LIV in matter

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We summarize recent work dealing with the characterization of effective actions giving the electromagnetic response of some topological materials, arising from their microscopic structure. The case of weakly tilted Weyl semimetals in the limit of zero temperature, but nonzero chemical potential, is presented as a subset of a specific choice of terms in the fermionic sector of the SME.

1. Introduction

The extension of the constant background tensors in the SME to spacetime dependent variables\(^1\) yields new possibilities of studying Lorentz invariance violations (LIV) which might result in effective theories describing macroscopic behavior of matter. Here we focus on the CPT-odd sector of the photon sector of the SME where we promote the standard constant vector \(k_{\mu F}^\theta\) to \(\partial^\mu \theta(x)\). The resulting coupling has been previously introduced in axion electrodynamics, where \(\theta(x)\) is the axion field.\(^2\) The nondynamical version of this theory has gained recent revival in the description of the electromagnetic (EM) response of topological matter such as topological insulators (TIs) and Weyl semimetals (WSMs), for example. In these cases, the LIV parameters are provided by the microscopic model of the matter in question and a crucial difference with the study of LIV in the fundamental interactions arises: in the former approach there is an underlying microscopic theory providing a Lorentz symmetry breaking pattern, while this possibility is missing in the second case because an established unified theory admitting LIV is still lacking. This is particularly notorious in some calculation of radiative corrections in the particle case yielding finite but indeterminate results,\(^3\) which nevertheless can be fixed in the matter calculation precisely due to the underlying theory. This basic theory further motivates the challenge of determining how an effective macroscopic response arises from the microscopic interactions describing the material.
The analogy with the studies of LIV in the fundamental interactions is enhanced because the linearized approximation of the lattice Hamiltonians of TIs and WSMs close to the Fermi energy can be formulated in terms of massless chiral fermionic quasiparticles. This similarity brings about the presence of anomalies together with the question of their relevance in the construction of the corresponding effective actions.

2. Nondynamical axion electrodynamics (θ-ED)

Perhaps the simplest way of introducing this modification of standard electrodynamics is by recalling Maxwell equations in a linear nondispersive media in terms of the vectors $D, H, E, B$ in standard notation. To be complete they require constitutive relations $D = D(E, B)$ and $H = D(E, B)$ which define the medium to be considered. In this case we take $D = \varepsilon E - \frac{\theta}{\pi} B$ and $H = \frac{1}{\mu} B + \frac{\alpha}{\pi} E$, where $\alpha \approx 1/137$ is the fine structure constant. The new ingredient here is the magnetoelectric polarizability $\theta(x)$ (MEP) which is an additional parameter of the medium in the same footing as $\varepsilon(x)$ and $\mu(x)$, i.e., the axion field is nondynamical. The resulting equations are the standard Maxwell equation in a medium having the additional field dependent sources (in Gaussian units and $c = 1$)

$$\rho_\theta = \frac{\alpha}{4\pi^2} \nabla \theta \cdot B, \quad J_\theta = -\frac{\alpha}{4\pi^2} \left( \nabla \theta \times E + \frac{\partial \theta}{\partial t} B \right).$$

(1)

The usual Maxwell action plus the additional coupling $-\frac{\alpha}{4\pi^2} \theta(x) E \cdot B$ reproduces the above equations. We remark that $E \cdot B$ is proportional to the Pontyagin density $\epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$, which precludes topological properties of the system. A most remarkable consequence of Eqs. (1) is the so called magnetoelectric effect (MEE), encapsulated in the additional sources $\rho_\theta$ and $J_\theta$. They summarize the ability of the magnetic (electric) fields to produce charge (current) densities, respectively. θ-ED describes the EM response of materials such as: magnetoelectrics (θ piecewise constant but arbitrary), TIs (θ piecewise constant but quantized) and chiral matter ($\theta = b_\mu x^\mu$, with $b_\mu$ constant), among others.

