State of Charge Estimation for Lithium-ion Battery Based on Cubature Kalman Filter with Fading Square-foot Factor

Yulong Zhang *, Sida Zhou, Yang Hua, Xinan Zhou, Shichun Yang and Xinhua Liu

School of Transportation Science and Engineering, Beihang University, 37 Xueyuan Road, Beijing 100191, China.
Email: by1213108@buaa.edu.cn

Abstract. The state of charge (SOC) is the key issue to ensure the safe operation of the battery, but the battery system’s state estimation often has low accuracy and robustness under complex conditions or uneven measurement problem. To solve this problem, this paper proposes an SOC estimation method based on the combination of cubature Kalman filtering, fading factor and square root filtering. In addition, the second-order equivalent model is adopted, and parameters are identified. To verify the success of the algorithm, various operation tests were conducted. The simulation results agree well with the experimental results, and the maximum error can reach 3%. Numerical analysis shows that the method has good convergence and robustness under the initial error disturbance.

1. Introduction
Lithium ion batteries have been widely used for their advantages of high-power density and long life. However, the battery management system (BMS) has always faced security issues. The state of charge (SOC) is the main state to ensure the safety and healthy work of the battery and is a very critical hidden state. However, due to fluctuations in environmental conditions, especially when the driver’s driving behavior is poor or driving under extreme conditions, the accuracy and robustness of SOC estimation for electric vehicles based on embedded systems are limited[1, 2].

Although SOC cannot be directly measured during battery operation, there are various methods to estimate SOC, such as coulomb counting (CC) and model-based estimation methods[3, 4]. The CC method is a common method for SOC estimation in BMS systems. The SOC is estimated according to the definition. The CC method usually works with other techniques such as model-based methods to improve the accuracy of SOC estimation. Model-based methods include equivalent circuit models, electrochemical models, and black box models. Black box models are derived from real data generated by neural networks or other networks. This depends on the extrapolation of training data to a comprehensive area under various operating conditions. Ability. Similarly, due to the strictness of the electrochemical theory, the electrochemical model can be used for high-precision estimation of SOC, but it still depends on the experimental parameters, reducing the possibility of online identification. Equivalent circuit models (ECMs) can use lumped circuit parameters such as resistance, capacitance, voltage source to describe the behavior of the battery, because of its simple structure and few parameters, it has been widely used[5, 6]. ECMs can provide a suitable balance in terms of computational cost and model accuracy and have been widely promoted as the main online model.

In the model-based SOC estimation method, the Kalman filter algorithm has been widely used[7-9]. Kalman filtering mainly solves the linearity problem in Gaussian noise environment. When dealing with non-linear systems such as batteries, the accuracy will be severely reduced or even diverged. Therefore,
scholars have proposed various nonlinear Kalman filters, such as Extended Kalman Filter (EKF)[10, 11], Unscented Kalman Filter (UKF) [12, 13], and Cubature Kalman Filter (CKF)[2, 14]. They can obtain better estimation accuracy at an acceptable computational cost. However, in some specific cases, when there is a model mismatch or noise interference, the aforementioned Kalman filters may encounter accuracy and robustness issues. In addition, due to the non-uniformity of sampling and measurement, interference always exists and is difficult to eliminate. To obtain better filtering performance, more improvements need to be combined with the algorithm.

2. Battery Model

2.1. Second-order Equivalent Circuit Model

Equivalent circuit models are widely used when focusing on the dynamic characteristics of the battery because of its simple structure and small computational cost. A second-order RC equivalent circuit model shown in Figure 1 is adopted, which can better describe the polarization characteristics of the battery.

![Figure 1 Second-order RC equivalent circuit model](image)

The second-order RC model consists of open circuit voltage (OCV, \( U_{oc} \)), the ohmic resistance \( R_o \) and two RC networks. \( R_{p1} \) and \( C_{p1} \) are the activation polarization resistance and capacitance respectively, \( R_{p2} \) and \( C_{p2} \) are the concentration polarization resistance and capacitance respectively. \( U_t \) is the terminal voltage, \( I_t \) is the current (assumed positive for charge, negative for discharge). \( U_{p1} \) and \( U_{p2} \) describe the diffusion voltage over the RC network.

