Searching for New Physics in $b \to s$ Hadronic Penguin Decays

Luca Silvestrini

Dip. di Fisica, Univ. di Roma “La Sapienza” and INFN, Sez. di Roma
P.le A. Moro, 2, I-00185 Rome, Italy

We review the theoretical status of $b \to s$ hadronic penguin decays in the Standard Model and beyond. We summarize the main theoretical tools to compute Branching Ratios and CP asymmetries for $b \to s$ penguin dominated nonleptonic decays, and discuss the theoretical uncertainties in the prediction of time-dependent CP asymmetries in this processes. We consider general aspects of $b \to s$ transitions beyond the Standard Model. Then we present detailed predictions in supersymmetric models with new sources of flavor and CP violation.

I. INTRODUCTION

New Physics (NP) can be searched for in two ways: either by raising the available energy at colliders to produce new particles and reveal them directly, or by increasing the experimental precision on certain processes involving Standard Model (SM) particles as external states. The latter option, indirect search for NP, should be pursued using processes that are forbidden, very rare or precisely calculable in the SM. In this respect, Flavor Changing Neutral Current (FCNC) and CP-violating processes are among the most powerful probes of NP, since in the SM they cannot arise at the tree-level and even at the loop level they are strongly suppressed by the GIM mechanism. Furthermore, in the quark sector they are all calculable in terms of the CKM matrix, and in particular of the parameters $\rho$ and $\eta$ in the generalized Wolfenstein parametrization [1]. Unfortunately, in many cases a deep understanding of hadronic dynamics is required in order to be able to extract the relevant short-distance information from measured processes. Lattice QCD allows us to compute the necessary hadronic parameters in many processes, for example in $\Delta F = 2$ amplitudes. Indeed, the Unitarity Triangle Analysis (UTA) with Lattice QCD input is extremely successful in determining $\rho$ and $\eta$ and in constraining NP contributions to $\Delta F = 2$ amplitudes [2–6].

Once the CKM matrix is precisely determined by means of the UTA (either within the SM or allowing for generic NP in $\Delta F = 2$ processes), it is possible to search for NP con-
tributions to $\Delta F = 1$ transitions. FCNC and CP-violating hadronic decays are indeed the most sensitive probes of NP contributions to penguin operators. In particular, penguin-dominated nonleptonic $B$ decays can reveal the presence of NP in decay amplitudes \cite{7–9}. The dominance of penguin operators is realized in $b \to s q \bar{q}$ transitions.

Thanks to the efforts of the BaBar and Belle collaborations, $B$-factories have been able to measure CP violation in several $b \to s$ penguin-dominated channels with an impressive accuracy \cite{10–20}. To fully exploit this rich experimental information to test the SM and look for NP, we need to determine the SM predictions for each channel. As we shall see in the following, computing the uncertainty in the SM predictions is an extremely delicate task. Only in very few cases it is possible to control this uncertainty using only experimental data; in general, one has to use some dynamical information, either from flavor symmetries or from factorization. Computing CP violation in $b \to s$ penguins beyond the SM is even harder: additional operators arise, and in many cases the dominant contribution is expected to come from new operators or from operators that are subdominant in the SM. In the near future, say before the start of the LHC, we can aim at establishing possible hints of NP in $b \to s$ penguins. With the advent of the LHC, two scenarios are possible. If new particles are revealed, $b \to s$ penguin decays will help us identify the flavor structure of the underlying NP model. If no new particles are seen, $b \to s$ penguins can either indirectly reveal the presence of NP, if the present hints are confirmed, or allow us to push further the lower bound on the scale of NP. In all cases, experimental and theoretical progress in $b \to s$ hadronic penguins is crucial for our understanding of flavor physics beyond the SM.

This review is organized as follows. In Sec. II we quickly review the basic formalism for $b \to s$ nonleptonic decays, and the different approaches to the calculation of decay amplitudes present in the literature. In Sec. III, we present the predictions for Branching Ratios (BR’s) and CP violation within the SM following the various approaches, and compare them with the experimental data. In Sec. IV, we discuss the possible sources of NP contributions to $b \to s$ penguins and how these NP contributions are constrained by experimental data on other $b \to s$ transitions. In Sec. V, we concentrate on SUSY extensions of the SM, discuss the present constraints and present detailed predictions for CP violation in $b \to s$ penguins. In Sec. VI we briefly discuss $b \to s$ penguins in the context of non-SUSY extensions of the SM. Finally, in Sec. VII we summarize the present status and discuss future prospects.
II. BASIC FORMALISM

A. Generalities

The basic theoretical framework for non-leptonic $B$ decays is based on the Operator Product Expansion (OPE) and renormalization group methods which allow to write the amplitude for a decay of a given meson $B=B_d$, $B_s$, $B^+$ into a final state $F$ generally as follows:

$$
\mathcal{A}(B \to F) = \langle F | \mathcal{H}_{\text{eff}} | B \rangle = \left( \frac{G_F}{\sqrt{2}} \sum_{i=1}^{12} V_i^{\text{CKM}}(\mu) + C_i^{\text{NP}}(\mu) \right) \langle F | Q_i(\mu) | B \rangle \\
+ \sum_{i=1}^{N_{\text{NP}}} \tilde{C}_i^{\text{NP}}(\mu) \langle F | \tilde{Q}_i(\mu) | B \rangle.
$$

Here $\mathcal{H}_{\text{eff}}$ is the effective weak Hamiltonian, with $Q_i$ denoting the relevant local operators which govern the decays in question within the SM, and $\tilde{Q}_i$ denoting the ones possibly arising beyond the SM. The CKM factors $V_{i}^{\text{CKM}}$ and the Wilson coefficients $C_i(\mu)$ describe the strength with which a given operator enters the Hamiltonian; for NP contributions, we denote with $C_i^{\text{NP}}(\mu)$ and $\tilde{C}_i^{\text{NP}}(\mu)$ the Wilson coefficients arising within a given NP model, which can in general be complex. In a more intuitive language, the operators $Q_i(\mu)$ can be regarded as effective vertices and the coefficients $C_i(\mu)$ as the corresponding effective couplings. The latter can be calculated in renormalization-group improved perturbation theory and are known including Next-to-Leading order (NLO) QCD corrections within the SM and in a few SUSY models [21–23]. The scale $\mu$ separates the contributions to $\mathcal{A}(B \to F)$ into short-distance contributions with energy scales higher than $\mu$ contained in $C_i(\mu)$ and long-distance contributions with energy scales lower than $\mu$ contained in the hadronic matrix elements $\langle Q_i(\mu) \rangle$. The scale $\mu$ is usually chosen to be $O(m_b)$ but is otherwise arbitrary.

The effective weak Hamiltonian for non-leptonic $b \to s$ decays within the SM is given by:

$$
\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \left\{ V_{ub} V_{us}^* \left[ C_1(\mu) \left( Q_1^s(\mu) - Q_1^l(\mu) \right) + C_2(\mu) \left( Q_2^s(\mu) - Q_2^l(\mu) \right) \right] \\
- V_{tb} V_{ts}^* \left[ C_1(\mu) Q_1^s(\mu) + C_2(\mu) Q_2^s(\mu) + \sum_{i=3,12} C_i(\mu) Q_i(\mu) \right] \right\},
$$

(2)
decays were written down. For our purpose, we just need to recall a few basic facts about where the RGI’s were identified and the decay amplitudes for several two-body nonleptonic
equations in terms of RGI’s. This exercise was performed in ref. [24],

convenient to identify the basic renormalization group invariant parameters (RGI’s) and to 
coefficients is canceled by the analogous dependence in the matrix elements. It is therefore
on the renormalization scheme for the operators. The scale and scheme dependence of the
non-penguin contractions of the current-current operators.

operators.

Then, we have four parameters containing only penguin contractions of the current-current
operators \(Q_{1,2}\) in the GIM-suppressed combination \(Q_{1,2}^c - Q_{1,2}^u\): \(P_{1,2}^{\text{GIM}}\) and Zweig suppressed \(P_{2-4}^{\text{GIM}}\). Finally, we have four parameters containing penguin contractions of current-current
operators \(Q_{1,2}^c\) (the so-called charming penguins [25]) and all possible contractions of penguin
operators \(Q_{3-12}: P_{1,2}\) and the Zweig-suppressed \(P_{3,4}\).

Let us now discuss some important aspects of \(b \to s\) penguin nonleptonic decays. First
of all, we define as pure penguin channels the ones that are generated only by \(P_i\) and \(P_i^{\text{GIM}}\)
parameters. Pure penguin \(b \to s\) decays can be written schematically as:

\[
A(B \to F) = -V_{ub}^* V_{us} \sum P_i^{\text{GIM}} - V_{tb}^* V_{ts} \sum P_i.
\]

Neglecting doubly Cabibbo suppressed terms, the decay amplitude has vanishing weak phase.
Therefore, there is no direct CP violation and the coefficient \(S_F\) of the \(\sin \Delta m t\) term in the
time-dependent CP asymmetry (for \( F \) a CP eigenstate with eigenvalue \( \eta_F \)) measures the phase of the mixing amplitude: \( S_F = \eta_F \text{Im} \lambda_F = -\eta_F \sin 2\phi_M \), where \( \lambda_F \equiv \frac{q_F}{p_F} \bar{A} = e^{-2i\phi_M} \), \( A = \mathcal{A}(B \to F) \), \( \bar{A} = \mathcal{A}(\bar{B} \to F) \) and \( \phi_M = \beta (\beta_s) \) for \( B_d (B_s) \) mixing. Comparing the measured \( S_F \) to the one obtained from \( b \to c \bar{c}s \) transitions such as \( B_d (s) \to J/\Psi K_s (\phi) \) can reveal the presence of NP in the \( b \to s \) penguin amplitude. However, to perform a precise test of the SM we need to take into account also the doubly Cabibbo suppressed terms in Eq. (4). The second term then acquires a small and calculable weak phase, leading to a small and calculable \( \Delta S = -\eta_F S_F - \sin 2\phi_M \). Furthermore, we must consider the contribution from the first term, \( i.e. \) the contribution of GIM penguins. An estimate of the latter requires some knowledge of penguin-type hadronic matrix elements, which can be obtained either from theory or from experimental data. Let us define this as the “GIM-penguin problem”: we shall come back to it in the next Section after introducing the necessary theoretical ingredients.

Besides pure penguins, we have \( b \to s \) transitions in which emission, annihilation or emission-annihilation parameters give a contribution to the decay amplitude. Let us call these channels penguin-dominated. Then we can write schematically the decay amplitude as:

\[
\mathcal{A}(B \to F) = -V_{ub}^* V_{us} \sum \left( T_i + P_i^{\text{GIM}} \right) - V_{tb}^* V_{ts} \sum P_i , \tag{5}
\]

where \( T_i = \{ E_i, A_i, EA_i \} \). Also in this case, neglecting doubly Cabibbo suppressed terms the decay amplitude has vanishing weak phase, so that \( \Delta S = 0 \) at this order. However, we expect \( T_i > P_j \) so that the double Cabibbo suppression can be overcome by the enhancement in the matrix element, leading to a sizable \( \Delta S \). Once again, the evaluation of \( \Delta S \) requires some knowledge of hadronic dynamics. Let us define this as the “tree problem” and return to it in the next Section.

### B. Evaluation of hadronic matrix elements

The last decade has witnessed remarkable progress in the theory of nonleptonic \( B \) decays. Bjorken’s color transparency argument has been put on firm grounds, and there is now a wide consensus that many \( B \) two-body decay amplitudes factorize in the limit \( m_b \to \infty \) and are therefore computable in this limit in terms of few fundamental nonperturbative quantities. Three different approaches to factorization in \( B \) decays have been put forward: the so-called
QCD factorization [26–30], perturbative QCD (PQCD) [31–33] and Soft-Collinear Effective Theory (SCET) [34–38]. A detailed discussion of these approaches goes beyond the scope of this review; for our purpose, it suffices to quickly describe a few aspects that are relevant for the study of $b \to s$ penguin nonleptonic decays.

Unfortunately, as suggested in ref. [25] and later confirmed in refs. [39–58], it turns out that subleading corrections to the infinite mass limit, being doubly Cabibbo-enhanced in $b \to s$ penguins, are very important (if not dominant) in these channels, so that they reintroduce the strong model dependence that we hoped to eliminate using factorization theorems. While different approaches to factorization point to different sources of large corrections, no approach is able to compute from first principles all the ingredients needed to test the SM in $b \to s$ penguins. Therefore, it is important to pursue, in addition to factorization studies, alternative data-driven approaches that can in some cases lead to model-independent predictions for CP violation in $b \to s$ penguins.

Let us now quickly review the main tools that are available for the study of $b \to s$ penguins.

1. QCD factorization

The first step towards a factorization theorem was given by Bjorken’s color transparency argument [59]. Let us consider a decay of the $B$ meson in two light pseudoscalars, where two light quarks are emitted from the weak interaction vertex as a fast-traveling small-size color-singlet object. In the heavy-quark limit, soft gluons cannot resolve this color dipole and therefore soft gluon exchange between the two light mesons decouples at lowest order in $\Lambda/m_b$ (here and in the following $\Lambda$ denotes a typical hadronic scale of order $\Lambda_{QCD}$).

