On $\theta_{23}$ Octant Measurement in 3 + 1 Neutrino Oscillations in T2HKK

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Abstract

It has been pointed out that the mixing of an eV-scale sterile neutrino with active flavors can lead to loss of sensitivity to the $\theta_{23}$ octant (sign of $\sin^2 \theta_{23} - 1/2$) in long baseline experiments, because the main oscillation probability $P_0 = 4 \sin^2 \theta_{23} \sin^2 \theta_{13} \sin^2 \Delta_{31}$ can be degenerate with the sum of the interferences with the solar oscillation amplitude and an active-sterile oscillation amplitude in both neutrino and antineutrino oscillations, depending on CP phases. In this paper, we show that the above degeneracy is resolved by measuring the same beam at different baseline lengths. We demonstrate that Tokai-to-Hyper-Kamiokande-to-Korea (T2HKK) experiment (one 187 kton fiducial volume water Cerenkov detector is placed at Kamioka, $L = 295$ km, and another detector is put in Korea, $L \sim 1000$ km) exhibits a better sensitivity to the $\theta_{23}$ octant in those parameter regions where the experiment with two detectors at Kamioka is insensitive to it. Therefore, if a hint of sterile-active mixings is discovered in short baseline experiments, T2HKK is a better option than the plan of placing two detectors at Kamioka. We also consider an alternative case where one detector is placed at Kamioka and a different detector is at Oki Islands, $L = 653$ km, and show that this configuration also leads to a better sensitivity to the $\theta_{23}$ octant.
Introduction

The octant of neutrino mixing angle $\theta_{23}$, namely, the sign of $\sin^2 \theta_{23} - 1/2$, has profound implications for SO(10) grand unification theory with renormalizable Yukawa couplings (see, e.g., Refs. [1, 2, 3]). Since $\nu_e$ and $\nu_\mu$ disappearance channels are almost solely sensitive to $\sin^2 (2\theta_{23})$, the best way to determine the octant is to use $\nu_\mu \rightarrow \nu_e$ channel in long baseline experiments. For the standard 3-flavor oscillations, the octant measurement has been investigated in Refs. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

It has been pointed out that if an eV-scale light sterile neutrino mixes with active flavors, it can considerably deteriorate the sensitivity of long baseline experiments to the $\theta_{23}$ octant [16].

This is based on the following observation: In the standard 3-flavor oscillations, the part of the $\nu_\mu \rightarrow \nu_e$ oscillation probability sensitive to the $\theta_{23}$ octant, $P_0 = 4 \sin^2 \theta_{23} \sin^2 \theta_{13} \sin \Delta_{31} (\Delta_{31} \equiv \Delta m^2_{31} L/(4E))$, is possibly mimicked by the interference of the solar and atmospheric oscillation amplitudes, $P_1 \simeq 4 \sin \theta_{13} \sin \theta_{12} \cos \theta_{12} (\Delta m^2_{21}/\Delta m^2_{31}) \Delta_{31} \sin \Delta_{31} \cos ( \Delta_{31} \pm \phi_{13} )$, in either neutrino oscillations or antineutrino oscillations depending on the CP phase $\phi_{13}$, but not in both oscillations. Hence, a combination of neutrino- and antineutrino-focusing operations resolves the above degeneracy. In the 3+1 oscillations, however, the interference of atmospheric and active-sterile oscillation amplitudes generates a new term, $P_2 \simeq 4 \sin \theta_{14} \sin \theta_{24} \sin \theta_{13} \sin \theta_{23} \sin \Delta_{31} \sin ( \Delta_{31} \pm \phi_{13} \mp \phi_{14} )$. The sum $P_1 + P_2$ can mimic $P_0$ in both neutrino and antineutrino oscillations for some values of $\phi_{13}$ and $\phi_{14}$, leading to loss of sensitivity to the $\theta_{23}$ octant.

In this paper, we pursue the possibility that a combination of neutrino- and antineutrino-focusing operations and measurements of the same beam at different baseline lengths resolves the degeneracy of $P_0$ and $P_1 + P_2$, thereby resurrecting the sensitivity to the $\theta_{23}$ octant in the 3+1 oscillations. For concreteness, we concentrate on Tokai-to-Hyper-Kamiokande-to-Korea (T2HKK) experiment [25] (for early proposals, see Refs. [26, 27, 28, 29, 30, 5, 31, 32, 33]), where one 187 kton fiducial volume water Cerenkov detector is placed at Kamioka ($L = 295$ km) and another 187 kton detector is in Korea ($L \sim 1000$ km), which is a proposed extension of Tokai-to-Hyper-Kamiokande (T2HK) experiment [34]. We conduct a comparative study of T2HKK experiment with the plan of placing two 187 kton detectors at Kamioka. We will demonstrate that for some values of CP phases $\phi_{13}$ and $\phi_{14}$, the sensitivity to the $\theta_{23}$ octant is lost in the latter experiment while the sensitivity is maintained in T2HKK, in spite of smaller statistics of T2HKK (as the baseline to Korea is longer than that to Kamioka). We further consider an alternative plan of placing a modest detector at Oki Islands [35, 36, 37, 38] ($L = 653$ km) in addition to one 187 kton detector at Kamioka, and study how the sensitivity to the $\theta_{23}$ octant

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1 Recent studies on the impact of sterile-active mixings on long baseline experiments are Refs. [17, 18, 19, 20, 21, 22, 23, 24].
changes in this case.

Previously, the sensitivity of T2HK and T2HKK to the \(\theta_{23}\) octant in the presence of mixings of an eV-scale sterile neutrino has been studied in Ref. [21]. Our study differs from it in that we consider both cases with \(\theta_{34} = 0\) and \(\theta_{34} \neq 0\), and further separately investigate the dependence of the sensitivity on \(\theta_{34}\) and that on \(\theta_{24}\). In contrast, Ref. [21] only assumes substantially large values for \(\theta_{34}\) and varies \(\theta_{34}, \theta_{14}, \theta_{24}\) only simultaneously. As we show in the main text, non-zero values of \(\theta_{34}\) tend to increase the sensitivity to the \(\theta_{23}\) octant, and hence it is important to scrutinize the case with \(\theta_{34} = 0\), which is the ‘worst situation’ for the \(\theta_{23}\) octant measurement. Also, since the sensitivity tends to increase with \(\theta_{34}\) and decrease with \(\theta_{24}\), the dependences on \(\theta_{34}\) and \(\theta_{24}\) must be analyzed separately.

This paper is organized as follows: In Section 2, we write the \(\nu_\mu \to \nu_e\) oscillation probability in the 3 + 1 oscillations, spot the terms that can lead to loss of sensitivity to \(\theta_{23}\) octant, and qualitatively state that this problem is mitigated in T2HKK experiment. In Section 3, we confirm the above qualitative statement through a numerical analysis. Section 4 summarizes the paper.

## 2 3 + 1 Oscillation Probability

We postulate the presence of one isospin-singlet neutrino, \(\nu_s\), that mixes with the three active flavors (\(\nu_e, \nu_\mu, \nu_\tau\)) and yields four mass eigenstates (\(\nu_1, \nu_2, \nu_3, \nu_4\)). We also assume that the mass of \(\nu_4\) is around 1 eV and thus \(\nu_4\) participates in oscillations in a J-PARC beam.

