Noise-Robust Quantum Teleportation with Counterfactual Communication

MUHAMMAD ASAD ULLAH, SAW NANG PAING, AND HYUNDONG SHIN, (Fellow, IEEE)
Department of Electronics and Information Convergence Engineering, Kyung Hee University, 1732 Deogyang-daero, Gichteung-gu, Yongin-si, Gyeonggi-do 446-701 Korea

Corresponding author: Hyundong Shin (e-mail: hshin@khu.ac.kr).

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ABSTRACT
Direct counterfactual communication (DCC) based on the dynamic quantum Cheshire cat effect is an interesting protocol for particle-less information transfer. In this paper, we propose a noise-robust, completely counterfactual, teleportation scheme based on the modified DCC. We show that our scheme does not require any a priori shared phase reference; an essential requirement for conventional teleportation schemes. For our modified DCC, we investigate the noise acting on the remotely controlled properties of a photon, i.e., path and polarization. We show that our proposed modification and resulting teleportation scheme is robust against channel delay and dephasing noise. While for photonic loss, depolarizing, and bit-flip noise, it is possible to obtain the correct state given only the channel state information.

INDEX TERMS
Quantum Cheshire Cat effect, counterfactual communication, interferometric invisibility, quantum depolarizing channel, quantum teleportation, shared phase reference.

I. INTRODUCTION
Throughout human history, information transfer between two distant locations required messengers. For the majority of human civilization, these messengers were actual people. In the last two centuries, information transfer has probably been revolutionized more than any other field. Now, we can communicate huge amounts of information at unprecedented rates through radio and microwave particles as messengers. Recently, a counterintuitive information transfer method has been discovered which does not require any messenger particles between the two parties for information transfer [1]–[11]. This method referred to as counterfactual communication, relies on the dynamic quantum Cheshire cat effect, wherein a controlled property of a particle can be modified at a location which the particle never visited [1]. To date, counterfactual communication has been utilized to transfer classical information, entanglement, and quantum information; which collectively are revolutionizing the next generation of information transfer through technologies like teleportation and super-dense coding to name a few [5], [12]–[14].

Direct counterfactual communication (DCC) is the first counterfactual protocol for particle less communication [2]. It relies on the interaction-free measurement and quantum Zeno effect to transfer information without sending any photon over the channel [15]–[19]. This provides security and information concealing ability without requiring conventional quantum secure schemes [15]–[19]. The system setup consists of two cascaded Michelson interferometers, at one party (say Alice), acting on a photon’s polarization degree of freedom. While an absorptive object (e.g., an electron) is available at the other party (say Bob). In each cycle of the system, the polarization and path controlled properties of a photonic wave-function are modified based on the presence or absence of an absorptive object [1]. It is to be noted that although there is no information-carrying particle over the channel, counterfactual communication requires a quantum channel between the two parties on which the controlled property of the photon (but not the photon itself) interacts with the electron. The DCC and subsequent protocols based on DCC have been utilized for communication, remote computation, entanglement generation and clock synchronization [12]–[14]. Recently, DCC has been experimentally implemented through a
Quantum teleportation (QT) is a mechanism for transfer of quantum state between remote parties, say Alice and Bob, through classical communication and prior shared entanglement [29], [30]. Since entanglement is preshared, teleportation reduces the communication overhead to just two bits per qubit. However, the fidelity of preshared entanglement relies on the common phase reference between Alice and Bob [31], [32]. In practice, this requirement of sharing nonfungible information is hard to achieve [31]. Therefore, a quantum teleportation protocol that works even in the absence of shared phase reference becomes an interesting prospect.

In this paper, we propose a completely counterfactual teleportation (CT) scheme, based on the DCC, that does not require any shared phase reference. To make our scheme robust against noise, we modify the original DCC by introducing a quantum noise mitigation setup as shown in Fig. 1. We then investigate the proposed scheme for the noise suffered by the remotely controlled properties (path and polarization) in the counterfactual interferometric system. We analyze the performance of our scheme in the presence of phase diffusion, interferometric invisibility, and photonic dispersion losses affecting the path DOF alongside the polarization DOF noise models, including dephasing, bit-flip, and depolarizing noise. We show that our proposed scheme is robust against channel delay and dephasing noise all the while outperforming the conventional teleportation for the remaining noise models as well.

