Some mathematical considerations about
generalized Yang-Mills theories

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Abstract

Generalized Yang-Mills theories are constructed, that can use fields other than vector as gauge fields. Their geometric interpretation is studied. An application to the Glashow-Weinberg-Salam model is briefly review, and some related mathematical and physical considerations are made.

1 Introduction

Yang-Mills theories have been tremendously useful in high energy physics, where they have served to successfully model the electroweak and strong interactions. The fundamental idea of a Yang-Mills theory is that its mathematical expression must be invariant under a local compact Lie group of transformations. By local here it is meant that the group element varies with the point in Minkowski space that is being considered. In a typical physical theory the quantum fields often appear differentiated with respect to space or time. This means that, when the field transforms under the local Lie group, the differentiation operator is going to act on the transforming group element as well as on the field itself, so there is no invariance of this
term under the Lie group. Invariance is reestablished substituting the differentiation operators by covariant derivatives. A covariant derivative is the sum of the differential operator and the Yang-Mills field. These fields are required to transform by adding terms that precisely cancel the extra terms brought in by the differentiation operators. Such a transformation is called a gauge transformation and the theory is left unmodified by it. Fields that perform this kind of service are generically called gauge fields.

Since the differentiation of a scalar field $\partial_\mu \varphi$, $\partial_\mu \equiv \partial/\partial x_\mu$, $\mu = 0, 1, 2, 3$, results in a four-vector, the gauge fields have also been taken to be four-vectors. In this article we are going to review\[1\] how it is possible to generalize the concept of a covariant derivative in a Yang-Mills theory, so that fields other than vector can be used as gauge fields.\[2\] The idea of putting the leptons in a triplet and using the graded group $SU(2/1)$ goes back to Ne’eman and Fairlie.\[3\] The Lie group $SU(3)$ almost has the right quantum numbers to embed the Glashow-Weinberg-Salam (GWS) Model in it, but not quite. The right group seems to be $U(3)$ in a special representation. An extra boson appears, but it automatically uncouples from the rest of the particles.\[4\] However, the emphasis of this paper is on the mathematical aspect, rather than on high energy applications.

2 The kinetic energy of a vector field transforming spinorially

The quantum electrodynamics Lagrangian is

$$\mathcal{L}_{QED} = \bar{\psi}i\slashed{D}\psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

where $D_\mu \equiv \partial_\mu + ieA_\mu$ and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu = -ie^{-1}[D_\mu, D_\nu]$, and where we are using a metric $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. This Lagrangian is invariant under a local transformation group based on the $U(1)$ Lie group. If $U = e^{-i\alpha(x)}$ is an element of this group, then the electrically charged fermion field transforms as $\psi \rightarrow U\psi$. The vector field is required to obey the gauge transformation law

$$A_\mu \rightarrow A_\mu + e^{-1}(\partial_\mu \alpha),$$

where $\alpha$ is a function of the coordinates $x$.
so that, recalling the definition of $D_\mu$, the covariant derivative must transform as
\[ D_\mu \rightarrow UD_\mu U^{-1}. \] (3)

In the previous equation the derivative that is part of the covariant derivative is acting indefinitely to the right. We call such operators *unrestrained*, while the ones that act only on the immediately following object, such as the partial in (2), we call *restrained*, and use a parenthesis to emphasize that the action of the differentiation operator does not extend any further. We do not find it admissible to have unrestrained operators in a Lagrangian, because, first, they are not gauge invariant, and, second, what they could physically or mathematically mean is not clear. While the operators $D_\mu$ and $D_\mu D_\nu$ are unrestrained, the operator $[D_\mu, D_\nu]$ is restrained, and it is entirely appropriate that the kinetic energy of a vector boson can be constructed using this commutator. The way the commutator becomes restrained is as follows:

\[ [D_\mu, D_\nu]f = \partial_\mu A_\nu f - A_\nu \partial_\mu f - \partial_\nu A_\mu f + A_\mu \partial_\nu f = (\partial_\mu A_\nu)f - (\partial_\nu A_\mu)f, \] (4)

where $f = f(x)$ is some differentiable function and it is seen how four unrestrained operators result in two restrained ones, thanks to Leibnitz’ rule.

