A pp-Wave With 26 Supercharges

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Abstract

A pp-wave solution to 11-dimensional supergravity is given with precisely 26 supercharges. Its uniqueness and the absence of 11-dimensional pp-waves which preserve (precisely) 28 or 30 supercharges is discussed. Compactification on a spacelike circle gives a IIA configuration with all 26 of the supercharges. For this compactification, D0 brane charge does not appear in the supersymmetry algebra. Indeed, the 26 supercharge IIA background does not admit any supersymmetric D-branes. In an appendix, a 28 supercharge IIB pp-wave is presented along with its supersymmetry algebra.

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I. INTRODUCTION

Recently, supergravity solutions with the number of supercharges between 16 and 32 non-inclusive have been presented.\[1, 2, 3, 4, 5\] The existence of such solutions is a priori surprising, although perhaps not completely unexpected considering the results of\[6, 7, 8, 9\]. It was shown in those references that the central charge matrix appears in the supersymmetry algebra in a way that allows for 3/4 BPS states, and it was speculated that it should be possible to preserve any fraction, n/32, of the supercharges. However, to date the only system (to my knowledge) in which more than one-half—but not all—the supercharges have been concretely observed to be preserved is the pp-wave system; there the number of supercharges is always even\[4\] but has so far been capped at 24\[1, 2, 3, 4\] (again not counting the maximally supersymmetric solution\[10\] or flat space).\[1\] The pp-wave system is also very interesting because it provides a class of models that are exactly solvable in string perturbation theory\[12, 13, 14\], and because of connections to Yang-Mills theory, as first noticed in\[15\].

It is the purpose of this paper to present a pp-wave solution to 11-dimensional supergravity with 26 supercharges. This is unique and there are no 11-dimensional pp-waves which

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1 After this paper was written,\[11\] appeared which presents a family of 28 supercharge type IIB solutions, including the one given here in appendix\[A\].
preserve 28 or 30 supercharges. Furthermore, compactification can give a IIA superstring with all 26 supercharges.

A general discussion of supersymmetry in 11-dimensional supergravity was recently given in [3, 4]. In section II we review these results, following [4], with emphasis on the facts we will need later. In section III we present the pp-wave which preserves 26 supercharges, and discuss the supersymmetry algebra for the 26 supercharges.

Compactification of this background is discussed in section IV. The emphasis is in section IV A, on the spacelike compactification which preserves all 26 supersymmetries—and a related compactification which preserves only 16 supersymmetries. Compactifications and finite order orbifolds which break some of the supersymmetry are briefly discussed in sections IV B and IV C, respectively. The Green-Schwarz type IIA string with 26 supercharges, is discussed in more detail in section V. In particular, it is shown that the background admits no supersymmetric D-branes.

Uniqueness of the pp-wave with 26 supercharges, and a no-go statement for 11-dimensional pp-waves with 28 and 30 supercharges, is presented in section VI. In appendix A, a type IIB solution with 28 supercharges is given along with its supersymmetry algebra.

II. THE SETUP

In ref. [3, 4], the pp-waves of 11-dimensional supergravity were analyzed in detail. Here I will (partially) follow [4]. Therein is written the ansatz

$$ds^2 = 2dx^+dx^- + \sum_{i,j} A_{ij} x^i x^j (dx^+)^2 + \sum_i dx^i dx^i, \quad (2.1a)$$

$$F = dx^+ \wedge \Theta, \quad (2.1b)$$

where $i = 1 \ldots 9$ and $\Theta$ is a 3-form which obeys the equation of motion $\text{Tr} A = -\frac{1}{2} \|\Theta\|^2$ and furthermore, without loss of generality, takes either of the forms

$$\Theta = m_1 dx^1 dx^2 dx^9 + m_2 dx^3 dx^4 dx^9 + m_3 dx^5 dx^6 dx^9 + m_4 dx^7 dx^8 dx^9, \quad (2.2a)$$

or

$$\Theta = n_1 dx^1 dx^2 dx^3 + n_2 dx^1 dx^4 dx^5 + n_3 dx^1 dx^6 dx^7 + n_4 dx^2 dx^4 dx^6 + n_5 dx^2 dx^5 dx^7 + n_6 dx^3 dx^4 dx^7 + n_7 dx^3 dx^5 dx^6. \quad (2.2b)$$

These will be referred to as the 4-parameter or 7-parameter solutions, respectively. The 4-parameter solutions were analyzed in detail in [3], and it was shown that there are solutions that preserve 16, 18, 22, 24 or 32 supercharges. However, the 7-parameter solution is more difficult to analyze except in the case that, e.g. $n_4 = n_5 = n_6 = n_7 = 0$, for which it becomes equivalent to a subset of the 4-parameter solution.

The analysis is conducted roughly as follows. The Killing spinor equation is

$$\mathcal{D}_\mu \epsilon \equiv \nabla_\mu \epsilon - \Omega_\mu \epsilon = 0, \quad (2.3a)$$

where $\mu$ is a spacetime index, and

$$\Omega_\mu = \frac{1}{24} (3PT_\mu - \Gamma_\mu P); \quad P \equiv \frac{1}{4!} F_{\sigma \tau \lambda \rho} \Gamma^{\sigma \tau \lambda \rho}. \quad (2.3b)$$
The $\Gamma$-matrices obey $\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu}$, and are more properly written in terms of the elfbein
\[ e^\gamma = dx^- - \frac{1}{2} \mu^2 [(x^I)^2 + \frac{1}{4}(x^A)^2] dx^+, \quad e^{\dagger} = dx^+, \quad e^{\bar{\dagger}} = dx^i; \] (2.4)
the hat distinguishes tangent space from spacetime indices, but sloppiness will prevail and the hat will often be dropped. Also, $\Gamma^{\mu\nu\cdots}$ are antisymmetrized $\Gamma$-matrices with unit weight.

For $F = dx^+ \wedge \Theta$, it is convenient to define
\[ \theta \equiv \Theta \equiv \frac{1}{3!} \Theta_{ijk} \Gamma^{ijk}, \] (2.5)
and
\[ \theta_{(i)} \equiv \Gamma^i \Theta \Gamma^i. \] (2.6)

Note that $\theta_{(i)}$ has the same form as $\theta$ and so $\theta$ and $\theta_{(i)}$ all commute with each other. With this notation,
\[ \Omega_- = 0, \quad \Omega_+ = -\frac{1}{12} \Theta (\Gamma^+ \Gamma^- + \mathbb{1}), \quad \Omega_i = \frac{1}{24} \Gamma^i (3\theta_{(i)} + \theta) \Gamma^+. \] (2.7a,b,c)

The integrability condition for the Killing spinor equation (2.3a) is
\[ [D_{\mu}, D_{\nu}] \epsilon = 0, \quad \Leftrightarrow \quad \frac{1}{4} R_{\mu\nu\alpha\beta} \Gamma^{\alpha\beta} \epsilon = - [\Omega_{\mu}, \Omega_{\nu}] \epsilon. \] (2.8)

Since the only non-zero components of the Riemann tensor are
\[ R_{++ij} = A_{ij}, \] (2.9)
and since $(\Gamma^+)^2 = 0$, the only nontrivial components of the integrability condition are those with $\mu = +, \nu = i$, which gives
\[ -144 \sum_j A_{ij} \Gamma^{2^+} \epsilon = \Gamma^i (3\theta_{(i)} + \theta)^2 \Gamma^+ \epsilon. \] (2.10)

From equation (2.10) it is clear that all pp-wave solutions of the form (2.1) preserve at least 16 supersymmetries \[ 4 \quad 4 \quad 4 \quad 14 \]; the corresponding Killing spinors are annihilated by $\Gamma^+$. Additional supersymmetries are obtained via the following eigenvalue analysis.

