VARIABLE GAMMA-RAY EMISSION INDUCED BY ULTRA-HIGH ENERGY NEUTRAL BEAMS: APPLICATION TO 4C +21.35

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ABSTRACT

The flat-spectrum radio quasar 4C +21.35 (PKS 1222+216) displays prominent nuclear infrared emission from \( \approx 1200 \) K dust. A 70–400 GeV flare with \( \approx 10 \) minute variations during half an hour of observations was found by the MAGIC telescopes, and GeV variability was observed on sub-day timescales with the Large Area Telescope on Fermi. We examine 4C +21.35, assuming that it is a source of ultra-high energy cosmic rays (UHECRs). UHECR proton acceleration in the inner jet powers a neutral beam of neutrinos, neutrons, and \( \gamma \)-rays from \( p\gamma \) photopion production. The radiative efficiency and production spectra of neutrals formed through photohadronic processes with isotropic external target photons of the broad-line region (BLR) and torus are calculated. Secondary radiations made by this process have a beaming factor \( \propto \delta_D^3 \), where \( \delta_D \) is the Doppler factor. The pair-production optical depth for \( \gamma \)-rays and the photopion efficiency for UHECR neutrons as they pass through external isotropic radiation fields are calculated. If target photons come from the BLR and dust torus, large Doppler factors, \( \delta_D \gtrsim 100 \), are required to produce rapidly variable secondary radiation with isotropic luminosity \( \gtrsim 10^{47} \) erg s\(^{-1}\) at the pc scale. The \( \gamma \)-ray spectra from leptonic secondaries are calculated from cascades initiated by the UHECR neutron beam at the pc-scale region and fit to the flaring spectrum of 4C +21.35. Detection of \( \gtrsim 100 \) TeV neutrinos from 4C +21.35 or other very high energy \( \gamma \)-ray blazars with IceCube or KM3NeT would confirm this scenario.

Key words: galaxies: jets – gamma rays: general – radiation mechanisms: non-thermal

Online-only material: color figures

1. INTRODUCTION

Three flat-spectrum radio quasars (FSRQs), 3C 279 (Albert et al. 2008; Aleksi´ce et al. 2011a), PKS 1510−089 (Wagner et al. 2010; Cortina 2012), and PKS 1222+216 (Aleksi´ce et al. 2011b), have been detected in the very high energy (VHE; \( \gtrsim 100 \) GeV) regime. This last object, a blazar at redshift \( z = 0.432 \) also known as 4C +21.35, is remarkable for day-scale GeV activity (Carrasco et al. 2010). Moreover, the Major Atmospheric Gamma Imaging Cherenkov (MAGIC) telescopes observed a VHE flaring episode during a period of observation lasting about one-half hour on 2010 June 17 that showed \( \approx \)factor-of-two flux variations on a 10 minute timescale (Aleksi´ce et al. 2011b). The rapidly variable \( \gamma \)-rays reached large apparent isotropic \( \gamma \)-ray luminosities \( L_\gamma \gtrsim 10^{47} \) erg s\(^{-1}\) in the VHE band and \( \gtrsim 10^{48} \) erg s\(^{-1}\) above 100 MeV.

The intense and highly variable \( \gamma \)-ray fluxes measured from blazars, including 4C +21.35, demonstrate that they are powerful accelerators of energetic particles. Energetic leptons make intense fluxes of synchrotron emission, with associated \( \gamma \)-rays formed by Compton processes from the same electrons. As shown by Fermi-LAT observations (Ackermann et al. 2011), the two-year time-averaged values of \( L_\gamma \) of dozens of FSRQs exceed \( 10^{48} \) erg s\(^{-1}\). During flaring episodes, values of \( L_\gamma \gtrsim 10^{49} \) erg s\(^{-1}\) are not unusual (the record-setting value is \( L_\gamma \approx 2 \times 10^{50} \) erg s\(^{-1}\) from the 2010 November outburst of 3C 454.3; Abdo et al. 2011a). In addition to relativistic leptons, hadrons are also likely to be accelerated in blazar jets and make secondary \( \gamma \) radiations by interacting with target photons of the internal and surrounding radiation fields. This emission will contribute to the formation of the spectral energy distribution (SED; see Böttcher 2010, for a recent review), and the accelerated hadrons or secondary neutrons could escape from the acceleration region to become ultra-high energy cosmic rays (UHECRs) with energies \( \gtrsim 10^{18} \) eV.

Typical quasars have broad optical and ultraviolet emission lines that originate from the broad-line region (BLR) near the supermassive black hole. The rapid variability, as short as a few hours for 4C +21.35 as found in the Fermi-LAT data (Foschini et al. 2011a, 2011b), indicates that the high-energy \( \gamma \)-rays are formed close to the black hole within the BLR. But if this is the case, such high-energy photons would be strongly attenuated through the \( \gamma\gamma \rightarrow e^+e^- \) process against scattered line and accretion-disk radiation at the sub-pc scale. We argue that the contradictory behaviors of rapid variability, large luminosity, and a production region distant from the central nucleus can be reconciled if the inner jet accelerates UHECRs that escape as neutrons, which then undergo photohadronic production in the infrared (IR) field of the dusty torus to make rapidly variable synchrotron \( \gamma \)-rays at GeV–TeV energies on the pc scale. Furthermore, UHE \( \gamma \)-rays made in the inner jet can contribute to the cascade at the pc scale.

The recent observations of 4C +21.35 are summarized in Section 2. An overview of the model, with estimates of the radiative efficiency and secondary photohadronic production spectra, is given in Section 3. A detailed treatment of secondary production of neutrals is presented in Section 4, giving beaming...
The discovery of VHE emission between \( \approx 70 \) and 400 GeV with the MAGIC telescopes (Mariotti 2010; Aleksić et al. 2011b) was anticipated by the report of VHE photons found in Fermi-LAT data during a high state of 4C +21.35 in 2010 April (Neronov et al. 2010). The source was targeted with the MAGIC telescopes from 2010 May 3. The spectrum of the major VHE flare observed with MAGIC on 2010 June 17, which observed for only \( \approx 30 \) minutes, is well described by a single power law with photon number index \( \alpha_f \approx 3.75 \) that is hardened by about one unit when corrected for attenuation with a low-intensity model (Kneiske & Dole 2010) of the extragalactic background light (EBL) at infrared and optical frequencies. The remarkably short 10 minute variability (in comparison, 3C 279 displayed weak VHE variability on timescales of days; see Albert et al. 2008) points to a production site near the central supermassive black hole where collimation by extreme processes can occur (Tavecchio et al. 2010). Variability associated with the dynamical timescale of a 10\(^8\)M\(_{\odot}\) solar mass black hole is \( t_{\text{dyn}} = (1 + z)r_{\text{sh}}/(c = (1 + z)10^5M_{\odot}\), where \( r_{\text{sh}} \) is the Schwarzschild radius of the black hole, whereas variability on timescales shorter than \( t_{\text{dyn}} \) by factors of 10 or more has been observed in the rapidly variable BL Lac objects Mrk 501 (Albert et al. 2007), PKS 2155–304 (Aharonian et al. 2007), and Mrk 421 (Fossati et al. 2008). For 4C +21.35, with a black-hole mass estimated at \( \approx 1.5 \times 10^8\) M\(_{\odot}\) (Wang et al. 2004), the corresponding timescale is \( \approx 2000\) s, so the VHE measurements indicate a marginally hyper-variable source.

A higher black-hole mass is obtained from recent optical spectroscopy of J1224+2122 by Shaw et al. (2012), using scaling relations derived from reverberation mapping of radio-quiet active galactic nuclei (AGNs). From analysis of the FWHM of the H\(\beta\) line, they deduce a black-hole mass for 4C +21.35 of 7.8(\(\pm 2.3\)) \(\times 10^9\) M\(_{\odot}\), which does not include uncertainties in applying scaling relations to blazars. This value is considerably higher than the mass reported by Wang et al. (2004) and would relax the energetics based on the Eddington luminosity but make the short timescale variability more problematic. Here we use the lower mass and return to this point in Section 7.

The BLR radius \( R_{\text{BLR}} \) can be written in terms of the accretion-disk luminosity \( L_{\text{disk}} \) (see Ghisellini & Tavecchio 2008)

\[
R_{\text{BLR}} \approx 10^{17} \sqrt{L_{\text{disk}} / 10^{45} \text{ erg s}^{-1}} \text{ cm.} \tag{1}
\]

Based on the H\(\beta\) luminosity, \( L_{\text{H}\beta} \approx 2 \times 10^{43}\) erg s\(^{-1}\), Tanaka et al. (2011) derive from the scaling relations of Wang et al. (2004) and Fan et al. (2006) that the BLR luminosity \( L_{\text{BLR}} \approx 5 \times 10^{44}\) erg s\(^{-1}\), and that the accretion-disk luminosity \( L_{\text{disk}} \) is an order of magnitude greater, \( L_{\text{disk}} \approx 5 \times 10^{45}\) erg s\(^{-1}\). In comparison, Tavecchio et al. (2011) estimate that \( L_{\text{disk}} \approx 5 \times 10^{46}\) erg s\(^{-1}\), assuming that the Swift UVOT spectrum from 4C +21.35 is the optically thick accretion-disk radiation field, in which case \( R_{\text{BLR}} \approx 7 \times 10^7\) cm \(\approx 0.2\) pc. This represents super-Eddington luminosities, given that \( L_{\text{Edd}} \approx 2 \times 10^{46}\) erg s\(^{-1}\) unless the black-hole mass is \( \gtrsim 4 \times 10^8\) M\(_{\odot}\). We use the smaller disk luminosity here, noting that a larger \( L_{\text{disk}} \) and BLR radius will improve photodissociation efficiency.

More precise relations depend on specific lines and assumptions about the geometry of the BLR (see, e.g., the recent discussion by Foschini 2011). In particular, we could use the rest-frame 5100 Å or 1350 Å continuum, or the high-ionization Ly\(\alpha\) line, as seen strongly in, e.g., 3C 454.3 (Bonnoli et al. 2011). For definiteness, we focus on the Ly\(\alpha\) line to illustrate BLR line effects, which typically carries \(\gtrsim 20\)% of the BLR luminosity (Francis et al. 1991). Approximating the Ly\(\alpha\)/BLR line luminosity \( L_{\text{Ly}\alpha} / L_{\text{BLR}} \approx 0.1 \) \(L_{\text{disk}}\) and defining

\[
\phi \equiv \frac{L_{\text{Ly}\alpha}}{L_{\text{disk}} (R_{\text{Ly}\alpha}/R_{\text{BLR}})^2} \equiv 0.1 \phi_{-1} \tag{2}
\]

gives a Ly\(\alpha\) photon field with energy density \( u_{\text{Ly}\alpha} \approx L_{\text{Ly}\alpha} / 4\pi R_{\text{Ly}\alpha}^2 c \approx 0.026 \phi_{-1} \text{ erg cm}^{-3}\).

The magnetic field \( B' \) in the comoving jet frame can be determined if the GeV \(\gamma\)-rays are Compton-scattered external radiation and the lower energy optical radiation is synchrotron radiation made by the same nonthermal electrons making the \(\gamma\)-rays at GeV energies. The Compton dominance parameter \( A_C \) is defined as the ratio of the \(\gamma\)-ray luminosity (assumed to originate from external Compton scattering) to the radio/X-ray synchrotron luminosity. For 4C +21.35, \( A_C \) reaches values as large as \(\approx 100\) (see multiwavelength SED using contemporaneous data in Tavecchio et al. 2011). If the external radiation fields scattered by these electrons are the Ly\(\alpha\) photons, then \( A_C \approx u_{\text{Ly}\alpha}/u_{\gamma} \), with \( u_{\text{Ly}\alpha} \approx 4\pi^2 E_{1\text{Ly}\alpha}/3 \) and \( u_{\gamma} = B^2/8\pi \) (e.g., Sikora et al. 2009). Thus, the comoving magnetic field for...
a blazar jet with bulk Lorentz factor $\Gamma$ is

$$B' \cong 9.3 \left( \frac{\Gamma}{100} \right) \sqrt{\frac{\Phi_{-1}}{(A_{\text{c}}/100)}} \text{ G.} \quad (3)$$

Modeling 5–35 $\mu$m Spitzer, Sloan Digital Sky Survey, Two Micron All Sky Survey, and Swift UVOT data of 4C +21.35, Malmrose et al. (2011) decomposes its spectrum into a nonthermal power law and two-temperature dust model. Hot dust with $T \approx 1200$ K radiates $\approx 8 \times 10^{45}$ erg s$^{-1}$ from a pc-scale region, and a second warm dust component radiates $\approx 10^{45}$ erg s$^{-1}$ at $T \approx 660$ K on the same scale. The IR component, which has been proposed as a target in external Compton scenarios (Blazejowski et al. 2000; Sikora et al. 2009), is especially important for UHECR scenarios, noting that the relative energy densities of the BLR, the dust fields, and the cosmic microwave background radiation (CMBR) are, in units of erg cm$^{-3}$, $u_{\text{BLR}} \approx 0.026 \Phi_{-1}$, $u_{\text{dust}} \approx 3 \times 10^{-3} (L_{\text{IR}}/10^{46}$ erg s$^{-1})/(R/\text{pc})^2$, and $u_{\text{CMBR}} \approx 4 \times 10^{-13}$, respectively. We use the same parameters as derived by Malmrose et al. (2011) in our model.

