Some connectivity eccentric indices and modified eccentric indices of Benzenoid $H_k$ system

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Abstract
The topological indices correlate certain physicochemical properties such as boiling point, stability of compounds. In this paper, we define the eccentricity based connectivity eccentric index, the product connectivity eccentric index, sum connectivity eccentric index, sum line connectivity eccentric index and product line connectivity eccentric index and Modified eccentric indices, we evaluate these indices for benzenoid $H_k$ system.

Keywords
Connectivity Eccentric Index, Sum Connectivity Eccentric Index, Product Connectivity Eccentric Index, Modified Eccentric Index.

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1. Introduction
In this paper, we consider finite simple undirected graphs. Let $G$ be a graph with a vertex set $V(G)$ and an edge set $E(G)$. The $e = uv \in E(G)$. Let $e \in E(G)$, where $L(G)$ is the line graph of $G$. The vertices and edges of a graph are called the elements of $G$.

One of the best known and widely used topological index is the product connectivity index or Randić index introduced by Randić [8]. The product connectivity index of a graph $G$ is defined in [5] as

$$ (G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}. $$

Motivated by the definition of the product connectivity index, the multiplicative product connectivity index, multiplicative sum connectivity index were very recently proposed in [7]. They are defined as follows:

The multiplicative product connectivity index of a graph $G$ is defined as

$$ \chi_{II}(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}. $$

The multiplicative sum connectivity index of a graph $G$ is defined as

$$ \chi_{II}(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}. $$

The multiplicative atom bond connectivity index of a graph $G$ is defined as

$$ ABCII = \prod_{uv \in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}. $$

The multiplicative geometric-arithmetic index of a graph $G$ is defined as

$$ GAI\!I(G) = \prod_{uv \in E(G)} \frac{2d_G(u)d_G(v)}{d_G(u)+d_G(v)}. $$

1. Introduction
In this paper, we consider finite simple undirected graphs.
We define the sum connectivity eccentric index of a graph $G$. This topological index is defined as follows:

$$GA^{mII}(G) = \left( \prod_{ue \in E(G)} 2\sqrt{d_G(u)d_G(v)} \right)^a$$

The modified first and second Zagreb indices are respectively defined by Kulli [5] as

$$^mM_1(G) = \sum_{u \in VG} \frac{1}{d_e(u)^2}, \quad ^mM_2(G) = \sum_{u \in VG} \frac{1}{d_e(u)d_e(v)}$$

The modified first and second $K$ eccentric indices of a graph are defined as

$$^mB_1(G) = \sum_{ue} \frac{1}{d_G(u) + d_G(v)}, \quad ^mB_2(G) = \sum_{ue} \frac{1}{d_G(u)d_G(e)}$$

The harmonic index of a graph $G$ is defined as

$$H(G) = \sum_{ue \in VG} \frac{2}{d_G(u) + d_G(v)}$$

this index was studied by Favaron et. al. [5].

Kulli introduced the harmonic $K$-Banhatti index of a graph $G$ as

$$H_b(G) = \sum_{ue} \frac{2}{d_G(u) + d_G(e)}$$

where in all the cases $ue$ means that the vertex $u$ and edge $e$ are incident with $u$ in $G$.

2. Some Connectivity Eccentric indices and Modified Eccentric indices of a graph $G$

In this paper, we define the eccentricity based connectivity indices and modified indices of a graph $G$.

