Stellar wind accretion and accretion disk formation: applications to neutron star high mass X-ray binaries

Shigeyuki KARINO¹, Kenji NAKAMURA², and Ali TAANI³

¹Faculty of Science and Engineering, Kyushu Sangyo University, 2-3-1 Matsukadai, Higashi-ku, Fukuoka 813-8503, Japan
²Department of Mechanical Engineering, Kyushu Sangyo University, 2-3-1 Matsukadai, Higashi-ku, Fukuoka 813-8503, Japan
³Physics Department, Faculty of Science, Al-Balqa Applied University, 19117 Salt, Jordan

∗E-mail: karino@ip.kyusan-u.ac.jp, nakamura@ip.kyusan-u.ac.jp, ali.taani@bau.edu.jo

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Abstract
Recent X-ray observations have revealed the complexity and diversity of high-mass X-ray binaries (HMXBs). This diversity challenges a classical understanding of the accretion process onto the compact objects. In this study, we reinforce the conventional concept of the nature of wind-fed accretion onto a neutron star considering the geometrical effect of radiatively accelerated wind, and re-evaluate the transported angular momentum by using a simple wind model. Our results suggest that even in an OB-type HMXB fed by stellar wind, a large amount of angular momentum could be transported to form an accretion disk due to the wind-inhomogeneity, if the binary separation is tight enough and/or stellar wind is slow. We apply our model into actual systems such as LMC X-4 and OAO 1657-415, and discuss the possibility of disk formations in these systems.

Key words: accretion, accretion disks — stars: neutron — X-rays: binaries– X-rays: individual (LMC X-4, OAO 1657-415)

1 Introduction
X-ray binary systems involving neutron stars are classified into two classes, according to their donor mass: high-mass X-ray binaries (HMXBs) and low-mass X-ray binaries (LMXBs). Among them, HMXBs are further classified into OB-type and Be-type (Corbet 1984; Corbet 1986; Bhattacharya & van den Heuvel 1991; Bildsten et al. 1997; Pfahl et al. 2002) according to the type of the donor. In general, in an OB-type system a neutron star captures quasi-spherical wind matter ejected by a massive donor. In this case, the accretion geometry around the neutron star takes almost spherical Bondi flow (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944). If the accreting compact object is a magnetized neutron star, accreted matter would be captured by the magnetosphere at a certain radius. It is generally considered that the quasi spherical accretion flow could be trapped by strong magnetic field of a neutron star before a formation of an accretion disk in an OB-type HMXB. With the recent growth of number of observed systems, however, the diversity of the accretion mode in HMXBs has been revealed and peculiar systems which do not follow the traditional understanding have been found (Walter et al. 2015). For instance, the accretion modes in supergiant fast X-ray transients (SFXTs) are still under active discussions: several theories have been verified such as clumpy wind accretion, magnetic gating mechanism due to strong field, and/or settling spherical accretion shell (in’t Zand 2005; Walter & Zurita Heras 2007; Bozzo et al. 2008; Shakura et al. 2014). Furthermore, recently, discussions about the accretion and emission mechanisms of pulsating ultra-
luminous X-ray sources involving neutron stars has been started (Bachetti et al. 2014; Eksi et al. 2015; Furst et al. 2016; Israel et al. 2017a; Israel et al. 2017b).

Although the study of OB-type HMXBs which are persistent bright X-ray sources has long history, nature of OB-type HMXBs have not yet been understood completely. OB-type systems have typically orbital periods of several days, and their X-ray luminosities are relatively persistent. They occupy the upper-left region of Corbet diagram; their spin periods are systematically long even their orbits are rather shorter than Be-type HMXBs (Corbet 1984; Corbet 1986). It means that the transportation rate of the angular momentum via wind accretion is much lower than disk accretion. In general, the spherical Bondi accretion is assumed in the wind-fed X-ray binaries and/or axisymmetric accretion is often considered (Hoyle & Lyttleton 1939; Foglizzo & Ruffert 1997; Edgar 2004). This simplification, however, cannot be valid in certain situations. Especially, in tight binary systems with slow wind, an effect due to Coriolis force cannot be negligible (Huaire-Espinosa et al. 2013). In such systems, the relative velocity vector of the stellar wind becomes inclined to the binary axis. Furthermore, if the donor is massive star, the wind matter could be accelerated via the line-driven mechanism (Castor et al. 1975), and has steep gradient of the velocity and density in radial direction. Due to this inclined accelerated wind, the accretion flow captured by a neutron star could have significant inhomogeneities of density and velocity, as shown in Fig. 1. These inhomogeneities bring a certain amount of angular momentum onto the accreting neutron star, and if it is enough large, an accretion disk may be formed around the neutron star. Here, we examine the transferred angular momentum due to the inhomogeneity of the stellar wind in tight binary systems and investigate the possibility of the disk formation in wind-fed X-ray binary systems.

