Denoising of Uncertain Type Noise Images by Spatial Feature Classification in Nonsubsampled Shearlet Transform

ZHIXU LYU1,2, MIN HAN1, (Senior Member, IEEE), AND DECAI LI2
1Department of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian 116024, China
2State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang 110016, China

Corresponding author: Zhiyu Lyu (lyuzhiyu@mail.dlut.edu.cn)

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ABSTRACT Most denoising methods are designed to deal standard images with specific type noise, which do not perform well when denoising real noisy images contain uncertain types of noise. However, underwater image is a typical uncertain type noise image. To solve this problem, this paper presents a method using spatial feature classification jointing nonsubsampled shearlet transform (NSST) for denoising uncertain type noise images. Justifiable granule is employed to solve the problem of parameter selection. The raw image was decomposed by using the NSST to get one low frequency subband and several high frequency subbands. Then, the preliminary binary map is built, the binary map is employed to decide whether a coefficient contains spatial feature or not. And we employ justifiable granule to solve the difficulty of parameter selection. The high subbands coefficients are classified into two classes by fuzzy support vector machine classification: the texture class and the noise class. At last, the adaptive Bayesian threshold is used to shrink the coefficients. Simulation results show the proposed method is effective in uncertain type noise images (also have good performance in specific type noise). The method we proposed has been compared with other popular denoising methods and get excellent subjective performance and PSNR improvement.

INDEX TERMS Uncertain type noise image, denoising, nonsubsampled shearlet transform (NSST), spatial feature, justifiable granularity.

I. INTRODUCTION

The corruption of images is common during its acquisition, processing, compression and transmission. The goal of image denoising is to preserve the detail information such as edges and textures to promote visual quality and to improve the peak-signal-to-noise-ratio (PSNR). Over the past decades, a great deal of denoising methods have been developed in the image processing and computer vision communities. However, many denoising methods were designed to deal with specific type noise, when we want to denoise images contain uncertain type noise (such as underwater images), these algorithms do not work well.

Underwater image is a typical uncertain type noise image. The 21st century is the century of great ocean development. All countries in the world have focused their strategic focus on the oceans and ocean resources. Autonomous underwater vehicles and maritime competition are becoming increasingly fierce [1]–[3]. Under this background, the study of underwater imaging technology is of great significance. We often use video or image to obtain the detail information of water quality, such as predicting water quality by calculating the percentage of suspended matter in underwater images. However, the underwater environment is much more complicated than the land, so many difficulties are encountered in the underwater image processing. For example, light will be absorbed in water, which will cause great attenuation and scattering. At the same time, a large number of organisms, such as plankton and other tiny particles, will cause the PSNR to decrease, the contrast will be poor, and the details will be blurred [4]–[6]. The image shows a fog effect. The degraded image seriously affected peoples exploration of the underwater environment. Therefore, the research on underwater imaging technology [7], [8] has a significant meaning.
For underwater images, the scattering process contains a variety of direction changes of light after collision with particles in the water, which is the main sources of underwater image noise. Underwater scattering consists of forward scattering and backward scattering, forward-scattering is the scattering along the direction of light transmission, causes the contrast of the image to decrease, which induces great harm to the underwater imaging [9]. On the other hand, back-scattering is more serious, light meets with suspended solids before reflects to the imaging device, and these suspended solids in turn scatter the light. The image receiver not only saturates but also does not obtain the clean image information. The quality of the underwater image is severely reduced. It is the main reason for the decrease of contrast and peak-signal-to-noise ratio (PSNR) of underwater images.

At present, the research on the blind restoration technology of low PSNR images is still largely blank. Although the general laws of forward and backward scattering noises of underwater light can be theoretically analyzed, the actual imaging environment is complex and varied. Different with other noisy image which usually corrupted by a specific type noise, we do not sure which type of the noise existed in the underwater image, even worse, both the ground truth and noise density of underwater images are unknown. Therefore, many denoising methods which designed to deal with specific noise are not appropriate with underwater image at the same time.

We use a new settlement of NSST coefficients to denoise the uncertain type noise image by using the threshold shrink. For the threshold shrink based denosing algorithms, researchers often use one threshold value for the whole coefficients. It is obviously that the threshold value is up to both the textures and the image noises degree, nevertheless, textures and noise level are contradict with each other. The textures need small threshold value, large threshold value is used to deal with high level of the noise. Therefore, setting only one threshold for the whole image is inappropriately. This is because in most cases images have both textures and smooth areas. An only one threshold may have good effect for smooth areas but does not effective for the textures as well.

In this paper, to denoise the uncertain type noise image, we propose a novel denoising algorithm by spatial feature classification in NSST. We first decomposed the raw image by using the NSST to get one low frequency subband and several high frequency subbands. Then, the preliminary binary map is built, the binary map is employed to decide whether a coefficient contains spatial feature or not. And then, we employ justifiable granularity to solve the problem of parameter selection. The high subbands coefficients are classified into two classes by fuzzy support vector machine classification: the texture class and the noise class. At last, the adaptive Bayesian threshold is used to shrink the coefficients. Our major contributions may be described as follows.

