FlashMatrix: Parallel, Scalable Data Analysis with Generalized Matrix Operations

Da Zheng, Disa Mhembere, Joshua T. Vogelstein, Carey E. Priebe, and Randal Burns

1Department of Computer Science, Johns Hopkins University
2Department of Applied Mathematics and Statistics, Johns Hopkins University
3Institute for Computational Medicine, Department of Biomedical Engineering, Johns Hopkins University

Abstract—FlashMatrix is a matrix-oriented programming framework for general data analysis with high-level functional programming interface. It scales matrix operations beyond memory capacity by utilizing solid-state drives (SSDs) in non-uniform memory architecture (NUMA). It provides a small number of generalized matrix operations (GenOps) and reimplements a large number of matrix operations in the R framework with GenOps. As such, it executes R code in parallel and out of core automatically. FlashMatrix uses vectorized user-defined functions (VUDF) to reduce the overhead of function calls and fuses matrix operations to reduce data movement between CPU and SSDs. We implement multiple machine learning algorithms in R to benchmark the performance of FlashMatrix. The execution of the R implementations in FlashMatrix has performance comparable to optimized C implementations. When scaling beyond memory capacity on a large parallel machine, the out-of-core execution of these R implementations in FlashMatrix has performance comparable to in-memory execution on a billion-scale dataset. Both in-memory and out-of-core execution significantly outperform the in-memory execution of Spark MLlib.

I. INTRODUCTION

The National Strategic Computing Initiative (NSCI [1]) puts forth a critical problem as we move to exascale: “Increasing coherence between the technology base used for modeling and simulation and that used for data analytic computing.” A key challenge lies in providing statistical analysis and machine learning tools that are simple and efficient. Simple tools need to be programmable, interactive, and extensible, allowing scientists to encode and deploy complex algorithms. Tools need to be programmable, interactive, and extensible, allowing scientists to encode and deploy complex algorithms.

Large-scale data analysis requires a large parallel machine or a cluster to gain computation power and memory capacity. Currently, there are two approaches of implementing parallel algorithms to process large datasets. One can write an efficient implementation with low-level parallel primitives such as the ones provided by MPI [2] or OpenMP [3]. This approach requires expertise in parallel programming and significant effort from programmers. The other approach is to use high-level programming frameworks that provide high-level operations to reduce the burden of programmers. In general, the second approach is less computationally efficient but can significantly increase productivity and lower the barrier to writing parallel implementations. The second approach is preferred in the rapidly evolving fields of machine learning and data mining.

It is challenging to provide a programming framework that has a high-level programming interface and achieves both generality and efficiency. Some highly-optimized linear algebra libraries [4], [5], [6], [7], [8] provides a matrix programming interface that has a limited set of matrix operations with efficient implementations, e.g. BLAS provides only matrix multiplication and not integer operations or row/column operations. Users have to parallelize the remaining matrix operations themselves that are not supported by the libraries. High-level programming frameworks, such as R and Matlab, provide a general programming interface for users to express varieties of algorithms, but do not produce efficient parallel code.

We present FlashMatrix, a programming framework that provides a high-level matrix-oriented functional programming interface and supports automatic parallelization and out-of-core execution for large-scale data analysis; users write R programs and FlashMatrix executes them efficiently. Unlike most of the linear algebra libraries, FlashMatrix provides a small set of highly-optimized generalized matrix operations (GenOps) to achieve generality. GenOps represent common data access patterns. By accepting different functions that define operations on individual elements in the input matrices, each GenOp covers a very large number of matrix operations. FlashMatrix reimplements many matrix operations from the R base package with GenOps to execute R code in parallel and out of core automatically. FlashMatrix focuses on optimizations in a single machine and scales matrix operations beyond memory capacity by utilizing solid-state drives (SSDs). This design choice conforms with a current trend of hardware design that scales up a single machine for high performance computing [9], including analysis of data stored on SSDs of I/O burst buffers [10].

We overcome many technical challenges to move data from SSDs to CPU efficiently, overcoming the large speed disparity between CPU and memory, as well as between memory and SSDs. The speed disparity of CPU and DRAM has increased exponentially over the past decades [11]. While SSDs have high IOPS and sequential I/O throughput, they are still an order of magnitude slower than DRAM. Many analysis tasks are data-intensive. Matrix formulation further increases data movement between CPU and SSDs because a matrix compu-
sition framework typically performs an operation on the entire input matrices before moving to the next operation.

Another challenge in FlashMatrix is to reduce computation overhead. A GenOp in FlashMatrix takes some functions as additional arguments that define operations on individual elements in the input matrices. To support the matrix operations in the R base package, a GenOp accepts functions at run time. As such, each operation on an element potentially results in a function call, incurring computational overhead.

To move data efficiently, FlashMatrix evaluates expressions lazily and fuses operations aggressively in a single parallel execution job. FlashMatrix builds a directed acyclic graph (DAG) to represent all operations in a single execution. When evaluating the computation in a DAG, FlashMatrix performs two levels of matrix partitioning to improve data utilization in memory hierarchy and reduce data movement between memory and SSDs as well as between CPU and memory. To access data stored on SSDs, FlashMatrix streams data from SSDs to maximize I/O throughput.

To reduce computation overhead, we deploy vectorized user-defined functions (VUDFs), which operates on a vector of elements, instead of an individual element. We define multiple forms for each VUDF and automatically select the right form for each GenOp to amortize the function call overhead. When invoking VUDFs, GenOps choose the right vector length to balance the amortization of the function call overhead and CPU cache misses. Inside VUDFs, we use vector CPU instructions, such as AVX [12], to further improve performance.

We implement multiple machine learning algorithms, including KMeans [13] and Gaussian Mixture Models [14] in FlashMatrix with its R programming interface to benchmark its performance. On a large parallel machine with 48 CPU cores and fast SSDs, the out-of-core execution of these R implementations in FlashMatrix achieves performance comparable to the in-memory execution, while significantly outperforming the same algorithms in Spark MLlib [15]. FlashMatrix effortlessly scales to datasets with billions of data points and its out-of-core execution uses a small fraction of resources required by in-memory implementations. When running in a single thread, the FlashMatrix implementations outperform the C and FORTRAN optimized implementations of the R framework. In addition, FlashMatrix achieves almost linear speedup in a multicore NUMA machine for all algorithms. We believe FlashMatrix significantly lowers the requirements for writing parallel and scalable implementations of data analysis algorithms; it also offers new design possibilities for data analysis clusters, replacing memory with larger and cheaper SSDs and processing bigger problems on fewer nodes.

II. RELATED WORK

Basic Linear Algebra Subprograms (BLAS) defines a small set of vector and matrix operations commonly used in scientific computing. There exist a few highly-optimized BLAS implementations such as MKL [4], OpenBLAS [5], GotoBLAS [6] and ATLAS [7]. However, these libraries optimize the vector and matrix operations in shared memory. BLAS provides only a small number of vector and matrix operations.

