Parametric diagnosis of the adaptive gas path in the automatic control system of the aircraft engine

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Abstract. The paper dwells on the adaptive multimode mathematical model of the gas-turbine aircraft engine (GTE) embedded in the automatic control system (ACS). The mathematical model is based on the throttle performances, and is characterized by high accuracy of engine parameters identification in stationary and dynamic modes. The proposed on-board engine model is the state space linearized low-level simulation. The engine health is identified by the influence of the coefficient matrix. The influence coefficient is determined by the GTE high-level mathematical model based on measurements of gas-dynamic parameters. In the automatic control algorithm, the sum of squares of the deviation between the parameters of the mathematical model and real GTE is minimized. The proposed mathematical model is effectively used for gas path defects detecting in on-line GTE health monitoring. The accuracy of the on-board mathematical model embedded in ACS determines the quality of adaptive control and reliability of the engine. To improve the accuracy of identification solutions and sustainability provision, the numerical method of Monte Carlo was used. The parametric diagnostic algorithm based on the LPr - sequence was developed and tested. Analysis of the results suggests that the application of the developed algorithms allows achieving higher identification accuracy and reliability than similar models used in practice.

1. Introduction

An aircraft gas-turbine engine (GTE) is a complex dynamic system. Its equations of motion are defined by the regular component of the non-stationary process of parameters changes and the random variations of characteristics caused by external and internal interference. The effect of noise is created by a wide range of destabilizing factors, producing additional errors and reducing the engine life.

Engine reliability and flight safety are affected by the quality of automatic control systems (ACS) of GTE, which main task is the automatic control of dynamic modes in interference conditions. The extension of the ACS functionality is the only way of fulfilling the reliability and quality requirements. This extension can be provided by an increase of the ACS adaptation ability. The main problem of optimal in-flight automatic control is to obtain the real-time reliable data on the current engine health. So, the adaptive GTE health monitoring and fault diagnosis requires the use of identification methods.

In modern digital ACS GTE, the reliability growth is achieved by the creation of algorithmic information redundancy based on the built-in mathematical model of the aircraft engine (GTE BMM) [1]. The structure and the accuracy of the BMM determine the quality of the in-flight identification.

As the interference (noise) effect exists in the channel of BMM and measurement sensor channel, an actual task is the improvement of model identification accuracy taking into account the current on-
board measurements. Authors [2, 3] offer to solve this problem by the use of the high-level nonlinear engine model. The disadvantage of this method is inability of in-flight operation.

The paper dwells on the design of the reliable multimode mathematical model of the civil aircraft engine embedded in the electronic controller (ACS), based on the dynamic and throttle engine performances, characterized by high accuracy of engine parameters identification in stationary and dynamic modes.

2. Materials and methods
The proposed on-board linearized adaptive engine model is based on automatic identification of numerical algorithms which include gas path parametric diagnosis. The basic control (immeasurable) parameters are: the fuel flow; gas temperature after the turbine; the outlet pressure of the high pressure compressor (HPC); the rotational speed of the high (HPT), and low (LPT) pressure turbine.

2.1. The On-board linearized GTE model design
The on-board linearized low-level mathematical engine model was designed by the state space method [4, 5]:

\[
\begin{align*}
\delta \dot{X} &= A(x)\delta X + B(x)\delta u \\
\delta Y &= C(x)\delta X + D(x)\delta u \\
\end{align*}
\]

where: \( \delta u = G_T \) – the control signal (fuel flow); \( \delta X \) – the state-space vector with coordinates \( \delta x_1 = \delta n_{LPT} \) (LPT rotational speed deviation); \( \delta x_2 = \delta n_{HPT} \) (HPT rotational speed deviation); \( \delta Y \) – the output vector with coordinates: \( \delta y_1 = \delta P_{HPC} \) (HPC outlet pressure deviation); \( \delta y_2 = \delta T_T \) (gas temperature after the turbine deviation); \( A, B, C, D \) – connection matrices with dimensions \( A = [2 \times 2], B = [2 \times 1], C = [2 \times 2], D = [2 \times 1] \).