The antisymmetric matrix $\theta$ has eight (doubly degenerate\(^2\)) skew-eigenvalues $\lambda_\Lambda$. For

\(^2\) In [4] it was convenient for the $\Gamma$-matrices to be SO(9) $\Gamma$-matrices, but here it has been convenient to use SO(1,10) $\Gamma$ matrices at the cost of doubling the degeneracy of the eigenvalues.
example, for the 7-parameter solution, 

\[ \lambda_1 = n_1 + n_2 + n_3 - n_4 + n_5 + n_6 + n_7, \quad (2.11a) \]
\[ \lambda_2 = n_1 + n_2 + n_3 + n_4 - n_5 - n_6 - n_7, \quad (2.11b) \]
\[ \lambda_3 = -n_1 + n_2 + n_3 + n_4 - n_5 + n_6 + n_7, \quad (2.11c) \]
\[ \lambda_4 = n_1 - n_2 - n_3 + n_4 - n_5 + n_6 + n_7, \quad (2.11d) \]
\[ \lambda_5 = n_1 - n_2 + n_3 + n_4 + n_5 - n_6 + n_7, \quad (2.11e) \]
\[ \lambda_6 = -n_1 + n_2 - n_3 + n_4 + n_5 - n_6 + n_7, \quad (2.11f) \]
\[ \lambda_7 = n_1 + n_2 - n_3 + n_4 + n_5 + n_6 - n_7, \quad (2.11g) \]
\[ \lambda_8 = -n_1 - n_2 + n_3 + n_4 + n_5 + n_6 - n_7; \quad (2.11h) \]

the skew eigenvalues for the 4-parameter solution were given in [4]. Defining the signs \( s_{\Lambda a} \), \( \Lambda = 1 \ldots 8, a = 1 \ldots 7 \) by

\[ \lambda_{\Lambda} = \sum_{a} s_{\Lambda a} n_a, \quad (2.12) \]

it is straightforward to see that the projection operators onto the eigenspinors of \( \theta \) with eigenvalues \( \pm i \lambda_{\Lambda} \) are

\[ P_{\Lambda} = \frac{1}{16} (\mathbb{1} - s_{\Lambda 1} s_{\Lambda 2} \Gamma^{2345}) (\mathbb{1} - s_{\Lambda 1} s_{\Lambda 3} \Gamma^{2367}) (\mathbb{1} + s_{\Lambda 1} s_{\Lambda 4} \Gamma^{1346}) (\mathbb{1} + s_{\Lambda 1} s_{\Lambda 5} \Gamma^{1357}). \quad (2.13) \]

(There is, of course, a similar result for the 4-parameter ansatz.)

Then, to obtain additional Killing spinors, one defines the matrix

\[ U_{(i)} = 3 \Gamma^i \theta \Gamma^i + \theta. \quad (2.14) \]

The eigenvalues of \( U_{(i)}^2 \) are doubly\(^3\) degenerate and given by \(-\rho_{\Lambda(i)}^2 (\Lambda = 1 \ldots 8)\). The matrices \( U_{(i)} \) all commute with each other and with \( \theta \), so the corresponding eigenspinors are again those which survive the projections \( (2.13) \). It follows from the integration condition \( (2.10) \) that if for all \( i \) and some choice of \( \Lambda \), we choose the matrix \( A_{ij} \) in the metric \( (2.1a) \) to be diagonal and given by \( A_{ij} = \mu_i^2 \delta_{ij} = \frac{1}{144} \rho_{\Lambda(i)}^2 \), then there are two more Killing spinors, namely those not annihilated by \( \Gamma^+ \) but preserved by \( P_{\Lambda} \). For each \( \Lambda \) for which this is satisfied, there are two additional Killing spinors; thus the necessary and sufficient condition for the pp-wave to have \( 16 + 2N \) Killing spinors is

\[ \rho_{\Lambda 1(i)}^2 = \rho_{\Lambda 2(i)}^2 = \cdots = \rho_{\Lambda N(i)}^2, \quad \forall i = 1 \ldots 9. \quad (2.15) \]

A necessary and useful, though insufficient, condition that follows from this, via \( i = 9 \), is

\[ \lambda_{\Lambda 1}^2 = \lambda_{\Lambda 2}^2 = \cdots = \lambda_{\Lambda N}^2. \quad (2.16) \]

The Killing spinors are given by

\[ \epsilon(\psi) = [\mathbb{1} + x^i \Omega_i] e^{-\frac{1}{12} x^i (\Gamma^+ \Gamma^- + \mathbb{1}) \theta} \psi \quad (2.17) \]

where \( \psi \) is a constant spinor which parametrizes the Killing spinor and which obeys

\[ \frac{1}{2} (P_{\Lambda 1} + \cdots + P_{\Lambda N}) \Gamma^- \Gamma^+ \psi + \frac{1}{2} \Gamma^+ \Gamma^- \psi = \psi; \quad (2.18) \]

that is, it survives the projection onto the subspace of the spinors that are annihilated by \( \Gamma^+ \) and/or are associated with the eigenvalues \( \lambda_{\Lambda 1}, \ldots, \lambda_{\Lambda N} \).

\(^3\) And doubled again for SO(1,10) \( \Gamma \)-matrices; see footnote [3].
III.  A 26 SUPERCHARGE PP-WAVE

A special case of the preceding is the pp-wave

\[ ds^2 = 2dx^+dx^- - \mu^2 \left[ (x^I)^2 + \frac{1}{4}(x^A)^2 \right] (dx^I)^2 + (x^I)^2 + (x^A)^2, \quad I = 1, \ldots, 7 \]
\[ A = 8, 9 \]  
\[ F = \mu dx^+ \left[ -3dx^1dx^2dx^3 + dx^1dx^3dx^5 - dx^1dx^6dx^7 - dx^2dx^4dx^6 ight. 
\left. - dx^2dx^5dx^7 - dx^3dx^4dx^7 + dx^3dx^5dx^6 \right]. \]

As usual, \( \mu \neq 0 \) can be set to any convenient value by rescaling \( x^\pm \). For this solution, \( \lambda_1 = \lambda_2 = -\lambda_3 = \lambda_4 = \lambda_8 = -3\mu \), so the condition (2.16)—and, in fact, (2.15)—is satisfied. Thus this solution has 26 Killing spinors, namely those preserved by the projection operator

\[ \mathbb{P}_{12348} = \frac{1}{16} \left( 5 \cdot 1 - \frac{1}{\mu} \right) \Gamma^{-}\Gamma^+ + \frac{1}{2} \Gamma^+\Gamma^-, \]

where

\[ \tilde{\theta} = \Gamma^{12345678} \Theta; \quad F = dx^+ \wedge \Theta. \]

That the projection operator (2.18) takes this simple form is related to equation (2.16) but still requires some magic. Precisely the Killing spinors (2.17) annihilated by \( \Gamma^+ \) are independent of \( x^i \); the others depend on all of the \( x^i \).

