Numerical analysis of a vibro-impact system with ideal and non-ideal excitation

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Abstract. This study is concerned with modelling and analyses of a vibro-impact system consisting of a crank-slider mechanism and one oscillator attached to it, where the system can be exposed to ideal or non-ideal excitation. The impact occurs during the motion of the oscillator when it hits a base, and the excitation of the driving source is affected by this behaviour. The aim is to determine the interaction between a driving torque and the motion of the oscillator. To achieve this aim in a methodologically sound manner, both vibro-impact systems with ideal and non-ideal excitation are analysed. For these system differential equations are formed and the impact model is provided in the paper. The impact causes a strong nonlinearity in the system. The mathematical model of the vibro-impact system with ideal excitation is presented as a second order differential equation where the vibro-impact system with non-ideal excitation is given as a coupled system of nonlinear second order differential equations. Numerical simulations are carried out for the two systems and the results obtained are shown in terms of frequency response diagrams as well as in terms of time-displacement diagrams. The results found for different systems are compared mutually, and the differences between them are pointed out. Impact solutions for different regions of the excitation frequency are shown. For a specific value of the excitation frequency in the frequency response diagram where multiple solutions are found, basin of attractor diagrams are formed. Average value of the excitation frequency is used for the vibro-impact system with non-ideal excitation.

1. Introduction

Vibro-impact systems appear in many engineering applications. They are of particular importance for achieving a desired work regime of certain machines, such as hand-held percussion machines, pile driving machines, cutting and grinding machines, etc. Extensive research has been conducted in the field of vibro-impact systems where the excitation of the system is constant as well as in the field of dynamical systems with non-ideal excitation without impact [1-14]. However, only few papers have been published related to vibro-impact systems with non-ideal excitation [15-17]. In [15], a dynamical system with ideal and non-ideal excitation is analysed, where the vibro-impact system is used as a dynamic absorber. In [16], a system with non-ideal excitation with three degrees of freedom is investigated, where the impact is modelled via a stiff spring. The motion observed is a relative motion between two bodies impacting each other. Frequency-response diagrams of the relative displacement are found numerically, but no run
up or close down simulations are distinguished. The necessity to carry out a run up and close down numerical simulations for obtaining all possible results is also pointed out in [1-6]. In [17], a possible use of vibro-impact systems in chaos control is analysed for dynamical systems with non-ideal excitation. Similarity between the research in [17] and [15] is that the vibro-impact system is used as a dynamical absorber attachment. From a wide span of literature, the analysis of a vibro-impact system with non-ideal excitation is not found in the manner that the main analysis is focused on the impacting body and the influence of the excitation source on it. In [18,19] analysis of the Sommerfeld effect of oscillatory system with non-ideal excitation, as stability analysis of such system is conducted. Different models of impact are described and analysed in [20]. The influence of the stall torque on the behaviour of a vibro-impact system with non-ideal excitation is done in [21].

This study is concerned with modelling and analyses of a crank-slider mechanism containing a vibro-impact system having the form of a spring-mass oscillator that can hit a base. The aim is to determine the interaction between a driving torque and the motion of the oscillator from the viewpoint of the influence of the excitation frequency on the regions of impact solutions in reference to the excitation frequency.

The paper is organized in four sections. Section 2 contains the description of a mechanical model of the system under consideration of the corresponding equations of motion. Section 3 presents the results and discussion for vibro-impact systems with ideal and non-ideal excitation. The paper ends up with the conclusions summarised in Section 4.

2. Mechanical model and equations of motion

A vibro-impact system under consideration (figure 1 a) consists of a crank-slider mechanism and a mass-spring system (oscillator) attached to it. The crank-slider mechanism consists of an eccentric drive that is assumed to be symmetrical or balanced. The system parameters are given in figure 1 a): the lengths of the rods are $OA = a$, $AB = l$; mass of rod AB is neglected; the moment of inertia of the rod OA with respect to the axis through $O$ is $J$; the mass of the slider is $m_B$ and of the oscillator $m_C$; the stiffness of the spring is $c$ and the length of an underformed spring is $l_0$. The mass $m_C$ can collide with a stop (base) in certain regimes. It will be assumed that the driving torque $T(\dot{\varphi})$ depends on the angular velocity of the eccentric drive.

