Collevecchio, Andrea; Elçi, Eren Metin; Garoni, Timothy M.; Weigel, Martin
On the coupling time of the heat-bath process for the Fortuin-Kasteleyn random-cluster model. (English) Zbl 1428.82008 J. Stat. Phys. 170, No. 1, 22-61 (2018).

Summary: We consider the coupling from the past implementation of the random-cluster heat-bath process, and study its random running time, or coupling time. We focus on hypercubic lattices embedded on tori, in dimensions one to three, with cluster fugacity at least one. We make a number of conjectures regarding the asymptotic behaviour of the coupling time, motivated by rigorous results in one dimension and Monte Carlo simulations in dimensions two and three. Amongst our findings, we observe that, for generic parameter values, the distribution of the appropriately standardized coupling time converges to a Gumbel distribution, and that the standard deviation of the coupling time is asymptotic to an explicit universal constant multiple of the relaxation time. Perhaps surprisingly, we observe these results to hold both off criticality, where the coupling time closely mimics the coupon collector’s problem, and also at the critical point, provided the cluster fugacity is below the value at which the transition becomes discontinuous. Finally, we consider analogous questions for the single-spin Ising heat-bath process.

MSC:
82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
82M31 Monte Carlo methods applied to problems in statistical mechanics
60J22 Computational methods in Markov chains
65C05 Monte Carlo methods
65C40 Numerical analysis or methods applied to Markov chains
60K35 Interacting random processes; statistical mechanics type models; percolation theory

Keywords:
coupling from the past; relaxation time; random-cluster model; Markov-chain Monte Carlo

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References:
[1] Aizenman, M; Duminil-Copin, H; Sidoravicius, V, Random currents and continuity of Ising model's spontaneous magnetization, Commun. Math. Phys., 334, 719-742, (2015) · Zbl 1315.82004 · 10.1007/s00220-014-2093-y
[2] Baxter, RJ, Solvable eight-vertex model on an arbitrary planar lattice, Philos. Trans. R. Soc. A, 289, 315-346, (1978) · 10.1098/rsta.1978.0062
[3] Beffara, V; Duminil-Copin, H, The self-dual point of the two-dimensional random-cluster model is critical for \( q \geq 1 \), Probab. Theory Relat. Fields, 153, 511-542, (2012) · Zbl 1257.82014 · 10.1007/s00440-011-0353-8
[4] Billingsley, P: Probability and Measure. (Wiley Series in Probability and Statistics), 3rd edn. Wiley, New York (1994) · Zbl 0411.60001
[5] Cesi, F; Guadagni, G; Martinelli, F; Schonmann, RH, On the two-dimensional stochastic Ising model in the phase coexistence region near the critical point, J. Stat. Phys., 85, 55-102, (1996) · Zbl 0937.82004 · 10.1007/BF02175556
[6] Chayes, L; Machta, J, Graphical representations and cluster algorithms II, Phys. A, 254, 477-516, (1998) · 10.1016/S0378-4371(97)00637-7
[7] Chow, Y.S., Teicher, H.: Probability Theory: Independence, Interchangeability, Martingales. Springer, New York (1978) · Zbl 0399.60001 · 10.1007/978-1-4684-0062-5
[8] Deng, Y; Blöte, H, Simultaneous analysis of several models in the three-dimensional Ising universality class, Phys. Rev. E, 68, 036125, (2003) · 10.1103/PhysRevE.68.036125
[9] Deng, Y; Garoni, T; Machta, J; Ossola, G; Polin, M; Sokal, A, Critical behavior of the chayes-machta-swendsen-Wang dynamics, Phys. Rev. Lett., 99, 055701, (2007) · 10.1103/PhysRevLett.99.055701
[10] Deng, Y; Garoni, TM; Sokal, AD, Critical speeding-up in the local dynamics of the random-cluster model, Phys. Rev. Lett., 98, 230602, (2007) · 10.1103/PhysRevLett.98.230602
[11] Duminil-Copin, H., Gagnebin, M., Harel, M., Manolescu, I., Tassion, V.: Discontinuity of the phase transition for the planar...
random-cluster and Potts models with $q > 4$. arXiv:1611.09877 (2016) · Zbl 1430.60080

[12] Duminil-Copin, H., Sidoravicius, V., Tassion, V.: Continuity of the phase transition for planar random-cluster and Potts models with $1 \le q \le 4$. arXiv:1505.04159 (2015) · Zbl 1357.82011

