Φ-variable calculated for the mixture of thermal sources

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Abstract. The Φpt-variable, a measure of statistical correlation of particle momenta, introduced by Mrówczyński and Gaździcki, has been calculated for the events simulated with a simple thermal toy model. It is shown that mixing two sources of unequal temperature can result in negative values of Φpt.

PACS. 25.75.Gz Particle correlations and fluctuations – 24.60.-k Statistical theory and fluctuations

1 Introduction

The Φ-variable (or function) was proposed by Mrówczyński and Gaździcki [1]. It is considered to be a good measure of equilibration. The value of Φ equals zero if a given type of equilibration (chemical or thermal in terms of momentum distribution) is reached. If a nucleus-nucleus collision is just a superposition of individual nucleon-nucleon collisions (without any further interaction that would lead towards equilibration) its value should be the same as for the elementary nucleon-nucleon collisions. Intermediate values of Φ would give some insight into the degree of thermalisation in the system.

Φ is not necessarily positive definite, but the majority of published papers report positive values of Φ, as would be expected from the “equilibration” arguments. There are papers (for instance [2]), where negative values of Φ are reported. This effect may be caused by a number of reasons like two-particle correlations or conservation laws. In this paper it is argued that mixing particles coming from sources of arbitrary temperatures. In particular, two or more equilibrated sources of different temperature negative values of Φpt are obtainable.

In the following the Φ-variable is defined and discussed. The applied toy model is described in more detail. The results are presented and finally some conclusions are drawn.

2 Φ-variable

The Φ-variable is a measure of fluctuations of a certain observable x (e.g. transverse momentum pt, charge, even baryonic number). Its idea is based on statistical considerations. The distribution of x is constructed and Φ is defined as the difference between widths of this distribution calculated event-wise and over the entire data set.

One can define z as the difference between x and its average, zk,j = xk,j − x, where k marks an event number, j counts the particles in the event, and the bar denotes global inclusive averaging. By construction z equals 0. For each event one may then define Zk = ∑ Njk=1 zk,j, by construction (Z) equals 0 (⟨x⟩ denote averaging over events).

Finally, one defines Φ as

\[ Φ = \sqrt{ \frac{(Z^2)}{N} - \sqrt{z^2} } . \]  

(1)

The averages of Z^2 and z^2 correspond to the variances of their respective distributions.

As a quick exercise Φpt may be “calculated” for the nucleon-nucleon elastic scattering. In each collision one obtains two nucleons with pt of the same size, therefore for each event Zk = 2 · zk and Nk = 2. By averaging and substituting those values in (1) one obtains Φ^{NN}_{pt} = (√2 − 1)√z^2. It is a positive, exact number depending on the energy.
3 Model description

In order to test the hypothesis that mixing of thermal sources can lead to negative values of $\Phi$ a simple toy model has been used. This model essentially allows to fold various distributions using a Monte Carlo technique.

The model generates “particles” with momentum according to simplest, non-relativistic, thermal-like distribution:

$$P(p) \sim p^2 \exp \left(-\frac{p^2}{2T}\right),$$

where $P$ denotes the probability and $T$ is a parameter (henceforth called “temperature”). Then two emission angles are drawn from the isotropic distribution, all 3 components of $p$ are calculated, and $p_1$ is deduced.

The events can contain particles from any number of sources, but in this study the number of sources was limited to two. The sources can be of any temperature. $N_i$, the number of particles coming from the $i$-th source, can either be the same in all events of a sample (“constant multiplicity”), or can be within each event selected according to the Poisson distribution (“Poisson multiplicity”). For this second case the $N_i$ shown is the mean of Poisson distribution.

This model contains no explicit correlations, the conservation laws are also not imposed.

Once a sample of events is generated, $\Phi_{pt}$ is calculated. Its uncertainty is estimated using the method of splitting the sample randomly into sub-samples of roughly equal sizes, calculating $\Phi_{pt}$ for each sub-sample, and finally calculating the variance of their distribution.

Formula (2) implies a universal mass of produced particles. As this formula is non-relativistic, the mass must fulfill the condition $M c^2 \gg T$.

