Implications of LEP results for SO(10) grandunification with two intermediate stages

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ABSTRACT

We consider the breaking of the grand unification group SO(10) to the standard model gauge group through several chains containing two intermediate stages. Using the values of the gauge coupling constants at scale $M_Z$ derived from recent LEP data, we determine the range of their intermediate and unification scales. In particular, we identify those chains that permit new gauge structure at relatively low energy ($\sim 1\,\text{TeV}$).
Recently, $SO(10)$ breaking chains with one-intermediate stage have been examined in light of the latest LEP data. This data gives

\[
\begin{align*}
\alpha_1(M_Z) &= 0.016887 \pm 0.000040 \\
\alpha_2(M_Z) &= 0.03322 \pm 0.00025 \\
\alpha_3(M_Z) &= 0.120 \pm 0.007
\end{align*}
\]

where the $\alpha_i$s are normalized such that they would be equal when $SO(10)$ is a good symmetry and refer to $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ respectively. Our conclusion was that if $SO(10)$ breaks through a single intermediate scale to the standard model, then this scale is in the range of $10^9$ to $10^{11}$ GeV. In this report, we extend our analysis to two intermediate stage breaking schemes. Such analysis has been done previously. Our analysis differs from these in the use of the most recent data given above. We are primarily interested in identifying those chains that permit low energy gauge groups containing the standard model as a subgroup. We find that it is possible to have extra neutral gauge bosons in the low energy regime, but definitely no extra charged ones below about $10^7$ GeV.

We start by noting that all possibilities with grand unified $SU(5)$ in the intermediate stage are already ruled out by the data. So we look at symmetry breaking chains where the intermediate level gauge groups are either $\{2_L 2_R 4_c P\}$ or any of its subgroups, where $2_L$, for example, stands for the group $SU(2)_L$ and $P$ denotes an unbroken $L \leftrightarrow R$ parity symmetry. All such chains are listed below, where we have also indicated the representation of Higgs multiplet responsible for breaking at each stage.

\[
\begin{align*}
I : & \quad SO(10) \xrightarrow{210} \{2_L 2_R 4_c\} \xrightarrow{45} \{2_L 2_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
II : & \quad SO(10) \xrightarrow{54} \{2_L 2_R 4_c P\} \xrightarrow{210} \{2_L 2_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
III : & \quad SO(10) \xrightarrow{54} \{2_L 2_R 4_c P\} \xrightarrow{45} \{2_L 2_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
IV : & \quad SO(10) \xrightarrow{54} \{2_L 2_R 1_X 3_c P\} \xrightarrow{210} \{2_L 2_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
V : & \quad SO(10) \xrightarrow{210} \{2_L 2_R 4_c\} \xrightarrow{45} \{2_L 1_R 4_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
VI : & \quad SO(10) \xrightarrow{54} \{2_L 2_R 4_c P\} \xrightarrow{45} \{2_L 1_R 4_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
VII : & \quad SO(10) \xrightarrow{54} \{2_L 2_R 4_c P\} \xrightarrow{45} \{2_L 2_R 4_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
VIII : & \quad SO(10) \xrightarrow{45} \{2_L 2_R 1_X 3_c\} \xrightarrow{54} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
IX : & \quad SO(10) \xrightarrow{54} \{2_L 2_R 1_X 3_c P\} \xrightarrow{45} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
X : & \quad SO(10) \xrightarrow{210} \{2_L 2_R 4_c\} \xrightarrow{210} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
XI : & \quad SO(10) \xrightarrow{54} \{2_L 2_R 4_c P\} \xrightarrow{210} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\} \\
XII : & \quad SO(10) \xrightarrow{45} \{2_L 1_R 4_c\} \xrightarrow{45} \{2_L 1_R 1_X 3_c\} \xrightarrow{h} \{2_L 1_Y 3_c\}
\end{align*}
\]
Intermediate gauge group | Higgs contribution $T(S_i)$
--- | ---
{2L1R4C} | $T_{2L} = 1\phi^{10}$
$T_{1R} = 1\phi^{10} + 20\Delta_R^{126} + 2\delta_R^{16}$
$T_{4C} = 6\Delta_R^{126} + 1\delta_R^{16} + 4\Lambda^{45} + 4\Lambda^{210}$

{2L2R4C} | $T_{2L} = 2\phi^{10} + 40\Delta_L^{126} + 4\delta_L^{16} + 2\Sigma_L^{45} + 30\sigma_L^{210}$
$T_{2R} = 2\phi^{16} + 40\Delta_R^{126} + 4\delta_R^{16} + 2\Sigma_R^{45} + 30\sigma_R^{210}$
$T_{4C} = 18\Delta_R^{126} + 18\Delta_L^{126} + 2\delta_R^{16} + 2\delta_L^{16} + 12\sigma_R^{210} + 12\sigma_L^{210}$
$+ 4\Lambda^{45} + 4\Lambda^{210}$

