New Proposal for a 5-dimensional Unified Theory of Classical Fields of Kaluza-Klein type

Paulo G. Macedo
Centro de Astrofisica da U.P.
R. das Estrelas, s/n - 4150 Porto - PORTUGAL

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Abstract

A new 5-dimensional Classical Unified Field Theory of Kaluza-Klein type is formulated using 2 separate scalar fields which are related in such a way as to make the 5-dimensional matter-geometry coupling parameter constant. It is shown that this procedure solves the problem of the variability of the gravity coupling parameter without having to assume a conformal invariance.

The corresponding Field equations are discussed paying particular attention to the possible induction of scalar field gradients by Electromagnetic Fields.

A new correspondence limit in which the field equations lead to the usual Einstein-Maxwell equations is obtained. This limit does not require the usual condition that the usual scalar field be constant.

1 Introduction

The aim of formulating a self consistent theory which unifies gravitation and electromagnetism has been a long standing objective of Theoretical Physics. In particular, Albert Einstein devoted a great deal of his life to it.

It is our opinion that although we now know that there are other fields in Nature apart from these two, namely the ones related with the strong and weak interactions (which in the long run one aims to unify with these two if one wants to formulate a complete Unified Field Theory), this is still a useful
exercise. In fact, as we shall show in later publications, this more simple and completely classical construction still enables us to find unexpected and interesting new experimental and observational predictions\(^1\).

The first steps in trying to unify gravity with electromagnetism were given by T. Kaluza\(^1\) and O. Klein\(^2\). The theories constructed by them describe the unified field in a 5-dimensional Space-Time, whose metric, apart from the 4-dimensional metric, contains the electromagnetic potential 4-vector. These theories contain as their field equations the Einstein-Maxwell set and correctly describe the movement of charged particles in the presence of Gravitational and Electromagnetic fields.

However, although being mathematically very elegant, they were unable to predict new effects which were unknown in the previous ones.

Later on, Jordan\(^3\), Thiry\(^4\) and others like Bergmann\(^5\), noticed that in order for the theory to be self-consistent, a Scalar Field also had to come into play along with the Gravitational and Electromagnetic fields. Their theory although not completely 5-dimensionally covariant, was invariant under the action of the SO(3,1) and U(1) groups. This smaller symmetry (more restricted than the SO(4,1) 5-dimensional covariance) is due to the assumption of the existence of a Killing vector field along the 5th coordinate (which is considered compact). This is usually referred to as the cylindricity condition. Unlike the theory of Kaluza and Klein these theories contain new information (in the form of a 15th field equation) unaccounted in the Einstein-Maxwell theory. In particular they describe a complete coupling of the 3 (Gravitational, Electromagnetic and Scalar) fields (for a good review of the subject we suggest the reading of the review article by Overduin and Wesson\(^6\) and its bibliography).

These, more recent, compactified Kaluza-Klein type theories contain a scalar field \(\phi\) as well as a constant \(\alpha\) which plays the role of making the Electromagnetic potential 1-form \(A\) dimensionless in order to be a part of the 5-dimensional metric. These theories contain 15 field equations instead of the 14 contained in the original Kaluza-Klein formulation.

In the limit when this Scalar field \(\phi\) is constant, the first 14 field equations reduce to the ones in Einstein-Maxwell theory. However, these theories suffer from 2 main problems, which, in our opinion, haven’t yet been solved in a satisfactory way, namely:

1) In the absence of a scalar field gradient the 15th field equation re-
duces to the usual electromagnetic null field condition. This condition is incompatible with the existence of electromagnetic fields in vacuum like the pure electric field encountered in a planar capacitor, between the two plates or the pure magnetic field such as the one in a solenoid. Therefore, one is confronted with the problem that either one assumes \( \phi \) to be constant and recovers the Einstein-Maxwell theory, but is left with the electromagnetic null field condition, or alternatively, one assumes that the scalar field \( \phi \) can vary and is faced with a second problem, namely:

2) In the 4-dimensional field equations which spring from the 5-dim Einstein-Hilbert action, the geometry to matter source coupling \( 16\pi G/c^4 \) depends on the product \( \alpha^2 \phi \). Therefore, according to these theories, this coupling should vary with the local scalar field value. This raises the problem of the variability of Newton’s Gravitational coupling constant \( G \). In fact, such variability has never been detected up to now in our laboratory and Solar system experiments.

