Unified Residue Method for Design of Compact Wide-area Damping Controller Based on Power System Stabilizer

Jun Qi, Qian Wu, Youbing Zhang, Guoqing Weng, and Dan Zhou

Abstract—A wide-area damping controller (WADC) is effective in damping inter-area low-frequency oscillation (LFO), if the time delay in a wide-area control loop can be properly handled. In order to simplify the WADC design and enlarge the delay adaptation range, the classic power system stabilizer (PSS) is adopted, and a new unified residue (UR) method is proposed for compact WADC design. The strategy of control loop selection is also improved by modifying the relative residue index based on a few dominant oscillation modes. The designed PSS-based compact WADC is as simple as classic PSS with no more than two lead-lag phase compensation units. Case studies are carried out on an IEEE 16-machine 68-bus power system. Simulation results demonstrate that the control loop selection before the WADC design is necessary and that the proposed selection strategy can easily pick out the suitable candidate control loops. In addition, it is feasible for the UR method to design WADCs with different time delays in the selected control loops. All the designed WADCs are effective in damping inter-area LFO and robust to time delay variations under operation conditions. Comparisons among five design methods for PSS-based WADC show that the proposed UR method is superior in delay adaptation range, the conciseness of WADC structure and computation speed of parameters.

Index Terms—Wide-area measurement system (WAMS), time delay, wide-area damping controller (WADC), power system stabilizer (PSS).

I. INTRODUCTION

LOW-FREQUENCY oscillation has been a serious problem in the stabilization of inter-area connected power systems for a long time. The infrastructure construction and application development of the wide-area measurement system (WAMS) [1] in a smart grid provide technical possibilities in damping inter-area low-frequency oscillation (LFO) [2], [3]. However, new problems like time delay in the WAMS have emerged, which may vary from tens to several hundred ms or even more [4], and will affect the damping control performance [5], [6].

In wide-area damping control, it is necessary to balance the processing of time delay and the enhancement of damping capability. This problem can be solved by directly adopting the time-delay control theory which is usually complex and obscure [7]-[11], or using the Pade approximation [12]-[14] or delay compensation [15]-[20] to simplify the controller design. Wide-area damping controllers (WADCs) can be classified into the following two categories according to the differences in WADC structure shown in Fig. 1.

Fig. 1. Structure comparison of two WADC types. (a) Compact WADC. (b) Combinatorial WADC.

1) Compact WADC. As shown in Fig. 1(a), the structure of a compact WADC is very concise. Since the transfer function of time delay in WAMS block is exponential, the design of a WADC is more difficult than it would be under delay-free conditions. Two ways have been formed for the compact design of WADC. One way is to draw the support from the latest findings on time-delay control theories, among which the most widely used are based on Lyapunov stability theory and linear matrix inequality (LMI) technology [8]-[11]. Simulation results in [8], [9] report that these WADCs can tolerate time delay up to 300 ms. In [10], two $H_\infty$ control schemes are developed for time-varying multiple delayed systems, where the upper bound of time delay is set to 150 ms in the simulation. In [11], LMI-based methods are...
applied to design the WADC, and can tolerate slight time delays below 100 ms. Conservatism and complexity are still the two major disadvantages of these methods. Another way is to transform the exponential time-delay function into a normal rational function by the Pade approximation [12]-[14]. Then conventional delay-free control theory can be put into use again. However, there will be some irrelevant oscillation modes introduced into the object system model during the transformation, and the order of the Pade function has to increase with large-scale time delays. The time delay of power system simulations in [12]-[14] is around 500 ms.

2) Combinatorial WADC. When there is a delay compensation block in the WADC as shown in Fig. 1(a), the delay-free controller block can use conventional control methods like the residue-based approach, the pole placement method, H2/H∞ robust control strategy, etc. The delay compensation block can be a signal prediction module based on a linear polynomial [15] or exponential function [16], [17], system models [18], [19], or a transfer function for lead-lag phase compensation [20]. The tolerance to a large-scale time delay above 1000 ms is reported in [18], [19], and most of the simulation cases in [15]-[17], [20] are carried out under a time delay around 500 ms. The performance of time-delay compensation has great influence on the damping performance. In addition, the error of signal prediction may be enlarged when the delay in a control loop becomes larger, and the additional transfer function for delay compensation may complicate the structure of WADC or even make the subsequent design of a delay-free controller sink into a dilemma.

For simplicity, understandability and robustness, the classic power system stabilizer (PSS) has been widely used in power systems all over the world in recent decades. It has been confirmed by practical engineering applications that the classic PSS is outstanding in damping LFO and can improve electro-mechanical transient stability in conventional power systems [21]-[23]. Many engineers and researchers have made great efforts to apply classic PSS structure to WADC. In [13] and [24], the time-delay controller block of Fig. 1(a) is replaced by PSS, and the Pade approximation is used to assist the PSS parameter calculation. In [15], [20] and [25], the combinatorial structure of a WADC in Fig. 1(b) is adopted, and a delay compensation block is delicately designed to offset the delay effect on PSS performance. In this paper, the conventional residue method will be modified to adapt the classic PSS to the applications of compact WADC of Fig. 1(a), without Pade approximation or delay compensation.

The remainder of this paper is organized as follows. In Section II, the modified relative residue index for control loop selection is discussed and derived from eigenvalue analysis. The unified residue (UR) method for designing a PSS-based compact WADC is clarified in Section III. Section IV demonstrates the performance of the proposed WADC designed by the UR method applied on an IEEE 16-machine 68-bus test system compared to several residue-based WADC design methods. Finally, conclusions are presented in Section V.