We parametrize the mixing of \((\nu_e, \nu_\mu, \nu_\tau, \nu_s)\) as follows:

\[
|\nu_\alpha\rangle = \sum_{k=1}^{4} (U)_{\alpha k} |\nu_k\rangle \quad (\alpha = e, \mu, \tau, s),
\]

with \(U\) being a 4\(\times\)4 unitary matrix defined as

\[
U = U_{34} U_{24} U_{14} U_{23} U_{13} U_{12},
\]

\[
(U_{ab})_{ij} = \delta_{ia} \delta_{ja} \cos \theta_{ab} + \delta_{ib} \delta_{jb} \cos \theta_{ab} + \delta_{ia} \delta_{ja} e^{-i\phi_{ab}} \sin \theta_{ab} - \delta_{ib} \delta_{ja} e^{i\phi_{ab}} \sin \theta_{ab}.
\]

\((a, b = 1, 2, 3, 4; a < b)\)

\(U_{23} U_{13} U_{12}\) corresponds to Pontecorvo-Maki-Nakagawa-Sakata matrix [39] [40]. Henceforth, we fix the phase convention such that \(\phi_{12} = \phi_{23} = \phi_{24} = 0\). Also, we take \(\theta_{14} \geq 0, \theta_{24} \geq 0, \theta_{34} \geq 0\).

\footnote{Since the impact of sterile-active mixings on the \(\theta_{23}\) octant measurement appears only in the combinations \(\sin \theta_{14} \sin \theta_{24}\) and \(\sin \theta_{14} \sin \theta_{34}\), we do not need to vary \(\theta_{14}\) in addition to \(\theta_{34}\) and \(\theta_{24}\).}
The probability of $\nu_\mu \rightarrow \nu_e$ oscillation, which is crucial for the $\theta_{23}$ octant measurement, is given by

$$P(\nu_\mu \rightarrow \nu_e) = \left| \left| \exp \left( -i \int_0^L dx \ H(x) \right) \right| \right|^2,$$

where $L$ is the baseline length, $E$ is the neutrino energy, and $\Delta m^2_{21} = m_2^2 - m_1^2$. $V_{cc}$ and $V_{nc}$ denote the potentials generated by charged and neutral current interactions, respectively, which are evaluated as $V_{cc} = -2V_{nc} \simeq 0.193 \times 10^{-3} (\rho/(\text{g/cm}^3))/\text{km}$ with $\rho$ being the matter density. The antineutrino oscillation probability $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ is obtained by flipping the signs of $\phi_{13}$, $\phi_{14}$, $\phi_{34}$, $V_{cc}$, $V_{nc}$.

To gain physical insight, we expand Eq. (2) in the leading order of $\Delta m^2_{21}L/E$, $\theta_{14}$, $\theta_{24}$, $\theta_{34}$ and the matter effect (approximated to be uniform), and further take an average over $\Delta m^2_{21}L/(4E)$ oscillations. The $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation probabilities are then written as

$$P^{(\nu)}(\nu_\mu \rightarrow \nu_e) \simeq 4 \sin^2 \theta_{13} \sin^2 \theta_{23} \sin^2 \left( \frac{\Delta m^2_{31}L}{4E} \right)$$

$$+ 4 \sin \theta_{13} \sin \theta_{12} \cos \theta_{12} \sin(2\theta_{23}) \sin \left( \frac{\Delta m^2_{31}L}{4E} \right) \sin \left( \frac{\Delta m^2_{31}L}{4E} \right) \cos \left( \frac{\Delta m^2_{31}L}{4E} \pm \phi_{13} \right)$$

$$+ 4 \sin \theta_{14} \sin \theta_{24} \sin \theta_{13} \sin \theta_{23} \sin \left( \frac{\Delta m^2_{31}L}{4E} \right) \sin \left( \frac{\Delta m^2_{31}L}{4E} \pm \phi_{13} \mp \phi_{14} \right)$$

$$\pm 4L \sqrt{\Delta m^2_{21}} \sin^2 \theta_{13} \sin^2 \theta_{23} \left\{ \frac{4E}{\Delta m^2_{31}L} \sin^2 \left( \frac{\Delta m^2_{31}L}{4E} \right) - \frac{1}{2} \sin \left( \frac{\Delta m^2_{31}L}{2E} \right) \right\}$$

$$\pm \sin \theta_{13} \sin \theta_{23} \sin(2\theta_{23}) \sin \theta_{14} \sin \theta_{34} \Delta m^2_{31} \sin \theta_{34} \sin \left( \frac{L \Delta m^2_{31}}{4E} \pm (\phi_{13} - \phi_{14} + \phi_{34}) \right),$$

$$= 4 \sin^2 \theta_{13} \sin^2 \theta_{23} \sin^2 \left( \frac{\Delta m^2_{31}L}{4E} \right)$$

$$+ \left\{ \mp A \sin \phi_{13} + B \cos(\phi_{13} - \phi_{14}) \right\} \sin^2 \left( \frac{\Delta m^2_{31}L}{4E} \right)$$

$$+ \frac{1}{2} \left\{ A \cos \phi_{13} \pm B \sin(\phi_{13} - \phi_{14}) \right\} \sin \left( \frac{\Delta m^2_{31}L}{2E} \right)$$

$$\pm 4L \sqrt{\Delta m^2_{21}} \sin^2 \theta_{13} \sin^2 \theta_{23} \left\{ \frac{4E}{\Delta m^2_{31}L} \sin^2 \left( \frac{\Delta m^2_{31}L}{4E} \right) - \frac{1}{2} \sin \left( \frac{\Delta m^2_{31}L}{2E} \right) \right\}$$

$$\pm \sin \theta_{13} \sin \theta_{23} \sin(2\theta_{23}) \sin \theta_{14} \sin \theta_{34} \frac{L \Delta m^2_{31}}{4E}, \pm (\phi_{13} - \phi_{14} + \phi_{34}) \right\},$$
where
\[
A = 4 \sin \theta_{13} \sin \theta_{12} \cos \theta_{12} \sin(2\theta_{23}) \frac{\Delta m^2_{21} L}{4E},
\]
\[
B = 4 \sin \theta_{14} \sin \theta_{24} \sin \theta_{13} \sin \theta_{23},
\]
and the upper signs in $\pm, \mp$ refer to the neutrino oscillation, and the lower signs to the antineutrino oscillation.

First, we find that in the absence of sterile-active mixings, we have $B = 0$ and the impact of $A$ on the $\theta_{23}$ octant measurement can be reduced by a combination of neutrino- and antineutrino-focusing beams, since $A$ term in Eq. (4) changes sign for neutrino and antineutrino.

Second, we observe that in the presence of sterile-active mixings (i.e. $B \neq 0$), a combination of neutrino- and antineutrino-focusing beams and two different baseline lengths revives the sensitivity to the $\theta_{23}$ octant, because a different oscillating function, $\sin(\Delta m^2_{31} L/(2E))$, enters Eq. (5). In the rest of the paper, we will confirm this qualitative observation through simulations of T2HK, T2HKK, and T2HK+Oki experiments.

The term Eq. (6) represents the ordinary matter effect that is present in the standard oscillations, and does not affect the measurement of the $\theta_{23}$ octant.

