Experimental Study for Aerodynamic Performance of Quadrotor Helicopter*

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Wind tunnel experiments are performed to investigate the aerodynamic characteristics of a quadrotor helicopter in a uniform flow. At first, the thrust and drag coefficients of a single rotor under low-Reynolds number conditions are measured to deduce the basic design parameters of the rotor. Second, the aerodynamic interference between two rotors in a tandem configuration is investigated. It is experimentally found that the thrust coefficient of the rear rotor is a maximum of 11% lower than that of the front rotor during forward flight. Moreover, it is found that the interference effect of the front rotor on the rear rotor can be predicted qualitatively using a theoretical formula on the basis of the Biot-Savart Law. Finally, it is shown that the theoretical predictions of the thrust coefficient and drag coefficient agree well with the experimental results of a quadrotor helicopter model within limited ranges of the inflow ratio and advance ratio.

Key Words: Quadrotor Helicopter, Momentum Theory, Blade Element Theory, Thrust Coefficient, Interference Effect

1. Introduction

Today, multi-rotor helicopters (MRHs) are being utilized in society. The applications cover a broad range of fields, such as construction, observation, and logistics. A MRH is propelled by multiple (more than two) rotors, each of which is driven by an electric motor. The attitude is stabilized autonomously by controlling the rotational speed of each motor. The maneuvering of a MRH is much easier than conventional helicopters, and autonomous stable hovering has been achieved by introducing GPS signals to the control circuit. On the other hand, in real flight, accidental fly-away and crashing for unexplained reasons occurs frequently. The safety of MRH flight is a matter of great concern. The main reasons for the accidents might be mechanical or internal electrical defects, external electro-magnetic interference, or wind gust. In order to access the gust response, it is necessary to clarify the unsteady aerodynamic characteristics, which should involve steady aerodynamic characteristics such as thrust coefficient and drag coefficient.

The aerodynamics of a single-rotor conventional helicopter has been investigated using the momentum theory and blade element theory.1) This should be true for the MRHs as well. Conventional theories have been applied to predict the performance of MRHs in previous studies.2,3) However, to the best of the authors’ knowledge, few studies have been published regarding systematic wind tunnel experiments to verify the application of conventional theories to MRH aerodynamics. In order to predict the performance of MRHs on the basis of conventional theories, the following two issues should be taken into account.

First, is the low-Reynolds flow effect. MRHs used broadly today are smaller in size and fly slower than conventional manned helicopters. The Reynolds number of flow passes through a rotor is from $10^3$ to $10^4$ for MRHs, while it is around $10^6$ for conventional helicopters. It is well known that

Nomenclature

- $A$: rotor disk area
- $B$: tip loss factor
- $c$: blade chord
- $l$: distance between rotors
- $N$: number of blades
- $R$: rotor radius
- $T$: thrust
- $V$: total velocity
- $V_{sc}$: free-stream velocity
- $v$: induced velocity
- $C_T$: thrust coefficient
- $C_D$: drag coefficient
- $C_Q$: torque coefficient
- $C_{H}$: induced drag coefficient
- $C_{R}$: profile drag coefficient
- $a$: 2-D lift curve slope
- $a$: angle of attack of rotor-disk
- $a_i$: angle of attack of blade section
- $\beta$: flapping angle
- $\gamma$: wake skew angle
- $\theta_{avg}$: rotor average pitch angle
- $\rho$: air density
- $\mu$: advance ratio
- $\lambda$: inflow ratio
- $\psi$: azimuth angle
- $\sigma$: solidity
- $\Gamma$: vortex of strength
- $\omega$: rotational speed
- $\zeta$: lock number

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*Received 15 January 2017; final revision received 7 March 2017; accepted for publication 5 September 2017.
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flows of such low-Reynolds number are entirely different from flows of higher Reynolds number and are quite sensitive to the Reynolds number. For example, an unstable separation bubble is generated on the suction side of the airfoil.4) Airfoil aerodynamics in such low-Reynolds flows have been actively studied recently.5) However, it is still difficult to predict the aerodynamic characteristics of arbitrarily shaped rotors, and it is necessary to measure them carefully in wind tunnel experiments.

