Unitarity Constraints on Anomalous Quartic Couplings

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We obtain the partial-wave unitarity constraints on the lowest-dimension effective operators which generate anomalous quartic gauge couplings but leave the triple gauge couplings unaffected. We consider operator expansions with linear and nonlinear realizations of the electroweak symmetry and explore the multidimensional parameter space of the coefficients of the relevant operators: 18 dimension-eight operators in the linear expansion and 5 $O(p^4)$ operators in the derivative expansion. We study two-to-two scattering of electroweak gauge bosons and Higgs bosons taking into account all coupled channels and all possible helicity amplitudes for the $J=0,1$ partial waves. In general, the bounds degrade by factors of a few when several operator coefficients are considered non-vanishing simultaneously. However, this requires to consider constraints from both $J=0$ and $J=1$ partial waves for some sets of operators.

PACS numbers: 11.80.Et, 11.25.Db, 12.15.-y

I. INTRODUCTION

The structure of the triple (TGC) and quartic (QGC) electroweak gauge-boson interactions in the Standard Model (SM) is determined by the gauge symmetry $SU(2)_L \otimes U(1)_Y$. Therefore, it is important to measure both TGC and QGC, not only to further test the SM or have indications of new physics, but also to determine whether the gauge symmetry is realized linearly or nonlinearly in the low energy effective theory of the electroweak symmetry breaking sector [1].

Generically, deviations from the SM predictions for TGC and QGC are generated by higher-order operators parametrizing indirect effects of new physics. Collider experiments probe TGC in the pair production of electroweak gauge bosons while the study of QGC requires the production of three electroweak vector bosons, the exclusive production of gauge-boson pairs or the vector-boson-scattering production of electroweak vector boson pairs [2–17]. Therefore, the Wilson coefficients of effective operators that contain both TGC and QGC are more strongly constrained through the study of its TGC component.

For this reason most of present LHC searches for effects of QGC focus on the so-called genuine QGC operators, that is, operators generating QGC but that do not have any TGC associated with them. In a scenario where the $SU(2)_L \otimes U(1)_Y$ is realized linearly the lowest-order QGC are given by dimension-eight operators [18]. Alternatively, if the gauge symmetry is implemented nonlinearly the lowest-order QGC appear at $O(p^4)$ [19, 20].

It is well known that departures of the TGC and QGC from the SM predictions lead to the growth of scattering amplitudes [21], signaling the existence of new physics. Thus, when probing anomalous QGC one must verify whether perturbative partial-wave unitarity is satisfied to guarantee consistency of the analyses. This is all well established and it has been previously addressed in the literature [4, 22–24]. It is also implemented in some form in the QGC searches by both ATLAS and CMS collaborations, see for instance Refs. [9–13, 15, 17], either by introducing ad-hoc form factors or unitarization procedures, or by directly evaluating the maximum center-of-mass energy allowed by unitarity as obtained from the VBFNLO framework [25]. However, these unitarity studies are not complete since they consider just a few scattering channels, a limited set of QGC effective operators or restricted the analysis to the $J=0$ partial wave.
In this work we complement the existing literature on the subject by systematically presenting the unitarity bounds in the multidimensional parameter space of the coefficients of the relevant operators in both linear and nonlinear realizations of the electroweak symmetry. We study two-to-two scattering of electroweak gauge bosons and Higgs bosons taking into account all coupled channels, and all possible helicity amplitudes for the \( J = 0, 1 \) partial waves. Indeed we find that \( J = 1 \) partial wave unitarity effects are relevant to derive the most stringent limits in some scenarios when the effects of several operators are considered simultaneously.

This paper is organized as follows: we present in Section II the QGC operators that we consider in our analyses, as well as basic expressions of partial-wave unitarity needed for our studies. Section III contains our results that are discussed in Section IV.

II. ANALYSES FRAMEWORK

Here, we introduce the effective interactions considered in this work, as well as the unitarity relations that we use to constrain them.

