Towards mass-deformed ABJM compactified on a spindle

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Abstract

By following the work of Arav, Gauntlett, Roberts and Rosen constructing the gravity dual of 4d Leigh-Strassler theory compactified on a spindle, we take the first step towards the gravity dual of 3d mass-deformed ABJM theory on a spindle. We derive the BPS equations necessary to construct the $AdS_2 \times \Sigma$ solutions asymptotic to the Warner $\mathcal{N} = 2$ $AdS_4$ vacuum dual to the mass-deformed ABJM where $\Sigma$ is a spindle. We find the flux quantization through the spindle. However, we were not able to fix the boundary conditions of the solution to explicitly find solutions numerically. We leave this problem for the future study.

November, 2022
1 Introduction

Topological twist is a systematic way of realizing supersymmetry in field theory, [1], and in gravity, [2], with a wide range of applications. Recently, a novel class of anti-de Sitter solutions were discovered which realize supersymmetry in different ways from the topological twist. The solutions are obtained from branes wrapping an orbifold, namely, a spindle, [3]. The spindle, \( \Sigma \), is an orbifold, \( \mathbb{C}P^1_{[n_-,n_+]} \), with conical deficit angles at two poles. The spindle numbers, \( n_- \), \( n_+ \), are arbitrary coprime positive integers. The solutions were first constructed from D3-branes, [3, 4, 5], and then applied to other branes: M2-branes, [6, 7, 8, 9], M5-branes, [10], D4-branes, [11, 12], mass-deformed D3-branes, [13], and D2-branes, [14]. Furthermore, two possible ways of realizing supersymmetry, topologically topological twist and anti-twist, were classified, [15, 16].
The orbifold solutions with a single conical deficit angle, namely, a half spindle, is topologically a disk. The $AdS_5$ solutions from M5-branes wrapped on a topological disk were first constructed in \cite{17,18}, and were proposed to be the gravity dual to a class of 4d $\mathcal{N}=2$ Argyres-Douglas theories, \cite{19}. See also \cite{20,21} for further studies. Solutions from other branes wrapped on a topological disk were constructed: D3-branes, \cite{22,23}, M2-branes, \cite{24,25}, and D4-branes, \cite{26}. The disk solutions are locally identical to spindle solutions, but originate from a different global completion. See also \cite{26} for more disk solutions and \cite{27} for defect solutions from the other different completion of global solutions.

The line of study was soon generalized to branes wrapping higher-dimensional orbifolds, in particular, four-dimensional orbifolds: the direct product of a spindle and a constant curvature surface. M5-branes on $\Sigma \times \Sigma$, \cite{5,28}, D4-branes on $\Sigma \times \Sigma$, \cite{12,11,28}, and M5-branes on $\Sigma \times H^3$, \cite{14}, were constructed where $\Sigma$ is a spindle or a disk and $\Sigma$ is a Riemann surface of genus, $g$. Furthermore, branes wrapped on a spindle fibered over another spindle or a Riemann surface were also constructed. In \cite{29} M5-branes wrapped on $\Sigma_1 \times \Sigma_2$ and $\Sigma \times \Sigma$ were constructed. Then, D4-branes wrapped on $\Sigma_1 \times \Sigma_2$ and $\Sigma \times \Sigma$ were found in \cite{30,31}. See \cite{11,31,32,33} also for the recent development of gravitational blocks, \cite{34}, and entropy functions for spindle solutions.

A common feature of the examples we have listed is that they could asymptote to the maximally supersymmetric vacuum in their respective dimensions. Via the AdS/CFT correspondence, \cite{35}, they are dual to 3d ABJM theory, 4d $\mathcal{N}=4$ super Yang-Mills theory, 5d $USp(2N)$ gauge theory and 6d $(2,0)$ theory, respectively. However, in \cite{13} by Arav, Gauntlett, Roberts and Rosen, it was shown that we can construct orbifold solutions asymptote to the $AdS$ vacua with lower supersymmetries: $AdS_3 \times \Sigma$ solution asymptotic to the $\mathcal{N}=2$ AdS$_5$ vacuum, \cite{37,38,39}, dual to the Leigh-Strassler SCFTs, \cite{40}, was constructed. Furthermore, holographically, central charge was computed and matched with the field theory result from the Leigh-Strassler theory on $\mathbb{R}^{1,1} \times \Sigma$.

In this work, we make the first step to construct the $AdS_2 \times \Sigma$ solutions asymptotic to the $\mathcal{N}=2$ AdS$_4$ vacuum, \cite{41,42}, which is dual to the 3d mass-deformed ABJM theory, \cite{43,44}. The holographic RG flow from the $\mathcal{N}=8$ AdS$_4$ vacuum dual to the ABJM theory to the $\mathcal{N}=2$ AdS$_4$ vacuum, \cite{41,42}, dual to the mass-deformed ABJM was constructed in \cite{45,46} and uplifted to eleven-dimensional supergravity in \cite{47}. In light of the discovery of ABJM theory, \cite{36}, the mass-deformed ABJM theory was further understood in \cite{43,44,48}. See \cite{49} also for the gravity calculation of three-sphere free energy of mass-deformed ABJM theory. The supersymmetric black hole solutions interpolating the $\mathcal{N}=2$ AdS$_4$ vacuum and the horizon of $AdS_2 \times \Sigma$ was constructed in \cite{50}.

We will employed the $U(1)^2$-invariant truncation, \cite{50}, of $SO(8)$-gauged $\mathcal{N}=8$ supergravity.
in four dimensions, \[51\] which is a consistent truncation of eleven-dimensional supergravity, \[52\], on a seven-dimensional sphere, \[53\]. We will derive the BPS equations for the \(AdS_2 \times \Sigma\) solutions asymptotic to the \(\mathcal{N} = 2\) \(AdS_4\) vacuum dual to the mass-deformed ABJM. We will study the BPS equations and find the flux quantizations through the spindle, \(\Sigma\). However, we were not able to fix the boundary values of the fields, \(e.g.,\) the scalar fields and the warp factors in the metric. Thus we could not construct the explicit numerical solutions and calculate the Bekenstein-Hawking entropy of the presumed black holes. We will elaborate the difficulties in section 3.3.1. We leave all these important questions for the future study. On the other hand, in the subtruncation to minimal gauged supergravity associated with the Warner vacuum, we find a class of analytic \(AdS_2 \times \Sigma\) solutions and their Bekenstein-Hawking entropy is calculated.

For the structure of the work, we will closely follow \[13\] as it is well organized and also to facilitate the comparison.

In section 2 we review the \(U(1)^2\)-invariant truncation. In section 3 we study the BPS equations and consider the flux quantizations. In section 3, for minimal gauged supergravity associated with the Warner fixed point, we obtain an analytic solution of \(AdS_2 \times \Sigma\) and calculate the Bekenstein-Hawking entropy. In section 4 we conclude. In appendix A we review the construction of the \(U(1)^2\)-invariant truncation. In appendix B we present the derivation of the BPS equations.

## 2 The supergravity model

We consider the \(U(1)^2\)-invariant truncation of \(SO(8)\)-gauged \(\mathcal{N} = 8\) supergravity in four dimensions, \[51\], which was studied in \[50\]. See appendix A for details. The field content is the metric, four \(U(1)\) gauge fields, \(A^a, a = 0, \ldots, 3\), and four complex scalar fields, \((\chi, \psi)\) and \((\lambda_i, \varphi_i), i = 1, \ldots, 3\). We set \(\varphi_i = \pi\). We employ mostly plus signature. The bosonic Lagrangian of the truncation is given by

\[
e^{-1} \mathcal{L} = \frac{1}{2} R - \partial_\mu \chi \partial^\mu \chi - \frac{1}{4} \sinh^2 (2\chi) D\chi D^{\mu} \chi - \sum_{i=1}^{3} \partial_\mu \lambda_i \partial^\mu \lambda_i - g^2 \mathcal{P} - \frac{1}{4} \left[ e^{-2(\lambda_1 + \lambda_2 + \lambda_3)} F_{\mu\nu} F^{0\mu\nu} + e^{-2(\lambda_1 - \lambda_2 - \lambda_3)} F_{\mu\nu}^{1} F^{1\mu\nu} + e^{2(\lambda_1 - \lambda_2 + \lambda_3)} F_{\mu\nu}^{2} F^{2\mu\nu} + e^{2(\lambda_1 + \lambda_2 - \lambda_3)} F_{\mu\nu}^{3} F^{3\mu\nu} \right],
\]

(2.1)

where we define

\[
D\chi \equiv d\chi + g \left( A^0 - A^1 - A^2 - A^3 \right).
\]

(2.2)

