Unparticle effects in photon-photon scattering

Chun-Fu Chang\textsuperscript{1}, Kingman Cheung\textsuperscript{1,2}, and Tzu-Chiang Yuan\textsuperscript{2}

\textsuperscript{1}Department of Physics, National Tsing Hua University, Hsinchu 300, Taiwan
\textsuperscript{2}Physics Division, National Center for Theoretical Sciences, Hsinchu 300, Taiwan

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Abstract

Elastic photon-photon scattering can only occur via loop diagrams in the standard model and is naturally suppressed. Unparticle can induce tree-level photon-photon scattering through the operator $F_{\mu\nu}F^{\mu\nu}O_{\text{U}}$ for spin-0 unparticle or $F^{\mu\alpha}F_{\nu}^{\alpha}O_{\text{U}}^{\mu\nu}$ for spin-2 unparticle. Due to the peculiar CP-conserving phase $\exp(-id_U\pi)$ associated with the $s$-channel unparticle propagator, its interference effects with the $t$- and $u$-channels on the total cross section and the angular distribution are found to be some significance. In addition, we show that the cross sections via unparticle exchange can be substantially larger than the standard model contribution.

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I. INTRODUCTION

Recently, Georgi [1] pointed out an interesting possibility for the existence of a scale-invariant sector with a continuous mass distribution. This scale invariant stuff was coined the term “unparticle” to describe a possible scale-invariant hidden sector sitting at an infrared fixed point at a high scale $\Lambda_U$. The scale-invariant sector may be weakly or strongly interacting but its effects on the standard model (SM) is assumed to be weakly interacting. In Georgi’s scheme [1], the hidden sector communicates with the SM content via a messenger sector characterized by a high mass scale $M$. At energy below $M$, one can integrate out the messenger sector and ends up with effective operators suppressed by inverse powers of $M$ in the following form

$$\frac{1}{M^{d_S + d_U - 4}} O_S O_U ,$$

where $O_S$ and $O_U$ represent local operators of the SM and hidden sector with scaling dimensions $d_S$ and $d_U$, respectively. As one scales down the theory from $M$, the hidden sector may flow to an infrared fixed point at the scale $\Lambda_U$, which is generated by quantum effects via dimensional transmutation for example. At the fixed point the hidden sector becomes scale invariant, the above operator Eq. (1) has to be replaced by a new set of operators of similar form

$$C_{O_U} \frac{\Lambda_U^{d_U - d_U}}{M^{d_S + d_U - 4}} O_S O_U ,$$

where $O_U$ is the unparticle operator with a scaling dimension $d_U$ and $C_{O_U}$ is the unknown coefficient. Due to the underlying theory is a scale invariant interacting theory, the scaling dimension $d_U$ needs not having the canonical values of integer or half-integer like the free boson or free fermion cases. Besides its scaling dimension, the unparticle operator $O_U$ can be characterized by scalar, vector, tensor, or spinor etc according to its Lorentz group representation.

Despite the scale invariant sector remains unspecified, the 2-point function [1] and the Feynman propagator [2, 3] of the unparticle field operator $O_U$ can be determined by scale invariance. The normalization of the 2-point function of unparticle operator of scaling dimension $d_U$ was fixed by Georgi [1] to be the same as the phase space of $d_U$ massless particles. On the other hand, the most peculiar feature of the unparticle propagator is a
phase factor \( \exp(-idU\pi) \) associated only with time-like momenta. This CP-conserving phase has been shown to have interesting interference effects at high energy experiments \([2, 3]\) and other phenomenology.

In this work, we consider photon-photon scattering via unparticle exchanges. The SM contribution to photon-photon scattering can only arise from loop diagrams with all charged particle running around the loop and thus is highly suppressed. It is anticipated that the cross section due to unparticle exchange can easily surpass the SM cross section at high enough energies, because exchanges of unparticle are at the tree-level. Moreover, photon scatters via unparticle exchanges in all s-, t-, and u-channels. The peculiar phase \( \exp(-idU\pi) \) associated with the s-channel exchange gives rise to interesting interference with the t- and u-channel amplitudes. Similar effects had been studied in the model of large extra dimensions \([4]\).

Note that similar ideas for the spin-0 unparticle have been pursued recently in Refs. \([5, 6]\). However, our analytic results disagree with Ref. \([5]\). We suspect that the phase factor \( \exp(-idU\pi) \) associated with the s-channel unparticle propagator was not taken care of properly. Our results are consistent with Ref. \([6]\) where we overlap. In addition, we extend these previous calculations to the spin-2 unparticle exchange, which is highly nontrivial.

