Dark matter and dark energy from a single scalar field: the cosmic microwave background spectrum and matter transfer function

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Abstract. The dual axion model (DAM), yielding both dark matter and dark energy forming a Peccei–Quinn-like scalar field solving the strong $CP$ problem, is known to allow a fair fit of cosmic microwave background (CMB) data. Recently, however, it was shown that its transfer function exhibits significant anomalies, causing difficulties for fitting deep galaxy sample data. Here we show how the DAM can be modified to agree with the latter data set. The modification follows the pattern suggested for reconciling any Peccei–Quinn-like approach with gravity. The modified DAM allows precise predictions which could be tested against future CMB and/or deep sample data.

Keywords: CMBR theory, dark matter, dark energy theory, axions
1. Introduction

A tenable cosmological model should include at least two dark components, cold dark matter (DM) and dark energy (DE), whose nature is still hypothetical. If DM and DE are physically unrelated, their similar density, in today’s world and just in it, is purely accidental.

Attempts to overcome this conceptual deadlock led, first of all, to dynamical DE [1] (for a review see [2] and references therein), then to considering a possible DE–DM interaction [3]. These options imply new parameters, the hope being that phenomenological limits on them guide one to a gradual understanding of the microphysics involved.

An opposite pattern is followed in the proposed dual axion model (DAM hereafter) [4], which widens the idea of DM made of axions. In the DAM, both DM and DE derive from a single complex scalar field $\Phi$, being its quantized phase and modulus, respectively. $\Phi$ assures CP conservation in strong interactions, according to Peccei and Quinn’s scheme [5] (see also [6,7]), yielding dynamical DE coupled to DM. But, once the DE potential is selected, model parameters are fixed.

It is then quite appealing that CMB data [8] are naturally fitted, yielding reasonable values for standard model parameters such as the primeval spectral index $n_s$, the cosmic opacity $\tau$ and the density parameters. In contrast, the DAM predicts rather high values for $H_0$.

More recently [9], however, it was recognized that the DAM leads to a significant weakening of the Meszaros effect in the early fluctuation evolution, causing an insufficient slope of the transferred spectrum for $k \gtrsim 0.1h$ Mpc$^{-1}$. In this paper, we show how the DAM can be improved on this point. The modified DAM predicts a phenomenology closer to $\Lambda$CDM, according to the value of a suitable parameter. As the available data agree with $\Lambda$CDM reasonably well, limits on such extra parameters can be set.
However, at variance from dynamical DE or coupled DE models, which introduce one or two new parameters on top of ΛCDMs, the whole modified DAM scheme involves the same parameter budget of ΛCDM, with the advantage that each parameter bears a specific physical interpretation.

Furthermore, in principle, the value of the matter density parameter Ω_{om} fixes all parameters in the DE potential of the DAM. But, even though these parameters cannot be stringently constrained by shortly forthcoming data, the range of the DE parameters is predicted by the model, and this is susceptible to more immediate testing.

2. The PQ scheme

The strong CP problem arises from the existence of multiple vacuum states |0_n⟩ in QCD: the set of the gauge transformations Ω(x_μ) can be subdivided into classes Ω_n(x_μ) whose asymptotic behaviors depend on n [10]. At fixed n, the transformations Ω_n(x_μ) can be distorted into each other with continuity, while this is impossible between the Ω_n(x_μ) with different n values. Although in classical field theory no communication between different-n gauge sectors is allowed, in quantum field theory, tunneling is possible, thanks to instanton effects. Any vacuum state is therefore a superposition

|0_θ⟩ = ∑_n |0_n⟩ exp(inθ) \quad (1)

with a suitable θ phase. The effects of varying θ can be recast into variations of a non-perturbative term in the QCD Lagrangian

L_θ = \frac{α_s}{2π} θ G · \tilde{G}; \quad (2)

here α_s is the strong coupling constant, G and \tilde{G} are the gluon field tensor and its dual. However, chiral transformations also change the vacuum angle, so that, when the quark mass matrix M is diagonalized, the parameter θ receives another contribution from the EW (electro-weak) sector, becoming

