Study on calculation method of submarine mud temperature

Li Jing

1Tianjin Research Institute for Water Transport Engineering, M.O.T., Tianjin, 300456, China
2Corresponding author’s e-mail: 543462098@qq.com

Abstract. In order to solve the problem of difficulty in obtaining and low accuracy of submarine mud temperature parameters for offshore marine engineering design, based on the latest IAP global water temperature data which released by the Institute of Atmospheric Physics of the Chinese Academy of Sciences, using one-dimensional heat conduction equation and Gumbel extreme value I type distribution method in this paper, a method of calculating the extreme value of mud temperature at different depths below sea bottom is discussed. The results of monthly mud temperature calculation are compared with the historical data at a certain station in the Bohai Sea and show that, the difference in autumn and winter is between 0.16 to 1.04, while in spring and summer is between 0.96 to 2.47. The results of mud temperature extremum calculation for different return periods in 1 to 100 years are compared with the regression statistical analysis results of measured values in an oilfield block in Liaodong Bay and show that, when the normal distribution function is less than 0.2, the grouping difference is between -0.45 to 1.55℃. Using this method, the design parameters of submarine mud temperature can be calculated quickly and conveniently, and can be applied to most of the sea areas in the world.

1. Research status of submarine mud temperature

Submarine mud temperature field is an important environmental parameter for the design, construction and operation of marine engineering. It has certain influence on the insulation, current carrying, fatigue, corrosion and biological attachment of seabed structures. Due to the particularity of the marine environment, it is expensive and difficult to detect the interannual periodic changes of the submarine mud temperature field through hundreds of meters of seawater [1]. As early as the 1990s, offshore oil development departments recognized the importance of mud temperature parameters, and jointly conducted several short-term or long-term joint observations with marine scientific research institutes on ocean water temperature and submarine mud temperature, and obtained some regular knowledge. Liu Wentong and Song Shan [2] based on the analysis of the interannual water temperature and mud temperature data at two observation points in the Bohai Sea, the relationship between the vertical distribution characteristics, seasonal variation of mud temperature and water temperature are preliminarily discussed. Liu Wentong et al. [3] regard the change of mud temperature with depth as a vertical wave, analyze the change characteristics of mud temperature phase with depth, and approximate determine the phase and value of seabed thermostat. Yin Xunfu et al. [4] have taken the temperature of the submarine mud-water interface as the boundary condition, by solving the heat conduction equation and utilizing the historical water temperature data near the ocean station, have calculated the design factors of mud temperature extreme value in the near-shore area for many years. Yang Kun [5] Based on the measured data of water temperature and mud temperature of a project in Liaodong Bay, the mud temperature extreme design parameters of different return periods are
calculated by regression analysis method. Since then, there have been few literatures on the calculation of design parameters of mud temperature, but the results of numerical simulation considering the coupling effects of mud temperature, water temperature and structure temperature have gradually increased.

Specifications for Submarine cable and pipeline route investigation (GB/T 17502 in Chinese) [6] pays attention to design parameters of submarine mud temperature. The specification of 1998 edition requires that important pipeline projects should be tested for sludge temperature. When there is no geothermal probe in China, mud temperature can be measured immediately when sediment is taken to the ship. Or measure the bottom water temperature and mud temperature at the same time, establish the relationship between them, and then collect the bottom water temperature data to calculate the mud temperature. The specification of 2009 edition requires that the mud temperature can be calculated by the relationship between the existing bottom water temperature and the mud temperature, or measured in time after the soil sample is taken on board the ship. The two editions have simpler suggestions on the determination of mud temperature design parameters, and have not described in detail the specific method for establishing the regression relationship between water temperature and mud temperature. This reflects the fact that the conditions for detecting the annual mud temperature in the field of marine engineering are not yet ripe.

In this paper, the theory and application of mud temperature estimation based on heat conduction equation are introduced, and the design parameters of submarine mud temperature in arbitrarily designated sea area can be quickly calculated without measured data by introducing the spatial-temporal distribution database of water temperature field.

2. Calculation method of submarine mud temperature based on heat conduction equation

2.1. One dimensional heat conduction equation

\[
\frac{\partial T}{\partial t} = a \left( \frac{\partial^2 T}{\partial z^2} \right)
\]  

(1)

\(T\): mud temperature; \(a\): temperature coefficient; \(t\): time; \(z\): seabed depth.

The thermal diffusivity \(a\) can be given based on the empirical value, and it can also be calculated based on the measured data.

\[a = (z_1 - z_2)\pi / (\ln T_1 - \ln T_2) \tau \]

(2)

\(T_1\) and \(T_2\): measured mud temperature at depth of \(z_1\) and \(z_2\) respectively, \(\tau\): the change cycle of mud temperature.