First, we define connectivity eccentric index of a graph $G$ as

$$\chi E(G) = \sum_{ue} \frac{1}{e_G(u)e_{LG}(e)}$$

We define the product connectivity eccentric index of a graph $G$ as

$$\chi_p E(G) = \prod_{ue} \frac{1}{e_G(u)e_{LG}(e)}$$

We define the sum eccentric connectivity index of a graph $G$ as

$$XE(G) = \sum_{ue} \frac{1}{\sqrt{e_G(u)+e_{LG}(e)}}$$

We define the multiplicative sum eccentric connectivity index of a graph $G$ as

$$X_p E(G) = \prod_{ue} \frac{1}{\sqrt{e_G(u)+e_{LG}(e)}}$$

We define the sum line connectivity eccentric index of a graph $G$ as

$$SLCEII = \sum_{ue} \sqrt{\frac{e_{LG}(e)}{e_G(u) + e_G(v)}}$$

We define the product line connectivity eccentric index of a graph $G$ as

$$PLCEII = \prod_{ue} \sqrt{\frac{e_{LG}(e)}{e_G(u) + e_G(v)}}$$

We define the modified eccentric first and second $K$ eccentric indices of a graph $G$ as

$$^mB_1(G) = \sum_{ue} \frac{1}{e_G(u) + e_{LG}(e)}$$

$$^mB_2(G) = \sum_{ue} \frac{1}{e_G(u)e_{LG}(e)}$$

We define the harmonic index and harmonic $K$-eccentric index of a graph $G$ as

$$H(G) = \sum_{ue \in VG} \frac{2}{e_G(u) + e_G(v)}$$

$$H_b(G) = \sum_{ue} \frac{2}{e_G(u) + e_{LG}(e)}$$

We define the modified multiplicative first and second $K$-eccentric indices of a graph $G$ as

$$^mB_1(G) = \prod_{ue} \frac{1}{e_G(u) + e_{LG}(e)}$$

$$^mB_2(G) = \prod_{ue} \frac{1}{e_G(u)e_{LG}(e)}$$

We define the harmonic eccentric index and harmonic $K$-eccentric index of a graph $G$ as

$$H(G) = \prod_{ue} \frac{2}{e_G(u) + e_G(v)}$$

$$H_b(G) = \prod_{ue} \frac{2}{e_G(u) + e_{LG}(e)}$$

where in all the cases $ue$ means that the vertex $u$ and edge $e$ are incident in $G$ and $e_{LG}(e)$ is the eccentricity of $e$ in the line graph $L(G)$ of $G$. Here, we evaluate these indices for benzenoid $H_b$ system.

2.1 Connectivity Eccentric indices and Modified eccentric indices of Benzenoid $H_b$ system:

The circumcoronene homologous series of benzenoid also belongs to the family of molecular graphs that has several copy of benzene $C_6$ on its circumference. Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$. The eccentricities of $u, v \in V(G)$ are denoted by $e(u), e(v)$ respectively and for $e = uv \in E(G)$, denote the eccentricities of the end vertices of the edge by $(e(u), e(v))$. 

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Theorem 2.1. Let $G$ be a Circumcoronene Series of Benzenoid $H_k (k \geq 1)$. The connectivity eccentric index of $G$ is given by

$$\chi E(G) = 6 \sum_{ r=1 }^{ k } \left[ \frac{ 2 }{ 2k + 2r - 1 } \right]$$

$$+ 6 \sum_{ r=1 }^{ k-1 } \left[ \frac{ r }{ 2k + 2r - 1 } + \frac{ r }{ \sqrt{2k + 2r}(2k + 2r - 1) } \right]$$

$$+ 12 \sum_{ r=1 }^{ k-1 } \left[ \frac{ r }{ 2k + 2r + \sqrt{2k + 2r + 1}(2k + 2r) } \right]$$

Proof. Let $G$ be a Circumcoronene Series of Benzenoid $H_k$. By using the definition of connectivity eccentric index, we have:

$$\chi E(G) = \sum_{ u \in E(G) } \frac{ 1 }{ \sqrt{ e_G(u)e_{L(G)}(e) } }$$

$$= \sum_{ e = uv \in E(G) } \left[ \frac{ 1 }{ \sqrt{ e_{H_k}(u)e_{L(H_k)}(e) } } + \frac{ 1 }{ \sqrt{ e_{H_k}(v)e_{L(H_k)}(e) } } \right]$$

