$p - \text{air}$ production cross-section and uncorrelated mini-jets processes in pp-scattering

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For the $p - \text{air}$ production cross-section, we use a Glauber formalism which inputs the $pp$ inelastic cross-section from a mini-jet model embedded in a one-channel eikonal expression, which provides the needed contribution of uncorrelated processes. It is then shown that current LO parton density functions for the $pp$ mini-jet cross-sections, with a rise tempered by collinearity induced by soft gluon re-summation, are well suited to reproduce recent cosmic ray results. By comparing results for GRV, MRST72 and MSTW parametrizations, we estimate the uncertainty related to the low-x behavior of these densities.

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I. INTRODUCTION

In this paper we address the problem of how to relate \( p - air \) production cross-section measurements from cosmic rays, to accelerator data for \( proton - proton \) scattering. This is a very old question and very ingenious ways to do so have been developed through the years [1]. The issue is often obfuscated by the need to estimate the contribution from elastic and diffractive processes, both in \( p - air \), but mostly in \( pp \) collisions.

The question of whether it is \( \sigma_{pp}^{total} \) or \( \sigma_{pp}^{inel} \) which is input to the Glauber formalism was discussed in [2]. Presently, most current analyses define a \( \sigma_{p-air}^{prod} \) through the inelastic cross-section, and a \( \sigma_{p-air}^{inel} \) through the total \( pp \) cross-section. In either case, elastic and quasi-elastic contributions need to be subtracted and a degree of uncertainty can arise from their parametrization. The definition of inelastic cross-section is also affected by uncertainties, both theoretically and experimentally, as seen in LHC experiments with different cuts in the forward region [3].

Here we shall show that the total \( p - air \) production cross-section can be obtained in a very direct way through the inelastic \( pp \) cross-section resulting from one-channel eikonal models. This formalism for the inelastic cross-section provides a description of non-correlated inelastic processes [4], and thus avoids the problem of how to model diffraction and elastic cross-section. The description of the latter, including the elastic differential cross-section, is still not resolved, and is obtained through various parametrizations. A recent suggestion by the Telaviv group [5] has made efforts in this direction. Here we shall follow a different path.

It is important to stress that in the case of cosmic rays, first and foremost one needs an eikonal function which gives a description of the total \( pp \) cross-section, through a good understanding of the underlying physics. In this paper we describe \( proton - air \) production cross-section up to the recent AUGER measurement [6], using the inelastic \( pp \) cross-section obtained from a QCD mini-jet model with soft gluon re-summation [7, 8].

We have long advocated QCD mini-jets as the driving mechanism for the rise of all total cross-sections [9] and have proposed a saturation mechanism based on infrared gluon resummation to tame the excessive rise with energy of the mini-jet cross-sections [10]. Thus, the emphasis of the present work is two fold. First to provide a good phenomenological description of cosmic \( p - air \) production cross-sections through a successful well accepted formalism, such as in the Glauber theory [11]. The second is to reconfirm that the rise of all total, elastic and inelastic cross-sections of protons on protons, or protons on nuclei and other hadrons, have the same origin: a rising contribution from the increasing number (with energy) of low-x gluons excited in the collision [12].

Since the ’80s, many models have used mini-jets in total cross-section physics [13, 15] and more recently in [16]. In most cases, the parton density functions [PDFs] are chosen or parametrized \textit{ad hoc}. However, we believe that mini-jets can give interesting information only if used in connection with current LO parton densities, such as available through updated PDF libraries. As in any perturbative QCD calculation, this LO effect needs then to be complemented by other QCD effects, such as that of very soft gluons arising from the QCD confinement potential [10].

We shall use the Glauber model [17], with the following basic hypothesis when the target is a nucleus: i) for low transverse momentum collisions \( p_t \lesssim (1 \div 2) \text{ GeV} \), the incoming proton does not penetrate the air nucleus and basically scatters off the surface, whereas, as the transverse momentum increases, the proton penetrates the nucleus of atomic number \( A \) and scatters off all the protons in the volume occupied by the nucleus. Thus the nuclear density seen by the incoming protons will only be proportional to \( A^{2/3} \) for the soft collisions, and to \( A \) for the hard part. (ii) For interactions with transverse momenta \( p_t \gtrsim (1 \div 2) \text{ GeV} \), we shall employ QCD effects in the form of mini-jets and soft gluon emission as in the model developed in [7, 8, 10].

