Complete Two-Loop Corrections to $H \to \gamma\gamma$

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In this paper the complete two-loop corrections to the Higgs-boson decay, $H \to \gamma\gamma$, are presented. The evaluations of both QCD and electroweak corrections are based on a numerical approach. The results cover all kinematical regions, including the $WW$ normal-threshold, by introducing complex masses in the relevant (gauge-invariant) parts of the LO and NLO amplitudes.

Key words: Feynman diagrams, Multi-loop calculations, Higgs physics

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1 Introduction

In the intermediate mass range of the Higgs-boson, its decay into photons is of great phenomenological interest. At hadron colliders the decay $H \rightarrow \gamma\gamma$ provides precious informations for the discovery in the gluon-gluon production channel [1]. An upgrade option at the ILC will allow for a high precision measurement of the partial width into two photons [2] with a quantitative test for the existence of new charged particles.

The QCD corrections to $H \rightarrow \gamma\gamma$ have been computed in the past and analytic results at next-to-leading order are available in Ref. [3] and in Ref. [4] (see also Ref. [5]). Electroweak two-loop corrections have been computed by suitable expansions of the two-loop Feynman diagrams [6]. Master integrals for the two-loop light fermion contributions have been analyzed in Ref. [7] and two-loop light fermion contributions to Higgs production and decays in Ref. [8].

In our approach we have generated the full amplitude (up to two-loops and including QCD) in a completely independent way and we have used the techniques of Ref. [9] to produce a numerical evaluation of the partial width $\Gamma(H \rightarrow \gamma\gamma)$. Since we are not bound to rely on expansion techniques, not even in the bosonic sector and in the top-bottom one, we can produce results with very high accuracy for any value of the Higgs-boson mass, taking into account the complete mass dependence of the $W$-boson, $Z$-boson, Higgs-boson and top-quark. A consistent and gauge-invariant treatment of unstable particles made it also possible to produce very accurate results around the $WW$-threshold.

2 Method of calculation and technical issues

Our calculation builds upon the numerical approach of Ref. [9] where two-loop, two and three point functions have been investigated in the most general case. In this project we have developed a set of routines which go from standard $A_0$, . . . , $D_0$ functions up to diagrams needed for a two-loop $1 \rightarrow 2$ process. This new ensemble of programs will succeed to the corresponding Library of TOPAZ0 [10]. The whole collection of codes also uses the NAG-library [11].

The generation as well as the manipulation of Feynman diagrams has been performed with the use of the FORM [12] code GraphShot [13]. Diagrams are generated, simplified and a FORTRAN interface is created. Furthermore, the code checks for the validity of the relevant Ward identities. Renormalization is performed according to the scheme developed in Ref. [14]. In this paper, we shall follow the same notations and conventions for two-loop diagrams as defined in Ref. [15]. In the following we will give a short outline of the techniques used for the calculation.

Before evaluating the two-loop integrals arising after generating the Feynman diagrams, two main simplifications are done recursively. At first, reducible scalar products are removed and secondly, the symmetries of the diagrams are taken into account. The integrals are then assigned to scalar-, vector- and tensor-type integrals, according to the number of irreducible scalar products in the numerator and form-factors are introduced. The cancellation of scalar products is performed by expressing the scalar products in the numerator in terms of their associated propagators. This procedure can lead to removing lines in a diagram, so that each diagram produces a set of daughter-families with at least one line less. Apart from the reduction of scalar products, the consideration of the symmetries of a given diagram is important in order to reduce the number of integrals, which will be evaluated numerically at the end of the calculation. A simple example, showing the exploitation of the symmetries, is given in Fig. 1 for a scalar diagram.

We now discuss briefly the extraction of collinear logarithms from Feynman diagrams. It is worth noting that the amplitude for $H \rightarrow \gamma\gamma$ is collinear-free and one could adopt the approach where all light fermions are massless, then collinear behavior of single components is controlled in dimensional regularization and collinear poles cancel in the total. We prefer another approach where collinear singularities are controlled by light fermion masses. Although the total amplitude is collinear-free, our procedure of reduction $\otimes$ symmetrization introduces a sum of several terms, of which some are divergent. Of course, we check that all logarithms of collinear origin cancel and, as a matter of fact, they cancel family by family of diagrams.