Assuming that in $B$ decays to two light pseudoscalars perturbative Sudakov suppression is not sufficient to guarantee the dominance of hard spectator interactions, QCD Factorization (QCDF) states that all soft spectator interactions can be absorbed in the heavy-to-light form factor [26]. Considering for example $B \to \pi\pi$ decays, the following factorization formula holds at lowest order in $\Lambda/m_b$:

$$
\langle \pi(p')\pi(q)|Q_i|\bar{B}(p)\rangle = \int_0^{q^2} dx T^I_i(x)\phi(\pi(x)) + 
\int_0^{1} d\xi \int_0^{1} dy T^{II}_i(\xi, x, y)\phi_B(\xi)\phi(\pi(x))\phi(\pi(y)),
$$

(6)
where \( f^{B \to \pi}(q^2) \) is a \( B \to \pi \) form factor, and \( \phi_{\pi} (\phi_B) \) are leading-twist light cone distribution amplitudes of the pion (\( B \) meson). \( T_{I,I}^{I,II} \) denote the hard scattering amplitudes. Notice that \( T^I \) starts at zeroth order in \( \alpha_s \) and at higher order contains hard gluon exchange not involving the spectator, while \( T^{II} \) contains the hard interactions of the spectator and starts at order \( \alpha_s \).

The scheme and scale dependence of the scattering kernels \( T_{i}^{I,II} \) matches the one of Wilson coefficients, and the final result is consistently scale and scheme independent.

Final state interaction phases appear in this formalism as imaginary parts of the scattering kernels (at lowest order in \( \Lambda/m_b \)). These phases appear in the computation of penguin contractions and of hard gluon exchange between the two pions. This means that in the heavy quark limit final state interactions can be determined perturbatively.

A few remarks are important for the discussion of CP violation in \( b \to s \) penguins:

- Penguin contractions (including charming and GIM penguins) are found to be factorizable, at least at one loop.

- Subleading terms in the \( \Lambda/m_b \) expansion are in general non-factorizable, so that they cannot be computed from first principles. They are important for phenomenology whenever they are chirally or Cabibbo enhanced. In particular, they cannot be neglected in \( b \to s \) penguin modes. This introduces a strong model dependence in the evaluation of \( b \to s \) penguin BR’s and CP asymmetries.

- Power suppressed terms can invalidate the perturbative calculation of strong phases performed in the infinite mass limit. Indeed, in this case subleading terms in the \( \Lambda/m_b \) expansion can dominate over the loop-suppressed perturbative phases arising at leading order in \( \Lambda/m_b \).

2. **PQCD**

The basic idea underlying PQCD calculations is that the dominant process is hard gluon exchange involving the spectator quark. PQCD adopts the three-scale factorization theorem [60] based on the perturbative QCD formalism by Brodsky and Lepage [61], with the inclusion of the transverse momentum carried by partons inside the meson. The three different scales are the electroweak scale \( M_W \), the scale of hard gluon exchange \( t \sim O(\sqrt{\Lambda m_b}) \), and
the factorization scale $1/b$, where $b$ is the conjugate variable of parton transverse momenta. The nonperturbative physics at scales below $1/b$ is encoded in process-independent meson wave functions. The inclusion of transverse momentum leads to a Sudakov form factor which suppresses the long distance contributions in the large $b$ region, and vanishes as $b > 1/\Lambda$. This suppression renders the transverse momentum flowing into the hard amplitudes of order $\Lambda m_b$. The off-shellness of internal particles then remains of $O(\Lambda m_b)$ even in the end-point region, and the singularities are removed.

Notice that:

- Contrary to QCD factorization, in PQCD all contributions are assumed to be calculable in perturbation theory due to the Sudakov suppression. This item remains controversial (see refs. [62] and [63]).

- The dominant strong phases in this approach come from factorized annihilation diagrams.

- Also in this case, there is no control over subleading contributions in the $\Lambda/m_b$ expansion.

3. SCET

Soft-collinear effective theory is a powerful tool to study factorization in multi-scale problems. The idea is to perform a two-step matching procedure at the hard ($O(m_b)$) and hard-collinear ($O(\sqrt{m_b}\Lambda)$) scales. The final expression is given in terms of perturbative hard kernels, light-cone wave functions and jet functions. For phenomenology, it is convenient to fit directly the nonperturbative parameters on data using the following expression for the decay amplitude, valid at leading order in $\alpha_s$ [56–58]:

$$A(B \to M_1 M_2) \propto f_{M_1} \zeta^{BM_2} [1 - \frac{1}{\alpha_s}] \int_0^1 du \phi_{M_1}(u) T_{1J}(u) + f_{M_1} \zeta^{BM_2} T_{1\zeta} + 1 \leftrightarrow 2 + A^{M_1 M_2}_{cc},$$

where $T$’s are perturbative hard kernels, $\zeta$’s are nonperturbative parameters and $A_{cc}$ denotes the “charming penguin” contribution.

We notice that:

- Charming penguins are not factorized in the infinite mass limit in this approach, contrary to what obtained in QCD factorization;
• Phenomenological analyses are carried out at leading order in $\alpha_s$ and at leading power in $\Lambda/m_b$;

• No control is possible on power corrections to factorization.

4. $SU(3)$ flavor symmetry

An alternative approach that has been pursued extensively in the literature is to use $SU(3)$ flavor symmetry to extract hadronic matrix elements from experimental data and then use them to predict $SU(3)$-related channels [64–87]. In principle, in this way it is possible to eliminate all the uncertainties connected to factorization and the infinite mass limit. On the other hand, $SU(3)$-breaking must be evaluated to obtain reliable predictions.

A few comments are in order:

• In some fortunate cases, such as the contribution of electroweak penguins $Q_{9,10}$ to $B \to K\pi$ decays, $SU(3)$ predicts some matrix elements to vanish, so that they can be assumed to be suppressed even in the presence of $SU(3)$ breaking [88–90].

• Explicit nonperturbative calculations of two-body nonleptonic $B$ decays indicate that $SU(3)$-breaking corrections to $B$ decay amplitudes can be up to 80%, thus invalidating $SU(3)$ analyses of these processes [91].

• To take partially into account the effects of $SU(3)$ breaking, several authors assume that symmetry breaking follows the pattern of factorized matrix elements. While this is certainly an interesting idea, its validity for $b \to s$ penguins is questionable, given the importance of nonfactorizable contributions in these channels.

5. General parameterizations

The idea developed in Refs. [42, 92] is to write down the RGI parameters as the sum of their expression in the infinite mass limit, for example using QCD factorization, plus an arbitrary contribution corresponding to subleading terms in the power expansion. These additional contributions are then determined by a fit to the experimental data. In $b \to s$ penguins, the dominant power-suppressed correction is given by charming penguins, and the corresponding parameter can be determined with high precision from data and is found to
be compatible with a $\Lambda/m_b$ correction to factorization [42]. However, non-dominant corrections, for example GIM penguin parameters in $b \to s$ decays, can be extracted from data only in a few cases (for example in $B \to K\pi$ decays) [92]. However, predictions for $\Delta S$ depend crucially on these corrections, so that one needs external input to constrain them. One interesting avenue is to extract the support of GIM penguins from $SU(3)$-related channels ($b \to d$ penguins) in which they are not Cabibbo-suppressed, and to use this support, including a possible $SU(3)$ breaking of 100%, in the fit of $b \to s$ penguin decays. Alternatively, one can omit the calculation in factorization and fit directly the RGI parameters from the experimental data, instead of fitting the power-suppressed corrections [93, 94].

We remark that:

- Compared to factorization approaches, general parameterizations have less predictive power but are more general and thus best suited to search for NP in a conservative way.

- This method has the advantage that for several channels, to be discussed below, the predicted $\Delta S$ decreases with the experimental uncertainty in $BR$'s and CP asymmetries of $b \to s$ and $SU(3)$-related $b \to d$ penguins.

We conclude this Section by remarking once again that neither the "GIM-penguin problem" nor the "tree problem" can be solved from first principles and we must cope with model-dependent estimates. It then becomes very important to be able to study a variety of channels in several different approaches. In this way, we can hope to be able to make solid predictions and to test them with high accuracy. In the following, we quickly review the present theoretical and experimental results, keeping in mind the goal of testing the SM and looking for NP.

III. $BR$'S AND CP ASYMMETRIES WITHIN THE SM

The aim of this Section is to collect pre- and post-dictions for $BR$'s and CP asymmetries of $b \to s$ penguin decays obtained in the approaches briefly discussed in the previous Section. The main focus will be on $\Delta S$, but $BR$'s and rate CP asymmetries will play a key role in assessing the reliability and the theoretical uncertainty of the different approaches.
A. \textit{BR’s and rate CP asymmetries}

In Tables I-III we report some of the results obtained in the literature for $B$ decay $BR$'s and CP asymmetries. For QCD Factorization (QCDF) results, the first error corresponds to variations of CKM parameters, the second to variations of the renormalization scale, quark masses, decay constants (except for transverse ones), form factors, and the $\eta - \eta'$ mixing angle. The third error corresponds to the uncertainty due to the Gegenbauer moments in the expansion of the light-cone distribution amplitudes, and also includes the scale-dependent transverse decay constants for vector mesons. Finally, the last error corresponds to an estimate of the effect of the dominant power corrections. For PQCD results from refs. [54, 55], the error only includes the variation of Gegenbauer moments, of $|V_{ub}|$ and of the CKM phase. For PQCD results from ref. [95], the errors correspond to input hadronic parameters, to scale dependence, and to CKM parameters respectively. For SCET results, the analysis is carried out at leading order in $\alpha_s$ and $\Lambda/m_b$ assuming exact $SU(3)$. The errors are estimates of $SU(3)$ breaking, of $\Lambda/m_b$ corrections and of the uncertainty due to SCET parameters respectively. SCET I and SCET II denote two possible solutions for SCET parameters in the fit [58]. For General Parametrization (GP) results, the errors include the uncertainty on CKM parameters, on form factors, quark masses and meson decay constants, and a variation of $\Lambda/m_b$ corrections up to 50\% of the leading power emission amplitude. The values in boldface correspond to predictions (\textit{i.e.} the experimental value has not been used in the fit).

First of all, we notice that all approaches are able to reproduce the experimental $BR$'s of $B \to PP$ penguins, although QCDF tends to predict lower $BR$'s for $B \to P\eta'$, albeit with large uncertainties. Concerning $BR$'s of $B \to PV$ penguins, QCDF is always on the low side and reproduces experimental $BR$'s only when the upper range of the error due to power corrections is considered. PQCD shows similar features for $K^*$ and $\rho$ modes, while it predicts much larger values for $BR$'s of $B \to K\omega$ decays.

The situation for rate CP asymmetries is a bit different. Both QCDF and SCET predict $A_{CP}(\bar{B}^0 \to \pi^0\bar{K}^0) \sim -A_{CP}(\bar{B}^0 \to \pi^0K^-)$ while experimentally the two asymmetries have the same sign. PQCD reproduces the experimental values, although it predicts $A_{CP}(\bar{B}^0 \to \pi^0\bar{K}^0)$ on the low side of the experimental value. It is interesting to notice that the GP approach is able to predict the correct value and sign of $A_{CP}(\bar{B}^0 \to \pi^0\bar{K}^0)$ in spite of the
TABLE I: Results for CP-averaged $BR$’s (in units of $10^{-6}$) and CP asymmetries (in %) in several approaches for $B \to PP$ decays. Experimental averages from the Heavy Flavor Averaging Group (HFAG) are also shown.

