Entropy of Intermediate-Mass Black Holes

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ABSTRACT

Observational searches for Intermediate-Mass Black Holes (IMBHs), defined to have masses between 30 and 300,000 solar masses, provide limits which allow up to ten percent of what is presently identified as halo dark matter to be in the form of IMBHs. These concentrate entropy so efficiently that the halo contribution can be bigger than the core supermassive black hole. Formation of IMBHs is briefly discussed.

1. Introduction

Surely the most spectacular confirmed prediction of general relativity is the black hole. Black holes are generally characterized by mass, charge and angular momentum. In the present paper I shall focus on mass because the electric charges of black holes in Nature are likely to be extremely small while angular momentum will here be suppressed for no better reason than simplicity.

I shall first introduce acronyms for three mass ranges of black hole in terms of the solar mass ($M_{\odot}$). Supermassive black holes (SMBHs) have $M_{\text{SMBH}} > 3 \times 10^5 M_{\odot}$. Intermediate-mass black holes (IMBHs) are defined by $3 \times 10^5 M_{\odot} > M_{\text{IMBH}} > 30 M_{\odot}$. Because most black holes are in the mass range I designate as Normal-mass black holes (NMBHs) $30 M_{\odot} > M_{\text{NMBH}} > 3 M_{\odot}$.

The compelling evidence for the existence of both SMBHs and NMBHs is well known and need not be repeated here. The status of IMBHs is reviewed in (Miller & Colbert 2004) with the conclusion that their existence in Nature is not absolutely certain. In the present paper, I shall suggest on the basis of entropy arguments[1] that many IMBHs exist in galactic halos.

[1] In an otherwise comprehensive discussion it is noteworthy that the word entropy occurs only once in (Miller & Colbert 2004).
and it is the job of astronomers to detect them.

2. Entropy Preamble

Entropy has at least two interesting properties. Firstly there is probably an upper limit \[2\] which arises from the holographic principle (Hooft 1994). Second, there is the idea that it never decreases, the second law of thermodynamics.

I shall employ throughout dimensionless entropy which means that Boltzmann’s formula is divided by his eponymous constant. Entropy is a powerful concept in theoretical physics generally and may have found a ubiquitous application in cosmology.

In discussing the dimensionless entropy of the universe I encounter very large numbers like a googol \((10^{100})\) and it will become clear how numbers below one googol are negligible. In discussing the number of microstates of the universe any number below one googolplex \((10^{10^{100}})\) is insignificant.

3. Entropy of Everything Except Black Holes

Black holes focus entropy density so efficiently that the entropy of everything else in the universe is negligible compared to its black holes.

Since this is crucial in justifying my focus on black holes, let me begin by considering briefly the entropy of other components which contribute to the energy of the universe. These are approximately 72\% dark energy, 24\% dark matter and 4\% normal matter.

I shall throughout make the assumption that the dark energy possesses no entropy \[3\].

It is convenient to introduce the notation where \(10^x\) denotes within one order of magnitude. \(10^x\) means an integer greater than \(10^{x-1}\) and less than \(10^{x+1}\).

Normal matter, other than relic photons and neutrinos, has entropy \(10^{80}\) nowhere near to one googol. The photons of the Cosmic Microwave Background (CMB) have entropy \(10^{88}\), orders of magnitude bigger, still negligible. The relic neutrinos have entropy comparable in order of magnitude to the CMB.

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2 The holographic principle has not, to my knowledge, been rigorously proven.

3 If dark energy entropy were discovered, the discussion would be modified.
What about dark matter? Unlike dark energy it must possess statistical properties like temperature and entropy and, other than black holes, its dimensionless entropy must fall far short of one googol.

I conclude that black holes overwhelmingly dominate cosmological entropy.

4. Entropy of Black Holes

From work \cite{Parker1969, Bekenstein1973, Hawking1974} (PBH) beginning four decades ago it is well established that the dimensionless entropy $S_{BH}$ of a black hole with mass $M_{BH} = \eta M_\odot$ is $10^{78} \eta^2$.

As a first (unrealistic) example consider the Sun which is normal matter with entropy $10^{57}$. Now take a black hole with the same mass: following the PBH analysis it has entropy $10^{78}$. Although the Sun is too light ever to collapse to a black hole, this gedankenexperiment illustrates the compression of entropy by 21 orders of magnitude such that in any entropy inventory with black holes everything else can be jettisoned with impunity.

Observed stellar NMBHs are at least three times the mass of the Sun and have masses between $3M_\odot$ and $30M_\odot$. For the sake of simplicity I assume every NMBH has $10M_\odot$ and therefore entropy $10^{80}$.

I shall assume the visible universe contains $10^{11}$ galaxies each with halo mass $10^{12} M_\odot$.

As a working estimate of the total entropy of NMBHs, I assume there are $10^{10}$ of them per galaxy and hence $10^{21}$ in the universe. Their total PBH entropy is $10^{101} = 10$ googols.

