RELATIVE NET UTILITY AND THE ST PETERSBURG PARADOX

DANIEL MULLER AND TS Hilidzi MARWALA

The famous St Petersburg Paradox shows that the theory of expected value does not capture the real-world economics of decision-making problem. Over the years, many economic theories were developed to resolve the paradox and explain the subjective utility of the expected outcomes and risk aversion. In this paper, we use the concept of the relative net utility to resolve the St Petersburg paradox. The reason why the principle of absolute instead of relative net utility does not work is because it is a first order approximation of some unknown utility function. Because the net utility concept is able to explain both behavioral economics and the St Petersburg paradox it is deemed a universal approach to handling utility. Finally, this paper explored how an artificial intelligent (AI) agent will make choices and observed that if AI agent uses the nominal utility approach it will see infinite reward while if it uses the relative net utility approach it will see the limited reward that human beings see.

KEYWORDS: St Petersburg paradox, artificial intelligence, net utility, reference point, expected utility theory, bounded utility.

1. INTRODUCTION

The St Petersburg Paradox is a well known problem in the probability and decision theory. St Petersburg Paradox shows that the theory of expected value does not capture the real-world economics of decision-making problems. The problem was formulated by Nicolas Bernoulli in 1713 and introduced by his cousin, Daniel Bernoulli (1738/1954) to the Imperial Academy of Sciences in St Petersburg as follows.

Peter tosses a coin and continues to do so until it lands on "heads" when it comes to the ground. He agrees to give Paul one ducat if he gets "heads" on the very first throw, two ducats if he gets it on the second, four if on the third, eight if on the fourth, and so on. With each additional throw the number of ducats he must pay is doubled. Suppose we seek to determine the value of Paul’s expectation.

In his paper, “Exposition of a New Theory on the Measurement of Risk” Bernoulli (1738/1954) suggests a solution to the paradox which is based on the subjectivity of utility of the expected outcome of the game. St Petersburg Paradox, is referred to by Samuelson (1977) as "a dramatic and even over-dramatic case", shows that a real world economics of decision making problem is not captured by the theory of expected value. In the history of economics literature, St Petersburg Paradox has been a key factor in the development of the utility function theories among many other ideas that was born from its resolution attempts. Over the years the question regarding a general utility function that captures arbitrary payoff functions draws attention in a wide range of scientific and industrial fields, and has been addressed by many experts in many fields. In his analysis of the financial and economic aspects of St Petersborg Paradox Libor
(2011) states that using the analysis and applications of this paradox we could avoid some catastrophic financial situations. Analyzing high-tech stocks Székely and Richards (2004) came to the same conclusion regarding the run-up in stock prices in the late 1990s and the subsequent declines in 2000.

Over the years, most of the solutions and attempts to explain the paradox that have been presented were dealing with definitions of subjective utility function and manipulations of the utility function to fit the real world observed behavior. In this paper, we take several perspectives on the paradox including the cost-benefit and time perspective. We start with an analysis of the paradox and present the solution attempts to the paradox and the theories of expected utility that were developed during the years from the resolution attempts. We define the notion of net utility and define a break-even point as a reference point for net utility inference. We continue with a definition of investment (or game) position and discuss the net utility of changing position of investment and show how the reference point of break-even point along with the net utility notion allows us to capture the utility of alternative positions in the game. We then define a dynamic reference point and present an incremental net utility evaluation procedure to evaluate the alternative game positions. This allows us to present several new properties in the structure of the problem defined in the St Petersburg Paradox and to establish the equilibrium position that maximizes the utility while preserving the invested resources. We formulate the Theorem of Indifference with respect to the expected utility payoff. This allows us to discuss the opportunity cost and the time factor in investment (and St Petersburg Paradox game). Incorporating time factor, the objective in the evaluation of the game becomes maximizing the expected net utility while minimizing the time invested to create a net utility. We treat time as resource and extend our observations to a general set of resources, establishing resources based criteria for tie breaking points where there is indifference with respect to the expected value of the game.

