A Robust Poincare Maps Method for Damage Detection based on Single Type of Measurement

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Abstract. The classical vibration-based structural health monitoring evaluates damage via the analysis of single type measurement. The relationships between different types of measurements, however, are usually neglected. To address this problem, vibration data is to be analyzed in state space in this paper based on the Poincare map method. The Poincare map method is an effective approach in damage detection. This method, however, requires the dependent measurements for displacement, velocity and acceleration, respectively. Based on the Fourier transform properties, a robust Poincare map method for damage detection is proposed based on the single type of measurement. Numerical and experimental verifications are employed to verify the effectiveness of the developed method. Compared with the classical numerical differential method, the present method is more accurate and robust to noise.

1. Introduction

Vibration-based damage assessment is one of the most crucial issues in structural health monitoring. Due to the on-line property of vibration measurement, the corresponding technologies are widely used in many engineering fields [1-4]. During the past decade, many efficient damage detection or assessment methods are proposed based on different measurements or evaluations. Zhou et al. [5] proposed a novel damage detection method based on the transmissibility. Based on their approach, the damage in early stage can be identified rapidly. To reveal the damage effect on structure, Khatir et al. [6, 7] investigated the dynamic responses of structures with single- or multiple-damage. Yang et al. [8, 9] investigated the entropy and fractural dimension methods for damage detection. These methods paved some novel damage detection paths in different aspects. However, only limited types of measurements are employed. The relationships between measurements are usually neglected. Different from the common approaches, the Poincare map method provides a novel view in damage detection, that is, damage are evaluated in state space rather than based on only one measurement. In the vibration-based system, the state space can be spanned by displacement, velocity and acceleration.
It has been proved that the difference before and after the damage appearance in the state space is an effective damage index. However, the displacement, velocity and acceleration in these works are measured independently in applications. Thus, the compensation for measurement may be expensive. The natural solution for this problem is to derive the velocity and acceleration from displacement, but the poor performance of numerical derivative make this way unsubstantial. Focusing on this problem, a robust Poincare map method based on single type of measurement is proposed in this paper to overcome the problems in numerical derivative-based methods. It will be proved in the following parts that the proposed method is effective in noisy conditions.

2. Damage Detection Method and Simple Verification

2.1. Model and state space

\[ \begin{bmatrix} m_1 & \frac{\partial F(t)}{\partial x_1} \\ \frac{\partial F(t)}{\partial x_1} & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -c_2 \\ c_1 + c_2 \end{bmatrix} \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} + \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \frac{d^2 x_1}{dt^2} \\ \frac{d^2 x_2}{dt^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \sin(2\pi f_t) \end{bmatrix} \]

where \( k \) is the stiffness, \( c \) is the damping, \( m \) is the mass, \( w \) is the displacement and \( t \) is the time. The subscript means the number of DOF for all symbols. In order to model the nonlinear defect, the stiffness of spring 1 \( (k_1) \) is assigned as the variable yields:
\[ k_1 = 1 + \cos(2\pi f t) \]

The other parameters are: \( k_2 = 1, m_1 = m_2 = 1, c_1 = c_2 = 1 \), and \( 2\pi f_1 = 1, 2\pi f_2 = 5 \). The natural frequencies of the intact system are 0.618 and 1.618. Obviously, the configurations of parameter avoid the resonance of system. By aid of the Newmark method, the calculated displacement \( w \), velocity \( \frac{dw}{dt} \) and acceleration \( \frac{d^2w}{dt^2} \) responses before and after the occurrence of damage are obtained and shown in Fig. 2. The red and gray lines depict the responses of the intact structure and damaged structure, respectively. The acceleration response changes sharply when damage appears and the corresponding differences of displacement and velocity responses are not evident. Since the defect \( k_1 \) is connected with mass 1 directly, the change of node 2 is smaller than that of node 1.

To enhance the nonlinear damage feature, Manoach and Trendafilova [10-12] proposed to express these responses in state space and identify damage based on the Poincare map method. In their works, the state space \( \mathbf{P} = [w, \frac{dw}{dt}]^T \) is employed and the two basic components of state space are provided by independent measurements. In this work, we extend the state space as \( \mathbf{P} = [w, \frac{dw}{dt}, \frac{d^2w}{dt^2}]^T \), and the three terms of state space will be provided by single measurement considering the differential relationships of them.

