Relativistic quantum speed limit time in dephasing noise

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Abstract

The behavior of quantum speed limit time (QSLT) for a single free spin $-1/2$ particle described by Gaussian wavepackets in the framework of relativity under dephasing noise is investigated. The dephasing noise acts only on the spin degrees of freedom of the spin$-1/2$ particle. In particular, the effects of initial time parameter, rapidity, average momentum and the size of the wavepackets in the presence of the dephasing noise on the dynamics of evolution process are studied. In general, the effects of relativity monotonically decrease the QSLT in time. In the range of large values of average momentum, critical values of both the rapidity and the size of the wavepackets exist at which the QSLT has its minimum value. In the range of small values of the average momentum, the QSLT monotonically decreases with both rapidity and the size of the wavepackets. The decrease of QSLT in a particular range of rapidity and with other relative parameters may be of great interest in employing fast quantum communication and quantum computation.

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I. INTRODUCTION

In quantum information theory, entanglement plays a vital role due to which investigating its dynamics under a variety of situations has been the focus for a long time [1]. The study of Quantum information processing in the framework of relativity is a challenging task and is presently under exploration. Initially, focused on the study of the dynamics of bipartite entanglement under different circumstances [2–6], the theoretical analysis of its effects to other scenarios have also been lately extended [7–9]. The output of such studies is an established fact that entanglement is a relative quantity and degrades with the acceleration of the observer frame. Now the question arises whether its only the entanglement that depends on the frame of reference or others quantum mechanical properties of a quantum system also behave this way. Keeping this in mind, we wish to investigate the effect of relativity on quantum speed limit time (QSLT) of a single spin−1/2 particle described by Gaussian wavepackets in the presence of dephasing noise that acts only on its spin degrees of freedom.

The evolution of a quantum system from an initial state to one of its allowed orthogonal states is not instantaneous rather the laws of quantum mechanics put a speed limit to the evolution within the Hilbert space of the system. The role of such limits has already been studied in different setups, such as, the identification of precision bounds in quantum metrology [10], quantum computation [11], the formulation of computational limits of physical systems [12] and the development of quantum optimal control algorithms [13]. The QSLT is the minimum time a quantum system takes to evolve within its Hilbert space from one to another allowed state. Different lower bounds on QSLT for isolated system have been obtained [14–17] and are then extended to nonorthogonal states as well as to driven systems [18–20]. The QSLT of a pure initial state as an open system for a given driving time and with non-Markovian dynamics has also been investigated [21–23]. Recently, based on the relative purity as the distance measure, a scheme has been developed to measure QSLT for the nonunitary evolution of open mixed initial state [24].

In this paper we use the approach of Ref. [24] and find the effects of different parameters on the QSLT of the spin degrees of freedom of spin −1/2 particle in the framework of relativity. We consider that the system is initially in a coherent superposition state and is coupled to an Ohmic-like dephasing environment. Our investigation focuses on the dynamics
of QSLT both under Ohmic and super-Ohmic dephasing environment. We show that the effect of the relative motion of the observer on the evolution process of the system is alike irrespective of the initial coherence in the state of the system. Our findings show that in the Markovian regime the relative motion of the detector speeds up the evolution process in time. There exists a critical value $\alpha_c$ of the rapidity in the limit of large normalized averaged momentum at which the speed of the evolution process is maximum both under the action of Ohmic and super-Ohmic reservoirs. Beyond $\alpha_c$, it slows down until it reaches a saturation value and becomes static. It is found that at constant rapidity a similar behavior exists against the width $W$ of the wavepacket.

