The Price Linkage between Oil, Gold, Stock and Exchange Rate Based on Vine Copula

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Abstract. With the development of China's economy and the deepening of the reform of the socialist market economic system, the relationship between different markets has increased and the relationship is becoming more complex, which poses new challenges to the portfolio and risk management of the market. Therefore, this paper empirically analyses the relationship between the price of China's oil market, the gold market, the stock market and the foreign exchange market by using the correlation measure based on Vine copula. The results show that the oil market occupies the dominant position in the four markets, and the price changes in other markets have a driving effect, and the lower tail correlation among the markets is generally higher than that of the upper and tail correlation, showing asymmetric correlation and thick tail. These results show that the risk price linkage among markets is more closely and the risk is more contagious in the extreme cases.

1. Introduction

The Copula function has certain advantages in the characterization of correlations, and in particular it can accurately describe the nonlinearity and thick-tailedness in the correlation. Therefore, it is widely used in the financial field. Li and Yang [1], Nguyen et al. [2] used the Copula theory to analyze the correlation between two financial markets. However, due to its dimensionality constraints, the binary Copula function cannot measure the correlation between multiple variables. Joe [3] defined the Vine Copula function based on the Copula function proposed by Sklar[4] to describe correlation between multivariables. Bedford and Cooke [5, 6] further refined Vine Copula's related theories. The Vine Copula model has both Vine structure and Copula model characteristics. The Vine structure is used to incorporate multiple variables under a unified research framework. Then the node relationships in the Vine structure are characterized by the binary Copula function to achieve a unified measurement of multivariable relationships.

Researches on Vine Copula function has gradually become a hot topic in academic circles in recent years. Aas and Czado [7] used the Vine Copula model to study the multivariate correlation structure; Mendes [8] used the Vine Copula model to study the correlation between multiple markets. Righi [9] used a new Pair Copula structure to characterize the correlation between several financial markets in Brazil and calculated the VaR of the portfolio.

Therefore, this article intends to empirically analyze the price linkage relationship between petroleum, gold, stocks and foreign exchange markets based on the correlation measure of Vine Copula, with a view to provide valuable participation for investment decision-making and risk management in the China market.
2. Correlation Measurement based on Vine Copula

2.1. Vine Copula Function

Vine Copula has obvious advantages when modeling the marginal distribution of multidimensional random variables and related structures. It does not require that the marginal distribution of variables has the same pattern. From the decomposition process of the Pair-Copula function, we know that an n-dimensional Copula function can be decomposed into \( \frac{p(p-1)}{2} \) binary Copula functions, and there are Pair-Copula construction methods. In order to simplify the number of models, Cooke and Bedford (2001) proposed the theory of “Rule Vine”. Canonical vine (C-vine, C-Vine for short) is the most commonly used rattan structure. Therefore, this paper uses C-Vine Copula function to fit the data in the vine structure separately so as to consider the difference of the vine structure. The C-Vine Copula function is constructed as follows:

Step 1: Verify orders of \( n \) variables in \( T_1 \), and rearrange the order and denote \((u_1, u_2, ..., u_n)\).

Step 2: Because there are \( n-1 \) edges between the \( n \) variables in the vine structure, \( n-1 \) binary Copula functions can be used to link the sorted adjacent variables, creating the first layer of Pair-Copula \( C_{1,2}, C_{1,3}, ..., C_{1,n} \).

Step 3: Take the \( n-1 \) dimensional variable \((C_{1,2}, C_{1,3}, ..., C_{1,n})\) in the second step as a random variable in \( T_2 \) and continue with the second step. The calculation step calculates the second layer Pair-Copula \( C_{2,3|1}, C_{2,4|1}, ..., C_{2,n|1} \).

Step 4: Repeat steps 2 and 3 until the end of the calculation.

2.2. Correlation measure

Joe (1997) gives the definition of the tail correlation coefficient, which is used to express the probability that when one variable's implementation value is extremum, another variable also has extremum, it is a measure widely used in extremum theory. First, the definition of quantile correlation measure of relevance is given. If \( u^* \in [0,1] \), define

\[
\lambda(u^*) \equiv P[U > u^*|V > u^*] = \frac{\bar{C}(u^*,u^*)}{1-u^*},
\]

as a quantile correlation measure of relevance, whereas \( \bar{C}(u,v) = 1-u-v-C(u,v) \).