A. Effective Lagrangian

1. Linear realization of the gauge symmetry

Assuming that the new state observed in 2012 is in fact the SM Higgs boson and that it belongs to an electroweak scalar doublet, we can construct a low-energy effective theory where the \( SU(2)_L \otimes U(1)_Y \) gauge symmetry is linearly realized \([24, 32]\) which takes the form

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{n=5}^{\infty} \sum_{i} \frac{f_i^{(n)}}{\Lambda^{n-4}} \mathcal{O}_i^{(n)},
\]

where the dimension–\( n \) operators \( \mathcal{O}_i^{(n)} \) involve gauge-bosons, Higgs doublets, fermionic fields, and covariant derivatives of these fields. Each operator has a corresponding Wilson coefficient \( f_i^{(n)} \) and \( \Lambda \) is the characteristic energy scale at which new physics (NP) becomes apparent.

Here, we are interested in operators that leads to QGC without a TGC counterpart. The lowest dimension of such genuine QGC operators is eight \([18]\). In what follows, we consider the bosonic dimension-eight operators relevant to two-to-two scattering processes involving Higgs and/or gauge bosons at tree level, and that conserve \( C \) and \( P \) \([33]\). Moreover, we classify them by the number of gauge-boson strength fields contained in the operator.

In the first class of genuine QGC, the operators contain just covariant derivatives of the Higgs field:

\[
\mathcal{O}_{S,0} = \left[ (D_\mu \Phi)^\dagger D_\nu \Phi \right] \times \left[ (D^\mu \Phi)^\dagger D^\nu \Phi \right], \quad \mathcal{O}_{S,1} = \left[ (D_\mu \Phi)^\dagger D^\mu \Phi \right] \times \left[ (D_\nu \Phi)^\dagger D^\nu \Phi \right],
\]

\[
\mathcal{O}_{S,2} = \left[ (D_\mu \Phi)^\dagger D_\nu \Phi \right] \times \left[ (D^\mu \Phi)^\dagger D^\nu \Phi \right],
\]

where \( \Phi \) stands for the Higgs doublet, the covariant derivative is given by \( D_\mu \Phi = (\partial_\mu + igW_{\mu j}^j \sigma_j^I + ig' B_{\mu j}^j) \Phi \) and \( \sigma_j^I \) \((j = 1, 2, 3)\) represent the Pauli matrices.

In the second class of genuine QGC the operators exhibit two covariant derivatives of the Higgs field, as well as two field strengths:

\[
\mathcal{O}_{M,0} = \text{Tr} \left[ \mathcal{W}_{\mu \nu} \mathcal{W}^{\mu \nu} \right] \times \left[ (D_\beta \Phi)^\dagger D^\beta \Phi \right], \quad \mathcal{O}_{M,1} = \text{Tr} \left[ \mathcal{W}_{\mu \nu} \mathcal{W}^{\nu \beta} \right] \times \left[ (D_\beta \Phi)^\dagger D^\mu \Phi \right],
\]

\[
\mathcal{O}_{M,2} = [B_{\mu \nu} B^{\mu \nu}] \times \left[ (D_\beta \Phi)^\dagger D^\beta \Phi \right], \quad \mathcal{O}_{M,3} = [B_{\mu \nu} B^{\nu \beta}] \times (D_\beta \Phi)^\dagger D^\mu \Phi, \quad \mathcal{O}_{M,4} = \left[ (D_\mu \Phi)^\dagger \mathcal{W}_{\beta \nu} D^\mu \Phi \right] \times B^{\beta \nu} + \text{h.c.},
\]

\[
\mathcal{O}_{M,5} = \left[ (D_\mu \Phi)^\dagger \mathcal{W}_{\beta \nu} D^\nu \Phi \right] \times B^{\beta \mu} + \text{h.c.},
\]

where \( \mathcal{W}_{\mu \nu} = W_{\mu \nu}^j \sigma_j^I \) is the \( SU(2)_L \) field strength while \( B_{\mu \nu} \) stands for the \( U(1)_Y \) one.
In addition to the above operators, there are also genuine QGC ones that contain just field strengths:

\[
\begin{align*}
\mathcal{O}_{T,0} &= \text{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] \times \text{Tr}[\hat{W}_{\alpha\beta}\hat{W}^{\alpha\beta}], \\
\mathcal{O}_{T,1} &= \text{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\alpha\mu}] \times \text{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\beta\nu}], \\
\mathcal{O}_{T,2} &= \text{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\alpha\mu}] \times \text{Tr}[\hat{W}_{\beta\nu}\hat{W}^{\beta\nu}], \\
\mathcal{O}_{T,5} &= \text{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] \times B_{\alpha\beta}B^{\alpha\beta}, \\
\mathcal{O}_{T,6} &= \text{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\alpha\mu}] \times B_{\beta\nu}B^{\beta\nu}, \\
\mathcal{O}_{T,7} &= \text{Tr}[\hat{W}_{\alpha\mu}\hat{W}^{\alpha\mu}] \times B_{\beta\nu}B^{\beta\nu}, \\
\mathcal{O}_{T,8} &= B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta}. \\
\end{align*}
\]

These 18 operators induce all possible modifications to vertices \{VVVV, VVVH and VVHH (V = W, Z and A)\} that are compatible of electric charge, C and P conservation; for further details on the anomalous vertices generated by each dimension-eight operator see Ref. [33].

2. Nonlinear \(O(p^4)\) realization of the gauge symmetry

In dynamical scenarios, the Higgs boson is a composite state, \(i.e.\) it is a pseudo-Nambu-Goldstone boson of an exact global symmetry. Therefore, the gauge symmetry of the low energy effective lagrangian is realized nonlinearly [34–37] and the effective lagrangian is a derivative expansion. In this case, the effective Lagrangian is written in terms of the SM fermions and gauge bosons and of the physical Higgs \(h\). The building block at low energies is a dimensionless unitary matrix transforming as a bi-doublet of the global symmetry \(SU(2)_L \otimes SU(2)_R\):

\[
U(x) = e^{i\sigma_3 \pi^a(x)/v}, \quad U(x) \to L U(x) R^\dagger,
\]

where \(L, R\) denote \(SU(2)_L, R\) global transformations, respectively and \(\pi^a\) are the Goldstone bosons. Its covariant derivative is given by

\[
\mathbf{D}_\mu U(x) \equiv \partial_\mu U(x) + ig \frac{\sigma^i}{2} W^i_\mu U(x) - \frac{ig}{2} B_\mu(x) U(x) \sigma_3.
\]

From this basic element it is possible to construct the chiral field

\[
V_\mu \equiv (\mathbf{D}_\mu U) U^\dagger,
\]

and the scalar chiral field \(T \equiv U \sigma_3 U^\dagger\). For further details see Ref. [33].

The lowest operators respecting \(C\) and \(P\) and exhibiting genuine QGC are of order \(p^4\), that in the notation of Refs. [19, 20] are

\[
\mathcal{P}_6 = \text{Tr}[V^\mu \nu \mu]\text{Tr}[V^\nu \nu \nu] F_6(h), \quad \mathcal{P}_{11} = \text{Tr}[V^\mu \nu \nu] \text{Tr}[V^\nu \nu \nu] F_{11}(h),
\]

that respect the \(SU(2)_c\) custodial symmetry and

\[
\mathcal{P}_{23} = \text{Tr}[V^\mu \nu \mu][\text{Tr}[TV^\nu]] F_{23}(h), \quad \mathcal{P}_{24} = \text{Tr}[V^\mu \nu \nu][\text{Tr}[TV^\mu]] F_{24}(h),
\]

\[
\mathcal{P}_{26} = (\text{Tr}[TV^\mu][\text{Tr}[TV^\nu]] F_{26}(h),
\]

which violate \(SU(2)_c\). \(F_i(h)\) are generic functions parametrizing the chiral-symmetry breaking interactions of \(h\). As we are looking for operators whose lowest order vertex contain four gauge bosons, we take \(f_i = 1\). So, the most general Lagrangian at \(O(p^4)\) for genuine QGC is

\[
\mathcal{L}^{p=4}_{QGC} = \sum_{i=6,11,23,24,26} f_i \mathcal{P}_i.
\]

It is interesting to notice that the above nonlinear operators do not contain photons.

