Arrest of three-dimensional gravity-confined shear flow of wet granular matter

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We study the arrest of three-dimensional flow in wet granular matter subject to a sinusoidal external force and a gravitational field confining the flow in the vertical direction. The minimal strength of the external force that is required to keep the system in motion is determined by considering the balance of injected and dissipated power. This provides a prediction whose excellent quality is demonstrated by a data collapse for an extensive set of event-driven molecular dynamics simulations where we varied the system size, particle number, the energy dissipated upon rupturing capillary bridges, and the bridge length where rupture occurs. The three parameters of the theoretical prediction all lie within narrow margins of theoretical estimates.

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I. INTRODUCTION

Sudden arrest of granular flows is an eminent problem in the engineering sciences as well as a challenge to the theoretical description of granular flows in a hydrodynamic setting. From the latter perspective it involves two challenges: (a) Appropriately incorporating the role of dissipation arising from the particle interactions into the framework of the balance equations underlying hydrodynamic transport equations. And, (b) Addressing the role of shear stresses, of the spatial distribution of stress, and of yield stress in systems where the flow is spatially anisotropic.

For granular systems with purely repulsive interactions recent studies put severe constraints on hydrodynamic descriptions of dense flows by pointing out a lack of scale separation of microscopic and relevant hydrodynamic time and length scales. Among others this gives rise to a severe dependence of the effective material properties on the preparation history. In contrast, hydrodynamic and continuum-mechanics considerations appear to provide a good description for granular systems where the hard-core collisions with restitution are augmented by (reversible) short-range attraction between particles. Arguably this is due to the separation of connectivity and rigidity percolation in response to attractive interactions. This idealization of the particle interactions applies as long as high-impact particle collisions with high capillary numbers dominate the dynamics. On the other hand, recent experimental and numerical work on slowly moving shear flow in dense granular systems clearly underline the important impact of dissipation due to the hysteretic formation and breaking of capillary bridges. Rather than accounting for the finite restitution in collisions and assuming reversible attractive forces, the present work therefore takes a complementary point of view: We explore slow flows in wet systems where dissipation is arising solely from the hysteretic nature of the capillary interaction between the wetting liquid and the particles, i.e., it is due to the formation and rupturing of capillary bridges between particles. The hard-core collisions are elastic.

Shear forces that drive the flow can be modeled in various forms. Experimentally studying shear forces in granular systems can be done, for instance, by constructing two counter-rotating cylindrical walls (see and references therein), by exploring a flow down an inclined plane. Numerical models also addressed the shear flow induced by applying a cosine force field. Similarly to the method of images, this may be used to implement zero flow velocity at the positions envisioned for the walls.

Here, we will focus on the arrest of flow when the force driving the flow falls below a threshold value \( F_{\text{cr}} \). In it was suggested that this critical force results from the power balance between the energy-injection rate resulting from particles motion in the external force field, and the dissipation rate accounting for the rupture of capillary bridges between the particles. We will show that this approach might universally model the arrest in vastly different settings. To this end we describe the arrest of flow of wet, hard spheres in three dimensions where the motion in the third dimension is constrained by a hard wall at the bottom and a gravitational field in the vertical direction. For external driving forces close to \( F_{\text{cr}} \) all particles accumulate at the bottom of the cell such that the packing density lies only slightly above random close packing. In the following ways this setting is fundamentally different from two-dimensional flows at fixed density that were considered in Ref. :  

- In the previously studied setting the particle density is prescribed by hard walls. Consequently, flow requires long-range rearrangements when the density is fixed close to random close packing. In that case the passing of particles in the flow requires a cooperative large scale rearrangement of large portions of the systems which is accompanied by the breaking of a large number of capillary bridges.
- In contrast, the present study focuses on gravity-
confined granular beds of particles, where particles can pass each other via a slight, local expansion of the bed in the vertical direction. Only few capillary bridges must be ruptured in the resulting fixed-pressure setting, even though the density of the bed is always very close to random close packing.

In spite of these considerable differences an informed implementation of the theoretical considerations developed in [30] provides an excellent description of arrest in both settings.

