Scalar dark matter in inert doublet model with scalar singlet

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Abstract. The simplest scalar representations under $SU(2)_{L}$ symmetry are considered to extend the scalar sector of the Standard Model. The main motivation is including a dark matter candidate, which could arise from the additional scalar fields. We also include an additional $U(1)$ symmetry in order to control the stability of the dark matter candidate.

1. Introduction
The most convincing evidence for Dark Matter (DM) came from the observation that luminous objects such as stars, gas clouds, globular clusters, or entire galaxies move faster than one would expect if they only felt the gravitational attraction of other visible objects [1]. DM is estimated to constitute about 23% of the total matter in the universe. However, the origin of DM still remains a mystery. The existence of dark matter (DM) is now essentially established [2]. The Standard Model (SM) of particle physics successfully explains experimental results observed in colliders, but none of the SM particles can be a good candidate for the dark matter. One of the main reasons is the lack of particles with scarce or null interactions with other SM particles. We assume that dark matter candidates are weakly interacting massive particles (WIMP). The DM relic density is around the observed value (Planck Collaboration) [3, 4]:

$$\Omega h^{2}_{\text{CDM}} = 0.1199 \pm 0.0027.$$  \hspace{1cm} (1)

In the case of scalars in different representations under $SU(2)_{L}$ as DM candidates in models beyond the SM the simplest version includes an additional singlet or doublet [5, 6, 7, 8, 9, 10, 11].
2. The model

In this section the field content and gauge bosons interactions are introduced. We will follow closely the notation introduced by Lee and Sher [12, 13, 14]. We consider the extension to SM with gauge group defined as $G_{1DMUX} = G_{SM} \otimes U(1)_X$ where $G_{SM}$ is the gauge group symmetry of the SM. Basically, this model is a Inert Doublet Model with $U(1)_X$. The coupling constant associated to $U(1)_X$ is denoted as $g_X$.

A mixing among the kinetic energy terms for the gauge fields, $U(1)_Y$ and $U(1)_X$, is allowed given the gauge invariance in $G_{1DMUX}$. This mixing term is parametrized by the weak mixing angle $\theta_W$ and a new parameter denoted by $\varepsilon$ [12, 13, 14]. The kinetic terms are written explicitly as

$$\mathcal{L}_{Kin} = -\frac{1}{4} \hat{B}_\mu \hat{B}^{\mu \nu} + \frac{1}{2} \varepsilon \cos \theta_W \hat{B}^{\mu \nu} \hat{Z}'_{0 \mu \nu} - \frac{1}{4} \hat{Z}'_{0 \mu \nu} \hat{Z}'_{\mu \nu}, \quad (2)$$

where, $\hat{B}^{\mu \nu}$ and $\hat{Z}'_{0 \mu \nu}$ are the field strength tensors of the $U(1)_Y$ and $U(1)_X$ gauge bosons, respectively. The gauge field of $Z'$ is redefined by the rotation

$$\begin{pmatrix} Z'_{0 \mu} \\ B_{\mu} \end{pmatrix} = \begin{pmatrix} \sqrt{1 - \varepsilon^2 / \cos^2 \theta_W} & 0 \\ -\varepsilon / \cos^2 \theta_W & 1 \end{pmatrix} \begin{pmatrix} \hat{Z}'_{0 \mu} \\ \hat{B}_{\mu} \end{pmatrix}, \quad (3)$$

and therefore the mixing term in Equation (2) is cancelled. The mixing parameter $\varepsilon$ will appear in all terms where $Z'$ is redefined. The magnitude of $\varepsilon$ has been constrained to $\varepsilon \leq 10^{-3}$ [13, 15, 16].

In the IDM with $U_X$ the scalar field content is given in the two doublets and one singlet defined as follows

$$\begin{align*}
\Phi_1 &= \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}} (v + \phi_1 + i \eta_1) \end{pmatrix}, \\
\Phi_2 &= \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}} (v + \phi_2 + i \eta_2) \end{pmatrix}, \\
\Phi_x &= \frac{1}{\sqrt{2}} (v_x + \phi_x + i \eta_x),
\end{align*} \quad (4)$$

where $v$ and $v_x$ are the vacuum expectation values (VEV), with $v = 246$ GeV. The SSB is achieved as $G_{1DMUX} \xrightarrow{\langle \Phi_x \rangle} G_{SM} \xrightarrow{\langle \Phi_1 \rangle} SU(3)_C \otimes U(1)_{EM}$, where $\langle \Phi_1 \rangle^T = (0, v_1 / \sqrt{2})$ and $\langle \Phi_X \rangle = v_x / \sqrt{2}$. The charges and representations for the scalar fields under the symmetry group $G_{1DMUX}$ can be written as follows

$$\begin{align*}
\Phi_1 &\sim (1, 2, 1/2, x_1), \\
\Phi_2 &\sim (1, 2, 1/2, x_2), \\
\Phi_x &\sim (1, 1, 0, x),
\end{align*} \quad (5)$$

where two first entries denote the representation under $SU(3)_C$ and $SU(2)_L$, meanwhile the hypercharge and charge $U(1)_X$ are written in the last two entries.

