Four quark processes at LEP 200

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Abstract

In this paper I investigate the production of four quarks at LEP 200 energies. Effects due to initial state QED corrections and background diagrams, including QCD contributions, are studied and examples of results obtained with an event generator presented.
1 Introduction

At LEP 200, four jet events will be used to perform a direct reconstruction of the $W$ mass [1]. The mass of the charged boson can be determined by looking at the maximum of the detected invariant mass distribution in the process

$$e^+ e^- \rightarrow W^+ W^- \rightarrow 4 \text{ quarks} .$$

(1)

Since the energy of the jets is not accurately known, one has to perform fits to the measured distribution using, as a constraint, information about the center of mass energy of the colliding electrons. This knowledge is crucial. If $\epsilon$ is the average energy radiated by the beams, one expects a shift

$$\Delta M = \epsilon \frac{M_W}{\sqrt{s}} .$$

(2)

in the reconstructed mass. The value of the maximum of the theoretically predicted distribution is not affected by initial state QED radiation or by the inclusion of electroweak and QCD background diagrams, but the average energy loss is very sensitive to them and to the experimentally applied cuts. Therefore, taking into account only the three basic diagrams of reaction (1) is not sufficient for comparison with experiment and a flexible tool is needed, that includes

- the possibility to implement any desired cut and to study any experimental distribution;
- all electroweak background diagrams, leading to the same four quark final state;
- initial state QED radiation;
- QCD background diagrams.

In ref. [2] a Monte Carlo program was described that satisfies all these requirements. The inclusion of electroweak background diagrams for all possible four fermion final states was performed in ref. [3], and QED initial state radiation introduced in ref. [4]. Here, I discuss the effects of QCD production channels on four quark processes, also showing numerical results obtained with the above Monte Carlo.
2 Implementing QCD contributions

There are two types of QCD contributions. On one hand, when considering a specific four quark final state, the particles can be produced either by decaying gauge bosons or gluons, and the respective terms interfere when squaring the amplitude. Therefore, this gluonic contribution constitutes an interfering QCD background for the process (1). On the other hand, the measured signal is a four jet final state, thus also the non-interfering cross section due to $e^+e^- \rightarrow g g q \bar{q}$ has to be included.

The implementation of the interfering QCD diagrams is trivial in a program like that in ref. [2], which already contains all possible electroweak Feynman diagrams. It is enough to add gluons wherever photons connect quark lines. In fact, if $Q_q$, $Q_{q'}$ are the charges of the quarks and $i,j,l,m$ colour labels, the amplitude due to photon exchange between quarks can be written as follows

$$M_\gamma = \alpha^2 Q_q Q_{q'} \left[ A^{(1)} \delta_{ij} \delta_{lm} + A^{(2)} \delta_{im} \delta_{jl} \right].$$

(3)

Substituting

$$\delta_{ij} \delta_{lm} \rightarrow \left(1 - \frac{\alpha_r}{6}\right) \delta_{ij} \delta_{lm} + \frac{\alpha_r}{2} \delta_{im} \delta_{jl}$$

$$\delta_{im} \delta_{jl} \rightarrow \left(1 - \frac{\alpha_r}{6}\right) \delta_{im} \delta_{jl} + \frac{\alpha_r}{2} \delta_{ij} \delta_{lm}$$

$$\alpha_r = \frac{\alpha_s}{\alpha Q_q Q_{q'}}$$

in the previous formula takes also gluons into account, including the correct QCD coupling and colour structure. Since there is no need to evaluate new Feynman diagrams, this approach requires almost no additional CPU time.

As for the the non-interfering $e^+e^- \rightarrow g g q \bar{q}$ process, the relevant amplitude can be efficiently computed by using the recursion relations of ref. [5]. A Monte Carlo program with the same structure of that of ref. [2] has been built in order to get this contribution.

Finally, comparisons have been made with results of refs. [6, 7], always finding a very good agreement.
3 Results

I shall now discuss a number of results obtained with the program of ref. [4]. Fragmentation effects are neglected. The chosen values for the input parameters are \( \alpha = 1/129, \alpha_s = 0.103, \sin^2 \theta_W = 0.23, M_W = 80.5, \Gamma_W = 2.3, M_Z = 91.19 \) and \( \Gamma_Z = 2.5 \) (all GeV).

