VSFT revisited

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May 7, 2014

Abstract

We argue validity of purely ghost kinetic operator in open string field theory from the perspective of the modern analytic method based on the $KBc$ subalgebra. A purely ghost kinetic operator is obtained as a result of gauge fixing string field theory around the identity based tachyon vacuum solution. It is shown that the obtained kinetic operator is not equivalent to the midpoint insertion of the conformal ghost which is extensively studied in literature. We also find that the equation of motion does not allow any nontrivial solutions.

1 Introduction

The discovery of the analytic solution in open string field theory [1] and subsequent developments have provided a cornerstone to understand the nonperturbative nature of string field theory. Recently, it has been recognized that multiple D-branes solutions [3, 2, 4, 5, 6, 7] suffer from an anomaly. It reflects nontrivial nature of the space of the string field, which is not yet fully understood. With such a hard problem, it might be helpful to understand the underlying structure of the space of D-brane solutions. It would be worth recalling that vacuum string field theory (VSFT) [8], whose kinetic operator is purely ghost [9, 10], provides very simple description of D-branes. The equation of motion can be factorized into matter and ghost parts, and the matter part obeys the projector equation $\Psi^2_m = \Psi_m$ [11]. With the help of the simplicity, many important works such as ratio of the D-brane tension [10, 11], surface states as projectors [11, 12], and relevance to noncommutative field theory [13, 14, 15, 16] had been done. If such simple prescription remains valid and can be realized in modern analytic context, it will be helpful to understand the nature of D-branes in string field theory.

Historically, VSFT was conjectured as a candidate for the tachyon vacuum before the analytic description became available. Now the analytic and closed expressions are already available [11, 17, 18] so we can apply them to examine whether VSFT is valid description of string field theory around the tachyon vacuum. At first look, a kinetic operator derived from an analytic solution contains matter pieces hence contradicts those of VSFT. On the other hand, some aspects of the analytic solutions seem to be relevant to VSFT. One of them is the appearance of a sliver-like phantom term in some analytic solutions, which implies underlying projector-like structure [11, 19, 20, 21]. The other aspect, which will be studied in this letter, is existence of identity based solutions whose kinetic operator is very close to but not exactly pure ghost [18, 21, 22, 23]. In this letter, we derive VSFT from the SFT expanded around an identity based solutions $c - cK$. Our results are summarized as below.

- A purely ghost kinetic operator can be obtained by gauge fixing SFT around an identity based solution. No singular reparametrization [24] is required.

\footnote{Hata and Kojita have identified a number of D-branes as a winding number [5, 6], while their solutions are also singular for large winding number.}
The kinetic operator is given by an insertion of conformal ghost at the boundary of open string world sheet, hence is not equivalent to the midpoint insertion studied in past [24].

There are no nontrivial classical solution. This is due to the gauge condition imposed on the string field.

Contrary to our motivation, these results indicate that the VSFT derived from the analytic solution is inconsistent.

2 Gauge invariant action

The VSFT conjecture [8] claims that a pure ghost kinetic operator can be obtained from string field theory expanded around the tachyon vacuum solution even before gauge fixing. We would like to reexamine the conjecture with the help of the analytic solutions based on the KBc technique [17]. We first assume that the tachyon vacuum solution $\Psi_V$ that realizes VSFT is gauge equivalent to an analytic solution $\Psi_O$ written in Okawa form $FcK(1-F^2)^{-1}BcF$ [17] or its real form [18]. Therefore we can write

$$\Psi_V = U^{-1}Q_BU + U^{-1}\Psi_O U, \quad (1)$$

where $U$ is a gauge element. The VSFT action should be obtained from the SFT action expanded around $\Psi_V$. The action can be evaluated as

$$S_{Q_B}[\Psi + \Psi_V] = S_{Q_B}[\Psi + U^{-1}Q_BU + U^{-1}\Psi_O U] \quad (2)$$

$$= S_{Q_B}[U^{-1}Q_BU + U^{-1}(\Psi_O + U\Psi U^{-1})U]$$

$$= S_{Q_B}[\Psi_O + U\Psi U^{-1}]$$

$$= S_{Q_O}[U\Psi U^{-1}],$$

where $S_{Q_B}$ and $S_{Q_O}$ are SFT actions whose kinetic operators are $Q_B$ (the usual BRST operator) and $Q_O$ (the kinetic operator derived from $\Psi_O$), respectively. We have used the gauge invariance of the SFT action between second and third lines in (2). The last line of (2) means that the VSFT action is equivalent to the action defined by the kinetic operator $Q_O$ up to field redefinition. Therefore we next discuss whether the kinetic operator $Q_O$ can be pure ghost. The expression of the kinetic operator $Q_O$ is given by

