A comprehensive photometric and kinematical characteristic of the newly discovered QCs clusters with Gaia EDR3

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Abstract. This study reports the first comprehensive astrometric, photometric, and kinematical analysis of four newly discovered open clusters; namely QC1, QC2, QC3, and QC4, using astrometric and photometric data from the most recent Gaia EDR3 for G < 17 mag. Utilizing the ASteCA code, we identified the most probable (P ≥ 50%) star candidates and found the numbers of star members (N) to be 118 (QC1), 142 (QC2), 210 (QC3), and 110 (QC4). By fitting King’s density profile to the cluster’s RDPs, we found the internal structural parameters of each cluster such as the cluster radii that are in the range 7.00–11.00 arc-min. For each cluster, we constructed the CMD and by fitting them with suitable isochrones we found that the metallicity range is (0.0152–0.0199) which is in line with the Solar value, the logarithmic age (in yrs) range between 6.987 and 8.858. The distances derived from CMD are 1674 ± 41, 1927 ± 44, 1889 ± 43, and 1611 ± 40 (pc) for QC1, QC2, QC3, and QC4, respectively, and they are in good agreement up to 85% with the values obtained from the astrometric data. In addition, from the MLR of the clusters, we obtained a total mass, $M_C$ (in Solar units), of 158, 177, 232, and 182 and an absolute magnitude $M_G$ (mag) of 4.33, 3.80, 4.25, and 4.10 for QC1, QC2, QC3, and QC4, respectively. The dynamical analysis and evolution parameters of the cluster members indicated that all the four clusters are dynamically relaxed; except QC1 which has an evolution parameter $s$ > 0.82 that indicates a dynamical activity within the cluster. From the kinematical analysis of the cluster data, we computed the space velocity, the coordinates of the apex point (A, D) using the AD—diagram method, as well as the Solar elements ($S_\odot$, $l_\odot$, $b_\odot$, $x_\odot$, $\delta_\odot$).

Keywords. Open clusters: Cygnus clouds—Gaia EDR3—ASteCA package—color-magnitude diagrams (CMDs)—velocity ellipsoid parameters (VEPs)—kinematics.

1. Introduction

Open clusters (OCs) are formed within giant molecular clouds (GMCs) that are located in the disk of the Milky Way Galaxy (Lada & Lada 2003; Portegies et al. 2010). OCs that possess simple populations with relatively easy determined ages are known as associations. They are among the best tracers of the spiral arm structure and the evolution of the Galactic disk (Trumpler 1930; Moffat & Vogt 1973; Janes & Adler 1982; Friel 1995; Moitinho 2010; Moraux 2016). OCs are excellent laboratories to examine stellar physical and dynamical evolution from which we explore the mechanisms of star creation and its recent history (Vandenberg 1983; Barnes 2007; Bertelli et al. 2017; Marino et al.
OCCs are important in the calibration of the distance scale because of the accurate determination of their distances (Perren et al. 2015) and in constraining both the initial luminosity and mass functions in aggregations of stars. Moreover, radial velocities of OCs are used to trace the local kinematics such as the velocity ellipsoid parameters (VEPs), Oort’s constants (A & B), and the rotation curve. Old distant clusters are used to define disk abundance gradients, construct the age-metallicity relation, understand the complex history of chemical enrichment, and the mixing processes in the disk (Friel 1995).

The constellation Cygnus is located in the Galactic plane within the spiral arm that hosts our Galaxy (Bochkarev & Sitnik 1985). Cygnus is a place of star formation where a number of young stars, associations and open clusters have been detected with a wide range of masses, ages, and uncertainties in their distances. One of the open clusters that lies in the region of Cygnus is NGC 7062 which is classified as II2p (Trumpler 1930). NGC 7062 is located at a heliocentric distance of about 1600 pc with an estimated age 6.9 × 10^8 years (Kharchenko et al. 2013). The cluster’s equatorial coordinates are (α = 21^h 23^m 27^s.00, δ = 46° 23’ 24”.00) while its Galactic coordinates are (l = 89°.967, b = −2°.740) (Carrera et al. 2019).

Qin et al. (2021) analyzed the region surrounding NGC 7062 and the eastern part of Cygnus, (77° ≤ l ≤ 90° and −3° ≤ b ≤ 4°), using data from Gaia DR2 (Gaia Collaboration et al. 2016, 2018). The authors reported the discovery of four open clusters known as: QC1 (l = 77°.635, b = 1°.932), QC2 (l = 84°.802, b = 3°.278), QC3 (l = 84°.976, b = 2°.513), and QC4 (l = 85°.036, b = 3°.654). Among the 2443 star cluster candidates observed and listed in the online catalog1 by Liu & Pang (2019), the location of the two clusters with IDs 506 (l = 84°.959, b = 2°.529) and 600 (l = 84°.81, b = 3°.258) is very close to that of QC3 and QC2, respectively, discovered by Qin et al. (2021). Qin et al. (2021) retrieved astrometric and photometric data for the four newly discovered clusters from the Gaia DR2 and computed their structure and photometric parameters. The authors concluded that future observations as well as investigations are required to characterize the properties of the four clusters.

Most recently, the Gaia mission collaborations released the early data release 3 (Gaia EDR3; Gaia Collaboration 2020). Similar to DR2, the EDR3 provides five astrometric parameters; Galactic position (l, b), proper motion (μ_x cos δ, μ_y), parallax (π) and the photometric magnitude parameters in 3 filters (G, G_BP, and G_RP) for a larger number of sources up to 1.8 billion sources with brightness larger than 21 (i.e., numerically < 21). The EDR3 is complemented with data of the radial velocity (V_r) for about 7 million stars from DR2 (Gaia Collaboration 2020). The source list has a slight change to DR2 with some notable changes. The significant advance of EDR3 over DR2 is the large improvement in the accuracy of the astrometric parameters; a factor 2 in proper motion accuracy and a factor of about 1.5 in the parallax accuracy. Astrometric errors were suppressed by 30–40% for the parallax and by a factor of 2.5 for the proper motion.

In addition, advances in photometry exist with a better homogeneity for the magnitude and color due to the significant improvement in several aspects related to the photometric reductions and calibrations lead to reduce the errors of the photometric magnitudes estimates (Gaia Collaboration 2020; Torra et al. 2021; Riello et al. 2021). All of these improvements lead to a more accurate measurement of the blue (G_BP) and red (G_RP) photometric magnitudes. As a consequence, for all of these improvements, reflections on the astrophysical results are expected and astronomers are optimistic to obtain a better characterization of different observed objects from different perspectives.

Motivated by the findings of Qin et al. (2021) and the huge improvement granted by EDR3, we conducted the present comprehensive astrometric, photometric, and kinematic study in an attempt to fully characterize the four newly discovered clusters; QC1, QC2, QC3, and QC4.

