Dimensional optimization of completely restrained positioning cable driven parallel manipulator with large span

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1. Introduction

Cable driven parallel manipulator develops rapidly from 1980s which is also called cable robot, cable driven robot or wire driven parallel robot. Cable driven parallel manipulator has lots of desirable characteristics, including low inertial properties, high payload-to-weight ratios, potentially vast workspace, transportability, ease of disassembly/reassembly, reconfigurability, and economical construction and maintenance (Alp & Agrawal, 2002). At present, giving full play to its advantage on large workspace, more and more large span cable driven parallel manipulator are used in Large radio telescopes or Wind tunnel support system (Lambert et al., 2007).

Since the cable driven parallel manipulator can only bear tension but not compression, a cable system with n end-effector Degrees of Freedom (DOFs) requires at least m=n+1 cables. When m=n+1, the cable driven parallel manipulator is Completely Restrained Positioning (CRP) (Verhoeven et al., 1998; Hiller & Fang, 2005). When a cable length is longer than 10m in a cable driven parallel manipulator, it is called a large span cable driven parallel manipulator. In this paper, we will take a six-cable driven parallel manipulator as an example to study the modeling and dimensional optimization method of a CPR cable driven parallel manipulator with large span.

The optimization target of the previous work on dimensional optimization design is workspace requirement and constraint condition is cable tension, stiffness or motion Index (Hassan & Khajepour, 2008; Yang et al., 2006; Fang et al., 2004), which will greatly influence the accuracy and vibration of a cable driven parallel manipulator. So, the purpose of this paper is to optimize dimensions of the six-cable driven parallel manipulator to meet the workspace requirement of constraint condition in terms of cable tension. So far, great progress has been made in this field. However, two difficulties arose during the dimensional optimization of cable driven parallel manipulator: cable catenary algorithm (Yao et al., 2007) and optimization target of a cable driven parallel manipulator. Firstly, the weight of cable itself results in catenary. But most of previous research neglected the cable's catenary because they thought the weight of cable is inconsiderable, and did not consider the deformation of cable (Duan et al., 2008). For a CPR cable driven parallel manipulator with
large span, the deformation and catenary greatly influence the modeling and control accuracy (Kozak et al., 2006). Many of the current study on this topic can be introduced from bridge and construction fields (Krishna & Prem, 1978; Yang & Chen, 2003). Based on previous improvements, modeling method for a CPR cable driven parallel manipulator is adopted in this paper.

Secondly, cable tension is always used as the optimization target in a cable driven parallel manipulator because it is a tension redundant mechanism. If one of the cable tensions is negative or small, cable driven parallel manipulator will be uncontrollable. So it is necessary to optimize the geometric parameters by adopting a cable tension as a constraint condition in the required workspace.

In China, a Five Hundred meter Aperture Spherical radio Telescope (FAST) employs a cable driven parallel manipulator with large span (more than 300m) as the first adjustable feed support system (Tang, et al.). For the design of the cable driven parallel manipulator in FAST, several design proposals are made, one of which is the six-cable driven parallel manipulator (Nan, 2006). This chapter will study the modeling and dimensional optimization method of this six-cable driven parallel manipulator.

In this chapter, section 2 introduces the classification of cable driven parallel manipulator. Section 3 gives the modeling method for a large cable drive parallel manipulator. The tension static equilibrium equation is set up by cable catenary equation with cable deformation. In Section 4, the six-cable driven parallel manipulator in FAST is taken as an example. The description and modeling method of the six-cable driven parallel manipulator is presented, and Tilt-and-Torsion Angles is adopted for modeling. Section 5 gives a dimensional optimization method. Taking the six-cable driven parallel manipulator in FAST as an example, the optimization target is pose angle of the six-cable driven parallel manipulator and constraint condition is cable tensions. Finally, a set of optimized dimensional parameters of the six-cable driven parallel manipulator is available for building the feed support system in FAST.

The result indicates that the methods in this chapter are feasible for modeling and optimizing a CPR cable driven parallel manipulator with large span. More importantly, it provides a theoretical basis for further study.