In this study, we show that an accretion disk could be formed around the magnetized neutron star even in a wind-fed binary system, when the binary separation is narrow and/or wind velocity is slow. This result will play an important role to understand some peculiarities and diversities of observed HMXBs. We apply our analysis of inhomogeneous wind accretion to the actual observed sources as following. For example, LMC X-4, that is a well-known OB-type HMXB, has an accretion disk around the neutron star and often shows large X-ray flares exceeding the Eddington luminosity (Ilovaisky et al. 1984; Levine et al. 1991; Moon et al. 2003). Additionally, OAO1657-415 shows peculiar binary parameters which cannot be explained by any binary evolution scenario (Mason et al. 2012). The nature of these systems could be somehow different from the typical figure of OB-type HMXBs fed via stellar wind (Taani et al. 2018a; Taani et al. 2018b). In past, these systems have been considered to be fed via Roche lobe over-flow (RLOF) accretion (Frank et al. 2002). According to recent studies, however, it is suggested that the donors of these systems are rather small and cannot fulfill their Roche lobe (van der Meer et al. 2007; Rawls et al. 2011; Falanga et al. 2015). In addition to this, it is needed to discuss the stability of the binary system that RLOF mass transfer proceeds from a massive donor to relatively less-massive neutron star (Eggleton 2006). In order to understand the nature of these peculiar systems, we need to revisit the nature of wind-fed accretion from massive donors. Therefore, we apply our model to these systems, and show that neutron stars in these systems could receive enough amount of angular momentum from the stellar wind to form accretion disks.

In the next section, we briefly show the method of our analysis. In Section 3, we show the results. In Section 4, we discuss the validity and limits of our analysis. Then we show some results of further applications to observed systems. The final section is devoted to the conclusion.

2 Transportation of angular momentum via stellar wind

Transportation processes of angular momentum via stellar wind in X-ray binaries have been studied not only by analytic approaches but also by numerical approaches. For example, in Shapiro & Lightman (1976), the authors suggested that a black hole fed by asymmetric stellar wind can form an accretion disk when the orbital motion is taken into account. On the other hand, they showed that a neutron star cannot have an accretion disk since the inner radius of the disk is too small. Their work had been extended to investigate the spin-up/down evolution of the compact accreting object fed by stellar wind by Wang (1981). Angular momentum transportation via non-uniform stellar wind has been studied numerically by many authors, and especially after ‘90s, where the numerical simulations of wind accretion onto compact objects have been actively studied (Theuns & Jorissen 1993; Ruffert & Anzer 1995; Ruffert 1999; Blondin & Pope 2009). After the notable findings of the
Highly resolved wind computations have revealed that, under certain conditions, angular momentum of the asymmetric wind could be transported to the circumference of the accreting object, and an accretion disk could be formed (Blondin & Pope 2009; Blondin 2013). Furthermore, when the wind is dense and slow, such as the slow wind emanating from giant/AGB donors, it is suggested that a static accretion disk could be formed without the flip-flop behavior (Theuns & Jorissen 1993; Soker 2004; Huarte-Espinosa et al. 2013). Several such hydrodynamical studies of the interaction between giant donor and compact accretor have been performed covering a wide range of phenomena (Jahanara et al. 2005; Hadrava, P. & Čechura 2012; Liu et al. 2017; El Mellah & Casse 2017). In these works, however, they have not supposed a magnetized neutron stars as an accreting object.

On the other hand, in the context of X-ray binaries with OB-type donors, (quasi-)spherical wind accretion is considered in most cases (Shakura et al. 2012; Postnov et al. 2017); it is considered that the angular momentum is rather withdrawn from the neutron star due to the interaction between the neutron stellar field and the wind matter (Davies & Pringle 1981; Bozzo et al. 2008; Giménez-García et al. 2016). However, in most of previous studies, the line-acceleration of the wind from the OB-type donor (Castor et al. 1975) is omitted, except for some simple analyses (Wang 1981). The rapid acceleration produces a steep gradient of the wind velocity and density in radial direction. When the orbital velocity is comparable to the wind velocity, the relative velocity vector of the wind could be inclined significantly (see Fig. 1); and this radial gradient of the wind velocity could remarkably eliminate the symmetry of the wind to the accreting object (Huarte-Espinosa et al. 2013).

Hence in this study, we consider wind accretion processes and the consequent angular momentum transport taking such an asymmetry of the wind into account, using the simple wind model. When the orbital motion is comparable to the wind velocity, the stellar wind is no longer symmetric about the binary axis. Additionally, if the stellar wind suffered from line-driven acceleration, the wind density decreases rapidly. As a result of this acceleration, the velocity and density distribution of the stellar wind passing the neutron star neighborhood becomes further asymmetric. These asymmetries bring some amount of angular momentum to the accreting neutron star. In this study, we aim to evaluate the angular momentum transported due to this asymmetry.