Distributed memory matrix computation libraries [7], [8], [6] speed up computation and scale to larger vectors and matrices. These libraries in general build on top of BLAS and distribute computation with MPI. They provide a limited set of predefined matrix operations and require users to manually parallelize the remaining matrix operations. Instead of providing predefined matrix operations, the core of FlashMatrix provides a few GenOps that represent some common data access patterns. Thus, each GenOp covers a very large number of matrix operations.

There are many distributed data processing frameworks. MapReduce [13] is a general large-scale data processing framework. It provides a single primitive that takes two user-defined functions. Due to the lack of efficient primitives for varieties of data access patterns, algorithms implemented in MapReduce are inefficient. Dryad [19] and Naiad [20] provide more primitives than MapReduce to support various data access patterns more efficiently.

Due to complexity of programming in the distributed execution engines, many programming frameworks have been developed on top of the distributed execution engines. Pig Latin [21] and FlumeJava [22] build on top of MapReduce to provide high-level operations for general data analysis. SystemML [23] builds on top of MapReduce with a focus on machine learning. DryadLINQ [24] builds on top of Dryad and exposes a high-level language to express data analysis tasks. The performance of these programming frameworks is bound by the underlying distributed execution engines.

Spark [15] is a distributed in-memory data processing framework. It provides a highly-optimized machine learning library called MLlib [25]. Spark also provides an R programming interface called SparkR, which focuses on computation on data frames, a table-like data structure in R.

Both academia and industry are making significant effort to bring parallelization to array programming languages and scale them to large datasets. Revolution R [26] and parallel computing toolbox in MatLab [27] provide parallel linear algebra and data analysis routines as well as explicit parallel programming interface such as MPI and MapReduce. Other works bring implicit parallelization to programming frameworks. Presto [28] extends R to support sparse matrix operations in distributed memory for graph analysis. Ching et. al [29] parallelizes APL code by compiling it to parallelized C code. Accelerator [30] compiles data-parallel operations on the fly to execute programs in GPU.

III. DESIGN

FlashMatrix is a matrix-oriented programming framework for general data analysis. It supports both sparse matrix operations and dense matrix operations. This work mainly focuses on dense matrices and scales dense matrix operations beyond memory capacity by utilizing fast I/O devices, such as solid-state drives (SSDs), in a non-uniform memory architecture (NUMA). The implementation and optimization of sparse
matrix multiplication is described in [31]. FlashMatrix uses R as its main programming interface and executes R code automatically in parallel and out of core.

Figure 1 shows the architecture of FlashMatrix. The core of FlashMatrix provides a small number of generalized matrix operators (GenOps) to simplify the implementation and improve expressiveness of the framework. The optimizer in FlashMatrix aggressively merges operations to reduce CPU cache misses and I/O accesses and achieve better parallelization. FlashMatrix stores large matrices on SSDs through SAFS [32], a user-space filesystem for a large SSD array, to fully utilize high I/O throughput of SSDs and deploys a set of I/O optimizations to improve its sequential I/O throughput [31].

A. Programming interface
FlashMatrix provides a matrix-oriented functional programming interface. The main interface is a small set of GenOps, implemented with C++. These operators take matrices and functions that define computation on individual elements in input matrices, and output new matrices that store computation results. As such, matrices in FlashMatrix are immutable. FlashMatrix exposes GenOps in its R interface and reimplements many functions from the R base package with GenOps.

The R interface provides many functions to support varieties of data analysis algorithms. We categorize the functions into three classes.

- The generalized matrix operators (GenOps), listed in Table I.
- Utility functions include functions that construct FlashMatrix vectors and matrices; functions that convert between FlashMatrix objects and R objects; functions that transform the shape of a matrix; functions that provide additional control on computation and data storage in FlashMatrix. Examples are shown in Table II.
- R matrix computation functions implemented with GenOps. Examples are shown in Table III.

B. Dense matrices
Dense matrices are the main data types in FlashMatrix. A vector is stored as a one-column dense matrix. In FlashMatrix, a dense matrix can be stored physically in memory or on SSDs or represented virtually by a sequence of computation. FlashMatrix supports dense matrices of different shapes (Figure 2). It specifically optimizes tall-and-skinny matrices

| Class | Function | Description |
|-------|----------|-------------|
| Construct | fm.rep.int | Create a vector with a repeated value |
| | fm.seq.int | Create a vector with sequence numbers |
| | fm.runif.matrix | Create a uniformly random matrix |
| | fm.rnorm.matrix | Create a random matrix under a normal distribution |
| Convert | fm.conv.F2R | Convert an FM matrix to an R matrix |
| | fm.conv.R2FM | Convert an R matrix to a FM matrix |
| Transform | t | Matrix transpose |
| | fm.rbind | Combine multiple matrices by rows |
| | fm.cbind | Combine multiple matrices by columns |
| Control | fm.conv.layout | Convert the data layout of a matrix |
| | fm.set.mate.level | Set the materialization level of a virtual matrix |
| | fm.materialize | Force to materialize a virtual matrix |
| | fm.conv.store | Move a matrix to a specified memory storage |

| Class | Function | Description |
|-------|----------|-------------|
| Element-wise | CC = AA + BB | CC\text{,ij} = AA\text{,ij} + BB\text{,ij} |
| | CC = AA - BB | CC\text{,ij} = AA\text{,ij} - BB\text{,ij} |
| | CC = AA * BB | CC\text{,ij} = AA\text{,ij} * BB\text{,ij} |
| | CC = AA/BB | CC\text{,ij} = AA\text{,ij} / BB\text{,ij} |
| | CC = pmin(AA, BB) | CC\text{,ij} = \text{pmin}(AA\text{,ij}, BB\text{,ij}) |
| | CC = pmax(AA, BB) | CC\text{,ij} = \text{pmax}(AA\text{,ij}, BB\text{,ij}) |
| | CC = sqrt(AA) | CC\text{,ij} = \text{sqrt}(AA\text{,ij}) |
| | CC = abs(AA) | CC\text{,ij} = \text{abs}(AA\text{,ij}) |
| | CC = exp(AA) | CC\text{,ij} = \text{exp}(AA\text{,ij}) |
| Aggregate | c = sum(AA) | c = \sum_{i=1}^{n} AA\text{,ij} |
| | C = rowSums(AA) | C_{i} = \sum_{j=1}^{n} AA\text{,ij} |
| | C = colSums(AA) | C_{j} = \sum_{i=1}^{n} AA\text{,ij} |
| | C = any(AA) | true if any element is true |
| | c = all(AA) | true if all elements are true |

TABLE III: Some of the R functions implemented with GenOps.
and short-and-wide matrices, and view tall matrices and wide matrices as groups of tall-and-skinny matrices and short-and-wide matrices, respectively.

1) Tall-and-skinny matrices: FlashMatrix optimizes for tall-and-skinny (TAS) dense matrices (Figure 2(a)) due to their frequent occurrence in data analysis. In this field, many data matrices contain a large number of samples with a relatively few features, so data matrices are usually tall and skinny. If a data matrix has many features, the first step is often dimension reduction [33], which results in a TAS matrix. FlashMatrix specifically optimizes for TAS dense matrices with tens of columns or fewer.

