Uncertainties in testing QCD with the photon structure function

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Abstract

We review the different approaches used to treat the photon structure function. We suggest that despite some uncertainties it should remain sensitive to $\Lambda$. 
I. INTRODUCTION

As the years go by, QCD seems to remain the only serious candidate for a theory of strong interactions. It has not encountered yet any unsurmountable disagreement. But this is in part because, from a practical point of view, it has a weakness, the non-perturbative sector of the theory which is still largely out of theoretical reach. Our inability to reach a non-perturbative solution is indeed frustrating. Most, if not all, QCD tests has to deal with some important non-perturbative component. This leaves us with two choices: improve the non-perturbative methods (the final test of QCD may eventually come from there), or isolate the non-perturbative effects, describing them with phenomenologically inspired models or simple parameterizations, to base the discussion mainly on the calculable parts. Of course, the credibility of QCD tests has suffered from this uncertainty especially with regard to the dynamical properties, in hadron collisions for example.

There is actually one process which does not quite fit this description in the sense that QCD predicts its shape, normalization as well as $Q^2$-evolution. This process is $\gamma\gamma^*$ deep-inelastic scattering where a real photon structure function can be extracted. The non-perturbative effects are not entirely absent though but they contribute to higher orders in $\alpha_s$, the strong coupling constant. One of the peculiarity of this process is that the structure function has a strong dependence on $\alpha_s(Q)$ which makes it a good candidate to the determination of the QCD scale parameter, $\Lambda$, as was pointed out by Witten [1].

Two major contributions to the photon structure function can be identified: the point-like part, which dominates in the high energy limit and provides the predictions for shape and normalization, and the hadronic piece, on which our knowledge is more limited. One of the problem in the past had to do with the separation of these two pieces, which share cancelling singularities. There are two prevailing attitude with that respect: One can try to exploit as much as possible the QCD prediction by introducing a proper regularization technique. This method introduce some uncertainties which has been discussed in several papers [2]. On the other hand, the singularities can also be ignored if one is only interested in the $Q^2$-evolution of the structure function by means of a subtraction at a given momentum scale $Q^2_0$ or $p_T \gg \Lambda^2$. In this more conservative approach however, the sensitivity to $\Lambda$ and the chance to test QCD are almost completely lost.

In recent years, experimentalists have been led to analyze their data only with respect to the second alternative. Clearly, deep-inelastic $ep$ scattering is better placed to provide a test of QCD’s prediction for the $Q^2$-evolution of structure functions. In this talk, I will try to argue that one should also look at the first approach and discuss briefly about some misconceptions regarding both techniques.

II. WITTEN’S ARGUMENT ON SENSITIVITY TO $\Lambda$

The QCD prediction for $F^\gamma_{2,n}(Q^2)$ (moments of the real photon structure function) is

$$F^\gamma_{2,n}(Q^2) \underset{Q^2 \gg \Lambda^2}{\simeq} \frac{a_n}{\alpha_s^2} + b_n + \text{(higher orders in } \alpha_s)$$

comes from the point-like part

where $\alpha_s \equiv \alpha_s(Q/\Lambda)$. The first two terms comes from the so-called "point-like" part.
As mentioned above, the photon structure function can be separated into two pieces. First and most important, is the point-like contribution. This part is essentially due to the point-like nature of the photon and is absent in the structure functions encountered in deep-inelastic e-p scattering. The second contribution is the so-called hadronic contribution and, hopefully, includes all non-perturbative QCD effects that may contribute. What characterizes a point-like object is that no matter how hard you hit it with a probe the object shows no structure, in other words, the structure is independent of any momentum scale. However, turning on strong interactions implies the presence of at least two mass scales, the QCD Λ parameter and the $Q^2$ of the probe through the running coupling constant $\alpha_s(Q)$. These are also the only scales possible in a “point-like QCD” process. Of course, strong interactions may be characterized by other intermediate mass scale. Such a point of view is advocated in [3] where the authors argue that some high-$p_T$ processes give indications of an intermediate scale, $Q_0$, above which perturbative effects seem to dominate. They then proceed by saying that only contributions with $Q^2 \geq Q_0^2$ participate to the point-like structure function. Needless to say that this redefinition of “point-like” reduces this component with respect to the hadronic component enough to jeopardize almost all hopes of testing QCD in $\gamma\gamma$-scattering [2].

III. THE HADRONIC COMPONENT AND THE SINGULARITIES

Let us rewrite the photon structure function in more details:

$$F_{2,n}^{\gamma}(Q^2) = \frac{a_n}{\alpha_s} + b_n + \cdots + A_n [\alpha_s]^d_n$$

Here, for simplicity we have omitted higher orders in $\alpha_s$ in the point-like part (denoted here by $[\cdots]$), higher orders corrections to the hadronic part which would result in a power series multiplying the last term and finally that the hadronic part do in fact get three distinct contributions $A_i^n[\alpha_s]^d_n$ usually denoted by $i = +, -, NS$ (non-singlet).