To be precise, we need some universal representation for the coefficient of the collinear logarithms, which allows us to show their analytical cancellation, and a method to compute the remaining collinear-free parts. The first task is achieved by introducing integrals of one-loop functions and using their well-known properties.
Figure 1: Symmetries of the $V^E$-family: The first diagram represents the $V^E$-family (a). Its integral remains unchanged by exchanging $m_1 \leftrightarrow m_2$ (b) as well as if one interchanges $m_3 \leftrightarrow m_4$ and $p_2 \leftrightarrow -P$ simultaneously (c). The last diagram (d) is a combination of the first (b) and the second (c) symmetry. One can also perform a total reflection of all external momenta, which is not shown in the figure and leaves the integral also unchanged.

to make the cancellation explicit. Using the techniques of Ref. [9] the collinear finite contribution is first written in terms of smooth integrands and then evaluated numerically; an example is shown in Fig. 2.

Figure 2: Example of a collinear-divergent two-loop vertex diagram. Dashed lines represent particles with a small mass $m$ and the wavy (external) line is massless. We introduced $L_m = \text{ln}(m^2/|P|^2)$.

3 Conceptual issues

We will now apply our formalism to the computation of the amplitude for $H(-P) + \gamma(p_1) + \gamma(p_2) \rightarrow 0$, $(P = p_1 + p_2)$ which will be written as

$$A^{\mu\nu}(H \rightarrow \gamma\gamma) = \frac{g^3}{16\pi^2} s_0 \left[ F_D \, \delta^{\mu\nu} + \sum_{i,j=1}^2 F_p^{(ij)} p_i^\mu p_j^\nu + F_\epsilon(\mu, \nu, p_1, p_2) \right].$$

The form factor $F_\epsilon$ is absent at $O(g^3)$ and only arises at $O(g^5)$ but for a decay width with accuracy $O(g^8)$ (which includes one-loop $\otimes$ two-loop) its contribution is again zero. Bose symmetry and Ward identities (doubly-contracted, simply-contracted but with physical sources, simply-contracted with off-shell photons and unphysical sources) allow us to write the amplitude as

$$A = \frac{g^3}{16\pi^2} s_0^2 e_\nu(p_1) \left[ F_D \, \delta^{\mu\nu} + F_p \, p_1^\mu p_1^\nu \right] e_\nu(p_2) = \frac{g^3 s_0^2}{16\pi^2} A, \quad F_p = F_p^{(21)},$$

where the form factors are expanded up to two-loops,

$$F_i = F_i^{(1)} + \frac{g^2}{16\pi^2} F_i^{(2)}, \quad i = D, P; \quad A = A^{(1)} + \frac{g^2}{16\pi^2} A^{(2)}.$$
To proceed we need to include the relations between renormalized masses (small letters) and experimental, on-shell, ones (capital letters). Finite renormalization is then completed by introducing external wave-function factors \( Z_{H}^{1/2} Z_{\alpha}^{-1} \) and the renormalization of the coupling constants. All needed relations are collected in Eq. (3).

\[
m^2_{H} = M^2_{\mu} \left[ 1 + \frac{G_{F} M^2_{W}}{2 \sqrt{2} \pi^2} \text{Re} \Sigma^{(1)}_{H}(M^2_{\mu}) \right] \quad B = W, H, \quad m^2_{t} = M^2_{t} \left[ 1 + \frac{G_{F} M^2_{W}}{\sqrt{2} \pi^2} \text{Re} \Sigma^{(1)}_{t}(M^2_{t}) \right]
\]

\[
g^2 s^2_{w} Z_{\alpha}^{-1} = 4 \pi \alpha, \quad \text{g} \ Z_{H}^{-1/2} = 2 ( \sqrt{2} G_{F} M^2_{W})^{1/2} \left[ 1 - \frac{G_{F} M^2_{W}}{4 \sqrt{2} \pi^2} \Pi_{H}(M^2_{\mu}) \right],
\]