The scalar potential is given by

\[
\mathcal{P} = \frac{1}{2} \left( \frac{\partial \mathcal{W}}{\partial \chi} \right)^2 + \frac{1}{2} \sum_{i=1}^{3} \left( \frac{\partial \mathcal{W}}{\partial \lambda_i} \right)^2 - \frac{3}{2} \mathcal{W}^2,
\]

(2.3)
where the superpotential is
\[ W = \frac{2e^{2(\lambda_1 + \lambda_2 + \lambda_3)} \sinh^2 \chi - \cosh^2 \chi \left( e^{2(\lambda_1 + \lambda_2 + \lambda_3)} + e^{2\lambda_1} + e^{2\lambda_2} + e^{2\lambda_3} \right)}{2e^{\lambda_1 + \lambda_2 + \lambda_3}}. \quad (2.4) \]

The BPS equations are obtained by setting the fermionic supersymmetry variations to vanish. The spin-3/2 field variations reduce to
\[ \left[ 2\nabla_\mu - iB_\mu - \frac{g}{\sqrt{2}} W\gamma_\mu - i \frac{1}{2\sqrt{2}} H_{\nu\rho} \gamma^{\nu\rho} \gamma_\mu \right] \epsilon = 0, \quad (2.5) \]

where \( \epsilon \) is a complex Dirac spinor and we define
\[ H_{\mu\nu} \equiv F_{\mu\nu}^{78} = \frac{1}{2} \left( e^{-\lambda_1 - \lambda_2 - \lambda_3} F_{\mu\nu}^{0} + e^{-\lambda_1 + \lambda_2 + \lambda_3} F_{\mu\nu}^{-1} + e^{\lambda_1 - \lambda_2 + \lambda_3} F_{\mu\nu}^{-2} + e^{\lambda_1 + \lambda_2 - \lambda_3} F_{\mu\nu}^{-3} \right), \]
\[ B_\mu \equiv - g \left( A_\mu^0 + A_\mu^1 + A_\mu^2 + A_\mu^3 \right) - \frac{1}{2} (\cosh (2\chi) - 1) D_\mu \psi. \quad (2.6) \]

The R-symmetry gauge field which charges the supersymmetry parameters is
\[ A_\mu^R \equiv - g \left( A_\mu^0 + A_\mu^1 + A_\mu^2 + A_\mu^3 \right). \quad (2.7) \]

From the spin-1/2 field variations, we obtain
\[ \left[ \gamma^\mu \partial_\mu \lambda_1 + \frac{g}{\sqrt{2}} \partial_\lambda_1 W + i \frac{1}{2\sqrt{2}} \gamma^{\mu\nu} F_{\mu\nu}^{-12} \right] \epsilon = 0, \]
\[ \left[ \gamma^\mu \partial_\mu \lambda_2 + \frac{g}{\sqrt{2}} \partial_\lambda_2 W + i \frac{1}{2\sqrt{2}} \gamma^{\mu\nu} F_{\mu\nu}^{-34} \right] \epsilon = 0, \]
\[ \left[ \gamma^\mu \partial_\mu \lambda_3 + \frac{g}{\sqrt{2}} \partial_\lambda_3 W + i \frac{1}{2\sqrt{2}} \gamma^{\mu\nu} F_{\mu\nu}^{-56} \right] \epsilon = 0, \]
\[ \left[ \gamma^\mu \partial_\mu \chi + \frac{g}{\sqrt{2}} \partial_\chi W + i \frac{1}{2\sqrt{2}} \partial_\chi B_\mu \gamma^\mu \right] \epsilon = 0. \quad (2.8) \]

The scalar potential has the maximally supersymmetric \( SO(8) \)-invariant \( AdS_4 \) vacuum with all scalar fields vanishing and \( P_* = -6 \). The vacuum uplifts to the \( AdS_4 \times S^7 \) solution of eleven-dimensional supergravity and is dual to the ABJM theory, [36]. There is another \( N = 2 \) supersymmetric \( SU(3) \times U(1)_R \)-invariant \( AdS_4 \) vacuum known as the Warner fixed point, [41, 42],
\[ \tanh \chi = \frac{1}{\sqrt{3}}, \quad \psi = \frac{\pi}{2}, \quad \tanh \lambda_i = 2 - \sqrt{3}, \quad \varphi_i = \pi, \quad (2.9) \]
with \( P_* = -\frac{9\sqrt{3}}{2} \). The vacuum uplifts to eleven-dimensional supergravity, [47], and is dual to the mass-deformed ABJM theory, [43, 44]. There is a holographic RG flow from the \( SO(8) \) vacuum to the \( SU(3) \times U(1)_R \) vacuum, [45, 46, 48]. The radius of the \( AdS_4 \) is given by
\[ L_{AdS_4}^2 = -\frac{3}{g^2 P_*} = \begin{cases} 
\frac{1}{2g^2}, & \text{SO}(8), \\
2, & \text{SU}(3) \times U(1)_R, \\
\frac{3\sqrt{3}g^2}{2}, & \text{SU}(3) \times U(1)_R.
\end{cases} \quad (2.10) \]
Further details of the vacua in this truncation can be found in [54].

From the AdS/CFT correspondence, the free energy of pure AdS$_4$ with an asymptotic boundary of $S^3$ is given by e.g., [50],

$$ F_{S^3} = \frac{\pi L_{AdS_4}^2}{2G_N^{(4)}}, \quad (2.11) $$

where $G_N^{(4)}$ is the four-dimensional Newton’s constant. This matches with the field theory free energy of the ABJM theory and the mass-deformed ABJM theory at large $N$, respectively,

$$ F_{S^3}^{\text{ABJM}} = \frac{\sqrt{2}\pi}{3} N^{3/2}, $$
$$ F_{S^3}^{\text{mABJM}} = \frac{4\sqrt{2}\pi}{9\sqrt{3}} N^{3/2}. \quad (2.12) $$

Furthermore, the ratio below is universal, [55],

$$ \frac{F_{S^3}}{L_{AdS_4}^2} = \frac{2\sqrt{2}\pi g}{3} N^{3/2}. \quad (2.13) $$

When the complex scalar field, $(\chi, \psi)$, is vanishing, the truncation reduces to the STU model with three scalar fields and four gauge fields. By setting all gauge fields equal and all scalar fields vanish, the truncation further reduces to minimal gauged supergravity. See appendix A for details.

3 \quad AdS$_2$ ansatz

We consider the background with the gauge fields,

$$ ds^2 = e^{2V} ds_{AdS_2}^2 + f^2 dy^2 + h^2 dz^2, \quad A^\alpha = a^\alpha dz, \quad (3.1) $$

where $ds_{AdS_2}^2$ is a unit radius metric on $AdS_2$ and $V$, $f$, $h$, and $a^\alpha$, $\alpha = 0, \ldots, 3$, are functions of the coordinate $y$ only. The scalar fields, $\chi$ and $\lambda_i$, $i = 1, \ldots, 3$, are also functions of the coordinate $y$. In order to avoid partial differential equations from the equations of motion for gauge fields, we take the scalar field, $\psi$, to be $\psi = \bar{\psi}z$ where $\bar{\psi}$ is a constant. This brings

$$ B_\mu dx^\mu \equiv B_z dz, \quad (3.2) $$

where $B_z$ is a function of the coordinate $y$.

We employ an orthonormal frame,

$$ e^a = e^V e^a, \quad e^2 = f dy, \quad e^3 = h dz, \quad (3.3) $$
where $\bar{e}^\alpha$ is an orthonormal frame for $dS^2_{AdS_2}$. The frame components of the field strengths are given by
\[ F^\alpha_{23} = f^{-1} h^{-1} (a^\alpha)', \]
(3.4)

The equations of motion for gauge fields are combined and integrated to give the integrals of motion,
\[ e^{2V} \left( e^{-2\lambda_1+2\lambda_2+2\lambda_3} F^0_{23} - e^{2\lambda_1-2\lambda_2+2\lambda_3} F^2_{23} \right) = \mathcal{E}_{F_1}, \]
\[ e^{2V} \left( e^{2\lambda_1-2\lambda_2+2\lambda_3} F^0_{23} - e^{2\lambda_1+2\lambda_2-2\lambda_3} F^2_{23} \right) = \mathcal{E}_{F_2}, \]
\[ e^{2V} \left( e^{2\lambda_1+2\lambda_2-2\lambda_3} F^3_{23} - e^{-2\lambda_1+2\lambda_2+2\lambda_3} F^1_{23} \right) = \mathcal{E}_{F_3}, \]
(3.5)
and
\[ e^{2V} \left( e^{-2\lambda_1-2\lambda_2-2\lambda_3} F^0_{23} + e^{-2\lambda_1+2\lambda_2+2\lambda_3} F^1_{23} \right) = \mathcal{E}_{R_1}, \]
\[ e^{2V} \left( e^{-2\lambda_1-2\lambda_2-2\lambda_3} F^0_{23} + e^{2\lambda_1-2\lambda_2+2\lambda_3} F^2_{23} \right) = \mathcal{E}_{R_2}, \]
\[ e^{2V} \left( e^{-2\lambda_1-2\lambda_2-2\lambda_3} F^0_{23} + e^{2\lambda_1+2\lambda_2-2\lambda_3} F^3_{23} \right) = \mathcal{E}_{R_3}, \]
(3.6)
with
\[ (e^{2V-2\lambda_1+2\lambda_2-2\lambda_3} F^3_{23})' = -e^{2V} fh^{-1} \frac{1}{2} \sinh^2 (2\chi) D_z \psi, \]
(3.7)
where $\mathcal{E}_{F_i}$ and $\mathcal{E}_{R_i}$ are constant. Among the six integrals of motion in (3.5) and (3.6), only three of them are independent, e.g., three in (3.6), and others can be obtained by combining the three independent ones.

