The effect of the Gaussian profile of the new Higgs doublet on the radiative lepton flavor violating decay

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Abstract

We study the branching ratios of the lepton flavor violating processes \( \mu \to e\gamma \), \( \tau \to e\gamma \) and \( \tau \to \mu\gamma \) by considering that the new Higgs scalars localize with Gaussian profile in the extra dimension. We see that the BRs of the LFV decays \( \mu \to e\gamma \), \( \tau \to e\gamma \) and \( \tau \to \mu\gamma \) are at the order of the magnitude of \( 10^{-12} \), \( 10^{-16} \) and \( 10^{-12} \) in the considered range of the free parameters. These numerical values are slightly suppressed in the case that the localization points of new Higgs scalars are different than origin.

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1 Introduction

Lepton flavor violating (LFV) interactions reach great interest since they are rich from the theoretical point of view. In the standard model (SM), these decays are allowed by introducing the neutrino mixing with non zero neutrino masses. However, their branching ratios (BRs) are much below the experimental limits due to the smallness of neutrino masses. Therefore, they are sensitive the physics beyond the SM. Since, theoretically, the loop effects are necessary for the existence of LFV decays, it would be possible to predict the free parameters of the underlying theory if one studies the measurable quantities of them.

Among the LFV processes, the radiative LFV $\ell_i \rightarrow \ell_j \gamma (i \neq j; i, j = e, \mu, \tau)$ decays deserve to analyze and there are various experimental and theoretical works in the literature. The current limits for the (BRs) of $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ decays are $1.2 \times 10^{-11}$ [1] and $3.9 \times 10^{-7}$ [2], respectively. A new experiment at PSI has been described and aimed to reach to a sensitivity of $BR \sim 10^{-14}$ for $\mu \rightarrow e\gamma$ decay [3] and at present the experiment (PSI-R-99-05 Experiment) is still running in the MEG [4]. For $\tau \rightarrow \mu\gamma$ decay an upper limit of $BR = 9.0 \times 10^{-8}$ at 90% CL has been obtained [5] (6), which is an improvement almost by one order of magnitude with respect to previous one. From the theoretical point of view, there is an extensive work on the radiative LFV decays in the literature [7]-[14]. In [7] these decays were analyzed in the supersymmetric models. [8, 9, 10, 11, 12] and [13] were devoted to the radiative LFV decays in the framework of the two Higgs doublet model (2HDM) and in a model independent way, respectively. In another work [14], they are analyzed in the framework of the 2HDM and in the supersymmetric model.

In this work, we study the LFV processes $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ in the 2HDM with the inclusion of a single extra dimension. In the 2HDM the radiative LFV decays are induced by the internal new neutral Higgs bosons $h^0$ and $A^0$ and the extension of the Higgs sector results in enhancement in the BRs of these decays. In addition to this, the inclusion of a single extra dimension causes to modify the BRs. The extra dimension scenario is based on the string theories as a possible solution to the hierarchy problem of the SM. The effects of extra dimensions on various phenomena have been studied in the literature [15]-[26]. In the extra dimension scenarios the procedure is to pass from higher dimensions to the four dimensions by compactifying each extra dimension to a circle $S^1$ with radius $R$, which is a typical size of corresponding extra dimension. This compactification leads to appear new particles, namely Kaluza-Klein (KK) modes in the theory. If all the fields feel the extra dimensions, so called universal extra dimensions (UED), the extra dimensional momentum, therefore the KK number
at each vertex, is conserved. If the extra dimensions are accessible to some fields but not all in the theory, they are called non-universal extra dimensions. In this case, the KK number at each vertex is not conserved and tree level interaction of KK modes with the ordinary particles can exist. If the fermions are assumed to locate at different points in the extra dimension with Gaussian profiles, the hierarchy of fermion masses can be obtained from the overlaps of fermion wave functions and such scenario is called the split fermion scenario [27–34].

