Nonlinear plasma density modification by the ponderomotive force of ULF pulsations at the dayside magnetosphere

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Abstract We investigate analytically and numerically a nonlinear modification of the magnetospheric plasma density under the action of the ponderomotive force induced by ULF traveling waves, using the nonlinear stationary force balance equation. This equation is applied to both the dipole and dayside magnetosphere having one and two minima of the geomagnetic field near the magnetospheric boundary. The separate and joint actions of the ponderomotive, centrifugal, and gravitational forces on the density distribution are shown.

Keywords Earth · magnetic fields · plasmas · ponderomotive force · waves

1 Introduction

The ultra-low frequency (ULF) electromagnetic pulsations ranging from tenths of Hz up to a few Hz are a common phenomenon in the Earth’s magnetosphere. These waves are electromagnetic in character, i.e., have the electric and magnetic components. This is confirmed by both ground measurements (e.g. Troitskaya 1961; Tepley 1961; Heacock and Akasofu 1973; Bortnik et al. 2008) and in-situ observations by satellites (e.g. Young et al. 1981; Erlandson et al. 1990; Halford et al. 2010; Kim et al. 2010). The ULF pulsations are duct along geomagnetic field lines in the direction of which the parameters
of magnetosphere are nonuniform. This inhomogeneity must be taken into account when considering the influence of the wave action on the magnetospheric plasma to obtain a more realistic picture of physical processes.

The theory of electromagnetic ion cyclotron waves in the ULF range has two aspects. The first one is the problem of sources of these waves related to the physics of the magnetosphere. This aspect is important not only for explaining the origin of these waves and mechanisms by which they are generated, but also for processes being responsible for the acceleration, diffusion and precipitation of magnetospheric particles. These problems have got a great deal of attention in past years (see e.g. Kennel and Petschek 1966; Cornwall 1966; Troitskaya and Guglielmi 1967; Feygin and Yakimenko, 1971; Gendrin et al. 1971; Tverskoy 1971; Nekrasov 1987; Feygin and Yakimenko, 1971; Guglielmi and Pokhotelov 1996). Another important aspect of the theory is the investigation of the influence of ULF pulsations on the background space plasma. Ponderomotive forces induced by these waves can significantly contribute to the plasma balance in the Earth’s magnetosphere (Allan 1992; Guglielmi et al. 1993; Guglielmi and Pokhotelov 1994; Witt et al. 1995; Allan and Manuel 1996; Pokhotelov et al. 1996; Feygin et al. 1998; Nekrasov and Feygin 2005). In the papers by Guglielmi et al. (1993, 1995), it was demonstrated for the dipole magnetosphere that a pronounced maximum of plasma density is formed in the vicinity of the minimum magnetic field force along the wave trajectory, if the amplitude of electromagnetic ion cyclotron waves exceeds a certain critical value. Near the dayside boundary of the magnetosphere, the geometry of geomagnetic field lines changes from the structure with only one magnetic field force minimum to the structure with two minima (so-called magnetic holes) along the field line (Antonova and Shabansky 1968). Pokhotelov et al. (1996) have shown that the ponderomotive force can lead to plasma density accumulation in the vicinity of these magnetic holes.

In some papers cited above (e.g. Guglielmi et al. 1993, 1995), it has been assumed that
the equation of the force balance along the magnetic field lines contains the total thermal pressure together with the nonlinear ponderomotive force derived for small perturbations. However, it is not correct from the point of view of the nonlinear theory, in which all terms must have the same order of magnitude in one equation (for the correct approach see e.g. Allan and Manuel 1996; Nekrasov and Feygin 2005, 2011). Therefore, conclusions made in the corresponding papers are doubtful.

In this paper, we consider analytically and numerically the nonlinear modification of the magnetospheric plasma density, using the nonlinear stationary force balance equation. We apply our investigation to the dipole and dayside magnetosphere having one and two minima of the geomagnetic field near the magnetospheric boundary, respectively.