Second, is the aerodynamic interference between adjacent rotors. In MRHs, multiple rotors are adjoined with a small gap on a single plane. The aerodynamic interference between rotors is not understood well. The basic features of tandem rotor interference have been studied in special configurations such as the Chinook CH-47 tandem rotor helicopter.6) Especially for the CH-47, the rear rotor disk is located above the front rotor disk. In a top-down view, the front and rear rotor disks have a large overlap with each other. Experimental results show that overlapping of the disks increases the induced power required for hovering.6) Even without overlapping, as is the case for most MRHs, inflow to the rear rotor should be influenced by the induced flow of the front rotor during forward flight. However, to the best of the authors’ knowledge, few studies are devoted to clarifying the interference effects of MRHs through wind tunnel experiments. Hence, it is very useful to model the interference effects in a simple analytical fashion and to verify the model experimentally.

The overall aerodynamic performance of a MRH is determined as a consequence of the features mentioned above. In the present study, a series of wind tunnel experiments is performed to study the aerodynamic characteristics of single-rotor, tandem-rotor, and quadropter helicopters in a uniform flow. Through comparing conventional theory and experimental results, the aerodynamic coefficients of a quadropter helicopter are formulated taking into account the low-Reynolds flow ratio is

\[ \lambda = \frac{V_{\infty} \sin \alpha + v}{\rho R} \]  

and the inflow ratio is \( \lambda \) \[ \mu = \frac{V_{\infty} \cos \alpha}{\rho R} \] 

and

The dimensionless velocity components are defined as

\[ u_T = r + \mu \sin \psi \]  
\[ u_R = \mu \cos \psi \]  
\[ u_P = \lambda + r \theta / \psi + \beta \mu \cos \psi \]  

where \( r \) is defined as \( y/R \).  

### 2. Analytical Model

The model is based on the momentum theory and blade element theory. In the momentum theory, the flow is assumed to be incompressible and inviscid, and blade loading is assumed to be distributed uniformly over the blade. The induced velocity, \( v \), and thrust, \( T \), are formulated as:

\[ v = \frac{u_T}{\sqrt{(V_{\infty} \cos \alpha)^2 + (V_{\infty} \sin \alpha + v)^2}} \]  
\[ T = 2 \rho A V v \]  

where the induced velocity when hovering, \( u_h \), and the total velocity, \( V \), are formulated as:

\[ v_h = \frac{T}{2 \rho A} \]  
\[ V = \sqrt{(V_{\infty} \cos \alpha)^2 + (V_{\infty} \sin \alpha + v)^2} \]  

The blade element theory is based on the lifting-line assumption neglecting stall.\(^3\) The aerodynamic forces are calculated at each infinitesimal trip of width, \( dy \), as illustrated in Fig. 1. Figure 2 defines the geometry, velocities, and aerodynamic forces of the blade section. The blade section has the pitch angle, \( \theta \), and inflow angle, \( \phi \), measured from the plane of rotation. The angle of attack of the blade section is \( \alpha_i = \theta - \phi \). The element forces act on a rotor blade are described in Fig. 3. Figure 4 shows the components of drag force. The advance ratio, \( \mu \), is defined as

\[ C_T = \frac{1}{2} \int_{0}^{\theta} \sigma \rho (\frac{1}{2\pi} \int_{0}^{2\pi} (\theta u_T^2 - u_P u_T) d\psi) d\theta \]  

where \( B \) is the tip loss factor, and is smaller than unity. The pitch angle is assumed linearly twisted as

\[ \theta = \theta_0 + \theta_{tw} r \]  

