r-MODE RUNAWAY AND RAPIDLY ROTATING NEUTRON STARS

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ABSTRACT

We present a simple spin-evolution model that predicts that rapidly rotating accreting neutron stars will be confined mainly to a narrow range of spin frequencies: \( P = 1.5\text{–}5 \text{ ms} \). This is in agreement with current observations of neutron stars in both the low-mass X-ray binaries and the millisecond radio pulsars. The main ingredients in the model are (1) the instability of \( r \)-modes above a critical spin rate, (2) the thermal runaway that is due to the heat released as viscous damping mechanisms counteract the \( r \)-mode growth, and (3) a revised estimate of the strength of the dissipation that is due to the presence of a viscous boundary layer at the base of the crust in an old and relatively cold neutron star. We discuss the gravitational waves that are radiated during the brief \( r \)-mode–driven spin-down phase. We also briefly touch on how the new estimates affect the predicted initial spin periods of hot young neutron stars.

Subject headings: dense matter — gravitation — stars: neutron — stars: oscillations — stars: rotation

1. INTRODUCTION

The launch of the Rossi X-Ray Timing Explorer (RXTE) in 1995 heralded a new era in our understanding of neutron star physics. Detailed observations of quasi-periodic phenomena at kilohertz frequencies in more than a dozen low-mass X-ray binaries (LMXBs) strongly suggest that these systems contain rapidly spinning neutron stars (for a recent review, see van der Klis 2000), providing support for the standard model for the formation of millisecond pulsars (MSPs) via spin-up due to accretion.

Despite these advances, several difficult questions remain to be answered by further observations and/or theoretical modeling. For example, we still do not know the reason for the apparent lack of radio pulsars at shorter periods than the 1.56 ms of PSR 1937+21 (for a review of recent progress in the modeling of rotating neutron stars, see Stergioulas 1998). The recent RXTE observations provide a further challenge for theorists. Various models suggest that the neutron stars in LMXBs spin rapidly, perhaps in the narrow range of 260–590 Hz (van der Klis 2000). Three different models have been proposed to explain this surprising result. The first model (from White \& Zhang 1997) is based on the standard magnetosphere model for accretion-induced spin-up, while the remaining two models are rather different, both being based on the idea that gravitational radiation balances the accretion torque. In the first such model for the LMXBs (proposed by Bildsten 1998 and recently refined by Ushomirsky, Cutler, \& Bildsten 2000), the gravitational waves are due to a quadrupole deformation induced in the deep neutron star crust because of accretion-generated temperature gradients. The second gravitational-wave model relies on the recently discovered \( r \)-mode instability (see Andersson \& Kokkotas 2000 for a review) to dissipate the accreted angular momentum from the neutron star.

In this Letter, we reexamine the idea that gravitational waves from unstable \( r \)-modes provide the agent that balances the accretion torque. This possibility was first analyzed in detail by Andersson, Kokkotas, \& Stergioulas (1999; but also see Bildsten 1998). Originally, it was thought that an accreting star in which the \( r \)-modes were excited to a significant level would reach a spin equilibrium, very much in the vein suggested by Papaloizou \& Pringle (1978) and Wagoner (1984). Should this happen, the neutron stars in LMXBs would be prime sources for detectable gravitational waves. However, as was pointed out by Levin (1999) and Spruit (1999), the original idea is not viable since, in addition to generating gravitational waves that dissipate angular momentum from the system, the \( r \)-modes will heat the star up (via the shear viscosity that counteracts the \( r \)-mode at the relevant temperatures). Since the shear viscosity gets weaker as the temperature increases, the mode heating triggers a thermal runaway, and in a few months the \( r \)-mode would spin an accreting neutron star down to a rather low rotation rate. Essentially, this conclusion rules out the \( r \)-modes in galactic LMXBs as a source of detectable gravitational waves since they will only radiate for a tiny fraction of the systems lifetime.

