Critical current in SFIFS junctions

A. A. Golubov‡, 1) M. Yu. Kupriyanov*, and Ya. V. Fominov‡‡

‡ Department of Applied Physics, University of Twente, 7500 AE, Enschede, The Netherlands
* Nuclear Physics Institute, Moscow State University, 119899 Moscow, Russia
‡ L. D. Landau Institute for Theoretical Physics RAS, 117940 Moscow, Russia

Submitted 15 January 2002

Quantitative theory of the Josephson effect in SFIFS junctions (S denotes bulk superconductor, F — metallic ferromagnet, I — insulating barrier) is presented in the dirty limit. Fully self-consistent numerical procedure is employed to solve the Usadel equations at arbitrary values of the F-layers thicknesses, magnetizations, and interface parameters. In the case of antiparallel ferromagnets’ magnetizations the effect of the critical current $I_c$ enhancement by the exchange field $H$ is observed, while in the case of parallel magnetizations the junction exhibits the transition to the $\pi$-state. In the limit of thin F layers, we study these peculiarities of the critical current analytically and explain them qualitatively; the scenario of the $0-\pi$ transition in our case differs from those studied before. The effect of switching between 0 and $\pi$ states by changing the F-layers’ mutual orientation is demonstrated.

PACS: 74.50.+r, 74.80.Dm, 75.30.Et

Josephson structures involving ferromagnets as weak link material are presently a subject of intensive study. The possibility of the so-called “$\pi$-state” (characterized by the negative sign of the critical current $I_c$) in SFS Josephson junctions was predicted theoretically [1–6, 8, 9, 10]. The first experimental observation of the crossover from 0- to $\pi$-state was reported by Ryazanov et al. [9] and explained in terms of temperature-dependent spatial oscillations of induced superconducting ordering in the diffusive F layer.

More recently a number of new phenomena were predicted in junctions with more than one magnetically ordered layer. First, the possibility of the critical current enhancement by the exchange field in SFIFS Josephson junctions with thin F layers and antiparallel magnetization directions was discussed in the regimes of small S layer thicknesses [10] and bulk S electrodes [11, 12]. Second, the crossover to the $\pi$-state was predicted in Ref. [11] for the parallel case even in the absence of the order parameter oscillations in thin F layers. Still, the physical explanation of these effects and accurate calculation of their magnitude have not been given so far. To make such estimates in the model with thin S electrodes, one must consider KO-1 type solutions [13] and take into account spatial variation of superconducting state in the SF bilayers; at the same time, in the bulk S case an approximate method was used in Ref. [11] beyond its applicability range [12]. This problem is of rather general nature, since one may expect from the previous knowledge (see, e.g., review [14]) that the supercurrent in a short weak link is $H$-independent.

The above intriguing scenario motivated us to attack the problem of the Josephson effect in SFIFS junctions by self-consistent solution of the Usadel equations for arbitrary thicknesses of the F layers, barrier transparencies and exchange field orientations. Below we show that the $0-\pi$ transition in the case of parallel $H$ orientation or enhancement of $I_c$ by $H$ in the antiparallel case with thin F layers occurs when the effective energy shift in the ferromagnets (due to the exchange field) becomes equal to a local value of effective energy gap induced into a F layer. Under this condition a peak in the local density of states (DoS) near the SF interfaces is shifted to zero energy. In the models with DoS of the BCS type this leads to logarithmic divergency of $I_c$ in antiparallel case at zero temperature, similarly to the well known Riedel singularity of ac supercurrent in SIS tunnel junctions at voltage $eV = 2\Delta$. We also describe the general numerical method to solve the problem self-consistently and apply it for quantitative description of the $0-\pi$ transition and $I_c$ enhancement in SFIFS junctions.

The model. We consider the structure of SFIFS type, where I is an insulating barrier of arbitrary strength. We assume that the S layers are bulk and that the dirty limit conditions are fulfilled in the S and F metals. Although our method is applicable in the general situation of different ferromagnets and superconductors, for simplicity below we illustrate our results in the case when equivalent S and F materials are used on both sides of the structure (although the directions of the exchange field in the two F layers may be different), both F layers have the thickness $d_F$, and the two SF interfaces have the same transparency. At the

1) e-mail: a.golubov@tn.utwente.nl
same time, we do not put any limitations on \( d_F \) and the transparency.

