Some aspects regarding the roughness of the railway surface and rolling noise at locomotives

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Abstract. Railway noise is activated by irregularities on the running surfaces of rails and wheels, these being more significant for the most part of the wavelength and the frequency range of interest. The rolling noise generation mechanism consists of two major structural components: the wheel and rail. The contribution to the noise of the track can be further separated into noise radiation on the rails and in the sleepers. To mitigate the rolling noise, these components must be analysed. To direct and manage mitigation methods within Faurei Railway Testing Center, the rail surface roughness in accordance with EN 15610 was determined in a line of alignment, this being a direct method of measuring the surface roughness of the rail, in association with the rolling noise introduction. In this paper, it is proposed a fractal analysis of the microgeometry of the rail surface, with the possibility of using it as an acoustic parameter.

1. Introduction
Noise is an unwanted sound due to its volume and can be harmful. As well as noise, vibration from railways can cause annoyance. This may be due to vibration, usually in the range 2 to 80 Hz, or due to the radiation of low frequency sound transmitted through the ground, usually in the range 30 to 250 Hz. Noise is an important and annoying side-effect of the process of rolling the wheel on the rail.

Different types of noise are generated by the wheel-rail interface and means of controlling them. They have as main source of rolling noise that occurs in a line of alignment and has a wide frequency content [-1-3].

The first step in noise control is to identify the dominant source. There are many different sources of noise from a railway, and in different situations the dominant source may vary [1,2].

- **Rolling noise** is the most important source of rail noise caused by wheel/rail vibrations induced at wheel/rail contact.
- **Curve squeal** noise is caused by the interaction between wheel and curve rails. It is strongly tonal, being associated with the vibration of the wheel in one of its resonances.
- **Bridge noise** is caused by dynamic forces acting on it from the track. Depending on the type of bridge, when a train runs over it, the emitted noise increases considerably.
- **Aerodynamic noise** is caused by a non-uniform airflow over the train compared to other railway noise sources, which are caused by the sound radiation through the vibrations of solid structures. Aerodynamic noise sources generally increase much faster with the train speeds than the mechanical sources.
- **Ground-borne vibration and noise** can be experienced in two ways. Low frequency vibration
between about 2 and 80 Hz is perceived as vibration and tends to be associated most with heavy freight trains. Higher frequency ground-borne vibration from about 30 to 250 Hz is associated more with trains in tunnels in urban areas such as metro operations but can also be significant for surface railways, particularly where noise barriers block out the direct airborne sound.

- Other sources of railway noise: Traction noise, warning signals from trains, track maintenance equipment, shunting noise.

2. The rolling noise and roughness

The rolling noise from the wheel-rail contact is the noise generated by the vehicle-track interaction. The most important form of noise generated at the wheel/rail interface is the rolling noise which is caused by the roughness of the wheel and rail running surfaces [3-4]. Significant rolling noise mechanisms attributable to wheel and rail roughness are:

(a) continuous deformation from gradually varying wheel and rail profiles,
(b) impact of wheel and rail asperities and
(c) parametric excitation associated with spatially varying rail and, possibly, wheel impedance.

Wheel and rail vibrations are induced by wheel-rail contact, in particular by irregularities on the surface of the wheel and rail (roughness and corrugation). Severe quasi-periodic roughness, known as corrugation, may also develop in some situations. This may have peak-to-trough amplitudes of typically 50 µm at a wavelength of 50 mm; at longer wavelengths the amplitudes can be greater than this. [1]

At the wavelengths of relevance to rolling noise, roughness may be caused by manufacturing variability in the rail section or variable wear of the wheel and rail surfaces. Roughness of the wheel and rail surface is a major cause of wheel-rail noise at all speeds. Typical wavelengths of roughness relevant to rolling noise are between about 5 and 500 mm.

When crossed at typical train speeds, the corrugation of wheel and rail surfaces with wavelengths between 5 and 250 mm produces vibrations in the acoustic frequency range.

Figure 1. Reduction of railway noise pollution of the rolling noise [Thompson, 2009].

