Dynamics of a heavy-duty vehicle in a driving cycle

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Abstract. The analysis of a heavy-duty vehicle moving in a driving cycle is presented. The different forces in the equation of motion are explained. In a driving cycle a target speed of the vehicle is prescribed as a function of time and then the revolutions per minute and the torque of the engine necessary to move the vehicle close to the prescribed speed are calculated. The driving cycle used in this paper is part of one of various cycles used by the Greenhouse Gas Emissions Model developed by the United States Environmental Protection Agency to certify the fuel consumption of heavy duty vehicles. The results using the software of that agency and one developed by the authors are reported for a vehicle with a 10-gear transmission, a 455 HP engine, three axes tractor and two axes trailer which have a total mass of 12742 kilograms and transport a load of 17236 kilograms.

1. Introduction

Due to the carbon dioxide emissions in the production, transformation and use of energy, energy efficiency regulations for heavy duty vehicles have been enacted in some countries of the world in order to reduce the impact of those emissions to climate change [1]. In Mexico, there is not yet an energy efficiency regulation for heavy duty vehicles despite the urgency to implement it. Despite that, it is necessary to start constructing or adapting the tools to certify its compliance after such Mexican regulation is promulgated. The energy efficiency regulations require that the heavy duty vehicles comply with certain fuel consumption limits [1].

The fuel consumption and the carbon dioxide emissions of a vehicle depend on its engine, transmission, final drives, tires, aerodynamic shape, anti-contaminant devices, electronic devices and many other components. The fuel consumption of the heavy-duty vehicles is evaluated mainly using software that simulates their motion. Among the mathematical models to evaluate the fuel consumption and carbon dioxide emissions are the Greenhouse Gas Emissions Model (GEM) [1][2], the European Vehicle Energy Consumption Calculation Tool (VECTO) [3] and UAMmero, which is being developed in Mexico [4]. In this paper the vehicle dynamics is detailed, which is the theoretical basis for the evaluation of the fuel consumption and of the energy efficiency of the heaviest duty vehicles which are the tractors that tow trailers. The procedure used to obtain the results presented in this paper is the same one set in UAMmero. In GEM as well as in VECTO the fuel consumption of a heavy-duty vehicle is obtained assuming that it moves in a driving cycle, that is, a target speed for the vehicle as a function of time is prescribed [4]. In

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UAMmero it is modeled that the vehicle moves in a driving cycle but it has also an option to simulate that the vehicle travels in some Mexican highways [4]. The modeling of the motion in driving cycles simulates the typical road trips of the vehicles along the regions where they travel. Therefore, the driving cycles used for Japan, United States of America and Europe are different. In the simulation, a vehicle will not follow the prescribed speed if it carries a heavy load because the engine will not be able to provide the necessary torque [4]; neither if the vehicle moves along a road with a steep slope; in this case the vehicle speed must be reduced. In the simulation cases in which a vehicle moves in a driving cycle the slope of the road is considered constant during the trip and that the vehicle is moving in a straight line.

In this paper the vehicle dynamics is detailed for a vehicle which moves in a driving cycle. The driving cycle considered is one in which the vehicle starts with zero speed, then increases it to the cruise speed (104.60736 km/h), maintains this speed for some time and finally deaccelerates up to zero speed. The mathematical operations are performed by UAMmero. In previous works by the authors, results for other cycles and for simulations of heavy-duty vehicles along Mexican roads have been presented [4], although the vehicle dynamics has never been presented as it is being done in this paper.

