THE NUCLEAR EFFECT ANALYSIS IN RELATIVISTIC HEAVY ION COLLISIONS AT BNL ENERGIES

M. AYAZ AHMAD

Department of Physics, Faculty of Science, P. O. Box 741, University of Tabuk, 71491, Saudi Arabia

Copyright © 2021 the author(s). This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract In this article, an attempt has been made to understand the behavior of the secondary charged particles multiplicity distributions produced during the heavy ion collisions at ultra-relativistic energies by using the Hurst exponent of the two dimensional (2D) factorial moments, $F_q$. For this purpose the experimental data have been analyzed by using the “Hurst exponent” in the original intermittency formula by considering different values of Hurst exponent ($H = 1.0, 1.5, 2.0, 2.5$). The investigations reveal the power law behavior, exhibited in self-affine or nuclear effect analysis, better than that in self-similar analysis. Finally, the described works were found very much significant and also it was within good agreement with some other works.

Keywords: multiplicity distribution; cumulant factorial moments; QGP; simulations, high energy density.

2010 AMS Subject Classification: 82D10.

1. INTRODUCTION

The primary objective of particle physics is to discover the fundamental forces and symmetries, and the elementary particles in Nature. A hierarchy of constituents of matter has been observed: macroscopic matter consists of molecules and atoms, the atoms consist of nucleons which in turn are formed of quarks, anti-quarks and gluons (partons). These results have been obtained by

*Corresponding author
E-mail address: mayaz.alig@gmail.com
Received November 30, 2020
scattering experiments at higher and higher energies, as required to achieve information on smaller and smaller objects. At the moment the hierarchy ends at quarks: no substructure has been observed for them, so they are regarded as point like particles. Isolated single free quarks have never been observed, and therefore it is conjectured that quarks are confined together with other quarks to form hadrons.

The study of multiplicity correlations and fluctuations of produced charged particles in high energy ion collisions has been well known for few decades. This has never been more apparent than in recent years where these measurements helped to mark the discovery of new state of matter, so called Quark-Gluon Plasma (QGP) [1-5]. Various types of correlations and fluctuations present in heavy ion collisions at relativistic energies can provide us with valuable knowledge regarding the microscopic interactions inside the high density medium. In particular, the non-perturbative aspects of the strong interaction are difficult to study experimentally and probing the hot and dense QGP is one of the few avenues we have available.

The main aim of scaled factorial moments \( F_q(M) \) is to look for the possible existence of the dynamical fluctuations so called “Intermittency” in high energy heavy ion collisions [6-8 and references therein]. While studying the behavior of factorial moments in h-h and A-A collisions, two different phenomenon have been observed.

(i) In one dimension (1D) phase space of pseudorapidity, the rise of \( \ln \langle F_q \rangle \) with the increasing phase space partition number M is much weaker for A-A collisions than for h-h collisions and the heavier the colliding nuclei are, the weaker is the rising of \( \ln \langle F_q \rangle \).

(ii) In 2D or 3D, the \( \ln \langle F_q \rangle \) vs. \( \ln M \) plot for A-A collisions is bending upwards strongly, much stronger than for h-h collisions and the heavier the colliding nuclei are, the stronger is the upward bending of \( \ln \langle F_q \rangle \) vs. \( \ln M \) plot.

Here, we illustrate various steps needed to obtain a reliable measurement of the “Hurst exponent” so called \( H_q \) moments. This includes the evaluation of both statistical and systematic errors, followed by a short study of the truncation in the tail of the charged-particle multiplicity

...
distribution. The outcomes were compared to the numerous analytical QCD predictions which exist up to the Next to Next to Leading Logarithm Approximation (NNLLA) based on the study of various Monte Carlo models.

2. EXPERIMENTAL DETAILS
The experiment data has been collected in the present work by FUJI type nuclear emulsion stacks, those were irradiated horizontally with a beam of $^{28}$Si nuclei (like projectile) and hit the heterogeneous mixture of nuclear emulsion (fixed target) at 14.6A GeV at Alternating Gradient Synchrophasotron (AGS) of Brookhaven National Laboratory (BNL), NewYork, USA. The scanning of the exposed emulsion stacks was performed with the help of Leica DM2500M microscope with a 10× objective and 10× ocular lens provided with semi-automatic scanning stages. The method of line scanning was used to collect the inelastic $^{28}$Si-Em interactions. The interactions collected from line scanning were scrutinized under an optical microscope (Semi-Automatic Computerized, Leica DM6000M) with a total magnification of 10 * 100 using 10× eyepiece and 100× oil immersion objective. The measuring system associated with it has 1 μm resolution along X and Y axes and 0.5 μm resolution along the Z-axis. The detailed discussion about the present experiment can be found in our earlier publications [9-13].