Figure 1 shows that between 30 and 200 km/h the rolling noise is the dominant source. Reduced speed can only be found in shunting yards or near stations. Speeds above 200 km/h are found only on high-speed lines. Speed between 30 and 200 km/h is also the speed at which freight trains operate (approximately 100 km/h). Many sources identify freight trains as the noisy trains.

2.1. Mechanism of rolling noise generation

The mechanism of rolling noise generation consists of two structural components: the wheel and rail. To mitigate rolling noise, these components need to be analysed. To direct and manage mitigation methods, it is desirable to be able to determine the roughness of the wheels and rail.

From the first series of works published by Remington, the rolling noise generation mechanism has
been proven to be due to vibrations of the wheel and the rail, excited by their surface heat. This is also known as roughness in the rail system (figure 2).

Figure 2. Noise generation mechanism [Remington, 1976].

3. Roughness standard parameters
The roughness of the contact surfaces is determined by the simultaneous action of several phenomena, among which the most important are the following: elastoplastic deformations of the material, vibrational phenomena of the wheel-rail system and friction phenomena.

The interaction of these phenomena leads to the conclusion that microgeometry, in terms of the height of the roughness, is characterized as a random size. Roughness is of great importance in surface response in relation to mechanical solicitations. The original topography is modified in friction phenomena.

Surface roughness is a component of surface texture. This is quantified by deviations in the direction of the normal vector of a real surface from its ideal shape. If these deviations are high, the surface is harsh; if deviations are small, the surface is smooth. In general, roughness is considered to be a high frequency component on short wavelengths of the micro-irregularities of the surface to be measured. Roughness is the microgeometric measurement of the height variations of a physical surface (according to surface metrology). This measurement is different from the determination of deviations from the correct geometric shape that characterizes surface geometry or undesirable misalignments. Rigidity with high values may be an undesirable feature, as it causes increased friction and wear. The effect of wear can be assessed by surface topography analysis.

The purpose of this study was to examine the microgeometry of the rail surface by different methods for a sequence of the thread 1 outer ring center line, such as:

- Statistical sizes (average, mean square deviation, centered moments);
- Limit curve according to EN 15610 and EN ISO 3095 standards;
- Abbott-Firestone bearing area curve;
- Fractal features.

By recording the results on the height of the roughness, the main statistical characteristics are determined and the Abbott-Firestone bearing curve and fractal character are derived.

Random character of roughness height is evidenced by experimental tests and different types of post-processing. Roughness on the wheel and rail surface gives rise to the vertical vibrations of the wheel and rail systems, depending on their dynamic properties. The main roughness wavelengths that are relevant to rolling noise are between approximately 5 and 500 mm (Thompson, 2009). [1] Determination of rail and wheel roughness according to Regulation no. 1304/2014 of the EU Commission; EN ISO 3095: 2013 [5].

The rolling noise of railway vehicles depends on the roughness of rails and wheels. Therefore, with
regard to the acoustics of railway vehicles, it is essential to have a detailed knowledge of the unevenness of the running surface.

Rail and wheel roughness measurement shows the system requirements for high precision measurement because irregular size of a micrometer or less be determined reliably. For the roughness of the rail, the tests on the Faurei Railway Testing Centre were carried out. This is managed by Romanian Railway Authority and it is used for dynamic testing of rolling stock.

Tests carried out at the Centre for Railway Faurei Testing (Figure 3) as well as those from the specialized laboratories Romanian Railway Notified Body (RRNB) are performed by technical staff that meets the highest standards of training and experience.

![Figure 3. Railway testing center graphic presentation.](image)

To determine the roughness of the rail, a segment in a straight line of a ballasted track with concrete sleepers, with no visible joints or defects, was chosen on the line segment between 0 + 100 m and 0 + 200 m, without sound impact caused by line welding or any faulty gripping of the sleepers. The surface roughness of the rail was determined in accordance with EN 15610 standard, which shows a direct method of measuring the surface roughness of the rail associated with rolling noise.

The test area was chosen so that the rail does not present localized geometric marks (rail defects, skid marks, etc.), which can lead to rolling noise amplification.