We stress that the arrest of flow which is addressed here is conceptually different from the jamming transition [15, 31–33]. Jamming is prevented here because the system can expand in the vertical direction where it is only bounded by gravity (see, for example [34] and references therein). Furthermore, the transition is also distinct from those observed in earlier studies on cohesive granular materials, because we focus on the role of dissipation due to the hysteretic nature of capillary bridge ruptures, while previous studies [15–17] addressing the transition between fluidized and arrested states dealt with conservative attractive forces. They implemented dissipation via grain friction and a restitution coefficient smaller than one [16, 17], or indirectly by treating the suspending fluid of attractive colloidal particles as an inert background [19].

Our paper is organized as follows. In Sec. II we describe the system, our numerical approach, and the dimensionless units adopted for modeling. Subsequently, in Sec. III we work out the power injected into the systems by the external field, and the dissipated power due to the breaking of capillary bridges. The power balance provides a prediction for the critical force $F_{\text{ex}}$. When larger forces are applied, energy injection dominates and there is sustained flow [20]. Otherwise, the systems relaxes into an arrested state where the capillary bridges form a static network. In Sec. IV the predicted parameter dependence of $F_{\text{ex}}$ is compared to the numerical findings. Based on only three scalar constants we can fully describe the dependence of the stability border in the four-dimensional parameter space spanned by the system size $L$, the particle number $N$, the energy $E_{\text{cb}}$ dissipated upon rupturing capillary bridges, and the critical bridge length $s_{\text{cb}}$ where rupture occurs. (Here and in the following the subscript cb refers to capillary bridge.) The values of the three constants are interpreted and derived from the model in Sec. V. Finally, in the concluding Sec. VI we summarize and interpret our main results.

II. THE MODEL

The system is confined in a rectangular cuboid of size $L \times 2L \times H$ with periodic boundary conditions in $x$ and $y$ direction, and solid, reflecting walls in the $z$ direction. This box contains $N$ particles whose motion is confined in the vertical direction by a gravitational field of uniform acceleration, $\vec{g} = -g \hat{z}$, where $\hat{z}$ is the unit vector along the $z$-axis (see Fig. 1). For the shear flows studied in the present work particles never touch the upper wall of the container due to the gravitational confinement.

A. Particle interactions

In the present study we consider monodisperse spheres of diameter $d$, in order to suppress any additional dynamics arising from different particle sizes. After all, polydisperse beads, which are subject to small shearing forces,
segregate according to their size \[28\].

The dynamics in the simulation is calculated using a standard event-driven molecular-dynamics method which has been described in detail in Refs. \([29, 37, 39]\). For the sake of a self-contained exposition we only briefly summarize the particle interactions. Following \[25\] the effect of particle adhesion due to the capillary bridges is modeled as follows

i. Capillary interaction gives rise to pair forces between particles only.

ii. When particles are not connected by a capillary bridge they feel no force when they approach each other.

iii. They collide elastically, and upon collision a capillary bridge is formed instantaneously. When the particle separate, this bridge gives rise to an attractive force which is modeled as a potential with a depth \(E_{\text{cb}}\), and a finite range \(s_{\text{cb}}\).

iv. The liquid bridge ruptures at the critical rupture separation, \(s_{\text{cb}}\), is modeled by removing the potential well. When the particles approach the next time they feel no force again. In this manner the capillary bridge energy, \(E_{\text{cb}}\), is dissipated whenever a capillary bridge is removed (i.e., ruptures) after a collision.

Work focusing on individual collisions \([22, 30, 43]\) reports a multitude of features for particle collisions involving capillary interactions that can not fully be captured by this model. On the other hand, the positions of phase boundaries of wet granular fluids appear to be universal in the sense that they only depend on \(E_{\text{cb}}\) and \(s_{\text{cb}}\), and not on other details of the particle interaction \([38]\). For computational convenience all simulations shown in the present paper therefore adopt a hysteretic square-well potential, i.e., we use an event-driven algorithm where the potential takes the form of a square-well with hysteresis as outlined in i.–iv.