2. Classification of a cable driven parallel manipulator

A cable driven parallel manipulator, according to its number of cables (m) and degrees of freedom (DOFs) of end-effector (n), can be classified as follows (Verhoeven et al., 1998; Hiller & Fang, 2005):

IRPMs: Incompletely Restrained Positioning Mechanisms. The number of cables is less than or equal to the number of DOFs, namely,

\[ m \leq n \]

CRPMs: Completely Restrained Positioning Mechanisms. There has a one extra cable for an n DOFs end-effector,

\[ m = n + 1 \]
RRPMs: Redundantly Restrained Positioning Mechanisms.

\[ m > n + 1 \]

Then, R stands for rotational, T for translational DOFs. The possible DOF classes are as follows,

| DOFs   | n  | Type of motion |
|--------|----|----------------|
| 1T     | 1  | Linear motion of a body |
| 2T     | 2  | Planar motion of a point |
| 1R2T   | 3  | Planar motion of a body |
| 3T     | 3  | Spatial motion of a point |
| 2R3T   | 5  | Spatial motion of a beam |
| 3R3T   | 6  | Spatial motion of a body |

Table 1. possible DOF classes

3. Tension static equilibrium equation for a large cable drive parallel manipulator

3.1 Cable static tension equilibrium equation with catenary and elastics deformation

In small cable driven mechanism, due to the relatively inconsiderable weight of the cable, a cable is often analyzed as a two-force member. However, the cable span of a large cable driven parallel manipulator is no less than 10m, so the cable weight and deformation are non-negligible. The catenary and deformation of the cable must be taken into account in the modeling of a large cable driven parallel manipulator. A meaningful work for analyzing a large cable driven parallel manipulator with catenary and elastic deformation is to describe the cable model mathematically.

For setting up the cable model and cable static equilibrium equation, the symbols used in Fig. 1 are defined as:

- \( l_0 \) is the unstrained length of the cables;
- \( A \) (the stain of the cable);
- \( T \) the force applied to the....
fixed end of the cable; $E$ the elastic modulus; $A_0$ the unstrained cross-sectional area. Using the variables and coordinate system above, we will briefly reproduce Irvine’s derivation in this paper (Irvine, 1981).

To begin with, the cable must satisfy the geometric constraint:

$$
\sum x = 0 \quad H + dH - H = 0
$$

$$
\sum y = 0 \quad H \frac{dz}{dx} + d \left( H \frac{dz}{dx} \right) + \rho g dl_0 - H \frac{dz}{dx} = 0
$$

where,

$$
\frac{dl}{dl_0} = \frac{T}{EA_0} + 1
$$

$$
T = H \sqrt{1 + \left( \frac{dz}{dx} \right)^2}
$$

From $dl = dx \sqrt{1 + \left( \frac{dz}{dx} \right)^2}$, Eq.(2) can be expressed as:

$$
d \left( H \frac{dz}{dx} \right) + \rho g \frac{EA_0}{T + EA_0} ds = 0 \Rightarrow \frac{d^2z}{dx^2} + \frac{\rho g EA_0}{H} \frac{\sqrt{1 + \left( \frac{dz}{dx} \right)^2}}{H \sqrt{1 + \left( \frac{dz}{dx} \right)^2} + EA_0} = 0
$$

assuming $\frac{dz}{dx} = p$ Eq.(5) can be written as:

$$
\frac{dp}{dx} + \frac{\rho g EA_0}{H} \frac{\sqrt{1 + p^2}}{H \sqrt{1 + p^2} + EA_0} = 0
$$

it yields:

$$
\frac{dx}{dp} = - \frac{H^2}{\rho g EA_0} - \frac{H}{\rho g \sqrt{1 + p^2}}
$$

Therefore,
Dimensional optimization of completely restrained positioning cable driven parallel manipulator with large span

\[ x = - \frac{H}{\rho g} sh^{-1} \left( \frac{dz}{dx} \right) - \frac{H^2}{\rho g E A_0} \frac{dz}{dx} + c_i \]  

