Strong Anti-Gravity
Life in the Shock Wave

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Abstract

Strong anti-gravity is the vanishing to all orders in Newton’s constant of the net force between two massive particles at rest. We study this phenomenon and show that it occurs in any effective theory of gravity which is obtained from a higher-dimensional model by compactification on a manifold with flat directions. We find the exact solution of the Einstein equations in the presence of a point-like source of strong anti-gravity by dimensional reduction of what is a shock-wave solution in the higher-dimensional model.
1 Introduction.

A distinctive feature of gravity is that it is always attractive and therefore impossible to shield. This property is currently understood in terms of the spin of the graviton, which is two and therefore an even number—the rule being that the exchange of particles of even spin gives rise to forces that are always attractive, whereas the exchange of particles of odd spin gives forces which are attractive for charges of opposite sign and repulsive for charges of the same sign.

By contrast, the term anti-gravity has come to represent all those physical phenomena in which the usual gravitational potential—the static limit of which is Newton’s inverse-square law—is modified to accommodate repulsive gravitational forces. A simple example of such a modification is a theory in which anti-matter gravitationally repels ordinary matter the same way as electric charges of the same sign do.

This is clearly a fascinating subject with many implications: from the possible check of anti-gravity against experiments to the several theoretical issues that are involved, the principle of equivalence and energy conservation among others (see [1] for a recent and comprehensive review of the subject).

However, in the present paper the term anti-gravity is used in the more circumscribed sense proposed by J. Scherk [2]: it stands for a theory in which the gravitational potential between two masses at rest vanishes in the Newtonian approximation because in addition to the attractive exchange of spin-2 gravitons there also exists a repulsive contribution coming from the exchange of spin-1 particles, the graviphotons. Matter and anti-matter behave in the ordinary manner under the exchange of the gravitons but, whenever appropriately charged, also couple to the graviphotons (and, possibly, to spin-0 graviscalars as well). The respective couplings are arranged to give a vanishing net force.

Such a fine tuning of the coupling strengths, as contrived as it may seem at first, was found to take place inside $N = 2, D = 4$ supergravity by Zachos [3]. Other examples were soon pointed out within the supergravity family by Scherk [2], and anti-gravity was accordingly promoted from being a mere curiosity to a potentially interesting phenomenon.

In this paper we reconsider Scherk’s anti-gravity. We show, first of all, that it actually comes in two varieties: weak anti-gravity, like in the $N = 2$ model, in which the vanishing of the static potential does not persist in the non-linear theory,
and *strong* anti-gravity, like in the $N = 8$ model, in which it does. Whereas the weak kind seems to be of a more accidental nature, the strong one is a widespread phenomenon. It takes place (at sufficiently small distances) whenever:

- the $D$ dimensional theory is obtained after compactification of some higher dimensional model; and

- the states for which it occurs are massless and neutral in the higher-dimensional theory but gain mass and charge in the process of compactification.

From supergravity to string theory, most modern theories of quantum gravity belong to such a class of models—a fact that speaks for the relevance of anti-gravity.

Supersymmetry, as we shall discuss, is not among the requirements necessary for anti-gravity to take place, even though (when present) it allows an elegant characterization of the phenomenon. It also played a historical role inasmuch as all models of anti-gravity were first found inside supersymmetrical theories.

The very general behavior outlined above, as well as the extent to which it applies, cry out for a simple physical explanation. This can be found in the higher dimensional theory—as it was realized already by Scherk himself [2]: gravitons, graviphotons and graviscalars are just different components of the higher-dimensional metric tensor and the anti-gravity phenomenon follows directly from the well-known laws of gravitational interactions of massless particles. In particular, the exact solution of an anti-gravitating source is just a generalized shock wave moving in one of the compact directions. The $D$ dimensional world lives so-to-speak inside such a shock wave. By turning the argument around, one can say that strong anti-gravity is only a complicated way to describe in $D$ dimensions what is a very simple metric in $D + E$. Nonetheless, in the process something remarkable has occurred: we have found an exact solution of Einstein’s equations in $D$ dimensions for a theory in which graviphotons and graviscalars are present together with the gravitons. In four dimensions it is a solution of the complete field equations of $N = 8$ (or $N = 4$) supergravity.

This anti-gravity solution—which we arrived at through the shock-wave analogy outlined above—corresponds in $D$ dimensions to a static and spherically symmetric solution of gravity coupled to Maxwell and scalar fields for an extremal value of the respective charges. These charged black hole solutions have been discussed in refs. [1, 2, 3] and in [4, 5] with reference to anti-gravity. Ref. [7] discusses the Newtonian limit of the solutions of [5] to check for the presence of weak anti-gravity.
In this paper we study the motion of a charged test particle in the anti-gravity background and find a complete and very simple solution to the equations of motion. In particular, we prove that the static potential vanishes not just in the Newtonian limit but at arbitrarily short distances from the source, that is, compactification of pure gravity does indeed lead to a theory of strong anti-gravity.

We proceed to establish the most general conditions under which strong anti-gravity persists at long distances. In most cases the graviphotons and graviscalars will be massive below the scale of compactification. While the existence of a Killing vector on the compact manifold ensures the presence of a massless graviphoton, the appropriate graviscalar will be massive unless the manifold has a flat direction. More specifically, anti-gravity is a long-distance phenomenon in $D$ space-time dimensions if

- the extra $E$ dimensions are compactified on a manifold that is Ricci flat and of holonomy $SO(E - 1)$; only compact manifolds with a flat direction satisfy these requirements.

The physical viability of anti-gravity can be discussed in the light of our results. Calabi-Yau manifolds, the holonomy of which is $SU(3)$, do give a mass to the graviphotons [8] and, therefore, in these models anti-gravity is limited to energies larger than the compactification scale. Below that scale, graviphotons and graviscalars give rise to Yukawa-like short-range forces and effectively decouple from the gravitational interactions.

The content of the paper is as follows. In section 2 we review the two models first proposed by Scherk, that is, $N = 2$ and $N = 8$ supergravity and add to them a model inspired by the superstring. Section 3 contains a study of anti-gravity in the more general setting outlined above: the Einstein-Hilbert action is compactified on tori and the matter fields are taken to be massless multiplets with momenta in one or more of the compact directions. We find anti-gravity by considering the Newtonian potential for the interaction of these states. In section 4 we write the exact solution of such an anti-gravitating source and discuss the corresponding motion of a test particle. In section 5 we consider the most general compactification scheme that allows long-range anti-gravity. Finally, section 6 contains our conclusions. We have tried to include all the material necessary to make the paper as self-contained as possible.
2 Three examples of anti-gravity.

In this section we briefly review the two models of anti-gravity originally discussed by Scherk [2] and add one of our own: type-II superstring theory with toroidal compactification.

Of the terms entering the static potential, only those linear in Newton’s constant $G_N$ are here taken into account. A discussion of the same models in the fully non-linear theory is postponed to section 4.

2.1 The N=2 model.

This model was first discussed by Zachos [3]. The graviton multiplet in $D = 4$, $N = 2$ supergravity contains, besides the graviton and the two gravitinos, a single vector field. Within the framework of field theory the only way to introduce massive matter is by means of a multiplet with central charge

$$ Z = 2m, \quad (2.1) $$

where $m$ is the mass of the multiplet. This is precisely the value of the central charge which reduces the dimension of the representation to that of the massless representation [4] and thus ensures that only matter fields of spin 0 and spin 1/2 enter. The vector field gauges the central charge. The corresponding value of the charge is

$$ q = \kappa m / \sqrt{2}, \quad (2.2) $$

where $\kappa^2 = 8\pi G_N$. The value (2.2) for the charge is exactly the one needed in the static limit to obtain a cancellation between gravitational and “electric” forces. The static Newtonian potential is

$$ V(r) = -\frac{\kappa^2 m_1 m_2}{8\pi} \frac{1}{r} (1 - \epsilon_1 \epsilon_2), \quad (2.3) $$

where $\epsilon = +1$ for particles and $-1$ for antiparticles. In eq. (2.3) the cancellation is between the attractive spin-2 graviton and the repulsive spin-1 graviphoton. Because no other particles partake in the interaction, $N = 2$ is the simplest instance of anti-gravity one can think of.
2.2 The N=8 model.

The second example of a theory with anti-gravity is $N=8$, $D=4$ supergravity, a model extensively studied by Scherk [2].

The $N=8$, $D=4$ supergravity theory [10] can be obtained by dimensional reduction from $N=1$ supergravity in eleven dimensions [11]. All massless particles are unified in the graviton multiplet, which contains, besides the graviton and the 8 gravitinos, 28 spin 1, 56 spin 1/2 and 70 spin 0 particles. It is not possible (within the framework of supersymmetric quantum field theory) to couple this theory to any kind of matter, because any massive multiplet contains (at least) particles with spins up to two and it is not known how to write consistent field theories for massive particles with spin greater than one.

Scherk [2] obtains massive matter from the graviton multiplet itself by means of a generalized dimensional reduction [12, 13] that breaks the $N=8$ supersymmetry. The theory is first reduced to unbroken $N=8$ supergravity in five dimensions by ordinary dimensional reduction. Then—in going from five to four dimensions—the global SO(6) invariance present in the spectrum of that theory (which describes global rotations of the six coordinates already compactified) can be used to introduce a dependence on the coordinate $x^5$:

$$\phi(x^\mu, x^5) = \exp(iM x^5)\phi(x^\mu), \quad (2.4)$$

where $M$ is an element of the SO(6) Lie algebra containing three arbitrary parameters with the dimension of a mass. The field $\phi$ becomes multivalued in going around the fifth direction but the ambiguity only amounts to a symmetry transformation. This way, any field in $D=5$ that transforms non-trivially under SO(6) acquires a mass in $D=4$ [13].

