Electron-phonon induced spin relaxation in InAs quantum dots

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Abstract

We have calculated spin relaxation rates in parabolic quantum dots due to the phonon modulation of the spin-orbit interaction in presence of an external magnetic field. Both, deformation potential and piezoelectric electron-phonon coupling mechanisms are included within the Pavlov-Firsov spin-phonon Hamiltonian. Our results have demonstrated that, in narrow gap materials, the electron-phonon deformation potential and piezoelectric coupling give comparable contributions as spin relaxation processes. For large dots, the deformation potential interaction becomes dominant. This behavior is not observed in wide or intermediate gap semiconductors, where the piezoelectric coupling, in general, governs the spin relaxation processes. We also have demonstrated that spin relaxation rates are particularly sensitive to the Landé g-factor.

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The ability to manipulate and control processes that involve transitions between spin states is, at the moment, of extreme importance due to the recent applications in quantum computation and quantum communication. Quantum dots (QD’s) of diverse geometries are good candidates for implementation of semiconductor quantum communication devices because the electronic, magnetic and optical properties can be controlled in the modern grown and nanofabrication techniques. The spin dephasing time is important because it sets the length-scale on which coherent physics can be observed. It is therefore important to understand the origin of decoherence so that ultimately it may be reduced or controlled. At the moment remains in discussion which, between

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these processes, is dominant in semiconductors in low dimensional systems. Some experimental results have shown good agreement with the theoretical predictions in 2-D systems [1] but, in general, the identification of the processes through direct comparison with the experimental results can become a formidable task. This problem is more critical in QD’s, since few experimental results exist and the theoretical discussion of the spin relaxation mechanisms is still an open subject. Khaetskii and Nazarov [2] studied two kind of processes of spin relaxation induced by phonons in GaAs QD’s: i) The first mechanism is due to the Dresselhaus admixture of states with an opposite spin. Without the spin-orbit interaction the Zeeman sublevels correspond to the orbital state with “pure” spin states. The addition of spin-orbit terms provides a small admixture of the states with opposite spin to each sublevel and, thus, enables the phonon-assistant transition between them; ii) The second process is related with the direct coupling between the spin and the strain field as produced by the acoustic phonons. With admixture process (i), scattering rates of the order of $10^3$ s$^{-1}$ were obtained. The authors also showed that the admixture process is the dominant one. Recently, Woods et al, [3] have studied spin relaxation considering the coupling between spins and phonons arising from interface motion. This mechanism arises when interface motion due to acoustic phonons changes a parameter of the system. In this case, small ($10^3$ s$^{-1}$) and strongly size dependent relaxation rates have been found by the authors. In these works, the spatial dependence of the $g$-factor was not considered. Also, due to the small transition energies, only the coupling by piezo-phonons were included in the rate calculation. These approximations are not necessarily valid for InAs QD’s, since: i) Experimental measurements have shown that the electron $g$-factor depends strongly on the dot size [4,5], ii) In InAs based QD’s, the Zeeman transition energies can be considerably larger than for GaAs QD’s and, therefore, the coupling due to the deformation potential can contribute significantly to the relaxation rates.

Our approach is based on the model of Pavlov and Firsov[9,10]. In this model, the Hamiltonian describing the transitions with spin reversal, in the scattering of electrons by phonons, can be written in a general form, $H_{e-ph}^{e} = U_{ph} + \beta [\sigma \times \nabla U_{ph}] \cdot (p + eA)$, where $U_{ph}$ is the phonon operator, $(\hbar/2)\sigma$ is the spin operator, $p$ is the momentum operator and $A$ is the vector potential for the magnetic field $B$. This interaction Hamiltonian depends on spin variables and, thus, can lead to spin-flip transitions between pure spin states. In this work we have calculated spin relaxation rates in InAs parabolic quantum dots, considering the phonon modulation of the spin-orbit interaction within the Pavlov-Firsov model. We also study the effects of the spatial dependence of the electron $g$-factor and evaluate the contributions of the deformation potential and piezoelectric couplings on the spin relaxation rates.