FlashMatrix supports row-major and column-major matrix layout (Figure 3). As such, we avoid data copy for common matrix operations such as matrix transpose. FlashMatrix optimizes matrix operations for both data layouts. Each GenOp has its own preferred matrix layout and determines the layout of an output matrix based on the input matrices.

FlashMatrix uses two-level horizontal partitioning on TAS matrices for efficient data access to SSDs and main memory (Figure 3). FlashMatrix partitions both in-memory and external-memory TAS matrices horizontally into I/O-level partitions. All elements in an I/O-level partition are stored contiguously regardless of the data layout in the matrix. A for in-memory matrix, the partition size determines the size of contiguous memory required in memory allocation (see Section III-B5). For an external-memory matrix, the partition size determines an I/O size, usually on the order of megabytes, because each I/O access reads an entire I/O-level partition. The number of rows in an I/O-level partition is always $2^i$. This produces column-major TAS matrices whose data are well aligned in memory to help CPU vectorization. For an in-memory TAS matrix, FlashMatrix maps I/O-level partitions to NUMA nodes to achieve data locality and full utilization of memory bandwidth. We map I/O-level partitions of different vectors/matrices involved in computation to the same NUMA node to reduce remote memory access. We further split an I/O-level partition horizontally into CPU-level partitions for computation. We use a small CPU-level partition (on the order of kilobytes) so that it fits in CPU L1/L2 cache to reduce CPU cache misses when evaluating a sequence of matrix operations (Section III-E). FlashMatrix determines the number of rows in a CPU-level partition based on the number of columns in a matrix.

2) Virtual matrices: In many cases, we do not need to store the data of a matrix physically. Instead, we compute and generate its data on the fly. Virtual matrices store computation and potentially the reference to some other matrices required by computation. A simple example is a matrix with all elements having the same value. For such a matrix, we only need to store a single value and construct its matrix partitions during computation.

Virtual matrices are essential for lazy evaluation (Section III-E). All GenOps may output virtual matrices that represent computation results by storing only the computation of GenOps and the references to input matrices. This strategy is essential for both in-memory and external-memory optimizations to improve performance. It significantly reduces data access to memory and SSDs as well as memory allocation overhead for creating new matrices.

3) Cached matrix: Memory cache is necessary for external-memory matrices on a machine with substantial memory. Even though SSDs have substantial I/O performance, they are still an order of magnitude slower than DRAM. Unfortunately, we cannot rely on the page cache in SAFS [34] to buffer some portion of a dense matrix because streaming an entire matrix to memory evicts existing data in the page cache and generates zero cache hits. Therefore, FlashMatrix allows users to explicitly cache part of a dense matrix.

We store a matrix cache as an in-memory dense matrix. To effectively cache data in a dense matrix, we store a tall matrix in column-major and cache the first few columns; we store a wide matrix in row-major and cache the first few rows. As such, when computation requests an I/O-level partition of a matrix, we only need to issue a single I/O request to read
4) A group of dense matrices: FlashMatrix represents a tall matrix with a group of tall-and-skinny matrices and a wide matrix with a group of short-and-wide matrices (Figure 2 (c) and (d)). We construct a special virtual matrix to represent a group of dense matrices. To take advantage of the optimizations on matrix operations on TAS matrices, we decompose a matrix operation on a group of matrices into operations on individual matrices in the group (Section III-H). Coupled with the two-level partitioning on TAS matrices, this strategy enables 2D-partitioning on a dense matrix and each partition fits in main memory or CPU cache.

5) Memory management: Creating in-memory matrices requires large memory allocations, which are expensive operations in Linux because Linux uses mmap to allocate large memory and relies on page faults to populate the memory. Frequent page faults prevent computation from fully utilizing CPUs in a large parallel machine and cause significant performance degradation. The functional programming interface of FlashMatrix leads to more frequent memory allocation, because each matrix operation generates a new matrix. FlashMatrix manages large memory allocation itself to reduce overhead.

FlashMatrix stores an in-memory matrix in fixed-size memory chunks and recycles memory chunks to reduce memory allocation overhead (Figure 4). FlashMatrix requires only an I/O-level partition to be stored in contiguous memory. As long as a memory chunk is sufficient to store an I/O-level partition, FlashMatrix can store a matrix in a set of fixed-size memory chunks. The size of a memory chunk is a global parameter and is the same for all matrices. As such, FlashMatrix reuses memory chunks allocated for matrices of different shapes. We choose the memory chunk size to be much larger than the I/O-level partition size to increase memory utilization. We use 64MB as the default memory chunk size.

C. Generalized computation operations

To improve generality and simplify the implementation, FlashMatrix provides only four generalized operators (GenOps) on matrices: inner product, apply, aggregation and groupby. Each operator represents a data access pattern and accepts some functions as additional arguments that define computation on individual elements in matrices (Section III-D).

Inner product is a generalized matrix multiplication (fm.inner.prod). It replaces multiplication and addition in matrix multiplication with two functions. We define many operations with inner product. For example, we use it to compute various pair-wise distances, such as Euclidean distance and Hamming distance, between data points. For dense matrices, we mainly focus on optimizing two cases: inner product of a wide matrix and a tall matrix and inner product of a tall matrix and a small matrix. It is impractical to materialize inner product of a large tall matrix and a large wide matrix owing to space complexity. This holds for all matrix algebra frameworks. When evaluating an inner product expression, FlashMatrix uses the BLAS implementation of matrix multiplication for floating-point matrices. This achieves the speed and precision required by numeric libraries, such as eigensolvers [35], [36].

Apply is a generalized form of element-wise operations and has multiple variants. The simplest variant (fm.sapply) is an element-wise unary operation. We use it to implement many unary operations such as negation, square root or element type casting on a matrix. The second variant (fm.mapply) is an element-wise binary operation. We use it to implement many binary matrix operations such as matrix addition and subtraction. The third (fm.mapply.row) and the fourth variants (fm.mapply.col) perform element-wise binary operations on the input vector with every row or column of the input matrix and output a matrix of the same shape as the input matrix.

Aggregation takes multiple elements and outputs a single element. It has three variants on a matrix. The first variant (fm.agg) aggregates over all elements on a matrix, e.g., matrix summation. The second (fm.agg.row) and the third variants (fm.agg.col) compute aggregation over each individual row or column. rowSums and colSums in R are examples.

Similarly, Groupby on a matrix has two variants. The first variant (fm.groupby.row) groups rows and the second variant (fm.groupby.col) groups columns of a matrix based on a vector of categorical values and performs aggregation on the rows or columns associated with the same categorical value. Matrix groupby is used in classification and clustering algorithms that compute aggregation on the data points in a class or in a cluster.

D. Vectorized user-defined function

GenOps take functions that define computation on individual elements. Computing on each element individually would result in significant function call overheads. Instead, all of the GenOps take vectorized user-defined functions (VUDFs) that operate on a vector of elements instead of an individual element. By transforming the operations on individual elements
to the ones on a vector, we amortize the overhead of function calls significantly.

