Kinetics of large scale vertical flow generation by weakly noncollinear nonlinear gravitational waves

A V Poplevin¹,², S V Filatov¹, M Yh Brazhnikov¹, A.M. Likhter²

¹Institute of Solid State Physics, Russian Academy of Sciences, 142432, Chernogolovka, Moscow Region, Russia
²Astrakhan State University, 414056, Astrakhan, Russia

E-mail: faraldos@issp.ru

Abstract. The generation of a large-scale vortex flow by gravity waves on the water surface has been experimentally studied. It is shown that large-scale flows are formed as a result of the interaction between nonlinear low-frequency modes which intersect at small angles. The amplitude of vorticity at large scales increases with increasing angle between pump waves. The experimental data agree well with the developed theoretical model.

1. Introduction

Vortex motion, along with wave motion, plays a key role in many global processes which occur in both the atmosphere and oceans of Earth and even on other planets. The understanding of processes in turbulent motions in these systems is a challenge to modern science. That is why the research of these motions has long been one of the main tasks.

The generation of vortices on the water surface was observed for the first time in the experiments with Faraday instability in a vessel performing vertical oscillations [1]. The work demonstrated that these flows are solenoidal and with increasing wave amplitude they can be intensive enough to form a turbulent energy cascade [2] like an inverse cascade in two-dimensional turbulence. In [3] it was found out experimentally that the interaction between nonlinear waves propagating at an angle to each other is responsible for vortex formation. The theoretical model describing vortex formation by waves and proposed in [4] shows that the interaction between nonlinear surface waves propagating at an angle to each other is responsible for vorticity generation on the surface of a liquid.

The vorticity Ω on the surface of a liquid is determined by the formula Ω = (rot V), where V is the velocity in X and Y directions. If standing waves with the frequency ω are formed on the surface in two mutually perpendicular directions, vorticity on the water surface calculated from particle tracks, depends on the wave amplitudes H₁, H₂, the wave number k, the phase difference between plungers ψ, and is described by the formula [4, 5]

\[ \Omega = - (1+\sqrt{2}) \sin \psi \frac{H_1 H_2 \omega}{k^2} \sin (kx) \sin (ky). \]  

Eq. (1) shows that vorticity is the sum of two terms. The first term describes Stokes’ drag [6], and the second term corresponds to the nonlinear interaction of waves [4].

The experiments [7] demonstrated that over time, an ideal vortex lattice is destroyed by the intense vortex interaction. The interaction between waves which propagate at small angles to each other and
which may appear in a bath due to their nonlinear interaction is a possible mechanism of the formation of a large-scale flow.

In this work, we study the formation process of a large-scale vortex flow generated by surface gravity waves which are formed by plungers mounted at small angles to each other.

2. Experimental methods

The investigations were performed on an experimental setup, the scheme of which is presented in Fig. 1. The setup consists of a bath with a length of 70 cm, a width of 70 cm, and a height of a 25 cm which was made from 10-mm-thick glass. The bath was placed on a Standa vibration isolation table with an air suspension and was filled with distilled water with a total volume of about 50 l to a depth of 10 cm. Wave generators each consisting of an actuator and a plunger were mounted on a supporting frame; they excite waves on the water surface.

The plungers were made of a stainless steel rod with a diameter of 10 mm and a length of 200 mm. They were placed at a small angle to the bath walls. The distance from the plunger center to the wall was 3.5 cm. Pioneer TS-W254R subwoofers with a rated power of 250W were used as the plunger actuators. A sinusoidal signal generated by a dual-channel generator and amplified by a signal amplifier was supplied to the subwoofers.

To visualize vortex motion, a white powder of polyamide particles with an average diameter of about 30 μm was deposited on the water surface. The particles on the surface were illuminated by the LEDs placed along the bath perimeter. An oscillating surface was recorded by a Canon EOS 70D camera at a rate of 24 fps. Such a frame rate permits choosing the images of the oscillating surface being in one wave phase and excluding an oscillating component of displacement of a test particle floating on the surface from the following processing. Then, the frames obtained were summed in order to identify tracks of motion of the particles on the surface.

