Research on design method of time-varying uncertainty of bolted connections

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Abstract. For the change of uncertainty caused by the influence of time on the bolt connection during use, this paper is based on a time-varying uncertainty design method for several parameters that change with time. The method consists of stochastic differential equations. The bolt-on time-varying design combines mathematical models of the distribution parameters of multivariate random variables. The design method of bolting connection with uncertainties of various influencing factors with time is proposed, and the time-varying design of tightly connected bolts subjected to axial working load is taken as an example. The method considers the dynamic changes of the stress and the allowable stress of the bolted joint, so that the safety of the bolted product is more accurately evaluated for dynamic reliability. Through this method, combined with the set initial values of the parameters and the historical data obtained by the test, the reliability of a certain time can be calculated. The design of the example uses Matlab to simulate the data, and the reliability of the specified time is obtained, which is the test of the bolting result. And assessment provides an important reference.

1. Introduction
The calculation of bolt strength at home and abroad has a very strong research background, especially in developed countries such as Germany and Japan, which have been leading the research and application fields such as bolting. The calculation method of the stress cross-sectional area and the bearing area of the threaded coupling is given, and the basic criteria and relationship of the threaded fastener are given. However, the strength of the joint of the bolt is inevitably changed with time. This uncertainty factor is often not taken into consideration, and it is easy to cause the calculation to be inaccurate. Moreover, the problem that the strength and stress of the bolted joints change over time has been neglected. In practical applications, the strength and working stress state of the bolted joint is a process that evolves over time during its useful life. Therefore, the reliability of the bolted connection changes dynamically with time, and the chronological order in which the uncertainty occurs has a different influence on the reliability. Recently, research experts and scholars have made preliminary explorations on the time-varying reliability and time-varying design methods of mechanical parts and mechanical structures from various aspects [1, 7].
At present, considering the influence of time-varying uncertainty factors, there is no new theory for the time-varying design method of bolted joints. At present, domestic and foreign expert researchers have studied the influence of uncertainties on some important instruments such as airplanes, automobiles and trains and high-speed trains [8, 10].

As long as the safety factor of a mechanical part is greater than a certain value according to the actual regulations, it is considered safe [11]. This has not always solved the value of the reliability or failure probability of the products involved. At present, the stress and strength of mechanical components that people think and use are multi-valued and have a certain distribution state [12]. It is generally assumed that the intensity distribution and the stress distribution are normally distributed, and the intensity mean and the stress mean are the same, and the average safety factor value is also the same. This method takes into account the working load of the bolted connection and the randomness of the bolt tightening condition and the bolt damage process, making the bolted joint safe or ineffective. That is to say it depends on the large standard deviation of the intensity and stress distribution. This method is more in line with the specific practice of mechanical engineering and is now widely used [13-16]. This paper is based on a time-varying uncertainty design method for several parameters that varies with time. The method is a bolt-join time-varying design of stochastic differential equations, combined with a mathematical model of the distribution parameters of multivariate random variables. The design method of bolting connection with uncertainties of various influencing factors changes with time is established. The time-varying uncertain mathematical model and the pre-tightening force of tightly connected bolts subjected to axial working load and preloading force are established. The time-varying uncertainty model of the tightly connected bolts is illustrated by a single axial load and a time-varying design of the bolts. This method not only considers the influence of the uncertainty factor in the use of the product, but also considers the reliability of the product over time. It can calculate the reliability of a certain time in advance, and provide reference for the maintenance and maintenance of the actual situation. Good application prospects.

2. Mathematical model for calculating time-varying uncertainty

2.1. Drift rate calculation A subsection

Assume that the requirement for the part is 1a later reliability is \( R(t) \), Stress and strength are independent random variables, known \((\lambda_\sigma, \delta_\sigma)\) and \((\lambda_\sigma, \delta_\sigma)\). We can find the design parameters relationship between \( R(0) \) and \( S(0) \). Because reliability is \( R(t) \), so we can ask for \( Z_{R(t)} \) and because

\[
\mu_{\ln[S(t)]} - \mu_{\ln[\sigma(t)]} = -Z_{\mu_\sigma(t)} \sqrt{\sigma_{\ln[\sigma(t)]}^2 + \sigma_{\ln[S(t)]}^2},
\]

among them, \( \sigma_{\ln[S(t)]} = \delta_S \sqrt{t}, \sigma_{\ln[\sigma(t)]} = \delta_\sigma \sqrt{t} \). So we can ask for the

\[
\mu_{R(t)} - \mu_{S(t)}:
\]

\[
\frac{d}{dt} \mu_{\ln[S(t)]} = \ln S(0) + \left(\frac{\lambda_\sigma}{2} - \frac{\delta_S^2}{2}\right)t,
\]

\[
\frac{d}{dt} \mu_{\ln[\sigma(t)]} = \ln \sigma(0) + \left(\frac{\lambda_\sigma}{2} - \frac{\delta_\sigma^2}{2}\right)t.
\]

