Quantum phase transition in the one-dimensional compass model

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We introduce a one-dimensional model which interpolates between the Ising model and the quantum compass model with frustrated pseudospin interactions $\sigma_i^x \sigma_{i+1}^x$ and $\sigma_i^z \sigma_{i+1}^z$, alternating between even/odd bonds, and present its exact solution by mapping to quantum Ising models. We show that the nearest neighbor pseudospin correlations change discontinuously and indicate divergent correlation length at the first order quantum phase transition. At this transition one finds the disordered ground state of the compass model with high degeneracy $2 \times 2^{N/2}$ in the limit of $N \to \infty$.

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I. INTRODUCTION

Recent interest in quantum models of magnetism with exotic interactions is motivated by rather complex superexchange models which arise in Mott insulators with orbital degrees of freedom. The degeneracy of 3d orbitals is only partly lifted in an octahedral environment in transition metal oxides, and the remaining orbital degrees of freedom are frequently described as $T = 1/2$ pseudospins. They occur on equal footing with spins in spin-orbital superexchange models and their dynamics may lead to enhanced quantum fluctuations near quantum phase transitions and to entangled spin-orbital ground states. The properties of such states are still poorly understood, so it is of great interest to investigate first the consequences of frustrated interactions in the orbital sector alone.

The orbital interactions are intrinsically frustrated — they have much lower symmetry than the usual SU(2) symmetry of spin interactions, and their form depends on the orientation of the bond in real space, so they may lead to orbital liquid in three dimensions. Although such interactions are in reality more complicated, a generic and simplest model of this type is the so-called compass model introduced long ago when the coupling along a given bond is Ising-like, but different spin components are active along different bond directions, for instance $J_x \sigma_i^x \sigma_j^x$ and $J_y \sigma_i^y \sigma_j^y$ along $a$ and $b$ axis in the two-dimensional (2D) compass model. The compass model is challenging already for the classical interactions. Although the compass model originates from the orbital superexchange, it is also dual to recently studied models of $p + ip$ superconducting arrays. It was also proposed as a realistic model to generate protected cubits so it could play a role in the quantum information theory.

So far, the nature of the ground state and pseudospin correlations in the compass model were studied only by numerical methods. It has been argued that the eigenstates of the 2D compass model are twofold degenerate. In contrast, a numerical study of the 2D compass model suggests that the ground state is highly degenerate and disordered. In fact, the numerical evidence suggests a first order quantum phase transition at $J_z = J_x^{1/2}$ with diverging spin fluctuations and a discontinuous change in the correlation functions.

The purpose of this paper is to show by an exact solution a mechanism of a first order quantum phase transition in quantum magnetism. Such a transition from quasiclassical states with short range order to a disordered state occurs in the one-dimensional (1D) XX–ZZ model, with antiferromagnetic interactions between $\sigma_i^x$ and $\sigma_i^z$ pseudospin components, alternating on even/odd bonds. The model is solved analytically by mapping its different subspaces to the exactly solvable quantum Ising model (QIM), which plays a prominent role in understanding the paradigm of a quantum phase transition, including recent rigorous insights into the transition through its quantum critical point at a finite rate. Note that the model introduced below is generic and by no means limited to the orbital physics. For instance, the QIM is realized by the charge degrees of freedom in NaV$_2$O$_5$, where it helped to explain the temperature dependence of optical spectra.

The paper is organized as follows: In Sec. II we introduce the pseudospin (orbital) model which realizes a continuous interpolation between the classical Ising model and the compass model in one dimension. This model is next solved exactly in its subspaces which are separated from each other. The properties of the model and the mechanism of the quantum phase transition are elucidated in Sec. III. At the transition point we demonstrate the discontinuous change of correlation functions (Sec.
II. PSEUDOSPIN XX–ZZ MODEL

In order to understand the nature of a quantum ground state with high degeneracy found in the 1D compass model with alternating superexchange interactions between \( \sigma_x^r \) and \( \sigma_x^l \) pseudospin components, it is important to investigate how the pseudospin correlations develop when this state is approached. Therefore, we start below from the classical Ising model with interacting \( \sigma_x^l \) pseudospins and modify gradually the interactions on every second bond by replacing them by the ones between \( \sigma_x^r \).

Although this does not correspond to any deformation of interacting orbitals in a crystal, in this way we keep a constant value of the total pseudospin coupling constant \( J \).