The track area on which the test was carried out, in accordance with ISO 3095 was conducted.

Thus, the level of the soil surface from the upper part of the rail was within the [0 - 2] m limits.

No access was allowed in the area and there were no sound absorbing or reflective surfaces.

A test surface length of 30 m was considered. As for the width of the reference surface, since it was greater than 30 mm, the acoustic roughness was measured on the center line of the reference surface as well as on two additional lines located at left and right of the center line at a distance of 10 mm from this, for both rails, over the reference surface length, both on the right and on the left side of the reference surface.

3.1. Roughness Measurement

Tests for rail roughness were performed with m | rail trolley (Figure 4). The m | rail trolley is a high-precision measuring system to continuously record the rail roughness. The m | rail trolley is a track acoustic rail roughness measurement system using an interdependent acceleration sensor with a mobile holder that is moved manually along the rail (carriage);

The system can measure amplitude deficiencies below a micrometer at wavelengths from a few millimeters to a few meters.

In Figure 5a, a stretch of the roughness profile (y1) evaluated on the rail in the area 0+150m-0+152.5m for thread 1 outer ring center on the Faurei Railway Testing Center is shown, on a 2.5 m length and with a 1 mm mesh (2500 points).

The acoustic rail roughness of the test section shall be assessed according to EN 15610:2009. For type test measurements, the default upper limit provided in Figure 5b shall apply, taking into account.
For train speeds up to 190 km/h, the wavelength bandwidth shall be at least 0.003 m to 0.10 m. For higher speeds at least 0.003 m to 0.25 m.

**Figure 4.** Rail trolley.

Acoustic roughness \( r(x) \) is defined as variation in the height of the rail running surface associated with rolling noise excitation expressed as a function of distance \( x \) along the rail.

The acoustic roughness expressed in dB is defined as:

\[
L_{r(x)} = 10 \log \left( \frac{r(x)}{1 \mu m} \right) \tag{1}
\]

*Figure 5. Evaluated rail roughness meter (a) and acoustic (b) from the area 0+150m-0+152.5m for thread 1 outer ring center.*

Subsequently, in order to analyze the state of the rail roughness, we selected and analyzed the \( r(x) = y(x_i) \) experimental sequence for \( N \) points measured with the equipment m | rail trolley.

Experimental results \( y(x_i) \) are analyzed according to standardized criteria specific to rolling wheel/rail (EN 15610: 2009, ISO 3095: 2013), according to general statistical criteria and by criteria used in fractal analysis of microgeometry of contact surfaces. The results are analyzed with an original program developed in MATCHAD software. In this sense, according to EN 15610: 2009, the data set is corrected by spikes and pits [5, 6]. Thus, the aberrant peak is the coordinate point \( (x_s, y_s) \) for which the second derivative conditions are simultaneously satisfied and derived first:

\[
\frac{d^2}{dx^2} y \leq S_{\alpha} \frac{\mu m}{m^2}, \text{ where } S_{\alpha} = -10^7 \tag{2}
\]
and product of derivative dy/dx, for x = x_i and x = x_{i+1}, is negative or zero.

Derivatives are determined numerically based on the coordinates (x_i + 1, y_i + 1) and (x_i, y_i) of the neighbouring points. If these conditions are fulfilled, the spike is considered aberrant and ordered (y_i) is corrected by linearization with the order of the neighbouring points y_i = (y_{i+1} + y_{i-1}) / 2 (figure 6, (b)- ycor (j...k) is the corrected profile).

The valley (hole, pit) is considered excessive (aberrant) if the intersection of a circle of radius R = 0.375 m and the coordinate center (x_i, y_i + R) with the roughness profile (y_i) will be in the point (x_k, y_k) than (x_i, y_i). The explicit equation of circle C is:

\[ y_{ck} = y_i + R - \sqrt{R^2 - (k\Delta x - i\Delta x)^2} . \]  

The correction of the profile at the abscissa point x_i is:

\[ y_{cpi} = y_i + \max(y_k - y_{ck}) . \]  

After correcting peaks (spikes) and aberrant valleys, the roughness profile (yc_{pi}) analyzes the general statistical parameters and functional parameters.

