11-dimensional curved backgrounds for supermembrane in superspace

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Abstract

We compute part of the superfield in terms of the component fields of 11-dimensional on-shell supergravity by using ‘Gauge completion’ in 2nd-order formalism. The result is the same as was derived recently in 1.5-order formalism by B. de Wit, K. Peeters and J. Plefka. We use 2nd-order formalism because in order to hold \( \kappa \)-invariance generally 2nd-order formalism is more hopeful and simpler than 1.5-order formalism.
1 Introduction

Some years ago, T. Banks, W. Fischler, S. H. Shenker and L. Susskind (BFSS) proposed that Matrix theory gives a complete description of light-front M-theory [1]. It had been proposed as a theory of D0-branes by E. Witten [2].

In two years following the BFSS conjecture, it has become clear that Matrix model encodes a remarkable amount of the structure of M-theory and 11-dimensional supergravity (for reviews, see [3]). The interaction between gravitons in Matrix theory has been shown to agree with supergravity to some extent [4].

However, this theory is constructed on flat spacetime, therefore Matrix theory on curved backgrounds is required. For single D0-branes, the theory on curved backgrounds is expected to be described by Born-Infeld action [5]. For multi-particle system of D0-branes, namely Matrix model, the theory on curved backgrounds is as yet unknown. There are many trials to this problem. For example, starting from flat Matrix theory, backgrounds are produced by many D0-branes [6]. The other idea is that it is expected as supermembrane on curved backgrounds [7]. In this paper we adopt the later idea.

The theory of supermembrane is described as nonlinear sigma model [8]. Supermembrane consistently couples to 11-dimensional superspace backgrounds that satisfy a number of constraints which are equivalent to 11-dimensional on-shell supergravity [9]. After light cone gauge fixing and $\kappa$-symmetry gauge fixing, supermembrane theory on flat backgrounds is equivalent to a quantum-mechanical model with supersymmetric U(N) gauge symmetry in the large N limit by use of matrix regularization [10]. It has a continuous mass spectrum and instability [11], therefore it is expected that supermembrane matrix theory describes second quantization of D0-branes [12]. From the beginning of sigma model, it couples to general backgrounds, therefore it is expected that sigma model on curved backgrounds is candidate of Matrix theory on curved backgrounds. Actually curved backgrounds for supermembrane were investigated [7]. In this reference, they used ‘1.5-order formalism’.

In this paper, we use ‘2nd-order formalism’ because in order to hold the $\kappa$-invariance generally ‘2nd-order formalism’ is simpler and more hopeful than ‘1.5-order formalism’. We obtain the result that is the same as that was driven in this reference [7] up to obtained components. And the equations for higher order components in ‘2nd-order formalism’ are different from those in ‘1.5-order formalism’.

The paper is organized as follows. In section 2, we review the supermembrane theory and the condition of $\kappa$-symmetry. In section 3, we explain our notations of the 11-dimensional supergravity and obtain the full algebra of transformations in component formalism. In section 4, we explain our notations of the superspace geometry and ob-
tain the full algebra of transformations in superspace. In section 5, we explain ‘gauge completion’ and compute part of the superfields. In section 6, we discuss importance of 2nd-order formalism. Other notations and conventions used throughout this paper are summarized in Appendix.

2 Supermembrane theory

Super membrane theory is described as nonlinear sigma model [8]. It is written in terms of superspace embedding coordinates \( Z^{M}(\xi) = (X^{m}(\xi), \theta(\xi)) \), which are functions of the three world-volume coordinate \( \xi^{i}(i = 0, 1, 2) \).

The action is

\[
I = \int d^{3}\xi(-\frac{1}{2}\sqrt{-g}g^{ij}\Pi_{i}^{a}\Pi_{j}^{b}\eta_{ab} + \frac{1}{2}\sqrt{-g} - \frac{1}{6}\epsilon^{ijk}\Pi_{i}^{A}\Pi_{j}^{B}\Pi_{k}^{C}B_{CBA}),
\]

where \( g^{ij} \) is the metric of the world-volume, \( g = \text{det}(g_{ij}) \) and \( \Pi_{i}^{A} \equiv \partial_{i}Z^{M}E_{\alpha}^{M} \). \( E_{\alpha}^{M} \) is supervielbein, and the 3-form \( B = \frac{1}{6}E^{A}E^{B}E^{C}B_{CBA} \) is potential for the closed 4-form \( H = dB \).

