Percolation approach to quark gluon plasma
in high energy pp collisions

J. Dias de Deus and A. Rodrigues

CENTRA and Departamento de Física (IST),
Av. Rovisco Pais, 1049-001 Lisboa, Portugal

We apply continuum percolation to proton-proton collisions and look for
the possible threshold to phase transition from confined nuclear matter
to quark gluon plasma. Making the assumption that J/ψ suppression is a
good signal to the transition, we discuss this phenomenon for pp collisions,
in the framework of a dual model with strings.

In recent years, high energy heavy ion collision experiments have been trying
to collect information on the possible existence of the plasma of quarks and
 gluon (QGP). One of the strategies has been to look for differences in particle
production between high density central heavy ion collisions, and low density
ion collisions, nucleon-nucleus collisions and nucleon-nucleon collisions. At the
CENR/SPS (√s ≈ 19 GeV) and now at Brookhaven/RHIC (√s ≈ 130 – 200
GeV) several important general results have been obtained.

The charged particle density was found to increases with energy and the num-
ber of participating nucleons. The average transverse momentum also increases
with energy and particle density, <p_T> increases as well with the mass of the
produced particle. Strangeness increases with energy and particle density [4].
All these results, naturally excluding the dependence on the number of par-
ticipants, are qualitatively similar to results obtained in nucleon-nucleon and
nucleon-nucleus collisions [2]. None of them, separately, can then be taken as
clear evidence for the formation of the QGP.

On the other hand, the anomalous suppression of the ratio J/ψ over Drell-
Yan production [3], at large associated transverse energy E_T, has been widely
accepted as good signal for QGP formation [4]. In fact, no such effect was seen
in lower density nucleus-nucleus, nucleon-nucleus or nucleon-nucleon collisions.

In this note, we shall argue that if the J/ψ suppression has its origin in the
creation of an extended colour conducting region, as in percolation, the same

1 work at: Escola Secundária da Ramada, Portugal
kind of suppression should occur even in nucleon-nucleon ($pp$ or $p\bar{p}$) collisions at high enough energy.

We shall work here in the framework of multi-collision models, namely the Dual Parton Model (DPM) \[5\], but try to be as general as possible. The basic ideas are: 1) nucleus-nucleus collisions can be built, in a non trivial manner, from nucleon-nucleon collisions; 2) nucleon-nucleon collisions occur with formation of $2k$ intermediate strings, strings are always in pairs, $k \geq 1$ being a function of energy; and 3) strings may fuse and percolate \[6\] a process destroying naive additivity of elementary collisions.

The key parameter in transverse plane string percolation is the dimensionless tranverse density $\eta$, with

$$\eta = \frac{r^2}{R^2} N_s,$$  \hfill (1)

where $r$ is the tranverse radius of the string (we shall take $r = 0.2$ fm), $R$ the radius of the interaction area, and $N_s$ the number of strings. Percolation occurs, in the $R \to \infty$, $N_s \to \infty$ limit, for $\eta \geq \eta_c \approx 1.15$.

As fusion and percolation of strings also occur in nucleon-nucleon collisions it is clear that the $J/\psi$ over D.Y. ratio will be strongly affected as $\eta$ approaches $\eta_c$. In the $pp(p\bar{p})$ case \[4\] becomes

$$\eta = \left( \frac{r_s}{R_p} \right)^2 2k$$ \hfill (2)

where $R_p$ is the effective proton radius. As the observed increase of particle densities with energy is mostly due to the increase in the number of formed strings, $k$ is also an increasing function of energy. Thus $\eta$ may reach $\eta_c$ and become larger than $\eta_c$. This implies anomalous $J/\psi$ suppression.

