The Persistent Radical Effect in Living Radical Polymerization – Borderline Cases and Side-Reactions

Hanns Fischer* and Marc Souaille

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Keywords: Living radical polymerization · Radical reaction rates · Reaction kinetics

Introduction

Living polymerizations provide polymers which are able to grow whenever additional monomer is supplied [1]. Hence, the chains are extendable, and block-copolymers can be produced. Today, this is mainly achieved by anionic and cationic processes. These suffer little from terminations [2], and, hence, the polymer chains retain reactive ion pair end groups. Initiations are fast compared to the overall polymerization time. Therefore, all chains start to grow essentially instantaneously, and the degree of polymerization increases linearly with monomer conversion and is inversely proportional to the initiator concentration. The width of the chain length distribution grows slower than the degree of polymerization, i.e. the polydispersity index decreases with time. Besides living, such polymerizations may also be more correctly called regulated or controlled.

Conventional free radical polymerizations employ slow continuous initiation, and the polymers are formed by termination of the growing chain radicals in coupling or disproportionation reactions. These two features lead to unreactive polymers with essentially time invariant degrees of polymerization and broad molecular weight distributions [2].
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Introduction

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Conventional free radical polymerizations employ slow continuous initiation, and the polymers are formed by termination of the growing chain radicals in coupling or disproportionation reactions. These two features lead to unreactive polymers with essentially time invariant degrees of polymerization and broad molecular weight distributions [2].
In recent years, ways have been found to combine the ease and robustness of radical polymerizations with the advantages of the ionic reactions. One successful scheme involves the reversible combination of growing chain radicals $R_n^·$ with persistent radical species $Y^·$ to dormant polymer chains $R_n-Y$, i.e. the mechanism

$$R_n-Y \rightarrow k_d R_n+Y$$
$$R_n+Y \rightarrow k_c R_n-Y$$
$$R_n+M \rightarrow k_p P_{n+m}$$

The products of irreversible terminations, such as

$$R_n+R_m \rightarrow k_t P_{n+m}$$

are virtually absent. The concentration of the transient growing radicals $R_n^·$ is low, and there is a remarkably large excess of the persistent species $Y^·$. Solomon et al. [3] introduced alkoxyamines $R_n-Y$ as initiators where $R_n$ is an alkyl and $Y$ is a nitroxide group. Later, Georges et al. [4] found that the polymerization can also be started with a conventional radical initiator in the presence of a stable nitroxide $Y^·$. In this case, the alkoxyamines $R_n-Y$ are formed in situ. Today, many variants are known which use different initiators, different persistent radicals and different procedures.

This radical polymerization resembles the infiner technique of Otsu et al. [5] and is intrinsically related to the atom transfer radical polymerization (ATRP) of Matyjaszewski, Percec and Sawamoto [6]. In ATRP, the dormant chains are capped by halogen atoms which are reversibly transferred to metal complexes in the lower oxidation state. This generates the transient growing radicals and the complexes in the higher oxidation state, and the latter are the persistent species.

The Basic Mechanism: Optimum Rate Constants and Borderline Cases

We consider reactions (1–4) with the simplification of chain length independent rate constants. This allows to sum up all $R_n-Y$ as $R-Y$, all $R_n^·$ as $R^·$ and all $P_{n+m}$ as $P$ and to drop the indices. Self-termination is by disproportionation only, but termination by combination does not change the principles, since it simply doubles the degree of polymerization of the unreactive polymers. Initially, there are only dormant molecules $R-Y$ which are abbreviated as $I$. The kinetic equations for the radical concentrations (radical dots omitted)

$$\frac{d[R]}{dt} = k_d ([I] - [Y]) - k_c [Y] [Y] - k_j [I] [I]$$
$$\frac{d[Y]}{dt} = k_c [I] [Y] - k_j [I] [I]$$

are simple but non-linear and have no closed analytic solutions. The validity of good approximative solutions can be proven by using the mathematical technique of phase-space analysis [9]. At early times, both radical concentrations increase linearly with time ($[R] = [Y] = k_j [I] t$). Then, typically in a few 100 milliseconds or seconds, a short complicated transition region is reached due to the onset of the radical-radical reactions. If the rate constants obey

$$k_d \leq k_c [I]$$
$$k_c \ll k_j [I]$$
$$k_c / k_j = K \ll k_j [I] / k_i$$

this is followed by a long-lasting stage with very unusual features: The concentration of the persistent species increases proportional to $t^{1/3}$, the concentration of the transient species decreases as $t^{1/3}$, and there is an equilibrium relation for the reversible dissociation.

