Unique thickness-dependent properties of the van der Waals interlayer antiferromagnet MnBi$_2$Te$_4$ films

M. M. Otrokov,$^{1,2}$ I. P. Rusinov,$^{2,3}$ M. Blanco-Rey,$^{4,5}$ M. Hoffmann,$^6$ A. Yu. Vyazovskaya,$^{2,3}$ S. V. Eremeev,$^{7,2,3}$ A. Ernst,$^{5,8}$ P. M. Echenique,$^{1,4,5}$ A. Arnau,$^{1,4,5}$ and E. V. Chulkov$^{1,4,5,3}$

$^1$Centro de Física de Materiales (CFM-MPC), Centro Mixto CSIC-UPV/EHU, 20018 Donostia-San Sebastián, Basque Country, Spain
$^2$Tomsk State University, 634050 Tomsk, Russia
$^3$Saint Petersburg State University, 198504 Saint Petersburg, Russia
$^4$Departamento de Física de Materiales UPV/EHU, 20080 Donostia-San Sebastián, Basque Country, Spain
$^5$Donostia International Physics Center (DIPC), 20018 Donostia-San Sebastián, Basque Country, Spain
$^6$Institut für Theoretische Physik, Johannes Kepler Universität, A 4040 Linz, Austria
$^7$Institute of Strength Physics and Materials Science, Russian Academy of Sciences, 634021 Tomsk, Russia
$^8$Max-Planck-Institut für Mikrostrukturphysik, Weinberg 2, D-06120 Halle, Germany

(Dated: October 15, 2018)

Using density functional theory and Monte Carlo calculations, we study the thickness dependence of the magnetic and electronic properties of a van der Waals interlayer antiferromagnet in the two-dimensional limit. Considering MnBi$_2$Te$_4$ as a model material, we find it to demonstrate a remarkable set of thickness-dependent magnetic and topological transitions. While a single septuple layer block of MnBi$_2$Te$_4$ is a topologically trivial ferromagnet, the thicker films made of an odd (even) number of blocks are uncompensated (compensated) interlayer antiferromagnets, which show wide bandgap quantum anomalous Hall (zero plateau quantum anomalous Hall) states. Thus, MnBi$_2$Te$_4$ is the first stoichiometric material predicted to realize the zero plateau quantum anomalous Hall state intrinsically. This state has been theoretically shown to host the exotic axion insulator phase.

After the isolation of graphene, the field of two-dimensional (2D) van der Waals (vdW) materials has experienced an explosive growth and new families of 2D systems and block-layered bulk materials, such as tetradymite-like topological insulators (TIs) [1,2], transition metal dichalcogenides [3], and others [4–6] have been discovered. The remarkable electronic properties, along with the possibility of their tuning via thickness control, doping, intercalation, proximity effects, etc., make the layered vdW materials attractive for both fundamental and applied research. In particular, vdW antiferromagnets are expected to be of great interest. Indeed, recently it has been reported that the layered vdW compound MnBi$_2$Te$_4$ is the first ever antiferromagnetic (AFM) TI [3]. This state of matter is predicted to give rise to exotic phenomena such as quantized magnetoelectric effect [14], axion electrodynamics [15], and Majorana hinge modes [16]. Moreover, the combination of magnetism with spin-orbit coupling, along with strong thickness dependence of electronic structure in the 2D limit suggest that vdW compounds like MnBi$_2$Te(Se)$_4$ and MnSb$_2$Te$_4$ might be attractive for both fundamental and applied research. Finally, a novel type of energetically stable and universal interface between (A)FM and topological insulators has been proposed recently [18]. At such an interface, the film of magnetic material, that does not show vdW bonding intrinsically, turns out to be vdW-coupled to a TI as a result of immersion below the surface of the latter. Incidentally, the axion insulator state could also be achieved in such heterostructures [19]. These and other AFM systems appear to be interesting candidates to couple the emerging fields of AFM spintronics [20] and layered vdW materials [1,8].

Here, using state-of-the-art ab initio techniques and...
the Monte Carlo method, we study the magnetic, electronic and topological properties of the layered vdW AFM TI compound MnBi$_2$Te$_4$ in the 2D limit. We find a unique set of thickness-dependent magnetic and topological transitions, which drive the MnBi$_2$Te$_4$ thin films through FM and (un)compensated AFM phases (see Fig. 1), as well as QAH and zero plateau QAH states.