θ_{eff} = θ + \text{Arg det } M. \quad (3)

The Lagrangian term (2) can be reset in the form of a 4-divergence and causes no change of the equations of motion. It does however violate CP and yields a neutron electric moment d_n ≈ 5 × 10^{-16} θ_{eff} e cm, conflicting with the experimental limit d_n ≲ 10^{-25} e cm, unless θ_{eff} ≲ 10^{-10}. The point is that the two contributions to θ_{eff} are uncorrelated, so there is no reason for their sum to be so small.

PQ suppress this term by imposing an additional global chiral symmetry U(1)_{PQ}, spontaneously broken at a suitable scale F_{PQ}. The axion field is a Goldstone boson which turns out to be suitably coupled to the quark sector. The details of this coupling depend on the model and may require the introduction of an ad hoc heavy quark [11]. The U(1)_{PQ} symmetry suffers from a chiral anomaly, so the axion acquires a tiny mass because of non-perturbative effects, whose size has a rapid increase around the quark–hadron transition scale Λ_{QCD}. The anomaly manifests itself when a chiral U(1)_{PQ} transformation is performed on the axion field, giving rise to a Lagrangian term of the same form as the one in equation (2), which provides a potential for the axion field.
As a result, \( \theta \) is effectively replaced by the dynamical axion field. Its oscillations about the potential minimum yield axions. This mechanism works independently of the scale \( F_{\text{PQ}} \). Limits on it arise from astrophysics and cosmology, requiring that \( 10^{10} \text{ GeV} \lesssim F_{\text{PQ}} \lesssim 10^{12} \text{ GeV}; \) in turn, this yields an axion mass which lies today in the interval \( 10^{-6} \text{ eV} \lesssim m_{\text{A}} \lesssim 10^{-3} \text{ eV}. \)

In most axion models, the PQ symmetry breaking occurs when a complex scalar field \( \Phi = \phi \hat{e}^{i\theta}/\sqrt{2}, \) falling into one of the minima of an NG potential:

\[
V(\Phi) = \lambda |\Phi|^2 - F_{\text{PQ}}^2, \tag{4}
\]

develops a vacuum expectation value \( \langle \phi \rangle = F_{\text{PQ}}. \) The CP-violating term, arising around the quark–hadron transition when \( \bar{q}q \) condensates break the chiral symmetry, reads

\[
V(\theta) = \left[ \sum_q \langle 0(T)|\bar{q}q|0(T)\rangle m_q \right] (1 - \cos \theta) \tag{5}
\]

(\( \sum_q \) extends over all quarks), so \( \theta \) is no longer arbitrary, but will be ruled by a suitable equation of motion. The term in square brackets, at \( T \simeq 0, \) approaches \( m^2_\pi \pi f^2_{\pi} (m_\pi \text{ and } f_\pi: \pi \text{-meson mass and decay constant}). \) In this limit, for \( \theta \ll 1 \) and using \( A = \theta F_{\text{PQ}} \) as the axion field, equation (5) reads

\[
V(\theta) \simeq \frac{1}{2} q^2 (m_q) m^2_\pi f^2_{\pi} A^2 \frac{F^2_{\text{PQ}}}{F_{\text{PQ}}^2}; \tag{6}
\]

here \( q(m_q) \) is a function of the quark masses \( m_q; \) in the limit of two light quarks (u and d), \( q = \sqrt{m_u/m_d(1+m_u/m_d)^{-1}}. \) Here below, instead of using \( A, \) the axion degrees of freedom will be described through \( \theta \) itself. Equation (6), however, shows that, when \( \langle \bar{q}q \rangle \) is no longer zero (since \( T \ll \Lambda_{\text{QCD}} \)), the axion mass decreases with temperature approaching the constant value \( m_{\text{A}} = m_\pi \pi f^2_{\pi} (m_\pi / F_{\text{PQ}} \text{ for } T \ll \Lambda_{\text{QCD}}. \) Accordingly, the equation of motion, in the small \( \theta \) limit, reads

\[
\ddot{\theta} + 2 \frac{\dot{a}}{a} \dot{\theta} + a^2 m^2_{\text{A}} \theta = 0 \tag{7}
\]

