Quantization of the Hall conductivity well beyond the adiabatic limit in pulsed magnetic fields

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We measure the Hall conductivity, $\sigma_{xy}$, on a Corbino geometry sample of a high-mobility AlGaAs/GaAs heterostructure in a pulsed magnetic field. At a bath temperature about 80 mK, we observe well expressed plateaux in $\sigma_{xy}$ at integer filling factors. In the pulsed magnetic field, the Laughlin condition of the phase coherence of the electron wave functions is strongly violated and, hence, is not crucial for $\sigma_{xy}$ quantization.

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On recognizing the crucial role of the edge channels in two-dimensional (2D) electron transport in a quantizing magnetic field [1], it became pretty clear that the quantization of the Hall resistance, $R_{xy}$, in Hall bar samples, which corresponds to the quantum Hall effect [2], is not directly connected with that of the Hall conductivity, $\sigma_{xy}$. Even if the longitudinal resistance, $R_{xx}$, is negligible, the measured resistance tensor cannot be converted into the conductivity one: the net Hall current is a sum of the bulk and edge currents while the conductivities $\sigma_{xx}$ and $\sigma_{xy}$ are related to the bulk of the 2D system. Therefore, the conductivity tensor and the accuracy of $\sigma_{xy}$ quantization should be investigated independently using the Corbino geometry which allows separation of the bulk contribution to the measured current. Such an arrangement was described in the Laughlin [3] and Widom-Clark [4] gedanken experiments. A (Hall) charge transfer below the Fermi level between the coasts of a Corbino sample is induced by magnetic field sweep and thus the shunting effect of the edge currents is completely excluded. The concept of Ref. [3] based on gauge invariance leads to the conclusion that at integer filling factor the conductivity $\sigma_{xy}$ will be quantized if the magnetic field, $B$, is changed adiabatically so as to keep the phase coherence of the wave functions on the sample size. The quantization of $\sigma_{xy}$ follows from the fact that an integer number of electrons is transferred between the ring edges if the magnetic flux changes by one quantum. It is clear that the phase coherence should be the case at field sweep rates when the magnetic flux change, $\Delta \Phi = \tau L^2 dB / dt$, in a sample with size $L$ within the settling time, $\tau$, of the wave function phase is small compared to the flux quantum, $h/e$:

$$\frac{dB}{dt} < 2\pi \Omega_c B (l / L)^4,$$

where $\Omega_c$ is the cyclotron frequency, $l$ is the magnetic length, and the phase settling time is estimated as the ratio of the sample size and the phase velocity of an electron, $\tau = L^2 / e^2 \Omega_c$.

Doubts about the correctness of the gauge invariance approach were expressed in Ref. [4] and were thought to be supported by results of the microwave studies, e.g., of Ref. [5]. In fact, those studies as well as edge magnetoplasmon [6] and related [7] experiments are not free of edge current contribution so that they do not yield the pure $\sigma_{xy}$ and cannot be an argument against the approach of Ref. [3]. The value $\sigma_{xy}$ can be measured in the arrangement of the above gedanken experiments which was employed in the work of Refs. [8,9], even though the quantization accuracy was about 1%.

Here, we study the charge transfer in a Corbino geometry sample subjected to a pulsed magnetic field with sweep rate up to $5 \times 10^2 \ T/s$ at low temperatures. The conductivity $\sigma_{xy}$ has been found to be quantized at integer filling factor. This result is very similar to the data obtained in quasistatically changing magnetic fields, although at such high sweep rates of the pulsed magnetic field, the phase coherence of the electron wave functions is strongly broken. So, the condition of adiabaticity is sufficient but not necessary for $\sigma_{xy}$ to be quantized.

