Nonlinear electrodynamics and modification of initial singularities, and dark matter and dark energy affecting structure formation in the early and later universe

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Abstract. We find that having the scale factor close to zero due to a given magnetic field value in an early universe magnetic field affects how we would interpret Mukhanov’s chapter on “self reproduction” of the universe in his reference. The stronger an early-universe magnetic field is, the greater the likelihood of production of about 20 new domains of size $1/H$, with $H$ the early-universe Hubble constant, per Planck time interval in evolution. We form DM from considerations as to a minimum time step, and then generate DM via axions. Through Ng’s quantum infinite statistics, we compare a DM count, giving entropy. The remainder of the document is in terms of DE as well as comparing entropy in galaxies versus entropy in the universe, through a lens of Mistra’s quantum theory of the big bang.

1. Introduction
This paper takes several routes to identifying nonlinear electrodynamics (NLED) phenomena pertinent to cosmological structure formation. First, we look at what Mukhanov writes as far as structure formation: Mainly, there is a so-called self-reproduction of inhomogeneity in terms of early universe conditions [1]. In this, the starting point is if one used the meme of chaotic inflation—i.e., inflation generated by a potential given by [1]:

$$V(\text{potential}) \sim \phi^2.$$ (1)

In this, Mukhanov writes that one can look at a scalar field at the end of (chaotic) inflation, with an amplitude given by $\phi$, and the initial value of the inflaton such that

$$\phi_{\text{max}}^\text{in} \sim m \cdot \phi_i^2,$$ (2)

where $m$ will be determined by NLED inputs. In terms of the initial inflaton, inhomogeneities do not form if the initial inflaton is bounded [1] as given by

$$m^{-1} > \phi_i > m^{-\frac{1}{2}}.$$ (3)

This leads to (low?) inhomogeneity in spacetime generated by inflation. Inflation is eternal [1] if there is only the inequality

$$\phi_i > m^{-\frac{1}{2}}.$$ (4)
2. NLED applied to Eq. (4) plus details of structure formation added

What we will do is to look at the following treatment of mass, and this will be our starting point. That is, we will be looking at if \( l_p \) is a Planck length, and \( \alpha > 0 \), then

\[
m \sim 10^\alpha \cdot l_p^\alpha \cdot \rho(\text{density}).
\]  

Then we can consider the following formulation of density given below. If we do not wish to consider a rotating universe, then Camara et al. [2] has an expression as to density, with a \( B \) field contribution to density, and we also can use the Weinberg result [3] of scaling density with one over the fourth power of a scale factor, which we remark upon in the general section, as well the Corda and Cuesta result of [4] for density (note [4] is for a star, [2] is for a universe). In addition, Corda and Cuesta [4] use quintessential density to falsify the null energy condition of a Penrose theorem cited in [4, 5]. Further details of what Penrose was trying to do as to this issue of GR can be seen in [4, 5]. To answer how to violate the null energy condition, one should go to [5] for quintessential density defined, with the constant in Eq. (4) greater than zero. Then in both the massive star and the early universe, the density result below is applicable.

\[
\rho_\gamma = \frac{16}{3} \cdot c_1 \cdot B^4.
\]  

Keeping in mind what was said as to choices of what to do about density, and its relationship to Eq. (5) above, we then can reference what Mukhanov [1] says about structure formation as follows, namely look at how a Hubble parameter changes with respect to cosmic evolution. It changes with respect to \( H_{\text{today}} \) being the Hubble parameter in the recent era, and the scale factor \( a \), with this scale factor being directly responsive to changes in density according to [3]—i.e.

\[
\rho \sim a^{-4}.
\]  

In the next section, we will examine how [2] suggests how to vary the scale factor cited in Eq. (7), and we will in this section take note of what the scale factor cited in [2] does to the Hubble parameter given in Eq. (8) below, and then in the section afterwards review a possible reconciliation of what Eqs. (6) and (7) say about defining early-universe parameters. But to know why we are doing it, we should take into consideration what happens to the Hubble parameter, as given below.

\[
H \sim \frac{H_{\text{today}}}{a^2}.
\]  

According to [1], if Eq. (4) holds, then inhomogeneous patches of spacetime appear in a causal region of spacetime for which

\[
\text{Causal – domain} \sim H^{-1} \sim \frac{1}{H_{\text{today}} a^2}.
\]  