Other recent results would (at first sight) seem to emphasize the conclusion that the \( r \)-modes are not relevant for the LMXBs. Bildsten \& Ushomirsky (2000) investigated the effect that the presence of a solid crust would have on the \( r \)-mode oscillations. They estimated that the dissipation associated with a viscous boundary layer that arises at the base of the solid crust in a relatively cold neutron star would greatly exceed that of the standard shear viscosity. Thus, Bildsten \& Ushomirsky concluded that the \( r \)-mode instability would only be relevant for very high rotation rates and could therefore not play a role in the LMXBs.

We have reassessed the effect of the viscous boundary layer (correcting an erring factor in the estimates of Bildsten \& Ushomirsky 2000). Our new estimates show that the presence of the crust is important but that the instability operates at significantly lower spin rates than suggested by Bildsten \& Ushomirsky. Once we combine our estimates with the thermal runaway (which is now due to the heating caused mainly by the presence of the viscous boundary layer) that results when
the star is spun up to the point at which the instability sets in, we arrive at a model for the spin evolution of accreting neutron stars. Remarkably, this simple model agrees well with existing observations of rapidly rotating neutron stars, covering both the LMXB and MSP populations.

2. DISSIPATION DUE TO A VISCOUS BOUNDARY LAYER

The $r$-mode instability follows after a tug-of-war between (mainly current multipole) gravitational radiation that drives the mode and various dissipation mechanisms that counteract the fluid motion. In the simplest model, the mode is dominated by shear viscosity at low temperatures, while bulk viscosity may suppress the mode at high temperatures. At intermediate temperatures, the $r$-mode sets an upper limit on the neutron star spin rate. In an interesting recent paper, Bildsten & Ushomirsky (2000) estimate the strength of dissipation due to the solid crust of an old neutron star and find that the presence of a boundary layer at the base of the crust leads to a very strong damping of the $r$-modes.

While we agree with the main idea and the various assumptions made by Bildsten & Ushomirsky, we would like to point out one important difference between their results and ones used previously in the literature. Their assumed timescale for gravitational radiation reaction differs significantly from, for example, the uniform density result derived by Kokkotas & Stergioulas (1999; subsequently used by several authors [see Andersson & Kokkotas 2000]). This is surprising since the uniform density result, which can be written as

$$t_{gw} \approx -22 \left( \frac{1.4}{M} \right) \left( \frac{10 \, \text{km}}{R} \right) \left( \frac{P}{1 \, \text{ms}} \right)^{1/2} \text{s}$$

(where the minus sign indicates that the mode is unstable), has been shown to be close (within a factor of 2) to the results for $n = 1$ polytropes. $M$, $R$, and $P$ represent the mass, radius, and spin period of the star, respectively. In contrast, Bildsten & Ushomirsky (2000) use the $n = 1$ polytrope result and argue that it corresponds to $t_{gw} \approx -146$ s for a canonical neutron star rotating with a period of 1 ms; i.e., they assume that the radiation reaction is almost 1 order of magnitude weaker than in equation (1). This difference occurs because Bildsten & Ushomirsky have only rescaled the fiducial rotation frequency $\Omega_n \approx (\pi G \rho)^{1/2}$ (where $\rho$ represents the average density) in terms of which the $n = 1$ polytrope results of Owen et al. (1998) were expressed [$t_{gw} \approx -3.26(\Omega_n/\Omega)^4$ for a specific polytropic stellar model]. Unfortunately, this procedure is not correct. From the fundamental relations, e.g., the formula for the gravitational-wave energy radiated via the current multipoles, one can see that the gravitational-wave timescale should scale with $M$, $R$, and $P$ in the way manifested in equation (1). Thus, we believe that Bildsten & Ushomirsky underestimate the strength of the radiation reaction significantly, which motivates us to reassess the relevance of the viscous boundary layer.

We should, of course, emphasize at this point that our current understanding of the $r$-mode instability is based on crude estimates of the various timescales. In order to understand the role of the instability in an astrophysical context, we must improve our modeling of many aspects of neutron star physics such as the effect of general relativity on the $r$-modes, cooling rates, viscosity coefficients, magnetic fields, potential superfluidity, the formation of a solid crust, etc. (see Andersson & Kokkotas 2000 for a description of recent progress in these various directions).

