Topological Analysis of Fibrations in Multidimensional (C, R) Space

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Abstract: A holomorphically fibred space generates locally trivial bundles with positive dimensional fibers. This paper proposes two varieties of fibrations (compact and non-compact) in the non-uniformly scalable quasinormed topological (C, R) space admitting cylindrically symmetric continuous functions. The projective base space is dense, containing a complex plane, and the corresponding surjective fiber projection on the base space can be fixed at any point on real subspace. The contact category fibers support multiple oriented singularities of piecewise continuous functions within the topological space. A composite algebraic operation comprised of continuous linear translation and arithmetic addition generates an associative magma in the non-compact fiber space. The finite translation is continuous on complex planar subspace under non-compact projection. Interestingly, the associative magma resists transforming into a monoid due to the non-commutativity of composite algebraic operation. However, an additive group algebraic structure can be admitted in the fiber space if the fibration is non-compact variety. Moreover, the projection on base space supports additive group structure, if and only if the planar base space passes through the real origin of the topological (C, R) space. The topological analysis shows that outward deformation retraction is not admissible within the dense topological fiber space. The comparative analysis of the proposed fiber space with respect to Minkowski space and Seifert fiber space illustrates that the group algebraic structures in each fiber spaces are of different varieties. The proposed topological fiber bundles are rigid, preserving sigma-sections as compared to the fiber bundles on manifolds.

Keywords: topological spaces; fiber bundles; group; projection; norm

MSC: 54F65; 55R15; 55R65

1. Introduction

The Minkowski space is a four-dimensional topological vector space over reals (i.e., four-manifold admitting Poincare symmetry group of isometries) with applications in physical and mathematical sciences [1,2]. In general, the Minkowski space is not well behaved if the corresponding Euclidean topological space is considered to be a locally homogeneous space [3]. The reason is that the Minkowski topological space gets decomposed into two locally homogeneous Euclidean subspaces, where the two topological subspaces are separated in nature. Note that the finest topology in \( n \)-dimensional Minkowski space is Zeeman topology, which is separable, Hausdorff, locally non-compact, and also non-Lindeloff in nature. Moreover, the Zeeman topology generates a first countable topological space. On the other hand, the \( n > 1 \) dimensional Minkowski space equipped with t-topology (\( M_t \)) is not completely Euclidean in nature [4]. The topological space on \( M_t \) is first countable, where the compactification and continuity of a function can be maintained through the
Zeno sequences. In view of general topology, the 4D Minkowski space equipped with s-topology is not a normal topological space, and it is a non-compact Hausdorff space. The axial rotations of a Minkowski space generate various geometric hypersurfaces in space. For example, the three types of helicoidal hypersurfaces are generated by axial rotation of 4-dimensional Minkowski space [5].

In this paper, the construction of topological fiber space in a non-uniformly scaled quasinormed space and the corresponding topological, as well as algebraic analysis of fibration varieties are presented. First, the brief descriptions about the related concepts, such as topological fiber spaces and immersion of manifolds are presented in Sections 1.1 and 1.2, respectively. Next, the motivation for this work and contributions made in this paper are explained in Section 1.3. In this paper, the symbols $R$, $C$ and $Z$ represent sets of extended real numbers, complex numbers, and integers, respectively.

1.1. Topological Fiber Spaces

The topological fiber bundles over a sphere exhibit a set of interesting topological properties if the respective fiber space is Euclidean. It is shown that if $M$ is a closed and compact manifold maintaining Hausdorff topological property, then the function space $X$ containing functions $m : M \times R \rightarrow R$ without interchanging the ends in $R$ is a contractible space [6]. In view of algebraic topology, the structure $G_n$ represents a monoid of homotopy self-equivalences of $(n - 1)$–spheres denoted by $S^{n-1}$ [6]. Note that in this case, the topological space is in the compact-open category.

However, from the viewpoint of differential geometry, the structure of a fiber space can be considered to be rigid under specific conditions, such as orientations [7,8]. Specifically, a Seifert fiber space is profinitely rigid if it is an oriented variety [8]. Otherwise, the Seifert fiber space is based on three-manifold, which is classified depending upon a set of invariants, and it incorporates rigidity of infinite $\pi$, with a hyperbolic two-orbifold base of fibration [9]. Similarly, the Haken orientable hyperbolic three-manifold is an irreducible variety, and it supports the rigidity of fibration [10].

1.2. Manifolds and Immersions

In view of geometric topology, the immersion of surfaces into the null submanifolds in 3D offers various interesting observations. If the structure $(M, g)$ is a Lorentzian manifold under tensor $h$ with $i : N \rightarrow M$ immersion function, then $(N, \tilde{h})$ is a degenerative null submanifold [11]. Interestingly, the Lorentzian manifolds are not metrizable, preserving isometry in reference to the space of immersion. It is known that the immersion of a hypersurface $S$ into a Euclidean space with normal vector field $V_N$ is self-adjoint in the presence of a suitable shape operator. In general, a Lorentz space can be finitely covered by a circle bundle if it is a compact space [12]. Note that topologically the spaces with Lorentzian geometry (such as a torus bundle) are locally Hausdorff, and the corresponding manifold is not a globally Hausdorff topological space. The immersion of a $k$–dimensional manifold in a $k + l$–dimensional Euclidean space is given by $f_i : M_k \rightarrow E^{k+i}$ induces the one-to-one map in tangent space on the manifold [13]. The immersion space is considered to be a regular topological space, and the manifold is a connected topological space with orientation.