II. MODE ANALYSIS AND CONTROL LOOP SELECTION

A. Power System Model

A power system is a typical multi-input and multi-output high-order system. The selection of control loops may determine whether the WADC design will be successful or not, and how strong the damping effect of the WADCs will be. The n-order dynamic model of a power system with I inputs and O outputs can be described by a set of differential-algebraic equations as:

\[ \dot{x} = Ax + Bu \]
\[ y = Cx \]

where \( x \in R^{n×1} \), \( y \in R^{o×1} \), and \( u \in R^{i×1} \) are the vectors of state variables, output variables, and control variables, respectively; \( A \in R^{n×n} \) is the state matrix; \( B \in R^{n×i} \) is the input matrix; and \( C \in R^{o×n} \) is the output matrix.

In order to express the eigen properties of state matrix \( A \), right eigenvector \( M \) and left eigenvector \( N \) are introduced, and the following equations hold for \( A \):

\[ AM = MA \]
\[ N^T M = I \]

where \( A \) is a diagonal matrix with the eigenvalues \( \lambda_1, \lambda_2, \ldots, \lambda_r \) as diagonal elements.

A new state vector \( z \) is defined by a linear transformation:

\[ x = Mz \]

Substituting the above expression for \( x \) into the state-space (1), we have:

\[ \dot{z} = M^{-1} AMz + M^{-1} Bu = Az + N^T Bu \]
\[ y = CMz \]

Supposing that the \( o^\alpha \) system output \( y_\alpha \) is selected as the feedback signal for a WADC whose output is connected to the \( i^\beta \) system input \( u_\beta \), the transfer function of the system corresponding to the control loop \( u_\beta y_\alpha \) can be written as:

\[ \frac{y_\alpha}{u_\beta} = C_i M (sI - A)^{-1} M^{-1} B = \sum_{j=1}^o \frac{\text{Obs}_{\alpha,j} \text{ Con}_{\beta,j}}{s - \lambda_j} + \sum_{j=1}^i \frac{\text{R}_{\alpha,j}}{s - \lambda_j} \]

where \( C_i \) and \( B \) are the \( o^\alpha \) row of \( C \) and the \( i^\beta \) column of \( B \), respectively; \( \text{Obs}_{\alpha,j} \), \( \text{Con}_{\beta,j} \), and \( \text{R}_{\alpha,j} \) are the observability of \( u_\beta \) and residue of the loop from \( u_\beta \) to \( y_\alpha \) respectively, with respect to the \( j^\text{th} \) mode \( \lambda_j \); and \( s \) is a complex variable. In order to evaluate the comparative strength of a control loop against a given mode \( \lambda \), the relative residue index \( RI \) in [24] is defined as:

\[ RI = \left| \frac{\text{R}_{\alpha,j}}{\sum_{j=1}^i \text{R}_{\alpha,j}} \right| \]

The larger the \( RI \) of the selected control loop \( u_\beta y_\alpha \) is, the weaker the interaction between WADC and other modes is. However, the model order of a wide-area interconnected power system is usually very high, and there are a large number of dynamic modes. It is time-consuming for the small signal analysis method and difficult for system identification technology to obtain all the modes, while the modes with large residue and rapid decay may induce interference.
and misjudgment into \( RI \). Therefore, a new simplified \( RI \) is proposed in the following sub-section, which only considers residues of the \( m \) dominant modes rather than all the \( n \) modes in (6).

**B. Simplified \( RI \) Based on Dominant Oscillation Modes**

For (4), if there is a WADC installed between \( y_o \) and \( u_c \), the relationship between \( u_c \) and \( y_o \) can be defined as:

\[
u_i = KA(s)e^{-\tau}y_o = H(s)y_o
\]

(7)

where \( K \) and \( A(\cdot) \) are the feedback gain and the phase compensator of the WADC, respectively; \( \tau \) is the total time delay aggregated in the control loop \( u_i \rightarrow y_o \); and \( H(s) \) is the transfer function of the WADC with the time delay \( \tau \).

Put the feedback control (7) into (4), and divide all the \( n \) modes of (4) into two groups as \( m \) dominant modes \((z_{\text{MI}} = [z_1, z_2, \ldots, z_m])\) and \( n - m \) non-dominant modes \((z_{\text{ML}} = [z_{m+1}, z_{m+2}, \ldots, z_n])\). The dominant modes contain all the LFO modes adjacent to the immediate axis or even in the right \( s \) plane, while the rest of the modes are classified as the non-dominant modes. Then the close-loop state-space model of the power system can be decomposed as:

\[
\begin{bmatrix}
\dot{z}_{\text{MI}} \\
\dot{z}_{\text{ML}}
\end{bmatrix} =
\begin{bmatrix}
A_{\text{MI}} & A_{\text{ML}} \\
A_{\text{LM}} & A_{\text{LL}}
\end{bmatrix}
\begin{bmatrix}
z_{\text{MI}} \\
z_{\text{ML}}
\end{bmatrix}
\]

(11)

\[
A_{\text{ML}} \approx 0
\]

(12)

By substituting (11) and (12) into the decomposed system model (8), the eigenvalue deviation of dominant modes caused by the WADC in (7) can be approximated as:

\[
\Delta \lambda_i \approx 0 \quad k = 1, 2, \ldots, m, k \neq j
\]

(13)

In the WADC design, a damping ratio between 0.05 and 0.2 is often suitable for inhibiting oscillations [21]. Since the value of \( R_{\text{adj}} \) in the selected control loop is comparatively large, \( KA(s) \) does not have to be very large to achieve the eigenvalue shift \( \Delta \lambda_i \) of the \( p \) mode. Although the eigenvalue shift of non-dominant modes may be non-negligible, it is hard for a WADC to change the stability of these fast decaying modes.