(here \( a \) is the scale factor and dots indicate differentiation with respect to conformal time; see the next section), so the axion field undergoes (nearly) harmonic oscillations as soon as \( m_{\text{A}} \) exceeds the expansion rate; then, the mean pressure vanishes, leaving the axion as a viable cold DM candidate [7].

This appealing scheme has been subject to various criticisms, in connection with quantum gravity effects and supersymmetries (SUSY). According to [12], in order to fulfill the no-hair theorem [13,14], essentially stating that black holes cannot exhibit global charges, one or more potential terms of the form

\[
\tilde{V} = m^4_{\text{P}} \frac{(\Phi\Phi^*)^q}{m^2_{\text{P}}^{q+p}} \left( g\Phi^p + g^*\Phi^{*p} \right) \tag{8}
\]

should be added. Among them, terms with \( p = 0 \) would yield just a \( \Phi \) field self-interaction. They are however needed, in association with \( p \neq 0 \) terms, to build potentials of the form

\[
\tilde{V}_n(\phi, \theta) = g_n \phi^4 \left( \phi/\sqrt{2} m_\text{P} \right)^n (1 - \cos \theta), \tag{9}
\]
Dark matter and dark energy from a single scalar field breaking the $U_{\text{PQ}}(1)$ invariance; $g = |g| \exp(i\delta)$, in principle, could also be complex, but $\delta \neq 0$ causes problems discussed in [12], that we avoid by taking $\delta = 0$.

In order to fulfil the no-hair theorem, a term of this kind with $n \geq 1$ should exist. The physical correction could be a function which can be expanded as a sum of terms like (9), with various $n$ values, however including $n = 1$. This might help us to recover consistency between the PQ approach and gravity, without explicitly requiring the fine-tuning $g_1 \lesssim 10^{-56}$, without which $CP$ violations reappear. The point is that the axion mass, $\sim \Lambda_{\text{QCD}}^2/F_{\text{PQ}}$, is naturally small, while gravitational corrections must have the same order of magnitude starting from the Planck mass scale, and a prescription for achieving this in a natural way has not been introduced yet. This problem will not be easier in the (modified) DAM approach.

3. The DAM scheme

In the DAM scheme, the NG potential in equation (4) is replaced by a potential $V(\Phi)$ admitting a tracker solution [1,16]. The field $\Phi$ is complex and $V(\Phi)$ is $U(1)$ invariant. In the modified DAM scheme a small symmetry breaking term, similar to equation (9), will also be added, which will be entirely irrelevant at large $T$.

At variance from the PQ scheme, in the DAM models there is no transition to a constant value $F_{\text{PQ}}$, which is replaced by the modulus $\phi$ itself, slowly evolving over cosmological times. At a suitable early time, $\phi$ settles on the tracker solution and, when chiral symmetry breaks, the dynamics becomes relevant also for the $\theta$ degree of freedom, as in the PQ case. In the modified DAM scheme, the potential (9) will also contribute to the $\theta$ dynamics, but this will occur at much smaller energy scales.

The $\Phi$ field, therefore, besides providing DM through its phase $\theta$, whose dynamics solves the strong $CP$ problem, also accounts for DE through its modulus $\phi$. Therefore, below, the $\theta$ and $\phi$ components will often be indicated by the indices c, de.

In principle, this scheme holds for any DE potential admitting tracker solutions. Here we use the SUGRA potential [16]

$$V(\Phi) = \frac{\Lambda^\alpha \phi^4}{\phi^\alpha} \exp[4\pi(\phi/m_p)^2]$$

found to fit available observational data, such as uncoupled DE data, even slightly better than $\Lambda$CDM [17], without severe restriction on the energy scale $\Lambda$ (and/or the exponent $\alpha$).