Here, we consider that the new Higgs doublet is localized in the extra dimension with a Gaussian profile, by an unknown mechanism, however, the other particle zero modes have uniform profile in the extra dimension. The Higgs localization in the extra dimension has been considered in several works. The work [12] was devoted to the branching ratios of the radiative LFV decays in the split fermion scenario, with the assumption that the new Higgs doublet is restricted to the 4D brane or to a part of the bulk in one and two extra dimensions, in the framework of the 2HDM. The idea of the localization of the SM Higgs, using the localizer field, has been studied in [35]. In [36] the new Higgs scalars were localized in the extra dimension and the SM Higgs was considered to have a constant profile. The localization of new Higgs scalars depended strongly on the strength of the small coupling of the localizer to the new Higgs scalar. In the present analysis, we first consider that the new Higgs scalars localize with Gaussian profiles around origin in the extra dimension. Second, we assume that the localization point is different than the origin but near to that. We see that the BRs of the LFV decays $\mu \rightarrow e\gamma$, $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ are at the order of the magnitude of $10^{-12}$, $10^{-16}$ and $10^{-12}$ in the given range of the free parameters. These numerical values are slightly suppressed in the case that the localization points of new Higgs scalars are different than origin.

The paper is organized as follows: In Section 2, we present the lepton-lepton-new Higgs scalar vertices and the BRs of the radiative LFV decays with the assumption that the new Higgs doublet is localized with a Gaussian profile in the extra dimension in the 2HDM. Section 3 is devoted to discussion and our conclusions.

2 The effect Gaussian profile of new Higgs scalars on the radiative LFV decays in the 2HDM

The radiative LFV $l_i \rightarrow l_j \gamma$ decays are rare decays in the sense that they exist at loop level in the SM. Since the numerical values of the BRs of these decays are extremely small in the framework of the SM, one goes the models beyond where the particle spectrum is extended and the additional contributions result in an enhancement in the numerical values of the physical
parameters. Due to the extended Higgs sector, the version of the 2HDM, permitting the existence of the FCNCs at tree level, is one of the candidate to obtain relatively large BRs of the decays under consideration. Furthermore, we take into account the effects of the inclusion of a single spatial extra dimension which causes to enhance the BRs due to the fact that the particle spectrum is further extended after the compactification. Here, we consider the effects of the additional Higgs sector with the assumption that the new Higgs scalar zero modes are localized in the extra dimension with Gaussian profiles by an unknown mechanism, on the other hand, the zero modes of other particles have uniform profile in the extra dimension. The Yukawa Lagrangian responsible for the LFV interactions in a single extra dimension reads,

$$L_Y = \xi^E_{5 \ i j} \bar{l}_i L \phi_2 E^R_j + h.c.$$ ,

where $L$ and $R$ denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, $\phi_2$ is the new scalar doublet and $\xi^E_{5 \ i j}$ are the FV complex Yukawa couplings in five dimensions, where $i, j$ are family indices of leptons, $\phi_i$ for $i = 1, 2$, are the two scalar doublets, $l_i$ and $E_j$ are lepton doublets and singlets respectively. These fields are the functions of $x^\mu$ and $y$, where $y$ is the coordinate represents the fifth dimension.

We choose the Higgs doublets $\phi_1$ and $\phi_2$ as

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2} \chi^+ \\ i \chi^0 \end{pmatrix} ; \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} H^+ \\ H_1 + iH_2 \end{pmatrix} ,$$

and their vacuum expectation values read:

$$<\phi_1> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ; <\phi_2> = 0 .$$

In this case, it is possible to collect the SM (new) particles in the first (second) doublet and $H_1$ and $H_2$ becomes the mass eigenstates $h^0$ and $A^0$, respectively since no mixing occurs between two CP-even neutral bosons $H^0$ and $h^0$ at tree level.

