IMPROVED CONSTRAINTS ON PRIMORDIAL NON-GAUSSIANITY FOR THE WILKINSON MICROWAVE ANISOTROPY PROBE 5-YEAR DATA

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1. INTRODUCTION

The cosmic microwave background (CMB) offers a picture of the early universe when it was only 400,000 years old. The CMB photons last scattered off electrons at that time and since then they traveled free through the space. The primordial perturbations set up during inflation are imprinted in both radiation and matter distribution. The CMB temperature anisotropies, related with the primordial perturbations, can be used to test some assumptions of the so-called standard model. In particular, we can test the prediction of the standard, single-field, slow roll inflation (Guth 1981; Albrecht & Steinhardt 1982; Linde 1982, 1983) which states that the anisotropies are Gaussian in the anisotropies (see, e.g., Bartolo et al. 2004). Primordial non-Gaussianity of the local form is characterized by the nonlinear coupling parameter \( f_{nl} \) (Salopek & Bond 1990; Gangui et al. 1994; Verde et al. 2000; Komatsu & Spergel 2001)

\[
\Phi(x) = \Phi_L(x) + f_{nl}\left(\Phi_L^2(x) - \langle \Phi_L^2(x) \rangle\right),
\]

where \( \Phi(x) \) is the primordial gravitational potential and \( \Phi_L(x) \) is a linear random field which is Gaussian distributed and has zero mean.

There are many studies based on different statistical tools to constrain the local \( f_{nl} \) parameter from the CMB anisotropies, using the data of different experiments. We can mention the analyses using the angular bispectrum and wavelets on the Cosmic Background Explorer (COBE) data (Komatsu et al. 2002; Cayón et al. 2003), the angular bispectrum on MAXIMA data (Santos et al. 2003), the angular bispectrum on Wilkinson Microwave Anisotropy Probe (WMAP) data (Komatsu et al. 2003, 2009; Creminelli et al. 2006, 2007; Spergel et al. 2007; Yadav & Wandelt 2008; Smith et al. 2009), different kind of wavelet analyses on WMAP data (Mukherjee & Wang 2004; Cabella et al. 2005; Curto et al. 2009), the Minkowski functionals on BOOMERANG data (De Troia et al. 2007), the Minkowski functionals on Archeops data (Curto et al. 2007, 2008), the Minkowski functionals on WMAP data (Komatsu et al. 2003; Spergel et al. 2007; Gott et al. 2007; Hikage et al. 2008; Komatsu et al. 2009) among others. We can also mention new promising techniques as for example one based on the \( n \)-point probability density distribution (Vielva & Sanz 2009), and other based on needlets (Pietrobon et al. 2009; Rudjord et al. 2009). Other works are based on the use of the large-scale structure to constrain \( f_{nl} \) (Slosar et al. 2008).

This work is a continuation of the wavelet-based analysis by Curto et al. (2009) of the WMAP data. We use high-resolution WMAP data maps, and compute the wavelet coefficients for 12 angular scales logarithmically spaced from 6.9 arcmin to 500 arcmin. With these wavelet coefficients, we compute all the possible third-order moments involving these scales.

The article is organized as follows. Section 2 presents the estimators used to test Gaussianity and to constrain \( f_{nl} \), the data maps, and the simulations. Section 3 summarizes the main results of this work and the conclusions are in Section 4.

2. METHODOLOGY

This analysis is based on the spherical Mexican hat wavelet (SMHW) as defined in Martínez-González et al. (2002). For references about the use of the SMHW to test Gaussianity in

\[\text{http://map.gsfc.nasa.gov/}\]
the CMB, see for example the review by Martínez-González (2008). We compute the wavelet coefficient maps at several scales $R_{l}$ logarithmically separated ($R_{l+1}/R_{l}$ constant). The considered scales are: $R_{1} =$ 6.9 arcmin, $R_{2} =$ 10.6 arcmin, $R_{3} =$ 16.3 arcmin, $R_{4} =$ 24.9 arcmin, $R_{5} =$ 38.3 arcmin, $R_{6} =$ 58.7 arcmin, $R_{7} =$ 90.1 arcmin, $R_{8} =$ 138.3 arcmin, $R_{9} =$ 212.3 arcmin, $R_{10} =$ 325.8 arcmin, $R_{11} =$ 500 arcmin. We also include the unconvolved map, which will be represented by the scale $R_{0}$ as in Curto et al. (2009). For each possible combination of three scales $R_{i}$, $R_{j}$, and $R_{k}$ (where the indices $i$, $j$, and $k$ can be repeated), we define a third-order statistic