(8)

where, \( sh^{-1}(x) = \ln \left( x + \sqrt{1 + x^2} \right), x \in (-\infty, +\infty) \)

\[ x = - \frac{H}{\rho g} \ln \left( \frac{dz}{dx} + \sqrt{1 + \left( \frac{dz}{dx} \right)^2} \right) - \frac{H^2}{\rho g E A_0} \frac{dz}{dx} + c_i \]  

(9)

Integrating and applying the boundary conditions as follows:

\[ x = 0, z = 0; \]

\[ x = L, z = h; \]

The length of cable is \( l \), the unstained length of the cable \( l_0 \), and \( \Delta l \) represents the stain of the cable, respectively. The relationship can be expressed as \( l = l_0 + \Delta l \).

\[ l = \int_0^L \sqrt{1 + \left( \frac{dz}{dx} \right)^2} \, dx \]  

(10)

\[ l_0 = \int_0^L \frac{1}{\frac{T}{E A_0} + 1} \, dl \]  

(11)

The numerical method is presented in the flow chart below:
Set the initial value of the integral constant \( c=0 \) in Eq. (8).

Calculate \( \frac{dz}{dx} \) in Eq. (9) with specified \( x \) using Newton's method.

Get the catenary equation \( z(x) \) using numerical integration or continuation method.

If \( |z(L)-h| < \varepsilon \)

Calculate force \( V \) and \( T \) with current \( z(x) \).

Adjust integral constant \( c \) with \( c = c - \frac{z(L) - h}{d(z(L)-h)/dc} \)

Calculate the change of \( z(L)-h \) to evaluate \( \frac{d(z(L)-h)}{dc} \)

Change integral constant \( c \) slightly

Fig. 2. Solution of a cable catenary with deformation

### 3.2 Static tension equilibrium equation for a cable driven parallel manipulator

In this paper, the CPR cable driven parallel manipulator with large span has following characteristics. The manipulator is static or moving slowly enough to be considered static. Gravity is considerable. The cables are assumed to be elastic. When cable catenary is not neglected, the static tension equation for a cable driven parallel manipulator becomes quite difficult to compute. In this section, the cable static tension equation is posed, and a solution method is presented for the CPR cable driven parallel manipulator.

![Fig. 3. A n DOFs cable driven parallel manipulator](image-url)
Fig. 3 is a \( m \) DOFs cable driven parallel manipulator, and two coordinates are set up: an inertial frame \( \mathcal{R} : O - XYZ \) is located at the center of the fixed platform. Another moving frame \( \mathcal{R}': O' - X'Y'Z' \) is located at the center of the moving platform. \( B_i \) \((i = 1, 2, \ldots, m)\) are connected points of the cables and fixed platform, and \( A_i \) \((i = 1, 2, \ldots, m)\) are hinge center of the cables and moving platform.

For analysis, the symbols used in this section are defined as:

- \( O^{\mathcal{R}} \) is the \( O' \) expressed in the inertial frame;
- \( B_i^{\mathcal{R}} \) the vector \( B_i \) expressed in the inertial frame;
- \( A_i^{\mathcal{R}} \) the vector \( A_i \) expressed in the inertial frame;
- \( A_i^{\mathcal{R}'} \) the vector \( A_i \) expressed in the moving frame;
- The symbols with subscript \( i \) express the parameters of the \( i \)-th cable.

According to Fig. 3, the vector of the cables can be defined as:

\[
A_i^{\mathcal{R}} = R \cdot A_i^{\mathcal{R}'} + O^{\mathcal{R}}
\]  

(12)

Where \( R \) is the coordinate-axis rotation matrix.

