On Modeling of Lifetime Data Using One Parameter Akash, Lindley and Exponential Distributions

Abstract
The analysis and modeling of lifetime data are crucial in almost all applied sciences including medicine, insurance, engineering, and finance, amongst others. In the present paper an attempt has been made to discuss applications of Akash distribution introduced by Shanker [1], Lindley distribution and exponential distributions for modeling lifetime data from various fields. Firstly a table for values of the various characteristics of Akash distribution and Lindley distribution has been presented for various values of their parameter which reflects their nature and behavior. The expressions for the index of dispersion of Akash, Lindley and exponential distributions have been obtained and the conditions under which Akash, Lindley and exponential distributions are over-dispersed, equi-dispersed, and under-dispersed has been given. Several lifetime data from medical science and engineering have been fitted using Akash distribution along with Lindley and exponential distributions to study the advantages and disadvantages of these distributions for modeling lifetime data.

Keywords: Akash distribution; Lindley distribution; Exponential distribution; Index of dispersion; Estimation of parameter; Goodness of fit

Introduction
The time to the occurrence of event of interest is known as lifetime or survival time or failure time in reliability analysis. The event may be failure of a piece of equipment, death of a person, development (or remission) of symptoms of disease, health code violation (or compliance). The modeling and statistical analysis of lifetime data are crucial for statisticians and research workers in almost all applied sciences including engineering, medical science/biological science, insurance and finance, amongst others.

Recently Shanker [1] has introduced a one parameter continuous distribution named, "Akash distribution" for modeling lifetime data from engineering and medical science and medical and its various mathematical properties, estimation of its parameter, and its applications. A number of continuous distributions for modeling lifetime data have been introduced in statistical literature including exponential, Lindley, gamma, lognormal and Weibull, amongst others. The exponential, Lindley and the Weibull distributions are more popular in practice than the gamma and the lognormal distributions because the survival functions of the gamma and the lognormal distributions cannot be expressed in closed forms and both require numerical integration. Though Akash, Lindley and exponential distributions are of one parameter, Akash and Lindley distributions have advantage over the exponential distribution that the exponential distribution has constant hazard rate and mean residual life function whereas the Akash and Lindley distributions have increasing hazard rate and decreasing mean residual life function. Further, Akash distribution of Shanker [1] has flexibility over both Lindley and exponential distributions.

Exponential, Lindley and Akash Distributions
Exponential distribution
In statistical literature, exponential distribution was the first widely used lifetime distribution model in areas ranging from studies on the lifetimes of manufactured items [Davis [2], Epstein & Sobel [3]], Epstein [4] to research involving survival or remission times in chronic diseases [Feigl & Zelen [5]]. The main reason for its wide usefulness and applicability as lifetime model is partly because of the availability of simple statistical methods for it [Epstein & Sobel [3]] and partly because it appeared suitable for representing the lifetimes of many phenomena such as various types of manufactured items [Davis [2]].

Lindley distribution
The Lindley distribution is a two-component mixture of an exponential distribution having scale parameter $\theta$ and a gamma distribution having shape parameter 2 and scale parameter $\theta$ with mixing proportions $\frac{\theta}{\theta+1}$ and $\frac{1}{\theta+1}$ and is given by Lindley [6] in the context of fiducial Statistics as a counter example of fiducial Statistics. A detailed study about its various mathematical properties, estimation of parameter and application showing the superiority of Lindley distribution over exponential distribution for the waiting times before service of the bank customers has been done by Ghitany et al. [7]. The Lindley distribution has been generalized, extended, mixed, modified and its detailed...
applications in reliability and other fields of knowledge by different researchers including Sankaran [8] Hussain [9], Zakerczadhe & Dolati [10], Nadarajah et al. [11], Deniz & Ojeda [12], Mazucheli & Achcar [13], Bakouch et al. [14], Shanker & Mishra [15,16], Shanker et al. [17], Shanker & Amanuel [18], Elbatal et al. [19], Ghitany et al. [20], Merovci [21], Liyanage & Pararai [22], Ashour & Eltehiwy [23], Oluyede & Yang [24], Singh et al. [25], Shanker & Amanuel [18], Elbatal et al. [19], Ghitany et al. [20], Merovci [21], Liyanage & Pararai [22], Ashour & Eltehiwy [23], Oluyede & Yang [24], Singh et al. [25], Shanker et al. [27], Alkarni [28], Pararai et al. [29], Abouammoh et al. [30] are some among others.

Although the Lindley distribution has been used to model lifetime data by many researchers and Hussain [9] has shown that the Lindley distribution is important for studying stress-strength reliability modeling, it has been observed that there are many situations in the modeling of lifetime data where the Lindley distribution may not be suitable from a theoretical or applied point of view. In fact, Shanker et al. [27] has detailed comparative study about the applicability of Lindley and exponential distributions for modeling various types of lifetime data and observed that none is a suitable model in all cases.

**Akash distribution**

Shanker [1] introduced a new distribution named, ‘Akash distribution’ which is flexible than the Lindley distribution for modeling lifetime data in reliability and in terms of its hazard rate shapes. Akash distribution is a two- component mixture of an exponential distribution having scale parameter $\theta$ and a gamma $\theta$ distribution having shape parameter 3 and scale parameter $\theta$ with mixing proportions $\frac{\theta}{\theta+1}$ and $\frac{1}{\theta+1}$ and has been shown by Shanker [1] that Akash distribution gives better fit than Lindley and exponential distributions in modeling some lifetime data.