As statistical parameters useful for the qualitative and quantitative assessment of rolling noise, the following are analyzed:

- Arithmetic mean deviation of the assessed profile (Ra): defined on the sampling length. Ra
is used as a global evaluation of the roughness amplitude on a profile; for the corrected roughness profile in example figure 5,

\[ R_a := \frac{1}{N} \sum_i (y_{cp_i}) ; \quad R_a = 81.18 \ \mu m, \]  

where \( N \) is the number of meshing points of the profile;

- **Root mean square deviation of the assessed profile** (Rq): corresponds to the standard deviation of the height distribution, defined on the sampling length. Rq provides the same information as Ra;

\[ R_q := \left( \frac{1}{N} \sum_i (y_{cp_i})^2 \right)^{\frac{1}{2}} ; \quad R_q = 85.7 \ \mu m. \]  

**Ten point height**: defined on the sampling length (Rz): this parameter is frequently used to check whether the profile has protruding peaks that might affect static or sliding contact function:

\[ R_{z_{ISO}} := (y_{sl_{N-2}} + y_{sl_{N-5}} + y_{sl_{N-6}} + y_{sl_{N-3}} + y_{sl_{N-4}} - |y_{sl_1}| - |y_{sl_2}| - |y_{sl_3}| - |y_{sl_4}| - |y_{sl_5}|)^{\frac{1}{5}} ; \]  

\[ R_z = 67.98 \ \mu m, \]

where \( y_{s_i} \) is the roughness vector sorted in ascending order.

- **Maximum profile peak height** (Rp): height of the highest peak from the mean line,

\[ R_p := \max(y_{cp}) - \text{mean}(y_{cp}) ; \quad R_p = 71.82 \ \mu m \]  

- **Maximum profile valley depth** (Rv): depth of the deepest valley from the mean line,

\[ R_v := \min(y_{cp}) - \text{mean}(y_{cp}) ; \quad R_v = -81.17 \ \mu m. \]  

- **Skewness of the assessed profile** (Rsk): asymmetry of the height distribution, defined on the sampling length. It is the third central moment of the profile amplitude probability density function, measured over the assessment length:

\[ R_{sk} := \sum_i (y_{cp_i})^3 , R_{sk} = 1.114 \ \mu m. \]  

- **Kurtosis of the assessed profile** (Rku): sharpness of the height distribution, defined on the sampling length. The kurtosis coefficient is the fourth central moment of a profile amplitude

\[ R_{ku} := \frac{1}{N R_q^4} \sum_i (y_{cp_i})^4 ; \quad R_{ku} = 1.288 \ \mu m. \]  

An important parameter of rail and wheel microgeometry used to evaluate rolling noise is acoustic roughness as a function of wavelength. For the evaluation of the roughness (y) dependence on the wavelength (\( \lambda \)), based on the experimental record y (x_i), the discrete Fourier Transform (CFFT) of the vector y (x_i) is used.

The corrected roughness (ycp) acoustic level (\( y_{\lambda_c} \)) is:

\[ y_{\lambda_c} = 10 \log \left( \frac{ycp^2}{\text{stdev}(ycp)^2} \right). \]
and the wavelength \( \lambda_c \) is the complex number of the discrete Fourier transform (CFFT) (implicit function in the Matchad program)

\[
\lambda_c = \left| Y_{Tc} \right| \quad \text{where} \quad Y_{Tc} = \text{CFFT}(y_c).
\]

(13)

\[ \Delta x = 1 \times 10^{-3} m, \quad k = 1, 2, ..., N - 2. \]

(14)

Thus, in Figure 8 shows the acoustic level of the spikes and pits \((y\lambda c \text{ and sorted values of } y\lambda c, y_{\text{sort}})\) of the rail running surface (Figure 5) as a function of the wavelength.

![Figure 8](image)

**Figure 8.** Acoustic roughness levels vs wave length.

This roughness in the surface profile has significant wavelengths between about 10 and 300 mm and amplitudes between about 0.1 \( \mu \text{m} \) and 30 \( \mu \text{m} \) greater for severe corrugation.

