Effect of an upper layer of viscous liquid on breaking surface gravity waves

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Abstract. The effect of the upper layer of water-immiscible oil on the process of regularization of breaking standing gravity waves on the free surface of a two-layer system in a rectangular vessel has been experimentally investigated for the first time. A comparison is made with the case of standing gravity waves on the free surface of homogeneous liquids (water and seed oil).

1. Introduction
The dynamics of a limited volume of liquid with a free surface constitute one of the most complex and practically important problems of fundamental and applied hydrodynamics [1]. Special difficulties are associated with taking into account an important property of real liquids— their viscosity. The main results in the theory of wave motions of a viscous fluid are obtained for the case of low viscosity in the approximation of the periodic boundary layer. In experimental hydrodynamics, the influence of viscosity is quantitatively estimated by the wave damping coefficient and frequency shift [2].

The suppression of intense liquid sloshing is the most important applied problem, solved by using different types of dampers [1]. Our previous experiments [3, 4] showed the significant effect of vertical baffles on violent water sloshing. Recently, the effect of viscosity on the regularization of collapsing Faraday waves was discovered and investigated [5, 6].

It is known that a thin layer of oil leads to the stilling of capillary ripples on the surface of the sea, e.g. [7–9]. As we know, the effect of the oil layer on gravity waves has not been studied.

The object of our experimental study is the influence of the upper layer of water-immiscible oil on the regularization of surface gravity waves excited by the parametric resonance. In experiments, a layer of oil of finite thickness was placed on the free surface of water and the effect of regularization of a breaking standing gravity wave was estimated. The two-layer system oscillates like a homogeneous fluid, which, in terms of [10], refers to the barotropic mode of oscillations.

2. Experimental procedure
The effect of the upper layer of water-immiscible oil on liquid sloshing was studied for the second mode (n = 2) of standing gravity waves on a free surface of two-layer system – (seed oil – water) or (paraffin oil– water) – in a rectangular acrylic vessel with a length of L = 50 cm and a width of W = 4 cm.

The vessel was placed on the platform of an electromechanical shaking table providing harmonic oscillations in a vertical direction. Two-dimensional wave motions were investigated in the regime of the main Faraday resonance [11], when the vessel oscillation frequency Ω is twice the frequency ω.
of excited waves ($\Omega \sim 2\omega$). With fixed vessel amplitude $s = 0.75$ cm the variations of $\Omega$ provided a change in the wave steepness $\Gamma = H/\lambda$ within 0.005 – 0.5 at a wavelength of $\lambda = 50$ cm, where $H$ is the wave height. The range of overload changes $\varepsilon = s\Omega^2 / g$ was estimated at $0.2 – 0.4$. We note that the value $s = 0.75$ cm was chosen in order to compare the results of these experiments with the data [5, 6].

In the experiments, immiscible liquids were used to obtain a two-layer system: paraffin oil – water and seed oil – water. The total depth of the two-layer system was unchanged ($h = 15$ cm), and the thickness $h_1$ of the upper layer ranged from 0.25 to 1.5 cm (figure 1). Seed oil and paraffin oil as water-immiscible liquids were chosen to evaluate the effect of their density and viscosity, as well as various interfacial tensions on breaking surface gravity waves.

![Figure 1](image)

**Figure 1.** (a) Two-layer system in a rectangular vessel. (b) Parametric excitation of a standing surface gravity wave (second mode). Here $h_1$ – upper layer thickness (varied from 0.25 to 1.5 cm with a constant total depth of immiscible liquids ($h = 15$ cm); $H$ - wave height

Degassed tap water, seed oil and paraffin oil were used in the experiments as working liquids; the corresponding values of density $\rho$, kinematic viscosity $\nu$ and surface tension $\sigma$ are given in table 1. Note that the table shows the values of surface tension for seed oil and paraffin at the interface with air and water.

| Liquid        | $\rho$ (g/cm$^3$) | $\nu$ (cSt) | $\sigma$ (dynes/cm) | $\sigma$ (dynes/cm) |
|---------------|-------------------|-------------|---------------------|---------------------|
|               |                   |             | (in air)            | (in water)          |
| Water         | 1.00              | 1           | 73                  | –                   |
| Seed oil      | 0.93              | 61          | 40                  | 24                  |
| Paraffin oil  | 0.78              | 1.3         | 27                  | 42                  |

The wave pattern was registered with digital cameras DIMAGE Z2 and Canon PowerShot SX50HS (frame rates were 30 and 120 fps) in a moving reference system rigidly connected with the vessel. The resolution of the video images was 0.15 mm/pixel. The subsequent processing of video frames was carried out by the software ImageJ. All experiments were conducted at room temperature 21–22°C.

