Process Adjustment and Monitoring in the Gear Grinding Process

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Abstract:
When a process demonstrates complex cause-and-effect relationships, process adjustments are often used, such as automatic process control (APC). On the other hand, a statistical process control (SPC) is used to identify the causes of abnormality. For example, we use control charts for process monitoring. However, in a process with process adjustments, careful consideration should be taken toward choosing control characteristics as abnormalities may go undetected through the sole monitoring of output.

In many cases, the complexity of the gear grinding process complicates the identification of abnormal causes. Therefore, we indicate a decreased deviation from a fixed target value by using feedback adjustments.

There are correlations among multivariate quality characteristics in cases where feedback adjustments are not used. However, in the process with feedback adjustments, the relationships are disappeared as a result of the controlled quality characteristics. In such a process, we cannot detect changes in the relationship by monitoring quality characteristics, and therefore, we allow abnormalities to continue in the process.

This research shows whether feedback adjustment is effective for the process and then shows appropriate control characteristics, other than quality characteristics, for the use of $T^2 - Q$ control charts to monitor the relationships among variables using the case study of the gear grinding process.

Keywords
APC, Control characteristic, SPC, $T^2 - Q$ control charts

1. Introduction

Manufacturing processes are often controlled using process adjustments and monitoring. Process adjustments are methods that reduce variation by adjusting manipulated variables. This is called engineering process control (EPC). When EPC is performed automatically, it is called automatic process control (APC). It is an effective approach for processes when the cause of variation cannot be identified or eliminated. On the other hand, statistical process control (SPC) is a method that identifies the cause of an abnormality, and takes action to prevent reoccurrence. A control chart is one of the typical methods used in SPC and is often used for process monitoring. However, the passive and conventional methods of the use of control charts, which monitor quality characteristics, may not be able to detect abnormalities in process using process adjustments. Since quality characteristics are controlled by process adjustments, they may not be suitable as control characteristics in monitoring. Therefore, determining what to monitor is crucial.

Box and Kramer (1992) focused on the integration of APC and SPC and indicated the importance of combining them. Furthermore, Nakajo (1995) and Kawamura et al. (2012) examined control characteristics in monitoring processes with process adjustments. These studies discussed monitoring with single variable control charts in processes where process adjustments, such as single-input and single-output systems, were being performed.

Yang and Shue (2006) compared and examined multivariate control charts in processes with multiple-input and multiple-output control systems. In addition, Siddiqui et al. (2015) focused on the control characteristics when using multivariate control charts in multiple-input and multiple-output processes. They showed that “simultaneous monitoring of quality characteristics and manipulated variables” are effective in that case.

This case study about gear grinding process has two purposes. We indicate the effectiveness of using process adjustments with different parameters depending on machining positions for decreasing process variations. In
addition, current study proposes control characteristics monitored by multivariate control chart when relationships were originally shown among multivariate quality characteristics but they lost by process adjustments.

2. Gear Grinding Process

2.1 Process by Thread-Shaped Grinding Wheel

This study focused on the gear grinding process by a thread-shaped grinding wheel, with three of its teeth used to grind one gear. As shown in Figure 1, it becomes possible to process at a plurality of the machining positions from p1 to p7 in one grinding wheel, and high-efficiency processing can be realized. Moreover, accounting for processing precision, the process of one gear is divided into nine and grinds gradually. When the process of one gear finishes, the next gear is processed at the next position. Gears are then ground one by one at all seven machining positions, and the grinding wheel is maintained. This maintenance is performed each time seven pieces of gears are completed.

![Figure 1: The gear grinding process](image)

Seven gears between the maintenance of the grinding wheel are considered one lot (Figure 2). Maintenance is performed for the grinding wheel every time gears at each position are completed, and grinding wheels are replaced after repeating the maintenance about 100 times. In addition, the grinding wheel wears through processing and maintenance.

![Figure 2: The relationship between the maintenance of the grinding wheels and lots](image)

2.2 Features of the Process

There was an increased tendency, as shown in Figure 3, in the diameter of the tip circle, which is the diameters of gears to measure in the grinding machine at end of the seventh grinding and one of the quality characteristics (Figure 1). This trend is a result of the influence the grinding power decrease as the grinding wheel wears. The diameter of the tip circle’s variation from the targets also caused variation in the post-process. The increasing tendency of this differs depending on the machining position from p1 to p7. For example, at the position of p7, the diameters of the tip circle sharply increased toward the end of the grinding wheel.
Figure 3: Transition of the diameter of the tip circle (data on three grinding wheels: A, B, and C)

In a lot, there are relationships among quality characteristics, which are the diameters of the tip circle. The diameters of the tip circle increase from p1 to p7. For example, the diameters of the tip circle of the gear processed at p2 were bigger than at p1, and the diameters of the tip circle of gear processed at p6 were bigger than at p5. This is a result of the grinding wheel with the bearing on only one side (Figure 1). Therefore, the gears processed at p7 become more susceptible to vibration than them at p1.