The battery SOC can be defined as equation (1):

\[
SOC_{k+1} = SOC_k - \frac{\eta_t}{Q_{cur}} \int_{k}^{k+1} I_t dt
\]

Where, \( k \) is the iteration time, \( \eta_t \) is the Coulomb efficiency, and it is assumed to be 1 in this paper, \( Q_{cur} \) is the capacity of current state of health, and \( I \) is the sampling current based on the sensor. The state-space model for second-order ECM can be depicted as equation (2), thanks to the Laplace transform.

\[
\begin{bmatrix}
SOC_{k+1} \\
U_{p1,k+1} \\
U_{p2,k+1}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & e^{-T_s/R_{p1}C_{p1}} & 0 \\
0 & 0 & e^{-T_s/R_{p2}C_{p2}}
\end{bmatrix} \begin{bmatrix}
SOC_k \\
U_{p1,k} \\
U_{p2,k}
\end{bmatrix} + \begin{bmatrix}
-\frac{\eta T_s}{Q_{cur}} \\
R_{p1}(1-e^{-T_s/R_{p1}C_{p1}}) \\
R_{p2}(1-e^{-T_s/R_{p2}C_{p2}})
\end{bmatrix} I_{t,k}
\]

(2)

2.2. Model Parameters Identification

As shown in Figure 1, the model parameters that need to be identified are \( R_o, R_{p1}, C_{p1}, R_{p2}, C_{p2} \) and \( U_{oc} \). The particle swarm optimization (PSO) method is applied to solve the parameter identification problem similarly to the optimization problem. Therefore, the fitness function should be used as the optimization goal, and the root mean square error (RMSE) of the measured voltage based on the simulated voltage should be used as equation (3). The states will be updated as equation (4). The specific
pulse power discharge process and experimental results are shown in Figure 2. Due to the polarization effect, a long enough rest time is needed to observe the recovery period, which helps parameter identification. The model parameters determined by curve fitting are partly shown in Figure 3.

\[ J_{\text{optimize}} = a^* \sum_{i=1}^{N} (U_{t,i} - Z_i)^2 / N + b^* \text{ME} \]  

(3)

\[ v(k+1) = \omega v(k) + c_1 r_1 [p(k) - x(k)] + c_2 r_2 [g(k) - x(k)] \]  

(4)

Figure 2. Pulse power discharge experiment results.  
Figure 3. Identification of parameters.

3. FSCKF Algorithm for SOC Estimation

3.1. The Fading Factor
In practical applications, the update status includes model prediction and measurement correction. The accuracy of the prediction model depends on the priori interpolation results of the pre-identified parameters, and it is difficult to guarantee the accuracy of the SOC estimation. In addition, in some special cases, inappropriate historical measurement data may cause filter distortion or even divergence. Therefore, to reduce the heavy reliance on past data and minimize divergence, a fading factor is used to emphasize new measurements. The key issue of the fading factor method is to reduce the prediction of historical measurements while increasing the weight of new measurements. The method is to multiply the covariance matrix by the fading factor.

When considering the filtering time domain N, for the arbitrary time (masked as k) from 1 to (N-1), the covariance matrix \( P \) can be written as equation (5) according to the traditional Kalman Filter:

\[ P(k \mid k-1) = F_{k-1} P(k-1) F_{k-1}^T + Q_{k-1} \]  

(5)

where, \( F \) is the measurement update matrix, and \( Q \) the covariance matrix of noise. To introduce the fading factor(masked as s), multiply \( s^{-(N-k)} \) to the both sides of equation(5):

\[ s^{-(N-k)} P(k \mid k-1) = s^{-(N-k)} F_{k-1} P(k-1) F_{k-1}^T + s^{-(N-k)} Q_{k-1} \]  

(6)

where s is a constant greater than 1, and N is the filtering time domain, and k is the operating time. The equation(6) can be rewritten as:

\[ s^{-(N-k)} P(k \mid k-1) = F_{k-1} [s^{-(N-(k-1))} P(k-1)] F_{k-1}^T + s^{-(N-k)} Q_{k-1} \]  

(7)

Define \( P^*(k \mid k-1) \) as equation(8):

\[ P^*(k \mid k-1) = s^{-(N-k)} P(k \mid k-1) \]  

(8)

Then equation(5) can be written as:
Use the equation (9) to replace the covariance matrix $P$ of the traditional Kalman Filter. And then, the covariance matrix can be enlarged by $s$ times, which is equivalent to weakening the model prediction component.