The structure of the article is as follows: Section 2 describes the raw data from Gaia EDR3 and the adopted methodology in this study. Results of the analysis of the EDR3 data for the new open clusters in which the basic astrometric and photometric as well as the dynamical and kinematical parameters are derived, are discussed in Section 3. Finally, our main conclusions are summarized in Section 4.

2. Data from Gaia EDR3 and analysis tools

The data retrieved for the four clusters from EDR3 includes the astrometric parameters (α, δ, l, b, μ, π), the three photometric magnitudes (G, G_BP, G_RP) and the radial velocity (V_r) with their uncertainties. For each cluster, we downloaded data sheets of a surrounding space centered at the position of the cluster centers (α, δ)
Table 1. The initial conditions adopted in this work to obtain data from Gaia EDR3 as taken from Qin et al. (2021).

| Parameters | QC1            | QC2            | QC3            | QC4            |
|------------|----------------|----------------|----------------|----------------|
| $\alpha$   | 20°20'25''.40  | 20°37'23''.70  | 20°41'31''.40  | 20°36'28''.00  |
| $\delta$   | 39°52'15''.60  | 46°28'08''.40  | 46°08'31''.20  | 46°52'55''.20  |
| $l$        | 77°.635        | 84°.802        | 84°.976        | 85°.036        |
| $b$        | 1°.932         | 3°.278         | 2°.513         | 3°.654         |
| $r_{cl}$   | 8.76           | 8.75           | 12.82          | 11.79          |

Figure 1. The spatial distribution of the four newly discovered open clusters together with the position of the well-known open cluster NGC 7062, for comparison. The five cluster positions are taken from Qin et al. (2021).

Figure 2. Uncertainties in the three photometric magnitude bands; $\sigma_G$, $\sigma_{BP}$, and $\sigma_{RP}$ (top panel) and the proper motion and its uncertainties ($\mu$, $\sigma_{\mu}$) in both directions (bottom panel), for the data used in this study, with respect to the photometric $G$ magnitude.

computed by Qin et al. (2021) and has an arbitrary chosen radius that is more than twice the cluster radius, $r_{cl}$, obtained by Qin et al. (2021). In this way, we initiate the calculations with a more realistic guess that would reduce the output uncertainties. The radius of QC1 and QC2 was adapted to be 20 arc-min while that for QC3 and QC4 was 25 arc-min. Table 1 lists the initial boundary conditions adopted in this study taken from Qin et al. (2021). The spatial distribution of the four OCs with the well-known cluster NGC 7062, for comparison, is illustrated in Figure 1 while errors in both the photometric $G$-magnitude (upper panels) and the astrometric proper motion (lower panels) in Gaia EDR3 with respect to the photometric $G$-magnitude for the retrieved data of this study are illustrated in Figure 2. The figure shows that the maximum error in the proper motion data components is 1.6 mas/yr for $G \leq 20$ mag and 0.71 mas/yr for $G \leq 17$ mag.

This study is intended to provide a comprehensive characterization of the four new OCs; QC1 through QC4. Therefore, the Automated Stellar Cluster Analysis (ASteCA) code (Perren et al. 2015) is a suitable tool to achieve our goal because it is designed with many functions that use positional and photometric data to get the basic parameters of a cluster with minimal user intervention. In addition and unlike other codes in literature, ASteCA is available online as an open-source for the public with full documentation.

Briefly, the code enables us to automatically compute most of the characteristic parameters of OCs such as
center coordinates, radius, and both the mass and the luminosity functions. The code is integrated with Bayesian field star decontamination algorithm to assign membership probabilities using photometric and astrometric data. Moreover, the presence of an isochrone fitting process, allows ASteCA to provide accurate estimates for a cluster’s metallicity, age, extinction, and distance values, as well as their uncertainties, by generating synthetic clusters from theoretical isochrones and selecting the best fit using a genetic algorithm. A full description of the code is available in Perren et al. (2015) and on the code website.\(^2\)

3. Results and discussion

3.1 Astrometric structural analysis

3.1.1 Re-determination of the cluster’s centers In order to determine the position of the new centers of the four open clusters, it is important to know the stellar distribution within the space of these clusters by finding the number of stars (the star count) in this space. Since the cluster diameters we used for our sample are \(>10\) arc-min (see Section 2), then we can divide the space along the right ascension (RA) and declination (Dec.) into equal bins each of size \(1.00\) arc-min (0\(^\circ\).017) following Maciejewski & Niedzielski (2007) and Maciejewski et al. (2009). The star counts we obtained are 17,208 for QC1, 28,765 for QC2, 52,319 for QC3, and 37,117 stars in QC4.

Figure 3 represents the Gaussian distribution for the stars within the space of each cluster along RA and Dec. directions. The peak of the distribution in each direction, where star counts concentrate, marks the new position of the center of the cluster. The results showed that the positions of the new centers of the four clusters are in good agreement with those obtained by Qin et al. (2021) in the RA direction while they show a minimal change in the Dec. direction (\(\Delta \delta \leq 1/17\)). The new estimated centers of the clusters QC1, QC2, QC3, and QC4 in the equatorial (\(\alpha, \delta\)) and Galactic (\(l, b\)) coordinate systems are listed in Table 2.

3.1.2 The radial density profiles (RDP) The slight change in the position of a cluster center may lead to a change in the star count of the cluster. Therefore, we extracted a new data sheet for each of the four clusters to re-estimate its star count, within the same radii used in identifying the new center (in Section 2), but centered at the new position of these centers. The new counts are 17,604 for QC1, 29,044 for QC2, 53,389 for QC3 and 38,041 for QC4. From these new star counts, we constructed the radial density profile (RDP) from which we computed the cluster radii by obtaining the best fitting to King’s density profile given in Equation (1) (King 1962). Figure 4 illustrates the four RDPs of the four clusters fitted by King’s profile (green dash line and shaded area).

To know the radial density distribution we divided the cluster into a number of concentric zones, centered at the new center coordinates, with equal sizes and computed the star density within each of these zones. The zone widths are \(0.017, 0.017, 0.017, 0.017\) for QC1, QC2, QC3, and QC4, respectively. For each zone, the stellar number density \((\rho)\) is the number of stars \((N)\) per zone area \((A)\), and hence \(\rho = N/A\). The variation of the stellar number density along the cluster radius defines the radial density profile of the cluster.

With the aid of ASteCA code, we generated four RDP, one for each cluster, and fitted it with King’s profile in Equation (1), to estimate the internal cluster structural parameters such as the cluster core radius \((r_c)\), limiting radius \((r_l)\), tidal radius \((r_t)\), central surface density \((\rho_o)\), and background surface density \((\rho_{bg})\). All radii are measured in arcminutes (arc-min) while all densities are in stars/arc-min\(^2\).

\[
\rho(r) = \rho_{bg} + \frac{\rho_o}{1 + (r/r_c)^{2}}. \tag{1}
\]

The core radius \(r_c\) is the distance at which the value of \(\rho(r)\) becomes half that of the central density \(\rho_o\). We estimated core radii: \(6.10^{\pm0.30}\), \(7.67^{\pm0.64}\), \(7.51^{\pm0.65}\), \(7.52^{\pm0.64}\) arc-min for QC1, QC2, QC3, and QC4, respectively. Our results for all clusters, apart from QC2 (despite the uncertainty), are not in agreement with results of Qin et al. (2021).