1) Different from other the-state-of-the-art image denoising algorithms which designed to deal with certain type noise, the method we proposed has ability to denoise the uncertain type noise images such as underwater images. Due to the flexible multi-scale, multi-direction, and shift-invariant properties of NSST, detail features of images can be well preserved.

2) We build a binary map and employ the binary map to decide whether a coefficient contains spatial feature or not in nonsubsampled shearlet transform domain. The selection of parameter is the key of denoising effect. However, the parameter is difficult to determine. We are the first using justifiable granularity to formally solve the problem of parameter selection.

3) We use SVM to classify coefficients into two class (information class and noise class), the classification can shrink the noise no matter what type the noise is. Due to the SVM is sensitive to the outliers, we selected fuzzy SVM to be classifier to category the coefficients.

The rest of this paper is organized as follows. We first overview related works in Section II, Section III we presents the foundation of NSST and briefly introduce the SVM and fuzzy SVM. Section IV contains the denoising method by spatial feature classification in NSST decomposition. We show the simulation results in Section V. And at last, we conclude the paper in the Section VI.

II. RELATED WORK
A. IMAGE DENOISING METHOD
A great deal of denoising methods have been proposed in the recent years [10], [11]. Although each methods are different and have diverse property, they all work with the same purpose: reserving the useful information like texture and edges, and eliminate the noise. The image denoising algorithms can be classify into following categories: Nonlocal Methods [12], [13], Transform Based Methods [14], Bilateral Filtering [16]–[18], Anisotropic Diffusion [19]–[21], Statistical Model [22], Conditional Random Fields [23] and Deep Learning Methods [24], [25]. Transform Based Methods mainly includes wavelet transform, curvelet transform, contourlet transform, and shearlet transform. In this paper, we use shearlet transform to denoising the uncertain type noise image. We briefly introduce the Transform Based Methods as follows:

1) WAVELET PACKET TRANSFORM
The Wavelet packet transform is based on wavelet transform and its further generalized. It can decompose the high-frequency of the wavelet without subdivision, and adaptively select the corresponding frequency band. It is precisely because the wavelet packet can further subdivide the high frequency and low frequency parts of the image at the same time, so the information of the original image can be better preserved, and the noise can be effectively removed.

2) CURVELET TRANSFORM
The Curvelet transform first applies a band-pass filter to decompose the image into multiple sub-bands, and then separates the different sub-band images, and then uses the Ridgelet transform to analyze the line singularities in each
block. Curvelet transform is essentially a multi-scale square Ridgelet transform, so it can well approximate the singular curve in the image.

3) NONSUBSAMPLED CONTORULET TRANSFORM
The NSCT is a deeper development of the contourlet transform (CT) which has a shift invariant characteristics [26]. The CT use the Laplacian pyramid structure to decompose the image into multi-scale, and then the directional filter bank (DFB) decompose the image into multi-directional. The NSCT is constituted by two components: nonsubsampled pyramid structure (NSP) and nonsubsampled Directional Filter Bank (NSDFB).

4) NONSUBSAMPLED SHEARLET TRANSFORM
The Shearlet transform preserves the advantages of Curvelet and Contourlet and overcomes its disadvantages. It not only has multi-scale, time-frequency locality, multi-directionality, but also can be decomposed in any direction. The support domain of its kernel function has anisotropy. Compared to other multi-scale geometric decompositions, the Shearlet transform theoretical system has a low computational complexity and is the only analytical method that can realize continuous and discrete domain unification in multi-scale domains [27].

B. SUPPORT VECTOR MACHINE
Support vector machine (SVM) is a better machine learning method that has emerged in recent years [28]–[31]. It is mainly used to solve small sample, nonlinear and high dimensional problems. It is a concrete realization of the VC dimension theory and structural risk minimization principle of statistical learning theory. It seeks the best compromise between the complexity of the model (learning accuracy for a particular training sample) and the learning ability (ability to identify arbitrary samples without errors) based on the limited sample information in order to obtain the best generalization ability due to the support vector. The outstanding learning performance of the machine has become an important direction in the field of machine learning. It has made great progress in the fields of pattern recognition, classification [32], regression analysis, function estimation, and has been successfully applied in many aspects, such as face recognition, text recognition [33], data mining, object classification, time series predictive data compression, etc.