Neglecting momentarily the above surface/volume effect, we begin with the usual Glauber expression for the production cross-section in the impact parameter representation, as given by

\[
\sigma_{prod}^{p-air}(E_{lab}) = \int d^2b [1 - e^{-n_{p-air}(b,s)}]
\]  

(1)
with
\[ n_{p \rightarrow \text{air}}(b, s) = T_N(b)\sigma_{\text{inel}}^{pp}(s) \] (2)
wherein \( T_N(b) \) is the nuclear density, for which we start by choosing a standard gaussian distribution,
\[ T_N(b) = \frac{A}{\pi R_N^2}e^{-b^2/R_N^2}, \] (3)
properly normalized to
\[ \int d^2b T_N(b) = A. \] (4)
The parameters used in the profile (3), namely the average mass number of an “air” nucleus, \( A \), and the nuclear radius, \( R_N \), are the following:
\[ A = 14.5, \quad R_N = (1.1 \text{ fermi}) A^{1/3}. \] (5)
The inelastic \( pp \) cross-section, \( \sigma_{\text{inel}}^{pp} \), is obtained from \( pp \) scattering, with
\[ \sigma_{\text{inel}}^{pp} = \int d^2b \left[ 1 - e^{-2\chi_I(b, s)} \right] \] (6)
\[ \sigma_{\text{tot}}^{pp} = 2 \int d^2b \left[ 1 - \Re e^{i\chi(b, s)} \right] \] (7)
where \( \chi_I(b, s) = 3m\chi(b, s) \) is the imaginary part of the eikonal function that defines the elastic amplitude. At high energy, it is a good approximation to neglect a possible real part of the eikonal function in Eq. (7) and write
\[ \sigma_{\text{tot}}^{pp} = 2 \int d^2b \left[ 1 - e^{-\chi_I(b, s)} \right] \] (8)
This formalism gives both the total and the inelastic non-correlated cross-section, once the quantity \( \chi_I(b, s) \) is known. The latter is an important point in the discussion of \( p \text{--air} \) processes. The one-channel eikonal formalism for the inelastic cross-section given by Eq. (6) includes only non-correlated, Poisson distributed independent collisions. This can be seen easily by comparing this equation with a sum over all independent Poisson like distributions, as discussed in [4]. Thus the above one-channel eikonal has the virtue of identifying all non-correlated processes, which we argue (and later verify phenomenologically) are all the non-diffractive processes contributing to the \( p \text{--air} \) production cross-section. We notice here that this property of the one-channel eikonal is a hindrance when one wants to separate the purely elastic from the diffractive part, but it is exactly what one needs for \( p \text{-air} \) shower initiated measurements. We shall return to this point again later.

In the following, we shall first consider \( pp \) scattering and give a brief summary of the physics content of our model and determine the parameters which give an optimal description of \( pp \) data up to LHC. We shall then use the one-channel eikonal to calculate the inelastic non-diffractive \( pp \) cross-section and obtain the \( p \text{--air} \) production cross-section to compare with data.

II. PROTON-PROTON TOTAL AND INELASTIC NON-DIFFRACTIVE CROSS-SECTION

The eikonal function of the mini-jet model of [7, 8] is given by
\[ 2\chi_I(b, s) = n_{\text{soft}}^{pp}(b, s) + n_{\text{jet}}^{pp}(b, s) = A_{FF}\sigma_{\text{soft}}^{pp}(s) + A_{BN}^{pp}(p; b, s)\sigma_{\text{jet}}(PDF, p_{\text{tmin}}; s) \] (9)
where $A_{FF}$, the impact parameter distribution in the non-perturbative term, is obtained through a convolution of two proton form factors, whereas for the perturbative term, the distribution $A_{BN}^{pp}(p; b, s)$, multiplying the mini-jet contribution, is given by the Fourier transform of overall soft gluon re-summation, i.e. we have

$$A_{BN}^{pp}(p; b, s) = \frac{e^{-h(p, b, s)}}{\int d^2b e^{-h(p, b, s)}}$$  \hspace{1cm} (10)

$$h(p; b, s) = constant \int_0^{q_{max}} \frac{dk_t}{k_t} \alpha_s(k_t) \log \frac{2q_{max}}{k_t} \left[ 1 - J_0(bk_t) \right]$$  \hspace{1cm} (11)

$$\alpha_s(k_t) \simeq \left( \frac{k_t}{\Lambda_{QCD}} \right)^{-2p} \quad k_t \to 0$$  \hspace{1cm} (12)

