Robust fault detection and isolation for dynamics of high-speed train with uncertainties based on descriptor systems

Runze Wang, Tiantian Liang, Xiang Zheng and Kexin Li

Abstract
This paper proposes a new type of robust fault detection and isolation filter for dynamics of HST based on descriptor systems with uncertainties in finite frequency. This filter is designed based on the unknown input filter to decouple the non-linear variables due to the aerodynamic drag pressing on the trains. The exogenous disturbance is partitioned into two parts-the decoupling one is regarded as the augmented variables of the non-linear part of the systems, and the non-decoupling one is seen as the augmented disturbance along with the uncertainties. Concurrent faults of different positions are considered, residual evaluation functions and adaptive threshold are given to judge if the faults occur. Fault isolation is implemented by a set of detection subspaces associated with every different fault which is assigned to its own detection subspace. The residual is not only sensitive to the fault, but also has a robustness against the non-decoupling disturbance and uncertainties in finite frequency. Simulation examples are given to demonstrate the effectiveness of this method.

Keywords
High-speed train, descriptor system, fault detection and isolation, uncertainties, finite frequency

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Introduction
Safety technology related to railway operation is urgently required with the rapid growth of high-speed railway mileage. In addition to technical factors such as human operation, the hidden dangers of system equipment failure and incomplete safety monitoring mechanisms are the main factors impacting on the safe operation of high-speed trains (HSTs). It is well known that the fault diagnosis technique is a key technology to improve system safety, reliability, and maintainability. Therefore, combined with advanced fault diagnosis technology such as fault tolerant control and fault detection, it is of far-reaching practical significance to improve the safety performance of HST operation.

Due to the requirements of railway transportation safety, the incipient faults of the train must be detected in time to avoid disastrous consequences and economic losses. In view of this, plenty of fault detection methods have been investigated for HST systems in recent years. Among these, research has been investigated for detecting the component faults of the traction devices such as inverter, rectifier, and traction motor. Aiming at the fault diagnosis of the brake cylinder subsystem of the EMU, a fault detection and fault isolation strategy

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based on variable variance has been proposed and verified by experiments.\textsuperscript{3} In addition, some other fault detection results have been reported for suspension systems,\textsuperscript{4} pantograph,\textsuperscript{5} and navigation systems\textsuperscript{6} of HST. However, these methods only focus on the structure devices, which ignore the complicated exogenous disturbance of HST.

Considering the trains as a whole, single particle model has been proposed for describing the dynamics of the HST\textsuperscript{7}. Treating each car of the trains as a rigid point, considering the mutual force between car interconnected, the multi-particle model is proposed further.\textsuperscript{8} Based on these models, adaptive algorithm and learning methods have been proposed to deal with the fault diagnosis of dynamics of HST.\textsuperscript{9–11} In the above literature, the research on HST’s fault diagnosis is more realistic by considering the changes in speed and position of the trains and the influence of the exogenous disturbance. However, these methods pay attention to speed and position tracking control under multiple faults with known boundary value rather than fault detection. To implement fault detection for dynamics of HST based on multi-particle, some prominent methods have been proposed to detect the actuator fault\textsuperscript{12} and sensor fault\textsuperscript{13,14} under the premise that the distribution matrix of the fault is known. Nevertheless, there are still some problems unsolved in these studies. One is that fault detection methods in these studies only focus on the actuator or sensor fault occurring separately but not simultaneously. The other is that these studies ignore exogenous disturbance existing in measurement. Specific to the fault detection of HST, the measurement disturbance or noise has a great influence on the detection index and is difficult to process by state space model, which calls for a system model with better descriptive characteristics.

Regarding the measurement disturbance or noise as an auxiliary variable, the dynamics of the HST are seen as a descriptor system. Descriptor system, which is also named as the singular system, has received widespread attention as it has a better description than the state space one, and has widely used for describing a large class of actual systems such as attitude control systems of satellites (ACSs),\textsuperscript{15} DC motor model,\textsuperscript{16} vehicle,\textsuperscript{17} and power network.\textsuperscript{18} Many significant results have been reported for fault detection of descriptor systems in the time domain.\textsuperscript{19–22} Among these, the $H_\infty$ filtering method has been widely used and extended to the finite frequency domain for fault descriptor systems. Nevertheless, specific to fault detection for dynamics of HST based on descriptor systems, there exist some difficulties for $H_\infty$ filtering. On the one hand, the disturbance of HST has complicated frequency characteristics. For disturbance coupled with fault, the fault is attenuated simultaneously when the disturbance attenuated so that the fault detection sensitivity is reduced. On the other hand, the essence of existing research on fault detection for nonlinear descriptor systems in frequency domain is still the linearization of nonlinear systems so that the Generalized KYP Lemma (GKYP)\textsuperscript{23} can be used, which inevitably lead to linearization errors. Due to these difficulties, although the descriptor system model has been applied to the fault detection for the structure component of HST, results for dynamics of HST in frequency, especially in the finite frequency domain, are rarely seen.

Based on these reasons, this paper proposed a new fault detection and isolation (FDI) method for the dynamics of HST based on non-linear descriptor systems in finite frequency considering the system uncertainties and the wind gust. The proposed method is to design a nonlinear decoupled filter to achieve the decoupling of nonlinear state variables, partially decoupling disturbances, and faults. Superior to the former studies, the proposed method not only considers the actuator fault, plant fault and sensor fault, simultaneously, but also avoids the linearization of the nonlinear variables that implementing more accurate fault detection results. Moreover, by designing detection subspaces, the filter has a better sensitiveness to the fault compared with the conventional $H_\infty$ one. Specifically, the descriptor system is established by regarding the wind gust and the measurement noise as the variables of the dynamics of the HST firstly. Then, the exogenous disturbance is divided into two parts-the decoupling part and the non-linear part of the system is seen as the augmented decoupling variables, the other part of the disturbance and the system uncertainties are seen as the augmented disturbance. Distribution matrix of each fault is allocated to its corresponding subspace to guarantee the fault signal is not truncated when the augmented disturbance is attenuated. Linear Inequality Matrix (LMI) technology is introduced for calculating the gain matrices of the filter associated with these subspaces. Moreover, by introducing parameters, the design freedom of the filter is increased for dealing with the singular matrix that exists in the descriptor system. The main contributions of this paper are concluded as follows.

1. The descriptor system is introduced for the fault diagnosis of the dynamics of HST innovatively. The wind gust and the system uncertainties are considered simultaneously. The established model is closer to the real train model, and the fault detection for HST based on non-linear descriptor system with uncertainties extends the fault diagnosis theory for descriptor systems in finite frequency domain.
(2) A decoupling filter avoiding the linearization of the non-linear systems is proposed. The decoupled disturbance and the non-linear variables of the dynamics of the HST are decoupled together which makes the filtering of non-linear systems be much easier.

(3) Different kinds of faults are considered simultaneously, and an adaptive threshold is proposed. Not only the concurrent faults are detected rapidly, but also the miss alarm rate and the false alarm rate are decreased effectively comparing with fixed threshold. At the same time, effective fault isolation is implemented by the designed detection subspaces.

The remainder of this paper is organized as follows. In section 2, the measurement noise and the mass of HST are seen as the variables and the uncertainties of the dynamics of HST, respectively. Then an uncertain descriptor system for the dynamics of HST is established. In section 3, a new detection filter is proposed to decouple the non-linear part of the descriptor systems, detection subspaces is presented to guarantee the sensitiveness of the filter to the fault. By this way, filter design is formulated as the LMI feasible problem, then. Further, residual evaluation function and adaptive threshold are introduced to judge if the faults occur. Section 4 gives simulation example considering that the actuator fault, sensor fault and plant fault occur simultaneously, and miss alarm rate and false alarm rate are decreased effectively comparing with fixed threshold. At the same time, effective fault isolation is implemented by the designed detection subspaces.