$$Q_O \Psi = Q_B \Psi + \Psi_O \Psi + \Psi_O \Psi. \quad (3)$$

Since matter part contributions in (3) only appear through the matter Virasoro generators $L_n^X$, $Q_O$ should satisfy $\delta Q_O/\delta L_n^X = 0$ for each $n$ if it is pure ghost. This leads to the following condition

$$c_n \Psi + \frac{\delta \Psi_O}{\delta L_n^X} \Psi + \Psi \frac{\delta \Psi_O}{\delta L_n^X} = 0, \quad (4)$$

where $c_n$ is a mode of the conformal ghost $c(z)$. In order to satisfy (4), $\Psi_O$ should be linear in $L_n^X$ hence $K$. This arrows only few candidates from the variety of $F$ in $\Psi_O$, i.e., the identity based solutions discussed in [25]. However, it is easily understood that they do not satisfy (4) since they involve $K$ only through terms such as $cK$, $Kc$ or $cKBC$, and variations of these terms with respect to $L_n^X$ are proportional to $c \sim c(1)$, not $c_n$. Therefore, there is no chance to obtain pure ghost kinetic operator in gauge invariant action under the assumption that the tachyon vacuum is gauge equivalent to Okawa’s solution.
3 Purely ghost kinetic operator in gauge fixed theory

While pure ghost kinetic operator can not be realized in the gauge invariant action, there still be a chance to obtain pure ghost kinetic term in gauge fixed theory. Most suitable candidate is the identity base solution

$$\Psi_0 = c - cK. \quad (5)$$

While it has a drawback of being singular without regularization, it still has been believed to be a consistent description of the tachyon vacuum [22, 25]. The kinetic operator around this solution is defined by

$$Q \Psi = Q_B \Psi + \Psi_0 \Psi - (-1)^\Psi \Psi_0$$

$$= Q_B \Psi + c(1 - K) \Psi - (-1)^\Psi c(1 - K). \quad (6)$$

Obviously, the kinetic operator $Q$ is not pure ghost due to the $cK$ pieces and the original BRST charge. However, it is quite suggestive that the kinetic operator acts on $c$ and $B$ as

$$Qc = 0, \quad QB = 1, \quad (7)$$

as if $Q$ is replaced by an adjoint action of purely ghost operator $\{c, \ast\}$. In addition, the latter equation of (7) indicates that the homotopy operator of $Q$, which ensures trivial cohomology at the tachyon vacuum, is just given by $B$. These facts tempt us to identify $Q$ as $\{c, \ast\}$. To realize this identification, it should be noticed that equations (7) do not define $Q$ uniquely. For example, a replacement $Q \rightarrow Q + \{c', \ast\}$ does not alter the equations as long as $c'$ anticommutes with both $c$ and $B$. It is expected that such redundant components of $Q$ can be removed by suitable gauge fixing. Suppose that we find a set of operators which satisfies

$$\{Q, B\} = 1, \quad \{C, B\} = 1, \quad (8)$$

where $C$ and $B$ are linear combinations of $c_n$ and $b_n$ respectively. We also require $B$ to be Hermite. Then we impose ‘linear $B$ gauge’ condition [26] on the string field as

$$B \Psi = 0. \quad (9)$$

Then, inserting $\{C, B\} = 1$ in the kinetic term, we arrive at a purely ghost kinetic operator

$$\text{Tr}[\Psi Q \Psi] = \text{Tr}[\Psi C \Psi], \quad (10)$$

as desired. A solution of (8) is easily found and is expressed in the $KBc$ language as

$$B \Psi = \frac{1}{2} \{B \Psi + (-1)^\Psi B\}, \quad C \Psi = c \Psi - (-1)^\Psi c. \quad (11)$$

4 Comparison with the VSFT conjecture

While the kinetic operator we found is indeed pure ghost, there is a significant difference between our result and the earlier version of VSFT [25]. Namely, our action is gauge fixed but the latter is not. Therefore, in principle, these two results can not be compared on an equal footing. However, it also should be remembered that the VSFT conjecture was made when the details of the tachyon vacuum solution is not available. There still be a possibility such that, VSFT predicts the kinetic term of gauge fixed theory correctly while the interpretation of the space of string fields is wrong. If so, earlier results derived from VSFT should be reconsidered according to suitable gauge condition consistent with the kinetic operator. In order to address such possibility,

