Extraction of $a_1$ and $a_2$ from $B \to \psi K(K^*)$, $D(D^*)\pi(\rho)$ Decays

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Abstract

Based on the factorization approach, we show that the CLEO data for the ratio $\Gamma(B \to \psi K^*)/\Gamma(B \to \psi K)$ and the CDF measurement of the fraction of longitudinal polarization in $B \to \psi K^*$ can be accounted for by the heavy-flavor-symmetry approach for heavy-light form factors provided that the form factor $F_0$ behaves as a constant, while the $q^2$ dependence is of the monopole form for $F_1$, $A_0$, $A_1$, and of the dipole behavior for $A_2$ and $V$. This $q^2$ extrapolation for form factors is further supported by $B \to K^*\gamma$ data and by a recent QCD-sum-rule analysis. We then apply this method to $B \to D(D^*)\pi(\rho)$ decays to extract the parameters $a_1$ and $a_2$. It is found that $a_1(B \to D^{(*)}\pi(\rho)) = 1.01 \pm 0.06$ and $a_2(B \to D^{(*)}\pi(\rho)) = 0.23 \pm 0.06$. Our result $a_2/a_1 = 0.22 \pm 0.06$ thus significantly improves the previous analysis that leads to $a_2/a_1 = 0.23 \pm 0.11$. We argue that, contrary to what anticipated from the leading $1/N_c$ expansion, the sign of $a_2(B \to \psi K^{(*)})$ should be positive and $a_2(B \to \psi K^{(*)}) > a_2(B \to D^{(*)}\pi(\rho))$. 

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1. Introduction  The fact that the $D^+$ meson has a longer lifetime than the $D^0$ is already manifest at the exclusive two-body decay level: The number of two-body $D^+$ decay modes is about two times less than that of the $D^0$ and there exists a large destructive interference in the Cabibbo-allowed decays $D^+ \rightarrow \bar{K}^0\pi^+$, $\bar{K}^0\rho^+$, $K^{*0}\pi^+$, which is also known as the Pauli interference at the inclusive level. The recent CLEO data on $B \rightarrow D\pi, D\rho, D^*\pi, D^*\rho$ decays exhibit a rather unexpected result [1]: The interference between the two different amplitudes contributing to exclusive two-body $B^-$ decays are evidently constructive, contrary to the charmed meson case. This feature is quite stunning since the rule of retaining only the leading terms in the $1/N_c$ expansion ($N_c$ being the number of color degrees of freedom) [2], which is empirically operative in charm decays, fails in $B \rightarrow D^{(*)}\pi(\rho)$ decays. Quantitatively, the ratio of the parameters $a_1$ and $a_2$ corresponding to the external and internal $W$-emission amplitudes is found to be positive with the magnitude $a_2/a_1 = 0.23 \pm 0.04 \pm 0.04 \pm 0.10$ [1].

Under the factorization assumption, the spectator meson decay amplitude is characterized by the parameters $a_1$ and $a_2$ [3] which are related to the Wilson coefficients $c_1$ and $c_2$ by

$$a_1 = c_1 + \xi_1 c_2, \quad a_2 = c_2 + \xi_2 c_1. \quad (1)$$

Naive factorization implies that $a_1$ and $a_2$ are universal, namely they are channel independent in $D$ or $B$ decays. Recently, we have shown from the analysis of experimental data that $a_2$ is not universal at least in charm decays [4]. We found $a_2 \sim -0.51, -0.66 \sim 0.78, -0.85 \sim 0.91$ respectively for $D \rightarrow \bar{K}\pi, \bar{K}^*\pi$ and $\bar{K}^*\rho$ decays [4]. Physically, this can be understood as follows. Writing

$$\xi_1 = \frac{1}{N_c} + \frac{r_1}{2}, \quad \xi_2 = \frac{1}{N_c} + \frac{r_2}{2}, \quad (2)$$

where $r_{1,2}$ denote the contributions of color octet currents arising from the Fierz transformation relative to the factorizable ones [5], it is natural to expect that the nonperturbative effect is such that $|r_2(D \rightarrow VV)| > |r_2(D \rightarrow VP)| > |r_2(D \rightarrow PP)|$ ($V$: vector meson, $P$: pseudoscalar meson) since soft-gluon effects become more important when final-state particles move slower, allowing more time for significant final-state interactions. Numerically, it follows from Eq. (2) that $r_2 \sim -0.67, -0.9 \sim 1.1, -1.2 \sim 1.3$ respectively in $D \rightarrow \bar{K}\pi, \bar{K}^*\pi$ and $\bar{K}^*\rho$ decays [4], in accordance with the theoretical expectation. It is clear that the leading $1/N_c$ expansion works most successfully for $D \rightarrow \bar{K}\pi$ as the subleading $1/N_c$ factorizable contribution is almost compensated by the soft-gluon effect so that $\xi_2(D \rightarrow \bar{K}\pi) \approx 0$.