Uncertain descriptor system of HST

Dynamics of HST

In this section, the dynamics of HST is analyzed. To simplify the force analysis of HST, each car of the trains is seen as a rigid point, and the force between each car caused by the coupler is reduced as a spring model. Basically speaking, the train is affected by basic running resistance and additional resistance during operation. The former includes the rolling mechanical resistance, aerodynamic, and the latter includes bounded disturbance caused by curve resistance, slope resistance, tunnel resistance, etc. The resistance has non-ignorable influence on HST running. Thus, taking them into account, the force diagram of HST is shown in Figure 1.

Where: $s_i(t)$ denotes the relative distance between connected car, the mass, the speed and the effort of $i$ car are represented as $m_i$, $v_i$, and $u_i$. The dynamic of coupler is given as:

$$ f(s_i(t)) = ks_i(t) + d\dot{s}_i(t) \quad (1) $$

Where: $k$ and $d$ denote the spring constants and damping constants, respectively. When $s_i(t)<0$, the coupler is compressed, and braking force is generated. Otherwise, the coupler is stretched, and the traction force is generated.

Due to the Kinetic characteristics of HST, aerodynamic drag is only considered for the first car, as aerodynamic drag mainly acts on the first car.\textsuperscript{24} Moreover, rolling resistance is impacted on each car. Then the aerodynamic drag and rolling resistance are formulated by Davis formula given as follows:

$$ R_r = c_0 + c_v v \quad (2) $$

$$ R_a = c_a v^2 \quad (3) $$

Where: $c_0$, $c_v$, and $c_a$ are Davis formula coefficients, and the rolling resistance and the aerodynamic are represented as $R_r$ and $R_a$, respectively.

Commonly, there are two ways for system modeling of dynamics of HST. On the one hand, the aerodynamic and rolling resistance are seen as the disturbance of the train together with the curve resistance, slope resistance, and the tunnel resistance.\textsuperscript{25} On the other hand, some others saw the aerodynamics and rolling resistance as the state variables of the train,\textsuperscript{24,26,27} which is also adopted in this study.

Define the additional resistance as $\delta_i(t)$ that is affecting on each car, then a multi-particle model of n-car HST is formulated as:

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Force diagram of HST.}
\end{figure}
Where: \( \dot{s}_i = s_i' - s_i'' \), \( \dot{v}_i = v_i' - v_i'' \), \( \ddot{u}_i = u_i' - u_i'' \), \( x_i', v_i' \), and \( u_i'' \) are the equilibrium state of relative position, speed and efforts, \( v_i' \) satisfies that \( v_i'' = v_i' \).

Define \( x(t) = \begin{bmatrix} \dot{s}(t) & \cdots & \dot{s}_{n-1}(t) & \dot{v}_1(t) & \cdots & \dot{v}_n(t) \end{bmatrix}^T, u(t) = \begin{bmatrix} \ddot{u}_1(t) & \cdots & \ddot{u}_n(t) \end{bmatrix}^T \), considering that there exist uncertainties in the dynamics of HST such as the changes of number of passengers and the change of the traveler luggage, based on equation (4), an uncertain state space system is established as:

\[
\begin{align*}
\dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) + E_p h(x(t)) + E_\theta \delta(t) \\
y(t) &= Cx(t) + p(t)
\end{align*}
\]  

(5)

Where: \( E_d \) is the distribution matrix of \( \delta(t) \), \( p(t) \) is the measurement noise and \( C \) is a constant matrix with compatible dimension, the rest of system matrices are given as:

\[ A = \begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, A_{12} = \begin{bmatrix} 1 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & -1 \end{bmatrix}, \]

\[ A_{21} = \begin{bmatrix} -c_{x} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -c_{x} \end{bmatrix}, A_{22} = \begin{bmatrix} \Omega_{i} & \frac{d}{m_{i}} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \Omega_{n-1} & \frac{d}{m_{n-1}} \end{bmatrix}, \]

\[ \Omega_{i} = \begin{cases} -c_{x} - \frac{d}{m_{i}} - 2 \sum_{i=1}^{n} \frac{m_i}{m_j} c_{v_j}, & i = 1, 2, \ldots, n \\ -c_{x} - \frac{d}{m_{i}}, & i = 2, \ldots, n \end{cases} \]

\[ B_{1} = \begin{bmatrix} 0_{n(n-1) \times n} \\ 0_{1 \times (n-1)} \end{bmatrix}, B_{2} = \text{diag} \left( \frac{1}{m_{1}}, \ldots, \frac{1}{m_{n}} \right), \]

\[ E_{p} = \begin{bmatrix} 0_{1 \times (n-1)} & 1 & 0_{1 \times (n-1)} \end{bmatrix}^T, h(x(t)) = \chi_{n}^2(t), \]

\[ \Delta A = M_{1} F_{1}(t) N_{1}, \Delta B = M_{2} F_{2}(t) N_{2}, \]

\( M_{1}, M_{2}, N_{1}, N_{2} \) are constant matrices with compatible dimensions, and \( F_{1}(t) \) is an unknown matrix satisfying \( F_{1}(t)^\top F_{1}(t) \leq I \).

**Remark 1.** Until now, the state space model for dynamics of HST based on multi-particle model is established as equation (5). Comparing with the single-particle model, the multi-particle model (4) based on Newton’s second law is further improved as the forces between cars are considered in equation (4). In this way, a better description for dynamics of HST is obtained.

**Fault model**

In order to consider the faults occurring in different positions of HST comprehensively for the proposed method, three types of common faults, namely, actuator fault, coupler fault, and sensor fault, are considered and modeled in this section.

Actuator fault usually occurs in the input of actuator, and its mathematical model can be expressed as:

\[ f(t) = F_{p} f_{p}(t), F_{p} = B_{p} \]

(6)

Where: \( f_{p}(t) \) denotes the \( p \)th fault of the actuator, and the distribution matrix \( F_{p} \) has the same direction with the \( p \)th column of \( B \), \( 1 \leq p \leq n \).

Coupler fault occurs on the coupler between adjacent cars due to the changes of the spring constant \( k \), and its mathematical model can be expressed as:

\[ f(t) = F_{c} f_{c}(t), f_{c}(t) = x_{c}(t) \]

(7)

Where: \( 1 \leq c \leq n \) denotes the \( c \)th elements of \( x(t) \), as the coupler fault is caused by the weaken of the spring constant \( k \) in \( c \)th car. The distribution matrix \( F_{c} = \Delta k \) denotes the change of spring constant \( k \) in \( c \)th car.

Sensor fault occurs in speed or position sensor and can be modeled as the additional term of the measurement variables in HST. Its mathematical model in measurement equation can be expressed as:

\[ y(t) = Cx(t) + E_{s} \theta(t) \]

(8)

Where: \( E_{s} \) denotes the distribution matrix, \( \theta(t) \) denotes sensor fault.

**Remark 2.** From equations (6) and (7), it is obvious that the actuator fault and the coupler fault are both happened in the dynamic equation of (5). That is, the coupler fault has the same form with the actuator fault and can be seen as the actuator-like fault. In this paper, the actuator fault and the coupler fault are considered together as the actuator-like fault of HST with the mathematical model. Considering that \( 1 \leq p \leq n, 1 \leq c \leq n \), then

\[ F_{f}(t) = \sum_{i=1}^{N} F_{f_{i}}(t), 1 \leq N \leq n \text{ with } p \neq c. \]
Descriptor system modeling

In actual working conditions, wind gust is also an important external force affecting HST. Thus, the disturbance \( \phi(t) \) caused by wind gust for HST system is modeled as:

\[
\begin{align*}
\phi(t) &= Gg(t) \\
g(t) &= L_1w(t) + L_2 \\
\dot{w}(t) &= Ww(t)
\end{align*}
\]

(9)

Where: \( g(t) \) denotes the wind gust and \( G \) denotes the distribution matrix of \( g(t) \). \( L_1, L_2 \), and \( W \) are constant matrices with compatible dimensions.

Remark 3. For equation (9), due to the characteristic of the wind gust, \( g(t) \) is a harmonic signal and can be seen as a matching disturbance. Thus, In this equation, \( L_1, L_2 \) are known as matrices associated with the amplitude of \( g(t) \), and \( W \) is a known matrix related to the frequency of \( g(t) \).

