Relating the anisotropic power spectrum to the CMB hemispherical anisotropy

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Abstract

We relate the observed hemispherical anisotropy in the cosmic microwave radiation data to an anisotropic power spectrum model. The hemispherical anisotropy can be parameterized in terms of the dipole modulation model. This model also leads to correlations between spherical harmonic coefficients corresponding to multipoles $l$ and $l + 1$. We extract the $l$ dependence of the dipole modulation amplitude, $A$, by making a fit to the $l$ dependence of the correlations between harmonic coefficients using PLANCK CMBR data. We propose an anisotropic power spectrum model which also leads to correlations between different multipoles. This power spectrum is determined by making a fit to data. We find that the spectral index of the anisotropic power spectrum is consistent with zero.

1 Introduction

The cosmic microwave background radiation (CMBR) shows a hemispherical power asymmetry [1–9]. The signal is seen both in WMAP and PLANCK data and indicates a potential violation of the cosmological principle. The hemispherical anisotropy can be parametrized in the form of dipole modulation [10–13] of the CMBR temperature field, such that

$$\Delta T(\hat{n}) = g(\hat{n}) \left(1 + A \hat{\lambda} \cdot \hat{n}\right)$$

(1)

where $g(\hat{n})$ an intrinsically isotropic and Gaussian random field, $\hat{\lambda}$ is the preferred direction and $A$ the amplitude of the dipole modulation. Taking the preferred direction along the z-direction, we have $\hat{\lambda} \cdot \hat{n} = \cos \theta$. Using the WMAP data, the dipole amplitude was found to be $A = 0.072 \pm 0.022$ and the dipole direction in the galactic coordinate is given by, $(\theta, \phi) = (224^\circ, 112^\circ) \pm 24^\circ$ for $l \leq 64$ [1–4, 6, 7]. PLANCK results [8] confirmed this anisotropy with significance $3\sigma$. For SMICA maps the dipole amplitude, $A = 0.073 \pm 0.010$ and the direction, $(\theta, \phi) = (217^\circ, 110^\circ) \pm 15^\circ$. These values are nearly same for NILC, SEVEM and COMMANDER RULER maps. Hence the result obtained by WMAP and PLANCK observations are consistent with one another. Later it was shown [14] that the power asymmetry persists for the small angular scales in the multipole range $l = 601 - 2048$ with significance of $6.5\sigma$. There also exist
other observations which indicate a potential violation of the cosmological principle \[15\]-\[21\]. There are many theoretical models, such as, \[22\]-\[34\], which aim to explain the observed large scale anisotropy. It has been suggested that this anisotropy may not really be in disagreement with the inflationary Big Bang cosmology, which may have a phase of anisotropic expansion at very early time. The anisotropic modes, generated during this early phase may later re-enter the horizon \[35\],\[36\] and lead to the observed signals of anisotropy.

In a recent paper, we showed that the dipole modulation model, Eq. \[1\], leads to several implications for CMBR. The CMBR temperature field may be decomposed as,

\[
\Delta T(\hat{n}) = \sum_{l,m} a_{lm} Y_{lm}(\hat{n})
\]

If we assume statistical isotropy, the spherical harmonic coefficients must satisfy,

\[
\langle a_{lm} a_{l'm'}^* \rangle_{iso} = C_l \delta_{l,l'} \delta_{m,m'}.
\]

where, \(C_l\) is the power spectrum. However in the presence of dipole modulation, Eq. \[1\], statistical isotropy is violated and one finds \[37\],

\[
\langle a_{lm} a_{l'm'}^* \rangle = \langle a_{lm} a_{l'm'}^* \rangle_{iso} + \langle a_{lm} a_{l'm'}^* \rangle_{dm}
\]

where, \(\langle a_{lm} a_{l'm'}^* \rangle_{iso}\) is the correlation given in Eq. \[3\] and the anisotropic term can be expressed as,

\[
\langle a_{lm} a_{l'm'}^* \rangle_{dm} = A (C_{l'} + C_{l}) \xi_{lm,l'm'}^0
\]

where,

\[
\xi_{lm,l'm'}^0 = \int d\Omega Y_{lm}^*(\hat{n}) Y_{l'm'}^* (\hat{n}) \cos \theta
\]

\[
= \delta_{m',m} \left[ \frac{(l - m + 1)(l + m + 1)}{(2l + 1)(2l + 3)} \delta_{l',l+1} + \frac{(l - m)(l + m)}{(2l + 1)(2l - 1)} \delta_{l',l-1} \right].
\]