### 3.1 BPS equations

We employ the gamma matrices,
\[ \gamma^m = \Gamma^m \otimes \sigma^3, \quad \gamma^2 = \mathbb{1}_2 \otimes \sigma^1, \quad \gamma^3 = \mathbb{1}_2 \otimes \sigma^2, \]
(3.8)
where $\Gamma^m$ are two-dimensional gamma matrices of mostly plus signature. The spinors are given by
\[ \epsilon = \psi \otimes \chi, \]
(3.9)
and the two-dimensional spinor on $AdS_2$ satisfies
\[ D_m \psi = \frac{1}{2} \kappa \Gamma_m \psi, \]
(3.10)
where $\kappa = \pm 1$ fixes the chirality. The spinor, $\chi$, is given by
\[ \chi = e^{V/2} e^{isz} \begin{pmatrix} \sin \frac{\xi}{2} \\ \cos \frac{\xi}{2} \end{pmatrix}, \]
(3.11)
where the constant, $s$, is the gauge dependent charge under the action of azimuthal Killing vector, $\partial_z$.

We consider the case of $\sin \xi \neq 0$. The complete BPS equations are derived in appendix B and are given by

\[
\begin{align*}
  f^{-1} \xi' &= \sqrt{2} g W \cos \xi + \kappa e^{-V}, \\
  f^{-1} V' &= \frac{g}{\sqrt{2}} W \sin \xi, \\
  f^{-1} \chi'_i &= -\frac{g}{\sqrt{2}} \partial_{\chi_i} W \sin \xi, \\
  f^{-1} \chi' &= -\frac{g}{\sqrt{2}} \partial_{\chi} W, \\
  f^{-1} h' &= \frac{1}{\sin \xi} \left( \kappa e^{-V} \cos \xi + \frac{g W}{\sqrt{2}} (1 + \cos^2 \xi) \right),
\end{align*}
\]

(3.12)

with two constraints,

\[
\begin{align*}
  (s - B_z) \sin \xi &= -\sqrt{2} g W h \cos \xi - \kappa h e^{-V}, \\
  2g \partial_{\chi} W \cos \xi &= \partial_{\chi} B_z \sin \xi h^{-1}.
\end{align*}
\]

(3.13)

The scalar-field dressed field strengths are given by

\[
\begin{align*}
  F^{-12}_{23} &= -g \partial_{\lambda_1} W \cos \xi, \\
  F^{-34}_{23} &= -g \partial_{\lambda_2} W \cos \xi, \\
  F^{-56}_{23} &= -g \partial_{\lambda_3} W \cos \xi, \\
  H_{23} &= -g W \cos \xi - \sqrt{2} \kappa e^{-V}.
\end{align*}
\]

(3.14)

We have checked that the BPS equations solve the equations of motion from the Lagrangian in (2.1) as presented in appendix A.1.

### 3.2 Integrals of motion

Following the observation made in [13], we find an integral of the BPS equations,

\[
  h e^{-V} = k \sin \xi,
\]

(3.15)

where $k$ is a constant. Hence, at the poles of the spindle solution, $h = 0$, we also have $\sin \xi = 0$. From (3.12) and (3.13), we obtain

\[
  \xi' = -k^{-1} \left( s - B_z \right) \left( e^{-V} f \right),
\]

(3.16)
and then the two constraints in (3.13) can be written as

\[
(s - B_z) = -k \left[ \sqrt{2} g W e^V \cos \xi + \kappa \right],
\]

\[
2g \partial_\chi W \cos \xi = k^{-1} e^{-V} \partial_\chi B_z. \tag{3.17}
\]

By employing the field strengths in (3.14), we can express the integrals of motion by

\[
\mathcal{E}_{R_1} = e^V \left[ 2g e^V \cos \xi - \sqrt{2} k e^{-\lambda_1} \cosh (\lambda_2 + \lambda_3) \right],
\]

\[
\mathcal{E}_{R_2} = e^V \left[ 2g e^V \cos \xi - \sqrt{2} k e^{-\lambda_2} \cosh (\lambda_3 + \lambda_1) \right],
\]

\[
\mathcal{E}_{R_3} = e^V \left[ 2g e^V \cos \xi - \sqrt{2} k e^{-\lambda_3} \cosh (\lambda_1 + \lambda_2) \right], \tag{3.18}
\]

and

\[
\mathcal{E}_{F_1} = \sqrt{2} k e^V e^{\lambda_3} \sinh (\lambda_1 - \lambda_2),
\]

\[
\mathcal{E}_{F_2} = \sqrt{2} k e^V e^{\lambda_1} \sinh (\lambda_2 - \lambda_3),
\]

\[
\mathcal{E}_{F_3} = \sqrt{2} k e^V e^{\lambda_2} \sinh (\lambda_3 - \lambda_1). \tag{3.19}
\]

### 3.3 Boundary conditions for spindle solutions

We fix the metric to be in conformal gauge,

\[
f = e^V, \tag{3.20}
\]

and the metric is given by

\[
ds^2 = e^{2V} \left[ ds^2_{\text{AdS}_2} + ds^2_{\Sigma} \right], \tag{3.21}
\]

where we have

\[
ds^2_{\Sigma} = dy^2 + k^2 \sin^2 \xi dz^2, \tag{3.22}
\]

for the metric on a spindle, \( \Sigma \). For the spindle solutions, there are two poles at \( y = y_{N,S} \) with deficit angles of \( 2\pi \left( 1 - \frac{1}{n_{N,S}} \right) \). The azimuthal angle, \( z \), has a period which we set

\[
\Delta z = 2\pi. \tag{3.23}
\]

#### 3.3.1 Analysis of the BPS equations

Following the argument in section 3.3.1 of 13 we study the BPS equations to find the spindle solutions. At the poles of the spindle solution, \( y = y_{N,S} \), as we have \( k \sin \xi \to 0 \), we find \( \cos \xi \to \pm 1 \) if \( k \neq 0 \). Thus, we express \( \cos \xi_{N,S} = (-1)^{t_{N,S}} \) with \( t_{N,S} \in \{0,1\} \). We choose \( y_N < y_S \) and \( y \in [y_N, y_S] \). We assume that the deficit angles at the poles are \( 2\pi \left( 1 - \frac{1}{n_{N,S}} \right) \) with
\( n_{N,S} \geq 1 \). Then we require the metric to have \( |(k \sin \xi)'|_{N,S} = \frac{1}{n_{N,S}} \). From the symmetry of BPS equations in (B.32) and (3.15), we further choose

\[
h \geq 0 \quad \Leftrightarrow \quad k \sin \xi \geq 0.
\] (3.24)

Then we find \( (k \sin \xi)'|_{N} > 0 \) and \( (k \sin \xi)'|_{S} < 0 \). Hence, we impose

\[
(k \sin \xi)'|_{N,S} = \frac{(-1)^{l_{N,S}}}{n_{N,S}} \quad \text{and} \quad l_{N} = 0, l_{S} = 1.
\] (3.25)

Two different classes of spindle solutions, the twist and the anti-twist, are known, [15]. Spinors have the same chirality at the poles in the twist solutions and opposite chiralities in the anti-twist solutions as

\[
\cos \xi|_{N,S} = (-1)^{t_{N,S}}; \quad \text{Twist:} \quad (t_{N}, t_{S}) = (1,1) \quad \text{or} \quad (0,0), \quad \text{Anti-Twist:} \quad (t_{N}, t_{S}) = (1,0) \quad \text{or} \quad (0,1).
\] (3.26)

