Envelope solitons induced by high-order effects
of light-plasma interaction

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The nonlinear coupling between the light beams and non-resonant ion density perturbations in a plasma is considered, taking into account the relativistic particle mass increase and the light beam ponderomotive force. A pair of equations comprising a nonlinear Schrödinger equation for the light beams and a driven (by the light beam pressure) ion-acoustic wave response is derived. It is shown that the stationary solutions of the nonlinear equations can be represented in the form of a bright and dark/gray soliton for one-dimensional problem. We have also present a numerical analysis which shows that our bright soliton solutions are stable exclusively for the values of the parameters compatible with of our theory.

I. INTRODUCTION

Recently, the investigations concerning the nonlinear dynamics governed by a multidimensional cubic-quintic nonlinear Schrödinger equation (NLSE) received a great deal of attention. In this context, both localized vortex solitons and nonlocalized optical vortices were studied\(^1\). Cubic-quintic (2 + 1)-dimensional NLSE were used to study the stability of spinning ring solitons\(^2\) and theoretical investigations to find solitary solutions for the cubic-quintic (1+1)-dimensional NLSE have been carried out. Dark solitary waves in the limit of small amplitude have been found, where the NLSE was reduced to a Korteweg-de Vries equation (KdVE)\(^3\). Moreover, both algebraic solitary wave solutions\(^4\) and traveling-wave solutions\(^5\) have been found and criteria for existence and stability of soliton solutions have been established\(^6\). Additionally, a theory which connects envelope solitons of a wide class of generalized NLSEs with solitons of a wide class of generalized KdVE have been recently carried out for arbitrary amplitudes\(^7\); in particular, the theory was applied to find analytical bright, gray and dark envelope soliton solutions of the cubic-quintic NLSE and some other types of nonlinearities\(^8\)\(^-\)\(^9\).

It is well known that nonlinear interactions between intense laser beams and a plasma are responsible for numerous nonlinear phenomena including parametric instabilities\(^10\), density cavitation, self-focusing and filamentation of light\(^11\)\(^-\)\(^13\), as well as the generation of wake fields\(^14\). Intense laser beams can cause density modifications through the ponderomotive force, enhance the electron mass due to relativistic effects and produce electron Joule heating. The interplay between the ponderomotive, relativistic and Joule heating nonlinearities has been examined\(^15\) in the context of laser plasma experiments and also in ionospheric modifications of the Earth’s ionosphere by powerful radar beams.

In this Letter, we investigate nonlinear interactions between the light beams and the non-resonant density perturbations, taking into account the combined effects of the light pressure induced ion density fluctuations and increased electron mass. We show that, under suitable physical conditions for which our system can be described by a (1+1)-dimensional cubic-quintic NLSE for the complex electromagnetic field amplitude, bright, gray and dark envelope solitons are analytically found. Finally, a stability analysis is carried out, which shows that our bright soliton solutions are stable.

II. BASIC EQUATIONS

We consider the propagation of a large amplitude circularly polarized electromagnetic wave with an electric field \(E = E(\hat{x} + i\hat{y})\exp(-i\omega t + ikR)\), where \(\omega\) is the wave frequency and \(k\) is the wavevector. The light equation in the presence of electron density perturbations in a plasma is obtained from

\[
\nabla B = \frac{4\pi}{c} J + \frac{1}{c} \partial_t E, 
\]

with
\[ \mathbf{J} = -e(n_0 + n_1)\mathbf{v}_e, \quad (2) \]

\[ \mathbf{B} = \nabla \mathbf{A}, \quad (3) \]

\[ \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{A}, \quad (4) \]

and

\[ \partial_t \mathbf{p}_e = -e\mathbf{E}, \quad (5) \]

where \( \mathbf{B} \) is the wave magnetic field, \( \mathbf{A} \) is the vector potential, \( n_0 \) and \( n_1 \) are the unperturbed and perturbed electron number densities, \( \mathbf{v}_e \) is the particle quiver velocity induced by the photons, \( \mathbf{p}_e = m_e \mathbf{v}_e \) is the momentum, \( m_e = m_0/(1 - v_e^2/c^2)^{1/2} \) is the mass, \( m_0 \) is the rest electron mass, \( e \) is the magnitude of the electron charge, and \( c \) is the speed of light in vacuum. The perturbation of the number density \( n_1 \) is reinforced by the light ponderomotive force. For our purposes, we have