1.3. Motivation and Contributions

The fiber bundles are geometric as well as topological objects, which can be simulated and visualized in computer models. The computer visualization of fiber bundles as geometric objects has opened up a wide array of applications in various domains of physical sciences as well as computational sciences [14]. In the topological spaces of fiber bundles, the determination of equivalence between fiber bundles is a challenging task. The fiber bundle space reduction theorem indicates that two topological fiber bundle spaces are equivalent if and only if the corresponding Ehresmann bundles have cross-sections over common base space [15]. Interestingly, if we consider
subspace $X_0 \subset X$ in the corresponding topological base space, then the cross-sections of an automorphic bundle within the subspace form an algebraic group structure. This paper proposes the construction and analysis of fiber space in the non-uniformly scalable multidimensional topological $(C, R)$ space [16]. One of the interesting aspects of multidimensional topological $(C, R)$ space is that the space is quasi-normed, admitting cylindrically symmetric continuous functions, and does not always preserve compactness under topological projections. Hence, the interesting and motivating questions are: what are the possible varieties of fibrations in such space, and is it possible to establish any algebraic structures within the respective fiber space? Moreover, what are the topological properties of the resulting fiber space within the quasi-normed multidimensional $(C, R)$ space if the holomorphic condition of complex subspace is relaxed? These questions are addressed in this paper. The presented analysis considers algebraic as well as topological standpoints as required. The elements of functional analysis are employed whenever necessary.

The main contributions made in this paper can be summarized as follows. The topological $(C, R)$ space is a non-uniformly scalable and quasi-normed space, where the cylindrical open sets form the topological basis. The proposed fibrations within the space can be constructed in two varieties, such as compact fibration and non-compact fibration. The fiber space is considered to be dense, and it can admit the concept of a special category of fibers called contact fibers. The fiber space is equipped with finite linear translation operation. The resulting fiber space in the topological $(C, R)$ space supports the expansion and orientations of multiple singularities of a piecewise continuous function on contact fibers. It is shown that a composite algebraic operation comprised of linear translation, and arithmetic addition prepares an associative magma in the non-compact fiber space. The associative magma space is commutative under linear translation within the magma space of fibers, and it resists the formation of a monoid under composite algebraic operation. However, an additive group algebraic structure can be admitted in the fiber space and in projective base space under specific conditions. Interestingly, the proposed fiber space does not support outward deformation retraction in a dense subspace.

The rest of the paper is organized as follows. Section 2 presents a set of preliminary concepts enhancing the completeness of the paper. The definitions related to topological fiber spaces and fibrations are presented in Section 3. The algebraic and topological properties are presented in Section 4. Section 5 illustrates the concepts of expansion and singularities in the proposed topological fiber space. A detailed comparative analysis of this work with respect to other contemporary works in the domain is presented in Section 6. Finally, Section 7 concludes the paper.

2. Preliminary Concepts

Let $v = (x_0, x_1, \ldots, x_{n-1})$ and $w = (y_0, y_1, \ldots, y_{n-1})$ be two vectors in a $n$-dimensional vector space represented as $(V_n, +)$ . The Lorentzian inner product between $v, w$ is given as $L(v, w) = -x_0 y_0 + \sum_{i=1}^{n-1} x_i y_i$ . The real vector space in $\mathbb{R}^n$ endowed with symmetric Lorentzian inner product with non-degenerate bi-linear form is called a $n$-dimensional Minkowski space $\mathbb{M}_n$ . In the Minkowski space, the linear operator $T : \mathbb{M}_n \rightarrow \mathbb{M}_n$ preserves $L(., .)$ because it maintains the condition represented as $\forall v \in \mathbb{M}_n, L(v, v) = L(T(v), T(v))$ generating a Lorentz group structure.

The Euclidean $e$-topology in $\mathbb{M}_n$ is formulated by the topological basis elements given as a set of neighborhoods $B = \{N(x, \epsilon) : x \in \mathbb{M}_n, \epsilon > 0\}$ . Note that $s$-topology on $\mathbb{M}_n$ is strictly finer than the $e$-topology. Moreover, in the Minkowski space $\mathbb{M}_n$ , the geometry of the space cone in the Euclidean topology maintains $\sigma = e = open$ criteria [3] .

The Steenrod formulation of fiber bundles is represented as an algebraic structure $F = \{B, X, p, Y, G, \phi\}$ , where $B$ is the bundle space, $X$ is a topological base space,
$p : B \rightarrow X$ is a projection function, $Y$ is a topological space called fiber, and $G$ is a bundle group [15]. In the formulation, the elements $V_j, \varphi_j$ are coordinate neighborhoods and the corresponding coordinate functions, respectively. The coordinate functions must satisfy a condition which is given by $\varphi_j : V_j \times Y \rightarrow p^{-1}(V_j)$. Note that a four-dimensional Minkowski space is essentially a real vector space preserving Hausdorff topological property [17].

3. Topological Fiber Space and Fibrations

In this section, a set of definitions related to fibrations and the resulting generation of fiber spaces in a quasinormed topological $(C, R)$ space is formulated. A point $x_p$ in the quasinormed topological $(C, R)$ space $(X, \tau_X)$ is represented as $x_p = (z_p, r_p)$, and the origin of $(X, \tau_X)$ is denoted as $z_0 = (z_0, 0)$, where $z_0$ is the Gauss origin. The corresponding topological projections are given by $\pi_p : X \rightarrow R$ and $\pi_c : X \rightarrow C$ on the real subspace and complex subspace, respectively. For the simplicity of algebraic representation $\{z_p\} \times I \equiv (z_p, I)$ is used to denote a sectional subspace within the topological fiber space, where $I$ is an interval (either open or closed).

In this paper, $A^0$ and $\overline{A}$ represent the interior and closure of an arbitrary set $A$ such that $\overline{A} = A^0 \cup \partial A$. Moreover, if $A$ is homeomorphic to $B$ then it is denoted as $\text{hom}(A, B)$. First, we define the projective base of a fiber space in the topological $(C, R)$ space.

3.1. Projective Base in $(C, R)$ Tangent Space

Let $A_m \in \tau_X$ be an open cylindrical subspace in the topological $(C, R)$ space $(X, \tau_X)$ such that $A_m = D(z_m, \varepsilon > 0) \times I$, where $(z_m, r) \in C \times \{r\}$ and $I \subseteq R$ is open. Suppose we determine $\{\alpha, \varepsilon\} \subset R$ maintaining the ordering relation given by $0 < \alpha < \varepsilon$.

