M-theory and the string genus expansion

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Abstract

The partition function of the membrane is investigated. In particular, the case relevant to perturbative string theory of a membrane with topology $S^1 \times \Sigma$ is examined. The coupling between the string world sheet Euler character and the dilaton is shown to arise from a careful treatment of the membrane partition function measure. This demonstrates that the M-theory origin of the dilaton coupling to the string world sheet is quantum in nature.
1 Introduction

The basis for string perturbation theory is the coupling of the world sheet Euler character, $\chi$, to the dilaton, $\Phi$ [1]. It is this coupling that determines the relationship between the topology of the string world sheet and the string coupling constant and allows string theory at small coupling to be organised in terms of a genus expansion. To see this, consider how the string coupling constant $g_s$ controls the amplitude for a closed string to split into two. This is the so-called “trousers” diagram. To increase the genus of any amplitude by one results in an extra multiplicative factor of $g_s^2$ for that amplitude. An amplitude corresponding to a diagram with $e$ external legs and $l$ loops, will therefore contain a factor of $g_s^{e-2+2l}$. Orientable two-dimensional surfaces are classified by their genus $g$ or Euler character $\chi$ together with the number of ends (corresponding to external legs) where $\chi = 2(1-g)$ and $g = l + e/2$. Thus each amplitude is weighted by a factor of $g_s^{-\chi}$.

Despite the well known connection between membranes, fundamental strings and D-branes [2–5], the M-theory origin of the string world sheet Euler character coupling to the dilaton has been regarded as something of a puzzle. To appreciate why the M-theory interpretation for this coupling has so far remained mysterious, let us make a few observations that will be important in understanding the M-theory origin of this term. In what follows, we will consider only string world sheets without boundaries i.e. vacuum diagrams; the extension to include boundaries is essentially trivial. Type IIA string theory is obtained from M-theory by compactification on a Kaluza-Klein circle of radius $R_{11}$. In string theory, the coupling of the dilaton to the world sheet $\Sigma$ is given by a contribution to the string action of

$$S_\Phi = \frac{1}{4\pi} \int_\Sigma d^2\sigma \sqrt{\tilde{\gamma}} R^{(2)} \Phi$$  \hspace{1cm} (1)$$

where $R^{(2)}$ is the Ricci scalar of the world-sheet metric $\tilde{\gamma}$. Choosing a constant dilaton

\[3\] There is interesting work on the quantum membrane and U-duality in string theory [6] which looks into quantum aspects of the membrane and the relation to string dualities.
and using the Gauss-Bonnet theorem

\[ \frac{1}{4\pi} \int_{\Sigma} d^2\sigma \sqrt{\tilde{\gamma}} R^{(2)} = \chi(\Sigma) \]  

reduces the action to

\[ S_\Phi = \chi(\Sigma) \Phi . \]  

Now, when one makes a Kaluza-Klein reduction of 11-dimensional supergravity on a circle of radius \( R_{11} \), down to 10-dimensional type IIA supergravity, one finds that the dilaton is given by

\[ e^\Phi = \left( \frac{R_{11}}{l_p} \right)^\frac{3}{2} , \]  

Hence, in M-theory language,

\[ g_s = \left( \frac{R_{11}}{l_p} \right)^\frac{3}{2} , \]  

and after choosing units such that \( l_p = 1 \)

\[ S_\Phi = \frac{3}{2} \chi \log(R_{11}) . \]  

Both the absence of a factor of \( \alpha' \) in the above action and the logarithmic dependence on \( R_{11} \) suggest that the origin of this term is not classical in nature. It is more likely that such a term arises from a quantum effective action. There is also the rather trivial observation that one would like to be able to lift the Euler character, \( \chi(\Sigma) \) to three dimensions, so as to be able to give a membrane interpretation. Finding the correct quantity in three dimensions that when evaluated in \( S^1 \times \Sigma \), where \( \Sigma \) is a Riemann surface, is therefore part of the puzzle.

The approach that we will adopt is to describe the fundamental string as a membrane with world volume topology \( S^1 \times \Sigma \) with \( \Sigma \) being some Riemann surface. The membrane will be restricted to wrap once around the M-theory circle such that the world volume \( S^1 \) will be identified with the M-theory circle. We will truncate to the zero mode sector of the circle. That is there will be no dependence of any of the fields on circle direction, other than the winding mode. This is certainly justified if the M-theory circle is small since any excitations will be very heavy. One can also
interpret this as isolating the pure fundamental string sector since any dependence on
the M-theory circle will be associated with D0 branes in the string theory.