Finally we present the universal theory of net utility and show the role of net utility in rational decision making process and how the value of net utility allows for a more informed decision making process. We conclude with the applications of our observation from St Petersburg Paradox and net utility in artificial intelligence (AI) and Economic systems, and point out some future research directions.

2. SAINT PETERSBURG PARADOX

We begin by presenting a table summarizing the details of the game in Table I. In Table I each row shows the number of coin tosses until Paul gets 'heads' for the first time and the game ends. The first column in table I represents the number, k, of coin tosses. The second column represents the probability of getting 'heads' on k coin toss. The third column represents the reward derived from getting 'heads' on toss k. The forth column represents the expected payoff from getting 'heads' on toss k, and it is calculated by multiplying the probability of getting "heads" on toss k with the respective value of the obtained reward.
The mathematical value of a fair price of playing in the game is the accumulation of expected payoffs,

\[ \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \ldots = \infty. \]

This implies that a player will be willing to pay any price, up to infinity to participate in the game. However, most of the agents will pay small amount of money to participate in the game unlike in the prediction of the expected value theory. This is a paradox. The literature talks about the paradox resulting in a solution of the expected utility of infinity. There is an additional paradox which has been ignored that even though the minimum win if one chooses to participate in this game is 1 ducat even though utility theory expects a win of a half a ducat. This is because the definition of expected utility is a first order approximation of the real utility. This can be viewed as the first order components of a Taylor expansion of the real utility.

The failure of the expected value theory to predict the real economic value of the game has attracted many researchers over the years and served the important role in the development of economic theories. Survey of the history of ideas, theoretical and empirical, to explain and solve the paradox was conducted by Samuelson (1977); Neugebauer et al. (2010); Seidl (2013); Cox et al. (2019). The explanations of this old challenge of the expected utility theory concentrated on diminishing marginal utility, treating small probabilities as zero and unbounded utility function or flip coins.

The utility function concept was first introduced by Bernoulli (1738/1954) to represent the subjective value of winnings, also refereed to as the moral value or the moral hope by Gabriel Cremer in Bernoulli’s paper. Gabriel Cremer and
Daniel Bernoulli explained the paradox by decreasing marginal utility and suggested concave transformation to solve the paradox. Cramer suggested a square root and Bernoulli suggested natural logarithm. The subjectivity assumption was experimentally confirmed Weber (1834); Fechner (1860) and formalized in the Expected Utility model Von Neumann and Morgenstern (1947); Von Neumann et al. (2007). Experimental work to support Bernoulli’s observation of risk-aversion tendency in decision making showed that the concept of marginal utility and perception of small probabilities cannot explain the paradox but support the observation of risk-aversion tendency in decision making Cox et al. (2011); Neugebauer et al. (2010); Cox et al. (2019). Utility or coin flips bounded assumption remains the most favored explanation of the paradox up to date.

Menger (1934) work on the St. Petersburg Paradox showed that the marginal diminishing utility transformation is not sufficient since the rewards can increase faster than the rate at which the utility diminish. This observation lead to the conclusion that boundedness of the utility function or coin flips is a necessary condition to prevent the occurrence of a St. Petersburg Paradox Menger (1934, 1979); Arrow (1970); Aumann (1977); Seidl (2013). Time boundedness assumption Brito (1975); Aumann (1977); Cowen and High (1988) suggests that limited time will bind the utility function. Even infinite rewards will create finite utility with the natural bound of individual life. In this Paper, in the context of boundedness in the St. Petersburg paradox, we discuss the basic bounds of the budget and time of the player and provide a framework to trace and set bounds on the net utility change between alternative game scenarios, i.e., the boundedness of the incremental change in net utility rather then the boundedness of the utility function outcomes.

Non-Expected Utility theories were developed due to the paradox in the expected utility theory, and these include the dual theory of choice under risk Yaari (1987), prospect theory Kahneman and Tversky (1979) and the cumulative prospect theory Tversky and Kahneman (1992). These theories do not address the St Petersburg Paradox, however, they provide several important observations that can be leveraged to solve the paradox.

