Comments on
Brane Configurations with Semi-infinite D4 Branes

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Abstract

We consider four dimensional supersymmetric gauge field theories from brane configurations with the matter content given by semi-infinite D4 branes ending on both sides of NS branes. In M theory configuration, we discuss the splitting of the M5 brane into infinite cylindrical M5 branes (which decouple) and transversal M5 brane. The splitting condition appears naturally from the consistency of the different projections of the Seiberg-Witten curve.
1 Introduction

One of the main developments in the recent years is clarification of the idea that gauge theory and gravity are complementary descriptions of a single theory. Configurations in string/M theory have been very useful tools to study supersymmetric gauge field theories in different dimensions and with different amounts of unbroken supersymmetry. (see [1] for a detailed review and a complete set of references up to February 1998. For more recent works, see [2, 3, 24, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]).

The usual construction of a four dimensional field theory contains D4 branes with one finite space direction, two NS branes (parallel for \( \mathcal{N} = 2 \) supersymmetry or non-parallel for \( \mathcal{N} = 1 \) supersymmetry) and possible D6 branes. The gauge gluons are given by strings between D4 branes. If there are \( N \) D4 branes, the gauge group is \( SU(N) \) (\( SO(N) \) or \( Sp(N) \) in the presence of an orientifold O4 or O6 plane). The matter content can be given by either strings between D4 branes and D6 branes or strings between D4 branes and semi-infinite D4 branes ending on the NS branes [19]. To describe the Higgs moduli space one can use both approaches. For massive matter the discussion was initiated in [20] with semi-infinite D4 branes ending on one of the NS branes, and for massless matter the problem was solved in [21].

In this paper we will discuss a brane configuration with semi-infinite D4 branes ending on both NS branes (similar configurations are considered in [22, 23]). The Seiberg-Witten curve is derived, and we observe that it can be decomposed into reducible curves and this will determine a split of the M5 brane obtained after we raise the brane configuration to M theory.

In section 2 we consider the case of the group \( SU(N) \) and we explain in detail the M5 brane splitting. In section 3 we do the same thing for the case of orientifolds (when the gauge group is \( SO(N) \) or \( Sp(N) \)).

2 \( SU(N_c) \) Gauge Theories and M5 branes Splitting

Consider a brane configuration for \( SU(N_c) \) gauge group with \( N_f \) hypermultiplets in the fundamental representation. In the type IIA theory on a flat space-time with time \( x^0 \) and space coordinates \( x^1, \ldots, x^9 \), the brane configuration consists of two NS5 branes with worldvolume coordinates \( x^0, x^1, x^2, x^3, x^4, x^5, N_c \) D4 branes suspended between them in the \( x^6 \)-direction, \( N_r \) semi-infinite D4 branes on the right of the right NS brane (which we call the right semi-infinite D4 branes), and \( N_l \) semi-infinite D4 branes on the left of
the left NS brane (which we call the *left* semi-infinite D4 branes). This configuration preserves \( N = 2 \) supersymmetry in four dimensions. In the models described before in the literature (see [19, 20]), for simplicity, the semi-infinite D4 branes end on only one of the NS branes.

We are interested in the M theory interpretation of these brane configurations. As usual, we introduce the complex variables:

\[
v = x^4 + ix^5, \quad w = x^8 + ix^9, \quad y = \exp(-(x^6 + ix^{10})/R)
\]

where \( x^{10} \) is the eleventh coordinate of M theory which is compactified on a circle of radius \( R \). Now we rotate the right NS5 brane towards \( w \) direction. The new location of the right NS5 brane becomes \( u := w - \mu v = 0 \) and the left NS5 brane is still located at \( w = 0 \). Moreover we assume that \( M_r \) of the right semi-infinite D4 branes are massless (in the sense that they are located at \( w = 0 \)) and \( M_l \) of the left semi-infinite D4 branes are massless (in the sense that they are located at \( u = 0 \)). In order to be able to rotate the NS5 brane, the M-theory Seiberg-Witten curve must be rational. Since \( u \) and \( w \) are two rational functions on this rational curve, they are related by a linear fractional transformation. Thus, after suitable constant shifts, we have

\[
uw = \zeta
\]

where \( \zeta \) is a constant. Now we project this curve to \((y, u)\)-space to obtain:

\[
w^{M_l} \prod_{i=1}^{N_l-M_l} (u - u_i)y - P(u) = 0,
\]

where

\[
P(u) = u^{N_c} + p_1 u^{N_c-1} + \cdots + p_{N_c}
\]

is some polynomial of degree \( N_c \) because there are \( N_c \) finite D4 branes between the two NS branes and the the number of finite D4 branes gives the degree of \( P \).

