Large $N$ Superconformal Gauge Theories and Supergravity Orientifolds

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Abstract

Maldacena’s duality between conformal field theories and supergravity is applied to some conformal invariant models with 8 supercharges appearing in the F-theory moduli space on a locus of constant coupling. This includes $Sp(2N)$ gauge theories describing the world-volume dynamics of D3-branes in the presence of D7-branes and an orientifold plane. Other examples of this kind are models with exceptional global symmetries which have no perturbative field theory description. In all these cases the duality is used to describe perturbations by primary marginal and relevant operators.

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1 Introduction

Maldacena has proposed a new kind of “duality” between large N conformal field theories in various dimensions and supergravity on anti-de Sitter spaces. The idea advocated in [1] is based on the construction of gauge theory as the world volume theory of N D-branes in the $\alpha' \to 0$ limit. The limit is taken in such a way as to keep the masses of field theory excitations finite. On the other hand this system can be described by the black brane solutions of supergravity. In the $\alpha' \to 0$ limit one is probing the near-horizon geometry of the black branes which is typically an anti-de Sitter space times a compact manifold. Since supergravity is expected to provide a reliable description only if the curvature is small (so $\alpha'$ corrections can be neglected), one is lead to consider a limit in which the radius of curvature is large which corresponds to the limit of a large number of branes. This observation has attracted a great deal of attention. Apart from comparing the spectra, it was also possible to calculate correlation functions in the conformal field theory using this correspondence.

The original conjecture referred to theories with 16 supersymmetries, however it was quickly realized that one could also use it in theories with less supersymmetry. Here a family of superconformal field theories with 8 supersymmetries is considered: the $Sp(2N)$ gauge theory with an antisymmetric tensor hypermultiplet and four fundamental hypermultiplets. This system is an orientifold of type IIB theory in which D7-branes are localized at the orientifold fixed planes with D3-branes. The field theory of interest describes the D3-brane dynamics. This model has a vanishing beta function and is believed to be exactly superconformal. Since the field theory description is known, one can analyze the spectrum and make a comparison much as in [5, 11]. The theory has a global $SO(8)$ symmetry, due to gauge symmetry enhancement in the D7-brane worldvolume.

This theory can also be regarded as a particular F-theory compactification in a region where the coupling is constant. This way of looking at it is revealing, and it leads one to consider other superconformal quantum systems, which do not have any known field theory description. While in the $Sp(2N)$ case the orientifold involves a $Z_2$ reflection, the cases with $Z_3$, $Z_4$, $Z_6$ lead to worldvolume field theories on the D3-branes with $E_6$, $E_7$, $E_8$ global symmetry. While these theories have no known field theory description, they can in the present context be treated in the same way as the $Z_2$ case which does. All of these theories can be obtained...
by compactifying an M-theory 5-brane \[24\] on a torus in the presence of an end-of-the-world 9-brane. The 5-brane theory has \(E_8\) global symmetry due to the presence of the \(E_8\) gauge symmetry living on the 9-brane. This suggests the possibility that the non-perturbative 4-dimensional \(E_n\) theories obtained from the six-dimensional theory may not be ordinary field theories but could inherit tension-less strings from its progenitor.

In the limit where the correspondence advocated in \[1, 6, 5\] is expected to hold these theories turn out to be described by \(Z_n\) orientifolds\(^1\) of \(AdS_5 \times S^5\). The twist acts only on the \(S^5\) factor, so that the resulting theory on the boundary of \(AdS_5\) is conformal. It is straightforward to analyze what happens to the Kaluza-Klein spectrum of the \(\mathcal{N} = 4\) theory on \(AdS_5 \times S^5\) and to identify the chiral primary operators. This note presents the results for the low-lying scalar operators. In the \(Sp(2N)\) case it is also easy to identify the chiral primaries in terms of the fields appearing in the perturbative description, using their group theoretical properties which follow from the calculation.

While the \(Sp(2N)\) case is fairly well understood since it has a perturbative formulation\[15\], the remaining cases have no known field theory description: they are inherently non-perturbative. Nevertheless, the methods developed in \[3, 11\] allow one to identify the quantum numbers of the relevant and marginal perturbations of these theories even though no gauge field theory description is known.

Due to multiplet shortening \[27, 28, 29\] it is expected that information about the spectrum of primary operators should be exact. In this sense supergravity allows the study of inherently non-perturbative phenomena on the gauge theory side which are inaccessible using standard field theory methods.

