A quark-meson coupling model based on Bogoliubov’s model of the nucleon

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Abstract

The quark-meson coupling model due to Guichon is formulated on the basis of the independent quark model of the nucleon proposed by Bogoliubov and is applied to a phenomenological description of symmetric nuclear matter. The model predicts, at saturation density, the compressibility \( K = 249 \) MeV and the quark effective mass \( m_q^* = 249.1 \) MeV, the effective nucleon mass being \( M^* = 747.3 \) MeV. The predicted nucleon mass radius is \( r = 0.93 \) fm.
1 Introduction

About almost half a century ago, Bogoliubov proposed an interesting model of baryons [1], which assumes that they are composed of quarks bound by a linearly raising potential, as suggested by gauge theories. With the help of a single phenomenological parameter, the string tension $\kappa$, this model is able to qualitatively account for the (dynamically generated) mass of the nucleon, for the corresponding magnetic moment, and for the mass-radius. The quark-meson coupling model due to Guichon [2] incorporates successfully the quark degrees of freedom into a many-body effective Hamiltonian, inspired on QCD. The aim of the present note is to obtain a phenomenological description of hadronic matter in the framework of a combination of both models.

The quark potential has been derived by Baker et al [4] using a dual-superconductor picture of QCD. The distribution of gluon fields has been investigated on the lattice by Bissey et al. [5], who have shown that the potential increases linearly with the length of the string, and that the Y shape configurations of the gluon flux-tube distribution is more favorable than the L or T configurations. In the Y shape the strings join at some point localized inside the triangle defined by the quarks. In the L shape, one of the quarks is at the point where the strings join. In the T shape, the point where the strings join is on the line segment defined by two quarks. There are similarities and differences with Bogoliubov’s model. Since this model is an independent quark model, in it all shapes, L,T,Y, defined by the positions of the quarks, have the same energy, provided the sum of the distances to the origin is the same, this being the difference. However, the potential energy increases linearly with the length of the string, or with the distance to the origin, this being the similarity. Moreover, in Bogoliubov’s model, the phenomenological string tension turns out to be about $1/4$ of the string tension in the lattice theory.

2 Bogoliubov’s independent quark model of the nucleon

According to Bogoliubov’s proposal, the nucleon, regarded as a system of three independent valence quarks is described by the Hamiltonian [1]

$$H = \sum_{j=1}^{3} \alpha_j \cdot p_j + \kappa \sum_{j=1}^{3} \beta_j |r_j|, \quad (1)$$
where the components of $\alpha_j$ and $\beta_j$ denote the Dirac matrices related to the quark $j$ and $\kappa$ is the string tension. For simplicity, a Coulomb term $1/|r_j|$, which is included in the so called quarkonium Cornel potential, has been omitted. The model is admittedly incomplete since it does not accommodate the hyperfine structure and so is unable to describe the nucleon – $\Delta$ mass splitting. However, such a refinement is beyond the scope of the present note. In the presence of a static magnetic field the Hamiltonian becomes

$$H = \sum_{j=1}^{3} \alpha_j \cdot (p_j - q_j A(r_j)) + \kappa \sum_{j=1}^{3} \beta_j |r_j|,$$

where $q_j$ is the charge of the quark $j$ and $A$ is the potential vector.

### 2.1 The Dirac Hamiltonian and its square

The square of the Dirac Hamiltonian $h = \alpha \cdot p + \beta |r|$ reads

$$h^2 = p^2 + \kappa^2 r^2 - i\beta \frac{\alpha \cdot r}{|r|} \kappa.$$  

(3)

It is convenient to introduce the operator

$$\tilde{h}^2 = (p^2 + \kappa r^2) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \sigma_r & 0 \\ 0 & \sigma_r \end{bmatrix} \kappa,$$

which is related to $h^2$ by a unitary transformation. The operators $h^2$ and $\tilde{h}^2$ have the same eigenvalues. Indeed, we may write

$$h^2 = \begin{bmatrix} p^2 + \kappa^2 r^2 & -i\sigma_r \kappa \\ i\sigma_r \kappa & p^2 + \kappa^2 r^2 \end{bmatrix}.$$  

So, if $\begin{bmatrix} \chi \\ i\chi \end{bmatrix}$ is an eigenvector of $\tilde{h}^2$, then $\begin{bmatrix} \chi \\ i\chi \end{bmatrix}$ is an eigenvector of $h^2$.