For a convenient discussion, the sensor fault is temporary ignored here in descriptor system modeling. Define \( \tilde{x}(t) = [x^T(t) \ t^T(t) \ p^T(t)]^T \) and considered equations (6) and (7), a descriptor system based on (5) and (9) is established as:

\[
\begin{align*}
\dot{\tilde{x}}(t) &= (\bar{A} + \Delta \bar{A})\tilde{x}(t) + (\bar{B} + \Delta \bar{B})u(t) + \bar{E}_p h(\tilde{x}(t)) + E_1 \\
&\quad + \sum_{i=1}^{N} \bar{F}_i q_i + \bar{E}_d \delta(t) \\
y(t) &= \bar{C}\tilde{x}(t)
\end{align*}
\]

(10)

Where:

\[
E = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A & GL_1 & 0 \\ 0 & W & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix},
\]

\[
\bar{F}_i = \begin{bmatrix} F_i \\ 0 \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} GL_2 \\ 0 \\ 0 \end{bmatrix}, \quad \bar{E}_p = \begin{bmatrix} E_p \\ 0 \\ 0 \end{bmatrix}, \quad \bar{E}_d = \begin{bmatrix} E_d \\ 0 \\ 0 \end{bmatrix},
\]

\[
\bar{C} = [C \ 0 \ I], \quad \Delta \bar{A} = \begin{bmatrix} \Delta A & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta \bar{B} = \begin{bmatrix} \Delta B \\ 0 \end{bmatrix}
\]

So far, a descriptor model is established for dynamics of HST by seeing the noise as an auxiliary variable, and the filter design for fault detection of HST is formulated as the fault detection filter design for the descriptor system (10).

Robust fault detection filter design

The main aims of this paper are to design a fault detection filter for HST based on uncertain descriptor system (10). The proposed filter can not only guarantee the residual which is sensitive to the fault signals, but also is robust to the system disturbance if the following aims are satisfied.

Aim 1. Define \( \delta(t) = [\delta^T(t) \ g^T(t)]^T \), given positive scalars \( \gamma_a, \gamma_b, \gamma \), for all non-zero \( \delta(t) \), the following inequality is satisfied under zero initial condition:

\[
\int_0^\infty r^T(\tau)\nu(\tau)d\tau \leq \gamma^2 \int_0^\infty \delta^T(\tau)\delta(\tau)d\tau + \gamma_a^2 \int_0^\infty \tilde{x}^T(\tau)\tilde{x}(\tau)d\tau + \gamma_b^2 \int_0^\infty u^T(\tau)u(\tau)d\tau
\]

(11)

Aim 2. Residual signals \( r(t) \) are diagonally affected promptly by the fault signals.

Fault detection filter

Lemma 1 is given to attenuate the influence of disturbance \( \delta(t) \).

Lemma 1. The unknown disturbance \( E_d\delta(t) \) in system (10) is partitioned into two parts. One part \( E_{di}\delta_i(t) \) is decoupled from the residual \( r(t) \) completely and another part \( \bar{E}_{di}\delta_2(t) \) is attenuated by finite frequency \( H_\delta \) technology. If there exists a matrix \( T \), such that the following conditions hold:

\[
TE_{di} = 0, \quad T\bar{F}_i \neq 0, \quad i = 1, \ldots, N.
\]

Where: \( \bar{E}_{di} \) contains all the column vectors of \( E_d \) which satisfy the above conditions.

Based on Lemma 1, a fault detection filter is proposed with a decoupled structure as:

\[
\begin{align*}
\dot{z}(t) &= Nz(t) + T\bar{B}u(t) + Ky(t) + TE_1 \\
\dot{x}(t) &= z(t) + H\bar{y}(t) \\
r(t) &= G\bar{y}(t) - \bar{C}\tilde{x}(t)
\end{align*}
\]

(12)

Where: \( N, T, K, H, \) and \( G \) are constant matrices to be determined with compatible dimensions.
Define the state estimation error \( e(t) \) as \( e(t) = \tilde{x}(t) - \bar{x}(t) \), combining (10) and (12), the dynamic error is formulated as:

\[
\dot{e}(t) = [(I - H_3 \bar{C})T_1 \bar{A} - K_t \bar{C}]e(t) + [(I - H_3 \bar{C})T_1 - T]\bar{B}u(t) + (I - H_3 \bar{C})T_1 \bar{E}_h \dot{b}(t) + (K_t - NH)e(t) + (I - H_3 \bar{C})T_1 \bar{E}_d \delta(t) + \dot{\delta}(t) = \left[ \delta^T_1(t) \delta^T_1(t) \right]^T, \quad \dot{h}(t) = \left[ h^T(t) \delta^T_1(t) \right]^T.
\]

Then the gain matrices \( T, H_3, \) and \( K_t \) are obtained as:

\[
T = (I - H_3 \bar{C})T_1, \quad T_0 = 0, \quad A_1 = TA_1 \quad \text{and} \quad K_t = NH.
\]

Substituting the above equations into equation (13), the error system is formulated as:

\[
\left\{ \begin{array}{l}
\dot{e}(t) = (\tilde{A}_1 - K_t \bar{C})e(t) + T(\tilde{E}_h \delta(t) + T \Delta \tilde{x}(t)) + T \Delta \bar{B}u(t) \\
r(t) = G \tilde{x}(t)
\end{array} \right.
\]

Moreover, Lemma 2 is given to calculate matrices \( T_1 \) and \( H_1 \).

**Lemma 2.** There exists a non-singular matrix \( T_1 \) and matrix \( H_1 \) which satisfy:

\[
T_1 E + H_1 \bar{C} = I
\]

and the matrix \( T_1 \) and matrix \( H_1 \) are obtained by the following equation:

\[
[T_1 \ H_1] = \begin{bmatrix} E & 0 \end{bmatrix}^T + S \left( I - \begin{bmatrix} E & 0 \end{bmatrix}^T \begin{bmatrix} E & 0 \end{bmatrix} \right)
\]

Where: \( S \) is an arbitrary matrix representing the design freedom.

Assumption 1 is proposed for calculating \( H_3 \) related to the gain matrix \( T \).

**Assumption 1.** For a known matrix \( \bar{C} \), if the following conditions is satisfied:

\[
\text{rank}(\tilde{C}T_1 \bar{E}_h) = \text{rank}(T_1 \bar{E}_h)
\]

Where: \( T_1 \) is a non-singular matrix, then \( H_3 \) is chosen as:

\[
H_3 = T_1 \bar{E}_h [(\tilde{C}T_1 \bar{E}_h)^T (\tilde{C}T_1 \bar{E}_h)]^{-1} (\tilde{C}T_1 \bar{E}_h)^T
\]

such that \( T \tilde{E}_h = 0 \) holds.

**Remark 4.** It is shown that in Equation (14), matrix \( T \) is obtained based on equations (16) and (18). Thus, \( K_t \) and \( G \) are gain matrices to be determined for the filter (12). That is, the filter design is formulated as finding gain matrices \( K_t \) and \( G \), which makes \( r(t) \) in equation (14) be sensitive to \( f_i(t) \) and be robust to the non-decoupling disturbance and the uncertainties.

**Composite fault isolation**

In this section, detection subspaces related to the distribution matrices of the faults are given, and each of the fault is allocated to its corresponding subspace to implement the fault isolation.