Furthermore, [1] states that about 20 such domains are created in a Hubble time interval \( \Delta t_H \propto H^{-1} \). As a function of say \( 10^8 \) times Planck time, for a domain size given by Eq. (9) above and that this requires then a clear statement as to how the scale factor changes, due to considerations given by [3] and reconciling the density expression given in Eq. (6) and Eq. (7) above.
3. Showing a nonzero initial radius of the universe due to nonlinear spacetime E&M

What we are asserting is in [1] there exists a scaled parameter \( \lambda \), and a parameter \( a_0 \), which is paired with \( \alpha_0 \). For the sake of argument, we will set the \( a_0 \propto \sqrt{t_{\text{Planck}}} \), with \( t_{\text{Planck}} \sim 10^{-44} \) seconds. Also, \( \Lambda \) is a cosmological “constant” parameter which is described later, as in quintessence, via [6], and as in [2] via:

\[
a_0 = \sqrt{\frac{4\pi G}{3\mu_0 c}} B_0 \quad \text{and} \quad \lambda = \frac{\Lambda c^2}{3},
\]

(10)

(11)

Then if, initially, Eq. (11) is large, due to a very large \( \Lambda \) the time, given in Eq. (53) of [2] is such that we can write, most likely, that, even though there is an expanding and contracting universe, the key time parameter may be set, due to very large \( \Lambda \) as

\[
t_{\text{min}} \approx t_0 \equiv t_{\text{Planck}} \sim 10^{-44} \text{s}.
\]

(12)

Whenever one sees the coefficient like the magnetic field, with the small 0 coefficient, for large values of \( \Lambda \), this is the initial coefficient at the beginning of spacetime which helps us make sense of the nonzero but tiny minimum scale factor [2]

\[
a_{\text{min}} = a_0 \cdot \left[ \frac{\alpha_0}{2\lambda} \left( \sqrt{\alpha_0^2 + 32\lambda \mu_0 \omega B_0^2} - \alpha_0 \right) \right]^\frac{3}{2}.
\]

(13)

The minimum time, as referenced in Eq. (12), most likely means, due to large \( \Lambda \) that Eq. (13) is of the order of about \( 10^{-55} \) —i.e., 33 orders of magnitude smaller than the square root of the Planck time. We next will be justifying the relative size of \( \Lambda \).

4. Showing how to obtain a varying \( \Lambda \) with a large initial value and its relationship to obtaining a scale factor value for the early universe via NLED methods

Notwithstanding the temperature variation in reference [2] for the cosmological Hubble parameter, we can also write

\[
\Lambda(t) \sim (H_{\text{inflation}})^2.
\]

(14)

In short, what we obtain, via looking at due to [6] that Eq. (14) is also equivalent to

\[
\Lambda_{\text{max}} \sim c_2 \cdot T^3
\]

(15)

where \( T \) is temperature. Comparing Eq. (6) and Eq. (7) above, leads to the following constraints—i.e., if we extend [1, 2] we would get

\[
(\rho \sim a^{-4})^{-1} \sim a^4 \sim \frac{16}{3} \cdot c_1^{-1} \cdot B^{-4} \sim \frac{a_0^{-4}}{2\lambda} \left( \sqrt{\alpha_0^2 + 32\lambda \mu_0 \omega B_0^2} - \alpha_0 \right)
\]

(16)

The above relationship will argue in favor of a large value for Eq. (15) and Eq. (16) \( B \) field and also the cosmological “constant” parameterized in Eq. (14) and Eq. (15). That is, once fully worked out, the allowed values of \( B \), for initial conditions will be large but tightly constrained, and this in turn will allow for Eq. (9) having initially extremely small inhomogeneity behavior, in line with being proportional to the inverse of an allowed Hubble parameter based upon Eq.
(8). Note that Eqs. (11) to (13) are arguing in favor of a very small scale factor, implying a large initial density while Eq. (16) appears to give credence to a large Hubble parameter. Further work will come up with a set of constraints as to admissible early universe quintessence. Further refinements of NLED, and the relationship in Eq. (16), as to structure formation may give credence, or help falsify the conclusions of reference [1], with great refinements needed in defining the suggested relationships implied in Eq. (16).

