Spin-orbit splitting in low-\(j\) neutron orbits
and proton densities in the nuclear interior

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(Dated: March 30, 2022)

On the basis of relativistic mean field calculations, we predict that the spin-orbit splitting of \(p_{3/2}\) and \(p_{1/2}\) neutron orbits depends sensitively on the magnitude of the proton density near the center of the nucleus, and in particular on the occupation of \(s_{1/2}\) proton orbits. We focus on two exotic nuclei, \(^{46}\text{Ar}\) and \(^{206}\text{Hg}\), in which the presence of a pair of \(s_{1/2}\) proton holes is predicted to cause the splitting between the \(p_{3/2}\) and \(p_{1/2}\) neutron orbits near the Fermi surface to be much smaller than in the nearby doubly-magic nuclei \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\). We note that these two exotic nuclei depart from the long-standing paradigm of a central potential proportional to the ground state baryon density and a spin-orbit potential proportional to the derivative of the central potential.

PACS numbers: 21.10.-k,21.10.Ft,21.10.Pc

One of the primary motivations for the study of exotic nuclei is to search for novel shell structure effects. A large amount of attention has been paid to the possibility that the spin-orbit force on high-\(j\) neutron orbits weakens in nuclei near the neutron drip line. The neutron magic numbers for stable nuclei rely on the effect of the proton density near the center of the nucleus, and in particular on the occupation of \(s_{1/2}\) proton orbits. We focus on two exotic nuclei, \(^{46}\text{Ar}\) and \(^{206}\text{Hg}\), in which the presence of a pair of \(s_{1/2}\) proton holes is predicted to cause the splitting between the \(p_{3/2}\) and \(p_{1/2}\) neutron orbits near the Fermi surface to be much smaller than in the nearby doubly-magic nuclei \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\). We make these predictions using the magic numbers in neutron-rich nuclei. The possibility of this force has the potential to change the neutron skin of heavy nuclei and a variety of neutron-star properties.

In both doubly-magic nuclei \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\), the highest lying proton orbits below the Fermi surface (or the lowest energy proton hole states in \(^{47}\text{K}\) and \(^{207}\text{Tl}\)) are \(s_{1/2}\) orbits. The effect of removing a pair of \(s_{1/2}\) protons from \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\) is illustrated in Fig. 1, which compares the proton densities of \(^{46}\text{Ar}\) and \(^{48}\text{Ca}\) (upper panel), and the proton densities of \(^{206}\text{Hg}\) and \(^{208}\text{Pb}\) (lower panel). It is important to note that the predicted root-mean-square charge radii for \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\) are in excellent agreement with experiment.

As the \(s_{1/2}\) wavefunctions are strongly peaked in the center of the nucleus, the removal of these protons from \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\) results in sharply reduced proton densities in the centers of \(^{46}\text{Ar}\) and \(^{206}\text{Hg}\). This, in turn, causes a sharp increase in the magnitude of the spin-orbit interaction in the nuclear interior. Figure 2(a) illustrates this effect in \(^{208}\text{Pb}\) and \(^{206}\text{Hg}\): inside of 2 fm, \(V_{\text{so}}\) is much stronger — and of the opposite sign — in the \(s_{1/2}\) nucleus \(^{206}\text{Hg}\) than in the doubly-magic \(^{208}\text{Pb}\) core. This unconventional behavior of the spin-orbit potential is intimately related to the Lorentz structure of the Dirac mean fields. While the depletion of \(s_{1/2}\) proton strength manifests itself in both the vector and scalar densities, the “proton-hole” disappears from the central potential as a result of the sensitive cancellation between the attractive scalar and the repulsive vector potentials. In contrast, (the derivatives of) the scalar and vector potentials add constructively in the spin-orbit potential and the development of a non-trivial spin-orbit structure in the interior of the nucleus ensues.

Figures 2(b)-(c) illustrate the effect of folding the spin-orbit potential in \(^{208}\text{Pb}\) and \(^{206}\text{Hg}\) with the \(3p_{3/2}\) and \(3p_{1/2}\) neutron wavefunctions. That is, we display

\[
\Delta V_{\text{so}}(r) \equiv \int_0^r dr' V_{\text{so}}(r') \left[ 2u_{p_{3/2}}^2(r') + u_{p_{1/2}}^2(r') \right].
\]
Expressions for the Schrödinger-equivalent spin-orbit potential and wavefunctions \([u(r)]\) may be found in Ref. \([31]\). For the purposes of this study, normalized wavefunctions have been used. Note that the above quantity, while not exact, provides an accurate (first-order) estimate of the \[p_{3/2}/p_{1/2}\] spin-orbit splitting \[\Delta V_{so} \equiv \Delta V_{so}(r \to \infty)\].

