Effect of Phase Shift on Mixed Convection in a Rectangular Vented Cavity filled with a Nanofluid and Submitted to Periodic Heating

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Abstract. The characteristic of flow and heat transfer of Al\textsubscript{2}O\textsubscript{3}-water nanofluids flowing through a horizontal ventilated cavity is investigated numerically. The bottom wall is subjected to a sinusoidal hot temperature profile, whereas the other boundaries are assumed to be thermally insulated. In fact, the flow comes forcefully into the system from the bottom of the left vertical wall and leaves it from the top of the right one by suction. The simulations focus on the effects of different key parameters, such as Reynolds number ($200 \leq Re \leq 5000$), nanoparticle’s concentration ($0 \leq \phi \leq 0.05$) and phase deviation of the sinusoidal heating ($\gamma$ equal 0 or $\pi$), on the flow and thermal patterns and heat transfer performances. The obtained results show that the presence of nanoparticles increases the heat transfer and the mean temperature within the cavity. In addition, the phase shift of the heating temperature may lead to periodic solutions for weaker values of Re and contributes to an increase or a decrease of heat transfer depending on the value of $\phi$ and the convection regime.

1. Introduction

Combined free and forced convection heat transfer in a vented cavity is present in many practical engineering applications, such as solar energy systems, design of the heat exchangers, and cooling of electronic equipment. These systems have been widely used throughout the world. Thus few percent improvement on their performance could have significant impact on the energy saving. On the other hand, low thermal conductivity of conventional fluids such as water, oil, and ethylene glycol mixture that have been usually used in these systems is a serious limitation in improving the performance and compactness of these devices. So it would be very useful to improve the thermal characteristics of fluids applied in such devices. An innovative way of increasing the thermal conductivity of fluid is to disperse small solid particles into the base fluid; the mixture is called “nanofluid” [1].

Since higher heat transfer rates are required in modern industrial applications, mixed convection mechanism is often recommended in such situations, where natural or forced convection is not able to provide cooling effectiveness. Over the past recently years, mixed convection problem in nanofluids with varying heating temperature profile is mostly associated to lid-driven cavity. In that context, it conveys to mention the numerical study of Arani et al [2] in the case of a lid-driven square cavity filled with Cu-water nanofluid and exposed to sinusoidal heating on sidewalls. The obtained results
indicate that for a constant Grashof number/(Reynolds number) the rate of heat transfer increases/(decreases) while decreasing the Richardson number. On the contrary, the volume fraction of nanoparticles affects positively such a quantity. The same problem was analyzed by Kefayati [3] for non-Newtonian aluminum-water nanofluids by using finite difference Lattice-Boltzmann method. It was observed that an augmentation of the power-law index or the Richardson number influences negatively the heat transfer enhancement by the nanoparticles. Other authors investigated mixed convection of nanofluids in a lid-driven shallow enclosure heated from one side. In this context, Goodarzi et al. [4] and Karimipour et al. [5] studied numerically mixed convection of Cu-water nanofluids, with cold and hot moving wall respectively. Their results showed that the average Nusselt number is augmented by increasing the volume fraction of nanoparticles.

Nanofluid mixed convection heat transfer occurs in other ways (case of ventilated cavities in semi-confined media). Over the last decades, few investigations on mixed convection of nanofluids in ventilated cavities with uniform heating (by imposing either temperature or heat flux on the heat source) have been realized. Such a problem is of great importance in various technological applications. Hence, in the case of applied uniform heat flux, Shahi et al. [6] conducted a numerical study of mixed convection in a partially heated square ventilated cavity, filled with copper-water nanofluid. The reported results show an increase of heat transfer rate at the heat source surface and a decrease in the mean bulk temperature with the solid concentration. The same problem was reconsidered by Mahmoudi et al. [7] for different locations of inlet and outlet ports in the case of total constant heating. It was found that the presence of nanoparticles is more effective in Bottom-Top configuration than in the other considered configurations, while the solid concentration increase has the least effects in Top-Top arrangement. In the case of a uniform imposed temperature, the problem of unsteady mixed convection flows through an Al$_2$O$_3$-water nanofluid in a square cavity with inlet and outlet ports due to incoming flow oscillation was performed numerically by Sourtiji et al. [8] for the Top-Bottom configuration. The results presented show an enhancement of heat transfer accompanying the addition of nanoparticles for all the Strouhal and Richardson numbers considered.

