Reliability of geotechnical structures

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ABSTRACT

ISO2394:2015 contains a new informative Annex D on “Reliability of Geotechnical Structures”. The emphasis in Annex D is to identify and characterize critical elements of the geotechnical reliability-based design (RBD) process, while respecting the diversity of geotechnical engineering practice. The most important element is the characterization of geotechnical variability and the next in line is arguably the characterization of model uncertainty. The key features of the first element are: (1) coefficient of variation (COV) of a geotechnical design parameter does not take a unique value, (2) multivariate nature of geotechnical data can be exploited to reduce the COV, and (3) spatial variability affects the limit state beyond reduction in COV due to spatial averaging. The second element is commonly carried out by taking the ratio of the measured result to the calculated result (called a model factor). This approach is entirely empirical, but it is practical. Because of its empirical basis, it is not surprising that there are limitations to this approach. The notable ones are lack of applicability to the serviceability limit state and correlation between the model factor and the input parameters in the prediction model.

Keywords: coefficient of variation, multivariate distribution, spatial variability, model factor, serviceability limit state

1 INTRODUCTION

The fourth edition of ISO2394 (General Principles on Reliability of Structures) has been scheduled for publication in 2015. ISO2394 is meant to be used as a basis for national/international code committees to draft codes of practice where the principles of risk and reliability are utilized for design and assessment of structures over the entire service life.

ISO2394:2015 contains a new informative Annex D on “Reliability of Geotechnical Structures”. The author is the chair of the task group responsible for drafting Annex D. The need to achieve consistency between geotechnical and structural reliability-based design is explicitly recognized for the first time in ISO2394 with the inclusion of Annex D. The emphasis in Annex D is to identify and characterize critical elements of the geotechnical reliability-based design (RBD) process, while respecting the diversity of geotechnical engineering practice. These elements are applicable to any implementations of RBD, be it in a simplified form such as the Load and Resistance Factor Design (LRFD), the Multiple Load and Resistance Factor Design (MRFD) (Phoon et al. 2003a), and the Quantile Value Method (QVM) (Ching and Phoon 2011), or in a full probabilistic form such as the expanded RBD approach (Wang et al. 2011). It is recognized that geotechnical engineering practice is less amendable to standardization compared to structural engineering practice, because there are diverse site conditions and diverse local practices that grew and adapted over the years to suit these conditions. For example, the compressive strength of concrete is defined by one standardized unconfined compression test on a cube or cylindrical specimen. The undrained shear strength of clay can be evaluated by many laboratory tests (Ching and Phoon 2013a) and many correlations with field tests (Kulhawy and Mayne 1990). Different field tests have been developed over the years to suit different ground conditions as well. Another example is the availability of many calculation methods for the same problem. Phoon and Kulhawy (2005) characterized the model uncertainties associated with calculation methods for the lateral capacity of a rigid drilled shaft. Five methods for the undrained mode and four methods for the drained mode were studied.

In the opinion of the author, the most important element is the characterization of geotechnical variability and the next in line is arguably the characterization of model uncertainty. After all, site investigation and the interpretation of site data are necessary aspects of sound geotechnical practice. Any design methodology, be it RBD or otherwise, should place site investigation as the cornerstone of the methodology. Site investigation is also the key feature...
that distinguishes geotechnical from structural design practice. There are two obvious differences. One, in contrast to structural materials such as steel and concrete, naturally occurring geo-materials such as soil and rock are not manufactured to meet prescribed quality specifications. Two, spatial variability is an intrinsic feature of a site profile. The availability of data to characterize geotechnical variability remains a source of contention between proponents and critics of geotechnical reliability-based design, despite significant inroads made into the compilation of soil data since 1999 (Phoon and Kulhawy 1999a, 1999b). Geotechnical variability is now routinely discussed in texts (e.g., Chapter 10, Look 2014). The first objective of this paper is to present an overview of the soil databases compiled to date and to highlight the key features of geotechnical variability that should be considered to bring RBD closer to geotechnical practice. It is more challenging to characterize model uncertainty associated with a calculation method, because full-scale test data are more limited than soil data. Model uncertainty always exists and can be significant, because of our geotechnical heritage that is steeped in empiricism. Designs obtained from different calculation methods can be compared rationally using RBD, but it is essential to characterize the model uncertainty associated with each calculation method to realize this advantage. The second objective of this paper is to present an overview of some load test data that can be exploited to reduce COV, and (3) spatial variability affects the limit state beyond reduction in COV due to spatial averaging. Geotechnical RBD must take cognizance of these features to avoid gross oversimplification of “ground truths”.