To estimate the dissipative properties of a two-layer system, we used the logarithmic decrement (the rate of decay), defined as $\delta = \ln H_m / H_{m+1}$ where $H_m$ and $H_{m+1}$ are wave heights taken in wave
period and estimated from the video materials of wave damping after removal of the excitation – shaking table turned off.

To interpret some experimental data, we used the theoretical model [11, 12], in which the asymptotic solution of the nonlinear problem of Faraday surface waves was constructed in Lagrange variables by the Krylov-Bogolyubov method.

3. Results and discussion
In the case of water, as shown in [11], the observed Faraday waves can be divided into three categories: regular, irregular and breaking waves.

Waves whose profile has a temporal periodicity and spatial symmetry relative to the vertical planes carried through the wave antinodes are regular Faraday waves. It was established that the maximum steepness of the regular waves on the water surface is \( \Gamma_m = H_m / \lambda \sim 0.22 \). Waves that have temporal and spatial symmetry violated, but the volume of the oscillating fluid retains connectivity, are classified as irregular Faraday waves. Finally, waves in which either individual droplets of liquid or jets are torn from the free surface are attributed to the breaking Faraday waves – see figure 2a.

![Figure 2](image)

**Figure 2.** Envelopes of the free surface in the case of (a) water and 1 cm – layer of paraffin oil (b) and seed oil (c) at a frequency of \( \Omega = 21.44 \text{s}^{-1} \)

Experiments have shown that a paraffin oil layer (\( h_1 = 0.25 – 1.5 \text{ cm} \)) practically does not change the nature of oscillations of a two-layer system in the barotropic mode; we observed wave motion similar to water – figure 2b. Breaking waves on the free surface of a two-layer fluid is similar to splashes on water. Note that the maximum steepness of the regular waves for paraffin – water system was \( \Gamma_m \sim 0.22 \).

However, placing a layer of seed oil on top of water drastically changes the wave pattern – we observe regularization of collapsing waves (figure 2c). The wave shown in the photo has a height of \( H = 12.6 \text{ cm} \), there are no signs of breaking of the standing gravity wave.

In experiments, the resonant dependences of the steady-state wave height \( H = H(\Omega) \) on the vessel vertical oscillation frequency \( \Omega \) were used as the integral wave characteristics of barotropic mode – figure 3.
Figure 3. Resonance dependences of $H(\Omega)$ in the case of a two-layer fluid (seed oil-water): 1-5 – $h_1 = 0.25, 0.5, 0.75, 1$ и 1.5 cm; 6 – water; 7 – seed oil.

It can be seen that the frequency range of the resonance dependences for the seed oil thicknesses used in the experiment (1-5) does not differ from water (6). The resonance dependence of the second mode on the surface of the seed oil (7) is shifted to low frequencies due to a 60-fold increase in oil viscosity compared to water. The second mode on the free surface of the kerosene-water system for all $h_1$ used in the experiment had resonance dependences identical to the case of water.

Using seed oil as a highly viscous fluid in a two-layer system significantly increases limiting steepness of a regular wave and the logarithmic decrement, as shown in figure 4.

Figure 4. Dependencies of limiting steepness $\Gamma = H/h$ (a) and the wave decrement $\delta$ (b) of the surface gravity wave on upper layer thickness $h_1$ at the total depth of a two-layer liquid of $h = 15$ cm: 1 – water; 2 – seed oil–water; 3 – paraffin oil–water.

In the case of seed oil, the wave steepness increases with increasing thickness ($h_1$) of the upper layer and reaches a value of 0.33, which is 1.5 times higher than the steepness of the regular wave on the surface of the water or the kerosene-water system figure 4a.

Analysis of the decay process of the second wave mode showed that in the case of seed oil, the decrement increases almost by an order of magnitude – figure 4b. The decrement of the wave on the free surface of the kerosene-water system is almost independent of the thickness of the upper layer and is equal to the decrement of water waves.

