Core Structure of Global Vortices in Brane World Models

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We study analytically and numerically the core structure of global vortices forming on topologically deformed brane-worlds with a single toroidally compact extra dimension. It is shown that for an extra dimension size larger than the scale of symmetry breaking the magnitude of the complex scalar field at the vortex center can dynamically remain non-zero. Singlevaluedness and regularity are not violated. Instead, the winding escapes to the extra dimension at the vortex center. As the extra dimension size decreases the field magnitude at the core dynamically decreases also and in the limit of zero extra dimension size we reobtain the familiar global vortex solution. Extensions to other types of defects and gauged symmetries are also discussed.

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I. INTRODUCTION

The idea of extra dimensions has been an appealing concept in theoretical physics since the 1920’s when it was first considered by Kaluza[1] and Klein[2]. They proposed that our universe may be embedded in a five dimensional spacetime where the fifth dimension is not detected because it is compact and small. They achieved a unification of gravitational and electromagnetic forces by inducing four dimensional gauge interactions solely by the five dimensional geometry. Isometries of the extra-dimensional compact manifold are then realized as gauge symmetries of an effective four dimensional theory. In such a framework the extra components of the metric play the role of the gauge fields of the four dimensional theory.

Later, in the context of string theory (for a good review see [3]) it was shown that the existence of extra dimensions is crucial not only for the unification of gravity with other interactions but also for its quantization. String theory is the only consistent quantum theory of gravity and its consistency requires the existence of extra dimensions. Recently, the possibility of having large extra dimensions has been considered as a solution of the hierarchy problem[1, 2, 3, 4, 5, 6, 7]. According to this proposal, the fundamental Planck scale of the $4+D$ dimensions is close to the TeV scale not far from the electroweak and strong interaction scales. The much larger value of the four dimensional Plank mass $M_4$ would be due to the large size of the extra dimensions according to the relation

\[ M_4^2 = b^D M_{4+D}^{D+2} \]  \hspace{1cm} (1.1)

where $b$ is the radius of the compact extra dimensions. Consistency with phenomenology then requires that whereas gravity can propagate in the $4+D$ dimensional bulk space, the ordinary matter fields and gauge bosons be bound to live on a 3 dimensional brane which would constitute the usual spatial dimensions.

The localization of the brane in the compact $D$-dimensional manifold spontaneously breaks the corresponding Kaluza-Klein (KK) isometries and defines a set of Goldstone boson fields which parametrize the location of the brane in the compact manifold $\mathbb{S}^n[4, 5, 10, 11, 12, 13]$. These zero modes can in general depend on the position on the brane and their excitations have been called ’branons’ in previous studies[11].

Topologically non-trivial configurations of these geometrical Goldstone fields can produce a new set of topological states: vortices, monopoles[10] and skyrmions[13]. These defects form when the isometry group $G$ of the $D$-dimensional compact manifold $M$ breaks down to a subgroup $H$ due to the localization of the brane. If now the vacuum manifold $G/H$ has $n$-dimensional closed surfaces that can not be contracted to a point in $G/H (\pi_n(G/H) \neq I)$ then topologically non-trivial configurations can form for $n = 1$ (vortices), $n = 2$ (monopoles) and $n = 3$ (skyrmions). These form by mapping the surfaces $S^1$, $S^2$ or $S^3$ from the physical space on the brane to the vacuum manifold. They correspond to a constant change of the position of the brane on the $D$-dimensional compact space (vacuum manifold) as we travel a closed surface in $3+1$ dimensions. For example for $n = 1$, the compact space is $S^1$ and a geometrical ‘cosmic string’ forms when travelling around such a string a four dimensional observer makes a full circle along the extra dimension $S^1$. The condition for such a geometrical string to form is

\[ \theta_2(\theta_1) = \theta_1 \]  \hspace{1cm} (1.2)

where the Goldstone field $\theta_2$ parametrizes the position of the brane along the extra dimension $S^1$ while $\theta_1$ is the usual azimuthal angle on the brane. On the other hand no string would form if $\theta_2 = const$ and the Goldstone field is independent of position on the brane.