II. THEORETICAL MODEL

We consider the time evolution of the spin degrees of freedom of a spin $-1/2$ particle (qubit), observed by a moving detector, interacting with a dephasing environment. Such a scenario can well be described in terms of the following Hamiltonian [25–27]

$$H = \frac{1}{2} \omega_0 \sigma_z + \sum_j \omega_j a_j^\dagger a_j + \sigma^z \sum_j (g_j a_j^\dagger + g_j^* a_j),$$

where the first and the second terms describe the independent evolution of the qubit and the environment, respectively. The third term of the Hamiltonian describes the interaction between the qubit and the environment whose strength with the $j$th mode of the field of frequency $\omega_j$ is specified by the constant $g_j$. The creation and annihilation operators of the $j$th mode of the environment obey the usual commutation relation $[a_j, a_{j'}^\dagger] = \delta_{j,j'}$. In Eq. (1), the $\omega_0$ and $\sigma_z$, respectively, represent the transition frequency and the evolution operator of the qubit. It is easy to check that Eq. (1) commutes with $\sigma_z$ thereby limiting the off-diagonal elements of the density operator, describing the coherence of the system, to zero and hence leaving the population terms unchanged. We begin from a factorized initial composite state of the qubit and the environment with the environment being in its vacuum state at zero temperature. In the limit of very large environment, the sum over the discrete coupling constants $g_j$ between the different modes of the environment and the qubit can be replaced by an integral over a continuous distribution of frequencies of the environment modes, that is, $\sum_j |g_j|^2 \to \int_0^\infty J(\omega) d\omega$, where $J(\omega)$ stands for the spectral density of the
environmental modes. For an Ohmic like dephasing model, it can be expressed as follows

\[ J(\omega) = \frac{\omega^n}{\omega_c^{n-1}} \eta e^{-\omega/\omega_c}, \]

where \( \omega_c \) is the cutoff frequency and \( \eta \) is a dimensionless coupling constant. The spectral density is called sub-Ohmic, Ohmic and super-Ohmic for \( n < 1 \), \( n = 1 \) and \( n > 1 \), respectively. We will limit our analysis only to the last two cases.

The evolved state of an open qubit at any time \( t \) can be written in terms of Kraus operators as follows

\[ \rho(t) = \sum_{k=1}^{2} E_k \rho(\tau) E_k^\dagger, \]

where \( \rho(\tau) \) is the initial density matrix of the system at any initial time \( \tau \) and \( E_k \) are the single qubit Kraus operators which are given by

\[ E_1 = |0\rangle\langle 0| + p_t |1\rangle\langle 1|, \quad E_2 = \sqrt{1 - p_t^2} |1\rangle\langle 1|, \]

where \( p_t = e^{-\gamma(t)} \) with \( \gamma(t) \) being the dephasing rate and is given by

\[ \gamma(t) = \eta \omega_c \Gamma(n) \int_0^t \sin[n \arctan(\omega_c t')](1 + (\omega_c t')^2)^{-n/2} dt', \]

where \( \Gamma(n) \) is the Euler Gamma function. For an Ohmic reservoir, it takes the form \( \gamma(t) = \eta \ln(1 + (\omega_c t)^2) \). Also, the nonunitary generator of the reduced dynamics of the system for the quantum dephasing channel is given by

\[ L\rho(t) = \frac{\gamma(t)}{2} [\sigma_z \rho(t) \sigma_z - \rho(t)]. \]

Next, we specify the state of the qubit as our system. The generic state, in the momentum representation, of a qubit in the laboratory frame can be expressed as \[28, 29]\n
\[ |\psi(p)\rangle = f^w_k(p) \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \]

where \( \theta = [0, \pi/4] \) and describes the initial level of coherence between the two allowed states, \( f^w_k(p) = \pi^{-3/4} w^{-3/2} \exp[-(p - k)^2/2w^2] \) with \( w \in \mathbb{R}_+ \) is a measured dispersion in momentum and \( k = (k,0,0) \) represents the average momentum. In the proper frame of the detector, which is moving with relativistic speed \( v \) with respect to the laboratory frame, the state of the qubit transforms through a unitary transformation given by \[30, 31]\n
\[ |\psi(p)\rangle \rightarrow |\varphi(p)\rangle = U(\Lambda) |\psi(p)\rangle, \]
with
\[ U(\Lambda) |\psi(p)\rangle = \sqrt{\frac{q_0}{p_0}} D \left( \Lambda, \Lambda^{-1} p \right) |\psi(p)\rangle. \] (9)