This action has the following symmetries,

\[ \text{world-volume reparametrization } \eta^{i}(\xi) \]
\[ \delta Z^{M} = \eta^{i}\partial_{i}Z^{M}, \]
\[ \delta g_{ij} = \eta^{k}\partial_{k}g_{ij} + 2\partial_{(i}\eta^{k}g_{j)k}, \]

\[ \kappa\text{-symmetry } \kappa^{\alpha}(\xi) \]
\[ \delta Z^{M}E_{\alpha}^{a} = 0, \]
\[ \delta Z^{M}E_{\alpha}^{a} = (1 + \Gamma_{\alpha}^{\beta})\kappa^{\beta}, \]
\[ \delta(\sqrt{-g}g^{ij}) = -2(1 + \Gamma_{\alpha}^{\beta})\kappa^{\beta}\Gamma_{ab}^{\alpha}\Pi_{n}^{a}g^{n(i}\epsilon^{j)kl}\Pi_{k}^{a}\Pi_{l}^{b} \]
\[ + \frac{-2}{3\sqrt{-g}}\kappa^{\alpha}\Gamma_{e}^{\alpha\beta}\Pi_{k}^{\beta}\Pi_{k}^{\gamma}\epsilon^{mn(\epsilon\sigma)pq} \]
\[ (\Pi_{m}^{a}\Pi_{pa}\Pi_{n}^{b}\Pi_{qb} + \Pi_{m}^{a}\Pi_{pa}g_{nq} + g_{mp}g_{nq}), \]

where \( \kappa^{\alpha}(\xi) \) is anticommuting space time spinor and the matrix \( \Gamma \) is defined by

\[ \Gamma = \frac{1}{6\sqrt{-g}}\epsilon^{ijk}\Pi_{i}^{a}\Pi_{j}^{b}\Pi_{k}^{c}\Gamma_{abc}. \]
Up to surface terms the $\kappa$-invariance of this action imposes the following constraints on the 11-dimensional superspace geometry [4].

\[
T^{a}_{\alpha\beta} = -2\Gamma^{a}_{\alpha\beta}, \\
H_{ab} = -2\Gamma_{ab}^{\alpha\beta}, \\
H_{\alpha\beta\gamma\delta} = H_{\alpha\beta\gamma d} = H_{abcd} = 0, \\
T^{a}_{\beta\gamma} = T^{a}_{bc} = T^{a}_{b\gamma} = 0. 
\] (2.5)

The equations of motion which follow from this action are

\[
0 = g_{ij} - \Pi^{a}_{i} \Pi^{b}_{j} \eta_{ab}, \\
0 = \partial_{i}(\sqrt{-g}g^{ij}\Pi^{a}_{j}) + \sqrt{-g}\Pi^{b}_{i}\Pi^{C}_{b} \Omega_{C}^{a} + \epsilon^{ijk}\Pi_{ib}^{a}(\Pi^{c}_{j}\Gamma^{ab}_{\alpha\beta}\Pi^{\beta}_{k}) + \frac{1}{6}\epsilon^{ijk}\Pi^{b}_{i}\Pi^{c}_{j}\Pi^{d}_{k} H^{a}_{bcd}, \\
0 = ((1 - \Gamma)\Pi^{a}_{i}\Gamma^{a}_{j} \Omega_{ABC}^{i}). 
\] (2.6)

Using (2.6), the action (2.1) can be rewritten as

\[
I = \int d^{3}\xi (\sqrt{-g} - \frac{1}{6} \epsilon^{ijk}\Pi_{i}^{A}\Pi_{j}^{B}\Pi_{k}^{C} B_{ABC}). 
\] (2.9)

Flat superspace is characterized by

\[
E^{a}_{m} = \delta^{a}_{m}, \\
E^{a}_{\mu} = -(\Gamma^{a}\theta)_{\mu}, \\
E^{a}_{m} = 0, \\
E^{a}_{\mu} = \delta^{a}_{\mu}, \\
B_{\gamma\beta\alpha} = \bar{\theta}\Gamma_{mn(\gamma}\bar{\theta}\Gamma_{\beta}^{m}\bar{\theta}\Gamma_{\alpha)}^{n}, \\
B_{c\beta\alpha} = -\bar{\theta}\Gamma_{cd(\beta}\bar{\theta}\Gamma_{\alpha)}^{d}, \\
B_{cba} = (\bar{\theta}\Gamma_{cb})_{\alpha}, \\
B_{cba} = 0. 
\] (2.10)

In flat superspace, the action (2.9) becomes

\[
I = \int d^{3}\xi \{-\sqrt{-g} - \epsilon^{ijk}\bar{\theta}\Gamma_{mn(\gamma}\bar{\theta}\Gamma_{\beta}^{m}\bar{\theta}\Gamma_{\alpha)}^{n}(\frac{1}{2}\partial_{i}X^{m}(\partial_{j}X^{n} + \bar{\theta}\Gamma^{n}\partial_{j}\theta) + \frac{1}{6}\bar{\theta}\Gamma^{m}\partial_{i}\bar{\theta}\bar{\theta}\Gamma^{n}\partial_{j}\theta)\}. 
\] (2.11)

