Non-Transferability in Communication Channels and Tarski’s Truth Theorem

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Abstract:
This article aims to study transferability issues in communication channels. The transferability of certain situations is not possible through communication channels. As a result, it is highlighted that a communication channel trying to transmit its error situation is in a non-transferrable state. I argue in this paper that Tarski’s Truth Undefinability Theorem and non-transferability in communication channels are equivalent. Due to this new approach, the other aspects of this famous theorem can be expressed more clearly.

Non-Transferability; Channel Theory; Tarski’s Truth Theorem

1. Introduction

In modern telecommunication, data transmission is done based on the standard model of communication as proposed by Shannon et al (1964). In this traditional model as in figure 1, there are three main parts as below:

- **Source/Channel Coding**: It is responsible for getting data \( D \) and converting it to the suitable string \( X_I \).
- **Channel**: It takes String \( X_I \) and sends it toward the receiver.
- **Source/Channel Decoding**: It constructs the final data \( D' \) based on the received signal \( Y_I \) from the channel.

An ideal communication channel must be so that \( D' = D \) or the final received data must be the same as the transmitted one. From an engineering perspective, the encoder includes source coding and channel coding. Source coding is used to minimize the number of symbols of the source data and channel coding is used to change the data alphabet into suitable symbols to cope with the properties of the channel and to maximize the channel capacity. The opposite transformations in the decoder side is done in the receiver to recover the source data \( D' \) (to be as similar as possible to \( D \)). (Cover et al, 2006)

This scheme is used to model just syntax issues. It is not known where the origin of data \( D \) is. So, it cannot say anything about the meaning and semantical issues, unless we determine where the data \( D \) come from. To cover the meaning and semantical issues it is necessary to generalize the model in a suitable way.

This article is going to focus on the semantical issues in order to introduce a new concept: “non-transferability of meaning”. This concept has not covered by any other research up to now. Henceforth, it has been introduced for the first time here. Also, an answer is given to a fundamental question regarding the “existence of non-transferable situations in communication

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1 Based on source coding and channel coding theories, input messages must be first compressed and then converted to suitable codes that avoid channel errors as much as possible.

2 The term “non-transferability” is used in this article in a new meaning for the first time. A formal definition of it is at the end of section 1.
channels”. The study of the non-transferability of meaning in communication channels is dependent on fundamental researches conducted into the semantical and logical concept of meaning. Fortunately, the concept of meaning has been studied well from logical and semantical point of view (Reed, 1994). In this article, after investigating the non-transferability issues in communication channels, its relation to one of the famous works in mathematical logic has found and proved.

One of the fundamental limitative results in mathematical logic is Tarski’s undefinability theorem (Tarski, 1956); accordingly, there is no formal method to define “Truth” in the language of arithmetic. This theorem was first known in the research on semantical concepts of truth in mathematical logic. But this article is the first attempt to find the equivalence of this theorem in another field of study. The result of this research is very similar to the relation that exists between Gödel’s incompleteness theorem and the halting problem in Turing machines. Based on Gödel’s incompleteness theorem, there are some true propositions in arithmetic that cannot be proved by arithmetic itself. This is equivalent to the existence of some codes that if given to the input of a universal Turing Machine, it does not halt. This equivalence relates mathematical logic to computer science. Based on the results of this article, Tarski’s truth theorem is related to non-transferability issues in communication channels. Therefore, this article argues that the undefinability of truth in the field of logic relates to a similar limitation in the field of communication theory.

Section 1 contains some basic definitions that are necessary for establishing the prerequisite of the article. For instance, a generalization of the standard model of communication is introduced to help us model semantical issues in communication.

The definition of transferability in communication channels is provided at the end of section 1. The existence of non-transferable situations in communication channels is proved in section 2. The equivalence of non-transferability in communication channels and the undefinability of truth in arithmetic is proved as the main point of this article in section 3. In addition, an immediate corollary of the theorem, the translation of the Liar Paradox into the field of communication theory, is given in section 3. In section 4 the conclusions and results of this research have summed up.

