Radiation Transfer of Models of Massive Star Formation. IV. The Model Grid and Spectral Energy Distribution Fitting

Yichen Zhang1© and Jonathan C. Tan2©

1 The Institute of Physical and Chemical Research (RIKEN), Hiroshima 2-1, Wako-shi, Saitama, 351-0198, Japan; yczhang.astro@gmail.com
2 Departments of Astronomy & Physics, University of Florida, Gainesville, FL 32611, USA; jctan.astro@gmail.com

Received 2017 August 29; revised 2017 December 13; accepted 2017 December 14; published 2018 January 18

Abstract

We present a continuum radiative transfer model grid for fitting observed spectral energy distributions (SEDs) of massive protostars. The model grid is based on the paradigm of core accretion theory for massive star formation with pre-assembled gravitationally bound cores as initial conditions. In particular, following the turbulent core model, initial core properties are set primarily by their mass and the pressure of their ambient clump. We then model the evolution of the protostar and its surrounding structures in a self-consistent way. The model grid contains about 9000 SEDs with four free parameters: initial core mass, the mean surface density of the environment, the protostellar mass, and the inclination. The model grid is used to fit observed SEDs via \( \chi^2 \) minimization, with the foreground extinction additionally estimated. We demonstrate the fitting process and results using the example of massive protostar G35.20-0.74. Compared with other SED model grids currently used for massive star formation studies, the properties of the protostar and its surrounding structures are more physically connected in our model grid, which reduces the dimensionality of the parameter spaces and the total number of models. This excludes possible fitting of models that are physically unrealistic or are not internally self-consistent in the context of the turbulent core model. Thus, this model grid serves not only as a fitting tool to estimate properties of massive protostars, but also as a test of core accretion theory. The SED model grid is publicly released with this paper.

Key words: dust, extinction – ISM: clouds – radiative transfer – stars: formation – stars: massive

1. Introduction

Massive stars impact many areas of astrophysics, yet there is still no consensus on how they form. Theories range from core accretion models, i.e., scaled-up versions of low-mass star formation, to competitive accretion models at the crowded centers of forming star clusters (e.g., Bonnell et al. 2001; Wang et al. 2010), to protostellar collisions (Bonnell et al. 1998; Bally & Zinnecker 2005). Such confusion is partly due to the observational difficulties caused by the relative rarity and typically large distances (> 1 kpc) of massive protostars, highly crowded environments, and high extinctions. The environments of massive star formation are observed to be massive, dense gas clumps with mass surface densities of \( \Sigma_{cl} \approx 1 \, \text{g cm}^{-2} \), which corresponds to a visual extinction of about \( A_V \approx 200 \, \text{mag} \) (e.g., see Tan et al. 2014 for a review).

Analysis of broadband spectral energy distributions (SED) composed of total fluxes from NIR to FIR/sub-mm of massive protostars, via radiative transfer (RT) modeling, is the primary way to understand the properties of massive protostars, being efficient for large samples. A number of RT models have been developed to compare with observations. Robitaille et al. (2006, 2007) developed a large model grid to fit the SEDs of young stellar objects (YSOs). However, their model grid was mainly developed for low-mass star formation, without coverage of the parameter space needed for massive star formation, such as very high accretion rates resulting from high mass surface density environments. A similar RT model grid has also been developed by Molinari et al. (2008), which focused on massive YSOs. However, the components in their model are relatively simple. A massive YSO involves complicated structures such as the protostar, accretion disk, envelope and outflow, each of which may need multiple parameters to define their properties that may affect the resulting SED. Therefore, fitting an observed SED usually requires setting a large number of independent parameters. This is the method that the abovementioned model grids have adopted. While the wide choice of free parameters can generate good fits to the observations, such a method usually also generates results that are physically less realistic or not self-consistent (see, e.g., De Buizer et al. 2017). Large numbers of free parameters will also lead to higher susceptibility to degeneracies.

In this paper, the fourth of our series, we aim to use a different approach to build the model grid. Instead of large numbers of free parameters, we make the components more physically connected, in order to reduce the number of independent parameters. Our model grid is based on a particular model of massive star formation, the turbulent core model (McKee & Tan 2002, 2003). In the turbulent core model, massive stars are formed from pre-assembled massive pre-stellar cores, supported by internal pressure that is provided by a combination of turbulence and magnetic fields. The pressure at the core surface is assumed to be approximately the same as that of the surrounding large-scale star-cluster-forming clump, with a typical mean mass surface density of \( \Sigma_{cl} \approx 1 \, \text{g cm}^{-2} \). We construct the model grid from two initial conditions: the initial core mass, \( M_c \), and environmental mass surface density, \( \Sigma_{cl} \). With various analytical or semi-analytical solutions, we calculate the properties of different components, including the protostar, disk, envelope, outflow, and their evolutions, self-consistently from the initial conditions. The main free parameters in this model grid are the initial conditions, i.e., \( M_c \) and \( \Sigma_{cl} \), and the protostellar mass, \( m_\ast \), indicating the evolutionary stage, as well as the inclination and foreground extinction from the larger clump. In such a method, the model grid will exclude certain combinations of the components that are not supported by the core accretion theory.
By fitting the observed SEDs, this model will allow us to see whether the observed variety of massive protostars can be explained by a scenario of core accretion in different evolutionary stages and initial/environmental conditions.

In the previous papers in our series (Zhang & Tan 2011, hereafter Paper I; Zhang et al. 2013b, hereafter Paper II; Zhang et al. 2014, hereafter Paper III), we studied a fiducial case of a massive protostar growing inside a core with an initial mass of $60 M_\odot$, in a 1 g cm$^{-2}$ environment, and a few variants thereof. In this paper, we now present the full model grid covering a large parameter space and investigate how the initial conditions and evolution affect the SEDs of massive protostars (Section 2). We develop an SED fitting tool to fit observed SEDs with this model grid (Section 3). In Section 4, we demonstrate the fitting process and results, using the SED of the massive protostar G35.20-0.74 as an example. We discuss our results and present conclusions in Section 5.

2. Model Grid

2.1. Physical Model

We first briefly describe the physical assumptions used in our models, which have been introduced in the previous papers in this series. For detailed derivation and discussion of these points, please refer to Papers I, II, and III. Following McKee & Tan (2003), a star-forming core is defined as a region of a molecular cloud that forms a single star or a close binary via gravitational collapse. We can define such cores to contain a single, central rotationally supported disk. The initial core is assumed to be quasi-spherical, self-gravitating, in near-virial equilibrium, and in pressure equilibrium with the surrounding star-cluster-forming clump. The size of such a core is determined by the mean mass surface density of the surrounding clump $\Sigma_{cl}$ (which sets the pressure on the boundary of the core) by (McKee & Tan 2003)

$$R_e = 5.7 \times 10^{-2} (M_c / 60 M_\odot)^{1/2} (\Sigma_{cl} / g \text{ cm}^{-2})^{-1/2} \text{ pc}. \quad (1)$$

In the following text, $\Sigma_{cl}$ is also referred to as the mass surface density of the star-forming environment. The density distribution in the initial core is described by a power law in spherical radius, $\rho \propto r^{-k_c}$. Observations suggest that $k_c$ has a mean value of 1.3–1.6 (Butler & Tan 2012; Butler et al. 2014). Therefore, we adopt a fiducial value of $k_c = 1.5$ for the whole model grid, which is also consistent with our previous studies and the turbulent core model of McKee & Tan (2003).

The collapse of the core is described by an inside-out solution (Shu 1977; McCaughlin & Pudritz 1996, 1997), together with the effect of rotation (Ulrich 1976). Disks around massive protostars are also expected to be massive, due to the high accretion rates of about $10^{-4}$–$10^{-3} M_\odot$ yr$^{-1}$ (e.g., Beltrán & de Wit 2016). We assume the mass ratio between the disk and the protostar is a constant $f_d = M_d / m_\star = 1/3$, considering the rise in effective viscosity due to disk self-gravity at about this value of $f_d$ (Kratter et al. 2008). Disk size is calculated from the rotating collapse of the core to be $r_d(M_{ad}) = 0.684 \beta_c (M_{ad} / m_\star)^{1/2} R_c$, where $M_{ad}$ is the mass of the star-disk system in the limit of no feedback, as calculated from the collapse solution, and $m_\star$ is the actual mass in the star-disk system (see Paper III). The rotational-to-gravitational energy ratio of the initial core $\beta_c$ is assumed to be 0.02, which is a typical value from observations of low- and high-mass prestellar cores (e.g., Goodman et al. 1993; Li et al. 2012; Palau et al. 2013).

The disk structure is described with an “$\alpha$-disk” solution (Shakura & Sunyaev 1973), with an improved treatment to include the effects of the outflow and the accretion infall to the disk (Paper II). Half of the accretion energy is released when the accretion flow reaches the stellar surface (i.e., the boundary layer luminosity, $L_{acc} = G M_\star m_\star / (2 R_\star)$). We assume this part of the luminosity is radiated together with the intrinsic stellar luminosity isotropically as a single blackbody, i.e., the total luminosity from the protostar is $L_{prot} = L_\star + L_{acc}$ and the surface temperature of the protostar is $T_{acc} = [L_{prot} / (4 \pi R_\star^2 \sigma)]^{1/4}$. The other half of the accretion energy is partly radiated from the disk during accretion and partly converted to the kinetic energy of the disk wind. The total amount and detailed distribution of the accretion energy radiated from the disk are simultaneously derived from the disk solution.

The density distribution of the disk wind is described by a semi-analytic solution, which is approximately a Blandford & Payne (1982) wind (see Appendix B of Paper II), and the mass loading rate of the wind relative to the stellar accretion rate is assumed to be $f_w = m_w / m_\star = 0.1$, which is a typical value for disk winds (Königl & Pudritz 2000). Such a disk wind carves out polar cavities in the core, which gradually open up as the protostar evolves. The opening angle of the outflow cavity is estimated, following the method of Matzner & McKee (2000), by comparing the wind momentum and that needed to accelerate the core material to its escape velocity (Paper III). The accretion rate to the protostar is regulated by such outflow feedback. Note that we allow the existence of dust in some regions of the outflow cavity if the disk winds in these regions originate from the disk outside of the dust sublimation front.

The evolution of the protostar is solved using the model of Hosokawa & Omukai (2009) and Hosokawa et al. (2010). The model solves the detailed internal structure of the protostar, such as the deuterium-burning region, convective zone, and radiative zone, from the accretion history calculated above (see Paper III for more details). A photospheric boundary condition, which is usually associated with the situation of disk accretion, is used in the protostellar evolution calculation. Several outputs of this calculation that are important for setting up our grid of physical models for radiative transfer computation include the evolution of protostellar radius, luminosity, and surface temperature with the protostellar mass.

2.2. Parameter Space

In such a framework, the evolution of the core, protostar, disk, and outflow cavity are self-consistently calculated from two main initial conditions of the core: its initial mass ($M_c$) and the mean mass surface density of the clamp that the core is embedded in ($\Sigma_{cl}$). We refer to the evolutionary history of a protostar from a given set of initial conditions as an evolutionary stage, and a particular moment on such a track as an evolutionary stage, which is specified by a third parameter, the protostellar mass $m_\star$. We refer to the entire set of tracks as the model grid. Therefore, this model grid is of three dimensions ($M_c - \Sigma_{cl} - m_\star$). In the current model grid, $M_c$ is sampled at 10, 20, 30, 40, 50, 60, 80, 100, 120, 160, 200, 240, 320, 400, 480 $M_\odot$; and $\Sigma_{cl}$ is sampled at 0.1, 0.32, 1, 3.2 g cm$^{-2}$, forming 60 evolutionary tracks. $m_\star$ is sampled at 0.5, 1, 2, 4, 8, 12, 16, 24, 32, 48, 64, 96, 128, 160 $M_\odot$. Note that not all of these $m_\star$ are sampled for each track. In particular, the maximum protostellar mass is limited by the final stellar mass achieved in a given evolutionary track (see Figure 3). As a result,
there are 432 totally different physical models defined by different sets of \((M_c, \Sigma_{cl}, m_*)\).

We note that other initial conditions may affect the models, such as the initial rotational-to-gravitational energy ratio of the core \(\beta_c\) and magnetic field strength in the core, both of which are expected to influence the size of the accretion disk. However, the SEDs are not significantly affected by variations of the disk size around its fiducial values, given that the disk is always large compared to the size of the star and small compared to the extent of the core and outflow cavities (see Paper III).

