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ABSTRACT

The conversion relationships, including the scaling of frequencies and sizes, between the scattering of a uniform plane electromagnetic wave by a dielectric cylinder and the scattering of a uniform plane acoustic wave by a non-shear-stress cylinder are studied. Owing to the dependence of electromagnetic scattering on polarization, especially for dielectric materials, a simulation method using two acoustic materials is proposed for cases with complex polarization, such as circular polarization. The scaling relationships are verified by simulation. This approach proves to be an effective supplement to the indirect method of measuring electromagnetic scattering by acoustic scattering. A preliminary study of scattering by three-dimensional objects is also performed for the simplest case of spheres. Although the results are not as good as those for two-dimensional objects, the errors after processing are sufficiently small for these results to be accepted as estimates.

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Acoustic and electromagnetic waves are the types of waves that are most commonly encountered in daily life. Although acoustic waves are longitudinal waves and electromagnetic waves are transverse waves, they share similar wave equations and boundary conditions. A series of methods in acoustics can also be used for electromagnetic waves because appropriate mathematical models are similar in both cases. Based on these similarities, a method for measuring electromagnetic radar cross sections (RCSs) using underwater acoustic technology is proposed here to deal with the difficulties that arise with some measurements.

The RCS is a very important parameter for electromagnetic radar and indicates the ability of a target to reflect electromagnetic waves. RCS measurements are generally divided into two categories: far-field outdoor measurements and microwave anechoic chamber measurements. For large targets, direct measurement in a microwave anechoic chamber is obviously not feasible. The scaling method is also limited by the materials involved (it is only applicable to metals) and by the excessively high frequency after scaling. For outdoor measurements, one problem is to ensure protection of targets from unauthorized observation, such as from satellites. On the other hand, for small targets such as cloud particles, the preparation and maintenance of the objects with certain components and shapes to be studied can be a difficult task. In addition, in such cases, metal scaling theory cannot be used. The use of acoustic simulations and the scaling up of acoustic measurements represent an effective supplement to electromagnetic scattering measurements of various objects.

There have been a number of investigations of this approach. Most of these have focused on metallic media under high-frequency approximation. Zeng deduced an exact relationship between electromagnetic scattering by a perfect electric conductor (PEC) and acoustic scattering by a rigid body in the case of a two-dimensional cylindrical geometry. In a previous paper, our group has also derived a scaling relationship between acoustic
scattering by a rigid sphere and electromagnetic scattering by a PEC sphere.

In this paper, we extend our previous work. The medium is changed from a PEC to a dielectric to provide a more realistic representation of the small particles of interest in practical measurements, such as the coatings on modern military aircraft. The scaling relationships between acoustic and electromagnetic scattering for a dielectric are studied. One of the significant differences between acoustic and electromagnetic waves is that the latter exhibit polarization, the effects of which are simulated by using two different acoustic media. The main focus of the paper is on two-dimensional scattering by using dielectric spheres as the simplest three-dimensional objects.

We take the origin of coordinates as the center of a cylinder of radius \( a_p \), the direction of propagation of an incident acoustic plane wave as \(-x\), the time factor as \( e^{j\omega t}\), and the \( z \) axis as the axis of the cylinder, as shown in Fig. 1.

The pressure of the incident acoustic plane wave can be expressed as a series of Bessel functions \( J_n(x) \) as follows:

\[
p_i = p_0 e^{jkx} = p_0 \sum_{n=-\infty}^{\infty} a_n J_n(k_p r) e^{-j\omega \theta},
\]

(1)

where \( a_n = (-j)^n \), \( k_p \) is the wavenumber of the acoustic wave outside the cylinder, and \( x = r \cos \theta \). The boundary conditions between the liquid outside the cylinder and the cylinder (assumed to be a non-shear-stress solid), which are the same as those between two liquids, can be expressed as

\[
\begin{align*}
p_i + p_s &= p_2, \\
u_{ir} + u_{ir} &= u_{r},
\end{align*}
\]

(2)

where \( p_i \) is the pressure of the scattered acoustic wave, \( p_s \) is the pressure of the acoustic wave inside the cylinder, and \( u_r \) is the particle velocity in the \( r \) direction. The pressure of the scattered acoustic wave is then obtained as

\[
p_s = p_0 \sum_{n=-\infty}^{\infty} b_{p,s} J_n(k_p r) e^{-j\omega \theta},
\]

(3)

with

\[
b_{p,s} = -a_n \frac{J_n(k_p a_p) - \frac{\mu_n}{\mu_p} J_n(k_{p,s} a_p)}{H_n^{(2)}(k_p a_p) - \frac{\mu_n}{\mu_p} J_n^{(2)}(k_{p,s} a_p)},
\]

(4)

where \( H_n^{(2)}(x) \) is the Hankel function of the second kind.