According to the prerequisites of (9) and (10) in the analysis of system mode, two indices are defined in (14) and (15) for choosing control loops suitable for damping the critical oscillation mode \( \lambda_c \).

\[
RI = \frac{|R_{\text{adj}}|}{\sum_{k=1}^{m} |R_{\text{adj}}|}
\]

(14)

\[
OI = \frac{|\text{Obs}_{\text{adj}}|}{\sum_{k=1}^{n} |\text{Obs}_{\text{adj}}|}
\]

(15)

Notice that the denominator of \( RI \) is the sum of all the residues of the \( m \) dominant modes, while the denominator of observability index \( OI \) is the sum of observability of all the modes in control loop \( u_i \rightarrow y_o \).

**C. Procedure of Control Loop Selection**

In practical power system, the procedure of control loop selection mainly includes five steps:

- **Step 1**: determine the critical and dominant modes in the power system, and form the residue-type transfer functions for all the candidate control loops.
- **Step 2**: calculate the simplified \( RI \) in (14) for every candidate loop.
- **Step 3**: pick out control loops with relatively high value of \( RI \) from Step 2.
- **Step 4**: calculate the value of \( OI \) in (15) for the selected candidate control loops from Step 3.
- **Step 5**: choose the control loops with relatively high \( OI \) from Step 4 to install a WADC.

According to the analysis in Section II-B, it is hard for a WADC designed for the critical mode with usual gain value to move eigenvalues of the non-dominant modes to cause new instabilities. In addition, the selection indices in (14) and (15) ensure that only the eigenvalue of the critical mode will be changed significantly by the WADC among all dominant modes, as can be seen from (13). Therefore, the control loop selected above can guarantee an evident improvement in damping the critical oscillation mode and a limited influence on other mode dynamics. Since a larger \( RI \) usually means a relatively larger \( OI \), the control loop selection proce-
III. PROPOSED UR METHOD FOR WADC DESIGN

The time-delay inherent in phasor measurement unit (PMU), communication and controller is inevitable in a WAMS, and all these delays in a control loop can be aggregated as a single delay, denoted as $r$. The impact of time delay on damping control is mainly reflected in the phase lag of a feedback signal. For a designated mode, the larger the delay is, the bigger the phase lag is. Time delay has less influence on the magnitude of a feedback signal owing to the relatively limited range of aggregated delay in practical WAMS.

Figure 2 is the block diagram of the classic PSS, including blocks of washout, gain, phase compensation and limiter in series, which is the core module of many types of engineering applied PSSs [21]. It is going to be applied in the compact WADC in Fig. 1(a) to damp LFO of the power system considering time delay in WAMS. For a selected control loop, the PSS-based WADC is going to push the eigenvalue of the critical mode $\lambda_j$ toward the left s plane horizontally. The change of $\lambda_j$ under time-delay feedback can be described as:

$$
\Delta \lambda_j = e^{-jRT_s}KA(\lambda_j)
$$

(16)

where $\sigma$ and $\omega_j$ are the real and imaginary part of $\lambda_j$, respectively; and $R$ is the corresponding residue of $\lambda_j$.

![Fig. 2. Block diagram of PSS-based compact WADC.](image)

The eigenvalue variation $\Delta \lambda_j$ in (16) should be designed as a negative real value in order to enhance the damping ability with little frequency change. According to the conventional residue method [21]-[24], the gain $K$ in (16) should be positive, and thus $A(\lambda_j)$ must try to adjust the right part of (16) to a phase of $-180^\circ$.

Figure 3 illustrates the principle of the proposed UR for the PSS-based compact WADC design. The phase shift of residue and time delay on the $j^{th}$ mode is marked as $\theta_1$ and $\theta_2$, respectively. There are four possibilities for the location of total phase shift $\theta$. The equations for these phase shifts are defined in (17).

$$
\begin{align*}
\theta_1 &= \angle R_j \\
\theta_2 &= -180^\circ \cdot \omega_j \cdot \tau \\
-180^\circ < &\theta_1 + \theta_2 + k \cdot 360^\circ \leq 180^\circ
\end{align*}
$$

(17)

where $k$ is an integer to adjust the total phase shift $\theta$ in the appointed range ($-180^\circ$, $+180^\circ$).

When $\theta$ is in the first or fourth quadrant, apparently a negative value of $K$ is beneficial to reduce the order $N$ of phase compensation function compared with a positive $K$. According to the design principle of UR, the compensation phase angle $\angle A(\lambda_j)$ and $K$ should be calculated for the four quadrants as:

$$
\begin{align*}
\angle A(\lambda_j) &= -\theta & 0^\circ < \theta \leq 90^\circ \\
K &= -|\Delta \lambda_j| / \left| e^{-\sigma jRT_s}A(\lambda_j) \right| & 90^\circ < \theta \leq 180^\circ \\
\angle A(\lambda_j) &= 90^\circ - \theta & -180^\circ < \theta \leq -90^\circ \\
K &= -|\Delta \lambda_j| / \left| e^{-\sigma jRT_s}A(\lambda_j) \right| & -90^\circ < \theta \leq 0^\circ
\end{align*}
$$

(18)-(21)

It is worth noting that the $|\angle A(\lambda_j)|$ is restricted within $90^\circ$ as indicated by (18)-(21). For the phase compensation function $A(s)$ in (22), every lead-lag unit is suitable for phase shift compensation up to $45^\circ$. Clearly, the number of lead-lag units needed by the WADC will be no more than two, that is $N \leq 2$, no matter how long the time delay is.

$$
A(s) = \frac{1 + T_1s}{1 + T_2s}^N
$$

(22)

Drawing on the experience of the conventional residue method [21]-[24], the coefficients $T_1$, $T_2$ and $N$ of (22) can be calculated similarly. The signal washout block serves as a high-pass filter with the time constant $T_w$ in the range of 1-20 s [21]. The output limiter is employed to avoid excessive influence on the regulating device like excitation system or flexible AC transmission system (FACTS), and it is often set as $\pm0.05$-0.10 p.u. [23]. In the following case study, $T_w$ and
output limiter are set to 5 s and ±0.05 p.u., respectively.

