A Format for Instantons and Their Characterizations

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Abstract

A characterization of instanton contributions to gauge field theory is given. The dynamics of instantons with the gluon field is given in terms of 'classical' instanton contributions, i.e. on the same footing as tree amplitudes in field theory. This parameterization depends on the kinematics of the local instanton operators and their coupling dependence. A primary advantage of this 'classical' formulation is that the weak-strong duality and its relations with the perturbative sector are possibly made more manifest.
1 Introduction

The classical dynamics of gauge theory has been investigated for many years. Obstacles associated with the computation of these tree amplitudes have been circumvented for a variety of reasons, including spinor helicity, color ordering, analyticity, KLT relations with gravity, the string-inspired derivation, and the recent twistor formulation of classical gauge theory. The derivative expansion is useful for finding results in the quantum regime [1]-[16].

Potential simplifications based on a well-ordered formulation of the instanton contributions to an $n$-point scattering amplitude can be useful in finding the total correction to the quantum amplitudes. These terms to the amplitude take on the form at $n$-point,

$$f_{m,n} = g_{m,n}(g; N) \prod_{\ell[p], a_{i,p}} \frac{1}{m_{\frac{4\pi}{g^2} + m_{\frac{g^2}}}} K(\varepsilon_i, k_i),$$

with $a_{i,p}$ labeling the power of the invariant (both negative and positive). The helicity content containing the factors $\varepsilon_i \cdot k_j$ and $\varepsilon_i \cdot \varepsilon_j$ is represented by the function $K(\varepsilon_i, k_j)$. The kinematic invariants $t_i^{[p]}$ are defined by

$$t_i^{[p]} = (k_i + k_{i+1} + \ldots + k_{i+p-1})^2,$$

for an ordering of the external legs in the cyclic fashion $(1, 2, \ldots, n)$.

The coefficients $g_{m,n}$ enter as a result of the integration of the non-trivial gauge field configuration at the instanton number $m$ and at $n$-point. The integration produces the kinematic invariants. There is expected to be symmetry between the numbers $g_{m,n}$ at the various orders spanned $m$ and $n$.

The classical scattering of the gauge field has much symmetry, which is not evident in the usual perturbative approach. This symmetry is partly based on the collection $\phi_n$ of numbers as given in [4]-[5]. The $n$-point scattering can be formulated set-theoretically in terms of $n-2$ numbers which range from 2 to $n$, for a sub-amplitude with the color-ordering from 1 to $n$. These numbers are such that the maximum number $n$ may occur $n-2$ times, with the minimum number 1 not occurring at all. The multiplicative times the numbers appear is denoted as $p_i$ (for $i = 1, \ldots, n$) and the collection $p_1, \ldots, p_n$ completely label the amplitude. (In another set theoretic form these numbers give way to a Hopf algebra of the tree diagrams [17].) All combinations
of these numbers $\phi_n$ or $p_i$ generate the tree contributions to the gauge scattering, in a given color ordering of the sub-amplitudes.

The suggestive form of these simple groups of numbers suggests that the ‘classical’ (or semi-classical) contributions of the multi-instanton configurations to the scattering should also be labeled by a partition of the same numbers. These partitions label the kinematic prefactors in (1.1). The symmetry of the classical form and that of the partitions labeling the instanton terms should be relevant to duality information in the gauge theory.

In previous work the classical perturbative tree diagrams are used to find the full quantum scattering by sewing them together; the integrals are performed and the primary complication is the multiplicity of the indices. These tree diagrams are used as nodes, as essentially vertices. In this work the instanton contributions are formulated to achieve the same result; although the exact functional dependence is not yet known, the nodes and sewing may be formulated in the same manner as the perturbative results. The full quantum scattering, containing also the non-perturbative physics may be illustrated as in Figure 1. The coefficients of the individual vertices (labeled by $G_1$ and $G_2$)

$$V(G_1, G_2) = \sum_{n=0}^{\infty} \chi_n(G_1, G_2) z^n = e^{-\frac{4\pi}{g^2} - i \frac{\theta}{4\pi}}$$

are required to be determined, i.e. $\chi_n(G_1, G_2)$. These numbers in principle are found from a unifying function via the pertinent number partitions that are similarly analyzed in the perturbative gauge context. The perturbative nodes required to find the scattering are given by

$$V(G_p) = \chi(G_p) g^{m-2},$$

for an $m$-point tree represented by $\chi_n(G_p)$ (one of many trees required to specify an $m$-point classical amplitude).