2.2. Target Radiation Fields

Candidate target radiation fields are the CMBR, with dimensionless temperature $\Theta_{\text{CMBR}} \equiv k_B T_{\text{CMBR}}/m_e c^2 \cong 4.6 \times 10^{-10} (1 + z)$, 1200 K and 660 K dust ($\Theta_{\text{dust}_1} \cong 2 \times 10^{-7}$ and $\Theta_{\text{dust}_2} \cong 1.1 \times 10^{-7}$, respectively), and quasi-thermal scattered 10 eV disk emission ($\Theta_{\text{acrdisk}} \cong 2 \times 10^{-5}$). For Ly$\alpha$ line radiation—neglecting broadening, which is unimportant for these calculations—$\epsilon_{\text{Ly}\alpha} = 10.2$ eV$/m_e c^2 = 2 \times 10^{-5}$. Using the Greisen-Zatsepin-Kuzmin (GZK) energy of $\approx 6 \times 10^{19}$ eV as a yardstick where $\Delta$ isobar excitation by 2.725 K CMBR photons is a source of opacity and energy loss, the IR dust and accretion-disk/Ly$\alpha$ radiation are effective targets for photopion production from cosmic-ray proton and neutron (nucleon) interactions with $E_p \gg 10^{17}$ eV and $E_p \gg 10^{15}$ eV, respectively. The energy densities of the target radiation fields are defined by the usual relations $u_{\text{eff}} \propto L/4\pi R^2 c$, where $R$ is the distance from the black hole.$^5$

The luminosity of photopion secondaries is proportional to the effective photon number density for photopion production. For monochromatic line sources, $n_{\text{eff}} = u_0 / \epsilon_0 m_e c^2$, and for thermal blackbody or graybody radiation fields, $n_{\text{eff}} = u_0 / \Theta m_e c^2$, as shown below. Here, $\epsilon$ is the mean energy of photons in the unit of $m_e c^2$. The effective photon number density for the CMBR is

$$n_{\text{eff}}^{\text{CMBR}} = \frac{\mu_{\text{CMBR}}}{m_e c^2 \Theta_{\text{CMBR}}} \cong 1.1 \times 10^3 (1 + z)^3 \text{ cm}^{-3} \quad (4)$$

assuming that the temperature of the CMBR at the present epoch is 2.725 K. The effective photon density for $p\gamma$ scattering of the Ly$\alpha$ field is

$$n_{\text{eff}}^{\text{Ly}\alpha} = \frac{\mu_{\text{Ly}\alpha}}{m_e c^2 \epsilon_{\text{Ly}\alpha}} \cong 1.6 \times 10^9 \Phi_{-1} \text{ cm}^{-3}. \quad (5)$$

The BLR radiation field due to the accretion-disk radiation scattered by BLR gas is, when approximated as a graybody, given by

$$n_{\text{eff}}^{\text{BLR}} = \frac{u_{\text{BLR}}}{m_e c^2 \Theta_{\text{acrdisk}}} \cong 1.7 \times 10^3 \Phi_{-1} \text{ cm}^{-3} \quad (6)$$

assuming an effective Thomson scattering depth of 0.1$\tau$-1. For the two dust radiation fields, the effective photon densities for photopion $p\gamma$ interactions are

$$n_{\text{eff}}^{\text{dust}_1} = \frac{u_{\text{dust}_1}}{m_e c^2 \Theta_{\text{dust}_1}} \cong 1.7 \times 10^{10} \frac{L_{46}}{R_{\text{pc}}^2} \text{ cm}^{-3} \quad (7)$$

and

$$n_{\text{eff}}^{\text{dust}_2} = \frac{u_{\text{dust}_2}}{m_e c^2 \Theta_{\text{dust}_2}} \cong 3.1 \times 10^9 \frac{L_{45}}{R_{\text{pc}}^2} \text{ cm}^{-3}. \quad (8)$$

Here the luminosity of the IR radiation, assumed to be radiated on a size scale of $R_{\text{pc}}$ pc, is denoted $10^9 L_{1} \text{ erg s}^{-1}$. To order of magnitude, $n_{\text{eff}} = u_0 / m_e c^2 \epsilon \sim u_0 / m_e c^2 \Theta \sim 10^9–10^{10}$ cm$^{-3}$, and the CMBR is negligible.

For sufficiently energetic cosmic-ray nucleons, the warm dust radiation field provides the densest and most important target photon field for photopion production. For dissipation of energy on the pc scale, only the highest energy nucleons with $E \gtrsim 10^{18}$ eV lose energy effectively by photohadronic processes with IR dust photons, because the BLR radiation does not extend to the pc scale, and there is no other sufficiently strong radiation field at the pc scale to extract energy efficiently from these lower energy protons.

3. MODEL AND ESTIMATES

The model is outlined, followed by a simple estimate of photohadronic efficiency and a simple derivation of secondary neutral production spectra made by photohadronic interactions of cosmic-ray jet protons in the inner jet.

3.1. Model Description

This model is motivated by the hypothesis that blazars and radio galaxies accelerate UHECRs (e.g., Mannheim & Biermann 1992; Berezhinsky et al. 2006; Dermer et al. 2009; Murase & Takami 2009); confirmation of this hypothesis is the detection of UHE neutrinos during blazar flares. Electromagnetic signatures can also provide crucial evidence in support of UHECR production in blazars, and the presence of distinct hadronic emission signatures in the variable $\gamma$-ray SEDs is consistent with the SED shape but not unambiguous (see Böttcher 2010 or, e.g., Abdo et al. 2011b, for Mrk 421). Detection of a multi-TeV radiation signature in extreme high-synchrotron-peak blazars like 1ES 0229+200 (Murase et al. 2012) at energies where the EBL should suppress this emission would support emission generated by UHECR protons accelerated by blazars (Essey & Kusenko 2010; Essey et al. 2010, 2011).

Figure 1 shows a schematic illustrating the underlying basis of this model. UHECR protons and ions are accelerated in the outflowing relativistic plasma formed in the inner jet. Interactions with internal synchrotron and external quasi-isotropic radiation field cause ultrarelativistic protons accelerated in the outflowing jet to undergo photopion losses, with the production of escaping neutrons, $\gamma$-rays, and neutrinos (Atoyan & Dermer 2003). Subsequent interactions of the escaping neutrals with IR torus photons on the pc scale make highly beamed leptons produced as secondaries of $\gamma\gamma$ interactions of UHE photons and as secondaries of photopion interactions of UHE neutrons. The relativistic leptons make beamed synchrotron and Compton-scattered radiation. These emissions, being highly beamed, will vary on the production timescale determined by the behavior in the inner jet. Off-axis emission from misdirected leptons will
not smear and enhance the timescale because it is produced by ultrarelativistic electrons that are not significantly deflected, so the contribution to the observer originates only from the particles traveling directly toward the observer.

### 3.2. Radiation Efficiency Estimates

We make simple estimates of the radiative efficiency for the production of photodisruptive secondaries when UHE protons interact with photons from isotropic radiation fields external to the jet. In our estimates, the energy density of the external radiation field is denoted $u_0$ and the characteristic spatial size scale of the external radiation field is $R$. For thermal radiation fields, $\epsilon_\gamma \approx 2.70\Theta$.

Suppose that a relativistic jet moving with bulk Lorentz factor $\Gamma$ is filled with cosmic-ray protons and ions. The comoving dynamical time for traveling through an external radiation field is $t_{\mathrm{dyn}} \approx R/\Gamma c$. In the jet frame, the energy density of external radiation field is $u_0 \approx \Gamma^2 u_\gamma$, and the mean photon energy is $\epsilon_\gamma \approx \Gamma \epsilon_\gamma^*$. The threshold Lorentz factor $\gamma_p^\prime$ for photomeson production by protons with comoving Lorentz factor $\gamma_p^\prime$ interacting with photons with energy $\epsilon_\gamma^*$ is given by $\gamma_p^\prime \approx \epsilon_\mathrm{thr}/\epsilon_\gamma^*$, where $\epsilon_\mathrm{thr} \approx m_\pi/m_e$ and $m_\pi \approx 140$ MeV is the pion mass.

The energy-loss rate of ultrarelativistic protons through photopion processes is therefore

$$t_{\gamma p}^{-1} (\gamma_p^* > \gamma_p^\prime) \approx \frac{(K_{\gamma p} \sigma_{\gamma p}) u_0}{m_e c^2 \epsilon_\gamma^*} \approx \frac{(K_{\gamma p} \sigma_{\gamma p}) u_0}{m_e c^2 \epsilon_\gamma^*} \Gamma \left( \frac{u_0}{m_e c^2 \epsilon_\gamma^*} \right),$$

where $\sigma_{\gamma p} \approx 70 \mu$b is the product of the inelasticity and the cross section for photomeson processes (Begelman et al. 1990; Atoyan & Dermer 2003). Note that the energy-loss timescale is proportional to the effective photon density $n_{\gamma p} \approx u_0/m_e c^2 \epsilon_\gamma^*$. Employing the approximation of Atoyan & Dermer (2003) here and in the following, we take $\epsilon_\mathrm{thr} \approx 390$. The photopion production efficiency is therefore simply

$$\eta_{\gamma p} (\gamma_p^*) \approx t_{\gamma p}^{-1}/t_{\gamma p}^{-1} (\gamma_p^* < \gamma_p^\prime) \approx \hat{\sigma} R \left( \frac{u_0}{m_e c^2 \epsilon_\gamma^*} \right),$$

provided that $\gamma_p^* > \gamma_p^\prime$. The Lorentz factor of protons measured in the stationary frame of the black-hole jet system (as if they had escaped the jet) is $\gamma \approx \Gamma \gamma_p^\prime$, so the threshold Lorentz factor is $\gamma_p^\prime \approx \epsilon_\mathrm{thr}/\epsilon_\gamma^*$.

Following Equations (5)–(8), we write $\eta_{\gamma p} \approx (u_0/m_e c^2 \epsilon_\gamma^*)^{-1} \equiv [(n_{\gamma p} / 10^9 R_{17})]$. For interactions in the BLR with Ly$\alpha$ photons or scattered accretion-disk radiation, $\eta_{\gamma p} (\gamma_p) \approx 0.007 n_{\gamma p} R_{17}$ for escaping protons with energy $E_p \gtrsim 2 \times 10^{16}$ eV. Here, $R_{17} = 10^{17}$ R$_{17}$ cm. For interactions with IR photons from the dust torus, $\eta_{\gamma p} (\gamma_p) \approx 0.07 n_{\gamma p} R_{17}$ for protons with $E_p \gtrsim 7 \times 10^{17}$ eV. For larger dissipation radii, one has $\eta_{\gamma p} (\gamma_p) \approx 2 n_{\gamma p} R_{17}$, which means that UHECR protons in the jet plasma can dissipate essentially all of their energy into secondaries while traveling through the radiation field of the dust torus. But note that in this case, the UHE proton beam has to be maintained throughout the entire pc-scale length of the external radiation field, in which case the emission would vary on timescales no shorter than $\sim R/\Gamma^2 c \sim 10^4 s R_{17}/(\Gamma 100)^{-2}$. Thus, we see that the efficiency to produce photomeson secondaries through interactions of UHECR jet protons with photons of an external radiation field is several percent in the inner jet, counting both line and scattered photons, and noting that $L_{\mathrm{disk}} \approx 5 \times 10^{45}$ erg s$^{-1}$ (Tanaka et al. 2011), so that $R_{\mathrm{BLR}} \approx 2 \times 10^{17}$ cm, from Equation (1). This holds for $\gtrsim 10^{16}$ eV protons. Under optimistic conditions, the photopion efficiency of cosmic-ray protons in a blazar jet is of order unity in interactions with the IR dust photons, but this applies only to protons with escaping energies $\gtrsim 10^{18}$ eV. Note furthermore that this is a conservative estimate of photodisruptive efficiency, because it does not take into account internal synchrotron photons, or photons from a sheath-spine structure or from different portions of the jet (compare Ghisellini et al. 2005; Georganopoulos & Kazanas 2003, for TeV blazars, though similarly structured jets could be found in FSRQs) that could significantly enhance the target radiation field and the photodisruptive efficiency.

It is interesting that all the $\Gamma$ factor dependencies in Equation (10) drop out. This shows that we can equivalently calculate the photodisruptive efficiency for a proton traversing the volume of the external radiation field without transforming to and from the comoving frame. We can furthermore make a simple derivation of the secondary neutron production by only considering interactions in the frame of the black-hole and external radiation fields.

With a photopion energy-loss cross section of $\approx 70 \mu$b, the column density of photons above threshold required for unity radiation efficiency is $N \approx 1/ (K_{\gamma p} \sigma_{\gamma p}) \sim 1.4 \times 10^{28}$ cm$^{-2}$. The column density of Ly$\alpha$ photons in the BLR is $N_{\mathrm{BLR}} \sim n_{\gamma p} R_{17} \sim 10^{26} n_{\gamma p} R_{17}$ cm$^{-2}$. The column density of IR photons from the pc-scale warm dust torus is $N_{\mathrm{dust}} \sim n_{\gamma p} R_{17} \sim 3 \times 10^{28} n_{\gamma p} R_{17}$ cm$^{-2}$. The IR photon field of the warm dust clearly presents the largest column density for photodisruptive production, for the most energetic nucleons above the photopion reaction threshold.

### 3.3. Simple Derivation of Secondary Neutron Production Spectra

The estimate above implicitly assumes that when protons undergo photopion interactions, they remain protons. In about 50% of the time, however, neutrons are formed that escape from the jetted plasma in a collimated outflow. The escaping neutrons deposit energy throughout the length of the jet by subsequent photodisruptive interactions. We now estimate the energy-loss...
rate of protons into neutrons via photopion production and use that result to make a simple derivation of the secondary neutron production spectrum, anticipating the more detailed derivation in the next section.

The inclusive energy-loss rate of protons into neutrons only is given by

\[ -\frac{dE_p}{dt}|_{p \rightarrow n} \approx \rho \chi \xi \sigma_{p\gamma} E_p, \]  

(11)

where \( \rho \) is the cross-section ratio, \( \chi \) is the fractional energy of the incident proton that is deposited into secondary neutrons, and \( \xi \) is the neutron multiplicity. For neutron production near threshold, \( \rho \approx 0.65, \chi \approx 0.8, \xi \approx 1/2 \), and the maximum photopion cross section is \( \sigma_{p\gamma} = 520 \pm 30 \mu b \). Here, the proton energy \( E_p = m_p c^2 \gamma_p \) and \( m_p \approx m_n \).

The rate for a primary proton to lose energy into secondary neutrons is therefore simply

\[ t_{p \rightarrow n}^{-1}(\gamma_p) = \int_0^\infty dE_p \frac{|dE_p|}{E_p} \approx \frac{c}{\rho \chi \xi \sigma_{p\gamma}} \int_0^\infty d\mu (1 - \mu) \times \rho(\tilde{E}_p)(\chi(\tilde{E}_p)\xi(\tilde{E}_p)\sigma_{p\gamma}(\tilde{E}_p)), \]

(12)

where \( \tilde{E}_p = \gamma_p E_p (1 - \mu) \) is the invariant interaction energy. Solving for a monochromatic radiation source \( n_{ph}(\epsilon) = u_0 \delta(\epsilon - \epsilon_s)/(m_e c^2 \epsilon) \) gives

\[ -\frac{dE_p}{dt}|_{p \rightarrow n} \approx \frac{m_p c \chi}{m_e} \gamma_p \frac{u_0}{\epsilon_s} \left[ 1 - \left( \frac{\tilde{E}_{th}}{2 \gamma_p \epsilon_s} \right)^2 \right] \times H(\gamma_p - \tilde{E}_{th}/2 \epsilon_s), \]

(13)

where \( \Upsilon = \rho \chi \xi \). 

