Spin waves in one-dimensional bi-component quasicrystals

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We studied finite Fibonacci sequence of Co and Py stripes aligned side-by-side and in direct contact with each other. Calculations based on continuous model including exchange and dipole interactions were performed for structures feasible for fabrication and characterization of main properties of magnonic quasicrystals. We have shown the fractal structure of the magnonic spectrum with a number of magnonic gaps of different width. Also localization of spin waves in quasicrystals and existence of surface spin waves in finite quasicrystal structure is demonstrated.

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Quasicrystals1,2 are aperiodic structures with long range order which can be constructed in deterministic way by self-similar replication or by partial projection of appropriate periodic structure from higher dimensions.3 This hidden symmetry can be revealed in diffraction spectrum2 and allows for advanced tailoring of band structure which exceeds possibilities offered by homogeneous material or artificial crystals.1,4

One-dimensional (1D) quasicrystals in the form of Fibonacci sequence have already been studied for photonic3,7, electronic8,9 and phononic10 systems. However, research of SWs in 1D magnonic quasicrystals (MQ) is so far mainly limited to theoretical investigations of multilayered structures.11 Attention was focused on lattice models for exchange SWs12,13 and in continuous model for magnetostatic SWs14. Experimentally, SW excitations in quasicrystals were investigated only recently regarding their localization properties in two-dimensional structures made of thin veins of Py15,16 and in a larger scale for enhancement of nonlinear effects in thick yttrium-iron-garnet film with Fibonacci sequence of etched groves.17 These studies have shown interesting properties of MQs useful for applications and fundamental studies. However, there is an evident gap in studies of SW dynamics in MQs, especially in studies of structures feasible for experimental realization. This area can be explored theoretically by considering finite structure of planar geometry and including both, exchange and dipole interactions. Therefore, we focus our study on planar bi-component MQs modulated in nanoscale, i.e., on structures preserving all features characteristic for wave dynamics in quasiperiodic structures which are possible for fabrication and characterization.

We considered a thin plate of 1D MQ in the form of finite Fibonacci sequence consisting of Co and permalloy (Py: Ni80Fe20) stripes. Ferromagnetic stripes were in direct contact which ensures exchange coupling between neighboring stripes, additional to long range dipole interactions. We investigated system in saturation state with static magnetization aligned along infinite stripes. The strong dynamical coupling makes system dispersive for propagating SWs with interesting features like a fractal SW spectra and localized properties of SWs. Moreover, the finite Fibonacci sequence allowed us to study surface SWs localized at edges of the MQ. To find frequencies of SWs and their spatial profiles we solve linearized Landau-Lifshitz equation with finite element method (FEM) in frequency domain.19,21

We have investigated fractal SW spectra for 1D MQs obtained according to Fibonacci inflation rule and made from long Co and Py stripes of 91 nm width and 30 nm thickness. Saturation magnetization and exchange constant for Co and Py are: \( M_{\text{Py}} = 0.86 \times 10^8 \) A/m, \( A_{\text{Py}} = 1.3 \times 10^{-11} \) J/m, \( M_{\text{Co}} = 1.445 \times 10^8 \) A/m, \( A_{\text{Co}} = 3.0 \times 10^{-11} \) J/m.20,22 For each material gyromagnetic ratio \( \gamma = 176 \) rad GHz/T is assumed the same. In accordance with the inflation rule, Co and Py stripes are arranged in Fibonacci sequence using the following recursion: \( S_n = [S_{n-1}S_{n-2}] \) where \( S_1 \) and \( S_2 \) are initial structures consisting of single Co and Py stripe, respectively. \([S_{n-1}S_{n-2}]\) means concatenation of two subsequences \( S_{n-1} \) and \( S_{n-2} \) of the stripes. We obtained in first subsequences: Co, Py, PyCo, PyCoPy, PyCoPyPyCo [Fig. 1(a)]. In each next step the structure total width is growing in \( x \) direction, e.g., for the quasicrystal made from 55 stripes the width of the whole structure is 5 \( \mu \)m. We have analyzed structures \( S_{10}, S_{11}, \ldots S_{16} \), made of 55, 89, 144, 233, 377, 610 and 987 number of stripes. With \( n \to \infty \) the filling fraction of Py approach to the golden ratio \( \sigma = (\sqrt{5} - 1)/2 \) and the sequence of stripes \( S_\infty \) becomes rigorously quasiperiodic. We use the coordinate system as defined in Fig. 1. Along stripes structure is infinite and saturated by external magnetic field \( \mu_0 H_0 = 0.1 \) T. To emphasize interesting properties of SWs in quasicrystals we will make reference to magnonic spectra calculated for MC composed of the same stripes of Py and Co as Fibonacci sequence [Fig. 1(b)].

