Detecting neutrino mass difference with cosmology

Anže Slosar

Faculty of Mathematics and Physics, University of Ljubljana, Slovenia

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Cosmological parameter estimation exercises usually make the approximation that the three standard neutrinos have degenerate mass, which is at odds with recent terrestrial measurements of the difference in the square of neutrino masses. In this paper we examine whether the use of this approximation is justified for the cosmic microwave background (CMB) spectrum, matter power spectrum and the CMB lensing potential power spectrum. We find that, assuming $\Delta m^2_{13} \sim 2.5 \times 10^{-3} \text{eV}^2$ in agreement with recent Earth based measurements of atmospheric neutrino oscillations, the correction due to non-degeneracy is of the order of precision of present numerical codes and undetectable for the foreseeable future for the CMB and matter power spectra. An ambitious experiment that could reconstruct the lensing potential power spectrum to the cosmic variance limit up to $\ell \sim 1000$ will have to take the effect into account in order to avoid biases. The degeneracies with other parameters, however, will make the detection of the neutrino mass difference impossible. We also show that relaxing the bound on the neutrino mass difference will also increase the error-bar on the sum of neutrino masses by a factor of up to a few. For exotic models with significantly non-degenerate neutrinos the corrections due to non-degeneracy could become important for all the cosmological probes discussed here.

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INTRODUCTION

Standard cosmological measurements offer an excellent probe of neutrino physics \[\text{(1)}.\] In contrast to Earth based measurements of neutrino oscillations that measure the difference in the square of neutrino mass eigen states, the cosmology is sensitive to the absolute mass of neutrinos. At the moment, cosmology is the only viable alternative to the beta-decay experiments \[\text{(2)}\] in this field and exceeds it in accuracy.

The most natural and accurate way to measure neutrino masses with cosmology is via measurements of the power spectrum of fluctuations in the cosmic microwave background (CMB) and the power spectrum of matter fluctuations, which is one of the basic products of the galaxy redshift surveys. In the future, the non-Gaussianity introduced by the lensing of the CMB fluctuations by the intervening structures between the last scattering surface and us will prove to be an important tool to constrain the power spectrum of matter fluctuations and consequently neutrino masses \[\text{(3)-(5)}\].

Many of the recent parameter estimation papers have put bounds on the sum of the neutrino masses \[\text{(6)-(9)}\], with the upper bound on the sum of neutrino masses of the order $0.5 \text{eV}$. The three neutrino species were assumed to be degenerate in mass.

However, Earth based measurements of neutrino oscillations suggest not only that neutrinos have mass, but also that these masses are not equal \[\text{(18)-(19)}\]. Neutrino that is produced in a flavour eigenstate, which is some linear superposition of mass eigenstates, can latter be measured as being of a different flavour, due to different mass eigenstates acquiring a different phase during propagation. In particular, oscillations of atmospheric neutrinos suggest a squared mass difference of $|\Delta m^2_{23}| = |m^2_3 - m^2_2| \sim 2.5 \times 10^{-3} \text{eV}^2$ \[\text{(20)-(21)}\], while solar neutrino observations, together with results from the KamLAND reactor neutrino experiment, point towards $\Delta m^2_{12} = m^2_2 - m^2_1 \sim 5 \times 10^{-5} \text{eV}^2$ \[\text{(22)-(23)}\]. Note that in the former case, the sign of the difference is not known, while in the latter it is. Given these constraints, it is possible construct two hierarchies of masses. Assuming the lightest neutrino to be of a negligible mass, the neutrino masses can be either in a so-called normal hierarchy with masses around $\sim 0$, $\sim \sqrt{\Delta m^2_{23}}$, $\sim \sqrt{\Delta m^2_{12}}$ or in an inverted hierarchy, in which case masses are $\sim 0$, $\sim \sqrt{|\Delta m^2_{23}|}$, $\sim \sqrt{|\Delta m^2_{12}|}$. This means that it is possible to rule out the inverted hierarchy simply by measuring the sum of neutrino masses and excluding the $\sum m > 2\sqrt{|\Delta m^2_{23}|}$ region.

