Supercurrent induced domain wall motion

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We study the dynamics of a magnetic domain wall, inserted in, or juxtaposed to, a conventional superconductor, via the passage of a spin polarized current through a FSF junction. Solving the Landau-Lifshitz-Gilbert equation of motion for the magnetic moments we calculate the velocity of the domain wall and compare it with the case of a FNF junction. We find that in several regimes the domain wall velocity is larger when it is driven by a supercurrent.

I. INTRODUCTION

Spintronics has attracted considerable attention both from the fundamental point of view and from the point of view of applications\textsuperscript{11-12}. Related to applications in magnetic registers, the manipulation of magnetic domain walls by spin polarized currents has been a topic of interest, as opposed to direct manipulation by application of local magnetic fields. Domain walls are present in magnetic materials and thin magnetic films. Their function is to lower the magnetostatic stray field energy\textsuperscript{6}. The basic phenomena of the DW motion occur in a submicron-size ferromagnetic strip.\textsuperscript{6} The manipulation of the location of the domain walls results from the action of spin torques on the magnetic moments,\textsuperscript{9} due to a polarized current,\textsuperscript{6,8} This has been analyzed theoretically\textsuperscript{10-11} and experimentally.\textsuperscript{12-13} Typically the magnetic domain walls are inserted in magnetic semiconductors but interest in other types of structures has arisen lately involving superconductors.\textsuperscript{14}

We have been interested in the possible interplay between magnetic moments and superconductors,\textsuperscript{15-16} specifically the possibility of ordered magnetic moments in the superconductor or in its vicinity.\textsuperscript{17,18} Magnetism and superconductivity typically compete and, therefore, most heterostructures considered tend to separate the ferromagnetic and superconducting regions (often separated by an insulator to prevent proximity effects). The presence of randomly located magnetic impurities in a conventional superconductor destroys the superconducting order for small impurity concentrations,\textsuperscript{15} but we have shown\textsuperscript{19,20} that, if the magnetic moments are correlated, superconductivity is much more robust and prevails for much higher concentrations. Therefore, if the magnetic moments are somewhat diluted we expect that superconducting order should remain. We will consider here diluted magnetic moments.

The effects of the polarization of the electrons due to the magnetic moments is well known to be very local. Therefore, one may expect that passing a spin polarized current through the superconductor (as in a normal metal) will induce a spin interaction between the magnetic moments and the spins of the conduction electrons.

In a conventional superconductor, with singlet pairing, the spin density dies out inside a clean superconductor at a distance of the order of the coherence length. Therefore, in the case of an infinite system one does not expect a spin torque on the magnetic moments due to the incident spin polarized current. However, the size of the nanostructure may be comparable to the coherence length and, therefore, the spin density may not die out inside the superconductor. Also, due to the polarization effect of the local magnetic moments on the spins of the conduction electrons, a smaller decay of the spin density occurs. We have shown recently\textsuperscript{21} that there is an appreciable spin torque induced on the magnetic moments that is comparable to the one observed in a normal metal. Clearly, one expects that considering system sizes of the order of the coherence length will enhance the effect. Also, spin triplet superconductors will circumvent this issue altogether.

It is therefore interesting to further explore a junction of the type ferromagnet-superconductor-ferromagnet (FSF) to find regimes where the domain wall motion induced by the passage of the supercurrent may be more effective in the FSF junction with respect to the ferromagnet-normal-ferromagnet (FNF) junction. Here we study the motion of the domain wall solving the Landau-Lifshitz-Gilbert (LLG) equations for the magnetic moments. We find that often the velocity of motion of the domain wall is actually larger for the supercurrent, as compared to a normal metal.

