Electroweak vacuum stability in classically conformal $B - L$ extension of the standard model

Arindam Das$^a$, Nobuchika Okada$^b$, Nathan Papapietro$^c$

Department of Physics and Astronomy, University of Alabama, Tuscaloosa, Alabama 35487, USA

Received: 9 February 2016 / Accepted: 8 February 2017 / Published online: 23 February 2017
© The Author(s) 2017. This article is published with open access at Springerlink.com

Abstract We consider the minimal $U(1)_{B-L}$ extension of the standard model (SM) with the classically conformal invariance, where an anomaly-free $U(1)_{B-L}$ gauge symmetry is introduced along with three generations of right-handed neutrinos and a $U(1)_{B-L}$ Higgs field. Because of the classically conformal symmetry, all dimensional parameters are forbidden. The $B - L$ gauge symmetry is radiatively broken through the Coleman–Weinberg mechanism, generating the mass for the $U(1)_{B-L}$ gauge boson ($Z'$ boson) and the right-handed neutrinos. Through a small negative coupling between the SM Higgs doublet and the $B - L$ Higgs field, the negative mass term for the SM Higgs doublet is generated and the electroweak symmetry is broken. In this model context, we investigate the electroweak vacuum instability problem in the SM. It is well known that in the classically conformal $U(1)_{B-L}$ extension of the SM, the electroweak vacuum remains unstable in the renormalization group analysis at the one-loop level. In this paper, we extend the analysis to the two-loop level, and perform parameter scans. We identify a parameter region which not only solve the vacuum instability problem, but also satisfy the recent ATLAS and CMS bounds from search for $Z'$ boson resonance at the LHC Run-2. Considering self-energy corrections to the SM Higgs doublet through the right-handed neutrinos and the $Z'$ boson, we derive the naturalness bound on the model parameters to realize the electroweak scale without fine-tunings.

1 Introduction

The stability of the electroweak scale is one of the biggest mysteries in the standard model (SM), since the self-energy of the SM Higgs doublet field receives quantum corrections which are quadratically sensitive to the ultraviolet cutoff of the SM. A fine-tuning of the Higgs mass parameter is required to reproduce the correct electroweak scale if the ultraviolet cutoff scale is far above the electroweak scale (the gauge hierarchy problem). This problem can be solved if new physics beyond the SM makes the self-energy of the SM Higgs doublet insensitive (or logarithmically sensitive) to the ultraviolet cutoff. It is well known that supersymmetric extension of the SM can achieve this insensitivity. Despite lots of efforts of searching for supersymmetry at the large hadron collider (LHC) experiments, current LHC data include less indications for productions of supersymmetric particles. Hence we may seek other possibilities to solve the gauge hierarchy problem without supersymmetry.

According to the argument by Bardeen [1] once the classical conformal invariance and its minimal violation by quantum anomalies are imposed on the SM (or the general Higgs model), the model can be logarithmically sensitive to the ultraviolet cutoff. If this is the case, we introduce the classically conformal symmetry to the SM to make the model free from the quadratic corrections. In this system, there is no mass parameter in the original Lagrangian, and the mass scale must be generated by quantum corrections. The massless $U(1)$ Higgs model discussed by Coleman and Weinberg [3] nicely fits this picture, where the model is defined as a massless, conformal invariant theory, and the $U(1)$ gauge symmetry is radiatively broken by the Coleman–Weinberg (CW) mechanism, generating a mass scale through the dimensional transmutation.

Recently, the extension of the SM with the classically conformal invariance has received a fair amount of attention, and

\[^{1}\text{In terms of the ultraviolet completion, one may consider a conformal model into which the SM is embedded. Based on a toy model, it has been shown in [2] that the SM Higgs mass is sensitive to a scale at which the SM merges into a conformal field theory. Since conformal field theories in four dimensions have not yet been completely understood, it is highly non-trivial to verify if this sensitivity is inevitable. Hence, we leave this issue in this paper and assume that the SM Higgs does not receive quadratic corrections to its self-energy.}\]

$^a$e-mail: adas8@ua.edu
$^b$e-mail: okadan@ua.edu
$^c$e-mail: npapapietro@ua.edu
many models in this direction have been proposed [4–27]. Among them, the classically conformal U(1)$_{B-L}$ extension of the SM [28, 29] is a very simple and well-motivated model, since the $B - L$ (baryon number minus lepton number) is a unique anomaly-free global symmetry and it can easily be gauged. Once the U(1)$_{B-L}$ is gauged, we need new chiral fermions to cancel the U(1)$_{B-L}$ gauge and the mixed gravitational anomalies. The simplest possibility is to introduce three right-handed neutrinos, which are nothing but the particles that we need to incorporate the neutrino mass in the SM. In this conformal symmetric model, the $B - L$ gauge symmetry is broken by the vacuum expectation value (VEV) of the $B - L$ Higgs field developed by the CW mechanism, and the masses for $Z'$ boson and three right-handed neutrinos are generated. This radiative $B - L$ gauge symmetry breaking is the sole origin of the mass scale in this model, and the negative mass squared for the SM Higgs doublet is generated by this symmetry breaking [28].

The SM Higgs boson was finally discovered at the LHC, and the experimental confirmations of the Higgs properties in the SM have just begun. According to the SM, we can read off the value of the quartic Higgs coupling at the electroweak scale from the measured Higgs boson mass, and we can investigate the behavior of the Higgs potential toward high energies by extrapolating the quartic coupling through its renormalization group evolution. It turns out that the running quartic coupling becomes negative around $10^{10}$ GeV, which in turn generates a large mass term for the SM Higgs boson and the mixed gravitational anomalies. The simplest possibility is to introduce the right-handed neutrino $N^i_H$ (denoted with the generation index) and a complex scalar $\Phi$ are introduced in the SM Higgs doublet from the heavy states, the $Z'$ boson and the right-handed neutrinos associated with the $B - L$ symmetry breaking, and find naturalness bounds to reproduce the electroweak scale without any fine-tunings of model parameters.

This paper is organized as follows. Our model is defined in the next section. In Sect. 3, we discuss the radiative $B - L$ symmetry breaking through the CW mechanism and the electroweak symmetry breaking triggered by it. In Sect. 4, we analyze the renormalization group evolutions of the couplings at the two-loop level, and find a parameter regions which can keep the quartic SM Higgs coupling to be positive anywhere between the electroweak scale and the Planck scale. We also consider the current collider bounds of the model parameters, in particular, the recent ATLAS and CMS results of search for $Z'$ boson resonance at the LHC Run-2 are interpreted to our $B - L$ model. In Sect. 5, we evaluate self-energy corrections to the SM Higgs doublet from the heavy states, the $Z'$ boson and the right-handed neutrinos associated with the $B - L$ symmetry breaking, and find naturalness bounds to reproduce the electroweak scale without any fine-tunings for the model parameters. We summarize our results in Sect. 6. Formulas we used in our analysis are listed in the appendices.

### 2 Classically conformal U(1)$_{B-L}$ extended SM

We investigate the minimal U(1)$_{B-L}$ extension of the SM with the classically conformal invariance, where the model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. The particle contents of the model are listed in Table 1. In addition to the SM particle contents, the right-handed neutrino $N^i_H$ (denoted with the generation index) and a complex scalar $\Phi$ are introduced. In this conformal symmetric model, the $B - L$ gauge symmetry breaking scale from the LEP electroweak precision measurements, and a lower bound on the $Z'$ boson mass from the recent ATLAS [34] and CMS [35] results at the LHC Run-2. In addition, we evaluate self-energy corrections to the SM Higgs doublet from the heavy states, the $Z'$ boson and the right-handed neutrinos associated with the $B - L$ symmetry breaking, and find naturalness bounds to reproduce the electroweak scale without any fine-tunings of model parameters.

