Gravity field modelling for the Hannover 10 m atom interferometer

Manuel Schilling¹,³ · Étienne Wodey² · Ludger Timmen³ · Dorothee Tell² · Klaus H. Zipfel² · Dennis Schlippert² · Christian Schubert¹,² · Ernst M. Rasel² · Jürgen Müller³

¹Institute for Satellite Geodesy and Inertial Sensing, DLR
²Institute of Quantum Optics, Leibniz University Hannover
³Institute of Geodesy, Leibniz University Hannover
Interferometry with cold atoms and lasers

- Atom interferometry is a versatile tool in
  - Fundamental physics (e.g. test of GR)
  - Geodesy and Geophysics (improving sensors e.g. for Earth observation)

- Current developments
  - Gravimeters (for air, sea, land deployment)
  - Inertial sensors (for navigation and accelerometry)
  - Demonstrator missions in microgravity / space (e.g. (BEC)CAL, MAIUS, QUANTUS)

This work focuses on combining classical gravimetry with a large-scale atom interferometer for
- Determination of the AI error budget
- Realising a gravimetric reference
Atom interferometry concept

Cold atoms as test masses in an interferometer

Leading order phase shift $\Delta \Phi$

$$\Delta \Phi = k_{\text{eff}} \cdot a T^2$$

Gravimeter / VLBAI: $k_{\text{eff}} \parallel g$

$$\Delta \Phi = k_{\text{eff}} \left( g - \frac{\alpha}{k_{\text{eff}}} \right) T^2$$

Frequency chirp $\alpha$ (partly) compensates acceleration of atoms

Measurement: population $P$ of atoms per state

$$P_e = \frac{1}{2} \left( 1 - \cos \Delta \Phi \right)$$

Mach-Zehnder light-pulse atom interferometer
Very Long Baseline Atom Interferometry

- Atomic sources: Rb and Yb
  - Drop: $T=400$ ms
  - Launch: $T=1.2$ s
- Baseline: 10 m magnetic shield and vacuum system [Wodey2020]
- Region of interest: defined by magnetic field gradient
- Inertial reference: based on gravitational wave detector vibration isolation [Wanner2012]

The Very Long Baseline Interferometry facility is part of the Hannover Institute of Technology (HITec) [Schlippert2020].
Model of HITec

Model includes
- Building (concrete, drywall, insulation…)
- Equipment
  - Support structure (VSS) ≈ 5800 kg
  - Baseplate and vacuum tank (VTS) ≈ 2800 kg
  - Optical tables ≈ 600 kg (some nm/s²)
  - Einstein Elevator ≈ 160 t (some 10 nm/s²)
- Environment
  - Basements for estimation of groundwater effect and gradient

Heights refer to the baseplate and are verified by levelling.
Model of HITec

HITec Building
- Rectangular prisms (>500 Elements)

VLBAI: VSS and VTS
- Polyhedral bodies of uniform density
- Triangular mesh of surface, e.g. export from CAD
- Calculate attraction [Pohanka1988]

See also [Schilling2020]

Gravitational attraction of building, laboratory equipment, VSS and VTS in the xz-plane and on two vertical profiles
Gravimetric network 2019

- Three gravimeters
  - Scintrex CG3M-4492, CG6-0171, ZLS B-64
  - 16 levels on VLBAI main axis
  - 9 levels on secondary profile
  - 439 gravity differences

- Least squares adjustment
  - Adjusted $g$: $\bar{\sigma}_g = 9 \text{ nm/s}^2$ ($7 - 19 \text{ nm/s}^2$)
  - Single gravity tie: $\sigma_{dg} = 15 - 60 \text{ nm/s}^2$
Monte Carlo simulations

**Density** of building materials and surrounding soil
- ±5% variation of density of each element
- Normal distribution
- 50000 runs
No simulation for VSS/VTS density

**Position** of VLBAI main axis and gravimeter
- Horizontal position ±3 cm
- Vertical position ±2 mm

Standard deviations of model and observations

\[
\sigma_{\text{mod}} = \sqrt{\sigma_{\text{MC}}^2 + \sigma_{\text{Hz,mod}}^2}
\]

\[
\sigma_{\text{obs}} = \sqrt{\sigma_g^2 + \sigma_{\text{Hz,geo}}^2 + \sigma_{\text{z,mod}}^2 + \sigma_{\text{grad}}^2}
\]

Subset (left) and heatmap (right) of all density-simulations with respect to model-density
Results 2019: main axis

$$\sigma_{\text{mod}} = \sqrt{\sigma_{\text{MC}}^2 + \sigma_{\text{hz,mod}}^2} \approx 6 - 11 \text{ nm/s}^2$$

$$\sigma_{\text{obs}} = \sqrt{\sigma_{g}^2 + \sigma_{h,\text{geo}}^2 + \sigma_{z,\text{mod}}^2 + \sigma_{\text{grad}}^2} \approx 14 - 36 \text{ nm/s}^2$$

Statistical test 95% confidence level

Null hypothesis: $$\delta g_{\text{omc},i} = \delta g_{\text{obs},i} - \delta g_{\text{mod},i} = 0$$

Alternative hypothesis: $$\delta g_{\text{omc},i} \neq 0$$

Test statistics: $$t_i = \frac{|\delta g_{\text{omc},i}|}{\sqrt{\sigma_{\text{obs},i}^2 + \sigma_{\text{mod},i}^2}}$$

Rejection criteria: $$t_i > N(0,1,1 - \frac{\alpha}{2})$$

Test fails for points at 1.72 m and 12.99 m. → outside of region of interest

$$\sigma_{\text{residuals}} = 20 \text{ nm/s}^2$$
Results 2019: secondary axis

- Results of 2017 show better match compared to main axis
- 2019: statistical test passed
- $\sigma_{\text{residuals}} = 34 \text{ nm/s}^2$

Also used as constraint, e.g. improvement on main axis do not degrade results on secondary axis.

To do improving current model:
- Estimate more diverse densities (building)
- More complex geometry (soil around building)
Conclusions and outlook

• Modelling of local gravity field demonstrated
• Verification with gravimetric methods
• Agreement on 95% confidence level

Next steps

• Add VLBAI-baseline to model
• Gravimetric measurements for verification
• Characterisation of temporal gravity changes (e.g. groundwater level variations)
• Determine ‘transfer function’ between main and secondary axis
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