2 UNCERTAINTY REPRESENTATION OF GEOTECHNICAL DESIGN PARAMETERS

The purpose of this section is to highlight that soil databases, both univariate and multivariate versions are available in the literature, and any development or implementation of geotechnical RBD should be firmly grounded on these databases. It is not reassuring to engineers when RBD is calibrated based on assumed statistics. The key features of geotechnical data in the context of variability are: (1) coefficient of variation (COV) of a geotechnical design parameter does not take a unique value, (2) multivariate nature of geotechnical data can be exploited to reduce the COV, and (3) spatial variability affects the limit state beyond reduction in COV due to spatial averaging. Geotechnical RBD must take cognizance of these features to avoid gross oversimplification of “ground truths”.

2.1 Soil databases

Phoon and Kulhawy (1999a) presented an extensive compilation of univariate soil data, covering data produced by many common laboratory and field tests. Soil databases covering more than one parameter have also been compiled recently. A summary is provided in Table 1. Contrary to popular belief, there are in fact plenty of data for RBD. These databases are exceedingly useful. They can be used by the engineer as prior information to update more limited site specific data (Ching and Phoon 2014b). They can be used by the code writer for reliability calibration of resistance/partial factors. The correlation matrices associated with the clay databases shown in Table 1 can be exploited to reduce COV in the presence of multiple data sources as well (refer to Section 2.3).

Table 1. Summary of multivariate clay databases.

| Database      | Reference                  | Parameters of interest | # data points | # sites/ studies | Range of properties                                      |
|---------------|----------------------------|------------------------|---------------|------------------|----------------------------------------------------------|
| CLAY/5/345    | Ching and Phoon            | LI, s0, s0\textsuperscript{e}, \sigma_p, \sigma_v \textsuperscript{q} | 345           | 37 sites         | OCR 1–4                                                  |
|               | (2012a)                    |                        |               |                  | Low to very high plasticity                              |
| CLAY/7/6310   | Ching and Phoon            | CIUC, CK0UC, CK0UE, DSS, FV, UU, UC | 6310          | 164 studies      | Low to very high plasticity                              |
|               | (2013a)                    | \(s_v/\sigma_v\), OCR, \((q_u-\sigma)/\sigma_v\), \((u_2-u_0)/\sigma_v\), \(B_q\) |               |                  | Insensitive to quick clays                              |
| CLAY/6/535    | Ching et al.               | LL, PI, LI, \sigma_v/P_v, \(s_v/\sigma_v\), \(s_2\), low resistant, \((q-u)/\sigma_v\) \textsuperscript{\textsuperscript{\prime}} | 535           | 40 sites         | Low to very high plasticity                              |
|               | (2014a)                    |                        |               |                  | Insensitive to quick clays                              |
| CLAY/10/7490  | Ching and Phoon            | S_1, OCR, PI, \(s_0\), \((q-u)/\sigma_v\), \(B_q\) | 7490          | 251 studies      | Low to very high plasticity                              |
|               | (2014a)                    |                        |               |                  | Insensitive to quick clays                              |