As an approximation, we may identify the mass of the quark with the square root of the lowest eigenvalue of $p^2 + \kappa^2 r^2$, that is, with $\sqrt{3\kappa}$. However, we must correct for the center of mass motion. If this is done, as explained later, the corrected mass is $\sqrt{5\kappa}/2$.

The relevant eigenvectors of $p^2 + \kappa^2 r^2$ read

$$\Psi_{(1, \frac{1}{2}, 0, \frac{1}{2})} = \left( \frac{\kappa}{\pi} \right)^{\frac{3}{4}} e^{-\frac{1}{2} \kappa r^2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$  

(3)
\[ \Psi_{(1;\frac{1}{2},1;\frac{1}{2})} = \left( \frac{\kappa}{\pi} \right)^{\frac{3}{4}} \left( \frac{2\kappa}{3} \right)^{\frac{1}{4}} e^{-\frac{i}{4}\kappa r^2} \begin{bmatrix} z \\ x + iy \end{bmatrix}, \]

\[ \Psi_{(2;\frac{1}{2},0;\frac{1}{2})} = \left( \frac{\kappa}{\pi} \right)^{\frac{3}{4}} \left( \frac{3\kappa}{2} \right)^{\frac{1}{4}} e^{-\frac{i}{4}\kappa r^2} \left( 1 - \frac{2}{3} \kappa r^2 \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \]

\[ \Psi_{(2;\frac{1}{2},1;\frac{1}{2})} = \left( \frac{\kappa}{\pi} \right)^{\frac{3}{4}} \left( \frac{5\kappa}{3} \right)^{\frac{1}{4}} e^{-\frac{i}{4}\kappa r^2} \left( 1 - \frac{2}{5} \kappa r^2 \right) \begin{bmatrix} z \\ x + iy \end{bmatrix}, \]

where \( \Psi_{(n;j,\ell,m_j)} \) explicitly indicates the important quantum numbers. Only the first two components of the 4-spinors are shown. In the subspace spanned by these vectors, \( \tilde{h}^2 \) is represented by the matrix

\[
\begin{bmatrix}
3 & \frac{2\sqrt{2}}{\sqrt{3}\pi} & 0 \\
\frac{2\sqrt{2}}{\sqrt{3}\pi} & 5 & \frac{2}{3\sqrt{\pi}} \\
0 & \frac{2}{3\sqrt{\pi}} & 7 \\
0 & 0 & \frac{7}{3\sqrt{\pi}}
\end{bmatrix} \kappa
\]

which has the following eigenvalues

9.30521\kappa, 6.77892\kappa, 5.28004\kappa, 2.63584\kappa.

If we had considered the space spanned by the first two vectors, the eigenvalues would have been 5.35972\kappa, 2.64028\kappa, showing a quick convergence. If center of mass corrections are not considered, the lowest eigenvalue is identified with the quark mass squared.

\[ m_q^2 = \left( 4 - \sqrt{1 + \frac{8}{3\pi}} \right) \kappa, \quad \kappa = 34087.2 \text{ MeV}^2, \quad R_M = 1.38398 \text{fm}. \]

### 2.2 Calculation of \( \langle \Phi_0 | (p_1 + p_2 + p_3)^2 | \Phi_0 \rangle \)

Center of mass corrections must be considered. As previously observed, the correction for the center of mass (CM) motion is implemented by subtracting the expectation value of the CM momentum squared from the expression for the square of the nucleon mass. We need, therefore, \( \langle \Phi_0 | (p_1 + p_2 + p_3)^2 | \Phi_0 \rangle \), where \( | \Phi_0 \rangle \) is the wave function of the three-quark system.