|                  | QCDF [50] | PQCD [54, 55] | SCET [58] | GP [92] | exp  |
|------------------|-----------|---------------|-----------|---------|------|
| $BR(\pi^- \bar{K}^0)$ | $19.3^{+1.9+11.3}_{-1.9-7.8}^{+13.2}_{-21-5.6}$ | $24.5^{+13.6}_{-8.1}$ | $20.8 \pm 7.9 \pm 0.6 \pm 0.7$ | $24.1 \pm 0.7$ | $23.1 \pm 1.0$ |
| $A_{CP}(\pi^- \bar{K}^0)$ | $0.9^{+0.2+0.3}_{-0.3-0.1}^{+0.6}_{-0.1-0.5}$ | $0 \pm 0$ | $< 5$ | $1.2 \pm 2.4$ | $0.9 \pm 2.5$ |
| $BR(\pi^0 K^-)$ | $11.1^{+1.8+5.8+0.9}_{-1.7-4.0}^{+6.9}_{-1.0-3.0}$ | $13.9^{+10.0}_{-5.6}$ | $11.3 \pm 4.1 \pm 1.0 \pm 0.3$ | $12.6 \pm 0.5$ | $12.8 \pm 0.6$ |
| $A_{CP}(\pi^0 K^-)$ | $7.1^{+1.7+2.0}_{-1.8-2.0}^{+0.8+9.0}_{-0.6-9.7}$ | $-1^{+3}_{-5}$ | $-11 \pm 9 \pm 11 \pm 2$ | $3.4 \pm 2.4$ | $4.7 \pm 2.6$ |
| $BR(\pi^- K^-)$ | $16.3^{+2.6+9.6+1.4}_{-2.3-6.5}^{+11.4}_{-1.4-4.8}$ | $20.9^{+15.6}_{-8.3}$ | $20.1 \pm 7.4 \pm 1.3 \pm 0.6$ | $19.6 \pm 0.5$ | $19.4 \pm 0.6$ |
| $A_{CP}(\pi^+ K^-)$ | $4.5^{+1.1+2.2}_{-1.1-2.5}^{+0.5+8.7}_{-0.6-9.5}$ | $-9^{+6}_{-8}$ | $-6 \pm 5 \pm 6 \pm 2$ | $-8.9 \pm 1.6$ | $-9.5 \pm 1.3$ |
| $BR(\pi^0 \bar{K}^0)$ | $7.0^{+0.7+4.7+0.7+5.4}_{-0.7-3.2-0.7-2.3}$ | $9.1^{+5.6}_{-3.3}$ | $9.4 \pm 3.6 \pm 0.2 \pm 0.3$ | $9.5 \pm 0.4$ | $10.0 \pm 0.6$ |
| $A_{CP}(\pi^0 \bar{K}^0)$ | $-3.3^{+1.0+1.3+0.5+3.4}_{-0.8-1.6-1.0-3.3}$ | $-7^{+3}_{-3}$ | $5 \pm 4 \pm 4 \pm 1$ | $-9.8 \pm 3.7$ | $-12 \pm 11$ |

TABLE II: Results for two-body $b \to s$ penguin decays to $\eta$ or $\eta'$ CP-averaged $BR$’s (in unit of $10^{-6}$) and CP asymmetries (in %) in several approaches. Experimental averages from HFAG are also shown.

|                  | QCDF [50] | SCET I [58] | SCET II [58] | exp  |
|------------------|-----------|-------------|-------------|------|
| $BR(\bar{K}^0 \eta')$ | $46.5^{+4.7+24.9+12.3+31.0}_{-4.4-15.4-6.8-13.5}$ | $63.2 \pm 24.7 \pm 4.2 \pm 8.1$ | $62.2 \pm 23.7 \pm 5.5 \pm 7.2$ | $64.9 \pm 3.5$ |
| $A_{CP}(\bar{K}^0 \eta')$ | $1.8^{+0.4+0.3+0.1+0.8}_{-0.5-0.3-0.2-0.8}$ | $1.1 \pm 0.6 \pm 1.2 \pm 0.2$ | $-2.7 \pm 0.7 \pm 0.8 \pm 0.5$ | $9 \pm 6$ |
| $BR(\bar{K}^0 \eta)$ | $1.1^{+0.1+2.0+0.4+1.3}_{-0.1-1.3-0.5-0.5}$ | $2.4 \pm 4.4 \pm 0.2 \pm 0.3$ | $2.3 \pm 4.4 \pm 0.2 \pm 0.5$ | $< 1.9$ |
| $A_{CP}(\bar{K}^0 \eta)$ | $-9.0^{+2.8+5.4+2.8+8.2}_{-2.1-12.6-7.8}$ | $21 \pm 20 \pm 4 \pm 3$ | $-18 \pm 22 \pm 6 \pm 4$ | $-18 \pm 22 \pm 6 \pm 4$ |
| $BR(K^- \eta')$ | $49.1^{+5.1+26.5+13.6+33.6}_{-4.9-16.3-7.4-14.6}$ | $69.5 \pm 27.0 \pm 4.3 \pm 7.7$ | $69.3 \pm 26.0 \pm 7.1 \pm 6.3$ | $69.7^{+2.8}_{-2.7}$ |
| $A_{CP}(K^- \eta')$ | $2.4^{+0.6+0.6+0.3+3.4}_{-0.7-0.8-0.4-3.5}$ | $-1 \pm 0.6 \pm 0.7 \pm 0.5$ | $0.7 \pm 0.5 \pm 0.2 \pm 0.9$ | $3.1 \pm 2.1$ |
| $BR(K^- \eta)$ | $1.9^{+0.5+2.4+0.5+1.6}_{-0.5-1.6-0.6-0.7}$ | $2.7 \pm 4.8 \pm 0.4 \pm 0.3$ | $2.3 \pm 4.5 \pm 0.4 \pm 0.3$ | $2.2 \pm 0.3$ |
| $A_{CP}(K^- \eta)$ | $-18.9^{+6.4+11.7+4.8+25.3}_{-6.9-17.5-8.5-21.8}$ | $33 \pm 30 \pm 7 \pm 3$ | $-33 \pm 39 \pm 10 \pm 4$ | $29 \pm 11$ |

Notice also that $B \to K \pi$ data in Tab. I are perfectly reproduced in the GP approach, thus showing on general grounds the absence of any “$K \pi$ puzzle”, although specific dynamical assumptions may lead to discrepancies between theory and experiment [80, 97–100].
TABLE III: Results for CP-averaged $BR$'s (in units of $10^{-6}$) and CP asymmetries (in %) in several approaches for $B \to PV$ decays. Experimental averages from HFAG are also shown.

|               | QCDF [50]       | PQCD [54, 55] | GP [92]  | exp  |
|---------------|-----------------|---------------|----------|------|
| $BR(\pi^- K^{*0})$ | $3.6^{+0.4+1.5+1.7+2.7}_{-0.3-1.4-1.2-2.3}$ | $6.0^{+2.8}_{-1.5}$ | $11.3 \pm 0.9$ | $10.7 \pm 0.8$ |
| $ACP(\pi^- K^{*0})$ | $1.6^{+0.4+0.6+0.5+2.5}_{-0.5-0.4-1.0}$ | $-1^{+1}_{-0}$ | $-7 \pm 6$ | $-8.5 \pm 5.7$ |
| $BR(\pi^0 K^-)$ | $3.3^{+1.1+1.0+0.6+4.4}_{-1.0-0.9-0.6-1.4}$ | $4.3^{+5.0}_{-2.2}$ | $7.3 \pm 0.6$ | $6.9 \pm 2.3$ |
| $ACP(\pi^0 K^-)$ | $8.7^{+2.1+5.0+2.9+4.1}_{-2.6-4.3-3.4-4.4}$ | $-32^{+21}_{-28}$ | $-2 \pm 13$ | $4 \pm 29$ |
| $BR(\pi^+ K^+)$ | $3.3^{+1.4+1.3+0.8+6.2}_{-1.2-1.2-0.8-1.6}$ | $6.0^{+6.8}_{-2.6}$ | $8.5 \pm 0.8$ | $9.8 \pm 1.1$ |
| $ACP(\pi^+ K^+)$ | $2.1^{+0.6+0.8+0.5+6.2+2.7}_{-0.8-0.7-0.9-5.8-6.4}$ | $-60^{+32}_{-19}$ | $-4 \pm 13$ | $-5 \pm 14$ |
| $BR(\pi^0 K^0)$ | $0.7^{+0.1+0.5+0.3+2.6}_{-0.1-0.4-0.3-0.5}$ | $2.0^{+1.2}_{-0.6}$ | $3.1 \pm 0.4$ | $0.0^{+1.3}_{-0.1}$ |
| $ACP(\pi^0 K^0)$ | $-12.8^{+4.0+4.7+2.7+31.7}_{-3.2-7.0-4.0-35.3}$ | $-11^{+7}_{-5}$ | $-11^{+15}_{-14}$ | $-1 \pm 27$ |
| $BR(K^0 \rho^-)$ | $5.8^{+0.4+7.9+1.5+10.3}_{-0.6-3.3-1.3-3.2}$ | $8.7^{+6.8}_{-4.4}$ | $7.8 \pm 1.1$ | $8.0^{+1.5}_{-1.4}$ |
| $ACP(K^0 \rho^-)$ | $0.3^{+0.1+0.3+0.2+1.6}_{-0.1-0.4-0.1-1.3}$ | $1 \pm 1$ | $0.02 \pm 0.17$ | $12 \pm 17$ |
| $BR(K^- \rho^0)$ | $2.6^{+0.9+3.1+0.8+4.3}_{-0.9-1.4-0.6-1.2}$ | $5.1^{+4.1}_{-2.8}$ | $4.15 \pm 0.50$ | $4.25^{+0.55}_{-0.56}$ |
| $ACP(K^- \rho^0)$ | $-13.6^{+4.5+6.9+3.7+62.7}_{-5.7-4.4-3.1-55.4}$ | $71^{+25}_{-25}$ | $29 \pm 10$ | $31^{+14}_{-10}$ |
| $BR(K^- \rho^+)$ | $7.4^{+1.8+7.1+1.2+10.7}_{-1.9-3.6-1.1-3.5}$ | $8.8^{+6.8}_{-4.5}$ | $10.2 \pm 1.0$ | $15.3^{+3.7}_{-3.5}$ |
| $ACP(K^- \rho^+)$ | $-3.8^{+1.3+4.4+1.9+34.5}_{-1.4-2.7-1.6-32.7}$ | $64^{+24}_{-30}$ | $21 \pm 10$ | $22 \pm 23$ |
| $BR(K^0 \rho^0)$ | $4.6^{+0.5+4.0+0.7+6.1}_{-0.5-2.1-0.7-2.1}$ | $4.8^{+4.3}_{-2.3}$ | $5.2 \pm 0.7$ | $5.4^{+0.9}_{-1.0}$ |
| $ACP(K^0 \rho^0)$ | $7.5^{+1.7+2.3+0.7+8.8}_{-2.1-2.0-0.4-8.7}$ | $7^{+8}_{-5}$ | $1^{+15}_{-14}$ | $-64 \pm 46$ |
| $BR(K^- \omega)$ | $3.5^{+1.0+3.3+1.4+4.7}_{-1.0-1.6-0.9-1.6}$ | $10.6^{+10.4}_{-5.8}$ | $6.9 \pm 0.5$ | $6.8 \pm 0.5$ |
| $ACP(K^- \omega)$ | $-7.8^{+2.6+5.9+2.4+39.8}_{-5.0-3.6-1.9-38.0}$ | $32^{+15}_{-17}$ | $5 \pm 6$ | $5 \pm 6$ |
| $BR(K^0 \omega)$ | $2.3^{+0.3+2.8+1.3+4.3}_{-0.3-1.3-0.8-1.3}$ | $9.8^{+8.6}_{-4.9}$ | $4.6 \pm 0.5$ | $5.2 \pm 0.7$ |
| $ACP(K^0 \omega)$ | $-8.1^{+2.5+3.0+1.7+11.8}_{-2.0-3.3-1.4-12.9}$ | $-3^{+2}_{-4}$ | $-5 \pm 11$ | $21 \pm 19$ |
| $BR(K^- \phi)$ | $4.5^{+0.5+1.8+1.9+11.8}_{-0.4-1.7-2.1-3.3}$ | $7.8^{+5.9}_{-1.8}$ | $8.39 \pm 0.59$ | $8.30 \pm 0.65$ |
| $ACP(K^- \phi)$ | $1.6^{+0.4+0.6+0.5+3.0}_{-0.5-0.5-0.3-1.2}$ | $1^{+1}_{-1}$ | $3.0 \pm 4.5$ | $3.4 \pm 4.4$ |
| $BR(K^0 \phi)$ | $4.1^{+0.4+1.7+1.8+10.6}_{-0.4-1.6-1.9-3.0}$ | $7.3^{+5.4}_{-1.6}$ | $7.8 \pm 0.9$ | $8.3^{+1.2}_{-1.0}$ |
| $ACP(K^0 \phi)$ | $1.7^{+0.4+0.6+0.5+1.4}_{-0.5-0.5-0.3-0.8}$ | $3^{+1}_{-2}$ | $1 \pm 6$ | $-1 \pm 13$ |

We conclude that factorization approaches in general show a remarkable agreement with experimental data, but their predictions suffer from large uncertainties. Furthermore, QCDF
TABLE IV: Results for CP-averaged $BR$'s (in units of $10^{-6}$) and CP asymmetries (in %) in several approaches for $B_s \to PP$ decays. The only available experimental result is $BR(B_s \to K^+K^-) = (24.4 \pm 4.8) \cdot 10^{-6}$ [96].

