Non-Hermitian quantum many-body scar

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(Dated: February 18, 2022)

Ergodicity depicts a thorough exploration of phase space to reach thermal equilibrium for most isolated many-body systems. Recently, the discovery of persistent revivals in the Rydberg-atom quantum simulator has revealed a weakly ergodicity breaking mechanism dubbed as quantum many-body scars, which are a set of nonthermal states storing the initial quantum information through long-time dynamics in the otherwise thermal spectrum. However, until now, the quantum scars have only been studied in closed systems with Hermiticity. Here, we establish the non-Hermitian quantum many-body scars in open systems and characterize their nature from the entanglement entropy, the physical observable and the energy level statistics. Notably, these signatures of the non-Hermitian scarred states switch from the real-energy axis to the imaginary-energy axis after a real-to-complex spectra transition, where an exceptional point and a quantum tricritical point emerge simultaneously. We further examine their stability against external fields and establish the whole phase diagram. In the real-spectrum region, we reveal that the non-Hermitian scars are more fragile than Hermitian counterparts when increasing the strength of non-Hermiticity. Our results may offer insights on ergodicity-breaking and clues for realizing long-lived coherent states in open quantum many-body systems.

Introduction.— The eigenstate thermalization hypothesis (ETH) governs an ergodic isolated quantum many-body system to locally evolve into equilibrium statistical ensemble [1–4], and plays a fundamental role in bridging quantum physics and statistical mechanics. However, in the presence of an extensive number of conserved quantities, such as the integrable systems [5–8] and many-body localized (MBL) phase [9, 10], the systems fail to thermalize and thus strongly deviate from ETH. In contrast, the quantum many-body scar (QMBS) systems [11–15], which have much fewer numbers of conserved quantities and are free of disorder, exhibit a distinct ergodicity breaking mechanism [16–41]. The QMBS system consists of both thermal and nonthermal eigenstates, and is distinguished by specific initial states experiencing periodic revivals, as first observed in an ultra-cold platform based on Rydberg atoms [42].

The above ergodicity and its breakdown are mainly considered for ideal isolated systems. Nevertheless, in many realistic situations, the systems inevitably contact thermal or nonthermal environments, raising the same issues with equal significance for the open systems. The system coupled to a thermal bath usually relaxes to a Gibbs ensemble with the same temperature as the bath [43–46], while the system coupled to a nonthermal bath may host distinct thermalization mechanisms [47, 48] due to the arbitrary nonunitary process. Unlike thermalization, the study of the ergodicity breaking in open systems, or more specifically, in non-Hermitian systems, remains in the early stage. For the strong ergodicity breaking, such as the non-Hermitian integrable systems and the non-Hermitian MBL phases [49, 50], the systems persist in showing the Poisson distribution of the uncorrelated complex random variables, in contrast to the Ginibre distribution in non-Hermitian ergodic systems [51–55]. Nevertheless, whether the weak ergodicity breaking exists in the presence of non-Hermiticity is still elusive both theoretically and experimentally.

In this Letter, we propose a weak ergodicity breaking mechanism in open systems from QMBS in a non-perturbative way, named the non-Hermitian QMBS, and characterize their nature via the entanglement entropy, the physical observable and the energy-level statistics using the bi-orthogonal eigenstates. Interestingly, we find that the fingerprints of non-Hermitian scarred states switch from the real-energy axis to the imaginary-energy axis after a real-to-complex spectra transition, where an exceptional point and a quantum tricritical point emerge simultaneously.