Let $T$ be a continuous random variable representing the lifetimes of individuals in some population. The expressions for probability density function, $f(t)$, cumulative distribution function, $F(t)$, survival function, $S(t)$, hazard rate function, $h(t)$, mean residual life function, $m(t)$, mean $\mu_1$, variance $\mu_2$, third moment about mean $\mu_3$, fourth moment about mean $\mu_4$, coefficient of variation (C.V), coefficient of Skewness ($\sqrt[3]{\beta_1}$), coefficient of Kurtosis ($\beta_2$), and index of dispersion ($\gamma$) of exponential, Lindley and Akash distributions are summarized in Table 1.

**Table 1:** Characteristics of Exponential, Lindley and Akash Distributions.

|                      | Exponential Distribution | Lindley Distribution | Akash Distribution |
|----------------------|--------------------------|----------------------|--------------------|
| $f(t)$               | $\theta e^{-\theta t}$  | $f(t) = \frac{\theta^3}{\theta^2 + 2} \left(1 + t^2\right) e^{-\theta t}$ | $f(t) = \frac{\theta^3}{\theta^2 + 2} \left(1 + t^2\right) e^{-\theta t}$ |
| $F(t)$               | $1 - e^{-\theta t}$     | $F(t) = 1 - \frac{\theta + 1 + \theta t}{\theta + 1} e^{-\theta t}$ | $F(t) = 1 - \frac{\theta t (\theta t + 2)}{\theta^2 + 2} e^{-\theta t}$ |
| $S(t)$               | $e^{-\theta t}$         | $S(t) = \frac{\theta + 1 + \theta t}{\theta + 1} e^{-\theta t}$ | $S(t) = \frac{\theta t (\theta t + 2)}{\theta^2 + 2} e^{-\theta t}$ |
| $h(t)$               | $\theta$                | $h(t) = \frac{\theta^3 (1 + t)}{\theta + 1 + \theta t}$ | $h(t) = \frac{\theta^3 (1 + t^3)}{\theta t (\theta t + 2) + (\theta^2 + 2)}$ |
| $m(t)$               | $\frac{1}{\theta}$     | $m(t) = \frac{\theta + 2 + \theta t}{\theta (\theta + 1 + \theta t)}$ | $m(t) = \frac{\theta^3 t^2 + 4 \theta t + (\theta^2 + 6)}{\theta [\theta t (\theta t + 2) + (\theta^2 + 2)]}$ |

Citation: Shanker R, Fesshaye H, Selvaraj S (2016) On Modeling of Lifetime Data Using One Parameter Akash, Lindley and Exponential Distributions. Biom Biostat Int J 3(2): 00061. DOI: 10.15406/bbij.2016.03.00061
| \(\tilde{\mu}_1^i\) = \(\frac{1}{\theta}\) | \(\tilde{\mu}_1^i\) = \(\frac{\theta + 2}{\theta(\theta + 1)}\) | \(\tilde{\mu}_1^i\) = \(\frac{\theta^2 + 6}{\theta(\theta^2 + 2)}\) |
| --- | --- | --- |
| \(\tilde{\mu}_2^i\) = \(\frac{1}{\theta^2}\) | \(\tilde{\mu}_2^i\) = \(\frac{\theta^2 + 4\theta + 2}{\theta^2(\theta + 1)^2}\) | \(\tilde{\mu}_2^i\) = \(\frac{\theta^4 + 16\theta^2 + 12}{\theta^2(\theta^2 + 2)^2}\) |
| \(\tilde{\mu}_3^i\) = \(\frac{2}{\theta^3}\) | \(\tilde{\mu}_3^i\) = \(\frac{2(\theta^3 + 6\theta^2 + 6\theta + 2)}{\theta^3(\theta + 1)^3}\) | \(\tilde{\mu}_3^i\) = \(\frac{2(\theta^3 + 30\theta^4 + 36\theta^2 + 24)}{\theta^3(\theta^2 + 2)^3}\) |
| \(\tilde{\mu}_4^i\) = \(\frac{9}{\theta^4}\) | \(\tilde{\mu}_4^i\) = \(\frac{3(3\theta^4 + 24\theta^3 + 44\theta^2 + 32\theta + 8)}{\theta^4(\theta + 1)^4}\) | \(\tilde{\mu}_4^i\) = \(\frac{3(3\theta^8 + 128\theta^6 + 408\theta^4 + 576\theta^2 + 240)}{\theta^4(\theta^2 + 2)^4}\) |

\[CV = \frac{\sigma}{\mu_1^i} = 1\]  
\[CV = \frac{\sigma}{\mu_1^i} = \frac{\sqrt{\theta^3 + 4\theta + 2}}{\theta + 2}\]  
\[CV = \frac{\sigma}{\mu_1^i} = \frac{\sqrt{\theta^3 + 16\theta^2 + 12}}{\theta^2 + 6}\]  

\[\sqrt{\beta_1} = 2\]  
\[\sqrt{\beta_1} = \frac{2(\theta^3 + 6\theta^2 + 6\theta + 2)}{(\theta^2 + 4\theta + 2)^{3/2}}\]  
\[\sqrt{\beta_1} = \frac{2(\theta^3 + 30\theta^4 + 36\theta^2 + 24)}{(\theta^4 + 16\theta^2 + 12)^{3/2}}\]  

\[\beta_2 = 9\]  
\[\beta_2 = \frac{3(3\theta^4 + 24\theta^3 + 44\theta^2 + 32\theta + 8)}{(\theta^2 + 4\theta + 2)^2}\]  
\[\beta_2 = \frac{3(3\theta^8 + 128\theta^6 + 408\theta^4 + 576\theta^2 + 240)}{(\theta^4 + 16\theta^2 + 12)^2}\]  

