Gauge invariance of color confinement due to the dual Meissner effect caused by Abelian monopoles

Tsunee Suzuki,1,2 Masayasu Hasegawa,1,2 Katsuya Ishiguro,1,2 Yoshiaki Koma,4,2 and Toru Sekido1

1Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan
2RIKEN Nishina Center for Accelerator-Based Science, Wako, Saitama351-0198, JAPAN
3Integrated Information Center, Kochi University, Kochi 780-8520, Japan
4Numazu College of Technology, Numazu 410-8501, Japan

(Dated: September 8, 2009)

The mechanism of non-Abelian color confinement is studied in SU(2) lattice gauge theory in terms of the Abelian fields and monopoles extracted from non-Abelian link variables without adopting gauge fixing. Firstly, the static quark-antiquark potential and force are computed with the Abelian and monopole Polyakov loop correlators, and the resulting string tensions are found to be identical to the non-Abelian string tension. These potentials also show the scaling behavior with respect to the change of lattice spacing. Secondly, the profile of the color-electric field between a quark and an antiquark is investigated with the Abelian and monopole Wilson loops. The color-electric field is squeezed into a flux tube due to monopole supercurrent with the same Abelian color direction. The parameters corresponding to the penetration and coherence lengths show the scaling behavior, and the ratio of these lengths, i.e., the Ginzburg-Landau parameter, indicates that the vacuum type is near the border of the type 1 and type 2 (dual) superconductor. These results are summarized and supports the existence of a gauge-invariant mechanism of color confinement due to the dual Meissner effect caused by Abelian monopoles.

PACS numbers: 12.38.AW, 14.80.Hv

I. INTRODUCTION

Color confinement in quantum chromodynamics (QCD) is still an important unsolved problem [1]. 't Hooft [2] and Mandelstam [3] conjectured that the QCD vacuum is a kind of a dual superconducting state caused by condensation of magnetic monopoles. The color charges are then confined inside hadrons due to formation of the color-electric flux tube through the dual Meissner effect. However, in contrast to the Georgi-Glashow model [4, 5] or SUSY QCD [6] with scalar fields, it is not straightforward to identify the color-magnetic monopoles in QCD.

An interesting idea to realize this conjecture is proposed by 't Hooft [7], such that SU(3) QCD can be reduced to an Abelian [U(1)]² theory by adopting a partial gauge fixing, and the color-magnetic monopoles appear to π2(SU(3)/[U(1)]²) = Z². The role of monopoles for the confinement mechanism is investigated extensively on the lattice by applying Abelian projection in the maximally Abelian (MA) gauge [8, 9], where monopoles are extracted a la DeGrand-Toussaint [10] as in compact U(1) lattice gauge theory. It is then found that the results strongly support the dual superconducting scenario [11, 12, 13, 14, 15, 16, 17, 18]. The confining properties are dominated by the Abelian fields [11, 13, 19] and monopoles [12, 20, 21, 22, 23, 24], which are called Abelian dominance and monopole dominance, respectively. The color-electric flux is squeezed by the dual Meissner effect [12, 13, 14, 18, 23]. Moreover monopole condensation is confirmed by the energy-entropy balance of the monopole trajectories [22, 26, 27, 28]. These results indicate that there must exist a dual Ginzburg-Landau type theory as an infrared effective theory of QCD [23, 30, 31].

However, there are still serious problems to prove this scenario. Firstly, there are infinite ways of the partial gauge fixing. Since the behavior of the monopoles can depend on the gauge choice, it is not clear if the lattice results in the MA gauge are universal. Note that in the Polyakov (PL) gauge, 't Hooft’s color-magnetic monopoles [7] propagate only in the time direction, which cannot confine static color charges [32]. Secondly, as the 't Hooft scheme essentially uses the Abelian degrees of freedom, it is not explained how non-Abelian color charges are confined.

Recently, we have obtained clear numerical evidences of Abelian dominance and the dual Meissner effect in local unitary gauges such as the F12 and the PL gauges in SU(2) lattice gauge theory [33], where we have used the DeGrand-Toussaint monopoles [10] as in the MA gauge. These results provide us with the following ideas.

1. The DeGrand-Toussaint monopoles on the lattice [10] can be different from the 't Hooft color-magnetic monopoles [7].

2. There must exist a gauge-invariant mechanism of color confinement due to Abelian monopoles [34, 32].

In this paper, we aim to show detailed numerical evi-
dence how these ideas are realized. We investigate the confining properties in SU(2) lattice gauge theory in terms of the gauge-variant Abelian fields and monopolies extracted from non-Abelian link variables without adopting any spatially local or non-local gauge fixing. We find a gauge-invariant Abelian mechanism of color confinement due to Abelian monopoles work even in the continuum limit of SU(2) QCD, although we consider gauge-variant Abelian operators. The results may also apply to SU(3) gauge theory, since the essential features are not altered.

