Orbits around the dwarf planet Haumea

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Abstract.

The dwarf planet Haumea is a very interesting celestial body due to the characteristics of its physical form and also the recently observed ring. A Kuiper Belt object, Haumea is a triaxial ellipsoid with dimensions of approximately $513 \times 852 \times 1161$ (km), with a mass of $4.006 \times 10^{21}$ kg and a rotation period of $3.915341$ h. The dwarf planet Haumea has its system formed by two natural satellites, the moons Namaka and Hi’iaka. We have presented an analysis of orbits around the dwarf planet Haumea taking into account the influences of the perturbations of its nonsphericity ($J_2, J_4, C_{22}$). We have found that the $C_{22}$ term and the rotation rate of Haumea have contributed strongly to reduce the variation rate of the periapsis radius of the spacecraft.

We have calculated the spherical harmonics of Haumea taking into account the most current values for the semi-axes of the ellipsoid and we have presented a comparison with the values of the harmonics found in other works.

1. Introduction

Our solar system has a great collection of celestial bodies besides those we have already known. The dwarf planets are new in the planetary classification, defined by the International Astronomical Union, in 2006, as bodies that have the mass of the magnitude similar to Plutos mass. The dwarf planet Haumea has a system formed by two natural satellites, the moons Namaka and Hi’iaka. Nowadays a great discovery, see reference [1], has identified a ring around the dwarf planet. Haumea is a triaxial ellipsoid, where the most current values of the semi-axes of the ellipsoid were presented in [1]. In this work, a study of the dynamics of a spacecraft around Haumea has been presented, through formulations of mathematical models and elaboration of computational programs for the realization of numerical simulations of the nonlinear differential equations systems that were developed in Maple software. We have presented an analysis of orbits around the dwarf planet Haumea, taking into account the influence of the perturbations of its nonsphericity.

2. Nonsphericity of the central body

In this section, the disturbing potential due to non-uniform distribution of mass of the central body is developed up to the fourth order for the zonal terms and more the $C_{22}$ term (Haumea equatorial ellipticity). Considering the equatorial plane of the planet, as the reference plane. The spacecraft orbits about the central body with semimajor axis $a$, eccentricity $e$, inclination $i$, right ascension of the ascending node $\Omega$, argument of the periapsis $\varpi$ and mean motion $n$. The disturbing potential due to nonsphericity of the planet can be written in the form:
\[ U_M = -\frac{\mu}{r} \sum_{n=2}^{4} \left( \frac{R_H}{r} \right)^n J_n P_n(\sin \phi) - \left( \frac{R_p}{r} \right)^2 C_{22} P_{22}(\sin \phi) \cos(2\lambda), \]  

(1)

where \( \mu \) is the gravitational constant of the planet, \( R_H \) is the reference radius of Haumea, \( P_n \) is the Legendre polynomials, \( P_{nm} \) the associated Legendre polynomials, the angle \( \phi \) is the latitude of the orbit with respect to the equator of the planet, the angle \( \lambda \) is the longitude measured from the direction of the longest axis of the planet, and \( \lambda_22 \) is the longitude of the Planet’s longitude measured from the same fixed direction. However, \( \lambda_22 \) contains the time explicitly [2]. Using spherical trigonometry we have \( \sin \phi = \sin i \sin(f + g) \). The Legendre polynomials for the term zonal up to the \( J_4 \) and the Legendre associated functions for sectorial \( C_{22} \) term, can be written in the form [2]

\[ \begin{align*}
P_2(\sin \phi) &= \frac{1}{2}(3s^2 - 1), \\
P_4(\sin \phi) &= \frac{35}{8}s^4 - \frac{15}{4}s^2(\sin(f + g))^2 + 3/8, \\
P_{22}(\sin \phi) \cos 2\lambda &= 6(\xi^2 \cos f + \chi^2 \sin f + \xi \chi \sin 2f) - 3(1 - s^2(\sin(f + g))^2),
\end{align*} \]