5. Density in the case of certain strengths for the magnetic field: the weak energy condition versus a more specialized expression which violates the null energy condition of the Penrose theorem

In [2], there is a generalized density

\[ \rho - \frac{1}{2\mu_0} \cdot B^2 \cdot (1 - 8 \cdot \mu_0 \cdot \omega \cdot B^2) . \]  

(17)

This density has a positive value only if [2]

\[ B < \frac{1}{2 \cdot \sqrt{2\mu_0 \cdot \omega}} . \]  

(18)

This density expression should be compared with the Corda and Cuesta [4] value of “quintessential density” given in Eq. (4). This Eq. (4) is how Corda and Cuesta violated the null energy condition of the Penrose theorem [4, 5]. Eq. (4) is part of the Corda and Cuesta pressure equation [4]:

\[ p = \rho - \frac{\rho}{3} - \rho \gamma . \]  

(19)

We do believe that Eq. (16) is more general, although the magnetic-field dependence is far more complicated. The importance of the pressure in Eq. (19) goes back to temperature: The higher the pressure is, the more likely extreme temperatures exist that will let us talk about the role of entropy and DM/DE next. We assert that large pressure will influence a large temperature, which in turn has implications as to the following relationship. From [2], we derived the following quark–gluon result

\[ \varepsilon(\text{energy} - \text{density}) \sim \varepsilon_{\text{QGP}} \simeq 47.5 \times \frac{\pi}{30} \times (k_B T)^4 \]  

\[ p = \frac{\varepsilon(\text{energy} - \text{density})}{3} \sim 47.5 \times \frac{\pi}{30} \times (k_B T)^4 \]  

(20)

Our large scale value of temperature applies, next to the entropy of both DM and DE.

6. We discuss DM which we identify with axions, this after the following identification

In [7, 8], one million or more BHs in the center of an equal number of galaxies leads to an entropy

\[ S_{\text{Total}} \sim 10^6 \times 10^{66} \sim \text{Galaxies}(\text{SMBH} - \text{center}) \times [\text{entropy}, \text{SMBH}] \]  

(21)

To understand what this means, we will review our version of Mishra’s [9] quantum theory of the big bang to find linkages to axions, for DM and other issues. Begin now with Mach’s principle, with \( M \) being the mass (of the universe), and \( R_0 \) being a radii of a presumed spherical space, then if \( R_0 \) is the presumed radii of the universe, and also the total following value of entropy:

\[ \frac{GM}{R_0 c^2} \approx 1, \]  

(22)
\[ M_{\text{Total}} \sim M_{DM} + M_{\text{Baryon}} + M_{DE} = N_{DM}m_{DM} + N_{\text{Baryon}}m_{\text{Baryon}} + N_{DE}m_{DE} \]
\[ \approx S_{DM}m_{DM} + S_{\text{Baryon}}m_{\text{Baryon}} + S_{DE}m_{DE}. \] (23)

Here, total entropy of the universe is assumed to be in the present era
\[ S_{\text{Total}} \sim S_{DM} + S_{\text{Baryon}} + S_{DE}. \] (24)

The use of entropy as akin to particle count comes from two sources. First source is due to “infinite quantum statistics” as given by Ng [10]:
\[ Z_N \sim \left( \frac{1}{N!} \right) \cdot \left( \frac{V}{\lambda^3} \right)^N. \] (25)

This, according to Ng, leads to entropy of the limiting value of
\[ S \approx N \cdot \left( \log \left[ \frac{V}{N\lambda^3} \right] + \frac{5}{2} \right). \] (26)

But \( V \approx R_H^3 \approx \lambda^3 \), so unless \( N \) in Eq. (26) above is about 1, \( S \) (entropy) would be less than zero, which is a contradiction. Now this is where Jack Ng introduces removing the \( N! \) term in Eq. (25) above. That is, inside the Log expression we remove the expression of \( N \) in Eq. (26) above. This is a way to obtain what Ng refers to as quantum infinite statistics, so then we obtain for sufficiently large \( N \).
\[ S \approx N. \] (27)

Alternately, but in the limit of late time cosmological constant behavior, Cai writes [11]
\[ S \leq N. \] (28)

We will refer to a bound value as referenced by Cai [11], and also Bousso [12, 13], given by
\[ N \equiv \left[ \frac{3\pi}{G\Lambda} \right]. \] (29)

Note, that the \( N \) of Eq. (29) refers to degrees of FREEDOM, which is interesting. The total degrees of freedom will be shown to become enormous for sufficiently small \( \Lambda \) [12, 13]. Pending a review of the situation, the following could be entertained.
\[ N \equiv \left[ \frac{3\pi}{G\Lambda} \right] \leftrightarrow \Lambda(\text{large}) \Rightarrow N(\text{small}) \& \Lambda(\text{small}) \Rightarrow N(\text{large}) \] (30)

