Cosmological Supergravity from a Massive Superparticle and Super
Cosmological Black Holes

M.E. Knutt-Wehlau

Physics Department, McGill University, Montreal, PQ CANADA H3A 2T8

and

R. B. Mann

Physics Department, University of Waterloo, Waterloo, ON CANADA N2L 3G1

ABSTRACT

We describe in superspace a classical theory of two dimensional (1, 1) dilaton supergravity with a cosmological constant, both with and without coupling to a massive superparticle. We give general exact non-trivial superspace solutions for the compensator superfield that describes the supergravity in both cases. We then use these compensator solutions to construct models of two-dimensional supersymmetric cosmological black holes.
1 Introduction

In recent years, the study of D-branes \[1\] has led to a resurgence of interest in theories of superparticles in various dimensions, as D0-branes are massive superparticles. Of particular interest in this context is the development of exact solutions of supergravity theories that contain a superparticle. The list of solutions is quite meagre: very few exact non-trivial classical solutions to supergravity theories are known (a list is given in ref. \[2\]) and exact superspace supergravity solutions were non-existent.

This latter situation has recently changed \[2\]. By considering a theory of a massive superparticle coupled to a version of two-dimensional (1, 1) dilaton supergravity, we obtained exact classical superspace solutions for the superparticle worldline and the supergravity fields. Finding non-trivial solutions (those that cannot be reduced by infinitesimal local supersymmetry transformations to purely bosonic solutions) to classical supergravity theories is a difficult task, even in two dimensions. The (non)-triviality of a solution can be determined by the method given in \[3\], which requires solving a differential equation for an appropriately well-behaved infinitesimal spinor. However, it is possible to sidestep this procedure by examining classical supergravity problems in superspace \[4\]. A bona fide superspace supergravity solution – one which satisfies the constraints – has nonzero torsion beyond that of flat superspace. The torsion is a supercovariant quantity, and as such its value remains unchanged under a gauge transformation. Hence any exact superspace solution with non-zero torsion must necessarily be non-trivial in this sense.

We take this approach in the present paper. Motivated by the above, we consider (1, 1) supergravity in (1+1) dimensions with a cosmological constant, both with and without coupling to a massive superparticle. The theory then essentially becomes that of a superLiouville field (a dilatonic theory of supergravity), with a possible superparticle coupling.

For the supergravity part of the theory, we consider as before a supersymmetric generalization of the (1 + 1) dimensional “R=T” theory, in which the evolution of the supergravitational fields are determined only by the supermatter stress-energy (and vice versa). This ensures that the dilaton field classically decouples from the evolution of the supergravity/matter system, resulting in a two-dimensional theory that is most closely aligned with general relativity. We obtain non-trivial solutions for the supergravity compensator for the cases of cosmological (1, 1) supergravity (with and without a massive superparticle). For the case with the superparticle, we construct a model of a supersymmetric cosmological black hole.

The outline of our paper is as follows. In section 2, we review the bosonic solution for cosmological dilaton gravity (with and without coupling to a massive particle)
and discuss the respective spacetimes. In section 3, we outline the compensator solution for cosmological dilaton (1, 1) supergravity, with no superparticle coupling. In section 4 we describe the superparticle coupled to supergravity in superconformal gauge. In section 5 we give the solution for the supergravity compensator in the presence of this superparticle (details can be found in the appendix) and discuss the results. In section 6 we give the supercoordinate transformations from superconformal to “super” Schwarzschild gauge, from which we construct the super black hole models. In section 7 we present some concluding remarks.

2 Cosmological Dilaton Gravity

We start by presenting the bosonic case, that of a massive point particle interacting with dilaton gravity. This has been done in [5], but we briefly review the situation here as it is simpler than the supersymmetric case, and illustrates the basic ideas. We then transform the result from Schwarzschild gauge to conformal gauge, and explicitly exhibit the requisite coordinate transformation in order to make contact later on with the supersymmetric case. We find it is necessary to solve the corresponding supersymmetric theory in superconformal gauge from the beginning, and transform in the opposite direction to “super” Schwarzschild gauge afterwards.

We consider the action for $R = T$ theory, which is

$$S = S_G + S_M = \frac{1}{2\kappa} \int d^2 x \left[ \sqrt{-g}(\psi R + \frac{1}{2}(\nabla \psi)^2) - \kappa L_M \right] \tag{2.1}$$

where the gravitational coupling $\kappa = 8\pi G$. The action (2.1) ensures that the dilaton field $\psi$ decouples from the classical equations of motion which, after some manipulation, are

$$R = \kappa T_{\mu} \tag{2.2}$$

$$\frac{1}{2} \left( \nabla_{\mu} \psi \nabla_{\nu} \psi - \frac{1}{2} g_{\mu\nu}(\nabla \psi)^2 \right) - \nabla_{\mu} \nabla_{\nu} \psi + g_{\mu\nu} \nabla^2 \psi = \kappa T_{\mu\nu} \tag{2.3}$$

where $T_{\mu\nu} = \frac{1}{\sqrt{-g}} \delta L_M{g_{\mu\nu}}$ is the stress-energy tensor and $R_{\mu\nu} = \partial_{\lambda} \Gamma^\lambda_{\mu\nu} - \partial_{\nu} \Gamma^\lambda_{\mu\lambda} - \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\lambda\nu} + \Gamma^\lambda_{\lambda\sigma} \Gamma^\sigma_{\mu\nu}$ is our convention for the Ricci scalar.

Here we take the matter Lagrangian to be that of a cosmological constant, $L_M = -\sqrt{-g} \Lambda$, so that $T_{\mu\nu} = \frac{1}{2} g_{\mu\nu} \Lambda$. In conformal coordinates where $ds^2 = e^{2\rho} dx^+ dx^-$, equation (2.2) becomes

$$\partial_{\pm} \partial_{\pm} \rho = -\frac{\kappa}{8} \Lambda e^{2\rho} \tag{2.4}$$

which is Liouville’s equation. This equation also follows directly from the action
(2.1) written in conformal coordinates:

\[ S = \frac{1}{2\kappa} \int d^2x \left[ -4\psi \partial_+ \partial_- \rho + \partial_- \psi \partial_+ \psi + \frac{\kappa}{2} \Lambda e^{2\rho} \right] \] (2.5)

This representation will be useful later on in comparing our results to the supersymmetric case.

Eq. (2.4) has the solution

\[ \rho = \frac{1}{2} \ln \left[ \frac{1}{\pi G |\Lambda|} \left( f_+ f_- - \text{sgn}(\Lambda) f_- \right)^2 \right] \] (2.6)

where \( f_\pm = f_\pm(x^\pm) \) are two arbitrary functions of the coordinates \( (x^+, x^-) \) (where \( x^\pm = (x \pm t)/2 \)), and \( ' \) means derivative with respect to the functional argument. The arbitrariness in the functions \( f_\pm \) corresponds to the freedom to choose coordinates. Indeed, the metric associated with (2.6) is

\[ ds^2 = \frac{dX^+dX^-}{\pi G |\Lambda| (X^+ - \text{sgn}(\Lambda)X^-)^2} \] (2.7)

where the coordinates \( X^\pm \) have been chosen so that \( X^\pm = f_\pm \).

