Modeling nonlinear dielectric properties of laminated composites

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Abstracts. The article is devoted to the development of a method for predicting the nonlinear dielectric properties of layered composites with complex dependences of the dielectric constant on the electric field strength, which consider the effects of a sharp increase and subsequent decrease in the dielectric constant with increasing field strength. The method is based on the use of the theory of asymptotic averaging proposed by N.S. Bakhvalov, E. Sanchez-Palencia, in relation to the nonlinear problem of electrostatics in a layered medium. As a result of applying this method, local nonlinear problems of electrostatics on a periodicity cell are formulated, and an algorithm for calculating effective nonlinear dielectric constants is proposed. It is shown that a layered composite is a transversely isotropic nonlinear dielectric material if its layers are isotropic materials. A numerical example of calculating the nonlinear properties of a 2-layer composite based on barium titanate and ferroelectric ceramic varicond VK4 is considered. Calculations have shown that the developed method for calculating the nonlinear-dielectric ratios of composites is quite effective and makes it possible to predict the dielectric properties of composites at various values of the electric field strength, including the regions where the dielectric permittivities of the layer materials are saturated. The developed technique can serve as a basis for designing new nonlinear dielectric composite materials with unusual properties.

Keywords: nonlinear dielectrics, laminated composites, modeling, asymptotic averaging method, barium titanate, varicond

1. Introduction.

Materials with nonlinear dielectric properties, in particular ferroelectrics, pyroelectrics, etc., are a promising class of materials for creating various devices and electrical devices [1-3]. In particular, these materials are used for various types of nonlinear varicond capacitors, which are used “for frequency multiplication and division; for signal detection; to create a modulation frequency in time relay circuits; in memory cells; to create dielectric amplifiers and other devices [4]. One of the promising directions is the creation of computer memory elements for recording and storing information [3,5]. In this regard, the problem of creating methods for predicting the properties of composite materials with nonlinear dielectric properties, which would make it possible to design materials with predetermined dielectric properties, is very urgent. The design of nonlinearly dielectric heterogeneous structures and materials is currently being actively carried out, including using a domain structure at the nanostructural level [5]. However, as a rule, approximate calculation methods are used in this case, which do not always take into account the anisotropy of the properties of the composite in the nonlinear region.

The aim of this work is to use the asymptotic averaging method [6-9] for composites with nonlinear dielectric properties, which was previously applied for a large number of different problems [10-13], including linear problems of electrodynamics and piezoelasticity [14,15].

2. Statement of the quasi-static problem of nonlinear electrostatics for polarizable composites.

Let us consider in three-dimensional space a composite (an inhomogeneous medium V with a periodic structure, consisting of N isotropic phases $V_\alpha$, $\alpha = 1, \ldots, N$, which is under the influence of an alternating electric field. All phases $V_\alpha$ of the composite will be assumed to be polarized dielectrics [16] with nonlinear dielectric strength diagrams, and the distribution of the fields of electric strength and electric induction in the composite will be assumed to obey Maxwell's quasi-static equations [16].
Then, under the indicated assumptions, in the Cartesian coordinate system $x^i$, the formulation of the problem of nonlinear electrostatics for the indicated type of composite will take the form [16, 17]:

$$\nabla_i d_i = 0, \quad x^i \in V,$$

$$d_i = \varepsilon_\alpha (|e|) e_i, \quad x^i \in V \cup \Sigma,$$

$$e_i = \nabla_i \varphi, \quad x^k \in V \cup \Sigma,$$

$$[\varphi] = 0, \quad d_i |_{\Sigma_i} = 0, \quad x^k \in \Sigma_{\alpha \beta},$$

$$\varphi |_{\Sigma_i} = \varphi_\alpha, \quad d_i n_i |_{\Sigma_i} = d_e,$$

where are designated: $d_i$ - components of the electric field induction vector; $e_i$ - components of the vector of the electric field strength; $\nabla_i$ - nabla operator; $f_i$ - components of a nonlinear vector function describing the relationship between $e_i$ and $d_i$ for the composite; $\varphi$ - electric potential; $n_i$ - components of the normal vector; $[\varphi]$ - is the jump of functions at the interface $\Sigma_{\alpha \beta}$ between the phases of the composite with numbers $\alpha$ and $\beta$, $\alpha, \beta = 1, ..., N$, $\varphi_\alpha$ - is the given potential on a part of the composite surface, $d_e$ is the given normal component of the electric field induction vector on a part of the surface $\Sigma_2$, $|e| = \sqrt{\delta_{ij} e_i e_j}$ is the modulus of the electric field strength vector, $\delta_{ij}$ is the Kronecker symbol.