The combined effect of a strong increase in \(V_{so}\) in the interior of \(^{206}\text{Hg}\) together with neutron wavefunctions that are much larger at small radii than larger \(j\) orbits, yields a big effect on the integrated spin-orbit energy and, therefore, on the spin-orbit splitting for the \(p\)-neutrons. Indeed, this effect leads to the collapse of the \(p_{3/2}/p_{1/2}\) spin-orbit splitting: from \(-0.60\) MeV in \(^{208}\text{Pb}\) to \(-0.02\) MeV in \(^{206}\text{Hg}\). A similar effect occurs in \(^{46}\text{Ar}\) relative to \(^{48}\text{Ca}\): the spin-orbit splitting is reduced by almost an order of magnitude, as shown in Fig. 3. Figure 4 displays the exact values. We conclude that in the two exotic nuclei \(^{46}\text{Ar}\) and \(^{206}\text{Hg}\), the spin-orbit interaction ceases to be a surface-dominated phenomenon.

Figure 1 summarizes the calculations and compares the experimental and calculated binding energies for \(p_{3/2,1/2}\) neutron orbits in \(^{48}\text{Ca}\) and \(^{208}\text{Pb}\). It should be emphasized that while models with nonlinear couplings between the isoscalar and isovector mesons change the energy of the individual \(p_{3/2}\) and \(p_{1/2}\) orbitals slightly, the prediction for their spin-orbit splitting is largely model independent. Thus, we limit ourselves to the original NL3 set of Ref. \([24]\). The experimental binding energies for \(^{48}\text{Ca}\) are taken from the \(^{48}\text{Ca}(d,p)\) measurement of Uozumi et al. \([32]\) and the mass compilation of Audi and collaborators \([33]\). The \(^{208}\text{Pb}(p,d)\) data used to extract the binding energies for \(^{208}\text{Pb}\) are taken from the compilation of Martin \([34]\); the mass data are taken from Ref. \([33]\).

For stable nuclei, the standard experimental technique for mapping single neutron strength in a nucleus is to use a stripping reaction such as \((d,p)\). To differentiate between spin-orbit partners (such as \(p_{3/2}\) and \(p_{1/2}\)) a polarized deuteron beam would be used (as in \([32]\)). For the exotic nucleus \(^{46}\text{Ar}\), such a measurement would be performed in inverse kinematics with a \(^{46}\text{Ar}\) beam and polarized deuteron target. The measurement would further be complicated by the likelihood that the \(p_{3/2}\) and \(p_{1/2}\) strengths would be somewhat fragmented, as they are in \(^{51}\text{Ti}\), \(^{53}\text{Cr}\) and \(^{55}\text{Fe}\) \([32]\). In \(^{206}\text{Hg}\), the \(p_{3/2,1/2}\) orbits would be observed as holes, requiring the use of the pickup reaction \((p,d)\) in inverse kinematics, once again.
FIG. 3: (a) The Schrödinger-equivalent spin-orbit potential for $^{48}$Ca (solid line) and $^{46}$Ar (dashed line). Panels (b) and (c) display the effect of folding the spin-orbit potential with the Schrödinger-equivalent $p$-orbitals, as defined in Eq. (1). The arrows point to a first-order estimate of the spin-orbit splitting.

with a polarized target to differentiate between spin-orbit partners. For example, the normal kinematics experiment $^{208}$Pb($p,d$) with a polarized beam is reported in [36].

In summary, we have used relativistic mean field calculations to predict that the spin-orbit splitting of $p_{3/2}$ and $p_{1/2}$ neutron orbits depends sensitively on the magnitude of the proton density near the center of the nucleus, and in particular on the occupation of $s_{1/2}$ proton orbits. The quenching (or collapse) of the spin-orbit splitting in high-$j$ neutron orbits has been advertised as the hallmark for novel nuclear-structure effects in neutron-rich nuclei. This collapse is associated with the development of a diffuse neutron-rich surface. In this communication we have proposed a new mechanism for the collapse of the spin-orbit splitting — but among low-$j$ neutron orbits. This mechanism is based, not on a rearrangement of the neutron density at the surface of the nucleus, but rather, on a depletion of the proton density in the nuclear interior. Two exotic nuclei, $^{46}$Ar and $^{206}$Hg, may be accessible for the study of this effect. In these nuclei we predict that the presence of a pair of $s_{1/2}$ proton holes causes the splitting between the $p_{3/2}$ and $p_{1/2}$ neutron orbits near the Fermi surface to be much smaller than in the nearby doubly-magic nuclei $^{48}$Ca and $^{208}$Pb. Thus these two exotic nuclei, only two protons away from being doubly magic, deviate from a long-standing paradigm that has been applied with enormous success in both structure and reaction calculations, namely, that of a central potential proportional to the ground state baryon density and a spin-orbit potential proportional to the derivative of the central potential. Finally, while the effects proposed here are likely to be modified by correlations that go beyond the mean-field approximation, we trust that most of its novel qualitative features will hold true.