B. Partial-wave unitarity

In the two-to-two scattering of electroweak gauge bosons \(V\)

\[
V_{1\lambda_1} V_{2\lambda_2} \to V_{3\lambda_3} V_{4\lambda_4}
\]
the corresponding helicity amplitude can be expanded in partial waves in the in the center–of–mass system as \[38\]

\[
\mathcal{M}(V_{1\lambda_1}V_{2\lambda_2} \rightarrow V_{3\lambda_3}V_{4\lambda_4}) = 16\pi \sum_J (2J+1) \sqrt{1+\delta_{V_{1\lambda_1}}^{V_{1\lambda_2}} \sqrt{1+\delta_{V_{1\lambda_3}}^{V_{1\lambda_4}}}} \lambda \mu \phi T^J \left(V_{1\lambda_1}V_{2\lambda_2} \rightarrow V_{3\lambda_3}V_{4\lambda_4}\right), \tag{12}
\]

where \(\lambda = \lambda_1 - \lambda_2, \mu = \lambda_3 - \lambda_4, M = \lambda_1 - \lambda_2 - \lambda_3 + \lambda_4\), and \(\theta (\phi)\) is the polar (azimuth) scattering angle. \(d\) is the usual Wigner rotation matrix. For processes where we substitute a vector boson by a Higgs, this expression can be used by setting the correspondent \(\lambda\) to zero.

Partial-wave unitarity for a given elastic channel requires that

\[
|T^J(V_{1\lambda_1}V_{2\lambda_2} \rightarrow V_{1\lambda_1}V_{2\lambda_2})| \leq 1, \tag{13}
\]

where we considered the limit \(s \gg (M_{V_5} + M_{V_6})^2\). More stringent bounds can be obtained by diagonalizing \(T^J\) in the particle and helicity space and then applying the condition in Eq. (13) to each of the eigenvalues.

In our analysis we evaluated \(T^0\) and \(T^1\) amplitude matrices in particle and parameter space as a function of the Wilson coefficients of the dimension-eight operators and the nonlinear ones. These matrices are formed with the \(s\)-divergent parts of the amplitudes corresponding to all combinations of gauge boson and Higgs pairs with a given total charge \(Q = 2, 1, 0\) with possible projections on a given partial wave \(J\) which are:

| \((Q, J)\) | States | Total |
|-----------|--------|-------|
| (2, 0)    | \(W_+^2 W_+ W_0 W_0^+\) | 3     |
| (2, 1)    | \(W_+^2 W_+^+ W_0 W_0^+\) | 6     |
| (1, 0)    | \(W_+^+ Z_0 W_0^+\) | 6     |
| (1, 1)    | \(W_0^+ Z_0 W_0 W_0^+\) | 14    |
| (1, 2)    | \(W_0^+ W_0 W_0^+\) | 12    |
| (2, 1)    | \(W_0^+ W_0 W_0^+\) | 18    |

where upper indices indicate charge and lower indices helicity. We also display in Eq. (14) the dimensionality of the particle and helicity matrix for each independent \((Q, J)\) channel. Parity conservation at tree level leads to the reduction of number of independent helicity amplitudes once we take into account the relation

\[
T^J(V_{1\lambda_1}V_{2\lambda_2} \rightarrow V_{3\lambda_3}V_{4\lambda_4}) = (-1)^{\lambda_1 - \lambda_2 + \lambda_3 + \lambda_4} T^J(V_{1\lambda_1}V_{2\lambda_2} \rightarrow V_{3\lambda_3}V_{4\lambda_4}). \tag{15}
\]

Furthermore, time-reversal invariance also reduces the number of helicity amplitudes that need to be evaluated.

As an illustration, let us study the \(Q = 2\) and \(J = 0\) channel. The leading term in the center-of-mass energy \(\sqrt{s}\) of the unitarity violating amplitudes is \(\mathcal{O}(s^2)\) as expected from a naive dimensional analysis\(^1\). Working in the basis \((W_+^2, W_0^+, W_0^+\), \(W_+^+\), \(W_0^+\)) of the 3 \(\times\) 3 matrix in helicity particle space reads

\[
\frac{1}{96\pi \Lambda^2} \left(\begin{array}{c}
6f_{T_1} + 3f_{T_2} \\
0 \\
0
\end{array}\right) \left(\begin{array}{c}
0 \\
3f_{S_0} + f_{S_1} + f_{S_2} \\
0 \\
4f_{T_0} + 8f_{T_1} + f_{T_2}
\end{array}\right). \tag{16}
\]