6.1. Do Eq. (28) and Eq. (30) pertain to DM, to DE, or both entropy (numerical) counts?
What is being referred to is, the applicability of [12, 13]. Specifically, [12, 13] refers to DM, and we should in answering our question ascertain if DM is the preferred venue for explaining the behavior of Eq. (30). In fact, if there is quintessence in terms of the cosmological constant parameter, as by Eq. (9), then this may explain why there is
\[ \Lambda(\text{field-theory}) \sim 10^{122} \times \Lambda(\text{actual-today}). \] (31)

As given by [14], there IS a linkage of black hole entropy with \( \Lambda \) as vacuum energy, and \( L \) as a spatial length associated with black holes. Then
\[ L^3\Lambda^3 \leq S_{BH} = \pi L^2 M_p^2 \]
\[ L^3\Lambda^4 \leq LM_p^2 \]
\[ \Leftrightarrow S_{\text{max}} \sim S_{BH}^{\frac{3}{2}}. \] (32)
Furthermore, Figure 1 of page 10 of [7], as well as [15], gives one an indication that DE may not be created in the beginning of spacetime evolution, but as an artifact of later cosmological evolution. If DE is not due to massive gravitons, then either quintessence (i.e., a varying vacuum energy over spacetime due to a background average temperature) should be considered with DM and DE as different facets of the same evolutionary cosmological dynamic. The multiverse is a way of making sense of the light value of a graviton mass and Mistra’s quantum-cosmology conjecture. The quantum-cosmology conjecture is also dependent upon if entropy is determined by a counting algorithm. We argue that if there is a temperature dependence, in vacuum energy as given by Eq. (9) that high temperature means a different vacuum energy than today’s “cosmological constant value.” Also, that if $N$, as a bound to entropy, is inversely proportional to vacuum energy, that according to Figure 1 of [7], that DM will not be affected, but that DE is an artifact of vacuum energy. The author deduces from the above that the bound to entropy, which is called $N$, as given by Eq. (29) with Eqs. (9) and (11) as backup, is an artifact of DE, not DM. and that, as a result, Figure 1 of reference [7] is saying that a bound to entropy which changes over time is due to quintessence, at least in the beginning, as given by the dynamics of the aforementioned Figure 1 above. As of 13.7 billion years ago, the background temperature given by first light about 380,000 years after the big bang was $10^5$ K, according to [15, 16]. As opposed to 3 K today [16]. So, if Eq. (9) for vacuum energy is used, then if we associate $\Lambda$ with DE

$$\text{Eq. (9)} \Rightarrow \Lambda_{\text{DE}}(10^5 \text{ K}) \ll \Lambda_{\text{DE}}(3 \text{ K}) \Leftrightarrow N_{\text{DE}}(10^5 \text{ K}) \gg N_{\text{DE}}(10^5 \text{ K})$$  \hspace{1cm} (33)

The temperature scaling given in Eq. (9) plus Figure 1 of reference [7], argues strongly against DM being created by $\Lambda$.

6.2. Refining Eq. (23) in lieu of Eq. (33)

To summarize so far, based upon our modification of [9]: As of about 380,000 years after the big bang

$$M_{\text{Total}} \sim M_{\text{DM}} + M_{\text{Baryon}} + M_{\text{DE}}(\text{no contribution 380,000 years after the big bang})$$

$$\sim M_{\text{DM}} + M_{\text{Baryon}} \sim N_{\text{DM}}m_{\text{DM}} + N_{\text{Baryon}}m_{\text{Baryon}} \approx S_{\text{DM}}m_{\text{DM}} + S_{\text{Baryon}}m_{\text{Baryon}}.$$  \hspace{1cm} (34)

The summed mass is our adaptation of [9], which is in the case that 380,000 years after the big bang, there was essentially a DM dominated universe, before DE became significant.

6.3. Filling in DM contribution to entropy, 380,000 years after big bang

From Subodha Mistra [9] his quantum model of the big bang has the following $R$ is the presumed present radius of the universe, $m$ is the mass of an “average” constituent particle, and $N$ is the number of particles in a (model) universe, with $\tau$ being the time after the big bang, to the present era.

| $m \times 10^{-35}$ g | $R \times 10^{28}$ cm | $N \times 10^{91}$ | $M_{\text{Total}} \times 10^{66}$ gm | $\tau_0 \times 10^9$ yr |
|-----------------------|------------------------|------------------|---------------------------|-------------------------|
| 1.07299               | 1.896                  | 2.38429          | 2.5582                    | 20                      |
| 1.23891               | 1.422                  | 1.54865          | 1.91875                   | 15                      |
| 1.51744               | 0.948                  | 0.84297          | 1.27916                   | 10                      |
| 2.14598               | 0.474                  | 0.29804          | 0.639588                  | 5                       |