### 2 F-theory Orientifolds with Constant Coupling

F-theory \[30\] is believed to describe non-perturbative type IIB theory. It is a powerful method for generating non-trivial type IIB backgrounds with varying dilaton and Ramond-Ramond scalar fields. By definition F-theory on an elliptically fibered manifold is type IIB string theory on the base manifold

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\(^1\)Orbifolds of \(AdS_5 \times S^5\) have recently been discussed in \[8, 9, 10, 11, 12, 13, 14\], and orientifolds in \[25, 26\].
of the fibration with the complexified coupling $\tau = \chi + i \exp(-\phi)$ identified with the modular parameter of the elliptic fiber. At a point on the base where the fiber degenerates one has a localized 7-brane.

In [15, 16] F-theory compactifications on elliptic K3 with constant type IIB coupling were considered. These K3’s are described by orbifolds $T^4/Z_n$ with $n = 2, 3, 4, 6$ and give 8-dimensional gauge symmetry of SO(8), $E_6$, $E_7$, $E_8$ respectively which can be understood from the point of view of coincident 7-branes at the orientifold point [31, 32, 33, 34]. The base of the elliptic fibration is an orbifold $T^2/Z_n$. The fiber above a fixed point degenerates and there is a nontrivial monodromy in the fiber [15, 16], which means that apart from the $Z_n$ projection on the base there is also a $Z_n$ action on the complexified coupling (dilaton and axion pair) and antisymmetric field (NS-NS and R-R two-form gauge fields). This monodromy is $-1$ in the $Z_2$ case, and $S, ST$ and $(ST)^2$ in the $Z_3, Z_4$ and $Z_6$ cases.

These models can also be described as supergravity solutions with metric [35]:

$$ds^2 = \tau_2 \mid \eta^2(\tau) \Delta^{-1/12}dz \mid^2 + \delta_{ij}dx_idx_j,$$

(1)

where $\tau_2$ is the imaginary part of the coupling $\tau$, $\Delta$ is the discriminant of the elliptic fiber

$$\Delta(z) = 4f^3(z) + 27g^2(z)$$

(2)

and $\eta$ is the Dedekind function. The metric is modular invariant. The K3s are described by the Weierstrass equation for the elliptic fiber as a function of $z$, the coordinate on the base manifold $P^1$:

$$y^2 = x^3 + f(z)x + g(z)$$

(3)

with $f$ a polynomial of degree 8 and $g$ a polynomial of degree 12 [30]. $\tau$ is determined from the $j$-invariant of the equation:

$$j(\tau(z)) = \frac{4(24f(z))^3}{\Delta(z)} = \frac{(\theta_1^8(\tau) + \theta_2^8(\tau) + \theta_3^8(\tau))^3}{\eta^{24}(\tau)}.$$ 

(4)

K3 manifolds with constant $\tau$ are the ones for which $j$ is constant. There are 4 cases for which a type IIB orientifold description exists [15, 16].
(i) $T^4/Z_2$:

\[
\begin{align*}
f(z) & = \alpha \prod_{i=1}^{4}(z - z_i)^2, \\
g(z) & = \prod_{i=1}^{4}(z - z_i)^3.
\end{align*}
\]  

(5)

The coupling constant depends on the parameter $\alpha$ and can be fixed to be any value by choosing an appropriate $\alpha$. In this case there are 4 points ($z = z_i$) at which the fiber degenerates. The K3 has four $D_4$ singularities resulting in the gauge group $SO(8)^4$. Each $SO(8)$ can be understood from the type IIB point of view as 4 D7-branes coinciding with an orientifold fixed plane [15].

(ii) $T^4/Z_3$

\[
\begin{align*}
f(z) & = 0, \\
g(z) & = \prod_{i=1}^{3}(z - z_i)^4.
\end{align*}
\]  

(6)

In this case [16], $j = 0$ thus the coupling constant is fixed to be $\tau = \exp i\pi/3$. The singularity type close to $z = z_i$ is $E_6$, resulting in the gauge group $E_6 \times E_6 \times E_6$.

(iii) $T^4/Z_4$

\[
\begin{align*}
f(z) & = (z - z_1)^3(z - z_2)^3(z - z_3)^2, \\
g(z) & = 0.
\end{align*}
\]  

(7)

In this case [16], $j = (24)^3$; thus the coupling constant is fixed to be $\tau = i$. The singularity type close to $z = z_1, z_2$ is $E_7$ and close to $z = z_3$ it is $D_4$ resulting in the gauge group $E_7 \times E_7 \times SO(8)$.

