Probing Non-standard Top Couplings Using spin-correlation

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Abstract

Top spin correlation has been shown to be nontrivial in hadronic top-pair production, and can be realized by the asymmetries of the decay products of the top quark and antiquark. We show in this work that the top spin correlation is a sensitive probe to the anomalous top-quark couplings beyond the standard model. Specifically, we look at the anomalous chromomagnetic and chromoelectric dipole moments, as well as a right-handed component in the weak decay of the top quark. We found a few measurable asymmetries formed by the decay products of the top-quark pair that vary in accord with the spin correlation.

I. Introduction

The top quark is very different from the other five quarks because it has a mass close to the electroweak symmetry breaking scale. The top quark can therefore provide useful avenues to probe the physics beyond the standard model (SM) through its decay, direct production, scattering, and the associated production with other particles. Since the structure or even the symmetry of the correct high energy theory is not known, the effective Lagrangian approach can be used to study low energy phenomena. Deviations from the SM can be studied systematically by including higher dimensional operators, which are made up of the SM fields, into the interaction Lagrangian. Such higher dimensional operators are suppressed by powers of the scale $\Lambda$ of the new physics.

In this paper, we study some anomalous couplings of the top quark, which often appear in extensions of the SM. We shall limit ourselves to as low dimension as possible because

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the effects of the high dimensional operators are very much suppressed at low energies. Here, we concern only the dim-4 and dim-5 operators. For dimension 4 we include a right-handed component into the weak decay of the top quark. Although the \( V - A \) structure in light quarks are more or less established, the \( V - A \) structure of the top quark should be confirmed. The presence of a right-handed component, even small, will signify some new physics beyond the SM. For dimension 5 we concentrate on the anomalous chromomagnetic and chromoelectric dipole moments of the top quark. They are particularly interesting because they are directly involved in the subprocesses \( gg, q\bar{q} \rightarrow t\bar{t} \) of top-pair production, and they are often nonzero on loop-level or even tree level in many extensions of the SM, e.g., the multi-Higgs doublet model \([2]\). Furthermore, a nonzero value for the chromoelectric dipole moment is a clean signal of CP violation. The effects of these anomalous couplings have been studied in \( t\bar{t} \) production \([3, 4, 5]\), \( b\bar{b} \) production \([4]\), inclusive jet production \([6]\), and prompt photon production \([7]\).

In this work, we study the effects of the above mentioned anomalous couplings on the spin correlation in top-pair production. Recently, a few studies \([8, 9, 10]\) showed that top-pair production is highly spin-correlated, which means that the top quark and antiquark have preferential spin polarizations. A theoretically interesting variable to quantify the spin correlation is \([8]\)

\[
C = \frac{\sigma(t_L \bar{t}_L + t_R \bar{t}_R) - \sigma(t_L \bar{t}_R + t_R \bar{t}_L)}{\sigma(t_L \bar{t}_L + t_R \bar{t}_R) + \sigma(t_L \bar{t}_R + t_R \bar{t}_L)},
\]

(1)

where the subscripts \( L \) and \( R \) denote the helicities of the top quark and antiquark. At the Tevatron, the dominant subprocess of top-pair production is \( q\bar{q} \rightarrow t\bar{t} \) and \( C \approx -0.4 \), while at the LHC energy \( gg \rightarrow t\bar{t} \) will dominate and \( C \approx +0.3 \) \([8]\). The measurement of \( C \) can then serve as an indirect probe to the underlying production mechanism. Although this variable is a true measure of the spin correlation, it is not directly measured experimentally. Fortunately, the top quark is so heavy that it decays before it hadronizes, and, therefore, almost all the spin information is retained in the decay products of the top quark. The heavy top quark will decay preferentially into a longitudinally polarized \( W^+ \) boson and a left-handed \( b \) quark. Because of the conservation of angular momentum the \( W^+ \) boson prefers to go in the same direction as the top polarization in the rest frame of the top quark. The antilepton \( \ell^+ \) decaying from the \( W^+ \) boson also goes in the direction of the \( W^+ \) boson,
i.e., in the same direction as the spin polarization of the top quark. On the other hand, the lepton decaying from the top antiquark prefers to go in the opposite direction as the spin polarization. Thus, by discriminating the directions of the lepton and antilepton we can select particular polarizations of the top quark and antiquark. Since the top-quark pair is spin-correlated, the asymmetries formed by the lepton and antilepton could be nontrivial. We shall introduce a few asymmetries in the next section.

