Persistent Fine-Tuning in Supersymmetry and the NMSSM

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We examine the use of modified Higgs sectors to address the little hierarchy problem of supersymmetry. Such models reduce weak-scale fine-tuning by allowing lighter stops, but they retain some fine-tuning independent of the Higgs. The modified Higgs sector can also introduce model-dependent tunings. We consider the model-independent constraints on naturalness from squark, gluino, chargino and neutralino searches, as well as the model-dependent tuning in the PQ- and R-symmetric limits of the NMSSM, where cascade decays or additional quartic interactions can hide a Higgs without relying on heavy stops. Obtaining viable electroweak symmetry breaking requires a tuning of $\sim 1 - 10\%$ in the PQ limit. A large A-term is also necessary to make charginos sufficiently heavy, and this introduces an additional weak-scale tuning of $\sim 10\%$. The R-symmetric limit requires large marginal couplings, a large singlet soft mass, and a tuning of $\sim 5 - 10\%$ to break electroweak symmetry. Hiding MSSM-like Higgs states below $\sim 112$ GeV requires additional tunings of A-terms at the $\sim 1 - 10\%$ level. Thus, although the NMSSM has rich discovery potential, it suffers from a unique fine-tuning problem of its own.

I. INTRODUCTION

The little hierarchy problem of supersymmetry is a blight on an otherwise impressive new-physics framework. Supersymmetry may explain both the mechanism of electroweak symmetry breaking and the weakness of gravity [1]. Moreover, in the minimal supersymmetric Standard Model (MSSM), gauge coupling unification occurs with striking accuracy near $M_{GUT} \approx 2 \times 10^{16}$ GeV and the lightest superpartner is a cold dark matter candidate.

Yet the heaviness of superpartners has created an apparent fine-tuning problem. $M_Z^2$ is determined by the Higgs soft masses, and depends indirectly on stop and gluino masses. A hierarchy between the $Z$ and the stop and gluino masses implies fine-tuning of the spectrum. The non-discovery of a Higgs at LEP, which is only consistent with the MSSM if the stop mass is $\gtrsim 500$ GeV, hints at such a hierarchy. For this reason, many attempts to reduce this little hierarchy have focused on the Higgs sector, either hiding a light Higgs or adding new Higgs quartic interactions so that $m_H > M_Z$ is consistent at tree-level.

But two sources of fine-tuning still plague models with a hidden or heavy Higgs. First, the Higgs is just one of many indications of a Z-stop hierarchy and the resulting tuning—the absence of other superpartners at LEP and Tevatron also implies a small hierarchy. Second, the modified Higgs sectors may also require additional fine-tuning to be consistent with data. The latter question is model-dependent, and we consider it in the context of the NMSSM.

In Section II, we discuss the implications of experimental limits on superpartner masses for fine-tuning of the $Z$-mass. These constraints come not only from Higgs exclusions, but also from gluino, squark, chargino and neutralino bounds. Abandoning standard theoretical assumptions can reduce the fine-tuning implied by these results, but at the cost of greater complexity and lessened predictive power.

In Section III, we consider the impact of a minimally extended Higgs sector (the NMSSM) on fine-tuning. In the context of the NMSSM, lifting the Higgs mass through new quartic couplings and opening up cascade decay channels for a light Higgs so that decay to $b\bar{b}$ is suppressed [2, 3] have been proposed as alternate means of explaining the non-discovery of the Higgs, alleviating fine-tuning. Either scenario can potentially be natural in the limit of an approximate $U(1)_{PQ}$ or $U(1)_R$ symmetry. Away from these limits, electroweak symmetry breaking is tuned and the Higgs sector must be tuned to allow cascade decays.

The pseudo-Goldstone boson of the broken $U(1)_{PQ}$ naturally opens a cascade decay channel that can hide the lightest MSSM-like Higgs below $\approx 110$ GeV for $\kappa \lesssim 0.05$. Large $\lambda \gtrsim 0.65$ and $A_{\chi} \gtrsim 300 - 350$ are required to raise the singlino and higgsinos above LEP bounds. The large $A$-term reintroduces a weak scale tuning of $\sim 10\%$. Electroweak symmetry only breaks when the up-type and singlet soft masses are made unnaturally small, thereby introducing an additional tuning of $\sim 1 - 10\%$.

In the $U(1)_R$ limit, severe tuning of the spectrum and mixing angles is required to hide an MSSM-like Higgs below $\approx 112$ GeV. The A-terms must be much smaller than typical radiative corrections, and so a tuning of $\sim 1 - 10\%$ is required. This is not true of the points discussed in [4], which naturally explain the excess of $2b$ events associated with a Higgs mass of $\approx 100$ GeV [5]. Rather, the tuning we discuss is what is required for the Higgs to go undetected, generically, given the present experimental constraints. For large singlet sector couplings where the Higgs states are heavier, direct search constraints are more readily avoided, but a tuning of $\sim 10\%$ is still required for electroweak symmetry to break.

Therefore, the NMSSM suffers from its own fine-tuning
problem. The discrepancy between these conclusions and the finding in \(2\) of points with mildly tuned \(M_Z\) arises from their narrower notion of fine-tuning and broader focus in parameter space. Points considered in \(2\) outside of the PQ or R limits may indeed have less fine-tuned \(Z\) masses, but they ignore the tunings required for viable electroweak symmetry breaking and disregard tunings of Higgs branching ratios and pseudoscalar masses necessary to hide the Higgs.

Because the strongest constraints on the NMSSM PQ limit are related not to the Higgs bosons but to electroweak symmetry breaking and higgsino masses, the fine-tuning may be reduced by adding additional couplings. This could present a new avenue for high-scale SUSY model-building, parallel to that of explaining the splittings. This could present a new avenue for high-scale fine-tuning may be reduced by adding additional couplings.

These relations suggest that the \(Z\) should be found near the middle or even the top of the superpartner spectrum—an expectation that has not been borne out by experiment. Fine-tuning is required to make the \(Z\) light compared to superpartners, and is minimized when the superpartners are as light as possible or when the relations \((1)-(3)\) do not apply.