It is always possible to write the metric (2.7) in static coordinates by making the coordinate transformation

\[ X^\pm = \pm e^2 \sqrt{\pi G |\Lambda|} (f(y))^\pm \] (2.8)

yielding

\[ ds^2 = \frac{-4\pi G |\Lambda| f^2(y) dt^2 + (f'(y))^2 dy^2}{\pi G |\Lambda| (f^2 + \text{sgn}(\Lambda))^2} \] (2.9)

To write the metric in Schwarzschild-type coordinates we must require

\[ \frac{2ff'}{\sqrt{\pi G |\Lambda|(f^2 + \text{sgn}(\Lambda))^2}} = \pm 1 \] (2.10)

which after solving for \( f(y) \) gives

\[ ds^2 = \mathcal{A} dt^2 + \frac{dy^2}{\mathcal{B}} \] (2.11)

where

\[ \mathcal{B} = -\mathcal{A} = 4[(c - \text{sgn}(\Lambda)c^2) \pm \sqrt{\pi G |\Lambda| y(1 - 2\text{sgn}(\Lambda)c) - \pi G Ay^2}] \] (2.12)
and where $c$ is an arbitrary constant of integration. Choosing $c = \text{sgn}(\Lambda)/2$ to eliminate the term linear in $y$ yields

$$ds^2 = -(\text{sgn}(\Lambda) - \frac{\kappa}{2}\Lambda y^2)dt^2 + \frac{dy^2}{\text{sgn}(\Lambda) - \frac{\kappa}{2}\Lambda y^2}$$

(2.13)

which is the generic form of the metric of a constant curvature spacetime in Schwarzschild-type coordinates.

For positive $\Lambda$ this is the metric of de Sitter spacetime

$$ds^2 = -(1 - \frac{\kappa}{2}\Lambda y^2)dt^2 + \frac{dy^2}{1 - \frac{\kappa}{2}\Lambda y^2}$$

(2.14)

whereas for negative $\Lambda$, we have

$$ds^2 = -\left(\frac{\kappa}{2}|\Lambda|y^2 - 1\right)dt^2 + \frac{dy^2}{\frac{\kappa}{2}|\Lambda|y^2 - 1}$$

(2.15)

which is the metric of a $(1 + 1)$ dimensional anti de Sitter black hole [3, 8]. Locally this metric is equivalent to

$$ds^2 = -\left(\frac{\kappa}{2}|\Lambda|Y^2 + 1\right)dT^2 + \frac{dY^2}{\frac{\kappa}{2}|\Lambda|Y^2 + 1}$$

(2.16)

which is the usual representation of the anti de Sitter metric in static coordinates.

We can extend the previous solutions to include the effects of a single point particle, whose stress energy is given by

$$T_{\mu\nu} = m \int d\tau \frac{1}{\sqrt{-g}} g^{\mu\alpha} g^{\nu\beta} \frac{dz^\alpha}{d\tau} \frac{dz^\beta}{d\tau} \delta^{(2)}(x - z(\tau))$$

(2.17)

with $z^\mu(\tau)$ being the worldline of the particle. Choosing a frame at rest with respect to the particle, the trace of the stress energy is

$$T^\mu_\mu = -2m e^{-2\rho}\delta(x)$$

(2.18)

in conformal coordinates, $x = 0$ being the location of the particle. The field equations (2.2) then become

$$\partial_+ \partial_- \rho = -\frac{\kappa}{8}\Lambda e^{2\rho} + \frac{km}{4}\delta(x)$$

(2.19)

or more simply

$$\rho''(x) = -\frac{\kappa}{8}\Lambda e^{2\rho} + M\delta(x)$$

(2.20)
with $M = 2\pi Gm$.

Setting $a^2 = \frac{4}{8}|\Lambda|$, equation (2.20) has for $\Lambda > 0$ the solution

$$\rho = -\ln (\cosh(a|x| + b)) \quad (2.21)$$

where $\tanh(b) = -\frac{M}{2a}$. In Schwarzschild-type coordinates the metric (2.21) becomes

$$ds^2 = -\left( -\frac{\kappa}{2} \Lambda y^2 + 2M|y| + C \right) dt^2 + \frac{dy^2}{-\frac{\kappa}{2} \Lambda y^2 + 2M|y| + C} \quad (2.22)$$

where $C = 1 - \frac{M^2}{4a^2}$. If $\Lambda < 0$, the solution can be chosen either as

$$\rho = -\ln (\cos(a|x| + b)) \quad (2.23)$$

where $\tan(b) = \frac{M}{2a}$, or as

$$\rho = -\ln (\sinh(a|x| + b)) \quad (2.24)$$

where $\coth(b) = -\frac{M}{2a}$. In Schwarzschild-type coordinates these become respectively

$$ds^2 = -\left( -\frac{\kappa}{2} |\Lambda| y^2 + 2M|y| + C \right) dt^2 + \frac{dy^2}{-\frac{\kappa}{2} |\Lambda| y^2 + 2M|y| + C} \quad (2.25)$$

where $C = 1 + \frac{M^2}{4a^2}$ and

$$ds^2 = -\left( -\frac{\kappa}{2} |\Lambda| y^2 + 2M|y| - C \right) dt^2 + \frac{dy^2}{-\frac{\kappa}{2} |\Lambda| y^2 + 2M|y| - C} \quad (2.26)$$

where $C = 1 - \frac{M^2}{4a^2}$.

The solutions (2.22), (2.25) and (2.26) have been discussed previously in ref. [5]. For positive $M$ and $C$, the metric (2.22) describes a point mass in de Sitter spacetime. If $M > 0$ but $C < 0$, then (2.22) describes the $(1+1)$-dimensional analogue of Schwarzschild de Sitter spacetime: there is both a cosmological and an event horizon. The metric (2.25) is that of a point mass in anti de Sitter space, and the metric in (2.26) is the $(1+1)$-dimensional analogue of a Schwarzschild anti de Sitter black hole provided $C > 0$.

3 Cosmological $(1, 1)$ Dilaton Supergravity

We consider next in superspace a theory of $(1, 1)$ dilaton supergravity in two dimensions with a cosmological constant, $L$. We use light-cone coordinates $(x^\pm, x^\mp) = \frac{1}{2}(x^1 \pm x^0)$ and $(\theta^+, \theta^-)$. The action is given by

$$I_C = -\frac{2}{\kappa} \int d^2 x d^2 \theta E^{-1}(\nabla_+ \Phi \nabla_- \Phi + \Phi R - 4L) \quad (3.1)$$
where $\Phi$ is the dilaton superfield, $R$ is the scalar supercurvature, $E = \text{sdet}E_A^M$, the superdeterminant of the vielbein, and $\nabla_\pm$ is the supercovariant derivative, defined below. (As in [3], we choose this particular action since the dilaton decouples from the evolution of the matter system in this case, giving a theory that most closely resembles a supersymmetric analogue of two-dimensional general relativity.)