Source: Subhodha Mistra [9], p. 212
Assume there is a situation analogous to the Figure 1 [7] circumstance 380,000 years after the big bang, assume then that $M$, as a total mass of the universe does not change. Then, according to Mistra [9], the average particle of the quantum universe is of the order of magnitude of an axion DM particle: $1.23 \times 10^{-35} \text{ g} \sim 5.609 \times 10^{-2} \text{ eV}$, whereas we are assuming that the entropy is similar to a numerical count of “average” particles. Then axions have a range of $10^{-30} - 20 \text{ eV}$ in value and by Figure 1 [7] and Table 1,

$$S_{\text{DM}} \approx 10^{90} - 10^{91}. \quad (35)$$

Should we use axions for DM, we have to look at the role of DE. That is, DE is roughly 3–4 times more massive than DM (Figure 1 [7]).

6.4. The DE we identify with gravitons. How feasible is this choice?

Then there are several alternatives. If, say, massive gravitons are an active source for DE, as has been postulated by [17, 18]: If a graviton has a mass of $2 \times 10^{-62} \text{ g} \sim 2.8 \times 10^{-30} \text{ eV}$, then

$$S_{\text{DE}} \approx 10^{117} - 10^{118}. \quad (36)$$

This value for Eq. (35) is for the present era. That is, if one is looking at say $N$ in Eq. (29)–(31), with an initial temperature of, say 3 K, then Eq. (35) would hold. If $10^{32} \text{ K}$ is used in Eqs. (29)–(31), then

$$S_{\text{DE}}(10^{32} \text{ K}) \approx \epsilon^+ \ll S_{\text{DE}} \approx 10^{117} - 10^{118}(\text{present}). \quad (37)$$

If there is no mass connected with gravitons, then they cannot be conflated with DE.

7. Conclusion

A graviton mass of $2 \times 10^{-62} \text{ g} \sim 2.8 \times 10^{-30} \text{ eV}$, will lead to gravitons as a candidate for DE. If the gravitons are massless, then Mistra’s procedure [9], with summation of the mass and information of Table 1 does not apply to DE. If so, then one can look at another representation of DE. The DM which is tabulated is consistent. We identify DM with axions, using the Mistra’s formulation, and afterwards investigate what is done with the total entropy. Using Eq. (34), the DE term would increase entropy, if a graviton is of $2 \times 10^{-62} \text{ g} \sim 2.8 \times 10^{-30} \text{ eV}$ then using Mistra’s figure of mass given in Table 1, there conceivably could be the following entropy tally, that there are possibly many more than 1 million galaxies—i.e., $10^6$ to $10^{20}$ super-massive black holes in the center of galaxies—and that up to a point, the entropy of a super-massive black hole in the center of a galaxy is at most $10^{112}$. To get an idea of what is going on, look at [19],

$$S_{\text{BH}}^{\text{Total}} \sim 3.2 \times 10^{101} \times \left( \frac{N}{10^{41}} \right) \times \left( \frac{M}{10^7 \cdot M_{\text{Sun}}} \right) \sim S_{\text{universe}} \quad (38)$$

If so, then does the following make cosmological sense?

$$M \sim 10^{10} \cdot M_{\text{Sun}} \quad N \approx 10^{20} - 10^{23} \quad (39)$$

That is, the numerical factor is then so high, if there are indeed many more than 1 million galaxies contributing to entropy, we may be indeed looking at the confuence of multiple universe contributions to our present entropy, if gravitons have a small mass. The figure to consider is, if there are gravitons with mass, and this entropy from DE is so large that there may be a need to investigate multiverse contributions to our universe’s evolution.

$$S_{\text{DE}}(10^{32} \text{ K}) \approx \epsilon^+ \ll S_{\text{DE}} \approx 10^{117} - 10^{118}(\text{present}). \quad (40)$$
The final take away from our investigation, as in [9] that, if gravitons have mass, there could be by [9] reasons to investigate if there are multiple universes for entropy. If not, then Table 1 argues for an axion type of DM contribution to entropy of the universe, with a value as given by Eq. (35) of $S_{\text{DM}} \approx 10^{90} - 10^{91}$. More than that, Eq. (35) almost certainly precludes a universe, which would put a premium upon really understanding how Eq. (9) and Eq. (13) affect, or could affect, a choice between Eq. (40) which may lead to multiverses, as given by the end of [9], or a single repeating universe.

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