$$q_{ijk} = \frac{1}{N_{i,j,k}} \sum_{p=0}^{N_{pix}-1} \frac{w_{p,i} w_{p,j} w_{p,k}}{\sigma_i \sigma_j \sigma_k},$$

where $N_{pix}$ is the total number of pixels of the map, $N_{i,j,k}$ is the number of pixels available after combining the extended masks corresponding to the three scales $R_{i}$, $R_{j}$, and $R_{k}$. $w_{p,i} = w_{p}(R_{i})$ is the wavelet coefficient in the pixel $p$ evaluated at the scale $R_{i}$, and $\sigma_i$ is the dispersion of $w_{p,i}$. Each map $w_{p,i}$ is masked out with the corresponding extended mask at the scale $R_{i}$, as in Curto et al. (2009). For a set of $n$ scales, we have $n_{stat} = (n + 3 - 1)!/[3!(n - 1)!]$ third-order statistics such as the one defined in Equation (2). We have tested with simulations that these statistics have a Gaussian-like distribution. We can construct a vector $q$ of dimension $n_{stat}$

$$q = [q_{0,0,0}; q_{0,0,1}; \cdots ; q_{0,0,11}; q_{0,1,1}; \cdots ; q_{11,11,11}].$$

With this vector, we can perform two different analyses using a $\chi^2$ statistic: one to test Gaussianity and a second one to constrain $f_{al}$ (Equations (7) and (8) of Curto et al. 2009).

We use the 5-yr WMAP foreground reduced data, available in the Legacy Archive for Microwave Background Data Analysis (LAMBDA) Web site.4 We combine the maps of different radiometers using the inverse of the noise variance as an optimal weight (Bennett et al. 2003). In particular, we analyze the $V$+W, $Q$, $V$, and $W$ combined maps at a resolution of 6.9 arcmin, corresponding to a HEALPix (Gorski et al. 2005) $N_{side} =$ 512. We use the $KQ75$ mask and also a set of extended masks for the wavelet coefficient maps. We use the same masks as the ones described in Curto et al. (2009) for a threshold of 0.01. This corresponds to an available fraction of the sky from 71.2% for the $R_{1}$ scale to 31.4% for the $R_{11}$ scale. Note that larger scales have the restriction of a lower available area, which means a lower sensitivity to $f_{al}$.

Finally, we analyze the data and compare them with Gaussian and non-Gaussian simulations. The Gaussian simulations are performed using the best-fit power spectrum $C_{l}$ for WMAP provided by LAMBDA and the instrumental white noise of each WMAP radiometer. The non-Gaussian simulations with the $f_{al}$ contribution are computed following the algorithms described in Liguori et al. (2003, 2007) and transformed into WMAP maps with the instrumental noise included. We also estimate the unresolved point-source contribution to $f_{al}$ for the V+W case by analyzing point-source simulations. These simulations have been generated as in Curto et al. (2009) following the source number counts $dN/dS$ given by de Zotti et al. (2005).

3. RESULTS

In this section, we present the Gaussianity analysis of the WMAP data for the combined V+W, $Q$, $V$, and $W$ data maps. We also constrain $f_{al}$ for these maps through a $\chi^2$ test. We estimate the contribution of point sources to the V+W map. Finally, we constrain $f_{al}$ for northern and southern pixels separately.