The electronic structure calculations were carried out within density functional theory using the projector augmented-wave method [21] (VASP code [22, 23]). The exchange-correlation energy was treated using the generalised gradient approximation [24]. The Heisenberg exchange coupling constants $J_{ij}$ were determined using Monte Carlo simulations based on a classical Heisenberg Hamiltonian parametrised with the magnetic anisotropy energies (MAEs) and $J_{ij}$ constants obtained from $ab$ initio calculations. The Chern numbers were determined using Z2Pack [32, 33] and $ab$-initio-based tight-binding calculations [24, 35]. The edge electronic band structure was calculated within the semi-infinite medium Green function approach. More details on methods can be found in the Supplementary Note I.

MnBi$_2$Te$_4$ is built of SL blocks stacked one on top of another along the [0001] direction and held together by vdW forces [13, 36] (Fig. 1a). As far as its magnetic order is concerned, it appears to be an interlayer antiferromagnet, where the FM Mn layers of neighboring blocks are coupled antiparallel to each other [13, 17]. Although the recently synthesized single crystals show some degree of statistical Mn/Bi disorder, our $ab$ initio calculations indicate that such an intermixing is less favorable than the ideal structure (Supplementary Note II). Therefore, in what follows we consider the ordered structure of MnBi$_2$Te$_4$.

We start with the magnetic characterization of a single free-standing MnBi$_2$Te$_4$ SL. The exchange coupling constants $J_{0ij}$, calculated as a function of the Mn-Mn distance $r_{Mn(i)-Mn(j)}$, show the same trend as in bulk MnBi$_2$Te$_4$ (Fig. 1b). Namely, that the FM interaction between first nearest neighbors, $J_{01} \approx 0.08$ meV/µ$_B^2$, strongly dominates over all other. Thus, there is a stable tendency towards the FM ground state in MnBi$_2$Te$_4$ SL, as confirmed by the total-energy calculations, which show a preference of the FM state over the 120° AFM state by 14.77 meV per Mn pair (Table I). This result is consistent with those of Refs. [13, 37], where it has been experimentally shown that each MnBi$_2$Te$_4$ SL orders ferromagnetically. An FM ground state has also been predicted for a single MnBi$_2$Te$_4$ SL placed on different tetradymite-like TI substrates [11, 12]. Therefore, we conclude that the intrafilm FM order is not sensitive to the thickness of the MnBi$_2$Te$_4$ film, as well as to the formation of the vdW interface with other block-layered compounds.

For the 2-SL-thick film, the total-energy calculations show that the interlayer coupling is AFM, leading to the compensated AFM (cAFM; Fig. 1c) ordering as in the bulk material [13, 17]. Thickness increase up to 3 SLs keeps the interlayer exchange coupling antiferromagnetic, but, since the number of blocks is odd, an uncompensated AFM (uAFM; Fig. 1c) state arises. Similarly to the 2-SL and 3-SL-thick films cases, we also predict cAFM and uAFM states for the thicker films made of even and odd number of SLs, respectively (Table I).

It is well known that the magnetic anisotropy and the interlayer exchange coupling play a crucial role in (quasi)2D magnets. Indeed, if a purely 2D magnet has an easy-plane magnetic anisotropy, it features no magnetic order at any temperature except 0 K according to the Mermin-Wagner theorem [38, 39]. The reason for this is the Goldstone mode of the gapless long-wavelength excitations, whose destructive role increases with decreasing dimensionality of the system. In the limit of strong easy-plane anisotropy such systems, instead of a second
Table I. Thickness dependence of the MnBi$_2$Te$_4$ films magnetism. $\Delta E_{AFM} = E_{AFM} - E_{FM}$ is the total energy difference of the AFM and FM states, where AFM refers to the intralayer 120° state in the case of the single SL, while for the thicker films and bulk it means the interlayer AFM state (Fig. 1). cAFM (uAFM) stands for the compensated (uncompensated) AFM state. $T_C$ denotes the Curie or Néel temperature in the FM or AFM cases, respectively. The numbers in brackets indicate the error bar. Details of the Monte Carlo simulations can be found in Supplementary Note I.

