Role of Tensor Force in Light Nuclei with Tensor-Optimized Shell Model

1 Introduction

The nucleon–nucleon ($NN$) interaction has strong tensor forces at long and intermediate distances caused by the pion exchange, which emerges large momentum transfer, and also strong central repulsions at short distance caused by the quark dynamics [1,2]. It is important to investigate the nuclear structure by treating these characteristics of the $NN$ interaction. There are two theoretical developments for this purpose. One is to find out that the strong tensor force is of intermediate range, and we are able to express the tensor correlation in a reasonable shell model space [3,4]. We name this method as tensor-optimized shell model (TOSM), in which the wave function is constructed in terms of the shell model basis states with full optimization of the two particle-two hole ($2p2h$) states. Here the spatial shrinkage of the particle states is essential to obtain the convergence of the tensor contribution involving high momentum components [5,6], and then we do not put any truncation to describe the particle states in TOSM. The other is the unitary correlation operator method (UCOM) to treat the short-range correlation [8]. We shall combine two methods, TOSM and UCOM, to describe nuclei using bare interaction and see how this new method works.

For $s$-shell nuclei, the validity of TOSM was confirmed using a few-body framework by taking only the $D$-wave component connected with the $S$-wave state directly via the tensor force [7]. They call this method as the tensor-optimized few-body model (TOFM). So far, we have obtained successful results using TOSM for the investigation of the tensor correlations in He and Li isotopes. In $^4$He, we have confirmed the selectivity of $(p_{1/2})^2(s_{1/2})^{-2}$ configuration in the $2p2h$ space with the $pn$ pair induced by the tensor force. This correlation is recognized as the deuteron-like state [3]. The specific $2p2h$ excitation plays a decisive role to reproduce the energy spectra of neutron-rich He and Li isotopes [9,10], such as the effect on the $p$-wave splitting energy in $^5$He [11], and the neutron halo formation in $^{11}$Li due to the breaking the neutron magic number $N = 8$ by the tensor and pairing correlations [12,13]. In this paper, we perform the systematic analyses of He and Li isotopes using TOSM+UCOM and discuss their structures focusing on the roles of the tensor force on the energies and configurations.
2 Tensor-Optimized Shell Model (TOSM)

We begin with a many-body Hamiltonian having the bare nucleon–nucleon interaction, AV8′ [1]. We explain the TOSM wave function $\Psi$ as

$$\Psi = \sum_{\nu} \sum_{i} A_{i} \left| 0_{p0h} \right\rangle_{\nu} + \sum_{k} A_{k} \left| 1_{plh} \right\rangle_{i} + \sum_{k} A_{k} \left| 2_{p2h} \right\rangle_{k}. \quad (1)$$

Here, $\left| 0_{p0h} \right\rangle$, $\left| 1_{plh} \right\rangle$ and $\left| 2_{p2h} \right\rangle$ are the $0p0h$, $1p1h$ and $2p2h$ states with various radial components for particle states $p$ which are distinguished by the labels $\nu$. The variational coefficients are given as $\{ A_i \}$. The hole states $h$ are described by the harmonic oscillator basis states in TOSM. To construct the $0p0h$ states in TOSM within the $0s+0p$ space of the hole states, we allow up to the two particle excitations from the $0s$ orbit to the $0p$ orbits due to the presence of the two-body interactions. From each configuration in the $0p0h$ states, up to two nucleons can be excited to the particle states to make the $1p1h$ and $2p2h$ states in TOSM. The orbits of particle states are taken as much as possible until we get the convergence of the solutions. In this treatment, there is no truncation of the particle states.