As we find \( (k \sin \xi)' = -\cos \xi (s - B_{z}) \) from the BPS equation in (3.16), we obtain

\[
(s - B_{z})|_{N,S} = \frac{1}{n_{N,S}} (-1)^{l_{N,S} + t_{N,S} + 1}.
\] (3.27)

We consider the flux quantization for R-symmetry flux. From (2.6) we have \( F^{R} = dB + d \left( \frac{1}{2} \cosh (2\chi) - 1 \right) D\psi \). At the poles, as \( \chi = 0 \) unless \( D\psi = 0 \), the second term on the right hand side of \( F^{R} \) does not contribute to the flux quantization. Then, we find the R-symmetry flux quantized to be

\[
\frac{1}{2\pi} \int_{\Sigma} F^{R} = \frac{1}{2\pi} \int_{\Sigma} -g \left( F^{0} + F^{1} + F^{2} + F^{3} \right) = \frac{n_{N}(-1)^{l_{S} + 1} + n_{S}(-1)^{l_{N} + 1}}{n_{N}n_{S}}.
\] (3.28)

We have \( \partial_{z} B = -\frac{1}{2} \sinh (2\chi) D_{z}\psi \). Once again, as \( \chi = 0 \) unless \( D\psi = 0 \) at the poles, we find \( \partial_{\chi} B_{z} = 0 \) at the poles. From the constraint in (3.17) we also find \( \partial_{\chi} W = 0 \) at the poles. Hence, we have

\[
\partial_{\chi} B_{z}|_{N,S} = \partial_{\chi} W|_{N,S} = 0.
\] (3.29)

We further assume that the complex scalar field, \( (\chi, \psi) \), is non-vanishing at the poles and we find

\[
\chi|_{N,S} \neq 0, \quad \Rightarrow \quad D_{z}\psi|_{N} = D_{z}\psi|_{S} = 0.
\] (3.30)

Thus, we find that the flux charging the complex scalar field should vanish,

\[
\frac{1}{2\pi} \int_{\Sigma} g \left( F^{0} - F^{1} - F^{2} + F^{3} \right) = (D_{z}\psi)|_{y_{N}} = 0.
\] (3.31)
Note that, from (3.30) and the second condition in (3.29), we find
\[ (e^{2\lambda_1} + e^{2\lambda_2} + e^{2\lambda_3} - e^{2\lambda_1+2\lambda_2+2\lambda_3})|_{N,S} = 0, \quad \Rightarrow \quad W|_{N,S} = -e^{\lambda_1+\lambda_2+\lambda_3}|_{N,S}. \quad (3.32) \]

In order to find the values of the integrals of motion, \( E_{R_i} \) in (3.18), we introduce two quantities,
\[ M(1) \equiv ge^{\lambda_1+\lambda_2+\lambda_3}e^V, \quad M(2) \equiv 2M(1)^2 \cos \xi, \quad (3.33) \]
and note that \( M(1) > 0 \). The integrals of motion, (3.18), are functions of \( V, \lambda_i \) and \( \cos \xi \). We can eliminate \( V \) by using the first equation in (3.17) and \( \cos \xi \) by (3.26). Then we find the integrals of motion to be
\[ E_{R_i} = \frac{M(2)}{g} e^{-2(\lambda_1+\lambda_2+\lambda_3)} - \frac{\kappa}{\sqrt{2g}} M(1) \left( e^{-2\lambda_i} - e^{-2(\lambda_1+\lambda_2+\lambda_3)} \right), \quad (3.34) \]
with
\[
M(1)|_{N,S} = \frac{1}{\sqrt{2}}((-1)^{t_{N,S}}k - \frac{1}{\sqrt{2kn_{N,S}}})(-1)^{t_{N,S}}, \\
M(2)|_{N,S} = (-1)^{t_{N,S}} + \frac{1}{k^2n_{N,S}^2}(-1)^{t_{N,S}} - \frac{2\kappa}{kn_{N,S}}(-1)^{t_{N,S}}. \quad (3.35)
\]

Finally, we can eliminate one of the scalar fields, \( \lambda_i \), say \( \lambda_3 \), by the condition on the left hand side of (3.32). Hence, we have three independent integrals of motion in terms of two scalar fields, \( \lambda_1 \) and \( \lambda_2 \). As the integrals of motion have identical values at the poles, we find three algebraic equations with four unknowns, \( (\lambda_{1N}, \lambda_{1S}, \lambda_{2N}, \lambda_{2S}) \),
\[
E_{R_1}(\lambda_{1N}, \lambda_{2N}) = E_{R_1}(\lambda_{1S}, \lambda_{2S}), \\
E_{R_2}(\lambda_{1N}, \lambda_{2N}) = E_{R_2}(\lambda_{1S}, \lambda_{2S}), \\
E_{R_3}(\lambda_{1N}, \lambda_{2N}) = E_{R_3}(\lambda_{1S}, \lambda_{2S}). \quad (3.36)
\]

Unlike the case for the Leigh-Strassler theory in [13], we are one equation short to solve for the values of all scalar fields at the poles.\(^1\) We need to find an additional constraint to determine all the values of the fields at the poles. Furthermore, even if we find an additional constraint, the associated equations in (3.36) are quite complicated and it appears to be not easy to solve them. We leave this crucial problem for the future study.

\(^1\)In [13], in the five-dimensional supergravity model, three \( U(1) \) gauge fields, \( F^i \), \( i = 1, 2, 3 \), produce two conserved quantities, \( E_R(V, \alpha, \beta) \) and \( E_F(V, \alpha, \beta) \), where \( V \) is again the warp factor and \( \alpha \) and \( \beta \) are the scalar fields. One can fix \( V \) from the flux quantization of \( (s - Q_z) \) and fix \( \beta \) from \( \partial W = 0 \). Then \( E_R(\alpha_S) = E_R(\alpha_N) \) and \( E_F(\alpha_S) = E_F(\alpha_N) \) fix the values of the scalar field at the poles, \( (\alpha_S, \alpha_N) \). The numbers of unknowns and the numbers of constraints from the conserved quantities match. The values of all the other fields at the poles are determined subsequently.
3.3.2 Fluxes

In appendix B we have obtained the expressions of field strengths in terms of the scalar fields, warp factors, the angle, \( \xi \), and \( k \),

\[
F_{yz}^\alpha = (a^\alpha)' = (T^\alpha)',
\]

where we have

\[
T^0 \equiv gke^V \cos \xi e^{\lambda_1 + \lambda_2 + \lambda_3}, \\
T^1 \equiv gke^V \cos \xi e^{\lambda_1 - \lambda_2 - \lambda_3}, \\
T^2 \equiv gke^V \cos \xi e^{-\lambda_1 + \lambda_2 - \lambda_3}, \\
T^3 \equiv gke^V \cos \xi e^{-\lambda_1 - \lambda_2 + \lambda_3}.
\]

(3.38)

Thus we find that the fluxes are solely given by the data at the poles,

\[
\frac{p_i}{n_N n_S} \equiv \frac{1}{2\pi} \int_\Sigma gF^\alpha = gT^\alpha|_N.
\]

(3.39)

3.3.3 The Bekenstein-Hawking entropy

The \( AdS_2 \times \Sigma \) solution would be the horizon of a presumed black hole which asymptotes to the \( \mathcal{N} = 2 \) \( AdS_4 \) vacuum dual to the mass-deformed ABJM theory. We calculate the Bekenstein-Hawking entropy of the presumed black hole solution.

From the AdS/CFT dictionary, (2.11) and (2.13), for the four-dimensional Newton’s constant, we have,

\[
\frac{1}{2G^{(4)}_N} = \frac{2\sqrt{2}}{3} g^2 N^{3/2}.
\]

(3.40)

Then the two-dimensional Newton’s constant is given by

\[
\left( G^{(2)}_N \right)^{-1} = \left( G^{(4)}_N \right)^{-1} \Delta z \int_{y_N} \left| f h \right| dy.
\]

(3.41)

Employing the BPS equations, we find

\[
f h = ke^V f \sin \xi = -\frac{k}{\kappa} \left( e^{2V} \cos \xi \right)'.
\]

(3.42)

Hence, the Bekenstein-hawking entropy is expressed by the data at the poles,

\[
S_{BH} = \frac{1}{4G^{(2)}_N} = \frac{2\sqrt{2\pi}}{3} N^{3/2} g^2 \left( -\frac{k}{\kappa} \right) \left[ e^{2V} \cos \xi \right]^S_N
\]

\[
= -\frac{2\sqrt{2N^{3/2}k}}{3\kappa} \left( M_{(1)}^2 \right)_S e^{-2(\lambda_1 + \lambda_2 + \lambda_3)} (-1)^t_S - M_{(1)}^2 |_N e^{-2(\lambda_1 + \lambda_2 + \lambda_3)} (-1)^t_N,
\]

where we expressed the Bekenstein-Hawking entropy in terms of \( M_{(1)} \).
4 Solving the BPS equations

In order to numerically construct the $\text{AdS}_2 \times \Sigma$ solutions, the knowledge on the values of all fields at the poles are required to give the boundary conditions for the BPS equations. However, we have left this problem for the future study in section 3.3.1. Thus, in this section, we restrict ourselves to the subtruncation to minimal gauged supergravity associated with the Warner $\mathcal{N} = 2$ $\text{AdS}_4$ vacuum. Refer appendix A.2 for the construction of the subtruncation.