The Usadel functions \( G, F \) obey the normalization condition \( G_ω^2 + F_ω F_ω^* = 1 \), which allows the following parameterization in terms of the new function \( Φ \):

\[
G_ω = \frac{\tilde{ω}}{\sqrt{ω^2 + Φ_ω Φ_ω^*}}, \quad F_ω = \frac{Φ_ω}{\sqrt{ω^2 + Φ_ω Φ_ω^*}}.
\] (1)

The quantity \( \tilde{ω} = ω + iH \) corresponds to the general case when the exchange field \( H \) is present. However, in the \( S \) layers \( H = 0 \) and we have simply \( \tilde{ω} = ω \).

We choose the \( x \) axis perpendicular to the plane of the interfaces with the origin at the barrier \( I \). The Usadel equations \([13]\) in the \( S \) and \( F \) layers have the form

\[
ξ_S^2 \frac{πT_c}{ωG_S} \frac{∂}{∂x} \left[ G_S^2 \frac{∂}{∂x} Φ_S \right] - Φ_S = −Δ, \quad \frac{ξ_F^2}{ωG_F} \frac{∂}{∂x} \left[ G_F^2 \frac{∂}{∂x} Φ_F \right] - Φ_F = 0,
\] (2, 3)

where \( T_c \) is the critical temperature of the superconductors, \( Δ \) is the pair potential (which is nonzero only in the \( S \) layers), \( ω \) is the Matsubara frequency, and the coherence lengths \( ξ \) are related to the diffusion constants \( D \) as \( ξ_{S(F)} = \sqrt{D_{S(F)}}/2πT_c \). The pair potential satisfies the self-consistency equations

\[
Δ \ln \frac{T}{T_c} + \pi T \sum_ω \frac{Δ − G_S Φ_S |\text{sgn} ω|}{|ω|} = 0.
\] (4)

In the present paper we restrict ourselves to the cases of parallel and antiparallel orientations of the exchange fields \( H \) in the ferromagnets.

The boundary conditions at the SF interfaces \( x = ±d_F \) have the form \([14]\) (see Ref. \([14]\) for details)

\[
\frac{ξ_S^2 G_S^2}{ω} \frac{∂}{∂x} Φ_S = \frac{ξ_F^2 G_F^2}{ω} \frac{∂}{∂x} Φ_F, \quad ±γ_B \frac{ξ_F G_F}{ω} \frac{∂}{∂x} Φ_F = G_S \left( \frac{Φ_F}{ω} - \frac{Φ_S}{ω} \right),
\] (5, 6)

with \( γ_B = R_B A/ρ_F ξ_F \), \( γ = ρ_S ξ_S/ρ_F ξ_F \),

where \( R_B \) and \( A \) are the resistance and the area of the SF interfaces; \( ρ_{S(F)} \) is the resistivity of the \( S \) (\( F \)) layer.

At the I interface \( x = 0 \) the boundary conditions read

\[
\frac{G_{F1}^2}{ω_1} \frac{∂}{∂x} Φ_{F1} = \frac{G_{F2}^2}{ω_2} \frac{∂}{∂x} Φ_{F2}, \quad γ_{B,I} \frac{ξ_F G_F}{ω_1} \frac{∂}{∂x} Φ_{F1} = G_S \left( \frac{Φ_{F2}}{ω_2} - \frac{Φ_{F1}}{ω_1} \right),
\] (7, 8)

with \( γ_{B,I} = R_{B,I} A/ρ_F ξ_F \),

where the indices \( 1, 2 \) refer to the left and right hand side of the I interface, respectively.

In the bulk of the \( S \) electrodes we assume a uniform current-carrying superconducting state

\[
Φ(x = ±∞) = Δ_0 \exp \left( i(ϕ/2 + 2mν_s x) \right)/1 + 2D_S m ν_s^2/\sqrt{ω^2 + Δ_0^2},
\] (9)

where \( m \) is the electron’s mass, \( ν_s \) is the superfluid velocity, and \( ϕ \) is the phase difference across the junction.