Note: LL = liquid limit; PI = plasticity index; LI = liquidity index; \(\sigma_v\) = vertical effective stress; \(\sigma_p\) = preconsolidation stress; \(s_0\) = undrained shear strength; \(s_0\) = remoulded shear strength; CIUC = \(s_0\) from isotropically consolidated undrained compression test; CK0UC = \(s_0\) from Ko-consolidated undrained compression test; CK0UE = \(s_0\) from Ko-consolidated undrained extension test; DSS = \(s_0\) from direct simple shear test; FV = \(s_0\) from field vane; UU = \(s_0\) from unconsolidated undrained compression test; UC = \(s_0\) from unconfined compression test; SI = sensitivity; OCR = overconsolidation ratio, \((q_u-\sigma)/\sigma_v\) = normalized cone tip resistance; \((q_u-\sigma)/\sigma_v\) = effective cone tip resistance; \(u_0\) = hydrostatic pore pressure; \((u_2-u_0)/\sigma_v\) = normalized excess pore pressure; \(B_q\) = pore pressure ratio = \((u_2-u_0)/(q-u_\sigma)\); and \(P_a\) = atmospheric pressure = 101.3 kPa.
2.2 Coefficient of variation of geotechnical design parameter

In our current geotechnical practice, a design soil parameter is typically estimated using a transformation (correlation) model that relates the measured (test) parameter with the desired design parameter. There are many different tests (e.g., standard penetration test, cone penetration test) and even different transformation models linking the same measured parameter (e.g., cone tip resistance) and design parameter (e.g., undrained shear strength). There is a large variety of transformation models, because many were developed for a specific geomaterial type and/or a specific locale. Site-specific models are generally more precise than “global” models calibrated from data covering many sites (Ching and Phoon 2012b). However, site-specific models can be significantly biased when applied to another site. This “site-specific” limitation is a distinctive and prevalent feature of geotechnical engineering practice.

The upshot is that a design soil parameter and its probability distribution must depend on the site condition, the measurement method, and the transformation model. The COV is merely one aspect of the probability distribution and it must also depend on the site variability, measurement precision, and transformation quality. This aspect is emphasized in Section D.2 of ISO2394:2015. Typical site variability of strength properties, index parameters, and field measurements are given in Tables 1-3 in Phoon and Kulhawy (1999a). Typical measurement error of laboratory tests and field tests are given in Tables 5 and 6 in Phoon and Kulhawy (1999a). These uncertainty components can be combined consistently with the transformation uncertainty using a simple second-moment approach described in Phoon and Kulhawy (1999b) or by more sophisticated uncertainty analysis methods. First-order approximate guidelines for COVs of some design soil parameters as a function of the test measurement, correlation equation, and soil type are presented in Table 5 in Phoon and Kulhawy (1999b).

Table 2. Three-tier classification scheme of soil property variability for reliability calibration (Source: Table 9.7, Phoon and Kulhawy 2008)

| Geotechnical parameter | Property variability | COV (%) |
|------------------------|----------------------|---------|
| Undrained shear strength | Low<sup>a</sup> | 10 - 30 |
|                        | Medium<sup>b</sup> | 30 - 50 |
|                        | High<sup>c</sup> | 50 - 70 |
| Effective stress friction angle | Low<sup>a</sup> | 5 - 10 |
|                                        | Medium<sup>b</sup> | 10 - 15 |
|                                        | High<sup>c</sup> | 15 - 20 |
| Horizontal stress coefficient | Low<sup>a</sup> | 30 - 50 |
|                                        | Medium<sup>b</sup> | 50 - 70 |
|                                        | High<sup>c</sup> | 70 - 90 |

<sup>a</sup> - typical of good quality direct lab or field measurements
<sup>b</sup> - typical of indirect correlations with good field data, except for the standard penetration test (SPT)
<sup>c</sup> - typical of indirect correlations with SPT field data and with strictly empirical correlations