The strongest unitarity limits from this channel comes from the eigenvalues of the above matrix:

\[
\left|\frac{3f_{S_0} + f_{S_1} + f_{S_2}}{\Lambda^4} \right| \leq 96\pi, \quad \left|\frac{2f_{T_0} + f_{T_1} - f_{T_2} s^2}{\Lambda^4} \right| \leq 48\pi, \quad \left|\frac{2f_{T_0} + 7f_{T_1} + 2f_{T_2} s^2}{\Lambda^4} \right| \leq 48\pi. \tag{17}
\]

Clearly this allows to constraint the coefficients only under the assumption of no cancellations between the different coefficients. So in order to obtain the most stringent bounds on the full set of coefficients we diagonalize the six \(T^J\) matrices and impose the constraint Eq. (13) on each of their eigenvalues.

\(^1\) Since genuine QGC do not have a TGC counterpart, gauge invariance does not lead to the cancellation of the \(s^2\) terms \[39\] in contrast to what happens for dimension-six QGC \[64\].
| Wilson Coefficient | 1 operator | all 3 operators |
|--------------------|------------|----------------|
| $\frac{f_{S,0}}{\Lambda}$ | $32\pi s^{-2}$ | $20 (1.2) \text{ TeV}^{-4}$ | $48\pi s^{-2}$ | $30 (1.9) \text{ TeV}^{-4}$ |
| $\frac{f_{S,1}}{\Lambda}$ | $96\pi s^{-2}$ | $8.5 (0.53) \text{ TeV}^{-4}$ | $288\pi s^{-2}$ | $35 (2.2) \text{ TeV}^{-4}$ |
| $\frac{f_{S,2}}{\Lambda}$ | $96\pi s^{-2}$ | $8.5 (0.53) \text{ TeV}^{-4}$ | $288\pi s^{-2}$ | $35 (2.2) \text{ TeV}^{-4}$ |

TABLE I: Unitarity constraints on the Wilson coefficients of the $O_{S,j}$ operators (Eq. 2) when just one operator coefficient is non-vanishing (second and third column), as well as, when all coefficients are included (last two columns). For convenience, in the third and fifth columns, we give the numerical value of the bounds for maximal subprocess center-of-mass energy of 1.5 and 3 TeV.

### III. RESULTS

We start our analysis studying the operators that contain four covariant derivatives of the Higgs field, which are given in Eq. (2). The strongest unitarity limits for these operators originate from the $J = 0$ partial wave. When we diagonalize the helicity-particle matrices for the three charges ($Q = 0, 1, 2$) we obtain three distinct non-vanishing eigenvalues given by

$$
\frac{s^2}{96\pi} \left( 3f_{S,0} + f_{S,1} + f_{S,2} \right), \quad \frac{s^2}{96\pi} \left( f_{S,0} + f_{S,1} + 3f_{S,2} \right), \quad \frac{s^2}{96\pi} \left( 3f_{S,0} + 7f_{S,1} + 5f_{S,2} \right),
$$

where we kept only the leading term in the center-of-mass energy.

These eigenvalues allow us to obtain limits on the three Wilson coefficients $f_{S,j}/\Lambda^4$. In order to explore the dependence of the bounds on the possible relations among operators imposed by specific forms of the ultraviolet physics, we consider two scenarios: in the first one, we assume that only one Wilson coefficient is non-vanishing. The second case assumes that all Wilson coefficients of the subset of operators considered are non-vanishing and we look for the possible largest values of each coefficient in the unitarity region. Notice that in this second scenario the limits for all couplings cannot be achieved simultaneously as they are simply extreme points in the three dimension ($f_{S,0}/\Lambda^4, f_{S,1}/\Lambda^4, f_{S,2}/\Lambda^4$) region delimited by Eqs. (13) and (18). We present in Table I the bounds on $f_{S,j}/\Lambda^4$ for these two scenarios. Expectedly, the limits in the second case are weaker since the undisplayed Wilson coefficients can be adjusted to mitigate unitarity violation but as explicitly shown this is only an effect of $O(1.5-4)$.