Many authors have developed image denoising algorithms based on SVM [34]. When we employ SVM to deal with classification problem, each classes are classified as equal. However, in uncertain noise type images and underwater image processing cases, some pixel points are misplaced on the wrong side due to the noise. The training model will also derive from the optimal hyperplane. In order to remove the noise of underwater image, based on Fuzzy SVM [35], [36], Yang et al. [37] presented a wavelet SVM denoising method employing an optimal solution to denoising image by using Morlet wavelet kernel function. Li [38] presented an effective method in image denoising based on SVs regression. The method identified support vectors and calculated weights after training raw image, then using the support vectors and weights in denoising degradation images. Wang et al. [39], [40] proposed a series of denoising methods by employing intelligence classifier jointing with the wavelet transformation or the multi-scale geometric transformation. These methods at first decomposed raw image into different frequency subbands through the multi-scale geometric transformation. Then, each subbands coefficients are classified into two classes by intelligence classifier: the texture class and the noise class. Finally, the subbands coefficients are shrinked by the threshold. However, they do not have a good solution to the difficulty of parameter selection.

III. PRELIMINARY WORK

A. NONSUBSAMPLED SHEARLET TRANSFORM
The Shearlet transform has a complete structural theory framework and strict mathematical logic support [18]. It is the most effective method for the affine system to extract geometric features of multi-dimensional signals. As a new type of multiscale transform, this transform inherits the advantages of Contourlet transform and Curvelet transform. It has the same order of image approximation as the Curvelet transform, but is simpler to implement and the scale direction is more flexible than the Contourlet transform. For 2D signals, the Shearlet transform can not only detect all its singularities, but also can adaptively track the direction of singular curves.

Shearlet transform is derived from the synthesis of wavelet theory, when the dimension. For $n = 2$, the affine system with synthetic expansion is defined as,

$$\Psi_{AB}(\varphi) = (\varphi_{j,k,l}(X) = |\text{det}A|^{1/2} \varphi(B^t A^t X - k); j,l \in \mathbb{Z}, k \in \mathbb{Z}^2).$$

The $\Psi_{AB}$ is called composite wavelet, the scaling matrix $A^t$ is associated with the scale transformation, $B^t$ is associated with an area-invariant geometric transformation. When

$$A = A_0 = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}; \quad B = B_0 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

Its form is the Shearlet transform. Its frequency decomposition and support area are shown in Figure 1.

The geometric properties of Shearlet transform are more intuitive in the frequency domain. For different scales, the support is in a pair of trapezoidal regions with $l$ as the slope and origin symmetry, and in the rotation process with the transformation of $j$. The area of the trapezoid does not change with the change of $l$.

In order to make the Shearlet transform have translation-invariant characteristics, the non-subsampled shearlet transform (NSST) method is constructed. The non-subsampled Laplacian pyramid algorithm is used to replace the Laplacian pyramid algorithm. Nonsubsampled Shearlet Transform (NSST) was constructed. The NSST decomposition process is shown in Figure 2.
The mathematical expression of SVM can be described as follows:

1) SUPPORT VECTOR MACHINE (SVM)

The mathematical expression of SVM can be described as follows:

The parameters of SVM are $C$, where each $X_i \in \mathbb{R}^N$ and $y_i \in \{-1, 1\}$ denotes the class label. The SVM split these samples with a margin between the two classes. It can be realized by dealing with the following quadratic program:

$$
\begin{align*}
\text{min} & \quad \|w\|^2 \\
\text{s.t.} & \quad y_i(w^T X_i + b) \geq 1, \quad i = 1, 2, \ldots, N \\
& \quad \xi_i \geq 0, \quad i = 1, 2, \ldots, N
\end{align*}
$$

where $w$ is the weight parameter and $b$ is the bias term. However, in nonlinearly separable case, it cannot satisfy with all constrains in Equation (1). Therefore, slack variables $\xi_i$, $i \in \{1, 2, \ldots, N\}$ are employed to evaluate the magnitude of violation of the constraints. The quadratic program becomes:

$$
\begin{align*}
\text{min} & \quad \|w\|^2 + C \sum_{i=1}^{N} \xi_i \\
\text{s.t.} & \quad y_i(w^T X_i + b) \geq 1 - \xi_i, \quad i = 1, 2, \ldots, N \\
& \quad \xi_i \geq 0, \quad i = 1, 2, \ldots, N
\end{align*}
$$

where $C$ is a trade-off parameter which is decided previously to describe the error and margin. However, in most cases it cannot be settled; thus, a nonlinear spread is used to settle this disadvantage. We map the input variable $X_i$ into a high dimensional feature space which is linear classification, and then solve it in high dimensional feature space. The function $\Phi(X)$ is satisfied with Mercer’s condition. The in the feature space is related with the $\Phi(X_i)$ in the original space. To work out the quadratic program, we compute the scalar products $\Phi(X_i) \cdot \Phi(X_j)$ which no need to acquire the characteristic of the $\Phi(X_i)$. It is appropriate to use the kernel function $K(X_i, X_j) = \Phi(X_i) \cdot \Phi(X_j)$. By using the Lagrange multiplier method, we can build the quadratic program:

$$
\begin{align*}
\text{min} & \quad \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} y_i y_j \alpha_i \alpha_j K(X_i, X_j) - \sum_{j=1}^{N} \alpha_j \\
\text{s.t.} & \quad \sum_{i=1}^{N} y_i \alpha_i = 0 \\
& \quad 0 \leq \alpha_i \leq C \\
& \quad i = 1, 2, \ldots, N
\end{align*}
$$