Now come back to the $B$ meson case. A priori there is no reason to expect that $a_2$ extracted from $B \rightarrow \psi K^{(*)}$ should be the same as that in $B \rightarrow D^{(*)}\pi(\rho)$ decays. Recall

\footnote{Soft gluon contributions are of course nonfactorizable. The fact that $r_{1,2}$ are channel dependent reminds us of that they are originally nonfactorizable.}
that the c.m. momentum in $B \to D\pi$ is 2306 MeV, while it is only 1682 MeV in $B \to \psi K$ decay. Hence a direct determination of $a_2$ from $B \to D^{(*)}\pi(\rho)$ data is desirable and it is expected that $|r_2(D \to PP)| > |r_2(B \to \psi K)| \gtrsim |r_2(B \to D\pi)|$. However, it is easily seen from Tables XX and XXI of Ref.[1] that, based on the modified Bauer-Stech-Wirbel model [6], an individual fit of $a_2/a_1$ to the CLEO data of $B \to D^{(*)}\pi(\rho)$ gives rise to

$$
\frac{a_2}{a_1} = \begin{cases} 
0.31 \pm 0.12, & B \to D\pi; \\
0.44 \pm 0.23, & B \to D\rho; \\
0.32 \pm 0.13, & B \to D^*\pi; \\
0.68 \pm 0.26, & B \to D^*\rho.
\end{cases}
$$

(3)

Since $r_{1,2}$ are not supposed to vary significantly from $B \to D\pi$ to $D^*\rho$ decays, a sizeable discrepancy among some of the values of $a_2/a_1$ shown in (3) determined from various $B$ decay modes is certainly unexpected. Recall that the aforementioned value $a_2/a_1 = 0.23 \pm 0.11$ cited in the CLEO paper [1], which is substantially different from the individual fits given in (3), is obtained by a global least squares fit to the CLEO data. Of course, it is more complicated to extract $a_2$ from $B \to D^{(*)}\pi(\rho)$ decays than from $B \to \psi K^{(*)}$ since the former involve final-state interactions and $W$-exchange contributions. Nevertheless, even in the absence of the above two effects, a better improvement on the previous theoretical calculations for $B \to D^{(*)}\pi(\rho)$ is called for.

It is tempting to argue that the above-mentioned difficulty for extracting $a_2$ is circumvented in the case of $B \to \psi K^{(*)}$ decays as they are free of final-state strong interactions and nonspectator effects. Indeed, an individual fit of $a_2$ to the CLEO data of $B^- \to \psi K^-, \psi K^{*-}, B^0 \to \psi K^0, \psi K^{*0}$ yields $|a_2| = 0.25 \pm 0.02, 0.25 \pm 0.04, 0.20 \pm 0.03, 0.24 \pm 0.03$, respectively (see Tables IX and XX of Ref.[1]). However, it was pointed out recently that there are two experimental data for $B \to \psi K^{(*)}$ which cannot be accounted for simultaneously by all commonly used models [7,8]. That is, all the known models in the literature fail to reproduce the data of either the production ratio $R = \Gamma(B \to \psi K^*)/\Gamma(B \to \psi K)$ or the fraction of longitudinal polarization $\Gamma_L/\Gamma$ in $B \to \psi K^*$ or both. This casts doubt on the estimate of $a_2$ from $B \to \psi K^{(*)}$ and even on the validity of the factorization approach.

In this paper, we will explore the possibility of accounting for the data of $B \to \psi K^{(*)}$ without giving up the factorization hypothesis. We found that the data of $R$ measured by CLEO and $\Gamma_L/\Gamma$ by CDF for $B \to \psi K^{(*)}$ decays can be accommodated by the heavy flavor symmetry approach for heavy-light form factors with an appropriate choice for their $q^2$ dependence. This method for heavy-light form factors is then applied to $B \to D^{(*)}\pi(\rho)$ decays and compared with experiment. We shall see that $a_2/a_1$ or $a_2$ extracted in this way is improved quite significantly. A discussion on its implication is then presented.
\section{Implications from $B \rightarrow \psi K^{(*)}$ decays}

The experimental data of interest for $B \rightarrow \psi K^{(*)}$ are the vector to pseudoscalar production ratio \cite{1}

$$R = \frac{\mathcal{B}(B \rightarrow \psi K^{*})}{\mathcal{B}(B \rightarrow \psi K)} = 1.71 \pm 0.34,$$  

(4)

and the fraction of longitudinal polarization in $B \rightarrow \psi K^{*}$

$$\left( \frac{\Gamma_L}{\Gamma} \right)_{B \rightarrow \psi K^{*}} = \begin{cases} 0.97 \pm 0.16 \pm 0.15, & \text{ARGUS \cite{9}}; \\ 0.80 \pm 0.08 \pm 0.05, & \text{CLEO \cite{1}}; \\ 0.66 \pm 0.10 \pm 0.10, & \text{CDF \cite{10}}. \end{cases}$$  

(5)

Based on factorization, it was shown recently that currently used $B \rightarrow K^{(*)}$ form factors fail to explain the data of $R$ and $\Gamma_L/\Gamma$ simultaneously \cite{7,8}. This might be attributed either to a failure of the factorization method (see e.g. Ref.\cite{11}) or to a wrong choice of form factors or both.

In what follows we would like to investigate if it is possible to “derive” a set of $B \rightarrow K^{(*)}$ form factors to account for the $B \rightarrow \psi K^{(*)}$ data within the factorization framework. Since the existing models lead to form factors excluded by data \cite{7,8}, we prefer to follow Ref.\cite{12} to relate the $B \rightarrow K^{(*)}$ and $D \rightarrow K^{(*)}$ form factors at the same heavy quark velocity $v$ via model-independent heavy flavor symmetry, as elaborated on in Ref.\cite{7}. So long as the momentum of the light meson $p_{K^{(*)}}$ does not scale with $m_{c,b}$ or $v \cdot p_{K^{(*)}} < < m_{c,b}$, one has the relations \cite{12} \footnote{Empirically, we found that the heavy-light form-factor relations (6) using the heavy quark masses $m_B$ and $m_c$ work better for $B \rightarrow \psi K^{(*)}$ decays than that using the meson masses $m_B$ and $m_D$.}

