1 Introduction.

Very soon after publication of the famous BCS work explaining the puzzle of the superconductivity, N.N. Bogolyubov and coworkers [1] have proposed their version of the theory. One of new results they have obtained was the discovery of a collective mode - an oscillation of the Cooper pair density with the energy smaller than $2\Delta$. P.W. Anderson has indicated that this collective mode can not be found experimentally since the Coulomb forces neglected in the above mentioned work shift its energy to the plasma frequency, i.e., to the high-ultraviolet range in which the superconductivity is inessential. The interest for plasmons in superconductors revived a little in the search for the mechanism of the High-$T_c$ superconductivity. However, a real surge of interest to this problem occurred after experimental observations of the plasma edge in the reflectivity of High-$T_c$ superconductors $La_{2-x}Sr_xCuO_4$ and $YBa_2Cu_3O_{8-y}$ [2], [3]. It is worthwhile to mention that a theoretical prediction have preceded the experiment [4]. For external reasons, the paper [4] has been published only a long time after its completion.

Here we present a brief review of the experiments and theoretical developments in the field. The theoretical works will be presented in more detail, given the author’s specialization. I hope that this imbalance will be compensated by the papers by O.K.C. Tsui, N.P. Ong and J.B. Peterson, and by P.M. Mueller.

The content of the review is as follows. In the second section we review the most general relationships for plasma frequency (PF) and dielectric function of the electron plasma in normal metals and their modification for layered normal metals. In the third section we review the experimental observations
of the plasma edge in High-\(T_c\) superconductors (HTSC). In the fourth section a simple two-fluid model of the plasma oscillations is presented. The fifth section is devoted to the BCS theory of the same phenomenon. Peculiarities of Josephson-linked layered superconductors are considered in Section 6. The experimental observations of the resonance plasma attenuation are discussed in Section 7. Mechanisms of dissipation in the Josephson-linked superconductors are discussed in Section 8. A novel effect of selectional transparency is discussed in Section 9. The conclusions are collected in Section 10.

We employ abbreviations SC for superconductors and OP for the order parameter.

### 2 Plasma resonance in normal metals.

The plasma oscillation is one of the simplest and most fundamental manifestations of the Coulomb interaction. Due to the long-range character of the Coulomb forces, its spectrum starts with a rather high frequency given by the well-known formula:

\[
\Omega_{pl}^2 = \frac{4\pi ne^2}{m\epsilon_\infty} \tag{1}
\]

where \(n\) and \(m\) are the electron density and mass respectively, \(\epsilon_\infty\) is the dielectric constant contributed by atomic and ion reminder shells. For usual values in solids \(n \sim 10^{22} \div 10^{23} cm^{-3}\), \(m = 10^{-27}g\) and \(\epsilon_\infty \sim 1\), the PF \(\Omega_{pl} \sim 10^{15} \div 10^{16} s^{-1}\) lies in the high-ultraviolet range. The variation of \(n\) and \(m\) in metals are not large. Therefore the characteristic value of the PF is always rather high. This is the reason why plasma oscillations do not play an important role, either in the thermodynamics or in transport phenomena in conventional metals.

The situation changes in layered metals, if the electron propagation perpendicular to the layers (the \(c\)-axis) is mediated by tunneling processes. Applying the tight-binding approximation for the \(c\)-direction one finds a following dispersion law for electrons:

\[
\epsilon(p, p_z) = \epsilon_\parallel(p) + t(1 - \cos \frac{p_z d}{\hbar}) \tag{2}
\]

where \(p, p_z\) are in-plane and perpendicular-to-plane components of the momentum, \(t\) is the overlap integral and \(\epsilon_\parallel(p)\) is the in-plane dispersion. Assuming it to be \(\epsilon_\parallel(p) = p^2/2m\) for simplicity, we find that for \(t \ll \epsilon_F\) the
Fermi-surface is a slightly corrugated circular cylinder. Solving the kinetic equation with such a Fermi-surface, we find that Eq. (1) holds for the electric field in-plane, but for the perpendicular-to-plane electric field the PF is much smaller \[ \Omega_{pl}^2 = \frac{4\pi n_e^2 m}{\epsilon_\infty M M} \left( \frac{\hbar}{p_F d} \right)^2 \] (3) where \( M = \frac{\hbar^2}{td^2} \gg m \) and \( d \) is the lattice constant in the \( c \)-direction.

Note that the ratio \( \Omega_c/\Omega_{ab} \propto m/M \) instead of \( \sqrt{m/M} \) as it could be naively expected and what is correct for an elongated ellipsoid. The reason is the change of topology: the cylinder is an open surface. This simple observation allows one to understand a drastic reduction of the PF which has been observed experimentally \[2, 3\]: \( \Omega_{pl} \) for \( \text{LaSrCuO} \) and for \( \text{YBCO} \) is in the far infrared region. This agrees well with a comparatively low carriers densities found in the Hall-effect measurements and high ratio \( M/m \sim 100 \div 2,500 \) found in experiments with the tork produced by the vortex lattices \[4\]. The anisotropy parameter in \( \text{Bi 2:2:1:2} \) is even larger \[5\], \( M/m \sim 40,000 \). Therefore we predicted that plasma frequency in this compound lies in the microwave region \[6\]. This theoretical prediction has been confirmed in the experiment \[7\], though the experimenters were not aware of the theoretical prediction.