The Hamiltonians (11) and (2) are related by symmetry: simultaneous exchange of \( \sigma_x^r \leftrightarrow \sigma_x^l \), \( 2i \leftrightarrow 2i-1 \) \( \forall \), and \( (1-\alpha) \leftrightarrow (\alpha-1) \), maps the Hamiltonians into each other. Thanks to this symmetry, we need to solve only the model (11) for \( \alpha \in [0,1] \), with modulated interactions for odd pairs of pseudospins \( \{2i-1,2i\} \) (on odd bonds). The Hamiltonian (11) can be conveniently solved in the basis of eigenstates of the ZZ Ising model at \( \alpha = 0 \). We start with one of the two degenerate ground states,

\[
|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle ,
\]

focusing on the consequences of increasing frustration. Right in the middle between the two classical cases (3) and (4), i.e., for \( \alpha = 1 \), it becomes

where the last state is the second degenerate ground state of the ZZ Ising model. This set of states (6)–(7) is a convenient basis in the subspace where all odd pairs of pseudospins are antiparallel, and give a constant energy contribution of \( -(1-\alpha)J \) per bond. By construction, the Hamiltonian (11) does not mix this subspace with the rest of the Hilbert space. Therefore, in this subspace the energy due to the even \( \{2i,2i+1\} \) bonds in Eq. (11) depends on the pseudospin orientation on the two neighboring odd bonds, and may be expressed by a product \( \tau^z \tau^z_{i+1} \) of two spin operators, with \( \tau^z \equiv \sigma^z_{2i-1} \sigma^z_{2i} \).

In this representation it is clear that the Hamiltonian (11) reduces in this subspace to the exactly solvable QIM. In general the Hilbert space can be divided into subspaces where different odd pairs of spins are either parallel or antiparallel. Each subspace can be labelled by a vector \( \vec{s} = (s_1, \ldots , s_{N'}) \), with \( s_i = 1 \) (\( s_i = 0 \)) when two pseudospins of the odd bond \( \{2i-1,2i\} \) are parallel (antiparallel). In each subspace \( \vec{s} \) the terms \( \alpha (1-\alpha) \) for odd bonds in Eq. (11) give a constant energy contribution

\[
C_s(\alpha) = (1-\alpha)J \sum_{i=1}^{N'} \sigma^z_{2i-1} \sigma^z_{2i} = -(1-\alpha)J(N' - 2s) ,
\]

where \( s = \sum_{i=1}^{N'} s_i \) is the number of parallel odd pairs of spins. With a convenient choice of spin operators for the
antiparallel \{2i - 1, 2i\} pairs of pseudospins,
\[ -\tau_i^z = (\langle \uparrow \downarrow \rangle \langle \downarrow \uparrow | + \langle \downarrow \downarrow \rangle \langle \uparrow \uparrow |), \]
\[ -\tau_i^z = (1 - \sum_{j=1}^{i-1} s_j) (\langle \uparrow \downarrow \rangle \langle \downarrow \uparrow | - \langle \downarrow \downarrow \rangle \langle \uparrow \uparrow |), \] (9)
and for the parallel pairs
\[ -\tau_i^z = (\langle \uparrow \uparrow \rangle \langle \uparrow \uparrow | + \langle \downarrow \downarrow \rangle \langle \downarrow \downarrow |), \]
\[ -\tau_i^z = (1 - \sum_{j=1}^{i-1} s_j) (\langle \uparrow \uparrow \rangle \langle \uparrow \uparrow | - \langle \downarrow \downarrow \rangle \langle \downarrow \downarrow |), \] (10)
the Hamiltonian (11) reduces to
\[ H_{\alpha} = -J \sum_{i=1}^{N'-1} \left[ \alpha \tau_i^z + \tau_i^z \tau_{i+1}^z \right] \]
\[ - J \left[ \alpha \tau_i^z + (1 - \sum_{j=1}^{i-1} s_j) \tau_i^z \right] + C_s(\alpha). \] (11)
For even \(s\) the Hamiltonian (11) is simply the ferromagnetic QIM, but when \(s\) is odd, the interaction on the last \(\{N', 1\}\) bond is antiferromagnetic.