3. MATHEMATICALLY TOOLS
It has been found that the above two apparently contradictory observations are due to the superposition effect of the contribution from the large number of elementary collisions in a nuclear collision process.

To characterize the phase space partition in 2D, a quantity known as “Hurst exponent” [14-15] is used and is defined such as:

$$H = \frac{\ln M_n}{\ln M_p}$$  \hspace{1cm} (1)
where $M_\eta$ and $M_\phi$ are the number of partitions along the two perpendicular directions. The behavior of factorial moments $F_q$ depends on the value of “H”. Therefore, to study the possible abnormal behavior of $F_q$ on the bin size, analysis should be performed with a suitable value of H. Otherwise, the observed trend of the calculated factorial moments will always be bending upwards, even if there is no fluctuation in the multi-particle production. The upward bending of the $F_q$ moments can be weakened or totally removed if the exponents, H are given a proper value.

For the present investigations of nuclear effect in $^{28}$Si-emulsion collision at total energy $\approx 409$ GeV, we have used different values of Hurst exponent (H = 1.0, 1.5, 2.0, 2.5). The intervals $\Delta \eta$ and $\Delta \phi$ have been divided into sub-cells with the widths such as:

$$
\delta_\eta = \frac{\Delta_\eta}{M_\eta} \\
\delta_\phi = \frac{\Delta_\phi}{M_\phi}
$$

(2)

The scale factors of $M_\eta$ and $M_\phi$ are connected to each other by the relation:

$$
M_\phi = \left\{M_\eta\right\}^{1/\eta}
$$

(3)

It is clear from eqn. (3) that $M_\eta$ and $M_\phi$ cannot be integers simultaneously. Therefore the size of elementary phase space cell cannot take continuously varying values. This problem can be solved by applying the method of L. Lianshou et. al., such as given:

$$
M_\phi = N + \alpha
$$

(4)

where N is the integer part and $0 \leq \alpha < 1$ represents the fractional part.

4. RESULTS AND DISCUSSIONS

In the present work, we used $\Delta \eta = -2 \leq \eta_{\text{max}} \leq +2$ and $\Delta \phi = 0 - 2\pi$. The $M_\eta$ was varied from 2-30. Further, to reduce the effect of non flat particle density distributions, the cumulative variables $X_\eta$ and $X_\phi$ were used to make it in the corresponding regions 0-1. By using the above partition scheme, the values of $\ln < F_2 >$ were calculated with the help of general adaptation of Intermittency / scaled factorial moments $F_q(M)$. 


The behavior of $\ln < F_2 >$ vs. $\ln M$ have been shown in Fig. 1(a-d) for the collisions of $^{28}\text{Si}$ with emulsion nuclei at an energy 409 GeV for different values of exponent $H$. From this figure it has been observed that there is strong upward curve bend in Fig. 1 (a). However, when $H$ increases, the upward bending is found weakened in Fig. 1(b-d).

**Figure 1(a-d):** The dependence of $\ln < F_2 >$ on $\ln M$ at energy $\approx 409\text{GeV}$.
We observe that the two dimensional second order factorial moment exhibits an upward bending as a function of partition of space, which is in turn means the superposition of contributions from the elementary collisions in the nucleus-nucleus collisions. This upward bending could, however, be removed by choosing proper partition along the longitudinal and perpendicular directions, that is, the right value of Hurst exponent “H”. Moreover, it has been observed that heavier the colliding are, the strong the upward bending is. It is consistence with the fact that the number of elementary collisions is more for heavier nuclei.

5. Conclusion and Final Remarks

It is worth mentioning that if QGP is formed, then there will be no elementary collisions. This in turn will lead to vanishing of the superposition effect due to the contribution of elementary collisions in nucleus- nucleus (A-A) collisions. Under such conditions, the upward bending in the two dimensional second factorial moment plots is not likely to be seen. Hence the study of the nuclear effect in nucleus- nucleus (A-A) collisions could be used as another indirect test of QGP formation.