\begin{align*}
F_{1}^{BK}(q_B^2) & = \frac{C_{bc}}{2\sqrt{m_Bm_c}} \left\{ (m_b + m_c)F_{1}^{DK}(q_D^2) - (m_b - m_c)\frac{m_B^2 - m_K^2}{q_D^2} \left[ F_{0}^{DK}(q_D^2) - F_{1}^{DK}(q_D^2) \right] \right\}, \\
V_{BK}^{*}(q_B^2) & = C_{bc}\sqrt{\frac{m_c}{m_b}} \frac{m_B + m_K^*}{m_B + m_K^*} V_{DK}^{*}(q_D^2), \\
A_{1}^{BK}^{*}(q_B^2) & = C_{bc}\sqrt{\frac{m_c}{m_b}} \frac{m_D + m_K^*}{m_B + m_K^*} A_{1}^{DK}^{*}(q_D^2), \\
A_{2}^{BK}^{*}(q_B^2) & = \frac{1}{2}C_{bc}\sqrt{\frac{m_c}{m_b}} \left\{ \left( 1 + \frac{m_c}{m_b} \right) \frac{m_B + m_K^*}{m_D + m_K^*} A_{2}^{DK}^{*}(q_D^2) + \left( 1 - \frac{m_c}{m_b} \right) \frac{m_B + m_K^*}{q_D^2} \times \left[ 2m_K^* A_{0}^{DK}^{*}(q_D^2) - (m_D + m_K^*) A_{1}^{DK}^{*}(q_D^2) + (m_D - m_K^*) A_{2}^{DK}^{*}(q_D^2) \right] \right\}.
\end{align*}

(6)

where $C_{bc} = (\alpha_s(m_B)/\alpha_s(m_c))^{-6/25}$, $q_B = m_b v - q$, $q_D = m_c v - q$, and we have followed Ref.\cite{13} for the definition of form factors.

It is customary to make a monopole ansatz for all the form factors $F_0$, $F_1$, $A_0$, $A_1$, $A_2$, $V$ \cite{13}. However, a careful study based on the scaling argument indicates that not all the form...
factors share the same $q^2$ behavior. A consideration of the heavy quark limit behavior of the form factors leads to \cite{14}

\[ F_{0BK}(q^2) = F_{1BK}(q^2) \left( 1 - \frac{q^2}{m_B^2 - m_K^2} \right), \]

and

\[ A_{0BK^*}(q^2) = \frac{m_B + m_K^*}{2m_K^*} A_{1BK^*}(q^2) - \frac{m_B^2 - m_K^2 - q^2}{2m_K^*(m_B + m_K^*)} A_{2BK^*}(q^2) \]

\[ A_{0BK^*}(0) = \frac{m_B + m_K^*}{2m_K^*} A_{1BK^*}(0) - \frac{m_B - m_K^*}{2m_K^*} A_{2BK^*}(0), \]

to the leading order in the heavy quark limit. Note that at $q^2 = 0$, (7) and (8) are precisely the constraints \cite{13}

\[ F_{0BK}(0) = F_{1BK}(0), \]

\[ A_{0BK^*}(0) = \frac{m_B + m_K^*}{2m_K^*} A_{1BK^*}(0) - \frac{m_B - m_K^*}{2m_K^*} A_{2BK^*}(0), \]

necessary for avoiding unphysical poles on the r.h.s. of Eq.(6). It is evident that the $q^2$ dependence of $F_1$ is different from that of $F_0$ by an additional pole factor. Several theoretical arguments and many QCD sum rule calculations \cite{15-18} indicate that $F_1(q^2)$ has a monopole behavior. This in turn implies an approximately constant $F_0$.\footnote{It was pointed out in Ref.\cite{19} that if the single pole behavior holds for both $F_0$ and $F_1$, one is led to $f_{B^\pi}(0) = -0.21 f_{B^\pi}(0)$, which is inconsistent with the heavy-quark-symmetry relation $f_{B^\pi} \approx -f_{B^\pi}$. A nearly constant behavior of $F_0$ in $q^2$ is confirmed by a recent QCD-sum-rule calculation \cite{17}.}

If $A_0$ and $A_1$ have the same $q^2$ dependence, Eq.(8) will lead to an interesting $q^2$ behavior for $A_2$ \cite{14}. Assuming

\[ A_0(q^2) = \frac{A_0(0)}{(1 - \frac{q^2}{m^2})^n}, \quad A_1(q^2) = \frac{A_1(0)}{(1 - \frac{q^2}{m^2})^n}, \]

where the pole mass $m$ is the same for $A_0$ and $A_1$ in the heavy quark limit, we see from Eq.(8) that, by neglecting $m_{K^*}$ relative to $m_B$, the $q^2$ dependence of $A_2$ is different from that of $A_0$ and $A_1$ by an additional pole factor \cite{14}:

\[ A_2(q^2) = \frac{A_2(0)}{(1 - \frac{q^2}{m^2})^{n+1}}. \]

In practice, we will take $n = 0, 1$.

In the following we will calculate $B \to K^{(*)}$ form factors from Eq.(6) using the experimental input on $D \to K^{(*)}$ ones \cite{20}:\footnote{The average experimental values given in the Particle Data Group \cite{21} are $F_1^{DK}(0) = 0.75 \pm 0.03$, $V^{DK^*}(0) = 1.1 \pm 0.2$, $A_1^{DK^*}(0) = 0.56 \pm 0.04$, $A_2^{DK^*}(0) = 0.40 \pm 0.08$.}

\[ F_1^{DK}(0) = 0.77 \pm 0.04, \quad V^{DK^*}(0) = 1.12 \pm 0.16, \]

\[ A_1^{DK^*}(0) = 0.61 \pm 0.05, \quad A_2^{DK^*}(0) = 0.45 \pm 0.09. \]
As for the $q^2$ dependence, since $A_2$ has one more pole factor than $A_0$ and $A_1$, as just discussed, two possibilities of interest are:

(i) a monopole form for $F_1$, $A_2$, $V$, and an approximately constant for $F_0$, $A_0$, $A_1$. Recall that a slowly varying $A_1(q^2)$ with $q^2$ is strongly advocated in Ref.[8].