B. Dimensionless units

Masses are measured in units of the particle mass, \(m\), distances in units of the particle diameter, \(d\), and the time unit is fixed by measuring forces in terms of \(mg\). Non-dimensionalised quantities are denoted by a hat. Unless stated otherwise the system size is \(L \times 2L \times H = 60 \times 120 \times 7.5\), and the number of monodisperse particles is \(4.39 \times 10^4\), resulting in a filling height of \(\hat{h} = 5.0\). Further, the capillary interaction amounts to \(\hat{E}_{\text{cb}} = 3/8\) and \(\hat{s}_{\text{cb}} = 1/16\).

\[\begin{align*}
\text{FIG. 2. Time evolution of the kinetic energy, } E_y, \text{ of motion parallel to } \vec{F}_s \text{ (upper set of lines), and the one in transverse direction, } E_x \text{ (lower set of lines) for a system of size } L = 50 \\
\text{driven by shear forces with amplitude } F_{\text{max}} \approx 0.235 \text{ and } 0.245, \text{ respectively.}
\end{align*}\]

C. Shear flow and arrest

A shear flow is induced by applying a space-dependent external force field

\[
\vec{F}_s(x) = F_s(x) \hat{y}
\]

with \(F_s(x) = F_{\text{max}} \cos \frac{2\pi x}{L}\) \((1)\)

to the system \([28, 30, 46, 48]\), which accelerates particles along the \(\hat{y}\) direction. The particles are initially homogeneously distributed within the system with a Gaussian velocity distribution of mean granular temperature \(T_g/E_{\text{cb}} = 40.0\).

For external forces with an amplitude, \(F_{\text{max}}\), slightly larger than the critical forcing, \(F_{\text{ex}}\), the system settles down into a stationary fluid flow whose local center-of-mass velocity follows the external field. In Fig. (1b) we show a system with a relatively large filling height, \(\hat{h} \simeq 8.8\), and an external force only 11\% above \(F_{\text{ex}}\). In that situation shear bands form such that the capillary bridges in the region around \(L_x/4\) and \(3L_x/4\) are ruptured, while in the other parts the network of capillary bridges evolves only slowly. For shear forces smaller than \(F_{\text{ex}}\) the system eventually arrests in a solid state with a frozen network of nearest neighbors.

D. Measuring \(F_{\text{ex}}\)

The kinetic energy, \(E_y\), of motion parallel to the driving shear force will serve as the order parameter to distinguish the dynamics \([49]\). In Fig. (2) its time evolution is shown together with the one of the kinetic energy, \(E_x\), of the motion transverse to the external field. When started
in a state with high kinetic energy, the fluid first cools down in a manner closely reminiscent to free cooling [37]. Starting at $t \approx 10$ it has cooled so far that the acceleration due to the external field becomes noticeable over the initial kinetic energy. Depending on whether the external field is stronger or weaker than a sharp critical value, $F_{\text{ex}} \simeq 0.240$, the fluid either gains sufficient energy to remain in the fluid state forever (cf. [25] for the dynamics in that state), or it settles into the arrested state. The phase boundary, $F_{\text{ex}}$, is calculated as the mean value between the neighboring values of shear forces, $F_{\text{max}}$, which approach different states. For the data shown in Fig. 2 it amounts to $F_{\text{ex}} \approx 0.240$.

The phase boundary hence corresponds to the smallest external force that still leads to sustained shear flow. We demonstrate in the following that the flow at this threshold corresponds to motion in a liquid state where the energy injected by the external field is exactly balanced by dissipation due to rupture of capillary bridges when particles move past each other. For all investigated systems the flow remains translationally invariant in $y$-direction; in accordance with the symmetry of the forcing. Moreover, density of the liquid can not be distinguished from the random close packing density, $\phi_{NCP}$, and the height of the layer is spatially uniform. Consequently, the number of particles, $n(x)$, in a thin rectangular cuboid of size $2L \times h \times dx$ aligned parallel to the external field takes a constant value, $N/L$, where $N$ denotes the number of particles in the system and $L$ the system size transverse to the flow. This finding is in line with the expectation that a flow breaking the symmetries of the system or noticeably expanding the bed would give rise to higher dissipation.

### III. FLUIDISATION POINT OF SHEARED WET GRANULAR MATTER

In this section we calculate the power, i.e., the energy injected per unit time into the kinetic energy of the particles due to their motion in the external force field. In a steady state this power is balanced by the energy dissipation rate due to the inelastic particle collisions. For external forces close to $F_{\text{ex}}$ the granular temperature is small. Therefore, effects due to the thermal motion may be neglected, and the energy dissipation rate is dominated by the rupture of capillary bridges when particles pass each other in the flow field.

