Radiative Splitting of Three Degenerate Neutrinos

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Abstract

We propose a radiative origin of the two mass splittings of three degenerate Majorana neutrinos. It can be achieved by extending the standard model to have the usual effective dimension 5 operators generating SO(3)-invariant tree-level masses, and a charged scalar singlet coupling with the leptons preserving the U(1) subgroup of SO(3). The mass splittings for the atmospheric and solar neutrino oscillations then arise from one-loop corrections due to the charged scalar singlet coupling and the usual tau Yukawa coupling, respectively.

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Current data from atmospheric and solar neutrino observations and terrestrial neutrino experiments provide meaningful constraints on neutrino masses and mixing. When one takes also into account cosmological indications for the existence of hot dark matter, neutrinos are required to be degenerate in mass. To accommodate all of these neutrino data, at least four neutrinos are required. If we, however, leave out the not yet confirmed LSND results, the atmospheric and solar neutrino anomalies can be explained through neutrino oscillations among three active species, \( \nu_e, \nu_\mu \) and \( \nu_\tau \). The atmospheric neutrino oscillation indicates the maximal mixing between \( \nu_\mu \) and \( \nu_\tau \) with a mass squared difference \( \Delta m^2_{\text{atm}} \approx 10^{1.5} \text{ eV}^2 \). The solar neutrino anomaly can be explained through matter enhanced neutrino oscillation if \( 3 \times 10^{-6} \leq \Delta m^2_{\text{sol}} \leq 10^{-5} \text{ eV}^2 \) and \( 2 \times 10^{-3} \leq \sin^2 2\theta_{\text{sol}} \leq 2 \times 10^{-2} \) (small angle MSW), or \( 10^{-5} \leq \Delta m^2_{\text{sol}} \leq 10^{-4} \text{ eV}^2, \sin^2 2\theta_{\text{sol}} \geq 0.5 \) (large angle MSW), \( \Delta m^2_{\text{sol}} \sim 10^{-7} \text{ eV}^2, \sin^2 2\theta_{\text{sol}} \sim 1.0 \) (LOW solution) and through long-distance vacuum oscillation if \( 5 \times 10^{-11} \leq \Delta m^2_{\text{sol}} \leq 10^{-9} \text{ eV}^2, \sin^2 2\theta_{\text{sol}} \geq 0.6 \). Furthermore, combination of the cosmological requirement and non-observation of neutrinoless double-beta decay singles out a specific pattern of three Majorana neutrino mass matrix with almost degenerate mass eigenvalues and bimaximal mixing for the atmospheric and solar neutrino oscillations. In the leading term, this mass matrix in the charged-lepton flavor basis is given by

\[
M_0^\nu \sim m_0 \begin{pmatrix}
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\
\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2}
\end{pmatrix},
\]

where \( m_0 \sim 2 \text{ eV} \) is needed for neutrino hot dark matter. This brings us to a theoretical challenge to answer the questions: what is the origin of such a mass pattern?, and how can one obtain naturally the desired tiny mass differences?

In this letter, we will suggest a simple model in which both atmospheric and solar neutrino mass splittings are generated from radiative corrections while keeping almost bimaximal mixing among three active neutrinos. The degeneracy of the three neutrinos would be a consequence of non-Abelian family symmetry, like SO(3) with three lepton doublets (and
right-handed neutrinos) transforming as a triplet. Then, there must be some sector to break the family symmetry to produce the tiny mass splittings at the level of $\Delta m^2_{\text{atm}}/m_0^2$ and $\Delta m^2_{\text{sol}}/m_0^2$ for the atmospheric and solar neutrino oscillations, respectively. There exist in the literature \cite{11} several models to explain both splittings by some textures at tree level or the solar mass splitting by loop corrections. Our proposal here is to generate both splittings radiatively at one loop level, and thus we do not require any undesirable fine-tuning of parameters. In particular, the finer splitting, $\Delta m^2_{\text{sol}}/m_0^2$, for the solar neutrinos arises from the inevitable one-loop correction due to the small tau Yukawa coupling \cite{12}. For the generation of the larger splitting $\Delta m^2_{\text{atm}}/m_0^2$, we introduce a charged Higgs singlet which couples to the leptonic sector in the same way appeared in Zee model \cite{13}.