We balance the amortization of function call overhead and CPU cache misses. To reduce latency of accessing data in VUDFs, the input data has to be small enough to fit in the CPU L1 cache. On the other hand, passing a longer vector to a VUDF amortizes the overhead of function calls more aggressively. We use 128 as the maximum length of the input vector of a VUDF.

We have three types of VUDFs to support the GenOps in FlashMatrix. Each VUDF type may have multiple forms to allow GenOps to transform operations to increase the length of an input vector to a VUDF and reduce function call overhead.

- **A unary VUDF** ($uVUDF$) takes a vector as input and outputs a vector of the same length.
- **A binary VUDF** has three forms: the first form ($bVUDF1$) takes two vectors of the same length and outputs a vector of the same length as the input vectors; the second form ($bVUDF2$) takes a vector as the left argument and a scalar as the right argument and outputs a vector with the same length as the input vector; the third form ($bVUDF3$) takes a scalar as the left argument and a vector as the right argument and outputs a vector. The second and third forms support non-commutative binary operations such as division and subtraction.
- **An aggregation VUDF** consists of two functions: aggregate and combine. Both functions may have two forms: the first one ($aVUDF1$) takes a vector and outputs a scalar; the second one ($aVUDF2$) takes two vectors of the same length and outputs a vector. For many aggregation VUDFs such as summation, aggregate and combine are the same and have both $aVUDF1$ and $aVUDF2$ forms. For some aggregation such as count, aggregate and combine are different.

FlashMatrix provides many commonly used VUDFs. These VUDFs wrap basic operations built in many programming languages and libraries. For example, FlashMatrix provides arithmetic operations (addition and subtraction), relational operations (equal to and less than), logical operations (logical AND and logical OR), as well as commonly used math functions (absolute value and square root). FlashMatrix also provides a set of VUDFs to cast primitive element types.

For each basic operation, FlashMatrix provides multiple VUDF implementations to support different element types. To reduce the number of binary VUDF implementations, FlashMatrix only provides the ones that take two input arguments of the same type. If a GenOp gets two matrices with different element types, it first casts the element type of one matrix to match the other. Type casting operations are implemented with $fn.sapply$ and are performed lazily.

FlashMatrix allows programmers to extend the framework by registering new VUDFs. Like built-in VUDFs, a new VUDF needs to provide multiple implementations to support different element types; based on the type of the VUDF, it may need to provide different forms as described above.

FlashMatrix currently requires a VUDF to be implemented with C/C++.

We use CPU vector instructions such as AVX [12] to accelerate the computation in a VUDF. The current implementation of FlashMatrix heavily relies on auto-vectorization of a compiler, such as GCC, to vectorize computation. FlashMatrix provides hints and transforms code to help auto-vectorization. For example, a VUDF in FlashMatrix frequently operates on vectors with data aligned in memory and of the length defined at compile time, so we inform the compiler of the data alignment and the vector length. Some compilers do not automatically vectorize aggregation operations well. In this case, we manually create a small vector of reduction variables, flatten the loop and transform the original aggregation operation into aggregation onto the vector of reduction variables to help auto-vectorization.

Figure 5 shows an example of using GenOps and VUDFs to compute standard deviation on a dataset with missing values, a common computation in R. In the presence of missing values, we need to exclude the missing values from the matrix. The first step is to apply $sapply$ with the unary VUDF $isna$ to test which elements are missing values. The second step is to apply $mapply$ with the binary VUDF $ifelse$ on $X$ and $isna.X$ to replace missing values in $X$ with 0 and on $X^2$ and $isna.X$ to replace missing values in $X^2$ with 0. The third step is to apply $agg$ with + to compute summation of $X$, $X^2$ and $isna.X$. Eventually, we compute the mean of $X$ and $X^2$ and the standard deviation, excluding missing values.

E. Lazy evaluation

FlashMatrix evaluates matrix operations lazily. Evaluation of individual matrix operations results in significant data movement between CPU and SSDs as well as frequent memory allocation. Lazy evaluation enables matrix operation fusion to minimize data movement, reduce memory allocation and improve parallelization [29].

FlashMatrix constructs a directed acyclic graph (DAG) to represent a sequence of matrix operations evaluated lazily (Figure 5 (a)). A lazily evaluated GenOp outputs a virtual matrix to capture matrix computation and the input matrices. A DAG comprises a set of virtual matrix nodes (shown as rectangles) and computation nodes (shown as ellipses). We refer to the input matrices of a computation node as the parent matrices and the output matrix as the child matrix. The computation nodes may contain some immutable computation state, such as scalar variables and small matrices involved in the matrix computation. A matrix node can be used by multiple computation nodes as an input matrix. Although the virtual matrices do not need to have the same shape, FlashMatrix requires all virtual matrices in a DAG to have the same long dimension (Figure 5 (b)) to simplify evaluation and data flow of a DAG (a long dimension refers to the matrix dimension with the larger size).

FlashMatrix allows lazy evaluation on all GenOps. There are two types of GenOps in a DAG. The first type, such as
Fig. 5: A directed acyclic graph of computing standard deviation on a dataset with missing values.

**F. Matrix materialization**

Lazy evaluation postpones computation in matrix operations, but we eventually have to materialize some virtual matrices to perform actual computation.

FlashMatrix can materialize any virtual matrix in a DAG and can materialize multiple virtual matrices together. Typically, we materialize only sink matrices in a DAG. In the example shown in Figure 5, we materialize the three sink matrices together to reduce data movement in the memory hierarchy. Many iterative algorithms need to materialize some non-sink matrices in a DAG to avoid redundant computation and I/O if these non-sink matrices are used across iterations. FlashMatrix allows programmers to set a flag on the non-sink matrices to inform FlashMatrix to save materialized data of these matrices to memory or SSDs during computation.

We partition matrices on a DAG in the long dimension for materialization and parallelization (Figure 5 (b)). All virtual matrices except sink matrices on a DAG have the same long dimension size. To simplify materialization, they all share the same partition size in the long dimension. As such, a partition \( i \) of a virtual matrix only requires data from partitions \( j \) of the parent matrices and FlashMatrix materializes partitions in a matrix independently. When materializing a sink matrix, each thread computes partial aggregation results independently on the partitions of the parent matrix assigned to the thread. In the end of computation, FlashMatrix merges the partial aggregation results to construct the sink matrix.

FlashMatrix takes advantage of the two-level partitioning on dense matrices to reduce data movement between SSDs and CPU. It assigns I/O-level partitions (Section II-B1) to a thread as computation tasks for parallelization. We choose a relatively small partition size to balance the overhead of accessing a partition, skew and memory consumption. A thread further splits an I/O-level partition into CPU-level partitions at run time and materializes one CPU-level partition at a time. After materializing a CPU-level partition, the thread passes the partition to the subsequent operation in the DAG, instead of materializing the next CPU-level partition in the same matrix. A CPU-level partition is sufficiently small to fit in the CPU cache so that the partition still resides in CPU cache when the subsequent operation consumes it. This significantly reduces data movement between CPU and memory.