Among the fields that remain massless is the five-dimensional graviton field. In four dimensions this decomposes into a graviton, a spin-1 graviphoton and a spin-0 graviscalar. The static potential between any massive states of like charges vanishes by virtue of the repulsion of the graviphoton balancing the total attraction due to the graviton and graviscalar:

$$V(r) = -\frac{\kappa^2 m_1 m_2}{8\pi r} (1 - 4\epsilon_1 \epsilon_2 + 3), \quad (2.5)$$

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1 The global SO(6) symmetry can actually be extended to a global E(6) with maximal compact subgroup Sp(8) of rank four. This way, a fourth mass parameter can be introduced [14].
where the terms in the bracket arise from spin-2, spin-1 and spin-0 exchange, respectively.

2.3 The type-II superstring.

Modern string theory was still to come when anti-gravity first appeared. It is however straightforward to add to the preceding two examples a model based on the superstring. This boils down to another way of introducing massive matter in the $N = 8, D = 4$ model.

The $N = 8$ model can be obtained from an $N_+ = N_- = 1$ supergravity in $D = 10$ dimensions. This theory in turn can be considered the low-energy limit of a type-II superstring [8]. Now, states with non-vanishing momentum in the compact direction will be seen as massive states in four dimensions, as it is always the case in Kaluza-Klein theories.

At this point, it is important to distinguish between states which are massive already in ten dimensions (i.e. massive excitations of the string) and states that are massless in ten dimensions and have only a mass in four dimensions by virtue of a nonzero compact momentum.

The difference between these two kinds of states shows up in their static interactions, which are easily obtained from the four-point Veneziano amplitudes in the limit $t \to 0$ (where $t$ and $s$ below are the usual Mandelstam variable). For instance, the elastic scattering of two massive string states

$$|\Psi_i\rangle = B_{\mu\nu\rho\bar{\mu}\bar{\nu}} \left( \psi_{-1/2}^\mu \psi_{-1/2}^\nu \right) \left( \bar{\psi}_{-1/2}^\rho \bar{\psi}_{-1/2}^{\bar{\mu}} \right) |k; 0\rangle; \quad i = 1, 2 \quad (2.6)$$

with mass $\alpha' m_{10}^2 = 4$ gives the amplitude

$$A(1234) \sim -\kappa^2 \frac{(s - 2m_{10}^2)^2}{t} B_1 \cdot B_2 B_3 \cdot B_4, \quad (2.7)$$

where all the momenta are taken to be four-dimensional. In the static limit the amplitude (2.7) is nonzero and these states do not produce any anti-gravity.

If we consider instead the elastic scattering of two massless states

$$|\Psi_i\rangle = \epsilon^i_{\mu\bar{\nu}} \psi^\mu_{-1/2} \bar{\psi}^{\bar{\nu}}_{-1/2} |k; 0\rangle; \quad i = 1, 2 \quad (2.8)$$

with a conserved compact momentum $p^5$ in the fifth direction, we obtain in four dimensions and in the static limit

$$A(1234) \sim -\frac{4\kappa^2}{t} m_1^2 m_2^2 (1 - \epsilon_1 \epsilon_2)^2 \epsilon_1 \cdot \epsilon_2 \epsilon_3 \cdot \epsilon_4, \quad (2.9)$$

which is equivalent to (2.7). The states (2.8) therefore do give rise to anti-gravity.
3 The essence of anti-gravity.

The three theories considered in the previous section are very different, differing in the content of their massless multiplet as well as in the way the massive states are introduced. Yet all three of them contain anti-gravity. This fact indicates that some general principle is at work. In this section we prove that anti-gravity is a general feature of any theory in $D$ dimensions which is obtained by toroidal compactification from a theory in $D + E$ dimensions with the following two properties:

- It contains gravity, described at long distances by the Einstein-Hilbert action;
- the low-dimensional massive states descend from massless states which in the Born approximation couple only gravitationally to each other.

3.1 Compactification of Einstein theory.

To prove the above statement we first consider the dimensional reduction of the Einstein-Hilbert action in $D + E$ dimensions:

$$\hat{S} = \frac{1}{2\kappa^2} \int \frac{\text{d}^{D+E} \hat{x}}{\rho(E)} \sqrt{-\hat{g}} \hat{R}. \quad (3.1)$$

Before discussing the action (3.1) it is necessary to introduce some notations. Here, and in the following sections, $(D+E)$-dimensional objects carry a hat. The $(D+E)$-dimensional vector $\hat{x}^\mu$ is decomposed as

$$\hat{x}^\mu = (x^\mu; y^\alpha) \quad (3.2)$$

with $\mu = 0, \ldots, D - 1$ and $\alpha = 1, \ldots, E$. We shall refer to $D$-dimensional objects as external and $E$-dimensional as internal. External indices are denoted by letters ($\mu$, $\nu$, $\rho$, ...) from the middle of the Greek alphabet; internal ones by letters ($\alpha$, $\beta$, $\gamma$, ...) from the beginning of it. $\rho(E)$ is the coordinate volume of the toroidal manifold. If the coordinate $y^\alpha$ takes values in the interval $[0; L^\alpha]$ (which is taken to be fixed under reparametrization) then

$$\rho(E) = \prod_{\alpha=1}^{E} L^\alpha. \quad (3.3)$$

This volume factor is inserted into (3.1) to ensure that $\kappa^2$ is the gravitational constant in $D$ rather than in $D + E$ dimensions. Our conventions for the metric and curvature are those of [15].
The $y$-dependence of the $(D + E)$-dimensional metric $\hat{g}_{\mu\nu}$ can be expanded on Fourier modes. The zero mode (corresponding to no dependence on $y$) gives massless states in $D$ dimensions. As in ref. [13], we introduce the following decomposition for the metric tensor:

$$
\hat{g}_{\mu\nu} = \begin{pmatrix}
\delta^\gamma g_{\mu\nu} + 2\kappa^2 A^\alpha_{\mu} A^\beta_{\nu} \phi_{\alpha\beta} & -\sqrt{2}\kappa A^\beta_{\mu} \phi_{\alpha\beta} \\
-\sqrt{2}\kappa A^\alpha_{\nu} \phi_{\alpha\beta} & \phi_{\alpha\beta}
\end{pmatrix}
$$

$$
\hat{g}^{\mu\nu} = \begin{pmatrix}
\delta^{-\gamma} g^{\mu\nu} & \sqrt{2}\kappa\delta^{-\gamma} A^\mu_{\alpha} \\
\sqrt{2}\kappa\delta^{-\gamma} A^\nu_{\beta} & \phi^{\alpha\beta} + 2\kappa^2 \delta^{-\gamma} A^\alpha_{\mu} A^\beta_{\mu}
\end{pmatrix}
$$

(3.4)

Apart from the $D$-dimensional metric $g^{\mu\nu}$, the decomposition (3.4) introduces a total of $E$ graviphotons $A^\alpha_{\mu}$ and $\frac{1}{2}E(E + 1)$ graviscalars $\phi_{\alpha\beta}$—the latter being the components of the compact space metric (whose inverse is $\phi^{\alpha\beta}$). In eqs. (3.4) all external indices are raised and lowered by means of $g^{\mu\nu}$, the internal ones by $\phi^{\alpha\beta}$. Furthermore,

$$
\delta \equiv \det(\phi_{\alpha\beta})
$$

(3.5)

and

$$
\gamma \equiv -\frac{1}{D-2}.
$$

(3.6)

It is straightforward to insert the decomposition (3.4) into the Einstein-Hilbert action (3.1) (see [13] for details) and obtain the following effective theory for the massless fields:

$$
S = \int d^D x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{4} \delta^{-\gamma} F^{\mu\nu\rho\sigma} F_{\mu\nu}^{\rho\sigma} \phi_{\alpha\beta} + \frac{1}{8\kappa^2} g^{\mu\lambda} \partial_{\mu} \phi_{\alpha\beta} \partial_{\nu} \phi^{\alpha\beta} - \frac{1}{8\kappa^2(D-2)} g^{\mu\lambda} \log \delta \partial_{\mu} \log \delta \right\}.
$$

(3.7)

Since we have compactified on tori, the vacuum expectation values of the graviscalars are taken to be

$$
\langle \phi_{\alpha\beta} \rangle = \delta_{\alpha\beta}.
$$

(3.8)
The diagonal graviscalars—$\phi_{\alpha \alpha}$ for $\alpha = 1, \ldots, E$—describe fluctuations in the radii of the tori and the off-diagonal ones describe fluctuations away from orthogonality of the compact directions.

The determinant (3.3) describes variations of the volume of the compact space. As such it is an effective space-time variation of the $D$-dimensional Newton constant. However, the conformal rescaling
\[ \hat{g}^{\mu \nu} = \delta^{-\gamma} g^{\mu \nu}, \] (3.9)
that was introduced in (3.4), ensures that the action for $g_{\mu \nu}$ is the canonical one (as in (3.7)). The volume factor $\delta^{-\gamma}$ now appears as a coupling to matter instead of as a variation of Newton’s constant.