Figure 1 shows the range of the initial conditions in our model grid. The mass surface density of the star-forming environment, \(\Sigma_{cl}\), covers a range from 0.1 to \(3 \times 10^2\) \(\text{g cm}^{-2}\). This range is similar to the values found in most Galactic massive star-forming regions, including: infrared dark clouds (IRDCs, dark green squares) and their internal clumps/cores (dark green crosses) that are thought to represent the initial stage of massive star formation; massive star-forming clumps/cores (red squares and light green circles), including those at Galactic Center (e.g., the “Brick”); some massive star clusters (e.g., the Orion Nebula Cluster [ONC]) and even more massive “super star clusters” (e.g., Westerlund 1, Arches, and Quintuplet). The initial core mass, \(M_c\), in the model grid covers a range from 10 to about 500 \(M_\odot\), which is similar to those of individual pre-stellar and protostellar cores inside IRDCs and massive clumps. Therefore, our model grid covers a wide range of initial conditions that are suitable to form individual stars from intermediate to high mass. We note that, as the first release of the model grid, the current version does not have very fine sampling over the initial conditions, especially the surface density of the star-forming environment \(\Sigma_{cl}\). While the sampling of \(\Sigma_{cl}\) in the current model grid, which covers most of the relevant range of local massive star formation, is sufficient to understand the differences between low and high \(\Sigma_{cl}\) models, as we will show in the examples of SED model fitting in Section 4, there can be degeneracies that span the full range of \(\Sigma_{cl}\). Therefore, we note that the constraints placed on \(\Sigma_{cl}\) at this point are still relatively limited.

Figure 2 shows all the evolutionary tracks in the current model grid. Panel (a) shows how the growth of the protostellar mass corresponds to time, i.e., protostellar age. The evolutionary stages we sample in the grid (marked by the crosses) cover a range of age from about \(7 \times 10^3\) year to about \(7 \times 10^4\) year. Panels (b) to (f) show the evolution of the outflow cavity opening angle, accretion rate, protostellar radius, luminosity, and surface temperature, with the growth of protostellar mass. All the models show similar trends in these figures. As the protostar grows, the outflow cavity gradually opens up (Panel (b)) due to the interaction between the outflow and the core, i.e., outflow feedback. The accretion rate increases with protostellar mass for most of the time, except at the final stages when outflow feedback causes the accretion rate to decrease (Panel (c)). The moments when the accretion...
rates start to decline happen around 0.5–0.9 of the total formation times, and up to such moments, the protostars have grown to 0.5–0.8 of their final masses. The protostellar evolutions can be divided into several stages, which are clearly seen in the change of protostellar radius (Panel (d)). Note that the radius is calculated from the protostellar evolution model (Section 2.1), which then helps determine the photospheric properties of the protostar. At the lowest protostellar masses that the model grid covers (~0.5 $M_\odot$), the protostellar radius steadily grows with mass, due to deuterium burning. From about several $\times M_\odot$, the radius increases drastically with protostellar mass, caused by the redistribution of entropy in the protostar. The radius reaches its peak at $m_\ast = 4-10 M_\odot$, after which the protostar enters the Kelvin–Helmholtz (KH) contraction stage. The main-sequence stage starts from $\gtrsim 10 M_\odot$ in the low $\Sigma_\mathrm{cl}$ cases and $\gtrsim 30 M_\odot$ in the high $\Sigma_\mathrm{cl}$ cases. The luminosity $L_{\ast,\mathrm{acc}}$, which combines the intrinsic protostellar luminosity and boundary-layer accretion luminosity, almost monolithically increases with protostellar mass (Panel (e)). It is worth noting that the accretion luminosity is dominant before the fast swelling phase of the protostar, after which the protostellar luminosity from nuclear (hydrogen and/or deuterium) burning becomes dominant. The surface temperature of the protostar is significantly affected by protostellar radius, especially around the fast-swelling phase and the KH contraction phase (Panel (f)).

While the general trends of the evolutionary tracks in the model grid are similar, they are affected by the initial conditions, especially the mass surface density of the environment $\Sigma_\mathrm{cl}$. If a core of the given mass is embedded in an environment with a higher surface density, then it is more pressurized and becomes denser and more compact, and thus collapses more quickly, leading to a shorter star formation timescale, a higher accretion rate, and a higher luminosity. Such a core is also more resistant to outflow feedback, so the outflow cavity opens up more slowly with $m_\ast$, compared to a core in a low $\Sigma_\mathrm{cl}$ environment. In such a case, with the higher accretion rate, the protostar also enters the fast-swelling phase and main-sequence stage at a higher mass; it also reaches a larger radius during the swelling phase, which affects the stellar surface temperature.

Figure 3 shows the star formation efficiencies from the initial cores to the final stars, $\epsilon_{\text{sf}} \equiv m_{\text{sf}}/M_\ast$, in the model evolutionary tracks. Such efficiencies are calculated for each evolutionary track and then expanded to the whole initial condition parameter space via two-dimension linear regression (in log space). The fitted relation between $\epsilon_{\text{sf}}$ and the initial conditions is

$$\epsilon_{\text{sf}} = 0.44 - 0.083 \log \left( \frac{M_\ast}{60 M_\odot} \right) + 0.14 \log \left( \frac{\Sigma_\mathrm{cl}}{1 \, \text{g cm}^{-2}} \right)$$

(2)

which agrees with the data points (the upper panel of Figure 3) within 17%. In the model grid, the star formation efficiency ranges from about 0.2 to about 0.6. Such values are consistent with the scenario that the stellar initial mass function (IMF) is inherited from the core mass function with a relatively constant core-to-star conversion efficiency (e.g., Alves et al. 2007; André et al. 2010; Offner et al. 2014), but now somewhat dependent on the distribution of $\Sigma_\mathrm{cl}$ for the global population of pre-stellar cores.

For a core of a given mass, the star formation efficiency is higher in an environment with a higher mass surface density. This is because cores are denser in such an environment, and so it is harder for the outflow to disperse the envelope and stop the accretion. On the other hand, in the same environment, the star formation efficiency decreases as the core mass increases. This is expected because the feedback becomes stronger as the protostar grows. In our model grid, we only include mechanical feedback from the outflow momentum, but ignore other
feedback mechanisms such as radiation pressure, photovaporation, and stellar winds. For several fiducial models in our model grid, Tanaka et al. (2017) have included these effects and found that, while the mechanical feedback from the outflow is always dominant, the other effects—especially radiation pressure—can significantly affect the star formation efficiency for the most massive cores in our model grid (forming >100 $M_\odot$ stars) in low mass surface density environments ($\Sigma_\text{cl} \lesssim 0.3$ g cm$^{-2}$). Following this trend, the relatively low efficiencies in the lower-right corner of Figure 3 will become even lower. However, Tanaka et al. (2017) did not find any sudden decrease of the efficiency with core mass at the high-mass end, even with the additional feedback included, suggesting that such feedback does not lead to a truncation of the high end of the stellar IMF.

Figure 4 shows the evolutionary tracks of the model grid in the $L_{\text{bol}}$–$M_\text{env}$ plane. Here, $M_\text{env}$ is the current envelope mass, which is different from the initial mass of the core $M_*$. As the protostar evolves, $M_\text{env}$ gradually decreases due to the accretion to the protostar and widening of the outflow cavity. The $L_{\text{bol}}$–$M_\text{env}$ diagrams have been used to identify the evolutionary stages of massive protostars (e.g., Molinari et al. 2008; Elia et al. 2010; König et al. 2017). As expected, the model evolutionary tracks start in the lower right of the figure and gradually move to the upper left. Although the models cover a wide range of $L_{\text{bol}}$ and $M_\text{env}$, models with similar evolutionary stages (defined by $m_\text{sf}/M_\text{env}$) form strips on this diagram. We fit two groups of models at different evolutionary stages. One group is around $m_\text{sf}/M_\text{env} = 1$, which is usually used to mark the end of the main accretion phase and the start of the envelope clear-up phase, and corresponds to the Class 0 to Class I transition in low-mass star formation. Another group is around $m_\text{sf}/M_\text{env} = 0.1$, i.e., typical sources in the main accretion phase. The fitting results to these two groups are $\log(L_{\text{bol}}/L_\odot) = 2.5 + 1.8 \log(M_\text{env}/M_\odot)$ and $\log(L_{\text{bol}}/L_\odot) = 0.54 + 1.9 \log(M_\text{env}/M_\odot)$, which have similar slopes but one is two orders of magnitude below the other in $L_{\text{bol}}$. Compared with the results of similar fits to the observed data by Molinari et al. (2008) and König et al. (2017), the slopes predicted by our models are steeper than the observations, which are in the range of 0.5–1.3. It is worth noting that only the core mass is included in our model, while in real observations, additional clump material will usually be included due to the low resolutions of single-dish FIR observations. This effect will tend to cause the observed slopes to be shallower if more luminous, typically more distant sources tend to have more contamination from surrounding clump material. However, Baldeschi et al. (2017) found that increasing distance, although it causes source positions to shift in the $L$–$M$ diagram, does not change the slope, on average. In addition, the luminosity used in this diagram is the true total luminosity from the source, which may differ from the luminosity directly integrated from the SEDs by an order of magnitude or more depending on the viewing inclination, because more radiation is emitted in the polar direction due to low-density outflow cavities (see Section 2.3.3).

2.3. Radiative Transfer Simulations and Resultant SED Grid

The Monte-Carlo continuum radiative transfer simulation is performed for the protostellar cores in the model grid using the HOCHUNK3d code by Whitney et al. (2003, 2013). Different dust opacity models are assigned to different regions, including the envelope, the low-density regions of the disk ($n_\text{H} < 2 \times 10^{10}$ cm$^{-3}$), the high-density regions of the disk, the part of outflow launched from the disk outside of the dust sublimation radius, and the foreground ISM (to calculate the foreground extinction; see below). Details regarding these opacity models were described in Paper I and II. The dust models and setups are same as those used by Robitaille et al. (2006, 2007). The code was updated to also include gas opacities (which is important in high-temperature regions around massive protostars), adiabatic cooling/heating, and advection (Papers I and II). For each model, the temperature profile is calculated and SEDs at 20 viewing inclinations are produced. The inclination is sampled at $\mu_{\text{view}} \equiv \cos \theta_{\text{view}} = 0.976, 0.925, ..., 0.025$, i.e., equally distributed between 1 (face-on) and 0 (edge-on). Therefore, there are 8640 SEDs in total on the current model grid, determined by four independent parameters $M_*$, $\Sigma_\text{cl}$, $m_\text{sf}$, and $\theta_{\text{view}}$. Note that these SEDs include all the emission from the source, i.e., with an aperture that is large enough to cover the whole core.

In order to compare with the observation, the model SEDs need first to be scaled by the distance, and then adjusted by

---

**Figure 3.** Dependence of the star formation efficiency, $\epsilon_{\text{sf}} \equiv m_{\text{sf}}/M_*$ (shown in color scale), on the initial core mass, $M_*$, and the mass surface density of the ambient clump, $\Sigma_\text{cl}$. The efficiencies are calculated for each evolutionary track in the model grid (upper panel) and then expanded via two-dimension linear regression to the whole initial condition parameter space, i.e., $\log M_* - \log \Sigma_\text{cl} - \epsilon_{\text{sf}}$ (lower panel). The $\epsilon_{\text{sf}}$ data are missing for models with the most massive cores in highest $\Sigma_\text{cl}$ environments (gray area in upper-right of upper panel), due to how difficult it is for the protostellar evolution calculation at very high accretion rates to reach the final stellar mass. The contours in the lower panel are $\epsilon_{\text{sf}} = 0.3, 0.4,$ and 0.5.
additional foreground extinction described by the parameter $A_V$, 

$$F_{\nu,\text{mod,ext}}(\lambda) = F_{\nu,\text{mod}}(\lambda) \times 10^{-0.4A_V(\lambda)/\cos\theta},$$  

(3)

where $\kappa(\lambda)$ and $\kappa_V$, the dust opacities at the wavelengths $\lambda$ and in the V-band, respectively, are from the extinction law of the dust model, and $F_{\nu,\text{mod}}$ are the distance-scaled model fluxes. When fitting actual data (see Section 3), the model SEDs are further convolved with the transmission profiles of the instrument filters to simulate the fluxes detected in observational bands of various instruments. Therefore, in principle, it is not necessary to perform color correction for the observed fluxes before fitting with the model grid.