$$= \sum_{ e = uv \in E(G) } \left[ \frac{ 1 }{ \sqrt{ e_{H_k}(u)e_{L(H_k)}(e) } } + \frac{ 1 }{ \sqrt{ e_{H_k}(v)e_{L(H_k)}(e) } } \right] + \ldots$$

$$+ \sum_{ e = uv \in E(H_k) } \left[ \frac{ 1 }{ \sqrt{ e_{H_k}(u)e_{L(H_k)}(e) } } + \frac{ 1 }{ \sqrt{ e_{H_k}(v)e_{L(H_k)}(e) } } \right]$$

$$= 6 \left( \frac{ 1 }{ \sqrt{(2k + 1)(2k + 1)} } + \frac{ 1 }{ \sqrt{(2k + 1)(2k + 1)} } \right)$$

$$+ 6 \left( \frac{ 1 }{ \sqrt{(2k + 2)(2k + 1)} } + \frac{ 1 }{ \sqrt{(2k + 2)(2k + 1)} } \right) + \ldots$$

$$+ 6 \left( \frac{ 1 }{ \sqrt{(2k + 2(k - 1) + 1)(2k + 2(k - 1) + 1)} } + \frac{ 1 }{ \sqrt{(2k + 2(k - 1) + 1)(2k + 2(k - 1) + 1)} } \right)$$

$$\chi E(G) = 6 \sum_{ r=1 }^{ k } \left[ \frac{ 2 }{ 2k + 2r - 1 } \right]$$

$$+ 6 \sum_{ r=1 }^{ k-1 } \left[ \frac{ r }{ 2k + 2r - 1 } + \frac{ r }{ \sqrt{2k + 2r}(2k + 2r - 1) } \right]$$

$$+ 12 \sum_{ r=1 }^{ k-1 } \left[ \frac{ r }{ 2k + 2r + \sqrt{2k + 2r + 1}(2k + 2r) } \right]$$

$\blacksquare$

Corollary 2.2. Eccentric based connectivity index of $H_1, H_2$ and $H_3$ are given by

$$\chi E(H_1) = 4, \chi E(H_2) = 10.2614, \chi E(H_3) = 15.998$$

Theorem 2.3. Let $G$ be a Circumcoronene Series of Benzenoid $H_k (k \geq 1)$. The product connectivity eccentric index of $G$ is given by

$$\chi p E(G) = 6 \prod_{ r=1 }^{ k } \left[ \frac{ 1 }{ (2k + 1)^2 } \right] \times \prod_{ r=1 }^{ k-1 } \left[ \frac{ r }{ (4k + 4r - 1)^2 \sqrt{2k + 2r + 1}(2k + 2r) } \right]$$

Proof. Let $G$ be a Circumcoronene Series of Benzenoid $H_k$. By using the definition of multiplicative product connectivity eccentric index, we have,

$$\chi p E(G) = \prod_{ e \in \text{edge set in } G } \frac{ 1 }{ \sqrt{ e_G(u)e_{L(G)}(e) } }$$

$$= \prod_{ e = uv \in \text{edge set in } G } \left[ \frac{ 1 }{ \sqrt{ e_{H_k}(u)e_{L(H_k)}(e) } } \times \frac{ 1 }{ \sqrt{ e_{H_k}(v)e_{L(H_k)}(e) } } \right] \times \ldots$$

$$= \prod_{ e = uv \in \text{edge set in } G } \left[ \frac{ 1 }{ \sqrt{ e_{H_k}(u)e_{L(H_k)}(e) } } \times \frac{ 1 }{ \sqrt{ e_{H_k}(v)e_{L(H_k)}(e) } } \right] \times \ldots$$

$$= 6 \prod_{ r=1 }^{ k } \left[ \frac{ 1 }{ (2k + 1)^2 } \right] \times \prod_{ r=1 }^{ k-1 } \left[ \frac{ r }{ (4k + 4r - 1)^2 \sqrt{2k + 2r + 1}(2k + 2r) } \right]$$

