STUDY ON OPTIMAL COMBINATION SETTLEMENT PREDICTION MODEL BASED ON LOGISTIC CURVE AND GOMPERTZ CURVE

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ABSTRACT

The Logistic and Gompertz embankment settlement prediction models have poor prediction accuracy for the late settlement of high-filled soil. This study proposes a combination of the two models based on their common characteristics and individuality, and their respective advantages and specific limitations. The minimum logarithmic error square sum of the combined model was used as the objective function to solve the optimal weighting coefficient. The optimal weighted geometric mean combination prediction model was deduced, to improve the confidence of the prediction accuracy of the settlement of high-filled soil. By fitting and analysing the measured settlement data of the engineered high-filled soil with each prediction model, the feasibility of the proposed optimal combination prediction model in the settlement prediction of high-filled soil was tested. It was found that the proposed optimal combination forecasting model was more accurate and adaptable compared to any single model, and was more reliable. Therefore, the proposed combination forecasting model could be used as an effective method to predict the settlement of high-filled soil in the later stages of settlement.

KEYWORDS

High-filled soil, Optimal combination model, Settlement prediction

INTRODUCTION

The central and western regions of China are occupied with cities in mountainous terrain. The continuous development of these urbanized areas has resulted in large cities expanding into surrounding areas, resulting in new urban districts, airports and railway and (highway) roads being built on land formed using high-filled soil. Uneven settlement and deformation of high-filled soil in hilly and gully terrain is of concern to geotechnical experts. Many structures built on high-filled soil have been destroyed or condemned due to the settlement of the foundations. The settlement of high-filled soil results in the deformation of the filling soil and the settlement of the foundation. Reported data indicate compression of the soil caused by the weight of the fill often accounts for 80%-90% of the total settlement [1-2]. After completion of the construction of the fill soil, uneven settlement often occurs over a long period to varying degrees. The settlement is primarily influenced by the filling speed, filling height, filling properties and local climatic conditions. This multi-dimensional problem based on time and space cannot be solved using empirical formulas [3]. The final settlement calculated using the finite element method based on the consolidation theory is often significantly different to the measured value. Therefore, it is necessary to understand the dynamic process of settlement of high-filled soil to accurately predict trends in settlement using data during the early period, to ensure safety and quality [4]. To date, there are three main methods used globally for predicting settlement of high-filled soil: the empirical formula deduction method, numerical simulation and the curve fitting method. The most commonly used curve fitting
methods include hyperbolic, modified hyperbolic, grey prediction, grey Theory-Markov, power function, exponential equations and Asaoka methods [5-14]. In recent years, the Logistic and Gompertz curves have been favoured as they reflect more accurately the relationship between settlement and time [15-18], however, the prediction accuracy is often limited by the amount of monitoring data, geographical conditions and construction technology. It is difficult to use a single prediction model to make an accurate prediction of settlement, resulting in low confidence of prediction precision.

The common characteristics and individuality of Logistic and Gompertz prediction models, together with their advantages and specific limitations are used in this study which combines the two models to calculate the optimal weighting coefficients. The sum of the minimum logarithmic error squares of the combined model was used as the objective function which derived the optimal weighted geometric mean of the combination prediction model. The prediction model was used to improve the prediction accuracy of the curve fitting method. The combined prediction model could be used as an effective method for predicting the late settlement of high-filled soil.

THE TWO SETTLEMENT PREDICTION MODELS

The Logistic model

In 1938, Pierre-Francois Verhulst proposed the Logistic model for predicting world population growth (Verhulst-Pearl model). Due to the development of inter-discipline fields, this model is widely used in ecology, medicine, economics and management. The model can be used for settlement prediction and its differential form can be rewritten as:

\[
\frac{dy}{dt} = k\frac{y(1 - \frac{y}{A})}{A}
\]

(1)

where: \( y = \)settlement; \( t = \)time; \( k = \)immediate settlement rate; \( A = \)final settlement.

The integral of equation (1) is:

\[
\int \left(\frac{1}{y} + \frac{1}{A - y}\right)dy = \int kdt
\]

(2)

Solution:

\[
y = \frac{A}{1 + Be^{-kt}}
\]

(3)

where: \( B = \)parameter to be sought; \( e = \)Euler number.

The Gompertz model

The Gompertz model was put forward by the British statistician Benjamin Gompertz. It was originally used to describe the law of the growth and change of nature. Since its birth, the Gompertz model has played an important role not only in the field of biology, but also in market economy forecasting and traffic forecasting. The model can be used for settlement prediction and its differential form can be rewritten as:

\[
\frac{dy}{dt} = -k \ln \frac{y}{A}
\]

(4)

where: \( y = \)settlement; \( t = \)time; \( k = \)immediate settlement rate; \( A = \)final settlement.