1. Communication Channel Model

In the real setup of a communication channel, the data comes from a real situation or set of facts. For example, it could refer to a switching status in a utility or some alarms in a telecommunication system, or maybe some of the measurements gathered by the sensory network. Also, it could be the signal sampled from a real voice, image, or video. Although the origin of some forms of data is analog, they will be changed into digital streams to transmit on a digital channel.

\begin{equation}
\begin{align*}
&M = \{ P_1(1), \neg P_1(1), P_1(2), \neg P_1(2), \ldots, P_n(m), \neg P_n(m), \ldots \} \\
&P_n(m): \text{Object } \#m \text{ has property } P_n; \quad \neg P_n(m): \text{Object } \#m \text{ has not property } P_n
\end{align*}
\end{equation}

In figure 2, the set or situation S refers to some state of affairs in the world. The communication channel must be able to send those states of affairs to the receiver. There are many ways to represent these states of affairs, but in this paper I just need to see them as a set of simple first-order sentences written by 1-ary relations or properties for different objects \#m:
Some simple properties are Device-OK(1), Battery-Low(4), AC-Fail(14), Switch-On(145) to represent the validity state of device #1, Low alarm of Battery #4, Fail of AC system #14, or the On State of Switch #145 respectively. To represent complicated facts in the world like those that are in large databases, we must use n-ary relations and maybe some sophisticated logical structures like quantifiers and modals, but simple properties and their negations as above are enough in this article.

To send the information content of a situation $S$, it is modeled by a set of 1-ary properties as in 1.1. Then each constituent proposition is represented by a frame as will be described in definition 1 below. Finally, these frames are sent to the receiver one by one. In this article, each proposition of the message is encoded by a binary frame. A suitable alphabet to represent all symbols is taken and then the binary numbers or binary codes (like ASCII) are used to convert each symbol to a binary string. For numbers, the binary equivalent of the numbers is used. Here, I define the encoding procedure for a 1-ary predicate.

**Definition 1:** The proposition "$P_n(m) = \text{Object } #m \text{ has property } P_n\" is encoded with the data frame $D = F_{P_n(m)}$ by the following rules:

1. $D = F_{P_n(m)}$ is the binary code of $P_n(m)$. That is the concatenation of strings: Polarity, $\text{Code}(P_n)$, and $\text{Code}(m)$.
2. $\text{Code}(m)$ is the binary representation of the number $m$ to code Object #m.
3. $\text{Code}(P_n)$ is the binary representation of the ASCII code for $P_n$ or the number n to code predicate.
4. Polarity is the truth value of $P_n(m)$. It is 1 if $P_n(m)$ is true and is 0 if $P_n(m)$ is false.
5. $x_0$ or 0 is to refer to all possible objects m.
6. $P_n(0)$ is the proposition to represent "all objects have property $P_n\".
7. Another simple representation of this frame is the triple $D=[\text{Polarity}, \text{Code}(P_n), \text{Code}(m)]$.

To ensure unambiguous decoding, the codes of $m$ and $P_n$ should be unique. This can be achieved by taking ASCII symbols for representing properties or using binary representations of objects. Also, a unique string ($x_0$ or 0) is required to refer to all values in predicates which is very common in networking and communication protocols. Do note that the $F_{P_n(m)}$ is itself a string. It can be obtained, if we agree on a method to scan the frame, say from left to right. In real communication and networking setup, there are many other parts in the frame to give more information about the nature of data and are suitable for different communication tasks like frame aligning, error detection and correction, data types, frame length and so on.

As an example of a frame, let’s encode the proposition $\text{ON}(112)$ to show the fact that System#112 is active. The ASCII code of the property $\text{ON}$ is 4F|4E = 01001111|01001110. The code of object #112 is 1110000.