In electronic boards, sometimes there is an electronic component disposed along a plate and which dissipates heat while the temperature of its surface remains either uniform or varies periodically. The modeling of such a heating mode could be done by assimilating this component by a heating plate having a sinusoidally variable temperature. In this context, such problem was treated by the authors. Thus, the aim and scope of the current study is to simulate numerically mixed convection heat transfer characteristics of water-based Al$_2$O$_3$ nanofluid flowing through a rectangular cavity, with inlet and outlet ports. The buoyancy driven flow is due to imposed sinusoidal hot temperature profile on the lower horizontal wall of the cavity and the external incoming cold flow. The consequence of varying Reynolds number, nanoparticle concentration and phase deviation on the dynamical and thermal characteristics of the flow and heat transfer performances are investigated and discussed.

2. Problem definition and mathematical formulation

Figure 1 shows a schematic of the case considered in the present study. The problem deals with a two-dimensional vented enclosure of length $L'$, height $H'$ and having an aspect ratio $A = 2$, and filled with water-Al$_2$O$_3$ nanofluid. The enclosure is heated from the bottom wall by a sinusoidal hot temperature (Figure 2) whereas the other solid boundaries are assumed to be thermally insulated. The cavity is subjected to a sucked imposed external flow entering the cavity from the left-bottom opening and leaving from the right-upper one. These openings have a constant relative dimension, $B = 1/4$. Thus, the size of solid particles are assumed to be uniform. Also, the thermo-physical properties of the nanofluid are constant except for the density variation, which is approximated by the Boussinesq model.

Therefore, using the following dimensionless variables:

\[ A = L'/H', \quad B = h'/H', \quad x = x'/H', \quad y = y'/H', \quad u = u'/u'_o, \quad v = v'/u'_o, \quad t = t'u'_o/H', \quad \Psi = \Psi/h'_oH', \quad T = (T' - T'_c)/(T'_h - T'_c), \quad \Omega = \Omega' H'/u'_o, \quad Pr = \nu/\nu_r, \quad Ra = g \beta(T'_h - T'_c)H'â/\alpha_\nu, \quad Re = u'_oH'/\nu_r, \quad Ri = Ra/Re Pr \]
The mixed convection governing equations using the vorticity-stream function formulation \((\Omega, \Psi)\), can be written in the non-dimensional form as follows:

\[
\frac{\partial \Omega}{\partial t} + u \frac{\partial \Omega}{\partial x} + v \frac{\partial \Omega}{\partial y} = \frac{Ra}{Re^2 Pr} \left( \frac{\phi}{1-\phi} \frac{\rho_f}{\rho_i} + \phi \right) \frac{\beta_i}{\beta_f} + \frac{1}{(1-\phi) \rho_f + \phi \rho_i} \left( \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} \right) \tag{1}
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{Re Pr} \left( \frac{\beta_i}{\lambda_i} + \frac{\beta_f}{\lambda_f} \right) \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{2}
\]

\[
\frac{\partial \Omega}{\partial x} + \frac{\partial \Omega}{\partial y} = -\Omega \tag{3}
\]

The dimensionless tangential and normal velocities are converted to:

\[
u = \frac{\partial \Psi}{\partial y} : \nu = -\frac{\partial \Psi}{\partial x} \quad \text{and} \quad \Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{4}
\]