Experimental measurements and numerical calculations [6] have indicated that in lens-shaped quasi-two dimensional self-assembled quantum dots the bound
states of both electrons and holes can be understood assuming an effective parabolic potential, \( V(\rho) = \frac{1}{2} m \omega_0^2 \rho^2 \), where \( \hbar \omega_0 \) is the characteristic confinement energy and \( \rho \) is the radial cylindrical coordinate. By using a one-band effective mass approximation and considering the presence of a magnetic field \( B \), applied normal to plane of the dot, one can write the electron wavefunctions as [7]

\[
f_{n,l,\sigma} = \left[ \frac{n!}{\pi(n + |l|)!} \right]^{1/2} \frac{\rho^{|l|}}{a^{n+1}} e^{-\frac{\rho^2}{2a^2}} e^{i|l|} L_n^{||} \left( \frac{\rho^2}{a^2} \right) \chi(\sigma). \tag{1}
\]

In the above expression \( L_n^{||} \) denotes the Laguerre polynomials, \( n \) is the principal quantum number, \( l \) is the azimuthal quantum number and \( \chi(\sigma) \) is the spin wavefunction for spin variable \( \sigma \). The corresponding eigen-energies are \( E_{n,l,\sigma} = (2n + |l| + 1) \hbar \Omega + (l/2) \hbar \omega_c + (\sigma/2) g \mu_B B \), where \( \Omega = (\omega_0^2 + \omega_c^2/4)^{1/2} \), \( \mu_B \) is the Bohr magneton, \( a = (\hbar/m \Omega)^{1/2} \) is the effective length and \( \omega_c = eB/m \).

The Landé \( g \)-factor and the effective mass \( m \) are expressed in second-order \( \mathbf{k} \cdot \mathbf{p} \) perturbation. [8]

\[
g = 2 - \frac{4m_0 P^2}{3\hbar^2} \frac{\Delta}{(E_g + E)((E_g + E) + \Delta)}, \tag{2}
\]

\[
\frac{1}{m} = \frac{1}{m_0} + \frac{2P^2}{3\hbar^2} \frac{3(E_g + E) + 2\Delta}{(E_g + E)((E_g + E) + \Delta)}. \tag{3}
\]

Here, \( \Delta \) is the spin-orbit splitting, \( E_g \) is the energy band gap, \( E \) is the electron energy measured from the bottom of the conduction band and \( P = (\hbar/m_0) \langle iS | p_z | Z \rangle \) represents the interband matrix element. For InAs the value of \( \Delta \) is comparable with the fundamental gap \( E_g \), thus we can expect significant variations of the electron \( g \)-factor with the size parameters.

For an assisted acoustic-phonon spin-flip process, the matrix element, \( M \), for electron spin-flip between initial ( \( |nl \uparrow\rangle \) ) and final ( \( |n'l' \downarrow\rangle \) ) states with emission of a phonon of momentum \( \mathbf{q} \) and energy \( \hbar \nu q \), can be obtained, from the Pavlov-Firsov spin-phonon Hamiltonian [9,10] as

\[
M_{nl\uparrow \rightarrow n'l'\downarrow} = d(q) \left( \frac{\hbar}{\rho_M \mathbf{V} \nu q} \right)^{1/2} \chi_{z}(\uparrow) \left( \begin{array}{cc} 0 & \hat{n}^- \times \hat{e}_q \\ \hat{n}^+ \times \hat{e}_q & 0 \end{array} \right) \chi_{z}(\downarrow) \cdot \int d^3 \mathbf{r} f_{n'l'} e^{-i\mathbf{q} \cdot \mathbf{r}} \left( \frac{\mathbf{P}}{\hbar} + \frac{e\mathbf{A}}{\hbar c} + \mathbf{q} \right) f_{nl}, \tag{4}
\]
where $\chi_z$ are the spin wavefunctions quantized along the $z$ axis, $f_{nl} = \langle r | n, l \rangle$ is the electron envelope wavefunction, the magnetic vector potential $A$ is obtained in the symmetric gauge considering that the orientation of $B$ coincide with the $z$-axis, $\hat{n}^\pm = \hat{x} \pm i\hat{y}$, $\hat{e}_q = q/q$ is the polarization vector of the longitudinal acoustic phonons, $v$ is the average sound velocity, $\rho_M$ is the mass density, $V$ is the system volume and $d$ is a coupling constant that depends on the electron-acoustic phonon coupling mechanism. Detailed expressions for the parameter $d$ can be found in Ref. [10].

The spin-flip transition rate $W$ is calculated from the Fermi Golden Rule

$$W = \frac{2\pi V}{\hbar (2\pi)^3} \int d^3q |M_{nl\uparrow \rightarrow n'l'\downarrow}|^2 \delta(hvq - \Delta E),$$

(5)

where $\Delta E = E_{nl\uparrow} - E_{n'l'\downarrow}$ is the transition energy.