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2The gauge condition $B \Psi = 0$ is equivalent to $(B_0 + B_b^0) \Psi = 0$, in which perturbation theory is first discussed in [27] and further elaborated in [26].
it is convenient to rewrite $\mathcal{C}$ in operator formalism as $c(1) - c(-1) = 2 \sum_{n=0}^{\infty} c_{2n}$. This linear combination indeed belongs to the general class of pure ghost kinetic operator suggested in [10]. However, in the subsequent developments of VSFT, most attentions were payed to the specific choice of ghost insertion on the midpoint of an open string, i.e., $c(i) - c(-i)$, which does not coincide with our result. We still have a chance to relate them by a suitable conformal transformation. More explicitly, two kinetic operators can be related by a conformal map $U$ such that

$$U(c(1) - c(-1))U^{-1} \sim c(i) - c(-i).$$

where $U = \exp(i\pi/2 L_0)$, which gives a rotation around the unit circle.

is not a symmetry of the gauge fixed theory since only those generated by $K_n = L_n - (-1)^n L_{-n}$ leave gauge fixed action invariant.

Therefore, our result $c(1) - c(-1)$, insertion of conformal ghosts on the boundary, is not equivalent to $c(i) - c(-i)$ those on the midpoint even in gauge fixed theory.

5 Classical solutions

Another implication comes from a study of the equation of motion. In our prescription, the pure ghost kinetic operator is obtained by gauge fixing. Therefore, a gauge condition on the classical solution is given from the beginning and can not be chosen arbitrary. This situation is in contrast with those in most literature, where Siegel gauge is chosen irrespective with the choice of the kinetic operators. With this in mind, let us consider the equation of motion

$$\mathcal{C}\Psi + \Psi^2 = 0$$

where the string field obeys the gauge condition $B\Psi = 0$, or equivalently $B\Psi = \Psi B$ in the $KBc$ notation. Multiplying $\mathcal{C}$ by $B$ from the left we have

$$\Psi + 2B\Psi^2 = 0.$$ (14)

Multiplying $\mathcal{C}$ further by $B$ from the left yields $B\Psi = 0$, and plugging this back to $\mathcal{C}$ yields $\Psi = 0$. In this way, we arrive at the striking result that the equation of motion does not allow any nontrivial solution. Obviously this is pathological since we know that the nontrivial solution $\Psi = -\Psi_0$ which represents original D25-brane is available before gauge fixing. One may guess that this pathology is due to the choice of too simplified kinetic operator. However, this is not the case. Consider more general background than (5) according to Okawa [17],

$$\Psi_O = Fc\frac{K}{1-F^2}BcF,$$ (15)

where $F$ is a function of $K$. In this case, the homotopy operator is given by $\mathcal{A} = B(1 - F^2)/K$.

It should be noted that we still have $A^2 = 0$ even though $A$ depends on $K$ nontrivially. A gauge condition analogous to (11) is

$$\mathcal{A}\Psi = \Psi^2 = 0.$$ (16)

Then, repeating same process as seen in [13] and [14] for the kinetic operator $\mathcal{Q}$ defined by (15), we obtain the trivial solution $\Psi = 0$ again. Therefore, the gauge condition $\mathcal{A}\Psi = 0$, which looks honest with the trivial cohomology, does not allow nontrivial solution. Above augment shows that the origin of the pathology is not due to too simplified kinetic term but due to too strong gauge condition.
6 Discussions

We study a pure ghost kinetic operator in open string field theory by gauge fixing a theory around identity based analytic solution. The obtained pure ghost operator is placed on the boundary therefore is not equivalent to the midpoint insertion. It is also shown that the gauge fixed equation of motion does not have any nontrivial solution. The latter result indicates that the VSFT obtained here cannot explain the ‘original’ configuration which represents D25-brane. Above results indicate that the early implications form VSFT can not be applied directly to the string field theory around the analytic solutions.

While the gauge fixing examined in this letter fails to explain D-branes, it may be still useful as a local description around the tachyon vacuum. For example, derivation of the effective action around tachyon vacuum is possible along the line with [10]. Derivation of closed string amplitude [21, 23, 29, 30] will also be interesting. In either case, a regularization will be required for vanishing area of the world sheet.

7 Acknowledgements

We thank H. Isono and I. Kishomoto for helpful discussions.

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