Enhancement of CP Violating terms for Neutrino Oscillation in Earth Matter

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Abstract

We investigate the $\nu_e \rightarrow \nu_\mu$ oscillation in the framework of three generations when neutrinos pass through the earth. The oscillation probability is represented by the form, $P(\nu_e \rightarrow \nu_\mu) = A \cos \delta + B \sin \delta + C$ in arbitrary matter profile by using the leptonic CP phase $\delta$. We compare our approximate formula in the previous paper with the formula which includes second order terms of $\alpha = \Delta m^2_{21} / \Delta m^2_{31}$ and $s_{13} = \sin \theta_{13}$. Non-perturbative effects of $\alpha$ and $s_{13}$ can be taken into account in our formula and the precision of the formula is rather improved around the MSW resonance region. Furthermore, we compare the earth matter effect of $A$ and $B$ with that of $C$ studied by other authors. We show that the magnitude of $A$ and $B$ can reach a few ten $\%$ of $C$ around the main three peaks of $C$ in the region $E > 1$ GeV by numerical calculation. We give the qualitative understanding of this result by using our approximate formula. The mantle-core effect, which is different from the usual MSW effect, appears not only in $C$ but also in $A$ and $B$, although the effect is weakened.

1 Introduction

The first evidence of neutrino oscillation have been discovered in the atmospheric neutrino experiments and the mass squared difference $|\Delta m^2_{31}|$ and the 2-3 mixing angle $\theta_{23}$ [1] have been measured. Also the deficit of solar neutrino strongly suggests the neutrino oscillation with the Large Mixing Angle (LMA) solution for $\Delta m^2_{21}$ and $\theta_{12}$ [2]. This has been confirmed by the KamLAND experiment by using the artificial neutrino beam emitted from several reactors [3]. On the other hand, only the upper bound $\sin^2 2\theta_{13} \leq 0.1$ is obtained for the 1-3 mixing angle [4]. Thus, the values of the mass differences and the mixing angles are gradually clarified. Our aim in the future is to determine the unknown parameters like the sign of $\Delta m^2_{31}$, $\theta_{13}$ and the leptonic CP phase $\delta$.

The simple analytic formula for estimating the matter effects is useful in order to study these parameters because neutrinos pass through the earth in most experiments and receive the matter potential represented by $a = \sqrt{2}G_F N_e$, where $N_e$ is the electron number density and $G_F$ is the Fermi constant. In the case of short baseline length, we can approximate the density as constant because the variation of $N_e$ is small. However, the longer the baseline is, the larger the matter effect is. In previous papers, several

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approximate formulas have been proposed in order to include the effect of varying density. Classified by
the neutrino energy $E$, there are following approximate formulas: low energy formulas by the expansion
in the small parameter $2Ea/|\Delta m^2_{31}| \ll 1$ or $s_{13} = \sin \theta_{13} \ll 1$ [5], high energy formulas by the expansion
in $\Delta m^2_{21}/2Ea \ll 1$ or $\alpha = \Delta m^2_{21}/\Delta m^2_{31} \ll 1$ [6], and the formulas by the expansion in $2E\delta a/\Delta m^2_{31} \ll 1$
[7], where $\delta a$ is the deviation from the average matter potential.

On the other hand, there is the method to approximate the earth matter density as three constant
layers in the case of mantle-core-mantle [8]. It was discussed in refs. [9] how the probability is enhanced
when neutrinos pass through periodically varying density. Then, it was pointed out in ref. [10] that the
mantle-core effect, which is different from the usual Mikheyev-Smirnov-Wolfenstein (MSW) effect [11],
appears in the oscillation probability. More detailed analysis has been in refs. [12, 13]. This effect is
interesting because the large enhancement of the probability can occur even if both the effective mixing
angles in the mantle and the core are small. In recent papers [14], the possibility for measuring the $\theta_{13}$
in atmospheric neutrino experiments has been discussed by using this mantle-core effect. They concluded
that the value of $\theta_{13}$ can be measured in some cases.