The last term Eq. (7) represents a synergy of the matter effect and sterile-active mixings. Here, $F$ is a complicated function of $L\Delta m^2_{31}/(4E)$ and $\pm(\phi_{13} - \phi_{14} + \phi_{34})$, and is expected to be $O(1)$. Since $\theta_{34}$ is less constrained than $\theta_{24}$, this term can lead to interesting phenomenology in some long baseline experiments with large matter effect. We will study the impact of the term Eq. (7) numerically in the next section.

3 Numerical Study

We simulate the propagation of neutrinos and antineutrinos, their $3+1$ oscillations, and their measurements at Kamioka, Oki and Korea. We calculate $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, $\nu_\mu \rightarrow \nu_\mu$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$ oscillation probabilities using Eq. (2) without approximation.

3.1 Oscillation Parameters

We make a simplifying assumption that $\theta_{12}$, $\theta_{13}$, $\Delta m^2_{21}$, $|\Delta m^2_{32}|$ and the true mass hierarchy have been measured precisely prior to a $\theta_{23}$ measurement. Accordingly, we fix the values of $\theta_{12}$, $\theta_{13}$, $\Delta m^2_{21}$, $|\Delta m^2_{32}|$ at their Particle Data Group values [41] in the simulation, and fit the simulation results with the true values. Likewise, we perform the simulation for the normal
hierarchy case ($\Delta m_{32}^2 > 0$) and fit the results by assuming the normal hierarchy, and do analogously for the inverted hierarchy case ($\Delta m_{32}^2 < 0$).

For sterile-active mixing parameters, we consider a situation where sterile-active mixings have already been discovered and $\Delta m_{24}^2, \theta_{14}, \theta_{24}, \theta_{34}$ have been measured precisely in short baseline experiments.

The full analysis is multi-dimensional, depending on benchmark values of $\theta_{14}, \theta_{24}, \theta_{34}$ as well as unknown values of $\theta_{23}$ and the three CP phases $\phi_{13}, \phi_{14}, \phi_{34}$. Such a complicated analysis does not provide clear physical insight. Therefore, first we restrict our study to the case with $\theta_{34} = 0$, and assume the largest values for $\theta_{14}, \theta_{24}$ that are consistent with the current experimental bounds in order to assess the maximal impact of $\theta_{14}, \theta_{24}$ mixings on the $\theta_{23}$ octant measurement. The reason for taking $\theta_{34} = 0$ is that the $\theta_{34}$ mixing affects the octant measurement only via the matter effect and so its impact should be studied separately. Here we fix $\Delta m_{21}^2$ at a value around $\sim 1$ eV$^2$, since the results do not depend on the precise value of $\Delta m_{41}^2$. In contrast, we consider various values for $\phi_{13}, \phi_{14}$ because the impact of $\theta_{14}, \theta_{24}$ mixings on the $\theta_{23}$ octant measurement depends keenly on $\phi_{13}, \phi_{14}$. We take two benchmark values for $\theta_{23}$, one in the lower octant and the other in the higher octant. The first benchmark parameter set, which is motivated by the above argument, is presented in Table 1.

| Table 1: Parameters in the first benchmark. | value in our simulation |
|-------------------------------------------|-------------------------|
| $\sin^2 \theta_{12}$                       | 0.304                   |
| $\sin^2 \theta_{13}$                       | 0.0219                  |
| $\sin^2 \theta_{23}$                       | 0.44 or 0.60            |
| $\phi_{13}$ (rad)                           | $[0, 2\pi]$             |
| $\Delta m_{21}^2$                           | $7.53 \times 10^{-5}$ eV$^2$ |
| $\Delta m_{32}^2$ (normal hierarchy)       | $2.51 \times 10^{-3}$ eV$^2$ |
| $\Delta m_{32}^2$ (inverted hierarchy)     | $-2.56 \times 10^{-3}$ eV$^2$ |
| $\sin^2 \theta_{14}$                       | 0.04                    |
| $\sin^2 \theta_{24}$                       | 0.006                   |
| $\sin^2 \theta_{34}$                       | 0                      |
| $\phi_{14}$ (rad)                           | $[0, 2\pi]$             |
| $\phi_{34}$ (rad)                           | unphysical              |
| $\Delta m_{41}^2$                           | 3 eV$^2$                |

In Table 1, the values of $\sin^2 \theta_{14}$ and $\Delta m_{41}^2$ are marginally compatible with the 90% CL bound obtained by a reanalysis [42] of Bugey-3 [43] experiment, and the values of $\sin^2 \theta_{24}$ and $\Delta m_{41}^2$ are marginally consistent with the 90% CL bound of the MINOS and MINOS+ results [44] and are outside the 90% CL bound from the IceCube [45]. The asymmetric benchmark values
of $\sin^2 \theta_{23}$ are motivated by the $3\sigma$ range reported by NuFIT 4.1 [46, 47], which is tilted towards the higher octant region. Note that $\phi_{34}$ is an unphysical phase when $\theta_{34} = 0$.

After the analysis of the first benchmark Table 1, we vary $\sin^2 \theta_{34}$, $\phi_{34}$ and study the impact of a synergy of the matter effect and sterile-active mixings described by Eq. (7). Here we fix $\phi_{13}$, $\phi_{14}$ at the values for which sterile-active mixings have most afflicted the $\theta_{23}$ octant measurement in the first benchmark. The second benchmark parameter set, which is motivated by the above argument, is presented in Table 2.

Table 2: Parameters in the second benchmark. The values of $\phi_{13}, \phi_{14}$ are such that sterile-active mixings most afflict the $\theta_{23}$ octant measurement in the first benchmark Table 1, where $\sin^2 \theta_{34} = 0$.

| parameter | value in our simulation |
|-----------|-------------------------|
| $\sin^2 \theta_{12}$ | 0.304 |
| $\sin^2 \theta_{13}$ | 0.0219 |
| $\sin^2 \theta_{23}$ | 0.44 or 0.60 |
| $\phi_{13}$ (rad) | 0 when $\sin^2 \theta_{23} = 0.44$, $\pi$ when $\sin^2 \theta_{23} = 0.60$ |
| $\Delta m^2_{21}$ | $7.53 \times 10^{-5}$ eV$^2$ |
| $\Delta m^2_{32}$ (normal hierarchy) | $2.51 \times 10^{-3}$ eV$^2$ |
| $\Delta m^2_{32}$ (inverted hierarchy) | $-2.56 \times 10^{-3}$ eV$^2$ |
| $\sin^2 \theta_{14}$ | 0.04 |
| $\sin^2 \theta_{24}$ | 0.006 |
| $\sin^2 \theta_{34}$ | [0, 0.18] |
| $\phi_{14}$ (rad) | 0 |
| $\phi_{34}$ (rad) | [0, $2\pi$] |
| $\Delta m^2_{41}$ | 3 eV$^2$ |

In Table 2, the range of $\sin^2 \theta_{34}$ is chosen to satisfy the 90% CL bound on the sterile-tau neutrino mixing reported from Super-Kamiokande atmospheric neutrino measurement [48].