Solidity, \( \sigma \), is defined as

\[ \sigma = \frac{N_c}{\pi R} \]
After lengthy calculations, Eq. (10) is transformed to
\[
C_T = \frac{1}{2} \sigma a \left( \frac{1}{3} B^3 \theta_0 + \frac{1}{4} B^2 \theta_w + \frac{1}{2} B \mu^2 \theta_0 \right) + \frac{1}{4} B^2 \mu^2 \theta_0 - \frac{1}{2} \frac{B^2}{\lambda} \right) \tag{13}
\]
Assuming that constant twist is \( \theta = \theta_{avg} \), then Eq. (13) reduces to
\[
C_T = \frac{1}{2} \sigma a \left( \frac{1}{3} B^3 \theta_{avg} + \frac{1}{2} B \mu^2 \theta_{avg} - \frac{1}{2} \frac{B^2}{\lambda} \right) \tag{14}
\]
**Inflow ratio calculation**

The thrust coefficient based on momentum theory is
\[
C_T = 2(\lambda - \mu \tan \alpha) \sqrt{\mu^2 + \lambda^2} \tag{15}
\]
By equating the right-hand sides of Eqs. (14) and (15), the formula for the inflow ratio is achieved as
\[
\lambda = \frac{2}{B^2} \left( \frac{1}{3} B^3 \theta_{avg} + \frac{1}{2} B \mu^2 \theta_{avg} \right) \nonumber - \frac{8}{B^2 \sigma a} (\lambda - \mu \tan \alpha) \sqrt{\mu^2 + \lambda^2} \tag{16}
\]
The Newton-Raphson procedure can be used to solve for \( \lambda \) iteratively
\[
\lambda_{n+1} = \lambda_n - \left[ \frac{f(\lambda)}{f'(\lambda)} \right]_n \tag{17}
\]
where \( n \) is the iteration number, and
\[
f(\lambda) = \lambda - \frac{2}{B^2} \left( \frac{1}{3} B^3 \theta_{avg} + \frac{1}{2} B \mu^2 \theta_{avg} \right) \nonumber + \frac{8}{B^2 \sigma a} (\lambda - \mu \tan \alpha) \sqrt{\mu^2 + \lambda^2} \tag{18}
\]
\[
f'(\lambda) = 1 + \frac{\frac{4}{B^2 \sigma a} \mu^2 + 2 \lambda^2 - \lambda \mu \tan \alpha}{\sqrt{\mu^2 + \lambda^2}} \nonumber \tag{19}
\]
The criteria for convergence is the error estimator, \( \varepsilon \), less than 0.05%
\[
\varepsilon = \left| \frac{\lambda_{n+1} - \lambda_n}{\lambda_{n+1}} \right| < 0.05% \tag{19}
\]
2.2. **Drag coefficient**

The drag force can be split into three components: induced drag, profile drag and parasite drag.

The horizontal force component is
\[
dH = N(dF_x \sin \psi - dF_y \cos \psi) \tag{20}
\]
Substituting the expression for \( dF_x \) and \( dF_y \) yields
\[
dH = NdD \sin \psi + NdL(\phi \sin \psi - \beta \cos \psi) \tag{21}
\]
The first term in Eq. (21) corresponds to the profile drag and the second term to induced drag.

The profile drag coefficient, \( C_{Hd} \), is defined as
\[
C_{Hd} = \int_0^1 \left( \frac{1}{2 \pi} \int_0^{2\pi} \frac{1}{2} \sigma C_{Dd} \psi d\psi \right) d\theta \tag{22}
\]
Assuming \( C_d = C_{d0} \) is constant, then
\[
C_{Hd} = \frac{\sigma C_{d0}}{4} \mu \tag{23}
\]
The induced drag coefficient is defined as
\[
C_d = \int_0^1 \left( \frac{1}{2 \pi} \int_0^{2\pi} \frac{1}{2} \sigma u_t^2 C_d(\psi \sin \beta - \cos \psi)d\theta \right) d\psi \tag{24}
\]
The flapping angle can be expressed as the first harmonic function of an azimuth angle
\[
\beta = \beta_0 + \beta_1 \cos \psi + \beta_1 \sin \psi \tag{25}
\]
Different from most conventional helicopters, the stiff, fixed-pitch rotor blades are used on quadrotors. In this study, a thin airfoil is utilized (Fig. 10) and the approximate calculation of rotor blade flapping is based on the formula derived for helicopter rotor blades\(^7\)
\[
\beta_0 = \frac{1}{2} \zeta \left[ \frac{\theta_{0.5}}{4} (1 + \mu^2) - \frac{\lambda}{3} \right] - \frac{M_w}{\rho \omega^2} \tag{26}
\]
\[
\beta_1 = 2 \mu \left( \lambda - \frac{4}{3} \theta_{0.5} \right) \quad \beta_1 = - \frac{4}{3} \beta_0 \mu \nonumber \tag{26}
\]
where \( \zeta \) is the lock number, \( M_w = m_{blade} \rho R^2 / 2 \) is the moment caused by the rotor blades and \( I = m_{blade} R^3 / 3 \) is the inertia moment of the rotor blades. More precise calculations of the flapping coefficients for stiff, fixed-pitch rotor blades are provided in Pound\(^2\) and Hoffman et al.\(^8\)