The inclusive energy loss by primary protons with Lorentz factor \( \gamma_p \) that is transformed into secondary neutrons with Lorentz factor \( \gamma_n \) is conserved; therefore,

\[ -\int_1^\infty d\gamma_p \left( \frac{dE_p}{d\gamma_p} \right)_{p \rightarrow n} = \int_1^\infty d\gamma_n \frac{dE_n}{d\gamma_n}, \]

(14)

and \( \gamma_n = \gamma_n/\chi \). The neutron production luminosity is \( L_{p \rightarrow n} = \gamma_p(\sigma_{p\gamma}d\gamma_p/dt)_{p \rightarrow n} \). For a stationary-frame distribution in the effective volume (where the retarded time is considered), \( N_p(\gamma_p, \Omega_p) \) of protons differential in \( \gamma_p \) and direction \( \Omega_p \), the secondary production spectrum of neutrons, assuming that the neutrons have the same direction as the original protons, is

\[ \gamma_n L_n(\gamma_n, \Omega_n) = \frac{m_p}{m_e} \gamma_p \frac{u_0}{\epsilon_s} \gamma_p^2 N_p(\gamma_p, \Omega_p) \times \left[ 1 - \left( \frac{\tilde{E}_{th}}{2 \gamma_p \epsilon_s} \right)^2 \right] \times H(\gamma_p - \tilde{E}_{th}/2 \epsilon_s). \]

(15)

Using the method of Georganopoulos et al. (2001, 2004), we transform the comoving jet frame proton spectrum \( N_p'(\gamma_p', \Omega_p') \) to the stationary-frame jet proton spectrum \( N_p(\gamma_p, \Omega_p) \) through the relation

\[ N_p(\gamma_p, \Omega_p) \equiv \frac{dN_p}{d\gamma_p d\Omega_p} = \frac{\delta_{p}^3}{4 \pi} N_p'(\gamma_p', \Omega_p'), \]

(16)

where the last relation is a consequence of the assumption of proton isotropy in the proper fluid frame, noting that \( \gamma_p = \gamma_n/\chi = \delta_p \gamma_p' \). Thus,

\[ 4\pi \gamma_p L_n(\gamma_p, \Omega_n) \approx \frac{m_p c \sigma_{p\gamma} u_0}{m_e} \frac{\epsilon_s}{\epsilon_s} \Upsilon \delta_p^5 \left[ \gamma_p^2 N_p'(\gamma_p') \right] \times \left[ 1 - \left( \frac{\tilde{E}_{th}}{2 \gamma_p \epsilon_s} \right)^2 \right] H(\gamma_p - \tilde{E}_{th}/2 \epsilon_s). \]

(17)

Note the \( \delta_p \) beaming factor arising from the transformation of the proton spectrum, Equation (16), and one power each from energy and time.

From Equation (17), we can determine the absolute amount of energy in protons, \( \epsilon_p = \Gamma \epsilon_p' \), that, while traveling through a radiation field characterized by \( u_0/\epsilon_s m_e c^2 \), will produce an apparent isotropic luminosity in neutrons \( L_n = 10^{48} \text{erg s}^{-1} \).

If the spectrum of protons is \( \sim 2 \), that is, having equal energy per decade in protons, and extends well above the threshold \( \gamma_{p,\text{thr}} = \tilde{E}_{th}/2 \epsilon_s \) for neutron production, then the fractional energy is proportional to the ratio of two logarithmic factors, which has a value of order \( 1/2 \), implying

\[ \frac{1}{2} \frac{m_p c \sigma_{p\gamma} u_0}{m_e} \frac{\epsilon_s}{\epsilon_s} \Upsilon \delta_p^5 \frac{\epsilon_p}{\Gamma} \delta_p \delta_p \geq 10^{48} L_{48} \text{erg s}^{-1}. \]

(18)

There is no fundamental reason why the engine should be Eddington-limited during outbursts, but this provides a fiducial amount of energy that could be generated by the central engine. Therefore, we assume that the absolute nonthermal proton energy \( \epsilon_p \approx 2 \times 10^{46} \text{ ergs} / (1+z) \) \( = 8.4 \times 10^{48} \text{ ergs} \) for the 4C +21.35 flares (\( t_{\text{var}} \approx 600 \text{ s} \) is the variability timescale of the flare and \( z = 0.432 \)), which implies that

\[ \delta_p \Upsilon \geq 10 \geq 10^2 \text{ erg s}^{-1} \text{ in neutrons.} \]

The \( \Gamma \) factor limit can be relaxed to \( \delta_p \geq 50 \) if we happen to be viewing exactly down the jet (\( \delta_p = 2 \Gamma \)) or if the energy content in protons is not limited by the Eddington luminosity during this outburst but is instead one or two orders of magnitude larger. Alternatively, small \( \Gamma \) is possible if the photomeson production occurs mainly by target photons produced in inner jets (see below). In principle, there is no real reason that such large values of the Doppler factor or \( \Gamma \) factor are not allowed. Arguments based on \( \gamma' \gamma \) opacity applied to the 2006 July/August flares from PKS 2155−304 (Aharonian et al. 2007) require \( \delta_p \geq 60 \) (Begelman et al. 2008), while a full synchrotron/synchrotron-synchrotron self Compton (SSC) model fit, including \( \gamma' \gamma \) and EBL effects, implies \( \delta_p \geq 100 \) for PKS 2155−304 and \( \delta_p \geq 80 \) for Mrk 501 with a variability timescale of \( \approx 10^3 \text{ s} \) (Finke et al. 2008). One-zone leptonic models for the VHE emission from 3C 279 (Albert et al. 2008; Aleksic et al. 2011a) furthermore require large, \( \Gamma \geq 100 \), bulk Lorentz factors (Böttcher et al. 2009). Standard arguments relating variability time and location of emission site through the expression \( t_{\text{var}} \geq (1+z) R/\Gamma c \) imply \( \Gamma \geq 500 \) for emission produced at the pc scale, which already undermines naive expectations about \( \Gamma \) and the size of the emitting region. Measurements of superluminal motion never reach such values, but this is based on radio observations, which
becomes adapted to other secondaries, noting that the dimensionless fraction of the mean energy fraction of a secondary neutron compared to the original proton energy for interactions associated with segment \( i \).

Substituting Equation (21) into (20) and solving gives

\[
\gamma_n L_n(y_n, \Omega_n) = \left( \frac{m_p}{m_e} \right) c \sigma_{p\gamma} \gamma_n^2 \sum_{i=1}^{N} \frac{\xi_i \rho_i}{\chi_i} \int d\Omega \ (1 - \cos \psi) \times \int_0^{\infty} d\epsilon \ u(\epsilon, \Omega) N_p \left( \frac{\gamma_n}{\chi_i}, \Omega_n \right) H \left( \tilde{\epsilon}_i, \tilde{\epsilon}_i, \tilde{\epsilon}_{i+1} \right),
\]

where

\[
\tilde{\epsilon}_i = \frac{\gamma_n}{\chi_i} (1 - \cos \psi)
\]

and \( \cos \psi = \mu \mu_n + \sqrt{1 - \mu^2} \sqrt{1 - \mu_n^2} \cos \phi \), taking \( \phi_n = 0 \) without loss of generality.

Using Equation (16), we have

\[
\gamma_n L_n(y_n, \Omega_n) = \left( \frac{m_p}{m_e} \right) c \sigma_{p\gamma} \frac{4\pi}{\delta \rho} \gamma_n \sum_{i=1}^{N} \frac{\xi_i \rho_i}{\chi_i} \times \int d\Omega (1 - \cos \psi) \int_0^{\infty} d\epsilon \ u(\epsilon, \Omega) H \left( \tilde{\epsilon}_i, \tilde{\epsilon}_i, \tilde{\epsilon}_{i+1} \right)
\]

where \( \gamma'_i \equiv \gamma_i / \chi_i \delta \rho \).

We now specialize to the case of a surrounding external radiation field that is isotropic in the stationary frame of the black-hole/accretion-disk system. Therefore, \( u(\epsilon, \Omega) = u(\epsilon)/4\pi \), and we can let \( m_n = 1 \) without loss of generality. After some straightforward manipulations, we obtain

\[
\gamma_n L_n(y_n, \Omega_n) = \frac{c \sigma_{p\gamma} m_p}{4\pi m_e} \gamma_n \sum_{i=1}^{N} Y_i \left[ y_i^2 N_i(\gamma'_i) \right] \times \left[ \int_{\gamma_i}^{\gamma_{i+1}} d\epsilon \ u(\epsilon) - \frac{\gamma_i^2}{\gamma_{i+1}^2} \int_{\gamma_i}^{\gamma_{i+1}} d\epsilon \ u(\epsilon) e^2 + y_{i+1}^2 \int_{\gamma_i}^{\gamma_{i+1}} d\epsilon \ u(\epsilon) e^3 \right],
\]

where \( Y_i \equiv \rho_i \xi_i \chi_i \) and

\[
Y_i(\epsilon) \equiv \frac{\chi_i \tilde{\epsilon}_i(\epsilon)}{2\gamma_n}.
\]

Or, we have

\[
\gamma_n L_n(y_n, \Omega_n) = \frac{c \sigma_{p\gamma} m_p}{4\pi m_e} \gamma_n \sum_{i=1}^{N} Y_i \left[ y_i^2 N_i(\gamma'_i, \Omega_n) \right] \times \left[ \int_{\gamma_i}^{\gamma_{i+1}} d\epsilon \ u(\epsilon) - \frac{\gamma_i^2}{\gamma_{i+1}^2} \int_{\gamma_i}^{\gamma_{i+1}} d\epsilon \ u(\epsilon) e^2 + y_{i+1}^2 \int_{\gamma_i}^{\gamma_{i+1}} d\epsilon \ u(\epsilon) e^3 \right].
\]

4.1. External Isotropic Monochromatic Radiation Field

For a monochromatic external radiation field, we write \( u(\epsilon) = u_0 \delta(\epsilon - \epsilon_0) \). In this case,

\[
\gamma_n L_n^{\text{line}}(y_n, \Omega_n) = \left( \frac{m_p}{m_e} \right) c \sigma_{p\gamma} u_0 \delta \rho \sum_{i=1}^{N} Y_i \left[ y_i^2 N_i(\gamma'_i) \right] \times H(\epsilon_0, \gamma_i) + \frac{\gamma_i^2}{\epsilon_0^2} H(\epsilon_0) - \frac{\gamma_i^2}{\epsilon_0^2} H(\epsilon_0, \gamma_i),
\]
This can also be written as
\[4\pi \gamma_n L_n^{\text{inc}}(\gamma_n, \Omega_n) = \frac{c^2 \sigma_T u_0 m_p}{m_e c^*} \delta_D^n \sum_{i=1}^{N} \frac{\chi_i e_i}{\epsilon_*^2} H \left( \gamma_n - \frac{\chi_i e_i}{2 \epsilon_*} \right) - \frac{\gamma_i^2}{\epsilon_*^2} H \left( \gamma_n - \frac{\chi_i e_i}{2 \epsilon_*} \right), \quad \text{(27)}
\]
The single step-function approximation with \(\varepsilon_2 \rightarrow \infty\) takes the form
\[4\pi \gamma_n L_n^{\text{inc}}(\gamma_n, \Omega_n) \approx \frac{c^2 \sigma_T u_0 m_p}{m_e c^*} \delta_D^n \sum_{i=1}^{N} \frac{\chi_i e_i}{\epsilon_*^2} H \left( \gamma_n - \frac{\chi_i e_i}{2 \epsilon_*} \right), \quad \text{(28)}
\]
with \(\gamma_1 = \chi_1 e_1 \rho_1, \gamma_{p1} = \gamma_n / \chi_1 d_0, \) and \(\gamma_1 = \chi_1 e_1 / 2 \gamma_n.\) This confirms the approximate derivation, Equation (17), and also shows that secondary production of photohadronic secondaries has a beaming factor \(\delta_D\).

4.2. External Graybody Radiation Field

The graybody radiation field, defined as a blackbody radiation field times the graybody factor \(g\) giving the ratio of energy densities of the radiation field under consideration to that of a blackbody radiation field with the same effective temperature \(T = m_e c^2 \Theta / k_B,\) is expressed as a spectral energy density in the form
\[u_{eb}(\epsilon; \Theta) = g \frac{8\pi m_e c^3}{\lambda_C^3} \epsilon^3 \exp(\epsilon / \Theta) - 1, \quad \text{(29)}
\]
The electron Compton wavelength \(\lambda_C = h / m_e c = 2.42 \times 10^{-10} \text{ cm} \) is the energy density of blackbody radiation
\[u_{bb}(\Theta) = \frac{8\pi^4 m_e c^2}{15 \lambda_C^3} \Theta^4, \quad \text{(30)}
\]
and \(g = u_0 / u_{bb},\) by definition. Substituting Equation (29) into (24) and solving gives
\[
4\pi \gamma_n L_n^{\text{gb}}(\gamma_n, \Omega_n) \approx \frac{15 c^2 \sigma_T u_0 m_p}{\pi^4 m_e^2} \delta_D^n \sum_{i=1}^{N} \frac{\chi_i e_i}{\epsilon_*^2} N_i(\gamma_{p1}) \left[ \frac{\gamma_i^2}{\epsilon_*^2} \right] I(x), \quad \text{(35)}
\]
where \(x = \chi_i e_i / 2 \gamma_n \Theta\) and
\[I(x) \equiv I_2(x) - x^2 I_0(x) \rightarrow \begin{cases} 2(3)^x + x^2(\ln x - 1/2), & x < 1 \\ (2 + 2x) \exp(-x), & x \gg 1 \end{cases}, \quad \text{(36)}
\]
Integrals \(I_1(x) = \int_x^\infty \frac{d w w^2}{\exp(w) - 1} \) also arise in the study of anisotropic Compton scattering of stellar blackbody radiation (Dermer & Böttcher 2006). The large-\(x\) asymptote is accurate to within 20% even for small \(x\) (Figure 2).

4.3. Secondary Photopion Production Cross Section

An \(N = 2\) segmented cross section was used in Atoyan & Dermer (2003) and Dermer & Menon (2009), which accounted for \(\Delta(1232)\) resonance and multi-particle production in \(p + \gamma \rightarrow \pi + X\) interactions. We extend the \(N = 2\) segment approximation to \(N = 3\), which now better accounts for the production of heavy resonances. Parameters are given in Table 1. The number of segments in the approximation can be made arbitrarily large, but energy dispersion, amounting to another integration, is required to reproduce the accuracy of Monte Carlo simulations. The minor loss in accuracy is compensated by code flexibility. Detailed numerical calculations of neutrinos in relativistic jet sources, where the multi-pion production is taken into account, are found in, e.g., Mücke & Protheroe (2001) and Murase (2007).