Some popular kernel functions which we often use are RBF, sigmoid and Gaussian function. In this paper, we use Gaussian function, Gaussian function can be expressed as follow:

$$
K(x_i, x_j) = \exp \left( -\gamma \|x_i - x_j\|^2 \right)
$$

The parameters of SVM are $C = 80$, $\gamma = 0.5$, Fig. 3 shows a brief theory of SVM.

2) FUZZY SUPPORT VECTOR MACHINE

The mathematical expression of fuzzy SVM can be described as follows:

Suppose the training samples are:

$$(X_i, y_i, \mu(x_i)), \quad i = 1, 2, \ldots, n, X_i \in \mathbb{R}^d, y \in \{-1, +1\}$$

B. FUZZY SVM CLASSIFICATION

SVM is a learning machine different from traditional neural network proposed by Vapnik. It is on the bias of the structural misclassification risk minimization, while other methods often employ the empirical risk minimization. Otherwise, SVM improve the generalization properties. Furthermore, using SVM can get the global optimal solution for a classification or prediction program. It has been a useful and effective method for data classification and prediction. SVM has been used in various fields like data prediction and face identification.

One of the main disadvantages of the SVM is the sensitivity of outliers points in the training sets. Fuzzy SVM is presented to overcome this disadvantage. Every samples will be distributed a membership degree by the relationship with its class. The contribution of every instance to the total error term is weighted by its membership functions in the fuzzy SVM. Simulation results verify that dealing with the NSST subband coefficients by using the fuzzy SVM can yield better classification than traditional SVM and get higher PSNR of the denoised image.

1) SUPPORT VECTOR MACHINE (SVM)

The mathematical expression of SVM can be described as follows:

Setting a training samples are $\{(X_1, y_1), (X_2, y_2), \ldots, (X_N, y_N)\}$, where each $X_i \in \mathbb{R}^N$ and $y_i \in \{-1, 1\}$ denotes

![FIGURE 1. Frequency decompositions by the shearlet transform and frequency support of a shearlet; (a) The Shearlet decomposition (b) The Shearlet support area.](image)

![FIGURE 2. Two layer decomposition process of NSST.](image)
The brief theory of SVM.

where each $X_i \in R^N$ and $y_i \in \{-1, +1\}$ denotes the class label. $0 \leq \mu(x_i) \leq 1$ is the degree of membership. Easy to be seen that it includes two categories. One category involves sample point $X_i$ with $y_i = 1$, denoting this class by $C^+$, then, $C^+ = \{X_i|X_i \in S \text{ and } y_i = 1\}$.

The other category includes sample point $X_i$ with $y_i = -1$; denoting this class by $C^-$, then, $C^- = \{X_i|X_i \in S \text{ and } y_i = -1\}$.

Note that $Q = C^+ \cup C^-.$

The objective function of fuzzy SVM can be expressed as:

$$\min \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{n} \mu(x_i) \varepsilon_i$$

s.t. $y_i[(w \cdot x_i) + b] \geq 1 - \varepsilon_i, \quad \varepsilon_i \geq 0 \quad (5)$

where $C$ is a constant, the fuzzy membership $\mu(x_i)$ is the degree of the corresponding point $X_i$ toward one class, $\xi_i$ is an evaluation index of error in the SVM.

The Lagrangian transformation of (1) is used to obtain the dual problem:

$$\max \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \alpha_i \alpha_j y_i y_j K(x_i \cdot x_j)$$

s.t. $\sum_{i=1}^{n} y_i \alpha_i = 0$

$0 \leq \alpha_i \leq C \mu(x_i) \quad (6)$

The decision function is:

$$F(x) = \text{sgn} \left[ \sum_{i=1}^{n} \alpha_i y_i K(x_i \cdot x) + b \right] \quad (7)$$

The quality of the fuzzy SVM is mainly determined by the selection of the membership function. The membership function is usually determined by the distance between the sample point and the sample center in F SVM. The initial membership degree is determined by the sample center point, then the membership degree of K neighborhoods around the sample points is computed to determine the new membership degree. This method is robust and the training time is short, and it has been widely used. It can be expressed as follows:

Let the sample set $X = \{x_1, x_2, \ldots, x_k\}, x_c$ is the sample center. The distance between the sample data point and the sample center is $R = \max \|x_i - x_c\|$. The initial membership degree is expressed as:

$$\mu_0(x_i) = 1 - \frac{\|x_i - x_c\|}{R + \sigma}, \quad \sigma > 0 \quad (8)$$

For a sample point, its neighbors are respectively expressed as $\{x_1, x_2, \ldots, x_k\}$, then the membership of the sample is expressed as:

$$\mu(x_i) = \frac{\sum_{j=1}^{k} \mu(\|x_i - x_j\|)^b}{\sum_{j=1}^{k} \|x_i - x_j\|^b} \quad (9)$$

The fuzzy SVM with the membership function described above can acquire satisfiable performance because it use average algorithm to deal with training samples. The influence of a single sample in the training set is very little so that the effect of outliers can be removed.