We should mention however that this does not necessarily mean that in very large scales, outside the reach of our today’s experimental range, \( G \) or even the speed of light \( c \), can not vary. Indeed, to our opinion, there has been a very wide spread misunderstanding related to the constancy of the speed of light \( c \). In fact this constancy, which springs from experiments like the Michelson-Morley experiment is a well established fact, but the great majority of authors has lost sight that such experiments are local ones. In fact, this experiment only shows evidence that the in a given Space-Time region the speed of light \( c \) observed by two inertial observers is the same irrespectively to their state of relative motion to each other. To our knowledge, there is no evidence to acert that \( c \) is a universal constant (i.e. with the same value for every Space-Time region) and not a local one. In fact, the speed of light \( c \) is related to the electromagnetic coupling, fine structure ”constant” which recent results seem to indicate to be not a constant at all but indeed vary with cosmic time.

The usual way out to this \( G \) variability problem consists in conformally rescaling the metric in such a way as to make the \( 16\pi G/c^4 \) coupling a constant.

In this paper, we suggest a different approach, in which the \( \alpha \) parameter is not necessarily a constant but instead an auxiliary scalar field which, for reasons that will become clear later on this paper, we shall assume to be related with the \( \phi \) field in such a way that \( \alpha^2 \phi = 16\pi G/c^4 = \text{const.}/R(5) \),
where $R(5)$ is the radius of the 5th dimension. The reason for this assumption has to do with the fact that in a 5-dimensional gravity theory with 5-dim matter sources, the coupling of the 5-dim Space-Time Geometry to its sources is given, as we shall later see, by $\chi = 16\pi GR(5)/c^4$. The above assumption only states that this coupling is a constant.

The assumption that the coupling parameter of the 5-dim Space-Time Geometry to its sources $\chi$ is a constant of Nature is very important from the conceptual point of view. From the point of view of such a Unified theory, it is perfectly conceivable that the parameters $G$ and $c$ which characterize the couplings of the gravitational and electromagnetic interactions separately could vary in different regions of Space-Time. However, it seems to us much less reasonable to accept that the very parameter $\chi$ which describes the Unified Field coupling to its sources could vary.

We shall further on in this paper show that this assumption leads to a correspondence condition (occurring in the limit in which this theory is equivalent to the Maxwell-Einstein Theory) which is the same as the one obtain in the assumption that the 4-dimensional coupling parameter $\chi$ is constant. This is a different and much more general correspondence condition then the one usually assumed, that $\phi = const.$.

In this way there is no need for a conformal rescaling of the metric.

In this approach, $\alpha$ is a parameter which measures the coupling between the 5-dimensional Space-Time geometry and the Electromagnetic field strength.

On the other hand, however, the dynamics of the scalar field has, to our knowledge, received up till now a lesser degree of attention, particularly in the presence of electromagnetic fields $[7,10]$.

Given the coupling between these fields which is clear in the field equations $[6]$, there exists the possibility of gradients of the scalar field being induced by an electromagnetic field and therefore a scalar force being generated $[6,10]$.

In the context of the theory that we propose in this paper, it is also possible that, in specific circumstances, electromagnetic fields generate local gradients of the scalar field. We shall show in a later publication that this induction can result in several new effects like, in particular, a slight modification of the dispersion relation of light waves propagating in vacuum which would possibly explain the Ralston-Nodland claimed effect $[12]$.

Some of these effects have already been discussed by other authors $[10,11]$ in the context of usual Kaluza-Klein theories.

This work, whose publication starts with this paper, shall be composed
of three papers:

1) In this first one we describe the proposed Classical Unified Field Theory and discuss the correspondence limit in 5-dimensional vacuum.

2) In the second paper we shall concentrate specifically on wave solutions and the propagation of electro-scalar waves in vacuum permeated by an underlying uniform magnetic field looking in detail at the rotation of the plane of polarization undergone by them as they propagate.

3) In the third paper, we shall try to formulate the Theory in the presence of 5-dimensional Matter Sources and study the equation of motion of particles subject to this Unified Field.