(ii) a monopole extrapolation for $F_1$, $A_0$, $A_1$, a dipole behavior for $A_2$, $V$, and an approximately constant for $F_0$. This is precisely the pole behavior shown by a recent QCD sum rule analysis [18].

In addition to the above two cases, we also consider case (iii) in which all the form factors are extrapolated in a monopole form, as employed in the Bauer-Stech-Wirbel (BSW) model [13]. Given a set of extrapolation procedures, the $D \to K^{(*)}$ form factors are first extrapolated from $q^2 = 0$ to maximum $q^2_D$, and then related to the $B \to K^{(*)}$ form factors at zero recoil via Eq.(6). Using $m_b = 5$ GeV and $m_c = 1.5$ GeV, the calculated $B \to K^{(*)}$ form factors at $q^2 = 0$ and $m_\psi^2$ are summarized in Table I. For a comparison, we have included in Table I two other model predictions: (1) the BSW model (BSWI) [13] in which the $B \to K^{(*)}$ form factors are first calculated at $q^2 = 0$ and then extrapolated to finite $q^2$ using a monopole behavior for all the form factors, and (2) the modified BSW model (BSWII) [5], which is the same as BSWI except for a dipole $q^2$ dependence for form factors $F_1$, $A_0$, $A_2$, $V$, inspired by the $q^2$ behavior for heavy-heavy meson transitions [5].

Table I. $B \to K^{(*)}$ form factors evaluated at $q^2 = 0$ and $m_\psi^2$ in various form-factor models.

|        | (i)       | (ii)      | (iii)     | BSWI   | BSWII    |
|--------|-----------|-----------|-----------|--------|----------|
| $F_1^{BK}(0), F_1^{BK}(m_\psi^2)$ | 0.56 0.84 | 0.56 0.84 | 0.47 0.70 | 0.38 0.56 | 0.38 0.83 |
| $V^{BK^*}(0), V^{BK^*}(m_\psi^2)$ | 0.70 1.04 | 0.33 0.72 | 0.70 1.04 | 0.37 0.55 | 0.37 0.81 |
| $A_1^{BK^*}(0), A_1^{BK^*}(m_\psi^2)$ | 0.55 0.55 | 0.29 0.41 | 0.29 0.41 | 0.33 0.46 | 0.33 0.46 |
| $A_2^{BK^*}(0), A_2^{BK^*}(m_\psi^2)$ | 0.26 0.36 | 0.19 0.36 | 0.31 0.43 | 0.33 0.46 | 0.33 0.64 |

With the $B \to K^{(*)}$ form factors given in Table I, it is straightforward to compute the quantities $R$ and $\Gamma_L/\Gamma$ (see [7,8] for their kinematic expressions). The results are tabulated in Table II. We see that the heavy-flavor-symmetry approach for heavy-light form factors with type (ii) of $q^2$ dependence gives a satisfactory agreement with the CLEO measurement of $R$ and CDF data of $\Gamma_L/\Gamma$. However, if the recent observation of a large fraction ($\geq 70\%$) of the longitudinal polarization in $B \to \psi K^*$ by ARGUS [9] and CLEO [1] is confirmed in the future, then case (ii) will be ruled out either. Note that the factorization hypothesis

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5 The unusual $q^2$ behaviors for the form factors $A_1$ and $A_2$ obtained in existing QCD-sum-rule calculations [16] are no longer seen in a recent similar work [18].
leads to a theoretical upper bound 0.83 for $\Gamma_L/\Gamma$ (see e.g. Ref.[7]). This means that if the measured $\Gamma_L/\Gamma$ is larger than 0.83, one can conclude that the factorization approach fails irrespective of the choice of form factors. Hence, a refined measurement of the fraction of longitudinal polarization in $B \to \psi K^*$ is urgently called for.

Table II. Predictions for $R$ and $\Gamma_L/\Gamma$ in various form-factor models.

|       | (i) | (ii) | (iii) | BSWI   | BSWII   | Experiment       |
|-------|-----|------|-------|--------|--------|-----------------|
| $R$   | 4.00| 1.84 | 2.83  | 4.23   | 1.61   | 1.71 ± 0.34 [1] |
| $\Gamma_L/\Gamma$ | 0.61 | 0.56 | 0.41  | 0.57   | 0.35   | 0.97 ± 0.22 ARGUS [9] |
|       |     |      |       |        |        | 0.80 ± 0.09 CLEO [1] |
|       |     |      |       |        |        | 0.66±0.13 CDF [10] |

Before proceeding further, we would like to give a brief summary on how many parameters are being fitted in this procedure. To describe $B \to \psi K(K^*)$ decays we need six form factors: $F_{0,1}^{BK}, A_{0,1,2}^{BK^*}$ and $V^{BK^*}$ at $q^2 = m_\psi^2$. They are related to $D \to K^{(*)}$ form factors at the same heavy quark velocity but near zero recoil by model-independent heavy flavor symmetry. Using the experimental input on $D \to K^{(*)}$ form factors at $q^2 = 0$, we are able to determine $B \to K^{(*)}$ form factors at any finite $q^2$ provided that their $q^2$ dependence is known. Since $F_1$ ($A_2$) has one more pole factor than $F_0$ ($A_0$ and $A_1$) and many model calculations indicate that $F_1$ is of monopole form, it follows that only two of the form factors, say $A_2$ and $V$, are needed to fit data to determine their $q^2$ behavior. In short, the $q^2$ extrapolation of $F_1$ is fixed by models, while $A_2$ and $V$ by data.