The samples are Corbino disks fabricated from two wafers of AlGaAs/GaAs heterostructures containing a 2D electron gas with mobility $1.2 \times 10^6$ and $4 \times 10^5 \ cm^2/Vs$ at 4.2 K and density $3.6 \times 10^{11}$ and $3.2 \times 10^{11} \ cm^{-2}$, respectively. Each sample has a circular gate covering a part of the sample area so that the gated region of the 2D electron system is separated from the contacts by guarding rings, see Fig. 1. The radii of the Corbino ring are $r_1$ and $r_2$, and the circular gate is restricted by radii $r_{1g}$ and $r_{2g}$; two sets of the radii are employed: (i) $r_1 = 0.2 \ mm$, $r_2 = 0.5 \ mm$, $r_{1g} = 0.305 \ mm$, and $r_{2g} = 0.39 \ mm$ and (ii) $r_1 = 1.013 \ mm$, $r_2 = 1.119 \ mm$, $r_{1g} = 1.025 \ mm$, and $r_{2g} = 1.108 \ mm$. The sample is placed into the mixing chamber of a dilution refrigerator with a base temperature of 80 mK. In each pulse, the magnetic field sweeps up to 34 T (or
lower) with rising and falling times of about 50 ms and 1 s, respectively (inset to Fig. 1). The azimuthal electric field induced by magnetic field sweep gives rise to an electric current only in the radial direction if \( \sigma_{xx} \rightarrow 0 \).

In the experiment we study the charge, \( Q \), brought out of the gated region, which is equal to the difference between the charge exiting and entering the gated region [10]

\[
Q = \pi (r^2_{2g} - r^2_{1g}) \sigma_{xy} \Delta B. \tag{2}
\]

This charge induces the voltage, \( V = Q/C \), across a sufficiently large capacitance, \( C \), connected in parallel to the gate, which is measured using a preamplifier and a digitizer. The capacitance \( C \) allows one to restrict the induced voltage in order to avoid the breakdown of the dissipationless quantum Hall state [11]. The equilibrium \((B = 0)\) electron density in the gated region can be changed by using a gate bias, \( V_g \). All data we show in the paper refer to the gate voltage \( V_g = 0 \); we have checked that for gate voltages between 0 and \(-80 \text{ mV}\) (the threshold voltage \( V_{th} \approx -0.3 \text{ V} \)), the results discussed below are not sensitive to \( V_g \). In the experiment with quasistatically changing magnetic fields (with sweep rates in the range \((1 - 5) \times 10^{-3} \text{ T/s}\)), we measure the charge \( Q \) using an electrometer.

Typical experimental traces of the voltage induced on the sample in a pulsed magnetic field in the vicinity of filling factor \( \nu = 1 \) and \( \nu = 2 \) are shown in Fig. 2 for up and down sweeps. When sweeping the magnetic field up, at small \( \sigma_{xx} \) the voltage rises linearly with \( B \), in accordance with Eq. (2), until it drops above a certain value of the magnetic field thereby signaling the breakdown of the dissipationless quantum Hall state. On changing the sweep direction, the voltage polarity reverses so that the up and down traces form a hysteresis loop. The asymmetry between its top and bottom parts is caused by larger overheating of the sample in sweeping the field up, which leads to a more pronounced narrowing of the quantum plateaux. A change in the background signal below and above the hysteresis loop originates from chemical potential oscillations [10]. Similar dependences \( V(B) \) are observed also at higher integer \( \nu \leq 6 \) \((\nu \leq 10)\) for up (down) sweeps; below we discuss the two lowest filling factors at which the observed structures occupy the widest magnetic field intervals.

It is important that the slope in the linear interval of the dependence \( V(B) \) is in excellent agreement with the calculated one using Eq. (2) with the quantized
value \( \sigma_{xy} = \nu e^2/\hbar \), see Fig. 3. As the magnetic field is increased within the linear interval of \( V(B) \), electrons are brought into the 2D system, and the electron density increases, in accordance with Eq. (6), by \( \Delta n_s = (\nu e/\hbar)\Delta B \) (where \( \nu = 1, 2, ... \)). As a result of the aligned change of magnetic field and electron density, the filling factor remains approximately constant: it is about 10% larger (smaller) than the integer \( \nu \) for up (down) sweep of the magnetic field (Fig. 2). Thus, the observed dependences \( V(B) \) yield well expressed plateaux in \( \sigma_{xy} \) as a function of filling factor.

