Two-dimensional electrostatic solitary structures in electron–positron plasmas

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Abstract
Due to the inertia symmetry the issue of whether electrostatic solitons may possibly form in pair plasmas has been addressed in a number of papers. Recently we have shown that pair solitons with interlacing electron and positron holes in electron–positron plasmas may form by means of streaming instability based on one-dimensional particle-in-cell simulations Jao and Hau (2012 Phys. Rev. E 86 056401). In this paper we present the first simulation results of two-dimensional electrostatic solitons in pair plasmas imbedded in a stationary background magnetic field. It is shown that the features of electrostatic solitary structures may depend on the ratio of \( \Omega_p \) to \( \omega_p \), where \( \Omega_e \) and \( \omega_p \) denote the electron cyclotron and plasma frequency, respectively. In particular, for weakly magnetized or unmagnetized plasmas with \( \Omega_e / \omega_p < 0.5 \), both parallel and transverse electric field with the same order of magnitude may first develop and be dissipated in the nonlinear stage such that electrostatic solitons are unable to form by the streaming instability, while for \( \Omega_e / \omega_p > 0.5 \) pair solitons resembling those occurring in one-dimensional simulations may possibly form. Comparisons between linear fluid theory and particle-in-cell simulations are made.

Introduction
In light of the important applications in certain astrophysical environments, the physics of electron–positron plasmas has been widely studied in recent years [e.g., 1–3]. Among which, a fundamental issue of whether electrostatic solitons may actually form in pair plasmas with inertia symmetry has been addressed in several papers [e.g., 4–10]. Recently we have demonstrated for the first time the formation of pair solitons in electron–positron plasmas by means of streaming instability based on one-dimensional particle-in-cell simulations [11]. In particular, it is shown that periodic positive and negative potential with interlacing electron and positron holes in phase space may be generated by current-free electron and positron beams streaming in stationary electron–positron background plasmas. These positron solitons with single polarity in electric potential may also form in a stable manner by means of bump-on-tail instabilities as recently shown by Jao and Hau [12]. Despite the progress made by several authors up to date the study of electrostatic solitons in pair plasmas is based on one-dimensional models [5, 6, 11, 12] which do not permit the growth of electrostatic solitary structures after hundreds of electron plasma periods [18]. For electron–ion plasmas past studies of electrostatic solitons have shown that electron hole structures may not necessarily exist in a steady manner in multi-dimensional simulations of unmagnetized [13] and magnetized plasmas [14–18]. For the latter cases the evolutionary features of solitary structures may depend on the parameter \( \Omega_e / \omega_p \), where \( \Omega_e \) and \( \omega_p \) denote electron cyclotron and plasma frequency, respectively. In particular, for \( \Omega_e / \omega_p = 0.2 \) corresponding to a very weak magnetic field and the condition in Earth’s plasmas sheet boundary layer, only structures with small amplitudes could exist in the simulation system [14]. While for \( \Omega_e / \omega_p = 1.0 \), two-dimensional electrostatic solitons excited by the lower hybrid waves may form during evolutionary process which become one-dimensional structures after hundreds of electron plasma periods [18]. For \( \Omega_e / \omega_p = 5.0 \) one-dimensional solitary structures may tend to break due to the generation of electrostatic whistler waves or oblique Langmuir modes [16, 17]. Assuming immobile ions in the simulation system, Miyake...
et al [15] have shown that two-dimensional solitons may be excited in the linear stage of the streaming instability which in the nonlinear phase may become one-dimensional structures for strong magnetic field case but can be dissipated with no significant electric structures for weak magnetic field case.

