Stellar Collisions and Black Hole Formation in Dense Star Clusters

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Abstract. Close encounters and physical collisions between stars in young dense clusters may lead to the formation of very massive stars and black holes via runaway merging. We examine critically some details of this process, using N-body simulations and simple analytical estimates to place limits on the cluster parameters for which it expected to occur. For small clusters, the mass of the runaway is effectively limited by the total number of high-mass stars in the system. For sufficiently dense larger clusters, the runaway mass is determined by the fraction of stars that can mass segregate to the cluster core while still on the main sequence. The result is in the range commonly cited for intermediate-mass black holes, such as that recently reported in the Galactic center.

1. Introduction

The past decade has seen the discovery of a large number of massive young star clusters throughout the local universe. These systems are large and young enough that they contain statistically significant numbers of massive stars, affording us key insights into their initial mass function and structural properties. Of greatest interest to dynamicists are those systems in which stellar dynamical time scales are short enough that the cluster can undergo significant structural change during the lifetimes of the most massive stars. In such clusters, dynamical evolution opens up entirely new avenues for stellar and binary evolution, allowing the formation of stellar species completely inaccessible by standard stellar and binary evolutionary pathways.

From this perspective, the clusters listed by Portegies Zwart and McMillan (these proceedings) represent an ideal combination of properties, having ages of less than a few million years and relaxation times of less than a few tens of millions of years. In these clusters, dynamical evolution, traditionally regarded as a “slow” process, actually occurs much more rapidly than the stellar evolution of even the most massive stars. Thus the dynamics controls the early phases of these stars’ lives. We focus here on perhaps the most dramatic environmental modification of standard stellar evolution—repeated physical collisions between stars. The scenario described below is fast becoming the “standard” paradigm by which runaway stellar mergers might occur, perhaps forming intermediate-mass black holes (IMBHs) in sufficiently dense systems.
The possibility of multiple mergers of massive stars in dense stellar systems was first demonstrated in N-body simulations of R136 by Portegies Zwart et al. (1999). Subsequently Ebisuzaki et al. (2001) suggested that the ultraluminous X-ray source M82-X1 might be the result of such a process; this possibility was explored in detail by Portegies Zwart et al. (2004). Portegies Zwart & McMillan (2002) and McMillan & Portegies Zwart (2003) have explored the possibility of IMBH formation in young clusters in the Galactic Center. Recently, Gürkan et al. (2004) have reaffirmed the basic process using Monte-Carlo simulations, and have carried out systematic studies of runaway mergers in galactic nuclei.

In this paper we consider how a young star cluster might come to be in such a high-density state, and look critically at the key physical processes needed for a runaway merger to occur. We then turn briefly to the possibility of an IMBH in the center of the Milky Way Galaxy (Maillard et al. 2004), showing how theory and recent observations may provide a consistent picture of the Galactic center.

2. Stellar Collisions

Consider a massive object of mass $M = m M_\odot$ and radius $R = r R_\odot$ moving through a field of background stars of total mass density $\rho = 10^6 \rho_6 M_\odot/pc^3$ and velocity dispersion $v = 10 v_{10} \text{km/s}$. If $M$ and $R$ are large compared to the masses and radii of other stars, and all velocities are small enough that gravitational focusing dominates the total cross section, the object’s collision cross section is $\sigma \approx 2\pi G M R / v^2$, nearly independent of the properties of the other stars. The rate of increase of the object’s mass due to collisions then is

$$\frac{dM}{dt} \approx \rho \sigma v \approx 2\pi G M R \rho / v = 6 \times 10^{-11} m r \rho_6 v_{10}^{-1} M_\odot/\text{yr}. \quad (1)$$

If the object initially has $m = 100 m_{100}$ and we adopt a simple mass–radius relation $r = 3 m^{1/2}$, then for the object to reach $m \gg 10^3$ in 3 Myr (to form an IMBH within the lifetime of a massive star), the local density must satisfy

$$\rho_6 \gtrsim 350 m_{100}^{-1/2} v_{10} = \rho_{\text{crit}}, \quad \text{say}. \quad (2)$$

Such a density is much higher than the mean density of any known star cluster, young or old. For comparison, the average density of the Arches cluster is $\rho_6 \sim 0.6$, that of a fairly compact globular cluster is $\rho_6 \sim 0.01$, while even the most concentrated globular cluster cores have $\rho_6 \lesssim 1 - 10$.

