Simple Dataset for Proof Method Recommendation in Isabelle/HOL
(Dataset Description) *

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Abstract. Recently, a growing number of researchers have applied machine learning to assist users of interactive theorem provers. However, the expressive nature of underlying logics and esoteric structures of proof documents impede machine learning practitioners from achieving a large scale success in this field. In this data description, we present a simple dataset that represents the essence of choosing appropriate proof methods in Isabelle/HOL. Our simple data format allows machine learning practitioners to try machine learning tools to predict proof methods in Isabelle/HOL, even if they are unfamiliar with theorem proving.

1 Introduction

As our society relies heavily on software systems, it has become essential to ensure that our software systems are trustworthy. Interactive theorem provers (ITPs), such as Isabelle/HOL [18], allows users to specify desirable functionalities of a system and prove that the corresponding implementation is correct in terms of the specification, serving as platforms for such software verification projects.

A crucial step in developing proof documents in ITPs is to choose the right tool for a proof goal at hand. Isabelle/HOL, for example, comes with more than 100 proof methods. Proof methods are sub-tools inside Isabelle/HOL. Some of these are general purpose methods, such as auto and simp. Others are special purpose methods, such as intro_classes and intro_locales. The Isabelle community provides various documentations [18] and on-line support to help new Isabelle users learn when to use which proof methods.

Previously, we developed PaMpeR [13], a proof method recommendation tool for Isabelle/HOL. Given a proof goal specified in a proof context, PaMpeR recommends a list of proof methods likely to be suitable for the goal. PaMpeR learns which proof method to recommend to what kind of proof goal from proof documents in Isabelle’s standard library and the Archive of Formal Proofs [10].

* This work was supported by the European Regional Development Fund under the project AI & Reasoning (reg.no.CZ.02.1.01/0.0/0.0/15_003/0000466).
The key component of PaMpeR is its elaborate feature extractor. Instead of applying machine learning algorithms to Isabelle's proof documents directly, PaMpeR first applies 108 assertions to the pair of a proof goal and its underlying context. Each assertion checks a certain property about the pair and returns a boolean value. Some assertions check if a proof goal involves certain constants or types defined in the standard library. Others check the meta-data of constants and types appearing in a goal. For example, one assertion checks if the goal has a term of a type defined with the codatatype keyword.

When developing PaMpeR, we applied these 108 assertions to the proof method invocations appearing in the proof documents and constructed a dataset consisting of 425,334 unique data points.

We trained PaMpeR by applying a regression tree construction algorithm to this dataset. Even though our tree construction is based on a fixed height and we did not take advantage of modern development of machine learning research, our cross evaluation showed PaMpeR can correctly predict experts’ choice of proof methods for many cases. However, decision tree construction based on a fixed height is an old technique that tends to cause overfitting and underfitting. We expect that one can achieve better performance by applying other algorithms to this dataset.

In the following we present the simple dataset we used to train PaMpeR. Our aim is to provide a dataset that is publicly available and easily usable for machine learning practitioners without backgrounds in theorem proving, so that they can exploit the latest development of machine learning research without being hampered by technicalities of theorem proving.

The Format and Nature of PaMpeR Dataset

Each data point in the dataset consists of the following three entries:

- the location of a proof method invocation,
- the name of proof method used there,
- an array of 0s and 1s expressing the proof goal and its context.

The following is an example data point:

Functors.thy119 simp 1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,...

This data point describes that in the theory file named Functors.thy, a proof author applied the simp method in line 119 to a proof goal represented by the sequence of 1s and 0s where 1 indicates the corresponding assertion returns true while 0 indicates the otherwise.

This dataset has important characteristics worth mentioning. Firstly, this dataset is heavily imbalanced in terms of occurrences of proof methods. Some general purpose methods, such as auto and simp, appear far more often than other lesser known methods: each of auto and simp accounts more than 25% of

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3 https://archive.org/details/PaMpeR_2018
all proof method invocations in the dataset, whereas no proof methods account for more than 1% of invocations except for the 15 most popular methods.