where \( D \left( \Lambda, \Lambda^{-1} p \right) \) is the well known Wigner rotation and is explicitly given by
\[ D \left( \Lambda, \Lambda^{-1} p \right) = \left( \frac{p_0 + m}{(q_0 + m)(p_0 + m)} \right)^{1/2} \]
\[ \begin{bmatrix}
\cosh \alpha & \sinh \alpha & 0 & 0 \\
\sinh \alpha & \cosh \alpha & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}. \] (10)

In Eq. (10), \( \alpha = -\tanh^{-1}(v/c) \) stands for the rapidity, \( e = (e_x, 0, 0) \) is the direction of the boost, \( q = (q_x, q_y, q_z) \) gives the spatial part of the four-vector with \( q = \Lambda^{-1} p \), \( \sigma_0 \) and \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) are 2x2 identity and Pauli matrices, respectively. Note that from this point onward we will set \( c = 1 \).

For the boost in the \( x \) direction, we have
\[ \Lambda = \begin{bmatrix}
\cosh \alpha & \sinh \alpha & 0 & 0 \\
\sinh \alpha & \cosh \alpha & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}. \] (11)

The explicit form of the transformed wavefunction of the qubit in the detector frame is obtained through back substitution of Eq. (10) into Eq. (9), the result of which along with Eq. (7) into Eq. (8) and is given by
\[ |\varphi(p)\rangle = \cos \theta \begin{pmatrix} a_1(p) \end{pmatrix} + \sin \theta \begin{pmatrix} -a_2(p) \\ a_1(p)^\ast \end{pmatrix}, \] (12)

with
\[ a_1(p) = kf_k(q) \left[ C(q_0 + m) + S(q_x + iq_y) \right], \]
\[ a_2(p) = kf_k(q) Sq_z, \]
\[ k = \left[ \left( \frac{q_0}{p_0} \right) / (q_0 + m)(p_0 + m) \right]^{1/2}, \] (13)

where the asterisk shows complex conjugate and the new parameters are defined as \( C = \cosh \alpha/2 \), and \( S = \sinh \alpha/2 \).

In order to investigate the behavior of QSLT for our system, we first want to review the main results for the different measures of the bounds of QSLT both for pure and mixed initial states proved in [14–17, 24]. Especially, we focus our review on the recent measure of QSLT [24] derived in terms of relative purity \( f(t) \) between the initial and the final states of a quantum system which is given as
\[ f(t) = \frac{\text{tr} (\rho(t)\rho(\tau))}{\text{tr} (\rho(\tau)^2)}, \] (14)
where \( \rho(\tau) \) is the initial density matrix at a time \( \tau \), and \( \rho(t) \) represents the density matrix at a later time \( t = \tau + \Delta \tau \) with \( \Delta \tau \) being the driving time. Differentiating Eq. (14) with respect to \( t \) and using \( \dot{\rho}(t) = \mathcal{L}\rho(t) \) with \( \mathcal{L} \) being the superoperator representing the nonunitary reduced dynamics of the system, we get

\[
\dot{f}(t) = \frac{tr(\mathcal{L}\rho(t)\rho(\tau))}{tr(\rho(\tau)^2)}.
\]  (15)

If \( \lambda_j \) stand for the singular values of the initial density matrix \( \rho(\tau) \) and \( \mu_j \) represent the singular values of \( \mathcal{L}\rho(t) \), then with the help of von Neumann trace inequality, the absolute of the numerator of Eq. (15) can be expressed as follows

\[
|tr(\mathcal{L}\rho(t)\rho(\tau))| \leq \sum_{j=1}^{m} \lambda_j \mu_j.
\]  (16)