In this cosmology, a critical stage occurs at the quark–hadron transition, when the invariance for phase rotations in the $\Phi$ field is broken by a chiral symmetry violating term:

$$V(\theta) = m^2(T, \phi) \phi^2(1 - \cos \theta)$$

$(m(T, \phi)$ is the mass of the $\theta$ field which is discussed in section 5). The $\theta$ phase is then driven to move about its minimum energy configuration, starting from a generic value. The amplitude of oscillations then gradually decreases and, when the small $\theta$ regime is achieved, this component will behave as CDM.

This stage can be suitably described through numerical integration, the essential point being that it sets the initial amount of DM. A fair value of DM today will then arise if
the modulus $\phi$ has a suitable value at the transition and a suitable evolution of $\phi$ and $\theta$, from then to the present epoch, will then occur.

In particular, the value of $\phi$ at the transition will exceed the $F_{\text{PQ}}$ energy scale by about three orders of magnitude, but to yield a significant DE amount it must increase to $\sim m_P$ when approaching today.

A fair evolution of both $\phi$ and $\theta$ is then achieved by setting $\Lambda \sim 10^{10}$ GeV in the SUGRA potential. $\theta$ is then also driven to values even smaller than in the PQ case, so that $CP$ is apparently conserved in strong interactions. Even more significantly, $\Omega_{o,c}$ and $\Omega_{o,\text{de}}$ (DM and DE density parameters) are allowed to take fair values.

In $\Lambda$CDM models, $\Omega_{o,c}$, $\Omega_{o,\text{de}}$, $\Omega_{o,b}$ (baryon density parameter) are free parameters. In uncoupled dynamical DE models, e.g. with a SUGRA potential, a further free parameter exists, $\alpha$ or $\Lambda$. When a constant DM–DE coupling is added, it must be weighted with a further parameter $\beta$; a variable coupling case needs at least a further parameter $\epsilon$, altogether setting that the coupling intensity $C = (\beta/m_P)(\phi/m_P)\epsilon$ (see also equation (24) below).

In the DAM scheme, once the $\Lambda$ scale is assigned, fair values of $\Omega_{o,c}$ and $\Omega_{o,\text{de}}$ naturally and unavoidably arise. They can be modified just by modifying the $\Lambda$ value. The parameter budget of this scheme is similar to that of SCDM.

4. The modified DAM

The need to modify this scheme, as already outlined, arises from the damping of the stagnation or Meszaros effect that it causes. Let us recall first what happens, in the absence of DM–DE coupling, when fluctuations approach the horizon before radiation–matter equality. Before entering the horizon, DM and photon–baryon fluctuations ($\delta_c$ and $\delta_{\gamma b}$) are the same. As soon as we consider inside the horizon, instead $\delta_c$ and $\delta_{\gamma b}$ have different behaviors: $\delta_{\gamma b}$ starts to fluctuate as a sonic wave, so $\langle \delta_{\gamma b} \rangle = 0$. In contrast, CDM fluctuations do not take part in sonic waves (CDM is non-collisional), do not ‘free stream’ (CDM particles are non-relativistic), and fail to increase significantly because of self-gravity, as then $\Omega_c \ll 1$ and the photon–baryon fluid, whose density parameter is $1 - \Omega_c$ (neglecting neutrinos), is no gravity source, because $\langle \delta_{\gamma b} \rangle = 0$. Therefore $\delta_c$ stagnates or has just a marginal growth. If we assume that $\delta_c$ between horizon entry and equality roughly grows $\propto a^4$ (outside the horizon, in a synchronous gauge, we then have $\delta_c \propto a^2$), we obtain the basic shape of the transfer function $T(k)$. Its dependence on $k$ simply arises from the varying duration of the stagnation period.

