Properties of the $a_0$ resonances

Agnieszka Furman and Leonard Leśnіak

Henryk Niewodniczański Institute of Nuclear Physics, PL 31-342 Kraków, Poland

We present results following from the coupled channel model of two $a_0$ resonances decaying into the $\pi\eta$ and $KK$ mesons. The $a_0(980)$ resonance can be described by two distinct poles. It is shown that the discrepancy in the $a_0(980)$ mass position between the Crystal Barrel Collaboration and the E852 Group can be explained and removed. In our model with parameters fixed by the present experimental data the $a_0(980)$ cannot be interpreted as a bound KK state although the KK forces in the S-wave isovector channel are attractive.

1. Introduction

Among the scalar mesons there are only two isovector resonances $a_0(980)$ and $a_0(1450)$. Physical properties of both mesons are, however, not well known, for example the widths and branching ratios are poorly determined. Also the mass determination is problematical. The proper interpretation of the resonant states can be obtained in terms of the S-matrix poles but the data are rarely analysed in such a way. Two KK thresholds at $m_{K^+} + m_{K^-} = 987.4$ MeV and at $2m_{K^0} = 995.3$ MeV lie close to $a_0(980)$. Therefore a description of the $a_0(980)$ line shape using the simple Breit-Wigner formula leads to a distortion of the $a_0(980)$ mass and width (compare, for example differences between the results given in [2, 3]). Montanet indicated important differences of the $a_0(1450)$ masses and widths found in the $p\bar{p}$ annihilation by the Crystal Barrel Collaboration [4] and by the OBELIX Collaboration [5]. Recently the WA102 Collaboration has observed the $a_0(980)$ production in the pp central collisions at 450 GeV, however the $a_0(1450)$ resonance has not been seen [6].

The main decay channels of the $a_0$ resonances are $\pi\eta$ and $KK$. In the first part of this paper the $a_0(980)$ production and decays will be discussed using the Flatté model [7]. Next we shall present the results following from the coupled channel model formulated in 1996 [8] and further developed in [9].

2. How the $a_0(980)$ splits into two poles?

Two experimental collaborations have used the Flatté model to analyse their data [2, 3]. Two different KK thresholds have not been distinguished. In the Flatté model the effective mass distribution in the $j-$channel ($j = \pi\eta$, KK) is given by

$$\frac{d\sigma_j}{dm} = c|F_j|^2,$$

(1)

where $c$ is a constant and the production amplitudes are given by

$$F_j = \frac{\sqrt{\Gamma_j}}{m_{K^2} - m^2 - i m_R(\Gamma_{\pi\eta} + \Gamma_{KK})}.$$

(2)

The $\pi\eta$ partial width is a product of the coupling constant $g_{\pi\eta}$ and the $\pi\eta$ momentum $k_1$: $\Gamma_{\pi\eta} = g_{\pi\eta}k_1$. Similarly above the KK threshold $\Gamma_{KK} = g_{KK}k_2$. However, below the threshold this width becomes imaginary: $\Gamma_{KK} = i g_{KK}|k_2|$, where $|k_2| = \sqrt{m_{K^2} - m^2}$ and $m_K$ denotes the kaon mass. Near the KK threshold $\Gamma_{\pi\eta}$ varies slowly and $\Gamma_{KK}$ varies rapidly. Therefore the $\pi\eta$ distribution is narrowed on both sides of the KK threshold as compared to the Breit-Wigner shape characteristic for the constant width (independent on $m$). The second important feature of the Flatté model is the existence of two complex poles of the production amplitudes $F_i$ at different energies corresponding to the same meson $a_0(980)$. 

---

*Talk given by L. Leśnіak at the QCD 2002 Conference, Montpellier, France, July 2–9 2002
Let us derive the pole positions following from the parameter values used by the E852 Group in their analysis of the reaction $\pi^- p \rightarrow \eta \pi^0 \pi^- n$. At the beginning one has to correct the coupling constants for the finite experimental energy resolution. Then the $\pi\eta$ coupling constant reduces to the value $g_{\pi\eta} = 0.210 \pm 0.015$ from which we have found two poles at $E_1 = (1006 - i\,25)$ MeV on sheet II and $E_2 = (988 - i\,44)$ MeV on sheet III. These values are in very good agreement with the pole positions of the Crystal Barrel Collaboration recalculated and corrected by us: $E_1 = (1005 - i\,25)$ MeV on sheet II and $E_2 = (985 - i\,46)$ MeV on sheet III. The energy differences are indeed quite large: $ReE_1 - ReE_2 = 20$ MeV; the corresponding difference of the total widths is $\Gamma_2 - \Gamma_1 = 43$ MeV. In Fig. 1 we show crosses with errors corresponding to our two pole determination of the real energy parts for the experiments [3] and [4]. One can notice that about 3σ discrepancy between the E852 value, based on the Breit-Wigner form, and the mean value determination by the Particle Data Group [1] disappears when one considers the pole value on sheet II at 988 MeV.