We have discussed this distribution in many publications, its main characteristic is to include soft gluon re-summation down to $k_t = 0$, and regulate the infrared singularity so as correspond to a dressed gluon potential $V \sim r^{2p-1}$ for $r \to \infty$. We have also shown an important consequence of an expression such as the above for $\alpha_s(k_t \to 0)$ [18], namely that asymptotically the regularized and integrated soft gluon spectrum of Eq. (11) is seen to rise as

$$h(p; b, s) \to (b\bar{\Lambda})^{2p}$$  \hspace{1cm} (13)

and thus the $b-$ distribution exhibits a cut-off in $b-$space strongly dependent on the parameter $p$, i.e.

$$A_{BN} \to e^{-(b\bar{\Lambda})^{2p}} \quad b \to \infty$$  \hspace{1cm} (14)

with $\bar{\Lambda} \propto \Lambda_{QCD}$. Since the mini jet cross-sections at low-x are parametrized so as to rise as $s^\epsilon$, the behavior of Eq. (14) leads to a high energy behavior for the total cross-section given as

$$\sigma_{tot}^{pp} \sim \frac{2\pi}{(\bar{\Lambda})^2} [\epsilon \log s]^{1/p}$$  \hspace{1cm} (15)

The parameter $1/2 < p < 1$: the lower limit so as to have a confining potential, the upper limit to insure convergence of the integral over the soft gluon spectrum of Eq. (11). An immediate consequence of this model is that the cross-section will never rise more than $[\log s]^2$, the saturation of the Froissart limiting behavior being obtained for $p = 1/2$. Notice, that, in this model, the mini-jet contribution, just as in hard Pomeron models [19], rises as $\sigma_{jet} \sim s^\epsilon$, with $\epsilon \sim 0.3-0.4$ depending on the low-x parametrization of the PDF. However the strong cut-off in $b$-space (saturation) brought in by the singular, but integrable, effective quark − soft − gluon coupling constant leads only up to a (logarithmic)$^2$ rise with energy. For more details, we refer the reader to Ref. [18].

The low energy term includes collision with $p_t \leq p_{t_{min}} \sim 1 - 2$ GeV, and the cross-section $\sigma_{soft}^{pp}(s)$ is not predicted by this model so far, thus we parametrize it here with a constant and one or more decreasing terms. The result is shown in Fig. 1. The perturbative, mini-jet, part is defined with $p_t^{parton} \geq p_{t_{min}}$ and is determined through a set of perturbative parameters for the jet cross-section, namely a choice of PDF and $p_{t_{min}}$. Since the soft gluon re-summation includes all order terms in soft gluon emission, as in previous publications we have used only LO densities. An important point of our approach is that we use the same, library distributed PDF, as used for jet physics. Previously used PDFs were GRV [20–22], or MRST72 [23]. Both still give a good description of data up to LHC results, as shown here and in the next section. In Fig. 2 we show the results obtained through a more recent set of LO densities, MSTW [24], for both the total and the inelastic pp cross-sections.
FIG. 1. Low energy parametrization of $pp$ total cross-section

$\sigma_{\text{soft}} = \sigma_0 + \frac{\sigma_1}{E_{\text{lab}}}$

$\sigma_0 = 47.9 \text{ mb}$

$\sigma_1 = 51.0 \text{ mb}$

$\alpha = 1.45$

FIG. 2. QCD mini-jet with soft gluon resummation model and $pp$ total cross-section (full line). Accelerator data at LHC include TOTEM [25, 26] and ATLAS measurements, as from [ATLAS-CONF-2014-040, ATLAS-COM-CONF-2014-054]. The inelastic uncorrelated cross-section is given by the dashed curve and compared with central collisions results at LHC by ATLAS [27], CMS [28] and ALICE [29].

