Non-Newtonian gravity confronting models with large extra dimensions

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We discuss the interplay between direct constraints on non-Newtonian gravity and particle-physics bounds in models with large extra dimensions. Existing and future bounds and the most effective ways of further testing these models in gravitational experiments are compared and discussed.

PACS numbers: 04.80.-y,04.50.+h,12.60.-i

Additional space dimensions large enough to be observable have been suggested\(^1\),\(^2\) as a possible solution to the problem of the large scale difference between gravity and the standard model. In this approach the standard-model particles are bound within four-dimensional space-time\(^3\) and—along the lines of brane-world models\(^4\)—only gravity inhabits the extra dimensions\(^\delta\).

Gravity is modified at short distances and becomes much stronger: its experimental long-distance weakness is explained by the space volume of the large extra dimensions by which the coupling must be divided in four dimensions. The length scale \(R^\delta\) at which the long distance regime is recovered is model dependent, and all possibilities between the millimeter and the inverse Planck mass are in principle available.

Such a scenario has spurred new interest in experimental tests of the short-distance behavior of gravity in the hope of either finding deviations or at least setting bounds on the fundamental coupling of gravity at lengths below the millimeter.

In this letter we discuss the complementary role of two classes of such experimental tests: particle-physics measurements—sensitive to the strength of the coupling independently of distance—and short-distance measurements of Newton law—sensitive to the range of non-Newtonian effects down to nanometers. In the discussion, the different role played by bounds derived assuming a specific compactification geometry and those that are compactification independent is stressed.

Our conclusion is that particle-physics measurements already rule out for \(\delta > 2\) the possibility that non-Newtonian effects induced by models with large extra dimension can be found in gravimetric experiments below \(10^{-5}\) m. This result is independent of the compactification model used. Also the cases \(\delta = 1\) and 2 are excluded if compactification on a \(\delta\)-torus is chosen. The best window for confronting non-Newtonian gravity and models with large extra dimensions is therefore at, and just below, the mm range.

Even though cosmological and astrophysical constraints can also be important, and very strong for the case of two large extra dimensions\(^5\), we will not discuss them here. Similarly, we do not discuss models in which the standard-model particles are also allowed to propagate into the extra dimensions\(^6\) or models with non-factorizable geometries\(^7\).

Searches for non-Newtonian gravity. While Newtonian gravity above the centimeter is well confirmed\(^8\), its short distance behavior is still under active scrutiny. All experiments, regardless of the actual apparatus, set a bound on non-Newtonian interactions from the absence of deviations between the force measured at distance \(r^\ast\) and the predicted one.

The bound is usually given in terms of the parameters \(\alpha\) and \(\lambda\) according to the two-body potential

\[ V(r)|_{r^\ast} = \frac{G N m_1 m_2}{r} \left[ 1 + \alpha G e^{-r/\lambda} \right] \]  

where \(G_N\) is the Newton constant in four space-time dimensions. Because of the exponential behavior, the best sensitivity is achieved in the range \(\lambda \sim r^\ast\). Currently, experiments testing Van der Waals forces are sensitive to the range \(r^\ast \sim 1.5 \pm 130\) nm\(^9\); Casimir-force experiments explore \(r^\ast \sim 0.02 \pm 6\) \(\mu\)m, \(r^\ast\) being here the distance between dielectrics or metal surfaces\(^10\),\(^11\) or up to mm by means of a torsion pendulum\(^12\). Cavendish-type experiments, in which the gravitation force is directly measured, are sensitive to \(r^\ast > 1\) mm\(^13\).

The exclusion regions thus determined are convex curves around the distance \(r^\ast\) at which the experiment is performed, rapidly becoming less sensitive at smaller separations. The combined exclusion regions obtained by these searches, for the relevant distances, are shown as grey areas delimited by black curves in Figs.\(^14\).

Gravitational potential in models with large extra dimensions. The two-body potential in models with extra dimensions is parameterized (for \(r\) less than \(R^\delta\), the characteristic compactification length) as\(^2\)

\[ V_\delta(r) = \frac{G N m_1 m_2}{r} \left( \frac{a_\delta}{r} \right)^\delta. \]
In Eq. (3)

\[ a_\delta = \left( G^{(\delta)} / G_N \right)^{1/\delta} = \frac{2\pi}{M_f} \left( \frac{4\pi M_0^2}{\Omega_5 M_f^2} \right)^{1/\delta}, \]

where we define \( M_f = \sqrt{1/G_N} = 1.22 \times 10^{16} \text{ TeV} \). In Eq. (3), \( \Omega_5 = \frac{2\pi^{(3+\delta)/2}}{\Gamma(3+\delta)/2} \) and \( M_f \) is the scale of the effective theory. For distances larger than \( R^* \), the potential in Eq. (3) is replaced by the usual Newtonian potential plus exponentially small corrections:

\[ V_\delta(r) = \frac{G_N m_1 m_2}{r} \left[ 1 + a_\delta e^{-r/R^*} + \cdots \right]. \]

In Eq. (4), the value of \( a_\delta \) depends on the compactification choice and is of the order of the number of extra dimensions \( \delta \).