Let us first discuss neutrino mass matrix at tree level from which the degenerate neutrino spectra can be obtained. Such a mass matrix can be constructed from symmetry principle. In this work, we impose $SO(3)$ family symmetry for the Majorana neutrino sector. Let $L_i = (\nu_i, l_i)$ be the lepton doublet, where the subscript $i$ refers to the $(+, -, 0)$ component of an $SO(3)$ triplet. Then, the $SO(3)$ invariant Majorana neutrino mass matrix can come from the effective dimension five operator,

$$
L_{\text{eff}} = \frac{h_{\nu}}{2M} (2L_+L_- + L_0L_0)(\bar{H}H) + h.c.
$$

(2)

where $H$ is the Higgs doublet coupling to the up-type quarks and $M \sim 10^{13}$ GeV is the seesaw scale. Note that the effective operator (2) arises below the scale $M$ through the see-saw mechanism endowed with heavy right-handed neutrinos \cite{14} or a heavy triplet scalar \cite{13}. Our discussions do not depend on either types of heavy fields at the scale $M$. The effective Majorana neutrino mass matrix in the $SO(3)$ flavor basis is

$$
M'_{\nu} = \begin{pmatrix}
0 & m_0 & 0 \\
m_0 & 0 & 0 \\
0 & 0 & m_0
\end{pmatrix}
$$

(3)

with $m_0 = h_{\nu}\langle H^0 \rangle^2/M$. 

3
The SO(3) symmetry has to be broken badly in the charged-lepton Yukawa sector. To obtain the neutrino mass matrix \([\Phi]\) in the charged-lepton flavor basis, we require that the SO(3) symmetry breaking is arranged to yield \([\Phi]\)

\[ L_{Yuk} = h^\tau (s_1 L_\mu + c_1 L_0) \tau c H + \cdots , \]  

(4)

where \(c_1 = \cos \theta_1\), etc. Here we omitted the smaller Yukawa couplings for the first two generations which are irrelevant for our discussions. The required SO(3) symmetry breaking in the charged-lepton sector \([\Phi]\) would be obtained by introducing some SO(3) “flavon” fields \([\Phi]\). The lepton doublet fields in the SO(3) basis is then related to the fields in the charged-lepton flavor basis as follows;

\[ L_+ = L_\mu, \quad L_- = c_1 L_\mu - s_1 L_\tau, \quad L_0 = s_1 L_\mu + c_1 L_\tau. \]  

(5)

This leads to the neutrino mass matrix in the charged-lepton flavor basis,

\[ M^\nu_0 = R_{23}(\theta_1) \cdot \begin{pmatrix} 0 & m_0 & 0 \\ m_0 & 0 & 0 \\ 0 & 0 & m_0 \end{pmatrix} \cdot R^T_{23}(\theta_1). \]  

(6)

Then, we have the required bimaximal mixing matrix,

\[ U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c_1}{\sqrt{2}} & \frac{c_1}{\sqrt{2}} & s_1 \\ \frac{s_1}{\sqrt{2}} & -\frac{s_1}{\sqrt{2}} & c_1 \end{pmatrix} \]  

(7)

for \(c_1 \approx s_1\), and the degenerate mass eigenvalues \((-m_0, m_0, m_0)\).

A degenerate mass pattern at tree level can be modified significantly by radiative corrections. The one-loop corrected neutrino mass matrix due to divergent wave function renormalization takes the form,

\[ M^\nu = M^\nu_0 + \frac{1}{2} (I \cdot M^\nu_0 + M^\nu_0 \cdot I), \]  

(8)

where \(I\) is a matrix of regularized one-loop integrals of neutrino self-energy diagrams. One of the important contribution to the one-loop correction comes from the renormalization
group evolution below the see-saw scale $M$ thanks to the tau Yukawa coupling \[18\]. This gives the nonzero component in the charged-lepton flavor basis,

$$I_{\tau\tau} \approx \frac{h_{\tau}^2}{32\pi^2} \ln \frac{M}{M_Z} = \epsilon_\tau.$$  \hspace{1cm} (9)