We use the Gaussian expansion method to describe the various radial basis states in the particle states [4]. In this method, we construct the ortho-normalized single-particle wave function for the orbit $a$ using a linear combination of Gaussian bases $\{ \phi_a \}$ with length parameter $b_{a,v}$.

$$\phi_a(r, b_{a,v}) = N_l(b_{a,v}) r^l e^{-(r/b_{a,v})^2} \left[ Y_l(\hat{r}), \chi_1^{\sigma}, \chi_2^{\tau} \right] \chi_{t_z}, \quad (2)$$

where $v$ is the index that distinguishes the bases with different lengths $b_{a,v}$. The quantum numbers $l$ and $j$ are the orbital and total angular momenta of the single-particle states, respectively, and $t_z$ is the isospin component. The normalization factor is denoted as $N_l(b_{a,v})$. We superpose the Gaussian bases with a sufficient number and having various length parameters, which are used in each configuration in TOSM. This description fully describes the radial components of the particle states of every configuration of TOSM independently. We eliminate the center-of-mass excitations using the projection technique of the lowest HO state for the center-of-mass motion.

We minimize the total energies by optimizing the length parameters of the bases for all the hole states and the particle states in each $J^\pi$ state of particular nucleus. The variation of the energy expectation value with respect to the total wave function $\Psi$ in Eq. (1), leads to the following equations:

$$\frac{\partial \langle \Psi | H - E | \Psi \rangle}{\partial b_{a,v}} = 0, \quad \frac{\partial \langle \Psi | H - E | \Psi \rangle}{\partial A_{k_i}} = 0 \quad \text{for} \quad i = 0, 1, 2. \quad (3)$$

The total energy is represented by $E$. We solve two variational equations in Eq. (3) in the following steps. First, fixing the length parameters $b_{a,v}$ and the partial waves of the basis states up to $L_{\text{max}}$, we solve the linear equation for $\{ A_{k_i} \}$ as an eigenvalue problem for $H$. We thereby obtain the eigenvalue $E$, which is functions of $\{ b_{a,v} \}$ and $L_{\text{max}}$. Next, we try to adopt various sets of the length parameters $\{ b_{a,v} \}$ and increase $L_{\text{max}}$ in order to find a better solution which minimizes the total energy $E$.

We use short-range part of UCOM [8] to include the short-range correlation in TOSM. In UCOM, the shift operator, which is unitary, is introduced for every nucleon pair in nuclei. This operator reduces the short-range amplitude of the relative pair wave functions. The amounts of shifts are determined variationally for four spin-isospin channels to minimize the total energy of the system. The detailed forms of shift operators and their parametrization are explained in our recent papers [4,9,10].

3 Results of TOSM+UCOM

3.1 $^4$He

We show first the results of $^4$He using TOSM+UCOM with the bare AV8′ interaction. The Hamiltonian components are shown in Table 1 in comparison with the stochastic variational method (SVM) [14], which is one of the rigorous calculations, and also the TOFM proposed by Horii et al. [7], the concept of which is very similar to TOSM to treat the tensor forces. The matter radius of $^4$He is obtained as 1.52 fm in TOSM+UCOM.
When we introduce the partial-wave dependence in the function $s(r)$ of UCOM, called as S-UCOM, the energy gain is about 2 MeV in total energy and the total energy gets closer to the rigorous value [4].

The dominant configurations of $^4$He in TOSM are listed in Table 2. It is found that the specific $2p2h$ states such as $(0s)^2(0p_{1/2})^2$ show the large probabilities and these configurations are essential to produce the tensor correlation in $^4$He because of the coupling by the tensor operator [3, 5]. These $2p2h$ states commonly correspond to the excitations of a $pn$ pair. This feature of $2p2h$ excitations plays an important role to determine the structures of heavier He and Li isotopes as will be discussed later.

We also list the occupation numbers of nucleon in $^4$He using AV8' in Table 3. It is shown that the $p_{1/2}$ orbit has the largest contribution among the particle states according to the large $2p2h$ mixing including the $p_{1/2}$ component shown in Table 2. For comparison, we show the results using the effective Minnesota force (MN) [15] consisting of central and LS force only. In MN case, it is found that the component of the $0s$ orbit is larger than the AV8' case, and the enhancement of the $p_{1/2}$ orbit is not seen. These differences indicate that the tensor force brings the specific excitations from the $s$-shell to the $p$- and $sd$-shells in $^4$He.