**Model of nonlinear dielectric properties of composite phases.** All phases of the composite will be assumed to be isotropic and nonlinearly dielectric. Let us take the following dependences as specific functions $\varepsilon_\alpha (|e|)$ of the nonlinear dielectric constant of phases:

$$\varepsilon_\alpha = f_\alpha (|e|) = \left\{ \begin{array}{ll}
\varepsilon_{\alpha 0}, & 0 \leq |e| \leq \varepsilon_{a S}, \\
\varepsilon_{\alpha 0} + A_\alpha \left(1 - \exp\left(-B_\alpha (|e| - \varepsilon_{a S})^2\right)\right), & \varepsilon_{a S} \leq |e| \leq \varepsilon_{a R}, \\
(\varepsilon_{\alpha 0} + A_\alpha \left(1 - \exp\left(-B_\alpha (|e| - \varepsilon_{a S})^2\right)\right)) \frac{\varepsilon_{a R}}{|e|}, & |e| > \varepsilon_{a R}
\end{array} \right. \quad \alpha = 1, ..., N$$

where $\varepsilon_{\alpha 0}, A_\alpha, B_\alpha, \varepsilon_{a S}, \varepsilon_{a R}$, - constants characterizing the material of each phase of the composite.

All equations in system (1) - (6) will be assumed to be dimensionless.

Model (6) describes 3 sections of the dielectric constant: 1) linear, when $\varepsilon_\alpha (|e|)$ does not change with increasing field strength, 2) - a section of $\varepsilon_\alpha (|e|)$ increase, and 3) - a section of $\varepsilon_\alpha (|e|)$ decrease (saturation) with increasing field strength.

**Linearization of the problem.** Applying the method of successive approximations for the nonlinear relation (6), we linearize the constitutive relations, where $m$ - is the number of the approximation.

Then the statement of problem (1) - (6) takes the form

$$\nabla_i d_i^{(m)} = 0, \quad x^i \in V,$$

$$d_i^{(m)} = \varepsilon_\alpha (|e_i^{(m-1)}|) e_i^{(m)}, \quad x^i \in V \cup \Sigma,$$

$$e_i^{(m)} = \nabla_i \varphi^{(m)}, \quad x^i \in V \cup \Sigma,$$

(7)
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\[
\begin{bmatrix}
\varphi^{(m)}
\end{bmatrix} = 0, \quad \begin{bmatrix}
d_i^{(m)}
\end{bmatrix} n_i = 0, \quad \chi' \in \sum_{\alpha \beta}
\]

\[
\varphi^{(m)}\bigr|_{\xi_1} = \varphi_e, \quad d_i^{(m)} n_i \bigr|_{\xi_2} = d_e, \quad m = 1, 2, ..., 
\]

Asymptotic expansions of the system of nonlinear Maxwell equations

Following the general concept of the asymptotic averaging method [6-9], we introduce a small parameter \( \kappa = \frac{l}{L} \equiv 1 \), where \( l \) is the characteristic size of the periodicity cell (PC), and also introduce the following dimensionless coordinates: \( \chi' \) – global coordinates and \( \xi = \frac{x^3}{\kappa} \) – local coordinate. The coordinate \( \xi \) changes in the interval \( -0.5 \leq \xi \leq 0.5 \) within the area \( V_\xi \) – the periodicity cell. Let us denote \( V_{\alpha} = \{ \xi : \xi_{\alpha-1} \leq \xi \leq \xi_{\alpha} \} \) the area of the \( \alpha \)-layer in the PC, and \( \xi_{\alpha} \) – the coordinates of the interfaces of the layers in the PC, \( \alpha = 1, ..., N \). All equations in this system (7) are considered dimensionless. The differentiation of the functions \( h(x', \xi) \) included in this system is carried out according to the following rule:

\[
\nabla_j h = h_j + \kappa^{-1} \delta_{ij} h_3,
\]

where are the derivatives \( h_j = \frac{\partial h}{\partial x^j}, h_3 = \frac{\partial h}{\partial \xi} \).