The organization is as follows. In Sec. II we introduce the asymmetries and calculate them in the SM. In Sec. III we study the effects of the anomalous chromomagnetic and chromoelectric dipole moments of the top quark on the variable $C$ and the asymmetries, and in Sec. IV we study the effects of the right-handed component in the weak decay of the top quark. We then discuss and conclude in Sec. V. In this work, we concentrate on the Run II of the Tevatron ($\sqrt{s} = 2$ TeV) with a yearly luminosity of order $2 \text{fb}^{-1}$ \[1\]. The top quark mass is chosen to be 175 GeV \[1\]. We use the parton distributions of CTEQ3L \[12\].

II. Spin Correlation and Asymmetries within the SM

The dominant subprocesses for the hadronic production of top quark are

\[
q\bar{q} \rightarrow t\bar{t} \\
gg \rightarrow t\bar{t}.
\]

The $q\bar{q}$-initiated subprocess dominates at the Tevatron energies ($\sqrt{s} = 1.8$ TeV for Run I and 2 TeV for Run II), while $gg$-initiated one dominates at the LHC, because of the increasing gluon luminosity with increases in energy. At the Tevatron energies, since most of the times the top pair is produced near threshold and in the dominant subprocess $q\bar{q} \rightarrow t\bar{t}$ the top pair is produced via a $J = 1$ s-channel gluon, most of the $t\bar{t}$ pairs are in a $^3S_1$ state. By counting the spin eigenstates the ratio of the top-quark pair in the same helicities to the top-quark pair in opposite helicities is 1:2, so the value of $C$ is $-\frac{1}{3}$. Far above the threshold, helicity conservation requires $t$ and $\bar{t}$ in opposite helicities, so $C = -1$. We found that at the Tevatron ($\sqrt{s} = 1.8 - 2.0$ TeV) $C \approx -0.4$, which was first given in Ref. \[8\]. On the other hand, $gg \rightarrow t\bar{t}$ dominates at the LHC. Near threshold the $t\bar{t}$ pair is produced in a $^1S_0$ state,
so the $t$ and $\bar{t}$ have the same helicities, with $C = +1$. Hence, there is a dramatic change of $C$ from negative to positive when energy increases from 1.8 TeV to 14 TeV.

As mentioned above that although $C$ is not directly measured, it can be revealed in the decay products of the top-quark pair. The best analyzer of the top spin polarization is the semileptonic decay, because in other decay modes it is difficult to distinguish the up-type or down-type quark in the decay of the $W$ boson, and leptons are much easier to be detected. We will study three different asymmetries, two of which have been shown to be very effective [8, 10]. All three of them involve semileptonic decays of the top quark and antiquark. The first one, denoted by $A$, is \[ A = \frac{\sigma(z_1 z_2 > 0) - \sigma(z_1 z_2 < 0)}{\sigma(z_1 z_2 > 0) + \sigma(z_1 z_2 < 0)}, \] where $z_i = \cos \theta_i$ ($i = 1, 2$) is the cosine of the angle of the lepton in the rest frame of the parent top quark with respect to the momentum of the top in the $t\bar{t}$ center-of-mass frame. The second one, denoted by $B$, is \[ B = \frac{\sigma(p_1 p_2 > 0) - \sigma(p_1 p_2 < 0)}{\sigma(p_1 p_2 > 0) + \sigma(p_1 p_2 < 0)}, \] where $p_1$ and $p_2$ are the $z$-component of the 3-momenta of the lepton and antilepton in the $t\bar{t}$ center-of-mass frame. The positive $z$ direction is defined as the direction of incoming proton beam. In other words, this asymmetry $B$ counts the number of events that the lepton and antilepton go to the same or opposite side of the plane, which is normal to the beam direction, in the $t\bar{t}$ center-of-mass frame. The third one, which we denote by $B'$, is given by \[ B' = \frac{\sigma(p'_1 p'_2 > 0) - \sigma(p'_1 p'_2 < 0)}{\sigma(p'_1 p'_2 > 0) + \sigma(p'_1 p'_2 < 0)}, \] where $p'_1$ ($p'_2$) is the $z$-component of the 3-momentum of the antilepton (lepton) in the rest frame of the top quark (top antiquark). The only difference between $B$ and $B'$ is that the 3-momenta of the lepton and antilepton are in different frames.

