Design of a 1-3 smart viscoelastic composite layer for augmented constrained layer damping treatment of plates

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Abstract: In the present work, damping performance of plate with constrained 1-3 smart viscoelastic composite (SVC) layer is studied. The 1-3 SVC is formed through the inclusion of piezoelectric patches within pure viscoelastic material (VEM) phase. The overall structural plate is modelled using finite element method. First, the characteristics of in-plane normal strain and transverse shear strain of overall structural plate are examined. Then, in order to study the dynamic performance, the optimal dimensions of present 1-3 SVC strips are determined using direct search method. There is significant improvement in damping with respect to pure VEM. Both extensional and shear counterparts of modal loss factor increase. These results express that 1-3 SVC may be a useful damping layer in the constrained layer damping treatment of overall structural plates.

1. INTRODUCTION

Vibration is usually an unwanted phenomenon in engineering structures because of its harmful consequences, including the damaging effect. Therefore, vibration control is one of the significant aspects in the design of engineering structure. In the majority of these cases, vibration control is mostly achieved through the modification of system parameters through integrating passive damping materials like viscoelastic materials (VEMs) and/or active damping materials like piezoelectric materials to the host structures. In this concern, constrained layer damping (CLD) treatment is the most popular damping treatment that is used to control the structural vibration. In this treatment, VEM layer is constrained between the surface of host structure and constraining layer. It is also named passive constrained layer damping (PCLD) when a passive constraining layer is used and an active constrained layer damping (ACLD) treatment when active piezoelectric constraining layer is used. Many works have been done for the augmentation of damping using PCLD treatment, as reported in the literature [1–3]. Some popular PCLD configurations using stand-off-layer[4], multi-layered [5], and partial CLD layer[6] were also studied. Besides PCLD treatment, ACLD treatment was also utilized for the enhancement of damping in structural vibration [7–9]. Some improved actively controlled structural damping configurations, ACLD using stand-off-layer [10], self-sensing active constrained layer (SACL) [11], and pre-compressed layer [12] were also studied. It has been observed that active-passive damping can also be enhanced by using active and passive patches in an optimum manner [13]. It may be noticed from the aforesaid literature that enhancement of damping in CLD (both PCLD and ACLD) treatment arises mainly because of shear deformation only.

Recently, 1-3 viscoelastic composite (VEC) was introduced by Kumar and Panda[14], for the improvement of damping in CLD treatment of the overall structural beam. The graphite blocks are embedded in pure VEM phase. Therefore, in-plane extension/compression deformation and transverse...
shear deformation in pure VEM enhance the damping of overall structural beam. Thus present work is also in the same direction; a 1-3 smart viscoelastic composite (SVC) layer is used where piezoelectric patches are embedded in pure VEM. So both in-plane extension/compression deformation and transverse shear deformation in pure VEM may enhance the overall damping of structural plate. The schematic diagram of 1-3 SVC is illustrated in figure 1(a). Initially, the characteristics of in-plane normal strain and transverse shear strain of overall structural plate are examined. Then, in order to study the dynamic performance, the optimal dimensions of present 1-3 SVC strips are determined using a direct search method. A comparative study is carried out to assess the damping performance of overall structural plate. Both extensional and shear counterparts \((\eta_e, \eta_s)\) of modal loss factor \((\eta)\) of plate with constrained 1-3 SVC strips are evaluated and compared with pure VEM layer. The frequency response and corresponding maximum control-voltage of overall structural plate are also appraised in order to comprehend the fruitfulness of present 1-3 SVC.

