Turbulence Characteristics of Internal Waves in an Improved 3D Numerical Wave Tank

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Abstract. A three-dimensional unsteady mathematical model was applied to simulate the generation and propagation of the internal solitary waves in an improved numerical wave tank. The turbulence characteristics of the internal waves were investigated by means of the turbulence model, a concept of a fractional volume of fluid (VOF) was employed to track the internal interface of two-layer water. The effect of irregular topography on turbulence was described by the spatial and temporal distributions of turbulence kinetic energy, its dissipation rate. Simulation results show that the highest values of turbulence kinetic energy and its dissipation rate occur at the interface of two-layer water when waves propagate in both flat and irregular terrains. The values are higher in irregular topography cases than those in flat topography cases, which indicates that the turbulence associated with abrupt topographies is more intensive. The paper provides a reasonable approach for understanding the turbulence characteristics of internal waves in density-stratified waters.

1. Introduction

Thermal stratification is one of the most important environmental issues for deep water bodies [1-3]. In summer, the typically stratified lakes usually have layered structure consisting of an epilimnion, thermocline, and hypolimnion. The thermocline can be regarded as a transition layer with a sharp temperature gradient [4], where there is a ubiquitous phenomenon of internal wave motion. Steepened internal waves with amplitudes similar to the thermocline depth have long been noted as a striking feature of long, narrow lakes [5]. The first scientific observations of internal waves were reported by an explorer Fridtjof Nansen. Subsequent researchers found that internal waves could play a vital role in the energy distribution of fluids [6]. A 3D numerical wave tank in a regular topography based on a model of computation fluid dynamics was established to simulate the propagation and evolution processes of internal solitary wave [7]. Boegman et al. [8] discovered that the energy was lost due to the high-frequency internal wave breaking along sloping topographies. Stevens et al. [9] inferred basin-scale internal wave can damp as much as 80% of the available potential energy in the initial internal wave setup in one internal wave period.
Mechanistic theoretical models of describing the internal wave motions are grouped into two fundamental classes: linear stability models and weakly nonlinear models. Internal waves are also observed near the regions with steep underwater topographies in the oceans [10-11]. The KdV (Korteweg-de Vries) theory has been shown to provide a valid description of the behavior of internal solitary waves for small amplitude motions [12-13].

Previous studies on the turbulence characteristics of internal waves were mainly based on the field measurements and laboratory experiments were also useful in researches of the interactions between stratified flows and topographies [14-15]. However, numerical methods had been scarce adopted.

At present, numerical wave tanks are widely used in the study of wave propagation, the interaction of waves and hydraulic structures, and so on [16-21]. This paper, we establish a non-hydrostatic 3D numerical wave tank by a CFD (Computational Fluid Dynamics) model with the KdV theory that can effectively simulate the generation and propagation of non-linear internal solitary wave in two different types of topographies. Then the turbulence kinetic energy, its dissipation rates in the process of wave propagation were described and analyzed in detail, which provides a reliable and reasonable approach for the study of the energy transformation in the density-stratified water.

2. Description of the mathematical model

2.1. The KdV equation

The non-linear KdV equation can describe the internal solitary wave:

\[
\frac{\partial \eta}{\partial t} = c_0 \frac{\partial \eta}{\partial x} + a \eta \frac{\partial \eta}{\partial x} + \beta \frac{\partial^3 \eta}{\partial x^3} = 0
\]  

(1)

where, \( t \) is the time, \( x \) is horizontal coordinate, \( \eta \) is internal solitary wave amplitude; \( c_0 \) is the speed of linear long-wave; \( \alpha, \beta \) are KdV coefficients.

A detailed description of the formula is given in reference [7].

2.2. The CFD model

2.2.1. Basic equations. The governing equations includes: continuity and momentum equations, turbulence transport equations and a turbulence viscosity equation:

\[
\frac{\partial u_i}{\partial x_i} = 0 \quad i = 1, 2, 3
\]  

(2)

\[
\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nabla' \frac{\partial u_i}{\partial x_j} \right) - \frac{\partial P}{\partial x_i}
\]  

(3)

\[
\frac{\partial k}{\partial t} + \frac{\partial (u_i k)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \nabla' \left( \frac{\nu_e}{\sigma_k} \frac{\partial k}{\partial x_i} \right) \right) + \nu_e \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} - \varepsilon
\]  

