Uniformly hot nightside temperatures on short-period gas giants

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Short-period gas giants (hot Jupiters) on circular orbits are expected to be tidally locked into synchronous rotation, with permanent daysides that face their host stars and permanent nightsides that face the darkness of space. Thermal flux from the nightside of several hot Jupiters has been detected, meaning energy is transported from day to night in some fashion. However, it is not clear exactly what the physical information from these detections reveals about the atmospheric dynamics of hot Jupiters. Here we show that the nightside effective temperatures of a sample of 12 hot Jupiters are clustered around 1,100 K, with a slight upward trend as a function of stellar irradiation. The clustering is not predicted by cloud-free atmospheric circulation models. This result can be explained if most hot Jupiters have nightside clouds that are optically thick to outgoing longwave radiation and hence radiate at the cloud-top temperature, and progressively disperse for planets receiving greater incident flux. Phase-curve observations at a greater range of wavelengths are crucial to determining the extent of cloud coverage, as well as the cloud composition on hot Jupiter nightsides.

We collected published full-orbit, infrared phase curves for 12 hot Jupiters: CoRoT-2b, HAT-P-7b, HD 149026b, HD 189733b, HD 209458b, WASP-12b, WASP-14b, WASP-18b, WASP-19b, WASP-33b, WASP-43b, and WASP-103b. We also included the brown dwarf KELT-1b. We calculated the nightside brightness temperatures from the phase-curve parameters, and used Gaussian process regression to estimate each planet’s bolometric flux, and subsequently its disk-integrated nightside effective temperature. Several of the published phase-curve fits imply negative nightside disk-integrated flux, which is unphysical, because it means that the planets have negative brightness at some longitudes on their surface. We explain how we handled these cases in the Methods. Future phase-curve observations should be fit with the constraint that flux is non-negative everywhere on the planet. We also inferred nightside temperatures by considering and modifying negative brightness maps, which is similar in spirit to demanding positive phase curves and brightness maps when fitting the data. The mapping approach yielded a nightside temperature trend consistent with that of the disk-integrated approach.

In Fig. 1, we show the dayside and nightside effective temperatures plotted against the stellar irradiation temperature, \( T_0 \), where \( T \) is the nightside effective temperature, \( R \) is the stellar radius, and \( a \) is the semi-major axis. The nightside temperatures are all around 1,100 K and exhibit a slight upward trend with stellar irradiation. We tabulate the dayside effective temperature, nightside effective temperature, Bond albedo and heat recirculation efficiency for each planet in Table 1. While this paper was under review, a similarly flat trend for nightside brightness temperatures was reported.

Various theories have suggested that reradiation, advection, wave propagation, molecular dissociation, Coriolis forces, and magnetic drag could all have a role in atmospheric circulation on hot Jupiters. Models predict that the amount of day–night heat recirculation depends sensitively on planetary properties and the amount of stellar irradiation each planet receives, which vary between individual hot Jupiters.

We fit the nightside temperatures using two models of atmospheric heat transport. The qualitative behavior of each model is shown in Supplementary Figs. 5 and 6. The first model is a semi-analytic energy balance model incorporating atmospheric radiation and advection, and predicts nightside temperatures given the planetary and stellar properties. We fit for two parameters: a common wind velocity, and \( P/g \) (the characteristic pressure scaled by acceleration due to gravity), where \( P/g \) is the mass per unit area of the active layer of the atmosphere, that is, the layer that responds to instellation. The model was updated recently to include the effects of hydrogen dissociation and recombinination, by solving the Saha equation to determine the amount of hydrogen dissociated at a given atmospheric temperature and the resulting heat sinks and sources. Hydrogen dissociation and recombinination is predicted to significantly increase heat transport in ultrahot Jupiters.

The second model is an analytic, dynamical model incorporating radiation, advection, magnetic drag, Coriolis forces and gravity waves. This model predicts—rather than prescribes—the wind velocities for each hot Jupiter, and was shown to qualitatively match predictions of day–night temperature contrast from general circulation models. The model was recently updated to include the effects of hydrogen dissociation, albeit in a greatly simplified form. As the magnetic field strengths of hot Jupiters are unknown to orders of magnitude, we chose to neglect magnetic drag, but note that magnetic drag can potentially depress day–night heat transport for planets with very strong magnetic fields (~100 G). We fit for a universal \( P/g \), hence this model has only one fit parameter.