**B. Exact solution**

Each effective model (11) can be solved using the Jordan-Wigner transformation for spin operators,
\[ \tau_i^+ = 1 - 2c_i^+ c_i, \]
\[ \tau_i^z = -(c_i + c_i^+) \prod_{j<i} (1 - 2c_j^+ c_j). \] (12)
Here \(c_i\) is a fermionic annihilation operator at site \(i\). After this transformation the Hamiltonian (11) becomes
\[ H_{\alpha}(\alpha) = J \sum_{i=1}^{N'-1} \left[ \alpha c_i^+ c_i - c_i^+ c_{i+1} - c_{i+1} c_i + h.c. \right] \]
\[ + J \left[ \alpha c_i^+ c_{N'} - c_{N'}^+ c_i - \tilde{c}_i c_{N'} + h.c. \right] + C_s(\alpha), \] (14)
with
\[ \tilde{c}_i = c_1 (1 - \sum_{j=i}^{N'-1} c_j^+ c_j), \] (15)
depending on parity of the number of \(c\)-quasiparticles. This parity is a good quantum number because \(c\)-quasiparticles can only be created or annihilated in pairs, as can be seen in Eq. (14).

In order to extend the sum in the first line of Eq. (14) to \(i = N'\) and, at the same time, to include the second line into this extended sum, we split the Hamiltonian (14) as
\[ H_{\alpha}(\alpha) = P^+ P_+^\alpha P^+ + P^- P_-^\alpha P^-, \] (16)
where
\[ P^\pm = \frac{1}{2} \left[ 1 \pm \prod_{i=1}^{N'} \tau_i^z \right] = \frac{1}{2} \left[ 1 \pm \prod_{i=1}^{N'} (1 - 2c_i^+ c_i) \right], \] (17)
are projectors on the subspaces with even (+) and odd (−) numbers of \(c\)-fermions and
\[ H_{\alpha}^\pm(\alpha) = J \sum_{i=1}^{N'} \left[ \alpha c_i^+ c_i - c_i^+ c_{i+1} - c_{i+1} c_i + h.c. \right] + C_s(\alpha), \] (18)
are corresponding reduced quadratic Hamiltonians. Definition of the Hamiltonians (18) is not complete without boundary conditions which depend both on the choice of subspace \(\pm\) and on the parity of \(s\), see Eq. (13). When both \(s\) and the number of \(c\)-fermions have the same parity, then the boundary condition is antiperiodic, i.e., \(c_{N'+1} \equiv -c_1\), but when on the contrary the two numbers have opposite parity — it is periodic, i.e., \(c_{N'+1} \equiv c_1\). With these boundary conditions the Hamiltonians (18) and (14) are the same.

The Hamiltonian (18) is simplified by a Fourier transformation, \(c_i \equiv \frac{1}{\sqrt{N'}} \sum_k c_k e^{i k i}\). Here \(k\)'s are quantized pseudomomenta. In the following we assume for convenience that \(N'\) is even (i.e., \(N = 4m, m\) integer). For periodic boundary conditions \(k\)'s take “integer” values (recall that \(N = 2N'\) \(k = 0, \pm \frac{\pi}{2N'}, \pm \frac{\pi}{N'}, \ldots, \pi\), and in the antiperiodic case they are “half-integer”, i.e., \(k = \pm \frac{\pi}{2N'}, \pm \frac{\pi}{N'}, \ldots, \pm \frac{\pi}{2(N'-1)}\).

As a result, the Hamiltonians (18) describe independent subspaces labelled by \(k\), with mixed \(k\) and \(-k\) quasiparticle states,
\[ H_{\alpha}^\pm(\alpha) = J \sum_k \left[ 2(\alpha - \cos k) c_k^+ c_k + \sin k \left( c_k^+ c_{-k}^+ + h.c. \right) \right] + C_s(\alpha). \] (19)