To calculate these asymmetries we have to put in the decay matrix elements of the top quark and antiquark with full spin correlation. We use the helicity amplitude method [13].

\[ B' \] is exactly the same as $A_4$ described in Ref. [10], in which $A_4$ is the best out of the four asymmetries considered.
to calculate the spin-polarized cross sections, and the asymmetries \( C, A, B, \) and \( B' \). With this helicity amplitude method the subsequent decays of the top quark and the \( W \) boson can be included straightforwardly. The necessary formulas for top-pair production and the decays are given in Ref. [3]. We found that the asymmetry \( A \) is about +0.1 in accord with the results of Ref. [8], the asymmetry \( B \) is about −0.3, and \( B' \) is about −0.2, which also agrees with the results of Ref. [10]. Although the asymmetries \( B \) and \( B' \) are larger than \( A \) numerically, \( A \) is more sensitive to the anomalous chromomagnetic dipole moment that we are going to discuss in the next section. With the asymmetries \( C, A, B, \) and \( B' \) we can now study the new physics associated with top-gluon vertex and the weak decay vertex of the top quark. They are, respectively, studied in the next two sections.

### III. Chromomagnetic and Chromoelectric Dipole Moments

The effective Lagrangian for the interactions between a top quark and a gluon that include the chromomagnetic (CMDM) and chromoelectric (CEDM) form factors is

\[
\mathcal{L} = g_s \bar{T}^a \left[ -\gamma^\mu G^a_\mu + \frac{\kappa}{4m_t} \sigma^{\mu\nu} G^a_{\mu\nu} - \frac{i\kappa}{4m_t} \sigma^{\mu\nu} \gamma^5 G^a_{\mu\nu} \right] t_i ,
\]

where \( \kappa/2m_t \) (\( \bar{\kappa}/2m_t \)) is the CMDM (CEDM) of the top quark. The Feynman rules for the interactions of the top and gluon can be written down:

\[
\mathcal{L}_{t_i t_j g} = -g_s \bar{T}^a \left[ \gamma^\mu + \frac{i}{2m_t} \sigma^{\mu\nu} p_\nu (\kappa - i\bar{\kappa} \gamma^5) \right] t_i G^a_\mu t_j ,
\]

where \( t_i (t_j) \) is the incoming (outgoing) top and \( p_\nu \) is the 4-momentum of the outgoing gluon. The Lagrangian in Eq. (5) also induces a \( ttgg \) interaction given by

\[
\mathcal{L}_{t_i t_j g g} = \frac{ig_s^2}{4m_t} \bar{T}^a \left[ T^b T^c - T^b T^c \right]_{ji} \sigma^{\mu\nu} (\kappa - i\bar{\kappa} \gamma^5) t_i G^b_\mu G^c_\nu ,
\]

which is absent in the SM.