A.  Killing Vectors

The solution (3.1) has isometry group (roughly) \( H^9 \times SU(2) \times SU(2) \times U(1) \), where \( H \) is the Heisenberg group. In fact, the nine copies of the Heisenberg group share their central element; there is also one more isometry which acts as an outer automorphism on the \( H \). This Heisenberg group appears for all pp-wave solutions [10].

For the solution (3.1), the “Heisenberg” Killing vectors are [when there is a possibility of confusion as to whether \( e_I \) is a component or an object, we will write \( k_{e_I} \) for the Killing vector]

\[ e_+ = -\partial_+, \quad e_- = -\partial-, \]
\[ e_I = -\cos \mu x^+ \partial_I - \mu x^I \sin \mu x^+ \partial_, \quad e^*_I = -\mu \sin \mu x^+ \partial_I + \mu^2 x^I \cos \mu x^+ \partial_, \]
\[ e_A = -2\cos \frac{\mu}{2} x^+ \partial_A - \mu x^A \sin \frac{\mu}{2} x^+ \partial_, \quad e^*_A = -\mu \sin \frac{\mu}{2} x^+ \partial_A + \frac{\mu^2}{2} x^A \cos \frac{\mu}{2} x^+ \partial_. \]

The algebra is

\[ [e_I, e^*_J] = \mu^2 \delta_{IJ} e_-, \quad [e_A, e^*_B] = \mu^2 \delta_{AB} e_-, \]
\[ [e_+, e_I] = e^*_I, \quad [e^+, e_A] = e^*_A, \]
\[ [e_+, e^*_I] = -\mu^2 e_I, \quad [e^+, e^*_A] = -\frac{\mu^2}{4} e_A. \]

with all other commutators vanishing. In particular, the element \( e_- \) is central.
The rotational symmetry group of the metric is $SO(7) \times SO(2)$; however, the field strength breaks the $SO(7)$ to $SO(4)$. Thus the symmetry group of the field configuration is isomorphic to $SU(2) \times SU(2) \times U(1)$. Writing the rotational (not necessarily Killing) vectors

$$M_{ij} = x^i \partial_j - x^j \partial_i,$$  \hspace{1cm} (3.6)

the rotational Killing vectors are

$$J = M_{89}, \hspace{1cm} (3.7a)$$

$$J_1^+ = -M_{23} + \frac{1}{2}(M_{45} - M_{67}), \hspace{1cm} J_1^- = -\frac{1}{2}(M_{45} + M_{67}), \hspace{1cm} (3.7b)$$

$$J_2^+ = M_{31} + \frac{1}{2}(M_{46} - M_{75}), \hspace{1cm} J_2^- = -\frac{1}{2}(M_{46} + M_{75}), \hspace{1cm} (3.7c)$$

$$J_3^+ = M_{12} + \frac{1}{2}(M_{47} - M_{56}), \hspace{1cm} J_3^- = -\frac{1}{2}(M_{47} + M_{56}). \hspace{1cm} (3.7d)$$

The $U(1)$ generator is $J$, and $J_x^\pm$ obey the $SU(2) \times SU(2)$ algebra

$$[J_x^+, J_y^+] = \epsilon_{xyz} J_z^+, \hspace{1cm} [J_x^+, J_y^-] = 0, \hspace{1cm} [J_x^-, J_y^-] = \epsilon_{xyz} J_z^-.$$

Observe that the $SO(7)$ symmetry of the metric has decomposed as

$$SO(7) \rightarrow SO(3) \times SO(4) \cong SU(2) \times SU(2) \times SU(2) \rightarrow SU(2)_D \times SU(2).$$ \hspace{1cm} (3.9)

In the first decomposition, $SO(3)$ rotates $x^1, x^2, x^3$ and $SO(4)$ rotates $x^4, x^5, x^6, x^7$. In the second decomposition, $SU(2)_D$ is the diagonal subgroup of the $SO(3)$ and one of the $SU(2) \subset SO(4)$. While one can find that these are the rotational isometries in a straightforward way, a nice way\(^4\) of seeing this symmetry of the field strength is to notice that

$$F = \mu dx^+ \left[ -3dx^1dx^2dx^3 - \sum_{y=1}^3 dx^y \wedge \omega_y^- \right], \hspace{1cm} (3.10)$$

where $\omega_y^-$ are the anti-selfdual two-forms of the $\mathbb{R}^4$ spanned by $x^4, x^5, x^6, x^7$:

$$\omega_1^- = -dx^4dx^5 + dx^6dx^7, \hspace{1cm} \omega_2^- = dx^4dx^6 + dx^5dx^7, \hspace{1cm} \omega_3^- = dx^4dx^7 - dx^5dx^6. \hspace{1cm} (3.11)$$

Note that $\mathcal{L}_{J_x^-} \omega_y^- = 0$ and $\mathcal{L}_{J_y^-} \omega_y^- = \epsilon_{xyz} \omega_z^-$. Equation (3.10) is now manifestly invariant under the diagonal $SO(3)$ (which simultaneously rotates the $x^i$s and the $\omega^-_i$s), and under the $SO(3)$ that preserves the anti-selfdual two-forms.

Finally, we remark that the rotational group $SU(2) \times SU(2) \times U(1) \subset SO(7) \times SO(2) \subset SO(9)$ rotates the generators $e_i, e_i^*$ of the Heisenberg group in the obvious way.

**B. The Action of the Isometries on the Killing Spinors**

We now want to see how the isometries act on the Killing spinors. This is useful, for example, to see what the effect of orbifolding is on the supersymmetry. Recalling that the Lie derivative on spinors is given by

$$\mathcal{L}_k \epsilon = k^\mu \nabla_\mu \epsilon + \frac{1}{4} \nabla_\mu k_\nu \Gamma^{\mu\nu} \epsilon, \hspace{1cm} (3.12)$$

\(^4\) I thank C. Hofman for this observation.
the reader will not be surprised to learn that the rotations act in the usual way, namely,

$$\mathcal{L}_J \epsilon(\psi) = \epsilon \left( \frac{1}{2} \Gamma^{89} \psi \right),$$  \hspace{1cm} (3.13a)

$$\mathcal{L}_{J_1^+} \epsilon(\psi) = \epsilon \left( \frac{1}{2} \left[ -\Gamma^{23} + \frac{1}{2} (\Gamma^{45} - \Gamma^{67}) \right] \psi \right), \quad \mathcal{L}_{J_1^-} \epsilon(\psi) = \epsilon \left( -\frac{1}{4} [\Gamma^{45} + \Gamma^{67}] \psi \right),$$  \hspace{1cm} (3.13b)

$$\mathcal{L}_{J_2^+} \epsilon(\psi) = \epsilon \left( \frac{1}{2} \left[ \Gamma^{31} + \frac{1}{2} (\Gamma^{46} - \Gamma^{75}) \right] \psi \right), \quad \mathcal{L}_{J_2^-} \epsilon(\psi) = \epsilon \left( -\frac{1}{4} [\Gamma^{46} + \Gamma^{75}] \psi \right),$$  \hspace{1cm} (3.13c)

$$\mathcal{L}_{J_3^+} \epsilon(\psi) = \epsilon \left( \frac{1}{2} \left[ \Gamma^{12} + \frac{1}{2} (\Gamma^{47} - \Gamma^{56}) \right] \psi \right), \quad \mathcal{L}_{J_3^-} \epsilon(\psi) = \epsilon \left( -\frac{1}{4} [\Gamma^{47} + \Gamma^{56}] \psi \right),$$  \hspace{1cm} (3.13d)