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3 In many networking protocols, to refer to all quantities of a variable, all-0 or all-1 is used. For example, the IP address with all 1 or 255.255.255.255 is used to refer to all IP addresses.
Then \( F_{ON(112)} = 1|010011101001110|1110000 \) or the triple \((1,4F4E,112)\).

For the purpose of this article, we must be sure that there are mechanisms in the Situation Coder-Decoder to distinguish three parts of the frame unambiguously\(^4\).

**Definition 2**: An active Communication Channel \( C \) is a triple \((Encode(.),TS(.),Decode(.))\) in which:

- \( Encode(.) \) is a function from the set of 1-ary predicates \( P_n(m) \) into the data streams \( D = F_{P_n(m)} \) (based on definition 1.)
- \( TS(.) \) or Transmission System is the function that takes data String \( D = F_{P_n(m)} \) and deliver it to receiver as \( D' \).
- \( Decode(.) \) is the function that takes received data \( D' \) and builds a proposition \( P'_n(m') \) according to rules in definition 1.
- \( TS(.) \) can transmit symbols of data. It means that both channel coding and source coding can be selected in such a way that a one-to-one relationship between \( X_i \) and \( Y_i \) is possible. This condition ensures the activeness of the channel. \( TS(.) \), in fact, includes both source coding and channel coding.

**Definition 3**: A proposition is called **transferable** over a channel if it is logically equivalent to the received proposition from the channel on which it has been transmitted. If the received proposition is not equivalent to the transmitted one, then that proposition is **non-transferable** over that channel:

Let’s channel \( C \) is \((Encode(.),TS(.),Decode(.))\)

Then a proposition \( P_n(m) \) is **transferable** over the channel \( C \) if and only if:

\[
\text{Decode}(TS(Encode(P_n(m)))) \leftrightarrow P_n(m)
\]

(2.1)

### 2. Non-Transferability Theorems in Communication Channels

After defining the necessary Coding-Decoding mechanisms in communication channels, we are ready to provide the theorem concerning the transferability in channels. In this section, both the informal form of the proof and the formal one is provided.

**Theorem 1**: An active communication channel cannot encode and send every proposition toward the receiver correctly, or not all propositions are transferable over an active communication channel.

**Proof A, (Informal setup)**:

Based on the structure of the introduced communication channel, every proposition in the system can be encoded and transmitted on the channel. Let’s construct a proposition named \( NT(0) \) and send it toward the receiver. The meaning of \( NT(x) \) is “\( x \) is the code of Non-transferable proposition”. (Also define \( Tr(x) \) as “\( x \) is the code of transferable proposition”).

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\(^4\) The coding methods that has described, is based on a simple model of data frames. There are many techniques in networking and telecommunication to reach an unambiguous coding-decoding and I prevent referring to them.
Let’s consider all different propositions or data frames and arrange all of them in a table as below. According to figure 2:

\[
NT(F_{P_n(m)}) \text{ is equivalent to } \neg(\text{Decode}(TS(CODE(P_n(m)))) \leftrightarrow P_n(m) \])
\]

\[
Tr(F_{P_n(m)}) \text{ is equivalent to } \text{Decode}(TS(CODE(P_n(m)))) \leftrightarrow P_n(m)
\]

(2.2)

This means that the proposition \(Tr(F_{P_n(m)})\) is true if and only if the received \(P_n(m)\) after coding and passing through the channel and decoding is equivalent to \(P_n(m)\), otherwise, \(P_n(m)\) is not transferrable on the channel or \(NT(F_{P_n(m)})\) is true.

Suppose that transmitter has prepared \(NT(0)\) and has sent it toward the receiver. There are two statuses’ in the receiver:

- \(NT(0)\) is transferrable: The decoded received proposition is equivalent to the \(NT(0)\), so the receiver interprets it as the non-transferability of all propositions including \(NT(0)\) itself and this is a contradiction.
- \(NT(0)\) is not transferrable: The decoded received proposition is not equivalent to the \(NT(0)\).