IV. THE DENOISING METHOD BY FUZZY SVM WITH NSST

For the threshold shrink based denoising algorithms, researchers often use one threshold value for the whole coefficients. It is obviously that the threshold value is up to both the textures and the image noises degree, nevertheless, textures and noise level are contradict with each other. The textures need small threshold value and larger threshold value be used to deal with high level of the noise. Therefore, setting only one threshold for the whole image is inappropriately. This is because in most cases images have both textures and smooth areas. An only one threshold may have good effect for smooth areas but does not effective for the textures as well. Therefore, we overcome this problem by setting the adaptively threshold value for textures and smooth areas simultaneously in one image.

For the fuzzy SVM classification, we categorize NSST coefficients into two classes: texture class and noise class which associated with the texture and smooth areas. Therefore, NSST coefficients at every scale corresponded to textures can be distinguished from corresponded to smooth areas, so that we can set different thresholds in the denoising method simultaneously. The thresholds which through the spatial regularity detection can deal with textures and smooth areas more effective.

Due to the spatial regularity, the NSST subbands include contiguous coefficients, which present features of the original image. Therefore, we take advantage of this characteristic to decide whether a coefficient contains spatial feature or not. Thus, we compute the binary map $I_{k,s}(x, y)$ and...
The brief construction of the image denoising by spatial feature classification in NSST, noise mainly includes in high frequency subbands, therefore we classify high frequency coefficients and do not handle with low frequency subband. The decomposition level of NSST is three.

**FIGURE 4.** The brief construction of the image denoising by spatial feature classification in NSST, noise mainly includes in high frequency subbands, therefore we classify high frequency coefficients and do not handle with low frequency subband. The decomposition level of NSST is three.

**FIGURE 5.** (a) NSST coefficient coordinates, (b) preliminary binary map, (c) vectors map, (d) the set of training samples.

The support value \( V_{k,s}(x, y) \), where \( x \) and \( y \) denote pixel positions in the original image. The binary map \( I_{k,s}(x, y) \) can be defined as:

\[
I_{k,s}(x, y) = \begin{cases} 
1, & \text{when } |D_{k,s}(x, y)| > \tau_k \\
0, & \text{else}
\end{cases}
\]

(10)

where \( \tau \) is a threshold for determining whether a NSST coefficient is valid or not in the binary NSST coefficient map which we set by ourselves due to coefficients magnitude.

Then, spatial regularity is used to inspect whether the NSST coefficient is an isolated noisy point or a spatial feature. Fig. 5 gives a coefficient map, the support value

Assume \( U_1 = 4 \)

\[
F_1 = \{C_{x,y}, C_{y,x}, C_{x,y}, C_{y,x}, C_{x,y}, C_{y,x}\}
\]

\[
F_2 = \{C_{1,2}, C_{2,1}, C_{1,2}, C_{2,1}, C_{1,2}, C_{2,1}\}
\]

\[
O_1 = \{1,1,1,1,1,1\}
\]

\[
O_2 = \{0,0,0,0,0,0\}
\]
$V_k(x, y)$ are highlighted. A coefficient is spatially connected to another if there exists a continuous path of valid NSCT coefficients between the two. From Fig. 5(b), we can observe that coefficients $C_B, C_C, C_G, C_I,$ and $C_L$ do not link with any other support value in the binary map. However, coefficients $C_E, C_F, C_H, C_J, C_K, C_M,$ and $C_N$ support each another. Coefficients $C_O, C_P,$ and $C_Q$ support each other. Fig. 5(c) shows the number of $V[x, y]$ for summing up the coefficients given in Fig. 5(b). As shown in Fig. 5(d), $U_k$ means NSST coefficients with the sufficient support value are chosen as support value. $F_k^1$ is constituted by coefficients which support value $V[x, y]$ is more than $U_k$, $F_k^2$ is constituted randomly by coefficients which value is zero. $O_k^1$ and $O_k^2$ denote feature class and noisy class which the number is depended on $F_k^1$ and $F_k^2$. Then, we use $F_k^1$ and $F_k^2$ for training, $O_k^1$ and $O_k^2$ are the training objective. Thus, the training samples are $\{F_k^1, F_k^2, O_k^1, O_k^2\}$. Then, we use well trained fuzzy SVM to classify the high subband coefficients into two categories: texture-related category and noise-related category.