The state space projections of the responses shown in Fig. 2 are presented in Fig. 3. In order to interpret the effectiveness of Poincare map method, velocity and acceleration responses in these figures are calculated by Newmark method directly instead of the estimation by a single measurement. Compared with the Poincare maps of node 2, the Poincare maps correspond to node 1 show a structural change before and after the occurrence of damage. This is the foundation of damage detection based on Poincare maps. For clarity, these Poincare maps are also called as the 1st \( (w, \frac{dw}{dt}) \), the 2nd \( (w, \frac{d^2w}{dt^2}) \), and the 3rd \( (\frac{dw}{dt}, \frac{d^2w}{dt^2}) \) projections in this paper.

\[ \text{(a)} \]

\[ \text{(b)} \]

**Figure 3.** Poincare maps of 2-DOFs system before and after damage, the projections of node 1 (a), node 2 (b). Red lines are for damaged conditions, gray lines are for intact conditions.

### 2.2. Poincare Maps obtained by single type of measurement

As mentioned above, Poincare maps can be generated by a single measurement referring to the differential relationships among displacement, velocity and acceleration. It is well-know that the numerical integration is stable, thus deriving displacement and velocity from acceleration will not be discussed in this paper. Instead, we will investigate the procedure of deriving velocity and acceleration from displacement via numerical differential, which is ill-conditioned.
The responses of node 1 in state space, velocity and acceleration responses are estimated by numerical derivative: (a) noise-free, (b) 70 dB. Red lines are for calculated signals, gray lines are for reference signals.

The central difference method, which is usually used in numerical differential, is employed to accomplish this procedure firstly. The related Poincare maps (red lines) of node 1 with different signal to noise ratios (SNR) are presented in Fig. 4, where the noisy signal is generated by invoking Matlab function "awgn" and the gray lines provide the reference signals calculated by model directly without noise. In the noise-free condition, the central difference method performances well. However, only the 1st projection agrees with the reference signal when noise exists in displacement $w$. This verifies the unstable problem in velocity and acceleration estimations by central difference method.
To address the similar problem, the derivative estimation based on Fourier spectrum was proposed by authors in [10-12] for modal curvature. Then the method is extended to Poincare maps estimation by single measurement in this paper. For completeness, the technological process of the method is briefly introduced in this section. The Fourier transform (FT) and the inverse Fourier transform (IFT) of displacement response $w(t)$ is given as:

$$W(\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} w(t) dt, \quad w(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} W(\omega) d\omega$$  \hspace{1cm} (3)

where $t \in \mathbb{R}$ and $\omega \in \mathbb{R}$. In the Fourier frame, velocity and acceleration can be obtained by:

$$\frac{dw}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \omega e^{j\omega t} \hat{w}(\omega) d\omega, \quad \frac{d^2 w}{dt^2} = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 e^{j\omega t} \hat{w}(\omega) d\omega$$  \hspace{1cm} (4)

For the discrete signal, the discrete Fourier transform (DFT) is the natural choice for velocity and acceleration calculations. Without the loss of generality, we assume $w(t)$ is defined on the interval $[0, 2\pi]$, and the interval is divided by $N$ grid points as $N$-1 terms with the length $\Delta$. By the DFT synthesis, Eq. (4) is rewritten as:

$$\frac{dw}{dt} = \frac{\Delta}{2\pi} \sum_{\omega} \omega e^{j\omega t} \left( \sum_{t} e^{-j\omega t} w(t) \right), \quad \frac{d^2 w}{dt^2} = -\frac{\Delta}{2\pi} \sum_{\omega} \omega^2 e^{j\omega t} \left( \sum_{t} e^{-j\omega t} w(t) \right)$$  \hspace{1cm} (5)

Based on Eq. (5), velocity and acceleration can be calculated from displacement directly. The corresponding results are plotted in Fig. 5. Compared with the classical central difference method, the Fourier spectrum based estimation is more robust to noise.