Similarly if we project the curve to \((y, w)\)-space, we get

\[
Q(w)y - A w^{M_r} \prod_{i=1}^{N_r-M_r} (w - w_i) = 0
\]

where

\[
Q(w) = w^{N_c} + q_1 w^{N_c-1} + \cdots + q_{N_c},
\]

and \( A \) is a normalization constant.
In order for the equations (2), (3) and (5) to hold simultaneously, it is required that

$$P(u)Q(\zeta/u) \equiv Au^{M_l} \prod_{i=1}^{N_l-M_l} (u - u_i)(\zeta/u)^{M_r} \prod_{i=1}^{N_r-M_r} (\zeta/u - w_i)$$

(7)

for all $u \in \mathbb{C}$.

The general solutions for $P$ and $Q$ are of the form

$$P(u) = u^{N_c+M_l-N_r} P'(u)$$

(8)

$$Q(w) = w^{N_c+M_r-N_l} Q'(w).$$

(9)

assuming $N_c > N_r$ and $N_c > N_l$. The possible solutions for $P'$ and $Q'$ are

$$P'(u) = \prod_{i=1}^{M} (u - u_{\alpha_i}) \prod_{i=1}^{N_r-M_l-M} (u - \zeta/w_{\beta_i})$$

(10)

$$Q'(w) = \prod_{j \neq \alpha_i} (w - \zeta/u_j) \prod_{j \neq \beta_i} (w - w_j).$$

(11)

If we plug (8) into (3), we obtain

$$u^{M_l} \prod_{i=1}^{N_l-M_l} (u - u_i)y - u^{N_c+M_l-N_r} \prod_{i=1}^{M} (u - u_{\alpha_i}) \prod_{i=1}^{N_r-M_l-M} (u - \zeta/w_{\beta_i}) = 0,$$

(12)

$$w^{N_c+M_r-N_l} \prod_{j \neq \alpha_i} (w - \zeta/u_j) \prod_{j \neq \beta_i} (w - w_j)y - Aw^{M_r} \prod_{i=1}^{N_r-M_r} (w - w_i) = 0.$$  

(13)

Now we can factorize these equations into

$$u^{M_l} \prod_{i=1}^{M} (u - u_{\alpha_i}) \left( \prod_{j \neq \alpha_i} (u - u_j)y - u^{N_c-N_r} \prod_{i=1}^{N_r-M_l-M} (u - \zeta/w_{\beta_i}) \right) = 0$$

(14)

$$w^{M_r} \prod_{j \neq \beta_i} (w - w_j) \left( w^{N_c-N_l} \prod_{j \neq \alpha_i} (w - \zeta/u_j)y - A \prod_{i=1}^{N_r-M_l-M} (w - w_{\beta_i}) \right) = 0.$$  

(15)

These equations suggest that the Seiberg-Witten curve in the $(u, w, y)$-space may be decomposed into irreducible curves. In fact, this is possible when $M_l = M_r$ and $M = M_l + M_r$. In this case, we can see that the curve $u^{M_l} \prod_{i=1}^{M} (u - u_{\alpha_i}) = 0$ in $(u, y)$-space and the curve $w^{M_r} \prod_{j \neq \beta_i} (w - w_j) = 0$ in $(w, y)$ are the images of a curve which lies on a threefold $uw = \zeta$. They describe parts of the original M5 brane without the NS branes. This means that the NS branes plus a part of the D4 branes turn into an M5 brane as usual (which we call a transversal M5 brane) and the D4 branes which are lined on the opposite sides of the NS branes turn into M5 branes that go through the transversal M5 brane (which we call a cylindrical M5 brane). So the M5 brane is decomposed into
a cylindrical M5 brane located at $u = w = 0$, $M$ cylindrical M5 branes located at $u = u_{\alpha i}, w = w_{i}$ and the transversal M5 brane given by the last terms in (14) and (15). This is a phenomenon described in [22] in the discussion of the conifold compactification. A very similar discussion appears in [23] where semi-infinite D4 branes ending on both sides of NS branes are also considered. In their paper there is another type of splitting i.e. the M5 brane splits into a flat one (without D4 branes ending on it) and a non-trivial one. In field theory this is obtained at the root of the baryonic Higgs branch.