Another way of distinguishing between the two hierarchies is to try to measure the neutrino mass difference directly with cosmology. The above results imply that the three neutrino families are non-degenerate at the level of $\sim 5\%$ if the sum of neutrino masses is $0.5 \text{eV}$. Decreasing the sum to $0.2 \text{eV}$, the two neutrinos are non-degenerate at the level of $25\%$. Hence, it is timely to investigate what are the biases introduced by assuming degenerate neutrino masses in cosmological probes of neutrinos and this is exactly what this paper is set to do.

Significant amounts of work in this direction was done before in \[\text{(24)}\]. This paper focuses on the ability of future Large Scale Structure (LSS) and CMB experiments to constrain neutrino masses by performing the Fisher matrix analysis. They were the first to perform the numerical integration of the CMB and LSS power spectra by independently integrating more than one neutrino species.
We extend their work in several aspects. Firstly, we also include the lensing potential reconstruction in the set of datasets used to constrain neutrino masses. This has been studied in another recent paper [4], which shows that it would be impossible to distinguish between normal and inverted hierarchies using even a very optimistic future experiment. Nevertheless, the authors of that paper do not use the multi-neutrino code and do not discuss degeneracy assumption further. Secondly, rather than expanding around a fiducial model, we study the general parameter space (although we also provide Fisher matrix analysis in order to check the effect of degeneracies). Finally, we provide a confirmation of their results by an independent implementation of a linear code with more than one neutrino species. An analysis of future sensitivities of high redshift galaxy surveys and CMB data to measure neutrino masses and number of neutrino species can be found in [23].

Assuming that the standard physics of the early universe applies, the energy density and mass of a given neutrino species are related through

\[ w_\nu = \frac{m_\nu}{94.2 \text{eV}}. \]  

Interactions in the early universe before neutrino decoupling ensure decoherence and so flavour physics does not enter cosmology. On the other hand, changing energy density (or mass) of a given neutrino species, affects the CMB and matter power spectra in two ways. First, it changes the redshift of the matter-radiation equality, thus affecting the position and height of the peaks in the CMB power spectrum and the maximum in the matter power spectrum. Second, it damps the power spectrum on small scales, because relativistic neutrinos in the early universe behave as radiation and effectively stream away from over-dense regions. The free streaming wave-vector is given by

\[ k_{fs} \sim 0.01 \frac{m_\nu}{\text{1eV}} \text{Mpc} \]  

and the matter power spectrum is damped in scales \( k > k_{fs} \). For neutrinos of a small mass the second effect is considerably more important.

In general, the neutrinos become non-relativistic at a redshift of

\[ z_{nr} \sim \frac{m_\nu c^2}{3k_{B} T_\nu} \sim 2 \times 10^4 \left( \frac{m_\nu}{\text{1eV}} \right) \]  

This means that neutrinos lighter that about 0.5eV will become non-relativistic after the recombination and thus have a very small effect on the CMB fluctuations power spectrum. The neutrino mass can, however, be inferred from the gravitational lensing of the CMB fluctuations. The quantity of interest here is the projected gravitational potential (see e.g. [27])

\[ \phi(n) = -2 \int_{0}^{r_*} \Psi(n \nu) \frac{r_* - r}{r r_*} \, dr, \]  

where \( r \) are conformal distances, \( r_* \) is the conformal distance to the surface of the last scattering and the integration is performed along our past light cone. The spherical power spectrum of \( \phi \), \( C_\ell^{\phi} \), can be recovered using a variety of methods [28, 29, 30, 31, 32] and essentially contains information similar to that of the matter power spectrum.

### NON-DEGENERATE NEUTRINOS

If one relaxes the degeneracy assumption, the neutrinos of different masses become non-relativistic at different times and have different free-streaming lengths, resulting in small corrections to the various power spectra discussed above. Since the difference in the squares neutrino masses \( \Delta m^2_{23} \) is over an order of magnitude larger than \( \Delta m^2_{12} \), we will, for the time being, assume the latter is zero. We thus have two neutrinos of the same mass \( m_1 = m_2 \) and the third one of a different mass \( m_3 \). We parametrise the masses in terms of the sum of neutrino masses \( \sum m_i \) and the fraction \( \alpha \) of the total mass in the third neutrino mass eigenstate, so that

\[ m_3 = \alpha \sum m_i \]  

This particular parametrisation has been chosen, because it allows for both extreme possibilities, namely that all of the mass is in the third neutrino (\( \alpha = 1 \)) or that third neutrino is massless (\( \alpha = 0 \)). The value of \( \alpha = 1/3 \) corresponds to the degenerate case.