Assuming \( L_i = B_i^{\mathcal{R}} A_i^{\mathcal{R}}, u_i = \frac{L_i}{\|L_i\|}, r_i = O^{\mathcal{R}} A_i^{\mathcal{R}} \), static equilibrium equations of a cable driven parallel manipulator can be written as:

\[
F = J^T \sigma
\]  

(13)

Where \( \sigma \) is the vector quantity of the cable tension (like \( T \) in Fig. 1); \( J^T \) the tension transmission matrix of the cable driven parallel manipulator; \( F \in \mathbb{R}^n \) the wrench of the moving platform.

\[
\sigma = [\sigma_1, \sigma_2, \ldots, \sigma_m]^T
\]  

(13)

\[
J = \begin{bmatrix}
    u_1 & \ldots & u_m \\
    r_1 & \ldots & r_m \times u_m
\end{bmatrix}_{n \times m}
\]  

(14)

According to the section 3.1, the cable vertical tension can be expressed as:

\[
V_i = H_i \frac{dz}{dx}
\]  

(15)

The static tension equilibrium equation for a cable driven parallel manipulator can be derived as:

\[
\begin{aligned}
\sum_{i=1}^{m} \sigma_{ix} &= 0; & \sum_{i=1}^{m} \sigma_{iy} &= 0; & \sum_{i=1}^{m} \sigma_{iz} &= 0; \\
\sum_{i=1}^{m} M_{ix} &= 0; & \sum_{i=1}^{m} M_{iy} &= 0; & \sum_{i=1}^{m} M_{iz} &= 0;
\end{aligned}
\]  

(16)

So the solution flow chart can be shown in Fig. 4.
4. Description and modelling of the six-cable driven parallel manipulator in FAST

4.1 Description of the six-cable driven parallel manipulator

China began to build a Five-hundred meter Aperture Spherical radio Telescope (FAST). Due to the largeness in FAST, no solid support system is available for orientation of the feed. Therefore, a cable driven parallel manipulator is introduced for the first adjustable feed support system to provide a primary orientation. For the design of the cable driven parallel manipulator in FAST, several design proposals are made, one of which is the six-cable driven parallel manipulator.

In Figure 5, the feed support system of FAST is a hybrid manipulator which includes two parts: a cable driven parallel manipulator is the first adjustable feed support system and a feed mechanism is the second adjustable feed support system which includes an A-B platform and a Stewart platform with a feed on its moving platform.
The six-cable driven parallel manipulator in FAST draws the feed mechanism to primary orientation. The A-B platform, as the moving platform of the six-cable driven parallel manipulator, can provide accurate pose angle of feed to track celestial bodies. The Stewart platform, fixed on the A-B platform, is to improve the orientation accuracy of the feed. Due to the distribution of the cables, spin angle of this six-cable driven parallel manipulator is small, and the magnitude order of spin angel is 0.1°. The spin angle will influence the motion stability of the moving platform, so we restrict the spin angle. Therefore, we suppose the six-cable driven parallel manipulator is a CPR cable driven parallel manipulator with 5 DOFs.

4.2 The coordinate-axis rotation matrix based on Tilt-and-Torsion Angles
As shown in Fig.6, pose angle of the six-cable driven parallel manipulator is the tracking angle of feed. The tracking angle can be described as an azimuth angle and a tilt angle, which is difficult to be discriminated by Euler-angles description. Gosselin C.M (Gosselin, 2002) presented a T&T (Tilt-and-Torsion) angle to set up the coordinate-axis rotation matrix R, which can directly discriminate the azimuth and tilt angles.
In Fig. 7, three angles are defined: azimuth $\theta$, tilt $\phi$ and torsion $\omega$. These new angles are easier to interpret geometrically and allow simple computation and representation of the 3D orientation workspace.