The third part, i.e. the transverse M5 brane, is a brane configuration for $SU(N_c - 3M_l)$ with $(N_l - M_l - M)$ flavors on the left and $(N_r - M_r - M)$ flavors on the right. It obvious that when there is no massless flavor (i.e. $M_l = M_r = 0$), the first two curves do not exist and, thus there is no factorization.

## 3 Orientifold O6 plane, SO (Sp) Groups and M5 Brane Splitting

- **$SO(2N_c)$ Case**

This part will be a generalization of the case considered in [24] where we have considered the case of fundamental flavor given by semi-infinite D4 branes and we have considered only the case when no flavor was massless. Here we go one step further and we consider both massive and massless flavor. The discussion is the same as in the $SU$ case.

To obtain the $SO$ group, we are going to introduce and orientifold O6 plane described by:

$$xy = \Lambda^{4N_c - 4 - 2N_f} v^4.$$  \hspace{1cm} (1)

Let us rotate the left NS5 brane toward $w$ and this determines a rotation of right brane in the mirror direction (towards $-w$) Thus the left NS5 brane is located at $w_+ = 0$ and the right NS5 brane is located at $w_- = 0$ where

$$w_{\pm} = w \pm \mu v.$$  \hspace{1cm} (2)

Let $\Sigma$ be the corresponding M-theory Seiberg-Witten curve. On $\Sigma$, the function $w_+$ will go to infinity only at one point and $w_+$ has only a single pole there, since there is only one NS5 brane i.e. the right NS5 brane. Thus we can identify $\Sigma$ with the punctured complex $w_+$-plane possibly after resolving the singularity at $x = y = v = 0$. Similarly, we can argue that $w_-$ has a single pole on $\Sigma$. Since these two functions are rational on
a rational curve, they are related by a linear fractional transformation. After suitable constant shifts, the functions \( w_\pm \) on \( \Sigma \) are related by

\[ w_+ w_- = \zeta \]  

(3)

where \( \zeta \) is a constant. Now we project this curve to \((y, w_+)-space. Then we obtain

\[ w_+^{M_f} \prod_{i=1}^{N_f-M_f} (w_+ - m_i)y - P(w_+) = 0, \]  

(4)

where

\[ P(w_+) = w_+^{2N_c} + p_1 w_+^{2N_c-1} + \cdots + p_2N_c \]  

(5)

is some polynomial of degree \( 2N_c \). Similarly if we project the curve to \((y, w_-)-space, we get

\[ Q(w_-)y - A w_-^{M_f} \prod_{i=1}^{N_f-M_f} (w_- - m_i) = 0 \]  

(6)

where

\[ Q(w_-) = w_-^{2N_c} + q_1 w_-^{2N_c-1} + \cdots + q_2N_c, \]  

(7)

and \( A \) is a normalization constant. In order for the equations (3), (4) and (6) to hold simultaneously, it is required that

\[ P(w_+)Q(\zeta/w_+) \equiv A w_+^{M_f}(\zeta/w_+)^{M_f} \prod_{i=1}^{N_f-M_f} (w_+ - m_i)(\zeta/w_+ - m_i) \]  

(8)

for all \( w_+ \in \mathbb{C} \). The solutions will be of the form

\[ P(w_+) = w_+^{2N_c-N_f+M_f} \prod_{i=1}^{M} (w_+ - m_{\alpha_i}) \prod_{i=1}^{N_f-M_f-M} (w_+ - \zeta/m_{\beta_i}) \]  