\[ \mathbf{v}_e = \frac{e}{m_0 c} \frac{\mathbf{A}}{\gamma_e} \quad (6) \]

in view of (4) and (5). Here we have denoted \( \gamma_e^2 = \sqrt{1 + \epsilon^2 A^2/m_0^2 c^2} \). Combining (1), (2), (3) and (6) we obtain

\[ \partial_t^2 \mathbf{A} - c^2 \nabla^2 \mathbf{A} + \omega_p^2 (1 + N) \frac{\mathbf{A}}{\gamma_e} = 0, \quad (7) \]

where \( \omega_p = (4\pi n_0 e^2/m_0)^{1/2} \) is the plasma frequency, \( N = n_1/n_0 \), and where we have introduced the Coulomb gauge \( \nabla \mathbf{A} = 0 \). We note that the term proportional to \( n_1 \) comes from the beating of the light quiver velocity with the electron density perturbations of the plasma slow motion, while the nonlinear term in \( \gamma_e \) arises due to the electron mass increase in the light wave fields. Supposing that \( \mathbf{A} = \mathbf{A}_s(r, \tau) \exp(i k \mathbf{r} - i \omega t) + \text{complex conjugate} \), where \( r \) and \( \tau \) represent slowly varying space and time coordinates, we obtain from (7)

\[ 2i\omega(\partial_r + v_g \nabla_r)\mathbf{A}_s + c^2 \nabla^2 \mathbf{A}_s + \Omega^2 \mathbf{A}_s - \frac{\omega_p^2 (1 + N) \mathbf{A}_s}{\gamma_e} = 0, \quad (8) \]

where \( v_g = kc^2/\omega \) is the group velocity of the light wave, and \( |\partial_r \mathbf{A}| \ll \omega \mathbf{A} \) has been invoked in view of the WKB approximation. We have denoted \( \Omega^2 = \omega^2 - c^2 k^2 \). We now derive the equation for low-phase velocity (in comparison with the electron thermal speed) density perturbations that are driven by the light wave ponderomotive force. The governing equations are the inertialess electron momentum equation

\[ 0 = e\nabla_e \phi - m_0 c^2 \nabla_e \gamma_e - T_e \nabla_e \ln(n_e/n_0), \quad (9) \]

the ion continuity equation

\[ \partial_t n_i + \nabla_r (n_i \mathbf{u}_i) = 0, \quad (10) \]

and

\[ \partial_t \mathbf{u}_i + (\mathbf{u}_i \nabla_r) \mathbf{u}_i = -\frac{e}{m_i} \nabla_r \phi - \frac{T_e}{m_i} \nabla_r \ln(n_i/n_0), \quad (11) \]

where \( \phi \) is the ambipolar potential, \( \mathbf{u}_i \) is the fluid velocity associated with the plasma slow motion, and \( T_e(T_i) \) is the electron (ion) temperature. The second term on the right-hand side of (9) represents the light pressure. Equations (9) to (11) form a closed system when the quasi-neutrality \( n_e = n_i \) is invoked. The light ponderomotive force acting on the ion fluid is insignificant. Equation (9) shows that the electrons are pushed away from the region of maximum light intensity, and reinforce a space charge field \(-\nabla \phi\) and the associated density fluctuations. The light ponderomotive force is transmitted to ions through the space charge field/ambipolar potential. Adding (9) and (11) and letting \( n_{e,i} = n_0 + n_1 \), \( \mathbf{u}_i = \mathbf{u}_{i0} + \mathbf{u}_{i1} = \mathbf{u}_{i1} \), we obtain for \( n_1 \ll n_0 \) and \( |(\mathbf{u}_{i1} \nabla_r)\mathbf{u}_{i1}| \ll |\partial_r \mathbf{u}_{i1}| \)