A detailed analysis of the video recording data made it possible, in the case of water, to identify short-scale disturbances on the wave profiles; their characteristic dimension did not exceed 6 cm –
These disturbances determine the initiation, development, and collapse of a cavity with the subsequent jet launch from the wave crest.

A similar picture is observed on the free surface of the kerosene-water system – figure 6. One can see that the upper layer of kerosene does not interfere with the wave breakdown process: the formation and collapse of a cavity with subsequent ejection and destruction of the jet can be traced.

In the case of seed oil – water system, the wave (excited at the same frequency $\Omega = 21.44s^{-1}$) is regular – figure 7. At the height of $H_o = 16.6$ cm, the wave profile is smooth, and there are no signs of wave breaking.

The only difference between the wave patterns in figures 6 and 7 is the use of paraffin and seed oil as the top fluid in a two-layer system. These fluids differ in density, viscosity and interfacial (with water) tension – see Table 1. Since the seed oil provided an increase in the limiting steepness of the wave, its dissipation and regularization of wave motion, below we will focus on the seed oil-water system.

**Figure 5.** Sequence of snapshots of water free surface at the stage of transition from the wave trough to wave crest at vessel center (half wave period); $\Omega = 21.44s^{-1}$, video ~ 120 fps. The time (in seconds) is indicated in the upper left corner of each frame.
Figure 6. Sequence of snapshots of free surface of two-layer system paraffin oil – water \((h_1 = 1 \text{ cm})\) during half wave period at \(\Omega = 21.44 \text{s}^{-1}\); video – 120 fps

Figure 7. Sequence of snapshots of free surface of two-layer system seed oil – water \((h_1 = 1 \text{ cm})\)
First of all, let us look at the dynamics of the upper layer with increasing wave height of $H$—figure 8. This can be done using the video materials of the process of establishing stationary oscillations of a two-layer fluid at the frequency of oscillation of the vessel $\Omega = 22.60 \text{ s}^{-1}$. The initial thickness of the layer of seed oil was $h = 1 \text{ cm}$.

With a wave height of $H = 2.1 \text{ cm}$, the wave profile is linear and the thickness of the upper layer is the same along the entire length of the vessel—figure 8a. Increasing height up to $H = 6.7 \text{ cm}$ leads to a thickening of the layer in crest and in troughs during its thinning in the nodal areas—figure 8b. At $H = 9.4$ and $10.1 \text{ cm}$ there is a flow of the entire upper fluid in the region of the wave crest and trough—figure 8c and d.

Using the nonlinear surface Faraday wave model [12, 13], we can plot not only the free surface profile ($b = 0$), but also determine the coordinates $(x, y)$ of the fluid particles inside a layer of a given depth $h$.

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**Figure 8.** The configuration of the upper layer (vegetable oil) at different values of wave height $H$: 

- $a - d - H = 2.1, 6.7, 9.4$ and $10.1 \text{ cm}$  

**Figure 9.** Calculated wave profiles showing deformation of the near-surface layer (1 cm) of a homogeneous liquid (water) at different wave phases $\psi = \pi, \pi / 2, 0$ ($a - c$); $b = 0$, -0.5 and -1 cm; $H = 12.7 \text{ cm}$ $\Omega = 21.74 \text{ s}^{-1}$
Let us define $b$ – isolines as lines of identical values of the initial vertical coordinate $b$ of fluid particles. Isolines $b = 0, -0.5$ and $-1$ cm for different values of the wave phase are shown in figure 9. Initially, the distance between these three lines is 0.5 cm, with each line being set by 50 points spaced 1 cm apart. Due to the wave nonlinearity (asymmetry of the wave profile), we observe a contraction of points on the wave crest and their rarefaction in its trough. This causes a change in the distance between $b$ – isolines – an increase in the crest and a decrease in the trough. This most likely explains the change in the thickness of the upper layer of an immiscible fluid in the case of barotropic waves if we consider the isolines $b = 0$ and $-1$ cm as the boundaries of the upper fluid.

If in figure 8 there is an interphase boundary between two immiscible liquids, then for waves of greater height ($H > 13$ cm), an oil-in-water emulsion is formed – see figure 10. In this case, we have a kinetically unstable emulsion, which is characterized by gravitational floating (creaming) of particles of the dispersed phase (seed oil) and the formation of a polydisperse layer of densely packed oil droplets.

