Acoustical Analysis of Circular Straight Mufflers with Bias Inlet/outlet Ducts

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Abstract. In order to overcome the drawback of the accuracy for the Plane wave theory in the muffler’s acoustical calculation in the region of higher frequencies, we use a theoretical analysis of eigen function in conjunction with a four-pole system matrix to evaluate the TL of a circular straight muffler with bias inlet/outlet duct. With this method, the acoustical calculation can be speeded up comparing to traditional simulation of FEM and BEM method. Consequently, results reveal that the bias effect will tremendously influence the acoustical performance of the muffler.

1. Nomenclature
This paper is constructed on the basis of the following notations:
a: the radius of the expansion chamber (m).
a$_1$: the radius of the inlet tube (m).
a$_2$: the radius of the outlet tube (m).
c$_0$: sound speed (m/s).
f: frequency (Hz).
f(x,y): the step function.
k: wave number of sound wave $\left( \frac{\omega}{c_0} \right)$.
k$_x$: wave number of sound wave in x-axis.
k$_y$: wave number of sound wave in y-axis.
k$_z$: wave number of sound wave in z-axis $\sqrt{k^2 - k_x^2 - k_y^2}$.
L: the length of the expansion chamber (m).
m,n: acoustical mode.
$\overline{p}_m$: averaged acoustical pressure (Pa).
$\overline{p}'_m$: averaged interactive acoustical pressure (Pa).
2. Introduction

Ih and Lee [1, 2], in 1985-1987, developed a theoretical method used for predicting the acoustical performance of a circular-sectioned expansion muffler. In 1987, Munjal [3] presented a numerical analysis method that simplified the calculation of an expansion muffler with rectangular and circular sections; however, the ratio of the expansion area to the inlet/outlet area needed an integer value. In addition, analysis proved to be difficult using the analytic method if the angle between inlet and outlet was 90 degrees. Abom [4], in 1990, provided a theoretical method in conjunction with a four-pole matrix to predict the acoustical performance of an expansion chamber muffler equipped with an extended inlet and outlet.

Other analytic methods used for mufflers such as the finite element method [5, 6, 7, 8, 9] and the two-dimensional boundary element method [10, 11, 12, 13] prolonged the calculation time. Ih [14], in 1992, analyzed the expansion muffler (equipped with a circular/rectangular section and an inlet/outlet duct) using a numerical method. However, the study of acoustical influence with respect to the bias parameters was still insufficient. In order to initiate the acoustical influence of a one-chamber circular muffler equipped with misaligned inlet/outlet tubes when varying the bias parameters, an eigen function method used in the acoustical simulation will be adopted and further deduced.

3. Theoretical background [14]

Based on higher order wave propagation, the mathematical model of a one-chamber straight rectangular muffler shown in Fig. 1 and a one-chamber straight circular muffler shown in Fig. 2 with different allocations of inlets and outlets is derived as below:
Figure 1. A straight rectangular muffler with different allocations of inlets and outlets.

Figure 2. A straight circular muffler with different allocations of inlets and outlets.

The three-dimensional governing equation of the sound wave for a one-chamber rectangular muffler shown in Fig. 1 is

\[ \nabla^2 \phi = \left(\frac{1}{c^2}\right) \frac{\partial^2 \phi}{\partial t^2} \] (1)

Assuming the simple harmonic motion of the piston, it yields

\[ \phi(x, y, z, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \left[ A_{mn} e^{-jkz} + B_{mn} e^{jkz} \right] e^{j\omega t} \] (2)

Doing a partial differential for \( y \) and setting the boundary condition at \( y=L \) yields

\[ \frac{\partial \phi}{\partial y} \bigg|_{y=L} = 0 \] (3)

Combining Eq. (2) and Eq. (3) yields

\[ \phi(x, y, z) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} 2A_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi z}{b}\right) \cos \left(k_z(l-y)\right) e^{-jk, t} \] (4)

Doing a partial differential for \( y \) and setting the boundary condition at \( y=0 \), it becomes
\[ \frac{-\partial \phi}{\partial y} \bigg|_{y=0} = V_i f(x, y) \]  

(5)

where \( V_i \) is the average particle velocity and \( f(x, y) \) is the step function of 1.