For each model fit, we performed a grid search in parameter space to find the parameters that minimize \( \chi^2 \). Models that allow for hydrogen dissociation provide better fits to the data than those without, even though this does not increase the number of parameters. The best-fit model predictions can be seen in Fig. 2. The semi-analytic energy balance model incorporating hydrogen dissociation yielded the best fit of all the models we considered. In the context of the energy balance model, the trend in observed nightside temperatures suggests that all hot Jupiters have similar wind velocities, contrary to predictions.
Fig. 1 | Dayside and nightside effective temperatures for 12 hot Jupiters, and one brown dwarf (KELT-1b). Top: dayside temperatures for the hot Jupiters in our analysis are proportional to the planets’ irradiation temperatures, \( T_{\text{eq}} \equiv T_{\star} \sqrt{R_{\star}/a} \), where \( T_{\star} \) is the stellar effective temperature, \( R_{\star} \) is the stellar radius, and \( a \) is the semi-major axis. The error bars correspond to the 1σ confidence intervals. To guide the eye, we plot the equilibrium temperature \( T_{\text{eq}} \). Bottom: nightside effective temperatures. The error bars correspond to the 1σ confidence intervals. Nightside temperatures are all around 1,100 K with a slight upward trend.

Alternatively, dynamical predictions may be correct, but ultimately overshadowed by optically thick nightside clouds. Clouds are predicted to be present on the nightsides of all hot Jupiters25–27, but the cloud composition depends on the temperature, pressure, and cloud formation physics. Observationally, nightside clouds have been previously invoked to explain non-detections of nightside flux13,16,28. The nightside temperature trend—or lack thereof—implies that the hot Jupiters in our study all have nightside clouds that emit at similar temperatures. Vertical mixing sets the cloud-top pressure, so in principle, we could be seeing cloud tops from different cloud species that all happen to have similar vertical cloud-top temperatures.

A simpler explanation is that hot Jupiters all have the same species of nightside clouds, which condense at a similar cloud-base temperature. The emitting temperature corresponds to the cloud-top temperature, which would be slightly cooler than the condensation temperature. These clouds would emit thermal radiation around the same effective temperature, and block outgoing longwave radiation from below, requiring clouds with large grains. Potential cloud species include manganese sulfide or silicate clouds, based on condensation curves25. As we show in Fig. 3, the nightside infrared colours are roughly isothermal. The similarity of the brightness temperature between Spitzer bandpasses implies that they are probing parts of the atmosphere with similar temperatures, consistent with optically thick clouds.

Incorporating radiative feedback and detailed cloud microphysics is computationally intensive, which is the reason many studies have used cloud-free general circulation models, and post-processed clouds after forward-suspending the resulting temperature–pressure profiles and cloud condensation curves. However, post-processing of exoplanet clouds can lead to different predictions of cloud coverage, phase offsets and day–night temperature contrasts than more intricate models26,27. Fully three-dimensional models incorporating realistic cloud physics and heat transport due to hydrogen chemistry are clearly needed to properly understand hot Jupiters spanning the full range of irradiation temperatures. Realistic treatments of magnetic effects may be necessary for the hottest planets24,29. On the observational front, spectroscopic phase-curve observations at longer wavelengths, with the Mid-InfraRed Instrument onboard the James Webb Space Telescope and with the Atmospheric Remote-sensing Infrared Exoplanet Large-survey, will make it possible to characterize the dominant cloud species on hot Jupiter nightsides.

Methods
We estimated nightside effective temperatures with two different methods. We outline the methods in the sections that follow.

Method 1. Disk-integrated flux. Our fiducial analysis used the disk-integrated flux, from phase curves, to estimate effective temperatures. Previous efforts have
used weighted averages or linear interpolation\(^{31,32}\). We used Gaussian process regression, to estimate the bolometric flux, and subsequently effective temperature and uncertainty given a handful of brightness temperatures, as it has recently been shown to produce more accurate uncertainty estimates (E. Pass et al., manuscript in preparation).

The disk-integrated nightside brightness temperature is given by

\[
T_{b,\text{nightside}}(\lambda) = \frac{hc}{k^2} \left[ \ln \left( 1 + \frac{I(\lambda)}{F_{\text{nightside}}} \right) \right]^{-1}
\]

where \( h \) is Planck’s constant, \( c \) is the speed of light, \( k \) is the Boltzmann constant, \( \lambda \) is the wavelength of the observation, \( T_b \) is the brightness temperature of the star at that wavelength, \( \Delta \theta \) is the transit depth and \( F_{\text{nightside}} \) is the planet-to-star flux ratio at a phase angle of \( \pi \), where phase angle is defined to be 0 at secondary eclipse.