Table 1 shows a correlation matrix among the diameters of the tip circle in a lot, and Figure 4 shows a scatterplot matrix among the variables. Before we calculate correlations coefficients, we standardized the diameters of the tip circle’s data by equation (1) to eliminate the bias among rows which are lots.

\[ \bar{y}_{lp} = y_{i,p} - \bar{y}_{l}. \]  

\( y_{i,p} \) is the diameter of an tip circle at lot No. \( i \), and machining position \( p \) and \( \bar{y}_{l} \), is the averages of the diameters of the tip circles at lot No. \( i \), and \( \bar{y}_{lp} \) is standardized data.

From Table 1 and Figure 4, there are strong relationships among the quality characteristics in a lot.

|     | p1   | p2    | p3    | p4    | p5    | p6    | p7    |
|-----|------|-------|-------|-------|-------|-------|-------|
| p1  | 1.0000 | 0.5958 | 0.5274 | 0.1591 | -0.3497 | -0.6557 | -0.8103 |
| p2  | 0.5958 | 1.0000 | 0.3669 | 0.1102 | -0.4317 | -0.6177 | -0.6108 |
| p3  | 0.5274 | 0.3669 | 1.0000 | -0.0408 | -0.3568 | -0.5409 | -0.5818 |
| p4  | 0.1591 | 0.1102 | -0.0408 | 1.0000 | -0.1766 | -0.3143 | -0.3107 |
| p5  | -0.3497 | -0.4317 | -0.3568 | -0.1766 | 1.0000 | 0.2308 | 0.1289 |
| p6  | -0.6557 | -0.6177 | -0.5409 | -0.3143 | 0.2308 | 1.0000 | 0.4796 |
| p7  | -0.8103 | -0.6108 | -0.5818 | -0.3107 | 0.1289 | 0.4796 | 1.0000 |
3. Reducing the Variation by Process Adjustment

3.1 Feedback Adjustments

Many factors affect the accuracy of gears. Therefore, it is difficult to identify the cause of the variation. Process adjustment can be an effective approach in processes like these.

In this section, we aim to suppress the increasing tendency of the diameter of the tip circle and reduce the variation using process adjustments. The diameter of the tip circle tends to rise due to the wear of the grinding wheel. This trend differs depending on the seven machining positions. Accordingly, process adjustments used predict the diameter of the tip circle \((y_{i,p})\) at the position \(p\) of lot No. \(i\) from the diameter of the tip circle at \(p\) of the lot No. \(i - 1\). We used the rotation speed of the grinding wheel as manipulated variables because the diameter of the tip circle related to it as equation (2) though it was not adjusted before process adjustments. The diameter of the tip circle decreases by increasing the rotation speed of the grinding wheel. Manipulated variables \((w_{i,p})\) in the process is the rotation speed of the grinding wheel at \(p\) of lot No. \(i\). The relationship between quality characteristics and manipulated variables is expressed as equation (2).

\[
y_{i,p} = a + b_p w_{i,p}
\]  

(2)

The process adjustment algorithm used is as follows:

\[
w_{i,p} = \frac{1}{b_p} \times (y_0 - y_{i-1,p}) \times \beta_p + w_{i-1,p}
\]  

(3)

where \(b_p\) is the constant of process gain at the machining position \(p\) and is determined through a regression analysis, such as equation (2). \(y_0\) is the target value of the diameter of tip circle, and \(\beta_p\) is the discount factor at machining position \(p\). The discount factor \(\beta_p\) is a parameter to prevent the hunting phenomenon in process adjustments. The discount factors were calculated by a method Montgomery (2001) introduced, which uses an exponentially weighted moving average (EWMA). In this method, quality characteristics \(y_{i,p}\) in the uncontrolled process can be predicted by equation (4).

\[
y_{i,p} = \beta_p y_{i-1,p} + (1 - \beta_p) \hat{y}_{i-1,p}
\]  

(4)

where \(0 < \beta_p \leq 1\) is the weighting factor for the EWMA, \(y_{i-1,p}\) is the real data of the quality characteristics, and \(\hat{y}_{i-1,p}\) are the predicted values of the quality characteristics.