(iv) $T^4/Z_6$

\[
\begin{align*}
f(z) & = 0, \\
g(z) & = (z - z_1)^5(z - z_2)^4(z - z_3)^2.
\end{align*}
\]  

(8)

In this case [16], $j = 0$; thus the coupling constant is fixed to be $\tau = \exp i\pi/3$. The singularity type close to $z = z_1$ is $E_8$, close to $z = z_2$ it is $E_6$ and close to $z = z_3$ it is $D_4$, resulting in the gauge group $E_8 \times E_6 \times SO(8)$.
The gauge groups above can also be understood using Chan-Paton factor analysis of coinciding 7-branes using multi-pronged strings [32, 33, 34].

Let us focus our attention on the vicinity of one of the points where the elliptic fiber degenerates, i.e. where the 7-branes are localized. One can introduce D3-branes [18] in the above backgrounds and reduce the supersymmetry to 8 real supersymmetries corresponding to $\mathcal{N} = 2$ in 4 dimensions. The theory living on the 3-branes will be an $\mathcal{N} = 2$ Seiberg-Witten theory with the same flavour symmetry group as the 7-brane gauge group [18]. As discussed above the Seiberg-Witten theory of the 3-brane probing the 7-brane geometry can have a perturbative description only in case (i).

If one introduces $N$ 3-branes close to a $D_4$ singularity and takes the other singularities to be far away, then the resulting theory on the probe is a $\mathcal{N} = 2$ gauge theory with gauge group $Sp(2N)$ coupled to a second rank antisymmetric tensor hypermultiplet and 4 fundamental hypermultiplets [19]. The four fundamental hypermultiplets arise from strings stretched between the D7-branes and the D3-branes. The anti-symmetric tensor arises from the action of the orientifold projection on the fields transverse to the D3-branes but parallel to the D7-branes. The theory has a Coulomb branch parameterized by the expectation value of the complex scalar field in the $\mathcal{N} = 2$ vector multiplet and corresponds to moving D3-branes away from the fixed plane. There is also a Higgs branch parameterized by the expectation values of the two complex scalars in the anti-symmetric tensor hypermultiplet, they correspond to the motion of a subset of the D3-branes parallel to the D7-branes. Finally, there is a second Higgs branch which is parameterized by the scalars in the fundamental representation and corresponds to dissolving some D3-branes inside D7-branes.

3 Supergravity Solutions

This section describes adding D3-branes to the D7-brane backgrounds described above. Let us take $N$ D3-branes with the worldvolume along $x^0, x^1, x^2, x^3$ and the appropriate number of D7-branes (according to the discussion in the previous section) with worldvolumes along $x^0, x^1, \ldots, x^7$. Let $z \equiv x^8 + ix^9, v \equiv x^4 + ix^5, \tilde{v} \equiv x^6 + ix^7$. The position of the D7-brane is given by $z$, while the D3-brane position is given by $z, v, \tilde{v}$. From the point of view of the field theory on the D3-branes $z$ parameterizes the Coulomb branch of the theory while $v, \tilde{v}$ parameterize the Higgs branch. Thus $z$ corresponds
to the vacuum expectation value of the adjoint Higgs, while \( v, \tilde{v} \) correspond
to the expectation values of the \( \mathcal{N} = 1 \) chiral multiplets comprising the
hypermultiplet in the antisymmetric representation of the gauge group.

Let us focus our attention on the vicinity of an F-theory K3 singularity
which corresponds to coinciding 7-branes in type IIB theory. For the present
cases, where \( \tau \) is a constant, the part of the metric describing the D7-branes
(and \( \Omega_7 \)) backgrounds with zero net 7-brane charge is (up to a constant
normalization):

\[
\text{SO}(8): \quad ds^2 = |z^{-\frac{1}{2}} dz|^2, \\
E_6: \quad ds^2 = |z^{-\frac{2}{3}} dz|^2, \\
E_7: \quad ds^2 = |z^{-\frac{3}{4}} dz|^2, \\
E_8: \quad ds^2 = |z^{-\frac{5}{6}} dz|^2.
\]

These can be described as orbifolds \( \mathbb{C}/\mathbb{Z}_n \) with \( n = 2, 3, 4, 6 \) respectively. The
covering space is the complex u-plane, with \( z = u^n \). In these coordinates
the metric is \( ds^2 = |d u|^2 \). The action of the orientifold group on \( u \) is
\( u \rightarrow u^{\exp(2\pi i/n)} \).