Self-consistent steady state model for electron holes in magnetized electron–ion plasmas has been constructed and its stability is examined by Muschietti et al [19–21] who have proposed the so-called transverse instability. By loading the electron phase–hole velocity distribution in the two-dimensional particle simulation system, the relation between \( \Omega_e \) and \( \omega_p \) is shown to be the key factor for determining the stability of the hole structures, where \( \omega_p \) is the bounce frequency of trapped electrons. In particular, for \( \omega_p < \Omega_e \) electrostatic solitons are shown to be stable while for \( \omega_p > \Omega_e \) they may be destroyed by the transverse instability. Wu et al [22] have conducted the detailed studies by varying the magnitude of ambient magnetic field and the amplitude of electric potential structure. The correlation between the parameter \( \Omega_e / \omega_p \) and the stability of electric structures can also be calculated from the simulation results of streaming instability [18, 23, 24].

In this paper we present the first study of electrostatic solitary structures in electron–positron plasmas based on two-dimensional relativistic full particle simulations. The effects of background magnetic field on the evolutionary process of streaming instability are examined. The two-dimensional streaming instability is also examined by the linear fluid theory which is compared with the simulation results of earlier stage. The characteristics of evolutionary features are also analyzed by the relation between electron/positron cyclotron frequency and plasma frequency as well as the bounce frequency of trapped electrons/positrons.

### Simulation model and parameters

In this study we focus on the generation mechanism of electron–positron solitons in a four-component pair plasma with background stationary electrons and positrons (denoted by the quantities with subscripts \( e1 \) and \( p1 \), respectively) and streaming electrons and positrons (denoted by the quantities with subscripts \( e2 \) and \( p2 \), respectively) along the background magnetic field which is the condition for the formation of pair solitons in electron–positron plasmas [11]. In the simulation dimensionless units are used with the thermal velocity of background electrons being \( v_{th} = 1.0 \) and the plasma frequency of total electrons being \( \omega_p = 1.0 \). The size of the simulation domain is chosen to be \( L_x \times L_y = 256 \lambda_D \times 256 \lambda_D \) with periodic boundary in both \( \hat{x} \) and \( \hat{y} \) directions. The grid size \( \Delta x \times \Delta y = 1 \lambda_D \times 1 \lambda_D \) and the time interval \( \Delta t = 0.01 \omega_p^{-2} \) are adopted for all cases. The number density is the same for all four components \( n_{e1} = n_{e2} = n_{p1} = n_{p2} \) with 64 pairs per cell over the system. The initial particle velocity of all components are isotropic and described by the Maxwellian distribution with the same thermal velocity \( v_{th} \). The drift velocity of streaming electrons and positrons is \( u_{e2} = u_{p2} = 20.0 v_{th} \) along the background magnetic field direction \( \hat{x} \). To study the effect of background magnetic field on the formation of electric solitons the parameter \( \Omega_e / \omega_p \) is set from 0 to 10 in various cases. For comparison we have carried out the simulations for both one and two-dimensional models with the same parameters which can help to resolve the effects of multi-dimensions on the formation of electrostatic solitons in pair plasmas.

### Linear theory and model calculations

Before showing the simulation results we shall first examine the oblique streaming instability based on the linear fluid theory. By linearizing the continuity, momentum, adiabatic energy equations and Poisson equation for oblique electrostatic waves, we obtain the following dispersion relation for electrostatic streaming instability in multi-component plasmas

\[
\sum_a \left\{ \left( \omega^2 - k_x u_{a,0} \cos \theta \right)^2 - \Omega_a^2 \cos^2 \theta \right\} / \left[ \left( \omega^2 - k_x u_{a,0} \cos \theta \right)^2 - \chi_a k^2 v_{th,a}^2 \right] \\
\times \left[ \left( \omega^2 - k_x u_{a,0} \cos \theta \right)^2 - \Omega_a^2 \right] \\
- \Omega_a^2 k^2 v_{th,a}^2 \sin^2 \theta \right\} = 1,
\]

where \( v_{th,a}^2 \equiv k_B T_a / m_a, \omega_{p,a}^2 \equiv n_{e0} q_a^2 e^2 / \epsilon_0 m_a, \) and \( \Omega_a \equiv |q_a| B / m_a. \) The fluid variables such as the number density and drift velocity with subscript \( 0 \) denote the equilibrium quantities, and \( \theta \) is the angle between the wave vector \( \hat{k} \) and the background magnetic field \( \hat{B} \).

