Meson wave function from holographic models

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We consider the light-front wave function for the valence quark state of mesons using the AdS/CFT correspondence, as has been suggested by Brodsky and Téramond. Two kinds of wave functions, obtained in different holographic Soft-Wall models, are discussed.

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\section{I. INTRODUCTION}

The hadronic wave function in terms of quark and gluon degrees of freedoms plays an important role in QCD process predictions. For example, knowledge of the wave function allows to calculate distribution amplitudes and structure functions or conversely these processes can give phenomenological restrictions on the wave functions.

In principle the Bethe-Salpeter approach \cite{1} and discrete quantization in the light-front formalism \cite{2} allow to obtain hadronic wave functions but in practice several problems present to realize this \cite{3,4}. Therefore approximate solutions for hadronic bound states are usually considered using in a first step specific quarks models to obtain the valence quark wave function.

There are several non-perturbative approaches to obtain properties of distribution amplitudes and/or hadronic wave functions from QCD, and now we have possibility to include techniques based on the Anti-de Sitter space/conformal field theory (AdS/CFT) correspondence.

Although a rigorous QCD dual is unknown, a simple approach known as Bottom-Up allows to built models that have some essential QCD features, including counting rules at short and confinement at long distances. This model has been successful in several QCD applications such as hadronic scattering processes \cite{5,6,7,8}, hadron spectrum \cite{9,10,11,12,13}, hadronic couplings and chiral symmetry breaking \cite{14,15,16}, quark potentials \cite{17,18,19} and hadron decays \cite{20}.

Together with these applications it is possible to set up a mapping between specific properties of the AdS description for hadrons and the Hamiltonian formulation for quantized QCD in the light-front formalism. Latter approach allows to obtain an excellent first approximation to the valence wave function for mesons \cite{21,22}. Wave functions obtained using the AdS/CFT correspondence can be used as an initial ansatz for a variational treatment or as basis states to diagonalize the light-front QCD Hamiltonian.

In this work meson wave functions obtained in the context of AdS/CFT ideas \cite{21,22} are studied considering two kinds of holographic Soft-Wall models. First we consider the more usual model with a quadratic dilaton \cite{10,14,22}. Then we discuss predictions of a recent model which considers a logarithmic dilaton as suggested by Einstein’s equations for an AdS metric. It also includes anomalous dimensions \cite{13} and allows to reproduce the Regge behavior even in the baryonic sector.

The work is structured as follows. Sec. II is devoted to the extraction of wave functions for scalar/pseudoscalar mesons using the two holographic models. In Sec. III we concentrate on the pion wave function discussing the adjustment of the model parameters. Distribution amplitudes and parton distributions for the valence state are calculated in both models. In the pion case we consider both current and constituent quark masses. In Sec. IV we calculate decay constants in the simplified case when the valence component is dominant. Conclusions are presented in Sec. V.

\section{II. MESON WAVE FUNCTION IN HOLOGRAPHICAL MODELS}

The comparison of form factors calculated both in the light-front formalism and in AdS offers the possibility to relate AdS modes to light-front wave functions (LFWF) \cite{21,22}. Below we briefly discuss the derivation of this matching procedure.

In the light-front formalism the electromagnetic form...
A factor of pion can be written as \[ F(Q^2) = 2\pi \int_0^1 dx \frac{1-x}{x} \int d\zeta J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta), \]

where \( \tilde{\rho}(x, \zeta) \) is the effective transverse distribution of partons; \( Q^2 \) is the spacelike momentum transfer squared; \( J_0 \) is the Bessel function. Here we introduced the variable

\[
\zeta = \sqrt{\frac{x}{1-x}} \sum_{j=1}^{n-1} x_j b_{\perp j},
\]

which represents the \( x \)-weighted transverse impact coordinate of the spectator system.