![Figure 1. Vibro-impact system:](image)

- a) mechanical model of the system under consideration,
- b) geometry of the system and generalized coordinates
The driving torque is assumed to be a linear function of the angular velocity of the eccentric drive:

\[ T(\dot{\varphi}) = \kappa T_0 \left( 1 - \frac{\dot{\varphi}}{\kappa \Omega_0} \right) \]  

Equation (1) describes how the driving torque changes with the angular velocity, and it represents the linear characteristic of the driving DC electromotor [18,19]. This is presented in figure 2, and the influence of the parameter \( \kappa \) is labelled. It is seen that the increase of parameter \( \kappa \) translates the characteristic upwards. The increase of the driving torque parameter (stall torque) \( T_0 \), as shown in figure 2 b), rotates clockwise the characteristic of the DC electromotor. For a certain value of \( T_0 \), this characteristic becomes vertical (dashed line), which represents the case of ideal excitation.

![Figure 2](image.png)

**Figure 2.** Characteristics of driving torque as a function of the angular velocity \( \varphi \) for different values of the parameter \( \kappa \)

The system has two degrees of freedom with the generalized coordinates being the displacement of the oscillator \( x \) and the angle of rotation of the eccentric drive \( \varphi \), as shown in figure 1 b). The equations of motion are formed based on Lagrange's equations of motion:

\[
\ddot{x} + 2 \delta \dot{x} + \omega^2 x = \omega^2 l_0 + \omega^2 u(\varphi) \\
(m_B u'(\varphi)^2 + f) \dot{\varphi} + m_B \dot{\varphi}^2 u'(\varphi) u''(\varphi) = \kappa T_0 \left( 1 - \frac{\dot{\varphi}}{\kappa \Omega_0} \right) + c u'(\varphi) (x - l_0 - u(\varphi))
\]  

where:

\[
\omega^2 = \frac{c}{m_c}, \quad 2 \delta = \frac{b}{m_c}
\]  

The motion of the slider B, \( u(\varphi) \) figure 3, and the corresponding derivatives are given by:

\[
u(\varphi) = a \cos \varphi + l \sqrt{1 - \left( \frac{a}{l} \right)^2 \sin^2 \varphi} = a \cos \varphi + l \sqrt{1 + \frac{1}{2} \left( \frac{a}{l} \right)^2 (\cos 2 \varphi - 1)}
\]
\[ u'(\phi) = -a \sin \phi - \frac{l}{2} \left( \frac{a}{l} \right)^2 \frac{\sin 2\phi}{\sqrt{1 + \frac{1}{2} \left( \frac{a}{l} \right)^2 (\cos 2\phi - 1)}} \]

\[ u''(\phi) = -a \cos \phi - l \left( \frac{a}{l} \right)^2 \frac{\cos 2\phi}{\sqrt{1 + \frac{1}{2} \left( \frac{a}{l} \right)^2 (\cos 2\phi - 1)}} - \frac{l^4}{4} \left( \frac{a}{l} \right)^4 \frac{\sin 2\phi}{\left[ 1 + \frac{1}{2} \left( \frac{a}{l} \right)^2 (\cos 2\phi - 1) \right]^{3/2}} \]

For the sake of analytical considerations, it is of interest to find the approximation of the function \( u(\phi) \). This can be done by developing it into series as follows:

\[
    u(\phi) = l + (l \cos \phi) \epsilon - \left( \frac{l}{4} (1 - \cos 2\phi) \right) \epsilon^2 + O(\epsilon^3)
\]

where the following small parameter \( \epsilon \) is introduced:

\[
    \epsilon = \frac{a}{l}
\]

The first approximation of the function \( u(\phi) \) is given by:

\[ u(\phi) = l + a \cos \phi \]

while its derivatives are:

\[ u'(\phi) = -a \sin \phi, \quad u''(\phi) = -a \cos \phi \]

In the case of the first approximation, a simplified model of the system can be formed as shown in figure 3.