Relation \((1)\) above follows from minimization of the Higgs potential, from which

\[
m_Z^2 = \frac{1 - \cos(2\beta)}{\cos(2\beta)} m_{H_u}^2 - \frac{1 + \cos(2\beta)}{\cos(2\beta)} m_{H_d}^2 - 2\mu^2 \quad (4)
\]

where \(\tan(\beta) = \frac{v_\tau}{v}\). Relation \((2)\) stems from RG-dependence of \(m_{H_u}^2\) on the soft masses \(m_{Q_3}^2, m_{U_3}^2\) controlling \(m_t^2\) and of \(m_{Q_3}^2, m_{U_3}^2\) on \(m_W^2\) which are enhanced by the large couplings \(y_t\) and \(g_3\). The one-loop corrections to these soft masses are

\[
\delta m_{H_u}^2 \simeq -\frac{3g_3^2}{8\pi^2} \frac{m_{U_3}^2}{\sin^4 \beta} \ln \left( \frac{\Lambda}{m_S} \right),
\]

\[
\delta m_{Q_3,U_3}^2 \simeq \frac{8\alpha_3 M_3^2}{3\pi} \ln \left( \frac{\Lambda}{m_S} \right),
\]

where \(M_3\) is the gluino soft mass, and \(\Lambda\) and \(m_S\) are the mediation scale and superpartner mass scale respectively \(\delta\). If the cutoff of the theory is high (e.g. \(\Lambda \sim M_{GUT}\)), then the logarithmic enhancement nearly cancels the loop factors. Relation \((3)\) follows from the assumption of universal gaugino masses, with the numerical factors derived from ratios of gauge couplings.

Fine-tuning in SUSY is often attributed to the non-discovery of a light Higgs at LEP, but in fact this is only one of several sources of tension. The non-observation of gluinos and squarks below \(\sim 200 - 300\) GeV suggests tuning at the \(10-20\%\) level or worse. If gaugino masses unify, the non-observation of neutralinos and charginos leads to even greater tuning.

Additional constraints on SUSY also appear as fine-tunings. We do not consider here the tuning required for reasonable dark matter abundance (below closure density) or the smallness of non-minimal flavor violation. These are largely decoupled from the tuning problems we consider here.

Two sources of tension that are highly dependent on the Higgs sector, which we will discuss in the next section, are stable electroweak symmetry breaking and Higgs and higgsino phenomenology with modified Higgs sectors.

This category of tuning has received little attention in the literature, but is morally similar to the familiar \(Z\)-mass fine-tuning. Such fine-tuning occurs when phenomenology requires a cancellation among larger and unrelated parameters. We can parameterize \(Z\)-mass tuning involving a parameter \(a\) by \(T \sim \left( \frac{d \log M_Z^2}{d \log a} \right)^{-1}\). More generally, if a criterion \(f(a, b) < f_c\) imposes a constraint on \(a\) and \(b\), then the local variation allowed without exceeding \(f_c\) defines fine-tuning \(T \sim d \log a\), which quantifies the degree of cancellation between parameters necessary to satisfy a criterion.

A. Tension with a Heavy Higgs

At tree-level, the lightest MSSM Higgs mass is bounded by \(m_H < M_Z |\cos(2\beta)|\), yet masses \(m_H < 114\) GeV are excluded by LEP \(\delta\). Lifting \(m_H\) above \(M_Z\) at one-loop requires a large stop loop contribution to \(m_H\) given by \(\delta\)

\[
\delta m_H^2 = \frac{3}{4\pi^2} y_t^4 \sin^4 \beta \ln \left( \frac{m_t m_{\tilde{t}}}{m_{\tilde{t}}^2} \right),
\]

which pushes the Higgs above the LEP bound for \(m_{\tilde{t}} > 500\) GeV. If SUSY breaking is mediated at a moderately high scale, then by eq. \((5)\) \(m_{H_u}^2\) also receives radiative corrections of \(\sim -(400)^2\) GeV\(^2\) that must be cancelled by a tree-level term. This implies a tuning worse than 5%.

This tuning can be remedied in several ways, of which we list only a few. (i) The Higgs mass can be protected from tree-level sources, so that it is naturally lighter than sparticles by a loop factor and the tree-level relation \((\delta)\) is irrelevant \(\delta\). (ii) The Higgs mass can be increased by new contributions to its quartic coupling beyond the SM D-term source \(\delta\). (iii) The Higgs decay modes can be
modified so that LEP and CDF are insensitive to a Higgs below 114 GeV \cite{15}. (iv) The SUSY-breaking mediation scale can be lowered, making \( m_{\tilde{g}L} \) less sensitive to \( m_{Q3} \) and \( m_{U3} \) \cite{14}. Other approaches are developed in \cite{14}.

All of these cures, especially (ii) and (iii), can require additional tuning to produce viable phenomenology or stable electroweak symmetry breaking. As we discuss in the next section, this must be counted against the gain in naturalness of \( M_Z \). Options (i) and (iv) often require significant and seemingly ad hoc extensions of the MSSM that may introduce new hidden tunings.

B. Tension with Heavy Gluinos and Squarks

Direct gluino and first- and second-generation squark searches constrain the soft masses \( m_{Q3}, m_{U3} \), and \( m_{\tilde{g}} \) regardless of the Higgs mass. If the gluino is heavy (\( m_{\tilde{g}} \gtrsim 500 \) GeV), squark masses as low as \( m_{\tilde{g}} \gtrsim 200 \) GeV are experimentally allowed. For heavier squarks, \( m_{\tilde{g}} \gtrsim 380 \) GeV, the gluino can be as light as \( m_{\tilde{g}} \approx 230 \) GeV \cite{11}. If \( m_{Q3,U3} \approx 330 \) GeV as well, then the \( Z \) mass is tuned at the 10% level or worse.