**G. Implementation of GenOps with VUDF**

GenOps invoke VUDFs on the elements of CPU-level partitions intelligently to increase the length of vectors passed and to reduce the overhead of function calls. Different GenOps choose different forms of VUDFs based on the data layout and the shape of the input matrices.

Some GenOps invoke VUDFs on the elements of matrices efficiently regardless of data layout and matrix shape. For example, \( \text{fm.sapply} \) and \( \text{fm.mapply} \) only require the input matrices and the output matrix to have the same data layout. For tall column-major matrices and wide row-major matrices, each CPU-level partition has long columns and long rows, respectively. These GenOps invoke a VUDF on the long columns and rows. For tall row-major matrices and wide column-major matrices, all rows and columns in a CPU-level partition are stored in a single piece of memory. These GenOps invoke a VUDF only once on all elements in a partition.

Most of the GenOps require a matrix with a specific data layout to reduce function call overhead. Many of the GenOps favor the column-major order for a tall-and-skinny matrix and the row-major order for a short-and-wide matrix. These data layouts increase the length of a vector passed to a VUDF and align data in memory. For example, the column-major order ensures that each column in a partition of a tall matrix is aligned in memory, regardless of the number of columns in the matrix. A GenOp, such as inner product, converts the data layout of a CPU-level partition to the preferred layout if an input matrix does not have the preferred layout.

Given a matrix with the preferred data layout, a GenOp selects different forms of a VUDF automatically based on the shape of the input matrix. For example, for a tall column-major matrix, \( \text{fm.mapply} \) invokes the \( b\text{VUDF1} \) form of the binary VUDF on a column from the input matrix and the input vector; for a wide row-major matrix, \( \text{fm.mapply} \) invokes the \( b\text{VUDF2} \) form on a row from the input matrix and an
element from the vector. We apply the similar strategy to other GenOps. When applying inner product on a tall column-major matrix, FlashMatrix uses the bVUDF2 form of the first VUDF to computes the outer product of a column from the left matrix and a row from the right matrix, and uses the aVUDF2 of the second VUDF to compute the final result. Because inner product operates on a CPU-level partition, all intermediate results in the computation reside in CPU cache. Inner product on a wide matrix and a tall matrix invokes the bVUDF1 form of the first VUDF on a row from the left matrix and a column from the right matrix, and invokes the aVUDF1 form of the second VUDF on the output from the first VUDF to compute an element in the output matrix for the input partitions.

H. Implementation of GenOps on a group of matrices

When applying a GenOp on a group of matrices (Section III-B4), we decompose the computation into multiple GenOps and apply them to individual matrices in the group if the GenOp supports decomposition. Decomposing computation to individual matrices reduces memory copies and increases CPU cache hits. For the GenOps that cannot be decomposed, we combine the individual matrices on the fly and apply the GenOps on the combined matrix directly.

We apply some of the GenOps to individual matrices directly without transformation. For example, fm.sapply and fm.agg run on individual matrices directly regardless of the shape and data layout of the matrices. Other GenOps may be applied to individual matrices directly if the input matrices have certain shape. For example, we apply fm.mapply.col and fm.agg.col to individual matrices in a group of tall matrices directly. Similarly, we apply fm.mapply.row and fm.agg.row to individual matrices in a group of wide matrices directly.

Applying other GenOps to a group of matrices requires transformation. If an aggregation VUDF provides a combine function, applying fm.agg.row to a group of tall matrices is transformed into two steps: apply the aggregate function on each row of individual matrices and apply the combine function on the partial aggregation results. When applying fm.mapply.row to a group of tall matrices, we break the input vector into parts to match the number of columns in the individual matrices in the group and apply fm.mapply.row to individual matrices separately. We apply the same strategies to fm.agg.col and fm.mapply.col on a group of wide matrices.

IV. EXPERIMENTAL EVALUATION

We evaluate the performance of FlashMatrix with statistics and machine learning algorithms both in memory and on SSDs. We compare their performance with the implementations in Spark MLlib [25], a highly-optimized parallel machine learning library, and the C and FORTRAN implementations in the R framework. We further illustrate the effectiveness of the optimizations deployed in FlashMatrix when running both in memory and on SSDs.

We conduct experiments on a non-uniform memory architecture machine with four Intel Xeon E7-4860 processors, clocked at 2.6 GHz, and 1TB memory of DDR3-1600. Each processor has 12 cores. The machine has three LSI SAS 9300-8e host bus adapters (HBA) connected to a SuperMicro storage chassis, in which 24 OCZ Intrepid 3000 SSDs are installed. The 24 SSDs together are capable of delivering 12 GB/s for read and 10 GB/s for write at maximum. The machine runs Linux kernel v3.13.0. By default, we use 48 threads for both in-memory and out-of-core execution of FlashMatrix. We use Spark v1.5.0 and R v3.2.4.

A. Statistics and Machine learning algorithms

We implement multiple important algorithms in the field of statistics and machine learning. We implement these algorithms completely with the R interface of FlashMatrix and rely on FlashMatrix to perform computation in parallel and out of core.

• Multivariate statistical summary: this computes column-wise minimum, maximum, mean, L1 norm, L2 norm, the number of non-zero values and variance on a data matrix.
• Correlation: this computes pair-wise Pearson’s correlation [37] among multiple series of data and is commonly used in statistics.
• Singular value decomposition (SVD) factorizes a matrix into three matrices: $U$, $\Sigma$ and $V$ such that $A = U\Sigma V^T$, where $U$ and $V$ are orthonormal matrices and $\Sigma$ is a diagonal matrix with non-negative diagonals in descending order. To compute SVD on a $n \times p$ matrix $A$ ($n \gg p$), we first compute Gramian matrix $A^TA$ and compute eigenvalues and eigenvectors to derive singular values and singular vectors of the matrix $A$. SVD is commonly used for dimension reduction. In the experiments, we compute 10 singular values.
• K-means [13] is an iterative algorithm of partitioning a set of data points into $k$ clusters so that each cluster has minimal mean of distances between the data points in the cluster and the cluster center. K-means is one of the most popular clustering algorithms and is identified as one of the top 10 data mining algorithms [38]. In the experiments, we run k-means to split a dataset into 10 clusters by default.
• Gaussian Mixture Model (GMM) [14] is another iterative clustering algorithm that assumes data points are sampled from a mixture of Gaussian distributions and use expectation maximization (EM) [14] algorithm to fit the model. This algorithm is also identified as one of the top 10 data mining algorithms [38]. In the experiments, we run GMM to split a dataset into 10 clusters by default.