**Figure 6.** The responses of node 1 in state space after EEMD filtering (SNR = 70 dB), velocity and acceleration responses are estimated by DFT. Red lines are for calculated signals, gray lines are for reference signals.

In order to improve the present result further, the EEMD (Ensemble Empirical Mode Decomposition) is introduced as the filter. The filtered projections (SNR = 70 dB) are presented in Fig. 6. An obvious improvement validates the effectiveness of EEMD in this methodology. To remove the boundary effect induced by EEMD filtering, several sampling points locating at the beginning and end have been deleted.

2.3. Damage index

Some damage indices have been presented and validated in references [10-12]. They are employed in this paper with some necessary extensions to fulfill the requirements of the extended state space. The damage index $I_i$ is defined as:

$$I_i = \frac{1}{3} \sum_{k=1}^{3} \left| S_{i,k}^d - S_{i,k}^u \right|$$  \hspace{1cm} (6)

$$S_{i,k}^u = \frac{1}{N} \sum_{j=1}^{N} \left| \mathbf{p}^u_{i,j,k} - \mathbf{p}^d_{i,j,k} \right|, \quad S_{i,k}^d = \frac{1}{N} \sum_{j=1}^{N} \left| \mathbf{p}^d_{i,j,k} - \mathbf{p}^d_{i,j,k} \right|$$  \hspace{1cm} (7)

The above damage index is based on the comparison between the intact and inspected Poincare maps. The spikes presented in damage indices will be considered as damage feature. In applications, threshold or outlier analysis is required to determine the presence of damage and its location. To evaluate the effectiveness of the combination of proposed method and damage index, a noise immunity test con-
taining 100 samples is conducted and presented in terms of box plots (as shown in Fig. 7). The meanings of the symbols in box plots could be found in the "Help" of Matlab. The increase of noise level expands the sample distribution of each node. The distributions of node 1 and 2 overlap with each other when SNR = 60 dB, and the value of $I_1$ is also not large enough to be deemed as the damage feature. When SNR is larger than 65 dB, the sample distributions of nodes separate from each other, and the value of $I_1$ also increases with SNR. $I_2$ changes slightly comparing with $I_1$. These tendencies and phenomenon verify that the combination of the proposed method and damage index provides a robust damage evaluation for the inspected system.

![Figure 7. Box plots of damage index with different SNRs: (a) 60 dB, (b) 65 dB, (c) 70 dB, (d) 80 dB, (e) 90 dB.](image)

### 3. Experimental Verification

A supplementary experimental validation is conducted on a cracked steel cantilever beam with the following dimensions: length 600 mm, crack location 350 mm from the fixed edge, cross-section 20 mm × 20 mm. Another intact beam with the same dimensions serves as the baseline. A shaker attached on the tip of the cantilever beam serves as the actuator, and the voltage acts on the shaker is assigned high enough to generate the nonlinear effect (the crash/rub within the crack). To register the response signal simultaneously, eight acceleration sensors (PCB) are implemented at 90 mm, 170 mm, 254 mm, 333 mm, 425 mm, 485 mm and 545 mm from the clamped end. For clarity, they are denoted as node 1-8 simply. Obviously, nodes 4 and 5 locate near the crack location. Damage characteristic should be featured at these two nodes if any. Restricted by the equipment, acceleration sensors are adopted to work as the displacement sensors in this experiment, namely, the derivatives are conducted on acceleration responses. To evaluate the algorithm in a wide range of frequency domain, four single-frequency excitations are selected, 50 Hz, 100 Hz, 150 Hz and 200 Hz.

![Figure 8. Damage indices for: (a) 50 Hz, (b) 100 Hz, (c) 150 Hz and (d) 200 Hz excitations.](image)

The damage indices obtained from different excitations are presented in Fig. 8. Compared with the other damage indices, the damage index obtained from the 150 Hz excitation feature the crack clearly. The related Poincare maps are given in Fig. 9. Compared with the 100 Hz response, it is observed that the nonlinear effect become more evident under the 150 Hz excitation, which is close to the natural frequency of the inspected structure. The resonance enhances the nonlinear effect, and hence makes...
the damage feature distinct. Besides the 150 Hz result, damage indices given by the other excitations present the highest peak at node 4 or node 5. This is helpful to pinpoint the damage location to some extent.