The interactions between the scalar and gauge bosons are given by

$$\mathcal{L}_{\text{scalar}} = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 + |D_\mu \Phi_x|^2, \quad (6)$$

where the covariant derivative $D_\mu$ is defined as

$$D_\mu = \left( \partial_\mu + ig' Y \hat{B}_\mu + ig T_3 \hat{W}_3 \mu + ig_x Q' \hat{Z}'_{0 \mu} \right), \quad (7)$$
The most general, renormalizable and gauge invariant potential is

\[ Q \]

Then, the mass matrices for charged and neutral scalars are

\[ M \]

Proper selection of the values of the terms, the mixing term between arises. This new mixing term is proportional to \( \Delta^2 = \frac{1}{2} g_Z v^2 \cos^2 \beta + \frac{1}{4} \frac{\varepsilon}{\cos \theta_W} g g' v^2 \). In order to cancel the mixing term the following rotation is required

\[
\begin{pmatrix}
Z \\
Z'
\end{pmatrix}
= 
\begin{pmatrix}
\cos \xi & -\sin \xi \\
\sin \xi & \cos \xi
\end{pmatrix}
\begin{pmatrix}
Z^0 \\
Z'^0
\end{pmatrix},
\]

where the mixing angle \( \xi \) satisfy the expression \( \tan 2\xi = \frac{2\Delta^2}{m_{Z^0}^2 - m_{Z'}^2} \), and has been constrained to \( |\xi| < 10^{-3} \) [17]. Therefore the \( Z' \) mass is

\[
m_{Z'}^2 = g_Z^2 (v^2 \cos^2 \beta + v_S^2) + \frac{\varepsilon}{\cos \theta_W} g g' v^2 \cos^2 \beta + \frac{1}{4} \left( \frac{\varepsilon}{\cos \theta_W} \right)^2 g g' v^2,
\]

meanwhile the \( Z \) mass retains the same value set by the SM,

\[
m_Z^2 = \frac{1}{2} g v^2.
\]

### 3. Scalar Sector

The most general, renormalizable and gauge invariant potential is

\[
V = \mu^2 \Phi_1^\dagger \Phi_1 + \mu^2 \Phi_2^\dagger \Phi_2 + \mu^2 \Phi_3^\dagger \Phi_3 + \left[ \mu^2 \Phi_1^\dagger \Phi_2 + h.c. \right] + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_3^\dagger \Phi_3)^2 + \lambda_4 (\Phi_4^\dagger \Phi_4)^2
\]

\[
+ \lambda_5 (\Phi_5^\dagger \Phi_5)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_3^\dagger \Phi_3) + h.c.
\]

\[
+ (\Phi_4^\dagger \Phi_4) \left[ \lambda_1 (\Phi_1^\dagger \Phi_1) + \lambda_2 (\Phi_2^\dagger \Phi_2) + \lambda_3 (\Phi_3^\dagger \Phi_3) + \lambda_4 (\Phi_4^\dagger \Phi_4) + h.c. \right]
\]

\[
+ \lambda_{1x} (\Phi_1^\dagger \Phi_1) \Phi_x + \lambda_{2x} (\Phi_2^\dagger \Phi_2) \Phi_x + \lambda_{3x} (\Phi_3^\dagger \Phi_3) \Phi_x + h.c.
\].

The terms proportional to \( \lambda_{1x} \), \( \lambda_{2x} \), \( \lambda_{3x} \) do not guarantee the stability of the DM candidate. Proper selection of the values of the \( U(1)_X \) charges succeeds in eliminating these terms. We fix the charges as \( x_1 = x_2 \) and \( x_1 \neq x \). Other selection has been considered previously [7, 18]. Then, the mass matrices for charged and neutral scalars are

\[
M_{H^\pm}^2 = 
\begin{pmatrix}
\mu^2 + \frac{1}{2}(\lambda_3 v^2 + \lambda_2 v_x^2) & 0 \\
0 & \mu^2 + \frac{1}{2}(\lambda_3 v^2 + \lambda_2 v_x^2)
\end{pmatrix}
\]