These algorithms have various ratios of computation complexity and I/O complexity (Table V), which helps to evaluate the performance of FlashMatrix on SSDs thoroughly. The first three algorithms only require a constant number of passes over the input matrix. K-means and GMM run iteratively and we show their computation and I/O complexity in a single iteration. GMM typically run on a dataset with a small number of features. Therefore, the first term of its computation complexity dominates the computation. Although correlation and SVD have lower asymptotic complexity than GMM, they
TABLE IV: The computation and I/O complexity of the algorithms for the five algorithms. $n$ is the number of data points in the dataset, $p$ is the number of features in a data point and $k$ is the number of clusters k-means and GMM partition the dataset. We assume $n \gg p$.

| Algorithm         | Computation | I/O          |
|-------------------|-------------|--------------|
| Summary           | $O(n \times p)$ | $O(n \times p)$ |
| Correlation       | $O(n \times p^2)$ | $O(n \times p)$ |
| SVD               | $O(n \times p^2)$ | $O(n \times p)$ |
| K-means (1 iteration) | $O(n \times p \times k)$ | $O(n \times p)$ |
| GMM (1 iteration) | $O(n \times p^2 \times k + p^2 \times k)$ | $O(n \times p \times n \times k)$ |

TABLE V: Datasets ($n \times p$ matrices) for performance evaluation.

| Data Matrix       | n   | p   | size  |
|-------------------|-----|-----|-------|
| Friendster-32     | 65M | 32  | 16GB  |
| MixGaussian-1B    | 1B  | 32  | 251GB |
| Random-65M        | 65M | 8-512 | 4-248GB |

may run on datasets with many features and, thus, may have very high computation overhead.

K-means and GMM typically run on a dataset with a small number of features in each data point due to curse of dimensionality [33] while the other algorithms may be applied to datasets with various numbers of features. We use the datasets in Table V for performance evaluation. In all datasets, the number of data points is far more than the number of features. We run k-means and GMM on the Friendster-32 matrix, constructed from 32 eigenvectors of the Friendster graph [39], as well as the MixGaussian-1B matrix with one billion data points and 32 features in each data point, sampled from 10 mixtures of multivariate Gaussian distributions with the identity covariance matrix and different means. We measure the performance of the other three algorithms on all of the matrices in Table V including the random matrices with 65 million rows and the number of columns varying from 8 to 512.

B. Comparative performance of FlashMatrix

We compare the performance of the FlashMatrix implementations with the ones in Spark MLlib [25] and the R framework. We run the MLlib implementations with their native Scala interface and use a very large heap size to ensure that all input data is cached in memory. We use 48 threads for both FlashMatrix and MLlib to run on the MixGaussian-1B matrix. The R framework provides C implementations for correlation, SVD and k-means. The R package mcclus [40] provides a FORTRAN implementation of GMM. These implementations run in a single thread. We run the FlashMatrix implementations in a single thread and compare their performance with the C and FORTRAN implementations on the Friendster-32 matrix.

FlashMatrix both in memory and on SSDs outperforms Spark MLlib significantly in all algorithms (Figure 6(a)). For some algorithms such as correlation, SVD and GMM, even though both FlashMatrix and MLlib implementations heavily rely on BLAS for matrix multiplication, FlashMatrix outperforms MLlib significantly owing to our heavy optimizations on GenOps such as aggressive matrix operation fusion and VUDFs. In contrast, MLlib materializes operations such as aggregation separately and implements non-BLAS operations with Scala.

Even though FlashMatrix provides a matrix-oriented functional programming interface, it easily scales to datasets with billions of data points and its scalability is bound by the capacity of SSDs (Figure 6(b)). For out-of-core execution, FlashMatrix keeps large matrices on SSDs and has a very small memory footprint. The functional programming interface generates a new matrix in each matrix operation, which potentially leads to high memory consumption. Owing to lazy evaluation, FlashMatrix does not store majority of matrices in the computation physically. As such, its in-memory execution barely increases memory consumption from the minimum memory requirement of the algorithms. This indicates that the out-of-core execution consumes small space on SSDs, which leads to very high scalability.

FlashMatrix running both in memory and on SSDs significantly outperforms R even with a single thread in all of these algorithms (Figure 7). We exclude statistic summary in the experiment because R does not provide a C or FORTRAN implementation of computing the same statistics. The performance results indicate that FlashMatrix executes R code efficiently to even outperform some optimized C and FORTRAN implementations when processing large datasets.
The in-memory execution of FlashMatrix achieves almost linear speedup in all algorithms while the out-of-core execution only starts to flatten out after 32 threads (Figure 8). Owing to operation fusion in CPU cache, FlashMatrix significantly reduces data movement between CPU and main memory. As such, memory bandwidth is no longer the bottleneck for in-memory execution and the algorithms speed up linearly with more CPU cores. For out-of-core execution, I/O is the bottleneck for the algorithms with lower computation complexity when they run with 48 threads. However, GMM still speeds up almost linearly even when running on SSDs, due to its high computation complexity. The performance results in Figure 8 and Figure 9 indicate that FlashMatrix can potentially execute R code with performance comparable to parallel C or FORTRAN implementations.

C. Performance of FlashMatrix in memory and on SSDs

We further measure the in-memory and external-memory performance of FlashMatrix thoroughly with different datasets and different parameters. We run the first three algorithms on random-65M matrices with the number of columns varying from 8 to 512. We run k-means and GMM on the Friendster-32 matrix and vary the number of clusters from 2 to 64. As the number of features or the number of clusters increases, the performance gap between in-memory and external-memory execution narrows and eventually the external-memory performance gets almost 100% of in-memory performance for some algorithms (Figure 9 and 10). This observation conforms with the computation and I/O complexity of the algorithms in Table IV. When the number of features gets larger, the computation of matrix multiplication in correlation and SVD grows more rapidly than I/O and eventually CPU becomes the bottleneck. The current implementation of correlation requires an additional pass on the input matrix to compute column-wise mean values, which results in lower external-memory performance. Similarly, as the number of clusters increases, the computation of k-means and GMM increases rapidly and these algorithms are dominated by their CPU computation as the number of clusters gets larger. Given an I/O throughput of 10 GB/s, the algorithms do not require many features or clusters to have their external-memory performance close to their in-memory performance.

D. Effectiveness of optimizations

In this section, we illustrate the effectiveness of our memory and CPU optimizations in FlashMatrix. To reduce memory overhead, we focus on three main optimizations: (i) reusing memory chunks for new in-memory matrices and I/O access to reduce large memory allocation (mem-alloc), (ii) matrix operation fusion in main memory to reduce data movement between SSDs and main memory (mem-fuse), (iii) matrix operation fusion in CPU cache to reduce data movement between main memory and CPU cache (cache-fuse). To reduce computation overhead, we illustrate the effectiveness of using VUDFs.

Each memory optimization has significant performance improvement on most of the algorithms when FlashMatrix runs
The number of clusters

KMeans  GMM

Fig. 10: The relative performance of FlashMatrix on SSDs for clustering algorithms with different numbers of clusters, normalized by its performance in memory. As the number of clusters increases, the external-memory performance of these implementations approach to their in-memory performance.

on SSDs (Figure 11 (a)). Operation fusion in main memory achieves the highest performance improvement in almost all algorithms, even in GMM, which has the highest asymptotic computation complexity. Even though the SSDs deliver an I/O throughput of 10GB/s, materializing every matrix operation separately causes SSDs to be the main bottleneck in the system. Fusing matrix operations in memory significantly reduces I/O access to SSDs and improves performance by a large factor. Operation fusion in the CPU cache also has very positive performance impact on some algorithms even when the algorithms run on SSDs. This suggests that with sufficient I/O optimizations, many machine learning algorithms that run on fast SSDs can be bottlenecked by the bandwidth of main memory, instead of I/O. Even though it is less noticeable, reducing large memory allocation improves I/O performance and almost doubles the overall performance of all algorithms.