We seek the solution of problem (7) in the form of asymptotic expansions in the small parameter

\[
\varphi^{(m)}(x', \xi') = \varphi^{(m)(0)}(x') + \sum_{n=1}^{\infty} \kappa^n \varphi^{(m)(n)}(x', \xi'), \quad \xi \in V_\xi, \quad x' \in V
\]

Substituting (9) into the equations of system (7), we obtain asymptotic expansions for the components of the electric field strength vector \( e_i^{(m)} \), dielectric constant and electric field induction vector \( d_i^{(m)} \)

\[
e_i^{(m)} = \sum_{n=0}^{\infty} \kappa^n e_i^{(m)(n)},
\]

\[
\varepsilon(\xi) e^{(m-1)}(\xi) = \sum_{n=0}^{\infty} \kappa^n e_i^{(m-1)(n)}, \quad d_i^{(m)} = \sum_{n=0}^{\infty} \kappa^n d_i^{(m)(n)}
\]

Local problems

Substituting expansions (9) – (11) into the system of equations (7), and collecting terms at the same powers of the small parameter, we obtain a recurrent sequence of local problems. For the zero approximation \( n = 0 \), we have the following problem

\[
d_i^{(m)(0)} = 0, \quad \xi \in V_\xi,
\]

\[
d_i^{(m)(0)} = \varepsilon_\alpha (\varepsilon e^{(m-1)(0)}(\xi)), \quad \xi \in V_{\alpha},
\]

\[
e_i^{(m)(0)} = \varphi_j^{(m)(0)} + \varphi_j^{(m)(1)}, \quad \xi \in V_\xi \cup \Sigma_\xi,
\]

\[
\xi = \xi_{\alpha}, \quad \begin{bmatrix}
\varphi^{(m)(1)}
\end{bmatrix} = 0, \quad \begin{bmatrix}
d_i^{(m)(0)}
\end{bmatrix} = 0,
\]

\[
< \varphi^{(m)(1)} > = 0, \quad \begin{bmatrix}
\varphi^{(m)(1)}
\end{bmatrix} = 0, \quad m = 1, 2, ..., 
\]

(12)
Here are designated $\Sigma$ – the boundary of the PC, \[
\left[ \varphi^{(m)(1)} \right] = 0
\] – the condition of periodicity on the boundary of the PC, \(< \varphi^{(m)(1)} > = \int_{\partial \Sigma} \varphi^{(m)(1)} d\xi\) – the operation of averaging over the PC.

The solution to problem (12) is sought with respect to functions. The input data of this problem is a gradient, so the solution can be presented in a formal form
\[
\varphi^{(m)(1)} = N_i^{(1)(0)} \varphi_j^{(m)(0)}
\]
where functions $N_i^{(1)(0)}$ depend on the following arguments
\[
N_i^{(1)(0)} = N_i^{(1)(0)} \left( |e^{(m-1)(0)}|, \varphi \right)
\]
Substituting (14) into the third equation of system (12), we find the intensity vector in the zero approximation
\[
e_i^{(m)(0)} = (\delta_j^g + \delta_j^s N_i^{(1)(0)}) \varphi_j^{(m)(0)}
\]

**Average problems**

After substituting asymptotic expansions (10) – (11) into the first equation of (11), and averaging it over PC, we obtain the following averaged equation describing the distribution of the average electric field induction in the composite
\[
\overline{d}_{ij}^{(m)} = 0
\]
where the notation is introduced
\[
\overline{d}_{ij}^{(m)} = \sum_{n=0}^{n} \kappa^{(n)} < d_j^{(m)(n)} >
\]
Substituting the functions $d^{(m)(n)}$ expressed in terms of $e_i^{(m-1)(n)}$, into equation (16), then we get
\[
\overline{d}_{ij}^{(m)} = < \overline{e}_j \left( |e^{(m-1)(0)}| e_i^{(m)(0)} >
\]
Taking into account (15), this relation can be written in the form
\[
\overline{d}_{ij}^{(m)} = \overline{e}_j \left( |e^{(m-1)}| \right) \overline{e}_i^{(m)}
\]
Here is the averaged composite stress vector
\[
\overline{\sigma}_i^{(m)} = < \overline{e}_i^{(m)} > = \varphi_j^{(m)(0)}
\]
And
\[
\overline{\sigma}_i \left( |e^{(m-1)}| \right) = < \overline{e}_j \left( |e^{(m-1)(0)}| \right) (\delta_j^g + \delta_j^s N_i^{(1)(0)} \left( |e^{(m-1)(0)}|, \varphi \right)) >
\]
is the effective tensor of the dielectric constant of the composite.