The concept of reference point and relative utility is observed in the Prospect theory. Prospect theory assumes the existence of a reference point for perception of decisions outcomes as losses or gains. Recent work by Werner and Zank (2019) provides a survey on models with reference points and suggest a model to reveal reference point in the prospect theory. In the next section we leverage Prospect Theory observations on the reference point and relative utility to build a framework to reveal dynamic reference points for net utility change of game positions in the St Petersburg paradox, which we exploit to choose policy and evaluate the game. The theoretic contribution of this paper suggests an explanation for the experimental observations in the expected utility theory and behavioral economics on reference point and relative utility as well as risk-aversion and diminishing marginal utility.
2.1. *Petersburg Game Decision Making Process Diagram*

Representation of the game as in Table I is common in most of the discussions about St Petersburg Paradox, although it is missing few important details regarding the decision process of participating in the game and the evaluation of possible scenarios in the game. In what follows we take a deeper look on the decision process in the St Petersburg paradox game and provide a diagram to illustrate it.

Figure 1 illustrates an event-decision diagram of a single St Petersburg paradox game that Peter suggests to Paul. The diagram represents the game scenarios and the decision process that Paul has to go through in the game that Peter suggests. Generally speaking, the process constitutes the states in the game, one decision, and one deterministic action followed by a sequence of stochastic events. The process starts from an initial state as illustrated with a rectangle and represents the position before the game starts. A decision state is illustrated with a rhombus and represents Paul’s decision to choose whether to participate in the game or not. An action state is illustrated with a circle and represents the action of Paul’s payment to Peter in case he decides to participate in the game. A sequence of stochastic events illustrated with dashed circles represents the sequence of tosses until the coin lands on 'heads'.

The game as illustrated in Figure 1 can be analyzed from a different time, value and budget perspectives. In what follows we will cover some of the main perspectives to analyze the game and to estimate the expected value of the game.

We now take a closer look at the different game scenarios in St Petersburg paradox which is represented by the diagram in Figure 1. Paul’s budget at the initial state of the game is $B = b$ ducats. Peter suggests to Paul to participate in the game which leaves Paul with two choices. If Paul declines Peter’s offer, he stays in his initial position with the initial budget of $b$ ducats. If he accepts Peter’s offer, he has to pay $c$ ducats to participate in the game. Once Paul decides to buy in the game, the decision phase is done, the action of paying for the game is applied and the reward is determined according to the outcome of a sequence of stochastic events. By stochastic events we refer to the sequence of coin tosses until the coin lands on 'heads'.

Given the terms of the game, Paul can calculate the expected brake-even point for the amount of money $c$ that was paid to participate in the game, within his budget restrictions $c < B$, that he is willing and able to pay to participate. Based on the fact that only one scenario of events from the presented scenarios can occur in one game, Paul chooses his expected position among different prospective scenarios, i.e., his payoff expectation for brake-even or an improved position from participation in the game. As illustrated in this example, the expected-value of each scenario of sequence of tosses that ends with 'heads' is $\frac{1}{2}$ which calculated by the multiplication of scenario probability and the related prize as illustrated in Table I. Hence for each sequence of tosses that ends with 'heads' the break-even expected value is $\frac{1}{2}$. This is without taking the cost of time.
Figure 1 — An illustration of a single game as described in St. Petersburg Paradox. The game illustrated with a decision-event diagram on a scale of time, budget, expected value, and expected total budget associated with achievable positions in the game.

Taking into account the invested time in the game (which can be enrolled into monetary value through lost alternatives of investments also referred to as the opportunity cost), we can formulate a decreasing utility function. The rate of the decrease is dependent on the alternative investments Paul is aware of.
3. NET UTILITY AND ST PETERSBURG PARADOX

3.1. Evaluation with Respect to Break-Even Point

In simple words the break-even point is where the costs we pay for participating in the game equals to the benefits that are derived from participating in the game. The first and the most basic break-even point in the game is the initial state which is achieved with the decision of not participating in the game.

