Conventions in relativity theory and quantum mechanics

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Abstract

The conventionalistic aspects of physical world perception are reviewed with an emphasis on the constancy of the speed of light in relativity theory and the irreversibility of measurements in quantum mechanics. An appendix contains a complete proof of Alexandrov’s theorem using mainly methods of affine geometry.

1 Know thyself

This inscription on the Oracle of Apollo at Delphi, Greece, dates from 6th century B.C., and it is still of tremendous importance today. For we do not and never will see the world “as is,” but rather as we perceive it. And how we perceive the world is mediated by our senses which serve as interfaces to the world “out there” (if any); but not to a small extend also by what we project onto it. Conventions are projections which we have to adopt in order to be able to cope with the phenomena “streaming in” from the senses. Conventions are a necessary and indispensable part of operationalizable[1] phenomenology and tool-building. There

1 In what follows we shall adopt Bridgman’s concept of “operational” as one of quite simple-minded experimental testability, even in view of its difficulties which this author approached later on [1, 2, 3, 4, 5].
is no perception and intervening without conventions. They lie at the very foundations of our world conceptions. Conventions serve as a sort of “scaffolding” from which we construct our scientific worldview. Yet, they are so simple and almost self-evident that they are hardly mentioned and go unreflected.

To the author, this unreflectedness and unawareness of conventionality appears to be the biggest problem related to conventions, especially if they are mistakenly considered as physical “facts” which are empirically testable. This confusion between assumption and observational, operational fact seems to be one of the biggest impediments for progressive research programs, in particular if they suggest postulates which are based on conventions different from the existing ones.

In what follows we shall mainly review and discuss conventions in the two dominating theories of the 20th century: quantum mechanics and relativity theory.

2 Conventionality of the constancy of the characteristic speed

Suppose two observers called Alice and Bob measure space and time in two coordinate frames. Operationally their activities amount to the following. They have constructed “identical” clocks and scales of “equal” length which they have compared in the distant past; when Bob lived together with Alice. Then they have separated. Alice has decided to depart from Bob and, since then, is moving with constant speed away from him. How do Bob’s and Alice’s scales and clocks compare now? Will they be identical, or will they dephase?

These are some of the questions which “relativity” theory deals with. It derives its name from Poicaré’s 1904 “principle of relativity” stating that [6, p. 74] “the laws of physical phenomena must be the same for a stationary observer as for an observer carried along in a uniform translation; so that we have not and can not have any means of discerning whether or not we are carried along in such a motion.” Formally, this amounts to the requirement of form invariance or covariance of the physical equations of motion.

One of the seemingly mindboggling features of the theory of relativity is the fact that simultaneity and even the time order of two events needs no longer be
conserved. It may indeed happen that Alice perceives the first event before the second, while Bob perceives both events as happening at the same time; or even the second event ahead of the first. Simultaneity can only be defined “relativ” to a particular reference frame. If true there, it is false in any different frame.

The first part of Einstein’s seminal paper [7] is dedicated to a detailed study of the intrinsically operational procedures and methods which are necessary to conceptualize the comparison of Alice’s and Bob’s reference frames. This part contains certain “reasonable” conventions for defining clocks, scales, velocities and in particular simultaneity, without which no such comparison could ever be achieved. These conventions appear to be rather evident and natural, almost trivial, and yield a convenient formalization of space-time transformations, but they are nevertheless arbitrary. The simultaneity issue has been much debated in the contemporary discussions on conventionality [8, 9, 10].

There is another element of conventionality present in relativity theory which has gotten less attention [11]. It is the assumption of the constancy of the speed of light. Indeed, the International System of units assumes light to be constant. It was decided in 1983 by the General Conference on Weights and Measures that the accepted value for the speed of light would be exactly 299,792,458 meters per second. The meter is now thus defined as the distance traveled by light in a vacuum in 1/299,792,458 second, independent of the inertial system.

Despite the obvious conventionality of the constancy of the speed of light, many introductions to relativity theory present this proposition not as a convention but rather as an important empirical finding. Indeed, it is historically correct to claim that experiments like the ones of Fizeau, Hoek and Michelson-Morley, which produced a null result by attempting to measuring the earth’s motion against some kind of “ether,” preceded Einstein’s special theory of relativity.

But this may be misleading. First of all, Einstein’s major reason for introducing the Lorentz transformation seems to have been the elimination of asymmetries which appeared in the electromagnetic formalism of the time but are not inherent in the phenomena, thereby unifying electromagnetism. Secondly, not too much consideration has been given to the possibility that experiments like the one of Michelson and Morley may be a kind of “self-fulfilling prophesy,” a circular, closed tautologic exercise. If the very instruments which should indicate a change
in the velocity of light are themselves dilated, then any dilation effect will be effectively nullified. This possibility has already been imagined in the 18th century by Boskovich [12] and was later put forward by FitzGerald [13] (see also John Bell [14, 15]), Lorentz, Poincaré and others in the context of the ether theory [6].

But what is the point in arguing that the constancy of the speed of light is a convention rather than an empirical finding? Is this not a vain question; devoid of any operational testability?

The answer to this concern is twofold. First, a misinterpretation might give rise to a doctrinaire and improper preconception of relativity theory by limiting the scope of its applicability. Indeed, as it turns out, for reasons mentioned below [13], the special theory of relativity is much more generally applicable as is nowadays appreciated. It applies also to situations in which the velocity of light needs not necessarily be the highest possible limit velocity for signaling and travel. Secondly, it is not totally unreasonable to ask the following question. What if one adopts a different convention by assuming a different velocity than that of light to be the basis for frame generation? Such a velocity may be anything, sub- but also superluminal. What will be the changes to Alice’s and Bob’s frames, and how do these new coordinates relate to the usual “luminal” frames?

