Development of the planning model of the production and cutting of the paper web ensuring the quality of the finished products

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Abstract. The problem of the planning of pulp and paper production has been investigated, a complex of model and algorithmic programs as well as a software of an automated system of planning and calculating of the cutting methods on a longitudinal slitting machine (LSM) is expected to be a result of this planning. An integrated model for planning paper production and multi-stage cutting of rolled and sheet products has been proposed based on methods, models and algorithms of optimal cutting from the condition of the ensuring the products orders fulfillment with minimal losses. A scientifically based method and algorithm of generating a set of acceptable ways of rolls cutting, criteria of generating a set of acceptable ways of rolls cutting as well as an integer algorithm of the result of the problem solving have been considered. All real restrictions encountered in the manufacturing sector have been taken into account. The results of the planning models usage for real production process have been presented.

1. Introduction
Modern paper and cardboard industry is characterized by high productivity, a wide nomenclature of products and a large assortment of the paper sizes, corresponding to the customer needs. The production of paper and cardboard consists of a number of processes, resulting in the quality of products depends on many factors that determine these processes. Achieving high quality products is impossible without automated process control systems (ACS), which, in turn, meet the requirements of the functioning quality. In this case, the mathematical program and software of the ACS are developed based on methods, algorithms, mathematical models for rolled and sheet materials, taking into account the quality indicators of products [1, 2, 3], as well as meeting the economic indicators of the enterprise's activity [4].

A significant task in the production of paper (cardboard) products is the integration of an automated system of the optimal cutting, since the quality for customers is ensured, in addition to
traditional indicators of finished product quality, by the convenience of acquiring the given formats, with a wide range of size, weight, quantity and quality.

Indeed, ensuring the quality of products at the stage of the optimal cutting consists of the technical level of the technological process organization, the specification of the cutting shape, the reliability of the functioning of the automatic maintenance systems (failure-free or maintainability, durability), the meeting of the requirements of ergonomics and ecology, is a complicated task, as in theoretical as well as in practical terms. In addition, the technical and the software resources that are applied must have a high level of measurement accuracy, stimulated by modern standards of the international level and by metrological support [5].

The problems of production planning and optimization of the sheets and rolls cutting are still a priority task to be deal with at the pulp and paper industry [6, 7, 8, 9]. Some approaches to construct the models for solving the problem of optimizing sheet cutting using integer programming are presented in the articles [10, 11, 12, 13], however there is no generally accepted solution of this problem in the paper making industry [14, 15].

The aim of the study is a model development of the planning paper production and cutting of rolled and sheet products based on the methods and the algorithms of the optimal cutting from the condition of ensuring the fulfillment of product orders with minimal losses.

2. Methods

The problem of optimal cutting of paper (cardboard) materials can be solved in several stages. Firstly, a mathematical model is being developed for planning the optimal cutting of paper web rolls using linear programming methods, and secondly, the problem of optimal planning of sheet material is solved similarly from a mathematical point of view.

2.1. A mathematical model of the planning of optimal rolls cutting.

The development of the model using linear programming methods is carried out in order to find the minimum of the target function

$$F = \min \left\{ \sum_{r=1}^{N} q_r \left( L_{\max} - \sum_{j=1}^{k} X_r F_{rj} \right) \right\},$$

where $j = [1,k]$, $k$ – is a number of paper web rolls formats; $r = [1,N]$, $N$ – is a variety of the acceptable cutting methods; $L_{\max}$ – is a cutting width of the tambour; $X_r$ – is $F_{rj}$ format multiplicity in the r-th cutting method; $F_{rj}$ – is j-th roll format; $q_r$ – is the intensity of the application of the r-th cutting method;

meeting the restrictions:

✓ on the number of rolls of $F_{rj}$ format

$$R_{j\min} \leq \sum_{r=1}^{N} X_{rj} q_r \leq R_{j\max}, j = [1,k]$$

where $R_{j\min}$ и $R_{j\max}$ – is the allowable range of variation of the number of rolls of the j-th format, obtained from the ratio

$$R_{j\min} = \text{Int} \left( \frac{B_j}{W_{\max} \cdot F_{rj}} \right),$$

$$R_{j\max} = \text{Int} \left( \frac{B_j}{W_{\min} \cdot F_{rj}} \right),$$

where $W_{\min}$ и $W_{\max}$ – are lower and upper bounds of the confidence interval of the paper web rolls weight $W_j$; $B_j$ – is a weight per 1 cm of roll width calculated on the basis of statistics of finished products; $B_j$ – are the orders for products in tons;

✓ on the multiplicity of the $F_{rj}$ format in cutting methods

$$0 \leq X_{rj} \leq X_{rj\max},$$

where $X_{rj\max}$ – maximum possible $F_{rj}$ format multiplicity;
\[ X_{\text{max}} = \ln \left( \frac{\text{Lmax}}{\text{Frm}_j} \right), j = [1, k]; \]

on the intensity of methods application

\[ q_r \geq 0, r = [1, N]. \]  

(5)

The essential features of the optimal cutting planning problem are that the set of acceptable cutting methods is not known in advance and the intensities of the \( q_r \) methods application are integer.

A method of generating ways for cutting a paper web (cardboard) into rolls is proposed. This method is based on the ideas of the branch and bound method known in discrete mathematical programming.

The essence of the method is as follows:

Let \( XM \) be the set of all possible cutting methods, \( X_r^* \in XM \) be the \( r \)-th cutting method. \( X_r^* = \{X_{r1}, X_{r2}, ..., X_{rn}, X_{r max}\} \), with coordinates \( X_{rj} \in [0, X_{r max}] \). Each vector \( X_r^* \) is associated with an element \( L_{r \text{ost}} \in \text{Lost} \), where \( \text{Lost} \) – is a vector of losses from the application of cutting methods.

According to the procedure of the branch and bound method, the set \( XM \) is divided into the subsets (Fig. 1.).

As a result of decomposition, the vectors are included in the set of valid cutting methods (SVCM) if the value of the \( L_{r \text{ost}} \) element satisfies the selected criteria, i.e. it does not exceed some upper bound of \( L_{kp} \).

The criterion of the inclusion of \( X_j^* \) vectors in the SVCM, which determines the upper boundary of \( L_{kp} \), is formed in various ways. Two methods for defining the boundary of \( L_{kp} \) have been proposed and analyzed in this article. In the first method, the vectors included in the SVCM, have the value of the \( L_{r \text{ost}} \) element that does not exceed a small percentage \( m \% \) of the cutting width of the tambour of the paper / cardboard \( \text{Lmax} \): \( L_{kp} = m(\%) \cdot 0.01 \cdot \text{Lmax} \), where \( m = 1, 2, 5 \% \). In the second method – \( L_{kp} = \max \{L_{r \text{ost}}\} \) for those vectors \( X_j^* \) with \( X_j^* = \{0, ..., X_{j \text{max}}, 0, ..., 0\} \in XM \), that has the indices of nonzero elements \( \{X_{j \text{max}}\} \), corresponding to the indices of nonzero elements of the vector \( X_r^* \).

For the practical realization of the task of optimal cutting, it is necessary to guarantee the existence of a solution. Among all kinds of vectors of cutting methods, there are homogeneous vectors, which include \( X_j^* \) with components other than \( j \)-th , with equal to zero \( X_j^* = \{0, ..., X_{j \text{max}}, 0, ..., 0\} \in XM \). Accordingly, for \( k \) of different formats, there are \( k \) of homogeneous methods of cutting. A set of \( k \) homogeneous vectors is necessarily included in the SVCM for any remainder.

An analysis of the dependence of the results of optimal cutting on the criterion of the formation of the SVCM showed that for the operating conditions of a particular enterprise, one should choose a criterion for the formation of the SVCM, depending on the set of initial data, their correspondence with the parameter \( \text{Lmax} \) and the amount of permissible losses during the cutting stage adopted in the industry.

The result obtained after solving the linear programming problem determines the set of optimal methods of cutting and the intensity of the application of each of them when performing the cutting task.

The article proposes an heuristic algorithm of the integer mechanism of the solution of a linear programming problem, the purpose of which is to find integer values of the intensity of the cutting methods taking into account the restrictions on the number of rolls of each format.