4.4. Jet Power and Energetics

The apparent isotropic jet power during a short flaring episode of a black-hole jet can be large and must be \(> 10^{48} \text{ erg s}^{-1}\) in...
order to reproduce the value of $L_{\gamma} \approx 10^{47}$ erg s$^{-1}$ observed in VHE $\gamma$-rays from 4C +21.35 during the flare (the GeV $\gamma$-rays can be and are likely to be produced in the inner jet; see Section 7). For a continuous jet, which makes persistent emission over long periods of time, the jet power is likely to be smaller but still has to exceed $\approx 2.5 \times 10^{47}$ erg s$^{-1}$, which is the two-year time-averaged $\gamma$-ray luminosity measured with the Fermi-LAT for 4C +21.35 (Ackermann et al. 2011).

For a continuous jet, the total jet power supplied by the black hole is composed of three terms from the magnetic field, particle, and photon power. In the naive standard model, we write the total jet power for a two-sided jet as

$$P_\star = P_{\star,B} + P_{\star,par} + P_{\star,\gamma}$$

$$= 2\Omega J_\beta c R^2 \beta^2 \left( \frac{B^2}{8\pi} + \frac{\xi}{V_b} \right) + \frac{8G}{3d^3_b} L_{\text{syn}} + \frac{32G^4}{5d^6_b} L_{\text{EC}}$$  \hspace{1cm} (37)

(Celotti & Fabian 1993; Celotti & Ghisellini 2008). The final two terms in this expression represent the photon power from synchrotron and Compton processes, respectively, and are derived in Appendix A. Here, $L_{\text{syn}}$ and $L_{\text{EC}}$ are the measured apparent bolometric luminosities of the synchrotron and Compton processes, respectively, noting that the synchrotron term applies to emission that is isotropic in the comoving frame with beaming factor $\delta_b$ and so would also apply to SSC emission, while the Compton term applies to the external Compton process with beaming factor $\delta_b$, which applies both in the Thomson (Dermer 1995) and Klein–Nishina regimes (Georganopoulos et al. 2001, 2004).

The comoving spherical blob volume is, of course, $V_b = 4\pi r_b^3 / 3$, and the particle power, assumed to be dominated by the hadrons (in particular, the accelerated protons), is just

$$\xi_{\text{par}} = m_pc^2 \int_1^{\infty} d\gamma' \gamma' N_p(\gamma')$$  \hspace{1cm} (38)

The jet opening solid-angle $\Omega_J \equiv \pi r_b^2 / R^2$.

The comoving magnetic field $B$ required to accelerate particles to energy $E$ after escaping the jet is restricted by the Hillas (1984) condition that the comoving particle Larmor radius $r'_L = E / Q B' \approx E / \Gamma Ze B' < r'_b$. The causality condition restricts the blob radius to be

$$r'_b \lesssim \frac{\epsilon_\delta\tau_{\text{var}}}{1 + z}$$  \hspace{1cm} (39)

where the measured variability time is denoted by $\tau_{\text{var}} \equiv 10^3 \tau_{\text{var},3}$ s. Thus, the minimum magnetic field $B'_{\text{min}}$ required for proton acceleration to energy $10^{20} \ Е_{20}$ eV is given by

$$B'_{\text{min}} = \frac{(1 + z)E_{20}}{\Gamma e c \delta \tau_{\text{var}}} \approx \frac{E_{20}(1 + z)}{(\Gamma/100)(\delta_p/100)\tau_{\text{var},3}} G.$$  \hspace{1cm} (40)

The accelerated proton distribution in the fluid frame is assumed to take the form

$$\gamma_{\nu}^2 N_p(\gamma'_p) = K' H(\gamma'_p - \gamma_{\min}^p) \gamma_{\nu}^{2-\delta} \exp\left(-\frac{\gamma'_p}{\gamma_{\max}^p}\right),$$  \hspace{1cm} (41)

so that $\gamma_{\min}^p \approx 1.1 \times 10^{11} E_{20}/\Gamma$ when $B' = B'_{\text{min}}$. The normalization for $K'$ in terms of the jet-frame particle energy content

$$\xi_{\text{par}} \approx \frac{m_pc^2 K' \gamma_{\min}^{2-\delta}}{s - 2} \left[1 - \left(\frac{\gamma_{\max}^p}{\gamma_{\min}^p}\right)^{2-s}\right]^{\gamma_{\max}^p} \approx 2 m_pc^2 K' \ln G,$$  \hspace{1cm} (42)

where $G = \gamma_{\max}^p / \gamma_{\min}^p$. The total particle power, assumed to be dominated by accelerated protons, is from Equation (37) given by

$$P_{\star,\text{par}} = \frac{3}{2b} \beta c \Gamma^2 \xi_{\text{par}}.$$  \hspace{1cm} (43)

The total magnetic-field power $P_{\star,B} = \beta cr_b^2 \Gamma^2 (B'^2 / 4)$. Using Equation (40) with $B' \approx B'_{\text{min}}$ gives

$$P_{\star,B} = \frac{c}{4Z^2} E_{20}^2 \approx 8.3 \times 10^{44} \beta E_{20}^2 / Z^2 \text{ erg s}^{-1}$$  \hspace{1cm} (44)

for relativistic outflows, which recovers a familiar result (Waxman 2004; Farrar & Grunzinov 2009; Dermer & Razzaque 2010). Note that unlike the magnetic-field power, the particle power $P_{\star,\text{par}} \propto r_b^{-3}$ and becomes larger for smaller $\tau_{\text{var}}$.

### 4.5. Maximum Particle Energy and Bulk Lorentz Factor

The minimum bulk Lorentz factor from $\gamma\gamma$ attenuation is given by $\Gamma_{\min} \approx [\sigma_T d_2^2 (1 + z^2) f_{\text{par}} e_1 / 4\pi \tau_{\text{var}} m_e c^4]^{1/6}$, where $f_\nu = 10^{-12} f_{-12}$ erg cm$^{-2}$ s$^{-1}$ is the $\nu F_\nu$ flux of the target photons (assumed to be produced cospatially with the highest energy photon with energy $m_ec^2 e_1$) evaluated at $\epsilon = 2\Gamma_{\min}^2 / (1 + z^2) e_1$. For 4C +21.35,

$$\Gamma_{\min} \approx 14.1 \left[ f_{-12}(E/100 \text{ GeV})^{1/6} \right].$$  \hspace{1cm} (45)

Photons measured with energies of 100 GeV preferentially interact through $\gamma\gamma$ processes with photons with energies $E_\nu = m_e c^2 \epsilon \approx 0.5$ keV, where $f_{-12} \approx 1$ (Tavecchio et al. 2011). This estimate assumes cospatiality of the emission and only takes into account $\gamma\gamma$ absorption by photons in the jet.

The Hillas (1984) condition, with $B'$ given by Equation (3) and the emission region size scale $r'_b$ given by Equation (39), implies a maximum energy

$$E_{\max} \approx Ze\Gamma B'r'_b \approx 4 \times 10^{20} Z \left(\frac{\Gamma}{100}\right)^3 \left(\frac{\tau_{\text{var}}}{(1 + z) \text{600 s}}\right) \times \sqrt{\frac{\phi_{-1}}{(A_c/100)}} \text{ eV}$$  \hspace{1cm} (46)
of particles accelerated by and escaping from the jet. Here \( \delta_D \approx \Gamma \) is used. Thus, though other losses such as photodisintegration cooling may be relevant, acceleration of UHECR protons is feasible in the inner jet of 4C +21.35 provided that \( \Gamma \geq 100 \) and \( B' \approx 10^2 \). There is no conflict between this value and Equation (45).

4.6. Single Blob versus Continuous Jet

In our formulation of the problem, the secondary production spectrum was derived for a single "one-zone blob," but the jet power was derived for a continuous jet. The energy density in the blob, which is mainly in the form of energetic particles, is equated with the product of the energy density used to determine the power of the continuous jet and the blob volume determined through Equation (39). Consequently, the single blob represents only a single slice with comoving width \( r_0 \) and stationary-frame width \( \approx c\Delta t_{var} / (1 + z) \) of the continuous jet and has a comoving particle energy content given by Equation (43).

This is not, however, the only or the most intuitive normalization. As outlined in the estimates, Sections 3.2 and 3.3, we can alternately constrain the total energy in an outburst that lasts for \( t_{var} \) to be Eddington-limited, so that the total particle energy \( E_{par} \sim L_{\text{Edd}} t_{var} / (1 + z) \), or the comoving particle energy

\[
E_{par}^c \sim \frac{L_{\text{Edd}} t_{var}}{\Gamma (1 + z)}.
\]

(47)

Comparing with Equation (43) and requiring \( \mathcal{P}_{s, par} \leq L_{\text{Edd}} \) shows that the two normalizations differ by a factor \( 2 \delta_D / 3 \Gamma \), which is of order unity for emission within the Doppler beaming cone.

We first derive the neutron production luminosity for a single blob traveling through the external radiation field. Because it represents only one zone of the continuous jet, it can severely underestimate the possible secondary power for a persistent jet. Afterward we consider secondary emission from a continuous jet, noting that this produces a nonvariable/continuous outflow with duration of \( \approx (1 + z) R / c \), whereas the single blob gives an outgoing pulse of secondaries that preserves the engine variability timescale.

4.6.1. Discrete Blob

The apparent isotropic luminosity of secondary neutron produced by cosmic-ray protons in a jet traveling through a background external isotropic monochromatic radiation field characterized by \( u_0 / \epsilon_* \) is, from Equation (28), given by the expression

\[
4\pi L^\text{line}_n(\Omega_n) = 4\pi \int_1^\infty dy_n \, L^\text{line}_n(y_n, \Omega_n) = c \sigma_{\gamma n} \frac{4 \pi \mu_0}{m_0 \epsilon_*} \frac{\Gamma^4 Y_1 \delta_D^5 K'}{s - 2} \left( \frac{\epsilon_1}{\epsilon_*} \right)^{2-s} \times \left[ 1 - \left( \frac{2 \delta_D \epsilon_* \gamma_{\text{max}}}{\epsilon_1} \right)^{s-2} \right]^{1/2} c \sigma_{\gamma n} \times \frac{4 \pi \mu_0}{m_0 \epsilon_*} \frac{\Gamma^4 \delta_D^5 K'}{s - 2} \ln \left( \frac{2 \delta_D \epsilon_* \gamma_{\text{max}}}{\epsilon_1} \right),
\]

(48)

restricting to the \( s \rightarrow 2 \) limit in the final expression. Here, the term \( [1 - (\gamma_1 / \epsilon_*)^2] \) in Equation (28) is neglected because it makes only a small correction to the integral.

If a single blob explains the 4C +21.35 observations, then a minimal requirement is that the apparent secondary power in neutrons exceeds the radiant power \( L_{\gamma} = 10^{48} L_{48} \) erg s\(^{-1}\) measured in the observer direction, which is subsequently converted to an apparent isotropic luminosity of \( \approx 10^{37} \) erg s\(^{-1}\) in \( \gamma\)-rays at the pc scale. The condition \( 4\pi L^\text{line}_n(\Omega_n) \geq 10^{48} L_{48} \) erg s\(^{-1}\) implies, when \( s = 2 \),

\[
K'_{\text{line}} \geq \frac{3 \times 10^{59} L_{48}}{\gamma_{\text{var}}^2 (u_0 / \epsilon_*)^3 \delta_D^5 \ln \Gamma},
\]

(49)

where \( \gamma_{\text{var}} \equiv 10^{-1} \gamma_{\text{var}} \) and \( A \equiv (2 \delta_D \epsilon_* \gamma_{\text{max}} / \epsilon_1) \). From Equations (42), (43) and (49), the absolute jet power is required to be at least

\[
\mathcal{P}_{s, par} > \frac{8 \times 10^{45}}{t_{\text{var}, 3}} \frac{\ln G}{L_{48}} \left( \Gamma / 100 \right)^2 \left( \delta_D / 100 \right)^6 \frac{u_0 / \epsilon_*}{\gamma_{\text{var}}^2 (u_0 / \epsilon_*)^3} \gamma_{\text{var}} \text{ erg s}^{-1}.
\]

(50)

The analogous expression for a thermal greybody radiation field, from Equation (31), is

\[
4\pi L^\text{gb}_n(\Omega_n) \geq \frac{30}{\pi^4} \frac{m_0 \mu_0 c \sigma_{\gamma n}}{m_e \Theta} \frac{\Gamma^4 Y_1 \delta_D^5}{s - 2} \left( \frac{\epsilon_1}{\epsilon_*} \right)^{2-s} \times \left[ 1 - \left( \frac{2 \delta_D \epsilon_* \gamma_{\text{max}}}{\epsilon_1} \right)^{s-2} \right]^{1/2} c \sigma_{\gamma n} \times \frac{30}{\pi^4} \frac{m_0 \mu_0 c \sigma_{\gamma n}}{m_e \Theta} \frac{\Gamma^4 \delta_D^5}{s - 2} \ln \left( \frac{2 \delta_D \epsilon_* \gamma_{\text{max}}}{\epsilon_1} \right),
\]

(51)

where \( C \equiv (2 \delta_D \epsilon_* \gamma_{\text{max}} / \epsilon_1) \), implying

\[
K'_{\text{gb}} \geq \frac{1.2 \times 10^{59} L_{48}}{\gamma_{\text{var}}^2 (u_0 / \Theta) \delta_D^5 \ln \Gamma}.\]

(52)

From Equations (42) and (43), the absolute jet power when \( s = 2 \) is at least

\[
\mathcal{P}_{s, par} > \frac{3 \times 10^{45} (1 + z) L_{48} \left( \Gamma / 100 \right)^2 \ln G}{t_{\text{var}, 3} u_0 / \Theta} \frac{\left( \delta_D / 100 \right)^6}{\gamma_{\text{var}}^2 (u_0 / \epsilon_*)^3} \gamma_{\text{var}} \text{ erg s}^{-1}.
\]

(53)

Using the alternate normalization for the single blob comoving energy content, Equation (47), amounts to multiplying these estimates by the \( 2 \delta_D / 3 \Gamma \) factor with no impact on these jet-power estimates for jets observed within the Doppler beaming cone.

Equations (50) and (53) can be read as saying that to produce apparent isotropic neutron luminosity of \( \approx 10^{38} \) erg s\(^{-1}\) from an outburst lasting \( \sim 10^3 \) s to the observer requires a jet with \( \delta_D \approx \Gamma \approx 10^2 \) and absolute jet power of \( \sim 10^{46} \) erg s\(^{-1}\) primarily in the form of cosmic-ray protons with an \( s = 2 \) spectrum, confirming the estimates of Section 3. The large Doppler factors and associated jet collimation, with jet opening angle \( \theta \sim 1 / \Gamma \), overcome the photodisintegration inefficiency and preserve the original timescale of variability to be consistent with sub-Eddington luminosities and rapid variability.