(9)

\[ Q(w_-) = w_-^{2N_c-N_f+M_f} \prod_{j \neq \alpha_i} (w_- - \zeta/m_{j}) \prod_{j \neq \beta_i} (w_- - m_{j}) \]  

(10)

if \( 2N_c \geq N_f - M_f \). With the choice of these \( P \) and \( Q \), the normalization constant \( A \) will be

\[ A = (-1)^{N_f-M_f-M} \frac{(\zeta)^{2N_c-M}}{\prod_{j \neq \alpha_i} m_{j}} \prod_{i=1}^{N_f-M_f-M} m_{\beta_i}. \]  

(11)
If we plug (9) into (4), we obtain

\[ w_{+}^{N_{f} - M_{f}} \prod_{i=1}^{M_{f}} (w_{+} - m_{i})y - w_{+}^{2N_{c} - N_{f} + M_{f}} \prod_{i=1}^{M_{f}} (w_{+} - m_{\alpha_{i}}) \prod_{i=1}^{N_{f} - M_{f} - M} (w_{+} + \zeta/m_{\beta_{i}}) = 0 \] (12)

\[ w_{-}^{-2N_{c} - N_{f} + M_{f}} \prod_{j \neq \alpha_{i}} (w_{-} - \zeta/m_{j}) \prod_{j \neq \beta_{i}} (w_{-} + m_{j})y - A w_{+}^{N_{f} - M_{f}} \prod_{i=1}^{N_{f} - M_{f} - M} (w_{-} - m_{i}) = 0. \] (13)

After factoring out the terms \( w_{+}^{M_{f}} \prod_{i=1}^{M_{f}} (w_{+} - m_{\alpha_{i}}) \) and \( w_{-}^{M_{f}} \prod_{j \neq \beta_{i}} (w_{-} - m_{j}) \), we obtain

\[ \prod_{j \neq \alpha_{i}} (w_{+} - m_{j})y - w_{+}^{-2N_{c} - N_{f} + M_{f}} \prod_{i=1}^{N_{f} - M_{f} - M} (w_{+} - \zeta/m_{\beta_{i}}) = 0 \] (14)

\[ w_{-}^{-2N_{c} - N_{f} + M_{f}} \prod_{j \neq \alpha_{i}} (w_{-} - \zeta/m_{j})y - A \prod_{i=1}^{N_{f} - M_{f} - M} (w_{-} - m_{\beta_{i}}) = 0. \] (15)

This is nothing but a brane configuration for \( SO(2N_{c} - M) \) theory with \( N_{f} - M_{f} - M \) flavors. The gauge group \( SO(2N_{c}) \) with \( N_{f} \) flavors is broken to the gauge group \( SO(2N_{c} - M) \) with \( N_{f} - M_{f} - M \) flavors which agrees with QCD. In brane geometry, the \( M \) semi-infinite D4 branes are matched together with finite D4 branes to move in \( w_{+} = 0 \) complex plane and in its mirror complex plane. The parts of (14) and (15) that decouple are again interpreted as infinite cylindrical M5 branes that go through the transverse M5 brane. For the case \( M = 0 \) we obtain the result of [24], so our results are consistent.

The terms which are factored out before arriving to the formulas (14) and (15) represent the cylindrical M5 branes which pass through the transverse M5 brane.

The extension to \( SO(2N_{c} + 1) \) is trivial. In the odd case the equations (4) and (3) become:

\[ \prod_{i=1}^{N_{f}} (w_{+} - m_{i})y - P(w_{+}) = 0 \] (16)

\[ Q(w_{-})y - A \prod_{i=1}^{N_{f}} (w_{-} + m_{i}) = 0 \] (17)

where

\[ P(w_{+}) = w_{+}^{2N_{c}} + p_{1}w_{+}^{2N_{c}-1} + \cdots + p_{2N_{c}} \] (18)

\[ Q(w_{-}) = w_{-}^{2N_{c}} + q_{1}w_{-}^{2N_{c}-1} + \cdots + q_{2N_{c}}. \] (19)