3.2 Simulation Using Real Data

Discount factors \(\beta_p\) were calculated using the data of grinding wheel A in Figure 3. We performed simulation experiments using the process adjustment algorithm given by equation (3) on the data of grinding wheel B and C.
When process adjustments were performed in the simulation, the diameter of the tip circle was expressed by equation (5).

\[
\hat{y}_{i,p} = y_{i,p} + \sum_{n=2}^{i} b_p(w_{n,p} - w_{n-1,p}) \tag{5}
\]

where \( y_{i,p} \) is real data of the diameter of the tip circle and \( \hat{y}_{i,p} \) is value obtained by simulation.

The simulation results of the feedback adjustments to the data of grinding wheel B are shown in Figure 5. Feedback adjustments controlled the tendency to increase. The ratio of the mean square error after adjustment to before adjustment was 1:12. In addition, the average variance in a lot after process adjustment was half of that before the process adjustment. Not only decreased the variation between lots decrease but also the variation in a lot did using different feedback adjustments depending on the machining positions. Similar results were obtained for the data of grinding wheel C.

Consequently, the feedback adjustments with the rotation speed of the grinding wheel as manipulated variables contribute to variation reductions.

![Figure 5: Transition of the diameter of the tip circle using process adjustments (data from grinding wheel B)](image_url)

4. Control Characteristics for Monitoring Relationships Using the Multivariate Control Chart

4.1 Changes in Relationships Among Variables in a Lot by Process Adjustments

There were relationships among quality characteristics as such in Table 1 and Figure 4. The diameter of the tip circle relates to other gears processed at other positions in the lot. Therefore, these relationships change when an abnormality is caused. For this reason, this process should be monitored changes in relationships in a lot.

However, comparing Table 2 and Figure 6, which showed correlations among the diameter of the tip circles after process adjustment, with Table 1 and Figure 4, which were before process adjustment, we found that the correlation became weak through the process adjustment. In the case of using feedback adjustments dependent on the machining position to reduce the process variations, the original relationship among quality characteristics in the lot was lost. This is a result of the variances of quality characteristics controlled by process adjustments. In this case, when monitoring the behavior of quality characteristics in a lot using control charts, there is a possibility that the abnormality, such as the change of the relationship, cannot be detected as the process abnormality does not appear as a change in the quality characteristics.

Conversely, we focused on the manipulated variables used in the process adjustment, which is the rotation speed of the grinding wheel as shown in Table 3 and Figure 7. In comparison with Table 2 and Figure 6, the correlations among variables were strong and similar to Figure 4, which nearly showed correlations among quality characteristics before process adjustment. In the case that used process adjustments, the relationships that originally existed among the quality characteristics appear among the manipulated variables. The tendency of the quality characteristics in a lot before adjustment was lost after adjustment. However, the manipulated variables, that is the rotation speed of the grinding wheel, better grasp the aspects of the relationships among the quality characteristics in a lot before adjustment.

Therefore, we used a multivariate control chart to monitor the manipulated variables. In the case of using process adjustments, the relationships in the variables before adjustment were monitored using the manipulated variables
as the control characteristic (instead of the quality characteristic).

We compared the following three variables as control characteristics to show appropriate control characteristics:

(a) Quality characteristic before adjustment, which is the diameter of the tip circle
(b) Quality characteristic after adjustment, which is the diameter of the tip circle
(c) Manipulated variable in process adjustment, which is the rotation speed of the grinding wheel

We evaluated whether it is possible to monitor the condition of the process represented in the control chart using (a) as the control characteristic using the control chart using (b) or (c) as the control characteristic.

Table 2: Correlation matrix among the diameters of the tip circle in a lot after adjustment

|   | p1    | p2    | p3    | p4    | p5    | p6    | p7    |
|---|-------|-------|-------|-------|-------|-------|-------|
| p1| 1.0000| 0.2735| 0.1142|-0.1001|-0.2850|-0.2942|-0.4041|
| p2| 0.2735| 1.0000|-0.0882|-0.0480|-0.1871|-0.3250|-0.3366|
| p3| 0.1142|-0.0882|1.0000|-0.2125|-0.1310|-0.2303|-0.2989|
| p4| -0.1001|-0.0480|-0.2125|1.0000|-0.1452|-0.2157|-0.1707|
| p5| -0.2850|-0.1871|-0.1310|-0.1452|1.0000|-0.0201|-0.1898|
| p6| -0.2942|-0.3250|-0.2303|-0.2157|-0.0201|1.0000|-0.0714|
| p7| -0.4041|-0.3366|-0.2989|-0.1707|-0.1898|-0.0714|1.0000|

Figure 6: Scatterplot matrix among the diameters of the tip circle in a lot after adjustment