4.6.2. Continuous Jet

The differential secondary neutron production luminosity \( \gamma_n L_n(y_n, \Omega_n) \) applies to a single blob traveling through a background external radiation field of spatial extent \( R \). This emission persists for a time \( \Delta t = R (1 + z) / \beta \gamma_{\text{var}} \delta_D \) for an observer at angle \( \theta \) with respect to the jet axis, so we can write the time-dependent secondary production spectrum for a single zone as \( \gamma_n L_n(y_n, \Omega_n, t) = \gamma_n L_n(y_n, \Omega_n) H(t; t_0, t_0 + \Delta t) \), where \( t_0 \) is some arbitrarily chosen zero of time.
For a succession of blobs that constitute the continuous jet, the time-averaged spectral luminosity is

\[ \langle \gamma_n L_n(\gamma_n, \Omega_n) \rangle = (\rho_s \Delta_t) \gamma_n L_n(\gamma_n, \Omega_n) \]

\[ \lesssim (1+z)^2 \frac{R_{\text{jet}}}{c t_{\text{jet}}} \gamma_n L_n(\gamma_n, \Omega_n), \]  

(54)

where \( \rho_s \) is the rate at which blobs are ejected. The largest possible value of \( \rho_s \) corresponds to the ejection of blobs at the rate \( \beta c/\Delta t \), where \( \Delta t = c t_{\text{jet}}/(1+z) \) is the stationary-frame width of the blob, that is, when the time between blob ejection is just \( \Delta t/\beta c \), corresponding to the continuous jet limit. This explains the coefficient in the final term of Equation (54). There is reduction by one power in the beaming factor from a discrete blob to a continuous jet, as is well known (Lind & Blandford 1985).

4.6.3. Internal Synchrotron Photons

An alternative to assuming large Doppler factors is to have small Doppler factors \( (\delta_D \ll 100) \), so that the internal synchrotron photons present a dense target radiation field (Atoyan & Dermer 2001, 2003); see Appendix B. But the Doppler amplification of the received luminosity would be much reduced by such small Doppler factors, so that absolute jet powers larger than the Eddington luminosity would be required. Consequently, the large Doppler factor solution seems preferable unless an alternative mechanism to make rapid variability at the pc scale is devised. Note furthermore that photons from the direct accretion-disk radiation field (Mücke & Protheroe 2001), which provide an additional important target photon source that has not been considered here, are more important for jetted emitting regions far from the black hole when the Doppler factor is larger, because aberration of accretion-disk photons is greater (Dermer & Schlickeiser 2002).

4.7. Calculations

We use the formalism developed here to make calculations of secondary neutron and neutrino production spectra from the interactions between a power-law distribution of UHECR protons, isotropic in the jet rest frame, that interact with BLR radiation in the inner jet to make secondary neutrons, pions, and neutrinos. The results are expressed in the form \( 4 \pi L(E, \Omega) = 4 \pi E \rho P_{\text{jet}}(E, \Omega)/dEd\Omega \) (units of erg \( s^{-1} \)), so as to appear isotropic luminosity for the production of secondaries with that energy. Given that we are scaling to 4C +21.35 with \( 4 \pi d_L^2 = 6.52 \times 10^{56} \text{ cm}^2 \) and bolometric energy fluxes during flares exceeding \( 10^{-9} \text{ erg cm}^{-2} \text{ s}^{-1} \), this corresponds to apparent bolometric \( \gamma \)-ray luminosities \( > 10^{48} \text{ erg s}^{-1} \).

Flaring results to model the SED of 4C +21.35 from photobehadronic proton and neutrino emissions made by UHECR particle acceleration in the inner jet are shown in Figures 3 and 4, respectively. In the picture studied here, cosmic-ray protons in a single-zone blob with \( \Gamma = 100 \) and \( \delta_D = 200 \)—hence looking directly down the jet—interact with photons of the BLR and the dust torus. Here we normalize to a total jet power \( P_{\text{jet}} = 2 \times 10^{46} \text{ erg s}^{-1} \), which is the Eddington luminosity of a 1.5 \( \times 10^9 M_\odot \) black hole. We use \( t_{\text{jet}} = 600 \text{ s} \) for the variability timescale and Equation (47) to normalize the energy content of jet protons. This jet energy is fed into magnetic field, photons, particles, and photons, limited by \( P_{\text{jet}} = L_{\text{Edd}} \). To accelerate protons to \( 10^{20} \text{ eV} \), \( B' \gtrsim 1 \text{ G} \), from Equation (40). Most of the power goes into particles; the magnetic field required to accelerate \( 10^{20} \text{ eV} \) protons leads to a total jet magnetic-field power of \( \approx 10^{25} \text{ } E_{20} \text{ erg s}^{-1} \), from Equation (44), which is \( \approx 5\% \) of \( L_{\text{Edd}} \). The photon power is smaller still, due to the small beaming factor \( \approx 1/T^2 \). We let \( \gamma_{\text{min}} = \Gamma \), corresponding to the minimum Lorentz factor of protons swept into the jet. The effective photon densities for the \( \text{Ly} \alpha \) and scattered accretion-disk radiation field

Figure 3. Black solid curve shows the secondary production spectrum of neutrons made from cosmic-ray protons accelerated in the inner jet of a blazar. The jet is assumed to have bulk Lorentz factor \( \Gamma = 100 \), be aligned along the jet axis \( (\theta_{\text{obs}} = 0^\circ \text{ and } \delta_D = 200) \), and interact with BLR and torus photons. The separate components from interactions with Ly\( \alpha \), scattered accretion disk, and warm and cool dust radiations are labeled. The contributions from the different resonance and multi-pion interactions are shown separately for the scattered accretion-disk radiation component.

(A color version of this figure is available in the online journal.)

Figure 4. Black solid curve shows the secondary production spectrum of neutrinos made from cosmic-ray protons accelerated in the inner jet of a blazar, using the same parameters as in Figure 3. The separate components for neutrino production from interactions with Ly\( \alpha \), scattered accretion disk, and warm and cool dust radiations are labeled for the channel \( (p + \gamma) \rightarrow \pi \rightarrow \nu \). The corresponding line styles are shown for the production of \( \beta \)-decay neutrinos denoted by \( n \rightarrow \nu \), which is formed when the neutrons, assumed to escape the production site and travel rectilinearly until decay, make neutrinos through the interactions between a power-law distribution of UHECR protons, isotropic in the jet rest frame, that interact with BLR radiation in the inner jet to make secondary neutrons, pions, and neutrinos. The results are expressed in the form \( 4 \pi L(E, \Omega) = 4 \pi E \rho P_{\text{jet}}(E, \Omega)/dEd\Omega \) (units of erg \( s^{-1} \)), so as to appear isotropic luminosity for the production of secondaries with that energy. Given that we are scaling to 4C +21.35 with \( 4 \pi d_L^2 = 6.52 \times 10^{56} \text{ cm}^2 \) and bolometric energy fluxes during flares exceeding \( 10^{-9} \text{ erg cm}^{-2} \text{ s}^{-1} \), this corresponds to apparent bolometric \( \gamma \)-ray luminosities \( > 10^{48} \text{ erg s}^{-1} \).

(A color version of this figure is available in the online journal.)
are given by Equations (5) and (6), respectively. The total power that is not in magnetic field or photons is assumed to be mostly in the form of a nonthermal particle distribution of the form of Equation (41), with \( s = 2.1 \).

Figure 3 shows the neutron production spectrum for a head-on (\( \theta_{\text{obs}} = 0^\circ \)), \( \Gamma = 100 \) jet. The large apparent neutrino luminosities, exceeding \( 10^{50} \text{ erg s}^{-1} \) from an AGN with absolute jet power of just \( 2 \times 10^{46} \text{ erg s}^{-1} \), show that photodrastic processes in the inner jet can be very efficient (contrary to Nalewajko et al. 2012), with the warm dust more effective than accretion-disk radiation for a hard cosmic-ray proton spectrum that extends to \( 10^{20} \text{ eV} \). The two peaks found in the production/injection of neutrons into the pc-scale region where only IR dust photons are present mean that only the highest energy neutrons with \( E_n \gtrsim 10^{18} \text{ eV} \) are effective at making photopion secondaries beyond the inner jet within the BLR.

Figure 4 is a calculation of the neutrino production spectra for an inner-jet system with the same parameters as in Figure 3. No mixing corrections are made. The apparent bolometric luminosity is \( \gtrsim 10^{48} \text{ erg s}^{-1} \), which, as noted before, requires large Lorentz factors, preferential Doppler factors, and production in the inner jet for short variability. The components labeled by \( \pi \rightarrow \nu \) refer to neutrino production in photopion interactions, \( p + \gamma \rightarrow \pi \rightarrow \nu \). The low-energy features in the neutrino production spectra labeled \( n \rightarrow \nu \) are \( \beta \)-decay electron neutrinos, when the bulk of the neutrons escape and decay in intergalactic space. Both the neutron and neutrino spectra become very steep, harder than +2 in \( E(L,E) \), below PeV energies, as a result of threshold effects. Detailed calculations (e.g., Murase 2007) find softer spectra below PeV energies than calculated here due to the spread in the energy distribution of low-energy pions, especially from \( \pi / \mu \) decay kinematics and partially from multi-pion production.

Figure 5 compares the apparent isotropic luminosities (or production spectra) of neutrons (blue curves) with neutrinos (red curves). Using the same parameters as Figure 3, this figure shows production spectra calculated for \( \Gamma = 100, 50, \) and 20 at observing angles \( \theta = 0 \) and \( \theta = 1/\Gamma \). The comparison reveals the strong, \( \delta_\parallel \) beaming factor of this process (Equation (35)) when viewing off-axis, leading to a factor \( 2^5 = 32 \) reduction of the production luminosity at the Doppler beaming angle compared to the luminosity along the jet axis. The relative fluxes as a function of \( \Gamma \) at the same observing angle (which would be peculiar except for \( \theta = 0 \)) are closer to \( \alpha \Gamma^{-4} \), due to kinematic effects and normalization. Neutrons are made as secondaries of protons with \( E_n \approx 0.8E_p \) and carry the bulk of the energy of the incident protons. Neutrinos, by comparison, carry only \( \approx 5\% \) and \( \beta \)-neutrinos only \( \approx 0.1\% \) of the energy of the original neutron.

Figures 3–5 apply to single one-zone blobs traveling through a beam production region of extent \( R_{\text{BLR}} = 2 \times 10^{17} \text{ cm} \). The duration of the neutron production for on-axis observers is \( \approx (1 + z)R_{\text{BLR}}/2\Gamma c \approx 500 \text{ s} \), and shorter if the accelerated proton distribution decays before it traverses the length of the jet.

Calculations of secondary neutron and neutrino production spectra from a continuous jet allow for larger secondary luminosities at the expense of variability. On the other hand, smaller values of \( \Gamma \) might be more typical during these extended periods, and it is less likely that we would be looking directly along the jet axis. Even for smaller (\( \Gamma \approx 20 \)) jet Lorentz factors, detection of neutron-induced or neutrino emission might be compensated by its longer duration, though compromised by the larger backgrounds for neutrino detection for the longer time windows.

5. Pair-Production Opacity and Photopion Efficiency for Isotropic Radiation Fields

Our results to now are fairly intuitive. Cosmic-ray protons with energies \( p \gtrsim \epsilon_{\text{min}}/\epsilon_\epsilon \), interacting with an external radiation field with mean photon energy \( m_\epsilon c^2 \epsilon_\epsilon \), have a photodrastic neutron production efficiency \( \approx K_p\sigma_{p\gamma}R_{\text{th}}/m_\epsilon c^2 \epsilon_\epsilon \sim 1\%–10\% \) in the inner jet consisting of the BLR and IR dust torus radiation field local to the supermassive black hole in 4C +21.35 (Equation (10) and Sections 3.2 and 3.3). With the stipulation that the absolute jet luminosity is Eddington-limited and that the bulk of this power is transformed to an \( s = 2 \) spectrum of protons with maximum escaping energies \( \approx 10^{20} \text{ eV} \), an impulsive jet has to have \( \delta_D \gtrsim 100 \) to produce a highly collimated beam of outflowing UHE neutrons with apparent luminosity \( \approx 10^{48} \text{ erg s}^{-1} \) in a pulse of particles \( \lesssim 10^{13} \text{ cm} \) in width measured in the black-hole frame. The large Doppler factor collimates the beam and preserves the rapid variability. For a continuous jet, apparent isotropic neutron luminosities of \( \approx 10^{46} \text{ erg s}^{-1} \) are formed by a jet with more modest Doppler factors. The outflowing neutron or neutrino fluxes will persist for longer times, though without short timescale variability.

We now turn our attention to the \( \gamma\gamma \) opacity and photopion efficiency of \( \gamma \)-rays and neutrons in the radiation field formed by the BLR and dust torus, for application to 4C +21.35 or other blazars. We work under the assumption of local isotropy; the paper by Gould (1979) calculates the energy density of the photon field with this assumption relaxed.