The solution to the constraints on the covariant derivatives is simplest in conformal gauge. The covariant derivatives, $\nabla_A = E_A^M D_M + \omega_A M$ where $\omega_A$ is the spin connection, are expanded with respect to the standard flat supersymmetry covariant derivatives, $D_A = (D_+, D_-) = (\partial_+ + i\theta^+ \partial_\Phi, \partial_- + i\theta^- \partial_\Phi)$. However when we consider the superparticle coupling, the natural description will be in terms of forms, and so we choose as a basis the ordinary derivatives $\partial_M = (\partial_m, \partial_\mu)$, where we write $\nabla_A = \mathcal{E}_A^M \partial_M + \omega_A M$. We solve the constraints in conformal gauge in terms of the $D$'s and change to the other basis afterwards. The (1,1) supergravity constraints [9, 10] are

$$\{\nabla_+, \nabla_+\} = 2i\nabla_+, \quad \{\nabla_-, \nabla_-\} = 2i\nabla_-$$

$$\{\nabla_+, \nabla_-\} = RM$$

$$T^A_+ = T^-_A = 0$$ (3.2)

where the covariant derivatives are determined in conformal gauge in terms of the compensator superfield $S$, as

$$\nabla_+ = e^S [D_+ + 2(D_+ S) M] , \quad \nabla_- = e^S [D_- - 2(D_- S) M]$$

$$R = 4e^{2S} D_- D_+ S$$ (3.3)

from which we can read off the elements of $E_A^M$ and compute $E^{-1} = e^{-2S}$.

In the preferred basis we calculate the elements of $\mathcal{E}_A^M$, and inverting this matrix we obtain

$$\mathcal{E}_M^A = \begin{bmatrix} e^{-2S} & 0 & 2ie^{-S}D_+ S & 0
0 & e^{-2S} & 0 & 2ie^{-S}D_- S
-ie^{-2S}\theta^+ & 0 & e^{-S}(1 - 2(D_+ S)\theta^+) & 0
0 & -ie^{-2S}\theta^- & 0 & e^{-S}(1 - 2(D_- S)\theta^-) \end{bmatrix}$$ (3.4)

Therefore in conformal gauge, the cosmological dilaton supergravity action becomes

$$I_C = -\frac{2}{\kappa} \int d^2x d^2\theta (D_- \Phi D_+ \Phi + 4\Phi D_- D_+ S - 4e^{-2S} L)$$ (3.5)

The equations of motion are

$$\Phi = -2S$$ (3.6)

$$D_- D_+ S = e^{-2S} L$$ (3.7)
Thus the dilaton is expressible in terms of the compensator $S$, which satisfies the usual superLiouville equation.

In order to recover the component action of (2.5), we identify the components of the superfields by the theta expansion (dropping the fermionic fields),

\[
S = -\frac{1}{2} \rho + \sigma \theta^+ \theta^- \tag{3.8}
\]

\[
\Phi = -\frac{1}{2} \psi + \varphi \theta^+ \theta^- \tag{3.9}
\]

which yields

\[
I_C = \frac{2}{\kappa} \int d^2 x \left[ -\psi \partial_\theta \partial_\theta \rho + \frac{1}{4} \partial_\theta \psi \partial_\theta \psi - \varphi^2 - 4 \sigma \varphi - 8 \sigma \Lambda e^{\rho} \right] \tag{3.10}
\]

from the superspace action (3.5). Upon elimination of the auxiliary fields $\varphi, \sigma$ via their equations of motion, (3.10) becomes

\[
I_C = \frac{1}{2\kappa} \int d^2 x \left[ -4 \psi \partial_\theta \partial_\theta \rho + \partial_\theta \psi \partial_\theta \psi - 16 \Lambda e^{2\rho} \right] \tag{3.11}
\]

which is equivalent to (2.5) provided $\Lambda = -\frac{32}{\kappa} \Lambda L^2$. Hence only the action for anti de Sitter spacetimes is recovered. Alternatively, inserting the superfield expansions (3.8,3.9) into eqs. (3.6) and (3.7) yields after some manipulation

\[
\partial_\theta \partial_\theta \rho = 4 \Lambda e^{2\rho} = -\frac{\kappa}{8} \Lambda e^{2\rho} \tag{3.12}
\]

which is the Liouville equation (2.4).

The form of the solution to the superLiouville equation (3.7) that we find most useful is given by [7]. We write the result so that it corresponds more closely with the bosonic result in [8] and find

\[
S = -\frac{1}{2} \ln \left[ -i \frac{(D_+ F_+)(D_- F_-)}{2 \Lambda (F_+ - F_- - i F_+ F_-)} \right] \tag{3.13}
\]

where $F_+ = F_+(x^+, \theta^+)$ and $F_- = F_-(x^-, \theta^-)$ are spinor superfields, with $D_- F_+ = D_+ F_- = 0$, and $F_{\pm/\mp}$ satisfying

\[
D_+ F_\pm = i F_+ D_+ F_\mp \\
D_- F_\mp = i F_- D_- F_\pm \tag{3.14}
\]

These equations are solved by

\[
F_\pm = f_\pm \pm i \theta^+ \lambda^+ \sqrt{\partial_\mp f_\pm}
\]
\[ F_+ = \pm \sqrt{\partial_+ f_\mp} \left[ 1 + \frac{\lambda^+ \lambda^+}{\partial_+ f_\mp} \right] \theta^+ + \lambda^+ \]
\[ F_- = \frac{f_\pm}{\sqrt{\partial_\pm f_\mp}} \sqrt{\partial_\pm f_\mp} \]
\[ F_\mp = f_\pm \pm i\theta^- \lambda^- \left[ 1 + \frac{i\lambda^- \lambda^-}{\partial_\pm f_\mp} \right] \theta^- + \lambda^- \]

where \( f_\pm = f_\pm(x^\pm), \lambda^+ = \lambda^+(x^\pm), f_\mp = f_\mp(x^\mp) \) and \( \lambda^- = \lambda^-(x^\mp) \). It is straightforward to show using eqs. (3.13) and (3.8) that the leading term in the superfield expansion of (3.13) is given by (2.6).

Since the functions \( \lambda^\pm \) and \( f_\pm/\mp \) are arbitrary, we consider the following special case: we simplify the expression (3.13) by choosing coordinates so that

\[ D_+ F_+ = 1 = -iD_- F_- \]

Upon insertion of (3.15) this yields \( f_\pm = \pm x^\pm, \lambda^+ = \lambda^+_0 \) and \( \lambda^- = i\lambda^-_0 \), where \( \lambda^+_0 \) and \( \lambda^-_0 \) are anticommuting constants. This gives

\[ S = \frac{1}{2} \ln \left[ 2L(x^\pm + x^\mp) + i(\theta^+ \lambda^+_0 + \theta^- \lambda^-_0) + (\theta^+ + \lambda^+_0)(\theta^- + \lambda^-_0) \right] \]

for the compensator. The significance of this expression is perhaps more easily understood if we examine it in terms of component fields. We follow the procedure given in [2, 10] in which we choose a Wess-Zumino gauge such that the superconformal gauge \( (E_\pm = e^S D_\pm) \) is compatible with the ordinary \( x \)-space conformal gauge \( (e^{a}_m = e^{\rho} \delta^m_a \) in appropriate coordinates). Using these results, we write \( E_\alpha \) in component conformal gauge as

\[ E_\pm = e^S D_\pm \]

\[ = [e^{-1/4} + \frac{i}{2} (\theta^+ \psi_\pm^+ - \theta^- \psi_\mp^-) + \frac{i}{4} \theta^+ \theta^- e^{1/4}(iA + \psi_\pm^- \psi_\pm^+)]D_\pm \]

and then Taylor-expand \( e^S \) so that we can identify the gravitino \( (\psi_\pm^+, \psi_\mp^-) \) and the component auxiliary field of the supergravity multiplet, \( A \), by comparing powers of \( \theta \).