Note that because the rotations preserve $F$, therefore the linear combinations of $\Gamma^{ij}$ that appear on the right-hand side of equation (3.13) commute with $\theta$ and the projection operator (3.2), and so indeed the right-hand side is another Killing spinor.

The Heisenberg algebra acts on the spinors as

$$\mathcal{L}_{e_{-}} \epsilon(\psi) = 0, \quad \mathcal{L}_{e_{+}} \epsilon(\psi) = \epsilon \left( \frac{1}{12} \theta [1 + \Gamma^+ \Gamma^-] \psi \right),$$  \hspace{1cm} (3.14a)

$$\mathcal{L}_{e_{I}} \epsilon(\psi) = \epsilon \left( \frac{1}{24} \Gamma^I U(I) \Gamma^+ \psi \right), \quad \mathcal{L}_{\bar{e}_{I}} \epsilon(\psi) = \epsilon \left( \frac{\mu^2}{2} \Gamma^I \Gamma^+ \psi \right),$$  \hspace{1cm} (3.14b)

$$\mathcal{L}_{e_{A}} \epsilon(\psi) = \epsilon \left( -\frac{1}{12} \Gamma^A U(A) \Gamma^+ \psi \right), \quad \mathcal{L}_{\bar{e}_{A}} \epsilon(\psi) = \epsilon \left( \frac{\mu^2}{4} \Gamma^A \Gamma^+ \psi \right),$$  \hspace{1cm} (3.14c)

where $U(I)$ was defined in equation (2.14). Again, it is easy to see that the algebra closes on Killing spinors.

**C. The Supersymmetry Algebra**

So far we have examined the action of the bosonic symmetries. One can check that the square of the Killing spinors is

$$\bar{\epsilon}(\psi_1) \Gamma^\mu \epsilon(\psi_2) \partial_\mu = - (\bar{\psi}_1 \Gamma^- \psi_2) e_- - (\bar{\psi}_1 \Gamma^+ \psi_2) (e_+ + \frac{\mu}{2} J)$$

$$- \frac{1}{24} \sum_{x=1}^{3} (\bar{\psi}_1 R^{+x} \Gamma^+ \psi_2) J^+_x + \mu \sum_x (\bar{\psi}_1 \Gamma^{123} R^{-x} \Gamma^+ \psi_2) J^-_x$$

$$- \sum_{I=1}^{7} (\bar{\psi}_1 \Gamma^I \psi_2) e_I + \frac{1}{24 \mu^2} \sum_I [\bar{\psi}_1 (\Gamma^I U(I) \Gamma^- \Gamma^+ + U(I) \Gamma^I \Gamma^+ \Gamma^-) \psi_2] e^*_I$$

$$- \sum_{A=8}^{9} (\bar{\psi}_1 \Gamma^A \psi_2) \left( \frac{1}{2} e_A + \frac{1}{\mu} \sum_{B=8}^{9} e_{AB} e^*_B \right)$$  \hspace{1cm} (3.15)
using the constant matrices

\[
\begin{align*}
R^+ &= - \{ \Gamma^{23}, U_{(3)} \}, \\
R^+ &= \{ \Gamma^{31}, U_{(1)} \}, \\
R^+ &= \{ \Gamma^{12}, U_{(2)} \}, \\
R^- &= \frac{1}{2} (\Gamma^{45} + \Gamma^{67}), \\
R^- &= \frac{1}{2} (\Gamma^{46} + \Gamma^{75}), \\
R^- &= \frac{1}{2} (\Gamma^{47} + \Gamma^{56}).
\end{align*}
\]

Note that the \( \Gamma \)-matrices carry tangent space indices, although this really only affects the first term. Although the derivation has been completely suppressed, it should be noted that the sign convention \( \Gamma^{1234567} = \Gamma^{89} \Gamma^{+} \) has been used to simplify the coefficient of \( e^*_A \); see also equation \( (3.14) \) below.

From equation \((3.13)\) we see that the Killing spinors square to isometries, as required. Note that every isometry—and no other vector—appears on the right-hand side. That precisely the rotational Killing vectors appear is quite nontrivial. However, that \( e_+ \) and \( J \) appear only in the combination \( e_+ + \frac{\mu}{2} J \), and that \( e_A, e_A^* \) appear only in the combinations \( \frac{1}{2} e_8 + \frac{1}{2} e_8^* \) and \( \frac{1}{2} e_9 - \frac{1}{\mu} e_8^* \) will have important implications for the IIA superstring; see section \( \text{[1A]} \).

The full supersymmetry algebra can now be written.

\[
\{ Q_\alpha, Q_\beta \} = - (\Gamma^{-1} C^{-1})_{\alpha\beta} e_- - (\Gamma^+ P_{12348} C^{-1})_{\alpha\beta} (e_+ + \frac{\mu}{2} J)
\]

\[
\begin{align*}
&- \frac{1}{24} \sum_{x=1}^{3} (R^+ x \Gamma^+ P_{12348} C^{-1})_{\alpha\beta} J^+_x + \mu \sum_{x=1}^{3} (\Gamma^{123} R^+ x \Gamma^+ C^{-1})_{\alpha\beta} J^-_x \\
&- \sum_{I=1}^{7} (\Gamma^I P_{12348} C^{-1})_{\alpha\beta} e_I + \frac{1}{24 \mu^2} \sum_{I=1}^{7} \left[ (\Gamma^I U_{(I)} \Gamma^{-1} + U_{(I)} \Gamma^I \Gamma^{-1} C^{-1})_{\alpha\beta} e^*_I \right] \\
&- \sum_{A=8}^{9} (\Gamma^A P_{12348} C^{-1})_{\alpha\beta} \left( \frac{1}{2} e_A + \frac{1}{\mu} \sum_{B} \epsilon_{AB} e^*_B \right),
\end{align*}
\]

\[(3.17)\]