5.1. \( \gamma\gamma \) Opacity

The opacity \( \tau_{\gamma\gamma}(E) \) for a \( \gamma \)-ray with energy \( m_\epsilon c^2 \epsilon_1 \) to travel a distance \( R \) through an isotropic radiation field with energy density \( u_0 = m_\epsilon c^2 \int_0^\infty \rho E \epsilon n_{\text{ph}}(\epsilon) \) consisting of photons with
The energy \(m_e c^2 \epsilon\) described by the number density distribution \(n_{ph}(\epsilon)\) is given through the relation

\[
d\tau_{\gamma\gamma} = \frac{\pi \gamma^2 R}{\epsilon^2} \int \frac{d\epsilon}{\epsilon} \epsilon^{-2} n_{ph}(\epsilon) \bar{\phi}(s_0),
\]

where \(s_0 = \epsilon \epsilon_1\),

\[
\bar{\phi}(s_0) = 2 \int_1^{s_0} ds \frac{s \sigma_{\gamma\gamma}(s)}{\pi \gamma^2},
\]

and \(\sigma_{\gamma\gamma}(s)\) is the \(\gamma\gamma\) pair-production cross section. Thus,

\[
\tau_{\gamma\gamma}(\epsilon_1) \equiv \frac{\pi \gamma^2 R}{\epsilon^2} \int \frac{d\epsilon}{\epsilon} \epsilon^{-2} n_{ph}(\epsilon) \bar{\phi}(s_0).
\]

The function

\[
\bar{\phi}(s_0) = \left(2s_0 - 2 + \frac{1}{s_0}\right) \ln w_0 + 2(1 - 2s_0)\sqrt{1 - s_0^{-1}}
+ \ln w_0[4\ln(1 + w_0) - 3 \ln w_0]
- \frac{1}{3} \pi^2 + 4 \sum_{n=1}^{\infty} (-)^{n-1} n^{-2} w_0^{-n},
\]

where \(w_0 \equiv 2s_0 \sqrt{1 - s_0^{-1}} - 1\). The asymptotes of \(\bar{\phi}(s_0)\) are

\[
\bar{\phi}(s_0) \rightarrow \left\{\begin{array}{ll}
\frac{(2s_0 + \ln 4s_0)(\ln 4s_0 - 2)}{3} & \text{if } s_0 \gg 1 \\
\frac{4}{3}(s_0 - 1)^{3/2} + \frac{2}{3}(s_0 - 1)^{1/2} & \text{if } 253/70(s_0 - 1)^{1/2} + \ldots \\
\end{array}\right.

s_0 \gg 1.
\]

Figure 6. Function \(\bar{\phi}(s)/s^2\), calculated from Equation (58), is given by the solid red curve, along with asymptotes in the low- and high-energy limits from Equation (59). The dark blue curve is the function \(\mathcal{F}(\nu)\), Equation (63), with asymptotes given by Equation (64).

(A color version of this figure is available in the online journal.)

target photon energy \(m_e c^2 \epsilon_*\) is therefore

\[
\tau_{\gamma\gamma}(\epsilon_1) \equiv \frac{\bar{\tau}_{\gamma\gamma}}{s_0^2}, \quad \bar{\tau}_{\gamma\gamma} = \frac{\pi \gamma^2 R u_0}{m_e c^2 \epsilon_*},
\]

where

\[
\int_{s_0}^{\infty} ds \frac{\bar{\phi}(s)}{s^2} \frac{\epsilon}{\epsilon^2} = \frac{3 \sigma_\gamma R u_0}{8 m_e c^2 \epsilon_*}.
\]

A graph of the function \(\bar{\phi}(s_0)/s_0^2\) used to calculate \(\tau_{\gamma\gamma}(\epsilon_1)\) in an isotropic photon field, including asymptotes from Equation (59) at small and large values of \(s_0\), is shown in Figure 6. Note the \(\ln(\epsilon_1)/\epsilon_1\) dependence at \(s_0 \gg 1\) because \(\bar{\phi}(s_0)/s_0^2 \rightarrow 2\ln(0.5s_0)/s_0\) in the limit \(s_0 \gg 1\). The function \(\bar{\phi}(s_0)/s_0^2\) reaches a maximum value of \(\approx 0.56\) at \(s_0 \approx 3.54\) and falls by a factor of \(\approx 20\) when \(s_0\) goes from 3.5 to 300. For Ly\(\alpha\) line photons, the peak opacity corresponds to about 90 GeV in the stationary frame. Neglecting photon broadening processes, there is a sharp lower limit to the opacity at \(\epsilon_1 = 1/\epsilon_1\), that is, at 25.6 GeV in the stationary frame for opacity from Ly\(\alpha\) photons.

Substituting the graybody spectrum, Equation (29), into Equation (55) gives

\[
\tau_{\gamma\gamma}^{\phi\phi}(\epsilon_1) \equiv \frac{2\gamma R}{\lambda_c} \int_{1/\epsilon_1}^{\infty} d\epsilon \frac{\bar{\phi}(s_0)}{\exp(\epsilon/\Theta) - 1} \approx \frac{2\gamma R}{\lambda_c} \Theta^2 \mathcal{F}(\nu) = \frac{15\gamma R}{\pi^3} \frac{u_0}{m_e c^2 \Theta} \mathcal{F}(\nu);
\]

for the energy-dependent \(\gamma\gamma\) opacity through a graybody radiation field, where the fine-structure constant \(\alpha = e^2/\hbar c \approx 1/137\), and

\[
\nu \equiv \frac{1}{\epsilon_1 \Theta}.
\]

This function reaches its maximum value at \(\mathcal{F}_{pk} \equiv \mathcal{F}(\nu_{pk}) \approx 1.076\) at \(\nu_{pk} \approx 0.51\). The asymptotes of \(\mathcal{F}(\nu)\) are

\[
\mathcal{F}(\nu) \rightarrow \begin{cases} \sqrt{\pi \nu} \exp(-\nu) (1 + \frac{9}{\nu^2}) & \nu \gg 1 \text{ or } \epsilon_1 \ll 1/\Theta \\ \frac{2}{3} \nu \ln(0.47/\nu) & \nu \ll 1 \text{ or } \epsilon_1 \gg 1/\Theta \end{cases}.
\]

Figure 6 is a graph of the function \(\mathcal{F}(\nu)\) along with its asymptotes.

### 5.2. Photopion Efficiency

The derivation of the photopion energy-loss rate of an ultra-relativistic neutron with energy \(E_n = \gamma n m_n c^2 = 10^{20} E_{20}\) eV in an external isotropic radiation field described by the function \(n_{ph}(\epsilon)\) follows closely the derivation sketched in Section 3.3 for cosmic-ray protons in a jet and is given by

\[
t_{\phi\phi}^{-1}(\gamma_n) = \frac{c}{2 \gamma_n^2} \int_0^{\infty} d\epsilon \frac{n_{ph}(\epsilon)}{\epsilon^2} \int_0^{2\epsilon \gamma_n} d\epsilon_\phi e^{\epsilon_\phi} \sigma_{\phi\phi}^{\phi\phi}(\epsilon_\phi) K_{\phi\phi}(\epsilon_\phi) \approx \frac{c \hat{\Theta}_{thr}^{\phi\phi}}{\epsilon_n/2\epsilon_{thr}} \int_{\epsilon_n/2\epsilon_{thr}}^{\infty} d\epsilon \frac{n_{ph}(\epsilon)}{\epsilon^2} \left[ 1 - \left(\frac{\epsilon_{thr}}{2\epsilon_{thr}}\right)^2 \right].
\]

using the approximation \(\sigma_{\phi\phi}^{\phi\phi}(\epsilon) K_{\phi\phi}(\epsilon) \approx \hat{\Theta} H(\epsilon_n - \epsilon_{thr})\), where, as before, \(\epsilon_{thr} \approx 400\) is the photopion threshold photon
energy and $\hat{\sigma} = 70 \mu b$ is the product of the photopion energy-loss proton cross section and inelasticity (Atoyan & Dermer 2003). In a graybody photon field given by Equation (29) with graybody factor $g$,

$$
I_{\phi\gamma}(\gamma_n) \equiv \frac{8\pi c g \hat{\sigma} \Theta^3}{\lambda_c^3} I(\omega),
$$

where $I(\omega) \equiv I_2(\omega) - \omega^2 I_0(\omega)$, from Equation (36); see Figure 7. The parameter

$$
\omega = \frac{\bar{\epsilon}_{\text{thr}}}{2 \gamma_n \Theta}
$$

characterizes the different regimes of photopion interaction of an ultrarelativistic cosmic-ray proton in a blackbody photon distribution, and whether the proton interacts with the exponentially declining number of photons in the Wien regime ($\omega \gg 1$; $\gamma_n \ll 200/\Theta$ or $E_n \ll 10^{18} \text{eV}/(\text{T}_{\text{BLR}}/1200 \text{K})$), or with the bulk of the distribution ($\omega \ll 1$; $\gamma_n \gg 200/\Theta$; $E_n \gg 10^{18} \text{eV}/(\text{T}_{\text{BLR}}/1200 \text{K})$).

Expressing the photopion efficiency

$$
\eta_{\phi\gamma} = \frac{t_c}{t_{\phi\gamma}},
$$

where the light-crossing timescale $t_c = R/c$ across a region of size $R$, then

$$
\eta_{\phi\gamma}(E_n) = \frac{8\pi g \hat{\sigma} \Theta R^3}{\lambda_c^3} I(\omega) = \frac{15}{\pi^4} \frac{\hat{\sigma} R \tau_0}{m_c e^2 \Theta} I(\omega).
$$

The photopion efficiency for monochromatic line radiation is simply

$$
\eta_{\phi\gamma}(E_n) = \frac{u_0 \hat{\sigma} R}{m_c e^2 \epsilon_e}(1 - x^{-2}) H(x - 1), \quad x \equiv \frac{2 \gamma_n \epsilon_e}{\bar{\epsilon}_{\text{thr}}},
$$

and approaches a constant value at $\gamma_n \gg \bar{\epsilon}_{\text{thr}}/2 \epsilon_e$ (compare Equation (13)). We show $\eta_{\phi\gamma}$ in Figure 8 using $R_{\text{BLR}} = 2 \times 10^{17} \text{cm}$. One should keep in mind that $R = 1 \text{pc}$ should be used for the photomeson production between the neutron beam escaping from the BLR region and target photon fields from the dusty torus, and beamed neutrons are essentially depleted in the dust torus.

![Figure 7](image-url) Figure 7. Function $I(\omega)$ and asymptotes, from Equation (36), that describe the photopion energy-loss rate of protons or neutrons with photons of a blackbody or graybody radiation field.

(A color version of this figure is available in the online journal.)

For isotropic monochromatic radiation, the ratio of the peak $\gamma^\gamma$ opacity to the photopion efficiency in the limit of high-energy neutrons is

$$
R_{\text{line}} = \frac{\tau_{\gamma\gamma}(E_1)}{\eta_{\phi\gamma}(x \gg 1)} = \frac{\pi \gamma_e^2}{\Delta} \left[ \frac{\phi(\lambda_0)}{\delta_0} \right] \approx 0.56 \frac{\pi \gamma_e^2}{\Delta} \approx 2000.
$$

(71)

The ratio of neutron energy at $x = 1$ and photon energy at the opacity peak is

$$
\varepsilon_{\text{line}} = \frac{E_n(x = 1)}{E_1} = \frac{m_n}{m_e} \frac{\epsilon_{\text{thr}}}{(2 \times 3.54)} \approx 10^5.
$$

(72)

For thermal blackbody or graybody radiation,

$$
R_{\text{thermal}} = \frac{\pi \gamma_e^2}{\Delta} \frac{\tau_{pk}}{t_{pk}} \approx 3600 \frac{1.076}{\Gamma(3) \zeta(3)} \approx 1600,
$$

(73)

and the ratio of neutron energy at $x = 1$ and photon energy at the opacity peak for the thermal photon distribution is

$$
\varepsilon_{\text{thermal}} = \frac{m_n}{4 m_e} \epsilon_{\text{thr}} \simeq 2 \times 10^5.
$$

(74)

Corresponding relations in relativistic jet sources are given by Dermer et al. (2007).
5.4. Application to 4C +21.35

From Equations (5) and (60), we find that the quasar BLR opacity to Lyα photons is

$$
\tau_{\gamma\gamma}(\epsilon_1) \cong 40R_1^2\phi_{-1}\frac{\Phi(s_0)}{s_0}.
$$

(75)

The Lyα opacity with $R_1\phi_{-1} = 2$ is shown in Figure 8. The large opacity in the VHE range precludes the photons detected with MAGIC from 4C +21.35 from being made in the BLR.

Using the Malmrose et al. (2011) parameters for the quasi-thermal IR emission from the hot dust, the graybody factor was determined for the scattered radiation fields by substituting Equations (79) into Equation (65), using Equations (78) and (81) into Equation (65), using Equations (79) and (68). For comparison, the dotted curves show the opacity and photopion efficiency when the scattered accretion-disk radiation field is approximated by a thermal graybody spectrum using the parameters from Equation (6). Note that radiation fields with higher effective temperatures but comparable luminosities provide negligible additional opacity because of the smaller density of target photons.

6. SYNCHROTRON RADIATION FROM UHECR SECONDARIES

6.1. Analytical Considerations

The results in Figure 8 show that the multi-GeV $\gamma$-rays detected from 4C +21.35 with MAGIC cannot be made in the BLR, and that inner jet radiation should be strongly attenuated above several GeV from scattered accretion-disk radiation. For an accretion-disk power of $5 \times 10^{45}$ erg s$^{-1}$, $R_1 \approx 2$ from Equation (1), and if the accretion-disk power is even larger, as indicated by the Swift data (Tavecchio et al. 2011), the $\gamma\gamma$ attenuation is likely to be even larger than shown in Figure 8. Thus, the emission region for VHE photons must at least be situated beyond the edge of the BLR, but even then photons with energies $\gtrsim$200 GeV are strongly absorbed by the IR radiation from the torus. The actual attenuation depends precisely on deviations of the dust spectrum from a graybody in the Wien regime, but even with the exponentially declining number of photons in the near-infrared spectrum, there is still large opacity for VHE emission above a few hundred GeV. The lack of attenuation in the MAGIC data to $\sim$400 GeV ($\sim$600 GeV in the source frame) implies that the VHE $\gamma$-rays have to be made at and beyond the pc scale.

Escaping neutrons easily leave the BLR but suffer strong photopion losses when passing through the infrared field of the dust torus, where neutrons would deposit a large fraction of their energy during escape. Hence, we should expect formation of beams of $UHE$ $\gamma$-rays, pairs, and neutrinos. The photopion production efficiency $\eta_{\gamma\alpha}(\gamma_n)$ of an escaping neutron or proton with Lorentz factor $\gamma_n$ passing through a graybody of beams consisting of $UHE$ $\gamma$-rays, pairs, and neutrinos. The photopion production efficiency $\eta_{\gamma\alpha}(\gamma_n)$ of an escaping neutron or proton with Lorentz factor $\gamma_n$ passing through a graybody of beams consisting of $UHE$ $\gamma$-rays, pairs, and neutrinos.

In the case of 4C +21.35, efficient photopion losses apply to neutrons or protons with $\omega < 1$, for which hot dust with $\Theta \approx 2 \times 10^{-7}$ implies a range of energies

$$
m_p c^2 \frac{\epsilon_{\text{max}}}{20} \lesssim 6 \times 10^{17} \text{eV} \lesssim \epsilon_p \lesssim \epsilon_{\text{max}} \equiv 10^{20} E_{20} \text{eV}
$$

(82)

for which photopion losses are large, as can be seen from Figure 8.

