Amplification of the coupling strength in a hybrid quantum system

Wei Xiong¹, Yueyin Qiu²,³, Lian-Ao Wu⁴,⁵ and J Q You²,¹

¹ Quantum Physics and Quantum Information Division, Beijing Computational Science Research Center, Beijing 100193, People’s Republic of China
² Department of Physics, Zhejiang University, Hangzhou 310027, People’s Republic of China
³ School of Science, Chongqing University of Posts and Telecommunications, Chongqing 400065, People’s Republic of China
⁴ Department of Theoretical Physics and History of Science, The Basque Country University (EHU/UPV), E-48080 Bilbao, Spain
⁵ IKERBASQUE, Basque Foundation for Science, E-48011 Bilbao, Spain

E-mail: jqyou@zju.edu.cn

Keywords: superconducting circuit, squeezing transformation, nitrogen–vacancy center, strong coupling regime

Abstract

Realization of strong coupling between two different quantum systems is important for fast transferring quantum information between them, but its implementation is difficult in some hybrid quantum systems. Here we propose a scheme to enhance the coupling strength between a single nitrogen–vacancy center and a superconducting circuit via squeezing. The main recipe of our scheme is to construct a unitary squeezing transformation by directly tuning the specifically-designed superconducting circuit. Using the experimentally accessible parameters of the circuit, we find that the coupling strength can be largely amplified by applying the squeezing transformations to the system. This provides a new path to enhance the coupling strengths in hybrid quantum systems.

1. Introduction

Hybrid quantum systems, with the goal of harnessing the advantages of different subsystems to better explore new phenomena and potentially bring about novel quantum technologies (see [1, 2] for a review), can have versatile applications in quantum information. Among various hybrid systems, the nitrogen–vacancy (NV) center in a diamond coupled to a superconducting circuit has attracted special attention (see, e.g. [3–11]), because it has distinct advantages, such as high tunability, long coherence time, and stable energy levels. In addition, superconducting circuits exhibit macroscopic quantum coherence, promise good scalability, and can be conveniently controlled and manipulated via external fields (see, e.g. [12, 13]).

However, the coupling strength between a single NV center and a superconducting circuit is too small to coherently exchange mutual quantum information [6, 7, 9, 14]. One solution to overcome this drawback is the use of an ensemble containing a large number (e.g., \(N \sim 10^{12}\)) of NV centers, where two lowest collective excitation states of the ensemble encode a qubit (i.e., a pseudo-spin). Thus, the coupling strength between the NV center ensemble and the superconducting circuit can be effectively enhanced by a factor of \(\sqrt{N}\) [15–17]. This makes it possible to reach the strong coupling regime of the hybrid system. However, it is difficult for the ensemble to implement direct single-qubit manipulation and also the coherence time is greatly shortened due to the inhomogeneous broadening [18–20]. Therefore, significantly coupling a single NV center to a superconducting circuit has been longed for.

Here we propose an experimentally feasible method to effectively amplify the coupling strength between a single NV center and a superconducting circuit. The main recipe of our scheme is to prepare the unitary one-mode squeezing transformations. After applying these squeezing transformations to the hybrid system, the effective coupling strength can be enhanced by two orders of the magnitude using the experimentally accessible parameters of the circuit.

The methodology dates back to the amplification of Kerr effect [21], where a rather complicated circuit was exploited. Recently, a simpler squeezing transformation circuit has been proposed for the cavity mode to amplify the coupling in an optomechanical system [22], but the generation of the squeezing terms in the system...
Hamiltonian requires an additional driven nonlinear medium. Here we specifically design a superconducting circuit that enables one to engineer the squeezing transformations by directly tuning the circuit.

The paper is organized as follows. Section 2 introduces the Hamiltonian of the proposed hybrid quantum system. In section 3, we design two basic gates by tuning the magnetic flux through the smaller loop of the circuit. In section 4, we use these two basic gates to construct the squeezing operator and then apply the squeezing transformations to amplify the coupling strength between the single NV center and the superconducting circuit. Finally, we give a brief discussion and conclusion in section 5.

