Flat structures on Frobenius manifolds in the case of irrelevant deformations

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Abstract

In this paper we use a recently suggested conjecture about the integral representation for flat coordinates on Frobenius manifolds, connected with isolated singularities, to compute the flat coordinates and Saito primitive form on the space of deformations of Gepner chiral ring $SU(3)_k$. We verify this conjecture comparing the expressions for flat coordinates obtained from the conjecture with the one found by direct computation. The considered case is of a particular interest since apart from the relevant and marginal deformations, it also has an irrelevant one.

Keywords: string theory, topological field theory, Frobenius manifolds, Saito primitive form

1. Introduction

This paper continues the series of works [1–3] and is dedicated to a new approach to computations of flat coordinates of the Frobenius manifolds connected with isolated singularity [4, 5].

Frobenius structure arises in three kinds of models of quantum field theory (QFT) and in string theory. Namely, in the models of two-dimensional topological conformal field theory (CFT) [6], in models of space–time supersymmetric compactifications of string theory on Calabi–Yau manifolds [7–10] and in models of Polyakov noncritical string theory [11].

One of the key ingredients in solutions of these models is the knowledge [6, 12, 13] of flat coordinates on the corresponding Frobenius manifolds.

A new method to compute flat coordinates, based on a conjecture about an integral representation, has been suggested for the models connected with the simple ADE singularities in [1]. In [2, 3] this method and the conjecture itself were formulated for general isolated
singularity. Also in these works the conjecture was verified for a model with a number of relevant and one marginal deformations.

The aim of this work is to use and verify the conjecture in a computation of flat coordinates on Frobenius manifolds of deformations of the Gepner chiral ring $SU(3)_4$. This model has one marginal and one irrelevant deformation. We verify the results obtained by the use of the conjecture by comparison with those of the direct computation.

In section 2 we briefly review the Dubrovin–Saito theory. In section 3 we formulate the conjecture about integral representation for flat coordinates in the case when the Saito primitive form is not trivial, which happens if the marginal and irrelevant deformations take place. In section 4 we review some necessary facts about the deformation of the Gepner chiral ring $SU(3)_4$, for which our computations are performed. In section 5 we explain how one can find metrics and flat coordinates in a direct way from the Frobenius manifold structure axioms; we also compare the results of both computations and find that they coincide. We provide more detailed expressions for the flat coordinates and the primitive form in the appendix.

2. Preliminaries

In this section, we review the role of flat coordinates of the Frobenius manifold in the case of topological CFT which comes from the Witten twist and restriction of space of states on the chiral sector of the $N = 2$ SCTF Landau–Ginzburg model $[14]$. In these models the superpotential $W_0[\Phi_1, ..., \Phi_n]$ depends on $n$ fundamental chiral fields. These fields generate the chiral ring $R_0$ and we will denote a basis in it by $F_a$ for $a = 1, ..., n$. Here $M = \dim R_0$, the first fields with $a = 1, ..., n$ will be the generators of the ring and $F = 1$ is the unit operator.

The chiral ring $R_0$ is isomorphic to the ring of the polynomials of $x_i$

$$R_0 = \mathbb{C}^n[x_1, ..., x_n]/\left\{\frac{\partial W_0}{\partial x_i}\right\},$$

(2.1)

where $\left\{\frac{\partial W_0}{\partial x_i}\right\}$ denotes the ideal generated by the partial derivatives of the polynomial $W_0[x_i]$.

In $[6]$ it was shown that to compute correlation functions of the fields $\Phi_a$ and its superpartners $\Phi_a^{(1,1)} = G_{-1,1}G_{-1,1}^{*}\Phi_a$ it is necessary and sufficient to know the two-point functions

$$\eta_{\alpha\beta} = \langle \Phi_\alpha \Phi_\beta \rangle,$$

(2.2)

together with the perturbed three-point function

$$C_{\alpha\beta\gamma}(s_1, ..., s_M) \overset{\text{def}}{=} \left\{ \Phi_\alpha \Phi_\beta \Phi_\gamma \exp\left(\sum_{\lambda=1}^{M} s_\lambda \int \phi^{(1,1)} \cdot d^2z\right) \right\}. $$

(2.3)

