Quark, gluon and ghost anomalous dimensions at $O(1/N_f)$ in quantum chromodynamics.

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Abstract. By considering the scaling behaviour of various Feynman graphs at leading order in large $N_f$ at the non-trivial fixed point of the $d$-dimensional $\beta$-function of QCD we deduce the critical exponents corresponding to the quark, gluon and ghost anomalous dimensions as well as the anomalous dimensions of the quark-quark-gluon and ghost-ghost-gluon vertices in the Landau gauge. As the exponents encode all orders information on the perturbation series of the corresponding renormalization group functions we find agreement with the known three loop structure and, moreover, we provide new information at all subsequent orders.
One of the more difficult exercises in understanding the quantum field theories, such as quantum chromodynamics, which underpin the dynamics of the particles we observe in nature, is in gaining knowledge of their perturbative structure to very high orders. For example, following the early one and two loop work of [1, 2, 3] the renormalization group functions of QCD are only known to third order, [4, 5]. To a large degree one is hindered from pushing such calculations to a further order, which will be necessary due to the increase in experimental energies envisaged at LHC and the SSC, by the huge numbers of Feynman diagrams one would have to compute, a subclass of which possess an intricate integral structure. Indeed such calculations can only be tackled by ingenious algorithms which are executed on computer. Whilst the basic functions of the renormalization group equation are ordinarily calculated at successive orders in the perturbative coupling constant, which we will denote by $g$, it is not the only parameter with which one can compute with. For theories, such as QCD which possess $N_f$ flavours of quarks one can also expand the functions in powers of $1/N_f$ and compute various coefficients. The technique to achieve this was introduced in [6, 7] for the $O(N)$ bosonic $\sigma$ model and later extended to the $\sigma$ model on $CP(N)$, which is a $U(1)$ gauge theory, and then to QED, [9, 10]. Basically, it entails computing various critical exponents at the $d$-dimensional fixed point of the theory, defined as the non-trivial zero of the $\beta$-function, where the theory has a conformal symmetry, [3]. The exponents which emerge depend on the space-time dimension, $d$, and are computed order by order in powers of $1/N_f$. Further, as they are calculated at a fixed point the renormalization group equation takes a simpler form there to the extent that exponents have simple relations with the corresponding renormalization group equations, such as the wave function or vertex renormalization, at criticality, [11]. Moreover, one can undo such relations to deduce information on the perturbative structure at non-critical $g$. In particular the coefficients of each power of the coupling constant, at the order in large $N$ one is working to, can be determined and they correspond to those of the $\overline{MS}$ scheme. This can be understood from the fact that at the critical point there is a conformal symmetry and thus the exponents must correspond to a mass independent scheme. Another success of the method of [6, 7] is the possibility of pushing the calculations to $O(1/N_f^2)$, [10], which is a depth not possible by conventional large $N$ methods.

The purpose of this letter is to set up the analogous approach for QCD, extending the earlier QED work of [9]. The aim is to deduce non-trivial exponents for various fields in QCD and demonstrate their agreement with
the known three loop perturbative $\overline{\text{MS}}$ calculations, and then to deduce
information at higher orders. By non-trivial we mean determining exponents
which are not merely deduced by examining the analogous QED results and
multiplying those by the appropriate non-abelian group factors. Such non-
trivial results will be derived either from graphs involving ghost fields, the
triple gluon vertex or those which would otherwise be absent in QED by
Furry's theorem. In particular, we will deduce the anomalous dimensions
of the quark, gluon and ghost fields at leading order in large $N_f$ as well
as the vertex anomalous dimensions of the quark-quark-gluon (qqg) and
ghost-ghost-gluon (ggc) vertices, in the Landau gauge. Although these field
anomalous dimensions are gauge dependent they must be computed initially
before attempting to determine gauge independent quantities such as the $\beta$-
function which will be a considerably more involved computation and is our
long term goal. This is akin to the perturbative approach when the wave
function renormalization of each field must be determined first.

