Highly-mixed measurement-based quantum computing and the one clean qubit model

Tomoyuki Morimae

ASRLD Unit, Gunma University, 1-5-1 Tenjin-cho Kiryu-shi Gunma-ken, 376-0052, Japan

(Dated: March 10, 2014)

We show that a highly-mixed state in terms of a large min-entropy is useless as a resource state for measurement-based quantum computation in the sense that if a classically efficiently verifiable problem is efficiently solved with such a highly-mixed measurement-based quantum computation then such a problem can also be classically efficiently solved. We derive a similar result also for the DQC1k model, which is a generalized version of the DQC1 model where k output qubits are measured. We also show that the measurement-based quantum computing on a highly-mixed resource state in terms of the von Neumann entropy, and DQC1k model are useless in another sense that the mutual information between the computation results and inputs is very small.

One of the most fundamental questions in quantum information science is whether a quantum computing model truly outperforms classical computing or not. In particular, to clarify the power of quantum computing models that are highly mixed is important both from the fundamental and practical points of view. In this paper, we consider two highly-mixed quantum computing models, namely, the measurement-based quantum computing on a highly-mixed resource state, and the one clean qubit model with many-qubit measurements. We show that if a classically efficiently verifiable problem is efficiently solved with these models, then such a problem can also be classically efficiently solved. In this sense, these two models are not useful. We also show that for these highly-mixed quantum computing models, the mutual information between the computation outputs and inputs is very small. This means that these models are useless for problems where inputs and outputs should be highly correlated like a search problem.

Measurement-based quantum computation (MBQC) by Raussendorf and Briegel [1] is a model of quantum computing where universal quantum computation can be done with only local measurements on a certain quantum many-body state, which is called a resource state, and a classical processing of the measurement results. The computational power of MBQC is equivalent to the traditional circuit model of quantum computation, but the clear separation between the quantum phase (i.e., the preparation of the resource state) and the classical phase (i.e., local adaptive measurements) has inspired many new results over the last decade, which would not be obtained from the circuit model mind. For example, new resource states for MBQC which are closely connected with condensed matter physics have been proposed [2,12]. Furthermore, relations between MBQC and partition functions of classical spin models were pointed out [13,17]. These discoveries have established new bridges between quantum information and condensed matter physics. MBQC has also offered a new framework of fault-tolerant quantum computing, namely, the topological measurement-based quantum computation which achieves dramatically high error thresholds [10,21]. New protocols of secure cloud quantum computing, so called the blind quantum computing, were also developed by using MBQC [22,38].

MBQC on mixed resource states have been studied by several researchers [21,21,31,41]. In Refs. 21,21, some condensed-matter physically motivated two-body Hamiltonians were proposed whose equilibrium states at sufficiently low temperatures can be used as resource states for the topologically-protected MBQC. In Ref. 39, the thermal three-dimensional cluster state was considered, and it was shown that the entanglement length can be infinite if the temperature is below a certain threshold, whereas it becomes finite if the temperature is higher than another certain threshold. In Ref. 40, a two-body qubit Hamiltonian was proposed whose low-temperature equilibrium states can be adiabatically brought to useful resource states for the topologically-protected MBQC. In Ref. 41, thermal cluster states are considered, and it was shown that at a low-temperature region these thermal states can be universal resources whereas at a high-temperature region MBQC on these thermal states can be classically efficiently simulated. However, all these results consider only equilibrium states, or assume specific forms of Hamiltonians or resource states.

In this paper, we obtain a general result that highly mixed states in terms of a large min-entropy are useless resource states for MBQC in the sense that if a classically efficiently verifiable problem is efficiently solved with MBQC on such states, then such a problem can also be classically efficiently solved. The result is general: we do not make any assumption on Hamiltonians, resource states, or the way of measurements, etc. The min-entropy \( H_{\min}(\rho) \) of an \( N \)-qubit state \( \rho \) is defined by \( H_{\min}(\rho) \equiv -\log_2 \lambda_1 \), where \( \lambda_1 \) is the largest eigenvalue of \( \rho \). The min-entropy quantifies the amount of random bits that can be extracted [42]. Our result is derived by using a similar argument of Ref. 43 that shows that highly-entangled pure states in terms of the geometric measure of entanglement [44,46] are useless resource states for MBQC in a similar sense. As in Ref. 43, we first assume that a quantum computing model can efficiently solve a problem whose solution can
be classically efficiently verifiable. Then we show that we can construct a classical random computing model that can efficiently solve the same problem. Note that the uselessness of a randomly chosen pure state as a resource state of MBQC was shown in Ref. \[43\].