$\blacksquare$

Corollary 2.4. Eccentric based connectivity index of $H_1, H_2$ and $H_3$ are given by $\chi p E(H_1) = 0.6667, \chi p E(H_2) = 0.0020, \chi p E(H_3) = 2.9289e^{-0.07}$. 
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## Table 1

| Edge set $E_k$ | No. of edges $e=uv$ | Eccentricity of end vertices $(e(u), e(v))$ | Eccentricity of $e$ in $L(G)_{e(G)(e)}$ |
|---------------|---------------------|---------------------------------------------|------------------------------------------|
| $E_1$         | 6                   | $(2k+1, 2k+1)$                             | $2k+1$                                   |
| $E_2$         | 6                   | $(2k+1, 2k+2)$                             | $2k+1$                                   |
| $E_3$         | 12                  | $(2k+2, 2k+3)$                             | $2k+2$                                   |
| $E_4$         | 6                   | $(2k+3, 2k+3)$                             | $2k+3$                                   |
| $E_5$         | 12                  | $(2k+3, 2k+4)$                             | $2k+3$                                   |
| $E_6$         | 24                  | $(2k+4, 2k+5)$                             | $2k+4$                                   |
| $E_7$         | 6                   | $(2k+5, 2k+5)$                             | $2k+5$                                   |
| $E_8$         | 18                  | $(2k+5, 2k+6)$                             | $2k+5$                                   |
| $E_9$         | 36                  | $(2k+6, 2k+7)$                             | $2k+6$                                   |
| $E_{3(k-2)}$  | $2$                 | $2$                                        | $2$                                       |
| $E_{3(k-2)}$  | $6$                 | $2$                                        | $2$                                       |
| $E_{3(k-2)}$  | $12$                | $2$                                        | $2$                                       |
| $E_{3(k-1)}$  | $6$                 | $2$                                        | $2$                                       |
| $E_{3(k-1)}$  | $12$                | $2$                                        | $2$                                       |

### Theorem 2.5

Let $G$ be a Circumcoronene Series of Benzenoid $H_k$ $(k \geq 1)$. The sum connectivity eccentric index of $G$ is given by

$$
XE(G) = 6 \sum_{k=1}^{k} \left[ \frac{1}{2\sqrt{(2k+2r-1)}} \right] + 6 \sum_{r=1}^{k-1} \left[ \frac{r}{\sqrt{4k+4r+2}} + \frac{r}{\sqrt{4k+4r+1}} \right] + 12 \sum_{r=1}^{k-1} \left[ \frac{r}{\sqrt{4k+4r+2}} + \frac{r}{\sqrt{4k+4r+1}} \right]
$$

### Proof

Let $G$ be a Circumcoronene Series of Benzenoid $H_k$. By using the definition of sum connectivity eccentric index, we have,

$$
XE(G) = \sum_{e=uv \in E(G)} \frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)}} = \sum_{e=uv \in E(G)} \left[ \frac{1}{\sqrt{e_{H_1}(u)+e_{L(H_1)}(e)}} + \frac{1}{\sqrt{e_{H_2}(v)+e_{L(H_2)}(e)}} + \ldots \right] + \ldots
$$

By using the definition of multiplicative sum connectivity eccentric index, we have,

$$
XE(G) = \prod_{e=uv \in E(G)} \frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)}} = \prod_{e=uv \in E(G)} \left[ \frac{1}{\sqrt{e_{H_1}(u)+e_{L(H_1)}(e)}} \times \frac{1}{\sqrt{e_{H_2}(v)+e_{L(H_2)}(e)}} \right]
$$

### Corollary 2.6

Eccentric based connectivity index of $H_1, H_2$ and $H_2$ are given by $XE(H_1) = 4.8990, XE(H_2) = 17.5006, XE(H_3) = 30.1729$.