It has been pointed out recently by Grinstein et al \([7]\) that the vector and tensor unparticle propagators for a conformal invariant hidden sector differ from a scale invariant ones. Unitarity constraints \([8, 9]\) on the scaling dimensions of the unparticle operators with conformal symmetry are also emphasized in their work \([7]\). In this work, we follow the original Georgi’s scheme by assuming just scale invariance in the derivation of the unparticle propagators. Integrating out the heavy messenger sector can also lead to contact interactions among SM fields of the form \( O_{SM}O'_{SM}/M^{d_{SM}+d_{SM}-4} \) and they can compete with the effects from unparticle exchanges \([7]\). For example, the following two dimension 8 operators \( (F^{\mu\nu}F_{\mu\nu})^2/M^4 \) and \( (F^{\mu\nu}F^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta})/M^4 \) can be induced and they can also contribute to the elastic photon-photon scattering. We assume the coefficients of these operators are minuscule and ignore them in our analysis. A complete analysis for the photon-photon scattering including all the interference effects among the SM contribution, unparticle exchanges as well as these contact interactions is interesting but beyond the scope of this work.

The organization of the paper is as follows. In the next section, we give in details the scattering amplitudes for \( \gamma\gamma \rightarrow \gamma\gamma \) via spin-0 as well as spin-2 unparticle exchange. In
Sec. III, we compare the unparticle contribution with the SM contribution in the angular distribution and in the total cross section. We also look into the nontrivial effects of the phase $\exp(-id_\ell \pi)$ of the $s$-channel propagator. We conclude in Sec. IV.

II. PHOTON-PHOTON SCATTERING

The interaction of spin-0 unparticle $U$ with the photon can be parameterized by \[ \mathcal{L}_{\text{eff}} \ni \lambda_0 \frac{1}{\Lambda_\ell^2} F_{\mu\nu} F^{\mu\nu} O_U, \] where $\lambda_0$ is an unknown coefficient of order $O(1)$, and $F_{\mu\nu}$ is the field strength of the photon field. The unparticle propagator is \[ \Delta_F(P^2) = \frac{A_{d_\ell}}{2\sin(d_\ell \pi)} (-P^2)^{d_\ell - 2}, \]
where $A_{d_\ell}$ is given by \[ A_{d_\ell} = \frac{16\pi^2 \sqrt{\pi}}{(2\pi)^{2d_\ell}} \frac{\Gamma(d_\ell + \frac{1}{2})}{\Gamma(d_\ell - 1)\Gamma(2d_\ell)}. \]

The peculiar phase associated with the propagator arises from the negative sign in front of $P^2$ in Eq.(4):
\[ (-P^2)^{d_\ell - 2} = \begin{cases} |P^2|^{d_\ell - 2} & \text{if } P^2 \text{ is negative and real,} \\ |P^2|^{d_\ell - 2}e^{-id_\ell \pi} & \text{for positive } P^2 \text{ with an infinitesimal } i0^+. \end{cases} \]

Therefore, the $s$-channel propagator has the nontrivial phase $\exp(-id_\ell \pi)$ while the $t$- and $u$-channel propagators do not.

There are three Feynman diagrams contributing to $\gamma(p_1)\gamma(p_2) \rightarrow \gamma(k_1)\gamma(k_2)$ with the unparticle exchanges in $s$-, $t$-, and $u$-channels. The sum of amplitudes for these three diagrams is given by
\[ \mathcal{M} = -16\lambda_0^2 Z_{d_\ell} \frac{1}{\Lambda_\ell^3} (\mathcal{M}_s + \mathcal{M}_t + \mathcal{M}_u)^{\mu\nu\rho\sigma} \epsilon_\sigma^* (k_1) \epsilon_\rho^* (k_2) \epsilon_\nu^* (p_1) \epsilon_\mu^* (p_2), \]
where
\[ \mathcal{M}_s^{\mu\nu\rho\sigma} = \left( -\frac{s}{\Lambda_\ell^2} \right)^{d_\ell - 2} (-k_2 \cdot k_1 g^{\rho\sigma} + k_2^{\mu} k_1^{\sigma}) (-p_1 \cdot p_2 g^{\mu\nu} + p_1^{\mu} p_2^{\nu}); \]
\[ \mathcal{M}_t^{\mu\nu\rho\sigma} = \left( -\frac{t}{\Lambda_\ell^2} \right)^{d_\ell - 2} (k_2 \cdot p_2 g^{\mu\rho} - k_2^{\mu} p_2^{\rho}) (k_1 \cdot p_1 g^{\nu\sigma} - k_1^{\nu} p_1^{\sigma}); \]
\[ \mathcal{M}_u^{\mu\nu\rho\sigma} = \left( -\frac{u}{\Lambda_\ell^2} \right)^{d_\ell - 2} (k_2 \cdot p_1 g^{\nu\rho} - k_2^{\nu} p_1^{\rho}) (k_1 \cdot p_2 g^{\mu\sigma} - k_1^{\mu} p_2^{\sigma}). \]
In the above amplitude, we can write the Mandelstam variables as