Diagonalization of \(H_{\alpha}^\pm(\alpha)\) is completed by a Bogoliubov transformation, \(c_k = u_k \gamma_k + v_k \gamma_k^\dagger\), where the Bogoliubov modes \(\{u_k, v_k\}\) are eigenmodes of the Bogoliubov-de Gennes equations:
\[ \epsilon u_k = 2J(\alpha - \cos k) u_k + 2J \sin k v_k, \]
\[ \epsilon v_k = 2J \sin k u_k - 2J(\alpha - \cos k) v_k. \] (20)
For each value of \(k\) there are two eigenstates with eigenenergies \(\epsilon_k\) and \(-\epsilon_k\). Positive eigenenergies
\[ \epsilon_k = 2J \sqrt{1 + \alpha^2 - 2\alpha \cos k}, \] (22)
define quasiparticles \(\gamma_k\) for each \(k\) which bring the Hamiltonian (19) to the diagonal form
\[ H_{\alpha}^\pm(\alpha) = \sum_k \epsilon_k \left( \gamma_k^\dagger \gamma_k - \frac{1}{2} \right) + C_s(\alpha), \] (23)
being a sum of fermionic quasiparticles. However, thanks to the projection operators \(P^\pm\) in Eq. (16) only states with, even or odd numbers of Bogoliubov quasiparticles belong to the physical spectrum of \(H_{\alpha}(\alpha)\).

In order to find if the number of Bogoliubov quasiparticles in a given subspace must be even or odd we must
find first the parity of the number of c-quasiparticles in the Bogoliubov vacuum in this subspace. This parity depends on the boundary conditions. Indeed, when they are antiperiodic, then sin \( k \neq 0 \) for any allowed \( k \) and the ground state of the Hamiltonian (19) is a superposition over states with pairs of quasiparticles \( c_0^\dagger c_0^\dagger \). This Bogoliubov vacuum has even number of c-quasiparticles. On the other hand, for periodic boundary conditions we have two special cases with sin \( k = \pi \) when \( k = 0 \) or \( \pi \). When 0 ≤ \( \alpha \) < 1 the Bogoliubov vacuum contains the quasiparticle \( c_0 \) but not the quasiparticle \( c_{\pi} \) and the number of c-quasiparticles is odd. In short, (anti-)periodic boundary conditions imply (even)odd parity of the Bogoliubov vacuum.

Taking further into account that any \( \gamma_k \) changes parity of the number of c-quasiparticles, we soon arrive at the simple conclusion that even (odd) \( s \) implies that only states with even (odd) numbers of Bogoliubov quasiparticles belong to the physical spectrum of \( H_\xi(\alpha) \). Note that for odd \( s \) the lowest energy state is not the Bogoliubov vacuum but a state with one Bogoliubov quasiparticle of minimal energy \( \epsilon_k \). Its pseudomomentum is \( k = 0 \) in a + subspace, but in a − subspace there is a choice between two degenerate quasiparticle states with \( k = \pm \frac{4\pi}{N} \).

III. QUANTUM PHASE TRANSITION

A. Correlation functions

For any \( \alpha \in (0, 1) \) the ground state is found in the \( s = 0 \) (+) subspace. Pseudospin correlators in the ground state can be expressed by the spin correlators of the effective QIM. For odd pairs of pseudospins:

\[
\langle \sigma_{2i-1}^z \sigma_{2i}^z \rangle = -1 ,
\]

\[
\langle \sigma_{2i-1}^z \sigma_{2i+1}^z \rangle = -1 + \frac{2}{N} \sum_k |v_k|^2 .
\]

Surprisingly, in spite of decreasing interaction \( \propto (1 - \alpha) \) in Eq. (1), the odd \( \langle \sigma_{2i-1}^z \sigma_{2i}^z \rangle \) correlator [21] shows the same perfect pseudospin order as that found at \( \alpha = 0 \), while the \( \langle \sigma_{2i-1}^z \sigma_{2i+1}^z \rangle \) one [24] gradually decreases from 0 at \( \alpha = 0 \) to \( -\frac{2}{N} \) when \( \alpha \to 1^- \) (at \( N \to \infty \)), see Fig. 1.

For different odd pseudospin pairs one finds (\( \alpha < 1 \)):

\[
\langle \sigma_{2i-1}^z \sigma_{2j-1}^z \rangle = 0 ,
\]

\[
\langle \sigma_{2i-1}^z \sigma_{2j-1}^z \rangle = (-1)^{m+n} \langle \tau_i^x \tau_j^x \rangle ,
\]

when \( i \neq j \) and for \( m, n = 0, 1 \). As is well known in the QIM, the correlator on the right hand side of Eq. (27) is Toeplitz determinant [16],

\[
\langle \tau_i^x \tau_{i+r}^x \rangle = \begin{vmatrix} f_1 & f_2 & \ldots & f_r \\ f_0 & f_1 & \ldots & f_{r-1} \\ \ldots & \ldots & \ldots & \ldots \\ \end{vmatrix}
\]