### Theorem 2.7

Let $G$ be a Circumcoronene Series of Benzenoid $H_k$ $(k \geq 1)$. The sum connectivity eccentric index of $G$ is given by

$$
X_e(G) = 6 \prod_{r=1}^{k} \left[ \frac{2}{(2k+2r-1)} \right]
$$

$$
X_e(G) = \prod_{r=1}^{k} \left[ \frac{2}{(2k+2r-1)} \times \frac{1}{\sqrt{(2k+2r-1)}(2k+2r)\sqrt{(2k+2r-1)}} \right]
$$

### Proof

Let $G$ be a Circumcoronene Series of Benzenoid $H_k$. By using the definition of multiplicative sum eccentric connectivity index, we have,

$$
X_e(G) = \prod_{e=uv \in E(G)} \frac{1}{\sqrt{e_G(u)+e_{L(G)}(e)}}
$$

$$
X_e(G) = \prod_{e=uv \in E(G)} \left[ \frac{1}{\sqrt{e_{H_1}(u)+e_{L(H_1)}(e)}} \times \frac{1}{\sqrt{e_{H_2}(v)+e_{L(H_2)}(e)}} \right]
$$
Theorem 2.9. Let $G$ be a Circumcoronene Series of Benzenoid $H_k (k \geq 1)$. The sum line connectivity eccentric index of $G$ is given by

$$SLCEII = 6 \sum_{r=1}^{k} \left[ \frac{1}{2} + \frac{2}{4k+4r-1} \right] + 12 \sum_{r=1}^{k} \left[ \frac{2k+2r}{4k+4r-1} \right]$$

Proof. Let $G$ be a Circumcoronene Series of Benzenoid $H_k$. By using the definition of sum line connectivity eccentric index, we have,

$$SLCEII = \sum_{e=uv \in E(G)} \frac{e_{L(G)}(e)}{e_G(u) + e_G(v)}$$

$$= 6 \sqrt{\frac{(2k+1)}{(2k+1) + (2k+1)}} + 6 \sqrt{\frac{(2k+1)}{(2k+1) + (2k+2)}} + \ldots + 6 \sqrt{\frac{(2k+2(k-1)+1)}{(2k+2(k-1)+1) + (2k+2(k-1)+1)}}$$

$$= 6 \sum_{r=1}^{k} \left[ \frac{1}{2} + \frac{2}{4k+4r-1} \right] + 12 \sum_{r=1}^{k} \left[ \frac{2k+2r}{4k+4r-1} \right]$$

Corollary 2.10. Eccentric based connectivity index of $H_1, H_2$ and $H_3$ are given by $SLCEII (H_1) = 4.2426$, $SLCEII (H_2) = 20.6829$, $SLCEII (H_3) = 49.8749$.

Theorem 2.11. Let $G$ be a Circumcoronene Series of Benzenoid $H_k (k \geq 1)$. The product line connectivity eccentric index of $G$ is given by

$$PLCEII = 6 \prod_{r=1}^{k} \left[ \frac{1}{2} + \frac{1}{4k+4r-1} \right] \prod_{r=1}^{k-1} \sum_{r=1}^{r} \left[ \frac{2k+2r-1}{4k+4r-1} \right]$$

Proof. Let $G$ be a Circumcoronene Series of Benzenoid $H_k$. By using the definition of product line connectivity eccentric index, we have,

$$PLCEII = \prod_{e=uv \in E(G)} \sqrt{\frac{e_{L(G)}(e)}{e_G(u) + e_G(v)}}$$

$$= 6 \sqrt{\frac{(2k+1)}{(2k+1) + (2k+1)}} \times 6 \sqrt{\frac{(2k+1)}{(2k+1) + (2k+2)}} \times \ldots \times 6 \sqrt{\frac{(2k+2(k-1)+1)}{(2k+2(k-1)+1) + (2k+2(k-1)+1)}}$$

$$= 6 \sum_{r=1}^{k} \left[ \frac{1}{2} + \frac{2}{4k+4r-1} \right] + 12 \sum_{r=1}^{k} \left[ \frac{2k+2r}{4k+4r-1} \right]$$

Corollary 2.12. Eccentric based connectivity index of $H_1, H_2$ and $H_3$ are given by $PLCEII (H_1) = 4.2426$, $PLCEII (H_2) = 593.6051$, $PLCEII (H_3) = 3.5245e^{+005}$.