In this purpose, we need an orbital radius $R_{\text{orb}}$, and we obtain this from the Keplerian low:

$$R_{\text{orb}} = \left[ \frac{G(M_d + M_{\text{NS}}) P_{\text{orb}}^2}{4\pi^2} \right]^{1/3}. \quad (1)$$

Here, we assume that the donor mass is much larger than that of the neutron star ($M_d \gg M_{\text{NS}}$) and the system takes a circular orbit. In addition, the orbital velocity of the neutron star, $\tilde{v}_{\text{orb}} = v_{\text{orb}} \hat{e}_R$ can be obtained by

$$v_{\text{orb}} = \sqrt{\frac{GM_d}{R_{\text{orb}}}} \quad (2)$$

$\hat{e}_R$ denotes a unit vector in the direction of the orbital motion of the neutron star. At the neutron star position ($R_{\text{orb}}$), the wind velocity accelerated via line force $\tilde{v}_w = v_w \hat{e}_R$ is computed by

$$v_w = v_{\text{inf}} \left( 1 - \frac{R_d}{R_{\text{orb}}} \right)^{\beta}, \quad (3)$$

where $R_d$ denotes the radius of the donor (Castor et al. 1975; Kudritzki & Puls 2000). $\hat{e}_R$ denotes a unit vector in the radial direction seen from the center of the donor (see Appendix). The acceleration parameter $\beta$ is fixed as $\beta = 1$. $v_{\text{inf}}$ denotes the wind velocity at the infinity. From Eqs. (2) and (3), the relative velocity between the neutron star and the wind can be obtained as following:

$$v_{\text{rel}}^2 = v_w^2 + v_{\text{orb}}^2. \quad (4)$$

Here we introduce a reference surface $S$, including the neutron star to evaluate the accretion rate of mass and angular momentum. We set this surface to have its normal vector coincides with the relative velocity vector of the wind, $\tilde{v}_{\text{rel}}$, at the neutron star position.

The gravitational potential of the neutron star at each position on the $S$ surface is

$$U = -\frac{GM_{\text{NS}}}{r}, \quad (5)$$

where $r$ is the distance from the neutron star on the $r$ surface. If this potential overcomes the kinetic energy of the wind $K = v_{\text{rel}}^2/2$, the wind matter passing this point will be trapped by the neutron star. Using an analog from Hoyle-Lyttleton accretion theory (Hoyle & Lyttleton 1939; Foglizzo & Ruffert 1997), the area where the accretion matter would be trapped in the reference surface $S$ can be shown by the maximum radius of such an area, $r_{\text{acc}}$. The shape of $r_{\text{acc}}$ gets distorted depending on the wind and orbital parameters, as shown in Fig. 2. In this figure, the locus of $U + K = 0$ on the $S$ surface is shown by the solid curve, while an accretion radius evaluated by the wind velocity at the neutron star position is shown by the dashed line (Hoyle-Lyttleton theory). At the same time, the wind density on the $S$ surface is indicated by the gray value. Clearly, the accretion area defined by $U + K = 0$ shifts toward the dense side. Note that such a deformation effect of accretion region has been neglected in previous theoretical works (Hoyle & Lyttleton 1939; Wang 1981; Foglizzo & Ruffert 1997).

The accretion rate of mass and angular momentum given by the accretion matter passing the area $r < r(U + K = 0)$ are computed as
The accretion flow with its angular momentum obtained by our model can form an accretion disk. The procedure described in Appendix. larized at the so-called circularization radius \( \xi \). Hence, to form an accretion disk, the above circularization radius should be larger than the radius which the magnetic force becomes dominant (Frank et al. 2002). Hence, to form an accretion disk, above circularization radius should be larger than the radius which the magnetic force becomes dominant (Frank et al. 2002); Fig. 2. Oblate accretion area defined by the balance between wind kinetic energy and the neutron star potential is shown by solid curve. Density of the wind matter is also shown by gray scale: dark gray region corresponds to thick wind density. The entire range from the pale end to the dark end covers 1 order magnitude of the wind density.

\[
\dot{M} = \int_{r < r(J = 0)} \rho_w v_{rel,x} dS
\]

and

\[
\dot{j} = \int_{r < r(U + K = 0)} \rho_w v_{rel,x} x dS,
\]

respectively. Here, \( x \) is the projected length measured from the donor onto the orbital plane. \( v_{rel,x} \) is the normal component of the relative velocity vector to the \( S \) surface, and it is given in the procedure described in Appendix.

From Eqs. (6) and (7), the specific angular momentum of the accretion flow can be computed as

\[
\ell = J \dot{M}^{-1}.
\]

3 Results

Next, we examine whether the accretion flow with the angular momentum obtained by our model can form an accretion disk. The accretion flow with its angular momentum \( \ell \) will be circularized at the so-called circularization radius \( r_{circ} \) that is defined as

\[
r_{circ} = \frac{\ell^2}{GM_{NS}},
\]

and this radius corresponds to the initial size of the accretion disk (Frank et al. 2002). If the accretion flow is captured by the magnetic field out of this radius, however, the flow cannot form an accretion disk and will be fall along the field line toward the polar regions of the neutron star (Basko & Sunyaev 1976; Frank et al. 2002). Hence, to form an accretion disk, above circularization radius should be larger than the radius which the magnetic force becomes dominant (Frank et al. 2002); Fig. 2. Oblate accretion area defined by the balance between wind kinetic energy and the neutron star potential is shown by solid curve. Density of the wind matter is also shown by gray scale: dark gray region corresponds to thick wind density. The entire range from the pale end to the dark end covers 1 order magnitude of the wind density.