2. The hybrid quantum system

We propose a hybrid system which is composed of a superconducting loop embedding a superconducting quantum interference device (SQUID) and encircling a single NV center (see figure 1). Here we consider a symmetric SQUID with identical junction capacitances and Josephson coupling energies, i.e., $C_1 = C_2 = C$, and $E_{J1} = E_{J2} = E_J$. In addition, we suppose that the main loop of the superconducting circuit is fabricated with a non-negligible inductance $L$, while the SQUID loop is small enough to have a negligible inductance. Also, two static magnetic fields in opposite directions are applied, respectively, to the small and main loops. The fluxoid quantization conditions for these two loops are

$$\varphi_1 - \varphi_2 + 2\pi f_s = 0, \quad \varphi_2 - 2\pi f_m + 2\pi IL/\Phi_0 = 0,$$

(1)

where $f_{s(m)} = \Phi_{s(m)}/\Phi_0$ with $\Phi_0 = h/2e$ being the flux quantum, $\varphi_i$ ($i = 1, 2$) is the phase drop across the $i$th Josephson junction in the SQUID, and $I$ is the total circulating current in the main loop.

The kinetic energy of the superconducting circuit corresponds to the electrostatic energy stored in the capacitors [23]:

$$T = \frac{1}{2}C(V_1^2 + V_2^2),$$

where $V_i = (\Phi_0/2\pi)\varphi_i$ is the voltage across the $i$th Josephson junction in the SQUID. Using the fluxoid quantization conditions for these two loops, the kinetic energy can be written as

$$T = \frac{1}{2}C\left(\frac{\Phi_0}{2\pi}\right)^2(\varphi_1^2 + \varphi_2^2)$$

$$\quad = C\left(\frac{\Phi_0}{2\pi}\right)^2[\varphi^2 + (\varphi_f)^2],$$

(2)

where $\varphi \equiv (\varphi_1 + \varphi_2)/2$. We consider a static external flux for $\Phi_0$, so $f_s = 0$. Then, the kinetic energy $T$ is reduced to $T = C(\Phi_0/2\pi)^2\varphi^2$. Also, it follows from equation (1) that

$$I = -\frac{\Phi_0}{2\pi L}(\varphi + \pi f_s - 2\pi f_m).$$

(3)

The inductive energy related to the inductance $L$ is given by

$$U_L = \frac{1}{2}LI^2 = E_L(\varphi + \pi f_s - 2\pi f_m)^2,$$

(4)

where $E_L = \Phi_0^2/(8\pi^2 L)$. When including this inductive energy, the total potential energy of the superconducting circuit is...
\[ U = -E_f (\cos \varphi_1 + \cos \varphi_2) + U_L \]
\[ = -E_f (f_0) \cos \varphi + U_L, \]  
(5)

where \( E_f (f_0) = 2E_f \cos(\pi f_0) \) is the flux-dependent effective Josephson energy. The Lagrangian of the superconducting circuit is \( \mathcal{L} = T - U \). Assigning \( \varphi \) as the canonical coordinate, we have the canonical momentum \( p \equiv \hbar \dot{\varphi} / \partial \varphi = 2C(\phi_0/2\pi)^2 \varphi \). Hence the Hamiltonian of the superconducting circuit is given by

\[ H_{\text{SC}} = E_c n^2 - E_f (f_0) \cos \varphi + E_L (\varphi + \pi f_0 - 2\pi f_m)^2, \]  
(6)

where \( E_c = (2e)^2/2C \) is the charging energy of a single Cooper pair and \( n = -i\partial / \partial \varphi \) is the number operator of Cooper pairs.

An NV center consists of a substitutional nitrogen atom next to a vacancy in the diamond lattice [24]. It has a spin triplet ground state and a zero-field splitting \( D \approx 2.87 \text{ GHz} [25] \) between the sublevels with the spin \( z \) components \( m_s = 0 \) and \( m_s = \pm 1 \). The strain-induced splitting is negligible in comparison with the Zeeman effect [26]. In our proposal, the crystalline axis of the NV center is set as the \( z \) direction. By applying a weak static magnetic field \( B^\text{ext}_z \) along the \( z \) direction, the two degenerate sublevels \( m_s = \pm 1 \) are split due to the Zeeman effect. The sublevels \( m_s = 0 \) and \( -1 \) can be well isolated from other levels by tuning \( B^\text{ext}_z \) and they act as a pseudo-spin. The pseudo-spin Hamiltonian is (we set \( h = 1 \) hereafter)

\[ H_{\text{NV}} = \frac{1}{2} \omega_{\text{NV}} \tau_z, \]  
(7)

where

\[ \omega_{\text{NV}} = D - g_e \mu_B B^\text{ext}_z \]  
(8)

is the energy difference between the lowest two sublevels with \( m_s = 0 \) and \( -1 \), respectively. The corresponding Pauli operators are \( \tau \equiv (\tau_x, \tau_y, \tau_z) \).

