Computational-experimental model of fatigue growth of the welded joint crack

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Abstract. Welded joints are commonly used in various industries, and in many cases, they contain an initial crack, which could lead to huge damage. For this reason, it is essential to have a model to predict fatigue evolution of the welded joint crack. This paper is dedicated to the method of constructing the computational-experimental model of the fatigue growth of the welded joint crack. The described method is based on a numerical calculation using finite element modelling and Paris-Erdogan equation of the crack growth. Besides, the fatigue experiments of the test sample with a welded joint were conducted and the model of crack growth was constructed with the use of the obtained experimental data.

1. Introduction
Welded joints are widely used in all industries from automotive to aerospace industry. At the same time, a lot of welded joints contain local defects, which is hardly identified and could lead to construction damage [1, 2]. Apart from that, it is important to diagnose a growth of the crack and to predict its critical condition for timely service.

One of the most common diagnostic methods is an experimental modal analysis by means of which we could find changes in construction state by changes in its dynamic response [3, 4]. To achieve it we need to know a relation between the changes in structure dynamic response and its residual life [5], which could be found by fatigue tests. A problem of the fatigue test is its duration. An acceleration of the tests can be reached by increasing a load frequency, which could be made by resonance testing on a shaker. The disadvantage of such testing is unstable loading caused by changing of construction stiffness and damping, which leads to computational difficulties.

In this paper, the method of conducting such tests using finite element (FE) simulation of resonance vibrations of tested samples and obtained computational-experimental model of fatigue growth of the welded joint crack are described.

2. Description of experimental apparatus and methods used
2.1. Investigated object
In this work we investigated the growth of initial crack in welded joint in steel beam. Beams used in experiment are shown in Figure 1. Beams were made from two steel strip, which the width and thickness of 30 mm and 4 mm, accordingly, with the use of single butt-joint seam. Further, beams were clamped by the short strip on the experimental stand and the long strip hangs as cantilever so the butt joint was on its base.
On the other side of the seam there was the lack of fusion, which had various length $l_{\text{init}}$. The lack of fusion was an initial crack, which was growing during the experiment.

2.2. Experimental stand
Experiments were held on the SignalForce electrodynamic shaker by Data Physics. The tested beams were fixed on the shaker by a rigid holder with accurate machined contact surfaces to reduce influence of the bolted joint on dynamic behaviour of the system [6]. Tests were conducted in the resonance for increasing of the beam response amplitude. Test samples were designed with the first natural frequency near 120Hz to reduce a testing time.

2.3. Stress intensity factor and crack growth rate
Accordingly to the Paris-Erdogan law [7, 8, 9], the evolution of crack could be obtained as the solution of differential equation:

$$
\frac{dl}{dN} = C(\Delta K)^m,
$$

(1)

Figure 1. Beam used in experiments.

Figure 2. Scheme of the experimental stand: 1 – shaker excitation feedback; 2 – beam response feedback; 3 – hammer excitation feedback.

Figure 3. Shaker with mounted test sample.
where $l$ — a crack length, $\Delta K = K_{max} - K_{min}$ — an amplitude of the stress intensity factor, which depends on a stress in the crack area and the crack length, $C$ and $m$ — empirical coefficients for the certain material.

The investigated beam could be considered as flat plate under the bending moment with crack of a 1st mode [7]. In case of loading by the periodically changing bending moment, the minimum of the stress intensity factor will be equal to zero and its amplitude could be calculated by the following equation [10]:

$$K_I = F_I(a) \frac{6M}{wh^2} \sqrt{\pi l}$$  \hspace{1cm} (2)

where $M$ is an amplitude of the bending moment in the crack area, $w$ and $h$ are strip width and thickness correspondingly, $F_I(a) = 1.122 - 1.40a + 7.33(a)^2 - 13.08(a)^3 + 14.0(a)^4$ for $a = \frac{l}{h}$.

2.4. Vibration simulation using FEM

To calculate the bending moment we used a finite elements method in the statement of flat dynamical problem with beam elements. According to the [11, 12] vibration of the body under harmonic load with circular frequency $\omega$ can be described by differential equation

$$M \ddot{u} + C \dot{u} + Ku = fe^{i\omega t},$$  \hspace{1cm} (3)

where $M$, $C$, $K$ – global mass, damping and stiffness matrices of the body, $u$ – vector of displacements. Assuming that the solution of the (3) is limited by the excitation force frequency, we will find it in a form

$$u(t) = \sum_j v_j e^{i\omega t},$$  \hspace{1cm} (4)

where $v_j$ is a normalized vector of displacements corresponded to $j^{th}$ body eigenform (or mode shape) and $v_j$ — its corresponding amplitude. Eigenvectors could be approximately obtained from eigendecomposition of the matrix $A$ [13], which is

$$A = M^{-1}K.$$  \hspace{1cm} (5)