To calculate SW spectra we solve the Landau-Lifshitz equation (LLE):

\[
\frac{\partial \mathbf{M}(r, t)}{\partial t} = \mu_0 \gamma \left[ \mathbf{M}(r, t) \times \mathbf{H}_{\text{eff}}(r, t) \right] + \frac{\alpha}{|\mathbf{M}|} \mathbf{M}(r, t) \times \left( \mathbf{M}(r, t) \times \mathbf{H}_{\text{eff}}(r, t) \right),
\]

(1)
of states IDOS, defined as:

\[
\text{IDOS}(f_m) = \sum_{i=0}^{m} \text{DOS}(f_i),
\]

where DOS is density of SW modes and \(f_i\) is a frequency of the \(i\)-th SW mode, which are ordered according with increased frequency. [38] IDOS as a function of frequency calculated for successive Fibonacci sequences from \(S_{10}\) to \(S_{16}\) are shown in Fig. (c).

Due to the finite width of structures used in calculations, we always observe discrete set of frequencies. The finer steps and faster increase of IDOS are observed for wider structures (i.e., composed of larger number of nanostripes). By finding plateaus in IDOS we are able to identify magnonic gaps. The width of these plateaus converges for larger Fibonacci structures and can approximate width of magnonic gaps in infinite MQs [see the gray areas marked in Fig. (c)]. Within some plateaus of IDOS, we can also find surface modes of MQ. In IDOS shown in Fig. (c) these surface SWs are indicated by separated steps (marked with circles for \(S_{10}\)) inside the magnonic gap, discussed in further part of the paper.

With the increase (decrease) size of the structure, the spectrum of IDOS reveals more (less) details of spectrum characteristic for quasicrystals (see Fig. (c)). For larger structures, we are able to identify the fine structure of magnonic gaps, whereas gaps found for shorter sequences still exist. [39] This provides mechanism to explore fractal nature of SW spectrum in MQs. In Fig. (a) we present in details the spectrum for structure consisting of 377 nanostripes. The inset in this figure shows magnified region of spectrum with subtle, multilevel structure of magnonic gaps, resembling property of self-similarity.

In order to validate the fractal nature of SW spectrum we have calculated its Hausdorff dimension. [4] We divided the whole investigated frequency range into intervals of equal lengths \(\Delta f\) and then we counted the number \(N(\Delta f)\) of these intervals which are included in or partially overlap with magnonic bands. The number \(N(\Delta f)\) increases with decrease of length \(\Delta f\) and dependence of \(\log [N(\Delta f)]\) on \(\log (\Delta f)\) should be linear (\(f_0\) is a frequency of the first mode in spectra, its choice is arbitrary and does not influence results). The Hausdorff dimension of the spectrum is defined as the derivative:

\[
D_H = \frac{d \log [N(\Delta f)]}{d \log (\Delta f)}.
\]

Numerically we have calculated \(D_H\) as a coefficient of regression for dependence of \(\log [N(\Delta f)]\) on \(\log (\Delta f)\).

We have obtained value \(D_H = 0.9603\) with the standard deviation 0.0011 and the coefficient of regression \(R^2 = 0.99991\), which points close to linear dependence. This non integer value of \(D_H\) proves fractal property of SW
spectra in MQ. Its value is close to the $D_H$ obtained for plasmonic Fibonacci structures.[8]

FIG. 2: (Color online) (a) IDOS for Fibonacci sequence $S_{14}$. Gray areas mark the most pronounced magnonic gaps. Inset presents magnified region of IDOS plot where the complex bandgap structure is visible. The Hausdorff dimension $D_H$ is 0.9602. (b) IDOS for MC consisting of 144 nanostripes, $D_H = 1$.