![Phase diagram](image_url)

FIG. 1. (Color online.) Phase diagram. We identify three distinct phases as a function of the external fields \( \gamma \) and \( h \): the phase (I) with non-Hermitian many-body scarred states and real energy spectra, the \( \mathbb{Z}_2 \) symmetry breaking phase (II) and the phase (III) with complex energy spectra. The triangles and circles are critical points of continuous and first-order quantum phase transitions, respectively. The red star denotes the quantum tricritical point. Here we consider systems with \( N = 26 \) and data steps \( dh = 0.005, d\gamma = 0.005 \).
phase diagram as a function of the model parameters. We subsequently examine the stability of the non-Hermitian QMBS in the presence of external fields, reveal the non-Hermitian quantum criticality, and finally establish the whole phase diagram, as illustrated in Fig. 1. We find that the non-Hermitian QMBS become fragile as the non-Hermiticity strength increases in the real-spectrum region, but they still exhibit substantial stability. Our construction of the non-Hermitian QMBS may also be probed experimentally based on the newly developed measurement of the non-Hermiticity in ultracold-atom platforms [42, 56–62].

Theoretical Setup.— Before establishing the non-Hermitian QMBS, we first introduce the Hermitian counterparts realized in a Rydberg-atom quantum simulator [42] with the Hamiltonian

\[ H_{\text{Ryd}} = \sum_{i=1}^{N} \left( \frac{\Omega}{2} \sigma_i^z + \Delta n_i \right) + \sum_{i<j} V_{i,j} n_i n_j. \] (1)

Here, \( \sigma_i^z = |g_i\rangle\langle r_i| + |r_i\rangle\langle g_i| \) couples an atom between the ground state \( |g_i\rangle \) and the Rydberg excited state \( |r_i\rangle \) at position \( i \), which is realized by a two-photon transition and driven at Rabi frequency \( \Omega \). \( \Delta \) denotes the strength of the laser detuning and \( n_i = |r_i\rangle\langle r_i| \). The potential \( V_{i,j} \approx 1/R_{i,j}^{6} \) characterizes the van der Waals interaction between atoms in Rydberg states at a distance \( R_{i,j} \). In the limit of strong nearest-neighbor interactions \( V_{i,i+1} \gg \Omega \), the system can be effectively described by \( H_{\text{PXP}} = \sum_i^{N} P_{i-1} \sigma_i^z P_{i+1} \) without simultaneous Rydberg excitation of nearest neighbors [11, 12], where \( P_i = (1 - \sigma_i^z) / 2 \) is a projection operator with \( \sigma_i^z = |r_i\rangle\langle r_i| - |g_i\rangle\langle g_i| \).

Here we introduce the non-Hermiticity by generalizing the symmetric coupling between \( |r_i\rangle \) and \( |g_i\rangle \) to be non-symmetric, i.e., \( |g_i\rangle\langle r_i| + |r_i\rangle\langle g_i| \rightarrow (1 - \gamma)|g_i\rangle\langle r_i| + (1 + \gamma)|r_i\rangle\langle g_i| \), then the Hamiltonian can be written as

\[ H_{\text{ah-PXP}} = \sum_{j}^{N} P_{j-1} \left( \sigma_j^z + i\gamma \sigma_j^y \right) P_{j+1}, \] (2)

where the parameter \( \gamma \in \mathbb{R} \) denotes the strength of non-Hermiticity. We will show the establishment and the characteristics of the non-Hermitian QMBS states, and further examine their stability against an external magnetic field \( h \in \mathbb{R} \).

The whole Hamiltonian can be written as

\[ H = H_{\text{ah-PXP}} + \sum_{j}^{N} h \sigma_j^z. \] (3)

We employ the exact diagonalization (ED) method to study the model Hamiltonian (3). Under the periodic boundary condition (PBC), we take advantage of both the translational symmetry and the spatial inversion symmetry to fully diagonalize the system up to \( N = 32 \) sites.

The Phase Diagram.— We begin with establishing the phase diagram as a function of the model parameters \( \gamma \) and \( h \). In Fig. 1, we identify three distinct phases in the \( \gamma - h \) plane: the phase (I) with non-Hermitian QMBS states and real energy spectra, the phase (II) with \( \mathbb{Z}_2 \) symmetry breaking ground states and the phase (III) with complex energy spectra. The black triangles and the circles denote the phase boundaries. A quantum tricritical point emerges at the merging point \( (\gamma, h) = (0, 0) \) that is highlighted by the red star in Fig. 1, which is also an exceptional point (EP). Compared to the PXP model in a closed system at \( (\gamma, h) = (0, 0) \), the non-Hermitian QMBS in phase (I) appear more fragile with increasing non-Hermitian strength \( \gamma \), though still persist until suffering the quantum tricritical point.