The integral of equation (4) is:
\[
\int \frac{d(ln y - ln A)}{ln y - ln A} = -\int kdt
\]

Solution:

\[y = Ae^{-Be^{-kt}}\]  \hspace{1cm} (6)

where: \(B\) = parameter to be sought; \(e\) = Euler number.

Then, the first derivative of equations (3) and (6) can be obtained:

\[
\begin{align*}
\frac{dy}{dt} &= \frac{ABke^{-kt}}{(1 + Be^{-kt})^2} > 0 \\
\frac{dy}{dt} &= ABk^2e^{-kt} \cdot e^{-kt} > 0
\end{align*}
\]

The second derivative of equations (3) and (6) are:

\[
\begin{align*}
\frac{d^2y}{dt^2} &= \frac{ABke^{-kt}(Be^{-kt} - 1)}{(1 + Be^{-kt})^3} \\
\frac{d^2y}{dt^2} &= ABk^2e^{-kt} \cdot e^{-kt} \cdot (Be^{-kt} - 1)
\end{align*}
\]

The limits of equations (3) and (6) are:

\[
\begin{align*}
\lim_{t \to \infty} \frac{A}{1 + Be^{-kt}} &= A \\
\lim_{t \to \infty} Ae^{-Be^{-kt}} &= A
\end{align*}
\]

Using the definition of consolidation degree \(U\), equations (3) and (6) can be written:

\[
\begin{align*}
U &= \frac{y_t - y_0}{y_\infty - y_0} = \frac{(A/1 + Be^{-kt}) - (A/1 + B)}{A - A/1 - B} \\
U &= \frac{y_t - y_0}{y_\infty - y_0} = \frac{Ae^{-Be^{-kt}} - Ae^{-B}}{A - Ae^{-B}}
\end{align*}
\]

Equation (7) demonstrates that the Logistic and Gompertz curves increase monotonously, Equation (8) demonstrates that when \(Be^{-kt} = 1\) (that is, \(t = ln B/k\)) there is an inflection point in the curve, and Equation (9) demonstrates that when time is infinite, the maxima in both curves equals \(A\). According to the properties of the two curves, the shape of the Logistic curve is an "S" shape, and the Gompertz curve has the characteristics of an "anti-S" shape distribution. This is in agreement with the proposal by Mei et al. that it is theoretically demonstrated that the settlement-time curve is an "S" shape when the load is linear or approximately linear [19]. Equation (10) demonstrates that when time equals zero, \(U=0\), and when time approaches infinity, \(U= 1\). Therefore, the Logistic and Gompertz models can be used to predict the variation in settlement of high-filled soil over time.
SOLUTION OF THE MODEL

Using the Logistic and Gompertz models, settlement can be predicted for any time when the values of the parameters A, B and k are known. In order to improve the fit, nonlinear Gompertz and Logistic curves were converted into modified exponential curves using the basic principles of linear regression. The transformation of the modified exponential equations is shown in Table 1.

Tab. 1 - Transformation of the modified exponential equation

| Model type | Model equation | Transformation process | Transformation model |
|------------|----------------|------------------------|---------------------|
| Logistic   | \[ y = \frac{A}{1 + Be^{-kt}} \] | if \( Y = \frac{1}{Y}, \ A' = \frac{1}{A}, \ B' = B/A \), \( k' = e^{-k} \) | \( Y = A' + B'k^d \) |
| Gompertz   | \[ y = Ae^{-Be^{-kt}} \] | \( \ln y = \ln A - Be^{-kt}, \text{ if } Y = \ln y, \ A'' = \ln A, \ B'' = -B, \ k' = e^{-k} \) | \( Y = A'' + B''k^d \) |

This study proposes the use of the Bryant method to solve the parameters of the curve. The three parameters in the model require that the measured y~t in the settlement data be of equal time interval; the measured data were generally non-isometric. Therefore, before using the two curves to fit the data, measured y~t data must be converted into an isochronous y~ sequence that meets the requirements. In this study, the Lagrange interpolation method was used to perform the isochronous transformation of non-isochronous settlement time series data. The equations of each parameter were calculated using the Bryant method as follows:

\[
k = \frac{(n-1)\sum_{i=1}^{n-1} Y_i Y_{i+1} - \sum_{i=1}^{n-1} Y_i \sum_{i=1}^{n-1} Y_{i+1}}{(n-1)\sum_{i=1}^{n-1} Y_i^2 - (\sum_{i=1}^{n-1} Y_i)^2} \quad (11)
\]

\[
B = \frac{n\sum_{i=1}^{n} k^i Y_i - \sum_{i=1}^{n-1} k^i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} k^{2i} - (\sum_{i=1}^{n-1} k^i)^2} \quad (12)
\]

\[
A = \frac{(\sum_{i=1}^{n} Y_i - B\sum_{i=1}^{n} k^i)}{n} \quad (13)
\]