The paper is organized as follows. In Sec. II we explain how to extract the Abelian fields and the monopolies from non-Abelian link variables without gauge fixing. In Sec. III and Sec. IV we compute the static quark-singlet state, the dual Meissner effect for the Abelian monopole Polyakov loop correlators, and find that the string tensions exhibit Abelian dominance and monopole dominance. These potentials also show the scaling behavior with respect to the change of lattice spacing. In Sec. V we discuss implications of our results, i.e., the Abelian fundamental charge defined in an arbitrary color direction is confined by the dual Meissner effect. As the color-neutral state in any Abelian color direction corresponds to the physical color-singlet state, the dual Meissner effect for the Abelian fundamental charge can also explain confinement of non-Abelian color charges. The final section VII is devoted to conclusion and remarks. Our preliminary results are already published in Ref. 36.

II. ABELIAN PROJECTION AND EXTRACTION OF MONOPOLIES

We explain how to extract the Abelian fields and the color-magnetic monopolies from the thermalized non-Abelian SU(2) link variables, 

\[ U_\mu(s) = U_\mu^0(s) + i\vec{\sigma} \cdot \vec{U}_\mu(s) , \tag{1} \]

where \( \vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3) \) is the Pauli matrix. Abelian link variables in one of the color directions, for example, in the \( \sigma^1 \) direction are defined as 

\[ u_\mu(s) = \cos \theta_\mu(s) + i\sigma^1 \sin \theta_\mu(s) , \tag{2} \]

where 

\[ \theta_\mu(s) = \arctan \left( \frac{U_\mu^1(s)}{U_\mu^0(s)} \right) . \tag{3} \]

correspond to the Abelian fields. Without gauge fixing the Abelian fields in any color directions should be equivalent.

We then define the Abelian field strength tensors as 

\[ \Theta_{\mu\nu}(s) = \theta_\mu(s) + \sigma_\nu(\hat{s} + \hat{\mu}) - \theta_\mu(s + \hat{\nu}) - \sigma_\nu(s) \]

\[ = \bar{\Theta}_{\mu\nu}(s) + 2\pi n_{\mu\nu}(s) , \tag{4} \]

where \( \Theta_{\mu\nu} \in [-\pi, \pi] \) and \( n_{\mu\nu}(s) \) is an integer corresponding to the number of the Dirac strings piercing the plaquette. The monopole currents are then defined by 

\[ k_\nu(s) = \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \partial_\mu \bar{\Theta}_{\rho\sigma}(s + \hat{\nu}) \]

\[ = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial_\mu n_{\rho\sigma}(s + \hat{\nu}) \in \mathbb{Z} , \tag{5} \]

where \( \partial_\mu \) is regarded as a forward difference.

III. ABELIAN DOMINANCE

We show the result of the Abelian static potential 36. We generate thermalized gauge configurations using the SU(2) Wilson action at a coupling constant \( \beta = 2.5 \) on the lattice \( N_s^3 \times N_t = 24^3 \times 24 \), where the lattice spacing \( a(\beta = 2.5) = 0.0836(8) \) [fm] is fixed by assuming \( \sqrt{\sigma} = 440 \) [MeV].

By using the multi-level noise reduction method 37, we evaluate the Abelian static potential \( V_A \) from the correlation function of the Abelian Polyakov loop operator

\[ P_A = \exp[i \sum_{k=0}^{N_t-1} \theta_4(s + k\hat{4})] , \tag{6} \]

separated at a distance \( R \) as 

\[ V_A(R_I) = -\frac{1}{aN_t} \ln \langle P_A(0)P_A^*(R) \rangle . \tag{7} \]

The \( q-\bar{q} \) distance \( R \) is improved to \( R_I = (4\pi G(R))^{-1} \) in order to reduce the lattice artifact due to finite-lattice spacing, where \( G(R) \) is the Green function of the lattice Laplacian in three dimensions 38 39. For the multi-level method, the number of sub-lattices adopted is 6 and the sublattice size is 4a.

The result is plotted in Fig. 11 where the non-Abelian static potential computed from the ordinary Polyakov loop correlation function is also plotted for comparison. The number of independent gauge configurations is \( N_{\text{conf}} = 10 \) in both cases, but the number of internal updates in the multi-level method is 15000 for the non-Abelian case and 160000 for the Abelian case. The statistical errors are determined by the jackknife method.

We fit the potential to the usual functional form 

\[ V_{\text{fit}}(R) = \sigma R - c/R + \mu , \tag{8} \]

where \( \sigma \) denotes the string tension, \( c \) the Coulombic coefficient, and \( \mu \) the constant. The result is summarized
FIG. 1: The Abelian static potential in comparison with the non-Abelian one. The lines denote the best fitting curve to the function $V_{\text{fit}}(R)$.