The calculations were performed at $T \sim 0$ K and we only have considered transitions between ground state Zeeman levels. The temperature dependence for one-phonon emission rate is determined from $W = W_0(n_B + 1)$, where $n_B$ is the Bose distribution function and $W_0$ is the rate at $T = 0$ K. In the temperature regime $T \lesssim 10$ K and considering typical values of magnetic field ($B \sim 2$ T), the Bose function is $n_B + 1 \approx 1$ and $W \approx W_0$. For temperatures larger than few Kelvin degrees, two-phonon processes should be considered as the dominant spin relaxation mechanism. These type of processes have not been considered in the present calculation.

In general, we obtain that $W \sim (g\mu_B B)^k$, $k$ being an integer number that depends on the electron-phonon coupling process ($k = 7$ for deformation potential and $k = 5$ for piezoelectric coupling). This strong dependence with the transition energy and, in consequence with the $g$-factor, demands that this parameter should be determined taking in account the effects of the quantum confinement. Spin splitting measurements in InAs self-assembled QD’s [4,5] have revealed $g$-factors showing clear dependence on the dot size. Values in the range of 0.8 - 1.29 were reported for QD’s in strong confinement regime and they differ strongly from the value of bulk InAs ($g_{\text{bulk}} = - 14.4$). In order to include the spatial dependence of the $g$-factor in the rate calculation we have used the Roth formula given in Eq. (2).

These facts are clearly illustrated in Fig. 1, where we show calculated InAs QD spin relaxation rates, for piezoelectric and deformation potential couplings, as a function of the lateral dot size $R$. Rates with $g = g_{\text{bulk}}$ are shown in dashed lines, and those with $g$ given by Eq.(2), in solid lines.

Our results show that the effect of confinement on the $g$-factor produces significant variations in the relaxation rates. We observe that the rates obtained
Fig. 1. Spin-relaxation rates, for InAs QD’s, as a function of the lateral dot size $R$ : a) piezoelectric and b) deformation potential mechanisms. Rates calculated from Eq. (2) ( for $g = g_{\text{bulk}}$) are shown in solid (dashed) lines.

for $g = g_{\text{bulk}}$ are one order of magnitude larger than rates calculated from Eq. (2) for deformation potential mechanism, $B = 1$ T and $R = 5$ nm.

The behavior of the rates with the QD size depends directly on the spatial overlap integral and the magnitude and on the sign of the $g$-factor. For negative values of $g$, the rate diminishes as the dot size is increased. This behavior can be illustrated in the rates calculated with $g = g_{\text{bulk}} = -14.4$ (dashed lines in Fig. 1). The Eq.(2) provides positive values of $g$ for $R \lesssim 10$ nm, in this case the rate increases as the dot size increases (solid lines in Fig. 1). The $g$-factor dependence on size can be neglected in the relaxation rates, for dots with $R > 15$ nm, since the energy of Zeeman level becomes very small.

According with the general relation, $W \sim (g\mu_B B)^k$, we can also observe that the rates will depend strongly on the magnetic field and can increases several order of magnitude when values of $B$ are swept from 0.5 T to 1 T. Our results
Fig. 2. Spin-relaxation rates for an GaAs QD as a function of the lateral dot size \( R \) for piezoelectric (dashed lines) and deformation potential (solid lines) mechanisms.

Demonstrate that for narrow-gap materials, the piezoelectric [Fig. 1a)] and deformation potential [Fig. 1b)] coupling present comparable contribution to the spin relaxation process. Indeed, for large dots and large magnetic fields, the deformation potential interaction becomes the dominant one. This result is not observed in GaAs (see Fig. 2), where the PE coupling, in general, governs the relaxation process.

For GaAs case, the confinement does not produce important modifications in the \( g \)-factor and the scattering rates shown in the Fig. 2 reveal the negative character of the \( g \)-factor for the considered dot sizes. Also, the Fig. 2 shows that the GaAs rates diminish really fast as the dot size increases. For \( R > 10 \) nm, the relaxation times can be as large as seconds and the rates become almost independent of \( R \). This behavior as well as the magnitude of rates are similar to those obtained by Woods [3] when considered other relaxation mechanisms mediated by acoustic phonons.

In conclusion, we have studied the spin relaxation of electrons in InAs and GaAs parabolic quantum dots by considering the phonon modulation induced on the spin-orbit interaction as the relaxation process. For dots based on narrow-gap materials, we have found that the size dependence (spatial localization) of the \( g \)-factor cannot be neglected in the calculation. Also, the deformation potential mechanism can become dominant, especially for large \( B \).
Acknowledgments

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