In the following, we will mainly consider uniform density stars; i.e., we will use the gravitational-wave timescale given by equation (1). In estimating the dissipation timescale $t_{\text{diss}}$ that is due to the presence of a viscous boundary layer at the base of the crust, we need to evaluate $t_{\text{diss}} \approx -2E/(dE/dt)$, where $E$ is the mode energy and $dE/dt$ follows from an integral over the surface area at the crust-core boundary (assumed to be located at radius $R_b$; see eq. [3] of Bildsten & Ushomirsky 2000). To evaluate this integral, we use the standard result for the shear viscosity in a normal fluid. To incorporate our uniform density model, we make the reasonable assumption that the density of the star is constant ($\sim MR$) inside radius $R$. Then it falls off rapidly in such a way that the base of the crust (corresponding to a density $\rho_b = 1.5 \times 10^{14}$ g cm$^{-3}$) is located at a radius only slightly larger than $R$. Hence, it makes sense to use $R_b \approx R$. If we neglect the small mass located outside radius $R$, we can then immediately compare the result for the viscous boundary layer with the timescales used by Andersson et al. (1999). In the end, our estimate for the dissipation that is due to the presence of the viscous boundary layer is

$$t_{\text{diss}} \approx 200 \left( \frac{M}{1.4 \, M_\odot} \right) \left( \frac{10 \, \text{km}}{R} \right)^2 \left( \frac{T}{10^9 \, \text{K}} \right) \left( \frac{P}{1 \, \text{ms}} \right)^{1/2} \text{s},$$

which is a factor of 2 larger than that of Bildsten & Ushomirsky. This difference arises simply because the mode energy $E$ is this factor larger for uniform density models. The star is assumed to have a uniform temperature distribution, with core temperature $T$.

In Figure 1, we show the instability window obtained from our revised estimate. As is clear from this figure, the presence

![Figure 1](image-url)
of a viscous boundary layer in an old, relatively cold neutron star is indeed important. However, Bildsten & Ushomirsky's conclusion that the \( r \)-mode instability is irrelevant for the LMXBs cannot be drawn from Figure 1. On the contrary, the figure suggests that the instability may well be limiting the rotation of these systems.

3. THERMAL RUNAWAY IN RAPIDLY SPINNING NEUTRON STARS

The fact that our revised instability curve for \( r \)-modes damped by dissipation in a viscous boundary layer agrees well with the fastest observed neutron star spin frequencies (see Fig. 1) motivates us to speculate further on the relevance of the instability. We want to model how the potential presence of an unstable \( r \)-mode affects the spin evolution of rapidly spinning, accreting neutron stars. To do this, we use the phenomenological two-parameter model devised by Owen et al. (1998), which is centered on evolution equations for the rotation frequency \( \Omega \) and the (dimensionless) \( r \)-mode amplitude \( \alpha \). Complete details of our particular version of this model will be given elsewhere.

At the qualitative level, our results are not surprising. Accreting stars in the LMXBs are expected to have core temperatures in the range of \( (1–4) \times 10^8 \) K (Brown & Bildsten 1998). For such temperatures, the dissipation that is due to the viscous boundary layer gets weaker as the temperature increases. Consequently, the situation here is essentially identical to that considered by Levin (1999; see also Spruit 1999 and Bildsten & Ushomirsky 2000). After accreting and spinning up for something like \( 10^7 \) yr, the star reaches the period at which the \( r \)-mode instability sets in. For our particular estimates, this corresponds to a period of \( 1.5 \) ms (at a core temperature of \( 10^8 \) K). It is notable that this value is close to the \( 1.56–6 \) ms period of PSR 1937+21. Once the \( r \)-mode becomes unstable (point \( A \) in Fig. 1), viscous heating (now mainly due to the energy released in the viscous boundary layer) rapidly heats the star up to a few times \( 10^9 \) K. The \( r \)-mode amplitude increases until it reaches a prescribed saturation level (amplitude \( \alpha \) at which unspecified nonlinear effects halt further growth (point \( B \) in Fig. 1). Once the mode has saturated, the neutron star rapidly spins down as excess angular momentum is radiated as gravitational waves. When the star has spun down to the point where the mode again becomes stable (point \( C \) in Fig. 1), the amplitude starts to decay, and the mode plays no further role in the spin evolution of the star (point \( D \) in Fig. 1) unless the star is again spun up to the instability limit. Two examples of such \( r \)-mode cycles (corresponding to \( \alpha_r = 0.1 \) and \( 1 \), respectively) are shown in Figure 1.