The cluster limiting radius \(r_l\) is the radius at which the line represents the value of the background density (dash black lines in Figure 4) intersects the King profile model fitting curve. At this point, the background star density \(\rho_{bg}\) is given by \((\rho_{bg} + 3\sigma_{bg})\) where \(\sigma_{bg}\) is the uncertainty of \(\rho_{bg}\). Bukowiecki et al. (2011) derived an expression for the limiting radius to be

\[
r_{cl} = r_c \left(\frac{\rho_o}{3\sigma_{bg}} - 1\right). \tag{2}
\]

Our calculations showed that the cluster radii are \(7.28, 8.30, 10.52, 10.64\) for QC1, QC2, QC3, and QC4, respectively.
and QC4, respectively. These values are in good agreement (83–95%) with those obtained by Qin et al. (2021). The large uncertainties in the computed values of the cluster radii may imply a non-spherical geometry of these open clusters. Therefore, the clusters might be extended along their radii.

Other parameters may be used to characterize the structure of open clusters such as the density contrast parameter ($d_c$) and the concentration parameter ($C$) (King 1966; Peterson & King 1975; Bonatto & Bica 2009; Santos-Silva & Gregorio-Hetem 2012; Maurya et al. 2021). So far, the present study is the first to compute these two parameters for the four open clusters QC1–QC4. The $d_c$ is the stellar density contrast of these clusters against the background population. It is a measure of the compactness of the

| Cluster | Equatorial coordinates | Galactic coordinates |
|---------|------------------------|----------------------|
|         | $\alpha$ | $\delta$ | $l$ | $b$ |
| QC1     | 20$^h$20$^m$14$.^s$31 | 39$^\circ$52'56''.41 | 77$^\circ$.6240 | 1$^\circ$.9678 |
| QC2     | 20$^h$37$^m$30$.^s$79 | 46$^\circ$28'40''.34 | 84$^\circ$.8011 | 3$^\circ$.2673 |
| QC3     | 20$^h$41$^m$41$.^s$68 | 46$^\circ$07'24''.51 | 84$^\circ$.9793 | 2$^\circ$.4785 |
| QC4     | 20$^h$36$^m$35$.^s$37 | 46$^\circ$51'07''.36 | 85$^\circ$.0249 | 3$^\circ$.6190 |

Figure 3. The Gaussian fitting (solid lines) for the stars occupied within the space of each open cluster. The coordinates where the density peaks along the right ascension (RA) and the declination (Dec.) refers to the position of the new center of the cluster. The identification maps of the four QCs are taken from L.Eicester Database and Archive Service (LEDAS) Digitized Sky Survey (DSS) available at (https://www.ledas.ac.uk/DSSimage).
cluster (Bonatto & Bica 2009) and can be expressed mathematically as
\[ d_c = 1 + \frac{\rho_0}{\rho_{bg}}. \]

For QC1 through QC4, the estimated \( d_c \) is 2.60, 3.91, 1.87, and 2.68 which indicates that the four clusters are scattered with respect to their background density. This finding is supported by Bonatto & Bica (2009) who found that for compact clusters (nearly spherical in geometry) the contrast parameter lies in the range \( (7 \leq d_c \leq 23) \). In addition, this result may give support to our explanation of the large obtained uncertainty in the estimated radii of the clusters.

King (1966) introduced the concentration parameter \( C \) to be the ratio between the cluster limiting, \( r_{cl} \), and core, \( r_c \), radii
\[ C = \frac{r_{cl}}{r_c}, \]
which may reflect the central concentration of the cluster. On the other hand, Santos-Silva & Gregorio-Hetem (2012) defined the \( C \) parameter to be the reciprocal of King (1966) definition. Santos-Silva & Gregorio-Hetem (2012) concluded that young clusters are expected to have small \( C \) values because most of their members are still concentrated around the center and did not have enough time to spread away towards the edge of the cluster. The authors obtained a \( C \) range \((0.04–1.03) \) which is equivalent to \((0.97–25) \) using King’s definition (adopted in this study). Our obtained \( C \) range is \((0.60–2.18) \) which is in fair agreement with Santos-Silva & Gregorio-Hetem (2012) and about 2 to 4 times lower than the range \((2.45–5.75) \) obtained by Qin et al. (2021); derived from their Table 2.

Moreover, Maciejewski & Niedzielski (2007) found that the limiting radius is \((2–7) \) times the value of the core radius (see Table 3). These values are in line with our estimation of only QC2 and larger than the values of the rest of the clusters, however, this range is in good agreement with Qin et al. (2021) results.

The results of QC4 are irregular, therefore more analysis is needed to draw a solid conclusion about it. However, given the small value of the \( C \) parameter \((0.60) \) due to the core radius \((17.52) \) is greater than the cluster radius \((10.64) \) which is coming from the fact that both the cluster density and the field density for QC4 have nearly the same level, then we may conclude that QC4 might not be a genuine cluster. The numerical results of both parameters for the four studied QCs are given in Table 3.

3.1.3 Astrometric parameters and distance determination In order to proceed with the analysis and parameter investigations, it is very important to identify the true (most probable) stellar membership
of each of the clusters in question. Thus, we performed a membership analysis using the available astrometric parameters (proper motion and parallax) from the Gaia EDR3 database. In order to identify the cluster members, we applied the selection criteria of Qin et al. (2021) to retrieve stellar data from Gaia EDR3. We obtained a full data sheet for all stars that have a photometric magnitude ($G > 17$ mag) which corresponds to an uncertainty of 0.20 mas/yr and 0.10 mas in the proper motion ($\mu$) and the parallax ($\pi$), respectively. All the data points were entered into the ASteCA code which we used to assign membership probability by searching for a meaningful stellar over-density and compare them to the surrounding stellar field. For this study, only stars with probabilities ($P \geq 50\%$) are assigned as cluster most probable members. The results of the ASteCA code revealed that the number of the most probable members for the four clusters are 118 (QC1), 142 (QC2), 210 (QC3), and 110 (QC4). This result is what we are relying on throughout the rest of this study in computing the other parameters that require knowledge of the true membership of the cluster.