There was a contrast stencil with LED illumination under the transparent bath bottom. The stencil was printed on a film with a thickness of 100 μm. Transparent dots with a diameter of 0.5 mm and a total number of 10 000 were randomly distributed over the film. The dots were located on a dark background which did not transmit light. An average distance between the dots was 7 mm, and an average transparency of the stencil was about 0.5%. The method allows measuring amplitudes of the waves propagating on the water surface. The PIVLab code [8] for MATLAB was used to process resulting images. It allows calculating the field of displacements between the images by the method of
cross-correlation processing of two images. Recording processing, as well as processing algorithm, is presented in [9].

3. Experimental results and discussion

Figure 2 shows tracks of polyamide particles on the water surface 690 s after switching on pumping at the frequency $f = 3$ Hz, the angle between plungers is $\alpha = 13.7^\circ$.

Figure 2. Tracks of polyamide particles on the water surface 690 s after switching on pumping by plungers at a frequency of 3 Hz. The angle between plungers is $\alpha = 3.1^\circ$.

A large vortex elongated along the cell wall is clearly seen in the tracks. Two smaller vortices are also seen which rotate in the opposite direction to the large vortex. The total vorticity on the water surface is zero. The formation mechanism of large-scale vortices is related to the interaction between low-frequency nonlinear waves on the water surface in a finite-size bath [9]. The generation of one or two large vortices is possible depending on the symmetry of plunger placement. Here, the plungers were placed not symmetrically relative to the right bath wall.

Figure 3. Distribution of the wave motion energy in the $k$ space. The pump frequency is $f = 2.96$ Hz, the angle between the plungers is $\alpha = 2.6^\circ$.

Figure 3a shows the distribution of wave motion energy in the k-space at a pump frequency of 2.96 Hz. Four peaks with coordinates $(\pm 0.32, 0.1) \text{ cm}^{-1}$ and $(\pm 0.32, -0.13) \text{ cm}^{-1}$ are clearly seen which correspond to the waves propagating in two directions and located at angles of $14^\circ \pm 2^\circ$ relative to the horizontal X axis. The modulus of the wave vector connecting two peaks is $0.23 \text{ cm}^{-1}$. 
Two peaks are seen on vorticity distribution in the k space (Fig. 3b) which correspond to a large-scale vortex flow. The wave vector directed from the coordinate origin to the peak center has coordinates \((0, \pm 0.23) \text{ cm}^{-1}\).

Thus, it is established experimentally that the modulus of the wave vector connecting two peaks on wave energy distribution is equal to a half of the modulus of the wave vector connecting two peaks on vorticity distribution in the k space. Figure 3a demonstrates four more peaks corresponding to waves with a frequency of 3 Hz propagating at small angles to the Y direction. The interaction between all nonlinear waves on the water surface apparently leads to the formation of a large-scale vortex.

Figure 4 shows the time dependence of vorticity modulus after switching on pumping by plungers. The pump frequency is 3 Hz, the angle between plungers is \(\alpha = 3.1^\circ\). An increase in vorticity modulus is observed approximately to 450 s. Then \(|\Omega|\) decreases non-monotonically. As the figure demonstrates, the experimental data in the range from 10 to 400 s can be described well by the exponential dependence \(|\Omega| \sim \exp(-\tau/t)\) with the characteristic time of about \(\tau = 520\) s. This result is in a good qualitative agreement with the model proposed in [4,5].

**Figure 4.** Time dependence of the vorticity modulus. The pumping frequency is \(f = 2.96\) Hz, the angle between the plungers is \(\alpha = 3.1^\circ\), blue curve 1 is the experimental time dependence of the vorticity modulus, red curve 2 is the exponential function approximation.

4. Conclusion

It has been shown experimentally that waves propagating at a small angle to each other can form a large-scale vortex flow. Vorticity modulus increases by the exponential law over time until the system reaches the stationary value. Experimental results can be well described in the frame of developed theory.

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