It has been shown[10] that in the core of such defects, the broken isometries of the compact space get restored and the four dimensional observer is free to move anywhere along the $D$-dimensional compact space. Another
interesting possibility which has not been investigated so far is the modification of the size of the extra dimensions at the geometrical defect core due to energetic reasons (the stabilization of extra dimensions size is dynamical). This could lead to spatially variable size of extra dimensions with interesting cosmological consequences.

These topological defects are geometrical since they form due to the spontaneous breaking of isometries of extra dimensions and are distinct from the usual defects formed by the spontaneous breaking of non-geometrical scalar field symmetries. However, non-geometrical defects do admit several new generalized properties in the context of brane-world models. For example the 3+1 dimensions of our brane-world can be identified with the core of topological defects residing in a higher dimensional space-time (a domain wall in 5d, string in 6d, monopole in 7d, or other defects) In this framework, the localization of matter fields on the defect core can be associated with the fermionic and scalar zero modes localized in defect cores. The scalar fields forming these defects are assumed to propagate in the extra dimensions (the bulk) and depend only on the extra coordinates. For example, the scalar field of a global vortex in 6d would be

\[ \phi(\rho_2, \theta_2) = f(\rho_2) e^{im\theta_2} \]  

(1.3)

where \( \rho_2 \) and \( \theta_2 \) are the bulk radius and bulk angle and \( m \) is the integer winding number (in what follows we set \( m = 1 \) for simplicity).

In addition to this brane-world generalization of non-geometrical defects we can consider another interesting generalization using just scalar fields that are localized on the brane \((t, \rho, \theta_1, z)\). Consider for example the brane localized complex scalar field ansatz:

\[ \phi(\rho, \theta_1, \theta_2) = f(\rho) e^{i\alpha \theta_1 + i(1-\alpha)\theta_2} \]  

(1.4)

where \( \phi \) is localized on the 3+1 brane of a 5d bulk with a toroidally compact extra dimension of size \( b \) parametrized by \( \theta_2 \) and \( \alpha(\rho) \) is a real function of \( \rho \) with

\[ \alpha(\rho \to \infty) = 1 \]  

(1.5)

for unit winding vortex. If the location of the brane in the bulk is independent of the position on the brane, the branons are not excited (locally or topologically) and we may set \( \theta_2 = \text{const} \). In this case, regularity of \( \phi \) at the origin \( \rho = 0 \) implies that

\[ \alpha(\rho) = 1 \quad f(0) = 0 \]  

(1.6)

and we obtain the well known global vortex.

A much more interesting case appears when we have topological excitations of branons. The simplest such case occurs when \( \theta_2 = \theta_1 \). In this case, the boundary condition does not imply the usual conditions. Instead, it is dynamics that will determine the core structure of the global vortex. This will be demonstrated in the next section. It will be seen that the global vortex generalizes in this case with a new dimensionless parameter \( \tilde{b} = \eta b \) which is the ratio of the size of the compact extra dimension over the scale of symmetry breaking of the vortex \( \eta^{-1} \). For \( \eta b \to 0 \) we dynamically reobtain the usual core structure of the global vortex \( f(0) = 0 \). For \( \tilde{b} \gg 1 \) we have a new structure with \( f(0) \approx \eta \) without violation of regularity since it is energetically favorable in this case for the winding to escape in the extra dimension with \( \alpha(0) = 0 \). The gauged case and the extension to other types of defects is discussed in section III. Finally in section IV we conclude and discuss the possible extensions and implications of these results.