![Figure 3. Simplified mechanical model of the system under consideration](image)

The equations of motion in the first approximation are:

\[
    \ddot{x} + 2 \delta \dot{x} + \omega^2 x = \omega^2 (l_0 + l) + \omega^2 a \cos \phi
\]

\[
    (m_B a^2 \sin^2 \phi + f) \ddot{\phi} + m_B \dot{\phi}^2 a^2 \sin \phi \cos \phi
    = \kappa T_0 \left( 1 - \frac{\phi}{\kappa \Omega_0} \right) - c a \sin \phi (x - l_0 - l - a \cos \phi)
\]
Given the form of Eq. (8) and the existing coupling between the generalized coordinates \( \varphi \) and \( x \), the angular velocity will not be constant, but it will change in time. The system of differential equations (8) represents a system of coupled nonlinear second order differential equations of motion. They cannot be solved analytically in the given exact form, but their solutions can be found by carrying out numerical simulations or utilizing approximate analytical methods.

For the sake of comparison, the previous equations for non-ideal excitation can be adjusted to represent a system with ideal excitation. In this case, one concludes that \( \dot{\varphi} \) is constant: \( \dot{\varphi} = \Omega_p = \text{const} \). Thus, the angle \( \varphi \) will be a linear function of time \( \varphi = \Omega_p t \), and it is independent from the motion of the system. These conclusions can be written as follows:

\[
\varphi = \Omega_p t, \quad \dot{\varphi} = \Omega_p = \text{const}, \quad \ddot{\varphi} = 0 
\]  

(9)

The equation of motion of the system with ideal excitation is derived using Eq. (9) and it is given in the following form:

\[
\ddot{x} + 2\delta \dot{x} + \omega^2 x = \omega^2 (l_0 + l) + \omega^2 a \cos(\Omega_p t) 
\]  

(10)

Equations (8) and (10) represent the equations of motion for a system with non-ideal excitation and a system with ideal excitation, respectively. Equation (10) is a linear differential equation of forced damped vibrations. In the case of impact, the system behaves according Eqs. (8) and (10) only between two impacts. Damping of the system is introduced by a linear viscous force from friction. The model of friction is predicted to be linear, where in reality this force can be nonlinear. Different linear and nonlinear models that are describing friction exist. In this research a linear model is chosen because of the strong nonlinearity that is caused by the impact, where in the case of the vibro-impact system with non-ideal excitation an additional nonlinearity comes from the non-ideal excitation.

As noted in figure 1 b) and figure 3, the oscillator can hit the base, where the coordinate marked by \( x_{\text{stop}} \) represents the position of a fixed wall at which the oscillator can impact. If the impact occurs periodically during the oscillating motion of the attachment, the system corresponds to a vibro-impact system. Depending on the type of excitation, this system can be an ideal forced system or a non-ideal forced system. To consider these cases in a methodologically sound manner, a subsequent analysis regards separately the following systems:

- Vibro-impact system with ideal excitation;
- Vibro-impact system with non-ideal excitation.

The system parameters used to in these considerations are given in table 1.

### Table 1. System parameters

| \( m_C \) | 0.35 | [kg] | \( k \) | 0.5 | [N/m] |
| \( m_B \) | 0.2 | [kg] | \( T_0 \) | 2-100 | [Nm] |
| \( c \) | 9\pi^2 | [N/m] | \( \kappa \) | 0.5 - 1.9 | nondimensional control parameter |
| \( b \) | 0.5 | [N/s/m] | \( l_0 \) | 0.36 | [m] |
| \( a \) | 0.04 | [m] | \( \omega \) | \( \sqrt{c/m_C} \) | [rad/s] |
| \( l \) | 0.12 | [m] | \( \Omega_0 \) | \( \omega \) | [rad/s] |
| \( J \) | 0.06 | [kg m²] | | | |

3. Vibro-impact systems with ideal and non-ideal excitation

When impact occurs, the additional nonlinearity is introduced into the system. Numerical analysis of vibro-impact system with ideal and non-ideal excitation is conducted subsequently to point out the basic difference between a system with ideal and non-ideal excitation. Note that although the numerical analysis is presented, a procedure for analytical analysis of the system with ideal excitation also exists and can be found in the literature [8].