Constraints on the soft masses for the third-generation squarks are much weaker than for the first two generations. This is particularly true for light-gluino scenarios, because only first-generation squarks can be produced through \( t \)-channel processes mediated by a gluino. Direct sbottom searches imply \( m_{\tilde{b}} \gtrsim 200 \) if the sbottom decays through a neutralino with \( m_{\chi^0_1} \lesssim 80 \) GeV \cite{12}. If the gluino is light, \( m_{\tilde{g}} \lesssim 280 \) GeV, then the constraint becomes a little stronger, \( m_{\tilde{b}} \gtrsim 230 \) GeV \cite{12}. Conservatively, this implies \( m_{\tilde{g}} \gtrsim (230)^2 \) GeV\(^2\).

The bound \( m_{\tilde{b}} \gtrsim 150 \) GeV from direct stop searches does not constrain the soft masses at all, as it is saturated by the contribution from \( m_{t} \) \cite{15}. However, we expect \( m_{U3,Q3} \) to be close to \( m_{\tilde{g}} \approx 230 \) GeV, allowing better than 10% tuning. UV sensitivity of the Higgs sector to a heavy gluino is also reduced if the gluino gets its mass through supersoft operators \cite{14}. However, this requires adding an extra \( SU(3) \) adjoint fermion to the MSSM. Barring this possibility, the most natural spectrum with a split third generation has the gluino and sbottoms near \( 200 - 250 \) GeV, stops near \( 250 - 300 \) GeV, and the first two generations of squarks near 500 GeV. Of course, no simple models exist that predict such a spectrum.

C. Tension with Gaugino Unification, Chargino Mass, and \( b \rightarrow s\gamma \)

The spectrum above, though natural, is badly at odds with gaugino unification, which requires \( M_U = M_{\tilde{g}} = M_{\lambda^+} \). Consequently, the optimistic bounds on the lightest chargino and neutralino of \( m_{\chi^+_1} \gtrsim 101 \) GeV and \( m_{\chi^0_1} \gtrsim 46 \) GeV respectively imply \( m_{\tilde{g}} \gtrsim 450 - 500 \) GeV for positive \( \mu \), and \( m_{\tilde{g}} \gtrsim 300 - 350 \) GeV for negative \( \mu \).

With gaugino unification, the \( Z \) mass tuning must be at least 5 – 10%, regardless of the Higgs sector.

An independent source of fine-tuning of \( M_Z \) is the \( \mu \)-term. Current limits are \( \mu \gtrsim 140 \) GeV or \( \mu \lesssim -100 \) GeV from LEP chargino searches \cite{15}. The numerical fine-tuning is negligible in either case, but again it is striking that the charginos and neutralinos, which should be near \( M_Z \), are, like all other superpartners, above it. In fact, positive \( \mu \) is preferred to match the observed \( b \rightarrow s\gamma \) branching ratio assuming minimal flavor violation, which is maximally at odds with gaugino unification if the gluino is light \cite{10}. In the NMSSM where the \( \mu \)-term is generated dynamically, the seemingly mild requirement of \( |\mu| \gtrsim M_Z \) can be very difficult to explain without severe tuning of parameters.

III. HIDDEN TUNINGS FROM A HIDDEN HIGGS

In evaluating the naturalness of a SUSY model, all sources of tuning should be accounted for. Models that address one source of tuning may aggravate others. For this reason, it is important to consider the new fine-tuning introduced by hiding or lifting the Higgs mass.

For concreteness, we restrict our attention to the NMSSM—the simplest model in which the non-discovery of the Higgs at LEP is consistent at tree-level. The NMSSM has been advocated as a model that has low fine-tuning in regions where the Higgs decays predominantly to two lighter pseudoscalars or where the Higgs is heavy at tree-level \cite{2}.

The parameter space of the NMSSM is large and the sources of fine-tuning differ from region to region, so a full analysis of the fine-tuning is not practical. Instead we restrict our attention to two regions where the NMSSM has the best naive chance of being natural. These are: (i) large \( \lambda \) so that the MSSM-like Higgs is heavy and (ii) moderate \( \lambda \) where a symmetry protects the mass of a light pseudoscalar that can open up cascade decays, making a significantly lighter Higgs consistent with LEP bounds. The two symmetries that accomplish this without decoupling the singlet are a \( U(1)_{PQ} \) and \( U(1)_{P} \) \cite{15}.

The tunings cited are intended to clarify the importance of various sources of tension in the models, and should be interpreted as order-of-magnitude estimates only.

In each limit, we find that several phenomenological constraints require tuning. Obtaining viable electroweak symmetry breaking constrains the model in both symmetry limits, though most severely in \( U(1)_{PQ} \). The \( Z \)-mass is once again somewhat fine-tuned, now primarily because the soft-breaking parameter \( A_{\lambda} \) must be large to evade LEP chargino limits. Moreover, unless \( \lambda \) is large, the singlino would have been seen at LEP. This limit has a naturally light pseudo-Goldston boson, which is a candidate for mediating cascade decays, but hiding the Higgs still requires mild tuning of masses and mixing angles. Electroweak symmetry breaking is less tuned in the
$U(1)_R$ limit. This symmetry is, however, broken by the Standard Model $A$-terms, so that the light pseudoscalar must be tuned against radiative corrections. We do not discuss dark matter or flavor signals in detail, though they should be considered as well to compare the NMSSM thoroughly to the MSSM.

We find that the Z-mass tuning in the NMSSM with a Higgs hidden by cascade decays is mildly improved compared to the MSSM. As we will show however, in the limits where cascade decays are natural, obtaining viable electroweak symmetry breaking requires a tune in the Higgs soft masses at the $\sim 1-10\%$ level. This problem is generic in regions of parameter space that can hide the Higgs, and elsewhere the tuning in the Higgs spectrum is at the $\sim 10\%$ level. Consequently, the NMSSM suffers from a fine-tuning problem of its own.