DM–DE coupling changes this scenario through two effects: it keeps DM and DE densities at close values; then, interactions carried by DE are significant and add to gravity. The stagnation period is then suppressed, mostly because of the enhancement of the effective self-gravity arising from $\delta_c$, able to ‘beat’ the low $\Omega_c$ value. In [9] the whole dynamics has been followed in detail, suitably modifying standard linear codes. A specimen of the modified behaviors of $\delta_c$ is given in figure 1, for different coupling intensities. The DAM case is $\beta = 0.244$ and $\epsilon = -1$. The cases $\beta = 0.1$ and 0 are the cases of smaller or vanishing coupling intensity.

To recover consistency with data, these anomalies must be prevented: $\phi$ and $\theta$ must decouple before the relevant scales enter the horizon. Adding a potential term

$$V_{\phi} = g\phi^2 m_P^2 (1 - \cos \theta)$$

(12)
Figure 1. Evolution of $\delta_c$ in the absence of coupling ($\beta = 0$), for constant coupling ($\epsilon = 0$) and for the DAM model ($\beta = 0.244$, $\epsilon = -1$), for different values of $k$. For increasing $k$ values, i.e. for smaller scales, which should undergo a longer stagnation period, the effects of (variable) coupling become more and more significant.

of the kind considered in equation (9), explicitly breaking the $U(1)$ invariance even before the quark–hadron transition, we succeed in doing this. The $g$ coefficient must and can be small enough, so this term does not perturb the whole PQ-like mechanism; it is however possible, even for very small $g$, that the potential (12) becomes significant when $\phi$ becomes large, so the $\phi$ and $\theta$ modes eventually decouple.

We will show that a general self-consistency is then recovered for $\Lambda$ values not far from those of the DAM, relegating the emergence of the $V_{-2}$ term at late times. This is however enough to let the Meszaros effect work, so allowing a fair fit of available data.

5. Lagrangian theory

To discuss the whole dynamics in a quantitative way, let us start from the Lagrangian

$$\mathcal{L} = \sqrt{-g} \left\{ g_{\mu \nu} \partial_\mu \Phi^* \partial_\nu \Phi - V(\Phi) \right\},$$

(13)
which is $U(1)$ invariant. Let us then add to it terms of the form (9)

$$V_n = 4g_n \left( \Phi^* \Phi \right)^{(n+1)/2} m_P^2 - 2g_n \left( \Phi^* \Phi \right)^{(n+3)/2} (\Phi + \Phi^*),$$

meant to fulfil the no-hair theorem, and the terms explicitly breaking the $U(1)$ symmetry when the chiral symmetry is broken. Altogether $L$ reads

$$L = \sqrt{-g} \left\{ \frac{1}{2} g_{\mu\nu} [\partial_\mu \phi \partial_\nu \phi + \phi^2 \partial_\mu \theta \partial_\nu \theta] - V(\phi) - \tilde{m}^2 (T, \phi) \phi^2 (1 - \cos \theta) \right\},$$

if $\phi$ and $\theta$ are explicitly used. Here $g_{\mu\nu}$ is the metric tensor; we assume that $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2 (d\tau^2 - \eta_{ij} dx_i dx_j)$, so $a$ is the scale factor, $\tau$ is the conformal time; Greek (italic) indices run from 0 to 3 (1 to 3); dots indicate differentiation with respect to $\tau$.

The equations of motion, for the $\phi$ and $\theta$ degrees of freedom, read

$$\ddot{\theta} + 2 \left( \dot{a} / a + \dot{\phi} / \phi \right) \dot{\theta} + a^2 \tilde{m}^2 \sin \theta = 0,$$

$$\ddot{\phi} + 2(\dot{a} / a) \dot{\phi} + a^2 \frac{\partial}{\partial \phi} \left( V(\phi) + \tilde{V}_n \right) = \phi \dot{\theta}^2.$$  \hspace{1cm} (16)