Carrying out the expansion, we find

\[ e^{-\rho/2} = \sqrt{2L(x^\pm + x^\mp)} \left( 1 + \frac{\lambda^+_0 \lambda^-_0}{2(x^\pm + x^\mp)} \right) \]
\[ \psi_\pm^+ = \sqrt{\frac{2L}{(x^\pm + x^\mp)}} (\lambda^+_0 + \lambda^-_0) \]
\[ \psi_\mp^- = \sqrt{\frac{2L}{(x^\pm + x^\mp)}} (\lambda^+_0 - \lambda^-_0) \]
\[ A = -4iL \]

\[ (3.19) \]
for the component fields. Note that the presence of both gravitini fields is needed in order for the correction to the conformal factor $\rho$ to be non-zero. The metric in Schwarzschild coordinates can be found by transforming to static coordinates

$$x^\# = \pm e^{4L_\Lambda} [f(y)]^\pm 1$$

as in the bosonic case, and transforming the component fields appropriately. For $f(y)$ given by

$$f(y) = \frac{8Ly + 1}{8Ly - 1}$$

which is the solution to (2.10) with $\Lambda < 0$, we obtain the supersymmetric analogue of the $(1 + 1)$-dimensional black hole (2.15).

4 Superparticle Coupled to Supergravity in Superconformal Gauge

We consider now the general action for a massive superparticle coupled to (1, 1) "R=T" dilaton supergravity in two dimensions. We reproduce here the relevant section of [2], as the necessary steps for obtaining the equation of motion for the compensator superfield are the same, in this case. The action is

$$I_P = m \int d\tau \left[ g^{-1} \dot{z}^M \mathcal{E}_M^+ \dot{z}^N \mathcal{E}_N^- + \dot{z}^M \mathcal{E}_M^A \Gamma_A + \frac{g}{4} \right]$$

where $\Gamma$ is a general gauge superfield for the Wess-Zumino type term, and $g$ is the einbein on the worldline of the superparticle. With \( \hat{\nabla}_A = \nabla_A + \Gamma_A \), including now the gauge field, we have

$$[\hat{\nabla}_A, \hat{\nabla}_B] = T_{AB}^C \hat{\nabla}_C + R_{AB}^M + F_{AB}$$

which defines the torsions, curvatures and gauge field strengths, respectively, where $\partial(M \mathcal{E}_N)^A = T_{NM}^A + \omega_{[MN]}^A$, and $F_{AB} = \nabla_{[A} \Gamma_{B]} - T_{AB}^C \Gamma_C$, with $\{ \Gamma_A, \Gamma_B \} = 0$. The constraints on $\Gamma$ are

$$\Gamma_+ = -i \nabla_+ \Gamma_+ \quad , \quad \Gamma_- = -i \nabla_- \Gamma_-$$

$$F_{++} = F_{--} = \nabla_+ \Gamma_- + \nabla_- \Gamma_+ = i$$

and all other $F$’s are zero, consistent with the Bianchi identities [3]. The action is $\kappa$-invariant under transformations of the coordinates in curved superspace provided that the supergravity constraints and those on $\Gamma$ are satisfied.

The action for the superparticle in superconformal gauge is

$$I_P = m \int d^4z \int d\tau \left[ g^{-1} \dot{z}_0^M \mathcal{E}_M^+ \dot{z}_0^N \mathcal{E}_N^- + \dot{z}_0^M \mathcal{E}_M^A \Gamma_A + \frac{g}{4} \right] \delta(z - z_0(\tau))$$
where \( z = (x, \theta) \) are the coordinates of the superspace, and \( z_0(\tau) = (x_0(\tau), \theta_0(\tau)) \) are the coordinates of the superparticle. Defining \( \{ \hat{\nabla}_+, \hat{\nabla}_+ \} \equiv 2i\hat{\nabla}_+ \) and similarly for \( \hat{\nabla}_- \), we find the constraints on \( \Gamma \) in conformal gauge to be

\[
\Gamma_+ = -ie^S[D_+ \Gamma_+ + (D_+ S) \Gamma_+] \\
\Gamma_- = -ie^S[D_- \Gamma_- + (D_- S) \Gamma_-] \\
0 = D_+(e^{-S} \Gamma_+) + D_-(e^{-S} \Gamma_-) - ie^{-2S}
\] (4.5)

Substituting for \( E \) and \( \Gamma \) we obtain

\[
I_P = m \int dt \int d\tau \left\{ g^{-1}e^{-4S}(\dot{x}_0^+ + i\theta_0^+ \dot{\theta}_0^+)(\dot{x}_- + i\theta_0^- \dot{\theta}_0^-) + ie^{-S}(\dot{x}_0^+ + i\theta_0^+ \dot{\theta}_0^+)[(D_+ S) \Gamma_+ - D_+ \Gamma_+] + ie^{-S}(\dot{x}_- + i\theta_0^- \dot{\theta}_0^-)[(D_- S) \Gamma_- - D_- \Gamma_-] + e^{-S}(\dot{\theta}_0^+ \Gamma_+ + \dot{\theta}_0^- \Gamma_-) + g \right\} \delta(z - z_0(\tau))
\] (4.7)

It is convenient to define \( G_\kappa = e^S \Gamma_\kappa \) and include (4.3) in the supergravity action by means of a lagrange multiplier, \( \lambda \). We obtain

\[
I_P = m \int dt \int d\tau \left\{ g^{-1}e^{-4S}(\dot{x}_0^+ + i\theta_0^+ \dot{\theta}_0^+)(\dot{x}_- + i\theta_0^- \dot{\theta}_0^-) + i(\dot{x}_0^+ + i\theta_0^+ \dot{\theta}_0^+ \dot{\theta}_0^+)D_+ G_+ + i(\dot{x}_- + i\theta_0^- \dot{\theta}_0^- \dot{\theta}_0^-)D_- G_- + \dot{\theta}_0^+ G_+ + \dot{\theta}_0^- G_- + g \right\} \delta(z - z_0(\tau))
\] (4.8)

and

\[
I_G = -\frac{2}{\kappa} \int d^2x d^2\theta [D_+ \Phi D_- \Phi + 4\Phi D_- D_+ S - 4e^{-2S}L - \kappa \lambda e^{-2S}(D_+ G_+ + D_- G_- - ie^{-2S})]
\] (4.9)