The graviphoton $A_{\mu}^\alpha$ is the gauge boson of the $U(1)$ group of the rigid translations in the direction $dy^\alpha$:
\[ y^\alpha \to \tilde{y}^\alpha = y^\alpha - \epsilon^\alpha(x), \] (3.10)
and $F_{\mu \nu}^\alpha$ is the usual gauge-invariant field strength
\[ F_{\mu \nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha. \] (3.11)
Another way of looking at the graviphotons is that they describe fluctuations away from orthogonality of the internal and external manifolds (see eq. (3.4)).

### 3.2 Coupling to massless matter and propagators.

In order to see anti-gravity in $D$ dimensions we have to couple the theory to some appropriately charged massive matter field. We do this by considering the canonical coupling in $D + E$ dimensions of a massless scalar field $\hat{\Phi}$ to pure gravity
\[ \hat{S}_\Phi = -\frac{1}{2} \int \frac{d^{D+E}\hat{x}}{\rho(E)} \sqrt{-\hat{g}} \hat{g}^{\mu \nu} \partial_\mu \hat{\Phi}(\hat{x}) \partial_\nu \hat{\Phi}(\hat{x}) + O(\hat{\Phi}^3). \] (3.12)
This is the generalization of the similar analysis performed in ref. [2] for the case $E = 1$.

A mass $m_D$ is generated in $D$ dimensions by giving the field $\hat{\Phi}(\hat{x})$ a momentum $p_\alpha$ in the compact directions:
\[ \hat{\Phi}(\hat{x}) = \Phi(x)e^{ip_\alpha y^\alpha}. \] (3.13)
Up to a constant, \( p_\alpha \) is the charge gauged by the graviphoton \( A^\alpha_\mu \). The mass is simply

\[
m^2_D = \delta^{\alpha\beta} p_\alpha p_\beta .
\] (3.14)

The action (3.12), once it has been decomposed into \( D \) dimensions, describes the coupling of \( \Phi \) not only to the massless theory (3.7) but also to the infinite tower of charged massive gravitons, graviphotons and graviscalars that are obtained by giving a \( y \)-dependence to \( \tilde{g}_{\mu\nu} \). Since we are only interested in long-distance physics we can safely restrict the \( y \)-dependence to the field \( \Phi(\hat{x}) \). Integration over the internal coordinates \( y \)'s then yields the conservation of the charges:

\[
p_{\alpha}^{in} + p_{\alpha}^{out} = 0 .
\] (3.15)

It is straightforward to rewrite (3.12) in terms of the decomposition (3.4) and the result is the action

\[
S_\Phi = -\frac{1}{2} \int d^Dx \sqrt{-g} \left\{ g^{\mu\nu} \left( \partial_\mu + i\sqrt{2}\kappa p^{in}_\alpha A^{\alpha}_\mu \right) \Phi \left( \partial_\nu + i\sqrt{2}\kappa p^{out}_\beta A^{\beta}_\nu \right) \Phi - \phi^{\alpha\beta} \left( p^{in}_\alpha \Phi \right) \left( p^{out}_\beta \Phi \right) \delta^{\gamma} \right\} .
\] (3.16)

If we insert the expansions

\[
\sqrt{-gg^{\mu\nu}} \equiv \eta^{\mu\nu} + 2\kappa \tilde{\phi}^{\mu\nu} \\
\phi_{\alpha\beta} \equiv \delta_{\alpha\beta} + 2\kappa h_{\alpha\beta}
\] (3.17)

into (3.16) we obtain a quadratic part describing the canonical propagation of a massive scalar field \( \Phi(x) \) and several cubic parts corresponding to the following Feynman rules (in imaginary time):

\[
\kappa \left\{ p^1_\mu p^2_\nu + p^1_\nu p^2_\mu - \frac{2}{D-2}\eta_{\mu\nu} m^2_D \right\} 
\] (3.18)

\[
q_\alpha \left( p^\mu_2 - p^\mu_1 \right) 
\] (3.19)

\[
\frac{1}{\kappa} \left( q_\alpha q_\beta + \frac{2}{D-2}\kappa^2 m^2_D \delta_{\alpha\beta} \right) .
\] (3.20)
Here the solid lines represent the massive state with charges \( q_\alpha (\alpha = 1, \ldots, E) \) given by

\[ q_\alpha = \sqrt{2}\kappa p_\alpha . \tag{3.21} \]

We see that the graviscalar couples to the matrix of the charges \( q_\alpha q_\beta \), and, because of the rescaling (3.9), also to the trace of the energy-momentum tensor.

The propagators of graviton, graviphoton and graviscalars can be extracted from (3.7) and are given by

\[
\langle \tilde{\phi}^{\mu_1 \nu_1} (p) \tilde{\phi}^{\mu_2 \nu_2} (-p) \rangle = \frac{1}{2p^2} \left\{ \eta^{\mu_1 \mu_2} \eta^{\nu_1 \nu_2} + \eta^{\mu_1 \nu_2} \eta^{\mu_2 \nu_1} - \eta^{\mu_1 \nu_1} \eta^{\mu_2 \nu_2} \right\}, \tag{3.22}
\]

\[
\langle A^\alpha_\mu (p) A^\beta_\nu (-p) \rangle = \frac{1}{p^2} \delta^{\alpha \beta} \eta_{\mu \nu} \tag{3.23}
\]

and

\[
\langle h^1_{\alpha_1 \beta_1} (p) h^2_{\alpha_2 \beta_2} (-p) \rangle = \frac{1}{2p^2} \left\{ \delta_{\alpha_1 \alpha_2} \delta_{\beta_1 \beta_2} + \delta_{\alpha_1 \beta_2} \delta_{\beta_1 \alpha_2} - \frac{2}{D + E - 2} \delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2} \right\}, \tag{3.24}
\]

respectively. We work in De Donder gauge

\[ \partial_\nu \tilde{\phi}^{\mu \nu} = 0 \tag{3.25} \]

for the graviton and Lorentz gauge

\[ \partial_\mu A^\mu_\alpha = 0 \tag{3.26} \]

for the graviphotons.

### 3.3 Anti-gravity at work.

It is now possible to compute in the Born approximation the elastic four-point amplitude for the static exchange between two charged massive states. It comes from the sum of the three Feynman diagrams in fig.1. For simplicity we assume that the two external states have only one non-zero charge

\[ q_i = \sqrt{2}\kappa \epsilon_i m_i \quad \epsilon_i = \pm 1 \quad i = 1, 2 , \tag{3.27} \]
corresponding to a compact momentum that points entirely in one direction, for example $dy^1$.

The static limit yields the following amplitude:

$$A(1234) = \frac{-4\kappa^2}{t} m_1^2 m_2^2 \left\{ \frac{D - 3}{D - 2} + \frac{1}{D - 2} + (\epsilon_1 \epsilon_2)^2 - 2\epsilon_1 \epsilon_2 \right\}$$

$$= \frac{-4\kappa^2}{t} m_1^2 m_2^2 (1 - \epsilon_1 \epsilon_2)^2. \quad (3.28)$$

We have displayed in the first line of eq. (3.28) the different contributions. The first one comes from the graviton exchange. Notice that this term cancels in $D = 3$, where there is no physical graviton. The second and third one are due to the graviscalar $\phi_{11}$, and the last one comes from the exchange of the graviphoton $A^1_{\mu}$. The amplitude turns out to be proportional to $(1 - \epsilon_1 \epsilon_2)^2$ and therefore we witness anti-gravity at work: the static potential vanishes between like charges and is enhanced between opposite charges.

The above balance will be upset if (in the Born approximation) the external states can interact in $D + E$ dimensions by other means beside pure gravity. We would then have to add extra Feynman diagrams to the ones depicted in fig.1. To prevent this from occurring, we can always use an excited mode of the $(D + E)$-dimensional graviton itself (see eq. (2.8)) as external states. This state is neutral with respect to all other gauge groups.

The cancellation (3.28), which may seem rather mysterious at first, has a very simple interpretation in $D + E$ dimensions \[2\]. The amplitude (3.28) is by con-
struction nothing but the amplitude for the gravitational scattering of two massless particles in the special case where the momenta are entirely in the compact direction dy. If \( \epsilon_1 = \epsilon_2 \), the two particles move in the same direction at the speed of light and will never meet, giving a vanishing amplitude (in the center-of-mass frame both carry zero energy). If \( \epsilon_1 = -\epsilon_2 \), the two particles are colliding and it is possible to have a non-zero amplitude.

In fact, if we consider the \((D + E)\)-dimensional four-point amplitude for the gravitational scattering of two massless states, which is given for \( t \to 0 \) by

\[
\hat{A}(1234) = -\frac{2\kappa^2}{t} \{ \hat{p}_1 \cdot \hat{p}_4 \hat{p}_2 \cdot \hat{p}_3 + \hat{p}_1 \cdot \hat{p}_3 \hat{p}_2 \cdot \hat{p}_4 - \hat{p}_1 \cdot \hat{p}_2 \hat{p}_3 \cdot \hat{p}_4 \},
\]

(3.29)

and evaluate it in the special kinematical situation

\[
\hat{p}_1 = -\hat{p}_2 = (m_1, 0; \vec{p}_1^0) \\
\hat{p}_4 = -\hat{p}_3 = (m_2, 0; \vec{p}_2^0)
\]

(3.30)

which corresponds to the static limit in \( D \) dimensions, we obtain

\[
A(1234) = -\frac{4\kappa^2}{t} m_1^2 m_2^2 \left( 1 - \frac{\sum_{\alpha=1}^{E} p_1^\alpha p_2^\alpha}{m_1 m_2} \right)^2.
\]

(3.31)

The amplitude (3.31) is the generalization of eq. (3.28) to the case where all \( E \) charges are non-zero.