\[ (-s)^{d_{ul}-2} = s^{d_{ul}-2} e^{-id_{ul} \pi}, \quad (-t)^{d_{ul}-2} = |t|^{d_{ul}-2}, \quad (-u)^{d_{ul}-2} = |u|^{d_{ul}-2} \]  \quad (7)

such that the phase \( \exp(-id_{ul} \pi) \) associates manifestly with the \( s \)-channel only. It is obvious that each channel is separately gauge invariant. The square of the amplitude averaged over initial polarizations is given by

\[
\sum |M|^2 = \frac{16 \lambda_2^4 Z_{d_{ul}}^2}{\Lambda_{d_{ul}}^4} \left\{ s^{2d_{ul}} + |t|^{2d_{ul}} + |u|^{2d_{ul}} + \cos(d_{ul} \pi) \left[ (s|t|)^{d_{ul}} + (s|u|)^{d_{ul}} \right] + (|t||u|)^{d_{ul}} \right\}. \quad (8)
\]

If the phase factor \( \cos(d_{ul} \pi) \) were removed, the amplitude squared would have been symmetric in \( s \leftrightarrow t \leftrightarrow u \). Note that we have written the Mandelstam variables as \( s, |t|, |u| \), where \( |t| = s(1 - \cos \theta)/2 \) and \( |u| = s(1 + \cos \theta)/2 \) and \( \theta \) is the central scattering angle. The angular distribution is given by

\[
\frac{d\sigma}{d\cos \theta} = \frac{1}{2\pi s} \sum |M|^2 \left\{ 1 + \frac{1}{2} \cos \theta + \frac{1}{2} - \frac{1}{2} \cos \theta \right\} s^{2d_{ul}-1} \left[ 1 + \frac{2 \cos(d_{ul} \pi)}{d_{ul} + 1} \right] + \cos(d_{ul} \pi) \left[ \frac{1}{2} \left( \frac{1 - \cos \theta}{2} \right)^{d_{ul}} + \frac{1}{2} \left( \frac{1 + \cos \theta}{2} \right)^{d_{ul}} \right]
\]

\[
+ \frac{\sqrt{\pi}}{2^d_{ul}+1} \Gamma(d_{ul}+1) \Gamma(d_{ul}+3/2) \right\}. \quad (9)
\]

where the range of integration for \( \cos \theta \) is from \(-1\) to \(1\). The total cross section can be obtained analytically in closed form by integrating Eq.(9) over \( \cos \theta \), viz.,

\[
\sigma = \frac{\lambda_2^4 Z_{d_{ul}}^2}{2\pi \Lambda_{d_{ul}}^4} s^{2d_{ul}-1} \right\{ 1 + \frac{2}{2d_{ul}+1} + \frac{2 \cos(d_{ul} \pi)}{d_{ul}+1} + \frac{\sqrt{\pi}}{2^{d_{ul}+1}} \Gamma(d_{ul}+1) \Gamma(d_{ul}+3/2) \right\}. \quad (10)
\]

The effective interaction of spin-2 unparticle with the photon is given by

\[
\mathcal{L}_{\text{eff}} \supset \lambda_2 \frac{1}{\Lambda_{d_{ul}}^4} F_{\mu \alpha} F^{\alpha \nu} \mathcal{O}^{\mu \nu}_{d_{ul}}, \quad (11)
\]

where \( \lambda_2 \) is an unknown effective coupling constant, of order \( O(1) \). Using the Feynman rules and the propagator derived in [3], the matrix element squared for elastic photon-photon scattering via spin-2 unparticle exchange is found to be