We will find in Section 3.5 that non-zero values of $\sin^2 \theta_{34}$ tend to mitigate the impact of sterile-active mixings on the $\theta_{23}$ octant measurement, i.e. one is more likely to obtain the correct octant when the $\theta_{34}$ mixing is present. Therefore, it is important to study how the impact of sterile-active mixings changes with $\theta_{14}$, $\theta_{24}$ in the ‘worst scenario’ for the octant measurement where $\theta_{34} = 0$. To this end, we vary $\theta_{24}$ while fixing $\theta_{34} = 0$. Here we fix $\sin^2 \theta_{14} = 0.04$ because, as seen in Eqs. (3)-(7), the effects of sterile-active mixings on the $\theta_{23}$ octant measurement appear only in the combination $\sin \theta_{14} \sin \theta_{24}$ when $\theta_{34} = 0$, and thus varying $\sin^2 \theta_{14}$ is equivalent to varying $\sin^2 \theta_{24}$. Here we take a specific combination of values of $\phi_{13}, \phi_{14}$ for which sterile-
active mixings have most afflicted the $\theta_{23}$ octant measurement in the first benchmark. The third benchmark parameter set, which is motivated by the above argument, is presented in Table 3.

Table 3: Parameters in the third benchmark. The values of $\phi_{13}, \phi_{14}$ are such that sterile-active mixings most afflict the $\theta_{23}$ octant measurement in the first benchmark Table 4, where we fix $\sin^2 \theta_{24} = 0.006$.

| parameter                          | value in our simulation |
|------------------------------------|-------------------------|
| $\sin^2 \theta_{12}$              | 0.304                   |
| $\sin^2 \theta_{13}$              | 0.0219                  |
| $\sin^2 \theta_{23}$              | 0.44 or 0.60            |
| $\phi_{13}$ (rad)                 | [0, $2\pi$]             |
| $\Delta m^2_{21}$                 | $7.53 \times 10^{-5}$ eV$^2$ |
| $\Delta m^2_{32}$ (normal hierarchy) | $2.51 \times 10^{-3}$ eV$^2$ |
| $\Delta m^2_{32}$ (inverted hierarchy) | $-2.56 \times 10^{-3}$ eV$^2$ |
| $\sin^2 \theta_{14}$             | 0.04                    |
| $\sin^2 \theta_{24}$             | [0, 0.006]              |
| $\sin^2 \theta_{34}$             | 0                       |
| $\phi_{14}$ (rad)                 | equals $\phi_{13}$ when $\sin^2 \theta_{23} = 0.44$, equals $\phi_{13} + \pi$ when $\sin^2 \theta_{23} = 0.60$ |
| $\phi_{34}$ (rad)                 | unphysical              |
| $\Delta m^2_{41}$                 | 3 eV$^2$                |

3.2 Experiments

We assume that J-PARC operates with 1.3 MW beam power, delivering $2.7 \times 10^{21}$ proton-on-target (POT) flux per year. We adopt the values in Table 4 as the baseline and detector parameters, where the matter density is approximated to be uniform [49, 50]. We consider three experiments, referred to as ‘T2HK’, ‘T2HKK’ and ‘T2HK+Oki’ in this paper. The configuration of each experiment is assumed as follows, and is summarized in Table 4.

- In ‘T2HK’, we assume that one 187 kton fiducial volume detector at Kamioka is exposed to a neutrino-focusing beam for 2.5 years and to an antineutrino-focusing beam for 2.5 years. Subsequently, two 187 kton detectors (374 kton in total) at Kamioka are exposed to a neutrino-focusing beam for 5 years and to an antineutrino-focusing beam for 5 years.

- In ‘T2HKK’, we assume that one 187 kton detector at Kamioka is exposed to a neutrino-focusing beam for 2.5 years and to an antineutrino-focusing beam for 2.5 years. Subsequently, one 187 kton detector at Kamioka and one 187 kton detector in Korea are
exposed to a neutrino-focusing beam for 5 years and to an antineutrino-focusing beam for 5 years.

- In ‘T2HK+Oki’, we assume that one 187 kton detector at Kamioka is exposed to a neutrino-focusing beam for 2.5 years and to an antineutrino-focusing beam for 2.5 years. Subsequently, one 187 kton detector at Kamioka and a smaller 100 kton detector at Oki Islands are exposed to a neutrino-focusing beam for 5 years and to an antineutrino-focusing beam for 5 years.

Table 4: Baseline and detector parameters. $L$, $FV$, $\rho$, ‘off-axis’, ‘$\nu$-focusing’ and ‘$\bar{\nu}$-focusing’ denote the baseline length, the fiducial volume, matter density, off-axis angle, the time of exposure to a neutrino-focusing beam, and that to an antineutrino-focusing beam, respectively.

| experiment   | site  | $L$   | $FV$  | $\rho$      | off-axis | $\nu$-focusing | $\bar{\nu}$-focusing |
|--------------|-------|-------|-------|-------------|----------|----------------|---------------------|
| T2HK         | Kamioka | 295 km | 187 kton | 2.60 g/cm$^3$ | 2.5$^\circ$ | 2.5 years | 2.5 years |
|              | Kamioka | 295 km | 374 kton | 2.60 g/cm$^3$ | 2.5$^\circ$ | 5 years | 5 years |
| T2HKK        | Kamioka | 295 km | 187 kton | 2.60 g/cm$^3$ | 2.5$^\circ$ | 2.5 years | 2.5 years |
|              | Kamioka | 295 km | 187 kton | 2.60 g/cm$^3$ | 2.5$^\circ$ | 5 years | 5 years |
|              | Korea  | 1000 km | 187 kton | 2.90 g/cm$^3$ | 1.0$^\circ$ | 5 years | 5 years |
| T2HK+Oki     | Kamioka | 295 km | 187 kton | 2.60 g/cm$^3$ | 2.5$^\circ$ | 2.5 years | 2.5 years |
|              | Kamioka | 295 km | 187 kton | 2.60 g/cm$^3$ | 2.5$^\circ$ | 5 years | 5 years |
|              | Oki    | 653 km | 100 kton | 2.75 g/cm$^3$ | 1.0$^\circ$ | 5 years | 5 years |

3.3 Number of Events

The number of $\nu_e + \bar{\nu}_e$ events and the number of $\nu_\mu + \bar{\nu}_\mu$ events in the bin of $0.4 + (i-1) \cdot 0.05$ GeV $< E < 0.4 + i \cdot 0.05$ GeV with a neutrino-focusing beam that are detected at a specific site, denoted by $N_{e,i,site}$ and $N_{\mu,i,site}$, and those with an antineutrino-focusing beam, denoted by $\bar{N}_{e,i,site}$ and
For $i=1, 2, 3, \ldots, 52,$

\[
N_{e,i}\text{site} = \int_{0.4+(i-1)-0.05}^{0.4+i-0.05} \text{GeV} \, dE \, \epsilon_{e,\text{site}} \, f_{\sigma e} \left\{ f_{\Phi_{\nu,\text{site}}(E)} \, P_{\text{site}}(\nu_{\mu} \rightarrow \nu_{e}; E) \, N_{\text{site}} \sigma(\nu_{e}n \rightarrow \ell^{-}p; E) + f_{\Phi_{\bar{\nu},\text{site}}(E)} \, P_{\text{site}}(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}; E) \, N_{\text{site}} \sigma(\nu_{\ell}p \rightarrow \ell^{+}n; E) \right\}, \tag{9}
\]

\[
N_{\mu,i}\text{site} = \int_{0.4+(i-1)-0.05}^{0.4+i-0.05} \text{GeV} \, dE \, \epsilon_{\mu,\text{site}} \, f_{\sigma \mu} \left\{ f_{\Phi_{\nu,\text{site}}(E)} \, P_{\text{site}}(\nu_{\mu} \rightarrow \nu_{\mu}; E) \, N_{\text{site}} \sigma(\nu_{\ell}n \rightarrow \ell^{-}p; E) + f_{\Phi_{\bar{\nu},\text{site}}(E)} \, P_{\text{site}}(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}; E) \, N_{\text{site}} \sigma(\nu_{\ell}p \rightarrow \ell^{+}n; E) \right\}, \tag{10}
\]