Hence the modes corresponding to multipoles, \(l\) and \(l+1\) are correlated. We define a correlation function \[37\],

\[
C_{l,l+1} = \frac{l(l+1)}{2l+1} \sum_{m=-l}^{l} a_{lm} a_{l+1,m}^*
\]

This leads to the statistic, \(S_H(L)\), by summing over a range of multipoles,

\[
S_H(L) = \sum_{l=l_{min}}^{L} C_{l,l+1}.
\]

The value of this statistic in data is obtained after maximizing over the preferred direction parameters, which correspond to the choice of \(z\)-axis. The resulting
statistic is labelled as $S_{H\text{data}}$. This may be compared with the estimate of the statistic based on the dipole modulation model, given by,

$$S_{H\text{dm}}(L) = \sum_{l=l_{\text{min}}}^{L} \frac{l(l+1)}{2l+1} \sum_{m=-l}^{l} \langle a_{lm}^* a_{l+1,m} \rangle \cdot dm .$$

(9)

where the correlation on the right hand side is given by Eq. 5. By comparing $S_{H\text{dm}}(L)$ with $S_{H\text{data}}(L)$ we can obtain an estimate of the dipole modulation parameter $A$.

In this paper, we relate the directional dependent primordial power spectrum with dipole modulated temperature field and hence the hemispherical anisotropy. The relationship between power spectrum and the observed anisotropy has also been considered earlier [35, 40]. However our analysis and results differ from these earlier papers. We determine the power spectrum which leads to the observed temperature anisotropy. The temperature anisotropy is parameterized by the two point correlations, Eq. 5, or equivalently the statistic, $S_{H}(L)$. We first determine $S_{H}(L)$ by directly computing $C_{l,l+1}$, Eq. 7, from data. The resulting statistic is called $S_{H\text{data}}$. We next use the prediction, Eq. 5, based on the dipole modulation model, Eq. 1, to compute $S_{H}(L)$. In this case $C_{l}$ and $C_{l}'$ in Eq. 5 are computed from data. We use the symbol, $S_{H\text{dm}}$, for this statistic. By relating $S_{H\text{data}}$ with $S_{H\text{dm}}$ we determine the variation of the parameter $A$ with $l$. We next determine the two point correlations, $\langle a_{lm}^* a_{l'm'} \rangle$, induced by an anisotropic power spectrum model. Hence we determine the power spectrum required to fit the observed values of $S_{H}(L)$. The resulting statistic is denoted as $S_{H\text{theory}}$. We first perform this analysis for the entire multipole range $l = 2 - 64$. We also repeat the calculation over three multipole ranges, $l = 2 - 22, 23 - 43, 44 - 64$. This allows us to determine the variation of $S_{H}(L)$ and hence the dipole modulation amplitude $A$ with the multipole bin. We also determine the primordial power spectrum which fits this $l$ dependence of the dipole modulation or equivalently the hemispherical anisotropy.

A preliminary analysis of the $l$ dependence of the dipole modulation amplitude has been performed in [37]. However in this paper we used a different procedure, which involved computing the dipole power of the temperature squared field. Furthermore in [37], we used the mask COM-MASK-gal-07 for the PLANCK data. In the present paper we use the more reliable, CMB-union mask. It turns out that the statistic $S_{H}(L)$ is much more reliable in comparison to temperature square power for probing dipole modulation anisotropy.

