Mechanical analysis of photovoltaic panels with various boundary condition

Y X Li¹, L Z Xie¹, ², T Y Zhang², Y P Wu³, Y Y Sun³, Z C Ni⁴, J Q Zhang⁵, B He²
¹College of Architecture and Environment, Sichuan University, Chengdu, 610065, P.R. China
²Institute of New Energy and Low-Carbon Technology, Sichuan University, Chengdu, 610207, P.R. China
³Department of Architecture and Built Environment, Faculty of Engineering, University of Nottingham, University Park, Nottingham NG7 2RD, UK
⁴Suzhou Talesun Solar Technologies Co., Ltd., Chang Shu, 215542, P. R. China
⁵College of Materials Science and Engineering, Sichuan University, Chengdu, 610065, P.R. China

E-mail: xielingzhi@scu.edu.cn

Abstract. The photovoltaic (PV) panels currently existed on market are a kind of laminated plate structure, which is composed of two stiff glass skins and a soft interlayer. Some of those panels are installed on the buildings and integrated as the components of the structure, such as wall and roof. In different locations, the installations of the panels are different and the boundary conditions are not simply supported any more. The generating electricity requirement of the panels needs the mechanic behaviour of them is stable with any boundary condition. In this paper the bending behaviour of PV panels with various boundary conditions is analysed and the influence of boundary condition is studied carefully. The Kirchhoff theory which is one of the classical lamination theory (CLT) is adopted to build governing equations of photovoltaic panels under static force. A Rayleigh-Rita method is modified to solve the governing equations and calculate the static deformation and stress. Different boundary condition requires different assumptions of the deflection function and a modified general function is developed to solve that problem. A theoretical solution is derived out and used to do the numerical calculation. A bending experiment of PV panel with two opposite edges simply supported and the other two free is used to verify the correctness and accuracy of the proposed solution. Finally, the influence of different boundary condition is stated by comparing the numerical results and some guides for the PV panel installation are proposed.

Keywords: photovoltaic panel; various boundary condition; classical lamination theory; Rayleigh-Rita method; modified deflection function; bending experiment

1. Introduction
U.S. Department of Energy stated that building consumes more than 40% of the electricity produced in U.S. every year. To reduce building energy consumption, building integrated photovoltaic (BIPV) has attracted much attention. It requires that the photovoltaic (PV) component must generate electricity for the building, and bear external loads to keep the safety of the building at the same time. Currently, only in the standards of PV module, such as IEC 61215 (2005) [1], there are several codes about its mechanical properties and the corresponding test methods. It is lack of specific codes to the mechanical characters of PV modules applied in BIPV. In present paper, it focuses on the bending behaviour of the PV panels under wind load or snow load. In BIPV, the double glass PV module with better photopermeability are widely applied. Therefore, the PV panels studied in here are double glass PV module which consists of two glasses and an interlayer. In buildings, different installation ways mean different boundary conditions for the PV module, and the mechanical behaviours of them are different. It is necessary to study the influences of different boundary conditions in here. From 1990s,
PV module started to be integrated into building, and some relative research works were promoted [2]. Naumenko and Eremeyev [3] used the layer-wise theory to analyze PV panel and they treated the PV panel as a layered composite with relatively stiff skin layer and relatively soft core. Vedrtnam and Pawar [4] made a review work on laminate composite, and laminate glass plate which is very like PV panel is mainly introduced. First order shear deformation theory (FSDT), Zig-zag theory and layer-wise theory (LWT) were applied in many previous research works. However, in many of those works, the boundary conditions of the plates are only four edges simply-supported. And there are very few works discussing the influence of different boundary conditions.

In present paper, the mechanical properties of double glass PV panels with two different boundary conditions are analysed by both experimental and theoretical research. A classical lamination theory, Hoff model, is applied to build the constitutive equations of whole panel. The specific boundary equations are given based on two boundary conditions. The Rayleigh-Riata method is modified to derive the closed form solution. By using water pressure, the bending experiment of PV panel was completed. And changing the test frame, two boundary conditions were realized in the experiments. Comparing the theoretical results with experimental results, the accuracy of the analytical solutions are verified. The influences of boundary condition are also concluded. The theoretical model and solutions obtained in this paper can be the foundations for the optimal design work in future.

2. Theoretical analysis

2.1. Mechanical model and basic hypothesis

In figure 1, it shows the basic components of PV panel. A laminate plate model (as shown in figure 2) is applied and several hypothesis are made at the beginning of the theoretical analysis.

1. The cover and backboard glasses are treated as top and bottom surfaces of the laminate plate, respectively. And both of them are simulated as isotropic plates with constant flexural rigidity.
2. The silicon solar cells layer is ignored and two EVA layers are merged as one layer.
3. The PV module is a soft core laminate plate and the stress of interlayer in x-y plane is ignored.
4. Only the anti-symmetrical deformation is studied in present paper, so \( \sigma_z = 0 \), \( \varepsilon_z = 0 \).

Figure 1. Structural diagram of monocrystalline silicon double glass PV panel.