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From Eqs. (6) and (7), the specific angular momentum of the accretion flow can be computed as

\[
\ell = J \dot{M}^{-1}.
\]

We investigate the condition where the above relation is settled, when we change various parameters.

In this study, we fix the mass, radius and magnetic field of the neutron star: \( M_{NS} = 1.4 M_{\odot} \), \( r_{NS} = 1.0 \times 10^{6} \text{cm} \), and \( B_{NS} = 2.0 \times 10^{12} \text{G} \), respectively. Though the mass range of a neutron star is distributed between roughly \( 1.2 M_{\odot} \) to \( 2 M_{\odot} \), the mass of the neutron star in HMXBs are not so far from \( 1.4 M_{\odot} \) (Schwab et al. 2010; Falanga et al. 2015; Suwa et al. 2018). Also the strength of the magnetic field of neutron stars in HMXBs are concentrated around a few of \( 10^{12} \text{G} \) (Christodoulou et al. 2016; Taani et al. 2018a). The wind velocity is computed by Eq. (3). First, to grasp the general tendencies, we assume the mass of the donor and radius as \( M_d = 15 M_{\odot} \) and \( R_d = 10 M_{\odot} \), respectively. However, later on, we use rather realistic values when we make comparisons with the observed results.

With these settings, we vary the spin period of the neutron star \( P_{spin} \), mass loss rate of the donor \( \dot{M}_* \), terminal velocity of the wind \( v_{\text{inf}} \), and the orbital period of the system \( P_{orb} \), respectively. As the typical result, in Fig. 3 we show the computed radii \( r_{circ}, r_m \) and \( r_{co} \) under the parameter setting as following: \( M_d = 1.0 \times 10^{-6} M_{\odot} \text{yr}^{-1}, v_{\text{inf}} = 1.0 \times 10^{8} \text{cm s}^{-1} \). In this case, the maximum orbital period that relationship of Eq. (12) is satisfied is \( 2.6 \text{d} \) regardless of the spin period. We name this maximum orbital period where the condition of relation Eq. (12) is satisfied as \( P_{orb,\text{max}} \). In the present case, the lines of propeller limits are well above of the magnetospheric radius in the region where \( P_{orb} < P_{orb,\text{max}} \) is satisfied. In a system with rapidly rotating neutron star, however, the disk formation condition could be limited by the propeller mass ejection (though fast spinning neutron stars have not been found in wind-fed HMXBs).

Even if the condition in Eq. (12) is satisfied, the subsonic accretion shell might rearranged the angular momentum before the disk is formed, when the accretion rate is too small (Shakura et al. 2012). This critical mass accretion rate is given as

\[
r_m < r_{co}, r_{circ}.
\]
On the other hand, when the mass loss rate is larger than this point, the disk formation is limited by the condition Eq. (12). These exchanges are shown with black dots in the same figure.

Possibility of a disk formation around a compact object fed by slow wind has been previously suggested (Wang 1981; Shakura et al. 2012; El Mellah et al. 2019a; Taani et al. 2018a). In Ducci et al. (2010), they considered the disk forming possibility in SFXTs which are accreted by slow wind, to explain burst activities of SFXTs. In our treatment of deformed accretion region (Fig. 2), we have found that further angular momentum could be transported and the possibility of disk formation becomes significant (this will be discussed further in section 4).

We can summarize our results as following:

- For systems with short orbital periods (for tight systems), the necessary condition for the disk formation \( r_{m} < r_{\text{circ}} \) could be satisfied.
- When the spin of the neutron star is fast, however, accretion could be inhibited due to propeller effect. For typical spin period of neutron stars in OB-HMXBs, however, it does not dominantly restrict the disk formation.
- When the wind velocity is slow, an accretion disk can be formed even in systems with large orbital period.
- When the mass loss rate of the donor is large, an accretion disk can be formed even in systems with large orbital period. When the mass loss rate of the donor is small, however, the disk formation could be limited by the subsonic shell forming condition.

4 Discussion

4.1 Deformation of the accretion region

In this study, we have shown that even in wind accretion regime enough amount of angular momentum to form an accretion disk is transported. Furthermore, we have shown that it is crucial to consider the orbital motion of the neutron star and the acceleration of the wind matter. If we consider these two effects, the accretion region shifts inward of the orbit as shown in Fig. 2. Because of this shift of accretion region, the neutron star can capture an extra dense wind matter from the inner region. And this makes possible to capture a large amount of angular momentum from the wind. At the same time, it could be understood in Fig. 2 that though the wind matter accreted from the outer region has a large velocity, its density is much lower and the accretion region is rather narrower. Namely, inward deformation of the accretion region is essentially important to estimate the accretion rate of the angular momentum.

In order to depict the effect of the deformation of the accretion region clearly, an illustration of the result is given in Fig. 6, where the transported specific angular momentum is shown in two cases. For the first case the deformation of the accretion region is not considered. This is the case which is shown by...
considered less serious in previous works (Hoyle & Lyttleton 1939; Shapiro & Lightman 1976; Wang 1981), this result shows that the deformation could play rather important role in the nature of accretion mode in tight binary systems.