Next, we focus on the unitarity constraints on the seven operators $O_{M,j}$ from their leading contributions ($O(s^2)$) to the scattering amplitudes. For these operators, the analysis of the $J = 0$ partial wave for the three charges yields two independent non-vanishing eigenvalues

$$
\frac{s^2}{64\pi\Lambda^4} (2f_{M,4} + f_{M,5}), \quad \frac{s^2}{256\pi\Lambda^4} \sqrt{32 (-4f_{M,2} + f_{M,3})^2 + 6 (8f_{M,0} - 2f_{M,1} + f_{M,7})^2}.
$$

They allow for constraining the Wilson coefficients when considering only one operator at a time. Nevertheless, they are not enough to bound all coefficients in the most general scenario with several non-vanishing operators entering the amplitudes simultaneously. Consequently, to obtain the limits from the leading $O(s^2)$ contribution in this case one must also consider the bounds from the $J = 1$ partial-wave unitarity. In so doing, we find seven additional independent non-zero eigenvalues in the $Q = 2, 1$ helicity-particle matrices

$$
\frac{s^2}{1536\pi\Lambda^4} C_{1,2,3}, \quad \frac{s^2}{6144\pi\Lambda^4} \left( C_4 \pm \sqrt{C_5^2 + C_6^2} \right), \quad \frac{s^2}{6144\pi\Lambda^4} \left( C_7 \pm \sqrt{C_8^2 + C_9^2} \right),
$$

with

$$
C_1 = 12f_{M,0} + 5f_{M,1}, \quad C_2 = 12f_{M,0} - 11f_{M,1} + 8f_{M,7}, \quad C_3 = 4f_{M,7},
$$

$$
C_{4,5} = \pm(24f_{M,0} + 10f_{M,1} - 15f_{M,7}) + 48f_{M,2} + 20f_{M,3}, \quad C_6 = 4\sqrt{3}(6f_{M,4} - 5f_{M,5}),
$$

$$
C_{7,8} = \pm(-24f_{M,0} + 22f_{M,1} - f_{M,7}) - 48f_{M,2} + 44f_{M,3}, \quad C_9 = 4\sqrt{3}(6f_{M,4} + 11f_{M,5}).
$$
Altogether, the total number of unitarity constraints originating from the unitarity condition Eq. (13) with Eqs. (19) and (20), allow for independently bound each of the $f_{M,j}/\Lambda^4$ Wilson coefficients even when all seven are considered simultaneously. In fact, due to the algebraic structure of the eigenvalues, it is technically possible to solve analytically the system of 9 constrains in the seven dimensional parameter space.

We present in Table II the unitarity bounds on the Wilson coefficients of the operators $O_{M,j}$ for the two scenarios described above. Even though the $J = 1$ partial waves have to be invoked to obtained bounds in the full seven dimensional parameter space and the bounds of higher angular momentum amplitudes are weaker, it is interesting that the constraints on the Wilson coefficients do not degrade substantially and become $\mathcal{O}(3-4)$ weaker than those obtained from the $J = 0$ partial waves under the assumption of only one non-vanishing coefficient.

The third class of dimension-eight operators exhibits only field strength tensors and it contains eight independent operators given in Eq. (1). Considering only leading contributions to the scattering amplitudes that grow as $s^2$, the diagonalization of the $J = 0$ helicity-particle matrices for the three $Q$ channels leads to eight distinct eigenvalues

$$\frac{s^2}{384 \pi \Lambda^4} D_{1,2,3,4} , \quad \frac{s^2}{384 \pi \Lambda^4} \left( D_5 \pm \sqrt{D_6^2 + D_7^2} \right) , \quad \frac{s^2}{384 \pi \Lambda^4} \left( D_8 \pm \sqrt{D_9^2 + D_{10}^2} \right) ,$$