As shown in figure 1, a single NV center is located at the coordinate \( z_{\text{NV}} \), starting from the left edge of the main loop and along the \( z \) direction on the midline. The interaction Hamiltonian \( H_{\text{int}} \) of the hybrid system is [27]

\[ H_{\text{int}} = -\frac{1}{\sqrt{2}} g_e \mu_B B_{\text{SC}}^z (z_{\text{NV}}) \tau_x, \]  
(9)

where the magnetic field \( B_{\text{SC}}^z (z_{\text{NV}}) \) is associated with the persistent current in the main loop. According to the Biot–Savart law, \( B_{\text{SC}}^z (z_{\text{NV}}) \) can be written as

\[ B_{\text{SC}}^z (z_{\text{NV}}) = IB_0(z_{\text{NV}}), \]  
(10)

where

\[ B_0(z_{\text{NV}}) = \frac{\mu_0}{4\pi} \left[ \frac{l^2 + 2z_{\text{NV}}^2}{l z_{\text{NV}} \sqrt{(l/2)^2 + z_{\text{NV}}^2}} \right. \]
\[ + \left. \frac{3l^2 - 4lz_{\text{NV}} + 2z_{\text{NV}}^2}{l(l - z_{\text{NV}}) \sqrt{(l - z_{\text{NV}})^2 + (l/2)^2}} \right] \]  
(11)

The total Hamiltonian \( H \) of the hybrid quantum system is given by \( H = H_{\text{SC}} + H_{\text{NV}} + H_{\text{int}} \).

### 3. Two basic gates

We tune the external magnetic field \( B^\text{ext}_z \) to have \( \omega_{\text{NV}} = 0 \), so as to achieve the two basic gates for constructing squeezing operations. Denote \( \omega_{\text{sc}} \) as the transition frequency between the lowest two energy levels of the superconducting circuit and \( g \) as the coupling strength between the single NV center and the superconducting circuit. Now the two subsystems become effectively decoupled due to \( |g / (\omega_{\text{sc}} - \omega_{\text{NV}})| = |g / \omega_{\text{sc}}| \ll 1 \). Also, we tune the two external magnetic fields in opposite directions to satisfy \( \Phi_m - \Phi_o = 0 \). Because \( \omega_{\text{NV}} = 0 \) and \( |g / \omega_{\text{sc}}| \ll 1 \), the total Hamiltonian can be approximately written as

\[ H \approx H_{\text{sc}} = E_c n^2 - E_f (f_0) \cos \varphi + E_L \varphi^2. \]  
(12)

Note that if \( L \to 0 \), \( E_L \to \infty \), so it is required that \( \varphi \to 0 \) in equation (12). However, \( L \neq 0 \) for a realistic circuit. Thus, in this nonzero \( L \) case, the phase drop \( \varphi \) is not constrained by the loop inductance but mainly by the effective Josephson energy of the SQUID.

By tuning the magnetic flux in the SQUID loop (now denoted as \( \Phi_0^{(0)} \) to \( \Phi_0^{(0)} = \Phi_0 / 2 \), i.e., \( f_0^{(0)} \equiv \Phi_0^{(0)} / \Phi_0 = 1/2 \), one has \( E_f (f_0^{(0)}) = 0 \), so the Hamiltonian in equation (12) is reduced to a harmonic
oscillator

\[ H_0 = E_c n^2 + E_L \varphi^2. \]  

(13)

In second quantization,

\[ \varphi = \sqrt{\frac{1}{2m\omega_0}} (a + a^\dagger), \quad n = i \sqrt{\frac{m\omega_0}{2}} (a^\dagger - a), \]  

(14)

where \( m = 1/(2E_c) \), and

\[ \omega_0 = 2\sqrt{E_c E_L} \]  

(15)

is the angular frequency of the harmonic oscillator. The creation (annihilation) operator \( a^\dagger (a) \) obeys the bosonic commutation relation \([a, a^\dagger] = 1\), and the Hamiltonian in equation (13) can be written as

\[ H_0 = \omega_0 a^\dagger a. \]  

(16)

Evolving the hybrid system for a time \( t \), a quantum gate

\[ U_0(t) \equiv e^{-iH_0 t} = e^{-i\omega_0 a^\dagger a t} \]  

(17)

is achieved.