The proposed UR method is used to compute parameters of the PSS-based compact WADC in Fig. 2, no matter whether there is time delay in the control loop. This method is also applicable in the design of other types of wide-area PSSs if they are based on the classic PSS structure of Fig. 2. In order to clarify the improvement of the proposed UR method, a comparison is illustrated in Table I, which contains the information on PSS parameters designed by the proposed UR and the conventional residue method with lead-lag delay compensation (CR+DC) [24]. Abviously, compared with CR+DC, the WADC structure is dramatically simplified by adopting UR, especially when the time delay becomes large.

| Design method | Quadrant | $\angle A(\lambda_i)$ | K | N | r = 0 | r > 0 |
|---------------|----------|----------------------|---|---|------|------|
| Proposed UR   | 1st      | $-\theta$            | <0|   |      |      |
|               | 2nd      | $180^\circ-\theta$   | >0| 1-2| 1-2  |      |
|               | 3rd      | $-180^\circ-\theta$  | >0| 1-2| 1-2  |      |
|               | 4th      | $-\theta$            | <0|   |      |      |
| CR+DC         | 1st      | $180^\circ-\theta_i$ | 3-4|   |      |      |
|               | 2nd      | $-\theta_i$          | >0| 1-2| 1-2  |      |
|               | 3rd      | $-180^\circ-\theta_i$| >0| 1-2| 1-2  |      |
|               | 4th      | $-\theta_i$          | <0|   |      |      |

Note: $\theta$ and $\theta_i$ are both between the range of $(-180^\circ, +180^\circ]$; $\lfloor \cdot \rfloor$ indicates the nearest integers greater than or equal to the value.

IV. CASE STUDY

To investigate the feasibility of the proposed UR for WADC design in a relatively complex and realistic power system, the IEEE 16-machine 68-bus test system shown in Fig. 4 is studied in this section.

![Fig. 4. IEEE 16-machine 68-bus test system.](image)

This system is a simplified interconnected system of five areas including New England, New York, and Areas 3-5. It has been used to test the effectiveness of a WADC in [8]. All generators are modeled as a 6-order model. The excitation system is adopted for generators G1-G15, and all loads are represented as constant impedance. In addition, PSS is equipped at G1-G12 to provide damping for local and some inter-area oscillation modes. The detailed description of this test system can be found in [26]. For this test system, bus 16 and bus 51 are equipped with a ±500 Mvar static var compensator (SVC) to maintain its voltage profile.

A. Control Loop Selection

There are 21 LFO modes in the IEEE 16-machine 68-bus test system as displayed in Table II. Mode 2 has the smallest damping ratio and lower oscillation frequency, so it is considered as the critical mode and should be damped first by WADC. Modes 1-4 are selected as the dominant modes as the real part of their eigenvalues is larger than the threshold set as −1. The smaller the threshold is, the more LFO modes the dominant-mode class contains. Nevertheless, over a certain threshold value, as the newly added LFO modes are relatively far away from the critical mode to be controlled, it is difficult for them to get near the imaginary axis after the WADC installation. In this study, −1 is sufficient for the threshold.

| Mode | Eigenvalue | Damping ratio | Frequency (Hz) |
|------|------------|--------------|----------------|
| 1    | −0.3037±2.4067j | 0.1252      | 0.3830         |
| 2    | −0.0794±3.1885j | 0.0249      | 0.5075         |
| 3    | −0.5617±4.1656j | 0.1336      | 0.6630         |
| 4    | −0.2930±4.9611j | 0.0590      | 0.7896         |
| 5    | −1.0235±6.9985j | 0.1447      | 1.1139         |
| 6    | −1.1231±7.4883j | 0.1483      | 1.1918         |
| 7    | −1.1127±8.0869j | 0.1363      | 1.2871         |
| 8    | −1.1686±8.0831j | 0.1693      | 1.0828         |
| 9    | −1.9061±7.3233j | 0.2519      | 1.1655         |
| 10   | −1.4781±3.8750j | 0.1738      | 1.3329         |
| 11   | −2.4333±0.0357j | 0.2596      | 1.4409         |
| 12   | −2.6278±9.4493j | 0.2679      | 1.5039         |
| 13   | −3.0382±10.1073j | 0.2879    | 1.6086         |
| 14   | −2.4283±11.2345j | 0.2113    | 1.7880         |
| 15   | −2.0119±11.7278j | 0.1691    | 1.8665         |
| 16   | −15.5515±3.5896j | 0.9744      | 0.5713         |
| 17   | −15.9409±7.7753j | 0.8988      | 1.2375         |
| 18   | −16.1808±9.6267j | 0.8594      | 1.5321         |
| 19   | −21.0274±11.2067j | 0.8825    | 1.7836         |
| 20   | −21.7132±11.9978j | 0.8753    | 1.9095         |
| 21   | −69.6504±1.7317j | 0.9997      | 0.2756         |

Generally speaking, automatic voltage regulators (AVR) on generators [21], FACTS equipment (e.g., SVC, static synchronous compensator) [8], HVDC [27], inverter-based power source (e.g., energy storage, doubly-fed induction generator, photovoltaic) [28], [29] all can be used for damping LFOs. Hence, the input signal of the power system model for a WADC design can be the corresponding auxiliary input of these above devices. They can be connected by an output...
port of WADC in series. For the IEEE 16-machine 68-bus power system with AVR and SVC, there are 17 potential system inputs including the reference voltage of AVR $V_{k}$ on G1-G15, and the reference voltage of SVC $V_{k}$ on SVC1 and SVC2. There are 326 potential system outputs including the active power $P$ on 86 transmission lines, and the relative machine angle $\delta$ and the relative machine speed $\omega$ between any two machines. Therefore, there are 5542 candidate control loops in this test system.

