Research on the Pricing of Chinese Artwork Based on Unascertained Measure and GM (1, N) Model

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Abstract. The Chinese art market has developed rapidly in recent years, but the problem of fuzzy and irregular art pricing has always perplexed the participants in the art market. This article proposes a new idea of dividing the influence factors of artworks according to the static and dynamic dimensions, so as to study the attributes and external factors of artworks and analyse the causes of price fluctuations of artworks better. The pricing model proposed in this paper applies the Unascertained Measure Model to the comprehensive evaluation of static attributes of artworks. In addition, the GM (1, N) Model is constructed by combining the static attribute comprehensive evaluation score with other dynamic influencing factors, so that the model has the characteristics of qualitative and quantitative combination, scientific and objective. Finally, this paper proves the validity of the model through empirical research.

1. Introduction
In recent years, the Chinese art market has been developing continuously. In 2019, the total auction turnover of pure art market in China was 4.102 billion US dollars.

As for the pricing of artworks, there are mainly Average Price Method, Repeated Sale Method and Hedonic Method in foreign research, while the main domestic research methods include Comparable Comparison Pricing and Hedonic Method[1-3].

Zhaoyan Wang applied the Grey System Theory to the pricing of works of art, which proved the feasibility of the Grey System Theory in the price prediction of works of art[4]. Shuailai Li[5]. Bin Liao and Lulin Xu[6] all used Unascertained Measure and other methods to predict the price of artworks, which enriched the theory of artworks pricing.

2. Art pricing model construction

2.1. Analysis of factors influencing art price
The artwork is a special product, and the factors that affect its price are also multi-angle. The intrinsic properties of artworks hardly change over time, but the external factors will constantly change over time. In order to better study the properties and external factors of artworks, and to better reflect the dynamic change process of the price of artworks, this paper puts forward a new idea to classify the influencing factors of artworks' price according to the two dimensions of static and dynamic.

The so-called static influence factor refers to the influence factor that hardly changes with time. These factors mainly reflect the intrinsic properties of artwork, and are more related to the artwork itself. The intrinsic properties of artworks are fixed the moment they are created and will hardly
change in a short period of time. Common static factors include: creation time, work size, content and subject matter, artistic creation level, material, mounting form, and product equality.

The so-called dynamic influencing factors refer to the influencing factors that constantly change dynamically over time. In the market, artworks will experience the process of trading, collection and exhibition, and the social, cultural, and economic environment people live in will also change. In this process, many external factors of artworks will lead to dynamic changes in the price of artworks. Common dynamic factors include: artist reputation index, GDP, and social education level.

2.2. Comprehensive evaluation of static attributes of artwork

2.2.1. Unascertained Measure Model and its characteristics. The Unascertained Measure Model is a comprehensive evaluation method based on the Unascertained Measure Theory. As a special commodity, artwork has a strong heterogeneity. Static factors of art, using Unascertained Measure Model to get the comprehensive evaluation score, can fully consider the opinions of the experts, at the same time, the introduction of information entropy to weight of indicators, can reduce the influence of subjective, and reflects the characteristics of art at the same time, make the art static attribute comprehensive evaluation is objective and scientific.

First of all, it is assumed that there are \( n \) sample artworks that need to be comprehensively evaluated by static attributes, respectively represented by \( x_1, x_2, \ldots, x_n \). There are \( m \) evaluation indexes reflecting the static attributes of artworks, which are respectively represented by \( i_1, i_2, \ldots, i_m \). There are \( p \) sample experts, represented by \( s_1, s_2, \ldots, s_p \) respectively. And \( x_{ij} \) is defined as the evaluation score of the sample artwork \( x_i \) given by the sample expert \( s_j \) under the evaluation index \( i_j \).

2.2.2. Evaluation space. Suppose that \( c_1, c_2, \ldots, c_z \) is the \( z \) evaluation interval corresponding to the expert evaluation score \( x_{ij} \), then the evaluation space vector \( c \) can be written as \( c = (c_1, c_2, \ldots, c_z) \), where \( (c_1, c_2, \ldots, c_z) \) is an ordered segmentation set of the evaluation space vector \( c \). According to the classification criteria of membership between each rank vector, the membership quantization standard \( a_{jk} \) can be obtained. Where, \( a_{jk} \) indicates that indicator \( i_j \) obtains the \( c_k \) standard of evaluation grade.