We now perform a change of variables in the superparticle action, by first explicitly writing it in terms of \( x_0^0 \) and \( x_0^1 \), and then making the gauge choice \( x_0^0 = \tau \) (static gauge) so that \( \frac{d\tau}{dx_0^0} = \frac{dx_0^1}{dx_0^0} \equiv \tilde{x}_0 \). We also relabel \( z^M = (x^0, x^1, \theta^\mu) = (s, \tilde{x}, \theta^\mu) \), so (4.8) becomes

\[
I_P = m \int dt \int d\tau \int dx_0^0 \left\{ g^{-1}e^{-4S} \left[ \frac{i}{2}(1 + \tilde{x}_0^0) + i\theta_0^0 \dot{\theta}_0^0 \right] \left[ \frac{i}{2}(1 - \tilde{x}_0^0) + i\theta_0^- \dot{\theta}_0^- \right] + i \left[ \frac{i}{2}(1 + \tilde{x}_0^0) + i\theta_0^0 \dot{\theta}_0^0 \right] D_+ G_+ + i \left[ \frac{i}{2}(1 - \tilde{x}_0^0) + i\theta_0^- \dot{\theta}_0^- \right] D_- G_- + \dot{\theta}_0^0 G_+ + \dot{\theta}_0^- G_- + g \right\} \delta(t - x_0^0)\delta(x - x_0(x_0^0))\delta(\theta^+ - \theta_0^+(x_0^0))\delta(\theta^- - \theta_0^-(x_0^0))
\] (4.10)
and doing the $x_0^0$ integration gives

$$I_P = m \int dt dx d^2 \theta \left\{ g^{-1} e^{-4S} \left[ \frac{1}{2} (1 + \dot{x}_0) + i \theta_0^+ \dot{\theta}_0^+ \right] \left[ \frac{1}{2} (1 - \dot{x}_0) + i \theta_0^- \dot{\theta}_0^- \right] + i \left[ \frac{1}{2} (1 + \dot{x}_0) + i \theta_0^+ \dot{\theta}_0^+ \right] D_+ G_+ + i \left[ \frac{1}{2} (1 - \dot{x}_0) + i \theta_0^- \dot{\theta}_0^- \right] D_- G_- + \theta_0^+ G_+ + \dot{\theta}_0^- G_- + \frac{g}{4} \right\} \delta (x - x_0 (t)) \delta (\theta^+ - \theta_0^+ (t)) \delta (\theta^- - \theta_0^- (t)) \right) \right) \tag{4.11}$$

From the sum of (4.9) and (4.11), we obtain for the equation of motion for $S$

$$D_- D_+ S(z) = e^{-2S} L = \frac{\kappa m}{4} \int dt' \left\{ g^{-1} e^{-4S} \left[ \frac{1}{2} (1 + \dot{x}_0) + i \theta_0^+ \dot{\theta}_0^+ \right] \left[ \frac{1}{2} (1 - \dot{x}_0) + i \theta_0^- \dot{\theta}_0^- \right] \right\} \delta (z - z_0 (t')) = \frac{\kappa m}{8} \sqrt{\pi^2} e^{-2S} \delta (x - x_0 (t)) \delta (\theta^+ - \theta_0^+ (t)) \delta (\theta^- - \theta_0^- (t)) \tag{4.12}$$

where $\sqrt{\pi^2} = \frac{1}{\sqrt{2}} \sqrt{1 - \dot{x}_0^2}$ for a free particle.

In solving the above equation for $S$, we strictly speaking must include the constraint on the G’s as well, in order to have a real solution. Unlike the previous superparticle case of [4], where the superparticle was allowed to move freely and $S$ vanished on the worldline of the superparticle, this case is more complicated. We now have a non-vanishing compensator, and as a consequence, the constraint on the G’s is now non-trivial. In the next section, we look only for solutions with the superparticle held fixed.

5 Solution for Compensator

To solve for the compensator $S$ that describes the supergravity generated by a superparticle in the presence of a cosmological constant, we consider the superparticle to be stationary and fixed at the origin ($x_0 = \theta_0 = 0$) to begin with. We find the solution for a superparticle located at a general point ($x_0, \theta_0$) by a supersymmetry transformation of this simpler result. In this case, (4.12) becomes

$$e^{2S} D_- D_+ S(z) - L = K \delta (x) \delta (\theta^+) \delta (\theta^-) \tag{5.1}$$

where $K = \frac{\kappa m}{16}$, and $\sqrt{\pi^2} = \frac{1}{\sqrt{2}}$ for $x_0 = 0$. We rewrite the equation in terms of $T = e^{2S}

$$TD_+ D_- T - D_+ TD_- T = -2T (L + K \delta (x) \delta^{(2)} (\theta)) \tag{5.2}$$

where we have set $x_0 = \theta_0 = 0$. We solve for $T$ by doing a $\theta$-expansion, $T(x, \theta) = A(x) + B_+ (x) \theta^+ + C_- (x) \theta^- + D (x) \theta^+ \theta^-$, substituting into (5.2), and matching powers
of $\theta$. (Note that $\delta^{(2)}(\theta) = \theta^+ \theta^-$. ) We find the following four equations as a result:

\[ \theta^0 : \quad -AD - B_+ C_- = -2LA \]  
\[ \theta^+ : \quad iAC' - 2DB_+ - iA' C_+ = -2LB_+ \]  
\[ \theta^- : \quad iAB_+ + 2DC_- - iB_+ A' = +2LC_- \]  
\[ \theta^+ \theta^- : \quad AA'' - 2iB_+ B' - 2iC_- C' + 2D^2 - A'^2 = 2LD + 2KA\delta(x) \]

where the prime denotes derivative with respect to $x$.

The detailed solution to these four equations for the component fields of $T$ is given in the appendix. One solution is found to be $T(x, \theta) = A(x) + B_+(x)\theta^+ + C_-(x)\theta^- + D(x)\theta^+\theta^-$, where

\[ A = 2Lc_0 \cos\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right) + \gamma \left[ 2Lc_0^2 + c_3|x| \sin\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right) + c_3c_0 \cos\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right) \right] \]  
\[ B_+ = 2Lc_0\left[i\alpha + \beta\varepsilon(x) \sin\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right)\right] \]  
\[ C_- = 2Lc_0\left[\alpha\varepsilon(x) \sin\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right) - i\beta\right] \]  
\[ D = 2L\left[1 - \gamma c_0 \cos\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right)\right] \]

where $\alpha$ and $\beta$ are arbitrary Grassmann constants of integration ($\gamma = \alpha\beta$) and the $c_i$'s are ordinary integration constants. This can be put in closed form, which we understand by a Taylor series expansion, as

\[ T(x, \theta) = 2L(\theta^+ - i\beta c_0)(\theta^- - i\alpha c_0) + 2Lc_0(1 + \frac{c_3\gamma}{2L}) \cos \left[ \left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right) - \beta \theta^+ - \alpha \theta^- - \frac{c_3\gamma}{2Lc_0} |x| \right] \]