In the subspace spanned by \( \Psi_{(1;\frac{1}{2},0;\frac{1}{2})} \) and \( \Psi_{(1;\frac{1}{2},1;\frac{1}{2})} \), \( \tilde{h}^2 \) is represented by the matrix

\[
\begin{bmatrix}
3 & \frac{2\sqrt{2}}{\sqrt{3}\pi} \\
\frac{2\sqrt{2}}{\sqrt{3}\pi} & 5
\end{bmatrix} \kappa
\]
The groundstate eigenvalue reads \( 4 - \sqrt{1 \pm \frac{8}{3\pi}}\kappa \) and the associated eigenvector may be written \( \Psi_0 = c_1 \Psi_{(1/2,0,1/2)} + c_2 \Psi_{(1/2,1,1/2)} \). The mass squared of the nucleon should be identified with \( 9 \left( 4 - \sqrt{1 \pm \frac{8}{3\pi}}\kappa \right) \). Taking into account center of mass correction, means subtracting \( \langle \Phi_0 | (p_1 + p_2 + p_3)^2 | \Phi_0 \rangle \). We obtain

\[
\langle \Psi_{(1/2,0,1/2)} | p^2 | \Psi_{(1/2,0,1/2)} \rangle = \frac{3\kappa}{2}, \quad \langle \Psi_{(1/2,1,1/2)} | p^2 | \Psi_{(1/2,1,1/2)} \rangle = \frac{5\kappa}{2}
\]

so that the correct expression reads

\[
\langle \Phi_0 | (p_1 + p_2 + p_3)^2 | \Phi_0 \rangle = \frac{3(3c_1^2 + 5c_2^2)}{2(c_1^2 + c_2^2)} \kappa = 9\Delta\kappa,
\]

where

\[
c_1 = -3\pi - \sqrt{3\pi(8 + 3\pi)}, \quad c_2 = 2\sqrt{6\pi},
\]

the quantity \( \Delta\kappa \) being the center of mass correction for each quark, so that, in this approximation, the quark mass becomes \( m_q^2 = \left( 4 - \sqrt{1 \pm \frac{8}{3\pi}}\kappa \right) \). Setting \( m_q = 300 \text{ MeV} \), we find \( \kappa = 43197.8 \text{ MeV}^2 \). The nucleon mass radius which is derived from the expectation value of \( r_1^2 \) is too big due to the fluctuation of the nucleon CM. A reasonable value is obtained if, instead of the expectation value of \( r_1^2 \), one compensates for the CM motion and considers the expectation value of \( (r_1 - (r_1 + r_2 + r_3)/3)^2 \). This is equal to the expectation value of \( 2(r_1^2 - r_2 \cdot r_3)/3 \). Now,

\[
\frac{2}{3} \langle \Phi_0 | r_1^2 - r_2 \cdot r_3 | \Phi_0 \rangle = \frac{2}{3} \left[ \frac{3}{2\kappa} c_1^2 + \frac{5}{2\kappa} c_2^2 + \frac{2}{6\kappa} c_1^2 c_2^2 \right]
\]

so that, for \( \kappa = 43197.8 \text{ MeV}^2 \) a mass radius equal to 0.930548 fm is obtained.

### 3 QMC model. Bogoliubov model with external scalar field

According to the quark-meson coupling (QMC) model \[2\], nuclear matter is a system of nucleons which behave like point-like particles, although they are constituted by quarks coupled to the scalar \( \sigma \) field, in the framework of an independent particle model. The QMC model, which has been proposed by Guichon \[2\] on the basis of the MIT bag model, has been considered
by other authors [3, 6]. Recently, it has also been implemented on the basis of a quadratically raising potential [10, 11]. Here, we wish to implement the QMC model based on the Bogoliubov quark model [1].

The energy density of quark matter reads

$$E = \frac{\gamma}{(2\pi)^3} \int_{k_F} d^3k \sqrt{k^2 + M^*^2} + \frac{1}{2} m_q^2 \sigma^2 - \frac{1}{2} m_q^2 \omega^2 + g_\omega \omega \rho_B$$ (4)

$$\rho_B = \frac{\gamma}{(2\pi)^3} \int_{k_F} d^3k, \quad M^* = 3m_q(\sigma),$$

where $\gamma = 4$ denotes the spin isospin degeneracy. Clearly, $\sigma$ and $\omega$ should be replaced by the values which minimize $E$.

