K-S Test for Crack Increment in Probabilistic Fracture Mechanics Analysis

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Abstract. Crack increment is a phenomenon in fracture mechanics. It is occurred due to the stress concentration at the imperfection material. Thus, it leads a crack to growth. Then, the crack will reach to a critical crack length before catastrophic failure could occur. Before the catastrophic failure occur, the cracked structure can be fully utilised until the crack reach to the critical crack length. Thus, it is crucial to investigate the behaviour of the crack increment in fracture mechanics. The main purpose of this paper is to model the crack increment in fracture mechanics analysis via Kolmogorov-Smirnov test. The modelling requires a collection of crack data through experimental work. Then, the data was evaluated based on Kolmogorov-Smirnov test. The results show that the crack increment can be modelled as a Gaussian distribution.

1. Introduction
Crack might occurs in engineering applications especially for aging structures [1]. Plus with the imperfection of material [2] during the manufacturing process, can lead to the initiation of the crack. Once the crack initiated, crack will growth before catastrophic failure will occur [3]. The period from crack growth to the catastrophic failure occur is the main loophole in fracture mechanics. Designer could use the period as an advantage to optimize the maintenance cost [4]. Prioritize the maintenance schedule while waiting the crack growth to reach critical length before catastrophic failure could occur.

Thus, the collection of crack increment data is crucial since the process could lead to reduce the maintenance cost. Moreover, representation of the data in statistical type of distribution will enhance the simulation analysis in engineering application. Thus, the main purpose of this paper is to model the crack increment via statistical analysis such as Kolmogorov-Smirnov (K-S) test [5].

2. Methodology
The K-S test was implemented in this paper. Before the implementation of K-S test took place, one need to collect data. Then, from the collection of the data, K-S test can be conducted. In this paper, the collection of data means the data for crack increment from beach mark to other beach mark. The crack increment was collected from experimental work. Thus, in this section, the experimental work will be explained. Then, followed by K-S test of the data.
2.1. Experimental Work
The experimental work starts with preparation of specimens. Specimens were prepared according to the dimension as shown in Figure 1 and the selection material was Aluminium Alloy 7075-T6. The initial surface crack was introduced using wire cut process to a specimen with a protuberance. Then, the specimen with a protuberance was loaded with a cyclic load. A cyclic load was stopped when the crack length grew to 2 mm from the protuberance. Then, the protuberance was milled off and the deeper surface crack remained in the work piece.

![Figure 1. Specimen dimension with crack surface.](image)

Then, the specimen was loaded under fatigue loading through servo hydraulic machine. As shown in Figure 2, the specimen was carried out based on the four-point bending fatigue test. Two stress ratio were used in this study (0.1 and 0.8). Stress ratio 0.8 for benchmark marking process. Stress ratio 0.1 for propagation of the crack. The mean load was setup at 24.75 kN and 45kN was maximum load with 20 Hz frequency.

The fatigue test was conducted until the specimen was fractured as shown in Figure 3. Once the specimen fractured, the benchmark will be appeared on the crack surface of the specimen. Figure 4 shows the benchmark on the surface of a crack. The crack length and depth were obtained from the measurement of the benchmark using Baty Vision System - Venture System. Then the fatigue test was repeated to collect the crack increment data. Thus, the calculation of mean and standard deviation of crack increment were calculated.
Figure 2. Four-point bending fatigue test using servo hydraulic machine.

Figure 3. Specimen fractured.

Figure 4. Beachmark on the surface of a crack.

2.2. Kolmogorov-Smirnov Test
The K-S test [6-7] was utilised to analyse a sample with a reference probability distribution. In this paper, the sample is referred to crack increment data and the reference probability distribution was selected as Gaussian distribution as an initial selection.
The K-S test will measure the difference among the empirical cumulative distribution function (ECDF) of the sample with the reference cumulative distribution function. In this case, the reference cumulative distribution function is Gaussian as shown in Table 1.

| Point | Mean (mm) | Standard deviation (mm) | Assumed type of distribution | Maximum difference, \( D_p \) |
|-------|-----------|-------------------------|------------------------------|-----------------------------|
| Z'    | 0.790     | 0.220                   | Gaussian                     | 0.0505                      |
| Z     | 0.750     | 0.200                   | Gaussian                     | 0.0460                      |
| Y     | 0.090     | 0.025                   | Gaussian                     | 0.0632                      |

Once the highest distance was obtained from the comparison between the ECDF and reference cumulative distribution function, the maximum distance was compared with the standard table of \( \alpha \) in K-S test. If the value is less than the permitted tolerance, then the referred type of distribution is acceptable.

3. Result and Discussion
The comparison between mathematical theoretical calculations with finite element software called Probabilistic S-version Finite Element Model (ProbS-FEM) as shown in Figure 5, Figure 6 and Figure 7. The mathematical calculations were based on the ECDF. For the assumed distribution, the ECDF is like an optimal CDF line. For example, as shown in Table 1, pretend that the crack depth increment fits a Gaussian distribution. Therefore, the ECDF (Gaussian CDF in this case) was a step function that has a step of one over total samples at each of the observed data points.

Furthermore, the dashed line presents the bounds of the ProbS-FEM’s results. It is for the reliability of the results. As the mathematical calculations were laid in between the bounds, it is sufficient to rely on developed software. As the ECDFs for Figure 5, Figure 6 and Figure 7 are in between the bounds, thus the developed method had shown its accuracy.

![Figure 5. Crack depth at Y via K-S test.](image-url)
Figure 6. Crack depth at Z' via K-S test.

Figure 7. Crack depth at Z via K-S test.

4. Conclusion
The crack increment in engineering material could be modelled as a Gaussian distribution. The result from K-S test shows that the crack increment data was fitted with Gaussian distribution.

5. References
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