The paper is organized as follows. In Sect. 2, we give the expression for the ponderomotive force of low-frequency waves traveling along the nonuniform geomagnetic field. The geomagnetic field at the dayside of the magnetosphere is discussed in Sect. 3. The stationary force balance equation is considered in Sect. 4. In Sect. 5, numerical calculations of balance equation in the curved geomagnetic field are given. Conclusive remarks are summarized in Sect. 6.

### 2 Ponderomotive force

In this paper, we use an expression for the ponderomotive force $F_{pi}$ along the background nonuniform magnetic field induced by the circularly-polarized electromagnetic WKB-waves traveling along field lines (see e.g. Nekrasov and Feygin 2005). In the case $N^2 \gg 1$, this expression has the form

$$F_{pi} = -\frac{N^2 E_i^2}{16\pi} \left\{ \nabla_{\parallel} \ln \rho + \left[ \frac{\sigma \omega}{\omega_i - \sigma \omega} + \left( \frac{2\omega_i}{\omega_i - \sigma \omega} \right) \frac{1}{N^2} \right] \nabla_{\parallel} \ln B \right\},$$

(1)
is the refractive index. Here, $\omega_{pi} = (4\pi ne^2/m_i)^{1/2}$ and $\omega_i = eB/m_ic$ are the ion plasma and cyclotron frequencies, respectively, $\rho = m_in$ is the equilibrium plasma mass density, $n$ is the local number density, $B$ is the local background magnetic field, $E_1$ is the wave amplitude, $\omega (> 0)$ is the wave frequency, $k_i$ is the wave number, $e$ and $m_i$ are the charge and mass of the ions, $\sigma$ denotes the left- (+1) and right- (−1) polarization, and $c$ is the speed of light in vacuum.

In the WKB approximation, we have $E_1 \sim N^{-1/2}$ or $E_1^2 N = E_{10}^2 N_0 = \text{const}$, where the subscript 0 here and below relates to the corresponding values at the magnetic equator. We will express the ponderomotive force through the amplitude of the wave magnetic field $B_{10}$ at the magnetic equator because satellite measurements give usually the magnitude of magnetic field perturbations. From Faraday’s equation for the traveling wave, it is followed that $B_1 = NE_1$. Thus, we obtain from these relations and (2)

$$N^2 E_1^2 = B_{10}^2 \frac{N}{N_0} = B_{10}^2 \left( \frac{\rho}{\rho_0} \right)^{1/2} \frac{B_0}{B} \frac{(1 - \nu_0)^{1/2}}{(1 - \nu_0 \frac{B_0}{B})^{1/2}},$$

(3)

where $\nu_0 = \sigma \omega / \omega_{i0}$. Substituting (3) in (1), we obtain

$$F_p = -\frac{B_{10}^2}{16\pi} \left( \frac{\rho}{\rho_0} \right)^{1/2} \frac{B_0}{B} \frac{(1 - \nu_0)^{1/2}}{(1 - \nu_0 \frac{B_0}{B})^{1/2}}$$

$$\times \left\{ \nabla_n \ln \rho + \frac{1}{1 - \nu_0 \frac{B_0}{B}} \left[ \nu_0 \frac{B_0}{B} + \frac{2}{N_0^2} \frac{(1 - \nu_0 \frac{B_0}{B}) \rho_0}{B^2} \frac{B^2}{B_0^2} \frac{\rho_0}{B_0^2} \right] \nabla_n \ln B \right\}. \quad (4)$$

3 Curved dayside geomagnetic field, field line equation and longitudinal gradient $\nabla_n$

The magnetic data collected by satellites and theoretical models show a complex structure of the geomagnetic field at the midday boundary of the Earth’s magnetosphere.
This structure is characterized by a smooth transition from "V" to a "W" magnetic field shape with two minima in the magnetic field along field lines. There exist several analytical (Hones 1963; Mead 1964; Antonova and Shabansky 1968) and quantitative models (Mead and Fairfield 1975; Tsyganenko 1987) of the magnetic field in the Earth’s magnetosphere based on satellite observations. The model by Antonova and Shabansky (1968) is quite convenient for an analysis of geophysical phenomena in the dayside magnetosphere and, in addition, is in a reasonable agreement with the magnetometer data provided by the HEOS 1, 2 satellites (Antonova et al. 1983). In this model, the structure of the geomagnetic field is determined by the existence of two dipoles: an original dipole having the Earth’s magnetic moment $M$ and an additional one which imitates the distortion of the magnetic field due to the solar wind pressure. The latter dipole has the magnetic moment $kM$, where $k$ is a constant parameter, and is shifted a distance $a$ (measured in the units of the Earth’s radius $R_E$) at the dayside along the Earth-Sun line from the position of the original dipole.