TABLE I: Best fitted values of the string tension $\sigma a^2$, the Coulombic coefficient $c$ and the constant $\mu a$. $V_{\text{NA}}$ denotes the non-Abelian static potential. $N_{\text{up}}$ is the number of internal updates in the multi-level method. FR means the fitting range. The $\chi^2$ for the central value is $\chi^2/N_{\text{df}} < 0.1$.

| $\sigma a^2$ | $c$ | $\mu a$ | FR(R/a) | $N_{\text{up}}$ |
|---------|-----|--------|---------|---------|
| $V_{\text{NA}}$ | 0.0348(7) | 0.243(6) | 0.607(4) | 3.92 - 9.97 | 15000 |
| $V_{\Lambda}$ | 0.0352(16) | 0.231(39) | 1.357(17) | 4.94 - 9.97 | 160000 |

TABLE II: Simulation parameters for the measurement of the static potential and the force from $P_{\Lambda}$, $P_{\text{ph}}$ and $P_{\text{mon}}$. $N_{\text{RGT}}$ is the number of random gauge transformations.

| $\beta$ | $N_s^2 \times N_t^2$ | $\alpha(\beta)$ | $N_{\text{conf}}$ | $N_{\text{RGT}}$ |
|---------|----------------|----------------|----------------|----------------|
| 2.20    | $24^2 \times 4$ | 0.211(7)       | 6000           | 1000           |
| 2.35    | $24^3 \times 6$ | 0.137(9)       | 4000           | 2000           |
| 2.35    | $36^3 \times 6$ | 0.137(9)       | 5000           | 1000           |
| 2.43    | $24^3 \times 8$ | 0.1029(4)      | 7000           | 4000           |

IV. MONOPOLE DOMINANCE

A. The monopole Polyakov loop

We investigate the monopole contribution to the static potential in order to examine the role of monopoles for confinement. The monopole part of the Polyakov loop operator is extracted as follows. Using the lattice Coulomb propagator $D(s - s')$, which satisfies $\partial_s \partial_{s'} D(s - s') = -\delta_{s,s'}$ with a forward (backward) difference $\partial_s \partial_{s'}$, the temporal component of the Abelian fields $\theta_4(s)$ are written as

$$\theta_4(s) = -\sum_{s'} D(s - s') [\partial_{s'} \Theta_{\nu A}(s') + \partial_4 (\partial_{s'} \Theta_{\nu A}(s'))] . \quad (11)$$

Inserting Eq. (11) (and then Eq. (10)) to the Abelian Polyakov loop (9), we obtain

$$P_{\Lambda} = P_{\text{ph}} \cdot P_{\text{mon}} ,$$

$$P_{\text{ph}} = \exp \{-i \sum_{k=0}^{N_t-1} \sum_{s'} D(s + k\hat{A} - s') \partial_{s'} \Theta_{\nu A}(s') \} ,$$

$$P_{\text{mon}} = \exp \{-2\pi i \sum_{k=0}^{N_t-1} \sum_{s'} D(s + k\hat{A} - s') \partial_{s'} n_{\nu A}(s') \} .$$

(12)

We call $P_{\text{ph}}$ the photon and $P_{\text{mon}}$ the monopole parts of the Abelian Polyakov loop, respectively [23]. The latter is due to the fact that the Dirac strings $n_{\nu A}(s)$ lead to the monopole currents in Eq. (5) [10]. Note that the second term of Eq. (11) does not contribute to the Abelian Polyakov loop in Eq. (9).

B. Simulation parameters

We then compute the static potential from the monopole Polyakov loop correlation function. However, since Eq. (12) contains the non-local Coulomb propagator $D(s - s')$ and the Polyakov loop is not written as a product of local operators along the time direction, the multi-level method cannot be applied. Without such a powerful noise reduction method, it is hard to measure the Polyakov loop correlation function at zero temperature with the present available computer resource. Thus
we consider a finite temperature $T \neq 0$ system in the confinement phase. We set $T = 0.8 \, T_c$. In order to examine the scaling behavior of the potential, we simulate the Wilson action on the $24^3 \times (N_t = 4, 6, 8)$ lattices. We choose the gauge coupling for each $N_t$ so as to keep the same temperature. We also investigate the spatial volume dependence of the potential for the $N_t = 6$ case. Simulation parameters are summarized in Table II. The lattice spacing $a(\beta)$ is determined by using the Sommer scale ($r_0 = 0.5 \, [\text{fm}]$) at zero temperature.

C. Noise reduction by gauge averaging

Since the signal-to-noise ratio of the correlation functions of $P_A$, $P_{\text{ph}}$ and $P_{\text{mon}}$ are still very small with no gauge fixing, we adopt a new noise reduction method [36]. For a thermalized gauge configuration, we produce many gauge copies applying random gauge transformations. Then we compute the operator for each copy, and take the average over all copies. It should be noted that as long as a gauge-invariant operator is evaluated, such copies are identical, but they are not if a gauge-variant operator is evaluated as in the present case. The results obtained with this method are gauge-averaged, thus, gauge-invariant.

In practice, we prepare a few thousand of gauge copies for each independent gauge configuration (see Table II). We also apply one-step hypercubic blocking (HYP) [42] to the temporal links for further noise reduction. The short-distance part of the potential may be affected by HYP.