II. THE GLOBAL BRANE VORTEX

Consider a spacetime with topology \( \mathcal{M}_4 \times S^1 \) where \( \mathcal{M}_4 \) is our four dimensional Minkowski space and \( S^1 \) is a toroidally compact extra dimension. In this spacetime we consider a Lagrangian of a complex scalar field \( \phi \) with a broken global \( U(1) \) symmetry

\[ \mathcal{L} = \frac{1}{2} \partial_M \phi^* \partial^M \phi - \frac{\lambda}{4}(\phi^* \phi - \eta^2)^2 \]  

(2.1)

where \( M = 0, \ldots, 4 \) and the extra coordinate \( x^4 \) is compact with size \( b \) and parametrized by an angle \( \theta_2 \). The corresponding energy density of a static scalar field configuration is of the form

\[ \mathcal{E} = \frac{1}{2} \partial_\rho \phi \partial_\rho \phi^* + \frac{1}{2} \partial_\theta_1 \phi \partial_\theta_1 \phi^* + \frac{1}{2} \partial_\theta_2 \phi \partial_\theta_2 \phi^* + \frac{\lambda}{4}(\phi^* \phi - \eta^2)^2 \]  

(2.2)

where \( (\rho, \theta_1, z) \) are the usual cylindrical coordinates of \( M_4 \).

We now use the generalized global vortex ansatz \( (1.4) \) with boundary conditions at infinity

\[ \alpha(\rho) \to 1 \quad f(\rho) \to \eta \]  

(2.3)

The energy density for this ansatz takes the form

\[ 2\mathcal{E} = f'^2 + \alpha^2(\theta_1 - \theta_2)^2 + \frac{\alpha^2 f^2}{\rho^2} + \frac{(1 - \alpha)^2 f^2}{b^2} + \frac{\lambda}{4}(f^2 - \eta^2)^2 \]  

(2.4)

We now distinguish between two topologically inequivalent brane configurations

1. The vacuum sector with \( \theta_2 = \text{constant} \) where all points of the brane are located at the same extra coordinate. In this case continuity and regularity of \( \phi \) at the origin \( \rho = 0 \) implies that \( f(0) = 0 \) and \( \alpha(\rho) = 1 \). We thus obtain the usual global vortex.

2. The topological sector with \( \theta_2 = \theta_1 \) where as we span a circle on the brane we at the same time span a circle in the extra dimension. Thus the brane is deformed along the extra dimension in a topologically non-trivial way. In this case the core properties of the global vortex can be modified in two important ways.
The function $\alpha(\rho)$ is not required to be integer and constant. Regularity and singlevaluedness of $\phi$ are not violated for any value of $\alpha(\rho)$. Thus $\alpha(\rho)$ is determined by dynamics (energy minimization).

If dynamical arguments lead to $\alpha(0) = 0$ then $\phi$ can be regular at the origin with $f(0) = c \neq 0$. This is a novel type of behavior for the global vortex core.

In what follows we focus on the case of the topological sector ($\theta_2 = \theta_1$) and investigate the properties of the global vortex by deriving the field functions $\alpha(\rho)$ and $f(\rho)$ with the boundary conditions (2.3). The analytical derivation of $\alpha(\rho)$ is simple since for $\theta_2 = \theta_1$ the energy density (2.14) is independent of $\alpha'$. Thus we simply demand

$$\frac{\delta \mathcal{E}}{\delta \alpha} = 0 \quad (2.5)$$

which leads to

$$\alpha(\rho) = \frac{\rho^2}{b^2 + \rho^2} \quad (2.6)$$

Thus $\alpha(\rho)$ has the expected asymptotic behavior at $\rho \to \infty$ ($\alpha \to 1$) and at $\rho \to 0$ ($\alpha \to 0$). This dynamical behavior of $\alpha(\rho)$ implies that the winding at the origin transfers from the brane coordinate $\theta_1$ to the extra dimension $\theta_2$. Since now $\phi$ is independent of $\theta_1$ at the origin $\rho = 0$, it does not have to be zero to maintain its singlevaluedness and regularity. Thus, the boundary condition for $f(\rho)$ at the origin is $f(0) = c$ (and $f'(0) = 0$ by symmetry considerations) where the values of $c$ and $f(\rho)$ are to be determined dynamically in terms of the single dimensionless parameter of the model $b = \eta \rho$. In order to find $f(\rho)$ we rewrite the energy density (2.4) in dimensionless form using $\theta_1 = \theta_2$. After setting