2 Formalism

2.1 The 5-dimensional metric

We shall start by assuming that a compactified point of view provides a good description of our present day local region of 5-dimensional Space-Time. I.e. the 5th dimension is therefore assumed to be topologically compact.

As we’ve already mentioned, in some limit, we expect that this theory should give us the Einstein - Maxwell Field Equations. In order to achieve that, following the usual procedure, we shall start by considering a 5-dimensional Space-Time whose metric $g_{AB}$ and the corresponding 5-dimensional line element:

$$d\sigma^2 = \bar{g}_{AB} \, dx^A \, dx^B$$

(1)

(where $A, B, C, \ldots = 0, 1, 2, 3, 5$) and the metric $\bar{g}_{AB} = \bar{g}_{AB}(x^C)$.

The extra dimension $x^5$ is assumed, as usual, to be space-like and therefore we shall use a metric signature $(-, +, +, +, +)$.

From now on, and unless otherwise specified, we shall denote the 4-dimensional geometric object which corresponds to a given 5-dimensional one by the same letter without a bar on top.

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2 This assumption does not in any way imply that its radius be very small. In fact one can think of the 5th dimension as topologically compact but with a characteristic curvature radius which can even be much larger than the characteristic curvature radius of the 3 usual space dimensions.
The 5-dimensional metric $\bar{g}_{AB}$ must be such that from the field tensors constructed out of it and its derivatives, one can obtain in a specific limit the Einstein - Maxwell Field Equations.

In order to achieve this, it must be such that splitting it in the usual 4+1 way, the metric components can be written in the matrix form as follows:

$$
\bar{g}_{AB} = \begin{pmatrix}
g_{\mu\nu} + \alpha^2 \phi A_\mu A_\nu & \alpha \phi A_\mu \\
\alpha \phi A_\nu & \phi
\end{pmatrix}
$$

(2)

and its inverse can be written as:

$$
\bar{g}^{AB} = \begin{pmatrix}
g^{\mu\nu} - \alpha A_\mu \\
-\alpha A_\nu & \alpha^2 A^\beta A_\beta + \frac{1}{\phi}
\end{pmatrix}
$$

(3)

where the greek indices are 4-dimensional ones (i.e. $\mu, \nu, \ldots = 0, 1, 2, 3$) and where $A_\mu$ is the 4-dimensional electromagnetic 4-potential 1-form field, $g_{\mu\nu}$ is the usual 4-dimensional metric tensor field and $\phi$ and $\alpha$ are 4-dimensional scalar fields. The $\phi$ field is dimensionless, whereas the $\alpha$ field needs to have the inverse dimensions of the electromagnetic potential in order for the metric itself to be dimensionless.

i.e.

$$
\phi \equiv \bar{g}_{55} \quad (4)
$$

$$
A_\mu \equiv \alpha^{-1} \frac{\bar{g}_{\mu5}}{\bar{g}_{55}} \quad (5)
$$

$$
g_{\mu\nu} \equiv \bar{g}_{\mu\nu} - \frac{\bar{g}_{\mu5} \bar{g}_{5\nu}}{\bar{g}_{55}} \quad (6)
$$

In order for this 5-dimensional theory to be a viable one, one would expect to find a specific limit where, in the case when $\bar{g}_{AB}$ does not depend on $x^5$, (i.e. $\bar{g}_{AB,5} = 0$, which characterizes compactified theories), the 5-dimensional vacuum Einstein Equations would lead to the Einstein-Maxwell’s Equations for the $g_{\mu\nu}(x^\rho)$ and $A_\mu(x^\rho)$ fields.