Figure 2. Mechanical model of PV panel and corresponding coordinate system.

2.2. Hoff model and governing equations

Reissner theory is modified by Hoff [5], and a Hoff model is developed for the laminated plate. We could obtain the governing equations of PV panel under uniformly distributed force as follows.

\[
\begin{align*}
    w &= \omega - \frac{D}{C} \nabla^2 \omega \\
    \left( D + 2D_f \right) \nabla^2 \nabla^2 \omega - \frac{2DD_f}{C} \nabla^2 \nabla^2 \omega &= q \\
    \frac{1}{2} D \left( 1 - \nu_f \right) \nabla^2 f - Cf &= 0
\end{align*}
\]
with 
\[ D = \frac{E_j (h + t)^3 t}{2(1 - \nu^2)}, \quad D_j = \frac{E_j t^3}{12(1 - \nu^2)}, \quad C = G_c \frac{(h + t)^3}{h}. \]

where \( E_j \) is the elastic modulus of the cover and the back glass plate, \( \nu \) is the Poisson’s ratio of the cover and the back glass plate, \( G_c \) is the shear modulus of EVA, \( t \) and \( h \) are the thickness of surface plate and EVA interlayer, respectively.

2.3. Boundary conditions and boundary equations

Two boundary conditions are studied in present paper to find the influence of boundary condition: one is four edges simply supported; one is two opposite edges simply supported and the other two edges free.

1. Boundary equations of four edges simply supported

\[
\begin{align*}
(M_x)_{y=0} &= 0, (w)_{x=0} = 0, (\varphi_x)_{x=0} = 0, (M'_x)_{y=0} = 0 \\
(M_y)_{x=b} &= 0, (w)_{y=b} = 0, (\varphi_y)_{y=b} = 0, (M'_y)_{y=b} = 0
\end{align*}
\]

(4)

(5)

2. Boundary equations of two opposite edges simply supported and the other two edges free

\[
\begin{align*}
(M_x)_{y=0} &= 0, (w)_{x=0} = 0, (\varphi_x)_{x=0} = 0, (M'_x)_{y=0} = 0 \\
(M'_y)_{y=b} &= 0, (M'_{xy})_{y=b} = 0, (Q_y)_{y=b} = 0, (M'_y)_{y=-b} = 0
\end{align*}
\]

(6)

(7)

2.4. Modified Rayleigh-Rita method and closed-form solutions

1. Closed-form solutions of four edges simply supported

In present paper, a modified Rayleigh-Rita method is applied to solve the governing equations and a modified general assumption is developed for the solutions. The deflection of PV panel under uniformly distributed force could be calculated based on equation (8).

\[
w = \omega - \frac{D}{C} \nabla^2 \omega = \sum_{n=1}^{\infty} \left\{ \sum_{j=1}^{6} \left[ \sum_{r=1}^{6} \left[ \sum_{s=1}^{6} \left[ \frac{2q}{n\pi} \left[ 1 - \cos(n\pi) \right] \frac{1}{A_k k_n^2} \right] \sin(k_n x) \right] \omega_{rs} + \sum_{r=1}^{6} \left[ \frac{2q}{n\pi} \left[ 1 - \cos(n\pi) \right] \frac{1}{A_k k_n^2} \right] \sin(k_n x) \right] \right]\]

(8)

2. Closed-form solutions of two opposite edges simply supported and the other two edges free

The modified Rayleigh-Rita method is also applied but eight unknown variables should satisfy the new boundary condition as equation (9) to equation (12).

\[
\sum_{j=1}^{6} \left[ \frac{2q}{n\pi} \left[ 1 - \cos(n\pi) \right] \frac{1}{A_k k_n^2} \right] \omega_{rs} + \sum_{r=1}^{6} \left[ \frac{2q}{n\pi} \left[ 1 - \cos(n\pi) \right] \frac{1}{A_k k_n^2} \right] \sin(k_n x) = 0
\]

(9)

(10)

(11)

\[
\sum_{j=1}^{6} \left[ \frac{2q}{n\pi} \left[ 1 - \cos(n\pi) \right] \frac{1}{A_k k_n^2} \right] \omega_{rs} + \sum_{r=1}^{6} \left[ \frac{2q}{n\pi} \left[ 1 - \cos(n\pi) \right] \frac{1}{A_k k_n^2} \right] \sin(k_n x) = 0
\]

(12)

The new eight unknown variables are solved, the deflection of PV panel is calculated by equation (8).
3. Finite element analysis
A finite element analysis is performed by the use of ANSYS. The material of each layer is simulated as isotropic material and the mechanical properties of them are shown in Table 1. Because it could be layered, SHELL181 composite shell element is used for modeling.

| Material                  | Modulus of elasticity /MPa | Poisson ratio | Thickness /mm |
|---------------------------|----------------------------|---------------|---------------|
| Reinforced glass          | 7.2E+4                     | 0.2           | 2             |
| Crystalline silicon battery | 1.44E+5                   | 0.28          | 0.2           |
| EVA                       | 3.5E+1                     | 0.3           | 0.8           |

4. Experimental analysis
The 6 specimens for bending test are all the double glass photovoltaic modules (as shown in figure 3) which are provided by Suzhou Tenghui Photovoltaic Technology Co., Ltd. The size is 1658×995×7.4 (unit: mm), in which the cover and back glasses are 3.2mm and the interlayer thickness is 1mm. The two boundary conditions are realized as figure 4 and figure 5, respectively. Water pressure is adopted in the tests. The test site with frame, PV panel and equipment are just shown as figure 6.