2 Theory

The temperature fluctuations, $\Delta T(\hat{n})$ arise due to primordial density perturbations $\delta(\vec{k})$. They can be expressed as,

$$\frac{\Delta T}{T_0}(\hat{n}) = \int d^3k \sum_{l} (i)^{l}(2l+1)\delta(k)\Theta_l(k)P_l(\hat{k} \cdot \hat{n})$$

(10)

where $P_l(\hat{k} \cdot \hat{n})$ are the Legendre polynomials and $\Theta_l(k)$ the transfer function. On large distance scales, small $l$, the main contribution to the temperature fluctuation is from the Sachs-Wolfe effect. Hence we approximate the transfer
function as $\Theta_i(k) = \frac{\mathcal{J}_i(k\eta)}{k\eta}$ \textbf{[11]}, where $\mathcal{J}_i$ is the spherical Bessel function. A more detailed analysis is postponed to future work. Let $\delta(\vec{x})$ represent the density fluctuations in real space. The two point correlation function, $F(\vec{\Delta}, \vec{X})$, in real space can be expressed as,

$$F(\vec{\Delta}, \vec{X}) = \langle \delta(\vec{\Delta}) \delta(\vec{X}) \rangle$$

where we have defined the variables, $\Delta = |\vec{x}|$ and $\vec{X} = (\vec{x} + \vec{x'})/2$. The corresponding correlation function of the Fourier transform, $\delta(\hat{k})$, may be expressed as,

$$\langle \delta(\hat{k}) \delta^*(\hat{k}') \rangle = \int d^3X d^3\Delta e^{i(\hat{k}+\hat{k}')} \cdot 2F(\vec{\Delta}, \vec{X})$$

If we assume that $F(\vec{\Delta}, \vec{X})$ depends only on the magnitude $\Delta \equiv |\vec{\Delta}|$, we obtain the standard form of the power spectrum, $\langle \delta(\hat{k}) \delta^*(\hat{k}') \rangle = P(k)\delta^3(\vec{k} - \vec{k}')$ where, $k \equiv |\vec{k}|$.

We next assume that the Universe is anisotropic but homogeneous. In this case $F(\vec{\Delta}, \vec{X})$ is independent of $\vec{X}$. Hence we obtain,

$$\langle \delta(\hat{k}) \delta^*(\hat{k}') \rangle = \delta^3(\vec{k} - \vec{k}') \int d^3\Delta e^{i(\hat{k}+\hat{k}')} \cdot 2F(\vec{\Delta})$$

Let us now assume a simple form of the anisotropic power spectrum in real space. We set,

$$F(\vec{\Delta}) = f_1(\Delta) + \hat{\lambda} \cdot \vec{\Delta} f_2(\Delta)$$

where $\hat{\lambda}$ is a unit vector pointing in the preferred direction and $f_1$ and $f_2$ depend only on the magnitude $\Delta$. After performing the integral in Eq. \textbf{[12]} we obtain,

$$\langle \delta(\hat{k}) \delta^*(\hat{k}') \rangle = \delta^3(\vec{k} - \vec{k}') P_{iso}(k)[1 + i(\hat{k} \cdot \hat{\lambda}) g(k)]$$

where the functions, $P_{iso}(k)$ and $g(k)$, are real and depend only on the magnitude $k = |\vec{k}|$. These can be related to the functions $f_1(\Delta)$ and $f_2(\Delta)$. The detailed form of this dependence can be obtained trivially but is not relevant for our purpose. The second term in Eq. \textbf{[15]} represent the violation of statistical isotropy.