#### Table 1

| $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_{B-L}$ |
|-----------|-----------|----------|-------------|
| $q_L^i$   | 3         | 2        | 1/6         | 1/2         |
| $u_R^i$   | 3         | 1        | $-1/2$      | $1/2$       |
| $d_R^i$   | 3         | 2        | $-1/2$      | $1/2$       |
| $e_L^i$   | 1         | 2        | $-1/2$      | 0           |
| $e_R^i$   | 1         | 1        | 0           | 0           |
| $H$       | 1         | 2        | $-1/2$      | 0           |
| $\Phi$    | 1         | 1        | 0           | 2           |
| $N^i_H$   | 1         | 1        | 0           | $-1$        |

The particle contents of the U(1)$_{B-L}$ extended SM. In addition to the SM particle contents, the right-handed neutrino $N^i_H$ (denoted with the generation index) and a complex scalar $\Phi$ are introduced.
relevant to \( U(1)_Y \times U(1)_{B-L} \) is given by

\[
D_\mu = \partial_\mu - i (Q_\mu \cdot Q_{B L}) \left( \begin{pmatrix} g_1 & g_2 \\ g_2 & g_3 \end{pmatrix} B_\mu \right),
\]

where \( Q_\mu \) and \( Q_{B L} \) are \( U(1)_Y \) and \( U(1)_{B-L} \) charges of a particle, respectively, and \( g_1 \) are the gauge couplings. Because of the kinetic mixing between the two \( U(1) \) gauge bosons, the off-diagonal elements \( (g_{2}=g_{3}) \) are introduced.

In the following analysis, we take the boundary condition, \( g_{YB} = g_{BB} = 0 \), at the \( B-L \) symmetry breaking scale, where the two \( U(1) \) gauge bosons are diagonal with each other, for simplicity.

The Yukawa sector of the SM is extended to have

\[
\mathcal{L}_{\text{Yukawa}} \supset -Y_d \bar{d} L H N_i^L - Y_N \bar{N}_R H N_i^R + \text{h.c.},
\]

where the mass term is all forbidden by the conformal invariance. The mass term for the SM Higgs doublet is generated through the \( g_1 g_3 \) Yukawa coupling. The Majorana Yukawa coupling is negligible compared to the \( U(1)_Y \) gauge coupling, the \( U(1)_Y \) symmetry is broken when we choose \( \lambda_3 \) small.

\[
\mathcal{L}_{\text{Yukawa}} \supset -Y_d \bar{d} L H N_i^L - \frac{1}{2} Y_N \bar{N}_R N_i^L N_i^R + \text{h.c.},
\]

where the first term is the neutrino Dirac Yukawa coupling, while the second term is the Majorana Yukawa coupling. Without loss of generality, we have already diagonalized the Majorana Yukawa coupling. The \( B-L \) gauge symmetry breaking generates the Majorana neutrino mass term in the second term. The seesaw mechanism [36–41] is automatically implemented in the model after the electroweak symmetry breaking.

We apply the classically conformal invariance to the model, and the scalar potential is given by

\[
V = \lambda (H^\dagger H)^2 + \lambda_2 (\Phi^\dagger \Phi)^2 + \lambda_3 (H^\dagger H)(\Phi^\dagger \Phi).
\]

Note that the mass terms are all forbidden by the conformal invariance. If \( \lambda_3 \) is negligibly small, we can analyze the Higgs potential separately for \( \Phi \) and \( H \). This will be justified in the next section. When the Majorana Yukawa coupling \( Y_N \) is negligible compared to the \( U(1)_{B-L} \) gauge coupling, the \( \Phi \) sector is identical to the original Coleman–Weinberg model

\[
M_{Z'} = 2 g_{B L} M, \quad M_N = \frac{Y_N}{\sqrt{2}} M.
\]

In this paper, we assume degenerate masses for the three Majorana neutrinos, \( Y_N^i = \frac{y_N}{3} \) (equivalently, \( M_N = M_N \)) for all \( i = 1, 2, 3 \), for simplicity. The \( U(1)_{B-L} \) Higgs boson mass is given by

\[
M_\phi^2 = \frac{d^2 V}{d \phi^2} \bigg|_{\phi = M} = \beta_\phi M_\phi^2 \simeq \frac{3}{8\pi^2} \frac{M_N^4}{Z'} - 2M_N^2.
\]

The Majorana Yukawa coupling is negligibly small, this reduces to the well-known relation derived in the radiative symmetry breaking by the CW mechanism [3]. For a sizable Majorana mass, this formula indicates that the potential minimum disappears for \( M_N > M_Z^2/24 \), leading to the upper bound on the right-handed neutrino mass in order for the \( U(1)_{B-L} \) symmetry to be broken radiatively.

When the Majorana Yukawa coupling is negligibly small, this reduces to the well-known relation derived in the radiative symmetry breaking by the CW mechanism [3]. For a sizable Majorana mass, this formula indicates that the potential minimum disappears for \( M_N > M_Z^2/24 \), leading to the upper bound on the right-handed neutrino mass in order for the \( U(1)_{B-L} \) symmetry to be broken radiatively.

Once the \( U(1)_{B-L} \) gauge symmetry is radiatively broken by the CW mechanism, the electroweak symmetry is subsequently triggered through the coupling \( \lambda_3 \). With \( \langle \phi \rangle = M \), the SM Higgs potential is given by

\[
V(h) = \frac{\lambda_2}{4} h^4 + \frac{\lambda_3}{4} M^2 h^2,
\]

where \( h = 1/\sqrt{2} (0 \ h)^T \) in the unitary gauge. Choosing \( \lambda_3 < 0 \), the electroweak symmetry is broken in the same way as in the SM [28]. However, the crucial difference from the SM is that in our model the electroweak symmetry breaking originates from the radiative breaking of the \( U(1)_{B-L} \) gauge symmetry. At the tree level, the stationary condition
$V|_{h=v} = 0$ leads to the relation $|\lambda_3| = 2\lambda(v/M)^2$, and the Higgs boson mass $m_h$ is given by

$$m_h^2 = \left. \frac{d^2V}{dh^2} \right|_{h=v} = |\lambda_3| M^2 = 2\lambda v^2. \quad (3.8)$$

In the following renormalization group analysis, this relation, $\lambda_3 = -m_h^2/M^2$, is used as the boundary condition for $\lambda_3$ at the normalization scale $\mu = M$. Since $M \gtrsim 3$ TeV by the LEP constraint [42–44], $|\lambda_3| \lesssim 10^{-3}$. With such a small $\lambda_3$, the back reaction to the $B-L$ Higgs sector through $\lambda_3 v^2$ is negligibly small, and this fact allows us to treat the two Higgs sectors separately.  

### 4 Electroweak vacuum stability

In the context of the classically conformal U(1)\(_{B-L}\) extended model discussed in the previous sections, we now investigate a possibility to solve the electroweak vacuum instability problem. The electroweak vacuum stability has been investigated in the minimal $B-L$ model [45,46] (see also [47]), and the parameter regions for which the electroweak vacuum is stable have been identified. A crucial difference in our analysis from the previous one is that our model is classically conformal and the gauge symmetry breaking originates from the CW mechanism. Hence, we have constraints on the initial values of $\lambda_2$ and $\lambda_3$ at the scale $M$, and it is non-trivial to solve the electroweak vacuum instability problem. In the classically conformal extension of the SM the electroweak vacuum stability has been investigated though the renormalization group analysis at the one-loop level in [26,27], it turns out that there is no parameter region to keep the electroweak vacuum stable. In the following, we extend the renormalization group analysis to the two-loop level, and examine if the vacuum instability can be resolved by the higher order corrections.

In our analysis, we employ the SM renormalization group (RG) equations at the two-loop level [30] from the top quark pole mass to the U(1)\(_{B-L}\) Higgs VEV ($M$), and connect the RG equations to those of the minimal U(1)\(_{B-L}\) extended SM at the two-loop level. All formulas used in our analysis are listed in the appendices. As is well known, the RG evolutions of the Higgs quartic coupling is sensitive to the input values of the Higgs boson and top quark masses. For inputs for the Higgs boson mass and top quark pole mass, we adopt the result from the combined analysis by the ATLAS and the CMS experiments for the Higgs boson mass measurement in the range of $m_h = 125.09 \pm 0.21$ (stat.) $\pm 0.11$ (syst.) GeV [32] and the recent result of top quark mass measurement by the CMS experiments [33] in the range of $m_t = 172.38 \pm 0.10 \pm 0.65$.

The RG evolutions of the Higgs quartic coupling are shown in Fig. 1 for two different values of $m_h = 125.09$ GeV (left panel) and 125.41 GeV (right panel) with a fixed $m_t = 171.63$ GeV. Here we have fixed the other parameters as $g_{BL} = 0.314$, $g_Y B = g_{BB} = 0$ and $\lambda_3 = 0$ at $\mu = M = 4$ TeV. The solid lines denote the RG evolutions of the Higgs quartic coupling in our model, while the dashed lines denote those in the SM. We can see that in our model, the Higgs quartic coupling remains positive up to the Planck scale, $M_{Pl} = 1.2 \times 10^{19}$ GeV, and therefore the electroweak vacuum becomes stable. As the same as in the SM [30], the situation becomes better with an increasing (decreasing) value of $m_h$ ($m_t$) for a fixed value of the $m_t$ ($m_h$).