At the same time, in the small box of Fig. 6, we show the ratio of the specific angular momentum between these two cases as a function of the orbital period. In the present parameter set, this ratio takes maximum at $P_{\text{orb}} = 3d$. This behavior could be understood as follows. When the orbital period is long (i.e. orbital separation is large), the wind from the donor has already well accelerated ($v_w = v_{\text{inf}} ≫ v_{\text{orb}}$). Hence, the wind could be considered like a planar uniform flow. Then the effect of the deformation of the accretion region could be smaller. On the other hand, when the orbital separation is small, the orbital velocity dominates in the relative velocity. Therefore, the acceleration of the wind does not have an important role. As the conclusion, the deformation of the accretion region becomes important at the intermediate orbital period around 3d.

### 4.2 Formation of accretion disk

Fig. 5 indicates that an accretion disk could be formed even when the orbital period of the system is rather large, if the
wind velocity is slow. The slow wind velocity is caused by slow terminal velocity due to certain reasons such as line-driven efficiency and X-ray photo-dissociations (Ducci et al. 2010; Krtíčka et al. 2012; Karino 2014). At the same time, wind velocity becomes slower also due to the increase of the relative radius of the donor to the orbital radius as seen in Eq. (3). In the above discussion, however, we have only shown that a certain amount of angular momentum could be transported inside the accretion radius. Deep inside of the accretion radius, it is required to consider whether an accretion disk could really be formed or not.

In general, the captured matter from the stellar wind forms a shock during its falling path to the neutron star. If the cooling is ineffective, falling matter could form a quasi-static settling shell below the shock (Shakura et al. 2012). Once such a shell is formed, the angular momentum is transported due to turbulence caused by hydrodynamic instabilities inside the shell. Hence, the transported angular momentum will be redistributed before it reaches deep inside of the accretion radius. When the accretion rate is enough large, \( \dot{M} > 4 \times 10^{16} \text{g s}^{-1} \), however, a settling shell is not formed and accretion matter could be fallen onto the neutron star quickly. This limiting accretion rate for the shell forming is shown by vertical dashed-dotted line in Figs. 3 and 4. This line shows the limiting case where the mass accretion rate evaluated by Eq. (6) becomes the critical value: \( 4 \times 10^{16} \text{g s}^{-1} \). In the right-hand side of this vertical line, a quasi-spherical subsonic shell will be formed and in the shell the accreted angular momentum will be redistributed. In this case, therefore axisymmetry recovers and the accretion disk cannot be formed. On the other hand, in the left-hand side of the vertical line, the mass accretion rate is enough high and a settling shell cannot be formed. Then the angular momentum will be transported to deep inside of the accretion region. Finally, if the condition given by Eq. (12) is satisfied, an accretion disk would be formed.

The above condition onto the mass accretion rate is also important to consider the nature of the formed accretion disk. When the mass supply is small, the accretion disk would be a radiatively inefficient accretion flow (RIAF), and its X-ray luminosity steeply decreases with mass accretion rate (Narayan & Yi 1994; Narayan & Yi 1995; Kato et al. 2008). The accretion disk will be RIAF-like only when the accretion rate is rather less and the temperature is high. Hence, if our disk formation condition is satisfied (high accretion rate), it seems reasonable to form a disk which behaves as a standard disk (Shakura & Sunyaev 1973). We hope future observations with more sensitive techniques can test this possibility.

To fix the position of the inner edge of the accretion disk, the effect of magnetic field is crucially important. Moreover, the neutron star magnetosphere plays an important role to onset the propeller mass ejection (Stella et al. 1986; Bozzo et al. 2008). In this study, we assume a constant magnetic field strength; \( B = 2 \times 10^{12} \text{G} \). Actually, the neutron star magnetic field could take from the lower limit (\( \sim 10^{10} \text{G} \)) seen in LMXB systems to the higher end (\( \sim 10^{14} \text{G} \)) observed in magnetars (Pfahl et al. 2002; Polsiadlowski et al. 2004; van den Heuvel 2009): it spans 5 orders of possibilities. According to the straightforward observations of cyclotron resonance features in HMXB systems, however, most of neutron stars in HMXBs show intermediate level of the field strength, \( \sim 10^{12} \text{G} \) (Taani et al. 2018a; Taani et al. 2018b). Although this stems to a certain observational bias, it is worth mentioning that the magnetic fields are still \( \sim 10^{13} \text{G} \) even in most active neutron X-ray sources such as LMC X-4 and NGC300 ULX-1 (Moon & Eikenberry 2001; Moon et al. 2003; Levine et al. 2000; Carpano et al. 2018). Therefore our assumed magnetic field \( 2 \times 10^{12} \text{G} \) is not an odd choice as the first attempt.