Figure 1 shows the imaginary part of wave frequency \( \omega_i \) as functions of wave number \( k_x \) and \( k_y \) with varying background magnetic field, \( \Omega_e / \omega_p = 0.1 \) (panel (a)), \( \Omega_e / \omega_p = 0.5 \) (panel (b)), \( \Omega_e / \omega_p = 1.0 \) (panel (c)), and
Figure 1. The imaginary part of the wave frequency $\omega_i$ as functions of wave number $k_x$ and $k_y$ derived from the linear fluid theory for the case of (a) $\Omega/\omega_p = 0.1$, (b) $\Omega/\omega_p = 0.5$, (c) $\Omega/\omega_p = 1.0$, and (d) $\Omega/\omega_p = 2.0$.

Figure 2. The time evolution of logarithm of maximum electric field $E_x$ (dashed line) and $E_y$ (dotted–dashed line) for the case of (a) $\Omega/\omega_p = 0.1$, (b) $\Omega/\omega_p = 0.5$, (c) $\Omega/\omega_p = 1.0$, and (d) $\Omega/\omega_p = 2.0$ from two-dimensional simulations. The dotted lines in each panel are the corresponding results from the one-dimensional simulation.
As indicated, the unstable mode with \( k_x = 0.095 \omega_p / v_{th} \) and \( k_y = 0 \) (i.e., parallel propagation) is the most unstable mode in all cases. This result is expected since the drift velocity has the maximum value at parallel propagation where the background magnetic field plays no role and Langmuir–Langmuir resonance between stationary and beam plasmas is the cause of streaming instability. For the weakly magnetized case as shown in figure 1(a) \( \Omega_{\omega} = 0.1 \), for \( \theta < 75^\circ \) the growth rate with the most unstable mode occurring at \( k_x = 0.095 \omega_p / v_{th} \) is nearly the same as the parallel propagation mode. As shown in figure 1(b) for the case of \( \Omega_{\omega} / \omega_p = 0.5 \), the growth rate for oblique propagation mode is decreased as compared to the case of \( \Omega_{\omega} / \omega_p = 0.1 \) for the same propagation angle. For the cases with even stronger magnetic field as shown in figures 1(c) and (d) \( \Omega_{\omega} / \omega_p = 1.0 \) and \( \Omega_{\omega} / \omega_p = 2.0 \), the growth rates of oblique propagation modes are much smaller than the parallel propagation mode and the most unstable region occurs only within \( \theta < 25^\circ \).

The linear fluid theory implies that due to the cyclotron motion the effect of strong background magnetic field is to suppress the oblique streaming instability where Langmuir–Langmuir, Langmuir–cyclotron as well as cyclotron–cyclotron resonances may all take place and interfere each other.

Figure 2 shows the time evolution of logarithm of maximum electric field \( E_x \) and \( E_y \) for four different cases \( \Omega_{\omega} / \omega_p = 0.1 \) (panel (a)), \( \Omega_{\omega} / \omega_p = 0.5 \) (panel (b)), \( \Omega_{\omega} / \omega_p = 1.0 \) (panel (c)), and \( \Omega_{\omega} / \omega_p = 2.0 \) (panel (d)).
based on one and two-dimensional particle simulations. We have found from two-dimensional simulation results that the evolution processes of streaming instability show distinct features for \( \Omega / \omega_p < 0.5 \) and \( \Omega / \omega_p > 0.5 \) corresponding to the relatively weaker and stronger magnetic field cases, respectively. In particular, for \( \Omega / \omega_p < 0.5 \), the electric field component \( E_y \) may grow to the same order as the \( E_x \) and both \( E_x \) and \( E_y \) are damped heavily after the peak value (see panel for \( \Omega = 0.1 \)), while for \( \Omega / \omega_p > 0.5 \) the evolution curves of \( E_x \) are in general similar in both one- and two-dimensional simulations with \( E_y \) much smaller than \( E_x \) in the final stage of nonlinear evolution (see panels (c) and (d) for \( \Omega = 1.0 \) and 2.0, respectively). Note that for \( \Omega / \omega_p = 0.5 \) (panel (b)) the time evolution of electric field also shows distinct features from the case of \( \Omega / \omega_p > 0.5 \) and \( \Omega / \omega_p < 0.5 \) in that both \( E_x \) and \( E_y \) are slightly damped only for a short period of time. As indicated in all panels, the electric field \( E_x \) in one-dimensional system (dotted line) grows and becomes saturated slightly earlier than the corresponding two-dimensional system (dashed line). It is also seen that the peak value of \( E_x \) approaches the same order in both one and two-dimensional simulations.