\[
\Pi_{H}(s) = \frac{M^2_{\mu}}{s - M^2_{H}} \text{Re} \left[ \Sigma^{(1)}_{H}(s) - \Sigma^{(1)}_{H}(M^2_{H}) \right] - \text{Re} \Sigma^{(1)}_{WW}(M^2_{W}) + \Sigma^{(1)}_{WW ; c}(0), \quad (4)
\]

where \( \Sigma^{(1)}_{WW} \), \( \Sigma^{(1)}_{H} \) and \( \Sigma^{(1)}_{t} \) are respectively the Higgs, \( W \) and top quark one-loop self-energies as defined in section 5.3 of the second paper of Ref. [14]; furthermore,

\[
\Sigma^{(1)}_{WW ; c}(0) = \Sigma^{(1)}_{WW}(0) + \delta_{c}, \quad \delta_{G} = 6 + \frac{7 - 4 s^2_{w}}{2 s^2_{w}} \ln c^2_{\theta}, \quad c^2_{\theta} = \frac{M^2_{W}}{M^2_{Z}}. \quad (5)
\]

The symbols \( M_{t}, M_{W}, Z_{\mu}, G_{F} \) and \( \alpha \) denote the mass of the top-quark, the \( W \)-boson, the \( Z \)-boson as well as the Fermi-coupling constant and the fine structure constant. Collecting all the ingredients we get the corresponding \( S \)-matrix completely written in terms of experimental data

\[
A_{\text{phys}} = \left( \sqrt{2} G_{F} M^2_{W} \right)^{1/2} \frac{\alpha}{2 \pi} \left\{ A^{(1)}_{\text{ex}} + \frac{G_{F} M^2_{W}}{2 \sqrt{2} \pi^2} \left[ A^{(2)}_{\text{ex}} - \frac{1}{2} A^{(1)}_{\text{ex}} \Pi_{H}(M^2_{H}) \right] + \frac{M^2_{H}}{m^2_{H}} \left[ \partial^{(3)}_{m^2_{H}} \text{Re} \Sigma^{(1)}_{H}(M^2_{H}) \right] + \frac{M^2_{W}}{m^2_{W}} \left[ \partial^{(1)}_{m^2_{W}} \text{Re} \Sigma^{(1)}_{WW}(M^2_{W}) + 2 M^2_{t} \partial^{(1)}_{m^2_{t}} \text{Re} \Sigma^{(1)}_{t}(M^2_{t}) \right] \right\}. \quad (6)
\]

The subscript “ex” indicates that all masses are the experimental ones and the mass-shell limit \( (s \to M^2_{H}) \) is taken only after the inclusion of finite renormalization. QCD corrections will appear in Eq. (4) and Eq. (6) multiplied by \( \pi \alpha_{s}(M_{H})/(\sqrt{2} G_{F} M^2_{W}) \).

In our calculation we prove the cancellation of the collinear logarithms and then set the light fermion masses to zero; therefore, due to Yukawa couplings, an imaginary part in \( A^{(1)} \) arises only if \( M_{H} > 2 M_{W} \). For two-loop terms imaginary parts are always present even for massless fermions. From Eq. (6) the total amplitude for \( H \to \gamma \gamma \) can be written symbolically as \( A^{\mu \nu}_{\text{phys}} = A^{\mu \nu}_{1} (1 + FR) + A^{\mu \nu}_{2} \). Finite renormalization (FR) amounts to expressing renormalized parameters in the one-loop amplitude in terms of data and in the insertion of the Higgs wave-function factor \( Z_{H} \) à la LSZ; both requires the notion of on-shell mass. There are two sources of inconsistency in this approach: the Higgs-boson is an unstable particle and this fact has a consequence which shows up at two-loops. When we compute the doubly-contracted Ward identity for the full two-loop amplitude we obtain

\[
p_{\mu} p_{2\nu} A^{\mu \nu}_{\text{phys}} = (G_{F} M^2_{W})^{3/2} \alpha \text{Im} W (M_{H}, M_{W}, \ldots).
\]