Then, the definition of the tail correlation coefficient is given. If \( X \) and \( Y \) are two consecutive random variables, their marginal distribution is \( F(\cdot) \), \( G(\cdot) \), and Copula function is \( C(\cdot,\cdot) \). Let

\[
\tau_U \equiv \lim_{u^* \to 1} P[Y > G^{-1}(u^*)|X > F^{-1}(u^*)] = \lim_{u^* \to 1} \frac{C(1-u^*,1-u^*)}{1-u^*},
\]

\[
\tau_L \equiv \lim_{u^* \to 0} P[Y < G^{-1}(u^*)|X < F^{-1}(u^*)] = \lim_{u^* \to 0} \frac{C(u^*,u^*)}{u^*},
\]

Respectively represent upper and lower tail correlation coefficients, and \( \bar{C}(1-u,1-v) = \bar{C}(u,v) \).

If the upper tail relationship \( \tau_U \) (or the lower tail correlation coefficient \( \tau_L \)) exists and is within the interval \((0,1]\), the upper tail (or lower tail) is related; if the upper tail relation \( \tau_U \) (or If the lower tail correlation coefficient \( \tau_L \)) is zero, the variables are independent of each other, which shows that the Copula function has a great advantage in measuring the tail correlation.

2.3. Parameter Estimation

The parameters of Copula model can use maximum likelihood estimation or moment estimation. In this paper, the maximum likelihood method is used to estimate the parameters. The likelihood function can be written as

\[
L(\xi; x) = \sum_{i=1}^{T} \left( \sum_{t=1}^{p} \log \left( f_i(x_{it}; \phi_t) \right) - \log \left( c(F_1(x_{i1}), ..., F_p(x_{ip}); \theta) \right) \right),
\]
where vector \( \xi = (\phi, \theta) \) include marginal distribution parameter \( \phi = (\phi_1, ..., \phi_p) \) and Copula parameter \( \theta \).

If the number of parameters to be estimated is too large, a two-stage estimation method is usually used to simplify the estimation process. The first step is to estimate the marginal distribution parameters and the second step is to estimate the Copula parameters. The maximum likelihood estimation of marginal parameters is as follows:

\[
mL(\phi; x) = \sum_{i=1}^{p} \sum \log f_{i}(x_{i}; \phi_{i}),
\]

(4)

The maximum likelihood estimation of the Copula parameter is as follows:

\[
cL(\theta; u, \phi) = \log c(F_1(x_{1,1}), ..., F_p(x_{p,1}); \theta),
\]

(5)

where \( u = (F_1(x_{1}), ..., F_p(x_{p})) \).

3. Empirical Analysis

3.1. Data Sources and Descriptive Statistics

The indexes are selected for the oil and gold markets are the FUL9 Fuel Index in the Shanghai Commodity Futures Market and the AUL9 Gold Index, and the Shanghai-Shenzhen 300 Index for the stock market index. These figures are derived from CITIC Securities. The foreign exchange market index is selected from the People’s Bank of China’s RMB exchange rate against the US dollar (RMB/100 US$). The data comes from the Bank of China. Since gold futures were formally traded on the Shanghai Futures Exchange in 2008, in order to ensure consistency in the timing of various market indicators, the daily data from 2008 to 2016 was selected and a total of 1,892 valid data were obtained.

First, the logarithmic returns of the four financial time series are calculated. The logarithmic daily return rate is shown in Figure 1. The four figures are from left to right, from top to bottom, the gold index, the fuel index, the CSI 300 Index, and the exchange rate.

![Figure 1. Logarithmic daily revenue rate fluctuation trend](image-url)
From Figure 1, it can see the fluctuation of the rate of returns of four markets. Except for the extreme value, the exchange rate benchmark price has the smallest volatility, and the CSI 300 index has the largest volatility. The volatility of the gold index and the fuel index is medium and the degree of volatility is similar. The further statistical analysis of the yield data are shown in Table 1.

| Table 1. Descriptive statistics of the daily return rate of data index |
|---------------------------------------------------------------|
| **Index** | **Gold Index Yield** | **Fuel Yield** | **Index** | **CSI 300 Index Yield** | **Exchange rate yield** |
|----------------|-----------------|----------------|----------------|-----------------|-----------------|
| minimum        | -0.0788         | -0.0945        | -0.0915       | -0.0108         |
| maximum        | 0.0567          | 0.0719         | 0.2599        | 0.0184          |
| average        | 1.02E-04        | 3.64E-05       | -2.79E-04     | -2.45E-05       |
| Standard       | 0.0130          | 0.0160         | 0.0209        | 0.0013          |
| deviation      |                 |                |               |                 |
| Skewness       | -0.48           | -0.29          | 0.75          | 3.57            |
| Kurtosis       | 3.92            | 3.55           | 15.28         | 47.54           |
| J-B statistics | 1273.63         | 1009.48        | 18470.66      | 181141.80       |
| ADF statistics | -44.17          | -27.43         | -43.10        | -39.90          |

Note: JB statistics and ADF statistics are significant at the 5% level.