$$f = \frac{\rho}{\eta} \to f$$

$$\lambda \eta^2 \rho^2 \to \rho^2$$

$$\theta_1 = \theta_2$$

$$\lambda \eta^2 b^2 \to b^2 \quad (2.10)$$

equation (2.4) becomes

$$\frac{2\mathcal{E}}{\lambda \eta^2} \equiv \bar{\mathcal{E}} = f'^2 + \frac{\alpha^2 f^2}{\rho^2} + \frac{(1 - \alpha)^2 f^2}{b^2} + \frac{1}{2}(f^2 - 1)^2 \quad (2.11)$$

The rescaled field equation for $f(\rho)$ is

$$f'' + \frac{f'}{\rho} - \frac{\alpha^2 f}{\rho^2} - \frac{f(1 - \alpha)^2}{b^2} - (f^2 - 1)f = 0 \quad (2.12)$$

For $b \to 0$ equation (2.10) gives $\alpha = 1$ and we obtain the usual global vortex field equation as we should. For $b \neq 0$ we may use equation (2.6) to eliminate $\alpha(\rho)$ from (2.11) and (2.12) leading to

$$\bar{\mathcal{E}} = f'^2 + \frac{f^2}{\rho^2 + b^2} + \frac{1}{2}(f^2 - 1)^2 \quad (2.13)$$

for the rescaled energy density and

$$f'' + \frac{f'}{\rho} - \frac{f}{\rho^2 + b^2} - (f^2 - 1)f = 0 \quad (2.14)$$

for the rescaled field equation. For small $\rho$ ($\rho < 1$ and $\rho < b$) equation (2.14) implies $f(\rho) = c$ where $c$ is a constant. For $b < 1$ and in the range $b < \rho < 1$, $f(\rho)$ is approximated by

$$f(\rho) = c \frac{\rho}{b} \quad (2.15)$$

By extrapolating equation (2.15) up to $\rho = \frac{b}{b}$ and setting $f = 1$ beyond that, we obtain a crude analytical approximation for the field function $f(\rho)$. This analytic approximation can be used to minimize the energy with respect to $c$ for various values of $b$. Thus, finding the value of $c$ that minimizes the energy for a given $b$ we have a crude analytic approximation of $c(b)$ to compare with the corresponding numerical result obtained by solving the differential equation (2.14) with boundary conditions $f(\infty) = 1$ and $f'(0) = 0$. This comparison is shown in Fig. 1. The corresponding forms of the field function $f(\rho)$ obtained numerically by solving equation (2.14) for various values of $b$ are shown in Fig. 2. As expected, for a size of the extra dimension much less than the scale of symmetry breaking ($b \ll 1$) $f(0)$ goes continuously to 0 and the usual global vortex solution is reached. At the other extreme where the extra dimension size is much larger than the scale of symmetry breaking ($b \gg 1$) the energy density approaches 0 everywhere. In this limit, $|\phi|$ remains at its vacuum everywhere while the winding is smoothly transferred to the extra dimension with practically no cost in gradient energy. This transition is shown in Fig. 3 where the numerically obtained energy...
The energy density of the static Abelian Higgs model embedded in a brane-world with a toroidally compact extra dimension is

$$\mathcal{E} = (D_i\phi)^*(D_i\phi) + \lambda \left( \phi^* \phi - \eta^2 \right)^2 + \frac{1}{2} F_{ij} F_{ij}$$  \hspace{1cm} (3.1)$$