In our previous papers, we have shown that the oscillation probability for $\nu_e \rightarrow \nu_\mu$ transition is represented
by the following form,

$$P(\nu_e \rightarrow \nu_\mu) = A \cos \delta + B \sin \delta + C$$  \hspace{1cm} (1)

in constant matter [15] and also in arbitrary matter [16]. By using this general feature for the CP dependence,
the method for solving the parameter ambiguity problem pointed out in refs. [17] is discussed in
ref. [18]. Each coefficients has an order $A = O(s_{13}\alpha)$, $B = O(s_{13}\alpha)$ and $C = O(s^2_{13}) + O(\alpha^2)$ on the two
small parameters $\alpha = \Delta m^2_{21}/\Delta m^2_{31} \sim 0.04$ and $s_{13} = \sin \theta_{13} < 0.2$. In the case of $\alpha < s_{13}$, the ratio of
$A$ to $C$ are given by $A/C = O(\alpha/s_{13})$ and $B/C = O(\alpha/s_{13})$. So, it is expected that the CP violating
effect due to $A$ and $B$ becomes large and can reach a few ten % of $C$ even for the case that neutrinos pass through the earth core. However, the effect due to the CP phase has not been taken into account in
previous works.

In this paper, as the preparation of studying earth matter effect, we review our approximate formula
introduced in ref. [19] as

$$A \simeq 2c_{23}s_{23}Re[S^e_{\mu e}S^h_{\nu e}],$$  \hspace{1cm} (2)
$$B \simeq -2c_{23}s_{23}Im[S^e_{\mu e}S^h_{\nu e}],$$  \hspace{1cm} (3)
$$C \simeq |S^e_{\mu e}|^2 c^2_{23} + |S^h_{\nu e}|^2 s^2_{23}$$  \hspace{1cm} (4)

where $S^e_{\mu e}$ and $S^h_{\nu e}$ are the amplitudes calculated from two-generation Hamiltonians $H_\ell$ and $H_h$. $H_\ell$ is
represented by $\Delta m^2_{21}$ and $\theta_{12}$ and $H_h$ is represented by $\Delta m^2_{31}$ and $\theta_{13}$. We show that our formula includes
the non-perturbative effect of $\alpha$ and $s_{13}$ and the precision is rather improved around the MSW resonance
regions compared to the well known simple formula [20, 21], which includes up to second order terms of
$\alpha$ and $s_{13}$. Furthermore, we compare the earth matter effect of $A$ and $B$ with that of $C$ by using the
Preliminary Reference Earth Model (PREM) [22] in the case of two reference baseline length. We show
that the magnitude of $A$ and $B$ can reach a few ten % of $C$ around the main three peaks of $C$ in the region
$E > 1$ GeV by numerical calculation. This means that the above perturbative estimation is valid even in
the case of including non-perturbative effect. We give the qualitative understanding of this result by using
our approximate formula. The mantle-core effect, which is different from the usual MSW effect, appears
not only in $C$ but also in $A$ and $B$, although the effect is weakened.
2 General Formulation for Neutrino Oscillation Probabilities

In this section, we review the exact formulation of neutrino oscillation in arbitrary matter profile based on ref. [16]. At first, let us parametrize the Maki-Nakagawa-Sakata (MNS) matrix $U$ [23], which connects the flavor eigenstate $\nu_\alpha$ with the mass eigenstate $\nu_i$, by the standard parametrization [24]

$$U = O_{23}\Gamma_3O_{13}\Gamma_1O_{12},$$

where $\Gamma_3 = \text{diag}(1, 1, e^{i\delta})$ and $O_{ij}$ is the rotation matrix between $i$ and $j$ generation. As the matter potential only appears in the $(ee)$ component of the Hamiltonian, $O_{23}$ and $\Gamma_3$ can be factored out. So, we can rewrite the Hamiltonian in matter as $H = O_{23}\Gamma_3H'(O_{23}\Gamma_3)$. $H'$ is the reduced Hamiltonian defined on the basis $\nu' = (O_{23}\Gamma_3)\nu$. $H'$ is a real symmetric matrix and the concrete expression is given by

