Supplementary Material: Modeling.

Here we summarize the results of the modeling. The predictions are shown in the graphs of the main text and the simulations underlying the modeling are shown in the spreadsheet that is also available with the supplementary material. We examine the predictions of models that have been proposed for the flash grab: extrapolation (van Heusden et al., 2019) and averaging (Cavanagh & Anstis, 2013; Sinico et al, 2009). We also examine models developed for the flash lag effect that can be applied to the flash grab as well. These models compute where the moving stimulus would be at the time of the flash. For the flash lag, this gives an offset or lag – a separation between the moving stimulus and the flash. In the case of the flash grab, the flash is instead seen transposed to the location of the moving stimulus, not separated from it. Nevertheless, the same computations can be used to predict where the flash will move to. These models have been proposed and extensively debated in the decade following Nijhawan’s 1994 revival of the flash lag effect. What is new here is that the motion path for the flash grab effect is more of a challenge for these models, especially the results of our Experiment 2. Some of the earlier flash lag models are specific to its dual task nature – that is, first detecting the flash and then attending to the motion (e.g., Baldo & Klein, 1995) so we will not consider these. But we will include the differential latency models which assume different temporal properties for the flash and the motion and the integration (Whitney et al., 2000; Öğmen et al., 2004) and averaging models (Krakelberg & Lappe, 2000).

This generalization of the flash lag models to the flash grab is likely valid for flashes presented at the motion reversal, as is true for our experiments here. In this case, the flash location appears to migrate to the location of the motion reversal. But this migration or “grabbing” process undoubtedly depends on the synchronicity of the flash and the motion reversal that binds them together (Cavanagh & Anstis, 2013). For a flash that is not at the reversal point in time and space,
the binding weakens and the flash shift decreases dramatically (Cavanagh & Anstis, 2013). The effect then becomes a separation of the no longer shifted flash from the motion – the flash lag. For the case where the flash is synchronous with the motion reversal, as it is in our experiments, we believe several flash lag models are relevant.

First, we describe how we model the illusion in the expanding and contracting stimulus. The stimulus produces a doubled effect as the blue flashed square becomes larger and the red flashed square, smaller. For simplicity, we will model only a single moving edge of the inner square and the superimposed red flash. We can calculate the size of the shift for that one edge that contributes to the combined illusion reported in experiment 1 and 2.

Figure S1. The expanding and contracting square version of the flash grab illusion.
To do so, we link the shift of the modelled flash to the measured illusion between both flashes. In the illusion, the red square gets smaller and the blue square larger. We assume that the size increase for the blue square is four times the decrease in the red square. That is because the outer background edge where the blue square flashes travels four times farther than the edge of the smaller, inner square where red flashes. In Figure S2, these differences are shown below as a change of +4X for the blue square from the starting value of 4 dva, but -X for the red square, again starting from size 4 dva.

The illusion magnitude, M, is the % increase of Blue relative to Red:

\[ M = 100 \times \left( \frac{B}{R} - 1 \right) = 100 \times \left( \frac{4 + 4X}{4 - X} - 1 \right) \]

\[ M = 500 \times \frac{X}{4 - X} \]

So the shift X needed to produce the illusion magnitude M is given by

\[ X = 4 \times \frac{M}{500 + M} \]

Differential latency.

The differential latency model deals with the neural delays for registering two signals – the flash and the position of the moving stimulus (e.g., Metzger, 1932; Whitney & Murakami, 1998;
Whitney, Murakami, & Cavanagh, 2000). The delay for the motion is shorter because it is a predictable signal that is already being processed. The difference in latencies creates a forward shift in the perceived location of the moving stimulus compared to the flash. Other than being delayed, this version here assumes no other changes in the neural representation of the position of the moving stimulus (some differential latency models apply an integration process to the motion path, e.g., Öğmen et al., 2004, which we will evaluate later). In the case of the flash lag, once the flash is detected, the position of the moving stimulus can be accessed, and the slower latency for the flash means that the position of the moving stimulus has moved farther along on its path. For the flash grab, in contrast, we assume that once the flash is registered, its position is assigned to the current position of the moving stimulus. Because of the longer delay for the flash, it will be assigned to a location farther along the motion path after the reversal, producing an offset (Fig. S3).
S3A). For Experiment 1, a delay difference of 50 ms produces an illusion of 40%, whereas a delay difference of 74 ms produces an illusion of 60% for Experiment 2. This transfer of the flash location to the position of the moving edge only involves events at and after the time of the flash, so it predicts no illusion for motion present only before the flash (flash terminated, Fig. S3B) and equal illusions for the conditions of motion both before and after the flash, and motion only after the flash (Fig. S3C). It therefore also predicts no change of illusion (Fig. S3D) for additional motion before the flash (Experiment 2).