These issues have been discussed by the author [11] on the basis of a geometrical theorem by Alexandrov [16, 17, 18, 19] and Borchers and Hegerfeldt [20] reviewed in [21, 22] (see also previous articles [23, 24]). Alexandrov’s theorem requires the convention that some speed is the same in Bob’s and Alice’s frames. Furthermore, if two space-time points are different in Alice’s frame, then these points must also be mapped into different points in Bob’s frame and vice versa; i.e., the mapping must be one-to-one, a bijection. It can be proven that under these conditions, the mapping relating Alice’s and Bob’s frames must be an affine Lorentz transformation, with some fundamental speed playing the role of light in the usual Lorentz transformations of relativity theory. The nontrivial geometric part of a proof uses the theorem of affine geometry, which results in the linearity of the transformation equations. No Poincaré’s 1904 “principle of relativity,” no relativistic form invariance or covariance is needed despite the postulate

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2 In stressing the conventionality aspect of these effects, the author would like to state that he does not want to promote any ether-type theory, nor is he against any such attempts.
or convention of equality of a single speed in all reference frames. The derivation uses geometry, not physics. The Appendix contains a detailed derivation of Alexandrov’s theorem which should be comprehensible for a larger audience.

To repeat the gist: it is suggested that the signalling velocity occurring in the Lorentz transformation is purely conventional. This effectively turns the interpretation of relativity theory upside down and splits it into two parts, one geometric and one physical, as will be discussed next.

So where is the physics gone? The claim of conventionality arouses suspicions. The proper space-time transformations cannot be purely conventional or even a matter of epistemology! After all, the Michelson-Morley experiment and most of its various pre- and successors actually yielded null results, which are valid physical observations as can be. The experimenters never explicitly acknowledged the conventionality of the constancy of the speed of light and approved their instruments according to these specifications. Just on the contrary, they first assumed to measure the unequality and anisotropy of the speed of light. And what if Alice and Bob assume, say, the constancy of the speed of sound instead of light? Would the mere assumption change the reading of the instruments in a Michelson-Morley experiment using sound instead of light? This seems to be against all intuitions and interpretations and the huge accumulated body of evidence.

The answer to these issues can be sketched as follows. First of all, the physics is in the form invariance of the electromagnetic equations under a particular type of Lorentz transformations: those which contain the speed of electromagnetic signals; i.e., light, as the invariant speed. Thus, merely the convention of the constancy of the speed of light in all reference frames yields the desirable relativistic covariance of the theory of electromagnetism. This is a preference which cannot be motivated by geometry or epistemology; it is purely physical.

However, any such Lorentz transformation will result in a non-invariance of the theory of sound or any other phenomena which are not directly dominated by electromagnetism. There, an asymmetry will appear, singling out a particular frame of reference from all the other ones.

Thus, one may speculate that the most efficient “symmetric” representation of the physical laws is by transformations which assume the convention of the invariant signalling velocity which directly reflects the phenomena involved. For
electromagnetic phenomena it is the speed of electromagnetic waves; i.e., light. For sound phenomena it is the speed of sound. For gravity it is the speed of gravitational waves. Thus the conventionality of the relativity theory not only relativizes simultaneity but must also reflect the particular class of phenomena; in particular their transmission speed(s). In that way, a general form invariance or covariance is achieved, satisfying Poicaré’s 1904 “principle of relativity” mentioned above, which is not only limited to electromagnetism but is valid also for a wider class of systems.

Secondly, it is not unreasonable to assume that in the particular context of the Michelson-Morley and similar experiments, all relevant physical system parameters and instruments are governed by electromagnetic phenomena and not by sound, gravity or something else. Thus, although not explicitly intended, the experiments are implicitly implementing the conventionality of the constancy of light. Of course, the experimenter could decide to counteract the most natural way to gauge the instruments and measure space and time differently than as suggested by the intruments. For instance, one may adopt scales to measure space which are anisotropic and velocity dependent. But this would be a highly unreasonable, unconvenient and complicating thing to do.

So, from a system theoretic standpoint, the proper convention suggests itself by the dominating type of interaction, and only in this way corresponds to a physical proposition. The result is a generalized principle of relativity.

3 Conventionality of quantum measurements

In what follows the idea is put forward and reviewed that measurements in quantum mechanics are (at least in principle) purely conventional. More precisely, it is purely conventional and subjective what exactly an “observer” calls “measurement.” There is no distinction between “measurement” and ordinary (unitary) quantum evolution other that the particular interpretation some (conscious?) agent ascribes to a particular process [25]. Indeed, the mere distinction between an “observer” and the “object” measured is purely conventional. Stated pointedly, measurement is a subjective illusion. We shall call this the “no measurement” interpretation of quantum mechanics.
The idea that measurements, when compared to other processes (involving entanglement), are nothing special, seems to be quite widespread among the quantum physics community; but it is seldomly spelled out publicly [26, 27]. Indeed, the possibility to “undo” a quantum measurement has been experimentally documented [28], while it is widely acknowledged that practical bounds to maintain quantum coherence pose an effective upper limit on the possibility to reconstruct a quantum state. We shall not be concerned with this upper bounds, which does not seem to reflect some deep physical truth but rather depends on technology, financial commitments and cleverness on the experimenter’s side.