A spin polarized current leads to an accumulation of spin density which interacts with the local moments inducing a torque and consequent rotation. This leads ultimately to the intended shift of the domain wall position by methods involving currents. These are more efficient and fast as compared to the application of local magnetic fields to flip the magnetic moments. For recent reviews see Refs.\textsuperscript{22,23} and references therein. Due to the finite size of the systems considered the motion of the domain wall is limited by the system size. When the domain wall approaches the boundary it gets distorted and stops its motion. We are therefore interested here in studying the early time regimes during which the domain wall is set into motion and has not yet distorted appreciably.
II. MODEL

In this work we consider a junction of either FNF or FSF types. For simplicity we consider a one-dimensional model system which is a good description of a narrow superconducting wire. Inside the superconducting region we place local magnetic fields. On the left-hand side the ferromagnet exchange field, $h$, points in the $z$-direction and on the right hand side the exchange field points in the $-z$ direction. We take the domain wall resulting from the diluted magnetic moments inserted in the superconductor centered at the midpoint of the SC (see Figs. 3, 4). We use a lattice formulation for a 1D model system oriented along the $x$ axis, with Hamiltonian:

$$\hat{H} = \hat{H}_c + \hat{H}_{c-S} + \hat{H}_S$$  

(1)

where

$$\hat{H}_c = - \sum_{n,\sigma} \left( \hat{c}_{n\sigma}^\dagger \hat{c}_{n+1\sigma} + \text{H.c.} \right) + U \sum_{n,\sigma} \left( \delta_{n,N_{SL}} + \delta_{n,N_{SR}} \right) \hat{c}_{n\sigma}^\dagger \hat{c}_{n\sigma} + \sum_n \left( \Delta_n \hat{c}_{n\uparrow}^\dagger \hat{c}_{n\downarrow} + \text{H.c.} \right)$$  

(2)

is the electronic part of the Hamiltonian, $\sigma = \uparrow, \downarrow$ denotes the spin projections along the $z$ axis, we set the hopping to unity and, thereby set the energy scale, and we choose the chemical potential to be zero.

$$\hat{H}_{c-S} = - \sum_{n,\sigma,\sigma'} JS_n \cdot s_n$$  

(3)

is the interaction between the spin density of the conduction electrons and the impurity spins, with $s_n = \sigma_{\sigma\sigma'} \hat{c}_{n\sigma'}^\dagger \hat{c}_{n\sigma}$, where $\sigma = (\sigma^x, \sigma^y, \sigma^z)$ are the Pauli matrices, we assume the local magnetic moments $S_n$ to be of unit length and to be Zeeman coupled to the electrons in the superconductor, behaving as local magnetic fields $JS_n \equiv \tilde{J}_n$ in the superconductor and

$$\hat{H}_S = - \sum_n J_{ex} S_n \cdot S_{n+1} + \frac{k_y}{2} \sum_n \left( S_n^z \right)^2$$  

(4)

It is convenient to introduce a planar anisotropy $k_y$, which for positive $k_y$ favors a state where the spins are in the $x-z$ plane (the plane of the initial domain wall). At the superconductor interfaces with the F or N systems ($N_{SL}, N_{SR}$) we introduce a potential term, $U$, that simulates the interface disorder. The local moments are distributed evenly inside the superconducting region and interact with each other via a nearest-neighbor ferromagnetic interaction $J_{ex}$. The superconducting region includes the magnetic moments that constitute the domain wall, as shown in Fig. 1 of Ref.\textsuperscript{24}. Often we take the number of sites $N = 160$, and $N_{SR} - N_{SL} = 100$. We are considering for simplicity a singlet $s$-wave superconductor.

The Bogoliubov-de Gennes (BdG) equations\textsuperscript{25} for $\hat{H}_c + \hat{H}_{c-S}$ determine both the equilibrium electronic states and the scattering states when a current is passing through. Their solution provide the wave functions in the usual way (see for instance Ref.\textsuperscript{22}). The BdG equations are solved self-consistently to find the profile of the gap function inside the superconductor, $\Delta_n$. The domain wall shape is calculated self-consistently in mean-field\textsuperscript{27} ensuring that the energy is minimized and a vanishingly small equilibrium torque. The procedure leads to a domain wall where the magnetization direction interpolates between the $z$-direction on the left hand side and the $-z$ direction in the right hand side, according to the orientation of the exchange fields in the two ferromagnets. The procedure is described in detail in Ref.\textsuperscript{24} and in Ref.\textsuperscript{23}.