Considering a system which orbital period is slightly shorter than the critical period, \( P_{\text{orb,max}} \), in such a system, the difference between two radii (\( r_{\text{in}} \) and \( r_{\text{circ}} \)) are small, and the size of formed disk may be very small. Once an accretion disk is formed, however, due to viscous effects the angular momentum of the disk matter could be transported from inside towards the outer edge. At the same time the matter near the outer edge would be pushed out and disk itself would expand. At the expanded disk edge, subsequent accreted matter will be attached and the size of the disk would be further extended. For accrual example, wind-fed HMXB system LMC X-4 has a warped accretion disk enough large to shade the X-ray from the neutron star, even though it is expected that \( r_{\text{circ}} \approx r_{\text{in}} \) and the disk size could be small (discussed later).

In observed systems, the orbital eccentricities are not exactly zero. The eccentricity changes the geometrical relation of the neutron star to the stellar wind and affects the results. In typical Be-type HMXBs, the orbital eccentricity is rather large; it is considered to be induced by the natal kick when the neutron star was born in core collapse supernova. On the other hand,
4.3 Diversity of the donor and neutron star

In this study, we have considered only a typical neutron star whose mass is $1.4M_\odot$. Recent studies have availed, however, that some neutron stars have larger mass, up to $2M_\odot$ (van Kirkwijk et al. 1995; Rawls et al. 2011; Falanga et al. 2015). Such heavy neutron stars made strong constraint onto the equation of state of the neutron stellar matter and play important role to understand the nature of condensed matter. In the present study, such a large mass of neutron star also changes the geometry of the accretion region. Deep potential broadens the accretion radius especially in slow wind side and enhances asymmetry of wind geometry. This asymmetric wind brings further extra angular momentum to the neutron star neighbor and relaxes the condition of disk formation. In order to evaluate this effect, we show the result for a heavy neutron star with $M_{\text{NS}} = 2.5M_\odot$ in Fig. 7. In this example, the other conditions are the same with the case shown in Fig. 3. The maximum orbital period, $P_{\text{orb}}$, is extended up to 3.5 d, in this case. Since the mass of neutron star cannot largely exceed $2M_\odot$, we can conclude that the effect of neutron stellar mass could be limited in the present context.

We have considered only a typical SG donor with $M_\text{d} = 15M_\odot$ and $R_\text{d} = 10R_\odot$. The wind velocity at the neutron star position, however, depends on the donor size (see, Eq. (3)). That is, when the donor size is large, the wind velocity becomes slower since the zone of acceleration effectively decreases. As the consequence, the possibility of disk-formation could be larger. To confirm the effect of donor size, we have additionally computed the same case study for a large donor with $R_\text{d} = 20R_\odot$. The result is shown in Fig. 8. In this case, the parameters except for the donor radius are the same with the case of Fig. 3. We can see that, in this case, the condition Eq. (12) is satisfied in a broader range $P_{\text{orb}} < 6.4d$ for all spin periods. This time, however, the limit of subsonic shell formation dominates the maximum orbital period for disk-formation. As the result, the system which orbital period is less than 5.3d could form an accretion disk around the neutron star. This limit will be pushed up due to high mass-loss rate of the donor (see Fig. 4). Since a large evolved star tends to increase its mass-loss rate at the same time, an evolved donor would result in a large possibility to form an accretion disk around the neutron star.

In this bright system, since the existence of the accretion disk around the neutron star is evident, it has been considered that the accretion matter is supplied via RLOF (Levine et al. 1991). The spin period, orbital period and orbital eccentricity is 13.5 s, 1.4 d and $< 0.003$, respectively (Li et al. 1978; White 1978; Kelley et al. 1983). From the spectral properties, the donor star is identified as an O8III type super-giant star, which mass is assumed to be $M_\text{d} = 18M_\odot$ (Chevalier & Ilovaisky 1977; Falanga et al. 2015). Besides the orbital period, it shows 30.5 d periodicity and it is suggested to be caused by a warped accretion disk around the neutron star (Ilovaisky et al. 1984; Levine et al. 1991).

4.4 Observed systems

4.4.1 LMC X-4

LMC X-4 is one of the most powerful persistent OB-type X-ray binary in our neighborhood. It emits luminous X-rays with $L_X = 3 \times 4 \times 10^{38}\text{erg}\text{s}^{-1}$ persistently, and often causes bright X-ray flares achieving $L_X \sim 10^{39}\text{erg}\text{s}^{-1}$ that exceeds the Eddington limit (Moon et al. 2003; Shtykovsky et al. 2018). The spin period, orbital period and orbital eccentricity is 13.5 s, 1.4 d and $< 0.003$, respectively (Li et al. 1978; White 1978; Kelley et al. 1983). From the spectral properties, the donor star is identified as an O8III type super-giant star, which mass is assumed to be $M_\text{d} = 18M_\odot$ (Chevalier & Ilovaisky 1977; Falanga et al. 2015). Besides the orbital period, it shows 30.5 d periodicity and it is suggested to be caused by a warped accretion disk around the neutron star (Ilovaisky et al. 1984; Levine et al. 1991).