Substituting asymptotic expansions (10) and (11) into the boundary conditions of problem (7), after averaging, we obtain
\[
\varphi^{(m)(0)} \bigg|_{\partial \Sigma} = < \varphi >, \quad \overline{d}_{ij}^{(m)} n_j^{\partial \Sigma} = < d_j >
\]
Here, it was considered that $< \varphi^{(m)(0)} > = \varphi^{(m)(0)}$ and $< \varphi^{(m)(e)} > = 0$ at $n > 1$, due to the normalization conditions imposed on local problems (12).
The system of equations (16), (19), (20) and conditions (21) forms the statement of the linearized averaged problem of composite electrostatics. This problem is considered with respect to the zero-order potential \( \phi^{(0)}(x) \).

Explicit solution to a local problem

Problem (12) is one-dimensional and admits an explicit solution with respect to functions \( N_i^{(1)(0)} \) (14)

\[
N_i^{(1)(0)}(| \textbf{e}^{(m-1)(0)} |, \xi) = N^{(10)}(| \textbf{e}^{(m-1)(0)} |, \xi) \delta_{ij},
\]

\[
N^{(10)}(| \textbf{e}^{(m-1)(0)} |, \xi) = (\epsilon_i(| \textbf{e}^{(m-1)(0)} |))^{-1} > (\epsilon_i(| \textbf{e}^{(m-1)(0)} |))^{-1} > \xi,
\]

Here we denote the integration operator

\[
\int_{-0.5}^{0.5} f(\bar{\xi}) \, d\bar{\xi} = \int_{-0.5}^{0.5} f(\bar{\xi}) \, d\bar{\xi}.
\]

From (15), (20) and (23), (24) we obtain the expression for the electric field strength vector in the zero approximation

\[
e_i^{(m)(0)} = W_{ij}^{(m-1)} \textbf{e}^{(m)}
\]

Where \( W_{ij}^{(m-1)} \) is the tensor of the concentration of the electric field strength in the layers of the composite

\[
W_{ij}^{(m-1)} = \delta_{ij} + \delta_{ij} W^{(m-1)},
\]

\[
W^{(m-1)} = N^{(10)} = \gamma_i - 1, \quad \gamma_i = (\epsilon_i(| \textbf{e}^{(m-1)(0)} |))^{-1} > (\epsilon_i(| \textbf{e}^{(m-1)(0)} |))^{-1}.
\]

It follows from (25) and (26) that the longitudinal components of the stress vectors \( e_i^{(m)(0)}, \textbf{e}_j^{(m)} \) coincide, and the transverse components differ.

\[
e_i^{(m)(0)} = \textbf{e}_j^{(m)}, \quad e_j^{(m)(0)} = (1 + W_{ij}^{(m-1)}) \textbf{e}_j^{(m)}
\]

From (25) we find the modulus of the vector \(| \textbf{e}^{(m)(0)} |\) and thus the recurrence relation between \(| \textbf{e}^{(m)(0)} |\) and \(| \textbf{e}^{(m-1)(0)} |\).

Substituting (25) into the second relation of system (12), we find the expression for the induction vector in the layers of the composite

\[
d_j^{(m)(0)} = \epsilon_i(| \textbf{e}^{(m-1)(0)} |) W_{ij}^{(m-1)} \textbf{e}^{(m)}
\]

Substituting (26) into formula (21), we find an explicit expression for the nonzero components of the effective tensor of the dielectric constant of the composite

\[
\epsilon_{11}(| \textbf{e}^{(m-1)} |) = \epsilon_{22}(| \textbf{e}^{(m-1)} |) = (\epsilon_i(| \textbf{e}^{(m-1)(0)} |))^{-1} > (\epsilon_i(| \textbf{e}^{(m-1)(0)} |))^{-1}.
\]

Formula (29) makes it possible to obtain a stable numerical algorithm for calculating the component \( \epsilon_{11}(| \textbf{e}^{(m-1)} |) \) with an iterative change in the average electric field strength. For a component \( \epsilon_{33}(| \textbf{e}^{(m-1)} |) \), a similar algorithm leads to computational instability in the 2 and 3 nonlinear regions \( \epsilon_i(| \textbf{e} |) \). The following algorithm has been proposed to fix this problem.