We next turn to the determination of the parameter $a_2$. Using $V_{cb} = 0.040$ [22], $\tau(B^-) = 1.54 \times 10^{-12}s$ [21], $f_\psi = 395$ MeV extracted from the measured width of $\psi \to e^+e^-$ [21], and assuming factorization, we find

\[
\mathcal{B}(B^- \to \psi K^-) = 2.85 \times 10^{-2} \left| a_2 F_1^{BK}(m_\psi^2) \right|^2 = 1.99 \times 10^{-2} |a_2|^2, \\
\mathcal{B}(B^- \to \psi K^{*-}) = 0.221 \left| a_2 A_1^{BK^*}(m_\psi^2) \right|^2 = 3.67 \times 10^{-2} |a_2|^2, \tag{13}
\]

where we have applied type-(ii) form factors given in Table I. From the CLEO data [1]

\[
\mathcal{B}(B^- \to \psi K^-) = (0.110 \pm 0.017)\%, \quad \mathcal{B}(B^0 \to \psi K^0) = (0.075 \pm 0.025)\%, \\
\mathcal{B}(B^- \to \psi K^{*-}) = (0.178 \pm 0.056)\%, \quad \mathcal{B}(B^0 \to \psi K^{*-}) = (0.169 \pm 0.036)\%, \tag{14}
\]

the respective $|a_2|$ is found to be $0.235 \pm 0.018, 0.192 \pm 0.032, 0.220 \pm 0.035, 0.212 \pm 0.023$, where use of $\tau(B^0) = 1.50 \times 10^{-12}s$ [21] has been made. Therefore, the combined value is

\[
|a_2(B \to \psi K^{(*)})| = 0.221 \pm 0.012. \tag{15}
\]
3. Moral from $B \to K^*\gamma$ decays. We see in the previous section that the approach of heavy flavor symmetry for heavy-light form factors with type (ii) of $q^2$ dependence is favored by the $B \to \psi K^{(*)}$ data. We shall see in this section that this method for heavy-light form factors also gives an excellent description of $B \to K^*\gamma$ decay, from which useful information on the form factors $V(0)$ and $A_1(0)$ can be extracted.

In the standard model, the weak radiative decay $B \to K^*\gamma$ is dominated by the short-distance penguin transition $b \to s\gamma$. The transition amplitude for $b \to s\gamma$ reads [23]

$$A(b \to s\gamma) = i\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}F_2(x_t)VtbV_{ts}^*\varepsilon^\mu k^\nu\bar{s}\sigma_{\mu\nu}[m_b(1 + \gamma_5) + m_s(1 - \gamma_5)]b,$$

(16)

where $F_2$ is a smooth function of $x_t \equiv m_t^2/M_W^2$ [23]; $F_2 \simeq 0.65$ for $m_t = 174$ GeV and $\Lambda_{QCD} = 200$ MeV. In the static limit of the heavy $b$ quark, we may use the equation of motion of $\gamma_0b = b$ to derive the relation [12]

$$\langle K^*|\bar{s}\sigma_{0i}(1 \pm \gamma_5)b|B\rangle = \langle K^*|\bar{s}\gamma_i(1 \mp \gamma_5)b|B\rangle.$$  

(17)

As a result, the form factors $f_1$ and $f_2$ in the matrix elements of the tensor current can be related to the form factors $A_1$ and $V$ at the same $q^2$. At $q^2 = 0$ we have (see e.g. Ref.[24])

$$f_1^{BK^*}(0) = -2f_2^{BK^*}(0) = -\left(\frac{m_B - m_{K^*}}{m_B}V^{BK^*}(0) + \frac{m_B + m_{K^*}}{m_B}A_1^{BK^*}(0)\right).$$

(18)

The decay rate is then given by

$$\Gamma(B \to K^*\gamma) = \frac{1}{32\pi}\left(\frac{m_B^2 - m_{K^*}^2}{m_B}\right)^3\left|\frac{G_F e}{\sqrt{2}8\pi^2}F_2VtbV_{ts}^*m_b\right|^2 \left(\left|f_1^{BK^*}(0)\right|^2 + 4\left|f_2^{BK^*}(0)\right|^2\right).$$

(19)

Hence, to the leading order in $1/m_b$ expansion

$$\mathcal{B}(B \to K^*\gamma) = 1.32 \times 10^{-4}\left|A_1^{BK^*}(0)\right|^2\left(1 + 0.711 V^{BK^*}(0)/A_1^{BK^*}(0)\right)^2 + \mathcal{O}(1/m_b^2),$$

(20)

where use has been made of the relation $V_{tb}V_{ts}^* \simeq -V_{cb}V_{cs}^*$.

The prediction for the branching ratio of $B \to K^*\gamma$ in the heavy-flavor-symmetry approach for form factors with various types of $q^2$ extrapolation is exhibited in Table III. The agreement between case (ii) and the CLEO experiment [25] is excellent. The measured branching ratio for $B \to K^*\gamma$ together with its theoretical prediction (20) thus provides useful constraints on the form factors $V^{BK^*}(0)$ and $A_1^{BK^*}(0)$.

Table III. Predictions for the branching ratio of $B \to K^*\gamma$ in various form-factor models.

|                      | (i)    | (ii) | (iii) | Experiment [25] |
|----------------------|--------|------|-------|-----------------|
| $\mathcal{B}(B \to K^*\gamma)$ | $1.5 \times 10^{-4}$ | $3.6 \times 10^{-5}$ | $8.5 \times 10^{-5}$ | $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ |
4. Analyses of $B \to D(D^*)\pi(\rho)$ decays We learn from previous two sections that $B \to \psi K^{(*)}$ and $B \to K^*\gamma$ decays are satisfactorily described by the heavy-flavor-symmetry approach for heavy-light form factors provided that $F_0$ behaves as a constant, $F_1$, $A_0$, $A_1$ have a monopole behavior, and the $q^2$ dependence of $A_2$ and $V$ is of the dipole form. In this section we will apply this method to $B \to D^{(*)}\pi(\rho)$ decays to extract $a_1$ and $a_2$. For recent analyses of $B \to D^{(*)}\pi(\rho)$ decays, see Ref.[26].