Overall, the model accounts for some aspects of the data but is unable to account for the contribution of motion before the flash seen in Experiment 2. It also shifts the perceived location of the flash away from the reversal point, something that is never reported. There are also additional drawbacks to the differential latency model that have been discussed in the flash-lag literature. For example, even though this model depends on a delayed perception of the flash relative to the moving bar, temporal order judgements do not reveal any relative delay (e.g., Eagleman, 2000). Given these failures, we can exclude differential latency as a plausible model.

**Extrapolation.**

The extrapolation model of Nijhawan (1994) proposes that the perceived location of a moving stimulus is extrapolated ahead to compensate for neural delays. The motion is then perceived at its current location. This extrapolation predicts an overshoot when motion stops or reverses whereas the data show an undershoot (Whitney et al., 2000). Hogendoorn and colleagues (Blom, Liang & Hogendoorn, 2019; van Heusden, Harris, Garrido, & Hogendoorn, 2019) have modified this extrapolation model for the flash grab stimulus by assuming that there is a compensation for this overshoot that actually overcompensates – producing an illusion in the correct direction. However, this extra step involves the unlikely process of extrapolation based on
an already extrapolated path. Moreover, it is not necessary as the extrapolation naturally corrects itself. Specifically, the extrapolation returns to zero as soon as the motion stops and it reverses when the motion reverses. These properties match closely the effects from van Heusden et al. (2019), although here without any smoothing of the extrapolated path. With a small adjustment for a slightly longer latency for the flash, extrapolation generates the same predictions as differential latency and has the same shortcomings (Fig. S4). It shifts the perceived location of the flash away from the reversal point, which is even earlier than the physical reversal due to the early overshoot before the motion stops. It is unable to account for the contribution of motion before the flash (Experiment 2).

**Figure S4. Extrapolation model.** The gray dashed line represents the delayed neural representation for the motion and the dashed green line the extrapolated positions shifted in the direction of the motion to compensate for the delay. During part of the motion trace, the green line lies on top of the physical motion path. Other conventions as in Fig. S3

**Averaging.**
Averaging (Krakelberg & Lappe, 2000; Cavanagh & Anstis, 2013; Whitney et al., 2000; Sinico et al., 2009) computes the average value of position within a time window and assigns that value as the perceived location for the time point of the leading edge of the window. Note that position averaging must operate on a higher order representation of position, not the image itself as the averaging would make the moving background appear as little more than a smear if it were applied to the image. When applied to back-and-forth motion, averaging shortens the apparent extent of the path (Whitney et al., 2000; Sinico, et al., 2009; Cavanagh & Anstis, 2013), shifting the location of the motion reversal and delaying it by half the window width (Fig. S5A). We assign the perceived location of the flash to the shifted location of the reversal. Because the averaging model is based on position, not motion, it answers the first of our three points and generates an offset that is opposite to the direction of the pre-flash motion and yet in the same direction as the post-flash motion.

Overall, the averaging model has several good points. It can assign the flash location to the perceived location of the motion reversal. It also explains why the illusion is in the direction of the motion following the flash but also in the direction of motion before the flash (because it is based on a position average, not motion). However, in Experiment 1, the averaging cannot account for the flash initiated or flash terminated results, both of which require separate assumptions about the delay in visibility (Fig. S5B and C). For the motion onset, averaging gives the veridical location. Öğmen et al. (2004) dealt with this for the similar integration model by delaying the point where the moving stimulus becomes visible (equivalent to the Fröhlich effect). Here we use a delay in visibility of half the window width, the same delay as for the reversal point. This matches the data reasonably well. For motion offset, the flash terminated condition, a delay of the full window width is required to give no illusion.
The width of the temporal window to predict the Both result is 370 ms in Experiment 1 and 450 ms in Experiment 2. This is shorter than the 600 ms averaging period reported by Krekelberg and Lappe (2000). The delay in visibility for the window centered on the flash (W/2 + 50) is 235 ms in the Motion After condition of Experiment 1 and 275 ms in Experiment 2. These values are long compared to published values for the equivalent Fröhlich effect (80 ms, Adamian & Cavanagh, 2017; 110 ms, Krischfield & Kammer, 1999). The averaging also predicts no illusion for motion only before the flash (flash terminated) as it continues to give the veridical location once only the motion reversal location is at the trailing edge of the window. To do so, however, it requires a delay in the perceived flash equal to the window width (370 to 450 ms), twice the delay it gives for the reversal point in the other conditions. The averaging model also predicts additional
illusory position shift for additional motion before the flash (Fig. S5D), but only over a restricted range – for up to 175 ms of additional motion before the flash (W/2-50). Its overall prediction for Experiment 2, even though better than the previous two models, still does not give a good match to the data (Fig. 8B, main text). Also, the result for 0 ms of motion before the flash (equivalent to the Motion Only After condition of Experiment 1) is again dependent on an arbitrary delay of one half the window width.