Consider the case of a two-layer composite \( N = 2 \). Then formula (30) can be written as follows
Where \( h_1 = 0.5 + \xi_1 \) and \( h_2 = 0.5 - \xi_1 \) are the relative thicknesses of the layers.

The dielectric permittivities of the layers are functions of; therefore, taking into account (26) and (27), these functions can be represented in the form

\[
\varepsilon_{33} = \frac{\varepsilon_1 \varepsilon_2}{h_1 \varepsilon_2 + h_2 \varepsilon_1} 
\]  

(31)

We introduce a parameter \( z = \varepsilon_1 / \varepsilon_2 \), then from (33) we obtain a nonlinear algebraic equation for calculating this parameter

\[
f_1 \left[ \sqrt{B^2 + \left( \frac{\varepsilon_{33}^{(m-1)}}{h_1 + h_2 z} \right)^2} \right] = z f_2 \left[ \sqrt{B^2 + \left( \frac{z \varepsilon_{33}^{(m-1)}}{h_1 + h_2 z} \right)^2} \right] 
\]

(34)

After finding the root \( z = \varepsilon_1 / \varepsilon_2 \) of this equation, we substitute this value in (34), and then, the resulting values \( \gamma_{1,2} \) in equation (33). As a result, we find the values \( \alpha \) corresponding to the values of the average intensity field \( \varepsilon_{33}^{(m-1)} \) at \( m-1 \) iterations. Using these values, we finally calculate the effective dielectric constant \( \varepsilon_{33}^{(m-1)} \) using formula (32).

### Explicit solution to a local problem

In accordance with the developed methodology, we calculated the effective dielectric constant of a two-layer nonlinear dielectric composite, one of the layers of which was barium titanate BaTiO \(_3\), and the other was a ferroelectric material – varicond VK4 [4,19,20]. Diagrams of nonlinear dielectric functions \( \varepsilon_{33}(|\varepsilon|) \) – dependences of dielectric permittivities on the electric field strength \(|\varepsilon|\) for BaTiO \(_3\) and VK4 varicons were taken from [19, 20]. These experimental functions were approximated \( \varepsilon_{33}(|\varepsilon|) \) using analytical functions (6). The values of the constants \( \varepsilon_{a0}, A_a, B_a, e_{ax} \) for BaTiO \(_3\) and BK4 obtained in this way are shown in Table 1. The values of all characteristics of the electric field, including constants \( \varepsilon_{a0}, A_a, B_a, e_{ax} \), are dimensionless. When dimensioning it out, the following characteristic values were introduced

\[
L = 1 \text{m}, \quad e_0 = 10^6 \text{B}/\text{m}, \quad e_0 = 8,854 \cdot 10^{-8} \text{Kl}/(\text{B} \cdot \text{m}), \quad d_0 = 8,854 \cdot 10^{-2} \text{Kl}/\text{m}^2, \quad \varphi_0 = 10^6 \text{B}.
\]

| Table 1. Values of constants in model (9) for barium titanate and varicond VK4 |
|-----------------|-----------|-----------|-----------|
|                 | \( \varepsilon_{a0} \) | \( A_a \) | \( B_a \) | \( e_{ax} \) |
| BaTiO \(_3\) \(( \alpha = 1 \) | 0,1 | 1,0 | 40 | 0,25 |
| BK4 \(( \alpha = 2 \) | 0,2 | 2,5 | 40 | 0,02 |

The relative thicknesses of the composite layers (phase concentrations) were denoted as \( h_1 \) and \( h_2 \) \(( h_1 + h_2 = 1, \quad 0 \leq h_j \leq 1 )\).

Figure 1 shows the plots of the function \( \varepsilon_{a} = f_a(|\varepsilon|) \) for both layers of the composite BaTiO \(_3\) \(( \alpha = 1 \) corresponds to BaTiO \(_3\), and \( \alpha = 2 \) to the VK4 varicond), as well as the effective dielectric constants \( \varepsilon_{33}^{(m-1)}(\xi_1) \), \( \varepsilon_{33}^{(m-1)}(\xi_2) \) of a 2-layer composite with the ratio of layers \( h_1 = h_2 = 0.5 \), calculated by formulas (29) and (32).
The dielectric constant of the VK4 ferroelectric increases with increasing electric field strength almost immediately. This increase reaches its maximum values (more than 10 times) compared to the initial value \( \varepsilon_2 = f_2(0) \). Approximately after the values \(|e| = 0.25\) of VK4, saturation occurs and the dielectric constant decreases. Barium titanate has a fairly wide range of values \(|e|\) in which the dielectric constant does not change. The increase in values \( \varepsilon_i = f_i(|e|) \) begins after \(|e| = 0.22\) and continues until \(|e| = 0.8\), then saturation occurs and the dielectric constant decreases.