It was also shown in $[6]$ that $\eta_{\alpha\beta}$ is non-degenerate and $s$-independent and $C_{\alpha\beta\gamma}(s)$ can be expressed in terms of a prepotential (or the free energy) $F$

$$C_{\alpha\beta\gamma} = \frac{\partial^3 F}{\partial s_\alpha \partial s_\beta \partial s_\gamma}. $$

(2.4)

At last $C_{\alpha\beta}^{jo} \overset{\text{def}}{=} \eta^{j\delta}C_{\alpha\beta\delta}$ are subject of the equation

$$C_{\alpha\eta\delta}^{jo} C_{\eta\beta\rho}^{\delta} = C_{\alpha\rho}^{\delta} C_{\beta\eta\delta}^{\mu}.$$  

(2.5)
These relations together with the evident property $C_{αβ} = C_{βα}$ mean that $C_{αβ}(s)$ are structure constants for a commutative, associative algebra or a ring $R$ with unity which depends on the parameters $\{s_{α}\}$. At $s_{α} = 0$ this ring coincides with the chiral ring $R_{0}$ defined by (2.1).

The properties of $η_{αβ}$ and $C_{αβ}(s)$ mean that we have indeed the Frobenius manifold structure [5] and $s_{α}$ are nothing but flat coordinates on this manifold, i.e. such coordinates in which the Riemannian metrics $η_{αβ}$ is constant.

The crucial fact found in [6], which makes possible to exactly solve the topological models of such kinds, is that the Frobenius manifold, defined by $C_{αβ}(s)$ and $η_{αβ}$, coincides with a Frobenius manifold defined by the versal deformation $W(x, t)$ of the superpotential $W_{0}$

$$W(x, t) \overset{\text{def}}{=} W_{0}(x) + \sum_{α=1}^{M} t_{α} e_{α}(x). \quad (2.6)$$

Here $\{e_{α}\}$ is a basis of the ring $R_{0}$ (2.1), and $e_{1}(x) = 1$ is a unit element of $R_{0}$. The corresponding ring, defined by $W(x, t)$ as the ring of polynomials of $x_{α}$ is given by

$$R_{W} = C[x_{1}, ..., x_{n}] \left\{ \frac{∂W}{∂x_{j}} \right\}.$$  \quad (2.7)

The structure constants $\tilde{C}_{αβ}^{γ}(t)$ of $R_{W}$ in the basis $\{e_{α}\}$ are defined by the relations

$$e_{α} e_{β} = \tilde{C}_{αβ}^{γ}(t) e_{γ} \mod \left\{ \frac{∂W}{∂x_{j}} \right\}. \quad (2.8)$$

The Riemannian metrics $g_{αβ}(t)$ are defined as a Grotendick residue in terms of the Saito primitive form [4]

$$Ω(x, t) = λ(x, t) dx_{1} ∧ ... ∧ x_{n} \quad (2.9)$$
as follows

$$g_{μν} = \text{Res}_{x=∞} e_{μ} e_{ν} Ω \prod_{i} \frac{∂W}{∂x_{i}}. \quad (2.10)$$

It was proved in [15] that the primitive form does exist. Namely, there exists such a differential form $Ω(x, t)$, so that the structure constants $\tilde{C}_{αβ}^{γ}(t)$ (2.8) and the Riemannian metrics $g_{αβ}(t)$ defined in (2.10) satisfy the Dubrovin–Frobenius manifold axioms:

$$\tilde{C}_{αβ}^{γ} \tilde{C}_{γρ}^{μ} = \tilde{C}_{μρ}^{α} \tilde{C}_{ρβ}^{γ}. \quad (2.11)$$

$$R_{\muλνρ} [g_{αβ}] = 0, \quad (2.12)$$

$$\nabla_{ν} \tilde{C}_{μλ}^{ρ} = \nabla_{ρ} \tilde{C}_{μλ}^{ν}, \quad (2.13)$$

$$\tilde{C}_{μλ}^{ρ} = \tilde{C}_{ρμλ} \quad (2.14)$$

The deformation parameters $\{t_{α}\}$ are coordinates on the Frobenius manifold. The coupling constants $s^{μ}$ are the flat coordinates on it. They are functions of the deformation parameters $\{t_{α}\}$.