The classical scattering is briefly reviewed in section 2. A generalization based on the partitions of numbers which gives more kinematic expressions other than perturbative tree diagrams is presented in section 3. The instanton analog is given in section 4. A simple graphical interpretation of these results is presented in section 5.
Figure 1: The product form solution to the recursive formulae defining the loop expansion. The nodes are classical scattering vertices. These vertices are both perturbative and non-perturbative.

2 Classical Perturbative Scattering

Classical gauge theory scattering has been formulated in terms of the sets $\phi_n$ in [4]. This representation of the gauge amplitudes has not included the spinor helicity technique, which with some further algebraic manipulations leads to maximally compact representations; some of the simplifications are due to the fact that the amplitudes have a symmetrical representation in terms of the sets of numbers $\phi_n$. The construction of the classical amplitudes in terms of these numbers is briefly reviewed.

Brief Review of the $\phi^3$ Labeling

The general scalar $\phi^3$ momentum routing of the propagators is presented. The gauge theory tree amplitudes are also formulated with the scalar momentum routing; their tensor algebra can be found with the use of the string inspired formulation combined with the number partitions labeling the scalar graphs.

A general $\phi^3$ scalar field theory diagram at tree-level is parameterized by the set of propagators and the momenta labeling them. In a color ordered form, consider the ordering of the legs as in $(1, 2, \ldots, n)$. The graphs are labeled by
\[ D_\sigma = g^{n-2} \prod_{t_{\sigma(i,p)}} \frac{1}{t_{\sigma(i,p)}} , \tag{2.1} \]

and the Lorentz invariants \( t_{\sigma(i,p)} \) are defined by \( t_i^{[p]} \),

\[ t_i^{[p]} = (k_i + \ldots + k_{i+p-1})^2 . \tag{2.2} \]

Factors of \( i \) in the propagator and vertices are placed into the prefactor of the amplitude. The sets of permutations \( \sigma \) are what are required in order to specify the individual diagrams. The full sets of \( \sigma(i,p) \) form all of the diagrams, at any \( n \)-point order. These \( \phi^3 \) diagrams and their labeling, via the sets \( \sigma \), are required in order to find the gauge theory amplitudes.

The compact presentation of \( \sigma \) can be obtained by a set of numbers \( \phi_n \), discussed next, that generate the individual diagrams. These sets are quite simple, and indirectly generate what is known as the Hopf algebra of the diagrams.

First, the vertices of the ordered \( \phi^3 \) diagram are labeled so that the outer numbers from a two-particle tree are carried into the tree diagram in a manner so that \( j > i \) is always chosen from these two numbers. The numbers are carried in from the \( n \) most external lines.

The labeling of the vertices is such that in a current or on-shell diagram the unordered set of numbers are sufficient to reconstruct the current; the set of numbers on the vertices are collected in a set \( \phi_m(j) \). For an \( m \)-point current there are \( m - 1 \) vertices and hence \( m - 1 \) numbers contained in \( \phi_m(j) \). These \( m - 1 \) numbers are such that the greatest number may occur \( m - 1 \) times, and must occur at least once, the next largest number may occur at most \( m - 2 \) times (and may or may not appear in the set, as well as the subsequent ones), and so on. The smallest number can not occur in the set contained in \( \phi_m(j) \). Amplitudes are treated in the same manner as currents. Examples and a more thorough analysis is presented in \([4],[5]\).

Two example permutation sets pertaining to 4- and 5-point currents are given in \([5]\). The five point amplitudes have sets of numbers such as \((5, 5, 5, 5)\) and \((5, 4, 3, 2)\), as an example.