4.1 Analytic solutions for minimal gauged supergravity via $W$

In minimal gauged supergravity associated with the Warner $\mathcal{N} = 2$ $\text{AdS}_4$ vacuum, utilizing the class of $\text{AdS}_2 \times \Sigma$ solutions in [6], we find solutions in the anti-twist class to the BPS equations in (3.12), (3.13) and (3.14). The scalar fields take the values at the Warner $\mathcal{N} = 2$ $\text{AdS}_4$ vacuum,

$$\tanh \chi = \frac{1}{\sqrt{3}}, \quad \psi = \frac{\pi}{2}, \quad \tanh \lambda_i = 2 - \sqrt{3}, \quad \varphi_i = \pi.$$  (4.1)

The metric and the gauge field are given by

$$ds^2 = \frac{2}{3\sqrt{3}g^2} \left[ \frac{y^2}{4} d\text{AdS}_2 + \frac{y^2}{q(y)} dy^2 + \frac{q(y)}{4y^2} c_0^2 dz^2 \right],$$

$$\frac{1}{3} A^0 = A^1 = A^2 = A^3 = -\left[ \frac{2c_0 \kappa}{3g} \left( 1 - \frac{a}{y} \right) + \frac{s}{g} \right] dz,$$  (4.2)

and we have

$$\sin \xi = -\frac{\sqrt{q(y)}}{y^2}, \quad \cos \xi = \kappa \frac{2y - a}{y^2}.$$  (4.3)

Note that for the overall factor in the metric, we have $L^2_{\text{AdS}_4} = \frac{2}{3\sqrt{3}g^2}$ for the Warner vacuum from (2.10). The quartic function is given by

$$q(y) = y^4 - 4y^2 + 4ay - a^2,$$  (4.4)

and the constants are

$$a = \frac{n_S^2 - n_N^2}{n_S^2 + n_N^2},$$

$$c_0 = \frac{\sqrt{n_S^2 + n_N^2}}{\sqrt{2n_S n_N}}.$$  (4.5)

We set $n_S > n_N$. For the two middle roots of $q(y)$, $y \in [y_N, y_S]$, we find

$$y_N = -1 + \sqrt{1 + a}, \quad y_N = 1 - \sqrt{1 - a}.$$  (4.6)
The Bekenstein-Hawking entropy is calculated to give

\[ S_{BH} = \frac{\sqrt{2} \sqrt{n_S^2 + n_N^2} - (n_S + n_N)}{n_S n_N} \pi \frac{n_S n_N}{4G_N^{(4)}} \]

\[ = \frac{\sqrt{2} \sqrt{n_S^2 + n_N^2} - (n_S + n_N) 3\sqrt{3}g^2}{4} F_{S^3}^{\text{mABJM}}, \] (4.7)

where we employed (2.10) and (2.11) and \( F_{S^3}^{\text{mABJM}} = \frac{4\sqrt{2}}{9\sqrt{3}} N^{3/2} \), (2.12), is the free energy of mass-deformed ABJM theory.

5 Conclusions

In this work, we studied the \( AdS_2 \times \Sigma \) solutions which asymptote to the Warner \( N = 2 \) \( SU(3) \times U(1) \) \( AdS_4 \) vacuum dual to the mass-deformed ABJM theory where \( \Sigma \) is a spindle. We derived the complete BPS equations required to construct the solution. We find the flux quantization through the spindle from the analysis of the BPS equations. However, we were not able to fix the boundary conditions of the solution, i.e., the values of the fields at the poles of the spindle. We leave this crucial problem for the future study. It would be most interesting i) to obtain the full boundary conditions and use them ii) to construct solutions numerically and iii) to calculate their Bekenstein-Hawking entropy. It would allow us iv) to characterize the solutions to be in the twist or anti-twist class. On the other hand, in the subtruncation to minimal gauged supergravity associated with the Warner vacuum, we found a class of analytic \( AdS_2 \times \Sigma \) solutions and their Bekenstein-Hawking entropy was calculated.

Acknowledgements

This research was supported by the National Research Foundation of Korea under the grant NRF-2020R1A2C1008497.

A The \( U(1)^2 \)-invariant truncation

In this appendix we review the \( U(1)^2 \)-invariant truncation of \( SO(8) \)-gauged \( N = 8 \) supergravity in four dimensions which was studied in [50] and we closely follow [50]. The \( U(1)^2 \) is the Cartan subgroup of the standard \( SU(3) \subset SO(6) \subset SO(8) \). This truncation is a generalization of the \( SU(3) \)-invariant truncation, [41, 42, 43, 44, 45, 46, 47, 48], and also the \( U(1)^4 \)-invariant truncation, also known as the STU model, [56, 57, 58], where \( U(1)^4 \) is the Cartan subgroup of \( SO(8) \).

The four standard Cartan generators of \( SO(8) \) can be denoted by \( T_{12}, T_{34}, T_{56} \) and \( T_{78} \) where \( T_{ij} \) is the generator of rotation in the \( (ij) \)-plane with a unit charge. Then the two symmetry
generators are
\[ \frac{1}{2} (T_{12} - T_{34}) \, , \quad \frac{1}{\sqrt{3}} (T_{12} + T_{34} - 2T_{56}) \, , \]
(A.1)
under which the eight gravitini, \( \psi^i \), and the supersymmetry parameters, \( \epsilon^i \), transform. The two invariant gravitini and the supersymmetry parameters are the chiral \( \psi^{7,8} \) and \( \epsilon^{7,8} \) and their complex conjugates, \( \psi_{\bar{7},\bar{8}} \) and \( \epsilon_{\bar{7},\bar{8}} \) of opposite chirality.

From the commutant of the generators in (A.1), the unbroken gauge symmetry is the Cartan subgroup, \( U(1)^4 \), of \( SO(8) \). The gauge fields are the graviphoton and three gauge fields from vector multiplet and they are the gauge fields in the STU model. The gauge fields in the symplectic frame, \( A^{ij} \), \( i,j = 1, \ldots, 8 \), are specified by the canonical gauge fields, \( A^\alpha \), \( \alpha = 0, \ldots, 3 \),
\[ A^{12} = \frac{1}{2} (A^0 + A^1 - A^2 - A^3) \, , \quad A^{34} = \frac{1}{2} (A^0 - A^1 + A^2 - A^3) \, , \]
\[ A^{56} = \frac{1}{2} (A^0 - A^1 - A^2 + A^3) \, , \quad A^{78} = \frac{1}{2} (A^0 + A^1 + A^2 + A^3) \, . \] (A.2)

The scalar 56-bein in the symmetric gauge is given by
\[
V \equiv \begin{pmatrix}
  u_{ij}^{IJ} & u_{ijKL} \\
v^{klIJ} & u^{klKL}
\end{pmatrix} = \exp \begin{pmatrix}
  0 & \phi_{ijkl} \\
  \phi_{ijkl}^* & 0
\end{pmatrix} \in E_7(7)/SU(8) ,
\] (A.3)
where we define
\[ \phi_{ijkl} = \frac{1}{24} \epsilon_{ijklmnpq} \phi^{mnpq} , \quad \phi_{ijkl}^* = (\phi_{ijkl})^* , \] (A.4)
which are completely antisymmetric complex self-dual scalar fields. Then the \( U(1)^2 \)-invariant \( \phi_{ijkl} \) are given by
\[ \phi_{1278} = -\frac{1}{2} \lambda_1 e^{i\varphi_1} , \quad \phi_{3478} = -\frac{1}{2} \lambda_2 e^{i\varphi_2} , \quad \phi_{5678} = -\frac{1}{2} \lambda_3 e^{i\varphi_3} , \]
\[ \phi_{1234} = -\frac{1}{2} \lambda_3 e^{-i\varphi_3} , \quad \phi_{1256} = -\frac{1}{2} \lambda_2 e^{-i\varphi_2} , \quad \phi_{3456} = -\frac{1}{2} \lambda_1 e^{-i\varphi_1} , \]
\[ \phi_{1357} = -\phi_{1467} = -\phi_{2367} = -\phi_{2457} = \frac{1}{4} (\chi_1 \cos \psi_1 + i \chi_2 \sin \psi_2) , \]
\[ \phi_{1367} = \phi_{1457} = \phi_{2357} = -\phi_{2467} = -\frac{1}{4} (\chi_1 \sin \psi_1 - i \chi_2 \cos \psi_2) , \]
\[ \phi_{1368} = \phi_{1458} = \phi_{2358} = -\phi_{2468} = -\frac{1}{4} (\chi_1 \cos \psi_1 - i \chi_2 \sin \psi_2) , \]
\[ \phi_{1358} = -\phi_{1468} = -\phi_{2368} = -\phi_{2458} = -\frac{1}{4} (\chi_1 \sin \psi_1 + i \chi_2 \cos \psi_2) , \] (A.5)
where we introduced the scalar fields and they corresponds to the scalar fields, \( (\lambda_i, \varphi_i) \), from three \( \mathcal{N} = 2 \) vector multiplets and, \( (\chi_r, \psi_r) \), from the universal hypermultiplet where \( i = 1, 2, 3 \) and
They parametrize the special Kähler and quaternionic Kähler manifolds, respectively,