![Figure 1. Schematic diagrams of the (a) 1-3 SVC; (b) plate with constrained 1-3 SVC strips.](image)

2. FINITE ELEMENT FORMULATION OF OVERALL STRUCTURAL PLATE

Figure 1(b) illustrates the schematic diagram of overall structural plate. The length, width and thickness of substrate plate is indicated by \(a\), \(b\) and \(h\), respectively. The reference plane is taken at middle plane of the substrate plate (figure 1(b)). The thicknesses of the constrained 1-3 SVC strips and passive constraining layer are indicated by \(h_d\) and \(h_c\), respectively while pure VEM thickness within 1-3 SVC is symbolized by \(h_v\). The in-plane axial gap between constrained 1-3 SVC strips and number of strips along the \(y\) direction is denoted by \(\Delta_s\) and \(n_s\), respectively. Whereas \(\Delta_p\) denotes the in-plane axial gap between the piezoelectric patches and \(n_p\) denotes the number of patches within a strip along the \(x\) direction. The stacking sequence of different layers within overall structural plate are shown in figure 1. These thin layers are made of different materials so based on layer-wise first-order shear deformation theory the displacement-fields of overall structural plate can be expressed as follows[15],

\[
\begin{align*}
\mathbf{u}^k &= u_0 + z^k \alpha_i; \\
\mathbf{v}^k &= v_0 + z^k \beta_i; \\
\mathbf{w}^k &= w_0
\end{align*}
\]

where \(\mathbf{u}^k / \mathbf{v}^k / \mathbf{w}^k\) are displacement of \(k^{\text{th}}\) layer along \(x/y/z\) direction; \(\alpha_i/\beta_i\) is the mid-plane rotation of \(i^{\text{th}}\) layer with respect to the \(y/x\) axis.

The thickness coordinate \((z^k)\) can be expressed as follows,

\[
\begin{align*}
z^1_k &= z \text{ or } (h/2) \quad \text{for } k = 1 \text{ or } k > 1 \\
z^2_k &= 0 \text{ or } (z-h/2) \quad \text{or } h_v \quad \text{for } k < 2 \text{ or } k = 2 \text{ or } k > 2
\end{align*}
\]
$z^k_3 = 0 \text{ or } (z-h/2-h_v) \text{ or } h_p \text{ for } k < 3 \text{ or } k = 3 \text{ or } k > 3$; where $h_p = (h_l - 2h_v)$ \hfill (2)

$z^k_4 = 0 \text{ or } (z-h/2-h_v-h_p) \text{ or } h_v \text{ for } k < 4 \text{ or } k = 4 \text{ or } k > 4$

$z^k_5 = 0 \text{ or } (z-h/2-2h_v-h_p) \text{ for } k < 5 \text{ or } k = 5$

From equation (1), the deformation ($d^k$) of $k^{th}$ layer is expressed as,

$$d^k = [u^k \ v^k \ w^k]^T, \quad d^k = (d_i + Z^k_i d_t), \quad d_i = [u_0 \ v_0 \ w_0]^T,$$

$$Z^k_i = [Z^k_1 \ Z^k_2 \ Z^k_3 \ Z^k_4 \ Z^k_5], \quad Z^k_i = \begin{bmatrix} z^k_i & 0 & 0 \\ 0 & z^k_i & 0 \end{bmatrix}^T,$$

$$d_i = [d_{i1} \ d_{i2} \ d_{i3} \ d_{i4} \ d_{i5}]^T, \quad d_{ii} = \begin{bmatrix} \alpha_i \ \beta_i \end{bmatrix}.$$

From equation (4), the generalized displacement vector ($d$) is expressed as,

$$d^k = (T_i + Z^k_i T_i)d,$$

$$d = [u_0 \ v_0 \ w_0 \ \alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_4 \ \alpha_5 \ \beta_1 \ \beta_2 \ \beta_3 \ \beta_4 \ \beta_5]^T$$

where, $T_i$ and $T_i$ represents transformation matrices. By considering plane stress assumption [15], the state of strain and stress at any point in overall structural plate is written as,

$$\varepsilon_b = [\varepsilon_x \ \varepsilon_y \ \varepsilon_{xy}]^T, \quad \varepsilon_b = [\varepsilon_{xz} \ \varepsilon_{yz}]^T,$$

$$\sigma_b = [\sigma_x \ \sigma_y \ \sigma_{xy}]^T, \quad \sigma_b = [\sigma_{xz} \ \sigma_{yz}]^T.$$