From this action, after gauge fixing of $\kappa$-symmetry and reparametrization invariance, using matrix regularization we obtain Matrix model [10]. In this paper we don’t struggle in more detail.
3 11-dimensional supergravity

Supergravity in 11-dimensional spacetime is based on ‘elfbein’ field $e_m^a$, a Majorana gravitino field $\psi_m^a$ and third rank antisymmetric gauge field $C_{klm}$. Its Lagrangian can be written as follows [13][7].

\[
L = -\frac{1}{2} eR - 2e\bar{\psi}_m \Gamma^{mnl} D_n(\frac{1}{2}(\omega + \tilde{\omega}))\psi_l - \frac{1}{96} e F^2
- \frac{1}{41472} e^{m_1...m_{11}} F_{m_1...m_4} F_{m_5...m_8} C_{m_9...m_{11}}
- \frac{1}{96} e(\bar{\psi}_n \Gamma^{m_1...m_{11}} \psi_l + 12 \bar{\psi}^{m_1} \Gamma^{m_2 m_3} \psi^{m_4})(F + \hat{F})_{m_1...m_4}.
\] (3.1)

where $e = \text{d}e a e_m$, and $\omega^a_{mb}$ denotes the spin connection

\[
\omega^a_{mb} = -e^n d \Gamma_{memb} + e^n d \Gamma_{mc e} \Gamma_{eb c} + e^n d \Gamma_{mb e}^a
+ 2(\psi_m \Gamma_b \psi^a + \bar{\psi}_b \Gamma_m \psi^a - \bar{\psi}_m \Gamma^a \psi_b) - \frac{1}{2} \bar{\psi}_n \Gamma^a_m b np \psi^p.
\] (3.2)

and $F_{klmn}$ denotes the field strength of the antisymmetric tensor

\[
F_{klmn} = 4d[C_{lmn}].
\] (3.3)

The derivative $D_m$ is covariant with respect to local Lorentz transformations,

\[
D_m(\omega) = (\partial_m - \frac{1}{4} \omega_{mb} \Gamma^{ab})\epsilon.
\] (3.4)

Supersymmetry transformations are equal to

\[
\begin{align*}
\delta_s e^a_m &= 2\epsilon \Gamma^a \psi_m, \\
\delta_s \psi_m &= D_m(\omega) + T^r_{stu} \epsilon \hat{F}_{rstu} \equiv \hat{D}\epsilon, \\
\delta_s C_{klm} &= -6\epsilon \Gamma_{[k \psi_m]}, \\
\text{with, } T^r_{stu} &\equiv \frac{1}{288}(\Gamma^r_{stu} - 8\delta^r_m \Gamma_{stu}),
\end{align*}
\] (3.5)

where $\hat{F}$ is the supercovariant field strength,

\[
\hat{F}_{klmn} = F_{klmn} + 12 \bar{\psi}_l \Gamma_{lmn} \psi^l_n.
\] (3.7)
and \( \hat{\omega} \) is the supercovariant spin connection
\[
\hat{\omega}_m{}^a{}_b = \omega_m{}^a{}_b + \frac{1}{2} \bar{\psi}_n \Gamma_m{}^a{}_b \psi_p. \tag{3.8}
\]

Note that the spin connection \( \omega \) has supersymmetry variation according to elfbein and gravitino's variation in 2nd-order formalism \[14\]. While in 1.5-order formalism, it is defined as a dependent field determined by its equation of motion, whereas its supersymmetry variation is treated as if it were an independent field \[15\]. In this paper we use 2nd-order formalism.

The gauge transformations are equal to
\[
\delta_s C_{mnl} = 3 \partial_{[m} \xi_{n]}.
\tag{3.9}
\]

The local Lorentz transformations are equal to
\[
\begin{align*}
\delta_l e_m{}^a &= \lambda_a^b e_m, \\
\delta_l \psi_m &= \frac{1}{4} \lambda_a^b \Gamma^b_{\alpha} \psi_m, \\
\delta_l \omega_m{}^b &= \partial_m \lambda_a^b + \lambda_a^c \omega_m{}^b - \omega_m{}^c \lambda_b.
\tag{3.10}
\end{align*}
\]

The general coordinate transformation are equal to
\[
\begin{align*}
\delta_g e_m &= \xi^n \partial_n e_m + \partial_m \xi_n e_n, \\
\delta_g \omega_m &= \xi^n \partial_n \omega_m + \partial_m \xi_n \omega_n, \\
\delta_g \psi_m &= \xi^n \partial_n \psi_m + \partial_m \xi_n \psi_n, \\
\delta_g C_{mnl} &= \xi^n \partial_n C_{mnl} + 3 \partial_{[m} \xi^l C_{lnl]}.
\tag{3.11}
\end{align*}
\]