Propagation of uncertainties. As the most common Spitzer decorrelation techniques have been shown to produce accurate, reproduced results\(^{33}\), we chose to take all positive phase curves at face value. To estimate uncertainties on each planet’s disk-integrated nightside flux and brightness temperature, we propagated uncertainties on planetary and stellar properties, and phase-curve parameters, using a 1,000-step Monte Carlo. The relevant physical properties are: the stellar effective temperature, stellar surface gravity, stellar metallicity, transit depth and ratio of semi-major axis to stellar radius. We took the most up-to-date values from the literature. For each draw, we randomly sampled each parameter from a Gaussian centred on each best-fit published value, with the width given by the published uncertainty. This gives an approximately Gaussian probability density function for the nightside flux \( \text{Prob}(F_{\text{nightside}}) = \frac{1}{F_{\text{nightside}}} \frac{dF_{\text{nightside}}}{dT_{b,\text{nightside}}} \), where \( F_b \) is the nightside flux.

To calculate the brightness temperature for each nightside flux value, we inverted the Planck function for each flux to obtain a probability density function for nightside brightness temperature, \( T_{b,\text{nightside}} \). This can be thought of as transforming the nightside flux probability density function to a function of nightside temperature through a change of variables. We have

\[
\text{Prob}(T_{b,\text{nightside}}) = g(F_b(T_{b,\text{nightside}})) \frac{dF_n}{dT_{b,\text{nightside}}}
\]

where \( F_b(T_{b,\text{nightside}}) \) is the Planck function (as a function of temperature, holding wavelength fixed), and \( \frac{dF_n}{dT_{b,\text{nightside}}} \) is the derivative of the Planck function with respect to temperature. This transformation is only defined for positive fluxes and temperatures.

For most of the planets in our study, the nightside flux distribution is well above zero. The nightside temperature probability distribution also has a Gaussian-like shape, so we used the peak and width for our best fit and uncertainty values. We took the average of the upper and lower limits when using the brightness temperature to infer effective temperatures.

For planets with low or negative nightside flux, parts of the nightside flux probability distribution do not correspond to physical temperatures. This is typically interpreted to be a strong non-detection of nightside flux. Mathematically this is allowed, but physically, negative fluxes and temperatures are impossible. An example is HAT-P-7b at 3.6\( \mu \)m, where the peak of the probability density function Prob(\( F \)) is negative. In this case, we set the best-fit flux, and hence brightness temperature, to zero, and used the width of the nightside flux distribution to calculate a 1σ upper limit on the brightness temperature, which we used as the error when estimating the bolometric flux. For planets with small but non-zero nightside flux (such as WASP-18b at 3.6\( \mu \)m), a significant part of the flux distribution is negative, and the lower part gets truncated when converting to errors in brightness temperature. In these cases, we used the upper limit on brightness temperature when estimating bolometric flux and effective temperature, which is more conservative than taking the average of the upper and lower limits.

**Brighter temperature difference plot.** To generate Fig. 3, we used a 1,000-step Monte Carlo. For each step in the Monte Carlo, we calculated the difference between the 3.6\( \mu \)m and 4.5\( \mu \)m brightness temperatures. We took the mean and standard deviation of the distribution of differences for each planet.

**Method 2. Mapping method.** We also calculated dayside and nightside temperatures by considering the brightness maps implied by each phase curve. For a planet on a circular, edge-on orbit, its orbital phase curve can be analytically inverted into a longitudinal brightness map\(^{35,36}\). WASP-14b has the highest eccentricity of the sample, \( e = 0.08 \). General circulation models using a small eccentricity (\( e = 0.15 \)) predict negligible differences in circulation patterns compared with circular orbits\(^{37,38}\). For our purposes, we treated the orbits of WASP-14b and the lower eccentricity planets in our sample as circular. We defined the phase curves to be \( f(\xi) \), where \( \xi \) is the planet’s phase angle (\( \xi = 0 \) at secondary eclipse, \( \xi = \pi \) at transit). The corresponding brightness maps are defined as \( f(\phi) \), where \( \phi \) is longitude from the substellar point. We set \( f(\xi = 0) = 0 \) equal to the eclipse depth, and obtained the map parameters analytically\(^{39}\).

Phase curve provides weak constraints on north–south asymmetry of planets\(^{40,41}\). It is possible to determine the latitudinal distribution using eclipse mapping, but so far this has only been done for HD 189733b at 8\( \mu \)m (refs. 42, 43). We therefore marginalize over the uncertainty in latitudinal brightness distributions when constructing the two-dimensional bolometric flux maps.

