Tetraquark Production in Hadronic Collisions

F Carvalho, E R Cazaroto, V. P. Gonçalves and F S Navarra

1 Departamento de Ciências Exatas e da Terra, Universidade Federal de São Paulo, Campus Diadema, Rua Prof. Artur Riedel, 275, Jd. Eldorado, 09972-270, Diadema, SP, Brazil

2 Instituto de Física, Universidade de São Paulo, CEP 05315-970 São Paulo, SP, Brazil

3 Department of Astronomy and Theoretical Physics, Lund University, 223-62 Lund, Sweden.

4 Instituto de Física e Matemática, Universidade Federal de Pelotas, CEP 96010-900, Pelotas, RS, Brazil

Abstract. We develop a formalism to study the tetraquark production in hadronic collisions. We focus on double parton scattering and formulate a version of the color evaporation model for the production of the $T_4c$ tetraquark, a state composed by the $car{c}car{c}$ quarks. We find that the production cross section grows rapidly with the collision energy $\sqrt{s}$ and the $T_4c$ might be observable at LHC energies.

1. Introduction

Over the last years the existence of exotic hadrons has been firmly established [1] and now the next step is to determine their structure. The most popular proposed configurations are meson molecule and tetraquark. So far almost all the experimental information about these states comes from their production in B decays. The production of exotic particles in proton proton collisions is one of the most promising testing ground for our ideas about the structure of the new states. It has been shown [1] that it is extremely difficult to produce molecules in p p collisions. In the molecular approach the estimated cross section for $X(3872)$ production is two orders of magnitude smaller than the measured one. The present challenge for theorists is to show that these data can be explained by the tetraquark model. In this contribution we give a first step in this direction, considering the production of the $T_4c$, a state composed by charm quark pairs: $car{c}car{c}$. This state was first discussed some time ago [2] and has triggered some attention in more recent works [3, 4]. This state is supposed to be a genuine tetraquark, since it is much more difficult to produce meson molecules with the $car{c}car{c}$ quark content.

We shall consider the events with two independent parton-parton scatterings with the production of the two $car{c}$ pairs. This is a particular case of double parton scattering (DPS) [5]. In [6] we have shown that DPS charm production is already comparable to single parton scattering (SPS) production at LHC energies. We shall generalize the color evaporation model (CEM) [7] of charmonium production to $T_4c$ production in DPS events. In order to obtain analytical estimates, in this contribution we shall use some simple expressions for the gluon distributions and for the gluon - gluon cross section. A more detailed analysis will be presented in [8].
2. A simple model for $T_{4c}$ production

In the CEM formalism one gluon from the hadron target scatters with one gluon from the hadron projectile forming a charmonium state, which can absorb (emit) additional gluons from (to) the hadronic color field to become color neutral. This is the usual (SPS) $c\bar{c}$ production. Now we are going to generalize the CEM to the case where two gluons from the hadron target scatter independently with two gluons from the hadron projectile as depicted in Fig. 1, where we show DPS production of $T_{4c}$. In the figure two gluons collide and form a $c\bar{c}$ state with mass $M_{12}$, while other two gluons collide and form a second $c\bar{c}$ state with mass $M_{34}$. The two objects bind to each other forming the $T_{4c}$. Additional gluon exchanges with the environment are not shown in the figure.

![Diagram](image)

**Figure 1.** The gluons with odd (even) label come from the upper (lower) hadron, and carry momentum fraction $x_i$. The “gluon 1” scatters with “gluon 2”, making the state $M_{12}$. Analogous processes occur with gluons 3 and 4. Finally $M_{12}$ and $M_{34}$ merge and form the $T_{4c}$ with mass $M$.

Working with the usual CEM one-dimensional kinematics, the rapidities of the objects $M_{12}$ and $M_{34}$ are respectively:

$$y_{12} = \frac{1}{2} \ln \frac{x_1}{x_2} \quad \text{and} \quad y_{34} = \frac{1}{2} \ln \frac{x_3}{x_4}$$

and their invariant masses are

$$M_{12} = \sqrt{x_1 x_2 s} \quad \text{and} \quad M_{34} = \sqrt{x_3 x_4 s}$$

The invariant mass of the $c\bar{c}c\bar{c}$ system is then given by:

$$M^2 = M_{12}^2 + M_{34}^2 + 2M_{12} M_{34} \cosh(y_{12} - y_{34})$$

2.1. Open charm production

Apart from a constant, the DPS cross section for the production of four charm quarks which hadronize independently (forming four heavy-light charm mesons) is given by:

$$\sigma_{4D} = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^1 dx_4 \ g(x_1, \mu^2) g(x_2, \mu^2) \sigma_{g_{12} \to c\bar{c}} g(x_3, \mu^2) g(x_4, \mu^2) \sigma_{g_{34} \to c\bar{c}}$$
\[ \times \Theta(1 - x_1 - x_3) \Theta(1 - x_2 - x_4) \Theta(M_{12}^2 - 4m_c^2) \Theta(M_{34}^2 - 4m_c^2) \] (4)

where \( g(x, \mu^2) \) is the gluon distribution in the proton at the gluon fractional momentum \( x \) and at the factorization scale \( \mu^2 \) and \( \sigma_{gg \rightarrow \bar{c}c} \) is the \( gg \rightarrow \bar{c}c \) elementary cross-section. The step functions \( \Theta \) enforce momentum conservation in the projectile and in the target. The step functions \( \Theta(M_{ij}^2 - 4m_c^2) \) guarantee that the invariant masses of the gluon pairs 12 and 34 are large enough to produce two charm quark pairs. Note that Eq. (4) needs a multiplicative constant with dimension of \( \sigma^{-1} \) in order to give the correct dimension for the cross section. In fact, in the DPS formalism this constant is usually chosen to be the inverse of an effective cross section \( (1/\sigma_{eff}) \).

In order to gain some insight on the problem and to determine the energy \( (\sqrt{s}) \) dependence of the cross section (4) we shall use simple expressions for the gluon distributions \( g(x) \) and gluon cross sections \( \sigma_{gg \rightarrow \bar{c}c} \), which are given by:

\[ g(x) = \frac{1}{x^{1+\lambda}} \] (5)

\[ \sigma_{gg \rightarrow \bar{c}c} = \frac{\alpha_s}{x_1 x_2 s} \] (6)

with \( \lambda \geq 0 \). With these simple expressions and in the high energy limit, \( s \gg 4m_c^2 \), the cross section (4) can be integrated giving:

\[ \sigma_{4D} = C \alpha_s^2 \frac{1}{(1+\lambda)^2} \left( \frac{1}{(4m_c^2)^{2+2\lambda}} \right) s^{2\lambda} \left\{ \ln \left( \frac{s}{4m_c^2} \right) \right\}^2 - \frac{2}{1+\lambda} \ln \left( \frac{s}{4m_c^2} \right) \] (7)

where the constant \( C \) represents \( 1/\sigma_{eff} \) and also normalization corrections due to the use of the approximate expressions (5) and (6). The expression above contains two constants, which are adjusted fitting (7) to the theoretical prediction for \( c\bar{c}c\bar{c} \) production in DPS events calculated in [6]. A good fit of the \( c\bar{c}c\bar{c} \) production cross section is obtained with \( \lambda = 0.45 \) and \( C \alpha_s^2 = 5 \times 10^{-3} \) GeV\(^{-2} \).

2.2. Tetraquark production

Having fixed the parameters and knowing that (7) reproduces the data on double open charm production we now proceed to calculate the cross section for \( T_{4c} \) production. All known charm tetraquarks have small binding energies, i.e., they lie right below the threshold of two open charm mesons. We shall assume that the \( T_{4c} \) mass \( (M_T) \) is smaller than the lightest two charm meson threshold, \( 2M_{\eta_c} \), i.e., \( M_T \leq 2M_{\eta_c} \). Looking at (3) one can check that the condition \( M^2 = M_T^2 \leq 4M_{\eta_c}^2 \) implies that \( y_{12} \simeq y_{34} \) even in the extreme case where \( M_{12} \simeq M_{34} \simeq 2m_c \) and \( m_c \simeq 1 \) GeV, and hence the total system \( M_{12} + M_{34} \) has the largest internal kinetic energy. We shall thus assume that the two objects (12 and 34) have the same rapidity, \( y_{12} = y_{34} \), which implies that:

\[ \frac{x_2}{x_1} = \frac{x_4}{x_3} \] (8)