(2)

where we use the shortcut \( \xi = \cos g \cos \Omega - c \sin g \sin \Omega, \chi = - \sin g \cos \Omega - c \cos g \sin \Omega, s = \sin i, \) and \( c = \cos i \). Note that in the case of the dwarf planet Haumea the \( J_3 = 0 \) term, this happens because of its triaxial ellipsoid form. Now, the potential given by Eq. (1) is written in function of the orbital elements, for this we have used the Legendre polynomials up to the fourth order (terms zonal). Invoking the Eq. (2) and the equation \( \mu = n^2a^3 \), we get

\[ \begin{align*}
U_{20} &= -\frac{1}{2} \frac{a^2}{r^3} \epsilon n^2(3s^2(\sin(f + g))^2 - 1) \\
U_{40} &= -\frac{a^2}{r^3} \epsilon_2 (\frac{35}{8}s^4 - \frac{15}{4}s^2(\sin(f + g))^2 + 3/8)
\end{align*} \]

(3)

(4)

where \( \epsilon = J_2R_H^2, \epsilon_2 = J_4R_H^4 \). Analogously, for the sectorial perturbation [2], we get

\[ \begin{align*}
U_{22} &= \frac{a^2}{r^3} \delta n^2(6\xi^2(\cos(f))^2 + 6\chi^2(\sin(f))^2 + 12\xi \chi \sin(2f) - 3 + 3s^2(\sin(f + g))^2)
\end{align*} \]

(5)

where \( \delta = C_{22}R_H^2 \). Here \( f \) is true anomaly of the artificial satellite. In order to write the long-period disturbing potential, we have applied the averaged analytical method (\( < F > = \frac{1}{2\pi} \int_0^{2\pi} Fdl \), where \( l \) is the mean anomaly of the spacecraft) to eliminate the short period terms. The development of the equations is carried out in closed form to avoid expansions in eccentricity and inclination. To accomplish this task, it is necessary to perform algebraic manipulations and we have used some equations known by the celestial mechanics, namely equations \( a/r = (1 + e \cos(f))/(1 - e^2) \) and \( dl = \frac{1}{\sqrt{1-e^2}} \frac{r^2}{a^2} df \). After performing the averaged model over the true anomaly of the spacecraft and after some algebraic manipulations, we get

\[ \begin{align*}
R_{J2} &= -\frac{1}{4} \frac{\epsilon}{(1-e^2)^{1/2}} n^2(3s^2 - 2) \\
R_{J4} &= \frac{3}{128} \left( \frac{r^2}{a^2} - \frac{3}{4} \right) n^2(140e^2s^4(\cos(g))^2 - 120e^2s^2(\cos(g))^2 - 175e^2s^4 + 180e^2s^2 - 70s^4 - 24e^2 + 80s^2 - 16)
\end{align*} \]

(6)

(7)

Now, using Eq. (5), the \( \xi \) and \( \chi \) variables to develop the potential due to the equatorial ellipticity of the planet, we get

\[ \begin{align*}
R_{C22} &= -\frac{3}{2} \frac{\delta}{(1-e^2)^{1/2}} n^2(c^2 - 1) \cos(2\Omega)
\end{align*} \]

(8)
In this equation, the $\Omega$ term is replaced by the expression $\Omega = h - \rho t$ given by reference [3], where $\rho$ is the rotation rate of Haumea, $h$ is the longitude of the ascending node and $t$ is the time. Thus, replacing this expression in Eq. (8), we get

$$R_{C22} = -\frac{3}{2} \frac{\delta}{(1-e^2)^{1/2}} n^2 (c^2 - 1) \cos(2h - 2\rho t) \quad (9)$$

Therefore, the long-period disturbing potential due to the non-uniform distribution of mass of the planet is written as $R = R_{J2} + R_{J4} + R_{C22}$. In [4] the equations of the $J_2$ and $C_{22}$ terms are presented without applying the averaged method (considering the terms of short-period) to investigate the dynamics of artificial satellites around Mercury. These equations may be useful for studying the Haumea ring.