The supercurrent density is constant across the system. In the \( F \) part it is given by the expression

\[
J = iπT \frac{2eρ}{ω} \sum_ω \frac{G^2(ω)}{ω^2} \left[ Φ_ω \frac{∂}{∂x} Φ_ω^* - Φ_ω^* \frac{∂}{∂x} Φ_ω \right],
\] (10)

while analogous formula for the \( S \) part is obtained if we substitute \( \tilde{ω} \to ω \). This expression, together with the boundary condition \([8]\) and the symmetry relation \( F(−ω; H) = F(ω; −H) \), yields the formula for the supercurrent across the I interface:

\[
I = \frac{πT}{eR_{B,I}} \sum_ω \text{Im} \left[ F_{F1}^* (-H_1) F_{F2} (H_2) \right]
\] (11)

[the functions \( F \) are related to \( Φ \) via Eq. \([1]\)].

The limit of small \( F \)-layer thickness: \( d_F \ll \min(ξ_π, \sqrt{D_F/2H}) \). Under the condition \( \gamma_B/γ \gg 1 \) we can neglect the suppression of superconductivity in the superconductors. We assume further that the transparency of the barrier \( I \) is small, \( γ_{B,I} \gg \max(1, γ_B) \), and the SF bilayers are decoupled (the exact criterion will be given below). In this case we can set \( ν_s = 0 \) and expand the solution of Eq. \([8]\) in the \( F \) layers up to the second order in small spatial gradients. Applying the boundary condition \([8]\), we obtain the solution in the form similar to that in SN bilayer \([13, 17]\):

\[
Φ_{F1,F2} = \frac{ω_1/I}{1 + γ_{BM}ω_1/2πT_c G_S} Δ_0 \exp(ℏϕ/2),
\] (12)

with \( γ_{BM} = γ_B d_F/ξ_F, \quad G_S = ω/√ω^2 + Δ_0^2 \).

Substituting Eq. \([12]\) into the expression for the supercurrent \([11]\) we obtain \( I(ϕ) = I_0 \sin ϕ \).

For the parallel orientation of the exchange fields, \( H_1 = H_2 = H \), the critical current is

\[
I_c(\varphi) = \frac{2πT}{eR_{B,I}} \sum_{Ω > 0} ω^2 \left( 1 − α + Ωγ_{BM} g_1 \right)^2 + 4ag_2.
\] (13)

where \( Ω = ω/π T_c, \quad δ = Δ_0/π T_c, \quad α = (hγ_{BM})^2, \quad h = H/π T_c, \quad g_1 = 2G_S + γ_{BM} Ω, \quad g_2 = (G_S + γ_{BM} Ω)^2 \).

For the antiparallel orientation, \( H_1 = −H_2 = H \), the critical current is given by

\[
I_c(\varphi) = \frac{2πT}{eR_{B,I}} \sum_{Ω > 0} ω^2 \sqrt{(1 − α + Ωγ_{BM} g_1)^2 + 4ag_2}.
\] (14)
At $h = 1/\gamma_{BM}$ and small $\Omega$ the expression under the sum in Eq. (14) behaves as $1/\Omega$, thus at low $T$ the critical current diverges logarithmically: $I_c^{(a)} \propto \ln(T_c/T)$. This effect was pointed out earlier in Refs. [10, 11].

The above results become physically transparent in the real energy $\varepsilon$ representation. Making analytical continuation in Eqs. (1), (12) by replacement $\omega \rightarrow -i\varepsilon$, we obtain the expression for the DoS per one spin projection (spin “up”) $N_F(\varepsilon) = \text{Re} \, G_F(\varepsilon)$ in the F layers

$$N_F(\varepsilon) = \left| \text{Re} \frac{\tilde{\varepsilon}}{\sqrt{\varepsilon^2 - \Delta_0^2}} \right|,$$

which demonstrates the energy renormalization due to the exchange field. Equation (13) yields $N_F(0) = \text{Re} \, (\gamma_{BM}h/\sqrt{(\gamma_{BM}h)^2 - 1})$, which shows that at $h = 1/\gamma_{BM}$ the singularity in the DoS is shifted to the Fermi level. Exactly at this value of $h$ the maximum of $I_c^{(a)}$ is achieved due to overlap of two $\varepsilon^{-1/2}$ singularities.