After lengthy calculation, Eq. (24) becomes
\[
C_d = \sigma a \left( \frac{\theta_{0.5}}{3} \left( -\beta_1 + \frac{3}{2} \mu \lambda \right) + \frac{\theta_{0.5}}{4} (-\beta_1 + \mu \lambda) \right) \nonumber + \frac{3}{4} \Lambda_{\beta_1} + 0 \beta_0 \beta_{1s} - \frac{1}{4} \frac{\mu (\beta_0^2 + \beta_{1s}^2 \right) \tag{27}
\]
The horizontal force coefficient is then
\[
C_H = C_{H0} + C_{Hd} \tag{28}
\]
Measurement of drag force is necessary for knowing the maximum forward speed that the quadrotor can achieve for
a specific angle of attack and rotational speed (RPM). For this purpose, the total drag force is expressed as

$$D_{\text{total}} = T \sin \alpha - H \cos \alpha - D_{\text{parasite}}$$  \hspace{1cm} (29)

where the thrust, $T$, and horizontal force, $H$, are

$$T = C_T \rho (\omega R)^2 4\pi R^2$$  \hspace{1cm} (30)

$$H = C_H \rho (\omega R)^2 4\pi R^2$$  \hspace{1cm} (31)

The parasite drag, $D_{\text{parasite}}$, should be formulated on the basis of the wind tunnel experiment. The total drag coefficient of a quadrotor helicopter is finally formulated as

$$C_D = \frac{D_{\text{total}}}{\rho (\omega R)^2 4\pi R^2}$$  \hspace{1cm} (32)

At a sufficiently high advance ratio, reverse flow is generated in a small circular region on the rotor disk. The diameter of the region of reverse flow corresponds to $\mu$, and it is known that Eqs. (23) and (27) are valid for an advance ratio, $\mu$, less than 0.4 where the effect of reverse flow is negligible.7

### 2.3. Interference Effect

The interference effect on the performance of a quadrotor helicopter can be modeled using the Biot-Savart Law, assuming the rear rotor operates in an additional uniform downwash induced by the front rotor. The induced velocity at the rear rotor is assumed constant over the rotor disc area. Considering a simple horseshoe vortex, as shown in Fig. 5, by calculating the ratio of the downward induced velocity at the center of the rear rotor to the induced velocity at the front rotor is de

$$k = \frac{\tan \alpha}{\kappa}$$  \hspace{1cm} (38)

When $\alpha$ increases, the wake generated by the front rotor is below that generated by the rear rotor. Hence, the interference from the front rotor to the rear rotor becomes smaller, indicated by a reduction in ratio $k$.

Accounting for the interference, the inflow ratio in Eq. (6) becomes

$$\lambda' = \frac{V_{\infty} \sin \alpha + (k + 1)v}{\omega R}$$  \hspace{1cm} (39)

Therefore, the thrust coefficient of the rear rotor is formulated as

Fig. 5. Horseshoe vortex of a tandem configuration.9

Fig. 6. Rotor interference factor as a function of wake skew angle.
\[ C_T = \frac{1}{2} \sigma a \left( \frac{1}{3} B^3 \theta_{\text{avg}} + \frac{1}{2} B \mu^2 \theta_{\text{avg}} - \frac{1}{2} B^2 \lambda \right) \] (40)

3. Experimental Method

An open-type wind tunnel facility is used. The cross-section is an octagon of 190 cm (width) \( \times \) 200 cm (height), and the size of the quadrotor model is 34 cm \( \times \) 8 cm. The flow velocity is varied from 0 m/s to 10 m/s. Prior to the experiment, the cross-sectional distribution of the velocity of the wind tunnel stream was measured. The mean velocity of stream was set at 8.0 m/s. As shown in Fig. 7, the difference between maximum and minimum velocity is 4\% at the maximum.