The overall structural plate is made of three different type of martials yields three different type of elemental stacking sequence in FE meshing. The element for substrate plate ($k = 1$) is denoted by Element#1 while for plate with constrained 1-3 SVC strips where the layers $k = 2,3,4$ within 1-3 SVC are made of pure VEM is denoted by Element#2 and for plate with constrained 1-3 SVC strips where the piezoelectric patches ($k = 3$) are inserted between two pure VEM layers ($k = 2,4$) is denoted by Element#3. The electric field is applied to activate the piezoelectric patches. The electric field components $E_x$, $E_y$, and $E_z$ along the $x$, $y$ and $z$ directions, respectively, can be assumed as, $E_x = 0, \ E_y \approx 0$ and $E_z = -k_d \hat{w}_s$, where $\hat{w}_s$ denotes transverse velocity and $k_d$ represents velocity feedback control gain of $s^{th}$ patch. So constitutive relations of $k^{th}$ layer is expressed as follows,

$$\sigma^k_b = C^k_b \varepsilon^k_b - e^k_b E_z, \quad \sigma^k_s = C^k_s \varepsilon^k_s, \quad k = 1,2,3,4,5$$

$$D^k_z = [e^k_b]^T \varepsilon^k_b + e^k_{33} E_z$$

where, bending and transverse shear stiffness matrix are represented by $C^k_b$ and $C^k_s$; $e^k_b$ is a vector of the piezoelectric coefficients; $D^k_z$ denotes electric displacement in $z$ direction; $e^k_{33}$ indicates electrical permittivity. Here, the substrate plate, VEM layer, and passive constraining layer are the piezoelectrically inactive materials ($e^k_b = 0, \ k = 1, 2, 4, 5$), hence Element#1 and Element#2 becomes piezoelectric inactive while the electric field is applied in Element#3 over piezoelectric material. The viscoelastic material is modelled using complex modulus method, correspondingly their stiffness matrices ($C^k_b, C^k_s, k = 2, 3, 4$ in Element#2 and $k = 2,4$ in Element#3) becomes complex matrices.
The overall structural plate is discretised using nine-noded quadrilateral elements. So displacement \((d_k^k)\) and strain vectors \((\varepsilon_{b0}, \kappa_b, \varepsilon_{a0}, \kappa_a)\) are represented as,

\[
d_k = (T_k + Z_k^k T_k) N d^e
\]

\[
\varepsilon_{bl} = B_{bl} d^e, \quad \kappa_b = B_{b\kappa} d^e, \quad \varepsilon_{sL} = B_{sL} d^e, \quad \kappa_s = B_{s\kappa} d^e
\]

\[
B_{bl} = L_{sL} T_k N, \quad B_{b\kappa} = L_{b\kappa} T_k N, \quad B_{sL} = L_{sL} T_k N, \quad B_{s\kappa} = L_{s\kappa} T_k N
\]  

(7)

where, \(N\) denotes shape function matrix, \(d^e\) is nodal displacement vector and \(L_{b0}, L_{b\kappa}, L_{s0}, L_{s\kappa}\) are the operator matrices. The transverse point-load \((p_0)\) is applied at a point \((a/2, b/2, -h/2)\). The Hamilton’s principle (equation (8)) is used for deriving the governing equation of motion.

\[
\int_{t_i}^{t_f} \left( \delta T_k - \delta T_p \right) \, dt = 0
\]  

(8)

\[
\delta T_p = \frac{\mu b}{2} \sum_{k=1}^{N_k} \int \left( \delta \varepsilon_{bl}^k \sigma_{bl}^k + (\delta \varepsilon_{sL}^k)^T \sigma_{sL}^k \right) dx - \frac{h_{k+1}}{2} \int (\delta E_z)^T D_z^k \right|_{(k-1, \text{element} \, 0)} \, dy \, dx
\]  

(9)

\[
\delta T_k = \frac{\mu b}{2} \sum_{k=1}^{N_k} \int \left( \delta u^k \delta v^k \delta w^k \right) \rho^k \left( \dot{u}^k \dot{v}^k \dot{w}^k \right)^T dz \, dy \, dx
\]  