This leads to logarithmic divergency of the critical current (14) in the limit $T \rightarrow 0$, similarly to the well known Riedel singularity of nonstationary supercurrent in SIS tunnel junctions at voltage $eV = 2\Delta_0$, where the energy shift is due to the electric potential. At the same value of the exchange field $h = 1/\gamma_{BM}$ the critical current changes its sign (i.e., the crossover from 0 to $\pi$ contact occurs) for parallel magnetizations in the F layers [see Eq. (13)]. We emphasize that the scenario of the 0-π transition in our case differs from those studied before (18) for the case the phase does not change in either layer; instead, it jumps at the SF interfaces. This scenario is most clearly illustrated in the limit of large $H$ where Eqs. (1), (12) yield $F_F \propto -i\Delta \text{sgn} H$ whereas $F_S \propto \Delta$; thus the phase jumps by $\pi/2$ at each of the SF interfaces, providing the total π-shift between $F_F1(-H)$ and $F_F2(H)$ [it is the phase difference between these two functions that determines the supercurrent according to Eq. (11)].

The considered effects take place only for sufficiently low I-barrier transparency. Indeed, it follows from Eq. (12) that $G_F(\Omega) \propto 1/\sqrt{\Omega}$ for small $\Omega$ under condition $h = 1/\gamma_{BM}$. As a result, the boundary condition (8) yields that at

$$\Omega \leq \min \left( \frac{\xi_F}{d_F\gamma_{B,I}}, \frac{\gamma_B}{\gamma_{B,I}} \right)$$

the solutions (12) are not valid, since in this frequency range the effective transparency of the I interface (the parameter $G_{F1}G_{F2}/\gamma_{B,I}$ [19]) increases and the spatial gradients in the F layers become large (the limit of large gradients is called “the KO-1 case” [3, 14]). In this case the nongradient term in Eq. (6) can be neglected and the general solution of the Usadel equation in the F layers has the KO-1 form [13]:

$$\frac{\Phi}{\omega} = C - iM \arctan \left[ \frac{M(Bx + Q)}{1 - \eta} \right],$$

where $M = \sqrt{(\eta^2 - 1) - C^2}$, while $C, B, Q$ and $\eta$ are integration constants. From Eqs. (6), (17) it follows that the Green functions $G, F$ and hence the contribution to the critical current from these frequencies are $H$-independent. As a result, the barrier transparency parameter $\gamma_{B,I}$ provides the cutoff of the low-temperature logarithmic singularity of $I_c^{(a)}$ at $h = 1/\gamma_{BM}$ [see Eq. (13)]. According to Eq. (14), the critical current saturates at low temperature $T_s = T_c \min(|\xi_F/dF\gamma_{B,I}, \gamma_B/\gamma_{B,I})$. We note that any asymmetry in the SFIFS junction will also lead to the cutoff of $I_c^{(a)}$ divergency [19]. The above estimates are done for the case of low barrier transparency, $\xi_F/dF\gamma_{B,I} \ll 1$ and $\gamma_B/\gamma_{B,I} \ll 1$. The opposite regime of high transparency deserves separate study.

The general case. For arbitrary F-layer thicknesses and interface parameters the boundary problem (6)–(9) has been solved numerically using iterative procedure. Starting from trial values of the complex pair potentials $\Delta$ and the Green functions $G_{S,F}$ we solve the resulting linear equations and boundary conditions for functions $\Phi_{S,F}$. After that we recalculate $G_{S,F}$ and $\Delta$. Then we repeat the iterations until convergence is reached. The self-consistency of calculations is checked by the condition of conservation of the supercurrent [10] across the junction. We emphasize that our method is fully self-consistent: in particular, it includes the self-consistency over the superfluid velocity $v_s$, which is essential (contrary to the constriction case) in the quasi-one-dimensional geometry. The details of our numerical method will be presented elsewhere [19].