3.1. Analysis of WMAP Data

We evaluate the wavelet coefficients at the 12 considered scales. With these coefficients, we compute the third-order estimators defined in Equation (2). For 12 scales, we have 364 possible third-order statistics. We compute these statistics for the data maps and for Gaussian simulations. The covariance matrix used in the $\chi^2$ statistics is constructed from 10,000 Gaussian simulations. For the considered cases, $V$+W, $Q$, $V$, and $W$, we have that the data are inside the 2$\sigma$ error bars, i.e., the data are compatible with Gaussian simulations. We compute a $\chi^2$ statistic by comparing the data with the expected value of Gaussian simulations following Curto et al. (2009). We use well compute the $\chi^2$ of an additional set of 1000 Gaussian simulations. The $\chi^2$ statistic of the data is compatible with the $\chi^2$ of the Gaussian simulations. This is presented in Table 1.

We have studied the properties of the covariance matrix for the V+W map (see the top panel of Figure 1) which is derived from 10,000 Gaussian simulations. We may wonder if that number of simulations is enough to achieve the convergence. The condition number, defined here as the ratio between the maximum and minimum eigenvalues (see the bottom panel of Figure 1) is Cond($C$) $\sim 10^{12}$. An upper limit for the relative errors in the inverse covariance matrix is Cond($C$) $\times \epsilon_{mach}$, where $\epsilon_{mach}$ is the computer error, $\epsilon_{mach} \sim 10^{-16}$ for double precision. Thus, the relative error for the inverse matrix coefficients is Cond($C$) $\times \epsilon_{mach} \sim 10^{-4}$ which is almost negligible. We have also computed the covariance matrix with two independent sets of 5000 simulations and constrained the best-fitting $f_{al}$ of the data with these two matrices. The best-fitting $f_{al}$ value of the data is almost the same for both matrices and agrees with the result obtained with 10,000 simulations suggesting that the convergence is almost reached with 5000 simulations.

We impose constraints on $f_{al}$ through a $\chi^2$ test. We calculate the expected values of the estimators for different $f_{al}$ cases using a set of 300 non-Gaussian simulations. In Figure 2, we plot the expected values of the 364 statistics $q_{i}$ for several $f_{al}$ cases for the V+W map. We have checked that the estimator is unbiased. We have evaluated the expected values of the estimator using 200 non-Gaussian simulations of the V+W map and analyzed the remaining 100 independent non-Gaussian simulations with $f_{al} = 50$. The result is an average best-fitting $f_{al}$ value of 50.6.

Table 1

| Map | $\chi^2$ | D.O.F. | $\langle \chi^2 \rangle$ | $\sigma$ | $P(\chi^2 \leq \chi^2_{data})$ |
|-----|----------|--------|-----------------|---------|-----------------|
| V+W | 349      | 364    | 379             | 49.2    | 0.29            |
| Q   | 384      | 364    | 378             | 48.7    | 0.63            |
| V   | 348      | 364    | 377             | 47.8    | 0.27            |
| W   | 354      | 364    | 376             | 47.9    | 0.35            |

Note: We also present the mean and the dispersion of the $\chi^2$ corresponding to 1000 Gaussian simulations, and the cumulative probability for the $\chi^2$ of the data obtained from the simulations.

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3 The level of discretization of the wavelet space is a balance between the minimum number of scales needed to extract the non-Gaussian signal from the data and acceptable computational requirements.

4 http://lambda.gsfc.nasa.gov
In Figure 3, we plot the values of the statistics for the data map and compare them with the expected values for the best-fitting \( f_{\text{nl}} \) model. We also analyze Gaussian simulations in order to obtain the frequentist error bars. Table 2 lists the best-fitting \( f_{\text{nl}} \) values for the \( V+W \), \( Q \), \( V \), and \( W \) combined maps and the main properties of the histograms of the best-fitting \( f_{\text{nl}} \) obtained from Gaussian simulations. In the top panel of Figure 4, we plot the \( \chi^2(f_{\text{nl}}) \) versus \( f_{\text{nl}} \) for the \( V+W \) map, and in the bottom panel of Figure 4 we plot the histogram of the best-fitting \( f_{\text{nl}} \) of 1000 Gaussian simulations. We have \(-12 < f_{\text{nl}} < +86\) for \( V+W \), \(-52 < f_{\text{nl}} < +77\) for \( Q \), \(-32 < f_{\text{nl}} < +82\) for \( V \), and \(+6 < f_{\text{nl}} < +123\) for \( W \) (all at 95% CL). Note that \( f_{\text{nl}} \) increases as the frequency grows from \( Q \) to \( V \) and \( W \) bands. This suggests the possible presence of foregrounds residuals as they are more important at low frequencies and they add a negative contribution to \( f_{\text{nl}} \) for the bispectrum estimator (Yadav & Wandelt 2008). To further check if this is also the case for the wavelet estimator, we have also studied the \( K \) and \( K_0 \) bands, where the foreground signal has an important contribution. We have obtained \( f_{\text{nl}} = -497 \pm 42 \) for \( f_{\text{nl}}^K \) and \( f_{\text{nl}}^K = -18 \pm 37 \). Taking this into account and the values of \( f_{\text{nl}} \) for the clean and raw \( Q \), \( V \), and \( W \) maps (see Tables 2 and 3), we can see that our estimator is sensitive to the presence of foregrounds (biasing the result toward lower values). The sensitivity is even more significant for the \( Q \), \( V \), and \( W \) bands using the bispectrum (see Table 6 in Komatsu et al. 2009).

