Local buckling of box-shaped beams due to skew bending

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Abstract. Local buckling efficiency of different types of beams was reviewed. For beams with non-linear walls and parameterized geometry local buckling analysis was made in ANSYS. Based on the results of ANSYS calculation analytic formula was evaluated in MathCAD using the least square method. Analysis of the influence of different geometrical parameters on local buckling stress value for beams with non-linear walls was completed.

1. Introduction
There are different structures of beams which are used in metal structures. The simplest of these are standard elements. These kinds of beams are usually included in simple structures of buildings and other standard constructions. More complex structures, like box-shaped beams, are usually used in complicated facilities. Such beams have upper and lower flanges, one or two walls and different rigidity-increasing elements such as longitudinal ribs and cross members. Cross members have cut-outs to avoid interference with longitudinal ribs (Figure 1, a). This design makes production of beams more complex and expensive. Simple metal structures do not have box-shaped beams. Usually engineers prefer beams with corrugated walls for these needs [1-4]. Such type of beam is mostly applicable to cargos and facilities. Beams with corrugated walls are not popular solutions for cranes and bridges [5]. It is economically profitable to use corrugated walls in mass production manufacturing, for instance in freight cars (Figure 1, b). A corrugated wall itself is not enough when the load-to-structure is intricate. In this case longitudinal ribs are required (Figure 1, c). However, welding of corrugated walls to longitudinal ribs makes production more complicated and leads to increased labor intensity. Due to this fact it is better to use box-shaped beams for low volume products. Wall cross members and longitudinal beams for box-shaped beams are made from standard straight metal panels. This increases the mass of the beam and quantity of welding seams which results in an increase in production cost. Moreover, hard mode usage of box-shaped beams raises the chances of deformations in welding seams [6].
Figure 1. Types of metal structures: a) box-shaped beam; b) freight cars; c) beam with corrugated wall.

Load is usually applied to the upper flange of the box-shaped beam so the load can make maximum impact on each point of the entire length of the beam. As a result, local buckling can be raised in any area of the beam’s wall. However, there are three calculating schemes for local buckling calculation (Figure 2). In these schemes, local buckling stress depends on normal stress, shear stress, or a combination of normal and shear stresses [7].

Figure 2. Calculation schemes of local buckling stress.

Calculation of local buckling stress for linear walls can be done based on the following formulas [8]:

\[
\sqrt{\left(\frac{\sigma}{\sigma_{cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2} \leq \frac{1}{n};
\]

(1)

\[
\sigma \leq \sigma_{cr} \frac{n}{\sigma_{cr}};
\]

(2)

\[
\tau \leq \tau_{cr} \frac{n}{\tau_{cr}};
\]

(3)

\(\sigma\) and \(\tau\) - normal and shear stresses in a beam, \(\sigma_{cr}\) and \(\tau_{cr}\) - critical normal and shear stresses (when a beam is buckled), \(n\) - safety factor.

Formula (1) is used when a load is applied to \(\frac{1}{4}\) of the beam’s length (Figure 2, c). When a load is applied to the center of the beam (Figure 2, a) only normal stress makes an impact on the local buckling stress. Formula (2) should be used in this case. Formula (3) is used when only shear stress affects due to the load on the ends of the beam (Figure 2, b).

The critical stress is expressed by the formula \(\sigma_{cr} = k_s K_{so} (t/d)^2\), where: \(k_s\) is the buckling factor reflecting the effect of fixing the conditions and distribution of stress across the width of the plate; \(K_{so}\) - the factor having the dimension MPa; \(t\) - plate thickness; and, \(d\) - height of the plate. The length of the plate is included in the determination of \(k_s\).

This calculating scheme was analyzed in the previous article [13]. In that article the influence of the flexion radius on local buckling was reviewed. The formula for calculating the local buckling stress value was found.

The second formula applies the load to the ends of the beam near the fixation areas (Figure 4, b). In this case the local buckling depends on the shear. The stability of the plate at stress is represented by the equation \(\tau \leq \frac{\tau_{cr}}{n=\tau_{cr}}\), where: \(\tau\) - the maximum stress in the plate; \(\tau_{cr}\) - the critical stress of stability; and,
n - the safety factor. The critical stress is expressed by the formula \( \tau_{cr} = k_q K_{so} (t/d)^2 \), where: \( k_q \) is the buckling factor reflecting the effect of fixing the conditions and distribution of shearing stress across the width of the plate; \( K_{so} \) - the factor having the dimension MPa, \( t \) - plate thickness; and, \( d \) - height of the plate. The length of the plate is included in the determination of \( k_q \). In this case a combination of normal stress and shear stress affects local buckling properties [18]. The second calculation scheme will be analyzed in this article.

Flection radius application to the walls of the beams (Figure 3) under the bending load is a solution to exclude cross members and longitudinal ribs. The production method of metal plates with a radius is not new. It is always used for production of pipes. There are patents which show beam structures with flection radius [9-11]. However, there is no research on how radius affects local buckling stress and how to calculate it for the beams with non-linear walls. Only the LLOYD standard [12] describes how to calculate stress values, but only for one loading scheme.