**Assumption 2.** The detection subspaces \( \eta_i \) associated with \( A_1 \) satisfies:

\[
(\tilde{A}_1 - K_t \bar{C}) \eta_i \subseteq \eta_i, \quad \lambda_i \subseteq \eta_i
\]

Define \( \bar{C} \eta_i \cap \sum_{j \neq i} \bar{C} \eta_j = \emptyset \)

Define \( Y_i \) which is spanned by a set of \( \eta_i \) as follows:

\[
Y_i = \begin{bmatrix} Y_{i1} & \cdots & Y_{i1} T \bar{F}_i & \cdots & \tilde{A}_1 T \bar{F}_i \end{bmatrix}
\]

\[
(\tilde{A}_1 + T \bar{F}_i \bar{K})Y_{ik} = z_{ik} Y_{ik}
\]

\[
\bar{C} Y_{ik} = 0
\]

Where: \( k_i \) is the smallest integer such that \( \bar{C} \tilde{A}_1^k T \bar{F}_i \neq 0 \) and \( z_{ik}, Y_{ik} \) are the invariant zero and associated zero direction. However, because of the special structure of \( \bar{C} \) and \( \tilde{A}_1 \) in this paper, none of \((\bar{C}, \tilde{A}_1, T \bar{F}_i)\) has invariant zeros. As a result, (19) is reduced as follows:

\[
Y_i = \begin{bmatrix} T \bar{F}_i & \tilde{A}_1 T \bar{F}_i & \cdots & \tilde{A}_1^k T \bar{F}_i \end{bmatrix}
\]

from (20) and assumption 2, it is obvious that:

\[
(\tilde{A}_1 - K_t \bar{C}) \tilde{A}_1^k T \bar{F}_i = Y_i [a_0 \ a_1 \ \cdots \ a_k]^T = Y_i g_i
\]

Define:

\[
\Pi = [ \bar{C} \tilde{A}_1^k T \bar{F}_1 \ \bar{C} \tilde{A}_1^k T \bar{F}_2 \ \cdots \ \bar{C} \tilde{A}_1^k T \bar{F}_N ],
\]

\[
\bar{Y}_i = [ T \bar{F}_1 \ \tilde{A}_1 T \bar{F}_i \ \cdots \ \tilde{A}_1^k T \bar{F}_i ],
\]

\[
\Pi C = \Pi \bar{C}, \quad \Pi^\dagger = (\Pi^T \Pi)^{-1} \Pi^T
\]

then the complementary subspace \( \bar{Y} \) satisfies that:

\[
\bar{Y}_1 \oplus \bar{Y}_2 \oplus \cdots \oplus \bar{Y}_N \oplus \bar{Y} = \ker(\Pi C)
\]
From (22), it is easy to verify that $Y_1 \oplus Y_2 \oplus \cdots \oplus Y_N \oplus \hat{Y} = R^r$, in particular, when $Y_1, Y_2, \ldots, Y_N$ have already spanned the whole space, $\hat{Y}$ does not exist.

Given matrices $V$ and $U$ satisfying $UV = I$:

$$V = [Y_1 \quad Y_2 \quad \cdots \quad Y_N \quad \hat{Y}],$$
$$U = [U_1^T \quad U_2^T \quad \cdots \quad U_N^T \quad \hat{U}^T]^T \quad (23)$$

and $(\bar{A}_i - K_i \bar{C})$ is multiplied by matrix $V$ and $U$, respectively. Then the following equation is obtained as:

$$U(\bar{A}_i - K_i \bar{C})V = \begin{bmatrix} \phi_1 & 0 & \cdots & 0 & U_1(\bar{A}_1 - K_1 \bar{C})\hat{Y} \\ 0 & \phi_2 & \cdots & 0 & U_2(\bar{A}_1 - K_1 \bar{C})\hat{Y} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \phi_N & U_N(\bar{A}_1 - K_1 \bar{C})\hat{Y} \\ 0 & 0 & \cdots & 0 & \bar{U}(\bar{A}_1 - K_1 \bar{C})\hat{Y} \end{bmatrix} \quad (24)$$

Where:

$$\phi_i = U_i(\bar{A}_1 - K_1 \bar{C})Y_i = \begin{bmatrix} 0 & 0 & \cdots & 0 & \alpha_0 \\ 1 & 0 & \cdots & 0 & \alpha_1 \\ 0 & 1 & \cdots & 0 & \alpha_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & \alpha_k \end{bmatrix} \quad (25)$$

The eigenvalues of $(\bar{A}_1 - K_1 \bar{C})$ are equal to the sum of eigenvalues of $\phi_i$ and $\bar{U}(\bar{A}_1 - K_1 \bar{C})\hat{Y}$, and the characteristic equation of $\phi_i$ is formulated as follows:

$$|\beta I - \phi_i| = \beta^{k+1} - \alpha_k \beta^k - \cdots - \alpha_1 \beta - \alpha_0 \quad (26)$$

Define $\Lambda_i$ associated with $\lambda_i$ which consists of eigenvalues of $\phi_i$ and dim $\Lambda_i = k_i$, substituting $\Lambda_i$ and $\phi_i$ into equation (26), the following equation is obtained as:

$$\begin{bmatrix} \beta^{k_1}_{i_1} & \beta^{k_1-1}_{i_1} & \cdots & \beta_{i_1} & 1 \\ \beta^{k_2}_{i_2} & \beta^{k_2-1}_{i_2} & \cdots & \beta_{i_2} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \beta^{k_{i_k}}_{i_k} & \beta^{k_{i_k}-1}_{i_k} & \cdots & \beta_{i_k} & 1 \end{bmatrix} \begin{bmatrix} \alpha_{k_1} \\ \alpha_{k_1-1} \\ \vdots \\ \alpha_1 \\ \alpha_0 \end{bmatrix} = \begin{bmatrix} \beta^{k+1}_{i_1} \\ \beta^{k+1}_{i_2} \\ \vdots \\ \beta^{k+1}_{i_k} \end{bmatrix} \quad (27)$$

from (27), the vector $\xi_i$ can be solved, and the unknown parameter vector $\xi_i$ are only related to the assigned elements of $\Lambda_i$ which is associated with $T_j(t)$, and the following equation holds:

$$\bar{A}_1 - K_1 \bar{C} = \begin{bmatrix} \bar{A}_1^{k_1} & \bar{A}_1^{k_1-1} & \cdots & \bar{A}_1^{1} \\ \bar{A}_1^{k_2} & \bar{A}_1^{k_2-1} & \cdots & \bar{A}_1^{1} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{A}_1^{k_{i_k}} & \bar{A}_1^{k_{i_k}-1} & \cdots & \bar{A}_1^{1} \end{bmatrix} \quad (28)$$

Then, the gain matrix $K_1$ is calculated as:

$$K_1 = \Theta \Pi^T + X_1(I - \Pi \Pi^T) \quad (29)$$

### Table 1. Values $\Psi$ for different frequency domain.

| Parameter | Low frequency | Middle frequency | High frequency |
|-----------|---------------|-----------------|---------------|
| $\omega$  | $|\omega| \leq \omega_0$ | $\omega_1 \leq \omega \leq \omega_2$ | $|\omega| \geq \omega_b$ |
| $\Psi$    | $[0, -10]$    | $[0, -1/2]$     | $[0, 10]$     |

Where: $\Theta = [x_1^{k_1}, x_2^{k_2}, \ldots, x_N^{k_N}] - [Y_1 \quad Y_2 \quad \cdots \quad Y_N] \cdot \text{diag}(\xi_1, \xi_2, \ldots, \xi_N)$, $X_1$ is an arbitrary matrix with compatible dimensions which denote the design freedom. Matrix $G$ in (12) is given as:

$$G = \Xi \Pi^T + X_2(I - \Pi \Pi^T) \quad (30)$$

Where: $\Xi$ is a constant diagonal matrix and $X_2$ is an arbitrary matrix with compatible dimensions.

### Finite frequency $H_{\infty}$ performance

To effectively describe the signal performance in different frequencies, the frequency domain is partitioned into the low, middle, and high frequency domain, respectively. Totally speaking, the finite frequency domain can be expressed as $\Theta_d = \{ \omega \in R : \sigma(\omega - \omega_0)(\omega - \omega_2) \leq 0 \}$, $\sigma \in \{1, -1\}$, $\omega$ represents the frequency of the signals, $\omega_1$ and $\omega_2$ are the given constants.

When $\sigma = 1$, $-\omega_1 = \omega_2 = \omega_1$, $\Theta_d$ indicates that the signal occurs at low frequency, then $\omega$ satisfies that $|\omega| \leq \omega_0$. When $\sigma = 1$, $\omega_1 < \omega_2$, $\Theta_d$ indicates that the signal occurs at middle frequency, then $\omega$ satisfies that $\omega_1 \leq \omega \leq \omega_2$. When $\sigma = -1$, $-\omega_1 = \omega_2 = \omega_1$, $\Theta_d$ indicates that the signal occurs at high frequency, then $\omega$ satisfies that $|\omega| \geq \omega_b$.