Let $S$ be an element of the spinor representation of the Lorentz group, so that, if $\psi$ is a spinor, then it transforms as $\psi \rightarrow S\psi$. Then, due to the homomorphism that exists between the vector and spinor representations of the Lorentz group, we have that $A / \rightarrow S A S^{-1}$. It was this homomorphism that allowed Dirac to write a spinorial equation that included the vector electromagnetic field. The first step in our road to a generalization of the covariant derivative will be to rewrite the Lagrangian (1) using the spinorial representation. To this effect we have the following

**Theorem.** Let $D_\mu = \partial_\mu + B_\mu$, where $B_\mu$ is some vector field. Then:

\[ (\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu) = \frac{1}{8} Tr p^2 - \frac{1}{2} Tr p^4, \] (5)

where the traces are to be taken over the Dirac matrices.

**Proof.** Notice the partials on the left of this identity are restrained, the ones on the right are not. To prove the theorem it is convenient to use the following trick, which makes the algebra manageable, in this and in more complicated cases to follow. Consider the differentiable operator $O \equiv \partial^2 + 2B \cdot \partial + B^2$. Notice that it does not contain any contractions with Dirac
matrices, so that $\text{Tr } O = 4O$, $\text{Tr } O (\partial B) = 4O (\partial \cdot B)$, etc. It is not difficult to see then that $\mathcal{O} = O + (\partial B)$, where the slashed partial is acting only on the succeeding slashed field. The trick is to use this form of $\mathcal{O}^2$ in the traces on the right of (5). With it one obtains

$$\frac{1}{8} \text{Tr} [O + (\partial B)] - \frac{1}{2} \text{Tr} [O + (\partial B)]^2 = 2 (\partial \cdot B)^2 - \frac{1}{2} \text{Tr} [(\partial B)(\partial B)]$$

$$= (\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu) \quad (6)$$
as we wished to demonstrate. The motivation for the additional trace term is the same as for taking the commutator of the covariant derivatives: to ensure that the differential operators be restrained.

With the aid of the Theorem we can rewrite the QED Lagrangian in the form

$$\mathcal{L}_{\text{QED}} = \bar{\psi}i\mathcal{D} \psi + e^{-2} \left( \frac{1}{32} \text{Tr } \mathcal{O}^2 - \frac{1}{8} \text{Tr } \mathcal{O}^4 \right), \quad (7)$$

whose Lorentz invariance can be easily proven using $\mathcal{O} \rightarrow S \mathcal{O} S^{-1}$ and the cyclic properties of the trace. As an example of the invariance, observe that $\text{Tr } \mathcal{O}^2 \rightarrow \text{Tr } S \mathcal{O} S^{-1} S \mathcal{O} S^{-1} = \text{Tr } \mathcal{O}^2$.

3 A scalar field functioning as a gauge field

We are going to construct an example of a theory that employs a scalar instead of a vector boson to maintain gauge invariance. To keep things simple we use $U(1)$ as our Lie group, as in the previous section, and so, again, (3) must hold. This time, however, we take the covariant derivative to be in the spinorial representation, as in (4), so it becomes possible to define it to be:

$$D_\varphi = \partial - e\gamma^5 \varphi \quad (8)$$

We now require the gauge field to transform as $\gamma^5 \varphi \rightarrow \gamma^5 \varphi - ie^{-1}\partial \alpha$. These equations immediately assure us that $D_\varphi \rightarrow UD_\varphi U^{-1}$, and so Lagrangian

$$\mathcal{L}_\varphi = \bar{\psi}iD_\varphi \psi + e^{-2} \left( \frac{1}{32} \text{Tr } D_\varphi^2 - \frac{1}{8} \text{Tr } D_\varphi^4 \right) \quad (9)$$

is gauge invariant. The trace terms in this Lagrangian can be simplified algebraically and the Lagrangian written in the more traditional form

$$\mathcal{L}_\varphi = \bar{\psi}i\partial \psi - e\bar{\psi}i\gamma^5 \varphi \psi + \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) \quad (10)$$

after a bit of algebra. This last calculation is similar to the one done last section, but with the $\gamma^5$ taking the place of the $\gamma^\mu$’s of that previous calculation.