In each $n + \gamma \rightarrow p + \pi^+$ interaction, roughly 20% of the neutron’s original energy is given to two pionic photons, $\approx 5\%$ is given to an electron, and $\approx 15\%$ is given to three neutrinos. Secondary electron energies therefore range in value from $\approx 3 \times 10^{10}$ eV to $2 \times 10^{12} E_{20}$ eV, implying Lorentz factors in the range

$$
6 \times 10^{10} \lesssim \gamma_e \equiv 10^{11} \gamma_{11} \lesssim 10^{13} E_{20},
$$

(83)

so $0.6 \lesssim \gamma_{11} \lesssim 100 E_{20}$. The synchrotron photons from these highly-relativistic$^7$ electrons are produced through secondary $e^+$

---

$^7$ These electrons are “hyper-relativistic” in the sense (Dermer & Atwood 2004) that such high-energy electrons cannot be directly accelerated by traditional Fermi acceleration mechanisms that compete directly with synchrotron losses.
This is not a good approximation when the photons pair produce in the magnetic field at the pc scale. Thus, the photon energies $\gamma e^{-}$ have observed energies

$$E_{\text{syn}} = mc^2 e_{\text{syn}} \approx \frac{3mc^2}{2B_{\gamma}(1+z)} \approx 120B_{\mu G, pc}\gamma_{11}^{\frac{3}{2}} \text{MeV}, \quad (84)$$

where $B_{\gamma}$ is the magnetic field at the pc scale. Thus, the photon energies of the first generation synchrotron spectrum are in the range

$$43\text{ MeV} \approx \frac{E_{\text{syn}}}{B_{\mu G, pc}} \lesssim 1.2E_{20}^{2} \text{TeV}. \quad (85)$$

The second-generation synchrotron spectrum from $e^{-}e^{-}$ pairs made when synchrotron $\gamma - \gamma$ forms pair through $\gamma \gamma$ attenuation is important for larger, $\sim \text{mG}$ fields. If a synchrotron $\gamma - \gamma$ forms a pair, with each electron and positron receiving half the energy, then the second-generation synchrotron spectrum ranges between

$$0.052 \text{ eV} \lesssim \frac{E_{\text{syn}}}{B_{\mu G, pc}} \lesssim 35E_{20}^{2} \text{MeV}. \quad (86)$$

However, those second-generation synchrotron photons are typically irrelevant for the highly variable emission (see below).

Synchrotron losses will dominate when the synchrotron energy-loss mean free path (MFP) is shorter than other energy-loss MFPs. Figure 9 shows the synchrotron energy-loss MFP in comparison with the Compton energy-loss MFP on the dust radiation fields and, for comparison, the CMBR field at $z = 0.432$. The electron synchrotron energy-loss rate assumes a randomly oriented magnetic field on the pc scale of strength $B_{\mu G, pc}$. The Compton energy-loss MFP of an electron passing through a graybody radiation field is approximated in Figure 9 by the expression

$$\lambda_{C}(\gamma') \equiv c \left[ \frac{\gamma'}{\gamma} \right]^{-1}$$

$$= \begin{cases} \frac{43\lambda_{\gamma}}{32\pi^{3/2}\epsilon^{1/2}B_{\gamma}^{3/2}T_{1200}^{1/2}} \text{pc}, & \gamma' < \gamma \equiv \gamma_{*} \equiv \frac{4.92}{\epsilon} \\ \frac{2\lambda_{\gamma}}{\pi^{3/2}\epsilon^{1/2}\ln(0.55\gamma\Theta)} \text{pc}, & \gamma > \gamma_{*} \equiv \frac{2.4\times10^{7}}{T_{1200}} \end{cases} \quad (87)$$

where the energy density of the blackbody radiation field with temperature $\Theta$ is given by Equation (30) and $T_{1200} \equiv T/1200 \text{K}$. This simple expression connects the Thomson and extreme KN asymptotes with a constant MFP in the intermediate regime determined by equating the Thomson MFP with the value of the KN MFP evaluated at the electron Lorentz factor $\gamma_{*}$ given by $\ln(0.55\gamma/\Theta) = 1$.

From Figure 9, one sees that synchrotron losses of electrons formed by neutron photopion production dominate Compton losses when $B_{pc} \gtrsim 3 \mu G$, and Equation (85) shows that first-generation synchrotron photons with energies exceeding a few GeV are formed when $B_{pc} \gtrsim 10 \mu G$. We now show that such a process can accommodate the short variability timescale observed with MAGIC if the inner engine generates pulses of UHE neutrinos that are modulated by activity of the inner jet on short timescales. In this case, synchrotron production at a distance $R$ from the central engine can vary on timescale $\Delta t$ if the emission is made within the angle $\theta_{e}$ of the line of sight according to the relation $(1 + z)R(1 - \cos \theta_{e})/c = \Delta t$. This is satisfied for electrons produced within the angle

$$\theta_{\text{eff}} \lesssim \theta_{e} = \sqrt{\frac{2c\Delta t}{R(1+z)}} \approx 2.9 \times 10^{-3} \sqrt{(\Delta t/600 \text{ s})/(1+z)R_{pc}}. \quad (88)$$

The electrons making the GeV/TeV synchrotron radiation are deflected by the angle $\theta_{\text{eff}} \approx \sqrt{2/3}\lambda_{\text{syn}}/r_{L} = \delta/\sqrt{3}\pi e/\sigma_{T}B_{\gamma}^{2}$, where $\lambda_{\text{syn}} = 6\pi m_{e}c^{2}/\sigma_{T}B_{\gamma}^{2}$ is the energy-loss length for synchrotron losses and $r_{L} = m_{e}c^{2}/eB$ is the electron Larmor radius. The criterion that $\theta_{\text{eff}} < \theta_{e}$ (Murase 2012) implies

$$B_{\mu G, pc}\gamma_{11}^{2} \gtrsim 330 \sqrt{(\Delta t/600 \text{ s})/R_{pc}} \quad (89)$$

to preserve observed variability on a timescale shorter than $\approx 600 \text{ s}$ for $4C +21.35$. Together with Equations (85) and (86), this variable synchrotron radiation will be detected at photon energies

$$\sqrt{(\Delta t/600 \text{ s})/R_{pc}} \lesssim E_{\text{syn}}(\text{GeV}) \lesssim 1.2 \times 10^{3}E_{20}^{2}B_{\mu G, pc}. \quad (90)$$

---

8 This is not a good approximation when the photons pair produce in the Klein–Nishina (KN) portion of the pair-production cross section (see, e.g., Murase 2009; Kotera et al. 2011). Escape of UHE $\gamma - \gamma$ depends on the internal target photon spectrum, which may be suppressed by self-absorption of the synchrotron spectrum or the Rayleigh–Jones portion of the blackbody spectrum (Razzaque et al. 2004; Murase 2009).
The constraints will be relaxed if the coherence length of the magnetic field is small in comparison with the synchrotron cooling length.

A number of important points can be made regarding this result. The synchrotron radiation frequency depends only on the product $B\gamma^2$ and not on $B$ or $\gamma$ separately. Hence, $\Delta \tau$ essentially depends on $E_{\text{syn}}$, without the explicit dependence on $B$ and $\gamma$ (Murase 2012). Thus, highly variable synchrotron emission is expected from VHE synchrotron $\gamma$-rays. Because the photohadronic production kinematics limits the lower bound of the secondary electron distribution to be at $\gamma_1 \approx 0.6$, from Equation (83) and Figure 8, the characteristic magnetic field at the pc scale is implied to be $\lesssim 3$ mG; otherwise, the energy range of synchrotron photons is out of the MAGIC band. Lower magnetic fields are allowed as long as the synchrotron cooling length is short enough, where lower $B$ corresponds to higher $\gamma$ at a given value of $E_{\text{syn}}$. A definite prediction of this model is that the variability times of the $\gamma$-ray synchrotron radiation become longer at lower energies, because a larger angular range of cooling and deflecting electrons can then contribute to the observed emission. Emission from the inner jet can, however, dominate at GeV energies, so this prediction applies only to the VHE synchrotron $\gamma$-rays. Inverse Compton cascade radiation should be more slowly variable because $\lambda_{\text{IR}} \gg \lambda_{\text{syn}}$, and second-generation synchrotron emission (Equation (86)) does not contribute to the highly variable emission since it is typically expected at lower energies.

6.2. Numerical Results

To demonstrate the resulting secondary spectra produced by the neutron beam launched from the BLR region, we also perform numerical calculations. In order to evaluate photon and pair yields from the photomeson production, we use SOPHIA (Mücke et al. 2000) and solve the kinetic equations for injected photons and pairs, as in Murase et al. (2012). We calculate cascades taking into account synchrotron and inverse Compton emissions, and we focus on beamed emissions such that $\theta_{\text{dust}} < \theta_e$ to evaluate variable emission components produced in the dust torus. In this work, we only show the highly variable emission component with $\Delta \tau < 600$ s, and we do not include emissions with longer timescales, though they, as well as radiation from the inner jet, partially contribute to the overall received flux (Murase 2012).

Figure 10 shows results for the case where the neutron beam is produced by photomeson interactions between UHE protons and external photons in the BLR. The spectrum used here is the same as that shown in the inner jet calculation of Figure 3, which includes target Ly\alpha, scattered accretion disk, and cool and warm torus radiation as target radiation fields. Then, we calculate the photomeson production by UHECR neutrons in the external photon fields of the dust torus, whose radius is set to $R_{\text{dust}} = 1$ pc. The magnetic field is set to 10 $\mu$G. In the calculations, the dust temperatures are set as, before, to $T_{\text{dust1}} = 1200$ K and $T_{\text{dust2}} = 660$ K, with luminosities $L_{\text{dust1}} = 8 \times 10^{45}$ erg s$^{-1}$ and $L_{\text{dust2}} = 10^{45}$ erg s$^{-1}$, respectively.

We found that the required absolute jet power is $\approx 10^{46}$ erg s$^{-1}$, which is consistent with analytical expectations. From Figure 8, one expects that UHE photons might escape from their emission regions, where they cascade in larger-scale magnetized regions, including elliptical galaxies, galactic winds, galaxy clusters, and filaments (Murase 2012). Since strong radio emission is presumably produced by kpc jets in FSRQs, UHE $\gamma$-rays can also be quickly depleted by such an additional target radio field, whose scale and intensity are uncertain. For demonstration, we therefore consider an additional radio field that would be provided by the kpc jet. From Tavecchio et al. (2010), the radio field is optimistically assumed to have a typical size of $R_{\text{radio}} = 10^{21}$ cm and a characteristic $\nu L_{\nu}$ radio luminosity $= 10^{43}$ erg s$^{-1}$($\nu/10^9$ Hz) at $\nu < 10^{11}$ Hz, with a naive extrapolation to lower frequencies.

To calculate this contribution, we also evaluate cascades initiated by UHE photons leaving the dust torus, using $c\Delta \tau \approx (1 + \cos \theta_{\text{dust}}) \min [\lambda_{\text{IR}}, R_{\text{radio}}]$. Rapidly variable emission is still possible for the UHE photon-induced emission as long as $\lambda_{\gamma \gamma}$ is short enough (Murase 2012). The steady VHE cascade emission induced by the kpc scale jet is predicted, from Figure 10, to be at a lower flux level than the rapidly variable VHE emission formed at the pc scale. The search for the low-level, high-energy plateau emission is feasible with the Cherenkov Telescope Array (CTA; Actis et al. 2011) in development. Besides the UHE photons and leptons generated by neutron photomeson processes at the pc scale (thin solid curves), the subsequent cascade $\gamma$-ray spectrum formed in the dust torus is shown by the dashed curve. The UHE photons escaping into the kpc jet can experience further cascades, which makes an additional high-energy component with rapid variability.

The results are not very sensitive to $B$ as long as the magnetic field is in the range 10 $\mu$G $\lesssim B \lesssim$ mG, as shown in Figure 11. Just for comparison, in Figure 12 we also show the case where the neutron beam is produced by photomeson production between UHE protons and nonthermal photons that can be generated by electrons accelerated at internal shocks. Here, we inject beamed neutrons with a power-law spectrum, $N_{\text{n}}(E_n) \propto E_n^{-0.8}$. Such a hard spectrum can be realized if the target photon spectrum in inner jets has $\beta = 2.2$ and the proton spectrum has $s = 2.0$ (Appendix B). The target photon spectrum is taken not to contradict the observations. As in Figure 11,
of many such events, or searches for PeV neutrinos from γ-ray bright FSRQs like 3C 454.3, 3C 279, or PKS 1510–089 with high-energy neutrino telescopes, are crucial for testing the model.

7. DISCUSSION AND SUMMARY

The sources of the UHECRs are unknown, but acceleration in the inner jets of blazars has been widely considered as a possible solution to this problem (Mannheim & Biermann 1992; Berezinsky et al. 2006; Dermer et al. 2009; Murase & Takami 2009). In this paper, we argue that the detection of VHE radiation from the FSRQ 4C +21.35 supports this scenario and provides a solution to the rapid VHE variability and large γ-ray luminosity as an effect of ultra-high energy protons accelerated in the inner jet.10

A new feature of this study is the derivation of beaming factors for secondaries formed by photodisgregation processes. Synchrotron and SSC emissions and radiations from photodisgression processes with internal isotropic radiation fields have a beaming factor \( \propto \delta_D^2 \) due to compression in solid angle and time and enhancement in energy. External Compton scattering has a \( \delta_D^2 \) beaming factor because Compton scattering is proportional to target photon energy density, which is boosted by two powers of \( \delta_D \) when transformed to the comoving frame. Photodisgregation processes with external isotropic photon sources have a beaming factor \( \propto \delta_D^2 \), because photodisgregation production is proportional to the target photon number density, which is increased by a single power of the Doppler factor in the comoving frame. Threshold and spectral effects complicate the beaming factors, and the more detailed relations, including a formalism to calculate production spectra of photodisgregation secondaries, are presented in this paper.

10 FSRQ blazars are scarce within the GZK volume, and their average emissivity is much less than blazar BL Lac objects, which may therefore be the preferred blazar class to accelerate most of the highest energy cosmic rays (Dermer & Razzaque 2010; Murase et al. 2012).
In Appendix A, we also derived accurate photon powers for bolometric $\gamma$-ray fluxes associated with synchrotron and SSC processes and for external Compton processes. These relations demonstrate the much stronger decline of flux for $\gamma$-rays from external Compton scattering compared to SSC fluxes, which may help explain the relative number of misaligned radio galaxies of different types (Abdo et al. 2010c).

The main goal of this paper is to propose a new mechanism to generate rapidly variable $\gamma$-rays at large distances from a black-hole engine in order to explain the puzzling observations of 4C +21.35. The variability timescale is determined by processes in the inner jet that relate to the dynamical timescale of the black hole. For black-hole masses of $(1.5-8) \times 10^8 M_\odot$ (Wang et al. 2004; Shaw et al. 2012), the dynamical timescale is $\approx 1-10 \times$ larger than the variability timescale. Some of the same solutions applied to rapid variability in TeV blazars (e.g., Begelman et al. 2008; Narayan & Piran 2012) could operate in the inner jet of 4C +21.35 if the larger black-hole mass is correct.