#### A. Energy injection rate

Let $v_y(x)$ denote the flow profile of the flow established when applying the critical force $F_s(x) = F_{\text{max}} \cos(2\pi x/L) = F_{\text{ex}} \cos(2\pi x/L)$. The input power that is injected by means of this external force acting on the particles is given by

$$\langle P_{\text{forcing}} \rangle = \int_0^L dx \ v_y(x) \ F_s(x) \ n(x)$$

$$= \frac{N F_{\text{ex}}}{L} \int_0^L dx \ v_y(x) \ cos(\frac{2\pi x}{L}) . \quad (2)$$

Up to the factor $N F_{\text{ex}}$ this amounts to the leading order Fourier coefficient of the expansion of the velocity profile $v_y(x)$. Consequently, the injected power takes the from

$$\langle P_{\text{forcing}} \rangle = C_f \ N \ \Delta v_y \ F_{\text{ex}} \quad (3)$$

where $\Delta v_y$ is the amplitude of the velocity field. Admissible values of $C_f$ lie in the narrow range $0.5 < C_f < \pi/4$. After all, the constant $C_f$ takes the value $0.5$ when the velocity profile is faithfully approximated by its first even harmonic, i.e., $v_y(x) \simeq \Delta v_y \cos(2\pi x/L)$, and $C_f = \pi/4$ in the other extreme case of plug flow.

#### B. Energy dissipation rate

For each particle the creeping flow enforces a change of neighbors (in the direction of the flow) with a rate $\dot{\gamma} = d v_y / d x$. Such a displacement goes along with an energy dissipation $\nu E_{cb}$ due to rupturing $\nu$ capillary bridges. The total power dissipated upon rupturing capillary bridges is thus given by

$$\langle P_{\text{bridge}} \rangle = \int_0^L dx \ n(x) \left| \frac{d v_y}{d x} \right| \nu E_{cb} . \quad (4)$$

For every $L$-periodic function $v_y(x)$ with a single maximum this integral yields

$$\langle P_{\text{bridge}} \rangle = \frac{4 N \Delta v_y}{L} \nu E_{cb} , \quad (5)$$

for spatially uniform, $n(x) \equiv N/L$.

For the present system rearrangements are achieved by a slight vertical expansion of the particle bed. Assuming that there is no height preference for the rearrangements, and that there are on average $h/2d$ particles in the column on top of the pair under consideration, a potential energy of

$$U = C_U \frac{h}{2d} m g \Delta h$$

is associated to the expansion. Here, $C_U$ accounts for the number of columns to be lifted, and $\Delta h$ to the expansion in height. Due to the frequent collisions in the dense bed the potential energy $U$ is immediately dissipated into thermal degrees of freedom of the granular fluid, and therefore it is not just a one-off investment. Rather, work has to be done against gravity each time particles move past each other. Multiplying this energy with the frequency of particle passages, $4N \Delta v_y/L$ [as given by Eq. (5)], therefore provides a second contribution to the energy dissipation,

$$\langle P_{\text{grav}} \rangle = \frac{4 N \Delta v_y}{L} U = \frac{4 N \Delta v_y}{L} C_U \frac{\Delta h}{2d} m g h . \quad (6)$$
C. Predicting the critical force $\hat{F}_{\text{ex}}$

We now observe that in a steady state the total dissipation rate due to rupturing bridges and relaxing $U$ into thermal degrees of freedom has to balance the input power. We hence obtain

\[
(\langle P_{\text{forcing}} \rangle = \langle P_{\text{bridge}} \rangle + \langle P_{\text{grav}} \rangle)
\]

\[
\Rightarrow \quad \hat{F}_{\text{ex}} \dot{\hat{L}} = a \hat{E}_{\text{cb}} + b m g h
\]

\[
\Rightarrow \quad \hat{F}_{\text{ex}} \dot{\hat{L}} = a \hat{E}_{\text{cb}} + \dot{h}.
\]  

(7a)

with

\[
a = \frac{4 \nu}{C_f}
\]

\[
b = \frac{2 C_b \Delta \dot{h}}{C_f}.
\]  

(7c)

We hence predict that $\hat{F}_{\text{ex}}$ is inversely proportional to the system size $\hat{L}$, and that $\hat{F}_{\text{ex}} \hat{L}$ is a linear function of $\hat{E}_{\text{cb}}$ and the filling height $\hat{h}$. In the following section we compare these predictions to the results of the molecular-dynamics simulations.