4 Non-abelian Yang-Mills theory with mixed gauge fields

Consider a Lagrangian that transforms under a non-abelian local Lie group that has $N$ generators. The fermion or matter sector of the non-abelian Lagrangian has the form $\bar{\psi}i\slashed{D}\psi$, where $D_\mu$ is a covariant derivative chosen to maintain gauge invariance. This term is invariant under the transformation $\psi \rightarrow U\psi$, where $U = U(x)$ is an element of the fundamental representation of the group. The covariant derivative is $D_\mu = \partial_\mu + A_\mu$, where $A_\mu = igA_\mu^a(x)T^a$ is an element of the Lie algebra and $g$ is a coupling constant. We are assuming here that the set of matrices $\{T^a\}$ is a representation of the groups generators. Gauge invariance of the matter term is assured if

$$A_\mu \rightarrow UA_\mu U^{-1} - (\partial_\mu U)U^{-1}, \quad (11)$$

or, what is the same,

$$\bar{A} \rightarrow U\bar{A}U^{-1} - (\partial U)U^{-1} \quad (12)$$

We have already seen how scalar fields can function as gauge fields. Our aim in this section is to construct a non-abelian theory that uses both scalar and vector gauge fields. We proceed as follows. For every generator in the Lie group we choose one gauge field, it does not matter whether vector or scalar. As an example, suppose there are $N$ generators in the Lie group; we choose the first $N_V$ to be associated with an equal number of vector gauge fields and the last $N_S$ to be associated with an equal number of scalar gauge fields. Naturally $N_V + N_S = N$. Now we construct a covariant derivative $D$ by taking each one of the generators and multiplying it by one of its associated gauge fields and summing them together. The result is

$$D = \slashed{\partial} + \slashed{A} + \Phi \quad (13)$$

where

$$\slashed{A} = \gamma^\mu A_\mu = ig\gamma^\mu A_\mu^a T^a, \quad a = 1,\ldots,N_V, \quad \Phi = \gamma^5 \varphi = -g\gamma^5 \varphi^b T^b, \quad b = N_V + 1,\ldots,N.$$
Notice the difference between $A_\mu$ and $A_\mu^a$, and between $\varphi$ and $\varphi^b$. We take the gauge transformation for these fields to be

$$A + \Phi \to U(A + \Phi)U^{-1} - (\partial U)U^{-1},$$  \hspace{1cm} (14)$$
from which one can conclude that $D \to UDU^{-1}$. The following Lagrangian is constructed based on the requirements that it should contain only matter fields and covariant derivatives, and that it possess both Lorentz and gauge invariance:

$$\mathcal{L}_{NA} = \bar{\psi}iD\psi + \frac{1}{2g^2} \tilde{\text{Tr}} \left( \frac{1}{8} \tilde{\text{Tr}} D^2 - \frac{1}{2} \text{Tr} D^4 \right)$$ \hspace{1cm} (15)$$
where the trace with the tilde is over the Lie group matrices and the one without it is over matrices of the spinorial representation of the Lorentz group. The additional factor of $1/2$ that the traces of (15) have with respect to (8) comes from normalization $\tilde{\text{Tr}} T^a T^b = \frac{1}{2} \delta_{ab}$, the usual one in non-abelian gauge theories.

Although we have constructed this non-abelian Lagrangian based only on the requirements just mentioned, its expansion into component fields results in expressions that are traditional in Yang-Mills theories. The reader who wishes to make the expansion herself can substitute (13) in (15), keeping in mind the derivatives are unrestrained, and aim first for the intermediate result

$$\frac{1}{16} \tilde{\text{Tr}} D^2 - \frac{1}{4} \text{Tr} D^4 = \left( (\partial \cdot A) + A^2 \right)^2 - \text{Tr} \left( (\partial A) + A^2 \right)^2$$
$$- \frac{1}{4} \text{Tr} \left( (\partial \Phi) + \{A, \Phi\} \right)^2,$$ \hspace{1cm} (16)$$
where the curly brackets denote an anticommutator. (We recommend to use here the trick explained in section 2.) Notice in this expression that the differentiation operators are restrained, and that the two different types of gauge fields appear only in an anticommutator. The $\gamma^5$ in the scalar boson term of the generalized covariant derivative $D$ ensures both that the partials become restrained and that these anticommutators become commutators once the properties of the Clifford matrices are taken into account. Substituting (13) in (16) and in the matter term of (15) we obtain the non-abelian Lagrangian in expanded form:

$$\mathcal{L}_{NA} = \bar{\psi}i(\partial + A)\psi - g\bar{\psi}\gamma^5\varphi^b T^b \psi + \frac{1}{2g^2} \tilde{\text{Tr}} (\partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu])^2$$
\[ + \frac{1}{g^2} \text{Tr} \left( \partial_\mu \varphi + [A_\mu, \varphi] \right)^2. \]