Mergers would be enhanced if the cluster were born very centrally concentrated, as suggested by Portegies Zwart et al. (2004) and Merritt et al. (2004). As a simple limiting model of such a cluster, consider the nearly isothermal system of total mass $M_c$ and half-mass radius $r_h$, described by the density profile

$$\rho(r) = \frac{M_c}{8\pi r_h r^2}, \quad (3)$$

$$M(r) = \frac{1}{2} M_c \left( \frac{r}{r_h} \right), \quad (4)$$

for $0 \leq r \leq 2r_h$. Densities exceeding $\rho_{\text{crit}}$ are found for $r < r_{\text{crit}}$, where

$$r_{\text{crit}} = \sqrt{\frac{M_c}{8\pi r_h \rho_{\text{crit}}}} = 2.3 \times 10^{-3} v_{10}^{1/2} m_{100}^{1/4} \text{pc}, \quad (5)$$
where the cluster velocity dispersion is \( v = \sqrt{GM_c/2r_h} \). However, the total mass contained within this radius is just

\[
M_{\text{crit}} \approx 50 \, v_{10}^{5/2} \, m_{100}^{1/4} \, M_\odot .
\]

(6)

We conclude that, for reasonable cluster parameters, there is too little initial mass in the high-density region to accomplish the task of forming a \( \sim 10^3 M_\odot \) object in the time available.

### 3. Cluster Dynamics

Thus collisions in a static cluster core cannot lead to the formation of an ultra-massive object. However, cluster dynamical evolution can result in conditions much more favorable for a runaway merger to occur. The evolution of a cluster is governed by its half-mass relaxation time, the time scale on which two-body encounters transport energy around the system:

\[
t_{rh} \approx \frac{0.14 M_c^{1/2} r_h^{3/2}}{G^{1/2} \langle m \rangle \ln \Lambda} \approx 0.5 \, v_{10}^3 / \bar{\rho}_6 \, \text{Myr}
\]

(7)

(Heggie & Hut 2003). Here, \( N \) is the number of stars in the system, \( \langle m \rangle = M_c/N \) is the mean stellar mass, taken here to be \( 0.5 M_\odot \), \( \bar{\rho} = M_c/8\pi r_h^3 = 10^6 \bar{\rho}_6 \, M_\odot \, \text{pc}^{-3} \) is the mean cluster density, and \( \ln \Lambda \sim \ln(0.1N) \sim 10 \). For an equal-mass system, the time scale for dynamical evolution—the core collapse time—is about \( 15 t_{rh} \), too long to cause significant structural change within a few million years. However, the presence of even a modest range in masses greatly accelerates the process of core collapse (Spitzer 1987). The time scale for a star of mass \( m \) to sink to the cluster center as equipartition reduces its velocity is

\[
t_s(M) \sim \frac{\langle m \rangle}{m} t_r ,
\]

(8)

where \( t_r \sim t_{rh} \bar{\rho}/\rho \) is the local relaxation time.

Portegies Zwart & McMillan (2002) find that the most massive \( (m \gtrsim 20 M_\odot) \) stars segregate rapidly to the cluster center, forming a dense stellar subcore on a time scale \( t_{cc} \sim 0.2 t_{rh} \). A central density increase of 2–3 orders of magnitude is typical, boosting even a relatively low-density core into the range where collisions become common, and greatly increasing the supply of raw material to form a collision runaway. In systems with \( t_{cc} \lesssim 5 \, \text{Myr} \) \( (t_{rh} \lesssim 25 \, \text{Myr}) \), essentially all the massive stars in the cluster reach the center before exploding as supernovae, and hence can participate in the runaway process. The Arches and Westerlund I fall into this category; the Quintuplet, NGC 3603, and R 136 all come close. In these cases, the maximum mass of the runaway is limited primarily by the total number of massive stars in the system—anywhere from a few percent to several tens of percent of the total, depending on the cluster mass function.

In less dense or more massive clusters, the longer half-mass relaxation time means that only a fraction of the massive stars initially present in the system can reach the center in the time available, but the total supply of mass may still ensure that a runaway can occur. We can estimate the amount of mass made
available by mass segregation as follows. Again adopting an isothermal model to simplify the calculation, and taking the segregation time scale from Eq. 8, it is easily shown that a star of mass \( m \) can sink to the center within time \( T \) if its initial distance \( r \) from the cluster center satisfies
\[
r \lesssim r_s(m) = r_h \left( \frac{m}{\langle m \rangle} \right)^{1/2} \left( \frac{T}{t_{rh}} \right)^{1/2}.
\] (9)
The fraction of stars of mass \( m \) satisfying this relation is
\[
f(m) = \frac{1}{2} \left( \frac{m}{\langle m \rangle} \right)^{1/2} \left( \frac{T}{t_{rh}} \right)^{1/2}.
\] (10)
Choosing (again for simplicity) a mass function \( dn/dm \sim m^{-2} \) for \( 0.1M_\odot < m < 100M_\odot \), we determine the total stellar mass potentially available for mergers as
\[
M_s = \int_{m_{min}}^{100M_\odot} dn/dm \, m \, f(m) \, dm,
\] (11)
where the lower mass limit \( m_{min} \) is somewhat greater than the mean stellar mass (0.7 \( M_\odot \) here), but is otherwise unimportant so long as it is much less than the upper limit of 100 \( M_\odot \). The result for the chosen mass function is
\[
M_s = 1.7 \left( \frac{T}{t_{rh}} \right)^{1/2} M_c
\] = 1.5 \times 10^4M_\odot \left( \frac{T}{3\,\text{Myr}} \right)^{1/2} \text{M}_\odot.
\]
comfortably above the value needed for IMBH formation.