It suffices to note that assigning a single COV value to a design soil parameter without reference to the property evaluation methodology is an example of gross over-simplification. For example, a COV of 30% for undrained shear strength may be appropriate for good quality laboratory measurements or direct correlations from field measurements such as the cone penetration test (SPT). It may not be appropriate for indirect correlations based on the standard penetration test (SPT). The practical impact of this observation is that it is unrealistic to calibrate a single value for each resistance/partial factor in a simplified RBD format. This practice is realistic for structural engineering, because the COV of manufactured materials can be controlled within a narrow range, say between 5 and 15%. The COV of the undrained shear strength, on the other hand, can vary between 10% and 70%. Based on reliability calibration studies for foundations (Phoon et al. 1995), Phoon and Kulhawy (2008) proposed a reasonably practical three-tier classification scheme (Table 2) for calibration of resistance/partial factors in simplified RBD. Based on this scheme, each resistance/partial factor can take a different numerical value depending on the level of property variability (low, medium, high) judged to be appropriate for a specific design scenario. The Quantile Value Method (QVM) (Ching and Phoon 2011) attempts to surmount this “broad COV range” challenge by using quantiles in the design equation. There is nothing new in the adoption of quantiles, but the conventional approach applies a fixed quantile (called a “characteristic value” in Eurocode and it is fixed at 5%) in conjunction with a partial factor. However, the quantile in QVM is calibrated to achieve the target reliability index. QVM has been further developed to take care of another significant challenge specific to geotechnical engineering, namely calibrating a simplified RBD format to handle a layered soil profile (Ching et al. 2015a).

2.3 Multivariate data

The preceding section focuses on the evaluation of COV for a single design soil parameter. However, it is more common to encounter multivariate information in a site investigation. For instance, when undisturbed samples are extracted for oedometer and triaxial tests, SPT/CPT may be conducted in close proximity. Moreover, basic index properties such as the unit weight, plastic limit (PL), liquid limit (LL), and liquidity index (LI) can be obtained from disturbed samples. It is generally known that data from these varied sources are not independent if they are measured in close physical proximity. The definition of “close” is related to the spatial variability of the site and the key concept of scale of fluctuation (to be discussed in the next section). These data sources are typically correlated to a design parameter such as the undrained shear strength (su). These correlations can be exploited...
to reduce the COV of the design parameter. The impact on RBD is obvious. In fact, the advantage of RBD is that it can respond to a change of COV in a rational way explicitly related to data, while the factor of safety approach cannot. This advantage is more pronounced if the multivariate nature of geotechnical data is considered. This aspect is emphasized in Section D.3 of ISO2394:2015.

Because bivariate correlations between soil parameters are more commonly available, for example $s_u$ versus SPT-N and $s_u$ versus OCR, the multivariate normal distribution is a sensible and practical choice to capture the multivariate dependency among soil parameters in the presence of transformation uncertainties (Phoon 2006). The astute reader will point out that soil parameters do not follow a normal distribution. Fortunately, it is possible to transform a non-normal physical random variable ($Y$) to a standard normal random variable ($X$) (Phoon et al. 2012). The popular solution is to assume that $\ln(Y)$ is normal, in which case the soil parameter $Y$ is lognormally distributed. Phoon and Ching (2013) highlighted that the Johnson system of distributions enjoys the same distributed. Phoon and Ching (2013) highlighted that the soil parameter $Y$ is lognormally distributed. The astute reader will point out that soil parameters do not follow a normal distribution. Fortunately, it is possible to transform a non-normal physical random variable ($Y$) to a standard normal random variable ($X$) (Phoon et al. 2012). The popular solution is to assume that $\ln(Y)$ is normal, in which case the soil parameter $Y$ is lognormally distributed. Phoon and Ching (2013) highlighted that the Johnson system of distributions enjoys the same distributed. Phoon and Ching (2013) highlighted that the soil parameter $Y$ is lognormally distributed.

\[
X = \begin{cases} 
\frac{b_X + a_X \ln \left( \frac{Y - b_Y}{a_Y} \right)}{1 + \left( \frac{Y - b_Y}{a_Y} \right)^2} & \text{SU} \\
\frac{b_X + a_X \ln \left( \frac{Y - b_Y}{a_Y} \right)}{1 - \left( \frac{Y - b_Y}{a_Y} \right)^2} & \text{SB} \\
\frac{b_X^* + a_X \ln (Y - b_Y)}{1} & \text{SL}
\end{cases}
\]

in which $X$ = standard normal random variable with mean $= 0$ and standard deviation $= 1$, $Y$ = Johnson random variable, and $(a_X, b_X, a_Y, b_Y) = $ distribution model parameters. Note that the shifted lognormal distribution appears in Eq. (1) under “SL”. When $b_Y = 0$, SL reduces to the conventional lognormal distribution.