| Parameters | QC1       | QC2       | QC3       | QC4       | Ref. |
|-----------|-----------|-----------|-----------|-----------|------|
| $r_c$ (arc-min) | 6.1$^{+5.50}_{-3.76}$ | 3.96$^{+5.67}_{-2.33}$ | 9.39$^{+12.7}_{-6.16}$ | 17.52$^{+19.95}_{-14.95}$ |      |
| $r_{cl}$ (arc-min) | 7.28      | 8.30      | 10.52     | 10.64     |      |
| $r_l$ (arc-min) | 8.76$^{+0.76}_{-0.8}$ | 8.75$^{+0.63}_{-0.9}$ | 12.82$^{+0.83}_{-1.9}$ | 11.79$^{+0.81}_{-1.9}$ | Q21  |
| $\rho_{bg}$ (stars arc-min$^{-2}$) | 1.047$^{+0.23}_{-0.5}$ | 1.783$^{+0.32}_{-0.6}$ | 2.029$^{+0.26}_{-0.5}$ | 1.666$^{+0.68}_{-0.5}$ | Q21  |
| $\rho_o$ (stars arc-min$^{-2}$) | 1.673$^{+0.08}_{-0.5}$ | 5.178$^{+0.05}_{-0.6}$ | 1.759$^{+0.08}_{-0.6}$ | 2.793$^{+0.06}_{-0.6}$ | Q21  |
| $\delta_c$ | 2.60      | 3.91      | 1.87      | 2.68      |      |
| $C$        | 1.18      | 2.18      | 1.07      | 0.60      | Q21  |

Figure 5. Upper panel: The distribution of the mean proper motion ($\mu$; mas/yr) in both directions of right ascension and declination. Lower panel: The normalized parallax distribution for all stellar member candidates in the cluster space.

Table 3. Our obtained structural properties for the 4 OCs as computed from the RDP fitted by King’s density profile, using ASteCA code, in comparison with the results of Qin et al. (2021) indicated as Q21 in the last column of the table.
Figure 6. The CMDs for star members of the open clusters QC1, QC2, QC3 (the three BSS are pointed by Asterisk), and QC4 fitted with the extinction isochrone corrected by Bressan et al. (2012) indicated by the red line. $G$ vs. $(G_{BP} - G_{RP})$ are shown (red solid curves).

For each cluster, we plotted its stellar distribution in the proper motion space ($\mu_x \cos \delta$, $\mu_\delta$), see upper panels in Figure 5, from which we determined the mean proper motion of the cluster by acquiring a Gaussian fitting along the corresponding directions. The lower panel of Figure 5 are four histograms that represent the parallax distribution of the candidate members of the four clusters with bin sizes (in mas) of 0.0060 (QC1), 0.0017 (QC2), 0.0020 (QC3), and 0.0034 (QC4). The black line on the histograms is the Gaussian fitting from which the mean parallax of each cluster was estimated.

At this stage of the analysis, we are capable of computing the distance to each cluster knowing its parallax. The calculated distances ($d_{\text{pix}}$, in pc) are: 1819 ± 43, 2151 ± 64, 2288 ± 48, and 2179 ± 47, for QC1, QC2, QC3, and QC4, respectively. These results are in good agreement with those obtained by Qin et al. (2021).

3.2 The photometric analysis

3.2.1 Age, reddening, and the distance modulus The color-magnitude diagram (CMD) is a valuable tool in photometric analysis because we can use it to infer the cluster reddening, distance modulus and age parameters knowing the number of true members. For each cluster, we drew the corresponding CMD using EDR3 photometric magnitudes ($G$, $G_{BP}$, $G_{RP}$) for the member stars (the blue dots in Figure 6). For each CMD of a cluster, we used the ASteCA code and applied the PARSEC v1.2S (Bressan et al. 2012) theoretical isochrones[^3] to obtain the cluster metallicity and age. The best fit values revealed a metallicity ($Z$) of 0.0152 ± 0.0010, 0.0150 ± 0.0004, 0.01520 ± 0.0004, and 0.0199 ± 0.0018 for QC1, QC2, QC3, and QC4, respectively. These values are in line with the standard Solar metallicity adopted by Qin et al. (2021). The logarithmic age, log(age/yr), of the four clusters QC1 through QC4 was estimated to be $6.987 \pm 0.022$, $8.524 \pm 0.046$, $8.858 \pm 0.114$, and $8.367 \pm 0.043$, respectively. For the QC3 cluster, we found that there are three member stars above the turn-off point of its CMD, so we suggest that those members are Blue Stragglers Stars (BSS) according to the criteria of Rain et al. (2021). These ages are in agreement with those previously calculated using Gaia DR2 (Qin et al. 2021).

The reddening parameter, $E(G_{BP} - G_{RP})$, was determined from the CMDs by using the formula $E(G_{BP} - G_{RP}) = 1.289E(B - V)$.

The observed data have been corrected for the reddening with a line-of-sight extinction coefficient ($A_G$) computed by Casagrande & VandenBerg (2018) and Zhong et al. (2019).

$$A_G = 2.74 \times E(B - V).$$

The reddening values for all star members, of each cluster, can be checked with Stilism[^4] 3D dustmaps (Capitanio et al. 2017).

Another important astrophysical parameter determined from the fitted CMDs is the distance modulus ($m - M$) from which the distance to the clusters can be estimated. The determined moduli for QC1, QC2, QC3, and QC4 are $13.24 \pm 0.27$, $13.28 \pm 0.28$, $13.18 \pm 0.28$, and $12.93 \pm 0.28$ mag, respectively, that are in agreement with those of Qin et al. (2021). However, we think that our data are more accurate than the previously estimated values due to the improvements that occurred in the EDR3 measurements for the different photometric parameters. The corresponding distances (in pc) for these distance moduli are 1674 ± 41, 1927 ± 44, 1889 ± 43, and 1611 ± 40. These distances are in agreement with those we obtained from astrometric measurements; i.e., using the cluster parallax.

[^3]: [http://stev.oapd.inaf.it/cgi-bin/cmd](http://stev.oapd.inaf.it/cgi-bin/cmd).
[^4]: [https://stilism.obspm.fr/](https://stilism.obspm.fr/).
Table 4. The calculated astrophysical and photometric parameters of the four open clusters in comparison with data taken from Qin et al. (2021) and indicated as Q21 in the last column of the table.