The surface of seabed and the bottom of the seawater are the same mud-water interface, and the values of the surface of submarine mud temperature and bottom of the seawater temperature are consistent. It can be judged from the periodic variation of the bottom of the seawater temperature that the surface of submarine mud temperature varies periodically. When the period is large to one year, the initial condition will be meaningless. In this case, the surface of seabed is considered as a plane, the depth of the seawater is considered as infinite, and the annual variation of mud temperature is regarded as a harmonic \(A \cos \omega t\). The problem of solving the variation of mud temperature with depth \(z\) and time \(t\) is simplified to a problem of solving the problem without initial conditions in semi-infinite space.

According to the boundary conditions, the general solution of one-dimensional heat conduction equation can be derived.

\[T(z, t) = T_0 + Ae^{-z\sqrt{\omega/2a}} \cos(\omega t - z\sqrt{\omega/2a})\]

(3)

\(T_0\): the mean value of surface of submarine mud temperature cycle; \(A\): the periodic variation of surface mud temperature; \(\omega\): circular frequency, \(\omega=2\pi/\tau\).

It is noteworthy that with the increase of depth, the mud temperature amplitude decreases and the phase lag occurs. The phase lag value of mud temperature at different depths can be calculated by the following empirical formula.

\[\varphi = \sqrt{\omega/2a} z\]

(4)
\( \phi \): the phase lag value of mud temperature, the unit is radian.

First, we should determine the change period of mud temperature by using the heat conduction equation to solve the temperature value of the bottom mud. Because the engineering needs to refer to the extreme mud temperature, the general maximum occurs in the summer and autumn season, the minimum occurs in the winter and spring season, therefore, choose to take the year as the cycle, the extreme mud temperature can appear in the solution interval. In this case, \( \tau \) is 365d, \( \omega \) is 0.0172d\(^{-1}\).

With the annual cycle, the unknown parameters remain only the annual mean \( T_0 \) and annual variation \( A \) of the surface of submarine mud temperature, which are equivalent to the annual mean and annual variation of the bottom of the seawater temperature. The annual mean and annual variations of the bottom of seawater temperature need to be calculated from the local bottom water temperature observations for at least one year. If the project needs the design factors of the surface of submarine mud temperature extremum with different return periods, it will need the support of multi-year periodic observation data, which is feasible only if there are ocean stations or fixed observation stations near the site area [7].

### 2.2. Calculation of mud temperature based on IAP water temperature field and heat conduction equation

In recent years, with the improvement of Argo buoy observation network, satellite thermal radiation remote sensing and other technical means [8], many international data centres have published their own lattice ocean temperature field data in turn. For example, EN data from the British Meteorological Agency, Ishii data from the Japan Meteorological Agency, WOD from NOAA in the United States, and various grid data based on Argo (IPRC, SCRIPPS).

In 2016, the Institute of Atmospheric Physics, Chinese Academy of Sciences, proposed a new spatial interpolation scheme, IAP water temperature field, which reproduced the climate mean state, interdecadal variation, interannual variability and long-term trend in the historical period from 1940 to 2015 [9]. In this paper, it is attempted to use it as the data base of the time-space distribution of the water temperature field and the heat conduction equation to calculate the design parameters of the surface of submarine mud temperature in the engineering site. In this paper, the actual observed data of mud temperature in the central Bohai Sea are selected as samples to verify the accuracy of the calculation of mud temperature. The coordinates of observation points are 39°42′N and 119°42′E. The observation periods are April to May 1992, August 1991, October to November 1991 and January 1992 respectively, including four seasons of spring, summer, autumn and winter.

Because the grid space scale of "IAP water temperature field" is 1 degree, this paper calculates the water temperature of the observation point by using the values of several lattice points which are close to the observation point, and calculates the predicted water temperature of the observation point by inverse distance weighting method according to longitude and latitude respectively. The thermal diffusivity is based on the measured data of each season. The calculation results show that the comparison between the calculated value and the measured value is shown in Figures 1 and Table 1.

It can be seen that the predicted values of mud temperature in autumn and winter are in good agreement with the measured values. The difference is between 0.16 to 1.04°C. The difference is between 0.96 to 2.47°C in spring and summer. The reason for the slight difference in spring may be that April and May are at the beginning of the annual cycle warming of seawater and mud, and the monthly average variation is larger, and the difference of the "IAP water temperature field" grid data between the two months is larger. The difference in summer may be due to the influence of thermal radiation of surface seawater and the heat conduction of bottom seawater and mud is lagging behind.