[13] Dyer, M; Greenhill, C; Ullrich, M, Structure and eigenvalues of heat-Bath Markov chains, Linear Algebra Appl., 454, 57-71, (2014) · Zbl 1291.15082 · doi:10.1016/j.laa.2014.04.018

[14] Elci, E.: Algorithmic and geometric aspects of the random-cluster model. Ph.D. thesis (2015)

[15] Elci, EM; Weigel, M, Efficient simulation of the random-cluster model, Phys. Rev. E, 88, 033303, (2013) · doi:10.1103/PhysRevE.88.033303

[16] Elci, EM; Weigel, M, Dynamic connectivity algorithms for Monte Carlo simulations of the random-cluster model, J. Phys., 510, 012013, (2014)

[17] Erdos, P; Renyi, A, On a classical problem of probability theory, Publ. Math. Inst. Hung. Acad. Sci. Ser. A, 6, 215-219, (1961) · Zbl 0102.35201

[18] Feller, W.: An Introduction to Probability Theory and Its Applications, vol. 1, 3rd edn. Wiley, New York (1968) · Zbl 0518.60021

[19] Friedli, S., Velenik, Y.: Statistical Mechanics of Lattice Systems. A Concrete Mathematical Introduction. Cambridge University Press, Cambridge (2016) · Zbl 1407.82001

[20] Gheissari, R., Lubetzky, E.: Mixing Times Of Critical 2D Potts Models. arXiv:1607.02182 (2016) · Zbl 1392.82007

[21] Glozzi, F, Simulation of Potts models with real q and no critical slowing down, Phys. Rev. E, 66, 016115, (2002) · doi:10.1103/PhysRevE.66.016115

[22] Graham, R.L., Knuth, D.E., Patashnik, O.: Concrete Mathematics. A Foundation for Computer Science, 2nd edn. Addison-Wesley Publishing Company, Boston (1994) · Zbl 0836.00001

[23] Grassberger, P, Damage spreading and critical exponents for “model A” Ising dynamics, Phys. A, 214, 547-559, (1995) · doi:10.1016/0378-4371(94)00285-2

[24] Grimmett, G.: The Random-Cluster Model. Springer, New York (2006) · Zbl 1122.60003 · doi:10.1007/978-0-387-74733-9

[25] Grimmett, G.: Probability on Graphs. Cambridge University Press, Cambridge (2010) · Zbl 1228.60003 · doi:10.1007/978-0-521-13112-8

[26] Guo, H., Jerrum, M.: Random cluster dynamics for the Ising model is rapidly mixing. arXiv:1605.00139 · Zbl 1419.82013

[27] Häggström, O.: Finite Markov Chains and Algorithmic Applications. Cambridge University Press, Cambridge (2003) · Zbl 0999.60001

[28] Hartmann, A, Calculation of partition functions by measuring component distributions, Phys. Rev. Lett., 94, 050601, (2005) · doi:10.1103/PhysRevLett.94.050601

[29] Holm, J; Lichtenberg, K; Thorup, M, Poly-logarithmic deterministic fully-dynamic algorithms for connectivity, minimum spanning tree, 2-edge, and biconnectivity, J. ACM (JACM), 48, 723-760, (2001) · Zbl 1127.68408 · doi:10.1145/502090.502095

[30] Jaeger, F; Vertigan, DL; Welsh, DJA, On the computational complexity of the Jones and Tutte polynomials, Math. Proc. Camb. Philos. Soc., 108, 35-53, (1990) · Zbl 0747.57006 · doi:10.1017/S0305004100006936

[31] Janson, S., Tail bounds for sums of geometric and exponential random variables (2014), http://www2.math.uu.se/~svante/papers/sjN14.pdf

[32] Jerrum, M.: Mathematical Foundations of the Markov Chain Monte Carlo Method. In: Probabilistic Methods for Algorithmic Discrete Mathematics, pp. 116-165. Springer, New York (1998) · Zbl 0920.60001

[33] Laanait, L; Messager, A; Miracle-Solé, S; Ruiz, J; Shlosman, S, Interfaces in the Potts model I: pirogov-Sinai theory of the Fortuin-Kasteleyn representation, Commun. Math. Phys., 140, 81-91, (1991) · Zbl 0734.60108 · doi:10.1007/BF02099291