The platform utilized in this experiment is a DJI-450f quadrotor with a diameter of 0.45 m and weighing 0.9 kg, as seen in Fig. 8. The control signals are generated by an Mbed microcontroller and controlled by a Flysky R/C transmitter. The force sensor is a 3D load cell, as seen in Fig. 9, with several characteristics, including rate capacity (R.C.): \( F_x, F_y: 500 \text{N}, M_z: 10 \text{Nm} \); rate output (R.O.): \( F_x, F_y: 0.5 \text{mV/V}, M_z: 0.5 \text{mV/V} \); non-linear error: \( \pm 0.3\% \) R.O., etc. The airfoil of rotor blade is a thin airfoil with geometry shown in Fig. 10.

A five-hole pitot tube was used to measure the velocity vector of the inflow to the tandem rotor. The 2D pressure coefficients, \( C_{p\text{pit}} \) and \( C_{p\text{yaw}} \), are defined as

\[ C_{p\text{pit}} = \frac{p_5 - p_4}{p_1 - \bar{p}}; \quad C_{p\text{yaw}} = \frac{p_2 - p_3}{p_1 - \bar{p}} \] (41)

where \( p_i \) is the pressure at the \( i \)-th hole, as illustrated in Fig. 11, and

\[ \bar{p} = (p_2 + p_3 + p_4 + p_5)/4 \] (42)

The probe was calibrated to determine its sensitivity to the pitch and yaw angles in a uniform flow in the wind tunnel. As a result, the pressure coefficients versus pitch and yaw angles are plotted in a contour graph shown in Fig. 12, which is used in turn to calculate the pitch and yaw angles of the flow relative to the probe.

Three different experiments, (E1) to (E3), were performed. The first experiment (E1) is illustrated in Fig. 13. The thrust and drag forces applied to a single rotor in a uniform flow were measured using a load cell to estimate the characteristic parameters of the rotor. The angle of attack, \( \alpha \), was kept constant at 0 deg, and the flow velocity, \( V \), was kept at 6 m/s. The rotation speed of the rotor was varied to range from 1000 to 5000 RPM. Among all of the characteristic parameters of the rotor used in Eq. (14), the rotor radius, \( R \), blade number, \( N \), blade chord, \( c \), and solidity, \( \sigma \), have been directly measured as tabulated in Table 1, while the 2D lift slope factor, \( \alpha \), rotor average pitch, \( \theta_{\text{avg}} \), and profile drag coefficient, \( C_{d\ell} \), were determined on the basis of the experiment.
The second experiment (E2) is illustrated in Fig. 14. The thrust and drag forces applied to the rear rotor of a couple in a tandem rotor were measured to investigate the inter-rotor interference effects. For a quadrotor helicopter, two couples of a tandem rotor of the same size are installed on a single plane. This experiment focuses on the interference effects on the tandem rotor couple with no disk overlap.

In the third experiment, (E3), as illustrated in Fig. 15, the thrust and drag forces applied to a quadrotor helicopter are measured in different flight modes, such as hover ($\alpha = 0^\circ$, $V = 6$ m/s), forward flight ($-18^\circ \leq \alpha \leq 0^\circ$), climb ($\alpha = -90^\circ$) and descent ($\alpha = 90^\circ$).