Thus

\[
\mu_{\ln[S(t)]} - \mu_{\ln[\sigma(t)]} = \left[ \ln S(0) + \left(\frac{\lambda_\sigma}{2} - \frac{\delta_S^2}{2}\right)t \right] - \left[ \ln \sigma(0) + \left(\frac{\lambda_\sigma}{2} - \frac{\delta_\sigma^2}{2}\right)t \right] =
\]

\[
-Z_{\mu_\sigma(t)} \sqrt{\sigma_{\ln[\sigma(t)]}^2 + \sigma_{\ln[S(t)]}^2} - \left(\frac{\lambda_\sigma}{2} - \frac{\delta_S^2}{2}\right) - \left(\frac{\lambda_\sigma}{2} - \frac{\delta_\sigma^2}{2}\right) \right]t
\]

So
\[
\ln \left( \frac{S(0)}{\sigma(0)} \right) = \exp \left\{ -Z_{\alpha(0)} \sqrt{\beta^2_{\alpha(0)} + \beta^2_{\beta(0)}} - \left( \lambda_0 - \frac{1}{2} \delta_0^2 \right) - \left( \lambda_0 - \frac{1}{2} \delta_0^2 \right) \right\} 
\]

(4)

2.1.1. Drift rate calculation. \( \lambda \) is the observed value obtained at the same time interval (such as day, week or year), then the drift rate

\[
\lambda = \frac{1}{n} \sum_{j=0}^{n} \ln \frac{X_{j+1}}{X_j} 
\]

(5)

Where \( n \) is the number of observations; \( X_j \) is the value of the \( j \) observation time point \( (j = 1, \ldots, n) \).

2.1.2. Volatility calculation. Make \( q_i = \ln \frac{X_{i+1}}{X_i}, i = 1, 2, \ldots, n \), \( \bar{q} \) is the average of \( q_i \), then

\[
\delta = \left[ \frac{1}{n-1} \sum_{i=1}^{n} (q_i - \bar{q})^2 \right]^{1/2}
\]

(6)

2.2. Time-varying uncertainty design of bolted joints

The bolts are only subjected to the pre-tightening force \( F_p \) and the tensile working load \( F_0 \) due to the axial working load. The total tensile force of the bolt is \( F \), and the thread strength condition is

\[
F \leq \frac{\pi d^2_1}{4 \times 1.3} \left[ \sigma \right] 
\]

thus,

\[
\sigma = \frac{1.3 \times 4 F}{\pi d^2_1} \leq \left[ \sigma \right] 
\]

(7)

Among them, \( [\sigma] \) - allowable stress, \( MPa \);

\( F \) - total axial tensile force of the bolt \( N \);

\( d_1 \) - the dangerous section diameter of the threaded part \( mm \).

According to (7), take the logarithm, \( \ln \sigma = \ln \frac{4 \times 1.3}{\pi} + \ln F - 2 \ln d_1 \)

\[
\frac{\partial \ln \sigma}{\partial F} = \frac{1}{F}, \quad \frac{\partial \ln \sigma}{\partial d_1} = \frac{2}{d_1}, \quad \frac{\partial^2 \ln \sigma}{\partial F^2} = -\frac{1}{F^2}, \quad \frac{\partial^2 \ln \sigma}{\partial d_1^2} = -\frac{2}{d_1^2}, \quad \frac{\partial \ln \sigma}{\partial F \partial d_1} = 0
\]

The mean and variance of \( \ln \sigma \) are...
\begin{equation}
\begin{aligned}
H_{\ln (t)} &= \ln \sigma(0) + \int_0^t \left( (\lambda_\varphi - 2\lambda_\sigma) + \frac{1}{2} \left( -\sigma_\varphi^2 + 2\sigma_\sigma^2 \right) \right) dt, \\
&= \ln \sigma(0) + \int_0^t \left( (\lambda_\varphi - 2\lambda_\sigma) + \frac{1}{2} \left( -\sigma_\varphi^2 + 2\sigma_\sigma^2 \right) \right) dt \\
\mathbf{y}_{\ln (t)} &= \left( \int_0^t \delta_\varphi \, dw_1 \right)^2 + \left( \int_0^t 2\delta_\sigma \, dw_2 \right)^2
\end{aligned}
\end{equation}

The allowable stress $[\sigma]$ can be regarded as obeying the geometric Brownian motion, with a drift rate of $\lambda_{[\sigma]}$ and a volatility of $\delta_{[\sigma]}$. According to Ito Lecture:

\begin{equation}
\ln[\sigma](t) = N[\ln[\sigma](0) + (\lambda_{[\sigma]} - \frac{1}{2} \delta_{[\sigma]}^2) t, (\delta_{[\sigma]} \sqrt{t})^2]
\end{equation}