$RI$ in (14) for all these loops is calculated and sorted in descending order. The control loops of the top 10 $RI$ value are given in Table III as No. 1-10. For comparison, control loop No. 11 with medium $RI$ value is also listed in Table III.

### TABLE III

**Selection Indices and WADC Parameters for 11 Control Loops Without Time Delay**

| Control loop | Selection index | WADC parameter |
|--------------|-----------------|----------------|
| No. $y$ $u$ | $RI$ $OI$ $N$ $T_{1}$ $T_{2}$ $K$ |
| 1 $\delta_{14-16}$ $V_{32}$ | 0.91 0.41 | 0.14 0.70 | 1.680 |
| 2 $\omega_{14-16}$ $V_{32}$ | 0.89 0.44 | 0.34 0.29 | -36.300 |
| 3 $\delta_{14-16}$ $V_{314}$ | 0.80 0.41 | 0.15 0.68 | -0.438 |
| 4 $P_{11-2-4}$ $V_{32}$ | 0.75 0.26 | 0.14 0.73 | 0.147 |
| 5 $\omega_{14-16}$ $V_{314}$ | 0.74 0.44 | 0.36 0.27 | 9.660 |
| 6 $\delta_{15-16}$ $V_{32}$ | 0.72 0.17 | 0.14 0.69 | 2.670 |
| 7 $P_{11-3-1}$ $V_{32}$ | 0.72 0.23 | 0.14 0.68 | 0.989 |
| 8 $P_{11-3-4}$ $V_{32}$ | 0.72 0.22 | 0.14 0.71 | -0.205 |
| 9 $P_{11-3-1}$ $V_{32}$ | 0.67 0.31 | 0.14 0.72 | 0.850 |
| 10 $P_{11-3-3}$ $V_{314}$ | 0.67 0.23 | 0.15 0.66 | -0.259 |
| 11 $\omega_{1,3}$ $V_{32}$ | 0.27 0.41 | 0.56 0.18 | -0.877 |

**B. Test of WADC Under Delay-free Condition**

WADCs are designed for all the control loops in Table III in order to increase the damping ratio of Mode 2 to 0.1. The eigenvalue shift $\Delta \lambda_{2}$ of Mode 2 is calculated, which is approximately equal to $-0.2395$. According to the principle of UR, the WADC parameters when $\tau = 0$ s are calculated and displayed in Table III. It can be seen that $K$ in some control loops may be negative, but only one or two lead-lag compensation units are needed by all the loops as $N$ shown in Table III.

Then these WADCs are put into the corresponding control loops. The damping ratio histogram of Modes 1-4 is presented in Fig. 5. Compared with the open loop marked as No.0, the damping ratios of Mode 2 in the closed loops No. 1 to No. 10 have been increased to the expected level of 0.1, while there is no evident change in the damping ratio of other dominant modes that are still larger than the acceptable level 0.05. For closed loop No.11, although there is obvious improvement in the damping ratio of Modes 1-2, Mode 3 almost halves and becomes the new underdamped oscillation mode. This is caused by the interference between the WADC and Mode 3 in the control loop with relatively small $RI$ of Mode 2, which is further discussed based on the root loci of Fig. 6(b).

![Figure 5](image1)

**Figure 5.** Damping ratio histogram of Modes 1-4 when WADC is installed in control loops No. 1-11 compared with open loop No. 0.

![Figure 6](image2)

**Figure 6.** Root loci comparison. (a) CL2. (b) CL11.

Figure 6 is the root loci for the open control loops No. 2 (CL2) and No. 11 (CL11) with the washout and phase compensation blocks in series. The negative $K$ on the root locus decreases along with the expected damping ratio of Mode 2 (M2), rising from the initial value 0.0249 to 0.2. The major differences between the two root loci lie in Modes 1-3 (M1-3). For CL2, accompanying the decrease of $K$, the damping ratio of M2 has been enhanced continuously, while six other oscillation modes that have been mentioned in Table II almost remain the same. By contrast, it is not possible to raise the damping ratio of M2 in CL11 to 0.2. Instead, the damping ratio will turn around to decrease when it has reached 0.16 following the decrease of $K$. In addition, M1 and M3 in CL11 are dramatically affected by the feedback control of the WADC. The eigenvalue of M3 moves towards the imagi-
nary axis quickly, and this is a non-negligible threat to system stability. Therefore, CL2 is more suitable for WADC installation than CL1. For comparison, another three modes (Modes 5-7) whose damping ratio ranks second only to Modes 1-4 are displayed in Fig. 6. They are relatively far away from the imaginary axis, and they move little during the change of $K$ value.

It can be concluded from Figs. 5 and 6 that control loop selection is very crucial to a successful design and a good performance of the WADC, and there is little damping difference among WADCs in the control loops with comparably large values of $RI$ as large $RI$ usually means large $OI$.