We obtain the full algebra of these transformations as follows
\[
[\delta_g(\xi_1) + \delta_s(\epsilon_1) + \delta_l(\lambda_1) + \delta_c(\xi_{1mn}), \delta_g(\xi_2) + \delta_s(\epsilon_2) + \delta_l(\lambda_2) + \delta_c(\xi_{2mn})] = \delta_g(\xi_3) + \delta_s(\epsilon_3) + \delta_l(\lambda_3) + \delta_c(\xi_{3mn}),
\tag{3.12}
\]

where
\[
\begin{align*}
\xi_3 &= \xi_2 \partial_n \xi_1 + \bar{\xi}_2 \Gamma_n \epsilon_1 - (1 \leftrightarrow 2), \\
\epsilon_3 &= -\bar{\xi}_2 \Gamma_n \epsilon_1 - \xi_1 \partial_n \epsilon_2 + \frac{1}{4} \lambda_{2c} \Gamma_n \epsilon_1 - (1 \leftrightarrow 2), \\
\lambda_3{}^a{}_b &= -\bar{\xi}_2 \Gamma_n \epsilon_1 - \xi_1 \partial_n \lambda_2{}^a{}_b + \lambda_2{}^a{}_c \lambda_1{}^b \\
&\quad + \frac{1}{144} \bar{\xi}_2 (\Gamma_{\rho}{}^a{}_{rstu} \hat{F}_{rstuv} + 24 \Gamma_{rs} \hat{F}_{b}{}^a{}_{rs}) \epsilon_1 - (1 \leftrightarrow 2), \\
\xi_3mn &= -\bar{\xi}_2 \Gamma^k \epsilon_1 C_{kmn} - \bar{\xi}_2 \Gamma_{mn} \epsilon_1 - \xi_1 \partial_k \xi_2 mn - 2 \xi_1 \partial_{[m} \xi_2 n]k \\
&\quad - (1 \leftrightarrow 2).
\tag{3.13}
\end{align*}
\]
4 Superspace representation

In this section, we explain notations of the superspace geometry and obtain the full algebra of transformations in 11-dimensional superspace. As usual, we suppose that the superspace has Lorentzian tangent space structure and the vielbein $E^A_M$ and the connection $\Omega_{MA}^B$ and the corresponding 1-forms

$$E^A = dz^M E^A_M, \quad \Omega_A^B = dz^M \Omega_{MA}^B. \quad (4.1)$$

The Lorentzian assumption implies

$$\Omega_{ab} = -\Omega_{ba}, \quad \Omega_{ab} = 0, \quad \Omega_{\alpha\beta} = \frac{1}{4} \Omega_{ab} \Gamma^{ab}_{\alpha\beta}. \quad (4.2)$$

There are also 3-form potential

$$B = \frac{1}{3!} dz^L dz^M dz^N B_{NML}, \quad (4.3)$$

and field strength 4-form

$$H = dB = \frac{1}{4!} dz^K dz^L dz^M dz^N H_{NMLK}. \quad (4.4)$$

From these basic fields we can define the torsion and curvature as follows

$$T^A = DE^A, \quad R_A^B = d\Omega_A^B + \Omega_A^C \Omega_C^B, \quad (4.5)$$

where covariant derivative $D$ is defined as follows

$$DE^A = dE^A + E^B \Omega_B^A. \quad (4.6)$$

One then has the Bianchi identities

$$DT^A = E^B R_B^A, \quad DR_A^B = 0, \quad DH = 0. \quad (4.7)$$
The supertransformation is equal to
\[ \delta_T X_{M_p...M_1} = \Xi^K \partial_K X_{M_p...M_1} + p \partial_M \Xi^K X_{[K|M_p...M_1]} \] (4.8)
for p-form’s components. The local Lorentz transformations are equal to
\[ \delta_L E^A = E^B \Lambda_B^A, \]
\[ \delta_L \Omega_B^A = -\Lambda_B^C \Omega_C^A + \Omega_B^C \Lambda_C^A - d \Lambda_B^A. \] (4.9)
The supergauge transformations are equal to
\[ \delta_G B_{LMN} = 3 \partial_L \Xi_{MN}. \] (4.10)
We obtain the full algebra of these transformations as follows
\[ [\delta_T (\Xi_1) + \delta_L (\Lambda_1) + \delta_G (\Xi_{1MN}), \delta_T (\Xi_2) + \delta_L (\Lambda_2) + \delta_G (\Xi_{2MN})] = \delta_T (\Xi_3) + \delta_L (\Lambda_3) + \delta_G (\Xi_{3MN}), \] (4.11)
where,
\[ \Xi_3^K = \Xi_2^K \partial_L \Xi_1^K + \delta_1 \Xi_2^K - (1 \leftrightarrow 2), \]
\[ \Lambda_{3A}^B = -\Xi_1^K \partial_K \Lambda_2^A + \delta_1 \Lambda_2^A + \Lambda_1^C \Lambda_2^B - (1 \leftrightarrow 2), \]
\[ \Xi_{3MN} = \delta_1 \Lambda_{2MN} - \Xi_1^K \partial_K \Xi_{2MN} - 2 \partial_M \Xi_{2N} \Xi_1^K - (1 \leftrightarrow 2). \] (4.12)

