Staggered currents in the mixed state

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The current pattern in the mixed state of high-$T_c$ superconductors is studied in the U(1) mean field theory of the t-J model. Our findings are the following. 1) In the absence of antiferromagnetism a robust staggered current pattern exists in the core of vortices if the doping is not too high. 2) At a fixed doping and with increasing magnetic field, the size of the staggered current core expands, and eventually percolates. 3) The polarity of the staggered current is pinned by the direction of the magnetic field. 4) Vortex cores locally modify the hole density - in a staggered (non-staggered) core, the excess charge is slightly negative (positive). 5) Gutzwiller projection does not wash out the staggered current. Finally we present two experimental predictions concerning neutron scattering and STM spectra that capture the signature of the staggered current induced by the vortices.

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Recently, there has been a renewal of interest in the so-called staggered flux phase of the t-J model \cite{1}, where a circulating current produces a staggered orbital magnetic moment. According to the mean-field theory, \cite{2} the staggered flux phase, although never stable, is a close competitor of d-wave superconductivity when antiferromagnetism is absent. This situation can be aptly described by a Ginzburg-Landau (GL) theory with two competing order parameters. \cite{3} A serious concern is whether the staggered flux phase actually exists in the high-$T_c$ phase diagram. \cite{4}

Even if the staggered flux phase does not exist, GL theory would predict that in the core of a vortex, where the superconducting order is suppressed, the staggered flux order has a chance to appear. This is first pointed out forcefully in a recent paper by Lee and Wen \cite{10} from the viewpoint of their SU(2) mean field theory of the t-J model. Recently we have shown that the staggered current core also exists in the U(1) mean-field solution of the same model. \cite{11} Moreover we demonstrated that such current pattern survives the Gutzwiller projection which removes any double occupation in the mean field order.

The purpose of this paper is to address several important issues concerning the staggered current in the mixed state of high-$T_c$ superconductors. One of our main results is the prediction for two experiments which are relevant to the existence of staggered current in vortex cores. Our conclusion is based on the U(1) mean-field solution and its Gutzwiller projection.

Our starting point is the t-J model with Coulomb interaction,

\begin{equation}
H = -t \sum_{(ij)\sigma} (C^\dagger_{i\sigma} C_{j\sigma} e^{-iA_{ij}} + \text{h.c.}) + J \sum_{(ij)} \langle S_i \cdot S_j - \frac{n_i n_j}{4} \rangle + \frac{V}{2} \sum_{i \neq j} \frac{1}{r_{ij}} n_i n_j - \mu \sum_i n_i, \tag{1}
\end{equation}

In the above $A_{ij} = (2\pi e/\hbar c) \oint_{\mathbf{R}_i} \mathbf{A} \cdot d\mathbf{l}$ is the link phase produced by the physical vector potential $\mathbf{A}$. All other notations are standard. In the rest of the paper we use the parameter set $t = V = 3J$.

In the U(1) slave-boson approach the following replacements are made in Eq. (1): 1) $C^\dagger_{i\sigma} C_{j\sigma} \rightarrow f^\dagger_{i\sigma} f_{j\sigma} b_i b_j$, 2) $S_i \rightarrow (1/2) f^\dagger_{i\sigma} f_{j\sigma} f_{j\beta} f^\alpha_{i\beta} f_{j\gamma} f^\delta_{i\gamma}$, 3) $n_i \rightarrow 1 - b_i^\dagger b_i$, and 4) the occupation constraint $b_i^\dagger b_i + \sum_{f} f^\dagger_{i\sigma} f_{i\sigma} = 1$. In the above $b_i$ is the boson and $f_{i\sigma}$ is the fermion annihilation operator respectively. Our calculation is performed at zero temperature where the bosons are assumed to have condensed. The spin-exchange term is replaced by the following mean-field decoupling

\begin{equation}
S_i \cdot S_j \rightarrow \frac{3}{8} \sum_{\sigma} [K^{\dagger}_{ij} f^\dagger_{i\sigma} f_{j\sigma} + \text{h.c.}] - \frac{3}{8} [\Delta_{ij} (f_{i\uparrow} f_{j\uparrow} - f_{i\downarrow} f_{j\downarrow}) + \text{h.c.}] + \frac{3}{8} (|K_{ij}|^2 + |\Delta_{ij}|^2), \tag{2}
\end{equation}