By multiplying the cosine functions and integrating Eq.(5) into the \( xy \) domain, Eq.(5) yields

\[
\int \int V_i \cos \left( \frac{m' \pi x}{a} \right) \cos \left( \frac{n' \pi z}{b} \right) \, dx \, dz = \sum_{m,m'=0}^{\infty} \sum_{n,n'=0}^{\infty} 2A_{mn} \int_0^a \int_0^b \cos \left( \frac{m \pi x}{a} \right) \cos \left( \frac{n \pi z}{b} \right) \cos \left( \frac{m' \pi x}{a} \right) \cos \left( \frac{n' \pi z}{b} \right) \, dx \, dz \sin (k_j l) e^{-k_j l} 
\]

(6)

For circular sectioned inlet and outlet tubes, the velocity potential energy at the piston’s inlet is expressed as

\[
\phi_i(x, y, z) = -V_i \left[ \frac{\pi r_i^2 \cos (l-y)}{ab k \sin kl} + \sum_{m} \sum' \frac{\pi r_i^2}{ab \nu_{mn}} J_1(w(m,n)r_i) \frac{1}{k_j \sin k_j l} \right. \\
\left. \times \cos \left( \frac{m \pi a}{a} \right) \cos \left( \frac{n \pi b}{b} \right) \cos \left( \frac{m' \pi x}{a} \right) \cos \left( \frac{n' \pi z}{b} \right) \cos (k_j (l-y)) \right] 
\]

(7)

where \( J_1 \) is the first order of the first kind of Bessel function.

Assuming a harmonic sound wave propagation, the acoustical pressure at the inlet becomes

\[
p_i(x, y, z) = j \omega \rho \phi_i(x, y, z) \\
= -j U_i Z_{au} \left[ \frac{\pi r_i^2 \cos (l-y)}{ab k \sin kl} + \sum_{m} \sum' \frac{\pi r_i^2}{ab \nu_{mn}} J_1(w(m,n)r_i) \frac{1}{k_j \sin k_j l} \right. \\
\left. \times \cos \left( \frac{m \pi a}{a} \right) \cos \left( \frac{n \pi b}{b} \right) \cos \left( \frac{m' \pi x}{a} \right) \cos \left( \frac{n' \pi z}{b} \right) \cos (k_j (l-y)) \right] 
\]

(8)

Concerning a straight circular muffler hybridized with a circular sectioned inlet and outlet tube shown in Fig. 2, the average acoustical pressures at inlet ( \( \overline{p}_i \) ) and outlet ( \( \overline{p}_o \) ) yield [3]

\[
\overline{p}_i = (-1)^{i-1} U_i Z_{au} \left[ \frac{1}{\tan kl} - \left( \frac{a}{a_i} \right)^2 \sum_m \sum_n \frac{4k_n J_n^2(\lambda_m a_i/a)}{J_n^2(\lambda_m a_i/a)} \frac{1}{\sin^2 \left( \frac{\pi n}{a_i} \right)} \tan \left( \frac{\pi n}{a_i} \right) J_n^2(\lambda_m a_i/a) \right] \\
= (-1)^{i-1} U_i Z E_{ii}
\]

(9)

where \( i = 1 \) or 2
where \((i=1,i'=2)\) or \((i=2,i'=1)\)

By using the superposition method, the overall potential energy at the chamber inlet is

\[
\vec{P}_1 = \vec{p}_{11} + \vec{p}_{12} = -j\pi_0 \left( U_1 E_{11} - U_2 E_{12} \right)
\]  

(11)

Similarly, the overall potential energy at the chamber outlet is

\[
\vec{P}_2 = \vec{p}_{21} + \vec{p}_{22} = -j\pi_0 \left( U_1 E_{21} - U_2 E_{22} \right)
\]  

(12)

The acoustical transfer matrix between nodes 1 and 2 is

\[
\begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P_2 \\ U_2 \end{bmatrix}
\]  

(13)

Developing Eq.(13) yields

\[
P_1 = T_{11} P_2 + T_{12} U_2
\]  

(14a)

\[
U_1 = T_{21} P_2 + T_{22} U_2
\]  

(14b)

where

\[
T_{11} = \left( \vec{P}_1 / \vec{P}_2 \right)_{U_2=0} = E_{11} / E_{12}, \quad T_{12} = \left( \vec{P}_1 / \vec{U}_2 \right)_{\vec{p}_{12}=0} = j\pi_0 \left( E_{12} - E_{11} E_{22} / E_{12} \right)
\]

(14c)

\[
T_{21} = \left( \vec{U}_1 / \vec{P}_2 \right)_{U_2=0} = j \left( \pi_0 E_{12} \right)^{-1}, \quad T_{22} = \left( \vec{U}_1 / \vec{U}_2 \right)_{\vec{p}_{12}=0} = E_{22} / E_{12}
\]

(14d)