The rotation matrix is derived as follows:

$$R = R_x(\theta)R_y(\phi)R_z(\omega) = R_z(\theta)R_y(\phi)R_z(-\phi)R_z(\omega) = R_y(\phi)R_z(\theta)R_z(\omega) = \begin{bmatrix} c\phi c\theta c(\omega - \phi) - s\phi s(\omega - \phi) & -c\phi c\theta s(\omega - \phi) - s\phi c(\omega - \phi) & c\phi s\theta \\ s\phi c\theta c(\omega - \phi) + c\phi s(\omega - \phi) & -s\phi c\theta s(\omega - \phi) - c\phi c(\omega - \phi) & s\phi c\theta \\ -s\phi c(\omega - \phi) & -s\phi c(\omega - \phi) & c\theta \end{bmatrix}$$  \hspace{1cm} (17)

Where $c(\cdot)$ and $s(\cdot)$ correspond to the $\cos(\cdot)$ and $\sin(\cdot)$, respectively.

The torsion angle of the six-cable driven parallel manipulator is restricted, the $\theta = 0$. The rotation matrix can be derived as:
Dimensional optimization of completely restrained positioning cable driven parallel manipulator with large span

Fig. 6. Pose angle of the six-cable driven parallel manipulator

In Fig. 7, three angles are defined: azimuth $\theta_0$, tilt $\theta$ and torsion $\theta_x$. These new angles are easier to interpret geometrically and allow simple computation and representation of the 3D orientation workspace.

$$
R = \begin{bmatrix}
c\phi c\theta c(-\phi) - s\phi s(-\phi) & -c\phi c\theta s(-\phi) - s\phi c(-\phi) & c\phi s\theta \\
s\phi c\theta c(-\phi) + c\phi s(-\phi) & -s\phi c\theta s(-\phi) - c\phi c(-\phi) & s\phi c\theta \\
-s\phi c(-\phi) & s\phi c(-\phi) & c\theta
\end{bmatrix}
$$

(18)

For the six-cable driven parallel manipulator, the azimuth $\theta_0$ is the tracking azimuth and tilt $\theta$ is the pitching angle of the feed. For a cable driven parallel manipulator without torsion angle, it is useful to set up the coordinate-axis rotation matrix $f_i$ by the Tilt-and-Torsion Angles.

4.3 modeling of the six-cable driven parallel manipulator

Fig. 8. Geometric parameter of the parallel mechanism

Fig. 8 is the six-cable driven parallel manipulator in FAST, and two coordinates are set up: an inertial frame $5R$: $0 - XYZ$ is located at the center of the reflector's bottom. Another moving frame $SR'$: $0' - XYZ$ is located at the center of the moving platform. $\beta_i$ ($i = 1, 2, ..., 6$) are connected points of the cables and cable towers, and $A_j$ ($i = 1, 2, 3$) are hinge center of the cables and moving platform.

For analysis, the symbols used in this section are defined as:

0$^M$ is the O expressed in the inertial frame; the vector $\beta_i$ expressed in the inertial frame; $A^\wedge$ the vector $A_j$ expressed in the inertial frame; the vector $A_j$ expressed in the moving frame; $r_b$ the radius of the cable towers' distributed circle; $r_a$ the radius of the moving platform; $D$ the aperture of the reflector; $h$ the height of the cable tower; $d$ the diameter of the cable; $m$ the weight of the feed mechanism. It is defined as:

$$
B_i^{gr} = \begin{bmatrix} r_b\cos(i-1)60^\circ & r_b\cos(i-1)60^\circ & h \end{bmatrix}^T (i = 1, 2, ..., 6)
$$

(19)

$$
A_j^{gr} = \begin{bmatrix} r_a\cos(4j-3)30^\circ & r_a\sin(4j-3)30^\circ & 0 \end{bmatrix}^T (j = 1, 2, 3)
$$

(20)

$$
A_j^{gr} = R \cdot A_j^{gr} + O^{gr}
$$

(21)

Where is $R$ is the coordinate-axis rotation matrix based on Tilt-and-Torsion Angles. Then,
according to the section 3, the static tension equilibrium equation for the six-cable driven parallel manipulator and its solution can be given.