\[
\begin{align*}
[e_-, Q] &= 0, & [e_+, Q] &= \frac{1}{12} \theta (\mathbb{1} + \Gamma^{-1} \Gamma^{+}) Q, \\
[e_I, Q] &= -\frac{1}{24} \tilde{P}_{12348} U_{(I)} \Gamma^I \Gamma^+ Q, & [e^*_I, Q] &= -\frac{\mu^2}{2} \tilde{P}_{12348} \Gamma^I \Gamma^+ Q, \\
[e_A, Q] &= -\frac{1}{12} \tilde{P}_{12348} U_{(A)} \Gamma^A \Gamma^+ Q, & [e^*_A, Q] &= -\frac{\mu^2}{4} \tilde{P}_{12348} \Gamma^A \Gamma^+ Q, \\
[J^+_1, Q] &= -\frac{1}{2} \left[ -\Gamma^{23} + \frac{1}{2} (\Gamma^{45} - \Gamma^{67}) \right] Q, & [J^-_1, Q] &= \frac{1}{4} (\Gamma^{45} + \Gamma^{67}) Q, \\
[J^+_2, Q] &= -\frac{1}{2} \left[ \Gamma^{31} + \frac{1}{2} (\Gamma^{46} - \Gamma^{75}) \right] Q, & [J^-_2, Q] &= \frac{1}{4} (\Gamma^{46} + \Gamma^{75}) Q, \\
[J^+_3, Q] &= -\frac{1}{2} \left[ \Gamma^{12} + \frac{1}{2} (\Gamma^{47} - \Gamma^{56}) \right] Q, & [J^-_3, Q] &= \frac{1}{4} (\Gamma^{47} + \Gamma^{56}) Q, \\
[J, Q] &= -\frac{1}{2} \Gamma^{89} Q, & (3.18a) & (3.18b) & (3.18c) & (3.18d) & (3.18e) & (3.18f) & (3.18g)
\end{align*}
\]
\[ e_I, e^*_I \] = \mu^2 \delta_{IJ} e_{-}, \quad [e_A, e^*_B] = \mu^2 \delta_{AB} e_{-}, \quad (3.19a) \\
\[ e_+, e_I \] = e^*_I, \quad [e_+, e_A] = e^*_A, \quad (3.19b) \\
\[ e_+, e^*_I \] = -\mu^2 e_I, \quad [e_+, e^*_A] = -\frac{\mu^2}{4} e_A, \quad (3.19c) \\
\[ J^+_x, J^+_y \] = \epsilon_{xyz} J^+_z, \quad [J^+_z, J^-] = \epsilon_{xyz} J^-_x. \quad (3.19d) \\
\[ J^+_x, e_I \] = \sum_{J=1}^7 R^+_I e_J, \quad [J^+_x, e^*_I] = \sum_{J=1}^7 R^+_I e^*_J, \quad (3.19e) \\
\[ J^-_x, e_I \] = \sum_{J=1}^7 R^-_I e_J, \quad [J^-_x, e^*_I] = \sum_{J=1}^7 R^-_I e^*_J, \quad (3.19f) \\
\[ J^+_x, e_I \] = \delta_{A,9} e_8 - \delta_{A,8} e_9, \quad [J^+_x, e^*_A] = \delta_{A,9} e^*_8 - \delta_{A,8} e^*_9, \quad (3.19g) \\
\]

where, in terms of the rotation matrices \( M^I_{KL} = \delta^I_K \delta^J_L - \delta^J_K \delta^I_L \), the matrices in (3.19a) are

\[ R^+_I = -M^I_{J,J} + \frac{1}{2} (M^I_{JJ} - M^I_{JJ}), \quad R^-_I = -\frac{1}{2} (M^I_{JJ} + M^I_{JJ}), \quad (3.20a) \]

\[ R^+_I = M^I_{J,J} + \frac{1}{2} (M^I_{JJ} - M^I_{JJ}), \quad R^-_I = -\frac{1}{2} (M^I_{JJ} + M^I_{JJ}), \quad (3.20b) \]

\[ R^+_I = M^I_{J,J} + \frac{1}{2} (M^I_{JJ} - M^I_{JJ}), \quad R^-_I = -\frac{1}{2} (M^I_{JJ} + M^I_{JJ}). \quad (3.20c) \]

All other commutators vanish. Here, \( \alpha, \beta, \ldots \) are spinor indices; \( C \) is the charge conjugation matrix; and \( \tilde{P}_{12348} \) is the projection operator

\[ \tilde{P}_{12348} = C^{-1} \tilde{P}_{12348}^T C = \frac{1}{16} \left( 5 \cdot \mathbb{1} - \frac{1}{\mu} \hat{\theta} \right) \Gamma^+ \Gamma^- + \frac{1}{2} \Gamma^- \Gamma^+. \quad (3.21) \]

The same symbols have been used here for the bosonic charges as were used for their Killing vectors, and \( Q \) are the supercharges; note that \( \{ \bar{\psi}_1 Q, \bar{\psi}_2 Q \} = \bar{\epsilon} (\psi_1) \Gamma^\mu \epsilon (\psi_2) \partial_\mu \), and \( [B, \bar{\psi} Q] = \bar{\psi} C^{-1} M^T C Q = \bar{\psi} \tilde{P}_{12348} C^{-1} M^T C Q \) if \( L_B \epsilon (\psi) = \epsilon (M \psi) \).

### IV. ORBIFOLDS

As in [2], we can use the isometries to find supersymmetric spatial compactifications of the pp-wave. Explicitly, define

\[ S^\pm_{AB} = \frac{1}{2} e_A \pm \frac{1}{\mu} e^*_B, \quad S^\pm_{IJ} = e_I \pm \frac{1}{\mu} e^*_J, \quad (4.1) \]

and note that \( \| S^\pm_{IJ} \|^2 = 1 = \| S^\pm_{AB} \|^2 \) \( (I \neq J, A \neq B) \). Of course one could compactify on any linear combination of the isometries; however, given the results of [2], these are the isometries for which one expects a chance of respecting some supersymmetry above the 16 annihilated by \( \Gamma^+ \).
A. A Compactification with 26 Supercharges

The simplest compactification is along $S_{\pm AB}^\pm$ ($A \neq B$); this is the simplest because $\theta$ is free of $\Gamma^A$. From this fact and equation (3.14),

$$\mathcal{L}_{S_{AB}^\pm} \epsilon(\psi) = \epsilon\left(\frac{1}{12} \Gamma^A(\theta \mp 3\mu \Gamma^B)\Gamma^+ \psi\right).$$

(4.2)

So if

$$\Gamma^+ \psi \neq 0 \text{ or } (\theta \Gamma^9 \mp 3\mu \epsilon_{AB}) \psi \neq 0,$$

(4.3)

then $\psi$ parametrizes a supersymmetry that is preserved by compactification on $S_{\pm AB}^\pm$, provided $\psi$ is also in the 26-dimensional subspace of constant spinors preserved by the projection operator $\epsilon(\psi) = (5 \cdot 1 - \frac{1}{\mu} \theta) \Gamma^- \Gamma^+ \psi = \psi \Leftrightarrow (3 - \frac{1}{\mu} \theta \Gamma^9) \Gamma^- \Gamma^+ \psi = 0$, (4.4)

using $\Gamma^{1234567} = \Gamma^8 \Gamma^9 \Gamma^+$. Thus we see that $S_9^+$, $S_9^-$ preserve the 16 supersymmetries annihilated by $\Gamma^+$, $S_9^+, S_9^-$ preserve all 26 supersymmetries.