The following conventions are used in the paper: subscripts denote source number, $T_i$ denotes the temperature of the $i$-th source and $N_i$ its multiplicity. $N = \sum_i N_i$ denotes the total (constant or average) multiplicity of the event.

The size of the event sample was chosen such that the estimated error is smaller than the size of the symbols or 1% relative.

4 Results

Figure 1 presents the results of an analysis of samples of events with the same total multiplicity of 100 split between two sources with constant multiplicities. The temperature $T_1$ was set to 100 MeV, $T_2$ was set to 8 values between 10 and 200 MeV, marked on the upper plot of fig. 1 with different symbols. The obtained values of $\Phi_{pt}$ are presented as a function of $N_2/N$ on the upper plot. For each $T_2$ the lowest value of $\Phi_{pt}$ is extracted and shown on the lower plot as a function of $T_2$.

One can see from fig. 1 that $\Phi_{pt}$ is negative definite, reaches a minimum for the admixture of around 55–60% of “softer” particles, and rises with rising difference between the source temperatures.

Figure 2 shows the data obtained for two sources with temperatures set to 100 and 15 MeV, respectively. The upper plot shows the dependence of $\Phi_{pt}$ on the total multiplicity, the results being presented in the same representation as in the upper plot of fig. 1. The symbols are spread in the horizontal direction to make the plot more readable, with different symbols denoting total multiplicities of 50, 100, 150 and 200. Within each sample $N_1$ and $N_2$ are constant.

The lower plot of fig. 2 shows how $\Phi_{pt}$ depends on the way multiplicities are obtained. $N_1$ and $N_2$ are either constant or drawn from a Poisson distribution.

The results for constant multiplicities are insensitive to the total multiplicity, $\Phi_{pt}$ depends on the differences in source temperatures and $N_2/N$. This is quite intuitive, as the widths of the $p_i$ sub-distributions should not depend on the number of particles taken into account from the same underlying distribution.

On the other hand, the $\Phi_{pt}$ value depends on the way source multiplicities are obtained, as shown in the lower plot of fig. 2. The circles follow the results from the upper plot, as the conditions are the same. However, once the event-wise distributions of the multiplicities $N_i$ approach a Poissonian, the $\Phi_{pt}$ values increase and actually reach zero for the case of both $N_i$ being Poisson-like distributions.

To simulate the influence of limited detector efficiency the constant source multiplicity was folded with the bino-
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Fig. 2. The values of $\Phi_{pt}$ for the mixing of two sources with temperatures of 100 and 15 MeV as a function of the percentage of particles coming from the lower-temperature source. Upper plot — different total multiplicities; lower plot — same total multiplicity (100), $N_1$ and $N_2$ for the data point are either constant, or coming from a Poisson distribution.

Fig. 3. The dependence of $\Phi_{pt}$ for the folding of the model events with binomial distribution, shown as a function of the particle acceptance probability $p$. $pN \approx 100$, $N_2/N = 0.6$, $T_1 = 100$ MeV and $T_2 = 15$ MeV.

5 Conclusions

It has been shown in principle that within the framework of a simple model of two equilibrated sources with different temperatures one can obtain negative values of the $\Phi_{pt}$-variable, while $\Phi_{pt} = 0$ for a single source. Therefore it is important to take into account possible contributions of different sources while analyzing the experimental data. In the SIS-18 energy range those sources may consist of participating nucleons and spectators, in the SPS energy range perhaps of created mesons and originally existing baryons. In this light one may notice that the results quoted in [2] show negative values of $\Phi_{pt}$ for all and for positively charged particles, while for negatively charged ones they show $\Phi_{pt} \approx 0$ (and all negative particles are created).

The effect of negative $\Phi_{pt}$ values is present only for fixed source multiplicities, which is an idealized case. It disappears if multiplicities are smeared, and this would be the case for any real-life detector system. No detector can measure simultaneously all reaction products and limited acceptance causes smearing of multiplicities. So the discussed model cannot be applied directly to the data analysis. On the other hand if event selection is made only on the basis of multiplicity cut artificial results may be obtained, and this should be kept in mind.

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