Nanofluid effective density, heat capacity, thermal expansion coefficient, effective dynamic viscosity, effective thermal conductivity and thermal diffusivity are, respectively, calculated as follow:

\[
\rho_{nf} = \rho_f \phi + (1-\phi) \rho_i \tag{5}
\]

\[
(pc_{nf})_{nf} = \phi (pc_{f})_{f} + (1-\phi)(pc_{i})_{i} \tag{6}
\]

\[
(\rho \beta)_{nf} = \phi \rho_f \beta_f + (1-\phi) \rho_i \beta_i \tag{7}
\]

\[
\mu_{nf} = \frac{\mu_f}{(1-\phi)^{m-1}} \tag{8}
\]

\[
\lambda_{nf} = \frac{\lambda_i + 2 \lambda_f - 2 \phi (\lambda_i - \lambda_f)}{\lambda_i + 2 \lambda_f + \phi (\lambda_i - \lambda_f)} \tag{9}
\]

\[
\alpha_{nf} = \frac{\lambda_{nf}}{(pc_{nf})_{nf}} \tag{10}
\]

Where “f”, “s” and “nf” indicate fluid, solid particles and nanofluid respectively.

### 2.1. Boundary Conditions

The boundary conditions applied to the problem are written as:

\[
u = \nu = 0 \quad \text{on the rigid walls}
\]

\[
\frac{\partial T}{\partial n} = 0 \quad \text{on the adiabatic walls (“n” indicates the normal direction to the considered adiabatic wall)}
\]

\[
\Psi = 0 \quad \text{on the walls below the two openings}
\]

\[
\Psi = B \quad \text{on the walls above the two openings}
\]

\[
T = 0 \quad \text{at the inlet port}
\]

\[
u = 1, \nu = 0, \Psi = y - (1 - B), \Omega = 0 \quad \text{at the outlet port}
\]

- Many extrapolation techniques are known for the last years in order to obtain the unknowns boundary conditions at the exit. Some technics are based on the incompressibility property of the fluid and others on the condition of an established flow. However, for a wide range of physical problems, the outlet flow has a complex flow structure. This led us to adopt an unrestrictive technique which consists of zero seconds derivatives at the outlet for the temperature. In the other hand, \(u, v, \Psi\) and \(\Omega\) are unknown at the inlet. These quantities are also extrapolated at each time step by assuming nil their second derivatives at this port.

![Figure 1. Geometry and coordinates system](image1)

![Figure 2. Sinusoidal temperature profile on the heated wall](image2)
2.2. Heat transfer

The estimation of the heat transfer enhancement is based on the average Nusselt number, Nu, calculated on the heated bottom wall of the cavity such as:

\[
\text{Nu} = -\frac{1}{A} \left( \frac{\lambda_{nf}}{\lambda_f} \right) \left. \frac{\partial T}{\partial y} \right|_{y=0} dx
\]

3. Numerical approach

Governing equations for continuity, momentum and energy equations associated with the boundary conditions in this investigation were calculated numerically based on the finite difference method using FORTRAN computer code. The first and second derivatives of the diffusive terms were approximated by a second-order central difference scheme. Furthermore, the advection terms were handled using a second-order upwind differencing scheme to avoid possible instabilities frequently encountered in mixed convection problems. Then, an alternating direction implicit procedure (ADI) was used to perform the time integration for Eqs. (1) and (2). At each time step, the Poisson equation, Eq. (3), was treated by using the point successive over-relaxation method (PSOR) with an optimum over-relaxation coefficient equal to 1.95 for the grid (201 × 101) retained in this study.

4. Results and discussion

The Rayleigh number was kept at a constant value (Ra = 10^6) while the Reynolds number, Re, ranging between 200 and 5000. Values of these parameters involve values of the Richardson number, Ri, varying in the range [6.45×10^{-3}, 4.03] (with Pr = 6.2 for water). In the following, effects of Re, φ and γ on flow and thermal fields and heat transfer effectiveness are examined.