In particular, our result implies that the equilibrium state, \(e^{-\beta H}/\text{Tr}(e^{-\beta H})\), of any Hamiltonian \(H\) with a high temperature (\(\beta \ll 1\)) are useless resource states for MBQC, where \(\beta \equiv 1/(kT)\), \(k\) is the Boltzmann constant, and \(T\) is the temperature. It is a generalization of the above mentioned previous results that assume specific Hamiltonians or resource states to any Hamiltonian and resource state. Furthermore, we can also obtain the following result: in order to change such a useless resource state into a useful one, \(O(N)kT\) of work is necessary for the isothermal process, where \(N\) is the number of particles of the resource state.

We also consider another model of highly-mixed quantum computing, namely, the DQC1\(_k\) model, and derive a similar result: if a classically efficiently verifiable problem is efficiently solved with the DQC1\(_k\) model then such a problem can also be classically efficiently solved. Here, the DQC1\(_k\) model is a generalized version of the deterministic quantum computation with one quantum bit (DQC1) model by Knill and Laflamme \[48\]. As is shown in Fig. 1 (a), a DQC1 circuit consists of the input state, \(\rho_{in} \equiv |0\rangle \langle 0| \otimes (\frac{1}{2})^{\otimes n}\), where \(I \equiv |0\rangle \langle 0| + |1\rangle \langle 1|\) is the two dimensional identity operator, polynomial number of quantum gates on it, and the computational basis measurement of the first qubit. The DQC1\(_k\) model is equivalent to the DQC1 model except that not the single but \(k\) output qubits are measured in the computational basis at the end of the computation (Fig. 1 (b)). Surprisingly, such highly-mixed quantum computing models can efficiently solve some problems for which no efficient classical algorithms are known, such as the spectral density estimation \[43\], testing integrability \[49\], calculation of the fidelity decay \[50\], and approximations of the Jones polynomials, the HOMFLY polynomials, and the Turaev-Viro invariant \[51–54\]. Furthermore, it was shown in Ref. \[55\] that if any output probability distribution of DQC1\(_k\) for \(k \geq 3\) can be classically efficiently sampled within a certain multiplicative error, the polynomial hierarchy \[50\] collapses at the third level, which is not believed to happen.

We show that a classically efficiently verifiable problem which can be efficiently solved with the DQC1\(_k\) model can also be classically efficiently solved. Note that there is another negative result about the power of the DQC1 model: the DQC1 model cannot simulate universal quantum computation under some reasonable assumptions \[57\].

Finally, we further show another negative results that MBQC on a highly-mixed resource state in terms of the von Neumann entropy, and the DQC1\(_k\) model are useless in another sense that the mutual information between the computation results and inputs is very small. This means that these models are not useful for problems where inputs and outputs should be strongly correlated like search problems.

**MBQC.—** Before giving our first result about MBQC, let us define the most general framework of MBQC. Let \(\sigma\) be the \(N\)-qubit resource state of MBQC. (We can also consider qudit states for \(d \geq 3\), but for the simplicity, we here consider qubit states. Generalizations to qudit states with \(d \geq 3\) are straightforward.) Note that \(\sigma\) is not necessarily the graph state. It can be any resource state, such as the Affleck-Kennedy-Lieb-Tasaki (AKLT) state \[44, 56\] or a general tensor-network state \[3\]. As is shown in Fig. 2, the resource state \(\sigma\) is divided into two subsystems, \(C\) and \(O\), which consists of \(N - n\) and \(n\) qubits, respectively. Qubits in the subsystem \(C\) are measured to perform the desired quantum computation.

