Research article

Estimating the time interval between transmission generations when negative values occur in the serial interval data: using COVID-19 as an example

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Abstract: The coronavirus disease 2019 (COVID-19) emerged in Wuhan, China in the end of 2019, and soon became a serious public health threat globally. Due to the unobservability, the time interval between transmission generations (TG), though important for understanding the disease transmission patterns, of COVID-19 cannot be directly summarized from surveillance data. In this study, we develop a likelihood framework to estimate the TG and the pre-symptomatic transmission period from the serial interval observations from the individual transmission events. As the results, we estimate the mean of TG at 4.0 days (95%CI: 3.3–4.6), and the mean of pre-symptomatic transmission period at 2.2 days (95%CI: 1.3–4.7). We approximate the mean latent period of 3.3 days, and 32.2% (95%CI: 10.3–73.7) of the secondary infections may be due to pre-symptomatic transmission. The timely and effectively isolation of symptomatic COVID-19 cases is crucial for mitigating the epidemics.

Keywords: coronavirus disease 2019; COVID-19; time of generation; serial interval; epidemic; modelling

1. Introduction

The transmission of infectious disease is commonly parameterized as a dynamical process that
is governed by the reproduction number and the time interval between the transmission generations [1–3]. The serial interval (SI) is the period of time between the onset of symptoms in an infector to that in an associated infectee [4–7], which is observable and determined with the onsets of symptoms. The time interval between transmission generations (TG) is defined as the period of time between the onset of the infectiousness in a primary case (i.e., infector) to the onset of the infectiousness in an associated secondary case (i.e., infectee) infected by the primary case. Given that an infected individual may commonly become infectious prior to the onset of symptoms, the TG is not observable based on symptoms [6,8], which indicates the pre-symptomatic transmission may occur. Figure 1 illustrates the difference between TG and SI in a transmission chain.

When the infectious period starts with the onset of symptoms, i.e., the latent and incubation periods perfectly match, the TG coincides with the SI, and as such, the pre-symptomatic transmission will not occur [6]. In most of the situations, the latent period may be shorter than the incubation period, and thus, the pre-symptomatic transmission may occur. In minor occasions, when the onset of symptoms in the infectee is earlier than that in its infector, the observed SI will be negative. By contrast, the TG is always non-negative. In the real-world situation, due to the TG data are unobservable, the data of SI are commonly used as a proxy to approximate the true patterns of TG. This approach may become biased and inefficient when there are negative SI observations in the SI data.

**Figure 1.** The demonstrative timeline of a transmission chain for a pair of infector and infectee.

Recently, the coronavirus disease 2019 (COVID-19), caused by the severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), emerged in Wuhan, China in the end of 2019 [9–11]. The COVID-19 spread to over 100 foreign countries in a short period of time [12,13]. The World Health Organization (WHO) declared the outbreak to be a public health emergency of international concern on January 30, 2020 [14]. By the end of March 2020, there are over 500000 COVID-19 confirmed cases globally [15]. In previous studies [16–18], negative SIs are observed in the transmission events of COVID-19, and considered as a support for the potential risk of pre-symptomatic transmission. Estimating the TG as well as the pre-symptomatic transmission period is essential for understanding the transmission patterns of the COVID-19 and future infectious disease modelling studies.
In this study, we developed a novel likelihood-based framework to estimate the generation time and the risk of pre-symptomatic transmission of COVID-19 by using the observed serial interval data.

2. Materials and Methods

The TG is defined on the basis of the infectiousness onset of the infector. Since the infectiousness onset represents the readiness (i.e., start time) of a case to generate other consecutive cases, the TG is the time interval between the start of having transmissibility of two consecutive cases in a transmission chain. Note that the definition of TG is different from that of the generation time (GT), and the latter is defined on a per the exposure of infector basis. All of GT, TG and SI measure the period of time required for an infector to generate an infectee, i.e., the ‘infector’ in the subsequent transmission generation. Although both GT and TG are non-negative (whereas the SI could be negative), the TG is focusing on the transmissibility, and thus more relevant to the spread of infectious diseases.