Here we consider a circuit with \(|E_1(f_s)/E_s| \gg 1\). For this circuit, we can define a quantity \( \alpha \) to characterize its anharmonicity:

\[ \alpha = \frac{E_{12} - E_{01}}{E_{01}}, \]  

(18)

where \( E_{01} \) is the energy level difference between the ground state energy \( E_0 \) and the first excited state energy \( E_1 \) of the circuit and \( E_{12} \) is the energy level difference between the first and second excited states energies \( (E_1 \) and \( E_L \) of the circuit. We use \( \alpha_f \) to denote the relative anharmonicity of the full Hamiltonian \( H_{\text{full}} \) in equation (12). Note that the phase \( \varphi \) is constrained to be small for the circuit with \(|E_1(f_s)/E_s| \gg 1\), so we can write

\[ \cos \varphi \approx 1 - \varphi^2/2 + \varphi^4/4! \]  

as a good approximation. Then, the Hamiltonian \( H_{\text{full}} \) in equation (12) is reduced to

\[ H_1 \approx E_c n^2 + \frac{1}{2} [2E_L + E_j(f_s)] \varphi^2 - \frac{1}{4!} E_j(f_s) \varphi^4. \]  

(19)

For this approximated Hamiltonian, we use \( \alpha_f \) to denote its relative anharmonicity. In figure 2(a), we show the lowest three energy levels of the circuit as a function of the normalized magnetic flux \( f_s \) in the SQUID loop, where the solid and dotted curves are calculated using the Hamiltonians in equations (12) and (19), respectively. The parameters are chosen to be \( E_L = 0.12 \) GHz, \( E_j = 58 \) GHz, and \( E_L = 58.6 \) GHz (corresponding to \( L = 1.4 \) nH [28]). In figure 2(b), we also show the dependence of the relative anharmonicity \( \alpha_f (\alpha_0) \) on the normalized magnetic flux \( f_s \). From these results, we can see that the approximate Hamiltonian in equation (19) well matches the Hamiltonian in equation (12).

Away from \( \Phi_c^{(0)} = \Phi_0/2 \), where the gate \( U_0(t) \) is achieved, we again tune the magnetic flux in the SQUID loop (now denoted as \( \Phi_c^{(1)} \)) to, e.g., \( \Phi_c^{(1)} \approx 0.9\Phi_0 \) (i.e., \( f_s^{(1)} \equiv \Phi_c^{(1)}/\Phi_0 = 0.9 \)) to obtain another quantum gate. As shown in figure 2, this flux is sufficiently away from \( \Phi_c^{(0)} \), and the Hamiltonian (12) can be well approximated by equation (19) at the flux \( \Phi_c^{(1)} \approx 0.9\Phi_0 \). Also, the Hamiltonian (19) has a larger relative anharmonicity at this flux. In second quantization, the quartic anharmonicity \( \varphi^4 \) in equation (19) corresponds to the Duffing terms [29] \((a + a^\dagger)^4\), where the main contributions arise from the double-photon scattering processes, \( a^\dagger a a^\dagger a \) and \( a^\dagger a^\dagger a a \). We neglect the high-order four-photon scattering processes \((a^\dagger)^4 \) and \( a^4 \), and use a mean-field approximation [30] \( a^\dagger a \sim \langle a^\dagger a \rangle = N_a \), where \( N_a = \left[ \exp \left( \omega_{\omega e}/k_B T \right) - 1 \right]^{-1} \) under the thermal equilibrium. At a very low temperature \( T \) (e.g., \( \sim 20 \) mK), \( \omega_{\omega e}/k_B T \gg 1 \) and therefore \( N_a \approx 0 \). The Hamiltonian (19) can then be reduced to

\[ H_1 = \omega_1 a^\dagger a - \eta_1 (a^2 + a^\dagger^2), \]  

(20)

where

\[ \omega_1 = \sqrt{2E_c [2E_L + E_j(f_s^{(1)})]} - \eta_0, \]

\[ \eta_1 = \frac{1}{4} \beta(f_s^{(1)})E_j(f_s^{(1)}), \]  

(21)

with

\[ \beta(f_s^{(1)}) = \frac{E_c}{2[2E_L + E_j(f_s^{(1)})]} \]  

(22)
Obviously, the two parameters $\omega_1$ and $\eta_1$ are both controllable by the magnetic flux $\Phi_s$. Owing to the presence of the inductance $L$, $E_J(f_s)$ can reach the regime of $E_J(f_s) < 0$ for a harmonic oscillator, where we only ensure $E_1 + \frac{1}{2}E_J(f_s) \geq 0$. However, the oscillator becomes unstable when $E_1 + \frac{1}{2}E_J(f_s) < 0$.