OPTIMAL WEIGHTED AVERAGE GEOMETRIC PREDICTION MODEL

If there were \( m \) kinds of settlement prediction models for a problem, \( \hat{y}_{1t}, \hat{y}_{2t}, \cdots, \hat{y}_{mt} \), \( t = 1,2,\ldots,N \), then at the same time, \( \hat{y}_{it} \) represents the \( i \) type of the prediction model and the predicted value of the \( t \) phase, \( \hat{y}_{jt} \) represents the weighted geometric average combination prediction model of the above \( m \) types of prediction models, and also represents the prediction value of the \( t \) phase of the model.
\[ \hat{y}_t = \prod_{i=1}^{m} y_{iu} \]  
(14)

If \( W = (w_1, w_2, \ldots, w_m) \in R^m \), and \( \sum_{i=1}^{m} w_j = 1, w_i \geq 0, i = 1, 2, \ldots, m \)
(15)

If \( y_t \) is used to represent the measured values of the actual settlement in the \( t \) period, \( s_{it} = \ln \hat{y}_{it} - \ln y_t \), and \( S_t = \ln \hat{y}_t - \ln y_t \) are used to denote the logarithmic errors of \( \hat{y}_{it} \) and \( \hat{y}_t \) respectively, then:

\[
S_t^2 = (\ln \hat{y}_t - \ln y_t)^2 = (\ln \prod_{i=1}^{m} y_{iu} - \ln y_t)^2 =
\]

\[
(\sum_{i=1}^{m} w_i \ln \hat{y}_t - \ln y_t)^2 = \left[ \sum_{i=1}^{m} w_i (\ln y_{iu} - \ln y_t) \right]^2 =
\]

\[
\begin{vmatrix}
\ln \hat{y}_{1t} - \ln y_t \\
\ln \hat{y}_{2t} - \ln y_t \\
\vdots \\
\ln \hat{y}_{mt} - \ln y_t
\end{vmatrix}
\]

\[
(w_1, w_2, \ldots, w_m)
\begin{vmatrix}
\ln \hat{y}_{1t} - \ln y_t \\
\ln \hat{y}_{2t} - \ln y_t \\
\vdots \\
\ln \hat{y}_{mt} - \ln y_t
\end{vmatrix} = W^{(s_{it}, s_{jt})} W
\]
(16)

where: \( W = (w_1, w_2, \cdots, w_m) \); \( (s_{it}, s_{jt}) \) is an \( m \) order square matrix.

Therefore, the sum of the square of logarithmic errors is:

\[
S^2 = \sum_{t=1}^{N} S_t^2 = \sum_{t=1}^{N} W^{(s_{it}, s_{jt})} W = W' \left[ \sum_{t=1}^{N} (s_{it}, s_{jt}) \right] W
\]
(17)

where matrix \( A = \sum_{t=1}^{N} (s_{it}, s_{jt}) \) is generally a positive definite matrix. Due to

(1) \[ \sum_{i=1}^{N} (s_{it}, s_{jt}) = \sum_{i=1}^{N} (s_{it}, s_{jt}) = \sum_{i=1}^{N} (s_{it}, s_{jt}) , \] matrix \( A \) is a symmetric matrix.

(2) For \( X = (x_1, x_2, \cdots, x_m) \in R^m - \{0\} \), there are:

\[
X' \left[ \sum_{i=1}^{N} (s_{it}, s_{jt}) \right] X = \sum_{i=1}^{N} X' (s_{it}, s_{jt}) X = \sum_{i=1}^{m} (\sum_{k=1}^{m} x_k s_{i_k}) \geq 0
\]
(18)
If \( X'AX \) is the constant zero, \( \sum_{k=1}^{N} x_k s_{k,t} = 0, t = 1,2,\ldots,N \). That is, for any nonzero \( m \) dimensional vector \( X \), it is the solution of the equations in (19).

\[
\begin{align*}
    s_1 x_1 + s_2 x_2 + \cdots + s_m x_m &= 0, \\
    s_1 x_1 + s_2 x_2 + \cdots + s_m x_m &= 0, \\
    \vdots \\
    s_1 x_1 + s_2 x_2 + \cdots + s_m x_m &= 0,
\end{align*}
\] (19)