### 3.4 Anti-gravity and supersymmetry.

Although the anti-gravity mechanism we have presented does not require supersymmetry, most known candidates for the quantum theory of gravity are endowed with some kind of space-time supersymmetry.

If the \((D + E)\)-dimensional theory is supersymmetric, the \( D \)-dimensional action (3.7) will be part of an extended supergravity theory. In these cases it is possible to give an elegant characterization of the charged massive states that give rise to anti-gravity, namely, they fall into massive multiplets with all central charges in the supersymmetry algebra equal and given by \( Z = \pm 2m \). This may be seen as follows. In \( D + E \) dimensions the states we are considering are massless and fall into a massless multiplet of the \( D + E \) dimensional supersymmetric algebra with no central charge (being massless there is no way to provide a dimensionful central charge). On compactification the \( D + E \) dimensional supersymmetric algebra is
reduced to the extended $D$ dimensional supersymmetric algebra with the compact momenta playing the role of central charges. The charged massive copies of the graviton multiplet will have all central charges $Z = \pm 2m$ simply because this is the only way to obtain a massive representation with the same dimension of the massless graviton multiplet \[1].

### 3.5 Section 2 revisited.

Let us now return to the three examples of anti-gravity presented in section 2 to point out how they fit into the general framework of the present section.

For the type-II superstring (compactified on tori) the situation is clear: if the state (2.8) has at least one compactified momentum, it will be exactly of the charged type we have considered here. Although in ten dimensions we have not only the gravitational field but also the dilaton and the rank two anti-symmetric tensor, neither of these give rise to diagrams of the type shown in fig.1 for the on-shell massless states we have considered\[2]. Hence, anti-gravity is an exact feature of the four dimensional low-energy effective field theory, which is $N = 8$, $D = 4$ supergravity.

The formulation in terms of central charges is also clearly exhibited by the string example. The charged states of the type (2.8) are essentially massive copies of the states in the massless multiplet. In general, those having a momentum $p_\alpha$ in the compact direction $\alpha (\alpha = 1, \ldots, 6)$ fall into a massless multiplet of the $N=8$ supersymmetry algebra with all four central charges equal to

$$Z = 2p_\alpha = \pm 2m$$ \hspace{1cm} (3.32)

as it can be understood by rewriting the ten-dimensional supersymmetry algebra of the superstring (which does not contain a central charge) in the four-dimensional language. Precisely because of (3.32) these multiplets have the same dimension as the massless one (which is 256).

On the contrary, the massive string excitations with mass $m_{10}$ fall into massive representations of the $N_+ = N_- = 1$ supersymmetry algebra. On compactification they too will have a central charge $Z = 2p_\alpha$ but since the 4-dimensional mass is

\[2\]The same is true for the vector field and the rank three anti-symmetric tensor field that describe the massless states in the $R - \overline{R}$ sector of the superstring.
now
\[ m_4 = \sqrt{(p_4)^2 + m_{10}^2}, \] (3.33)

the central charge is not equal to \( \pm 2m_4 \) and the multiplets will retain their higher dimensionality (which for the case of the scalar multiplet describing the first excited level of the string is \( 256 \times 256 \)).

In the model proposed by Scherk, the generalized dimensional reduction breaks the \( N = 8 \) supersymmetry completely and hence gives a mass to most of the 256 states in the \( D = 4 \) graviton multiplet. By construction, all these states are charged under the \( U(1) \) group of the fifth dimension (see eq. (2.4)). Since the corresponding gravivector \( A_{1\mu}^1 \) and graviscalar \( \phi_{11} \) remain massless (as does, of course, the graviton) the balance of forces in (3.28) remains intact.

This example shows that anti-gravity need not be a phenomenon restricted to very massive states coming from an excited momentum in one of the compact directions. Ordinary matter could, therefore, carry graviphoton charges. Unfortunately, the question whether it actually does will not be answered until we have a better understanding of how masses are generated in nature.

The \( N = 2 \) model of Zachos is somewhat a special case. It contains only the graviton multiplet (that is, the graviton, two gravitinos and a single graviphoton) and a massive multiplet with central charge \( Z = 2m \). There are no graviscalars. Even though a theory of \( N = 2 \) supergravity in \( D = 4 \) may be obtained by dimensional reduction of \( N_+ = N_- = 1 \) supergravity in six dimensions, this will contain further massless multiplets in which the second graviphoton and three graviscalars can be found. We see that the \( N = 2 \) model of Zachos cannot be obtained by compactification—the problem being that the \( N = 2 \) multiplets are too small for all graviphotons and graviscalars to fit inside.

4 How strong is anti-gravity?

As we have defined it, anti-gravity is just the cancellation of the Newtonian part of the static potential. It is natural to ask what happens if we take non-linear corrections into account. In this section we investigate this question.
4.1 Non-linear corrections.

The $N = 2$, $D = 4$ model of Zachos is easily dealt with by noting that the bosonic part of the massless theory is just gravity coupled to a single U(1) gauge field. The exact solution generated by a point-mass with charge $Q$ is the Reissner-Nordstrøm space-time geometry \[16\]

$$
\begin{align*}
    ds^2 &= -\left(1 - \frac{2MG_N}{r} + \frac{G_NQ^2}{4\pi r^2}\right) dt^2 + \left(1 - \frac{2MG_N}{r} + \frac{G_NQ^2}{4\pi r^2}\right)^{-1} dr^2 \\
    &\quad + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)
\end{align*}
$$

(4.1)

The solution of the corresponding Maxwell equations gives a static Coulomb potential:

$$
A^0 = -\frac{Q}{4\pi r} \quad ; \quad A^i = 0.
$$

(4.2)

The value of the charge for which anti-gravity occurs is

$$
Q = \frac{\sqrt{2}}{2} \kappa M.
$$

(4.3)

However, it is clear that the ensuing cancellation of forces in the static limit is only a feature of the linear approximation; thereby the theory is an example of weak anti-gravity. Notice that (4.1) contains a horizon (For a discussion of the Reissner-Nordstrøm space-time in this particular case, see [17]).

The $N = 2$ example strongly suggests that weak anti-gravity is associated to the case in which the theory cannot be derived by compactification. On the other hand, the $N = 8$, $D = 4$ model of the other two examples of section 2 belongs to the more general situation of section 3—where anti-gravity arise as a result of compactification—and, as we shall see, represents strong anti-gravity. In this case we expect no horizon to appear. The reason for this is simple. The charged massive state generating the gravitational field in four dimensions is nothing but a massless state moving in one of the compact directions. The exact solution of the metric generated by such a particle is well-known and contains no horizons. It is the Aichelburg-Sexl (AS) metric \[18\].

4.2 The exact solution based on the generalized AS metric.

The Aichelburg-Sexl metric is the solution of Einstein’s equations for a point source moving at the speed of light. It can be obtained by an infinite boost of the
Schwarzschild solution and in $D = 4$ is given by:

$$ds^2 = -du dv - (8G_N E \log r)\delta(u)du^2 + dx_\perp^2,$$  \hspace{1cm} (4.4)

where $u = t - y$, $v = t + y$ and $x_\perp$ is the vector of the components that are transverse with respect to $dy$, the direction of motion of the massless particle. $E$ is the energy of the particle.

What we will actually need is a straightforward generalization of (4.4) to include an arbitrary number of space-time dimensions and a more general energy profile. The solution becomes

$$ds^2 = -dt^2 + dy^2 + d\hat{x}_\perp^2 + f(\hat{x}_\perp, t-y)(dt - dy)^2.$$  \hspace{1cm} (4.5)

Here $y$ can be any one of the $E$ compact coordinates. The metric (4.5) is the most general solution that one can write. It describes the propagation in the direction $dy$ of a shock wave with a shape specified by the function $f(\hat{x}_\perp, t-y)$, which is related to the energy profile $\hat{\rho}(\hat{x}_\perp, t-y)$ by the Einstein equations

$$\nabla^2_{\hat{x}_\perp} f(\hat{x}_\perp, t-y) = -16\pi G_N \hat{\rho}(\hat{x}_\perp, t-y),$$  \hspace{1cm} (4.6)

where $\nabla^2_{\hat{x}_\perp}$ is the flat-space Laplacian in the transverse coordinates. If we take $f(\hat{x}_\perp, t-y) = f(\hat{x}_\perp)\delta(t-y)$, eq. (4.6) describes the shock wave due to a source completely localized in the beam direction $[19]$. The shock wave reaches out to infinity in the transverse directions.

The fact that Einstein equations take the linear form (4.6) is a remarkable feature of the shock wave solution. It means that we can superpose individual solutions to create any kind of profile in the beam direction. In particular, we can choose $f(\hat{x}_\perp, t-y)$ to be independent of $t-y$. This corresponds to a wave that is completely smeared out in the compact direction $dy$. We likewise smear out the wave in the transverse compact directions by taking $f(\hat{x}_\perp, t-y)$ to be a function of $\vec{x}$ only, the $D-1$ non-compact spatial coordinates.