\[\gamma = \frac{\sigma^2}{\mu_1^i} = \frac{1}{\theta}\]  
\[\gamma = \frac{\sigma^2}{\mu_1^i} = \frac{\theta^2 + 4\theta + 2}{\theta(\theta + 1)(\theta + 2)}\]  
\[\gamma = \frac{\sigma^2}{\mu_1^i} = \frac{\theta^4 + 16\theta^2 + 12}{\theta(\theta^2 + 2)(\theta^2 + 6)}\]
It can be easily verified that the Akash distribution is over-dispersed \((\mu < \sigma^2)\), equi-dispersed \((\mu = \sigma^2)\) and under-dispersed \((\mu > \sigma^2)\) for \(\theta < (\theta^*) = 1.515400063\) respectively. Further, Lindley distribution is over-dispersed \((\mu < \sigma^2)\), equi-dispersed \((\mu = \sigma^2)\) and under-dispersed \((\mu > \sigma^2)\) for \(\theta < (\theta^*) = 1.70086487\) respectively, whereas as exponential distribution is over-dispersed \((\mu > \sigma^2)\) equi-dispersed \((\mu = \sigma^2)\) and under-dispersed \((\mu < \sigma^2)\) for \(\theta < (\theta^*) = 1\) respectively.

A table of values for coefficient of variation (C.V.), coefficient of skewness \((\sqrt{\beta_1})\), coefficient of kurtosis \((\beta_2)\), and index of dispersion \((\gamma)\) for Akash and Lindley distributions for various values of their parameter for comparative study are summarized in Table 2.

### Table 2: Values of C.V., \(\sqrt{\beta_1}\), \(\beta_2\) and \(\gamma\) of Akash and Lindley Distributions for varying values of the parameter \(\theta\).

| Values of \(\theta\) for Akash Distribution | C.V 0.01 | 0.05 | 0.09 | 0.5 | 0.8 | 1.5 | 2 |
|------------------------------------------|---------|------|------|-----|-----|-----|---|
| C.V                                     | 0.577379| 0.578071| 0.579679| 0.641249| 0.716741| 0.882958| 0.959166|
| \(\sqrt{\beta_1}\)                      | 1.154643| 1.153268| 1.150133| 1.083974| 1.10564| 1.138077| 1.61372|
| \(\beta_2\)                             | 4.999867| 4.996681| 4.989352| 4.784948| 4.735717| 5.472724| 6.391304|
| \(\gamma\)                              | 100.0067| 20.03328| 11.17079| 2.284444| 1.615097| 1.1008913| 0.766667|

| Values of \(\theta\) for Lindley Distribution | C.V 0.01 | 0.05 | 0.09 | 0.5 | 0.8 | 1.5 | 2 |
|-----------------------------------------------|---------|------|------|-----|-----|-----|---|
| C.V                                          | 0.710607| 0.723943| 0.736298| 0.824621| 0.863075| 0.914732| 0.935414|
| \(\sqrt{\beta_1}\)                          | 1.414317| 1.416546| 1.421076| 1.512281| 1.580387| 1.698866| 1.756288|
| \(\beta_2\)                                 | 6.000294| 6.006807| 6.020488| 6.342561| 6.621505| 7.172516| 7.469388|
| \(\gamma\)                                  | 100.4926| 20.46458| 11.55007| 2.266667| 1.448413| 0.780952| 0.583333|

### Applications

The Akash, Lindley and exponential distributions have been fitted to a number of real lifetime data-sets to test their goodness of fit. Goodness of fit tests for sixteen real lifetime data-sets have been presented here. In order to compare Akash, Lindley and exponential distributions, \(-2 \ln L\), AIC (Akaike Information Criterion), AICC (Akaike Information Criterion Corrected), BIC (Bayesian Information Criterion), K-S Statistics (Kolmogorov-Smirnov Statistics) for all sixteen real lifetime data-sets have been computed and presented in Table 3. The formulae for computing AIC, AICC, BIC, and K-S Statistics are as follows:

**Citation:** Shanker R, Fesshaye H, Selvanaj S (2016) On Modeling of Lifetime Data Using One Parameter Akash, Lindley and Exponential Distributions. Biom Biostat Int J 3(2): 00061. DOI: 10.15406/bbij.2016.03.00061
| Data | Model     | Parameter Estimate | -2ln L    | AIC    | AICC   | BIC    | K-S Statistic |
|------|-----------|--------------------|-----------|--------|--------|--------|--------------|
| 1    | Akash     | 1.355445           | 163.73    | 165.73 | 165.79 | 169.93 | 0.355        |
|      | Lindley   | 0.996116           | 162.56    | 164.56 | 164.62 | 166.70 | 0.371        |
|      | Exponential | 0.663647         | 177.66    | 179.66 | 179.73 | 171.80 | 0.362        |
| 2    | Akash     | 0.043876           | 950.97    | 952.97 | 953.01 | 955.58 | 0.184        |
|      | Lindley   | 0.028859           | 983.11    | 985.11 | 985.15 | 987.71 | 0.242        |
|      | Exponential | 0.014635        | 1044.87   | 1046.87| 1046.91| 1049.48| 0.357        |
| 3    | Akash     | 0.04151            | 227.06    | 229.06 | 229.25 | 230.20 | 0.107        |
|      | Lindley   | 0.027321           | 231.47    | 233.47 | 233.66 | 234.61 | 0.149        |
|      | Exponential | 0.013945        | 242.87    | 244.87 | 245.06 | 246.01 | 0.263        |
| 4    | Akash     | 0.013514           | 1255.83   | 1257.83| 1257.87| 1260.43| 0.071        |
|      | Lindley   | 0.00897            | 1251.34   | 1253.34| 1253.38| 1255.95| 0.098        |
|      | Exponential | 0.004505        | 1280.52   | 1282.52| 1282.56| 1285.12| 0.190        |
| 5    | Akash     | 0.030045           | 794.70    | 796.70 | 796.76 | 798.98 | 0.184        |
|      | Lindley   | 0.019941           | 789.04    | 791.04 | 791.10 | 793.32 | 0.133        |
|      | Exponential | 0.010018        | 806.88    | 808.88 | 809.94 | 811.16 | 0.198        |
| 6    | Akash     | 0.11961            | 941.28    | 943.28 | 943.32 | 943.68 | 0.393        |
|      | Lindley   | 0.077247           | 1041.64   | 1043.64| 1043.68| 1046.54| 0.448        |
|      | Exponential | 0.040606        | 1130.26   | 1132.26| 1132.29| 1135.16| 0.525        |
| 7    | Akash     | 0.013263           | 803.96    | 805.96 | 806.02 | 810.01 | 0.298        |
|      | Lindley   | 0.008804           | 763.75    | 765.75 | 765.82 | 767.81 | 0.245        |
|      | Exponential | 0.004421        | 744.87    | 746.87 | 746.94 | 748.93 | 0.166        |
| 8    | Akash     | 0.013423           | 609.93    | 611.93 | 612.02 | 613.71 | 0.280        |
|      | Lindley   | 0.008891           | 579.16    | 581.16 | 581.26 | 582.95 | 0.219        |
|      | Exponential | 0.004475        | 564.02    | 566.02 | 566.11 | 567.80 | 0.145        |
| 9    | Akash     | 0.3105             | 887.89    | 889.89 | 889.92 | 892.74 | 0.198        |
|      | Lindley   | 0.196045           | 839.06    | 841.06 | 841.09 | 843.91 | 0.116        |
|      | Exponential | 0.106773        | 828.68    | 830.68 | 830.72 | 833.54 | 0.077        |
| 10   | Akash     | 0.050293           | 354.88    | 356.88 | 357.02 | 358.28 | 0.421        |
|      | Lindley   | 0.033021           | 323.27    | 325.27 | 325.42 | 326.67 | 0.345        |
|      | Exponential | 0.016777        | 305.26    | 307.26 | 307.40 | 308.86 | 0.213        |
| 11   | Akash     | 1.165719           | 115.15    | 117.15 | 117.28 | 118.68 | 0.156        |
|      | Lindley   | 0.823821           | 112.61    | 114.61 | 114.73 | 116.13 | 0.133        |
|      | Exponential | 0.532081        | 110.91    | 112.91 | 113.03 | 114.43 | 0.089        |
| 12   | Akash     | 0.295277           | 641.93    | 643.93 | 643.95 | 646.51 | 0.100        |
|      | Lindley   | 0.186571           | 638.07    | 640.07 | 640.12 | 642.68 | 0.058        |
|      | Exponential | 0.101245        | 658.04    | 660.04 | 660.08 | 662.65 | 0.163        |
| 13   | Akash     | 0.024734           | 194.30    | 196.30 | 196.61 | 197.01 | 0.456        |
|      | Lindley   | 0.01636            | 181.34    | 183.34 | 183.66 | 184.05 | 0.368        |
|      | Exponential | 0.008246        | 173.94    | 175.94 | 176.25 | 176.65 | 0.277        |
| 14   | Akash     | 1.156923           | 59.52     | 61.52  | 61.74  | 62.51  | 0.320        |
|      | Lindley   | 0.816118           | 60.50     | 62.50  | 62.72  | 63.49  | 0.341        |
|      | Exponential | 0.526316        | 65.67     | 67.67  | 67.90  | 68.67  | 0.389        |
| 15   | Akash     | 0.097062           | 240.68    | 242.68 | 242.82 | 244.11 | 0.266        |
|      | Lindley   | 0.062988           | 253.99    | 255.99 | 256.13 | 257.42 | 0.333        |
|      | Exponential | 0.032455        | 274.53    | 276.53 | 276.67 | 277.96 | 0.426        |
| 16   | Akash     | 0.964726           | 224.28    | 226.28 | 226.34 | 228.51 | 0.384        |
|      | Lindley   | 0.659000           | 238.38    | 240.38 | 240.44 | 242.61 | 0.390        |
|      | Exponential | 0.407941        | 261.74    | 263.74 | 263.80 | 265.97 | 0.434        |
On Modeling of Lifetime Data Using One Parameter Akash, Lindley and Exponential Distributions

\[ AIC = -2 \ln L + 2k, \quad AICC = AIC + \frac{2k(k+1)}{n-k-1} \]

\[ BIC = -2 \ln L + k \ln n \]

\[ D = \sup_x \left| F_n(x) - F_0(x) \right| \]

where \( k \) is the number of parameters, \( n \) is the sample size and \( F_0(x) \) is the empirical distribution function. The best distribution corresponds to lower values of \(-2 \ln L, \ AIC, \ AICC, \ BIC, \) and K-S statistics. The fittings of Akash, Lindley and exponential distributions are based on maximum likelihood estimates (MLE).

Let \( t_1, t_2, \ldots, t_n \) be a random sample of size \( n \) from exponential distribution. The likelihood function, \( L \) and the log likelihood function, \( \ln L \) of exponential distribution are given by

\[ L = \theta^n e^{-\theta \bar{T}} \quad \text{and} \quad \ln L = n \ln \theta - n \theta \bar{T}. \]

The MLE \( \hat{\theta} \) of the parameter \( \theta \) of exponential distribution is the solution of the equation \( \frac{d \ln L}{d \theta} = 0 \) and is given by \( \hat{\theta} = \frac{1}{\bar{T}} \), where \( \bar{T} \) is the sample mean.