The knowledge of these functions permit to express the perturbed three-point functions $C_{αβγ}$ and the prepotential $F$ in terms of $g_{μν}$ and $\tilde{C}_{αβ}^{γ}(t)$. Thus determination of the functions $s^{μ}(t)$ and the primitive form $Ω(x, t)$ is the major part of the solution of the topological Landau–Ginzburg model.
3. The flat coordinates via the oscillating integrals

We will assume that the superpotential $W_0(x)$ is a quasihomogeneous polynomial associated to an isolated singularity

$$W_0(A^\alpha x^\beta) = A^\rho W_0(x_i),$$

where integer weights $d = [W_0]$ and $\rho_i = [x_i].$

In this case we can choose the basis $e_\alpha$ of the ring $R_W$ to be quasihomogeneous. We will denote its weights by $e_\alpha \in \mathbb{Z}$. The elements of the basis are called relevant, marginal or irrelevant if their weights satisfy the relations $\deg e_\alpha < d$, $\deg e_\alpha = d$ or $\deg e_\alpha > d$ correspondingly.

It was conjectured in [1, 3] that the flat coordinates are given by the following integral expression

$$s_\mu(t) = \sum_{m_\alpha \in \Sigma_\mu} \left( \int_{\gamma_\mu} \exp(W_0(x)) \prod_{\alpha} e_\alpha^m \Omega \prod_{\alpha} \frac{t_{m_\alpha}}{m_\alpha} \right),$$

where $\Sigma_\mu$ is specified by requirement for the l.h.s. and r.h.s. of this equation to have the same weights. We will give their explicit expression for this below.

The cycles $\gamma_\mu$ form a basis for the homology $H_n(C^n, \text{Re} W_0 = -\infty)$ which defined as $\lim_{n \to +\infty} H_n(C^n/\{\text{Re} W_0 \leq -L\})^\delta$.

A simple example of the explicit choice of such cycles for $n = 1$ has been given in [1] on figure 2. It illustrates this notion.

We fix the normalization of the coordinates by the requirement for the first term of the decomposition to be $s_\mu = t_\mu + \ldots$.

We compute the integrals in (3.2) in the same way as in [3]. The main point of the computation is the following property of the oscillating integrals

$$\int_{\gamma} \exp(W_0(x)) P_1(x)dx = \int_{\gamma} \exp(W_0(x)) P_2(x)dx,$$

if there exists an $(n-1)$-form $U$ such that

$$P_1(x)dx = P_2(x)dx + D_W U,$$

where $D_W$ is the Saito differential

$$D_W = d + dW_0 \wedge .$$

The differential $D_W$ defines the Saito cohomology $H^n$ on the space of n-forms.

The forms $e_\mu dx$ for $\mu = 1, \ldots, M$ can be chosen as a convenient basis in $H^n$.

Let us define a pairing between the elements of the basis $e_\mu dx$ in $H^n$ and the cycles $\gamma_\mu$ as

$$r_{\mu,\nu} = \int_{\gamma_\mu} \exp(W_0(x)) e_\nu dx.$$
However, a more general possibility has to be considered. The reason for this is the appearance of resonances. We will call resonance the case when the weights of some coordinates satisfy
\[ m_n s \equiv 0 \mod d \] and
\[ \mu \equiv \nu \mod d. \]
We assume \( r_{\mu,\nu} = 1 \) for all \( \mu \) and \( r_{\nu,\nu} = 0 \) if coordinates \( s_\mu \) and \( s_\nu \) are not in resonance. Doing it in this way we obtain the expressions for flat coordinates, which depend on some extra parameters as was predicted in [17].

From dimensional reasoning the primitive form \( \Omega \) must be decomposed as
\[ \Omega = \sum_{n,l \in \omega} A(n, l) \prod_\alpha e^{m_\alpha + l_\alpha} dx, \] (3.8)
where the summation domain \( \omega \) is defined as
\[ \omega : \sum_\alpha (n_\alpha [e_\alpha] + l_\alpha [l_\alpha]) = 0, \quad n_\alpha \geq 0, \quad l_\alpha \geq 0. \] (3.9)

By substituting (3.8) into (3.2) one finds
\[ s_\mu(t) = \sum_{m+n=\mu} \int e^{W_0(x)} A(n, l) \prod_\alpha e^{m_\alpha + n_\alpha} dx \prod_\alpha \frac{m_\alpha + l_\alpha}{m_\alpha^!}. \] (3.10)

