Distillation and Visualization of Spatiotemporal Structures in Turbulent Flow Fields

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Abstract. Although turbulence suggests randomness and disorder, organized motions that cause spatiotemporal ‘coherent structures’ are of particular interest. Revealing such structures in numerically given turbulent or semi-turbulent flows is of interest both for practically working engineers and theoretically oriented physicists. However, as long as there is no common agreement about the mathematical definition of coherent structures, extracting such structures is a vaguely defined task. Instead of searching for a general definition, the data visualization community takes a pragmatic approach and provides various tool chains implemented in flexible software frameworks that allow the user to extract distinct flow field structures. Thus physicists or engineers can select those flow structures which might advance their insight best. We present different approaches to distill important features from turbulent flows and discuss the necessary steps to be taken on the example of Lagrangian coherent structures.

1. Feature-Based Visualization

Fluid flow datasets arising from experiments and simulation increase in size and complexity. During the past two decades, much efforts have been devoted to the development of computer-based visualization techniques for steady and unsteady flow fields, both in 2D and 3D. The resulting tools are widely used today for visual analysis of flow fields. However, even with advanced visualization tools, analyzing and understanding intricate flow structures is often difficult, since usually only raw data are displayed. For understanding of complex flows higher levels of abstraction are needed: the significant underlying structures must be revealed and visually conveyed. This is exactly the objective of feature-based visualization. In fluid dynamics text books and journals there is a long tradition of depicting masterpieces of hand-crafted principle flow sketches – revealing the main flow characteristics in a single picture. Research in feature-based visualization aims at developing computational methods for distilling similarly insightful structures from raw data in a semi-automatic or even automatic manner and depicting them in a perceptually and cognitively effective way.

This requires four steps: First, the structures of interest have to be identified and informally described. In the field of data analysis and visualization, this is done in cycles of (often extensive) discussion with fluid dynamics experts and reflection. Typical structures are those for which fluid mechanics has developed intuitive conceptions, such as stagnation points, vortex core lines, vortex regions, attracting or repelling manifolds. These structures are geometric objects (point, line, surface, volume, space-time volumes) in space-time.
In the second step, a unique mathematical characterization of these geometric objects has to be performed. This requires in-depth discussions with domain experts, too. Often an intuitive conception comprises a class of structures that is characterized by various attributes. On the other hand, it is often sensible to focus on some of these characteristics while neglecting others. Thereby, the feature definition can be adapted to the specific questions of the researchers – which can result in both general or very specific feature definitions.

The third step is the development of efficient and robust algorithms for extracting these mathematically characterized objects from a given, typically discretely sampled and noisy flow field. This is often a non-trivial task that requires a skillful combination of techniques from numerical and geometrical, sometimes also topological, combinatorial and statistical computing. For instance, only recently, after almost 20 years of research, an algorithm has been proposed for robust and accurate computation of integral surfaces in time-dependent vector fields (Garth et al., 2008). Results of structure extraction algorithms should be insensitive to algorithmic parameters. Ideally, extraction algorithms do not depend on any algorithmic parameter.

The last step in the pipeline is the design of effective visual mappings and corresponding algorithmic realizations, in order to convey the extracted features graphically in a perceptually efficient way – with the final goal to create insight or to provoke new questions.

2. Lagrangian Coherent Structures

2.1. Feature Identification

Many kinds of flow structures have been investigated, both experimentally and computationally. In the following, we consider Lagrangian coherent structures (LCS). Under this somewhat vague notion different flow structures are subsumed. The only consensus seems to be that LCS represent organized flow behavior in the chaotic movement of particles in a turbulent flow. Unfortunately, a unique and agreed mathematical characterization is not available. An extensive discussion that exhibited the diversity of opinions regarding the proper definition of coherent structures can be found in Lumley (1990). Some researchers even argue that “in principle, concepts like coherent structures are best left implicit” (Hussain, 1986). Such a point of view, however, is not satisfying, if the concept is considered as scientifically valuable.

Of course, many trials have been made to define LCS more precisely. Examples are the definition by Michalke (Hussain, 1986) or Farge (Farge et al., 1998) both proposing a coherence function. Others gave more conceptual definitions. Yule suggests that coherent structures should follow three requirements: (i) being repetitive, (ii) survive distance larger than structure size, (iii) significantly contribute to the kinetic energy (Yule, 1981). While Hussain considers these
conditions, at least partly, as “unnecessary and unrealistic”, he defines coherent structures as “connected turbulent fluid mass with instantaneous phase-correlated vorticity over its spatial extent” (Hussain, 1983). Farge states, that “the only definition of a coherent structure that seems objective is a locally meta-stable state, such that, in the reference frame associated with the coherent structure, the nonlinearity of Navier-Stokes equations becomes negligible” (Farge et al., 1998).