$$H' = O_{13}O_{12}\text{diag}(0, \Delta_{21}, \Delta_{31})(O_{13}O_{12})^T + \text{diag}(a(t), 0, 0),$$

where $\Delta_{ij} = \Delta m_{ij}^2/2E$. The number of parameters in the Hamiltonian $H'$ is fewer than that in the original Hamiltonian $H$ by two. This is useful to calculate the oscillation probability simply. If we define the amplitude $S'_{\alpha\beta}$ of $\nu_\alpha \rightarrow \nu'_{\beta}$ transition as $(\alpha\beta)$ component of time ordered product

$$S' = T \exp \left[ -i \int_0^L H'(t)dt \right],$$

the oscillation probability is given by

$$P(\nu_e \rightarrow \nu_\mu) = A \cos \delta + B \sin \delta + C,$$

$$A = 2c_{23}s_{23}\text{Re}[S'_{\mu e}S'_{\tau e}],$$

$$B = -2c_{23}s_{23}\text{Im}[S'_{\mu e}S'_{\tau e}],$$

$$C = |S'_{\mu e}|^2c_{23}^2 + |S'_{\tau e}|^2s_{23}^2,$$

as in [16]. From the eqns. (8)-(11), one can see that the probability for $\nu_e \rightarrow \nu_\mu$ transition is represented by two components of the reduced amplitude, $S'_{\mu e}$ and $S'_{\tau e}$. Namely, the matter effect for the oscillation probability is only contained in the two components.

3 Non-Perturbative Effect in Our Approximate Formula

In this section, we numerically calculate the amplitudes $S'_{\mu e}$ and $S'_{\tau e}$ introduced in the previous section by using the PREM. Then, it is explained how we obtain the hint for the basic concept on deriving our approximate formula. As an example, the approximate formula in constant matter is derived explicitly and is compared with the formula in refs. [20] [21], which includes up to second order of the small parameters $\alpha = \Delta_{21}/\Delta_{31}$ and $s_{13}$. As a result, it is shown that our approximate formula includes the non-perturbative effect which becomes important around the MSW resonance.

3.1 Behavior of Reduced Amplitudes in Earth Matter

Let us calculate the amplitudes $S'_{\mu e}$ and $S'_{\tau e}$ for the case that neutrinos pass through the earth. We use the PREM as the earth density model and choose two reference baselines, 6000 km and 12000 km. Fig. 1 shows how the matter density changes along the path of neutrinos. In fig. 2, we plot the values of the amplitudes $S'_{\mu e}$ and $S'_{\tau e}$ corresponding to the neutrino energy 0.03-20 GeV. Here, we use the parameters $\Delta m_{21}^2 = 7 \times 10^{-5}$eV$^2$ and $\sin^2 2\theta_{12} = 0.8$ as indicated from the solar neutrino experiments.
and the KamLAND experiment, $\Delta m^2_{31} = 2 \times 10^{-3}$eV$^2$ from the atmospheric neutrino experiments and the K2K experiment, and $\sin^2 2\theta_{13} = 0.1$ within the upper limit of the CHOOZ experiment.

![Matter density in the PREM](image1)

**Fig. 1.** Matter density in the PREM with baseline length 6000 km and 12000 km from left to right.

![Energy dependence of $S'_{\mu e}$ and $S'_{\tau e}$](image2)

**Fig. 2.** Energy dependence of $S'_{\mu e}$ and $S'_{\tau e}$ with baseline length 6000 km and 12000 km by using the PREM. The solid and dashed lines represent $S'_{\mu e}$ and $S'_{\tau e}$ respectively.

It is found from fig. 2 that $S'_{\mu e}$ and $S'_{\tau e}$ become large in low energy and high energy, respectively, for both baselines and the regions, where $S'_{\mu e}$ and $S'_{\tau e}$ dominantly contribute, are separated to each other.