We next calculate the two point temperature correlations,

$$\langle \Delta T(\hat{n}) \Delta T(\hat{n'}) \rangle = T_0^2 \int d^3k \sum_{l,l'=0}^{\infty} (i)^{l-l'} (2l+1)(2l'+1) \Theta_i(k) \Theta_{l'}(k) \times P_l(\hat{k} \cdot \hat{n}) P_{l'}(\hat{k} \cdot \hat{n'}) P_{iso}(k)[1 + ig(\hat{k} \cdot \hat{\lambda})]$$

We take the preferred direction along $z$-axis and set, $\hat{k} \cdot \hat{\lambda} = \cos \theta$. The two point correlation function of the spherical harmonic coefficient $a_{lm}$ can be computed as,

$$\langle a_{lm} a^*_{l'm'} \rangle = \int d\Omega_n d\Omega_{n'} Y_{lm}^*(\hat{n}) Y_{l'm'}(\hat{n'}) \langle \Delta T(\hat{n}) \Delta T(\hat{n'}) \rangle$$

Using,

$$P_l(\hat{n} \cdot \hat{n'}) = \frac{4\pi}{2l+1} \sum_m Y_{lm}(\hat{n}) Y_{lm}(\hat{n'})$$
and the directional dependent power spectrum we get,

$$\langle a_{lm} a_{l'm'}^* \rangle = \langle a_{lm} a_{l'm'}^* \rangle_{iso} + \langle a_{lm} a_{l'm'}^* \rangle_{aniso}. \quad (19)$$

The isotropic part is given by,

$$\langle a_{lm} a_{l'm'}^* \rangle_{iso} = (4\pi)^2 \frac{9T_0^2}{100} \delta_{ll'} \delta_{mm'} \int_0^\infty k^2 dk j_l^2(k\eta_0)P_{iso}(k) \quad (20)$$

and the anisotropic part

$$\langle a_{lm} a_{l'm'}^* \rangle_{aniso} = (i)^{l-l'+1} (4\pi)^2 \frac{9T_0^2}{100} G_{ll'} \xi_{lm; l'm'}' \quad (21)$$

where \( \xi_{lm; l'm'} \) is defined in Eq. 6 and

$$G_{ll'} = \int_0^\infty k^2 dk P_{iso}(k) j_l(k\eta_0) j_{l'}(k\eta_0) g(k) \quad . \quad (22)$$

Hence we find that the anisotropic power spectrum, Eq. 14, leads to a correlation between \( l \) and \( l \pm 1 \). This may be compared to the correlations, Eq. 5, we obtained from the dipole modulation model. The statistic \( S_H \) corresponding to the theoretical correlation, Eq. 21, is denoted as \( S_{Htheory} \).

We set \( P_{iso}(k) = k^{n-4} A_\phi / (4\pi) \), where the parameters, \( n = 1 \) and \( A_\phi = 1.16 \times 10^{-9} \). For the anisotropic power, we assume the form

$$g(k) = g_0 (k\eta_0)^{-\alpha}. \quad (23)$$

Here \( g_0 \) and \( \alpha \) are constant parameters. Using this, we obtain,

$$G_{ll'} = \frac{g_0 A_\phi}{4\pi} \int_0^\infty ds \frac{d\zeta}{s^{1+\alpha}} j_l(s) j_{l'}(s) \quad . \quad (24)$$

In our data analysis we first set \( \alpha = 0 \). This allows us to determine the mean value of \( g(k) \) which provides the best fit over the entire multipole range \( 2 \leq l \leq 64 \). We next determine the best fit values of both \( g_0 \) and \( \alpha \).

3 Data Analysis

We use the cleaned CMB maps, SMICA and COMMANDER RULER maps, provided by the PLANCK team [42]. We use the CMB-union mask to eliminate the contribution from galactic plane. We first generate a full sky map from the masked data by filling the masked portion with an intrinsically isotropic and Gaussian random field. This map is generated at high resolution with \( N_{side} = 2048 \). Subsequently we downgrade all the maps to lower resolution with \( N_{side} = 32 \) after applying appropriate Gaussian beam to smooth the mask boundary [37]. This procedure eliminates any breaks that might be introduced at the boundary due to the random filling procedure. We use this map to determine the statistic, \( S_H(L) \), over the multipole range \( 2 \leq l \leq 64 \). This statistic is maximized by search over the preferred direction parameters, i.e. the choice of our z-axis. We use polar coordinates and denote the axis parameters as, \( (\theta, \phi) \). Due to the random filling of the masked regions, we will find different results for different
realizations for the full sky maps. Hence the resulting parameters, i.e. maximum value of \( S_H(L) \) and \((\theta, \phi)\), are obtained by taking an average over a 100 maps. The significance of anisotropy is determined by comparing the maximum value of \( S_H(L) \) with that obtained from 4000 isotropic randomly generated full sky CMBR maps. The significance is quoted in terms of the P-value, which is defined as the probability that a random isotropic CMB map may yield a statistic larger than that seen in data.