The two-dimensional spatial distributions of electric potential at \( t = 16 \omega_p^{-1} \) when the electric field reaches the peak value are presented in figure 3. As indicated, the electric potential structures are oriented in the oblique direction relative to the background magnetic field for the case of \( \Omega = 0.1 \) (panel (a)). While for the cases of \( \Omega = 1.0 \) (panel (b)) and \( \Omega = 2.0 \) (panel (c)) the structures are similar to the typical one-dimensional simulation results. Figure 4 shows the wave number spectrum analysis \( (k_x - k_y) \) of electric potential at \( t = 16 \omega_p^{-1} \) for the case of (a) \( \Omega = 0.1 \), (b) \( \Omega = 1.0 \), and (c) \( \Omega = 2.0 \).
potential for the case of $\Omega_\omega/\omega_p = 0.1$, $\Omega_\omega/\omega_p = 1.0$, and $\Omega_\omega/\omega_p = 2.0$. It is shown that the wave mode with $k_x \approx 0.095\omega_p/v_{th}$ is the most unstable mode in all cases and for $\Omega_\omega/\omega_p = 0.1$ there is a pronounced $k_y$ as compared to the cases of $\Omega_\omega/\omega_p = 1.0$ and $\Omega_\omega/\omega_p = 2.0$. The results shown in figure 4 are consistent with the linear fluid theory shown in figure 1.

As shown in figure 2 for the time evolution of $E_x$ and $E_y$ in the weakly magnetized system (figure 2(a)) the fluctuations are damped heavily after the peak value which is in contrast with the strongly magnetized systems (figures 2(c) and (d)). Figure 5 shows the two-dimensional spatial distributions of electric potential at $t = 48\omega_p^{-1}$. As indicated, the amplitude of the electric potential for the case of $\Omega_\omega/\omega_p = 0.1$ (panel (a)) is smaller than the cases of $\Omega_\omega/\omega_p = 1.0$ (panel (b)) and $\Omega_\omega/\omega_p = 2.0$ (panel (c)) which is in agreement with the time evolution of the electric field (figure 2). It is also seen that for the case of $\Omega_\omega/\omega_p = 1.0$ the electric potential structures seemly have no variation between $t = 160\omega_p^{-1}$ (figure 5(b)) and $t = 48\omega_p^{-1}$, remaining to be one-dimensional (figure 5(b)) for $t < 100\omega_p^{-1}$ and becoming two-dimensional during $100\omega_p^{-1} < t < 150\omega_p^{-1}$ as a result of merging process, and for $t > 150\omega_p^{-1}$ the structure evolve to one-dimensional again. For the case of $\Omega_\omega/\omega_p = 2.0$ the electric potential structures also become two-dimensional for certain periods of time in the nonlinear stage (figure 5(c)) which may take place earlier than the case of $\Omega_\omega/\omega_p = 1.0$.