Figure 3. Box-shaped beam with non-linear walls.

Papers [13-17] show that beams with non-linear walls have a better local buckling performance than box-shaped beams with standard walls under a pressure and a bending load

2. Methods

Previous dependencies (1)-(3) are not applicable to beams with non-linear walls because they do not take into the account the curvature of the walls. An assessment of the local buckling of the beam wall was made in ANSYS Workbench. To do this, a cross-section of the beam was developed. Also, variable geometric parameters were selected (table 1) [19].

After that, a project in ANSYS Workbench was created. Structure of calculation program includes static analysis and linear buckling analysis (Figure 4). In this project the same 3D model with geometric parameters as in the table 1 was created. Previously discussed parameters were selected on the created 3D model. It provided an opportunity to calculate all possible variants of beams geometry [20-28].

Table 1. Geometric parameters

| Geometric parameter | Unit of measurement | Value               |
|---------------------|---------------------|---------------------|
| H                   | mm                  | 1000, 1600, 2000    |
| \( t_b \)           | mm                  | 10, 16, 20, 24      |
| \( t_w \)           | mm                  | 6, 10, 12, 16       |
| b                   | mm                  | 400, 600, 800       |
| \( R_w \)           | mm                  | 2000, 4000, 8000    |
| a                   | mm                  | 1000, 2000, 4000    |
The second step was to select beam material. It was performed in Ansys material manager. As a result, structural steel from Ansys engineering data was selected as the material of the beam. Tensile yield strength of this steel is 250 mPa and tensile ultimate strength is 460 mPa. The last step was to input the type of fixation of the beam (highlighted in pink on Figure 3), select the load method (F on Figure 3) and define the maximum size of the grid elements. Stress and displacement values were also entered as preferable results of the calculation. After that an evaluation of all geometry combinations of beams was completed.

Figure 4. Project schematic of buckling analysis in Ansys.

To analyze the results of the calculation in ANSYS, the received data can be approximated with the following formula

\[ \tau_{cr} = A_0 (X_1)^{a_1} (X_2)^{a_2} (X_3)^{a_3} (X_4)^{a_4} (X_5)^{a_5} \]  
(4)

To do it, value of parameters \(A_0, a_i\) should be found. Firstly, the logarithm of formula (4) should be taken

\[ \log \tau = \log(A_0 \prod_{i=1}^{N} X_i^{a_i}) = \log A_0 + \sum_{i=1}^{N} a_i \cdot \log X_i \]  
(5)

Where \(X_i\) is geometric parameter of beams, for example

\[ X_1 = \frac{t_b}{H}, X_2 = \frac{t_w}{H}, X_3 = \frac{R_w}{H}, X_4 = \frac{a}{H}, X_5 = \frac{b}{H}. \]

Secondly, representing \(Y\) as \(\tilde{y}\), log \(A_0 = a_0\), \(\log X_i = \tilde{x}_i\) formula (5) can be rewritten as:

\[ \tilde{y} = a_0 + a_1 \tilde{x}_1 + a_2 \tilde{x}_2 + \ldots + a_N \tilde{x}_N = a_0 + \sum_{i=1}^{N} a_i \tilde{x}_i \]  
(6)

Now, equation (6) can be solved with the least square method. This method is based on the fact that the difference between values of critical stresses \(y_m\) which were found in Ansys Workbench, and values of critical stresses, which can be found by solving the equation (6), should be approached to 0:

\[ \Delta = \frac{1}{n} \sum_{m=1}^{n} (y_m - \tilde{y}_m)^2 = \frac{1}{n} \sum_{m=1}^{n} \left( y_m - \left( a_0 + \sum_{i=1}^{N} a_i \tilde{x}_{im} \right) \right)^2 \rightarrow \min \]  
(7)

where \(n\) is the quantity of beams variants calculated in Ansys Workbench. Solution of the formula (7) is partial derivative, which should be alike 0:

\[ \frac{\partial \Delta}{\partial a_{0,i}} = 0. \]

Application of this solution to all parameters \(a_i\) can be found from a set of equations:
\[
\begin{align*}
\frac{\partial F}{\partial a_0} = & \sum_{m=1}^{n} \left( y_m - (a_0 + \sum_{i=1}^{N} a_i x_{i,m}) \right) = 0 \\
\frac{\partial F}{\partial a_1} = & \sum_{i=1}^{N} \left( y_m - (a_0 + \sum_{i=1}^{N} a_i x_{i,m}) \right) x_{i,m} = 0 \\
\frac{\partial F}{\partial a_2} = & \sum_{i=1}^{N} \left( y_m - (a_0 + \sum_{i=1}^{N} a_i x_{i,m}) \right) x_{i,m}^2 = 0 \\
\frac{\partial F}{\partial a_N} = & \sum_{m=1}^{n} \left( y_m - (a_0 + \sum_{i=1}^{N} a_i x_{i,m}) \right) x_{N,m} = 0
\end{align*}
\] (8)