If we now take the energy profile to be

$$\hat{\rho}(\vec{x}) = M \frac{1}{\rho(E)}\delta^{D-1}(\vec{x})$$  \hspace{1cm} (4.7)

this will correspond in $D$ dimensions to having a point-like source at the origin with mass $M$ and charge $q = \sqrt{2\kappa M}$. Accordingly, $f(\vec{x})$ is the spherically symmetric solution of Laplace equation in $D-1$ dimensions:

$$\sum_{i=1}^{D-1} \partial_i \partial_i f(\vec{x}) = -16\pi G_N M\delta^{D-1}(\vec{x}),$$  \hspace{1cm} (4.8)
that is, in terms of the radial coordinate \( r = \left( \sum_{i=1}^{D-1} x_i^2 \right)^{1/2} \),

\[
    f(r) = \frac{2\kappa^2 M r^{3-D}}{\Omega_{D-1}(D-3)},
\]

(4.9)

where

\[
    \Omega_D = 2\frac{\pi^{D/2}}{\Gamma(D/2)}
\]

(4.10)

is the solid angle in \( D \) dimensions.

What we have obtained is an exact solution of the Einstein theory in \( D + E \) dimensions for a fluid of massless particles moving in the direction \( dy \) with a total momentum \( p \). The fluid is homogeneously smeared out in all compact coordinates (hence our solution satisfies all periodicity requirements) but is localized at the origin in the \( D - 1 \) dimensional non-compact space. A \( D \)-dimensional observer is living so-to-speak inside the shock wave at all times.

By construction, then, we have also found the exact solution of the non-linear theory (3.7) corresponding to a charged point mass at rest at the origin. In fact, the solution (4.5), (4.9) can be recasted in \( D \)-dimensional language by using the decomposition (3.4). Taking the compact momentum in the direction \( dy \) one finds

\[
    ds^2 = (1 + f(r))^{-\gamma} \left[ -(1 + f(r))^{-1} \, dt^2 + d\vec{x}^2 \right]
\]

\[
    A^\alpha_\mu = \frac{\epsilon}{\sqrt{2\kappa}} \frac{f(r)}{1 + f(r)} \delta^0_\mu \delta^1_\alpha
\]

\[
    \phi_{\alpha\beta} = \delta_{\alpha\beta} + f(r) \delta^1_\alpha \delta^1_\beta.
\]

(4.11)

Here \( \epsilon \) is the sign of the charge \( q_1 \).

The simplicity of the solution (4.5),(4.9) should be contrasted to the more complicated (4.11). The \( D \)-dimensional field equations of which (4.11) is a solution are quite complicated too. We give them in the appendix. Because only the fields \( \phi_{11} \) and \( A^1_\mu \) are excited in (4.11), the action (3.7) reduces in this case to the simpler one

\[
    S = \int d^Dx \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{4} \exp\left( \frac{D-1}{D-2} \log \phi \right) (F_{\mu\nu})^2 
    - \frac{1}{8\kappa^2} \frac{D-1}{D-2} (\partial_\mu \log \phi)^2 \right\},
\]

(4.12)

where \( F_{\mu\nu} \equiv F^1_{\mu\nu} \) and \( \phi \equiv \phi_{11} \). The fields (4.11) are a solution of the field equations derived from this action.
Various theories of gravity coupled to scalar and Maxwell fields have been considered in the literature [4, 5, 6] of which the action (4.12) and our solution (4.11) are a special case. The idea that the solution could be obtained by boosting the Schwarzschild solution into a compact fifth direction was pointed out in [4].

It should be noted that if we take $D + E = 10$ and $D = 4$ the theory (3.7) is part of the four-dimensional Lagrangian for $N = 8$ (or alternatively $N = 4$) supergravity. Therefore, what we have found is also a solution to the field equations of $N = 8$ (or $N = 4$) supergravity in the case where only one of the graviphoton charges is nonzero and given by eq. (3.21). The other fields that are present in the $N = 2$ ($N = 1$) supergravity Lagrangian in ten dimensions, from which the $N = 8$ ($N = 4$) theory is obtained by dimensional reduction [20, 21], can be consistently put to zero in the ten dimensional equations of motion—as one can check by inspection. A similar observation was made in [4].

4.3 Discussion of the exact solution.

To study the non-linear corrections to anti-gravity we now consider the motion of a test particle of mass $m_D$ and charge $\epsilon' \sqrt{2} \kappa m_D$ in the above solution. From the $D$-dimensional point of view the equations of motion can be obtained by requiring covariant energy-momentum conservation of the combined system (test particle + field). This can be done, and the resulting equations are given in the appendix.

It is, however, much easier to take the $D + E$-dimensional point of view: the test particle is massless with compact momentum $p_1$ and its equations of motion are just the geodesic equations of motion in the shock-wave metric given by (4.3) and (4.3). These are easily obtained and reads:

\[
\begin{align*}
\dot{t} &= \frac{E}{m_D} (1 + f(r)) - \epsilon \epsilon' f(r) \\
\dot{y} &= \frac{E}{m_D} \epsilon f(r) + \epsilon' (1 - f(r)) \\
\dot{\theta} &= \frac{L}{(m_D r^2)} \\
\frac{1}{2} r^2 - \frac{1}{2} \frac{E^2}{m_D^2} \left(1 - \epsilon \epsilon' \frac{m_D}{E}\right)^2 f(r) + \frac{L^2}{2 m_D^2 r^2} = \frac{1}{2} \left(\frac{E^2}{m_D^2} - 1\right).
\end{align*}
\]

In eq. (4.13) the transverse compact directions decouple completely. We have also denoted $y \equiv y^1$. $E$ is the energy of the test particle in $D$ dimensions and $L$ is its
angular momentum. The motion is planar in $D$ dimensions and can be described by introducing polar coordinates $(r, \theta)$ in that plane. The dot denotes differentiation with respect to the proper time $\tau$. In the case of a massless test particle, where $m_D = \epsilon' = 0$, it is necessary to introduce the rescaled parameter $\sigma = \tau/m_D$. Alternatively one may take the limit $E/m_D \gg 1$ in the final results.

The canonical momentum in the $dy$-direction is

$$\pi = \hat{g}_{yt} \dot{t} + \hat{g}_{yy} \dot{y} = m_D \epsilon'. \tag{4.14}$$

By differentiating the radial equations of motion in (4.13) with respect to $\tau$ we obtain (in the case $L = 0$ of radial motion):

$$\ddot{r} = \frac{1}{2} \frac{E^2}{m_D} \left( 1 - \epsilon \epsilon' \frac{m_D}{E} \right)^2 f'(r). \tag{4.15}$$

The right-hand side is either negative or zero. In the static limit $E = m_D$ we recover anti-gravity:

$$\ddot{r} = \frac{1}{2} (1 - \epsilon \epsilon')^2 f'(r) \tag{4.16}$$

that is, the vanishing of the radial acceleration for like charges. It is now an exact result that holds to all orders! Even arbitrarily close to the singularity at $r = 0$ the test particle remains at rest when $\epsilon = \epsilon'$. There is no horizon, as it is also clear from the form (4.11) of the metric: $g_{tt}$ is negative everywhere.

However, as soon as the particle starts moving in the radial direction, $E/m_D > 1$ and gravity becomes slightly stronger than the other forces with the result that the acceleration becomes non-zero and towards the point-mass.

The entire problem of motion contained in (4.13) is in fact mathematically equivalent to the Kepler problem with an effective potential

$$\phi_{\epsilon \epsilon'}^N (r) = -\frac{1}{2} \frac{E^2}{m_D} \left( 1 - \epsilon \epsilon' \frac{m_D}{E} \right)^2 f(r) \tag{4.17}$$

and an effective Newtonian energy (kinetic + potential) which is

$$E_{\epsilon \epsilon'}^N = \frac{1}{2} m_D \left[ \left( \frac{E}{m_D} \right)^2 - 1 \right]. \tag{4.18}$$

Therefore, in $D = 4$ the test particle trajectory will be elliptic, parabolic or hyperbolic in the $(r, \theta)$-coordinate plane, depending on whether $E/m_D$ is less than, equal
to or greater than one. In the latter case, we can compute the deflection angle $\Delta \varphi$ to all orders in $G_N$. The result is

$$\sin \frac{\Delta \varphi}{2} = \frac{z}{\sqrt{1 + z^2}} \quad (D = 4),$$

(4.19)

where

$$z = \frac{2MG_N}{b} \frac{(1 - \epsilon \epsilon' \frac{m_D}{E})^2}{1 - \left(\frac{m_D}{E}\right)^2} \quad (D = 4)$$

(4.20)

and the impact parameter $b$ is defined by $L^2 = b^2(E^2 - m_D^2)$. For a neutral massless test particle (or for a very energetic charged one) we find

$$z = \frac{2MG_N}{b} \quad (D = 4),$$

(4.21)

and in the limit $b \to \infty$ we recover Einstein’s formula for the deflection of light:

$$\Delta \varphi = \frac{4G_N M}{b} \quad (D = 4).$$

(4.22)

Notice the absence in (4.19) of a term proportional to $G_N^2$, a characteristic feature of scattering in a shock-wave metric [19].