The five dimensional lepton doublets and singlets have both chiralities and the four dimensional Lagrangian is constructed by expanding these fields into their KK modes. Besides, the extra dimension denoted by $y$ is compactified on an orbifold $S^1/Z_2$ with radius $R$. The KK decompositions of the lepton and the SM Higgs fields read

$$\phi_1(x, y) = \frac{1}{\sqrt{2\pi R}} \left\{ \phi_1^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \phi_1^{(n)}(x) \cos(ny/R) \right\} ,$$

$$l_i(x, y) = \frac{1}{\sqrt{2\pi R}} \left\{ l_{iL}^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ l_{iL}^{(n)}(x) \cos(ny/R) + l_{iR}^{(n)}(x) \sin(ny/R) \right] \right\} ,$$

$$E_i(x, y) = \frac{1}{\sqrt{2\pi R}} \left\{ E_{iR}^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[ E_{iR}^{(n)}(x) \cos(ny/R) + E_{iL}^{(n)}(x) \sin(ny/R) \right] \right\} ,$$

(4)
where $\phi_1^{(0)}(x)$, $l_{iL}^{(0)}(x)$ and $E_{iR}^{(0)}(x)$ are the four dimensional Higgs doublet, lepton doublets and lepton singlets respectively. Here, we assume that the new Higgs scalars are localized in the extra dimension with Gaussian profiles,

$$S(x, y) = Ae^{-\beta y^2} S(x),$$  

by an unknown mechanism\(^1\). The normalization constant $A$ is

$$A = \frac{(2 \beta)^{1/4}}{\pi^{1/4} \sqrt{Erf[\sqrt{2 \beta \pi R}]}};$$  

and the parameter $\beta = 1/\sigma^2$ regulates the amount of localization, where $\sigma$, $\rho = \rho R$, is the Gaussian width of $S(x, y)$ in the extra dimension. Here the function $Erf[z]$ is the error function, which is defined as

$$Erf[z] = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt.$$  

The coupling of the new Higgs doublet to the leptons brings modified Yukawa interactions in four dimensions. To obtain the lepton-lepton-Higgs interaction coupling in four dimensions we need to integrate the combination $\bar{f}_i^{(0(\nu))}(x, y) S(x, y) f_j^{(n(0))}(x, y)$, appearing in the part of the Lagrangian (eq. (1)), over the fifth dimension. Using the KK basis for lepton fields (see eq. (4)), we get

$$\int_{-\pi R}^{\pi R} dy \bar{f}_i^{(0(\nu))}(x, y) S(x, y) f_j^{(n(0))}(x, y) = V_n \bar{f}_i^{(\nu(\nu))}(x) S(x) f_j^{(n(0))}(x),$$  

where the factor $V_n$ reads

$$V_n = A c_n,$$  

and the function $A$ is defined in eq. (6). Here, the fields $f_i^{(n(0))}, f_j^{(n(0))}$ are four dimensional left and right handed zero (n) mode lepton fields. The function $c_n$ in eq. (9) is obtained as:

$$c_n = e^{-\frac{\nu^2}{4\beta R^2}} \left( Erf\left[\frac{\nu n + 2 \beta \pi R^2}{2 \sqrt{\beta R}}\right] + Erf\left[\frac{-\nu n + 2 \beta \pi R^2}{2 \sqrt{\beta R}}\right]\right) \frac{4 \sqrt{\beta \pi R}}{4 \sqrt{\beta \pi R}},$$  

Notice that the Yukawa couplings $\xi_{ij}^E$ in four dimensions are

$$\xi_{ij}^E = A \xi_{ij}^E,$$  

\(^1\)We consider the zero mode Higgs scalars and we do not take into account the possible KK modes of Higgs scalars since the mechanism for the localization is unknown and we expect that the those contributions are small due to their heavy masses.
where $\xi_{5ij}^E$ are the Yukawa couplings in five dimensions (see eq. (11))\textsuperscript{2}.