To examine the phase boundaries, we compute both the derivatives of the ground-state energy and the fidelity susceptibility. In particular, we adopt the generalized fidelity \( F \),

\[ F(\lambda, \delta \lambda) = \langle \psi_L(\lambda)|\psi_R(\lambda + \delta \lambda)\rangle / \langle \psi_L(\lambda)|\psi_R(\lambda)\rangle, \] (4)

where \( |\psi_L(\lambda)\rangle \) is the bi-orthogonal left (right) eigenvectors satisfying \( H|\psi_R(\lambda)\rangle = \lambda|\psi_R(\lambda)\rangle \) and \( H^\dagger|\psi_L(\lambda)\rangle = \lambda^\dagger|\psi_L(\lambda)\rangle \). The corresponding fidelity susceptibility \( \chi \) can be obtained via \( \chi(\lambda) \approx (1 - F) / \delta \lambda^2 \). Physically, the divergence of \( \chi \) towards negative infinity signifies EPs [63–65].

We first probe the quantum phase transitions when tuning the non-Hermiticity strength \( \gamma \). At \( h = 0 \), as shown in Fig. 2 a, the real parts of the ground-state fidelity suscepti-
bility $\text{Re}(\chi) \to -\infty$ with increasing system length when approaching $\gamma_c = 1$ from its two sides, which demonstrates an EP. The first-order derivative curve $dE_0/d\gamma$ also displays a singularity behavior at $\gamma_c$ (see Fig. 2 b). These observations of EP are consistent with the evolution of the energy spectra, where the whole real spectra at $\gamma < 1$ develop into complex conjugate pairs at $\gamma > 1$, separated by a transition at $\gamma_c = 1$. For $\gamma > 1$, we take the states with the lowest imaginary parts of the eigenenergy as ground states since the real parts are zero. The quantum phase transitions at other fixed magnetic fields $h$ can also be identified similarly [see Fig. 2 b], giving rise to a line of EPs, as denoted by the circles in Fig. 1.

By contrast, we find the Hamiltonian exhibits a continuous phase transition when tuning the magnetic field $h$ at $\gamma < 1$. Figure 2 c shows $\text{Re}(\chi_0)$ as a function of $h$ for different system lengths $N$, where the smooth curve as well as the increasing maximal values of $\text{Re}(\chi_0)$ with $N$ indicate the confirmation of such a phase transition, which can also be confirmed by the absence of singularities in the first-order derivative curve $dE_0/dh$. We further analyze the critical exponent $\nu$ and the central charge $c$. The critical exponent $\nu$ can be directly extracted from the fidelity susceptibility via $\text{Re}(\chi_0)_{\max} = L^{2/\nu - 1}$, as shown in the inset of Fig. 2 c, where the linear fitting demonstrates $\nu = 1$. The central charge can be obtained from the generalized bipartite von Neumann entanglement entropies (definition see below) with different subsystem length $L_A$ through $S_{N} \sim c/3 \ln(\pi L_A/L) + \text{const}$. The logarithmic fitting in Fig. 2 d indicates a finite central charge $c = 1/2$. Here we notice that the critical exponent $\nu$ and the central charge $c$ are the same as the phase transition reported in the Hermitian limit (i.e., $\gamma = 0$) [66–69], indicating the same universality class might be generalized to the non-Hermitian case. When $\gamma = 0$ and $h \to \infty$, the ground states are two-fold degenerate due to the projection constraint, both breaking the $\mathbb{Z}_2$ symmetry and violating thermalization. In particular, we find a quantum tricritical point when the above two phase transitions meet at the EP ($\gamma, h$) = (1, 0).