Using the transfer matrix method we determine the wave functions at every site in the heterostructure and calculate the various physical quantities such as the local spin densities, the spin and charge currents and the spin torque at each site. The torque at point $n$ is defined by the cross product between the magnetic moments and the local spin density, $s_n$,

$$\tau(n) = 2JS_n \times s_n.$$  

(5)

Assuming spin conservation the spin torque may also be calculated by the difference in spin currents as

$$\tau_\beta(n) = j_\beta(n) - j_\beta^s(n),$$  

(6)

where $\beta = x, y, z$. The expressions for the spin currents are given in Ref.\textsuperscript{24}.

In experiments, a potential difference, $V$, is imposed between the two sides of the heterostructure in a standard way\textsuperscript{24,25} leading to a current that moves from left to right. Imposing a potential difference at the ends of the heterostructure leads to an overall torque on a given
magnetic moment, that is the result of an integral over the incident energies, up to the applied potential\textsuperscript{21}. This torque leads to the motion of the magnetic moments, governed by the Landau-Lifshitz-Gilbert equation\textsuperscript{29}.

### III. EQUILIBRIUM PROPERTIES OF THE HETEROSTRUCTURE

We begin by considering a situation with no current flowing through the system. In this case the BdG equations yield the equilibrium solution. In Fig. 1 we show the average magnetization of the electrons over the left half of the superconductor, $<s_z>$, along the $z$-direction, induced by the coupling to the local spins, as a function of $J$. We consider 15 spins distributed evenly in the superconductor (density 0.15). We see in the superconducting case that, as $J$ increases, the magnetization increases, but at some points it changes discontinuously between various plateaus. These discontinuities are due to quantum phase transitions in the system\textsuperscript{16,17}. In the case of one impurity and for small $J$ the average magnetization vanishes. Even though the local magnetic moment polarizes the spin density of the electrons at the impurity location, the spin density of the electrons has fluctuations (like Friedel oscillations) that compensate the perturbation. Above a certain critical $J_c$ the system is no longer able to shield the perturbation and the overall magnetization jumps discontinuously to 1/2. Various other discontinuities occur at this first order quantum phase transition due to a level crossing, such as the gap function changing sign at the impurity location, various entanglement\textsuperscript{30} measures and the partial state fidelity\textsuperscript{31}.

Increasing the number of magnetic impurities the discontinuities are still present, but at very small $J$ the magnetization is not strictly zero\textsuperscript{17}, as shown in Fig. 1. We also show in Fig. 1 the magnetization for the normal case. We see that there are no discontinuities and the magnetiza-
tion changes smoothly. At small $J$ the magnetization is smaller in the superconducting case. As the first transition to a plateau occurs, in the superconducting case, the magnetization becomes larger in the superconducting case.

The profile of the gap function is shown in Fig. 2 for various spin couplings, $J$. At small $J$ the gap function is approximately constant over the superconducting region. It vanishes outside the superconductor and it has very small fluctuations due to the finite size of the SC region and the finite number of impurity spins. As the coupling grows the fluctuations in the gap function increase considerably and are modulated by oscillations that increase in number as $J$ grows. Note that, interestingly, the number of peaks in the oscillations of the gap function as a function of space along the chain is related to the various plateaus of the total magnetization. As one goes from one plateau to the next the number of peaks increases by one. This is reminiscent of a LOFF state where the gap function has real space oscillations due to the finite momentum of the Cooper pairs resulting from the splitting of the up and down spin bands due to a magnetic field. We recall that in a superconductor with magnetic impurities, if $J$ is large enough, the gap function changes sign at the impurity locations. Here, however, we have an heterostructure. Interestingly, we find that even though the gap function decreases significantly at the impurity locations as $J$ grows, it only changes sign at very large $J \sim 2.5$. This is in contrast with an infinite superconductor where the gap function changes sign at the location of the first quantum phase transition. This is shown in Fig. 2.