The second solution for the superfield $T$ is $T(x, \theta) = A(x) + B_+(x)\theta^+ + C_-(x)\theta^- + D(x)\theta^+\theta^-$, where

\[ A = 2Lc_0 \sinh\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right) - \gamma \left[ 2Lc_0^2 + c_3|x| \cosh\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right) - c_3c_0 \sinh\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right) \right] \]  
\[ B_+ = 2Lc_0\left[i\alpha\varepsilon(x) \cosh\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right) - \beta\right] \]  
\[ C_- = -2Lc_0\left[\alpha + i\beta\varepsilon(x) \cosh\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right)\right] \]  
\[ D = 2L\left[1 - \gamma c_0 \sinh\left(\frac{|x| - \frac{c_1}{c_0}}{c_0}\right)\right] \]
In closed form, this is

\[ T(x, \theta) = 2L(\theta^+ - \alpha c_0)(\theta^- + \beta c_0) + 2Lc_0(1 + \frac{c_3}{2L} \gamma \sinh \left( \frac{|x| - c_1}{c_0} \right) + i\alpha \theta^+ - i\beta \theta^- - \frac{c_3}{2Lc_0}|x|) \]

(5.16)

As mentioned at the beginning of this section, general solutions for the case when the superparticle is located at an arbitrary point \((x_0, \theta_0)\) can be obtained by a supersymmetry transformation of the above results. Performing a finite supersymmetry transformation on (5.11) with the usual generators \(P\) and \(Q\) gives the new \(T\)

\[ e^{i(x, \theta)P + x, \theta)} e^{-i(x, \theta)Q} T(x, \theta) e^{i(x, \theta)P + x, \theta)} = T(x - x_0 - i(\theta^+ \theta_0^- + \theta^- \theta_0^+), \theta - \theta_0) \]

(5.17)

where \(T(x', \theta') = A(x') + B_+(x')\theta^+ + C_-(x')\theta^- + D(x')\theta^+\theta^-\). Defining \(X \equiv x - x_0 - i(\theta^+ \theta_0^- + \theta^- \theta_0^+)\), we can write the transformed solution as

\[ T(x, \theta) = 2L(\theta^+ - \theta_0^+ - i\beta c_0)(\theta^- + \theta_0^- - i\alpha c_0) + 2Lc_0(1 + \frac{c_3}{2L} \gamma \sinh \left( \frac{|x| - c_1}{c_0} \right) - \beta(\theta^+ - \theta_0^+) - \alpha(\theta^- - \theta_0^-) - \frac{c_3}{2Lc_0}|X|) \]

(5.18)

where

\[ |X| \equiv |x - x_0 - i(\theta^+ \theta_0^- + \theta^- \theta_0^+)| \]

(5.19)

The shifted hyperbolic solution is obtained in a similar manner.

The solution depends upon the parameters \(x_0, \theta_0, \alpha\) and \(\beta\). In particular, the gravitino field is found to be

\[ \psi_+^* = -4\theta_0^+ \frac{d}{dx} \sqrt{A - C_- \theta_0^-} + 2i \frac{B_+ + D\theta_0^-}{\sqrt{A - C_- \theta_0^-}} \]

\[ \psi^- = 4\theta_0^- \frac{d}{dx} \sqrt{A - B_+ \theta_0^+} - 2i \frac{C_- - D\theta_0^+}{\sqrt{A - B_+ \theta_0^+}} \]

(5.20)

where \(A - D\) are given by (5.10) with \(x\) replaced by \(x - x_0\). The parameters \(\alpha\) and \(\beta\) are arbitrary constants of integration. We note that choosing them to be zero makes the gravitino field proportional to \(\theta_0\). This is not surprising since the superparticle component supercurrent - a source for the gravitino field - is itself proportional to \(\theta_0\). For general \(\alpha, \beta\), it is possible to choose \(\theta_0\) so as to make one of the gravitino components vanish, but except for the case \(\alpha = \beta = \theta_0 = 0\), the gravitino field is nonvanishing.
The non-triviality of the solutions can be seen by calculating the torsion $T_{\pm,\mp}^\alpha$, which is proportional to $R$ and obviously not zero, or the torsion $T_{\ast\ast}^\alpha$, which is proportional to $\nabla_\pm R$. The latter can be shown to be non-zero, even when $\alpha = \beta = \theta_0 = 0$. Hence our result has the requisite non-zero torsion, and is therefore not just a gauge transform of a purely gravitational solution in dilaton gravity.

6 Schwarzschild Gauge

So far we have used the superconformal coordinates $z = (x, \theta)$ to parametrize the superspace. To facilitate comparison with the results of [3], we transform now to superspace coordinates $w = (u, \lambda)$ that correspond to Schwarzschild gauge, in which the vielbein of the bosonic subspace takes the form

$$e_m^a = \left[ \begin{array}{cc} \sqrt{\alpha} & 0 \\ 0 & \sqrt{\alpha}^{-1} \end{array} \right]$$

(6.1)

For simplicity we shall set $c_3 = 0$ throughout.

By analogy with the bosonic case, the supercoordinate transformation that takes us from $z = (x, \theta)$ to $w = (u, \lambda)$ for $T$ given by (5.11) is

$$2L(|x| - c_1) = \tan^{-1}[4L(|u| + u_0)]$$

(6.2)

$$\lambda^+ = \frac{\theta^+ - \frac{i\beta}{2L}}{4L \tan[2L(|x| - c_1)] + \alpha \theta^+ + \beta \theta^-}$$

(6.3)

$$\lambda^- = \frac{\theta^- - \frac{i\alpha}{2L}}{4L \tan[2L(|x| - c_1)] + \alpha \theta^+ + \beta \theta^-}$$

(6.4)

and the transformation from superconformal to Schwarzschild coordinates for $T$ of (5.16) is

$$2L(|x| - c_1) = \coth^{-1}[4L(|u| + u_0)]$$

(6.5)

$$\lambda^+ = \frac{\theta^+ - \frac{i\alpha}{2L}}{4L \coth[2L(|x| - c_1)] + \beta \theta^+ - i\alpha \theta^-}$$

(6.6)

$$\lambda^- = \frac{\theta^- + \frac{\beta}{2L}}{4L \coth[2L(|x| - c_1)] + \beta \theta^+ - i\alpha \theta^-}$$

(6.7)

The transformations (6.2) and (6.5) are those which bring the bosonic part of the vielbein into a form which reproduces the supersymmetric extension of the metrics (2.25) and (2.26) respectively. The latter case corresponds to that of a super black hole. When applied to remaining part of the vielbein, these transformations yield expressions for the gravitini and the auxiliary fields once a component expansion is carried out. We shall not reproduce these expressions here.
7 Summary

We have obtained several new exact solutions in (1 + 1) dimensional supergravity in superspace. By coupling a superdilaton to supergravity as in (3.1), we are able to construct a theory in which the stress-energy of supermatter generates super-curvature, without any influence (classically) from the superdilaton field. If the supermatter is chosen to be that associated with a cosmological constant, we find that the cosmological constant is necessarily negative. Two exact solutions to the field equations are then obtained. One is a supersymmetric version of anti de Sitter spacetime. The other is a supersymmetric version of the anti de Sitter black hole discussed in refs. [3, 8].