|          | QCDF [50]          | PQCD [95]          | SCET I [58]         | SCET II [58]        |
|----------|--------------------|--------------------|---------------------|---------------------|
| $BR(K^+K^-)$ | $22.7^{+3.5+12.7+2.0+24.1}_{-3.2-8.4-2.0-9.1}$ | $17.0^{+5.1+8.8+0.9}_{-4.1-5.0-0.3}$ | $18.2 \pm 6.7 \pm 1.1 \pm 0.5$ |
| $ACP(K^+K^-)$ | $4.0^{+1.0+2.0+0.5+10.4}_{-1.0-2.3-0.5-11.3}$ | $-25.8^{+1.1+5.2+0.9}_{-0.2-4.5-1.1}$ | $-6 \pm 5 \pm 6 \pm 2$ |
| $BR(K^0\bar{K}^0)$ | $24.7^{+2.5+13.7+2.6+25.6}_{-2.4-9.2-2.9-9.8}$ | $19.6^{+6.4+10.4+0.0}_{-4.9-5.4-0.0}$ | $17.7 \pm 6.6 \pm 0.5 \pm 0.6$ |
| $ACP(K^0\bar{K}^0)$ | $0.9^{+0.2+0.2+0.1+0.2}_{-0.2-0.2-0.1-0.3}$ | $0$ | $<10$ |
| $BR(\eta\eta)$ | $15.6^{+1.6+9.9+2.2+13.5}_{-1.5-6.8-2.5-5.5}$ | $14.6^{+4.0+8.9+0.9}_{-3.2-5.4-0.4}$ | $7.1 \pm 6.4 \pm 0.2 \pm 0.8$ | $6.4 \pm 6.3 \pm 0.1 \pm 0.7$ |
| $ACP(\eta\eta)$ | $-1.6^{+0.5+0.6+0.4+2.2}_{-0.4-0.6-0.7-2.2}$ | $-1.6^{+0.3+0.7+0.1}_{-0.3-0.6-0.1}$ | $7.9 \pm 4.9 \pm 2.7 \pm 1.5$ | $-1.1 \pm 5.0 \pm 3.9 \pm 1.0$ |
| $BR(\eta'\eta')$ | $54.0^{+5.5+32.4+8.3+40.5}_{-5.2-24.4-6.4-16.7}$ | $39.0^{+9.9+20.4+0.0}_{-7.8-13.1-0.0}$ | $24.0 \pm 13.6 \pm 1.4 \pm 2.7$ | $23.8 \pm 13.2 \pm 1.6 \pm 2.9$ |
| $ACP(\eta'\eta')$ | $0.4^{+0.1+0.3+0.1+0.4}_{-0.1-0.3-0.1-0.3}$ | $-1.2^{+0.1+0.2+0.1}_{-0.0-0.1-0.1}$ | $0.04 \pm 0.14 \pm 0.39 \pm 0.43$ | $2.3 \pm 0.9 \pm 0.8 \pm 7.6$ |
| $BR(\eta''\eta'')$ | $41.7^{+4.2+26.3+15.2+36.6}_{-4.0-17.2-8.5-15.4}$ | $29.6^{+5.2+14.0+0.0}_{-5.3-8.9-0.0}$ | $44.3 \pm 19.7 \pm 2.3 \pm 17.1$ | $49.4 \pm 20.6 \pm 8.4 \pm 16.2$ |
| $ACP(\eta''\eta'')$ | $2.1^{+0.5+0.4+0.2+1.1}_{-0.6-0.5-0.3-1.2}$ | $2.2^{+0.4+0.2+0.2}_{-0.4-0.4-0.1}$ | $0.9 \pm 0.4 \pm 0.6 \pm 1.2$ | $-3.7 \pm 1.0 \pm 1.2 \pm 5.6$ |

and SCET cannot reproduce rate asymmetries in $B \to K\pi$; this might be a hint that some delicate aspects of the dynamics of penguin decays, for example rescattering and final state interaction phases, are not fully under control. It is then reassuring that a more general approach as GP can reproduce the experimental data with reasonable (but not too small) values of the $\Lambda/m_b$ corrections to factorization. To quantify this statement, we report in Fig. 1 the results of the GP fit for $ACP(B \to K\pi)$ as a function of the upper bound on $\Lambda/m_b$ corrections [92]. It is clear that imposing a too low upper bound, of order 10%, would generate a spurious tension between theory and experiment.

For the reader’s convenience, we report in Tabs. IV and V the predictions obtained in several approaches for $BR$’s and CP asymmetries of $B_s$ penguin-dominated $b \to s$ decays.

### B. Predictions for $S$ and $\Delta S$ in $b \to s$ penguins

Keeping in mind the results of Sec. IIIA, we now turn to the main topic of this review, namely our ability to test the SM using time-dependent CP asymmetries in $b \to s$ penguin nonleptonic decays.
TABLE V: Results for CP-averaged BR’s (in units of 10^{-6}) and CP asymmetries (in %) in several approaches for $B_s \to PV$ decays. No experimental data are available yet.

| Channel | QCDF [50] | PQCD [95] |
|---------|-----------|-----------|
| $BR(K^+K^{*-})$ | $4.1^{+1.7+1.5+1.0+9.2}_{-1.5-1.3-0.9-23}$ | $7.4^{+2.1+1.9+0.9}_{-1.8-1.4-0.4}$ |
| $ACF(K^+K^{*-})$ | $2.2^{+0.6+8.4+5.1+68.6}_{-0.7-8.0-5.9-71.0}$ | $-40.6^{+2.9+2.2+1.8}_{-2.4-3.0-1.3}$ |
| $BR(K^0\bar{K}^*)$ | $3.9^{+0.4+1.5+1.3+10.4}_{-0.4-1.4-1.4-2.8}$ | $9.1^{+3.2+2.6+0.0}_{-2.2-1.5-0.0}$ |
| $ACF(K^0\bar{K}^*)$ | $1.7^{+0.4+0.6+0.5+1.4}_{-0.5-0.5-0.4-0.8}$ | $0$ |
| $BR(K^-K^{*+})$ | $5.5^{+1.3+5.0+0.8+14.2}_{-1.4-2.6-0.7-3.6}$ | $6.5^{+1.2+3.3+0.0}_{-1.2-1.8-0.1}$ |
| $ACF(K^-K^{*+})$ | $-3.1^{+1.0+3.8+1.6+47.5}_{-1.1-2.6-1.3-45.0}$ | $63.2^{+5.2+8.0+5.1}_{-5.8-10.2-2.6}$ |
| $BR(K^0\bar{K}^{*0})$ | $4.2^{+0.4+1.6+1.1+13.2}_{-0.4-2.2-0.9-3.2}$ | $5.9^{+0.9+2.8+0.0}_{-1.1-1.8-0.0}$ |
| $ACF(K^0\bar{K}^{*0})$ | $0.2^{+0.0+0.2+0.1+0.2}_{-0.1-0.3-0.1-0.1}$ | $0$ |
| $BR(\eta\omega)$ | $0.012^{+0.005+0.010+0.028+0.025}_{-0.004-0.005-0.006-0.006}$ | $0.10^{+0.02+0.03+0.00}_{-0.02-0.01-0.00}$ |
| $ACF(\eta\omega)$ | $3.2^{+6.1+15.2+0.3}_{-3.9-11.2-0.1}$ | $0$ |
| $BR(\eta'\omega)$ | $0.024^{+0.011+0.028+0.077+0.042}_{-0.009-0.006-0.010-0.015}$ | $0.66^{+0.23+0.22+0.01}_{-0.18-0.21-0.03}$ |
| $ACF(\eta'\omega)$ | $-0.1^{+0.7+3.9+0.0}_{-0.8-4.2-0.0}$ | $0$ |
| $BR(\eta\phi)$ | $0.12^{+0.02+0.95+0.54+0.32}_{-0.02-0.14-0.12-0.13}$ | $1.8^{+0.5+0.1+0.0}_{-0.5-0.2-0.0}$ |
| $ACF(\eta\phi)$ | $-8.4^{+2.0+30.1+14.6+36.3}_{-2.1-71.2-44.7-59.7}$ | $-0.1^{+0.2+2.3+0.0}_{-0.4-1.4-0.0}$ |
| $BR(\eta'\phi)$ | $0.05^{+0.01+1.10+0.18+0.40}_{-0.01-0.17-0.08-0.04}$ | $3.6^{+1.2+0.4+0.0}_{-0.9-0.4-0.0}$ |
| $ACF(\eta'\phi)$ | $-62.2^{+15.9+132.3+80.8+122.4}_{-10.2-84.2-46.8-49.9}$ | $1.2^{+0.1+0.4+0.1}_{-0.0-0.6-0.1}$ |

Starting from Eq. (5), we write down the expression for $S_F$ as follows:

$$S_F = \frac{\sin(2(\beta_s + \phi_M)) + |r_F|^2 \sin(2(\phi_M + \gamma)) + 2 \Re r_F \sin(\beta_s + 2\phi_M + \gamma)}{1 + |r_F|^2 + 2 \Re r_F \cos(\beta_s - \gamma)},$$

where $r_F = |V_{us}V_{ub}|/|V_{ts}V_{tb}| \times \sum(T_i + P^{GIM}_i)/\sum P_i$ with $T_i = 0$ for pure penguin channels. Since the angle $\beta_s$ is small and very well known ($\beta_s = (2.1 \pm 0.1)^\circ$), the problem is then reduced to the evaluation of $\kappa_F = \sum(T_i + P^{GIM}_i)/\sum P_i$ for each channel (notice that $T_i = 0$ for pure penguin channels). Factorization methods have been used to provide estimates of $\kappa_F$, $S_F$ and $\Delta S_F$ for $b \to s$ channels. The latter are reported in Tables VI and VII. A few remarks are important. First of all, the evaluation of $P^{GIM}_i$ relies on the factorization of penguin contractions of charm and up quarks, which is debatable even in the infinite mass limit. In addition to that, in factorization $P^{GIM}$ has a perturbative loop suppression
FIG. 1: $A_{CP}$ values for $B \to K\pi$ in the GP approach [92], obtained varying $\mathcal{O}(\Lambda_{QCD}/m_b)$ contributions in the range $[0, \text{UV}]$, with the upper value UV scanned between zero and one (in units of the factorized emission amplitude). For comparison, the experimental 68% (95%) probability range is given by the dark (light) band.

TABLE VI: Predictions for $S$ parameters in % for $B$ decays. Experimental averages from HFAG are also shown.

|               | PQCD [54, 55] | SCET I [58] | SCET II [58] | GP [92] | exp   |
|---------------|---------------|-------------|--------------|---------|-------|
| $S_{\pi^0 K_S}$ | $74^{+2}_{-3}$ | $80 \pm 2 \pm 2 \pm 1$ | $74.3 \pm 4.4$ | $33 \pm 21$ |
| $S_{\eta K_S}$  | $70.6 \pm 0.5 \pm 0.6 \pm 0.3$ | $71.5 \pm 0.5 \pm 0.8 \pm 0.2$ | $70.9 \pm 3.9$ | $61 \pm 7$  |
| $S_{\eta' K_S}$ | $69 \pm 15 \pm 5 \pm 1$ | $79 \pm 14 \pm 4 \pm 1$ |               |         |
| $S_{\phi K_S}$  | $71^{+1}_{-1}$ |               | $71.5 \pm 8.7$ | $39 \pm 18$ |
| $S_{\rho^0 K_S}$ | $50^{+10}_{-6}$ |               | $64 \pm 11$  | $20 \pm 57$ |
| $S_{\omega K_S}$ | $84^{+3}_{-7}$ |               | $75.7 \pm 10.3$ | $48 \pm 24$ |

so that it is likely to be dominated by power corrections. Furthermore, the contribution of $T_i$ and $P_i^{\text{GIM}}$ is particularly difficult to estimate for $\eta$ and $\eta'$ channels. Last but not least, the determination of the sign of $\Delta S_F$ relies heavily on the determination of the sign of $\text{Re} \kappa_F$. If $P_i^{\text{GIM}}$ is dominated by power corrections, there is no guarantee that the sign given by the perturbative calculation is correct.

With the above caveat in mind, from Tables VI and VII we learn that:

- Experimentally there is a systematic trend for negative $\Delta S$. This might be a hint of the presence of new sources of CP violation in the $b \to s$ penguin amplitude.

- The experimental uncertainty is dominant in all channels. In addition to that, the GP
TABLE VII: Predictions for $\Delta S$ parameters in % for $B$ decays. Experimental averages from HFAG are also shown.

| $\Delta S$        | QCDF [101] | SCET I [58] | SCET II [58] | GP [92] | exp |
|-------------------|------------|-------------|-------------|---------|-----|
| $\Delta S_{\phi^0 K_S}$ | $7^{+5}_{-4}$ | $7.7 \pm 2.2 \pm 1.8 \pm 1$ | $2.4 \pm 5.9$ | $-35 \pm 21$ |
| $\Delta S_{\eta' K_S}$ | $1^{+1}_{-1}$ | $-1.9 \pm 0.5 \pm 0.6 \pm 0.3$ | $-1.0 \pm 0.5 \pm 0.8 \pm 0.2$ | $-0.7 \pm 5.4$ | $-7 \pm 7$ |
| $\Delta S_{\eta K_S}$ | $10^{+11}_{-7}$ | $-3.4 \pm 15.5 \pm 5.4 \pm 1.4$ | $7.0 \pm 13.6 \pm 4.2 \pm 1.1$ |
| $\Delta S_{\phi K_S}$ | $2^{+1}_{-1}$ | |
| $\Delta S_{\rho^0 K_S}$ | $-8^{+8}_{-12}$ | $-6.2 \pm 8.4$ | $-48 \pm 57$ |
| $\Delta S_{\omega K_S}$ | $13^{+8}_{-8}$ | $5.6 \pm 10.7$ | $-20 \pm 24$ |

estimate of the theoretical uncertainty, which is certainly conservative, can be reduced with experimental improvements on $BR$’s and CP asymmetries.