IV. LANDAU-LIFSHITZ-GILBERT EQUATION AND DOMAIN WALL MOTION

Consider now that the spin current is turned on. The LLG equation for the motion of a magnetic moment $\mathbf{S}$ in the presence of an effective magnetic field $\tilde{\mathbf{H}}$ and subject to a spin current $\dot{j}^s$ (Slonczewski term) giving origin to a spin torque $\mathbf{\tau}$ is written as

$$\frac{d\mathbf{S}}{dt} = -\gamma \mathbf{S} \times \tilde{\mathbf{H}} + \frac{\alpha}{S} \mathbf{S} \times \frac{d\mathbf{S}}{dt}$$ (7)

Here $S$ is the length of the magnetic moment, $\gamma$ is the gyromagnetic ratio and $\alpha$ is the damping constant. The magnetic field $\tilde{\mathbf{H}}$ is a generic notation for an effective field defined as a derivative of the energy with respect to the magnetization (spin). Due to the spin torque the magnetic moments in general evolve out of the plane and therefore we assumed the system has an anisotropy term that favors the plane. Also, the interaction between the local magnetic moments leads to a term that contributes to the effective magnetic field. The effective magnetic field can be written as

$$\tilde{\mathbf{H}} = H^y + H^{ex}$$ (8)

The contributions of these two terms to the effective magnetic field are

$$H^y = -k_y S \dot{\mathbf{y}}$$ (9)

and

$$H^{ex}_\beta (n) = J_{ex} S_\beta (n + 1)$$ (10)

where $\beta = x, y$.

Taking the cross product of the LLG equation with the magnetic moment and using that its length is fixed ($\mathbf{S} \cdot \frac{d\mathbf{S}}{dt} = 0$), the LLG can be reformulated as

$$\frac{d\mathbf{S}}{dt} = -\gamma_L \mathbf{S} \times \tilde{\mathbf{H}} + \frac{\alpha \gamma_L}{S} \mathbf{S} \times \left( \mathbf{S} \times \tilde{\mathbf{H}} \right)$$

$$+ \frac{1}{1 + \alpha^2} \tau + \frac{\alpha}{1 + \alpha^2} \frac{1}{S} \mathbf{S} \times \tau$$ (11)

where

$$\gamma_L = \frac{\gamma}{1 + \alpha^2}$$ (12)

In terms of the spherical angles $\theta$, $\varphi$ it is written as

$$\frac{d\theta}{dt} = \gamma_L \left( \tilde{H}_\varphi + \alpha \tilde{H}_\theta \right) + \frac{1}{S} \frac{\alpha}{1 + \alpha^2} \tilde{\tau}_\theta - \frac{1}{S} \frac{\alpha}{1 + \alpha^2} \tilde{\tau}_\varphi$$

$$\sin \theta \frac{d\varphi}{dt} = -\gamma_L \left( \tilde{H}_\theta - \alpha \tilde{H}_\varphi \right) + \frac{1}{S} \frac{1}{1 + \alpha^2} \tilde{\tau}_\varphi + \frac{1}{S} \frac{\alpha}{1 + \alpha^2} \tilde{\tau}_\theta$$ (13)

where $\tilde{H}_\theta$, $\tilde{\tau}_\theta$ and $\tilde{H}_\varphi$, $\tilde{\tau}_\varphi$ are the spherical components of the vectors $\tilde{\mathbf{H}}$ and $\mathbf{\tau}$, respectively. The anisotropy and the exchange term imply extra terms in the LLG equations of the form

$$H^y_\theta = -k_y S \sin \theta \cos \theta \sin^2 \varphi$$

$$H^y_\varphi = -k_y S \sin^2 \theta \sin \varphi \cos \varphi$$ (14)

and

$$H^{ex}_\theta = J_{ex} [\cos \theta \sin \theta_{n+1} \cos (\varphi_n - \varphi_{n+1}) - \sin \theta_n \cos \theta_{n+1}]$$

$$H^{ex}_\varphi = -J_{ex} [\sin \theta_{n+1} \sin (\varphi_n - \varphi_{n+1})]$$ (15)

These equations are solved iteratively: for a given domain wall configuration, at a certain time, we calculate the electronic properties, such as the spin torque from Eqs. 5,6. This torque is then used to evolve the magnetic moment configuration, at a certain time, we calculate the new configuration. This new configuration is then used to calculate the new spin torque and so on. To solve the LLG equations we used a standard second order Runge-Kutta method or the Heun method, yielding similar results.

V. EVOLUTION OF THE DOMAIN WALL

We start with a domain wall that interpolates between opposite exchange fields in the ferromagnetic regions.
FIG. 3: Time evolution of a domain wall of 15 spins with parameters $J = 1$, $J_{ex} = 0.5$, $V = 1$. At the initial time ($t = 0$) the domain wall magnetic moments are contained in the $x - z$ plane. As time evolves the location of the center of the DW shifts to the right.