$$
\mathcal{M}_V \times \mathcal{M}_H = \left[ SU(1,1) / U(1) \right]^3 \times \frac{SU(2,1)}{SU(2) \times U(1)}.
$$

(A.6)

In the following, we will restrict ourselves to the case of

$$
\chi \equiv \chi_2, \quad \psi \equiv \psi_2, \quad \chi_1 = 0, \quad \psi_1 = \frac{\pi}{2}, \quad \varphi_1 = \varphi_2 = \varphi_2 = \pi.
$$

(A.7)

The bosonic Lagrangian of the truncation is given by

$$
e^{-1} \mathcal{L} = \frac{1}{2} R - \partial_\mu \chi \partial^\mu \chi - \frac{1}{4} \sinh^2 (2\chi) D_\mu \psi D^\mu \psi - \sum_{i=1}^{3} \partial_\mu \lambda_i \partial^\mu \lambda_i - g^2 \mathcal{P}
- \frac{1}{4} \left[ e^{-2(\lambda_1 + \lambda_2 + \lambda_3)} F^0_{\mu \nu} F^{0\mu \nu} + e^{-2(\lambda_1 - \lambda_2 - \lambda_3)} F^1_{\mu \nu} F^{1\mu \nu} + e^{2(\lambda_1 + \lambda_2 + \lambda_3)} F^2_{\mu \nu} F^{2\mu \nu} + e^{2(\lambda_1 + \lambda_2 - \lambda_3)} F^3_{\mu \nu} F^{3\mu \nu} \right],
$$

(A.8)

where we define

$$
D_\psi \equiv d\psi + g (A^0 - A^1 - A^2 - A^3).
$$

(A.9)

The scalar potential is given by

$$
\mathcal{P} = \frac{1}{2} \left( \frac{\partial W}{\partial \chi} \right)^2 + \frac{1}{2} \sum_{i=1}^{3} \left( \frac{\partial W}{\partial \lambda_i} \right)^2 - \frac{3}{2} W^2,
$$

(A.10)

where the superpotential is

$$
W = \frac{2 e^{2(\lambda_1 + \lambda_2 + \lambda_3)} \sinh^2 \chi - \cosh^2 \chi \left( e^{2(\lambda_1 + \lambda_2 + \lambda_3)} + e^{2\lambda_1} + e^{2\lambda_2} + e^{2\lambda_3} \right)}{2 e^{\lambda_1 + \lambda_2 + \lambda_3}}.
$$

(A.11)

The supersymmetry variations of fermionic fields are given by

$$
\delta \chi^{ijk} = - A_\mu^{ijkl} \gamma^\mu \epsilon_1 + \frac{3}{2} \gamma^\mu \gamma^\rho \gamma^\lambda \epsilon_1 - 2 g A_{2l}^{ijk} \epsilon^l,
$$

$$
\delta \psi^i = 2 D_\mu \epsilon^i + \frac{\sqrt{2}}{4} F^{-i j}_{\nu \rho} \gamma^\rho \gamma^\lambda \epsilon_j + \sqrt{2} g A_{1 i}^{ij} \gamma^\mu \epsilon_j.
$$

(A.12)

Refer [53] for the definitions of the tensors, $A_\mu^{ijkl}$, $A_{1 i}^{ij}$ and $A_{2l}^{ijk}$. The scalar-field dressed field strengths, $F^{-i j}$, are defined by

$$
F^{-i j} = (u_{i j} + v_{i j l}) F^{-i j}, \quad F^{-i j} = \frac{1}{2} (F^{i j} - i \ast F^{i j}),
$$

(A.13)

and the minus sign denotes the anti-self dual part of the field strength.

In the $U(1)^2$-invariant truncation, we find the supersymmetry parameters to be

$$
\epsilon^1 = \ldots = \epsilon^6 = 0,
$$

(A.14)
and the only non-trivial parameters are $\epsilon^7$ and $\epsilon^8$ with $\epsilon_i = (\epsilon^i)^*$. We find the real superpotential, \( (A.11) \), from the eigenvalues of $A_{ij}^{\ell}$ tensors,

$$ A_{ij}^{\ell} \epsilon_i = -\frac{1}{2} W \epsilon_i, \quad A_{ij}^{88} \epsilon_8 = -\frac{1}{2} W \epsilon_8. \quad (A.15) $$

We also define $B_{\mu}$ and $H_{\mu\nu}$ from

$$ B_{\mu} \equiv B_{\mu}^{\ell \ell} = -B_{\mu}^{\ell \ell}, \quad H_{\mu\nu} \equiv F_{\mu\nu}^{\ell \ell}, \quad (A.16) $$

where explicitly we have

$$ H_{\mu\nu} \equiv F_{\mu\nu}^{\ell \ell} = \frac{1}{2} \left( e^{-\lambda_1 - \lambda_2 - \lambda_3} F_{\mu\nu}^{-0} + e^{-\lambda_1 + \lambda_2 + \lambda_3} F_{\mu\nu}^{-1} + e^{\lambda_1 - \lambda_2 + \lambda_3} F_{\mu\nu}^{-2} + e^{\lambda_1 + \lambda_2 - \lambda_3} F_{\mu\nu}^{-3} \right), \quad (A.17) $$

Instead of the Weyl spinors, $\epsilon^i$, we rewrite the supersymmetry variations with a complex Dirac spinor,

$$ \epsilon = (\epsilon^7 + \epsilon^7) + i (\epsilon^8 + \epsilon^8). \quad (A.18) $$

The spin-3/2 field variation reduces to

$$ \left[ 2 \nabla_{\mu} - i B_{\mu} - \frac{g}{\sqrt{2}} W \gamma_{\mu} - i \frac{1}{2\sqrt{2}} H_{\mu\nu} \gamma^\mu \gamma_{\mu} \right] \epsilon = 0. \quad (A.19) $$

The spin-1/2 field variation reduces to the gaugino variations,

$$ \left[ \gamma^\mu \partial_{\mu} \lambda_1 + \frac{g}{\sqrt{2}} \partial_{\lambda_1} W + i \frac{1}{2\sqrt{2}} \gamma_{\mu\nu} F_{\mu\nu}^{-12} \right] \epsilon = 0, $$

$$ \left[ \gamma^\mu \partial_{\mu} \lambda_2 + \frac{g}{\sqrt{2}} \partial_{\lambda_2} W + i \frac{1}{2\sqrt{2}} \gamma_{\mu\nu} F_{\mu\nu}^{-34} \right] \epsilon = 0, $$

$$ \left[ \gamma^\mu \partial_{\mu} \lambda_3 + \frac{g}{\sqrt{2}} \partial_{\lambda_3} W + i \frac{1}{2\sqrt{2}} \gamma_{\mu\nu} F_{\mu\nu}^{-56} \right] \epsilon = 0, \quad (A.20) $$

and the hyperino variations,

$$ \left[ \gamma^\mu \partial_{\mu} \chi + \frac{g}{\sqrt{2}} \partial_{\lambda} W + i \frac{1}{2\sqrt{2}} \partial_{\lambda} B_{\mu} \gamma^\mu \right] \epsilon = 0, \quad (A.21) $$

where we have

$$ F^{-0} = \frac{1}{2} e^{\lambda_1 + \lambda_2 + \lambda_3} \left( F^{-12} + F^{-34} + F^{-56} + F^{-78} \right), $$