**Entanglement Entropy and Physical Observables.**—Below we will show that the non-Hermitian QMBS can be characterized by the lower entanglement entropy and the physical observables that violate thermalization.

In Hermitian systems, entanglement entropy is a complementary way to examine thermalization. We generalize the von Neumann entanglement entropy $S_N$ to non-Hermitian systems and show that $S_N$ indeed capture the thermalization property. We consider the non-Hermitian density matrix of the $n$th state $\rho_n$ defined by $\rho_n = |\psi_{R,n}\rangle \langle \psi_{L,n}|$ with the biorthonormal relation $\langle \psi_{L,n}| \psi_{R,m}\rangle = \delta_{nm}$ [70], and study a generic complex entanglement entropy $S_N = -\text{Tr}_A(\rho_{A,n} \ln(\rho_{A,n}))$ in the non-Hermitian system [71–73]. Here, $\rho_{A,n}$ is the reduced density matrix for subsystem $A$ after tracing out the rest of the system. Figure 3 a shows one typical example of the generalized $S_N$ in the phase (I) at $\gamma = 0.1$ and $h = 0$. In contrast to the highly entangled thermal state, the non-Hermitian quantum many-body scarred states exhibit abnormally low entanglement, as marked by the red dots. The energy intervals among these scarred states are nearly equal. Here, we point out that these non-Hermitian scarred states are all located at the $k = 0$ and $k = \pi$ momentum sectors, which resembles the Hermitian counterparts. Notably, we find large overlaps between the scarred states at $\gamma = 0$ and the corresponding left (right) scarred states at $\gamma = 1$ are written in black (green) color alongside the associated states. Here we consider $h = 0$. Panels b and d exhibit the expectation values of local observables $\langle m_z \rangle \equiv \langle \psi_L | \sigma_z | \psi_R \rangle$. All the calculations presented here are for the systems with length $N = 28$ in the momentum sector $k = 0$ at $h = 0$. FIG. 3. (Color online.) Entanglement entropies and physical observables. Panels a and c show the bipartite von Neumann entanglement entropies with respect to eigenenergies at $\gamma = 0.1$ (a) and $\gamma = 1.2$ (c) for all eigenstates. The non-Hermitian scarred states are highlighted by red dots. The values of overlaps between scar states at $\gamma = 0$ and the corresponding left (right) scar states at $\gamma = 1$ are written in black (green) color alongside the associated states. Here we consider $h = 0$. Panels b and d exhibit the expectation values of local observables $\langle m_z \rangle \equiv \langle \psi_L | \sigma_z | \psi_R \rangle$. All the calculations presented here are for the systems with length $N = 28$ in the momentum sector $k = 0$ at $h = 0$. Here, the expectation value of the magnetization $m_z \equiv \sum_{i} \sigma_{i}^z / N$ of all eigenstates. Here, the expectation value $\langle m_z \rangle \equiv \langle \psi_L | m_z | \psi_R \rangle$ with the biorthonormal eigenvectors is considered. Due to the translation symmetry, we have $\langle m_z \rangle = \langle \psi_L | \sigma_z | \psi_R \rangle$. As shown in Fig. 3 b for $\langle m_z \rangle$ versus energy, we find that a series of states (denoted by red dots) with maximal $\langle m_z \rangle$ and equal energy-level spacing are distinct from other states clearly, similar to the violation of ETH in Hermitian systems. In particular, these special states are exactly the non-Hermitian QMBS states characterized by the low entanglement entropy in Fig. 3 a.
Interestingly, in case of complex energy spectra at $\gamma > 1$, $h = 0$, we also find a set of eigenstates (marked as red dots in Fig. 3 c-d) with low entanglement entropy and maximal expectation values of $\langle m_r \rangle$ but with respect to the imaginary part of eigenenergy. These states are analogous to the non-Hermitian scarred states at $\gamma < 1$.

Energy Level Statistics.— Besides the above-mentioned entanglement entropy and physical observables that characterize the non-Hermitian QMBS, we further generalize the eigenenergy level-spacing distributions and ratios to reveal the chaotic nature of the non-Hermitian QMBS.