In other words, the MSW effect related to the 1-2 mixing and 1-3 mixing angles are mainly included in $S'_{\mu e}$ and $S'_{\tau e}$, respectively. We have derived the approximate formula for arbitrary matter profile by using this feature in ref. [19]. Concretely, $S'_{\mu e}$ and $S'_{\tau e}$ are calculated from two kinds of different Hamiltonians, which are given by the 1-2 and 1-3 subsystems, respectively.

### 3.2 Procedure of Deriving Approximate Formula

The idea introduced in the previous subsection is actually realized as follows. We use the two small parameters $\alpha = \Delta_{21}/\Delta_{31}$ and $s_{13}$. Then, our approximate formula is calculated by the following three steps.
1. We define two Hamiltonians in the 1-2 and 1-3 subsystems taking the limit of $s_{13} \rightarrow 0$ and $\alpha \rightarrow 0$ in (6) as

$$ H_\ell = \begin{pmatrix} \Delta_{21}s_{12}^2 + a(t) & \Delta_{21}c_{12}s_{12} & 0 \\ \Delta_{21}c_{12}s_{12} & \Delta_{21}c_{12}^2 & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix}, \quad H_h = \begin{pmatrix} \Delta_{31}s_{13}^2 + a(t) & 0 & \Delta_{31}c_{13}s_{13} \\ 0 & 0 & 0 \\ \Delta_{31}c_{13}s_{13} & 0 & \Delta_{31}c_{13}^2 \end{pmatrix}. \quad (12) $$

2. We calculate two amplitudes $S^\ell$ and $S^h$ from the Hamiltonians $H_\ell$ and $H_h$ by the equations

$$ S^\ell = T \exp \left[ -i \int_0^L H_\ell(t) dt \right], \quad S^h = T \exp \left[ -i \int_0^L H_h(t) dt \right]. \quad (13) $$

3. We replace the amplitudes in (9)-(11) as $S'_{\mu e} \rightarrow S^\ell_{\mu e}$ and $S'_{\tau e} \rightarrow S^h_{\tau e}$.

### 3.3 Approximate Formula in Constant Matter

Next, let us review the approximate formula in constant matter based on ref. [19]. According to the procedure in the previous subsection, we substitute the Hamiltonian $H_\ell$ in constant matter given by (12) into (13) and we obtain $S^\ell_{\mu e}$ as

$$ S^\ell_{\mu e} = [\exp(-iH_\ell L)]_{\mu e} = -i \sin 2\theta_\ell \sin \phi_\ell \exp \left( -i \frac{\Delta_{21} + a}{2} L \right), \quad (14) $$

where $\phi_\ell \equiv \Delta_\ell L/2$ and the subscript $\ell$ represents the quantities calculated from $H_\ell$. The concrete expressions for the mass squared difference and the 1-2 mixing angle in matter are given by

$$ \frac{\Delta_\ell}{\Delta_{21}} = \frac{\sin 2\theta_{12}}{\sin 2\theta_\ell} = \sqrt{ \left( \cos 2\theta_{12} - \frac{a}{\Delta_{21}} \right)^2 + \sin^2 2\theta_{12} }. \quad (15) $$

These are well known expressions in the framework of two generations. The contribution of the low energy MSW effect, which is dominant around the energy region determined by $a \sim \Delta_{21} \cos 2\theta_{12}$, is included in mainly $S^\ell_{\mu e}$. The phase factor in (14) does not contribute when we calculate the probability in two generations. However, this gives important contribution on the calculation of the terms dependent on the CP phase in three generations.