| Parameters               | QC1    | QC2    | QC3    | QC4    | Ref. |
|--------------------------|--------|--------|--------|--------|------|
| Number of members        | 118    | 142    | 210    | 110    | Q21  |
| 72                       | 114    | 124    | 130    |        |      |
| $\mu_x \cos \delta$ (mas/yr) | $-2.34$ | $-3.94$ | $-2.28$ | $-2.95$ | Q21  |
| $-2.30 \pm 0.09$         | $-4.05 \pm 0.06$ | $-2.25 \pm 0.10$ | $-2.51 \pm 0.17$ | Q21  |
| $\mu_y$ (mas/yr)         | $-3.55$ | $-4.36$ | $-4.39$ | $-4.40$ | Q21  |
| $-3.48 \pm 0.12$         | $-4.32 \pm 0.06$ | $-4.46 \pm 0.13$ | $-5.43 \pm 0.17$ | Q21  |
| $\sigma$ (mas)           | $0.548^{+0.570}_{-0.529}$ | $0.424^{+0.471}_{-0.459}$ | $0.437^{+0.443}_{-0.430}$ | $0.455^{+0.467}_{-0.444}$ | Q21  |
| $d_{phot}$ (pc)          | 1819   | 2151   | 2288   | 2179   | Q21  |
| $Z$                      | 0.0152 | 0.0150 | 0.0152 | 0.0199 | Q21  |
| $0.0010$                 | 0.00004 | 0.00004 | 0.0018 | 0.0018 |      |
| log (age/yr)             | 7.00   | 7.45   | 7.60   | 8.40   | Q21  |
| 6.987 ± 0.022            | 8.524 ± 0.046 | 8.858 ± 0.114 | 8.367 ± 0.043 | Q21  |
| $E(G_{BP} - G_{RP})$     | 0.955 ± 0.031 | 0.875 ± 0.017 | 0.847 ± 0.031 | 0.889 ± 0.027 | Q21  |
| $E(B - V)$               | 0.772 ± 0.024 | 0.679 ± 0.013 | 0.657 ± 0.024 | 0.690 ± 0.021 | Q21  |
| $0.756 ± 0.280$          | 0.552 ± 0.010 | 0.577 ± 0.019 | 0.556 ± 0.049 | Q21  |
| $A_G$                    | 2.12   | 1.86   | 1.80   | 1.89   | Q21  |
| 2.07                     | 1.51   | 1.58   | 1.52   |        | Q21  |
| $(m - M)$                | 13.24 ± 0.27 | 13.28 ± 0.28 | 13.18 ± 0.28 | 12.93 ± 0.28 | Q21  |
| 12.55                    | 13.24  | 13.42  | 13.26  |        | Q21  |
| $d$ (pc)                 | 1674   | 1927   | 1889   | 1611   | Q21  |
| 1261                     | 2223   | 2340   | 2230   |        | Q21  |
| $R_g$ (kpc)              | 8.010 ± 0.089 | 8.251 ± 0.091 | 8.252 ± 0.091 | 8.218 ± 0.091 | Q21  |
| 0.359 ± 0.018            | 0.174 ± 0.013 | 0.165 ± 0.013 | 0.139 ± 0.012 |        |      |
| $Y_0$ (kpc)              | 1.634 ± 0.040 | 1.916 ± 0.044 | 1.880 ± 0.043 | 1.602 ± 0.040 | Q21  |
| 0.057 ± 0.008            | 0.110 ± 0.010 | 0.082 ± 0.009 | 0.102 ± 0.010 |        |      |

$(d_{phot})$. All obtained results of the astrophysical and photometric parameters appeared in Table 4.

From the estimated photometric distances to the cluster, we can infer the distance to the Galactic center ($R_g$) using

$$R_g = \sqrt{R_o^2 + (d \cos b)^2 - 2R_o d \cos b \cos l},$$

where $R_o = 8.20 \pm 0.10$ kpc (Bland-Hawthorn et al. 2019). After that, the projected distances towards the Galactic plane ($X_0$, $Y_0$) and the distance above the Galactic plane ($Z_0$) can be computed by using the following relationships

$$X_0 = d \cos b \cos l, \quad Y_0 = d \cos b \sin l, \quad Z_0 = d \sin b.$$  \hspace{1cm} (3)

3.2.2 The luminosity and mass functions At this stage of calculations, we had estimated for each cluster the new center positions and photometric parameters. From these data, we can derive both the luminosity and the mass functions (LF and MF) because each cluster member is formed under similar physical conditions from the same molecular cloud at the same time. This process makes star clusters ideal objects to study the initial mass function (IMF; e.g., Scalo 1998; Phelps & Janes 1993; Yadav & Sagar 2004; Bisht et al. 2019). The IMF is an empirical relationship that refers to the initial stellar mass distribution in the cluster. The present time IMF can be inferred from the cluster LF and MF by using the mass–luminosity relation (MLR).

Salpeter (1955) described the IMF as a power law that relates the total stellar number density ($dN$) distributed over a mass scale in a mass bin ($dM$) with central mass ($M$) as $(dN/dM = M^{-\alpha})$ and found that $\alpha = 2.35$ for massive stars than our Sun. The expansion of Salpeter’s power law implies that the number of stars in each mass range (bin) decreases rapidly with increasing stellar masses. The MF slope can be derived by using Equation (4) (Bisht et al. 2020)

$$\log \left( \frac{dN}{dM_o} \right) = -(1 + \Gamma) \log (M_o) + \text{constant},$$  \hspace{1cm} (4)
where \( \Gamma = (x - 1) \) is a dimensionless parameter refers to the slope of the straight line that represents the MF (see lower panel of Figure 7) which equals to 1.35 for Salpeter (1955). Our computed slopes of each cluster from the least-square fitting to our MF data are in line with the findings of Salpeter (1955) for QC2 and QC3 but have larger values for QC1 and QC4.

The second-order polynomial function, in Equation (5), relates the cluster absolute magnitude \( M_G \) and its collective masses \( M_C \) that can be obtained by fitting the adopted isochrones for each cluster. From the LF of each cluster, we can estimate the cluster absolute magnitudes \( M_G \) like 4.33 (QC1), 3.80 (QC2), 4.25 (QC3), and 4.10 (QC4). While from MLR we concluded for each cluster both the total mass \( M_C \) and the average mass \( \overline{M}_C \) in Solar mass unit (M\(_{\odot}\)), i.e., \( (M_C, \overline{M}_C) = (158, 1.34; QC1), (177, 1.25; QC2), (232, 1.10; QC3), and (182, 1.65; QC4) \). These parameters are computed for the first time in the present work and are summarized in Table 5.

### Table 5. The estimated absolute magnitudes from LFs, masses with MLR fitting, and the slopes of MFs.

| Parameters | QC1    | QC2    | QC3    | QC4    |
|------------|--------|--------|--------|--------|
| \( M_G \) (mag) | 4.33 ± 0.75 | 3.80 ± 0.23 | 4.25 ± 0.01 | 4.10 ± 0.01 |
| \( a_0 \) | 3.567 ± 0.050 | 2.659 ± 0.062 | 2.170 ± 0.046 | 3.047 ± 0.090 |
| \( a_1 \) | −1.060 ± 0.050 | −0.506 ± 0.045 | −0.118 ± 0.008 | −0.650 ± 0.063 |
| \( a_2 \) | 0.115 ± 0.040 | 0.034 ± 0.031 | −0.029 ± 0.003 | 0.067 ± 0.001 |
| \( M_C(M_G) \) | 158 ± 13 | 177 ± 13 | 232 ± 15 | 182 ± 14 |
| \( \overline{M}_C(M_G) \) | 1.34 | 1.25 | 1.10 | 1.65 |
| \( \Gamma \) | 1.74 ± 0.02 | 1.66 ± 0.10 | 1.08 ± 0.05 | 1.68 ± 0.03 |

**Figure 7.** Upper panel: the true luminosity function (LF) for each of the studied clusters. Lower panel: The mass function (MF) for the cluster members fitted by Salpeter (1955) power-law (solid black line) to compute the slope \( \Gamma \) in Equation (4).