Rather, we shall discuss the differences between the two types of time evolution which are usually postulated in quantum mechanics: (i) unitary, one-to-one, isometric time evolution inbetween two measurements and (ii) many-to-one state reduction at the measurement.

Inbetween two measurements, the quantum state undergoes a deterministic, unitary time evolution, which is reversible and one-to-one. This amounts to arbitrary generalized isometries—distance-preserving maps—in complex Hilbert space. A discrete analogue of this situation is the permutation of states. An “initial message” is constantly being re-encoded. As such an evolution is reversible, there is no principle reason why any such evolution cannot be undone. (There may be insurmountable practical obstacles, though.)

Any irreversible measurement is formally accompanied by a state reduction or “wave function collapse” which is many-to-one. Indeed, this many-to-oneness is the formal mathematical expression of irreversibility.

What is a measurement? Besides all else, it is associated with a some sort of “information” transfer through a fictious boundary between some “measurement apparatus” and the “object.” In the following we shall call this fictious boundary the “interface.” The interface has no absolute physical relevance but is purely conventional. It serves as a scaffolding to mediate and model the exchange. In principle, it can be everywhere. It is symmetric: the role of the observer and the observed system is interchangeable and a distinction is again purely conventional.

In more practical terms, it is mostly rather obvious what is the observer’s side. It is usually inhabited by a conscious experimenter and his measurement device. It should be also in most cases quite reasonable to define the interface as the
location where some agent serving as the experimenter loses control of one-to-
oneness. This is the point where “the quantum turns classical.” But from the
previous discussion it should already be quite clear that any irreversibility in no
way reflects a general physical principle but rather the experimenter’s ability to
reconstruct previous states. Another “observer” equipped with another technology
(or just more money) may draw very different interface lines.

Let me add one particular scenario for quantum information. Assume as an
axiom that a physical system always consists of a natural number of $n$ quanta
which are in a single pure state among $q$ others. Any single such particle is thus
the carrier of exactly one $q$-it, henceforth called “quit.” (In the spin-one half case,
this reduces to the bit.) That is, encoded in such a quantum system are always $n$
quits of information. The quit is an irreducible amount of classical and quantum
information. The quits need not be located at any single particle (i.e., one quit per
particle), but they may be spread over the $n$ particles [29]. In this case one calls
the state of the particles “entangled.” According to Schrödinger’s own interpretation
[30], the quantum wave function (or quantum state) is a “catalogue of expectation
values” about this state; and in particular about the quits. Since an experimenter’s
knowledge about a quantum system may be very limited, the experimenter might
not have operational access to the “true” pure state out there. (In particular, it
need not be clear which questions have to be ask to sort out the valid pure state
from other ones.) This ignorance on the experimenter’s side is characterized by
a nonpure state. Thus one should differentiate between the “true” quantum state
out there and the experimenter’s “poor man’s version” of it. Both type of states
undergo a unitary time evolution, but their ontological status is different.

Why has the no-measurement interpretation of quantum mechanics been not
wider accepted and has attracted so little attention so far? One can only speculate
about the reasons.

For one thing, the interpretation seems to have no operational, testable conse-
quences. Indeed, hardly any interpretation does. So, what is any kind of interpre-
tation of some formalism good for if it cannot be operationalized?

Think of the Everett interpretation of quantum mechanics, which is nevertheless
highly appreciated among some circles, mainly in the quantum computation
community. It has to offer no operationalizable consequences, just mindboggling
scenarios.

Or consider Bohr’s “Copenhagen” interpretation, whatever that means to its successors or to Bohr himself. It is the canonical interpretation of quantum mechanics, a formalism co-created by people, most notably Einstein, Schrödiner and De Broglie, who totally disagreed with that interpretation. This does not seem to be the case for Heisenberg and von Neumann. The latter genius even attempted to state an inapplicable theorem directed against hidden parameters to support some of Bohr’s tendencies. Nowadays, many eminent researchers in the foundations of quantum mechanics still stick with Bohm’s interpretation or whatever sense they have made out of it. But does Bohr’s “Copenhagen” interpretation have any operational consequences?

With the advent of quantum information theory, the notion of information seems to be the main interpretational concept. Consequently, information interpretations of quantum mechanics begin to be widespread. Yet, despite the heavy use of the term “information,” the community does not seem to have settled upon an unambiguous meaning of the term “information.” And also in this case, the interpretations do not seem to have operational consequences.

Many recent developments in quantum information theory are consistent with the no-measurement interpretation. Unitarity and the associated one-to-oneness even for one quantum events seems to be the guiding principle. Take, for example, the no-cloning theorem, quantum teleportation, entanglement swapping, purification and so on [31, 32]. Actually, the no-measurement interpretation seems to promote the search for new phenomena in this regime, and might thus contribute to a progressive research program.

Indeed, it is the author’s belief that being helpful in developing novel theories and testing phenomena is all one can ever hope for a good interpretation. Any “understanding” of or “giving meaning” to the formalism is desireable only to the effect that it yields new predictions, phenomena and technologies. And in this sense, the no-measurement interpretation claiming the conventionality of quantum measurements should be perceived. It too cannot offer direct operationalizable consequences, yet may facilitate thoughts in new, prosperous directions.
4 Summary

We have reviewed conventions in two of the dominating theories of contemporary physics, the theory of relativity and quantum mechanics. In relativity theory we suggest to accept the constancy of one particular speed as a convention. Lorentz-type transformation laws can then be geometrically derived under mild side assumptions. In order for a generalized principle of relativity and thus generalized form invariance to hold, the particular signalling type entering the transformations should correspond to the dominating type of physical interactions.