Since \( 0 < \lambda_j \leq 1 \), therefore, \( \|\mathcal{L}\rho(t)\|_t = \sum_{j=1}^{m} \mu_j \geq \sum_{j=1}^{m} \lambda_j \mu_j \), where \( \|\cdot\|_t \) represents the trace norm of an operator. With the use of this condition, Eq. (15) and Eq. (16) give

\[
|\dot{f}(t)| \leq \frac{\sum_{j=1}^{m} \mu_j}{tr(\rho(\tau)^2)}.
\]  (17)

Integrating Eq. (17) with respect to \( t \) between \( t = \tau \) and \( t = \tau + \Delta \tau \), the result can be expressed in the form of the following inequality

\[
\tau \geq \max \left\{ \frac{1}{\sum_{j=1}^{m} \lambda_j \mu_j}, \frac{1}{\sum_{j=1}^{m} \mu_j} \right\} |f(t) - 1| tr(\rho(\tau)^2).
\]  (18)

In Eq. (18), the bar over the terms in the denominator represents the average over time and can be written as

\[
\overline{X} = \frac{1}{\Delta \tau} \int_{\tau}^{\tau + \Delta \tau} X \, dt.
\]  (19)

Using the fact that under unitary transformation, \( \sum_{j=1}^{m} \mu_j \) stands for the energy of the system averaged over time, the Margolus-Levitin (ML) \cite{ML1, ML2} type bound on the speed of evolution for closed systems from Eq. (18) can be expressed as

\[
\tau \geq \frac{|f(t) - 1| tr(\rho(\tau)^2)}{2\sum_{j=1}^{m} \mu_j}.
\]  (20)

On the other hand, using the inequality \( \sum_{j=1}^{m} \lambda_j \mu_j \leq \sum_{j=1}^{m} \mu_j \) or \( \frac{1}{\sum_{j=1}^{m} \lambda_j \mu_j} \geq \frac{1}{\sum_{j=1}^{m} \mu_j} \), Eq. (18) modifies to the following form

\[
\tau \geq \frac{|f(t) - 1| tr(\rho(\tau)^2)}{\sum_{j=1}^{m} \lambda_j \mu_j}.
\]  (21)
Eq. (21) gives the ML type bound on the speed of nonunitary dynamics of open systems with mixed initial states.

Similarly, taking absolute of Eq. (15) and employing the Cauchy-Schwarz inequality for operators, it is straightforward to arrive at the following result

\[ |\dot{f}(t)| \leq \sqrt{\text{tr}(\mathcal{L}\rho(t)\dagger\mathcal{L}\rho(t)) \text{tr}(\rho(\tau)^2) / \text{tr}(\rho(\tau)^2)} \quad (22) \]

For mixed initial state \( \rho(\tau) \), \( \text{tr}(\rho(\tau)^2) < 1 \), which implies that \( \sqrt{\text{tr}(\mathcal{L}\rho(t)\dagger\mathcal{L}\rho(t)) \text{tr}(\rho(\tau)^2) < \sqrt{\text{tr}(\mathcal{L}\rho(t)\dagger\mathcal{L}\rho(t))}}. \) Using this condition, the above equation can be reduced to the following form

\[ |\dot{f}(t)| \leq \sqrt{\sum_{j=1}^{m} \mu_j^2} \quad (23) \]

where the use of Hilbert-Schmidt norm given by \( \sqrt{\text{tr}(\mathcal{L}\rho(t)\dagger\mathcal{L}\rho(t))} = \|\mathcal{L}\rho(t)\|_{hs} = \sqrt{\sum_{j=1}^{m} \mu_j^2} \) is made. Integration of Eq. (23) leads to the following Mandelstam-Tamm (MT) \([14]\) type bound on QSLT for nonunitary dynamics of quantum systems

\[ \tau \geq \frac{|f(t) - 1| \text{tr}(\rho(\tau)^2)}{\sqrt{\sum_{j=1}^{m} \mu_j^2}} \quad (24) \]