The paper is arranged as follows. In Section II, we first introduce the modified DCC setup. Then, we provide the completely counterfactual teleportation mechanism. We establish the robustness of our modified DCC based counterfactual teleportation against the absence of shared phase reference and quantum noise in Section III. Finally, we provide a conclusion and future directions in Section IV.

II. MODIFIED DIRECT COUNTERFACTUAL COMMUNICATION (DCC)

Taking account of the combined logic of interaction-free measurement (IFM) and chained quantum Zeno (CQZ) effect, counterfactual communication enables the direct transmission of information between two communicators (say Alice and Bob) in a particle-less manner. In this section, we first review the original DCC setup before providing the modified DCC setup for noise-robust counterfactual teleportation.

A. DIRECT COUNTERFACTUAL COMMUNICATION SETUP

The practical setup of DCC is realized through two cascaded Michelson interferometers as shown in Fig. 1. Alice inputs a horizontally polarized photon ($|H⟩_A$) into her H-CQZ gate located at one end of the quantum transmission channel and Bob counterfactually controls the polarization of the photon through an absorptive object. The absorptive object either blocks or unblocks the photonic wave-function in the channel. The blocking and unblocking action by Bob controls the photon polarization state even though the photon never visited Bob. For logical 1, Bob blocks, leading to a $|H⟩_A → |V⟩_A$ rotation on the photon. For logical 0, Bob does not block, which leaves the photon state unchanged. $|↑⟩_B (|↓⟩_B)$ represents the block (unblock) state of AO.

Alice performs $M$ cycles of the outer interferometer. Inside each outer cycle, she performs $N$ cycles of the inner Michelson interferometer. The value of $M (N)$ is decided by the success probability of the counterfactual setup as discussed later in Section III-D. Alice starts the H-CQZ operation by inputting the $|H⟩_A$-polarized pho-
In order to achieve counterfactual communication, Alice performs Bell basis measurement on the message qubit and sends her QAO state. In the next step, Alice applies Bell basis measurement on the message qubit and her QAO setup, the SPR1 rotates the input state $|H\rangle_A$ to $|H\rangle_A + \sin \theta_K |V\rangle_A$. Then the PBS acts as

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\alpha |H\rangle_A |0\rangle_{\text{path}} \\
\beta |V\rangle_A |0\rangle_{\text{path}} \\
0 \\
0
\end{bmatrix}
= 
\begin{bmatrix}
\alpha |H\rangle_A |0\rangle_{\text{path}} \\
\beta |V\rangle_A |1\rangle_{\text{path}} \\
0 \\
0
\end{bmatrix}.
$$

The V-polarized component in $|1\rangle_{\text{path}}$ encounters SPR2 followed by the PBS1 which makes the H-component in $|1\rangle_{\text{path}}$ enter channel via $|2\rangle_{\text{path}}$. If Bob’s AO is present i.e., in the $|\uparrow\rangle_B$ state, it blocks the photonic wave-component in the channel collapsing the wavefunction. However, if AO is absent i.e., in the $|\downarrow\rangle_B$ state, the mirror $M_3$ reflects the wave-component. After this, the reflecting component interferes with V-component in $|1\rangle_{\text{path}}$ at PBS2. After interference, the photonic wavefunction in the inner interferometer loops back to start the next inner cycle. The state of the photon after the $n^{th}$ cycle for the unblocking case is

$$
|H\rangle_A \rightarrow \cos n\theta_N |H\rangle_A + \sin n\theta_N |V\rangle_A \xrightarrow{n=N} |V\rangle_A,
$$

while for the blocking case, the state is

$$
|H\rangle_A \rightarrow \cos^{n-1}(\cos \theta_N |H\rangle_A + \sin \theta_N |V\rangle_A) \xrightarrow{n=N} |H\rangle_A.
$$

After the completion of $N$ inner cycles, the polarization components form $|0\rangle_{\text{path}}$ and $|1\rangle_{\text{path}}$ interfere at PBS1. D3 absorbs any $|V\rangle_A$ component that comes out of from the inner interferometer. This completes one outer cycle and the photon loops back to start the next outer cycle. In order to achieve counterfactual communication, Alice completes $M$ times of the outer interferometer. The state of the photon for the unblocking case after the $m^{th}$ cycle is

$$
|H\rangle_A \rightarrow \cos^{m-1}(\cos \theta_M |H\rangle_A + \sin \theta_M |V\rangle_A) \xrightarrow{n=M} |H\rangle_A,
$$

while for the blocking case, the state is

$$
|H\rangle_A \rightarrow \cos n\theta_M |H\rangle_A + \sin n\theta_M |V\rangle_A \xrightarrow{n=M} |V\rangle_A.
$$