With the Hamiltonian in equation (20), by evolving the hybrid system for a time $t$, another quantum gate is then obtained. Note that a series of quantum gates are used to achieve the coupling amplification between the single NV center and the superconducting circuit (see the next section). To have a high fidelity for each quantum gate, sudden switching between successive gates is needed. In the present case, one should be able to fast tune the magnetic flux in the SQUID loop. Currently, it is easy to implement such sudden switch as quickly as in just $\sim 1$ ns using conventional techniques (see, e.g. [31]). With fast developing quantum technologies, much quicker sudden switch is expected to be implementable.

4. Amplification of the coupling strength

4.1. Squeezing operator
To enhance the coupling between the single NV center and the superconducting circuit, we need to construct a photon-squeezing operator using the two propagators in equations (17) and (23). With the annihilation operator $a$ and the creation operator $a^\dagger$, we can define three operators...
with commutation relations

\[ [\Gamma_1, \Gamma_2] = -2i\Gamma_3, \quad [\Gamma_2, \Gamma_3] = 2i\Gamma_1, \quad [\Gamma_3, \Gamma_1] = 2i\Gamma_2. \]

Therefore, three new operators in equation (24) can be regarded as the three generators of SU(1, 1) group that is non-compact and does not have any finite unitary representation.

Following the method used in [21], we can write these operators, in a simple two-dimensional non-Hermitian representation, as

\[ \Gamma_1 = i\tau_x, \quad \Gamma_2 = -i\tau_x, \quad \Gamma_3 = \tau_z, \]

where \( \tau = (\tau_x, \tau_y, \tau_z) \) are Pauli matrices, and then

\[
\begin{align*}
\exp(-i\gamma_3 \Gamma_1 t) &= \cos(\gamma_3 t) - i\gamma_3 \sin(\gamma_3 t), \\
\exp(-i\gamma_2 \Gamma_2 t) &= \cosh(\gamma_2 t) - i\gamma_2 \sinh(\gamma_2 t), \\
\exp(-i\gamma_1 \Gamma_3 t) &= \cosh(\gamma_1 t) - i\gamma_1 \sinh(\gamma_1 t),
\end{align*}
\]

where \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) are three parameters. Also, the propagators in equations (17) and (23) can be rewritten, respectively, as

\[ U_0(t) = \exp(-i\omega_0 \Gamma_1 t), \quad U_1(t) = \exp(-i\omega_1 \Gamma_3 t + i2\eta_\Gamma \Gamma_1 t). \]

Then, we can employ these two propagators to generate another propagator \( U(t) = \exp(i2\eta_\Gamma \Gamma_1 t) \), which can be approximately constructed using

\[ U^t_0(t) = \lim_{M \to \infty} [U_0^t(t'/M) U_1(t/M)]^M, \]

where \( t' = \omega_0 t / \omega_1 \), and \( M \) is the operation times of the gate \( U_0(U_1) \).

Considering the specific parameters used in figure 2, we numerically compare the four matrix elements of \( U_0^t \) with those of \( U_1 \) and find that \( U_0^t \) can be well approximated to \( U_1 \) in the regime of \( t/M \leq 0.15 \) (see figure 3). Once obtaining the propagator \( U_0^t \), we can combine it and the achieved propagator \( U_0 \) to produce the desired squeezing operator.
Without coupling amplification, the coupling strength $g$ between a single NV center and a superconducting circuit can reach $g \approx 2 \pi \times 10 \text{ kHz}$ in our proposed hybrid quantum system. This value of the coupling strength was also estimated in [5]. Experimentally, the coupling strength between a single NV center and the superconducting circuit was reported to be $g \approx 8.8 \text{ kHz}$ in [7] and $g \approx 4.4 \text{ kHz}$ in [9]. Note that either the theoretically estimated or experimentally achieved value of the coupling strength is larger than the decoherence rate of a single NV center (which is about $\gamma_{\text{NV}} \sim 1 \text{ kHz}$ in [32]), but it is still too weak in comparison with the decoherence rate of the superconducting circuit (which is $\gamma_{\text{s}} \sim 1 \text{ MHz}$ in [7]). This indicates that the coupling between the single NV center and the superconducting circuit is in the weak-coupling regime, and the decoherence time of this hybrid system is limited by the decoherence time of the superconducting circuit (i.e., $T_{\text{sc}} = 1/\gamma_{\text{sc}} \sim 1 \mu\text{s}$ for $\gamma_{\text{sc}} \sim 1 \text{ MHz}$ [7]).