| Thickness (SL) | $\Delta E_{AFM}$ (meV/(Mn pair)) | Order | MAE (meV/Mn) | $T_C$ (K) |
|---------------|----------------------------------|-------|--------------|-----------|
| 1             | 14.77                            | FM    | 0.125        | 12(1)     |
| 2             | -1.22                            | cAFM  | 0.236        | 24.4(1)   |
| 3             | -1.63                            | uAFM  | 0.215        |           |
| 4             | -1.92                            | cAFM  | 0.210        |           |
| 5             | -2.00                            | uAFM  | 0.205        |           |
| 6             | -2.05                            | cAFM  |              |           |
| 7             | -2.09                            | uAFM  |              |           |
| $\infty$ (bulk) | -2.80                           | cAFM  | 0.225        | 25.42(1)  |

order phase transition, were shown to undergo a so-called Berezinskii-Kosterlitz-Thouless transition [40, 41], which is manifested in a change of the spin-spin correlation function behavior from a power law below the crossover temperature $T_{BKT}$ to an exponential law above it. It is precisely the interlayer exchange coupling that stabilizes the long-range order at finite temperatures in such cases [42]. Alternatively, even a small gap in the excitation spectrum introduced by the easy-axis magnetic anisotropy can significantly reduce the impact of the low-energy excitations. In this case, the three-dimensional (3D) exchange contribution is expected to further enhance the critical temperature [43].

We then calculate the MAE for the MnBi$_2$Te$_4$ films from 1 to 5 SLs as well as for bulk (Table I). In all these cases, MAE is positive, indicating an out-of-plane easy axis in agreement with recent experiments [37]. The anisotropy of the SL-thick FM film turns out to be weaker than that of the thicker films with AFM interlayer coupling, for which the MAE was found to be close to the bulk value (see Table I). The magnitudes of the local magnetic moments are practically independent from the film thickness, being roughly equal to 4.6 $\mu_B$ in all cases. The Curie temperature of the SL-thick FM film, that represents a purely 2D magnetic system, appears to be approximately equal to 12 K. Due to appearance of the interlayer exchange coupling and an increase in the MAE, the Néel temperature of a double SL film enhances to $\approx 24.4$ K, which is just slightly lower than that of bulk (Table I).

Now we show that in the thin-film limit not only the magnetic, but also the electronic and topological properties of MnBi$_2$Te$_4$ are strongly thickness dependent. The bandstructure of the MnBi$_2$Te$_4$ single SL block is shown in Fig. 2a. In agreement with the experimental data [37], it shows an indirect bandgap of $\approx 0.32$ eV. The Chern number calculations reveal a $C = 0$ state, the system being a topologically trivial ferromagnet (Table II).

Upon increasing thickness up to 2 SLs the interlayer cAFM order sets in, leading to a doubly degenerate band spectrum (Fig. 2b) and $C = 0$ again. For this system, we find a bandgap of 107 meV. However, if calculated in the artificial FM phase of the 2-SL-thick film, the Chern number appears to be equal to $-1$ indicating a QAH insulator state. Accordingly, the edge band structure of the system shows a single 1D chiral mode (Fig. 3a). Reversing the magnetization of the FM 2-SL-thick MnBi$_2$Te$_4$ film yields the $C = +1$ QAH state. These results suggest

Table II. Thickness dependence of the MnBi$_2$Te$_4$ films topology and bandgap size. QAH and ZPQAH stand for the quantum anomalous Hall phase and its zero plateau state, respectively.

| Thickness (SL) | Topology | Bandgap (meV) |
|---------------|----------|--------------|
| 1             | Trivial  | 321          |
| 2             | ZPQAH    | 107          |
| 3             | QAH      | 66           |
| 4             | ZPQAH    | 97           |
| 5             | QAH      | 77           |
| 6             | ZPQAH    | 87           |
| 7             | QAH      | 85           |
| $\infty$ (bulk) | 3D AFM TI | 225        |
that the 2-SL-thick cAFM MnBi$_2$Te$_4$ film is likely to be in a so-called zero plateau QAH (ZPQAH) state. Up to now, the ZPQAH state was an artificial state of a QAH insulator that is realized in the process of the magnetization reversal by external magnetic field (i.e. during the transition between the two QAH states with Chern numbers of opposite signs). ZPQAH state manifests itself in the appearance of flat regions in the hysteresis-like dependence of the Hall conductivity on the external field $\sigma_{xy}(H)$. Namely, within certain range of $H$ close to the coercivity, the $\sigma_{xy} = 0$ plateau is observed, which corresponds to a fully gapped band structure. Outside this $H$ range, $\sigma_{xy}$ rapidly reaches a quantized value of either $+e^2/h$ or $-e^2/h$, depending on the magnetization direction. Such a situation can be achieved either in (i) a zero magnetization state of the magnetically-doped QAH insulator \cite{44} due to the coexistence of the upwards and downwards magnetized domains or (ii) the antiparallel magnetizations state of an FM1(↑)/TI/FM2(↓) QAH heterostructure, where FM1 and FM2 are two different FM insulators \cite{45}. To check whether the ZPQAH state is realized in the 2-SL-thick MnBi$_2$Te$_4$ film, we have calculated the edge band structure in the cAFM ground state of the system. We find a fully gapped spectrum (Fig. 3), corresponding to $\sigma_{xy} = 0$. Thus, the 2-SL-thick MnBi$_2$Te$_4$ film represents first ever example of an intrinsic ZPQAH phase.