From now on, we shall use instead of the $A_\mu$ field another related 4-dimensional 1-form field $k_\mu$ which we define as its dimensionless counterpart:

$$
k_\mu = \alpha \ A_\mu \quad (7)
$$
In this case, the 5-dimensional metric $\bar{g}_{AB}$ is such that splits in the usual 4+1 way:

$$\bar{g}_{AB} = \begin{pmatrix} g_{\mu\nu} + \phi k_\mu k_\nu & \phi k_\mu \\ \phi k_\nu & \phi \end{pmatrix}$$

its inverse being:

$$\bar{g}^{AB} = \begin{pmatrix} g^{\mu\nu} - k_\mu & -k_\mu \\ -k_\nu & k_\beta k_\beta + \frac{1}{\phi} \end{pmatrix}$$

where, therefore one can write:

$$k_\mu \equiv \frac{\bar{g}_{\mu5}}{g_{55}}. \quad (10)$$

### 2.2 The 5-dimensional Connection

Starting from the above mentioned 5-dimensional metric, one can write the corresponding 5-dimensional connection taking into account the metric and its derivatives as follows:

$$\Gamma^A_{BC} = \frac{1}{2} f^{AD} \left( f_{BD,C} + f_{CD,B} - f_{BC,D} \right) \quad (11)$$

On the other hand, we shall also define a new 2-form field $S_{\mu\beta}$ which is the dimensionless correspondent of the Faraday field. It is obtained form the dimensionless electromagnetic potential $k_\mu$ as follows:

$$S_{\mu\beta} = k_{\beta,\mu} - k_{\mu,\beta} \quad (12)$$

i.e., this field is related with the Faraday field in the following way:

$$S_{\mu\beta} = \alpha F_{\mu\beta} + \alpha_{,\mu} A_\beta - \alpha_{,\beta} A_\mu \quad (13)$$

One can therefore write the 5-dimensional connection coefficients arising from the above metric splitted as follows:

$$\Gamma^5_{55} = \frac{1}{2} k_\mu \phi_{,\mu} \quad (14)$$

$$\Gamma^\mu_{55} = -\frac{1}{2} \phi^{,\mu} \quad (15)$$
\[ \Gamma^\mu_{\beta 5} = \frac{1}{2} \left( \phi S^\mu_{\beta} - \phi^{\mu} k_{\beta} \right) \] (16)

\[ \Gamma^5_{\beta 5} = \frac{1}{2} \left[ \phi k^\mu S_{\mu \beta} + \frac{\phi_{\beta}}{\phi} + \phi_{\mu} k^{\mu} k_{\beta} \right] \] (17)

\[ \Gamma^\mu_{\beta \sigma} = \Gamma^\mu_{\beta \sigma} + \frac{\phi}{2} \left( k_{\beta} S^\mu_{\sigma} + k_{\sigma} S^\mu_{\beta} \right) - \frac{1}{2} \phi^{\mu} k_{\sigma} k_{\beta} \] (18)

\[ \Gamma^5_{\beta \sigma} = \frac{1}{2} \left\{ \frac{1}{\phi} \left[ \phi_{\sigma} k_{\beta} + \phi_{\beta} k_{\sigma} \right] + k^\mu \phi_{\mu} k_{\beta} k_{\sigma} + k_{\beta \sigma} + k_{\sigma \beta} \right\} \] (19)

### 2.3 The 5-dimensional Ricci Tensor.

From the above connection and its derivatives, one can construct the 5-dimensional Ricci Tensor:

\[ R_{BD} \equiv \overline{R} \equiv \overline{R}^A_{BAD} \equiv \Gamma^A_{BAD} - \Gamma^A_{BA,D} + \Gamma^E_{BD} \Gamma^A_{AE} - \Gamma^E_{BA} \Gamma^A_{DE} \] (20)

Whose components can be splitted as follows:

\[ \overline{R}_{55} = -\frac{1}{2} \phi^\rho \rho + \frac{1}{4} \frac{\phi^\rho \phi_{,\rho}}{\phi} + \frac{\phi^2}{4} S_{\rho \sigma} S^{\rho \sigma} \] (21)

\[ \overline{R}_{5\mu} = \frac{3}{4} \phi_{,\rho} S^\rho_{\mu} + \frac{1}{2} \phi S^\rho_{\rho} \rho + k_{\mu} \overline{R}_{55} \] (22)

\[ \overline{R}_{\mu \nu} = R_{\mu \nu} - \frac{1}{2} \left( \phi \frac{\phi_{,\mu \nu}}{\phi} - \frac{1}{2} \frac{\phi_{,\mu} \phi_{,\nu}}{\phi^2} \right) - \frac{1}{2} \phi S^\rho_{\mu} S_{\nu \rho} \]

\[ + \overline{R}_{5\mu} k_{\nu} + \overline{R}_{5\nu} k_{\mu} - \overline{R}_{55} k_{\mu} k_{\nu} \] (23)
2.4 5-Dimensional Vacuum Field Equations

We shall now look at 5-Dimensional Vacuum Field Equations and see what new information they contain. In particular we shall look at the coupling of the three (Gravitational, Electromagnetic and Scalar) fields.