As stressed in passing, there exist two complications for evaluating the $\bar{B}^0 \to D^{(*)}\pi^-(\rho^-)$ decay amplitudes. First, the $W$-exchange diagram contributes to $\bar{B}^0$ decay, but it is difficult to estimate its size. Second, there are final-state interactions (FSI) since the decay amplitudes involve isospin 1/2 and 3/2 channels. In what follows we will first analyze the data without considering these two effects, and then come back to them later on. Based on factorization, we obtain

$$
\mathcal{B}(\bar{B}^0 \to D^+\pi^-) = 0.80 \times 10^{-2} \left| a_1 F_0^{BD}(m^2_\pi) \right|^2,
$$

$$
\mathcal{B}(\bar{B}^0 \to D^+\rho^-) = 1.93 \times 10^{-2} \left| a_1 F_1^{BD}(m^2_\rho) \right|^2,
$$

$$
\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-) = 0.75 \times 10^{-2} \left| a_1 A_0^{BD^*}(m^2_\pi) \right|^2,
$$

$$
\mathcal{B}(\bar{B}^0 \to D^{*+}\rho^-) = 0.11 \times 10^{-2} \left| a_1 A_1^{BD^*}(m^2_\rho) \right|^2 H,
$$

and

$$
R_1 \equiv \frac{\mathcal{B}(B^- \to D^0\pi^-)}{\mathcal{B}(\bar{B}^0 \to D^+\pi^-)} = \frac{\tau(B^-)}{\tau(B^0)} \left( 1 + \frac{m^2_B - m^2_\pi f_D F_0^{B\pi}(m^2_D) a_2}{m^2_B - m^2_\rho f_\rho F_0^{BD}(m^2_\rho) a_1} \right)^2,
$$

$$
R_2 \equiv \frac{\mathcal{B}(B^- \to D^0\rho^-)}{\mathcal{B}(\bar{B}^0 \to D^+\rho^-)} = \frac{\tau(B^-)}{\tau(B^0)} \left( 1 + \frac{f_D A_0^{B\rho}(m^2_D) a_2}{f_\rho A_0^{BD}(m^2_\rho) a_1} \right)^2,
$$

$$
R_3 \equiv \frac{\mathcal{B}(B^- \to D^{*0}\pi^-)}{\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-)} = \frac{\tau(B^-)}{\tau(B^0)} \left( 1 + \frac{f_{D^*} F_1^{B\pi}(m^2_{D^*}) a_2}{f_\rho A_1^{BD^*}(m^2_\rho) a_1} \right)^2,
$$

$$
R_4 \equiv \frac{\mathcal{B}(B^- \to D^{*0}\rho^-)}{\mathcal{B}(\bar{B}^0 \to D^{*+}\rho^-)} = \frac{\tau(B^-)}{\tau(B^0)} \left( 1 + 2\eta \frac{H_1}{H} + \eta^2 \frac{H_2}{H} \right),
$$

with

$$
\eta = \frac{m_{D^*}(m_B + m_\rho)}{m_\rho(m_B + m_{D^*})} \frac{f_{D^*} A_1^{B\rho}(m^2_{D^*}) a_2}{f_\rho A_1^{BD^*}(m^2_\rho) a_1},
$$

$$
H = (a - bx)^2 + 2(1 + c^2 y^2),
$$

$$
H_1 = (a - bx)(a - b'x') + 2(1 + cc'yy'),
$$

$$
H_2 = (a - b'x')^2 + 2(1 + c^2 y'^2),
$$

and

$$
a = \frac{m_B^2 - m_{D^*}^2 - m_\rho^2}{2m_{D^*}m_\rho}, \quad b = \frac{2m_{BPc}^2}{m_{D^*}m_\rho(m_B + m_{D^*})^2}, \quad c = \frac{2m_{BPc}}{(m_B + m_{D^*})^2},
$$

9
\[
x = \frac{A_2^{BD^*}(m_{p_{c}}^2)}{A_1^{BD^*}(m_{p_{c}}^2)}, \quad y = \frac{V^{BD^*}(m_{p_{c}}^2)}{A_1^{BD^*}(m_{p_{c}}^2)},
\]

where \( p_{c} \) is the c.m. momentum, and \( b', c, x', y' \) are obtained from \( b, c, x, y \) respectively with the replacement \( D^* \leftrightarrow \rho \); for instance, \( x' = A_2^{B\rho}(m_{D^*}^2)/A_1^{B\rho}(m_{D^*}^2) \).

The heavy-heavy form factors e.g. \( F_0^{BD}, A_1^{BD^*} \) appearing in (21-22) can be related to a universal Isgur-Wise function \( \xi(v \cdot v') \) via heavy quark symmetry [5]. We will use the \( B \rightarrow D^{(*)} \) form factors evaluated in Ref.[5], which include \( 1/m_Q \) corrections. As for the heavy-light form factor \( F_1^{B\pi} \), we get \( F_1^{B\pi}_{0,1}(0) = 0.48 \) using \( F_0^{D\pi}_{0,1}(0) \approx 0.83 \) derived from \( D^+ \rightarrow \pi^+\pi^0 \) decay [27]. In the absence of experimental input for \( D \rightarrow \rho \) form factors, we will assume SU(3) flavor symmetry for \( D \rightarrow B \), or from \( B \) [21]. A fit to the measured branching ratios of \( \bar{B} \rightarrow D^{(*)} \pi \) determines the parameter \( a_1 \), while the ratio \( a_2/a_1 \) listed in Table IV is extracted either from the measured ratios \( R_1, \ldots, R_4 \) or from \( B^- \rightarrow D^{(*)} \pi(\rho) \) decays together with the \( a_1 \) obtained in the corresponding \( B^0 \) decays.