The deviations are defined as a relative divergences of parameters from the basic state (static characteristic of the engine). The static characteristic of the engine is defined as the solution of the gas-dynamic nonlinear equations system (the mathematical high-level model), in case of zeroing the left sides of the equations. Since we have one independent control variable (fuel flow), the static characteristic may be represented on a plane in the form of functions of one variable. The coefficients of connection matrix (partial derivatives) are obtained by the high-level nonleaner multimode mathematical model.

The linear structure of the model provides high speed and reliability of operation. The introduction of non-linear coefficients of matrices \( A(x), B(x), C(x), D(x) \) contributes to achieving the required identification accuracy in the wide range of the engine operation mode.

2.2. Gas path parametric GTE diagnostics based on an on-board mathematical model
The on-board mathematical model of the engine is effectively used for on-line monitoring and detecting of gas path defects [6]. Identification of the GTE health is performed by using the method of the general influence coefficient matrix [7]. The method is based on the mathematical model identification by using the measurement of GTE gas dynamic parameters. In the identification algorithm, the sum of squares of the deviation between the parameters of the mathematical model and real GTE parameters is minimized.

The defects arising in GTE lead to a change in the characteristics of the various nodes, which, in turn, affect the controlled gasdynamic parameters [8]. Therefore we can write that

\[
P_j = P_{i_k} + \frac{\partial f_j}{\partial x_1} \delta x_1 + \ldots + \frac{\partial f_j}{\partial x_k} \delta x_k.
\]

Here, \( P_j \) – the measured value of the controlled parameter, \( P_{i_k} \) – the model value of the controlled parameter, \( i \) – the number of the controlled parameter (total - \( m \)), \( j \) – the number of measurement of the supervised parameter (iteration), \( k \) – the number of unmeasured parameters, \( \frac{\partial f_j}{\partial x_k} \) – the influence of coefficients of relative variations of unmeasured (independent) variables \( x_k \) (the cross sectional area,
efficiency, etc.) on the measured (controlled) parameters, formally designated as \( f_i = P_i \).

The influence coefficients can be determined by using the GTE high-level mathematical model. The influence coefficients matrix is obtained experimentally. This matrix shows the influence of unmeasured varying parameters on the measured parameters in different engine operation modes.

To form the system of equations, difference \( S \), obtained from (1), is minimized:

\[
S = \sum_{j=1}^{n} \sum_{i=1}^{m} \left( \delta P_y - \left( \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial f_j}{\partial x_i} \delta x_i \frac{\partial f_j}{\partial x_k} \delta x_k \right) \right)^2,
\]

where \( m \) – the number of monitoring parameters, \( n \) – the number of measurements for each parameter. For finding \( \min S \), the partial derivatives are rated at nil:

\[
\frac{\partial S}{\partial \delta x_i} = \ldots = \frac{\partial S}{\partial \delta x_k} = 0.
\]

The system of equations, obtained from (2) and (3), has the following form:

\[
\sum_{j=1}^{n} \sum_{i=1}^{m} \left[ \frac{\partial f_j}{\partial x_i} \delta x_i \right] \delta P_y = \sum_{j=1}^{m} \sum_{i=1}^{n} \left[ \frac{\partial f_j}{\partial x_k} \delta x_k \right] \delta P_y,
\]

Substituting known influence coefficients \( \left[ \frac{\partial f_j}{\partial x_k} \right] \) and the deviation of the measured parameters of model \( \delta P_y \) into this system, we will find the deviations of unmeasured variables \( \delta x_k \) from the average statistical serviceable engine and determine the cause of deviations (we will identify the faulty engine element).

Grouping the terms in (4), we obtain a system of linear equations with several variables. For each \( j \)-iteration, it can be written in the matrix form as

\[
C_{ij} \delta x_j = a_i,
\]

where \( i = 1, m \), \( m \) – the number of measured parameters; \( l = 1, k \), \( k \) – the number of unmeasured parameters; \( C_{ik} \) – in general the rectangular matrix with coefficients, dependent on influence coefficients (with size \( m \times k \)); \( \delta x_l \) – the column matrix of the deviations of unmeasured parameters (with size \( k \times 1 \)); \( a_i \) – the column matrix with coefficients dependent on measured parameters (with size \( m \times 1 \)).