In the spherical coordinate system, magnetic field components for the two-dipole model in the meridional noon-midnight plane have the form (Antonova and Shabansky 1968)

$$B_r = -\frac{2B_{E} \alpha}{r^3},$$  \hspace{1cm} (5)

$$B_\varphi = \frac{B_E}{r^3} \sqrt{1-x^2} \beta,$$  \hspace{1cm} (6)

where $x = \sin \varphi$, $\varphi$ is the geomagnetic latitude, $r$ is measured in units $R_E$, and $B_E = 0.311$ G is the equatorial magnetic field at the Earth’s surface. Coefficients $\alpha$ and $\beta$ are the following (Antonova and Shabansky 1968):

$$\alpha = 1 - \frac{kr^3 \left(a^2 - 2r^2 + ar\sqrt{1-x^2}\right)}{2 \left(a^2 + r^2 - 2ar\sqrt{1-x^2}\right)^{5/2}},$$  \hspace{1cm} (7)

$$\beta = 1 + \frac{kr^3 \left[\sqrt{1-x^2} (a^2 + r^2) - ar(2 + x^2)\right]}{\sqrt{1-x^2} (a^2 + r^2 - 2ar\sqrt{1-x^2})^{5/2}}.$$  \hspace{1cm} (8)
The value of the magnetic field \( B \) in an arbitrary point of the field line can be defined from (5) and (6) as
\[
B = \frac{B_E}{r^3} \left[ 4x^2\alpha^2 + (1 - x^2\beta^2) \right]^{1/2}.
\] (9)

An equation for the field line is determined by \( dr/d\varphi = B_r/B_\varphi \) or
\[
\frac{dr}{dx} = -\frac{2xr\alpha}{1 - x^2\beta}.
\] (10)

The intensity of the magnetic field force along the near boundary field lines described by (9) can have two minima located symmetrically relative to the magnetic equator (Antonova and Shabansky 1968). When \( a \) tends to infinity, the values \( \alpha \) and \( \beta \) (see 7 and 8) approach to 1. In this case, we have a transition to the one-dipole approximation. Thus, all the expressions obtained in this paper can be transformed to those corresponding to the one-dipole model. Since a purpose of our paper is a qualitative description of the ponderomotive force action induced by geomagnetic pulsations in the dayside magnetosphere along field lines at different distances from the Earth (up to the magnetopause), we will not focus on details of the two-dipole model.

The operator \( \nabla_u \) is defined by relation \( \nabla_u = \mathbf{b} \cdot \nabla \), where \( \mathbf{b} \) is the unit vector along the magnetic field. Using (5), (6), (9) and (10), we find
\[
\nabla_u = 2R_E^{-1}\eta^{-1/2} \frac{d}{dx},
\] (11)

where
\[
\eta = \left( \frac{dr}{dx} \right)^2 + \frac{r^2}{1 - x^2}.
\] (12)

In order to investigate the ponderomotive force action induced by geomagnetic pulsations in the dayside magnetosphere along field lines and carry out analytical and numerical calculations at different distances from the Earth (up to the magnetopause), we
use the magnetic field model by Antonova and Shabansky (1968) described above. For the
equilibrium plasma mass density, we take the power law form to describe the longitudinal
field line distribution, $\rho \sim r^{-\gamma}$. For the large distances from the Earth’s surface which
we consider below, the best choice for $\gamma$ to be appropriate to experimental data is $\gamma = 1$
(Denton et al. 2006). Thus, we take approximately

$$\rho(x) = \rho_0 (1 - x^2)^{-1}. \quad (13)$$

This formula can be applied up to $\varphi \approx \pm 50 - 60^\circ$ (Denton et al. 2006). For simplicity, we
do not take into account in (13) the non-dipole form of the magnetic field (see 10) because
of an uncertainty of plasma mass distribution in this region.