If \( x_k = 1, x_i = 0, i \neq k, i = 1,2,\ldots,m \), there is \( s_{k,t} = 0, k = 1,2,\ldots,m, t = 1,2,\ldots,N \), \( s_{k,t} = \ln y_{k,t} - \ln y \) is sometimes not zero, therefore:

\[
X'AX \geq 0
\] (20)

From equations (1) and (2) the matrix \( A \) is a positive definite matrix, so that \( A \) is invertible and for any nonzero vector \( W \), \( S^2 > 0 \). \( Q = (1,1,\ldots,1)' \in R^m \), so the optimal weight coefficient \( W \) can be determined by minimizing the sum of the logarithmic error square \( S^2 = W'AW \) of the combined prediction model under the constraint conditions of \( W'Q = 1 \), \( W \geq 0 \).

Using the result from [20], the solution is:

\[
W = \left[ \sum_{i=1}^{N} (s_i, s_{j,t}) \right]^{-1} Q \left[ \sum_{i=1}^{N} (s_i, s_{j,t}) \right]^{-1} \] (21)

For the weighted geometric combination prediction model composed of Logistic and Gompertz models, the optimal weight coefficients are:

\[
\begin{align*}
    w_1 &= \left( \sum_{i=1}^{N} s_i^2 - \sum_{i=1}^{N} s_i s_{2,i} \right) \left[ \sum_{i=1}^{N} (s_i^2 + s_{2,i}^2 - 2s_i s_{2,i}) \right]^{-1} \\
    w_2 &= \sum_{i=1}^{N} (s_i^2 - s_i s_{2,i}) \left[ \sum_{i=1}^{N} (s_i^2 + s_{2,i}^2 - 2s_i s_{2,i}) \right]^{-1}
\end{align*}
\] (22)

Due to \( w_1 + w_2 = 1 \), only one of the \( w_1, w_2 \) was solved.

**ACCURACY OF THE COMBINED PREDICTION MODEL**

According to the square sum of absolute errors (SSE), square sum of relative errors (SSRE), standard error (SE), relative standard error (RSE) and mean absolute percentage error (MAPE), the accuracy of the optimal combination prediction model proposed in this study was tested using five parameters. The formulas of each parameter are as follows:
According to the value MAPE, the prediction accuracy can be divided, as shown in Table 2.

**Tab. 2 - MAPE classification of prediction accuracy**

| MAPE % | Prediction accuracy | MAPE % | Prediction accuracy |
|--------|---------------------|--------|---------------------|
| <10    | High accuracy       | 20~50  | Feasible            |
| 10~20  | Good                | >50    | Infeasible          |

**ENGINEERING EXAMPLE ANALYSIS**

High-filled soil engineering in Wuhan TianHe airport [21]

Located in the Wuhan City, Hubei Province, China, the Wuhan Tianhe Airport Second Channel Expressway has a total length of approximately 15.5 km. Most of the landforms are low mountains and hills with little structural erosion. The topography is undulating and there are some high-filled sections across the whole line. After the completion of the construction, the settlement deformation of the high-filled section of the whole line was monitored in real time. This study focuses on the S2 section, the pile number of the section is K4+200~K5+325 and total length is 1.215 km. Section K4+860 is a special subgrade section and the settlement deformation of this section was analysed and predicted. The embankment filling height of this section was 8.2 m and measurement period was from June 12 to December 15, 2016. Based on the measured settlement data across 153 days, using the Bryant method, logarithmic transformation and linearization equations listed in Table 1, the Logistic and Gompertz models were determined, respectively:

\[
\hat{y}_{1t} = \frac{132.2949}{1 + 4.2697 \times e^{-0.0356t}} \quad \text{and} \quad \hat{y}_{2t} = 133.8612 \times e^{-1.9656 \times e^{-0.0423t}}
\]

Therefore:

\[
\sum_{t=1}^{20} s_{1t} = 0.0548, \sum_{t=1}^{20} s_{2t} = 0.0581, \sum_{t=1}^{14} s_{1t} s_{2t} = 0.0539
\]

The Optimal weight coefficient \( w_1 = 0.8204, w_2 = 0.1796 \). Therefore, the optimal weighted geometric average combination prediction model was:

\[
\hat{y}_t = \hat{y}_{1t} \cdot 0.8204 + \hat{y}_{2t} \cdot 0.1796
\]

Using the Logistic model, Gompertz model and the proposed optimal weighted geometric average combination prediction model, the predicted settlement values and precision index values
were calculated (Tables 3 and 4). The relationship curve of settlement and time of section K4+860 is shown in Figure 1. The curve of relationship between measurement time and relative error of section K4+860 and a diagram of measurement time and residual scatter plot are shown in Figure 2 and Figure 3, respectively.