The projection of corresponding tangent subspace onto topological base space at $r \in R$ is defined as:

$$\forall D(z_n, \alpha) \subset D(z_m, \varepsilon), B_n = \partial D(z_n, \alpha) \times I,$n

$$\forall z_p \in \partial D(z_n, \alpha), \forall r \in R, \pi_p : B_n \rightarrow (\partial D(z_n, \alpha) \times \{r\}), \pi_c((z_p, I)) = (z_p, r).$$

Note that the projection $\pi_p(\cdot)$ is surjective in nature onto the dense base space $D(z_n, \varepsilon) \times \{r\}$, where the $\text{hom}(\partial D(z_n, \alpha), S^1)$ condition is maintained for every $z_n \in C$. Moreover, it preserves the standard notion of projection in the tangent space of a topological manifold $(M, \tau_M)$. Next, the definition of fiber bundles in the tangent space considering a base space is formulated.

3.2. Topological Fiber Bundle in $(C, R)$ Space

As stated earlier, let $A_m = D(z_m, \varepsilon) \times I$ be a subspace in the topological $(C, R)$ space for $\varepsilon > 0$. The topological fiber bundle in the corresponding tangent space $B_m = \partial D(z_m, \varepsilon) \times I$ is given by:

$$F_m = \{(z_p, I) : z_p \in \partial D(z_m, \varepsilon)\}, \forall E_p \subseteq \partial D(z_m, \varepsilon) \times \{r\}, \sigma : E_p \rightarrow F_m, (\pi_p \circ \sigma)(x_p \in E_p) = (z_p, r).$$

where $D(z_m, \varepsilon)$ is the open ball centered at $z_m$ with radius $\varepsilon$. The projection $\pi_p(\cdot)$ maps points in the fiber space $F_m$ to the corresponding base points $z_p$ in the tangent space $B_m$. This formulation allows for the study of fiber bundles in the context of topological spaces, which is fundamental in various areas of mathematics and physics, including differential geometry and gauge theory.
A fiber \((\sigma \circ \pi_\nu)((x, y), I)) = \mu_{p \times I} \equiv (z, I)\) where \((z, I) \in F_m\) is called a compact fiber, if and only if \(I \subset R \setminus \{-\infty, \infty\}\) and \(I = I^0 \cup \partial I\). Otherwise, it is a result of non-compact fibration in the topological \((C, R)\) space.

**Remark 1:** The topological fiber bundle \(F_m\) in \((C, R)\) space is more rigid as compared to the fiber bundle in \((M, \tau_M)\) because \(F_m\) preserves the \(\sigma\) - sections within the fiber bundles in the corresponding tangent \((C, \mathcal{R})\) space, where the topological base space is a dense set. The \(\sigma\) - sections in the topological manifold \((M, \tau_M)\) may not be preserved in every case within the fiber bundle of \((M, \tau_M)\). The reason is that a set of \(\sigma\) - sections of \((M, \tau_M)\) is constructed separately independent of projections onto topological base space.

### 3.3. Fiber Space

The fiber space in a topological \((C, \mathcal{R})\) space \((X, \tau_X)\) is formulated based on the sets of fiber bundles in the dense subspaces. Let us consider \(A_m = D(z_m, \varepsilon) \times I\) be a dense subspace in topological \((C, \mathcal{R})\) space for \(\varepsilon > 0\). The corresponding fiber space \(\Gamma_m\) in \(A_m\) considering \(0 \leq \varepsilon < k < +\infty\) is defined as:

\[
\forall z_a \in D(z_m, \varepsilon), A_a \subset A_m, \\
\Gamma_m = \bigcup_{z_a \in D(z_m, \varepsilon)} F_a = \bigcup_{\forall \varepsilon \in (0, k)} F_m.
\]  

(3)

In the above definition \(F_a\) is a local fiber bundle of corresponding topological subspace \(B_a = \partial D(z_a, \varepsilon) \times I\) in \((X, \tau_X)\). The entire fiber space of topological \((C, \mathcal{R})\) space can be generated from the sets of local fiber bundles as given below:

\[
\Gamma_{CR} = \bigcup_{z_a \in C} \Gamma_m.
\]  

(4)

Note that at \(\varepsilon = 0\) in the topological subspace, \(A_m\) the set \(\mu_{m \times I}\) is also a fiber at \(z_m \in C\). Earlier it is mentioned that there are two varieties of fibrations, which can be admitted into the topological \((C, \mathcal{R})\) space. However, it is interesting to observe that in any case, the fiber space in \((X, \tau_X)\) is not compact, which is presented in the following proposition.

**Proposition 1:** Every fiber space \(\Gamma_m\) in \((X, \tau_X)\) is not compact.

**Proof:** Let \((X, \tau_X)\) be a topological \((C, \mathcal{R})\) space and \(F_a \subset \Gamma_m\) be a topological fiber bundle in the topological subspace \(A_a \subset A_m\). The topological projection \(\pi_R : X \to \mathcal{R}\) prepares a real subspace in \((X, \tau_X)\). We will prove the proposition by considering two separate cases. In the first
case, suppose \( I \subset \mathbb{R} \setminus \{-\infty, +\infty\} \) such that \( I = I_0 \cup \partial I \) preparing a compact fibration in the topological space. However, recall that as \( A_a = D(z_a, \varepsilon > 0) \times I \), thus, the projection \( \pi_c(v \in A_a) \subset C \) is an open subspace. Otherwise, if \( I = \mathbb{R} \) in \( A_a \) then the fibration \( F_a \subset \Gamma_m \) is a non-compact variety. As a result, the projective subspace \( \pi_p(v \in A_a) = R \) is an open set. Hence, in every case \( F_a \subset \Gamma_m \) is not compact and as a result \( \Gamma_m = \bigcup_{I \subset \mathbb{R} \setminus \{z_0, \varepsilon\}} F_a \) is also not compact in \((X, \tau_X)\). □

**Remark 2:** The above proposition further indicates that the fiber space \( \Gamma_{CR} \) in \((X, \tau_X)\) is not a compact space.