It is easy to introduce 3-branes into the problem: this is done in such a
way as to respect the identification under the orientifold group. The black-
brane solution for 3-branes of \[36\] reads

\[
ds^2 = f^{-1/2} dx_{\|}^2 + f^{1/2} dx_\perp^2, \\
f = 1 + \frac{4\pi g N \alpha'^2}{r^4}
\]

where

\[
dx_{\|}^2 = -(dx^0)^2 + \sum_{k=0}^3 (dx^k)^2, \\
dx_\perp^2 = |du|^2 + |dv|^2 + |d\tilde{v}|^2.
\]

The solution in the presence of the 7-brane — orientifold system is obtained
simply by identifying \( u \rightarrow u \exp(i2\pi/n) \). One can now express the metric
in terms of the single valued \( z = u^n \). It is convenient to use the following
variables: let \( r \) be the radial distance away from the point where the N D3-
branes are located (in the 6 dimensional transverse space), and let \( R \) be the

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*In the exceptional cases these are actually (p,q) 7-branes.*
projection onto the the covering u-plane, i.e. \( R = r \cos \phi \) with \( \phi \in [0, \pi/2] \). Then one has \( z = R^n \exp i\theta \) with \( \theta \in [0, 2\pi] \). This leads to

\[
ds^2 = f^{-1/2} d^2 x + f^{1/2} (dr^2 + r^2 d\phi^2 + \frac{r^2}{n^2} \cos^2 \phi d\theta^2 + r^2 \sin^2 \phi d\Omega^2),
\]

\[
f = 1 + \frac{4\pi g N \alpha'^2}{r^4}.
\] (12)

Taking the near horizon limit as in [1], keeping \( r/\alpha' \equiv U \) fixed while taking \( \alpha' \to 0 \), one arrives at:

\[
ds^2 = \alpha' \left\{ \frac{U^2}{\sqrt{4\pi g N}} d^2 x + \sqrt{4\pi g N} \frac{dU^2}{U^2} + \sqrt{4\pi g N} (d\phi^2 + \frac{\cos^2 \phi}{n^2} d\theta^2 + \sin^2 \phi d\Omega^2) \right\}.
\] (13)

This shows that in the near-horizon limit the metric looks like \( AdS_5 \times S^5/Z_n \) as in [8, 9], except that now the orbifold group action is different. Consider \( SU(2) \times SU(2) \times U(1) \subset SU(4) \sim SO(6) \) where \( SO(6) \) is the isometry group of \( S^5 \). In [8, 9] the orbifold action was taken to act on one of the \( SU(2) \) factors, whereas in the present case the orbifolding acts only on the \( U(1) \) factor. The orbifold action does not break the \( U(1) \) but identifies \( \exp(i\theta) \) with \( \exp(i(\theta + 2\pi/n)) \).

The \( SO(9, 1) \) Lorentz group is broken by the D7-brane to \( SO(7, 1) \times SO(2) \) and the D3-branes break this further to \( SO(3, 1) \times SO(4) \times SO(2) \). Thus the global symmetry group of the theory on the D3-brane is \( SO(3, 1) \times SO(4) \times U(1) \sim SO(3, 1) \times SU(2) \times SU(2) \times U(1) \). This is the Lorentz group in 3 + 1 dimensions, an \( SU(2) \times U(1) \) R-symmetry group (as expected for \( \mathcal{N} = 2 \) supersymmetry) and an \( SU(2) \) (non-R) global symmetry.

4 The Spectrum

The spectrum of type IIB supergravity on \( AdS_5 \times S^5 \) was found in [37, 38]. It falls into representations of the \( SU(4) \) R-symmetry. The states of spin zero families of zero or negative mass can be summarized by the following table:
Here $\Delta$ is the dimension of the operator that the field couples to in the conformal field theory. It is related to the mass through the formula:

$$m^2 = (\Delta + p)(\Delta + p - d)$$

for $p$-form fields in $d$ dimensions, where the correct solution for $\Delta$ is the larger of the two roots.

Table 1

| $\Delta$ | $m^2$          | Range         | Dynkin Label | Irreps       |
|----------|----------------|---------------|--------------|--------------|
| $k$      | $k(k - 4)$     | $k \geq 2$   | $(0, k, 0)$  | $20', 50, 105, \ldots$ |
| $k+3$    | $(k - 1)(k + 3)$ | $k \geq 0$   | $(0, k, 2)$  | $10, 45, 126, \ldots$ |
| $k+4$    | $k(k + 4)$     | $k \geq 0$   | $(0, k, 0)$  | $1, 6, 20', \ldots$ |

The orientifold construction that one is lead to consider for the D3-D7 system involves modding out by $Z_n$ subgroups of the $U(1)$ in the decomposition $SU(4) \supset SU(2) \times SU(2) \times U(1)$. In addition, the orientifold acts non-trivially on the supergravity fields. As discussed in section 2, it affects the complexified coupling and two form gauge fields, leaving the other fields invariant\[13, 16\]. This will be discussed on a case by case basis below. Since it is only the second rank antisymmetric tensor fields that are non-trivially transformed, this will only affect the modes coming from the representations $10$ and $45$ of $SU(4)$\[37, 38\].