### 3. Calculation method of submarine mud temperature at different return periods

The design parameters of submarine mud temperature for marine engineering mainly include the maximum and minimum values of mud temperature at different depths in different return periods. Generally, the extreme values of return periods of 1, 10, 25, 50 and 100 years should be given. IAP published 78 years water temperature data from 1940 to 2017. When the mud temperature extreme
value is calculated based on the IAP water temperature field data and the heat conduction equation, benefit from the long-term, integral and continuity of the data the annual Mud temperature extreme value over 33 years can be calculated first, and then the extreme value of mud temperature at different return periods can be calculated applying Gumbel extreme value I type distribution [10].

Figure 1. Comparison of reckoned values of water and mud temperature with measured values

Table 1. Difference between reckoned values and measured values of water and mud temperature

| Layer                   | Spring | Summer | Autumn | Winter |
|-------------------------|--------|--------|--------|--------|
| Surface layer of seawater | 2.22   | 3.69   | 1.24   | 0.87   |
| Middle layer of seawater | 1.72   | 1.30   | 0.95   | 1.23   |
| Bottom layer of seawater | 1.30   | 0.45   | 0.64   | 1.19   |
| Surface r of seabed      | 1.28   | 0.81   | 0.36   | 1.17   |
| 0.5m below seabed        | 1.78   | 0.96   | 0.16   | 1.04   |
| 1.0m below seabed        | 1.83   | 2.15   | 0.22   | 0.36   |
| 2.0m below seabed        | 1.98   | 2.47   | 0.50   | 0.46   |

Gumbel extreme I type distribution function can be expressed as

\[ F(x) = \exp[-\exp(-\alpha(x - \beta))] \]  

(5)

The method of estimating \( \alpha \) and \( \beta \) in the equation is as follows: First, according to a set of random observation data \( x_1, x_2, ... x_n \), which obeys Gumbel extreme value I type distribution, calculates \( \hat{\mu} \) (Estimated value of mean value) and \( \hat{\sigma} \) (Estimated value of standard deviation)

\[
\begin{align*}
\hat{\mu} &= \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \\
\hat{\sigma} &= S = \left( \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{\frac{1}{2}}
\end{align*}
\]  

(6)
Next, estimate the parameters $\alpha$ and $\beta$ using the following formula

\[
\begin{aligned}
\bar{\alpha} &= \frac{1.2826}{\hat{\sigma}} \\
\beta &= \bar{x} - 0.5772/\hat{\alpha}
\end{aligned}
\] (7)

In this paper, an oil field in Liaodong Bay is selected as the validation area. The oil field has been in operation for many years. During the construction period, the mud temperature data of each layer, such as monthly mean value and annual extreme value, have been obtained by long-term multi-point test. After regression analysis with water temperature, the design parameters of mud temperature extremum have been established in the return periods of one year, ten years, 25 years, 50 years and 100 years.

According to the geographic location of the oilfield, the maximum and minimum values of mud temperature within 3 m below the surface of seabed are calculated year by year by using the data of the IAP water temperature field from 1985 to 2017. Then the mud temperature extremes of different return periods are calculated by using the Gumbel extreme value I distribution (Table 2).

Table 2. Comparison between reckoned values of mud temperature design parameters and original values at different return periods in an oil field block in Liaodong Bay (unit: °C)

| Layers      | Return periods | 100 years | 50 years | 25 years | 10 years | 1 year |
|-------------|----------------|-----------|----------|----------|----------|--------|
|             | Max | Min | Max | Min | Max | Min | Max | Min | Max | Min |
| Seabed surface | 23.69 | 0.04 | 23.28 | 0.47 | 22.86 | 0.92 | 22.29 | 1.51 | 21.98 | 2.70 |
| Original value | 23.20 | 0.42 | 23.16 | 0.50 | 23.12 | 0.57 | 22.99 | 0.69 | 22.91 | 0.77 |
| 0.5m below | 22.41 | 1.37 | 22.01 | 1.78 | 21.63 | 2.19 | 21.10 | 2.75 | 20.77 | 3.91 |
| Calculated value | 21.23 | 2.08 | 21.20 | 2.15 | 21.16 | 2.21 | 21.06 | 2.31 | 20.99 | 2.38 |
| Original value | 19.15 | 3.82 | 19.12 | 3.87 | 19.09 | 3.92 | 19.01 | 4.00 | 18.96 | 4.05 |
| 1.0m below | 20.35 | 3.51 | 19.99 | 3.88 | 19.64 | 4.26 | 19.16 | 4.76 | 18.79 | 5.89 |
| Calculated value | 19.11 | 3.97 | 19.09 | 4.02 | 19.06 | 4.07 | 18.97 | 4.15 | 18.92 | 4.20 |
| Original value | 17.38 | 5.49 | 17.36 | 5.54 | 17.34 | 5.57 | 17.27 | 5.63 | 17.23 | 5.68 |
| 1.5m below | 18.17 | 5.77 | 17.85 | 6.10 | 17.52 | 6.44 | 17.09 | 6.90 | 16.66 | 8.02 |
| Calculated value | 16.71 | 6.14 | 16.70 | 6.18 | 16.68 | 6.21 | 16.62 | 6.27 | 16.58 | 6.30 |