The version of the QCD lagrangian we use to compute exponents is

$$
L = -\frac{(F^a_{\mu\nu})^2}{4e^2} + i\bar{\psi}^i T^a_{\mu} \gamma^\mu \psi^i - 1 \frac{f^{abc}}{e^2} \partial_\mu A^a_\mu A^{\mu b} A^{\nu c} - \frac{1}{4e^2} f^{abc} f^{ade} A^b_\mu A^c_\nu A^{\mu d} A^{\nu e} - \frac{1}{2\xi e^2} (\partial_\mu A^{\mu a})^2 - \partial^\mu c^a \partial_\mu c^a + f^{abc} \partial_\mu c^a c^b A^{\mu c}
$$

(1)

where $\psi^i$ is the quark field, $1 \leq i \leq N_f$, $1 \leq I \leq N_c$, $A^a_\mu$ is the gluon field,
$1 \leq a \leq N_c^2 - 1$, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$, $c^a$ and $\bar{c}^a$ are the ghost fields, $\xi$
is the covariant gauge parameter, $T^a_{\mu}$ are the generators of the non-abelian
gauge group, $f^{abc}$ its structure constants and $e$ is the coupling constant. It
is important to note that we have absorbed a power of the coupling constant
into the definition of the gauge field so that the qqg vertex has the same
style of interaction as the electron photon vertex of QED, which was
primarily for applying the techniques of uniqueness of to computing
massless Feynman diagrams which arise at $O(1/N_f^2)$. To make more explicit
the notions already discussed we note that the fixed point where we will
concentrate our analysis is given by $\beta(g_c) = 0$, where $g_c$ is the non-trivial
zero of the $d = 4 - 2\epsilon$ dimensional $\beta$-function. It has been calculated in $\overline{\text{MS}}$
using dimensional regularization and in terms of the dimensionless coupling
constant $g = (e/2\pi)^2$, in the notation of, it is, $[1-5],$

$$
\beta(g) = (d - 4)g + \left[\frac{2}{3} T(R) N_f - \frac{11}{6} C_2(G)\right] g^2
$$

3
\[
\begin{align*}
+ \left[ \frac{1}{2} C_2(R) T(R) N_f + \frac{5}{6} C_2(G) T(R) N_f - \frac{17}{12} C_2^2(G) \right] g^3 \\
- \left[ \frac{11}{72} C_2(R) T^2(R) N_f^2 + \frac{79}{432} C_2(G) T^2(R) N_f^2 \\
+ \frac{1}{16} C_2^2(R) T(R) N_f - \frac{205}{288} C_2(R) C_2(G) T(R) N_f \\
- \frac{1415}{864} C_2^2(G) T(R) N_f + \frac{2857}{1728} C_2^3(G) \right] g^4 + O(g^5)
\end{align*}
\] (2)

from which it follows that

\[
g_c = \frac{3 \epsilon}{T(R) N_f} + \frac{1}{4 T^2(R) N_f^2} \left[ 33 C_2(G) \epsilon - (27 C_2(R) + 45 C_2(G)) \epsilon^2 \\
+ \left( \frac{99}{4} C_2(R) + \frac{237}{8} C_2(G) \right) \epsilon^3 + O(\epsilon^4) \right] + O \left( \frac{1}{N_f^3} \right)
\] (3)

The Casimirs for a general classical Lie group which appear in (2) and (3) are defined via

\[
\text{Tr}(T^a T^b) = T(R) \delta^{ab} , \quad T^a T^a = C_2(R) I , \quad f^{acd} f^{bde} = C_2(G) \delta^{ab}
\] (4)

and for \( SU(N_c) \), \( T(R) = \frac{1}{2} \), \( C_2(R) = (N_c^2 - 1)/2N_c \) and \( C_2(G) = N_c \).

At \( g_c \) the fields of (1) obey simple power law behaviour in analogy with ideas in statistical mechanics and therefore in momentum space the asymptotic scaling forms of the propagators of each field in the critical region as \( k^2 \to \infty \) are, \([8]\),

\[
\begin{align*}
\psi(k) & \sim \frac{A k}{(k^2)^{\mu-\alpha}} , \quad A_{\mu \nu} \sim \frac{B}{(k^2)^{\mu-\beta}} \left[ \eta_{\mu \nu} - \frac{k_\mu k_\nu}{k^2} \right] \\
c(k) & \sim \frac{C}{(k^2)^{\mu-\gamma}}
\end{align*}
\] (5)