The same memory optimizations also have very positive impact on the performance of most of the algorithms when FlashMatrix runs in memory (Figure 11 (b)). Operation fusion in CPU cache has performance improvement on all algorithms because reducing data movement between main memory and CPU cache in a sequence of matrix operations always improves performance. Operation fusion in main memory further reduces memory allocation overhead when FlashMatrix runs in memory. Thus, like the optimization of reducing large memory allocation, its effectiveness heavily depend on the computation patterns in the algorithms. Both of the optimizations are more effective for the algorithms such as statistical summary and k-means, which require to generate new large matrices but do not have very heavy computation.

VUDFs improve the performance of the algorithms that rely on GenOps for computation (Figure 12). In this experiment, the base implementations deploy all memory optimizations to avoid memory from being the bottleneck of the system, and invoke functions on individual elements. The FlashMatrix implementations of statistical summary and k-means solely rely on GenOps. Therefore, their performance is almost doubled when we use VUDFs. The main computation in correlation and GMM is matrix multiplication, but they still rely on GenOps for the remaining computation. As such, the use of VUDFs helps their performance. SVD solely uses matrix multiplication, so switching to VUDFs has no performance impact.

V. CONCLUSION

We present FlashMatrix, a matrix-oriented programming framework for general data analysis. FlashMatrix scales to large datasets by utilizing commodity SSDs. It provides a high-level functional programming interface for users to write data analysis algorithms in R and executes the R implementations in parallel and out of core automatically. For simplicity
and generality, the core of FlashMatrix only implements a small number of generalized matrix operators (GenOps). It reimplements many matrix operations in R base package with GenOps to provide a familiar programming environment to users. To improve performance, FlashMatrix uses vectorized user-defined functions (VUDFs) to reduce the overhead of function calls and fuses matrix operations to reduce data movement between CPU and SSDs.

We demonstrate that the matrix-oriented functional programming interface in FlashMatrix can achieve high performance and scalability for many data analysis algorithms. We implement multiple statistics and machine learning algorithms in R and compare their performance with Spark MLlib, a highly-optimized parallel machine learning library, on large datasets. The R implementations executed in FlashMatrix significantly outperform the implementations in Spark MLlib. We further demonstrate that the R implementations running FlashMatrix with a single thread can outperform the C and FORTRAN implementations in the R framework. In addition, FlashMatrix also achieves linear speedup with multithreading in all of these algorithms.

Even though SSDs are still an order of magnitude slower than DRAM, the external-memory execution of many data analysis algorithms in FlashMatrix can achieve performance comparable to their in-memory execution. We demonstrate that an I/O throughput of 10 GB/s is able to saturate CPU for many algorithms, even in a large parallel NUMA machine. As such, the external-memory execution also benefits from many in-memory optimizations.

FlashMatrix simplifies significantly the programming effort of writing parallel and out-of-core implementations for large-scale data analysis. It provides domain experts a familiar programming environment for implementing their algorithms in R and compare their performance with Spark MLlib, a highly-optimized parallel machine learning library, on large datasets. The R implementations executed in FlashMatrix can achieve performance in all of these algorithms.

We demonstrate that the matrix-oriented functional programming interface in FlashMatrix can achieve high performance and scalability for many data analysis algorithms. We implement multiple statistics and machine learning algorithms in R and compare their performance with Spark MLlib, a highly-optimized parallel machine learning library, on large datasets.

**ACKNOWLEDGMENT**

This work is partially supported by NSF ACI-1261715, DARPA GRAPHS N66001-14-1-4028 and DARPA SIMPLEX program through SPAWAR contract N66001-15-C-4041.

**REFERENCES**

[1] B. OBAMA, “Executive order – creating a national strategic computing initiative,” The White House Office of the Press Secretary, 2015.

[2] B. Barney, “Message passing interface (MPI),” https://computing.llnl.gov/tutorials/mpip/, 2015.

[3] “OpenMP,” http://openmp.org/wp/, 2016.

[4] “Intel math kernel library,” https://software.intel.com/en-us/intel-mkl Accessed 1/24/2016.

[5] Q. Wang, X. Zhang, Y. Zhang, and Q. Yi, “AUGEM: Automatically generate high performance dense linear algebra kernels on x86 cpus,” in High Performance Computing, Networking, Storage and Analysis (SC), 2013 International Conference for, Nov 2013, pp. 1–12.

[6] J. Poulson, B. Marker, R. A. van de Geijn, J. R. Hammond, and N. A. Romero, “Elemental: A new framework for distributed memory dense matrix computations,” ACM Trans. Math. Softw., vol. 39, no. 2, pp. 13:1–13:24, Feb. 2013.

[7] M. A. Heroux, R. A. Bartlett, V. E. Howle, R. J. Hoekstra, J. J. Hu, T. G. Kolda, R. B. Lehoucq, K. R. Long, R. P. Pawlowski, E. T. Phipps, A. G. Salinger, H. K. Thornquist, R. S. Tuminaro, J. M. Willenbring, A. Williams, and K. S. Stanley, “An overview of the Trilinos project,” ACM Trans. Math. Softw., 2005.

[8] S. Balay, S. Abhyankar, M. F. Adams, J. Brown, P. Brune, K. Buschelman, L. Dalcin, V. Eijkhout, W. D. Gropp, D. Kaushik, M. G. Knepley, L. C. McInnes, K. Rupp, B. F. Smith, S. Zampini, and H. Zhang, “PETSc: Web page,” http://www.mcs.anl.gov/petsc, 2015. [Online]. Available: http://www.mcs.anl.gov/petsc.

[9] J. A. Ang, R. F. Barrett, R. E. Benner, D. Burke, C. Chan, J. Cook, D. Donofrio, S. D. Hammond, K. S. Hemmert, S. M. Kelly, H. Le, V. J. Leung, D. R. Resnick, A. F. Rodrigues, J. Shalf, D. Stark, D. Unat, and N. J. Wright, “Abstract machine models and proxy architectures for exascale computing,” in Proceedings of the 1st International Workshop on Hardware-Software Co-Design for High Performance Computing, ser. Co-HPC ’14. Piscataway, NJ, USA: IEEE Press, 2014.

[10] N. Liu, J. Cope, P. Carns, C. Carothers, R. Ross, G. Grider, A. Crume, and C. Malztahn, “On the role of burst buffers in leadership-class storage systems,” in MSST/SNAPi, 2012.

[11] M. V. Wilkes, “The memory gap and the future of high performance memories,” SIGARCH Comput. Archit. News, vol. 29, no. 1, pp. 2–7, Mar. 2001.

[12] C. Lomont, “Introduction to intel advanced vector extensions,” https://software.intel.com/en-us/articles/introduction-to-intel-advanced-vector-extensions, 2011.

[13] S. Lloyd, “Least squares quantization in PCM,” IEEE Trans. Inf. Theor., vol. 28, no. 2, Sep. 2006.