On the other side the corresponding expression for scalars in AdS with a dilaton \( \varphi(z) \) is

\[
F(Q^2) = \int_0^z dz \Phi(z) J_\kappa(Q^2, z) \Phi(z),
\]

where \( \Phi(z) \) corresponds to modes that represent hadrons, \( J(Q^2, z) \) is the dual mode to the electromagnetic current, and the metric considered is

\[
ds^2 = \frac{R^2}{z^2} \eta_{\mu\nu} dx^\mu dx^\nu, \quad \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1),
\]

where \( z \) is the holographic coordinate and \( \kappa \) is the scale parameter characterizing the dilaton field. An important step is to set up the electromagnetic current as

\[
J(Q^2, z) = \int_0^1 dx f(x) J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right),
\]

Putting \( z = \zeta \) and comparing Eqs. 1 and 3 we get

\[
\tilde{\rho}(x, \zeta) = \frac{x f(x)}{1-x} \frac{|\Phi(\zeta)|^2}{2\pi \zeta}.
\]

Finally, considering the case with two partons \( q_1 \) and \( \bar{q}_2 \)

\[
\tilde{\rho}_{n=2}(x, \zeta) = \frac{|\tilde{\psi}_{q_1\bar{q}_2}(x, \zeta)|^2}{1 - \frac{1}{A^2}},
\]

where \( \zeta = x(1-x) b_\perp^2 \) and \( A \) is the normalization constant, we obtain the relation between the AdS modes and the meson LFWF \( \tilde{\psi}_{q_1\bar{q}_2}(x, \zeta) \)

\[
|\tilde{\psi}_{q_1\bar{q}_2}(x, \zeta)|^2 = A^2 x(1-x) f(x) \left| \frac{\Phi(\zeta)}{2\pi \zeta} \right|^2 .
\]

Here \( A \) is constrained by the probability condition

\[
P_{q_1\bar{q}_2} = \int_0^1 dx \int d^2 b_\perp |\tilde{\psi}_{q_1\bar{q}_2}(x, b_\perp)|^2 \leq 1
\]

with \( P_{q_1\bar{q}_2} \) being the probability of finding the valence Fock state \( |q_1\bar{q}_2\rangle \) in the meson \( M \). Note, in the case of massless quarks we have \( A = \sqrt{P_{q_1\bar{q}_2}} \), while this is not the case for massive quarks (see discussion in Sec. II.A). Next we consider two kinds of holographic models (Model 1 and Model 2) and their respective wave functions.

**A. Model 1**

Model 1 is based on the Schrödinger equation

\[
-\frac{d^2}{dz^2} + \frac{1 - 4L^2}{4\zeta^2} + \kappa \zeta^2 + 2\kappa_1^2(L + S - 1) \Phi_1(\zeta) = M_1^2 \Phi_1(\zeta),
\]

for the AdS modes \( \Phi(\zeta) \) that describe hadrons with integer spin \( S \) and the mass spectrum

\[
M_1^2 = 4\kappa_1^2 \left( n + L + \frac{S}{2} \right),
\]

where \( \kappa_1 \) is the coupling constant.
where \( n \) and \( L \) are the radial and orbital quantum numbers. Here subscript “1” indicates the solutions of Model 1.

In this model the function \( f(x) \) in matching condition (8) is fixed as \( f(x) = 1 \) for large values of \( Q^2 \gg 4\kappa^2 \). In this case the current \( J_2(Q^2, z) \) decouples from the dilaton field \( \phi \). The examples considered in this work correspond to mesons with \( n = L = 0 \), although both for scalars and vectors we find

\[
\Phi_1(\zeta) = \kappa_1 \sqrt{\zeta} \exp^{-\frac{1}{2}\kappa_1^2\zeta^2} \sim \sqrt{\zeta} \exp^{-\frac{1}{2}\kappa_1^2\zeta^2} . \tag{12}
\]

Using Eq. (12) and keeping in mind that \( \zeta^2 = x(1-x)b_1^2 \), the meson LFWF of this model is

\[
\tilde{\psi}_{q_1, q_2}(x, b_1) = \frac{k_1 A_1}{\sqrt{\pi}} \sqrt{x(1-x)} \exp\left(-\frac{1}{2}\kappa_1^2 x(1-x)b_1^2\right) . \tag{13}
\]

The wave function (13) does not consider massive quarks. We include the quark masses following the prescription suggested by Brodsky and Téramond [26]. First one should perform the Fourier transform of (13)

\[
\psi_{q_1, q_2}(x, k) = \frac{4\pi A_1}{\kappa_1 \sqrt{x(1-x)}} \exp\left(-\frac{k^2}{2\kappa_1^2 x(1-x)}\right) . \tag{14}
\]

In a second step the quark masses are introduced by extending the kinetic energy of massless quarks with \( K_0 = \frac{k_1^2}{x(1-x)} \) to the case of massive quarks:

\[
K_0 \rightarrow K = K_0 + \mu_{12}^2 , \quad \mu_{12}^2 = \frac{m_1^2}{x} + \frac{m_2^2}{1-x} . \tag{15}
\]