In general, the Seifert fiber space on an oriented manifold maintains profinite rigidity [8]. However, in the case of a holomorphic map \( f : M \rightarrow N \) between two complex manifolds \( M \) and \( N \), the rigidity of fibration allows sending a single fiber to another fiber, as along as the complex manifolds, are individually connected spaces [18]. In this paper, the topological \((C, R)\) space is a connected space, and the associated fiber space supports finite translation of fibers within the space.

### 3.4. Translation in Fiber Space

Let \( \Gamma_m \subset \Gamma_{CR} \) be a fiber subspace in the topological \((C, R)\) space \((X, \tau_X)\). The topological translation of a fiber subspace in \( \Gamma_m \) is defined as:

\[
\exists \Gamma_p : \Gamma_m \subset \Gamma_p \subset \Gamma_{CR},
\]

\[
T : \Gamma_{CR} \rightarrow \Gamma_{CR},
\]

\[
T(\mu_{a \times l}) = \mu_{p \times l} = T(z_a, l),
\]

\[
\forall \mu_{a \times l} \in \Gamma_m, \mu_{p \times l} \in \Gamma_p.
\]

**Remark 3:** Note that, the translation \( T : \Gamma_{CR} \rightarrow \Gamma_{CR} \) is continuous in \( C \) under projection \( \pi_c : X \rightarrow C \) within the open subspaces of fibers. However, the translation can be transformed into a strictly closed and convergent variety if the following restrictions are imposed on it: \( \Gamma_p = \Gamma_p^o \cup \partial \Gamma_p \) and \( \Gamma_p \subset \Gamma_{CR} \).

The properties of finite translations within the topological fiber space depend on the Baire categorization of a subspace (i.e., dense or meager) and compactness of the corresponding projective subspace. The following proposition presents such observation.
Proposition 2: If \( \Gamma_p \subset \Gamma_{CR} \) is dense and the topological projection on subspace \( \pi_R \circ \sigma(x_m \in X) \) is not compact, then \( T : (\Gamma_m \subset \Gamma_{CR}) \rightarrow (\Gamma_p \subset \Gamma_{CR}) \) is a continuous and finite fiber space translation.

Proof: Let \( \Gamma_m, \Gamma_p \) be two fiber subspaces in \((X, \tau_X)\) such that the translation is given as \( T : (\Gamma_m \subset \Gamma_{CR}) \rightarrow (\Gamma_p \subset \Gamma_{CR}) \). Note that \( \Gamma_m \subset \Gamma_p \) in the topological space and as a result \( T : \Gamma_{CR} \rightarrow \Gamma_{CR} \) is continuous in all monotone classes of projective subspaces \( \pi_c(\Gamma_m) \subset \pi_c(\Gamma_p) \) in \((X, \tau_X)\). If the topological projection \( \pi_R \circ \sigma(x_m \in X) \) is not compact, then the corresponding fibration is a non-compact variety resulting in the generation of fiber \( \mu_{x,s} \equiv (z_m, I = R) \). Suppose fibration is non-compact everywhere in the topological space \((X, \tau_X)\). Thus \( \forall \mu_{x,s} \in \Gamma_m, T(\mu_{x,s}) = \mu_{x,s} \) is maintained in the subspaces such that \( \mu_{x,s} \in \Gamma_p \) and \( \mu_{x,s} = T(z_a, I) \), where \( T(z_a) \in C \) and \( T(I = R) = I \). If \( \Gamma_p \subset \Gamma_{CR} \) is dense, then \( \Gamma_p \subset \Gamma_{CR} \) is a compact subspace of fibers. As a result, it is true that \( \forall \mu_{x,s} \in \Gamma_m, 0 \leq \| \pi_c(\mu_{x,s}) - z_a \|_2 < +\infty \) considering the inclusion of translation invariant fiber, which is given by \( \exists \mu_{x,s} \in \Gamma_m, T(\mu_{x,s}) = \mu_{x,s} \). Hence, the continuous translation of fiber subspace \( T : \Gamma_{CR} \rightarrow \Gamma_{CR} \) in \((X, \tau_X)\) is finite if \( \Gamma_p \subset \Gamma_{CR} \) is dense and the topological projection \( \pi_R \circ \sigma(x_m \in X) \) on real subspace is not compact. □

The proposed fibration in a topological \((C, R)\) space is equipped with a special category of fiber called contact category, as defined below. A contact category fiber admits multiple oriented singularities of a function within the fibered topological space under specific conditions.

3.5. Contact Category Fiber

Let \( \Gamma_m \subset \Gamma_{CR}, \Gamma_p \subset \Gamma_{CR} \) be two fiber subspaces in the topological \((C, R)\) space \((X, \tau_X)\). A fiber \( \mu_{x,s} \) is defined to be in the contact fiber category if the following conditions are maintained by it:

\[
\Gamma_m \cap \Gamma_p = \{ \mu_{x,s} \}, \\
\forall r \in R, \pi_R \circ \sigma((z_s, r)) = R. \tag{6}
\]

Note that the restriction for a contact fiber to be maintained is given as, \( z_s \in C \setminus \{-\infty, +\infty\} \). This restriction is required to retain the finiteness of projection of fiber space into the topological base.
symmetry of the contact fiber category. Once the contact fiber is defined, we can formulate the oriented singularities of a function in the topological space.

