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A control strategy for a dual-motor driving electric vehicle based on Pontryagin’s Minimum Principle

Zhenqiang Ma, Zhengfeng Chen*, Jinyong Shangguan, Xuemeng Li and Yugang Cheng
School of Mechanical and Automotive Engineering, Liaocheng University, China

*Corresponding author e-mail: chenzhengfeng_01@163.com

Abstract. For some dual-motor driving electric vehicles (EV) a method to realize the control of electric vehicles were proposed. Firstly, the control strategy was researched on the basis of typical urban conditions. The original data were analysed to classify the circulation condition by the stations, after which, the eigenvalues of each circulation condition and the comprehensive eigenvalues of each station were calculated and compared through the metrics algorithm. The typical driving condition of each station was chosen to build the typical driving condition of the global test path, carry out the study on globally optimal control strategy, and sum up the best shift schedule of the two-motor dynamic coupler. The schedule was applied in the algorithm of pontryagin’s Minimum Principle (PMP). The principle of minimization of the PMP was used to realize the best real-time control of electric vehicles. The simulation results demonstrate that the proposed optimize strategy could lead to the optimal control of the driven efficiency of electric vehicles and reduced power consumption by 4.16%. The results of this study could provide a certain theoretical basis for engineering application.

1. Introduction

Due to the increasingly serious severity of the environmental pollution and lacking of oil resources, and the development of electric vehicles has become the focus of the world and makes the research in the field of electric vehicle become a hotspot. Nowadays, the short driving range and the high prices are still the bottleneck of the development of the electric vehicles. The uniaxial driven electric vehicles have insufficient power and make massive waste of the electricity. Conversely, the technique of dual-motor driving electric vehicles can not only solve the problem of insufficient power, but also extend the driving distance. Moreover, the dual-motor driving method can lead the electric vehicles to be a cost-effective item.

The dual-motor driving EV proposed by Zhu Bo et al who could improve the economy of the
electric vehicles, and it had become an important research area in the field of the electric vehicles. In addition, the British Voices Company and BAIC BJEV also studied the related configurationally structure of electric vehicles. Shougao Xiao had accomplished the calculation of dynamic performance and the choice of the motor types and putted the foundation for the following study on dual-motor driving EV [6]. Based on the traditional EV, Shengbing Yang et al had invented a new type of EV, and the EV was driven by two separate motors [7]. They combined the CRUISE/SIMULINK software with setting up the new organization of the electric vehicle model and the traditional organization of the electric vehicle model and they simulated the electric vehicle performance.

In the paper, we aim to build the global optimal mathematical model for dual-motor driving EV. Our research uses a no power interruption coupler and the coupler is two-input and single output, and the coupler is selected as the study object. On the basis of DP optimization results we sum up the best shift schedule about the dual-motor dynamic couplers and apply it to the PMP model. Finally, we accomplish the minimizing the objective function by coordinating the efficiency of the global vehicle and SOC.

2. The model of coupler

For some dual-motor driving electric vehicles (EV), the no power interruption coupler with dual input and single output is selected as the study model. The optimal power and torque distribution of couplers are studied. Figure 1 is about the no power interruption coupler with dual input and single output. The coupler has four gears and two different types of synchronizers, two types of drive patterns, three work states. There are two types of work status: motor1 works and provides power, motor2 doesn’t work; mortor2 works and provides power, motor1 doesn’t work.