Unless the collision time for quasiparticles \( \tau \) is small enough, there is no substantial difference between the plasma oscillations in normal metals and superconductors. In both cases they are the charge density oscillations with the restoring force determined by the electrostatics. The situation is very different for small collision time \( \Omega_{pl} \tau \ll 1 \). Then normal carriers almost do not participate in the plasma oscillations. We return to the analysis of this problem later.

The PF may be alternatively determined as a frequency at which the dielectric function \( \epsilon(\omega) \) is zero. For the collisionless plasma, the dielectric function is:

\[ \epsilon(\omega) = \epsilon_\infty \left( 1 - \frac{\Omega_{pl}^2}{\omega^2} \right) \] (4)

The reflectivity for an EM wave is determined by the Fresnel formula:

\[ R = \left| \frac{\sqrt{\epsilon(\omega)} - 1}{\sqrt{\epsilon(\omega)} + 1} \right|^2 \] (5)
According to Eqs. (4, 5) \( R = 1 \) at \( \omega < \Omega_{pl} \) and \( R < 1 \) at \( \omega > \Omega_{pl} \). This effect is called the plasma edge. Its sharpness depends on the value \( \epsilon_\infty \). For \( \epsilon_\infty > 1 \), the graph of \( \epsilon(\omega) \) has a characteristic beak (Fig 1). Introducing a phenomenological collision time \( \tau \) and the electron attenuation \( \Gamma = 1/\tau \), one finds instead of (4):

\[
\epsilon(\omega) = \epsilon_\infty \left(1 - \frac{\Omega_{pl}^2}{\omega(\omega + i\Gamma)}\right).
\]

(6)

The plasma edge is smeared out at large \( \Gamma \gg \Omega_{pl} \) or small \( \tau \). Theoretical curves of \( R(\omega) \) are shown on Fig. 1.

3 Experimental observation of the plasma edge in superconductors.

The first observation of the plasma edge has been reported by Koch et al. [2] in 1990. They used a single crystal of \( YBCO \) specially grown in such a way that the layers were oriented perpendicularly to the surface of a sample and the \( c \)-axis was directed parallel to the surface. The electric vector in the incident EM wave was directed along the the \( c \)-axis. This geometry enabled the authors to observe the plasma edge. They have found a sharp decrease of the reflectivity at a frequency \( \omega \approx 12\text{meV} \) in the superconducting state (\( T < 90K \)) and very smooth decrease of reflectivity in the normal state (Fig. 2). Unfortunately, the plasma edge in \( YBCO \) occured to be close to other features of the spectrum associated with optic phonons, thus interfering with a thorough study of plasma oscillations.

Tamasaku et al. [3] have used very good single crystals of \( La_{2-x}Sr_xCuO_4 \) in the same geometry which enabled the authors to observe the plasma edge reliably (Fig. 3). They also did not observe any edge effect in the normal state and a rather sharp edge already at a temperature just slightly below the transition temperature. A systematic decrease of the PF with the temperature has been found. Their measurements are compatible with the assumption that \( \Omega_{pl}^2 \propto \Delta^2 \).

Two questions naturally arise in connection with these measurements.

1. Why there is no plasma edge in the normal state while it occurs immediately in the superconducting state?
2. Why PF depends strongly on the temperature?

At a qualitative level these questions can be answered without detailed calculations [3], [4]. The absence of the plasma edge in the normal state can be explained by strong collisions $\Omega_{pl} \tau \ll 1$. In the superconducting state a part of electrons is confined in Cooper pairs (superconducting carriers) which are not scattered. It explains why the plasma oscillations occur in the superconducting state. The same idea explains the temperature dependence of the PF: not the full, but only the superfluid density $n_s$ enters Eqs. (1, 3). The superfluid density in an anisotropic SC is a second-rank tensor. The superfluid density varies from the total electron density at $T = 0$ in a clean SC till zero at $T = T_c$.