Emulsification of oil in water requires a lot of energy. In the process of emulsification, the dispersion of the oil into droplets with a radius from 0.01 to 0.5 cm occurs. This process is counteracted by the surface tension force, which resists the formation of a new interface. Thus, the formation of an emulsion is that additional dissipative mechanism that suppresses small-scale perturbations on the free surface and provides the regularization of breaking waves.

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\begin{align*}
    x &= a - H \frac{\cosh b + h}{2 \sinh h} \sin ka \cos \psi + H^2 k \frac{\sin 2ka}{32 \sin^2 kh} (1 + \cos 2\psi) - \frac{3}{64} H^2 k \frac{\cosh b + h}{\sin^4 kh} \sin 2ka \cos 2\psi \\
    + & \frac{1}{16} H^2 k \frac{\cosh b + h}{\sin^2 2kh} \sin 2ka, \\
    y &= b + H \frac{\sinh b + h}{2 \sinh h} \cos ka \cos \psi + H^2 k \frac{\sinh 2(k + h)}{32 \sinh^2 kh} (1 + \cos 2\psi) + \frac{3}{64} H^2 k \frac{\sinh b + h}{\sin^4 kh} \cos 2ka \cos 2\psi \\
    - & \frac{1}{16} H^2 k \frac{\sinh 2(k + h)}{\sin^2 2kh} \cos 2ka, \\
    \psi &= \Omega t / 2, \quad k = 2\pi / \lambda, \quad a \in [0, L], \quad b \in [-h, 0]
\end{align*}
\]
We suggest the following possible mechanism for the formation of O/W emulsion. In the case of the second wave mode at the stage of crest formation, the central part of the liquid moves upwards – figure 11. At this time interval, the inertia force acting on the individual oil droplets is directed downwards and provides the separation and movement of the droplet into the second phase (water).

Let us estimate the threshold wave height necessary to initiate the emulsification process. In order for a single droplet of the upper liquid (dispersed phase) to pass through the interface into the lower liquid (dispersive phase), it must overcome the buoyancy force $\frac{4\pi R^3 \Delta \rho g}{3}$ and the interfacial tension $2\pi \sigma R$. This is possible only under the action of the inertial force $\frac{(4\pi R^3/3)\rho_1 (H/2) \omega^2}{2}$ determined by the moving interface. The threshold wave height is determined from the following balance equation for these three forces

$$H = \frac{2}{\omega^2} \left( \frac{3\sigma}{2\rho_1 R^2} + \frac{\Delta \rho}{\rho_1} g \right)$$

Here, $\omega$ is the wave frequency, $R$ is the drop radius, $\rho_{1,2}$ - oil and water density, $\Delta \rho = \rho_2 - \rho_1$, $\sigma$ is the O/W interfacial tension, and $g$ is the gravity acceleration. The presented estimate of the threshold wave height is very approximate, since it does not take into account the droplet resistance force. We suppose that this force is proportional to the droplet relative velocity and is equal to zero at the maximum acceleration of the interface $H \omega^2 / 2$. 

**Figure 11.** Emulsification due to inertial forces at the stage of crest formation; $H = 13.6$ cm $\Omega = 21.74$ s$^{-1}$. The numbers in the upper left corner are time in sec
The dependence of the threshold wave height depending on the radius of the emulsified droplet is shown in figure 12 for seed oil (1) and paraffin (2) as the upper fluid of the oil-water system. For seed oil (1), the formation of droplets with a radius of 0.2 to 0.4 cm is possible at a wave height of 4 – 13 cm, which corresponds to the experimental data. In the case of paraffin (2), emulsification is not possible, which is again confirmed by experiment. This is due to the large density difference between paraffin oil and water, as well as high interfacial tension between them (see Table 1).

4. Concluding remarks

We have presented novel experimental results on effects of the upper layer of viscous liquid on the process of regularization of breaking standing gravity waves on the free surface of a two-layer system.

It is shown that the use of seed oil as the upper layer significantly changes the dynamics of the wave mode – wave regularization is observed with complete suppression of the breaking mechanisms. The effects observed in experiments are associated with the formation of creamed emulsion layer, which provides additional dissipation of wave energy.

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