5. Dimensional optimization of the six-cable driven parallel manipulator in FAST

5.1 Geometric parameters of the six-cable driven parallel manipulator

According to the reference report of FAST provided by National Astronomical Observatories Chinese Academy of Science (Nan, 2005), the related geometric parameters are given in Table 2.

| Geometric Parameters                      | Symbol | dimension |
|------------------------------------------|--------|-----------|
| Aperture of the reflector                | D      | 500 (m)   |
| Radius of the moving platform            | r_a   | 5 (m)     |
| Weight of the feed mechanism             | m      | 30 (ton)  |
| Young’s modular                          | E      | 1.6 \times 10^{11} (Pa) |
| Radius of cable tower’s distributed circle | r_b  | To be optimized (m) |
| Height of cable tower                    | h      | To be optimized (m) |
| Diameter of cable                        | d      | To be optimized (mm) |
| Density of cable                         | P      | To be optimized (Kg/m) |

Table 2. related geometric parameters of the six-cable driven parallel manipulator

Fig. 9. required workspace of the feed

The required workspace of the feed is shown in Fig.9. It is a sphere crown with a radius of 160m. The center of the sphere and the reflector are concentric. Height of the required workspace is from 140m to 177m above the bottom of the reflector. The tracking angle of the feed is: azimuth $\phi$: 0° - 360°, tilt $\theta$: 0° -40°.

5.2 Dimensional optimization method for the six-cable driven parallel manipulator

Cable driven parallel manipulator is a tension redundant mechanism. If one of the cable tensions is negative or small, cable driven parallel manipulator will be uncontrollable. So it is necessary to optimize the geometric parameters by adopting a cable tension constraint condition in the required workspace.

The constraint conditions of cable tension are given as follows:

$$\sigma_{\text{min}} \leq \sigma_i \leq \sigma_{\text{max}} \quad (i = 1, 2, 3, \ldots, 6)$$ (22)
where $\sigma_{\text{min}}$ is pretightening tension of cable, $\sigma_{\text{max}}$ is the maximum tension of cable. Feed support system in FAST is a hybrid manipulator, and the six-cable driven parallel manipulator is used to primary orientation. The required tilt angle of feed is $0^\circ - 40^\circ$, and we hope the tilt angle of the six-cable driven parallel manipulator can closest to required title angle of feed. So, the optimization target is tilt angle of the six-cable driven parallel manipulator.

In the Six-cable driven parallel manipulator, assuming the tilt angle is $\theta$ and the required tilt angle is $\theta_{\text{req}}$, the optimization target and conditions are as follows: Optimization target:

$$\min |\theta - \theta_{\text{req}}|$$  \hspace{1cm} (23)

Optimization constraint conditions:

$$\sigma_{\text{min}} \leq \sigma_i \leq \sigma_{\text{max}} \quad (i = 1, 2, 3, ..., 6)$$  \hspace{1cm} (24)

$$(x, y, z, \phi) \in \mathbb{R}^{\text{required workspace}}$$  \hspace{1cm} (25)

5.3 Dimensional optimization for the six-cable driven parallel manipulator

The purpose of the dimensional design is to optimize the three important geometric parameters for the four-cable driven parallel manipulator: diameter of cable $d$, cable tower height $h$, and the radius of cable tower’s distributed circle $r$. Considering the maximum observation scope, the cable tower height should be less than 290m (ref). A given route is the boundary of feed required workspace. The required pose angle in this route is azimuth angle $\phi: 0^\circ - 360^\circ$, tilt angle $6:40^\circ$. For the sake of safety, the cable tension is roughly calculated in an interval solution $[80\text{KN}, 400\text{KN}]$.

So, $\theta_{\text{req}} = 40^\circ; \sigma_{\text{min}} = 80\text{KN} \ \sigma_{\text{max}} = 400\text{KN}$;

According to the equation (22)-(25), the influence of the dimensional parameters on the tilt angle of the six-cable driven parallel manipulator is shown in Fig.10-13.

Fig. 10. Influence of cable tower height on tilt angle
In Fig.10, assuming the radius of the cable tower's distributed circle $r_b$ is 290m and the cable's diameter $d$ is 40mm. Given the selective cable tower heights, it shows that cable tower height has influence on workspace. Tilt angle of the six-cable driven parallel manipulator is larger by increasing the radius of the cable tower height $h$. So, the cable tower height $h$ is designed as 285m.