From the Eq. (9) and Fig. 5(d), we need to note that the values of the parameter $\tau_k$ and the parameter $U_k$ are selected by ourselves. The conventional method [39] selected $\tau_k$ and $U_k$ by manual test, but the two parameters have a coupling relationship, and the calculation time is usually very long. Many manual experiments are needed which greatly reduces the efficiency of the method. The method [40] asserted that the relationship between $\tau_k$ and $U_k$ is almost linear. However, the relationship between the two parameters is not strictly linear. The parameters determined by the linear equation are inaccurate. Therefore, it is of great significance to solve this problem. In this paper, we employ the principle of justifiable granularity to determine the parameter $U_k$ [41]. Justifiable granularity is to organize support value in binary map into more meaningful and semantically. Consider a vector series $\{x_0, x_1, x_2, \ldots, x_N\}$ shown in Fig. 6, $N$ means the number of support value in continuous path. A suitable justifiable granule spreads over support value in interval $V(a)$. For all support value falling within the interval $V(a)$, an justifiable granule is formed by invoking the principle of justifiable granularity, for a given $\alpha \in [0, 1]$, the optimized $a$ maximize the interval $V(a)$, that is,

$$V(a) = f_1(\text{card}\{a \leq x_k < x_N\}) \cdot f_2(x_N - a)$$

(11)

Through several design alternatives, We constructed function $f_1$ and $f_2$ as the following forms,

$$f_1(u) = u$$

(12)

$$f_2(u) = \exp(-\alpha u)$$

(13)

We optimized justifiable granule and look for the resulting Pareto front, that is,

$$V(a) = \arg \max_{a \leq x_N} \{\text{card}\{a \leq x_k \leq x_N\} \times \exp(-\alpha(x_N - a))\}$$

(14)

We give the suitable value of the parameter $\tau_k$ as shown in Table 1 which can acquire the best PSNR value through our testing. $\tau_1$ denote the finest scale of the NSST decomposition, $\sigma_n = 0$ denotes the best PSNR results of underwater image shown in Fig. 7 and the rest of $\sigma_n$ denotes the best PSNR results with different noise density $\sigma_n$ of Lena shown in Fig. 8.

The steps of the denoising method by using fuzzy SVM jointing with NSST can be expressed as follows:

**Step 1:** Decompose the raw image by NSST to $J$ levels and $\gamma$ orientations as shown in Fig. 1 and get one low-frequency subband $A_1$ and several high-frequency subbands $D_k^s (k = 1, 2, \ldots, J; \ s = 1, 2, \ldots, \gamma).$ $J$ denotes the decomposition level, and $\{s = 1, 2, \ldots, \gamma\}$ represents the decomposition direction, $\gamma$ denotes the quantity of decomposition direction.

**Step 2:** Categorize the high-frequency subband coefficients $D_k^s$ into texture class and noise class by fuzzy SVM.

**Step 3:** Compute the threshold for high-frequency subband coefficients $D_k^s$. We employ the adaptive Bayesian threshold to shrink the coefficients. The adaptive Bayesian threshold can be expressed as follows:

(i) Evaluate the corresponded subband noise variance by the median estimator:

$$\hat{\sigma}_n = \text{Median}(|C(x, y)|)/0.6745$$

$$C(x, y) \in D_k^s$$

(15)

(ii) Use maximum likelihood (ML) Estimator to compute of the variance $\gamma_k^s (k = 1, 2, \ldots, J; \ s = 1, 2, \ldots, \gamma)$ for the noisy coefficients of each subband $D_k^s$:

$$\sigma_k^s = \max(0, \frac{1}{mn} \sum\sum C_{x=x}^y \sum C_{x}^y - \sigma_n^2)$$

(16)

where $C_{k}^y \in D_k^s; \ m$ and $n$ denote size of the image.

(iii) Compute threshold $\sigma_{th}$ of the NSST coefficients across scales:

$$\sigma_{th} = \frac{\sigma_n \cdot \sum_k \hat{\sigma}_k^s \cdot 2^{-k}}{\sum_k k^2 \cdot 2^{-k}}$$

(17)

where $k$ denotes the current scale.

![Justifiable Granularity Interval $V(\alpha)$](image)

**TABLE 1. The suitable value of the parameter $\tau_k$.**

| Noise $\sigma_n$ | 0    | 20   | 30   | 40   |
|-----------------|------|------|------|------|
| $\tau_1$        | 6    | 18   | 29   | 38   |
| $\tau_2$        | 10   | 25   | 40   | 55   |
| $\tau_3$        | 15   | 33   | 51   | 68   |
(iv) Compute denoising threshold \( T(k, \sigma_k^s) \) for each detail subband if \( \bar{\sigma}_k^s < \sigma_{th} \),

\[
T(k, \sigma_k^s) = 2^{-\frac{J - k}{2}} \cdot \frac{\sigma_n^2}{\sigma_k^s}
\]

where \( k \) denotes the current scale, \( J \) means the largest scale enduring denoising, \( \sigma_n \) denotes the noise standard deviation, \( \sigma_k^s \) represents the deviation of the current subband \( \mathcal{D}_k^s \).