Now, we consider that the new Higgs scalars are localized in the extra dimension at the point $y_H$, $y_H = \alpha R$ near to the origin, namely,

$$S(x, y) = A_H e^{-\beta(y-y_H)^2} S(x),$$

with the normalization constant

$$A_H = \frac{2 \beta^{1/4}}{(2\pi)^{1/4} \sqrt{\text{ Erf} \left[\sqrt{2\beta (\pi R + y_H)}\right] + \text{ Erf} \left[\sqrt{2\beta (\pi R - y_H)}\right]}}.$$

After integrating the combination $\bar{f}_{iL(R)}^{(n)}(x, y) S(x, y) f_{jR(L)}^{(n)}(x, y)$ over extra dimension, the factor $V_n$ in eq. (13) reads

$$V_n = A_H c_n,$$

with the function $A_H$ in eq. (13). The function $c_n$ in eq. (14) is calculated as:

$$c_n = e^{-\frac{m^2_{y_H}}{4\beta R}} \cos \left[ \frac{ny_H}{R} \right] \left( \frac{\text{ Erf} \left[\frac{in + 2\beta \pi R^2}{2\sqrt{\beta R}}\right] + \text{ Erf} \left[\frac{-in + 2\beta \pi R^2}{2\sqrt{\beta R}}\right]}{4\sqrt{\beta \pi R}} \right).$$

Similar to the previous case, we define the Yukawa couplings in four dimensions as

$$\xi_{ij}^E = A_H \xi_{5ij}^E.$$

Now, we will present the decay widths of the LFV processes $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$. These decays exist at least at one loop level in the 2HDM and there appear the logarithmic divergences in the calculations. These divergences can be eliminated by using the on-shell renormalization scheme\textsuperscript{3}. The decay width $\Gamma$ for the $l_i \to l_j \gamma$ decay reads

$$\Gamma(l_i \to l_j \gamma) = c_1 (|A_1|^2 + |A_2|^2),$$

for $l_i (l_j) = \tau; \mu (\mu$ or $e; e)$. Here $c_1 = \frac{G_F^2\alpha_{em} m^3}{32\pi^4},\ A_1 (A_2)$ is the left (right) chiral amplitude and taking only $\tau$ lepton for the internal line\textsuperscript{4}, they read

$$A_1 = Q_\tau \frac{1}{48 m^2_\tau} \left( 6 m_\tau \xi_{N,\tau}^{E*} \xi_{N,\tau}^{E*} c_0^2 \left( F(v_h) - F(v_{A^0}) \right) \right).$$

\textsuperscript{2}In the following we use the dimensionful coupling $\xi_{5ij}^E$ in four dimensions, with the definition $\xi_{N,ij}^E = \sqrt{\xi_{5ij}^E}$, where $N$ denotes the word "neutral".

\textsuperscript{3}In this scheme, the self energy diagrams for on-shell leptons vanish since they can be written as $\sum(p) = (\bar{p} - m_{\ell_i}) \sum(p)(\bar{p} - m_{\ell_j})$, however, the vertex diagrams (see Fig.1) give non-zero contribution. In this case, the divergences can be eliminated by introducing a counter term $V_\mu^C$ with the relation $V_\mu^{Ren} = V_\mu^0 + V_\mu^C$, where $V_\mu^{Ren} (V_\mu^0)$ is the renormalized (bare) vertex and by using the gauge invariance: $k^{\mu} V_\mu^{Ren} = 0$. Here, $k^{\mu}$ is the four momentum vector of the outgoing photon.

\textsuperscript{4}We take into account only the internal $\tau$-lepton contribution since, we respect the Sher scenario [37], results in the couplings $\xi_{5ij}^E (i,j = e, \mu)$ are small compared to $\xi_{N,ij}^E (i = e, \mu, \tau)$, due to the possible proportionality of them to the masses of leptons under consideration in the vertices.
\[ + 2 \sum_{n=1}^{\infty} c_n^2 \left( F(v_{n,h^0}) - F(v_{n,A^0}) \right) + m_{f_1} \xi_{N,f_2} \xi_{N,f_1} \left( c_0^2 \left( G(v_{h^0}) + G(v_{A^0}) \right) \right) \]
\[ + 2 \sum_{n=1}^{\infty} c_n^2 \left( G(v_{n,h^0}) + G(v_{n,A^0}) \right) \}