\[
M_0^2 = 
\begin{pmatrix}
M_{11} & M_{12} & M_{13} & 0 \\
M_{12} & M_{22} & M_{23} & 0 \\
M_{13} & M_{23} & M_{33} & M_{34} \\
0 & 0 & M_{34} & M_{44}
\end{pmatrix}
\]

where \( M_{11} = 2\lambda_1 v^2 \), \( M_{12} = \lambda_1 x v_x \), \( M_{13} = \frac{1}{2} \lambda_6 v^2 \), \( M_{22} = 2\lambda_2 v_x^2 \), \( M_{23} = \frac{1}{2} \lambda_9 v^2 \), \( M_{34} = \frac{1}{2} \lambda_9 v^2 \), \( M_{34} = -\text{Im}(\alpha) v^2 \). The \( M_{13} \) and \( M_{23} \) matrix elements are removed to avoid mixing between DM candidates and neutral scalars like Higgs. A discrete symmetry \( Z_2 \) is introduced to eliminate these matrix elements; if \( \Phi_1 \) is even and \( \Phi_2 \) is odd, then \( M_{13} = M_{23} = 0 \).
The $M_0^2$ matrix can be diagonalized by
\[
\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha_1 & -\sin \alpha_1 \\ \sin \alpha_1 & \cos \alpha_1 \end{pmatrix} \begin{pmatrix} \text{Re}[\phi_1^0] \\ \text{Re}[\Phi_x] \end{pmatrix}
\]
and
\[
\begin{pmatrix} H_3 \\ H_4 \end{pmatrix} = \begin{pmatrix} \cos \alpha_2 & -\sin \alpha_2 \\ \sin \alpha_2 & \cos \alpha_2 \end{pmatrix} \begin{pmatrix} \text{Re}[\phi_2^0] \\ \text{Im}[\phi_2^0] \end{pmatrix},
\]
where \( \tan \alpha_{1,2} = \frac{r_{1,2}}{1+\sqrt{1+r_{1,2}^2}} \) with \( r_1 = \frac{\lambda_1 v_x}{\lambda_1 v_1^2 - \lambda_1 v_x^2} \) and \( r_2 = \frac{-\text{Im}[\lambda_5]}{\text{Re}[\lambda_5]} \). Therefore the masses for the scalars are
\[
m_{H_1,H_2}^2 = \lambda_1 v^2 + \lambda_1 v_x v^2 \pm (\lambda_1 v^2 - \lambda_1 v_x v^2) \sqrt{1 + r_1^2},
\]
\[
m_{H_3,H_4}^2 = \mu_2^2 + \frac{1}{2} (\lambda_3 + \lambda_4) v^2 \pm \frac{1}{2} \lambda_2 v_x v^2 \pm v^2 \sqrt{\text{Re}[\lambda_0]^2 + \text{Im}[\lambda_0]^2},
\]
\[
m_{H^\pm}^2 = \mu_2^2 + \frac{1}{2} (\lambda_3 v^2 + \lambda_2 v_x v^2).
\]

4. Dark matter candidates and their portals
The scalars fields $\Phi_1$ and $\Phi_x$ have interactions with fermions and their VEVs are different from zero, therefore none of them could provide a DM candidate. However $\Phi_2$ has no Yukawa interactions and has VEV equal to zero, so this field can provide DM candidates. The $H_3$, or $H_4$ could play the role as dark matter candidate.

The Dark matter abundance can be obtained by solving the Boltzmann equation for the number density rate \cite{19}. The Boltzmann equation can be written as
\[
a^{-3} \frac{d}{dt} (na^3) = \langle \sigma v \rangle (n_{eq}^2 - n^2),
\]
where $n$ is the DM number density and $a$ is a scale factor. All information about the model interactions is contained in the thermally averaged cross section, defined as
\[
\langle \sigma v \rangle = \frac{(2\pi)^4}{(n_{eq})^2} \left| \mathcal{M} \right|^2.
\]

The contributions for scattering from this DM candidate with portals are shown in the Feynman diagrams in figures 1 and 2.

5. Conclusions
We consider an extended scalar sector with two doublet and one singlet. The DM candidates arise from the second doublet which has VEV equal to zero in order to guarantee the stability of DM. The scalar fields with non zero VEV could contribute to new portals for the DM annihilation.

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Figure 1. DM annihilation through neutral Higgs portals. The final states are for pair fermion in left-up, for $Z$ or $Z'$ in right-up, for charged Higgs in left-down and for $W^\pm$ bosons in right-down.

Figure 2. DM annihilation through $Z$ or $Z'$ portal. The final states are for pair fermion in left-up, for $Z$ or $Z'$ in right-up, for charged Higgs in left-down and for $W^\pm$ bosons in right-down.

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