One can now write the 5-dimensional Einstein’s Field Equations in vacuum as:

$$R_{BD} = 0$$  \hfill (24)

Using the components of the 5-dimensional Ricci tensor given by equations (21) to (23), one can write the field equations (24) as follows:

$$- \phi^{;\rho} \rho^{;\rho} + \frac{1}{2} \phi^{;\rho} \phi^{;\rho} + \frac{\phi^2}{2} S_{\rho\sigma} S^{\rho\sigma} = 0$$  \hfill (25)

$$\frac{3}{2} \phi^{;\rho} S_{\mu} \rho^{;\rho} + \phi S_{\mu}^{;\rho} = 0$$  \hfill (26)

and

$$R_{\mu\nu} - \frac{1}{2} \left( \frac{\phi_{;\mu\nu}}{\phi} - \frac{1}{2} \frac{\phi_{;\mu} \phi_{;\nu}}{\phi^2} \right) - \frac{1}{2} \phi S_{\mu} \rho S_{\nu\rho} = 0$$  \hfill (27)

We shall now try to find a correspondence limit where the last two sets of field equations (26) and (27) give us the usual Maxwell - Einstein set of field equations.

Substituting the dimensionless electromagnetic tensor $S_{\mu\nu}$, by its expression given in (12), in the above Einstein type Equations (27) one obtains:

$$R_{\mu\nu} = \frac{1}{2} \left( \frac{\phi_{;\mu\nu}}{\phi} - \frac{1}{2} \frac{\phi_{;\mu} \phi_{;\nu}}{\phi^2} \right) + \frac{1}{2} \phi \left[ \frac{\alpha^2 F_{\mu} \rho F_{\nu\rho} + \alpha F_{\mu} \rho (\alpha_{;\nu} A_{\rho} - \alpha_{;\rho} A_{\nu})}{\alpha F_{\nu\rho} + \alpha_{;\nu} A_{\rho} - \alpha_{;\rho} A_{\nu}} \right]$$  \hfill (28)

on the other hand, substituting (12) in the above Maxwell’s type equations (26), one obtains

$$0 = \phi \alpha F_{\mu} \rho^{;\rho} + \left( \frac{3}{2} \phi^{;\rho} + \phi \alpha_{;\rho} \right) F_{\mu} \rho$$
\[ + \frac{3}{2} \phi_{\rho} (\alpha, \mu A^\rho - \alpha^\rho A_\mu) + \phi (\alpha, \mu A^\rho - \alpha^\rho A_\mu); \rho \quad (29) \]

In order for equations (28) and (29) to give us the set of the usual Maxwell - Einstein field equations, one must have the 3 following correspondence conditions:

\[
\left( \frac{3}{2} \phi_{\rho} \alpha + \phi \alpha_{\rho} \right) F^{\rho}_{\mu} \\
+ \frac{3}{2} \phi_{\rho} (\alpha, \mu A^\rho - \alpha^\rho A_\mu) + \phi (\alpha, \mu A^\rho - \alpha^\rho A_\mu); \rho = 0, \quad (30)
\]

\[ \alpha F^{\rho}_{\mu} (\alpha, \nu A_\rho - \alpha, \rho A_\nu) + (\alpha, \mu A^\rho - \alpha^\rho A_\mu) (\alpha F_{\nu \rho} + \alpha, \nu A_{\rho} - \alpha, \rho A_\nu) = 0 \quad (31)\]

and

\[ \frac{\alpha^2 \phi}{2} = \chi = \frac{8\pi G}{c^4} \quad (32)\]

the first two above conditions (30) and (31) can be substituted by the following :

\[ \left( \frac{3}{2} \phi_{\rho} \alpha + \phi \alpha_{\rho} \right) F^{\rho}_{\mu} = 0 \quad (33)\]

and

\[ \alpha, \nu A_\rho - \alpha, \rho A_\nu = 0 \quad (34)\]