The damping effect of the above WADCs is tested through time-domain simulation on nonlinear and time-varying IEEE 16-machine 68-bus test system. A three-phase short-circuit fault is applied on one of the tie lines between Bus 1 and Bus 2. The fault occurs at $t = 1$ s and is cleared by disconnecting the fault line 0.2 s later. After the disturbance, the open loop system will oscillate for more than 20 s, which is reflected in the oscillations of the relative rotor speed $\omega_{14-16}$ and the active power $P_{L68-52}$ on line 68-52 shown in Fig. 7. When one of the control loops No. 1-3 (CL1-3) is equipped with the designed WADC, the oscillation of system dynamics is well inhibited within about 10 s. In fact, all the WADCs of control loop No. 1-10 in Table III have a similar damping effect.

### C. Test of WADCs Under Time-Delay Condition

Taking control loop CL1 as an example, the WADC is designed with different time delays. By close loop eigenvalue analysis, the damping ratios of the dominant modes Modes 1-4 are shown in the histogram of Fig. 8(a). In the time-delay range of 0-2 s, the damping ratio of Mode 2 is approximately equal to or larger than the expected value of 0.1. There is no evident deterioration in the damping ratios of other modes.

The key parameters of the WADC in CL1 with time delay 0.5 s, 1.0 s, 1.5 s and 2.0 s are listed in Table IV. The system dynamics under different time delays are given in Fig. 8 (b) when the corresponding control parameters are set for the WADC in CL1. The results show that the performance of the WADC with time delay is as good as that under the delay-free condition in Fig. 7, even when the time delay rises up to one oscillation cycle (about 2.0 s) of Mode 2 in Table II. As well as CL1, other control loops No. 2-10 listed in Table III have also been tested in this study, and this confirms that the proposed UR is feasible for WADC design in the selected control loops with relatively large $RI$ value.

According to the principle of the UR method, the deviation of the actual time delay will cause phase mismatch. Fortunately, only when the phase mismatch is very large will the damping performance be strikingly affected. The WADCs in Table IV designed with time delay 0.5 s and 1.5 s have been taken to test the robustness of WADC to delay deviations. It confirms that after the line fault disturbance, the rotor speed $\omega_{14-16}$ and active power $P_{L68-52}$ can return to steady state in about 10-15 s with the actual delay deviation $\Delta \tau$ up to ±0.2 s.
### C. Comparison of Design Methods of PSS-based WADC

To further show the advantage of the proposed UR method, three other types of residue-based method are tested in the normal IEEE 16-machine 68-bus test system to design the PSS-based compact WADC in CL1 for the same expected damping value of 0.1.

1) Conventional residue (CR) method in [21];
2) CR method with first- and second-order Pade approximation (CR+1st Pade, CR+2nd Pade);
3) Conventional residue method with LMI technology (CR+LMI) in [8].

Since the residue phase of Mode 2 in CL1 is $-96.14°$ in the third quadrant, the WADCs designed by these methods when $\tau=0$ s are the same as that with the UR method. As the time delay increases, Fig. 9(a) presents the damping ratios of Mode 2 when the WADC is designed by different methods. For CR, the damping ratio is relatively high when the time delay is near 0 s or 2 s, but it becomes negative during time delays of 0.5-1.5 s. For CR+LMI, the damping ratio cannot reach the expected value of 0.1 in most delay cases, and the worst case is that the damping ratio is equal to the value of 0.0249 for the open loop in Table II. CR+1st Pade is satisfactory in damping control for time delays less than 0.9 s. By increasing the order of the Pade approximation, the delay adaptation range of the CR+Pade method can be expanded, while there will be more irrelevant oscillation modes. From the comparison, it can be seen that the proposed UR can guarantee a satisfactory damping ratio for time delays from 0 s to 2 s.

Figure 9(b) shows another perspective to compare the damping ratio of Modes 1-4 when the WADC in CL1 is designed by different methods with a time delay of 1.5 s. The damping ratios of Modes 1, 3 and 4 change little after the installation of WADC, which is consistent with the conclusion of control-loop selection strategy in Section II-C. For a time delay of 1.5 s, only the proposed UR method is able to raise the damping ratio of Mode 2 to the expected value of 0.1.

![Fig. 9. WADC in CL1 designed by different methods. (a) Comparison of the damping ratio of Mode 2. (b) Damping ratio histogram of Modes 1-4 with time delay of 1.5 s.](image)

The parameters of WADCs designed with a time delay of 1.0 s by the four methods are displayed in Table V, accompanied by the relative computation time (RCT). Supposing that the RCT for CR method is considered as 100%, the proposed UR method is only a little longer than that of CR. However, the RCTs of CR+Pade and CR+LMI methods are larger than 300% and 1000%, respectively. The CR+LMI method is especially vulnerable to the system model order. In current research on the design of LMI-based WADC, it is a common technique to apply system model reduction to get a feasible LMI solution. By contrast, system model order has little influence on the application of the CR and UR methods.

| Parameters of PSS-based Compact WADCs Designed with Time Delay 1.0 s and Comparison of RCT |
|---|---|---|---|---|---|
| Design method | $K$ | $N$ | $T_1$ (s) | $T_2$ (s) | RCT |
| CR | 1.680 | 2 | 0.14 | 0.70 | 100% |
| CR+Pade | 0.181 | 1 | 0.57 | 0.17 | >300% |
| CR+LMI | 0.000 | 2 | 0.14 | 0.70 | >1000%* |
| UR | -1.450 | 2 | 0.14 | 0.68 | 119% |

Note: *For a set of PSS parameters and 7-order system model.

Putting these WADCs into the control loop CL1, the comparison of system dynamics after line fault is presented in Fig. 10. It is clear that the WADC designed by UR has com-
fortably damped out the system oscillation, while the WADC
designed by CR has unexpectedly caused system instability,
and WADCs designed by CR+Pade and CR+LMI have little
effect on damping enhancement.