The no-measurement interpretation of quantum mechanics suggests that there is no such thing as an irreversible measurement. In fact, there is no measurement at all, never. This kind of irreversibility associated with the measurement process is just an idealistic, subjective construction on the experimenter’s side to express the for-all-practical-purposes impossibility to undo a measurement.

Appendix. Proof of Alexandrov’s theorem

Alexandrov’s theorem states that, for $\mathbb{R}^n$, $n \geq 3$ with the metric diag$(+,-,-, \cdots)$ and a one-to-one map $r \mapsto r'$ preserving light cones (i.e., zero distance) such that for all $r,s \in \mathbb{R}^n$,

$$(r-s,r-s) = 0 \iff (r'-s',r'-s') = 0,$$

$r \mapsto r'$ is essentially a Lorentz transformation; i.e., it has the form $r \mapsto r' = \alpha L r + a$ for some nonzero $\alpha \in \mathbb{R}$. $a \in \mathbb{R}^n$, and a linear one-to-one map $L : \mathbb{R}^n \mapsto \mathbb{R}^n$ satisfying $(Lr, Ls) = (r,s)$ for all $r$ and $s$ in $\mathbb{R}^n$.

In what follows we shall review a complete proof of Alexandrov’s theorem very similar to the one sketched by Lester [22]. The proof consists of three stages:

(I) a proof that, given $\mathbb{R}^n$, $n \geq 3$ with the metric diag$(+,-,-, \cdots)$ and a one-to-one map preserving light cones (i.e., zero distance), all lines are mapped onto lines;

(II) a proof of the fundamental theorem of affine geometry stating that a one-to-one map from $\mathbb{R}^n$, $n \geq 2$ onto itself which maps all lines onto lines must be affine; i.e., must be a linear map and a translation;
a proof that, given $\mathbb{R}^n$, $n \geq 2$ with the metric diagonal $\operatorname{diag}(+, \cdots, +, -)$ and a linear one-to-one map preserving a single light cone (i.e., zero distance) must be essentially a Lorentz transformation (up to a translation and a dilatation); i.e., it has the form $r \mapsto \alpha Lr + a$ for some nonzero $\alpha \in \mathbb{R}$, $a \in \mathbb{R}^n$, and a linear one-to-one map $L : \mathbb{R}^n \mapsto \mathbb{R}^n$ satisfying $(Lr, Ls) = (r, s)$ for all $r$ and $s$ in $\mathbb{R}^n$.

In what follows, a constant translation is taken account of by addition of a vector $a \in \mathbb{R}^n$. The remaining transformation preserves the origin; i.e., $0 \mapsto 0'$. We shall often refer to this remaining transformation (after the constant parallel shift moving the map of the origin back into the origin) simply as (homogeneous) transformation. (Note that if $f : r \mapsto \alpha Lr + a$, then the homogeneous part is obtained by subtracting $a = f(0)$.) This constant shift $a$ has to be added to the final mapping.

The geometric proof of (I) proceeds in five steps, covering the mapping of
(i) lightlike (null) lines onto lightlike lines; (ii) lightlike (null) planes onto null planes; (iii) spacelike lines onto spacelike lines; (iv) timelike planes onto planes; and finally (v) timelike lines onto lines.

In what follows, the configurations are demonstrated for $\mathbb{R}^3$ with the metric $(r, s) = r_1s_1 + r_2s_2 - (1/c^2)r_3s_3$. For arbitrary dimensions we refer to [21]. In this section, the velocity of light $c$ will be set to unity; i.e., $c = 1$. The terms “null” and “lightlike” will be used synonymously.

To show (i) let us first define a null cone with vertex $a$ by
$$C(a) = \{ r \in \mathbb{R}^3 \mid (r - a, r - a) = 0 \}.$$ By assumption, light cones are preserved, i.e., $C(a) \leftrightarrow C(a')$.

As illustrated in Fig. [a], any null (lightlike) line is the intersection of two tangent null cones. Since null cones are preserved, so are null (lightlike) lines. Thus, null lines are mapped into null lines. The same is true for the inverse map. Hence, null lines are mapped onto null lines. (The same is true for the other proof steps as well but will not be mentioned explicitly.)

To show (ii), notice that, as illustrated in Fig. [b], a null cone with vertex on some null plane is tangent to that plane along a null line. Points of $\mathbb{R}^3$ are on the null plane if and only if they either lie on this null line or on no null cone with
Figure 1: Illustrations of the proof that (a) lightlike (null) lines into lightlike lines; (b) lightlike (null) planes into null planes; (c) spacelike lines into spacelike lines; (d) timelike planes map into planes.
vertex on this line. The latter sentence could be understood as follows. Imagine any point of \( \mathbb{R}^3 \) outside of the null plane (either “below” or “above”). Any such point is element of some null cone with vertex on the null line mentioned. On the contrary, any point on the null plane cannot be reached by such null cones (except the ones located on the null line mentioned), but by other null cones whose vertex is not on that null line. Null lines and cones are preserved; thus null planes are preserved as well.

To show (iii), notice that, as illustrated in Fig. 1(c), any spacelike line is the intersection of two null planes. Since null planes are preserved, spacelike lines are preserved.