(17)

The reader will recognize familiar structures: the first term on the right looks like the usual matter term of a gauge theory, the second like a Yukawa term, the third like the kinetic energy of vector bosons in a Yang-Mills theory and the fourth like the gauge-invariant kinetic energy of scalar bosons in the non-abelian adjoint representation. It is also interesting to observe that, if in the last term we set the vector bosons equal to zero, then this term simply becomes \( \sum_{b, \mu} \frac{1}{2} \partial_\mu \varphi^b \partial^\mu \varphi^b \), the kinetic energy of the scalar bosons. We have constructed a generic non-abelian gauge theory with gauge fields that can be either scalar or vector.

5 Applying these ideas to the GWS Model

These ideas were applied\[^1\] to the GWS Model of high energy physics. The Higgs fields of the GWS Model were used, along with the usual vector bosons, to construct a generalized covariant derivative. The original intention was to use \( SU(3) \) as the gauge Lie group, because it generates quantum numbers for the particles that are very close to the experimental ones. Eventually it was noticed that the complete leptonic and bosonic sectors of the GWS Model could be written in the form of Lagrangian (15) using the group \( U(3) \).

The covariant derivative \( D \) contains the gauge vector bosons of \( U(1) \times SU(2) \), the scalar Higgs bosons, and a new scalar boson. The triplet \( \psi = (\nu_L, e_L, e_R) \) contains the leptons and is transformed by the fundamental representation of \( U(3) \). All quantum numbers are correctly predicted, and an extra scalar boson, but it automatically decouples from the rest of the model and is thus unobservable except through its gravitational effects. The representation of the group generators is not the usual one, but instead, a special one where the generators are obtained as a linear combination of the usual ones.

The quarks have not been included so far into the scheme. There is one term of the pertinent sector of the GWS Model that is not predicted by this generalized derivative model, and it is the potential of the Higgs field \( V(\varphi) \) that could cause the spontaneous symmetry breaking.
6 A generalized curvature

In a Yang-Mills theory, be it abelian or not, the terms with physical content must be gauge invariant and not contain unrestrained derivatives. For example, if $D_\mu$ is the covariant derivative of a non-abelian theory, the usual expression for the kinetic energy of the gauge vector bosons is $\overline{\text{Tr}}[D_\mu, D_\nu][D^\mu, D^n]$, which satisfies both conditions. But even the expression $F_{\mu\nu} = [D_\mu, D_\nu]$ by itself does not have any unrestrained derivatives, while $\text{Tr} F_{\mu\nu}$ has the additional property of being gauge invariant. This quantity is the curvature in a principal vector bundle. The question arises if similar results as those for $F_{\mu\nu}$ hold also for a theory with a generalized covariant derivative. The answer to this question is in the affirmative. We proceed now to define a quantity which we shall call the generalized curvature $F$. Let $D$ be the generalized covariant derivative, as given in (13); then:

$$F \equiv \frac{1}{4} \text{Tr} D^2 - D^2,$$

(18)
or else, in terms of the derivative’s constituents,

$$F = 1 \partial \cdot A - (\partial A) + 1 A \cdot A - A A - \partial \Phi - \{A, \Phi\},$$

(19)

where $1$ is a $4 \times 4$ unit matrix, so that $\text{Tr} 1 = 4$. It can be seen that there are no unrestrained derivatives, and clearly $\text{Tr} F$ is gauge invariant.