- As discussed in Sec. II, the theoretical uncertainty estimated from first principles is much smaller for pure penguin decays such as $B \to \phi K_s$ than for penguin-dominated channels.

- In the model-independent GP approach, the theoretical uncertainty is smaller for $B \to \pi^0 K_s$ because the number of observables in the $B \to K \pi$ system is sufficient to constrain efficiently the hadronic parameters. This means that the theoretical error can be kept under control by improving the experimental data in these channels. On the other hand, the information on $B \to \phi K_s$ is not sufficient to bound the subleading terms and this results in a relatively large theoretical uncertainty that cannot be decreased without additional input on hadronic parameters. Furthermore, using $SU(3)$ to constrain $\Delta S_{\phi K_s}$ is difficult because the number of amplitudes involved is very large [64, 85–87].

The ideal situation would be represented by a pure penguin decay for which the information on $P_i^{GIM}$ is available with minimal theoretical input. Such situation is realized by the pure penguin decays $B_s \to K^{0(*)}\bar{K}^{0(*)}$. An upper bound for the $P_i^{GIM}$ entering this amplitude can be obtained from the $SU(3)$-related channels $B_d \to K^{0(*)}\bar{K}^{0(*)}$. Then, even adding a generous 100% $SU(3)$ breaking and an arbitrary strong phase, it is possible to have full control over the theoretical error in $\Delta S$ [94].
For the reader’s convenience, we report in Tab. VIII the predictions for the $S$ coefficient of the time-dependent CP asymmetry for several $B_s$ penguin-dominated decays.

Before closing this Section, let us mention non-resonant three-body $B$ decays such as $B \to K_s \pi^0 \pi^0$, $B \to K_s K_s K_s$ or $B \to K^+ K^- K_s$. In this case, a theoretical estimate of $\kappa_F$ is extremely challenging, and using $SU(3)$ to constrain $\kappa_F$ is difficult because of the large number of channels involved [85]. Nevertheless, they are certainly helpful in completing the picture of CP violation in $b \to s$ penguins.

To summarize the status of $b \to s$ penguins in the SM, we can say that additional experimental data will allow us to establish whether the trend of negative $\Delta S$ shown by present data really signals the presence of NP in $b \to s$ penguins. Theoretical errors are not an issue in this respect, because the estimates based on factorization can in most cases be checked using the GP approach based purely on experimental data. $B_s$ decays will provide additional useful channels and will help considerably in assessing the presence of NP in $b \to s$ penguins.

| TABLE VIII: Predictions for $S$ parameters for $B_s$ decays. |
|-------------------------------------------------------------|
| $B_s^0 \to K_s \pi^0$ | $0.46^{+0.14+0.19+0.02}_{-0.13-0.20-0.04}$ | $-0.16 \pm 0.41 \pm 0.33 \pm 0.17$ |
| $B_s^0 \to K_s \eta$ | $0.31^{+0.05+0.16+0.02}_{-0.05-0.17-0.03}$ | $0.82 \pm 0.32 \pm 0.11 \pm 0.04$ | $0.63 \pm 0.61 \pm 0.16 \pm 0.08$ |
| $B_s^0 \to K_s \eta'$ | $0.72^{+0.02+0.04+0.00}_{-0.02-0.03-0.00}$ | $0.38 \pm 0.08 \pm 0.10 \pm 0.04$ | $0.24 \pm 0.09 \pm 0.15 \pm 0.05$ |
| $B_s^0 \to K^- K^+$ | $0.28^{+0.04+0.04+0.02}_{-0.04-0.03-0.01}$ | $0.19 \pm 0.04 \pm 0.04 \pm 0.01$ |
| $B_s^0 \to \pi^0 \eta$ | $0.00^{+0.03+0.09+0.00}_{-0.02-0.10-0.01}$ | $0.45 \pm 0.14 \pm 0.42 \pm 0.30$ | $0.38 \pm 0.20 \pm 0.42 \pm 0.37$ |
| $B_s^0 \to \eta \eta$ | $0.03^{+0.00+0.01+0.00}_{-0.00-0.01-0.00}$ | $0.026 \pm 0.040 \pm 0.030 \pm 0.014$ | $-0.077 \pm 0.061 \pm 0.022 \pm 0.026$ |
| $B_s^0 \to \eta \eta'$ | $0.04^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$ | $0.041 \pm 0.004 \pm 0.002 \pm 0.051$ | $0.015 \pm 0.010 \pm 0.008 \pm 0.069$ |
| $B_s^0 \to \eta' \eta'$ | $0.04^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$ | $0.049 \pm 0.005 \pm 0.005 \pm 0.031$ | $0.051 \pm 0.009 \pm 0.017 \pm 0.039$ |
| $B_s^0 \to \omega \eta$ | $0.07^{+0.00+0.00+0.00}_{-0.01-0.11-0.00}$ |
| $B_s^0 \to \omega \eta'$ | $-0.19^{+0.01+0.04+0.01}_{-0.01-0.04-0.03}$ |
| $B_s^0 \to \phi \eta$ | $0.10^{+0.01+0.04+0.01}_{-0.01-0.03-0.00}$ |
| $B_s^0 \to \phi \eta'$ | $0.00^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$ |
| $B_s^0 \to K_s \phi$ | $-0.72$ |
IV. CP VIOLATION IN $b \to s$ PENGUINS BEYOND THE SM

We have seen that there is a hint of NP in CP-violating $b \to s$ hadronic penguins. In this Section, we would like to answer two basic questions that arise when considering NP contributions to these decays:

1. What are the constraints from other processes on new sources of CP violation in $b \to s$ transitions?

2. Are NP contributions to $b \to s$ transitions well motivated from the theoretical point of view?

We consider here only model-independent aspects of these two questions, and postpone model-dependent analyses to Section V.

A. Model-independent constraints on $b \to s$ transitions

The last year has witnessed enormous progress in the experimental study of $b \to s$ transitions. In particular, the TeVatron experiments have provided us with the first information on the $B_s - \bar{B}_s$ mixing amplitude [102], which can be translated into constraints on the $\Delta B = \Delta S = 2$ effective Hamiltonian. In any given model, as we shall see for example in Sec. V, these constraints can be combined with the ones from $b \to s\gamma$ and $b \to s\ell^+\ell^-$ decays to provide strong bounds on NP effects in $b \to s$ hadronic penguins.

Let us now summarize the presently available bounds on the $B_s - \bar{B}_s$ mixing amplitude, following the discussion of ref. [103]. General NP contributions to the $\Delta B = \Delta S = 2$ effective Hamiltonian can be incorporated in the analysis in a model-independent way, parametrizing the shift induced in the mixing frequency and phase with two parameters, $C_{B_s}$ and $\phi_{B_s}$, equal to 1 and 0 in the SM [104–108]:

$$C_{B_s} e^{2i\phi_{B_s}} = \frac{\langle B_s | H_{\text{full}}^{\text{eff}} | \bar{B}_s \rangle}{\langle B_s | H_{\text{SM}}^{\text{eff}} | \bar{B}_s \rangle}.$$  \hspace{1cm} (8)

As for the absorptive part of the $B_s - \bar{B}_s$ mixing amplitude, which is derived from the double insertion of the $\Delta B = 1$ effective Hamiltonian, it can be affected by non-negligible NP effects in $\Delta B = 1$ transitions through penguin contributions. Following refs. [4, 5], we thus introduce two additional parameters, $C_{s}^{\text{Pen}}$ and $\phi_{s}^{\text{Pen}}$, which encode NP contributions
FIG. 2: Constraints on the $\phi_{B_s}$ vs. $C_{B_s}$ plane [103]. Darker (lighter) regions correspond to 68% (95%) probability.

to the penguin part of the $\Delta B = 1$ Hamiltonian in analogy to what $C_{B_s}$ and $\phi_{B_s}$ do for the mixing amplitude.

The available experimental information is the following: the measurement of $\Delta m_s$ [102], the semileptonic asymmetry in $B_s$ decays $A^L_{s}\phi$ and the dimuon asymmetry $A_{CH}$ from DØ [109, 110], the measurement of the $B_s$ lifetime from flavor-specific final states [111–116], the determination of $\Delta \Gamma_s/\Gamma_s$ from the time-integrated angular analysis of $B_s \to J/\psi \phi$ decays by CDF [117], the three-dimensional constraint on $\Gamma_s$, $\Delta \Gamma_s$, and $B_s-\bar{B_s}$ mixing phase $\phi_s$ from the time-dependent angular analysis of $B_s \to J/\psi \phi$ decays by DØ [118].

Making use of this experimental information it is possible to constrain $C_{B_s}$ and $\phi_{B_s}$ [5, 103, 119–122]. The fourfold ambiguity for $\phi_{B_s}$ inherent in the untagged analysis of ref. [118] is somewhat reduced by the measurements of $A^L_{s}\phi$ and $A_{SL}$ [123], which prefer negative values of $\phi_{B_s}$. The results for $C_{B_s}$ and $\phi_{B_s}$, obtained from the general analysis allowing for NP in all sectors, are [103]

$$C_{B_s} = 1.03 \pm 0.29, \quad \phi_{B_s} = (-75 \pm 14)^\circ \cup (-19 \pm 11)^\circ \cup (9 \pm 10)^\circ \cup (102 \pm 16)^\circ .$$

Thus, the deviation from zero in $\phi_{B_s}$ is below the 1$\sigma$ level, although clearly there is still ample room for values of $\phi_{B_s}$ very far from zero. The corresponding p.d.f. in the $C_{B_s}$-$\phi_{B_s}$ plane is shown in Fig. 2.
The experimental information on $b \to s \gamma$ and $b \to s \ell^+ \ell^-$ decays [124–130] can also be combined in a model-independent way along the lines of refs. [131–134]. In this way, it is possible to constrain the coefficients of the $b \to s \gamma$, $b \to s \gamma^*$ and $b \to sZ$ vertices, which also contribute to $b \to s$ hadronic penguins. It turns out that order-of-magnitude enhancements of these vertices are excluded, so that they are unlikely to give large effects in $b \to s$ nonleptonic decays. On the other hand, the $b \to sg$ vertex is only very weakly constrained, so that it can still give large contributions to $b \to s$ hadronic penguins. Finally, the information contained in Eq. (9) can be used to constrain NP effects in $b \to s$ hadronic decays only within a given model, since a connection between $\Delta B = 2$ and $\Delta B = 1$ effective Hamiltonians is possible only once the model is specified. We shall return to this point in Sec. V.

B. Theoretical motivations for NP in $b \to s$ transitions

We now turn to the second question formulated at the beginning of this Section, namely whether on general grounds it is natural to expect NP to show up in $b \to s$ transitions. The general picture emerging from the generalized Unitarity Triangle analysis performed in ref. [4, 5, 103] and from the very recent data on $D - \bar{D}$ mixing [135–138] is that no new sources of CP violation are present in $B_d$, $K$ and $D$ mixing amplitudes. Conversely, large NP contributions to $s \to dg$, $b \to dg$ and $b \to sg$ transitions are not at all excluded. Therefore, although the idea of minimal flavor violation is phenomenologically appealing [21, 139–144], an equally possible alternative is that NP is contributing more to $\Delta F = 1$ transitions than to $\Delta F = 2$ ones. Within the class of $\Delta F = 1$ transitions, (chromo)-magnetic vertices are peculiar since they require a chirality flip to take place, which leads to a down-type quark mass suppression within the SM. On the other hand, NP models can weaken this suppression if they contain additional heavy fermions and/or additional sources of chiral mixing. In this case, they can lead to spectacular enhancements for the coefficients of (chromo)-magnetic operators. Furthermore, if the relevant new particles are colored, they can naturally give a strong enhancement of chromomagnetic operators while magnetic operators might be only marginally modified [145]. The electric dipole moment of the neutron puts strong constraints on new sources of CP violation in chirality-flipping flavor-conserving operators involving light quarks, but this does not necessarily imply the suppression of flavor-violating operators,
especially those involving $b$ quarks. Therefore, assuming that NP is sizable in hadronic $b \to s$ penguins is perfectly legitimate given the present information available on flavor physics.

From a theoretical point of view, a crucial observation is the strong breaking of the SM $SU(3)^5$ flavor symmetry by the top quark Yukawa coupling. This breaking necessarily propagates in the NP sector, so that in general it is very difficult to suppress NP contributions to CP violation in $b$ decays, and these NP contributions could be naturally larger in $b \to s$ transitions than in $b \to d$ ones. This is indeed the case in several flavor models (see for example Ref. [146]).

Another interesting argument is the connection between quark and lepton flavor violation in grand unified models [147–150]. The idea is very simple: the large flavor mixing present in the neutrino sector, if mainly generated by Yukawa couplings, should be shared by right-handed down-type quarks that sit in the same $SU(5)$ multiplet with left-handed leptons. Once again, one expects in this case large NP contributions to $b \to s$ transitions.