| \( \sqrt{s} \) (GeV) | 175 | 190 |
|----------------------|-----|-----|
| \( \sigma \)         | 3.0674 | 3.5136 |
| \( \epsilon \)        | 0.0074 | 0.0090 |
| \( WW \) signal      | 2.5622 | 3.1416 |
|                       | 0.0071 | 0.0089 |
| \( WW \) signal + ISR| 2.5922 | 3.3553 |
|                       | 0.0075 | 0.0097 |
| All EW diagrams + ISR| 2.6202 | 3.3789 |
| + ISR                | 0.0079 | 0.0100 |
| All EW diagrams + all QCD + ISR | 3.1155 | 3.8688 |
|                       | 0.0123 | 0.0146 |

Table 1: Cross section in picobarn and average energy loss (\( \epsilon \)) in GeV for \( e^+e^- \rightarrow 4 \) jets. \( E_{(all \, particles)} > 20 \) GeV, \( |\cos \theta_{(all \, particles)}| < 0.9, m_{(ij)} > 10 \) GeV and \( |\cos (i,j)| < 0.9 \) between all possible final state pairs. The second line in each entry is the estimated Monte Carlo error.

Table 1 shows results for \( e^+e^- \rightarrow 4 \) jets under inclusion of several QED and QCD contributions at LEP 200 energies. The cross section is lowered by the initial state QED radiation (ISR), while all the other contributions tend to raise it back to its Born value. The electroweak (EW) background is at the per cent level as well as the interfering QCD background. In the last row also \( e^+e^- \rightarrow 2 \) gluons + 2 quarks is included; it increases the cross section by 16% at \( \sqrt{s} = 175 \) GeV. As far as the energy loss is concerned, QCD interfering diagrams are more important than EW background. However, both contributions are small compared to the huge effect due to the inclusion of \( ggq\bar{q} \). Cuts on the invariant masses around the \( W \) mass strongly suppress...
these QCD events. For instance, with the additional requirement that at least one invariant mass falls in the interval \( M_W \pm 2.5 \text{ GeV} \), the \( ggq \bar{q} \) cross section goes down by a factor 4 and the \( WW \) signal only by 5%. Furthermore, the corrections to the energy loss become of the same order of magnitude as the interfering QCD contributions. Finally, it should be noted that \( \epsilon \) is also very sensitive to the experimental setup \([4]\). For example, with the additional cut described above, the value of \( \epsilon \) in the second row of table \([4]\) goes down from \( 1.091 \pm 0.005 \) to \( 1.039 \pm 0.007 \).

In figure 1 the effect of the combinatorial background on the invariant mass distribution for a specific final state at \( \sqrt{s} = 175 \text{ GeV} \) is shown. In (a) any possible invariant mass \( m_{(i,j)} \) is taken into account, so that the normalization of the histogram is 6 times the value of the cross section. In (b) only those combinations that can be produced by decaying \( W' \)s are plotted.

Figure 2 shows invariant mass distributions in four jet events under inclusion of QCD, QED and EW background contributions at two different energies. The curves have been obtained by fitting histograms as those given in figure 1. ISR lowers the distributions and \( ggq \bar{q} \) gives a roughly constant positive contribution between 30 and 100 GeV. Below the peak the latter effect is dominant, while at the peak the situation is reversed.

A more detailed analysis of the peak of the distribution at \( \sqrt{s} = 175 \text{ GeV} \) is contained in figure 3. The maximum is not affected by ISR, EW background and QCD contributions, and, after subtraction of constant effects due to combinatorial background and \( ggq \bar{q} \) production, also the width is unchanged.

### 4 Conclusions

At LEP 200 the \( W \) mass will be measured from invariant mass distributions by direct reconstruction. An accurate analysis shows that, in four jet events, the maximum and the width of the distributions are not strongly affected by the inclusion of electroweak and QCD background diagrams as well as QED initial state radiation. On the other hand, due to uncertainties in the measurement of the jet energy, a precise knowledge of the average energy loss is required, and this quantity is very sensitive to the mentioned effects and to the experimental setup. Therefore, a flexible program like that in ref. \([2]\) is needed, which includes all these corrections and the possibility to implement
cuts and compute any experimental distribution.

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Figure 1: Distributions of the invariant masses for \( e^+e^- \rightarrow u(3)\bar{d}(4)d(5)\bar{u}(6) \) with (a) and without (b) combinatorial background at \( \sqrt{s} = 175 \text{ GeV} \). Cuts like in table 1. All diagrams are taken into account. QED corrections and QCD background included.
Figure 2: Distributions of the invariant masses for $e^+e^- \rightarrow 4$ jets at $\sqrt{s} = 175$ GeV (a) and $\sqrt{s} = 190$ GeV (b). Cuts like in table 1.
Figure 3: Distribution of the invariant masses near the peak for 
$e^+e^- \rightarrow 4 \text{ jets}$ at $\sqrt{s} = 175$ GeV. Cuts like in table 1.