According to the statistical results in Table 3-1, the average return rate of the fuel index and the gold index is positive, and the average return rate of the CSI 300 index and the exchange rate benchmark price is negative, indicating that the oil market and the gold market show an upward trend and the stock market and the foreign exchange market show an downward trend. The standard deviation of the CSI 300 index is the largest, indicating that the stock market has the highest price volatility, followed by the volatility of the fuel index and the gold index, and the difference is not significant, indicating that the volatility of the oil market and the gold market are similar. The volatility of the exchange rate benchmark price is the smallest in the four indicators. From the perspective of skewness, the four markets are biased. The skewness of the fuel index and the gold index is negative and the performance is left deviation. The CSI 300 and the benchmark exchange rate are positive, showing a right deviation. Judging from the kurtosis value, the four markets are all presented with "spikes" and the performance of the exchange rate market is particularly obvious.

In addition to the basic descriptive statistics, the J-B statistics (J-B) and ADF unit root tests (ADF) were also performed for each index's rate of return. From the J-B statistics, each index refuses to obey the Gaussian normal distribution at a significant level of 5%, and accepts the assumption that each indicator's time series is stable at a significant level of 5%. Therefore, the Copula function can be used to mathematically modeling the first relations between markets.

3.2. Modeling Analysis of Price Interaction Based on Binary Copula

In the binary Copula model, the Clayton Copula function with asymmetry can quickly capture the relevant changes in the lower tail, and its representing parameters the Kendall rank correlation coefficient $\tau$. The SJC-Copula is a function that allows the asymmetry of the left tail and right tail, and its two parameters to be evaluated $\tau^U$ and $\tau^L$, which represent the upper tail correlation coefficient $\lambda^U$ and the lower tail correlation coefficient $\lambda^L$.

Therefore, the Kendall rank correlation coefficient $\tau$ is calculated using the Clayton Copula function, and the upper tail correlation coefficient $\lambda^U$ and the lower tail correlation coefficient $\lambda^L$ of the SJC-Copula function. The parameter estimation results are shown in Table 2.
Table 2. Non-conditional tail correlation coefficient estimation results

| combination                | Kendall's tau | upper dependence | tail          | lower tail dependence |
|----------------------------|----------------|------------------|---------------|-----------------------|
| Gold Index - Fuel Index    | 0.1594         | 0.0682           | 0.1797        |                       |
|                            | (0.014)        | (0.031)          | (0.031)       |                       |
| Gold Index - CSI 300 Index | 0.0623         | 0.0000           | 0.0394        |                       |
|                            | (0.014)        | (0.000)          | (0.028)       |                       |
| Gold Index - Exchange Rate Benchmark Price | 0.0062       | 0.0000           | 0.0002        |                       |
|                            | (0.012)        | (0.036)          | (0.042)       |                       |
| Fuel Index - CSI 300 Index | 0.1165         | 0.0386           | 0.1026        |                       |
|                            | (0.014)        | (0.026)          | (0.030)       |                       |
| Fuel Index - Exchange Rate | 0.0097         | 0.0000           | 0.0003        |                       |
|                            | (0.012)        | (0.056)          | (0.079)       |                       |
| CSI 300 - Exchange Rate Benchmark Price | 0.0096       | 0.0005           | 0.0000        |                       |
|                            | (0.013)        | (0.003)          | (0.000)       |                       |

Note: The value in brackets is the standard deviation.

From Table 2, we can see that the tail correlation coefficient estimated by the binary Copula model is positive and the relationship is weak. Among them, the tail correlation between the oil market and the gold market is most obvious among the six groups of related relations, and the tail market correlation between the oil market and the stock market is second. At the same time, in the two correlations, the values of lower tail correlation coefficient are significantly greater than the values of upper tail correlation coefficient, indicating that the tail price linkage between the oil market and the gold market and between the oil market and the stock market is strong. Especially when the market price plummets, the risk of infection is even more pronounced.

Under the condition of binary correlation (non-conditional relationship), a portfolio that can be related to other markets with weaker correlation, such as foreign exchange, can be constructed. When the entire market is in a bear market, or when price fluctuations are large, risks can be effectively avoided. However, the binary investment portfolio does not apply to China's complex financial market system. It should also incorporate the selected samples into a holistic model and comprehensively analyze the correlations among various markets.

3.3. Modeling Analysis of Multivariate Price Linkage Based on Vine Copula

For ease of description, in the modeling process below, 1 is used to represent the gold index, 2 is the fuel index, 3 is the CSI 300 index, and 4 is the exchange rate base price.