Similarly, we now take the origin as the center of a dielectric cylinder of radius \( a_e \), the direction of propagation of an incident electromagnetic plane wave as \(-x\), and the time factor as \( e^{j\omega t}\). We are now confronted by the difference between electromagnetic and acoustic waves: electromagnetic waves are characterized by their polarization, and different polarizations give different scattering results. Transverse magnetic (TM) polarization means that the magnetic field is perpendicular to the \( z \) axis, so the electric field is parallel to the \( z \) axis, as shown in Fig. 2. The \( E \) field of an incident electromagnetic wave can be expressed as a series of Bessel functions as follows:

\[
E_{TM,z}^{inc} = E_0 e^{jkx} = E_0 \sum_{n=-\infty}^{\infty} a_n J_n(k_{e} r) e^{-j\omega \theta},
\]

(5)

where \( k_{e} \) is the wavenumber of electromagnetic waves outside the cylinder. The boundary conditions between dielectric media can be expressed as

\[
\begin{align*}
E_0^{inc} + E_0^{sca} &= E_0^{TM}, \\
H_0^{inc} + H_0^{sca} &= H_0^{TM}, r = a_e.
\end{align*}
\]

(6)

The \( E \) field of the scattered electromagnetic wave is then obtained as

\[
E_{TM,z}^{sca} = E_0 \sum_{n=-\infty}^{\infty} b_{TM,s} H_n^{(2)}(k_e r) e^{-j\omega \theta},
\]

(7)

with

\[
b_{TM,s} = -a_n \frac{J_n(k_e a_e) - \frac{\mu_n}{\mu_e} J_n(k_{TM,s} a_e)}{H_n^{(2)}(k_e a_e) - \frac{\mu_n}{\mu_e} J_n^{(2)}(k_{TM,s} a_e)},
\]

(8)

Transverse electric (TE) polarization means that the electric field is perpendicular to the \( z \) axis, so the magnetic field is parallel to the \( z \) axis, as shown in Fig. 3. The \( H \) field of the incident wave is then obtained as

\[
H_{TM,z}^{sca} = H_0 \sum_{n=-\infty}^{\infty} b_{TM,t} J_n(k_e r) e^{-j\omega \theta},
\]

(9)

with

\[
b_{TM,t} = -a_n \frac{J_n(k_e a_e) - \frac{\mu_n}{\mu_e} J_n(k_{TM,t} a_e)}{H_n^{(2)}(k_e a_e) - \frac{\mu_n}{\mu_e} J_n^{(2)}(k_{TM,t} a_e)}. 
\]

(10)
electromagnetic wave can again be expressed as a series of Bessel functions as follows:

$$H_{\text{TE},z}^{\text{sca}} = H_0 e^{jkz} = H_0 \sum_{n=-\infty}^{\infty} a_n e^{i(k_n r)} e^{-jn\theta}. \quad (9)$$

With the same boundary conditions as for a TM wave, the $H$ field of the scattered electromagnetic wave is then obtained as

$$H_{\text{TE},z}^{\text{sca}} = H_0 \sum_{n=-\infty}^{\infty} b_{\text{TE},n} H^{(2)}(k_n r) e^{-jn\theta}, \quad (10)$$

with

$$b_{\text{TE},n} = -\frac{a_n}{H_n^{(2)}(k_n a_e)} \frac{i(k_n a_e)}{\eta_2(k_n a_e)} \frac{n_2 H_n^{(2)}(k_n a_e)}{n_1 H_0^{(2)}(k_n a_e) \eta_1(k_n a_e)}, \quad (11)$$

It is obvious that the three types of coefficients $b_{p,n}$, $b_{TM,n}$, and $b_{\text{TE},n}$ are similar in form. A conversion from acoustic scattering to electromagnetic scattering can then be achieved when the following conditions are met:

1. For TM electromagnetic wave scattering, where the strict correspondence is between the electric field and the pressure field,

$$\begin{align*}
\eta_2 &= \frac{\rho_1 k_p}{\rho_1 k_p}, \\
\eta_1 &= \frac{\rho_2 k_p}{\rho_2 k_p}, \\
k_{2a_e} &= k_{2a_p}, \\
k_{1a_e} &= k_{1a_p}.
\end{align*} \quad (12)$$

