Studies on coherent multi-bunch tune shifts with different bunch spacing at the J-PARC Main Ring

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Abstract. At a high-power proton synchrotron, betatron tune shifts induced by space charge effects cause beam loss which limits the beam intensity. To achieve further high beam intensity at the main ring of the Japan Proton Accelerator Research Complex, precise control of the tune shift is indispensable. When carrying out multi-bunch measurements, we observed that the dependence of the tune shift intensity on the number of bunches follow opposite slope trends for the horizontal and vertical directions. The dependence of the bunch spacing was also observed. We report on a simplified tune shift model reconstruction for understanding the origin of these phenomena and present a correction of the tune shifts for reducing beam loss up to 30 %.

1. Introduction
The Japan Proton Accelerator Research Complex (J-PARC) is a high frequency proton beam accelerator facility consisting of a 400 MeV linac [1], a 3 GeV rapid cycling synchrotron (RCS) [2] and a 30 GeV main ring synchrotron (MR) [3]. The MR is a synchrotron with three-fold symmetry, having a circumference of 1567.5 m. Up to eight buckets out of nine can be filled with proton bunches, where two bunches are injected from the RCS every 40 msec. The space charge effects are severe during the injection period for 130 msec, therefore quantitative evaluation of the tune shifts is necessary as the number of particles in the ring is increased.

2. Measurement of tune shifts
Coherent tune shifts were measured by a fast Fourier transform (FFT) analyzing the beam position monitor (BPM) signals with a strip-line kicker (exciter) [4, 5].

In previous measurements with a single bunch, the horizontal and vertical tune shifts decreased linearly with increasing intensity [6].

To achieve higher beam power operation, we measured the intensity dependence of the tune shifts with multi-bunches up to the maximum 8 bunches. The intensities were handled by changing the proton population per bunch by maintaining a constant bunching factor [7]. The bunches are injected from the 1st to the 8th RF bucket in order as a bunch train, thus the bunch spacing is non-equidistant.

Figure 1 shows the measured tune shift intensity dependence with various number of bunches for the horizontal (a) and vertical (b) directions. The dots represent measured points, and the lines are linear fits. The different colors indicate the number of bunches filled in the ring. The tune shift behavior was different for the horizontal and vertical directions, and the slopes became...
Figure 1. Measured tune shifts as a function of the bunch intensity, horizontal (a) and vertical (b) directions. The linear fits are also shown.

Figure 2. Measured tune shift slopes as a function of the number of bunches with fitted lines. The sign of the tune shift slope for horizontal and vertical directions is opposite.

gradually larger as the number of bunches were increased, as shown in Figure 2. The linear fitted slopes were $\Delta \nu_x = -0.000271 + 0.000101 \times M$ and $\Delta \nu_y = -0.000319 - 0.000121 \times M$ for the horizontal and vertical tune shifts, respectively. Here, $M$ represents the number of bunches.

The measured tunes plotted in Figure 1 are replotted as functions of the total intensities in Figure 3. The tune shift is a dependent on the number of bunches, even for the same number of circulating protons.

The dependence of the tune shift, which inversely proportional to the bunching factor, was measured with a fixed intensity, and the results are shown in Figure 4. The bunching factor $B_f$
Figure 3. Measured tunes as a function of the total proton intensity, horizontal (a) and vertical (b) directions. The same data as Figure 1 are replotted, but the abscissa is total proton intensity. The linear fits are also shown. The color indicates the proton population per bunch.

Figure 4. Measured bunching factor dependence of the tune shifts for horizontal (a) and vertical (b) directions with fitted curves.

is defined by the ratio of maximum and average current.

3. Tune shift model reconstruction
At the MR, both the space charge effects and the resistive wall impedance effects should be taken into consideration. There are various duct cross-sections [8], five extraction kickers, and two injection kickers. In previous studies, the transverse impedance effects of kickers and space charge effects with a single bunch were investigated [9, 10]. In our model reconstruction, the space charge effects and the resistive wall effects are estimated by considering the theories in the following sections [11].