Throughout this discussion, we assume that gaugino masses do not unify and third-generation squarks are split from the first two generations, allowing us to lower $m_{\tilde{q}_3}$ and $m_{\tilde{t}_3}$ near $\sim 200$ GeV. Moreover, we will assume that the electroweak gaugino soft masses $M_1$ and $M_2$ are decoupled except where otherwise noted. This loosens the constraints from chargino and neutralino searches at LEP. These are poorly motivated assumptions, but they allow us to divide the fine-tuning problem into manageable pieces. We use the Lagrangian conventions of [17] throughout.

A. Sources of Tuning in the NMSSM PQ Limit

1. Electroweak Symmetry Breaking

In this section, we attempt to resolve particular limits of the NMSSM in which viable EWSB can occur radiatively— that is, for which the origin is stable at $M_{GUT}$, but unstable at $M_Z$.

The potential for the Higgs and singlet vevs is given by

$$V(s, v_u, v_d) = \lambda^2 (h_u^2 s^2 + h_d^2 s^2 + h_3^2 h_2^2) + \kappa^2 s^4 - 2\lambda s h_u d s^2 - 2\lambda A h_u d s^2 + \frac{3}{2} \kappa A s^3 + m_{H_u}^2 h_u^2 + m_{H_d}^2 h_d^2 + m_2^2 s^2 + \frac{1}{4} g^2 (h_u^2 - h_d^2)^2,$$

where $g^2 = \frac{1}{2}(g_1^2 + g_2^2)$. Assuming a given $\lambda, \kappa, A_\lambda$, and $A_\kappa$, we wish to find the values of soft masses $m_{H_u}^2$ and $m_2^2$ such that electroweak symmetry breaking with $v_u^2 + v_d^2 = v^2 = (174\text{ GeV})^2$ is possible for some suitable choice of $m_{H_u}^2$. The size of this allowed region relative to the radiative corrections to these soft masses ($\sim 200\text{ GeV}^2$) is a measure of the tuning for electroweak symmetry breaking. If $m_{H_u}^2$ or $m_{H_d}^2$ is parametrically larger than $M_Z^2$, there is further tuning of the weak scale.

The $U(1)_R$ under which $H_u, H_d$ and $S$ have charges 1, 1, and -2 respectively requires $A_\kappa = 0, \kappa = 0$. In this case, the extremization conditions on $V$ imply

$$m_d^2 = \frac{\mu A_\lambda}{\sin(2\beta)} - \lambda^2 v^2 - 2\mu^2 - m_u^2,$$  

$$M_Z^2 = 1 - \frac{\cos(2\beta)}{\cos(2\beta)} m_u^2 - \frac{1 + \cos(2\beta)}{\cos(2\beta)} m_d^2 - 2\mu^2,$$  

$$\mu = \lambda s = \frac{A_\lambda \sin(2\beta)}{2(1 + \xi)},$$

where $\xi = m_3^2/(\lambda^2 v^2)$, and corrections of order $\xi^2$ are expected. For small $m_3^2$ and nonzero $h_u$ and $h_d$ vevs, the $\lambda A_\lambda$ term generates a minimum at nonzero $s$. Thus, $A_\lambda$ controls both $\mu_{eff} = \lambda s$ and $\mu_{B_{eff}} = \lambda A \lambda s$. Because of this relation, $-M_Z^2/2 \lesssim m_{H_u}^2 < 0$ for small $m_3^2$. $-(50\text{ GeV})^2 \lesssim m_2^2$ is also required for a stable electroweak symmetry-breaking vacuum to exist with $\lambda \approx 0.7$. $m_3^2 \gtrsim (30\text{ GeV})^2$ decreases $\mu$, and as we discuss in Section II A.2, raising $A_\lambda$ to compensate results in fine-tuning of $M_Z$. Because $m_{H_u}^2$ receives radiative corrections of order (100 GeV)$^2$ from stop soft masses and gluinos, and $m_3^2$ receives corrections of comparable size from $A_\lambda$ [20], being in this region of parameter space requires a tuning of a few percent. The $m_{H_u}^2$ tuning has no analogue in the MSSM because there, $\mu$ and $\mu B$ can be varied independently.

If the universe was ever at a temperature of $T \sim 100$ GeV, the negative $m_3^2$ is cosmologically dangerous. Although the electroweak symmetry breaking minimum is the global minimum of the potential, it is not necessarily the minimum to which a hot early universe rolls first. At a temperature $T \gtrsim M_Z$, the leading finite-temperature corrections to the soft masses are given by [19]

$$m_{H_u}^2(T) \approx m_{H_u}^2 + \frac{T^2}{12} \frac{6y_u^2}{\sin^2(\beta)}$$  

$$m_S^2(T) \approx m_S^2 + \frac{T^2}{12} 4\lambda^2.$$  

Consequently, we expect the singlet field to roll into the local extremum along the $s$-axis (which is typically a local minimum, not a saddle point) before the Higgs field rolls off when $m_S^2 < \frac{4\sin^2(\beta)\lambda^2}{6y_u^2} m_{H_u}^2$. The timescale for tunneling to the true EWSB vacuum is typically much longer than Hubble time scale $H(T)^{-1}$ at $T \sim M_Z$. If the cosmological constant is tuned to cancel the vacuum energy in the electroweak minimum, then the slow decay rate allows the universe to enter an inflationary phase below the weak scale. A detailed study of this cosmology would be necessary to check the viability of this scenario, but we assume that it is generically unacceptable. This argument implies further fine-tuning of $m_S^2$ for cosmologically safe EWSB.

Figure 1 shows the constraints on the Higgs soft masses for electroweak symmetry breaking in a typical slice of the PQ-symmetric limit. The white region indicates ranges of $m_{H_u}^2$ and $m_S^2$ for which an electroweak symmetry-breaking vacuum exists, for some $m_{H_u}^2$, with
$M_Z = 91$ GeV and $\tan \beta > 1$. In the black regions, no stable symmetry-breaking minimum exists that can reproduce $M_Z$. The calculations are done at tree-level. The line through the plot indicates the boundary of cosmological safety.