The parameter $p$, whose value is explicitly given in this figure, is related to the amount of saturation due to soft gluon emission, as discussed in [18]. Its value lies in the range $0.6 \lesssim p \lesssim 0.8$ depending on the PDF used. For MSTW, we find that the parameter set $\{p_{\text{min}} = 1.3 \text{ GeV}, p=0.66\}$ best reproduces the $pp$ cross-section up to LHC8.
We note the important result that the inelastic cross-section predicted by the parametrization of the total cross-section through a one-channel eikonal, reproduces very well the LHC data for non-diffractive collisions by ATLAS \cite{27}, CMS \cite{28} and ALICE \cite{29}. Such agreement had already been highlighted in \cite{4}. We shall return to comment on this point at the end of the paper.

II.1. A comment on the model parameters

The present focus of our model is the parametrization of the high energy behavior described by QCD processes. To this aim, we need a set of PDFs, a lower cut-off dividing the perturbative and non-perturbative regions, \( p_{\text{min}} \), and a saturation, parameter \( p \), which we also referred to as singularity parameter. The higher this parameter, the more saturation is present. Phenomenologically, its value is fixed in relation to the low-x behavior of the densities. The parameter \( p \) thus appears to be unrelated to the perturbative expression for the QCD coupling constant \( \alpha_s(Q^2) \). We however believe it be of more fundamental interest, and have made the ansatz \cite{30} that the actual expression to use in the integrand of Eq. (11) is

\[
\alpha_s^{BN}(Q^2) = \frac{1}{\ln[1 + (\frac{Q^2}{\Lambda^2})^{b_0}]} \tag{16}
\]

where \( b_0 = (33 - 2N_f)/12\pi \) and the suffix \( BN \) is used to indicate its applicability into the infrared region (the one first explored in QED by Bloch and Nordsieck \cite{31}), while coinciding with the usual one-loop asymptotic freedom expression at high \( Q^2 \). The above ansatz would imply that the infrared region description does not require introduction of an extra parameter \( p \): the behavior from \( Q^2 = 0 \) to \( Q^2 \rightarrow \infty \) is dictated only by the anomalous dimension factor. However, the present uncertainty about a fundamental calculation for the low-x behavior of the parton densities, prevents a full use of Eq. (16). Suffice to say that our phenomenological values for \( p \) are in the same range of variability of the anomalous dimension factor \( b_0 \).

III. THE PRODUCTION CROSS-SECTION FOR \( p - \text{air} \)

With the low energy part parametrized as shown in Fig. 1 and the mini-jet part, we can calculate the inelastic \( pp \) cross-section and thus the production \( p - \text{air} \) cross-section. The result is shown in Fig. 3 where our model is compared with cosmic ray data \cite{6, 32-38}. In this figure we have reduced the constants \( \sigma_{0.1} \) in the \( pp \) cross-section so as to comply with the surface/volume effect for the low transverse momentum collisions. Because of the uncertainty in this low energy region, the soft term in the \( pp \) cross-section has been included openly as a constant. However, we have also considered the full low-energy parametrization of Figs. 1,2, but in the energy range of Fig. 3 such low energy decreasing term makes no difference whatsoever.

To estimate the error of this procedure as well as check the stability of the model and its application to both \( pp \) and \( p - \text{air} \) cross-sections, we have done the following checks:

- after parametrizing the low energy part of \( pp \) data, the rise has been described through other available LO PDFs, namely MRST72 and GRV in addition to MSTW. For a given PDF set, the parameters \( p_{\text{min}} \) and \( p \) have been chosen to best reproduce LHC results for \( \sigma_{\text{tot}}^{pp} \) \cite{25, 26}.
- we have done an actual fit to both the low energy data and LHC (excluding cosmic rays extracted data), using GRV and MRST72, and with free saturation parameter \( p \).
- We have changed the nuclear density model, applying a Wood-Saxon potential, as in \cite{16}.