2.2. Possible Mathematical Descriptions

In the following, we will discuss two possible alternative approaches to extract Lagrangian features. We start with an approach to LCS, suggested by Haller, that has become quite popular in the past ten years. Haller (2001) proposed a concept of distinguished material lines or surfaces in the flow field that can be extracted on base of the finite-time Lyapunov exponent (FTLE), which measures the rate of separation of infinitesimal close particles over a finite time period. He states that the extremal structures of the FTLE field are possible boundaries of LCS, although they are not always perfect indicators (Haller, 2002). Shadden and Marsden (Shadden, 2006; Shadden et al., 2005) even define ridges of the FTLE scalar field as LCS. The FTLE field is now widely used in computational flow visualization, see, e.g., Garth et al. (2007), Sadlo & Peikert (2009), Kasten et al. (2009) and Sadlo et al. (2010).

Another approach to extract Lagrangian features of time-dependent flow fields is based on the acceleration field. It is inspired by the successful concept of vector field topology, but in contrast to this is Galilean invariant. Critical points in steady fields correspond to zeros of the acceleration magnitude (Kasten et al., 2010). These locations also define points of interest for time-dependent fields. Using the Navier-Stokes equations, it can be shown that these feature points are related to minima of the pressure. In the case of turbulent flow containing fast moving structures, one can apply a Lagrangian smoothing by averaging the values along pathlines. Minima of the averaged acceleration are related to vortex structures. In addition, the extent of the vortex can be described by the basins of the minima given by the scalar topology of the acceleration magnitude (Kasten et al., 2011b).

As can be seen from the long history, the search for a mathematical definition can be tedious. Moreover, a given definition does not always cover the whole range of interesting phenomena and intended applications. For example, the LCS definition based on FTLE ridges has some shortcomings, as we discussed recently in Kasten et al. (2011a).

2.3. Algorithmic Realization

The next step in the pipeline is to devise an algorithm for extracting the features. As stated above, many feature definitions are related to ridges, or critical points, which are components of the topological skeleton. In addition, the temporal evolution of such structures is of interest. In terms of scalar field topology, tracking of critical points depicts this evolution.

The scalar quantities representing the feature identifiers typically contain derivatives of the simulated (or measured) flow fields, which amplify noise contained in the data. Due to the ubiquitous noise in the numerical data, reliable extraction of topological skeletons and tracking of critical points are challenging tasks. Utilizing a derivative-free combinatorial approach, we have recently designed robust algorithms for extraction of topological skeletons and tracking of critical points (Reininghaus et al., 2011).

2.4. Visualization

The last step in the pipeline is to provide a meaningful visualizations of the extracted features. This includes a sensible depiction of the extracted structures to provide insight in the data with only a few images. Many more flow visualization techniques have been developed during the past two decades; for reviews on texture-based, topology-based and integration-based geometric
Figure 1. Evolution of the vortex skeleton of a flow over a cavity (yellow). The weakly compressible 2D flow at $Ma=0.38$ has been numerically simulated (data courtesy of M. Samimy). The flow passes from left to right; time is represented by the third dimension (note the three axes of the coordinate system at the left). The vortices (red vortex cores) are identified as minima of the acceleration magnitude. The influence regions of the vortices are depicted by the blue tubes. The spiraling curves represent fluid particle paths in the vortical regions. The volumetric smoke-like regions indicate a range of large acceleration magnitudes. The vortices originate at the leading edge and move through the cavity over the trailing edge.

Visualization techniques see, e.g., Laramee et al. (2004), Laramee et al. (2007) and McLoughlin et al. (2010), respectively. Due to the rising amount of data – in terms of temporal and spatial resolution – the number and resolution of the extracted structures is also increasing. Here, focus and context visualization can help to get an impression of the data, see e.g. Muigg et al. (2008). In our visualizations we are mostly relying on standard computer graphical techniques for depicting surfaces, scalar fields – via iso-surfaces and direct volume rendering. Furthermore, we utilize display techniques for 2D vector fields, like line integral convolution (Cabral & Leedom, 1993; Stalling & Hege, 1995), and 3D vector fields, like illuminated field lines (Zöckler et al., 1996; Stalling et al., 1997).

3. Conclusion

The computer-aided extraction of features hidden in flow field data is a complicated but worthwhile task. As an example in Fig. 1, we have depicted a flow over a cavity and features, such as vortex cores and their regions, that have been extracted using robust methods. Exemplary for the benefits of a cooperation between fluid mechanics and flow visualization, we would also like to mention the analysis of the coherent structures of a jet undergoing a vortex breakdown (Oberleithner et al., 2011; Petz et al., 2011). In summary, the field of computational flow visualization is on a good way to provide valuable analysis tools to the flow community.
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