FIG. 4: Comparison between the experimental and calculated $p_{3/2}$/p$_{1/2}$ spin-orbit splitting for the doubly-magic nuclei $^{48}$Ca and $^{208}$Pb. Also shown is the predicted collapse of the spin-orbit splitting in the two exotic nuclei $^{46}$Ar and $^{206}$Hg.

This work was supported in part by the U.S. Department of Energy through grant DE-FG05-92ER40750, the National Science Foundation through grant PHY-0139950, and the State of Florida.
[1] T.R. Werner et al., Nucl. Phys. A597, 327 (1996).
[2] J. Terasaki, H. Flocard, P.-H. Heenen, and P. Bonche, Nucl. Phys. A621, 706 (1997).
[3] J. Retamosa, E. Courrier, F. Nowacki, and A. Poves, Phys. Rev. C 55, 1266 (1997).
[4] G.A. Lalazissis et al., Nucl. Phys. A628, 221 (1998).
[5] D.J. Dean, M.T. Ressell, M. Hjorth-Jensen, S.E. Koonin, K. Langanke, and A.P. Zuker, Phys. Rev. C 59, 2474 (1999).
[6] J. Dufo and A.P. Zuker, Phys. Rev. C 59, R2347 (1999).
[7] P-G. Reinhard et al., Phys. Rev. C 60, 014316 (1999).
[8] G.A. Lalazissis et al., Phys. Rev. C 60, 014310 (1999).
[9] B.V. Carlson and D. Hirata, Phys. Rev. C 62, 054310 (2000).
[10] S. Péru, M. Girod, and J.F. Berger, Eur. Phys. J. A 9, 35 (2000).
[11] M. Del Estal, M. Centelles, X. Viñas, and S.K. Patra, Phys. Rev. C 63, 044321 (2001).
[12] T. Otsuka et al., Phys. Rev. Lett. 87, 082502 (2001).
[13] P. Mitra, G. Gangopadhyay, and B. Malakar, Phys. Rev. C 65, 034329 (2002).
[14] R. Rodríguez-Guzmán, J.L. Egido, and L.M. Robledo, Phys. Rev. C 65, 024304 (2002).
[15] T. Glasmacher et al., Phys. Lett. B 395, 163 (1997).
[16] F. Sarazin et al., Phys. Rev. Lett. 84, 5062 (2000).
[17] D. Sohler et al., Phys. Rev. C 66, 054302 (2002).
[18] M. Notani et al., Phys. Lett. B 542, 49 (2002).
[19] W. Nazarewicz and R.F. Casten, Nucl. Phys. A682, 295c (2001).
[20] R.F. Casten, B.M. Sherrill, Prog. Part. Nucl. Phys. 45, S171 (2000).
[21] B.G. Todd and J. Piekarewicz, Phys. Rev. C 67, 044317 (2003).
[22] C.J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. 86, 5647 (2001).
[23] C.J. Horowitz and J. Piekarewicz, Phys. Rev. C 64, 062802 (2001).
[24] G.A. Lalazissis, J. König, and P. Ring, Phys. Rev. C 55, 540 (1997).
[25] G.A. Lalazissis, S. Raman and P. Ring, At. Data Nucl. Data Tables 71, 1 (1999).
[26] H. Müller and B.D. Serot, Nucl. Phys. A606, 508 (1996).
[27] C.J. Horowitz and J. Piekarewicz, Phys. Rev. C 66, 055803 (2002).
[28] J. Carriere, C.J. Horowitz and J. Piekarewicz, nucl-th/0211015
[29] H. de Vries, C.W. de Jager, and C. de Vries, Atomic Data and Nucl. Data Tables. 36, 495 (1987).
[30] B.D. Serot and J.D. Walecka, Adv. in Nucl. Phys. 16, J.W. Negele and E. Vogt, eds. (Plenum, N.Y. 1986); Int. Jour. Mod. Phys. E6, 515 (1997).
[31] J. Piekarewicz, Phys. Rev. C 48, 2174 (1993).
[32] Y. Uozumi et al., Nucl. Phys. A576, 123 (1994).
[33] G. Audi et al., Nucl. Phys. A624, 1 (1997).
[34] M.J. Martin, Nucl. Data Sheets 70, 315 (1993).
[35] D.C. Kocher and W. Haeberli, Nucl. Phys. A196, 225 (1972).
[36] M. Matoba et al., Phys. Rev. C 55, 3152 (1997).