5. Discussion and conclusion

Outside the time periods for achieving squeezing transformations $S$ and $S^\dagger$, we tune $\omega_{\text{NV}}$ to be nonzero to satisfy the near-resonance condition $\omega_{\text{NV}} \sim \omega_{s}$. In contrast to the sudden switching between successive quantum gates, the tuning of $\omega_{\text{NV}}$ to satisfy the near-resonance condition $\omega_{\text{NV}} \sim \omega_{s}$ should be adiabatic. For the NV center given in equations (7) and (8), this adiabatic process can still be achievable by fast tuning the magnetic field on the NV center, because the level difference has a simple linear dependence on the applied magnetic field and no level anticrossing occurs there.

During this period of near-resonance, the coupling between the single NV center and the superconducting circuit becomes important. Also, the magnetic flux in the SQUID loop remains at $\Phi_s^{(0)}$. Therefore, the total Hamiltonian $H_{\text{tot}}$ of the hybrid system reads

$$H_{\text{tot}} = \omega_0 a^\dagger a + \frac{1}{2} \omega_{\text{NV}} \tau_z + g (a^\dagger + a) \tau_x,$$

where

$$g = \frac{g_s \mu_B \Phi_0 (2 \omega_{\text{NV}}) [\beta (f_{s}^{(0)})]^{1/4}}{2 \sqrt{2} \pi L}.$$

To estimate the value of the coupling strength $g$, we choose the experimentally accessible parameters of the superconducting circuit as in figure 2, i.e., $E_c = 0.12 \text{ GHz}$, $E_j = 58 \text{ GHz}$, and $L = 1.4 \text{ nH}$. Using $\omega_{\text{NV}} = 0.01 \text{ MHz}$ and $f_{s}^{(0)} = 0.5$, we have $g \sim 2 \pi \times 10 \text{ kHz}$.

Applying the unitary squeezing transformations $S$ and $S^\dagger$ to the Hamiltonian $H_{\text{tot}}$, we obtain an effective Hamiltonian for the hybrid system,

$$H_{\text{eff}} = SH_{\text{tot}}S^\dagger = \omega_0 a^\dagger a + \frac{1}{2} \omega_{\text{NV}} \tau_z + \chi (a^\dagger + a) \tau_x,$$

where $\omega_{\text{eff}} = \omega_0 \cosh (4 \eta_2)$ is the transformed frequency of the circuit, $\chi = \frac{1}{2} \omega_0 \sinh (4 \eta_2)$ is the strength for squeezing photons, and

$$g_{\text{eff}} = g \exp (2 \eta_2),$$

is the effective coupling strength between the single NV center and the superconducting circuit. Obviously, it is enhanced exponentially by a factor of $2 \eta_2$.

In figure 4, we plot the effective coupling strength $g_{\text{eff}}$ versus the normalized magnetic flux $f_s$ for different ratios of $E_j/E_c$ where $E_c = 0.12 \text{ GHz}$ and $t = 1 \text{ ns}$. It shows that the coupling enhancement is very sensitive to the ratio of $E_j/E_c$. Comparing four curves in figure 4, we can see that the coupling strength between the single NV center and the circuit can be enhanced by two orders of the magnitude. Namely, the enhanced coupling strength can approach a few megahertz. Also, it can be further improved by prolonging time or using larger $E_c$, but a long time requires more operation times.
For the superconducting circuit system governed by the Hamiltonian (20), the energy difference of the lowest two levels is about \( \Delta E = 1.5 \text{ GHz} \) at the point \( f_s^{(1)} \) (see figure 2(a)); the corresponding characteristic time of the system is \( T_c = 1/\Delta E \approx 0.67 \text{ ns} \). As demonstrated in [32], the typical \( \pi/2 \)- and \( \pi \)-pulse durations of manipulating an NV center are 15 ns and 30 ns, respectively, which are much longer than the characteristic time \( T_c \). Thus, manipulating the frequency of an NV center \( (\omega_{NV}) \) in resonance with the frequency of the superconducting circuit \( (\omega_{sc}) \) can be nearly adiabatic. Moreover, the typical pulse durations [32] are much shorter than the decoherence time of the superconducting circuit. Tuning \( \omega_{NV} \) to be resonant with \( \omega_{sc} \) can be implemented before the system decoheres. In our superconducting circuit, the Josephson coupling energy \( E_J \) should be larger than the charging energy \( E_c \) and a superconducting loop is introduced. These characteristics are analogous to those of a flux qubit. A recent experiment [33] shows that the decoherence time of the flux qubit can be increased to 85 \( \mu \text{ s} \) when shunting a large capacitor to the smaller Josephson junction of the circuit to reduce the effect of the charge noise [34]. This idea can be applied to the superconducting circuit here to improve its quantum coherence.