IDOS for MCs has a regular dependence on $f$. In Fig. 2(b) we have plotted the IDOS($f$) for MC structure consisting of 144 nanostripes. Here, $D_H$ is equal 1. The first magnonic band gap starts at 16.5 GHz, however in MQ gaps are present already at lower frequencies. Existence of low frequency gaps in MQs can be useful for application, for instance in magnon transistors.[22]

The other interesting issue of excitations in quasicrystals is a possibility for their localization.[7] In crystals without any defects, all modes are extended, but in random systems localization occurs.[22, 27] MQs are neither periodic nor random systems. Therefore, we can observe both extended and localized SW modes.

To discuss localization quantitatively, we need measure of the strength of localization. For this purpose we will use localization factor $\lambda$. The parameter $\lambda_i$ for $i^{th}$ mode is defined as:

$$\lambda_i = \frac{1}{L} \sum_{j=0}^{x=L} \log |m_{x,i}(x_j)|, \quad (4)$$

where the in-plane dynamical component of magnetization $m_{x,i}(x_j)$ is taken at the point $x_j$ and is normalized according to the norm:

$$\frac{1}{L} \sum_{j=0}^{x=L} |m_{x,i}(x_j)| = 1. \quad (5)$$

Summation runs over all $L$ equidistant points $x_j$ covering all the space between surfaces of structure along $x$-axis. The $\lambda = 0$ corresponds to uniform excitation. For all other modes $\lambda$ takes negative values. Modes with stronger localization are characterized by large absolute value of $\lambda$.

In considered systems, the separation of frequencies of SW modes quantized across the thickness exceeds the frequency range considered in this paper. Thus, for further analysis of localization factors we take SW amplitude from the middle plane of the structure.

FIG. 3: (Color online) Localization factors $\lambda$ of SW modes in (a) $S_{12}$ sequence of the Fibonacci MQ and (b) of MC composed of the same number of nanostripes in dependence on frequency. Numbers labeling points in the $(\lambda, f)$ plot denote successive numbers of modes ordered with increased frequency. Profiles of the modulus of $x$-component of magnetization along the structure are plotted in insets above the main plot. Region of selected band in (a) is magnified in separate inset. Grey areas mark magnonic gaps of the widest width.

Localization factors of SWs in MC are shown in Fig. 3(b). In general, the localization factor $\lambda$ increases in magnitude with growing frequency (with increasing number of nodal points), although these are extended modes. The rate of this change is relatively small. Even for SWs at 20 GHz the localization factor does not go below -1 (apart from mode no. 73 being a surface SW discussed later). Localization factors depending on the frequency in the Fibonacci quasicrystal ($S_{12}$) are shown in Fig. 3(a). The high frequency part of the spectrum forms a complex structure, which is clearly visible when we magnify selected region, e.g., marked with dashed frames in Fig. 3(a) and magnified in the inset. For MQ, similar as for MC, the localization strength of SWs increases with the increase of their frequencies. However, here values of $\lambda$ drop up to $-4$ at 20 GHz. Thus, the localization of bulk modes of high frequency makes their amplitude well concentrated at the particular position inside of the structure [e.g., profile of SW with no. 109 in Fig. 3(a)].

Secondarily there is group of modes whose profiles are rather chaotic [e.g., mode 40 in Fig. 3(a)]. These two groups of SW modes: localized and chaotic, are common with other types of waves in quasiperiodic structures.[22]

Surface states are supposed to be induced in quasicrystals more often than in periodic structures due to larger number of frequency gaps. In fact, in MC we have found only one surface mode up to 20 GHz [mode no. 73 in Fig. 3(b)] but in MQ, many surface modes can be identified [e.g., mode 55 shown in Fig. 3(a)]. They have $\lambda < -4$, which exceeds the factor of bulk modes. For
some modes with their frequency near the end of bunch of bulk modes (e.g. mode 102), distinguishing between bulk and surface character can be ambiguous. We can point out that, the strength of localization of surface modes increases if its frequency is located in the center of the gap, when this gap is wider, and when its frequency is high.

In low frequency limit, both periodic and Fibonacci structures exhibit similar properties: much the same profiles [see insets in Fig. 3(a) and (b)] for modes 1 and 2], frequencies and localization factors of the SWs [compare Fig. 3(a) and 3(b)]. In this limit both systems can be treated as metamaterials characterized by effective magnetic properties. It is worth noting, that in this limit, the IDOS(f) spectra of MQs and MCs have the features characteristic for the Damon-Eshbach wave in homogeneous film.