Now, we can give the expression for \( \Sigma_\mu \) more explicitly
\[ \Sigma_\mu : \sum_\alpha (m_\alpha + l_\alpha)[l_\alpha] = [s_\mu], \quad m_\alpha \geq 0. \] (3.11)

Since elements \( e_\mu dx \) form a basis of \( H^n \), any n-form can be decomposed in it. In particular,
\[ \prod_\alpha e_\alpha dx = \sum_\mu B_\mu(k) e_\mu dx + D_{W_0} U. \] (3.12)

From the homogeneity requirements only such elements \( e_\mu dx \) of the basis appear in the r.h.s of this equation whose weights are equal to those of the l.h.s module \( d \). In the case of the resonance several elements of the same weights can appear in the r.h.s. of (3.12). Their appearance in the oscillating integrals (3.6) is the reason for the emergence of the parameters \( r_{\mu,\nu} \) in the expressions for \( s_\mu \) when \( e_\mu \) and \( e_\nu \) are in a resonance.

In the case of our interest \( SU(3)_4 \), which is considered below, there are two resonances \([s_1] - [s_{14}] = 7 \) and \([s_2] - [s_{15}] = 7 \). We find that in this case two parameters \( r_{1,14} \) and \( r_{2,15} \), if they are not assumed to be equal zero, arise in the expressions for the flat coordinates derived from (3.2).

For given \( k_\mu \), we can solve the equation (3.12) and find the coefficients \( B_\mu(k) \). Substitution of them into (3.10) gives
\[ s_\mu(t) = \sum_{m+n=\mu} A(n, l) B_\mu(m + n) \prod_\alpha \frac{m_\alpha + l_\alpha}{m_\alpha^!}. \] (3.13)

This formula gives expressions for \( s_\mu \) which depend on the unknown parameters \( A(n, l) \) of the primitive form. We find these parameters using the normalisation conditions together with the equation [3]

5 In the case of our interest \( SU(3)_4 \) this gives four parameters \( r_{1,14}, r_{2,15}, r_{14,1}, r_{15,2} \). The last two are fixed by the normalisation condition and the equation (3.14). However, \( r_{1,14}, r_{2,15} \) stay to be free parameters.
In such a way we arrive to the explicit expression for flat coordinates. The final answer for the flat coordinates contains no free parameters besides those of \( r_{\mu, \nu} \), which correspond to the resonances.

4. The deformed chiral ring \( \tilde{S}U(3)_4 \)

In the topological CFT, which is connected with the deformed chiral ring \( \tilde{S}U(3)_4 \) \([14, 20]\), the superpotential is

\[
W_0(x_1, x_2) = \frac{q_1^2 + q_2^2}{7},
\]

where \( x_i = q_i + q_2 \), \( x_2 = q_1 q_2 \).

We choose the basis of the ring to be Schur polynomials \([3]\]

\[
e_1 \equiv e_0 = 1, \quad e_2 \equiv e_0 = q_1 + q_2, \quad e_3 \equiv e_0 = q_1 q_2, \quad e_4 \equiv e_0 = q_1^2 + q_1 q_2 + q_2^2,
\]

\[
e_5 \equiv e_0 = q_1 q_2 (q_1 + q_2), \quad e_6 \equiv e_0 = q_1^2 + q_1 q_2 + q_2^2, \quad e_7 \equiv e_0 = q_1^2 q_2^2,
\]

\[
e_8 \equiv e_0 = q_1 q_2 (q_1^2 + q_1 q_2 + q_2^2), \quad e_9 \equiv e_0 = q_1^2 + q_1 q_2 + q_2^2, \quad e_{10} \equiv e_0 = q_1^2 q_2^2 + q_1 q_2^2 + q_1 q_2 + q_2^2.
\]

The deformed superpotential is

\[
W(x_1, x_2) = \frac{q_1^2 + q_2^2}{7} + \sum_{\mu=1}^{15} t_\mu e_\mu
\]

The first 13 elements of this basis are related to relevant deformations. The elements \( e_{14} \) and \( e_{15} \) correspondingly related to the marginal and irrelevant deformations.

We computed the flat coordinates up to the 6th order in \( t \) by using the technique of the previous section. The expressions for them up to the 2nd order are presented in the appendix. We also include the answer for the primitive form \( \Omega \) up to the 2nd order in \( t \).