Based on the stop position shown in figure 4, a geometrical condition for the impact can be defined by Eqs. (11) and (12):

$$x = x_{\text{stop}} = l + l_0 + \Delta$$  \hspace{1cm} (11)

![Figure 4. Geometry defining the stop position $x_{\text{stop}}$ and the gap $\Delta$](image)

The boundary condition needed for solving Eqs. (8) and (10) can be expressed as

$$x(t^-) = x(t^+) = \Delta$$
$$\dot{x}(t^+) = -k \dot{x}(t^-)$$  \hspace{1cm} (12)

where $k$ appearing in Eq. (12) represents the coefficient of restitution. The energy loss of the system due to impact is determined by the coefficient of restitution. Numerous impact models have been derived by different research with the goal to achieve more accurate mathematical models. As mentioned in [20] during impact two phases are happening: the compression phase occurs when the bodies initially start to contact and compressed against each other and the restitution phase takes place when the bodies start to separate but still in contact. The latter phase ends when the bodies are completely separated. These two phases are used to theoretically derive impact models. In general impact models can be divided in several types: perfectly elastic (no energy loss), partially elastic (energy loss with no permanent deformation), perfectly plastic (all energy loss with permanent deformation), perfectly plastic (all energy loss with permanent deformation). From the described types of impact models, in the system described by this paper the partially elastic or inelastic impact model is used. The reason is the nature of the physical process where it is necessary to describe the real phenomenon where an impact occurs in which energy loss exist but no deformation is happening. An impact model is developed by combining a stiffness and/or damping element. If the elements used to design an impact model are linear or nonlinear, the impact model can be linear or nonlinear.

3.1. Vibro-impact systems with ideal excitation

In order to obtain frequency-response diagrams for vibro-impact systems with ideal excitation, numerical simulations are carried out of Eq. (10) based on Eqs. (11) and (12). Figure 5 a) is obtained for the run up (the corresponding solutions are plotted as the black circles) and figure 5 b) for close down (the corresponding solutions are plotted as the magenta crosses). These two diagrams are joint together in figure 5 c).

It can be seen that multiple solutions occur for the vibro-impact system with ideal excitation. The region of multiple solution is grey coloured and shown in figure 5 c).

Three different regions can be distinguished. The middle region of the diagram corresponds to $x_{\text{max}} = x_{\text{stop}}$ and indicates that the stationary stable solutions are actually impact solutions. This means that in the steady state regime of motion, the impact regime of motion exists. There are two regions on the left and on the right with non-impact solutions. The form of this non-impact motion can be either with no impact during the whole motion of the oscillator or with the impact motion at the beginning of motion,
so that lately in steady state regime it changes to non-impact motion. In the same frequency-response diagrams, two transition regions can be noted. Jumps in the solutions on the frequency response diagrams are marked with letters, A to B and C to D, respectively for the run up and close down simulation. The jumps are purely quantitative because only change in the value of oscillation amplitude is happening. The jump from C to D during the close down simulation is in fact no jump for the \( x_{\text{max}} \) value, where a jump exists for the \( x_{\text{min}} \) value. Jumps identified during the run up and close down simulation are depicted in red. Point C and D are at the same position in the frequency response diagram because in the diagram no jump is occurring for the \( x_{\text{max}} \) value and for the case when \( \Delta = 2a \).

**Figure 5.** Frequency-response diagrams for a vibro-impact system with ideal excitation when \( \Delta = 2a \)

The (grey) colored region in figure 5 c). is distinguishing a region of multiple solutions. Two stable solution that can happen in this region are obtained numerically and each of them is depending on initial conditions. One of the solutions is non-impact and the other one is an impact solution. How the oscillator
will behave, impact or non-impact motion, depends on the initial condition in the colored (grey) region, because of this basin of attraction diagram is formed.