To simplify the calculation we also require that the two objects have equal masses, i.e., \( M_{12} = M_{34} \). This condition is not really very restrictive. Assuming that \( m_c \simeq 1 \) GeV and \( M_{\eta_c} \simeq 3 \) GeV the \( T_{4c} \) mass must be in the range \( 4 \leq M_T(= M_{12} + M_{34}) \leq 6 \) GeV. The maximal difference between \( M_{12} \) and \( M_{34} \) will happen when, for example, \( M_{12} = 2 \) GeV and \( M_{34} = 4 \) GeV. In other words, in the worst case one mass is twice the other. Since we will always work
with \( m_c > 1 \) GeV, assuming \( M_{12} = M_{44} \) will not be a bad approximation. This implies that
\[
M_{12}^2 = M_{34}^2 = x_1 x_2 s = x_3 x_4 s \quad \text{and hence:}
\]
\[
x_1 x_2 = x_3 x_4 .
\]  
Combining (9) with (8) we find that \( x_3 = x_1 \) and \( x_4 = x_2 \). With these new conditions we go back to Eq. (4). With the help of (3) we multiply (4) by the unity, written as:
\[
\int_{16m_c^2}^{4M_{nc}^2} dM^2 \delta(M^2 - [M_{12}^2 + M_{34}^2 + 2M_{12}M_{34} \cosh(y_{12} - y_{34})]) = 1
\]  
We next implement the conditions (9) and (8) with delta functions and obtain:
\[
\sigma_{T_{1c}} = C \frac{\alpha_s}{\pi} \int_{16m_c^2}^{4M_{nc}^2} dM^2 \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^1 dx_4 \left( \frac{1}{x_1^{1+\lambda}} \frac{1}{x_2^{1+\lambda}} \frac{1}{x_3^{1+\lambda}} \frac{1}{x_4^{1+\lambda}} \frac{\alpha_s}{x_3 x_4 s} \right) \delta(M^2 - [M_{12}^2 + M_{34}^2 + 2M_{12}M_{34} \cosh(y_{12} - y_{34})]) \delta(x_3 - x_1) \delta(x_4 - x_2)
\]  
This equation can be readily integrated to give:
\[
\sigma_{T_{1c}} = C \alpha_s^2 \left( \frac{3 + 2\lambda}{3 + 2\lambda} \ln(1/16m_c^2) - 1 \right) - \frac{1}{(3 + 2\lambda)(4M_{nc}^2)^{3+2\lambda}} \left( \frac{3 + 2\lambda}{3 + 2\lambda} \ln(4M_{nc}^2) - 1 \right) + \frac{1}{(16m_c^2)^{3+2\lambda}} \ln(s) \right)
\]  
The constants \( C \) and \( \lambda \) are the same as before. The above cross section still has the double open charm cross section as a reference normalization. This derivation was made in the spirit of the CEM, emphasizing the kinematical aspects of hidden charm production and neglecting some microscopic aspects, especially those related to the quantum numbers of the produced particles.

It is instructive to compare the asymptotic limits of Eqs. (7) and (12). At very high energies we have:
\[
\sigma_{4D} \propto s^{2\lambda} (\ln s)^2 \quad \sigma_{T_{1c}} \propto s^{1+2\lambda} (\ln s)
\]  
From this comparison we may expect the tetraquark production cross section to grow much faster than the one for double open charm production.

In the usual CEM it is further assumed that the nonperturbative probability for the \( Q\bar{Q} \) pair to evolve into a quarkonium state \( H \) is given by a constant \( F_H \) that is energy-momentum and process independent. Once \( F_H \) has been fixed by comparison with the measured total cross section for the production of the quarkonium \( H \) at one given energy, the CEM can predict, with no additional free parameters, the energy dependence of the production cross section and the momentum distribution of the produced quarkonium. Following the CEM strategy we shall multiply (12) by the constant factor \( F_T \), which represents the nonperturbative probability that the four quark system \( cc\bar{c}\bar{c} \) (with the right mass) evolves to the \( T_{1c} \). Since we do not have experimental data to fix \( F_T \), we will estimate it as follows. In Table I we collect the available data on cross sections of open charm, \( J/\psi \), double open charm and double \( J/\psi \) production in high energy proton-proton collisions. From the table we can define the probability \( P_{2\rightarrow 1} \) that the two quarks \( c \) and \( \bar{c} \) get together to form a \( J/\psi \). It is given in terms of the cross sections as:
\[
P_{2\rightarrow 1} = \frac{\sigma_{J/\psi}}{\sigma_{cc\bar{c}}} \approx 10^{-3} .
\]
Table 1. Total inclusive cross sections measured in proton-proton collisions at $\sqrt{s} = 7$ TeV.