Let \( t_1, t_2, \ldots, t_n \) be a random sample of size \( n \) from Lindley distribution. The likelihood function, \( L \) and the log likelihood function, \( \ln L \) of Lindley distribution are given by

\[ L = \left( \frac{\theta^2}{\theta^2 + 2} \right)^n \prod_{i=1}^n (1 + t_i^2) e^{-\theta t_\bar{T}} \quad \text{and} \quad \ln L = n \ln \left( \frac{\theta^2}{\theta^2 + 2} \right) + \sum_{i=1}^n \ln (1 + t_i^2) - n \theta \bar{T}. \]

The MLE \( \hat{\theta} \) of the parameter \( \theta \) of Lindley distribution is the solution of the equation \( \frac{d \ln L}{d \theta} = 0 \) and is given by

\[ \hat{\theta} = \frac{4 - 4 \bar{T} + \bar{T}^2}{\bar{T}^3} + \frac{8 \bar{T}^2}{3\bar{T}}, \quad \bar{T} > 0, \quad \text{where } \bar{T} \text{ is the sample mean.} \]

Let \( t_1, t_2, \ldots, t_n \) be a random sample of size \( n \) from Akash distribution. The likelihood function, \( L \) and the log likelihood function, \( \ln L \) of Akash distribution are given by

\[ L = \left( \frac{\theta^2}{\theta^2 + 2} \right)^n \prod_{i=1}^n (1 + t_i^2) e^{-\theta t_\bar{T}} \quad \text{and} \quad \ln L = n \ln \left( \frac{\theta^2}{\theta^2 + 2} \right) + \sum_{i=1}^n \ln (1 + t_i^2) - n \theta \bar{T}. \]

It is obvious from the goodness of fit of Akash, Lindley and exponential distributions that the Akash distribution provides better fit than the Lindley and exponential distributions in data-sets 2, 3, 6, 14, 15, and 16; the Lindley distribution gives better fit than the exponential and Akash distributions in data-sets 1, 4, 5 and 12; the exponential distribution gives better fit than the Lindley and the Akash distributions in data sets 7, 8, 9, 10, 11, and 13 (Data sets 1-16).

**Data Set 1:** The data set represents the strength of 1.5 cm glass fibers measured at the National Physical Laboratory, England. Unfortunately, the units of measurements are not given in the paper, and they are taken from Smith & Naylor [31].

| Strength (kN) |
|---------------|
| 0.55          |
| 0.74          |
| 0.77          |
| 0.81          |
| 1.24          |
| 0.93          |
| 1.04          |
| 1.11          |
| 1.13          |
| 1.33          |
| 1.25          |
| 1.27          |
| 1.28          |
| 1.29          |
| 1.48          |
| 1.51          |
| 1.49          |
| 1.53          |
| 1.54          |
| 1.55          |
| 1.55          |
| 1.61          |
| 1.63          |
| 1.67          |
| 1.78          |
| 2             |
| 1.68          |
| 1.69          |
| 1.76          |
| 1.76          |
| 1.74          |
| 1.84          |
| 0.84          |
| 1.81          |

Citation: Shanker R, Fesshaye H, Selvaraj S (2016) On Modeling of Lifetime Data Using One Parameter Akash, Lindley and Exponential Distributions. Biom Biostat Int J 3(2): 00061. DOI: 10.15406/bbij.2016.03.00061
Data Set 2: The data is given by Birnbaum & Saunders [32] on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 101 observations with maximum stress per cycle 31,000 psi. The data (× 10^{-3}) are presented below (after subtracting 65).

|   | 5 | 25 | 31 | 32 | 34 | 35 | 38 | 39 | 40 | 42 | 43 | 43 |
|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 43 | 44 | 44 | 47 | 47 | 48 | 49 | 49 | 49 | 51 | 54 | 55 | 55 |
| 55 | 56 | 56 | 56 | 58 | 59 | 59 | 59 | 59 | 63 | 63 | 64 | 64 |
| 64 | 65 | 65 | 65 | 66 | 66 | 66 | 66 | 67 | 67 | 67 | 68 | 68 |
| 69 | 69 | 69 | 71 | 71 | 71 | 72 | 73 | 73 | 74 | 74 | 76 | 76 |
| 76 | 77 | 77 | 77 | 77 | 77 | 77 | 79 | 79 | 80 | 81 | 83 | 83 |
| 84 | 86 | 86 | 87 | 90 | 91 | 92 | 92 | 92 | 93 | 94 | 97 | 97 |
| 98 | 98 | 99 | 101 | 103 | 105 | 109 | 136 | 147 |  |  |  |  |

Data Set 3: The data set is from Lawless [33]. The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests and they are.

|   | 17.88 | 28.92 | 33 | 41.52 | 42.12 | 45.6 | 48.8 | 51.84 | 51.96 | 54.12 | 55.56 | 67.8 |
|---|-------|-------|----|-------|-------|------|------|-------|-------|-------|-------|------|
| 68.44 | 68.64 | 68.88 | 84.12 | 93.12 | 98.64 | 105.12 | 105.84 | 127.92 | 128.04 | 128.04 | 173.4 |

Data Set 4: The data is from Picciotto [34] and arose in test on the cycle at which the Yarn failed. The data are the number of cycles until failure of the yarn and they are.