A number of comments are in order regarding this last expression to clarify the separation between the point-like and hadronic parts: (1) The point-like and hadronic part, which we denote from here on by $F_{2,n}^{PL}(Q^2)$ and $F_{2,n}^{HAD}(Q^2)$ respectively, are both plagued by the presence of singularities. These are characterized by poles at given n’s for moments of the structure function or equivalently by singular behavior in the low x region in the x-dependent structure function. Fortunately, each singularity in $F_{2,n}^{PL}(Q^2)$ is cancelled by a similar singularity in $F_{2,n}^{HAD}(Q^2)$. The total $F_{2,n}(Q^2)$ is regular and physically measurable as it should. (2) The point-like part should be in principle independent of any scale since point-like objects look the same no matter how hard one probes them. This is nearly the case here since the only dependence on scale comes from $\alpha_s$ which it is caused by vacuum polarization not by a structured object. Note also that $\alpha_s$ depends on the only two scales defined in this problem $Q$ and $\Lambda$. In principle, one can introduce arbitrary intermediate scales in this problem but $F_{2,n}(Q^2)$ is clearly independent on these scales. For parameterization purposes, such scales are often introduced however one should keep in mind that QCD gives no clear indications on which scales should be preferred. (3) $A_n$ is the only non-perturbative (and in practice non-calculable) object in this equation. Unlike in the virtual photon case, it has
no scale dependence. Its a pure function of $n$ (or $x$ when the moments are inverted). (4) Finally, the $[\alpha_s]^d_n$ piece is the same as for the DIS on hadronic targets. Indeed, the $d_n$’s depend on the same anomalous dimensions.

**IV. DIFFICULTY WITH INTRODUCING VDM INTO THE PICTURE**

Since the theoretical uncertainty regarding the photon structure function comes from the hadronic part, it seems justified to use our knowledge of the hadronic structure functions, more precisely through Vector Meson Dominance (VDM) to get some insight on this question. There are indeed several models which invoke VDM to justify their parameterization. This should be done with great care since there are inherent difficulties in identifying the hadronic part with VDM.

(i) It is in fact tempting to associate the hadronic piece of the structure function directly with a VDM contribution

$$F_{2,n}^{HAD}(Q^2) \equiv A_n[\alpha_s]^d_n \equiv F_{2,n}^{VDM}(Q^2) = A_n^{VDM}[\alpha_s]^d_n$$

but formally, this is not possible. $F_{2,n}^{HAD}(Q^2)$ must be singular to cancel singularities in $F_{2,n}^{PL}(Q^2)$. The moments $a_n, b_n,...$are singular at specific values of $n$. For example, near $n = 2$, the moment $b_n$ is ill-behaved:

$$b_n \sim \frac{b}{n-2}$$

which leads to a negative singular behavior in the small $x$ region (Regge region)

$$b(x) \sim \frac{b}{x^2}$$

Clearly, $A_n$ must contain a similar singularities so that the total $F_{2,n}^{\gamma}$ is free of singularities \[2\]. For the $n = 2$ case,

$$A_n \sim \frac{b}{n-2}$$

so $A_n \neq A_n^{VDM}$ since $A_n^{VDM}$ is regular for all $n$’s.

(ii) Instead one could think of separating the singular and regular pieces of $A_n$ and identify the VDM contribution as follows:

$$A_n = A_n|_{\text{singular}} + A_n^{VDM}.$$

But there are some arbitrariness in the definition of the singular term. Adding any regular piece to the singular term gives an equally valid definition for the singular term. On the other hand $A_n^{VDM}$ is physically well defined and this implies that the singular term should be uniquely defined. In fact, there seems to be no convincing way to define the $A_n^{VDM}$ part in $A_n$. This kind of splitting can indeed lead to double counting i.e. part of $A_n|_{\text{singular}}$ may be contributed by VDM.

(iii) There exist a popular alternative \[3\] which suggest to consider the $Q^2$-evolution of $F_{2,n}^{\gamma}(Q^2)$ and assume that the total $F_{2,n}^{\gamma}(Q^2)$ is given at some lower $Q_0^2$ by $F_{2,n}^{VDM}(Q^2)$ alone.
Of course once $F_{\gamma}^{\gamma}(Q^2)$ is known at some $Q^2_0$ it only remains to perform the $Q^2$-evolution and check if this is in concordance with experiments. The choice of the scale $Q^2_0$ may be physically motivated to some extent. For example one popular choice is the scale at which QCD becomes perturbative (actually this scale is not so well defined).

But this procedure has some degree of arbitrariness which comes in part from the parameter $Q^2_0$. QCD gives no indications on the value of any intermediate scale $Q^2_0$. The only scale dependence of the predictions is through the running coupling $\alpha_s$. Furthermore $A_n$ which contains the hadronic contribution is scale independent. It is also not clear that the only contribution to the structure function below $Q^2_0$ origins from VDM.