\[ \partial_t \mathbf{u}_{i1} = -\frac{m_0}{m_i} c^2 \nabla_r \gamma_e c_1 - \frac{c^2}{n_0} \nabla_r n_{i1}, \quad (12) \]
where for consistency we have supposed \( e^2 A^2 / m_0 \gamma e^4 << 1 \) and, consequently, introduced a small perturbation \( \gamma_{e1} \) of the electron relativistic factor \( \gamma_e (\approx 1 + \gamma_{e1}) \), where \( \gamma_{e1} = e^2 A^2 / 2m_0 \gamma e^4 \). \( \gamma_{e1} \) = \([T_e + T_i] / m_i \]^{1/2} is the effective sound speed. According to the above approximations, Equation (8) becomes

\[
2i\omega (\partial_r + v_g \nabla_r) A_s + c^2 \nabla_r^2 A_s + (\Omega^2 - \omega_p^2) A_s - \omega_p^2 [(N - \gamma_{e1}) - N\gamma_{e1}] A_s = 0,
\]

and (12), combined with the linearized version of (10), yields

\[
\partial_r^2 N - C_s^2 \nabla_r^2 N = \frac{m_0 c^2}{m_i} \nabla_r^2 \gamma_{e1}.
\]

Equations (13) and (14) are the desired equations for coherent light beams that are coupled with non-resonant density perturbations in an electron-ion plasma. Note that, in principle, the quantities \((N - \gamma_{e1})\) and \(N\gamma_{e1}\), involved in Eq. (13), could be of the same order. In the following, this physical circumstance will be considered and, to this end, we seek possible stationary nonlinear solutions of equations (13) and (14) in the form of envelope solitons.

### III. ENVELOPE SOLITONS

We introduce \( \xi = r - V \tau \), where \( V \) is the velocity of the nonlinear waves, and assume \( A_s = a(\xi) \exp(-i\Omega_0 \tau) \), where \( \Omega_0 \) is a constant. Hence, we readily obtain from (13) and (14)

\[
2i\omega \left[ (-V + v_g) \cdot \nabla_\xi \right] a + c^2 \nabla_\xi^2 a + (\Omega^2 - \omega_p^2) a - \omega_p^2 [(N - \gamma_{e1}) - N\gamma_{e1}] a = 0,
\]

and

\[
(V \cdot \nabla_\xi)^2 N - C_s^2 \nabla_\xi^2 N = \frac{m_0 c^2}{m_i} \nabla_\xi^2 \gamma_{e1}.
\]

For the sake of simplicity, we consider here the one dimensional case for which can write \((V \cdot \nabla_\xi)^2 N = V^2 \partial_\xi^2 N\). Consequently, (16) can be immediately integrated, giving

\[
N = \frac{m_0 c^2}{m_i (V^2 - C_s^2)} \gamma_{e1}.
\]

By taking into account the explicit expression of \( \gamma_{e1} \), choosing \(|V| = |v_g| >> C_s\), and combining (15) and (17), we easily get

\[
\frac{1}{2} \partial_\eta^2 \Psi + \lambda^2 \Psi - \left[ q_1 |\Psi|^2 + q_2 |\Psi|^4 \right] \Psi = 0,
\]

where we have introduced the following dimensionless quantities

\[
\eta = c \xi / \omega_p,
\]

\[
\Psi = \frac{e a}{\sqrt{2} m_0 c^2},
\]

\[
\lambda^2 = \frac{\omega^2 - c^2 k^2 - \omega_p^2 + 2\omega \Omega_0}{2\omega_p^2},
\]

\[
q_1 = \frac{1}{2} (\mu - 1)
\]

and

\[
q_2 = -\frac{1}{2} \mu,
\]

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with \( \mu = m_0c^2/m_1(V^2 - C_e^2) \approx m_0c^2/m_1V^2 \).