In the grey area the right initial condition must be chosen so that a vibro-impact machine is in regime of impact motion. For this intention a basin of attraction diagram is shown in figure 6 c). This diagram is formed for the value of the frequency ratio $\Omega_p/\omega = 1.4$. The impact solution can be obtained by choosing the initial conditions in the blue area on the diagram, where the red area is related to non-impact solutions. The example how initial conditions are influencing the behavior of the oscillator is shown in figure 6 a) and b) through time-displacement diagrams. In the diagram a) impact solution is shown for a set of initial conditions of the oscillator, where in the diagram b) non-impact solution is shown for a different set of initial conditions where all other parameters of the system are the same. The black circle in the diagram shows the initial conditions $\dot{x} = 0$ and $x = l_0 + l$ and it is a reference point on the diagram for the analysis of the vibro-impact system with non-ideal excitation. This point is here as an orientation point that facilitates the comparison with the basin of attraction diagrams of the vibro-impact system with non-ideal excitation.

![Diagram](image)

**Figure 6.** a) Vibro-impact motion, $x_{t=0} = 0.4$ and $\dot{x}_{t=0} = 0.0$, b) non-impact motion, $x_{t=0} = 0.4$ and $\dot{x}_{t=0} = 1.2$, c) Basin of attraction diagram for impact and non-impact motion of the case of ideal excitation ($\Delta = 2a$, $\Omega_p = 1.4\omega$)
3.2. Vibro-impact systems with non-ideal excitation

Figure 7 shows numerically obtained frequency-response diagrams for the vibro-impact system with non-ideal excitations for the case $\Delta = 2a$. Figure 7 a) and b) present the run up (black circles) and close down (red crosses) solutions.

![Figure 7](image.png)

**Figure 7.** Frequency-response diagrams for a vibro-impact system with non-ideal excitation when $\Delta = 2a$

As in figure 5, three different regions can again be distinguished in figure 7. The first region covers the case when the motion amplitude increases with the increase of the excitation frequency. In this region, the $x_{\text{max}}$ value is lower than the value of $x_{\text{stop}}$, labelled by the upper horizontal solid green line. So, there are no impact solutions. At the frequency ratio close to 0.70, the value of $x_{\text{max}}$ reaches the value of impact and stays at that value until the next transition. All the solutions existing in this region are impact solutions. The next transition related to the $x_{\text{max}}$ value is different for ideal and non-ideal excitation.
Figure 8. Vibro-impact system with non-ideal excitation $\Delta = 2\alpha$, $\Omega_p = 1.40 \omega$; Basins of attraction:
a) $T_0 = 10 \, Nm$, b) $T_0 = 25 \, Nm$, c) $T_0 = 50 \, Nm$, d) $T_0 = 100 \, Nm$; e) Excitation frequency vs time diagram for different $T_0$. 
In systems with non-ideal excitation this region can be divided in three parts. In the darker (grey) region only impact or non-impact solutions occur, where in light colored (grey) region impact and non-impact solutions can occur. The characteristic of this region is that two numerical solutions exists, where one solution is an impact solution and the other is non-impact. The type of motion in which the oscillator will operate depends on the initial conditions. This is a qualitatively different phenomenon than in the earlier analysed system in this paper and it is not pointed in any literature or paper that can be found.

The region with impact solutions only (figure 7 c) dark (grey) region from point C to point D) is important for the practical use in vibro-impact machinery because only impact solution exist in this region. The light (grey) colored region, in which both impact and non-impact solution can occur is also interesting for vibro-impact machinery, but the initial condition must be chosen so that it operates in the impact regime. This region is from points A to C. Excitation frequencies in the region from point A to B, if chosen, are putting a vibro-impact machine in a vibrating regime of motion because only non-impact solution exists therein.

For the case of a system with non-ideal excitation, jumps are different in a way that they are not vertical, but appear as oblique, as noted in figure 7 by the (red) arrows.