There are ten possible bivariate correlation coefficients for a database such as CLAY/5/345 covering five soil parameters. These correlation coefficients ($\delta_{ij}$) can be presented in the form of a correlation matrix as shown in Table 3. In simple second-moment terms, a univariate random variable can be described by a mean and a COV, while a multivariate random vector can be described by the mean and COV of each component plus an additional correlation matrix. Hence, the key concept in modeling multivariate data is the correlation matrix and it is not surprising that this correlation matrix is the key input for a multivariate normal distribution. Although the application is straightforward, there are subtle theoretical challenges that deserve further research [refer to end of chapter in Ching and Phoon (2015) for a list of challenges].

The practical significance of constructing a multivariate distribution is that the COV of one soil parameter is reduced when information on a second parameter (or a group of parameters) is made available. For illustration, the variance of $X_4$ (related to $S_0$) after the value of $X_1$ (related to LI) has been measured is $(1 - \delta_{14}^2) = 1 - 0.68^2 = 0.54$. The original variance of $X_4$ is 1. Hence, the variance of $X_4$ is reduced by approximately 50% in the presence of one data source. If $X_1$ and $X_3$ (related to $\sigma_p$) are both measured, the variance of $X_4$ reduces further to $(1 - \rho_{14}^2 - \rho_{34}^2 - \rho_{13}^2 - 2\rho_{14}\rho_{34}\rho_{13})/(1 - \rho_{13}^2) = 0.38$. The variance of $X_4$ is reduced by approximately 60% in the presence of two data sources. It is possible to calculate this variance reduction in the presence of any number of data sources analytically for the multivariate normal distribution.

Based on this “updating” framework, site investigation can be viewed as an investment item rather than a cost item, because reduction of uncertainties through multivariate tests can translate directly to design savings through RBD (Ching et al. 2014b). This important link between the quality/quantity of site investigation and design savings cannot be addressed systematically in a deterministic design approach.

Table 3. Product-moment (Pearson) correlations among $(X_1, X_2, ..., X_5)$ for CLAY/5/345.

| X_1  | X_2  | X_3  | X_4  | X_5  |
|------|------|------|------|------|
| 1.00 | -0.28| -0.15| 0.68 | 0.19 |
| 1.00 | 0.77 | 0.14 | -0.15|      |
|      | 1.00 | 0.29 | 0.22 |      |
| 1.00 | 0.44 |      |      |      |
|      |      | 1.00 |      |      |

Note: $X_1, X_2, X_3, X_4, X_5$ are standard normal variables transformed from the soil parameters LI, $\ln(\sigma_v/P_s)$, $\ln(\sigma_p/P_s)$, $\ln(S_0)$, $\ln(\sigma_u/\sigma)$, respectively.

### 2.4 Spatial variability

Another important feature of geotechnical data is that they vary spatially both in the vertical and horizontal direction. As illustrated in Fig. 1, this spatial variation can be decomposed conveniently into a smoothly varying trend function $t(z)$ and a fluctuating component $w(z)$. This fluctuating component is typically modeled as a statistically homogeneous (or stationary) random field (Vanmarcke 2010). It is noteworthy that a physically homogeneous soil layer is not necessarily statistically homogeneous. Some reasonably practical methods have been proposed to identify these statistically homogeneous layers (Phoon et al. 2003b, Uzielli et al. 2005). The key concept used to describe spatial variability is the scale of fluctuation. This concept is presented in Section D.2.7 of ISO2394:2015. The scale of fluctuation provides an
indication of the distance within which the property values show relatively strong correlation. A simple but approximate method of determining the scale of fluctuation is shown as an insert in Fig. 1.

When the scale of fluctuation is very short, the property value at one point is nearly independent of the property value at another point, even if the distance apart is very short. This is manifested visually as properties varying rapidly with depth. This extreme case, called the independent case, is rare in a typical soil profile. When the scale of fluctuation is very long, the property value at one point is nearly equal to the property value at another point for a given random realization, even if the distance apart is very long. This is manifested visually as property values following a near constant trend with depth. This second extreme case, called the fully correlated (or random variable) case, is also rare in a typical soil profile.