D. Results

We obtain very good signals for the potentials and the forces defined by differentiating the potential with respect to $R$. The results at $\beta = 2.35$ on the $24^3 \times 6$ lattice and on the $36^3 \times 6$ lattice, and at $\beta = 2.43$ on the $24^3 \times 8$ lattice are plotted in Figs. 2 and 3 respectively. The $q$-$\bar{q}$ distances $R$ of the potentials and the forces are improved to $R_I$ and $\bar{R} = (G(R - a) - G(R))^{-\frac{1}{2}}$, respectively. We fit these potentials to the function $V_{\text{fit}}(R)$ in Eq. (8) and extract the string tension and the Coulombic coefficient, which are summarized in Table III. Since the potential and the force at $\beta = 2.20$ on the $24^3 \times 4$ lattice are already published in Ref. [36], we only present the fitting result for this data set.
Abelian dominance is seen again beautifully as in Sec. III Moreover, we observe monopole dominance, i.e., the string tension of the static potential from the monopole Polyakov loop correlation function is identical to that of the non-Abelian static potential, while the potential from the photon Polyakov loop correlation function contains no linear part. It is remarkable that Abelian dominance and monopole dominance for the string tension are almost perfect as explicitly shown in Fig. 5, which also show the good scaling behavior with respect to the change of lattice spacing. We do not see the volume dependence of the string tension as shown in Fig. 6.

These results suggest that although the lattice monopoles defined in Eq. (31) are gauge-dependent, they contain physical gauge-invariant pieces responsible for confinement, which show up after taking the gauge average.

### V. The Abelian Dual Meissner Effect

#### A. Correlation function for the field profile around the $q$-$\bar{q}$ system

We investigate the correlation function $\langle O(r) \rangle_W$ between a Wilson loop $W$ and a local Abelian operator $O$ connected by a product of non-Abelian link variables (Schwinger line) $L$,

$$\langle O(r) \rangle_W = \frac{\langle \text{Tr} \left[ L W(R, T) L^\dagger \sigma^1 O(r) \right] \rangle}{\langle \text{Tr} \left[ W(R, T) \right] \rangle}.$$  \hspace{1cm} (13)

A schematic figure is depicted in Fig. 7.

We shall use the cylindrical coordinate $(r, \phi, z)$ to parametrize the $q$--$\bar{q}$ system, where the $z$ axis corresponds to the $q$--$\bar{q}$ axis and $r$ to the transverse distance as shown in Fig. 8. We are interested in the field profile as a function of $r$ on the mid-plane of the $q$--$\bar{q}$ system.

#### B. Simulation parameters

In this computation, we employ the improved Iwasaki gauge action [45] with the coupling constants $\beta = 1.10$ and 1.28 on the $32^4$ lattice, and $\beta = 1.40$ on the $40^4$ lattice in order to investigate the scaling behavior of the correlation functions with less finite lattice cutoff effects. Simulation parameters are listed in Table IV. The lattice spacings are determined so as to reproduce the physical string tension $\sqrt{\sigma} = 440$ [MeV]. To improve the signal-to-noise ratio, the APE smearing is applied to the spatial

| $V$ | $\sigma_a^2$ | $c$ | $\mu_a$ | $\chi^2/N_{df}$ |
|-----|-------------|-----|---------|---------------|
| $24^3 \times 4$ | 0.18(1) | 0.25(15) | 0.54(7) | 3.9 - 8.5 | 1.00 |
| $V_{\text{NA}}$ | 0.183(8) | 0.20(15) | 0.98(7) | 3.9 - 8.2 | 1.00 |
| $V_{\text{A}}$ | 0.183(6) | 0.25(11) | 1.31(5) | 3.9 - 6.7 | 0.98 |
| $V_{\text{mon}}$ | $-2(1) \times 10^{-4}$ | 0.010(1) | 0.48(1) | 4.9 - 9.4 | 1.02 |
| $V_{\text{ph}}$ | 0.051(3) | 0.49(7) | 0.0132(1) | 1.10(2) | 0.99 |
| $24^3 \times 6$ | 0.072(3) | 0.49(6) | 0.53(3) | 4.0 - 9.0 | 0.99 |
| $V_{\text{NA}}$ | 0.073(4) | 0.41(7) | 1.09(3) | 3.7 - 10.9 | 1.00 |
| $V_{\text{A}}$ | 0.073(4) | 0.44(10) | 1.41(4) | 3.9 - 9.3 | 1.00 |
| $V_{\text{mon}}$ | $-17(3) \times 10^{-4}$ | 0.0131(1) | 0.4717(3) | 5.1 - 9.4 | 0.99 |
| $V_{\text{ph}}$ | 0.072(3) | 0.48(9) | 0.53(3) | 4.6 - 12.1 | 1.03 |
| $36^3 \times 6$ | 0.072(3) | 0.47(6) | 1.10(2) | 4.3 - 11.2 | 1.03 |
| $V_{\text{NA}}$ | 0.073(3) | 0.46(7) | 1.43(3) | 4.0 - 11.8 | 1.01 |
| $V_{\text{A}}$ | 0.073(3) | 0.46(7) | 1.43(3) | 4.0 - 11.8 | 1.01 |
| $V_{\text{mon}}$ | $-10(1) \times 10^{-4}$ | 0.0132(1) | 0.4770(2) | 6.4 - 11.5 | 1.03 |
| $V_{\text{ph}}$ | 0.0415(9) | 0.47(2) | 0.46(8) | 4.1 - 7.8 | 0.99 |
| $24^3 \times 8$ | 0.041(2) | 0.47(6) | 1.10(3) | 4.5 - 8.5 | 1.00 |
| $V_{\text{NA}}$ | 0.043(3) | 0.37(4) | 1.39(2) | 2.1 - 7.5 | 0.99 |
| $V_{\text{A}}$ | 0.043(3) | 0.37(4) | 1.39(2) | 2.1 - 7.5 | 0.99 |
| $V_{\text{mon}}$ | $-6(3) \times 10^{-5}$ | 0.0059(3) | 0.4664(6) | 7.7 - 11.5 | 1.02 |