2.2.3. Unascertained Measure of a single index. According to the Unascertained Measure Theory, due to the limitations such as different personal preferences, emotions, and knowledge of field experts, as well as insufficient comprehensiveness of information grasp, experts will have different evaluation levels for various static influencing factor indexes of artworks. At this point, \( u_{ijk} \) is defined as:

\[
 u_{ijk} = \frac{\text{number of experts rated } c_k \text{ for index } i_j \text{ of } x_i}{\text{Sample number of experts } p} \tag{1}
\]

\( u_{ijk} \) reflects the distribution of evaluation grade \( c_k \) obtained by sample artwork \( x_i \) on static attribute evaluation index \( i_j \), and also indicates the concentration degree of sample experts on sample artwork \( x_i \) on different evaluation criteria \( c_k \).

And \( u_{ijk} \) can be defined as Unascertained Measure of a single index. For sample artwork \( x_i (i = 1, 2, \ldots, n) \), its single-index measure matrix is:

\[
 \begin{pmatrix}
 u_{i11} & \cdots & u_{i1z} \\
 \vdots & \ddots & \vdots \\
 u_{in1} & \cdots & u_{inz}
\end{pmatrix}
\tag{2}
\]
2.2.4. **Indexes based on information entropy are objectively weighted.** When using information entropy to objectively assign static indexes of artworks, the information entropy of unascertained measure \( u_{ijk} \) should be obtained first:

\[
H_j = -\sum_{k=1}^{z} u_{ijk} \cdot \log u_{ijk}
\]  
(3)

Then, an intermediate variable \( v_j \) is introduced, which is defined as:

\[
v_j = 1 - \frac{H_j}{\log z}
\]  
(4)

Furthermore, the entropy weight coefficient \( w_j \), which represents the index weight of the sample artwork, is defined as follows:

\[
w_j = \frac{v_j}{\sum_{j=1}^{n} v_j}
\]  
(5)

It can be seen above that the smaller the information entropy \( H_j \) is and the closer the value of \( v_j \) is to 1, the larger the entropy weight coefficient \( w_j \) will be. This indicates that the model will assign higher index weight to the indexes with higher representativity of the disclosed information.

Thus, we can get the entropy weight coefficient vector \( w_j = (w_{j1}, w_{j2}, \cdots, w_{jm}) \), which represents the weight of each index of the sample artwork \( x_j \). Where, \( w \) can be defined as the weight vector of the sample expert's evaluation of each attribute of the sample artwork.

2.2.5. **Comprehensive evaluation grade score.** Through the above derivation, we can get:

\[
P_j = (u_{ijk})_{m \times z} \cdot \mathbf{c}^T = \left( \sum_{k=1}^{z} u_{ij1} \cdot c_z, \sum_{k=1}^{z} u_{ij2} \cdot c_z, \cdots, \sum_{k=1}^{z} u_{ijn} \cdot c_z \right)
\]  
(6)

Where, \( P_j \) is the evaluation score vector of order \( 1 \times m \), which represents the score of sample artwork \( x_j \) under each index, and the comprehensive score of sample artwork \( x_j \) under index \( i_j \) is \( \sum_{k=1}^{z} u_{ijk} \cdot c_z \).

Then, the evaluation score vector \( P_j \) is multiplied by the transpose of the weight coefficient vector \( w_i \), which represents the weight, and we can get:

\[
T_i = P_j \cdot w_i^T = \sum_{j=1}^{n} \sum_{k=1}^{z} w_j \cdot u_{ijk} \cdot c_z
\]  
(7)

\( T_i \) is the static attribute comprehensive evaluation score of sample artwork \( x_i \).

Through the Unascertained Measure Model, we can get the static attribute comprehensive evaluation score vector \( T = (T_1, T_2, \cdots, T_n) \) of some sample artworks.

2.3. **Art pricing model based on Grey System Theory**

2.3.1. **Grey System Theory and its characteristics.** Grey System Theory is a new method to study part of the information is known, part of the information is unknown, the internal relationship of the system is not clear, it is widely used in the study of the problem of few samples, few data. The problems of art pricing just coincide with the characteristics of Grey Forecasting Model. Therefore, this paper will apply the grey predictive GM (1, N) Model to construct the artwork pricing function.
2.3.2. GM (1, N) Model. The Grey Multivariable Forecasting Model, namely GM (1, N) Model, is adopted to construct the art pricing model. First, it is assumed that there are \( n \) sample artworks used to build the pricing model, and the corresponding prices are represented by \( y \), and \( m \) dynamic influencing factor indexes are considered, respectively \( x_1, x_2, \ldots, x_m \), one of the indicators is the static attribute comprehensive evaluation score of the sample artwork obtained from the discussion in Section 2.2.