There are several practical observations associated with spatial variability. First, the spatial average along a failure surface is more relevant for a limit state, rather than the property value at a general point in the soil mass. This is obvious. Second, for limit states with failure surfaces constrained along a fixed path (e.g., the COV of the spatial average is related to the COV of the property at a point and a variance reduction function (Vanmarcke 2010). This COV reduction effect increases with decreasing scale of fluctuation. Hence, the assumption of independent soil parameters will produce an excessive reduction of the point COV for the spatial average. The assumption of fully correlated soil parameters will not result in COV reduction for the spatial average, which is overly conservative. Third, Vanmarcke’s classical variance reduction function does not apply to unconstrained failure surfaces that are coupled to spatial variability (e.g., slope failure surfaces). The location and shape of such a surface are different for different random realizations. In contrast, the surface where side resistance is mobilized is always constrained along the shaft regardless of the spatial variation of the soil property. The COV of the spatial average along such “unconstrained” failure surfaces is more complicated. A full solution for this more general class of failure surfaces is not available at present, although some progress has been made (Ching and Phoon 2013b, Ching et al. 2014c). Fourth, there is limited work on non-stationary random fields, particularly in the form of one soil layer embedded in another or inclusion of pockets of different soil type within a more uniform soil mass. Qi et al. (2015) studied the applicability of the coupled Markov chain (CMC) model to model this type of geologic uncertainty. Finally, it is significantly more difficult to characterize correlated data statistically than independent data (Phoon et al. 2003b, Ching et al. 2015b). This aspect is of obvious practical significance and more research should be invested in this direction.

Fig. 1. Random field model for spatial variability (revised from Phoon and Kulhawy 1999a).
3 STATISTICAL CHARACTERIZATION OF MODEL FACTORS

The model factor for the capacity of a foundation is commonly defined as the ratio of the measured (or interpreted) capacity to the calculated capacity:

\[ M = \frac{Q_i}{Q_p} \]  

in which \( Q_i \) = “capacity” interpreted from a load test (e.g., hyperbolic capacity), \( Q_p \) = capacity generally predicted using limit equilibrium models, and \( M \) = model factor, assumed to be a log-normal random variable. This approach is entirely empirical, but it is practical. Engineers can easily relate to and adopt this model factor approach. It is obvious that \( M \) is a function of the prediction model (or calculation method) and to a lesser extent, the definition of the capacity on a measured load-displacement curve. Because of its empirical basis, it is potentially possible for the distribution and statistics of \( M \) to be related to the calibration load test database. It is highly recommended to validate the distribution of \( M \) or at least its statistics against an independent load test database to gauge their applicability beyond scenarios not covered in the calibration database. The model factor \( M \) is certainly not a function of a response such as bearing capacity or lateral capacity. Phoon and Kulhawy (2005) demonstrated using a large load test database that the mean of \( M \) for the lateral capacity of a rigid drilled shaft is function of the prediction model (5 models for undrained mode, 4 models for drained mode) and the capacity interpretation method (lateral or moment limit, hyperbolic limit). Reported model statistics for a response without reference to a specific prediction model are probably ball-park figures of unknown accuracy based on experience.

The first purpose of this section is to highlight that model factor databases are available in the literature, particularly for foundations. The second purpose is to discuss two important limitations to the model factor approach. One, it cannot be applied directly for the serviceability limit state. Two, it may be correlated with the input parameters in the prediction model, particularly if the prediction model is overly-simplified in some aspects. Over-simplification is deliberately adopted at times to reduce the solution to a simple analytical form.

3.1 Model factor databases

Phoon and Kulhawy (2005) reported the empirical distributions of model factors from a significant database containing more than 70 load tests for different ultimate lateral soil stress models under undrained and drained loading modes. Model factors for driven piles are reported in NCHRP Report 507 (Paikowsky et al. 2004), Dithinde et al. (2011), and Zhang and Chu (2009). Model factors for shallow foundations are reported in NCHRP Report 651 (Paikowsky et al. 2010). This review is not complete.