4. Results and Discussion

4.1. Single rotor experiment (E1)

Figure 16 shows the thrust coefficient, $C_T$ of the single rotor at $\alpha = 0^\circ$, $V = 6$ m/s. $C_T$ decreases monotonously with the rotation speed per minute of the rotor, RPM. Equation (14) is rearranged as

$$C_T = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$  \quad (43)

where

$$X_1 = \left( \frac{1}{6} \sigma \beta^3 + \frac{1}{4} \sigma \beta \mu^2 \right); \quad X_2 = -\left( \frac{1}{4} \sigma \beta^2 \lambda \right)$$  \quad (44)

and the unknown parameters to be estimated are

$$\theta_1 = a \theta_{\text{avg}}; \quad \theta_2 = a$$  \quad (45)

The 2D lift slope factor, $a$, and the rotor average pitch angle, $\theta_{\text{avg}}$, are determined as tabulated in Table 2 by fitting Eq. (43) to the experimental data assuming the pitch angle of the blade constant.

Figure 17 shows the drag force. The profile drag coefficient, $C_{d_p}$, is estimated as tabulated in Table 2 by fitting
The interference from the front rotor to the rear rotor is presented by increasing the inflow ratio, \( \lambda \), of the rear rotor, which can be computed using Eq. (39). Increasing \( \lambda \) results in decreasing the thrust coefficient.

For given \( V \) and \( \alpha \), when RPM increases, \( \mu \) decreases. The wake skew angle of the front rotor, \( \gamma \), therefore increases, resulting in a reduction in the interference factor, \( k \) (as seen in Fig. 6). Hence, the interference effect of the front rotor on the rear rotor decreases as RPM increases due to the front rotor wake having less influence.

In Fig. 19, the degree of similarity in flow distribution between the front rotor and the rear rotor increases as RPM increases. The degree of similarity is represented by the correlation coefficient, \( Corr \), computed using Eq. (46). The correlation coefficient increases as RPM increases, becoming 0.84, 0.85 and 0.88, which corresponds to RPM = 2500, 3600 and 4500, respectively.

\[
Corr = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{j=1}^{n} (y_j - \bar{y})^2}}
\]  

where \( x \) and \( y \) are the vector of angle of attack of the front rotor and rear rotor, respectively.

Figure 21 shows the influence of front rotor on rear rotor—flow distribution (\( V = 6 \text{ m/s}, \alpha = 0^\circ, l/R = 2.83, \text{RPM} = 4500 \)).

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\]  

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4.3. Quadrotor aerodynamics (E3)

A quadrotor includes two couples of tandem rotors. The rear rotors are under the interference effect of the front rotors only during forward flight, when the interference effects should be taken into account to calculate $CT$. In what follows, for the couple of the front rotors, Eq. (14) is utilized as the theoretical curve, while Eq. (40) is used for the couple of the rear rotors. The thrust coefficient of a quadrotor helicopter is the average value of the four rotors

$$CT = \frac{1}{4} \sum_{i=1}^{4} CT_i$$  \hspace{1cm} (47)

The aim of this section is to show the validity of Eq. (47) based on the wind tunnel experiment.

4.3.1. Hovering

The theoretical thrust when hovering is calculated using

$$T = CT \rho A (\omega R)^2 = kT \omega^2$$  \hspace{1cm} (48)

applying $CT$ formulated in Eq. (14). The quadrotor hovering thrust measured is shown in Fig. 23. The theoretical line agrees well with the experimental results. Hovering, $CT$, at $\alpha = 0^\circ$ in the presence of horizontal wind is shown in Fig. 24. $CT$ increases with $V$ due to the increase in $\mu$. The theory agrees well with experimental data.

4.3.2. Climbing

$CT$ in the climb mode ($\alpha = -90^\circ$) is shown in Fig. 25. $CT$ decreases as $V$ increases and RPM decreases due to the increase in $\lambda$. The theoretical curves agree well with the experimental results for a $\lambda$ smaller than 0.15, corresponding to $V = 2$, $V = 4$, $V = 6$ with RPM $\geq 4000$. A large discrepancy between theory and experiment happens at high-$\lambda$ (dotted region in Fig. 25). Figure 26 shows that the theoretical results agree well with the experimental results at $\lambda < 0.15$.

There is a possibility of flow separation or stall under high
climb rate conditions and a high angle of attack. Therefore, there is a discontinuity in the mass flow through the rotor disk. In the climb mode at a $\lambda$ larger than 0.15, it is better to use the empirical model of induced velocity rather than theoretical models (momentum theory).