We use a modified version of the CAMB linear solver [33] that can evolve two families of neutrinos of different masses separately [34]. The accuracy boost parameter was set to 2, which should result in an accuracy around 0.1%. In addition, we used transfers high precision option. We have checked that the results are the same regardless of whether a full hierarchy integration is performed or a switch to series in velocity weight once neutrinos become non-relativistic is used. Other parameters were set to their nominal values for a \( \Lambda \)CDM universe. The CMB power spectra discussed here were lensed using the algorithm discussed in [35].

In Figure 1 we plot the lensed CMB power spectrum, the matter power spectrum and the projected gravitational potential power spectrum for three models containing either three massless neutrinos or three massive neutrinos with \( \sum m_i = 2 \text{eV} \) and \( \alpha = 1/3 \) and \( \alpha = 1 \). The figures correspond to the standard flat \( \Lambda \)CDM cosmology. The energy densities of baryonic and cold dark matter as well as curvature were kept fixed so that hot
FIG. 1: The lensed CMB power spectrum (top), the matter power spectrum (middle) and the lensing potential power spectrum (bottom). Solid line correspond to the standard ΛCDM model. Other two models plotted have $\sum m \sim 2\text{eV}$ and $\alpha = 1/3$ (dotted) or $\alpha = 1$ (dashed). See text for discussion.

FIG. 2: This figure shows the change in the $\chi^2$ for a cosmic variance limited experiment (to $\ell = 2000$) if one wrongly assumes degenerate neutrinos. The contours are at $\Delta \chi^2$ of 1, 5, 25 and 125 (from $\alpha = 1/3$ line outwards). See text for discussion of other features on the plot.

The CMB peak positions (due to change in the sound horizon) and heights (via the early integrated Sachs-Wolfe effect). However, the correction due to non-degeneracy assumption is very small. The matter power spectrum and the lensing potential power spectrum show very similar trends. If one neutrino contains all the mass, it has a smaller free streaming length and consequently damping does not extend to scales as large as in the case where neutrinos have degenerate mass. However, because the total neutrino energy density is the same, the overall effect is the same for $k \gg k_{fs}$. The latter also implies that, contrary to what might be naively expected, the precision measurements at very small scales (such as those probed by Lyman alpha forest) will not be sensitive probes of neutrino mass differences.

RESULTS

How big are the discussed effects for realistic neutrino masses and experiments? To answer this question we calculated power spectra for a grid of models with $\sum m_i$ between 0 and 1eV and $\alpha$ between 0 and 1. In each case the power spectra were calculated using both the approximation that we have 3 degenerate neutrinos with a given $\sum m_i$ and the correct distribution of neutrino masses, evolving the two neutrino species separately.

For the CMB power spectrum, there exist a natural limit for the accuracy with which the power spectrum can ever be measured. The so called cosmic-variance is a result of a finite number of spherical harmonic modes on the sky and is (for $\ell \geq 50$) excellently approximated by a Gaussian distribution with an error given by...
the degeneracy condition (where $\Delta \chi^2$ parameter defined above. The dashed line at literature [9], that is 0.3eV. A few recent preprints find tighter limits using the latest cosmological data: 0.3eV in [37] and 0.17eV in [38]. Solid thin lines are contours of constant $\Delta \chi^2$ as stated in the caption. The two solid thick lines correspond to the value of $\alpha$ required to satisfy the $\Delta m^2_{23} = 2.5 \times 10^{-3} \text{eV}^2$ condition. The upper branch corresponds to the normal hierarchy, while the lower branch corresponds to the inverted hierarchy.

This results shows the expected result that the magnitude of correction due to the non-degeneracy is a function of both relative mass difference and the total sum of the masses. If one takes earth-based measurements of the mass square difference seriously, then it seems that the standard degenerate neutrinos assumption is a good one: the approximation is either saved by being a too small relative effect at larger sum of masses or being a too small absolute effect anyway, when the sum of masses is small. However, if one is not bound by the small mass square difference (when considering exotic neutrino models, for example), then the effect can be quite large and detectable with high precision in the future CMB experiments.