We conclude that the possibility of large NP effects in $b \to s$ penguin hadronic decays is theoretically well motivated on general grounds. The arguments sketched above can of course be put on firmer grounds in the context of specific models, and we refer the reader to the rich literature on this subject.

V. SUSY MODELS

Let us now focus on SUSY and discuss the phenomenological effects of the new sources of flavor and CP violation in $b \to s$ processes that arise in the squark sector [151–173]. In general, in the MSSM squark masses are neither flavor-universal, nor are they aligned to quark masses, so that they are not flavor diagonal in the super-CKM basis, in which quark masses are diagonal and all neutral current vertices are flavor diagonal. The ratios of off-diagonal squark mass terms to the average squark mass define four new sources of flavor violation in the $b \to s$ sector: the mass insertions $(\delta d_{23})_{AB}$, with $A, B = L, R$ referring to the helicity of the corresponding quarks. These $\delta$’s are in general complex, so that they also violate CP. One can think of them as additional CKM-type mixings arising from the SUSY sector. Assuming that the dominant SUSY contribution comes from the strong interaction sector, i.e. from gluino exchange, all FCNC processes can be computed in terms of the
SM parameters plus the four $\delta$’s plus the relevant SUSY parameters: the gluino mass $m_{\tilde{g}}$, the average squark mass $m_{\tilde{q}}$, $\tan \beta$ and the $\mu$ parameter. The impact of additional SUSY contributions such as chargino exchange has been discussed in detail in Ref. [166]. We consider only the case of small or moderate $\tan \beta$, since for large $\tan \beta$ the constraints from $B_s \rightarrow \mu^+\mu^-$ and $\Delta m_s$ preclude the possibility of having large effects in $b \rightarrow s$ hadronic penguin decays [163, 174–179].

Barring accidental cancellations, one can consider one single $\delta$ parameter, fix the SUSY masses and study the phenomenology. The constraints on $\delta$’s come at present from $B \rightarrow X_s\gamma$, $B \rightarrow X_s l^+l^-$ and from the $B_s - \bar{B}_s$ mixing amplitude as given in Eq. (9). We refer the reader to refs. [180–182] for all the details of this analysis.

Fixing as an example $m_{\tilde{g}} = m_{\tilde{q}} = |\mu| = 350$ GeV and $\tan \beta = 3$ or 10, one obtains the constraints on $\delta$’s reported in Figs. 3-5 [181, 182]. We plot in light green the allowed region considering only the constraint from the $C_B$ vs. $\phi_{B_s}$ p.d.f. of Fig. 2, in light blue the allowed region considering only the constraint from $b \rightarrow s \ell^+\ell^-$ and in violet the allowed region considering only the constraint from $b \rightarrow s \gamma$. The dark blue region is the one selected imposing all constraints simultaneously.

Several comments are in order at this point:

- Only $(\delta_{23}^d)_{\text{LL,LR}}$ generate amplitudes that interfere with the SM in rare decays. Therefore, the constraints from rare decays for $(\delta_{23}^d)_{\text{RL,RR}}$ are symmetric around zero, while the interference with the SM produces the circular shape of the $B \rightarrow X_s \gamma$ constraint on $(\delta_{23}^d)_{\text{LL,LR}}$.

- We recall that LR and RL mass insertions generate much larger contributions to the (chromo)magnetic operators, since the necessary chirality flip can be performed on the gluino line ($\propto m_{\tilde{g}}$) rather than on the quark line ($\propto m_b$). Therefore, the constraints from rare decays are much more effective on these insertions, so that the bound from $B_s - \bar{B}_s$ has no impact in this case.

- The $\mu \tan \beta$ flavor-conserving LR squark mass term generates, together with a flavor changing LL mass insertion, an effective $(\delta_{23}^d)_{\text{LR}}^{\text{eff}}$ that contributes to $B \rightarrow X_s \gamma$. For positive (negative) $\mu$, we have $(\delta_{23}^d)_{\text{LR}}^{\text{eff}} \propto +(-)(\delta_{23}^d)_{\text{LL}}$ and therefore the circle determined by $B \rightarrow X_s \gamma$ in the LL and LR cases lies on the same side (on opposite sides) of the origin (see Figs. 3 and 4).
FIG. 3: Allowed region in the \( \text{Re}(\delta_{23}^d)_{LL} \)-\( \text{Im}(\delta_{23}^d)_{LL} \) plane. In the plots on the left (right), negative (positive) \( \mu \) is considered. Plots in the upper (lower) row correspond to \( \tan \beta = 3 \) (\( \tan \beta = 10 \)). See the text for details.

- For \( \tan \beta = 3 \), we see from the upper row of Fig. 3 that the bound on \( (\delta_{23}^d)_{LL} \) from \( B_s - \bar{B}_s \) mixing is competitive with the one from rare decays, while for \( \tan \beta = 10 \) rare decays give the strongest constraints (lower row of Fig. 3). The bounds on all other \( \delta \)'s do not depend on the sign of \( \mu \) and on the value of \( \tan \beta \) for this choice of SUSY parameters.
- For LL and LR cases, \( B \to X_s \gamma \) and \( B \to X_s l^+ l^- \) produce bounds with different
FIG. 4: Allowed region in the $\text{Re}(\delta_{23}^{d})_{LR}$-$\text{Im}(\delta_{23}^{d})_{LR}$ (left) and $\text{Re}(\delta_{23}^{d})_{RL}$-$\text{Im}(\delta_{23}^{d})_{RL}$ (right) plane. Results do not depend on the sign of $\mu$ or on the value of $\tan \beta$.

FIG. 5: Allowed region in the $\text{Re}(\delta_{23}^{d})_{RR}$-$\text{Im}(\delta_{23}^{d})_{RR}$ plane. Results do not depend on the sign of $\mu$ or on the value of $\tan \beta$. See the text for details.

shapes on the $\text{Re} \delta$ – $\text{Im} \delta$ plane (violet and light blue regions in Figs. 3 and 4), so that applying them simultaneously a much smaller region around the origin survives (dark blue regions in Figs. 3 and 4). This shows the key role played by rare decays in
FIG. 6: Allowed region in the \( \text{Re}(\delta_{23})_{LL=RR} - \text{Im}(\delta_{23})_{LL=RR} \) plane. In the plots on the left (right), negative (positive) \( \mu \) is considered. Plots in the upper (lower) row correspond to \( \tan \beta = 3 \) (\( \tan \beta = 10 \)). See the text for details.

constraining new sources of flavor and CP violation in the squark sector.

- For the RR case, the constraints from rare decays are very weak, so that the only significant bound comes from \( B_s - \bar{B}_s \) mixing.

- If \( (\delta_{23})_{LL} \) and \( (\delta_{23})_{RR} \) insertions are simultaneously nonzero, they generate chirality-breaking contributions that are strongly enhanced over chirality-conserving ones, so
that the product \((\delta_{23}^d)_{\mathrm{LL}}(\delta_{23}^d)_{\mathrm{RR}}\) is severely bounded. In Fig. 6 we report the allowed region obtained in the case \((\delta_{23}^d)_{\mathrm{LL}} = (\delta_{23}^d)_{\mathrm{RR}}\). For \((\delta_{23}^d)_{\mathrm{LL}} \neq (\delta_{23}^d)_{\mathrm{RR}}\), this constraint can be interpreted as a bound on \(\sqrt{(\delta_{23}^d)_{\mathrm{LL}}(\delta_{23}^d)_{\mathrm{RR}}}\). We observe a very interesting interplay between the constraints from rare decays and the one from \(B_s - \bar{B}_s\) mixing. Increasing \(\tan \beta\) from 3 to 10, the bound from rare decays becomes tighter, but \(B_s - \bar{B}_s\) mixing still plays a relevant role.

- All constraints scale approximately linearly with squark and gluino masses.

---

![](image1.png)

**FIG. 7:** Probability density functions for \(S_{\phi K_s}\), \(S_{\pi^0 K_s}\), \(S_{\eta' K_s}\), and \(S_{\omega K_s}\) induced by \((\delta_{23}^d)_{\mathrm{LL}}\).
Having determined the p.d.f.'s for the four $\delta$'s, we now turn to the evaluation of the time-dependent CP asymmetries. As we discussed in Sec. II, the uncertainty in the calculation of SUSY effects is even larger than the SM one. Furthermore, we cannot use the GP approach since to estimate the SUSY contribution we need to evaluate the hadronic matrix elements explicitly. Following ref. [180], we use QCDF, enlarging the range for power-suppressed contributions to annihilation chosen in Ref. [50] as suggested in Ref. [42]. We warn the reader about the large theoretical uncertainties that affect this evaluation.

In Figs. 7-10 we present the results for $S_{\phi K_s}$, $S_{\pi^0 K_s}$, $S_{\eta' K_s}$ and $S_{\omega K_s}$. They do not show a
FIG. 9: Probability density functions for $S_{\phi K_s}$, $S_{\pi^0 K_s}$, $S_{\eta' K_s}$, and $S_{\omega K_s}$ induced by $(\delta^d_{23})_{RL}$.

sizable dependence on the sign of $\mu$ or on $\tan \beta$ for the chosen range of SUSY parameters. We see that:

- deviations from the SM expectations are possible in all channels, and the present experimental central values can be reproduced;

- they are more easily generated by $LR$ and $RL$ insertions, due to the enhancement mechanism discussed above.

- As noticed in refs. [183, 184], the correlation between $\Delta S_{PP}$ and $\Delta S_{PV}$ depends on
FIG. 10: Probability density functions for $S_{\phi K_s}$, $S_{\pi^0 K_s}$, $S_{\eta' K_s}$ and $S_{\omega K_s}$ induced by $(\delta_{2\delta}^d)_{RR}$.

the chirality of the NP contributions. For example, we show in Fig. 11 the correlation between $\Delta S_{K_s\phi}$ and $\Delta S_{K_s\pi^0}$ for the four possible choices for mass insertions. We see that the $\Delta S_{K_s\phi}$ and $\Delta S_{K_s\pi^0}$ are correlated for $LL$ and $LR$ mass insertions, and anticorrelated for $RL$ and $RR$ mass insertions.

An interesting issue is the scaling of SUSY effects in $\Delta S$ with squark and gluino masses. We have noticed above that the constraints from other processes scale linearly with the SUSY masses. Now, it turns out that also the dominant SUSY contribution to $\Delta S$, the chromomagnetic one, scales linearly with SUSY masses as long as $m_{\tilde{g}} \sim m_{\tilde{q}} \sim \mu$. This
means that there is no decoupling of SUSY contributions to $\Delta S$ as long as the constraint from other processes can be saturated for $\delta < 1$. From Figs. 3-5 we see that the bounds on $LL$ and $RR$ mass insertions quickly reach the physical boundary at $\delta = 1$, while $LR$ and $RL$ are safely below that bound. Chirality flipping $LR$ and $RL$ insertions cannot become too large in order to ensure the absence of charge and color breaking minima and unbounded from below directions in the scalar potential [185]. However, it is easy to check that the flavor bounds given above are stronger for SUSY masses up to (and above) the TeV scale. We conclude that $LR$ and $RL$ mass insertions can give observable effects for SUSY masses
within the reach of LHC and even above. This is shown explicitly in Figs. 12 and 13, where we present the p.d.f. for $S_{\phi K_s}$, $S_{\pi^0 K_s}$, $S_{\eta' K_s}$ and $S_{\omega K_s}$ for SUSY masses of 1 TeV.

![Probability density functions for $S_{\phi K_s}$, $S_{\pi^0 K_s}$, $S_{\eta' K_s}$ and $S_{\omega K_s}$ induced by $(\delta_{23})_{LR}$ for $m_{\tilde{g}} = m_{\tilde{q}} = \mu = 1$ TeV.](image)

**FIG. 12**: Probability density functions for $S_{\phi K_s}$, $S_{\pi^0 K_s}$, $S_{\eta' K_s}$ and $S_{\omega K_s}$ induced by $(\delta_{23})_{LR}$ for $m_{\tilde{g}} = m_{\tilde{q}} = \mu = 1$ TeV.

**VI. NON-SUSY MODELS**

In general, one expects sizable values of $\Delta S$ in all models in which new sources of CP violation are present in $b \to s$ penguins. In particular, models with a fourth generation, both vectorlike and sequential, models with warped extra dimensions in which the flavor structure
FIG. 13: Probability density functions for $S_{\phi K_s}$, $S_{\pi^0 K_s}$, $S_{\eta' K_s}$ and $S_{\omega K_s}$ induced by $(\delta_{23})_{RL}$ for $m_{\tilde{g}} = m_{\tilde{q}} = \mu = 1$ TeV.

of the SM is obtained using localization of fermion wave functions, and models with $Z'$ gauge bosons can all give potentially large contributions to $b \to s$ penguins [186–190].