\[ A_2 = Q_{\tau} \frac{1}{48 m_{\tau}^2} \left\{ 6 m_{\tau} \xi_{N,2\tau} \xi_{N,\tau} \left( c_0^2 \left( F(v_{h^0}) - F(v_{A^0}) \right) \right) \right. \]
\[ + 2 \sum_{n=1}^{\infty} c_n^2 \left( F(v_{n,h^0}) - F(v_{n,A^0}) \right) + m_{f_1} \xi_{N,f_2} \xi_{N,f_1} \left( c_0^2 \left( G(v_{h^0}) + G(v_{A^0}) \right) \right) \]
\[ + 2 \sum_{n=1}^{\infty} c_n^2 \left( G(v_{n,h^0}) + G(v_{n,A^0}) \right) \} \right\}, \quad (18) \]

where \( v_{n,S} = \frac{m_{\tau}^2 + m_{\tau}^2}{m_{\tau}^2} \), \( m_n = \frac{n}{R} \) and \( Q_{\tau} \) is the charge of \( \tau \) lepton. Here the vertex factor \( c_n \) is defined in eq. (10) (eq. (13)) and the functions \( F(w) \) and \( G(w) \) are

\[ F(w) = \frac{w (3 - 4 w + w^2 + 2 \ln w)}{(-1 + w)^3}, \]
\[ G(w) = \frac{w (2 + 3 w - 6 w^2 + w^3 + 6 w \ln w)}{(-1 + w)^4}. \quad (19) \]

### 3 Discussion

In the present work, we study the radiative LFV decays \( l_i \to l_j \gamma \) in the 2HDM with the addition of a single spatial extra dimension. Here, we consider that new Higgs scalars are localized in the extra dimension with Gaussian profiles by an unknown mechanism, on the other hand, the other particles have uniform zero mode profiles in the extra dimension which is compactified onto orbifold \( S_1/Z_2 \). Since these decays exist at least at one loop level, there appear free parameters related to the model used in the theoretical values of the physical quantities. The Yukawa couplings \( \xi_{N,ij} \), \( i, j = e, \mu, \tau \) are among those parameters. We consider that the couplings \( \xi_{N,ij} \), \( i, j = e, \mu \) are smaller compared to \( \xi_{N,\tau} \), \( i = e, \mu, \tau \) since latter ones contain heavy flavor. Furthermore, we assume that, in four dimensions, the couplings \( \xi_{N,ij} \) are symmetric with respect to the indices \( i \) and \( j \). For the Higgs masses we take the numerical values \( m_{h^0} = 100 \, GeV \), \( m_{A^0} = 200 \, GeV \). The compactification scale \( 1/R \) and the Gaussian width \( \sigma \) of new Higgs doublet are the additional free parameters which are chosen not to contradict with the experimental results. The direct limits from searching for KK gauge bosons imply \( 1/R > 800 \, GeV \), the precision electroweak bounds on higher dimensional operators generated by KK exchange place a far more stringent limit \( 1/R > 3.0 \, TeV \) and, from \( B \to \phi K_S \), the lower bounds for the scale \( 1/R \) have been obtained as \( 1/R > 1.0 \, TeV \), from \( B \to \psi K_S \) one got \( 1/R > 500 \, GeV \), and from the upper limit of the \( BR \), \( BR(B_s \to \mu^+ \mu^-) < 2.6 \times 10^{-6} \), the
estimated limit was $1/R > 800 \text{ GeV}$ \cite{31}. Here, we take the compactification scale $1/R$ in the range $200 \text{ GeV} \leq 1/R \leq 1000 \text{ GeV}$ and choose the Gaussian width at most $\rho = 0.05$.

Our analysis is based on the Higgs localization width and the compactification scale dependence of the BRs of the LFV decays. First, we consider that the new Higgs is localized around the origin in the extra dimension. Furthermore, we choose the localization point is near to the origin and study its effect on the BRs.