(21)

where

$$D_1 = 8(2f_{T,0} + f_{T,1} - f_{T,2}) , \quad D_2 = 8(2f_{T,0} + 7f_{T,1} + 2f_{T,2}) ,$$
$$D_3 = 8(2f_{T,5} + f_{T,6} - f_{T,7}) , \quad D_4 = 8(2f_{T,5} + 7f_{T,6} + 2f_{T,7}) ,$$
$$D_{5,6} = 4(2f_{T,0} + 4f_{T,1} - f_{T,2} \pm (8f_{T,8} - 4f_{T,9})) , \quad D_7 = 8\sqrt{3} f_{T,6} ,$$
$$D_{8,9} = 80f_{T,0} + 40f_{T,1} + 26f_{T,2} \pm (128f_{T,8} + 56f_{T,9}) , \quad D_{10} = 4\sqrt{3}(12f_{T,5} + 2f_{T,6} + 3f_{T,7}) .$$

In this case the total number of unitarity constraints originating from the unitarity condition Eq. (13), together with the $J = 0$ eigenvalues in Eq. (21) allow for independently bound each of the $f_{T,j}/\Lambda^4$ Wilson coefficients even when the eight are considered simultaneously. And again, it is technically possible to solve analytically the system of 8 constraints in the nine dimensional parameter space. The bounds emanating from the $J = 1$ partial wave are weaker than the ones from the $J = 0$ one, therefore, we neglected them in this analysis.

We list in Table III the corresponding bounds for the $f_{T,j}/\Lambda^4$ coefficients assuming the two scenarios described above. Comparing the results in Tables II and III we learn that the Wilson coefficients of the operators $O_{T,j}$ are subject to stronger unitarity bounds than the other QGC classes. Moreover, for only one non-vanishing $f_{T,j}/\Lambda^4 = 1$ TeV$^{-4}$, unitarity is not violated for subprocesses center-of-mass energies smaller than 1.5–2.8 TeV depending on the anomalous QGC.

We end by presenting the unitarity constraints on the $\mathcal{O}(p^3)$ QCG given in Eqs. (8) and (9) that originate in the nonlinear realization of the gauge symmetry. For this set of operators the most stringent limits stem from the $J = 0$
| Wilson Coefficient | 1 operator | all 8 operators |
|---------------------|------------|----------------|
| $\frac{|f_{\pi,0}|}{\Lambda}$ | $\frac{12}{5} \pi s^{-2}$ | $1.5 (0.093) \text{ TeV}^{-4}$ |
| $\frac{|f_{\pi,1}|}{\Lambda}$ | $\frac{24}{5} \pi s^{-2}$ | $3.0 (0.19) \text{ TeV}^{-4}$ |
| $\frac{|f_{\pi,2}|}{\Lambda}$ | $\frac{96}{5} \pi s^{-2}$ | $4.6 (0.29) \text{ TeV}^{-4}$ |
| $\frac{|f_{\pi,5}|}{\Lambda}$ | $\frac{8}{\sqrt{3}} \pi s^{-2}$ | $2.9 (0.18) \text{ TeV}^{-4}$ |
| $\frac{|f_{\pi,6}|}{\Lambda}$ | $\frac{48}{7} \pi s^{-2}$ | $4.1 (0.27) \text{ TeV}^{-4}$ |
| $\frac{|f_{\tilde{\pi},7}|}{\Lambda}$ | $\frac{32}{\sqrt{3}} \pi s^{-2}$ | $11 (0.72) \text{ TeV}^{-4}$ |
| $\frac{|f_{\tilde{\pi},8}|}{\Lambda}$ | $\frac{2}{\pi} \pi s^{-2}$ | $0.93 (0.058) \text{ TeV}^{-4}$ |
| $\frac{|f_{\tilde{\pi},9}|}{\Lambda}$ | $\frac{24}{\pi} \pi s^{-2}$ | $2.1 (0.13) \text{ TeV}^{-4}$ |

Table III: Same as Table II but for the operators $\mathcal{O}_{T,j}$ (Eq. (2)).