The numbers \( \phi_n(j) \) are used to find the propagators in the labeled diagram. The external momenta are \( k_i \), and the invariants are found with the algorithm,

1) \( i = \phi(m - 1), \; p = 2, \) then \( l_{am-1} + l_{am} \rightarrow l_{m-1} \)
2) $i = \phi(m-2)$, $p = 2$, then $l_{a_{m-2}} + l_{a_{m-1}} \rightarrow l_{m-2}$

\[ \ldots \]

$m - 1)$ $i = 1$, $p = m$

The labeling of the kinematics, i.e. $t_i^{[p]}$, is direct from the definition of the vertices.

Alternatively, the numbers $\phi_n(i)$ can be arranged into the numbers $(p_i, [p_i])$, in which $p_i$ is the repetition of the value of $[p_i]$; as an example, if the number $p_i$ equals zero, then $[p_i]$ is not present in $\phi_n$. The numbers can be used to obtain the $t_i^{[q]}$ invariants without intermediate steps with the momenta. The branch rules to determine the presence of $t_i^{[q]}$ is,

0) $l_{\text{initial}} = [p_m] - 1$

1) $r = 1$ to $r = p_m$

if $r + \sum_{j=1}^{m-1} p_j = [p_m] - l_{\text{initial}}$ then $i = [p_m]$ $q = [p_m] - l_{\text{initial}} + 1$

beginning conditions has no sum in $p_j$

2) else $l_{\text{initial}} \rightarrow l_{\text{initial}} - 1 :$ decrement the line number

$l_{\text{initial}} > [p_i]$ else $l \rightarrow l - 1 :$ decrement the $p$ sum

3) goto 1)

The branch rule has to be iterated to obtain all of the poles. The procedure uses the $\phi^3$ vertices and matches with the momentum flow to determine if a tree is present in a clockwise manner. If not, then the external line number $l_{\text{initial}}$ is changed to $l_{\text{initial}}$ and the tree is checked again. The $i$ and $q$ are labels to $t_i^{[q]}$.

The previous recipe pertains to currents and also on-shell amplitudes. There are $m - 1$ poles in an $m$-point current $J_\mu$ or $m - 3$ in an $m$-point amplitude. The comparison between amplitudes and currents is as follows: the three-point vertex is attached to the current (in $\phi^3$ theory), and then the counting is clear when the attached vertex has two external lines with numbers less than the smallest external
Brief Review of the Gauge Theory Labeling

The gauge theory contributions follow a similar labeling \[4\], but with the added complexity of the kinematics in the numerator, such as \(\varepsilon_i \cdot k_i\) and \(\varepsilon_i \cdot \varepsilon_j\). These pairings are determined set-theoretically from the integers in \(\phi_n\), where the latter labels the momentum flow of the individual tree diagrams.

The \(\kappa(a;1)\) and \(\kappa(b;2)\) set of primary numbers used on can be found via the set of string inspired rules for the numerator factors, and define their individual contributions by,

\[
\left(\frac{-1}{2}\right)^{a_1} \left(\frac{1}{2}\right)^{n-a_2} \prod_{i=1}^{a_1} \varepsilon(\kappa(i;1)) \cdot \varepsilon(\kappa(i;1))
\]

\[
\times \prod_{j=a_1+1}^{a_2} \varepsilon(\kappa(j;1)) \cdot k_{\kappa(j;2)} \times \prod_{p=a_2+1}^{n} k_{\kappa(p;1)} \cdot k_{\kappa(p;2)} ,
\]

(2.5)

(2.6)

together with the permutations of 1,\ldots,\(n\). The permutations extract all possible combinations as explained in \[4\].

The form of the amplitudes is then expressed as \(T_\sigma\) multiplying the propagators in (2.1). \(T_\sigma\) is given in (2.6), and the sets \(\kappa\) are determined in \[4\] (using the string inspired rules for amplitudes). The normalization is \(i(-1)^n\). The numbers \(a_1\) and \(a_2\) are summed so that \(a_1\) ranges from 1 to \(n/2\), with the boundary condition \(a_2 \geq a_1 + 1\). Tree amplitudes in gauge theory must possess at least one \(\varepsilon_i \cdot \varepsilon_j\). Also, the color structure is \(\text{Tr} (T_{a_1} \ldots T_{a_n})\), and the complete amplitude involves summing the permutations of 1,\ldots,\(n\).