Acknowledgements

The author would like to acknowledge the keen support in financial assistance for this work of the Vice Presidency / Studies and Scientific Research/Deanship of Scientific Research on behalf of the University of Tabuk, Kingdom of Saudi Arabia and Ministry of Higher Education, K.S.A. Under the research grant no. S-0263-1436/dated 15-03-1436 along with some recent publications based on COVID-19 [16]-[33].

Conflict of Interests

The authors declare that there is no conflict of interests.
REFERENCES

[1] J.D. Bjorken, Highly relativistic nucleus-nucleus collisions: The central rapidity region, Phys. Rev. D. 27 (1983), 140–151.
[2] M. Gyulassy, Signatures of new phenomena in ultrarelativistic nuclear collisions, Nuclear Phys. A. 418 (1984), 59–85.
[3] M. Stephanov, K. Rajagopal, E. Shuryak, Event-by-event fluctuations in heavy ion collisions and the QCD critical point, Phys. Rev. D. 60 (1999), 114028.
[4] R.C. Hwa, Fractal measures in multiparticle production, Phys. Rev. D. 41 (1990), 1456–1462.
[5] R.C. Hwa, Criticality, Erraticity and Chaoticity in Strong Interaction, Proc. 7th Int. Workshop on Multiparticle Production, Correlation and Fluctuations, Nijmegen, Netherlands, (1996), 303-312.
[6] M.A. Ahmad, A Study of Intermittency and Multifractality in 28Si-emulsion Collisions at 14.6A GeV, Ph.D. thesis, Aligarh Muslim University, Aligarh, India, 2010.
[7] A. Bialas, R. Peschanski, Intermittency in multiparticle production at high energy, Nuclear Phys. B. 308 (1988), 857–867.
[8] S. Ahmad, M. Ayaz Ahmad, Some observations related to intermittency and multifractality in 28Si and 12C-nucleus collisions at 4.5A GeV, Nuclear Phys. A, 780 (2006), 06-221.
[9] S. Ahmad, M. Ayaz Ahmad, M. Irfan, M. Zafar, Study of non-statistical fluctuations in relativistic nuclear collisions, J. Phys. Soc. Japan, 75(6) (2006), 064604.
[10] S. Ahmad, M. Ayaz Ahmad, A comparative study of multifractal moments in heavy ion collisions at high energies, J. Phys. G: Nuclear Part. Phys. 32 (2006), 1279-1293.
[11] M.A. Ahmad, S. Ahmad, Study of Angular Distribution and KNO Scaling in the Collisions of 28Si with Emulsion Nuclei at 14.6A GeV, Ukrainian J. Phys. 57(12) (2012), 1205-1213.
[12] M.A. Ahmad, M.H. Rasool, S. Ahmad, Scaling nature of target fragments in the 28Si-emulsion interactions at an energy 14.6A GeV, Ukrainian J. Phys. 58(10) (2013), 944-955.
[13] M.A. Ahmad, J.H. Baker, M.H. Rasool, S. Ahmad, R. Dobra, D. Pascalescu, C.R. Telles, Fluctuations in produced charged particle multiplicities in relativistic nuclear collisions for simulated events, J. Phys.: Conf. Ser. 1258 (2019), 012010.
[14] M.E. Gaddis, M.J. Zyda, The Fractal Geometry of Nature: Its Mathematical Basis and Application to Computer Graphics, Naval Postgraduate School, California, 1986.
[15] B.B. Mandelbrot, The fractal geometry of nature, W.H. Freeman, San Francisco, 1982.
NUCLEAR EFFECT ANALYSIS IN RELATIVISTIC HEAVY ION COLLISIONS

[16] M. Tariq, M.A. Ahmad, S. Ahmad, M. Zafar, Analysis of high N\textsubscript{s}-multiplicity events produced in relativistic heavy ion collisions at 4.5A GeV/c, Romanian Rep. Phys. 59 (3) (2007), 773-790.

[17] M.A. Ahmad, S. Ahmad, Study of non-thermal phase transition in \textsuperscript{28}Si - nucleus collisions at 14.6A GeV, Int. J. Mod. Phys. E, 7(8) (2007), 2241-2247.