[34] Leadbetter, M.R., Lindgren, G., Rootzen, H.: Extremes and related properties of random sequences and processes. Springer, New York (1983) · Zbl 0510.60001 · doi:10.1007/978-1-4612-5449-2

[35] Levin, D.A., Peres, Y., Wilmer, E.L.: Markov Chains and Mixing Times. American Mathematical Society, Providence (2009) · Zbl 1160.60001

[36] Lindgren, G., Rootzen, H.: Extremes and related properties of random sequences and processes. Springer, New York (1983) · Zbl 0510.60001 · doi:10.1007/978-1-4612-5449-2

[37] Madras, N., Slade, G.: The Self-Avoiding Walk. Birkhauser, Boston (1996) · Zbl 0872.60001 · doi:10.1007/978-1-4612-4132-4

[38] McCoy, B.M., Wu, T.T.: The Two-Dimensional Ising Model. Harvard University Press, Cambridge (1973) · Zbl 0194.25003 · doi:10.1103/PhysRevLett.26.4548

[39] Mitzenmacher, M., Upfal, E.: Probability and Computing. Randomized Algorithms and Probabilistic Analysis. Cambridge University Press, Cambridge (2005) · Zbl 1092.60001 · doi:10.1017/CBO9780511813603

[40] Nacu, S, Glauber dynamics on the cycle is monotone, Probab. Theory Relat. Fields, 127, 177-185, (2003) · Zbl 1068.60014 · doi:10.1007/s00440-003-0279-x

[41] Nienhuis, B., Critical behavior of two-dimensional spin models and charge asymmetry in the Coulomb gas, J. Stat. Phys., 34, 731-761, (1984) · Zbl 0595.60071 · doi:10.1007/BF01099437

[42] Nightingale, MP; Bloete, HWJ, Dynamic exponent of the two-dimensional Ising model and Monte Carlo computation of the subdominant eigenvalue of the stochastic matrix, Phys. Rev. Lett., 76, 4545-4551, (1996) · doi:10.1103/PhysRevLett.76.4548

[43] Posfai, A.: Approximation Theorems Related to the Coupon Collector’s Problem. Ph.D. thesis (2010)

[44] Propp, J; Wilson, D, Exact sampling with coupled Markov chains and applications to statistical mechanics, Random Struct. Algorithms, 9, 223-252, (1996) · Zbl 0859.60067 · doi:10.1002/(SICI)1098-2418(199608/09)9:1/2<223::AID-RSA14>3.0.CO;2-O

[45] Sinclair, A.B., Alistair, Sinclair, A.: Random-Cluster Dynamics in $Z^2$. In: Twenty-Seven Annual ACM-SIAM
Symposium on Discrete Algorithms, pp. 498-513 (2016) · Zbl 1369.60067

[45] Sokal, AD; DeWitt-Morette, C (ed.); Cartier, P (ed.); Folacci, A (ed.), Monte Carlo methods in statistical mechanics: foundations and new algorithms, 131-192, (1997), New York · doi:10.1007/978-1-4899-0319-8

[46] Sweeny, M, Monte Carlo study of weighted percolation clusters relevant to the Potts models, Phys. Rev. B, 27, 4445-4455, (1983) · doi:10.1103/PhysRevB.27.4445

[47] Swendsen, RH; Wang, J.S, Nonuniversal critical dynamics in Monte Carlo simulations, Phys. Rev. Lett., 58, 86-88, (1987) · doi:10.1103/PhysRevLett.58.86

[48] Wang, J.S; Kozan, O; Swendsen, R, Sweeny and gliozi dynamics for simulations of Potts models in the Fortuin-Kasteleyn representation, Phys. Rev. E, 66, 057101, (2002) · doi:10.1103/PhysRevE.66.057101

[49] Welsh, D.J.A.: Complexity: Knots, Colourings and Counting, London Mathematical Society Lecture Note Series, vol. 186. Cambridge University Press, Cambridge (1993) · Zbl 0799.68008 · doi:10.1017/CBO9780511752506

[50] Deng, Youjin, Garoni, Timothy, M., Sokal, Alan, Zhou, Zongzheng: Dynamic critical behavior of the Chayes-Machta random-cluster algorithm II: Three-dimensions. In preparation

[51] Young, P.: Everything You Wanted to Know about Data Analysis and Fitting but were Afraid to ask. Springer, New York (2015) · doi:10.1007/978-3-319-19051-8

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