We have proposed the MQ composed of Py and Co stripes suitable for experimental study of fractal properties in magnonics. Indeed, magnonic band structure have already been investigated in MCs composed of Py or Co and Py stripes with periodicity ≈ 500 nm and interesting physical properties, like magnonic band gaps and re-programmability have been demonstrated. The Brillouin light scattering (BLS) to measure dispersion relation of SWs, micro-BLS spectroscopy and time resolved magneto-optical Kerr effect (TR-MOKE) microscopy with spatial resolution down to 250 nm to visualize profiles of SW excitations have been used. These techniques can be directly use to study SWs in MQs and to confirm predicted properties. The area of SW localization in MQs under investigation spreads over few Py stripes [e.g., modes 55, 102 and 109 in Fig. 3(a)], which is above limits of spatial resolution in BLS and TR-MOKE microscopes. Moreover, properties of SWs in MQs presented above preserve also in larger structures, i.e., with stripe width extended up to few hundreds of nm. To confirm this we present in Fig. 4 IDOS as a function of frequency calculated for MQ composed of 250 nm width Py and Co stripes i.e., the size which exactly matches frequency calculated for MQ composed of 250 nm width Py and Co stripes arranged in periodic and Fibonacci structures where considered. We have shown that the spectrum of IDOS for MQs systems exhibits complex, multilevel structure of frequency gaps with finer details revealed for long Fibonacci sequences. Calculated magnonic spectra form fractal set with self-similarity property characterized by the Hausdorff dimension 0.9603. Computated localization factors allow us to discuss quantitatively the strength of localization for SW in MQs. We show the localization of bulk modes in quasiperiodic systems and that it is enhanced for higher frequencies, where spectrum of Fibonacci structures becomes substantially complex. The presence of multiple magnonic gaps supports also the existence of surface SWs which are observed numerously in MQs. In the long wavelength limit both MQs and MCs preserve similar effective properties. Obtained results show that quasicrystal structures can be investigated in magnonics with standard experimental techniques and their fractal properties can be explored.

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FIG. 4: (Color online) IDOS of Fibonacci sequence $S_{14}$ composed of wide Py and Co stripes–250 nm width and 30 nm thickness. Gray areas mark the most pronounced magnonic gaps. Inset presents the magnified region of IDOS plot where the complex bandgap structure is visible. Hausdorff dimension of the spectrum is $D_H = 0.9413$. 

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[1] D. Levine and P. J. Steinhardt, Phys. Rev. Lett. 53, 2477 (1984).

[2] C. Janot, Quasicrystals: a Primer, 2nd ed. (Oxford University Press, Oxford, 1994).

[3] Z. V. Vardeny, A. Nahata, and A. Agrawal, Nat. Photonics 7, 177 (2013).

[4] E. Macia, Rep. Prog. Phys. 69, 397 (2006).

[5] T. Janssen and A. Janner, Acta Cryst. B 70, 617 (2014).

[6] G. I. Zagoskin, A. Grudiev, K. Schünemann, and P. V. Turbin, Phys. Rev. Lett. 88, 195005 (2002).

[7] M. Kohn, B. Sutherland, and K. Iguchi, Phys. Rev. Lett. 58, 2436 (1987).

[8] R. Merlin, K. Bajema, Roy Clarke, F.-Y. Juang, and P. K. Bhattacharya, Phys. Rev. Lett. 55, 1768 (1985).