#### 3.3 Dynamical and kinematical structure

#### 3.3.1 The dynamical tidal radius

For any cluster, there are two gravitational forces act; one towards the Galactic center and another towards the cluster center; to keep it bound. The tidal radius is the distance at which a balance between these two forces is reached. Thus, it
may act as a separator between gravitationally bound and unbound stars to a cluster. Röser & Schilbach (2019) studied the effect of the gravitational massive bodies such as stellar clusters in the Galactic disk and expressed it in terms of the distance \(x_L\) between the Lagrangian points and the Galactic center. These distances are equivalent to the tidal radius \(r_t\) of the cluster (i.e., \(x_L \approx r_t\)) and can be expressed as follows:

\[
x_L = \left[ \frac{GM_C}{4A(A-B)} \right]^{1/3} = \left[ \frac{GM_C}{4\Omega^o - \kappa^2} \right]^{1/3},
\]

where \(M_C\) is the cluster mass (in \(M_\odot\)), \(G\) is the gravitational constant (= \(4.30 \times 10^{-6}\) kpc \(M_\odot^{-1}\) km\(^2\) s\(^{-2}\)), \(\Omega^o\) (= \(A - B\)) is the angular velocity, and \(\kappa\) (= \(\sqrt{-4B(A-B)}\)) is the epicyclic frequency at the position of the Sun; both of the \(\Omega^o\) and \(\kappa\) parameters are measured in km s\(^{-1}\) kpc\(^{-1}\) (Röser et al. 2011). The two constants \(A (= 15.6 \pm 1.6)\) and \(B (= -13.9 \pm 1.8)\) are the Oort constants (in km s\(^{-1}\) kpc\(^{-1}\)) with adopted values from Nouh & Elsanhoury (2020).

The calculated tidal radii (in pc) for the clusters QC1, QC2, QC3, and QC4 are \(7.17 \pm 1.68\), \(7.45 \pm 1.73\), \(8.16 \pm 1.86\), and \(7.52 \pm 1.75\), respectively.

3.3.2 Dynamical evolution times Unlike compact halo counterparts (i.e., globular clusters), open clusters have a looser spatial distribution, and the interaction among the stars in an open cluster leads to the energy exchange (Inagaki & Saslaw 1985; Baumgardt & Makino 2003). This phenomenon has been recently reported for many open clusters (Dib & Basu 2018; Bisht et al. 2020; Joshi et al. 2020). The dynamical relaxation time \(T_{\text{relax}}\), measured in years, is the characteristic time needed for a cluster to reach equilibrium. Spitzer et al. (1971) expressed this time, Equation (7), in terms of both the number of member stars of the cluster \(N\) and the cluster diameter \(D\)

\[
T_{\text{relax}} = \frac{8.9 \times 10^5 N^{1/2} R_h^{3/2}}{(\overline{M_C})^{1/2} \log (0.4 N)},
\]

where \(R_h\) (in pc) is the radius containing half of the cluster mass and \(\overline{M_C}\) is the average mass of all cluster members in Solar masses. Lada & Lada (2003) assumed that the cluster diameter is twice its limiting radius; \(D \approx 2 r_t\).

Estimating \(T_{\text{relax}}\) enables us to estimate the evaporation time, \(\tau_{\text{ev}}\) which is the time needed to eject all member stars due to internal stellar encounters (Adams & Myers 2001) and it is estimated to be about \(10^5 T_{\text{relax}}\). Low-mass stars continue to escape from the cluster, at low speeds through the Lagrange points ( Küpper et al. 2008).

For a cluster to remain bound, the escaping velocity \(V_{\text{esc}}\) of rapid gas removal from the cluster has to satisfy Equation (8) (Fich & Tremaine 1991; Fukushige & Heggie 2000)

\[
V_{\text{esc}} = R_{\text{esc}}^{1/2} \sqrt{2 GM_C/3 r_t^3}.
\]

Therefore, a bound group will emerge only if the star-formation efficiency (SFE) is greater than 50% (Wilking & Lada 1983).

The dynamical state of QCs clusters can be described by computing the dynamical evolution parameter, \(\tau (= \text{age}/T_{\text{relax}})\). The cluster is relaxed (i.e., no dynamical effects) if \(\tau \gg 1\) and vice versa. Our calculations showed that the dynamical parameter for the four clusters, apart from QC1, is larger than 1 which means that they are dynamically relaxed. We found that \(\tau\) QC1 \(\sim 0.82\). The result of QC1 supports our finding (in Table 4) that this cluster is the youngest (\(log(\text{age}) \sim 6.9\)) among the other four open clusters. Moreover, the stellar content of QC1 could be dominated by low-mass instead of massive star members and hence it will have, on average, larger random velocities that generally lead to more interactions with the surrounding space (Mathieu & Latham 1986). All of the calculations for the dynamical study are listed in Table 6.

3.3.3 Ellipsoidal motion and the kinematical structure To highlight the gravitationally bound system of the stellar groups in a limited volume of space within the Galactic system characterized by the parallelism and equality of their motions, we studied the velocity ellipsoid parameters VEPs and those kinematics using a computational algorithm developed by Elsanhoury et al. (2018) and Bisht et al. (2020).

For any cluster members with coordinates \((\alpha, \delta)\) at distance \(d\) and have a proper motion components \((\mu_x \cos \delta, \mu_y)\) and radial velocity \(V_r\), the space velocity vectors \((V_x, V_y, V_z)\) are given by:

\[
V_x = -4.74 d_\mu \cos \delta \sin \alpha - 4.74 d_\mu \sin \delta \cos \alpha + V_r \cos \delta \cos \alpha,
\]
\[
V_y = +4.74 d_\mu \cos \delta \cos \alpha - 4.74 d_\mu \sin \delta \sin \alpha + V_r \cos \delta \sin \alpha,
\]
\[
V_z = +4.74 d_\mu \cos \delta + V_r \sin \delta.
\]
where the average radial velocity \( V_r \) (in km/s) for the four clusters (taken from the Gaia EDR3 database) is \(-13.79 \pm 1.86 \) (QC1), \(-17.75 \pm 3.21 \) (QC2), \(-34.67 \pm 2.94 \) (QC3), and \(-22.82 \pm 2.39 \) (QC4).