FIG. 4: Time evolution of a domain wall of 41 spins with parameters $J = 1$, $J_{ex} = 0.5$, $V = 1$. 
FIG. 5: Time dependence of the (a) distance between the center of the DW, $x_{\text{DW}}$, and the midpoint of the SC and (b) velocity of the DW for different values of $J$ from $J = 0.1, \cdots, 2.5$. In black (full lines) are the results for the superconductor and in red (dashed lines) the results for the normal system.
Initially the magnetic moments form a Néel-type domain wall in the $x-z$ plane. As time evolves the domain wall moves to the right due to the spin torque exerted by the spin polarized current. The spin torque has in-plane and out of plane components that rotate the magnetic moments. The combined effect of the spin torque, the exchange coupling between the magnetic moments, the magnetic anisotropy and the damping term determines the time evolution of the spins. The magnetic moments tend to rotate to align with the spins on the left-hand side (leading to the intended displacement of the center of the DW) but also tend to move out of the $x-z$ plane (this effect is counteracted by the magnetic anisotropy and by the damping term).

In Figs. 3 and 4 we show the time evolution of the domain wall for different parameters. There are several parameters that need to be specified which implies a very large parameter space. As a standard case shown in Fig. 3 we take $J = 1, J_{ex} = 0.5, V = 1, k_y = 4, \alpha = 0.02, U = 0, \gamma = 2.2, h = 0.2$. The parameter $h$ is the exchange magnetic field in the ferromagnetic regions. We take $h = 0.2$ in the left F and $h = -0.2$ in the right F.

Here we are interested in studying the early time evolution, up to the moment the domain wall has come close to the edge of the spin configuration. In particular, we are interested in comparing the time evolution using a supercurrent with the one obtained from a standard normal metal. In Fig. 5a we show the displacement of the center of the DW as a function of time for various parameters. We define $x_{DW}$ as the distance between the center of the DW and the midpoint of the SC. To better quantify the motion of the domain wall we calculate the velocity shown in Fig. 5b. In the initial configuration the spins on the left hand side have an orientation such that $\theta < \pi/2$ while on the right hand side $\theta > \pi/2$. As the domain wall moves to the right, the orientation of spins on the right hand side will successively cross $\pi/2$. We define the center of the DW to be the spin with $\theta = \pi/2$. The velocity of the DW is $v = (dx_{DW}/dt)$. This is shown in Fig. 5b for different values of $J$, where we compare the velocities as a function of time for the superconductor and a normal system. This last case is simply obtained turning off the superconducting pairing.

Typically, at early times, the velocity is small. It increases over time and then starts to saturate. This is related to either basically all spins have already flipped or the DW starts to get distorted and the last spins do not flip at all. As discussed in Ref. the spin torque has in general a component that leads to the motion of the magnetic moments out of the original DW plane. This

FIG. 6: Dependence of the domain wall velocity with the various parameters. We consider the dependence with (a) the interface disorder, $U$, (b) the exchange between the spins, $J_{ex}$ and (c) the applied potential, $V$. In panel (d) and we compare the velocity for the superconductor and the normal metal for the parameters $J = 2, J_{ex} = 1.5, V = 0.5, \alpha = 0.5$. 
FIG. 7: Time evolution of domain wall of 15 spins and $J = 1, J_{ex} = 0.5, V = 1$ for later times. For the parameters considered here the DW center point reaches the right hand side border and the remaining non-aligned magnetic moments move out of the $x-z$ plane. The boundary condition imposed by the right hand side ferromagnet fixes the orientation of the last magnetic moment ($t = 1.25$). At later times $t = 1.5, 1.75$ the DW starts to distort and several spins rotate out of the original plane.