and the solutions will be of the form

\[ P(w_{+}) = w_{+}^{2N_{c}+1-N_{f}} \prod_{i=1}^{M} (w_{+} - m_{\alpha_{i}}) \prod_{i=1}^{N_{f}-M} (w_{+} + \zeta/m_{\beta_{i}}) \] (20)
Q(w) = w^{2N_c+1-N_f} \prod_{j \neq \alpha_i} (w - \zeta/m_j) \prod_{j \neq \beta_i} (w + m_j)  \tag{21}

if $2N_c \geq N_f$. When $M = 0$, we have a special solution

\begin{align*}
P(w+) &= w^{2N_c+1-N_f} \prod_{i=1}^{N_f} (w + \zeta/m_i) \\
Q(w-) &= w^{2N_c+1-N_f} \prod_{i=1}^{N_f} (w - \zeta/m_i)  \tag{22}
\end{align*}

which yields

\begin{align*}
\left(\prod_{i=1}^{N_f} m_i\right)^{y} \prod_{i=1}^{N_f} \left(\frac{w_+ - m_i}{w_+ + m_i}\right) = w^{2N_c+1}.  \tag{24}
\end{align*}

This agrees with [22] and, thus shows the consistency of our results.

The discussion for $M \neq 0$ goes the same as in the $SO(2N_c)$ case and one obtains the infinite cylindrical D4 branes which go through the transverse M5 brane.

- **$Sp(2N_c)$ Case**

The situation is similar to $SO$ case. Now the O6 plane is described by:

\[ xy = \Lambda^{2N_c-4-2N_f} u^{-4}. \tag{25} \]

Again the general solution is of the form

\begin{align*}
P(w+) &= w^{2N_c-N_f} \prod_{i=1}^{M} (w - m_{\alpha_i}) \prod_{i=1}^{N_f-M} (w_+ + \zeta/m_{\beta_i}) \\
Q(w-) &= w^{2N_c-N_f} \prod_{j \neq \alpha_i} (w_- - \zeta/m_j) \prod_{j \neq \beta_i} (w_- + m_j)  \tag{26}
\end{align*}

if $2N_c \geq N_f$. For $M = 0$, we get a special solution given by

\begin{align*}
xy &= \Lambda^{2N_c-4-2N_f} \mu^4 (w_+ - w_-)^4  \\
w_+w_- &= \zeta  \tag{27} \\
\left(\prod_{i=1}^{N_f} m_i\right)^{y} \prod_{i=1}^{N_f} \left(\frac{w_+ - m_i}{w_+ + m_i}\right) &= w^{2N_c}.  \tag{28}
\end{align*}

On a smooth surface

\[ x'y' = \Lambda^{2N_c-4-2N_f} \tag{29} \]
which maps onto the old surface via the map $x = x'v^{-2}$, $y = y'v^{-2}$, the special solution can be described by

$$
x' y' = \Lambda_{N=2}^{2N_c-4-2N_f} \quad (32)
$$

$$
w_+ w_- = \zeta \quad (33)
$$

$$
\left( \prod_{i=1}^{N_f} m_i \right) y' \prod_{i=1}^{N_f} \left( \frac{w_+ - m_i}{w_- + m_i} \right) = w_+^{2N_c+2}. \quad (34)
$$

If we map this curve to the old surface, then the last equation becomes:

$$
\left( \prod_{i=1}^{N_f} m_i \right) y \prod_{i=1}^{N_f} \left( \frac{w_+ - m_i}{w_- + m_i} \right) = v^{-2} w_+^{2N_c+2} \quad (35)
$$

which is the same as (5.27) of [25] after rescaling of variables.

The discussion for $M \neq 0$ is the same as before.

## 4 Conclusion

In this paper we have considered field theories obtained on the worldvolume of D4 branes suspended between two NS branes. The matter content is given by semi-infinite D4 branes ending on both NS branes (as opposed to the previously considered case when they end only on one NS brane [19, 20]). The Seiberg-Witten curve can be projected to $(y, u)$ and $(y, w)$ spaces and the requirement that both projections hold simultaneously suggests that the Seiberg-Witten curve may be decomposed into irreducible curves. This implies that in M theory, the M5 brane is split into reducible curves, one being the transversal M5 brane and the rest being infinite cylindrical M5 branes. We have discussed the case with or without an orientifold O6 plane.

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