Similarly, we obtain $S^h_{\tau e}$ by substituting $H_h$ in constant matter given by (12) into (13) as

$$ S^h_{\tau e} = [\exp(-iH_h L)]_{\tau e} = -i \sin 2\theta_h \sin \phi_h \exp \left( -i \frac{\Delta_{31} + a}{2} L \right), \quad (16) $$

where $\phi_h \equiv \Delta_h L/2$ and the subscript $h$ represents the quantities calculated from $H_h$. The concrete expressions are given by

$$ \frac{\Delta_h}{\Delta_{31}} = \frac{\sin 2\theta_{13}}{\sin 2\theta_h} = \sqrt{ \left( \cos 2\theta_{13} - \frac{a}{\Delta_{31}} \right)^2 + \sin^2 2\theta_{13} }. \quad (17) $$
One can see that these expressions correspond to those obtained by the replacement $\Delta_{21} \rightarrow \Delta_{31}$ and $\theta_{12} \rightarrow \theta_{13}$ in (14) and (15). The contribution of high energy MSW effect, which is dominant around the energy region determined by $a \sim \Delta_{31} \cos 2\theta_{13}$, is included in mainly $S_{\tau e}^h$. We can calculate $A$, $B$ and $C$ in constant matter as

$$P(\nu_e \rightarrow \nu_\mu) = A \cos \delta + B \sin \delta + C,$$

(18)

$$A \simeq \sin 2\theta_\ell \sin 2\theta_h \sin \phi_\ell \sin \phi_h \frac{\Delta_{32} L}{2},$$

(19)

$$B \simeq \sin 2\theta_\ell \sin 2\theta_h \sin \phi_\ell \sin \phi_h \frac{\Delta_{32} L}{2},$$

(20)

$$C \simeq c_{23}^2 \sin^2 2\theta_\ell \sin^2 2\theta_\mu + s_{23}^2 \sin^2 2\theta_h \sin^2 \phi_h$$

(21)

from these expressions. These approximate formulas are similar to the following well known formulas

$$A \simeq \frac{a}{a - \Delta_{31}} c_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \sin \frac{aL}{2} \sin \frac{(a - \Delta_{31})L}{2} \cos \frac{\Delta_{31} L}{2},$$

(22)

$$B \simeq \frac{a}{a - \Delta_{31}} c_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \sin \frac{aL}{2} \sin \frac{(a - \Delta_{31})L}{2} \sin \frac{\Delta_{31} L}{2},$$

(23)

$$C \simeq \frac{a^2}{a^2 - c_{23}^2} \sin^2 2\theta_{12} \sin^2 \frac{aL}{2} + \frac{\Delta_{31}^2}{(a - \Delta_{31})^2} s_{23}^2 \sin^2 2\theta_{13} \sin^2 \frac{(a - \Delta_{31})L}{2}. $$

(24)

These formulas are often used in order to analyze the property of neutrino oscillation because they have very simple form and approximate the exact values with a good precision.

In the following, we compare the probability calculated from our approximate formula (19)-(21) with that from the formula (22)-(24) in the case of constant matter. We calculate $P(\nu_e \rightarrow \nu_\mu)$ for two kinds of baselines, 3000 km and 6000 km. We use the parameters $\sin^2 2\theta_{23} = 1$ and $\delta = 0^\circ$ in addition to those introduced in fig. 2. Furthermore, $\rho = 4.7g/cm^3$ and $Y_e = 0.494$ are used as the matter density and the electron fraction. The result is given in fig. 3.
It is found that our approximate formula has good coincidence to the exact one even around the MSW resonance, compared with that from the formula (22)-(24). We consider the reason for the difference in the next section.

3.4 Comparison of Approximate Formulas

Let us explain the order counting of $\alpha$ and $s_{13}$ in our approximate formula. In the limit $\alpha \to 0$, we obtain $S'_{\mu e} = 0$. Therefore, we can write down the order of $S'_{\mu e}$ as

$$S'_{\mu e} = O(\alpha) + O(\alpha^2) + O(\alpha^3) + \cdots + O(s_{13} \alpha) + O(s_{13}^2 \alpha) + O(s_{13}^3 \alpha) + \cdots + \cdots. \quad (25)$$