It can be seen clearly that as the number of inner and outer cycles ($N$ and $M$) approaches to infinity, the chance of an information-carrying particle passing through the channel approaches to zero. As a result, a classical bit can be transmitted counterfactually depending on Bob’s action of blocking or unblocking the polarization property of the photon [1].

1) Counterfactual Entanglement Distribution

Recently, some counterfactual entanglement distribution mechanisms have been proposed based on the DCC protocol [5], [13], [28]. These utilize a quantum AO (QAO) which can exist in a superposition of the blocking ($|\uparrow\rangle_B$) and unblocking ($|\downarrow\rangle_B$) states.
Consider that Bob holds a QAO in the maximal superposition state \( \frac{1}{\sqrt{2}} (|\uparrow\rangle_B + |\downarrow\rangle_B) \). Initially, the joint state of Alice’s photon and Bob’s QAO is

\[
|\psi\rangle_1 = |H\rangle_A \otimes \frac{1}{\sqrt{2}} (|\uparrow\rangle_B + |\downarrow\rangle_B).
\]

As discussed before, if Bob’s QAO is in state \( |\uparrow\rangle_B \), it rotates the polarization of Alice’s photon from H to V after the complete H-CQZ operation. On the other hand, the polarization of Alice’s photon remains unchanged if Bob decides to put QAO in the \( |\downarrow\rangle_B \) state. Hence, the combined action of blocking and unblocking by Bob leads to

\[
|\psi\rangle_1 \rightarrow |\psi\rangle_2 = \frac{1}{\sqrt{2}} (|H\rangle_A |\downarrow\rangle_B + |V\rangle_A |\uparrow\rangle_B).
\]

Therefore, the entanglement has been shared between Alice and Bob, but no information-carrying particle passes through the channel.

B. NOISE-ROBUST COMPLETELY COUNTERFAC TUAL TELEPORTATION

Quantum teleportation is an intriguing protocol for transmitting quantum states through classical channels [29]. Conventional quantum teleportation protocol involves four steps: 1) Alice prepares a bipartite entangled state and sends one particle to Bob, 2) Alice applies the local Bell basis measurement on her particle and message qubit [29], 3) Alice sends two-bit message to Bob over the classical channel, 4) Bob obtains the original quantum message by transforming her half of the originally entangled state using the two received bits. Therefore, from the resource inequality, 1 ebit and 2 cbits are used to transmit 1 qubit of quantum information; where ebit is an entangled two-qubit state and cbit is a classical bit [33].

Previous protocols for counterfactual quantum teleportation based on DCC involve the aid of classical channel [5], [28]. However, the DCC setup itself cannot be directly utilized for practical communication systems due to several limitations. One such limitation is the imperfect interference due to the quantum channel noise. In this paper, we modify the DCC setup to make it noise-robust and develop a completely counterfactual teleportation system without requiring any classical channel. To account for the quantum channel noise, we modify the DCC through a quantum noise mitigation setup as highlighted in the Fig. 1. It is to be noted here that this modification becomes a necessity for polarization DOF noise on the channel component of the photon [7], [34]. We employ PBS\(_{QNM}^{\uparrow}\) with detector D\(_{QNM}\) to remove any V-component that appears due to polarization DOF noise. The purpose of PR\(_{QNM}\) is to transform the \( |H\rangle \) component in [2]\(_{path}\) to a suitable polarization given channel state information. The detailed utilization of noise mitigation setup has been provided in the next section.

We realize the task of teleportation through this modified DCC system by utilizing 2 consecutive H-CQZ\(_{QNM}\) gates as illustrated in Fig. 2. The first H-CQZ\(_{QNM}\) gate has QAO as the control qubit while the second H-CQZ\(_{QNM}\) gate has classical AO as the control bit.