Fig. 2 represents the BR($\mu \rightarrow e\gamma$) with respect to $\rho$, for different values of the Yukawa couplings $\bar{\xi}^{E}_{N,\tau\mu}$ and $\bar{\xi}^{E}_{N,\tau e}$. Here the lower-upper solid (dashed) line represents the BR for a single extra dimension without-lepton KK modes, the real couplings $\xi^{E}_{N,\tau\mu} = 50 \text{ GeV}$, $\xi^{E}_{N,\tau e} = 0.1 \text{ GeV}$ ($\xi^{E}_{N,\tau e} = 0.5 \text{ GeV}$) and $R = 0.005 \text{ GeV}^{-1}$. It is observed that the BR is strongly sensitive to the Gaussian width of the localized neutral Higgs scalars and it increases with the increasing values of the width. This enhancement is almost at the order of $10^4$ in the range $0.005 \leq \rho \leq 0.05$. The inclusion of lepton KK modes brings additional enhancement at one order. The BR is at the order of the magnitude of $10^{-14}$ ($10^{-12}$) for the intermediate values of the localization parameter $\rho$ and the coupling $\bar{\xi}^{E}_{N,\tau e} = 0.1 \text{ GeV}$ ($\bar{\xi}^{E}_{N,\tau e} = 0.5 \text{ GeV}$). A new experiment at PSI has been described and aimed to reach to a sensitivity of $BR \sim 10^{-14}$ and at present the experiment (PSI-R-99-05 Experiment) is still running in the MEG \cite{4}. The improvement of the numerical result of the BR would make it possible to search the effects of the extra dimensions and the possible localization of new Higgs bosons in the extra dimension.

In Fig. 3 and 4 we present the BR($\tau \rightarrow e\gamma$) and BR($\tau \rightarrow \mu\gamma$) with respect to $\rho$, for different values of the Yukawa couplings. Here the lower-upper solid (dashed), line represents the BR for a single extra dimension without-lepton KK modes, for $R = 0.005 \text{ GeV}^{-1}$, the real couplings $\bar{\xi}^{E}_{N,\tau\tau} = 100 \text{ GeV}$, $\xi^{E}_{N,\tau e} = 0.1 \text{ GeV}$ ($\xi^{E}_{N,\tau e} = 0.5 \text{ GeV}$) and $\bar{\xi}^{E}_{N,\tau\mu} = 50 \text{ GeV}$ ($\xi^{E}_{N,\tau\mu} = 80 \text{ GeV}$). It is observed that the BR is sensitive to the Gaussian width of the localized neutral Higgs scalars and there is an enhancement at the order of $10^4$ in the range $0.005 \leq \rho \leq 0.05$ similar to the previous decay. The BR is at the order of the magnitude of $10^{-18}$ ($10^{-16}$) and $10^{-13}$ ($10^{-12}$) for the intermediate values of the localization parameter $\rho$ and the coupling $\bar{\xi}^{E}_{N,\tau e} = 0.1 \text{ GeV}$ ($\xi^{E}_{N,\tau e} = 0.5 \text{ GeV}$) and $\bar{\xi}^{E}_{N,\tau\mu} = 0.1 \text{ GeV}$ ($\xi^{E}_{N,\tau\mu} = 0.5 \text{ GeV}$). Notice that the inclusion of lepton KK modes brings additional enhancement at one order, in both decays.

Figs. 5, 6, 7 are devoted to the BR($\mu \rightarrow e\gamma$); BR($\tau \rightarrow e\gamma$); BR($\tau \rightarrow \mu\gamma$) with respect to the compactification scale $1/R$. Here the lower-upper solid (dashed), line represents the BR for a single extra dimension without-lepton KK modes, the real couplings $\bar{\xi}^{E}_{N,\tau\mu} = 50 \text{ GeV}$,
\( \xi_{N,\tau e}^E = 0.1 \text{GeV} \ (\xi_{N,\tau e}^E = 0.5 \text{GeV}); \xi_{N,\tau \tau}^E = 100 \text{GeV}, \xi_{N,\tau e}^E = 0.1 \text{GeV} \ (\xi_{N,\tau e}^E = 0.5 \text{GeV}); \xi_{N,\tau \tau}^E = 100 \text{GeV}, \xi_{N,\tau \mu}^E = 50 \text{GeV} \ (\xi_{N,\tau \mu}^E = 80 \text{GeV}) \) and \( \rho = 0.01 \). These figures show that the enhancement of the BR with the addition of lepton KK modes is not so much sensitive to the compactification scale \( 1/R \) for its large values.