3.1. Space charge effects
Space charge effects are induced by the electric and magnetic image forces caused by the vacuum chambers and magnet pole faces [12]. The coherent tune shifts caused by a space charge can be estimated by the equation developed by B. Ng [13, 14], which is related to the chamber cross-
Figure 5. The typical cross-section of the bending magnet at the MR [8]. The red ellipse and the orange straight lines were used in the calculation.

For simplicity, in the electrical image coefficients calculation, the chamber cross-sections excluding the bending magnets were regarded as circles, and that of the bending magnets were taken as elliptic, as shown in Figure 5 (red). There are 96 bending magnets that occupy 35.8% of the ring circumference. Since the magnetic image coefficients cannot be solved for using elliptical cross-sections, a parallel plate approximation was applied, as shown in Figure 5 (orange). The calculated tune shift slopes are presented in Table 1.

| Tune shift slope       | Horizontal | Vertical |
|------------------------|------------|----------|
| Bending magnets        | $9.17 \times 10^{-5}$ | $-2.06 \times 10^{-4}$ |
| Circular ducts        | $-7.11 \times 10^{-5}$ | $-7.11 \times 10^{-5}$ |

3.2. Resistive wall effects

In the theory by Chao et al. [15], tune shift from a resistive wall impedance is mainly defined by the cross-section of the duct as shown in Eq. (16) in the reference [15]. It should be noted that the simplified equation ignores the wall thickness and assumes equidistant bunches. In this model reconstruction based on the theory by Chao et al., the chamber was assumed to be a perfect conductor.

On the other hand, the theory by Shobuda et al. [16] also considers equidistant bunches. While the theory is focuses on the coupled bunch modes, we used Eqs. (26) and (27) in the reference [16] for estimation of the resistive wall effects on the betatron tune. The frequency modes $\mu$ are defined as $\Delta \nu_{x,y}^{(\mu)} = \nu_{x,y} - \mu$. In the MR, the geometrical cross-section factors are $D_{1x,y} = 0.615, 0.884$ and $D_{2x,y} = -0.269$ for dipole and quadrupole wake fields, respectively. The stainless steel wall thickness is $d_w = 2 \times 10^{-3}$ m and its resistivity is $\rho_0 = 1.910 \times 10^{-9}$ m. The results of the calculations for each model (simplified model C, based on Chao et al. and simplified model S, based on Shobuda et al) are shown in Table 2. The difference between two models comes from neglected parameters and assumptions.
Table 2. Calculated Resistive Wall Tune Shift Slopes \( (d\nu_x/y/dN_b[\times 10^{12}]) \) based on the Models [15, 16].

| Model                     | Horizontal       | Vertical       |
|---------------------------|------------------|----------------|
| C (based on Chao) et al.  | \( 1.24 \times 10^{-4} \) | \(- 1.23 \times 10^{-4} \) |
| S (based on Shobuda) et al. | \( 2.51 \times 10^{-6} \) | \(- 2.51 \times 10^{-6} \) |

### 3.3. Bunch spacing effects

Since the aforementioned theories do not take the uneven distances between bunches into account, we measured intensity-dependent tune shifts with two different beam filling patterns. Three buckets out of nine were filled with beam bunches in both cases. In the first case the buckets are evenly filled (equidistant), whereas in the second case the buckets are filled sequentially (non equidistant). The measured results and their linear fits are shown in Figure 6. The filling patterns are also shown. The slopes are different for each filling pattern. Here, we emphasize that the bunch spacing is an effective and important factor for the tune slope in the MR.

### 3.4. Comparison with measurements

As shown in Figures 7 (a) and (b), the tune shifts with three equidistant bunches are roughly explained by models calculated with the slopes in Tables 1 and 2. Extrapolating from Figure 2, the tune slope for 9 bunches can be deduced. In Figures 7 (c) and (d), models are compared. These results suggest that the tune shifts are also dependent on the bunch spacing.

