A Closed-Loop Supply Chain under Retail Price and Quality Dependent Demand with Remanufacturing and Refurbishing

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ABSTRACT. The demand of a product is linearly dependent on the retail price and quality of the product. We address a closed-loop supply chain where the manufacturer manufactures products according to the demand and sells them through a retailer in the market. A third party collects the used products from costumers and sends to the manufacturer to increase the quality. If the products can retrieve the original quality, thus the process is called remanufacturing. Not every products can retrieve the original quality, thus manufacturer refurbish this products with lower price. We construct four different scenarios – centralized and decentralized led by manufacturer, retailer, and third party. From the comparison of the result obtained in the numerical example, we conclude that the joint profit obtained under centralized, manufacturer-led, and retailer-led policies is higher than third party-led policy.

1. Introduction

Reverse logistic management is process to recover the quality of used products by manufacturer. If the used products can retrieve the original quality as a new product, then the recovery process is called remanufacturing. If the used product cannot retrieve their original quality, the manufacturer can sell it with lower quality to the secondary market. This recovery method is called refurbishing. The usual forward supply chain associates with this reverse logistic management, creates a closed-loop supply chain (CLSC).

We address a CLSC where the manufacturer manufactures products according to the demand and sells it through a retailer in the market. A third party collects the used products from costumers and sends to the manufacturer. We use price and quality dependent demand [2,6,13].

Kampstra and Ashayeri [3] reviewed that supply chain model is invented around 1900 and developed until the middle of 1960. Then the research on CLSC [1,4-15] is widely expanding until now. There are plenty of available literatures on CLSC.

Until 2015, many in the literature assumed that all used items returned to the manufacturer can retrieved their original quality as “new product” [1,6]. In the reality, sometimes third party returned non-remanufacturable products, thus they cannot retrieve their original quality as “new product”. If manufacturer disposes them as mentioned in Chung et al [1], thus there will be additional cost for disposal management. To avoid this disposal cost, the process of refurbishing mentioned in Konstantaras et al [5] can be added to the model created by Maiti and Giri [6].

The next problem is how to decide the retail price and the product quality maintained by the manufacturer [1-2,4-14]. If we take the price too low, thus the obtained profit will be low. In contrary, if we set the price too high, thus the rate of the demand will drop drastically. Deciding the optimum value of the price is the main idea to obtain the maximum profit. The same problem occurs in deciding the product quality. The highest quality increases the rate of the demand along with the cost of quality improvement.

Our purpose is to construct a of CLSC model by Maiti and Giri [6] modified by adding the refurbishing process in the Konstantaras [5]. The notations and assumptions are listed in Section 2. Section 3 presents the formulation of the model. The numerical experiment and the analysis under four scenarios is in the Section 4 and this paper is concluded in Section 5.
2. Notations and Assumptions

The following notations are used for developing the proposed model:

- \( w \) Unit wholesale price
- \( v \) Unit wholesale price for the secondary market
- \( D \) Demand rate of the product in the market
- \( p \) Unit selling price
- \( q \) Product quality maintained by the manufacturer
- \( q_r \) Product quality to be sold to the secondary market
- \( q_f \) Product quality of refurbishable products
- \( \tau \) Collection rate of the third party channel
- \( \theta \) Proportion of remanufacturable items of returned products
- \( c_m \) Unit cost of manufacturing end product from raw materials
- \( c_r \) Unit cost of manufacturing end product from return products
- \( c_f \) Unit cost of refurbishing end product from return products
- \( c_i \) Quality improvement cost at the manufacturer
- \( c_q \) Unit cost of inspection process from return products
- \( d \) Basic market demand
- \( \alpha \) Sensitivity factor of selling price in the demand
- \( \beta \) Sensitivity factor of product quality in the demand
- \( A_t \) Average recycling cost of used products collected by the third party
- \( c_t \) Average recycling price of used products paid by the manufacturer to the third party
- \( \gamma \) Effective investment by the third party to collect return products
- \( g \) Inspection cost of unit product at the manufacturer

The following assumptions are made to develop the proposed model:

1. The market demand \( D \) is linearly dependent on the retail price and quality of the product. We take the function of the demand as \( D(p, q) = d - \alpha p + \beta q \).
2. The quality of the manufactured product is \( q \). It lies between 0 and 1, where 1 means the best quality manufacturer can afford. While the quality of returned product \( q_r \) (<\( q \)). That is, the returned products are acceptable for remanufacturing or refurbishing if those are attained minimum quality, i.e., \( q_r \). However, after remanufacturing the quality of remanufactured product becomes \( q \), and refurbished product becomes \( q_f > q \).
3. The return rate \( (r) \) of the third party is a fraction of the total demand, but only \( \theta \) of returned products are remanufacturable, the rest is refurbised to the secondary market. All the return products which do not qualify for remanufacturing is refurbished and sold in the secondary market with unit price \( v \). A series of processes is performed and only a part of the collected used products are remanufactured and resold.
4. Unit remanufacturing cost is lower than unit manufacturing cost, i.e., \( c_m > c_r \).
5. We assume that \( p > w > 0, c_m - c_r > c_r > A_t > 0 \).
6. There is no defective or expired product.
7. There is not learning process in the manufacturing system. The manufacturer keeps the amount of product as the rate of the demand to avoid unsold product in the retailer.
8. The amount of the product always satisfy the demand and there is not shortage in the system.

The difference between the model presented in this paper, Maiti and Giri [6], and Konstantaras [5] is shown in the Table 1.
Table 1. The comparison of the CLSC models

| Model                          | Maiti and Giri | Konstantaras | This paper |
|-------------------------------|----------------|--------------|------------|
| There are 3 parties in the system (manufacturer, retailer, and third party as collector) | Yes             | No (Manufacturer collects the used product) | Yes         |
| All of the used products are collected | No              | Yes          | Yes        |
| There is inspection of the returned products | No              | Yes          | Yes        |
| The system used remanufacturing process | Yes             | Yes          | Yes        |
| The system used refurbishing process | No              | Yes          | Yes        |
| The quality is used as decision variable and can be calculated | Yes             | No           | Yes        |
| The indirect cost (set-up cost, transportation cost, etc) is used in the system | No              | Yes          | No         |

3. Model Formulation and Analysis

The diagram consisting forward and reversed logistic of modified model of Maiti and Giri [6] added by refurbishing process is presented on the Fig. 1.

![Fig.1. Graphical representation of CLSC](image)

The third party returned $\tau D$ products to manufacturer, only $\theta \tau D$ products are remanufacturable, the rest $(1-\theta)\tau D$ products are sold to the secondary market with the lower price. Manufacturer must manufacture $(1-\theta \tau)D$ products to satisfy the demand rate. The serviceable products are sold through a retailer. Thus the profit of the manufacturer, retailer, and third party is

$$\Pi_M = Dw + (1-\theta)\tau D \left( v - c_f - c_q \left( q_2 - q_f \right) \right) - (1-\theta \tau)D \left( c_m + c_q q \right)$$

$$-\theta \tau D \left( c_r + c_q \left( q - q_r \right) \right) - \tau D (c_t + g),$$

$$\Pi_R = Dp - Dw,$$ and

$$\Pi_T = Dc_t - \tau D A_t - \gamma r^2$$ respectively.

There are four available scenarios, centralized policy, manufacturer-led policy, retailer-led policy and third party-led policy. In centralized policy all players cooperatively decide the value of $p$ and $q$ to maximize the whole joint profit of the system. In the led policies, the leading player decides the
value of the specific variable to maximize the profit, then the rest of the players follow the variable stated by the leading player.

3.1 Centralized Policy

The whole joint profit of the system is

\[ \Pi = Dp + (1 - \theta)\tau D \left( v - c_f - c_q(q_2 - q_f) \right) - (1 - \theta)\tau D(c_m + c_qq) \]

\[ -\theta \tau D \left( c_r + c_q(q - q_r) \right) - \tau D(A_t - g - \gamma \tau^2). \]  \hspace{1cm} (1)