|   | 86 | 146 | 251 | 653 | 98 | 249 | 400 | 292 | 131 | 169 | 175 | 176 | 76 |
|---|----|-----|-----|-----|----|-----|-----|-----|-----|-----|-----|-----|----|
| 264 | 15 | 364 | 195 | 262 | 88 | 264 | 157 | 220 | 42 | 321 | 180 | 198 | |
| 38 | 20 | 61 | 121 | 282 | 224 | 149 | 180 | 325 | 250 | 196 | 90 | 229 | |
| 166 | 38 | 337 | 65 | 151 | 341 | 40 | 40 | 135 | 597 | 246 | 211 | 180 | |
| 93 | 315 | 353 | 571 | 124 | 279 | 81 | 186 | 497 | 182 | 423 | 185 | 229 | |
| 400 | 338 | 290 | 398 | 71 | 246 | 185 | 188 | 568 | 55 | 55 | 61 | 244 | |
| 20 | 284 | 393 | 396 | 203 | 829 | 239 | 236 | 286 | 194 | 277 | 143 | 198 | |
| 264 | 105 | 203 | 124 | 137 | 350 | 193 | 188 |  |  |  |  |  |  |

Data Set 5: This data represents the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal [35].

|   | 12 | 15 | 22 | 24 | 24 | 32 | 32 | 33 | 34 | 38 | 38 | 43 | 44 |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 48 | 52 | 53 | 54 | 54 | 55 | 56 | 57 | 58 | 58 | 59 | 60 | 60 | |
| 60 | 60 | 61 | 62 | 63 | 65 | 65 | 67 | 68 | 70 | 70 | 72 | 73 | |
| 75 | 76 | 76 | 81 | 83 | 84 | 85 | 87 | 91 | 95 | 96 | 98 | 99 | |
| 109 | 110 | 121 | 127 | 129 | 131 | 143 | 146 | 146 | 175 | 175 | 211 | 233 | |
| 258 | 258 | 263 | 297 | 341 | 341 | 376 |  |  |  |  |  |  | |

Data Set 6: This data is related with behavioral sciences, collected by Balakrishnan N et al. [36]: The scale “General Rating of Affective Symptoms for Preschoolers (GRASP)” measures behavioral and emotional problems of children, which can be classified with depressive condition or not according to this scale. A study conducted by the authors in a city located at the south part of Chile has allowed collecting real data corresponding to the scores of the GRASP scale of children with frequency in parenthesis, which are.

|   | 19(16) | 20(15) | 21(14) | 22(9) | 23(12) | 24(10) | 25(6) |
|---|--------|--------|--------|-------|--------|--------|-------|
| 26(9) | 27(8) | 28(5) | 29(6) | 30(4) | 31(3) | 32(4) | |
| 33 | 34 | 35(4) | 36(2) | 37(2) | 39 | 42 | 44 |
**Data Set 7:** The data set reported by Efron [37] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using radiotherapy (RT).

| Survival Time (Months) | Value | Value | Value | Value | Value | Value |
|------------------------|-------|-------|-------|-------|-------|-------|
| 6.53                   | 7     | 10.42 | 14.48 | 16.1  | 22.7  | 34    |
| 64                     | 83    | 84    | 91    | 108   | 112   | 129   |
| 149                    | 154   | 157   | 160   | 160   | 165   | 146   |
| 173                    | 176   | 218   | 225   | 241   | 248   | 273   |
| 523                    | 583   | 594   | 1101  | 1146  | 1417  |       |

**Data Set 8:** The data set reported by Efron [37] represent the survival times of a group of patients suffering from Head and Neck cancer disease and treated using a combination of radiotherapy and chemotherapy (RT+CT).

| Survival Time (Months) | Value | Value | Value | Value | Value | Value |
|------------------------|-------|-------|-------|-------|-------|-------|
| 12.2                   | 23.56 | 23.74 | 25.87 | 31.98 | 37    | 41.35 |
| 74.47                  | 81.43 | 84    | 92    | 110   | 112   | 129   |
| 155                    | 159   | 173   | 179   | 194   | 195   | 209   |
| 519                    | 633   | 725   | 817   | 1776  |       |       |

**Data Set 9:** This data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee & Wang [38].

| Remission Time (Months) | Value | Value | Value | Value | Value | Value |
|-------------------------|-------|-------|-------|-------|-------|-------|
| 0.08                    | 2.09  | 3.48  | 4.87  | 6.94  | 8.66  | 13.11 |
| 9.02                    | 13.29 | 0.4   | 2.26  | 3.57  | 5.06  | 7.09  |
| 5.09                    | 7.26  | 9.47  | 14.24 | 25.82 | 0.51  | 2.54  |
| 0.81                    | 2.62  | 3.82  | 5.32  | 7.32  | 10.06 | 14.77 |
| 14.83                   | 34.26 | 0.9   | 2.69  | 4.18  | 5.34  | 7.59  |
| 5.41                    | 7.62  | 10.75 | 16.62 | 43.01 | 1.19  | 2.75  |
| 2.83                    | 4.33  | 5.49  | 7.66  | 11.25 | 17.14 | 79.05 |
| 1.4                     | 3.02  | 4.34  | 5.71  | 7.93  | 11.79 | 18.1  |
| 1.76                    | 3.25  | 4.5   | 6.25  | 8.37  | 12.02 | 2.02  |
| 20.28                   | 2.02  | 3.36  | 6.76  | 12.07 | 21.73 | 2.07  |

**Data Set 10:** This data set is given by Linhart & Zucchini [39], which represents the failure times of the air conditioning system of an airplane.

| Failure Time (Days) | Value | Value | Value | Value | Value | Value |
|---------------------|-------|-------|-------|-------|-------|-------|
| 23                  | 261   | 87    | 7     | 120   | 14    | 42    |
| 20                  | 5     | 12    | 120   | 11    | 3     | 14    |
| 1                   | 16    | 52    | 95    |