The values of the coefficients of the matrices in (5) are given by

\[
C_{ii} = \sum_{l=1}^{m} \frac{\partial f_i}{\partial x_j} \frac{\partial f_i}{\partial x_k} \quad a_i = \sum_{l=1}^{m} \frac{\partial f_i}{\partial x_k} \delta P_l.
\]

In practical application of the proposed method, the following problems occur:

- the multiple real-time measurements are required for fault detection;
- the ill-conditioning matrix of the equations system is possible (the determinant is near zero);
- the uncertain systems of equations are possible (the number of equations is less than the number of variables).

3. Results and Discussion

In this study, the controlled (measured) engine parameters \((f_1 \ldots f_m)\) are: the rotational speeds \( n_i = n_{iPR}, n_j = n_{iHPR}, \) outlet HPC pressure \( P_{C*}, \) and gas temperature after the turbine \( T_{*T}. \) The uncontrolled engine parameters, determined by the deviations of the measured parameters, are: efficiency \( \eta \) of a fan \((x_j), \) \( \eta \) LPC \((x_2), \) \( \eta \) HPC \((x_3), \) \( \eta \) HPT \((x_4), \) \( \eta \) booster \((x_5), \) leakage due to HPC \((x_6), \) air bleed of the third
stage for aircraft needs \((x_7)\) and bypassing after LPC \((x_8)\). Influence coefficients (partial derivatives) \(\frac{\partial f_i}{\partial x_t}\) are taken from the practical experimental matrix of influence coefficients obtained for different engine operation modes. For transitions between the engine operation modes, the experimental graphs of functions of the measured variables are used. The solution is based on the interpolation of these graphs. Substituting experimental influence coefficients into formula (6), we get \(C_{ii}\) and \(a_{ii}\). The resulting system is uncertain. In general, there are infinitely many solutions to this system.

The practiced method of parametric diagnosis is based on the reduction of uncertain and ill-conditioned systems of equations to the certain ones by selecting the optimal set of unmeasured parameters, the number of which is selected equal to the number of measured parameters.

The calculation algorithm is based on the choice of the set of basic (core) unmeasured variables. Then other unmeasured variables are treated as free (non-core) and zeroing. The main determinant of the system becomes a square. The solution in which all the main (basic) variables are unequal to zero is basic. If at least one basic variable is zero, this solution is called degenerative.

In our task, the basic solution system has the same number of measured and unmeasured parameters (equal to 4), so all the rest \((8 - 4 = 4)\) variables (non-core) are zeroing. If the main determinant is equal to zero (the matrix is singular), the system has no solutions (degenerative).

Let us take a variety of groups of four unmeasured variables as the basis. The result is a set of groups, including four unmeasured variables \(\delta x\). The maximum number of groups is defined as the number of combinations \(C_4^8 = \frac{8!}{4!(8-4)!} = 70\). Thus, the number of solved equations systems is not more than 70. Since some main determinants of equations systems for obtained sets of basis unmeasured variables are equal to zero, the basic solutions for such systems do not exist. In the discussed practical task, we have about 10 such groups. The obtained non-degenerate systems of equations were solved by the Gauss method. The total time of solutions in the modelling MatLab environment was 15 minutes, which does not satisfy practical requirements to ACS speed.

Another way is the use of optimization methods. The choice of the optimization method must be based on the assumption that the studied objective function of eight variables cannot be unimodal, and the considered space of solutions cannot have the required gradient for finding the unique solution. So, the use of non-linear programming methods (Powell method, conjugate gradient method, etc.) for these tasks is unreasonable. In this context, it was proposed to use the Monte Carlo methods - numerical methods for solving the mathematical problems based on simulation of random variables [9].

To find the optimal, the space of sought unmeasured parameters is probed, which boundaries are defined. The method is based on systematic viewing of multi-dimensional areas. The points of uniformly distributed sequences are used as the sampling points in the space of variables. For this purpose, the LP-\(\tau\) pseudo-random number sequences are applied. These sequences possess asymptotically the best characteristics of uniformity among all the presently known uniformly distributed sequences [10].

To solve the optimization problem, the possible changes of unmeasured parameters (the delivery range of \(\delta x\)) should be limits. Further, the extremum (minimum) of the selected criterion is determined. A criterion is defined by an objective function of eight variables.

For optimization, the C++ program was developed, which provides a possibility of modifying: the search space of solutions (desired range of parameters); the type of used optimization criteria; the confidence interval; the type of the objective function to be minimized; the number of iterations (sampling points) for probing the search space.