### 3.2. FSCKF Algorithm

CKF is derived from Bayesian estimation theory and is used to solve nonlinear filtering problems. By calculating the Gaussian weighted multidimensional nonlinear integration function, the difficulty of describing the nonlinear process can be simplified. The cubature criterion is one of the most representative criteria. It transforms the non-linear integral function into volume calculation of multi-dimensional geometry, with high calculation efficiency and numerical accuracy. Based on Bayesian estimation theory, the prediction process of CKF is expressed as equation (10) and equation (11).

$$\hat{x}(k) = E[x(k) | Z(k-1)] = \int f(x(k-1))N(x(k-1); \hat{x}(k), P(k-1))dx(k-1)$$

$$\hat{z}(k) = \int h(x(k))N(x(k); \hat{x}(k | k-1), P(k | k-1))dx(k)$$

where, $\hat{x}$ is the system status, $Z$ is the observation matrix, $f$ and $h$ are the nonlinear functions, and $N$ is the Gaussian probability density, which obeying the distribution where the mean is $\hat{x}$ and the variance is $P$. The equation above indicates that the prediction of both the state and measurement can be expressed as a Hammerstein nonlinear integral equation under the assumption of Gaussian probability density function as equation (12).

$$\int g(x)N(x; \bar{x}, P)dx$$

Based on the spherical-radial volume criterion, the nonlinear integral can be expressed as equation (13)

$$\int g(x)N(x; \bar{x}, P)dx = \frac{1}{2n} \sum_{i=1}^{n} (g(\sqrt{nP_{e_i}} + \bar{x}) + g(-\sqrt{nP_{e_i}} + \bar{x}))$$

where, $P_{e_i}$ is the variance matrix, and the $e_i$ is the unit vector where the $i$th component is 1, while others are zero. The equation above shows that the number of sample points for the spherical-radial volume criterion is $L=2n$, and the sampling points and their weight values can be expressed as equation (14).

$$\begin{cases}
\xi_i = \bar{x} + \sqrt{nP_{e_i}} \\
\omega_i = \frac{1}{2n} \\
\xi_{i+n} = \bar{x} - \sqrt{nP_{e_i}} \\
\omega_{i+n} = \frac{1}{2n}
\end{cases}$$

Through the strict mathematical derivation process, the validity of the sample points and the accuracy and stability of the CKF algorithm are guaranteed. Due to factors such as calculation rounding errors or noise, the covariance matrix of the state variables may be non-positive, which may cause divergence. To ensure the convergence of the algorithm, the square root form of the covariance matrix is used.

$$P(k | k-1) = S(k | k-1)S^T(k | k-1)$$
\[ P(k) = S(k)S^T(k) \]  

(16)

Here, \( S \) represents the square-root matrix of the covariance matrix, which can be obtained by SVD decomposition or QR decomposition. The square-root matrix can effectively reduce the possibility of divergence without affecting the algorithm precision.

4. Experimental Validation and Discussion

4.1. Experiment Setup

As shown in Figure 4, the 18650 lithium-ion battery has a nominal voltage of 3.7V and a nominal capacity of 2.6Ah, which was tested using a Neware battery tester. To validate the effectiveness of the proposed FSCKF SOC estimation method, Tests of constant current pulse discharge and constant current charge/discharge at ambient temperature are carried out.

Figure 4. working bench

![Figure 4. working bench](image_url)

Figure 5. Comparison of experimental and simulation results under constant current pulse discharge and constant current charging conditions.

![Figure 5. Comparison of experimental and simulation results](image_url)

Figure 6. Comparison of experimental and simulation results under constant current condition.

![Figure 6. Comparison of experimental and simulation results](image_url)

Figure 7. Error analysis of experimental data and simulation results under constant current pulse discharge conditions.

![Figure 7. Error analysis of experimental data and simulation results](image_url)

Figure 8. Error analysis of experimental data and simulation results under constant current charge/discharge conditions.

![Figure 8. Error analysis of experimental data and simulation results](image_url)
4.2. Validation Results and Discussion
The terminal voltage and battery capacity are record automatically by the test device. The experiment data and SOC estimation results are shown as the Figure.5 and Figure.6. Constant current pulse discharge and constant current charging conditions experiment is carried out first to verify the proposed algorithm. The SOC tracking ability and the error distribution are shown in Figure.5 and Figure.7. The maximum absolute error (MAE) is 2.4%. Constant current charge/discharge conditions were carried out subsequently and the MAE is 1.28% as shown in Figure.6 and Figure.8. It can be seen from the comparison results that the estimated error of FSCKF is less than 2.5%, and the computing cost is not much increased.