Suppose that Bob wants to send his qubit \( |\phi\rangle = \alpha |0\rangle + \beta |1\rangle \) to Alice. Unlike conventional communication, the receiver (not the sender) utilizes classical channel during communication. To initiate the protocol, Alice inputs her first H-polarized photon into the first H-CQZ\(_{QNM}\) gate while Bob holds QAO in a maximally superposition state. At the output of the first H-CQZ\(_{QNM}\), an EPR pair is created between Alice’s QAO and Bob’s first photon according to (9). Following this, Bob makes Bell basis measurement on \( |\phi\rangle \) and his QAO. The four measurement operators are

\[
\begin{align*}
\Pi_0 &= |\phi^+\rangle \langle \phi^+|,
\Pi_1 &= |\phi^-\rangle \langle \phi^-|,
\Pi_2 &= |\phi^+\rangle \langle \phi^-|,
\Pi_3 &= |\phi^-\rangle \langle \phi^+|,
\end{align*}
\]

where

\[
\begin{align*}
|\psi^+\rangle &= \frac{1}{\sqrt{2}} (|0\rangle |\uparrow\rangle_B \pm |1\rangle |\downarrow\rangle_B),
|\phi^\pm\rangle &= \frac{1}{\sqrt{2}} (|1\rangle |\uparrow\rangle_B \pm |0\rangle |\downarrow\rangle_B).
\end{align*}
\]

Depending on the result of Bell basis measurement \{\( \Pi_0, \Pi_1, \Pi_2, \Pi_3 \)\}, Bob sends two classical bits \( b_0b_1 \) from the corresponding set \{00, 01, 10, 11\} to Alice. For this purpose, Alice successively inputs two H-polarized photons into the second H-CQZ\(_{QNM}\) gate while Bob prepares classical AOs for each transmission of \( b_i \). Once Alice receives \( b_0 \) and \( b_1 \), she applies the Pauli operation \( Z^{b_0}X^{b_1} \) on her photon that exists at the output of the first H-CQZ\(_{QNM}\) gate, as shown in Fig.2, to transform it to the state \( |\phi\rangle \). Hence, Bob can successfully teleport her quantum state to Bob without transmitting any physical particles through the quantum channel and without utilizing the classical channel. Ignoring any photon losses, the total resources consumed for our counterfactual teleportation protocol are 3 photons, 1 QAO, and 1 AO; i.e., 4 qubits and 1 cbit. Conventional teleportation protocol requires 2 qubits (as 1 ebit) and 2 cbits [35]. So, the trade-off for not requiring any classical channel is the need for 2 extra non-traveling qubits.

III. PROTECTION OF COUNTERFAC TUAL SYSTEM AGAINST NOISE

Quantum noise has been the fundamental bottleneck in the implementation of practical quantum computation and communication systems. In this section, we investigate the performance of counterfactual quantum information transfer against both unspeakable and speakable quantum noise [31]. The unspeakable quantum noise refers to the absence of shared phase reference between the distributed parties. Shared phase reference refers to the existence of common definitions of superposition
quantum states e.g., $|+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$ and the operators like Hadamard operation

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$

for quantum information processing at distributed nodes (see Fig. 3). While speakable quantum noise constitutes channel and local quantum noise affecting the involved DOFs of a quantum system. For the counterfactual system, these are path and polarization DOFs of a photon on Alice’s end and QAOs at Bob’s end. For the counterfactual interferometric setup, Salih et al. discussed the detector and optical element inefficiency in their analysis of the DCC for classical communication [2]. Later, a counterfactual-like communication based on coherent light was analyzed by [36], [37]. However, this analysis is limited to the interferometric noise affecting multi-photon non-polarization states unlike single photon polarization state utilized in DCC [2], [7]. The single photon implementation almost inhibits phase diffusion via active phase stabilization to obtain near-perfect result [7]. For the active phase stabilization, two Piezoceramic translation stages are utilized. So far, there has been no noise analysis of the DCC encompassing the polarization and path analysis which as we shall see in the next subsection requires modifications to the original DCC based information transfer schemes. Furthermore, our analysis in not limited to classical information but also to the quantum information transfer involving QAOs. We now perform this analysis and show how these modifications make DCC based schemes robust compared to direct quantum communication based schemes.