In this bright system, since the existence of the accretion disk around the neutron star is evident, it has been considered that the accretion matter is supplied via RLOF (Levine et al. 2000; Preciado et al. 2002). Recent analysis, however, reveal that the radius of the donor remains confined to $7 \sim 8R_\odot$ (Rawls et al. 2011; Falanga et al. 2015). Assuming that the neutron star mass is $1.4M_\odot$, the Roche lobe size is much larger than the donor radius. Hence, the donor could remain well inside the
lobe and RLOF has not been started. On the other hand, also it has been suggested the ejection of strong wind from the donor (Vrtilek et al. 1997; Boroson et al. 1999). Then the wind-fed accretion in this system is suspected.

Given these circumstances, we consider the possibility of the disk formation due to angular momentum accretion via stellar wind in this system. In this purpose, we use the same analysis described in the previous sections. The used parameters are summarized in Table. 1. The corresponding radii ($r_{\text{circ}}, r_{m}$ and $r_{co}$) are shown in Fig. 9. From the condition, Eq. (12), in this system, disk formation via stellar wind could be possible if the orbital period is shorter than 2.5d. Since the orbital period of this system is 1.4d, and since the propeller effect could be avoided with its orbital period, it is strongly suggested that the disk formation via wind accretion could be possible.

Additionally, in this system, the difference between $r_{\text{circ}}$ and $r_{m}$ is rather small. It means that the accretion regime (direct wind accretion / disk accretion) could be converted easily if the situation varies. Observation shows that this system fluctuates between the spin-up and spin-down regime (Molkov et al. 2017). Such a spin evolution might be related to such a marginal situation. Since the exchange of accretion regime may change the spectral feature, multifaceted studies are highly required (Paul et al. 2002; Naik & Paul 2003).

### 4.4.2 OAO 1657-415

OAO 1657-415 was identified in 1970’s, and has been one of the most famous HMXBs. The orbital period and orbital eccentricity is 10.448 d and 0.107, respectively (Chakrabarty et al. 1993). The donor star was identified as Ofpe star, however, since hydrogen line is weak, it might be a Wolf-Rayet star, as like Cyg X-3 (Mason et al. 2012). By optical observations, the parameters of the donor have been measured as $M_d = 14.3 M_\odot$, $R_d = 24.8 R_\odot$, $M_\dot{\nu} = 2 \times 10^{-6} M_\odot \text{yr}^{-1}$, respectively (Mason et al. 2012). The wind velocity is suggested to be very slow ($v_{\text{inf}} = 2.5 \times 10^3 \text{cm s}^{-1}$), and it may due to a displacement from spherical symmetry caused by a neutron star. It is known that the standard binary evolution scenario cannot derive such a set of binary parameters of this system (Molteni et al. 2001; Mason et al. 2012). Although no firm evidence of an accretion disk has been reported, the shape of the cumulative luminosity distribution of this system is similar to disk-forming systems (Sidoli & Paizis 2018).

We apply our model to this system with the same procedure that is discussed above. (Since the radius of the donor and the size of the Roche lobe size is almost the same in this system, it is a sensitive issue whether RLOF proceeds or not.) The results of each radii computed with the parameters of OAO1657-415 is shown in Fig. 10. From this figure, if the orbital period is shorter than 23 d, the disk formation could be possible. In fact, the orbital period of this system is 10.4 d and it clearly satisfies this condition. Also we can see that the propeller effect and subsonic shell forming do not work in this parameter range. Therefore, we conclude that also in this system, the disk formation could be realized via wind-fed accretion.

In this system, the slow wind (due to slow wind and large donor size) plays an important role in the disk formation. Such a disk formation in slow-wind binary is also suggested by numerical computations (Huarte-Espinosa et al. 2013), and this consistency supports the robustness of our working hypotheses.

Very recently it has been reported that a disk-like structure could be formed even when the donor does not fill its Roche lobe under certain conditions, according to the numerical computations (El Mellah et al. 2019a; El Mellah et al. 2019b). Such a marginal accretion mode (wind RLOF / beamed wind) might be a key to understand the diversity and complexity of HMXBs.

### 5 Conclusion

In this study, we have revisited to the transportation rate of the angular momentum in the wind-fed neutron star X-ray binaries. We have taken into account the asymmetry due to the orbital motion of the neutron star and the wind acceleration. The asymmetry of the wind can lead to a deformation of the accretion region and due to this, a large amount of angular momentum could be transported inside the accretion radius even in wind
accretion. Furthermore, we have shown that an accretion disk could be formed around the neutron star under certain situations; when the wind velocity is slow because of a small terminal velocity and/or inefficient wind acceleration due to a large donor radius, the possibility of disk formation could be larger.

We applied our disk formation criteria to the observed peculiar systems hitherto-considered to be fed via RLOF. In our analysis, neutron stars in LMC X-4 and OAO1657-415 could have accretion disks, even if they were fed via stellar winds. For the LMC X-4, the orbital period is short enough to capture plenty amount of angular momentum from the wind of the donor. In addition, the propeller effect could be avoided with this orbital period. This results elucidates the source of a warped accretion disk in this object. While for the OAO1657-415, the slow wind enhances the transportation rate of the angular momentum and gains the possibility of the disk formation.