**Data Set 11:** This data set used by Bhaumik et al. [40], is vinyl chloride data obtained from clean up gradient monitoring wells in mg/L

| C110 (mg/L) | Value | Value | Value | Value | Value | Value |
|-------------|-------|-------|-------|-------|-------|-------|
| 5.1         | 1.2   | 1.3   | 0.6   | 0.5   | 2.4   | 0.5   |
| 0.4         | 2     | 0.5   | 5.3   | 3.2   | 2.7   | 2.9   |
| 1.8         | 0.9   | 2     | 4     | 6.8   | 1.2   | 0.4   |

**Data Set 12:** This data set represents the waiting times (in minutes) before service of 100 Bank customers and examined and analyzed by Ghitany et al. [7] for fitting the Lindley [6] distribution.

| Waiting Time (Minutes) | Value | Value | Value | Value | Value | Value |
|------------------------|-------|-------|-------|-------|-------|-------|
| 0.8                    | 0.8   | 1.3   | 1.5   | 1.8   | 1.9   | 1.9   |
| 3.3                    | 3.5   | 3.6   | 4.0   | 4.1   | 4.2   | 4.2   |
| 4.7                    | 4.8   | 4.9   | 5.0   | 5.3   | 5.5   | 5.7   |
| 6.3                    | 6.7   | 6.9   | 7.1   | 7.1   | 7.1   | 7.4   |
| 8.6                    | 8.6   | 8.8   | 8.9   | 8.9   | 9.5   | 9.6   |
| 11.0                   | 11.1  | 11.2  | 11.5  | 11.9  | 12.4  | 12.5  |
| 13.7                   | 13.9  | 14.1  | 15.4  | 15.4  | 17.3  | 17.3  |
| 20.6                   | 21.3  | 21.4  | 21.9  | 23.0  | 27.0  | 31.6  |

Citation: Shanker R, Fesshaye H, Selvanaj S (2016) On Modeling of Lifetime Data Using One Parameter Akash, Lindley and Exponential Distributions. Biom Biostat Int J 3(2): 00061. DOI: 10.15406/bbij.2016.03.00061
Data Set 13: This data is for the times between successive failures of air conditioning equipment in a Boeing 720 airplane, Proschan [41].

| 74 | 57 | 48 | 29 | 502 | 12 | 70 | 21 | 29 | 386 | 59 | 27 | 153 |
|----|----|----|----|-----|----|----|----|----|-----|----|----|-----|
| 26 | 326 |

Data Set 14: This data set represents the lifetime’s data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross & Clark [42].

| 1.1 | 1.4 | 1.3 | 1.7 | 1.9 | 1.8 | 1.6 | 2.2 | 1.7 | 2.7 | 4.1 | 1.8 | 1.5 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1.2 | 1.4 | 3   | 1.7 | 2.3 | 1.6 | 2   |

Data Set 15: This data set is the strength data of glass of the aircraft window reported by Fuller et al [43].

| 18.83 | 20.8 | 21.657 | 23.03 | 23.23 | 24.05 | 24.321 | 25.5 | 25.52 | 25.8 | 26.69 | 26.77 |
|--------|------|--------|-------|-------|-------|--------|------|-------|------|-------|-------|
| 26.78  | 27.05| 27.67  | 29.9  | 31.11 | 33.2  | 33.73  | 33.89| 34.77 | 35.75| 35.91 |
| 36.98  | 37.08| 37.09  | 39.58 | 44.045| 45.29 | 45.381 |

Data Set 16: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm, Bader and Priest [44].

| 1.312 | 1.314 | 1.479 | 1.552 | 1.7  | 1.803 | 1.861 | 1.865 | 1.944 | 1.958 | 1.966 | 1.997 |
|-------|-------|-------|-------|------|-------|-------|-------|-------|-------|-------|-------|
| 2.066 | 2.021 | 2.027 | 2.055 | 2.063 | 2.098 | 2.14  | 2.179 | 2.224 | 2.24  | 2.253 | 2.27  |
| 2.272 | 2.274 | 2.301 | 2.301 | 2.359 | 2.382 | 2.382 | 2.426 | 2.434 | 2.435 | 2.478 | 2.49  |
| 2.511 | 2.514 | 2.535 | 2.554 | 2.566 | 2.57  | 2.586 | 2.629 | 2.633 | 2.642 | 2.648 | 2.684 |
| 2.697 | 2.726 | 2.77  | 2.773 | 2.8  | 2.809 | 2.818 | 2.821 | 2.848 | 2.88  | 2.954 | 3.012 |
| 3.067 | 3.084 | 3.09  | 3.096 | 3.128 | 3.233 | 3.433 | 3.585 | 3.858 |

Acknowledgement
None.

Conflict of Interest
None.