4 Results

We first determine the best fit value of the dipole modulation parameter \( A \) over the entire multipole range, \( 2 \leq l \leq 64 \). We find the maximum value of \( S_H(L) \) by searching over all possible direction. The resulting value of data statistic, \( S_{Hdata} \), is given in Table 1 for the maps SMICA and COMMANDER. The best fit direction parameters are found to be in good agreement with those found by using hemispherical anisotropy. The anisotropy is found to be significant at a little more the 2\( \sigma \) for both the maps. The precise significance value depends on the mask used for analysis. In Table 1 the significance is given in terms of the P-value, defined in section 3. We next determine the value of the dipole modulation parameter, \( A \), by comparing \( S_{Hdata} \) with \( S_{Hdm} \) over the range \( 2 \leq l \leq 64 \). For SMICA, the best fit value of \( A \) is found to be \( 0.077 \pm 0.018 \), in good agreement with that found by hemispherical analysis. Finally we extract the value of the amplitude \( g \) of the anisotropic term in the power spectrum which fits the data over entire multipole range, \( 2 \leq l \leq 64 \). In this case we assume that the spectral index \( \alpha = 0 \). Hence the function, \( g(k) \) is equal to a constant, \( g_0 \). Comparison with data, leads to the value, \( g_0 = 0.23 \pm 0.06 \).

|          | \( \text{max. } S_H(L) \) (mK\(^2\)) | \( (\theta, \phi) \) | P-value                  |
|----------|--------------------------------------|----------------------|--------------------------|
| SMICA    | 0.021 \( \pm 0.005 \)                | \((105^\circ, 229^\circ)\) | 3.82\% (0.37\%)          |
| COMMANDER| 0.021 \( \pm 0.005 \)                | \((115^\circ, 232^\circ)\) | 4.22\% (0.42\%)          |

Table 1: The extracted maximum value of \( S_H(L) \) and the corresponding direction using masked analysis in the range \( l = 2 – 64 \) for both maps. The value given in the bracket represents the standard deviation in the value of \( S_H(L) \). The P-value given in the bracket do not account for the search over the two axes parameter.

We next study the variation of dipole modulation amplitude \( A \) with the multipole \( l \) using the map, SMICA. We divide the data into 3 bins, \( l = 2 – 22, 23 – 43 \) and \( l = 44 – 64 \), for this purpose. In each bin we determine the best fit value of \( S_{Hdata} \) and extract the value of \( A(l) \) by comparing with \( S_{Hdm} \). For this analysis we found it convenient to fix the direction to be same as that obtained by making the best fit over the entire multipole range \( 2 – 64 \). This direction parameters are given in Table 1. Alternatively, we may determine the best fit direction parameters, along with the amplitude, in each bin. We find that, in this case, the direction parameters show a mild dependence on the bin. The results for this case are shown in Table 2. Since the dependence is relatively small, we ignore it and fix the direction to be equal to the mean direction over the entire range. The resulting values of \( S_{Hdata}(l) \) for the three
bins, \( l = 22, 23 - 43 \) and \( l = 44 - 64 \), are found to be \( 0.0083 \pm 0.0026, 0.0082 \pm 0.0032 \) and \( 0.0069 \pm 0.0019 \) respectively. By comparing these with \( S_{Hdm} \) in the three bins we extract the \( l \) dependence of \( A \). This is plotted in Fig. 4. We find that \( A \) shows a monotonic decrease with \( l \).