Note, the change proposed in (15) is equivalent to the following change in (10)

\[
-\frac{d^2}{d\zeta^2} \rightarrow -\frac{d^2}{dz^2} + \mu_{12}^2 . \tag{16}
\]

Finally we obtain

\[
\psi_{q_1, q_2}^{(1)}(x, k) = \frac{4\pi A_1}{\kappa_1 \sqrt{x(1-x)}} \exp\left(-\frac{k^2}{2\kappa_1^2 x(1-x)} - \frac{\mu_{12}^2}{2\kappa_1^2}\right) . \tag{17}
\]

Note, in the case of massive quarks the normalization constant fulfills the relation

\[
A_1 = \sqrt{P_{q_1 q_2}} \left( \int_0^1 dx e^{-\mu_{12}^2/z_2}/\kappa_1^2 \right)^{-1/2} \tag{18}
\]

and \( A_1 \rightarrow \sqrt{P_{q_1 q_2}} \) when \( m_{1,2} \rightarrow 0 \).

### B. Model 2

Model 2 has originally been developed in Ref. [13]. It is based on the following equation of motion for the AdS modes

\[
\partial_\zeta^2 \varphi(\zeta) - \frac{2 - \beta}{\zeta} \partial_\zeta \varphi(\zeta) + \left(M_2^2 - \frac{m_2^2 R^2}{\zeta^2}\right) \varphi(\zeta) = 0 , \tag{19}
\]

where for mesons we have

\[
m_2^2 R^2 = (3 + L - S + \kappa_2^2)(L - S + \beta + \kappa_2^2) . \tag{20}
\]

Here subscript “2” indicates the solutions of the Model 2. From (19) we get the mass spectrum

\[
M_2^2 = 4\kappa_2^2 \left[n + L + \left(\frac{2 + \beta}{2} - S\right)\right] , \tag{21}
\]

where \( \beta = -3 \) is the value for scalar mesons and \( \beta = -1 \) for vector mesons [13]. To have consistency with the definition of the form factor of Eq. (5) the equation (19) should be changed into a Schrödinger type equation of

\[
\left[-\frac{d^2}{d\zeta^2} + \frac{4m_2^2 R^2 + \beta^2 - 6\beta + 8}{4\zeta^2}\right] \Phi_2(\zeta) = M_2^2 \Phi_2(\zeta) , \tag{22}
\]
In this model we have the condition between the LFWF and AdS modes reads by means of the following transformation
\[ \varphi(\zeta) = e^{(1-\beta/2)\ln\zeta} \Phi_2(\zeta). \] (23)

In this model we have \( f(x) = 2x \) and the matching condition between the LFWF and AdS modes reads
\[ |\psi_{q_1,q_2}^{(2)}(x,\zeta)|^2 = 2 A_2^2 x^2 (1-x) \frac{\Phi_2(\zeta)^2}{2\pi \zeta}. \] (24)

Again we restrict to the ground state case — \( n = L = 0 \) and as AdS mode \( \Phi_2(\zeta) \) similar to the one of Model 1:
\[ \Phi_2(\zeta) \sim \zeta^{-\frac{\alpha_+}{2}}. \] (25)

Finally applying the Brodsky and Téramond prescription, the meson momentum space LFWF including massive quarks is
\[ \psi_{q_1,q_2}^{(2)}(x,k_\perp) = \frac{4\pi A_2}{\kappa_2} \sqrt{\frac{2}{1-x}} \exp \left( -\frac{k_\perp^2}{2\kappa_2^2 x(1-x)} + \frac{\mu_2^2}{2\kappa_2^2} \right), \] (26)
where \( A_2 \) is the normalization constant constrained by the probability condition \( \Phi_2(\zeta)^2 \) in analogy to \( A_1 \).

III. EXAMPLE I: THE PION

A. Fixing the parameters

The wave functions we consider depend on parameters \( (A_i,m_{1,2},\kappa_i) \) which must be fixed. As a first application we consider some of the fundamental properties of the pion: leptonic and two-photon decay constants, distribution quantities. We work in the isospin limit, supposing that the masses of \( u \) and \( d \) quarks are equal to each other: \( m = m_u = m_d \). In this case we have a set of three free parameters \( (A_i,m,\kappa_i) \) which is the same number of parameters considered in other models \([27]\).