(10)

where \(\delta T_k\) and \(\delta T_p\) are first variations of the total potential and kinetic energy, respectively; \(\rho^k\) denotes mass density of \(k^{th}\) layer. The elemental equation of motion can be obtained in equation (11) using equation (5), (6) and (7) in equations (8), (9) and (10) as,

\[
M^E \ddot{d}^e + \left( K_b^e + K_s^e \right) d^e = P_E^c E_s^c + P_M^c p_0
\]  

(11)

Various matrices and vectors in equation (11) are as follows,

\[
K_b^e = \int_{A^e} \left( B_{bl} \right)^T \left( A_{b0} B_{bl} + B_{L1} B_{b\kappa} \right) \left( B_{b\kappa} \right)^T \left( B_{L2} B_{bL} + D_L B_{b\kappa} \right) \, dA^e,
\]

\[
K_s^e = \int_{A^e} \left( B_{sL} \right)^T \left( A_{sL} B_{sL} + B_{s1} B_{s\kappa} \right) \left( B_{s\kappa} \right)^T \left( B_{s2} B_{sL} + D_s B_{s\kappa} \right) \, dA^e,
\]

\[
P_E^c = \int_{A^e} \left( B_{b\kappa} \right)^T A_{bc} \left( B_{b\kappa} \right)^T \, dA^e, \quad P_M^c = \int_{A^e} \left[ \left( N^T T_i \right) T \left( 0 \quad 0 \right) \right] \, dA^e
\]

\[
M^e = \int_{A^e} \left[ \left( T_i \right)^T m_1 T_i + \left( T_i \right)^T m_2 T_i + \left( T_i \right)^T m_3 T_i + \left( T_i \right)^T m_4 T_i \right] N \, dA^e
\]  

(12)

In equation (12), different parameters are expressed as follows,

\[
A_{b0} = \sum_{k=1}^{N_k} \int_{h_k}^{h_{k+1}} C_b^k \, dz, \quad B_{L1} = \sum_{k=1}^{N_k} \int_{h_k}^{h_{k+1}} C_b^k \, dz, \quad B_{L2} = \sum_{k=1}^{N_k} \int_{h_k}^{h_{k+1}} \left( Z_b^k \right)^T C_b^k \, dz
\]

\[
D_L = \sum_{k=1}^{N_k} \int_{h_k}^{h_{k+1}} \left( Z_b^k \right)^T C_b^k \, dz, \quad A_s = \sum_{k=1}^{N_k} \int_{h_k}^{h_{k+1}} C_s^k \, dz, \quad B_{s1} = \sum_{k=1}^{N_k} \int_{h_k}^{h_{k+1}} C_s^k \, dz, \quad B_{s2} = \sum_{k=1}^{N_k} \int_{h_k}^{h_{k+1}} C_s^k \, dz
\]

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\[ B_{s2} = \sum_{k=1}^{N_h} \int \left( Z^k_s \right)^T C^k_s \, dz, \quad D_s = \sum_{k=1}^{N_h} \int \left( Z^k_s \right)^T Z^k_s \, dz, \quad m_i = \sum_{k=1}^{N_h} \int \rho^k \, dz \]  

(13a)

\[ m_2 = \sum_{k=1}^{N_h} \int \rho^k Z^k_s \, dz, \quad m_3 = \sum_{k=1}^{N_h} \int \left( Z^k_s \right)^T \rho^k \, dz, \quad m_4 = \sum_{k=1}^{N_h} \int \left( Z^k_s \right)^T \rho^k Z^k_s \, dz, \]  

(13b)

\[ A_{be} = \int \left\{ e^k \left( 1/h_b \right) \right\} \, dz, \quad B_{be} = \int \left\{ \left( Z^k_b \right)^T e^k \left( 1/h_b \right) \right\} \, dz \]  

(For Element#3)