3.2. The Decision-Choice of Not Participating in the Game

The decision to do nothing, depicted in decision nodes 'Buy In' in the game diagram in Figure 1, is always implicitly available. To evaluate the alternatives and the expected outcomes of the game, we set the decision-choice of doing nothing, i.e. not participating in the game, as the first and the most basic reference point. The reference point of 'doing nothing' will allow us to define the break-even point of the game. When we take into account of the costs of participating in the game and the budget of the participant, the reference point of 'doing nothing' is a break-even point. We define the break-even point as follow.

**Definition 3.1**  Given a sequence of events \( e_0, e_1, e_2 \ldots e_k \) a cost function \( c(e_i) \) and a value function \( u(e_i) \) for each \( i \leq k \) the break even point of the game \( BEP \) occurs when \( \sum_{i=1}^{k} u(e_i) = \sum_{i=1}^{k} c(e_i) \).

Note, the definition of the break-even point is on sequence of events even though the utility achieved only with the occurrence of the last event in the sequence, which is 'heads' in coin toss. The cost, however, are paid at the initial point of the sequence of events with the decision to participate in the game. The expected payoff depends on both "tails" and "heads". The number "tails" determines the payoff whereas the "heads" is a critical indicator of when the game is stopped and the payoff effected.

Table II extends Table I with the details of the initial position at the initial state, which is depicted in nodes 'init' in the game diagram in Figure 1. The additional row 'Init/Fold' in Table II represents the pre-decision state where there is no reward and no costs paid to participate in the game. It also represents the decision "Fold", that is the decision not to participate in the game in which case the reward as well as the expected payoff are 0.

3.3. Net Utility of Changing Position

In what follows we present an incremental evaluation approach to find optimal position in the Saint Petersburg Paradox game.
The St. Petersburg Paradox is a game in which a player repeatedly rolls a fair coin, starting with an initial state of value 0. For each flip, if the result is heads, the player receives a prize that is a power of 2, starting with 1 for the first flip, then 2 for the second, 4 for the third, and doubling for each subsequent flip. The player is allowed to choose whether to continue playing or to stop after any flip, at which point they receive their accumulated prize. The question is whether it is worth paying a finite amount to play this game, or if it is better to decline and receive nothing.

| k (num of throws) | $P(k) = \frac{1}{2^k}$ | Prize | Expected Payoff |
|-------------------|------------------------|-------|-----------------|
| Init/Fold         | 0                      | 0     | 0               |
| 1                 | $\frac{1}{2}$         | 1     | $\frac{1}{2}$  |
| 2                 | $\frac{1}{4}$         | 2     | $\frac{1}{4}$  |
| 3                 | $\frac{1}{8}$         | 4     | $\frac{1}{8}$  |
| 4                 | $\frac{1}{16}$        | 8     | $\frac{1}{16}$ |
| 5                 | $\frac{1}{32}$        | 16    | $\frac{1}{32}$ |
| 6                 | $\frac{1}{64}$        | 32    | $\frac{1}{64}$ |
| 7                 | $\frac{1}{128}$       | 64    | $\frac{1}{128}$|
| ...               | ...                    | ...   | ...             |

**Table II**

Illustration of the St. Petersburg Paradox game as in Table I with the initial position description added, colored in gray, which describes additionally the ‘init’ state and the decision of not participating the game.

Leaning on Oxford dictionary definition Dictionary (2008) we define a Game Position as follows:

**Definition 3.2** A Game Position is:

- A person’s point of view or attitude towards the game payoff
- The extent to which an investor, dealer, or speculator has made a commitment in the market by buying or selling securities.

**Definition 3.3** The utility of game position $i$ in St. Petersburg Paradox is the expected payoff of a game with $i$ coin flips.

Our initial reference point is the break-even point in initial state with the value of 0 (no costs nor benefit produced in the game). Having an initial reference point we can evaluate other applicable positions of Paul in the game with respect to the initial break-even point.

The net utility of game position $k$ is the change in the expected payoff with respect to the reference point, i.e., the change from the expected payoff in the initial position where $k = 0$.

Table III extends Table II with the details of the net utility of changing position with respect to the initial state.