Since the only QCD prediction tested in this approach is the $Q^2$-evolution (not the normalization) of $F_{\gamma}^{\gamma}(Q^2)$, the sensitivity to the $\Lambda$ parameter is almost completely lost. Unfortunately, this approach does ignore precious information coming from QCD on normalization and $\gamma\gamma$ processes are not the best place to test the $Q^2$-evolution. On the other hand, the above assumptions can be checked experimentally. But one must be aware that an analysis of data according to these models seems to have little chance of testing QCD itself. It will more likely test the assumptions and the choice of $Q^2_0$.

(iv) There exists an even more cautious approach: Test only the $Q^2$-evolution by finding $F_{\gamma}^{\gamma}(Q^2_0)$ experimentally at a given $Q^2_0$. It has the advantage of being free of any ambiguity and model independent. But it suffers from similar inconvenience namely loss of sensitivity to $\Lambda$, loss of information from QCD and difficulty of testing QCD from $\gamma\gamma$ processes due to small range of $Q^2$ spanned and lack of statistic.

V. QCD REGULARIZATION OF $F_{2,N}^{\gamma}(Q^2)$

There exists an approach which tries to exploit QCD theoretical predictions as much as possible. The procedure consist in regularizing the singular pieces from both the point-like and the hadronic part. Although it contains some uncertainties, the procedure suggest how they may be controlled. Let us rewrite

$$F_{2,n}^{\gamma}(Q^2) = F_{2,n}^{PL}(Q^2) + F_{2,n}^{HAD}(Q^2)$$

where

$$F_{2,n}^{PL}(Q^2) = \sum_{l=0}^{\infty} a_{l,n} [\alpha_s]^{l-1}$$

with $a_{l,n} = a_{l,n}^{\text{regular}} + \frac{a_l}{n - n_l}$

and

$$F_{2,n}^{HAD}(Q^2) = A_n [\alpha_s]^{d_n}$$

The regularization is similar to the one used for the virtual photon case where the ambiguities are absent. It consists of replacing the cancelling singular pieces by the following expression

$$\frac{a_l}{n - n_l} [1 - (\lambda_l \alpha_s)^{d_n+1-l}] [\alpha_s]^{l-1} = \frac{a_l [\alpha_s]^{l-1}}{n - n_l} - \frac{a_l \lambda_l^{d_n+1-l} [\alpha_s]^{d_n}}{n - n_l}$$

where the first and second in the brakets on RHS comes from the point-like and hadronic components respectively. The whole expression is regular since $d_n + 1 - l \to 0$ as $n \to n_l$. 
(Note that for the virtual photon case the cancellation, the parameter \( \lambda_l \) is replaced by \( \alpha_s^{-1}(p) \) where \( p^2 \) is the virtual photon mass). The parameters \( \lambda_l \) are in principle calculable from QCD and so are well-defined objects. In practice however these are non-perturbative parameters which escape as of now theoretical prediction.

What is interesting in the last expression is that it suggests to use the \( \lambda_l \) to parameterize the total \( A_n \):

\[
A_n = - \sum_{l=0}^{\infty} \frac{a_l}{n - n_l} \lambda_l^{d_l + 1 - l}
\]

In practice one only needs a finite number of \( \lambda_l \)'s to reach a good fit since the remaining regular pieces are suppressed by powers of \( \alpha_s \) at large \( l \)'s. The second interesting fact is that the \( \lambda_l \) parameters affects the region of small \( x \) so these parameter can be fitted from low \( x \) data.

This QCD regularization is a physical (not an arbitrary mathematical \([4]\)) parameterization. The parameters can be determined from a well-defined region of phase space and it corresponds to a convenient parameterization. Furthermore, it has been found that for a large range of values of \( \lambda_l \), the \( F_2^\gamma(x, Q^2) \) is still dominated by the regularized point-like part at not too small \( x \). Therefore the sensitivity to \( \Lambda \) is maintained in these regions. The procedure gives a well-defined approach to control the theoretical uncertainties associated with the cancellations of point-like and hadronic singularities.

VI. CONCLUSION

In summary, a message to experimentalists: Two attitudes can prevail in treating this problem. One may be cautious and, in view of uncertainties, consider only the \( Q^2 \)-evolution of the photon structure function. The uncertainties are then parameterized either by fitting \( F_2^\gamma(x, Q^2) \) to experimental data at given \( Q_0^2 \) or by using some other assumption. In this case, one cannot seriously expect to provide a conclusive test of QCD (it more appropriate to look at DIS for \( Q^2 \)-evolution) and sensitivity to \( \Lambda \) is almost completely lost. The second approach is more opportunist for it makes use of as much information as QCD can provide. In this case the uncertainties regarding the normalization can be parameterized by a finite number of parameters. But most important, it seems to indicate that a good determination of \( \Lambda \) is still possible. In any case, both parameterization should be analyzed until one is experimentally excluded.

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