TABLE III: Best fitted values of the string tension $\sigma_a^2$, the Coulombic coefficient $c$, and the constant $\mu_a$ for the potentials $V_{\text{NA}}, V_{\text{A}}, V_{\text{mon}}$, and $V_{\text{ph}}$. 

![Fig. 4](image-url)
FIG. 5: The $a(\beta)$ dependence of the square root of the non-Abelian, Abelian and monopole string tensions for the same temperature $T = 0.8T_c$. The bottom axis for a set of three data points at the same lattice spacing is slightly shifted to distinguish each other.

FIG. 6: The volume dependence of the square root of the string tensions on the $24^3 \times 6$ and $36^3 \times 6$ lattices at $\beta = 2.35$.

TABLE IV: Simulation parameters for the measurement of the field profile. $n$ and $\alpha$ are the number of smearing steps and the smearing parameter, which are optimized to obtain reasonable signals.

| $\beta$ | $V$   | $a(\beta)$ [fm] | $N_{\text{conf}}$ | $n$  | $\alpha$ |
|---------|-------|------------------|-------------------|------|----------|
| 1.10    | 32    | 0.1069(8)        | 5000              | 80   | 0.2      |
| 1.28    | 32$^a$| 0.0635(5)        | 6000              | 80   | 0.2      |
| 1.40    | 40$^a$| 0.0465(2)        | 7996              | 80   | 0.2      |

C. The penetration depth

We measure all cylindrical components of the color-electric fields $O(s) = E_{A_i}(s) = \Theta_{A_i}(s)$. The results are plotted in Fig. 9. We find that only $E_{A_z}$ has correlation with the Wilson loop. We then fit $\langle E_{A_z}(r) \rangle_W$ to a function $f(r) = c_1 \exp(-r/\lambda) + c_0$ and find that the profile of $\langle E_{A_z}(r) \rangle_W$ is well described by this functional form, i.e., the color-electric field is exponentially squeezed. The fitting curves are also plotted in Fig. 9. The parameter $\lambda$ corresponds to the penetration depth and the values for three gauge couplings are summarized in Table V and plotted in Fig. 10 as a function of lattice spacing $a(\beta)$.

TABLE V: The parameter $\lambda$ corresponding to the penetration depth.

| $\beta$ | $W(R, T)$ | $\lambda$ [fm] | $c_1$ | $c_0$ |
|---------|------------|-----------------|-------|-------|
| 1.10    | W(3, 5)    | 0.1075(13)      | 6.09(18) $\times 10^{-2}$ | 9(2) $\times 10^{-3}$ |
| 1.28    | W(5, 5)    | 0.1077(14)      | 1.02(14) $\times 10^{-2}$ | 4.6(8) $\times 10^{-6}$ |
| 1.40    | W(7, 7)    | 0.106(4)        | 3.40(17) $\times 10^{-3}$ | 1.6(8) $\times 10^{-5}$ |

links of the Wilson loop $[40]$. We use the Wilson loop $W(R = 3, T = 5)$ at $\beta = 1.10$, $W(R = 5, T = 5)$ at $\beta = 1.28$ and $W(R = 7, T = 7)$ at $\beta = 1.40$. Note that the physical $q\bar{q}$ distance is the same ($R = 0.32$ [fm]) for these Wilson loops.
D. The dual Ampère law

To see what squeezes the color-electric field, we study the Abelian (dual) Ampère law derived from the definition of the monopole current in Eq. (5),

$$\nabla \times \vec{E}_A = \partial_t \vec{B}_A + 2\pi \vec{k},$$

(14)

where $B_{A_i}(s) = (1/2)\epsilon_{ijk} \tilde{\Theta}_{jk}(s)$. The correlation of each term with the Wilson loop is evaluated on the same mid-plane of the $q\bar{q}$ system as for the profile measurements of the color-electric field. We find that only the azimuthal components are non-vanishing, which are plotted in Fig. 11. Note that if the color-electric field is purely of the Coulomb type, the curl of the electric field is zero. On the contrary, the curl of the electric field is non-vanishing and is reproduced mostly by the monopole currents. In any case, the dual Ampère law is satisfied, which is a clear signal of the Abelian dual Meissner effect. This result is quite the same as that observed in the MA gauge [17, 18].