Figures 4 a-c show the nearest-level-spacing distribution $P(s)$ of the statistics parameter $s_n = \min_m |E_m - E_n|$ on the real or imaginary energy axis for three phases. In phase (I) with non-Hermitian QMBS, as shown in Fig. 4 a, although the scarred states have low entanglement entropy, the bulk of the unfolded level statistics displays a Wigner-Dyson (WD) distribution [74, 75], indicating a prominent feature of the quantum chaos. Here, we remark that a chaotic non-Hermitian system is expected to follow Ginebre statistics. However, when the complex spectra become totally real, one may still expect the WD distribution for a chaotic system [51], as in our phase (I) case.

In phase (II), we find the level-spacing statistics resemble semi-Poisson (SP) statistics [c.f. Fig. 4 b], at least for the largest system we have reached. As non-universal statistics, SP distribution displays the intermediate statistics between Poisson and WD, and it is typical of the pseudointegrable systems [76]. Here the slight deviation from SP in phase (II) might be induced by the finite size effect. Remarkably, when the whole spectra become imaginary at $\gamma > 1$, $h = 0$ in phase (III), Fig. 4 c shows that the level statistics tend to approach the WD distribution with the increase of system size, exhibiting the same feature as the phase (I) with a non-Hermitian QMBS. In particular, the energy spectra become complex at finite external field $h$. To avoid the ambiguity of unfolding complex spectrum, we instead consider the complex level-spacing ratios [77] $z_n = (E_{NN}^n - E_n)/(E_{NNN}^n - E_n)$. Here, $E_n$ is referred to as the $n$th real or complex eigenenergy, of which the nearest neighbor and next-to-nearest neighbor in the complex plane are $E_{NN}^n$ and $E_{NNN}^n$ respectively. Moreover, when the distribution of $z$ is anisotropic, we also consider the radial and angular marginal distribution with the relation $z \equiv re^{i\theta}$.

As shown in Fig. 4 d for $h = -0.3$, $\gamma = 1.2$, we find strongly suppressed ratio density around the origin and for small angles of $z$, demonstrating the significant level repulsion in this phase. We further examine the averaged $\langle r \rangle \approx 0.737$ and $\langle \cos \theta \rangle \approx -0.181$ for the size of the system $N = 32$, which tend to approach the standard values of the Ginebre distribution $\langle r \rangle \approx 0.74$ and $\langle \cos \theta \rangle \approx -0.24$ instead of the complex Poisson distribution as the system size increases.

Summary and Outlook.— In this work, we study the exemplary mechanism of the weak ergodicity breaking in open quantum systems, named as non-Hermitian QMBS, in a non-perturbative manner. Using the bi-orthogonal eigenstates, we characterize the non-Hermitian QMBS from the perspectives of quantum information, quantum simulation, and statistical physics. Moreover, we also examine the robustness of non-Hermitian QMBS, reveal the non-Hermitian quantum criticality and establish the whole phase diagram. Our work might serve as a starting point to investigate the weak ergodicity breaking mechanism in open quantum many-body systems, and further stimulate future studies on the interplay among the thermalization, the non-Hermiticity, and the strong correlations.

For the Hermitian QMBS realized in the Rydberg-atom quantum simulator [42], the symmetric coupling between the ground state and the Rydberg state is induced by a two-photon process via an intermediate level. Recently, much effort has been devoted to realizing the non-Hermiticity in cold-atom platforms [56–62], notably the Rydberg atoms [59, 60]. When the coupling is experimentally tuned to be non-symmetric, such as through the three-level atoms with spontaneous decay [62], our results would be directly verified in experiments. The non-Hermitian QMBS proposed in this work may also stimulate more experimental activities to realize long-lived coherent states storing the initial quantum information in open quantum systems.

This work was supported by the National Natural Science...
Foundation of China (Grant No. 12074375) and the start-up funding of KITS at UCAS (Grant No.118900M026).

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