A. ABSENCE OF SHARED PHASE REFERENCE

Pre-shared entanglement (especially singlets $|\phi^-\rangle$) between Alice and Bob is a necessary ingredient for some of the groundbreaking quantum protocols. These include teleportation, entanglement-assisted quantum key distribution and the revolutionary quantum clock synchronization protocol that does not require any real-time communication [30], [32], [35], [38], [39]. Conventionally, entanglement sharing is a prepare-and-share approach in which one party (say Alice) prepares $X$ entangled states locally and then shares one particle of each entangled state with Bob [32], [39], [40]. However, this method of entanglement sharing is prone to quantum infidelity in the absence of the shared phase reference between Alice and Bob [31]. This is because the shared entanglement is in Alice’s basis definition and not the local basis definition of each party, i.e., [32]

$$\frac{1}{\sqrt{2}} (|10\rangle_A - |01\rangle_A) \neq \frac{1}{\sqrt{2}} (|11\rangle_A |0\rangle_B - |00\rangle_A |1\rangle_B).$$

(10)

Recently, entanglement purification has been identified as a possible solution to counter the absence of shared phase reference [29], [32]. In this procedure, using the quantum circuit method, Bennet et al.’s entanglement purification is iteratively used to obtain singlets in the local basis [29]. However, it leads to the loss of half population of entangled states in each purification cycle. Furthermore, the fidelity $F_n$ of the entangled pairs after $n$ rounds of purification is [29]

$$F_n = \frac{F_{n-1}^2 + \frac{1}{3} (1 - F_{n-1})^2}{F_{n-1}^2 + \frac{2}{3} F_{n-1} (1 - F_{n-1}) + \frac{2}{3} (1 - F_{n-1})^2},$$

(11)

where $F_{n-1}$ is the fidelity after $(n-1)$th round of purification. After the $n$th round, the $X$ entangled states are reduced to $X^2/2^n$ singlets. Furthermore, the probability of discarding the results of purification for a round is non-zero.

For counterfactual communication, the quantum operations on a qubit are always local. In the setup of Fig. 2, the QAO does not evolve during the operation. Meanwhile, the H-polarized photon undergoes quantum transformations at Alice’s end only. The only action from Bob’s QAO and AO on the photon is either reflection or
absorption of the H-component on the channel. Therefore, our setup allows independent local definitions of phase reference. This makes our setup practical for plug-and-play networks without requiring a prior handshake for quantum information transfer unlike previous quantum teleportation schemes. On the flip side, counterfactual teleportation is probabilistic due to the possibility of a photon ending up in the channel. We have discussed the success probability of our scheme in III-D.

B. POLARIZATION DOF NOISE IN DCC PROTOCOLS
The presence of noise on the quantum channel decoheres the quantum state which directly affects the counterfactual communication. We first discuss the noise affecting the polarization property of the photon over the channel. For this purpose, we consider the depolarizing noise, dephasing noise and, bit-flip noise models in the computational basis. Note that for all the noise models, we assume that the photon is in the computational basis where

\[
\rho_{\text{out}} = \sum_{i=0}^{1} \mathcal{K}_{A_i} \rho \mathcal{K}^\dagger_{A_i},
\]

where \(\mathcal{K}_{A_0} = \sqrt{1-p} I, \mathcal{K}_{A_1} = \sqrt{p} Z\) and \(p\) is dephasing noise parameter and \(0 \leq p \leq 1\). Here \(I\) is the identity operator on the quantum system while \(Z\) is the Pauli-Z quantum operator on a qubit. One primary advantage of the counterfactual setup is that the photonic component in the channel has no entropy because it is always H-polarized. Assuming we have the knowledge of the preferred polarization state over the channel, the PRQNM in Fig 1 converts H-polarized channel component to the preferred polarization and re-converts it back to H once it returns from Bob. Since only the preferred polarization basis enters the channel, therefore it will not be affected by the dephasing noise. For the case of dephasing noise in computation basis where \(\mathcal{K}_{A_0} \propto Z\), if the horizontal polarization corresponds to the lower level (level-0), the preferred basis is H-polarized [33]. Therefore, PRQNM will not cause any rotation. Hence, the effect of the dephasing channel noise is nullified.

In comparison to our setup, conventional quantum teleportation involves entanglement preparation and distribution (EP&D). The distribution involves one particle travelling over the channel which is susceptible to dephasing.