$$ F^{-1} = \frac{1}{2} e^{\lambda_1 - \lambda_2 - \lambda_3} \left( F^{-12} - F^{-34} - F^{-56} + F^{-78} \right), $$

$$ F^{-2} = -\frac{1}{2} e^{-\lambda_1 + \lambda_2 - \lambda_3} \left( F^{-12} - F^{-34} + F^{-56} - F^{-78} \right), $$

$$ F^{-3} = -\frac{1}{2} e^{-\lambda_1 - \lambda_2 + \lambda_3} \left( F^{-12} + F^{-34} - F^{-56} - F^{-78} \right). \quad (A.22) $$
We present the equations of motion from the Lagrangian in (A.8). The Einstein equations are
\[
\begin{align*}
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + g^2 \mathcal{P} g_{\mu\nu} - 2 \left( T^\chi_{\mu\nu} + T^\lambda_{\mu\nu} + T^\lambda_\mu + T^\lambda_\nu \right) - \frac{1}{2} \sinh^2 (2\chi) T^\psi_{\mu\nu} \\
- e^{-2(\lambda_1 + \lambda_2 + \lambda_3)} T^{A^0}_{\mu\nu} - e^{-2(\lambda_1 - \lambda_2 - \lambda_3)} T^{A^1}_{\mu\nu} - e^{2(\lambda_1 - \lambda_2 + \lambda_3)} T^{A^2}_{\mu\nu} - e^{2(\lambda_1 + \lambda_2 - \lambda_3)} T^{A^3}_{\mu\nu} = 0,
\end{align*}
\] (A.23)
with the energy-momentum tensors by
\[
\begin{align*}
T^X_{\mu\nu} &= \partial_\mu X \partial_\nu X - \frac{1}{2} g_{\mu\nu} \partial_{\rho} X \partial^\rho X, \\
T^{A^\alpha}_{\mu\nu} &= g^{\rho\sigma} F^\alpha_{\mu\rho} F^\alpha_{\nu\sigma} - \frac{1}{4} g_{\mu\nu} F^\alpha_{\rho\sigma} F^{\alpha\rho\sigma},
\end{align*}
\] (A.24)
where \( X \) denotes a scalar field. The Maxwell equations are
\[
\begin{align*}
\partial_\nu \left( \sqrt{-g} e^{-2(\lambda_1 + \lambda_2 + \lambda_3)} F^0_{\mu \nu} \right) - \frac{1}{2} \sinh^2 (2\chi) D_\mu \psi &= 0, \\
\partial_\nu \left( \sqrt{-g} e^{-2(\lambda_1 - \lambda_2 - \lambda_3)} F^{1}_{\mu \nu} \right) + \frac{1}{2} \sinh^2 (2\chi) D_\mu \psi &= 0, \\
\partial_\nu \left( \sqrt{-g} e^{2(\lambda_1 - \lambda_2 + \lambda_3)} F^{2}_{\mu \nu} \right) + \frac{1}{2} \sinh^2 (2\chi) D_\mu \psi &= 0, \\
\partial_\nu \left( \sqrt{-g} e^{2(\lambda_1 + \lambda_2 - \lambda_3)} F^{3}_{\mu \nu} \right) + \frac{1}{2} \sinh^2 (2\chi) D_\mu \psi &= 0,
\end{align*}
\] (A.25)
and the scalar field equations are given by
\[
\begin{align*}
\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \lambda_1 \right) - \frac{g^2}{2} \frac{\partial \mathcal{P}}{\partial \lambda_1} \\
+ \frac{1}{4} e^{-2(\lambda_1 + \lambda_2 + \lambda_3)} F^{0}_{\mu\nu} F^{0\mu\nu} - \frac{1}{4} e^{-2(\lambda_1 - \lambda_2 - \lambda_3)} F^{1}_{\mu\nu} F^{1\mu\nu} \\
- \frac{1}{4} e^{2(\lambda_1 - \lambda_2 + \lambda_3)} F^{2}_{\mu\nu} F^{2\mu\nu} - \frac{1}{4} e^{2(\lambda_1 + \lambda_2 - \lambda_3)} F^{3}_{\mu\nu} F^{3\mu\nu} &= 0,
\end{align*}
\] (A.26)
with
\[
\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \chi \right) - \frac{g^2}{2} \partial_\mu \mathcal{P} + \frac{1}{8} \sinh(4\chi) D_\mu \psi D^\mu \psi = 0. \tag{A.27}
\]

A.1 Truncation to the STU model

The truncation reduces to the STU model, [56, 57, 58], by setting the charged complex scalar field to vanish,
\[
\chi = 0, \quad \psi = \frac{\pi}{2}, \tag{A.28}
\]
and the scalar potential is
\[
\mathcal{P} = -2 \left[ \cosh(2\lambda_1) + \cosh(2\lambda_2) + \cosh(2\lambda_3) \right]. \tag{A.29}
\]

Reparametrizing \( \lambda_i \rightarrow \frac{1}{2} \varphi_i \) and \( F_\alpha \rightarrow \frac{1}{\sqrt{2}} F_i \) with \( g = \frac{1}{\sqrt{2}} \), we obtain the normalizations of the STU model used in [15].

When we further impose
\[
\lambda_i = 0, \quad \varphi_i = \pi, \quad A \equiv A^0 = A^1 = A^2 = A^3, \tag{A.30}
\]
we obtain the Lagrangian of minimal gauged supergravity,
\[
e^{-1} \mathcal{L} = \frac{1}{2} R - 6g^2 - F_\mu F^{\mu\nu}, \tag{A.31}
\]
where \( F = dA \). We set \( g = \frac{1}{\sqrt{2}} \) and \( F \rightarrow \frac{1}{\sqrt{2}} F \) and obtain the Lagrangian of the normalization of minimal gauged supergravity employed in [6].

A.2 Truncation to minimal gauged supergravity via W

There is an alternative truncation to minimal gauged supergravity associated with the Warner \( \mathcal{N} = 2 \) vacuum. We have the scalar fields to be at their values of the Warner \( \mathcal{N} = 2 \) vacuum and impose the gauge fields to be
\[
\tanh \chi = \frac{1}{\sqrt{3}}, \quad \psi = \frac{\pi}{2}, \quad \tanh \lambda_i = 2 - \sqrt{3}, \quad \varphi_i = \pi, \quad \frac{1}{3} A \equiv \frac{1}{3} A^0 = A^1 = A^2 = A^3, \tag{A.32}
\]
and find
\[
e^{-1} \mathcal{L} = \frac{1}{2} R - \frac{9\sqrt{3}}{2} g^2 - \frac{1}{12\sqrt{3}} F_\mu F^{\mu\nu}, \tag{A.33}
\]
where \( F = dA \). When we have \( g = \frac{\sqrt{2}}{3\sqrt{3}} \) and \( F \rightarrow \sqrt{2} 3^{3/4} F \), it reduces to the action of minimal gauged supergravity employed in [6].
B  Supersymmetry variations

B.1  Derivation of the BPS equations

Following the arguments in [13], we derive the BPS equations required to construct the spindle solutions. We consider the background with the gauge fields,

\[ ds^2 = e^{2V} ds_{AdS_2}^2 + f^2 dy^2 + h^2 dz^2, \]
\[ A^\alpha = a^\alpha dz, \]  
(B.1)

where \( V, f, h, \) and \( a^\alpha, \alpha = 0, \ldots, 3, \) are functions of coordinate \( y \) only. We employ the gamma matrices,

\[ \gamma^m = \Gamma^m \otimes \sigma^3, \quad \gamma^2 = \mathbb{1}_2 \otimes \sigma^1, \quad \gamma^3 = \mathbb{1}_2 \otimes \sigma^2, \]  
(B.2)

and the spinors,

\[ \epsilon = \psi \otimes \chi, \]  
(B.3)

where \( \Gamma^m \) are two-dimensional gamma matrices of mostly plus signature. The two-dimensional spinor satisfies

\[ D_m \psi = \frac{1}{2} \kappa \Gamma_m \psi, \]  
(B.4)

where \( \kappa = \pm 1. \)

From the directions tangent to \( AdS_2, \) the gravitino variation reduces to

\[ -i \left( \sqrt{2} \kappa e^{-V} + H_{23} \right) \gamma^{23} + \sqrt{2} V' f^{-1} \gamma^3 \right] \epsilon = gW \epsilon. \]  
(B.5)

It requires a projection condition,

\[ \left[ i \cos \xi \gamma^{23} + \sin \xi \gamma^2 \right] \epsilon = +\epsilon, \]  
(B.6)

where we introduce \( \xi \) from

\[ -\sqrt{2} \kappa e^{-V} - H_{23} = gW \cos \xi, \quad \sqrt{2} V' f^{-1} = gW \sin \xi. \]  
(B.7)