3. Radiation in θ-electrodynamics

The static limit of θ-ED in the piecewise constant realization of the MEP has been thoroughly studied in numerous references summarized in Ref. 5, either by the method of images or by constructing the Green’s function (GF) of the system. As a consequence of the MEE the GF turns out to be
a matrix operator. This method can be extended to the time dependent case and it is particularly useful in the far field approximation describing radiation. The evaluation of the GF in the radiation zone required to construct the electric and magnetic fields defining the Poynting vector involves integrals of highly oscillating functions, which can be dealt either with the stationary phase approximation or with the steepest descent method in order to obtain analytical results. A remarkable result is the reversed Cherenkov radiation (CHR) in naturally existing materials found when a charge at constant velocity $v > c/n$ impinges perpendicularly to the planar interface between two TIs with the same permittivity $\epsilon = n^2$. In this case the angular distribution of the radiation includes a term $\delta(1 - \frac{vn}{\cos \theta})$ which allows the contribution of angles $\pi/2 < \theta < \pi$, thus yielding an additional radiation cone in the backward direction with respect to the incident charge. The power output in the reversed direction is highly suppressed with respect to the forward direction by a factor of the order of $(\theta_{\alpha}/n)^2$, with $\theta$ being the difference between the MEPs of the two media. Reversed CHR has been observed only in metamaterials with negative permeability and permittivity. A similar approach is used in when dealing with radiation in chiral matter and the case of CHR also yields interesting results.

4. The effective action for weakly tilted Weyl semimetals

An important issue in the study of LIV in matter is the construction of the macroscopic EM response starting from the corresponding microscopic Hamiltonian coupled to the EM field. We illustrate this point taking as a model the Hamiltonian

$$H_{\chi}(p) = v_{\chi} \cdot (p + \chi \tilde{b}) - \chi \tilde{b}_0 + \chi v_F \sigma \cdot (p + \chi \tilde{b}),$$  \tag{2}

which describes a Weyl semimetal with two 3D band crossings (Weyl points) of chirality $\chi = \pm 1$ separated by $2 \tilde{b}$ and $2 \tilde{b}_0$ in momentum and energy, respectively. Equation (2) corresponds to the linearized approximation of the lattice Hamiltonian close to the Fermi energy and clearly shows the appearance of Weyl excitations around each Weyl point. Here $v_F$ is the isotropic Fermi velocity at each band crossing, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ is the triplet of spin-$1/2$ Pauli matrices, $v_{\chi}$ is the tilting parameter and $p$ is the momentum. In the Weyl (chiral) representation of the matrices $\gamma^\mu$, the Hamiltonian (2) can be shown to emerge from the Lagrangian density $\mathcal{L} = \bar{\Psi} (\Gamma^\mu i\partial_\mu - M) \Psi$ with

$$\Gamma^\mu = c^\mu \gamma^\nu + d^\mu \gamma^5 \gamma^\nu, \quad M = a_\mu \gamma^\mu + b_\mu \gamma^5 \gamma^\mu,$$  \tag{3}
which we recognize as a partial contribution from the fermionic sector in
the minimal QED extension of the SME.\textsuperscript{9} We follow the conventions of Ref. 10. The term $c_{\mu \nu}$ already includes the $\delta_{\mu \nu}$ contribution corresponding to the free Dirac action and we impose the restriction $\Gamma^0 = \gamma^0$. Let us remark that the choices in Equation (3) include also anisotropy in the Fermi velocity at each node (not shown in Equation (2)). We emphasize that the LIV parameters in (3) can be determined as functions of the parameters in the microscopic Hamiltonian in each case. Coupling the fermionic action arising from Equation (3) to the EM field and using standard field theory methods we obtain the effective EM response codified in the Lagrangian density $L(p) = \frac{1}{2} A_{\mu} (p) \Pi_{\mu \nu} (p) A_{\nu} (p)$ in momentum space. We require the calculation of the CPT-odd contribution to the vacuum polarization tensor (VPT) $\Pi_{\mu \nu}$, which has a well-known expression\textsuperscript{10} in terms of the exact fermion propagator $S(k) = i/(\Gamma^\mu k_\mu - M)$ including LIV modifications.