4 Two-fluid theory of the plasma resonance.

The two-fluid model of SC by Gorter and Kasimir (see for example [10]) is a simplest phenomenological way to understand the plasma oscillations in SC. It has been used in fact by T. Mishonov [4], and also in a great extent by D. van der Marel et al. [11] and later by Tachiki et al. [12]. In the framework of this model, the dielectric function is written in the following form:

$$\epsilon(\omega) - \epsilon_\infty \left( 1 - \frac{\Omega_s^2}{\omega^2} - \frac{\Omega_n^2}{\omega(\omega + i\Gamma)} \right).$$  (7)

The terms with $\Omega_s^2$ and $\Omega_n^2$ represent contributions of the superfluid and normal parts of the electron liquid, respectively. As it was emphasized in Section 3, the superfluid part of the liquid is not subject to any dissipation whereas the normal part contains the conventional dissipative coefficient $\Gamma$. If $\Omega_{pl} \tau \gg 1$, then $\Omega_{pl}^2 = \Omega_s^2 + \Omega_n^2$, i.e. all carriers participate in the plasma oscillation. In the opposite case $\Omega_{pl} \tau \ll 1$, one finds $\Omega_{pl} = \Omega_s$, in agreement with what was expected. Note that in the latter case the imaginary part of $\epsilon$ is small:

$$\mathcal{I}m \epsilon = \frac{\epsilon_\infty \Omega_n^2 \tau}{\Omega_s}; \quad (\omega \approx \Omega_s).$$  (8)

Thus, the stronger are collisions the weaker is the dissipation of the plasma oscillations. This paradoxical result is explained by different roles played by normal carriers at different $\Omega_{pl} \tau$. If this parameter is large, the normal
carriers create the coherent plasma wave on the same footing as the superconducting electrons. If $\Omega_{pl}\tau$ is small, the coherent motion is created by superconducting electrons only whereas the normal electrons, being partly involved into this motion, dissipate the energy. The sooner they are put in equilibrium by collisions the more adiabatically they follow an instant value of the OP, and the smaller are losses.

Though the two-fluid model describes qualitatively the plasma oscillations it has several important failures. First, the very notions of the superconducting and normal densities $n_s$ and $n_n$ can be defined reasonably only for a rather clean SC. These definitions presume the conservation of the momentum which is violated by impurities, phonons etc. Second, the phenomenological definition of the collision time does not enable one to calculate its temperature and frequency dependence. Third, the two-fluid model does not take into account the symmetry of the superconducting OP making no difference between $s$- and $d$-pairing.

5 BCS theory of plasma oscillations.

The BCS theory of plasma oscillations has been proposed in [5] to overcome the failures of the two-fluid model. It is a microscopic theory based on the BCS Hamiltonian with the dispersion law given by Eq. (2). The OP in the unperturbed SC is assumed to be uniform in the real space and very anisotropic in the relative momentum space. Only the elastic scattering by impurities has been considered. The scatterers were assumed to change the electron momentum in plane, but to be unable to transfer an electron from one plane to an adjacent plane. It means that the scattering amplitude does not depend on the $z$-component $p_z$ of the momentum. One more simplification is provided by an assumption of the full in-plane isotropy. The problem is to calculate the Pippard kernel $Q(\omega, q)$. The Pippard kernel relates the Fourier components of the current density $j$ and the vector potential $A$:

\[ j(\omega, q) = -Q(\omega, q)A(\omega, q) \] (9)

The relationship (9) is written for a special gauge $\nabla A = 0$. Once the Pippard kernel is known the dielectric function can be calculated as:

\[ \epsilon(\omega, q) = \epsilon_\infty - \frac{4\pi c}{\omega^2}Q(\omega, q) \] (10)
In the real space the relationship (9) reads:

$$j_\alpha(\omega, \mathbf{r}) = -\int Q_{\alpha,\beta}(\omega, \mathbf{r}, \mathbf{r}') A_\beta(\omega, \mathbf{r}') d^3x$$  \quad (11)

The relationship (11) is valid at a fixed configuration of impurities which violates the translational invariance. According to the Kubo formula, the response function (Pippard kernel) is expressed in terms of the retarded current-current Green function which in turn can be obtained as an analytical continuation of the Matsubara Green function:

$$Q_{\alpha,\beta}^{M}(\omega_n, \mathbf{r}, \mathbf{r}') = \frac{iT}{c} \int_0^{1/T} d\tau e^{i\omega_n \tau} \langle j_\alpha(\tau, \mathbf{r}) j_\beta(0, \mathbf{r}') \rangle$$  \quad (12)

with $\omega_n = 2\pi n T$ and integer $n$. The retarded response is the analytical continuation of this function from discrete points $\omega = i\omega_n$ in the upper half-plane of the complex plane $\omega$. The next averaging over the random location of impurities should be performed in the end.

Abrikosov and Gor’kov \cite{13} have developed a special diagrammatic technique for this averaging. In a simplest situation of weak scatterers, the problem is reduced to the solution of an analog of the Boltzmann kinetic equation for SC \cite{14}, \cite{15}. An essential complication for the superconducting case is the existence of several distribution functions instead of one in the normal case. Not only the occupation numbers of quasiparticles, but also the condensate wave function varies locally. The solution of the kinetic equation is generally nontrivial even in the normal case. Fortunately, our special geometry together with the fact that the penetration depth is much larger than any other geometrical scale in the problem makes this problem exactly solvable. More accurately, it can be solved in the leading order over small tunneling amplitude $t$ if one employs the independence of the impurity scattering amplitude on $p_z$. For the most interesting $zz$-component of the Pippard kernel the result is:

$$Q_{zz}(\Omega, T) = \frac{e^2 t^2 m d}{4 \pi \hbar^2} K(\omega, T)$$  \quad (13)

$$K(\omega, T) = \int_{-\infty}^{\infty} d\eta \left[ \left( 1 - \frac{\epsilon_R(\eta_+) \epsilon_A(\eta)}{\epsilon_R(\eta_+ + \Delta^2) / \epsilon_R(\eta_+) + \epsilon_A(\eta) + i/\tau} + \frac{\epsilon_R(\eta_-) \epsilon_A(\eta)}{\epsilon_R(\eta_-) \epsilon_A(\eta) + i/\tau} \right) \right]$$  \quad (14)
where $\eta_{\pm} = \eta \pm \omega$, $\epsilon_{R,A} = \sqrt{(\eta \pm i\delta)^2 - \Delta^2}$, the branch of the square root is chosen to be positive at $\eta > \Delta$. The reader is referred to the original work [5] for details.

One of the most important results in the BCS theory of the plasma oscillations is the vanishing of the dissipation in a pure SC at any $T < T_c$. Two conventional dissipation processes are the Cherenkov absorption of light by quasiparticles and the Cooper pair breaking, i.e., the creation of a pair of quasiparticles with the absorption of a photon. The first process is kinematically forbidden since the light velocity $c/\epsilon_{\infty}$ is much larger than the Fermi velocity. The second process, specific for SC, is kinematically allowed at $\omega > 2\Delta$. However, it is forbidden dynamically. The amplitude of the quasiparticle-photon interaction is proportional to the so-called Bogolyubov coherence factor $[10] \ (u_{p+q}v_p - v_{p+q}u_p)^2$ where $p$ is the electron momentum and $q$ is the photon momentum. For the extreme of the normal skin-effect $q \to 0$ this factor is zero. Both processes are allowed in the impure SC in which the distribution over momentum at a fixed energy looses its delta-like character and becomes a Lorenzian one. This argument explains why the plasma edge is observable in the HTSC which have been experimentally proven to have lines of nodes of the OP and zero energy gap in the spectrum [16], [17], [18], [19], [20].

Eqs. (10, 14) show that what is called $\Omega_s$ and $\Omega_n$ in the two-fluid model in the microscopic BCS theory are complicated functions of all variables. Even at very low temperature the analog of the superfluid density $n_s$ depends rather sharply on $\tau$ varying from the value of total electron density at $\tau \Delta \gg 1$ to a value less by a factor $\sqrt{\tau \Delta}$ at $\tau \Delta \ll 1$.

If $\Omega_{pl} \ll \Delta$, the PF is expressed in terms of the penetration depth: $\Omega_{pl} = c/\lambda_c$. This is just the case for Bi 2:2:1:2. However, in YBCO and LaSrCuO, $\Omega_{pl}$ is comparable with $\Delta$. In this case, the PF can be found as a solution of a following equation:

$$Re \ \epsilon(\Omega_{pl}) = 0$$

For this situation the exact result [14] is especially important.

In a clean SC, $ImQ = 0$, as it has been shown earlier. Therefore, $K = 1$ at $\tau = \infty$, independently on temperature and frequency. This is a generalization of the relationship $\Omega_{pl}^2 = \Omega_s^2 + \Omega_n^2$ at $\Gamma = 0$ in the two-fluid model. The transition between the static value $K \propto n_s(T)$ and the pure limit $K = 1$ occurs in a region of frequency $\omega \sim 1/\tau$, very narrow in clean SC.
Comparing BCS theory and the two-fluid theory of EM response in layered SC we find that quantitatively the two-fluid model is rather unsatisfactory. It is especially poor in evaluation of the dissipation for which its error (a deviation from the BCS theory) may be 1000% and more. The reason is that there is only one scale of frequency $1/\tau$ for dissipation in the two-fluid model whereas the BCS theory displays several equally important scales: $\Delta, T, 1/\tau, \Delta^2/\tau, (\Delta^2)^{-1}$.

Though the BCS theory is a step forward in our understanding of the phenomenon its premises are oversimplified. The most important of these simplifications are as follows:

i) The BCS theory is a theory of weakly interacting electrons. It may not work well for HTSC. Unfortunately the existing theories of strongly correlated fermions do not reach a level which allows for a reliable calculation of the EM response.

ii) The scattering in the BCS model has been assumed to be elastic scattering by weak impurity scatterers. It probably dominates in the LSCO which becomes a SC at a sufficiently high doping only. However, measurements of the Hall angle in YBCO at low temperatures [21] have shown a high purity of this compound. In combination with the plasma edge observation [2] this leads to the conclusion that the scattering rate is large near the transition temperature ($\Omega_\text{pl}\tau \ll 1$) and drops fast with decreasing the temperature.