1) Dephasing channel
We first consider the effect of dephasing channel noise in the polarization degree of freedom on our setup. Generally, the dephasing noise modifies a state \(\rho\) as

\[
\rho_{\text{out}} = \sum_{i=0}^{1} \mathcal{K}_{A_i} \rho \mathcal{K}^\dagger_{A_i},
\]

where \(\mathcal{K}_{A_0} = \sqrt{1-p} I, \mathcal{K}_{A_1} = \sqrt{p} Z\) and \(p\) is dephasing noise parameter and \(0 \leq p \leq 1\). Here \(I\) is the identity operator on the quantum system while \(Z\) is the Pauli-Z quantum operator on a qubit. One primary advantage of the counterfactual setup is that the photonic component in the channel has no entropy because it is always H-polarized. Assuming we have the knowledge of the preferred polarization state over the channel, the PRQNM in Fig 1 converts H-polarized channel component to the preferred polarization and re-converts it back to H once it returns from Bob. Since only the preferred polarization basis enters the channel, therefore it will not be affected by the dephasing noise. For the case of dephasing noise in computation basis where \(\mathcal{K}_{A_0} \propto Z\), if the horizontal polarization corresponds to the lower level (level-0), the preferred basis is H-polarized [33]. Therefore, PRQNM will not cause any rotation. Hence, the effect of the dephasing channel noise is nullified.

2) Bit-Flip Noise
Similar to dephasing, bit-flip noise is generally represented by the Kraus operators \(\mathcal{K}_{A_0} = \sqrt{1-p} I, \mathcal{K}_{A_1} = \sqrt{p} X\), where \(X\) is the Pauli-X operation. Once the
The vantage associated with the PBS under the bit-flip operation. Here again, we utilize the ad-channel component has been transformed to the mixed-photon component returns from the channel, H-polarized noise.

Counterfactual entanglement generation for counterfactual shared entangled state. Fig. 5 shows the purity of counterfactually teleported state for error-free qubit states according to the Haar measure. Here, the mean performance of the classical communication. Each line in the figure shows the fidelity of output states for depolarizing noise along

where

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Michelson interferometer, the channel delay information components in son’s interferometer. In this case, once the two reflected nent is reflected, the inner interferometer acts as Michel-}

\[
\rho_{\text{out}} = \frac{1}{2\pi} \int_{\theta_\ell} p_{\theta_\ell}(\theta_\ell) R_{\theta_\ell} \rho R_{\theta_\ell}^\dagger d\theta_\ell, \quad (18)
\]

For the combined photon-QAO system, the photon component in the channel \( |H(V)\rangle_A |2\rangle_{\text{path}} \) interacts with the present \( |\uparrow\rangle_B \) and absent \( |\downarrow\rangle_B \) components of QAO. The interaction between QAO present component \( |\uparrow\rangle_B \) with \( |H(V)\rangle_A |2\rangle_{\text{path}} \) results in \( |H(V)\rangle_A |2\rangle_{\text{path}} \)’s absorption. While the interaction between QAO absent component \( |\downarrow\rangle_B \) with \( |H(V)\rangle_A |2\rangle_{\text{path}} \) reflects the photon. In the latter case, the randomized rotation due to channel delay is applied between the \( |1\rangle_{\text{path}} \) and \( |2\rangle_{\text{path}} \) arms of the inner interferometer. If the photon component is reflected, the inner interferometer acts as Michelson’s interferometer. In this case, once the two reflected components in \( |1\rangle_{\text{path}} \) and \( |2\rangle_{\text{path}} \) meet in the inner Michelson interferometer, the channel delay information gets encoded into the polarization DOF. Therefore, without loss of generality, for \( i \)th Kraus operator, we consider \( R_{\theta_{\text{path}_{1,2}}} \) being applied between the paths \( |1\rangle_{\text{path}} \) and \( |2\rangle_{\text{path}} \) as

\[
\mathcal{K}_i = |\downarrow\rangle_B (|\downarrow\rangle_B \otimes I_A \otimes R_{\theta_{\text{path}_{1,2}}} + |\downarrow\rangle_B (|\downarrow\rangle_B \otimes I_A \otimes |0\rangle_{\text{path}} \langle 0|_{\text{path}} + |\uparrow\rangle_B (|\uparrow\rangle_B \otimes I_A \otimes I_{\text{path}}. \quad (19)
\]

Considering this, the effect of channel delay is to cause a path mismatch between the two arms of the inner interferometer for the case when QAO is absent. This causes periodic sudden drops in fidelity whenever the channel component of the photon is reflected back by Bob, as evident in Fig. 7. For given values of \( M \) and \( N \), fidelity within a tolerance level could be achieved in this case even if only approximate channel state information is available. Unlike counterfactual case, conventional entanglement distribution through EP&D method has a progressive drop in fidelity. For the case where deviation in path length corresponds to \( \phi = \pi \), fidelity is 0.