Step 4: Use soft-thresholding to shrink the noise coefficients in high-pass subbands \( \mathcal{D}_k^s \).

\[
\hat{C}_k^s = \begin{cases} 
\text{sgn}(C_k^s(|C_k^s| - T_{k,s})), & |C_k^s| \geq T_{k,s} \\
0, & \text{else} 
\end{cases}
\]

(19)

where \( k = 1, 2, \ldots, J; s = 1, 2, \ldots, \gamma; \hat{C}_k^s(x, y) \) denotes the NSST coefficients by soft-thresholding algorithm, \( C_k^s(x, y) \) denotes the original noise coefficients in high-pass subbands \( \mathcal{D}_k^s \), and \( T \) is the adaptive Bayesian threshold.

Step 5: Employ the inverse NSST transform to reconstruct the image.

V. SIMULATION RESULTS

In order to verify the noise reduction effect of the proposed denoising method, we have tested the various denoising methods for underwater images which obtained from a reservoir in China Liaoning Province, meanwhile, we have tested standard images like Lena, Barbara, Peper, Boat, Cameraman, House. Fig.6 Fig.7 and Fig.8 show some image simulation...
results with popular denoising methods. We used several popular methods as compared algorithms to denoising the image, such as ProbShrink [42], BLS-GSM [43], NL-means [44], MSEPLL [45] and NSCT-SVM [40]. We acquire underwater image through an image sensor. Image sensors are mainly divided into two major categories, CCD and CMOS. Both of these imaging principles convert optical signals into electrical signals that are usually affected by dark current noise, electronic shot noise, and thermal noise. These noises are proportional to the integration time of the imaging and the temperature of the imaging element of the image sensor, which generally can be seen as consisting of most of the Gaussian noise, a small part of other types of additive noise. Therefore, we put diverse level Gaussian white noise on standard images.

We employ the PSNR as an objective evaluation index for the denoising performance. The PSNR is defined as Eq.(19). We use visual effect to verify those may not be well evaluated by the PSNR. Table 2 shows the PSNR values for image denoising with different noise intensities. Fig. 10(a) shows the PSNR curve for underwater image in different depth, Fig. 10(b) shows the PSNR results for ProbShrink, BLS-GSM, NL-means, MSEPLL, NSCT-SVM, and the proposed method on different test images. (for standard grayscale images Lena, Barbara and others). The PSNR can be expressed as follows.

\[
PSNR = 10 \log(255^2/MSE)
\]

\[
MSE = (M \cdot N)^{-1} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (U(i,j) - U_1(i,j))^2
\]

where \(U(i,j)\) is the original image, \(U_1(i,j)\) is the image after the noise reduction, \((M \cdot N)\) is the size of image.

The proposed method display higher sensitivity than other popular denoising methods we compared. In fact, other popular denoising methods blur edges in the Lena and Barbara which were easy to visually detectable from Fig.8 and Fig.9.

From Fig.7, Fig.8 and Fig.9, we can find the method we proposed has the superior visual effect that is excellent to other methods we compared particularly in keeping the textures and detail information of the image. The proposed method does not include the quantity of distracting artifacts with texture that usually appear in other denoising methods. We observes that the denoising methods often appear distortions of the textures especially in edges and suffers from extensive loss of significant detail. Such as BLS-GSM [35] method, it seems like having good overall effects, whereas, it smooth the image too much to preserve substantial detail information very well, which are highly crucial in image denoising. At the meantime, it appears large small blemishes; if we improve threshold values to reduce these blemishes, it would lead to lost more structure information. The MSEPLL has higher PSNR value, but it filter too much valuable information details, subjective effect is not so good. Our method has better performance in uncertain type noise images such as underwater image. That is because we employed fuzzy SVM to classify the coefficients of preliminary binary map into texture category and noise category regardless the type of noise, that is the reason why our method can deal with uncertain type noise images denoising. When we manipulate a slightly clean image, we can denoise the image effectively by spatial feature classification. However, good results are not achieved when we process images with high noise density, this is because under high noise density conditions, noise coefficients may be connected into a continuous path, resulting in misclassification. Need to point that, the evaluation
TABLE 2. Comparison of some popular efficient denoising methods (PSNR).