The pressure is given by

$$-P = \frac{\gamma}{(2\pi)^3} \int_{k_F} d^3k \sqrt{k^2 + M^*^2} + \rho_B \left( g_\omega \omega - \sqrt{k_F^2 + M^*^2} \right) + \frac{1}{2} m_q^2 \sigma^2 - \frac{1}{2} m_q^2 \omega^2.$$ 

Clearly, $\sigma$ and $\omega$ should be replaced by the values which maximize $P$ for $k_F = \sqrt{\mu^2 - M^*^2}$. Notice that $M^*$ depends on $\sigma$, but not on $\omega$. In order to determine $m_q(\sigma)$, we introduce, following [2], an external sigma field acting on the quarks, so that, in eq. (1), the term $\kappa \sum_j |r_j|$ is replaced by $\sum_j \beta_j (g_\sigma \sigma + \kappa |r_j|)$. Next we determine $m_q(\sigma) = M^*/3$ and the binding energy of nuclear matter within two approximation schemes.

3.1 Operator $p^2 + (-a + |r|\kappa)^2$

It is natural to regard as a perturbation, in eq. (3), the term involving $\alpha \cdot r$, and to restrict our attention to the operator $p^2 + (-a + |r|\kappa)^2$, where $a = g_\sigma \sigma$. Then, a simplified model is obtained which may be interesting to consider because much of the basic physics is, indeed, already in it.

The quark groundstate wave function has angular momentum $\ell = 0$, and, in the presence of an external scalar field $\sigma$, is given by the ansatz

$$\Psi_{0,0} = \exp \left( -\frac{1}{2} \left( \sqrt{\kappa^2} - \frac{a}{\sqrt{\kappa}} \right)^2 \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$ (5)
Figure 1: Nucleon effective mass for the present QMC approach based on the Bogolyubov model, according to eqs. (10). Comparison with the original QMC model of [2].

Let

\[ F_{N,0}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r \psi_{0,0}^\dagger \psi_{0,0}, \]
\[ F_{K,0}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r \psi_{0,0}^\dagger (-\nabla^2) \psi_{0,0}, \]
\[ F_{P,0}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r \psi_{0,0}^\dagger (\kappa r - a)^2 \psi_{0,0}. \]

We find

\[ F_{N,0}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r e^{-\left(\sqrt{\kappa}r - \frac{a}{\sqrt{\kappa}}\right)^2}, \]
\[ F_{K,0}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r \left( 3\kappa - \frac{2a}{r} - (\kappa r - a)^2 \right) e^{-\frac{1}{2}\left(\sqrt{\kappa}r - \frac{a}{\sqrt{\kappa}}\right)^2}, \]
\[ F_{P,0}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r (\kappa r - \alpha)^2 e^{-\left(\sqrt{\kappa}r - \frac{a}{\sqrt{\kappa}}\right)^2}. \]
So that

\[ F_{N,0}(\kappa, \alpha) = \frac{\pi}{\kappa \sqrt{\kappa}} \left( 2ae^{-a^2} + (1 + 2a^2)\sqrt{\pi}(1 + \text{Erf}(\alpha)) \right) \]

\[ F_{P,0}(\kappa, \alpha) = F_{K,0}(\kappa, \alpha) = \frac{\pi}{2\sqrt{\kappa}} \left( 2ae^{-a^2} + (3 + 2a^2)\sqrt{\pi}(1 + \text{Erf}(\alpha)) \right), \]

where \( \alpha = a/\sqrt{\kappa} = g_\sigma \sigma/\sqrt{\kappa} \). The expression for the squared quark mass reads

\[ m_q^2(\kappa, \alpha) = \frac{F_{K,0}(\kappa, \alpha) + F_{P,0}(\kappa, \alpha)}{F_{N,0}(\kappa, \alpha)}, \] (6)

or, if the CM correction is considered

\[ m_q^2(\kappa, \alpha) = \frac{F_{K,0}(\kappa, \alpha) + F_{P,0}(\kappa, \alpha)}{F_{N,0}(\kappa, \alpha)} - \frac{F_{P,0}(\kappa, \alpha)}{3F_{N,0}(\kappa, \alpha)}. \] (7)

Minimization of (4) with respect to \( \sigma \) is easily performed and the minimizing value of \( \sigma \) is determined by requiring self-consistency.