We assume that the inner jets of blazars accelerate UHECRs that undergo photodissochromic losses with ambient radiation fields and generate UHE neutrinos, neutrinos, and $\gamma$-rays (Atoyian & Dermer 2003). Outflowing UHECR neutrinos undergo photodissochromic losses with IR torus photons to make pions that decay into hyper-relativistic leptons. If the magnetic field is $-0.01-1$ mG, as shown by the cross-hatched region in Figure 9, then the VHE synchrotron radiation preserves the rapid variability of the inner engine at the pc scale. This mechanism is proposed as the origin of the rapid variability of 70–400 GeV $\gamma$-rays observed with the MAGIC telescopes. Only a very narrow angular range of electrons will contribute to the observed VHE radiation, so that the variability of the central source is preserved even if the opening angle of the relativistic jet is not narrow. UHE photons escaping to larger distances, e.g., at the Mpc scale region in the large-scale structure surrounding the source, may furthermore produce a slowly variable VHE synchrotron pair echo with timescales of $\approx 1-1$ yr (Murase 2012), while UHE protons escaping to the magnetized regions can give almost non-variable signals (Gabici & Aharonian 2005; Kotera et al. 2011). These radiations are useful as characteristic signals of UHECR acceleration and are potentially observable with the planned CTA (Actis et al. 2011).

The $\gamma$-ray fluxes made principally by the synchrotron mechanism become less variable with decreasing energy, but at sufficiently low energies, $\gamma$-rays from the inner jet can avoid $\gamma\gamma$ absorption and additionally contribute to the observed flux. Indeed, Figure 8 shows that the $\gamma\gamma$ absorption for inner jet $\gamma$-rays becomes significant only above a few GeV. This corresponds precisely to the range of energies where the Fermi-LAT has discovered spectral breaks in FSRQs and low- and intermediate-synchrotron-peaked blazars (Abdo et al. 2009, 2010a, 2010b). Whether this is a coincidence or a cause of the GeV spectral breaks, it is pertinent that most models for the GeV breaks invoke production within the BLR (Poutanen & Stern 2010; Finke & Dermer 2010; Ackermann et al. 2010; Stern & Poutanen 2011). An inner jet origin of the GeV radiation is consistent with the model proposed here, and the synchrotron $\gamma$-rays at multi-GeV/TeV energies would appear as a separate spectral component emerging from the attenuated inner jet radiation spectrum, which furthermore display increased variability with energy.

An accurate fit to the combined Fermi and MAGIC $\gamma$-ray spectrum requires detailed modeling of the radiation in the inner jet in addition to the subsequent cascade radiations formed as the VHE synchrotron photons pass through the IR photon field. Besides the leptonic emissions in the inner jet, a model involving the UHECR protons that are accelerated and undergo photopion losses to make escaping UHECR neutrons must be included. As noted earlier, long-term average apparent $\gamma$-ray luminosities $\gtrsim 10^{48}$ erg s$^{-1}$ are common in FSRQs, so apparent jet powers must even be larger. The apparent luminosity in UHECRs accelerated in the inner jet of 4C +21.35 must be $\approx 10^{48}-10^{49}$ erg s$^{-1}$ to compensate for the inefficiencies to produce neutrons in the inner jet and pions at the pc scale. For the target photons from the external BLR and IR torus fields, the radiative efficiency for $\gamma\gamma$ photopion processes and the requirement of rapid variability from a single blob lead to the requirement of large Doppler factors, $\approx 100$ in order to make an apparent luminosity $\approx 10^{48}$ erg s$^{-1}$ in outflowing neutrons from a jet whose absolute power is Eddington-limited.

Internal shocks formed by colliding plasma shells ejected from the nuclear black hole are often considered as the mechanism that dissipates energy into broadband nonthermal radiation. The constraint $R \lesssim 10^8 \Gamma^2 D (1+z)$ on variable emission from colliding shells means that $R \lesssim 0.01 (\Gamma/50)^2 (D/600\text{ s})$ pc, for $\Gamma \approx 0.01$ (e.g., Tavecchio et al. 2010). At the location of the emission site, it is required to be at a distance larger than a few pc from the central black hole, then unless $\Gamma \gtrsim 10^3$, which exceeds by an order of magnitude the outflow Lorentz factor inferred from any blazar data, such a model for the MAGIC observations cannot explain the VHE radiation due to the severe attenuation.

The necessity to reconcile the contrary features of short-term variability, which points to an inner jet origin, and detection of VHE $\gamma$-rays that must originate at the pc scale has led to several proposed solutions. Tavecchio et al. (2011) consider a system with a compact zone outside the BLR and a second, possibly more extended zone either inside or outside the BLR to reproduce the complete SED. The compact emitting regions making the VHE radiation must have large Doppler factors, $\delta_D \approx 75$. Nalewajko et al. (2012) consider various energetic constraints on the 4C +21.35 system and also arrive at the requirement of compact emitting zones at pc scales. The question then is the origin of these compact regions. One possibility is the formation of compact recollimation shocks (Marscher 2006; Bromberg & Levinson 2009) that are produced when the external medium pressure overcomes the radiative jet, as proposed for M87 flares or the radio cores of blazars. The large powers from 4C +21.35 challenge such an origin of the strongly variable $\gamma$-ray fluxes. These behaviors might also be reconciled in highly magnetized Poynting jet models, mini-jet models, or turbulent cells (e.g., Giannios et al. 2009; Marscher & Jorstad 2010; Nalewajko et al. 2011) that have been considered in TeV BL Lac objects. Nalewajko et al. (2012) conclude that self-collimating jet structures might produce the conditions needed to explain the observations.

Here we propose a new technique to produce rapid variability of VHE $\gamma$-rays far from the central black hole as a result of UHECR neutron production in the inner jet, with the outflowing neutrons making cascade synchrotron radiation from leptons formed as secondaries from photopion interactions of the UHECR neutrons with the dust IR radiation field at the pc scale. The power in escaping neutrons is accompanied by $\gamma$-ray and neutrino radiations from the decay secondaries of neutral and charged pions. This approach to variability, which depends on rectilinear propagation of the particles to large distances, has similarities with an idea of Ghisellini et al. (2009).
where rapid variability of TeV blazars like PKS 2155–304 or Mkn 501 results from electrons forced to follow field lines. This magnetocentrifugal model would not likely work with 4C +21.35 at the pc scale, but it also involves a geometry essentially different from a relativistic plasma.

This model predicts neutrino fluxes detectable with IceCube from flares of FSRQs, not only PKS 1222+216, but also sources like 3C 279 and PKS 1510–089, that generate a VHE fluence reaching \( \approx 10^{-3} \) erg cm\(^{-2}\). For the one-half hour of activity of 4C +21.35, the total electromagnetic fluence is \( \approx 10^{-5} \) erg cm\(^{-2}\), but the flare itself could have lasted for a much longer time than the half hour during which MAGIC was observing. Moreover, the long-term jet radiation could accelerate UHECRs in the inner jet with associated neutrino production from FSRQs with VHE emission. Furthermore, UHECR protons and ions could escape upstream to produce emissions at the pc scale without significant \( \gamma \)-ray and neutrino production in the inner jet but would be strongly depleted by interactions in the dust torus.

In conclusion, the proposed model, if correct, would provide an important clue to the question of the origin of UHECRs. The feature of transport of inner jet energy to the pc scale via UHECRs would explain why the question of the location of the \( \gamma \)-ray emission site in blazars has been so puzzling. UHECR processes could be essential in fitting the SED of all blazars, not just the FSRQs from which VHE radiation has been detected.

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APPENDIX A
DERIVATION OF THE PHOTON POWER

The apparent isotropic bolometric synchrotron luminosity is related to \( L_{\text{syn}} \), the isotropic synchrotron luminosity in the comoving jet frame, according to the relation \( L_{\text{syn}}(\Omega) = \frac{\delta D^4}{\delta D^4} L_{\text{syn}}(\Omega) = \frac{\delta D^4}{\delta D^4} L_{\text{syn}}/4\pi \). The absolute synchrotron power radiated in all directions is, after multiplying by a factor of 2 for a two-sided jet, given by

\[
L_{\text{syn,abs}} = \int d\Omega L_{\text{syn}}(\Omega) = 2 \times 2\pi \left( \frac{L_{\text{syn}}}{4\pi} \right) \int_{-1}^{1} d\mu \delta D^4. \quad (A1)
\]

This can also be written as

\[
L_{\text{syn,abs}} = \frac{L_{\text{syn}}}{4\pi} \int_{-1}^{1} d\mu \frac{\delta D^4}{\delta D^4} L_{\text{syn}} = \frac{2\Gamma^2(3 + \beta^2)}{3\delta D^4} L_{\text{syn}} \rightarrow 8\Gamma^2 \frac{3\delta D^4}{\delta D^4} L_{\text{syn}}., \quad (A2)
\]

noting \( L_{\text{syn}} = 4\pi L_{\text{syn}}(\Omega) \). This beaming factor also applies to SSC emissions in the standard one-zone blazar model.

The apparent isotropic bolometric Compton luminosity resulting from an isotropic comoving electron distribution scattering an external isotropic radiation field transforms according to \( L_{\text{EC}} \propto \delta D^6 L_{\text{EC}} \). Following the same reasoning, therefore, the absolute Compton power radiated in all directions is given in terms of the apparent Compton luminosity by the relation

\[
L_{\text{EC,abs}} = \int_{-1}^{1} d\mu \frac{\delta D^6}{\delta D^6} L_{\text{EC}} = \frac{2\Gamma^4(3 + \beta^2 + \beta^4)}{5\delta D^6} L_{\text{EC}} \rightarrow 32\Gamma^4 \frac{5\delta D^6}{\delta D^6} L_{\text{EC}}. \quad (A3)
\]

These relations apply to bolometric luminosities. Measurements over a finite frequency range introduce integration limits that give corrections that depend on spectral parameters.

APPENDIX B
UHE NEUTRON BEAM PRODUCTION IN INNER JETS

UHECR acceleration may occur in inner jets via shock acceleration or possibly magnetic reconnection. For \( t_{\text{var}} = 600 \) s, the comoving size of the emission region is \( r_{\text{b}} = \delta D t_{\text{var}}/(1 + z) \simeq 3.6 \times 10^{14} (\delta D/20)(t_{\text{var}}/600 \text{ s})/(1 + z) \) cm. As long as we consider blob radii smaller than the radius of the BLR region, the photon field in inner jets is important. Though the nonthermal photon field in inner jets is uncertain, following Tavecchio et al. (2010), we take the maximum allowed photon field as

\[
L_{\text{syn}} = 10^{44.5} \text{ erg s}^{-1} \text{ at } v_b = 10^{14} \text{ Hz and a photon index } \beta = 2.2 \text{ at } v > v_b \text{ for demonstration. The effective photon energy density at } v_b \text{ is }
\]

\[
n_{\text{eff}} = \frac{\mu_{\text{syn}} b}{m_e c^2 e_b} \simeq 1.2 \times 10^{13} \text{ erg cm}^{-3} L_{\text{syn,45.5}}(\delta D/20)^{-5} v_{b,12}^{-1} \times (t_{\text{var}}/600 \text{ s})^{-2}(1 + z)^2. \quad (B1)
\]

Then, the photomeson production efficiency in inner jets is (e.g., Atoyan & Dermer 2003; Murase & Beacom 2010)

\[
\eta_{\text{p}\gamma} \simeq \frac{2\mu_{\text{syn}} b (e_b)}{(1 + \beta) m_e c^2} c^2 \left( \frac{E_{p}}{E_{p}} \right)^{1.2} \simeq 0.2(\delta D/20)^{-4} v_{b,14}^{-1}(t_{\text{var}}/600 \text{ s})^{-1} L_{45.5}(E/E_{p})^{1.2}, \quad (B2)
\]

where \( E_{p} \simeq 1.5 \times 10^{20} \text{ ev} (\delta D/20)^2 v_{b,14}^{-1} \). This implies that \( n_{\text{p}\gamma} \simeq 5 \times 10^{-5} \text{ at } E_{p} \simeq 10^{19} \text{ eV, which is not so efficient. Note that the corresponding photomeson production efficiency due to external photon fields (during the dissipation) would be low for smaller values of } \delta D. \text{ In fact, for interactions with the dust photon field, we have}
\]

\[
n_{\text{p}\gamma} \simeq 3 \times 10^{-3} (u_0/e_{\gamma,i})(R/10^{15.5} \text{ cm}). \quad (B3)
\]

where \( R \approx \Gamma_{r,b} \) is the distance to the blob.

If neutron production mainly occurs in such a compact region, we have the neutron conversion efficiency as

\[
\eta_{\text{p}\gamma} \rightarrow n \simeq 2.5\eta_{\text{p}\gamma} \simeq 0.48(\delta D/20)^{-4} v_{b,14}^{-1} \times (t_{\text{var}}/600 \text{ s})^{-1} L_{45.5}(E/E_{p})^{1.2}. \quad (B4)
\]

High-energy neutrons are depleted via conversion to protons or further photomeson production loss. The neutron absorption efficiency \( n_{\text{n}\gamma} \rightarrow p \) is expected to be comparable to \( n_{\text{p}\gamma} \rightarrow n \), so the critical energy (where \( n_{\text{n}\gamma} \rightarrow p = 1 \) is estimated to be

\[
E_a^* \simeq 2.3 \times 10^{20} \text{ ev} (\delta D/20)^{16/3} v_{b,14}^{-1/2} (t_{\text{var}}/600 \text{ s})^{5/6} (L_{45.5})^{-5/6}. \quad (B5)
\]
As a result, the resulting neutron spectrum can be written as

$$E_n^2 N_n(E_n) \approx \frac{\min[1, \eta_{\gamma \gamma - \gamma}]}{1 + \eta_{\gamma \gamma - p}} E_p^2 N_p(E_p) \propto E_p^{s - 2s \beta},$$  \hspace{1cm} (B6)

where we have assumed that the maximum proton energy is lower than $E_p^c$ for the last expression. The Hillas condition gives $E_{n}^\text{max} \approx 0.8 E_{p}^\text{max} \sim 10^{18.5}$ eV, below which we expect $E_n^2 N_n \propto E_\gamma^{1.2}$ for $s = 2$. However, in our cases, the neutron beam production in inner jets typically requires quite large cosmic-ray luminosities. On the other hand, one may also expect proton escape especially at the maximum energies. But details depend on the uncertain escape mechanism.

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