The quantities \( A, B \) and \( C \) are the respective \( k \)-independent amplitudes of each field and \( \alpha, \beta \) and \( \gamma \) are their dimensions which are built out of the canonical (or classical) dimension and the anomalous contribution. The latter arises as a result of quantum corrections such as radiative corrections to the underlying Green’s functions. In particular, we define the exponents to be

\[
\alpha = \mu - 1 + \frac{1}{2} \eta , \quad \beta = 1 - \eta - \chi , \quad \gamma = \mu - 1 + \frac{1}{2} \eta_c
\] (6)
where $d = 2\mu$ and $\eta, \eta_c$ and $\chi$ are respectively the quark, ghost and qqg vertex anomalous dimensions. Each is $O(1/N_f)$ and depends on $\mu$ and $N_c$ through the Casimirs of (4) and we will determine their leading order large $N_f$ structure here. Also their relation to the conventional renormalization group functions are, \[ \eta \leftrightarrow \gamma_2(g_c), \eta + \chi \leftrightarrow \gamma_3(g_c) \]
\[ \chi \leftrightarrow \gamma_1(g_c), \eta_c \leftrightarrow \tilde{\gamma}_3(g_c) \] (7)

where $\gamma_2(g)$ is the quark wave function renormalization, $\gamma_1(g)$ is the quark gluon vertex renormalization function, $\gamma_3(g)$ is the gluon wave function renormalization and $\tilde{\gamma}_3(g)$ corresponds to the ghost wave function renormalization. The third relation of (6) arises from analysing the ghost kinetic term in (1) but it can be related to other anomalous dimensions by examining the ghost interaction term to obtain
\[ \eta_c = \eta + \chi - \chi_c \] (8)

where $\chi_c$ is the gcc vertex anomalous dimension being defined in a similar way to $\chi$. This result, which we will use later to deduce $\eta_c$ and demonstrate its agreement with perturbation theory, is nothing more than an expression of one of the Slavnov Taylor identities for QCD in exponent language. A similar observation concerning the QED Ward identity was made in \[ \text{[15]} \]. As we are working in a gauge theory we must choose a particular gauge to calculate within. As the covariant gauge parameter remains unrenormalized in the Landau gauge, we have made the choice $\xi = 0$, \[ \text{[9]} \]. This is important for comparing the structure of gauge dependent exponents with perturbation theory which we do later.

The method to solve for (7) involves examining the Dyson equations of various Green’s functions truncated to $O(1/N_f)$ in the critical region where the propagators of the Feynman graphs are replaced by the expressions (5). Although the analysis of the quark and gluon 2-point functions is trivial in the sense defined earlier, it is worth discussing it to illustrate the simplicity of the method and the Dyson equations are shown in fig. 1 where dotted lines correspond to quarks and solid lines to ghosts. We use dressed propagators since the effect of including a non-zero anomalous dimension in (5) is to reproduce the infinite chain of bubble graphs one would ordinarily have to consider, which was the original approach to examining QED in the large $N_f$ expansion, \[ \text{[17]} \]. Using (5) means fewer Feynman graphs have to be analysed. Several points concerning the graphs of fig. 1 ought to be noted.
First, there are no graphs in the gluon 2-point function involving ghost loops, triple or quartic gluon vertices since these are $O(1/N_f)$ with respect to the quark loop. (A similar set of graphs were analysed in a different context in \cite{17}.) Second, the quantities $\psi^{-1}$, $A_{\mu\nu}^{-1}$ and $c^{-1}$ denote the respective 2-point functions and their asymptotic scaling forms can be deduced from (5) by simple momentum space inversion where the gluon is inverted on the transverse subspace, \cite{9}, as

$$
\psi^{-1}(k) \sim \frac{k}{A(k^2)^{\alpha - \mu + 1}}, \quad A_{\mu\nu}^{-1}(k) \sim \frac{1}{B(k^2)^{\beta - \mu}} \left[ \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right],
$$

$$
c^{-1}(k) \sim \frac{1}{C(k^2)^{\gamma - \mu}}
$$

Thus the first two graphs of fig. 1 can be represented in the critical region by

$$
0 = 1 + \frac{(2\mu - 1)(\beta + 1 - \mu)\alpha}{(\mu - \beta)(\alpha + \beta)} a(\mu - \beta)a(\mu - \alpha)a(\alpha + \beta)C_2(R)z \quad (10)
$$