Let me turn now to the SMBHs at galactic centers. From the Southern Hemisphere in the direction of the constellation Sagittarius, where the Milky Way appears densest to the naked eye, there is good radio astronomical evidence for a SMBH known as Sag A* with mass of about four million solar masses. For simplicity I assume there is one SMBH with mass $10^7 M_\odot$ in every galaxy. Each has entropy $10^{92}$ and their total cosmological entropy is therefore $10^{103} = 10^3$ googols.

Finally I arrive at my principal topic, the intermediate-mass black holes whose observational evidence is inadequate. As an example let me take all IMBHs to have mass $10^5 M_\odot$ and entropy $10^{88}$. According to Fig. 1 as much as 10 percent \cite{Yoo2004} of the $10^{12} M_\odot$ halo can be IMBHs so there can be $10^6$ per galaxy and $10^{17}$ in the universe giving cosmological entropy

\footnote{I am grateful to Gianfranco Bertone for bringing \cite{Yoo2004} to my attention.}
Fig. 1.— The vertical axis is the total intermediate-mass black hole (IMBH) mass as the percent of the halo dark matter; the horizontal axis is the individual IMBH mass in terms of the solar mass $M_\odot$. Note that for the range of masses between $30M_\odot$ and $3 \times 10^5M_\odot$ as much as ten percent of the halo mass can be IMBHs, as can be seen by drawing a horizontal line at 10% and noting that it avoids the disallowed regions for all $30M_\odot < M_{IMBH} < 3 \times 10^5M_\odot$. Taken from (Yoo et al. 2004)
of $10^{105} = 10^5$ googols and 99% of total entropy.

The holographic principle allows a total entropy of the universe up to $10^{123} = 10^{23}$ googols achievable only if the universe were to become one big black hole quite different from the present situation.

5. Cyclicity

One reason that it is so urgent to seek and discover IMBHs is that their contribution to cosmological entropy is germane to a better understanding of the correct alternative to the big bang.

In the cyclic model of (Baum & Frampton 2007; Baum & Frampton 2008) which solves the Tolman conundrum, the total entropy provides a lower limit on the number of universes spawned at turnaround (Baum, Frampton & Matsuzaki 2008) and is relevant to the infinite past (Frampton 2007; Frampton 2009). The cyclic model can solve purely classically the concern expressed in (Penrose 2005) about fine-tuning to less than a part in a googolplex at the start of an expansion era.

6. Formation of IMBHs

There are two formation mechanisms which suggest themselves:

Firstly, the IMBHs may be remnants of Population III stars as discussed in (Madau & Rees 2001). If so, the mass estimates for Population III stars have been too conservative and they may range up to $3 \times 10^5 M_\odot$.

Secondly, numerical simulations (Navarro et al. 1996) of dark matter halos could be tightened to much higher spatial resolution and include realistic inelasticity or equivalent dynamical friction. In this way IMBH formation could be studied assiduously. Note that the largest IMBH has a radius less than $10^{-7}$ parsecs so the improvement must be by orders of magnitude.

It is possible that IMBH formation results sequentially from both mechanisms.
7. Discussion

The situation with respect to cosmological entropy is as if with respect to cosmological energy there remained ignorance of dark matter and dark energy.

I have put stock in the second law of thermodynamics to predict that entropy is dominated by IMBHs. The applicability of the second law to the evolution of the universe ignores dynamics yet I believe the dynamics would need to be pathological if IMBHs do not prevail in the galactic halo.

From Fig. 1, Wide Binaries (Chaname & Gould 2004) offer an opportunity to tighten the constraint on, or to discover, IMBHs.

The MACHO searches (Alcock et al. 1997; Alcock et al. 2000) found many definite examples of microlensing all with longevities less than 2y.

| Table I | Intermediate Mass Black Holes (IMBHs) and Microlensing Longevity. |
|---|---|
| log$_{10} \eta$ | $t_0$ (years) |
| 3 | 6 |
| 4 | 20 |
| 5 | 60 |

Here $\eta$ is defined by the mass of the IMBH being $\eta M_\odot$ where $M_\odot$ is the solar mass and $t_0$ is the longevity of a hypothetical microlensing event produced by the putative IMBH.

From Table I we see, however, that higher microlensing longevities are pertinent to IMBHs. For the example of $M_{IMBH} = 10^5 M_\odot$, it is 60y.

The Disk Stability upper limits on $M_{IMBH}$ seem robust due to the useful and important work in (Lacey & Ostriker 1985; Xu & Ostriker 1994).

On the theoretical side the observed metallicity of intergalactic dust requires that Population III stars exist and little is established about their masses. I suggest that there may have once existed far higher-mass zero-metallicity stars than discussed in (Madau & Rees 2001).

For numerical simulations (Navarro et al. 1996) of the dark matter halo to become useful for the study of IMBH formation they must dramatically improve on spatial resolution.

In conclusion, I believe that the IMBH entropy make up of the universe can be understood in a just few years. My question is for astronomers. What is the percentage contribution by
intermediate-mass black holes to the total cosmological entropy?

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