The analytical form of \( W \) is known and the non-zero result comes from the fact that the pure two-loop contribution to the Ward identity gives \( W \) while finite renormalization gives the real part \( \text{Re} W \). Therefore, the Ward identity is violated above the \( WW \)-threshold. On top of this problem we find a second unphysical feature: let us analyze how the amplitude for \( H \to \gamma \gamma \) behaves around a normal-threshold, i.e. for \( M_{H} = 2 M_{W} = 2 M_{t} \). In particular we are interested in the question of possible square-root or logarithmic singularities. Even if present they are unphysical, although integrable. Both problems can be solved by using complex masses as discussed in subsection 5.3.
3.1 Square-root singularities

It is very simple to prove that derivatives (represented by a dot in Eq.(8)) of one-loop, two-point functions with equal masses ($m$) develop a square root singularity:

$$\dot{B}_0(p^2, m, m) = -\frac{1}{\beta} \ln \frac{\beta + 1}{\beta - 1}, \quad \beta^2 = 1 + \frac{4m^2}{p^2 - i\epsilon}. \quad (8)$$

The same argument can be repeated for all the one- and two-loop diagrams with any number of external legs where we can cut two and only two $m$-lines; normal-threshold will be a sub-(sub-... leading singularity, but a $1/\beta$-behavior shows up only if the reduced sub-graph responsible for the singularity can be reduced to a $\dot{B}_0$-function. Therefore the only two-loop vertex giving rise to a $1/\beta$-divergent behavior is the one depicted in Fig. 3. For this diagram it is possible to find a representation where the singular part is completely written in terms of one-loop diagrams, as shown in the figure. The remainder can be cast in a form suited for numerical integration.

Figure 3: Contraction of a $V^M$ configuration leading to a $\beta^{-1}$-behavior at the normal $m$-threshold.

In the decay $H \rightarrow \gamma\gamma$, the $1/\beta$-singularity ($\beta^2 = 1 - 4M_W^2/M_H^2$) arises from the two-loop diagram of Fig. 3 from Higgs-boson wave-function factor (derivative of a B-function) and from finite $W$-mass renormalization (derivative of a C-function). Our conclusion is that the unphysical $1/\beta$-behavior around some normal-threshold is induced by self-energy like insertion, a fact that is not surprising at all; those insertions, signaling the presence of an unstable particle, should not be there and complex poles should be used instead.

3.2 Logarithmic singularities

Let us consider the two-loop diagram of Fig. 4 with $P^2 = -s$ ($s > 0$). Writing the corresponding integral in parametric space we introduce the quadratic forms

$$\chi(x) = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} \beta^2, \quad \xi(x, y) = x \left(x - 1\right) y^2 + \frac{1}{4} \left(1 - \beta^2\right), \quad \beta^2 = 1 - \frac{4m^2}{s} \quad (9)$$

and obtain

$$V^K = \frac{2}{s^2} \int_0^1 \frac{dx \; dy}{y \chi(x)} \left[ \text{Li}_2 \left(1 - \frac{y \chi(x)}{\chi(xy)}\right) - \text{Li}_2 \left(1 - \frac{y \chi(x)}{\xi(x, y)}\right) \right]. \quad (10)$$

Since we are interested in the behavior around $\beta \rightarrow 0$, we split $V^K$ into a singular and regular part and find

$$V^K = V^K_{\text{sing}} + V^K_{\text{reg}} = \frac{2}{s^2} \int_0^1 \frac{dx \; dy}{y \chi(x)} \left[ \text{Li}_2 \left(1 - \frac{y \chi(x)}{\chi(xy)}\right) - \zeta(2) \right] + V^K_{\text{reg}}. \quad (11)$$
The singular part $V_{\text{sing}}^K$ will be written as \cite{10}

$$V_{\text{sing}}^K = \frac{2}{s^2} \int_0^1 dt \frac{\ln t}{1-t} I(t), \quad I(t) = \int_0^1 dx dy \left[(1-t)\chi(xy) + ty\chi(x)\right]^{-1} = \int_0^1 dx dy \left[a (x-X)^2 + \lambda\right]^{-1},$$