Finally, the existence of the accretion disk will affect the observed properties such as spectral hardness, and the spin evolution of the accreting neutron star. To reveal the accretion nature of HMXBs, further studies both in observations and theoretical sides are required.

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Appendix. Tips of computations

Here, the coordinate system which we have used to evaluate the accretion rate of angular momentum via line-driven stellar wind is described. At the beginning, we define a reference surface $S$ of the wind accretion to evaluate the accreted angular momentum. We find the relative velocity vector of the wind at the position of the neutron star, $\vec{v}_{\text{rel}}$. The relative velocity can be obtained by a simple addition of the orbital velocity $\vec{v}_{\text{orb}} = v_{\text{orb}} \vec{e}_T$ (see, Eq. (2)) and the wind velocity $\vec{v}_w = v_w \vec{e}_T$ (see, Eq. (3)). Then, we set a surface which normal vector coincides with $\vec{v}_{\text{rel}}$. We consider an accretion region in this surface and estimate the mass and angular momentum transported to the neighbor of the neutron star.

Now we set $x$-axis on the line of intersection between $S$ plane and the orbital plane. Then, the $x$-direction is inclined to the relative velocity vector by the angle $\theta$ given as

$$\theta = \frac{\pi}{2} - \tan^{-1} \left( \frac{v_w}{v_{\text{orb}}} \right).$$

We set the $z$-direction as the direction of the relative velocity of the wind at the neutron star position. For $x$- and $z$-axes, we set the positive direction as the direction where recedes away from the donor. Additionally, we set $y$ direction as $\vec{e}_y = \vec{e}_x \times \vec{e}_y$, where $\vec{e}_{x,y,z}$ denote the unit vectors in each directions. This situation is depicted in Fig. 11.

On the $S$ plane $ix- y$ plane, the relative velocity of the wind is obtained as

$$v_{\text{rel}}(x, y) = \left( v_w (R')^2 + v_{\text{orb}} (R')^2 \right)^{1/2}$$

where $R'$ is the distance from the center of the donor:

$$R' = \sqrt{R(x, 0)^2 + y^2}.$$  

The value of $R$ on the $x$-axis, $R(x)$ could be obtained as

$$R^2 = R_{\text{orb}}^2 + x^2 - 2R_{\text{orb}} x \cos(\pi - \theta).$$

Under such an setting of the coordinate system, we evaluate the balance between wind kinetic energy and the neutron star potential, and find the accretion region. In our $x - y$ coordinate system, the potential of the neutron star can be described as

$$U = \frac{-G M_{\text{NS}}}{\sqrt{x^2 + y^2}},$$

and the kinetic energy of the wind can be written as

$$K = \frac{1}{2} v_{\text{rel}}(x, y)^2.$$  

The wind matter passing each point on $S$ surface, $(x, y)$, will be trapped by the neutron star if $K + U < 0$. Making the dependence on the $x - y$ coordinate clear, Eqs. (6) and (7) can be written as

$$\dot{M} = \int_{K+U<0} \rho_w(x, y) v_{\text{rel}}(x, y) dx dy,$$

and

$$\dot{J} = \int_{K+U<0} \rho_w(x, y) v_{\text{rel}}(x, y)^2 dx dy,$$

respectively. $v_{\text{rel}}(x, y)$ is the $z$-component of the relative velocity vector of the wind at the point $(x, y)$ (namely, the normal component to the $S$ plane), and it is given as the following:

$$v_{\text{rel}}(x, y) = \frac{v_w R}{R'} \sin \left( \frac{\pi}{2} - \alpha \right) + |\vec{v}_{\text{orb}}| \sin \beta.$$  

$\alpha$ is the angle between the relative velocity vector of the wind at the position $(x, y)$ and the reference plane $S$, which is given by

| Table 1. Important parameters of HMXB systems: LMC X-4 and OAO1657-415 |
|-----------------|-----------------|-----------------|
| Name            | LMC X-4         | OAO 1657-415    |
| $M_d [\text{M}_\odot]$ | 18              | 17.5            |
| $R_d [\text{R}_\odot]$  | 7.7             | 25              |
| $P_{\text{spin}} [\text{s}]$ | 13.5            | 37.7            |
| $P_{\text{orb}} [\text{d}]$ | 1.4             | 10.4            |
| $v_{\text{in}} [10^8 \text{cm s}^{-1}]$ | 0.5             | 0.25            |
| $M_d [10^{-6} \text{M}_\odot \text{yr}^{-1}]$ | 50              | 2               |
| $B_{\text{NS}} [10^{12} \text{G}]$ | 11.2            | 4               |

The binary parameters that we have used in the analysis. References are in the text.
Fig. 11. Setting concept of axes. The wind, orbital, and relative velocity vectors are those at the neutron star position.

Fig. 12. Definition of angles. The wind, orbital, and relative velocity vectors are projected on the $x - z$ plane.

$$\alpha = \frac{\pi}{2} - \cos^{-1}\left(\frac{R^2 + x^2 - R_{\text{orb}}^2}{2Rx}\right)$$

(see Fig. 12).

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