where $D_i = \partial_i - i e A_i$ and $F_{ij} = \partial_i A_j - \partial_j A_i$ with $i,j = 1,2,3$. The spatial coordinate $x_3$ is taken to be toroidally compact ($x_3 = b \theta_3$). In analogy with the usual NO vortex in two spatial dimensions and with the results of the previous section we use the following generalization of the NO ansatz

$$\phi = f(\rho) e^{i (\alpha \theta_1 + (1-\alpha) \theta_2)}$$  \hspace{1cm} (3.2)$$

$$A_1 = -a_1(\rho) \sin \theta_1$$  \hspace{1cm} (3.3)$$

$$A_2 = a_1(\rho) \cos \theta_1$$  \hspace{1cm} (3.4)$$

$$A_3 = a_2(\rho)$$  \hspace{1cm} (3.5)$$

Using this ansatz and equation (3.11) we may express the energy density in terms of $f(\rho)$, $a_1(\rho)$ and $a_2(\rho)$. The result (for $\theta_2 = \theta_1$) in dimensionless form is

$$\frac{2\mathcal{E}}{\eta^4 \rho^2} = \tilde{\mathcal{E}} = f'^2 + \frac{f^2}{\rho^2} (\alpha - a_1)^2 + \frac{f^2}{\rho^2} ((1-\alpha) - a_1)^2 + \frac{\beta}{2} (f^2 - 1)^2 + \frac{a_{11}^2}{\rho^2} + \frac{a_{22}^2}{\beta^2}$$  \hspace{1cm} (3.6)$$

where we have set

$$\eta \rightarrow f$$  \hspace{1cm} (3.7)$$

$$ea_1 \rightarrow a_1$$  \hspace{1cm} (3.8)$$

$$ea_2 \rightarrow a_2$$  \hspace{1cm} (3.9)$$

$$e\eta \rho \rightarrow \rho$$  \hspace{1cm} (3.10)$$

$$e\eta \beta \rightarrow b$$  \hspace{1cm} (3.11)$$

$$\lambda \rightarrow \beta$$  \hspace{1cm} (3.12)$$

As in the global case the functional form of $\alpha(\rho)$ is obtained analytically by energy minimization

$$\frac{\delta \mathcal{E}}{\delta \alpha} = 0 \Rightarrow \alpha = \frac{\rho^2 + b^2 a_1 - \rho^2 a_2}{b^2 + \rho^2}$$  \hspace{1cm} (3.13)$$

which has the correct global limit. Using equation (3.13) for $\alpha(\rho)$ in (3.6) we may write the rescaled energy density as

$$\tilde{\mathcal{E}} = f'^2 + \frac{f^2 ((1-\alpha) - a_2)^2}{\rho^2 + b^2} + \frac{\beta}{2} (f^2 - 1)^2 + \frac{a_{11}^2}{\rho^2} + \frac{a_{22}^2}{b^2}$$  \hspace{1cm} (3.14)$$

By extremizing $\tilde{\mathcal{E}}$ with respect to $f$, $a_1$ and $a_2$ we find the corresponding field equations

$$f'' + \frac{f'}{\rho} = -\frac{(1-a_1 - a_2)^2}{\rho^2 + b^2} f - \beta (f^2 - 1) f = 0$$

$$a_1'' - \frac{a_1'}{\rho} + \frac{(1-\alpha - a_2)^2}{\rho^2 + b^2} f^2 = 0$$  \hspace{1cm} (3.15)$$