Once a superparticle is coupled to the system, the situation changes. The constraints on the gauge superfield $\Gamma$ are no longer trivial (as compared to the case with zero cosmological constant), and so solving for the compensator becomes somewhat more complicated. Two exact solutions are again obtained, which are supersymmetric analogues of their bosonic counterparts (2.23) and (2.24). The latter solution corresponds to a massive supersymmetric black hole in anti de Sitter spacetime; it can be considered the supersymmetric (1+1) dimensional analogue of Schwarzschild anti de Sitter spacetime.

We have been able to obtain exact superspace solutions for the cases we have considered. The role these solutions play in two-dimensional quantum supergravity remains to be studied.

Acknowledgments

This research was supported in part by the Ontario-Québec Projects of Exchange at the University Level, NSERC of Canada, and an NSERC Postdoctoral Fellowship.

A Appendix

We solve the four equations in the text for the component fields of $T$ ($A, B_+, C_-$ and $D$). In doing so, we find it convenient to rewrite (5.3)–(5.6) as follows:

\begin{align}
D - 2L &= -\frac{B_+ C_-}{A} \quad (A.1) \\
\left(\frac{C_-}{A}\right)' &= -2i(D - L)\frac{B_+}{A^2} \quad (A.2) \\
\left(\frac{B_+}{A}\right)' &= 2i(D - L)\frac{C_-}{A^2} \quad (A.3)
\end{align}
\[
\frac{A''}{A} - \frac{A'^2}{A^2} - 2i \frac{B_+ B'_+}{A} - 2i \frac{C_- C'_-}{A} = -2 \frac{D^2}{A^2} + 2L \frac{D}{A^2} + 2K \frac{1}{A} \delta(x) \quad (A.4)
\]

We stress that some care is required in performing algebraic manipulations on these equations, because of the nilpotent nature of many of the quantities involved.

Multiplying \((A.2)\) and \((A.3)\) by appropriate factors of either \(B_+ / A\) or \(C_- / A\), we find

\[
\left( \frac{C_-}{A} \right)' \frac{B_+}{A} = 0 \quad (A.5)
\]

\[
\left( \frac{B_+}{A} \right)' \frac{C_-}{A} = 0 \quad (A.6)
\]

The sum of these two equations can be written as

\[
\left( \frac{B_+ C_-}{A^2} \right)' = 0 \quad (A.7)
\]

which implies that

\[
\frac{B_+ C_-}{A^2} = \gamma ,
\]

where \(\gamma\) is a nilpotent constant, so that \((A.1)\) becomes

\[
D - 2L = -\gamma A \quad (A.9)
\]

We return now to the fourth of our original set of equations, \((A.4)\). Rewriting the ratios of the derivatives, and using \((A.2)\), \((A.3)\), and \((A.9)\), we obtain

\[
AA'' - A'^2 = -2(\gamma LA + 2L^2) + 2KA\delta(x) \quad (A.10)
\]

Following the bosonic example, we let \(A = A(|x|)\), and substituting (where now the prime denotes derivative with respect to the argument of \(A\))

\[
AA'' + 2AA'\delta(x) - A'^2 = -2\gamma LA - 4L^2 + 2KA\delta(x) \quad (A.11)
\]

Matching the \(\delta\)-functions, we get

\[
A(0)A'(0) = KA(0) \quad (A.12)
\]

\[
AA'' - A'^2 = -2\gamma LA - 4L^2 \quad (A.13)
\]

To solve \((A.13)\), we expand \(A\) into its ordinary and nilpotent parts, \(A = f(|x|) + \gamma g(|x|)\), and solve for \(f\) and \(g\) separately by matching coefficients of \(\gamma\). We find then at \(x = 0\)

\[
f(0)f'(0) = Kf(0) \quad (A.14)
\]

\[
f(0)g'(0) + g(0)f'(0) = Kg(0) \quad (A.15)
\]
In order to have a non-trivial solution, we assume that \( f(0) \neq 0 \) and \( g(0) \neq 0 \), and obtain
\[
f'(0) = K \quad , \quad g'(0) = 0 \quad (A.16)
\]
as well as
\[
\begin{align*}
ff'' - f'^2 & = -4L^2 \quad (A.17) \\
gg'' + gf'' - 2f'g' & = -2Lf \\
\end{align*}
\]
Note that (A.17) is just the bosonic Liouville equation for \( f \).

Using standard techniques, we find that there are two sets of solutions that give physically interesting results. We detail the first set here (the second one appears below). We find for \( f \) and \( g \)
\[
\begin{align*}
f(|x|) & = 2Lc_0 \cos\left(\frac{|x| - c_1}{c_0}\right) \quad (A.19) \\
g(|x|) & = 2Lc_0^2 + c_2 \sin\left(\frac{|x| - c_1}{c_0}\right) + c_3 \left[|x| + c_0 \cot\left(\frac{|x| - c_1}{c_0}\right)\right] \sin\left(\frac{|x| - c_1}{c_0}\right) \\
\end{align*}
\]
where the \( c_i \)'s are arbitrary constants of integration. From (A.16), we determine \( c_1 = \frac{1}{2L} \sin^{-1}\left(\frac{-8\sin}{32L}\right) \) and \( c_2 = 0 \). We find that \( c_3 \) is arbitrary. (Note that choosing \( c_0 = \frac{1}{2L} \) gives correspondence with the bosonic solution.)

Having determined \( A \), we can now find \( D \) in terms of \( f \). We find
\[
D = 2L - \gamma f \quad (A.21)
\]
We turn now to solving for \( B \) and \( C \). Using (A.1) in (A.2) and (A.3), and also that \( A = f + \gamma g \), we have
\[
\begin{align*}
\left(\frac{B_+}{f}\right)' & = 2iL\frac{C_-}{f^2} \quad (A.22) \\
\left(\frac{C_-}{f}\right)' & = -2iL\frac{B_+}{f^2} \quad (A.23)
\end{align*}
\]
By using the Liouville solution for \( f \), (A.19), and defining \( p = \frac{B_+}{f} \) and \( q = \frac{C_-}{f} \), (A.22) and (A.23) become
\[
\begin{align*}
p' & = \frac{i}{c_0 \cos\left(\frac{|x| - c_1}{c_0}\right)} q \\
q' & = -\frac{i}{c_0 \cos\left(\frac{|x| - c_1}{c_0}\right)} p \\
\end{align*}
\]

These can be solved through a change of variables, \( P = p + iq, Q = p - iq \) where

\[
\frac{P'}{P} = \frac{1}{c_0 \cos\left(\frac{|x| - c_1}{c_0}\right)}, \quad \frac{Q'}{Q} = -\frac{1}{c_0 \cos\left(\frac{|x| - c_1}{c_0}\right)} \tag{A.26}
\]

By looking separately at \( x < 0 \) and \( x > 0 \), we find that the solutions can be written as

\[
P = \chi \left[ \frac{1 + \varepsilon(x) \sin\left(\frac{|x| - c_1}{c_0}\right)}{\cos\left(\frac{|x| - c_1}{c_0}\right)} \right] \tag{A.27}
\]

\[
Q = \xi \left[ \frac{1 - \varepsilon(x) \sin\left(\frac{|x| - c_1}{c_0}\right)}{\cos\left(\frac{|x| - c_1}{c_0}\right)} \right] \tag{A.28}
\]

where \( \varepsilon(x) = \Theta(x) - \Theta(-x) \), \( \Theta(x) \) is the Heaviside function, and \( \chi \) and \( \xi \) are arbitrary nilpotent constants. We solve for \( B \) and \( C \) from this, defining \( \chi + \xi = 2i\alpha \) and \( \chi - \xi = 2\beta \), and find \( \gamma = \alpha\beta \).