In order to identify parameter regions to keep the electroweak vacuum stable, we perform parameter scans for the free parameters $M_Z$ and $M_N$ with fixed values of $M = 3.5$, 4.0 and 4.5 TeV. Here, we have used the same values for $m_h$ and $m_t$ as in Fig. 1. In this analysis, we impose the following conditions for the running couplings at $M \leq \mu \leq M_{Pl}$: the stability of the Higgs potential ($\lambda, \lambda_2 > 0$ and $|\lambda_3|^2 < 4\lambda\lambda_2$), and the conditions that all the running couplings remain in the perturbative regime, namely, $g^2_Y(1, 2, 3), g^2_{BL}, g^2_{BB}, g^2_Y < 4\pi$ and $\lambda, \lambda_2, \lambda_3 < 4\pi$. The results are shown in Fig. 2. In this Figure, we also show the $B-L$ Higgs boson mass by using Eq. (3.6). As we expect, the allowed region becomes larger as $m_h$ is increased.

We also perform parameter scan for various values of $m_h$ and $m_t$ in the ranges of $124.68 \leq m_h/\text{GeV} \leq 125.32$ and $171.63 \leq m_t/\text{GeV} \leq 173.13$, with fixed values of $M = 2$ and 4 TeV. The results are shown in Fig. 3 for $M = 2$ TeV (left panel) and $M = 4$ TeV (right panel). The parameter sets inside of the triangles satisfy all constraints of the electroweak vacuum stability and the perturbativity of the running couplings. For a fixed $m_h$, there is an upper bound on $m_t$, or equivalently, there is a lower bound on $m_h$ for a fixed $m_t$. The allowed region for $M = 4$ TeV is more restricted.

---

Footnote 3 continued

Footnote 3 continued diagrams, which changes the top Yukawa input at $M$ by $\mathcal{O}(0.01\%)$ for $\alpha_{BL} = 0.012$ (see Fig. 4), or equivalently $\mathcal{O}(0.01 \text{ GeV})$ in terms of top quark mass. Since we have neglected the threshold corrections in our analysis, our results in this paper have a theoretical uncertainty of $\mathcal{O}(0.01 \text{ GeV})$ in the top quark mass. As can be seen from Fig. 3, the uncertainty at this size is negligibly small.
Fig. 1 The renormalization group evolution of the Higgs quartic coupling ($\lambda$) in the $B - L$ model (solid lines), along with the one in the SM (dashed lines). We have taken $m_h = 125.09$ GeV (left panel) and $m_h = 125.41$ GeV (right panel) with the fixed values of $M = 4.0$ TeV and $m_t = 171.63$ GeV.

than the one for $M = 2$ TeV. When we increase the $M$ value further, the allowed region disappears (see Fig. 4).

Finally, we show in Fig. 4 the results of our parameter scans for various values of $g_{BL}$ and $M$, with $m_h = 124.77$ GeV (left panel) and 125.09 GeV (right panel) for $m_t = 171.63$ GeV. In this figure, we present the results with $g_{BL} = g_{BL}/(4\pi)$ and $M_{Z'}$ by using the mass formula $M_{Z'} = 2g_{BL}M$. Here we have considered not only the conditions of the electroweak vacuum stability and the perturbativity, but also the current collider bounds. The search for
The results of parameter scans for various values of $m_h$ and $m_t$ in the ranges of $124.68 \leq m_h/\text{GeV} \leq 125.32$ and $171.63 \leq m_t/\text{GeV} \leq 173.13$, with the fixed values of $M = 2 \text{ TeV}$ (left panel) and $M = 4 \text{ TeV}$ (right panel).

$$M_{Z'} \gtrsim 6.9 \text{ TeV}$$ \tag{4.1}

at 95% confidence level. The ATLAS and the CMS collaborations have searched for $Z'$ boson resonance at the LHC Run-1 with $\sqrt{s} = 8 \text{ TeV}$. The most stringent bounds on the $Z'$ boson production cross section times branching ratio have been obtained by using the dilepton final state. For the so-called sequential SM $Z'$ model [50], where the $Z'$ boson has exactly the same couplings with the SM fermions as those of the SM Z boson, the cross section bounds lead to lower bounds on the $Z'$ boson mass as $M_{Z'} \gtrsim 2.90 \text{ TeV}$ from the ATLAS analysis [51] and $M_{Z'} \gtrsim 2.96 \text{ TeV}$ from the CMS analysis [52,53], respectively. Very recently, these bounds have been updated by the ATLAS [34] and CMS [35] analysis with the LHC Run-2 at $\sqrt{s} = 13 \text{ TeV}$ as $M_{Z'} \gtrsim 3.4 \text{ TeV}$ (ATLAS) and $M_{Z'} \gtrsim 3.15 \text{ TeV}$ (CMS), respectively. We interpret these ATLAS and CMS results to the $B-L$ model. In our model, the $U(1)_{B-L}$ gauge coupling is a free parameter, and for a fixed gauge coupling we can read off the lower limit on the $Z'$ boson mass from the ATLAS and CMS cross section bounds. In this way, we can find an upper (lower) bound on the $U(1)_{B-L}$ gauge coupling $\alpha_{BL} = g_{BL}^2/(4\pi) (Z'$ boson mass $M_{Z'})$ as a function of $M_{Z'}$ ($\alpha_{BL}$). In interpreting the ATLAS and CMS results to the $B-L$ model, we follow a strategy presented in detail in [54] (see also [55,56]). In Fig. 4, the vertical solid lines correspond to the bounds from the LEP result, the ATLAS with the LHC Run-1, the CMS with the LHC Run-1, the CMS with the LHC Run-2 and the ATLAS with the LHC Run-2, respectively. The naturalness argument prefers the regions on the left sides of the diagonal dashed lines.
in the next section, is also shown as the dashed lines. In, for example, Ref. [57], the search reach of the $Z'$ boson at the LHC Run 2 with a 14 TeV collider energy and a 100/fb luminosity is obtained as $M_{Z'} \sim 5$ TeV for $\alpha_{BL} \simeq 0.01$. A large portion of the allowed regions presented in Fig. 4 can be tested in the near future. The (indirect) search reach of the future $e^+e^-$ linear collider with a 1 TeV collider energy can be as large as 10 TeV (see, for example, [29]), and almost of all allowed regions presented in Fig. 4 can be covered.

5 Constraints from naturalness

Once the U(1)$_{B-L}$ gauge symmetry is radiatively broken by the CW mechanism, the masses for the $Z'$ boson and the Majorana neutrinos are generated, which in general create self-energy corrections to the SM Higgs doublet. If the $B-L$ gauge symmetry breaking scale is very large, the self-energy corrections may exceed the electroweak scale and require us to fine-tune the model parameters in reproducing the correct electroweak scale. Two major corrections have been discussed in [28,29]: one is one-loop corrections with the Majorana neutrinos, and the other is two-loop corrections involving the $Z'$ boson and the top quark. In the calculations of the self-energy corrections in [29], the cutoff procedure with the Planck scale cutoff is applied to derive the naturalness bounds. Although this treatment is good for rough estimates, in order to derive more accurate naturalness bounds we will renormalize the loop corrections properly in this section.