Figure 3 shows $I_c(H)$ dependencies calculated at $T = 0.05T_c$ from the numerical solution of the boundary problem (9), (10) for the fixed value of $\gamma_{BM} = 1$ and a set of different F-layers thicknesses and the SF interface parameters $\gamma$. The normal junction resistance is $R_N = R_{B,I} + 2R_B + 2p_{D,F}/A$. The curves $dF/\xi_F = 0$ are the limits of vanishing $dF/\xi_F$ ratio at fixed $\gamma_{BM}$ and are calculated from Eqs. (13), (14). For thin F layers the results depend only on the combination $\gamma_M = \gamma F/\xi_F$. The enhancement of $I_c$ and the crossover to the π-state are clearly seen for the antiparallel and parallel orientations, respectively. In accor-
A. A. Golubov, M. Yu. Kupriyanov, and Ya. V. Fominov

In Ref. [11] (see [12]), the thickness $d$ in the F layers since the solution Eq. (12) loses its validity. This is illustrated in Fig. 2 by increasing the thickness $d_F$ or $\gamma_M$. In particular, in the case of large $\gamma_M$ the enhancement is absent, in contrast to the statement in Ref. [11] (see [2]).

Influence of temperature and barrier transparency on the critical current anomaly is shown in Fig. 4. One can see that, in accordance with the above estimate, the cutoff of $I_c^{(a)}$ singularity is provided by finite temperature or barrier transparency. Namely, with the decrease of the barrier strength parameter $\gamma_{B,I}$ the peak magnitude starts to drop when the ratio $d_F\gamma_{B,I}/\xi_F$ becomes comparable to $T/T_c$. With further decrease of $d_F\gamma_{B,I}/\xi_F$ the singularity disappears, while the transition to the $\pi$-state shifts to large values of $H$.

Figure 3 demonstrates the DoS in the F layers for one spin projection, calculated numerically in the limit of small I-barrier transparency. At $H = 0$ we reproduce the well-known minigap existing in SN bilayer. At finite $H$ the gap shifts in energy (asymmetrically) and the peak in the DoS reaches zero energy at $h = 1/\gamma_{BM}$. One can see that even for a small value $\gamma_M = 0.05$ the peaks are rather broad, this is the reason why the singularity in $I_c^{(a)}$ is suppressed by $\gamma_M$ very rapidly.

In the practically interesting limit of finite F-layer thickness (see Fig. 5) the numerical calculations show monotonic suppression of $I_c$ with increase of the exchange field $H$ for antiparallel magnetizations of the F layers and the $0-\pi$ crossover for the parallel case. One can see from Fig. 6 that for given temperature and thickness of the F layers it is possible to find the value of the exchange field at which switching between parallel and antiparallel orientations will lead to switching of $I_c$ from nearly zero to a finite value (or to switching between 0 and $\pi$ states). This effect may be used for engineering cryoelectronic devices manipulating spin-polarized electrons.
The case of parallel F-layers magnetizations corresponds to the standard SFS junction where the 0–π transition is possible due to spatial oscillations of induced superconducting ordering in the F layer. The thermally induced 0–π crossover in SFS junction was observed in Ref. 1, where simple theory based on the linearized Usadel equations was also presented. Here we show such a crossover (see the inset in Fig. 1) from the fully self-consistent solution in the range of the exchange fields corresponding to that of Ref. 2. Comparison to the experimental data and more detailed results of our model will be given elsewhere 19.

In conclusion, we have presented a general method to solve the Usadel equations in SFIFS junctions self-consistently. Using our method, we have investigated theoretically the Josephson current in SFIFS and SFS junctions as a function of relative F-layers magnetizations, thicknesses and parameters of the S/F and F/F interfaces. We have identified the physical mechanisms of the critical current enhancement and of the 0–π transition in these junctions.