### 4 Stationary force balance equation

From equations of motion for the ions and electrons in the second approximation on
the wave amplitude averaged over fast oscillations, we obtain the force balance equation in
the stationary state along the magnetic field line (e.g. Nekrasov and Feygin 2005). Taking
into account the gravitational and centrifugal forces (e.g. Lemaire 1989; Persoon et al.
2009), we have

$$\nabla \cdot p_2 - (g_i + n_{\Omega b} \Omega^2 R_E r \cos^2 \varphi) \rho_2 = F_{p\parallel}. \quad (14)$$

Here $p_2 = \rho_2 c_s^2$ and $\rho_2$ are nonlinear stationary perturbations of pressure and mass
density, respectively, $c_s = (2T/m_i)^{1/2}$ is the sound speed, $T$ is the temperature,$g_i = \mathbf{g} \cdot \mathbf{b} = -g_E B_r/r^2 B$ is the longitudinal gravitational acceleration ($g_E = 9.8 \text{ m sec}^{-2}$),
$\Omega$ is the Earth’s rotation frequency, and $n_{\Omega b} = \mathbf{n}_\Omega \cdot \mathbf{b}$, where $\mathbf{n}_\Omega$ is the unit vector along the
centrifugal force. Calculations show that $n_{\Omega b} = (1 + \beta/2\alpha)(B_r/B) \cos \varphi$. Substituting (4)
into (14) and using (5), (11), and (13), we find
\[
\frac{d}{dx} \rho_2 \rho_0 = -\frac{R_E}{2c_s^2 r^2} \eta^{1/2} B_r \left[ g_E - (1 + \beta/2\alpha) \Omega^2 R_E r^3 \cos^3 \varphi \right] \frac{\rho_2}{\rho_0} - \frac{B_{10}^2}{16\pi \rho_0 c_s^2} \frac{B_0}{B} \left( 1 - \frac{\nu_0 B_0}{c_s^2} \right)^{1/2} \rho_2 \rho_0
\]
\[
\times \left\{ \frac{2x}{(1 - x^2)^{3/2}} + \left[ \frac{1}{(1 - x^2)^{1/2}} \left( 1 - \frac{\nu_0 B_0}{c_s^2} \right) \right] B_0 + \frac{(1 - x^2)^{1/2}}{2\pi \rho_0 c_s^2} B \frac{dB}{dx} \right\}. \tag{15}
\]
This differential equation determines a nonlinear redistribution of the plasma density due to the action of the ponderomotive force.

Equation (15) can be rewritten in the form
\[
\frac{d}{dx} \rho_2 = A_1 \frac{\rho_2}{\rho_0} + A_2 (A_3 + A_4 + A_5). \tag{16}
\]

Coefficients \( A_i \), \( i = 1, 2, 3, 4, 5 \), are the following:
\[
A_1 = \frac{g_{\text{eff}} R_E x^\alpha}{(1 - x^2) r c_s^2 \beta}, \quad A_2 = -\frac{B_{10}^2}{16\pi \rho_0 c_s^2} \frac{B_0}{B} \left( 1 - \frac{\nu_0 B_0}{c_s^2} \right)^{1/2}, \quad A_3 = \frac{2x}{(1 - x^2)^{3/2}}, \tag{17}
\]
\[
A_4 = \frac{1}{(1 - x^2)^{1/2}} \left( 1 - \frac{\nu_0 B_0}{c_s^2} \right) B_0 \frac{dB}{dx}, \quad A_5 = \frac{(1 - x^2)^{1/2}}{2\pi \rho_0 c_s^2} B \frac{dB}{dx},
\]
where we have used (9) and (12) in the first term on the right-hand side of (16). The acceleration \( g_{\text{eff}} \) in the coefficient \( A_1 \) is equal to
\[
g_{\text{eff}} = g_E + g_c = g_E - (1 + \beta/2\alpha) \Omega^2 R_E r^3 \cos^3 \varphi, \tag{18}
\]
where \( g_E \) and \( g_c \) describe an influence of the gravitational and centrifugal forces, respectively.