The amplitude of $j^{th}$ eigenform could be found from solution of (3) or in case of uncertain damping matrix $C$ amplitudes $v_j$ it would be easier to find it directly from known response data:

$$v_j = \frac{a_{z0}}{\omega^2 v_j(z_0)},$$  \hspace{1cm} (6)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{flowchart.png}
\caption{Flow chart of the acquisition program algorithm.}
\end{figure}
where \( a_{z_0} \) — acceleration of the measured point with coordinate \( z_0 \), \( v_j(z_0) \) — displacement of that point corresponded to the \( j^{th} \) normalized eigenform in that point.

Vector of displacements of the certain point of the body is calculated by:

\[ v_j(z) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} N^e v^e_j \]  

(7)

where \( N^e \) — form function matrix of the \( e^{th} \) element and \( v^e_j \) is a local displacements vector of \( j^{th} \) normalized eigenform, components of which are corresponding to certain components of \( v_j \).

Computation of local displacements vector \( u^e \) from the global displacements vector \( u \) could be written as

\[ u^e = Tu, \]  

(8)

where \( T_{n \times m} \) is translate matrix with \( m \) - number of allowed element displacements and \( m \) - number of allowed body displacements. In (8) vector \( u \) could be changed by \( v^e_j \).

The element stiffness and mass matrices can be evaluated by following equations

\[
K^e = \frac{E I}{l} \begin{pmatrix}
12 & 6l & -12 & 6l \\
6l & 4l^2 & -6l & 2l^2 \\
-12 & -6l & 2 & -6l \\
6l & 2l^2 & -6l & 4l^2
\end{pmatrix}
\]  

and

\[
m^e = \frac{\rho Al}{4l^2} \begin{pmatrix}
156 & 22l & 54 & -13l \\
22l & 4l^2 & 13l & -3l^2 \\
54 & 13l & 156 & -22l \\
-13l & -3l^2 & -22l & 4l^2
\end{pmatrix},
\]  

where \( E \) — Young’s modulus of the beam material, \( \rho \) — density of the beam material, \( A \) — cross section area, \( I \) — cross section moment of inertia, \( l \) — element length. Then the global stiffness matrix can be simply evaluated as

\[ K_{n \times n} = \sum_{e=1}^{N} K^e_{n \times n}, \]  

(9)

where \( K^e_{n \times n} \) — local stiffness matrix \( K^e \) extended to the global one dimension, \( N \) — number of elements in mesh. The global mass matrix can be evaluated (assembled) by the same way.

Next step is calculation of the internal bend moment, caused by dynamical bending of the beam. The internal moment of bent beam is

\[ M(z) = EI \frac{\partial^2 u}{\partial z^2} \]  

(10)

The second derivative of the displacements for each element could be found as

\[ \frac{\partial^2 u}{\partial z^2} \bigg|_{z \in L^e} = Bu^e \]  

(11)

where \( B \) — matrix of deformations of the element and \( u^e \) is an elements displacements vector, which components are corresponding components of the global displacements vector \( u \). Accordingly, the bending moment will be simply computed by substitution of (11) into (10).

3. Tests results
3.1. Obtained experimental data
An example of obtained response amplitudes and frequencies time data is presented in Figure 5. Since the system damping had been increasing during the experiments due to crack growth, the response amplitude was decreased during the tests and we were needed to increase the amplitude of excitation to hold the high crack growth rate. For this reason, the test was hold by on five parts, each of them has a different excitation signal level (Figure5–a). And as a result, the total amplitude-time relation has a chaotically changing character.

Furthermore, as it is shown in Figure 5–b, the frequency changes in the stair-step manner due to the low accuracy of frequency identification methods used. Its makes us impossible to calculate the crack growth rate directly from the experiments.
Figure 5. Example of amplitude (a) and resonance frequency evolution (b) during the test.

Table 1. Identified model parameters.

| Parameter         | Beam no. 1   | Beam no. 2   |
|-------------------|--------------|--------------|
| $C$               | $1.024^{-11}$| $1.063^{-11}$|
| $m$               | 3.35         | 3.3          |
| $l_{init}$, mm    | 1.4          | 1.2          |

3.2. Preparing of the data and computation algorithm

First step in the calculation is replacement of time variable $t$ in obtained experimental sequences $a_{max}(t)$ and $f_r(t)$ (which equal to the first natural frequency) to cycle numbers $N$. This replacement could be made by the following:

$$N(t) = \int_0^t f_r(\tau) d\tau. \quad (12)$$

Further we can solve the DE (1) using Euler methods, so the crack length evolution would be found in iterative calculation:

$$\begin{cases} l_i = l_{i-1} + d l_{i-1} = l_{i-1} + C \left( K_1 (a_{i-1}, l_{i-1}) \right) d N_{i-1} \\ l_0 = l_{init} \end{cases} \quad (13)$$

There are three unknown parameters in (13) — $C$, $m$ and in general $l_{init}$ — which could be identified.