| multipole     | \( A(l) \)   | \( S_{Hdata}(l) \) | \((\theta, \phi)\) |
|---------------|-------------|---------------------|---------------------|
| 22            | 0.134 ± 0.034 | 0.0101 ± 0.0026     | (139°, 228°)        |
| 23 - 43       | 0.080 ± 0.024 | 0.0073 ± 0.0022     | (87°, 241°)         |
| 44 - 64       | 0.068 ± 0.016 | 0.0074 ± 0.0017     | (127°, 221°)        |

Table 2: The maximum value of \( S_H(L) \) in the multipole range \( 2 - 22, 23 - 43, 44 - 64 \) and the corresponding direction parameters for SMICA. The extracted values of the effective dipole modulation parameter, \( A(l) \), for these three bins are also shown.

We finally extract the function, \( g(k) \), using data in the three multipole bins, \( l = 22, 23 - 43 \) and \( l = 44 - 64 \). As explained above, the function, \( g(k) \), is parametrized in terms of the overall constant, \( g_0 \), and the spectral index \( \alpha \). We extract these two parameters by fitting \( S_{Hdata} \) in the three bins using the map SMICA. We first set \( \alpha = 0 \) and determine \( g_0 \) which best fits the data in three bins. We obtain \( g_0 = 0.26 \pm 0.05 \) with \( \chi^2 = 0.38 \). Hence this provides a good fit to data. The data clearly favors a zero spectral index for the anisotropic part of the power spectrum. The resulting fit is shown in Fig. 2 as the dotted line. Allowing a non-zero value of \( \alpha \) we find that the 1\( \sigma \) limit on this parameter is, \(-0.2 < \alpha < 0.4\).

![Figure 1: The extracted value of the dipole modulation parameter, \( A \), as a function of the multipole \( (l) \) after fixing the direction parameters for the three chosen multipole bins, \( 2 - 22, 23 - 43, 44 - 64 \), to be equal to the mean over the entire range, \( 2 \leq l \leq 64 \).](image)
Figure 2: The statistic, $S_{H\text{data}}$, as a function of the multipole $l$. Here the statistic in the three bins is extracted by fixing the direction parameters to be equal to the mean direction over the entire multipole range. The dotted line corresponds to the theoretical fit corresponding to $\alpha = 0, g_0 = 0.26 \pm 0.05$.

5 Conclusion

We have extended the results obtained in a recent paper [37], which showed that the dipole modulation model leads to correlations among spherical harmonic multipoles belonging to different $l$ values. In that paper, [37], we defined a statistic, $S_H$ which provides a measure of this correlation in a chosen multipole range. By making a fit to this statistic in three multipole bins, $l = 2 - 22, 23 - 43$ and $l = 44 - 64$, we find that the effective dipole modulation parameter $A$ slowly decreases with the multipole, $l$. We consider an anisotropic power spectrum model and show that it also leads to such a correlation. The anisotropic power spectrum is parameterized by the function, $g(k)$. We first fit the data by assuming that $g(k)$ is a constant equal to $g_0$. We determine the value of $g_0$ by making first making a fit over the entire multipole range, $2 - 64$. The best fit value is found to be $g_0 = 0.23 \pm 0.06$. We next assume a power law form of $g(k) = g_0 (k\eta_0)^{-\alpha}$ and extract the corresponding amplitude, $g_0$ and spectral index $\alpha$ by making a fit over the three multipole bins, $l = 2 - 22, 23 - 43$ and $l = 44 - 64$. Setting $\alpha = 0$, the best fit leads to $\alpha = 0.26 \pm 0.05$. This leads to a good fit to data with $\chi^2 = 0.38$. Hence the data suggests that the anisotropic power, $g(k)$, is independent of $k$. Furthermore we find the one sigma limit on $\alpha$ to be, $-0.2 < \alpha < 0.4$.

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