IV. PHASE DIAGRAMS

In Fig. 3(a–d) we explore the dependence of $\hat{F}_{\text{ex}}$ on the systems size, $L$, the filling height, $h$, the dissipated energy, $E_{\text{cb}}$, and the rupture length, $\hat{s}_{\text{cb}}$. Plotting $\hat{F}_{\text{ex}}$ as a function of the respective parameters provides sections through the phase diagram: there is sustained flow for values of $\hat{F}_{\text{ex}}$ larger than $\hat{F}_{\text{ex}0}$, and flow is arrested for smaller external forces.

A. Parameter dependence of $\hat{F}_{\text{ex}}$

(a) The variation of the system size $\hat{L}$ was done whilst keeping the aspect ratio of the container constant at $L_y/L_x = 2$. At the same time the average particle number density was kept constant at $\phi = 0.43$ which means that the number of particles is changing in order to provide a fixed filling height, $\dot{h} \approx 5$. As expected from Eq. (7) the value $\hat{F}_{\text{ex}} \hat{L}$ is constant. We find that $\hat{F}_{\text{ex}} \dot{\hat{L}} \simeq 12$ for fixed $\hat{E}_{\text{cb}} = 0.375$ and $\dot{h} \approx 5$.

(b) The filling height, $h$, was varied by changing the number of particles in the system whilst keeping the geometry of the simulation volume and particle interactions fixed. The filling height is estimated as the filling height in the solid state when assuming random close packing. As predicted by Eq. (7) the dependence of $\hat{F}_{\text{ex}} \hat{L}$ on $h$ for a fixed $\hat{E}_{\text{cb}}$ is linear. In the simulations we find

\[
\hat{F}_{\text{ex}} \dot{\hat{L}} \simeq 6.0 + 1.2 \dot{h}
\]

(8)

for $\hat{E}_{\text{cb}} = 0.375$.

(c) Whilst varying the capillary bridge energy, $E_{\text{cb}}$, the rupture separation, $\hat{s}_{\text{cb}} = 1/16$, filling height, $\dot{h} \approx 5$, and the system size, $\hat{L} = 60$, were kept constant. Fig. 3(c) shows the expected linear dependence of $\hat{F}_{\text{ex}} \hat{L}$ on $\hat{E}_{\text{cb}}$,

\[
\hat{F}_{\text{ex}} \dot{\hat{L}} \simeq 24 \hat{E}_{\text{cb}} + 2.8.
\]  

(9)

(d) The variation of the critical rupture separation, $\hat{s}_{\text{cb}}$, was done for a fixed aspect ratio of width vs. depth of the potential well, i.e. $E_{\text{cb}}$ was varied together with $\hat{s}_{\text{cb}}$ at a fixed ratio of $R = E_{\text{cb}}/s_{\text{cb}} = 6.0$. System size and filling height were fixed to $\hat{L} = 60$ and $\dot{h} = 5$, respectively. The resulting linear dependence

\[
\hat{F}_{\text{ex}} \dot{\hat{L}} \simeq 16 R \hat{s}_{\text{cb}} + 6.0
\]

(10)

is shown in Fig. 3(d).

B. Consistency checks

Merely finding linear dependencies of $\hat{F}_{\text{ex}}$ on $\dot{h}$, $\hat{E}_{\text{cb}}$, and $\hat{s}_{\text{cb}}$ is not sufficient to show that Eq. (7) faithfully describes the arrest of flow. One also has to verify that the linear functions are mutually consistent for all sections through the phase diagram.

(e) Observing that the $y$-intercept in Eq. (8) may be written as $6.0 \simeq 16 \hat{E}_{\text{cb}}$ for $\hat{E}_{\text{cb}} = 0.375$, and that the one in Eq. (9) is $2.8 \simeq 0.56 \dot{h}$ for $\dot{h} = 5$, one realizes, that the linear dependencies, Eqs. (8) and (9), can not be compatible with Eq. (7) at the same time. After all, 0.56 disagrees with 1.2, and 24 disagrees with 16. In Fig. 3(e) this is demonstrated by plotting $\hat{F}_{\text{ex}} \hat{L}$ versus $\hat{E}_{\text{cb}}/\dot{h}$: the data shown in Fig. 3(b) and Fig. 3(c) lie on different straight lines.