It is important to bear in mind that Eq. (4) depends on the way the extra dimensions are treated in the process of compactification while Eq. (3) only relies on Gauss law and is therefore compactification independent.

When the experimental bounds parameterized by Eq. (1) are plotted (on a logarithmic scale in the \((\alpha - \lambda)\)-plane) against Eq. (4), a single point is obtained at \( \alpha_G = a_\delta \) and \( \lambda = R^* \); on the other hand, when the same bounds are compared with Eq. (2), they give lines, the shape and position of which are controlled by \( M_f \) and the number of large extra dimensions.

**Compactification-independent bounds from particle physics.** Contrarily to short-distance gravity measurements, particle-physics measurements are independent of \( r \) and only constrain the effective gravitational coupling \( G^{(\delta)} \) by means of the bound on \( M_f \).

This independence from \( r \) is manifest in the \((4+\delta)\)-dimensional theory, which probes distances much smaller than the compactification radius \( R^* \), and recovered in the 4-dimensional computation after resuming over the Kaluza-Klein states.

For this reason, the relationship obtained by comparing Eq. (1) and Eq. (3), must be valid for any choice of \( r \) (as long as \( r \lesssim R^* \)) and gives the stringiest bound at the minimum. Therefore, the curve of exclusion is found to be

\[ \alpha_G(\lambda) = \min_r \left\{ \left[ \left( \frac{a_\delta}{r} \right)^\delta - 1 \right] e^{r/\lambda} \right\}. \]

To find the curve given by Eq. (5), we must solve the polynomial equations obtained by the minimalization procedure. Exact solutions exist for \( \delta < 4 \); however, for all practical purposes, approximated solutions can be found by elementary calculus for any \( \delta \). The exclusion region is given by the lines

\[ \alpha_G(\lambda) = \begin{cases} (a_\delta / \delta \lambda)^\delta - 1 & \text{for } \lambda < \lambda_{\text{max}} \\ \delta e^{\delta + 1} & \text{for } \lambda \geq \lambda_{\text{max}}, \end{cases} \]

where \( \lambda_{\text{max}} = (1 + \delta)^{-(1+\delta)/\delta \delta} \). The value \( \lambda_{\text{max}} \) is reached when no real solution can be found. The exclusion region is extended for \( \lambda > \lambda_{\text{max}} \) by taking smaller values of \( M_f \) (already excluded) for which the solution is translated to larger values of \( \lambda \) while still ending at the same (constant) value of \( \alpha_{\text{min}} \).

Recent calculations have considered precision measurements like the anomalous magnetic moment of the muon \( \mu \) and radiative oblique parameters \( \delta \), as well as collider physics \( \delta \). In collider physics, the most effective channel at both LEP and Tevatron is that in which virtual gravitons take part in dilepton or diphoton production. Production of real graviton gives less stringent bounds. Whenever the bound depends on the sign of the potential we have taken the lesser bound. Recent reviews of all these bounds can be found in Ref. [13].

We have summarized in Table I the best bounds from particle physics. While precision measurements and collider bounds from production of real gravitons depend on the number of extra dimensions \( \delta \), missing entries were not reported in the literature.

| \( \delta \) | \( g_\mu - 2) \) | \( \mu \) | \( \gamma \) | \( \text{Tevatron I} \) | \( \text{Tevatron IIb} \) | \( \text{LHC} \) |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 0.3 | 0.3 | 0.4 | 1.5 | - | - |
| 2 | 1.2 | 1.2 | 1.2 | 1.5 | 3.5 | 13 |
| 3 | 1.2 | 1.2 | 1.2 | 1.5 | 3.0 | 12 |
| 4 | 0.4 | 0.4 | 0.4 | 1.5 | 2.0 | 10 |

TABLE I: Particle physics bounds on \( M_f \). The numbers reported are the constrains in TeV for the first few large extra dimensions \( \delta \). Missing entries were not reported in the literature.
Non-Newtonian gravity and particle-physics constraints. Given the particle-physics bounds in Table I and Eq. (6), we obtain the curves in Fig. 1, where the respective exclusion regions (the area above the lines) are presented for the first few extra dimensions. In using these bounds, the values for $\alpha_G$ and $\lambda$ of a specific model must be plotted against the bounds of the corresponding effective theory at $r \sim \lambda$, the space dimension of which is not necessarily that of the fundamental theory.

Figure 1 shows that for $\delta > 2$ particle-physics bounds, in particular those coming from collider physics, are various orders of magnitude stronger than direct searches for non-Newtonian gravity below the mm. In other words, if any deviation is ever found in these experiments, it will not be possible to explain it in terms of large extra-dimension models.