Eqn. (9) differs from the usual law of mass action because it contains the initial concentration $[I]_0$ instead of the momentary value $[I]$. Also, the radical concentrations (8) are time dependent though this time dependence cancels out to produce a time independent expression (9).

Therefore, we call the stage a quasi-equilibrium. If the kinetic criteria (7) are not fulfilled, $Y^·$ continues to increase as in the initial phase, and $R^·$ attains a constant...
concentration \([R] = (k_d l_d / k_c)^{1/2}\) like in a conventional radical polymerization. The process ends when \([Y]\) approaches its final value \([Y] = [Y]_0\) with a period in which \([R]\) decays slowly in a bimolecular fashion, and all dormant species are converted to unreactive products [9].

As known from various studies involving nitroxide radicals the rate constants are in the ranges of \(10^6 < k_y < 1 \text{ s}^{-1}\), \(10^8 < k_c < 10^9 \text{ M}^{-1}\text{s}^{-1}\) and \(k_l = 10^9 \text{ M}^{-1}\text{s}^{-1}\) for usual reaction temperatures around 100–120 °C [10]. The initial concentration of R–Y is usually about \(10^3\) to \(10^4\) M. With these data, Eqns (7) reveal that for realistic cases the quasi-equilibrium always exists, and the \(t^{1/2}\)-dependence of \([Y]\) was indeed observed experimentally [11]. The lifetime prolongation of the radical precursor is given by the factor \(k^3/2l_y/k_l\) and amounts to many orders of magnitude.

To treat the polymerization one couples the Eqn for the concentration of the transient radical (8) with the usual rate law for monomer consumption (d[M]/dt = \(-k_y [M][R]\)) and with the rate equations for the moments of the molecular weight distribution [7][9]. Livingness and control can be achieved if the monomer conversion occurs in the quasi-equilibrium but not in the other time regimes. This requires that

\[k_y << 3k_l\]

and

\[k_y << 3k_c\],

and these conditions are normally also fulfilled. Then, the monomer conversion obeys

\[\ln([M]_0/[M]) = \frac{3}{2} k_d l_d = 3/2 k_d [R](0) t\]  (11)

This dependence of \(\ln([M]_0/[M])\) on time deviates from the familiar linear dependence in conventional radical and in ionic living polymerizations [2] because the concentration of the propagating species decreases in time, and this also accounts for the factor \(3/2\) on the r.h.s. of Eqn (11). This Eqn was shown to accommodate experimental data [12] but we notice that a linear time dependence has been found in many living radical polymerizations. It may be caused by a decrease of the termination constant with increasing chain length and conversion [9][13] and by additional reactions which produce constant radical concentrations (see below).

For the basic mechanism the degree of polymerization and the polydispersity are given by Eqns (12) and (13) [9] where

\[PDI = 1 - e^{-k_d l_d / k_c} + \frac{1}{2k_l/k_c} (\frac{[M]_0}{[M]})^2 \left(1 - e^{-k_d l_d / k_c}\right)^{1/2}\]

\[K = \frac{k_d}{k_c} \leq \frac{1}{2} \left(\frac{[M]_0}{[M]}ight)^{1/2}\]

\[K = \frac{k_d}{k_c} \geq \frac{2}{9} \left(\frac{[M]_0}{[M]}ight)^{1/2} T\]

\[\text{erf}\] is the error function, and the latter Eqn becomes Eqn (14) at long times.

Secondly, the residual polydispersity index \(PDI - 1 - [M][M]_0\) shall be smaller than or equal to an upper limit \(\delta\), which provides

\[k_d/E^2 \geq \frac{1}{9} \left(\frac{[M]_0}{[M]}ight)^{1/2} T\]

Thirdly, the time for large conversion (here 90%) shall not exceed a preset time \(T\), and this requires

\[K = \frac{k_d}{k_c} \geq \frac{2}{9} \left(\frac{[M]_0}{[M]}ight)^{1/2} T\]

Obviously, the optimum ranges of \(k_d\) and \(k_c\) depend on the monomer \((k_p)\) and the desired degree of polymerization \(([M]_0)\). Fig. 1 shows a double-logarithmic plot of \(k_d\) vs. \(k_c\) for the representative parameters \([M]_0 = 0.1\) M, \(k_y = 5000 \text{ M}^{-1}\text{s}^{-1}\) and \(k_l = 10^8 \text{ M}^{-1}\text{s}^{-1}\). The region where all three conditions are fulfilled is marked by a heavy frame. A larger equilibrium constant \(K\) provides faster polymerizations on the expense of less living polymer, and a lower product \(k_d/k_c\) leads to a larger final polydispersity.