Consequently, the transmission loss (TL) of the muffler is

\[
TL(f, \vec{X}_1) = 20 \log \left( \frac{T_{T11} + T_{T12} + T_{T21} Z_1 + T_{T22} (S_2 / S_1)}{2} \right)
\]  

(15a)

where

\[
Z_1 = \rho_0 c / S_1; \quad Z_2 = \rho_0 c / S_2; \quad S_1 = a_1^2; \quad S_2 = a_2^2; \quad \vec{X}_1 = (r, a_1, a_2, \alpha, \delta_1, \delta_2, \theta, L)
\]  

(15b)
4. Model check

Before completing the acoustical simulation and analysis for a one-chamber circular straight muffler equipped with a bias inlet/outlet tube shown in Fig. 2, an accuracy check of the mathematical models will be verified using Ih and Lee’s data [1]. Results in Figs. 3 and 4 reveal that they are in agreement for a circular muffler with a concentric inlet and outlet tube. Results in Figs. 5 and 6 also indicate that they are consistent for a circular muffler with a bias inlet and outlet tube.

Figure 3. Accuracy check of a one-chamber circular straight muffler equipped with a bias inlet/outlet tube using Ih and Lee’s data [1] (L=0.1665 m).

Figure 4. Accuracy check of a one-chamber circular straight muffler equipped with a bias inlet/outlet tube using Ih and Lee’s data [1] (L=0.0406 m).
5. Acoustical simulation

A one-chamber circular straight muffler equipped with a bias inlet and outlet tube is shown in Fig. 2. The related geometric data for the muffler is

- $a = 0.1 \text{ m}$;
- $L = 0.2 \text{ m}$;
- $a_1 = 0.01 \text{ m}$;
- $a_2 = 0.01 \text{ m}$;
- $\delta_1 = 0.04 \text{ m}$;
- $\delta_2 = 0.04 \text{ m}$

Figure 5. Accuracy check of a one-chamber circular straight muffler equipped with a bias inlet/outlet tube using Ih and Lee’s data [1] (bias, $L=0.1665 \text{ m}$).

Figure 6. Accuracy check of a one-chamber circular straight muffler equipped with a bias inlet/outlet tube using Ih and Lee’s data [1] (bias, $L=0.0406 \text{ m}$).
In order to expose the influence of the transmission loss with respect to various geometric parameters, a series of acoustical analyses using Eq. (15) is performed and described below:

**Strateg I — Varying the bias angle between the inlet tube and the outlet tube** ($\theta$). Three bias angles ($\theta$), including $180^\circ$, $120^\circ$, and $90^\circ$, are adopted in the acoustical simulation for the one-chamber circular straight muffler equipped with a bias inlet and outlet tube. As indicated in Fig. 7, an adjustment of the bias angle ($\theta$) is performed. The related geometric parameters of $a$, $L$, $a_1$, $a_2$, $\delta_1$, $\delta_2$ are fixed as 0.1, 0.2, 0.01, 0.01, 0.04.

![Figure 7](image)

**Figure 7.** Adjustment of mufflers with a bias angle ($\theta$) between the inlet tube and the outlet tube.

**Strateg II — Varying the distance between the center of the inlet/outlet tube and the center of the expansion chamber** ($\delta_1$ and $\delta_2$). As indicated in Fig. 8, an adjustment of the distance between the center of the inlet/outlet tube and the center of the expansion chamber ($\delta_1$ and $\delta_2$) is performed with an acoustical simulation where other geometric parameters of $a$, $L$, $a_1$, $a_2$ are fixed as 0.1, 0.2, 0.01, 0.01 and the bias angle ($\theta$) is set as $180^\circ$. In addition, an acoustical simulation is performed by varying $\delta_1$ and $\delta_2$ at $\theta = 120^\circ$. The resultant profile compared with the original TL profile is depicted in Fig. 9. Furthermore, the acoustical simulation is performed by varying $\delta_1$ and $\delta_2$ at $\theta = 90^\circ$. The resulting TL curve in conjunction with the original TL profile is plotted in Fig. 10.

![Figure 8](image)

**Figure 8.** Adjustment of mufflers with two distances ($\delta_1$ and $\delta_2$) between the center of the inlet/outlet tube and the center of the expansion chamber (at $\theta=180^\circ$).

![Figure 9](image)

**Figure 9.** Adjustment of mufflers with two distances ($\delta_1$ and $\delta_2$) between the center of the inlet/outlet tube and the center of the expansion chamber (at $\theta=120^\circ$).
Figure 10. Adjustment of mufflers with two distances ($\delta_1$ and $\delta_2$) between the center of the inlet/outlet tube and the center of the expansion chamber (at $\theta=90^\circ$).