Figure 3. Edge electronic band structures of the MnBi$_2$Te$_4$ 2-SL-thick film calculated for the (a) FM and (b) cAFM states. The regions with a continuous spectrum correspond to the 2D bulk states projected onto a 1D Brillouin zone. The edge crystal structure is shown in Supplementary Note I.

At a thickness of 3 SLs, the system enters in a $C = -1$ QAH insulator state with a bandgap of $\sim 66$ meV. Similarly to the 2-SL-thick (3-SL-thick) film cases, we also predict the intrinsic ZPQAH (QAH) states for the 4- and 6-SL-thick (5- and 7-SL-thick) films, respectively (Table \ref{table1}).

At this point, having described various phases realized in the MnBi$_2$Te$_4$ films, it is important to stress a crucial advantage of the here proposed (ZP)QAH insulators: they show ordered structures, inherent to the stoichiometric material, which guarantees them against disorder-related drawbacks such as the bandgap fluctuation \cite{16} or superparamagnetism \cite{17, 18}. This very fact, together with the large bandgaps of such systems (Table \ref{table1}), could facilitate the observation of the QAH state at temperatures notably higher than those achieved so far. This is all the more true since, from the 2 SLs thickness, the MAE is already close to that of the bulk, indicating that the MnBi$_2$Te$_4$-based QAH insulators should have critical temperatures comparable to the bulk Néel temperature of MnBi$_2$Te$_4$ (Table \ref{table1}). To be mentioned as well is a solid state realization of axion electrodynamics in a ZPQAH state proposed recently \cite{15}. Up to now the axion insulator state was being sought for in the FM1/TI/FM2 QAH heterostructures. In such systems, a relatively thick TI spacer enables magnetization reversal of the individual FM layers that have different coercivities, leading to the overall AFM alignment and, consequently, to a ZPQAH state \cite{49, 50}. In contrast to the latter heterostructures, the MnBi$_2$Te$_4$ thin films made of even number of SLs realize this state intrinsically, i.e. without need of magnetic field application.

In summary, using \textit{ab initio} and Monte Carlo calculations, we have scrutinized the magnetic, electronic and topological properties of the MnBi$_2$Te$_4$ AFM TI thin films. Belonging to the class of layered vdW compounds, in the 2D limit MnBi$_2$Te$_4$ shows a unique set of thickness-dependent transitions through various phases, being among them wide-bandgap QAH and ZPQAH states. Similar behaviour can possibly take place in other compounds of the MnBi$_2$Te$_4$ family, such as MnSb$_2$Te$_4$, MnBi$_2$Se$_4$, and others. We believe that our findings will stimulate intensive studies of thin films of vdW antiferromagnets as prospective materials for AFM spintronics.

**ACKNOWLEDGMENTS**

Authors thank J.I. Cerdá and V.N. Men’shov for stimulating discussions. We acknowledge the support by the Basque Departamento de Educacion, UPV/EHU (Grant No. IT-756-13), Spanish Ministerio de Economía y Competitividad (MINECO Grant No. FIS2016-75862-P), Academic D.I. Mendeleev Fund Program of Tomsk State University (Project No. 8.1.01.2018), and Fundamental Research Program of the State Academies of Sciences for 2013–2020, line of research III.23. The support by the Saint Petersburg State University for scientific investigations (Grant No. 15.61.202.2015) is also acknowledged. I.P.R. acknowledges support by the Ministry of Education and Science of the Russian Federation within the framework of the governmental program Megagrants (state task no. 3.8895.2017/P220). This study was supported by Russian Science Foundation No. 18-12-00169 in the part of the calculations within tight-binding method. A.E. acknowledges fi-
nancial support from DFG through priority program SPP1666 (Topological Insulators). The calculations were performed in Donostia International Physics Center, in the Research park of St. Petersburg State University Computing Center (http://cc.spbu.ru), and at SKIF Cyberia cluster of Tomsk State University.