The Galactic spatial velocity distribution of the stellar members of the four studied clusters is shown in Figure 8. The spatial velocity components \( (U, V, W) \) in the Galactic coordinates, Equation (10), were derived in light of the calculated space velocity components, given in Equation (9), by using an equatorial-Galactic transformation matrix based on the SPECFIN v2.0 catalog of radio continuum spectra; see Equation (14) in Liu et al. (2011):

\[
U = -0.0518807421V_x - 0.8722226427V_y - 0.4863497200V_z, \\
V = +0.4846922369V_x - 0.4477920852V_y + 0.7513692061V_z, \\
W = -0.8731447899V_x - 0.1967483417V_y + 0.4459913295V_z. \\
\]

The apex position \( (A, D) \) is a convergent point into which member stars of the cluster will be coherently directed. This point represents the intersection of the stellar spatial velocity vectors with the celestial sphere. Chupina et al. (2001, 2006) introduced a method to determine the apex components in the equatorial coordinates by knowing the average space velocity vectors as expressed in Equation (9). The apex coordinates can be determined as follows:

\[
A = \tan^{-1}\left(\frac{V_y}{V_x}\right), \quad D = \tan^{-1}\left(\frac{V_z}{\sqrt{V_x^2 + V_y^2}}\right). \\
\]

The cross mark in Figure 9 illustrates the apex position for each of the four open clusters and lists their coordinates in Table 6.

3.3.4 Other kinematic parameters

(a) The cluster center: For a cluster consisting of \( N \) members each has a location \((x_i, y_i, z_i)\) at a distance \( d_i \), the position of the cluster center \((x_c, y_c, z_c)\) can be calculated by finding the center of mass of the \( N \) member stars in equatorial coordinates following Equation (12):

\[
x_c = \frac{1}{N} \sum_{i=1}^{N} d_i \cos \delta_i \cos \lambda_i, \\
y_c = \frac{1}{N} \sum_{i=1}^{N} d_i \sin \delta_i \cos \lambda_i, \\
z_c = \frac{1}{N} \sum_{i=1}^{N} d_i \sin \delta_i. \\
\]

These coordinates are expressed in units of parsecs because they represent the distance between the cluster center and the observer.
The Solar elements: If the mean spatial velocity components for a cluster in the Galactic coordinates are \((U, V, W)\), then we may deduce the Solar space velocity components \((U_\odot, V_\odot, W_\odot)\) via the following relation:

\[
U_\odot = -\frac{U}{C_1}, \quad V_\odot = -\frac{V}{C_1} \quad \text{and} \quad W_\odot = -\frac{W}{C_1},
\]

from which we can determine the absolute value of the Solar space velocity with respect to the studied objects as follows:

\[
S_\odot = \sqrt{\left(\frac{U}{C_1}\right)^2 + \left(\frac{V}{C_1}\right)^2 + \left(\frac{W}{C_1}\right)^2}
\]  \(13\)

and then we may estimate the location of the Solar apex \((l_A, b_A)\) in the Galactic coordinates to be

\[
l_A = \tan^{-1}\left(\frac{-V}{U}\right) \quad \text{and} \quad b_A = \sin^{-1}\left(-\frac{W}{S_\odot}\right),
\]  \(14\)

Equations (13) and (14) can be expressed in terms of the Cartesian coordinates, \((x, y, z)\), assuming that the center of the axes is located at the Sun. Given the following relation:

\[
X_\odot^* = -\frac{V_x}{V}, \quad Y_\odot^* = -\frac{V_y}{V} \quad \text{and} \quad Z_\odot^* = -\frac{V_z}{V},
\]

where \((V_x, V_y, V_z)\) are the mean space velocity components of the cluster that can be determined from Equation (9). Hence, the Solar space velocity is

\[
S_\odot = \sqrt{\left(\frac{X_\odot^*}{V}\right)^2 + \left(\frac{Y_\odot^*}{V}\right)^2 + \left(\frac{Z_\odot^*}{V}\right)^2}
\]  \(15\)

and therefore, the position of the Solar apex point \((x_A, \delta_A)\) in the equatorial coordinates is

\[
x_A = \tan^{-1}\left(\frac{Y_\odot^*}{X_\odot^*}\right)
\]  \(16\)

The cluster 3D-morphology: The shapes of young star clusters must reflect the conditions in the parental molecular clouds and during the cluster formation process (Chen et al. 2004). The median age for clusters associated with clouds is 4 Myr, whereas it is 50 Myr for clusters that are sufficiently separated from a molecular cloud to be considered unassociated. After \(~6\) Myr, the
majority of the star clusters lose association with their molecular gas (Grasha et al. 2019).

We analyzed the 3D spatial position \((X, Y, Z)\) of the member stars in the heliocentric Cartesian coordinates \((x, y, z)\) by knowing the estimated distance to the cluster, \(d\):

\[
\begin{align*}
X &= d \cos \delta \cos \alpha, \\
Y &= d \cos \delta \sin \alpha, \\
Z &= d \sin \delta.
\end{align*}
\]

The 3D morphology for the four clusters is plotted in Figure 10. It is noticeable that cluster members show an elongated expansion in their cluster region. This expansion may be regarded as fast gas expulsion and virilization (Pang et al. 2021) and hence we may conclude that the birthplaces of those member stars are in the same region of the disk. This result, in particular for QC1, supports our calculations for the parameters \(d_c\) and \(C\) that unveiled that the cluster is not compact but slightly scattered.

4. Conclusion

We presented the first complete comprehensive astrometric, photometric, and kinematical study of the newly discovered open clusters namely; QC1, QC2, QC3, and QC4 using the most recent data from Gaia EDR3 with the aid of ASteCA code. We derived most of the fundamental astrophysical and dynamical parameters of these clusters after estimating their most probable members (118; QC1, 142; QC2, 210; QC3, and 110; QC4). Moreover, we computed some astrometric parameters for the first time such as the density contrast, \(d_c\), and the concentration, \(C\), parameters. We summarize the main results and conclusions as follows:

1. The new positions of the clusters’ centers are in agreement in the RA direction and slightly different in the Dec. direction with those obtained by Qin et al. (2021).
2. Employing the ASteCA code, for each cluster, we determined its radial density profile (RDP) from which we inferred the cluster internal spatial structure, the number of most probable member candidates, metallicity, \(\log(\text{age})\), reddening, distance modulus, and all of the astrophysical and photometric parameters of the cluster.
3. For the estimated MF and LF of those clusters, we constructed the MLR of individual member stars and estimated the total mass of each cluster to be 158 \(M_\odot\) (QC1), 177 \(M_\odot\) (QC2), 232 \(M_\odot\) (QC3), and 182 \(M_\odot\) (QC4). In addition, the slopes of the MF are in good agreement with Salpeter (1955).
4. For all clusters, the space velocity components \((U, V, W)\) relative to the Galactic coordinates and \((V_x, V_y, V_z)\) with respect to the Cartesian coordinates were computed, and the position of their corresponding apex coordinates \((A, D)\) were also determined. Moreover, the Solar elements for each cluster were also determined.
5. The dynamical evolution parameters showed that the four clusters are dynamically relaxed, the exception is QC1 which is dynamically active with \(\tau \sim 0.82\). In addition, the 3D morphology of the clusters showed that they are extended along the plane of the Galactic disc which is supported by our calculations of the contract parameter \(d_c\).