**Strateg III – Varying the diameters of the inlet and outlet tube ($a_1$ and $a_2$).** As indicated in Fig. 11, the acoustical simulation of the muffler is performed by adjusting two diameters ($a_1$ and $a_2$) of the inlet and outlet tube where other geometric parameters of $a$, $L$, $\delta_1$, $\delta_2$ are fixed as 0.1, 0.2, 0.04, 0.04 and the bias angle ($\theta$) is set as 180$^\circ$. Also, by adjusting $a_1$ and $a_2$ at $\theta$ of 120$^\circ$, the TL curve of the muffler is simulated. The TL profile is compared using the original TL profile and plotted in Fig. 12. Moreover, acoustical simulation of the muffler by adjusting $a_1$ and $a_2$ at $\theta$ of 90$^\circ$ is carried out and illustrated in Fig. 13.

Figure 11. Adjustment of mufflers with two diameters ($a_1$ and $a_2$) of the inlet and outlet tube (at $\theta=180^\circ$).

Figure 12. Adjustment of mufflers with two diameters ($a_1$ and $a_2$) of the inlet and outlet tube (at $\theta=120^\circ$).

Figure 13. Adjustment of mufflers with two diameters ($a_1$ and $a_2$) of the inlet and outlet tube (at $\theta=90^\circ$).
6. Discussions
The influence of TL with respect to the bias angle (θ) is shown in Fig. 7. As indicated in Fig. 7, the broadband TL of the muffler will be improved if the bias angle of θ increases. Three bias angles of θ including 90°, 120°, and 180° are tried. Results in Fig. 7 indicate that the TL profile at θ = 90° is superior to the other two bias angles. The TL at 1200 Hz will be improved by 2 dB when adjusting the bias angle (θ) from 180° to 120°. Also, The TL at 1200 Hz will be improved by 3 dB when adjusting the bias angle (θ) from 120° to 90°.

Additionally, the influence of TL with respect to the distance between the center of the inlet/outlet tube and the center of the expansion chamber (δ₁ and δ₂) is shown in Figs. 8-10. As can be seen in Fig. 8, the TL of the muffler at lower frequencies will be improved when δ₁ and δ₂ decrease at θ = 180°. Also, the TL profile will be shifted to the left side a bit. Likewise, as indicated in Fig. 9, the TL of the muffler at lower frequencies will also increase if δ₁ and δ₂ decrease at the bias angle of θ = 120°. However, as indicated in Fig. 10, the TL of the muffler will remain the same at lower frequencies and fluctuate at higher frequencies when δ₁ and δ₂ decrease at the bias angle of θ = 90°. As can be seen in Figs. 8-9, the TL at 1200 Hz will be improved by 2-3 dB when δ₁ and δ₂ are shortened from 0.06 m to 0.04 m.

The investigation of the TL with respect to the diameters of the inlet and outlet tubes (a₁ and a₂) is shown in Figs. 11-13. As can be seen in Fig. 11, the broadband TL of the muffler will be largely improved when a₁ and a₂ decrease and the bias angle of θ is 180°. Similarly, Fig. 12 indicates that the whole TL profile will be promoted if a₁ and a₂ decrease and the bias angle of θ is 120°. Also, as illustrated in Fig. 13, the improvement of the TL profile is obvious when decreasing the value of a₁ and a₂ at θ = 90°. Figs. 11-13 indicate that the peak of the TL curve will improved by 6-7 dB if the diameters of inlet and outlet tube (a₁ and a₂) are shortened from 0.015 m to 0.01 m.

7. Conclusion
It has been shown that the bias angle (θ) between the inlet tube and the outlet tube will influence acoustical performance. The decrement of the bias angle (θ) will increase the acoustical performance. In addition, the distance between the center of the inlet/outlet tube and the center of the expansion chamber (δ₁ and δ₂) will also influence the muffler’s acoustical performance. Results in Figs. 8-9 reveal that the TL at lower frequencies will be somewhat improved when δ₁ and δ₂ are shortened and the bias angle (θ) is between 120° and 180°. Moreover, the radius (a₁ and a₂) of inlet and outlet tube also play an essential role in improving the broadband TL curve. The peak vale of the TL curve will be obviously improved when a₁ and a₂ decrease.

Consequently, considering the higher order mode of the sound wave, the approach used for investigating the influence of the TL with respect to various geometric parameters is achieved.

8. References
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