The input state of the computation is included in \(C\). We perform a POVM \(\{M_j\}_{j=1}^r\) on \(C\) and obtain the result \(m \in \{1, 2, ..., r\}\), where \(\sum_{j=1}^r M_j = I^\otimes N - n\). (Local adaptive projective measurement used in usual MBQC \[1\] is a special case of the POVM. Here, we consider the most general way of MBQC, hence we use POVM, which can be global.) After the POVM, the output of the computation is encoded on qubits in \(O\). We measure the subsystem \(O\) in the computational basis in order to read out the output of the quantum computation. Let us denote the computational basis measurement on \(O\) by \(\{P_z\}_z\), where \(z \in \{0, 1\}^n\) is an \(n\) bit binary string, and \(P_z \equiv |z\rangle \langle z| \equiv \bigotimes_{j=1}^n |z_j\rangle \langle z_j|\) is the projection operator onto the computational basis \(|z\rangle \equiv \bigotimes_{j=1}^n |z_j\rangle\). Here, \(z_j \in \{0, 1\}\) is the \(j\)th bit of \(z\). Depending on the outcome \(m \in \{1, 2, ..., r\}\) of the previous POVM \(\{M_j\}_{j=1}^r\) on \(C\), the result \(z\) of the computational basis measurement on \(O\) is classically post-processed in order to correct the effect of the byproduct operators, which are unavoidable in MBQC \[1, 59\].

**Highly mixed states are useless resource states.—** Now let us show our first result that highly mixed states are useless resource states for MBQC. As in Ref. \[42\], we consider MBQC solving a classically efficiently verifiable problem, and require that the probability of obtaining a
correct result is larger than 1/2:

\[
\frac{1}{2} \leq \sum_{j=1}^{r} \sum_{z \in S_j} \text{Tr}[(M_j \otimes P_z)\sigma],
\]

(1)

where \(S_j \subseteq \{0,1\}^n\) is the set of correct results when the result of the POVM on \(C\) is \(j\). (Note that the set of correct results depends on the result of the previous POVM on \(C\) because of the byproduct operators.) Because we assume that our MBQC is deterministic, i.e., any byproduct operators are correctable, we require that \(|S_1| = |S_2| = \ldots = |S_r| = |S|\). It is interesting to consider non-deterministic MBQC, but it would be a subject of a future study.) Let \(\sigma = \sum_{k=1}^{2^n} \lambda_k |\lambda_k\rangle \langle \lambda_k|\) be the spectral decomposition (i.e., the diagonalization) of the resource state \(\sigma\) with the decreasing order \(1 \geq \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{2^n-1} \geq \lambda_{2^n} \geq 0\) of eigenvalues and corresponding eigenvectors \(\{|\lambda_k\rangle\}_{k=1}^{2^n}\). Then, Eq. (1) becomes

\[
\frac{1}{2} \leq \sum_{j=1}^{r} \sum_{z \in S_j} \lambda_k \text{Tr}[(M_j \otimes P_z)|\lambda_k\rangle \langle \lambda_k|] \leq \lambda_1 |S| 2^{N-n}.
\]

This means that \(|S| \geq 2^{-N+n-1}\lambda_1^{-1}\). Let us assume that we randomly generate an \(n\)-bit binary string \(a \in \{0,1\}^n\). Since the problem is classically efficiently verifiable, we can efficiently check whether the string \(a\) is a solution or not. If it is not a solution, we again generate another random \(n\)-bit binary string \(a' \in \{0,1\}^n\), and check whether it is a solution or not. We repeat this process until we finally obtain a correct solution. Let us assume that \(H_{\min}(\sigma) = N - \delta\), where \(\delta\) is at most \(O(\log N)\). This means that \(\lambda_1 = 2^{-N+\delta}\). Now we can use the result of Ref. [43]: the probability that we do not obtain any correct string after \(t\) repetitions of the above process is

\[
\left(1 - \frac{|S|}{2^n}\right)^t \leq \left(1 - 2^{-N-1}\lambda_1^{-1}\right)^t = (1 - 2^{-\delta-1})^t < e^{-t2^{\delta-1}}.
\]

If we take \(t = 2^{\delta+1} \ln(1/p_f)\), the probability of failure is less than \(p_f\). Therefore, a correct result can be efficiently classically obtained with a sufficiently large success probability.