One can rewrite this condition by splitting it in its space and time components as follows:

\[ \left\{ \begin{array}{l}
\varepsilon^{ijk} \alpha_j A_k = 0 \\
\alpha, 0 A_i = \alpha, i A_0
\end{array} \right\} \quad (35)\]

where \( \varepsilon \) is the Space 3-dimensional Levi-Civita tensor. This means that, in this classic correspondence limit, the spatial part of the \( \alpha \) scalar field gradient is aligned with the electromagnetic potential \( \mathbf{A} \).
On the other hand, looking at the 5-dim. metric, one can see that the scalar field $\phi$ is such that $\sqrt{\phi}$ plays the role of the 5th dimension scale factor. Therefore, the 5th dimension radius $R_{(5)}$, is related to the scalar field $\phi$ in the following way:

$$R_{(5)} \propto \sqrt{\phi}$$  

(36)

Since in this type of 5-dimensional Kaluza-Klein theories the 5-dimensional and 4-dimensional gravity coupling parameters $\chi$ and $\chi$ are related in such a way that:

$$\chi = \chi R_{(5)}$$  

(37)

Under the above assumption (that $\chi$ is constant), by substituting (32) and (36) on (37), one can write:

$$\alpha^2 \phi^\frac{3}{2} = \text{const.}$$  

(38)

Since both $\alpha$ and $\phi$ fields are always non null, one can differentiate this equation and divide both members by $\alpha \phi^\frac{3}{2}$, obtaining:

$$\frac{3}{2} \alpha \phi_\rho + 2 \alpha \phi \phi_\rho = 0$$  

(39)

One can therefore substitute (33) in the above equation (39), and divide by $\alpha$, obtaining that:

$$\phi_\rho F_\mu^\rho = 0$$  

(40)

Analyzing this above correspondence condition, one can see that:

For $\mu = 0$, one obtains:

$$\nabla \phi \cdot \mathbf{E} = 0$$  

(41)

This means that, in this correspondence limit, the gradient of the scalar field $\phi$ is orthogonal to the Electric field.

On the other hand, for $\mu = i$, one obtains:

$$\phi^0 \mathbf{E} = c \nabla \phi \times \mathbf{B}$$  

(42)
This means that, in this correspondence limit and in the absence of an electric field, or, in the case when the scalar field $\phi$ is stationary, the gradient of the scalar field $\phi$ is aligned with the magnetic field, and conversely, in the absence of either a magnetic field or a scalar field spatial gradient, either the electric field is null or the scalar field is stationary.

In fact, substituting (30) in (29) and dividing both members by $\alpha \phi$, one can also see that the set of field equations, in this correspondence limit, can be written as follows:

$$F_{\mu \beta}^{\beta} = 0$$  \hspace{1cm} (43)

$$R_{\mu \nu} = \frac{1}{2} \frac{\phi_{\mu \nu}}{\phi} - \frac{1}{4} \frac{\phi_{\mu} \phi_{\nu}}{\phi^2} - \frac{\alpha^2 \phi}{2} F_{\mu \beta}^{\beta} F_{\nu \beta}$$  \hspace{1cm} (44)

$$2 \frac{\phi_{\beta}}{\phi} - \frac{\phi_{\beta} \phi_{\beta}}{\phi^2} = \alpha^2 \phi F_{\mu \beta} F^{\mu \beta}$$  \hspace{1cm} (45)

and the correspondence condition is given by equation (40).

I.e., one can see that in this limit, one obtains the last 14 equations as the Einstein-Maxwell field Equations, for vacuum permeated by an electromagnetic field, provided that the identification given by equation (32) is carried out, where $\alpha$ and $\phi$ are the local space-time values of the corresponding $\alpha$ and $\phi$ scalar fields.