Figure 9. The Poincare maps of node 4 excited by (a) the 100 Hz and (b) 150 Hz input before (gray) and after (red) the occurrence of damage.

4. Conclusions
As validated by simulations, the developed method can construct the Poincare maps by single measurement, thus it is helpful to decrease the possible cost in measurement. However, it must be mentioned that the accuracy and effectiveness of the present method will never exceed the method based on the full measurement, as the algorithm proposed in this method only provide the more accurate approximations rather than the actual values [13-15].

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References
[1] Khatir A, Tehami M, Khatir S and Abdel Wahab M, Multiple damage detection and localization in beam-like and complex structures using co-ordinate modal assurance criterion combined with firefly and genetic algorithms Journal of Vibroengineering 2016; 18(8): 5063-5073.
[2] Zhou Y L, Maia N M M, Sampaio R P C, Wahab MA. Structural damage detection using transmissibility together with hierarchical clustering analysis and similarity measure. Structural Health Monitoring, 2016; doi: 10.1177/1475921716680849.
[3] Gillich G-R, Praisach Z-I, Abdel Wahab M, Gillich N, Mituletu IC and Nitescu C, Free vibration of a perfectly clamped-free beam with stepwise eccentric distributed masses, Shock and Vibration, 2016; http://dx.doi.org/10.1155/2016/2086274, 10 pages.
[4] Zhou Y L, Maia N M M, Wahab M A, Damage detection using transmissibility compressed by principal component analysis enhanced with distance measure. Journal of Vibration and Control, 2016; doi: 10.1177/1077546316674544.
[5] Zhou Y-L and Abdel Wahab M, Rapid early damage detection using transmissibility with distance measure analysis under unknown excitation in long-term health monitoring Journal of Vibroengineering, 2016; 18(7) 4491-4499.
[6] Khatir S, Belaidi I, Serra R, Abdel Wahab M and Khatir T, Numerical study for single and multiple damage detection and localization in beam-like structures using BAT algorithm, Journal of Vibroengineering, 2016; 18(1) 202-213.
[7] Khatir S, Belaidi I, Serra R, Abdel Wahab M and Khatir T, Damage detection and localization in composite beam structures based on vibration analysis, Mechanika, 2015; 21(6) 472-479.
[8] Yang ZB, Chen XF, Xie Y, Zhang XW. The hybrid multivariate analysis method for damage detection. Structural Control and Health Monitoring. 2016;23:123-43.
[9] Yang ZB, Chen XF, Xie Y, Miao HH, Gao JJ, Qi KZ. Hybrid two-step method of damage detection for plate-like structures. Structural Control and Health Monitoring. 2016;23:267-85.
[10] Manoach E, Samborski S, Mitura A, Warminski J. Vibration based damage detection in composite beams under temperature variations using Poincare maps. International Journal of Mechanical Sciences. 2012;62:120-32.
[11] Trendafilova I, Manoach E. Vibration-based damage detection in plates by using time series analysis. Mechanical Systems and Signal Processing. 2008;22:1092-106.
[12] Manoach E, Trendafilova I. Large amplitude vibrations and damage detection of rectangular plates. Journal of sound and vibration. 2008;315:591-606.
[13] Yang Z-B, Radzienski M, Kudela P, Ostachowicz W. Two-dimensional modal curvature estimation via Fourier spectral method for damage detection. Composite Structures. 2016;148:155-67.
[14] Yang Z-B, Radzienski M, Kudela P, Ostachowicz W. Scale-Wavenumber Domain Filtering Method for Curvature Modal Damage Detection. Composite Structures. 2016;154:396-409.
[15] Yang Z-B, Radzienski M, Kudela P, Ostachowicz W, Fourier spectral-based modal curvature analysis and its application to damage detection in beams. Mechanical Systems and Signal Processing. 2017;84:763–81.