**Example 1: equilibrium states.**— As an example, let us consider the special case that the resource state \(\sigma\) is the equilibrium state, \(\sigma = e^{-\beta H}/\text{Tr}(e^{-\beta H})\), of a Hamiltonian \(H\), where \(\beta = (kT)^{-1}\). Let us scale the energy spectrum of the Hamiltonian so that the lowest energy of \(H\) is 0. Then, the min-entropy of \(\sigma\) is given by \(H_{\min}(\sigma) = \log \text{Tr}(e^{-\beta H}) = -\beta F\), where \(F\) is the free energy. Let us assume \(H_{\min}(\sigma) = N - \delta\), where \(\delta\) is at most \(O(\log N)\). In other words, we assume that \(\sigma\) is a useless resource state. If we want to change \(\sigma\) into another state \(\sigma'\) that is useful, \(H(\sigma')\) must be \(N - \delta'\), where \(\delta' = O(N)\). Then, \(H_{\min}(\sigma) - H_{\min}(\sigma') = \delta' - \delta = O(N)\). This means that the free energy change is \(\Delta F = O(N)kT\), and therefore \(O(N)kT\) of work is necessary for the isothermal transformation.

**Example 2: thermal cluster state.**— Our next example is the \(N\)-qubit thermal cluster state \(\sigma_{cl} = e^{-\beta H_{cl}/\text{Tr}(e^{-\beta H_{cl}})}\), where \(H_{cl} = -\sum_{j=1}^{N} K_j\) is the cluster state Hamiltonian, \(K_i = X_i \otimes \otimes_{j \in N(i)} Z_j\) is a stabilizer operator for the cluster state. Here, \(X_j\) and \(Z_j\) are Pauli \(X\) and \(Z\) operators acting on site \(j\), and \(N(i)\) is the set of the nearest-neighbour sites of site \(i\). Note that \([K_i,K_j] = 0\) for all \(i\) and \(j\). Let \(|C\rangle\) be the ground state (i.e., the cluster state) of the Hamiltonian \(H_{cl}: H_{cl}(|C\rangle) = -N|C\rangle\). Then, the set of states \(|s\rangle \equiv \left(\otimes_{j=1}^{N} Z_j^{s_j}\right)|C\rangle\), for \(s = (s_1,...,s_N) \in \{0,1\}^N\) is an orthonormal basis of the \(2^N\)-dimensional Hilbert space. Therefore, we can easily calculate \(H_{\min}(\sigma_{cl})\) as

\[
H_{\min}(\sigma_{cl}) = -\log_2 \left(\frac{\text{Tr}(\sigma_{cl}^{N})}{\langle C|\sigma_{cl}|C\rangle}\right) = N \log_2(1 + e^{-2\beta}).
\]

If \(T \to \infty\), then \(\beta \to 0\), and therefore \(H_{\min}(\sigma_{cl}) \to N\). This means that the thermal cluster state with a high temperature is useless resource state for MBQC.

**DQC1\(_k\) model.**— Now let us show our second result about the DQC1\(_k\) model. We consider the DQC1\(_k\) model solving a classically efficiently verifiable problem. We again require that the probability of obtaining a correct result is larger than 1/2:

\[
\frac{1}{2} \leq \sum_{z \in S} \text{Tr} \left[(P_z \otimes I^{\otimes n+1-k}) U \rho_{in} U^\dagger\right];
\]

(2)

where \(P_z \equiv \langle z|z\rangle\) is the \(k\) qubit projection operator onto the computational basis, \(z \in \{0,1\}^k \) is a \(k\) bit string, and \(S \subseteq \{0,1\}^k\) is the set of correct results. Let \(U \rho_{in} U^\dagger = \sum_{j=1}^{2^{n+1}} \lambda_j |\lambda_j\rangle \langle \lambda_j|\) be the spectral decomposition (i.e., the diagonalization) of \(U \rho_{in} U^\dagger\) with the decreasing order \(1 \geq \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{2^{n+1}} \geq 0\) of eigenvalues and corresponding eigenvectors \(\{|\lambda_j\rangle\}_{j=1}^{2^{n+1}}\). Since the unitary \(U\) does not change the spectrum of \(\rho_{in}\),

\[
\lambda_j = \begin{cases} 2^{-n} (1 \leq j \leq 2^n) \\ 0 \quad (2^n + 1 \leq j \leq 2^{n+1}) \end{cases}
\]