There are a great number of component fields in superspace. Thus if we try to identify superspace representation as ordinary supergravity, there are a great number of unknown degrees of freedom. The method of this identification is known as ‘gauge completion’ [16]. We shall explain it in the next section.

5 Gauge completion

‘Gauge completion’ was introduced in order to identify superspace representation as on-shell supergravity [16]. In this section we review this method and compute part of the superfield in terms of the on-shell supergravity fields.

Using this method in 2nd-order formalism, up to first order in anticommuting coordinates, the superfield component was investigated by E. Cremmer and S. Ferrara [5]. And in 1.5-order formalism, part of component at second order in anticommuting coordinates was investigated by B. de Wit, K. Peeters and J. Plefka [4].
’Gauge completion’ is finding the superfields and superparameters which are compatible with ordinary supergravity. That is to say, supertransformations (4.8) - (4.10) are identified as transformations in 11-dimensional spacetime (3.5) - (3.11) and the \( \theta = 0 \) components of superfields and super parameters are identified as the fields and parameters of ordinary supergravity.

Firstly, a gauge is chosen as follows

\[
\begin{align*}
E^a_m(0) &= e^a_m, \\
E^\alpha_m(0) &= \psi^\alpha_m, \\
\Omega_{ab}^m(0) &= -\hat{\omega}^a_m b, \\
\Xi^m(0) &= \xi^m, \\
\Xi^{\alpha}(0) &= \epsilon^\alpha, \\
\Xi^{mn}(0) &= \xi^{mn}, \\
B_{mnl}(0) &= C_{mnl}.
\end{align*}
\] (5.1)

From (4.9) and (3.10), we obtain

\[
\Lambda^a_{b}(0) = \lambda^a_{b}.
\] (5.2)

And we introduce some assumptions as follows

\[
\Xi^{(0)}_{\mu N} = 0, \\
\Xi^{(1)}_{\mu N} = 0.
\] (5.3)

Then, the higher order component in anticommuting coordinates can be obtained by requiring consistency between the algebra of superspace supergravity and that of ordinary supergravity.

To make this procedure clear, we write one simple example explicitly. We take \( \theta = 0 \) component of vielbein.

According to superspace algebra,

\[
\delta E^a_m|_{\theta=0} = (\Xi^K \partial_K E^a_m + \partial_m \Xi^K E^a_K + E^a_m \Lambda^b_{b} )|_{\theta=0} = \xi^k \partial_k e^a_m + \partial_m \xi^k e^a_k + \epsilon^\nu \partial_\nu (E^a_m(a^{1})) + \partial_m \epsilon^\nu E^a_{m^{(1)}} + \epsilon^b_{m} \lambda^a_{b},
\] (5.4)

while in ordinary supergravity

\[
\delta e^a_m = \xi^k \partial_k e^a_m + \partial_m \xi^k e^a_k + 2\epsilon^{a}_{m} \psi^m + \lambda^a_{b} e^b_m.
\] (5.5)
Thus, we obtain
\[ E^{(0)}_\nu = 0, \]
\[ E^{a(1)}_m = 2\hat{\theta}^{a}\psi_m. \] (5.6)

By this procedure, we obtain the following results.

\[ E^{a}_m = e^{a}_m + 2\hat{\theta}^{a}\psi_m - \frac{1}{4}\hat{\theta}\Gamma^{acd}\hat{\omega}_{mca} + \frac{1}{72}\hat{\theta}\Gamma^{rst}\theta\hat{F}_{rst}^a + \frac{1}{288}\hat{\theta}\Gamma^{rstu}\hat{F}_{rstu}e^{a}_m + \frac{1}{36}\hat{\theta}\Gamma^{astu}\theta\hat{F}_{mstu} + O(\theta^3), \] (5.7)

\[ E^\alpha_m = \psi^\alpha_m - \frac{1}{4}\hat{\omega}_{mab}(\Gamma^{ab}\theta)^\alpha + (T^r_m\theta)^\alpha\hat{F}_{rstu} + O(\theta^2), \] (5.8)

\[ E^\alpha_\mu = -(\Gamma^\alpha\theta)_\mu + O(\theta^2), \] (5.9)