with constant diagonals \( f_r(\alpha) = \delta_{r,0} - 2a_r(\alpha) + b_r(\alpha) \), given by \( a_r(\alpha) = \frac{1}{N} \sum_k |v_k|^2 \cos(kr) \) and \( b_r(\alpha) = \frac{1}{N} \sum_k u_k v_k^* \sin(kr) \). This correlator is positive for all \( r \) and finite when \( r \to \infty \), indicating long range ferromagnetic order of \( \tau_i^x \) moments. When \( r \to \infty \) it decays exponentially towards finite long range limit with a correlation length \( \xi \) which diverges as \( \xi \sim (1 - \alpha)^{-1} \). As we have seen, in this limit the gap in the quasiparticle energy spectrum [22] tends to 0 and a quantum critical point is approached in the universality class of the QIM.

For nearest neighbor even pairs [when \( j = i + 1 \), \( m = 0 \) and \( n = 1 \) in Eq. (27)], the \( \langle \sigma_{2i}^z \sigma_{2i+1}^z \rangle \) pseudospin correlator is negative, see Eq. (27), as expected...
for an antiferromagnet. This correlator shows a complementary behavior to that of the \( \langle \sigma_{2i-1}^x \sigma_{2i}^z \rangle \) correlator for odd bonds \( \sigma_{2i} \) — it interpolates between \(-1\) at \( \alpha = 0 \) and \(-\frac{2}{\pi} \) when \( \alpha \to 1^- \) (see Fig. 1). In contrast, the \( \langle \sigma_{2i-1}^z \sigma_{2i}^z \rangle \) and \( \langle \sigma_{2i}^x \sigma_{2i+1}^z \rangle \) correlators change in a discontinuous way at the quantum critical point (\( \alpha = 1 \)), where the pseudospins become entirely disordered, and \( \langle \sigma_{2i-1}^z \sigma_{2i}^z \rangle = \langle \sigma_{2i}^x \sigma_{2i+1}^z \rangle = 0 \). Therefore, only \( \langle \sigma_{2i-1}^z \sigma_{2i}^z \rangle = \langle \sigma_{2i}^x \sigma_{2i+1}^z \rangle = -\frac{2}{\pi} \) are finite and contribute to the ground state energy of the compass model.

Using the symmetry of the model \( \mathbf{1} \)–\( \mathbf{2} \), the ground state correlators for \( \alpha \in (1, 2) \) can be obtained by mapping the correlators for \( \alpha \in [0, 1] \). In this way we obtain correlators that are well defined for any value of \( \alpha \) except \( \alpha = 1 \), where most of the correlators are discontinuous (Fig. 2). For example, when \( \alpha < 1 \) we have antiparallel odd pairs of pseudospins \( \langle \sigma_{2i}^z \sigma_{2i+1}^z \rangle = -1 \), but the same correlator tends to 0 when \( \alpha \to 1^+ \) (Fig. 1). This discontinuity of the correlators at the quantum phase transition is a manifestation of level crossing between ground states in different subspaces \( \vec{s} \) when \( \alpha \to 1^\pm \).

**B. Ground state energy and excitations**

The spectrum of the Hamiltonian \( \mathbf{1} \) changes qualitatively from a ladder spectrum with the width of \( 2JN \) at \( \alpha = 0 \) to a quasicontinuous spectrum with the width reduced to \( \frac{1}{2}JN \) at the \( \alpha = 1 \) point (Fig. 3). While the ground state has degeneracy \( d = 2 \) at \( \alpha = 0 \) (the + and - lowest energy states in the subspace with \( s = 0 \)) and \( d = 1 \) for \( 0 < \alpha < 1 \) (the + ground state with \( s = 0 \)), large degeneracy occurs at \( \alpha = 1 \).