The $SU(4)$ representations appearing in table 1 have to be decomposed under $SU(2) \times SU(2)_R \times U(1)_R$. The action of the orientifold group is a product of the (spacetime) orbifold phase determined by the $U(1)_R$ charge and the orientifold action on the supergravity fields. In the cases where the latter action is trivial the projection leaves the operators with $Q = 0$ mod $2n$ (for $\Gamma = Z_n$). These are all the cases which do not arise from the two-forms. The operators coming from the two-forms are in the $10$ and $45$ of $SU(4)$\[37, 38\], and there one has to account for the extra phase from the orientifold group. It turns out, as discussed below, that the operators which remain after the projection have $Q_R = \pm 2$ mod $2n$.

This leads to the spectrum of invariant operators in the $\mathcal{N} = 2$ theory. Only some of the resulting operators will be chiral primaries of the conformal field theory. This is due to the fact, that for a chiral primary operator\[\text{[37, 38]}\] of

\[\text{[3]}\]There will of course be antichiral primaries also, which are not shown in the tables below.
dimension $\Delta$ the following relation must be satisfied:

$$\Delta = \frac{1}{2}(R + 2J), \quad (15)$$

where $R$ is the R-charge of the operator, and $J$ is its $U(1)_J$ charge.

### 4.1 The $\mathbb{Z}_2$ case

In this case the field theory is known, so a detailed comparison can be made. The Kaluza-Klein harmonics can be obtained by performing the $\mathbb{Z}_2$ projection as described above. For example, using the results in table 1 we have in the $\mathcal{N} = 4$ theory modes in the $20'$ representation of $SU(4)$. This representation is decomposed as

$$20' = \begin{pmatrix} 3 \end{pmatrix}_0 \oplus \begin{pmatrix} 2 \end{pmatrix}_2 \oplus \begin{pmatrix} 2 \end{pmatrix}_{-2} \oplus \begin{pmatrix} 1 \end{pmatrix}_4 \oplus \begin{pmatrix} 1 \end{pmatrix}_{-4} \oplus \begin{pmatrix} 1 \end{pmatrix}_0 \quad (16)$$

under the global symmetry group $SU(2) \times SU(2)_R \times U(1)_R$. The modes get a phase $\exp(i\pi Q_K/2)$ under the orientifold action, so the invariant representations are those with the $U(1)$ charge $Q_R = 0 \mod 4$. Thus one finds that only 4 representations are left:

$$20' \supset \begin{pmatrix} 3 \end{pmatrix}_0 \oplus \begin{pmatrix} 1 \end{pmatrix}_4 \oplus \begin{pmatrix} 1 \end{pmatrix}_{-4} \oplus \begin{pmatrix} 1 \end{pmatrix}_0. \quad (17)$$

In case of the operators arising from the second rank gauge fields one has to take care to account for the "extra" phase coming from the orientifold action on these fields, as discussed earlier. These operators come from the representations $10$ and $45$ of $SU(4)$, which decompose as

$$10 = \begin{pmatrix} 1 \end{pmatrix}_2 \oplus \begin{pmatrix} 3 \end{pmatrix}_1 \oplus \begin{pmatrix} 2 \end{pmatrix}_0$$

$$45 = \begin{pmatrix} 4 \end{pmatrix}_2 \oplus \begin{pmatrix} 2 \end{pmatrix}_4 \oplus \begin{pmatrix} 2 \end{pmatrix}_2 \oplus \begin{pmatrix} 2 \end{pmatrix}_{-2},$$

$$\oplus \begin{pmatrix} 3 \end{pmatrix}_{-4} \oplus \begin{pmatrix} 1 \end{pmatrix}_4 \oplus \begin{pmatrix} 3 \end{pmatrix}_0 \oplus \begin{pmatrix} 1 \end{pmatrix}_0 \oplus \begin{pmatrix} 1 \end{pmatrix}_0. \quad (18)$$

The orientifold action on the two-form fields is in this case

$$\begin{pmatrix} B \\ \tilde{B} \end{pmatrix} = - \begin{pmatrix} B \\ \tilde{B} \end{pmatrix} \quad (19)$$

where $B$ is the NS-NS two-form field, and $\tilde{B}$ is the R-R two-form. Thus each representation in the decomposition (18) gets in total a phase $\exp(i\pi (Q_R + 2)/2)$.

\[\text{This is the } U(1) \text{ subgroup of } SU(2)_R \text{ as defined in [30].}\]
Thus in this case only operators with $Q_R = 2 \mod 4$ remain. Specifically, in the decomposition of 10 only the first two representations are invariant and the last one drops out, whereas in the case of the 45 the first four survive and the remaining ones are projected out.