Figure 3 shows the corresponding dependence \( Q(B) \) in a pulsed magnetic field. As seen from the figure, within the whole hysteresis loop, the behaviour of the charge \( Q \) brought out of the 2D electron system is independent of shunting capacitance \( C \). This implies that the observed linear \( B \) dependence of the charge \( Q \) is limited by a capacitance discharge that is controlled by the dependence of \( \sigma_{xx} \) on magnetic field [10]. The derivative \( dQ/dB \) yields plateaux in \( \sigma_{xy} \) as a function of magnetic field (inset to Fig. 3).

Typical curves of the voltage induced by the charge \( Q \) in the quasistatic measurement are displayed in Fig. 4. In addition to the curves obtained by sweeping the magnetic field up and down all way through the hysteresis loop, two more traces correspond to reversal of the sweep direction within hysteresis loop. The expected linear behaviour of \( V \) against \( B \) is shown for comparison by dashed lines. As seen from Fig. 4, the intervals of the upper and lower curves, in which \( V(B) \) is linear, are narrower as compared to the pulsed field data of Fig. 3.

This is undoubtedly caused by leakage currents emerging because of non-zero dissipative conductivity \( \sigma_{xx} \) whose influence is suppressed in pulsed measurements. As a result, the accuracy of \( \sigma_{xy} \) quantization for pulsed magnetic fields turns out to be the same or even higher than in quasistatic measurement.

One can easily see that the above mentioned adiabatic limit of the inequality (1) is not fulfilled in our experiments. This limit corresponds to the magnetic field sweep rate \( \sim 10^{-4} \) T/s if \( L = 0.5 \) mm, which is already an order of magnitude lower than sweep rates in the quasistatic measurement. Moreover, the conductivity \( \sigma_{xy} \) is still found to be quantized even at much higher sweep rates of the pulsed magnetic field, at least six orders of magnitude beyond the estimated adiabatic limit of the expression (2). This finding unambiguously shows that the condition of the phase coherence of the electron wave functions is not crucial for \( \sigma_{xy} \) quantization.

Apparently, our line of reasoning holds if the temperature-dependent dephasing time, \( \tau_{\phi}(T) \), is much larger than the phase settling time \( \tau \). The former can be evaluated from the balance condition for thermal electron excitation to the upper quantum level and relaxation of the excited electrons

\[
\tau_{\phi}^{-1} = n_0 \exp(-\Delta/2k_B T)\tau_{exc}^{-1},
\]

where \( n_0 \) is the quantum level degeneracy, \( \Delta \) is the level splitting, and \( \tau_{exc} \) is the lifetime of an excited electron. As known from optical studies (see, e.g., Ref. [15]), the lifetime \( \tau_{exc} \) exceeds \( \hbar/\Delta^{-1} \) for both spin and cyclotron splittings. Hence, we obtain \( \tau_{\phi} > \hbar/\Delta^{-1} \exp(\Delta/2k_B T) \gg \tau \) in our experiment.

From the expression (2) it follows that for our highest
sweep rates, the phase coherence of the wave functions is broken on the length $\sim 10 \mu m$, which is still much larger than the magnetic length. In other words, $\sigma_{xy}$ is found to be quantized when the adiabatic limit is not the case for the whole sample but still holds on macroscopic distances. Whether there exists a maximum sweep rate of the magnetic field for $\sigma_{xy}$ to be quantized remains to be seen.

In summary, we have measured the Hall conductivity in the arrangement of Laughlin’s gedanken experiment in pulsed magnetic fields. Well expressed plateaux in $\sigma_{xy}$ have been observed at integer filling factors, which is similar to the data obtained in quasistatic measurements. Although in the pulsed magnetic field, the phase coherence of the electron wave functions is strongly broken, the $\sigma_{xy}$ quantization is still the case. Therefore, the gauge-invariance-based argumentation [4] is sufficient but not necessary for $\sigma_{xy}$ quantization.

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