(12)

where we have introduced the shorthands

$$a = \tau y, \quad X = \frac{1}{2} \tau, \quad \lambda = \frac{1}{4} \frac{t(1-t)}{\tau} \left[(1-y)^2 - \beta^2 (y + T) \left(y + \frac{1}{T}\right)\right],$$

(13)

with $\tau = (1-t) y + t$ and $T = t/(1-t) > 0$. $I(t)$ can be split into two parts,

$$I(t) = B \left(\frac{1}{2}, \frac{1}{2}\right) \int_{y_{\min}}^{1} dy \ a^{-1/2} \lambda^{-1/2} - \frac{1}{2 \pi i} \sum_{i=1,2} \int_0^1 dx dy (-1)^i X_i x^{-1/2} (aX_i^2 + \lambda x)^{-1},$$

(14)

with $X_1 = -X,$ $X_2 = 1 - X$ and $B(x,y)$ is the Euler beta-function. While the second term of Eq. (14) is regular for $\beta = 0,$ the singularity of the first term follows from the fact that $\lambda \sim (1-y)^2$ for $\beta \to 0$; however, we have a singular behavior only if $0 \leq X \leq 1$ which requires $y \geq y_{\min} = \max\{0, \ (t-1/2)/(t-1)\}$. Since we are interested in the leading behavior for $\beta \to 0,$ we can extend the integration domain in the first term to $[0,1],$ without modifying the divergent behavior of the diagram. The singular part is then given by

$$I_{\text{sing}}(t) = 2 \pi \left[t(1-t)\right]^{-1/2} \int_0^1 dy y^{-1/2} \left[(1-y)^2 - \beta^2 (1 + T y) \left(1 + \frac{y}{T}\right)\right]^{-1/2} = 2 \pi \left[t(1-t)\right]^{-1/2} J(t),$$

$$J(t) = \frac{1}{2 \pi i} \int_{-i \infty}^{+i \infty} ds B \left(s, \frac{1}{2} - s\right) (-\beta^2 - i0)^{s-1/2} \int_0^1 dy y^{-1/2} \left[(1-y)^{-2s} (1 + T y)^{s-1/2} \left(1 + \frac{y}{T}\right)\right]^{-1/2}$$

$$= \frac{1}{2 \pi i} \int_{-i \infty}^{+i \infty} ds \Gamma(s) \Gamma(1/2-s) \Gamma(1-2s) \Gamma(3/2-2s) \left(-\beta^2 - i0\right)^{s-1/2} F_1 \left(\frac{1}{2}, \frac{1}{2} - s, \frac{1}{2} - s, \frac{3}{2} - 2s; -T, -\frac{1}{T}\right),$$

(15)

$0 < \Re s < 1/2.$ Here $F_1$ denotes the first Appell-function. To obtain the expansion corresponding to $\beta \to 0$ we close the integration contour over the right-hand complex half-plane at infinity. The leading (double) pole is at $s = 1/2.$ Therefore, we obtain

$$J(t) = - \frac{1}{2} \ln (-\beta^2 - i0) + \mathcal{O}(1), \quad \beta \to 0.$$  

(16)

Inserting it into Eq. (12) and using $\int_0^1 dt t^{-1/2} (1-t)^{-3/2} \ln t = -2 \pi,$ we get

$$V_{\text{sing}}^K = \frac{4 \pi^2}{s^2} \ln (-\beta^2 - i0) + \mathcal{O}(1), \quad \beta \to 0.$$  

(17)
All light fermion masses are set to zero and we define the $W$-boson complex masses. Our result, Eq. (17), is confirmed by the evaluation of $V^K$ of Ref. [17] in terms of generalized log-sine functions. Starting from Eq.(6.34) of Ref. [17] and using the results of Ref. [18] we expand around $\theta = \pi$, where $x = e^{i \theta} = (\beta - 1) (\beta + 1)$, with $0 < \theta < \pi$. This gives for the leading behavior of $V^K$ below threshold $(\pi^2/2) \ln(\theta - \pi)$, where $\ln(-\beta^2) = \ln(\theta - \pi)^2 - \ln 2$. The same behavior can also be extracted from the results of Ref. [4].