The problem of the scattering mechanism stays in a close connection to the problem of the strong electron interaction. The famous linear-temperature resistivity [22] can hardly be explained in the frameworks of the Fermi-liquid theory. For our purpose we need the $c$-axis resistivity which behavior resembles semiconductors more than the [23]. We can argue that the variable-distance hopping processes determining the static $c$-axis resistivity are irrelevant at plasma frequency since they require a long time. Nevertheless, the direct theoretical approach is much desirable.

iii) Our version of the BCS theory does not take into account a strong spatial modulation of the superconducting OP. However, this is rather important for explanation of a strong influence produced by the magnetic field on the PF. This question is discussed in the next section.

iv) We have already mentioned that the experiments indicate a nontrivial character of the pairing in HTSC with the nodes of the OP. The BCS theory should be reformulated for this situation.
6 Plasma oscillations in Josephson-linked SC.

In real HTSC the OP is deeply modulated in the $c$-direction with the lattice periodicity. The coherence length in the $c$-direction, $\xi_c$, for Bi, Tl and Hg based SC is much less than the lattice constant $d$ in the same direction practically at any $T < T_c$. For YBCO, $\xi_c$ becomes smaller than $d$ at $T \approx 75K$ while $T_c = 90K$. The OP in deeply modulated superconductors is concentrated in thin layers near conducting planes. The coupling between layers is a weak Josephson link. The modulus $|\psi_n|$ of the OP in $n$-th plane is a rigid variable which fluctuates only weakly except very close to the transition temperature, whereas the the phases $\varphi_n$ are Goldstone variables, which are strongly fluctuating and are easily influenced by external fields. In particular, the phase variables are drastically influenced by the magnetic field. This is a general mechanism for a strong dependence of the PF on the magnetic field.

Specifically, the magnetic field penetrates into a layered SC as an Abrikosov vortex lattice. We start with a geometry in which the magnetic induction is parallel to the layers. Then the vortices form a 2-dimensional lattice with central lines situated just in the middle between two adjacent planes. These vortices are similar to the Josephson vortices occurring in a Josephson junction [24]. Each of them creates an additional phase difference between adjacent layers $2\pi$ on a characteristic scale $\lambda_J = \gamma d$, the so called Josephson screening length (JSL), where $\gamma = \sqrt{M/m} = \xi_{ab}/\xi_c$ is the anisotropy coefficient. The latter is about 30 ÷ 50 for YBCO and not less than 200 for Bi 2:2:1:2, as it was established by several independent experiments. Thus the JSL for BiSCCO reaches a macroscopic value $\lambda_J \sim 3000\AA$. If the magnetic field is so strong that the vortices strongly overlap, i.e., the distance between their centers is much less than $\lambda_J$, the phase in the plane grows almost linearly with the distance [25]. In the opposite limiting case of weakly overlapping vortices, the phase difference $\Delta \varphi_{n,n+1}$ between $n$-th and $n+1$-th layers grows rapidly by $2\pi$ on the scale of JSL near each vortex and remains a constant until the next vortex line. The boundary between these two regimes is the value of magnetic field $B_J = \phi_0/\gamma d^2$.

To understand how the magnetic field influences the PF we first establish an important relationship between the PF and the maximal Josephson current $j_{\text{max}}$. The analog of the Pippard kernel $Q_{zz}$ or $n_s$ in the Josephson linked SC is a value proportional to the maximum Josephson current $j_{\text{max}}$. 

10
Indeed from the Josephson relationship:

\[ j_z = j_{\text{max}} \sin(\Delta \varphi_{n,n+1} - \frac{2A_z d}{\phi_0}) \]  

(16)
in a special gauge \( \Delta \varphi_{n,n+1} = 0 \) and at small \( A_z \) we discover the linear dependence:

\[ j_z = -Q_{zz} A_z; \quad Q_{zz} = \frac{2j_{\text{max}} d}{\phi_0}. \]  

(17)

By analogy with Eq. (10) we find:

\[ \Omega_{pl}^2 = \frac{8\pi c j_{\text{max}} d}{\epsilon_\infty \phi_0} \]  

(18)

The Josephson current can be found by differentiating the corresponding free energy:

\[ F_J = \frac{j_{\text{max}} \phi_0}{2\cd} \sum_n \int d^2x \left[ 1 - \cos \left( \Delta \varphi_{n,n+1} - \frac{2A_z d}{\phi_0} \right) \right]. \]  

(19)

A regular lattice of vortices stretched along the \( y \)-axis causes a regular variation of \( \Delta \varphi_{n,n+1} \) with \( x \). We consider first \( B \gg B_J \). Then

\[ \Delta \varphi_{n,n+1} \approx 2\pi \sqrt{\frac{B}{\gamma \phi_0}} x + \frac{B_J}{B} \sin \left( 2\pi \sqrt{\frac{B}{\gamma \phi_0}} x \right) \]  

(20)

After averaging of Eq. (19) over \( x \) with the \( \Delta \varphi_{n,n+1} \) given by Eq. (20) we find

\[ j_{\text{eff}} = j_{\text{max}} \frac{B_J}{2B} \]  