Therefore, Eq. (2) becomes

\[
\frac{1}{2} \leq \sum_{z \in S} \sum_{j=1}^{2^{n+1}} \lambda_j \text{Tr} \left[(P_z \otimes I^{\otimes n+1-k}) |\lambda_j\rangle \langle \lambda_j|\right] \leq \lambda_1 |S| 2^{n+1-k}.
\]
This means that $|S| \geq \lambda^{-2n+k-2}2^{k-2}$. Let us generate a random $k$ bit string and check whether it is a solution or not. Since the problem is classically efficiently verifiable, we can check efficiently. If it is not a solution, we again generate another random $k$ bit string. The probability that we do not obtain any correct solution after $t$ repetition of the process is $\left(1 - \frac{\lambda^2}{2}\right)^t \leq \left(1 - \frac{1}{4}t\right)^t = \left(\frac{1}{2}\right)^t$. Therefore, $t = \text{poly}(n)$ of the repetition is sufficient to obtain a correct solution with an exponentially small failure probability. In short, a classically efficiently verifiable problem which can be efficiently solvable with the DQC1 model can also be classically efficiently solved.

Finally, we further show that MBQC on a highly-mixed resource state in terms of the von Neumann entropy, and DQC1 model are useless in another sense that the correlation between computation outputs and inputs quantified by the mutual information is very small. Let us consider the bipartite quantum computing between Alice and Bob: Alice has the input, and she asks Bob to perform quantum computing. After the computation, Bob sends Alice the output of the computing. Alice’s classical input $i$ is represented by an $N_A$-qubit state $|i\rangle$. For example, if Bob does MBQC, $i$ is the instruction of how to measure each qubit, i.e., the measurement angles and the way of adaptation, etc. If Bob does the DQC1 computing, $i$ specifies the unitary that should be implemented, etc. Since different classical inputs $i$ and $i'$ must be distinguishable, we assume that $\{\langle i \rangle\}_{i=1}^{2^{N_A}}$ is an orthonormal basis. The initial state of the bipartite computing is

$$\rho_{AB} = \sum_{i=1}^{2^{N_A}} p_i |i\rangle\langle i| \otimes U_i \sigma_B U_i^\dagger,$$

where the classical input $|i\rangle$ is generated with the probability $p_i$, and $\sigma_B$ is Bob’s initial quantum state of $N_B$ qubits. Bob applies the unitary $U_i$ if Alice’s input is $i$. Then the output state of the bipartite computing is

$$\rho_{AB} = \sum_{i=1}^{2^{N_A}} p_i |i\rangle\langle i| \otimes U_i \sigma_B U_i^\dagger,$$

(Note that if Bob does MBQC, he further measures his part of this state. Since we are deriving a negative result, it is sufficient to consider the positive branch, i.e., the case when accidentally no feedforwarding was required.) The mutual information between $A$ and $B$ of $\rho_{AB}$ is $I_{AB} = S(\rho_B) - \sum_{i=1}^{2^{N_A}} p_i S(\sigma_B^i) \leq N_B - S(\sigma_B)$, where $\rho_B$ is the reduced density operator of $\rho_{AB}$ for Bob’s system, and $\sigma_B^i = U_i \sigma_B U_i^\dagger$. If $S(\sigma_B) = N_B - \delta$, for a certain small $\delta$, then $I_{AB} \leq \delta$, which means that the mutual information is bounded by $\delta$. For example, if Bob does MBQC, $\sigma_B$ is an $N$-qubit resource state, i.e., $N_B = N$, and the unitary $U_i$ is the rotation of each qubit according to the specification $i$. If $\delta$ is very small, the output of the MBQC is not sufficiently correlated with Alice’s inputs, and therefore in that sense such an MBQC is useless. On the other hand, if Bob does the DQC1 computing, $\sigma_B$ is the highly-mixed input state $\rho_B = |0\rangle\langle 0| \otimes (\frac{1}{2})^{\otimes N}$ of DQC1 model (and therefore $N_B = n + 1$). In the latter case, $S(\sigma_B) = n$, and therefore $I_{AB} \leq 1$.