E. The coherence length

Let us estimate the coherence length by evaluating the correlation function between the squared monopole density $\langle \mu^2(s) \rangle$ and the Wilson loop [47]. To measure such a correlation function, we use the disconnected correlation function, since the Schwinger lines are cancelled and the connected correlation functions for the squared monopole currents are automatically reduced to the disconnected ones. Simulation parameters, the lattice volume and the gauge couplings are the same as the measurements of the color-electric field profile, but the number of gauge configurations is increased, $N_{\text{conf}} = 5500$ for $\beta = 1.10$ and $N_{\text{conf}} = 11887$ for $\beta = 1.40$. For $\beta = 1.28$ we use the same number of configurations $N_{\text{conf}} = 6000$. The physical $q\bar{q}$ distance is again fixed to $R = 0.32\text{ [fm]}$. To reduce the noise, we further produce $N_{\text{RGT}} = 100$ gauge copies for each independent configuration by applying the random gauge transformations and take gauge-averaging.

The results are plotted in Fig. 12. We then fit the profile of $\langle k^2(\vec{r}) \rangle_W$ to the functional form $g(r) = c'_1 \exp(-\sqrt{2r}/\xi) + c'_0$, where the parameter $\xi$ corresponds to the coherence length. We obtain the values for $\xi$ as summarized in Table VI. The coherence length shows the scaling behavior as demonstrated in Fig. 13 as a function of lattice spacing $a(\beta)$. 
F. The vacuum type

Taking the ratio of the penetration depth and the coherence length, the GL parameter $\sqrt{2}\kappa = \lambda/\xi$ can be estimated, which characterizes the type of the superconducting vacuum. The results are plotted in Fig. 14 against lattice spacing $a(\beta)$. We obtain $\sqrt{2}\kappa = 1.04(7), 1.19(5)$ and $1.09(8)$ for $\beta = 1.10, 1.28$ and 1.40, respectively.

We find that the GL parameter shows the scaling behavior and the value is about one. This means that the vacuum type is near the border between the type 1 and 2 dual superconductor. However, we note that the physical spatial size of the Wilson loop used in the present simulations is still small ($R = 0.32$ [fm]). Clearly, further quantitative studies with larger Wilson loops are needed to determine the definite value.

VI. NON-ABELIAN COLOR CONFINEMENT

Let us consider what is induced from the above numerical results.

Since gauge fixing is not applied in these computations, Abelian fields in any color directions are equivalent. Thus, our result is interpreted as that the color-electric fields in all color directions are squeezed and the Abelian (monopole) string tensions in all color directions are the
same as the non-Abelian string tension. This indicates that QCD contains a gauge-invariant Abelian mechanism of confinement which is not related to the specific gauge fixing. Namely Abelian monopoles in three color directions are condensed in the vacuum of the confinement phase of SU(2) QCD.

Let us denote quark fields having charge $1/2$ and $-1/2$ in the $\sigma_3$ direction, respectively, as $u_3$ and $d_3$. Then local mesonic states, $u_3 \bar{u}_3$ and $d_3 \bar{d}_3$, are Abelian color neutral in the $\sigma_3$ direction. Consider next

\[
\begin{align*}
    u_1 &= \frac{u_3 + d_3}{\sqrt{2}}, & u_2 &= \frac{i u_3 + d_3}{\sqrt{2}}, \\
    d_1 &= \frac{u_3 - d_3}{\sqrt{2}}, & d_2 &= \frac{i u_3 - d_3}{\sqrt{2}}.
\end{align*}
\]

$u_1$ and $d_1$ ($u_2$ and $d_2$) are quark fields having charge $1/2$ and $-1/2$ in the $\sigma_1$ ($\sigma_2/2$) direction. Using these expressions, the quark-gluon coupling term is written as

\[
\begin{align*}
    \bar{\psi} \gamma^\mu g_\mu^a \psi A^a_\mu &= \frac{1}{2} (\bar{u}_3 \gamma_\mu d_3 + \bar{d}_3 \gamma_\mu u_3) A^{1}_\mu \\
    &\quad - \frac{1}{2} (\bar{u}_3 \gamma_\mu d_3 - \bar{d}_3 \gamma_\mu u_3) A^{2}_\mu \\
    &\quad + \frac{1}{2} (\bar{u}_3 \gamma_\mu u_3 - \bar{d}_3 \gamma_\mu d_3) A^{3}_\mu, \\
    &= \frac{1}{2} (\bar{u}_1 \gamma_\mu u_1 - \bar{d}_1 \gamma_\mu d_1) A^{1}_\mu \\
    &\quad + \frac{1}{2} (\bar{u}_2 \gamma_\mu u_2 - \bar{d}_2 \gamma_\mu d_2) A^{2}_\mu \\
    &\quad + \frac{1}{2} (\bar{u}_3 \gamma_\mu u_3 - \bar{d}_3 \gamma_\mu d_3) A^{3}_\mu,
\end{align*}
\]