We will then describe the wrapped membrane partition function. In particular,
we will be interested in calculating the measure for a membrane with the topology
relevant for the string, that is $S^3 \times \Sigma$. We will then show that a careful treatment
of this measure naturally gives rise to the correct dilaton coupling when interpreted
from a string world sheet point of view.

2 The membrane

The bosonic part of the action for the M-theory 2-brane is, in Howe-Tucker form,

$$
S_{M2}[X, \gamma; G, C] = \frac{T_{M2}}{2} \int_{M^4} d^3\sigma \sqrt{\gamma}(\gamma^{\mu\nu}\partial_\mu X^I\partial_\nu X^J G_{IJ} - 1
+ \epsilon^{\mu\nu\rho} \partial_\mu X^I \partial_\nu X^J \partial_\rho X^K C_{IJK})
$$

where $T_{M2}$ is the tension of the M2-brane, $\gamma_{\mu\nu}$ is the world-volume metric, $X^I(\sigma)$
specify the location of the brane in the target space. In what follows, upper case latin
indices $I, J, K, \ldots$ are 11-dimensional target space indices whereas greek indices are
world volume indices, and so $\sigma^\mu$ are the world volume coordinates. $G_{IJ}$ and $C_{IJK}$ are
respectively the background metric and three form potential of eleven dimensional
supergravity. There are known obstacles for treating the membrane as one would
the string. For example, there is no discrete spectrum of states that allows one to
identify membrane states with space time excitations [7]. It is also not obviously a
renormalisable theory in an arbitrary spacetime, even one that obeys the supergravity
equations of motion. Yet there are various consistency checks, mostly from the
supermembrane. $\kappa$ symmetry of the supermembrane is consistent with the eleven
dimensional supergravity equations, [2]. After world volume dualisation of one the
scalar fields, the membrane action can be identified with the D2 action [5]. The rela-
tion of the BPS sector of the membrane to dualities in string theory has been studied
in [6] and important work on M-theory loops and higher derivative corrections to the
string effective action corrections has been done in [8].
What we now propose, following analogous work of Polyakov for the string, is to take seriously the idea of a membrane partition function, \( Z \) given by:

\[
Z = \sum_{\text{topologies}} \int \frac{DX D\gamma}{\text{Vol}(\text{Diff}_0)} e^{-S_{M2}[X,\gamma;G,C]}.
\]

(8)

or more precisely its supersymmetric extension. (For the present we will ignore the Fermionic sector which appears to be irrelevant to our considerations regarding the origin of the dilaton coupling). As with the string, the integrals are taken over all \( X \) and world volume metrics, \( \gamma_{\mu\nu} \) and then summed over all topologies of the world volume. Since the action is diffeomorphism invariant one divides by the volume of three dimensional diffeomorphisms. (For the string it would be the product of diffeomorphisms and Weyl transformations.)

To carry out the integral over world volume metrics, \( \gamma \), one has to make an orthogonal decomposition of the deformations into those that are pure diffeomorphisms and those which are physical. The decomposition of the measure will then introduce a Jacobian which is the Faddeev-Popov determinant. One is then free to gauge fix and integrate over the pure diffeomorphism part of \( \gamma \) leaving only the physical degrees of freedom, the \textit{moduli} of the three manifold. (The division by the volume of the diffeomorphism group then cancels the integral over the pure diffeomorphism part of \( \gamma \).)

Thus after gauge fixing and integrating over pure diffeomorphisms one is left with the rather formal expression:

\[
Z = \sum \int J\{\text{moduli}\} DX e^{-S_{M2}[X,\text{moduli};G,C]}
\]

(9)

where \( J \) denotes the Jacobian associated with the decomposition into diffeomorphisms and moduli. Evaluating this for arbitrary membrane topologies is certainly problematic since we do not have a classification of three dimensional manifolds and their moduli. What we will do instead is consider evaluating the above for a class of fixed topologies where the moduli are known. The case relevant to the fundamental string is where \( M^3 = S^1 \times \Sigma \) and the membrane is wrapped once around the M-theory circle,
fixing the $S^1$ modulus. We will also truncate to the zero mode sector of the circle. The membrane moduli space is now isomorphic to the moduli space of Riemann surfaces.