| Image             | $\sigma_n$ | ProbShrink [42] | BLS-GSM [43] | NL-means [44] | MSEPLL [45] | NSST-SVM | NSCT-SVM [40] | Our method |
|-------------------|------------|-----------------|--------------|---------------|-------------|-----------|---------------|------------|
| Underwater Image(2m) | 0.08       | 72.29           | 72.68        | 71.34         | 73.21       | 73.86     | 73.58         | 74.23      |
| Underwater Image(5m) | 0.08       | 72.09           | 71.54        | 70.98         | 72.23       | 72.78     | 72.59         | 72.92      |
| Underwater Image(8m) | 0.08       | 71.65           | 71.29        | 71.08         | 71.54       | 71.67     | 71.59         | 71.97      |
| Underwater Image(10m) | 0.08      | 71.85           | 71.49        | 71.12         | 72.89       | 72.73     | 72.51         | 73.01      |

TABLE 3. The computation time of each methods (noise $\sigma_n = 30$ for Lena and Barbara, $\sigma_n = 0.08$ for underwater image).

| Methods          | Lena      | Barbara    | Underwater Image |
|------------------|-----------|------------|------------------|
| ProbShrink       | 38.23     | 36.86      | 45.03            |
| BLS-GSM          | 24.09     | 23.15      | 37.59            |
| NL-Means         | 1670      | 1602       | 3281             |
| MSEPLL           | 189       | 202        | 4281             |
| NSCT-SVM         | 287       | 323        | 531              |
| Our method       | 359       | 381        | 598              |

The training of fuzzy SVM classification, the time cost of our method is much longer than other methods we compared, the time cost of Lena and Barbara is usually a few minutes.
According to the Table 2, and Fig. 10(b), we can observe that the PSNR of proposed spatial feature classification in nonsubsampled shearlet transform method has superior performance in terms of objective evaluation over other state-of-the-art image denoising methods, it can remove noise but preserve significant detail information. When denoising standard noisy images, the proposed method also performs well when dealing with low-density noise. Through the PSNR results, it is easy to note that the denoising method we proposed can denoise image with higher quality output. The effectiveness of the denoising method we presented is proved, and the noises are effectively reduced indeed. In order to better verify the denoising performance of our algorithm for uncertain noisy images, we do more experiments on RENIOR dataset which consist of real noisy images, the results can be seen in Fig. 11.

VI. CONCLUSION

In this paper, we proposed an uncertain noise images denoising method by using fuzzy SVM classification jointing with the NSST. We build a binary map and employ the binary map to decide whether a coefficient contains spatial feature or not in nonsubsampled shearlet transform domain. We first use justifiable granule to formally solve the problem of parameter selection. The original input firstly decompose by NSST into one low frequency subband and several high frequency subbands. Then coefficients of each high frequency subband were classified into two classes by using fuzzy SVM: the texture class and the noise class, as spatial feature vector. Finally, the NSST coefficients are shrunk by using the adaptive Bayesian threshold. The test of the denoising simulation
results in uncertain noise images (underwater images) and specific type noise images (Lena and Barbara), compared with other popular denoising methods, our method shows the good performance in PSNR for most of the images especially the uncertain noise images (underwater images). The visual effect of the images which we denoised has good performance as well.

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ZHIYU LYU received the M.S. degree from the School of Electronic and Information Engineering, Kunming University of Science and Technology, Kunming, China, in 2015. He is currently pursuing the Ph.D. degree with the School of Electronic and Information Engineering, Dalian University of Technology.

MIN HAN (Senior Member, IEEE) received the B.S. and M.S. degrees from the Department of Electrical Engineering, Dalian University of Technology, Dalian, China, in 1982 and 1993, respectively, and the M.S. and Ph.D. degrees from Kyushu University, Fukuoka, Japan, in 1996 and 1999, respectively.

Since 2003, she has been a Professor with the Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology. She serves as the Deputy Director of the Chinese Society of Instrumentation Youth Work Committee, the Director of the Institute of Liaoning Province System Simulation, a Committee Member of the Chinese Society of Artificial Intelligence, a Consultant of Jiangsu Province Department of Science and Technology, and the Deputy Director of the Institute of Fuzzy Information Processing and Machine Intelligence, Dalian University of Technology. She has visited Washington University, St. Louis, USA, in 2009. She has authored five books and over 300 articles in international journals and conference proceedings. Her current research interests include complex system modeling and forecasting method, time series analysis and forecasting, artificial intelligence method, and neural networks and chaos and their applications to control and identification. She serves as the Organizing Chair for ISNN 2013, ICICIP 2014, and ICIST 2016.

DECAI LI was born in Anshan, China, in 1983. He received the B.S. degree in electronic and information engineering and the Ph.D. degree in electronic information and electrical engineering from the Dalian University of Technology, Dalian, China, in 2006 and 2012, respectively. He is currently an Associate Professor with the State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang, China. His main research interests include neural network modeling, machine learning, and unmanned vehicles.