In equation (9), (10) and (13a), \( N_L = 1 \) for Element#1 while \( N_L = 5 \) for Element#2 and Element#3; \( h_d = h_d - 2h_v \) is the piezoelectric patch thickness. After assembly of elemental matrices, the equation of motion of overall structural plate is represented as,

\[ M \ddot{d} + K \dot{d} = \sum_{s=1}^{n_p} P^s_M E_z^s P_0, \quad K = K_b + K_s \]  

(14)

In equation (14), global nodal displacement vector, global mass matrix, global stiffness matrix, global nodal mechanical load coefficient vector, global nodal electro-elastic coefficient vector, are denoted by \( \dot{d}, M, K, P_M, P^s_M \), respectively; \( E_z^s (= -k_d \dot{w}_s) \) is applied electric field for the \( s^{th} \) patch; \( K_b \) is bending and \( K_s \) is transverse shear matrix corresponding to global stiffness matrix \( (K) \); \( n_p \) indicates the number of patches in overall structural plate. By introducing the expression of \( \dot{w}_s \) from equation (15) in equation (14)

\[ \dot{w}_s = N_T \dot{X} \]  

(15)

\[ M \ddot{\dot{d}} + C \ddot{d} + K \dot{d} = P_M P_0 \quad C = \sum_{s=1}^{n_p} \left\{ P^s_M \dot{w}_s N_T \right\} \]  

(16)

2.1 Evaluation of active-passive damping

In case of free vibration \( (P_0 = 0) \), equation (16) can be represented as [16],

\[ \left\{ (K_b^R + K_b^I) + j(K_b^I + K_b^R + C) \right\} \phi_i = \omega_i^2 M \phi_i \]  

(17)

Equation (17) is the equation of complex quadratic eigen value problem, here real and imaginary counterpart are denoted by \( R \) and \( I \) superscripts; \( \phi_i \) is the complex nodal displacement vector for \( i^{th} \) natural mode having the complex natural frequency \( (\omega_i) \) as given in equation (18) [17].

\[ (\omega_i)^2 = (\omega_i^0)^2 (1 + j \eta_i); \quad \eta_i = \text{Im}(\omega_i)^2 / \text{Re}(\omega_i)^2 \]  

(18)

where \( \omega_i^0 \) and \( \eta_i \) denotes natural frequency and corresponding modal loss factor of \( i^{th} \) mode. The extensional modal loss factor \( (\eta_i) \) occurs due to in-plane strain so it can be computed by considering \( K_b^R \neq 0, K_s^R = 0 \) although shear modal loss factor \( (\eta_i) \) occurs due to shear strain so it can be computed by considering \( K_b^I = 0, K_s^I \neq 0 \) in equation (17) and it can be solve by direct iteration method as expressed in equation (19).
\[ (K^R_b + K^R_s) + j(K^I_b + K^I_s + C\rho^\text{prev})\),\psi_i = (\omega_i^\text{curr})^2 M\psi_i \]  
where, previous and current iterative values of natural frequency ($\omega_i$) corresponding to $i^{th}$ mode are denoted by $\omega_i^\text{prev}$ and $\omega_i^\text{curr}$, respectively.

### 3. RESULTS AND DISCUSSION

In this work, 1-3 SVC is formed for the improvement of damping in the plate with constrained 1-3 SVC strips. The 1-3 SVC is designed through the inclusion of piezoelectric patches in the VEM phase (figure 1(a)). Material properties of substrate plate and constraining strips are taken as Aluminium ($E = 68.9 \text{ GPa}, \nu = 0.3, \rho = 2740 \text{ kg/m}^3$) while VEM is formed with butyl rubber ($E = 15(1+0.6i) \text{ MPa}, \nu = 0.49, \rho = 920 \text{ kg/m}^3$) and piezoelectric patches are formed with PZT-5H ($C_{11} = 127 \text{ GPa}, C_{12} = 80.21 \text{ GPa}, C_{13} = 84.67 \text{ GPa}, C_{33} = 117.84 \text{ GPa}, C_{44} = 22.99 \text{ GPa}, C_{66} = 23.47 \text{ GPa}, \rho = 7500 \text{ kg/m}^3$, $e_{31} = -6.6228 \text{ C/m}^2$, $e_{33} = 23.24 \text{ C/m}^2$, $e_{33} = 17.03 \text{ C/m}^2$) [18], respectively. The boundary of overall structural plate is considered as simply-supported edges. Since the result for plate with constrained 1-3 SVC strips is unavailable in the literature. So the present FE formulation is validated by taken plate with constrained pure VEM layer. The first four natural frequencies and corresponding modal loss factor of overall plate are reported in Table 1 and compare with the similar results present in [19]. It may be remarked in Table 1 that the present results are good in agreement with the similar results available in [19]. The modelling of electro-elastic coupling is validated with free strain analysis by considering overall structural plate as the piezoelectric layer of thickness 0.25 mm. The free strain analysis results from present FE formulation is expressed in Table 2 and compared with the similar results calculated with analytical method. It may be remarked that the present results (Table 2) are good in agreement with analytical results.