Motor 1 is always driven. Motor 2 can reduce the force on the gear ring by zero-speed braking. It is convenient for brake to release. After the brake is removed, the motor 2 will be driven again and power coupling between two electric motors at high speed is realized; In the process of switching from high speed mode to low speed mode, Motor 2 rapidly decelerates gear ring speed to zero through regenerative braking. It is convenient for brake to lock the gear ring. Motor 2 is closed after gear ring lock. In the whole process of mode switching, the zero-speed brake of motor 2 is fully used to realize flexible mode switching. There is no speed difference between brake and gear ring, and the impact is small.

| Gear              | Synchronizer 1 | Synchronizer 2 | To adapt to the road conditions                      |
|-------------------|----------------|----------------|-------------------------------------------------------|
| Low gear          | Low speed      | Low speed      | The driving of the climbing section.                  |
| Medium speed block 1 | High speed    | Low speed      | Suitable for urban traffic.                           |
| Medium speed block 1 | Low speed     | High speed     | Suitable for urban traffic.                           |
| High speed gear   | High speed     | High speed     | A high-speed section for fewer pedestrians.           |

Table 1. Gear shift rule and application
3. The minimum optimization of the control strategy

The control strategy model adopted in this paper is shown in Figure 3. The control strategy consists of the optimization of DP and the optimization of PMP.

The optimization of DP uses the Dynamic programming algorithm to optimize the global vehicle model offline and gets the optimal offline result. The best shift curve is separated from the optimal offline result of DP and the best shift curve is the most suitable for global vehicle model. PMP optimization module uses the minimum principle to calculate the control model through the best shift curve. Finally, the optimal control strategy is applied to the real vehicle model and proceeded the real-time optimization.
3.1 Typical urban conditions

With the increase of car ownership in China, such problems as environmental pollution, traffic congestion and energy crisis were serious. The control strategy was researched on the basis of typical urban conditions. The control strategy was about the design of dual-motor driving electric vehicles. The vehicle’s dynamic performances, mileage, energy-saving of dual-motor driving electric vehicles had greatly improved. Therefore, the construction of typical urban driving conditions should be carried out to prepare for the design and optimization of dual-motor driving electric vehicles.

The experimental data were collected by the on-board GPS of city buses, and the experimental data were classified by stations. Typical driving conditions at each station were received by analysis and processing of the experimental data. Typical driving condition of the whole test path was received through linking typical driving conditions at each station together. The length of the test path selected in this test was about 34km and the test path had 22 stations. The steps of test data processing were as
follows: The first step was about acquisition and processing of original data. The experimental data were collected by the on-board GPS of city buses, and the test data were collected at a frequency of 1s or 0.5s. All the motion segments were defined as a drive cycle. (The motion segment refers to the speed of the vehicle from the starting speed of the vehicle at 0km/h to the stopping speed at 0km/h). Raw data processing refers to simple processing such as removal of the wrong value. The second step was about making drive cycles be classified by stations. All the experimental data were divided into 2948 motion segments and 660 drive cycles, and drive cycles were classified by stations to receive a matrix and the matrix was 22*30, (the rows of a matrix refer to amount of stations and the columns refer to each site have 30 drive cycles). The third step was about calculating the eigenvalues of each circulation condition and the comprehensive eigenvalues of each station, and there were 34 eigenvalues and those eigenvalues could be divided into 15 descriptive eigenvalues and 19 distribution eigenvalues. The fourth step was about calculation. The eigenvalues of each circulation condition and the comprehensive eigenvalues of each station were calculated and compared through the metrics algorithm. The typical driving condition of each station was chosen to build the typical driving condition of the global test path. In conclusion, typical operating conditions of driving distance were accomplished through those steps.

![Figure 4. Typical operating condition](image)

### 3.2. The mathematical model of Dynamic programming

The model of driven motor can be described to:

\[
P_m = \omega_m \cdot T_m = \begin{cases} 
\eta_m(\omega_m, P_b) \cdot P_b & \text{if } P_b \geq 0 \text{(motoringmode)} \\
\frac{1}{\eta_m(\omega_m, P_b)} \cdot P_b & \text{if } P_b \leq 0 \text{(generatingmode)} 
\end{cases} 
\]

\[P_{mot1} = \alpha \cdot P_m\]
\[P_{mot2} = P_m - P_{mot1}\]  \hspace{1cm} (3.2)

Like in the equation 3.1, \(P_m\) is the exporting power of motors and \(P_b\) denote exporting power of battery; \(\eta_m(\omega_m, P_b)\) is the efficiency of motors; \(\omega_m\) is the revolving speed of motors and \(T_m\) is the torque of motors; \(\alpha\) is the distributed ratio of power (the ratio of the first motor’s power to the demand power); \(P_{mot1}\) is the power of motor 1; \(P_{mot2}\) is the power of motor 2.