Since the original theory is classically conformal and defined as a massless theory, the self-energy corrections to the SM Higgs doublet originates from corrections to the quartic coupling $\lambda_3$. Thus, what we calculate to derive the naturalness bounds is quantum corrections to the term $\lambda_3 h^2 \phi^2$ in the effective Higgs potential. For the one-loop diagram involving the Majorana neutrinos (for the Feynman diagram, see Fig. 3 in [29]), we calculate the effective potential as

$$\Delta V_{1-\text{loop}} \supset -\frac{|Y_D|^2|Y_N|^2}{16\pi^2} h^2 \phi^2 \left(\ln[\phi^2] + C\right),$$

(5.1)

where the logarithmic divergence and the terms independent of $\phi$ are all encoded in $C$. By adding a counter term, we renormalize the coupling $\lambda_3$ with the renormalization condition,

$$\frac{\partial^4}{\partial h^2 \partial \phi^2} V_{\text{eff}} \bigg|_{h=0, \phi=M} = \lambda_3,$$

(5.2)

where $V_{\text{eff}}$ is the sum of the tree-level potential and $\Delta V_{1-\text{loop}}$, and $\lambda_3$ is the renormalized coupling. As a result, we obtain

$$V_{\text{eff}} \supset \left[\frac{1}{4} \lambda_3 - \frac{|Y_D|^2|Y_N|^2}{16\pi^2} \left(\ln\left[\frac{\phi^2}{M^2}\right] - 3\right)\right] h^2 \phi^2.$$  

(5.3)

Substituting $\phi = M$, we obtain the SM Higgs self-energy correction as

$$\Delta m_h^2 = \frac{3|Y_D|^2|Y_N|^2}{8\pi^2} M^2 \sim \frac{3m_v M_N^3}{4\pi^2 v^2}$$

(5.4)

where we have used the seesaw formula, $m_v \sim Y_D^2 v^2 / M_N$ [36–41]. If $\Delta m_h^2$ is much larger than the electroweak scale, we need a fine-tuning of the tree-level Higgs mass ($|\lambda_3| M^2/2$) to reproduce the correct Higgs VEV, $v = 246$ GeV. Here, we introduce the naturalness condition as

$$\delta = \frac{m_h^2}{|\Delta m_h^2|} \geq 1.$$  

(5.5)

For example, when the light neutrino mass scale is around $m_\nu \simeq 0.1$ eV after the seesaw mechanism, we have an upper bound for the Majorana mass as $M_N \lesssim 4 \times 10^6$ GeV.

For the two-loop diagrams involving $Z'$ boson and top quark (for the Feynman diagrams, see Fig. 4 in [29]), we have

$$\Delta V_{2-\text{loop}} \supset -\frac{2\alpha_B^2 M_\nu^2}{\pi^2 v^2} h^2 \phi^2 \left(\ln[\phi^2] + C\right),$$

(5.6)

where the logarithmic divergence and the terms independent of $\phi$ are all encoded in $C$. Following the same strategy as the above, we obtain

$$\Delta m_h^2 = \frac{3\alpha_B M_\nu^2}{4\pi^2 v^2} M_Z^2.$$  

(5.7)

The dashed lines shown in Fig. 4 are plotted by using the condition $\delta = 1$ in Eq. (5.5).

6 Conclusions

We have considered the minimal $B-L$ extension of the Standard Model, where the anomaly-free global $B-L$ symmetry in the Standard Model is gauged and three right-hand neutrinos and a $B-L$ Higgs field are introduced. This model is very simple and well motivated, since the right-handed neutrinos acquire their Majorana masses associated with the $B-L$ gauge symmetry breaking, and the seesaw mechanism for the neutrino mass generation is automatically implemented. Motivated by the argument that the Higgs model can be free from the gauge hierarchy problem once the classically conformal symmetry is imposed in the model, we have introduced the classically conformal symmetry to the minimal $B-L$ model. In this context, the $B-L$ symmetry is radiatively broken by the Coleman–Weinberg mechanism and this breaking is the sole origin of all mass parameters in the model. The electroweak symmetry breaking is realized by the negative mass term for the Higgs doublet, which is subsequently generated through the $B-L$ gauge symmetry breaking. Therefore, the electroweak symmetry breaking
originates from the radiative $B - L$ gauge symmetry breaking.

In the context of the classically conformal $B - L$ model, we have investigated the electroweak vacuum instability problem. With the measured Higgs boson mass around 125 GeV, it turns out that the electroweak vacuum is not the true minimum in the effective Higgs potential of the Standard Model. In other words, the electroweak symmetry is radiatively broken at some energy much higher than the electroweak scale. This ruins the theoretical consistency of our model that the radiative $B - L$ symmetry breaking is the sole origin of the mass. We have analyzed the renormalization group evolutions of the model couplings at the two-loop level with the recent results of the Higgs boson mass and top quark mass measurements at the LHC. We have identified parameter regions which satisfy the conditions of the stability of the electroweak vacuum and the perturbativity of the running couplings, as well as the current collider bounds from the search for the $B - L$ gauge boson, in particular, at the LHC Run-2.

In addition, we have considered the naturalness of the electroweak scale against self-energy corrections for the Higgs doublet. We have refined the previously obtained results in a theoretically consistent way for the Coleman–Weinberg effective potential, and derived the naturalness bounds on the $B - L$ gauge boson and the right-handed neutrino masses. The allowed regions satisfying the naturalness bounds can be tested in the future collider experiments.

**Acknowledgements** This work is supported in part by the United States Department of Energy Grant, No. DE-SC0013680.

**Open Access** This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

**Appendix A: The beta functions for the SM couplings**

A.1: The one-loop beta functions for the SM gauge couplings

We have

$$\beta_{g_i}^{(1)} = \frac{41}{10} g_i^3, \quad \beta_{g_2}^{(1)} = -\frac{19}{6} g_2^3, \quad \beta_{g_3}^{(1)} = -7 g_3^3. \quad (A.1)$$

A.2: The one-loop beta function for the top Yukawa coupling

We have

$$\beta_{y_t}^{(1)} = y_t \left( -\frac{17}{20} g_1^2 - \frac{9 g_2^2}{4} - 8 g_3^2 + \frac{9 y_t^2}{2} \right). \quad (A.2)$$

A.3: The one-loop beta function for the quartic Higgs coupling

We have

$$\beta_{\lambda}^{(1)} = -\lambda \left( \frac{9 g_1^2}{5} + g_2^2 \right) \left( \frac{2}{5} g_1^2 g_2^2 + \frac{3 g_1^4}{25} + g_2^4 \right) + 12 \lambda^2 + 12 \lambda y_t^2 - 12 y_t^4. \quad (A.3)$$

A.4: The two-loop beta functions for the gauge couplings

We have

$$\beta_{g_i}^{(2)} = g_i^3 \left( \frac{199 g_1^2}{50} + \frac{27 g_2^2}{10} + \frac{44 g_3^2}{5} - \frac{17 y_t^2}{10} \right),$$

$$\beta_{g_2}^{(2)} = g_2^3 \left( \frac{9 g_1^2}{10} + \frac{35 g_2^2}{6} + 12 g_3^2 - \frac{3 y_t^2}{2} \right),$$

$$\beta_{g_3}^{(2)} = g_3^3 \left( \frac{11 g_1^2}{10} + \frac{9 g_2^2}{2} - 26 g_3^2 - 2 y_t^2 \right). \quad (A.4)$$

A.5: The two-loop beta function for the top Yukawa coupling

We have

$$\beta_{y_t}^{(2)} = y_t \left( \frac{y_t^2}{y_t^2} \left( \frac{393 g_1^2}{80} + \frac{225 g_2^2}{16} + 36 g_3^2 \right) - \frac{9}{20} g_1^2 g_2^2 \right) + \frac{13}{15} g_1^2 g_2^2 + \frac{11 g_1^2 g_3^2}{600} + g_1^2 g_2^2 - \frac{23 g_2^4}{4} - 108 g_3^4 + \frac{3 g_3^4}{2} - 6 \lambda y_t^2 - 12 y_t^4. \quad (A.5)$$

A.6: The two-loop beta function for the Higgs quartic coupling

We have

$$\beta_{\lambda}^{(2)} = \left( 10 \lambda y_t^2 \left( \frac{17 g_1^2}{20} + \frac{9 g_2^2}{4} + 8 g_3^2 \right) + 18 \lambda^2 \left( \frac{3 g_1^4}{5} + 3 g_2^4 \right) - \lambda \left( -\frac{117}{20} g_1^2 g_2^2 - \frac{1887}{200} g_1^4 + \frac{73 g_2^4}{8} \right) - \frac{1677}{200} g_1^4 g_2^2 - \frac{289}{40} g_1^2 g_2^2 - \frac{3}{5} g_1^2 g_3^2 - \frac{37 g_1^2}{10} - 21 g_2^2 \right) - 3411 g_3^2 + 818 g_3^2 + 305 g_2^6 - \frac{9}{2} g_2^2 y_t^2 - 64 g_3^2 y_t^4 - 78 \lambda^3 - 72 \lambda^2 y_t^2 - 3 \lambda y_t^4 + 60 y_t^6. \quad (A.6)$$
In our analysis, we numerically solve the SM RG equations with the following boundary conditions at \( \mu = m_t \) [30].

\[
g_3(m_t) = \frac{\sqrt{5}}{3} \left[ 1.1666 + 0.00314 \left( \frac{\alpha_3(m_Z) - 0.1184}{0.0007} \right) - 0.00046 \left( \frac{m_t}{\text{GeV}} - 173.34 \right) \right],
\]

\[
g_2(m_t) = 0.64779 + 0.00004 \left( \frac{m_t}{\text{GeV}} - 173.34 \right) + 0.00011 \left( \frac{m_W - 80.384 \text{GeV}}{0.014 \text{GeV}} \right),
\]

\[
g_1(m_t) = 0.35830 + 0.00011 \left( \frac{m_t}{\text{GeV}} - 173.34 \right) - 0.00020 \left( \frac{m_W - 80.384 \text{GeV}}{0.014 \text{GeV}} \right),
\]

\[
y_1(m_t) = 0.93690 + 0.00556 \left( \frac{m_t}{\text{GeV}} - 173.34 \right) - 0.00042 \left( \frac{\alpha_3(m_Z) - 0.1184}{0.0007} \right),
\]

\[
\lambda(m_t) = 0.12604 + 0.00206 \left( \frac{m_t}{\text{GeV}} - 125.15 \right) - 0.00004 \left( \frac{m_t}{\text{GeV}} - 173.34 \right). \tag{A.7}
\]

We have used the inputs, \( \alpha_3(m_Z) = 0.1184 \) and \( m_W = 80.384 \text{ GeV} \).