To show (iv), notice that, as illustrated in Fig. 1(d), the points in a timelike plane are covered by infinitely many intersecting null and spacelike lines in that plane. By fixing, for instance, a triangle formed by the vertices \( a, b, c \) of three such lines (e.g., two null and one spacelike line) “spans” the timelight plane. Because of the one-to-oneness of the mapping, the image of the triangle with the vertices \( a', b', c' \) “spans” the transformed plane (different points are mapped onto different points). Therefore, the three lines forming the transformed triangle must be coplanar. In general, the images of all lines lying in the original timelight plane must be coplanar. Thus, timelike planes map into planes.

To show (v), notice that any timelight line is the intersection of two timelight planes. Since timelike planes are mapped onto planes, they intersect into a line. Thus, any timelight line is mapped into a line.

In summary, all three types of lines—lightlike (null), spacelike and timelike lines—are mapped onto lines. (Recall that the same arguments apply for the inverse transformation as well.)

The geometric proof of (II), in particular the linearity of the transformation proceeds from the preservation of lines essentially by utilizing the preservation of parallelism among lines. As will be demonstrated below, the preservation of parallelism implies that the transformation is additive. The associated transformation of the field \( \mathbb{R} \) is an automorphism. It then only remains to be proven that the only automorphism of \( \mathbb{R} \) is the identity function.

Let us first introduce some notation. For a much more comprehensive approach the reader is refered to the literature (e.g., the book by Gruenberg & Weir...
Let $a$ be a fixed “translation” vector of $\mathbb{R}^n$ and $M$ be a linear subspace of $\mathbb{R}^n$. [Recall that a subset $S \subset \mathbb{R}^n$ is called a (linear) subspace if $S$ is a vector space in its own right with respect to the same vector addition and scalar multiplication than $\mathbb{R}^n$.] Then $a + M$ denotes the set of all vectors $a + M = \{a + m \mid m \in M\}$. It is called translated subspace or coset or affine subspace of $\mathbb{R}^n$. The dimension of a translated subspace $a + M$ is the dimension of the linear subspace $M$; i.e., $\dim(a + M) = \dim(M)$. Translated subspaces of dimensions 0, 1, 2 are called points, lines and planes, respectively. Let the join $S_1 \circ S_2$ of two translated subspaces $S_1, S_2$ be the intersection of all translated subspaces in $\mathbb{R}^n$ which contain both $S_1$ and $S_2$. (The join is again a translated subspace.) Furthermore, if $S \subset \mathbb{R}^n$ is any set of vectors in $\mathbb{R}^n$, we denote by the (linear) span $\text{span}(S)$ the intersection of all the subspaces of $\mathbb{R}^n$ which contain $S$.

We shall call an automorphism a one-to-one mapping of $\mathbb{R}^n$ onto itself preserving all translated subspaces. The fundamental theorem of affine geometry (e.g., ref. [33, Theorem 5]) states that, for $\mathbb{R}^n, n \geq 2$, any automorphism induces a linear transformation $L$ and a translation vector $a$ such that $r \mapsto r' = \alpha Lr + a$.

In what follows, a proof of the fundamental theorem of affine geometry will be given for the case of the vector space $\mathbb{R}^n, n \geq 2$ with field $\mathbb{R}$. First, a proof will be given that any such automorphism of $\mathbb{R}^n$ implies an automorphism on the field of reals $\mathbb{R}$ (a definition will be given below). By invoking the preservation of parallelism one obtains both the uniqueness of the associated mapping of the field $\mathbb{R}$ onto itself and furthermore the additivity of the transformation as a whole.

Note that the automorphism preserves parallelism. This can be seen by “fixing” appropriate four points $a, b, c, d$ on two lines which are originally parallel, drawing two nonparallel lines through them which meet in another point $e$. Since by assumption all lines are preserved, so are their meeting points $a', b', c', d'$. Furthermore, because of bijectivity, two parallel lines have no point in common. Thus, the two lines which are originally parallel are mapped onto colpanar lines which are disjoint; i.e., they are again parallel. Hence, parallelism is conserved. A concrete configuration illustrating this geometrical argument is drawn in Fig. 2.

Consider an arbitrary nonzero vector $a \in \mathbb{R}^n$. According to the assumptions, any line $0 \circ a = \text{span}(a)$ is transformed into a line $0' \circ a' = \text{span}(a')$, thereby inducing a one-to-one mapping of all points of $\text{span}(a)$ onto the points of $\text{span}(a')$.
That is, the transformation defines a one-to-one mapping

$$\zeta : x \mapsto x'$$  \hspace{1cm} (1)

of the field of real numbers onto itself by the definition

$$(xa)' = x'a'.$$  \hspace{1cm} (2)

It immediately follows that $\zeta : 0 \mapsto 0'$ as well as $\zeta : 1 \mapsto 1'$. It will be shown that $\zeta$ is an automorphism; i.e., a one-to-one mapping of $\mathbb{R}$ onto itself with the properties that $\zeta(x + y) = \zeta(x) + \zeta(y)$, as well as $\zeta(xy) = \zeta(x)\zeta(y)$.