In such a model grid, we are explicitly linking the SEDs to the initial conditions and evolutionary stages of massive star formation. An SED to fit the observation is determined by six parameters: $M_*$, $\Sigma_\text{cl}$, $m_*$, $\theta_{\text{view}}$, $d$, and $A_V$. Such an approach assumes different components are physically connected to each other, thereby reducing the dimensionality of the parameter space—and thus the number of model that need to be computed. Meanwhile, by comparing such models with observations, especially through fitting a large sample of massive protostars, one can understand to what extent the observed variety in the infrared emission of massive protostars can be explained simply by different initial conditions and evolutionary stages, and ultimately test the turbulent core accretion theory of massive star formation.

### 2.3.1. Example SEDs

Figure 5 shows example SEDs from the model grid. From top to bottom, the four panels show how the SED is affected by the viewing inclination angle $\theta_{\text{view}}$, growth of the protostellar mass $m_*$, the mass surface density of the star-forming environment $\Sigma_\text{cl}$, and the initial core mass $M_*$.

Panel (a) compares the SEDs of the same physical model ($M_* = 60 \, M_\odot$, $\Sigma_\text{cl} = 1 \, \text{g cm}^{-2}$, $m_* = 8 \, M_\odot$), but viewed at different inclination angles. From an edge-on view ($\cos\theta_{\text{view}} = 0$) to a face-on view ($\cos\theta_{\text{view}} = 1$), the SED at shorter wavelengths increases, while the SED at wavelengths longer than about 100 $\mu$m (i.e., longer than the wavelengths of the SED peak) does not change. In this particular physical model ($M_* = 60 \, M_\odot$, $\Sigma_\text{cl} = 1 \, \text{g cm}^{-2}$, $m_* = 8 \, M_\odot$), the opening angle of the outflow cavity is $\theta_{\text{w,esc}} \approx 25^\circ$ (cos $\theta_{\text{w,esc}} \approx 0.9$). The SED becomes flat when the viewing inclination angle is smaller than this (i.e., when the line of sight toward the protostar goes through the outflow cavity).

Panel (b) shows how the growth of the protostar affects the SED along one evolutionary track ($M_* = 60 \, M_\odot$, $\Sigma_\text{cl} = 1 \, \text{g cm}^{-2}$). A typical viewing inclination angle of $\theta_{\text{view}} = 61^\circ$ is used here. As the protostar grows, the fluxes at all wavelengths increase, especially at the shorter wavelengths. This is not only because of the increase in total luminosity with the growth of protostar, but also because of the gradual widening of the outflow cavity. As the flux increases, the SED peak also moves from about 100 $\mu$m at early stages to about 50 $\mu$m at later stages. The SED becomes flat at wavelengths $<700 \mu$m once the outflow cavity becomes wider than the viewing inclination angle (the SED for $m_* = 24 \, M_\odot$).

Panel (c) compares the SEDs of models with the same $M_*$ and $m_*$, but in environments with different $\Sigma_\text{cl}$. The mass surface density of the environment affects the SED in several ways. First, for these four models, the luminosity is higher in a higher mass surface density environment, which affects the height of the far-IR peak of the SED. This is true in general, especially at earlier stages when the accretion luminosity is dominant, but is affected by detailed evolutionary histories (see Panel (e) of Figure 2). Second, the mid-IR fluxes are mainly determined by the emission from the accretion disk and also by dust inside the outflow cavity, after being extinguished by the envelope. Because the envelope extinction is lower in the low $\Sigma_\text{cl}$ case, higher mid-IR fluxes are seen. Third, the outflow cavity develops faster with the growth of the protostar in the low $\Sigma_\text{cl}$ case, which also makes the mid-IR fluxes higher and shortens the wavelengths of the far-IR peaks. Fourth, different protostellar evolutionary tracks cause the input stellar temperature to be significantly different among these mass surface conditions and evolutionary stages, and ultimately test the protostars can be explained simply by different initial
In this section, we discuss several characteristics of the SEDs and their distributions in the parameter space of the model grid. These characteristics include the wavelength of the SED peak, the bolometric temperature, the far-IR slope at wavelengths around 160–500 μm, and the mid-IR slope at wavelengths around 20–40 μm. As the results of the previous section have shown, these characteristics of the SED are directly affected by the initial conditions, evolutionary stages, and viewing inclination angles. While SED fitting works best with a fully sampled SED from MIR to FIR, these characteristics may help to constrain some of the conditions of massive protostars in circumstances where fully sampled SEDs are not available.

As shown in Figure 5, the wavelength of the SED peak λ_{peak} in the far-IR is not very sensitive to the viewing inclination or the foreground extinction (except for face-on sources), and only sensitive to the physical conditions of the source (in our case, determined by the initial conditions M*, Σcl, and the protostellar mass m*). Figure 6 shows the distributions of λ_{peak} in the parameter space of the model grid. The SED becomes flat, and therefore λ_{peak} ≤ 30 μm once the opening angle is large and the line of sight toward the protostar goes through the outflow cavity. Such SEDs are also highly sensitive to the foreground extinction. Because foreground extinction is common in massive star-forming regions, as well as quite uncertain, we expect to see the situation represented in the right panels more often in real observations. For the models with viewing inclination angles larger than the opening angle of the outflow cavity, the effect of the foreground extinction on λ_{peak} is minor.

Figure 7 shows the distributions of λ_{peak} in the mass surface density of the environment Σcl. In early stages, the high Σcl models have SED peaks at about 90 μm, while the low Σcl models have SED peaks at about 70 μm. In later stages, the high Σcl models have SED peaks at about 70 μm and the low Σcl models have SED peaks at about 50 μm, if the viewing inclination angle is larger than the opening angle of the outflow cavity. With a foreground extinction, the difference is even smaller. After averaging over inclination angles and mass surface densities of the environment (lower panels), it is evident that λ_{peak}, with or without foreground extinction, is dependent on the evolutionary stages indicated by m*/M* or m*/M_{env}, especially when the protostellar mass m* ≥ 8 M_⊙. At the stage of m*/M_{env} = 1, which marks the transition from the main accretion phase to the envelope clear-up phase, λ_{peak} is about 50–60 μm in the case of foreground extinction A_v = 100. At the stage that m*/M_{env} = 0.1, λ_{peak} is about 60–70 μm.

Figure 7 shows how the [160 μm]−[40 μm] color (defined as log[F_{160 μm}/F_{40 μm}]) depends on the initial conditions, protostellar mass, and viewing inclination angle. The [160 μm]−[40 μm] color is related to λ_{peak}, which is between these two wavelengths. In fact, the [160 μm]−[40 μm] color is easier to determine, because it does not require a well-sampled SED around the peak position, although it is more affected by the inclination or foreground extinction because the flux at 40 μm is used. However, as Figure 7 shows, the effects of inclination and possible foreground extinction on the [160 μm]−[40 μm]...
color are modest. The value of log\(\frac{F_{160\mu m}}{F_{40\mu m}}\) increases by \(\sim 1\) when varying the inclination from a face-on view to an edge-on view, and increases by \(\sim 0.3\) with a foreground extinction of \(A_V = 100\). Overall, it changes by up to \(\sim 4\) over the whole parameter space. The dependence of the \(\frac{F_{160\mu m}}{F_{40\mu m}}\) color on the mass surface density of environment \(\Sigma_{cl}\) is also relatively weak (the increase is by \(\lesssim 1\) from the high mass surface density case to the low mass surface density case). It is sensitive to the evolutionary stage, indicated by \(m_s/M_e\) or \(m_s/M_{env}\). Therefore, the \([160 \mu m]\)–[40 \mu m] color may be used as an indicator of the evolutionary stage of massive protostars, in addition to the peak position of the SED. The SEDs at the stage when \(m_s/M_{env} = 1\) have \(F_{160\mu m}/F_{40\mu m} \approx -1\) and those at the stage of \(m_s/M_{env} = 0.1\) have \(F_{160\mu m}/F_{40\mu m} \approx 0\).

Aside from the peak wavelength, one can also use the flux-weighted mean wavelength as an indicator of the evolutionary stage. The bolometric temperature, a related concept, is defined as the temperature of a blackbody having the same mean frequency as the observed SED (Myers & Ladd 1993), which can be written as \(T_{bol} = 1.25 \times 10^{-11}\langle \nu \rangle\) K \(\text{Hz}^{-1}\).

Here, for \(\langle \nu \rangle\), we use the flux-weighted mean frequency \(\int \nu F_\nu d\nu / \int F_\nu d\nu\) in the wavelength range of \(\lambda > 1\ \mu m\). This is because often only infrared data are available when constructing SEDs for massive protostars, and the fluxes at shorter wavelengths normally suffer from high levels of extinction because massive protostars are typically highly embedded in high mass surface density clouds. Compared to the peak wavelength of the SED, the bolometric temperature is not as affected by whether the SED is well-sampled around the peak. However, it is sensitive to the inclination and the foreground extinction, because the short-wavelength fluxes are used to determine \(T_{bol}\). Therefore, one should be cautious when using the bolometric temperature to estimate the evolutionary stage, especially for individual sources. For a sample with a large number of sources, we expect the effects of different viewing inclination angles will be averaged out.

Figure 8 shows how the bolometric temperatures are affected by the initial conditions, the evolutionary stages, the viewing inclination angles, and foreground extinction. From Panels (a)
and (b), we can see that $T_{\text{bol}}$ is distinctively different for the inclinations at which the line of sight toward the central protostar goes through the outflow cavity, compared to those at which the line of sight goes through the envelope. In the former case (low inclination angles), $T_{\text{bol}} \approx 300$ K without foreground extinction, and about 200 K if a foreground extinction of $A_V = 100$ is applied. In the latter case (high inclination angles), $T_{\text{bol}}$ is less affected by the foreground extinction. The value of $T_{\text{bol}}$ ranges from about 30 K to about 120 K without foreground extinction, and up to about 80 K with a foreground extinction of $A_V = 100$. The bolometric temperature is slightly dependent on the mass surface density of the environment, $\Sigma_{cl}$, which makes $T_{\text{bol}}$ increase by about 30 K from the highest $\Sigma_{cl} = 3.2$ g cm$^{-2}$ to the lowest $\Sigma_{cl} = 0.1$ g cm$^{-2}$ in the case without any foreground extinction, which will lower the effect. At a typical inclination of 60° and averaging over the mass surface density of the environment, the bolometric temperature is sensitive to the evolutionary stage indicated by $m_s/M_*$ or $m_*/M_{\text{env}}$, i.e., how embedded the protostar is, as Panels (c) and (d) show. At the stage where $m_*/M_{\text{env}} = 1$ (which corresponds to the transition from Class 0 to I in low-mass star formation), $T_{\text{bol}} \approx 200$–300 K in the case of no foreground extinction, and $T_{\text{bol}} \approx 100$ K with a foreground extinction of $A_V = 100$. These values are higher than $T_{\text{bol}} = 70$ K, which is commonly used as the boundary between the Class 0 and Class I sources in low-mass star formation studies (Chen et al. 1995). However, in the cases of low $\Sigma_{cl}$ and low $M_*$, which are closer to the situation of normal low-mass star formation, this transition does occur at about $T_{\text{bol}} = 70$ K.

Figure 9 shows the far-IR/sub-mm slope of the SED defined by $\log (F_{160\mu m}/F_{500\mu m})$ in the parameter space. As Figure 5 has shown, the slope in this wavelength range is not affected by the inclination or foreground extinction; therefore, we only consider its dependence on $M_*$, $\Sigma_{cl}$, and $m_*$. As Figure 9 shows, the far-IR slope of the SED has a clear dependence on the protostellar mass. As the protostar grows, the far-IR slope of the SED becomes steeper, which can be also seen in Panel (b) of Figure 5. Unlike the peak wavelength or colors of other
wavelength ranges discussed above, the far-IR slope has only a weak dependence on the initial core mass (or the current envelope mass), especially when the protostellar mass $m_\ast \gtrsim 10 M_\odot$. The mass surface density of the environment also affects the far-IR slope of the SED. In a high mass surface density environment, the higher column density of the envelope causes more shorter-wavelength emission from the inner hot regions, such as the disk or innermost envelope, to be shifted to longer wavelengths, leading to a steeper slope of the far-IR/sub-mm SED. Although the far-IR slope of the SED is not as affected by the viewing inclination angle or the foreground extinction—and thus is mostly dependent only on the physical conditions of the source—it is affected, in real observations, by the ambient clump material, which is not included in our models. Also, the exact value of the slope is affected by dust emissivity properties.