Finally, five design methods for PSS-based WADC are
compared from the following four aspects: ① consideration
of delay; ② WADC order; ③ computation time; ④ delay
adaptation range. From the comparison in Table VI, it can
be concluded that:

1) The CR method cannot deal with time delay, and the
designed WADC cannot adapt to evident delays.

2) The CR+DC method is used to design a PSS-based
combinatorial WADC, and the structure of the WADC will
complicate as the delay arises.

3) The ability of CR+Pade method to handle time delay
largely relies on the order of Pade approximation, and it is
relatively more time-consuming than CR and UR.

4) The greatest defect of CR+LMI method lies in the long
computation time but narrow range of delay adaptation.

5) The proposed UR method wins on all four aspects. It
can deal with time delay over a large range, its computation
time is comparable with CR, and the designed WADC struc-
ture is the most concise.

| Design method | Delay considered | WADC order | Computation time | Delay adaptation range |
|---------------|------------------|------------|------------------|------------------------|
| CR            | No               | 1-4        | Short            | Narrow                 |
| CR+DC         | Yes              | High for large delays | Short         | Wide                   |
| CR+Pade       | Yes              | 1-4        | Medium           | Medium-wide a          |
| CR+LMI        | Yes              | 1-4        | Long b          | Narrow                 |
| UR            | Yes              | 1-2        | Short            | Wide                   |

Note: a It depends on the order of Pade approximation; b This computation time increases faster than the rise of system model order.

V. CONCLUSION

This paper adopts a classic PSS structure for the design of
compact WADC, and proposes the UR method for PSS pa-
parameter calculation considering the time delay in WAMS. A
control loop selection index based on dominant oscillation
modes is suggested for simplicity. Results from eigenvalue
analysis and time-domain simulations carried out on the
IEEE 16-machine 68-bus test system have verified that:

1) Control loop selection is an indispensable step in the
design of WADC, and the proposed relative residue index
based on dominant oscillation modes is succinct and effec-
tive in loop selection. In the selected loops, the effort of
feedback control is primarily concentrated on the critical
mode.

2) The structure of WADC is greatly simplified. While at
the same time, the time-delay adaptability and robustness of
WADC is dramatically improved by the proposed UR meth-

3) A considerably large percentage of control loops are
suitable for WADC equipment, and the designed PSS-based
compact WADC is effective to damp out low-frequency pow-
er oscillations with different time delays.

Owing to its simplicity in design and effectiveness in
damping performance, the proposed PSS-based compact
WADC designed by the UR method has great potential for
engineering applications in wide-area interconnected power
systems.

REFERENCES
[1] R. B. Sharma and G.M. Dhole, “Wide area measurement technology
in power systems,” Procedia Technology, vol. 25, pp. 718-725, Sept.
2016.
[2] S. M. Ali, M. Jawad, B. Khan et al., “Wide area smart grid architec-
tural model and control: a survey,” Renewable & Sustainable Energy
Reviews, vol. 64, pp. 311-328, Oct. 2016.
[3] Y. Chompoobutrgool, L. Vanfretti, and M. Ghandhari, “Survey on pow-
er system stabilizers control and their prospective applications for pow-
er system damping using synchrophaser-based wide-area systems,” Eu-onean Transactions on Electrical Power, vol. 21, no. 8, pp. 2098-
2111, Nov. 2011.
[4] H. Wu, S. T. Konstantinos, and T. H. Gerald, “Evaluation of time-
delay effects to wide-area power system stabilizer design,” IEEE Trans-
actions on Power Systems, vol. 19, no. 4, pp. 1935-1941, Nov. 2004.
[5] J. He, C. Lu, X. Wu et al., “Design and experiment of wide area
HVDC supplementary damping controller considering time delay in
China southern power grid,” IET Generation, Transmission & Distribu-
tion, vol. 3, no. 1, pp. 17-25, Jan. 2009.
[6] F. Wilches-Bernal, J. P. Pierre, R. T. Elliott et al., “Time delay defini-
tions and characterization in the pacific DC intertie wide area damping
controller,” in Proceedings of IEEE PES General Meeting, Chicago,
USA, Jul. 2017, pp. 1-5.
[7] X. Zhang, C. Lu, S. Liu et al., “A review on wide area damping con-
trol to restrain inter-area low frequency oscillation for large-scale pow-
er systems with increasing renewable generation,” Renewable & Sus-
tainable Energy Reviews, vol. 57, pp. 45-58, May 2016.
[8] W. Yao, L. Jiang, J. Wen et al., “Wide-area damping controller of
FACTS devices for inter-area oscillations considering communication
time delays,” IEEE Transactions on Power Systems, vol. 29, no. 1, pp.
310-329, Jan. 2014.
[9] Y. Li, Y. Zhou, F. Liu et al., “Design and implementation of delay-de-
pendent wide-area damping control for stability enhancement of pow-
er systems,” IEEE Transactions on Smart Grid, vol. 8, no. 4, pp. 1831-
1842, Jul. 2017.
[10] J. Li, Z. Chen, D. Cai et al., “Delay-dependent stability control for
power system with multiple time-delays,” IEEE Transactions on Pow-
er Systems, vol. 31, no. 3, pp. 2316-2326, May 2016.
[11] S. Wang, X. Meng, and T. Chen, “Wide-area control of power systems
through delayed network communication,” IEEE Transactions on Con-
trol Systems and Technology, vol. 20, no. 2, pp. 495-503, Mar. 2012.
[12] J. Qi, Q. Jiang, G. Wang et al., “Wide-area time-delay robust damping
control for power system,” European Transactions on Electrical Pow-
er, vol. 19, no. 7, pp. 899-910, Oct. 2009.
[13] M. R. Shakarami and I. F. Davoudkhani, “Wide-area power system sta-
bilizer design based on grey wolf optimization algorithm considering
the time delay," Electric Power Systems Research, vol. 133, pp. 149-159, Apr. 2016.

