Back-tracking of primary particle trajectories for muons detected at the Earth surface

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Abstract. Investigations of cosmic rays on the surface of the Earth allow to derive information of applied character on the conditions of the interplanetary magnetic field and of the geomagnetic field. For this purpose, it is necessary to collate trajectories of particles detected in the ground-based detector to trajectories of primary cosmic rays in the heliosphere. This problem is solved by means of various back-tracking methods. In this work, one of such methods is presented.

1. Introduction
The heliospheric perturbations change the conditions of the interplanetary magnetic field (IMF). It results in a change of trajectories of charged primary cosmic rays (PCR) that arrive to the atmosphere of the Earth. Thus, the atmospheric muon flux formed at the interaction of PCR with nuclei of atoms of the atmosphere is to change. For comparison of muon trajectories registered in the ground detector with trajectories of PCR in the heliosphere it is necessary to construct the backward trajectories.

On the backward trajectories of particles from the detector to interplanetary space, it is possible to pick out the following spatial regions: the atmosphere, the Earth magnetosphere, and the interplanetary space. Particle motion in all these regions occurs in corresponding magnetic fields. For muon it is necessary to build a backward trajectory from the detector up to the altitude of its generation. Further, it is necessary to replace muon with a parent particle, for example, proton or helium nucleus and to build the backward trajectory for it. At passage of particles inside the atmosphere it is necessary to take into account the energy losses. In this work, the step-by-step method of construction of the backward trajectory is used.

2. Coordinate systems
At construction of the backward trajectory the following coordinate systems are used:
- Coordinate system of the detector, its center coincides with the center of the detector.
- Local coordinate system, which center also is in the center of the detector.
- Coordinate system WGS84 (World Geodetic System 1984) which is used by the systems of global positioning GPS and GLONASS.
- Coordinate system GEO (Geographic coordinate system) [1] – geocentric system.
- Coordinate system HEEQ (Heliocentric Earth Equatorial) [1].
In figure 1, the images of the axes in the HEEQ, GEO, the local system and the system of the detector for a certain position of the detector on a surface of the Earth are presented.

![Image of coordinate systems](image)

**Figure 1.** The images of the axes of coordinates in GEO, local system and system of the detector on a surface of the Earth (left); the axes of coordinates in HEEQ (right).

For coordinates conversion between GEO and HEEQ it is possible to use the library CXFORM [2].

### 3. Atmosphere

Propagation of charged cosmic ray particles through the atmosphere of the Earth is accompanied by their interaction with nuclei of atoms of the atmosphere and energy losses. At interactions of primary particles, rapidly decaying pions and kaons are formed, they generate the atmospheric muon flux. The probability of interaction depends on density of the atmosphere and length of way passed by particles. Energy losses - ionization and radiation - also depend on density of the atmosphere and length of the way. The density of the atmosphere in any point in geodetic coordinates WGS84 for the number of the day relative to the beginning of the year and the time of the day is given with empirical model of atmosphere NRLMSISE-00 [3].

The altitude at which the interaction occurs (a zone of generation) depends on the trajectory of the primary particle. At construction of the backward trajectory, this altitude can be estimated by means of numerical integration of density along a direction of backward motion of muon up to the atmosphere boundary, that is to calculate atmospheric depth (see figure 2). If the depth is less than a preset value (for example, 100 g·cm⁻²) muon is replaced with a parent particle (proton or helium nucleus).

![Diagram of atmospheric depth](image)

**Figure 2.** The altitude of the zone of muon generation is determined by atmospheric depth along the backward trajectory.

Ionization loss for muon, proton and helion, and also radiation loss of muon are taken into account according to known formulas [4]. The step along the trajectory $\Delta r$ for muon must be chosen small enough, for example, $\Delta r = 10$ m or $\Delta r = (20 \text{ m})/\cos \theta_0$ (see figure 2), but not more than 150 m.
4. Earth magnetosphere
The magnetic field of the Earth has a complicated spatial distribution. For its description, various models are used. In our case, the necessary model should allow to get an estimation of a vector of a magnetic field in all magnetosphere regions for the preset time. Such opportunity provide the models of the magnetic field of the Earth by N.A.Tsyganenko TS05 (distant field) and GEOPACK-2008 (near-field) [5].

For the beginning, it is necessary to prepare input parameters describing a solar wind and geomagnetic conditions for the preset time. For this purpose, it is possible to take advantage of the data [6] or to calculate them with the use of database OMNI [7] by the algorithm described there. After preparation of input parameters, initialization with the use of subroutine RECALC_08 is carried out.

As the trajectory of particles can pass through regions with a big gradient of the magnetic field (cusp, magnetic anomalies) it is necessary to use a step along trajectory $\Delta r = 200$ m.

At construction of the backward trajectory, it is necessary to check overrunning by the particle of the magnetosphere boundary. It is done with the help of subroutines GEOGSW_08 and SHUETAL_MGNP_08. After overrunning of the magnetosphere boundary, the particle appears in the interplanetary magnetic field.