| Final state | cross section ($\mu$b) | Reference |
|-------------|------------------------|-----------|
| $c\bar{c}$  | $8.5 \times 10^3$      | [9]       |
| $J/\psi$    | 10.7                   | [10]      |
| $c\bar{c}c\bar{c}$ | $3.5 \times 10^3$ | [6]       |
| $J/\psi J/\psi$ | $1.5 \times 10^{-3}$ | [11]      |

Analogously we define $P_{4\to 2}$, which is the probability that four quarks, $c\bar{c}c\bar{c}$ form two $J/\psi$'s. Using the numbers from Table I, we have:

$$P_{4\to 2} = \frac{\sigma_{J/\psi J/\psi}}{\sigma_{c\bar{c}c\bar{c}}} \approx 10^{-6}.$$  \hspace{1cm} (15)

These relations suggest that each time that two charm quarks coalesce we pay a penalty of a factor $10^{-3}$. This would explain the above observation, namely that $P_{4\to 2} \approx (P_{2\to 1})^2$. We shall assume that when two charmoniumlike objects coalesce to form the $T_{4c}$ the penalty factor is the same. Looking at Fig. 1 we see that $T_{4c}$ is a two step process: first we have a $4 \to 2$ process and then a $2 \to 1$ coalescence. With this assumption we can estimate $F_T$ as:

$$F_T = \frac{\sigma_{T_{4c}}}{\sigma_{c\bar{c}c\bar{c}}} = P_{4\to 2} \cdot P_{2\to 1} = (P_{2\to 1})^3 = 10^{-9}.$$ \hspace{1cm} (16)

Multiplying (12) by the $F_T$ found above we obtain the final cross section for the production of $T_{4c}$, which is shown in Fig. 2 for two values of the charm mass. The main feature of the curves is the rapid rise with $\sqrt{s}$, which might render the $T_{4c}$ easily observable already at 14 TeV. This same fast growing trend was observed in other estimates with DPS [5].

The next step would be to replace (5) by a parametrization of the gluon distribution obtained in the global analysis of the current experimental data and also replace (6) by the full charm production cross section. Finally it would be useful to study the decay channels of the $T_{4c}$, making more concrete predictions. We postpone this for a future work [8].

Acknowledgments
This work was partially financed by the Brazilian funding agencies CAPES, CNPq, FAPESP and FAPERGS.

References
[1] A. Esposito, A. L. Guerrieri, F. Piccinini, A. Pilloni and A. D. Polosa, Int. J. Mod. Phys. A 30, no. 04n05, 1530002 (2014).
[2] R. J. Lloyd and J. P. Vary, Phys. Rev. D 70, 014009 (2004).
[3] A. V. Berezhnoy, A. K. Likhoded, A. V. Luchinsky and A. A. Novoselov, Phys. Rev. D 84, 094023 (2011).
[4] A. V. Berezhnoy, A. V. Luchinsky and A. A. Novoselov, Phys. Rev. D 86, 034004 (2012).
[5] S. Bansal et al., arXiv:1410.6664 [hep-ph].
[6] E. R. Cazaroto, V. P. Goncalves and F. S. Navarra, Phys. Rev. D 88, 034005 (2013).
[7] See N. Brambilla et al., Eur. Phys. J. C 71, 1534 (2011) and references therein.
[8] F. Carvalho, E. R. Cazaroto, V. P. Gonçalves and F. S. Navarra, in preparation.
[9] B. Abelev et al. [ALICE Collaboration], JHEP 1207, 191 (2012).
[10] K. Aamodt et al. [ALICE Collaboration], Phys. Lett. B 704, 442 (2011); Phys. Lett. B 718, 692 (2012).
[11] V. Khachatryan et al. [CMS Collaboration], JHEP 1409, 094 (2014).
**Figure 2.** Cross sections as a function of the energy. Solid line: $\sigma_{4D}$ calculated in [6]. Dashed line: $\sigma_{4D}$ calculated with Eq. (7). Dot-dashed lines: $\sigma_{T_{4c}}$ calculated with two different masses for the charm quark.