A bilateral matching method of decision-making for customized buses

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Abstract. As the cornerstone for urbanization, public transportation needs to raise attraction. Hence, the key motivation of this research is about balancing the needs of customized-buses and commuters. Regarding satisfaction and fairness of matching schemes, this paper explores the characteristics of customized-bus services and combines actual needs to establish a multi-objective bilateral matching optimization model including maximum matching correlation degree, minimum correlation deviation and maximum matching logarithm. To show the indicators information, the methodologies introduces the grey correlation analysis method adopting four data types. The satisfaction level is obtained by solving the maximum matching scheme that matches the appropriate customized-bus for commuters. The proposed model meets the requirements of operators and commuters, may further improve the attraction of public transportation and its service quality, which can be the key contributions of this paper.

1. Introduction
Different from the stereotype of operation mode for conventional buses, customized buses can flexibly adjust operation lines according to the travel needs of commuters, making personalized travelling more convenient and efficient while keeping its conventional properties of low cost and sharp time remained. This paper comprehensively considers the subjective needs of both bus companies and commuters by introducing a bilateral matching method, through which a bilateral matching decision optimization model is therefore established with the goal of maximizing the overall satisfaction of passengers and bus companies.

2. Establishing a bilateral matching model of customized buses
Each subject sets the expected value for the attributes they concerned about, and the decision maker makes a matching decision based on the expected and the real value of both subjects. In this paper, clear number, interval number, triangular fuzzy number and intuitionistic fuzzy number are adopted to comprehensively represent various attribute information of the subject. grey correlation analysis method is adopted to obtain the matching satisfaction of both parties through simple and effective correlation degree calculation.

From the overall perspective of stability, fairness and satisfaction, this paper constructs a multi-objective optimization model of bilateral matching which maximizes matching correlation degree.

2.1. The process of model establishment
Let \( A \) be a set of commuters: \( A = \{A_1, A_2, \ldots, A_i, \ldots, A_n\} \), where \( A_i \) denotes the \( i \)th commuter. Let \( B \) be a set of customized buses: \( B = \{B_j, B_2, \ldots, B_j, \ldots, B_n\} \), where \( B_j \) denotes the \( j \)th customized bus. Let \( C \) be a
set of commuter evaluation attributes: \( C = \{ C_1, C_2, \ldots, C_i, \ldots, C_y \} \), where \( C_y \) denotes the \( k \)th evaluation attribute, \( k = 1, 2, \ldots, p \). Let \( D \) be a set of customized bus evaluation attributes: \( D = \{ D_1, D_2, \ldots, D_i, \ldots, D_y \} \), where \( D_i \) denotes the \( i \)th customized bus, \( l = 1, 2, \ldots, q \). \( w_k (k = 1, 2, \ldots, p) \) denotes the weight information of \( C_y \) evaluation attributes, \( v_i (l = 1, 2, \ldots, q) \) denotes the weight information of \( D_i \) evaluation attributes, \( w_i \) and \( v_i \) are not completely certain, but we suppose \( w_i = [w_{i1}^l, w_{i2}^l] \) and \( v_i = [v_{i1}^l, v_{i2}^l] \), where

\[
\sum_{l=1}^{q} w_{i1}^l \leq 1, \sum_{l=1}^{q} w_{i2}^l \leq 1, \sum_{l=1}^{q} v_{i1}^l \geq 1, \sum_{l=1}^{q} v_{i2}^l \geq 1, 0 \leq w_{i1}^l \leq w_{i2}^l \leq 1, 0 \leq v_{i1}^l \leq v_{i2}^l \leq 1.
\]

The evaluation information of commuter \( A \) under the evaluation attribute \( C_y \) is \( a_{ik} \), and the expected level is \( a'_{ik} \). The evaluation information of customized bus \( B \) under the evaluation attribute \( D_i \) is \( b_{ik} \), and the expected level is \( b'_{ik} \).

Clear number (\( Y_1 \)), interval number (\( Y_2 \)) and triangular fuzzy number (\( Y_3 \)) denote the quantitative attributes of commuters and customized buses, while intuitive fuzzy number (\( Y_4 \)) denotes the qualitative attributes. Therefore, the evaluation information set is represented as \( Y = Y_1 \cup Y_2 \cup Y_3 \cup Y_4 \), including benefit (\( Y^1 \)) and cost (\( Y^2 \)) attribute sets.