It is interesting to point out that the wavelet-based method has an intermediate dimension (\( n_{\text{stat}} = 364 \) for 12 scales) when compared with the bispectrum combinations (\( \ell_{\text{max}} \) where \( 2 < \beta < 3 \) and \( \ell_{\text{max}} \sim 10^3 \)) and the Minkowski functionals (usually several tens of combinations). The number of wavelet cubic combinations increases significantly with the number of scales.

We also estimate the contribution of the point sources for the \( V+W \) combined map as in Curto et al. (2009). We add the point-source simulations to the CMB plus noise simulations. For each one of them we compute its best-fitting \( f_{\text{nl}} \) and compare it with the obtained for the same case without including the point-source simulation. The difference returns an estimate of the contamination on \( f_{\text{nl}} \) due to point sources. For the \( V+W \) map we have \( \Delta f_{\text{nl}} = 6 \pm 5 \). Therefore our estimate taking into account the point sources is \(-18 < f_{\text{nl}} < +80 \) at 95% CL for the \( V+W \) map. Comparing with the best-fitting value for the \( V+W \) map given by Yadav & Wandelt (2008), \( f_{\text{nl}} = 87 \), our analysis excludes that value at \( \sim 99% \) CL. However, it is important to point out that Yadav & Wandelt (2008) used different choices for the data maps in the analysis (they used \( WMAP \) 3-yr data whereas we use \( WMAP \) 5-yr data, different weighting for the channels, etc.), which could also contribute to the found discrepancy. In particular, a simple average of the channels (instead of a noise-weighted combination) enhances the signal at high multipoles at the cost of having noisier maps.

**Table 2**

| Map     | Best \( f_{\text{nl}} \) | \( \sigma(f_{\text{nl}}) \) | \( X_{0.160} \) | \( X_{0.840} \) | \( X_{0.025} \) | \( X_{0.975} \) |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|
| V+W     | 39             | 1              | 25             | 26             | 24             | 51             | 47             |
| Q       | 11             | 0              | 30             | 31             | 34             | 63             | 66             |
| V       | 23             | 0              | 30             | 28             | 30             | 55             | 59             |
| W       | 65             | 4              | 30             | 33             | 26             | 59             | 58             |

Note. We also present the mean, dispersion, and some percentiles of the distribution of the best-fit \( f_{\text{nl}} \) values obtained from Gaussian simulations.
The W map best-fitting $f_{\text{al}}$ value is only compatible with zero at 99% CL. This result is in apparent discrepancy with the values obtained for the V and V+W maps, which are compatible with zero at 95% CL. The point sources add a low contribution to the W map, $\Delta f_{\text{al}} = 1 \pm 2$, and therefore they do not explain its best-fitting value of $f_{\text{al}} = 65$. We may wonder if that value can be obtained by a statistical fluctuation. Considering simulations with different models ($f_{\text{al}} = 0$, $f_{\text{al}} = 40$, and $f_{\text{al}} = 70$) we have confirmed that the best-fitting $f_{\text{al}}$ for W is compatible with the values obtained for the V and V+W maps.