Note that, since \( \mu > 0 \), \( q_2 \) is a negative quantity. Additionally, since \( N \) and \( \gamma_c \) must be of the same order, from (17) is evident that \( \mu \) is of the order of the unity (\( \mu \sim 1 \)), but slightly greater than 1. This circumstance is satisfied when we choose, consistently, a group velocity \( v_g = V \sim (m_0/m_1)^{1/2}/c \). This justifies why we have kept both the nonlinear terms in (18); accordingly, the terms \( q_1|\Psi|^2 \) and \( q_2|\Psi|^4 \) are of the same order. In particular, if \( \mu \) is exactly equal to 1 (i.e., we have exactly \( V = (m_0/m_1)^{1/2} \)), the (18) becomes

\[
\frac{1}{2} \partial^2_\eta \Phi + \lambda^2 \Phi - q_2|\Psi|^4 \Phi = 0,
\]

which shows that a part of the relativistic mass variation nonlinearity is exactly cancelled out by the light ponderomotive force driven supersonic electron density contribution.

If we put

\[
\Phi(\eta, s) = \Psi(\eta) \exp \left(-i\lambda^2 s\right),
\]

where \( s \) is a new dimensionless timelike variable, the (18) can be cast as

\[
i\partial_\eta \Phi + \frac{1}{2} \partial^2_\eta \Phi - \left[q_1|\Phi|^2 + q_2|\Phi|^4\right] \Phi = 0,
\]

Let us suppose that \( \mu \) is (slightly) larger than the unity. In this way \( q_1 > 0 \), (21) admits bright, gray and dark envelope solitonlike solutions. In fact, from the results of recent investigations \([7]\) that have found a wide class of envelope solitonlike solutions of (21), one can find, through (20), the following solitonlike solution for \( \Psi(\eta) \)

\[
\Psi(\eta) = i\sqrt{\pi}[1 + \left. \frac{2e}{\sqrt{1-e^2}} \right] \frac{\arctan \left( \frac{(\epsilon - 1) \tanh \left( \eta/2\Delta \right)}{\sqrt{1-e^2}} \right) + i\phi_0}{\sqrt{1 - \frac{64|\mu|^2V_0^2}{3(\mu - 1)^2}}} ,
\]

where \( \phi_0 \) is an arbitrary real constant, \( \pi = -3q_1/(8q_2) = 3(\mu - 1)/(8\mu) \), the constants \( \epsilon, \Delta \) and \( B \) are given, respectively, by

\[
\epsilon = \pm \sqrt{1 - \frac{32q_2|V_0^2|}{3q_1^2}} = \pm \sqrt{1 - \frac{64|\mu|^2V_0^2}{3(\mu - 1)^2}},
\]

\[
\Delta = \left( 2 \sqrt{\frac{2}{3q_1^2} - \frac{64|\mu|^2|V_0^2|}{128(\mu - 1)^2}} \right)^{-1} = \left( 2 \sqrt{\frac{3(\mu - 1)^2}{128(|\mu|) + \frac{V_0^2}{2}}} \right)^{-1},
\]

\[
B = V_0\Delta ,
\]

provided that

\[
\lambda^2 = \frac{15(\mu - 1)^2}{128|\mu|} + \frac{V_0^2}{2} ,
\]

and the real constant \( V_0 \) satisfies the condition

\[
-\sqrt{\frac{3(\mu - 1)^2}{64|\mu|}} < V_0 < \sqrt{\frac{3(\mu - 1)^2}{64|\mu|}} .
\]

Accordingly the definition of \( \lambda^2 \) implies that

\[
(\omega^2 - c^2k^2 + 2\omega\Omega_0 = \omega_p^2 \left( 1 + \frac{15(\mu - 1)^2}{64|\mu|} + \frac{V_0^2}{2} \right) ,
\]

which is a condition for the real constant \( \Omega_0 \). According to terminology and the results of Refs. \([8,9]\), we can distinguish the following four cases. (a). \( 0 < \epsilon < 1 \) \( (\Omega_0 \neq 0) \): up-shifted bright soliton.
that now the density fluctuation corresponds to the following solitonlike solution:

\[ u(\eta = 0) = \mp(1 + \epsilon) \], \quad \text{and} \quad \lim_{\eta \to \pm\infty} u(\eta) = \mp \]

which corresponds to a bright soliton of maximum amplitude \((1+\epsilon)\mp\) and up-shifted by the quantity \mp. (b). \(-1 < \epsilon < 0\)