3 Product Form of Invariants

The number partitions have been used to generate scalar \(\phi^3\) graphs in \[5\]. Their use is involved in generating the tensor algebra in gauge theory classical scattering as in \[2\] and \[4\]. The general instanton vertex requires generally non-tree like combinations of the kinematic invariants, and so a generalization of these number partitions is required.
to generate their form. The general combination of kinematics can also be used to label the quantum vertices in the scattering as they are not tree-like kinematically.

In the previous section it was shown how certain partitions of numbers generate the non-vanishing coefficients $C_{i,p}$ in the scalar scattering. In perturbative gauge theory these coefficients are determined classically from the sets $\phi_n$ as in \[1\]-\[5\].

In a similar vein to the perturbative gauge theory work, these numbers could be represented pictorially as in Figure 2. Each set of numbers $\sigma(n)$ corresponds to deleted entries in the color ordered set $(1, 2, \ldots, n)$

\[(1, 2, \ldots, n) \quad (1, 2; 3, \ldots, n-2; n-1, n) \quad (1; 2, 3; 4, \ldots, n)\] (3.1)

The deleted entries are grouped together with other numbers in the set as the semicolon indicates, but with the deleted entries neighboring the largest non-deleted number, such as $n-2$ with the 'deleted entries 3 to $n-3$ in \[3.1\]. These sets of numbers are in analog to labeling all diagrams with legs grouped together as in the pair in Figure 2; the second diagram has legs four and five grouped (corresponding to a set $(1, 2, 3; 4, 5)$). In this manner the sets of numbers $C_{i,p,\sigma(n)}$ span all $\phi^3$ diagrams with ordered sets of legs. The numbers $\sigma(n)$ can be put into one-to-one correspondence with the integers from 1 to

\[\sum_{i=1}^{n-1} \frac{n!}{i!(n-i)!} = 2^{n-1}, \quad (3.2)\]

as the number is either absent or present ($2^{n-1}$).

The sets of numbers span all possible ordered pole structures in

\[\prod C_{i,p,\sigma(n)} \frac{1}{t_{i,p}}. \quad (3.3)\]

As a trivial example, the second graph in Figure 2 could be $n$-point graph with the non-three last legs grouped with the leg $n$; the set $\sigma$ is $(4, \ldots, n-1)$ and the single invariant $s_{12}$ is obtained in the denominator. In general it is possible to construct any combinations by grouping into two blocks these numbers, but in doing so the symmetry of the tree and non-tree diagrams is lost. As in constructing the tree diagrams of both scalar and gauge field theory, the sets $\phi_n$ are very well-ordered sets of numbers and presumably this holds in the classical vertices of the instanton case as well.
The $2^{m-1}$ possible leg orderings and topologies labeled by the individual $\phi_m$ sets produce all possible sets of pole terms; these sets $\phi_m$ at $m$-point together with the $2^{m-1}$ possibilities inherit a pseudo-Hopf structure. The construction however is quite explicit.

The coefficients $C_{i,p,\sigma(n)}$ span the graphs generating the pole terms in (3.3). A sample set of pole terms is given in Figure 2. The perturbative sets of numbers $C_{i,p}$ is given by the sets $\phi_n$ as described in [5].

The labeling of the numbers $C_{i,p,\sigma(n)}$ is required to obtain the functions $g_{C_{i,p,\sigma(n)}}$ that multiply the individual terms. Assumed is that there is a group theory interpretation of the individual $g$ functions for a given set of terms at $n$-point order, spanned by the numbers $C_{i,p,\sigma(n)}$. This group theory relevance should have an interpretation in terms of the classical perturbative scattering, together with the $2^{n-1}$ global possible interpretations.

The determination of the vertex kinematics of both the perturbative and non-perturbative terms (i.e. instantons) in terms of the possible sets $\phi_n$ and $\phi_n, \sigma(n)$ might allow for a better understanding of the quantum duality from the classical configurations. The full quantum scattering is obtained from both of these contributions, and any duality must manifest itself at the level of the tree-like nodes. A kinematic duality in the partitions of the numbers, such as required from $\phi_n$ to generate the classical perturbative graphs, seems possible with the instanton contributions. The
kinematic structure of both are required to find for example, S-duality in the $\mathcal{N} = 4$ supersymmetric gauge theory.