A much more interesting picture emerges if one looks at the reconstructed lensing potential. The reference experiment discussed above could, ideally, reconstruct the lensing potential to a cosmic variance limit up to $\ell = 1000$ [30]. Figure 3 shows contours analogous to that of the Figure 2, but for the lensing potential instead. One can see that for $\sum m_i \lesssim 0.1 \text{eV}$, the non-degeneracy corrections can become important at high confidence. Our estimate is conservative since it assumes that information on lensing potential is not available beyond $\ell = 1000$. A real experiment will still have sensitivity to recover the lensing potential at $\ell > 1000$, albeit with a sub cosmic variance precision. We also note, that linear transfer functions were used, but it is unlikely that non-linear corrections would significantly destroy the sensitivity.

Next we turn to the matter power spectrum. The power spectrum is more difficult to consider as it is not clear what an idealised experiment can do in the presence of non-linear biasing and complications arising from astrophysical considerations. Therefore we calculate the relative change of the slope of the linear power spectrum at three nominal values of $k = 0.005h/\text{Mpc}, 0.01h/\text{Mpc}$ and $0.1h/\text{Mpc}$ (where $h$ is the reduced Hubble’s constant). These are plotted in the Figure 4. We see that the changes in the slope of the linear power spectrum are of the order of $\alpha \sim 0.1\%$ and very likely undetectable in the foreseeable future. Again, we note, however, that if the bound on the mass square difference is abandoned, the non-degeneracy correction can be quite large and exceed the 10% mark in some cases.

**FISHER MATRIX ANALYSIS**

In the above sections we have shown that in some parts of the parameters space the effect of the difference in neutrino masses can produce sizeable $\chi^2$ differences. This implies that neutrino mass difference must be taken into account in order to avoid biases in the data. However, it does not necessarily mean that neutrino mass difference will be detectable due to possible degeneracies with other parameters. In order to check for that effect we perform a Fisher Matrix analysis. We use the following parametrisation of the model

$$\theta_i = (\omega_b, \omega_{cdm}, h, \tau, n_s, A, \sum m, \alpha),$$

$$\sigma_{\text{CV}}(C_\ell) = C_\ell \sqrt{\frac{2}{2\ell + 1}}$$ (6)

For each error in our parameter space we calculated the change in $\chi^2$ induced by the correction stemming from non-degeneracy, i.e.

$$\Delta \chi^2 = \sum_\ell \frac{(C_{\ell, \text{degen}} - C_\ell)^2}{\sigma_{\text{CV}}(C_\ell)^2}$$ (7)

where $C_{\ell, \text{degen}}$ is the theoretical prediction if degeneracy is assumed, for an experiment that is cosmic-variance limited up to $\ell = 2000$. Such measurements could be performed with a future experiment with a beam size of 4', temperature sensitivity of $\Delta T \sim 1\mu\text{K}$ and polarisation sensitivity of $\Delta P = \sqrt{2}\Delta_T \sim 1.4\mu\text{K}$ [31]. The results are plotted in the Figure 2. This figure is worthy some discussion. The horizontal axis is the sum of the neutrino masses, while the vertical axis correspond to the $\alpha$ parameter defined above. The dashed line at $\alpha = 1/3$ shows the degeneracy condition (where $\Delta \chi^2$ is equal to zero by construction) and the vertical dashed line corresponds to the tightest limit on the sum of neutrino mass found in literature [3], that is 0.4eV. A few recent preprints find

FIG. 3: Same as Figure 2 but for the lensing potential reconstruction that is cosmic variance up to $\ell = 1000$. 

$$\sum m \ [\text{eV}]$$ 

$0$ $0.5$ $1$ 

$\alpha$ 

$\ell$ 

$0$ $0.5$ $1$ 

The thick solid lines correspond to the value of $\alpha$ required to satisfy the $\Delta m^2_{23} = 2.5 \times 10^{-3} \text{eV}^2$ condition. The upper branch corresponds to the normal hierarchy, while the lower branch corresponds to the inverted hierarchy.
where parameters have their usual meaning in the cosmological context. The Fisher matrix is given by

\[ F_{ij} = \sum \frac{\partial C_i}{\partial \theta_j} \frac{\partial C_j}{\partial \theta_i} (\text{Cov}^{-1})_{XY} \]

where \( XY \) is either TT, EE, TE (temperature, E polarisation and cross power spectra) and LL (lensing potential cross-spectrum). Since the exact experimental parameters are not the focus of this work, we simply assumed that the CMB TT power spectrum is known to cosmic variance for \( \ell < 2000 \), while TE, EE and lensing spectra are known to cosmic variance for \( \ell < 1000 \).