The projection condition is solved by

\[ \epsilon = e^{i\xi \gamma^3} \eta, \quad \gamma^2 \eta = +i \gamma^3 \eta. \]  
(B.8)

We find \( \partial_\xi \xi = 0 \) from (B.7). At \( \xi = 0, \pi, \) the spinors have definite chirality with respect to \( \gamma^{23}, \)

\[ \xi = 0, \pi, \quad \gamma^{23} \epsilon = \pm i \epsilon. \]  
(B.9)

From the \( y \) direction, the spin-3/2 field variation, (A.19), reduces to

\[ \left[ \partial_y - \frac{1}{2} V' + i \frac{1}{2} \left( \partial_y \xi + \sqrt{2} f H_{23} + \kappa f e^{-V} \right) \gamma^3 \right] \eta = 0 , \]  
(B.10)
where we employed (B.7). From the $z$ direction, we find
\[
\begin{align*}
2\partial_z - iB_z + if^{-1}h' \cos \xi - i\frac{1}{\sqrt{2}}H_{23}h \sin \xi \\
+ \left( f^{-1}h' \sin \xi - i\frac{1}{\sqrt{2}}gWh + \frac{1}{\sqrt{2}}H_{23}h \cos \xi \right) \gamma^3 \right) \eta = 0. 
\end{align*}
\] (B.11)

When we have $(a_1 + ia_2 \gamma^3) \eta = 0$, it requires $a_1^2 + a_2^2 = 0$. Hence, we find from (B.10) and (B.11),
\[
\eta = e^{V/2} e^{isz} \eta_0, 
\] (B.12)
where $\eta_0$ is independent of $y$ and $z$ and we obtain
\[
\begin{align*}
\partial_y \xi + \sqrt{2} f H_{23} + \kappa e^{-V} &= 0, \\
(s - B_z) + f^{-1}h' \cos \xi - \frac{1}{\sqrt{2}} H_{23}h \sin \xi &= 0, \\
f^{-1}h' \sin \xi - \frac{gWh}{\sqrt{2}} + \frac{1}{\sqrt{2}} H_{23}h \cos \xi &= 0. 
\end{align*}
\] (B.13)

Thus we find
\[
\begin{align*}
f^{-1}h' &= \frac{gWh}{\sqrt{2}} \sin \xi - (s - B_z) \cos \xi, \\
hH_{23} &= gWh \cos \xi + \sqrt{2} (s - B_z) \sin \xi, 
\end{align*}
\] (B.14)
and, from the first constraint in (B.7), we obtain
\[
(s - B_z) \sin \xi = -\sqrt{2} gWh \cos \xi - \kappa e^{-V}, 
\] (B.15)
and thus
\[
\begin{align*}
H_{23} &= -gWh \cos \xi - \sqrt{2} \kappa e^{-V}, \\
f^{-1} \partial_y \xi &= \sqrt{2} gWh \cos \xi + \kappa e^{-V}. 
\end{align*}
\] (B.16)

If we have $\xi \neq 0$, by solving for $(s - B_z)$, we also obtain
\[
f^{-1} \frac{h'}{h} \sin \xi = \kappa e^{-V} \cos \xi + \frac{gW}{\sqrt{2}} (1 + \cos^2 \xi). 
\] (B.17)

From the gaugino variations, (A.20), in a similar way, we obtain
\[
f^{-1} \lambda_i' + \frac{g}{\sqrt{2}} \partial_i W \sin \xi = 0, 
\] (B.18)
with
\[ g \partial_{\lambda_1} W \cos \xi + F_{23}^{-12} = 0, \]
\[ g \partial_{\lambda_2} W \cos \xi + F_{23}^{-34} = 0, \]
\[ g \partial_{\lambda_3} W \cos \xi + F_{23}^{-56} = 0. \]  
(B.19)

From the hyperino variations, (A.21), we obtain
\[ f^{-1} \chi' \sin \xi + \frac{g}{\sqrt{2}} \partial_\chi W = 0, \]
\[ 2g \partial_\chi W \cos \xi - \partial_\chi B_z \sin \xi h^{-1} = 0. \]  
(B.20)

**Summary:** If we have \( \sin \xi \neq 0 \), the complete BPS equations are given by
\[ f^{-1} \xi' = \sqrt{2} g W \cos \xi + \kappa e^{-V}, \]
\[ f^{-1} V' = \frac{g}{\sqrt{2}} W \sin \xi, \]
\[ f^{-1} \lambda_i' = - \frac{g}{\sqrt{2}} \partial_{\lambda_i} W \sin \xi, \]
\[ f^{-1} \chi' = - \frac{g}{\sqrt{2}} \frac{\partial_\chi W}{\sin \xi}, \]
\[ f^{-1} h' \sin \xi = \kappa e^{-V} \cos \xi + \frac{gW}{\sqrt{2}} (1 + \cos^2 \xi), \]  
(B.21)

with two constraints,
\[ (s - B_z) \sin \xi = - \sqrt{2} g W h \cos \xi - \kappa h e^{-V}, \]
\[ 2g \partial_\chi W \cos \xi = \partial_\chi B_z \sin \xi h^{-1}. \]  
(B.22)

The scalar-field dressed field strengths are given by
\[ T_{23}^{-12} = - g \partial_{\lambda_1} W \cos \xi, \]
\[ T_{23}^{-34} = - g \partial_{\lambda_2} W \cos \xi, \]
\[ T_{23}^{-56} = - g \partial_{\lambda_3} W \cos \xi, \]
\[ H_{23} = - g W \cos \xi - \sqrt{2} \kappa e^{-V}. \]  
(B.23)

We have checked that the BPS equations solve the equations of motion from the Lagrangian in (A.8) as presented in appendix A.1.

We also obtain
\[ \partial_y W = - \frac{g}{\sqrt{2}} f \sin \xi \left[ \sum_{i=1}^{3} (\partial_{\lambda_i} W)^2 + \frac{1}{\sin^2 \xi} (\partial_\chi W)^2 \right], \]  
(B.24)
and, thus, the superpotential, $W$, is monotonic along the BPS flow if the sign of $f \sin \xi$ does not change.

We find an integral of the BPS equations,
\[
he^{-V} = k \sin \xi,
\]  
where $k$ is a constant. Employing this to eliminate $h$, we find the BPS equations to be
\[
\begin{align*}
f^{-1} \xi' &= -k^{-1} (s - B_z) e^{-V}, \\
f^{-1} V' &= \frac{g}{\sqrt{2}} W \sin \xi, \\
f^{-1} \lambda'_i &= -\frac{g}{\sqrt{2}} \partial_\lambda W \sin \xi, \\
f^{-1} \chi' &= -\frac{g}{\sqrt{2}} \sin \xi,
\end{align*}
\]  
with the two constraints,
\[
\begin{align*}
(s - B_z) &= -k \left( \sqrt{2} g We^V \cos \xi + \kappa \right), \\
2g \partial_\chi W \cos \xi &= k^{-1} e^{-V} \partial_\chi B_z.
\end{align*}
\]  
From the definition of $B_z$ in (A.17), we find
\[
\partial_\chi B_z = -\sinh (2\chi) D_z \psi.
\]  
If we have $\chi \neq 0$, from the second constraint in (B.27), we find
\[
D_z \psi = -\frac{2gke^V \partial_\chi W \cos \xi}{\sinh (2\chi)},
\]  
and the right hand side is independent of $\chi$. By differentiating (B.29) we find expressions of fluxes in terms of the derivatives of other fields,
\[
F^{\alpha}_{yz} = (a^\alpha)' = (T^\alpha)',
\]  
where we have
\[
\begin{align*}
T^0 &\equiv gke^V \cos \xi e^{\lambda_1 + \lambda_2 + \lambda_3}, \\
T^1 &\equiv gke^V \cos \xi e^{\lambda_1 - \lambda_2 - \lambda_3}, \\
T^2 &\equiv gke^V \cos \xi e^{-\lambda_1 + \lambda_2 - \lambda_3}, \\
T^3 &\equiv gke^V \cos \xi e^{-\lambda_1 - \lambda_2 + \lambda_3}.
\end{align*}
\]  
There is a symmetry of the BPS equations,
\[
h \rightarrow -h, \quad z \rightarrow -z,
\]  
when we have $B_z \rightarrow -B_z$, $s \rightarrow -s$, $a^\alpha \rightarrow -a^\alpha$, $k \rightarrow -k$ and $F^\alpha_{23} \rightarrow +F^\alpha_{23}$. The frame is invariant under this transformation. We fix $h \geq 0$ by this symmetry in the main text.
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