We have assumed here that the cluster relaxation time is long enough that \( f(m) \leq 1 \) for all \( m \) of interest. This is the case for \( t_{rh} \geq 50 \text{ Myr} \). An appropriately modified version of this analysis for \( t_{rh} < 50 \text{ Myr} \) shows the available mass leveling off at a fixed fraction of the cluster mass as \( t_{rh} \to 0 \), as expected.

4. Mergers and Stellar Evolution

Given that dynamical evolution can concentrate enough mass in a cluster core for collisions to occur at a significant rate, we can then ask (i) if the collisions actually lead to mergers, and (ii) under what circumstances a runaway merger can occur. The answer to the first question is provided by the SPH simulations of Freitag & Benz (2001; see also Lombardi et al. 2003), who find that the low relative velocities typical of these systems ensure that the colliding stars usually merge with minimal mass loss. In small systems (containing less than a few tens of thousands of solar masses), collision rates are significantly enhanced by the fact that the massive object tends to form binaries, which are then perturbed into eccentric orbits by encounters with other stars (Portegies Zwart & McMillan 2002). Binary-induced mergers increase the collision cross section, but they still require high central densities before the (three-body) binary formation rates become significant. In larger systems, unbound collisions appear to be the norm.
Thus collisions naturally involve the most massive stars in the cluster, and lead to the production of even more massive objects. The merger products are generally out of thermal equilibrium and often rapidly rotating, with the result that their subsequent stellar evolution is currently poorly understood. In our simulations we generally assume that the merged object evolves along a suitably rejuvenated (non-rotating) track appropriate to its mass (Portegies Zwart et al. 1997). This is at best a crude approximation, but we note that it probably underestimates the radius of the merger product during the out-of-equilibrium phase and hence the likelihood of a runaway. Acting in the opposite sense is the fact that, while stellar mass loss rates are very uncertain, ultramassive stars probably have very strong winds. Van Beveren (these proceedings) points out that if the wind mass loss rate exceeds the accretion rate due to mergers, then the entire runaway process may fail. Our simulations generally yield net merger accretion rates of $\sim 10^{-3} M_\odot \, \text{yr}^{-1}$, suggesting that a high (but perhaps not impossibly so) mass-loss rate is needed to shut the process down.

Of course, it must be conceded that next to nothing is known about the detailed evolution and ultimate fate of stars hundreds or thousands of times more massive than the Sun. Nevertheless, the estimates presented here make it clear that dynamical evolution in dense stellar systems can easily produce conditions suitable for repeated stellar collisions. The collision runaway at the center of such a system should be extremely luminous and eminently observable during its short lifetime. Observations of the cores of dense young star clusters in our Galaxy and beyond may thus shed light on the structure and lifetimes of such ultramassive stellar objects.

5. An IMBH in the Galactic Center?

If our evolving cluster happens to reside close to the Galactic center, then dynamical friction will tend to drive it inward, raising the possibility that mass segregation can lead to a collision runaway en route, and that the resulting IMBH can be transported rapidly toward the center by the much more massive cluster (McMillan & Portegies Zwart 2003). Figure 1 illustrates this possibility. It shows the mass $M$ and Galactocentric distance $R$ of the runaway formed in a cluster of initial mass $4 \times 10^5 M_\odot$, placed in a circular orbit of radius 10 pc, as it spirals inward and is disrupted by the Galactic field. We clearly see the inward transport of the growing merger product, terminating with the dissolution of the cluster at $t \sim 7$ Myr. Subsequently, the black hole sinks more slowly to the center, eventually reaching $R = 0$ at $\sim 24$ Myr. The early ($t \leq 7$ Myr) portion of the figure is from an N-body simulation; the remainder is based on a simple analytical model of the Galactic potential (Sanders & Lowinger 1972).

The recent report by Maillard et al. (2004) of a possible IMBH near the Galactic center, at the heart of the swarm of stars known as IRS 13, provides the exciting prospect of confronting our models directly with reality. Portegies Zwart et al. (2005, in preparation) find that the observed properties of IRS 13 are completely consistent with theoretical expectations for a dense cluster remnant. They further estimate that the total IMBH mass resulting from runaway collisions in the cluster population which produced the inner bulge is consistent with the $\sim 3 \times 10^6 M_\odot$ supermassive black hole at the Galactic center.
Figure 1. Evolution of the Galactocentric radius $R$ (dotted line, left axis) and mass $M$ of the runaway merger product/IMBH (solid line, right axis) in a cluster of initial mass $4 \times 10^5 M_\odot$ moving in the Galactic field.

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