1. When the attribute types are clear number \( (C_i \in Y_1) \), interval number \( (C_i \in Y_2) \), triangular fuzzy number \( (C_i \in Y_3) \) and intuitionistic fuzzy number \( (C_i \in Y_4) \), the evaluation information \( a_{ik} \) and the expectation level \( a'_{ik} \) of commuters are expressed as:

\[
a_{ik} = \begin{cases} 
  \frac{d_{ik}}{d_{max}}, & C_i \in Y_1 \\
  \frac{e_{ik}}{e_{max}}, & C_i \in Y_2 \\
  (f_{ik}, g_{ik}, h_{ik}), & C_i \in Y_3 \\
  (r_{ik}, s_{ik}), & C_i \in Y_4
\end{cases} \quad (1)
\]

\[
a'_{ik} = \begin{cases} 
  \frac{d'_{ik}}{d_{max}}, & C_i \in Y_1 \\
  \frac{e'_{ik}}{e_{max}}, & C_i \in Y_2 \\
  (f'_{ik}, g'_{ik}, h'_{ik}), & C_i \in Y_3 \\
  (r'_{ik}, s'_{ik}), & C_i \in Y_4
\end{cases} \quad (2)
\]

where \( d_{ik} \geq 0, d'_{ik} \geq 0; 0 \leq e_{ik} < e_{max}; 0 \leq e'_{ik} < e_{max} ; 0 \leq f_{ik} \leq g_{ik} \leq h_{ik} \leq 1 ; 0 \leq f'_{ik} \leq g'_{ik} \leq h'_{ik} \leq 1 ; 0 \leq r_{ik} \leq s_{ik} \leq 1, 0 \leq r'_{ik} \leq s'_{ik} \leq 1.

2. The evaluated value and expected value of each attribute of commuter is normalized, and \( a_{ik} \) is taken as an example to be processed as:

- clear number \( a_{ik} = d_{ik} \) \((C_i \in Y_1)\) is normalized to:

\[
\zeta_{ik} = \begin{cases} 
  \frac{d_{ik}}{d_{max}}, & C_i \in Y^+ \\
  1 - \frac{d_{ik}}{d_{max}}, & C_i \in Y^-
\end{cases} \quad (3)
\]

- interval number \( a_{ik} = [e_{ik}, e_{max}] \) \((C_i \in Y_2)\) is normalized to:

\[
\zeta_{ik} = \begin{cases} 
  \frac{e_{ik}}{e_{max}}, & C_i \in Y^+ \\
  \left[1 - \frac{e_{ik}}{e_{max}}, 1 - \frac{e_{max}}{e_{max}}\right], & C_i \in Y^-
\end{cases} \quad (4)
\]

- triangular fuzzy number \( a_{ik} = (f_{ik}, g_{ik}, h_{ik}) \) \((C_i \in Y_3)\) is normalized to:

\[
\zeta_{ik} = \begin{cases} 
  (f_{ik}, g_{ik}, h_{ik}), & C_i \in Y^+ \\
  \left[1 - h_{ik}, h_{max}, 1 - f_{ik} / h_{max}\right], & C_i \in Y^-
\end{cases} \quad (5)
\]

\[
d_{max} = \max \{d_{ik}\}, e_{max} = \max \{e_{ik}\}, h_{max} = \max \{h_{ik}\} (i = 1, 2, \ldots, m, k = 1, 2, \ldots, p)
\]

- intuitionistic fuzzy number is \( a_{ik} = (r_{ik}, s_{ik}) \) \((C_i \in Y_4)\), where \( 0 \leq a_{ik} \leq 1 \), without performing standardization. For simplicity, the normalized symbol remains unchanged.
3. According to the grey correlation analysis method, the grey correlation coefficient is firstly calculated to determine the degree of similarity between the evaluation value of commuters \( a_i \) and the expected value of customized buses to commuters \( a'_j \) under various attributes of commuters, and then each attribute is weighted to comprehensively judge the degree of correlation between commuters and customized buses. Therefore, the grey correlation coefficient between commuter \( A_i \) and customized bus \( B_j \) under the attribute \( C_k \) is defined as:

\[
\alpha_{ik} = \frac{\min_{j} \left( \min_{i} O(a_i, a'_j) + \theta \max_{j} \max_{i} O(a_i, a'_j) \right)}{O(a_i, a'_j) + \theta \max_{j} \max_{i} O(a_i, a'_j)}
\]

(6)

where \( i = 1, 2, \ldots, m \); \( j = 1, 2, \ldots, n \), \( \theta \) is the resolution coefficient, where \( 0 \leq \theta \leq 1 \). Smaller \( \theta \) will induce larger gap among correlation coefficients and stronger discrimination ability. Generally, \( \theta = 0.5 \).

\( O(a_i, a'_j) \) is defined as the distance between the evaluation value \( a_i \) of the normalized commuter and the expected value \( a'_j \) of the customized bus, and \( O(a_i, a'_j) \) is the normalized distance under the attribute \( C_k \), which can be described as:

\[
O(a_i, a'_j) = \begin{cases} 
\sqrt{2} \left( |f_{ik} - f'_{jk}| + |g_{ik} - g'_{jk}| \right), & a_i, a'_j \in Y_1 \\
\frac{1}{3} \left( |f_{ik} - f'_{jk}| + |g_{ik} - g'_{jk}| + |h_{ik} - h'_{jk}| \right), & a_i, a'_j \in Y_2 \\
\frac{1}{3} \left( |f_{ik} - r'_{jk}| + |s_{ik} - s'_{jk}| + |\delta_{ik} - \eta'_{jk}| \right), & a_i, a'_j \in Y_3 \\
\frac{1}{3} \left( |f_{ik} - r'_{jk}| + |s_{ik} - s'_{jk}| + |\delta_{ik} - \eta'_{jk}| \right), & a_i, a'_j \in Y_4 
\end{cases}
\]

(7)

where \( \lambda > 0 \) is distance parameter, \( \delta_{ik} = 1 - r_{ik} - r'_{jk} \) and \( \eta'_{jk} = 1 - s_{ik} - s'_{jk} \) are the degree of hesitation.