\[
\tilde{N}_{e,i}\text{site} = \text{replace} \left( \Phi_{\nu,\text{site}}(E), \Phi_{\bar{\nu},\text{site}}(E), \epsilon_{e,\text{site}}, f_{\Phi_{\nu}}, f_{\Phi_{\bar{\nu}}}, f_{\sigma e} \right) \text{in } N_{e,i}\text{site}
\]

\[
\text{with } \left( \tilde{\Phi}_{\nu,\text{site}}(E), \tilde{\Phi}_{\bar{\nu},\text{site}}(E), \tilde{\epsilon}_{e,\text{site}}, \tilde{f}_{\Phi_{\nu}}, \tilde{f}_{\Phi_{\bar{\nu}}}, \tilde{f}_{\sigma e} \right)
\]

\[
\tilde{N}_{\mu,i}\text{site} = \text{replace} \left( \Phi_{\nu,\text{site}}(E), \Phi_{\bar{\nu},\text{site}}(E), \epsilon_{\mu,\text{site}}, f_{\Phi_{\nu}}, f_{\Phi_{\bar{\nu}}}, f_{\sigma \mu} \right) \text{in } N_{\mu,i}\text{site}
\]

\[
\text{with } \left( \tilde{\Phi}_{\nu,\text{site}}(E), \tilde{\Phi}_{\bar{\nu},\text{site}}(E), \tilde{\epsilon}_{\mu,\text{site}}, \tilde{f}_{\Phi_{\nu}}, \tilde{f}_{\Phi_{\bar{\nu}}}, \tilde{f}_{\sigma \mu} \right)
\]

where $\Phi_{\nu,\text{site}}(E), \Phi_{\bar{\nu},\text{site}}(E), \left( \tilde{\Phi}_{\nu,\text{site}}(E), \tilde{\Phi}_{\bar{\nu},\text{site}}(E) \right)$ respectively denote $\nu_{\mu}$ and $\bar{\nu}_{\mu}$ fluxes per energy in a neutrino-focusing beam (in an antineutrino-focusing beam) at a specific detector. $P_{\text{site}}$ denotes a transition probability calculated with Eq. (2) for a specific site. $N_{n,\text{site}}$ and $N_{p,\text{site}}$ are respectively the number of neutrons and protons in the water Cerenkov detector at a specific site. $\sigma$ denotes the cross sections for the $\nu_{\ell}n \rightarrow \ell^{-}p$ and $\nu_{\ell}p \rightarrow \ell^{+}n$ processes.

We employ the result of Ref. [51] for $\nu_{\mu}$ and $\bar{\nu}_{\mu}$ flux in neutrino-focusing and antineutrino-focusing beams emitted from J-PARC and detected at a water Cerenkov detector at Kamioka, Oki and in Korea if the neutrino oscillations were absent. For reference, we plot the flux in Fig. 1. The baseline length and beam off-axis angle of each detector are found in Table 4. In the plots, the blue lines correspond to a neutrino-focusing beam and the red lines to an antineutrino-focusing beam. The solid lines indicate $\nu_{\mu}$ flux and the dashed lines $\bar{\nu}_{\mu}$ flux.
Figure 1: Flux of $\nu_\mu$ and $\bar{\nu}_\mu$ in neutrino-focusing and antineutrino-focusing beams detected at a water Cerenkov detector at Kamioka, Oki and in Korea if the neutrino oscillations were absent, taken from Ref. [51]. The baseline length and beam off-axis angle of each detector are as given in Table 4. The upper plot, the lower-left plot and the lower-right plot correspond to Kamioka, Oki and Korea, respectively. In the plots, the blue lines correspond to a neutrino-focusing beam and the red lines to an antineutrino-focusing beam. The solid lines indicate $\nu_\mu$ flux and the dashed lines $\bar{\nu}_\mu$ flux.

$\nu_e$ and $\bar{\nu}_e$ components in the beams are neglected in our study.

The cross sections for charged current quasi-elastic scattering between a neutrino and a proton, $\nu_\ell n \rightarrow \ell^- p$, and that between an antineutrino and a neutron, $\bar{\nu}_\ell p \rightarrow \ell^+ n$, ($p$ and $n$ denote proton and neutron, respectively, and $\ell$ denotes $e$ or $\mu$) are obtained from Ref. [52].

$\varepsilon_{e,\text{site}}$, $\varepsilon_{\mu,\text{site}}$ respectively denote the efficiencies for electrons and muons of the far detector at a specific site, in a neutrino-focusing run. $f_{\Phi_\nu}$, $f_{\Phi_\bar{\nu}}$ account for the uncertainty of neutrino and antineutrino fluxes, respectively, in a neutrino-focusing beam. $f_{\sigma e}$ ($f_{\sigma \mu}$) accounts for the uncertainty of the weighted sum of charged current quasi-elastic scattering cross sections of $\nu_e$ and $\bar{\nu}_e$ ($\nu_\mu$ and $\bar{\nu}_\mu$) that is estimated from near detector data and theory, in a neutrino-focusing run. $\tilde{\varepsilon}_{e,\text{site}}$, $\tilde{\varepsilon}_{\mu,\text{site}}$, $\tilde{f}_{\Phi_\nu}$, $\tilde{f}_{\Phi_\bar{\nu}}$, $\tilde{f}_{\sigma e}$, $\tilde{f}_{\sigma \mu}$ are the corresponding quantities in an antineutrino-focusing run. In this paper, we neglect energy dependence of the above systematic parameters. For the
efficiencies of the far detectors, based on Table 9 of Ref. [53], we assume

\[
\varepsilon_{\text{e,site}} = 1 \pm 0.007, \quad \varepsilon_{\mu,\text{site}} = 1 \pm 0.010, \quad \tilde{\varepsilon}_{\text{e,site}} = 1 \pm 0.017, \quad \tilde{\varepsilon}_{\mu,\text{site}} = 1 \pm 0.011. \quad (11)
\]

\(\varepsilon_{\text{e,site}}\) and \(\tilde{\varepsilon}_{\text{e,site}}\) at each site are maximally correlated.

\(\varepsilon_{\mu,\text{site}}\) and \(\tilde{\varepsilon}_{\mu,\text{site}}\) at each site are maximally correlated.

For the flux uncertainty, based on Fig. 9 of Ref. [53], we approximate

\[
f_{\Phi_\nu} = 1 \pm 0.1, \quad f_{\Phi_{\bar{\nu}}} = 1 \pm 0.08, \quad \tilde{f}_{\Phi_\nu} = 1 \pm 0.1, \quad \tilde{f}_{\Phi_{\bar{\nu}}} = 1 \pm 0.1. \quad (12)
\]

For the cross section uncertainty, based on Table 9 of Ref. [53], we assume

\[
\begin{align*}
\sigma_{\text{e}} &= 1 \pm \sqrt{0.030^2 + 0.012^2}, \\
\sigma_{\mu} &= 1 \pm \sqrt{0.028^2 + 0.015^2}, \\
\tilde{\sigma}_{\text{e}} &= 1 \pm \sqrt{0.056^2 + 0.020^2}, \\
\tilde{\sigma}_{\mu} &= 1 \pm \sqrt{0.042^2 + 0.014^2}. \quad (13)
\end{align*}
\]

All uncertainties above are assumed to be uncorrelated unless otherwise stated.