| Mode ($m, n$) | Present FE results | Ref.[19] |
|---------------|-------------------|----------|
| $\omega_{m,n}$ | $\eta_{m,n}$ | $\omega_{m,n}$ | $\eta_{m,n}$ |
| (1,1) | 60.235 | 0.1901 | 60.3 | 0.19 |
| (1,2) | 115.224 | 0.2034 | 115.4 | 0.203 |
| (2,1) | 130.427 | 0.1991 | 130.6 | 0.199 |
| (2,2) | 178.463 | 0.1806 | 178.7 | 0.181 |

| Voltage (volt) | FE results | Analytical results |
|----------------|------------|--------------------|
| $\varepsilon_x (\times 10^{-4})$ | $\varepsilon_y (\times 10^{-4})$ | $\varepsilon_x (\times 10^{-4})$ | $\varepsilon_y (\times 10^{-4})$ |
| 100 | 1.1009 | -1.1009 | 1.1009 | -1.1009 |
| 250 | 2.7523 | -2.7523 | 2.7522 | -2.7522 |

In the present study, initially, the strain distributions of overall structural plate are examined to study the change in strain characteristics with the embedded piezoelectric patches in pure VEM phase. The dimensions of overall structural plate in this strain analysis are taken as, length $a = 0.4 \text{ m}$, width $b = 0.4 \text{ m}$, substrate thickness $h = 4 \text{ mm}$, constraining strip thickness $h_c = 0.5 \text{ mm}$, number of strips $n_s = 4$, gap between two consequent strips $\Delta_s = 10 \text{ mm}$, constrained 1-3 SVC thickness $h_3 = 1 \text{ mm}$, VEM layer thickness $h_v = 0.25 \text{ mm}$, number of piezoelectric patches in each strip $n_p = 4$ and gap between two consequent patches $\Delta_p = 5 \text{ mm}$ (figure 1). The transverse harmonic point-load ($p_0 = 125 \text{ N}$) is
applied at the center of the bottom surface of overall structural plate, while piezoelectric patches in 1-3 SVC are activated by supplying the 100-volt voltage at the top electrode surface of the patches. The strain of overall structural plate in $xy$-plane through the middle of the constrained 1-3 SVC strips and in $xz$-plane through the center of the plate with constrained 1-3 SVC strips are illustrated in figures 2 and 3, respectively. The distribution of in-plane normal strain ($\varepsilon_x$) and shear strain ($\gamma_{xy}$) in preceding $xy$-plane are presented in figures 2(e)-2(f). Similarly, the strain distributions in the absence of applied voltage in piezoelectric patches are illustrated in figures 2(c)-2(d), while for the plate with constrained pure VEM are illustrated in figures 2(a)-2(b). Similarly the distribution of normal strain ($\varepsilon_x$) and transverse shear strain ($\gamma_{xz}$) at aforesaid location in $xz$-plane are shown in figures 3(a)-3(b), 3(c)-3(d) and 3(e)-3(f) for plate with constrained pure VEM, plate with constrained 1-3 SVC strips without activated piezo patches and plate with constrained 1-3 SVC strips with activated piezo patches, respectively. It may be observed from figures 2 and 3(a), 3(c), 3(e) that there is an increase in in-plane normal strain ($\varepsilon_x$) and shear strain ($\gamma_{xy}$) of pure VEM in between two successive piezoelectric patches. Although, it may also be observed from the figures 3(b), 3(d) and 3(f) that transverse shear strain ($\gamma_{xz}$) in pure VEM phase is increased and it is more in the top and bottom surface of VEM phase within 1-3 SVC in case of active piezoelectric patches with respect to piezoelectric patches without applied voltage and plate with constrained pure VEM. It may be noticed that the maximum magnitude of strain in VEM is more in plate with constrained 1-3 SVC strips with respect to constrained pure VEM layer. The rise in strain is expected to boost damping in overall structural plate.
Figure 2. Distributions of normal strains ($\varepsilon_x$) and shear strain ($\gamma_{xy}$) for ((a)-(b)) plate with constrained VEM layer; ((c)-(d)) plate with constrained 1-3 SVC strips (without voltage); ((e)-(f)) plate with constrained 1-3 SVC strips (with voltage $V = 100$ volt), respectively, in the $xy$-plane (point load $p_0 = 125$ N).