Since the incubation period is not shorter than the latent period for an individual patient [6], we denote the difference of the incubation period minus the latent period as \( d \), and thus \( d \geq 0 \), see Fig 1. We name this period \( d \) as pre-symptomatic transmission period. Let \( d_1 \) and \( d_2 \) denotes the pre-symptomatic transmission periods for the infector and infectee respectively. The \( d_1 \) and \( d_2 \) are considered as independent and identically distributed (IID) random variables determined by a probability density function (PDF) \( h() \) with mean \( \mu_d \) and standard deviation (SD) \( \sigma_d \). The TG, denoted by \( g \), is determined by a PDF \( \delta() \) with mean \( \mu_g \) and SD \( \sigma_g \). Hence, the SI, denoted by \( s \), is defined by \( s = g + d_2 - d_1 \), which is determined by a PDF denoted by \( f() \). By convolution, the \( f() \) can be formulated in Eq (1).

\[
f(x) = \int \delta(x - y) \cdot [\int h(z) \cdot h(y + z) \, dz] \, dy.
\] (1)

Straightforwardly, the expectations of \( g \) and \( s \) are equal, i.e., \( E[g] = E(s) \), which indicates the expectation of TG and the expectation of SI are consistent.

With the PDF formulated in Eq (1), the associated likelihood profile can be adopted to estimate the parameters in PDFs of \( h() \) and \( \delta() \). The likelihood, \( l() \), is defined as in Eq (2).

\[
l(\Theta|x_1, ..., x_n) = \Sigma_i \log f(x_i|\Theta), \tag{2}
\]

where the \( \Theta \) denotes the vector of the PDF parameters to be estimated, and the \( x_i \)s are the SI observations. Following previous studies [10,17,19–21], Gamma distributions are presumed for both \( h() \) and \( \delta() \) for demonstration purpose. As such, by fitting to the SI data in Du et al. [16], the means and SDs of \( d \) and \( g \), i.e., \( \mu_d, \sigma_d, \mu_g \) and \( \sigma_g \), can be estimated by using the maximum likelihood estimation approach. The 95% confidence intervals (95%CI) are calculated by using the profile likelihood estimation framework with a cutoff threshold determined using a Chi-square quantile [22].

Furthermore, the percentage of the secondary infections, denoted by \( \eta \), due to pre-symptomatic transmission can be estimated by Eq (3).

\[
\eta = \left[ \int_{-\infty}^{0} \left[ \int \delta(z) \cdot h(x + z) \, dz \right] \, dx \right] \times 100\%.
\] (3)
3. Results and discussions

For COVID-19, we estimate the mean of TG ($\mu_g$) at 4.0 days (95%CI: 3.3–4.6) and SD ($\sigma_g$) at 3.6 days (95%CI: 2.6–4.5), see Figure 2. The $\mu_g$ estimate is consistent with mean SI at 4.0 days (95%CI: 3.5–4.4) estimated in Du et al. [16], which is also largely consistent with the mean SI estimated in [18,20,21]. The $\sigma_g$ estimate is smaller than the SD of SI estimated at 4.8 days (95%CI: 4.5–5.1) in Du et al. [16]. The smaller SD indicates that TG may be considered as a more efficient estimator of GT than SI. From a Gamma distribution with mean 4.0 days and SD 3.6 days, we report that the TG has median at 3.0 days, interquartile range (IQR) from 1.4 to 5.5 days, 95% centile from 0.2 to 13.5 days, and 95% percentile at 11.1 days.

The pre-symptomatic transmission period ($d$) is estimated with mean ($\mu_d$) 2.2 days (95%CI: 1.3–4.7) and SD ($\sigma_d$) 2.3 days (95%CI: 1.9–3.0), see Figure 2. From a Gamma distribution with mean 2.2 days and SD 2.3 days, we report that the $d$ has median at 1.5 days, IQR from 0.6 to 3.0 days, 95% centile from 0.1 to 8.4 days, and 95% percentile at 6.8 days. Since the $d$ is defined as the difference of the incubation period minus the latent period, by using the mean incubation period at 5.5 days and SD at 2.4 days estimated from previous studies on average [10,24–26], we approximate the mean latent period at 3.3 days.