with

\[ \sqrt{\sum_{j=1}^{m} \mu_j^2} = \frac{1}{\Delta t} \int_{\tau}^{t} \sqrt{\sum_{j=1}^{m} \mu_j^2} \mathrm{d}t, \quad (25) \]

as the time averaged variance of energy. A unified relation for QSLT of arbitrary mixed initial state interacting with environment can be obtained by combining Eqs. (21) and (24) as follows \([14]\)

\[ \mathcal{T}_{QSL} = \max \left[ \frac{1}{\sqrt{\sum_{j=1}^{m} \mu_j^2}} \frac{1}{\sum_{j=1}^{m} \lambda_j \mu_j} \right] |f(t) - 1| \text{tr}(\rho(\tau)^2). \quad (26) \]

Since the singular values \( \lambda_j \) of a pure initial state \( \rho(\tau) \) obey the condition \( \lambda_j = \delta_{j,1} \), therefore, \( \sum_{j=1}^{m} \lambda_j \mu_j = \mu_1 \leq \sqrt{\sum_{j=1}^{m} \mu_j^2} \). This result is in agreement with the one obtained in \([23]\).

With all the required tools in hand, we can now use them to find the dynamics of QSLT for a qubit in the relativistic framework. The initial density matrix we start from corresponds to the state given in Eq. (12) whose explicitly form becomes

\[ \rho(0) = \frac{1}{2} \begin{pmatrix} 1 + (1 - 2\chi) \cos 2\theta & (1 - 4\chi) \sin 2\theta \\ (1 - 4\chi) \sin 2\theta & 1 - (1 - 2\chi) \cos 2\theta \end{pmatrix}, \quad (27) \]
where the parameter $\chi = \chi(\alpha)$ represents the relativistic effect and is given by

$$\chi(\alpha) = \sinh^2(\alpha/2) \int \frac{q^2}{(q_0 + m)(p_0 + m)} |f_k^w(q)|^2 dq. \quad (28)$$

The analytical solution of Eq. (28) is difficult, however, we can solve it numerically by first transforming it into cylindrical coordinates with $q_x$ as the symmetry axis and defining $Q_r = q_r/m$, $Q_x = q_x/m$, $W = w/m$, $K = k/m$ and $Q_0 = \sqrt{Q_r^2 + Q_x^2 + 1}$ as the normalized nondimensional variables.

The time evolution of the density matrix is obtained by using Eq. (3) and can be expressed as

$$\rho(t) = \frac{1}{2} \begin{pmatrix} 1 + (1 - 2\chi) \cos 2\theta & p_t(1 - 4\chi) \sin 2\theta \\ p_t(1 - 4\chi) \sin 2\theta & 1 - (1 - 2\chi) \cos 2\theta \end{pmatrix}. \quad (29)$$

Similarly the nonunitary dynamics of the system are obtained by using Eq. (6) and can be written as

$$L\rho(t) = \frac{1}{2} \begin{pmatrix} 0 & p_t(4\chi - 1)\gamma(t) \sin 2\theta \\ p_t(4\chi - 1)\gamma(t) \sin 2\theta & 0 \end{pmatrix}. \quad (30)$$

In order to investigate the QSLT, we need to find the singular values of Eqs. (29) and (30). For Eq. (29) these are given as follows

$$\lambda_\pm = \frac{1}{2} \pm \frac{1}{2\sqrt{2}} \sqrt{p_t^2(1 - 4\chi)^2 + (1 - 2\chi)^2 - (p_t^2(1 - 4\chi)^2 - (1 - 2\chi)^2) \cos 4\theta}. \quad (31)$$

Similarly, the singular values of Eq. (30) can be expressed as follows

$$\mu_1 = \mu_2 = \frac{1}{2} |\gamma(t)p_t(1 - 4\chi)| \sin 2\theta. \quad (32)$$