From the bolometric flux maps for the 12 planets, we obtained an estimate of the dayside and nightside effective temperatures of each planet.

**Luminosity brightness profiles.** Longitudinal maps, \( f(\phi) \), are weighted by the visibility of the phase observer, since the phase curve measures the disk-integrated flux from the planet. For an equatorial observer (a zero-obliquity planet orbiting edge-on), the longitudinal maps are related to the two-dimensional brightness distribution as a function of planetary co-latitude and longitude, \( I(\theta, \phi) \), by

\[
I(\phi) = \int f(\phi, \theta) \sin^2 \theta \, d\theta
\]

One of the powers of sine comes from the area element in spherical coordinates, and the other comes from the visibility for an equatorial observer. The longitudinal
Fig. 3 | Difference in brightness temperatures at Spitzer wavelengths 3.6 μm and 4.5 μm (ch1 and ch2) for the ten planets with both 3.6 μm and 4.5 μm phase curves. The error bars correspond to the 1σ confidence intervals. Top: dayside brightness temperature colours. Cooler planets have blue Infrared Array Camera (IRAC) colours (consistent with H₂O absorption), while hotter planets are isothermal or slightly red. Ch1 and ch2 refer to channel 1 and channel 2 of IRAC, respectively. Bottom: nightside brightness temperature colours. Half of the planets have nightside brightness temperature colours consistent with zero, meaning their nightside brightness temperatures are similar at both wavelengths. The rest are within 2σ of zero.

map f(θ) effectively integrates over the latitudinal dependence of I(θ, ϕ). We adopted the simplifying assumption that I(θ, ϕ) is separable, and accounted for our ignorance of the latitudinal dependence of brightness by letting it vary as sinθ with a polar brightness I_pole. The exponent γ represents how the flux is distributed latitudinally—the higher the value of γ, the more the planetary flux is concentrated at the equator compared to the poles. The expression is

\[ I_\gamma(\theta, \phi) = \left( \frac{I_p(\phi) - \pi I_pole/2}{I_p \sin^{\gamma+1}(\theta)} \right) \sin^\gamma(\theta) + I_pole(1 - \gamma) \]  

where \( I_pole \) is a constant representing the intensity at the poles. The full derivation can be found at the end of the Methods. The γ = 0 case represents perfect poleward heat transport, or a constant temperature in the latitudinal direction. In the Rayleigh–Jeans limit of long wavelength, \( I(\theta) \propto T(\theta) \), and thus \( I(\theta) \propto T(\theta) \propto \sin^{1/4}(\theta) \) for no poleward heat transport, that is, \( γ = 1/4 \). To be conservative, we drew samples from the range 0 < γ ≤ 1, as we find that the value of γ doesn’t drastically affect our calculated quantities. In Supplementary Fig. 8, we show how the value of γ changes the latitudinal brightness profile.

The brightness temperature map is related to the intensity map by the inverse Planck function

\[ T_\gamma(\theta, \phi) = \frac{h_c}{\Delta k} \left( \ln \left( 1 + \frac{(\sin^\gamma(\theta) - 1)R_p}{\alpha T(\theta)} \right) \right)^{-1} \]  

where \( T(\theta) \) is the brightness temperature from Phoenix stellar models⁴⁴ and \( R_p \) is the radius of the planet.

**Brightness temperatures to effective temperatures.** From the wavelength-dependent brightness maps in equation (5), we inferred effective temperature maps. If the full spectrum at each location was known, one could integrate it to get the effective temperature at each location. Instead, we must estimate the bolometric flux by interpolating between, and extrapolating from, a few brightness temperatures. The Gaussian process regression used for the disk-integrated analysis is too computationally expensive to use at each location on the planet. We instead approximated the effective temperature via the error weighted mean of the brightness temperatures (or, in practice, the arithmetic mean of brightness temperatures, but embedded in a Monte Carlo). We adopted systematic uncertainties calibrated by performing such estimates on synthetic spectra (E. Parry et al., manuscript in preparation). We took the arithmetic mean of the individual brightness temperatures at each location as an estimate of the effective temperature

\[ T_{\text{eff}}(\theta, \phi) = \frac{1}{n} \sum_{n} T_{\text{ch}, n}(\theta, \phi) \]  

where \( n \) is the number of wavelengths. We propagated errors in a Monte Carlo fashion. From \( T_{\text{ch}, n}(\theta, \phi) \) we calculated the disk-integrated dayside and nightside effective temperatures, \( T_{\text{day}} \) and \( T_{\text{nigh}} \), using the Stefan–Boltzmann law

\[ T_{\text{day}} = \frac{1}{2k} \int_{-\pi/2}^{\pi/2} \int_{0}^{\pi} T_{\text{eff}}(\theta, \phi) \sin \theta d\phi d\theta \]  

and

\[ T_{\text{nigh}} = \frac{1}{2k} \int_{-\pi/2}^{\pi/2} \int_{0}^{\pi} T_{\text{eff}}(\theta, \phi) \sin \theta d\phi d\theta \]  