where the first equation (13) is expressed in terms of $u_3$ and $d_3$ alone. Consider local mesonic states $u_1 \bar{u}_1$ and $d_1 \bar{d}_1$ ($u_2 \bar{u}_2$ and $d_2 \bar{d}_2$) which are Abelian color neutral in the $\sigma_1$ ($\sigma_2$) direction. When we look at the states $u_1 \bar{u}_1$ and $d_1 \bar{d}_1$ in the $\sigma_3$ direction, they are written as the sum of color-neutral and color-charged states:

\[
\begin{align*}
    u_1 \bar{u}_1 &= \frac{1}{2} (u_3 \bar{u}_3 + d_3 \bar{d}_3 + u_3 \bar{d}_3 + d_3 \bar{u}_3), \\
    d_1 \bar{d}_1 &= \frac{1}{2} (u_3 \bar{u}_3 + d_3 \bar{d}_3 - u_3 \bar{d}_3 - d_3 \bar{u}_3).
\end{align*}
\]

The same observation applies to the color-neutral states $u_2 \bar{u}_2$ and $d_2 \bar{d}_2$ in the $\sigma_2$ direction. However, we find that

\[
\begin{align*}
    u_1 \bar{u}_1 + d_1 \bar{d}_1 &= u_2 \bar{u}_2 + d_2 \bar{d}_2 = u_3 \bar{u}_3 + d_3 \bar{d}_3
\end{align*}
\]

are Abelian color neutral in all color directions. The state (19) is nothing but the non-Abelian color singlet state.

This example tells us that the Abelian color-neutral state in any color directions corresponds to the physical non-Abelian color-singlet state. Hence, the confinement of non-Abelian color charges can be explained in terms of the Abelian dual Meissner effect due to Abelian monopoles. To the authors knowledge, this is the first paper that explains the confinement of non-Abelian color charges only in terms of the Abelian dual Meissner effect.

VII. CONCLUDING REMARKS

We make some concluding remarks. The Abelian gauge fields extracted from the thermalized non-Abelian link fields contain originally topological monopoles responsible for the confinement mechanism of non-Abelian color charges even in the continuum limit. Our results presented in this paper are almost the same as those obtained in the maximally Abelian gauge. This suggests that the MA gauge fixing is the easiest method to extract the physical ingredients of the monopoles, since we do not need very precise time-consuming simulations in the MA gauge as done here.

In the lattice Landau gauge, it is known that no monopoles exist [48] if one uses DeGrand-Toussaint definition and the magnetic displacement current takes a
role of monopoles in the dual Ampère law. How to interpret the existence of a gauge-invariant Abelian confinement mechanism in the framework of the Landau gauge? Abelian monopoles are as a whole gauge-variant without gauge fixing, but they may contain a gauge-invariant physical component and a gauge-variant unphysical one. The compatible interpretation would be that the Landau gauge is a special gauge in which the unphysical gauge variant piece apparently cancels the physical one in the DeGrand-Toussaint monopoles, but at the same time, the role of physical monopoles are carried by the color-magnetic displacement current, which is just a matter of definition of monopoles on the lattice. On the other hand, in the MA gauge, the main part of the DeGrand-Toussaint monopoles is a physical component.

If there exist physical gauge-invariant ingredients of Abelian monopoles, one could observe them in the real experiment [49]. To find the effect in the confinement and also in the deconfinement phases is a very interesting topic in the future.

Acknowledgments

The numerical simulations of this work were done using RSCC computer clusters in RIKEN and SX-8 computer at RCNP of Osaka University. The authors would like to thank RIKEN and RCNP for their support of computer facilities. The authors are also supported by JSPS and DFG under the Japan-Germany Research Cooperative Program. One of the authors (Y.K.) is partially supported by the Ministry of Education, Science, Sports and Culture, Japan, Grant-in-Aid for Young Scientists (B) (20740149).

[1] K. Devlin, The millennium problems: the seven greatest unsolved mathematical puzzles of our time, Basic Books, New York (2002).

[2] G. ’t Hooft, in Proceedings of the EPS International, edited by A. Zichichi, p. 1225, 1976.

[3] S. Mandelstam, Phys. Rept. 23, 245 (1976).

[4] G. ’t Hooft, Nucl. Phys. B79, 276 (1974).

[5] A. M. Polyakov, Nucl. Phys. B120, 429 (1977).

[6] N. Seiberg and E. Witten, Nucl. Phys. B426, 19 (1994).

[7] G. ’t Hooft, Nucl. Phys. B190, 455 (1981).

[8] A. S. Kronfeld, M. L. Laursen, G. Schierholz, and U. J. Wiese, Phys. Lett. B198, 516 (1987).

[9] A. S. Kronfeld, G. Schierholz, and U. J. Wiese, Nucl. Phys. B293, 461 (1987).