### 3.4 \(\chi^2\) analysis

We perform a \(\chi^2\) fit of the numbers of events in the bins with neutrino-focusing and antineutrino-focusing beams measured at Kamioka/Korea/Oki, \(N_{\text{e,i,site}}, N_{\mu,i,\text{site}}, \tilde{N}_{\text{e,i,site}}\) and \(\tilde{N}_{\mu,i,\text{site}}\) \((i = 1, 2, ..., 52; \text{site}=\text{Kamioka, Korea, Oki})\). In the fitting analysis, we vary \(\sin^2 \theta_{23}, \phi_{13}, \phi_{14}\) and \(\phi_{34}\) freely, while using true values for the other standard mixing angles \(\theta_{12}, \theta_{13}, \theta_{14}, \theta_{24}, \theta_{34}\), and the mass differences \(\Delta m^2_{21}, |\Delta m^2_{32}|, |\Delta m^2_{41}|\). Such an analysis is justifiable because \(\theta_{12}, \theta_{13}, \theta_{14}, \theta_{24}, \theta_{34}, \Delta m^2_{21}, |\Delta m^2_{32}|, |\Delta m^2_{41}|\) can be measured to a good accuracy in short baseline experiments and atmospheric neutrino measurements, while the octant of \(\theta_{23}\) and the CP phases \(\phi_{13}, \phi_{14}, \phi_{34}\) can be measured precisely only in long baseline experiments. Additionally, we fit the normal hierarchy events by assuming the normal hierarchy, and the inverted hierarchy events by assuming the inverted hierarchy, because the mass hierarchy may have been determined before T2HK starts operation.

We take into account the uncertainties of the efficiencies of far detectors \(\varepsilon_{\text{e,site}}, \varepsilon_{\mu,\text{site}}, \tilde{\varepsilon}_{\text{e,site}}, \tilde{\varepsilon}_{\mu,\text{site}}\), those of neutrino-focusing and antineutrino-focusing beam fluxes \(f_{\Phi_\nu}, f_{\Phi_{\bar{\nu}}}, \tilde{f}_{\Phi_\nu}, \tilde{f}_{\Phi_{\bar{\nu}}}\), and those of the cross sections (estimated from near detector data and theory) \(f_{\sigma\text{e}}, f_{\sigma\mu}, \tilde{f}_{\sigma\text{e}}, \tilde{f}_{\sigma\mu}\), which constitute systematic uncertainties. We vary the above parameters, with the inclusion of the maximal correlation of \(\varepsilon_{\text{e,site}}\) and \(\tilde{\varepsilon}_{\text{e,site}}\), and that of \(\varepsilon_{\mu,\text{site}}\) and \(\tilde{\varepsilon}_{\mu,\text{site}}\). Eventually, the \(\chi^2\) function

---

3 We consider that the same water Čerenkov detector as Kamioka is situated at Oki.
Thus, we evaluate the significance of rejecting the wrong octant, correlated.

\[ \chi^2(\sin^2 \theta_{23}, \phi_{13}, \phi_{14}, \phi_{34}, \Pi_{\text{sys}}) = \sum_{\text{site}} \sum_{i=1}^{52} \]

\[
\left[ \frac{(N_{e,i,\text{site}} - N_{e,i,\text{site}}^{\text{pred}}(\sin^2 \theta_{23}, \phi_{13}, \phi_{14}, \phi_{34}, \Pi_{\text{sys}}))^2}{N_{e,i,\text{site}}} + \frac{(N_{\mu,i,\text{site}} - N_{\mu,i,\text{site}}^{\text{pred}}(\sin^2 \theta_{23}, \phi_{13}, \phi_{14}, \phi_{34}, \Pi_{\text{sys}}))^2}{N_{\mu,i,\text{site}}} \right]^{2}
\]

\[
+ \sum_{\text{site}} \left\{ \left( \frac{\varepsilon_{e,\text{site}} - 1}{0.007} \right)^2 + \left( \frac{\varepsilon_{\mu,\text{site}} - 1}{0.010} \right)^2 \right\}
\]

\[
+ \left( \frac{f_{\Phi_\nu} - 1}{0.1} \right)^2 + \left( \frac{f_{\Phi_\nu} - 1}{0.08} \right)^2 + \left( \frac{\tilde{f}_{\Phi_\nu} - 1}{0.1} \right)^2 + \left( \frac{\tilde{f}_{\Phi_\nu} - 1}{0.1} \right)^2
\]

\[
+ \left( \frac{f_{\sigma_e} - 1}{0.032} \right)^2 + \left( \frac{f_{\sigma_\mu} - 1}{0.032} \right)^2 + \left( \frac{\tilde{f}_{\sigma_e} - 1}{0.059} \right)^2 + \left( \frac{\tilde{f}_{\sigma_\mu} - 1}{0.044} \right)^2
\]

where \( \Pi_{\text{sys}} = \{ \varepsilon_{e,\text{site}}, \varepsilon_{\mu,\text{site}}, f_{\Phi_\nu}, f_{\Phi_\nu}, \tilde{f}_{\Phi_\nu}, f_{\sigma_e}, f_{\sigma_\mu}, \tilde{f}_{\sigma_e}, \tilde{f}_{\sigma_\mu} \} \)
and \( \tilde{\varepsilon}_{e,\text{site}} - 1 = \frac{0.017}{0.007}(\varepsilon_{e,\text{site}} - 1), \quad \tilde{\varepsilon}_{\mu,\text{site}} - 1 = \frac{0.011}{0.010}(\varepsilon_{\mu,\text{site}} - 1). \) (15)

Here \( N_{e,i,\text{site}}, N_{\mu,i,\text{site}}, \tilde{N}_{e,i,\text{site}} \) and \( \tilde{N}_{\mu,i,\text{site}} \) are calculated for the central values of Eqs. (11), (12), (13). \( N_{\text{pred}}(\sin^2 \theta_{23}, \phi_{13}, \phi_{14}, \phi_{34}, \Pi_{\text{sys}}) \) denotes the number of events calculated as a function of \( \sin^2 \theta_{23} \), \( \phi_{13}, \phi_{14}, \phi_{34} \) and the systematic parameters \( \Pi_{\text{sys}} = \{ \varepsilon_{e,\text{site}}, \varepsilon_{\mu,\text{site}}, f_{\Phi_\nu}, f_{\Phi_\nu}, \tilde{f}_{\Phi_\nu}, f_{\sigma_e}, f_{\sigma_\mu}, \tilde{f}_{\sigma_e}, \tilde{f}_{\sigma_\mu} \} \), using the true values of \( \theta_{12}, \theta_{13}, \Delta m^2_{21}, |\Delta m^2_{32}|, \theta_{14}, \theta_{24}, \theta_{34} \) and the true mass hierarchy. The values of \( \tilde{\varepsilon}_{e,\text{site}}, \tilde{\varepsilon}_{\mu,\text{site}} \) are calculated from \( \varepsilon_{e,\text{site}}, \varepsilon_{\mu,\text{site}} \) through Eq. (15), since they are maximally correlated.