As Table III shows, in terms of the net utility with respect to our initial position, there is indifference with regard to each positions with $k > 0$. 

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3.4. Dynamic Reference Point for Net Utility of Changing Position

Having an initial reference point we can evaluate other applicable positions of Paul in the game and update the reference point dynamically when a reference point with better utility is found.

Incremental-Net-Utility-Evaluation

\[
\text{RefPos} := 0 \quad \text{// Initialize Reference Position}
\]

\[
\text{CurrPos} := 0 \quad \text{// Initialize Current Position}
\]

\[
\text{NUC} := 0 \quad \text{// Initialize Net Utility Change}
\]

\[
\text{NUC*} := 0 \quad \text{// Initialize Best Net Utility Change so far}
\]

\text{loop:}

\[
\text{CurrPos} = \text{CurrPos} + 1
\]

\[
\text{NUC} = \text{EU}(\text{CurrPos}) - \text{EU}(\text{RefPos}) \quad \text{// Calculate Expected Net Utility Change}
\]

if \( \text{NUC} > \text{NUC*} \):

\[
\text{NUC*} = \text{NUC}
\]

\[
\text{RefPos} = \text{CurrPos}
\]

if Termination-criteria:

return \( \text{NUC*}, \text{RefPos} \)

FIGURE 2.— Incremental net utility evaluation with dynamic reference point update

Table IV extends Table III with the details of the net utility of changing position with respect to the best position found so far. Figure 2 depicts a pseudo-code of an incremental net utility evaluation with dynamic reference point which puts into practice our observations on net utility change in St. Petersburg Paradox.
In next section we present the Theorem of Indifference and resource preserving tie-breaking decision criteria which allow us to define termination criteria for the incremental net utility evaluation procedure. The Theorem of Indifference will allow us to find the equilibrium in effective net utility change and the resource preserving tie-breaking decision criteria will allow us to define an efficient evaluation termination within the equilibrium zone.

4. THEOREM OF INDIFFERENCE AND RESOURCE PRESERVING TIE-BREAKING DECISION CRITERIA

In St Petersburg game we pursue an optimal position to improve the net utility. Table IV shows that the incremental increase of number of flip coins (incremental change in game position) after the initial flip of coin, results with no change in the expected utility (with respect to the reference point). Although the Payoff might be increasing with the increased number of coin flips, the probability to achieve the payoff is decreasing in a rate that preserves the same expected utility for each game position. This results in the increase of risk in the game, respectively to the number of flip coins (i.e. the respective game positions) without justification of improved expected utility. We now show how to find a game position that achieves equilibrium in the game that preserves resources without damaging the potential expected utility. Note that in what follows we optimize the evaluation process of choosing Paul’s game position and by doing that we find the resource preserving position among equally attractive positions in terms of the expected utility.

### Table IV

Illustration of the St. Petersburg Paradox game with the initial position details (colored in gray row) and a "Relative (dynamic) Net Utility" with respect to best position found so far (colored in gray column). Highlighted in yellow is the first (and last) beneficial changing position decision.

| k (num of throws) | \( P(k) = \frac{1}{2^k} \) | Prize | Expected Payoff | Net Utility | Relative Net Utility |
|-------------------|-----------------------------|-------|-----------------|-------------|----------------------|
| Fold              | –                           | –     | –               | 0           | 0                    |
| 1                 | \( \frac{1}{2} \)          | 1     | \( \frac{1}{2} \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) |
| 2                 | \( \frac{1}{4} \)          | 2     | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0                    |
| 3                 | \( \frac{1}{8} \)          | 4     | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0                    |
| 4                 | \( \frac{1}{16} \)         | 8     | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0                    |
| 5                 | \( \frac{1}{32} \)         | 16    | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0                    |
| 6                 | \( \frac{1}{64} \)         | 32    | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0                    |
| 7                 | \( \frac{1}{128} \)        | 64    | \( \frac{1}{2} \) | \( \frac{1}{2} \) | 0                    |
| ...               | ...                         | ...   | ...             | ...         | ...                 |