### Appendix B: The beta functions for the couplings in the U(1)\_B−L extended SM

#### B.1: The one-loop beta functions for the gauge couplings

We have

\[
\beta_{g_1}^{(1)} = \frac{1}{10} \left( g_1 \left( \frac{2}{3} \sqrt{5} g_{BLGBY} + 41 g_{BY}^2 + 180 g_{YB}^2 \right) + 32 \sqrt{10} g_1^2 g_{BY} + 4 g_1^3 g_{BY} \left( 15 \sqrt{6} g_{BL} + 4 \sqrt{10} g_{BY} \right) \right),
\]

\[
\beta_{g_2}^{(1)} = -19 \left( \frac{6}{3} g_2 \right)^3,
\]

\[
\beta_{g_3}^{(1)} = -7 g_3^3,
\]

\[
\beta_{g_{4\nu}}^{(1)} = \frac{1}{10} \left( 4 g_{1\nu} g_{BY} \left( 15 \sqrt{6} g_{BL} + 4 \sqrt{10} g_{BY} \right) + g_1^2 \left( \frac{2}{3} \sqrt{5} g_{BLGBY} + 41 g_{BY} \right) + g_{BY} \left( 64 \sqrt{5} g_{BLGBY} + 120 g_{BL} + 41 g_{YB} \right) \right),
\]

\[
\beta_{g_{4\nu}}^{(1)} = \frac{1}{10} \left( g_1 \left( \frac{2}{3} \sqrt{5} g_{BLGBY} + 16 \sqrt{10} \left( \frac{g_{BL}^2}{3} + g_{YB}^2 \right) \right) + 41 g_1^2 g_{YB} + 4 g_{YB} \left( 8 \sqrt{5} g_{BLGBY} + 45 \left( \frac{2 g_{BL}^2}{3} + g_{YB}^2 \right) \right) \right),
\]

\[
\beta_{g_{BB}}^{(1)} = \sqrt{\frac{3}{2}} \left( \frac{1}{10} \left( 4 \sqrt{5} g_{BL} \left( g_{YB} \left( 4 \sqrt{10} g_1 + 45 g_{YB} \right) + 30 g_{BL}^2 \right) + g_{BY} \left( 41 g_1^2 + 16 \sqrt{10} g_{YB} \right) \right) \right).
\tag{B.1}
\]

#### B.2: The one-loop beta function for the top Yukawa coupling

We have

\[
\beta_{\tilde{Y}}^{(1)} = \frac{9 \tilde{y}_t^3}{2} - y_t \left( \sqrt{\frac{5}{3}} g_{YB} g_{BL} + \frac{2}{3} g_{B}^2 g_{L} + \frac{5}{2} g_1 \sqrt{g_{YB}} \right) + \frac{17 g_1^2}{20} + \frac{9 g_2^2}{4} + 8 g_3^2 + \frac{17 g_{YB}^2}{20} + g_{YB}^2 \right). \tag{B.2}
\]

#### B.3: The one-loop beta function for the Majorana Yukawa coupling

We have

\[
\beta_{Y}^{(1)} = 2 \left( 10 \left( \frac{Y}{2} \right)^3 - 9 \frac{Y}{2} \left( \frac{g_{BL}^2 + g_{YB}^2}{3} \right) \right). \tag{B.3}
\]

#### B.4: The one-loop beta function for the scalar quartic couplings

We have

\[
\beta_{\lambda}^{(1)} = \frac{2}{3} \left( \frac{\lambda}{2} - \frac{9}{5} g_1^2 g_{YB} - \frac{9 g_{YB}^2}{5} + 12 \lambda_t^2 \right) + 24 \left( \frac{\lambda}{2} \right)^2 + \frac{9}{20} g_1^2 g_{YB} + \frac{27}{100} g_{YB}^2 \lambda_3^2 + \frac{27 g_1^2}{200} + \lambda_3^2 - 6 \lambda_t^2 \right),
\]

\[
\beta_{\kappa_2}^{(1)} = \frac{2}{3} \left( -36 \lambda_2 \left( \frac{g_{BL}^2 + g_{YB}^2}{3} \right) + 144 g_{YB}^2 \right) + \frac{48 g_{BL}^4 + 108 g_{YB}^4}{10} + \frac{10 \lambda_2^2 + \lambda_2^2 + 3 \lambda_2 Y_N^2 - 3 Y_N^2}{2} \right),
\]

\[
\beta_{\kappa_3}^{(1)} = -\lambda_3 \left( 24 g_{BL}^2 + \frac{9}{10} g_1^2 g_{YB} + \frac{9 g_{YB}^2}{10} + 36 g_{YB}^2 \right) + \frac{36 g_{BL}^4}{5} \sqrt{6} g_{YB} g_{YB} g_{BL} + \frac{36 g_{YB}^2 g_{BL}^2}{5} + \frac{54 g_{YB}^2 g_{BL}^2}{5} + \frac{4 g_{YB}^2}{3} + 6 \lambda_3^2 + 8 \lambda_2 \lambda_3 + \lambda_3 \left( 3 Y_N^2 + 6 Y_t^2 \right) \right) \tag{B.4}
\]