### 4.3.3. Forward flight

Figures 27–30 show $C_T$ during forward flight. The theoretical curves are very close to the experimental results. The interference effect can be accurately accounted for using Eq. (40) with $l/R = 2.83$. The ratio $C_{T,\text{rear}}/C_{T,\text{front}}$ is 0.89 applying the experimental data and 0.86 applying the theory, respectively. For given $V$ and $\alpha$, $C_T$ decreases with RPM. This originates from the decrease in $\mu$. For given $V$ and RPM, $C_T$ decreases with $|\alpha|$. For given RPM and $\alpha$, $C_T$ increases with $V$ due to the increase in $\mu$. In Fig. 30, at $\alpha = -18^\circ$ (high angle of attack), the theoretical curve agrees poorly with the experimental results for low-RPM (dotted region in Fig. 30). This is because the blade element theory is valid only for small values of $\alpha$ and $\lambda$.

The drag force is the sum of the profile and induced drag for the rotors, which is computed analytically using Eqs. (23)–(32), and parasitic drag for the body, which is specified by measuring without rotating the rotors in the wind tunnel. The parasite drag measured is as shown in Fig. 31. Because the body of the quadrotor model is in a compact capsule-like shape, it is natural that the parasite drag of the body is insensitive to $\alpha$.

$C_D$ during forward flight is shown in Figs. 32–35. The theory agrees well with the experimental results at a low advance ratio, $\mu$, which corresponds to the low airspeed and high RPM region. The reason for this is validating the theory at $\mu < 0.4$, as shown in Fig. 36. For $\mu \geq 0.4$ (corresponding to dotted region where $V = 8$, 10 and RPM $\leq 2000$ in Figs. 32–35), there exists a difference between theoretical and experimental results due to the reverse flow effect.

The maximum speed the quadrotor can achieve corresponds to positive drag coefficient. Those are 4, 6, 8 and 10 for angle of attack $\alpha = -5^\circ$, $-10^\circ$, $-15^\circ$ and $-18^\circ$, respectively.
4.3.4. Descent

Since a rotor working in the vortex ring state corresponds to the condition $0 < V_d < 2v_h$, where there is $V_d$ descent rate and $v_h$ velocity during hovering, no solution exists for momentum theory. The induced velocity in the vortex ring state is computed using an empirical model

$$v = v_h \left( k_0 + k_1 \frac{V_d}{v_h} \right)$$

where, the coefficients $k_0 = -2.896$ and $k_1 = 2.844$ are achieved by fitting the theoretical results to experimental results in Fig. 37.

The thrust coefficient during descent is computed using Eq. (14) with a modification in the inflow ratio

$$\lambda_d = \frac{V_d - v}{\omega R}$$

The results corresponding to the theory (using the empirical model of induced velocity) and experiment are shown in Fig. 37.
5. Conclusion

The wind tunnel data and theoretical modeling for the aerodynamic performance of a quadrotor helicopter have been presented. The agreement between experimental and theoretical modeling was good throughout the angle of attack range investigated, 0° to 18°, and airflow velocity of 0 to 10 m/s. Therefore, the experimental results have proven the effectiveness of the theoretical approach for aerodynamic modeling.

Additionally, the influence caused by the interference of multiple rotor configurations has been corroborated. The thrust coefficient of the rear rotor is lower than that of the front rotor by a maximum of 11%. The reduction in rear rotor thrust due to the interference of front rotors degrades hovering performance. This investigation regarding influence is also useful for choosing the distance between rotors when designing multi-rotor configurations.

In the process of doing this research, many useful things were learned; especially, how to enhance the systematic way of studying and implementing academic theories in practical problems. Our future work, a real flight test, will be conducted to fully investigate the aerodynamics of quad-rotor helicopters and make comparisons between real flight test results and wind tunnel test results.

Acknowledgments

We gratefully acknowledge financial support from the Japanese Government, which sponsored our research under a MEXT scholarship. A sincere thanks to Mr. Yang Sida as well for his support during the experiment.

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