The unpolarized \( t\bar{t} \) cross sections including the CMDM and CEDM couplings were calculated in Ref. [3] [4] [5]. Since we are interested in polarized cross sections here, we use the helicity amplitude method [13] to calculate the polarized cross sections including the CMDM
and CEDM couplings. Once we obtained the polarized cross sections for $t_L\bar{t}_L$, $t_R\bar{t}_R$, $t_L\bar{t}_R$, and $t_R\bar{t}_L$, we can then calculate the ideal spin-correlation variable $C$ of Eq. (1). We show the results of $C$ versus $\kappa$ and $\tilde{\kappa}$ by the solid curves in Fig. 1 and Fig. 2, respectively. As expected the behavior of the curve for $\tilde{\kappa}$ is symmetric about the $y$-axis because the cross sections contain only even powers of $\tilde{\kappa}$, while the curve for $\kappa$ is not symmetric about $y$-axis because the cross sections do depend on odd powers of $\kappa$. Since $C$ is so sensitive to the anomalous dipole moments, if one were able to measure $C$ directly and accurately enough, we could measure the CMDM and CEDM of the top quark easily. However, this is not so. But it is not impossible to have indirect measurements of $C$ by means of the asymmetries $A$, $B$, and $B'$ that we introduced before. Similar to how we obtained the SM results, we decay the top quark and antiquark semileptonically with full spin correlation. The variations of $A$, $B$, and $B'$ with $\kappa$ and $\tilde{\kappa}$ are also superimposed with $C$ onto Fig. 1 and Fig. 2. From these two figures we can see that the rise and fall of the curves $A$, $B$, and $B'$ are in accord with those of $C$. It is then clear that the asymmetries $A$, $B$, and $B'$ are, to a great extent, true representations of the spin-correlation $C$. Especially, $A$ is directly related to $C$ by $A = -C/4$ [8]. We also see that the asymmetry $A$ is more sensitive than the asymmetries $B$ and $B'$ to changes in $\kappa$, while all three are more or less equally sensitive to changes in $\tilde{\kappa}$. Nevertheless, the asymmetry $B$ has a larger numerical value than $A$ and $B'$ at the SM point ($\kappa = \tilde{\kappa} = 0$), which implies that it statistically needs fewer dilepton events to see the spin correlation.

IV. Right-handed Weak Decay

In dimension 4 the general charged-current couplings of the top quark is given by [14]

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \bar{b}\gamma^\mu \left[ \frac{1}{2}(1 - \gamma^5)(1 + \kappa_L) + \frac{1}{2}(1 + \gamma^5)\kappa_R \right] tW^-_{\mu} + \text{h.c.}$$

where the parameters $\kappa_L$ and $\kappa_R$ are used to denote the strength of the additional left-handed and right-handed couplings, respectively. In the SM, $\kappa_L = \kappa_R = 0$. Since the right-handed weak decay interaction only affects the decays of the top quark and antiquark, it has no effects on the variable $C$, as $C$ only depends on the top-gluon couplings. But it will affect the asymmetries $A$, $B$, and $B'$. The non-standard top decay can be implemented using the
helicity amplitude method \[13\], replacing the spinors of the top quark and top antiquark by

\[ \bar{u}(t) \rightarrow -\frac{g^2}{8} \frac{1}{W^2 - m_{W}^2 + i\Gamma_W m_W} \frac{1}{t^2 - m_t^2 + i\Gamma_t m_t} \bar{u}(\nu) \gamma_{\mu} (1 - \gamma_5) v(\ell^+) \]

\[ \times \bar{u}(b) \gamma_{\mu} \left( 1 + \kappa_L + \kappa_R + \gamma_5 (\kappa_R - 1 - \kappa_L) \right) (\ell + m_t) \]  