2. Vehicle dynamics

The vehicle dynamics is described in equation (1) [5].

\[(M + m_t)a = \frac{T_{e}R_{eff}}{T_{w}} - F_{d}F_{p}F_{r}\]  

(1)

In equation (1), \(F_d\), is the aerodynamics force, \(F_p\) is the force due to the slope of the road, \(F_r\) is the force due to the friction of the tires with the pavement of the road, \(M\) is the mass of the vehicle, \(m_t\) is the inertial mass, \(T_w\) is the magnitude of the linear acceleration of the vehicle, \(T_e\) is the radius of the tires and is \(T_e\) the torque provided by the engine. \(R_{eff}\) accounts for the combined ratios of the gears of the transmission and the final drives of the vehicle. \(R_{eff}\) is more complex for the heaviest vehicles and the vehicles with many tires and axles. \(R_{eff}\) and \(m_t\) are variables depending on time which change as the vehicle increases its speed and different gears of the transmissions are used (first gear, second gear, etcetera). The different gears of the transmission have different inertia moments and ratios. In this paper a vehicle with a 10-gear transmission and a 455 HP engine is considered. The value of the parameters of the heavy-duty vehicle of 5 axis including those of the transmission and the engine are found in the 2010 GEM User’s Guide [2] and in the GEM software used for certification during the first phase of the United States regulation.

The forces \(F_d\), \(F_p\) and \(F_r\), are written in the equations 2, 3 and 4.

\[F_d = \frac{1}{2}C_{air}A_L\rho_a V^2\]  

(2)

In equation (2), \(C_{air} = 0.69\), is the coefficient of aerodynamic drag, \(\rho_a = 1.1071 \frac{kg}{m^3}\), is the air density, \(A_L = 9.8 m^2\), is the effective frontal area of the vehicle and \(V\) is the speed or magnitude of the velocity of the vehicle.

\[F_p = M g \sin(\theta)\]  

(3)

\[F_r = C_r M g \cos(\theta)\]  

(4)

The in equations (1), (3) and (4), \(M\) is the sum of the masses of the tractor, trailer and cargo (including the masses of the transmission, engine, differentials, axis and fuel). In this example \(M = 31978\). In the equations (3) and (4), \(g = 9.8066 m/s^2\), is the acceleration of gravity. \(\theta\) is the angle of inclination of the road or street with respect to the horizontal, along which the vehicle moves. In the case in which the vehicle moves in a driving cycle the inclination angle is considered constant during all the time of the simulation.
In the example of this paper the inclination angle is considered zero. In equation (4), $C_r = 0.007205$, is the average of the tires’ coefficients of rolling resistance [2].

The inertial mass, $m_t$, depends on the parts which rotate, on the ratios of the gears of the transmission, on the ratios of the final drives and on the radii of the tires. In this paper in order to compare results with those out of GEM, the inertial mass model used in the GEM software is taken for the calculations carried out in this work. The values of the variables in the equations 1, 2, 3 and 4 are obtained from the reference [2] and from the source code of GEM.

Once that the forces are known, the acceleration, speed and travelled distance of the vehicle are obtained at any instant.

3. Calculation of the rpm and torque of the engine

In this section, the steps to obtain the revolutions per minute (rpm) and the torque of the engine to move the vehicle close to the driving cycle is explained.

The driving cycle is shown in figure 1. From the 800 seconds driving cycle the target speed of the vehicle at any time is obtained. The objective speed is reduced if the torque needed to reach that speed is over the maximum torque which the engine can provide; in this case the speed is diminished to the maximum speed that the maximum torque can provide.

![Figure 1. Target speed versus time.](image)

The speed of the vehicle, $V$, is the speed of any tire in the linear motion of the vehicle. From the linear speed of a tire the rotating frequency or revolutions per minute of the wheel is obtained.

$$\omega_w = \frac{V}{r_w}$$  \hspace{1cm} (5)

In Eq. 5, is the radius of one tire. In this paper the radii of all the tires are considered the same and equal to 0.489 m.