It is shown in figure 4 that the effectively coupling strength \( g_{\text{eff}} \) can be enhanced by two orders of the magnitude when using squeezing transformations. For example, given \( g \sim 2\pi \times 10 \text{ kHz} \), when the parameters in figure 4 are used, \( g_{\text{eff}} \) is enhanced to \( \sim 2\pi \times 4 \text{ MHz} \) at \( f_s \sim 1 \). It has reached a few megahertz.

In conclusion, we have proposed an experimentally feasible method to effectively enhance the coupling strength between a single NV center and a superconducting circuit. The main recipe of our scheme is to use the unitary squeezing transformations constructed by system evolution. This idea dates back to the amplification of Kerr effect and it can provide a new path to enhance the coupling strengths in hybrid quantum systems.

Acknowledgments

This work is supported by the National Key Research and Development Program of China (Grant No. 2016YFA0301200), the National Natural Science Foundation of China (Grant No. 11774022 and Grant No. 11404019), the National Basic Research Program of China (Grant No. 2014CB921401), the NSAF (Grant No. U1530401), and the Postdoctoral Science Foundation of China (Grant No. 2016M60905). L.A.W. is supported by the Basque Government (grant IT472-10) and the Spanish MICINN (Project No. FIS2012-36673-C03-03).

WX and YQ contributed equally to this work.

References

[1] Xiang Z.L., Ashhab S, You J Q and Nori F 2013 Hybrid quantum circuits: superconducting circuits interacting with other quantum systems Rev. Mod. Phys. 85 623
[2] Kurizki G, Bertet P, Kubo Y, Mohler K, Petrosyan D, Rahib P and Schmiedmayer J 2015 Quantum technologies with hybrid systems Proc. Natl Acad. Sci. USA 112 3866
[3] Kubo Y et al 2010 Strong coupling of a spin ensemble to a superconducting resonator Phys. Rev. Lett. 105 140502
[4] Schuster D I et al 2010 High-cooperativity coupling of electron-spin ensembles to superconducting cavities Phys. Rev. Lett. 105 140501
[5] Kubo Y et al 2011 Hybrid quantum circuit with a superconducting qubit coupled to a spin ensemble Phys. Rev. Lett. 107 220501

Figure 4. The effective coupling strength \( g_{\text{eff}} \) versus the normalized magnetic flux \( f_s \) for different ratios of \( E_L/E_s \), where \( E_c = 0.12 \text{ GHz} \) and the time \( t \) in equation (31) is chosen as \( t = 1 \text{ ns} \).
[6] Marcos D, Wubs M, Taylor J M, Aguado R, Lukin M D and Serensen A S 2010 Coupling nitrogen-vacancy centers in diamond to superconducting flux qubits Phys. Rev. Lett. 105 210501

[7] Zha X et al 2011 Coherent coupling of a superconducting flux qubit to an electron spin ensemble in diamond Nature 478 221

[8] Hoffman J E et al 2011 Atoms talking to SQIDs Rev. Mex. Fis. S 57 1

[9] Saito S, Zha X, Amusia R, Matsuzaki Y, Kakuyanagi K, Shimo-oka T, Mizuochi N, Nemoto K, Munro W J and Semba K 2013 Towards realizing a quantum memory for a superconducting qubit: storage and retrieval of quantum states Phys. Rev. Lett. 111 107008

[10] Twamley J and Barrett S D 2010 Superconducting cavity bus for single nitrogen-vacancy defect centers in diamond Phys. Rev. B 81 241202

[11] Qiu Y, Xiong W, Tian L and You J Q 2014 Coupling spin ensembles via superconducting flux qubits Phys. Rev. A 89 042321