3. Calculation of the spherical harmonics of Haumea

At present the values of the spherical harmonics of Haumea are not determined with precision, because we have found in the literature works with different values for the $J_2$ and $C_{22}$ terms and different formulas to calculate these terms. Here, we calculate the values of the $J_2$, $J_4$ and $C_{22}$ terms using the formulas presented in [5, 6, 7, 8, 9] and we have considered the most current values for the Haumea axes that are obtained in reference [1]. Haumea is a triaxial ellipsoid with principal semi-axes $A > B > C$ ($A = 1161$ km, $B = 852$ km, $C = 513$ km (see [1]). To calculate the reference radius ($R_H$) of Haumea we have used equation (16) of the reference [9], which is given by

$$R_H = \frac{\sqrt{3}}{\sqrt{\frac{A^2}{B^2} + \frac{B^2}{C^2} + \frac{C^2}{A^2}}} \quad (10)$$

we have found that $R_H = 711.907$ km. To calculate the value of $J_2$ we have used the equation (18) (oblateness) of the reference [9], as shown in Eq. (11)

$$J_2 = \frac{1}{4} \frac{A^2 + B^2 - 2C^2}{R_H^2} \quad (11)$$

and we have also used the equation of the references [5, 7], as shown in Eq. (12)

$$C_{20} = \frac{1}{10} \frac{-A^2 - B^2 + 2C^2}{R_H^2} \quad (12)$$

where $J_2 = -C_{20}$. Note that the Eq. (11) has a factor 4 in the denominator, but the Eq. (12) has a factor 10. In [8] this equation also has a factor 10 in the denominator. To calculate the value of $C_{22}$ we have used the equation (18) (elongation) of the reference [9], as shown in Eq. (13)

$$C_{22} = \frac{1}{2} \frac{A^2 - B^2}{R_H^2} \quad (13)$$

and we have also used the equation of the references [5, 7], as shown in Eq. (14)

$$C_{22} = \frac{1}{20} \frac{A^2 - B^2}{R_H^2} \quad (14)$$

see equation 67 in [5]. Comparing the Eqs. (13) and (14) we have noted that there is a difference of a factor 10 in the denominator. The Eq. (14) is in agreement with [8]. Now, to calculate the $J_4$ term we use the equation of the reference [7], as shown in Eq. (15)

$$C_{40} = 3(\frac{3A^4 + 3B^4 + 8C^4 + 2A^2B^2 - 8(A^2 + B^2)C^2}{140R_H^4}) \quad (15)$$

note that the Eq. (15), which is obtained of the reference [7], is missing a factor 2 in the denominator as commented in [6], where the author comments that given a constant density
triaxial ellipsoid, the $J_2$ and $C_{22}$ terms may be computed as in [10]. The Eq. (15), see reference [6], is written in the form

$$C_{40} = 3\left(\frac{3A^4 + 3B^4 + 8C^4 + 2A^2B^2 - 8(A^2 + B^2)C^2}{280R_H^4}\right)$$  \hspace{1cm} (16)$$

In [8] the author shows a more compact equation, to calculate the $C_{40}$ term, than the Eq. (16). According to [8] the Eq. (16) is written in the form

$$C_{40} = \frac{15}{7} (C_{20}^2 + 2C_{22}^2)$$  \hspace{1cm} (17)$$

To calculate the harmonic values $J_2$ and $C_{22}$ we have used the Eqs. (11) and (13) of the reference [9], we get $J_2 = 0.763$ and $C_{22} = 0.614$, which is in agreement with the value shown in the Table 1 of the reference [9]. We have also calculated the values of $J_2$ and $C_{22}$ using the Eqs. (12) and (14), we get $J_2 = 0.305$ and $C_{22} = 0.061$ (see Table 1). To calculate the value of $J_4$ we have used the Eq. (16) of the reference [6], we have found that $J_4 = -0.216$ (see Table 1). Now, using the Eq. (15), which is also used by [11], we have found $J_4 = -0.432$. Note that in this way the $J_4$ term would be larger than $J_2$. This happens because of factor 2 that was missing in the denominator of Eq. (15). Now, to calculate the value of $J_4$ using the Eq. (17), we have found $J_4 = -0.216$ which is exactly the same value found using the Eq. (16). Finally, in this present work, the Eqs. (10), (12), (14) and (16) were considered to calculate the values of the spherical harmonics of Haumea (see Table 1). Note that in [12], the authors have used the equations of [8], which are the Eqs. (12) and (14), but to calculate the reference radius of Haumea the authors have considered the equation $R_H = (ABC)^{1/3}$, here we have considered the Eq. (10). As there are different values for the Haumea reference radius, as shown in Table 1, thus, to improve the comparison, we have calculated the $R_H J_{nm}$ term and we have presented the results in Table 2. We have found that the values obtained in [12] are equal to those found in this work (see Table 2). Note that there is still a large uncertainty in the values of the spherical harmonics of Haumea (see Table 1 and Table 2). Thus, a scientific mission will be necessary to study the gravitational field of Haumea.