\[\text{Springer}\]
B.5: The two-loop beta functions for the gauge couplings

We have

\[ \beta(g) = \frac{1}{100} \left( 398g_1^2 + 328\sqrt{10}g_2g_1^4 + 398g_2g_1^4 \right. \]
\[ + 328\sqrt{10}g_1g_2^3 
+ 270g_2^3g_1^3 + 880g_2^3g_1^3 + 328\sqrt{10}g_2^2g_1^3 
+ 1318g_2^2g_2g_1^3 + 920g_2^2g_2g_1^3 - 170\gamma_1g_3 
+ 1840 
\div 3g_2^2g_1^3 + 328\sqrt{10}g_2g_2g_1^3 
\left. \right) \]
\[ + 360g_2g_2g_2g_1^3 + 656\sqrt{\frac{5}{3}}g_2g_2g_2g_1^3 
+ 328\sqrt{10}g_2g_2g_2g_1^3 + 1318g_2g_2g_2g_1^3 
+ 1120\sqrt{10}g_2g_2g_2g_1^3 + 920g_2g_2g_2g_1^3 
- 200g_2g_2g_2g_1^3 + 2240 
\div 3\sqrt{10}g_2g_2g_2g_1^3 
+ 1840 
\div 3g_2g_2g_2g_1^3 + 360\sqrt{10}g_2g_2g_2g_1^3 
+ 270g_2g_2g_2g_1^3 + 320\sqrt{10}g_2g_2g_2g_1^3 
+ 880g_2g_2g_2g_1^3 + 1120\sqrt{10}g_2g_2g_2g_1^3 
- 80g_2g_2g_2g_1^3 - 85\sqrt{10}g_2g_2g_2g_1^3 
\]
\[
\beta_{SB}^{(2)} = \frac{1}{100} \left( 920 g_{1SBY}^4 + 560 \sqrt{10} g_{SBY}^4 + 4800 \sqrt{6} g_{BLSBY}^4 + 328 \sqrt{10} g_{1SBY}^3 + 9600 g_{BLSBY}^3 + 1840 g_{1SBY}^3 + 1120 \sqrt{3} g_{1SBY}^3 + 920 \sqrt{3} g_{1SBY}^3 + 5600 \sqrt{3} g_{BLSBY}^3 + 380 g_{1SBY}^3 + 600 \sqrt{3} g_{BLSBY}^3 + 320 \sqrt{3} g_{1SBY}^3 + 164 \sqrt{3} g_{1SBY}^3 + 920 \sqrt{3} g_{1SBY}^3 \right).
\]
We have
\[
\beta_{\gamma}^{(2)} = \frac{1}{600} \gamma_{1} \left( 3600 \left( \frac{\lambda}{2} \right)^2 + 500 \sqrt{15} g_{YB} g_{BL} \right) + 1000 \gamma_{1} g_{BL} g_{YB} + 340 \sqrt{3} g_{1} g_{BYY} g_{gBL} + 50 \sqrt{3} g_{1} g_{BYY} g_{gBL} + 190 \sqrt{3} g_{1} g_{BL} + 270 \sqrt{3} g_{gBYY} g_{gBL} + 450 g_{1} g_{BYY} g_{gBL} + 1600 \frac{g_{gBYY} g_{gBL}}{3} + 1000 \sqrt{3} g_{gBYY} g_{gBL} + 26600 \frac{g_{gBYY} g_{gBL}}{3} + 10850 g_{gBYY} g_{gBL} + 4016 \frac{g_{gBYY} g_{gBL}}{3} + 40600 \frac{g_{gBYY} g_{gBL}}{9} + 750 \sqrt{10} g_{1} g_{BYY} g_{Y} + 1275 g_{1} g_{Y} + 3375 g_{1} g_{Y} + 12000 \frac{g_{gBYY} g_{Y}}{3} + 1500 g_{gBYY} g_{Y} + 405 \sqrt{10} g_{1} g_{gBYY} g_{gY} - 270 g_{1} g_{gY} + 760 g_{1} g_{g} + 25 \sqrt{10} g_{1} g_{gBYY} g_{gY} - 106 g_{1} g_{gY} + 2008 \sqrt{10} g_{1} g_{gY} + 16275 g_{1} g_{gY}
\]
We have
\[
\beta_{\nu N}^{(2)} = \frac{1}{20} \left( - \frac{3411 g_Y^6}{2000} - \frac{4221 g_{BY}^4 g_Y^4}{2000} + \frac{1887}{400} g_Y^4 \right) Y_N^2
+ \frac{128}{3} \left( \frac{Y}{g_{BY}} \lambda g_Y^3 + 70 g_{BY}^2 g_Y^2 \right) Y_N^2
+ \frac{7380 g_{BY}^2 g_Y^2}{3} + 2540 g_Y^4 + 45 Y_N^2
+ 96 \sqrt{10} g_Y g_{BY}^2 + 105 g_Y^2 g_{BY}^2 + 5715 g_Y^4
-80 \lambda_Y^2 - 20 \lambda_Y^3 \right) + 880 g_{BL Y}^2 Y_N^2 + 1320 g_{BY}^2 Y_N^2
-160 \lambda_Y^2 Y_N^2 - 10 Y_N^2. \tag{B.7}
\]

B.7: The two-loop beta functions for the heavy neutrino Yukawa coupling

We have
\[
\beta_{\alpha}^{(2)} = \frac{2}{5} \left( - \frac{3411 g_Y^6}{2000} - \frac{4221 g_{BY}^4 g_Y^4}{2000} + \frac{1887}{400} g_Y^4 \right) \lambda^2
- \frac{48}{5} \sqrt{5} g_Y g_{BY}^2 g_Y^3 + 12 \sqrt{\frac{7}{5}} g_{BY} \lambda g_Y^3
- \frac{10971 g_{BY}^4 g_Y^2}{2000} - \frac{8}{5} \left( \frac{Y}{g_{BY}} \right)^2 g_Y^2 \lambda g_Y
+ \frac{63}{10} Y_Y^2 \lambda g_Y^2 + 6 \frac{3}{2} g_{BY} \lambda g_Y^2
+ \frac{153}{10} g_{BY} \lambda g_Y^2 + 18 g_{BY} \lambda g_Y^2 \lambda g_Y^3
-2 \sqrt{10} \left( \frac{Y}{g_{BY}} \right)^2 g_{BY} \lambda g_Y^2 + 5 \sqrt{\frac{7}{2}} g_{BY} \lambda g_Y^2 \lambda g_Y^2
- \frac{12}{5} \sqrt{6} \left( \frac{Y}{g_{BY}} \right) g_{BY} \gamma g_{BL} g_{BL}^2 g_Y^2
+ \frac{12}{5} \sqrt{6} g_{BY} g_{BY} \lambda g_Y \lambda g_Y g_{BL} g_Y^2 g_Y^2
+ \frac{305 g_Y^6}{16} - \frac{4221 g_Y^6}{2000}
-39 \lambda_Y^3 - 4 \lambda_Y^3 - 32 \left( \frac{Y}{g_{BY}} \right)^2 g_Y^3 \lambda_Y^2 - \frac{8}{5} \left( \frac{Y}{g_{BY}} \right)^2 g_{BY} \lambda g_Y^2
-4 \left( \frac{Y}{g_{BY}} \right)^2 g_{BY} \lambda g_Y^2
+27 g_Y^2 \lambda_Y^2 - 36 \lambda_Y^2 \gamma g_{BY} \lambda g_Y^2 - 36 \lambda_Y g_{BY} \lambda g_Y^2
+ \frac{8}{3} \left( \frac{Y}{g_{BY}} \right)^2 g_{BY}^2 g_Y^2 + 32 \lambda_Y \lambda g_Y^2
+ \frac{51}{5} g_{BY} \lambda g_Y^2 + 12 g_{BY} \lambda g_Y^2
- \frac{289}{80} \left( g_{BY}^2 g_Y^2 + g_{BY} g_Y^2 \right) - \frac{1677}{400} \left( g_{BY}^2 g_Y^2 + g_{BY} g_Y^2 \right)
+ \frac{9}{4} \left( \frac{Y}{g_{BY}} \right)^2 g_{BY}^2 g_Y g_Y^2
+ \frac{171}{100} \left( \frac{Y}{g_{BY}} \right)^2 g_{BY}^2 g_Y g_Y^2
+ \frac{117}{400} \left( \frac{Y}{g_{BY}} \right)^2 g_{BY}^2 g_Y g_Y^2
+ \frac{45}{4} Y_Y^2 \lambda g_Y^2
+ 5 Y_Y^2 \lambda g_Y^2 + \frac{108}{5} \left( \frac{1}{4} \lambda_Y^2 + \frac{1}{4} g_{BY} \lambda g_Y^2 \right)^2
+ \frac{117}{20} \left( \frac{1}{2} g_Y^2 \lambda g_Y^2 + g_{BY} \lambda g_Y^2 \right) - 32 \frac{3}{5} \sqrt{3} g_{BY}^2 g_Y g_Y^2
- \frac{4}{5} \left( \frac{Y}{g_{BY}} \right)^2 g_{BY}^2 g_Y g_Y^2
+ 8 \left( \frac{3}{5} g_{BY} \lambda g_Y^2 + 5 \left( \frac{2}{3} \lambda_Y g_Y g_{BL} + 5 \sqrt{\frac{5}{3}} Y_Y^2 g_Y g_{BL} + \frac{5}{3} Y_Y^2 \lambda g_Y g_{BL}
-9 \left( \frac{2}{3} Y_Y^2 \gamma g_{BY} g_{BL} + \frac{2}{3} g_{BY} g_{BL}^2 g_Y^2 - g_{BY} g_{BL}^2 g_Y^2
+ \frac{2}{3} g_{BY} g_{BL}^2 g_Y^2
+ \frac{144}{25} \sqrt{\frac{2}{5}} \left( g_{BY}^2 g_Y^2 + g_{BY} g_Y^2 \right)
+ \frac{1}{5} Y_Y^2 \lambda g_Y g_{BL}^2 + \frac{1}{5} Y_Y^2 \lambda g_Y g_{BL}^2
+ \frac{1}{10} g_{BY}^2 \lambda g_Y^2 + \frac{1}{10} g_{BY}^2 \lambda g_Y^2
- \frac{117}{20} \left( \frac{1}{2} g_Y^2 \lambda g_Y^2 + g_{BY} \lambda g_Y^2 \right)
- \frac{351}{50} \left( g_{BY}^2 g_Y^2 + g_{BY} g_Y^2 \right) + \frac{2}{3} g_{BY}^2 g_Y^2 g_Y^2
+ \frac{2}{3} g_{BY}^2 g_Y^2 g_Y^2
\right)\right). \tag{B.7}
\]