5. Direct computation of flat coordinates

The expression for the flat coordinates (3.13) is a conjecture. This conjecture was tested in \([2, 3]\) for the topological CFT connected with chiral ring \( \tilde{S}U(3)_3 \), where one marginal deformation exists.

One of the main aims of this work is to check this conjecture for the case when there are also irrelevant deformations such as in the model connected with the deformed Gepner chiral ring \( \tilde{S}U(3)_4 \).

In order to do this, we have to compute the flat coordinates the direct way. We completed this perturbatively in overall \( t \) up to 4th order in \( t \). The final answers are too lengthy to be presented here. Therefore, we only outline the main steps of the calculation, giving as much detail as possible.
The metric on the Frobenius manifold is defined as

\[ g_{\mu\nu} = \text{Res}_{x=\infty} \frac{e_\mu e_\nu \Omega}{\prod_i \partial W / \partial x_i}. \]

Instead of computing this residue, we will follow the method used in [16]. Namely, we rewrite the metrics as

\[ g_{\mu\nu} = c^\lambda_{\mu\nu}(t) \text{Res}_{x=\infty} \frac{e_\lambda \Omega}{\prod_i \partial W / \partial x_i} = c^\lambda_{\mu\nu}(t) h_\lambda(t), \]

where \( h_\lambda(t) \) are some unknown functions of \( t \). These functions can be found from the Frobenius axioms

\[ R_{\mu\nu\lambda\rho} [g_{\alpha\beta}] = 0, \tag{5.3} \]

\[ \nabla_\mu C_{\nu\lambda\rho} = \nabla_\rho C_{\nu\lambda\mu}, \tag{5.4} \]

\[ C_{\mu\nu\lambda} = C_{\lambda\nu\mu} = C_{\mu\lambda\nu}, \tag{5.5} \]

where \( R_{\mu\nu\lambda\rho} \) is the Riemann curvature, and \( C_{\mu\nu\lambda} \) is structure constants with index lowered by \( g_{\alpha\beta} \).

By using a computer we found the expression for the metric up to the 4th order in \( t \). From equation (5.3) we have found expressions for \( h_\lambda(t) \) which still contain two parameters. After these equations (5.5) is automatically satisfied. The solution of (5.4) fixes the value of the one of the two parameters leaving only one. The fact that the solution of equations (5.3)–(5.5) has one parameter is in perfect agreement with [17].

Finally, one can find flat coordinates from the equation

\[ \frac{\partial^2 x_i}{\partial t_\mu \partial t_\nu} = \Gamma^\lambda_{\alpha\beta} \frac{\partial^2 x_i}{\partial t_\lambda}. \tag{5.6} \]

Since the metrics found from equations (5.3)–(5.5) contain one parameter, the flat coordinates will also contain a parameter. These results are in perfect agreement up to the fourth order with the one of section 3, if we impose the constraint \( r_{1,14} = r_{2,15} \).

6. Conclusion

The comparison of two computations, performed in this work, shows that after imposing the constraint on the parameters \( r_{1,14} = r_{2,15} \) the both expressions for the flat coordinates coincide. This coincidence confirms the correctness of conjecture (3.2), now for the case when there is one marginal and one irrelevant deformations in the addition to the relevant ones. In the same time, this comparison leads to an interesting question about the nature of the constraints which have to be imposed on the extra parameters \( r_{\mu,\nu} \), predicted in [17].

Once the correct way to fix the superfluous resonance parameter is determined, the conjectured method can be used to determine the flat coordinates on any homogeneous isolated singularity. For example, one can use it to determine the flat structures of the other Kasama–Suzuki models. Knowledge of flat coordinates is important in the study of the minimal models of two-dimensional \( W \) gravity, whose precise formulation is absent.

Recently, a new perturbative method to compute the flat coordinates and the primitive form has been suggested in [18, 19]. It would be interesting to understand the connection between this method and our approach. Using the results of [18, 19] can probably help to prove the conjecture (3.2) concerning the representation of flat coordinates through oscillating integrals.
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Appendix

Expressions for $s_\mu$ up to second order in $t$

Here we present expressions for the flat coordinates obeyed via the conjecture. In order for these answers to coincide with the direct computation of section 5 one must set $r_{1,14} = r_{2,15}$.