The discussion above shows that life inside the shock wave is so simple that the fully relativistic theory is effectively Newtonian. The reason for this remarkable fact lies in the linearity of the full Einstein equations on the class of shock wave metrics.

5 Anti-gravity in general.

In the previous section we have established the presence of strong anti-gravity in any theory of gravity that is obtained from a higher-dimensional theory by toroidal compactification. In this section we consider what happens if we try to relax the condition that the compact dimensions should be tori.

5.1 Propagation in compactified Einstein theory.

We consider again the Einstein-Hilbert action (3.1) in $D + E$ dimensions. But now we take a background more general than (3.8), namely a manifold of the form $M_D \times K$, where $M_D$ is flat $D$-dimensional Minkowski space-time and $K$ is some general compact $E$-dimensional manifold of volume $\rho(E)$. 

21
We may still decompose the $D+E$-dimensional metric into gravitational, gravivectorial and graviscalar fields along the lines of eq. (3.4). Again different $D$-dimensional particles can be identified with different modes on the compact manifold through the ansatz:

$$\begin{align*}
\tilde{\phi}^{\mu\nu}(x,y) &= \tilde{\phi}^{\mu\nu}(x)u(y) \\
A_\alpha^{\beta}(x,y) &= A_\alpha^{\beta}(x)u^{\beta}(y) \\
h^{\alpha\beta}(x,y) &= h(x)u^{\alpha\beta}(y) .
\end{align*}
$$

(5.1)

On the flat manifold $(S^1)^E$ made of tori, the mass as seen in $D$ dimensions was simply the eigenvalue of the Laplacian in $E$ dimensions. Corresponding to the one scalar, $E$ vectorial and $E(E+1)/2$ tensorial zero modes we had a similar number of massless gravitons, graviphotons and graviscalars.

On a curved manifold we still have a unique scalar zero mode—the one that is constant—that gives in $D$ dimensions a massless graviton. On the other hand, for the graviphoton and the graviscalar the situation is more complicated. Their mass is now given by the eigenvalue of a more general operator that involves not only the Laplacian but also the coupling of these modes to the background curvature. Furthermore, the concept of a constant mode becomes non-trivial for vectors and tensors because we must replace the ordinary derivatives by the covariant ones.

Our aim is to identify those compact manifolds $K$ for which the anti-gravity effect persist, that is, for which one graviphoton and one graviscalar remain massless.

To analyze this question we consider the propagating part of the Einstein-Hilbert action (3.1). We expand the $D+E$-dimensional metric (3.4) in the fluctuations $(\tilde{\phi}^{\mu\nu}, A_\alpha^{\beta}, h^{\alpha\beta})$ where, in analogy with (3.17), we take

$$\begin{align*}
\sqrt{-g}g^{\mu\nu} &\equiv \eta^{\mu\nu} + 2\kappa\tilde{\phi}^{\mu\nu} \\
\phi^{(B)}_{\alpha\beta} &\equiv g^{(B)}_{\alpha\beta} + 2\kappa h_{\alpha\beta} .
\end{align*}
$$

(5.2)

By inserting these expansions into eq.(3.4) we find

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \hat{g}^{(B)}_{\hat{\mu}\hat{\nu}} + \hat{h}_{\hat{\mu}\hat{\nu}} ,
$$

(5.3)

where the background metric (after a conformal rescaling of the coordinates in Minkowski space) is given by

$$\hat{g}^{(B)}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} \eta_{\mu\nu} & 0 \\ 0 & g^{(B)}_{\alpha\beta} \end{pmatrix} .
$$

(5.4)
and the fluctuation is parametrized as follows:

\[ \hat{h}_{\mu\nu} = 2\kappa \left\{ \hat{\phi}_{\mu\nu} + \frac{1}{D-2}(\tilde{\phi} - h)\eta_{\mu\nu} \right\} + 2\kappa^2 A^\alpha_{\mu} A^{\alpha}_{\nu} + 2\kappa^2 \left\{ \frac{1}{D-2}(h_{\alpha\beta})^2\eta_{\mu\nu} + \frac{1}{(D-2)^2}h^2\eta_{\mu\nu} \right\} \]

\[ + \frac{4\kappa^2}{D-2}h(\hat{\phi}_{\mu\nu} + \frac{1}{D-2}\tilde{\phi}\eta_{\mu\nu}) + \text{(higher order)} \]

In this section all external indices are raised and lowered with the Minkowski metric and all internal indices with the internal background metric \( g^{(B)}_{\alpha\beta} \). \( \hat{\phi} \) and \( h \) denote the trace of \( \tilde{\phi}_{\mu\nu} \) and \( h_{\alpha\beta} \), respectively. It is now straightforward to expand the action (3.1) in powers of the fluctuation \( \hat{h} \). One obtains [13]

\[ \hat{S} = \hat{S}^{(0)} + \hat{S}^{(1)} + \hat{S}^{(2)} + \ldots \] (5.6)

with

\[ \hat{S}^{(0)} = \frac{1}{2\kappa^2} \int \frac{d^{D+E}}{\rho(E)} \sqrt{-g^{(B)}(B)} \hat{R}^{(B)} \]

\[ \hat{S}^{(1)} = \frac{1}{2\kappa^2} \int \frac{d^{D+E}}{\rho(E)} \sqrt{-g^{(B)}(B)} \hat{h}^{\mu\nu} \left( -\hat{R}^{(B)}_{\mu\nu} + \frac{1}{2}\hat{g}^{(B)}_{\mu\nu} \hat{R}^{(B)} \right) \]

\[ \hat{S}^{(2)} = \frac{1}{2\kappa^2} \int \frac{d^{D+E}}{\rho(E)} \sqrt{-g^{(B)}(B)} \left\{ \frac{1}{8} \hat{h}^2 - \frac{1}{4} \hat{h}^{\mu\nu} \hat{h}_{\mu\nu} \right\} \hat{R}^{(B)} + \hat{h}^{\mu\nu} \hat{h}^{\rho\sigma} \hat{R}^{(B)}_{\mu\nu} \]

\[ + \frac{1}{2} \hat{h}^{\mu\nu} \hat{R}^{(B)}_{\mu\nu} + \frac{4}{4} \nabla_{\mu} \hat{h} \nabla_{\nu} \hat{h} - \frac{1}{2} \nabla_{\mu} \hat{h} \nabla_{\nu} \hat{h} \hat{\mu\nu} \]

\[ + \frac{1}{2} \nabla_{\rho} \hat{h}^{\nu\sigma} \nabla_{\nu} \hat{h}^{\rho\sigma} - \frac{1}{4} \nabla_{\rho} \hat{h}^{\rho\sigma} \nabla_{\nu} \hat{h}^{\mu\nu} \] (5.7)

The covariant derivatives are with respect to the background metric. Similarly \( \hat{h} \) is the trace with respect to \( \hat{g}^{(B)} \). Since we have considered only pure gravity in \( D + E \) dimensions, tadpole terms linear in the fluctuation will appear unless the background metric satisfies Einstein equations in the vacuum, i.e. unless the manifold \( K \) is Ricci flat:

\[ R^{(B)}_{\alpha\beta} = 0 . \] (5.8)
Even though we could imagine creating any kind of background metric by introducing appropriate classical sources, for the moment, we proceed and consider the part of the Einstein-Hilbert action quadratic in the fluctuations ($\tilde{\phi}, A, h$) without making any assumptions on the background curvature. We come back to this point at the end of the section.

5.2 The graviton.

Let us first consider the rather simple case of the graviton. All terms involving derivatives with respect to the coordinates on the compact space vanish and we are left with the following few terms:

$$S^{(2)}_\phi = \frac{1}{2} \int d^D x \left\{ 2 \nabla_\mu \tilde{\phi}_\nu \nabla_\nu \tilde{\phi}_\mu - (\nabla_\mu \tilde{\phi}_\nu)^2 + \frac{1}{D - 2} (\nabla_\nu \tilde{\phi})^2 \right\} - \Lambda \int d^D x \left\{ \frac{2}{(D - 2)^2} \tilde{\phi}^2 - \frac{2}{D - 2} (\tilde{\phi}_{\mu\nu})^2 \right\}. \quad (5.9)$$

The compact space variables have been integrated out (with the zero mode $u$ of eq. (5.1) being normalized to unity). The first three terms give simply the propagator of the graviton in $D$ dimensions after an appropriate gauge-fixing term is added to fix the invariance under external reparametrizations $x^\mu \rightarrow x^\mu - \epsilon^\mu(x)$.

The last terms in eq. (5.9) represent the coupling of the graviton to a cosmological constant

$$\Lambda = -\frac{1}{2} \int \frac{dE}{\rho(E)} \sqrt{g^{(B)}} R^{(B)} \quad (5.10)$$

that is produced by the curvature of the internal space. If we assume that the background satisfies the Einstein equations in vacuum, this cosmological constant vanishes.