If we take for estimation \( \alpha \sim \beta \sim 1 \) in (18) and substitute numerical values of parameters \( g_E, \Omega, \) and \( R_E \), we obtain
\[
\frac{g_E}{g_c} \sim 3 \times 10^2 \frac{3 \times 10^2}{r^3 \cos^3 \varphi}. \tag{19}
\]
where \( r \approx r_0 \cos^2 \varphi \) (see 10). Thus, for \( r_0 \cos^3 \varphi < 6.7 \), one can neglect the contribution of the centrifugal force. For example, the last condition for \( r_0 = 8 \) is satisfied at \( |\varphi| \gtrsim 20^\circ \). The same for \( r_0 = 10 \) is at \( |\varphi| \gtrsim 30^\circ \). In the region of the magnetic equator, the centrifugal force is dominant and leads to a local peak in mass density. An estimation for \( r_0 = 8 \) corresponds to the results by Denton et al. (2006) (see their Figs. 9 and 10).

The coefficient \( A_1 \) given by (17) will be the same in the equation for the background equilibrium. Its estimation for gravity, for example, at \( r_0 = 8 \) is the following:

\[
A_1 \sim 8 \times 10^{-3} x (1 - x^2)^{-2} \quad \text{(we have taken} c_s^2 = 10^9 \text{ m}^2 \text{ sec}^{-2} \text{ for} T = 10 \text{ eV). Thus, one could wait an essential contribution of the gravitational force to the equilibrium mass density distribution at} 1 - x^2 \lesssim 4 \times 10^{-3}. \text{ However, this result does not coincide with (13). This could be a consequence of additional factors except for the gravitational force which lead to the power law distribution of mass density.}
\]

We see from (16) that the effective gravity \( g_{eff} \) and ponderomotive force \( \sim A_2 \) influence on the nonlinear density redistribution. These two forces can be of the same order of magnitude or one force can dominate in the dependence on the wave amplitude. We note also that the nonlinear disturbance of density increases when \( \nu_0 B_0 \rightarrow B \) or \( \omega \rightarrow \omega_i \). This case can occur in magnetic holes.

5 Numerical results

For numerical calculations, we have chosen \( a = 33 \) and \( k = 13 \). Such parameters correspond to the dayside boundary of the magnetosphere at the distance \( 10R_E \) and the region of experimental data obtained by HEOS 1, 2 satellites (Antonova et al. 1983). Solution of Eq. (16) has been carried out by means of the Runge-Kutta method. In all the cases considered, we assumed that the square of the sound velocity \( c_s^2 = 10^9 \text{ m}^2 \text{ sec}^{-2} \).
As it has been mentioned above, this value corresponds to $T = 10$ eV. The plasma mass density at the equator $\rho_0$ was chosen equal to $1.67 \times 10^{-20}$ kg m$^{-3}$ for all $L$ ($L$ is McIlwain's parameter) near the midday boundary of the Earth's magnetosphere. These values are based on experimental data from measurements of the plasma density from the plasmapause up to the magnetosphere boundary. In these areas, the plasma density depends weakly on $L$ (Chappel 1974; Carpenter and Anderson 1992). Since the purpose of this paper is a qualitative study of the ponderomotive force effect of low-frequency perturbations on the redistribution of the background plasma density, we do not focus on details of the weak dependence of $\rho_0$ on $L$.

The dependence of the Earth’s magnetic field $B$ on the geomagnetic latitude $\varphi$ in the meridional plane of the dayside magnetosphere for different $L$ for the model by Antonova and Shabansky (1968) is given in Fig. 1. This figure shows a smooth transition from the "V" to "W" magnetic field shape with two minima in the magnetic field along the field lines. Fig. 2 demonstrates the separate and joint influence of the gravitational, centrifugal, and ponderomotive forces on the relative latitudinal density perturbation $\delta = \rho_2/\rho_0$ for $L = 6$ and $k = 13$ (see 16 and 17). The same is shown in Fig. 3 at $L = 10$. We see that the role of gravity decreases and the mass density is peaked due to ponderomotive and centrifugal forces. Such peaking is observed in the magnetosphere of the Earth (Denton et al. 2006). Figures 4 and 5 represent the difference in $\delta$ distribution for the dipole ($k = 0$) and non-dipole ($k = 13$) geomagnetic field at $L = 8$ and $L = 10$, respectively. We see that the essential difference is due to the presence of geomagnetic holes. We note that small variations in the density distribution given in Figs. 2-5 are because of the small value of the coefficient $A_1$ in (16). In order to show different dependence’s, we have taken a small value for $B_{10}$.