FIG. 1: White region: Allowed soft mass ranges for electroweak symmetry breaking with $M_Z = 91$ GeV for some $m_{\tilde{u},\tilde{d}}$, in the PQ limit of the NMSSM, with parameters $\lambda = 0.68$, $\kappa = 0.03$, $A_\lambda = 350$, and $A_{\kappa} = 50$. Below the black line, an electroweak symmetry-breaking global minimum exists, but cosmologically viable electroweak symmetry breaking is not guaranteed.

2. Neutralino and Chargino Masses

The NMSSM introduces a new neutralino and ties the mass of the chargino to the couplings of the Higgs sector in new ways. It is important, therefore, to consider the implications of experimental bounds on charginos and neutralinos for allowed regions of NMSSM parameter space. Charginos are constrained by the direct production bounds from LEP II [15, 21], $m_{\tilde{c}} \approx 95 - 105$ GeV. Neutralinos must evade bounds from the $Z$-pole width and associated production of two neutralinos off-shell. They must also exceed roughly $m_H/2$ so that Higgs invisible decays are not allowed. We consider each of these constraints in the PQ limit, and all expressions are corrected by terms of higher order in $\kappa$.

As discussed in the previous section, the LEP chargino bounds require $\mu \lesssim -100$ GeV or $\mu \gtrsim 140$ GeV. In the approximate-PQ, small-$m_S$ regime,

$$\mu \approx \frac{A_\lambda \sin(2\beta)}{(1 + \xi)} (1 + O(\kappa)). \quad (13)$$

From eqs. (3) and (13), the effective $\mu B$-term is $\mu B_{eff} = \mu A_\lambda$ and the $\mu$-term is controlled by $A_\lambda$. As $A_\lambda$ is increased to make $\mu$ sufficiently large, the first equation in (3) shows that either $m_{\tilde{u}}^2$ or $m_{\tilde{d}}^2$ must also increase. If electroweak symmetry breaking occurs radiatively, then $m_{\tilde{u}}^2 \sim M_Z^2$. Assuming this condition, large $A_\lambda$ requires a compensating large $m_{\tilde{d}}^2$ and increasing $\tan \beta$. Thus, large $A_\lambda \gtrsim 250 - 350$ GeV required to make $|\mu| \gtrsim 100 - 140$ GeV introduces a quadratic tuning of order $\sim 10\%$ to keep the $Z$ light.

The lightest neutralino bounds place a further constraint on NMSSM parameters. Ignoring mixing with gauginos, the lightest higgsino mass is given by

$$m_{\chi^0_1} \approx \frac{\lambda^2 (2\gamma \mu^2 + v^2 \sin^2 2\beta)}{\sqrt{\lambda^2 v^2 + \mu^2}} + O \left( \frac{v}{s} \right)^2 \gamma^2 + \left( \frac{v}{s} \right)^4, \quad (14)$$

where the small parameter $\kappa/\lambda \equiv \gamma$ quantifies breaking of the PQ symmetry. With $\gamma \ll 1$, the chargino and neutralino constraints together imply an approximate lower bound on $\lambda$:

$$\lambda \gtrsim \sqrt{\frac{m_{\chi^0_1, min}}{v^2}} \approx 0.5, \quad (15)$$

for $m_{\chi^0_1, min} \approx 45$ GeV, the rough bound from direct $Z$-pole production. Given that large $A_\lambda$ implies small $\sin(2\beta)$, larger $\lambda$ is required to keep $m_{\chi^0_1} > \frac{A_\lambda}{\mu}$ GeV in most of the parameter space of the PQ limit. It should also be noted that the bound from $h \rightarrow \gamma \gamma$ invisible typically requires heavier $\chi_1^0$.

Before considering the requirements for cascade decays, let us summarize the constraints on the PQ limit imposed by non-Higgs phenomenology: Large $A_\lambda$ is required to evade LEP chargino bounds, and in turn imposes a tuning to get a light $Z$ of about $\sim 10\%$. This is comparable to the tuning imposed by gluino constraints. Considering the neutralino mass bound as well forces us into the regime $\lambda \gtrsim 0.5$. Such large $\lambda$ raises the mass of the Higgs to $\approx 105$ GeV. Thus, in the PQ limit, where cascade decays could naturally hide the MSSM-like Higgs, the Higgs is always fairly heavy when neutralino and chargino constraints are satisfied. The only way to avoid this is to back out of the PQ limit, which reintroduces problems with EWSB and requires additional tuning to keep the pseudoscalar light.

3. Higgs Sector: Cascade Scenario

The experimental constraints on the MSSM with singlets added have recently been discussed in [15]. The most constraining bounds are on Higgs-strahlung production with decay modes $h \rightarrow bb$ and $h \rightarrow \gamma \gamma$. For SM-like Higgs-strahlung cross sections, these bounds exclude $m_h < 114.4$ GeV for $h \rightarrow bb$ and $m_h < 114$ GeV for $h \rightarrow \gamma \gamma$. This assumes $100\%$ branching to the decay mode in question. To hide a Higgs below 114 GeV, either the cross section for Higgs-strahlung and direct production must be lowered, or a visible but less constrained cascade decay channel of the form $h \rightarrow \phi\phi \rightarrow 4X$ must
be opened up. Of course, this requires a lighter boson φ (scalar or pseudoscalar) with $m_\phi \lesssim m_h/2 \lesssim 57$ GeV.

A light scalar $s$ is further constrained by direct bounds on Higgs-strahlung $Z \to sZ$ production, while a light pseudoscalar $a$ is constrained by bounds on direct production $Z \to ha$. The suggestive notation $s$ and $a$ refers to the lightest scalar or pseudoscalar of the Higgs spectrum, which must be primarily singlet to satisfy $m_s < m_h/2$ and be allowed by LEP, but will be mixed.