The real surprise here concerns the quantitative predictions of our model. As already mentioned, the model suggests that an accreting star will not spin up beyond \( 1.5 \) ms. This value obviously depends on the chosen stellar model, but it is independent of the \( r \)-mode saturation amplitude and only weakly dependent on the accretion rate (through a slight change in core temperature). In fact, the accretion rate only affects the time it takes the star to complete one full cycle. As soon as the mode becomes unstable, the spin evolution is dominated by gravitational radiation and viscous heating. Once the star has gone through the brief phase when the \( r \)-mode is active, it has spun down to a period in the range of \( 2.8–4.8 \) ms (corresponding to \( 0.01 \leq \alpha_r \leq 1 \)). Based on these results, we propose the following spin-evolution scenario: An accreting neutron star will never spin up beyond, say, \( 1.5 \) ms. Once it has reached this level, the \( r \)-mode instability sets in and spins the star down to a period of several milliseconds. At this point, the mode is again stable, and continued accretion may resume to spin the star up. Since the star must accrete roughly \( 0.1 M_\odot \) to reach the instability point and the LMXB companions have masses in the range of \( 0.1–0.4 M_\odot \), it can pass through several “\( r \)-mode cycles” during its lifetime.

Let us confront this simple model with current observations. To do this, we note that our model leads to one main prediction: Once an accreting neutron star has been spun up beyond, say, \( 5 \) ms, it must remain in the rather narrow range of periods \( 1.5–5 \) ms until it has stopped accreting and magnetic dipole braking eventually slows it down. Since a given star can go through several \( r \)-mode cycles before accretion is halted, one would expect most neutron stars in LMXBs and MSPs to be found in the predicted range of rotation rates. As is clear from Figure 1, this prediction agrees well with the range of rotation periods inferred from observed kilohertz quasi-periodic oscillations in LMXBs. The observed range shown in Figure 1 corresponds to rotation frequencies in the range of \( 260–590 \) Hz (see van der Klis 2000). Our model also agrees with the observed data for MSPs, which are mainly found in the range of \( 1.56–6 \) ms (see Fig. 2). In other words, our proposed model is in agreement with current observed data for rapidly rotating neutron stars.

Finally, it is worthwhile discussing briefly the detectability of the gravitational waves that are radiated during the relatively short time when the \( r \)-mode is saturated and the star spins down. As was argued by Levin (1999), the fact that the \( r \)-mode is active only for a small fraction of the lifetime of the system (something like \( 1 \) month out of the \( 10^7 \) yr it takes to complete one full cycle) means that even though these sources would be supremely detectable from within our Galaxy, the event rate is far too low to make them relevant. However, it is interesting to note that the spin evolution is rather similar to that of a hot young neutron star once the \( r \)-mode has reached its saturation amplitude. This means that we can analyze the detectability of the emerging gravitational waves using the framework of Owen et al. (1998). We then find that these events can be observed from rather distant galaxies. For a source in the Virgo Cluster (assumed to be at a distance of \( 15 \) Mpc), these gravitational waves could be detected with a signal-to-noise ratio of a few using the upgraded detector on the laser interferometer.
gravitational-wave observatory (LIGO II). However, even at the distance of the Virgo Cluster, these events would be quite rare. By combining a birthrate for LMXBs of $7 \times 10^{-6}$ yr$^{-1}$ galaxy$^{-1}$ with the fact that the volume of space out to the Virgo Cluster contains $\sim 10^7$ galaxies and the possibility that each LMXB passes through, say, four $r$-mode cycles during its lifetime, we deduce that one can only hope to see a few events per century in Virgo. In order to see several events per year, the detector must be sensitive enough to detect these gravitational waves from, say, 150 Mpc. This would require a more advanced detector configuration such as a narrowedband LIGO II. We will discuss this issue in more detail elsewhere.