In the case of a Yang-Mills theory the vector boson kinetic energy can be written exclusively in terms of the curvature $F_{\mu\nu}$, the commutator of the covariant derivative with itself. In the generalized case we study here, the kinetic energy can also be written in terms of the quantity $F$ defined above, a quantity, that, as previous examples of the curvature, is quadratic in $D$ and restrained. The relation between these two quantities is

$$\frac{1}{8} \overline{\text{Tr}} D^2 - \frac{1}{2} \text{Tr} D^4 = -\frac{1}{2} \text{Tr} F^2,$$

(20)

that uses only traces for the Dirac matrices.
7 The curvature in terms of the covariant derivative

Let us review the curvature concept using Riemannian geometry as example. It is well-known that geodesic deviation and parallel transport around an infinitesimally small closed curve in a Riemann manifold are two aspects of the same construction, and that trivial results do not occur in each case only for curved manifolds. Consider thus a four dimensional Riemann manifold with metric $g_{\mu\nu}$ and a vector $B^\mu$. The parallel transport of a vector $B^\beta$ around a closed curve $C$ is given by

$$\Delta B^\beta = \oint_C \Gamma^\beta_{\nu\sigma} B^\sigma \frac{dx^\nu}{ds} ds,$$

where $s$ is a parametrization of the curve and $\Gamma^\beta_{\nu\sigma}$ is the connection in a Riemann space, the Christoffel symbol. Let now $C$ be a small parallelogram made up of two short vectors $x^\mu$ and $y^\mu$, so that its area tensor is $\Delta S^{\mu\nu} = \frac{1}{2} x^\mu y^\nu$. The integral can be performed taking the vector field to be constant along the sides of the parallelogram, and expressing the values of the Christoffel symbol and the vector field at the curve as the first two terms of an expansion about, say, the center of the parallelogram. Thus, if the values of those quantities at a point in one of the sides of the parallelogram are $\tilde{\Gamma}^\alpha_{\beta\delta}$ and $\tilde{B}^\beta$, then, for one of the sides, $\tilde{\Gamma}^\alpha_{\beta\delta} = \Gamma^\alpha_{\beta\delta} + \frac{1}{2} x^\sigma$, and $\tilde{B}^\beta + (B^\beta_{\sigma} + \Gamma^\beta_{\sigma\tau} B^\tau) \frac{1}{2} x^\sigma$, where the quantities without tilde take their values at the center of the parallelogram. The contribution of this side is:

$$y^\delta \tilde{\Gamma}^\alpha_{\beta\delta} \tilde{B}^\beta |_{1} = y^\delta (\Gamma^\alpha_{\beta\delta} + \Gamma^\alpha_{\beta\delta,\sigma} x^\sigma / 2) [B^\beta + (B^\beta_{\sigma} + \Gamma^\beta_{\sigma\tau} B^\tau) d^\sigma / 2].$$ (22)

The contribution of this and its opposite side is:

$$y^\delta \tilde{\Gamma}^\alpha_{\beta\delta} \tilde{B}^\beta |_{1+3} = y^\delta dx^\sigma [\Gamma^\alpha_{\beta\delta} B^\beta_{\sigma} + \Gamma^\alpha_{\beta\delta} \Gamma^\beta_{\sigma\tau} B^\tau + \Gamma^\alpha_{\beta\delta,\sigma} B^\beta].$$ (23)

Summing over the four sides one obtains

$$\Delta B^\alpha = -R^\alpha_{\beta\gamma\delta} B^\beta x^\gamma y^\delta = -R^\alpha_{\beta\gamma\delta} B^\beta \Delta S^{\gamma\delta},$$ (24)

where $R^\alpha_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\gamma,\delta} - \Gamma^\alpha_{\beta\delta,\gamma} + \Gamma^\alpha_{\delta\tau} \Gamma^\gamma_{\beta\tau} - \Gamma^\alpha_{\gamma\tau} \Gamma^\gamma_{\beta\delta}$ is the Riemann tensor. The positive contributions to the tensor come from two opposite sides of the parallelogram, and the negative from the other two sides, resulting in
structure of commutators that can be seen to arise from a commutator of the covariant derivative with itself.

In the case of a Yang-Mills theory, the vector boson kinetic energy can be written exclusively in terms of the curvature $F_{\mu\nu}$, the commutator of the covariant derivative with itself. The parallel transport of a Yang-Mills field, in its role as a connection, around a closed path results in the curvature, the Yang-Mills connection squared. For two short vectors $x^\mu$ and $y^\nu$ that form a parallelogram parallel transport results, as can be shown similarly to the way it was done in last paragraph for the Christoffel symbol, in

$$\oint A_\mu dx^\mu = F_{\mu\nu}x^\mu y^\nu,$$

which gives a motivation for the interpretation of $F_{\mu\nu}$ as a curvature.