[10] T. A. DeGrand and D. Toussaint, Phys. Rev. D22, 2478 (1980).

[11] T. Suzuki, Nucl. Phys. Proc. Suppl. 30, 176 (1993).

[12] V. Singh, D. A. Browne, and R. W. Haymaker, Phys. Lett. B306, 115 (1993).

[13] S. Ejiri, S. Kitahara, T. Suzuki, and K. Yasuta, Phys. Lett. B400, 163 (1997), hep-lat/9608133.

[14] M. Chernodub and M. I. Polikarpov, (1997), hep-th/9710205.

[15] G. S. Bali, C. Schlichter, and K. Schilling, Prog. Theor. Phys. Suppl. 131, 645 (1998).

[16] T. Suzuki, Prog. Theor. Phys. Suppl. 131, 633 (1998).

[17] Y. Koma, M. Koma, E.-M. Ilgenfritz, T. Suzuki, and M. I. Polikarpov, Phys. Rev. D68, 094018 (2003).

[18] Y. Koma, M. Koma, E.-M. Ilgenfritz, and T. Suzuki, Phys. Rev. D68, 114504 (2003).

[19] S. Kitahara, Y. Matsubara, and T. Suzuki, Prog. Theor. Phys. 93, 1 (1995), hep-lat/9411036.

[20] J. D. Stack, S. D. Neiman, and R. J. Wensley, Phys. Rev. D50, 3399 (1994), hep-lat/9404014.

[21] H. Shiba and T. Suzuki, Phys. Lett. B333, 461 (1994), hep-lat/9404015.

[22] H. Shiba and T. Suzuki, Phys. Lett. B351, 519 (1995), hep-lat/9408004.

[23] T. Suzuki, S. Ilyar, Y. Matsubara, T. Okude, and K. Yotsuji, Phys. Lett. B347, 375 (1995).

[24] S. Ejiri, S. Kitahara, Y. Matsubara, and T. Suzuki, Phys. Lett. B343, 304 (1995), hep-lat/9407022.

[25] G. S. Bali and C. Schlichter, Prog. Theor. Phys. Suppl. 122, 67 (1996).

[26] S. Kato, N. Nakamura, T. Suzuki, and S. Kitahara, Nucl. Phys. B520, 323 (1998).

[27] M. Chernodub et al., Phys. Rev. D62, 094506 (2000).

[28] K. Ishiguro, T. Suzuki, and T. Yazawa, JHEP 01, 038 (2002), hep-lat/0112022.

[29] Z. F. Ezawa and A. Iwazaki, Phys. Rev. D25, 2681 (1982).

[30] T. Suzuki, Prog. Theor. Phys. 80, 929 (1988).

[31] S. Maedan and T. Suzuki, Prog. Theor. Phys. 81, 229 (1989).

[32] M. Chernodub, Phys. Rev. D69, 094504 (2004), hep-lat/0308031.

[33] T. Sekido, K. Ishiguro, Y. Koma, Y. Mori, and T. Suzuki, Phys. Rev. D76, 031501 (2007).

[34] J. M. Carmona, M. D’Elia, A. Di Giacomo, B. Lucini, and G. Paffuti, Phys. Rev. D64, 114507 (2001).

[35] P. Cea and L. Cosmai, Phys. Rev. D62, 094510 (2000).

[36] T. Suzuki, K. Ishiguro, Y. Koma, and T. Sekido, Phys. Rev. D77, 034502 (2008), 0706.4366.

[37] M. Lüscher and P. Weisz, JHEP 09, 010 (2001).

[38] S. Necco and R. Sommer, Nucl. Phys. B622, 328 (2002).

[39] M. Lüscher and P. Weisz, JHEP 07, 049 (2002).

[40] M. C. Ogilvie, Phys. Rev. D59, 074505 (1999).

[41] M. Faber, J. Greensite, and S. Olejnik, JHEP 01, 008 (1999).

[42] A. Hasenfratz and F. Knechtli, Phys. Rev. D64, 034504 (2001).

[43] P. Cea and L. Cosmai, Phys. Rev. D52, 5152 (1995).

[44] A. Di Giacomo, M. Maggiore, and S. Olejnik, Phys. Lett. B236, 199 (1990).

[45] Y. Iwasaki, Nucl. Phys. B258, 141 (1985).

[46] APE, M. Albanese, M. Maggiore, and S. Olejnik, Phys. Lett. B258, 199 (1990).

[47] A. Hasenfratz and F. Knechtli, Phys. Rev. D64, 034504 (2001).

[48] A. Hasenfratz and F. Knechtli, Phys. Rev. D64, 034504 (2001).

[49] T. Suzuki, Prog. Theor. Phys. 115 (1993).

[50] T. Suzuki, Prog. Theor. Phys. 115 (1993).

[51] T. Suzuki, Prog. Theor. Phys. 115 (1993).

[52] T. Suzuki, Prog. Theor. Phys. 115 (1993).
Phys. Rev. \textbf{D78}, 074021 (2008), 0807.5012.