We conclude that EDR3 might have improved the overall determined parameters of the clusters, in
particular those obtained from the astrometric and photometric observations, by reducing their uncertainties. This would lead to more accurate characteristics of these new open clusters. However, more studies on these clusters are necessary to validate the current findings and to support our conclusions.

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**References**

Adams F. C., Myers P. C. 2001, ApJ, 553, 744
Barnes S. A. 2007, ApJ, 669, 1167
Baumgardt H., Makino J. 2003, MNRAS, 340, 227
Bertelli Motta C., Salaris M., Pasquali A., Grebel E. K. 2017, MNRAS, 466, 2161
Bisht D., Yadav R. K. S., Ganesh S., et al. 2019, MNRAS, 482, 1471
Bisht D., Elsanhoury W. H., Zhu Q., et al. 2020, AJ, 160, 119
Bland-Hawthorn J., Sharma S., Tepper-Garcia T., et al. 2019, MNRAS, 486, 1167
Bochkarev N. G., Sitnik T. G. 1985, Ap&SS, 108, 237
Bonatto C., Bica E. 2009, MNRAS, 397, 1915
Bressan A., Marigo P., Girardi L., et al. 2012, MNRAS, 427, 127
Bukowiecki Ł., Maciejewski G., Konorski P., Strobel A. 2011, Acta Astronomica, 61, 231
Capitanio L., Lallement R., Vergely J. L., Elyajouri M., Monreal-Ibero A. 2017, A&A, 606, A65
Carrera R., Bragaglia A., Cantat-Gaudin T., et al. 2019, A&A, 623, A80
Casagrande L., VandenBerg D. A. 2018, MNRAS, 479, L102
Chen W. P., Chen C. W., Shu C. G. 2004, AJ, 128, 2306
Chupina N. V., Reva V. G., Vereshchagin S. V. 2001, A&A, 371, 115
Chupina N. V., Reva V. G., Vereshchagin S. V. 2006, A&A, 451, 909
Dib S., Basu S. 2018, A&A, 614, A43
Elsanhoury W. H., Postnikova E. S., Chupina N. V., et al. 2018, Ap&SS, 363, 58
Fich M., Tremaine S. 1991, Annu. Rev. A&A, 29, 409
Friel E. D. 1995, Annu. Rev. A&A, 33, 381
Fukushige T., Heggie D. C. 2000, MNRAS, 318, 753
Gaia Collaboration, Prusti T., de Bruijne J. H. J., et al. 2016, A&A, 595, A1
Gaia Collaboration, Brown A. G. A., Vallenari A., et al. 2018, A&A, 616, A1
Gaia Collaboration 2020, VizieR Online Data Catalog, I/350
Grasha K., Calzetti D., Adamo A., et al. 2019, MNRAS, 483, 4707
Inagaki S., Saslaw W. C. 1985, ApJ, 292, 339
Janes K., Adler D. 1982, ApJS, 49, 425
Joshi Y. C., Maurya J., John A. A., et al. 2020, MNRAS, 492, 3602
Kharchenko N. V., Piskunov A. E., Schilbach E., Röser S., Scholz R. D. 2013, A&A, 558, A53
King I. 1962, AJ, 67, 471
King I. R. 1966, AJ, 71, 64
Küpper A. H. W., MacLeod A., Heggie D. C. 2008, MNRAS, 387, 1248
Lada C. J., Lada E. A. 2003, Annu. Rev. A&A, 41, 57
Liu J. C., Zhu Z., Hu B. 2011, A&A, 536, A102
Liu L., Pang X. 2019, ApJS, 245, 32
Maciejewski G., Niedzielski A. 2007, A&A, 467, 1065
Maciejewski G., Mihov B., Georgiev T. 2009, Astronomische Nachrichten, 330, 851
Marino A. F., Milone A. P., Casagrande L., et al. 2018, , 863, L33
Mathieu R. D., Latham D. W. 1986, AJ, 92, 1364
Maurya J., Joshi Y. C., Elsanhoury W. H., Sharma S. 2021, AJ, 162, 64
Moffat A. F. J., Vogt N. 1973, A&A, 23, 317
Moitinho A. 2010, in Star clusters: Basic galactic building blocks throughout time and space, eds. de Grijs R., Lépine J. R. D., Vol 266, 106
Moraux E. 2016, in EAS Publications Series, Vol 80–81, EAS Publications Series, 73–114
Nouh M. I., Elsanhoury W. H. 2020, Astrophysics, 63, 179
Pang X., Yu Z., Tang S.-Y., et al. 2021, arXiv e-prints, arXiv:2106.07658
Perren G. I., Vázquez R. A., Piatti A. E. 2015, A&A, 576, A6
Peterson C. J., King I. R. 1975, AJ, 80, 427
Phelps R. L., Janes K. A. 1993, AJ, 106, 1870
Portegies Zwart S. F., McMillan S. L. W., Gieles M. 2010, Annu. Rev. A&A, 48, 431
Qin S.-M., Li J., Chen L., Zhong J. 2021, Research in Astronomy and Astrophysics, 21, 045
Rain M. J., Ahumada J. A., Carraro G. 2021, A&A, 650, A67
Riello M., De Angeli F., Evans D. W., et al. 2021, A&A, 649, A3
Röser S., Schilbach E., Piskunov A. E., Kharchenko N. V., Scholz R. D. 2011, A&A, 531, A92
Röser S., Schilbach E. 2019, A&A, 627, A4
Salpeter E. E. 1955, ApJ, 121, 161
Santos-Silva T., Gregorio-Hetem J. 2012, A&A, 547, A107
Scalo J. 1998, in Astronomical Society of the Pacific Conference Series, Vol 142, The Stellar Initial Mass Function (38th Herstmonceux Conference), eds. Gilmore G., Howell D., p 201
Spitzer, Lyman J., Hart M. H. 1971, ApJ, 164, 399
Torra F., Castañeda J., Fabricius C., et al. 2021, A&A, 649, A10
Trumpler R. J. 1930, Lick Observatory Bulletin, 420, 154
Vandenberg D. A. 1983, ApJS, 51, 29
Wilking B. A., Lada C. J. 1983, ApJ, 274, 698
Yadav R. K. S., Sagar R. 2004, MNRAS, 349, 1481
Zhong J., Chen L., Kouwenhoven M. B. N., et al. 2019, A&A, 624, A34