Including this effect, we get the one-loop corrected mass matrix in terms of the SO(3) eigenstates,

$$M^\nu = m_0 \begin{pmatrix} 0 & 1 + \frac{1}{2}s_1^2\epsilon_\tau & -\frac{1}{2}c_1s_1\epsilon_\tau \\ 1 + \frac{1}{2}s_1^2\epsilon_\tau & 0 & -\frac{1}{2}c_1s_1\epsilon_\tau \\ -\frac{1}{2}c_1s_1\epsilon_\tau & -\frac{1}{2}c_1s_1\epsilon_\tau & 1 + c_1^2\epsilon_\tau \end{pmatrix}$$ \hspace{1cm} (10)

where $\epsilon_\tau \approx 10^{-5}$. Diagonalizing the mass matrix $M^\nu$, one finds that the mass splittings for the solar and atmospheric neutrino oscillations are of the same order, that is, $\Delta m_{\text{sol}}^2 \approx \Delta m_{\text{atm}}^2 \approx m_0^2\epsilon_\tau$. Therefore, it is impossible to provide the relevant mass splittings for both solar and atmospheric neutrinos within this model.

To get the correct mass splittings, we need more corrections arising from some other flavor violating interactions in the lepton sector. In this work, we will show that it can be achieved by introducing a charged scalar singlet $\phi^+$, which allows for the couplings $f_{ij}L_iL_j\phi^+$ \[13\]. Note that these couplings cannot be SO(3)-invariant due to antisymmetry between lepton doublets, that is, $f_{ij} = -f_{ji}$. The relevant flavor violating interaction term for our purpose is then

$$L_{\text{add}} = fL_iL_-\phi^+$$ \hspace{1cm} (11)

which respects the U(1) subgroup of the SO(3) family symmetry. Conservation of this U(1) is crucial to maintain the degeneracy between the first two eigenvalues at the level of the desired degree, as will become clear in the following discussions. Let us recall that there may exist additional finite one-loop corrections in Eq. (8) arising from the interaction term (11) in the context of two Higgs doublet models \[19\]. With one Higgs doublet, we do not have these finite corrections.
In Fig.1, we present the one-loop diagram for neutrino masses generated from the above Lagrangian \([11]\). The resulting one-loop integrals are

\[
I_{++} = I_{--} \approx \frac{f^2}{32\pi^2} \ln \frac{M_{\phi^+}}{M_Z} = \epsilon_f.
\] (12)

Then, the one-loop corrected neutrino mass matrix becomes

\[
M' = m_0 \begin{pmatrix}
0 & 1 + \epsilon_f + \frac{1}{2} s_1^2 \epsilon_\tau & -\frac{1}{2} c_1 s_1 \epsilon_\tau \\
1 + \epsilon_f + \frac{1}{2} s_1^2 \epsilon_\tau & 0 & -\frac{1}{2} c_1 s_1 \epsilon_\tau \\
-\frac{1}{2} c_1 s_1 \epsilon_\tau & -\frac{1}{2} c_1 s_1 \epsilon_\tau & 1 + c_1^2 \epsilon_\tau
\end{pmatrix}.
\] (13)

In the leading order, the mass eigenvalues are

\[
m_1^2 = m_0^2(1 + \epsilon_f + \frac{1}{2} s_1^2 \epsilon_\tau)^2,
\]

\[
m_2^2 = m_0^2(1 + \epsilon_f + \frac{1}{2} s_1^2 \epsilon_\tau + \frac{s_1^2 c_1^2 \epsilon_\tau^2}{2 \epsilon_f})^2,
\]

\[
m_3^2 = m_0^2(1 + c_1^2 \epsilon_\tau)^2.
\] (14)

The atmospheric and solar neutrino mass-squared differences are then given by

\[
\Delta m_{atm}^2 = \Delta m_{32}^2 \approx 2m_0^2 \epsilon_f
\]

\[
\Delta m_{sol}^2 = \Delta m_{21}^2 \approx \frac{1}{4} m_0^2 \sin^2 2\theta_1 \frac{\epsilon_\tau^2}{\epsilon_f}.
\] (15)