where the 4-momenta of the particles are labeled by the particle symbols. We have used the narrow width approximation to handle the top and \( W \) boson propagators, and we used the SM values for the width of the top quark and the \( W \) boson. Since we are dealing with the asymmetries only (not the total cross sections), we do not need to include the non-standard interactions in calculating the top width \( \Gamma_t \). We have verified that changing \( \kappa_L \) alone while keeping \( \kappa_R = 0 \) would not change the asymmetries because the weak decay is still \( V - A \). Therefore, we only show the results versus \( \kappa_R \) while keeping \( \kappa_L = 0 \). In Fig. 3, we show the asymmetries \( A, B, \) and \( B' \) versus \( \kappa_R \) from \(-10\) to \(10\). We can see that all three asymmetries decrease for non-zero \( \kappa_R \), and are symmetric about \( \kappa_R = 0 \). Even at \( \kappa_R = 1 \) (vector coupling only) and at \( \kappa_R = -1 \) (axial-vector coupling only) the asymmetries are non-zero, because non-zero spin correlation is still being fed down from top-pair production (\( C \approx -0.4 \) in the SM). As \( \kappa_R \) increases further away from zero, the asymmetries \( A \rightarrow +0.01, B \rightarrow -0.18, \) and \( B' \rightarrow -0.02 \). It would also be interesting to check the case of an entirely right-handed coupling by putting \( \kappa_L = -1 \) and \( \kappa_R = 1 \), at which \( A, B, \) and \( B' \) are verified to be the same as the asymptotic values of the curves in Fig. 3. The reason why \( A \) and \( B' \) are not quite zero yet is that the \( t\bar{t} \) pairs being produced are still spin-correlated, i.e., \( C \approx -0.4 \neq 0 \). Nevertheless, the right-handed weak decay of the top quark and antiquark causes the asymmetries very difficult to be detected. We have also verified that at the point where \( C = 0 \), i.e., the spins of the top pair are uncorrelated, the values of \( A \) and \( B' \) are in fact zero for \( \kappa_L = -1 = -\kappa_R \).
V. Discussions

(a) So far, the calculation of the asymmetries are without experimental acceptance cuts, efficiencies of reconstructing the 4-momenta of the top quark and antiquark, or the smearing of momenta due to the detector. Typical acceptance cuts on the observed leptons, $b$ quarks, and missing transverse energy are

$$p_T(b, \ell) > 15 \text{ GeV}, \quad |y(b, \ell)| < 2, \quad \not{p}_T > 25 \text{ GeV},$$

(11)

which are also needed for eliminating backgrounds and for reconstructing the top quark and antiquark. We found that the cuts reduce the asymmetries by about 15–20% near the SM point ($\kappa = \tilde{\kappa} = \kappa_R = 0$). The effects of smearing and reconstruction of the top quark and antiquark rest frame have been studied in details in Ref. [10], so we do not repeat here. In general, the asymmetries near the SM point are reduced by another 10–15%. However, since it is only based on a parton-level Monte Carlo, a full Monte Carlo is needed to study the true effects.

(b) The SM values for the asymmetries $A$, $B$, and $B'$ are $+0.1$, $-0.3$, and $-0.2$, respectively. It would be important to check if these asymmetries can be observed above experimental uncertainties. Since we do not have a full Monte Carlo simulation, we only take into account the statistical error. The SM cross section for $t\bar{t}$ production with the dilepton decay mode at $\sqrt{s} = 2 \text{ TeV}$ under the cuts in Eq. (11) is about 0.14 pb, which gives about 280 dilepton events for a 2 fb$^{-1}$ luminosity in the Run II of the Tevatron [11]. The statistical error is then $\sqrt{280}/280 \approx 0.06 = 6\%$. Therefore, the values for $A = +0.1$, $B = -0.3$, and $B' = -0.2$ should be clean to be observed, even after taking into account other systematical uncertainties, especially, $B'$ shows a possible $3\sigma$ effect from an uncorrelated $t\bar{t}$ sample. Thus, the spin correlation in $t\bar{t}$ production can be tested cleanly in the Run II of the Tevatron.