Since the improvement in damping of overall structural plate depends on the constrained 1-3 SVC strips. The damping of overall structural plate is depended on dimensions ($h_v$, $n_p$, $\Delta_p$) of 1-3 SVC damping strips. So in order to maximize damping (modal loss factor, $\eta$), the optimal dimensions of constrained 1-3 SVC strips are determined using direct search method. The geometric properties of overall structural plate are taken as, length $a = 0.4$m, width $b = 0.4$m, substrate thickness $h = 4$mm, constraining strip thickness $h_c = 0.5$ mm, number of strips $n_s = 10$, gap between two successive strips $\Delta_s = 10$ mm, constrained 1-3 SVC thickness $h_d = 1$ mm. The feasible bounds of the dimensions of 1-3 SVC strips is decided as, $20 \mu$m $\leq h_c \leq 100 \mu$m, $4 \leq n_p \leq 20$, $50 \mu$m $\leq \Delta_p \leq 200 \mu$m based on manufacturing point of view. The values of modal loss factor ($\eta$) at different values of 1-3 SVC dimensions($h_v$, $n_p$, $\Delta_p$) and velocity feedback control-gains ($k_d$) are expressed in three-dimensional contour as illustrated in figure 4. The optimal dimensions of 1-3 SVC strips may be acquired from the figure (figure 4) to maximize damping ($\eta$) of overall structural plate.
Figure 3. Distributions of normal strains ($\varepsilon_x$) and shear strain ($\gamma_{xz}$) for ((a)-(b)) plate with constrained VEM; ((c)-(d)) plate with constrained 1-3 SVC strips (without voltage); ((e)-(f)) plate with constrained 1-3 SVC strips (with voltage $V = 100$ volt), respectively, in the $xz$-plane (point load $p_0 = 125$ N).

The optimal dimensions of 1-3 SVC strips at different velocity feedback control-gains ($k_d$) are shown in Table 3. The fundamental frequency ($\omega_n$) and corresponding modal loss factor ($\eta$) and its counterpart (extensional $\eta_e$ and shear $\eta_s$) of overall structural plate are computed and presented in Table 4, at optimal dimension of 1-3 SVC and different velocity feedback control-gains ($k_d$). Identical results for plate with constrained pure VEM layer are also illustrated in the same Table 4. The value of counterpart (extensional $\eta_e$ and shear $\eta_s$) of modal loss factor ($\eta$) of plate with constrained 1-3 SVC strips is higher than pure VEM layer. Both extensional ($\eta_e$) and shear ($\eta_s$) modal loss factor increases so that overall modal loss factor ($\eta$) is also increased. This improvement in overall damping may be because of an increase in normal strain and shear strain, as also described in aforesaid strain distribution results (figures 2 and 3). It may be noticed that modal loss factor ($\eta$) and its counterpart (extensional $\eta_e$ and shear $\eta_s$) increases with velocity feedback control-gain ($k_d$) (Table 4). So the augmentation in damping arises mainly because of active domination of piezoelectric patches and passive domination of VEM phase.