We acknowledge stimulating discussions with J. Aarts, N.M. Chcthelkatchev, K.B. Efetov, M.V. Feigel’man, V.V. Ryazanov, and M. Siegel. The research of M.Yu.K. was supported by the Russian Ministry for Industry and Technology. Ya.V.F. acknowledges financial support from the Russian Foundation for Basic Research (project 01-02-17759), and from Forschungszentrum Jülich (Landau Scholarship).

1. L.N. Bulaevskii, V.V. Kuzii, and A.A. Sobyanin, Pis’ma Zh. Eksp. Teor. Fiz. 25, 314 (1977) [JETP Lett. 25, 290 (1977)].
2. A.I. Buzdin, L.N. Bulaevskii, and S.V. Panyukov, Pis’ma Zh. Eksp. Teor. Fiz. 35, 147 (1982) [JETP Lett. 35, 178 (1982)].
3. A.I. Buzdin and M.Yu. Kupriyanov, Pis’ma Zh. Eksp. Teor. Fiz. 53, 308 (1991) [JETP Lett. 53, 321 (1991)].
4. A.I. Buzdin, B. Vujićić, and M.Yu. Kupriyanov, Zh. Eksp. Teor. Fiz. 101, 231 (1992) [Sov. Phys. JETP 74, 124 (1992)].
5. E.A. Koshina and V.N. Krivoruchko, Pis’ma Zh. Eksp. Teor. Fiz. 71, 182 (2000) [JETP Lett. 71, 123 (2000)].
6. L. Dobrosavljević-Grujić, R. Zikić, and Z. Radović, Physica C 331, 254 (2000).
7. N.M. Chcthelkatchev, W. Belzig, Yu.V. Nazarov, and C. Bruder, Pis’ma Zh. Eksp. Teor. Fiz. 74, 357 (2001) [JETP Lett. 74, 323 (2001)].
8. Yu.S. Barash and I.V. Bobkova, preprint cond-mat/0108206.
9. V.V. Ryazanov, V.A. Oboznov, A.Yu. Rusanov et al., Phys. Rev. Lett. 86, 2427 (2001); V.V. Ryazanov, V.A. Oboznov, A.V. Veretennikov et al., Usp. Fiz. Nauk (Suppl.) 171, 81 (2001).
10. F.S. Bergeret, A.F. Volkov, and K.B. Efetov, Phys. Rev. Lett. 86, 3140 (2001).
11. V.N. Krivoruchko and E.A. Koshina, Phys. Rev. B 63, 224515 (2001); 64, 172511 (2001).
12. In Ref. [1] there was an attempt to study the influence of the suppression parameter γM on the critical current in SFIFS junctions. The obtained Ic(H) dependencies only qualitatively show the effects discussed in Ref. [10] and in the present paper. To obtain quantitative accuracy in the small γM limit it is necessary to take into account self-consistently the corrections to the pair potential, as it was done in V.F. Lukichev, Fiz. Nizk. Temp. 10, 1219 (1984) [Sov. J. Low Temp. Phys. 10, 639 (1984)]. The calculation in the large γM limit in Ref. [1] is valid for T ≫ Tc/γM, while the effect of Ic(0) enhancement exists only at low T.
13. I.O. Kulik and A.N. Omelyanchuk, Fiz. Nizk. Temp. 3, 945 (1977) [Sov. J. Low Temp. Phys. 3, 459 (1977)].
14. K.K. Likharev, Rev. Mod. Phys. 51, 101 (1979).
15. K.D. Usadel, Phys. Rev. Lett. 25, 507 (1970).
16. M.Yu. Kupriyanov and V.F. Lukichev, Zh. Eksp. Teor. Fiz. 94, 139 (1988) [Sov. Phys. JETP 67, 1163 (1988)].
17. E.A. Koshina and V.N. Krivoruchko, Fiz. Nizk. Temp. 26, 157 (2000) [Low Temp. Phys. 26, 115 (2000)].
18. A.A. Golubov and M.Yu. Kupriyanov, Zh. Eksp. Teor. Fiz. 96, 1420 (1989) [Sov. Phys. JETP 69, 805 (1989)].
19. A.A. Golubov, M.Yu. Kupriyanov, and Ya.V. Fominov, in preparation.