In next section we present the Theorem of Indifference and resource preserving tie-breaking decision criteria which allow us to define termination criteria for the incremental net utility evaluation procedure. The Theorem of Indifference will allow us to find the equilibrium in effective net utility change and the resource preserving tie-breaking decision criteria will allow us to define an efficient evaluation termination within the equilibrium zone.
The evaluation process of choosing position in the game that is described in Figure 2 consists of evaluation actions of incremental change and evaluation of game positions. Treating the evaluation process as a sequential action application allows us to adopt action planning methods. Theorem 4.1 suggested by Muller and Karpas (2018) for solving problems of sequential action planning in which the objective is to choose a sequence of actions to maximize agent utility under budget restrictions. It allows us to optimize a sequence of actions that lead to effective net positive change in the utility by truncating actions that exploit resources (such as time) and not lead to the net positive utility value change. This will allow us to define an efficient termination criteria for the incremental net utility evaluation procedure (Figure 2). In plain words, the Theorem states that; for each sequence of actions $\pi$, there is a sub-sequence $\pi'$ that; (i) ends with a net positive utility value action (net utility of the action of changing game position in St. Petersburg Paradox), (ii) is at most as costly as $\pi$, and (iii) is at least as valuable as $\pi$.

**Theorem 4.1 (Theorem of Indifference)** Given a game $\Pi$ with an additive utility function $u$, for any policy $\pi$ for $\Pi$ such that $u(s[\pi]) > u(s_0)$, there is a prefix $\pi'$ of $\pi$ such that:

1. $u(s_0[\pi]) \leq u(s_0[\pi'])$, and
2. for the last action $a_{last}$ along $\pi'$, we have $u(a_{last}) > 0$.

**Proof:** The proof is by induction on the plan length $n$. For $n = 1$, we have $\pi = \langle a_1 \rangle$, and since $u(s[\pi]) > u(s_0)$, the action $a_1$ has a positive net utility value. Hence, $\pi' = \pi$ satisfies the claims. Assuming that the claim holds for $n \geq 1$, we now prove it for $n + 1$.

Considering a plan $\pi = \langle a_1, \ldots, a_{n+1} \rangle$, for $i \in [n+1]$, let $\pi_i$ denote the prefix of $\pi$ consisting of its first $i$ actions. If the last action $a_{n+1}$ along $\pi$ has a positive net utility value, then we are done with $\pi' = \pi$. Otherwise, if $a_{n+1}$ has either negative or zero net utility value, then $u(s[\pi]) > u(s_0)$ in particular implies $u(s[\pi_n]) > u(s_0)$. If $\pi'$ is a prefix of $\pi_n$ that satisfies the lemma by our assumption of induction, then $\pi'$ also satisfies the lemma with respect to $\pi$ since $u(\pi') \geq u(s[\pi_n]) \geq u(s[\pi])$.

Q.E.D.

### 4.1. Tie-Breaking Decision Criteria

Although the expected utility is the same for all alternatives on choosing positions in the game, it will be wrong to state that a rational player will play infinite game. In terms of the expected value there is no benefit from changing the position in the game, i.e., the net utility from changing the position from one toss is zero. At the same time, the implicit costs can be enormous in terms

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1 This can be seen as a process of planning a solution, i.e. planning a decision in the case of St Petersburg paradox.
of time. In any case, changing position from the position of one toss to any other position is not optimal in terms of time due to the increase the costs in terms of time. To summarize, by logical principles, rational choice among equal opportunities will optimize the cost of time invested in each opportunity.

4.2. Utility Tie Braking with Time Factor

As Table III shows, there is indifference in the dimension of the benefit among the alternatives that Paul can choose from. In such a case Paul can choose a random position or find a tie breaking criteria that is measured in different dimension, for instance, the dimension of time. Given two alternatives of optional St. Petersburg games, game $A$ and $B$, with equal expected utility, $u(A) = u(B)$, a rational player will choose the game that optimizes the factor of time.