[9] F. Laruelle and B. Etienne, Phys. Rev. B 37, 4816(R)}
(1988).
[10] W. Steurer and D. Sutter-Widmer, J. Phys. D. Appl. Phys. 40, R229 (2007).
[11] E. L. Albuquerque and M. G. Cottam, Phys. Rep. 376, 225 (2003).
[12] C. H. O. Costa and M. S. Vasconcelos, J. Phys. Condens. Matter 25, 286002 (2013).
[13] T. S. Liu and G. Z. Wei, Phys. Rev. B 48, 7154 (1993).
[14] D. H. A. L. Anselmo, M. G. Cottam, and E. L. Albuquerque, J. Appl. Phys. 85, 5774 (1999).
[15] V. S. Bhat, J. Sklenar, B. Farmer, J. Woods, J. T. Hastings, S. J. Lee, J. B. Ketterson, and L. E. De Long, Phys. Rev. Lett. 111, 077201 (2013).
[16] V. S. Bhat, J. Sklenar, B. Farmer, J. Woods, J. B. Ketterson, J. T. Hastings, and L. E. De Long, J. Appl. Phys. 115, 17C502 (2014).
[17] S. V. Grishin, E. N. Beginin, Yu. P. Sharaevskii, and S. A. Nikitov, Appl. Phys. Lett. 103, 022408 (2013).
[18] M. Krawczyk, D. Grundler, J. Phys.: Condens. Matter 26, 123202 (2014).
[19] M. Mruczkiewicz, M. Krawczyk, V. K. Sakharov, Yu. V. Khvintsev, Yu. A. Filimonov, S. A. Nikitov, J. Appl. Phys. 113, 093908 (2013).
[20] Z. K. Wang, V. L. Zhang, H. S. Lim, S. C. Ng, M. H. Kuok, S. Jain, A. O. Adeyeye, Appl. Phys. Lett. 94, 083112 (2009).
[21] M. Mruczkiewicz, M. Krawczyk, G. Gubbiotti, S. Tacchi, Yu. A. Filimonov, D. V. Kalyabin, I. V. Lisenkov, S. A. Nikitov, New J. Phys. 15, 113023 (2013).
[22] M. L. Sokolovskyy, M. Krawczyk, J. Nanopart. Res. 13, 6085 (2011).
[23] M. Kohmoto, B. Sutherland, and C. Tang, Phys. Rev. B 35, 1020 (1987).
[24] A. G. Gurevich and G. A. Melkov, Magnetization Oscillations and Waves (CRC Press, 1996).
[25] A. Chumak, A. A. Serga and B. Hillebrands, Nat. Commun. 5, 4700 (2014).
[26] W.-C. Xie and I. Elishakoff, Chaos, Solitons & Fractals 11, 1559 (2000).
[27] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979).
[28] P. E. Wolf and G. Maret, Phys. Rev. Lett. 55, 2696 (1985).
[29] M. Mruczkiewicz, M. Krawczyk, R. V. Mikhailovskiy, and V. V. Kruglyak, Phys. Rev. B 86, 024425 (2012).
[30] J. Topp, D. Heitmann, M. P. Kostylev, and D. Grundler, Phys. Rev. Lett. 104, 207205 (2010).
[31] J. Ding, M. Kostylev, A. O. Adeyeye, Phys. Rev. Lett. 107, 047205 (2011).
[32] V. Au, E. Ahmad, O. Dmytriiev, M. Dvornik, T. Davison, and V. V. Kruglyak, Appl. Phys. Lett. 100, 182404 (2012).
[33] M. Madami, G. Gubbiotti, S. Tacchi, and G. Carlotti, Solid State Physics 63, 79 (2012).
[34] A. Barman and A. Haldar, Solid State Physics 65, 1 (2014).
[35] S. Tacchi, G. Duerr, J.W. Klos, M. Madami, S. Neusser, G. Gubbiotti, G. Carlotti, M. Krawczyk, and D. Grundler, Phys. Rev. Lett. 109, 137202 (2012).
[36] According to experimental studies in MCs composed of polycrystalline Co and Py nanostructures deposited with electron beam lithography magnetocrystalline and magnetic anisotropy from the interface with the substrate is small and have negligible effect on SW dynamics. [20, 35] Thus, in our calculations devoted to similar structures we have neglected magnetic anisotropies.
[37] The imaginary part of $\omega$ is closer to the respective value of homogeneous Py film than of Co film for SWs investigated in this paper, because the main contribution to these excitations comes from Py. Similar as in planar MCs, [20] this damping shall not influence significantly experiments proposed at the end of this paper.
[38] IDOS defined according with Eq. (2) is in fact the sum of the number of SW frequencies up to the frequency $f_m$. In Fig. 1(c), we marked magnonic gaps (between frequencies of the last and the first bulk mode, in the band below and above given gap, respectively). Width of these frequency ranges can by slightly overestimated due to quantization of frequencies.