3.2 Serviceability limit state

At the ultimate limit state, if a consistent capacity interpretation method is adopted, then each measured load-displacement curve will produce a single “capacity”. It is natural to follow the same approach for the serviceability limit state (SLS). The capacity is replaced by an allowable capacity that depends on the allowable displacement. The distribution of the SLS model factor can be established from a load test database in the same way. The chief drawback is that the distribution of this SLS model factor has to be re-evaluated when a different allowable displacement is prescribed. A more fundamental limitation is that this approach cannot be applied when the allowable displacement is treated as a random variable (Zhang and Ng 2005).

A more general approach is to fit measured load-displacement data to a normalized hyperbolic curve as show below:

\[ \frac{F}{Q_i} = \frac{y}{(a+by)} \]

in which \( F = \) applied load, \( Q_i = \) failure load or capacity interpreted from a measured load-displacement curve, “\( a \)” and “\( b \)” = curve-fitting parameters, and \( y = \) pile butt displacement. Note that the curve-fitting parameters are physically meaningful, with the reciprocals of “\( a \)” and “\( b \)” equal to the initial slope and asymptotic value of the hyperbolic curve, respectively. The curve-fitting equation is empirical and other functional forms can be considered (Phoon and Kulhawy 2008). However, the important criterion is to apply a curve-fitting equation that produces the least scatter in the measured normalized load-displacement curves. Each measured load-displacement curve is thus reduced to two curve-fitting parameters. Based on “\( a \)” and “\( b \)” statistics estimated from load test database, one can easily construct an appropriate bivariate probability distribution for (\( a, b \)) that can reproduce the scatter in the normalized load over the full range of displacements. It is easy to extend this approach to curve-fitting equations described by more than two parameters, because the multivariate distribution discussed in Section 2.3 can be adopted in this application. Details are given in Phoon and Kulhawy (2008). It is evident that this approach can be used in conjunction with a random allowable settlement.

3.3 Correlation between M and input parameters

Ideally, a theoretical model should capture the key features of the physical system, and the remaining difference between the model and reality should be random in nature because it is caused by numerous minor factors that were left out of the model. The
statistics of model factors should capture these random differences resulting from model idealisations. In practice, the ratio between the measured result and the calculated result may not be random in the sense that it is systematically affected by input parameters such as the problem geometry. It is incorrect to model M as a random variable in this situation.

Zhang et al. (2015) studied the calculation of cantilever deflections in undrained clay using the mobilized strength design (MSD) method proposed by Osman and Bolton (2004). The displacement model factor \( \varepsilon \) is defined as:

\[
\delta_m = \varepsilon \times \delta_c
\]

(4)

in which \( \delta_m \) = measured wall top displacement either from case histories in the field or from model tests in the laboratory and \( \delta_c \) = calculated displacement. The authors found that this standard definition cannot be applied directly in the case of the MSD calculation method, because \( \varepsilon \) is a function six input parameters: (1) excavation width, (2) excavation depth, (3) wall thickness, (4) at-rest lateral earth pressure coefficient, (5) undrained shear strength ratio at mid-depth, and (6) ratio between undrained Young’s modulus and undrained shear strength at mid-depth.

It is not possible to remove these dependencies from \( \varepsilon \) using field data, because the values of these input parameters cannot be varied systematically for regression analysis. The authors proposed a rather novel approach consisting of: (1) removing these dependencies using the finite element method (FEM) where the input parameters can be freely varied and (2) characterizing the displacement model factor for the finite element method which is unlikely to suffer from the same dependency problem given that it is mechanically more consistent. Step (1) is carried out by defining the ratio between the FEM wall top displacement \( \delta_{c,FEM} \) and the corresponding MSD calculated displacement \( \delta_{c,MSD} \):

\[
\delta_{c,FEM} = \eta \times \delta_{c,MSD}
\]

(5)

The correction factor \( \eta \) in Eq. (5) can be decomposed into a systematic part \( f \) that is determined using multivariate regression and a residual random factor \( \eta^* \) (regression error) as follows:

\[
\eta = f \times \eta^*
\]

(6)