### 3.2 Operator \( p^2 + (-a + |r|\kappa)^2 + \sigma r \kappa \)

Going beyond the previous Section, we investigate now the effect of the term involving \( \alpha \cdot r \) which appears in eq. (3). We seek the matrix which represents the operator \( p^2 + (-a + |r|\kappa)^2 + \sigma r \kappa \) in a subspace spanned by \( \ell = 0 \) and \( \ell = 1 \) wave-functions.

In the presence of an external scalar field \( \sigma \), the wave function of the lowest quark state with angular momentum \( \ell = 1 \), is given by the ansatz

\[ \Psi_{0,1} = \exp \left( -\frac{1}{2} \left( \sqrt{\kappa}r - \frac{a}{\sqrt{\kappa}} \right)^2 \right) \begin{bmatrix} z \\ x + iy \end{bmatrix}. \] (8)

In the space spanned by the wave-functions (5) and (8), the operator \( p^2 + (-a + |r|\kappa)^2 + \sigma r \kappa \) is represented by the matrix

\[ A = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix}, \]

where

\[ a_{00} = \frac{F_{K,0}(\kappa, \alpha) + F_{P,0}(\kappa, \alpha)}{F_{N,0}(\kappa, \alpha)}, \]

\[ a_{10} = a_{01} = \frac{F_{C,01}}{\sqrt{F_{N,0}F_{N,1}}}, \]

\[ a_{11} = \frac{F_{K,1}(\kappa, \alpha) + F_{P,1}(\kappa, \alpha)}{F_{N,1}(\kappa, \alpha)}, \]
Figure 2: Binding energy for the QMC approach based on the Bogolyubov model, using the effective mass defined by means of eqs. (6), (7), (9) and (10). The curves corresponding to eqs. (6) and (7) coincide.

the functions $F_{K,0}$, $F_{P,0}$, $F_{N,0}$, $F_{K,1}$, $F_{P,1}$, $F_{N,1}$, $F_{C,01}$, being defined and given in the Appendix. The lowest eigenvalue of $A$ reads

$$m_q^2(\kappa, \alpha) = \frac{1}{2} \left( a_{00} + a_{11} - \sqrt{(a_{00} - a_{11})^2 + 4a_{01}^2} \right). \quad (9)$$

Here, CM corrections have not been yet included. In order to take into account the CM corrections, we need the integral

$$G_{C,01}(\kappa, \frac{a}{\sqrt{\kappa}}) = \frac{1}{3} \int d^3r r\rho(r) e^{-(\sqrt{\kappa r} - \frac{a}{\sqrt{\kappa}})^2}$$

$$G_{C,01}(\kappa, \alpha) = \frac{\pi}{2\kappa \sqrt{\kappa}} \left( 2\alpha e^{-\alpha^2} + (1 + 2\alpha^2) \sqrt{\pi} (1 + \text{Erf}(\alpha)) \right).$$

We may write

$$m_q^2(\kappa, \alpha) = \frac{1}{2} \left( a_{00} + a_{11} - \sqrt{(a_{00} - a_{11})^2 + 4a_{01}^2} \right) - \Delta_{CM}, \quad (10)$$
Figure 3: Binding energy for the present QMC approach based on the Bogolyubov model, according to eqs. [10]. Comparison with the original QMC model of [2].