We also calculated the Bond albedo, the fraction of incoming stellar power that the planet reflects to space

\[ A_B \equiv 1 - \frac{\pi T_{\text{eff}}^4(\phi, \theta) \sin \theta d\phi d\theta}{\pi T_0^4} \]  

Since the nightside absorbs no stellar radiation, this is the amount of heat that has moved from the dayside to nightside. A value of zero implies that no heat is transported to the nightside, and a value of 0.5 implies that half of the absorbed incoming stellar flux is recirculated to the nightside.

For each planet, we calculated \( T_{\text{day} \rightarrow \text{nigh}} A_B \) and \( P \) simultaneously for 10⁶ steps of a Monte Carlo. At each step, we randomly drew all measured physical planetary parameters and published phase-curve parameters from Gaussian distributions centred around their published values, with standard deviation given by their published uncertainties. We marginalized over our uncertainty of \( γ \) and \( I_pole \) by drawing these from a random uniform distribution where 0 ≤ γ ≤ 1 and 0 ≤ I_pole ≤ 20I_pole. The constraint on \( I_pole \) ensures that the poles are not hotter than the equator. Lastly, we added the systematic uncertainty associated with estimating effective temperatures using the mean of a limited number of brightness temperatures; the systematic uncertainties are estimated based on retrieval exercises with synthetic cloud-free dayside emission spectra. The 1σ systematic uncertainties are 23% for planets with a phase curve at just 4.5 μm, 13% for planets with phase curves at 3.6 μm and 4.5 μm, and 3% for planets with phase curves at 3.6 μm, 4.5μm and 1.4μm. This is a conservative estimate, as the observed nightside brightness temperatures are closer to isothermal than the dayside brightness temperatures (Fig. 3).

**Phase curves and brightness maps.** The first exoplanet map was of HD 189733b at 8 μm (ref. ⁴⁷). It showed an eastward-shifted hotspot on the planet, in line with theoretical predictions of equatorial, super-rotating jets. With the exception of HD 189733b⁴⁷, 55 Cancri e⁴⁸, CoRoT-2b⁴⁹, WASP-43b⁵⁰, WASP-103b⁵¹ and KELT-1b⁵², most phase curves have been fit and published without considering the brightness maps that could have produced them.

We distinguish between two problematic cases: negative phase curves, and positive phase curves that imply negative brightness maps, and explain how we handle these problematic cases. We summarize the suite of phase curves in Supplementary Table 3.

**Negative phase curves.** In a recent analysis of the phase curves of the brown dwarf KELT-1b, the authors suggest that negative phase curves can be fixed by adopting...
a non-sinusoidal brightness map\(^{20}\). However, a phase curve that is negative at any value of orbital phase guarantees that the underlying brightness distribution (map) is negative at some longitudes, because a phase curve measures the disk-integrated flux. Changing the functional form of the map does not change this fact. Every phase curve that goes negative at any point implies the planet has negative flux somewhere, regardless of how the map looks.

The planets HAT-P-7b, WASP-14b and WASP-43b have published phase curves that are negative on their nightsides, which ensures unpolar, negative brightness maps. By inverting the phase curve without redefining it, we obtain a brightness map that is always non-negative. To be physically possible, the brightness map must instead have non-sinusoidal maps. As we have shown, for some planets with first- and second-order sinusoids\(^{14}\). However, the published 3.6 μm phase curves for these planets are not unique, and they cannot have a negative brightness map at this wavelength. For each draw in the section ‘Positive phase curves but negative brightness maps’ . For HAT-P-7b, we were not able to obtain a good fit without allowing the phase curve to have significantly negative nightside flux. As the phase curve is strictly negative, this could potentially be due to some unmodelled stellar effect, such as non-uniform stellar brightness, or that the dock models are inadequate for these particular data. For the purposes of this study, we chose to treat the negative nightside flux as an upper limit when estimating the effective temperature.