\((V_0 \neq 0)\): \textit{gray soliton}. \[ u(\eta = 0) = \mp(1 - \epsilon) \], \quad \text{and} \quad \lim_{\eta \to \pm\infty} u(\eta) = \mp \]

which is a dark soliton with minimum amplitude \((1-\epsilon)\mp\) and reaching asymptotically the upper limit \mp. (c). \(\epsilon = 1\)

\((V_0 = 0)\): \textit{upper-shifted bright soliton}. \[ u(\eta = 0) = 2\mp \], \quad \text{and} \quad \lim_{\eta \to \pm\infty} u(\eta) = \mp \]

which corresponds to a bright soliton of maximum amplitude \(2\mp\) and up-shifted by the maximum quantity \mp. (d). \(\epsilon = -1\) \((V_0 = 0)\): \textit{standard dark soliton}. \[ u(\eta = 0) = 0 \], \quad \text{and} \quad \lim_{\eta \to \pm\infty} u(\eta) = \mp \]

which is a dark soliton (zero minimum amplitude), reaching asymptotically the upper limit \mp. Correspondingly, the (17) gives the following solitonlike solution for the density fluctuation:

\[ N(\eta) = \mu |\Psi(\eta)|^2 = \frac{3}{8} (\mu - 1) \left| 1 + \epsilon \sech(\eta/\Delta) \right| \]  \(\text{ (23)}\)

On the other hand, according to Refs \[1,3,8\], the (19) has the following bright envelope solitonlike solution:

\[ \Psi(\eta) = \left[ \frac{6|E_0|}{\mu} \right]^{1/4} \sech^{1/2} \left[ \sqrt{2|E_0|} \eta \right] \exp(i\phi_0) \]

\(\text{ (24)}\)

where \(\phi_0\) is an arbitrary real constant and \(E_0\) is a negative real constant satisfying the condition \(\lambda^2 = E_0\). The latter implies the following condition for \(\Omega_0 \omega^2 - c_0^2 k^2 + 2\omega \Omega_0 + \omega_p^2 \left(2|E_0| - 1\right) = 0\). Furthermore, the (17) implies that now the density fluctuation corresponds to the following solitonlike solution:

\[ N(\eta) = \mu |\Psi(\eta)|^2 = \left[ \frac{6|E_0|}{\mu} \right]^{1/2} \sech \left[ \sqrt{2|E_0|} \eta \right] \]

\(\text{ (25)}\)

We now investigate the stability of plane wave solutions of the one-dimensional equation (21). We first note that if we set \(q_2 = 0\), the corresponding equation is the well known defocusing cubic Schrödinger equation which is known to be stable. It is therefore interesting to study if the \(q_2\left|\Psi\right|^4\) term can modify the instability. The analysis is performed by seeking a solution corresponding to a uniform wave train perturbed by small disturbances:

\[ \Phi = [\Phi_0 + \rho(s,\eta)] \exp\{ [-q_1|\Phi_0|^2 - q_2|\Phi_0|^4] s + \theta(s,\eta) \}, \]

\(\text{ (26)}\)

where \(\rho\) and \(\theta\) are considered to be small amplitude and small phase perturbations. We then substitute the perturbed solution in (21) and retain only the linear terms in \(\rho\) and \(\theta\). Since the resulting equation is linear we can now assume a solution for the perturbation of the form \(\rho = \rho_0 \exp i[K\eta - \Omega s]\) and \(\theta = \theta_0 \exp i[K\eta - \Omega s]\). The resulting dispersion relation is the following:

\[ \Omega^2 = \frac{K^2}{4} (K^2 + 4q_1|\Phi_0|^2 + 8q_2|\Phi_0|^4) \]

\(\text{ (27)}\)

This shows that the wave train is unstable if the perturbation \(K\) lies in the range of \(0 < K < 2|\Phi_0|\sqrt{-q_1 - 2q_2|\Phi_0|^2}\). According to the definition of \(q_1\) and \(q_2\), the instability will occur only if \(\mu > 1/(1 - 2|\Phi_0|^2)\). The maximum instability occurs at \(K = |\Phi_0|\sqrt{-2q_1 - 4q_2|\Phi_0|^2}\).
IV. NUMERICAL SIMULATIONS

In this section we analyse numerically both the influence of the quintic nonlinearity on the modulational instability and on the stability of a class of solitonlike solutions obtained in the previous sections. Equation (21) is solved numerically using a standard pseudo-spectral code with a second order Runge-Kutta method for advancing in time. We recall that the use of pseudo-spectral code implies the assumption of periodic boundary conditions.