With the set of eigenvalues given above, the ML type bound is satisfied and the QSLT for a qubit in the relativistic framework becomes

$$\mathcal{T}_{QSL} = \frac{|(1 - 4\chi)(p_t p_r - p_r^2)|}{\Delta \frac{\tau + \tau^D}{\tau} |p_t| dt} \sin 2\theta. \quad (33)$$

Setting $\chi = 0$ and the initial coherence term $\sin 2\theta = [C(\rho_0)]^{1/2}$, Eq. (33) straightaway goes to the result of [24]. The presence of $\chi$ in Eq. (33), reveals that relativity affect the QSLT for quantum systems. Since the term describing the initial coherence factors out from the terms inside the brackets containing $\chi$, therefore, the effect of relativity on all initial states, regardless of the degree of initial coherence, is same. One can easily observe that with the increasing value of $\chi$ and freezing all the other variables to some constant values, first QSLT
decreases reaching a minimum equal to zero at the critical value of $\chi = 1/4$ and then start increasing. This means that at the critical value of $\chi$ the evolution of the quantum system is instantaneous. Nevertheless, we believe that observing this effect is not possible as $\chi$ is itself a function of other parameters such as $\alpha$, $K$ and $q$ that limit the instantaneous evolution of quantum system. The different choices of these parameters give rise to some novel results for QSLT, which we, next, demonstrate them graphically. In the Markovian regime \[33\], Eq. \[33\] reduces to the following form

$$T_{QSL} = p \tau \Delta \tau |1 - 4\chi| \sin 2\theta.$$  \[34\]

We use this equation to explicitly further investigate the effects of other parameters on the dynamics of QSLT by limiting our analysis to the Markovian regime.

III. DISCUSSION

To get a deep insight into the influence of relativity on the dynamics of QSLT, we plot it against different parameters under various conditions. In figure (1), we plot the QSLT against $\tau$ in the presence of environment both for relativistic (infinite rapidity) and nonrelativistic cases. The solid curves represent its behavior under the influence of Ohmic reservoir and the dashed curves show its dynamics under the effect of Super-Ohmic reservoir. One can see that the qualitative behavior of QSLT under the effect of relativity in both Ohmic and Super-Ohmic limits remains unchanged. However, quantitatively it is damped such that the damping itself becomes a function of $K$. For large $K$ the damping is moderate and for small $K$ it is strong. The notable difference between the effect of the two limits of the reservoir is that for Ohmic case it goes to zero at different times depending on the choice of the value of $K$ whereas for super-Ohmic case it reaches a stable and static value, different for every choice of $K$.

The behavior of QSLT against rapidity for different choices of $\tau$ is shown in figure (2a). Again the solid curves in both subfigures represent the effect of Ohmic reservoir and the dashed ones show the effect of super-Ohmic reservoir. Besides the relatively large damping caused by the Ohmic reservoir, the effect of both reservoirs is qualitatively identical. All the curves in figure (2a) correspond to $K = 100$ and that in figure (2b) correspond to $K = 0.01$. In figure (2b), regardless of the choice of $\tau$, there is a strict monotonous decrease in QSLT
FIG. 1: (Color Online) The $\mathcal{T}_{QSL}^R$ as a function of the initial time parameter $\tau$ by choosing the other parameters such that $\Delta\tau = 1$, $\eta = 1$, $\omega_c = 1$, $W = 4$ and $\theta = \pi/4$. The solid (dashed) curves correspond to Ohmic (super-Ohmic) reservoir. The red curves present zero relativistic effect ($\alpha = 0$) whereas the blue and the purple curves correspond to $K = 100$ and $K = 0.01$, respectively, in the limit of infinite rapidity.

with the increasing of rapidity which results to a near zero value at a critical value of $\alpha_c$ for all $\tau$ both for Ohmic and super-Ohmic reservoir. With further increase in rapidity, the QSLT monotonously increases reaching a saturation value different for each choice of $\tau$. In other words, the evolution process constantly speeds up for $\alpha < \alpha_c$ and then speeds down for $\alpha > \alpha_c$ until it becomes constant at large value of $\alpha$. On the other hand, for $K = 0.01$ (figure(2b)) there is no decelerating effect in the evolution process. The QSLT for every choice of $\tau$ decreases monotonously until it reaches a nonvanishing minimum constant value that results in uniform evolution process.