| Author                        | $A > B > C$ (km) | $R_H$ (km) | $J_2$ | $C_{22}$ | $J_4$ |
|-------------------------------|-----------------|-----------|-------|----------|-------|
| Ragozzine and Brawn (2009) [13]| 1000, 750, 500  | 652       | 0.244 | —        | —     |
| Lockwood et al. (2014) [14]   | 1920, 1540, 990  | 620       | —     | —        | —     |
| Sanchez (2015) [11]           | 960, 770, 495   | 960       | 0.111 | 0.0178   | -0.056|
| Kondratyve (2016) [15]         | 980, 759, 498   | 718.178   | —     | —        | —     |
| Ortiz et al. (2017) [1]        | 1161, 852, 513  | 816       | —     | —        | —     |
| Sicardy et al. (2018) [9]      | 1161, 852, 513  | 712       | 0.76  | 0.61     | —     |
| Kovác and Regály (2018) [16]   | 1161, 852, 513  | 816       | 0.1274| 0.0256   | —     |
| Winter et al. (2019) [12]      | 1161, 852, 513  | 797.6207  | 0.243 | 0.049    | —     |
| This work                     | 1161, 852, 513  | 711.9097  | 0.305 | 0.061    | -0.216|

4. Results
Considering the perturbations due to the nonsphericity of Haumea ($J_2$, $J_4$ and $C_{22}$) the disturbing potential is written in the form
Table 2. Comparison using $R_H^2 J_{nm}$ of Haumea.

| Author                                      | $R_H^2 J_2$ (km²) | $R_H^2 C_{22}$ (km²) |
|---------------------------------------------|-------------------|----------------------|
| Ragozzine and Brawn (2009) [13]             | 103725.376        | —                    |
| Sanchez (2015) [11]                        | 102297.600        | 16404.4800           |
| Sicardy et al. (2018) [9]                  | 385277.44         | 309235.84            |
| Kovács and Regály (2018) [16]              | 84830.0544        | 17045.9136           |
| Winter et al. (2019) [12]                  | 154748.7000       | 31100.85000          |
| This work                                  | 154748.7000       | 31100.85001          |

\[ R = R_J_2 + R_J_4 + R_{C_{22}} \] \quad (18)