The reliability of the tight joint bolt subjected to the axial working load is:

\begin{equation}
R(t) = P(Z_{t_{s0}} > 0) = P(\sigma \geq \sigma) = P(\ln[\sigma](t) - \ln \sigma(t) \geq 0)
= \Phi \left( \frac{\ln[\sigma](0) + (\lambda_{[\sigma]} - \frac{1}{2} \delta_{[\sigma]}^2)t - H_{\ln (t)}}{\sqrt{\sigma_{\ln (t)}^2 + \mathbf{y}_{\ln (t)}}} \right)
\end{equation}

3. Numerical study

For example, a tight connecting bolt whose mechanical load is subjected to an external load, the initial axial load of the bolt is $F = 50kN$, $[\sigma] = \sigma_b = 640MPa$, according to $\sigma = \frac{1.3 \times 4F}{\pi d_i^2} \leq [\sigma]$, Calculated $d_i \geq 11.374mm$, Use $M16$ bolt, $d_i = 13.835mm$. From the above, in the bolt connection structure, the logarithm of all random variables $X_i$ follows a normal distribution, and the logarithm of the half-axis $d_i$, $F, [\sigma]$ obeys the normal distribution of the expression (9), their drift rate $\lambda_{d_i}, \lambda_{F}, \lambda_{[\sigma]}$ and the volatility $\delta_{d_i}, \delta_{F}, \delta_{[\sigma]}$. It can also be calculated statistically based on statistical data (6) and (7).

The observed time interval is in days and the reliability after the required service life is greater than 0.95. Due to the conditional constraints, the numerical simulation method for obtaining the diameter of the bolt, the total tensile force of the bolt and the allowable stress of the bolt is obtained. The time-varying data of the diameter of the bolt is shown in Figure 1. The total tensile force of the bolt is shown. The time-varying data is shown in Figure 2. The time-varying data of the allowable stress of the bolt is shown in Figure 3.
According to the data, the drift rate and volatility statistical formula (6) and (7), available 
\( \lambda_0 = -0.000073939355 \), \( \delta_0 = 0.00010356796 \), \( \hat{\lambda}_0 = 0.00067945674 \), \( \hat{\delta}_0 = 0.022440897 \), \( \lambda_{\sigma_1} = -0.00089219698 \), \( \delta_{\sigma_1} = 0.019496877 \) From the formula (10): assuming that \( 2a \) is required \( R(730) \geq 0.95 \), then
The reliability of the bolt can be more accurately calculated based on the reliability of the bolt. Elasticity of the bolt is about due to .

\[
\sigma = 640 \text{ MPa},
\]

Got : \( \ln(\sigma)(0) = 6.461468 \), \( \sum_{i=0}^{730} \mu \ln(\sigma) = -0.79005059 \), \( \sum_{i=0}^{730} \sigma \ln(\sigma) = 0.52677661 \). According to the formula

\[
R(730) = \Phi \left( \frac{\mu_{\ln(\sigma)} - \mu_{\ln(\sigma)}}{\sigma_{\ln(\sigma)} + \sigma_{\ln(\sigma)}} \right) = \Phi(1.7421068) = 95.91%.
\]

Because \( R(730) \approx 95.91% > 95\% \), So meet the requirements.

4. Conclusion

Aiming at the time-varying uncertainty design of bolted joints, the time-varying uncertainty design of tightly connected bolts subjected to axial working loads and preloading forces considering uncertain factors and the time-varying tightening of bolts with pre-tightening force are proposed. Uncertainty design. Combine the distribution of random variables with multiple variables, and derive the allowable stress and the volatility and drift rate of each factor. Calculate the volatility and drift rate of the total external force parameters affected by the bolt diameter and bolt affected by the change of stress value. Therefore, the reliability of the bolt can be more accurately calculated based on the reliability requirements for the bolt assembly at a specific time. The simulation data is obtained by Matlab numerical simulation method. It is calculated that after two years, the reliability of the bolt is about 95.91%, which meets the reliability requirement. After a period of time, when the reliability drops to a certain reliability, the bolt needs to be carried out. Repair or replace. The results show that the method can effectively ensure the safety and reduce the workload of the workers and reduce the losses under the premise of meeting the requirements of use and reliability. It provides an important reference for proper selection and maintenance of bolted joints. This method takes into account the dynamic evolution of the strength and working stress of the bolted joints, and the reliability and reliability of each part can be calculated at any time. Provides a new time-varying design method for bolted parts to safely and reliably evaluate the reliability of bolted connections.

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