where $K_{ij}$ and $\Delta_{ij}$, the mean-field hopping and pairing amplitudes, are determined self-consistently, together with the condensate boson order parameter $\langle b_i \rangle$ and the Lagrange multiplier that enforces the occupancy constraint. Due to the internal U(1) symmetry we can always choose the gauge so that $\langle b_i \rangle$ is real and positive. It is important to stress that the above decoupling excludes the spin magnetic moment ($S_i \cdot S$) as an order parameter, and hence will not be able to describe low doping regime in which commensurate/incommensurate spin magnetic order exists. Aside from the above restriction, no other constraint is placed on the mean-field parameters. The actual calculation is performed numerically on a $N_x \times N_y$ lattice under twisted boundary condition. Much of our treatment of the vortex lattice is the same as that in Ref. \cite{12}. The average magnetic flux density is $f = \frac{1}{N_x N_y}$ flux quanta, in units of $\hbar c/e$, per plaquette.
The flux density $f$ causes the pairing amplitude $\Delta_{ij}$ to wind by $4\pi$ around each $N_x \times N_y$ unit cell. The question concerning whether it is energetically more favorable to nucleate two $hc/2e$ vortices or a single $hc/e$ vortex in the unit cell has been raised in the literature. Our result shows unambiguously that it is the $hc/2e$ vortex that is favored. More specifically we have checked that even when the initial condition corresponds to a single $hc/e$ vortex, i.e. two $hc/2e$ vortices on top of each other, the final self-consistent solution always exhibits two separated $hc/2e$ vortices. We emphasize, however, that such conclusion will be subject to change if the strength of the Coulomb potential is modified. The reason for that is because the vortex core is charged!

In Figs.1(a) and (b) we show two dimensional map of the hole density in the unit cell (with two vortices) at doping levels $x = 10\%$ (a), and $15\%$ (b). The blue region marks lower and the red region marks higher hole density respectively. Thus the vortex core is negatively charged in (a) and positively charged for (b). The charge difference between the $x = 10\%$ and the $x = 15\%$ vortices turns out to be related to the presence/absence of staggered currents within the vortex core as we will discuss below.

![Fig. 1. Two vortices in a $24 \times 24$ lattice. (a) Hole distribution with $x = 10\%$; (b) Hole distribution at $x = 15\%$; (c) Bond current pattern corresponding to (a); (d) Bond current pattern corresponding to (b). The crosses in (c) and (d) highlight the vortex cores.](image)

In Fig.1(c) we present the current pattern near the core of the lower-left vortex in Fig.1(b), where the staggered current has disappeared.

![Fig. 2. Bond current patterns at (a) $f = 1/576$, (b) $1/144$, (c) $1/64$, and (d) $1/36$.](image)

Let us return to Fig.1(c), and ask what is the correlation, if any, between the vorticity of the current pattern and the direction of the magnetic field. The answer is that the circulation around the central plaquette where the vortex center reside is always opposite to that of the magnetic field. (This is in contrast to the case in Fig.1(d).) Moreover, we intentionally start with an initial condition where the circulation of the central plaquette is the same as that of the magnetic field, and find in the final self-consistent solution that the vortex center moves by one lattice spacing, so as to make the circulation of the central plaquette and the magnetic field opposite. Thus the magnetic field pins the polarity of the staggered current.

In Figs.2(a)-(d) we illustrate the evolution of the staggered current pattern found at 10\% doping over a wide range of magnetic field. The field strength is (a) $f = 1/576$, (b) $f = 1/144$, (c) $f = 1/64$, and (d) $f = 1/36$, respectively. In physical units the fields considered here are very large ($f = 1/1600$ roughly corresponds to a magnetic field of 10 Tesla). It is clear that as the vortices get closer their staggered current cores overlap and permeate the entire lattice, as shown in Figs.2(c) and (d). The end result is a state with uniform staggered current. This is a magnetic field induced co-existing d-wave superconducting and staggered flux state.

Although the lowest field considered in this paper (roughly 30 Tesla) is still very high from the experimental standpoint, none of our experimental predictions (as will be discussed later) will change qualitatively at a lower field.

Just as in our earlier findings, the staggered cur-
rent in the mixed state also survives Gutzwiller projection. As an example, Fig. 3 shows the staggered current pattern at \( x = 6.25\% \) and \( f = 1/64 \), before (a) and after (b) the Gutzwiller projection. Although the staggered current is weakened by the projection, it certainly does not disappear.