(21)

where \( j_{\text{eff}} \) is the effective maximal current after the averaging. In the opposite limiting case \( B \ll B_J \) the in-plane distance between vortices is \( \sqrt{\gamma \phi_0 / B} \), the out-of-plane distance between vortices is \( \sqrt{\phi_0 / \gamma B} \) and the range of essential variation of phase near each vortex line is \( d\lambda_J \). Therefore

\[ j_{\text{eff}} = j_{\text{max}} (1 - 0(B/B_J)). \]  

(22)

One should expect a sharp dependence of the PF on the direction of the magnetic field. Such a dependence has been predicted in [26]. In particular
the authors of the work [26] have predicted a reentrant behavior of the PF vs the angle similar to the reentrant behavior of the ac magnetic susceptibility and resistivity discovered earlier [27], [28].

Next we consider the magnetic field directed perpendicularly to the layers. Each vortex in a layered SC consists of separate plane vortices – pancakes. If there is no irregularity in the location of pancakes, the phase difference \( \Delta \varphi_{n,n+1} \) created by them is zero and the magnetic field does not affect the PF until \( B \ll H_{c2} \). However, any disorder in the pancake arrangement creates a random phase difference and reduces effectively \( j_{\text{max}} \) and \( \Omega_{\text{pl}} \). We consider the case of the strong magnetic field when many pancakes enter a circle of the radius \( \lambda_J \) [29]:

\[
B \gg B_{J \perp} = \frac{\phi_0}{\lambda_J^2} = \frac{\phi_0}{\gamma^2 d}. \quad (23)
\]

The condition (23) is much less restrictive than the analogous condition for the parallel field: \( B_{J \perp} \approx 200 \) Gs for Bi 2:2:1:2. If the condition (23) is satisfied, the phase \( \Delta \varphi_{n,n+1} \) is effectively a sum of a large number \( N = B/B_{J \perp} \) of random terms. Therefore it obeys the Gaussian statistics. The effective maximal current is proportional to the Debye-Waller factor:

\[
j_{\text{eff}} = j_{\text{max}} \exp \left( -\frac{1}{2} \langle (\Delta \varphi_{n,n+1})^2 \rangle \right). \quad (24)
\]

The value \( \Delta \varphi_{n,n+1} \) in our case is:

\[
\Delta \varphi_{n,n+1}(r) = \sum_{\nu} [\Phi(r - r_{n,\nu}) - \Phi(r - r_{n+1,\nu})] = \int (\rho_n(r) - \rho_{n+1}(r)) \Phi(r) d^2x \quad (25)
\]

where \( \Phi(r) = \arctan(y/x) \) and \( r_{n,\nu} \) is the vector coordinates of the center of a \( \nu \)-th vortex in the \( n \)-th plane; \( \rho_n(r) = \sum_{\nu} \delta(r - r_{n,\nu}) \) is the vortex density in the \( n \)-th plane. After the averaging we find:

\[
\langle \Delta \varphi_{n,n+1}^2 \rangle = 2 \mu \ln(\lambda_J/a); \quad \mu = \frac{\pi}{4} \int r^2 K(r) d^2x \quad (26)
\]

where \( K(r) \) is a correlator:

\[
K(r) = \langle \rho_n(r) \rho_n(0) \rangle - \langle \rho_n(r) \rangle \langle \rho_{n+1}(0) \rangle \quad (27)
\]
If there is no correlation between adjacent layers, the correlator $\mathcal{K}(r)$ turns into the correlator of a two-dimensional pancake liquid. Thus, the measurement of the field dependence of the PF provides an important information on the statistical properties of the pancake liquid. Assuming the disorder within a plane to be so strong that the only characteristic scale for $\mathcal{K}(r)$ is the average distance between pancakes $a$, we find that the dimensionless value $\mu$ is a numerical constant. As a consequence we find a power law decrease of the PF with the magnetic field:

$$\Omega_{pl}^2 = \Omega_{pl0}^2 (B_{J\perp}/B)^\mu$$

(28)

A reason for a strong in-plane disorder may be either thermal fluctuations in the pancake liquid or a rather strong pinning in a glassy phase. The first case corresponds to a temperature above the irreversibility line, the second one – below this line. In a simplest irreversible situation a large number of the strong pinning centers provide a random location of the pancakes. A metastable state depends on the way of its preparation. If the magnetic field is frozen, the vortices find closest pinning centers and do not deviate much, at least locally, from the regular Abrikosov lattice. If the field switches on after the cooling, the vortices enter consequently at the field is increased. The resulting vortex arrangement may be highly chaotic.

Consider a situation when there is no correlation between positions of pancakes in adjacent planes, but the in-plane crystal order persists over a distance $R$ much larger than the in-plane lattice constant $a$. We also assume that $R \ll \lambda_J$. Then the exponent in Eq. (28) (see also Eq. (26)) becomes very sensitive to the model of disorder and may be either much larger or much smaller than one, as well as of the order of one. As a consequence, the reduction of the PF may be much stronger than that caused by a strong in-plane disorder. This striking result is explained by the accumulation of a random interplane phase shift on a large area $\propto R^2$.