By looking at Fig. 1 one can try to answer often asked question: where is located the $a_0(980)$, below or above the $K\bar{K}$ threshold? The most probable answer is: the first $a_0(980)$ pole on sheet III is located below and the second $a_0(980)$ pole on sheet II is located above the $K^0\bar{K}^0$ threshold. One remark is relevant here: it would be useful to print in the Review of Particle Physics the $a_0(980)$ masses and its widths in the form of the complex energy values corresponding to different determinations of the T-matrix poles. Such a presentation is already given for the $f_0(980)$ and the $f_0(1370)$ [1, 10].

The $a_0(980)$ coupling to the $K\bar{K}$ channel is responsible for the appearance of two $a_0(980)$ poles. If $g_{K\bar{K}} = 0$ then $E_2 = E_1$. If, however, $g_{K\bar{K}} \neq 0$ then

$$ReE_2 - ReE_1 \approx -g_{K\bar{K}} \frac{m_K q_1}{4 \, Req_2} \quad (3)$$

and

$$\Gamma_2 - \Gamma_1 \approx 2 \, g_{K\bar{K}} \, Req_2. \quad (4)$$

In these equations $q_1$ is the $\pi\eta$ relative momentum at the $K\bar{K}$ threshold and $q_2$ denotes the $K\bar{K}$ momentum at the $a_0(980)$ pole on sheet III.

### 3. Coupled channel model of the $a_0(980)$ and the $a_0(1450)$ resonances

Below we present results of the simple two-channel model of $a_0$ resonances applied in [3] to the analysis of experimental results. In this model four reactions: $\pi\eta \rightarrow \pi\eta$, $\pi\eta \rightarrow K\bar{K}$, $K\bar{K} \rightarrow \pi\eta$ and $K\bar{K} \rightarrow K\bar{K}$ are described simultaneously using the separable interactions in the form

$$\langle p | V_{ij} | q \rangle = \lambda_{ij} f_i(p) f_j(q), \quad i, j = 1, 2. \quad (5)$$

Here $\lambda_{ij}$ are the real coupling constants and $f_i$ are the Yamaguchi form factors inversely proportional to $p^2 + \beta_{2i}^2$, where $p$ is the c.m. momentum and $\beta_i$ are constants. The model has altogether only five independent parameters: the $\pi\eta$ coupling constant $\lambda_{11}$, the $K\bar{K}$ coupling constant $\lambda_{22}$, the interchannel coupling $\lambda_{12}$ and two range parameters $\beta_1$ and $\beta_2$. The T-matrix satisfies the Lippmann–Schwinger equation $T = V + VGT$, where $G$ is the propagator matrix.
We fix four model parameters by choosing the $a_0(980)$ pole of the T-matrix sheet II at (1005 $- i 24.5$) MeV and the $a_0(1450)$ pole on sheet III at (1474 $- i 132.5$) MeV. The fifth model parameter is constrained by the experimental $K\bar{K}/\pi\eta$ branching ratio near the $K\bar{K}$ threshold:

$$BR = \frac{\int_{m_{1}}^{m_{\text{max}}} \rho_2 |F_2(m)|^2 \, dm}{\int_{m_1}^{m_{\text{max}}} \rho_1 |F_1(m)|^2 \, dm}, \quad (6)$$

where $\rho_i = 2k_i/m$. We have chosen $m_1 = m_{\pi^0} + m_\eta$ and $m_2 = 2m_{K^0}$, corresponding to the branching ratio for the neutral $a_0(980)$ decays. The branching ratio depends very strongly on the upper integration limit $m_{\text{max}}$ as shown in Fig. 2. The Crystal Barrel result $BR = 0.23 \pm 0.05$ corresponds to large value of $m_{\text{max}} = 2m_\pi - m_{\pi^0} = 1741$ MeV. The upper limit of the WA102 group is lower as it is equal to 1147 MeV since this group studied the $f_1(1285)$ decay into $\pi\eta\eta$. Their measured value $0.166 \pm 0.01 \pm 0.02$ agrees well with the theoretical value 0.19 shown in Fig. 2. The OBELIX ratio $0.26 \pm 0.06$ is also in a good agreement with the theoretical curve.

