Non-perturbative renormalization in domain-wall QCD by Schrödinger functional scheme

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We formulate and numerically test the Schrödinger functional scheme for domain-wall QCD. We then apply it to a non-perturbative calculation of the renormalization factors for vector and axial-vector currents in quenched domain-wall QCD with plaquette and renormalization group improved gauge actions at $a^{-1} \approx 2$ GeV.

1. Introduction

Recent calculations with domain-wall QCD (DWQCD) have shown that the good chiral property of domain-wall fermions leads to good scaling behavior of physical observables such as quark mass and $B_K$. Aside from the quenched approximation, the use of perturbative renormalization factors is the largest source of the uncontrolled systematic errors in these calculations. Non-perturbative renormalization is required to reduce the total error to a few percent except for quenching, and we decide to employ the Schrödinger functional (SF) scheme, which is a gauge-invariant and finite volume method, toward this goal.

In this report we formulate the SF scheme for DWQCD and calculate the renormalization factors for vector and axial-vector currents as a first step to obtain the renormalization factors for the quark mass and $B_K$.

2. Schrödinger Functional in DWQCD

We consider DWQCD on an $L^3 \times T \times N_s$ lattice. The SF boundary condition for the domain-wall fermion is given by $\psi(\vec{x}, x_0 = 0, s) = P_L s \rho(\vec{x})$, $\bar{\psi}(\vec{x}, x_0 = 0, s) = \bar{\rho}(\vec{x}) P_R s P_-$, $\psi(\vec{x}, x_0 = T, s) = P_- s P_L s \rho'(\vec{x})$ and $\bar{\psi}(\vec{x}, x_0 = T, s) = \bar{\rho}'(\vec{x}) P_R s P_+$. Here $\rho, \bar{\rho}$ are the boundary quark fields, $P_L(s) = P_L \delta_{s,1} + P_R \delta_{s,N_s}$ and $P_R(s) = P_R \delta_{s,1} + P_L \delta_{s,N_s}$ with $P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$ and $P_{\pm} = \frac{1}{2}(1 \pm \gamma_0)$.

We follow the definitions and notations of ref. for the calculation of the renormalization factors.

3. Tests of the formulation

Since the SF boundary condition in time for $\rho = \bar{\rho} = 0$ is identical to the Shamir’s domain-wall(Dirichlet) boundary condition in the fifth direction, extra zero modes appear near $x_0 = 0$ and $T$. We check whether these unwanted zero modes induce extra contribution to the low en-
may be a better alternative as the measure of ex-
larger N[7]. Therefore calculated from the conserved axial vector current of m zero even at m xtte action. The x m dependence is also weak away 0 of the plaque-
terms at the bare quark in quenched DWQCD at β=6.
mination of the renormalization factors evaluated zero modes associated with the Dirichlet bound-
ary conditions at the bare quark mass, M[9]. The dependence
of amAWTI on the boundary condition, which is visible near the boundaries, disappears away from them. Therefore we conclude that the extra zero modes associated with the Dirichlet boundary condition gives negligible effects to the determination of the renormalization factors evaluated at x0 ≃ T/2. Further analytic investigations on this problem for the free case will be given elsewhere[3].

In Fig. 3, amAWTI for the SF scheme is plotted in quenched DWQCD at β = 6.0 of the plaquette action. The x0 dependence is also weak away from the boundaries. Interestingly mAWTI is non-zero even at m_f = 0, and becomes smaller for larger N_s. Moreover the value is consistent with m_{5q}, the explicit breaking of chiral symmetry calculated from the conserved axial vector current of DWQCD[3]. Therefore mAWTI for the SF scheme may be a better alternative as the measure of explicit chiral symmetry breaking in DWQCD, since it can be calculated directly at m_f = 0 with much less computational cost. Note also that the large explicit breaking in mAWTI at N_s = 8 (open circles) is well compensated if one takes a negative quark mass of m_f = −0.005 (filled circles). Hence the domain-wall fermion at N_s ≠ ∞ may be considered as a highly improved Wilson fermion[5].

The non-perturbative renormalization factors for vector and axial-vector currents are defined by Z_V(1 + b_V m_f) = f_V(x_0) and Z_A^2 = 2f_{AA}(x_0, x_0^+, x_0^-), where we fix x_0^+ = T/2 ± T/4 and put m_f = 0 for Z_A. In Fig. 3 Z_V and Z_A are plotted as a function of x_0 at β = 6.0. Similar to the case of amAWTI, a plateau is seen away from the boundaries. The relation Z_V = Z_A, exactly valid in perturbation theory, is satisfied within 1–2%. Moreover the magnitude almost agrees with the mean-field(MF) improved one-loop value. We also observe that Z_V is insensitive to boundary parameters such as the 2-loop boundary counter-terms for gauge fields and the parameter θ of the twisted boundary condition for quarks.

4. Results

We have calculated Z_V and Z_A in quenched DWQCD at a^{-1} ≈ 2 GeV with the plaquette action (β = 6.0) and with the renormalization group(RG) improved action (β = 2.6) on an 8^3 × 16 × 16 lattice with m_f = 0 for
$M = 1.0 \sim 2.2$. The results are summarized in Fig. 3 where $Z_V$ and $Z_A$ are plotted as a function of $M$, together with one-loop perturbative estimates with and without MF improvement. For both gauge actions, $Z_V \simeq Z_A$ holds, and they show a minimum at $M \simeq 1.7$ for the plaquette action or $M \simeq 1.6$ for the RG action. Perturbative estimates without MF improvement fail, particularly for the plaquette action for which the curve cannot be placed in the figure. The agreement becomes much better with MF improvement for both actions.

5. Future directions

We are encouraged with the present results to proceed to an extension of the present work to scale-dependent cases such as quark masses and four-quark operators needed for $B_K$.

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