The combined value of \( a_1 \) obtained from Table IV is
\[
a_1(B \rightarrow D^{(*)} \pi(\rho)) = 1.012 \pm 0.057 .
\]

We see that our values of \( a_2/a_1 \) are improved substantially over the previous results obtained using the BSWII model [see Eq.(3)]. We also note that the magnitude of \( a_2/a_1 \) determined in this manner is in general quite stable except for \( B^- \rightarrow D^{0}\rho^- \) or \( R_4 \). Therefore, a refined measurement of this decay mode is greatly welcome. Excluding the data from \( B^- \rightarrow D^{*0}\rho^- \), the combined value of \( a_2/a_1 \) extracted from the remaining \( B^- \) decay modes is found to be
\[
a_2/a_1(B \rightarrow D^{(*)} \pi(\rho)) = 0.224 \pm 0.058 .
\]

Note that the corresponding combined value of \( a_2/a_1 \) obtained in the BSWII model [see Eq.(3)] is \( 0.33 \pm 0.08 \). Combining (26) with (25) leads to
\[
a_2(B \rightarrow D^{(*)} \pi(\rho)) = 0.226 \pm 0.060 .
\]

\footnote{\textsuperscript{6}The prediction \( F_0^{D\pi}_{0,1}(0) = 0.48 \) is close to the value of 0.53 obtained in the framework of chiral perturbation theory that incorporates chiral and heavy quark symmetries [19]. Note that \( F_1^{D\pi}(0) > F_1^{DK}(0) \), whereas \( F_0^{D\pi}(0) < F_0^{DK}(0) \).}

\footnote{\textsuperscript{7}We have checked explicitly that, assuming \( A_2^{D\rho}(0) = A_2^{DK^+}(0) \) and \( V^{D\rho}(0) = V^{DK^+}(0) \), SU(3) flavor symmetry for \( B \rightarrow \rho \) and \( B \rightarrow K^+ \) form factors at finite \( q^2 \) obtained using Eq.(6) with type-(ii) \( q^2 \) dependence is better than 10\%.}
The main theoretical uncertainty comes from the form factors $A_{1,2}^{B\rho}$ and $V^{B\rho}$, for which we lack experimental input on $D \to \rho$ form factors. At this point, we wish to emphasize again that, in contrast to Ref.[1], our value for $a_2/a_1$ is not obtained by a least squares fit to the data. Recall that a global fit of the BSWII model to the data yields $a_2/a_1 = 0.23 \pm 0.11$ [1]. Our result (27) thus improves the previous error analysis by a factor of two. We also remark that the fraction of longitudinal polarization in $B^0 \to D^{*+}\rho^-$ is measured to be $0.93 \pm 0.05 \pm 0.05$ [1]. Unlike the $B \to \psi K^*$ case, this relative amount of longitudinal polarization is easily accounted for by theory, which is predicted to be 88%.

Table IV. Extraction of $a_1$ and $a_2/a_1$ from various $B \to D(D^*)\pi(\rho)$ decays by comparing the theoretical prediction for branching ratios with experiment.

|          | $\mathcal{B}(\%)_{\text{theory}}$ | $\mathcal{B}(\%)_{\text{expt}}$ [1] | $a_1$       | $a_2/a_1$ |
|----------|----------------------------------|----------------------------------|------------|-----------|
| $\bar{B}^0 \to D^+\pi^-$ | $0.270a_1^2$ | $0.29 \pm 0.07$ | $1.04 \pm 0.13$ | -         |
| $\bar{B}^0 \to D^+\rho^-$ | $0.673a_1^2$ | $0.81 \pm 0.21$ | $1.10 \pm 0.14$ | -         |
| $\bar{B}^0 \to D^{*+}\pi^-$ | $0.260a_1^2$ | $0.26 \pm 0.05$ | $1.00 \pm 0.10$ | -         |
| $\bar{B}^0 \to D^{*+}\rho^-$ | $0.806a_1^2$ | $0.74 \pm 0.17$ | $0.96 \pm 0.11$ | -         |
| $B^- \to D^0\pi^-$ | $0.277a_1^2(1 + 1.497a_2/a_1)^2$ | $0.55 \pm 0.07$ | $1.04 \pm 0.13$ | $0.24 \pm 0.10$ |
| $B^- \to D^0\rho^-$ | $0.690a_1^2(1 + 1.131a_2/a_1)^2$ | $1.35 \pm 0.19$ | $1.10 \pm 0.14$ | $0.24 \pm 0.14$ |
| $B^- \to D^{*0}\pi^-$ | $0.267a_1^2(1 + 1.918a_2/a_1)^2$ | $0.52 \pm 0.10$ | $1.00 \pm 0.10$ | $0.21 \pm 0.08$ |
| $B^- \to D^{*0}\rho^-$ | $0.827a_1^2(1 + 1.446a_2/a_1)^2$ | $1.68 \pm 0.35$ | $0.96 \pm 0.11$ | $0.34 \pm 0.13$ |