(4)

\[
\frac{\partial \varepsilon}{\partial t} + \frac{\partial (u_i \varepsilon)}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \nabla' \left( \frac{\nu_e}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_i} \right) \right) + c_k \nu_e \frac{\varepsilon}{k} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} - c_{2\varepsilon} \frac{\varepsilon^2}{k}
\]  

(5)

\[
\nu_e = \nu_t + \nu_m = \frac{k^2}{\varepsilon}
\]  

(6)

where \( i = 1, 2 \) and \( 3 \) respectively; \( x_1, x_2, x_3 \) are wave propagation direction, span-wise direction and vertical direction; \( u_1, u_2, u_3 \) are velocities in \( x_1, x_2, x_3 \) directions; \( t \) is the time; \( \nu_e \) is effective viscosity coefficient defined as \( \nu_e = \nu_t + \nu_m \), in which \( \nu_t \) is the turbulent eddy viscosity coefficient and \( \nu_m \) is the molecular viscosity coefficient; \( \rho \) is the water density; \( g \) is the acceleration of gravity; \( P \) is pressure; \( k \) is the turbulence kinetic energy; \( \varepsilon \) is the turbulence energy dissipation rate.
2.2.2. **Boundary and initial conditions.** The above flow equations are valid in the inner fully developed region of the flow field, not suitable near the wall region. So a wall function was employed as follows:

\[
\begin{align*}
    u^* &= y^+ & y^+ < 11.6 \\
    u^* &= \frac{1}{\kappa} \ln(Ey^+) & y^+ \geq 11.6 \\
    k &= \frac{u_*^2}{C_{\mu}} & y^+ \geq 11.6 \\
    \varepsilon &= \frac{u_*^3}{\kappa y}
\end{align*}
\]  

(7) (8)

where, \( \kappa \) is the von Karman constant; \( E \) is a coefficient accounting for the roughness of the solid wall; \( u_* \) is the friction velocity; \( u^* \) is normalized velocity by \( u_* \) near the wall; \( y^+ \) is normalized distance from the wall.

In order to generate an internal solitary wave at the wave inlet boundary, the solitary wave solutions of \( \eta(0, t) \) and \( u_{1,2}(0, t) \) based on the KdV equation were applied at the interface of the two-layer water.

Initial \( k \) and \( \varepsilon \) were set to be zero for the stationary ambient water. A horizontal interface of two-layer water was assumed and a hydrostatic pressure distribution was accordingly imposed.

3. **Setup of numerical wave tanks**

As is shown in Figure 1, the dimension of the numerical wave tank is 20m×1m×0.6m (length×width×height) in \( x \), \( y \) and \( z \) direction respectively. The initial upper and lower water layers are \( H_1 = 0.2 \) m with density \( \rho_1 = 994.06 \) kg/m\(^3\) and \( H_2 = 0.4 \) m with density \( \rho_2 = 999.73 \) kg/m\(^3\), respectively. The designed (incident) wave amplitude \( \eta_i \) is 0.05 m.

In nature, the river beds are always rugged, and the generation and propagation of the internal solitary wave with complex topographies are not clearly described in a flat-bed wave tank. In order to create a realistic looking terrain, the tank needs to be improved. In Figure 2 and Figure 3, three bulges at the bottom were established to simulate the irregular terrain.

![Figure 1. Schematic diagram of the numerical wave tank.](image1)

![Figure 2. Improved numerical wave tank in rough terrain.](image2)
Boundary conditions need to be specified on all surfaces of the numerical wave tank. The ABEG face was set to be velocity inlet, ADEF was set to be symmetry, ABCD, EFGH, BCGH and CDFH were set to be solid walls.

4. Results and Discussion

4.1. Validation of the model
The effects of the numerical wave tank had been verified, a three-dimensional numerical wave surface when $t=60s$ in the flat topography was shown in Figure 4, from which we could see that an internal solitary wave of depression had been generated at the moment.

Figure 5 showed the comparison of the numerical result and the theoretical solution of the KdV equation at the initial stage of the simulation. The numerical internal wave height was about 0.05 m, close to the designed wave amplitude, indicating a good agreement between the numerical and theoretical waveforms. Therefore, the proposed mathematical model was capable of simulating the propagation and evolution of internal solitary waves with high precision.
Figure 5. Comparison of numerical and theoretical waveform: on the vertical axis the variable is wave amplitude (i.e., the shape of the interface).