**Tab. 3 - Settlement datas of K4+860 by measured and predicted**

| Time/d | Measured data /mm | Predicted data/mm | Logistic model | Gompertz model | The combined model |
|--------|-------------------|-------------------|----------------|----------------|-------------------|
| 10     | 36.707            | 38.936            | 37.126         | 38.604         |
| 13     | 45.111            | 43.855            | 43.316         | 43.758         |
| 18     | 59.202            | 52.681            | 53.808         | 52.888         |
| 31     | 66.11             | 77.194            | 79.334         | 72.574         |
| 35     | 79.321            | 84.443            | 86.125         | 78.742         |
| 38     | 96.485            | 89.594            | 90.815         | 94.812         |
| 45     | 103.898           | 100.357           | 100.390        | 102.363        |
| 47     | 105.427           | 103.069           | 102.783        | 104.018        |
| 52     | 110.353           | 109.110           | 108.138        | 109.935        |
| 56     | 112.551           | 113.199           | 111.822        | 112.950        |
| 57     | 113.509           | 114.122           | 112.666        | 113.860        |
| 68     | 122.903           | 121.997           | 120.186        | 122.670        |
| 88     | 126.203           | 128.864           | 127.859        | 126.683        |
| 94     | 127.358           | 129.850           | 129.191        | 127.731        |
| 99     | 129.063           | 130.454           | 130.076        | 130.086        |
| 115    | 130.757           | 131.557           | 131.936        | 130.625        |
| 123    | 131.109           | 131.829           | 132.490        | 131.248        |
| 130    | 133.617           | 131.984           | 132.843        | 133.138        |
| 145    | 133.983           | 132.164           | 133.324        | 134.071        |
| 153    | 134.203           | 132.212           | 133.479        | 134.339        |

**Tab. 4 - Comparison of precision index values of each prediction model**

| Model type       | SSE    | SSRE   | SE     | RSE    | MAPE%  |
|------------------|--------|--------|--------|--------|--------|
| Logistic model   | 293.234| 0.057  | 3.829  | 0.054  | 3.461  |
| Gompertz model   | 330.611| 0.064  | 4.066  | 0.057  | 3.364  |
| The combined model| 96.776 | 0.025  | 2.200  | 0.036  | 1.866  |
Fig. 1 - Measured settlement and predicted settlement curves

Fig. 2 - Measurement time and relative error curves

Fig. 3 - Measurement time and residual scatter plots
The high-filled embankment project of Wuhan Tianhe Airport, the regularity of SSE and SE values of the accuracy index of the prediction model, and SSRE values and RSE values of the stability reliability index of the prediction model were similar to that of the high-filled embankment project of Yan Jiafeng Section (Table 8). However, in this study the MAPE values of the Logistic, Gompertz and proposed combined prediction models were 3.461%, 3.364% and 1.866%, respectively. All indicate a high precision prediction and the MAPE values in this study were smaller, suggesting greater accuracy. When the monitoring time was 38 days, the relative error was large, and the variation range was -11.02%-20.00% (Figure 5). However, when the monitoring time was longer than 38 days, the variation range of the relative error was - 3.41%-0.07%. The residual diagram was consistent with the randomness and unpredictability of probability theory, and the points described by the combined prediction model were scattered randomly about a line with residuals equal to 0 (Figures 3 and 6), suggesting that the proposed prediction model was effective.

CONCLUSION

(1) To address the deficiencies in single settlement prediction models in predicting the settlement of high-fill soil, this study combined the Logistic and Gompertz prediction models to solve optimal weighting coefficients. This was achieved taking the sum of the minimum logarithmic error squares of the combined model as the objective function, and then deriving the optimal weighted geometric mean combination prediction model.

(2) Selecting the engineer measured data for the fitting analysis, the comparative analysis indicated that the accuracy (SSE, SE and MAPE) and reliability (SSRE, RSE) of the optimal combination predicting model was better than any single model, and the adaptability was greater. When the monitoring time exceeded the curve inflection point, it was similar to the measured value, indicating the optimal combination predicting model was good for settlement prediction.

(3) The proposed combination prediction model could be used to optimize the combination of multiple models and buffer the advantages and disadvantages of each single model. When there is no negative weight ratio, the settlement prediction accuracy was improved significantly.

In this study, linear loading or approximate linear loading only was used for the analysis. The application of a non-linear growth model to settlement prediction of step loading and more complex loading processes requires further study and analysis.

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