The author thanks Keisuke Fujii for valuable discussion. This work was supported by the Tenure Track System MEXT Japan.

Appendix.— We here show that parallelization of DQC1 circuits does not change the result. We allocate $r$ DQC1 circuits in parallel. Let $o_i^j \in \{0, 1\}^k$ be $i$th outcome of $j$th DQC1 circuit, where $i = 1, 2, \ldots, 2^k$, and $j = 1, 2, \ldots, r$. We can repeat such a parallel computing in $v$ times. We assume that some sets $\{(o_1^{(1)}, j_1), (o_2^{(2)}, j_2), \ldots, (o_L^{(L)}, j_L)\}$ are solutions of our problem. If we denote $p_i = \sum_{s \in S_i} \text{Tr}(P_s \otimes I^{n+1-k})\sigma$, where $S_i = \{o_i^{(j)}, j\}$, then $p_i v$ must be increasing as a function of $v$, since the probability that we never obtain any element of $S_i$ in the $v$ time repetition is $(1 - p_i)^v \rightarrow e^{-p_i v}$. Then, we obtain $p_i \leq |S_i| |\lambda| 2^{n+1-k}$, which means that the failure probability of the classical sampling is $p_f = (1 - \frac{|S_i|}{2^n}) v \leq (1 - \frac{|\lambda|}{2^n}) v \leq e^{-\frac{p_i}{2^n}} \rightarrow 0$.