C-Vine copula model is a model with pilot node. According to the empirical research, the first layer consists of 2-1, 2-2, 2-4, the second layer consists of 1-3|2, 1-4|2 and the third layer consists of 3-4|1,2, with a 2-1-3-4 order. Hence, the sequence based on the relative importance of the four markets is oil market, gold market, stock market and exchange rates market. And it is clear that the oil market is the most dominant market among them.

Similar to the analysis of the binary correlation, the Kendall rank correlation coefficient was calculated using the Clayton Copula function, and the tail relationship and the tail-tail correlation coefficient were used on the SJC-Copula function. The parameter estimation results are shown in Table 3.
Table 3. C-Vine Copula Correlation Coefficient Estimate Results

| Decomposition | Kendall's tau(τ) | upper tail dependence | lower tail dependence |
|---------------|------------------|-----------------------|-----------------------|
| 2-1           | 0.1557           | 0.0715                | 0.1730                |
| 2-3           | 0.0939           | 0.0219                | 0.0757                |
| 2-4           | 0.0017           | 0.0000                | 0.0000                |
| 1-3|2         | 0.0635           | 0.0000                | 0.0411                |
| 1-4|2         | 0.0119           | 0.0000                | 0.0000                |
| 3-4|1,2      | 0.0105           | 0.0011                | 0.0000                |

The analysis of the first-level tree shows that the Kendall rank correlation coefficient between the oil market and the gold market is the highest, and it is most likely to rise or fall in the same direction; At the same time, the tail correlation coefficient between the oil market and the gold market is also the highest, indicating the possibility that oil market and the gold market soar or plunge simultaneously is greatest.

The Kendall rank correlation coefficient between the oil market and the stock market is relatively high, indicating that the overall correlation between them is also strong. Although the tail correlation is not as significant as the tail market correlation between the oil market and the gold market, it still has a certain tail correlation. However, the correlation coefficient between the two tails in the two groups of relations is significantly greater than the correlation coefficient of the last tail, indicating that the collapse of oil prices is more likely to cause the price of gold and stock markets to plummet.

The possibility of skyrocketing gold prices and stock prices caused by skyrocketing oil prices is not as obvious as when the oil market was in a bear market. The Kendall rank correlation coefficient between the oil market and the foreign exchange market is less than 0.002, and the overall correlation is the weakest.

The analysis of the second-tier tree shows that when taking the oil market's yield as a known condition, taking the gold market as the leading node and measuring the correlation between the gold market and the stock market and the foreign exchange market, the overall correlation between the gold market and the stock market is significantly larger than the overall correlation between the gold market and the foreign exchange market. And there is a certain degree of correlation between the gold market and the stock market, and the price plunge in the gold market is likely to cause a bear market in the stock market. However, the soaring price of gold is less likely to bring a bull stock market.

The analysis of the third-level tree shows that the overall correlation between the stock market and the foreign exchange market is small, compared with the result of the binary Copula model, when the oil market yield and the gold market yield are known conditions. However, there has been some improvement. The value of the Kendall rank correlation coefficient exceeds 0.01, and the tail-tail correlation between the two markets begins to show.

Overall, in these four market portfolios, the oil market dominates, and price fluctuations have the greatest impact. In extreme circumstances, especially the decline in the overall trend of the financial market brought about by the collapse of oil prices has the most significant impact. When there is more the known yield information, the conditional correlation between the two markets is less obvious. The correlation between the foreign exchange market and the tails of the other three markets has approached zero, indicating that even in extreme circumstances, the risk transmission and price linkage effects between the foreign exchange market and the other three markets are not significant.

4. Conclusion
This paper constructs a correlation measurement based on Vine copula to study the price linkage relationship about petroleum, gold, stocks and foreign exchange markets. Firstly, the correlation measurement model based on Vine Copula function and the parameter estimation method of its model are given. Then the correlation measurement models of binary Copula and Vine Copula functions are
used to empirically analyze price linkage relationship of the oil, gold, stock and foreign exchange markets.

The empirical results show that there is a positive correlation among the oil market, the gold market, the stock market and the exchange rate market, and the correlation between the oil market and the gold market is the strongest, and the correlation between the exchange rate market and the other three markets is relatively weak. The correlation between the two markets is significantly stronger than the correlation between the tails, indicating that there is a certain degree of risk contagion among the markets, especially in extreme cases; the price linkage relationship is more significant. And the oil market dominated the four sample markets, leading to price linkages in sample markets.

According to the comparison between the binary Copula model and the multivariate Vine Copula model, Vine Copula has more advantages than binary Copula in terms of intra-sample fitting and data analysis. Therefore, in practice, Vine Copula-based models can be used for risk analysis in the process of portfolio construction, and then it can improve the accuracy of risk measurement results.

5. References

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