\[ E^\alpha_\nu = \hat{\delta}^\alpha_\mu + O(\theta^2), \] (5.10)

\[ \Omega_{\mu b}^a = \frac{1}{144}\{(\Gamma^a_b\theta)\mu\hat{F}_{rstu} + 24(\Gamma^a_r\theta)\mu\hat{F}_{rsb}^a\} + O(\theta^2), \] (5.11)

\[ \Omega_{mab} = \hat{\omega}_{mab} + 2\hat{\bar{\omega}}\{e^n_a e^b_k (\Gamma^a_b\theta) + \Gamma^a_k D_{[n}\psi_{m]} + \Gamma_k D_{[m}\psi_{n]} + \Gamma_{m} D_{[n}\psi_{k]}\} \]
\[ -\bar{\psi}^\alpha_b T^r_m \Gamma_{rstu}\theta\hat{F}_{rstu} + \bar{\psi}^\alpha_m \Gamma_b T^r_{stu}\theta\hat{F}_{rstu} + \bar{\psi}^\alpha_m \Gamma_a T^r_{stu}\theta\hat{F}_{rstu} \]
\[ -\bar{\psi}^\alpha_m \Gamma_a T^r_{stu}\theta\hat{F}_{rstu} + \bar{\psi}^\alpha_b \Gamma_a T^r_{stu}\theta\hat{F}_{rstu} + O(\theta^2), \] (5.12)

\[ B_{mnl} = C_{mnl} - 6\hat{\theta}\Gamma_{[mn}\psi] + \frac{3}{4}\Gamma_{[l}^{cd}\hat{\theta}\Gamma_{mn]}^{cd}\theta - \frac{3}{2}\hat{\omega}_{[mn]l}\theta^2 \]
\[ -\frac{1}{96}\hat{\theta}\Gamma_{mnl}^{rstu}\hat{F}_{rstu} - \frac{3}{8}\hat{\theta}\Gamma_{l}^{rs}\theta\hat{F}_{[rs][mn]} - 12\hat{\theta}\Gamma_{a}^{l}[m\hat{\theta}\Gamma_{n}\psi]_{l} \]
\[ + O(\theta^3), \] (5.13)

\[ B_{l m r} = (\hat{\theta}\Gamma_{l m}^r)_\mu + O(\theta^2), \] (5.14)

\[ B_{m r p} = (\hat{\theta}\Gamma_{m r}^p)_\mu (\hat{\theta}\Gamma_{r p}^m)_\nu + O(\theta^2), \] (5.15)

\[ \Xi^m = \xi^m + \theta\Gamma^m\epsilon - \theta\Gamma^n\epsilon\psi_n + O(\theta^2), \] (5.16)

\[ \Xi^\mu = e^\mu + \frac{1}{4}\lambda_{cd}(\Gamma^{cd}\theta)^\mu - \theta\Gamma^n\epsilon\psi_n^\mu + O(\theta^2), \] (5.17)

\[ \Lambda^a_b = \lambda^a_b - \theta\Gamma^n\epsilon\hat{\omega}^a_{nb} + \frac{1}{144}\hat{\theta}(\Gamma^a_b\theta)\mu\hat{F}_{rstu} + 24\Gamma^a_r\theta\hat{F}_{rs}^a + O(\theta^2), \] (5.18)

\[ \Xi_{mn} = \xi_{mn} - (\hat{\theta}\Gamma^{p}\epsilon C_{pmn} + \hat{\theta}\Gamma_{mn}\epsilon) + O(\theta^2), \] (5.19)

\[ \Xi_{m\mu} = O(\theta^2), \] (5.20)
3-form fields are obtained up to first order in anticommuting coordinates. In order to compute this at second order in anticommuting coordinates, the superparameter $\Xi_{MN}$ at second order is needed. And in order to compute $\Xi_{MN}$ at second order in anticommuting coordinates, the superparameter $\Xi^M$ at second order is needed. Thus we can’t compute 3-form fields at second order. However, because flat geometry is known (2.10), we include the $\theta^3$ term in $B_{\mu\nu\rho}$ and the $\theta^2$ term in $B_{m\mu\nu}$ for completeness.

We obtained all components which was required in order to write the action (2.1) up to $\theta^2$ term.