The two conditions related to the decay amplitudes for \( \pi \to \mu \nu \) and \( \pi^0 \to \gamma \gamma \) \([28]\) read
\[ \int_0^1 dx \int d^2k_\perp \psi_{q\pi}(x,k_\perp) = \frac{F_\pi}{2\sqrt{3}} \] (27)
and
\[ \int_0^1 dx \psi_{q\pi}(x,k_\perp = 0) = \frac{\sqrt{3}}{F_\pi} \] (28)
where \( F_\pi = f_\pi/\sqrt{2} \approx 92.4 \text{ MeV} \) is the pion leptonic decay constant. Note, the second condition \([28]\) is the low-energy theorem relating the two-photon \( g_{\pi\gamma\gamma} \) and leptonic \( F_\pi \) decay constants as \( g_{\pi\gamma\gamma} = 1/(4\pi^2 F_\pi) \approx 0.274 \text{ GeV}^{-1} \).

On the other side, the average transverse momentum squared of a quark in the pion \( \langle k_{\perp}^2 \rangle_\pi \) is about \((300 \text{ MeV})^2 \) \([31]\). The average transverse momentum squared of a quark in the pion valence state is defined by
\[ \langle k_{\perp}^2 \rangle_\pi = \frac{1}{P_{\pi\gamma}} \int dx \int d^2k_\perp \frac{k_\perp^2}{16\pi^3} |\psi_{q\gamma}(x,k_\perp)|^2, \] (29)
which must be higher than \( \langle k_{\perp}^2 \rangle_\pi \). For this reason we consider a value of several hundreds of MeV for \( \sqrt{\langle k_{\perp}^2 \rangle_\pi} \). This can be used as a third restriction. When fixing the parameters we consider two cases for each wave function \([17]\) and \([26]\), current and constituent quark masses. The values used are \( 4 \text{ MeV} \) for current masses and \( 330 \text{ MeV} \) for constituent masses.

Since quarks masses are introduced in advance, the remaining parameters \( A_i \) and \( \kappa_1 \) or \( \kappa_2 \) can be fixed using...
and (28) with the value of $F_\pi = 92.4$ MeV. Then with the fixed parameters $A_1, m, \kappa$, we predict $\sqrt{\langle k_\perp^2 \rangle}_{\Sigma\Sigma}$ and the probability $P_{\Sigma\Sigma}$. Table I gives the values for $A_{1,2}$ and $\kappa_{1,2}$ including the predictions for $\sqrt{\langle k_\perp^2 \rangle}_{\Sigma\Sigma}$ and $P_{\Sigma\Sigma}$. One can see that our results for $\sqrt{\langle k_\perp^2 \rangle}_{\Sigma\Sigma}$ and $P_{\Sigma\Sigma}$ are in agreement with the predictions of Ref. [27]: $\sqrt{\langle k_\perp^2 \rangle}_{\Sigma\Sigma} \approx 356$ MeV and $P_{\Sigma\Sigma} \approx 0.296$.

The parameters $\kappa_{1,2}$ define the holographic model considered in Refs. [13, 22] and both are related to the Regge slope. Thus in principle both quantities could be fixed by spectral data. Unfortunately the pion mass is an exception since it falls outside the Regge trajectories. Therefore $\kappa_{1,2}$ have been usually fixed by using form factors [13, 22]. The values obtained in the present work differ somewhat from those values, which is understandable since the $\kappa_1$ and $\kappa_2$ found previously were obtained using (3), the form factor in AdS, which when compared with the light front expression gave [3]. Nevertheless, the wave functions (17) and (26) correspond to the case with only two quarks, and we therefore should expect a small change in the $\kappa_{1,2}$ values.

**B. Pion Distribution Amplitude**

The meson distribution amplitude is calculated using (32)

$$\phi(x, q) = \int d^2k_\perp 16\pi^3 \psi_{\text{val}}(x, k_\perp).$$ (30)

We remind that the pion $|\psi\rangle$ can be expanded into Fock states $|\psi\rangle = a_1|qq\rangle + a_2|q\bar{q}g\rangle + a_3|q\bar{q}gg\rangle + \ldots$. For large values of $q^2$ the dominant term is the first one and since our wave functions were obtained considering (7), which corresponds to the $qq$ configuration, we can calculate $\phi(x) \equiv \phi(x, Q \to \infty)$.