It might seem strange that $S_{AB}^+$ preserves a different number of supersymmetries from $S_{AB}^-$—naively they are equivalent via parity ($x^8 \rightarrow -x^8$, say)—so it should be emphasized that which of $S_{AB}^+$ or $S_{AB}^-$ is the fully supersymmetric circle depends on whether we choose $\Gamma^{1234567} = \pm \Gamma^8 \Gamma^9$. This partial chirality of the solution is rather curious.

The Type IIA configuration that preserves 26 supercharges is

$$ds^2 = 2dX^+ dX^- - \mu^2 (X^i)^2 (dX^+)^2 + (dX^i)^2, \quad i = 1 \ldots 8,$$

(4.5a)

$$F = -\mu dX^+ dX^8,$$

(4.5b)

$$F = -\mu dX^+ \left[3dX^1 dX^2 dX^3 + dX^g \wedge \omega_y\right],$$

(4.5c)

where $F$ is the Kaluza-Klein, Ramond-Ramond (RR), field strength. This is derived as follows. From equation (3.11), make the change of coordinates

$$x^+ = X^+, \quad x^- = X^- + \frac{\mu}{2} X^8 X^9, \quad x^I = X^I,$$

(4.6a)

$$X^8 = X^9 \cos\left(\frac{\mu}{2} X^9\right) + X^8 \sin\left(\frac{\mu}{2} X^9\right), \quad x^9 = -X^9 \sin\left(\frac{\mu}{2} X^9\right) + X^8 \cos\left(\frac{\mu}{2} X^9\right).$$

(4.6b)

Then $\frac{\partial}{\partial X^9} = -S_9^-$ is a manifest Killing direction; the metric reads

$$ds^2 = 2dX^+ dX^- - \mu^2 \sum_{i=1}^8 (X^i)^2 (dX^+)^2 + \sum_{i=1}^8 (dX^i)^2 + (dX^9 + \mu X^8 dX^+)^2.$$

(4.7)

This and standard dimensional reduction formulæ—namely $ds_{11}^2 = ds_{10}^2 + (dx^9 + A_\mu dx^8)^2$—then give the configuration (4.5).

The Killing spinors are independent of the internal $X^9$ coordinate. Thus the perturbative type IIA string in this background sees 26 supercharges. However, note that the Killing
vector which generates the circle $S^-_{89}$ (or $S^+_{98}$) does not appear on the right-hand side of the
supersymmetry algebra $(3.17)$. Thus, this IIA string does not admit supersymmetric D0 branes. Indeed if this 26 supercharge IIA string did admit supersymmetric D0 branes—
that is, if $S^-_{89}$ and $S^+_{98}$ appeared in the square of the supercharges—then there would be a
contradiction with the Jacobi identity since $[S^-_{89}, S^+_{98}] \neq 0$. Similarly, observe that the
combination $-e_+ - \frac{2}{7} J$ which appears in the algebra $(3.17)$ is just the Hamiltonian, $\frac{\partial}{\partial X}$, of
the new coordinates $(4.6)$, and the SO(2) generator $J$, that is broken by the compactification, does not appear in the 10-dimensional supersymmetry algebra.

Finally, note that we cannot compactify on another (commuting and spacelike) circle without breaking some of the supersymmetry (see section $[IVB]$).

### B. Other Compactifications

If we compactify on any of the individual isometries $S^\pm_{IJ}$, at least the 16 supercharges
annihilated by $\Gamma^+$ are preserved. Unfortunately, the general situation is rather difficult to
analyze. It appears, however, that some of the 26 supercharges are always broken.

Specifically, note that

$$ L_{S^\pm_{IJ}} \epsilon(\psi) = \epsilon \left( -\frac{1}{24} \Gamma^I (U_{(I)} \mp 12 \mu \Gamma^J) \Gamma^+ \psi \right). \quad (4.8) $$

Unlike the isometries discussed in section $[IVA]$, there is no “chirality” condition on the
spinors that allows this to vanish for all 26 supersymmetries, even upon taking linear combina-
tions. The simplest way to understand this is to note that every circle will break at least some of the SO(4) rotational symmetries. Since these appear on the right-hand side
of $(3.17)$, the Jacobi identity requires that some of the supersymmetry also be broken by
the circle. Generically, therefore, only 16 supersymmetries are preserved. However, note,
for example, that a circle along

- $S^+_{12}$ preserves 20 supercharges,
- $S^+_{45} + S^-_{67}$ preserves 20 supercharges,
- $S^+_{45} - S^-_{67}$ preserves 20 supercharges,
- $S^+_{45}$ preserves 18 supercharges,
- $S^+_{45} - S^-_{67}$ preserves 18 supercharges,
- $S^+_{12} + S^+_{45} + S^-_{67}$ preserves 18 supercharges,
- $S^+_{45} + S^+_{67}$ preserves 16 supercharges,
- $S^+_{12} + S^+_{45} - S^+_{67}$ preserves 16 supercharges.

### C. Finite Orbifolds

We can also find supersymmetric orbifolds by considering finite subgroups
$\Gamma \subset SU(2) \times SU(2)$. In flat space—as well as for pp-waves with more conventional (16 or 32)
numbers of supersymmetries $[IV]—\Gamma \subset SU(2)_-$ preserves half of the supercharges. There-
fore, it will preserve at least eight supercharges—that is half of the sixteen conventional
supercharges annihilated by $\Gamma^+$. For simplicity, we will only consider $\mathbb{Z}_N$ orbifolds here;
then direct computation shows that $\mathbb{Z}_N \subset SU(2)_-$ generically preserves the 10 supercharges
for which

$$ P_{12345} \psi = \psi, \quad \text{but}, \quad (\Gamma^{45} + \Gamma^{67}) \psi = 0, \quad (4.9) $$
having chosen the $Z_N$ to be along the $J_1^-$ orbit. Note that the second condition picks out 16 of the 32 SO(10,1) spinors, but that the first condition acts asymmetrically on these, and keeps 10 of them.

For $Z_N \subset SU(2)_+$, we should realize that each $J^+_x$ acts simultaneously on three two-planes. Therefore, we are only guaranteed 4 supercharges. In fact, the $Z_N$ generated by $J^+_1$ generically preserves 6 supercharges.

Finally, the $Z_N \subset SU(2)_- \subset SU(2)_+ \times SU(2)_-$, generated by $J_1^+ - J_1^- = -M_{23} + M_{45}$, generically preserves 14 supercharges, which is 6 more than the guaranteed eight.

V. THE 26 SUPERCHARGE IIA BACKGROUND

Equation (4.5) gives a type IIA background with 26 supersymmetries. Dimensionally reducing the Killing spinor equation (2.3a)—and, in the $X^\mu$ coordinate system, 5 setting the chirality matrix $\Gamma^{11} = \Gamma^9$—gives the IIA Killing spinor equation,

$$\mathcal{D}_\mu \epsilon = \nabla_\mu \epsilon + \frac{1}{4} \mathcal{F}_{\mu\nu} \Gamma^{11} \epsilon - \frac{1}{24} (3 \mathcal{F}^3 \Gamma - \Gamma_{\mu} \mathcal{F}^3) \epsilon \equiv \nabla_\mu \epsilon - \Omega_\mu \epsilon. \quad (5.1)$$

The integrability of the 26 Killing spinors is both guaranteed, and easy to check.