The cascade decays $h \to \phi \phi \to 4b$ are further constrained for $m_h \lesssim 110$ GeV. Hence a hidden Higgs requires either suppression of Higgs-strahlung (by a factor of $\sim 10$ over SM levels) or $m_\phi \lesssim 11$ GeV, so that $4b$ production is kinematically suppressed, and the less constrained $h \to \phi \phi \to 4\tau$ decay mode dominates.

In either the $4b$ or the $4\tau$ scenario, the Higgs sector appears tuned to keep a pseudoscalar light. This tuning is typically at the $1-10\%$ level to keep $m_\phi \lesssim 11$ GeV. If, however, the intermediate Higgs $\phi$ of the cascade is protected by an approximate symmetry, this tuning can be reduced. The $PQ$ pseudo-Goldstone boson is a good candidate, as the $PQ$ limit is also favored for EWSB.

The mass of the lightest pseudoscalar in the $PQ$ limit is given by

$$m_{A_1}^2 \approx \frac{6\mu^2(3\sin 2\beta \lambda^2 v^2 - 2A_\kappa \mu)}{4\mu^2 + \lambda^2 v^2 \sin^2 2\beta} + O(\gamma^2 \mu^2) \quad (16)$$

We note that the pseudoscalar can be made light by tuning $A_\kappa$, even when $\kappa$ is only moderately small. From the lower bounds $\lambda \gtrsim 0.5$ and $\mu \gtrsim 140$, the natural size of $m_{A_1}$ is

$$m_{A_1}^2 \gtrsim \frac{3\gamma}{2}(3\sqrt{\mu_{\min}^2 + \lambda^2 v^2 m_\chi^2 \lambda^2}), \quad (17)$$

or $m_{A_1} \sim \sqrt{\lambda \kappa}$ (250 GeV). When $\frac{\lambda}{\kappa} < 0.04$, $m_A \lesssim 50$ GeV is natural and the cascade $h \to 2 \to 4b$ opens up. When $\frac{\lambda}{\kappa} \lesssim 0.002$, $m_A < 11$ GeV and $h \to 2a \to 4b$ is kinematically forbidden, and the pseudoscalar naturally decays to $2\tau$.

The necessity of large $\lambda$ in the hidden Higgs scenarios discussed below suggest that, in the $PQ$ limit, the heavy Higgs is most natural. In fact, a heavy Higgs can be hidden quite generically for $\lambda \gtrsim 0.65$ in the $PQ$ limit. The tension with $A_\lambda$ and EWSB remains in this case, however.

4. Benchmark Parameter Scans

The plots below are generated using NMHDECAY 1.1 to study the phenomenology of the NMSSM for various choices of parameters [17]. Inputs are pre-processed by a Mathematica script that finds the derived parameters $\mu$ and $\tan \beta$, which are inputs to NMHDECAY, for given $m_{H_u}^2$ and $m_{H_d}^2$. As above, $m_{H_u}^2$ is determined by fixing $M_2$. The Mathematica front-end determines $\mu$ and $\tan \beta$ at tree-level, and NMHDECAY calculates at one-loop order, so the input values of $m_{H_u}^2$ and $m_{H_d}^2$ are not the same as those returned by NMHDECAY. In the tables below, we cite the input values. The ranges of $m_{H_u}^2$ ($m_{H_d}^2$) returned by NMHDECAY are roughly 500 (40) GeV$^2$ larger.

In Figures 2 and 3, we have scanned two parameters randomly while leaving the other four fixed, to demonstrate the dominant sources of fine-tuning in the $PQ$-symmetric limit. The parameter values and ranges used to generate these figures are given in Table I. Scanning the soft masses reveals the same contour as in Figure 1 for electroweak symmetry breaking. A substantial region near $m_\chi^2 = 0$ is consistent with phenomenology. Varying $\lambda$ and $\kappa$ reveals that large $\lambda$ is necessary to evade neutralino constraints, and that a hidden Higgs is not generic except in the $PQ$ limit. For $\lambda \gtrsim 0.7$, the MSSM-like Higgs is heavier than 114.4 GeV, and its decays are unconstrained. But this region, too, becomes smaller as $\kappa$ increases beyond $\sim 0.1$, because the Higgs searches constrain the lightest Higgs as well away from the $PQ$ limit.

| Figure | $\lambda$ | $\kappa$ | $A_\lambda$ | $A_\chi$ | $m_{H_u}^2$ | $m_{H_d}^2$ |
|--------|----------|----------|-------------|----------|-----------|-----------|
| 2      | 0.5-0.8  | 0.0-0.1  | 350, 50    | -1500    | 0         | 0         |
| 3      | 0.68     | 0.03     | 350, 50    | * - 0    | -2000 - 1000 |

TABLE I: Fixed parameter values and scanned ranges for Figures 2 and 3. All dimensionful quantities are given in GeV, and in the second line, * denotes that the lower limit of the scanned range is not input by hand but dictated by electroweak symmetry breaking.

![FIG. 2: Phenomenology constraints on the NMSSM as $\lambda$ and $\kappa$ are varied. Fixed parameters are listed in Table I. The red region at the top and black region just below it are phenomenologically allowed, with the second-heaviest (MSSM-like) Higgs heavier and lighter than 114.4 GeV, respectively. The center-right blue region is excluded by Higgs decays to nonsupersymmetric particles. The yellow region on the lower left is excluded by light neutralino searches, while the lower-right green region is excluded by both neutralino searches and Higgs decay searches.](image-url)
FIG. 3: Phenomenology of regions of $m_{H_d}$-$m_S^2$ parameter space where EWSB consistent with $M_Z$ is possible. Fixed parameters are listed in Table I. The black region on the right is phenomenologically allowed, and has the second-heaviest (MSSM-like) Higgs hidden below 114.4 GeV. The small red region directly below it is also allowed, but with the MSSM-like Higgs mass $\geq 114.4$ GeV. The lower blue region is excluded by Higgs decays to nonsupersymmetric particles. The yellow regions on the upper left and middle are excluded by light neutralino searches and Higgs decay searches. Only points above the white line are cosmologically viable if symmetry is restored at high temperatures. Points below the line roll first along the singlet axis, and typically do not roll off into an electroweak symmetry-breaking vacuum.