**Integration.**

The integration model was originally proposed for the flash lag (Öğmen et al, 2004) but is applicable for the flash grab as well given that the flash position does not lag the motion but is instead transferred to the shifted location of the motion reversal. The perceived location is the integral of difference of the input position and the current perceived location (adapted from Öğmen et al., 2004). This again blunts the reversal in the flash grab and produces a delay. In Experiment 1, the integration model, like the averaging model, predicts no effect for motion that starts with the flash, and a delay of 180 ms (Exp. 1) or 205 ms (Exp. 2) in the visibility of the onset is required to give the observed result (equivalent to the Fröhlich effect). Also, we have to assume that when motion is present only before the flash (flash terminated), there is a continuing activity of the motion offset location to drive the integration toward it (as in Öğmen et al, 2004 for flash terminated condition, Fig. 12). This persisting trace of the motion offset is problematic as it must be quite long, 370 ms, whereas Whitney et al. (2000) have demonstrated with masking that persistence does not contribute to the flash lag persistence.

Overall, the flash is seen at the reversal point, and there is some effect of motion before the flash as seen in Experiment 2. However, the Experiment 1 data require 3 parameters for 3 data points and two of these parameters are for delays unrelated to integration process. The effect for
additional motion before the flash is close but not fully linear like the data and also relies on a second, separate delay parameter for the 0 ms condition.

Figure S6. Integration model. The gray dashed line represents the integrated representation of position within the window in panels A and B. The perceived location of the flash is assigned to the maximum of the position trace, the perceived location of the reversal in direction in panels A and D. The flash location is set by delay parameters in panels B and C. Panel D shows the predicted motion traces for the 5 conditions and the flash shift for 250 ms of additional motion before the flash. Other conventions as in Fig. S3.
References

Adamian, N., & Cavanagh, P. (2017). Fröhlich effect and delays of visual attention. *Journal of Vision, 17*(1), 3-3.

Baldo, M. V. C., & Klein, S. A. (1995). Extrapolation or attention shift? *Nature, 378*(6557), 565-566.

Blom, T., Liang, Q., & Hogendoorn, H. (2019). When predictions fail: Correction for extrapolation in the flash-grab effect. *Journal of Vision, 19*(2), 3-3.

Cavanagh, P., & Anstis, S. (2013). The flash grab effect. *Vision Research, 91*, 8-20.

Kirschfeld, K., & Kammer, T. (1999). The Fröhlich effect: A consequence of the interaction of visual focal attention and metacontrast. *Vision Research, 39*(22), 3702-3709.

Krekelberg, B., & Lappe, M. (2000). A model of the perceived relative positions of moving objects based upon a slow averaging process. *Vision Research, 40*(2), 201-215.

Metzger, W. (1932). Versuch einer gemeinsamen theorie der phänomene fröhlichs und hazelhoffs und kritik ihrer verfahren zur messung der empfindungszeit. *Psychologische Forschung, 16*(1), 176-200.

Nijhawan, R. (1994). Motion extrapolation in catching. *Nature, 370*(6487), 256-257.

Öğmen, H., Patel, S. S., Bedell, H. E., & Camuz, K. (2004). Differential latencies and the dynamics of the position computation process for moving targets, assessed with the flash-lag effect. *Vision Research, 44*(18), 2109-2128.

Sinico, C., & Fadda, A. M. (2009). Vesicular carriers for dermal drug delivery. *Expert Opinion on Drug Delivery, 6*(8), 813-825.

van Heusden, E., Harris, A. M., Garrido, M. I., & Hogendoorn, H. (2019). Predictive coding of visual motion in both monocular and binocular human visual processing. *Journal of Vision, 19*(1), 3-3.

Whitney, D., & Murakami, I. (1998). Latency difference, not spatial extrapolation. *Nature Neuroscience, 1*(8), 656-657.

Whitney, D., Murakami, I., & Cavanagh, P. (2000). Illusory spatial offset of a flash relative to a moving stimulus is caused by differential latencies for moving and flashed stimuli. *Vision Research, 40*(2), 137-149.