Figure 5. The two-dimensional spatial distributions of electric potential at $t = 48\omega_p^{-1}$ for the case of (a) $\Omega_\omega/\omega_p = 0.1$, (b) $\Omega_\omega/\omega_p = 1.0$, and (c) $\Omega_\omega/\omega_p = 2.0$ from two-dimensional simulations.
By examining the particle’s motion along the background magnetic field at $Y = 199 \sim 201 \lambda_D$, we plot the phase diagram of positrons (top panel) and electrons (bottom panel) for three cases at $t = 48 \omega_p^{-1}$ in figure 6. As indicated, there is no clear solitary structure on the phase diagram for the case of $\Omega_e/\omega_p = 0.1$ (panel (a)) while for the case of $\Omega_e/\omega_p = 1.0$ (panel (b)) interlacing electron and positron holes are present which seems identical.

Figure 6. The phase diagram of positrons (top panel) and electrons (bottom panel) at $t = 48 \omega_p^{-1}$ and $Y = 200 \pm 1 \lambda_D$ for the case of (a) $\Omega_e/\omega_p = 0.1$, (b) $\Omega_e/\omega_p = 1.0$, and (c) $\Omega_e/\omega_p = 2.0$ from two-dimensional simulations.

Figure 7. The two-dimensional spatial distributions of electric potential at $t = 256 \omega_p^{-1}$ for the case of (a) $\Omega_e/\omega_p = 1.0$ and (b) $\Omega_e/\omega_p = 2.0$ from two-dimensional simulations.
to the one-dimensional simulation results [11]. Note that the merging process of the hole structures starts to take place which is not seen in our one-dimensional simulations [11] in the latter stage of nonlinear evolution (say, $t = 48\omega_p^{-1}$). The merging process occurs in earlier stage for the case of $\Omega_e/\omega_p = 1.0$ and $\Omega_e/\omega_p = 2.0$ at $t = 256\omega_p^{-1}$. (For the case of $\Omega_e/\omega_p = 0.1$ there exist virtually no solitary structures in the late stage of nonlinear evolution.) As indicated in figure 7 for $t = 256\omega_p^{-1}$, the electric potential structures for both cases become nearly one-dimensional and may sustain in the late stage of nonlinear evolution. But the number of solitary structures is decreased from three to two and one for the case of $\Omega_e/\omega_p = 1.0$ and $\Omega_e/\omega_p = 2.0$, respectively. The corresponding phase diagrams shown in figures 8(a) and (b) exhibit the interlacing electron and positron holes with different spatial length in a steady state manner. Since the relation between gyro-frequency $\Omega_e$ and bounce frequency $\omega_b$ of trapped particles may be used to determine the stability of the hole structures in electron–ion plasmas [15, 18, 20, 23, 24], we have also deduced the value $\omega_b/\Omega_e$ to examine the stability of formed electric soliton for electron–positron plasmas. We have estimated the $\omega_b$ value of trapped particles based on $\omega_b = \sqrt{ek_x |\vec{E}_x|/m}$, where $|\vec{E}_x|$ is the average amplitude of the parallel electric field $E_x$ and $k_x$ is the parallel wave number of the most unstable wave [15, 18, 23]. Based on the simulation data the estimated values $\omega_b/\Omega_e = 0.194$ and $\omega_b/\Omega_e = 0.071$ are derived for the case of $\Omega_e/\omega_p = 1.0$ and $\Omega_e/\omega_p = 2.0$, respectively. It is found that stable hole structures can exist in electron–positron plasmas provided that the parameter $\omega_b/\Omega_e$ is less than one.