Thus far we have neglected FSI and nonspectator effects. Experimentally, FSI can be tested by measuring the decay rates of $B^- \to D^{(*)0}\pi^- (\rho^-)$, $B^0 \to D^{(*)+}\pi^-(\rho^-)$, $D^{(*)0}(\rho^0)$ to deduce the isospin amplitudes $A_{1/2}$, $A_{3/2}$ and the phase-shift difference ($\delta_{1/2} - \delta_{3/2}$). Moreover, in the absence of $W$ exchange, $a_2/a_1$ can be determined from $A_{3/2}/A_{1/2}$; for example, in $B \to D\pi$ decay [30]

$$\frac{m_B^2 - m_\pi^2}{m_B^2 - m_D^2} \frac{f_D}{f_\pi} \frac{F_0^{B\pi}(m_D^2)}{F_0^{BD}(m_\pi^2)} \frac{a_2}{a_1} = 1.497 \frac{a_2}{a_1} = 2 \frac{A_{3/2}/A_{1/2} - \frac{1}{\sqrt{2}}}{A_{3/2}/A_{1/2} + \sqrt{2}}.$$ (28)

Unfortunately, an observation of $\bar{B}^0 \to D^0\pi^0$ is still not available yet.

The presence of $W$ exchange will affect the determination of $a_1$ from $\bar{B}^0$ decays. Theoretically, the $W$-exchange amplitude receives its main contribution from nonperturbative color octet currents [4], which is difficult to estimate. Though both nonspectator and FSI effects are known to be important in charm decays, it is generally believed that they do not play a significant role in bottom decay as the decay particles are moving fast, not allowing adequate time for FSI.
5. Discussion and conclusion  

We see from (15) and (27) that the magnitude of $a_2$ determined from $B \to D^{(*)} \pi(\rho)$ decays agrees well with that extracted from $B \to \psi K^{(*)}$. Then, can we conclude that $a_2$ extracted in the latter decay is positive? Recall that, as we have argued before, nonperturbative effects must be in such a way that $|r_2(B \to \psi K^{(*)})| \lesssim |r_2(B \to D^{(*)}\pi)|$ [4]. Since $a_2(B \to D^{(*)}\pi) = (c_2 + c_1/N_c) + \frac{1}{8} m_b^2 c_1$ is positive and $c_2 + c_1/N_c = 0.15 \sim 0.20$ at $\mu = m_b$ beyond the leading logarithmic approximation [31], it is clear that $r_2(B \to D^{(*)}\pi) = 0.05 \sim 0.14$ is positive. Now there are two possibilities for $a_2$ in $B \to \psi K^{(*)}$: (i) $r_2 > 0$; this implies a positive $a_2$ and that $a_2(B \to \psi K^{(*)}) \simeq a_2(B \to D^{(*)}\pi)$, and (ii) $r_2 < 0$; this together with (15) indicates a negative $a_2$ and $r_2(B \to \psi K^{(*)}) = -(0.72 \sim 0.80)$. We consider the case (ii) very unlikely since the magnitude of $r_2$ in $B \to \psi K^{(*)}$ should not deviate too much from that in $B \to D^{(*)}\pi$. Therefore, contrary to the previous publication [4], we believe that $a_2(B \to \psi K^{(*)})$ ought to be positive. In fact, it is not difficult to achieve the relation $a_2(B \to \psi K^{(*)}) \simeq a_2(B \to D^{(*)}\pi)$. Note that our extraction of $a_{1,2}$ from $B \to D^{(*)}\pi(\rho)$ so far is based on the assumption that FSI and $W$ exchange are negligible. It is likely that the inclusion of these two effects will reduce the present estimate of $a_2(B \to D^{(*)}\pi(\rho))$. In view of this, a measurement of $B^0 \to D^{(*)0}\pi^0(\rho^0)$ is urgently needed. Finally, the question of why $r_2$ is positive in exclusive $B$ decays whereas it is negative in $D$ decays remains an enigma. This will be a great challenge to both lattice and QCD-sum-rule practitioners. Nevertheless, the small magnitude of $r_2$ in $B \to D^{(*)}\pi(\rho)$ compared to that in exclusive two-body $D$ decays ($|r_2| = (0.67 \sim 1.3)$) is consistent with our expectation that nonfactorizable soft-gluon effects are much less significant in the former. This also explains why the $1/N_c$ approach, which is empirically known to be operative in charm decays, fails in $B \to D^{(*)}\pi(\rho)$ and $B \to \psi K^{(*)}$ decays.

To conclude, we have shown that, based on factorization, the heavy-flavor-symmetry approach for heavy-light form factors in conjunction with the type-(ii) $q^2$ dependence provides a satisfactory description of the CLEO data on $B(B \to \psi K^{(*)})/B(B \to \psi K)$ and the CDF data on the fraction of the longitudinal polarization in $B \to \psi K^{(*)}$ decays. However, if the measured $\Gamma_L/\Gamma$ is larger than 0.70, then our form factors are also ruled out. Furthermore, we will conclude that the factorization approach fails irrespective the choice of form factors if the relative amount of the transverse polarization is found to be less than 17%. Therefore, a refined measurement of $\Gamma_L/\Gamma$ in $B \to \psi K^{(*)}$ is urgently needed in order to test form-factor models and the factorization hypothesis. Armed with the above method, we have extracted the parameters $a_1$ and $a_2$ from $B \to \psi K^{(*)}$ and $B \to D^{(*)}\pi(\rho)$ decays. Our result $a_2/a_1 = 0.22 \pm 0.06$ improves the previous error analysis by a factor of two. Finally, we have argued that the sign of $a_2(B \to \psi K^{(*)})$ should be positive and we have discussed its

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8To the leading logarithmic approximation, $c_1(m_b) \sim 1.11$, $c_2(m_b) \sim -0.26$ and hence $c_2 + c_1/N_c \sim 0.11$. 

12
important implications.
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