As optimization criteria the following was selected:

1) \(Q\) – Euclidean norm of the solution vector of the system of eight equations in the sampling point of the search space, which has eight coordinates \((\delta x_1 ... \delta x_8)\):

\[
\min Q = \sqrt{\delta x_1^2 + \delta x_2^2 + \delta x_3^2 + \cdots + \delta x_8^2}
\]  \(\text{(7)}\)
2) \( \min F_1 \) - the sum of the absolute differences between the values of the right and left parts of the eight equations in the sampling point of the search space with coordinates (\( \delta x_1, \ldots, \delta x_8 \));

3) \( \min F_2 \) - Euclidean norm of deviations between the values of the right and left parts of the eight equations in the sampling point of the search space with coordinates (\( \delta x_1, \ldots, \delta x_8 \)).

It should be noted that in this study, the Hölder norm was used in addition to the Euclidean norm, but it has not given positive results.

The minimized objective function is integral criterion \( I \), formed by either the additive or multiplicative law by using \( F \) and \( Q \), defined by

\[
I = d_1 \cdot F + d_2 \cdot Q \quad \text{or} \quad I = F \cdot Q. 
\]  

(8)

The practical implementation of this method has been made for the civil aircraft engine in the ground operating mode: flight altitude \( H = 0 \), Mach number \( M = 0 \), atmospheric temperature \( t = 15 \) °C, fuel flow \( G_t = 3927 \text{ kg/h} = \text{const} \), the initial parameters of \( n_1 = 3778 \text{ rpm} \), \( n_3 = 15390 \text{ rpm} \), \( P^*c = 35.67 \text{ kgf/cm}^2 \), and \( T^*_t = 789^\circ K \). The given values of the measured parameters are deviations of: LPT rotational speed \( f_1 = \delta n_1 = -1.468\% \), HPT rotational speed \( f_2 = \delta n_3 = -0.42\% \), outlet compressor HPC pressure \( f_3 = \delta P^*c = -3.69\% \), temperature after the turbine \( f_4 = \delta T^*_t = +3.92\% \).

The numerical values of the influence coefficient matrix are shown in Table 1. Coefficients \( C_{il} \) and \( a_{il} \) are shown in Table 2. The main determinant of the system is \( \Delta = 3.06475 \times 10^{-46} \rightarrow 0 \), so we have the ill-conditioning main matrix.

| \( l \) | \( X_i \) | \( f_1 = \delta n_1 \) | \( f_2 = \delta n_3 \) | \( f_3 = \delta P^*c \) | \( f_4 = \delta T^*_t \) |
|---|---|---|---|---|---|
| 1 | \( \delta \eta_{\text{fan}} \) | -0.266 | +0.087 | -0.179 | +1.3 |
| 2 | \( \delta \eta_{\text{LPC}} \) | -0.065 | +0.077 | -0.072 | +1.3 |
| 3 | \( \delta \eta_{\text{HPC}} \) | -0.160 | -0.459 | -0.584 | +5.3 |
| 4 | \( \delta \eta_{\text{HPT}} \) | -0.210 | -0.565 | -0.955 | +7.1 |
| 5 | \( \delta \eta_{\text{booster}} \) | -0.344 | +0.110 | -0.232 | +5.3 |
| 6 | Air leakage (from HPC) | -0.399 | -0.266 | -1.388 | +6.7 |
| 7 | Air bleed for aircraft needs | -0.229 | +0.142 | -1.028 | +4.9 |
| 8 | Air redirection (after LPC) | -0.058 | +0.159 | -0.385 | +2.5 |