One can also see that in this correspondence limit, the 15th field equation takes the form:

$$\frac{\phi^{\rho}}{\phi} \phi^{\rho} - \frac{1}{2} \frac{\phi^{\rho} \phi_{\rho}}{\phi^2} = \frac{8\pi G}{c^4} F^{\rho \sigma} F_{\sigma \rho}$$  \hspace{1cm} (46)

which can also be written in terms of the electric and magnetic 3-dimensional vector fields in the following way:

$$\frac{\phi^{\rho}}{\phi} \phi^{\rho} - \frac{1}{2} \frac{\phi^{\rho} \phi_{\rho}}{\phi^2} = \frac{8\pi G}{c^4} \left( E^2 - c^2 B^2 \right)$$  \hspace{1cm} (47)

As an exercise, one can see that if instead one would assume, as usual, that $\chi$ is itself constant (instead of $\chi$), one would have, the following condition:

$$\alpha^2 \phi = const.$$

(48)
instead of the condition given by equation. (38). Therefore, differentiating this equation and dividing by $\alpha$, instead of equation (39), one would obtain:

$$\alpha \phi_{,\rho} + 2 \alpha_{,\rho}\phi = 0 \quad (49)$$

substituting this result in the above condition (33), one would again obtain the same correspondence conditions given by (34) and (40).

I.e., in both cases (namely, taking $\chi = \text{const.}$ or $\nabla = \text{const.}$) one obtains the same correspondence conditions. These conditions are in no way restricted to the usually assumed condition that $\phi = \text{const.}$.

One can see that, in this theory, since in the classical correspondence limit one can still have a varying $\phi$ field, one is therefore no longer bound by the electromagnetic null field condition ($F_{\mu\nu}F^{\mu\nu} = 0$) which in the usual formulations arise from the 15th field equation (46).

This electromagnetic null field condition ($E^2 - c^2B^2 = 0$) which is present in other Kaluza-Klein formulations is obviously too restrictive, since one knows that it is possible to generate in vacuum, pure stationary electric or magnetic uniform fields like the ones present in a plane plate condenser or inside a solenoid. These electromagnetic fields do not satisfy the above mentioned null field condition.

One can therefore see that, according to this theory, even uniform and stationary electromagnetic fields must induce local scalar field gradients.

In the next publication of this series of 3 papers, we shall explicitly describe the solutions of the field equations corresponding to these two simple cases.

3 Conclusions

To summarize, we conclude that:

In this proposed theory, the parameter $\alpha$ measures the coupling between the 5-dimensional Space-Time geometry and the Electromagnetic field strength and plays the role of rendering its product with the Electromagnetic potential 1-form $A$ dimensionless in order to be a part of the 5-dimensional metric. By assuming $\alpha$ to be an auxiliary scalar field instead of assuming it to be a constant, we were able to avoid the usual need for conformal invariance of the 5-dim. Kaluza-Klein type Theories. As we have recalled,
this need arised from the fact that it was thought that in the correspondence limit in which the 5-dimensional gravity theory would yield the usual Einstein–Maxwell theory, the $\phi$ scalar field would have to be constant. This in turn led to the fact that, in this limit, the 15th field equation implied the electromagnetic null field condition $c^2 B^2 - E^2 = 0$. This condition, for the above mentioned reasons, was shown to be too restrictive.

In this proposal, this condition is substituted by a much less restrictive correspondence condition, namely $\phi_{,\rho} F_{\mu}{}^{\rho} = 0$. This allows for this 5-dimensional gravity theory, in the correspondence limit, to describe such simple electromagnetic fields as the ones arising in solenoids or between the plates of a plane plate condenser. These were not possible to account for in the usual Kaluza-Klein type formulation.

The subject is however far from being thoroughly studied. In particular, one can ask what is the physical meaning and the physical reasons leading to this new correspondence condition $\phi_{,\rho} F_{\mu}{}^{\rho} = 0$. We would expect this condition to arise naturally from the initial assumption of the cylindricity condition (which amounts to the assumption of the existence of a Killing vector field of the 5-dimensional metric). This could be connected to the arisal of an anisotropy in 5-dimensional Space-Time in the Early Universe. This would in turn generate the splitting of the 4 space dimensions in 3+1 that we observe today.

This problem along with the analysis of the Equation of motion for particles subject to the electromagnetic, scalar and gravitational fields fields, as well as the analysis of wave solutions for the theory and the formulation of the theory in the presence of 5-dimensional matter sources will be the subject of a series of publications of which this is the first one.

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