$s_{15} = t_5 - (r_{1,14} + r_{2,15}) t_4 t_5$

$s_{14} = t_4 - r_{1,14} t_4^2 - r_{2,15} t_5 t_4 - r_{2,15} s_{15}$

$s_{13} = t_3 + (3 - r_{1,14}) t_3 t_4 + (3 - r_{2,15}) t_4 t_5 - r_{2,15} s_{15}$

$s_{12} = t_2 - r_{1,14} t_2 t_4 - r_{2,15} s_{15}$

$s_{11} = t_1 + t_1^2 + 2 t_2 t_3 + (2 - r_{1,14}) t_1 t_4 + 2 t_0 t_4 - r_{2,15} s_{15}$

$s_{10} = t_0 + \frac{3 t_1^2}{2} + 3 t_1 t_4 - r_{1,14} t_0 t_4 + 3 t_1 t_5 - r_{2,15} s_{15}$

$s_9 = t_9 + t_1 t_3 + t_0 t_3 + t_0 t_2 - r_{1,14} t_2 t_4 + t_0 t_4 + t_3 t_4 - r_{2,15} s_{15}$

$s_8 = t_8 + 2 t_1 t_3 + 2 t_1 t_2 + t_0 t_4 + 2 t_0 t_4 + (2 - r_{2,15}) t_8 t_4$

$s_7 = t_7 + 3 t_1 t_3 + 3 t_0 t_4 - r_{1,14} t_7 t_4 - r_{2,15} s_{15}$

$s_6 = t_6 + \frac{t_1^2}{2} + t_0 t_1 + \frac{t_0^2}{2} + t_0 t_3 + t_0 t_3 + t_0 t_2 + t_7 t_3 + (1 - r_{1,14}) t_6 t_4$

$s_5 = t_5 + t_1 t_3 + 2 t_0 t_2 + 2 t_0 t_3 + 2 t_2 t_4 + 2 t_0 t_4 - r_{2,15} s_{15}$

$s_4 = t_4 + t_0 t_2 + t_0 t_0 + t_0 t_1 + t_0 t_4 + t_4 t_4 + t_6 t_4 + t_6 t_4 + (1 - r_{1,14}) t_4 t_4 - r_{2,15} s_{15}$

$s_3 = t_3 - t_0 t_4 + 2 t_0 t_4 + 2 t_0 t_1 + 2 t_0 t_3 - r_{1,14} t_3 t_4 - r_{2,15} s_{15}$

$s_2 = t_2 + \frac{1}{2} r_{2,15} - 1 \left( (2 t_9 + t_9 t_9 + \frac{1}{2} r_{2,15} + \frac{1}{2}) t_9^2 \right.$

$+ t_9 t_9 + \frac{1}{2} r_{2,15} t_9^2 + (r_{2,15} + 1) t_9 t_1 + t_9 t_0 + t_5 t_1$

$+ r_{2,15} s_{15} t_0 + (r_{2,15} + 1) t_4 t_3 + r_{2,15} s_{15} t_2 + (r_{2,15} - r_{2,15}) t_2 t_4,$

$s_1 = t_1 + (r_{1,14} - 1) t_2 t_8 + (r_{1,14} - 1) t_9 t_9 + t_5 t_9 + r_{1,14} t_1 t_1 + r_{1,14} t_1 t_8 t_1,$

$+ r_{1,14} t_1 t_0 + r_{1,14} t_1 t_0 + r_{1,14} t_2 t_3 + r_{1,14} t_2 t_2$.
Primitive form up to second order in $t$

We present the expression for the primitive form up to the second order in overall $t$. Note that decomposition (3.8) is overdetermined since any polynomial of $e_\alpha$ can be reexpressed as polynomial of only $e_2$ and $e_3$. We used this freedom to express the primitive form linearly in $e_\alpha$.

$$\Omega = [1 - r_{1,14}t_4 + (r_{1,14}^2 - 1)t_4^2 + r_{1,14}r_{2,15}t_3t_5 + (r_{1,14}r_{2,15} - 2)t_2t_3]e_1$$
$$+ [-r_{2,15} + (r_{2,15}^2 + r_{1,14}r_{2,15} - 3)t_4]t_3e_2 - 3f_{15}^3e_3.$$ 

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