The characteristics of non-impact systems with non-ideal excitation is the gap or a non-existing numerical solution in a certain range of the $\Omega/\omega$ ratio. The nonlinearity of this system is two-fold: it exists due to the impact and due to the non-ideal excitation of the system. The fact that there are ‘missing’ solutions in the frequency-response diagram, or that they cannot be found using numerical simulations as they are non-existing, leads to the conclusion that it is necessary to analyse vibro-impact systems with non-ideal excitation separately. The corresponding run up solutions are shown in figure 7. It is seen that these solutions before and after the gap are moved to the left with respect to those presented in figure 5. It is also noticeable that the gap is smaller for the vibro-impact system. When the solutions are joined together, the gap disappears, which is different from the case with non-impact systems as given in [1-6].

As noticed in the middle (grey) region where two solutions exist it is necessary to adjust the right initial condition so that the mechanical system operates in a wanted regime of motion. Such. basin of attraction diagrams are formed for different values of $T_0$. These diagrams are shown in figure 8. For small values of $T_0$, which represent the case of non-ideal excitation, the red area of initial condition with non-impact solution is small.

As the value of $T_0$ increases, the system behaves more as a system with ideal excitation where the diagram d) figure 8 is identical to the diagram c) in figure 6. Given the fact that the blue area dominates for systems with non-ideal excitation, it implies that it is much easier to choose the appropriate initial conditions yielding that the machine operates as a vibro-impact system. Figure 8 e) shows how the angular velocity, average excitation frequency $\dot{\phi} = \Omega$, change with time for different values of $T_0$. The curve for the values of $T_0 = 100 \, Nm$ goes almost immediately to a constant value of $\Omega$, which then basically represents a system with ideal excitation. By decreasing the value of $T_0$, the average value of the excitation frequency changes slower in time and thus represents the case of a system with non-ideal excitation.

The blue bars are representing values of the excitation frequency difference for run up simulation, where the red bars represent excitation frequency differences for close down simulation. From the figures it is shown that by increasing the stall torque differences between the frequencies are decreasing and the vibro-impact system with non-ideal excitation behaves more identical to a vibro-impact system with ideal excitation. For a wide span of the frequency ratio the values of the difference are overlapping which is expected, where in the region of multiple solutions they are different. Some small region of the run up simulation (blue bars) has a gap, this region represents the position where the jump is happening in the frequency response diagram. Additionally in the figure 9 diagrams the gap where no solutions exist (stable solutions) is getting smaller by increasing the stall torque. Comparing the frequency response diagrams given for the vibro-impact system with ideal and non-ideal excitation this is expected.
Figure 9. Difference between the average excitation frequency of the vibro-impact system with non-ideal excitation and the excitation frequency of a vibro-impact system with ideal excitation in [%], $\Delta = 2\alpha$; a) $T_0 = 2\, Nm$, b) $T_0 = 5\, Nm$, c) $T_0 = 10\, Nm$, d) $T_0 = 25\, Nm$

4. Conclusions
A mechanical model of a vibro-impact system has been considered, consisting of a crank-slider mechanism with a mass-spring oscillator system attached to it. This oscillator can hit a base. Two cases of this model have been analysed: a vibro-impact system with ideal excitation and a vibro-impact system with non-ideal excitation.

The results have been presented in the form of frequency-response diagrams. The impact model has been assumed as inelastic, causing the change of velocity in sign and intensity by the restitution factor. All frequency-response diagrams found show the maximum and minimum values of the displacement in terms of the excitation frequency for systems with ideal excitation or in terms of the average value of the excitation frequency for the systems with non-ideal excitation. It has been concluded that the excitation frequency has a qualitative influence on the type of resulting motion: even small variations of the excitation frequency in certain regions can have a great effect on it. Frequency-response diagrams for vibro-impact systems with ideal and non-ideal excitation are characterised by the existence of impact motion regions (impact solutions) and jumps from it or to it. However, they are qualitatively different. A part of the region where only impact solution exist is found. In the system with ideal excitation, these jumps are vertical, which means that only the amplitude changes discontinuously, while the frequency of the steady-state response stays the same. In the system with non-ideal excitation, these jumps are oblique, implying that both the amplitude and the frequency of the response change. Different regions between the vibro-impact system with ideal and non-ideal excitation in the frequency response diagram are noticed. For regions with two coexisting solutions, the corresponding basins of attractor diagrams are formed. The influence and the importance of the right selection of the initial conditions is pointed out.
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