In this paper, FDI is studied in longitude dynamics of HST. As the human body is more sensitive to the horizontal vibrations of 1–2 Hz, and this frequency range belongs to the high frequency obviously. Thus the exogenous disturbance is assumed to belong to high frequency, which makes the study more practical.

The main purpose of this section is to find symmetric matrix $P_d$ and matrix $Q_d > 0$ to guarantee the following inequality:

$$\int_0^\infty \Sigma(t)(\Phi \otimes P_d + \Psi \otimes Q_d)\Sigma(t)dt \geq 0 \quad (31)$$

hold for asymptotically stable $e(t)$ belonging to $\Theta_d$ in (14). Additionally, in equation (31), $\Sigma(t) = [e(t) \quad e(t)]$, $\Phi = \text{diag}(1, 1)$, $\Psi$ is defined in Table 1. Moreover, as mentioned above, state uncertainties are considered in this paper. To analyze the performance of uncertainties in finite frequency domain, lemmas 3 and 4 are given.
Lemma 3. Let $x_1, x_2$, and $x_3(t)$ be real matrices with compatible dimensions which satisfy:

$$x_1^T(t)x_2(t) \leq I$$

then, there exist a scalar $\varepsilon > 0$ to make the following inequality hold:

$$x_1 x_1(t) + (x_1 x_2(t)) \leq \varepsilon x_1 x_1^T + \varepsilon^2 x_2 x_2$$

Lemma 4 (Projection lemma) The following conditions are equivalent:

(a) $x^T Q x < 0, \forall \Gamma \in \Re^{m \times n}, \Gamma + H e (\Gamma) T < 0$.
(b) $\exists \Gamma \in R^{m \times n}, Q + H e (\Gamma T) < 0$.

where $N_T$ is the null space of $\Gamma$.

To analyze the $H^\infty$ performance for error system (14), let $f(t) = 0$, then (14) becomes:

$$
\begin{aligned}
\dot{\tilde{e}}(t) &= (\tilde{A} \Gamma - \Gamma C) e(t) + \Delta e(t) \\
\Gamma &= G \tilde{C}(t)
\end{aligned}
$$

Where: $\tilde{E}_v = [T_{\tilde{E}_d2} T_{\Delta A} T_{\Delta B}], \tilde{\theta}(t) = [\delta_1^+(t) \times \Gamma (t) u(t)]^T$.

Theorem 1 is proposed to guarantee that the performance (31) is satisfied for an asymptotic stable signal $e(t)$ in (14).

Theorem 1. For given scalars $\gamma > 0, \gamma_a > 0, \gamma_b > 0$, and $\omega_0 > 0$, if there exist symmetric matrices $P_d > 0, Q_d > 0, P_s > 0$, and matrices $X_1, X_2$ such that:

$$He(\hat{x} - \hat{x}_1) < 0$$

$$
\psi = \begin{bmatrix}
\psi_{1,1} & \psi_{1,2} & \psi_{1,3} & \psi_{1,4} \\
\ast & \psi_{2,2} & 0 & 0 \\
\ast & \ast & \psi_{3,3} & 0 \\
\ast & \ast & \ast & \psi_{4,4}
\end{bmatrix} < 0
$$

Where:

$$
\psi_{1,1} = \begin{bmatrix}
-2P_d & P_d & P_d \hat{A} \\
* & He(P_d \hat{A}) & Q_d
\end{bmatrix}
$$

$$
\psi_{1,2} = \begin{bmatrix}
P_sT_{\tilde{E}_d2} & 0 \\
P_sT_{\tilde{E}_d2} & 0
\end{bmatrix},
$$

$$
\psi_{1,3} = \begin{bmatrix}
0 & 0 \\
0 & \Xi I^T \tilde{C} + X_2 O
\end{bmatrix},
$$

$$
\psi_{1,4} = \begin{bmatrix}
P_sT_{\tilde{M_1}} & P_sT_{\tilde{M_2}} \\
P_sT_{\tilde{M_1}} & P_sT_{\tilde{M_2}}
\end{bmatrix},
$$

$$
\psi_{2,2} = \begin{bmatrix}
-\gamma^2 I & 0 \\
* & -\gamma^2 I + e_1 I X_1^T X_1
\end{bmatrix},
$$

$$
\psi_{3,3} = \begin{bmatrix}
-\gamma^2 I + e_1 I X_1^T X_1 & 0 \\
0 & -I
\end{bmatrix},
$$

$$
\psi_{4,4} = \begin{bmatrix}
-e_1 I & 0 \\
0 & -e_2 I
\end{bmatrix}
$$

Therefore, the Following conditions are satisfied:

$$
\hat{N} = P_s(\hat{A} - \Theta \Omega^T \hat{C}), O = (I - \Omega \Omega^T) \hat{C}, X_1 = P_s^{-1} \tilde{X}_1, \\
\hat{A} = A_1 - K_1 \hat{C}, \hat{C} = G_0 \hat{C},
$$

where $K_1$ and $G_0$ are the initial values of $K_1$ and $G$ obtained by (29) and (30), respectively. Then inequality (31) is satisfied for an asymptotic stable signal $e(t)$.

The proof theorem 1 is given in Appendix.

Remark 5. For sensor fault given in equation (8), it can also be detected by the method proposed in theorem 1 after an equivalent transformation. According to Yan and Edwards sensor fault can be transformed into a pseudo-actuator fault. In this paper, using Yan’s method, dynamics of HST with actuator fault, coupler fault and sensor fault which is represented as:

$$
\begin{aligned}
\dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) + E_p(h(x(t))) \\
&+ \phi(t) + \sum_{i=1}^{N} \tilde{F}_i f_i(t) + E_d \delta(t) \\
y(t) &= Cx(t) + E_d \theta(t) + p(t)
\end{aligned}
$$

Where: $E_d \theta(t)$ is transformed into the one with pseudo-actuator fault $w_c(t)$ as

$$
\begin{aligned}
\dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) + E_p(h(x(t))) \\
&+ \phi(t) + E_d \delta(t) + \sum_{i=1}^{N} \tilde{F}_i f_i(t) + F_s w_s(t) \\
y(t) &= Cx(t) + p(t)
\end{aligned}
$$

Where: $E_s = C F_s^1, F_s^2 = A F_s^1, F_s = [F_s^1 F_s^2], w_s(t) = [\theta^T(t) \theta^T(t)]^T$, and can be further expressed as the descriptor system in the form of equation (10).

Based on the aforementioned process, the designing steps of the fault detection filter are summarized in algorithm 1.

Algorithm 1. The design method of gain matrix $N$, $T$, $K$, $H_c$ and $G$ in (12) are given in the following algorithm.

Step 1. Transform sensor fault to an equivalent actuator-like fault based on (37).

Step 2. Partition the disturbances into two parts $\tilde{E}_{d1}$ and $\tilde{E}_{d2}$.

Step 3. Calculate gain matrices $T_1$ and $H_1$ which are obtained by lemma 2, and calculate $H_2$ by equation (18).

Step 4. Define a series of detection subspaces $\eta_i, \{i \in [1, N]\}$ for faults.

Step 5. Define $\Lambda_i, \{i \in [1, N], \dim \Lambda_i = k_i \}$ as the set of distinct self-conjugate complex numbers, assign the elements of $\Lambda_i$, choose the initial value of $X_1$ and $X_2$ as $X_1^0$ and $X_2^0$, then according to (29)–(30),
Figure 2 is given to show the design flow of the proposed method. To detection the fault promptly and more accurately, an adaptive threshold is designed for comparing with the fixed threshold to show the effectiveness the proposed method.

Firstly, the $j$th residual evaluation function is given as:

$$J_{rj}(t) = \frac{1}{T} \int_0^T \|r_j(t)\|^2 \, dt$$

(41)

Where: $r_j(t), j = 1, \cdots, N$ denotes the $j$th residual.