Now, we study the effects of the position of the localization point of the new Higgs doublet on the BR of the considered decays.

Fig. 8 represents the BR(\( \mu \to e\gamma \)) with respect to \( \alpha \), for different values of the Yukawa couplings \( \xi_{N,\tau \mu}^E \) and \( \xi_{N,\tau e}^E \). Here the lower-upper solid (dashed), line represents the BR for a single extra dimension with lepton KK modes, for \( y_H = 0 - y_H = \alpha \sigma \), the real couplings \( \xi_{N,\tau \mu}^E = 50 \text{GeV}, \xi_{N,\tau e}^E = 0.1 \text{GeV} \ (\xi_{N,\tau e}^E = 0.5 \text{GeV}) \) and \( R = 0.005 \text{GeV}^{-1} \). This figure shows that the BR decreases with the increasing values of \( \alpha \), at the order of \( %45 \) in the interval \( 1 \leq \alpha \leq 10 \), for the large values of the Yukawa coupling.

In Fig. 9, 10 we present the BR(\( \tau \to e\gamma \)); BR(\( \tau \to \mu\gamma \)) with respect to \( \alpha \). Here the lower-upper solid (dashed), line represents the BR for a single extra dimension with lepton KK modes, for \( y_H = 0 - y_H = \alpha \sigma \), the real couplings \( \xi_{N,\tau \mu}^E = 50 \text{GeV}, \xi_{N,\tau e}^E = 0.1 \text{GeV} \ (\xi_{N,\tau e}^E = 0.5 \text{GeV}) \) and \( R = 0.005 \text{GeV}^{-1} \). These figures show that the BR decreases with the increasing values of \( \alpha \), at the order of \( %35; %45 \) in the interval \( 1 \leq \alpha \leq 10 \), for the large values of the Yukawa coupling.

At this stage we would like to summarize our results:

- The BR is strongly sensitive to the Gaussian width of the localized neutral Higgs scalars and it increases with the increasing values of the width for the decays under consideration. This enhancement is almost at the order of \( 10^4 \) in the range \( 0.005 \leq \rho \leq 0.05 \). The inclusion of lepton KK modes brings additional enhancement at one order. The BR for \( \mu \to e\gamma \ (\tau \to e\gamma, \tau \to \mu\gamma) \) is at most at the order of the magnitude of \( 10^{-12} \) \((10^{-16}, 10^{-12})\) for the intermediate values of the localization parameter \( \rho \) and the couplings taken.

- The BR decreases with the increasing distance, \( y_H = \alpha \sigma \), of the localization point of the new Higgs doublet from the origin in the extra dimension. With the increasing values of \( \alpha \), there is almost \( %50 \) suppression of the BRs of the decays under consideration in the interval \( 1 \leq \alpha \leq 10 \), for the large values of the Yukawa couplings.

The improvement of the experimental results of the radiative LFV decay BRs would make it possible to search the effects of the extra dimension and the possible localization of the new Higgs bosons in the extra dimension.
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Figure 1: One loop diagrams contribute to $l_1 \rightarrow l_2 \gamma$ decay due to the zero mode neutral Higgs bosons $h^0$ and $A^0$ in the 2HDM, for a single extra dimension. Here $l_i^n$ represents the internal KK mode charged lepton and $n=0,1,\ldots$
Figure 2: The BR($\mu \to e\gamma$) with respect to $\rho$. Here the lower-upper solid (dashed), line represents the BR for a single extra dimension without-with lepton KK modes, the real couplings $\tilde{\xi}^E_{N,\tau\mu} = 50 \text{ GeV}$, $\tilde{\xi}^E_{N,\tau e} = 0.1 \text{ GeV}$ ($\tilde{\xi}^E_{N,\tau e} = 0.5 \text{ GeV}$) and $R = 0.005 \text{ GeV}^{-1}$.