5.3 Conditions for a massless graviphoton.

Next we turn our attention to the graviphoton. The part of the action quadratic in the graviphoton field can be written as

$$S^{(2)}_A = \frac{1}{2} \int d^D x \left\{ \nabla_\mu A^\nu \nabla_\nu A_\mu - (\nabla_\mu A^\nu)^2 - m_V^2 A_\mu A_\mu \right\}, \quad (5.11)$$

where the mass $m_V^2$ is given by

$$m_V^2 = -\int \frac{dE}{\rho(E)} \sqrt{g^{(B)}} \left\{ 2 u^\alpha u^\beta R^{(B)}_{\alpha\beta} + \nabla_\gamma u^\beta \nabla_\beta u_\gamma - (\nabla_\gamma u^\alpha)^2 \right\}. \quad (5.12)$$
To arrive at (5.11) and (5.12) we have introduced the ansatz (5.1) and normalized the vector $u^\alpha$ in eq. (5.1) in such a manner that

$$\int \frac{d^E y}{\rho(E)} \sqrt{g^{(B)}_{\gamma\delta}} u^\gamma u^\delta g^{(B)}_{\alpha\beta} = 1.$$  \hspace{1cm} (5.13)

By using the commutation relation

$$[\nabla_\gamma, \nabla_\beta] u^\alpha = -R^{\alpha}_{\delta\beta\gamma} u^\delta , \hspace{1cm} (5.14)$$

we bring the mass term (5.12) into the form:

$$m^2_{V} = \frac{1}{2} \int \frac{d^E y}{\rho(E)} \sqrt{g^{(B)}} (\nabla_\alpha u_\beta + \nabla_\beta u_\alpha)^2 . \hspace{1cm} (5.15)$$

Notice that to arrive at (5.14) we used the gauge invariance $x^\mu \to x^\mu - \epsilon^\mu (y)$ to fix $\nabla_\alpha A^\alpha_\mu = 0$ (and therefore $\nabla \cdot u = 0$).

We thus recover the well-known result that the only way to obtain a massless vector by dimensional reduction of the Einstein-Hilbert action is by having a symmetry of the compact manifold. This symmetry is encoded in the existence of the Killing vector $V^\alpha$ satisfying the Killing vector equation:

$$\nabla_\alpha V_\beta + \nabla_\beta V_\alpha = 0 . \hspace{1cm} (5.16)$$

The Killing vector generates translations of the manifold $K$ along its symmetry direction. The graviphoton field describes a translation of this kind that varies from point to point in $D$-dimensional Minkowski-space. It is of course exactly this local gauge invariance that brings about the masslessness of the graviphoton.

If we restrict our attention to background field configurations without matter energy-momentum (i.e. background manifolds which are Ricci flat), the requirement that we have a Killing vector is very restrictive because

- Any compact Ricci-flat manifold that admits a Killing vector field $V^\alpha$ is flat in the direction of $V^\alpha$, i.e. satisfies

$$R_{\alpha\beta\gamma\delta} V^\alpha = R_{\alpha\beta\gamma\delta} V^\beta = R_{\alpha\beta\gamma\delta} V^\gamma = R_{\alpha\beta\gamma\delta} u^\delta = 0 ; \hspace{1cm} (5.17)$$

moreover,

- The Killing vector is covariantly constant:

$$\nabla_\alpha V^\beta = 0 . \hspace{1cm} (5.18)$$
The proof goes as follows. By assumption
\[ \int dE \sqrt{g(y)} V^\alpha V^\beta R_{\alpha\beta} = 0. \] (5.19)
Using the relation (5.14) we may rewrite this in terms of a commutator of two covariant derivatives:
\[ 0 = \int dE \sqrt{g} V^\alpha [\nabla_\beta, \nabla_\alpha] V^\beta \]
\[ = \int dE \sqrt{g} V^\alpha (\nabla_\beta \nabla_\alpha V^\beta - \nabla_\alpha \nabla_\beta V^\beta) \] (5.20)
Since \( V^\alpha \) is a Killing vector field, \( \nabla \cdot V = 0 \). Next we do a partial integration to obtain
\[ 0 = \int dE \sqrt{g} \nabla_\beta V^\alpha \nabla_\alpha V^\beta \] (5.21)
Since a compact manifold has no boundary, the surface term vanishes. Finally, using the Killing vector condition (5.16) eq.(5.21) is brought into the form:
\[ 0 = \int dE \sqrt{g} (\nabla_\beta V^\alpha)^2, \] (5.22)
from which it follows immediately that the Killing vector is covariantly constant. Such a result is very restrictive because it implies the integrability condition
\[ [\nabla_\alpha, \nabla_\beta] V^\gamma = R^\gamma_{\delta\alpha\beta} V^\delta = 0, \] (5.23)
that tells us that the manifold has to be flat in the direction of the Killing vector. Note that by the symmetries of the Riemann tensor, (5.23) implies all four equations (5.17). This concludes our proof.

If we now choose coordinates such that \( V^\alpha = (1, 0, \ldots, 0) \) the metric will not depend on \( y^1 \). By choosing Gaussian normal coordinates we can make \( g_{\alpha 1} = 0 \) for \( \alpha \neq 1 \). Moreover, the condition \( \nabla_1 V^\alpha \equiv 0 \) implies \( \partial_\alpha g_{11} = 0 \), i.e. by a rescaling of the Killing vector we can obtain \( g_{11} \equiv 1 \).

The resulting form of the metric shows that our compact Ricci-flat manifold \( K \) of dimension \( E \) decomposes into
\[ K = S^1 \times \tilde{K} \] (5.24)
—the direct product of a circle and a Ricci-flat manifold \( \tilde{K} \) of dimension \( E - 1 \).

The same result can be phrased by saying that we have a Ricci-flat manifold with \( \text{SO}(E - 1) \) holonomy. The vector representation \( E \) of \( \text{SO}(E) \) decomposes
under SO$(E - 1)$ into $(E - 1) + 1$, and the singlet corresponds to the existence of a covariantly constant vector. Being covariantly constant, it is a Killing vector. As such, it gives rise to a single massless graviphoton.

In the special case of a superstring being compactified from ten to four dimensions, the presence of a massless graviphoton requires SO(5) holonomy. It is well known that to have N=1 supersymmetry in four dimensions one needs instead SU(3) holonomy \[22\]. These two holonomy groups (both of rank two) are clearly incompatible—neither has the other as a subgroup. This is hardly a surprising result; after all, we knew that in a Calabi-Yau compactification there are no massless graviphotons \[8\].

5.4 The graviscalars.

Having analyzed the graviphotons in detail we now turn our attention to the graviscalars. Relying on Ricci flatness to remove all terms involving the background curvature, we arrive at the following action quadratic in the graviscalar field:

\[ S_h^{(2)} = S_{\text{kin}}^h + S_{\text{mass}}^h, \]  

where

\[ S_{\text{kin}}^h = -\frac{1}{2} \int \frac{d^E y}{\rho(E)} \sqrt{g(B)} d^D x \left\{ (\nabla_{\mu} h^{\alpha\beta})^2 + \frac{1}{D - 2} (\nabla_{\mu} h)^2 \right\} \]  

(5.26)

gives the kinetic part in $D$ dimensions, and

\[ S_{\text{mass}}^h = \frac{1}{2} \int \frac{d^E y}{\rho(E)} \sqrt{g(B)} d^D x \left\{ \frac{4 - D}{(D - 2)^2} (\nabla_{\alpha} h)^2 + \frac{4}{D - 2} \nabla_{\alpha} h \nabla_{\beta} h^{\alpha\beta} \\
+ 2 \nabla_{\gamma} h^{\beta} \nabla_{\gamma} h^{\alpha} - (\nabla_{\gamma} h^{\alpha\beta})^2 \right\} \]  

(5.27)

is the mass operator.

In order to find massless scalars, we should look for tensors $u^{\alpha\beta}$ that are zero modes of the mass operator. There are two obvious candidates: the background metric

\[ u^{\alpha\beta}_T \equiv g^{\alpha\beta}_{(B)}, \]  

(5.28)

and the tensor product of the Killing vector with itself:

\[ u^{\alpha\beta}_V \equiv V^\alpha V^\beta. \]  

(5.29)
Bearing in mind that the manifold has the structure $K = S^1 \times \tilde{K}$, (5.28) and (5.29) are easily seen to be the only zero modes that have non-vanishing components in the direction of the Killing vector. In addition to these we can also have some zero modes of the compact subspace $\tilde{K}$, which we will denote by $u_\alpha^\beta_i$; $(i = 1, 2, \ldots)$.

Corresponding to each zero mode we have massless graviscalar fields that we denote by $h_T, h_V$ and $h_i$, respectively. The only one of these graviscalars that couples to matter charged only under the U(1) gauge symmetry is $h_V$. Anti-gravity arises as a result of cancellation between exchange of graviton, graviphoton and this unique graviscalar.

To prove the decoupling of $h_T$ and $h_i$, we consider again the generic coupling to gravity of a massless field in $D+E$ dimensions, which is described by the action (3.12). We take the matter field $\Phi$ to be charged under the U(1) group of translations by the Killing vector, that is, we fix the $y$-dependence of the field to be of the form:

$$\tilde{\Phi}(x; y) = \Phi(x) \exp\{ipV^\alpha y_\alpha\}.$$ (5.30)

We may think of $pV^\alpha$ as the internal momentum in the direction of the Killing vector.

As in section 3 the $D$-dimensional scalar field $\Phi$ will have a mass $m = |p|$ and a charge $q = \sqrt{2}\kappa p$ and will couple to the graviton and the graviphoton in the canonical way given by eqs. (3.18) and (3.19).