Set of equations can be represented as:

\[
\begin{align*}
\sum_{m=1}^{n} y_m = & a_0 \sum_{i=1}^{N} x_{i,m} + a_1 \sum_{i=1}^{N} x_{i,m}^2 + a_2 \sum_{i=1}^{N} x_{i,m}^3 + \ldots + a_N \sum_{i=1}^{N} x_{i,m}^N \\
\sum_{m=1}^{n} y_m x_{1,m} = & a_0 \sum_{i=1}^{N} x_{i,m}^2 + a_1 \sum_{i=1}^{N} x_{i,m}^3 + a_2 \sum_{i=1}^{N} x_{i,m}^4 + \ldots + a_N \sum_{i=1}^{N} x_{i,m}^{N+1} \\
\sum_{m=1}^{n} y_m x_{N,m} = & a_0 \sum_{i=1}^{N} x_{i,m}^N + a_1 \sum_{i=1}^{N} x_{i,m}^{N+1} + a_2 \sum_{i=1}^{N} x_{i,m}^{N+2} + \ldots + a_N \sum_{i=1}^{N} x_{i,m}^{2N} \\
\end{align*}
\] (9)

and rewritten in the matrix form \( \vec{\hat{Y}} = A \cdot \vec{\hat{X}} \), where

\[
\vec{\hat{Y}} = \begin{pmatrix}
\sum_{m=1}^{n} y_m \\
\sum_{m=1}^{n} y_m x_{1,m} \\
\vdots \\
\sum_{m=1}^{n} y_m x_{N,m}
\end{pmatrix},
\quad
A = \begin{pmatrix}
a_0 \\
a_1 \\
\vdots \\
a_N
\end{pmatrix},
\quad
\vec{\hat{X}} = \begin{pmatrix}
\sum_{m=1}^{n} x_{1,m} \\
\sum_{m=1}^{n} x_{2,m} \\
\vdots \\
\sum_{m=1}^{n} x_{N,m}
\end{pmatrix}.
\]

From the matrix equation \( \vec{\hat{Y}} = A \cdot \vec{\hat{X}} \) parameters \( a_i \) can be found as

\[
A = \vec{\hat{X}}^{-1} \cdot \vec{\hat{Y}}
\] (10)

3. Results and Discussion
In ANSYS, 240 beams with different geometrical parameters were calculated. Diagrams of local buckling have the same shape: two big beams on both walls (Figure 5). Beams with linear walls have the same shape. However, beams with non-linear walls have a higher value of critical buckling stress.

Figure 5. Movements diagrams with loss of local stability for different beam geometry.

Evaluations based on the least square method from the previous article section can be used to find the analytic formula for local buckling stress. To do it, calculations were done in MathCad software. Solutions of parameters \( a_i \) from the formula (7) are shown in table 2.

| \( a_i \) | \( a_{0} \) | \( a_{1} \) | \( a_{2} \) |
|---------|----------|---------|---------|
|        | 6.035    | -0.926  | 2.32    |

Table 2. Values of parameters \( a_i \).
Finally, all parameters $a_i$ can be applied to the formula (4) and it can be represented as

$$\tau_{cr} = 1.086 \cdot 10^6 \left( \frac{b}{h} \right)^{0.483} \left( \frac{t_w}{h} \right)^{2.32} \left( \frac{a_h}{h} \right)^{0.41} \left( \frac{t_b}{h} \right)^{0.926}$$

(11)

Also, constant $K_o=0.759$ MPa for structural steels can be added to the formula (7) [8]

$$\tau_{cr} = 1.43 \cdot 10^6 K_0 \left( \frac{b}{h} \right)^{0.483} \left( \frac{t_w}{h} \right)^{2.332} \left( \frac{a_h}{h} \right)^{0.41} \left( \frac{t_b}{h} \right)^{0.926}$$

(12)

The assessed value of formula (11) error can be found as

$$\delta = \frac{\tau_{ansys} - \tau_{cr}}{\tau_{ansys}} \cdot 100\%$$

(13)

Average value of formula (11) error is 8.1%. Distribution of formula (12) results to calculate Ansys stress is represented on Figure 6.

Figure 6. Graphic of ansys and analytic formula (12) buckling stress.

A graph was created to understand how radius makes an impact on the value of buckling critical stress (Figure 7).
Figure 7. Impact of a radius on a stress value for different wall thickness.

4. Conclusions
In this study, we analyzed box-shaped beams with non-linear walls. This type of beam is not frequently used in metals structures but there are patents that describe the layout and geometry of such beams. The advantage of such beams is a larger value of local buckling stress. This property is important for main beams in traveling cranes, so that beams with non-linear walls can improve the robustness of current structures. This is the reason why it is essential to study beams with non-linear walls. Only a finite element method can be used to calculate the stress value of beams with non-linear walls. Mechanic properties variations of these beams are not studied due to the flexion radius increase.

Also, it was defined that local buckling critical force of beams with non-linear walls was bigger than the critical force of the standard box-shaped beams. The value of the critical force increases with the gain of flexions’ radius. The most important result of the study is the development of an analytic formula for local buckling stress. It can be used in earlier stages of metal structure engineering to get estimated values of stress and geometric parameters.

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