In this paper, we are interested in the danger of wrong measurement of the \( \theta_{23} \) octant. Thus, we evaluate the significance of rejecting the wrong octant, \( \sqrt{\Delta \chi^2} \), which is the square root of the difference between the minimum of \( \chi^2 \) for \( \sin^2 \theta_{23} > 1/2 \) and that for \( \sin^2 \theta_{23} < 1/2 \).

Specifically, for each benchmark with \( \sin^2 \theta_{23} = 0.44 \) or 0.60, \( \Delta \chi^2 \) is given by

\[
\Delta \chi^2 = \min_{\sin^2 \theta_{23} > 1/2} \chi^2(\sin^2 \theta_{23}, \phi_{13}, \phi_{14}, \phi_{34}, \Pi_{\text{sys}}) - \min_{\sin^2 \theta_{23} < 1/2} \chi^2(\sin^2 \theta_{23}, \phi_{13}, \phi_{14}, \phi_{34}, \Pi_{\text{sys}})
\]

when true \( \sin^2 \theta_{23} \) is 0.44, \( \Delta \chi^2 = \min_{\sin^2 \theta_{23} < 1/2} \chi^2(\sin^2 \theta_{23}, \phi_{13}, \phi_{14}, \phi_{34}, \Pi_{\text{sys}}) - \min_{\sin^2 \theta_{23} > 1/2} \chi^2(\sin^2 \theta_{23}, \phi_{13}, \phi_{14}, \phi_{34}, \Pi_{\text{sys}})
\]
when true \( \sin^2 \theta_{23} \) is 0.60. (16)
We have confirmed that for both $\sin^2 \theta_{23} = 0.44$ and 0.60, the $\chi^2$ function has local minima in both $\sin^2 \theta_{23} > 1/2$ and $\sin^2 \theta_{23} < 1/2$ regions.

### 3.5 Results

Before going to the main results, we show in Table 5 the total numbers of $\nu_\mu + \bar{\nu}_\mu$ events at the Kamioka, Oki and Korea detectors in T2HK, T2HKK and T2HK+Oki experiments when there were no oscillations, corresponding to the configurations of Table 4 and the assumption that J-PARC operates with 1.3 MW beam power.

Table 5: The total number of $\nu_\mu + \bar{\nu}_\mu$ events at each detector in each experiment when there were no oscillations, corresponding to the configurations of Table 4 and J-PARC operation with 1.3 MW beam power.

| experiment | detector | event number with neutrino-focusing beam ($\times 10^4$) | event number with antineutrino-focusing beam ($\times 10^4$) |
|------------|----------|----------------------------------------------------------|----------------------------------------------------------|
| T2HK       | Kamioka  | 13                                                       | 4.7                                                      |
| T2HKK      | Kamioka  | 7.9                                                      | 2.8                                                      |
|            | Korea    | 1.6                                                      | 0.86                                                     |
| T2HK+Oki   | Kamioka  | 7.9                                                      | 2.8                                                      |
|            | Oki      | 2.0                                                      | 1.1                                                      |

Now we present $\Delta \chi^2$ with $\sin^2 \theta_{23} = 0.44$ Eq. (16) and that with $\sin^2 \theta_{23} = 0.60$ Eq. (17) in the first benchmark Table 1. The results are presented as a function of $(\phi_{13}, \phi_{14})$ and shown separately for the normal and inverted mass hierarchies. Fig. 2 is the result when the mass hierarchy is normal, and Fig. 3 is the result when the mass hierarchy is inverted. The red, orange, green, cyan, blue and purple contours correspond to $\Delta \chi^2 = 1^2, 1.5^2, 2^2, 2.5^2, 3^2, 3.5^2$, respectively.
Figure 2: Significance of rejecting the wrong $\theta_{23}$ octant when the true $\sin^2 \theta_{23}$ is 0.44, Eq. (16) (upper three panels), and that when the true $\sin^2 \theta_{23}$ is 0.60, Eq. (17) (lower three panels), in T2HK, T2HKK and T2HK+Oki experiments, in the first benchmark Table 1. The mass hierarchy is normal. The red, orange, green, cyan, blue and purple contours correspond to $\Delta \chi^2 = 1^2, 1.5^2, 2^2, 2.5^2, 3^2, 3.5^2$, respectively.
The following observations are made from Figs. 2-3:

- In T2HK, the significance of rejecting the wrong $\theta_{23}$ octant can decrease as low as $\sqrt{\Delta \chi^2} < 1.5$ when $\sin^2 \theta_{23} = 0.44$, and as low as $\sqrt{\Delta \chi^2} < 2$ when $\sin^2 \theta_{23} = 0.60$, in a wide region of the ($\phi_{13}, \phi_{14}$) space for both mass hierarchies. This corroborates the claim of Ref. [16].

- In T2HK, when $\sin^2 \theta_{23} = 0.44$, the decrease of the significance is most prominent for $\phi_{13} \simeq \phi_{14}$, and in particular for $(\phi_{13}, \phi_{14}) = (0, 0), (\pi, \pi)$. When $\sin^2 \theta_{23} = 0.60$, the decrease of the significance is most prominent for $\phi_{13} \simeq \phi_{14} + \pi$, and in particular for $(\phi_{13}, \phi_{14}) = (0, \pi), (\pi, 0)$. The above results are quite consistent with Eqs. (3),(4).

Since $B \geq 0$ in our phase definition, the second part of Eq. (4) is positively maximal when $\phi_{13} \simeq \phi_{14}$, and is negatively maximal when $\phi_{13} \simeq \phi_{14} + \pi$. Hence, this part is most likely to lead to wrong $\theta_{23}$ octant measurement when $\sin^2 \theta_{23} < 1/2$ and $\phi_{13} \simeq \phi_{14}$, and when $\sin^2 \theta_{23} > 1/2$ and $\phi_{13} \simeq \phi_{14} + \pi$. Moreover, when $\phi_{13} = 0, \pi$, the first part

\[ \text{(5)} \]

The term Eq. (5) is small around the energy peak at the Kamioka detector and so plays no role in the following argument.
of Eq. (4) vanishes and thus the combination of neutrino and antineutrino oscillations cannot resolve the above degeneracy.

- T2HKK and T2HK+Oki give a larger significance of rejecting the wrong $\theta_{23}$ octant than T2HK in all cases, in spite of their smaller statistics than T2HK (see Table 5). This confirms the qualitative argument of Section 2 of the present paper.

- Comparing T2HKK and T2HK+Oki, we observe that T2HKK gives a larger significance, in spite of the fact that T2HKK has smaller statistics than T2HK+Oki (see Table 5). The reason that T2HK+Oki, in spite of its larger statistics, shows a smaller improvement of the octant sensitivity than T2HKK is understood as follows: As shown in Fig. 1 under our assumption that the beam off-axis angle at Oki is 1.0°, the flux at Oki has a (broad) peak around $E \approx 1.4$ GeV. Thus, the value of $L/E$ at the flux peak is 653 km/1.4 GeV for Oki, and it is 295 km/0.6 GeV for Kamioka. Since 653 km/1.4 GeV and 295 km/0.6 GeV are close values, a major portion of (anti)neutrinos oscillate in a similar manner at Oki and Kamioka, which spoils our attempt to resolve the degeneracy between Eq. (3) and Eqs. (4), (5) by combining different baselines.