FIG. 8: Time evolution of domain wall of 15 spins and $J = 2, J_{ex} = 1.5, V = 0.5, \alpha = 0.5$. Increasing the couplings between the magnetic moments and their coupling to the electron spin density, and the Gilbert damping, the domain wall distortion is smaller, even though some spins start to move out of the initial plane when the center of the DW approaches the right hand boundary ($t = 1.65$).
term implies an acceleration of the motion of the domain wall that leads to an increased velocity. The velocity then saturates both due to the presence of friction originated by the damping term, and by shear finite size effects due to the approach to the boundary of the system. As a function of the coupling between the magnetic moments and the electron spin density, $J$, we see various regimes. At small $J$ the DW moves faster for the superconductor, between $J = 0.7 - 1.2$ the normal metal is more efficient, and then it changes from one to the other. It may seem therefore that the velocity may be larger if the magnetization is smaller. However, there are regimes for larger $J$ for which the velocity is larger in the superconducting phase, such as $J = 1.5, 2$. Also, one would expect that the velocity should be larger if the spin torque is larger. The values of the velocities are similar for all cases but for small $J$ and for the case of $J = 1.5$ the velocity in the superconductor is particularly larger with respect to the normal metal. For this particular value of the coupling and for this system size the velocity does not seem to saturate before the domain wall gets distorted.

To establish in greater detail the influence of the various parameters we consider a few cases. Due to the large parameter space we fix the various parameters at the standard case and change one of the parameters to see its influence. In Fig. 6 we change the interface disorder $U$, the coupling between the magnetic moments, $J_{ex}$, and the external potential, $V$. As one expects, the velocity is typically larger for a clean interface, or at least for small interface disorder. The velocity increases as the exchange $J_{ex}$ increases and as the potential, $V$, increases. Small interface disorder is important to have larger currents, and larger potential has the same effect. Larger currents lead to larger spin torques. The effect of the exchange interaction is also clear because it tends to collectively move the spins.

From experimental results one knows that the domain walls either stop, distort or change into vortex configuration[19] and the process has to be restarted. This is not related however to the finite size effects of the wire. In any case we were mostly interested in describing the early times displacement of the DW.

Since the system sizes considered are finite, the motion of the domain wall is conditioned by the end of the system. Typically the domain wall moves from left to right and, when the spins close to the edge move appreciably, there is an important distortion of the domain wall since the magnetic moments rotate out of the original plane. This occurs near the right edge of the system but often it propagates to the left. In some cases it destroys the domain wall on the right hand side and the spins on the left hand side may start to precess and move out of the plane. Clearly, these situations are not what is intended since we want to move the center of the domain wall but maintain the spins aligned in the original plane (defined as $x - z$). This is illustrated in Fig. 4. Beyond the point for which the center of the DW approaches the right boundary the spins on the right hand side start to disorder (in the sense they loose the alignment with the neighboring spins) and the spins on the left hand side start to move out of the plane.

This problem may be avoided (for not very long times) by changing the parameters. We present in Fig. 6 the time evolution of the same domain wall with 15 spins (recall that the system size is 100 sites and therefore 15 spins is rather dilute) where we have changed some of the parameters to a set with $J = 2, J_{ex} = 1.5, V = 0.5, \alpha = 0.5$. The motion of the domain wall is now much better controlled, at least up to a time of the order of $t = 2$. Clearly, when all the spins have been flipped one should turn the current off. We have found that for these parameters it is important to increase the coupling between the magnetic moments and the spins of the conduction electrons, to increase the coupling between the magnetic moments and to increase the damping term, so that the motion of the magnetic moments does not lead to a large distortion of the DW. As shown in Fig. 6j in this case the SC velocity is considerably larger than the velocity in the normal phase.

Another way to make the evolution of the domain wall well controlled is to increase the density of the magnetic moments as shown in Fig. 4. The evolution is slower but well controlled up to $t = 2$, for which the domain wall has not yet reached the border.

VI. CONCLUSIONS

Following previous works where we considered the possibility of the effects of magnetic impurities, immersed or in the vicinity of a conventional superconductor, paying attention to both the effects of the impurities on the superconductor and of the superconductor on the magnetic impurities, we have solved the LLG equations for the motion of a domain wall due to the passage of a spin polarized current through a heterostructure of the type FSF. The new aspect is that the magnetic moments are distributed in the S region. Consistently with previous results, where the spin torque exerted on the magnetic moments may be larger in the superconductor compared to a normal metal, we have found that the velocity of the motion of the domain wall in the superconducting phase may be enhanced with respect to the normal phase.

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