4 Instanton Analogue

The perturbative scattering has been modeled through the use of classical tree diagrams sewn together to find the loop scattering. The non-perturbative scattering incorporating the $e^{-4\pi/g^2}$ effects requires the introduction of further terms in the nodes of Figure 1. These further terms are modeled with the generalized kinematics of the previous sections together with the full coupling dependence. The reduction of the instanton contributions to the nodes is an algebraic simplification, and the full quantum scattering requires the iteration of the interactions as in Figure 1. (The integrals and much of the tensor results have been performed in the latter.)

The instanton vertices are sufficient, without logarithmic modifications, to deduce the appropriate quantum analog of instantons interacting with the gluon field to any loop order. For example, the instanton vertex at $n$-point can be defined to compensate the quantum recursion between the vertices and the result that is truly non-perturbative gauge theory; this compensation can be seen as adding irrelevant operators to fine tune a theory, but in this case, the classical instanton vertices are defined to agree with the quantum gauge theory instantons. The appropriate matter is to deduce the kinematics and coupling dependence of the 'classical' instanton vertices.

The instanton terms to an $n$-point amplitude receive contributions from the $m$-instanton, and also at loop order $L$. The most general term, with a coupling structure $f(g_{YM})$ has the form,

$$f_{m,n,C}^L \ e^{-m\frac{4\pi}{g^2}} + im\frac{\theta}{2\pi} \prod C_{i,p;\sigma(n)} \frac{1}{\epsilon_j \epsilon_p ; a_i, a_p} , \quad (4.1)$$

with the coupling dependence $f$ arising from a quantum corrected instanton moduli space; $a_i,p$ denotes general powers of the invariants, both positive and negative. The helicity structure is,

$$\prod \varepsilon_\sigma(j) \cdot k\hat{\sigma}(j) \varepsilon_\kappa(j) \cdot \varepsilon\hat{k}(j) . \quad (4.2)$$
The coefficients $C_{i,p}$ are coefficients that take on the values 0 and 1. They are set theoretic numbers that label the non-vanishing of the invariants $t_i^{[p]}$. The logarithmic possibilities require another set of numbers $C'$ to label, but the classical vertices do not require the soft dimensions so we neglect this notation (which is relevant in final results for quantum scattering). The coefficients $C_{i,p}$ also generate the tensorial vectors $\sigma(j)$, $\tilde{\sigma}(j)$, and $\tilde{\sigma}'(j)$, which enter into the polarization inner products.

The function $f_{Lm,n,C}$ is expected to have an analogous determination as the coefficients describing perturbative gauge theory. (In the self-dual $N = 4$ supersymmetric gauge theory context these functions are determined from the self-dual mapping $g \to 1/g$; one formulation involves the ansatz of the Eisenstein functions.)

The pre-factor $f_{C_{i,p},a_{i,p}}$, multiplying each term containing the products of $\varepsilon_i \cdot k_j$ and $\varepsilon_i \cdot \varepsilon_j$ has the coupling expansion at $m$-point,

$$
\tilde{f}^{(m)}_{C_{i,p},a_{i,p}} = \sum b_n(C; g; m)e^{-n^2\pi/g^2+n\theta/2\pi}.
$$

(4.3)

The coefficients could in principle be determined from the appropriate expansion of a manifold.

The vertices associated with every kinematic term in the series is used to find the full scattering in the gauge field, after including the perturbative terms. The classical gauge field tree diagrams generates the classical vertices, as used in [1,2] to generate the full quantum scattering. These tree diagrams are then added with the instanton vertices to find the full nodes. The quantum theory is obtained by sewing both nodes together in the 'rainbow' graphs to determine the scattering, as illustrated in the Figure 1.

5 Compact graphical form

The notation can be further simplified graphically with the use of two diagrams for the non-helicity specified amplitudes, and only one diagram for the helicity amplitude. This graphical representation makes closer possible group theoretic and geometry in the interpretation of the instanton contributions, which could make a determination of the functions in (4.3) easier.