### 3.2 Energy Spectra of He and Li Isotopes

We explain the results of He and Li isotopes with AV8' interaction, where $L_{\text{max}}$ is commonly taken as 10 to get a sufficient convergence. The excitation energies of He isotopes are shown in Fig. 1. and also shown in Fig. 2 for Li isotopes. For the excitation energies, we see good agreement with the experimental ones in two isotopes, so that we can discuss the structure differences between energy levels. We also predict some energy levels, which have not observed experimentally yet or not settled to assign the spin of the states, in particular, in the neutron-rich side, $^7$He and $^8$Li.

In Fig. 2, it is also found that the resulting level spacing of the Li isotopes in TOSM+UCOM is good, but slightly more compact than the experimental spectra. For example, in $^9$Li, the small energy difference between the lowest $3/2^-$ and $1/2^-$ states in TOSM+UCOM in comparison with the experimental values. These characteristics are commonly obtained in the GFMC calculation within the two-body interaction level [1]. The additional genuine three-body interaction can be one of the components to reproduce the experimental situation.

In TOSM, the length parameters of the hole and particle states are determined variationally in each state, hence we can discuss the radial properties of particular nuclei including the neutron halo nuclei $^6$He. The matter radii of $^4$He and $^8$He are listed in Table 4. The results are good to explain the enhancement of radius of loosely
binding neutrons in two nuclei, which are, however, slightly smaller than the experiments. We also show the results of cluster model consisting of the $^4$He core and extra neutrons [16], in which the spatial extension of extra neutrons are fully described. We also show the matter radii of $^6$−$^9$Li in TOSM+UCOM in Table 5, which agree with experiments. We include the both results of He and Li isotopes in Fig. 3 and find the whole trend on the matter radii in two isotopes is very good.

We discuss the configuration properties of each level of He and Li isotopes. We here show two cases of $^5$He and $^6$He. More detailed and systematic discussion is given in the recent papers [9, 10]. For $^5$He, various energy components in the doublet states are listed in Table 6. In this calculation, we take the common length
Table 6 Various energy components in $^5$He measured from those of the $^4$He ground state

| $^5$He($J^P$) | Config. | Energy | Kinetic | Central | Tensor | $LS$ |
|---------------|---------|--------|---------|---------|--------|------|
| 3/2$^-$       | $p_{3/2}$ | 6.97   | 24.14   | -8.99   | -5.60  | -2.58|
| 1/2$^-$       | $p_{1/2}$ | 10.05  | 17.53   | -6.96   | -1.11  | 1.04 |

Energy units are given in MeV and $\hbar\omega = 18.43$ MeV. Neutron configuration is also listed.

Table 7 Various energy components in $^6$He in comparison with $^4$He

| $^6$He($J^P$) | Config. | $E$    | Kinetic | Central | Tensor | $LS$ |
|---------------|---------|--------|---------|---------|--------|------|
| $^4_1^-$      | ($p_{3/2}$)$^2$ | 8.95   | 53.04   | -27.75  | -12.02 | -4.04|
| $^2_0^+$      | ($p_{1/2}$)$^2$ | 21.90  | 34.30   | -14.06  | -0.17  | 2.11 |

Units are given in MeV.