5. Interplanetary magnetic field
Unlike the characteristics of the atmosphere and magnetosphere which can be approached in part to real, distribution of the magnetic field in the interplanetary space is not known. Only in the locations of space vehicles which are equipped with the corresponding equipment it is possible to get a real vector of the magnetic field. In the given work, the model of the interplanetary magnetic field considered in [8] was used.

6. Motion in magnetic field
Motion of the relativistic charged particles in the magnetic field is described by the formula:

$$\frac{d}{dt}\gamma \vec{v} = q \left( \vec{v} \times \vec{B} \right)$$

$$\gamma = \frac{E}{E_0} + 1 \quad \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

Here: $m, q$ are mass and charge of the particle; $\gamma$ is Lorentz factor; $\vec{v}$ is unit vector of velocity; $E, E_0$ are kinetic and rest energies; $\vec{B}$ is the vector of a magnetic field; $t$ is the time.

Let's notice, that movement in the magnetic field of particles of different types will be similar at equal values of $q / (m\gamma)$. Therefore, motion of proton (rest energy $E_{0p}$) with kinetic energy $E_p$ will be similar to motion of $^{4}\text{He}$ (α-particle with rest energy $E_{0\alpha}$) with kinetic energy $E_{\alpha}$:

$$E_{\alpha} = 2E_p + 2E_{0\alpha} - E_{0\alpha} \approx 2E_p - 1.85 \text{ [GeV]}$$

7. Calculation of backward trajectory
Initial parameters before the beginning of calculation are: date and time in UTC; coordinates of the detector on the surface of the Earth; charge of muon (positive or negative); kinetic energy of muon in the detector; the direction of motion of muon in the detector; atmospheric depth for definition of the zone of generation; the type of the parent particle (proton or α-particle), its charge and mass; kinetic energy of the parent particle; parameters of the interplanetary magnetic field.

Construction of the trajectory is carried out in GEO in a unified cycle. Coordinates of the first point of the trajectory $\vec{r}_1$ are calculated from coordinates of the detector. From the direction of motion of muon in the detector the initial unit vector of velocity $\vec{v}_1$ is calculated. Kinetic energy of muon in first point is $E_1$. And further, with step $\Delta r$ we travel on the backward trajectory. At reaching of the zone of
generation, the type of a particle varies, initial values $q, m, E_0, E_i$ are set and we return to the beginning of the cycle.

Condition of the loop exit can be, for example, reaching by the particle of a certain distance from the Sun, or the surface of the Earth.

8. Examples of backward trajectories
As examples, backward trajectories of protons with kinetic energy 20 GeV for positive muon with kinetic energy 3 GeV registered in a vertical direction at three points on the surface of the Earth have been constructed: Apatity (67.57° N, 33.39° E, 181 m a.s.l.), Khabarovsk (48.49° N, 135.08° E, 106 m a.s.l.), Moscow (55.651° N, 37.668° E, 173 m a.s.l.).

Trajectories were constructed for June 22, 2016 04:00 UT. In our method of construction of backward trajectories, the spatial distribution of air density and magnetic field of the magnetosphere of the Earth are simply determined by the date. Parameters for the model of the interplanetary magnetic field have been chosen manually:

- magnetic field at distance 1 AU: $B_0 = 5$ nT;
- solar wind velocity: $V_{SW} = 400$ km·s$^{-1}$;
- "tilt angle" of the heliospheric current sheet $\alpha_{HCS} = 10^\circ$; its estimations are accessible in [9];
- slewing of "skirt" of the current sheet $\phi_{HCS} = 15^\circ$;
- solar polarity: negative $A = -1$ and positive $A = +1$.

At the chosen date, inclination and displacement of the current sheet, the Earth is located above the heliospheric current sheet. In figure 3, the calculated backward trajectories near the Earth are shown.

![Figure 3. Examples of backward trajectories of protons near the Earth for vertical muons, registered in three points of Russia: 1 – Apatity, 2 – Khabarovsk, 3 – Moscow.](image)

In figures 4 and 5, continuations of these trajectories in a near heliosphere and at the distance up to 10 AU in HEEQ (a circle in the center is the Sun) are plotted.

In figure 4 it is seen that at the negative polarity of the Sun muons, registered in Apatity, Khabarovsk and Moscow, pass through different regions of the near heliosphere. And at the positive polarity of the Sun, protons behind the orbit of the Earth pass through close regions of the heliosphere.

In figure 5 it is seen that at the negative polarity of the Sun the trajectories nestle to the current sheet, and at positive polarity they leave to the pole of the heliosphere. This distinction is explained by the helicity of the interplanetary magnetic field.

9. Conclusion
The method of construction of the backward trajectories described in the present work, despite of its simplicity, allows analyzing in details passage of muons in the atmosphere, protons in the magnetosphere and the heliosphere. The described method is used for various purposes: construction of asymptotic directions for ground-based muon and neutron detectors; analysis of heliospheric and magnetospheric perturbations; analysis of variations of the muon flux; in educational process.

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**Figure 4.** Examples of backward trajectories of protons in the near heliosphere (up to 1.5 AU) for vertical muons: 1 – Apatity, 2 – Khabarovsk, 3 – Moscow. Solar polarity: a) negative; b) positive.

**Figure 5.** The same as in figure 4, but for distant heliosphere (up to 10 AU).

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