4.2. Turbulence characteristics

Turbulence kinetic energy $k$ and its dissipation rate $\varepsilon$ are the important indexes representing the turbulence intensity in water. In the irregular topography, the shape of the interface at $t=80s$ was shown in Figure 6, in addition, from Figure 7 and Figure 8, we could see that the contours of $k$ and $\varepsilon$ in two different types of topographies at the same time. In the flat topography, the maximum value of $k=5.16 \times 10^{-5} \text{m}^2/\text{s}^2$ appeared at the interface of two-layer water, indicating that there was a violent turbulence kinetic energy at the interface, and the minimum value of $\varepsilon=2 \times 10^{-6} \text{m}^2/\text{s}^2$ appeared at the top and bottom of the tank far away from the interface. In the irregular topography, the contour of $k$ was similar to that in the flat topography, however the maximum value of $k=2.5 \times 10^{-5} \text{m}^2/\text{s}^2$ was higher, which meant that when the internal wave propagated across the irregular topography, the turbulence energy would increase.

The contours of $\varepsilon$ were mostly consistent with the distributions of $k$ at $t=80s$, namely a violent dissipation of turbulence energy also appeared at the interface of the two-layer water, where the energy loss was higher, and over the rest of the interface the values were gradually weaker. In the flat topography, the maximum value of $\varepsilon$ was $5.76 \times 10^{-7} \text{m}^2/\text{s}^3$. Meanwhile, compared with that in the flat terrain, when the internal solitary wave propagated in the irregular topography, $\varepsilon$ was also larger, and the maximum value was $1.4 \times 10^{-6} \text{m}^2/\text{s}^3$.

Figure 6. Shape of the interface at $t=80 \text{s}$. 
Figure 7. Contours of $k$ at $t = 80$ s $(m^2/s^2)$.

(a) Flat topography

(b) Irregular topography

Figure 8. Contours of $\varepsilon$ at $t = 80$ s $(m^2/s^3)$.

(a) Flat topography

(b) Irregular topography

Time series of the maximum values of $k$, $\varepsilon$ in two different topographies were shown in Figure 9 and Figure 10, it was evident that the magnitude of $k$ and $\varepsilon$ gradually decreased as the simulation
time. For example, in the flat topography, the values of $k$ declined from $3.18 \times 10^{-5} \text{ m}^2/\text{s}^2 \ (t = 20 \text{ s})$ to $1.03 \times 10^{-5} \text{ m}^2/\text{s}^2 \ (t = 150 \text{ s})$; the values of $\varepsilon$ declined from $1.98 \times 10^{-6} \text{ m}^2/\text{s}^3 \ (t = 20 \text{ s})$ to $2.72 \times 10^{-7} \text{ m}^2/\text{s}^3 \ (t = 150 \text{ s})$. So it was known that in the process of the wave propagation, there was an attenuation trend of turbulence intensity in the viscous numerical wave tank.

However, in the irregular topography, when $t = 80 \text{ s}$, $100 \text{ s}$, $140 \text{ s}$, the values of $k$ were $2.7 \times 10^{-5} \text{ m}^2/\text{s}^2$, $2.01 \times 10^{-5} \text{ m}^2/\text{s}^2$, $1.52 \times 10^{-5} \text{ m}^2/\text{s}^2$, the values of $\varepsilon$ were $1.4 \times 10^{-6} \text{ m}^2/\text{s}^3$, $8.08 \times 10^{-7} \text{ m}^2/\text{s}^3$, $5.27 \times 10^{-7} \text{ m}^2/\text{s}^3$, and it was clear that the $k$ and $\varepsilon$ all appeared three jumps corresponding to the three bulges at the bottom. There were rapid increases in the turbulence kinetic energy and its dissipation rate due to the internal solitary wave propagation over the bulges, and therefore the irregular topography had a significant effect on the turbulence intensity.

5. Conclusions
The generation and propagation processes of internal solitary waves in an improved numerical wave tank have been investigated in this study, and the turbulence characteristics of waves in both flat and irregular topography were analyzed. The proposed mathematical model used a $k-\varepsilon$ turbulence closure scheme, which can describe the distributions of turbulence kinetic energy and its dissipation rate effectively. The results show that there was a intensified turbulence at the interface of the two-layer water when waves arrived. Compared with the case of the flat topography, when the internal solitary wave propagated in the irregular topography, the values of $k$ and $\varepsilon$ were higher. Therefore, the turbulence highly correlated with the topography, the variable topography was the main cause of the abrupt variations in $k$ and $\varepsilon$. 
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