[18] S. Ahmad, M.A. Ahmad, M. Tariq, M. Zafar, Charged multiplicity distribution of relativistic charged particles in heavy ion collisions, Int. J. Mod. Phys. E, 18(9) (2009), 1929-1944.

[19] M.A. Ahmad, S. Ahmad, M. Zafar, Intermittent and scaling behaviour of shower particles produced in the collisions of \textsuperscript{28}Si - Em at 14.6A GeV", Indian J. Phys. 84 (12) (2010), 1675-1681.

[20] M.H. Rasool, M.A. Ahmad, S. Ahmad, Signal of Unusual Large Fluctuations in \textsuperscript{32}S-Em Interactions at SPS Energies, J. Korean Phys. Soc. 67 (2015), 448-457.

[21] M.H. Rasool, M.A. Ahmad, S. Ahmad, Multifractal study and multifractal specific heat of singly charged particles produced in \textsuperscript{32}S-Em interactions at 200A GeV, Chaos Solitons Fractals, 81 (2015), 197-202.

[22] M.H. Rasool, M.A. Ahmad, O.V. Singh, S. Ahmad, Multiplicities of Forward-Backward Relativistic Charged Particles Produced in \textsuperscript{32}S-Emulsion Interactions at 200 AGeV/c, Chinese J. Phys. 53 (2015), 100302.

[23] M. Rîşteiu, R. Dobra, D. Pasculescu, A.A. Mohammad, Quality Engineering Tools Focused on Designing Remote Temperature Measurements for Inaccessible Locations by Using Light Components Parameterization of the Heated Materials, IOP Conf. Ser.: Mater. Sci. Eng. 133 (2016), 012059.

[24] M.L. Boca, R. Dobra, D. Pasculescu, M. A. Ahmad, Analysis and simulation of industrial distillation processes using a graphical system design model, Proc. SPIE, VIII (10010), (2016), 100102U.

[25] M.H. Rasool, M.A. Ahmad, M. Bhat, O.V. Singh, S. Ahmad, Multiplicity Characteristics of Forward-Backward Emitted Particles in Heavy-Ion Interactions at SPS Energies, in: B. Bhuyan (Ed.), XXI DAE-BRNS High Energy Physics Symposium, Springer International Publishing, Cham, 2016: pp. 61–66.

[26] M.J. Mbunwe, M.A. Ahmad, S.K. Mustafa, An effective energy saving design strategy to maximize the use of electricity, J. Math. Comput. Sci. 10 (2020), 1808-1833.

[27] I. Nevliudov, V. Yevsieiev, J.H. Baker, M.A. Ahmad, V. Lyashenko, Development of a cyber design modeling declarative Language for cyber physical production systems, J. Math. Comput. Sci. 11 (2021), 520-542.

[28] S.S. Safaai, S.L. Yap, S.V. Muniday, M.A. Ahmad, Some Aspects of Fluctuations Dynamics of Particles in Dusty Plasma, Commun. Math. Biol. Neurosci. in Press.

[29] V. Lyashenko, S.K. Mustafa, I. Tvoroshenko, M.A. Ahmad, Methods of Using Fuzzy Interval Logic During Processing of Space States of Complex Biophysical Objects, Int. J. Emerg. Trends Eng. Res. 8(2), (2020), 372-377.
[30] S.K. Mustafa, M.A. Ahmad, S. Sotnik, O. Zeleniy, V. Lyashenko, O. Alzahrani, Brief review of the Mathematical Models for Analyzing and Forecasting Transmission of COVID-19, J. Critical Rev. 7(19) (2020), 4206-4210.

[31] S.K. Mustafa, M.A. Ahmad, et. al. COVID-19 and Immune Function – “A Significant” Zinc, Oriental J. Chem. 36(6) (2020), 1026-1036.

[32] V. Pavlova, B.B. Popov, V.K. Hristova et al. Chemical characterization of VARUMIN (1 and 2) oral solutions as potential therapeutic beverages, Int. J. Pharm. Res. 12 (2020), 4454-4460.

[33] S.K. Mustafa, M.A. Ahmad, V. Baranova, et al. Using Wavelet Analysis to Assess the Impact of COVID-19 on Changes in the Price of Basic Energy Resources, Int. J. Emerg. Trends Eng. Res. 8(7) (2020), 2907-2912.