![Fig. 3. Bond current pattern at \( x = 6.25\% \) and \( f = 1/64 \). (a) Mean field result. (b) Result after Gutzwiller projection. The crosses mark the locations of the vortex cores.](image)

What are the experimental consequences of the staggered-current cores? Figures 1(c) and 2 suggest that in a neutron scattering experiment there will be a magnetic field induced Bragg peak near or even at \((\pi, \pi)\). Whether there is a peak precisely at \((\pi, \pi)\) depends on whether the vortex lattice is entirely contained in one of the bi-partite sublattices of the underlying \(CuO_2\) plane. The peaks around \((\pi, \pi)\) should be more robust. To make sure that the scattering near \((\pi, \pi)\) is induced by the vortex core, one can 1) study the dependence of the intensity and position of the Bragg peak on the magnetic field and 2) correlate the peak position near \((\pi, \pi)\) with that near \((0,0)\), which determines the structure of the vortex lattice. In Fig. 4 we present \(|m_k|^2\), where \(m_k\) is the Fourier transform of the lattice curl of the current \(i.e.,\) the directed sum of the bond current around a plaquette). In this case the vortex lattice is commensurate with one of the bi-partite sublattices of the underlying crystal structure. We expect elastic neutron scattering will show a similar pattern.

Seeing the magnetic field induced Bragg peaks near \((\pi, \pi)\) might be also consistent with vortex cores being spin-antiferromagnetic. Although it would be difficult to distinguish between the staggered orbital moments and staggered spin moments from the standpoint of neutron scattering, the latter is much less likely from theoretical considerations. Due to the presence of the external magnetic field and the vortex lattice, all symmetries that ensure the degeneracy between the two polarity of the staggered flux order parameter are broken. Thus the staggered flux in the vortex core is an induced, rather than spontaneous, order. This is not true for the spin antiferromagnetic order. Indeed, under the experimental condition, rotation of spins around the magnetic field direction remains an unbroken symmetry. As a result spin antiferromagnetic alignment in the vortex core requires a spontaneous symmetry breaking, which is not possible for a finite (small) system such as the core of a vortex.

![Fig. 4. \(|m_k|^2\) as a function of \(k\) corresponding to Fig. 2(a).](image)

In addition to the scattering experiment staggered vortex core might also cause detectable difference in the local tunneling spectra. In Fig. 5 we present the local tunneling density of state as a function of location. The arrows in (a) indicate the path (crosses mark the location of vortices) along which the local density of states is computed. Figure 5(b) is the result for \(x = 10\%\) where there is staggered current in the vortex core. Figure 5(c) is the result for \(x = 15\%\), where staggered current does not exist. The most significant difference occur at low ener-
gies. While there is a zero-bias peak in the core of the 15% vortex, there is no such peak in the core of the 10% vortex. Since the staggered current is known to open up a pseudogap the result is not surprising.

In general spin polarized tunneling experiment can be used to differentiate orbital versus spin antiferromagnetism. In the presence of the latter, the spin-dependent tunneling density of state should show a two-sublattice structure. We do not expect such effect for orbital antiferromagnetism, because a) the Zeeman splitting caused by the orbital moment is extremely small, and b) on a given site the staggered magnetic field originated from four neighboring plaquettes tends to cancel.

If the signatures of the staggered current core discussed above are seen in both the neutron and STM experiments it will constitute an extremely strong evidence for the existence of staggered current in the vortex core.

Finally, what do we learn from the existence/non-existence of the staggered current in the vortex core? From the beginning of high-$T_c$ physics, the t-J model has been identified as the model that captures the competing local interactions, i.e. charge hopping and spin antiferromagnetic exchange, of the cuprate materials. During the last fifteen years important progress has been made on numerically simulating this model. Nonetheless questions such as whether the ground state is homogeneous, and whether it is d-wave superconducting still remain controversial. On the other hand the mean-field theory of the t-J model has produced very tantalizing results. For example, it predicts that d-wave pairing is among the most pronounced ordering tendency of the model. Recently, to a limited extent, mean-field theory has also predicted the presence of stripe inhomogeneity.

In our opinion these successes give enough motivation to check whether another robust prediction of the mean-field theory of the t-J model has produced very tantalizing results. For example, it predicts that d-wave pairing is among the most pronounced ordering tendency of the model. Recently, to a limited extent, mean-field theory has also predicted the presence of stripe inhomogeneity. In our opinion these successes give enough motivation to check whether another robust prediction of the mean-field theory, the close competition of the staggered flux phase with d-wave pairing, is actually correct. The presence of staggered current in the core of a vortex is a direct consequence of this competition. If such an ordered current pattern is observed, it is reasonable to argue that we have understood the rudimentary ordering tendency of the model. On the other hand, in the face of negative experimental results we should seriously worry about the validity of the mean-field theory, or perhaps even about our understanding of the relevant local interactions.

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