![Figure 2](image)

**Figure 2.** Dependence of the $a_0(980)$ decay branching ratio on the effective mass upper limit.

## 4. Model prediction in the $a_0(1450)$ range

According to [10], we take the $a_0(1450)$ mass equal to $M = 1474$ MeV and its total width $\Gamma = 265$ MeV. Then the $K\bar{K}/\pi\eta$ branching ratio calculated in the limits of $m$ between $M - \Gamma/2$ and $M + \Gamma/2$ is equal to 0.98. When this ratio is evaluated in the slightly larger limits between 1300 MeV and 1741 MeV then it decreases to 0.78. Both numbers stay well within the experimental value $0.88 \pm 0.23$ found by the Crystal Barrel Collaboration. Thus the model provides us with the theoretical branching ratios consistent with the experimental findings for both $a_0$ mesons.

## 5. Other results following from the model

There are many other predictions coming from the coupled channel model described previously. One can calculate the elastic and transition amplitudes, the effective mass distributions in two channels, positions of other T-matrix poles not initially imposed in the procedure of fixing the model parameters and the coupling constants in two channels at each pole. Some of these predictions have been already published in [9]. We have calculated rather important energy difference of the $a_0(980)$ poles: $ReE_1 - ReE_2 = 13.5$ MeV and the width difference $\Gamma_2 - \Gamma_1 = 18$ MeV. These numbers are in qualitative agreement with the results found in Sect. 2 using the Flatté model.

Three interesting physical quantities are related to the diagonal $S$-matrix elements in two channels:

$$S_{jj} = \eta e^{2i\delta_j}, \quad j = 1, 2. \quad (7)$$

The inelasticity $\eta$ is common for both channels, what follows from the unitarity condition, however the phase shifts $\delta_j$ are different in both channels, as shown in Fig. 3. The sudden rise of the $\pi\eta$ phase shifts near the $K\bar{K}$ threshold is due to the presence of the $a_0(980)$ pole on sheet II. Rather weak variation of the $K\bar{K}$ phase shifts near 1 GeV is a result of the destructive interference between two related $a_0(980)$ poles lying on sheets II and III. The interference effects are also seen near 1450 MeV where in the $\pi\eta$ channel the $a_0(1450)$ pole interferes with the $S_{11}$ zero related to the second $a_0(1450)$ pole. In the $K\bar{K}$ channel, however, the interference is constructive and the $K\bar{K}$ phase shifts rise very quickly in vicinity of 1500 MeV. The presence of the $a_0$ resonances leads to two
characteristic dips of the inelasticity coefficient \( \eta \).

![Graph showing channel phase shifts and inelasticity versus the effective mass](image)

**Figure 3.** Channel phase shifts and inelasticity versus the effective mass

6. **Nonexistence of the bound \( KK \) isovector state**

The present model is suitable to study the question whether a \( KK \) pair can form a bound S-wave isospin 1 state. This question is relevant since the coupling between kaons in that state is negative so the forces in the isovector state are attractive. In order to answer this interesting question we have studied the evolution of the T-matrix poles in the limit of vanishing interchannel coupling constant \( \lambda_{12} \). In this limit one can find that the \( KK \) resonance exists at the complex effective mass equal to \( (1270 - i 77) \text{ MeV} \). If the interchannel coupling is switched on then this resonance evolves into the \( a_0(1450) \) pole and not into the \( a_0(980) \) pole. The energy of the bound state would be real and smaller than the sum of two kaon masses. Since this is not a case in our model the answer about the presence of the bound \( KK \) isovector state is negative.

7. **Conclusions**

We have found that the \( a_0(980) \) meson can be described in terms of two distinct poles lying near the \( KK \) threshold on sheets II and III. This result is a common feature of the two models, namely the Flatté model, often used to describe a single \( a_0 \) resonance, and the coupled channel model of two \( a_0 \) resonances, constructed recently by us. The second model has been constrained in the \( a_0(980) \) mass range and then successfully applied at higher effective \( \pi \eta \) or \( KK \) masses where the \( a_0(1450) \) resonance is present. The two channel model can be extended to treat more decay channels.

**REFERENCES**

1. K. Hagiwara, *et al.*, (Particle Data Group), Phys. Rev. D66 (2002) 010001.
2. A. Abele, *et al.*, (Crystal Barrel Coll.), Phys. Rev. D57 (1998) 3860.
3. S. Teige, *et al.*, (E852 Coll.), Phys. Rev. D59 (1999) 012001.
4. L. Montanet, Nucl. Phys. B (Proc. Suppl.) 86 (2000) 381.
5. A. Bertin, *et al.*, (OBELIX Coll.), Phys. Lett. B434 (1998) 180.
6. D. Barberis, *et al.*, (WA102 Coll.), Phys. Lett. B488 (2000) 225.
7. S. M. Flatté, Phys. Lett. B63 (1976) 224.
8. L. Leśniak, Acta Phys. Pol. B27 (1996) 1835.
9. A. Furman and L. Leśniak, Phys. Lett. B538 (2002) 266.
10. D. E. Groom, *et al.*, (Particle Data Group), Eur. Phys. J. C15 (2000) 1.