We assume that each toss has some time duration and we annotate the duration of a single toss with $\epsilon$. Given a St. Petersburg game $A$ that constitute of $k$ tosses, with $k > 0$, the duration of game $A$ is $k\epsilon$. Given St. Petersburg game $B$ that consists of $i$ tosses where $i < k$, the duration of game $B$ is $i\epsilon$ and $i\epsilon < k\epsilon$. Suppose that in terms of utility there is indifference between game $A$ and $B$, i.e., $u(A) = u(B)$. In terms of time it is straight-forward that game $B$ is preferable. Since in terms of the cost of time, game $B$ is preferable to game $B$, a rational player that both maximizes the utility and optimizes the time will choose game $B$ which is time preserving.

We now extend the tie breaking point to a set of game alternatives. Let a set

$$STP = X_1, X_2, X_3, ... X_k$$

be a set of St. Petersburg games with respective expected utilities of

$$u(X_1), u(X_2), u(X_3)...u(X_k)$$

and respective time duration

$$t(X_1), t(X_2), t(X_3)...t(X_k).$$

Let the expected utility of each two games in the set be equal, i.e for each $X_i, X_j \in STP$ holds $u(X_i) = u(X_j)$. Suppose

$$t(X_1) < t(X_2) < t(X_3)... < t(X_k)$$

a rational player will pursue the game $X_1$ which optimizes the time required to achieve the expected utility. In other words, among equally attractive games in terms of the expected utility, a rational player will pursue the time-preserving alternative which is the least number of tosses.
4.3. From Time to Resource Based Utility Tie Breaking Point

The tie breaking point that is based on the time factor can be easily extended to a general case of resource consuming actions or games. We treat time as resource when we define the tie-breaking point and pursue the resource preserving scenario among scenarios with equal net utility change. The logic of preserving time or any other resource is the same.

5. The Universal Theory of Net Utility

In 1901 Planck (1901) published a paper that explained the black body radiation problem by assuming that energy is in the form of little packets called quanta. In 1905 Einstein (1905) published a paper that explained the photoelectric effect using the concept of quanta that was proposed by Max Planck. Because the theory of quanta was able to explain two different phenomena then it was assumed to be a universal theory. The manner in which we solved the St Petersburg paradox is through the use of the concept of net utility. The same concept of net utility was used in behavioral economics by Kahneman and Tversky (1979). Because of the usefulness of this concept of net utility in two different disciplines which are the St Petersburg paradox and behavioral economics perhaps we should start viewing it as a universal concept of understanding economics the same way the theory of energy quanta was assumed to be a universal theory when it explained both the black body radiation problem and the photoelectric effect. In classical economics we define a rational agent as an agent that maximizes its utility. This is a wrong way of defining a rational agent. We define as a rational agent as an agent that maximizes its net utility.

6. AI and Saint Petersburg Paradox

Artificial intelligence (AI) is a technology that revolutionizing the field of decision making. Marwala and Hurwitz (2017) observed that the presence of AI agents in the market makes markets more efficient than when they are only populated by human agents. This is because AI agents remove human behavioral characteristics and reduce information asymmetry from the markets. If we are to create an AI system which is able to decide whether to play in the St Petersburg Paradox problem what choices will it make. We know that the human being will not invest too much resources despite the fact that the expected utility is infinite. If we program the AI system using the nominal utility values, the AI system unlike human agents will violate the St Petersburg paradox and see the expected utility of infinity. If, however, the AI system is programmed with the net utility approach, then it will act like a rational human being. In the discipline of artificial intelligence for automated planning, the concept of the net utility was applied by Muller and Karpas (2018) for over-subscription planning and was found to outperform the traditional utility measures of outcomes.
7. CONCLUSION

This paper proposed the use of the concept of the net utility to successfully explain the St Petersburg problem. The concept of net utility or relative utility was experimentally observed by Kahneman and Tversky and has had a profound impact in the field of behavioral economics. This paper furthermore concludes that the net utility concept more universally explains value than the nominal utility. The paper also explores the AI agent making decisions using the net utility concept versus the nominal utility concept. It concludes that the AI agent that is based on the nominal utility will see infinite value whereas the AI system that is based on the net utility will see limited value just like human beings.

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