Table 3: Correlation matrix among the rotation speed of the grinding wheel in a lot in adjustment

|   | p1    | p2    | p3    | p4    | p5    | p6    | p7    |
|---|-------|-------|-------|-------|-------|-------|-------|
| p1| 1.0000| 0.4303|-0.0568|0.1087|-0.5560|-0.5927|-0.2722|
| p2| 0.4303| 1.0000|0.5889|0.0847|-0.4284|-0.7752|-0.6616|
| p3| -0.0568|0.5889|1.0000|-0.0479|0.0258|-0.4723|-0.7647|
| p4| 0.1087|0.0847|-0.0479|1.0000|-0.0896|-0.2331|-0.2619|
| p5| -0.5560|-0.4284|0.0258|-0.0896|1.0000|0.4323|-0.1264|
| p6| -0.5927|-0.7752|-0.4723|-0.2331|0.4323|1.0000|0.4768|
| p7| -0.2722|-0.6616|-0.7647|-0.2619|-0.1264|0.4768|1.0000|
Figure 7: Scatterplot matrix among the rotation speed of the grinding wheel in a lot in adjustment

4.2 $T^2 - Q$ Control Charts

We used a $T^2 - Q$ control chart proposed by Jackson (1979) to monitor the variation in a lot. $T^2 - Q$ control charts are one of the multivariate control charts used to monitor a process using two statistics, the $T^2$ statistic and $Q$ statistic, which are calculated from a principal component analysis. The $T^2$ statistic and $Q$ statistic, respectively, are as follows:

$$T^2 = \frac{z_1^2}{\lambda_1} + \cdots + \frac{z_m^2}{\lambda_m}$$  \hspace{1cm} (6)

$$Q = z_{m+1}^2 + \cdots + z_p^2$$  \hspace{1cm} (7)

where $p$ is the total number of principal components, $m$ is the number of principal components used in $T^2$ statistic, $\lambda_k$ is the $k$th largest eigenvalue, and $z_k$ is the $k$th principal component score.

The $T^2$ statistic indicates variations that usually occur in a process, and the $Q$ statistic is the orthogonal direction of the $T^2$ statistic and indicates unusual process variation.

The control lines (the control limits and the center line) of the $T^2 - Q$ control charts are given as follows:

Control limits of $T^2$ charts:

$$UCL_a = \frac{m(n + 1)(n - 1)}{n(n - m)} F_a(m, n - m)$$  \hspace{1cm} (8)

where $F_a(\varphi_1, \varphi_2)$ is the upper 100 $\alpha\%$ percentile point of the $F$ distribution with $(\varphi_1, \varphi_2)$ degrees of freedom, and $n$ is the sample size.

The center line of the $T^2$ charts is calculated as follows:

$$CL = \frac{m(n + 1)(n - 1)}{n(n - m - 2)}$$  \hspace{1cm} (9)

The $Q$ statistic can be approximated to the standard normal distribution by transforming the statistic $c$ as follows:

$$c = \frac{\theta_i (Q_{\theta_1})^{h_0} - 1 - \left\{ \frac{\theta_i h_0 (h_0 - 1)}{\theta_i^2} \right\}}{\sqrt{2\theta_i h_0^2}}$$  \hspace{1cm} (10)

$$\theta_i = \sum_{j=m+1}^{p} \lambda_j^i \hspace{1cm} (i = 1, 2, 3)$$

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\[ h_0 = 1 - \left( \frac{\theta_2^2 \theta_3^2}{\theta_2^2} \right) \]

Therefore, the control limits of \( Q \) charts using the \( c \) statistic are obtained in the same fashion as the \( X \) control charts.

4.3 Comparison of Scree Plots

A scree plot was used to divide the \( T^2 \) and \( Q \) statistics. Therefore, we focused on the change of the scree plot (Figure 8). In the quality characteristic before the adjustment and the manipulated variable used in the process adjustment, the upper principal component demonstrated a large eigenvalue, and the lower principal component showed a small eigenvalue and was separated (Figures 8a and 8c). From these figures, the number of upper principal components are respectively decided to one and to two. However, in the quality characteristic after adjustment, the eigenvalue gradually decreased, and it was difficult to separate the upper principal component from the lower (Figure 8b) because the relationships among quality characteristics after adjustment were too weak. We decide to one in the number of upper principal components in the quality characteristic after the adjustment for comparison with before adjustment.