To understand the relatively large $f_{\text{al}}$ value found in the W map we have performed some additional tests. First of all, we have checked if this deviation could be due to the presence of residual foregrounds by studying the V–W map for the clean and the raw (before template subtraction) maps as well as the raw maps for the $Q$, $V$, $W$, and V+W cases (see Table 3). We find that there is a very significant (positive) deviation in the clean V–W map. Note that the V–W map has CMB residuals due to the different resolutions of the V and W bands, which have also been taken into account in the V–W simulations. In any case, the signal due to the CMB is very small and we do not expect it to affect the results given in Table 3 for the different maps. Since this combination contains mainly residual foregrounds and noise, both could be responsible for the deviation. Interestingly, when we repeat the test for the raw V–W map, where foreground contamination should be more important, the best $f_{\text{al}}$ value becomes compatible with the simulations. This indicates again that foreground emission tends to bias the estimated $f_{\text{al}}$ toward lower values. This is also observed for the best $f_{\text{al}}$ value estimated for the raw $Q$, $V$, $W$, and V+W maps, which is systematically lower than the one obtained for the clean maps. Therefore, for the case of the raw V–W map, some effect from systematics may be cancelled by foreground residuals.

If foregrounds are not responsible for the deviation found in the W and V–W maps, we may wonder if it is due to systematics present in the W radiometers. To test this possibility, we have studied two different combinations of the four W radiometers, where CMB and foregrounds are basically cancelled. One of these combinations is consistent with Gaussian simulations but the second one shows again a deviation at the 95% CL, indicating the possible presence of some spurious signal in the noise of one or several of the W radiometers. In order to localize further the origin of this signal, we have also studied each W radiometer separately, finding a deviation at the level of 98% for the W2 radiometer, with a best $f_{\text{al}}$ value of 91, while the rest of the radiometers are consistent with the zero value at the 95% CL. Finally, we have also considered the difference between the two V radiometers, which is found to be compatible with Gaussianity. This further indicates that, if a systematic is responsible of the V–W map, this would be present in the W frequency channel (see Table 3).

All these tests suggest that the relatively large $f_{\text{al}}$ value obtained for W may come from systematics present in the W radiometers. In any case, a more exhaustive study is necessary in order to establish the origin of this deviation.

### 3.3. $f_{\text{al}}$ for the North and South Hemispheres

The localization property of the wavelets allows a local analysis of the $f_{\text{al}}$ parameter. In particular, we test the Gaussianity and estimate the best-fitting $f_{\text{al}}$ value of the V+W combined map using only northern (Galactic latitude $b > 0$) and southern pixels ($b < 0$) in Equation (2). For both cases, the third-order statistics obtained for the data are compatible with Gaussian simulations (inside the $2\sigma$ error bars). In Table 4, we list the best-fitting $f_{\text{al}}$ values for the northern and southern hemispheres and the main properties of the distribution of the best-fitting $f_{\text{al}}$ obtained from 1000 Gaussian simulations. We estimated the contribution of the unresolved point sources as in the previous

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**Table 3**

| Map | Foreground | Best $f_{\text{al}}$ | $\langle f_{\text{al}} \rangle$ | $\sigma(f_{\text{al}})$ |
|-----|------------|---------------------|-----------------|------------------|
| V+W | Raw | 34 | -1 | 25 |
| Q   | Raw | -3 | 0 | 33 |
| V   | Raw | 16 | 0 | 30 |
| W   | Raw | 60 | 0 | 30 |
| V+W | Raw | -0.02 | 0.02 | 0.29 |
| V-W | Clean | 1.02 | 0.02 | 0.29 |
| W1  | Clean | 39 | 1 | 41 |
| W2  | Clean | 91 | -3 | 45 |
| W3  | Clean | 23 | 2 | 47 |
| W4  | Clean | 59 | 0 | 44 |
| W1 + W2 - W3 - W4 | Clean | -0.52 | 0.00 | 0.34 |
| W1 - W2 + W3 - W4 | Clean | 0.85 | 0.00 | 0.35 |
| V1 - V2 | Clean | -0.01 | 0.01 | 0.34 |

**Note.** We also present the mean and the dispersion of the best-fitting $f_{\text{al}}$ values obtained from Gaussian simulations.