First, let us choose a gauge where,

$$\sigma^3 = X^{11}, \quad \gamma_{33} = 1$$

(10)

and since we have a trivial bundle we can also fix $\gamma_{i3} = 0$. (We have also fixed the target space metric $G_{11,11} = 1$.) When the $S^1$ bundle is nontrivial we will not be able to make this gauge choice and it is an important question how to extend this analysis for non-trivial circle bundles. After this partial gauge fixing the world volume metric is now:

$$\gamma_{\mu\nu} = \left( \begin{array}{cc} \tilde{\gamma}_{ij} & 0 \\ 0 & 1 \end{array} \right)$$

(11)

where $\tilde{\gamma}_{ij}$ denotes the string world sheet metric, $i, j$ are two dimensional world sheet indices. The radius of the $S^1$ is fixed to be $R_{11}$ thus $\sigma^3 \in [0, R_{11}]$. The action then reduces to the usual action for the string, without the dilaton coupling term.

$$S = \frac{1}{4\pi \alpha'} \int d^2 \sigma \sqrt{\tilde{\gamma}} \left( \tilde{\gamma}^{ij} \partial_i X^I \partial_j X^J G_{IJ} + \epsilon^{ij} \partial_i X^I \partial_j X^J B_{IJ} \right)$$

(12)

with $\alpha' = 1/2\pi T_{M2} R_{11}$. Note, this action now has conformal symmetry. If one had taken a rather more general Kaluža-Klein ansatz then we would have found that the two-dimensional Weyl symmetry was indeed part of the three-dimensional diffeomorphism symmetry, as described in [9].

Therefore the two dimensional conformal symmetry may be regarded as part of the three-dimensional diffeomorphism symmetry, [9]. Note, if one were to include the Kaluža-Klein modes of the circle the conformal invariance would be broken.

Since the $S^1$ modulus of the membrane is fixed, one may now follow the usual procedure of string theory to calculate the Jacobian, $J$ and evaluate the path integral for a given topology of the Riemann surface. Following, [10] the space of Riemann surface metrics is parameterised by $\hat{\gamma} = \exp(\delta v) e^{2\sigma \hat{\gamma}}$. Where $\hat{\gamma}$ is transverse to the orbits of $\text{Weyl}(\Sigma) \times \widetilde{\text{Diff}}_0(\Sigma)$ and $\exp(\delta v)$ denotes a diffeomorphism generated by the vector field $\delta v$. $\widetilde{\text{Diff}}_0$ denotes two dimensional diffeomorphisms.
Infinitesimally, the joint action of Weyl transformations and diffeomorphisms on the two dimensional metric is given by:

$$\delta \gamma_{ij} = (2\delta \sigma + \nabla^k \delta v_k) \gamma_{ij} + 2(P_1 \delta v)_{ij}$$  \hspace{1cm} (13)$$

where:

$$P_1(\delta v)_{ij} = \frac{1}{2} (\nabla_i \delta v_j + \nabla_j \delta v_i - \gamma_{ij} \nabla^k \delta v^k) .$$ \hspace{1cm} (14)$$

which maps vectors into tracefree symmetric rank two tensors. We also introduce the adjoint map

$$P_1^\dagger(\gamma)_{ij} = -2\nabla^i \gamma_{ij}$$ \hspace{1cm} (15)$$

from a symmetric tracefree two tensor to a vector. It will be useful to treat the following three cases separately, genus $g \geq 2$, the torus, $g = 1$ and the sphere, $g = 0$. The case where $g \geq 2$ is the simplest because there are no conformal Killing vectors, i.e. the kernel of $P_1$ is null. The dimension of the moduli space is given by the kernel of $P_1^\dagger$ which for $g \geq 2$ is given by:

$$\text{Ker}(P_1^\dagger) = 6g - 6 = -3 \chi .$$ \hspace{1cm} (16)$$

The Jacobian $J$ is given by:

$$J = \det(P_1^\dagger P_1)^{\frac{1}{2}} .$$ \hspace{1cm} (17)$$

(Note, for the case $g \geq 2$ there are no zero modes of $P_1$). This leads to the partition function:

$$Z = \int D X \, d^{6g-6} m \, \det(P_1^\dagger P_1)^{\frac{1}{2}} \frac{dv \, d\sigma}{\text{Vol}(\widetilde{\text{Diff}}_0 \times \text{Weyl})} e^{-S[X,m,G,B]}$$ \hspace{1cm} (18)$$

with $d^{6g-6} m$, denoting the integration over the $6g - 6$ dimensional moduli space of the Riemann surfaces with genus $g$. The integration over $v$ and $\sigma$ will then cancel the factor of $\text{Vol}(\widetilde{\text{Diff}}_0 \times \text{Weyl})$ in the denominator. One now calculates the measure on moduli space, being especially careful with the normalisation, [10, 11]. So far the above has been exactly as for the string. However, the normalisation of the string and wrapped membrane moduli will be different due to the presence of $S^1$. 