Figure 10 shows the slope of the SED from 20 to 40 μm in the parameter space. The SED slope at this wavelength range can be significantly affected by the mass surface density of the environment, $\Sigma_{cl}$, from a relatively flat slope with $\log(F_{\nu}/F_{\nu,0}) \approx 0$ to a steep slope with $\log(F_{\nu}/F_{\nu,0}) \approx 3$, given the same initial core mass and protostellar mass. This is caused by the significant effects of the column density of the envelope on the fluxes around 10–20 μm which is mainly from either the disk, dust inside the outflow cavity, or the innermost hot envelope. After averaging over $\Sigma_{cl}$, one can clearly see that the SED slope at 20 to 40 μm depends on the evolutionary stages indicated by $m_\ast/M_\ast$, i.e., how embedded the protostar is, rather than simply on $m_\ast$, i.e., the growth of the protostar. The SED slope at this wavelength range is also affected by the viewing inclination angle, but normally within a range of about one order of magnitude, not considering the extreme cases such as a face-on source. The foreground extinction has a modest effect on the slope compared with that due to the evolution, changing $\log(F_{\nu}/F_{\nu,0})$ by about 0.6 for a foreground extinction of $A_V = 100$. 

Figure 8. Distributions of the bolometric temperature (shown in color scale). (a) Each small square is a group of models for each set of $M_\ast$ and $m_\ast$. Inside each square, the four rows from top to bottom are $\Sigma_{cl} = 0.1, 0.32, 1, \text{ and } 3.2 \, \text{g cm}^{-2}$. From left to right, each column gives the inclination angle, $\theta_{i,v}$, from an edge-on view to a face-on view, respectively. (b) Similar to Panel (a), but calculated from SEDs with a foreground extinction of $A_V = 100$. (c) Similar to Panel (a), but the color of each square shows the bolometric temperature at inclination angle of 60° and averaged over $\Sigma_{cl}$ at each $M_\ast$ and $m_\ast$. (d) Similar to Panel (c), but calculated from SEDs with a foreground extinction of $A_V = 100$. The dashed lines are where $m_\ast/M_\ast = 0.01, 0.03, 0.1, \text{ and } 0.3$. The solid lines are where $m_\ast/M_{env} = 0.01, 0.1, \text{ and } 1$. Here, $M_{env}$ is averaged over $\Sigma_{cl}$ for each $M_\ast$ and $m_\ast$. 

The Astrophysical Journal, 853:18 (24pp), 2018 January 20 Zhang & Tan
Figure 9. (a) Distribution of the [160 μm]−[500 μm] color (defined as log[F160 μm/F500 μm], shown in color scale). Each rectangle is a group of models for each set of $M_*$ and $m_*$. The four points inside each rectangle from left to right are $\Sigma_{\text{cl}} = 0.1, 0.32, 1, \text{and } 3.2 \text{ g cm}^{-2}$. At this wavelength range, the SEDs are not affected by the inclination or the foreground extinction. (b) Similar to Panel (a), but each rectangle shows the [160 μm]−[500 μm] color averaged over $\Sigma_{\text{cl}}$ at each $M_*$ and $m_*$. The dashed lines are where $m_*/M_* = 0.01, 0.03, 0.1$, and $0.3$. The solid lines are where $m_*/M_* = 0.01, 0.1$, and $1$. Here, $M_{\text{env}}$ is averaged over $\Sigma_{\text{cl}}$ for each $M_*$ and $m_*$. Figure 11 shows how the SED features discussed in this section, including the [160 μm]−[500 μm] color, [160 μm]−[40 μm] color, [40 μm]−[20 μm] color, and the SED peak wavelength correlate to each other. As discussed above, all these colors at different wavelength ranges can be used to estimate the evolutionary stages, but they are sensitive to different components of the source. The [160 μm]−[500 μm] color is mostly affected by the envelope, while the [160 μm]−[40 μm] or [40 μm]−[20 μm] colors are more sensitive to the inner warmer regions, including the outflow cavity wall. Panel (a) and (b) clearly show that the long- and short-wavelength colors correlate to each other, but the correlations are affected by the initial condition of the source, especially the environmental mass surface density $\Sigma_{\text{cl}}$. Panel (c) shows that the [160 μm]−[40 μm] color is well-correlated with the peak wavelength of the SED, except for some models at late stages or very wide outflow opening angles, for which the SED peaks are at wavelengths $\lesssim 20 \mu m$. As mentioned above, both of these features can be used to indicate the evolutionary stages, but the [160 μm]−[40 μm] color is easier to determine because it does not require a well-sampled SED around its peak position.

2.3.3. Flashlight Effect

As discussed above, the observed SED is highly dependent on the viewing inclination angle, because the existence of the low-density outflow cavity allows more radiation to escape from the polar direction, which is known as the “flashlight effect.” This causes the bolometric luminosity integrated from the observed SED to deviate from the true bolometric luminosity of the source by a factor that depends on the inclination. Panel (a) of Figure 12 shows the ratios between the bolometric luminosities inferred from SEDs by assuming isotropic radiation, $L_{\text{inc}}$, and the true bolometric luminosities, $L_{\text{tot}}$, in the model grid. As the figure shows, the degree of the flashlight effect is almost completely dependent on two factors: the opening angle of the outflow cavity, $\theta_{\text{w,esc}}$, and the viewing inclination angle, $\theta_{\text{view}}$. In most of the cases where $\theta_{\text{view}} < \theta_{\text{esc}}$, $L_{\text{inc}}$ will overestimate $L_{\text{tot}}$ by a factor up to 10, while in most of the cases where $\theta_{\text{view}} > \theta_{\text{esc}}$, $L_{\text{inc}}$ will underestimate the true luminosity, $L_{\text{tot}}$. For an opening angle of about $50^\circ$, the inferred luminosity $L_{\text{inc}}$ overestimates the true luminosity by a factor of 2−3 for a face-on view, and underestimates the true luminosity by a factor of 10 for an edge-on view. For an opening angle of about $20^\circ$ or smaller, the inferred luminosity is close to the true luminosity for most inclinations, except at a face-on view.

In real observations, the luminosity directly inferred from the SED further deviates from the true luminosity due to possible foreground extinction. Panel (b) of Figure 12 shows the ratios between the luminosities inferred from SEDs with $A_V = 100$ and the true bolometric luminosities of the sources. In such a case, most of the inferred luminosities underestimate the true luminosities. Because the foreground extinction mainly lowers the SEDs at shorter wavelengths, which mainly affects the SEDs at lower angles of viewing inclination, the flashlight effect is not affected as much by the foreground extinction, if the inclination angle is larger than the opening angle of the outflow cavity. In this case, the flashlight effect is significantly affected by the opening angle of the outflow cavity, but only very weakly affected by the inclination angle. With a foreground extinction of $A_V = 100$, for an opening angle of about $50^\circ$, the inferred luminosity underestimates the true luminosity by a factor of about 10, no matter the viewing inclination angle.

2.3.4. Temperature Evolution in the Envelope

The radiative transfer simulation also predicts the dust temperature profiles for the models in the grid. Panel (a) of Figure 13 shows the evolutions of the mass-weighted mean temperature in the whole envelope as a function of protostellar mass under different initial conditions. The envelope temperature has a clear dependence on the protostellar mass $m_*$ and the mass surface density of the star-forming environment, $\Sigma_{\text{cl}}$, but only a weak dependence on the initial core mass, $M_\odot$. In the low mass surface density cases ($\Sigma_{\text{cl}} = 0.1$ and $0.3 \text{ g cm}^{-2}$), at early stages, the core has a mean temperature of $\lesssim 20 \text{K}$, while with $\Sigma_{\text{cl}} = 1 \text{ g cm}^{-2}$, the mean temperature is about $30 \text{K}$, and in the high mass surface density case, the temperature reaches about $50 \text{K}$, even in the earliest stages. As the protostar grows, the envelope becomes warmer. There is a significant increase in the envelope temperature around $m_* \approx 4 M_\odot$. The peak temperature is reached at $m_* \gtrsim 20 M_\odot$. The peak temperature
of the high mass surface density core is about 100 K. The mass surface density of the star-forming environment affects the temperature in the envelope in several ways. As discussed in Section 2.1, in a high mass surface density environment, the core is more compact and collapses with a higher accretion rate, leading to a higher luminosity. The high luminosity and the small size of the core combined make the temperature higher in such a core. In some models, the temperature starts to decrease in the final stages. This is because of the wide opening angle of the outflow cavity starts to suppress the heating from the protostar due to escape of radiation through the low-density outflow cavity.

The different luminosities and envelope densities in the various $\Sigma_{cl}$ environments have a more significant impact on the temperature in the innermost region of the envelope. Panel (b) shows the mass-weighted mean temperature in the region of the envelope within 1000 au of the protostar. On this scale, the peak temperature of the high mass surface density core can reach about 400–600 K at around $m_{s} = 20 M_\odot$. For most of the models in low mass surface density environments ($\Sigma_{cl} = 0.1$ or 0.3 g cm$^{-2}$), the mean envelope temperature on the 1000 au scale is below 100 K at early stages ($m_{s} \lesssim 4 M_\odot$) and below about 200 K at later stages.

Hot core chemistry is initiated when the temperature reaches about 100 K, at which point dust-grain ice mantles are largely sublimated and various complex molecules are released to the gas phase, producing important observational diagnostics for the protostellar stage of massive star formation. Panel (c) of Figure 13 shows the evolution of the hot core size (defined as the size of the envelope that has a mass-weighted mean temperature of 100 K) with the growth of the protostar under different initial conditions. In the early stages, the hot core region is only present within a small zone of several hundreds of au. Later on, when $m_{s} \gtrsim 10 M_\odot$, the hot core is typically on the scale of several thousands of au. In addition, the size of the hot core has clear dependence on the mass surface density of the environment in the early stages. However, this dependence becomes weaker in the later stages, when the hot core is fully

---

**Figure 10.** Distributions of the [40 $\mu$m]–[20 $\mu$m] color (defined as $\log (F_{40 \mu m}/F_{20 \mu m})$, shown in color scale). (a) Each small square is a group of models for each set of $M_{c}$ and $m_{c}$. Inside each square, the four rows from top to bottom are $\Sigma_{cl} = 0.1, 0.32, 1, \text{ and } 3.2$ g cm$^{-2}$. From left to right, each column gives the inclination angle, $\theta_{view}$, from an edge-on view to a face-on view, respectively. (b) Similar to Panel (a), but each square shows the [40 $\mu$m]–[20 $\mu$m] color averaged over $\Sigma_{cl}$ and $\theta_{view}$ at each $M_{c}$ and $m_{c}$. (c) Similar to Panel (a), but calculated from SEDs with a foreground extinction of $A_{v} = 100$. The dashed lines are where $m_{s}/M_{c} = 0.01, 0.03, 0.1, \text{ and } 0.3$. The solid lines are where $m_{s}/M_{env} = 0.01, 0.1, \text{ and } 1$. Here, $M_{env}$ is averaged over $\Sigma_{cl}$ for each $M_{c}$ and $m_{c}$. (d) Similar to Panel (c), but calculated from SEDs with a foreground extinction of $A_{v} = 100$. The dashed lines are where $m_{s}/M_{c} = 0.01, 0.03, 0.1, \text{ and } 0.3$. The solid lines are where $m_{s}/M_{env} = 0.01, 0.1, \text{ and } 1$. Here, $M_{env}$ is averaged over $\Sigma_{cl}$ for each $M_{c}$ and $m_{c}$. 

---

The Astrophysical Journal, 853:18 (24pp), 2018 January 20

Zhang & Tan
developed. The dependence of the envelope temperature on the mass surface density of the environment suggests that—even for protostellar cores with the same mass, at the same evolutionary stages—the chemistry can be significantly affected by the star-forming environment, which is true not only for massive protostars, but also for the low-mass protostars forming alongside them (Zhang & Tan 2015).