Using (17) and (26) we get

$$\phi_1(x) = \frac{A_1\kappa_1}{2\pi} \sqrt{x(1-x)} \exp \left(-\frac{m^2}{2\kappa_1^2 x(1-x)}\right),$$ (31)

and

$$\phi_2(x) = \frac{A_2\kappa_2}{2\pi} x \sqrt{2(1-x)} \exp \left(-\frac{m^2}{2\kappa_2^2 x(1-x)}\right).$$ (32)

In Fig.4 both expressions are compared for current (c) and constituent (cs) quark masses to the prediction of PQCD using $\phi(x, Q \to \infty) = \sqrt{3}F_\pi x(1-x)$ [32]. Fig.4 shows that increasing quark masses reduces the differences between the two variants of LFWFs. Knowing the distribution amplitudes, it is possible to calculate the moments. Taking $\xi = 1 - 2x$ we have

$$\langle \xi^N \rangle = \frac{1}{\int_{-1}^{1} d\xi \phi(\xi)} \int_{-1}^{1} d\xi \xi^N \phi(\xi).$$ (33)

Table II contains a summary of the moments up to $\langle \xi^4 \rangle$.

**C. Parton distributions**

If the LFWF has the form

$$\psi_{q\bar{q}}(x, k_\perp) = \eta(x) \exp \left(\frac{k_\perp^2}{2\lambda^2 x(1-x)}\right),$$ (34)

then the parton distribution is given by [34]

$$f(x) = \frac{x(1-x)\lambda^2}{16\pi^2} \eta^2(x).$$ (35)

The LFWFs obtained from Models 1 and 2 have the form considered in [34] and then the two-body contribution to the parton distributions can be calculated in a direct way. In Fig.5 we display the product $xf(x)$ for both models again using current and constituent quark masses in the LFWF. We use the same parameters as in Table I.

In principle, contributions from higher Fock states should be added because they are not necessarily small. In fact, in the pion case that we are discussing here, the valence state component is around 25 percent as can be seen in Table I or for example in Refs. [27, 28].
TABLE I: Parameters defining LFWF given by Eqs. \(17\) and \(20\) and predictions for \(\sqrt{\langle k_z^2 \rangle_{\pi\pi}}\) and \(P_{\pi\pi}\). \[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Model} & \psi(x,k_{\perp}) & m \, (\text{MeV}) & \kappa \, (\text{MeV}) & \sqrt{\langle k_z^2 \rangle_{\pi\pi}} \, (\text{MeV}) & P_{\pi\pi} \\
\hline
1 & \psi_{1c} & 4 & 0.452 & 951.043 & 388.319 & 0.204 \\
& \psi_{1c}\times & 330 & 0.924 & 787.43 & 356.478 & 0.279 \\
2 & \psi_{2c} & 4 & 0.486 & 921.407 & 376.222 & 0.236 \\
& \psi_{2c}\times & 330 & 0.965 & 782.128 & 353.877 & 0.299 \\
\hline
\end{array}
\]

FIG. 5: Valence parton distribution \(x f(x)\) according to the LFWF considered in this work. The right graph corresponds to model 1 and the left graph is for model 2. In both cases the dashed line corresponds to the case with current masses, while the solid line correspond to the constituent mass case. The parameters involved are the same as displayed in Table I.

TABLE II: First moments of the distribution functions \(\langle \xi^N \rangle\) calculated using \(\phi_{\text{PHQCD}}\) and \(\phi\), given explicitly by \(31\) and \(32\), for \(m = 4\) MeV and \(m = 330\) MeV. For \(\phi_{2c,\times}\) we take \(m = 300\) MeV, which shows that odd moments are reduced when the quark mass quarks increases.

| \(\phi\)  | \(\langle \xi \rangle\) | \(\langle \xi^2 \rangle\) | \(\langle \xi^3 \rangle\) | \(\langle \xi^4 \rangle\) | \(\langle \xi^5 \rangle\) |
|----------|----------------|----------------|----------------|----------------|----------------|
| \(\phi_{\text{PHQCD}}\) | 1 | 0 | 0.2 | 0 | 0.086 |
| \(\phi_{1c}\) | 1 | 0 | 0.250 | 0 | 0.125 |
| \(\phi_{2c}\) | 1 | 0.143 | 0.238 | 0.073 | 0.116 |
| \(\phi_{1c}\times\) | 0 | 0.186 | 0 | 0.073 |
| \(\phi_{2c}\times\) | 1 | 0.102 | 0.179 | 0.040 | 0.068 |
| \(\phi_{2c,\times}\) | 1 | 0.106 | 0.187 | 0.044 | 0.073 |