Comparing the design extremum of mud temperature with the original extremum, the results show that the difference is between -1.30 to 2.16 °C, and the comparison results between the 10-year recurrence period and the 25-year recurrence period are in good agreement, between -0.70 to 1.10 °C, -0.62 to 1.49 °C. The normal distribution curve (Figure 2) which is drawn by the comparison result shows that when the frequency is less than 4 and the normal distribution function is less than 0.2, the grouping difference is between -0.45 to 1.55 °C in 60 samples. This proves that the method proposed in this paper can be used to calculate the design parameters of submarine mud temperature in any known sea area. Compared with the traditional method of long-term in-situ test and regression statistics, the results are in good agreement and consistency. It can be used as a quicker and more effective alternative method and can save a lot of money, manpower and time costs.

4. Conclusion

Both the one-dimensional heat conduction equation and the regression statistical analysis method can be used to calculate the design factors of submarine mud temperature. The regression statistical analysis method needs the support of the measured data of water and mud temperature. The one-dimensional heat conduction equation can calculate the mud temperature value under the condition of only water temperature data. After introducing "IAP water temperature field" data, the mud temperature values of any designated sea area in the world can be quickly calculated by using heat conduction equation without restriction of measured data.
The design factors of submarine mud temperature between the regression statistics results of measured values and calculated results by heat conduction equation based on IAP data in different return period show that the grouping difference is between -0.45 to 1.55 ℃ when the normal distribution function is less than 0.2, which indicates that the two methods are in good agreement. The above methods can be used to calculate the design factors of submarine mud temperature.

Due to the demand limitation from the measured data, the above methods have great advantages in cost saving and efficiency improvement, and benefit from the wide coverage of the "IAP water temperature field" data plane coverage, which can be applied to the prediction of the design factors of the submarine mud temperature in most parts of the world and has good popularization in serving "one belt and one road" and offshore deep water projects.

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References
[1] Zhang Honglei, Han Wenli, Jiang Linlin. (2012) Present situation of thermal-insulation technology for foreign and domestic subsea pipeline. Anticorrosion & Insulation Technology, 1:27-31.
[2] Liu Wentong, Song Shan. (1993) Vertical distribution of soil temperature (at sea bed ) and relationship with between soil and water temperatures in the central Bohai Sea. Oceanologia Et Limnologia Sinica, 24(6):584-591.
[3] Liu Wentong, Song Shan. (1995) Soil-temperature at the depth with constant temperature and phase characteristics of soil temperature profile in the central Bohai Sea. Oceanologia Et Limnologia Sinica, 26(5):460-465.
[4] Yin Xunfu, Li Fanhua. (1993) The variation in mud – temperature in the western liaodong bay and the deduction of its extreme. Coastal Engineering, 1: 11-19
[5] Yang Kun. (2009) Regression Analysis of Water and Mud Temperature in Marine Environment Survey. In : Symposium on Marine Surveying and Mapping. Chengdu. pp. 165-178
[6] GB/T 17502-2009, Specification for submarine cable and pileline route investigation [S].
[7] Hou Chawei. (2008) Representativeness analysis of water temperature data of China's oceanographic stations. Ocean University of China, Qingdao.
[8] Li Wen'e, Su Hua, Wang Xiaoqin. (2017) Estimation of global subsurface temperature anomaly
based on multisource satellite observations. Journal of Remote Sensing, 21 (6): 881-891.

[9] Cheng L, Zhu J.(2016) Benefits of CMIP5 Multi-model Ensemble in Reconstructing Historical Ocean Subsurface Temperature Variations. Journal of Climate, 29 (15): 10-15.

[10] Hou Ruike. (1993) Calculating the highest annual water level by using Gumbel extreme value distribution. Marine Science Bulletin, (3):126-129.