The original influencing factor series and price series are as follows:

\[
x^{(0)}(k) = (x_1^{(0)}(1), x_2^{(0)}(2), \ldots, x_n^{(0)}(n)) \quad i = 1,2,\ldots, m
\]

\[
y^{(0)}(k) = (y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(n))
\]

Then, an accumulated generating operation is done for \( x^{(0)}(k) \) and \( y^{(0)}(k) \) to obtain the accumulation generation series:

\[
x^{(1)}(k) = (x_1^{(1)}(1), \sum_{j=1}^{k} x_1^{(0)}(j), \ldots, \sum_{j=1}^{k} x_n^{(0)}(j)) = (x_1^{(1)}(1), x_2^{(1)}(2), \ldots, x_n^{(1)}(n)) \quad k = 1,2,\ldots, n
\]

\[
y^{(1)}(k) = (y^{(1)}(1), \sum_{j=1}^{k} y^{(0)}(j), \ldots, \sum_{j=1}^{k} y^{(0)}(j)) = (y^{(1)}(1), y^{(1)}(2), \ldots, y^{(1)}(n)) \quad k = 1,2,\ldots, n
\]

The time \( k \) of the above sequence is regarded as a continuous time variable \( t \), and the sequence is regarded as a function of time \( t \). If the sequence \( x^{(1)}(k), x_2^{(1)}(k), \ldots, x_n^{(1)}(k) \) influences the rate of change of \( y^{(1)}(k) \), then \( x^{(1)}(k) \) and \( y^{(1)}(k) \) satisfy the following first-order linear differential equation model:

\[
\frac{dy^{(1)}(k)}{dt} + ay^{(1)}(k) = b_1x_1^{(1)}(k) + b_2x_2^{(1)}(k) + \cdots + b_n x_n^{(1)}(k)
\]

The above equation is called the differential equation of GM (1, N) Model, also known as the shadow equation. Where, \( a \) is the development coefficient, \( b_i x_i^{(1)}(k) \) is the system driver term, and \( b_j \) is the driving coefficient.

According to the definition of derivative, there are:

\[
\lim_{\Delta t \to 0} \frac{y^{(1)}(t) - y^{(1)}(t-\Delta t)}{\Delta t}
\]

The above formula is discretized to obtain:

\[
\frac{\Delta y^{(1)}(k)}{\Delta t} = \frac{y^{(1)}(k) - y^{(1)}(k-1)}{k - (k - 1)} = y^{(1)}(k) - y^{(1)}(k-1) = y^{(1)}(k)
\]

In addition, \( z^{(1)}(k) \) is used to represent the accumulating generation sequence of \( y^{(1)}(k) \), where:

\[
z^{(1)}(k) = \frac{y^{(1)}(k) + y^{(1)}(k-1)}{2} \quad k = 2,3,\ldots, n
\]

Thus, the differential equation of GM (1, N) Model, namely equation (12), can be written as follows:

\[
y^{(1)}(k) + az^{(1)}(k) = b_1x_1^{(1)}(k) + b_2x_2^{(1)}(k) + \cdots + b_n x_n^{(1)}(k) \quad k = 2,3,\ldots, n
\]

The above equation is called the grey differential equation of the GM (1, N) Model and is also the basic form of the GM (1, N) Model. Write the parameter column as \( \mathbf{a} = (a, b_1, b_2, \cdots, b_n)^T \). Equation (16) is written in matrix form:

\[
\mathbf{Y} = \mathbf{B} \mathbf{a}
\]

According to the least square method, the parameter column can be solved:

\[
\hat{\mathbf{a}} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{Y}
\]
According to Equation (18), the parameter column \( \hat{\alpha} = (\hat{\alpha}_1, \hat{\alpha}_2, \ldots, \hat{\alpha}_n)^T \) can be obtained and substituted into the differential equation, i.e., Equation (12), and then solved:

\[
\hat{\gamma}^{(1)}(k + 1) = \left( y^{(1)}(0) - \frac{1}{\alpha} \sum_{i=1}^{n} \hat{b}_i x_i^{(1)}(k + 1) \right) e^{-\hat{\alpha}k} + \frac{1}{\alpha} \sum_{i=1}^{n} \hat{b}_i x_i^{(1)}(k + 1)
\]

Equation (19) is the approximate time response function of GM (1, N) Model, which is also the prediction function of art price. The grey prediction model of the original sequence \( y^{(0)}(k) \) can be obtained by Inverse Accumulated Generating Operation:

\[
\hat{y}^{(0)}(k + 1) = \hat{y}^{(1)}(k + 1) - \hat{y}^{(1)}(k)
\]

3. Empirical analysis

In order to verify the feasibility of the above-mentioned methods, this paper conducts an empirical study on the pricing model of artworks.