6
To calculate the measure on moduli space one typically writes the moduli in terms of quadratic differentials (see [10, 11]). Then the norm of each quadratic differential is given by:

\[ ||\gamma_{\mu\nu}||^2 = \int d^3\sigma \sqrt{\gamma}(\gamma^{\rho\sigma} - c\gamma^{\rho\nu}\gamma^{\sigma\nu})\delta\gamma_{\mu\nu}\delta\gamma_{\rho\sigma}. \]  
(19)
(c is a constant upon which nothing physical will depend and so is set to zero from now on). The measure on moduli space is then given by the product of norms (assuming an orthogonal basis for quadratic differentials). We now take the above expression and calculate the norm of the moduli space of the wrapped membrane in terms of the moduli space of the string. Evaluating the norm (19) with the metric (11) and integrating over the \( S^1 \) gives the relation between the norm of a quadratic differential for the wrapped membrane and that of the string:

\[ ||\delta\gamma_{\mu\nu}|| = \sqrt{R_{11}}||\tilde{\delta}\gamma_{ij}|| \]  
(20)
Thus, the relation between the wrapped membrane moduli space measure \( d^{6g-6}m \) and string moduli space measure \( d^{6g-6}\tilde{m} \) is given by:

\[ d^{6g-6}m = (R_{11})^{3g-3} d^{6g-6}\tilde{m}. \]  
(21)
Inserting this into (18) we write the wrapped membrane partition function in terms of string normalised moduli as follows:

\[ Z = \int D\mathcal{X} (R_{11})^{3g-3} d^{6g-6}\tilde{m} \det(P^\dagger_1 P_1)^{3/2} e^{-S[\mathcal{X};G,B]} \cdot \]  
(22)
We may then use the M-theory, string theory relations (4) to write the \( R_{11} \) factors arising from the wrapped membrane moduli space measure in terms of the dilaton, \( \phi \) and the string world sheet Euler character, \( \chi \). This gives,

\[ Z = \int D\mathcal{X} d^{6g-6}\tilde{m} \det(P^\dagger_1 P_1)^{3/2} e^{-S[\mathcal{X};G,B]-\phi\chi}. \]  
(23)
So remarkably the partition function of the wrapped membrane reproduces that of the string including the dilaton coupling to the world sheet Euler character. The latter is essential an effective action coming from a proper treatment of the wrapped membrane moduli space measure.
To complete the discussion we now need to consider the case of the sphere and the torus where $P_1$ has zero modes (i.e. there are conformal Killing vectors). Again we will follow [10, 11] treatment of the string.

The measure must now be modified so as not to integrate over zero modes:

$$
D\gamma_{\mu\nu} = (\text{det}' P_1^\dagger P_1)^{\frac{3}{2}} d' v^\mu d\sigma \text{dmoduli}
$$

where the prime denotes the restriction to the space orthogonal to the Kernel of $P_1$ ie. $(\text{Ker}(P_1))^\perp$. This may then be written as:

$$
D\gamma_{\mu\nu} = \frac{1}{\text{Vol} (\text{Ker} P_1)} (\text{det}' P_1^\dagger P_1)^{\frac{3}{2}} d' v^\mu d\sigma \text{dmoduli}
$$

where all reparameterisations are now included in $d' v^\mu$. We then must see how the norm of the conformal Killing vectors scales between the wrapped membrane and the string just as for the norms of the quadratic differentials ie. the moduli. Again using, (19) one sees that the relation (20) is also followed by the norm of the conformal Killing vectors. Thus,

$$
\text{Vol} (\text{Ker} P_1) = (R_{11})^{\frac{1}{2}} \text{Dim}((\text{Ker}(P_1)) \text{Vol}(\text{Ker} \tilde{P}_1)
$$

(26)

where the tilde denotes $P_1$ in string variables. Combining the above contributions of both the rescaling of the moduli space measure (21) and the rescaling of the volume of the kernel of $P_1$ ie. the volume of conformal Killing vectors (26) one obtains the overall scaling between the wrapped membrane partition function measure and the string partition function measure to be:

$$
\int \frac{D\gamma}{\text{Vol}(\text{Diff}_0)} = \int \frac{D\tilde{\gamma}}{\text{Vol}(\tilde{\text{Diff}}_0 \times \text{Weyl})} (R_{11})^{\frac{1}{2}} \text{Dim} \text{Ker} P_1^\dagger \text{Dim} \text{Ker} P_1
$$

(27)

where we have used that the dimension of moduli space is given by $\text{Dim} \text{Ker} P_1^\dagger$. We may now invoke the Riemann-Roch theorem:

$$
\text{Dim} \text{Ker} P_1^\dagger - \text{Dim} \text{Ker} P_1 = -3 \chi
$$

(28)

to write the total rescaling as:

$$
\frac{D\gamma}{\text{Vol}(\text{Diff}_0)} = \frac{D\tilde{\gamma}}{\text{Vol}(\tilde{\text{Diff}}_0 \times \text{Weyl})} (R_{11})^{-\frac{3}{2}} \chi = \frac{D\tilde{\gamma}}{\text{Vol}(\tilde{\text{Diff}}_0 \times \text{Weyl})} e^{-\phi \chi}. \tag{29}
$$
Thus we have again produced, as required the dilaton coupling to the world sheet Euler characteristic from the rescaling of the membrane measure.