\[
\sum |M|^2 = \frac{\lambda_2^4 Z_{d_{ul}}^2}{2\Lambda_{d_{ul}}^4} \left\{ s^{2d_{ul}-4} \left( t^4 + u^4 \right) + |t|^{2d_{ul}-4} \left( s^4 + u^4 \right) + |u|^{2d_{ul}-4} \left( s^4 + t^4 \right) + 2 \cos(d_{ul} \pi) s^{d_{ul}-2} \left[ (s|t|)^{d_{ul}-2} u^4 + |u|^{d_{ul}-2} t^4 \right] + 2 (tu)^{d_{ul}-2} s^4 \right\}. \quad (12)
\]
As $d_U \to 2$, the above expression is proportional to $s^4 + t^4 + u^4$ which is the familiar result [4] for the spin-2 Kaluza-Klein graviton exchange in the large extra dimensions model. However, the $s$-channel unparticle exchange contains a CP-conserving phase factor $\exp(-id_U\pi)$ that does not share with the $t$- and $u$-channels. Therefore, the expressions of Eq. (8) and Eq. (12) for the matrix element squared contain the factor $\cos(d_U\pi)$ in the interference terms between $s$- and $t$- and between $s$- and $u$-channels. This is a unique feature of the unparticle.

III. RESULTS

In Fig. 1 we show the normalized angular distributions $d\sigma/d\cos\theta$ at $\sqrt{s_{\gamma\gamma}} = 0.5$ TeV for spin-0 and spin-2 unparticle exchanges. Note that in the part for spin-0 the scale on the $y$-axis is linear while that for spin-2 the scale is logarithmic. Therefore, in general the spin-2 exchange will give much larger contributions in the forward region. Another interesting feature is that for spin-0 case when $d_U$ increases from 1.1 to 1.9 the distribution is becoming more forward. This is because the factors of $|t|$ and $|u|$ only appear in the numerator, and so when $d_U$ increases, more powers of $|t|$ and $|u|$ are contributing in the forward region. On the other hand, for spin-2 case more powers of $|t|$ and $|u|$ appear in the denominator as $d_U$ is closer to 1. Thus, the distribution is much more forward for small $d_U$. In fact, it diverges at $|\cos\theta| = 1$ for $d_U < 2$. We have also verified that the term containing the factor $\cos(d_U\pi)$ is affecting the distribution. If there were no such a factor, the distribution would have been different, especially for small $d_U$. This demonstrates the effect of the peculiar phase associated with the $s$-channel propagator only. If the phase were associated with all $s, t, u$ propagators, the effect would have been canceled out when we squared the amplitude.

In Fig. 2 we plot the integrated cross sections versus the center-of-mass energy $\sqrt{s_{\gamma\gamma}}$. We also show the expectation from the SM, using the results of Ref. [10] with the form factors from Ref. [11]. Since the SM cross section peaks in the forward and backward directions, we impose an angular cut of $|\cos\theta_\gamma| < \cos(30^\circ)$ to reduce the SM cross section. It is easy to see that the unparticle cross sections can surpass the SM one at high enough energy depending on the spin and scaling dimension of the unparticle. The factor containing $\cos(d_U\pi)$ also affects the total cross sections to some extent, especially for small $d_U$. 
FIG. 1: Normalized angular distributions of $\gamma\gamma \rightarrow \gamma\gamma$ via spin-0 and spin-2 unparticle exchanges for various $d_U$ at $\sqrt{s_{\gamma\gamma}} = 0.5$ TeV.

IV. CONCLUSIONS

One of the most peculiar features of unparticle is the phase factor $\exp(-id_U \pi)$ associated with the $s$-channel propagator. We have studied its effect in $\gamma\gamma \rightarrow \gamma\gamma$, which would have
FIG. 2: Total cross sections for $\gamma\gamma \rightarrow \gamma\gamma$ via (a) spin-0 and (b) spin-2 unparticle exchange versus center-of-mass energy for various $d_U$. The SM expectation is also shown.

been symmetric in $s$-, $t$-, and $u$-channels without the phase factor. However, since the phase is only associated with the $s$-channel, the effect will show up in the interference terms between $s$- and $t$- and between $s$- and $u$-channels. The effect of such a factor affects the angular distribution and total cross sections to some significance, especially for small $d_U$. We have also shown that the scattering cross sections due to unparticle exchanges easily surpass the SM contribution. Thus, the possibility of studying photon scattering in the future linear collider, using either laser backscattering technique or bremsstrahlung, is important to test the existence of any tree-level photon-photon scattering. Unparticle is a unique example that allows tree-level exchange and contains a special CP-conserving phase factor $\exp(-id_U\pi)$ solely in the $s$-channel propagator to facilitate interesting interference effects.

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