The interpretation of the Fisher matrix is straightforward: \( (F_{ii})^{-1/2} \) is the expected 1 – \( \sigma \) error on the measurement of the \( i \)-th parameter, assuming all other parameters to be fixed. The value of \( (F_{ii}^{-1})^{1/2} \) gives the expected 1 – \( \sigma \) error on the measurement of the \( i \)-th parameter taking into account possible degeneracies with other parameters while the direction of these degeneracies are given by the eigenvectors of \( F \). A Gaussian nature of the posterior is assumed throughout.

We have performed the Fisher matrix analysis, using the nominal values for most parameters, \( (\omega_m, \omega_{cdm}, h, \tau, n_s, A) = (0.02, 0.12, 0.7, 0.11, 1.0, 2.3 \times 10^{-9}) \). For the values of the remaining two parameters we took two representative points, one for the normal and one for the inverted hierarchy, which also satisfy the atmospheric neutrinos constraint. We have also performed the analysis by fixing \( \alpha = 1/3 \) and excluding that parameter from the analysis. The results are shown in the Table 1.

![Table 1](image)

| parameter | Normal | Hierarchy | Inverted | Degenerate 1 | Degenerate 2 |
|-----------|--------|-----------|----------|-------------|-------------|
| \( \sum m_i/eV \) | \( 0.055 \) | \( 0.105 \) | \( 0.055 \) | \( 0.105 \) | \( 0.055 \) |
| \( \alpha \) | \( 0.95 \) | \( 0.043 \) | \( 1/3 \) | \( 1/3 \) | \( - \) |

There are a few interesting conclusions to be drawn. Firstly, we find a reasonable agreement (given the crude approximation of experimental performance) with [4] for the degenerate case, where we find that the marginalised error on the sum of neutrino masses will be of the order \( 0.013 - 0.04 \) eV. Secondly, we note that the degeneracies with other parameters completely destroy the detection of the neutrino mass difference. From a modest few sigma detection, the error increases several-fold. Thirdly, assuming degeneracy severely decreases the accuracy with which the sum of neutrino masses can be measured. Indeed, the analysis of the eigenvectors of the Fisher matrix show that the two strongest degeneracies involve \( m, \alpha \) and \( h \) in one eigenvector and \( \omega_{cdm}, \alpha \) and \( h \) in the other. The latter is yet another face of the degeneracy discussed in [3].

**CONCLUSIONS**

In this paper we have examined the importance of the degeneracy assumption, which is often used in literature to constrain the sum of masses of neutrinos from cosmology. Since the combination of the measurements of neutrino oscillations from the earth-based experiments with the upper limit on neutrino masses from the cosmological experiments imply that the neutrinos are non-degenerate at the level of at least several percent, it is not obvious that this assumption is a justified one.

By comparing model predictions for a model with two different neutrino masses with that of an equivalent model with three degenerate neutrinos we were able to show that the degeneracy assumption is indeed valid for the constraints based on the CMB and galaxy power spectra. In the case of CMB, this is true even for the cosmic variance limited experiment, while the matter power spectra are unlikely to reach the accuracy required to detect neutrino mass difference due to various astrophysical constraints such as scale-dependent biasing. Moreover, the size of the effect is at the level of numerical precision of the present generation linear codes.
FIG. 4: This figure shows the relative change in the matter power spectrum slope at $k = 0.005h$/Mpc (top), $0.01h$/Mpc (middle) and $0.1h$/Mpc (bottom). The thin solid and dotted lines correspond to 0.1% (dotted), 1% and 10% difference and increase in value from the thin horizontal dashed line outwards.

The best hope for detecting neutrino mass difference with cosmology lies in the CMB lensing potential reconstruction. In an idealised future experiment such as that of [36], the neutrino mass difference could bias the results of parameter estimation for $\sum m_i \gtrsim 0.4$eV, since it significantly affects the $\chi^2$. However, recent analysis have shown that the number and masses of neutrino species are significantly constrained even with the present generation cosmological data and thus we will not explore this further.

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