[14] D. Reynolds, “Gaussian Mixture Models,” MIT Lincoln Laboratory, 1990.

[15] M. Zaharia, M. Chowdhury, T. Das, A. Dave, J. Ma, M. McCaully, M. J. Franklin, S. Shenker, and I. Stoica, “Resilient distributed datasets: A fault-tolerant abstraction for in-memory cluster computing,” in Presented as part of the 9th USENIX Symposium on Networked Systems Design and Implementation (NSDI 12). San Jose, CA: USENIX, 2012, pp. 15–28.

[16] K. Goto and R. A. v. d. Geijn, “Anatomy of high-performance matrix multiplication,” ACM Trans. Math. Softw., vol. 34, no. 3, pp. 12:1–12:25, May 2008.

[17] R. C. Whaley, A. Petitet, and J. J. Dongarra, “Automated empirical optimization of software and the atlas project,” PARALLEL COMPUTING, vol. 27, p. 2001, 2000.

[18] J. Dean and S. Ghemawat, “MapReduce: Simplified data processing on large clusters,” in Proceedings of the 6th Conference on Symposium on Operating Systems Design & Implementation, ser. OSDI’04. Berkeley, CA, USA: USENIX Association, 2004.

[19] M. Isard, M. Budiu, Y. Yu, A. Birrell, and D. Feitterly, “Dryad: Distributed data-parallel programs from sequential building blocks,” in Proceedings of the 2nd ACM SIGOPS/EuroSys European Conference on Computer Systems 2007. New York, NY, USA: ACM, 2007.

[20] D. G. Murray, F. McSherry, R. Isaacs, M. Isard, P. Barham, and M. Abadi, “Naiad: A timely dataflow system,” in Proceedings of the Twenty-Fourth ACM Symposium on Operating Systems Principles, ser. SOSP ’13. New York, NY, USA: ACM, 2013, pp. 439–455.

[21] C. Olston, B. Reed, U. Srivastava, R. Kumar, and A. Tomkins, “Pig Latin: A not-so-foreign language for data processing,” in Proceedings of the 2008 ACM SIGMOD International Conference on Management of Data. New York, NY, USA: ACM, 2008.

[22] C. Chambers, A. Ranawala, F. Perry, S. Adams, R. R. Henry, R. Bradshaw, and N. Weizenbaum, “FlumeJava: Easy, efficient data-parallel pipelines,” in Proceedings of the 31st ACM SIGPLAN Conference on Programming Language Design and Implementation. New York, NY, USA: ACM, 2010.

[23] A. Ghoting, R. Krishnamurthy, E. Pednault, B. Reinwald, V. Sindhwani, S. Tatikonda, Y. Tian, and S. Vairathathan, “SystemML: Declarative machine learning on mapreduce,” in Proceedings of the 2011 IEEE 27th International Conference on Data Engineering. Washington, DC, USA: IEEE Computer Society, 2011.

[24] Y. Yu, M. Isard, D. Fetterly, M. Budiu, U. Erlingsson, P. K. Gunda, and J. Currey, “DryadLINQ: A system for general-purpose distributed data-parallel computing using a high-level language,” in Proceedings of the 8th USENIX Conference on Operating Systems Design and Implementation. Berkeley, CA, USA: USENIX Association, 2008.
[25] X. Meng, J. Bradley, B. Yavuz, E. Sparks, S. Venkataraman, D. Liu, J. Freeman, D. Tsai, M. Amde, S. Owen, D. Xin, R. Xin, M. J. Franklin, R. Zadeh, M. Zaharia, and A. Talwalkar, “MLlib: Machine learning in Apache Spark,” CoRR, vol. abs/1505.06807, 2015.

[26] “Revolution R Enterprise,” http://www.revolutionanalytics.com/revolution-r-enterprise, Accessed 4/9/2016.

[27] “Parallel computing toolbox in MATLAB,” http://www.mathworks.com/products/parallel-computing/, Accessed 4/9/2016.

[28] S. Venkataraman, E. Bodzsar, I. Roy, A. AuYoung, and R. S. Schreiber, “Presto: Distributed machine learning and graph processing with sparse matrices,” in Proceedings of the 8th ACM European Conference on Computer Systems. New York, NY, USA: ACM, 2013.

[29] W.-M. Ching and D. Zheng, “Automatic parallelization of array-oriented programs for a multi-core machine,” International Journal of Parallel Programming, vol. 40, no. 5, pp. 514–531, 2012.

[30] D. Tarditi, S. Puri, and J. Oglesby, “Accelerator: Using data parallelism to program GPUs for general-purpose uses,” in Proceedings of the 12th International Conference on Architectural Support for Programming Languages and Operating Systems, New York, NY, USA, 2006.

[31] D. Zheng, D. Mhembere, V. Lyzinski, J. Vogelstein, C. E. Priebe, and R. Burns, “Semi-external memory sparse matrix multiplication on billion-node graphs in a multicore architecture,” CoRR, vol. abs/1602.02864, 2016.

[32] D. Zheng, R. Burns, and A. S. Szalay, “Toward millions of file system IOPS on low-cost, commodity hardware,” in Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis, 2013.

[33] A. K. Jain, R. P. W. Duin, and J. Mao, “Statistical pattern recognition: a review,” IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 22, no. 1, pp. 4–37, Jan 2000.

[34] D. Zheng, R. Burns, and A. S. Szalay, “A parallel page cache: IOPS and caching for multicore systems,” in Proceedings of the 4th USENIX Conference on Hot Topics in Storage and File Systems, 2012.

[35] C. G. Baker, U. L. Hetmaniuk, R. B. Lehoucq, and H. K. Thornquist, “Anasazi software for the numerical solution of large-scale eigenvalue problems,” ACM Trans. Math. Softw., vol. 36, no. 3, pp. 13:1–13:23, Jul. 2009.

[36] D. Zheng, R. Burns, J. Vogelstein, C. E. Priebe, and A. S. Szalay, “An SSD-based eigensolver for spectral analysis on billion-node graphs,” CoRR, vol. abs/1602.01421, 2016.

[37] K. Pearson, “Notes on regression and inheritance in the case of two parents,” in Proceedings of the Royal Society of London, 1895, pp. 240–242.

[38] X. Wu, V. Kumar, J. Ross Quinlan, J. Ghosh, Q. Yang, H. Motoda, G. McLachlan, A. Ng, B. Liu, P. Yu, Z.-H. Zhou, M. Steinbach, D. Hand, and D. Steinberg, “Top 10 algorithms in data mining,” Knowledge and Information Systems, vol. 14, no. 1, pp. 1–37, 2008.

[39] J. Yang and J. Leskovec, “Defining and evaluating network communities based on ground-truth,” in Proceedings of the ACM SIGKDD Workshop on Mining Data Semantics, 2012.

[40] C. Fraley, A. E. Raftery, T. B. Murphy, and L. Scrucca, “mclust version 4 for R: Normal mixture modeling for model-based clustering, classification, and density estimation,” Technical Report No. 597, Department of Statistics, University of Washington, 2012.