![FIG. 3: Eigenenergies \( E_n \) of the XX–ZZ model \( \mathbf{1} \)–\( \mathbf{2} \) as obtained for a chain of \( N = 8 \) sites with periodic boundary condition for increasing \( \alpha \). Level crossing at \( \alpha = 1 \) marks the quantum critical point of the compass model \( \mathbf{5} \).](image)

![FIG. 4: (Color online) Pseudospin excitation gap \( \Delta \) in the XX–ZZ model \( \mathbf{1} \)–\( \mathbf{2} \) for increasing \( \alpha \) (solid lines). The gap collapses at the quantum critical point (QCP) (in the compass model obtained at \( \alpha = 1 \)), which separates the disordered phase with finite pseudospin \( \langle \sigma_{2i}^z \sigma_{2(i+r)}^z \rangle \) correlations (left) from the one with finite \( \langle \sigma_{2i}^x \sigma_{2(i+r)}^x \rangle \) correlations (right).](image)

Lowest energies in different subspaces \( \vec{s} \) are:

\[
E_s^\pm (\alpha) = -\frac{1}{2} \sum_k e_k + C_s(\alpha) + O(1)
\]

\[
= E_s^0 (\alpha) + O(1)
\]

where for \( \alpha \leq 1 \)

\[
E_s^+ (\alpha) = -\frac{1}{2} (1 - \alpha) JN
\]

\[
- JN \frac{1}{2\pi} \int_0^\pi dk \sqrt{1 + \alpha^2 - 2\alpha \cos k}
\]

is the energy of the ground state found in the \( s = 0 \) (+) subspace in the limit of \( N \to \infty \). A similar formula to Eq. \( \mathbf{30} \) is easily obtained for \( \alpha \geq 1 \) by a substitution \( \alpha \to (2 - \alpha) \). For any \( \vec{s} \) the gap between the + and - lowest energy states is \( O(1/N) \), but the gaps \( 2(1 - \alpha) s \) between \( s = 0 \) and \( s > 0 \) are \( O(1) \) and survive when \( N \to \infty \). However, when \( \alpha \to 1^- \), then all ground state energies for +/− and for any \( \vec{s} \) become degenerate, level crossing between all \( 2 \times 2^N/2 \) lowest energy states occurs when \( N \to \infty \). This ground state degeneracy is much higher than \( 2 \times 2^5 \) found in the \( L \times L \) 2D compass model \( \mathbf{10} \) but the overall behavior is similar — it indicates that frustrated interactions, the discontinuity in the correlation functions, and the pseudospin liquid disordered ground state are common features of the 1D and 2D compass models.

The ground state energy \( E_s^+ (\alpha) \) \( \mathbf{30} \) increases with \( \alpha \) for \( \alpha \in (0, 1) \) (Fig. 3), as pseudospin interactions are gradually more frustrated when \( \alpha \to 1^+ \). The ground state is separated from the lowest energy pseudospin excitation by a gap \( \Delta = 2\epsilon_0 = 4J|1 - \alpha| \), which vanishes at \( \alpha = 1 \) (see Fig. 4). Note that this excitation corresponds to reversing a \( \sigma^z \) pseudospin component for \( \alpha < 1 \), and a \( \sigma^x \) pseudospin component for \( \alpha > 1 \). We have verified that the linear decrease of \( \Delta \) when \( \alpha \to 1 \) is well
reproduced by the energy spectra obtained by exact diagonalization of finite systems (as obtained, for instance, from the data of Fig. 3).

IV. CONCLUSIONS

We have shown by an exact solution of the 1D XX–ZZ model \(^{(1)}\)–\(^{(2)}\) that pseudospin disordered states are triggered already by an infinitesimal admixture of the interactions between other spin components than those used to construct classical Ising models at \(\alpha = 0 \) \(^{(3)}\) or \(\alpha = 2 \) \(^{(3)}\). The properties of the XX–ZZ model are summarized in Fig. 4 with two different types of pseudospin correlations, dictated by the 'dominating' interactions. These pseudospin correlations and finite excitation gap may be seen as precursor of the antiferromagnetic order induced by the respective classical Ising model, with either \(\sigma^z_i\) or \(\sigma^x_i\) pseudospins, at \(\alpha = 0\) or \(\alpha = 2\), respectively. These opposite trends become frustrated in the 1D compass model lying precisely at the quantum critical point. We anticipate that a similar quantum critical point determines the properties of the orbital liquid state in a 2D compass model \(^{(10)}\).

In conclusion, the 1D XX–ZZ model provides a beautiful example of a first order quantum phase transition between two different disordered phases, with hidden order of pairs of pseudospins on every second bond. When the pseudospin interactions become balanced at the quantum critical point, the pseudospin (orbital liquid) disordered state takes over. It is characterized by: (i) high \(2 \times 2^{N/2}\) degeneracy of ground state, and (ii) the gapless excitation spectrum.

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