parameters of the hole states as 1.5 fm for $s$ and $0p$ orbits. The corresponding $\hbar\omega$ is 18.43 MeV. This is done to exclude the continuum effect, which produces a few MeV energy gain in $^5$He, and to focus our discussion on the internal structures of He isotopes. In Table 6, we compare various energy components in 3/2$^-$ and 1/2$^-$ states of $^5$He measured from those of $^4$He. The $LS$ splitting energy is obtained as about 3 MeV. We discuss the effect of the tensor interaction on this splitting energy. A large difference is seen in the tensor energies of the 3/2$^-$ and 1/2$^-$ states. The larger contribution of the tensor interaction in 3/2$^-$ brings the enhancement of the kinetic energy, because of the involvement of high momentum components from the tensor interaction. The amount of the enhanced kinetic energy is 24 MeV, which is larger than $\hbar\omega$. For 1/2$^-$, on the other hand, the energy gain from the tensor interaction is small and the enhancement of the kinetic energy is 17.5 MeV, which is close to the value of $\hbar\omega$. These results are related to the larger amount of the $p_{1/2}$ component than the $p_{3/2}$ one in $^4$He as shown in Tables 2 and 3. When the last neutron in $^5$He occupies the $p_{3/2}$ orbit, this neutron does not disturb the $^4$He structure. Hence, the $p_{3/2}$ occupied state gains an additional tensor energy without disturbing the large energy gain in $^4$He. On the other hand, in case of the $p_{1/2}$ occupation for the 1/2$^-$ state, this neutron blocks some component of the spatially compact $p_{1/2}$ neutron in the $2p2h$ excitations of the $^4$He core configuration because of the small degeneracy of the $p_{1/2}$ orbit. This effect dynamically produces the Pauli-blocking and reduces the total binding energy of $^5$He. As a result, the last neutron located in the $p_{1/2}$ orbit should be orthogonal to the excited $p_{1/2}$ orbit in $^4$He and the tensor interaction does not gain the energy in $^5$He(1/2$^-$). Those dynamical coupling behaviors between $^4$He and a last neutron explain the difference in the Hamiltonian components in two states of $^5$He, which results in the $LS$ splitting energy as a net value.

We also consider the case of $^6$He, the ground and the excited 0$^+$ states. In Table 7, it is found that the 0$^+_1$ ground state has a larger tensor contribution than the 0$^+_2$ case. This result is very similar to the $^5$He case. When the last two neutrons occupy the $p_{3/2}$ orbit in $^6$He, these neutrons do not disturb the $^4$He structure so much. This configuration causes the gain of the tensor contribution in $^6$He(0$^+_1$), which enhances the kinetic energy due to the high momentum nature of the tensor interaction. In case of the $(p_{1/2})^2$ configuration in the 0$^+_2$ state, two neutrons are blocked to occupy the orbit owing to the excited $p_{1/2}$ neutron from $^4$He by the tensor interaction. This Pauli-blocking does not increase the tensor contribution in $^6$He(0$^+_2$) from that of $^4$He.
These results of $^5$He, $^6$He show that the tensor interaction plays a decisive role to create the $LS$-like splitting energy. The same mechanism has been confirmed in heavier He isotopes, such as the tensor energy differences between the ground and the excited states in $^7,^8$He [9]. Here, as seen in $^8$He($0^+_2$), the $(p_{1/2})^2$ occupation of last two neutrons is also occurred in $^{11}$Li, the halo nuclei. We have confirmed that in $^{11}$Li, the above blocking effect reduces the $p$-wave configuration and brings the enhancement of $s$-wave mixing relatively [12, 13]. This effect dynamically produces the halo structure in $^{11}$Li. There is also the discussion of the effect of the genuine three-body force on the splitting energy [22].

For Li isotopes, we discuss the configurations of the $p$-shell nucleons. In Table 8, we list the dominant configurations of $p$-shell nucleons in the ground states of Li isotopes. These are useful to understand the structures of Li isotopes. It is found that the $^4$Li ($1^+$) state with isospin $T = 0$ shows the $LS$ coupling behavior of last two nucleons with about a half weight. On the other hand, the $0^+$ state with isospin $T = 1$ corresponding to the isobaric analog state of $^4$He, shows the $jj$ coupling state occupying dominantly the $0p_{3/2}$ orbit. For heavier $^{7,8}$Li, those ground states show the $jj$ coupling structure with about half weights. From those results, only the $^6$Li ground state show the $LS$ coupling structure and this can be related to the $α+d$ clustering in the $T = 0$ state. The other heavier Li isotopes show the different $jj$ coupling structure in their ground states. The detailed analyses including the excited states are performed in our paper [10].

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