For prestellar core or early-stage protostellar cores, graybody fitting is often performed to estimate the temperature of the cores (e.g., Elia et al. 2017). Figure 14 shows how the mean mass-weighted temperature in the envelope differs from the graybody fitted temperature in our model grid. Here the graybody temperature is obtained by fitting the model fluxes at 100, 160, 250, 350, 500, and 850 μm following the graybody spectrum \( F_\nu \propto \nu^\beta B_\nu (T) \), where the dust emissivity index \( \beta \) is set to be 2, and \( B_\nu \) is the Planck function. The fitting gives higher \( T_{\text{graybody}} \) for later-stage protostellar sources, which are not shown in this figure. For models with the same \( M_*, \Sigma_{\text{cl}} \), and \( m_\nu \), only the one with an inclination of 60° is shown. The circles, triangles, squares, upside-down triangles are models with \( \Sigma_{\text{cl}} = 0.1, 0.32, 1, \) and 3.2 g cm\(^{-2} \), respectively.

![Figure 11](Figures/figure-11.png)

**Figure 11.** Correlations between different SED characteristics in the model grid. (a) The \([160 \mu m]−[500 \mu m]\) color and the \([160 \mu m]−[40 \mu m]\) color. (b) The \([160 \mu m]−[500 \mu m]\) color and the \([40 \mu m]−[20 \mu m]\) color. (c) The \([160 \mu m]−[40 \mu m]\) color and the SED peak wavelength. (d) The \([160 \mu m]−[500 \mu m]\) color and the mass-weighted mean temperature in the whole envelope. The colors of the points show the evolutionary stages indicated by the parameter \( m_\nu/M_* \), from blue for early stage sources to red for later stage sources. For models with the same \( M_*, \Sigma_{\text{cl}} \), and \( m_\nu \), only the one with an inclination of 60° is shown. The circles, triangles, squares, upside-down triangles are models with \( \Sigma_{\text{cl}} = 0.1, 0.32, 1, \) and 3.2 g cm\(^{-2} \), respectively.

For models with \( \Sigma_{\text{cl}} = 3.2 \) g cm\(^{-2} \), the graybody temperature follows \( T_{\text{env}} \) better in the early stages. We note that our model grid only covers the protostellar phase of massive star formation when a protostar \( \gtrsim M_\odot \) has formed, while the graybody fit works best for the prestellar or early protostellar phases.

Panel (d) of Figure 11 shows how the mean temperature in the envelope correlates with the \([160 \mu m]−[500 \mu m]\) color. These two properties are well-correlated, especially for models with \( \Sigma_{\text{cl}} = 0.1−3 \) g cm\(^{-2} \) and at evolutionary stages with \( m_\nu/M_* < 0.1 \). This correlation, taking into account the correlation between \( T_{\text{env}} \) and \( T_{\text{graybody}} \), is consistent with that found by Elia et al. (2017) from a large sample of cores/clumps observed in the Hi-GAL survey. Their results extend this correlation further to even earlier stages with \( T < 10 \) K, which is not covered by our model grid.

### 2.4. Caveats of the Model Grid

Here, we describe several important caveats and limitations of our SED model grid. First, the model grid is based on the turbulent core model of McKee & Tan (2003), in which massive cores are embedded in a larger ambient clump that will form the star cluster. The cores are assumed to be in pressure
including contributions from parts of the outflow that extend beyond the core. However, observations of real sources at different wavelengths by different instruments may be on different scales, which must also be considered when performing the SED fitting.

Third, the observed short-wavelength fluxes at ≤8 μm are affected by PAH emission and thermal emission from very small grains that are transiently heated by single photons, and these effects have not been included in our radiative transfer models. Therefore, we expect that the models are under-predicting the real fluxes at these wavelengths. In the example we show in Section 4, we set the observed fluxes at ≤8 μm to be upper limits. However, users can freely adjust which data points should be used as limits, according to their situation and needs, and the fitting program provides high flexibility for setting the upper/lower limits and uncertainties (see Section 3).

Finally, some detailed features of the SEDs, such as the peak wavelength and long-wavelength spectral index, may be affected by the particular dust models used in the radiative transfer simulations. Although the general trends of these features with the initial/environmental conditions (\(M_\ast, \Sigma_{cl}\)) and evolution (\(m_\ast\)) discussed above should not change, the exact values may be affected.
3. SED Fitting

We use $\chi^2$ minimization to find the best model to fit the observed SED. Assuming that we have observed flux densities, $F_{\nu,\text{obs}}$, with upper and lower uncertainties of $\sigma_l(F_{\nu,\text{obs}})$ and $\sigma_u(F_{\nu,\text{obs}})$ at wavelengths $\lambda_1$, ..., $\lambda_N$, and for each model (i.e., each set of $M_*, \Sigma_{\text{cl}}, m_*$, $\theta_{\text{view}}$, $d, A_V$) we have model flux densities $F_{\nu,\text{mod,ext}}$ (see Equation (3)) at these wavelengths, the reduced $\chi^2$ is defined as

$$\chi^2 = \frac{1}{N_{\text{total}}} \left\{ \sum_{F_{\nu,\text{mod,ext}} > F_{\nu,\text{fit}}} \left[ \frac{\log F_{\nu,\text{mod,ext}} - \log F_{\nu,\text{fit}}}{\sigma_l(F_{\nu,\text{fit}})} \right]^2 + \sum_{F_{\nu,\text{mod,ext}} < F_{\nu,\text{fit}}} \left[ \frac{\log F_{\nu,\text{mod,ext}} - \log F_{\nu,\text{fit}}}{\sigma_u(F_{\nu,\text{fit}})} \right]^2 \right\},$$

where $F_{\nu,\text{fit}}, \sigma_l(F_{\nu,\text{fit}})$ and $\sigma_u(F_{\nu,\text{fit}})$ are derived from $F_{\nu,\text{obs}}, \sigma_l(F_{\nu,\text{obs}})$, and $\sigma_u(F_{\nu,\text{obs}})$ (see Section 3.1). For $F_{\nu,\text{obs}}$ used as upper limits, $\sigma = \infty$, i.e., no contribution to the $\chi^2$ if $F_{\nu,\text{mod,ext}} < F_{\nu,\text{fit}}$, and for $F_{\nu,\text{obs}}$ used as lower limits, $\sigma = \infty$, i.e., no contribution to the $\chi^2$ if $F_{\nu,\text{mod,ext}} > F_{\nu,\text{fit}}$. The total number of data points $N_{\text{total}}$ contains both normal data points and upper/lower limits.

During each fitting, we first search for a minimum $\chi^2$ by varying the foreground extinction $A_V$ for each set of $(M_*, \Sigma_{\text{cl}}, m_*, \theta_{\text{view}}, d)$. We then compare these minimum $\chi^2$ values to find the best models in the five-dimensional parameter space formed by different $(M_*, \Sigma_{\text{cl}}, m_*, \theta_{\text{view}}, d)$ (four-dimensional, if an exact source distance $d$ is provided). We also select another group of best models. For each set of $(M_*, \Sigma_{\text{cl}}, m_*)$, we search for the minimum $\chi^2$ by varying $A_V$, $d$, and $\theta_{\text{view}}$, and then compare these minimum $\chi^2$ values to find the best models in the three-dimensional parameter space formed by different $(M_*, \Sigma_{\text{cl}}, m_*)$. Therefore, each member in this group of best models is a different physical model. In the released code, both groups are output and users can choose which to use according to their need.

While the $\chi^2$ defined in Equation (4) is used in the ranking and selection of the best-fitted models, we also define

$$\chi^2_{\text{nonlimit}} = \chi^2 \frac{N_{\text{total}}}{N_{\text{nonlimit}}},$$

where $N_{\text{nonlimit}}$ is the number of data points that have non-zero contributions to $\chi^2$. Note that, for the same observed SED, $N_{\text{nonlimit}}$ is dependent on the model SEDs. For example, for a data point used as an upper limit, if the model SED is higher than that data point, it is counted in $N_{\text{nonlimit}}$, while if the model SED is lower than that data point, it is not counted in $N_{\text{nonlimit}}$ because it is not contributing to $\chi^2$. Therefore, $\chi^2_{\text{nonlimit}}$ is the average deviation of the model SED from the constraining data points.

The SED fitting tool and model grid are available for download.\(^3\) Currently, the fitting tool is written in IDL—therefore, IDL software needs to be installed before running the fitting program. However, knowledge of the IDL language is not necessary. An instruction file including detailed information about the structure of the package, its installation, editing input parameters, running the fitting program, and output files and figures is included in the package. In Section 4, we will show an example of the SED fitting (the same example is also included in the package) and discuss the output results and figures. Currently, the fitting program is designed to fit SEDs of individual sources. However, it can be easily adapted to perform recursive fitting to a sample of SEDs. It takes about one minute to fit an SED by running the program on the processor of a typical current laptop or desktop computer.

3.1. Treatment of the Errors

The fitting is performed in logarithmic space because the fluxes are nonlinear with wavelength and most of the errors are best described as certain percentages of the fluxes. Assuming the observed flux and its error at a certain wavelength are $F_{\nu,\text{obs}}$ and $\sigma(F_{\nu,\text{obs}})$, i.e., $F_{\nu,\text{obs}} \equiv F_{\nu}^\text{obs}$ and $\sigma(F_{\nu,\text{obs}}) \equiv \sigma$, the expectation and variance of the fluxes in logarithmic space are related to these values by

$$\log F_{\nu,\text{fit}} \equiv \log \left\langle F_{\nu} \right\rangle,$$

$$= \log \left\langle F_{\nu} \right\rangle - \frac{1}{2 \ln 10} \left[ \frac{\operatorname{var}(F_{\nu})}{\left\langle F_{\nu} \right\rangle} \right]^2 + ...$$

$$= \log F_{\nu,\text{obs}} - \frac{1}{2 \ln 10} \left[ \frac{\sigma(F_{\nu,\text{obs}})}{F_{\nu,\text{obs}}} \right]^2 + ... \ , \quad (6)$$

and

$$\sigma_l(\log F_{\nu,\text{fit}}) = \sigma_l(\log F_{\nu,\text{fit}}) \equiv \sigma(F_{\nu})$$

$$= \frac{1}{\ln 10} \left[ \frac{\operatorname{var}(F_{\nu})}{\left\langle F_{\nu} \right\rangle} \right] + ...$$

$$= \frac{1}{\ln 10} \left[ \frac{\sigma(F_{\nu,\text{obs}})}{F_{\nu,\text{obs}}} \right] + ... . \quad (7)$$

This is only valid when the percentage errors are small, because it is a first-order approximation. In real observations, due to the

---

\(^3\) See https://doi.org/10.5281/zenodo.1134877.
uncertainties in brightness calibration, background subtraction, and selection of apertures to integrate the emission, the percentage uncertainties in the observed flux can easily be several \( \times 10\% \) or higher, in which case the error becomes asymmetric around the observed flux in the logarithmic space. Therefore we define the following fluxes and errors in logarithmic space:

\[
\log F_{v,\text{fit}} \equiv \log F_{v,\text{obs}},
\]

\[
\sigma_v(\log F_{v,\text{fit}}) \equiv \log \left[ 1 + \frac{\sigma_v(F_{v,\text{obs}})}{F_{v,\text{obs}}} \right],
\]

\[
\sigma_l(\log F_{v,\text{fit}}) \equiv -\log \left[ 1 - \frac{\sigma_l(F_{v,\text{obs}})}{F_{v,\text{obs}}} \right],
\]

which are simply obtained by converting the data points and error bars to logarithmic space. The log space flux densities and errors in these two methods start to differ significantly when the percentage error becomes \( \gtrsim 50\% \). We allow both types of conversion in the fitting program. If the first method is used, the input upper and lower errors need to be same, while if the second method is used, the users can input different upper and lower errors for each flux. We use the second method in the example discussed in Section 4.

In the second method, the data points with lower errors larger than 100% have infinite log space \( \sigma_l \), and therefore act as upper limits without constraints on the models below the observed fluxes. Therefore, we provide a third option so that certain constraints can still be applied to the models even in such a situation. If at some wavelength \( \sigma_l(F_{v,\text{obs}}) \gtrsim 100\% \), we set \( \sigma_l(\log F_{v,\text{fit}}) = \sigma_l \equiv 2 \). However, unlike the normal data points, the contribution of this data point to the total \( \chi^2 \) is set to be \( \text{arcsinh}(x^2) = \ln(x^2 + \sqrt{x^4 + 1}) \), where \( x = (\log F_{v,\text{mod,ext}} - \log F_{v,\text{fit}}) / \sigma_l \), instead of \( x^2 \). The reason to choose such a function is that \( \text{arcsinh}(x^2) \approx x^2 \) when \( x \) is small and \( \approx 2x^2 \) when \( x \) is large, so that, unlike a data point with \( \sigma_l = \infty \) that has no constraint on the model SED, it still tends to select the models with fluxes closer to \( F_{v,\text{fit}} \), but the constraint is not as strong as a normal data point with \( \sigma_l = \sigma_l \), because the contribution to the \( \chi^2 \) increases with \( x \) in a logarithmic form. This method thus tends to select models that are themselves upper limits of the range of possibilities.