2) Photonic loss/dispersion in Channel

In a lossy counterfactual interferometric setup, fictitious polarization beam splitters in the channel account for the photonic loss and dispersion in the channel [34]. In the case of this dispersion, the component in the channel disperses and is replaced by vacuum state. For counterfactual setup, we shall treat such dispersion as equivalent to the absorption of channel component due to noise. This corresponds to an amplitude damping channel with Kraus operators given as

\[
\mathcal{K}_{\text{path}_0} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\eta} \end{pmatrix}, \quad \mathcal{K}_{\text{path}_1} = \begin{pmatrix} 0 & \sqrt{\eta} \\ 0 & 0 \end{pmatrix}. \quad (20)
\]
where $\eta = 1 - e^{-\gamma t}$ is the decay probability and $\gamma$ is the loss rate. For these Kraus operators, path$_1$ is the non-decaying entry and path$_2$ is the decaying entry. The generalized noise model will therefore be

$$
\mathcal{K}_i = |\downarrow\rangle_B \langle \downarrow|_B \otimes I_A \otimes \mathcal{K}_{\text{path}},
$$

$$
+ |\downarrow\rangle_B \langle \downarrow|_B \otimes I_A \otimes |0\rangle_{\text{path}} \langle 0|_{\text{path}}
$$

$$
+ |\uparrow\rangle_B \langle \uparrow|_B \otimes I_A \otimes I_{\text{path}}.
$$

Fig. 8 shows the fidelity of the H-CQZ$_{\text{QNM}}$ operation in the presence of photonic losses for both CT and QT. In comparison to conventional schemes, phase diffusion does affect the fidelity of counterfactual teleportation. Although the loss of fidelity is more than conventional teleportation, output state recovery is possible given the channel state information due to no mixedness (see Fig. 5).

D. PROBABILITY OF SUCCESS FOR H-CQZ$_{\text{QNM}}$ OPERATIONS

For each of the three H-CQZ$_{\text{QNM}}$ operations in Fig. 2, if the photon ends up in the channel, the counterfactuality is lost. Therefore, we need to redo that H-CQZ$_{\text{QNM}}$ operation. The success probability for H-CQZ$_{\text{QNM}}$ operation with AO absent is $\lambda_0$ given by [2]

$$
\lambda_0 = \cos^2 M \theta_M,
$$

while for AO present case, we have

$$
\lambda_1 = \prod_{m=1}^{M} \left[ 1 - \sin^2 (m\theta_M) \sin^2 \theta_N \right]^N.
$$

For entanglement generation with QAO, the probability of success is [42]

$$
\lambda_2 = \left( 1 - \frac{1}{2} \sin^2 \theta_M \right)^M \prod_{m=1}^{M} \left[ 1 - \frac{1}{2} \sin^2 (m\theta_M) \sin^2 \theta_N \right]^N.
$$

Figures 9, 10 and 11 show the successful probability of H-CQZ$_{\text{QNM}}$ operation as a function of outer cycles $M$ and inner cycles $N$ of H-CQZ$_{\text{QNM}}$ gate for AO absence, AO presence and QAO in the channel respectively.

IV. CONCLUSION

We have proposed a completely counterfactual teleportation scheme based on the modified DCC. The modification allows us to achieve high-fidelity teleportation under noisy conditions compared to the conventional quantum teleportation. We have shown that our modified DCC based teleportation scheme is robust against channel delay and dephasing noise. While for photonic loss, depolarizing, and bit-flip noise, it is possible to retrieve the original state given only the channel state information. Furthermore, our scheme does not require any a priori shared phase reference which is a necessity for conventional quantum teleportation schemes.

Future works may include an experimental implementation of our proposed setup and investigating the hardware requirements and considerations for scaling up this setup to a network level.

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