As there exists noise in disturbance, residual is a non-stationary stochastic process, and is approximated to a Gaussian distribution. Thus, adaptive threshold is relative to the mean value $\mu$ and variance $\sigma$ of $J_{rj}(t)$. In this paper, $\mu$ and $\sigma$ are given as:

$$\mu(r_j, t_k) = \frac{1}{T} \sum_{k=1}^{T} r_j(t_k)$$

$$\sigma^2(r_j, t_k) = \frac{1}{T-1} \sum_{k=1}^{T-1} [r_j(t_k) - \mu(r_j, t_k)]^2$$

(42)

The confidence interval for the mean with a confidence level of $(1 - \alpha)$ is expressed as:

$$P(\hat{\mu} - z\alpha < \mu < \hat{\mu} + z\alpha) = 1 - \alpha$$

(43)

Where: $\alpha$ denotes the confidence level; $z$ denotes the coefficient associated with the confidence level. In practice, confidence $(1 - \alpha)$ is usually chosen as $(95\% - 99\%)$. When the confidence level is $95\%$, then $z$ is obtained as 1.96. Based on (41)-(43), the adaptive threshold is designed as:

$$J_{th,j} = \mu(r_j, t_k) \pm 1.96\sigma^2(r_j, t_k)$$

(44)

Moreover, the detection logical is given as:

$$J_{rj}(t) > J_{th,j} \Rightarrow \text{alarm, } J_{rj}(t) \leq J_{th,j} \Rightarrow \text{no faults}$$

(45)

Remark 6. In the former study, under the residual evaluation function (41), the fixed threshold $31,32$ is often given as:

$$J_{th} = \sup_{f(t) = 0, h(t) = 0} J_{rj}(t) + J_d$$

(46)

where $J_d$ a small prescribed.

From equation (46) it is seen that, although $J_d$ in the fixed threshold can improve the detection method to some extent, the improvement of the fixed threshold by $J_d$ is limited. To the opposite, the proposed adaptive threshold can track the residual evaluation function during the detection, which provides a better fault detection, especially in handling miss alarm and false alarm for dynamics of HST.
Table 2. Parameters of HST.

| Parameters | Value     | Unit       | Parameters | Value     | Unit       |
|------------|-----------|------------|------------|-----------|------------|
| $c_0$      | $1.12 \times 10^{-6}$ | Ns/mkg     | $c_1$      | $7.295 \times 10^{-5}$ | Ns^2/m^2kg |
| $c_0$      | 0.00863   | 0.00863    | $k$        | $80 \times 10^{3}$    | N/m        |

Simulation

Simulation parameters

In this section, a six-car HST model\textsuperscript{24} is given to illustrate the effectiveness of the proposed method. In this paper, parameters of the model are given in Table 2.

Moreover, the damping constant $d$ is given as 600 Ns/m, and the mass of the train is assumed to be the time-varying parameter, which satisfies that $m(t) = m + \Delta m$, denotes the body mass of the cars, and satisfies that $m_1 = \cdots = m_6 = 50 \times 10^3$ kg. $\Delta m$ denotes the uncertainties of the train caused by the change of the traveler luggage, and satisfies that $\Delta m = 10^3 \sin(t)$.

By calculation, the parameters which are related to uncertainties $A$ and $B$ are obtained as:

$$M_1 = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, N_1 = I_{11} F_1(t) = I_{11} \sin(t)$$

Where:

$$M_{11} = 0_{5 \times 5}, M_{12} = 0_{5 \times 6}, M_{21} = \begin{bmatrix} 0.032 & 0 & 0 & 0 & 0 \\ -0.032 & 0.032 & 0 & 0 & 0 \\ 0 & -0.032 & 0.032 & 0 & 0 \\ 0 & 0 & -0.032 & 0.032 & 0 \\ 0 & 0 & 0 & -0.032 & 0.032 \end{bmatrix},$$

$$M_{22} = \begin{bmatrix} 2.4 \times 10^{-4} & -2.4 \times 10^{-4} & 0 & 0 & 0 \\ -2.4 \times 10^{-4} & 2.4 \times 10^{-4} & -2.4 \times 10^{-4} & 0 & 0 \\ 0 & -2.4 \times 10^{-4} & 2.4 \times 10^{-4} & -2.4 \times 10^{-4} & 0 \\ 0 & 0 & -2.4 \times 10^{-4} & 2.4 \times 10^{-4} & -2.4 \times 10^{-4} \\ 0 & 0 & 0 & -2.4 \times 10^{-4} & 2.4 \times 10^{-4} \end{bmatrix},$$

$$M_2 = -4 \times 10^{-7} \begin{bmatrix} 0_{5 \times 5} \\ I_6 \end{bmatrix}, F_2(t) = I_6 \sin(t), N_2 = I_6$$

Assuming that the disturbance $\delta(t)$ belongs to the high frequency domain, its distribution matrices $E_d$ are given as

$$\begin{align*}
\delta_1(t) &= \cos(20\pi t + x_1(t)) + 10^{-3} \cos(400\pi t + x_1(t)) + w_{gn}(t) \\
\delta_2(t) &= \sin(25\pi t + x_2(t)) + 10^{-3} \sin(400\pi t + x_2(t)) + w_{gn}(t) \\
\delta_3(t) &= 0.5 \cos(20\pi t + x_3(t)) + 0.5 \sin 30\pi t + x_3(t) + 10^{-3} \sin(400\pi t + x_2(t)x_3(t)) + w_{gn}(t)
\end{align*}$$

$$E_d = \begin{bmatrix} E_{d1} & E_{d2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.4 & 1 & 0 & 0 & 0 & 0 \\ 2.46 & 0.8 & 0 & 0 & -1.24 & 0 & 0 & 1 \end{bmatrix}^T$$

In this paper, three types of fault are considered. The actuator fault is assumed to be the motor power loss of the third car, the coupler fault is assumed to be the coupler failure between the fifth and sixth car, and the sensor fault is assumed to be the speed sensor drift of the first car. According to their actual working conditions, the fault signals are given as:

and their distribution matrices are given as:

\[
\begin{align*}
F_p &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.85 & 0 & 0 & 0 \end{bmatrix}^T, \\
F_c &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.2 \end{bmatrix}^T, \\
E_s &= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T
\end{align*}
\]

By equation (38), the sensor fault is formulated as the pseudo-actuator fault with the distribution matrices:

\[
F_s = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T, \\
1 & 0 & 0 & 0 & -0.0009 & 0 & 0 & 0 & 0
\]

**Remark** 7. As the highly development of the measurement techniques in HST, speed and position information of each car has been real-time measured nowadays, so in the paper, the measurement coefficient matrix \( C \) is chosen as \( C = I_{11} \).

Then, matrices \( T \) and \( H \) are designed as:

\[
T = \begin{bmatrix}
I_{4 \times 4} & 0_{4 \times 2} & 0_{4 \times 18} \\
0_{2 \times 4} & 0_{2 \times 2} & 0_{2 \times 18} \\
0_{18 \times 4} & 0_{18 \times 2} & I_{18 \times 18}
\end{bmatrix},
\]

\[
H = \begin{bmatrix}
0_{4 \times 4} & 0_{4 \times 2} & 0_{4 \times 5} \\
0_{2 \times 4} & I_{2 \times 2} & 0_{2 \times 5} \\
0_{18 \times 4} & 0_{18 \times 2} & 0_{18 \times 5}
\end{bmatrix}
\]

By solving LMI (35) and LMI (36), \( \gamma \), \( \gamma_a \), and \( \gamma_b \) are obtained as \( \gamma = 0.16 \), \( \gamma_a = 0.08 \), \( \gamma_b = 0.2 \), and the gain matrices \( G \) is obtained as \( G = [G_1 \ G_2] \). \( G_1 \) and \( G_2 \) satisfies that:

\[
G_1 = \begin{bmatrix}
0.0010 & 0.5868 & 0.5897 & 0.0142 & 0.0063 & -0.0001 \\
-5.8404 & -4.3569 & -4.3384 & -755.1869 & -517.6000 & -9.1718 \\
0.0029 & -0.0016 & -0.0016 & -0.0075 & -0.0026 & -0.0001 \\
-0.0050 & -0.0098 & 0.0074 & -0.0031 & 0
\end{bmatrix}, \\
G_2 = \begin{bmatrix}
126.3218 & 252.6436 & -88.7851 & 57.8686 & -11423.3493 \\
-0.2484 & 0.1282 & -0.0023 & -0.0001 & 0
\end{bmatrix}
\]

**Table 3.** Alarm delay times of the fault detection in case 1.

| Fault type       | Derive fault | Coupler fault | Sensor fault |
|------------------|--------------|---------------|--------------|
| Alarm delay time | 0.294 s      | 0.213 s       | 0.0052 s     |

Case 1. To show the effectiveness of the proposed adaptive threshold, the actuator fault, coupler fault and sensor fault are assumed to occur simultaneously, that is, the concurrent faults detection is considered in this section. Using the adaptive threshold, the fault detection results are seen in Figure 3. The alarm delay times of the fault detection are given in Table 3.