To sum up, when the errors of the observed fluxes are small and symmetric, or the measurements and errors are statistically well-defined, the first option may be used. Otherwise the second option should be used. The third option provides a special treatment for the data points with \( >100\% \) lower errors. In the fitting program, different options can be assigned to different data points.

Here, we emphasize that the \( \chi^2 \) value we define is not statistically meaningful, especially given the assumption of normally distributed errors, because it is then not linked to a well-defined probability in the more general case. Often, we expect errors to be dominated by uncertain systematic effects, such as clump background subtraction. Thus, the method presented here only provides a way to compare different models in the model grid to select the ones that are close to the given observed data. That is to say, our focus is on comparing the \( \chi^2 \) values of models in one SED fitting, not on the absolute \( \chi^2 \) values, nor on comparing the \( \chi^2 \) values of fittings to different observations.

4. G35.20-0.74 as an Example of SED Fitting

4.1. Model Parameters and Degeneracies

We use the SED of the massive protostar G35.20-0.74 as an example to demonstrate the SED fitting program with our model grid. The SED is constructed based on the SOFIA FORCAST observations from 20 to 40 \( \mu m \), along with fluxes measured at other wavelengths from 3.6 to 500 \( \mu m \) using archived data of other instruments, including Spitzer-IRAC, Herschel-PACS/SPIRE, and other ground-based infrared instruments. The SOFIA observation of this source is part of the SOFIA Massive Star Formation (SOMA) survey (De Buizer et al. 2017). The SOFIA continuum images and SEDs of this source were first presented and analyzed by Zhang et al. (2013a), with the model grid (which was still under development at the time) and a limited, ad hoc exploration of parameter space. Later, the method used to derive the SED from the continuum imaging was improved by De Buizer et al. (2017). In that paper, the current model grid was used to fit the SED of G35.20-0.74 and seven other massive protostars, but without detailed discussion about the fitting process and the full results. Here, we focus on using this SED as an example to demonstrate the SED fitting program. The values of the fluxes and their errors are listed in De Buizer et al. (2017). For details about the observations and derivation of the SED, please also refer to De Buizer et al. (2017). Also, following De Buizer et al. (2017), in the example here, we set the short-wavelength fluxes at \( \lesssim 8 \mu m \) to be upper limits, as discussed in Section 2.4.

As described in the previous section, the fitting program produces two groups of best models. The first group contains models that are different in the five-dimensional parameter space comprised of \( M_*, \Sigma_{\text{cl}}, m_*, \theta_{\text{view}}, \) and \( d \) (or four-dimensional parameter space, if the exact value of \( d \) is provided). The second group further selects the best models that are different in the three-dimensional space comprised of the three primary parameters \( M_*, \Sigma_{\text{cl}}, m_* \). Therefore, the best models in the first group may share the same physical model but differ only because of different distances or viewing inclinations, while the models in the second group are actually different in their initial conditions or evolutionary stages, and have different physical structure and properties. Figure 15 shows all the model SEDs with \( \chi^2 < 50 \) in the two groups of results. Here, the distance to the source is set to be a fixed value of 2.2 kpc (Zhang et al. 2009; Wu et al. 2014). In this case, the best-fit model has \( \chi^2_{\text{min}} = 2.64 \). In the first group of results (upper panel), there are 171 model SEDs with \( \chi^2 - \chi^2_{\text{min}} < 3 \), 670 model SEDs with \( \chi^2 - \chi^2_{\text{min}} < 10 \), and 2441 model SEDs with \( \chi^2 < 50 \), among the total 8640 model SEDs. In the second group of results (lower panel), there are 27 models with \( \chi^2 - \chi^2_{\text{min}} < 3 \), 80 models with \( \chi^2 - \chi^2_{\text{min}} < 10 \), and 178 models with \( \chi^2 < 50 \), among the total 432 model SEDs. For the relatively well-fitted models with \( \chi^2 - \chi^2_{\text{min}} < 3 \), on average, a change of 0.3 in \( \cos \theta_{\text{view}} \) will not affect the rank of the models determined by the primary parameters of \( M_*, \Sigma_{\text{cl}} \) and \( m_* \). We can consider \( \pm 0.15 \) in the \( \cos \theta_{\text{view}} \) space to be the fitting uncertainty of the inclination in this case. However, as we will show below, this is actually dependent on the exact model.

As discussed in the previous section, in our fitting, the upper limits work as normal data points and contribute to \( \chi^2 \) if the model SED is higher than the upper limits. Therefore, it is possible that the best models are above one or more upper limits, which is the case shown here. However, users can assign
smaller errors to the upper limits in order to make them stronger constraints. In fact, very small errors for the upper limits practically exclude models with fluxes above the upper limits. It is also worth noting that, in this example, the group of best models also appear to underpredict the fluxes at longer wavelengths \( \gtrsim 160 \mu m \). Among the seven sources whose SEDs were fitted with our model grid by De Buizer et al. (2017) (eight sources were fitted in this paper, but one of them is without fluxes at wavelengths \( > 100 \mu m \); see also Section 4.3), two sources (including G35.2-0.74) show such deviation at the long wavelengths. Because the slopes at wavelengths \( > 200 \mu m \) of the model SEDs are similar to the observed slopes, the adopted dust models are not likely to be the reason for such deviation. It is more likely caused by the inclusion of additional clump material at these long wavelengths when deriving the SEDs from observations.

Figure 16 shows how the models deviate from the observed fluxes and colors (i.e., SED slopes) at different wavelengths. Only the models in the second group of results are shown. The best five models are also marked. Most of the models with \( \chi^2 - \chi^2_{\text{min}} < 3 (\chi^2_{\text{min}} = 2.64) \), which corresponds to an average deviation of \( < 2.4\sigma \) at each data point, are indeed located within the range of \( \lesssim 2\sigma \) at the wavelengths shown here, suggesting the data points of different wavelengths are evenly contributing to the \( \chi^2 \), and the fitting is constrained over the whole range of wavelengths. The fitting is constrained especially well in the wavelength range of 20–40 \( \mu m \), in terms of the absolute fluxes and the SED slope. The best models are underpredicting the 160 \( \mu m \) flux at levels of about 3\( \sigma \), and also underpredicting the 500 \( \mu m \) flux at levels of about 2\( \sigma \). In addition, most of the models are predicting a slightly steeper slope between 160 and 500 \( \mu m \) at levels within 1\( \sigma \). At short wavelengths, although the data points at 8 \( \mu m \) are set to be upper limits, most of the best-fitted models are still close to the data points.

Table 1 lists the parameters of the five best models in the second group of results, which are different in the primary parameter space of \( (M_*, \Sigma_{\text{cl}}, m_*) \). The additional independent parameters \( (\theta_{\text{view}}, d, \text{and } \alpha_V) \) to achieve these minimum \( \chi^2 \) are also listed. We also list several important parameters that are derived from the primary parameters based on the physical model, including: the core radius, \( R_c \), which only depends on the initial conditions, \( M_* \) and \( \Sigma_{\text{cl}} \), in the model, and is constant over time; the opening angle of the outflow cavity, \( \theta_{\text{out}} \); the current envelope mass, \( M_{\text{env}} \); the accretion rate from disk to protostar, \( \dot{m}_* \); and the total luminosity, \( L_{\text{tot}} \). Note that \( L_{\text{tot}} \) is different from the value directly integrated from the SED, due to the effect of inclination (i.e., the flashlight effect) and foreground extinction. In addition to the \( \chi^2 \) used to rank the models, \( \chi^2_{\text{nonlim}} \) is also listed, showing the average deviation between the model SEDs and the observed data points. Note that these results are slightly different (there is a slight change of ordering of the best models) from those shown by De Buizer et al. (2017). This is because we have improved the quality of the model SEDs here by using larger number of photon packets in the Monte-Carlo RT simulations and reducing the Monte-Carlo noise levels of the model SEDs. The improvement of the model SED qualities slightly affects the fitting, especially at short wavelengths, for more embedded or more edge-on sources. Among the best five models, the initial core mass \( M_* \) is constrained to be \( \gtrsim 100 M_\odot \) (most likely \( \gtrsim 200 M_\odot \)) and the protostellar mass is constrained to be 10–20 \( M_\odot \). The half-opening angle of the outflow cavity is constrained to be 15–30\( ^\circ \), the accretion rate is constrained to be between \( 10^{-4} \) and \( 10^{-3} M_\odot \text{ yr}^{-1} \) (most likely \( 1–2 \times 10^{-4} M_\odot \text{ yr}^{-1} \)), and the total luminosity is about \( (4–8) \times 10^4 L_\odot \). On the other hand, the clump environment mass surface density \( \Sigma_{\text{cl}} \) and the inclination angle are not well-constrained.

For these five best models, Figure 17 shows how the inclination would change the fitting when other parameters are kept the same. Note that a different foreground extinction, \( \alpha_V \), is adopted to minimize the \( \chi^2 \) for each inclination angle. For three of the five models (Models 1, 4, and 5 shown in the first, fourth, and fifth rows; see also Table 1), the fitting results are not as sensitive to inclination, except close to \( \cos \theta_{\text{view}} = 1 \), i.e., a face-on view. In this case, the inclination is not well-constrained. On the other hand, the fitting results in the other two models (Models 2 and 3) are highly sensitive to the inclination angle. The reason for this difference is that, in the former case, the unextinct SEDs of different inclinations are all above the observed data points at wavelengths \( \lesssim 70 \mu m \); by adjusting foreground extinction, \( \alpha_V \), the model SEDs of different inclinations except close to a face-on view can all have relatively good fits to the observations. In the latter case, the unextinct model SEDs of higher inclinations are below the observed fluxes and the fitting cannot be improved by adjusting \( \alpha_V \); therefore, relatively good fitting can be achieved only at a narrow range of inclinations. This suggests a certain degeneracy between the inclination \( \theta_{\text{view}} \) and foreground extinction \( \alpha_V \).

Figure 18 shows the distribution of the models in the primary parameter space comprised of \( M_*, \Sigma_{\text{cl}}, \) and \( m_* \). The first row shows the distribution of \( \chi^2 \) of the best models in the \( M_*–\Sigma_{\text{cl}} \) space in the first panel, and the distributions of \( m_* \), \( \theta_{\text{view}} \), \( \alpha_V \) to achieve these best models in the second to fourth panels.
Similarly, the second row shows the distribution of $c^2$ of the best models in the $M_*, \Sigma_{cl}$ space and the corresponding distributions of $S_{cl}$, $q_{\text{vie}}$, and $A_V$, and the third row shows the equivalent distributions in the $*_{\text{m}}-\Sigma_{cl}$ space. In the $*_{\text{m}}-\Sigma_{cl}$ space, the models with $c^2 < 52$ (inside the red contour) occupy a region with high $M_*$ but spanning the full range of $\Sigma_{cl}$ (first row, first panel). For these models, the initial core mass $M_*$ appears to be higher in a lower surface density environment. The range of $M_*$ of the best models ($\chi^2 - \chi^2_{\text{min}} < 5$) gradually increases from 60–200 $M_\odot$ at $\Sigma_{cl} = 3.2$ g cm$^{-2}$ to 240–480 $M_\odot$ at $\Sigma_{cl} = 0.1$ g cm$^{-2}$. The models are more constrained in $m_*$ (e.g., second row, first panel). All of the best models ($\chi^2 - \chi^2_{\text{min}} < 5$) have $m_*$ higher than about 10 $M_\odot$ and mostly have $m_* \leq 24 M_\odot$, but extend to higher $m_*$ for the models with $M_*$ in the range of 160–240 $M_\odot$. The fitted inclinations are mostly around $\cos \theta_{\text{view}} \approx 0.8$, i.e., $\theta_{\text{view}} \approx 37^\circ$ between the line of sight and the outflow axis, for the best models with $\Sigma_{cl} = 0.3–3$ g cm$^{-2}$ and $M_* = 60–200 M_\odot$ (e.g., first row, third panel). For the best models with low $\Sigma_{cl}$ (0.1 g cm$^{-2}$) and high $M_*$ (>200 $M_\odot$), $\theta_{\text{view}}$ is in the range of about $50^\circ$–$80^\circ$. For most of these best models, a foreground extinction of $A_V < 100$ mag is needed to achieve the minimum $\chi^2$ (e.g., first row, fourth panel). To summarize, in the example shown here, while $M_*$ and $m_*$ are relatively well-constrained, $\Sigma_{cl}$ is not. However, if the inclination can be independently determined from other observations, such as those of outflow kinematics, then this degeneracy can be further broken.