At the magnetic equator, the projection of the ponderomotive, gravitational, and
centrifugal forces on the geomagnetic field line is equal to zero. The longitudinal centrifugal force $f_{cf}$ is expressed via the longitudinal gravitational force $f_{g\parallel}$ at $\cos \varphi \sim 1$ and for $r_0 = 8 \div 10$ in the form

$$f_{cf} \sim 1.7 \div 3.3 f_{g\parallel}$$

(see 19). Thus roughly, these forces are of the same order of magnitude and small (see discussion below 19). Their influence is displayed when the ponderomotive force is also small (Figs. 2-5). At the wave amplitudes $B_{10}$ larger than $10^{-6}$ G, the gravitational and centrifugal forces are not important.

6 Conclusion

In this paper, we have performed an analytical and numerical study of the influence of the geomagnetic field structure near the dayside magnetospheric boundary on the plasma density redistribution due to the ponderomotive force action induced by ULF perturbations. We have shown that the ponderomotive force causes the nonlinear stationary plasma density redistribution, which is much smaller than the background plasma density. We note that the ponderomotive force is a nonlinear effect, which is found by using linear perturbations. The assumption that the ponderomotive force has the same order of magnitude as the background pressure and gravity is incorrect. However, such an assumption has been adopted in some papers (Guglielmi et al. 1993, 1995; Guglielmi and Pokhotelov 1994; Pokhotelov et al. 1996; Feygin et al. 1998). Ponderomotive force (1) is derived by the perturbation method. Therefore, its contribution can only disturb an equilibrium state. It is followed from (15) that $\rho_2/\rho_0 \sim B_{10}^2/4\pi \rho_0 c_s^2$ on the order of magnitude (see also Nekrasov and Feygin 2005). For parameters given above and for $B_{10} = 5 \times 10^{-6}$ G, the ratio $\rho_2/\rho_0$ is of the order of $10^{-2}$.

We have investigated the plasma density redistribution for the model of the geomagnetic
field $B$ by Antonova and Shabansky (1968) (Fig. 1). This choice is justified by its simplicity. The use of a more complex geometry for $B$ can be useful for an analysis of certain particular situations (for example, specific satellite measurements etc). The results, however, will be qualitatively the same. We have shown that the two-dipole structure of the Earth’s magnetic field (geomagnetic holes) leads to an essential modification in the plasma distribution (Figs. 4 and 5). In Figs. 2-5, we observe the appearance of two disturbed plasma density minima under the action of ULF pulsations (the curves 1 in Figs. 2 and 3 display the action of the centrifugal and gravitational forces). This two minima effect could be verified by observations.

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**Figure captions**

Fig. 1. The dependence of the Earth’s magnetic field $B$ on the geomagnetic latitude $\varphi$ in the meridional plane of the dayside magnetosphere for different $L$ in the model by Antonova and Shabansky (1968).

Fig. 2. The dependence of the relative density perturbation $\delta$ for $L = 6$, $k = 13$. The curve 1 corresponds to $g_{eff} \neq 0$, $B_{10} = 0$, curve 2 - $g_{eff} \neq 0$, $B_{10} = 10^{-6}$ G, and curve 3 - $g_{eff} = 0$, $B_{10} = 10^{-6}$ G.

Fig. 3. The same as in Fig. 2 for $L = 10$.

Fig. 4. The dependence of $\delta$ distribution on the dipole ($k = 0$) and non-dipole ($k = 13$) geomagnetic field for $L = 8$ ($g_{eff} \neq 0$, $B_{10} = 10^{-6}$ G).

Fig. 5. The same as in Fig. 4 for $L = 10$. 