From Figure 3 it is seen that, when the actuator fault, coupler fault and sensor fault occur at the same time \( t = 1.5s \), the adaptive thresholds of faults with their residual evaluation functions, respectively. From Table 3 it is seen that, the alarm starts 0.294 s after actuator fault occurs, and starts 0.213 s, 0.0052 s after coupler fault and sensor fault occurs respectively. The results of the figure and table show that the actuator fault, coupler fault and sensor fault are detected relatively quickly, especially for the sensor fault, the detected time is only about 0.005 s.

**Simulation results and analysis of fault detection**

To illustrate the effectiveness of proposed filter and the adaptive threshold for fault detection, two fault detection cases are considered. Moreover, for a convenient discussion, the residual evaluation function of the actuator fault, coupler fault and sensor fault occurrence dimensions are defined as \( Jr_1, Jr_2, \) and \( Jr_3 \).

From the above analysis, it is known that the designed filter and the proposed adaptive threshold implements prompt fault estimation even the faults occur simultaneously.
Case 2. To illustrate the advantages of the proposed adaptive threshold in handling the miss alarm and false alarm for fault compared with the fixed threshold, the coupler fault is given for an example, and simulation again.

Firstly, the disturbance in equation (47) is assumed to increase to 1.5 times, and the amplitude of the coupler fault is assumed to be decreased by 30% considering that the fixed threshold may lead to miss alarm for the fault with a smaller amplitude. By equation (47), the fixed threshold is calculated as $J_{th,i} = 0.3$, and the simulation result for miss alarm is seen in Figure 4.

Considering the actual working conditions of the train, the coupler connects the two adjacent cars, even short-time fault may cause trains to decouple. Thus, the acceptable detected delay is limited to 0.5 s in this paper. From Figure 4 it is seen that, for coupler fault, using the fixed threshold, the fault is detected as about $t = 2.3s$, that is, the fault alarm is about 0.8 s after it occurs, which means that the fault alarm is missed in limited time. To the opposite, by the proposed adaptive threshold, the coupler fault is detected at $t = 1.658s$, that is, the detected delay is 0.158 s, which means that for the coupler fault with a smaller amplitude and even under a higher level of disturbance, the adaptive threshold is more effective in handling the miss alarm.

Figure 3. Fault detection results for concurrent faults: (a) fault detection result for actuator fault (b) fault detection result for coupler fault and (c) fault detection result for sensor fault.

Figure 4. Miss alarm of the adaptive threshold and the fixed threshold.
threshold decreases the miss alarm rate greatly comparing with the fixed threshold.

In addition, it is seen that for the single coupler fault, even under a higher level of disturbance, the fault detection is quicker comparing with the simultaneous ones. Comparing with the detection result in case 1, the detected time for coupler fault is decreased by 44.72%. The simulation result further verifies the effectiveness of the proposed adaptive threshold.

Then the false alarm results are given to show the effectiveness of the proposed adaptive threshold. As shown in remark 7, the fixed threshold is mainly relative to the disturbance level. Thus, in this section, the disturbance is assumed to abruptly increase by 1.5 times at \( t = 1.5 \) s. By equation (49), the fixed thresholds for actuator fault, coupler fault and sensor fault are obtained as 0.036, 0.013, and 0.16, respectively. The simulation results for false alarm are seen in Figure 5. False alarm times for single actuator fault, single coupler fault and single sensor fault are given in Table 4.

From Figure 4 and Table 5 it is seen that, although the faults don’t occur, as the disturbance abruptly increases, the residual evaluation functions for three types of fault increase correspondingly, then the false alarms happen for each single fault. For single actuator fault, the false alarm happens at about \( t = 2.3 \) s, and for

| Fault type False alarm time | Derive fault | Coupler fault | Sensor fault |
|----------------------------|-------------|---------------|--------------|
| Fixed threshold            | 2.3         | 1.9           | 2.2          |
| Adaptive threshold         | Not occur   | Not occur     | Not occur    |

Table 5. Alarm delay times of the single fault detection.

| Fault type    | Derive fault | Coupler fault | Sensor fault |
|---------------|--------------|---------------|--------------|
| Alarm delay time (second) | 0.224        | 0.143         | 0.004        |

Figure 5. False alarm when disturbance abruptly changes: (a) false alarm for actuator fault (b) false alarm for coupler fault, and (c) false alarm for sensor fault.
single coupler and single sensor fault, the false alarms happen about $t = 1.9$ s and $t = 2.2$ s respectively. To the opposite, although the residual evaluation functions for three types of faults also increase, as the adaptive thresholds for three types of faults change correspondingly, all the residual evaluation functions do not exceed their corresponding thresholds, which means that the false alarm is avoided during the simulation.

By the above analysis, as the adaptive threshold tracks the residual evaluation function synchronously, the threshold of each single fault adjusts adaptively, the miss alarm rate and false alarm rate are both decreased effectively comparing with the fixed threshold, which means that the designed filter and the proposed adaptive threshold implements more accurate fault detection.

**Simulation results and analysis of fault isolation**

To show the effectiveness of the proposed filter for fault isolation, simulation results for residual evaluation functions $J_{r1}$, $J_{r2}$, and $J_{r3}$, and their adaptive thresholds when single fault occurs are given in this section.

Firstly, assume that only actuator fault occurs, and the simulation results are given in Figure 6.

From Figure 6 it is seen that, the residual evaluation functions $J_{r1}$ and $J_{r3}$ don’t exceed their adaptive thresholds during the whole simulation time. To the opposite, the residual evaluation function $J_{r1}$ doesn’t exceed its adaptive threshold during 0–1.5 s of the simulation, but rapidly exceeds the adaptive threshold at $t = 1.7224$ s. The results show that, as the actuator fault occurs at 1.5 s, the residual evaluation function $J_{r1}$ not only reflects the actuator fault information rapidly, but also has no influence on the other residual evaluation functions, which means that the proposed filter implements effective actuator fault isolation.

Then, the coupler fault isolation and sensor fault isolation results are given in Figures 7 and 8, respectively. Similar fault isolation results are obtained by analyzing Figures 7 and 8. From Figures 7 and 8 it is known that, single coupler fault and single sensor fault are effectively isolated respectively. Figure 7 shows that, for single coupler fault, residual evaluation function $J_{r2}$ rapidly exceeds its adaptive threshold at 1.643 s, while
Jr₁ and Jr₃ don’t exceed their adaptive threshold during the whole simulation time, which means that the coupler fault information only affects its own residual evaluation function and has no influence of the other ones. Figure 8 shows that, the residual evaluation function Jr₃ is very sensitive to the single sensor fault, and at t = 1.554s, Jr₃ exceeds its own adaptive threshold, while Jr₁ and Jr₂ are not affected by sensor fault information. The results in Figures 7 and 8 fully illustrate that the designed filter implements effectively coupler fault isolation and sensor fault isolation.

Moreover, alarm delay times of the single fault detection is given in Table 5.