A. Modulational instability

Accordingly to the linear stability analysis performed previously, initial conditions for our numerical simulations are given as follows:

$$\Phi(x,0) = \Phi_0[1 + \varepsilon \cos(Lx)],$$  \hspace{1cm} (28)

where $\varepsilon$ is the amplitude of the small perturbation and is taken as $10^{-2}$ the amplitude of the unperturbed wave. Without loss of generality in our simulations we have chosen $\Phi_0 = 1$ and $L = 1$ and have considered only one period of the perturbation. We have performed several numerical simulations with different values of the parameter $\mu$. For $\Phi_0 = 1$ the theory predicts stability for $\mu < -1$. In Fig. 1 we show the evolution of a plane wave in the $\eta-s$ plane for $\mu = -1.5$. The initial wave field persists for all times. For $\mu > 1$ modulational instability should occur. In Fig. 2 we show the case of $\mu = -0.2$; analogously with the standard modulational instability we observe a Fermi-Pasta-Ulam recurrence: periodically the perturbation grows, the wave reaches a maximum amplitude and then goes back to the initial condition. For $\mu > 0$ a completely different physics takes place: in Fig. 3, obtained for $\mu = 0.5$, we do not observe anymore a recurrence and as time passes the wave amplitude increases while the its width decreases. This phenomenon corresponds to the initial stage of a wave collapse (see [13]).

B. Stability of solitonlike solutions

It is well know that the cubic NLS equations ($q_2 = 0$) possess solitons solutions if $q_1$ is greater than zero. We here investigate numerically if the solitonlike solutions described previously are stable or not. In order to do that we simply consider a solitonlike solution at time $s = 0$ and we let evolve numerically the eq. (21). For semplicity we restrict our analysis to a sub class of solutions which corresponds to the case of bright solitons with $V_0 = 0$. We have performed many different numerical simulations with different values of the parameter $\mu$. The major result obtained is the following: if $\mu > 1$ solutions are stable and for $0 < \mu < 1$ are unstable. This is due to the fact that for $\mu > 1$ the coefficients in front of the cubic and quintic nonlinearities, respectively $q_1$ and $q_2$, have opposite sign and therefore there is a sort of balance between nonlinearities that stabilizes the solitonlike solution. This is not the case for $0 < \mu < 1$: both nonlinearities have the same sign and the dispersion is not strong enough to balance them. In Figs. 4 and 5 we give numerical evidence of the results presented for $\mu = 1.1$ and for $\mu = 0.9$ respectively. The first case, Fig. 4, corresponds to a stable solutions (the wave profile does not change as time $s$ passes). The second case, Fig. 5, is the unstable case: a clear increase in the wave amplitude is shown.

According to the above stability analysis we can conclude that our soliton solutions are stable in the range where $\mu > 1$ only; this inequality, according to section II, is consistent with the conditions for their existence in our problem.

V. CONCLUSIONS

To summarize, we have considered the nonlinear interaction between intense light beams and the non-resonant density perturbations, taking into account the relativistic mass increase of the electrons as well as the light beam ponderomotive force that reinforces the density perturbations in an electron-ion plasma. The nonlinear coupling is governed by a pair of equations which, in one space dimension admit stationary solutions in the form of a planar bright and dark/gray envelope solitons. The condition of stability of the bright soliton-like solutions has been found numerically and it has shown that they are stable just for the range of parameters required in our problem. Unfortunately, the numerical stability analysis for the family of dark and gray solitons, although revealed to be much more difficult, is on the way. Authors expect that the preliminary numerical analysis of the stability carried out in the present paper will be extended to the dark and gray solitons in a future work.
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