So we succeeded in constructing a proposition \(NT(0)\) that cannot be transferred in any case.

![Figure 5: The frame representation of \(NT(0)\) is prepared and sent over the channel.](image)

**Proof B, (Formal setup):**

Let’s consider all different propositions or data frames and arrange all of them in a table as below.

- \([1, P_n, m]\) is equal to \(P_n(m)\). It means that object \(#m\) has property \(P_n\).
- \([0, P_n, m]\) is equal to \(\neg P_n(m)\). It means that object \(#m\) has not property \(P_n\).

| \(NT(0)\): | 1 | 2 | 3 | 4 | \(m\) |
| --- | --- | --- | --- | --- | --- |
| Polarity=1 | \(P_1(m)\): | \([1, P_1, 1]\) | \([1, P_1, 2]\) | \([1, P_1, 3]\) | \([1, P_1, 4]\) | ... | \([1, P_1, m]\) | ... |
| \(\neg P_1(m)\): | \([0, P_1, 1]\) | \([0, P_1, 2]\) | \([0, P_1, 3]\) | \([0, P_1, 4]\) | ... | \([0, P_1, m]\) | ... |
| \(P_2(m)\): | \([1, P_2, 1]\) | \([1, P_2, 2]\) | \([1, P_2, 3]\) | \([1, P_2, 4]\) | ... | \([1, P_2, m]\) | ... |
| \(\neg P_2(m)\): | \([0, P_2, 1]\) | \([0, P_2, 2]\) | \([0, P_2, 3]\) | \([0, P_2, 4]\) | ... | \([0, P_2, m]\) | ... |
| \(P_n(m)\): | \([1, P_n, 1]\) | \([1, P_n, 2]\) | \([1, P_n, 3]\) | \([1, P_n, 4]\) | ... | \([1, P_n, m]\) | ... |
| \(\neg P_n(m)\): | \([0, P_n, 1]\) | \([0, P_n, 2]\) | \([0, P_n, 3]\) | \([0, P_n, 4]\) | ... | \([0, P_n, m]\) | ... |

\(B(n)\) \([1, NT, [1, P_1, 1]]\) \([1, NT, [1, P_2, 2]]\) \([1, NT, [1, P_3, 3]]\) \([1, NT, [1, P_4, 4]]\) ... \([1, NT, [1, P_n, m]]\)

\(B'(n)\) \([0, Tr, [0, P_1, 1]]\) \([0, Tr, [0, P_2, 2]]\) \([0, Tr, [0, P_3, 3]]\) \([0, Tr, [0, P_4, 4]]\) ... \([0, Tr, [0, P_n, m]]\)

*Table 1: The enumeration of all propositions in the communication channel*
Since Transferability and also Non-Transferability are definable as in formulas in 2.2, then we construct two special propositions or data frames named $B(n)$, $B'(n)$ based on them as below:

\[
B(n) \equiv NT(F_{P_n(n)}) \quad \equiv \quad [1, NT, F_{P_n(n)}] \equiv [1, NT, [1, P_n, n]]
\]

\[
B'(n) \equiv \neg Tr(F_{\neg P_n(n)}) \equiv [0, NT, F_{P_n(n)}] \equiv [0, Tr, [0, P_n, n]]
\]

(2.3)

For each $n$ we construct the frame $[1, P_n, n]$ (or $F_{P_n(n)}$) and evaluate its Non-Transferability by the predicate $NT(\cdot)$ and set $B(n)$ as the frame $[1, NT, [1, P_n, n]]$. Also, we construct the frame $[0, P_n, n]$ (or $F_{\neg P_n(n)}$) and evaluate its Transferability by the predicate $Tr(\cdot)$ and set $B'(n)$ as $[0, Tr, [0, P_n, n]]$.