IV. EXAMPLE II: DECAY CONSTANTS

Now we are in the position to calculate leptonic couplings of pseudoscalar \((f_P)\) and vector \((f_V)\) mesons which are given in our approach by

\[
f_P = f_V = 2\sqrt{6} \int_0^1 dx \int \frac{d^2k_{\perp}}{16\pi^3} \psi_{\pi\pi}(x,k_{\perp}). \tag{36}
\]

We use experimental values for the decay constants and the probability condition

\[
P_{\pi\pi} = \int_0^1 dx \int \frac{d^2k_{\perp}}{16\pi^3} |\psi_{\pi\pi}(x,k_{\perp})|^2 \leq 1, \tag{37}
\]

where the equality holds for the case when the valence part dominates. This procedure allows to fix the parameters \(\kappa_{1,2}\) and the normalization constants \(A_{1,2}\).

Holographic models usually give a relation between \(\kappa_{1,2}\) and the Regge slope fixed by spectroscopic data. Thus the only free parameter \(A_{1,2}\) can be fixed by the normalization condition. As an example we consider the decay constant for kaons and \(J/\psi\) assuming the valence contribution to be dominant, i.e we use \(37\) with \(P_{\pi\pi} = 1\). The quark masses used are

\[
m_u = m_d = 330 \text{ MeV},
m_s = 500 \text{ MeV},
m_c = 1500 \text{ MeV}.
\]

As already mentioned, the parameters \(\kappa_{1,2}\) can be fixed by using Regge slope data \(29, 30\): for kaon we take \(\kappa_1 = \kappa_2 = 524\) MeV \(29\), while for \(J/\psi\) we use \(\kappa_1 = \kappa_2 = 894\) MeV.

Now we can calculate the decay constants of \(K\) and \(J/\psi\) mesons. In Model 1 we obtain: \(f_K = 156.01\) MeV and \(f_{J/\psi} = 226.68\) MeV. Our predictions in Model 2 are: \(f_K = 156.35\) MeV and \(f_{J/\psi} = 224.97\) MeV. Our results for the \(\pi\) and \(K\) meson decay constants in both models are close to the experimental values of 155.5 and 277.6 MeV, respectively.

Further applications of the approach considered here to the mass spectrum and decay constants of light and heavy hadrons will be considered in Ref. \(37\).
V. CONCLUSIONS

We have considered two kinds of wave functions for mesons in the light-front formalism obtained by the AdS/CFT correspondence with two soft wall holographic models. By identifying in the momentum space wave function the kinetic energy in the massless case we could introduce the quark mass dependence as suggested by Brodsky and Teramond [26]. Both wave functions have a different $x$ dependence, which is less pronounced when the quark masses are increased as can be seen in Figs.1-3.

If we restrict ourselves to pions, there is a singularity in (26) which does not appear in (17), although this is reduced when using constituent masses. One motivation to use (26) is that it was obtained from a more general holographic soft wall model than the one that considers a quadratic dilaton.

But when other mesons are considered, it is important to note that the parameters $\kappa_{1,2}$ used in the holographic models can be fixed by spectroscopic data, since these parameters are related to the Regge trajectory. Taking quark masses as initial input only one parameter remains (the normalization constant $A_{1,2}$) which can be fixed by the normalization condition.

Due to the importance of the hadronic wave function in QCD the versions considered in this work represent a clear example of the usefulness of the AdS/CFT ideas in QCD applications. These wave functions can be used as initial ansatz in variational treatments or as a first step in order to diagonalize the light front QCD Hamiltonian.

Another aspect that was not considered here is related to the fact that the AdS modes dual to mesons have a dependence on $n$ and $L$ [13, 27], the radial and angular quantum numbers respectively. Thus in principle it should be possible to obtain LFWFs for radial and angular excitations also. The Gauge/Gravity dualities offer an interesting opportunity to consider different meson excitations and in future work we plan to see whether these models reproduce the corresponding data in these cases.

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