A labeled diagram $G_1$ with $m$ nodes ($m$-point vertex) is used to specify the contractions of the polarizations with the momenta, that is $\varepsilon_i \cdot k_j$ and $\varepsilon_i \cdot \varepsilon_j$. The nodes label the ordered set of lines, such as $1, 2, \ldots, n$. There are two lines, with
Figure 3: The parameterization of the node kinematics. The dashed and solid lines represent contractions of the polarizations with the momenta. The circles are the nodes 1 to 5.

a dashed or solid line, that represent the contraction $\varepsilon_i \cdot \varepsilon_j$ or $\varepsilon_i \cdot k_j$, respectively. The latter case requires an arrow orientation to label either the polarization or the momenta; the end of the arrow labels a momenta. The figure is represented in figure 3.

The second diagram $G_2$ is used to represent the various momentum invariants in the denominator and numerator. The line from node $i$ to node $j$ labels the invariant $(k_i + \ldots + k_{i+j-1})^2$, and the index on the line represents its power; for example, the index of 1 is a denominator and $-2$ is a double power in the numerator. This diagram is represented in figure 5.

Together these two diagrams label an individual contribution to the instanton vertex in figure 4. The functional form of the coupling dependence are given by the equation,

$$V^{(m)}(G_1, G_2) = \sum_{n=0}^{\infty} \chi_n^{(m)}(G_1, G_2; g)e^{-n\frac{4\pi}{g^2} - n\frac{\theta}{2\pi}}.$$  (5.1)

The graphical illustration alludes to a geometric derivation of the components $\chi_n(G_1, G_2)$. Indeed, writing the 'classical' multi-instanton contributions as

$$z = e^{-\frac{4\pi}{g^2} - i\frac{\theta}{2\pi}},$$  (5.2)

generates a holomorphic function,
Figure 4: The second diagram represents the contractions \((k_i + \ldots + k_{i+j-1})^2\); numbers represent the powers of these invariants. This graph represents \(\prod_{i=1}^{5} 1/s_{i,i+1}\).

\[
V^{(m)}(G_1, G_2) = \sum_{n=0}^{\infty} \chi_n^{(m)}(G_1, G_2; g) z^n ,
\]

(5.3)
similar to a Kähler potential. Presumably specifying this function through its analytic properties, based on the symmetries of the graphs \(G_1\) and \(G_2\), generates the instanton contributions. The latter allows the perturbative scattering of the quantum gauge theory to incorporate the non-perturbative terms.

In a spinor helicity format the two diagrams can be reduced to only one diagram \(G_s\). The lines are arrows, with each one either dashed or solid, representing the inner products \(\langle ij \rangle\) or \([ij]\). The arrow specifies the orientation from \(i\) to \(j\) in the graph with \(n\) nodes.

### 6 Discussion

The classical scattering within gauge theory has a simple formulation in terms of sets of numbers \(\phi_n\) that elucidates the instanton contribution formulation. There are simplifications, further due to the spinor helicity implementation in this approach which are also algebraic. It seems that a simple program will generate all contributions to obtain the full quantum amplitudes, with both perturbative and non-perturbative corrections.

The sets of numbers \(\phi_n\), or \(p_i\), formulate a direct representation of the non-helicity format classical amplitudes. The corresponding sets of numbers \(\tilde{\phi}_n\), and \(p_{i,n}\) generate
Figure 5: The parameterization of the helicity basis nodes. The numbers represent the powers of the invariants $\langle ij \rangle$ and $[ij]$. This graph represents $\prod_{i=1}^{5} 1/(i + 1, i)$ the instanton kinematics at leading order. The coefficients $V^{(n)}(G_1, G_2)$ are expected to follow a similar number theoretic, and an accordant differential representation. The group theory in association with these numbers should lead to a full determination of the instanton contributions to the $n$-point amplitude.

The instanton interactions with the gauge bosons are modeled with the use of classical interaction terms entering into the sewing relations, the latter of which generate the full quantum theory amplitudes. Duality transformations at the basic level should operate only on these vertices in the coupling constant, as full transformations are repetitive on the individual nodes. This property possibly simplifies the non-perturbative $g \to 1/g$ transformations within the quantum dynamics, when the appropriate transformations of the fields between the nodes are given such as a straight gluon to gluon map, or those in various theories. The appropriate transformations might generate the instantonic vertices and the non-perturbative contribution to the quantum theory.
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