For the optimal solution, the first order necessary conditions are \( \frac{\partial \Pi}{\partial p} = 0 \) and \( \frac{\partial \Pi}{\partial q} = 0 \), which given

\[ p = \frac{1}{2\alpha} \left( d + \alpha c_r + q\beta + \left( c_qc_f + A_t + g - v \right)\alpha \tau + \left( v - c_r - c_q(c_f + q_r) \right)\alpha \theta \tau - \alpha \gamma \tau^3 \right). \] \hspace{1cm} (2)

and

\[ q = \frac{1}{2c_q\beta} \left( -c_qd + c_qp\alpha - c_r\beta + p\beta - c_f c_q\beta \tau - A_t \beta \tau - g \beta \tau + v \beta \tau + c_f c_q \beta \theta \tau + c_r \beta \theta \tau + c_q q_r \beta \theta \tau - v \beta \theta \tau + \beta \gamma \tau^3 \right). \] \hspace{1cm} (3)

We checked that \( \frac{\partial^2 \Pi}{\partial p^2} = -2\alpha < 0 \) and \( \frac{\partial^2 \Pi}{\partial q^2} = -2c_q\beta < 0 \), thus the Eqs (2) and (3) is the optimum solution for Eqs (1).

3.2 Manufacturer-led Policy

This policy is usually applied in the industry which creates high quality of branded product. Manufacturer decides the optimum value of \( q \) thus they obtain the maximum profit.

\[ q = \frac{1}{2c_q\beta} \left( -c_qd + c_qp\alpha - c_m \beta + w \beta + c_m \beta \theta \tau - c_r \beta \theta \tau + c_q q_r \beta \theta \tau - c_f \beta \tau - c_r \beta \tau - g \beta \tau - c_q q_2 \beta \tau + c_q q_f \beta \tau + v \beta \tau + c_f \beta \theta \tau + c_q q_2 \beta \theta \tau - c_q q_f \beta \theta \tau - v \beta \theta \tau \right), q \in [0,1] \]

is obtained as the solution of \( \frac{d\Pi_M}{dq} = 0 \).

We checked that \( \frac{\partial^2 \Pi_M}{\partial q^2} = -2c_q\beta < 0 \), thus the solution is unique. The retailer and the third party follow the value of \( q \) stated by the manufacturer. If the obtained \( q > 1 \), thus we take \( q = 1 \). On the other hand, if the obtained \( q < 0 \), thus we take \( q = q_r \).

3.3 Retailer-led Policy

This policy is usually applied in the industry with high demand for unbranded products. Retailer decides the optimum value of \( p \) thus they obtain the maximum profit.

\[ p = \frac{d - wa + q\beta}{2\alpha} \]
is obtained as the solution of \( \frac{d\Pi_{p}}{dp} = 0 \).

We checked that \( \frac{d^{2}\Pi_{p}}{dp^{2}} = -2a < 0 \), thus the solution is unique. The manufacturer and the third party follow the value of \( p \) stated by the retailer.

### 3.4 Third party-led Policy

This policy is usually applied in the recycling or crafting industry where the value of \( AT \) is high. Third party decides the optimum value of \( \tau \) thus they obtains the maximum profit.

\[
\tau = \frac{(\gamma \cdot c_{t} - \gamma \cdot c_{q})(d - ap + \beta q)}{2\gamma}, \tau \in [0,1]
\]

is obtained as the solution of \( \frac{d\Pi_{\tau}}{d\tau} = 0 \).

We checked that \( \frac{d^{2}\Pi_{\tau}}{d\tau^{2}} = -2\gamma < 0 \), thus the solution is unique. The manufacturer and the retailer follow the value of \( \tau \) stated by the third party.

### 4. Numerical Experiment and Analysis

We construct a simulation with the costs \( c_{m} = 2,000, c_{q} = 100, c_{r} = 1,500, c_{f} = 800, g = 1, \), \( c_{t} = 350 \) and \( AT = 150. \) The used basic demand \( d = 100 \) with \( v = 900 \) and \( w = 300,000. \) We used \( \tau = 0.75 \), except for the third party-led policy. The rest of variable is \( q_{r} = 0.3, q_{f} = 0.1, q_{2} = 0.5 q, \alpha = 0.0015, \beta = 200, \gamma = 1, \) and \( \theta = 0.7. \) The obtained results in the maximum of 6 significant digits are shown in **Table 2**.