The coupling to the graviscalar is given by

$$\kappa m^2 \int d^Dx \phi^2(x)h(x)\mathcal{N},$$ (5.31)

where the number $\mathcal{N}$ depends on the assumptions made for the graviscalar zero mode $u_\alpha^\beta$:

$$\mathcal{N} = \int \frac{d^E y}{\rho(E)} \sqrt{g^{(B)}} \left( u_\alpha^\beta V_\alpha V_\beta + \frac{1}{D-2} g^{(B)}_{\alpha\beta} u_\alpha^\beta \right).$$ (5.32)

By choosing the following basis for the zero modes

$$u_\alpha^\beta_V = V^\alpha V^\beta$$

$$u_\alpha^\beta_T = g^{(B)}_{\alpha\beta} - \frac{D+E-2}{D-1} V^\alpha V^\beta$$

$$u_\alpha^\beta_i = u_\alpha^\beta_i - \frac{1}{E-1} (g^{(B)}_{\alpha\beta} - V^\alpha V^\beta) (g^{(B)}_{\gamma\delta} u_\gamma^\delta),$$ (5.33)
we find that all scalar states decouple (i.e. $\mathcal{N} = 0$), except the one related to the mode $u_V$. Furthermore, the graviscalar propagator given by eq.(3.24) is diagonalized by the choice of basis \((5.33)\). The propagator of this graviscalar is given by:

$$\langle h_V(p)h_V(-p) \rangle = \frac{1}{p^2} \frac{D-2}{D-1}. \quad (5.34)$$

Hence, we are back to the case of toroidal compactification even for the graviscalars and the computation of the four-point amplitude proceeds as in section 3, with the resulting eq.(3.28).

The solution of Einstein’s equations will not be Ricci flat if we introduce a classical matter distribution on the compact manifold. It is then possible to have Killing vectors which are not covariantly constant and form non-Abelian groups. The resulting massless vectors in $D$ dimensions will have couplings that depend on the details of the manifold, i.e. on the matter distribution introduced \([23]\). There will in general be no anti-gravity. This is particularly clear if we consider the graviscalar $h_V$, because the mode \((5.29)\) from which it is constructed is not a zero mode of the mass operator \((5.27)\) unless $V^\alpha$ is covariantly constant.

We conclude that the anti-gravity phenomenon is a special property of toroidal compactification.

6 Conclusions.

We have seen that anti-gravity is a feature of any four-dimensional effective theory of gravity obtained from a higher dimensional theory by compactification on a manifold with flat directions.

Anti-gravity occurs between states that start out by being massless in the higher-dimensional theory but obtain on compactification a mass and a charge under the U(1) group of rigid translations in the flat direction.

If these states obtain their U(1) charge by virtue of a non-vanishing compact momentum, they are going to be very heavy (of the order of, say, $10^{16}$ GeV) and hence of little relevance to experimental physics (except, perhaps, as dark-matter candidates). However, as it has been shown by Scherk \([2]\), U(1) charged states need not be very heavy and can appear in the process of supersymmetry breaking. This leaves open the most important question about anti-gravity, that is, whether ordinary matter is charged or not under the internal U(1) group of the graviphoton. If ordinary matter turns out to be charged, the mass of the graviscalar and
the graviphoton cannot be truly zero, since this would violate many experimental bounds on the principle of equivalence [24]. Among fifth-force candidates, anti-gravity is singled out by the prediction that the new force couples with the same strength of gravity. While a more complete discussion of the experimental relevance of anti-gravity is outside the scope of this paper, we want to stress here that, as we have seen in the previous sections, anti-gravity is the low-energy signature of having a flat compact dimension. Flat compact directions are incompatible with Calabi-Yau compactifications and will tend to produce too much supersymmetry in four dimensions, making it very hard to accommodate the chiral asymmetry of the real world.

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A Field equations of compactified Einstein theory.

In this appendix we present the full field equations of the theory (3.7) in the presence of point-like sources.

First, there are the Einstein equations for the gravitational field:

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 T_{\mu\nu}, \]  
(A.1)

where the energy-momentum tensor splits into two parts. The first part is due to the energy-momentum of the graviscalar and graviphoton fields.

\[ T^{(\phi,A)}_{\mu\nu} = \delta^{-\gamma} \left( F_{\mu\rho}^\beta F^{\nu\rho}_{\nu} \phi_{\alpha\beta} - \frac{1}{4} g_{\mu\nu} F^{\rho\sigma}_{\rho\sigma} \phi_{\alpha\beta} \right) \]
\[ + \frac{1}{4\kappa^2} \phi^{\alpha_1\alpha_2} \phi^{\beta_1\beta_2} \left( \partial_{\mu} \phi_{\alpha_1\beta_1} \partial_{\nu} \phi_{\alpha_2\beta_2} - \frac{1}{2} g_{\mu\nu} \partial_{\rho} \phi_{\alpha_1\beta_1} \partial_{\sigma} \phi_{\alpha_2\beta_2} \right) \]
\[ + \frac{1}{4\kappa^2(D-2)} \left( \partial_{\mu} \log \delta \partial_{\nu} \log \delta - \frac{1}{2} g_{\mu\nu}(\partial_{\rho} \log \delta)^2 \right). \]  
(A.2)

The second part is due to the motion of a point-particle source of canonical momentum \( \hat{p}_\mu \) and charge \( q_\alpha \):

\[ T^{(PP)}_{\mu\nu} = \delta^{-\gamma} \int d\sigma (\hat{p}_\mu + q_\beta A^\beta_{\mu})(\hat{p}_\nu + q_\alpha A^\alpha_{\nu}) \delta^D(x - x(\sigma)) \frac{\delta^D(x - x(\sigma))}{\sqrt{-g}}. \]  
(A.3)
The trajectory \( x(\sigma) \) of the particle is related to the canonical momentum by
\[
g_{\mu\nu} \frac{dx^\nu}{d\sigma} = \delta^{-\gamma}(\tilde{\pi}_\mu + q_\beta A^\beta_\mu). \tag{A.4}
\]
Here \( \sigma = \tau/m_D \), \( \tau \) being the proper time and \( m_D \) the rest mass of the point particle.

The equations for the graviphoton field are
\[
\nabla_\rho (\delta^{-\gamma} F^{\rho\mu\beta} \phi_{\alpha\beta}) = \delta^{-\gamma} J^\mu_\alpha, \tag{A.5}
\]
where the charge current density of the source is given by
\[
J^\mu_\alpha = \int d\sigma q_\alpha (\tilde{\pi}_\mu + q_\beta A^\beta_\mu) \frac{\delta^D(x - x(\sigma))}{\sqrt{-g}}. \tag{A.6}
\]
Finally, we have the graviscalar equations of motion
\[
\Box \phi_{\alpha\beta} + \frac{1}{D-2} \phi_{\alpha\beta} \Box \log \delta - \phi^{\alpha_1\alpha_2} \nabla_\mu \phi_{\beta\alpha_2} \nabla^\rho \phi_{\alpha_11} + \kappa^2 \delta^{-\gamma} \left( F^{\alpha_1\mu} F^{\beta_1\nu} \phi_{\alpha_3\beta_3} \phi_{\alpha_4\beta_4} - F^{\mu\nu\alpha_1} F^{\beta_1\mu} \phi_{\alpha_3\alpha_4} \phi_{\beta_3\beta_4} \right) = \nabla_\rho \left( T^{(PP)} - \Pi_{\alpha\beta} \right), \tag{A.7}
\]
where \( \Box \) is the curved-space Laplacian and the charge matrix density is given by
\[
\Pi_{\alpha\beta} = \int d\sigma q_\alpha q_\beta \frac{\delta^D(x - x(\sigma))}{\sqrt{-g}}. \tag{A.8}
\]
In the above equations \( \delta \) and \( \gamma \) are given by eqs.(3.5) and (3.6), respectively.

The solution (4.11) corresponds to a point-particle source of mass \( M \) and charge \( q_\alpha = \epsilon \sqrt{2} \kappa M \delta^1_\alpha \tag{A.9} \)
stationary at the origin. It is a solution of the full field equations of \( N = 8 \) (or \( N = 4 \)) supergravity because, of the many fields appearing in that theory, only those contained in (3.7) are excited by the source (A.9).

The equations of motion of a test particle of charge \( q_\beta \) and mass \( m_D \) in the above theory are derived within the \( D \)-dimensional framework by requiring conservation of energy-momentum of field + test-particle:
\[
\nabla_\nu (T^{\nu\mu}_{(\phi,A)} + T^{\nu\mu}_{(PP)}) = 0. \tag{A.10}
\]
The equations are the following ones:
\[
\ddot{x}^\mu + \Gamma^\mu_{\rho\tau} \dot{x}^\rho \dot{x}^\tau = -\frac{q_\beta}{m_D} \delta^{-\gamma} F^{\mu\beta} \dot{x}^\rho \\
+ \frac{1}{2(D-2)} \phi^{\alpha\beta} (\delta_\alpha \partial_\rho + \delta_\beta \partial_\rho - g_{\rho\tau} \partial^\mu) \dot{\phi}_{\alpha\beta} \dot{x}^\tau - \frac{1}{4\kappa^2 m^2_D} q_\alpha q_\beta \partial^\mu \phi^{\alpha\beta}, \tag{A.11}
\]
where the dot denotes differentiation with respect to \( \tau \).
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