The optimal plan obtained by solving the linear programming problem can be written as follows:

\[ \bar{R}_{\text{min}} \leq \sum_{j=1}^{k} X_j q_j^n + \sum_{j=1}^{k} X_j q_j^n + ... + \bar{X}_1 q_1^n + ... + \bar{X}_k q_k^n \leq \bar{R}_{\text{max}}, \]

(6)

where \( \bar{X}_j \), \( j= [1, k] \) – vector of the \( j \)-th cutting method, \( \bar{X}_j \) of the SVCM included in the optimal plan; \( q_j^n \) – the elements of non-integer vector \( q_j^n = \{q_1^n, ..., q_j^n, ..., q_k^n\} \) intensity of the application of the \( j \)-
The optimal method of cutting, $\overline{R_{\text{min}}}, \overline{R_{\text{max}}}$ – the vectors of the boundaries of the acceptable variation range of the rolls number of each format.

![Decision tree of the problem of forming a set of acceptable methods for cutting paper web roll products](image)

**Figure 1.** Decision tree of the problem of forming a set of acceptable methods for cutting paper web roll products

The integer algorithm includes the following operations.
First of all, the rows are selected from the system of inequalities (6) that have only one nonzero element. If such lines are found, then the integer value of the intensity $q_j$ is determined from the conditions:

$q_j = q^n_j$, if $q^n_j$ – is an integer;

$q_j = [q^n_j]$, if the total number of rolls of the $j$-th format is on the upper boundary of the permissible variation range of the number of rolls

$$\sum_{i=1}^{k} X_i q^n_j = R_{j,\max},$$

where $[q^n_j]$ – the integer part of the number; $q_j = [q^n_j] + 1$, if the total number of rolls of the $j$-th format is on the lower boundary of the permissible variation range of the number of rolls

$$\sum_{i=1}^{k} X_i q^n_j = R_{j,\min}.$$

Second of all, the system of remaining inequalities is solved taking into account the calculated values at the previous stage. In this case, inequalities are sorted by increasing values of $R_{j,\min}$ or $R_{j,\max}$, and the solution begins with inequality with the smallest variation range of the number of rolls. When solving the system, the intensity values are determined as the nearest integers to the obtained ones, to satisfy inequalities (6). The proposed algorithm provides more accurate results of the integer mechanism in the sense of satisfying system (6) than simple rounding to the nearest integer.

2.2. The solution of the problem of optimal sheet material planning.

Planning of the sheet cutting is carried out in several stages.

During the first stage, a lot of spools of all acceptable sizes are determined for cutting on a sheet cutter. The maximum spool width is limited by the cutting width of the sheet cutter.

During the second stage, many ways of laying out the spools along the width of the tambour of the paper $L_{\max}$ for a longitudinal slitting machine (LSM) are formed.

In the third stage, the problem of choosing the optimal plan for cutting the tambour is solved, in which a lot of ways of spools laying out along the cutting width of the tambour are used as an initial data that generated in the second stage.

In the fourth stage, the obtained optimal cutting results of the spools are recalculated and presented in the form of an optimal cutting plan for sheet cutter.

The task of cutting sheet materials is reduced to the task of cutting rolls and the total multiplicity of the $j$-th format in the $r$-th cutting method of $X_{jr}$ is estimated as the number of conditional rolls of $W_{dj}$ width in the $r$-th cutting method. The mathematical model of the task of cutting the spools in terms of conditional rolls is written as follows:

$$F = \sum_{r=1}^{N} q_r \left( L_{\max} - \sum_{j=1}^{k} X_{jr} W_{dj} \right) \rightarrow \min_{q_r, r = [1, N]} \quad (7)$$

while limiting

$$K_{jr,\min} \leq \sum_{r=1}^{N} X_{jr} q_r \leq K_{jr,\max}, j = [1, k], \quad (8)$$