3 Discussion

The first question to address is, given what we know about M-theory, to what extent was the above observation inevitable. The usual relations between M-theory and string theory are made in the classical sector and then (often using BPS type arguments) extrapolated into the quantum regime where we know little about M-theory. The key relation between M-theory and string theory is between the string coupling and the radius of the eleventh dimension, \( g_s = \left(R_{11}\right)^{\frac{3}{2}} \). This is usually derived from the identification of the classical action of the wrapped membrane with the fundamental string and the momentum around the M-theory circle with the D0 brane.

From the point of view presented here, that relation (in particular the power of \( \frac{3}{2} \)) is a consequence of two facts. First, the dimension of moduli space of Riemann surfaces scales like \(-3\chi\) and secondly, the measure of the moduli space of \( S^1 \times \Sigma \) is related to the measure of the moduli space of a Riemann surface, \( \Sigma \) by a factor of \( \left(R_{11}\right)^{\frac{3}{2}} \text{dim(modulispace)} \). This then gives the appropriate coupling between the dilaton and the Euler characteristic of the string word sheet in M-theory language. It did not have to be this way. The M-theory, string theory identification could have been anomalous for the membrane, that is, although the classical actions are equivalent; the partition functions need not be. What we have shown here is that the measure of the wrapped membrane partition function does indeed match the string theory partition function measure.

Importantly, the string coupling to the world sheet Euler character is not put in by hand but arises naturally from the membrane partition function measure as one would hope for in a nonperturbative theory.

One might also ask, what is the justification for our ad hoc restriction of the membrane topology and the truncation to the zero modes of \( \partial_\theta \), ie. none of the fields have dependence on the M-theory circle.
Consider the expansion of the world volume fields on a circle,

\[ X^{11} = N\theta + \sum_k (e^{ik\theta} + e^{-ik\theta}) + p\tau \] \hspace{1cm} (30)

\[ X^i = \sum_l (e^{il\theta} + e^{-il\theta}) . \] \hspace{1cm} (31)

\( N \) is the membrane winding number and corresponds to the number of fundamental strings. \( p \) is the momentum around the M-theory circle and corresponds to the D0 brane charge. In order to satisfy the boundary conditions \( k \) and \( l \) must be integer. \( k \) corresponds to the \( D0 - \bar{D}0 \) states while \( l \) corresponds to the states of the non wrapped membrane. We can ignore these modes if the M-theory circle is small with respect to the Planck scale. That is weak coupling in string theory language. The action for the non wrapped membrane will be larger than for the wrapped membrane and so the wrapped membrane will dominate. The D0 brane states will also be more massive than the states we are considering. Thus we truncate to the \( k = 0, l = 0, p = 0, N = 1 \) sector. Essentially leaving a single fundamental string. This is of course consistent with what we know about string theory in that the string is a consistent object at weak coupling but at strong coupling the contribution of nonperturbative objects such as D-branes becomes important. Redoing the above calculation of the membrane measure but including these truncated states may give interesting couplings between the dilaton and D-branes states bound to the fundamental string.

Lastly, we ignored the possibility of the world volume having boundaries, as would be expected when there are external states attached to the string worldsheet. All of the results above can be trivially extended to cover that possibility.

Integrating over metrics modulo diffeomorphisms, is obviously a topological invariant. It would be interesting to know if this can be evaluated in other circumstances, i.e. for membranes with more nontrivial three dimensional topologies. This would then give more credence to membrane instantons.
4 Acknowledgements

We wish to thank the following people for discussions: James Bedford, Jan de Boer, Nick Dorey, Michael Green, Sean Hartnoll, Jim Liu, Lubos Motl, Is Singer, Paul Townsend, Neil Turok and in particular Robert Helling for initial discussions. DSB is supported by EPSRC grant GR/R75373/02 and would like to thank DAMTP and Clare Hall Cambridge for continued support. This work was in part supported by the EC Marie Curie Research Training Network, MRTN-CT-2004-512194.

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