Based on the assumption that all propositions are in the list, then these two apparently new propositions must coincide with two rows in the table. There must be $k$ and $k'$ that:

\[
B(n) \equiv P_k(n)
\]

\[
B'(n) \equiv \neg P_{k'}(n)
\]

(2.4)

So there are $k$ and $k'$ that for every $n$:

\[
P_k(n) \equiv NT(F_{P_n(n)}) \quad \text{or} \quad [1, P_k, n] \equiv [1, NT, [1, P_n, n]]
\]

\[
\neg P_{k'}(n) \equiv \neg Tr(F_{\neg P_n(n)}) \quad \text{or} \quad [0, P_{k'}, n] \equiv [0, Tr, [0, P_n, n]]
\]

(2.5)

By selecting $n = k$ in the first formula and $n = k'$ in the second one:

\[
P_k(k) \equiv NT(F_{P_k(k)}) \quad \text{or} \quad [1, P_k, k] \equiv [1, NT, [1, P_k, k]]
\]

\[
\neg P_{k'}(k') \equiv \neg Tr(F_{\neg P_{k'}(k')}) \quad \text{or} \quad [0, P_{k'}, k'] \equiv [0, Tr, [0, P_{k'}, k']]
\]

(2.6)

The first formula means that $P_k(k)$ as a true proposition implies that it is not transferable. Also, the second formula means that $\neg P_{k'}(k')$ as a true proposition results in that $\neg P_{k'}(k')$ is not transferable. So we succeeded in constructing propositions that are not transferable. The structure of this proof is similar to the famous diagonal method in set theory.

3. Non-Transferability and Tarski’s Truth Theorem

The theorem 1 declares in communication theory what Tarski’s argument shows in mathematical logic. Based on the channel non-transferability theorem, there is no encoding-decoding mechanism for all propositions together with their truth value state to send from transmitter to receiver. This is equal to the non-existence of a suitable arithmetical function, encoder-decoder, for representing all truth states of the propositions in the system that is what Tarski’s truth theorem states. Also, the structure of proof is based on the fixed-point method as a famous method in many theorems in logic. Both theorems are declaring the impossibility of finding a method to ‘express’ or ‘transfer’ the truth value of all facts of the related system. This equivalence is proved in next theorem.

**Theorem 2**: The Non-Transferability of an active communication channel and Tarski’s truth undefinability theorem are equivalent.

**Proof**:

Suppose an arbitrary active channel $C = (\text{Encode}(\cdot), TS(\cdot), \text{Decode}(\cdot))$ including an encoder and decoder to represent and send every proposition in the system. The channel is active, so there could be a one-to-one
correspondence between encoder output $D$ and decoder input $D'$. The presence of suitable coding for representing each proposition and its truth value in the system is at the core of the proof. I use the method of “proof by contradiction” as below:

a) If $C$ has Non-Transferability and Tarski’s truth theorem is not the case:

Since it is supposed that Tarski’s theorem is not valid, there is a 1-ary arithmetic predicate $T(x)$ as the truth function. That is, the truth value of each proposition $P$ can be evaluated by the truth value of $T(n)$ in which $n$ is Gödel’s number or code of $P$.

$$T(n) \leftrightarrow P \quad \text{and} \quad n = \text{Encode}(P)$$

For each proposition $P$:

$$T(\text{Encode}(P)) \leftrightarrow P \quad (3.1)$$

Suppose, we are going to send an arbitrary proposition $P$ toward the receiver. To do this, the same coding mechanism as is used to build $n$ from $P$ in 3.1 is selected. After inputting $n$ to the channel, it delivers a corresponding unique $n'$ to the input of the decoder based on the one-to-one function $TS(.)$ (This derives from the activeness of the channel).