The results of this exercise for different densities are shown in the two panels of Fig. 4 where the bands highlight the uncertainty related to the the low-x behavior of the parton densities used for the mini jet calculation. As we are not so much interested in understanding right now the low
energy part, the constant $\sigma_0$ has simply been reduced adjusting it to the data. As expected, the contribution from the low energy part gets weaker and weaker for very high energies. The results are also shown in Table I. In the table, the low energy part of the eikonal function, $n_{soft}$, is fitted to the low energy data alone, as in Fig. 3 whereas the QCD part $n_{hard}$ is chosen so as to best describe the $pp$ accelerator data. As for the other check, non reproduced in this table, namely fitting at the same time both the low and the high energy accelerator data in order to determine the best $p$-value, for a given choice of PDF and $p_{tmin}$, we have found the result to be consistent with above, for $p \approx 0.6$ for MRST72 densities and $p_{tmin} \approx 1.3 - 1.4$ GeV. Using the Wood-Saxon potential slightly lowers the curves for $p - air$ with respect to the standard nuclear potential of Eq. (3).

Before concluding this paper, we would like to return to an important physics point, namely...
TABLE I. Total and inelastic uncorrelated pp cross-sections (second and third column). Fourth column is the uncorrelated inelastic pp cross-section for input to the Glauber formula for $\sigma_{p\text{--air}}$ with low energy part reduced for nuclear area/volume effect. Last column shows the resulting $p\text{--air}$ cross-section. Different parameter sets are as indicated.

| Parameter set | $\sqrt{s}$ (GeV) | $\sigma_{pp}^{\text{tot}}$ with $\sigma_0 = 48$ mb | $\sigma_{pp}^{\text{uncorr}}$ with $\sigma_0 = 48$ mb | $\sigma_{pp}^{\text{uncorr}}$ with $\sigma_0 = 32$ mb | $\sigma_{p\text{--air}}^{\text{prod}}$ with $\sigma_0 = 32$ mb |
|---------------|------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| GRV, $p_{\text{min}} = 1.2$ GeV, $p = 0.69\sqrt{s}$ | 5                | 39.9                                         | 33.2                                          | 24.9                                          | 255.8                                         |
|               | 10               | 38.3                                         | 32.0                                          | 24.0                                          | 248.9                                         |
|               | 50               | 41.9                                         | 34.0                                          | 26.7                                          | 268.7                                         |
|               | 100              | 46.7                                         | 36.1                                          | 29.7                                          | 288.6                                         |
|               | 500              | 63.2                                         | 43.0                                          | 38.6                                          | 340.9                                         |
|               | 1000             | 71.7                                         | 46.9                                          | 43.1                                          | 364.1                                         |
|               | 1800             | 79.5                                         | 50.5                                          | 47.2                                          | 383.5                                         |
|               | 7000             | 98.9                                         | 59.8                                          | 57.4                                          | 426.1                                         |
|               | 8000             | 100.9                                        | 60.7                                          | 58.4                                          | 430.0                                         |
|               | 14000            | 109.3                                        | 64.8                                          | 62.8                                          | 445.9                                         |
|               | 30000            | 121.3                                        | 70.7                                          | 69.0                                          | 467.0                                         |
|               | 60000            | 132.0                                        | 76.0                                          | 74.6                                          | 484.3                                         |
| MRST72, $p_{\text{min}} = 1.25$ GeV, $p = 0.62\sqrt{s}$ | 5                | 39.9                                         | 33.2                                          | 24.9                                          | 255.8                                         |
|               | 10               | 38.3                                         | 32.0                                          | 24.0                                          | 249.1                                         |
|               | 50               | 43.1                                         | 34.6                                          | 27.6                                          | 274.5                                         |
|               | 100              | 48.4                                         | 36.9                                          | 30.8                                          | 295.8                                         |
|               | 500              | 63.8                                         | 43.7                                          | 39.3                                          | 344.6                                         |
|               | 1000             | 71.3                                         | 47.1                                          | 43.3                                          | 365.1                                         |
|               | 1800             | 78.1                                         | 50.3                                          | 46.9                                          | 382.3                                         |
|               | 7000             | 98.2                                         | 60.4                                          | 58.0                                          | 428.3                                         |
|               | 8000             | 100.7                                        | 61.7                                          | 59.4                                          | 433.6                                         |
|               | 14000            | 112.2                                        | 67.7                                          | 65.7                                          | 456.2                                         |
|               | 30000            | 129.1                                        | 76.5                                          | 75.0                                          | 485.7                                         |
|               | 60000            | 144.2                                        | 84.4                                          | 83.3                                          | 509.1                                         |
| MSTW, $p_{\text{min}} = 1.3$ GeV, $p = 0.66\sqrt{s}$ | 5                | 39.21                                        | 32.7                                          | 23.7                                          | 240.8                                         |
|               | 10               | 38.60                                        | 32.3                                          | 23.1                                          | 242.6                                         |
|               | 50               | 42.2                                         | 34.2                                          | 25.9                                          | 263.4                                         |
|               | 100              | 46.9                                         | 36.4                                          | 29.2                                          | 285.5                                         |
|               | 500              | 62.0                                         | 43.3                                          | 38.1                                          | 338.6                                         |
|               | 1000             | 71.0                                         | 47.5                                          | 43.1                                          | 364.4                                         |
|               | 1800             | 77.5                                         | 50.5                                          | 46.6                                          | 381.2                                         |
|               | 7000             | 98.3                                         | 60.5                                          | 57.8                                          | 428.0                                         |
|               | 8000             | 101.3                                        | 62.0                                          | 59.4                                          | 434.0                                         |
|               | 14000            | 113.7                                        | 68.2                                          | 66.1                                          | 457.7                                         |
|               | 30000            | 129.4                                        | 76.0                                          | 74.3                                          | 483.8                                         |
|               | 60000            | 150.3                                        | 86.3                                          | 85.1                                          | 514.3                                         |