In any given NP model, it is possible to perform a detailed analysis along the lines of Sec. V, considering the constraints from $B_s - \bar{B}_s$ mixing and from rare $B$ decays, plus the constraints from all other sectors if they are correlated with $b \to s$ transitions in the model. On general grounds, the dominant contributions to $b \to s$ hadronic decays are expected to come from electroweak or chromomagnetic penguins. The correlation between the induced
$\Delta S_{PP}$ and $\Delta S_{PV}$ can give a handle on the chirality of the NP-generated operators. NP effects in electroweak penguin contributions are in general correlated with effects in $b \to s\ell^+\ell^-$, in $b \to s\gamma$ and possibly in $Z \to b\bar{b}$. Depending on the flavor structure of NP, other effects might be seen in $K \to \pi\nu\bar{\nu}$ or in $\varepsilon'/\varepsilon$. NP effects in the chromomagnetic penguin might also show up in $b \to s\gamma$, in $B_s - \bar{B}_s$ mixing and, if there is a correlation between the $B$ and $K$ sectors, in $\varepsilon'/\varepsilon$.

VII. CONCLUSIONS AND OUTLOOK

We have reviewed the theoretical status of hadronic $b \to s$ penguin decays. We have shown that, in spite of the theoretical difficulties in the evaluation of hadronic matrix elements, in the SM it is possible to obtain sound theoretical predictions for the coefficient $S_F$ of time-dependent CP asymmetries, using either models of hadronic dynamics or data-driven approaches. Experimental data show an interesting trend of deviations from the SM predictions that definitely deserves further theoretical and experimental investigation.

From the point of view of NP, the recent improvements in the experimental study of other $b \to s$ processes such as $B_s - \bar{B}_s$ transitions or $b \to s\gamma$ and $b \to s\ell^+\ell^-$ have considerably restricted the NP parameter space. However, there are still several NP models, in particular SUSY with new sources of $b \to s$ mixing in squark mass matrices, that can produce deviations from the SM in the ballpark of experimental values. In any given model, the study of hadronic $b \to s$ penguins and of the correlation with other FCNC processes in $B$ and $K$ physics is a very powerful tool to unravel the flavor structure of NP.

Any NP model with new sources of CP violation and new particles within the mass reach of the LHC can potentially produce sizable deviations from the SM in $b \to s$ penguins. It will be exciting to combine the direct information from the LHC and the indirect one from flavor physics to identify the physics beyond the SM that has been hiding behind the corner for the last decades. In this respect, future facilities for $B$ physics will provide us with an invaluable tool to study the origin of fermion masses and of flavor symmetry breaking, two aspects of elementary particle physics that remain obscure in spite of the theoretical and experimental efforts in flavor physics.
Acknowledgments

I am grateful to M. Ciuchini, E. Franco and M. Pierini for carefully reading this manuscript and for useful discussions. I acknowledge partial support from RTN European contracts MRTN-CT-2004-503369 “The Quest for Unification”, MRTN-CT-2006-035482 “FLAVIANet” and MRTN-CT-2006-035505 “Heptools”.

[1] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).
[2] M. Bona et al. [UTfit Collaboration], JHEP 0507, 028 (2005) [arXiv:hep-ph/0501199].
[3] M. Bona et al. [UTfit Collaboration], JHEP 0610, 081 (2006) [arXiv:hep-ph/0606167].
[4] M. Bona et al. [UTfit Collaboration], JHEP 0603, 080 (2006) [arXiv:hep-ph/0509219].
[5] M. Bona et al. [UTfit Collaboration], Phys. Rev. Lett. 97, 151803 (2006) [arXiv:hep-ph/0605213].
[6] J. Charles et al. [CKMfitter Group], Eur. Phys. J. C 41, 1 (2005) [arXiv:hep-ph/0406184].
[7] Y. Grossman and M. P. Worah, Phys. Lett. B 395, 241 (1997) [arXiv:hep-ph/9612269].
[8] M. Ciuchini, E. Franco, G. Martinelli, A. Masiero and L. Silvestrini, Phys. Rev. Lett. 79, 978 (1997) [arXiv:hep-ph/9704274].
[9] D. London and A. Soni, Phys. Lett. B 407, 61 (1997) [arXiv:hep-ph/9704277].
[10] K. F. Chen et al. [Belle Collaboration], Phys. Rev. Lett. 98, 031802 (2007) [arXiv:hep-ex/0608039].
[11] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0607112.
[12] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 98, 031801 (2007) [arXiv:hep-ex/0609052].
[13] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0702046.
[14] K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0609006.
[15] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0607096.
[16] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 98, 051803 (2007) [arXiv:hep-ex/0608051].
[17] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0607101.
[18] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0408095.
[19] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0702010.

[20] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0507016.

[21] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B 534, 3 (1998) [arXiv:hep-ph/9806308].

[22] C. Bobeth, M. Misiak and J. Urban, Nucl. Phys. B 567, 153 (2000) [arXiv:hep-ph/9904413].

[23] G. Degrassi, P. Gambino and G. F. Giudice, JHEP 0012, 009 (2000) [arXiv:hep-ph/0009337].

[24] A. J. Buras and L. Silvestrini, Nucl. Phys. B 569, 3 (2000) [arXiv:hep-ph/9812392].

[25] M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Phys. B 501, 271 (1997) [arXiv:hep-ph/9703353].

[26] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999) [arXiv:hep-ph/9905312].

[27] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000) [arXiv:hep-ph/0006124].

[28] M. Beneke and M. Neubert, Nucl. Phys. B 651, 225 (2003) [arXiv:hep-ph/0210085].

[29] M. Beneke and S. Jager, Nucl. Phys. B 751, 160 (2006) [arXiv:hep-ph/0512351].

[30] M. Beneke and S. Jager, Nucl. Phys. B 768, 51 (2007) [arXiv:hep-ph/0610322].

[31] H. n. Li and H. L. Yu, Phys. Rev. Lett. 74, 4388 (1995) [arXiv:hep-ph/9409313].

[32] H. N. Li and H. L. Yu, Phys. Lett. B 353, 301 (1995).

[33] H. n. Li and H. L. Yu, Phys. Rev. D 53, 2480 (1996) [arXiv:hep-ph/9411308].

[34] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D 65, 054022 (2002) [arXiv:hep-ph/0109045].

[35] C. W. Bauer, S. Fleming, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 66, 014017 (2002) [arXiv:hep-ph/0202088].

[36] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. 87, 201806 (2001) [arXiv:hep-ph/0107002].

[37] J. g. Chay and C. Kim, Phys. Rev. D 68, 071502 (2003) [arXiv:hep-ph/0301055].

[38] J. Chay and C. Kim, Nucl. Phys. B 680, 302 (2004) [arXiv:hep-ph/0301262].

[39] M. Ciuchini, R. Contino, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Phys. B 512, 3 (1998) [Erratum-ibid. B 531, 656 (1998)] [arXiv:hep-ph/9708222].

[40] M. Ciuchini, R. Contino, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Instrum. Meth. A 408, 28 (1998) [arXiv:hep-ph/9801420].
[41] C. Isola, M. Ladisa, G. Nardulli, T. N. Pham and P. Santorelli, Phys. Rev. D 64, 014029 (2001) [arXiv:hep-ph/0101118].
[42] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, Phys. Lett. B 515, 33 (2001) [arXiv:hep-ph/0104126].
[43] C. Isola, M. Ladisa, G. Nardulli, T. N. Pham and P. Santorelli, Phys. Rev. D 65, 094005 (2002) [arXiv:hep-ph/0110411].
[44] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, arXiv:hep-ph/0208048.
[45] C. Isola, M. Ladisa, G. Nardulli and P. Santorelli, Phys. Rev. D 68, 114001 (2003) [arXiv:hep-ph/0307367].
[46] P. Zenczykowski, Phys. Lett. B 590, 63 (2004) [arXiv:hep-ph/0402290].
[47] M. Ciuchini, E. Franco, G. Martinelli, A. Masiero, M. Pierini and L. Silvestrini, arXiv:hep-ph/0407073.
[48] H. Y. Cheng and K. C. Yang, Phys. Rev. D 64, 074004 (2001) [arXiv:hep-ph/0012152].
[49] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606, 245 (2001) [arXiv:hep-ph/0104110].
[50] M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003) [arXiv:hep-ph/0308039].
[51] M. Beneke, J. Rohrer and D. Yang, arXiv:hep-ph/0612290.
[52] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D 63, 054008 (2001) [arXiv:hep-ph/0004173].
[53] X. q. Li and Y. d. Yang, Phys. Rev. D 72, 074007 (2005) [arXiv:hep-ph/0508079].
[54] H. n. Li, S. Mishima and A. I. Sanda, Phys. Rev. D 72, 114005 (2005) [arXiv:hep-ph/0508041].
[55] H. n. Li and S. Mishima, Phys. Rev. D 74, 094020 (2006) [arXiv:hep-ph/0608277].
[56] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 70, 054015 (2004) [arXiv:hep-ph/0401188].
[57] C. W. Bauer, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 74, 034010 (2006) [arXiv:hep-ph/0510241].
[58] A. R. Williamson and J. Zupan, Phys. Rev. D 74, 014003 (2006) [Erratum-ibid. D 74, 03901 (2006)] [arXiv:hep-ph/0601214].
[59] J. D. Bjorken, Nucl. Phys. Proc. Suppl. 11, 325 (1989).
[60] C. H. Chang and H. n. Li, Phys. Rev. D 55, 5577 (1997) [arXiv:hep-ph/9607214].
[61] G. P. Lepage and S. J. Brodsky, Phys. Rev. D 22, 2157 (1980).

[62] S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B 625, 239 (2002) [arXiv:hep-ph/0109260].

[63] Y. Y. Keum, H. n. Li and A. I. Sanda, AIP Conf. Proc. 618, 229 (2002) [arXiv:hep-ph/0201103].

[64] D. Zeppenfeld, Z. Phys. C 8, 77 (1981).

[65] M. J. Savage and M. B. Wise, Phys. Rev. D 39, 3346 (1989) [Erratum-ibid. D 40, 3127 (1989)].

[66] L. L. Chau, H. Y. Cheng, W. K. Sze, H. Yao and B. Tseng, Phys. Rev. D 43, 2176 (1991) [Erratum-ibid. D 58, 019902 (1998)].

[67] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 50, 4529 (1994) [arXiv:hep-ph/9404283].

[68] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 52, 6374 (1995) [arXiv:hep-ph/9504327].

[69] M. Gronau and J. L. Rosner, Phys. Rev. D 53, 2516 (1996) [arXiv:hep-ph/9509325].

[70] B. Grinstein and R. F. Lebed, Phys. Rev. D 53, 6344 (1996) [arXiv:hep-ph/9602218].

[71] C. W. Chiang, M. Gronau, Z. Luo, J. L. Rosner and D. A. Suprun, Phys. Rev. D 69, 034001 (2004) [arXiv:hep-ph/0307395].

[72] C. W. Chiang, M. Gronau, J. L. Rosner and D. A. Suprun, Phys. Rev. D 70, 034020 (2004) [arXiv:hep-ph/0404073].

[73] C. W. Chiang and Y. F. Zhou, JHEP 0612, 027 (2006) [arXiv:hep-ph/0609128].

[74] R. Fleischer, Phys. Rev. D 58, 093001 (1998) [arXiv:hep-ph/9710331].

[75] R. Fleischer, Phys. Lett. B 435, 221 (1998) [arXiv:hep-ph/9804319].

[76] A. J. Buras and R. Fleischer, Eur. Phys. J. C 11, 93 (1999) [arXiv:hep-ph/9810260].

[77] R. Fleischer, Phys. Lett. B 459, 306 (1999) [arXiv:hep-ph/9903456].

[78] A. J. Buras and R. Fleischer, Eur. Phys. J. C 16, 97 (2000) [arXiv:hep-ph/0003323].

[79] R. Fleischer and J. Matias, Phys. Rev. D 66, 054009 (2002) [arXiv:hep-ph/0204101].

[80] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Eur. Phys. J. C 32, 45 (2003) [arXiv:hep-ph/0309012].

[81] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Phys. Rev. Lett. 92, 101804 (2004) [arXiv:hep-ph/0312259].
[82] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Acta Phys. Polon. B 36, 2015 (2005) [arXiv:hep-ph/0410407].

[83] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, Eur. Phys. J. C 45, 701 (2006) [arXiv:hep-ph/0512032].

[84] R. Fleischer, S. Recksiegel and F. Schwab, arXiv:hep-ph/0702275.

[85] G. Engelhard, Y. Nir and G. Raz, Phys. Rev. D 72, 075013 (2005) [arXiv:hep-ph/0505194].

[86] G. Engelhard and G. Raz, Phys. Rev. D 72, 114017 (2005) [arXiv:hep-ph/0508046].

[87] G. Raz, arXiv:hep-ph/0509125.

[88] M. Neubert and J. L. Rosner, Phys. Rev. Lett. 81, 5076 (1998) [arXiv:hep-ph/9809311].

[89] M. Gronau, D. Pirjol and T. M. Yan, Phys. Rev. D 60, 034021 (1999) [Erratum-ibid. D 69, 119901 (2004)] [arXiv:hep-ph/9810482].

[90] M. Neubert, JHEP 9902, 014 (1999) [arXiv:hep-ph/9812396].

[91] A. Khodjamirian, T. Mannel and M. Melcher, Phys. Rev. D 68, 114007 (2003) [arXiv:hep-ph/0308297].