Figure 3: The BR($\tau \to e\gamma$) with respect to $\rho$. Here the lower-upper solid (dashed), line represents the BR for a single extra dimension without-with lepton KK modes, for $R = 0.005 \text{ GeV}^{-1}$, the real couplings $\tilde{\xi}^E_{N,\tau\tau} = 100 \text{ GeV}$, $\tilde{\xi}^E_{N,\tau e} = 0.1 \text{ GeV}$ ($\tilde{\xi}^E_{N,\tau e} = 0.5 \text{ GeV}$).
Figure 4: The BR($\tau \to \mu \gamma$) with respect to $\rho$. Here the lower-upper solid (dashed), line represents the BR for a single extra dimension without-with lepton KK modes, for $R = 0.005 \, GeV^{-1}$, the real couplings $\bar{\xi}^E_{N,\tau\tau} = 100 \, GeV, \bar{\xi}^E_{N,\tau\mu} = 50 \, GeV (\bar{\xi}^E_{N,\tau\mu} = 80 \, GeV)$.

Figure 5: The BR($\mu \to e \gamma$) with respect to the compactification scale $1/R$. Here the lower-upper solid (dashed), line represents the BR for a single extra dimension without-with lepton KK modes, the real couplings $\bar{\xi}^E_{N,\tau\mu} = 50 \, GeV, \bar{\xi}^E_{N,\tau e} = 0.1 \, GeV (\bar{\xi}^E_{N,\tau e} = 0.5 \, GeV)$ and $\rho = 0.01$. 
Figure 6: BR(\(\tau \rightarrow e\gamma\)) with respect to the compactification scale 1/R. Here the lower-upper solid (dashed) line represents the BR for a single extra dimension without-with lepton KK modes, the real couplings \(\xi_{E,\tau\tau} = 100 GeV\), \(\xi_{E,\tau\mu} = 0.1 GeV\) (\(\xi_{E,\tau\mu} = 0.5 GeV\)) and \(\rho = 0.01\).

Figure 7: BR(\(\tau \rightarrow \mu\gamma\)) with respect to the compactification scale 1/R. Here the lower-upper solid (dashed) line represents the BR for a single extra dimension without-with lepton KK modes, the real couplings \(\xi_{N,\tau\tau} = 100 GeV\), \(\xi_{N,\tau\mu} = 50 GeV\) (\(\xi_{N,\tau\mu} = 80 GeV\)) and \(\rho = 0.01\).
Figure 8: The $\text{BR}(\mu \to e\gamma)$ with respect to $\alpha$. Here the lower-upper solid (dashed), line represents the BR for a single extra dimension with lepton KK modes, for $y_H - y_{\nu} = \alpha \sigma$, the real couplings $\bar{\xi}^{E}_{s_{N,\tau\mu}} = 50 \text{GeV}$, $\bar{\xi}^{E}_{s_{N,\tau e}} = 0.1 \text{GeV}$ ($\bar{\xi}^{E}_{s_{N,\tau e}} = 0.5 \text{GeV}$) and $R = 0.005 \text{GeV}^{-1}$.

Figure 9: The $\text{BR}(\tau \to e\gamma)$ with respect to $\alpha$. Here the lower-upper solid (dashed), line represents the BR for a single extra dimension with lepton KK modes, for $y_H - y_{\nu} = \alpha \sigma$, the real couplings $\bar{\xi}^{E}_{s_{N,\tau\tau}} = 100 \text{GeV}$, $\bar{\xi}^{E}_{s_{N,\tau e}} = 0.1 \text{GeV}$ ($\bar{\xi}^{E}_{s_{N,\tau e}} = 0.5 \text{GeV}$) and $R = 0.005 \text{GeV}^{-1}$. 

16
Figure 10: The BR(τ → μγ) with respect to α. Here the lower-upper solid (dashed), line represents the BR for a single extra dimension with lepton KK modes, for $y_H = 0 - y_H = α\sigma$, the real couplings $\bar{\xi}^E_{N,\tau\mu} = 50\text{ GeV}$ ($\bar{\xi}^E_{N,\tau\mu} = 80\text{ GeV}$) and $R = 0.005\text{ GeV}^{-1}$. 