Following previous work \[3,4\] we treat fusion and percolation of strings as a two dimensional continuum percolation problem \[8\]. We performed computer simulation by throwing $N$ discs (of radius 0.2 fm) into a uniform region (of radius of order of the radius of the proton, 1 fm) and counting the fraction of events $f(\eta)$ with percolation. In the $R \to \infty$, $N \to \infty$ limit $f(\eta)$ becomes a step function with a sharp change at $\eta = \eta_c$. As $r/R$ is not so small, finite size effects are important, affecting mostly the slope ($a$) of the function at $\eta_c$, but the value of $\eta_c$ itself. The computer simulation results were fitted by the function,

$$f(\eta) = \left( 1 + e^{-(\eta-\eta_c)/a} \right)^{-1}$$ \hfill (3)
and the following values were found for the parameters: $a = 0.1666 \pm 0.0067$ and $\eta_c = 1.3584 \pm 0.0116$ (see Fig. 1)

Assuming now, as in [7], that the $J/\psi$ production is prevented in a plasma of colour charges [4] and that such situation corresponds to percolation and creation of a large conducting area [6] we obtain for the $J/\psi$ over Drell-Yan ratio,

$$R(\eta) = K(1 - f(\eta)) = K \left[ e^{(\eta - \eta_c)/a} + 1 \right]^{-1},$$

(4)

where $K \approx 55$ [9] is the value of the ratio at moderate energy.

The problem now is simply the problem of relating $\eta$ to $\sqrt{s}$, (2). In other words, we need to obtain a reasonable estimate for the energy dependence of $k$. This is what we shall attempt now.

If in nucleus-nucleus collisions $<\nu>$ is the average number of nucleon-nucleon collisions and $2k$ is the number of strings per nucleon-nucleon collision, the average number of strings $N_s$ is given by,

$$N_s = <\nu> \cdot 2k.$$

(5)

In nucleon-nucleon collisions $<\nu> = 1$ and $N_s = 2k$. From (4) and (5) the
condition for the percolation transition, in the case of $r/R \ll 1$, is

$$\eta = \eta_c = \left(\frac{r}{R}\right)^2 <\nu> 2k \approx 1.15.$$  \(6\)

By interpreting the NA50 anomalous $J/\psi$ suppression at $\sqrt{s} \approx 19$ GeV as the result of percolation, one can try to estimate the number of formed strings in nucleon-nucleon collisions. The basic information is that anomalous suppression is absent in S-U central collisions, but it is present in Pb-Pb central collisions \[3\]. This means, $\eta_{S-U} < 1.15$ and $\eta_{Pb-Pb} > 1.15$.

At relatively low energy, it is known, from NA49 and WA98, SPS experiments, that the number of nucleon-nucleon collisions in central nucleus-nucleus collisions is, roughly,(see, for instance [11]).

$$<\nu> \approx \frac{1.5 N_p}{2}, \quad (7)$$

where $N_p$ is the number of participants in a central AB, $A \leq B$, collision,

$$N_p \approx A^{\frac{2}{3}} (A^{\frac{1}{3}} + B^{\frac{1}{3}}). \quad (8)$$

On the other hand

$$R \approx A^{\frac{1}{3}} \text{fm}. \quad (9)$$

By using (7), (8) and (9) in (6) we obtain, from S-U

$$k(\sqrt{s} \approx 19) \lesssim 1.7, \quad (10)$$

and, from Pb-Pb,

$$k(\sqrt{s} \approx 19) \gtrsim 1.6. \quad (11)$$

As we are not so confident with these estimates we shall include an error of the order of 15% and study the ratio $J/\psi$ over $D\bar{Y}$ in the range

$$1.4 \lesssim k(\sqrt{s} \approx 19) \lesssim 1.9. \quad (12)$$

If we look now at the charged particle densities in $pp(p\bar{p})$ collisions, in the spirit of DPM, we have that particles are emitted from two kinds of strings: valence strings(V), always 2 from valence quark interactions and shorter sea
Fig. 2. Pseudo-rapidity density as function of c.m. energy. The solid line represents parameterization used to fit the data.

strings(S), from sea parton interactions, in a number growing with energy, 2(k – 1). The central charged particle density is written as (see, for instance, [10] and [11]),

\[
\frac{dN}{dy} \bigg|_{pp} = 2 \frac{dN}{dy} \bigg|_{V} + 2(k - 1) \frac{dN}{dy} \bigg|_{S}.
\]  (13)

On the right hand side of (13) we have both contributions, from V and S strings.