[92] M. Ciuchini et al., in preparation.

[93] M. Ciuchini, M. Pierini and L. Silvestrini, Phys. Rev. Lett. 95, 221804 (2005) [arXiv:hep-ph/0507290].

[94] M. Ciuchini, M. Pierini and L. Silvestrini, arXiv:hep-ph/0703137.

[95] A. Ali, G. Kramer, Y. Li, C. D. Lu, Y. L. Shen, W. Wang and Y. M. Wang, arXiv:hep-ph/0703162.

[96] M. Morello [CDF Collaboration], arXiv:hep-ex/0612018.

[97] S. Baek, P. Hamel, D. London, A. Datta and D. A. Suprun, Phys. Rev. D 71, 057502 (2005) [arXiv:hep-ph/0412086].

[98] C. S. Kim, S. Oh and C. Yu, Phys. Rev. D 72, 074005 (2005) [arXiv:hep-ph/0505060].

[99] S. Baek and D. London, arXiv:hep-ph/0701181.

[100] R. Fleischer, arXiv:hep-ph/0701217.

[101] M. Beneke, Phys. Lett. B 620, 143 (2005) [arXiv:hep-ph/0505075].

[102] A. Abulencia et al. [CDF Collaboration], Phys. Rev. Lett. 97, 242003 (2006) [arXiv:hep-ex/0609040].

[103] M. Bona et al. [UTfit Collaboration], in preparation.

[104] J. M. Soares and L. Wolfenstein, Phys. Rev. D 47, 1021 (1993).
[105] N. G. Deshpande, B. Dutta and S. Oh, Phys. Rev. Lett. 77, 4499 (1996) [arXiv:hep-ph/9608231].

[106] J. P. Silva and L. Wolfenstein, Phys. Rev. D 55, 5331 (1997) [arXiv:hep-ph/9610208].

[107] A. G. Cohen, D. B. Kaplan, F. Lepeintre and A. E. Nelson, Phys. Rev. Lett. 78, 2300 (1997) [arXiv:hep-ph/9610252].

[108] Y. Grossman, Y. Nir and M. P. Worah, Phys. Lett. B 407, 307 (1997) [arXiv:hep-ph/9704287].

[109] V. M. Abazov et al. [D0 Collaboration], arXiv:hep-ex/0701007.

[110] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. D 74, 092001 (2006) [arXiv:hep-ex/0609014].

[111] D. Buskulic et al. [ALEPH Collaboration], Phys. Lett. B 377, 205 (1996).

[112] F. Abe et al. [CDF Collaboration], Phys. Rev. D 59, 032004 (1999) [arXiv:hep-ex/9808003].

[113] P. Abreu et al. [DELPHI Collaboration], Eur. Phys. J. C 16, 555 (2000) [arXiv:hep-ex/0107077].

[114] K. Ackerstaff et al. [OPAL Collaboration], Phys. Lett. B 426, 161 (1998) [arXiv:hep-ex/9802002].

[115] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 97, 241801 (2006) [arXiv:hep-ex/0604046].

[116] E. Barberio et al. [Heavy Flavor Averaging Group (HFAG)], arXiv:hep-ex/0603003.

[117] D. Acosta et al. [CDF Collaboration], Phys. Rev. Lett. 94, 101803 (2005) [arXiv:hep-ex/0412057].

[118] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 98, 121801 (2007) [arXiv:hep-ex/0701012].

[119] Z. Ligeti, M. Papucci and G. Perez, Phys. Rev. Lett. 97, 101801 (2006) [arXiv:hep-ph/0604112].

[120] P. Ball and R. Fleischer, Eur. Phys. J. C 48, 413 (2006) [arXiv:hep-ph/0604249].

[121] Y. Grossman, Y. Nir and G. Raz, Phys. Rev. Lett. 97, 151801 (2006) [arXiv:hep-ph/0605028].

[122] A. Lenz and U. Nierste, arXiv:hep-ph/0612167.

[123] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 96, 251802 (2006) [arXiv:hep-ex/0603053].

[124] A. Ishikawa et al. [Belle Collaboration], Phys. Rev. Lett. 91, 261601 (2003) [arXiv:hep-
[125] P. Koppenburg et al. [Belle Collaboration], Phys. Rev. Lett. 93, 061803 (2004) [arXiv:hep-ex/0403004].

[126] M. Iwasaki et al. [Belle Collaboration], Phys. Rev. D 72, 092005 (2005) [arXiv:hep-ex/0503044].

[127] A. Ishikawa et al. [Belle Collaboration], Phys. Rev. Lett. 96, 251801 (2006) [arXiv:hep-ex/0603018].

[128] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 93, 081802 (2004) [arXiv:hep-ex/0404006].

[129] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 72, 052004 (2005) [arXiv:hep-ex/0508004].

[130] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 73, 092001 (2006) [arXiv:hep-ex/0604007].

[131] A. Ali, G. F. Giudice and T. Mannel, Z. Phys. C 67, 417 (1995) [arXiv:hep-ph/9408213].

[132] A. Ali, E. Lunghi, C. Greub and G. Hiller, Phys. Rev. D 66, 034002 (2002) [arXiv:hep-ph/0112300].

[133] G. Hiller and F. Kruger, Phys. Rev. D 69, 074020 (2004) [arXiv:hep-ph/0310219].

[134] P. Gambino, U. Haisch and M. Misiak, Phys. Rev. Lett. 94, 061803 (2005) [arXiv:hep-ph/0410155].

[135] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0703020.

[136] M. Staric et al. [Belle Collaboration], arXiv:hep-ex/0703036.

[137] K. Abe et al. [BELLE Collaboration], arXiv:0704.1000 [hep-ex].

[138] M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, V. Porretti and L. Silvestrini, arXiv:hep-ph/0703204.

[139] E. Gabrielli and G. F. Giudice, Nucl. Phys. B 433, 3 (1995) [Erratum-ibid. B 507, 549 (1997)] [arXiv:hep-lat/9407029].

[140] M. Misiak, S. Pokorski and J. Rosiek, Adv. Ser. Direct. High Energy Phys. 15, 795 (1998) [arXiv:hep-ph/9703442].

[141] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B 500, 161 (2001) [arXiv:hep-ph/0007085].

[142] G. D’Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B 645, 155 (2002)
[arXiv:hep-ph/0207036].

[143] C. Bobeth, M. Bona, A. J. Buras, T. Ewerth, M. Pierini, L. Silvestrini and A. Weiler, Nucl. Phys. B 726, 252 (2005) [arXiv:hep-ph/0505110].

[144] M. Blanke, A. J. Buras, D. Guadagnoli and C. Tarantino, JHEP 0610, 003 (2006) [arXiv:hep-ph/0604057].

[145] M. Ciuchini, E. Gabrielli and G. F. Giudice, Phys. Lett. B 388, 353 (1996) [Erratum-ibid. B 393, 489 (1997)] [arXiv:hep-ph/9604438].

[146] A. Masiero, M. Piai, A. Romanino and L. Silvestrini, Phys. Rev. D 64, 075005 (2001) [arXiv:hep-ph/0104101].

[147] S. Baek, T. Goto, Y. Okada and K. i. Okumura, Phys. Rev. D 63, 051701 (2001) [arXiv:hep-ph/0002141].

[148] R. Harnik, D. T. Larson, H. Murayama and A. Pierce, Phys. Rev. D 69, 094024 (2004) [arXiv:hep-ph/0212180].

[149] J. Hisano and Y. Shimizu, Phys. Lett. B 565, 183 (2003) [arXiv:hep-ph/0303071].

[150] C. S. Huang, T. j. Li and W. Liao, Nucl. Phys. B 673, 331 (2003) [arXiv:hep-ph/0304130].

[151] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477, 321 (1996) [arXiv:hep-ph/9604387].

[152] R. Barbieri and A. Strumia, Nucl. Phys. B 508, 3 (1997) [arXiv:hep-ph/9704402].

[153] A. L. Kagan and M. Neubert, Phys. Rev. D 58, 094012 (1998) [arXiv:hep-ph/9803368].

[154] S. A. Abel, W. N. Cottingham and I. B. Whittingham, Phys. Rev. D 58, 073006 (1998) [arXiv:hep-ph/9803401].

[155] A. Kagan, arXiv:hep-ph/9806266.

[156] R. Fleischer and T. Mannel, Phys. Lett. B 511, 240 (2001) [arXiv:hep-ph/0103121].

[157] T. Besmer, C. Greub and T. Hurth, Nucl. Phys. B 609, 359 (2001) [arXiv:hep-ph/0105292].

[158] E. Lunghi and D. Wyler, Phys. Lett. B 521, 320 (2001) [arXiv:hep-ph/0109149].

[159] M. B. Causse, arXiv:hep-ph/0207070.

[160] G. Hiller, Phys. Rev. D 66, 071502 (2002) [arXiv:hep-ph/0207356].

[161] S. Khalil and E. Kou, Phys. Rev. D 67, 055009 (2003) [arXiv:hep-ph/0212023].

[162] G. L. Kane, P. Ko, H. b. Wang, C. Kolda, J. h. Park and L. T. Wang, Phys. Rev. D 70, 035015 (2004) [arXiv:hep-ph/0212092].

[163] S. Baek, Phys. Rev. D 67, 096004 (2003) [arXiv:hep-ph/0301269].
[164] K. Agashe and C. D. Carone, Phys. Rev. D 68, 035017 (2003) [arXiv:hep-ph/0304229].

[165] J. F. Cheng, C. S. Huang and X. h. Wu, Phys. Lett. B 585, 287 (2004) [arXiv:hep-ph/0306086].

[166] D. Chakraverty, E. Gabrielli, K. Huitu and S. Khalil, Phys. Rev. D 68, 095004 (2003) [arXiv:hep-ph/0306076].

[167] S. Khalil and E. Kou, Phys. Rev. Lett. 91, 241602 (2003) [arXiv:hep-ph/0303214].

[168] S. Khalil and E. Kou, In the Proceedings of 2nd Workshop on the CKM Unitarity Triangle, Durham, England, 5-9 Apr 2003, pp WG305 [arXiv:hep-ph/0307024].

[169] J. F. Cheng, C. S. Huang and X. H. Wu, Nucl. Phys. B 701, 54 (2004) [arXiv:hep-ph/0404055].

[170] S. Khalil and E. Kou, Phys. Rev. D 71, 114016 (2005) [arXiv:hep-ph/0407284].

[171] E. Gabrielli, K. Huitu and S. Khalil, Nucl. Phys. B 710, 139 (2005) [arXiv:hep-ph/0407291].

[172] S. Khalil, Mod. Phys. Lett. A 19, 2745 (2004) [Afr. J. Math. Phys. 1, 101 (2004)] [arXiv:hep-ph/0411151].

[173] S. Khalil, Phys. Rev. D 72, 035007 (2005) [arXiv:hep-ph/0505151].

[174] G. Isidori and A. Retico, JHEP 0111, 001 (2001) [arXiv:hep-ph/0110121].

[175] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B 659, 3 (2003) [arXiv:hep-ph/0210145].

[176] J. Foster, K. i. Okumura and L. Roszkowski, JHEP 0603, 044 (2006) [arXiv:hep-ph/0510422].

[177] J. Foster, K. i. Okumura and L. Roszkowski, Phys. Lett. B 641, 452 (2006) [arXiv:hep-ph/0604121].

[178] G. Isidori and P. Paradisi, Phys. Lett. B 639, 499 (2006) [arXiv:hep-ph/0605012].

[179] G. Isidori, F. Mescia, P. Paradisi and D. Temes, arXiv:hep-ph/0703035.

[180] M. Ciuchini, E. Franco, A. Masiero and L. Silvestrini, Phys. Rev. D 67, 075016 (2003) [Erratum-ibid. D 68, 079901 (2003)] [arXiv:hep-ph/0212397].

[181] M. Ciuchini and L. Silvestrini, Phys. Rev. Lett. 97, 021803 (2006) [arXiv:hep-ph/0603114].

[182] M. Ciuchini et al., in preparation.

[183] A. L. Kagan, arXiv:hep-ph/0407076.

[184] M. Endo, S. Mishima and M. Yamaguchi, Phys. Lett. B 609, 95 (2005) [arXiv:hep-ph/0409245].

[185] J. A. Casas and S. Dimopoulos, Phys. Lett. B 387, 107 (1996) [arXiv:hep-ph/9606237].
[186] D. Atwood and G. Hiller, arXiv:hep-ph/0307251.

[187] G. Burdman, Phys. Lett. B 590, 86 (2004) [arXiv:hep-ph/0310144].

[188] G. Buchalla, G. Hiller, Y. Nir and G. Raz, JHEP 0509, 074 (2005) [arXiv:hep-ph/0503151].

[189] K. Agashe, M. Papucci, G. Perez and D. Pirjol, arXiv:hep-ph/0509117.

[190] W. S. Hou, H. n. Li, S. Mishima and M. Nagashima, Phys. Rev. Lett. 98, 131801 (2007) [arXiv:hep-ph/0611107].