On the contrary, for $\delta = 1$ in the range $\lambda \gtrsim 1$ nm Casimir and Cavendish-like experiments are the most sensitive and rule out a large amount of parameter space, while particle physics is relevant only at much shorter distances. Notice that the bounds still allow a strong gravity coupling (of the order of $1/(\text{TeV})^3$) up to few hundred $\mu$m as long as it then decreases fast enough to match the long distance regime. This possibility could be important in the framework of sterile neutrino physics [22].

The case of $\delta = 2$ is special and is discussed below.

The comparison depicted in Fig. 1 seems to suggest that the most effective way of testing models with large extra dimensions in experiments of non-Newtonian gravity is not by going to shorter and shorter distances in the nm regime by means of Casimir experiments but rather to improve the sensitivity of Cavendish-like experiments just below the nm regime or Casimir experiments at the $\mu$m. For work in progress in this directions and the first results, see [23].

Non-Newtonian gravity and compactification-dependent bounds. Once a particular compactification scheme is chosen, bounds coming from non-Newtonian gravity tests can become more stringent because of the relationship between $R^*$ and $M_f$. As we shall see, this is the case for $\delta \leq 2$.

The comparison must be performed independently for the two ranges $r^* < R^*$ (SD) and $r^* \gtrsim R^*$ (LD), where the predicted non-Newtonian behaviors are different.

In the LD regime, the bound is obtained by finding the intersection of the stright line corresponding to $\alpha_G = \alpha_\delta$ with the bound coming from Cavendish-like experiments. The value of $\lambda$ at the intersection is the upper bound on $R^*$, from which one obtains a lower bound (model dependent) on $M_f$.

Each model is one point in the $(\alpha, \lambda)$-plane. Changing the shape of the compactification manifold implies only an $O(1)$ correction factor to these bounds [14].

The simplest example is depicted in Fig. 2, where the model of Ref. 2 (ADD) is used and a specific potential derived from compactification on a $\delta$-torus. In this case we have

$$M_f^{2+\delta} = \frac{M_P^2}{(R^*)^\delta}. \quad (7)$$

The resulting bounds are $M_f \gtrsim 5.3 \times 10^5$ TeV for $\delta = 1$, $M_f \gtrsim 3.7$ TeV and $M_f \gtrsim 3.1$ GeV, respectively, in $\delta = 2$ and 3. The bound for $\delta = 1$ is much stronger than those from particle physics and often quoted to exclude altogether $\delta = 1$ as an observable case. This conclusion however depends on a specific way of treating the extra dimensions.

In the SD regime, things are different: the experiment and the potential (2) must be compared at the distance explored by the experiment. This comparison requires computing the force in the actual set-up of the apparatus, and yields a direct bound on $M_f$. We have checked that
these bounds, derived in the Casimir experiment [1], are two orders of magnitude weaker than those found in the LD regime. For these experiments to be competitive in testing large extra dimensions, a further improvement of many orders of magnitude in sensitivity would be necessary. Therefore, we use the results of the LD regime to redo our analysis and obtain the exclusion regions, shown in Fig. 3, in the case of compactification-dependent bounds.

More complicated compactification schemes, in particular those in which different extra dimensions are treated in different manners, can be discussed along the same lines by means of the corresponding effective theories.

Contrarily to the case of particle-physics bounds, we have now weaker bounds for larger numbers of extra dimensions. Only the cases δ = 1 and 2 are more stringent than in the previous case.

The case of two large extra dimensions. The case of δ = 2 is particularly interesting because, as shown in Fig. 3, the different approaches give comparable bounds. In the model dependent case, the exclusion region is given by the area above the curve denoted ADD in Fig. 3.

Particle physics provides very stringent model-independent bounds in the whole range; the best of them is denoted by Tevatron in Fig. 3. In the same figure, for reference, we also included the bound coming from the measurement of the anomalous magnetic moment of the muon (µAMM).

In the near future, LHC is expected to reach a sensitivity similar or better than that of the model-dependent analysis ADD (see the curve LHC in Fig. 3). In confronting non-Newtonian gravity experiments, LHC will be competitive over the whole range of distances except in the mm region where an improvement of the sensitivity of Cavendish-like experiments could be an important source of information.

FIG. 3: SD bounds αG vs. λ. The curves are computed at Mf > 5.3 × 10^5 TeV for δ = 1, Mf > 3.7 TeV and Mf > 3.1 GeV, respectively, in δ = 2 and 3. The change of slope takes place at λmax.

FIG. 4: Bounds αG vs. δ for δ = 2: comparison between the sensitivity of all available techniques, including planned LHC reach.

We must also bear in mind that the case δ = 2 is strongly disfavoured by astrophysical constraints [5].

We thanks L. Sorbo for comments on the manuscript.

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