First it is shown that $\zeta$ does not depend on the particular choice of $a \in \mathbb{R}^n$. (i) Case 1: Consider two linearly independent vectors $a, b$ of $\mathbb{R}^n$, $(xa)' = x'a'$ and $(xb)' = x''b'$, $x' \neq x''$. Since $0 = 0' = 0''$, one can assume that $x \neq 0$. The join $xa \circ xb$ is the intersection of all the subspaces of $\mathbb{R}^n$ containing both $xa$ and $xb$. Since $xa$ and $xb$ are vectors, this is just the subspace spanned by the line joining them. $xa \circ xb$ is parallel to $a \circ b$. (Rescaling does not affect parallelism; cf. Fig. 3.) The transformation preserves parallelism, and therefore $a' \circ b'$ must also be parallel to $x'a \circ x'b'$ and $x'a \circ x'b'$, the lines connecting $x'a$ with $x'b'$ and $x'a$ with $x''b'$. This can only be satisfied for $x' = x''$. Hence, $\zeta$ is independent of the argument and only depends on the transformation.

(ii) Case 2: Consider two linearly dependent vectors $a, b$ of $\mathbb{R}^n$. In this case, choose a third vector $c$ which does not lie in the linear subspace span$(a)$ spanned...
by $a$ and $b$. Then, by the argument used in case 1, $\zeta$ is the same for $a, c$ and $b, c$; thus $\zeta$ is also the same for $a, b$. Hence, to sum up the finding in the two cases, $\zeta$ is independent of the argument vector and only depends on the transformation.

We shall pursue the proof that, given the preservation of lines, the associated mapping is additive (up to translations). A geometric interpretation of this proof is drawn in Fig. 4. (i) Case 1: If $a$ and $b$ are linearly independent nonzero vectors (the zero vector case is trivial) of $\mathbb{R}^n$, then the parallelogram $a, 0, b, a+b$ is mapped into the parallelogram $a', 0', b', a' + b'$ and

$$(a + b)' = a' + b'.$$  (3)

This is also true if $a$ or $b$ is the zero vector.

(ii) Case 2: If $a$ and $b$ are linearly dependent and nonzero, choose a third vector $c \not\in \text{span}(a)$ (so that $c$ is linearly independent of $a$ and $b$), and apply the above considerations for the pairs $a + b & c$ rendering $a + b + c \mapsto (a + b)' + c'$, $a & b + c$ rendering $a + b + c \mapsto a' + (b + c)'$, $b & c$ rendering $b + c \mapsto b' + c'$, such that $(a + b)' + c' = a' + b' + c'$, which is satisfied only if again $(a + b)' = a' + b'$.

Two further properties assuring that $\zeta$ is an automorphism can be deduced from the uniqueness of Eq. (2) and Eq. (3) and the usual axioms of linear vector spaces. (i) Automorphism property 1: Let $a' \neq 0$, then for all $x, y \in \mathbb{R}$,

$$(x + y)'a' = [(x + y)a]' = (xa + ya)' = (xa)' + (ya)' = x'a' + y'a' = (x' + y')a'$$  (4)

and thus

$$(x + y)' = x' + y'.$$  (5)
(ii) Automorphism property 2: By the assumption of vector spaces, \((xy)a = x(ya)\) for all \(x, y \in \mathbb{R}\) and \(a \in \mathbb{R}^n\). Therefore,
\[
(xy)'a' = x'(ya)' = x'(y'a') = (x'y')a' \quad (6)
\]
and thus
\[
(xy)' = x'y' \quad (7)
\]

In order to complete the proof of linearity, it will be shown that the only automorphism of the field \(\mathbb{R}\) into itself is the identity function \(id : x \mapsto x\). This can be demonstrated by realizing that the algebraic properties of neutral elements 0, 1 with regard to addition and multiplication have to be preserved; i.e., \(0 \mapsto 0\) and \(1 \mapsto 1\). Furthermore, since 1 has to be preserved and any natural number \(n \in \mathbb{N}\) is the sum of \(n\) 1’s, \(\mathbb{N}\) has only a single automorphism—the identity function \(id\). A very similar argument holds for \(\mathbb{Z}\). Since any element of the positive rationals can be represented by the quotient \(n/m\) with \(n, m \in \mathbb{N}\), again \(\mathbb{Q}\) has only a single automorphism—the identity function \(id\). In order to be able to obtain the same result for \(\mathbb{R}\), one has to make sure the Dedekind construction of the reals works; in particular the preservation of Dedekind sections. This requires the preservation of the order relation “<” in \(\mathbb{R}\), which is equivalent to the preservation of positivity, since \(x < y\) can always be rewritten into \(0 < y - x\). Notice that every positive \(0 < x \in \mathbb{R}\) can be written as \(x = y^2\), \(y \in \mathbb{R}\) \(y \neq 0\). Since \(x = y^2\) is mapped onto \(x' = (y^2)' = (y')^2\) with \(y' \neq 0\) (recall that \(0 \mapsto 0\), \(x' > 0\). This allows the Dedekind construction of the reals using the rationals, which in turn yields the desired fact.
that \( \mathbb{R} \) has only a single automorphism--the identity function \( \text{id} \). (This is not true for example for \( \mathbb{C} \), since for example \( x + iy \mapsto x - iy \) is an automorphism but not the identity.)

A way to get rid of the factor \( \alpha \) is by considering the tangent hyperboloid \( x^2 + y^2 - z^2 = 1 \) of the null cone \( x^2 + y^2 - z^2 = 0 \), translating it once and then back to the original figure. The requirement that this should result in the same hyperboloid fixes \( \alpha \).