$$a_2'' + \frac{a_2'}{\rho} + \frac{(1-\alpha - a_2)^2}{\rho^2 + b^2} f^2 = 0$$

III. GENERALIZED NIELSEN-OLESEN VORTEX

In this section we attempt a generalization of the Nielsen-Olesen [29] (NO) vortex along the lines of the previous section where the global vortex was generalized by an embedding to topologically non-trivial brane-worlds. The energy density of the static Abelian Higgs model embedded in a brane-world with a toroidally compact extra dimension is
The boundary conditions at infinity are
\[ f \to 1, \quad a_1' \to 0 \quad a_2' \to 0 \] (3.16)
and at the origin
\[ f' \to 0, \quad a_1 \to 0 \quad a_2' \to 0 \] (3.17)
The next step would be to solve the system (3.15) with the above boundary conditions to obtain the field functions \( f(\rho), a_1(\rho) \) and \( a_2(\rho) \). However, the existence of a trivial zero energy solution satisfying the boundary conditions saves the effort. The solution is
\[ f = 1 \quad a_1 = 0 \quad a_2 = 1 \] (3.18)

ie there is a non-singular gauge transformation that transforms the vortex to a 0 energy configuration. Notice that we are free to take \( a_2 \neq 0 \) at \( \rho = 0 \) without violating singlevaluedness of the gauge field \( A \) since \( A \) does not wind along the extra dimension and also \( b \neq 0 \). We conclude that the gauged analog of the generalized global vortex discussed in the previous section is trivial.

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The brane-world defect generalizations discussed in this and in the previous section can be extended to the cases of monopoles and skyrmions. For example the case of global monopoles would require two extra dimensions with spherical compactification parametrized by the angles \((\theta_2, \varphi_2)\). The corresponding generalized global monopole ansatz is
\[ \phi(\rho, \theta_1, \varphi_1, \theta_2, \varphi_2) = f(\rho) \begin{pmatrix} \sin \Theta \cos \Psi \\ \sin \Theta \sin \Psi \\ \cos \Theta \end{pmatrix} \] (3.19)
where
\[ \Theta \equiv \alpha_\theta \theta_1 + (1 - \alpha_\theta) \theta_2 \]
\[ \Psi \equiv \alpha_\varphi \varphi_1 + (1 - \alpha_\varphi) \varphi_2 \] (3.20)
\[ \theta_1, \varphi_1 \] are the usual spherical coordinates on the brane and \( \alpha_\theta(\rho), \alpha_\varphi(\rho) \) generalize the function \( \alpha(\rho) \) of equation (12).

The detailed study and classification of all generalized brane-world defects is an interesting open issue.

IV. CONCLUSION - OUTLOOK

We have studied field configurations of vortices in braneworlds which are topologically deformed along a bulk with a toroidally compact extra dimension. We have found that the field configuration of generalized global vortices depends on a single dimensionless free parameter \( b = \eta b \): the ratio of the size of the extra dimension \( b \) over the scale of symmetry breaking \( \eta^{-1} \) that gives rise to the vortex. For \( \eta b \to 0 \) the effects of the extra dimension can be ignored and the field configuration is that of the well known global vortex. For \( \eta b \simeq O(1) \) the core of the vortex acquires novel properties: the complex scalar field remains non-zero and singlevalued at the origin while its winding escapes to the extra dimension. In addition, the energy density of the vortex gets reduced at the origin since the potential (and gradient) energies get smaller. For \( \eta b \gg 1 \) the energy density is negligible on scales \( \rho < b \) since the winding resides exclusively on the large extra dimension on these scales, contributing negligible gradient energy. Thus it is possible to have topologically non-trivial configurations with negligible energy density at the core. The total energy however was shown to remain divergent logarithmically like \( \ln(\frac{\rho}{b}) \). It should be noted that for large extra dimensions \((\mathcal{O}(\geq 1nm))\) and vortices produced at symmetry breaking energy scales larger than the electroweak scale we anticipate \( \eta b \gg 1 \).

For the gauged case we showed that the extra component of the gauge field corresponding to the extra dimension is able to eliminate the energy density of the generalized NO vortex even though its topological properties remain non-trivial \( [31] \). If however only the coordinates on the brane are gauged then the energy density of the vortex can not be eliminated and its core properties will be similar to those of the brane-world global vortex. The extension of this study to other types of defects in topologically non-trivial braneworlds is an issue worth of further investigation.

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