where

$$\Delta_{CM} = \frac{1}{3(c_1^2 + c_2^2)} \left( \frac{F_{K,0}(\kappa, \alpha)}{F_{N,0}(\kappa, \alpha)} c_1^2 + \frac{F_{K,0}(\kappa, \alpha)}{F_{N,0}(\kappa, \alpha)} c_2^2 \right) + \frac{6c_1^2c_2^2(G_{C,01}(\kappa, \alpha))^2}{9(c_1^2 + c_2^2)^2F_{N,0}(\kappa, \alpha)F_{N,1}(\kappa, \alpha)},$$

with

$$c_1 = a_{11} - a_{00} + \sqrt{(a_{11} - a_{00})^2 + 4a_{01}^2}, \quad c_2 = -2a_{01}.$$

Minimization of (4) with respect to $\sigma$ is easily performed, and the minimizing value of $\sigma$ is determined by requiring self-consistency.

3.3 Discussion

In Fig.1 the nucleon effective mass for the present QMC approach based on eqs. [10] is represented and is compared with the original QMC model of [2], showing that the Bogolyubov model leads to a smoother decrease of $M^*$ with the baryonic density. In Fig. 2 the binding
energy for the QMC approach based on the Bogolyubov model, using the effective mass defined my means of eqs. (6), (7), (9) and (10) is shown. We observe that the curves corresponding to eqs. (6) and (7) coincide. The effect of the $\kappa\sigma_r$ term is to stiffen the EOS, as expected. The curve displayed in Figure 3, representing the binding energy vs. the baryon density, was obtained using (10) and shows that the Bogolubov model leads to a slightly less stiff EOS than the original QMC model of [2]. The inputs are $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV, $m_q = 313$ MeV. The coupling constants $g_\sigma$, $g_\omega$ were chosen so as to reproduce the binding energy and density at equilibrium, that is, $\mathcal{E}/\rho_B - M_N = -15.7$ MeV at saturation (pressure $P=0$), being $M_N = 939$ MeV the free nucleon mass. The value of $\kappa$ is determined by the quark mass in vacuum, $m_q$. The effective quark mass at saturation density is denoted by $m_q^*$. The outputs are summarized in Table 1.

4 Comparison with the Walecka and Zimanyi-Moszkowski models

It is challenging to compare the nucleon effective mass, as predicted by the present version of the QMC model, and by the Walecka and the Zimanyi-Moszkowski models [13, 14], with respect to the dependence on the scalar field. We consider the expression of the effective mass in terms of the scalar coupling $g_\sigma\sigma$. In the Walecka model, the relation $M_{eff} = M(\sigma) = 1 - g_\sigma\sigma$ holds. The mass of the free nucleon is set equal to 1. In the derivative coupling model (cf. [14]), we find

$$M_{eff} = 1/(1 + g_\sigma\sigma) = 1 - g_\sigma\sigma + (g_\sigma\sigma)^2 - (g_\sigma\sigma)^3 + \ldots.$$ 

In the simple version of the QMC model obtained in the present formulation, eq. (6), we have, to a good approximation, a close expression to the previous one, if cubic and higher terms in $g_\sigma\sigma$ are neglected. According to both the present version of QMC and the bag version of QMC (cf. [2]) these terms are very small. In particular, in the present version, it turns out that the coefficient of the squared term in an expansion of the effective mass is an order of magnitude less than it is in the ZM model. Specifically, keeping in mind that in the groundstate the
expectation values of $p^2$ and $\kappa(|r|-a)^2$ are equal, and taking $\kappa = b^{-2}$, we find that:

$$M_{\text{eff}} = \sqrt{\frac{2 \int_0^\infty r^2(r-a)^2 \exp(-(r-a)^2b^{-2})dr}{3b^2 \int_0^\infty r^2 \exp(-(r-a)^2b^{-2})dr}}$$

$$= 1 - 0.3761\alpha + 0.1113\alpha^2 - 0.00294\alpha^3 + \ldots$$

where $\alpha = a/b$. We have for this case,

$$g_\sigma\sigma = \frac{2\alpha}{3\sqrt{\pi}} \approx 0.3761\alpha, \quad M_{\text{eff}} = 1 - g_\sigma\sigma + 0.788(g_\sigma\sigma)^2 - 0.131(g_\sigma\sigma)^3 + \ldots$$