3.1. Influencing factor index classification

According to the availability and substitutability of data, the indicators shown in Table 1 are selected for empirical analysis, and are divided according to the static and dynamic dimensions.

| Static Factor Index                  | Dynamic Factor Index                  |
|-------------------------------------|---------------------------------------|
| Creation Time                       | Artist Reputation Index               |
| Work Size                           | Overall Level Index of Art Market     |
| Content and Subject Matter          | GDP                                   |
| Artistic Creation Level             | CPI                                   |
| Mounting Form                       |                                       |

This paper will collect relevant information of art auction records from Artron Artnet. Considering the availability of data, we selects some representative artists from the auction records from 2015 to 2019 according to the amount of transactions and the number of transactions. After that, 5 painting artworks were randomly selected. The 5 artworks are Feng Chaoran’s "Imitation of Li Xi's Ancient Landscape", Pu Hua’s "Zhitian's Birthday", Pan Tianshou’s "Unlimited life", Wu Changshuo’s "Two-color Chrysanthemum", Qi Baishi’s "Aquatic Diving", They are marked as A1, A2, A3, A4, A5.

3.2. Static attribute comprehensive evaluation score

According to the availability of data, this article considers the comprehensive evaluation scores of static attributes of artworks from 5 dimensions: Creation Time, Work Size, Content and Subject Matter, Artistic Creation Level, and Mounting Form. According to the Unascertained Measurement Theory, this paper divides the evaluation criteria into five levels: general, medium, upper-middle, excellent, and superb, and each level has a corresponding score level interval, from which the evaluation space vector can be obtained:

\[
c = \left( (0,1],(1,2],(2,3],[3,4],[4,5] \right)
\]

The information of the above five works of art is classified and sorted out, and the evaluation scores of 12 experts in the field of art are collected in the form of questionnaire. Calculate the single index measurement matrix \( u_{ij} \), information entropy \( H_y \) and weight \( w_j \), and then according to equations (6) and (7), the static attribute comprehensive evaluation scores of 5 sample artworks can be calculated: \( T = (1.35,1.72,1.82,2.29,3.60) \)
3.3. GM (1, N) Model
According to the availability and substitutability of the data, select 4 influencing factors of Artist Reputation Index, GDP, CPI, and the Overall Level Index of Art Market as dynamic influencing factor indicators. This article selects the Current Artist Auction Index compiled by Artron Artnet as the Artist Reputation Index, and the Chinese Painting 400 Index of Artron Artnet as the Overall Level Index of Art Market. The GDP and CPI data are obtained from the website of the National Bureau of Statistics. At the same time, the comprehensive evaluation score of each artwork obtained above is added to the model as one of the dynamic variables, and finally the GM (1,5) Model is constructed.

For the solution of the GM (1,5) Model, this paper uses the Grey System software GSTAV8.0 developed by the Grey System Institute of Nanjing University of Aeronautics and Astronautics.

Finally, the predicted prices of 5 sample artworks are obtained, as shown in Table 2:

|          | A1     | A2     | A3     | A4     | A5     |
|----------|--------|--------|--------|--------|--------|
| Actual   | 282492 | 402500 | 437000 | 649635 | 1011250|
| Predicted| 282500 | 161700 | 544800 | 537300 | 1115700|

3.4. Analysis
The prediction error of the pricing model is generally about 10% to 20%, but there may be unstable situations, and the error will be close to 60%. Artwork has a strong heterogeneity, and there are serious fake auctions and deliberate hype in the art market, which will cause large errors in the pricing.

The artwork pricing model mentioned in this paper has a good effect on the valuation of Chinese artworks. With more comprehensive data on art transaction records and a more complete art market supervision system in the future, the effect of this model is expected to be further improved.

4. Conclusion
This paper divides the factors affecting the price of artworks into two dimensions, static and dynamic. And the method of constructing GM (1, N) Model by combining the static attribute comprehensive evaluation score obtained from unascertained measure model and other dynamic influencing factors makes the model have the characteristics of qualitative and quantitative combination and scientific and objective. The empirical analysis also shows that the pricing model has a good effect. At the same time, there is room for further improvement of the model.

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