3. Modified eccentric first and second K-eccentric index of Benzenoid $H_k$, system

Theorem 3.1. For any positive integer number $k$, let $H_k$ be the general form of circumcoronene series of benzenoid system, then

(i) $mB_1 (H_k) = 6 \sum_{r=1}^{k-1} \left[ \frac{1}{2k+2(r-1)+1} \right]$

(ii) $mB_2 (H_k) = 6 \sum_{r=1}^{k-1} \left[ \frac{2}{(2k+2(r-1)+2) \cdot (2k+2(r-1)+1)^2} \right]$
Proof. Consider the General form of \( H_k \) - Circumcoronene graph, Using Table 2.1, we obtain the following:

(i) \( \sum_{uv \in E(G)}^\r
6 \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

(ii) \( \sum_{uv \in E(G)}^\r
6 \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

(iii) \( \sum_{uv \in E(G)}^\r
6 \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\( H_b(H_k) = 12 \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ + 12 \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ + 24 \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ + 12 \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ + 24 \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

\[ \sum_{r=1}^{k-1} \left[ \frac{r}{(2k+2r)} \right]^2 + \frac{r}{(2k+2r)(2k+2r-1)+1} \]

Corollary 3.2. \( H_1 \) be the first terms of this Circumcoronene series of Coronene \( H_k \). Then \( mB_1(H_1) = 2 \), \( mB_2(H_1) = 1.3333, H_b(H_1) = 4 \).

Corollary 3.3. \( H_2 \) be the second terms of this Circumcoronene series of Coronene \( H_k \). Then \( mB_1(H_2) = 5.1257 \), \( mB_2(H_2) = 1.7839, H_b(H_2) = 10.2513 \)

Corollary 3.4. \( H_3 \) be the third terms of this Circumcoronene series of corone \( H_k \). Then \( mB_1(H_3) = 7.9948, mB_2(H_3) = 1.8156, H_b(H_3) = 15.9896 \).

4. Modified Multiplicative eccentric first and second \( K \)-eccentric indices of Benzenoid \( H_k \) system

Theorem 4.1. For any positive integer number \( k \), let \( H_k \) be the general form of circumcoronene series of benzenoid system, then

(i) \( mB_1 \prod_{r=1}^{k} \left[ \frac{1}{(2k+2r-1)^2} \right] \)

(ii) \( mB_2 \prod_{r=1}^{k} \left[ \frac{1}{(2k+2r-1)^2} \right] \)

(iii) \( H_b \prod_{r=1}^{k} \left[ \frac{1}{(2k+2r-1)^2} \right] \)

Proof.

(i) \( mB_1 \prod_{r=1}^{k} \left( \frac{1}{e_{H_k}(u)} + \frac{1}{e_{L(H_k)}(v)} \right) \)

(ii) \( mB_2 \prod_{r=1}^{k} \left( \frac{1}{e_{H_k}(u)} + \frac{1}{e_{L(H_k)}(v)} \right) \)

(iii) \( H_b \prod_{r=1}^{k} \left( \frac{1}{e_{H_k}(u)} + \frac{1}{e_{L(H_k)}(v)} \right) \)
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... some connectivity eccentric indices and modified eccentric indices of Benzenoid $H_k$ system.

5. Conclusion

The purpose of this paper is to discuss the eccentric indices of chemical structure $H_k$. We have determined the connectivity eccentric index, sum connectivity eccentric index and product connectivity eccentric index, sum and product line connectivity eccentric index and Modified eccentric first and second $K$-eccentric index and Modified Multiplicative eccentric first and second $K$-eccentric indices of Benzenoid $H_k$ system. This can be extended for other structures also.

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ISSN(P):2319 – 3786
Malaya Journal of Matematik
ISSN(O):2321 – 5666
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