3.3 Complex masses

Our pragmatisal solution to the problems induced by unstable particles has been to remove the Re label in those terms that, coming from finite renormalization, give Re $W$ in the Ward identity of Eq. (7). Furthermore, we decompose Eq. (13) according to:

$$A_{\text{phys}} = \left(\sqrt{2} G_F M_W^2 \right)^{1/2} \frac{\alpha}{2 \pi} A_{\text{phys}}, \quad A_{\text{phys}} = A^{(1)}_{\text{ex}} + \frac{G_F M_W^2}{2 \sqrt{2} \pi^2} \left[ A^{(2)}_{\bar{n}} / \beta + A^{(2)}_{\bar{L}} \ln(-\beta^2 - i 0) + A^{(2)}_{\text{reg}} \right].$$

and prove that, as expected, $A^{(2)}_{\bar{n}}, A^{(2)}_{\bar{L}}$ and $A^{(2)}_{\text{reg}}$ satisfy (separately) the Ward identity. The latter fact allows us to – minimally – modify $A^{(2)}_{\bar{n}}$, by working in the complex-mass scheme of Ref. [19], i.e. we include complex masses in the, gauge-invariant, leading part of the two-loop amplitude as well as in the one-loop part.

The decomposition of Eq. (18) deserves a further comment. There are three sources of $1/\beta$-terms: a) pure two-loop diagrams of the $V_{\mu}$-family, i.e. bubble insertions on the internal lines of the one-loop triangle; b) $W$-mass renormalization, i.e. on-shell $W$-self-energy × the mass squared derivative of the one-loop $W$-triangle (the latter giving rise to $1/\beta$); c) Higgs wave-function renormalization × lowest order (the former giving rise to $1/\beta$).

One can easily prove that only c) survives and a,b) that are separately singular add up to a finite contribution ($\beta \to 0$); their divergency is an artifact of expanding Dyson resummed propagators.

The $\ln \beta$-term originates from pure two-loop diagrams (the $V_{\mu}$-family) and it is a remnant of the one-loop Coulomb singularity of one-loop sub-diagrams.

4 Numerical results

The partial width of the Higgs-boson decay into two photons can be written as

$$\Gamma(H \to \gamma \gamma) = \frac{\alpha^2 G_F M_W^2}{32 \sqrt{2} \pi^3 M_H} |A_{\text{phys}}|^2.$$  \hspace{1cm} (19)

The relative correction $\delta$ induced by two-loop (NLO) effects is given by $\Gamma = \Gamma_0(1+\delta)$, where $\Gamma_0$ is the lowest order result. It can be split into electroweak and QCD contributions, $\delta = \delta_{\text{EW}} + \delta_{\text{QCD}}$.

For the numerical evaluation we use the following set of parameters:

$$M_W = 80.398 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad m_t = 170.9 \text{ GeV}, \quad \Gamma_W = 2.093 \text{ GeV},$$

$$G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}, \quad \alpha(0) = 1/137.0359911, \quad \alpha_s(M_Z) = 0.118.$$  

All light fermion masses are set to zero and we define the $W$-boson complex pole [20] by

$$s_W = \mu_W (\mu_W - i \gamma_W), \quad \mu_W^2 = M_W^2 - \Gamma_W^2, \quad \gamma_W = \Gamma_W \left(1 - \frac{r_{\mathcal{R}}^2}{2 r_{\mathcal{R}}^2} \right).$$  \hspace{1cm} (20)

The one-loop $H \to \gamma \gamma$ amplitude, with a complex $W$-mass, is shown in Fig. 5 around the $WW$-threshold including a comparison with the real $W$-mass amplitude.
Figure 5: Real and imaginary part of the one-loop $H \rightarrow \gamma \gamma$ amplitude with real and complex $W$-boson mass.