| Wilson Coefficient | 1 operator | all 5 operators |
|---------------------|------------|----------------|
| $\frac{|f_{p,6}|}{\Lambda}$ | $\frac{12}{11} \pi v^4 s^{-2}$ | $6 \pi v^4 s^{-2}$ |
| $\frac{|f_{p,11}|}{\Lambda}$ | $\frac{12}{7} \pi v^4 s^{-2}$ | $6 \pi v^4 s^{-2}$ |
| $\frac{|f_{p,23}|}{\Lambda}$ | $\frac{2(\sqrt{43} + 5)}{3} \pi v^4 s^{-2}$ | $17 (1.1) \times 10^{-3}$ |
| $\frac{|f_{p,24}|}{\Lambda}$ | $\frac{6(3\sqrt{3} + 5)}{5} \pi v^4 s^{-2}$ | $8.7 (0.54) \times 10^{-4}$ |
| $\frac{|f_{p,26}|}{\Lambda}$ | $\frac{3\pi v^4}{5} s^{-2}$ | $1.4 (0.08) \times 10^{-4}$ |

Table IV: Same as Table III but for the operators in the nonlinear representation of the electroweak symmetry (Eqs. (8) and (9)).

The partial wave. After diagonalizing the $Q = 0, 1, 2$ channels, we obtain four non-vanishing eigenvalues:

$$\frac{s^2}{384M_W^4} F_1, \quad \frac{s^2}{384M_W^4} F_2, \quad \frac{s^2}{384M_W^4} \left[ F_3 \pm \sqrt{8 F_4^2 + F_5^2} \right],$$

where

$$F_1 = 4(f_{p,6} + 2f_{p,11}), \quad F_2 = f_{p,6} + 2f_{p,11} + f_{p,23} + 2f_{p,24}, \quad F_3 = 13f_{p,6} + 11f_{p,11} + 10f_{p,23} + 10f_{p,24} + 20f_{p,26}, \quad F_4 = 3f_{p,6} + f_{p,11} + 3f_{p,23} + f_{p,24}, \quad F_5 = 3f_{p,6} + f_{p,11} - 10f_{p,23} - 10f_{p,24} - 20f_{p,26}.$$ 

Again, the structure of the four eigenvalues allows for independently constraining the five $f_{p,i}$ coefficients even when considered all non-zero simultaneously. Table IV contain the corresponding bounds on the coefficients. Notice that these results indicate that the present experimental analyses require the introduction of a unitarization procedure as the one in Ref. [11].
IV. DISCUSSION

Exploration of the structure of the quartic couplings of electroweak gauge bosons is at the forefront of the tests of the SM in general, and of its mechanism of symmetry breaking in particular. Parametrizing deviations from the SM predictions in terms of effective operators is the standard methodology followed in such studies in the present experimental searches at LHC [6–17]. Notwithstanding, the contribution of effective operators leads to unitarity violation at high energies, and therefore the methodology must be applied only in the energy regime in which this is not the case. For the specific case of genuine QGC operators, this has been partially addressed in the literature by studying the bounds imposed by partial-wave unitarity of gauge-boson scattering in specific channels and/or waves.

In this work we have presented a complete partial-wave analyses of two-to-two scattering of electroweak gauge bosons and Higgs bosons all for the charged channels in Eq. (14). We have considered operator expansions with linear and nonlinear realizations of the electroweak symmetry. The leading anomalous contribution is proportional to $s^2$ and we studied the conditions to obtain the most stringent limits for all couplings.

Quantitatively our results are summarize in Tables I, II, III, and IV. In the minimal scenario with just one non-vanishing QCG Wilson coefficient our analyses show that the strongest unitarity constraints can be obtained from the analyses of the $J = 0$ partial wave for $Q = 0, 1, 2$. However, in more realistic scenarios where more than one QGC operator contributes, the $J = 0$ partial-wave analysis do not lead to the strongest unitarity bounds for all Wilson-coefficient combinations. In this case, we must also take into account the $J = 1$ partial wave. Once all waves are considered the bounds on each Wilson coefficient become a factor of a few weaker than in the minimal scenario.

Acknowledgments

O.J.P.E. is supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) grant 2019/04837-9; E.S.A. thanks FAPESP for its support (grant 2018/16921-1). M.C.G-G is supported by NSF grant PHY-1620628, by the MINECO grant FPA2016-76005-C2-1-P, by EU grant FP10 ITN ELUSIVES (H2020-MSCA-ITN-2015-674896), and by AGAUR (Generalitat de Catalunya) grant 2017-SGR-929.

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