In general

$$\tilde{m}^2 = m^2(T, \phi) + g_n \phi^2 (\phi / \sqrt{2} m_P)^n$$  \hspace{1cm} (17)

is made up of two terms. According to [18], at $T > \Lambda_{\text{QCD}}$ the former term exhibits a rapid rise,

$$m(T, \phi) \simeq 0.1 \left( \Lambda_{\text{QCD}} / T \right)^{3.8} m_\phi(\phi),$$  \hspace{1cm} (18)

as $T$ approaches $\Lambda_{\text{QCD}}$; here

$$m_\phi(\phi) = q(m_q) m_\pi f_n / \phi$$  \hspace{1cm} (19)

so $m^2(T, \phi)$ is $\phi$ independent. At $T < \Lambda_{\text{QCD}}$, this term reduces to $m_\phi(\phi)$. Figure 2 shows the rise and decline of $m(T, \phi)$. When it prevails, DM and DE are coupled.

In the latter mass term we will then consider just a $\tilde{V}_2$ correction, so that

$$\tilde{m}^2 = m^2(T, \phi) + g \tilde{m}^2.$$  \hspace{1cm} (20)

Any value $n \neq -2$ clearly complicates the second term on the rhs, yielding a $\phi$ dependent mass. In turn, such dependence should be taken into account in the equation of motion, where the $\partial \tilde{V}_n / \partial \phi$ term would become more intricate. Only the case $n = -2$ will be treated here.

In what follows, equations (17) will be written in the form allowed by the restriction $\theta \ll 1$. This regime is reached soon after the quark–hadron transition, as is shown in figure 3. In particular, for $\theta \ll 1$, the energy densities $\rho_{\theta, \phi} = \rho_{\theta, \phi, \text{kin}} + \rho_{\theta, \phi, \text{pot}}$ and the pressures $p_{\theta, \phi} = p_{\theta, \phi, \text{kin}} - p_{\theta, \phi, \text{pot}}$ are then obtainable by combining the terms

$$\rho_{\theta, \phi, \text{kin}} = \frac{\phi^2}{2a^2} \dot{\theta}^2,$$

$$\rho_{\theta, \phi, \text{pot}} = \frac{\tilde{m}^2}{\phi^2} \theta^2,$$

$$\rho_{\phi, \text{kin}} = \frac{\dot{\phi}^2}{2a^2};$$

$$\rho_{\phi, \text{pot}} = V(\phi).$$  \hspace{1cm} (21)

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Figure 2. Evolution of the mass term. The breaking of PQ $U(1)$ symmetry causes the rise of the mass term $m(T, \phi)$. While it exceeds the tiny mass term $g^{1/2}m_p$, DM and DE are dynamically coupled.

Figure 3. When chiral symmetry breaks down, the PQ $U(1)$ symmetry is also broken by the rise of the mass term $m(T, \phi)$. As a consequence, $\theta$ becomes a significantly dynamical variable and begins its oscillatory behavior. Here we show the results of a numerical integration of the stages leading to $\theta$ oscillations.

When $\theta$ undergoes many (nearly) harmonic oscillations within a Hubble time, $\langle \rho_{\theta, \text{kin}} \rangle \simeq \langle \rho_{\theta, \text{pot}} \rangle$ and $\langle p_\theta \rangle$ vanishes [7]. Under such a condition, using equations (16) and (17), it is easy to see that

$$\dot{\rho}_\theta + 3 \frac{\dot{a}}{a} \rho_\theta = \frac{\dot{m}}{m} \rho_\theta, \quad \dot{\rho}_\phi + 3 \frac{\dot{a}}{a} (\rho_\phi + p_\phi) = -\frac{\dot{m}}{m} \rho_\phi.$$  \hspace{1cm} (23)

Let us note that the right-hand sides of these equations will be non-vanishing only when $m^2(T, \phi)$ yields the dominant contribution to the mass. This will occur at about the quark–hadron transition, when $\dot{m}/m = -\dot{\phi}/\phi - 3.8 \dot{T}/T$. When $T$ approaches 0, however,
the constant mass term dominates and the dark component coupling fades. The exchange of energy between DM and DE, indicated by the right-hand sides of the equations (23), puts the modified DAM scheme among the set of coupled models treated in [19].