We shall now concentrate on a proof of (III). Let us first note that, in the case of a linear map, the preservation of a single light cone is a sufficient condition for the preservation of all of them. For, given the transformation \( x \mapsto \alpha Lx + a \), any shift of the null cone \( C(p) \) with vertex \( p \) by a vector \( s = q - p \) results in a null cone \( C(q) = C(p) + s \) with vertex \( q = p + s \). The latter null cone \( C(q) \) is mapped onto the null cone

\[
C(q) \mapsto \alpha LC(q) + a = \alpha L(C(p) + s) + a = \alpha LC(p) + \alpha Ls + a = (\alpha LC(p) + a) + \alpha Ls = C(p') + \alpha Ls,
\]

which again is a null cone.

Recall that in Einstein’s original work \([7, \text{par } 3]\), linearity was never derived but was assumed for physical reasons. “In the first place it is clear that the equations must be linear on account of the properties of homogeneity which we attribute to space and time.” \([\text{“Zunächst ist klar, daß die Gleichungen linear sein müssen wegen der Homogenitätseigenschaften, welche wir Raum und Zeit bei-legen.”}]\) In what follows we shall closely follow Einstein’s original argument rendering \( L \) to be the Lorentz transformations.

Take the standard four dimensional space-time case \( \mathbb{R}^4 \), and consider, for the sake of simplicity, the quasi-twodimensional case (one space and the time coordinate) of the constant motion along the \( x \)-axis of \( K \) with velocity \( v \) of a coordinate frame \( K' \) with the components \( (x', y', z', t') \) against another coordinate frame “at rest” \( K \) with the components \( (x, y, z, t) \). (Otherwise, \( K \) can be rotated such that the direction of motion is along the \( x \) axis.) Again, \( c \) stands for the velocity of light.
Figure 5: Generation of radar coordinates by a light clock following Einstein’s procedures and conventions (a) from within the system $K'$; (b) the same procedure seen from the system $K$.

Now define a particular series (in time) of points $\bar{x} = x - vt$. Notice that the “worldlines” $(x = vt, 0, 0, t)$ just mark the parametrization by the time parameter $t$ of all points at rest with respect to the moving frame $K'$. That is, any such point has constant $\bar{x}, y, z$ throughout all times $t$. It is sometimes convenient (cf. below) to write the parameters of events in terms of $(\bar{x}, y, z, t)$ instead of $(x, y, z, t)$.

Let us construct “radar coordinates” of $K'$ by utilizing a light clock starting at some arbitrary point $\bar{x} = 0$ at $t'_0$, traveling some distance $\Delta \bar{x}$ to a mirror, where it arrives and is instantly reflected at $K'$-time $t'_1$ towards the original source mirror and arrives there at $K'$-time $t'_2$ [cf. Fig. 5(a)]. If one adopts the usual conventions for synchronization, $t'_1$ is just the arithmetic mean of the two times $t'_0$ and $t'_2$; i.e.,

$$t_1 = \frac{1}{2}(t'_0 + t'_2).$$  \hspace{1cm} (8)

In order to find the transformation mapping $K$ onto $K'$, rewrite the transformed coordinates as functions of the original system; e.g., $t' = t'(\bar{x}, y, z, t)$. In
this parametrization, the coordinates are given by

\[ t_0' = t'(0, 0, 0, t), \quad (9) \]

\[ t_1' = t' \left( \Delta \bar{x}, 0, 0, t + \frac{\Delta \bar{x}}{c^2 - v^2} \right), \quad (10) \]

\[ t_2' = t' \left( 0, 0, 0, t + \frac{\Delta \bar{x}}{c^2 - v^2} + \frac{\Delta \bar{x}}{c^2 + v^2} \right), \quad (11) \]

where

\[ t_0 = t, \]

\[ t_1 = t + \frac{\Delta \bar{x}}{c^2 - v^2}, \]

\[ t_2 = t + \frac{\Delta \bar{x}}{c^2 - v^2} + \frac{\Delta \bar{x}}{c^2 + v^2} \]

results from the following consideration. The $K$-time $\Delta t_1 = t_1 - t_0$ it takes for light to arrive at the first mirror is given by the total distance it takes for light to travel to it, divided by the velocity of light. Since the mirror travels with velocity $v$,

\[ \Delta t_1 = \frac{\Delta \bar{x} + v \Delta t_1}{c} = \frac{\Delta \bar{x}}{c - v}. \quad (12) \]

A similar argument yields

\[ \Delta t_1 = t_2 - t_1 = \frac{\Delta \bar{x} - v \Delta t_1}{c} = \frac{\Delta \bar{x}}{c + v}. \]

Inserting Eqs. (9)–(11) into (8) yields

\[ t' \left( \Delta \bar{x}, 0, 0, t + \frac{\Delta \bar{x}}{c^2 - v^2} \right) = \frac{1}{2} \left[ t'(0, 0, 0, t) + t' \left( 0, 0, 0, t + \frac{\Delta \bar{x}}{c^2 - v^2} + \frac{\Delta \bar{x}}{c^2 + v^2} \right) \right]. \quad (13) \]

\( \Delta \bar{x} \) can be arbitrarily small. A partial derivation of (13) by $\frac{\partial t'}{\partial \Delta \bar{x}}$ in the limit of infinitesimal $\Delta \bar{x}$ yields

\[ \frac{\partial t'}{\partial \Delta \bar{x}} + \frac{1}{c - v} \frac{\partial t'}{\partial t} = \frac{1}{2} \left( \frac{1}{c - v} + \frac{1}{c + v} \right) \frac{\partial t'}{\partial t}. \quad (14) \]
and thus
\[ \frac{\partial t'}{\partial \bar{x}} + \frac{1}{c^2 - v^2} \frac{\partial t'}{\partial t} = 0. \] (15)