which is close to $M_{\text{eff}} = 1/(1 + g_\sigma\sigma) = 1 - g_\sigma\sigma + (g_\sigma\sigma)^2$. For large number of dimensions $D \gg 1$, that is, if in the previous integrations over $r$, rdr is replaced by $r^{D-1}dr$, we obtain:

$$M_{\text{eff}} = 1 - g_\sigma\sigma + \frac{1}{2}(g_\sigma\sigma)^2,$$

where

$$g_\sigma\sigma = \frac{1}{\sqrt{2D}}\alpha.$$  

Still, it is most remarkable that the present expression is very close to the corresponding Zimanyi-Moszkowski expression. Actually, both expressions are in agreement up to the mentioned cubic order. The difference shows up in the ratio $K/|W_0|$, between the incompressibility and the binding energy $|W_0| = |E/\rho_B - M_N|$. In the weak coupling limit, we have in lowest order, $K/|W_0| = 18(1 - 2\sqrt{|W_0|/(M_Nc^2)})$ for the present version of QMC, and $K/|W_0| = 18(1 - 3\sqrt{|W_0|/(M_Nc^2)})$ for Zimanyi-Moszkowski model. Thus for the same binding energy, the present version of QMC leads to a slightly larger $K$ than Zimanyi-Moszkowski model. The incompressibility of 235 MeV which is found, is close to the value of 225 MeV of the derivative coupling model, and the effective mass is 0.85 for both models. For the more realistic version, eq. (10), things are very close.

## 5 Conclusions

In the present work we have proposed an effective relativistic nuclear model that takes into account the internal structure of the nucleon explicitly, in the philosophy of the QMC model of Guichon [2]. Matter at low densities and temperatures is a system of nucleons composed of quarks bound by a linearly raising potential, as suggested by gauge theories, according to
Table 1: Outputs for nuclear matter, which have been determined from the binding energy at equilibrium density, being $M^*_N$ the corresponding effective nucleon mass.

The string tension which is obtained with Bogoliubov model turns out to be less than one fourth of the string tension which is found in the lattice calculation. However, this model is incomplete since it does not account for the mass splitting between the nucleon and the resonance. If the model is refined in this direction, following, for instance, a higher string tension, closer to $0.5 \text{ GeV fm}^{-1}$ may be obtained. Such improvement is left for a future publication.

On the other hand, it may be observed that this model is suggested by the string concept and leads essentially to a 3-dimensional harmonic oscillator, since the operator $\hbar^2$ may be approximated by $p^2 + \kappa^2 r^2$. The failure of the model in accounting for the correct string tension is, perhaps, the price one has to pay for the approximate treatment of the string force. Indeed, the Bogoliubov model is based on the idea that the quarks move independently in a three dimensional potential. Now, a string is a one-dimensional object. If we replace the

|         | eq (6) | eq (7) | eq (9) | eq (10) |
|---------|--------|--------|--------|---------|
| $g_\sigma$ | 3.876  | 4.246  | 3.696  | 4.024   |
| $g_\omega$ | 6.492  | 6.492  | 7.818  | 8.159   |
| $\sigma$ (MeV) | 20.57  | 20.57  | 23.76  | 24.60   |
| $\omega$ (MeV) | 12.20  | 12.20  | 14.69  | 15.33   |
| $M^*_N$ (MeV) | 803.84 | 803.84 | 766.30 | 755.49  |
| K (MeV)     | 235.5  | 235.5  | 245.9  | 249.1   |
| $\kappa$ (MeV$^2$) | $3.27 \times 10^4$ | $3.92 \times 10^4$ | $3.71 \times 10^4$ | $4.60 \times 10^4$ |
| $m^*_q/m_q$ | 0.95   | 0.85   | 0.81   | 0.80    |

The Bogoliubov model of baryons. The parameters of the model have been fitted to the saturation density and the binding energy of symmetric nuclear matter at this density, and the quark mass in vacuum. As output the incompressibility of matter and the effective nucleon mass at saturation were calculated respectively with values 249.1 MeV and 0.8 $M$.