it is obvious that

$$q_r \geq 0, r = [1, N], \quad (9)$$

where $r = [1, N], N$ – is a multitude of acceptable cutting methods; $q_r$ – is an intensity of the $r$-th method; $j = [1, k], k$ – is a number of sheets of the following formats $W_{dj} \times L_{dj}$; $W_{dj}$ – is a sheet width; $L_{dj}$ – is a sheet length; $K_{jr,\min}$ to $K_{jr,\max}$ – the permissible variation range of the number of conditional rolls, it is calculated similarly to the boundaries of the permissible variation range of the number of rolls $R_{j,\min}, R_{j,\max}$ from the ratio (3).
The method of forming SVCM of spools cutting is developed based on the method of branches and borders. The partition of the set of all conceivable layouts of JM spools is performed similarly to that shown on the Figure 1.

Each spool contains conditional rolls of only one width. In addition, the edges, subsequently cut on the sheet cutter, are further included in the calculation of the width of the spool.

The r-th cutting method can be represented by a vector \( J_r = \{J_{r_1}, J_{r_2}, \ldots, J_{r_m}\} \in JM \), where each element \( J_{ri} \) – is a format number equal to the width \( W_dj \) of the sheet stacked in the i-th spool, \( J_{ri} = [1, k]; i=[1,m], m – is a total number of spools. Each vector \( J_r = \{J_{r_1}, J_{r_2}, \ldots, J_{r_m}\} \) is associated with a multiplicity vector of the conditional rolls \( X_r = \{X_{r_1}, X_{r_2}, \ldots, X_{r_n}\} \).

In the process of decomposition, it is necessary to take into account that the total width of the spools \( SH_k \) does not exceed the cutting width of the paper tambour \( L_{max} \), where

\[
SH_k = \sum_{i=1}^{m} L_a (W_d) .
\]

As a result of decomposition of the set of JM, subsets, containing one or more vectors, are formed. The resulting vectors \( J_r \) of the numbers of the sheet width in the spool are recalculated into the number of conditional rolls in each cutting method. When recalculating the spool cutting vectors into conditional roll vectors, duplicate vectors can be obtained, which should be excluded from consideration.

Only those cutting methods that satisfy the relevant criterion are included in the resulting SVCM, and a set of homogeneous cutting method vectors is also included. The criterion of vectors selection is the maximum allowable residue from the cutting into sheets, adopted in the industry \( L_{lost} \leq L_{lost_{max}} \).

3. Results and Discussion

The main interest was in the comparative analysis of the results of solving the optimal cutting problem for two mathematical models proposed in the study (Model 1) and a model in which the result is calculated on the basis of minimizing the amount of waste calculated in tons, when cutting a unit of material in each way and then recalculating it into an integer number of samples, taking into account customers’ requirements for roll sizes (Model 2).

The format of the roll is determined by the customer of the product. For the same initial data of one type of the product, for example, the Ladoga brand (weight of 1 m² is 220 g, sleeve diameter 150 mm, roll diameter 1000 mm), the roll format can take any valid value with an accuracy of 1 cm.

Let a portfolio of orders for paper/cardboard rolls be formed at paper/cardboard production, according to which for a certain type of product and combination of sleeve diameter - roll diameter, a sample of orders for rolls of certain format \( s \) in the volume determined by the application is obtained (Table 1). When calculating in constraints (2), (4), the customer’s requirements for the roll dimensions were taken into account, based on which, the solution result is obtained in the form of the number of samples, that is, units of the source material that are cut by each of the optimal methods.

| Roll format, cm | Order, t | Results from Model 1 | Results from Model 2 |
|----------------|---------|----------------------|----------------------|
|                |         | Estimated weight, t  | Absolute deviation from the task, t | Estimated weight, t | Absolute deviation from the task, t |
| 58             | 22      | 22.1                 | 0.1                  | 20.4                | 1.6                      |
| 60             | 40      | 38.8                 | 1.2                  | 38.1                | 1.9                      |
| 70             | 120     | 118.9                | 1.1                  | 118.5               | 1.5                      |
| 90             | 42      | 42.3                 | 0.3                  | 40.2                | 1.8                      |
| Total:         |         | 2.7                  | -                    | 6.6                 |                          |
A comparative analysis of the results of solving the optimal roll cutting problem has shown that the cutting model proposed (Model 2) in this work gives a greater degree of approximation of the result to the task. For the further convenience, the discrepancy between the order on the cutting each type of product and the result of solving the problem, taken in absolute value, will be called the accuracy of solving the cutting problem.