5. Verifications and discussions

5.1. PV panel deflection
In figure 7, the deflection calculated by both ANSYS and equation (8) is changed linearly with water pressure but the test results are changed nonlinearly. It is because the Hoff model is based on linear elastic deformation hypothesis, but the real deformation of PV panel with four edges simply supported is a nonlinear elastic deformation. A nonlinear theory should be applied in future study. The figure 8 states clearly the linear elastic deformation of PV panel with two opposite edges simply supported and the other two edges free. The results from equation (8) are closer to test results than ANSYS, and the accuracy of the proposed equations is verified. All the specimens were checked carefully after the test, and there was not any cracks on the surface glass. The deformation of PV panels is in safe range.
5.2. PV panel stress

As shown in figure 9, the central stress of PV panel behaves same as the central deflection discussed in figure 7 and it is indeed a nonlinear elastic deformation for the PV panels with four edges simply supported. The data from proposed equations are better when the water pressure is small, but the ones from ANSYS are better under the large water pressure. It is still different in the second group as shown in figure 10, all the stress data has a linear relationship to the water pressure. The deformation of those PV panels is indeed a linear elastic deformation which is also concluded by the deflection data in section 5.1. Although both proposed equation data and ANSYS date match the experimental data very well, the errors are smaller in proposed equation than ANSYS. Moreover, as shown in both figure 9 and figure 10, the maximum stresses on the surface glass of PV panels are all smaller than the limit stress of reinforced glass, so it is safe for the PV panels when they are utilized under those loads.

5.3. Influence of boundary condition

Some discussions about the influence of boundary condition are made. One boundary condition is four edges simply supported, which is marked as SSSS in the figures. The other boundary condition is two opposite edges simply supported and the other two edges free, which is marked as SSFF in figures.

In figure 11, the deflections of PV panels with SSSS are much better. It shows the SSSS can bear a larger uniformly distributed force with a smaller deflection. The central stresses are shown in figure 12 and it can be seen clearly that the maximum central stresses of two cases are both close to 50mPa, which is the limit strength of the reinforced glass. It means that the final damage of the PV panel is because the central stress of panel exceeds the limit stress of surface glass. Under the limit stress of
surface glass, SSSS has a larger ultimate load so its mechanical behaviour is proved better than SSFF again. Based on those, SSSS is a better choice for engineers in future BIPV design work.

![Figure 11. Central deflection of PV panels with two boundary conditions.](image1)

![Figure 12. Central stress of PV panels with two boundary conditions.](image2)

6. Conclusions and recommendations
The aim of this paper is to study the mechanical properties of the double glass PV panel with two different boundary conditions. Both experimental and theoretical works are completed.

Based on the results we may conclude as follows:

- The deformation of PV panels with SSFF is a linear elastic deformation. But it is a nonlinear elastic deformation to the ones with SSSS. A nonlinear elastic theory should be applied later.
- As to the PV panels with SSFF, the calculation results from ANSYS and proposed equations are all close to the experimental results. The deflection and stress calculated by proposed equations are even more accurate than ANSYS. It can be used in the optimal design work.
- As to the PV panel with SSSS, the ANSYS and proposed method are based on linear elastic deformation theory but the real deformation of PV panels is nonlinear. In central deflection, the proposed equations are more accurate for the maximum deflection. In central stress, it is suitable to calculate the stress when the PV panel is under small deformation.
- Comparing the central deflection and central stress from different boundary conditions, the PV panels with SSSS have better effects. The PV panels with SSSS can bear a larger load with a smaller deformation. It should be considered as the primary choice in the BIPV.

Acknowledgments
The authors are grateful for the financial support from the National Key Research and Development Program of China: Newton Fund - China-UK Research and Innovations Bridges (No. 2016YFE0124500).

References
[1] IEC 61215-2005, Crystalline Silicon Terrestrial Photovoltaic (PV) Modules[S]. International Electrotechnical Commission, 2005
[2] Patrina Eiffert, An Economic Assessment of Building Integrated Photovoltaics, Oxford Brookes School of Architecture, 1998
[3] Konstantin Naumenko and Victor A. Eremeyev, A layer-wise theory for laminated glass and photovoltaic panels, Composite Structures, 112(1), 2014, pp 283-291
[4] Ajitanshu Vedrtnam and S. J. Pawar, Laminated plate theories and fracture of laminated glass plate-A review, Engineering Fracture Mechanics, 186(1), 2017, pp 316-330
[5] N. J. Hoff, Bending and buckling of rectangular sandwich plates, NACA, TN2225, 1950