear regression method has been found to provide better results than the maximum likelihood method in stands of Pinus pinaster (Álvarez González and Ruiz González, 1998); the percentile method in Pinus pinea (García Güemes et al., 2002), and both the percentile, moment and maximum likelihood methods in Betula alba (Gorgoso et al., 2007).

The regression relating the scale parameter with the quadratic mean diameter under cork accounted for a very high proportion of the variability in “b” ($R_{adj}^2 = 0.99$). These results are consistent with previously works as regards both the favourable impact of using the mean square diameter as input variable (Rennols et al., 1985; Kilki et al., 1989; Lejeune, 1994; Álvarez González, 1997; Torres Rojo et al., 2000; García Güemes et al., 2002; Gorgoso et al., 2007), and the high resulting correlation (Álvarez González, 1997; Torres Rojo et al., 2000; García Güemes et al., 2002; Gorgoso et al., 2007).

Modelling the shape parameter with Method 1 involved using the maximum diameter, minimum diameter and plot elevation as predictors; by contrast, modelling with Method 2 involved the quadratic mean diameter and $P_{25}$. Álvarez González (1997) used the same independent variables as in our Method 2 for Pinus pinaster, and so did Gorgoso et al. (2007) for Betula alba. As in these studies, our $P_{25}$ was explained by the mean tree height and number of trees per hectare; in our case, however, basal area was also influential. The low correlation for Quercus suber is comparable to that previously observed by other authors (Maltamo et al. 1995; García Güemes et al. 2002) and may have resulted from the special characteristics of cork oak forests in Los Arconocales Natural Park. In fact, such characteristics are highly variable, largely as a consequence of the absence of systematic, sustained use of appropriate silvicultural practices (Montero and López, 2008).

Although stand age was previously used as a variable to predict parameter “c” (Maltamo et al. 1995; Nanang 1998), incorporating it into our model for Quercus suber would have been rather complicated since the traditionally growth ring counting method has very often proved inaccurate for estimation age owing to their ill-definition in stripped cork oaks (Pereira, 2007).

A proportion of 95% of the plots had “c” values in excess of 3.6 (the mean was 5.9). The diameter distribution was mound-shaped and most of the plots exhibit negative asymmetry. These results are consistent with the general characteristics of cork oak forests in the study area, which consist of largely even-aged stands—but also, occasionally, two-aged stands—of highly variable age occurring together in each compartment. Low thinning and/or low competition generally cause negative asymmetry (García Güemes et al., 2002). Los Alcornocales is under very conservative silvicultural practices including highly restricted felling and targeted at the lower quality trees, which are usually the thinnest ones, in order to preserve those with better production characteristics. In this work, “c” was positively correlated with tree mean diameter; as a result, an increase in diameter was invariably accompanied by one in negative asymmetry. This is consistent with previous results of White and Harper (1970), who found overmature populations to frequently exhibit a negative bias in their distributions.

The input variables used to model parameter b, and those used to model parameter “c” (Method 2), were

![Figure 1. Absolute histogram of observed diameter distribution and distribution fitted by Method 1 of a plot with average results in terms of goodness of fit.](image-url)
those typically used in whole-stand models. These two equations were used to complete the cork yield tables developed until then (Montero and Cañellas, 1999) by including a new tool (viz. the diameter distribution, which provides a knowledge of the stand structure at different growth stages). However, using these equations in the cork production model entails updating it by incorporating the mean height and circumference under cork —instead of that over cork— as input variables.

Forest stand management in Cortes de la Frontera (Málaga) therefore seems to be a suitable choice. With this approach, the inventory should provide appropriate information about each individual stand. Application of traditional inventory methods leads to higher costs owing to small size of the stands. Using inventories with a reduced number of measured variables essentially relying on relascopic measurements would be more cost-effective. Thus, by measuring the number of trees per hectare and the basal area with a Bitterlich relascope, as well as the maximum and minimum diameter in each plot and the cork thickness of the mean tree, would allow the diameter distribution to be reproduced by the proposed models for parameters “$b$” and “$c$” (Method 1). Applying the models for these two parameters in Method 2 would entail measuring not only the basal area and number of trees per hectare, but also the height and cork thickness of the mean tree. These two methods would allow one to reproduce the diameter distribution in terms of rapid measurements.

It should be noted that, although measuring the number of trees per hectare and basal area with a Bitterlich relascope involves measuring diameters in a sample of trees in the plot and provides an approximation to the diameter distribution, the relative small number of trees measured would be inadequate to accurately reproduce the diameter distribution. Also, the number of trees per hectare could be determined ocularly classifying trees into broad diameter groups (Bitterlich, 1984). Thus, Palahí et al. (2006a) proposed cheaper forest inventory method based on relascopic measurements of the number of trees per hectare and basal area for the major forest species in Catalonia —cork oak included.

The significance of a knowledge of diameter distributions in a cork oak forests lies not only in its usefulness towards predicting cork production, but also in that of the information it provides about the structure of the stand, which is essential to assessing stability and choosing the most suitable management forest. Also, knowing the diameter distribution enables the application of available individual-tree models for cork oak. Thus, the growth model of Sánchez-González et al. (2006) would allow one to predict the future diameter distribution of a stand in terms of site index and stand density. Also, knowledge of diameter distribution would allow one to estimate cork production by using the model of Ribeiro y Tomé (2002), which includes the circumference under cork as input variable; however, this model should be used with caution since it was developed for cork oak forests in central-southern Portugal. Also applicable here would be the model of Montero et al. (1996), which, however, require using an equation relating the circumference to the stripping height as input variables.

In summary, the two-parameter Weibull function proved effective towards fitting diameter distributions of cork oak forests in Los Arconocales Natural Park. The non-linear regression method was more accurate than the moment, percentile and maximum likelihood-based methods. The independent variable providing the best fit of parameter “$b$” was the mean square diameter under cork, whereas those for parameter “$c$” (in terms of the Kolmogorov-Smirnov test) were the maximum plot diameter, minimum diameter and plot elevation. This work has substantially increased our knowledge of diameter distributions of cork oak forests in Los Arconocales Natural Park. In fact, this is the first study of this type on cork oak in the area. The proposed model diameter distribution can be a highly useful tool for the inventorying and management of cork oak forests.

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