### 3.6. Oriented Singularities of Function

Let \( f : [a \in R, b \in R] \to (X, \tau_X) \) be a continuous function in the topological \((C, \mathcal{R})\) space. The function \( f(.) \) is defined to have positively oriented singularity in the space if the following conditions are maintained by it on a fiber:

\[
\begin{align*}
I & \subset R \setminus \{-\infty, +\infty\}, \exists \mu_{f,r} \subset X, \\
\exists c \in (a, b) : \forall x_\mu \in f([a, b] \setminus \{c\}), x_\mu = (z_f, r \in I), \\
\lim_{t \to +\infty} f(t) &= (z_f, r), r \to +\infty.
\end{align*}
\]

**Remark 4:** It is straightforward to verify that the negatively oriented singularity of a continuous function can be defined by following a similar concept, as stated above, with a change in orientation. Moreover, a planar and symmetric variety of the function with multiple oriented singularities can be formulated if the following restrictions are maintained by it:

\[
\begin{align*}
\Gamma_m \cap \Gamma_p &= \{\mu_{z,r}\}, \\
D(z_m, c) \cap D(z_p, c) &= \{z_r\}, \exists r \in R \setminus \{-\infty, +\infty\}, \\
f([a, b]) &\in D(z_m, c) \times \{r\}, f((c, b)) \in D(z_p, c) \times \{r\}, \\
\lim_{t \to +\infty} f(t) &= (z_f, r), r \to +\infty, \\
\lim_{t \to -\infty} f(t) &= (z_f, r), r \to -\infty.
\end{align*}
\]

It is important to note that a planar and symmetric function with multiple oriented singularities in the fiber space of topological \((C, \mathcal{R})\) space is a discontinuous variety (i.e., the function is piecewise continuous). Furthermore, the fiber admitting multiple singularities of a planar as well as symmetric function is in the contact fiber category, and the corresponding fibration is non-compact type.

### 4. Algebraic and Topological Properties

In this section, the algebraic and topological properties of fiber space in the quasinormed topological \((C, \mathcal{R})\) space are presented. First, we show that the fiber space \( \Gamma_{CR} \) is an associative magma under the algebraic composite operation \((+T)\) as given below:

\[
\begin{align*}
\forall \mu_{a+1}, \mu_{b+1} &\in \Gamma_{CR}, \\
\mu_{a+1} (+T) \mu_{b+1} &= \mu_{(a+T)b+1}.
\end{align*}
\]

The above representation indicates that \((+T)\) is a composite algebraic operation, which is comprised of functional translation and arithmetic addition within the topological space. Note that the linearity of translation operation is considered to be maintained for generality. It is interesting to observe that the algebraic structure \((\Gamma_{CR}, (+T))\) constructs an associative magma in the topological space \((X, \tau_X)\) if and only if the translation is invariant to first-order and the fibration is non-compact type.
Theorem 1: The non-compact fiber space \((\Gamma_{CR}, (+T))\) is an associative magma if and only if \(T : \Gamma_{CR} \rightarrow \Gamma_{CR}\) is linear and \(\forall n \geq 2, T^n = T, n \in \mathbb{Z}^+\).

Proof: Let \((\Gamma_{CR}, (+T))\) be a fiber space associated with the algebraic operation in the topological space \((X, \tau_X)\). If \(T : \Gamma_{CR} \rightarrow \Gamma_{CR}\) is linear and the fibration is non-compact type, then a \(\sigma -\)
section preserves the following condition involving corresponding topological base space, \(\forall \mu_{a;1} \in \Gamma_{CR}, (\pi_R \circ \sigma)(x_a \in X) = R\). Moreover, due to the linearity of translation of fibers in the space it can be verified that \(\forall \{k_1, k_2\} \subset R, T(k_1\mu_{a;1} + k_2\mu_{b;1}) = k_1T(\mu_{a;1}) + k_2T(\mu_{b;1})\), where \(\{\mu_{a;1}, \mu_{b;1}\} \subset \Gamma_{CR}\). Recall that \(\forall \mu_{a;1} \in \Gamma_{CR}, T(\mu_{a;1}) = \mu_{T_0a;1}\) condition is maintained in the corresponding fiber space. Thus we can derive the following equations considering the algebraic operation \((+T)\):

(10) \[
\{\mu_{a;1}, \mu_{b;1}, \mu_{c;1}\} \subset \Gamma_{CR}, \\
(\mu_{a;1}(+T)\mu_{b;1})(+T)\mu_{c;1} = \mu_{(a+Tb+Tc);1} \in \Gamma_{CR}, \\
\mu_{a;1}(+T)(\mu_{b;1}(+T)\mu_{c;1}) = \mu_{(a+T(b+Tc));1} \in \Gamma_{CR}.
\]

This indicates that the fiber space is associative if \(T^2 = T\) in \((\Gamma_{CR}, (+T))\). Moreover, if \(T^2 = T\) then \(\forall n \in \mathbb{Z}^+, T^n = T^{n-2}(T^2) = T^{n-1} = \ldots = T\) and it is closed in \((\Gamma_{CR}, (+T))\). Hence, the fiber space \((\Gamma_{CR}, (+T))\) is an associative magma under the non-compact fibration in the topological space. □

Corollary 1: It is important to note that the fiber space \((\Gamma_{CR}, (+T))\) is not a monoid as no fixed fiber (i.e., original fiber) can be identified within the space. This indicates that it is a relatively rigid structure because even if the invariance condition of translation is further relaxed at the origin of \((X, \tau_X)\) so that \(\exists x_0 \in X, T(\mu_{0;1}) = \mu_{0;1} = T(z_0, I)\) then also the algebraic structure \((\Gamma_{CR}, (+T))\) is not transformed into a monoid, where \(z_0\) is Gauss origin. The main reason is that the algebraic operation \((+T)\) is not commutative in nature.

Interestingly, although the fiber space \((\Gamma_{CR}, (+T))\) is an associative magma under non-commutative algebraic operation \((+T)\); however, an additional linear translation of the magma of fiber space is a commutative space. This observation is presented in Theorem 2.
Theorem 2: In \((\Gamma_{CR}, (+T))\) associative magma of fiber space, the translation \(T : (\Gamma_{CR}, (+T)) \to (\Gamma_{CR}, (+T))\) admits commutativity in magma under \(T\).