References
1. Shanker R (2015) Akash distribution and Its Applications. International Journal of probability and Statistics 4(3): 65-75.
2. Davis DJ (1952) An analysis of some failure data. Journal of American Statistical Association 47(258): 113-150.
3. Epstein B, Sobel M (1953) Life testing. Journal of American Statistical Association 48: 486-502.
4. Epstein B (1958) The exponential distribution and its role in Life-testing. Ind Qual Control 15: 2-7.
5. Feigl P, Zelen M (1965) Estimation of exponential survival probabilities with concomitant information. Biometrics 21(4): 826-838.
6. Lindley DV (1958) Fiducial distributions and Bayes’ Theorem. Journal of the Royal Statistical Society Series 20(1): 102-107.
7. Ghitany ME, Atieh B, Nadarajah S (2008) Lindley distribution and its Applications. Mathematics and Computers in Simulation 78(4): 493-506.
8. Sankaran M (1970) The discrete Poisson-Lindley distribution. Biometrics 26(1): 145-149.
9. Hussain E (2006) The non-linear functions of Order Statistics and Their Properties in selected probability models. Pakistan.
10. Zakerzadeh H, Dolati A (2009) Generalized Lindley distribution. Journal of Mathematical extension 3(2): 13-25.
11. Nadarajah S, Bakouch HS, Taha MBS (2011) A generalized Lindley distribution. Sankhya Series 73(2): 331-359.
12. Deniz EG, Ojeda EC (2011) The discrete Lindley distribution-Properties and Applications. Journal of Statistical Computation and Simulation 81(11): 1405-1416.
13. Mazuchelli J, Achar JA (2011) The Lindley distribution applied to competing risks lifetime data. Comput Methods Programs Biomed 104 (2): 188-192.
14. Bakouch SH, Al-Zahrani BM, Al-Shomrani AA, Marchi VAA, Louzada F (2012) An extended Lindley distribution. Journal of Korean Statistical Society 41(1): 75-85.
15. Shanker R, Mishra A (2013 a) A quasi Lindley distribution. African Journal of Mathematics and Computer Science Research 6(4): 64-71.
16. Shanker R, Mishra A (2013 b) A two-parameter Lindley distribution. Statistics in transition new series 14 (1): 45-56.
17. Shanker R, Sharma S, Shanker R (2013) A two-parameter Lindley distribution for modeling waiting and survival times data. Applied Mathematics 4(2): 363-368.
18. Shanker R, Amanu M (2014) A new quasi Lindley distribution. International Journal of Statistics and Systems 9(2): 143-156.
19. Elbatal I, Menra F, Elgarhy M (2013) A new generalized Lindley distribution. Mathematical theory and Modeling 3(13): 30-47.
20. Ghitany M, Al-Mutairi D, Balakrishnan N, Al-Enezi I (2013) Power Lindley distribution and associated inference. Computational Statistics & Data Analysis 64: 20-33.

21. Meruvi F (2013) Transmuted Lindley distribution. International Journal of Open Problems in Computer Science and Mathematics 6: 63-72.

22. Liyanage GW, Pararai M (2014) A generalized Power Lindley distribution with applications. Asian Journal of Mathematics and Applications 1-23.

23. Ashour S, Eltehiwy M (2014) Exponentiated Power Lindley distribution. Journal of Advanced Research 6(6): 895-905.

24. Oluveye BO, Yang T (2014) A new class of generalized Lindley distribution with applications. Journal of Statistical Computation and Simulation 85(10): 2072-2100.

25. Singh SK, Singh U, Sharma VK (2014) The Truncated Lindley distribution-inference and Application. Journal of Statistics Applications & Probability 3(2): 219-228.

26. Sharma V, Singh S, Singh U, Agiwal V (2015) The inverse Lindley distribution-A stress-strength reliability model with applications to head and neck cancer data. Journal of Industrial & Production Engineering 32(3): 162-173.

27. Shaker R, Hagos F, Sujatha S (2015) On modeling of Lifetimes data using exponential and Lindley distributions. Biometrics & Biostatistics International Journal 2(5): 1-9.

28. Alkarni S (2015) Extended Power Lindley distribution-A new Statistical model for non-monotone survival data. European journal of statistics and probability 3(3): 19-34.

29. Pararai M, Liyanage GW, Oluveye BO (2015) A new class of generalized Power Lindley distribution with applications to lifetime. Theoretical Mathematics & Applications 5(1): 53-96.

30. Aboobamoo MH, Alshangiti AM, Ragab IE (2015) A new generalized Lindley distribution. Journal of Statistical Computation and Simulation preprint.

31. Smith RL, Naylor J (1987) A comparison of Maximum likelihood and Bayesian estimators for the three parameter Weibull distribution. Applied Statistics 36(3): 358-369.

32. Birnbaum ZW, Saunders SC (1969) Estimation for a family of life distributions with applications to fatigue. Journal of Applied Probability 6(2): 328-347.

33. Lawless JF (1982) Statistical models and methods for lifetime data. John Wiley and Sons, New York, USA.

34. Picciotto R (1970) Tensile fatigue characteristics of a sized polyester/viscose yarn and their effect on weaving performance. North Carolina State, University at Raleigh, USA.

35. Bjerkedal T (1960) Acquisition of resistance in guinea pigs infected with different doses of virulent tubercle bacilli. Am J Hyg 72(1): 130-148.

36. Balakrishnan N, Victor L, Antonio S (2010) A mixture model based on Birnbaum-Saunders Distributions, A study conducted by Authors regarding the Scores of the GRASP (General Rating of Affective Symptoms for Preschoolers), in a city located at South Part of the Chile.

37. Efron B (1988) Logistic regression survival analysis and the Kaplan-Meier curve. Journal of the American Statistical Association 83(402): 414-425.

38. Lee ET, Wang JW (2003) Statistical methods for survival data analysis. (3rd edn), John Wiley and Sons, New York, USA.

39. Linhart H, Zucchini W (1986) Model Selection. John Wiley, New York, USA.

40. Bhaumik DK, Kapur K, Gibbons RD (2009) Testing Parameters of a Gamma Distribution for Small Samples. Technometrics 51(3): 326-334.

41. Proschan F (1963) Theoretical explanation of observed decreasing failure rate. Technometrics 5(3): 375-383.

42. Gross AJ, Clark VA (1975) Survival Distributions: Reliability Applications in the Biometrical Sciences, John Wiley, New York, USA.

43. Fuller EJ, Frieman S, Quinn J, Quinn G, Carter W (1994) Fracture mechanics approach to the design of glass aircraft windows: A case study. SPIE Proc 2286: 419-430.

44. Bader MG, Priest AM (1982) Statistical aspects of fiber and bundle strength in hybrid composites. In: Hayashi T, et al. (Eds.). Progress in Science in Engineering Composites, ICCM-IV, Tokyo, Canada, pp. 1129-1136.