(f) On the other hand the data shown in panels (d) and (b) are compatible. Eq. (10) is commensurate with Eq. (8) since the latter has a $y$-intercept as $6.0 \simeq 16 \hat{E}_{\text{cb}} = 16 \dot{h} \hat{s}_{\text{cb}}$ with $R = 6.0$ and $\dot{s}_{\text{cb}} = 1/16$, and since the $y$-intercept of Eq. (10) is $6.0 \simeq 1.2 \dot{h}$ with $\dot{h} = 5.0$.

These findings suggest that $\hat{E}_{\text{cb}}$ and $\dot{h}$ are not independent variables — as put forward by Eq. (8) when assuming that $a$ and $b$ take constant values. In the following we show that a consistent description of the numerical data can be achieved, however, by assuming that $a$ has a weak, linear dependence on $\dot{h}$. It reflects that the number of capillary bridges, $\nu$, ruptured in an exchange event depends on the filling height $\dot{h}$.

V. DATA COLLAPSE

Equation (10) can only be consistent with Eq. (8) if its $y$-intercept comprises a contribution proportional to $R$, and if the slopes are adjusted accordingly, $i.e.,$ by decomposing the $y$-intercept as $6.0 \simeq 1.2 \dot{h} \simeq (0.55 + R/10) \dot{h}$ with $\dot{h} = 5$ and $R = 6.0$. Observing that $R \equiv$
FIG. 3. The parameter dependence of the critical driving forces, $F_{\text{ex}}$, separating regions that lead to either solid or fluidized behavior. The symbols indicate the phase boundary between the two phases, and straight lines are the theoretical predictions described in the main text. For most of the data points the numerical error in the phase boundary is smaller than the size of the symbol. Panels (a)–(d) show the phase boundary between the solid and the fluidized state upon varying different parameters: (a) system size, $\hat{L}$, (b) the filling height $\hat{h}$, (c) dissipated energy, $\hat{E}_{\text{cb}}$, and (d) critical rupture separation, $\hat{s}_{\text{cb}}$. In panel (e) $\hat{F}_{\text{ex}} \hat{L}/\hat{h}$ is plotted as a function of $\hat{E}_{\text{cb}}/\hat{h}$ in order to demonstrate that the data of panels (b) and (c) are not compatible with the prediction, Eq. (7). Panel (f) shows a data collapse of the data of panels (b) and (d), when $\hat{F}_{\text{ex}} \hat{L}/\hat{h}$ is plotted as a function $\hat{s}_{\text{cb}}/\hat{h}$ in order to demonstrate that $\hat{s}_{\text{cb}}$ and $\hat{h}$, rather than $\hat{E}_{\text{cb}}$ and $\hat{h}$ may considered independent parameters to predict $\hat{F}_{\text{ex}} \hat{L}$.

The method for performing the simulations is discussed in the text.

$E_{\text{cb}}/s_{\text{cb}}$ this provides an improved prediction

$$\hat{F}_{\text{ex}} \hat{L} \simeq \left( 1 + \tilde{\chi} \frac{\hat{h}}{\hat{s}_{\text{cb}}} \right) \tilde{a} \hat{E}_{\text{cb}} + \hat{b} \hat{h}, \quad (11a)$$

with $\tilde{a} \simeq 16$, \quad (11b)

$\tilde{b} \simeq 0.55$ \quad (11c)

$\tilde{\chi} \simeq 1/160$ \quad (11d)

which takes into account the increase of the number of bridge ruptures upon increasing $h/s_{\text{cb}}$ due to the possibility to rupture bridges in the vertically displaced column of particles on top of the site where a swapping event occurs. As demonstrated in Fig. 4 this prediction is in excellent agreement with all data. As a final step of data analysis we interpret the values of the fitting parameters $\tilde{a}$, $\tilde{b}$ and $\tilde{\chi}$ entering Eq. (11).