Fig. 11. Influence of radius of cable tower's distributed circle on tilt angle

As shown in Fig.11, the radius of the cable tower's distributed circle has not great influence on tilt angle. But, the increasing radius of the cable tower's distributed circle will slightly decrease the tilt angle. Therefore, the radius of the cable tower's distributed circle is designed $r_b$ as 290m.

Fig. 12. Influence of diameter of cable on tilt angle

Fig.12 shows that the diameter of cable has almost no influence on tilt angle. It indicates that all the given diameter of cable can be used in FAST. However, For the sake of safety, cable's diameter $d$ is designed as 42mm and its density is 7.35kg/m.

According to the dimensional optimization method, a set of final optimized dimensional parameters are obtained in Table 3.

| Geometric Parameters       | Symbol | Optimized Dimension |
|----------------------------|--------|---------------------|
| Aperture of the reflector  | $D$    | 500 (m)             |
| the radius of the moving platform | $r_a$  | 5 (m)               |
| Weight of the feed mechanism | $m$    | 30 (ton)            |
| Young's modular            | $E$    | $1.6 \times 10^{11}$ (Pa) |
| Radius of cable tower's distributed circle | $r_h$  | 290 (m)             |
| Height of cable tower      | $h$    | 285 (m)             |
| Diameter of cable          | $d$    | 42 (mm)             |
| Density of cable           | $P$    | 7.35 (Kg/m)         |

Table 3. Optimized geometric parameters of the six-cable driven parallel manipulator

6. Conclusion

This paper addressed several important issues related to a large CPR cable driven parallel manipulator. In conclusion, we emphasize the following:

Firstly, in section 3 we have introduced an effective modeling method for a large CPR cable driven parallel manipulator. This method set up the effective catenary formula with cable elastic deformation into the tension equilibrium equation to work out the cable tensions.

Secondly, according to the modeling method, a coordinate-axis rotation matrix based on Tilt-and-Torsion Angles is adopted to set up the tension equilibrium equation for the six-cable driven parallel manipulator in FAST. This rotation matrix can directly discriminate the tracking angle of feed.

Thirdly, based on the requirement workspace and tension condition, a set of optimization dimensional parameters is obtained to build the feed support system of FAST. The dimensional optimization method can avoid control invalidity for a cable driven parallel manipulator.

7. Acknowledge

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parameters of the six-cable driven parallel manipulator are obtained in Table 3. This set of optimized dimensional parameters can be applied in the construction of the feed support system in FAST.

| Geometric Parameters                  | Symbol | Optimized dimension |
|--------------------------------------|--------|---------------------|
| Aperture of the reflector            | $D$    | 500 (m)             |
| the radius of the moving platform    | $r_a$  | 5 (m)               |
| Weight of the feed mechanism         | $m$    | 30 (ton)            |
| Young's modular                      | $E$    | $1.6 \times 10^{11}$ (Pa) |
| Radius of cable tower's distributed circle | $r_h$ | 290 (m)            |
| Height of cable tower                | $h$    | 285 (m)             |
| Diameter of cable                    | $d$    | 42 (mm)             |
| Density of cable                     | $P$    | 7.35 (Kg/m)         |

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Thirdly, based on the requirement workspace and tension condition, a set of optimization dimensional parameters is obtained to build the feed support system of FAST. The dimensional optimization method can avoid control invalidity for a cable driven parallel manipulator.

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Robot manipulators are developing more in the direction of industrial robots than of human workers. Recently, the applications of robot manipulators are spreading their focus, for example Da Vinci as a medical robot, ASIMO as a humanoid robot and so on. There are many research topics within the field of robot manipulators, e.g. motion planning, cooperation with a human, and fusion with external sensors like vision, haptic and force, etc. Moreover, these include both technical problems in the industry and theoretical problems in the academic fields. This book is a collection of papers presenting the latest research issues from around the world.

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