The validity of the linear relationship assumed between roughness and noise depends on the quality and completeness of roughness measurements.

This is illustrated by comparing different rail roughness systems. Particular attention should also be paid to processing the roughness data before a representative excitation spectrum can be derived.

![Figure 9](image)

**Figure 9.** Comparison of acoustic roughness spectra thread 1 outer ring centre.

Future research will investigate and perform experimental determinations from a range of sources to confirm the linear relationship between roughness and noise, at least for roughness amplitudes that are
not too severe. Consideration will also be given to the validity of the assumption made in the models that the roughness of the wheels and rails can be added to an equal basis to give the total roughness spectrum.

Acoustic roughness was initially performed using equipment software m | rail trolley. It has been chosen that the acoustic roughness on the outer ring 1 string center; it is determined on the three parallel lines: left, center and right (figure 9).

4. Bearing Area Curve (BAC)
Bearing Area Curve Abbott-Firstone represents the intersection of the profilogram with parallel planes and the cumulation at the same level of the lengths of the intersection of the profile with the respective plane.

BAC of the roughness profile on a certain direction deduced by Abbott-Firstone highlights the microgeometry's behaviour during the processing of external loads.

BAC can be determined by intersecting the roughness profile with n equidistant planes relative to a reference plane (generally the roughness base plane) as shown in Figure 10 [7, 8, 9].

![Figure 10. -Bearing Area Curve (BAC).](image)

The roughness profile from figure 5 corresponds BAC from figure 10. It should also be noted that the area above the inflection point F, i.e. the area above the plane of F point, would represent the roughness that will elastically deform; below this line would follow the plastic deformation regime.

The bearing area curve can be used as a local stiffness parameter and as the source of excitation for torsional and flexural vibrations of the wheel.

5. Characterization geometry of surface topography by fractal
Fractal geometry as a tool for the characterization of surface topography has gained much attention in recent years [10-13]. This is due in part to the observations that fractal geometry can reflect the natural and intrinsic properties of random phenomena and that it can overcome several disadvantages of conventional statistics and random process methods of surface analysis.

Several methods have been developed to characterize the dimension of a fractal set, such as the compass dimension, box dimension, mass dimension and area-perimeter dimension [9-10]. If a homogeneous and isotropic rough surface has fractal dimension \( D \), and its profile in an arbitrary direction has fractal dimension \( D \), then a simple relation exists between them; namely, \( D = 1 + D \) [9].

We use the structure function method to calculate fractal parameters dimension (D) and topothesy length (L_0). It is considered a signal \( y_1(x) \) whose statistical properties for a delay are to be investigated.

The increment of signal \( y_1(x+t) - y_1(x) \) is assumed to have a gaussian distribution with zero mean and the variance is the sums of the squares of the delayed amplitude differences. This function is structure...
function in continuum form:

\[ S(t) = E \left[ \left( y_1(x) - y_1(x+t) \right)^2 \right] = c \left[ \left( \Delta y \right)^{4-2D} \right] \]  

(15)

Where, \( E \left[ \left( y_1(x) - y_1(x+t) \right)^2 \right] \) denotes an expectation, \( C \) is a constant, \( D \) is the fractal dimension of the \( y_1(x) \).

The chord joining \( y_1 \) values separated by a distance \( t \) has a finite mean-square slope. If there is a displacement \( t = L_t \) such that the string has an r.m.s. slope of unity statistically:

\[ S(L_t) = L_t^2 \]  

(16)

where \( L_t \) is a characteristic parameter of the fractal function called the topothesy.

Thus, the structure function for fractal roughness has an expression

\[ S(t) = L_t^{2D-1} \left[ \left( \Delta y \right)^{4-2D} \right] \]  

(17)

The fractal parameters (\( D, L_t \)) can be determined by graphical form (log-log) of the structure function. The signal \( y_1(x_i) \) is discretized in \( N \) equal intervals, \( \Delta x = x_p / N \), where \( x_p \) is the reference length of the rail profile as standard (2.5m). Thus, an “i” point of the signal has the coordinates \( x_i = \Delta x \cdot i \) and \( y_i = y_1 \).