![Comparison of tune shift slope for two filling patterns in the horizontal (a) and vertical (b) directions.](image)

**Figure 6.** Comparison of tune shift slope for two filling patterns in the horizontal (a) and vertical (b) directions.

### 4. Tune shift correction

In our measurement, the deviations of the tune from the low intensity measurement were clearly observed as shown in Figure 8 (a), when injecting the bunches at every 40 msec. Due to the tune shift, a fraction of the bunch hit the resonance lines, and the beam losses were increased. The fluctuations come from the main magnet power supply ripples and bunching factors at the bunch injection timings. The tunability enabled by the quadrupole magnet power supplies [17] allowed...
Figure 7. Comparison of the measured (dots) and the modeled (lines) tune shifts for three equidistant bunches in the horizontal (a) and vertical (b) directions. Comparison of the extrapolation from measured (dashed) and modeled (solid) tune shifts with nine equidistant bunches for the horizontal (c) and vertical (d) directions. Model L represents the model based on space charge effects. Models S and C are resistive wall effects based on Shobuda et al. and Chao et al., respectively.

the initial tune point for both the horizontal and vertical directions to be maintained during the injection period. Obviously the betatron tune are corrected as presented in Figure 8 (b); beam loss was reduced by approximately 30 %. By applying this correction and with additional tuning, a beam power of 500 kW ($2.6 \times 10^{14}$ ppp in 2.48 s cycle) was achieved [18, 19].

5. Conclusion
To achieve higher beam intensity at the MR, we studied the tune shift in detail with a multi-bunch approach. Different tune shift behavior was observed in the horizontal and vertical directions. The influence of the number of bunches as well as that of the bunch spacing were investigated. The reconstructed models based on space charge and the resistive wall effects qualitatively explains the tune shift slopes. The discrepancy between the measurements and the models is considered to be from the simplified models and lack of other impedance sources than the resistive wall. To understand the model assumptions, we measured the dependence on the bunch spacing. The impact due to kicker impedance is currently under estimated. By applying the correction of the tune shift, the output beam power 500 kW was achieved. The correction will be more important for future operation at 1.3 MW ($3.3 \times 10^{14}$ ppp in 1.16 s cycle).
Figure 8. Before (a) and after (b) correction that considers bunch train tune shifts for the vertical (blue) and horizontal (red) directions as a function of time after injection. Beam loss was reduced by 30% after correction.

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References
[1] Ikegami M et al 2012 Progress of Theoretical and Experimental Physics 2012 1 02B002
[2] Hotch H et al 2012 Progress of Theoretical and Experimental Physics 2012 1 02B003
[3] Koseki T et al 2012 Progress of Theoretical and Experimental Physics 2012 1 02B004
[4] Toyama T et al 2016 Proceedings of the 13th Annual Meeting of Particle Accelerator Society of Japan pp 1081-1085
[5] Yamada S 2012 Proceedings of the 9th Annual Meeting of Particle Accelerator Society of Japan pp 313-316
[6] Toyama T 2015 internal documents
[7] Tamura F et al 2007 Proceedings of PAC07
[8] Uota M 2011 internal documents
[9] Chin Y H 2008 Proceedings of HB2008 WGA01 p 40
[10] Rendon B Y et al 2018 Proceedings of IPAC2018
[11] Kobayashi A et al 2018 Proceedings of the 15th Annual Meeting of Particle Accelerator Society of Japan pp 60-64
[12] Laslett L J 1963 Proceedings of the 1963 Summer Study on Storage Rings, Accelerators and Experimentation at Super-High Energies pp 324-367
[13] Ng Y B 2006 World Scientific Pub Co Inc
[14] Martins M A and Ng K Y 1994 FERMILAB-TM-1889
[15] Chao A, Heifets S and Zotter B 2002 Phys. Rev. A. B. 5 111001
[16] Shobuda Y and Yokoya K 2003 Phys. Rev. E 66 056501
[17] Shimogawara T et al 2017 Proceedings of the 14th Annual Meeting of Particle Accelerator Society of Japan pp 73-76
[18] Sato Y et al 2018 Proceedings of IPAC2018
[19] Igarashi S et al 2018 Proceedings of HB2018