The coupling here depends on $\phi$ and is therefore time dependent. However, if we set

$$C(\phi) = \frac{1}{\phi} \frac{m^2(T, \phi)}{m^2},$$

so that equations (17) and (23) read, respectively,

$$\ddot{\phi} + 2\frac{a}{a} \dot{\phi} + a^2 V'(\phi) = C(\phi) \rho_\theta a^2,$$

$$\dot{\rho}_\theta + 3\frac{a}{a} \rho_\theta = -C(\phi) \dot{\phi} \rho_\theta,$$

we see that, when the main contribution to $\tilde{m}$ is given by $m(T, \phi)$, the coefficient of $1/\phi$ in $C(\phi)$ approximates unity. In this case the time dependence is evident. According to equations (19) and (20), however, we have $m(T, \phi) \propto \phi^{-1}$ and the second term in equation (21) will eventually take over. When this occurs, $m^2(T, \phi)/\tilde{m}^2$ becomes negligible and the coupling between DM and DE vanishes.

In figure 2 we describe the total mass behavior, starting from the stage before the quark–hadron transition, in the regime when chiral symmetry is still unbroken; then $m^2(T, \phi)$ becomes dominant, to be overcame again by the $g m_P^2$ term at late times.

Altogether, DE is coupled to DM at large $z$, but gradually decouples at late times. The unified scheme is responsible for producing fair amounts of DM and DE, whose origins are no longer unrelated. But one of the extra terms with which we are aiming to reconcile the PQ approach with GR is doomed to hide the coupling when approaching the present epoch.

6. Using the SUGRA potential

If the SUGRA potential (10) is used, in the radiation dominated era, until the eve of the QH transition, $\phi$ evolves according to the tracker solution

$$\phi^{\alpha+2} = g(\alpha) \Lambda^{\alpha+4} a^2 \tau^2,$$

with $g(\alpha) = \alpha(\alpha + 2)^2/4(\alpha + 6)$. This high $z$ tracker solution is abandoned when the coupling switches on. Then $\theta$ becomes significant so that $\phi \theta^2$ exceeds $a^2 V'$, and the field enters a different tracking regime:

$$\phi^2 = \frac{3}{7} \rho_\phi a^2 \tau^2.$$  

For very small or vanishing $g$ values, this regime covers the transition from radiation dominated to $\phi$ matter dominated expansion, which would actually result just in a change of the coefficient from 3/2 to 9/10.

The mass $\sqrt{g} m_P$ must however be tuned to overcome the $m(T, \phi)$ contribution to $\tilde{m}$ before the epoch when the cosmologically significant mass scales enter the horizon, so to avoid the suppression of the Meszaros effect.

In figure 4 we report the density parameters of the different components in the various evolutionary stages, for axion mass $g^{1/2} m_P = 10^{-20}$ GeV and $\Omega_{oc} = 0.25$, $\Omega_{ob} = 0.04$, $H_0 = \ldots$
70 km s\(^{-1}\) Mpc\(^{-1}\). For the same density parameters and \(H_0\), in figure 5 we show also the time dependence of the \(\phi\) field, for a variety of \(g^{1/2}m_P\) values.

The value of the energy scale \(\Lambda\), yielding the preferred dark matter density parameter at \(z = 0\), exhibits a dependence on the axion mass, as is shown in figure 6.

The final comparison with data, however, is to be based on CMB angular spectra and the matter fluctuation spectrum. When considering figure 7 it must be taken into account that all plots are obtained with the same density parameters, \(n_s\) and \(H_0\); only normalization is slightly shifted to improve the fit. A best-fit procedure, allowed to adapt
Figure 6. Different values of $g^{1/2}m_P$ require different values of the energy scale $\Lambda$. Below $\sim 10^{-22}-10^{-23}$ GeV, we recover the value of the DAM. We report also the related values of $\alpha$.