For \(m_0 \sim 2\) eV, we get the right value of mass splitting for the atmospheric neutrinos with \(\epsilon_f \sim 10^{-3}\). Let us remark that if one introduces the terms which break the U(1) subgroup of the SO(3) family symmetry like \(f' L_\pm L_0 \phi^+\), it will produce a too large splitting, \(\Delta m_{21}^2 \sim m_0^2 \epsilon_f\), unless the coupling \(f'\) is suppressed enough. The relation in Eq. \([15]\) reproduces the simple connection between the atmospheric and solar neutrino oscillations \([10]\)

\[
\frac{\Delta m_{atm}^2 \Delta m_{sol}^2}{m_0^4 \sin^2 2\theta_1} \approx \frac{1}{2} \epsilon_\tau^2
\] (16)

without resorting to \emph{ad hoc} tree-level splitting between the first two and third neutrino masses. From the above relation, it turns out that our model picks out the MSW solution
with lower mass-squared difference, $\Delta m^2_{sol} \sim 10^{-7}$ eV$^2$, which is often disregarded in discussions. Contrary to the conclusion in Ref. [16], it is rather hard to get the vacuum oscillation solution to the solar neutrino problem due to the $\epsilon_\tau$ effect with logarithmic enhancement.

There is more freedom in the two Higgs doublet model. In this case, the expression for $\epsilon_\tau$ (9) contains the additional factor $2 \tan^2 \beta$ where $\tan \beta$ is the ratio between the vacuum expectation values of two Higgs fields. Therefore, the relation (16) is modified to

$$\frac{\Delta m^2_{atm} \Delta m^2_{sol}}{m_0^4 \sin^2 2\theta_1} \sim 10^{-10} \tan^4 \beta.$$  

(17)

For $m_0 \sim 2$ eV and $\tan \beta \sim 3$, we get $\Delta m^2_{sol} \sim 2 \times 10^{-5}$ eV$^2$ which is in the right range for the large mixing angle MSW solution. With two Higgs doublets, there could arise large finite one-loop masses through Zee mechanism [13,19] in the presence of the coupling $\mu H_1 \bar{H}_2 \phi^+$. This can be suppressed when the $\mu$ term is absent, or the charged scalar singlet has a mass at the see-saw scale, $M_{\phi^+} \sim M$.

Let us finally consider the change of mixing angles from their tree level values in Eq. (7) due to the radiative corrections. The neutrino mixing matrix $U$ which is obtained from re-diagonalization of the one-loop corrected mass matrix (8) can be parameterized by

$$U = R_{23}(\theta_1 + \delta\theta_1) \cdot R_{13}(\theta_2 + \delta\theta_2) \cdot R_{12}(\theta_3 + \delta\theta_3)$$  

(18)

where $\theta_1 \approx \pi/4$, $\theta_2 = 0$ and $\theta_3 = \pi/4$ coming from Eq. (7). The mass pattern (13) leads to the vanishing corrections $\delta\theta_{2,3}$ as long as the $\mu$ and $e$ Yukawa couplings are neglected. Furthermore, thanks to the hierarchy of $\epsilon_f$ and $\epsilon_\tau$, the angle $\delta\theta_1$ comes out to be as small as $\delta\theta_1 \sim \epsilon_\tau/\epsilon_f \sim \sqrt{\Delta m^2_{sol}/\Delta m^2_{atm}}$, which can be estimated by the see-saw diagonalization of (23)-submatrix of $R_{12}M^\nu R_{12}^T$. Thus, the neutrino mixing matrix is quite stable against the above quantum corrections.

In conclusion, we presented a simple way to understand the tiny mass splittings of three degenerate Majorana neutrinos which are good candidates for hot dark matter of the universe. Our proposal is to extend the standard model by introducing two additional sectors.
One sector consists of the usual effective dimension 5 operators for tree-level neutrino masses arising from the see-saw mechanism. Here we impose the non-Abelian family symmetry, SO(3), to enforce the degeneracy of three neutrino species. The other sector contains a charged scalar singlet coupling with the lepton doublets which breaks SO(3) but preserves its U(1) subgroup. As a consequence, the mass splitting for the atmospheric neutrino oscillation is generated through (the U(1) preserving) one-loop corrections with a coupling of $\mathcal{O}(0.1)$. The tiny mass splitting for the solar neutrino oscillation arises then from the renormalization group effect due to the usual tau Yukawa coupling. In the case of one Higgs doublet, or two Higgs doublets with $\tan \beta \sim 1$, the lower mass-squared differences of the MSW solution can be realized. The large mixing angle MSW solution can be obtained only for two Higgs doublets with $\tan \beta \sim 3$. It also turns out that the vacuum oscillation solution is disfavored in our scheme.
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FIG. 1. Neutrino self-energy diagram coming from the charged singlet interactions.