The string tension which is obtained with Bogoliubov model turns out to be less than one fourth of the string tension which is found in the lattice calculation. However, this model is incomplete since it does not account for the mass splitting between the nucleon and the resonance. If the model is refined in this direction, following, for instance, a higher string tension, closer to $0.5 \text{ GeV fm}^{-1}$ may be obtained. Such improvement is left for a future publication.

On the other hand, it may be observed that this model is suggested by the string concept and leads essentially to a 3-dimensional harmonic oscillator, since the operator $\hbar^2$ may be approximated by $p^2 + \kappa^2 r^2$. The failure of the model in accounting for the correct string tension is, perhaps, the price one has to pay for the approximate treatment of the string force. Indeed, the Bogoliubov model is based on the idea that the quarks move independently in a three dimensional potential. Now, a string is a one-dimensional object. If we replace the
3-dimensional oscillator by a 1-dimensional oscillator, the value of the string tension which reproduces the quark mass increases. Indeed, consider the 1-dimensional harmonic oscillator $p_z^2 + \kappa^2 z^2$. The lowest eigenvalue of this operator, which must be identified with the quark mass squared, is $\kappa$. Since the average nucleon−Delta resonance mass is $(1232 + 939)/2 \text{ MeV} = 1085.5 \text{ MeV}$, the quark mass is $\sqrt{\kappa} = 361.8 \text{ MeV}^2 = 0.655 \text{ GeV/fm}$, so that 0.655 GeV/fm, which is a sizable improvement over the simple 3D model.

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Appendix

Let

\[ F_{N,1}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r \Psi_{0,1}^\dagger \Psi_{0,1}, \]
\[ F_{K,1}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r \Psi_{0,1}^\dagger (-\nabla^2) \Psi_{0,1}, \]
\[ F_{P,1}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r \Psi_{0,1}^\dagger (\kappa r - a)^2 \Psi_{0,1}, \]
\[ F_{C,01}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r \Psi_{0,0}^\dagger \frac{\kappa}{r} \begin{bmatrix} z & x - iy \\ x + iy & -z \end{bmatrix} \Psi_{0,1}. \]

We find

\[ F_{N,1}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r r^2 e^{-\left(\sqrt{\kappa r} - \frac{a}{\sqrt{\kappa}}\right)^2}, \]
\[ F_{K,1}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r r^2 \left(5\kappa - \frac{4a}{r} - (\kappa r - a)^2\right) e^{-\frac{1}{2}\left(\sqrt{\kappa r} - \frac{a}{\sqrt{\kappa}}\right)^2}, \]
\[ F_{P,1}(\kappa, \frac{a}{\sqrt{\kappa}}) = \int d^3 r r^2 (\kappa r - a)^2 e^{-\left(\sqrt{\kappa r} - \frac{a}{\sqrt{\kappa}}\right)^2}, \]
\[ F_{C,01}(\kappa, \frac{a}{\sqrt{\kappa}}) = \kappa \int d^3 r r e^{-\left(\sqrt{\kappa r} - \frac{a}{\sqrt{\kappa}}\right)^2}, \]

so that,

\[ F_{N,1}(\kappa, \alpha) = \frac{\pi}{2\kappa^{5/2}} \left((10\alpha + 4\alpha^3) e^{-\alpha^2} + (3 + 12\alpha^2 + 4\alpha^4) \sqrt{\pi}(1 + \text{Erf}(\alpha))\right) \]
\[ F_{K,1}(\kappa, \alpha) = F_{P,1}(\kappa, \alpha) = \frac{\pi}{4\kappa^{3/2}} \left((34\alpha + 4\alpha^3) e^{-\alpha^2} + (15 + 36\alpha^2 + 4\alpha^4) \sqrt{\pi}(1 + \text{Erf}(\alpha))\right) \]
\[ F_{C,01}(\kappa, \alpha) = \frac{\pi}{\kappa} \left(2(1 + \alpha^2) e^{-\alpha^2} + (3\alpha + 2\alpha^3) \sqrt{\pi}(1 + \text{Erf}(\alpha))\right) \]