This point differs from another located at a distance \( x_k = \Delta x \cdot k \) and with order \( y_k = y_{i+k} \). The structure function in discrete form, \( S_1(N,k) \), of a signal \( y_1(x_i) \) is:

\[ S_1(N,k) = \frac{1}{N-k} \sum_{i=1}^{N-k} \left( y_{i+k} - y_i \right)^2 \]  

(18)

This function is independent of the mean plane; thus, any profile structure function, irrespective of its mean line, is a section of the surface structure function.

For example, in Figure 11 shows the structure function of the roughness profile corrected for the rail with the roughness of Fig. 5.

![Figure 11. Structure function of roughness rail.](image)

If the curve representing the structure function in the logarithmic coordinate, is a right with a non-integer slope, then the roughness can be considered as a fractal character [7-14].

On the basis of the analysis of the curve, \( \left( \log(S(N,k)) - \log(x(k)) \right) \), it is inferred by the smallest square method that the rightmost approximates the function of the structure having slope \( \beta = 1.476 \).

Thus, the fractal parameters are evaluated in Matchad program:
\[ D = \frac{4 - \beta}{2} = 1.262 \text{ and} \]
\[ L_1 = 3\Delta X = 0.003 \text{ m} \]  

For fractal character roughness (parameters D and L), we can associate a continuous and undifferentiated Weierstrass-Mandelbrot (W-M) function \( y(x) \) [7, 12, 14, 16]:

\[ y(x, D, \gamma) = G^{D-1} \sum_{n=0}^{\infty} \frac{\cos(2\pi \gamma^n x)}{\gamma^{(2-D)n}}, \quad 1 < D < 2 \text{ and } \gamma > 1 \]  

where \( G \) is a scaling constant and the frequency modes \( \gamma^n \) correspond to the reciprocal of the wavelength \( (\lambda_n) \) of roughness as \( \gamma^n = 1/\lambda_n \).

The scaling constant \( G \) can be defined and the hypotheses that increment of signal \( y_1(x+t) - y_1(x) \) is assumed to have a gaussian distribution with zero mean:

\[ G(D) = \frac{L_1}{C_4(D)} \]

where \( C_4(D) = \left[ \frac{\Gamma(2D-3)\sin\left[\pi\frac{D-3}{4}\right]}{2^{D-3}} \right]^{\frac{1}{D-1}} \), \quad 1 < D < 2 \text{ and } \gamma > 1

where \( \Gamma(x) \) is Gamma function at the \( x \) argument.

In the case of the corrected roughness rail from Figure 5, the scaling constant \( G = 1.377 \text{ mm} \).

The W-M associated function can be used to characterize the height of the roughness at any scale regardless of the radius of the probe for experimental determination.

The state of stresses and deformations between the rail and wheel roughness as well as the wear behavior can be assessed using the W-M function [7, 9, 11].

Figures 12 a, b exemplify the fractal evaluation of the rail roughness by the Weierstrass-Mandelbrot (W-M) function and the acoustic roughness level for small part of rail (0.7---0.72 m).
In a future paper we will compare the acoustic roughness and the fractal roughness in terms of the rolling noise of a driving wheel to the railroad locomotives.

6. Conclusions
Microgeometric deviations (roughness), corrugations and circularity of wheel are the major cause of rolling noise in locomotives. The current parameters characterizing microgeometry are acoustic roughness (dB) and wavelength (cm) and compare to an acoustic-curve-wavelength curve standard limit.

The microgeometry of the rails at the Faurei ring of testing corresponds to the EN 15610-2011 standard for wavelengths longer than 0.8 cm and it has the fractal character.

To validate the results, it is necessary to remove or correct peaks and pits with the criteria of standard EN 15160-2011.

Matchad's computational program allows statistical analysis, correction of aberrant spikes and pits, deduction of the bearing area curve, and fractal parameters of the microgeometry of railway tracks.

Standard experimental attempts on locomotive and wagon noise will be correlated with fractal parameters and bearing curves of roughness in the next paper.

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