$$
0 = 1 + \frac{8\alpha^3 T(R)N_f}{(2\alpha + 1)} a^2(\mu - \alpha)a(2\alpha + 1)z \quad (11)
$$

where only the transverse part of the gluon equation is physically relevant, \cite{9}, tr1 = 4, $z = A^2B$ (and $y = BC^2$) and we have set $a(\alpha) = \Gamma(\mu - \alpha)/\Gamma(\alpha)$ for all $\alpha$. In (10) and (11) the only unknowns at leading order in $1/N_f$ are $\eta_1$ and $z_1$, where for example $\eta = \sum_{i=1}^{\infty} \eta_i/N_f$ with $\eta_i = \eta_i(\mu, N_c)$. Although it may appear that $\chi_1$ also arises through the factor $a(\alpha + \beta)$ in (10), in the momentum space approach to solving the Dyson equations in this critical point method, one sets $\chi_1 = 0$ at leading order so that $\beta = 1 - \eta$ here. This is partly to ensure consistency between the momentum and coordinate space solutions for $\eta_1$, which can easily be verified by examining the $x$-space QED solution, \cite{14}, or considering the same calculation in the toy $O(N)$ $\sigma$ model of \cite{18}. More importantly, though, since the method we are using relies very much on the conformal nature of the fixed point, the qgg 3-point vertex is in fact only conformal or unique, \cite{10}, at leading order when $\chi_1 = 0$, \cite{8, 9, 10}, which justifies this point. Further, the factors $(p^2)^{-\chi}$ have been omitted from the second terms of (10) and (11) partly for this reason but also because $\chi_1 = O(1/N_f)$ and so its $1/N_f$ expansion, ie $1 - (\chi_1 \ln p^2)/N_f$, means it will only be important for deducing $\eta_2$. The treatment of this can be seen, for example, in the QED work of \cite{14}. Thus eliminating $z_1$ between
\( \eta_1 = \frac{(2\mu - 1)(\mu - 2)\Gamma(2\mu)C_2(R)}{4\Gamma^2(\mu)\Gamma(\mu + 1)\Gamma(2 - \mu)T(R)} \) \hspace{1cm} (12)

which corresponds to the QED result of [9] aside from the non-abelian factor of \( C_2(R)/T(R) = 8/3 \) for QCD and agrees with the 3-loop perturbative expansion of \( \gamma_2(g_c) \).

To deduce the vertex anomalous dimensions one carries out a similar analysis using instead the truncated leading order large \( N_f \) graphs contributing to the qqq and gcc 3-point functions which are illustrated in figs 2 and 3 respectively. However, naively computing each individual graph with the leading order exponents one would discover that they are infinite and therefore need to be regularized. This is achieved by shifting the gluon exponent by an infinitesimal amount \( \Delta, \beta \to \beta - \Delta \), with \( \Delta \) playing a similar role to the \( \epsilon \) in dimensional regularization, [10]. The aim now is, following the ideas of [18], to compute each graph with the shifted exponent to observe that they have the following structure at \( O(1/N_f) \)

\[
\frac{P}{\Delta} + Q + R \ln p^2 + O(\Delta)
\] \hspace{1cm} (13)

where \( P, Q \) and \( R \) will depend on \( \mu \) and \( N_c \) and \( p \) is the external momentum flowing through the external quark or ghost fields. The simple pole of (13) is absorbed into the usual vertex counterterm, [18]. This leaves a \( \Delta \)-finite Green’s function but the scaling behaviour would be spoiled by the presence of the \( \ln p^2 \) term. However, general arguments, [8, 18], imply that the overall scaling behaviour of the qqq vertex, for instance, has to be \( (p^2)^{\chi/2} \) and therefore the \( \ln p^2 \) terms at each order in \( 1/N_f \) must re-sum to this form which allows one to determine an expression for \( \chi \).