From the frequency of the tires, the frequency or rpm of the engine’s crankshaft is obtained, as it is shown in Eq.6.

$$\omega_e = R_{eff} \omega_w$$  \hspace{1cm} (6)

In this work the gear changes of the transmission are considered to be instantaneous; that is, the clutch is always attached to the engine.
From the equations (5) and (6), equation (7) is obtained.

$$V = r_w \omega_w = \frac{r_w}{R_{eff}} \omega_e$$  \hspace{1cm} (7)

From equation (7), figure 2 is obtained. In figure 2, the horizontal lines correspond to the instantaneous gear changes in the transmission (from first to second gear, from second gear to third gear, etcetera, up to the change from the ninth gear to the tenth gear). From equation (7) (or from figure 2) the gear of the transmission being used is obtained at any time because the speed of the vehicle and the speed at which the gear changes occur are known. The rpm of the engine is also obtained from equation (7) (or from figure 2).

In figure 2, the ordinate axis corresponds to the speed of the vehicle and the abscise axis to the rpm of the engine. From equation (7), the slopes of the straight lines (those different from zero) in figure 2, correspond to the values of $\frac{r_w}{R_{eff}}$.

The straight line at the bottom in figure 2 corresponds to the speed at which the vehicle moves when the first gear of the transmission is used. As the vehicle increases its speed there is an instantaneous gear change in the transmission (to the second gear) which in figure 2 is represented by the lowest parallel line to the abscise axis.

![Figure 2. Vehicle speed versus rpm.](image)

As the vehicle further increases its speed the other gears are used. The assumption of instantaneous changes of gears is a good approximation because those changes are relatively few in a highway where the heaviest duty vehicles are used the most; and the time in which the changes occur are very short compared to the duration of the trips. In this model, the gears are used progressively (beginning with the first gear, followed by the second gear, and so on) without jumping or skipping any of them. At low speed the vehicle travels using the lowest gear of the transmission (the first gear), while at cruise speed (65 miles per hour or 104.60736 km/h) the vehicle uses the highest gear of the transmission (the tenth gear).

When the vehicle reduces its speed from 65 miles per hour to zero, the gears of the transmission are used progressively from the 10th gear to the 1st gear; in this case the changes of gears of the transmission occur at different values of the speed than those used when the vehicle accelerates from zero speed to cruise speed.
The torque needed from the motor to move the vehicle is obtained according to equation (8).

\[ T_e = \frac{(F+Ma)r_w}{Reff} \]  \hspace{1cm} (8)

Where \( F \) is defined in equation (9);

\[ F = F_r + F_a + F_p + m_i{a} \]  \hspace{1cm} (9)

The inertial mass in equations (1), (8) and (9) is defined in equation 10;

\[ m_i = \frac{I_{eff}}{r_w} \]  \hspace{1cm} (10)

In equation (10), is the effective inertia moment which includes the inertia moments of the transmission, the clutch and the engine as well as the ratios of the transmission and the final drives and all axle inertia [2]. A figure of as a function of time is shown below.

From the torque of the vehicle, the power needed to move the vehicle is obtained according to equation (11).

\[ P_e = 0.745699872 \left( T_e{\omega_e} \right) \]  \hspace{1cm} (11)

4. Comparison of results between GEM and UAMmero

GEM is written in the C programming language and uses the commercial MATLAB package to solve equation (1). UAMmero is also written in the C programming language but it does not use MATLAB. The results for \( m_i, F, T_e \) and \( P_e \) as a function of time obtained from GEM and UAMmero are shown in figures 3, 4, 5 and 6 for \( \theta = 0 \), cargo of 17236 kilograms and a total mass of the vehicle of 31978 kilograms.

The results using UAMmero are shown with solid lines and those from GEM with point lines. The results are closed except for the torque during the changes of gear of the transmission. The results from GEM present spikes in the torque during the changes of the transmission because a traditional Proporcional Integral Diferencial control mechanism (PID) is used in that model during those changes. UAMmero already has a PID to change the gears in the transmission smoothly but it has not been used to obtained the results presented in this paper.

![Figure 3. Comparison of inertial mass.](image1)

![Figure 4. Comparison of force.](image2)
5. Conclusions
UAMmero is a software useful to calculate the rpm and the torque of the engine necessary to move a vehicle in a driving cycle and their results are close to those obtained using the GEM of the United States of America Environmental Protection Agency for the example studied.

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