* Electronic address: morimae@gunma-u.ac.jp

[1] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. 86, 5188 (2001).
[2] F. Verstraete and J. I. Cirac, Phys. Rev. Lett. 70, 060502(R) (2003).
[3] D. Gross and J. Eisert, Phys. Rev. Lett. 98, 220503 (2007).
[4] G. K. Brennen and A. Miyake, Phys. Rev. Lett. 101, 010502 (2008).
[5] J. Cai, W. Dür, M. Van den Nest, A. Miyake, and H. J. Briegel, Phys. Rev. Lett. 103, 050503 (2009).
[6] A. Miyake, Phys. Rev. Lett. 105, 040501 (2010).
[7] A. Miyake, Ann. Phys. 326, 1656 (2011).
[8] T. C. Wei, I. Affleck, and R. Raussendorf, Phys. Rev. Lett. 106, 070501 (2011).
[9] J. Cai, A. Miyake, W. Dür, and H. J. Briegel, Phys. Rev. A 82, 052309 (2010).
[10] Y. J. Chiu, X. Chen, and I. L. Chuang, Phys. Rev. A 87, 012305 (2013).
[11] T. Morimae, Phys. Rev. A 83, 042337 (2011).
[12] T. Morimae, Phys. Rev. A 85, 062328 (2012).
[13] S. Bravyi and R. Raussendorf, Phys. Rev. A 76, 022304 (2007).
[14] M. Van den Nest, W. Dür, and H. J. Briegel, Phys. Rev. Lett. 98, 117207 (2007).
[15] M. Van den Nest, W. Dür, and H. J. Briegel, Phys. Rev. Lett. 100, 110501 (2008).
[16] R. Raussendorf, J. Harrington, and K. Goyal, New J. Phys. 9, 199 (2007).
[17] S. D. Barrett and T. M. Stace, Phys. Rev. Lett. 105, 200502 (2010).
[18] K. Fujii and Y. Tokunaga, Phys. Rev. Lett. 105, 250503 (2010).
[19] Y. Li, S. D. Barrett, T. M. Stace, and S. C. Benjamin,
Phys. Rev. Lett. 105, 250502 (2010).
[20] Y. Li, D. E. Browne, L. C. Kwek, R. Raussendorf, and T. C. Wei, Phys. Rev. Lett. 107, 060501 (2011).
[21] K. Fujii and T. Morimae, Phys. Rev. A 85, 010304(R) (2012).
[22] A. Broadbent, J. Fitzsimons, and E. Kashefi, Proc. of the 50th Annual IEEE Sympo. on Found. of Comput. Sci. 517 (2009).
[23] J. Fitzsimons and E. Kashefi, arXiv:1203.5217.
[24] S. Barz, E. Kashefi, A. Broadbent, J. Fitzsimons, A. Zeilinger, and P. Walther, Science 335, 303 (2012).
[25] V. Dunjko, E. Kashefi, and A. Leverrier, Phys. Rev. Lett. 109, 200502 (2012).
[26] T. Morimae, V. Dunjko, and E. Kashefi, arXiv:1009.3486.
[27] T. Morimae and K. Fujii, Nature Comm. 3, 1036 (2012).
[28] T. Morimae, Phys. Rev. A 87, 050301(R) (2013).
[29] T. Morimae and K. Fujii, Phys. Rev. A 87, 050301(R) (2013).
[30] T. Sueki, T. Koshiba, and T. Morimae, Phys. Rev. A 87, 060301(R) (2013).
[31] V. Giovannetti, L. Maccone, T. Morimae, and T. G. Rudolph, Phys. Rev. Lett. 111, 230501 (2013).
[32] A. Mantri, C. A. Pérez-Delgado, and J. F. Fitzsimons, Phys. Rev. Lett. 111, 230502 (2013).
[33] S. Barz, J. F. Fitzsimons, E. Kashefi, and P. Walther, Nature Phys. 9, 727 (2013).
[34] T. Morimae, Nature Phys. 9, 693 (2013).
[35] R. Raussendorf, S. Bravyi, and J. Harrington, Phys. Rev. A 71, 062313 (2005).
[36] T. H. Kyaw, Y. Li, and L. C. Kwek, arXiv:1307.5140.
[37] S. D. Barrett, S. D. Bartlett, A. C. Doherty, D. Jennings, and T. Rudolph, Phys. Rev. A 80, 062328 (2009).
[38] R. König, R. Renner, and C. Schaffner, IEEE Trans. Inf. Theor. 55, 4337 (2009).
[39] D. Gross, S. T. Flammia, and J. Eisert, Phys. Rev. Lett. 102, 190501 (2009).
[40] A. Shimony, Ann. N. Y. Acad. Sci. 755, 675 (1995).
[41] H. Barnum and N. Linden, J. Phys. A 34, 6787 (2001).
[42] T. C. Wei and P. M. Goldbart, Phys. Rev. A 68, 042307 (2003).
[43] M. Brenner, C. Mora, and A. Winter, Phys. Rev. Lett. 102, 190502 (2009).
[44] E. Knill and R. Laflamme, Phys. Rev. Lett. 81, 5672 (1998).
[45] D. Poulin, R. Laflamme, G. J. Milburn, and J. P. Paz, Phys. Rev. A 68, 022302 (2003).
[46] D. Poulin, R. Blume-Kohout, R. Laflamme, and H. Ollivier, Phys. Rev. Lett. 92, 177906 (2004).
[47] P. W. Shor and S. P. Jordan, Quant. Inf. Comput. 8, 681 (2008).
[48] G. Passante, O. Moussa, C. A. Ryan, and R. Laflamme, Phys. Rev. Lett. 103, 250501 (2009).
[49] S. P. Jordan and P. Wocjan, Quant. Inf. Comput. 9, 264 (2009).
[50] S. P. Jordan and G. Alagic, Proc. of the Sixth Conference on Theory of Quantum Computation, Communication and Cryptography (TQC 2011); arXiv:1105.5100.
[51] T. Morimae, K. Fujii, and J. F. Fitzsimons, arXiv:1312.2496.
[52] S. Toda, SIAM J. Comput. 20, 865 (1991).
[53] A. Ambainis, L. J. Schulman, and U. V. Vazirani, Proc. of the 32nd Ann. ACM Sympo. on Theor. of Comput. pp. 697 (2000).
[54] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Comm. Math. Phys. 115, 477 (1988).
[55] T. Morimae, arXiv:1208.5714.
[56] Note that here we consider the classical simulation of only the output port (i.e., the subsystem $O$). In Ref. [43], the classical simulation of all measurement histories, i.e., measurement results on both $C$ and $O$, is considered. Our result is not changed if we consider such a case, since $\sum_{s \in S} \text{Tr}(P_s \sigma) \leq \frac{|S|}{2N} \geq \lambda_1^{-1} 2^{-1}$, where $S \subseteq \{0, 1\}^N$ is the set of “correct paths”.