In the generalized case we have been studying in this paper the kinetic energy can also be written in terms of a curvature, precisely the quantity $F$ already defined, which was quadratic in $D$ and restrained. The equality does not need the taking of the Lie group trace, and one simply has

$$\frac{1}{8} \text{Tr} D^2 - \frac{1}{2} \text{Tr} D^4 = -\frac{1}{2} \text{Tr} F^2.$$

This equality can be shown to be true substituting $F$ as given by (18) on the right in the equation above. Thus the kinetic energy goes as the square of the curvature in the generalized case, too.

8 Studying the geometric background

Let us study the possible geometric interpretations of the formalism. Let then $x^\mu$ and $y^\nu$ be two small vectors that form a parallelogram, as before. To calculate the parallel transport, the dot product between vectors has to be done taking a trace over the Dirac matrices, but otherwise the procedure is the same as before. Using the generalized covariant derivative and the contracted forms $\hat{f}$ and $\hat{y}$ for the two vectors, and taking the small parallelogram as a path to perform the parallel transport integral, one obtains

$$\frac{1}{4} \text{Tr} D D \hat{f} \hat{y} = F_{\mu\nu} x^\mu y^\nu,$$
where $F_{\mu\nu}$ is constructed as usual with the vector fields that make up the covariant derivative. While this result is reasonable, it is unsatisfactory in that the scalar fields are left as useless bystanders.

One possible attempt to use the formalism we have developed to its fullest, is adding other terms to $\mathcal{F}$ and $\mathcal{G}$. Let then $x^5$ and $y^5$ be two numbers and define

$$d_x \equiv \mathcal{F} + x^5 \gamma^5 \quad \text{and} \quad d_y \equiv \mathcal{G} + y^5 \gamma^5.$$  \hfill (28)

It is possible now to generalize the parallel transport integral, and substitute (28) in the trace expression of (26). The result is:

$$\frac{1}{4} \text{Tr} DD\mathcal{F}\mathcal{G} = \left( \partial_{[\mu} A_{\nu]} + [A_{\mu}, A_{\nu}] \right) x^\mu y^\nu + (\partial_\mu \varphi + [A_\mu, \varphi]) \left( y^5 x^\mu - x^5 y^\mu \right)$$

$$+ \partial_5 A_\mu (x^5 y^\mu - x^\mu y^5).$$ \hfill (29)

The geometric interpretation of this equation is straightforward: we are dealing with a five dimensional manifold. The metric is $\text{Tr} d_x d_y = 4 (x \cdot y + x^5 y^5)$.

The previous result is interesting from a mathematical point of view, but it does not seem to be very enlightening when it comes to an understanding of the geometric meaning of the formalism as applied to the GWS Model. The reason is that, once we have promoted the linearly independent term $x^5 \gamma^5$ in (28) to represent a new dimension, the covariant derivative (13) has to include an extra term, a derivative with respect to the new dimension, and become $D \equiv \partial + \partial_5 \gamma^5 + A + \Phi$. This could, in principle, add several new terms to the generalized derivative, and we would not have the GWS Model anymore. As it is, all the new terms vanish except one; but one term alone is bad enough, and we conclude that this five dimensional model cannot represent the GWS Model. For the record we present curvature due to the generalized derivative with the extra dimension:

$$F = \partial \cdot A - (\partial A) + A \cdot A - A A - (\partial \Phi) - \{A, \Phi\} - (\partial_5 A);$$ \hfill (30)

the sole remaining new term is the last one.

9 Final remarks

We have reviewed the construction of a generalized Yang-Mills theory that uses fields other than vector as gauge fields. We have shown a possible geometric interpretation that requires an additional length parameter, but it
requires, to be consistent, an additional term in the covariant derivative with respect to the new dimension. The results are interesting from a mathematical point of view, but not very helpful if one is trying to understand the geometry of the formalism as applied to the GWS Model, since this dimension is not observed.

In this model some of the generators of the Lie group are vectorial, while other are scalar. From a mathematical point of view it make no difference which ones are which, but if one is aiming to reproduce the GWS Model, one is has no free choice in this matter. This is interesting because it relates the symmetries of the base manifold to the internal symmetry space of the particles, the Lie group. One would expect that eventually an association of this kind should follow from first principles.

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