B. Sources of Fine-Tuning in the NMSSM R Limit

1. Electroweak Symmetry Breaking

The $U(1)_R$ limit where $H_u, H_d$ have charge 1 and $S$ is neutral corresponds to $A_\lambda = A_\kappa = 0$. With $h_u^2 + h_d^2 = v^2$, the singlet potential is stable at the origin unless $m_S^2 \leq -(\lambda^2 v^2 - \lambda \kappa v^2 \sin(2\beta))$. When the singlet origin is destabilized,

$$s^2 = \frac{- (m_S^2 + \lambda^2 v^2 - \lambda \kappa v^2 \sin(2\beta))}{2 \kappa v^2},$$

$$\mu = \frac{\lambda s}{\kappa} = \frac{- \lambda (m_S^2 + \lambda^2 v^2 - \lambda \kappa v^2 \sin(2\beta))}{\kappa \sqrt{2}}.$$  

Thus, very large negative $m_S^2$ is necessary to generate a sufficiently large $\mu$-term. In this case, the singlet vev rolls away from the origin first, but $h_u$ and $h_d$ can be unstable along the $s$-axis. The Higgs directions are unstable at the s-minimum at $s^2 = -\frac{\lambda^2}{2 \kappa} m_S^2$ for $m_{H_d}^2 + m_{H_u}^2 \approx 0$ (this expression is valid for $\lambda \approx \kappa, |m_{H_d}^2|, |m_{H_u}^2| \ll |m_S^2|$). This, too, is consistent with radiative symmetry breaking. Although $m_S^2$ must be quite large and negative at high scales, the vacuum on the singlet axis is stable until $m_{H_u}^2 + m_{H_d}^2$ runs negative.

Figure 4 shows the regions of Higgs soft masses for which electroweak symmetry breaks in a typical slice of the $U(1)_R$ limit of the NMSSM. The white regions indicate values of $m_{H_d}$ and $m_S^2$ for which a tree-level electroweak symmetry-breaking vacuum exists, for some $m_{H_u}^2$, with $M_Z = 91$ GeV and $\tan \beta > 1$. In the black regions, no stable symmetry-breaking minimum exists that can reproduce $M_Z$. $m_{H_u}^2$ must be somewhat smaller than $\sim (200 \text{GeV})^2$, the size of its radiative corrections, so roughly 10% tuning is required for electroweak symmetry breaking.

FIG. 4: White region: Allowed soft mass ranges for electroweak symmetry breaking with $M_Z = 91$ GeV for some $m_{H_u}^2$, in the R-symmetric limit of the NMSSM, with $\lambda = \kappa = 0.63, A_\lambda = 0.5, A_\kappa = 0$.

2. Higgs Mass Constraints

The chargino and neutralino tensions that are problematic in the PQ limit are unimportant in the R-symmetric limit, where the higgsinos are generally heavier. The primary source of tuning in the R-symmetric limit is Higgs decays. In particular, whereas it is technically natural to assume $m_{A_1}$ small in the PQ-symmetric limit because of the approximate symmetry, $U(1)_R$ is broken by the Standard Model $A$-terms and gaugino masses, so that $A_\lambda$ receives radiative corrections of order 100–200 GeV. To evade LEP bounds without significantly tuning the Higgs couplings to the $Z$ small, a Higgs below 112 GeV must decay to $4\tau$, requiring $m_{A_1} < 12$ GeV.

The mass of $U(1)_R$ pseudo-scalar is

$$m_{A_1}^2 = \frac{9 \lambda^2 \kappa v^2 A_\lambda}{2 \mu} - \frac{3 \kappa \mu A_\kappa}{\lambda},$$

(20)

so that tuning of $A_\lambda$ against $A_\kappa$ is necessary. Given that radiative corrections to $A_\lambda$ are $\delta A_\lambda \sim 100$ GeV when the Higgs sector soft masses are large and the cancellation required is $\sim 0.5$ GeV, this tuning is at the $\sim 1 \to 10\%$ level. Cascade decays to $4\tau$ are severely constrained by
the data \(\mathcal{A}\), so tuning to suppress Higgs-strahlung of order \(\sim 1 - 10\%\) or more is required.

In \(\mathcal{B}\), it was claimed that for small \(\lambda \sim \kappa \sim 0.2\), \(|\mathcal{A}_1| \sim 100\text{ GeV}, \mathcal{A}_\kappa \sim 3\text{ GeV},\) and moderate \(\tan(\beta)\), there are technically natural points in the R-limit that hide a MSSM-like Higgs near \(\approx 100\text{ GeV}\). The only fine-tuning required there is in obtaining electroweak symmetry breaking. If the LEP excess of \(2\mathcal{b}\) events near this mass range is a signal, then the parameter choices required to explain the spectrum in these regions are constrained only by the positive signal. The tunings we consider are necessary to avoid predicting a signal, and therefore should be considered phenomenologically unnatural.

For a heavier Higgs (which requires \(\lambda \gtrsim 0.63\)), cascades to \(4\mathcal{b}\) are not constrained, and this essentially reduces to the large quartic scenarios of \(\mathcal{B}\). In these cases, the remaining tuning is dominated by requiring viable electroweak symmetry breaking (\(\sim 5 - 10\%\)).

Figures \(\mathcal{A}\) and \(\mathcal{B}\) display representative cross-sections of the parameter space, with the allowed parameter space for a heavy (> 112 GeV, red) or light hidden Higgs (< 112 GeV, black), and the region where the Higgs is excluded by Higgs decay searches at LEP (blue). The parameter ranges used are given in Table \(\text{II}\). All points allowed by the experimental constraints have Landau poles below \(M_{GUT}\).