An example of solving the problem of the optimal cutting of sheet products is represented below (Tables 2, 3).

**Table 2** Initial data for solving the problem of the optimal cutting of sheet products

| Sheet width, cm | Sheet length, cm | Order, t |
|----------------|-----------------|---------|
| 79.0           | 70.0            | 12.0    |
| 77.0           | 62.5            | 12.5    |
| 66.0           | 91.0            | 26.0    |
| 66.0           | 92.0            | 16.0    |
| 60.0           | 84.0            | 38.0    |
| 56.0           | 98.0            | 6.0     |
| 52.5           | 99.0            | 65.0    |

The optimal cutting plan for LSM, proposed in solving problems by models, is represented in the Table 3.

**Table 3** The results of solving the problem of the optimal cutting of sheet products

| Number of method | Layout | Balance, cm | Amount of tons | Balance, t |
|------------------|--------|-------------|----------------|------------|
| 1                | 183· (60·3) + 135· (66·2) + 108· (52.5·2) | 0.0 | 93.0 | 0.0 |
| 2                | 213· (52.5·4) + 213· (52.5·4) | 0.0 | 25.0 | 0.0 |
| 3                | 201· (66·3) + 115· (56·2) + 108· (52.5·2) | 2.0 | 23.0 | 0.1 |
| 4                | 161· (79·2) + 157· (77·2) + 108· (52.5·2) | 0.0 | 34.0 | 0.0 |
| Total balance:   |        |             |                | 0.1        |

What is more, the structure of the automated optimal cutting system of rolled and sheet materials is proposed.

The automated optimal cutting system is integrating and it includes the following hierarchy levels:
- optimal planning of cutting rolled and sheet products;
- longitudinal slitting machine control.

The functional structure of the system and its place in the factory ACS is shown on the Figure 2.

The subsystem of the optimal planning of cutting paper/cardboard web includes a database system; user interface; database management system (DMS); external applications in the form of exe-files for generating a set of acceptable ways of cutting and calculating optimal plans.

DMS software modules are used to calculate the initial data when performing preparatory cutting operations, to display the results in a user-friendly form for roll and sheet cutting, as well as to create an optimal cutting plan for rolls, spools and sheets.

An experimental study of the automated operating modes of the LSM, which were carried out on the actual production of printed papers, revealed the problem of varying the weight of rolls when they come out from LSM [16]. The reasons of this phenomenon were the unsatisfactory operation of the machine (as equipment) and the dispersion of the weight of the paper web on the paper machine [17]. It confirms the need to ensure product quality of the paper web at all stages to the final one - so-called, the optimal cutting of paper products.
The proposed model can be taken into account when solving the problem of choosing a supplier of paper products [18].

If a manufacturing defect occurs on the paper machine that affects the uniform distribution of 1 m² of the mass, and a region of heterogeneous density is formed on the paper web, which is not suitable to be sold to the customer, then tambours with such paper can be used for the needs of the factory or can be sent for further processing. However, the use of the tambours with such paper during the cutting stage is possible if the defective part of the web is cutting out [19]. It can significantly reduce the amount of defected paper, as well as it can lead to the use of such tambours to obtain finished rolls of small formats.

In the development of the modern approaches in the field of digitalization, the solution of the problem of optimal production planning can be implemented in the information system using so-called “cloud” technologies [20, 21, 22].

4. Conclusions
A method, mathematical models, algorithms of the optimal cutting of rolls and sheets to ensure the quality of products and to increase production efficiency have been proposed. The dependences of the accuracy of solving the cutting problem on the fluctuations in the weight of the rolls have been revealed. Suggestions to improve the LSM and its management, based on the created automated control system, have been made. At the same time, it was recorded that losses from the implementation of the developed automated control system of the LSM, unlike previously used manual cutting, have been reduced by 5-6 times.

Figure 2. Functional structure of an automated system of optimal cutting
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