After the crack length calculated we can calculate natural frequency of the beam. The relation between crack length $l$ and the first natural frequency of the beam $f_1$ was obtained by parametric numerical modal analysis of flat beam with transverse crack, with use of ANSYS Workbench. The mesh used is shown in Figure 6. The elements PLANE183 are used for meshing. Size of the elements was approximately 0.2 mm in the crack area and 1 mm beyond the crack. The overall number of the elements was 3092. The obtained relation is plotted in Figure 7.
Figure 6. Finite element mesh.

Figure 7. Relation between crack length $l$ and first natural frequency of the beam $f_1$.

3.3. Results
The calculated evolution of the crack length and caused changes in natural frequency during the destruction process are shown in Figure 8. Values of identified parameters in Paris-Erdogan model (1) presented in table 1. In the current work these parameters were identified manually.

As we can see in Figure 8-b, proposed method gives a good propagation of the first natural frequency evolution, except the last curve section, where curve character remains, but quantitative values give an error. According to the authors opinion, this effect can be explained by an increased non-linearity of vibration of the strip with large crack.

Figure 8. Evolution of the crack length (a) and first natural frequency (b).

4. Conclusion
In this work the computational-experimental diagnostic model of fatigue growth of the crack in welded joint root of the flat strip based on Paris-Erdogan low was proposed. This model allow to estimate residual life of the welded plate by known current first natural frequency, which could be obtained by the experimental modal analysis of the part. Calculation of crack growth shows a good convergence with the experimental data.

It is planned to proceed further works in three directions:
(i) Develop the method to determine the model parameters by means of optimization;
(ii) Consider the non-linear behaviour of the body with large crack under the cyclic loading;
(iii) Consider a non-linear fatigue crack growth, which could be described by the more complicated models instead of Paris-Erdogan model.

References
[1] Zakharov M N, Morozov E M and Nasonov V A 2015 Assessing the Risk of Welding Defects Russian Engineering Research 35/11 pp 846–849
[2] Makhutov N A, Pokrovskii A M and Dubovitskii E I 2019 Analysis of Crack Resistance of an Oil Trunk Pipeline Considering the Varying Failure Viscosity in the Neighborhood of a Welded Joint Journal of Machinery Manufacture and Reliability 48/1 pp 35–42
[3] Geradin M and Rixen D J 2015 Mechanical vibrations: theory and application to structural dynamics (John Wiley & Sons) p 598
[4] Zhulev V and Kuts M 2018 Contact models verification by the finite element model updating method based on the calculation of the sensitivity coefficient MATEC Web of Conferences pp 02008
[5] Akulenko L D, Gavrikov A A, and Nesterov S V 2018 Natural Vibrations of a Liquid-Transporting Pipeline on an Elastic Base Mechanics of Solids 53/1 pp 101–110
[6] Kuts M and Zhulev V 2019 Two approaches for determination of contact layer stiffness Proceedings of the 8th IOMAC - International Operational Modal Analysis Conference Kopenhagen
[7] Pestrikov V M and Morozov E M 2012 Mekhanika razrushenija. Konspekt lektsij (Fracture mechanics. Lectures) (St.-Petersburg: Professija) p 552
[8] Chernyatin A S, Matvienko Y G and Razumovsky I A 2018 Fatigue Surface Crack Propagation and Intersecting Cracks in Connection With Welding Residual Stresses Fatigue & Fracture of Engineering Materials & Structures 41/10 pp 2140–2152
[9] Pokrovskii A M, Dubin D A and Vdovin D S 2019 Fatigue-Crack Propagation in Torsional Shafts Within the Suspension of High-Speed Caterpillar Vehicles Russian Engineering Research 39/7 pp 548–555
[10] Murakami Y, Aoki S, Hasebe N, Itoh Y, Miyata H, Miyazaki N, Terada H, Tohgo K, Toya M and Yuuki R 1987 Stress intensity factors handbook vol 1 (Pergamon Press) p 1464
[11] Felippa C A 2004 Introduction to Finite Element Methods (University of Colorado) p 620
[12] Kim N H 2014 Introduction to Nonlinear Finite Element Analysis (N.Y.:Springer) p 430
[13] Zienkiewicz O C, Taylor R L and Zhu J Z 2005 The Finite Element Method: Its Basis and Fundamentals (Oxford: Elsevier Butterworth-Heinemann) p 752