[12] You J Q and Nori F 2011 Atomic physics and quantum optics using superconducting circuits Nature 474 589

[13] Wendin G 2017 Quantum information processing with superconducting circuits: a review Rep. Prog. Phys. 80 106001

[14] Jin P Q, Marthaler M, Shnirman A and Schön G 2012 Strong coupling of spin qubits to a transmission line resonator Phys. Rev. Lett. 108 190506

[15] Raizen M G, Thompson R J, Brecha R J, Kimble H J and Carmichael H J 1989 Normal-mode splitting and linewidth averaging for two-state atoms in an optical cavity Phys. Rev. Lett. 63 240

[16] Petersson K D, McFaul L W, Schroer M D, Jung M, Taylor J M, Houck A A and Petta J R 2012 Circuit quantum electrodynamics with a spin qubit Nature 490 380

[17] Rose B C, Tsyryshkin A M, Riemann H, Abrosimov N V, Becker P, Polh H-J, Thewalt M L W, Itoh K M and Lyon S A 2017 Coherent Rabi dynamics of a superradiant spin ensemble in a microwave cavity Phys. Rev. X 7 011002

[18] Dobrovitski V V, Feiguin A E, Awschalom D D and Hanson R 2008 Decoherence dynamics of a single spin versus spin ensemble Phys. Rev. B 77 245212

[19] Westonberg J H, Ardavan A, Briggs G A D, Morton J J L, Schoelkopf R J, Schuster D I and Mølmer K 2009 Quantum computing with an electron spin ensemble Phys. Rev. Lett. 103 070502

[20] Julsgaard B, Grezes C, Bertet P and Mølmer K 2013 Quantum memory for microwave photons in an inhomogeneously broadened spin ensemble Phys. Rev. Lett. 110 259503

[21] Bartkowiak M, Wu L A and Miranowicz A 2014 Quantum circuits for amplification of Kerr nonlinearity via quadrature squeezing J. Phys. B: At. Mol. Opt. Phys. 47 145501

[22] Li X Y, Wu Y, Johansson J R, Jing H, Zhang J and Nori F 2015 Squeezed optomechanics with phase-matched amplification and dissipation Phys. Rev. Lett. 114 093602

[23] Makhlin Y, Schön G and Shnirman A 2001 Quantum-state engineering with Josephson-junction devices Rev. Mod. Phys. 73 357

[24] Doherty M W, Manson N B, Delaney P, Jelezko F, Wrachtrup J and Hollenberg L C L 2013 The nitrogen-vacancy colour centre in diamond Phys. Rep. 528 1

[25] Loubser J H N and van Wyk J A 1978 Electron spin resonance in the study of diamond Rep. Prog. Phys. 41 1201

[26] Neumann P et al 2009 Excited-state spectroscopy of single NV defects in diamond using optically detected magnetic resonance New. J. Phys. 11 013017

[27] Xiang Z L, Li X Y, Li T F, You J Q and Nori F 2013 Hybrid quantum circuit consisting of a superconducting flux qubit coupled to a spin ensemble and a transmission-line resonator Phys. Rev. B 87 144516

[28] Sullivan D F, Dutta S K, Dreyer M, Gubrud M A, Roychowdhury A, Anderson J R, Lobb C J and Wellstood F C 2013 Asymmetric superconducting quantum interference devices for suppression of phase diffusion in small Josephson junctions J. Appl. Phys. 113 183905

[29] Li X Y, Liao J Q, Tian L and Nori F 2015 Steady-state mechanical squeezing in an optomechanical system via Duffing nonlinearity Phys. Rev. A 91 013834

[30] Genes C, Vitali D, Tombesi P, Gigan S and Aspelmeyer M 2008 Ground-state cooling of a micromechanical oscillator: comparing cold damping and cavity-assisted cooling schemes Phys. Rev. A 77 033804

[31] Silveri M P, Kumar K S, Tuorila J, Itoh K M, Vepsäläinen A, Thuneberg E V and Paraoanu G S 2015 Stückelberg interference in a superconducting qubit under periodic latching modulation New. J. Phys. 17 043058

[32] Gaebel T et al 2006 Room-temperature coherent coupling of single spins in diamond Nat. Phys. 2 408

[33] Yan F et al 2016 The flux qubit revisited to enhance coherence and reproducibility Nat. Commun. 7 12964

[34] You J Q, Hu X, Ashhab S and Nori F 2007 Low-decoherence flux qubit Phys. Rev. B 75 140515