Figure 7. CMB anisotropy spectra for the modified DAM, compared with $\Lambda$CDM anisotropy spectra. All parameters, would surely come up with even better curves. CMB data are not a problem for the DAM.

In figure 8, then, we show the transferred spectrum for a set of models; $n_s = 1$ and $\sigma_8 = 0.89$ are taken for all of them. $gm_P^2$ mass values of $\lesssim 10^{-38}-10^{-39}$ GeV$^2$ allow us to recover a $\Lambda$CDM-like behavior; the residual discrepancy appearing in the figure is mainly due to the use of dynamical DE instead of $\Lambda$. The figure exhibits a progressive decrease of the transferred spectrum steepness when greater $g$ values are taken.
7. Discussion and conclusions

Reconciling PQ axion models with GR risks spoiling their elegance. The PQ model motivation is to avoid a fine-tuning of the $\theta$ angle. Apparently, to correct for GR, fine-tuning of $g^{1/2}m_P$ is needed. This problem affects both standard PQ axions and the DAM. However, although the natural mass scale is $m_P \simeq 1.22 \times 10^{19}$ GeV, ordinary particles are many orders of magnitude lighter. Tuning a mass scale, therefore, is more acceptable than tuning an angle.

Another fine-tuning problem however exists in any dynamical DE approach. As is known, the DE field mass, obtainable from the derivative $\partial^2 V(\phi)/\partial \phi^2$, in the present epoch when $\phi \sim m_P$, is

$$m_\phi^2 \simeq V(m_P)/m_P^2 \sim G\rho_{\text{o,cr}} \sim H_o^2$$

($\rho_{\text{o,cr}} \sim V(m_P)$ is the present critical density). Such an extremely tiny mass, $\sim \mathcal{O}(10^{-42}$ GeV), allows us to consider DE as a field, instead of quanta. This fine-tuning is put under further strain when potential terms $V$ (equation (8)) with $p = 0$ are taken, but just in association with terms with $p \neq 0$ (equation (14)). A term with $p = 0$, with a coupling constant $g$ of the order needed to yield axion mass, if considered autonomously, would prevent the modulus of $\Phi$ from behaving as DE.

It is true that the term we need to modify the DAM, turning it into a model quite close to $\Lambda$CDM, has the shape of terms reconciling PQ with GR, and that a similar tuning is however necessary also with this aim (instead of a term with $p = 0$, one could then tune a ‘cosmological constant’ term). But here we need a $V_n$ potential with $n = -2$, while fulfilling the no-hair theorem requires $n > 0$. (It is however fair to add that, in the case of large wormhole effects, the need of a $V_{-2}$ potential has also been discussed [13].)

It might then well be that quantum gravity does not prescribe a single $V_n$ correction, but a combination of such corrections. For instance, instead of a power of $\phi/\sqrt{2}m_P$ the right potential could naturally include a polynomial. Then, while making the PQ approach coherent with GR, the correction would also include terms explaining why DM
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and DE, after a period when they interact, gradually re-decouple, while they are driven to have similar densities in the present epoch.

However, once discrepancies between the DAM and standard dynamical DE models are fully avoided, the possibility of falsifying the modified DAM might seem limited to its particle aspects. This is only partially true: in fact the modified DAM, with a SUGRA potential, makes a prediction on the energy range for the scale $\Lambda$ (and/or the exponent $\alpha$) in the potential. Available data do not provide very stringent constraints on the energy scale $\Lambda$ and the DAM value is still consistent with them. More precise CMB data, however, may soon be available and the energy scale $\Lambda$ will be more stringently constrained.

But the model goes farther, predicting a relation between the precise $\Lambda$ value, in the above scale range, and $\Omega_{oc}$. Testing this prediction requires still higher precision cosmological data, which could be achievable with next generation experiments. Meanwhile, however, if consistency is confirmed, a precise $\Lambda$ value can be predicted from $\Omega_{oc}$.

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