The state variable is taken as the SOC of the electric vehicle battery and the change of battery charge determine the SOC; \(C\) is the electric quantity of the dynamic battery pack and the relationship between the charged state and the battery capacity can be expressed as:
SOC(t) = 1 - \frac{1}{C} \int_{t_0}^{t_f} I_{bat}(t) \, dt \quad (3.3)

The battery power management of pure electric cars makes battery as the energy source, and the battery power management of pure electric cars uses the strategy of power consumption. Because of all the consumption are provided by the battery, the SOC value of the electric vehicle must be higher than 0.35 at the end of the work. It is required of optimal design.

\[ SOC_F \geq 0.35 \quad (3.4) \]

Except for the terminal restrictions of the above formula, the output power of the battery should also satisfy the following relation.

\[ P_{b,\min} \leq P_b \leq P_{b,\max} \quad (3.5) \]

For the global optimization of discrete and continuous variables, offline analysis should be carried out through DP. So this paper takes gears and battery SOC as state variables and the discrete state equation as shown in equation (3.6) is established.

\[
\begin{align*}
SOC(k + 1) &= SOC(k) - \frac{V_{oc}(k) - \sqrt{V_{oc}^2(k) - 4R_b(k)P_b(k)}}{2P_b(k)Q_{nom}} \Delta t \\
g(k + 1) &= \begin{cases} 
4 & g_x(k) + Dega(k) \geq 4 \\
1 & g_x(k) + Dega(k) \leq 1 \\
g_x(k) + Dega(k) & \text{otherwise}
\end{cases}
\end{align*}\]

\quad (3.6)

In the equation 3.6, SOC(k) is the status variable of battery and \( g_x(k) \) is the status variable; \( Dega(k) \) is the instruction of gears; \( 1 \) represents upshifting and \( 0 \) represents maintain current gear position and \( -1 \) represents downshifting.

The model makes the power distribution ratio and the instruction of gears as control variants. The global optimization objective is to minimize the output power of the battery SOC, and the method can establish a formula (3.7) is shown in the global optimal discrete mathematical model. In addition, in order to ensure that the higher the SOC state value at the end of the mileage of electric vehicles is, the additional cost function is set as \( \alpha(SOC(k) - SOC_F)^2 \); to avoid frequent shifting of coupler gears, a buffer band is reserved for the current lifting range of gears, and additional cost function is set as \( \beta|D\text{gear}(k)| \).

\[
\begin{align*}
\text{Min} J_{\pi}(x_0) &= \lim_{N \to \infty} E\{\sum_{k=0}^{N-1}[eff(k) + \alpha(SOC(k) - SOC_F)^2 + \beta |D\text{gear}(k)|]\}
\end{align*}\]

At the same time, given the following constraints

\[
\begin{align*}
\text{S. t.} \quad \begin{cases} 
\omega_{m,\min} \leq \omega_m(k) \leq \omega_{m,\max} \\
SOC_{\min}(k) \leq SOC(K) \leq SOC_{\max}(k) \\
P_{m,\min}(\omega_m(k)) \leq P_m(k) \leq P_{m,\max}(\omega_m(k))
\end{cases}
\end{align*}\]

\quad (3.8)

In the equation 3.8, \( \omega_m(k) \) is the revolving speed of motor; \( \omega_{m,\min} \) and \( \omega_{m,\max} \) are the limiting revolving speeds of motor; \( P_m(k) \) is the power of motor; \( P_{m,\min}(\omega_m(k)) \) and \( P_{m,\max}(\omega_m(k)) \) are the limiting powers of motor.
3.3. The regular model of the shifting gears

As shown in figure 5, Gears distribution during vehicle operation has strong regularity, and gears switch frequently in the low speed range. Vehicles tend to drive in high gear. The first gear is used to output large torque and low output power. The fourth gears are mainly used in high-power and high-speed driving conditions. The second and third gears are mainly applicable to urban road environment.