$$n' = TS(n) \quad TS \text{ is a one-to-one function}$$

So

$$n' = TS(\text{Encode}(P)) \quad (3.2)$$

The decoder function is selected as below:

$$\text{Decode}(.) \equiv T(TS^{-1}(.) ) \quad (3.3)$$

The final output of the system is:

$$P' \leftrightarrow \text{Decode}(n')$$

$$\leftrightarrow T(TS^{-1}(n'))$$

$$\leftrightarrow T(TS^{-1}(TS(\text{Encode}(P)))) \quad (3.4)$$

$$\leftrightarrow T(\text{Encode}(P))$$

$$\leftrightarrow P$$

So the truth value of $P$ after transferring on the channel can be determined based on the arithmetical predicate $T(x)$. In another word, that channel has transferability for all propositions. $\Rightarrow$ Contradiction.

b) If Tarski’s truth theorem is the case and $C$ has Transferability:

In this case, every proposition $P$ in the system can be encoded in a frame, through arithmetic operations, and can be decoded unambiguously in the receiver as $P'$ whose truth value is the same as $P$:

$$P' \leftrightarrow \text{Decode}(TS(\text{Encode}(P))) \quad (3.5)$$

So by this framing and coding-decoding mechanism, we can represent every proposition of the system containing its truth value by a number $n$:

$$n = \text{Encode}(P)$$

$$n = F_p \quad n \text{ is like a binary representation of the Frame } F_p \quad (3.6)$$
Let’s define predicate $T(n)$ as below:

$$T(n) \equiv \text{Decode}(TS(n)) \quad (3.7)$$

Then:

$$T(n) \equiv \text{Decode}(TS(n))$$
$$T(n) \equiv \text{Decode}(TS(\text{Encode}(P)))$$
$$T(n) \equiv P' \quad (3.8)$$

But since $P' \leftrightarrow P$ then:

$$P \leftrightarrow T(n)$$
$$n = \text{Encode}(P) \quad (3.9)$$

So we have an arithmetic function $T(n)$ that can give the truth value of each proposition and this is not what Tarski’s theorem is saying. → Contradiction.

The framing mechanism is a method of representing each proposition that contains its truth value. So, having a channel that could send every proposition together with its truth value is equal to having an arithmetic function for representing a truth table for all propositions. ■

In the following part, I demonstrate an immediate consequence of Non-Transferability in the channels. It gives us a better understanding of the relation between non-transferability in communication channels and Tarski’s theorem. The Liar Paradox as a famous self-referential situation is chosen for this demonstration. First, a Liar paradox situation is modeled in a system. Then it is checked, whether the communication channel can handle it or not. Before doing this in theorem 3 below, let’s take a look at a more fundamental relation between a communication channel and Tarski’s definition of Truth.

Based on figure 2, it is going to inform the receiver about the set of facts $S$. Also, $S'$ is the set of facts constructed by the receiver based on massage $M'$. If $S' = S$ then the communication channel and related systems are errorless. The receiver, in fact, is going to check the truth value of the elements of $S$ and it does it through checking the received $S'$:

$$S \text{ is true if and only if } S'. \quad (3.10)$$

(Here “$S$ is true” means “all elements of $S$ are true”)

This sentence is similar to the famous T-scheme definition of truth by Tarski:

$$P \text{ is true if and only if } P \text{ is the case.} \quad (3.11)$$

The communication channel, in fact, behaves like an interpretation system. The receiver is like a cognitive agent that is going to have all truth about all propositions and he does it through the information that gets from the transmitter using the channel. So, the Tarski’s definition of truth can be expressed by the channel model of communication system.

Below theorem introduces an un-transferable situation in a channel that is similar to the Liar Paradox.
Theorem 3: In a communication channel, the error situation of the channel is not transferable.