Thus substituting (5) into the graphs of fig. 2 the first graph involves multiplying the QED result of [13] by the appropriate group factor, so that it contributes

\[
- \left[ C_2(R) - \frac{C_2(G)}{2} \right] \frac{\eta_1^0}{T(R)}
\] \hspace{1cm} (14)

to \( \chi_1 \) where \( \eta_1 \equiv C_2(R)\eta_1^0/T(R) \). Useful in computing the contribution from the two loop graphs of fig. 2 is the result

\[
\int_k k_\mu k_\nu \frac{k^2}{(k^2)^\alpha((k-p)^2)^\beta} = \frac{(\alpha + \beta - \mu - 2)a(\alpha - 1)a(\beta - 1)a(2\mu - \alpha - \beta + 2)}{2(\alpha - 1)(\beta - 1)(p^2)^{\alpha + \beta - \mu - 4}}
\times \left[ \eta_{\mu\nu} + \frac{2p_\mu p_\nu (\mu - \alpha + 1)(\alpha + \beta - \mu - 1)}{(\mu - \beta)} \right]
\] \hspace{1cm} (15)
valid for all $\alpha$ and $\beta$ and they and the second graph of fig. 2 are proportional to $C_2(G)$ and contribute

$$\left(\mu - 1\right)C_2(G)\eta_1^0 \over 2(\mu - 2)T(R)$$

(16)

to $\chi_1$. Thus, we have

$$\chi_1 = - \left[ C_2(R) + {C_2(G) \over 2(\mu - 2)} \right] \eta_1^0 \over T(R)$$

(17)

from which we deduce the gluon anomalous dimension as

$$\eta_1 + \chi_1 = - {C_2(G)\eta_1^0 \over 2(\mu - 2)T(R)}$$

(18)

To check that (17) and (18) are in agreement with perturbation theory we recall that the three loop $\overline{\text{MS}}$ result of in arbitrary covariant gauge, in our notation, is

$$\gamma_1(g) = \left[ \xi C_2(R) + {3 + \xi \over 4} C_2(G) \right] g^2 \over 2$$

$$+ \left[ {17 \over 2} C_2(R)C_2(G) - {3 \over 2} C_2^2(R) + {67 \over 24} C_2^2(G) - 2C_2(R)T(R)N_f \right.$$  

$$- {5 \over 6} C_2(G)T(R)N_f - \left(1 - \xi\right)C_2(G) \left( {5 \over 2} C_2(R) + {15 \over 16} C_2(G) \right)$$

$$+ {1 - \xi^2 \over 4} C_2(G) \left( C_2(R) + {1 \over 2} C_2(G) \right) \right] g^2 \over 8$$

$$+ \left[ {3 \over 2} C_2^3(R) + C_2^2(R)C_2(G) \left( 12\zeta(3) - {143 \over 4} \right) \right.$$  

$$+ C_2(R)C_2^2(G) \left( {10559 \over 144} - {15 \over 2}\zeta(3) \right) + {3 \over 4} \zeta(3) + {10703 \over 864} \right) C_2^3(G)$$

$$+ 3C_2^2(G)T(R)N_f + 2C_2(R)C_2(G)T(R)N_f \left( 6\zeta(3) - {853 \over 36} \right)$$

$$- C_2^2(G)T(R)N_f \left( 9\zeta(3) + {205 \over 108} \right) + {20 \over 9} C_2(R)T^2(R)N_f^2$$

$$- {35 \over 27} C_2(G)T^2(R)N_f^2 - \left(1 - \xi\right) \left( C_2(R)C_2^2(G) \left( {3 \over 2}\zeta(3) + {371 \over 32} \right) \right.$$  

$$+ {C_2^3(G) \over 32} \left( 18\zeta(3) + 127 \right) - C_2(G)T(R)N_f \left( {17 \over 4} C_2(R) + C_2(G) \right) \right)$$

8
\[
+ (1 - \xi)^2 C_2^2(G) \left( \frac{3}{8} \zeta(3) + \frac{69}{32} \right) + \frac{C_2(G)}{32} (3\zeta(3) + 27) \\
- (1 - \xi)^3 C_2^2(G) \left( \frac{5}{16} C_2(R) + \frac{7}{64} C_2(G) \right) \left[ g^3 \right] \frac{1}{32} \frac{C_2(G)}{T(R) N_f} \left[ g^3 \frac{9}{8} \left( \frac{1}{N_f^2} \right) \right] 
\]

which with (3) gives, in the Landau gauge,

\[
\gamma_1(g_c) = \frac{C_2(G)}{T(R) N_f} \left[ \frac{9}{8} \epsilon - \frac{15}{16} \epsilon^2 - \frac{35}{32} \epsilon^3 + O(\epsilon^4) \right] \\
- \frac{C_2(R)}{T(R) N_f} \left[ \frac{9}{4} \epsilon^2 - \frac{15}{8} \epsilon^3 + O(\epsilon^4) \right] + O \left( \frac{1}{N_f^2} \right) \tag{20} 
\]