Collecting all the component fields, we reconstruct the superfield \( T \) as \( T(x, \theta) = A(x) + B_+(x)\theta^+ + C_-(x)\theta^- + D(x)\theta^\theta^- \), where

\[
A = 2Lc_0 \cos\left(\frac{|x| - c_1}{c_0}\right) + \gamma \left[ 2Lc_0^2 + c_3|x| \sin\left(\frac{|x| - c_1}{c_0}\right) + c_3c_0 \cos\left(\frac{|x| - c_1}{c_0}\right) \right] \tag{A.29}
\]

\[
B_+ = 2Lc_0[i\alpha + \beta \varepsilon(x) \sin\left(\frac{|x| - c_1}{c_0}\right)] \tag{A.30}
\]

\[
C_- = 2Lc_0[\alpha \varepsilon(x) \sin\left(\frac{|x| - c_1}{c_0}\right) - i\beta] \tag{A.31}
\]

\[
D = 2L[1 - \gamma c_0 \cos\left(\frac{|x| - c_1}{c_0}\right)] \tag{A.32}
\]

This can be put in closed form, which we understand by a Taylor series expansion, as

\[
T(x, \theta) = 2L(\theta^+ - i\beta c_0)(\theta^- - i\alpha c_0) + 2Lc_0(1 + \frac{c_3\gamma}{2L}) \cos \left[ \left(\frac{|x| - c_1}{c_0}\right) - \beta \theta^+ - \alpha \theta^- - \frac{c_3\gamma}{2Lc_0} |x| \right] \tag{A.33}
\]

The second solution for \( f \) and \( g \) is given by

\[
f(|x|) = 2Lc_0 \sinh\left(\frac{|x| - c_1}{c_0}\right) \tag{A.34}
\]

\[
g(|x|) = -2Lc_0^2 + c_2 \cosh\left(\frac{|x| - c_1}{c_0}\right) + c_3 \left[ c_0 \tanh\left(\frac{|x| - c_1}{c_0}\right) - |x| \right] \cosh\left(\frac{|x| - c_1}{c_0}\right) \tag{A.35}
\]
and again the $c_i$’s are arbitrary constants of integration. As before we find $c_1 = \frac{1}{2L} \cosh^{-1} \left( \frac{\kappa m}{2\pi L^2} \right)$ and $c_2 = 0$. In this case

\[
P = \chi \left[ \frac{e(x) \cosh \left( \frac{|x| - c_1}{c_0} \right) - 1}{\sinh \left( \frac{|x| - c_1}{c_0} \right)} \right]
\]

\[
Q = \xi \left[ \frac{e(x) \cosh \left( \frac{|x| - c_1}{c_0} \right) + 1}{\sinh \left( \frac{|x| - c_1}{c_0} \right)} \right]
\]

(A.36, A.37)

with $\chi, \xi$ arbitrary nilpotent constants. We solve for $B$ and $C$ as above, with $\alpha$ and $\beta$ defined as previously.

Putting it all together, we obtain the second superfield $T$ as $T(x, \theta) = A(x) + B_+(x)\theta^+ + C_-(x)\theta^- + D(x)\theta^+\theta^-$, where

\[
A = 2Lc_0 \sinh \left( \frac{|x| - c_1}{c_0} \right) - \gamma \left[ 2Lc_0^2 + c_3 |x| \cosh \left( \frac{|x| - c_1}{c_0} \right) - c_3c_0 \sinh \left( \frac{|x| - c_1}{c_0} \right) \right]
\]

(A.38)

\[
B_+ = 2Lc_0 \left[ i\alpha e(x) \cosh \left( \frac{|x| - c_1}{c_0} \right) - \beta \right]
\]

(A.39)

\[
C_- = -2Lc_0 \left[ \alpha + i\beta e(x) \cosh \left( \frac{|x| - c_1}{c_0} \right) \right]
\]

(A.40)

\[
D = 2L \left[ 1 - \gamma c_0 \sinh \left( \frac{|x| - c_1}{c_0} \right) \right]
\]

(A.41)

and once again $\gamma = \alpha\beta$. In closed form, this is

\[
T(x, \theta) = 2L(\theta^+ - \alpha c_0)(\theta^- + \beta c_0) + 2Lc_0 \left( 1 + \frac{c_3\gamma}{2L} \right) \sinh \left( \frac{|x| - c_1}{c_0} \right) + \frac{i\alpha \theta^+ - i\beta \theta^- - c_3\gamma}{2Lc_0} |x|
\]

(A.42)

References

[1] M. Aganagic, C. Popescu and J. Schwarz, Phys. Lett. B393 (1997) 311, for example, and references therein.

[2] M.E. Knutt-Wehlau and R.B. Mann, Nucl. Phys. B 514 (1998) 355; see also references therein.

[3] P.C. Aichelburg, Phys. Lett. B91 (1980) 382.

[4] S. J. Gates, Jr., M.T. Grisaru, M. Roček, and W. Siegel, Superspace, Benjamin/Cummings, Reading, MA 1983; I.L. Buchbinder and S.M. Kuzenko,
Ideas and Methods of Supersymmetry and Supergravity, Institute of Physics Publishing, Philadelphia, 1995.

[5] R.B. Mann, A. Shiekh and L. Tarasov, Nucl. Phys. B 341 (1990) 134; J.D. Brown, M. Henneaux and C. Teitelboim, Phys. Rev. D 33 (1986) 319; J.D. Brown, Lower Dimensional Gravity (World Scientific, Singapore, 1988).

[6] R.B. Mann, Nucl. Phys. B 418 (1993) 231.

[7] J.F. Arvis, Nucl. Phys. B 212 (1983) 151; Nucl. Phys. B 218 (1983) 309.

[8] J.D. Christensen and R.B. Mann, Class. Quant. Grav. 9 (1992); K.C.K. Chan and R.B. Mann, Class. Quant. Grav. 10 (1993) 913.

[9] S. James Gates, Jr. and H. Nishino, Class. Quant. Grav. 3 (1986) 391.

[10] M. Roček, P. van Nieuwenhuizen, and S.C. Zhang, Ann. Phys. 172 (1986) 348.