The final result is that the EM effective action will keep the form of $\theta$-ED for chiral matter, with all the parameters entering through a new vector $B_\lambda$ to be determined. The link between the relevant expressions is

$$
\Pi_{\lambda}^{\mu \nu} (p) = -i(e^2/2\pi^2) B_\lambda p_\epsilon \epsilon^{\mu \nu \lambda \epsilon}, \quad \theta(x) \rightarrow \Theta(x) = 2B_\lambda x^\lambda. \quad (4)
$$

The action resulting from the choice (3) is chiral invariant, which simplify enormously the calculations allowing the splitting of the VPT into the chiral contributions $\Pi_{\lambda, \chi}$, $\chi = \pm 1$,

$$
\Pi_{\lambda, \chi}^{\mu \nu} (p) = -2\chi e^2 (\det m_\chi) (m_\chi^{-1})^\rho_\lambda \epsilon^{\mu \nu \lambda \epsilon} p_\epsilon I_\rho (C_\chi). \quad (5)
$$

Here we have $(m_\chi)^{\mu \nu} = e^{\mu \nu} - \chi \delta^{\mu \nu}$ together with $C_\chi = a_\rho - \chi b_\rho$, and

$$
I_\rho (C_\chi) = \int \frac{d^4k}{(2\pi)^4} g_\rho^{(\chi)} (k_0, k), \quad g_\rho^{(\chi)} (k_0, k) = \frac{(k^{(\chi)} - C_\chi)^\rho}{[(k^{(\chi)} - C_\chi)^2]^2}, \quad (6)
$$

with $k^{(\chi)} = k_\alpha (m_\chi)^\alpha_{\mu}$. In order to extend the phenomenological applications of our approach we consider the case of nonzero chemical potential $\mu$ in the limit where the temperature $T$ goes to zero. To this end we adopt the imaginary time regularization where $\int dk_0 f(k_0)$ is replaced by the Matsubara sum $\sum_n 2\pi T f(k_0)$ with the replacement $k_0 \rightarrow i(2n + 1)\pi T + \Lambda$. Here $\Lambda$ is the chemical potential measured from the band-crossing points, i.e., $\Lambda = \mu - E_\chi (p_\chi)$, where $p_\chi$ is the location in momentum of the node with chirality $\chi$, and $E_\chi (p_\chi)$ the corresponding energy. The required limit
\( T \to 0 \) splits into the contributions \( B^{(1)}_\lambda \) and \( B^{(2)}_\lambda \) to \( B_\lambda \) in Eq. (4)

\[
B^{(1)}_\lambda = -\frac{1}{2} \sum_\chi \chi C^{(\chi)}_\rho (m_\chi^{-1})^\rho \sigma_\lambda, \quad B_\lambda = B^{(1)}_\lambda + B^{(2)}_\lambda,
\]

\[
B^{(2)}_\lambda = -\sum_\chi \chi \Lambda_\chi N_\chi V^{(\chi)}_i (m_\chi^{-1})_i^\sigma \lambda, \quad \nu^{(\chi)}_i = (m_\chi^{-1})_i^j (m_\chi)^j_0
\]

\[
\Lambda_\chi = \mu - \left( V^{(\chi)}_i C^{(\chi)}_i + C^{(\chi)}_0 \right), \quad N_\chi = \frac{1}{2} |V^{(\chi)}_i| - \frac{\text{arctanh}(|V^{(\chi)}_i|)}{|V^{(\chi)}_i|}
\]

where \( |V^{(\chi)}_i| \) is the modulus of the vector \( V^{(\chi)}_i \). Equations (7) provide the full vector \( B_\lambda \) which defines the EM effective action corresponding to the fermionic sector of the SME stemming from the choice in Eq. (3). Applications of this EM response will be considered in a following work. For further implementation of the methods and techniques motivated by the SME into the realm of topological matter see Refs. 12.

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