After obtaining the grey correlation coefficient of commuter \( A_i \), the matching correlation degree \( \tau_{ij} \) can be obtained by weighting the attributes \( C_k \):

\[
\tau_{ij} = \sum_{k=1}^{p} w_k \alpha_{ik}^j (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)
\]

(8)

Similarly, the matching correlation coefficient \( \beta_{ji}^l \) of customized bus \( B_j \) and commuter \( A_i \) under the attribute \( D_l \) is obtained. The matching correlation degree \( \tau_{ji} \) can be obtained by weighting the attributes \( D_l \):

\[
\tau_{ji} = \sum_{l=1}^{q} v_l \beta_{ji}^l (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)
\]

(9)

2.2. Bilateral matching model

Correlation degree reflects the connection degree of the two sides under the different attribute index. In order to acquire the correlation degree, weights \( w_k \) and \( v_l \) need to be determined in advance. If the maximum value of the correlation degree \( \tau_{ij} \) and \( \tau_{ji} \) is required, and both parties are in the same position in the matching process, a simple weighting method can be adopted (the priority of each subject is equal) to establish the following single-objective optimization model:
\[
\max \phi = \sum_{i=1}^{m} \sum_{j=1}^{n} w_i \alpha_{ij} + \sum_{j=1}^{n} \sum_{i=1}^{m} v_i \beta_{ji} \\
\text{s.t. } \sum_{i=1}^{m} w_i = 1, \quad w_i \geq 0
\]

\[
\sum_{i=1}^{m} v_i = 1, \quad v_i \geq 0
\]

\[i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n; \quad w_k = [w_k^I, w_k^J]; \quad v_j = [v_j^I, v_j^J] \]

The weight vectors \(w_k\) and \(v_j\) is solved, and the correlation degrees \(\tau_{ij}\) and \(\tau_{ji}\) are acquired. When matching stability is taken into account, attention should be paid to a minimum degree of satisfaction for the whole matching system, that is, the minimum degree of correlation which denotes the critical value entering the matching system. Only when the correlation degree is not lower than the critical value can the matching subject be willing to choose. Otherwise, the subject is not willing to participate in the matching decision. Set \(\sigma (\sigma > 0)\) as the critical value for matching the subject's satisfaction, if \(\tau_{ij} \geq \sigma\), \(A\) can accept \(B\); if \(\tau_{ji} \geq \sigma\), \(B\) can accept \(A\).

Set \(x_{ij}\) as 0-1 decision variable. If \(x_{ij} = 0\), commuter and customized bus cannot be matched; If \(x_{ij} = 1\), the two sides are matched. In order to maximize the overall matching correlation degree between the two parties and minimize the matching scheme correlation degree deviation, a multi-objective bilateral matching model is established as follows:

\[
\max Z_i = H_1 \sum_{j=1}^{m} \tau_{ij} x_{ij} + H_2 \sum_{i=1}^{n} \tau_{ji} x_{ij}
\]

\[
\max Z_2 = \sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij}
\]

\[
\min Z_3 = \sum_{j=1}^{m} \sum_{i=1}^{n} \left| \tau_{ij} - \tau_{ji} \right| x_{ij}
\]

\[\text{s.t. } \sum_{j=1}^{m} x_{ij} \leq 1, \quad i = 1, 2, \ldots, m;
\]

\[\sum_{i=1}^{n} x_{ij} \leq \mu, \quad j = 1, 2, \ldots, n;
\]

\[\sigma \leq \tau_{ij}, \sigma \leq \tau_{ji};
\]

\[x_{ij} \in [0,1], \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n.
\]

The above model includes three objective functions. Objective function (14) represents maximizing the correlation degree between commuters and customized buses, which maximizes their satisfaction. \(H_1\) and \(H_2\) denote the weight information of both parties during the matching. To ensure the fairness of matching, this paper assumes that both parties are in the same position, where \(H_1 = H_2 = 0.5\). The objective function (15) represents maximizing the number of the successful matches between commuters and customized buses. Objective function (16) indicates minimizing the deviation of correlation degree between commuters and customized buses, so as to minimize the deviation of satisfaction degree. Equation (17) to (20) are the constraint condition. Lingo and/or other software can be used to process the model.

3. Conclusions
Based on the customized public transport network, this paper analyzed and optimized the theoretical framework of multi-dimensional route planning, the path selection model aiming at the cost of operators, as well as the customized public transport network design with fixed demand in advance,
thereby proposing a bilateral matching mechanism on which three elements of satisfaction, stability and fairness are focused.

However, in this paper there were still some shortcomings in parameter verifications and algorithms design. The effect of distance parameter change on decision-makers' psychological expectations, the impact of traffic jams, walking distance and other factors on the matching results were not thoroughly considered; the algorithms solving the model may be further optimized, on which future researches will be carried out concerning these short boards.

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