Figure 4. Three-dimensional contour of modal loss factor ($\eta$) of overall structural plate at different 1-3 SVC dimensions ($h_v$, $n_p$, $\Delta_p$) and control-gain (a) $k_d = 0$, (b) $k_d = 100$, (c) $k_d = 500$. 

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Table 3: Optimal dimensions \((h_v, n_{p}, \Delta_{p})\) of overall 1-3 SVC structural plate

| \(k_d\) | \(h_v\) (\(\mu \text{m}\)) | \(n_{p}\) | \(\Delta_{p}\) (\(\mu \text{m}\)) |
|--------|------------------|--------|------------------|
| 0      | 20               | 18     | 50               |
| 100    | 20               | 12     | 50               |
| 500    | 20               | 4      | 50               |

Table 4: Comparative study of fundamental frequency \((\omega_n)\) and corresponding modal loss factor \((\eta)\) and its counterpart (extensional \(\eta_e\) and shear \(\eta_s\)) of overall structural plate.

| Treatment                                      | \(\eta\)  | \(\eta_e\) | \(\eta_s\) |
|------------------------------------------------|-----------|------------|------------|
| Plate with constrained pure VEM layer (Passive) | 0.0775    | 4.885 \times 10^{-5} | 0.0775    |
| Plate with constrained 1-3 SVC strips (Only passive \(k_d=0\)) | 0.0975    | 0.0063     | 0.0930    |
| Plate with constrained 1-3 SVC strips (Active-passive \(k_d=100\)) | 0.1151    | 0.0216     | 0.1122    |
| Plate with constrained 1-3 SVC strips (Active-passive \(k_d=500\)) | 0.2075    | 0.1157     | 0.2049    |

The frequency response and corresponding maximum control-voltage of overall structural plate are also appraised in order to comprehend the fruitfulness of present 1-3 SVC under transverse point-load \((p_0 = 125 \text{ N})\) and at different velocity feedback control-gain \((k_d)\). Similarly, frequency response for the plate with a constrained pure VEM layer is also obtained, as illustrated in figures 5(a)-(b). Figures 5(a)-(b) illustrate that maximum transverse displacement-amplitude \((W_{\text{max}}/h)\) of overall structural plate decreases while corresponding maximum control-voltage \((V_{\text{max}})\) increases with the velocity feedback control-gain \((k_d)\). It may also be noticed that there is a reduction in maximum transverse displacement-amplitude of plate with constrained 1-3 SVC strips with respect to pure VEM layer. Thus, preceding results conclude that there may be a remarkable improvement in damping using 1-3 SVC with respect to pure VEM.

Figure 5. (a) Frequency responses and (b) the corresponding maximum control voltage of overall structural plate \((p_0 = 125 \text{ N})\).
4. CONCLUSIONS

Damping performance of plate with constrained 1-3 smart viscoelastic composite (SVC) layer is studied. The 1-3 SVC is formed through the inclusion of piezoelectric patches within pure viscoelastic material (VEM) phase. The overall structural plate is modelled using finite element method. 1-3 SVC is utilized in the form of strips that are constrained betweena substrate plate and a passive constraining layer to improve the constrained layer damping (CLD) of overall structural plate. First, the characteristics of in-plane normal strain and transverse shear strain of overall structural plate are examined. Itadmits that there is a significant improvement in the magnitude of in-plane normal strain and transverse shear strain in VEM phase of 1-3 SVC strips with respect to pure VEM layer. In order to achieve the maximum attainable damping of overall structural plate, the optimal dimensions of the present 1-3 SVC strips are determined using a direct search method. It is noticed that in the case of the plate with constrained 1-3 SVC strips, modal loss factor and its counterpart(extension and shear) enhancesynchronously with respect to pure VEM layer. It happen because of increament of in-plane normal strain and transverse shear strain in VEM phase by embedded piezoelectric patches. Thus, the results described above express that 1-3 SVC may be useful damping layer in the constrained damping treatment of overall structural plates.

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