### Table 2. The numerical result of the optimum values under difference policies

| Optimal decisions | Centralized | Manufacturer-led | Retailer-led | Third party-led |
|-------------------|-------------|------------------|-------------|-----------------|
| \( p \)            | 408,229     | 408,229          | 408,229     | 364,100         |
| \( q \)            | 1           | 1                | 1           | 0.3             |
| \( \tau \)         | 0.75        | 0.75             | 0.75        | 1               |
| \( \Pi_{M} \)      | \( 1.38547 \times 10^{8} \) | \( 1.38547 \times 10^{8} \) | \( 1.38547 \times 10^{8} \) | \( 1.53147 \times 10^{8} \) |
| \( \Pi_{R} \)      | \( 8.835 \times 10^{7} \) | \( 8.835 \times 10^{7} \) | \( 8.835 \times 10^{7} \) | \( 3.2978 \times 10^{7} \) |
| \( \Pi_{T} \)      | \( 69,749.4 \) | \( 69,749.4 \) | \( 69,749.4 \) | \( 102,769 \) |
| \( \Pi \)          | \( 2.30227 \times 10^{8} \) | \( 2.30227 \times 10^{8} \) | \( 2.30227 \times 10^{8} \) | \( 1.76128 \times 10^{8} \) |

We obtained \( q > 1 \) under centralized, manufacturer-led and retailer-led policy. It means that the industry gain the maximum profit if the best quality of product is sold. In contrary, we obtained \( q < 0 \) under the third party-led policy. That means the manufacturer sold the product with the lowest quality as if it is returned by third party and resold to the retailer directly.

As expected, manufacturer receives the highest profit, i.e. more than 50% of the joint profit in the CLSC system. This is the most common result to be considered as manufacturer plays the greatest role in the system. Third party receives the lowest profit due to the least role in the system.

The sensitivity analysis of \( p \) from the obtained numerical result with maximum of 3 significant digits is shown in the **Table 3** and **Fig 2**. While the variable \( q \) does not require sensitivity analysis since it is in the maximum value under centralized, manufacturer-led and retailer-led policy, and in the minimum value under third party-led policy.
Table 3. The sensitivity analysis of $p$ from the obtained numerical result of $\Pi$ under different policies

| $p$   | Centralized, Manufacturer-led and Retailer-led policy | Third party-led policy |
|-------|------------------------------------------------------|------------------------|
| 250,000 | $1.92 \times 10^8$                                  | $1.57 \times 10^8$    |
| 300,000 | $2.13 \times 10^8$                                  | $1.70 \times 10^8$    |
| 350,000 | $2.25 \times 10^8$                                  | $1.76 \times 10^8$    |
| 400,000 | $2.30 \times 10^8$                                  | $1.74 \times 10^8$    |
| 450,000 | $2.28 \times 10^8$                                  | $1.65 \times 10^8$    |
| 500,000 | $2.18 \times 10^8$                                  | $1.48 \times 10^8$    |
| 550,000 | $2.00 \times 10^8$                                  | $1.24 \times 10^8$    |
| 600,000 | $1.75 \times 10^8$                                  | $9.26 \times 10^7$    |

Fig 2. Graphical representation of sensitivity analysis of $p$ from the obtained numerical result of $\Pi$ under (a) centralized, manufacturer-led, retailer-led policy and (b) third party-led policy

Table 2 and Fig 2 show that the results in the Table 1 are definitely the optimum solution of $\Pi$ under the four scenarios. We find out that centralized, manufacturer-led and retailer-led policy obtain the same result for this numerical simulation. The profit obtained by third party as the leading player in the third party-led policy is definitely the highest compared to the other scenarios, consequently it has the lowest joint profit.

5. Conclusions

In this paper we have constructed a CLSC model with manufacturer, retailer and third party. Manufacturer manufactures products and sell it to the customer through a retailer. The third party collects all the used product and send it to the manufacturer to be remanufactured or refurbished. We construct four scenarios, they are centralized policy, manufacturer-led policy, retailer-led policy and third party-led policy. The numerical experiment is conducted to find out the best scenarios. From the numerical simulation, we obtained the same results under centralized, manufacturer-led and retailer policy. The joint profit obtained under third party-led policy is lower than the other policies.

Several studies can be undertaken for future research. Profit sharing under stochastic demand and including more than one retailer who sells the product. would be more realistic to consider multiple retailers in the system. Further, one may consider the retailer to act as a collector of return product or recast the current problem as a multi-period decision making problem from more realistic point of view.
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