B.8: The two-loop beta functions for the scalar quartic couplings

We have
\[
\beta_{\nu}^{(2)} = 2 \left( \frac{2}{5} \left( \frac{Y}{g_{BY}} \right)^2 g_{BY} \lambda g_Y^2 \right) Y_N^2
+ \frac{12}{5} \sqrt{6} \left( \frac{Y}{g_{BY}} \right) g_{BY} \gamma g_{BL} g_Y^2 g_Y^2
+ \frac{12}{5} \sqrt{6} g_{BY} g_{BY} \lambda g_Y \lambda g_Y g_{BL} g_Y^2 g_Y^2
+ \frac{305 g_Y^6}{16} - \frac{4221 g_Y^6}{2000}
-39 \lambda_Y^3 - 4 \lambda_Y^3 - 32 \left( \frac{Y}{g_{BY}} \right)^2 g_Y^3 \lambda_Y^2 - \frac{8}{5} \left( \frac{Y}{g_{BY}} \right)^2 g_{BY} \lambda g_Y^2
-4 \left( \frac{Y}{g_{BY}} \right)^2 g_{BY} \lambda g_Y^2
+1440 g_{BY}^2 g_Y^2 g_Y^2 Y_N Y_Y^2 + 75 \lambda g_Y g_{BL} g_Y^2 Y_Y^2
+ 90 g_{BL} Y_Y^2 \left( \frac{Y}{g_{BY}} \right)^2 + 480 g_{BL} Y_Y Y_N Y_Y^2
+ \frac{60}{5} \sqrt{6} g_{BY} g_{BY} \lambda g_Y \lambda g_Y g_{BL} g_Y^2 g_Y^2
-6144 \sqrt{10} g_{BY} g_{BY} g_{BY} g_{BY} g_Y^2 g_Y^2
\]
\[\frac{2}{5} \left( 1440 g_{BY}^2 g_Y^2 g_Y^2 Y_N Y_Y^2 + 75 \lambda g_Y g_{BL} g_Y^2 Y_Y^2
+ 90 g_{BL} Y_Y^2 \left( \frac{Y}{g_{BY}} \right)^2 + 480 g_{BL} Y_Y Y_N Y_Y^2
+ \frac{60}{5} \sqrt{6} g_{BY} g_{BY} \lambda g_Y \lambda g_Y g_{BL} g_Y^2 g_Y^2
-6144 \sqrt{10} g_{BY} g_{BY} g_{BY} g_{BY} g_Y^2 g_Y^2 \right)\]
\begin{align*}
\rho_\lambda^2 &= \frac{-6417}{50} g^2_{YB} \gamma_1^4 + \frac{1671}{400} \lambda^3 g_1^4 - \frac{2304}{5} \sqrt{\frac{2}{5}} g^3_{YB} g_1^3 \\
&+ 12 \sqrt{\frac{2}{5}} g_{YB} \lambda g_3 \gamma_1^2 \\
&- \frac{1137}{25} \sqrt{6} g_{BY} g_{YB} g_{BL} g_1^3 - \frac{4428}{5} g^4_{YB} g_1^2 \\
&- \frac{81}{2} g^2_{YB} g_1^2 - \frac{81}{10} g_{YB} g_{BL}^2 \\
&+ 3 \frac{\lambda^2 g_1^2}{5} - \frac{27}{5} g^2_{YB} g_1^2 - 216 g^2_{YB} g^{2}_{BL} g_1^2 \\
&- \frac{342}{5} \gamma_1^2 g_{YB} g_1^2 + 108 g_{YB} \lambda g_2^2 \\
&+ 72 g^2_{YB} \lambda g_2^2 + \frac{9}{8} g^2_{YB} g_2^2 + \frac{81}{40} g_{YB} \lambda g_3^2 \\
&+ \frac{1941}{10} g^2_{YB} \lambda g_3^2 + \frac{36}{5} g_{YB} \lambda g_3^2 \\
&- \frac{44608 g_{BY} g^2_{YB} g^2_{BL} g_1^2}{5\sqrt{15}} - \frac{1424}{5} \sqrt{6} g_{BY} g_{YB} g^3_{BL} g_1 \\
&- \frac{1536}{5} \sqrt{\frac{2}{5}} g_{YB} g_{YB} g_2^2 g_1 g_1^2 \\
&- 48 \sqrt{10} \gamma_1^2 g_{YB} g_{YB} g_2^2 g_1 g_1^2 - 72 \sqrt{10} \gamma_1^2 g_{YB} g_1^2 g_1^2 \\
&+ 96 \sqrt{10} g^3_{YB} \lambda g_1^2 - \frac{2016}{5} \sqrt{6} g_{YB} g^3_{YB} g_{BL} g_1 \\
&- \frac{1137}{25} \sqrt{6} g^3_{YB} g_{YB} g_{YB} g_1^3 - 27 \sqrt{6} g^3_{YB} g_{YB} g_{YB} g_1^3 \\
&- \frac{228}{5} \sqrt{6} \gamma_1^2 g_{YB} g_{YB} g_1^2 g_1^2 \\
&+ 72 \sqrt{6} g_{YB} g_{YB} g_1^2 g_1^2 + 48 \sqrt{6} g_{YB} g_{YB} \lambda g_2 g_1 g_1^2 \\
&+ \frac{24}{5} \sqrt{6} g_{YB} g_{YB} \lambda g_3 g_1 g_1^2 \\
&- \frac{1968}{5} g^2_{YB} g_1^2 - 64 \gamma_1^2 \lambda g_4^4 + 672 \lambda^3 g_4^4 - 11 \lambda_3 \\
&- \frac{1024}{5} \sqrt{\frac{3}{5}} g^3_{YB} g_1^3 - 32 \sqrt{15} \gamma_1^2 g_{BY} g_1^2 g_4^3 \\
&+ 128 \frac{\sqrt{3}}{5} g_{YB} g_1^3 g_2 g_1 + 3 \gamma_1^2 \lambda g_3^2 + \frac{3}{5} g^2_{YB} \lambda^2_3 - 36 \lambda_3^2 \\
&- 48 \lambda_2 \lambda_3^2 - \frac{2139}{25} g^2_{YB} g_1^2 g_4^2 - 27 g^2_{YB} g_1^2 g_4^2 \\
&- 216 g^2_{YB} g_{YB} g_4^2 g_1^2 + 16 \lambda_2 \lambda_3^2 - \frac{228}{5} \gamma_1^2 g_{BY} g_4 g_1^2 \\
&- \frac{192}{5} \gamma_1^2 g_{YB} g_1^2 + 72 \gamma_1^2 g_{YB} g_1^2 \\
&+ 48 g^2_{YB} \lambda g_2^2 g_4^2 + \frac{497}{5} g^2_{YB} \lambda g_3^2 g_4^2 + 1200 g^2_{YB} \gamma g_4^2 \\
&+ 256 \gamma_1^2 \lambda g_3^2 - \frac{2}{3} \lambda^3 g_3 \\
&- 144 \gamma_1^2 \lambda g_1^2 - \frac{145}{16} g^2_{YB} g_1^2 + 1671 g_{YB} g_1^2 \\
&+ 1512 g^2_{YB} g_1^2 g_1^2 + \frac{9}{8} g^2_{YB} \lambda g_1^2 + \frac{27}{2} \left( \frac{\lambda^2}{5} \right)^2 \\
&- 15 \lambda^2 g_3 - 40 \lambda_2 \lambda_3^2 + \frac{17}{4} \left( \gamma_1^2 \lambda g_3^2 \\
&+ 36 g^2_{YB} g_1^2 + 36 \frac{2}{5} g^2_{YB} \lambda g_1^2 \\
&+ 384 g^2_{YB} \lambda^2 g_1^2 - 48 \sqrt{15} \gamma_1^2 g_{BY} g^2_{YB} g_1 g_4^2 \\
&+ 8 \sqrt{\frac{3}{5}} g_{YB} \lambda g_3 g_4^2 + 24 \lambda_3 \left( g_{YB} + \frac{2}{3} g^2_{YB} \\
&+ 5 \gamma_1^2 g_1^2 \left( \frac{9}{4} g^2_{YB} + 8 g^2_{YB} + \frac{2}{3} g^2_{YB} + \sqrt{\frac{5}{2}} g_{YB} \\
&+ \frac{36}{5} \lambda^2 - \frac{12 \lambda_3}{5} + \frac{\sqrt{5}}{5} g_{YB} g_4 \\
&+ \frac{1}{4} \gamma_1^2 \left( \frac{72}{5} g^2_{YB} + 30 \lambda_3 g^2_{YB} \\
&+ 48 \sqrt{6} g_{YB} g_{YB} g_{YB} g_4 + 8 \lambda^2 + \frac{48}{5} g^2_{YB} g_1^2 \\
&+ 20 \lambda_3 g^2_{YB} - 32 \lambda_2 \lambda_3 \right) \gamma_1^2 
\right) Y_1^0.
\end{align*}