Thus the theory predicts a power-law decrease of the PF at $B > B_{J\perp}$ and a strong in-plane disorder with the exponent of the order of unity. The same result may be correct in the absence of the interplane correlation, but I cannot exclude a possibility of the strong reduction of the PF in this case. Finally, one should expect much weaker magnetic field effect, if the interplane correlation is strong.

The description of the Josephson-linked SC accepted in this section is a kind of a long-wave-length phenomenology basically the same as in the
Lawrence-Doniach model \cite{30}. A microscopic approach has been proposed by Artemenko and Kobel’kov \cite{31}.

7 Experimental observation of the resonance attenuation at the PF.

The first observation of the resonance attenuation has been reported by O.K. Tsui et al. \cite{9}. A sample of $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi 2:2:1:2) has been placed into a waveguide. The sample was grown in the conventional way with layers parallel to its surface. The resonance has been observed in a range of frequencies between 30 and 500GHz and magnetic fields from 1 to 7T. The absorption was recorded by a resonance bolometric technique. During each experiment, the frequency was fixed and the magnetic field varied. Rather sharp lines of attenuation have been observed in the temperature interval between 2.5 and 20K. The effect has been initially ascribed to the cyclotron resonance. However, an additional study has shown that it occurs only if the electric field has a non-zero $c$-component, and that the resonance frequency decreases with the magnetic field \cite{32}, \cite{33}. This fact can be considered as a convincing evidence in favor of the plasma-resonance origin of the phenomenon.

Accurate measurements of the resonance \cite{32} may be summarized by an empirical formula valid at $T$ below the irreversibility line:

$$\Omega^2_{pl}(B,T) = AB^{-\mu} \exp(T/T_0)$$

where $A$ is a constant, $\mu \approx 0.8 \div 1.0$ and $T_0 \approx 12.5K$. In Section 6 we have derived the power low for $\Omega^2_{pl}$ at a condition of strong in-plane disorder. Surprisingly the PF grows with the temperature whereas according to a naive idea it should decrease as the superfluid density. This growth can be qualitatively explained in terms of partial thermal depinning of the vortices. Depinned vortices tend to restore the positional order destroyed by the pinning. Quantitatively we were able to prove that the linear in $T$ correction to the PF is positive \cite{29}.

In Bi 2:2:1:2 the contribution of quasiparticles to the PF is probably always small. The reason is that they are strongly damped at $T$ close to $T_c$ whereas the normal density is very small at $T \ll T_c$. Besides of that $\Omega_{pl} \ll \Delta$ in Bi 2:2:1:2.
8  Dissipation mechanisms in Josephson-linked SC.

The resonance attenuation at $\omega = \Omega_{pl}$ requires an explanation. A most obvious effect is the amplification of the electric field inside the SC. In the experimental geometry the sample surface was parallel to the layers, the electric field in the wave-guide was perpendicular to the surface and the layers. From the continuity of the component of the electric induction $D_z$ normal to the surface, it follows that $E_{int} = E_{ext}/\epsilon(\omega)$, where $E_{int}$ is the field inside the sample and $E_{ext}$ is the field in the wave-guide. Since $\epsilon(\omega)$ is almost zero at $\omega = \Omega_{pl}$, the electric field is strongly amplified inside the sample. It creates a necessary premise for the resonance attenuation, but this is not sufficient. The problem is that the electric field in the $z$-direction creates the ac Josephson current in the same direction, which is not subject to dissipation. A mechanism transforming the electric field perpendicular to layers into the parallel one and the dissipation mechanism for the parallel component of the electric field or current should be found. Though this question is not fully solved we present here preliminary ideas due to L. Bulaevsky and S. Pokrovsky without detailed calculations.

Strongly pinned randomly located pancakes create large fluctuations of the dielectric constant $\epsilon_{zz}$. The amplitude of this fluctuations can be estimated as $1/\langle\cos \Delta \varphi_{n,n+1}\rangle$, the inverse Debye-Waller factor calculated in Section 6. The fluctuating dielectric constant leads to a strong and random refraction which represents just the necessary transformation mechanism [39].

As for dissipation mechanism two ideas are discussed. The first [39] is the dissipation by the normal component which exists in SC with the nodes of the OP in magnetic field even at zero temperature [34]: the normal carriers appear near vortices along special directions. The density of normal carriers is estimated as $n\sqrt{B/H_{c2}}$. This mechanism leads to a linewidth $\Delta \omega/\omega \sim 0.1$ in agreement with the experiment. However, it is not clear whether the growth of the relative width of the resonance line with decreasing temperature predicted by this mechanism is compatible with the experimental data [4, 32, 33].