In this section, we have presented the numerical simulations considering the spherical harmonics that were calculated in this work ($J_{2} = 0.305$, $J_{4} = -0.216$, $C_{22} = 0.061$). Replacing Eq. (18) into Lagrange’s planetary equations and numerically integrating a set of nonlinear differential equations (using the Software Maple) we present Figs. 1-8. To analyze the effect of the nonsphericity of the dwarf planet Haumea on the orbit of the spacecraft, we have considered the following semimajor axis: $1.3R_H$, $1.6R_H$ and $2R_H$ (low altitude orbits). As [12] and [9] pointed, for this distance, the third-body perturbation can be neglected. The initial conditions that are used in all the figures are: $i = 90^\circ$, $g = 270^\circ$ and $h = 90^\circ$. In Fig. 1, the $C_{22}$ term is not taken into account. The figure shows the behavior of the periapsis radius with respect to time. Note that for different initial values of the eccentricity, the orbits collide rapidly with the surface of Haumea, which is represented by the horizontal line. However, when the $C_{22}$ term is taken into account, the orbits have presented small variations and the spacecraft has not collided with the Haumea surface as shown in Fig. 2. It is known that the $C_{22}$ term primarily influences the orbital inclination, but also influences the rate of change in argument of the periapsis and the longitude of the ascending node (see reference [17]). The $C_{22}$ term has not affected the directly eccentricity, but since the differential equation system is coupled, the $C_{22}$ term, when it has combined with other perturbations, has affected the indirectly eccentricity. As we can see in Fig. 2, the $C_{22}$ term contributed to reduce the growth of the periapsis radius ($r = a(1-e)$). Note that the term due to the rate of rotation of Haumea ($\cos(2h-2\rho t)$) has appeared in the Eq. (9). The rate of rotation of Haumea is very rapid, of the order of $10^{-4}$, when compared to the rate of the mean motion of Haumea around Namaka, which is of the order of $10^{-8}$. Assuming the motion of Haumea synchronous with Namaka we have noticed that the inclination is less affected, but since this motion is not synchronous, the rotation rate contributed to increase inclination growth. Comparing Figs. 3 and 4, we have had that the $C_{22}$ term has amplified the increase of the inclination and has reduced the period. Figure 5 shows the behavior of the eccentricity for different initial conditions considering the perturbations due to nonsphericity $J_{2}, J_{4}, C_{22}$. Note that the orbit with eccentricity 0.01 is the one with the smaller variation. Now, Fig. 6 shows the behavior of the argument of the periapsis, which has presented a precession motion. Note that this orbital element has been strongly disturbed, and this precession has made it difficult to find the frozen orbit for satellites near the dwarf planet. Figures 7 and 8 show the behavior of the eccentricity to orbits with a slightly higher height than in the previous case, as we could see in the caption of the figures. Note that, distancing the spacecraft of the dwarf planet, the orbits are less disturbed due to the decrease in the effect of the nonsphericity of the central body.
Figure 1. Position of the periapsis versus $t$. Initial conditions: $a = 1.3R_H = 925.483$ km. Potential: $R_{J_2} + R_{J_4}$. Horizontal line represents the surface of Haumea.

Figure 2. Position of the periapsis versus $t$. Initial conditions: $a = 1.3R_H = 925.483$ km. Potential: $R_{J_2} + R_{J_4} + R_{C_{22}}$. Horizontal line represents the surface of Haumea.

Figure 3. $i$ (deg) versus $t$. Initial conditions: $a = 1.3R_H = 925.483$ km.

Figure 4. $i$ (deg) versus $t$. Initial conditions: $a = 1.3R_H = 925.483$ km.

5. Conclusions
The dynamics of a spacecraft around Haumea is studied taking into account the nonsphericity of the dwarf planet, including the $J_2$, $J_4$ and $C_{22}$ terms. Considering the most recent data for the semi-axes of the ellipsoid, we calculate the values of the harmonics $J_2$, $C_{22}$ and $J_4$ and the reference radius of Haumea. The values of the spherical harmonics found in this work may contribute to future analyzes of the dwarf planet Haumea. We present a comparison of the spherical harmonics with other works. Here, in this work, we use the equations of the references [5] and [7] to calculate the $J_2$ and $C_{22}$ terms. The values found agree with reference [12]. But these values do not agree with the reference [9]. The parameters $f$ and $e$ of the equation 18 in [9] are probably defined by $f = 2.5J_2$ and $e = 10C_{22}$, if defined in this way, the results are in agreement with our calculations. We shown that the $C_{22}$ term contribute strongly to reduce the
Figure 5. $e$ versus $t$. Initial conditions: $a = 1.3R_H = 925.483$ km. Potential: $R_{J_2} + R_{J_4} + R_{C_{22}}$.

Figure 6. $g$ versus $t$. Initial conditions: $a = 1.3R_H = 925.483$ km. Potential: $R_{J_2} + R_{J_4} + R_{C_{22}}$.

Figure 7. $e$ versus $t$. Initial conditions: $a = 1.6R_H = 1139.055$ km. Potential: $R_{J_2} + R_{J_4} + R_{C_{22}}$.

Figure 8. $e$ versus $t$. Initial conditions: $a = 2R_H = 1423.819$ km. Potential: $R_{J_2} + R_{J_4} + R_{C_{22}}$.

growth of the periapsis radius, avoiding collisions of the spacecraft with the surface of Haumea in low altitude orbit. We also show that the $C_{22}$ term amplifies the increase of the inclination and causes a precession motion in the argument of the periapsis. In the continuity of this research will be analyzed the dynamics of debris in orbit around the dwarf planet Haumea based on [4], [12] and [18].

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