The components of vielbein and 3-form field obtained above is the same as was constructed before [4]. Thus this result holds the invariance of $\kappa$-symmetry. However the other components and the equations which components of vielbein and 3-form and superparameter at second order in anticommuting coordinates must obey are different from those in reference [4]. These can be written explicitly as follows,

\begin{equation}
\Xi_{\mu\nu} = \mathcal{O}(\theta^2).
\end{equation}

\begin{align}
0 &= X^k \partial_k e_m^a + \partial_m X^k e_k^a - \bar{\epsilon}_2 \Gamma^k \epsilon_1 (\psi^\mu \partial_\mu E_m^a) + \hat{\psi}_{m|cd} \bar{\theta} \Gamma^a \Gamma^{cd} \psi_k \\
&+ 2\bar{\theta} \Gamma^n \psi_m \left\{-\bar{\epsilon}_2 \Gamma^n \epsilon_1 \hat{\omega}_m^a + \frac{1}{144} \bar{\epsilon}_2 (\Gamma^b_{\ rstu} \hat{F}_{rstu} + 24 \Gamma_{rs} \hat{F}_{b}^{rs}) \epsilon_1 \right\} \\
&- \frac{1}{288} \bar{\epsilon}_2 (\Gamma^c_{d rstu} \hat{F}_{rstu} + 24 \Gamma_{rs} \hat{F}_{d}^{rs}) \epsilon_1 \bar{\theta} \Gamma^a \Gamma^{cd} \psi_m + \epsilon_m Y_a^b \\
&+ 2\{\epsilon_2 \nu \partial_\nu \Xi_1^{(2)} - \bar{\theta} \Gamma^k \epsilon_2 \bar{\psi}_k \Gamma^n \epsilon_1 \psi_n^\mu + \frac{1}{4} \bar{\theta} \Gamma^k \epsilon_2 \hat{\omega}_{kcd}(\Gamma^{cd} \epsilon_1)^\mu \\
&- \bar{\theta} \Gamma^k \epsilon_2 \hat{F}_{rstu}(T_{k rstu} \epsilon_1)^\mu \} (\Gamma^a \psi_m)_\mu \\
&- (1 \leftrightarrow 2),
\end{align}

where

\begin{align}
X^k &= \epsilon_2 \partial_v \Xi_1^{(2)} + \bar{\theta} \Gamma^n \epsilon_1 \bar{\epsilon}_2 \Gamma^k \psi_n + \epsilon_2 \Gamma^n \epsilon_1 \bar{\theta} \Gamma^k \psi_n, \\
Y_b^a &= -\bar{\theta} \Gamma^n \epsilon_1 \epsilon_2 \Gamma^k \psi_n \hat{\omega}_m^a - \epsilon_1 \nu \partial_\nu \Lambda_{2b}^a \\
&+ \frac{1}{144} \bar{\theta} \Gamma^k \epsilon_1 \bar{\psi}_k (\Gamma^a_{b rstu} \hat{F}_{rstu} + 24 \Gamma_{rs} \hat{F}_{b}^{rs}) \epsilon_2 \\
&- \frac{1}{18} \bar{\epsilon}_1 \Gamma^r \psi_n \bar{\Gamma}^{a ustu} \epsilon_2 \hat{F}_{rstu} - \frac{2}{3} \bar{\epsilon}_1 \Gamma^r \psi_k \bar{\Gamma}^{ku} \epsilon_2 \hat{F}_{b tu} \\
&- \frac{1}{3} \bar{\theta} \Gamma^{tu} \epsilon_2 \bar{\epsilon}_1 \Gamma^s \psi_b \hat{F}_{stu} \\
&- \frac{1}{3} \bar{\theta} \Gamma^n \epsilon_2 \epsilon_{b c l} \hat{D}_{[n k]} \psi_{l]} \\
&- 2\bar{\theta} \Gamma^n \epsilon_2 \epsilon_{b c l} \hat{D}_{[n k]} \psi_{l]} \bar{\epsilon}_1 \Gamma^l \hat{D}_{[k]} \psi_{l]}
\end{align}
\[-\frac{1}{3} \bar{\theta} \Gamma^n \epsilon_2 \Gamma^n \psi_n \hat{F}_{bst} \]
\[-\frac{1}{108} (\bar{\theta} \Gamma^n_b r_{stu} \epsilon_2 + 24 \bar{\theta} \Gamma^n_t u \epsilon_2 e^{r_a} e^{s_b}) \epsilon_1 \Gamma_{(r_s w_{xy} \psi_{[v} \hat{F}_{u]w_{xy}}} \]
\[-\frac{1}{6} (\bar{\theta} \Gamma^n_b r_{stu} \epsilon_2 + 24 \bar{\theta} \Gamma^n_t u \epsilon_2 e^{r_a} e^{s_b}) \epsilon_1 \Gamma_{[r_s \bar{D}_{l} \psi_u]} \]
\[-\frac{1}{18} (\bar{\theta} \Gamma^n_b r_{stu} \epsilon_2 + 24 \bar{\theta} \Gamma^n_t u \epsilon_2 e^{r_a} e^{s_b}) \epsilon_1 \Gamma^w \psi_{[v} \hat{F}_{stu]w}. \]

(5.25)

There are $\partial_m$ terms in $Y^a_b$. Thus $\Xi^{k(2)}$ is different from that in reference [7]. The other components are also different from that in reference [4].