Positive phase curves but negative brightness maps. It is also possible to measure a strictly positive phase curve, yet infer a brightness map that is not strictly positive, and this is the case for WASP-12b, WASP-18b and WASP-103b. Although it is mathematically possible to obtain a non-negative phase curve from a brightness map that is not strictly positive, such a brightness map is physically impossible. This was pointed out long ago in the case of reflected light curves from asteroids\(^{20}\), which is mathematically similar to the thermal emission case. Brightness maps obtained from inverting sinusoidal phase curves are not unique, as there is a nullspace of the transformation from map to light curve—excluding the fundamental mode, any odd sinusoidal mode present in the brightness map of a synchronously rotating planet on a circular, edge-on orbit will integrate to zero over a hemisphere, and will thus be invisible in the phase curves\(^{20}\). If a measured phase curve implies a negative brightness map, then it may be possible to add higher-order harmonics to correct the map—indeed, if a solution exists, then odd brightness map harmonics are necessary to ensure a physically possible solution. For example, WASP-18b has strictly positive phase curves that were fit with first- and second-order sinusoids\(^{14}\). However, the published 3.6μm phase-curve parameters imply a negative brightness map at this wavelength. For each draw in our Monte Carlo, if the phase curve is positive but the brightness map is negative at any location for any planet, we numerically solve for the smallest amplitude third-order harmonic that makes the brightness map non-negative. If no such solution exists, we reject the draw. We demonstrate this in Supplementary Fig. 4.

The brown dwarf KELT-1b also has positive phase curves that imply negative brightness maps\(^{20}\). The authors showed that a smoothed trapezoidal brightness map integrates to give an approximately sinusoidal phase curve close to their fiducial phase curve for KELT-1b, and conclude that KELT-1b’s map must be non-sinusoidal. They argue that this solves the problem of negative brightness maps, and implies that all planets with seemingly negative sinusoidal brightness maps must instead have non-sinusoidal maps. As we have shown, for some planets with positive phase curves, the brightness map can be made non-negative by adding the third harmonic to the brightness map. In fact, adding higher-order sinusoids allows for trapezoidal temperature maps, or any other continuous function (in other words, Fourier analysis). The odd harmonic method is elegant and more robust than adopting a specific non-sinusoidal parameterization, and does not alter the phase curve. It may be necessary to fit for the odd map harmonics when fitting phase curves—even though they are not visible in the phase curve, they may be needed to ensure a physically possible map.

Lastly, the phase curves of WASP-12b are contentious—if the fiducial, polynomial fit for the 4.5μm phase curve is taken to be solely due to brightness variations of a spherical planet, the map is negative and unpolar\(^{20}\). The authors note that part of the second harmonic could be due to ellipsoidal variations. Reanalysis of the same data, and a second set of phase observation at the same wavelengths, found that the 4.5μm results were consistent with the previous results\(^{16,17}\). We adopt the interpretation ultimately chosen by the authors of the first paper: some of the second harmonic in the 4.5μm phase curve is due to the planet’s inhomogeneous temperature map, but the rest is due to ellipsoidal variations. To be consistent with their interpretation, we set the planet’s aspect ratio to 1:5, and calculated the resulting amplitude\(^{16}\) and subtracted it from the second-order amplitude to yield a non-negative brightness map.

Dynamical model. The radiative timescale in the analytic, dynamical model is scaled by pressure at the base of the radiatively active layer of the atmosphere, and equilibrium temperature\(^{20,21}\). Supplementary Fig. 5 shows a version of the model where all the planets have the same physical properties, but the radiation temperature varies.

We updated the radiative timescale formulation to scale with $P_0g$, as with the energy balance model. We neglected magnetic drag. Magnetic drag can decrease nightside temperature more significantly than previously reported\(^{16,17}\). The Spitzer phase curves for WASP-43b were refit by using a different instrument sensitivity model, shown to be better at removing residual red noise due to intra-pixel sensitivity, also resulting in much higher nightside temperatures\(^{20}\). We use the reanalysed Spitzer phase curves for our analysis. For the WFC3 phase curve, we treated the negative nightside flux as an upper limit, rather than simply setting the negative parts of the map to zero. We did not use the reanalysed WFC3 phase curves as they were not fit with sinuosoids, and hence could not be treated in a consistent manner to the other phase curves.