{2L2RX3c} | $T_{2L} = 2\phi^{10} + 4\delta_L^{126} + 1\delta_R^{16} + 2\Sigma_L^{45}$
$T_{2R} = 2\phi^{10} + 4\delta_R^{126} + 1\delta_R^{16} + 2\Sigma_R^{45}$
$T_{1X} = 9\Delta_L^{126} + 9\Delta_R^{126} + \frac{3}{2}\delta_L^{16} + \frac{3}{2}\delta_R^{16}$
$T_{3c} = 0$

{2L1RX3c} | $T_{2L} = 1\phi^{10}$
$T_{1R} = 1\phi^{10} + 2\delta_R^{126} + \frac{1}{2}\delta_R^{16}$
$T_{1X} = 3\Delta_R^{126} + \frac{3}{2}\delta_R^{16}$
$T_{3c} = 0$

| Table 1: Expressions for $T(S_i)$ for different intermediate gauge groups. |

In the above equations, $X = \frac{B-L}{2}$. In all cases, $\{2L1Y3c\}$ breaks to $\{3c1Q\}$ with a complex 10. The breaking to the standard model is done with $h$, where $h$ can be a 16 or 126 dimensional representation of $SO(10)$. In either case, we can achieve see-saw mechanism to generate small neutrino masses, at the tree level [5] with 126, or through loops [6] using a 16.

To examine these cases, we use the one-loop renormalization group equations (RGE’s)

$$\mu \frac{\partial \alpha_i}{\partial \mu} = \frac{b_i}{2\pi} \alpha_i^2,$$  \hspace{1cm} (3)

which gives

$$\alpha_i^{-1}(M_2) = \alpha_i^{-1}(M_1) - \frac{b_i}{2\pi} \ln \frac{M_2}{M_1}.$$  \hspace{1cm} (4)

Here,

$$b_i = \frac{4}{3} n_g - \frac{11}{3} N + \frac{T(S_i)}{6}$$  \hspace{1cm} (5)

where $n_g$ is the number of fermion generations which we take as three and $N$ is the value of $N$ in $SU(N)$ with $N = 0$ for $U(1)$. The necessary expressions for $T(S_i)$ are
Table 2: The submultiplet, whose representation under the subgroup is shown, contributes to the evolution of the gauge coupling of that subgroup.

| SO(10) multiplet | Relevant submultiplet in SO(10) subgroup | Notation |
|------------------|----------------------------------------|----------|
|                  | \{2L1R1X_{c}\}                      | \phi_{16} |
| 10               | (2, \frac{1}{2}, 1)                  | \delta_{R}^{16} |
| 16               | (1, \frac{1}{2}, 4)                  | \delta_{L}^{16} |
| 16               | (2, 2, 1)                            | \Delta_{R}^{126} |
| 126              | (2, 1, 4)                            | \Delta_{L}^{126} |
| 126              | (1, 3, 10)                           | \Lambda^{45} |
| 45               | (1, 1, 10)                           | \Lambda^{210} |
| 210              | (1, 1, 15)                           | \Sigma_{R}^{45} |
| 45               | (1, 3, 1)                            | \Sigma_{L}^{45} |
| 45               | (3, 1, 1)                            | \sigma_{R}^{210} |
| 210              | (1, 3, 15)                           | \sigma_{L}^{210} |
| 210              | (3, 1, 15)                           |          |

given in Table 1. If the Higgs fields are complex then the value of $T(S_i)$ has been multiplied by a factor of two. We use the hypothesis of minimal fine-tuning \[7\], which fixes the masses of Higgs bosons according to their transformation under the unbroken subgroups at any scale. The Higgs fields that enter Table 1 are those submultiplets that have masses below the energy level of interest and contribute to evolution of the couplings. (The Greek symbols take on the values of the numbers of corresponding submultiplets with masses less than the scale of interest.) These submultiplets are defined in Table 2.

In our analysis, we match couplings at each stage of symmetry breaking. We assume that all fermions have masses less than $M_Z$. Although the mass of the $t$-quark is expected to be slightly larger than $M_Z$, and the mass of $\nu_R$ could be much larger than $M_Z$, the corrections due to these are negligible for the purposes of our calculations.