Comparisons between results of pure water (φ = 0) and nanofluid (φ = 0.05), expressed in terms of streamlines and isotherms, are displayed in Figs. 3a and 3b for moderate value of Re (Re = 700) and two values of γ. As shown in Fig. 3a, for heating with zero phase shift (γ = 0), the open lines of the forced flow are quasi-parallel to the heated wall in the entry area allowing more thermal interaction between the fluid and this wall and forcing the lower closed cell to be cloistered in a restricted area in contact with the right vertical wall. At the same time, the corresponding isotherms are tightened at the level of the heated horizontal wall, from the inlet to its center, indicating a good convective heat transfer, characterized by a limited increase of the thermal boundary layer thickness from the inlet toward the center of the lower wall. In another side, three-quarters of the cavity space is at the ambiance temperature. Note also that the addition of nanoparticles has a limited qualitative effect on the streamlines but their effect is more visible on the temperature distribution in the heated area. For γ = π (Fig. 3b), the change of phase shift leads qualitatively to similar flow structure. The corresponding isotherms indicate a progressive growth/decay of the zone with high temperature gradients in the right/left side of the heated wall from right to left. Since the growth in the right side occurs at a faster rate than the decay in the left side, we assist to a heat rate uniformly distributed over the whole lower wall. This behavior results from the fact that a predominant forced convection in the left side is balanced by a predominant natural convection in the right part of the heated wall.

For Re = 5000, streamlines and isotherms, obtained for two values of γ, are presented in Figs. 4a and 4b, for both pure water and nanofluid. According to Fig. 4a, obtained for γ = 0, the dynamical structure shows that highest values of Re favor the closed natural convection cell, located in the right bottom corner, due to the rising assisting effect of natural and forced convections. Also, forced convection amplifies the closed cell located in the left top corner by reducing the main one surmounting the open lines. Remark that, even for large values of Re, the open lines of forced convection remain curved near the heated bottom wall. The isotherms indicate generally a slight increase of the thermal boundary layer, as a disadvantage of the cold zone, which testifies of the increasing effect of natural convection occasioned by the greater lower cell. On the other hand, while referring to Fig. 4b, for γ = π, keeps unchanged the flow structure but modifies the thermal one. It is noteworthy that, the dynamical field is almost insensitive to any change of φ or γ, for the high values of Re, as depicted in Fig.4.
In order to illustrate the performance of the imposed flow mode on the process of heat removal, the variations of Nu, with Re, along the heated wall are presented in Fig. 5, for both heating temperature profiles (the case of periodic heating with zero phase shift \( \gamma = 0 \) and \( \gamma = \pi \) (temperature profile opposed to that corresponding to \( \gamma = 0 \)) and two values of \( \phi \). Overall, Nu increases monotonously with Re, for both cases of heating temperature profile, because of the flow intensification by the forced inertial effects. For a fixed value of Re, an augmentation of \( \phi \) up to 0.05 affects positively the effective thermal conductivity of the nanofluid, which leads to a manifest increasing effect of convection, either for both heating modes. It should be noted that, the case of periodic heating without phase shift (\( \gamma = 0 \)) enhances more the heat transfer in comparison with the other case of heating, either in case of a cavity filled with a base fluid (pure water) or with a nanofluid. These results allow concluding that a better heat removal from the heating wall is ensured by applying the first (\( \gamma = 0 \)) case of heating mode.