* mikhail.otrokov@gmail.com

[1] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
[2] S. V. Eremeev, G. Landolt, T. V. Menshchikova, B. Slomski, Y. M. Koroteev, Z. S. Aliev, M. B. Babanly, A. Ernst, L. Patthey, et al., Nat. Commun. 3, 635 (2012).
[3] X. Xu, W. Yao, D. Xiao, and T. F. Heinz, Nat. Phys. 10, 343 (2014).
[4] K. Ishizaka, M. Bahramy, H. Murakawa, M. Sakano, T. Shimojima, T. Sonobe, K. Koizumi, S. Shin, H. Miyahara, A. Kimura, et al., Nature 510, 521 (2011).
[5] L. Li, Y. Yu, G. J. Ye, Q. Ge, X. Ou, H. Wu, D. Feng, X. H. Chen, and Y. Zhang, Nat. Nanotech. 9, 372 (2014).
[6] A. Banerjee, C. Bridges, J.-Q. Yan, A. Acel, L. Li, M. Stone, G. Granroth, M. Lumsden, Y. Yiu, J. Knolle, et al., Nat. Mater. 15, 733 (2016).
[7] C. Gong, L. Li, Z. Li, H. Ji, A. Stern, Y. Xia, T. Cao, W. Bao, C. Wang, et al., Nature 546, 265 (2017).
[8] B. Huang, G. Clark, E. Navarro-Moratalla, D. R. Klein, R. Cheng, K. L. Seyler, D. Zhong, E. Schmidgall, M. A. McGuire, D. H. Cobden, et al., Nature 546, 270 (2017).
[9] T. Hirahara, S. V. Eremeev, T. Shirasawa, Y. Okuyama, T. Kubo, R. Nakanishi, R. Akiyama, A. Takayama, T. Hajiri, S. Ideta, et al., Nano Lett. 17, 3493 (2017).
[10] J. A. Hagmann, X. Li, S. Chowdhury, S.-N. Dong, S. Rouvimov, S. J. Pookpanratana, K. M. Yu, T. A. Orlova, T. B. Bolin, C. U. Segre, et al., New J. Phys. 19, 085002 (2017).
[11] M. M. Otrokov, T. V. Menshchikova, I. P. Rusinov, M. G. Vergniory, V. M. Kuznetsov, and E. V. Chulkov, JETP Lett. 105, 297 (2017).
[12] M. M. Otrokov, T. V. Menshchikova, M. G. Vergniory, I. P. Rusinov, A. Y. Vyzavskaya, Y. M. Koroteev, G. Bihlmayer, A. Ernst, P. M. Echenique, A. Arnau, et al., 2D Mater. 4, 025082 (2017).
[13] M. M. Otrokov, I. I. Klimovskikh, H. Bentmann, A. Zeugner, Z. S. Aliev, S. Gass, A. B. Wolter, A. V. Koroleva, D. Estuyin, A. M. Shikin, et al., arXiv:1809.07389 (2018).
[14] R. S. K. Mong, A. M. Essin, and J. E. Moore, Phys. Rev. B 81, 245209 (2010).
[15] R. Li, J. Wang, X.-L. Qi, and S.-C. Zhang, Nat. Phys. 6, 284 (2010).
[16] Y. Peng and Y. Xu, arXiv:1809.09112 (2018).
[17] S. V. Eremeev, M. M. Otrokov, and E. V. Chulkov, J. Alloys Compd. 709, 172 (2017).
[18] S. V. Eremeev, M. M. Otrokov, and E. V. Chulkov, Nano Lett. 18, 6521 (2018).
[19] Y. S. Hou and R. Q. Wu, arXiv:1809.09265 (2018).
[20] V. Baltz, A. Manchon, M. Tsoi, T. Moriyama, T. Ono, and Y. Tserkovnyak, Rev. Mod. Phys. 90, 015005 (2018).
[21] P. E. Blöchl, Phys. Rev. B 50, 17953 (1994).
[22] G. Kresse and J. Furthmüller, Phys. Rev. B 54, 11169 (1996).
[23] G. Kresse and D. Joubert, Phys. Rev. B 59, 1758 (1999).