Proof: Consider below situation that I am going to transmit over the channel:

\[ S_e = \{ \text{The channel has error} \} \]

\[ S_e = \{ D = \text{Encode}(M) \land TS(D) = D' \land M' = \text{Decode}(D') \land M' \neq M \} \] \hspace{1cm} (3.12)

- \( D = \text{Encode}(M) \) means that \( D \) is the encoded form of the message \( M \).
- \( M' = \text{Decode}(D') \) is the inverse function of \( \text{Encode}(.) \) Function.
- \( TS(D) = D' \) means that \( D' \) is the received string of the Transmission System with input \( D \).
- \( M' \neq M \) means that the channel has an error and the message \( M \) is not transferable on it.

This situation is used when the transmitter wants to inform the receiver that the channel has an error in transmitted data. To model this situation, let us make a simple representation of it. Consider a simple proposition named \( \text{Err}(x) \) and it means that channel has an error in sending string \( x \):

- \( \text{Err}(a) \) means that channel has error in sending a special string \( a \).
- \( \text{Err}(0) \) means that channel has error in sending each string.  

\[ D = \begin{cases} \text{Err} & \text{Polatility=1} \\ \text{Err}(0) & \text{Polatility=0} \end{cases} \quad x_0 = 00 \ldots 00 \]

Now suppose that the transmitter encodes the proposition \( \text{Err}(0) \) and sends it toward the receiver. So the string is decodable and the receiver extracts the message \( \text{Err}(0) \) with polarity 1. So the received frame is saying that the received message \( \text{Err}(0) \) is true.

\[ \Rightarrow \text{Err}(0) \text{ is true } \Rightarrow \text{Channel had error for all strings } \Rightarrow \text{Err}(0) \text{ itself is not true} \]

So, the status of the channel is not known or decidable to the receiver at all. That’s the way, we succeeded in finding a situation that is not transferable at all.■

As revealed in this proof, a communication system cannot transmit some situations that are referring to its error status. The theorem that is proved is the communication counterpart of the liar paradox reasoning.

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5 It is common in telecommunication systems to model the error condition of the channel by a special code. For example, in SDH systems, this condition is shown by a frame with all 1 in its payload.
4. Conclusion

As proved in this article, the existence of Non-Transferrable codes, as showed in the field of Communication Theory, is equivalent to the undefinability of truth based on Tarski’s theorem in mathematical logic. Similarly, the existence of Non-Computable functions, as has been proved in computer science, is equivalent to the result of the first Incompleteness theorem of Gödel in mathematical logic. These two equivalences have depicted in the picture below:

| Computer Science:          | Communication Theory:          |
|----------------------------|--------------------------------|
| Gödel’s Incompleteness Theorem | Tarski’s Undefinability Theorem |
| Non-Computability in Turing Machines | Non-Transferability in Communication Channels |

Figure 7: Famous logical theorems and their practical counterparts.

It has been known to us that Gödel’s incompleteness theorem demonstrates the inherent limitations in any kind of computing system. Tarski’s un-definability of truth is the most famous theorem in mathematical logic after Gödel’s incompleteness theorem. I have shown that this theorem applies another fundamental limitation on systems. However, it is not on the computing and reasoning abilities of the systems but on their communication capabilities. As Gödel’s incompleteness theorem denotes the limitations of computers, Tarski’s un-definability of truth make us aware of the inherent limitations in communication systems. Also, Gödel’s result refers to the fundamental limitations of human reasoning in handling formal systems. Similarly, Tarski’s work refers to similar limitations of human language in transferring and grasping the truth state of affairs in the world. The cognitive abilities of humans are based on many mental mechanisms, among them reasoning and language are confined to the limits that are drawn by the exceptional works of Gödel and Tarski. From a system theoretic point of view, the existence of Non-Computable functions and Non-Transferrable situations make similar limitations on the computational and communicational power of any system.

This article not only makes clear the existence of a fundamental limitation in communication systems but also gives us a new point of view for thinking about Tarski’s undefinability theorem. Based on theorems 2 and 3 of this article, both the Non-Transferability issue and the un-definability of truth are pointing to the same perceptual and lingual restrictions in a cognitive system.

5. References

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