For practical purposes, the evaluation of the nanofluid mean temperature, \( T_0 \), inside the cavity is of a great interest. Hence, the variations of this quantity with Re are plotted in Fig. 6, for two values of \( \phi \) and both heating temperature profiles. In the case of periodic heating with zero phase shift, the evolution of \( T_0 \), with Re, is characterized by a weak decrease as long as Re is lower than a value around 500. Above this threshold value, the rate of decrease becomes more obvious. For the case of periodic heating with phase shift equal to \( \pi \), the evolutions of \( T_0 \) are characterized first by an increase with Re up to a critical value situated between 500 and 600 (depending on \( \phi \)). This trend is attributed to the assisting flows of natural and forced convections due to the competition between them. Above this threshold the evolution is reversed and is characterized by a clear decrease of \( T_0 \) with Re as a consequence of the forced flow predominance which favors the heat evacuation through the outlet. For a given value of Re, it can be observed that the addition of nanoparticles enhances notably \( T_0 \) for both heating temperature profiles. Moreover, it is retained from this figure that the case of periodic heating with zero phase shift (\( \gamma = 0 \)) involves globally to a better cooling inside the cavity since the related values of \( T_0 \) are weaker in comparison with the other case of heating (for \( \gamma = \pi \)), if we except the case corresponding to weaker values of Re (\( Re \leq 400 \)) for which the case of periodic heating with phase shift (\( \gamma = \pi \)) gives the most cooling efficiency. The solutions obtained are unsteady periodic in the case of periodic heating with phase shift (\( \gamma = \pi \)), (full circles in Figs. 5 and 6) by increasing Re from its lower value up to a certain limit that depends on the volume fraction of nanoparticles.

Variation of Nu divided by \( \lambda_{nf}/\lambda_f \) with Re, presented in Fig. 7, is qualitatively similar to the evolution of Nu with Re (shown in Fig. 5). Quantitatively, a weak difference is observed when the nanoparticles are added to the base liquid (\( \phi \neq 0 \)). Precisely, for a fixed value of Re, \( Nu(\lambda_{nf}/\lambda_f) < Nu \).

Finally, it is to underline that a comparison between the case of a sinusoidal temperature heating and a uniform temperature heating was performed (Fig. 8). The findings showed that the heating by sinusoidal temperature profile is more favorable to the heat transfer.
5. Conclusion

In this paper, a numerical study is carried out in order to investigate the effect of Al$_2$O$_3$-water nanofluid on mixed convection heat transfer in a vented rectangular cavity provided with two openings and submitted to a sinusoidal heating from below. The study focused on the suction mode of imposed external flows and two types of heating (periodic heating with or without phase shift). The obtained results indicate that Re affect significantly the thermal characteristics and the flow filed, which are found more sensitive to any change in $\phi$ and $\gamma$ only for small values of Re. It is also seen that the presence of the nanoparticles in the base fluid acts to increase heat transfer rate and mean temperature inside the enclosure. On the other hand, it is to remember that the case of periodic heating without phase shift gives a better thermal efficiency than the case with phase shift ($\gamma = \pi$) by leading to more heat transfer across the cavity even with or without presence of nanoparticles. In addition, it is found that a better cooling of the cavity is generally reached by applying the case of periodic heating without phase shift (for moderate and high values of Re) and the case of periodic heating with phase shift (for low values of Re) since these cases lead to lower values of the mean temperature.

References

[1] Jana S, Khojin A S and Zhong W H 2007 Thermochim. Acta 462 45-55.
[2] Arani A A A, Sebdati S M, Mahmoody M, Ardehshir A and Aliakbari M 2012 Superlat and Microstr. 51 893-911.
[3] Kefayati G H R 2014 Powder Technology 266 268-281.
[4] Goodarzi M, Safaei M R, Vafaik K, Ahmadi G, Dahari M, Kazi S N and Jomhari N 2014 Inter Jour of Ther Scien 75 204-220.
[5] Karimpour A, Esfe M H, Safaei M R, Semiromi D T, Jafari S and Kazi S N 2014 Phys A: Sta Mec and its App 402 150-168.
[6] Shahi M, Mahmoudi A H, and Talebi F 2010 Int Com in Heat and Mass Transfer 31 201-213.
[7] Mahmoudi A H, Shahi M, and Talebi F 2010 Int Com in Heat and Mass Transfer 37 1158-1173.
[8] Sourihi E, Hosseinizadeh S F, Gorji-Bandpy M and Ganji D D 2011 Int Com in Heat and Mass Transf 38 1125-1134.