![Figure 8: Scree plot based on principal components](image)

(a) (b) (c)

The number of the principal components

4.4 Comparison of Control Characteristics

We created control charts and compared them. Figure 9 shows the control charts based on the quality characteristics of the data of grinding wheel C shown in Figure 3. Figure 10 shows the control charts with the quality characteristics as the control characteristics after the process adjustment was used on grinding wheel C data. Figure 11 shows the control charts with the manipulated variable, which is the rotation speed of the grinding wheel, as the control characteristics. The control limit line was determined using the data of grinding wheel B. The \( Q \) control chart shows the \( c \) statistic that approximates the \( Q \) statistic to a standard normal distribution.

We added the shift change of 3\( \sigma \) (standard deviation) to the data of lot No. 80, or later in the machining position of p2 in grinding wheel C, and examined whether abnormality was detected by the control charts. Evidently, as shown in Figure 9, this abnormality of the shift changes was detected by control charts. However, when process adjustments were used, it could not be detected by monitoring the behavior of the quality characteristics, as shown in Figure 10. On the other hand, as shown in Figure 11, it is found that the plot at No. 80 or later in the \( Q \) control chart fluctuated greatly, and an abnormality was detected using the manipulated variable as the control characteristic. Where process adjustments were used, the control charts with quality characteristics as control characteristics could not detect abnormality as a result of the variation suppressed by the process adjustment because the relationship among quality characteristics was lost.

In addition, the plots of Figure 9 and Figure 11 show similar behavior in \( T^2 \) control chart. However, as shown in Figure 10, the fluctuation is different from them.

Therefore, when process adjustments are used and relationships among the variables on machining positions are monitored, we propose the use of the manipulated variables as the control characteristics because the relationships that originally existed among quality characteristics appear in the relationships among the manipulated variables.
Figure 9: $T^2 - Q$ control charts monitoring quality characteristics before adjustment

Figure 10: $T^2 - Q$ control charts monitoring quality characteristics after adjustment
5. Discussion

We proposed manipulated variables as control characteristics for process monitoring in case process adjustments were used for the gear grinding process. Nakajo (1995) and Montgomery (2001) also indicated that manipulated variables are appropriate for control characteristics. Different from the current study, they assumed that process adjustments and monitoring were univariate, and these studies were not multivariate.

Current study focused on monitoring the relationship between multivariate variables. We used $T^2 - Q$ control charts to monitor these relationships. The $Q$ control chart was used to capture the collapse of the correlation, and the $T^2 - Q$ control charts demonstrated the changes in the relationship.

Siddiqui et al. (2015) recommended simultaneous monitoring of the quality characteristics and manipulated variables when integrating multivariate SPC and multivariate EPC. That study stated that a control chart on quality characteristics is more effective than that of manipulated variables when the shift magnitude is higher. However, a control chart on manipulated variables shows better results for small magnitudes. In current study, when there were relationships among quality characteristics, they appear among manipulated variables by process adjustments. Thus, this feature in the process can be grasped through monitoring manipulated variables. Therefore, monitoring manipulated variables is ideal for this process.

Also, Kawamura et al. (2012) commented that manipulated variables pose a risk in not involving information about quality characteristics, i.e., it cannot detect abnormal causes if the control mechanism breaks down and the manipulated variables behave normally in spite of non-suitable adjustments. In other words, in current study, it may be necessary to monitor the behavior of quality characteristics at the same as that Siddiqui et al. (2015) mentioned. However, as proposed, in multivariate monitoring, monitoring manipulated variables can be effective with regard to focusing on the relationship among variables.

Furthermore, the learning data used for the control limit line of the control charts to increase the power should be considered. It is necessary to examine whether it can be used in actual processes. It is desirable to accumulate and universalize cases in various processes other than the gear grinding process.
6. Conclusion

In the current study, we examined the effectiveness of process adjustments in a gear grinding process. Also, we proposed the use of manipulated variables as control characteristics when using $T^2 - Q$ control charts for the cases using process adjustments.

In the variation reduction using process adjustment, we showed that the variation reduced by the process adjustment, which was calculated using EWMA and depended on machining positions, through simulation experiments using the data. In addition, we indicated that not only lot-to-lot variations but also intra-lot variations could be suppressed by process adjustments based on different algorithms depending on the machining positions.

However, when the assumed process adjustments were used, the relationships between quality characteristics in a lot was lost. In this process, it was necessary to monitor the change of the relationship among the variables in a lot, but if the multivariate control chart monitoring the quality characteristics as control characteristics was used, the features in this process could not be grasped nor monitored. Therefore, as control characteristics, we proposed $T^2 - Q$ control charts monitoring manipulated variables, instead of the quality characteristics, as it allows monitoring of the relationship among quality characteristics lost by feedback adjustments.

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