In the study of each chain, we solve analytically in one loop order for allowed scales, $n_G = \log_{10} \left( \frac{M_G}{\text{GeV}} \right)$ at grand unification, $n_2 = \log_{10} \left( \frac{M_2}{\text{GeV}} \right)$, the higher of the intermediate scales, and $n_1 = \log_{10} \left( \frac{M_1}{\text{GeV}} \right)$, the lower of the intermediate scales. The graphs for each case considered are drawn with 1σ errors from LEP data. Only those portions of the graphs are meaningful where $n_G \geq n_2 \geq n_1$. Further $n_G$ has to be sufficiently high to escape the constraint from non-observation of proton decay. We
| Chain  | Allowed values of $n_1$ |         |         |
|--------|-------------------------|---------|---------|
|        | Lowest                  | Highest |         |
| Ia     | 8.2 ± 0.2               | 10.6 ± 0.2 |
| Ib     | 10.0 ± 0.2              | 13.5 ± 0.2 |
| IIa    | 8.6 ± 0.2               | 13.6 ± 0.2 |
| IIb    | 10.0 ± 0.2              | 13.6 ± 0.2 |
| IIIa   | 8.0 ± 0.4               | 13.6 ± 0.2 |
| IIIb   | 9.8 ± 0.2               | 13.6 ± 0.2 |
| IVa    | 8.2 ± 0.2               | 10.8 ± 0.3 |
| IVb    | 9.8 ± 0.2               | 12.3 ± 0.3 |
| Va     | 11.0 ± 0.2              | 11.2 ± 0.2 |
| Vb     | 12.2 ± 0.2              | 13.6 ± 0.2 |
| VIa    | 11.2 ± 0.1              | 13.8 ± 0.2 |
| VIb    | 12.3 ± 0.1              | 13.6 ± 0.2 |
| VIIa   | 11.3 ± 0.2              | 13.6 ± 0.2 |
| VIIb   | 13.6 ± 0.2              | 13.8 ± 0.2 |
| VIIIa  | 2.0                     | 7.7 ± 0.1 |
| VIIIb  | 2.0                     | 7.5 ± 0.1 |
| IXa    | 2.0                     | 10.0 ± 0.2 |
| IXb    | 2.0                     | 10.6 ± 0.2 |
| Xb     | 2.0                     | 12.2 ± 0.2 |
| XIa    | 2.0                     | 13.5 ± 0.2 |
| XIb    | 2.0                     | 13.6 ± 0.2 |
| XIIa   | 2.0                     | 5.3 ± 0.1 |
| XIIb   | 2.0                     | 12.1 ± 0.2 |

Table 3: Acceptable domains of $n_1$ for all chains. The chains are defined in Eq. (2), and $a$, $b$ refer to the choice of $h$ being 126 or 16 respectively.
take this constraint to be approximately

\[
\left( \frac{\alpha_G^{-1}(M_G)}{40} \right) \left( \frac{M_G}{10^{15} \text{GeV}} \right)^2 > 2.5 \tag{6}
\]

Therefore, we consider the portion of any chain where \( M_G < 10^{15} \text{GeV} \) to be unacceptable. We show our findings in the graphs of Fig. 1, and in Table 3 we present the acceptable regions of the lower intermediate scale \( n_1 \) for these graphs. For each chain we refer to case (a) where \( h = 126 \) and case (b) where \( h = 16 \). Chain Xa is not featured because it has no meaningful solution.

We find no chain where extra light charged bosons can occur. In all chains with \( SU(2)_R \) or \( SU(4)_C \) in the lower intermediate scale, the allowed regions of \( n_1 \) tend to be small with both intermediate scales at very high values. We find that additional gauge bosons in the range of TeV’s are permitted only in the chains VIII through XII except for chain Xa which has no meaningful solution. All of these chains have \( \{2_L1_R1_X3_e\} \) as the lower intermediate gauge group. This means that there is only one extra gauge boson in the TeV range, and this one is neutral. Even in this set, the chains XIa and XIIb are of very marginal acceptability due to the constraint of Eq. (6) set by experiments on proton decay.

In see-saw models of neutrino mass, ordinary neutrinos are light due to a large Majorana mass of the right-handed neutrinos \([5]\). This large Majorana mass cannot be generated unless the \( \{1_R\} \) symmetry is broken. Thus the magnitude of this Majorana mass is expected to be similar to the scale of the \( \{1_R\} \) breaking. Our conclusions stated above thus show that it is possible to obtain this Majorana mass in the TeV range. However, the full parity symmetry is not restored until at much higher energy since \( \{2_R\} \)-breaking always occurs at high scale.

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Figure captions

Fig. 1: $n_G = \log_{10} \left( \frac{M_G}{\text{GeV}} \right)$ and $n_2 = \log_{10} \left( \frac{M_2}{\text{GeV}} \right)$ plotted vs. $n_1 = \log_{10} \left( \frac{M_1}{\text{GeV}} \right)$ for chains I through XII. For each chain we refer to case (a) where $h = 126$ and case (b) where $h = 16$. The constraint $n_G, n_2 \geq n_1$ is violated by $n_G$ or $n_2$ being in the shaded portion of the graphs. The acceptable domain for $n_1$ in each case is given in Table 3.