Figure 19 shows the distribution of all the second group models (different in the $M_*, \Sigma_{cl}-m_*$ space) with $\chi^2 < 50$ and various secondary parameters. In the model grid, while all the SEDs are determined by the three primary parameters $M_*$, $\Sigma_{cl}$,
and $m_8$, along with the additional independent parameters $\theta_{\text{view}}$, $d$, and $A_V$, the secondary parameters, which are derived from the primary parameters and describe the properties of the protostar, envelope, outflow cavity, and disk, are of more interest in understanding the physical conditions of the observed massive protostars. Panels (a) and (b) show the distribution of the models in the $M_{\text{env}}$–$L_{\text{tot}}$ and $M_{\text{env}}$–$m_8$ diagrams. As discussed in previous sections, the location in the $M_{\text{env}}$–$L_{\text{tot}}$ diagram and the parameter $m_8/M_{\text{env}}$ are often used as indicators of the evolutionary stage. Most of the models with $\chi^2 < 50$ are located between $m_8/M_{\text{env}} = 0.1$ and $1$, which corresponds to the turning points of the evolutionary tracks in the $M_{\text{env}}$–$L_{\text{tot}}$ diagram (see Figure 4), and the best models that have $\chi^2 - \chi_{\text{min}}^2 < 3$ (red and orange colors) are located around $m_8/M_{\text{env}} = 0.1$. This suggests that this source is still highly embedded and in the main accretion phase.

Panels (c), (d), and (e) show the relation of the total luminosity $L_{\text{tot}}$, accretion rate $m_8$, and age since the start of star formation, with the protostellar mass $m_8$ in the fitted models. These relations are determined by the evolutionary models (see Figure 2). The total luminosity is highly dependent on the protostellar mass, but not so much on the other primary parameters. Because $m_8$ is relatively well-constrained, as discussed above, the total luminosity is also constrained well. The best models with $\chi^2 - \chi_{\text{min}}^2 < 3$ have a luminosity ranging from several $\times 10^4$ to $\lesssim 10^5 L_{\odot}$. The accretion rates of the models with $\chi^2 - \chi_{\text{min}}^2 < 3$ are constrained to be $10^{-4}$–$10^{-3} M_\odot$ yr$^{-1}$, and the protostellar age from several $\times 10^4$ to several $\times 10^5$ year. They are affected by the environmental mass surface density $\Sigma_{\text{cl}}$, which is less well-constrained in this case.

Panel (f) shows the relation between the inclination and the opening angle of the outflow cavity of the fitted models. All the models have an inclination angle larger than the outflow cavity opening angle, i.e., the line of sight toward the protostar goes through the envelope. As discussed above, the inclination angle is not well-constrained in this case, but interestingly, for the models with $\chi^2 - \chi_{\text{min}}^2 < 3$, the opening angles are either close to the inclination angle up to about $40^\circ$ or around $20^\circ$, despite the wide range of inclinations. All five of the best models have opening angles around $20^\circ$.

Panel (g) shows the distribution of the fitted models in the space of the two additional independent parameters $\theta_{\text{view}}$ and $A_V$. The models with $\chi^2 - \chi_{\text{min}}^2 < 3$ have $A_V \lesssim 150$ mag, with most of them within 100 mag. Panel (h) further compares the fitted $A_V$ to the values that correspond to the mean mass surface densities of the ambient clumps $\Sigma_{\text{cl}}$. On average, the contribution of the ambient clump to the foreground extinction to the core should correspond to $\Sigma_{\text{cl}}/2$, which we define as $A_V,\Sigma_{\text{cl}}/2$. However, the foreground extinction in the real situation will differ from $A_V,\Sigma_{\text{cl}}/2$, due to clumpy and/or anisotropic structures in the ambient clump, as well as additional foreground extinction that is not related to the host star-forming clouds. For the best-fitted models that have $\chi^2 - \chi_{\text{min}}^2 < 3$, with $\Sigma_{\text{cl}} = 0.3$–3 g cm$^{-2}$, the fitted $A_V$ is within $2A_V,\Sigma_{\text{cl}}/2$, but for models with $\Sigma_{\text{cl}} = 0.1$, the fitted $A_V$ can be as high as $3A_V,\Sigma_{\text{cl}}/2$, which indicates that the foreground extinction needed to fit the observed SED may not be only that expected from the ambient clump. Thus, if a constraint is imposed that the foreground extinction should be no more than that expected given the value of $\Sigma_{\text{cl}}$, then some low $\Sigma_{\text{cl}}$ models would be excluded in this case.

Panels (i) and (j) compare the fitted total luminosities, $L_{\text{tot}}$, with $L_{\text{inc}, A_V}$, the bolometric luminosities directly integrated from the observed SED (see also Figure 12). The latter luminosity is also slightly dependent on the model fitting because of the uncertainties of the SED at the wavelength ranges not covered by the observation. For the models with $\chi^2 - \chi_{\text{min}}^2 < 3$, $L_{\text{inc}, A_V}/L_{\text{tot}}$ is in the range of 0.2–0.8. This is caused by a combined effect of the inclination (flashlight effect) and the foreground extinction (see Section 2.3.3). According to Figure 12, such a ratio between $L_{\text{inc}, A_V}$ and $L_{\text{tot}}$ is consistent with an outflow opening angle of about $20^\circ$–$30^\circ$.

In the end, we note that the above discussion is based on a specific example and may not be general (see Section 4.3). However, during each fitting, in addition to giving just a few best models, the program will generate similar figures to help the users to better understand the results. The above discussion serves as an example of what information we can expect from these figures.

### 4.2. Discussion of the Source

G35.20-0.74 is a massive protostar in a broader region of star formation located at a distance of 2.2 kpc (Zhang et al. 2009; Wu et al. 2014). CO(1-0) and (2-1) observations have revealed a wide outflow structure in the northeast–southwest direction, extending to $>1$ from the central source (Gibb et al. 2003; Birks et al. 2006). Perpendicular to this, CS(2-1) observations have revealed a ridge-like structure (Dent et al. 1985a) that has been further resolved into a 15$^\prime$ long filament with a string of cores embedded by ALMA (Sánchez-Monge et al. 2013, 2014).
and SMA (Qiu et al. 2013) in the sub-mm continuum. Heaton & Little (1988) observed this source in the centimeter radio continuum and were able to resolve three compact sources arranged north–south. They concluded that the central source was likely an UCHII region, while the north and south sources had spectral indices consistent with free–free emission from a collimated, ionized, bipolar jet. Since then, it has been debated whether the NE–SW CO outflow is caused by the ionized jet undergoing precession or if they are composed of separate outflows driven by different sources. This elongated radio continuum emission was further resolved into 17 individual knots lying along the N–S direction by the VLA (Gibb et al. 2003; Beltrán et al. 2016), with the driving source identified as a UC/HCHII region. This radio source is coincident with one of the embedded cores identified in sub-mm observations (Core B identified by Sánchez-Monge et al. 2013, 2014; MM1b identified by Qiu et al. 2013). The N–S outflow is also seen in NIR and MIR observations (Dent et al. 1985b; De Buizer 2006). At these wavelengths, the emission is elongated in the N–S direction but peaked to the north of the identified radio source and continuum core. It has been argued that the outflow/jet is blueshifted to the north and the emissions at NIR and MIR are dominated by the northern near-facing outflow cavity. However, at longer wavelengths of 30–40 μm, the SOFIA-FORCAST observations have revealed a southern, far-facing outflow cavity (Zhang et al. 2013a).

In our previous fitting of the SED of this source (Zhang et al. 2013a) with an earlier version of our radiative transfer...
models (which had fixed outflow cavity-opening angles) and using a limited, ad hoc exploration of model parameter space, this source was estimated to be a protostar with \( m_\star \approx 20-34 M_\odot \), accreting at rates of \( \dot{m}_\star \approx 10^{-4} M_\odot \text{yr}^{-1} \). The total luminosity was estimated to be \((7-22) \times 10^4 L_\odot \), and the opening angle of the outflow cavity was estimated to be 35°–50°. Compared with these earlier results, our completed model grid and SED fitting program presented in this paper have estimated a protostellar mass of \( m_\star = 10-20 M_\odot \) and a total luminosity of \((4-8) \times 10^4 L_\odot \). Sánchez-Monge et al. (2013) has identified a Keplerian disk in Core B with rotation corresponding to a central mass of 18 \( M_\odot \), and they argued that the disk is around a binary based on the total luminosity. Indeed, a binary system of UC/HCHII regions has been observed by Beltrán et al. (2016) at the position of Core B. Our model is based on a single protostar, which underpredicts the
total protostellar mass if the luminosity is from a binary; therefore, our new estimation is quite consistent with the mass estimation from gas kinematics. Compared to the previous estimation, our new estimation of the opening angle of the outflow (~20°) is also more consistent with the MIR observations, which suggests a narrow outflow cavity (e.g., De Buizer 2006).

Our SED fitting also estimates the current envelope mass to be about 100–400 $M_\odot$, and a ratio of protostellar mass to envelope mass of $m_\ast/M_{\text{env}} \approx 0.1$. The total mass of the filament has been estimated to be about 160 $M_\odot$, and the mass of the core that hosts the driving source of the N–S outflow/jet was estimated to be about 18 $M_\odot$ (Qiu et al. 2013). However, these masses are concentrated to fragments with sizes of about 1", and the MIR emission shows that there is a narrow outflow cavity that exists on a scale of about 10", suggesting envelope material extends at least to such a scale. This indicates that the total mass of the gas envelope surrounding this N–S outflow should be higher than 160 $M_\odot$, which is consistent with our estimation of the current envelope mass. However, unlike our model, which has a highly idealized spherical core with smooth density distribution, the observations show that the envelope is actually highly fragmented and may be feeding several protostars and close binary systems. Therefore, if one of the protostars or close binary systems is contributing most of the IR emission (e.g., Core B in this case), our model grid is still able to generate relatively accurate estimates about this main source and overall properties of the large envelope. However, due to the fragmentation, the mass reservoir for each protostar or close binary system is smaller than what the model suggests; therefore, the sources may be at a later evolutionary stage than indicated in the models.

We note also that our previous model fitting of G35.20-0.74 (Zhang et al. 2013a) used not only the SED, but also the MIR to FIR flux intensity profiles along the outflow axis. In a follow-up paper, we plan to extend our presented model grid to include multi-wavelength images, which can be used in such ways to further constrain the protostellar properties.

4.3. Standard Format Output for SOMA Survey Protostars

Finally, to illustrate simplified outputs from the SED fitting tool, in Figure 20 we show a standard format output of three panels, i.e., $\Sigma_{\text{cl}}$ versus $M_\ast$, $m_\ast$ versus $M_\ast$, and $M_\ast$ versus $\Sigma_{\text{cl}}$, which were already presented in Figure 18. These results are shown for G35.20-0.74, as well as for seven other massive protostars from the first SOFIA Massive (SOMA) Star Formation survey data release (De Buizer et al. 2017). We do not discuss these other sources individually in detail here, but these results complement the simpler SED fitting results (i.e., the lists of the top five SED models) for these sources presented by De Buizer et al. 2017 and the discussion of these sources in this paper.

Concerning general trends, we can see from Figure 20 that, among the three primary parameters, protostellar mass $m_\ast$ is the best constrained. Most of the sources have $m_\ast$ around 10–20 $M_\odot$. However, in G45.47+0.05, $m_\ast$ is clearly higher around 30–40 $M_\odot$, and in IRAS 07299, $m_\ast$ is slightly lower (around 8–16 $M_\odot$). In the $M_\ast$–$\Sigma_{\text{cl}}$ space, for all the sources, the best models ($\chi^2 - \chi^2_{\text{min}} < 5$, within the red contours) occupy a region with lower $M_\ast$ at higher $\Sigma_{\text{cl}}$ and higher $M_\ast$ at lower $\Sigma_{\text{cl}}$, similar to what we found in G35.20-0.74 in the previous sections. However, there is a clear difference in the ranges of $M_\ast$ of the best models from source to source, with G45.47-0.05 having the highest $M_\ast$ and AFGL4029 having a relatively low $M_\ast$. As discussed above, $\Sigma_{\text{cl}}$ is the least constrained, with most of the sources having best models spanning over the full range of $\Sigma_{\text{cl}}$ in the model grid. The constraint of $\Sigma_{\text{cl}}$ is slightly better in IRAS 20126 and IRAS 07299 in which the best models concentrate in $\Sigma_{\text{cl}} = 0.1$–0.3 g cm$^{-2}$, and G45.47+0.05, where the best models concentrate in $\Sigma_{\text{cl}} \gtrsim 1$ g cm$^{-2}$.