(c) In the following, we are going to estimate the sensitivities to or the bounds on $\kappa, \tilde{\kappa}$, and $\kappa_R$ using the various spin-correlation asymmetries if assuming the SM is correct. To estimate the $1\sigma$ sensitivity to $\kappa, \tilde{\kappa}$, and $\kappa_R$, we assume the SM values for $A$, $B$, and $B'$ are correct and put a $\pm 0.06$ onto them to get the corresponding ranges for $\kappa, \tilde{\kappa}$, and $\kappa_R$. The bounds
are then given by

\[-0.7 < \kappa < +0.6\]
\[-0.5 < \tilde{\kappa} < +0.5\]
\[-0.5 < \kappa_R < +0.5\]

(12)

where we have combined, for each of the \(\kappa\)'s, the three ranges given by \(A\), \(B\), and \(B'\).

Although these estimates are rather crude, they do give a feeling of how well the limits can be obtained using the spin-correlation. In reality, the limits should be weaker than the above because there are also systematical errors which have to be taken into account. Nevertheless, the limits obtained above are comparable to those using total cross sections. Moreover, using spin-correlation is better than using the total cross section in controlling the uncertainties coming from higher order corrections, parton distribution functions, and the strong coupling constant.

Higher luminosities, e.g., an integrated 10 fb\(^{-1}\) in the stretched run of the Run II or even 100 fb\(^{-1}\) in TeV33 plan \([11]\) will certainly reduce the statistical error by the square root of the increase in luminosity. Using a 10 (100) fb\(^{-1}\) luminosity compared to a 2 fb\(^{-1}\) luminosity reduces the statistical error by a factor of \(\sqrt{5}\) (\(\sqrt{50}\)). Thus, the limits can also be improved substantially. There is also a possibility of using other decay modes of the top quark and antiquark, which can result in a larger number of events, thus reducing the statistical error. However, it is very difficult experimentally to distinguish the quark and antiquark in the \(W\) boson decay, which then reduces the asymmetries significantly \([8]\).

(d) Other facilities to study the top quark are the \(e^+e^-\) colliders at 0.5 TeV and the LHC. In \(e^+e^-\) collisions, since the production of \(t\bar{t}\) pair is via \(s\)-channel exchanges of \(\gamma\) and \(Z\), the production rate of \(t\bar{t}\) actually decreases with increase in energy when the energy is well above the threshold. Therefore, it would not be advantageous to study \(t\bar{t}\) production in very high energy \(e^+e^-\) colliders. The \(t\bar{t}\) pair produced in \(e^+e^-\) collisions will also be spin-correlated, and the studies in this paper can be applied. We can further study the variation versus the center-of-mass energy of the collisions.

The LHC will be a copious source of \(t\bar{t}\) pair, of order \(10^6 - 10^7\). There should be large number of dilepton events to measure the spin-correlation asymmetries down to one percent
accuracy. Furthermore, the dominant production process changes to $gg \to t\bar{t}$, thus also changing the spin-correlation substantially, as discussed in the Introduction.

In conclusions, we have studied the spin correlation of $t\bar{t}$ production and the asymmetries formed by the decay products of the top quark and antiquark at the Fermilab Tevatron. We also studied the effects of anomalous chromomagnetic and chromoelectric dipole moments of the top quark, and a right-handed component in the weak decay of the top quark on the spin correlation. We also estimated the limits on $\kappa$, $\tilde{\kappa}$, and $\kappa_R$ that can be statistically obtained in the Run II of the Tevatron if assuming the SM is correct.

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Figure 1: Variations of the spin correlation $C$, and the asymmetries $A$, $B$, and $B'$ versus $\kappa$. 

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Figure 2: Variations of the spin correlation $C$, and the asymmetries $A$, $B$, and $B'$ versus $\bar{\kappa}$.

Figure 3: Variations of the asymmetries $A$, $B$, and $B'$ versus $\kappa_R$. 