Fig. 1. Range of rate constants \(k_d\) and \(k_c\) for optimized living radical polymerizations in 5 h for 90% conversion with less than 2% unreactive polymer and a polydispersity smaller than 1.2. Other parameters, see text.
Fig. 1 also contains a line QE. Above this line the quasi-equilibrium conditions are not met, i.e. there can be no living and controlled radical polymerization. The points marked A, B, and C refer to the following cases: For A the rate constants are in the optimum range. B and C lie outside but in a region where the quasi-equilibrium still exists, and it is easy to predict the outcome of a polymerization for the parameters corresponding to these cases:

For B and A the equilibrium constants \( K \) are equal, which means equal conversion times and equal fractions of unreactive polymer. However, the product \( k_yk_d \) is smaller for B, and this leads to a larger polydispersity. Therefore, in case B one obtains a polymer which is mainly living but not well controlled. In fact, simulations reveal that for A and B, 90% conversion are reached within the same 5 h, and the fraction of unreactive polymer is below 2%. For A the final PDI is 1.1 but for B it increases to a high value of 3. Moreover, for case B and opposite to case A the degree of polymerization decreases with increasing conversion.

For case C the product \( k_yk_d \) is the same as for A but the equilibrium constant \( K \) is larger. Hence, one obtains a faster polymerization and a larger fraction of unreactive polymer. On the other hand, the polydispersity does not change from A to C. This means that for case C the polymer is largely controlled but less living. This is supported by simulations which show the same controlled behavior of the degree of polymerization and the polydispersity for A and C, but for C the final fraction of unreactive polymer amounts to about 50%. Obviously, in living radical polymerizations the properties of livingness and control do not mutually imply each other.

**Side Reactions**

**Cross-disproportionation**

Radical-radical reactions often occur not only by coupling but also by disproportionation if the \( \beta \)-hydrogen transfer is possible. Hence, for usual monomers the cross-coupling reaction (2) may be accompanied by the reaction

\[
\text{R}_n + \text{Y} \rightarrow \text{R}_n(\text{H}) + \text{YH}. \quad (18)
\]

This removes both the transient and the persistent radicals. Products of disproportionation have been observed in model systems, and it has been speculated that (18) drastically reduces the attainment of the theoretical degree of polymerization of 2000 is not reached but if the polymerization were stopped at about 20% conversion a living and controlled polymer would still be obtained. Eqn (20) shows that the effect of disproportionation depends strongly on the other rate parameters. We predict that 90% conversion to a living and controlled polymer is still achieved if the fraction of disproportionation obeys

\[
f_d < \frac{3}{(2 \ln(10))^2} \text{PDI} \quad (21)
\]

which allows seldom more than a very few percent. Propagating radicals of methylmethacrylates react with TEMPO mainly by disproportionation, and this explains the failure to achieve good results for this monomer by TEMPO-mediation [15].

**True Equilibria**

Two situations have been analyzed by the phase-space method which may lead to steady-state radical concentrations and linear time dependencies of \( \ln([M]/[M]) \). They have also been treated to some extent by Fukuda et al. [16].

We consider first the initial presence of additional persistent species in a concentration \([Y]_0 \) for a system which also initially contains \( I = R-Y \) and obeys the conditions (7). If the initial concentration \([Y]_0 \) is very small, more specifically if
[I]_0 << (k, k_0[I]_0)^(1/2)/k_t, there is no effect, and the previous equations remain valid. For (k, k_0[I]_0)^(1/2)/k_t << [I]_0 << [I]_0 relation (9) still holds but [Y] is the sum of the concentrations of persistent species initially present and those formed due to the Persistent Radical Effect. For times smaller than t = [I]_0/3k^2[I]_0^2 one has [Y] = [Y]_0, i.e. [R] is constant. Monomer conversion in this regime leads to a linear dependence of the conversion index ln([M]/[M]) on time. For larger times, [Y] is dominated by the fraction arising from the Persistent Radical Effect, and one has to apply the relations for the basic system with the quasi-equilibrium. If one has [Y]_0 > [I]_0, the quasi-equilibrium does not develop. In general, the initial persistent species slows down the polymerization but it may improve livingness and control quite significantly.

Secondly, we consider the action of a conventional initiator which is added to the basic system to speed up conversion and which produces transient radicals with a constant rate R_t during the whole polymerization process. If the constant rate is smaller than the rate of radical formation by decomposition of I = R - Y, i.e. R_t < k, [I]_0 and if the conditions (7) are fulfilled, the system first behaves as if R = R - Y, and the previous equations remain valid. Interestingly, even a transient species has the same effect.

Moreover, a slow decay of persistent to transient species has the same effect.

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