Proceeding this way one finds the results summarized in table 2a below:

| $\Delta$ | $SU(4)$ | $SU(2) \times SU(2)_R \times U(1)_R$ |
|----------|--------|-----------------------------------|
| 2        | 20'    | $(3, 3)_0 \oplus (1, 1)_4 \oplus (1, 1)_{-4} \oplus (1, 1)_0$ |
| 3        | 10     | $(3, 1)_{-2} \oplus (1, 3)_2$ |
| 3        | 50     | $(2, 2)_4 \oplus (2, 2)_{-4} \oplus (4, 4)_0 \oplus (2, 2)_0$ |
| 4        | 1      | $(1, 1)_0$ |
| 4        | 45     | $(4, 2)_{-2} \oplus (2, 4)_2 \oplus (2, 2)_2 \oplus (2, 2)_{-2}$ |
| 4        | 105    | $(1, 1)_0 \oplus (1, 1)_4 \oplus (1, 1)_{-4} \oplus (1, 1)_8 \oplus (1, 1)_{-8}$ $\oplus (3, 3)_0 \oplus (3, 3)_{-4} \oplus (3, 3)_{-4} \oplus (5, 5)_0$ |

Table 2a: The $Z_2$ case: invariant operators.

The $(1, 1)_0$ operator here is the dilaton.

For dimension $\Delta = 2$ only the $(3, 3)_0$ and $(1, 1)_4$ modes have charges satisfying (15), so only they correspond to chiral primary fields. In fact one can easily identify these operators in the field theory. In the present case one can use the perturbative field theory description of the worldvolume dynamics [15, 19]. The low energy degrees of freedom of this field theory include (in $\mathcal{N} = 1$ language) the vector multiplet $W_\alpha$ and the chiral multiplets $\phi$ in the adjoint representation of $Sp(2N)$ and $V$, $\tilde{V}$ in the antisymmetric representation. These fields correspond to the geometric distances $z$, $v$, $\tilde{v}$ introduced in section 3. Under the global symmetry group $SU(2) \times SU(2)_R \times U(1)_R$ these transform as

$$\phi \sim (1, 1)_2$$
$$V \sim (2, 2)_0$$
$$W_\alpha \sim (1, 2)_1$$

(20)

The operators $(3, 3)_0$ and $(1, 1)_4$ can be represented as $Tr(V^2)$ and $Tr(\phi^2)$. For example $Tr((3, 3)_0$ has $R = 0$ and $J = 2$, so (15) is satisfied. In the case of $(1, 1)_{-4}$ however one has $R = -4$, $J = 0$, so (15) is not satisfied.

The normalization here is such that for the fundamental of $SU(2)$ $J$ assumes values $-1, 1$. 

10
Carrying out this argument for dimensions up to 4 one reaches the conclusions summarized in Table 2b. This table, as well as the following ones, contains only chiral primaries; anti-chiral primary operators will also be present in the spectrum (and protected) and are conjugates of the ones listed.

| Δ  | $SU(2) \times SU(2)_R \times U(1)_R$ | Field content |
|----|----------------------------------|---------------|
| 2  | $(1,1)_4$                        | $Tr(\phi^2)$  |
| 2  | $(3,3)_0$                        | $Tr(V^2)$     |
| 3  | $(1,3)_2$                        | $Tr(W^2)$     |
| 3  | $(4,4)_0$                        | $Tr(V^3)$     |
| 3  | $(2,2)_4$                        | $Tr(V\phi^2)$ |
| 4  | $(2,4)_2$                        | $Tr(W_\alpha W^\alpha V)$ |
| 4  | $(1,1)_8$                        | $Tr(\phi^4)$  |
| 4  | $(5,5)_0$                        | $Tr(V^4)$     |
| 4  | $(3,3)_4$                        | $Tr(V^2\phi^2)$ |

Table 2b: The $Z_2$ case: chiral primary fields.

As in other cases studied in the literature, it was possible to identify uniquely all the surviving Kaluza-Klein states with appropriate operators on the gauge theory side.

### 4.2 The Cases with Exceptional Global Symmetry

This section presents the results for the cases of $Z_3$, $Z_4$ and $Z_6$, for which the theory on the D3-brane worldvolume has global exceptional symmetry $E_6$, $E_7$, $E_8$ respectively.