Proof: Let \(\{\mu_{ax}, \mu_{bx}\} \subset \Gamma_{CR}\) be a subset of fibers in associative magma \((\Gamma_{CR}, (+T))\). According to the principle of commutativity, the following equations can be derived in the magma of fiber space:

\[
\begin{align*}
\mu_{ax}(+T)\mu_{bx} &= \mu_{(a+T_b)x}, \\
\mu_{bx}(+T)\mu_{ax} &= \mu_{(b+T_a)x}.
\end{align*}
\]

Thus, further translation in the associative magma of fiber space leads to the following conclusions considering \(T^2 = T\).

\[
\begin{align*}
T(\mu_{ax}(+T)\mu_{bx}) &= \mu_{(a+T_b)x}, \\
T(\mu_{bx}(+T)\mu_{ax}) &= \mu_{(b+T_a)x}.
\end{align*}
\]

However, the algebraic structure \((C, +)\) is commutative in nature, indicating that \(T(z_b) + T(z_a) = T(z_a) + T(z_b)\). This leads to the following algebraic identity:

\[
T(\mu_{ax}(+T)\mu_{bx}) = T(\mu_{bx}(+T)\mu_{ax}).
\]

Hence, the function \(T : (\Gamma_{CR}, (+T)) \to (\Gamma_{CR}, (+T))\) admits commutativity within the associative magma of fiber space in \((X, \tau_X)\). □

From the algebraic standpoint, the above theorem illustrates that the linear translation in the associative magma of fiber space admits commutativity within the space. However, the space does not support a group structure under such given conditions. On the other hand, if the fibration is a non-compact variety in topological \((C, R)\) space, then the algebraic structure \((\Gamma_{CR}+, +)\) in the respective fiber space attains a group structure as presented in the next theorem.

Theorem 3: The structure \((\Gamma_{CR}+, +)\) is a fiber group in \((X, \tau_X)\) if and only if \(\forall \mu_{ax} \in \Gamma_{CR}, \forall r \in R\) the fibration maintains \((\pi_R \circ \sigma)(x_a \in X) = R\).

Proof: Let \(\Gamma_{CR}\) be a fiber space in the topological \((C, R)\) space \((X, \tau_X)\). Suppose we consider a subspace \(M_p = \{\mu_{ax}, \mu_{bx}, \mu_{cx}\} \subset \Gamma_{CR}\) such that the projection on base space is \(M_v = \{\pi_r (v \in M_p)\}\) and the corresponding topological projection maintains \((\pi_R \circ \sigma)(v \in M_v) = R\). This leads to the following result maintaining commutativity under non-compact fibration.
\[ I = R, z_a + z_b = z_{a+b} \in C, \]
\[ \mu_{a+1} + \mu_{b+1} = \mu_{(a+b)+1} = (z_{a+b}, I), \]
\[ (\mu_{a+1} + \mu_{b+1}) + \mu_{c+1} = \mu_{a+1} + (\mu_{b+1} + \mu_{c+1}). \]  

Moreover, at the origin \( x_0 \) of \((X, \tau_X)\) the fiber \( \mu_{0+1} \) is an identity fiber (i.e., original fiber) because \( \forall \mu_{a+1} \in \Gamma_{CR}, \mu_{a+1} + \mu_{0+1} = \mu_{0+1} + \mu_{a+1} = \mu_{a+1} \). Furthermore, if we consider fibers such that \( \forall \mu_{a+1} \in \Gamma_{CR}, \exists \mu_{a+1} \in \Gamma_{CR} \) maintaining \( \mu_{a+1} + \mu_{a+1} = \mu_{-a+1} + \mu_{a+1} = \mu_{0+1} \) then \( \mu_{-a+1} \) is the inverse of \( \mu_{a+1} \). Hence, the algebraic structure \( (\Gamma_{CR},+) \) is a fiber group if the fibration is a non-compact variety. \( \square \)

**Lemma 1:** In the topological \((C, R)\) space \((X, \tau_X)\), the algebraic structure \( (\pi, (\Gamma_{CR}),+) \) is a group, if and only if \( r = 0 \).

**Proof:** The proof is relatively straightforward. Suppose we select an arbitrary \( r \in R \setminus \{0\} \) to prepare a topological base space for projection. In this case, the \( \{x_0\} \cap \pi_r(\Gamma_{CR}) = \phi \) condition is satisfied within the projective space of fibers. This leads to the conclusion that \( \forall z_a, z_b \in (\pi_c \circ \pi_r)(\Gamma_{CR}), z_a + z_b \neq z_a \neq z_b \) if \( z_a \neq z_b \neq z_0 \) and in addition the following condition is maintained, \( \{(\pi_r \circ \pi_r)(v \in \Gamma_{CR})\} \cap \{\pi_r(x_0)\} = \phi \). Thus, the algebraic structure \( (\pi, (\Gamma_{CR}),+) \) is not a group because \( \forall x_a, x_b \in \pi_r(\Gamma_{CR}), x_a + x_b \neq x_a \neq x_b \) indicating that the additive identity element does not exist in \( (\pi, (\Gamma_{CR}),+) \) if \( r \neq 0 \). Hence, the algebraic structure \( (\pi, (\Gamma_{CR}),+) \) is a group, if and only if \( r = 0 \). \( \square \)

**Remark 5:** If \( \psi : \Gamma_{CR} \rightarrow \Gamma_{CR} \) is a function in fiber space such that \( \psi(\mu_{a+1}) = \mu_{a+\psi(1)} \) with \( (\pi_R \circ \sigma)(x_a \in X) = R \), then it can be observed that \( \psi(\mu_{a+1}) \equiv \mu_{a+\psi(1)} \), if and only if \( \mu_{a+\psi(1)} \equiv \mu_{a+1} \).