A. Determine $\tilde{a}$

In view of Eq. (7b) $a$ is related to the number, $\nu$, of capillary bridge ruptures in every particle exchange

$$a = \frac{4 \nu}{C_f} \simeq 8 \nu \quad (12)$$

where we used the estimate $C_f \simeq 0.5$, as argued upon introducing this constant in Eq. (3). Moreover, in order to take into account the correction term for the height dependence introduced in Eq. (11) another factor $1 + \hat{h}/160\hat{s}_{\text{cb}}$ must be accounted for. For the standard choice of parameters $\hat{h} = 5$ and $\hat{s}_{\text{cb}} = 1/16$ we hence find

$$\tilde{a} = \frac{4 \nu}{C_f} \left( 1 + \frac{\hat{h}}{160\hat{s}_{\text{cb}}} \right)^{-1} \simeq \frac{16}{3} \nu \quad (13)$$

Finally, in a granular bed that is expanding to allow particles to pass each other the number $\nu$ of rupture events must be larger than 1, but still small. For $\nu = 3$ we hence recover the value $\tilde{a} = 16$ reported in Eq. (11b).
close-packed structure are separated by a height distance ∆h = 0.065 that is larger than the height displacement, ∆h = 0.05, of the column. After all, in such a situation only pre-stretched bonds are likely to rupture. Keeping this in mind, the small value of χ may be understood as a result of incorporating a factor of ∆h and a probability of about 1/4 to encounter a pre-stretched bond in a layer of a column that is vertically displaced. In this interpretation the average number of layers in the column amounts to ĥ/2.

VI. SUMMARY AND OUTLOOK

The present study substantiates the finding that the transition from a fluidized to an arrested state in wet granular matter arises when the dissipation rate due to the rupture of capillary bridges in the shear flow can no longer be balanced by the power injection from the external field. Earlier work [30] showed that this approach provides a comprehensive understanding of the transition in the setting of a two-dimensional flow of bidisperse disks where the density is fixed by a confining box. The present work addressed the flow of a bed of monodisperse spheres that are confined in the vertical direction by a gravitational field. Closely above the transition the flow is reminiscent to a slow plastic flow of the bed in the direction of the applied field. The granular bed expands only minimally in vertical direction. For the small flow velocities gravity still keeps density, filling height, and pressure to values observed in an arrested packing. Also for this setting, which fundamentally differs from systems where the particles are confined by walls, the power balance provides an excellent prediction, Eq. (11), of the critical force where flow ceases. This is demonstrated by a master plot, Fig. 4, showing an excellent data collapse of numerical data obtained by varying four different characteristics of the system: the system size, the filling height, as well as the strength and critical rupture separation of capillary bridges. This data collapse suggests

i. When the granular flow is confined by gravity, typically only 2–3 capillary bridges are ruptured upon swapping neighboring particles moving with slightly different speed in the direction of the external forcing. This is a striking difference to wall-bounded flows [30] where this number diverges when the density approaches close packing.

ii. In a gravity-confined granular bed the effortless passing of the particles is facilitated by a slight ex-
pansion of the granular bed where 2–3 columns of particles are lifted by a small amount to let the particles pass between neighboring potential wells arising from the corrugations formed by the layer below. The associated potential energy is also dissipated.

iii. With a small probability additional capillary bridges are broken due to the slight expansions in vertical direction.

The prediction of the flow-threshold involves only three constants characterizing the processes i.–iii. The values of these constants have been determined to a very good accuracy in Sec. V.

The most remarkable finding of our study is that the critical force \( F_{cr} \) does not depend on the specific form of the flow profile. It can be calculated without specifying the hydrodynamic equations of the flow and determining their solution. We therefore conclude that the approach of balancing the energy input rate (due to the external force inducing the flow) and the dissipation rate (due the rupture of capillary bridges, when particles move past each other) provides a powerful framework to study the arrest of flow in wet granular materials where dissipation is dominated by capillary bridge ruptures. This approach provides a universal framework to predict the threshold for the arrest of flow, and it can be applied without need to determine flow profiles. Forthcoming work will explore this intriguing possibility also for flows in other geometries and due to other forcing.

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[49] In [30] we rather used the amplitude of the velocity profile in the direction of the external field (i.e., the amplitude of the velocity response in reaction to the applied force field, $F_{\text{max}}$, as an order parameter. This has advantages when following the hysteresis loop of the response upon slowly decreasing and subsequently increasing $F_{\text{max}}$. However, in the present study, where we focus on the arrest of flow, the kinetic energy, $E_{\text{y}}$, turned out to be a numerically stable and easier accessible order parameter.