In a 2-layer composite VK4 + BaTiO\(_3\) with a 50% titanate content of barium, the longitudinal dielectric constant \( \varepsilon_{11} \) basically follows the nature of the change in the dielectric constant of VK4: an increase in values \( \varepsilon_{11} \) begins almost immediately with an increase in values \( \varepsilon_{11}^{(0)} \).

![Figure 1](image.png)

**Figure 1.** The dependence of the dielectric constant on the electric field strength: 1) function \( \varepsilon_1 = f_1(|e|) \) for BaTiO\(_3\), 2) function \( \varepsilon_2 = f_2(|e|) \) for VK4, 3) function \( \varepsilon_{33}(\varepsilon_3) \) for 2-layer composite VK4 + BaTiO\(_3\), 4) function \( \varepsilon_{11}(\varepsilon_1) \) for 2-layer composite VK4 + BaTiO\(_3\).

At the same time, the change in the dielectric constant \( \varepsilon_{33} \) of the 2-layer composite VK4 + BaTiO\(_3\) is closer in character to the analogous dependence of barium titanate: there is a sufficiently large range of values \( \varepsilon_{33} \) at which \( \varepsilon_{33} \) practically does not change (Fig. 2).

In the interval \( 0 < |e| < 0.25 \) where both functions \( \varepsilon_i = f_i(|e|) \) are monotonically increasing for VK4 and BaTiO\(_3\), the graphs of functions \( \varepsilon_{11}(\varepsilon_1) \) and \( \varepsilon_{33}(\varepsilon_3) \) are arranged in a "classical" way:

\[ f_1(|e|) < \varepsilon_{11}(|e|) < \varepsilon_{33}(|e|) < f_3(|e|) \]

In the interval \( 0.35 < |e| < 0.75 \), the functions \( \varepsilon_i = f_i(|e|) \) are monotonically increasing, and \( \varepsilon_i = f_i(|e|) \) – monotonically decreasing, as a result, the graphs of the functions \( \varepsilon_{11}(\varepsilon_1) \) and \( \varepsilon_{33}(\varepsilon_3) \) are arranged in a "non-classical" way:

\[ f_1(|e|) \leq \varepsilon_{11}(|e|) \leq \varepsilon_{33}(|e|) \leq f_3(|e|) \]

that is, the transverse dielectric constant \( \varepsilon_{33}(\varepsilon_3) \) becomes larger than the longitudinal one \( \varepsilon_{11}(\varepsilon_1) \).

At \( 0.5 < |e| \), the dielectric constant \( \varepsilon_{33}(\varepsilon_3) \) practically coincides with \( f_3(|e|) \).
At $0.8 < |\mathbf{e}|$, the following ratio of dielectric constants takes place

$$f_2(|\mathbf{e}|) = \varepsilon_{33}(|\mathbf{e}|) < \varepsilon_{15}(|\mathbf{e}|) < f_1(|\mathbf{e}|)$$

Thus, a nonlinear dielectric composite exhibits unusual substantially nonlinear properties that cannot be predicted by classical mixing methods - in the form of Khashin – Strickman forks.

The developed method makes it possible to predict the nonlinear dielectric properties of composites with different phase contents, and thereby design materials with specified nonlinear dielectric properties.

3. Conclusion

Using the method of asymptotic averaging of periodic layered structures, a method is proposed for calculating the effective constitutive relations for nonlinear dielectric composites with complex dependences of the dielectric constant, including saturation regions. The technique is based on the construction of asymptotic solutions of the system of quasi-stationary Maxwell equations for the electric field.

A local nonlinear problem of electrostatics on the periodicity cell of the composite is formulated, with the help of which the tensor of the effective nonlinear dielectric properties of layered composites with transverse isotropy is calculated.

An example of solving local problems and calculating nonlinear dielectric relations for layered composites: barium titanate - varicond VK4 is considered. Calculations have shown that the developed method for calculating the nonlinear-dielectric ratios of composites is quite effective and makes it possible to predict the dielectric properties of composites at different values of the electric field strength, including the regions where the dielectric permittivities of the layer materials are saturated.

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