Comparing with the alarm delay times of the concurrent faults which are shown in Table 3, the alarm delay time of the single actuator fault is decreased by 23.81%, and that of the coupler fault and sensor fault are decreased by 32.86% and 25.93%, respectively, which is consistent with the simulation results for single coupler fault detection of miss alarm simulation scenario. The simulation results in this section further verify that the proposed filter and the adaptive threshold not only implement rapid fault detection, but also implement effective fault isolation at the same time.

**Simulation summary**

Based on the results and analysis in FDI, for three types of fault, it is easy to know that the proposed decoupled filter not only implements single fault detection rapidly, but also implements concurrent faults detection promptly. At the same time, the proposed adaptive threshold decreases the miss alarm rate for faults with smaller amplitude even in a higher level of disturbance, and greatly decreases false alarm rate when disturbance increases abruptly. Comparing with the fixed threshold, the proposed adaptive threshold implements fault detection more accurately. Moreover, the residual evaluation function of the fault occurrence dimension of the proposed filter is sensitive to its own adaptive threshold, but has no influence on the other residual evaluation function, which means that the designed subspaces of the filter implement fault isolation for faults effectively. Simulation results and
analysis show that, the proposed FDI method is effective for dynamics of HST with uncertainties, wind gust, exogenous disturbance and measurement noise by a descriptor system approach.

Moreover, it should be noticed that this descriptor approach is not only suitable for dynamics of HST, but also for other vehicles with concurrent faults and measurement disturbance/noise. In spite of this, there is still room for improvement in this method. For example, in actual working conditions, there may exist time-varying delays for state variables of the train due to severe working conditions. This delay may have an influence on simulation results. This provides some ideas for the study in the future.

**Conclusion**

In this paper, FDI problem in finite frequency for dynamics of HST is studied. The mathematical model for the dynamics of HST is established based on nonlinear descriptor systems considering the system uncertainties, wind gust, exogenous disturbance as well as measurement noise. FDI filter is proposed which is sensitive to the concurrent faults and has a robustness against the exogenous disturbance and uncertainties. To decouple the nonlinear variable and attenuate the disturbance from residual signals, the nonlinear variable and part of disturbance are seen as the decoupled variable, and the rest of disturbance is attenuated by $H_\infty$ performance in finite frequency domain. The eigenvector assignment technique is exploited to confine the composite faults into a series of detection subspaces. Such that the composite faults can be identified by observing the directional residuals. An adaptive threshold is designed to decrease the miss alarm rate and false alarm rate. Simulation results illustrate the effectiveness of the proposed method.

The proposed method not only has broad application prospects in fast FDI of high-speed trains, but also improves the fault diagnosis theory for descriptor systems.
systems. Moreover, this method can be extended to FDI for uncertain dynamics of HST based on descriptor systems with time-varying delay in the future.

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Data availability statement
All data, models, and code generated or used during the study appear in the submitted article.

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Appendix

Proof. Assuming that $\tilde{\delta}(t) = 0$, a Lyapunov function is chosen as:

$$V(t) = e^T(t)P_1e(t)$$ (A1)

Where: $P_1$ is a symmetric positive-definite matrix, then the following equation is hold.

$$\dot{V}(t) = e^T(t)(A_1-K_1C)^TP_1e(t) + P_1(A_1-K_1C)e(t)$$ (A2)

from (35), it is easy to verify that $\dot{V}(t)<0$ and system (34) is asymptotic stable.

By Schur complement and lemma 3, (36) is equivalent to

$$\psi = \begin{bmatrix} -2P_{s} & P_{d} + P_{s}(A_1 - K_1C) \\ * & He(P_1(A_1 - K_1C)) \end{bmatrix} + \Psi \otimes Q_d,$$ (A3)

where:

$$\psi_{1.1} = \begin{bmatrix} -2P_{s} & P_{d} + P_{s}(A_1 - K_1C) \\ * & He(P_1(A_1 - K_1C)) \end{bmatrix} + \Psi \otimes Q_d,$$

$$\psi_{1.2} = \begin{bmatrix} P_{T}\tilde{E}_d & P_{T}\tilde{M}_1F_1(t)\tilde{N}_1 \\ \end{bmatrix},$$

$$\psi_{1.3} = \begin{bmatrix} P_{T}\tilde{M}_2F_2(t)\tilde{N}_2 \\ P_{T}\tilde{M}_1F_1(t)\tilde{N}_1 \end{bmatrix},$$

$$\psi_{2.1} = \begin{bmatrix} -\gamma^2I & 0 \\ 0 & -\gamma^2I \end{bmatrix},$$

$$\psi_{2.2} = \begin{bmatrix} -\gamma^2I & 0 \\ 0 & -\gamma^2I \end{bmatrix}.$$

Define $\tilde{E}_w = \begin{bmatrix} T\tilde{E}_d & T\Delta A & T\Delta B \end{bmatrix}$, $Z = \text{diag}(-\gamma^2I, -\gamma^2I, -\gamma^2I)$, the following equation is obtained as:

$$\begin{bmatrix} -2P_s & P_\tilde{A} + P_d & P_\tilde{E}_w \\ * & He(P_1 \tilde{A}) & P_\tilde{E}_w (GC)^T \\ * & * & Z \end{bmatrix} + \begin{bmatrix} \Psi \otimes Q_d \\ * & * & 0 \end{bmatrix} < 0$$ (A4)

Let:

$$F_d = \begin{bmatrix} \tilde{A} & \tilde{E}_w \end{bmatrix}, U_d = \begin{bmatrix} P_s \\ P_s \end{bmatrix}, N_{F_d} = \begin{bmatrix} -I \\ \tilde{A}^T \end{bmatrix},$$

$$\xi(t) = \begin{bmatrix} e^T(t) & \delta_c^T(t) \end{bmatrix}^T,$$

Where: $N_{F_d}$ is a null space of $F_d$, it is obvious that (A4) is equivalent to:

$$\begin{bmatrix} 0 & P + P_d & 0 \\ * & C^T C & 0 \end{bmatrix} + \begin{bmatrix} \Psi \otimes Q_d & 0 \\
* & * & Z \end{bmatrix} > 0$$ (A5)

from lemma 4, (A5) is equivalent to the following equation:

$$\xi^T(\tau)F_d \begin{bmatrix} 0 & P_\tilde{A} + P_d & (GC)^T (GC) \\ * & * & Z \end{bmatrix} + \begin{bmatrix} \Psi \otimes Q_d \\ * & * & 0 \end{bmatrix} < 0$$ (A6)

from lemma 5, under zero-initial condition, it is obvious that:
from (A7), the following inequality is satisfied:
\[
\int_0^\infty [r^T(\tau) r(\tau) - \gamma_2^2 \tilde{\vartheta}^T(\tau) \tilde{\vartheta}(\tau) - \gamma_2^2 \tilde{x}^T(\tau) \tilde{x}(\tau) - \\
- \gamma_2^2 \tilde{u}^T(\tau) \tilde{u}(\tau)] d\tau
\]

\[
\int_0^\infty [\tilde{r}^T(\tau) r(\tau) - \gamma_2^2 \tilde{\vartheta}^T(\tau) \tilde{\vartheta}(\tau) - \gamma_2^2 \tilde{x}^T(\tau) \tilde{x}(\tau) - \\
- \gamma_2^2 \tilde{u}^T(\tau) \tilde{u}(\tau) + \dot{V}(\tau)] d\tau
\]

\[
+ \int_0^\infty \Sigma^T(\tau) (\Phi \otimes P_d + \Psi \otimes Q_d) \Sigma(\tau) d\tau
\]

\[
+ \int_0^\infty \xi^T(\tau) F_d^T \left\{ \begin{bmatrix} 0 & P_s + P_d & 0 \\
* & \hat{C}^T \hat{C} & 0 \\
* & * & Z \end{bmatrix} + \begin{bmatrix} \Psi \otimes Q_d & 0 \\
* & 0 \end{bmatrix} \right\} F_d \xi(\tau) d\tau
\]

(A7)

The proof is completed.