that the experimentally measured $\sigma_{p\text{--air}}^{\text{prod}}$ differs from the total $\sigma_{p\text{--air}}^{\text{tot}}$ through the exclusion of elastic $\sigma_{p\text{--air}}^{el}$ as well as quasi-elastic $\sigma_{p\text{--air}}^{q\text{--el}}$. An example of $\sigma_{p\text{--air}}^{q\text{--el}}$ is given by processes such as $p+p\to\Delta(1238)+p$. In general, the measured cosmic cross-section does not include (single as well as double) diffractive processes. This acquires a particular significance (and endows a certain simplicity) to (one-channel) mini-jet models when applied to an analysis of cosmic ray cross-sections. As has been noticed by several authors [4, 39], single-channel mini-jet models overestimate the elastic cross-section by including in it the diffractive processes. Otherwise said, the inelastic cross-section in such models does not include the diffractive part. It is thus best suited for calculations of the production $p\text{--air}$ cross-section from cosmic ray measurements. For this purpose, we have employed parameters (such as $p$) suitable for describing the total cross-section well and by default giving us the inelastic part devoid of diffraction. A posteriori, such a description seems to work quite well.
The most remarkable result that we find is that we reproduce very well the AUGER point, in addition to have a reasonably good description of all the more recent cosmic ray measurements.

IV. CONCLUDING REMARKS

In this paper, we have seen that the Glauber formula in conjunction with an inelastic $pp$ cross-section obtained through a one-channel eikonal formalism provides a very good description of the cosmic ray extracted ($p - \text{air}$) cross-section. Thus, we might ask, whether a one-channel eikonal expression adequately representing the $pp$ total cross-section is also sufficient to describe high energy elastic scattering. Obviously not, unfortunately. It is fair to say that the momentum transfer ($t$)-dependence of the elastic differential cross-section from the forward ($t = 0$) up to after the dip still escapes a fundamental QCD explanation. For this, and thus for the diffractive part of the cross-section, a multi channel formalism [39–41] is still required. However, it is our ansatz that a viable multichannel formalism must be geared to reproduce the results from a single term at the optical point (that is at $t = 0$).

For the present, we may reiterate that a good one-channel eikonal representation for the total cross-section should be sufficient to describe the cosmic ray $p - \text{air}$ production cross-section data and conversely, that models which reproduce $\sigma_{\text{production}}^{pp}$ can be trusted to extrapolate correctly $\sigma_{\text{inel--non--diff}}^{pp}$, and thus the total $\sigma_{\text{tot}}^{pp}$ in a one-channel eikonal model. However, very high energy predictions are affected by an uncertainty related to the low-$x$ behavior of the PDFs used in the phenomenological calculation of the mini-jet cross-sections. It may thus be very important to include the forthcoming LHC data at $\sqrt{s} > 10$ TeV to reduce such uncertainty and hopefully be able to extract information on $\sigma_{\text{tot/incl}}^{pp}$ from the even higher energy cosmic ray measurements to be expected from cosmic rays.

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