Hence, the function \( \psi(.) \) is a \( 2\pi \) - radian rotation of a fiber maintaining symmetry property. Moreover, it preserves the identity in a \( \sigma \) - section as \( (\psi \circ (\sigma \circ \pi_r)) = (\sigma \circ \pi_r) \) due to symmetry.

**Theorem 4:** In the fiber space \( \Gamma_{CR} \), there is no deformation retract
\[ \eta_D : D(z_m, \varepsilon) \times I \rightarrow \partial D(z_m, \varepsilon) \times I, \text{ where } \varepsilon > 0, \varepsilon \in R^+ \].
Proof: Let \( \Gamma_{CR} \) be a fiber space in the topological \((C,R)\) space \((X,\tau_X)\). If we consider \( \varepsilon > 0, \varepsilon \in R^+ \) then \( D(z_m,\varepsilon) \times I \) is an open subspace in \((X,\tau_X)\). Note that in the corresponding subspace \( \text{hom}(\partial D(z_m,\varepsilon), S^1) \) condition is maintained. According to the deformation retraction theorem, there can be no retraction from convex \( D(z_m,\varepsilon) \) to \( \partial D(z_m,\varepsilon) \) maintaining continuity of retraction function within the topological space. As a result, there can be no deformation retract function \( \eta_D : D(z_m,\varepsilon) \times I \to \partial D(z_m,\varepsilon) \times I \) admissible within \((X,\tau_X)\). \( \square \)

Remark 6: Although \( \eta_D : D(z_m,\varepsilon) \times I \to \partial D(z_m,\varepsilon) \times I \) outward deformation retract is not admissible in \((X,\tau_X)\); however, the inward deformation retract given by \( \eta_E : D(z_m,\varepsilon) \times I \to D(z_m,\alpha) \times I \) can be admitted, if and only if \( \alpha < \varepsilon \) and \( \alpha > 0, \alpha \in R^+ \).

5. Expansion and Singularity

The fiber space \( \Gamma_{CR} \) in a quasinormed topological \((C,R)\) space \((X,\tau_X)\) supports the continuous and uniform expansion of a subspace \( \Gamma_m \) under the translation function \( T : \Gamma_{CR} \to \Gamma_{CR} \). First, we define the uniformity of \( T \) in \( \Gamma_{CR} \) as given below, considering the Euler representation of a point \( z_m \in C \):

\[
z_m = \alpha_m e^{i\theta_m}, \quad \forall z_m \in C, \mu_{m \times I} = (\alpha_m e^{i\theta_m}, I).
\] (15)

If we equip the fiber subspace \( \Gamma_m \subset \Gamma_{CR} \) with uniform \( T : \Gamma_{CR} \to \Gamma_{CR} \) then the algebraic structure \((\Gamma_m,T)\) admits continuous and uniform expansion in \((X,\tau_X)\), within the monotone class of fiber space. In other words, the fiber subspace \( \Gamma_m \subset \Gamma_{CR} \) admits continuous and uniform expansion within the fiber space if it maintains the following properties:

\[
\forall \Gamma_m \subset \Gamma_{CR}, \exists \Gamma_{\mu m} \subset \Gamma_{CR}, \Gamma_m \subset \Gamma_{\mu m}, \\
\forall \mu_{m \times I} \in \Gamma_m, n > 1, T^n(\mu_{m \times I}) \in \Gamma_{\mu m}, \\
T^n \neq T.
\] (16)

Note that it indicates \( \Gamma_{\mu m} \) is dense in \( \Gamma_{CR} \) because \( \Gamma_{\mu m} \cup \partial \Gamma_{\mu m} = \Gamma_{\mu m} \) within \((X,\tau_X)\). Moreover, the continuous and uniform expansion of a fiber subspace is finite within a compact subspace. However, this restriction may not be valid in the entire fiber space \( \Gamma_{CR} \) within \((X,\tau_X)\), where \( X \in \tau_X \) is open.

The contact category fiber in \((X,\tau_X)\) admits multiple singularities of a function \( f:[0,1] \to (X,\tau_X) \). In this case, the function is piecewise continuous in \((X,\tau_X)\). Let
$\Gamma_m, \Gamma_p \subseteq \Gamma_{CR}$ be two fiber subspaces such that $\Gamma_m \cap \Gamma_p = \mu_{\mu_1}$ is a fiber in the contact fiber category. Suppose $a \in (0,1]$ such that $\forall x \in [0, a), f(x) \in D(\Gamma_m, \varepsilon) \times I$ and $\forall x \in (a, 1), f(x) \in D(\Gamma_p, \varepsilon) \times I$ for some $\varepsilon > 0$. Consider that fibration is a non-compact variety, and as a result the $(\pi_\varepsilon \circ \sigma)(f(x)) = R$ condition is maintained under projection. If $\lim_{x \to a^+} f(x) \to (z_a, +\infty)$ and $\lim_{x \to a^-} f(x) \to (z_a, -\infty)$ conditions are maintained by the function, then $f : [0, 1] \to (\Gamma_m \cup \Gamma_p)$ admits multiple singularities in the fiber space. Furthermore, the proposed structure does not exclude the possibility of the existence of oriented singularity within the space under the additional requirement. If $\lim_{x \to a^-} f(x) \equiv \lim_{x \to a^+} f(x)$ equivalence relation is maintained by the limiting values of the function, then $\lim_{x \to a^\pm} f(x) \to (z_a, +\infty)$ is a strictly positively oriented singularity of the function on the respective contact fiber in $(X, \tau_X)$.

6. Comparative Analysis

There are varieties in the construction of fiber bundles and associated singularities based on various parameters, such as the connectedness of topological spaces, locality of homeomorphism, and different types of projection maps. The comparison of various properties associated with different varieties of fiber spaces is summarized in Table 1.