4. ADDITIONAL REMARKS

Before concluding our discussion, we recall that the initial excitement over the $r$-mode instability was related to the fact that it provided an explanation for the relatively slow inferred spin rates for young pulsars. In view of this, it is natural to digress somewhat and discuss how the picture of the $r$-mode instability in hot, newly born neutron stars is affected by the possible formation of a solid crust. Hence, we consider the evolution of a neutron star just after its birth in a supernova explosion. At first glance, we might expect to model its evolution of a neutron star just after its birth in a supernova explosion. At first glance, we might expect to model its evolution in the standard way (see Owen et al. 1998) using the normal (crust-free) fluid viscous damping times for stellar temperatures above the melting temperature of the crust ($T_m$) and the viscous boundary layer damping time for temperatures below $T_m$. However, the situation is a little more complicated than this. Recall that the latent heat (i.e., the Coulomb binding energy) of a typical crust is $E_{\text{lat}} \sim 10^{44}$ ergs, while the $r$-mode energy is $E_r \approx 2\alpha^2 (1 \text{ ms/P})^2 \times 10^{51}$ ergs. Provided that the time taken for the star to cool to $T_m$ is sufficiently long, the energy in the $r$-mode (which grows exponentially on a timescale $t_{\text{gw}} \approx 20$ s) will exceed $E_{\text{lat}}$, preventing the formation of the crust, even when $T < T_m$. Then the star will spin down in the manner described by, e.g., Owen et al. (1998). This phase will end either because the star leaves the instability region of the $\Omega$-$T$ plot (see, e.g., Fig. 1 of Owen et al. 1998) or because the mode energy in the outer layers of the star (where the crust is going to form) has fallen below the crustal binding energy. We can estimate that this would happen at a frequency $\approx 70$ Hz/$\alpha$, by equating $E_{\text{lat}}$ to roughly 10% of $E_r$. A more accurate treatment would take into account the local kinetic energy of fluid elements, and since the latter is smaller near the poles than near the equator, the crust might form earlier at the poles. Clearly, the problem of crust formation in an oscillating star requires further investigation. The final spin period will be around 15 ms, if the $r$-mode grows to an amplitude of $\alpha_r \sim 1$, which is consistent with the extrapolated initial spin rates of many young pulsars. On the other hand, if the mode is not given time to grow very large, it will not prevent crust formation at $T_m$. Such a scenario was described by Bildsten & Ushomirsky (2000), who noted that the $r$-mode instability would then not spin the star down beyond a much higher frequency. Using our estimated timescales, the resultant spin period would be 3–5 ms in this scenario. Which scenario applies depends sensitively on the early cooling of the star, the crustal formation temperature, and, perhaps most importantly, the initial amplitude of the $r$-mode following the collapse. It is in fact possible that both routes are viable and that a bimodal distribution of initial spin periods results. A likely key parameter is whether or not the supernova collapse leads to a large initial $r$-mode amplitude $\sim \alpha_r$. A period of $\sim 15$ ms at the end of the $r$-mode spin-down phase would fit the long-established data for the Crab, while the recently discovered 16 ms pulsar PRS J0537-6910 (Marshall et al. 1998) requires a considerably shorter period of a few milliseconds.

In conclusion, we have reexamined the effect that the dissipation due to a possible viscous boundary layer in a neutron star with a solid crust has on the stability of the $r$-modes. By combining our new estimates with the thermal runaway introduced by Levin (1999) and Spruit (1999), we arrive at a spin-evolution model that agrees with present observations for rapidly spinning neutron stars. In particular, our predictions agree well with observations of both LMXBs and MSPs. Furthermore, the model can potentially explain the extrapolated spin periods of the young pulsars. Since it brings out this unified picture, our simple model has many attractive features, and we are currently investigating it in more detail.

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