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
\(\lambda\) & \(\kappa\) & \(A_\lambda\) & \(A_\kappa\) & \(m_{\tilde{t}}\) & \(m_{\tilde{\nu}}\) \\
\hline
\(\mathcal{A}\) & 0.4-0.8 & 0.4-0.8 & 0.5 & 0.0 & \(-100^2\) & \(-200^2\) \\
\(\mathcal{B}\) & 0.4-0.8 & 0.65 & 0.0-1.5 & 0.0 & \(-100^2\) & \(-200^2\) \\
\hline
\end{tabular}
\caption{Fixed parameter values and scanned ranges for Figures \(\mathcal{A}\) and \(\mathcal{B}\). All dimensionful quantities are given in GeV.}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{Phenomenology constraints on the NMSSM as \(\lambda\) and \(\kappa\) are varied in the R-symmetric limit. Fixed parameters are listed in Table \(\text{II}\). The red region at the top and black region just below it are phenomenologically allowed, with the lightest Higgs heavier and lighter than 112 GeV, respectively. The blue region at the bottom is excluded by Higgs decays.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig6.png}
\caption{Phenomenology constraints on the NMSSM as \(\lambda\) and \(\Lambda\) are varied in the R-symmetric limit. Fixed parameters are listed in Table \(\text{II}\). The red region at the right and black region in the center-bottom are phenomenologically allowed, with the lightest Higgs heavier and lighter than 112 GeV, respectively, and the blue region at the left is excluded by Higgs decay searches.}
\end{figure}

\section{IV. CONCLUSIONS}

In this paper, we have reviewed the constraints on the naturalness of low-scale SUSY imposed by experimental limits. Many recent attempts to address the little hierarchy problem have focused on modifying the MSSM Higgs sector, but it is unclear whether such modifications reduce the fine-tuning required for a consistent SUSY completion of the Standard Model or merely move the fine-tuning around.

Experimental bounds on the squark, gluino, and electroweak gaugino sectors all suggest fine-tuning of \(M_Z\). This tuning exists independently of the Higgs sector, but can be alleviated by abandoning gaugino unification and splitting the third-generation squark masses from the first and second generations.

In addition to the fine-tuning independent of Higgs physics, we considered tuning associated with a modified Higgs sector, taking the NMSSM as an example and further specializing to the PQ- and R-symmetric limits with positive \(\mu\) to avoid Br\((b \to s\gamma)\) constraints \(\mathcal{E}\). These limits can naturally open up cascade decay channels for the MSSM-like Higgs, permitting a hidden Higgs.

In the PQ-symmetric limit, tuning is required to evade Higgs bounds, though not directly worse than in the MSSM. Cosmologically safe radiative electroweak symmetry breaking requires that the soft masses be tuned small at the \(\sim 1 - 10\%\) level. The Z-mass depends on \(A_\kappa\), which must be large to raise the mass of charged higgsinos above experimental limits. This generates an independent tuning of \(\sim 10\%\). Further tension comes from evading direct searches for the Higgs and the lightest neutralino, which in this case is mostly singlino. The neutralino bounds are only satisfied for relatively large \(\lambda \gtrsim 0.5\), so that the MSSM-like Higgs state must be heavier than 100 GeV. With small \(\kappa\), this generically keeps all Landau poles above the unification scale.
In the R-symmetric limit, large $\kappa, \lambda \gtrsim 0.6$ is typically required so a Landau pole below the unification scale is typically present. To generate a sufficiently large $\mu$ term, the singlet soft mass must be large and negative, causing mild tension with electroweak symmetry breaking. Consequently, tuning in Higgs sector soft masses at the $\sim 10\%$ level is required for electroweak symmetry to break. In cases where the lightest MSSM-like Higgs states are below $\approx 110$ GeV, the $A$-terms are required to be unnaturally small or tuned against each other in order to evade Higgs search constraints. This introduces a tuning at the $\sim 1-10\%$ for parameter ranges that would generically go unseen. This is not apply to the points discussed in [3] that are used to explain the excess of $2b$ events associated with a Higgs mass of $\approx 100$ GeV.

Away from the PQ- or R-symmetric limit, other regions of NMSSM parameter space may allow more natural $M_Z$, but require comparable or worse tuning for light pseudoscalars or electroweak symmetry breaking.

As the NMSSM illustrates, the lack of technical naturalness required by demanding a $Z$ at the bottom of the SUSY spectrum can be partially exchanged for tunings in the Higgs and higgsino spectra. It is not clear whether this fine-tuning is less mysterious than $Z$-mass tuning or whether it is a comparable cause for concern.

Tuning peculiar to the NMSSM may be reducible by further modifying the Higgs sector, though certain extensions may merely obscure this tuning, so they must be considered with care. For example, an explicit $\mu$ term allows smaller $A_{\lambda}$ in the PQ-limit and so can reduce the $Z$-mass tuning, but may also increase the tuning to open cascade decay channels. Other MSSM-singlet models suggested in [8] may reduce tuning more successfully, but they too must be considered in detail to address this question.

The possibility of a tuned but light superpartner spectrum also merits study because it is a very different scenario for LHC physics. The light spectrum is clearly promising, as even the weakly interacting superpartners will easily be within reach of the LHC.

By itself, the NMSSM does not eliminate the fine-tuning of SUSY, but it does change the nature of this tuning and suggest an alternate direction for model-building. If the conventional scenario of heavy superpartners is correct, solutions to the little hierarchy problem must explain the relative lightness of the $Z$ and Higgs. If the spectrum is light, models will still need to explain the surprising order of the spectrum and the fine-tuning of the $Z$-mass suggested by current superpartner bounds. They must also, of course, consistently explain electroweak symmetry breaking, heavy higgsinos, and Higgs phenomenology that is invisible to LEP. Contrary to some claims, the NMSSM does not accomplish these tasks without a fine-tuning problem of its own. It would be interesting to construct models that do hide the Higgs states from detection at LEP and do not suffer from this problem.

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