5. Discussion and Conclusions

We have presented a model grid for fitting the SEDs of massive protostars. The model grid is based on the turbulent core model of massive star formation (McKee & Tan 2002, 2003). The initial conditions of the model grid are pressurized, dense, massive cores embedded in high mass surface density.
“clump” environments. These initial conditions are parameterized by the initial mass of the core, \( M_c \), and the mean mass surface density of the clump, \( S_{cl} \). Using analytical and semi-analytical solutions, we self-consistently calculate the properties and evolutions of the rotating collapsing core, the accretion disk, the protostar, and the disk wind that gradually opens up the outflow cavity, from different sets of initial conditions. The model grid covers a parameter space with \( M_c = 10^{-4} - 480 M_\odot \) and \( S_{cl} = 0.1 - 3 \) g cm\(^{-2} \), which is consistent with the observed environments of massive star formation. By sampling at different protostellar masses, \( m_\ast \), we end up with a total of 432 different physical models in the current model grid. SEDs are generated via Monte-Carlo radiative transfer simulation at 20 inclinations between an edge-on view and a face-on view for each of these models, making a total of 8640 SEDs in the model grid. These model SEDs, which also allow for foreground extinction, are used to fit the observed SED via \( \chi^2 \) minimization.

In such a model grid, the properties and evolutions of the protostar and its surrounding structures are more physically connected, which reduces the dimensionality of the parameter spaces and the total number of models. It also helps to rule out possible fitting results that are physically unrealistic or are not internally self-consistent. Therefore, this model grid serves not only as a fitting tool to estimate properties of massive protostars from observed SEDs, but also as a test of core accretion theory. Its use tells us whether or not the observed SEDs of various massive protostars can be explained by the core accretion theory, with different initial conditions and evolutionary stages.

We studied how the parameters \( M_c, S_{cl}, m_\ast, \) inclination \( \theta_{\text{view}} \), and foreground extinction \( A_V \) affect the various features of the SEDs, especially the peak wavelength, the 20–40 \( \mu \)m slope, the 160–500 \( \mu \)m slope, and the bolometric temperatures. All these features show clear dependencies on the evolutionary stages. Among these features, the peak wavelength of the SED and the 160–500 \( \mu \)m slope are not as sensitive to the inclination or possible foreground extinction, except at an inclination close to face-on, while the 20–40 \( \mu \)m slope or the bolometric temperature are highly sensitive to the inclination. The environmental mass surface density, \( S_{cl} \), also strongly affects the 20–40 \( \mu \)m slope, while the other features are only weakly dependent on \( S_{cl} \). We found that the degree of the flashlight effect (the difference between the inferred luminosity from the SED and the true total luminosity) is dependent almost solely on the viewing inclination and the opening angle of the outflow cavity. With outflow cavities at typical opening angles, the inferred luminosity can be higher or lower than the true total luminosity by a factor of a few from a low inclination to a high inclination. However, with a foreground extinction, the inferred bolometric luminosity almost always underestimates the true luminosity by a factor almost solely dependent on the outflow cavity opening angle.

We used the massive protostar G35.20-0.74 as an example of SED fitting with our model grid (see also De Buizer et al. 2017 for a less detailed application to eight sources, including G35.20-0.74). The fitting program not only provides information of a few best-fitted models, but also shows the distribution of the fitted models in the parameter space to understand constraints and degeneracies. The fitting yields a protostellar mass \( m_\ast \approx 10^{-2} - 20 M_\odot \), a total luminosity of \((4-8) \times 10^4 L_\odot \), an accretion rate of a few \( \times 10^{-4} M_\odot \) yr\(^{-1} \), and a half-opening angle of the outflow cavity of about 20°, all of which are

---

**Figure 20.** Short-format SED model fitting outputs for the eight sources of the SOMA survey (De Buizer et al. 2017). See text for detailed discussion of the properties of these protostars. The panels are similar to those in the first column of Figure 18.
consistent with those estimated from other observations. The fitting also yields an initial core mass of $\geq 100 \, M_\odot$, while $\Sigma_{cl}$ is not well-constrained. There are certain degeneracies caused by combined effects of $\Sigma_{cl}$ and $\theta_{view}$. Further breaking these degeneracies will require additional observational constraints, such as using predictions of image intensity profiles (e.g., Zhang et al. 2013a) or radio continuum emission that traces ionized gas (e.g., Tanaka et al. 2016). Compared with the widely used Robitaille et al. (2006, 2007) model grid (results presented by De Buizer et al. 2017), our model grid yields slightly lower protostellar mass and similar total luminosities, but much higher accretion rates (accretion rates of a few $\times 10^{-7} \, M_\odot \, yr^{-1}$ are estimated using Robitaille et al. 2006, 2007 model). We believe that these differences are due, at least in part, to there being a wider choice of free parameters in the Robitaille et al. (2006, 2007) grid, which can lead to models that we consider less physically realistic, i.e., high mass infall rates in the core envelope but small disk accretion rates. Our model grid, on the other hand, is designed to include the different components more consistently with fewer free parameters, to yield results that are more physically realistic.

Future papers in this series will present the multi-wavelength imaging data, which, as mentioned, may be helpful to break model degeneracies. Extension to lower core masses is also planned (see Zhang & Tan 2015 for some preliminary examples). These physical models, i.e., for the time evolution of density and temperature, are also the necessary boundary conditions for astrochemical computations and eventual line radiative transfer simulations to predict molecular line emission properties of the protostars.

J.C.T. acknowledges NSF grant AST-1411527. We also acknowledge the UF HPC and RIKEN HOKUSAI GreatWave for supporting computational resources.

ORCID iDs
Yichen Zhang © https://orcid.org/0000-0001-7511-0034
Jonathan C. Tan © https://orcid.org/0000-0002-3389-9142

References
Alves, J., Lombardi, M., & Lada, C. J. 2007, A&A, 462, L17
André, P., Men'shchikov, A., Bontemps, S., et al. 2010, A&A, 518, L102
Baldecki, A., Molinari, S., Elia, D., Pezzuto, S., & Schisano, E. 2017, MNRAS, 472, 1778
Bally, J., & Zinnecker, H. 2005, AJ, 129, 2281
Beltrán, M. T., Cesaroni, R., Moscadelli, L., et al. 2016, A&A, 593, A49
Beltrán, M. T., & de Wit, W. J. 2016, A&ARv, 24, 6
Birks, J. R., Fuller, G. A., & Gibb, A. G. 2006, A&A, 458, 181
Blandford, R. D., & Payne, D. G. 1982, MNRAS, 199, 883
Bonnell, I. A., Bate, M. R., Clarke, C. J., & Pringle, J. E. 2001, MNRAS, 323, 785
Bonelll, I. A., Bate, M. R., & Zinnecker, H. 1998, MNRAS, 298, 93
Butler, M. J., & Tan, J. C. 2012, ApJ, 754, 5
Butler, M. J., Tan, J. C., & Kainulainen, J. 2014, ApJL, 782, L30
Chen, H., Myers, P. C., Ladd, E. F., & Wood, D. O. S. 1995, ApJ, 445, 377
De Buizer, J. M. 2006, ApJL, 642, L57
De Buizer, J. M., Liu, M., Tan, J. C., et al. 2017, ApJ, 843, 33
Dent, W. R. F., Little, L. T., Kaifu, N., Ohishi, M., & Suzuki, S. 1985a, A&A, 146, 375
Dent, W. R. F., Little, L. T., Sato, S., Ohishi, M., & Yamashita, T. 1985b, MNRAS, 217, 217
Elia, D., Molinari, S., Schisano, E., et al. 2017, MNRAS, 471, 100
Elia, D., Schisano, E., Molinari, S., et al. 2010, A&A, 518, L97
Gibb, A. G., Hoare, M. G., Little, L. T., & Wright, M. C. H. 2003, MNRAS, 339, 1011
Goodman, A. A., Benson, P. J., Fuller, G. A., & Myers, P. C. 1993, ApJ, 406, 528
Heaton, B. D., & Little, L. T. 1988, A&A, 195, 193
Hosokawa, T., & Omukai, K. 2009, ApJ, 691, 823
Hosokawa, T., Yorke, H. W., & Omukai, K. 2010, ApJ, 721, 478
König, C., Urquhart, J. S., Csengeri, T., et al. 2017, A&A, 599, 139
Königl, A., & Pudritz, R. E. 2000, in Protostars and Planets IV, ed. V. Mannings (Tucson, AZ: Univ. Arizona Press), 759
Kratter, K. M., Matthews, C. D., &Krumbholz, M. R. 2008, ApJ, 681, 375
Li, J., Wang, J., Gu, Q., Zhang, Z.-Y., & Zheng, X. 2012, ApJ, 745, 47
Matzner, C. D., & McKee, C. F. 2000, ApJ, 545, 364
McKee, C. F., & Tan, J. C. 2002, Natur, 416, 59
McKee, C. F., & Tan, J. C. 2003, ApJ, 585, 850
McLaughlin, D. E., & Pudritz, R. E. 1996, ApJ, 469, 194
McLaughlin, D. E., & Pudritz, R. E. 1997, ApJ, 476, 750
Molinari, S., Pezzuto, S., Cesaroni, R., et al. 2008, A& A, 481, 345
Myers, P. C., & Ladd, E. F. 1993, ApJL, 413, L47
Offner, S. R. S., Clark, P. C., Hennebelle, P., et al. 2014, in Protostars and Planets VI, ed. H. Beuther et al. (Tucson: AZ: Univ. Arizona Press), 53
Palau, A., Fuente, A., Girart, J. M., et al. 2013, ApJ, 762, 120
Qiu, K., Zhang, Q., Menten, K. M., Liu, H. B., & Tang, Y.-W. 2013, ApJ, 779, 182
Robitaille, T. P., Whitney, B. A., Indebetouw, R., & Wood, K. 2007, ApJls, 669, 328
Robitaille, T. P., Whitney, B. A., Indebetouw, R., Wood, K., & Denzmore, P. 2006, ApJS, 167, 256
Sánchez-Monge, Á, Beltrán, M. T., Cesaroni, R., et al. 2014, A&A, 569, A11
Sánchez-Monge, Á, Cesaroni, R., Beltrán, M. T., & et al. 2013, A&A, 552, L10
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Shu, F. H. 1977, ApJ, 214, 488
Tan, J. C., Beltrán, M. T., Caselli, P., et al. 2014, in Protostars and Planets VI, ed. H. Beuther et al. (Tucson: AZ: Univ. Arizona Press), 149
Tanaka, K. E. I., Tan, J. C., & Zhang, Y. 2016, ApJ, 818, 52
Tanaka, K. E. I., Tan, J. C., & Zhang, Y. 2017, ApJ, 835, 32
Ulrich, R. K. 1976, ApJ, 210, 377
Wang, P., Li, Z.-Y., Abol, T., & Nakamura, F. 2010, ApJ, 709, 27
Whitney, B. A., Robinetaille, T. P., Bjorkman, J. E., et al. 2013, ApJL, 768, 30
Whitney, B. A., Wood, K., Bjorkman, J. E., & Wolff, M. J. 2003, ApJ, 591, 1049
Wu, Y. W., Sato, M., Reid, M. J., et al. 2014, A&A, 566, A17
Zhang, B., Zheng, X. W., Reid, M. J., et al. 2009, ApJ, 693, 419
Zhang, Y., & Tan, J. C. 2011, ApJ, 733, 55
Zhang, Y., & Tan, J. C. 2015, ApJL, 802, 15
Zhang, Y., Tan, J. C., De Buizer, J. M., et al. 2013a, ApJ, 767, 58
Zhang, Y., Tan, J. C., & Hosokawa, T. 2014, ApJ, 788, 166
Zhang, Y., Tan, J. C., & McKee, C. F. 2013b, ApJ, 768, 86