The second dissipation mechanism is via the excitations of the phase oscillations in superconductors with disordered vortex arrangement. As it was discussed above the disordered vortices generate strong static phase fluc-
tutions. Due to nonlinearity of the Josephson energy, these fluctuations may create inhomogeneous localized phase oscillations. They are the same plasma oscillation in an inhomogeneous situation. One can expect a continuous spectrum of these oscillations starting with zero energy. The excitation of these oscillations represents a kind of the inhomogeneous line broadening. Estimates made by L.N Bulaevsky et al., [40] give the relative line-width $\approx 0.1$. They also predict that the relative line-width does not depend on the temperature.

In the same preprint, Bulaevsky et al. have also considered the well-known Bardeen–Stephen mechanism [35] applied to the vortices oscillating near their equilibrium pinned positions. It should be taken into account that this dissipation is reduced by the quantization of quasiparticles in the vortex core. The quantization is insubstantial for low-temperature SC with the core size $\xi \sim 1,000\text{Å}$. However, in HTSC $\xi_{ab} \sim 30\text{Å}$ and the characteristic level spacing is about $100K$. Since the experiments have been performed starting from $2K$ the excitation of quasiparticles bound to vortex cores seems to be irrelevant.

9 Selectional transparency in magnetic field.

This is a novel effect which, as we hope, can be observed at comparatively weak magnetic field $B < B_J = \phi_0 / \gamma d^2$ parallel-to-layers. It was shown experimentally [36], [27], [28] that the pinning of parallel vortices is much weaker than the pinning of pancakes. Therefore, it can be expected that the parallel vortices form a highly regular lattice even at fields $B < B_J$. As it was discussed earlier, the PF is not influenced much by the magnetic field in this region. An incident EM wave is modulated by the vortex lattice. The modulated wave has the same frequency as the incident one, but its wave-vector coincides with one of the vortex lattice reciprocal vectors. A most important situation is that when the reciprocal vector is parallel to layers. Neglecting the in-plane anisotropy, an in-plane wave-vector is determined by an integer $k$ and has the length $q = 2\pi k/a$. Thus the resonance is shifted to a frequency;

$$\omega = \sqrt{\Omega^2_{pl} + \alpha v_F^2 q^2}$$

(30)

where $\alpha$ is a constant of the order of unity. The resonance frequency can be regulated by a variation of the magnetic field. If the frequency is fixed, the
centers of transparency windows correspond to following values of fields:

\[
B_k = \frac{(\omega^2 - \Omega_{pl}^2)\phi_0}{(2\pi v_F\alpha k)^2} \quad (31)
\]

The transparency can be observed if the electric vector has a component perpendicular to the layers. This phenomenon can be used for the diagnostics of the vortex lattice. For example, if the vortex lattice is pinned by the periodic potential of the crystal lattice [37], [38], then, instead of Eq. (31), the resonance frequency obeys a following equation:

\[
B_k = \frac{\sqrt{(\omega^2 - \Omega_{pl}^2)\phi_0 B_0}}{2\pi v_F\alpha k} \quad (32)
\]

where \(B_0\) is a value of the field at which the vortex lattice has been quenched. The effect can be used for creating magnetic field regulated infrared and micro-wave optical gates.

10 Conclusions.

1. In layered structures the PF is strongly reduced, if electric field is perpendicular to the layers.
2. The plasma oscillations in SC can propagate even if they are overdamped in the normal state.
3. The experiments cited in the text convincingly show the existence of the plasma edge and the resonance plasma attenuation in three HTSC.
4. The plasma oscillations can propagate in rather clean SC with nodes of the OP, since the Cooper-pair breaking is dynamically forbidden.
5. In Josephson-linked SC, sufficiently high magnetic fields reduce the PF dramatically.
6. At weaker fields, parallel to the layers, an effect of the selectional transparency should be observable.
7. Measurement of the magnetic field dependence of the PF can be used for the diagnostics of the vortex state in HTSC.

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Figure Captions

Fig. 1.
Theoretical curves for the reflectivity vs frequency measured in units $\Omega_{pl}$. The high-frequency dielectric constant $\epsilon_\infty = 25$. Different curves correspond to different values of the scattering rate: $\Omega_{pl}\tau = 0.01; 0.1; 0.5; 1.0; 5.0$.

Fig. 2.
The plasma edge in $YBCO$. Far infrared reflectivity at 300 K (solid line), 110 K (dashed line), 60 K (dotted line), and 10 K (dash-dotted line). Upper panel: electric field parallel to the $ab$-plane. Lower panel: electric field parallel to the $c$-axis.

Fig. 3.
The plasma edge in $La_{2-x}Sr_xCuO_4$. The electric field is parallel to the $c$-axis. The values of the concentration $x$ and temperature $T$ are indicated on the graphs.

Fig. 4.
The resonance attenuation in $Bi_2Sr_2CaCu_2O_{8+\delta}$. The surface resistance $R_s$ vs magnetic field observed at 30 GHz and 2.8 K. A remarkable hysteresis is caused by flux trapping in the crystal. In the inset the geometry of the experiment is shown.