For the $Z_3$ case the invariant operators are those with the $U(1)$ charge $Q = 0 \mod 6$, except for the operators coming from the 10 and 45. In the present case the orientifold action on the two-form fields is

$$
\begin{pmatrix}
B \\
\tilde{B}
\end{pmatrix} =
\begin{pmatrix}
-1 & -1 \\
1 & 0
\end{pmatrix}
\begin{pmatrix}
B \\
\tilde{B}
\end{pmatrix}.
$$

(21)

The surviving modes will be linear combinations of these:

$$
B' = e^{-2\pi i/3} B + \tilde{B},
$$

$$
\tilde{B}' = e^{+2\pi i/3} B + \tilde{B}.
$$

(22)
These combinations transform multiplicatively. The total phase is then \( \exp(i\pi(Q_R \pm 2)/3) \), so the surviving operators in the \( 10 \) and \( 45 \) are those with \( Q_R = \pm 2 \) mod 6. This way one finds the invariant operators given in Table 3a below:

| \( \Delta \) | \( SU(4) \) | \( SU(2) \times SU(2)_R \times U(1)_R \) |
|---|---|---|
| 2 | 20' | \((3,3)_0 \oplus (1,1)_0\) |
| 3 | 10 | \((3,1)_{-2} \oplus (1,3)_2\) |
| 3 | 50 | \((1,1)_6 \oplus (1,1)_{-6} \oplus (2,2)_0 \oplus (4,4)_0\) |
| 4 | 45 | \((4,2)_{-2} \oplus (2,4)_2 \oplus (2,2)_{2} \oplus (2,2)_{-2} \oplus (3,1)_{-4} \oplus (1,3)_4\) |
| 4 | 105 | \((1,1)_0 \oplus (2,2)_6 \oplus (2,2)_{-6} \oplus (3,3)_0 \oplus (5,5)_0\) |

Table 3a: The Z\(_3\) case: invariant operators.

As in the \( Z_2 \) case, to identify the chiral primaries one has to check whether the relation (15) is satisfied. This leads to the conclusions summarized in Table 3b.

| \( \Delta \) | \( SU(2) \times SU(2)_R \times U(1)_R \) |
|---|---|
| 2 | \((3,3)_0\) |
| 3 | \((1,1)_6\) |
| 3 | \((1,3)_2\) |
| 3 | \((4,4)_0\) |
| 4 | \((1,3)_4\) |
| 4 | \((2,4)_2\) |
| 4 | \((2,2)_6\) |
| 4 | \((5,5)_0\) |

Table 3b: The Z\(_3\) case: chiral primary fields

The analysis for the remaining cases proceeds analogously. In the \( Z_4 \) case the invariant operators are those with the \( U(1) \) charge \( Q = 0 \) mod 8, again with the exception of operators coming from the \( 10 \) and \( 45 \). In this case the orientifold action on the two-form fields is

\[
\begin{pmatrix}
B \\
\tilde{B}
\end{pmatrix} =
\begin{pmatrix}
0 & -1 \\
1 & 0
\end{pmatrix}
\begin{pmatrix}
B \\
\tilde{B}
\end{pmatrix}.
\] (23)
The surviving modes will be linear combinations of these:

\[ B' = B + i\tilde{B}, \]
\[ \tilde{B}' = B - i\tilde{B}. \]  

(24)

The total phase is \( \exp \left( i\pi \left( \frac{QR \pm 2}{4} \right) \right) \), so the surviving operators in the \textbf{10} and \textbf{45} are those with \( QR = \pm 2 \mod 8 \). This leads to the results in tables 4a and 4b.

| \( \Delta \) | \( SU(4) \) | \( SU(2) \times SU(2)_R \times U(1)_R \) |
|---|---|---|
| 2 | 20' | (3,3)\(_0\) \(\oplus\) (1,1)\(_0\) |
| 3 | 10 | (3,1)\(_{-2}\) \(\oplus\) (1,3)\(_2\) |
| 3 | 50 | (2,2)\(_0\) \(\oplus\) (4,4)\(_0\) |
| 4 | 45 | (4,2)\(_{-2}\) \(\oplus\) (2,4)\(_2\) \(\oplus\) (2,2)\(_2\) \(\oplus\) (2,2)\(_{-2}\) |
| 4 | 105 | (1,1)\(_0\) \(\oplus\) (1,1)\(_8\) \(\oplus\) (1,1)\(_{-8}\) \(\oplus\) (3,3)\(_0\) \(\oplus\) (5,5)\(_0\) |

Table 4a: The \( Z_4 \) case: invariant operators.

| \( \Delta \) | \( SU(2) \times SU(2)_R \times U(1)_R \) |
|---|---|
| 2 | (3,3)\(_0\) |
| 3 | (1,3)\(_2\) |
| 3 | (4,4)\(_0\) |
| 4 | (2,4)\(_2\) |
| 4 | (1,1)\(_8\) |
| 4 | (5,5)\(_0\) |