[14] P. Saraf, K. Balasubramaniam, R. Hadidi et al., “Design of a wide area damping controller based on partial right eigenstructure assignment,” Electric Power Systems Research, vol. 134, pp. 134-144, May 2016.

[15] Y. Liu, X. Li, Z. Liu et al., “A compensation design for wide-area power system stabilizer distributed time delay based on grey prediction,” Automation of Electric Power Systems, vol. 39, no. 12, pp. 44-49, Jun. 2015.

[16] B. Yang, Y. Li, Q. Jia et al., “A compensation method for time delay of wide area measurement signals applying improved proxy algorithm,” Power Systems Technology, vol. 40, no. 6, pp. 1778-1784, Jun. 2016.

[17] M. Beiraghi and A. M. Ranjbar, “Adaptive delay compensator for the robust wide-area damping controller design,” IEEE Transactions on Power Systems, vol. 31, no. 6, pp. 4966-4976, Nov. 2016.

[18] W. Yao, L. Jiang, J. Wen et al., “Wide-area damping controller for power system interarea oscillations: a networked predictive control approach,” IEEE Transactions on Control System Technology, vol. 23, no. 1, pp. 27-36, Jan. 2015.

[19] B. P. Padhy, S. C. Srivastava, and N. K. Verma, “A wide-area damping controller considering network input and output delays and packet drop,” IEEE Transactions on Power Systems, vol. 32, no. 1, pp. 166-176, Jan. 2017.

[20] F. Bai, L. Zhu, Y. Liu et al., “Design and implementation of a measurement-based adaptive wide-area damping controller considering time delays,” Electric Power Systems Research, vol. 130, pp. 1-9, Jan. 2016.

[21] P. Kundur, Power System Stability and Control. New York: McGraw-Hill, 1994.

[22] Q. Liu, Power System Stability and Generator Excitation Control. Beijing: China Electric Power Press, 2007.

[23] J. Zhang, C. Y. Chung, C. Lu et al., “A novel adaptive wide area PSS based on output-only modal analysis,” IEEE Transactions on Power Systems, vol. 30, no. 5, pp. 2633-2642, Sept. 2015.

[24] C. Lu, J. Zhang, and Y. Han, Wide-area Dynamic Stability Identification and Control of Power System. Beijing: Science Press, 2015.

[25] X. Zhang, C. Lu, X. Xie et al., “Stability analysis and controller design of a wide-area time-delay system based on the expectation model method,” IEEE Transactions on Smart Grid, vol. 7, no. 1, pp. 520-529, Jan. 2016.

[26] G. Rogers, Power System Oscillations. Norwell: Kluwer, 2000.

[27] Y. Cao, W. Wang, Y. Li et al., “A virtual synchronous generator control strategy for VSC-MTDC systems,” IEEE Transactions on Energy Conversion, vol. 33, no. 2, pp. 750-761, Jun. 2018.

[28] L. Wang, Q. S. Vo, and A. V. Prokhorov, “Stability improvement of a multimachine power system connected with a large-scale hybrid wind-photovoltaic farm using a supercapacitor,” IEEE Transactions on Industrial Applications, vol. 54, no. 1, pp. 50-60, Jan. 2018.

[29] Y. Zhu, C. Liu, B. Wang et al., “Damping control for a target oscillation mode using battery energy storage,” Journal of Modern Power Systems and Clean Energy, vol. 6, no. 4, pp. 833-845, Jul. 2018.

[30] C. Lu, Y. Zhao, K. Men et al., “Wide-area power system stabiliser based on model-free adaptive control,” IET Control Theory & Applications, vol. 9, no. 13, pp. 1996-2007, Aug. 2015.

[31] S. Zhang and V. Vittal, “Wide-area control resiliency using redundant communication paths,” IEEE Transactions on Power Systems, vol. 29, no. 5, pp. 2189-2199, Sept. 2014.

Jun Qi received her B.S. and Ph.D. degrees from Zhejiang University, Hangzhou, China, in 2004 and 2009, respectively. Now she is an associate professor in Zhejiang University of Technology, Hangzhou, China. Her research interests include renewable energy, power system analysis and control.

Qian Wu received his M.S. degree from Zhejiang University of Technology, Hangzhou, China, in 2019. His research interests are optimal load control of wind power, power system stability and control, HVDC.

Youheng Zhang received his B.S. and M.S. degrees from Hunan University, Changsha, China, and Ph.D. degree from Huazhong University of Science and Technology, Wuhan, China, in 1993, 1996 and 2003, respectively. He has been a professor of Zhejiang University of Technology, Hangzhou, China, since 2011. His research interests are smart grid, distributed generation and microgrid, electrical vehicles, power quality monitoring.

Guoqing Weng received his B.S. and M.S. degrees from Southwest Jiaotong University, Chengdu, China, in 2000 and 2003, respectively, and Ph.D. degree from Zhejiang University of Technology, Hangzhou, China, in 2010. Currently, he is an associate professor in Zhejiang University of Technology, Hangzhou, China. His research interests include power quality monitoring and control, smart energy, and intelligent algorithms applied in smart grid.

Dan Zhou received his B.S. and Ph.D. degrees from Zhejiang University, Hangzhou, China, in 2006 and 2011, respectively. From 2011 to 2016, he was a Senior Engineer, a Project Manager, and the Director with State Grid Zhejiang Electric Power Co., Ltd. Electric Power Research Institute, Hangzhou, China. From 2016, he is with the Department of Electrical Engineering, Zhejiang University of Technology, Hangzhou, China. His current research interests include distributed generation and power system, especially planning of microgrids, optimization methods, and demand response.