References

1. W.A. Bardeen, Naturality in the Standard Model. FERMILAB-CONF-95-391-T

2. G. Marques Tavares, M. Schmaltz, W. Skiba, Higgs mass naturalness and scale invariance in the UV. Phys. Rev. D 89(1), 015009 (2014) doi:10.1103/PhysRevD.89.015009. arXiv:1308.0025 [hep-ph]

3. S. Coleman, E. Weinberg, Radiative corrections as the origin of spontaneous symmetry breaking. Phys. Rev. D 7, 1888 (1973)

4. R. Hempfling, The next-to-minimal Coleman-Weinberg model. Phys. Lett. B 379, 153 (1996). arXiv:hep-ph/9604278

5. K.A. Meissner, H. Nicolai, Conformal symmetry and the standard model. Phys. Lett. B 648, 312 (2007). arXiv:hep-th/0612165
6. J.R. Espinosa, M. Quiros, Novel effects in electroweak breaking from a hidden sector. Phys. Rev. D 76, 076004 (2007). arXiv:hep-ph/0701145

7. W.F. Chang, J.N. Ng, J.M.S. Wu, Shadow Higgs from a scale-invariant hidden U(1) model. Phys. Rev. D 75, 115016 (2007). arXiv:hep-ph/0701254

8. R. Foot, A. Kobakhidze, R.R. Volkas, Electroweak Higgs as a pseudo-Goldstone boson of broken scale invariance. Phys. Lett. B 655, 156 (2007). arXiv:0704.1165 [hep-ph]

9. R. Foot, A. Kobakhidze, K.L. McDonald, R.R. Volkas, Neutrino mass in radiatively-broken scale-invariant models. Phys. Rev. D 76, 075014 (2007). arXiv:0706.1829 [hep-ph]

10. R. Foot, A. Kobakhidze, K.L. McDonald, R.R. Volkas, A solution to the hierarchy problem from an almost decoupled hidden sector within a classically scale invariant theory. Phys. Rev. D 77, 035006 (2008). arXiv:0709.2750 [hep-ph]

11. K.A. Meissner, H. Nicolai, Effective action, conformal anomaly and the issue of quadratic divergences. Phys. Lett. B 660, 260 (2008). arXiv:0710.2840 [hep-th]

12. K.A. Meissner, H. Nicolai, Neutrinos, axions and conformal symmetry. Eur. Phys. J. C 57, 493 (2008). arXiv:0803.2814 [hep-th]

13. M. Holthausen, M. Lindner, M.A. Schmidt, Radiative symmetry breaking of the minimal left-right symmetric model. Phys. Rev. D 82, 055002 (2010). arXiv:0911.0710 [hep-ph]

14. S. Iso, Y. Orikasa, TeV Scale B-L model with a flat Higgs potential. Phys. Rev. Lett. B 79, 493 (2007). arXiv:0704.1165 [hep-ph]

15. S. Oda, N. Okada, Y. Sato, Higgs boson mass in radiatively-broken scale-invariant models. Phys. Rev. D 89, 055020 (2014). arXiv:1307.8428 [hep-ph]

16. C.D. Carone, R. Ramos, Classical scale-invariance, the electroweak scale and vector dark matter. Phys. Rev. D 88, 055020 (2013). arXiv:1305.0444 [hep-ph]

17. C.D. Carone, R. Ramos, Dark chiral symmetry breaking and the origin of the electroweak scale. Phys. Lett. B 746, 424 (2015). arXiv:1505.0435 [hep-ph]

18. A. Farzinnia, H.J. He, J. Ren, Natural electroweak symmetry breaking from scale invariant higgs mechanism. Phys. Lett. B 727, 141 (2013). arXiv:1308.0295 [hep-ph]

19. E. Gabrielli, M. Heikinheimo, K. Kannike, A. Racioppi, M. Raidal, C. Spethmann, K. Tuominen, Physical naturalness and dynamical breaking of classical scale invariance. Mod. Phys. Lett. A 29, 11450077 (2014). arXiv:1304.7006 [hep-ph]

20. A. Farzinnia, J. Ren, Higgs partner searches and dark matter phenomenology in a classically scale invariant Higgs Boson sector. Phys. Rev. D 90(1), 015017 (2014). arXiv:1405.0498 [hep-ph]

21. M. Lindner, S. Schmidt, J. Smirnov, Neutrino masses and conformal-electroweak symmetry breaking. JHEP 1410, 177 (2014). arXiv:1405.6204 [hep-ph]

22. W. Allmannaeh, W.A. Bardeen, M. Bauer, M. Carena, J.D. Lykken, Light dark matter, naturalness, and the radiative origin of the electroweak scale. JHEP 1501, 032 (2015). arXiv:1408.3429 [hep-ph]

23. F. Goertz, Electroweak symmetry breaking without the $\mu^2$ term. arXiv:1504.00355 [hep-ph]

24. A. Farzinnia, H. Yamaguchi, Vacuum stability in the $U(1)_Y$ extended model with vanishing scalar potential at the Planck scale. arXiv:1504.05609 [hep-ph]

25. N. Haba, H. Ishida, N. Okada, Y. Yamaguchi, Bosonic seesaw mechanism in a classically conformal extension of the standard model. arXiv:1508.06828 [hep-ph]

26. V.V. Khoze, C. McCabe, G. Ro, Higgs vacuum stability from the dark matter portal. JHEP 1408, 026 (2014). arXiv:1403.4953 [hep-ph]

27. S. Oda, N. Okada, D.S. Takahashi, Classically conformal U(1) extended standard model and Higgs vacuum stability. Phys. Rev. D 92(1), 015026 (2015). arXiv:1504.06291 [hep-ph]

28. S. Ito, N. Okada, Y. Orikasa, Classically conformal $B - L$ extended standard model. Phys. Lett. B 676, 81 (2009). arXiv:0902.4050 [hep-ph]

29. S. Ito, N. Okada, Y. Orikasa, The minimal B-L model naturally realized at TeV scale. Phys. Rev. D 80, 115007 (2009). arXiv:0909.0128 [hep-ph]

30. F. Staub, SARAH 3.2: Dirac Gauginos, UFO output, and more. Comput. Phys. Commun. 184, 1792 (2013). arXiv:1207.0906 [hep-ph]

31. F. Staub, SARAH 4: a tool for (not only SUSY) model builders. Comput. Phys. Commun. 185, 1773 (2014). arXiv:1309.7223 [hep-ph]
50. V.D. Barger, W.Y. Keung, E. Ma, Doubling of weak gauge bosons in an extension of the standard model. Phys. Rev. Lett. 44, 1169 (1980). doi:10.1103/PhysRevLett.44.1169

51. G. Aad et al. [ATLAS Collaboration], Search for high-mass dilepton resonances in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector. Phys. Rev. D 90(5), 052005 (2014). arXiv:1405.4123 [hep-ex]

52. CMS Collaboration [CMS Collaboration], Search for resonances in the dilepton mass distribution in pp collisions at $\sqrt{s} = 8$ TeV. CMS-PAS-EXO-12-061;

53. V. Khachatryan et al. [CMS Collaboration], Search for physics beyond the standard model in dilepton mass spectra in proton-proton collisions at $\sqrt{s} = 8$ TeV. JHEP 1504, 025 (2015). arXiv:1412.6302 [hep-ex]

54. N. Okada, S. Okada, $Z'_BL$ portal dark matter and LHC Run-2 results. arXiv:1601.07526 [hep-ph]

55. S. Patra, F.S. Queiroz, W. Rodejohann, Stringent dilepton bounds on left-right models using LHC data. arXiv:1506.03456 [hep-ph]

56. A. Alves, A. Berlin, S. Profumo, F.S. Queiroz, Dirac-Fermionic dark matter in $U(1)_X$ models. arXiv:1506.06767 [hep-ph]

57. L. Basso, A. Belyaev, S. Moretti, G.M. Pruna, Probing the Z-prime sector of the minimal B-L model at future linear colliders in the $e^+e^- \rightarrow \mu^+\mu^-$ process. JHEP 0910, 006 (2009). arXiv:0903.4777 [hep-ph]