We assume that \( \frac{dN}{dy} \bigg|_{V} \) and \( \frac{dN}{dy} \bigg|_{S} \) are constant (“Feynman scaling”) and that the observed rise of the plateau is determined by the increase in the number of strings, i.e., by increase of \( k \). In the low energy limit, \( k \to 1 \), we thus have Feynman scaling with, from data [11,12],

\[
\frac{dN}{dy} \bigg|_{pp} \to V \quad k \to 1 \quad 2 \frac{dN}{dy} \bigg|_{V} \simeq 1.45 \pm 0.05
\]  (14)

In order to determine the energy dependence of \( k \) we do the following. Solve de equation (13) for \( k \), with \( \frac{dN}{dy} \bigg|_{V} \) fixed by (14) and using for \( \frac{dN}{dy} \bigg|_{pp} \) a parameterization to the \( pp(p\bar{p}) \) data [11],

\[
\frac{dN}{dy} \bigg|_{pp} = 0.957 + 0.0458 \ln \sqrt{s} + 0.0494 \ln^2 \sqrt{s}
\]  (15)
For each of the limiting values of $k(\sqrt{s} \approx 19)$ (12), combined with the limiting values of $\frac{dN}{dy}|_{V}$ (14), we adjust the constant $\frac{dN}{dy}|_{S}$ to obtain agreement with (15). The fit to $\frac{dN}{dy}|_{pp}$ high energy data for $\sqrt{s} \geq 20$ GeV is shown in (Fig. 2).

In Fig. 3 we present the $\sqrt{s}$ dependence of the $J/\psi$ over DY ratio, equation (4) and (2), for the two limiting values of $k$. In conclusion: we expect a fast drop of $J/\psi$ over DY ratio in $pp$ ($p\bar{p}$) collisions in the energy range $150 \lesssim \sqrt{s} \lesssim 1000$ GeV/nucleon. These are energies of RIHC and Tevatron.

Our work can be criticized from several different points of view:

1) The $J/\psi$ over DY ratio is also affected by internal absorption and as the number of strings increases with energy, absorption makes the ratio continuously decrease with energy. This effect was not included.

2) As charm production probability increases with energy the $J/\psi$ over DY ratio should have a tendency to increase with energy. This correction was also not included. One should, perhaps, in future consider the ratio $J/\psi$ over $c\bar{c}$ production as the reference quantity.

3) As in $pp$ ($p\bar{p}$) collisions the interaction radius increases slowly with energy the critical value of $\eta, \eta_c$, should decrease with energy. However the important effect is the actual decrease of $\eta \sim \frac{1}{R^2}$ with the consequence that the percolation transition occurs at higher value of energy.

4) The model is purely soft model and hard effects related to the increase of $<p_T>$ and changes in multiplicities were not included. These effects...
can be accounted for with fusion of strings but were not considered here. They are being studied now.
5) Non-uniform distributions in impact parameter (like gaussian distributions) give rise to an increase of $\eta_c$ (see last paper in [4]), and consequently the percolation transition will tend to be displaced to higher energy.
6) One may question the validity of continuum percolation arguments when the ratio $r/R$ is so large, $r/R \simeq 1/5$. Technically there is no problem, but we are not sure of the validity of the treatment.

While finishing this paper we became aware of the work of T. Alexopoulos et al. [13] dealing with evidence for deconfinement at Tevatron ($\sqrt{s} = 1.8$ TeV).

Acknowledgments

We would like to thank Roberto Ugoccioni for help at several stages in this research project.

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