This regression can be carried out, because a large number of design scenarios described by different combinations of the six input parameters can be analyzed using FEM and MSD. No field data is involved in Eq. (5). Field data is involved in Step (2) where the model factor for FEM (\( \varepsilon_{FEM} \)) is characterized in the usual way:

\[
\delta_m = \varepsilon_{FEM} \times \delta_{c,FEM}
\]

(7)

Zhang et al. (2015) showed that \( \varepsilon_{FEM} \) is indeed not plagued by the dependency problem. Combining Step (1) and (2), it is quite clear that the model factor for MSD (\( \varepsilon_{MSD} \)) is:

\[
\delta_m = \varepsilon_{MSD} \times \delta_{c,MSD}
\]

(8)

\[
\varepsilon_{MSD} = \varepsilon_{FEM} \times \eta^* \times f
\]

The critical observation here is that \( \varepsilon_{MSD} \) is not a random variable because of the deterministic function \( f \), although it follows the standard model factor definition. More specifically, \( \varepsilon_{MSD} \) is the product of a random variable \( (\varepsilon_{FEM} \times \eta^*) \) and a deterministic function \( f \). One can also take the alternate view that the calculated displacement from MSD should be modified by \( f \). By rearranging Eq. (8), it is easy to see that the model factor for this revised MSD method \( (\delta_{c,MSD} \times f) \) will be a random variable \( = (\varepsilon_{FEM} \times \eta^*) \).

4 CONCLUSIONS

ISO2394:2015 contains a new informative Annex D on “Reliability of Geotechnical Structures”. The emphasis in Annex D is to identify and characterize critical elements of the geotechnical reliability-based design (RBD) process, while respecting the diversity of geotechnical engineering practice. The overall emphasis on practice rather than theory is sound as the final product of ISO2394 is a code of practice that geotechnical engineers are willing to use. It is unlikely that a code of practice that deviate significantly from sound geotechnical engineering principles widely proven in practice will be widely accepted by engineers, notwithstanding the advantages of applying reliability principles.

In the opinion of the author, the most important element is the characterization of geotechnical variability and the next in line is arguably the characterization of model uncertainty. After all, site investigation and the interpretation of site data are necessary aspects of sound geotechnical practice. Any design methodology, be it RBD or otherwise, should place site investigation as the cornerstone of the methodology.

The key features of geotechnical data in the context of variability are: (1) coefficient of variation (COV) of a geotechnical design parameter does not take a unique value, (2) multivariate nature of geotechnical data can be exploited to reduce the COV, and (3) spatial variability affects the limit state beyond reduction in COV due to spatial averaging. Geotechnical RBD must take cognizance of these features to avoid gross oversimplification of “ground truths”.

The model factor for the capacity of a foundation is commonly defined as the ratio of the measured (or interpreted) capacity to the calculated capacity. This approach is entirely empirical, but it is practical. Engineers can easily relate to and adopt this model.
factor approach. Because of its empirical basis, it is not surprising that there are limitations to this approach. Some of them are highlighted in this paper:

1. It is potentially possible for the distribution and statistics of M to be related to the calibration load test database.
2. It is obvious that M is a function of the prediction model (or calculation method) and to a lesser extent, the definition of the capacity on a measured load-displacement curve. Reported model statistics for a response without reference to a specific prediction model are probably ball-park figures of unknown accuracy based on experience.
3. It cannot be applied directly for the serviceability limit state. It is more sensible to fit the entire load-displacement using an empirical curve, say a hyperbolic curve. The curve-fitting parameters can be modelled as a bivariate random vector for 2 parameters or a multivariate random vector if there are more than 2 parameters.
4. It may be correlated with the input parameters in the prediction model, particularly if the prediction model is overly-simplified in some aspects. Over-simplification is deliberately adopted at times to reduce the solution to a simple analytical form. This dependency should be removed by regression. However, field data cannot be regressed with any degree of confidence, because data are too limited in the context of multivariate regression. The proposed strategy is to replace field data by a mechanically consistent numerical method (such as FEM) for regression and to apply field data only for model characterization of the numerical method.

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