**Table 4**

| Region | Best $f_{\text{al}}$ | $\langle f_{\text{al}} \rangle$ | $\sigma(f_{\text{al}})$ | $X_{0.160}$ | $X_{0.840}$ | $X_{0.025}$ | $X_{0.975}$ |
|--------|---------------------|-----------------|------------------|----------|----------|----------|----------|
| North  | 46 | 2 | 37 | -35 | 40 | -71 | 74 |
| South  | 35 | -1 | 38 | -39 | 37 | -80 | 69 |

**Note.** We also present the mean, dispersion, and some percentiles of the distribution of the best-fit $f_{\text{al}}$ values obtained from Gaussian simulations.
subsection, and the values are $\Delta f_{nl} = 7 \pm 7$ for the north and $\Delta f_{nl} = 5 \pm 7$ for the south. Taking into account this, the results are $-32 < f_{nl} < 113$ for the north and $-50 < f_{nl} < 99$ for the south at 95% CL. These values are compatible with zero at 95% CL. We also study the compatibility of the best-fitting value for the north and south hemispheres between them. We compute the difference of the best-fitting $f_{nl}$ value $\Delta f_{nl} = f_{nl}^{(N)} - f_{nl}^{(S)}$ for the north and south hemispheres for the set of 1000 $V+W$ Gaussian simulations. The difference is $\Delta f_{nl}^{(\text{data})} = 11$ for the data, and for the simulations is $\Delta f_{nl} = 3 \pm 55$. The cumulative probability is $P(\Delta f_{nl} \leq \Delta f_{nl}^{(\text{data})}) = 0.57$ and therefore the difference for the data is compatible with the results obtained from simulations. This means that we do not find any asymmetry in the north–south $f_{nl}$ value.

4. CONCLUSIONS

We have tested the Gaussianity and constrained the $f_{nl}$ parameter with the 5-yr WMAP data. We use an optimal wavelet-based test. We have considered the $V+W$, $Q$, $V$, and $W$ combined maps at high resolution. We have used a set of 300 realistic non-Gaussian simulations and thousands of Gaussian simulations for the analysis. We have computed the wavelet coefficient maps at scales from 6.9 arcmin to 500 arcmin and computed all the possible third-order moments (Equation (2)) using appropriate extended masks.

The data are compatible with Gaussian simulations for the considered combined maps (see Table 1). We have imposed constraints on the nonlinear coupling parameter $f_{nl}$ by using non-Gaussian simulations with $f_{nl}$. The results show that $f_{nl}$ increases when we go from the $Q$ to the $V$ and $W$ combined maps. This frequency dependence also appears in the results by Yadav & Wandelt (2008), Komatsu et al. (2009), and Curto et al. (2009). The results are compatible with zero at 95% CL for the $V+W$, $Q$, and $V$ combined maps, but not for the $W$ map (which is compatible at 99% CL). This value cannot be explained by unresolved point sources since their contribution is $\Delta f_{nl} = 1 \pm 2$ for the $W$ map. We have estimated the probability of having those values with simulations and the results do not show incompatibility among different channels. We have also seen that the relatively large $f_{nl}$ value obtained for the $W$ band may come from systematics in one or several radiometers of this band.

We have also estimated the contribution of unresolved point sources to $f_{nl}$ for the $V+W$ map using a realistic model given by de Zotti et al. (2005). The results are $\Delta f_{nl} = 6 \pm 5$. Taking into account this value, our best estimate for $f_{nl}$ is $-18 < f_{nl} < +80$ at 95% CL. The use of new scales and all the possible third-order moments has returned better constrains to $f_{nl}$ and lower error bars compared with the results by Curto et al. (2009) and previous works. Our best estimate is compatible with the values obtained by Komatsu et al. (2009) and excludes the best-fitting $f_{nl}$ value obtained by Yadav & Wandelt (2008) at the $\sim$99% CL.

Finally, we have constrained $f_{nl}$ for the north and south hemispheres and the results give two best-fitting values that are compatible with zero at 95% CL and also are compatible between them. Therefore, we do not find any north–south asymmetry for this parameter.

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