Next, we present $\Delta \chi^2$ with $\sin^2 \theta_{23} = 0.44$ Eq. (16) and that with $\sin^2 \theta_{23} = 0.60$ Eq. (17) in the second benchmark Table 2. The results are presented as a function of $(\sin^2 \theta_{34}, \phi_{34})$ and shown separately for the normal and inverted mass hierarchies. Fig. 4 is the result when the mass hierarchy is normal, and Fig. 5 is the result when the mass hierarchy is inverted.
Figure 4: Significance of rejecting the wrong $\theta_{23}$ octant when the true $\sin^2 \theta_{23}$ is 0.44, Eq. (16) (upper three panels), and that when the true $\sin^2 \theta_{23}$ is 0.60, Eq. (17) (lower three panels), in T2HK, T2HKK and T2HK+Oki experiments, in the second benchmark Table 2. The mass hierarchy is normal. The orange, green, cyan, blue and purple contours correspond to $\Delta \chi^2 = 1.5^2$, $2^2$, $2.5^2$, $3^2$, $3.5^2$, respectively.
The following observations are made from Figs. 4, 5:

- T2HKK and T2HK+Oki give a larger significance of rejecting the wrong $\theta_{23}$ octant than T2HK in all cases. Also, T2HKK shows a larger significance than T2HK+Oki in all cases.

- Non-zero values of $\theta_{34}$ tend to increase the significance of rejecting the wrong $\theta_{23}$ octant in both cases with $\sin^2 \theta_{23} = 0.44$ and 0.60, for both mass hierarchies, in all the experiments. Interestingly, even though the matter effect is small in T2HK, we still obtain an increase in the significance in T2HK. The above results are because the term Eq. (7) is proportional to $\sin \theta_{23}$ and flips sign for neutrinos and antineutrinos. Hence, a measurement of this term using the combination of neutrino-focusing and antineutrino-focusing operations offers a new probe for the $\theta_{23}$ octant.

- In the case with $\sin^2 \theta_{23} = 0.60$ and for small $\sin^2 \theta_{34}$, there is a tiny parameter region where the significance of rejecting the wrong $\theta_{23}$ octant decreases with $\theta_{34}$. The presence of such a region is probably because the term Eq. (7) mimics the first part of Eq. (4).
(remember that $\phi_{13} = \pi$ in the second benchmark and so the true value of the first part of Eq. (4) is 0), which makes it harder to distinguish the term Eq. (4) from the term Eq. (3).

Finally, we present $\Delta \chi^2$ with $\sin^2 \theta_{23} = 0.44$ Eq. (16) and that with $\sin^2 \theta_{23} = 0.60$ Eq. (17) in the third benchmark Table 3. The results are presented as a function of $(\sin^2 \theta_{24}, \phi_{13} = \phi_{14})$ for $\sin^2 \theta_{23} = 0.44$ and $(\sin^2 \theta_{24}, \phi_{13} = \phi_{14} - \pi)$ for $\sin^2 \theta_{23} = 0.60$, and shown separately for the normal and inverted mass hierarchies. Fig. 6 is the result when the mass hierarchy is normal, and Fig. 7 is the result when the mass hierarchy is inverted.

![Figure 6](image)

Figure 6: Significance of rejecting the wrong $\theta_{23}$ octant when the true $\sin^2 \theta_{23}$ is 0.44, Eq. (16) (upper three panels), and that when the true $\sin^2 \theta_{23}$ is 0.60, Eq. (17) (lower three panels), in T2HK, T2HKK and T2HK+Oki experiments, in the third benchmark Table 3. The mass hierarchy is normal. The orange, green, cyan, blue and purple contours correspond to $\Delta \chi^2 = 1.5^2, 2^2, 2.5^2, 3^2, 3.5^2$, respectively.
Figure 7: Same as Fig. 6 except that the mass hierarchy is inverted. The red contour corresponds to $\Delta \chi^2 = 1$.

The following observations are made from Figs. 6, 7:

- T2HKK and T2HK+Oki give a larger significance of rejecting the wrong $\theta_{23}$ octant than T2HK in all cases. Also, T2HKK shows a larger significance than T2HK+Oki in all cases.

- The significance of rejecting the wrong $\theta_{23}$ octant mostly decreases with $\sin^2 \theta_{24}$ in both cases with $\sin^2 \theta_{23} = 0.44$ and 0.60, for both mass hierarchies, in all the experiments. The significance in T2HK is above 3 for $\sin^2 \theta_{24} < 0.0015$ (with $\sin^2 \theta_{14} = 0.004$) when $\sin^2 \theta_{23} = 0.44$, and for $\sin^2 \theta_{24} < 0.003$ (with $\sin^2 \theta_{14} = 0.004$) when $\sin^2 \theta_{23} = 0.60$. This suggests that the sensitivity of T2HK to the $\theta_{23}$ octant is recovered for such small values of sterile-active mixings.

- When $\sin^2 \theta_{23} = 0.44$, for $\phi_{13} \approx 3\pi/2$ and for large $\sin^2 \theta_{24}$, there is a tiny parameter region where the significance increases with $\sin^2 \theta_{24}$. The presence of such a region is probably because for larger $\sin^2 \theta_{24}$, the first part of Eq. (4) is less likely to mimic the second part of Eq. (5) (remember that $\phi_{13} - \phi_{14} = 0, \pi$ in the third benchmark and so
the true value of the second part of Eq. (5) is zero), which makes it easier to distinguish the first part of Eq. (4) from the other terms. Then the combination of neutrino- and antineutrino-focusing operations can more easily resolve the degeneracy between Eq. (3) and Eq. (4).

We note in passing that in the $\chi^2$ minimum in the wrong octant region, the fitted value of $\phi_{13}$ is close to its true value. So, even if the sensitivity to the $\theta_{23}$ octant is lost, the measurement of the standard CP phase is relatively unaffected.

4 Summary

We have confirmed that in the presence of sterile-active mixings with non-zero $\theta_{14}$ and $\theta_{24}$, T2HK experiment with two 187 kton detectors at Kamioka can lose sensitivity to the $\theta_{23}$ octant, depending on values of CP phases. We have revealed that T2HKK exhibits a better sensitivity to the $\theta_{23}$ octant in all parameter regions including those where the experiment with two detectors at Kamioka loses its sensitivity, in spite of smaller statistics of T2HKK. The better sensitivity of T2HKK is because measurements of the same beam at Kamioka and Korea, which involve distinctively different baseline lengths, resolve the degeneracy between the atmospheric oscillation probability and the sum of the interferences with the solar oscillation amplitude and active-sterile oscillation amplitude. We have further studied the impact of non-zero $\theta_{34}$ mixing and found that the sensitivity to the $\theta_{23}$ octant tends to increase with $\theta_{34}$ in all parameter regions in all the experiments.

Our results suggest that if a hint of sterile-active mixings with $\theta_{14} \neq 0$ and $\theta_{24} \neq 0$ is discovered but $\theta_{34}$ is 0 or substantially smaller than the current experimental bound, T2HKK is a preferable option compared to the plan of placing the two detectors at Kamioka, for the determination of the $\theta_{23}$ octant.

Additionally, we have studied a case where one 187 kton detector is at Kamioka and a 100 kton detector is at Oki Islands. This case also shows a better sensitivity to the $\theta_{23}$ octant in those parameter regions where the plan of two 187 kton detectors loses its sensitivity, but the improvement is quite mild compared to T2HKK, in spite of its larger statistics than T2HKK.

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