5. Conclusion
To solve the battery SOC estimation problem, an adaptive method based on Cubature Kalman filtering with fading factor is proposed. The mathematical methods are discussed and compared with the experimental results. The results show that under working conditions, the error between the simulation voltage and the measured value is less than 3%. The real-time estimation of lithium-ion battery SOC can be solved with a Cubature Kalman filter, and the noise adaptive method under dynamic conditions is improved. This article emphasizes that, whether static or dynamic conditions, FSCKF can achieve satisfactory accuracy and satisfactory robustness.

6. Acknowledgments
This work is supported by the National Key R&D Program of China under the grant number of 2018YFB0104100.

7. References
[1] X. Lai, C. Qin, W. Gao, Y. Zheng, W. Yi, A State of Charge Estimator Based Extended Kalman Filter Using an Electrochemistry-Based Equivalent Circuit Model for Lithium-Ion Batteries, Applied Sciences 8(9) doi:10.3390/app8091592
[2] B. Xia, S. Zhen, Z. Ruifeng, L. Zizhou, A Cubature Particle Filter Algorithm to Estimate the State of the Charge of Lithium-Ion Batteries Based on a Second-Order Equivalent Circuit Model, Energies 10(4) (2017), doi:10.3390/en10040457
[3] S. Li, J. Li, h. Hongwen, H. Wang, Lithium-ion battery modeling based on Big Data, Energy Procedia 159 (2019), pp 168-173.doi:10.1016/j.egypro.2018.12.046
[4] I. Wahyuddin, P. Priambodo, H. Sudibyo. State of Charge (SoC) Analysis and Modeling Battery Discharging Parameters. In Proceedings of 2018 4th International Conference on Science and Technology (ICST), Sri Lanka;doi:10.1109/ICSTC.2018.8528631
[5] G. Pebriyanti. A Lithium-ion Battery Modeling For A HIL-Battery Simulator. In Proceedings of 2013 International Conference on Computer, Control, Informatics and Its Applications, Jakarta; pp. 185-190
[6] H. Zhang, M.Y. Chow, On-line PHEV battery hysteresis effect dynamics modeling, IECON 2010 - 36th Annual Conference on IEEE Industrial Electronics Society, 2010.
[7] X. Yan, Y. Yang, G. Qi, H. Zhang, Q. Wei. Electric vehicle battery SOC estimation based on fuzzy Kalman filter. In Proceedings of International Symposium on Instrumentation & Measurement, Ottawa.doi:10.1109/IMNSA.2013.6743414
[8] H. Wang, H. Zhang. SOC estimation and simulation of electric vehicle lead-acid storage battery with Kalman filtering method, 2013 IEEE 11th International Conference on Electronic Measurement & Instruments (ICEMI), 2013.
[9] Liu, Z. Cheng, F. Yi, T. Qiu. SOC calculation method based on extended Kalman filter of power battery for electric vehicle. pp. 1-4.doi:10.1109/ISKE.2017.8258840
[10] Z. Zhang, S. Li, J. Li, H. He, Online estimation for parameters and state-of-charge of LiMn2O2 batteries with a modified adaptive Kalman filter, in: J. Yan, J.K. Kaldellis, P.E. Campana (Eds.), Renewable Energy Integration with Mini/Microgrid.2019, pp. 497-502.
[11] X. Lai, Y. Zheng, T. Sun, A comparative study of different equivalent circuit models for estimating state-of-charge of lithium-ion batteries, Electrochimica Acta 259 (2018), pp 566-577. doi:10.1016/j.electacta.2017.10.153

[12] Z. Chen, L. Yang, X. Zhao, Y. Wang, Z. He, Online state of charge estimation of Li-ion battery based on an improved unscented Kalman filter approach, Applied Mathematical Modelling 70 (2019), pp 532-544. doi:10.1016/j.apm.2019.01.031

[13] S. Fan, Z. Yan, Y. Lin, Adaptively tuning sampling weights of the unscented Kalman filter in starlight refraction navigation, OPTIK 148 (2017), pp 300-311. doi:10.1016/j.ijleo.2017.08.097

[14] R.V. Garcia, P.C.P.M. Pardal, H.K. Kuga, M.C. Zanardi, Nonlinear Filtering for Sequential Spacecraft Attitude Estimation with Real Data: Cubature Kalman Filter, Unscented Kalman Filter and Extended Kalman Filter, Advances in Space Research 63(2) (2019), pp 1038-1050. doi:10.1016/j.asr.2018.10.003