[24] J. P. Perdew, K. Burke, and M. Ernzerhof, Phys. Rev. Lett. 77, 3865 (1996).
[25] D. D. Koelling and B. N. Harmon, J. Phys. C: Sol. St. Phys. 10, 3107 (1977).
[26] S. Grimme, J. Antony, S. Ehrlich, and H. Krieg, J. Chem. Phys. 132, 154104 (2010).
[27] S. Grimme, S. Ehrlich, and L. Goerigk, J. Comput. Chem. 32, 1456 (2011).
[28] V. I. Anisimov, J. Zaanen, and O. K. Andersen, Phys. Rev. B 44, 943 (1991).
[29] S. L. Dudarev, G. A. Botton, S. Y. Savrasov, C. J. Humphreys, and A. P. Sutton, Phys. Rev. B 57, 1505 (1998).
[30] E. Wimmer, H. Krakauer, M. Weinert, and A. J. Freeman, Phys. Rev. B 24, 864 (1981).
[31] FLEUR site: http://www.flapw.de
[32] A. A. Soluyanov and D. Vanderbilt, Phys. Rev. B 83, 235014 (2011).
[33] D. Gresch, G. Autès, O. V. Yazyev, M. Troyer, D. Vanderbilt, B. A. Bernevig, and A. A. Soluyanov, Phys. Rev. B 95, 075146 (2017).
[34] N. Marzari and D. Vanderbilt, Phys. Rev. B 56, 12847 (1997).
[35] A. A. Mostofi, J. R. Yates, Y.-S. Lee, I. Souza, D. Vanderbilt, and N. Marzari, Comput. Phys. Commun. 178, 685 (2008).
[36] D. S. Lee, T.-H. Kim, C.-H. Park, C.-Y. Chung, Y. S. Lim, W.-S. Seo, and H.-H. Park, CrystEngComm 15, 5532 (2013).
[37] Y. Gong et al., arXiv:1809.07926 (2018).
[38] N. D. Mermin and H. Wagner, Phys. Rev. Lett. 17, 1133 (1966).
[39] M. Bандер and D. L. Mills, Phys. Rev. B 38, 12015 (1988).
[40] V. Berezinskii, Sov. Phys. JETP 32, 493 (1971).
[41] J. M. Kosterlitz and D. J. Thouless, J. Phys. C: Solid State Phys. 6, 1181 (1973).
[42] A. A. Katanin and V. Y. Irkhin, Phys.-Uspekhi 50, 613 (2007).
[43] M. M. Otrokov, G. Fischer, P. Buczek, A. Ernst, and E. V. Chulkov, Phys. Rev. B 86, 184418 (2012).
[44] J. Wang, B. Lian, and S.-C. Zhang, Phys. Rev. B 89, 085106 (2014).
[45] J. Wang, B. Lian, X.-L. Qi, and S.-C. Zhang, Phys. Rev. B 92, 081107 (2015).
[46] I. Lee, C. K. Kim, J. Lee, S. J. L. Billinge, R. Zhong, J. A. Schneeloch, T. Liu, T. Valla, J. M. Tranquada, G. Gu, et al., Proc. Natl. Acad. Sci. U.S.A. 112, 1316 (2015).
[47] E. O. Lachman, A. F. Young, A. Richardson, J. Cuppens, H. Naren, Y. Anahory, A. Y. Meltzer, A. Kandala, S. Kempinger, et al., Sci. Adv. 1, e1500740 (2015).
[48] J. A. Krieger, C.-Z. Chang, M.-A. Husanu, D. Sostina, H. Naren, Y. Anahory, A. Y. Meltzer, A. Kandala, S. Kempinger, et al., Sci. Adv. 1, e1500740 (2015).
[49] M. Mogi, M. Kawamura, A. Tsukazaki, R. Yoshimi, K. S. Takahashi, M. Kawasuki, and Y. Tokura, Sci. Adv. 3, eaa01669 (2017).
[50] D. Xiao, J. Jiang, J.-H. Shin, W. Wang, F. Wang, Y.-F. Zhao, C. Liu, W. Wu, M. H. Chan, N. Samarth, et al., Phys. Rev. Lett. 120, 056801 (2018).