The published HAT-P-7b and WASP-14b 3.6μm phase curves are negative on their nightsides. This can occur when not enforcing positive brightness maps when fitting the data. We refit both phase curves by enforcing physically possible phase values, and the polynomial function to which it is fit. For WASP-14b, we were able to obtain a good fit. The nightside temperature we infer is 4K lower when using the Monte Carlo rejection method, described in the section ‘Positive phase curves but negative brightness maps’. For HAT-P-7b, we were not able to obtain a good fit without allowing the phase curve to have significantly negative nightside flux. As the planet cannot have a non-negative brightness, this could potentially be due to some unmodelled stellar effect, such as non-uniform stellar brightness, or that the dock models are inadequate for these particular data. For the purposes of this study, we chose to treat the negative nightside flux as an upper limit when estimating the effective temperature.
The longitudinal brightness maps in equation (14) are related to the brightness maps as a function of latitude and longitude, $I(\phi, \theta)$, by

$$I(\phi) = \int_0^\pi I(\phi, \theta) \sin^2 \theta d\theta$$

(15)

where the integral over latitude is weighted by $\sin \theta$—the visibility of an equatorial observer.

Given just a phase curve, it is impossible to infer the latitudinal dependence of the corresponding brightness map. In other words, given $I(\phi)$, there are multiple possible $I(\theta, \phi)$. However, the map must be continuous at the poles. A reasonable parameterization is a sinusoidal latitudinal dependence. This would cause the planetary flux to be greatest at the equator, where the planet receives the most stellar flux, and drop to zero (or a constant) at the poles. This brightness map takes the form

$$I(\phi, \theta) = (I_{eq}(\phi) - I_{pole}) \sin^2 \theta + I_{pole}$$

(16)

where $\gamma > 0$ and $I_{eq}$ is a constant with units of intensity, and $I_{eq}(\phi) = \Pi(\phi, \pi/2)$ is the brightness profile along the equator of the planet. Higher values of $\gamma$ give lower values of flux for a given infinitesimal longitudinal slice of the planet. This essentially amounts to the integral of $I^2 d\theta$, which decreases as $\gamma$ increases.

We can constrain $I_{pole}$ by noting that $I(\phi, \theta)$ must be positive, and that the poles shouldn’t be brighter than the equator. This yields $0 \leq I_{pole} \leq I_{eq}(\phi)$.

We observe $I(\phi)$, but need $I(\phi, \theta)$ to properly estimate the planet’s energy budget. Combining equation (16) with equation (15) yields

$$I(\phi) = (I_{eq}(\phi) - I_{pole}) \int_0^{\pi/2} \sin^2 \theta d\theta + I_{pole}$$

Rearranging this gives

$$I_{eq}(\phi) = \frac{I(\phi) - \sin I_{pole}/2}{\int \sin^2 \theta d\theta} + I_{pole}$$

(18)

or

$$I(\phi, \theta) = \frac{I(\phi) - \sin I_{pole}/2}{\int \sin^2 \theta d\theta} \sin^2 \theta + I_{pole}(1 - \sin^2 \theta)$$

(19)

These expressions give the planet’s brightness distribution given some observed longitudinal map $I(\phi)$, for an assumed latitudinal dependence parameterized by $\gamma$ and $I_{pole}$. Given an inferred $I(\phi)$, greater $I_{pole}$ means a greater $I_{eq}(\phi)$, and greater $\gamma$ means a greater $I_{pole}$. See Supplementary Fig. 8 for an illustration.

For example, if we assume a brightness map where $\gamma = 1$ and $I_{pole} = 0$, we obtain

$$I_{eq}(\phi) = \frac{3I(\phi)}{4}$$

(20)

If the effective temperature of the host star and the transit depth are known, then by combining equation (11) and the equatorial brightness map of equation (20), it is possible to invert the Planck function and obtain a longitudinal temperature map for a planet. For the above example we get:

$$B_\lambda(T_\gamma, \phi) = \frac{3\pi(\phi)B_\lambda(T_\gamma)}{4K(R_\gamma R_\gamma^3)}$$

(21)

where we have multiplied by $\pi$ to account for the fact that we are now comparing the flux from infinitesimal, longitudinal slices on the planet to the entire flux of the star. We get an expression for the brightness temperature of a planet, $T_\gamma$, as a function of latitude:

$$T_\gamma(\phi) = \frac{hC}{4K} \left[ \ln \left( 1 + \frac{(\phi/\kappa)^2 - 1}{\ln(R_\gamma/R_\gamma^2)^2} \right) \right]^{-1}$$

(22)

The general expression is:

$$T_\gamma(\phi, \theta) = \frac{hC}{4K} \left[ \ln \left( 1 + \frac{(\phi/\kappa)^2 - 1}{\ln(R_\gamma/R_\gamma^2)^2} \right) \right]^{-1}$$

(23)

with $I(\phi, \theta)$ given by equation (19).