2. For TE electromagnetic wave scattering, where the strict correspondence is between the magnetic field and the pressure field,

$$\begin{align*}
\eta_2 &= \frac{\rho_1 k_p}{\rho_2 k_p}, \\
\eta_1 &= \frac{\rho_2 k_p}{\rho_2 k_p}, \\
k_{2a_e} &= k_{2a_p}, \\
k_{1a_e} &= k_{1a_p}.
\end{align*} \quad (13)$$

where

$$k = \frac{2\pi f}{v}, \quad \eta = \sqrt{\frac{\mu}{\epsilon}}, \quad \nu_e = \frac{1}{\sqrt{\mu \epsilon}}.$$
TABLE IV. Relative scaling for simulation.

| $ka$ (mm) | Electromagnetic | Acoustic |
|----------|-----------------|----------|
|          | Frequency       |          |
| 5        | 23.873 GHz      | 10       |
| 10       | 23.873 GHz      | 10       |
| 15       | 71.619 GHz      | 10       |
|          | 119.37 kHz      | 100      |
|          | 59.683 kHz      | 40       |
|          | 35.810 kHz      | 100      |

To demonstrate the conversion relationship between acoustic and electromagnetic scattering, acoustic and electromagnetic scattering by an infinite cylinder were simulated by the COMSOL Multiphysics 5.4 software. The parameters of the media used in the simulation are listed in Table III. Once the spatial medium has been specified, the scaling relationship between frequency and size is also determined. Several $ka$ parameters were simulated, as shown in Table IV.

When the incident electromagnetic wave is a combination of TE and TM waves, such as a circularly polarized wave, two different acoustic media can be used to simulate electromagnetic scattering through decomposition of the complex polarization into TE and TM waves.
polarization of electromagnetic waves needs to be considered when applying the acoustic simulation method. The scattering results for circular polarization show a good match with those for an acoustic combination, which proves the feasibility of using two acoustic materials to simulate electromagnetic scattering for waves with complex polarization. The matching between electromagnetic and acoustic results is good for different wave frequencies and object sizes, proving that scattering of electromagnetic waves by a dielectric cylinder can be simulated by scattering of acoustic waves from a non-shear-stress cylinder with an appropriately scaled size.
Following on from the abovementioned solution of the two-dimensional problem, we now turn to three-dimensional objects. As in the two-dimensional case, the scattering behavior of electromagnetic waves depends on their polarization, and the simulation of electromagnetic scattering in the three-dimensional case will again be based on acoustic scattering in two different media. For a preliminary study of scattering by three-dimensional objects, we shall consider the simplest case of a dielectric sphere.

Acoustic and electromagnetic scattering by spheres were simulated in the COMSOL Multiphysics 5.4 and MATLAB software. The parameters of the medium used for the spheres are the same as those for the cylinder, as shown in Table III.

The normalized results for sphere simulation are shown in Figs. 7–10. The electromagnetic and acoustic results do not fully match, especially for backward scattering; although, for forward scattering, there are still good matches in terms of shape. As $ka$ increases, the results for backward scattering improve, and, at larger scales, the error in backward scattering decreases to an acceptable level. From the results at small scales, it can be seen that the two types of acoustic scattering give more and less errors for backward electromagnetic scattering. Thus, a combination of the two types of acoustic scattering can be used to reduce the error. The difference between the RCS obtained from the different acoustic models and the RCS of the dielectric sphere is shown in Fig. 11. Compared with the results for a single medium, the combined results obtained from two acoustic media show fewer errors and greater stability. The average error of the combination mode is less than 1 dB for $ka > 3$. For target sizes less than one wavelength, the simple combination mode...
of two acoustic media is no less accurate, and a combination function is going to be studied in a future work. These results also show that it is feasible, and indeed necessary, to use two acoustic media to deal with three-dimensional objects.

In this paper, the conversion relationship between the scattering of an acoustic wave by a non-shear-stress cylinder and the scattering of an electromagnetic wave by a dielectric cylinder under the far-field condition has been derived. It should be noted that the polarization of the electromagnetic wave will affect the choice of the acoustic cylinder medium, with TM and TE polarizations corresponding to different media. For complex polarizations, such as circular polarization, a joint simulation method using two acoustic materials has been proposed. The scaling transformations for sizes and frequencies between different waves have been discussed. The conversion theory has been validated by simulation. The proposed theory provides a promising approach for the measurement of electromagnetic scattering by dielectric cylinders, which can be transformed into the measurement of acoustic scattering. The study of two-dimensional special cases can help us to find features that can be extended to and provide inspiration for the study of three-dimensional problems. A preliminary investigation of scattering by using spheres as the simplest three-dimensional objects has also been performed. Although the corresponding results are not as good as those in the two-dimensional case, the errors are sufficiently low after processing, and these results can be taken as estimates. More general three-dimensional objects will be considered in future investigations. We hope that the simulation of electromagnetic scattering by acoustic scattering will provide an effective supplementary method for electromagnetic scattering measurements of a variety of objects.

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