The effect of the width $W$ of the wavepacket for the same values of the time parameter $\tau$ as in figure (2) on the QSLT is shown in figure (3). Again figure (3a) corresponds to $K = 100$ and figure (3b) to $K = 0.01$. Similarly, the solid and the dashed curves in both subfigures, respectively, represent the influence of Ohmic and super-Ohmic reservoirs. As in figure (2b), here in figure (3b) also exists a critical value $W_c$ of the width of the wavepacket at which the QSLT, regardless of the value of $\tau$, reduces to a nonvanishing minimum value and then start increasing with the increasing value of $W$. However, the comparison of the
FIG. 2: (Color Online) The $T_{QSL}^R$ as a function of rapidity by choosing the other parameters such that $\Delta \tau = 1$, $\eta = 1$, $\omega_c = 1$, $W = 30$, $\alpha = \infty$, $\theta = \pi/4$ and (a) $K = 100$, (b) $K = 0.01$. The red, the blue, the purple and the green solid (dashed) curves correspond to the initial time parameter $\tau = 0, 0.5, 1, \infty$ for Ohmic(super-Ohmic) reservoir, respectively.

two figures immediately reveals that in figure (3a) the fall in QSLT in the region $W < W_c$ is sharper than in figure (2a). Similarly, in the region $W > W_c$, the behavior of QSLT is completely different than in figure (2a). Here is no saturation point, rather, beyond $W_c$ there is a relatively slower increase, reaching a maximum that happens at the half of its initial value and then falls gradually to a nonvanishing value for each choice of $\tau$. Unlike the effect of $W$ for large $K$, its effect for small $K$ (figure(3b)) is to relatively slow down the decrease in QSLT as compared to figure(2b).

IV. CONCLUSION

We investigate the effect of relative motion on the dynamics of quantum speed limit time for a single free spin–$1/2$ particle initially in a superposition state and is coupled to an Ohmic-like dephasing environment. In particular, the effects of rapidity, normalized momentum, the size of the wavepackets and the initial time parameter in the Markovian regime both for Ohmic and super-Ohmic reservoirs are considered. It is found that in the
FIG. 3: (Color Online) The $T_{QSL}^R$ as a function of the width of the wavepacket $W$ by choosing the other parameters such that $\Delta \tau = 1$, $\eta = 1$, $\omega_c = 1$, $\alpha = \infty$, $\theta = \pi/4$ and (a) $K = 100$ (b) $K = 0.01$. The red, the blue, the purple and the green solid (dashed) curves correspond to the initial time parameter $\tau = 0, 0.5, 1, \infty$ for Ohmic (super-Ohmic) reservoir, respectively.

The presence of relative motion, the coupling with the Ohmic reservoir constantly speeds up the evolution process without having an upper bound there by reducing the QSLT to zero as the evolution time increases, whereas the coupling with super-Ohmic reservoir has an upper limit on the speed at which the evolution process proceeds uniformly. The effect of rapidity on QSLT is not alike for the whole range of the average momentum. In the range of small $K$, it decreases monotonically to a constant minimum. On the other hand, in the range of large $K$ a critical value of rapidity exists at which it reduces to a nonvanishing minimum and then increases back until it becomes stationary. Although quantitatively different, the effect of the width of the wavepackets on QSLT is qualitatively parallel in some respects to that of the rapidity both in the small and the large ranges of the values of $K$ in the intermediate range of values of the two parameters. Nevertheless, in the limit of large values the analogy breaks, such that in the case of $W$ it again decreases after reaching a maximum. The results of our study may prove useful for exploring the speed of evolution of more complex quantum systems consists of many marginal system in the frame work of relativity that can be used
in quantum information processing in the presence of noisy environment.

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