Secondly, this dataset only serves to learn what proof methods to apply, but it does not describe how to apply a proof method. None of our 108 assertions examines arguments passed to proof methods. For some proof methods, notably the \texttt{induct} method, the choice of arguments is the hardest problem to tackle, whereas some methods rarely take arguments at all. We hope that Isabelle users can learn what arguments to pass to proof methods from the use case of these methods in existing proof documents once they learn which methods to apply to their goal.

Thirdly, it is certainly possible that \texttt{PaMPeR}'s feature extractor misses out certain information essential to accurately recommend some methods. This dataset was not built to preserve the information in the original proof documents: we built the dataset so that we can effectively apply machine learning algorithms to produce recommendations.

Finally, this dataset shows only one way to prove a given goal, ignoring alternative possible approaches to prove the same goal. Consider the following goal: "True \lor False". Both \texttt{auto} or \texttt{simp} can prove this goal equally well; however, if this goal appeared in our dataset our dataset would show only the choice of proof author, say \texttt{auto}, ignoring alternative proofs, say \texttt{simp}.

One might guess that we could build a larger dataset that also includes alternative proofs by trying to complete a proof using various methods, thus converting this problem into a multi-label problem. That approach would suffer from two problems. Firstly, there are infinitely many ways to apply methods since we often have to apply multiple proof methods in a sequence to prove a conjecture. Secondly, some combinations of methods are not appropriate even though they can finish a proof in Isabelle. For example, the following is an alternative proof for the aforementioned proposition:

\begin{verbatim}
lemma "True \lor False" apply(rule disjI1) apply auto done
\end{verbatim}

This is a valid proof script, with which Isabelle can check the correctness of the conjecture; however, the application of the \texttt{rule} method is hardly appropriate since the subsequent application of the \texttt{auto} method can discharge the proof without the preceding \texttt{rule}. For these reasons we take the proof methods chosen by human proof authors as the correct choice while ignoring other possibilities.

## 3 The Task for Machine Learning Algorithms

The task for machine learning algorithms is to predict the name of a promising proof method from the corresponding array of boolean values. Since we often have multiple equivalently suitable methods for a given proof goal, this learning task should be seen as a multi-output problem: given an array of boolean values machine learning algorithms should return multiple candidate proof methods rather than only one method. Furthermore, this problem should be treated as a regression problem rather than a classification problem, so that users can see
numerical estimates about how likely each method is suitable for a given proof goal.

4 Conclusion and Related Work

We presented our dataset for proof method recommendation in Isabelle/HOL. Its simple data format allows machine learning practitioners to try out various algorithms to improve the performance of proof method recommendation.

Kaliszyk et al. presented HolStep [9], a dataset based on proofs for HOL Light [7]. They developed the dataset from a multivariate analysis library [8] and the proof of the Kepler conjecture [6]. They built HolStep for various tasks, which does not include proof method prediction. While their dataset explicitly describe the text representations of conjectures and dependencies of theorems and constants, our dataset presents only the essential information about proof documents as an array of boolean values.

Blanchette et al. mined the Archive of Formal Proofs [2] and investigated the nature of proof developments, such as the size and complexity of proofs [12]. Matichuk et al. also studies the Archive of Formal Proofs to understand leading indicators of proof size [12]. Either of their projects aimed at suggesting how to write proof documents: to the best of our knowledge we are the first to mine a large repository of ITP proofs using hand crafted feature extractors.

Our dataset does not contain information useful to predict what arguments to pass to each method. Previously we developed, smart_induct [14], to address this problem for the induct method in Isabelle/HOL, using a domain-specific language for logical feature extraction [13].

Recently a number of researchers have developed meta-tools that exploit existing proof methods and tactics and bring stronger proof automation to ITPs [1][5][11][16][17]. We hope that our dataset helps them improve the performance of such meta-tools for Isabelle/HOL.

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