The electron velocity distributions at various times in one- and two-dimensional simulations are shown in figure 9. Note that the velocity distributions of electrons and positrons are almost identical in the simulations. Top panels show $v_x$ distributions at $t = 256\omega_p^{-1}$, indicating that only for the weak magnetic field case $\Omega_e/\omega_p = 0.1$ (panel (a)) the differences between one- (dashed line) and two-dimensional (solid line) simulation results are evident due to the fact that particle trapping and electric soliton do not occur in the weakly magnetized system. Bottom panels show $v_y$ (in the direction perpendicular to the background magnetic field) distributions, indicating that particle heating is evident in the case of $\Omega_e/\omega_p = 0.1$ (panel (a)). Comparing $v_y$ distributions at $t = 16\omega_p^{-1}$ (dotted–dashed line) and $t = 256\omega_p^{-1}$ (solid line) shows that particle heating occurs in the nonlinear stage when the electric field energy is severely damped. While for stronger magnetic field case

![Figure 8](image_url)

**Figure 8.** The phase diagram of positrons (top panel) and electrons (bottom panel) at $t = 256\omega_p^{-1}$ and $Y = 200\pm1\lambda_D$ for the case of (a) $\Omega_e/\omega_p = 1.0$ and (b) $\Omega_e/\omega_p = 2.0$ from two-dimensional simulations.
(solid line in panel (b)), slight heating also occurs in the perpendicular direction as shown in the bottom panel of figure 9(b). It is found that for $\Omega_p/\omega_p \gtrsim 10.0$ (panel (c)) the particle heating in the perpendicular direction no longer occurs.

The results presented in figures 2–9 consistently show that the electron/positron hole structures and solitary waves seen in one-dimensional simulations may possibly exist in two-dimensional simulations only for relatively stronger magnetic field case which can be interpreted as follows. In accordance with the linear theory, oblique streaming instability with significant growth rate may occur for relatively weak magnetic field while oblique streaming instability may become insignificant for relatively strong magnetic field case. In two-dimensional simulations the fastest growing mode takes place naturally which for relatively stronger magnetic field case is the parallel propagation but for weaker magnetic field case may also occur at oblique propagation. As a result, stronger magnetic field case tends to develop one-dimensional structures while weaker magnetic field case with weak cyclotron motion has more freedom to develop oblique structures which however are not in favor of hole structure formation due to the relatively weak Langmuir–Langmuir resonance.

Conclusion

Based on one-dimensional particle-in-cell simulations, we have recently demonstrated the formation of electrostatic solitons in three- and four-component electron–positron plasmas [11, 12]. In this study we further examine the formation of electrostatic solitons and the effects of background magnetic field on the generation mechanism based on two-dimensional particle simulations and compare the results with the corresponding one-dimensional cases. The linear fluid theory is employed and compared with the earlier evolutionary process of particle simulation results. According to the simulation results, the electrostatic soliton can only form and steadily exist in electron–positron plasmas for relatively stronger background magnetic field.

For weakly magnetized or unmagnetized electron–positron plasmas no electron or positron hole structures may possibly form in two-dimensional system. While in weakly magnetized or unmagnetized electron–ion plasmas the hole structures may form first and be destroyed in the later nonlinear stage [13, 15, 17]. The simulations show that in the linear stage the transverse electric field may develop with the same order as the electric field parallel to the background magnetic field. This result is consistent with the linear fluid theory which predicts a relatively larger growth rate for oblique propagation in weakly magnetized plasma systems. It is also found that in the nonlinear stage the saturated electric fields are strongly damped with the electric field energy being transformed to the particle heating in the direction perpendicular to background magnetic field.

For relatively stronger magnetic field, one-dimensional interlacing electron and positron hole structures may form in the earlier nonlinear stage with the same wavelength as predicted by the linear fluid theory.
two-dimensional system, the electron and positron hole structures however may merge with each other in the nonlinear stage. Similar to the evolutionary process in electron–ion plasmas [15], the structure of electrostatic solitons may develop transient two-dimensional features and become one-dimensional again after about a hundred of electron plasma periods. It is also found that the relation between electron cyclotron and bounce frequencies of trapped electrons [15, 18, 20, 23, 24] may also determine whether electrocylinder and bounce frequencies of trapped electrons [15, 18, 20] may also determine whether electrostatic solitons may exist steadily in the two-dimensional system for electron–positron plasmas.

Acknowledgments

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