Expanding (17) in powers of \( \epsilon = 2 - \mu \) to \( O(\epsilon^3) \), it is easy to see that it is in complete agreement with (20) which is a stringent check on our results. More importantly, the higher \( O(1/N_f) \) coefficients of (19) can now be deduced by extending this argument. If we write the leading order part of (19) as

\[
\gamma_1(g) = \frac{3}{8} C_2(G) g + \sum_{n=2}^{\infty} a_n (T(R) N_f)^{n-1} g^n \tag{21} 
\]

then (17) implies

\[
a_4 = \frac{1}{36} \left[ \frac{35}{36} C_2(R) + \left( \zeta(3) - \frac{83}{144} \right) C_2(G) \right] \\
a_5 = -\frac{1}{108} \left[ C_2(R) \left( 2\zeta(3) - \frac{83}{72} \right) - \frac{C_2(G)}{2} \left( 3\zeta(4) - \frac{5\zeta(3)}{3} - \frac{65}{48} \right) \right] \tag{22} 
\]

It is interesting to note that the first appearance of the \( \xi \) parameter in (19) and also in the other three loop renormalization group functions of (11) is at \( O(1/N_f^2) \). In fact this is true to all orders so that the leading order large \( N_f \) coefficients are gauge independent. Thus the exponents we have written down encode gauge independent information, at all orders, of part of the renormalization group functions.

Repeating the analysis for the gcc vertex graphs of fig. 3, one finds that each is \( \Delta \)-finite and therefore at \( O(1/N_f) \)

\[
\chi_{c1} = 0 \tag{23} 
\]

which implies from (8) that

\[
\eta_{c1} = -\frac{C_2(G) \eta_1^0}{2(\mu - 2) T(R)} \tag{24} 
\]
Expanding (24) in powers of $\epsilon$ as before and comparing with the two loop arbitrary gauge calculation of [3] one again finds agreement. The three loop ghost anomalous dimension has been calculated in [4]. Although that calculation was carried out in the Feynman gauge the three loop $O(1/N_f)$ coefficient is in agreement with the $O(\epsilon^3)$ term of the expansion of (24) in the Landau gauge and our observation concerning the non-appearance of $\xi$ at third order at $O(1/N_f)$ in previous renormalization group functions also hold at this order.

Finally, it is worth recording the leading order form of a gauge independent exponent which encodes all orders coefficients on the perturbative structure of the renormalization of the quark mass, $\gamma_m(g)$. It is trivial in the sense we mentioned earlier and is deduced by computing the anomalous dimension of the composite operator $\bar{\psi}\psi$ within the Green’s function $\langle \psi [\bar{\psi}\psi] \bar{\psi} \rangle$. This was discussed originally in [16] for QED and including the structure which arises from the non-abelian nature of (1), we find that the exponent is

$$\gamma_m(g_c) = -\frac{2C_2(R)\eta_1^0}{(\mu - 2)T(R)N_f}$$

and its $\epsilon$-expansion is in agreement with the $\overline{\text{MS}}$ perturbation series of [19].

We conclude by remarking that we have provided the first stage in the analysis of QCD in the large $N_f$ expansion where the aim is to deduce the coefficients of the perturbative renormalization group functions in order to provide new results as well as checking existing results. It ought now to be possible to deduce the $\beta$-function in the same approximation as well as building on the $O(1/N_f^2)$ QED work of [10] to gain a deeper insight into the theory.

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**Figure Captions.**

**Fig. 1.** Leading order skeleton Dyson equations where dotted lines are quarks and solid lines ghosts.

**Fig. 2.** Leading order graphs for $A^a_\mu \bar{\psi} \gamma^\mu T^a \psi$.

**Fig. 3.** Leading order graphs for $f^{abc} A^a_\mu \partial^\mu \bar{c}^b c^c$. 