![Shift curve](image)

**Figure 5. Shift curve**

3.4. The algorithmic model of PMP

Global optimal control problem is transformed into local optimal control problem through PMP. In the other words, the state of the battery SOC reflects the different consumption of double motors power. The optimal strategy is obtained by finding the minimum value of Hamiltonian function at each time.

\[ u^*(t) = \arg \min \{ H(x(t), u(t), t) \} \] (3.9)

The corresponding Hamiltonian function is introduced:

\[ H(t, x, u, \lambda) = L(t, u(t)) + \lambda(t)f(t, x, u) \] (3.10)

Dual-motor driving system based on PMP algorithm model, \( L(t, u(t)) = \alpha_{\text{efficiency}} \) represents battery efficiency and \( f(t, x, u) = \beta|D_{\text{gear}}| \) represents the changing state of the motor; \( \lambda(t) \) is the coordination factor, and the coordination factor aims to harmonize the optimal efficient. When \( H(x(t), \lambda(t), u(t), t) \) are determined, the coordination factor can change the battery efficient. In the actual working condition, the overall change of battery SOC is decreasing, and when \( \lambda(t) \approx 0 \), and the coordination factor is a variable constant. The equation can be described as:

\[ \lambda(t) = -\frac{\partial H(x(t), u(t), \lambda(t), t)}{\partial x} = -\lambda(t) \frac{\partial \text{SOC}(t)}{\partial \text{SOC}} \] (3.11)

\( \text{SOC}(t) \) can be expressed as:
\[
\begin{align*}
I_b &= \frac{V_{oc}(SOC) - \sqrt{V_{oc}^2(SOC) - 4P_bR_0(SOC,T_b)}}{2R_0(SOC,T_b)} \\
\sin(t, SOC(t), u(t)) &= \frac{V_{oc}(SOC) - \sqrt{V_{oc}^2(SOC) - 4R_b(SOC)P_b(u^*)}}{2R_b(SOC)^2Q} \\
SOC(t) &= \frac{\partial H(x(t), u(t), \lambda(t), t)}{\partial \lambda} = f(t, SOC(t), u(t))
\end{align*}
\] (3.12)

4. Simulation results and discussion

Figure 5 shows the change curve of battery SOC, and the change curve of battery SOC is based on DP global optimization results. Under the known working conditions, the change curve of battery SOC conforms to the change rule of pure electric vehicle SOC. The battery is always in the state of discharge and the discharge meets the design requirements of battery SOC.

![Figure 6. Battery SOC curve](image)

The figure 7 shows the demanded power of the motor 1. The figure 8 shows the demanded power of the motor 2. If the value of the power is plus, it explains the battery outputs power to driving motor, and if the value of the power is subtractive, it explains the ground produces a braking force and the braking force reacts on the motor. The motor generates braking energy and recycles it.

![Figure 7. Motor 1 required power](image) ![Figure 8. Motor 2 required power](image)

5. Conclusion

DP algorithm and global optimal control strategy for typical urban conditions are studied, and the optimal gear shifting rule of dual motor dynamic coupler is extracted from the study. The shifting rule
is applied to the PMP algorithm to minimize the battery efficiency, and the coordination factor is used to control the power consumption of the vehicle, and the optimal real-time control is realized for the dual motor driven vehicle. The simulation results demonstrate that the proposed optimize strategy can lead to the optimal control of the driven efficiency of electric vehicles and reduce power consumption by 4.16%. The results of this study can provide a certain theoretical basis for engineering application.

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