| \( C_{il} \) | \( l=1 \) | \( l=2 \) | \( l=3 \) | \( l=4 \) | \( l=5 \) | \( l=6 \) | \( l=7 \) | \( l=8 \) |
|---|---|---|---|---|---|---|---|---|
| \( i=1 \) | 0.1375 | 0.0641 | 0.2180 | 0.3262 | 0.2535 | 0.4715 | 0.3597 | 0.1505 | 1.66 |
| \( i=2 \) | 0.0641 | 0.0426 | 0.1280 | 0.1874 | 0.1584 | 0.2455 | 0.2023 | 0.0960 | 0.98 |
| \( i=3 \) | 0.2180 | 0.1280 | 1.0289 | 1.4555 | 0.5916 | 1.5671 | 0.9891 | 0.3742 | 5.22 |
| \( i=4 \) | 0.3261 | 0.1874 | 1.4555 | 2.0854 | 0.8365 | 2.3237 | 1.5085 | 0.5753 | 7.60 |
| \( i=5 \) | 0.2534 | 0.1584 | 0.5916 | 0.8365 | 0.6358 | 1.0005 | 0.7502 | 0.3398 | 3.95 |
| \( i=6 \) | 0.4715 | 0.2455 | 1.5670 | 2.3237 | 1.0005 | 2.8773 | 2.0077 | 0.7844 | 9.15 |
| \( i=7 \) | 0.3597 | 0.2023 | 0.9891 | 1.5085 | 0.7502 | 2.0077 | 1.5150 | 0.6285 | 6.50 |
| \( i=8 \) | 0.1504 | 0.0960 | 0.3742 | 0.5753 | 0.3398 | 0.7844 | 0.6285 | 0.2773 | 2.68 |

The right solution is known from practice (in %): \( \delta x_1 = 0, \delta x_2 = 0, \delta x_3 = -2, \delta x_4 = 0, \delta x_5 = -2, \delta x_6 = 0, \delta x_7 = +2, \delta x_8 = 0. \)

The best results of numerical solution of equations system (5) under different objective functions are summarized in Table 3. The best optimisation parameters: the number of iterations in the first run (rough approximation) is \( N_1 = 5000 \); the number of iterations in the second run (exact solution) is \( N_2 = 1000 \); the coefficients of objective function \( I \) in formula (8) during the first run: \( d_1 = 0.3, d_2 = 0.7 \), during the second run: \( d_1 = d_2 = 0.5 \); the confidence interval is ±0.5; the search space of solutions is ±10%. The standard deviation of the known right solutions \( \sigma \) was used as the criteria of the best
solutions.

Table 3. The best results of numerical solution (in %)

| Criterion | $\delta_1$ | $\delta_2$ | $\delta_3$ | $\delta_4$ | $\delta_5$ | $\delta_6$ | $\delta_7$ | $\delta_8$ | $\sigma$ |
|-----------|------------|------------|------------|------------|------------|------------|------------|------------|---------|
| $I = d_1 \cdot F_1 + d_2 \cdot Q$ | -0.338 | -0.1 | -1.593 | -0.272 | -1.867 | 0.457 | 0.806 | 1.280 | 0.678 |
| $I = d_3 \cdot F_2 + d_4 \cdot Q$ | -0.370 | -0.283 | -0.984 | -0.722 | -1.086 | 0.388 | 1.599 | 0.428 | 0.623 |
| $I = F_2 \cdot Q$ | -1.042 | 0.639 | -2.047 | -0.143 | -2.211 | 0.266 | 0.691 | 1.096 | 0.754 |

According to the performed experiments, the selection of criterion influences the prediction error of the engine health. We can distinguish three groups of homogeneous parameters from the eight sought parameters defining the engine health: the efficiency of the low-pressure rotor (fan, low-pressure compressor, low pressure turbine), the efficiency of the high-pressure rotor (high-pressure high-pressure compressor and turbine) and air bleed from the engine path (leakage, redirection and bleed for aircraft needs). Depending on the type of applicable criteria, we obtain a different distribution of the deviations within groups. Comparing rows 1 and 2 in table 3, with standard deviations close to a minimum, shows that in the first case, we had a better group prognosis of the distribution efficiency of the rotor components, and in the second case, the group prognosis of distribution of the air bleed was better. For the final selection of a preferred criterion (statistically adequate), the evaluation of the ensemble of possible deviations is necessary.

4. Conclusion
The analysis of the results showed the operability of this approach for engine health estimation on the limited set of parameters (number of measured parameters is less than the number of health parameters - state variables). Furthermore, it would be expected that the expansion of the state space of the engine model (n) with a limited amount of measured parameters (m), i.e. $n > m$, increases the stability of the solution due to some numerical errors of estimation under real conditions of the noisy signal in the measurement channel. The stability of solutions is an important quality of the automatic control systems with the built-in mathematical model of the object. So, the proposed technology makes it possible to build robust dynamic models of control objects in real time. The application of the developed algorithms allows to achieve the higher identification accuracy and reliability than similar models used in practice.

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