Bolometric flux maps. Applying the above technique to a single phase curve yields a brightness temperature map for a single wavelength. Brightness temperature maps can be combined to infer an effective temperature map of the planet. Taking the arithmetic mean of the brightness temperatures at each point on the planet gives an estimate of the effective temperature at each point.

Using the Stefan–Boltzmann law with this effective temperature map gives the flux of the planet at each longitude and latitude

$$F_{bol} = \sigma_{SB} T^4_{eff}(\phi, \theta)$$

(24)

and integrating this over each infinitesimal unit of area on the planet yields the total power radiated by the planet

$$P_{bol} = \sigma_{SB} \frac{1}{4} T^4_{eff}(\phi, \theta) R_\gamma^2 \sin \theta d\theta d\phi$$

(25)

where $\sigma_{SB}$ is the Stefan–Boltzmann constant.

The stellar flux that arrives at the planet is given by

$$F_{stellar} = \sigma_{SB} T^4_{eff} \pi \gamma$$

(26)

Dividing equation (25) by equation (26) gives the amount of incoming radiation absorbed by the planet. Subtracting this from unity and simplifying yields an expression for—the definition of—the planet’s Bond albedo:

$$A_B \equiv 1 - \frac{P_{bol}}{P_{stellar}}$$

(27)

To infer the day–night heat recirculation, we calculate the ratio of the power radiated by the nightside of a planet to the total power radiated

$$P = \frac{P_{bol, night}}{P_{bol}}$$

(28)

Note that $R_\gamma^2$ appears in the numerator and denominator of both equations (27) and (28), and hence doesn’t affect the energy budget.

Negative brightness maps. A negative phase curve ensures that a planet’s brightness map must be negative at some longitude, as the phase curve is obtained by simply integrating over the brightness map that is visible at each value of orbital phase. But even a strictly positive phase curve may require negative map values at certain locations on the planet.

We can think of the sinusoidal components of phase curves in terms of a sum of odd and even harmonics:

$$F = F_{odd} + F_{even}$$

(29)

where $F(\phi) = \sum_{n=1}^{\infty} C_n \cos(n \phi) + D_n \sin(n \phi)$. The sum of the odd harmonics is given by $F_{odd}$, and likewise the sum of the even harmonics is given by $F_{even}$. For a planet viewed equator-on ($\phi = 90^\circ$), all of the odd harmonics above the first harmonic, $n = 1$, are invisible in the measured light curve. If we let $F_1$ denote the first harmonic, we get:

$$F = F_1 + F_{even}$$

(30)

Analogously we can represent the longitudinal brightness maps as

$$I(\phi) = I_{odd}(\phi) + I_{even}(\phi)$$

(31)

where $I(\phi) = \sum_{n=1}^{\infty} A_n \cos(n \phi) + B_n \sin(n \phi)$. We can obtain a brightness map by inverting a phase curve, but the solution will not be unique. It will be $I(\phi) = I_{odd}(\phi) + I_{even}(\phi)$. The brightness maps may have any number of odd harmonics, as long as the map remains non-negative, since odd harmonics greater than $n = 1$ are in the nullspace of the transformation from brightness map space to phase curve space in equation (5). They may be present in the map, but integrate to zero in the phase curve. If a planet has an inclination less than 90° and north–south asymmetry or a time-variable map, or just an eccentric orbit, then the light curves can exhibit odd harmonics too.

If a positive phase curve is inverted using the measured sinusoidal modes and the brightness map is negative at any longitude, then it may be possible to add enough odd harmonics to obtain a positive brightness map. If this is the case, then it implies that the planet’s brightness map has odd harmonics, which by their very nature are invisible in the phase curve of the planet: so-called latent variables.

We make two recommendations of fitting conditions for future phase curves: (1) the phase curves themselves must be positive and (2) the corresponding brightness maps must be positive. For the second condition, it may be necessary to fit for the latent variables, odd brightness map harmonics, to achieve positive brightness maps. These latent variables should count when estimating, for example, the Bayesian information criterion to compare models.

Data availability

The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

Code availability

The Gaussian process regression code used is publicly available and can be found at https://github.com/ekpass/gp-telf. The Spitzer Phase Curve Analysis pipeline is publicly available and can be found at https://github.com/lissadang27/SPCA.
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Author contributions
D.K. led the data analysis and wrote the manuscript. N.B.C. discussed ideas and contributed to writing the manuscript. I.D. provided the Spitzer data analysis pipeline and helped with reducing Spitzer phase curves.

Competing interests
The authors declare no competing interests.

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