6 Discussion

Using 2nd-order formalism we obtained the same results as that was given in reference [7] up to obtained components, but at higher order it seems to be different from that in 1.5-order formalism. In order to obtain superspace geometry which holds $\kappa$-invariance 2nd-order formalism may be much simpler and more hopeful than 1.5-order formalism. As seen in section 2, the condition for $\kappa$-invariance is given on torsion. From the definition of torsion (4.5),

\[E_m^\beta E_n^\gamma T^a_{\gamma \beta} = 2 \partial_{[n} E_m^a + 2 E_{[m}^b \Omega_n]b^a, \]

at $\theta = 0$ component,

\[\psi_m^\beta \psi_n^\gamma T^a_{\gamma \beta} = 2 \partial_{[n} e_m^a - 2 e_{[m}^b \hat{\omega}_n^a b. \]

(6.1)

This is compatible with definition of spin connection (3.2) and constraint for $\kappa$-invariance (2.5).

If we use 1.5-order formalism, the constraint (2.3) is not invariant under supersymmetry transformations. Thus $\kappa$-invariance of action (2.1) will be realized in very complicated form at higher order in anticommuting coordinates.

But, if we use 2nd-order formalism we can obtain superfields which hold the conditions (2.5). Thus for higher components in order to hold $\kappa$-invariance 2nd-order formalism is expected to be more hopeful than 1.5-order formalism.
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Appendix

Our notations are almost same as that used in a text by J. Wess and J. Bagger \[17\].

A Indices

We use Greek indices for spinorial components and Latin indices for vector components. And we use former alphabet for the tangent space indices and later for general coordinates indices: $a, b, c, \ldots$ for tangent vector indices and $k, l, m, \ldots$ for general vector indices, and $\alpha, \beta, \ldots$ for tangent spinorial indices and $\mu, \nu, \ldots$ for general spinorial indices.

Superspace coordinates $(x^m, \theta^n)$ are designated $Z^M$, where later capital Latin alphabet $M, N, \ldots$ are collective designations for general coordinate indices. While for mer capital Latin alphabet $A, B, \ldots$ are collective designations for tangent space indices.

B p-form superfield

Vielbein is represented by $E^A_M$ and its inverse is $\tilde{E}^A_M$, which is defined as follows,

$$
\tilde{E}^A_M E^B_M = \delta^B_A,
E^A_N \tilde{E}^M_A = \delta^M_N.
$$

(B.1)

We introduce p-form superfields as follows,

$$
X \equiv \frac{1}{p!} dz^M dz^M_1 \cdots dz^M_p \cdots M_1 X_{M_p \cdots M_1}
\equiv \frac{1}{p!} E^A_p \cdots E^A_1 X_{A_p \cdots A_1},
$$

(B.2)

$$
X_{A_p \cdots A_1} \equiv \sum_{i=1}^{32} X_{A_p \cdots A_1}^{(i)}.
$$

(B.3)
$X_{A_p...A_i}$ is component at $i$-th order in anticommuting coordinates.

## C  Convention

We use the mostly plus metric; $\eta_{ab} \sim (- + ... +)$. Symmetrization bracket $( )$ and antisymmetrization bracket $[,]$ is defined as follows,

\[
[M_1...M_N] = \frac{1}{N!} (M_1...M_N + \text{antisymmetric terms}),
\]
\[
(M_1...M_N) = \frac{1}{N!} (M_1...M_N + \text{symmetric terms}).
\] (C.1)

## D  Gamma matrices (11-dimensional)

Since we use the Majorana representation, all components are real. Gamma matrix $\Gamma^a \alpha_\beta$ is defined as follows,

\[
\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}.
\] (D.1)

We lower the spinorial indices by charge conjugation matrix $C_{\alpha\beta}$.

\[
\bar{\psi}_\beta = \psi^\alpha C_{\alpha\beta},
\]
\[
\Gamma^a_{\alpha\beta} = C_{\alpha\gamma} \Gamma^a_{\gamma\beta}.
\] (D.2)

The symmetric matrices are

\[
\Gamma^a, \Gamma^{a_1...a_2}, \Gamma^{a_1...a_5}, \Gamma^{a_1...a_6}, \Gamma^{a_1...a_9}, \Gamma^{a_1...a_{10}},
\] (D.3)

and antisymmetric matrices are

\[
C, \Gamma^{a_1...a_3}, \Gamma^{a_1...a_4}, \Gamma^{a_1...a_7}, \Gamma^{a_1...a_8}, \Gamma^{a_1...a_{11}}.
\] (D.4)

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