In the case of Seifert fiber space, the fiber bundles are $S^1$-bundles of a three-manifold $M_3$ [7,8]. The Seifert fiber space admits local compactness along with the fundamental group $\pi_1(M_3)$. The fundamental group $\pi_1(M_3)$ includes a unique normal subgroup structure. However, the fiber space of topological $(C, R)$ space is a set of elements given by $\{\mu_{\mu_1} : (z_a \in C, I \subseteq R) \subset X\}$. As a result, the fiber space in topological $(C, R)$ space admits oriented singularity and two respective varieties of fibrations. Moreover, unlike Seifert space, the topological $(C, R)$ space is a quasinormed space with non-uniform scaling of points in space. The fiber space of topological $(C, R)$ space forms an additive group algebraic structure under certain conditions. Furthermore, the composite algebraic operation comprised of arithmetic addition and functional translation forms an associative magma in the fiber space of topological $(C, R)$ space.

| Table 1. Summary of comparative analysis of various spaces and fibrations. |
|---------------------------------------------------------------|
| Space/Fibration | Geometric Property | Topological Decomposed Subspaces | Local Compactness | Group Structure |
| Seifert fiber space | three-Manifold | $S^1$-fiber space | Yes | Fundamental group with unique normal subgroup |
| Minkowski space | four-Manifold | 3D real, 1D real spaces | Yes | Lorentz group in fiber sub-bundle |
| Complex fiber bundle | Holomorphic | nD normal complex space ($n \geq 2$) | Yes | Lie group |
| $(C, R)$ space | Quasinormed | 2D complex, 1D real spaces | Yes | Two additive group varieties in fiber space |

The generalization of singular fibration considers that the topological space $(X, \tau_X)$ is connected in nature [19]. Specifically, if $A_1 \cup A_2 = X$ then the singular fibration in $(X, \tau_X)$ maintains the condition that $A_1 \cap A_2 \neq \emptyset$. Moreover, the function $h : (B \subset A_1) \to A_2$ is a homeomorphism, and for each component, $E \subset A_2 \setminus h(B)$, the projection map is given by $h \setminus h^{-1}(E)$ for a fiber bundle, where the base space is $E \subset X$ [19]. The similarity of singularities
between the topological \((C, R)\) space and the generalized singular fibration is that in both cases, the topological space is a connected type. However, the main distinction between the two constructions is that the function of admitting multiple singularities in a topological \((C, R)\) space does not require the existence of local homeomorphism within the topological space. There are varieties of constructions to preserve compactness. In the case of a generalized singular fibration, the functions \(f : A_1 \rightarrow A_2\) and \(f^{-1} : A_2 \rightarrow A_1\) maintain compactness [19]. On the contrary, the compactness in a topological \((C, R)\) space is not always preserved. For example, in the topological \((C, R)\) space, \(\pi_c(B \subset X)\) is compatible but \(\pi_R(B \subset X)\) is not compatible in nature.

The fibration and fiber bundles in Minkowski space consider four-manifold \((M_4)\) structure [20, 21]. Interestingly the fibration in Minkowski space is possible if the space is Hausdorff and connected topological space, which is locally Euclidean on \(M_4\). The fibration in topological \((C, R)\) space considers similar topological properties (i.e., space is connected \(T_2\) space). However, the fibration in Minkowski space is four-dimensional, whereas the fibration in topological \((C, R)\) space is three dimensional in nature. Furthermore, the topological \((C, R)\) space contains a complex subspace under projection, and the Minkowski \(M_4\) is a completely real manifold. Note that the Minkowski space and topological \((C, R)\) space both support the formation of sub-bundles within the respective fiber spaces. The fiber space of topological \((C, R)\) space admits two varieties of additive group algebraic structures under projections. However, the Minkowski fiber space admits a sub-bundle with Lorentz group structure [20]. Interestingly, the holomorphic and principal fiber bundle of a compact complex manifold supports a complex Lie group structure [22]. The topological decomposition indicates that Minkowski space can be completely decomposed into two components, such as 3D real Euclidean subspace and 1D real Euclidean subspace [21]. On the contrary, the topological \((C, R)\) space is a quasinormed space supporting non-uniform scaling, and it can be topologically decomposed into two components, such as 2D complex subspace and 1D real subspace.

7. Conclusions

A fiber space is a topological space where it locally behaves as a product space, but globally the space has a different topological structure. The analytical properties of fibrations and resulting fiber spaces vary depending upon the constructions as well as the structures of topological spaces. The multidimensional topological \((C, R)\) space is a non-uniformly scaled quasinormed space admitting a topological group under composite algebraic operations. The fibrations in multidimensional topological \((C, R)\) space have two varieties in view of compactness. The non-compact fiber space forms associative magma under composite algebraic operation comprised of linear translation and addition within the complex subspace. In this case, the translation is invariant to first order. However, it resists the formation of a monoid structure under composite algebraic operation, and the magma space supports commutativity under linear translation operation within the space of fibers. The fiber space forms an additive group structure if the fibration is a non-compact variety. The projective base space admits an additive group structure if and only if the base space contains the origin of real subspace. The fiber subspace is dense, and it does not allow outward deformation retraction. The fiber space of multidimensional topological \((C, R)\) space allows expansion under uniform translation and preserves multiple oriented singularities of a piecewise continuous function on contact fibers. The structures of Minkowski space and multidimensional topological \((C, R)\) space have different dimensions and partial similarities. The comparative analysis of fiber spaces of multidimensional topological \((C, R)\) space, Minkowski space, and Seifert fiber space illustrates that the supporting group algebraic structures are different for each space. Moreover, the topologically decomposed subspaces have different dimensions as well as properties in each space. Finally, this is
to note that the proposed constructions of topological fibrations may find applications in mathematical sciences (i.e., manifold immersion and surface classifications) and in physical sciences (i.e., supersymmetry, topological string theory, cosmology, and analyzing symmetry fibration in biological networks).

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