Figure 6: Percentage electroweak corrections to $H \rightarrow \gamma \gamma$ with real (dashed) and complex (solid) $W$-boson mass, below the $WW$-threshold.

A comparison of the percentage electroweak corrections, with and without complex $W$-masses, is shown in Fig. 6 for a Higgs mass range below the $WW$-threshold showing the unphysical growth of the real case and also some sizable difference in a region of about two GeV below the threshold.

We have also analyzed the effect of (artificially) varying the imaginary part of the $W$-boson complex mass; results are given in Fig. 6 showing that our complex result reproduces the real one in the limit $\Gamma_W \rightarrow 0$. Fig. 6 clearly demonstrates the large but artificial effects arising at normal-thresholds of unstable particles when their masses are kept real.

Finally, in Fig. 8 we show both QCD and electroweak percentage corrections to the decay width $\Gamma(H \rightarrow \gamma \gamma)$, including the region around the $WW$-threshold. A running $\alpha_s$ has been used for the computation of the QCD corrections. The remaining cusp of $\delta_{ew}$ at the $WW$-threshold, whose details are shown in the blow-up of Fig. 8, is due to our minimal scheme where the $W$-mass is kept real in $A^{(2)}_{\text{reg}}$, the regular part of the amplitude (see Eq. 15). The relatively small error bars in a region so close to the threshold serve as evidence for the efficiency of our numerical algorithms.

Our result for $\delta_{ew}$ in the region $100 \text{ GeV} < M_H < 150 \text{ GeV}$ is in substantial agreement with those of Ref. [6]. In conclusion, we observe a cancellation of the two corrections below the threshold whereas, above
Figure 7: The effect of varying the $W$-boson width in the real part of $A^{(2)}_K$ containing the $1/{\beta}$ terms of the two-loop amplitude (left) and in the real part of $V^K$ of Fig. 4 containing the $\ln{\beta}$ terms (right) is shown. We also show the effect of using a real $W$-boson mass but removing Re-labels in finite renormalization.

Figure 8: Electroweak (solid), QCD (dashed) and total (dashed-dotted) correction in percent for the partial width of the decay $H \to \gamma\gamma$.

It, both $\delta_{\text{QCD}}$ and $\delta_{\text{EW}}$ are positive leading to a sizable (up to 4.5%) total correction to the decay width. The perturbative expansion for the decay rate, supplemented with the complex-mass scheme, gives reliable and accurate predictions in a wide range of values for the Higgs-boson mass, typically $-1\% < \delta_{\text{tot}} < 4\%$ in the range $100\text{ GeV} < M_H < 170\text{ GeV}$. 
5 Conclusions

In this paper we provide a stand-alone numerical calculation of the full two-loop corrections to the decay width $\Gamma(H \rightarrow \gamma\gamma)$. Since no expansion is involved in the calculation we can produce results for all values of the Higgs-boson mass, as shown in Fig. 8 including the WW-threshold. The techniques introduced in this context are general enough to be used for all kinematical configurations of $1 \rightarrow 2$ processes at the two-loop level.

To deal with normal-threshold singularities, specifically the WW-threshold, we have introduced complex $W$-masses in a gauge-invariant manner; our minimal scheme selects gauge-invariant components, typically LO (one-loop) amplitude and divergent parts of the NLO (two-loop) amplitude, and perform the replacement of Eq. (20). Details of our approach will be described in a forthcoming publication.

The main result obtained in this paper can be summarized by saying that the NLO percentage corrections to the decay width $\Gamma(H \rightarrow \gamma\gamma)$, $\delta_{\text{QCD}}$ and $\delta_{\text{EW}}$, compensate below threshold leading to a small total correction; however, above the WW-threshold they are both positive, leading to a sizable overall effect of $\approx 4\%$.

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