It is now straightforward, using the shortcut of [13]—which can be checked against the results of [17]—to write down the lightcone gauge-fixed Green-Schwarz action in this background. It is convenient to set $\alpha' = 1 = p^+$. Since the two Majorana-Weyl spinors, $S^A$, of the Green-Schwarz string are of opposite chirality, we combine them into a single Majorana spinor $S$. Lightcone gauge is defined by $\Gamma^+ S = 0$ and $X^+ = \tau$; then the gauge-fixed Green-Schwarz action is given by

$$S = \int d^2 \sigma \left\{ (\dot{X}^i)^2 - (X'^i)^2 - \mu^2 (X^i)^2 - 2i \partial_a X^\mu \bar{S} \Gamma_\mu (\delta^{ab} - \epsilon^{ab} \Gamma^{11}) D_b S \right\}, \quad (5.2)$$

where $a, b, \ldots$ are worldsheet indices, $D_b$ is the pullback to the worldsheet of the supercovariant derivative (5.1), and the overdot and prime respectively denote differentiation with respect to $\tau$ and $\sigma$. Explicitly,

$$S = \int d^2 \sigma \left\{ (\dot{X}^i)^2 - (X'^i)^2 - \mu^2 (X^i)^2 - 2i \left[ \bar{S} \Gamma^- (\partial_\tau + \Gamma^{11} \partial_\sigma) S - \frac{\mu}{4} \bar{S} \Gamma^- \Gamma^8 \Gamma^{11} S + \frac{1}{4} \bar{S} \Gamma^- \Theta S \right] \right\}. \quad (5.3)$$

The bosonic part of the action is the action of 8 massive bosons of equal mass $\mu$; this is quite familiar by now [4, 12, 13, 14, 13].

The fermionic equation of motion is

$$(\partial_\tau + \partial_\sigma \Gamma^{11}) S - \frac{\mu}{4} \bar{S} \Gamma^8 \Gamma^{11} S + \frac{1}{4} \Theta S = 0. \quad (5.4)$$

5 One should beware that because equation (4.6b) is an improper rotation, there is a sign change in the $X^\mu$ coordinate system to $\Gamma^{1234567} = -\Gamma^{8910}$ as compared to the discussion surrounding equation (4.4). As a result, $\Gamma^{11} = \Gamma^{+-12345678} = +\Gamma^9$, in these conventions, yet equation (4.4) reads $(3 + \frac{1}{\mu} \bar{S} \Gamma^8 \Gamma^{11}) \Gamma^- \psi = 0.$
Multiplying by \((\partial_\tau - \Gamma_{11}\partial_\sigma)\) gives
\[
(\partial^2_\tau - \partial^2_\sigma)S - \frac{1}{16}(\mu \Gamma^8\Gamma_{11} - \Theta)^2S = 0.
\] (5.5)
Remarkably, it is precisely for \(\Gamma^+S = 0\) that \((\mu \Gamma^8\Gamma_{11} - \Theta)^2S = -16\mu^2S\); thus
\[
(-\partial^2_\tau + \partial^2_\sigma)S - \mu^2S = 0,
\] (5.6)
so the fermions have degenerate mass with the bosons, as required given the supersymmetry. [As an aside, note that for an \(S^+\) compactification, the only difference is the sign of the Kaluza-Klein gauge field. This compactification preserves only the 16 supersymmetries annihilated by \(\Gamma^+\), and so the gauge-fixed action (5.3) then has no worldsheet (or dynamical) supersymmetries. Indeed it is straightforward to see that the 16 physical fermions for this less-symmetric background have mass eigenvalues of \(\frac{\mu}{2}\) (with degeneracy 10) and \(\frac{3\mu}{2}\) (with degeneracy 6).]

To quantize open strings on this background, set
\[
S\big|_{\text{boundary}} = MS\big|_{\text{boundary}},
\] (5.7)
where \(M\) is a matrix determined by the open string. Clearly \(M^2 = I\), and since \(M\) should relate the negative chirality part of \(S\) to its positive chirality part, it should anticommute with \(\Gamma_{11}\). Specifically,
\[
M = \Gamma^{-i_1\ldots i_{p-1}}(\Gamma_{11})^{p/2+1},
\] (5.8)
gives boundary conditions appropriate to a \(D_p\)-brane oriented parallel to the “light-cone” directions, and \(X^{i_1}\ldots X^{i_{p-1}}\). The factor of \(\Gamma_{11}\) is included for certain values of \(p\) in order to ensure that \(M^2 = I\).\(^6\) Of course, since \(M\) anticommutes with \(\Gamma_{11}\), it must contain an odd number of \(\Gamma\)-matrices, so \(p\) is even for type IIA.

To find supersymmetric D-branes, we demand zero-modes of the fermions \(S\) by requiring that the boundary condition (5.7) respect the equation of motion (5.4) when there are no excitations along the string: \(\partial_\sigma S = 0\). This results in the condition
\[
[M, \mu \Gamma^8\Gamma_{11} - \Theta] = 0.
\] (5.9)
There do not appear to be any solutions of the form (5.8) to this equation. It would be interesting, perhaps using techniques of [20] or [21, 22], to find the central extension to the algebra (3.17). This would provide an additional proof of the absence of supersymmetric D-branes in this background.

VI. DISCUSSION

We have presented an 11-dimensional pp-wave that preserves 26 supercharges. On compactification to 10 dimensions, it gives a 26 supercharge IIA string that does not admit supersymmetric D0-branes—or any supersymmetric D-branes. Interestingly, for a different

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\(^6\) For additional details on fermionic boundary conditions, see e.g. [18]. The method described here was used to find supersymmetric D-branes of the IIB maximally supersymmetric pp-wave in [19].
compactification (on $S^9$ instead of $S_{−89}$, say), D0-brane charge does appear in the supersymmetry algebra, even though that IIA theory only has 16 supercharges.

It is natural to wonder if the special pp-wave introduced here is in any sense generic, and if there are solutions with 28 and 30 supercharges. The pp-wave given in equation (3.1) was found via a general analysis of the eigenvalues $\rho_i$, with the aid of Mathematica. Although there are, of course, many solutions with 24 or fewer supercharges, upon demanding at least 26 supercharges, the analysis found only the solution (3.1), and the maximally supersymmetric solution [10], albeit in several coordinate systems. Thus, the pp-wave (3.1), and the pp-wave [10], are the only M-theoretic pp-waves that preserve at least 26 supercharges. In particular, there are no M-theory pp-waves preserving 28 or 30 supercharges.

Note that the condition (2.15) is essential for this. It is straightforward to satisfy the necessary but not sufficient condition (2.16) for up to six eigenvalues (or all eight). For example, setting $n_1 = n_2 = n_3 = 0$ and $n_5 = n_6 = n_7 = −n_4 = \mu$ sets $\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8 = 0$. However, this does not give a solution with 28 supercharges; for example, $\rho_5^2(2) = 144\mu^2$ but $\rho_6^2(2) = 0$ so the necessary condition (2.15) is not satisfied. Indeed, it is straightforward, if tedious, to show that if $\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = \lambda_8$ then the condition (2.15) implies that the corresponding supersymmetric solution is either the maximally supersymmetric pp-wave [10], or flat space, and thus has 32 supercharges, not 28.