Likewise, \((\partial t'/\partial y) = (\partial t'/\partial z) = 0\). As a result of this and Eq. (15), \(t'\) must be a linear function of \(t\) and \(\bar{x}\) of the form
\[ t'(\bar{x},y,z,t) = \alpha(v) \left( t - \frac{v}{c^2 - v^2} \bar{x} \right). \] (16)

\(\alpha(v)\) is a yet arbitrary scale factor depending only on \(v\). Note that, without loss of generality, the origins of \(K\) and \(K'\) has been chosen such that \(t = t' = 0\). By substituting the explicit parameters for \(\bar{x} = x - vt\) one obtains
\[ t'(\bar{x},y,z,t) = \alpha(v) \frac{1}{1 - \frac{v^2}{c^2}} \left( t - \frac{v}{c^2} x \right). \] (17)

The transformation rules of the \(x'\) parameter can be obtained by considering the propagation of a light ray in \(K'\) which starts at the origin of \(K\) and \(K'\) (same origins) and moves along the \(x\) and \(x'\)-axes. The convention of the constancy of the speed of light requires
\[ x' = ct' = \alpha(v)c \left( t - \frac{v}{c^2 - v^2} \bar{x} \right). \] (18)

Now recall that, in terms of the \(K\)-parameters, this propagation of this light ray is given by Eq. (12); i.e., by \(t = \bar{x}/(c - v)\) (the differences \(\Delta\) can be omitted because of the ray starting at the coordinate origins). By substituting \(t\) in (18) one obtains
\[ x' = \alpha(v) \frac{c^2}{c^2 - v^2} \bar{x} = \alpha(v) \frac{1}{1 - \frac{v^2}{c^2}} \bar{x} = \alpha(v) \frac{1}{1 - \frac{v^2}{c^2}} (x - vt). \] (19)

Let us now turn to the transformation of coordinates \(y, z\) perpendicular to the direction of motion \(x\). Consider a light ray propagating along the \(y'\)-axis, and hence \(\bar{x} = 0\). Inside the system \(K\), the \(y\)-component of the light propagation follows from the Pythagorean theorem, which is illustrated in Fig. [3]; i.e., \(v_y = \sqrt{c^2 - v^2}\). Hence,
\[ y' = ct' = \alpha(v)c \left( t - \frac{v}{c^2 - v^2} \bar{x} \right), \] (20)
Figure 6: Velocity $v_y$ of a light ray propagating along the positive $y'$ axis of a system traveling with velocity $v$ along the $x$- and $x'$-axes.

for $\bar{x} = 0$ and $t = y/v_y = y/\sqrt{c^2 - v^2}$

$$y' = \alpha(v) \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} y.$$ (21)

The same consideration applies to the transformation of the $z$- and $z'$-axes. Summing up, we obtain a transformation of the coordinates $x \mapsto x' = Lx$ given by

$$L(v) = \alpha(v) \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \begin{pmatrix}
\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} & 0 & 0 & -\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-\frac{v}{c^2\sqrt{1 - \frac{v^2}{c^2}}} & 0 & 0 & \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} 
\end{pmatrix}$$ (22)

We now fix the factor $\alpha(v)$ by the conventional requirement that a back-transformation should recover the original coordinates. For this purpose we invent a third coordinate frame $K''$ which propagates with the reverse (relative to $K'$) velocity $-v$ (measured in $K$) along the $x$, $x'$, and its $x''$-axes. The successive application of the transformation (23) with $L(v)$ and $L(-v)$ should bring back the coordinates to their original form; i.e.,

$$L(v)L(-v) = \mathbb{I}_4,$$ (23)

where $\mathbb{I}_4 = \text{diag}(1, 1, 1, 1)$ stands for the four-dimensional unit matrix. After evaluating the matrix product and comparing the coefficients, one obtains

$$\alpha(v)\alpha(-v) = 1 - \frac{v^2}{c^2}.$$ (24)
That \( \alpha(v) = \alpha(|v|) \) only depends on the absolute value of the velocity can be seen by symmetry and isotropy arguments. For the length \( l' \) of a rod \( \{p' \in \mathbb{R}^4 \mid x' = 0, 0 \leq y' \leq l, z' = 0 \} \) which is at rest along the \( y' \)-axis with respect to the system \( K' \) traveling along the \( x \)-axis should not depend on the direction of motion; i.e., should only depend on the absolute magnitude of the velocity. If this is granted, one obtains

\[
\alpha(v) = \sqrt{1 - \frac{v^2}{c^2}}, \tag{25}
\]
and finally the transformation laws \( x \mapsto x' = Lx \) with

\[
L(v) = \begin{pmatrix}
\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} & 0 & 0 & -\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}} \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-\frac{v}{c^2 \sqrt{1 - \frac{v^2}{c^2}}} & 0 & 0 & \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{pmatrix} \tag{26}
\]

up to constant translations \( a \in \mathbb{R}^4 \). As can be easily checked, \( L \) preserves the distance of any two points; i.e., \((Lr, Ls) = (r, s)\) for all \( r \) and \( s \) in \( \mathbb{R}^4 \).

It would be nice to have a more general result using a more general metric and/or relaxation of bijectivity.

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