Table 4b: The \( Z_4 \) case: chiral primary fields

Finally in the \( Z_6 \) case the invariant operators are those with \( U(1) \) charge \( Q = 0 \mod 12 \), again with the exception of operators coming from the \textbf{10} and \textbf{45}. The orientifold action on the two-form fields is

\[
\begin{pmatrix}
B \\
\tilde{B}
\end{pmatrix} =
\begin{pmatrix}
0 & -1 \\
1 & 1
\end{pmatrix}
\begin{pmatrix}
B \\
\tilde{B}
\end{pmatrix}.
\]  

(25)

The total phase turns out to be \( \exp \left( i\pi \left( \frac{QR \pm 2}{6} \right) \right) \), so the surviving operators in the \textbf{10} and \textbf{45} are those with \( QR = \pm 2 \mod 12 \). This leads to the results in tables 5a and 5b.
Table 5a: The \( Z_6 \) case: invariant operators.

| \( \Delta \) | \( SU(4) \) | \( SU(2) \times SU(2)_R \times U(1)_R \) |
|-------------|-------------|----------------------------------|
| 2           | 20'         | \((3, 3)_0 \oplus (1, 1)_0\)       |
| 3           | 10          | \((3, 1)_{-2} \oplus (1, 3)_2\)    |
| 3           | 50          | \((2, 2)_0 \oplus (4, 4)_0\)       |
| 4           | 45          | \((4, 2)_{-2} \oplus (2, 4)_{2} \oplus (2, 2)_{2} \oplus (2, 2)_{-2}\) |
| 4           | 105         | \((1, 1)_0 \oplus (3, 3)_0 \oplus (5, 5)_0\) |

Table 5b: The \( Z_6 \) case: chiral primary fields.

| \( \Delta \) | \( SU(2) \times SU(2)_R \times U(1)_R \) |
|-------------|----------------------------------|
| 2           | \((3, 3)_0\)                     |
| 3           | \((1, 3)_{2}\)                   |
| 3           | \((4, 4)_0\)                     |
| 4           | \((2, 4)_{2}\)                   |
| 4           | \((5, 5)_0\)                     |

This is a prediction for the marginal and relevant perturbations, based on the correspondence advocated in \[1\]. Since there is no known perturbative description of these fixed points it is not possible to give a representation of these operators here. One feature which is easy to check is the presence of the operator which corresponds to the parameter on the Coulomb branch \[^6\] (which appears in the Seiberg-Witten curves of these theories\[22, 23\]). In the \( Sp(2N) \) case it is \( tr(\phi^2) \), so it is the \( \Delta = 2, (1, 1)_4 \) entry in Table 2b. In the \( E_6, E_7, E_8 \) cases the corresponding operator has dimension 3, 4 and 6 (essentially for symmetry reasons). Indeed, in the \( E_6 \) case (Table 3b) one finds the dimension 3 operator \((1, 1)_6\), and in the \( E_7 \) case (Table 4b) one finds the dimension 4 operator \((1, 1)_8\). There is no \((1, 1)_{12}\) operator in Table 5b, since only relevant and marginal operators are listed there.

5 Conclusions

It is clearly of interest to make use of Maldacena’s duality in contexts, where other methods are not available. This note reported an analysis of spectra of superconformal field theories with 8 supersymmetries describing the...
worldvolume dynamics of D3-branes in the presence of D7-branes and an orientifold plane. These models can be viewed as points in F-theory moduli space characterized by a constant expectation value of the dilaton and Ramond-Ramond scalar. At these points the base of the K3 fibration becomes $T^2/Z_n$ ($n = 2, 3, 4, 6$). The worldvolume theories on the D3-branes have flavour symmetry groups $SO(8)$, $E_6$, $E_7$ and $E_8$. In the $Z_2$ case where the field theory is known one can make a detailed comparison of the supergravity prediction with what is expected in the field theory, and as in other cases studied previously the spectra match. It should be noted however that the operators considered here account only for the open strings stretched between D3-branes. In addition to these operators there are those which account for strings stretched between D3- and D7-branes. They correspond to states appearing on the supergravity side. For instance there are gauge bosons coming from the coinciding 7-branes which couple to flavour currents in the CFT.

The $Z_3$, $Z_4$, $Z_6$ cases provide examples of superconformal systems without known field theory descriptions. Yet it is still possible to apply Maldacena’s duality! The predictions reported here cannot be compared with anything known at this time, but perhaps these and similar calculations may provide information which will help to find a description of such theories.

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