It is, however, possible to find a 28 supercharge pp-wave in the type IIB theory. One such solution and its superalgebra is given in the appendix.

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**APPENDIX A: A 28 SUPERCHARGE IIB PP-WAVE**

In this section, we present a type IIB pp-wave and its 28 supercharge superalgebra. The field configuration is

\[ ds^2 = 2dx^+ dx^- - \frac{1}{4} \mu^2 (x^i)^2 (dx^+)^2 + (dx^i)^2, \ i = 1 \ldots 8, \]  
\[ (3)F = \mu dx^+ \wedge (dx^1 dx^2 + dx^3 dx^4 - dx^5 dx^6 + dx^7 dx^8) \equiv dx^+ \wedge \Theta, \]

where for definiteness in the following, $(3)F$ is the RR field strength. (Turning on the NS-NS field strength instead is equally good, but modifies some equations below that involve spinors.)

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7 The Mathematica® program and its output—which lists all 7-parameter solutions which preserve at least 20 supercharges—can be found at [http://www.rci.rutgers.edu/~jmich/pp/all7.html](http://www.rci.rutgers.edu/~jmich/pp/all7.html) and [http://schwinger.harvard.edu/~jeremy/pp/all7.html](http://schwinger.harvard.edu/~jeremy/pp/all7.html).

8 This solution, and its $SL(2,\mathbb{R})$ cousins, was also recently presented in [11].
The 28 Killing spinors are parameterized by a doublet of positive chirality Majorana-Weyl spinors \( \psi^\Lambda, \Lambda = 1, 2 \), which are preserved by the projection operator

\[
P = \frac{1}{16} (\Gamma^{12} - \Gamma^{56} + \Gamma^{78})^2 \Gamma^- \Gamma^+ + \frac{9}{16} \Gamma^- \Gamma^+ + \frac{1}{2} \Gamma^+ \Gamma^- . \tag{A2}\]

Note that this rank 14 projection operator does not distinguish between or mix \( \psi^1 \) and \( \psi^2 \). The Killing spinors are

\[
\epsilon^\Lambda(\psi) = \left( \mathbb{1} \delta^\Lambda_{\Sigma} - x^i \Omega^\Lambda_{i \Sigma} \right) \left( e^{\frac{1}{16} (2^i \Theta (\Gamma^+ \Gamma^- + 2) \rho_1) \Sigma} \right) \Pi \psi^\Pi, \tag{A3a}\]

where

\[
\Omega^\Lambda_{i \Sigma} = \frac{1}{16} \left[ 2 (2^i \Theta \Gamma^i - \Gamma^i (2^i \Theta) \Gamma^+ \rho_1^\Lambda_{\Sigma} \right], \tag{A3b}\]

and \( \rho_1 \) is the Pauli matrix \( \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \). The Killing spinors obey the differential equation (see e.g. [23] and for recent applications to pp-waves, e.g. [3, 24, 25])

\[
\nabla_\mu \epsilon^\Lambda - \frac{1}{16} (2 (3^i F^i + \Gamma^i (3^i F)) \rho_1^\Lambda_{\Sigma} \epsilon^\Sigma = 0, \tag{A4}\]

as well as the constraint, from the dilatino variation,

\[
(3^i F) \epsilon^\Lambda = 0. \tag{A5}\]

Recognizing that \( (2^i \Theta) \) is a simple symplectic form on flat \( \mathbb{R}^8 \), we see that the solution \( [A1] \) has rotational isometry group \( U(4) \subset SO(8) \). In particular, the central \( U(1) \) is

\[
J = M_{12} + M_{34} - M_{56} + M_{78}, \quad \text{where} \quad M_{ij} = x^i \partial_j - x^j \partial_i. \quad \text{Also, we have the usual Heisenberg isometries}
\]

\[
e_+ = -\partial_+, \quad e_- = -\partial_-, \tag{A6a}\]

\[
e_i = -\cos \frac{\mu}{2} x^+ \partial_i - \frac{\mu}{2} x^i \sin \frac{\mu}{2} x^+ \partial_-, \quad e_i^* = -\frac{\mu}{2} \sin \frac{\mu}{2} x^+ \partial_i + \frac{\mu^2}{4} x^i \cos \frac{\mu}{2} x^+ \partial_. \tag{A6b}\]

In addition to the Killing spinors, \( e_i \) and \( e_i^* \), transforming in the standard way under the \( U(4) \subset SO(8) \) rotations, the supersymmetry algebra is

\[
\bar{e}_\Lambda (\psi_1) \Gamma^\mu \epsilon^\Lambda (\psi_2) \partial_\mu = - (\bar{\psi}_1 \Lambda  \Gamma^- \psi_2^\Lambda) e_- - (\bar{\psi}_1 \Lambda  \Gamma^+ \psi_2^\Lambda) e_+ + \frac{\mu}{2} (\bar{\psi}_1 \Lambda  \Gamma^+ \rho_1^\Lambda_{\Sigma} \psi_2^\Sigma), J 
\]

\[
+ \frac{1}{2 \mu^2} \left( \bar{\psi}_1 \Lambda  [2^i \Theta \Gamma^i (\mathbb{1} + \frac{1}{2} \Gamma^- \Gamma^+)] - \Gamma^i (2^i \Theta (\mathbb{1} + \frac{1}{2} \Gamma^- \Gamma^+) \rho_1^\Lambda_{\Sigma} \psi_2^\Sigma) \right) e_i^* - \bar{\psi}_1 \Lambda  \Gamma^i \psi_2 e_i, \tag{A7}\]

\[
\mathcal{L}_{e_-} \epsilon^\Lambda (\psi) = 0, \quad \mathcal{L}_{e_+} \epsilon^\Lambda (\psi) = \epsilon^\Lambda (\frac{1}{16} (2^i \Theta (\Gamma^+ \Gamma^- + 2) \rho_1 \psi), \quad \mathcal{L}_{e_i} \epsilon^\Lambda (\psi) = \epsilon^\Lambda (\frac{\mu}{2} \Gamma^+ \rho_1 \psi), \quad \mathcal{L}_{e_i^*} \epsilon^\Lambda (\psi) = \epsilon^\Lambda (\frac{\mu^2}{8} \Gamma^+ \psi), \tag{A8}\]

\[16\]
\[ [e_i, e_j^*] = \frac{\mu^2}{4} \delta_{ij} e_-, \quad \text{(A9a)} \]
\[ [e_+, e_i] = e_i^*, \quad \text{(A9b)} \]
\[ [e_+, e_i^*] = -\frac{\mu^2}{4} e_i. \quad \text{(A9c)} \]

In particular, note that the supergroup has a semidirect product structure \( G \rtimes SU(4) \).

Also, note that all spacelike compactifications break at least some supersymmetry. This is in accord with the no-go statement for 11-dimensional solutions with 28 supercharges.

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