By using Eq (3), we estimate the percentage of the secondary infections due to pre-symptomatic transmission ($\eta$) at 32.2% (95%CI: 10.3–73.7). This estimate appears largely in line with the range from 4% to 44% found in [17]. We also compare with 37% (95%CI: 28–45) in [27] and 44% (95%CI: 25–69) in [28], respectively. Although our estimate appears the lowest among the three existing
estimates, we note the 95% CIs are largely aligned. On the one hand, this indicates that there will still be 32.2% of the secondary COVID-19 cases if immediate isolation is only implemented on the symptomatic cases (i.e., no quarantine for the asymptomatic close contacts). This figure increases to 72.1% if the isolation of the symptomatic cases delayed for 1 day after the symptom onset. On the other hand, if the isolation is implemented on all cases immediately after the onset of symptoms, the basic reproduction number larger than \((1/32.2\%) = 3.1\) is required for sustaining the COVID-19 outbreak. Given the basic reproduction number of COVID-19 is less likely to reach 3.1 \([3,10–12,16,29–31]\), timely and effectively isolation of symptomatic cases is crucial for mitigating the outbreaks.

We further explore the difference in the reproduction number \((R)\) calculation based on TG or SI. By using the Lotka-Euler equation \([8]\), we formulate \(R = 1/M(−γ|f)\). The \(γ\) is the intrinsic growth rate of the epidemic curve, which can be estimated from the disease surveillance data at the early phase of the outbreak \([1,10,11,23]\). The function \(M(\cdot|f)\) represents the Laplace transform, known to statisticians as the moment generating function, of the distribution function of SI or TG, denoted by \(f(\cdot)\). Here, we discuss three scenarios for comparison the \(R\) calculation. Those include considering \(f(\cdot)\) as

- (naive scenario:) a shifted Dirac delta function with mean at 4.0 days;
- the estimated of Gamma distribution of TG with mean at 4.0 days and SD at 3.6 days; and
- the empirical distribution of SI directly from the SI data in Du et al. \([16]\).

In Figure 3, difference in the reproduction number calculation can be observed when using different metrics to measure the interval between transmission generations. There are considerable differences among the three calculations when the \(γ\) increases, which is also found in \([8]\) previously. Since the \(R\) is expected to be an increasing function of \(γ\) theoretically, we remark that using TG for the reproduction number calculation may be more reasonable.

The study has limitations. First, the dataset used in this analysis might be biased toward more severe cases, as pointed out in \([16]\), ‘the rapid isolation of such case-patients might have prevented longer serial intervals’, which might, together with the recalling biasness, potentially lead to an underestimation of the mean TG comparing to that in a COVID-19 outbreak without control measures. Two recent studies also used public available COVID-19 surveillance data and had the sample mean of SI at 5.0 days in \([27]\) and 5.8 days in \([28]\), respectively, which appear higher than the sample mean of SI at 4.0 days in \([16]\) so as in this study. Second, as a data-driven analysis, our estimates are relying on both statistical framework and quality of the SI observations. And we remark that the differences in the datasets of \([16,27,28]\) might lead to differences in the TG estimates, which is unsurprisingly. Third, the estimation of \(η\) is conducted under the assumption that individuals are equally infectious before and after symptoms begin. In practice, the individual infectiousness is not necessarily identical, but we presume this difference is unlikely to be large. Furthermore, if the isolation date of each individual infector were available, our likelihood function, \(l\), in Eq (2) could be extended to a right-censoring version as below.

\[
l(\Theta|x_1, ..., x_n; τ_1, ..., τ_n) = \sum_l \log[f(x_l|\Theta) \cdot (\Delta(τ_l)\Theta)],
\]

where the \(τ_l\) is the day of isolation for the \(i\)-th infector since the onset of symptoms. The \(\Delta(\cdot)\) is the cumulative distribution function (CDF) of \(δ(\cdot)\), and other notations are the same as in Eq (2).
Figure 3. The comparison of the reproduction numbers calculated based on TG or SI estimated in this study. The grey shading area represents the situation when the reproduction number becomes less than unity. The yellow shading area highlights the range of the intrinsic growth rate from 0.1 to 0.2 days, which is considered as the range for COVID-19 from the existing literatures.

4. Conclusions

In summary, we estimate the mean of TG at 4.0 days (95%CI: 3.3–4.6), and the mean of pre-symptomatic transmission period at 2.2 days (95%CI: 1.3–4.7). We estimate the basic reproduction number of 1.6 (95%CI: 1.4–1.9), and there are 32.2% (95%CI: 10.3–73.7) of the secondary infections are due to pre-symptomatic transmission. We approximate the mean latent period of 3.3 days.

Availability of materials

All data used in this work were publicly available via the data source in Du et al. [16].

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Conflict of interest

The author declares no conflict of interest.
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