Note that $\alpha$ is included in all terms. Here, if we take the $s_{13} \to 0$, only the first line is remaining. In these terms, all orders of $\alpha$ are included and the first line is considered to have a larger contribution compared to the following lines, because of the increasing exponent of $s_{13}$. This is confirmed by the comparison with the exact formula in fig. 3. In the same way, we obtain $S'_{\tau e} = 0$ in the limit $s_{13} \to 0$. Therefore, we can write down the order of $S'_{\tau e}$ as

$$S'_{\tau e} = O(s_{13}) + O(s_{13}^2) + O(s_{13}^3) + \cdots + O(s_{13} \alpha) + O(s_{13}^2 \alpha) + O(s_{13}^3 \alpha) + \cdots + \cdots. \quad (26)$$

Here, if we take the limit $\alpha \to 0$, only the first line is remaining. In these terms, all orders of $s_{13}$ are included and the first line is considered to have a larger contribution compared to the following lines, because of the increasing exponent of $\alpha$.

Our method includes both, the terms of higher order of $\alpha$ in (25) and also those of $s_{13}$ in (26). So, this new approach is not a systematic expansion. However, our method is not in contradiction to the well known formula (22)-(24), which takes only the first order term of $\alpha$ and $s_{13}$ in (25)-(26) regarding them as small parameters. In addition, higher order terms of the perturbative expansion, which are not included in the formula (22)-(24), are now also included in our formula.

In the following, let us investigate the difference between these two methods more concretely. As seen in fig. 2, the contribution of $S'_{\tau e}$ is dominant in the energy region $E > 1 \text{ GeV}$. So, we can roughly consider as

$$P(\nu_e \to \nu_\mu) \simeq C \simeq s_{23}^2 \sin^2 2\theta_h \sin^2 \left( \frac{\Delta_h L}{2} \right). \quad (27)$$

Note that we have the relation (17) between the mass squared differences and the mixing angles in vacuum and in matter. Expanding the right hand side of (17) on the mixing angle in vacuum, we obtain

$$\frac{\Delta_h}{\Delta_{31}} = \frac{\sin 2\theta_{13} \sin 2\theta_h}{2 \theta_h} \simeq \left| 1 - \frac{a}{\Delta_{31}} \right| \left( 1 + \frac{2a \Delta_{31}}{(\Delta_{31} - a)^2} s_{13}^2 + \frac{a^2 \Delta_{31}^2 s_{13}^2}{2(\Delta_{31} - a)^4} s_{13}^4 + \cdots \right). \quad (28)$$

The condition for convergence is given by

$$\frac{4a \Delta_{31} s_{13}^2}{(\Delta_{31} - a)^2} < 1. \quad (29)$$
This condition is not satisfied around the MSW resonance region, where $\Delta_{31} \sim a$. Namely, the perturbative expansion becomes inconvergent. However, substituting the above expression (28) into (27) of the oscillation probability and taking only the first term, we obtain

$$ P(\nu_e \to \nu_\mu) \simeq s_{23}^2 \Delta_{31}^2 \sin^2 2\theta_{13} \sin^2 \frac{\Delta_{31} - a}{2} L. $$

(30)

It gives the finite value in the limit $\Delta_{31} \to a$ as

$$ P(\nu_e \to \nu_\mu) \simeq s_{23}^2 c_{13}^2 (s_{13} \Delta_{31} L)^2. $$

(31)

This is due to the product of the infinity of effective mixing angle and the zero of the effective mass squared difference in the probability (30). If we take the limit $\Delta_{31} \to a$ directly in the non-perturbative expression (27), we obtain

$$ P(\nu_e \to \nu_\mu) = s_{23}^2 c_{13}^2 \sin^2 (s_{13} \Delta_{31} L). $$

(32)

We find that the difference between the perturbative formula (31) and our formula (32) is the sin factor. In the case of the short baseline length $L$, the perturbation gives a good approximation, but the longer the baseline $L$ is, the worse the perturbation becomes, as shown in fig. 3, although the probability has a finite value. Concretely, if the condition

$$ L < \frac{1}{s_{13} \Delta_{31}} $$

(33)

is satisfied, the perturbation gives a good approximation. Around the MSW resonance region, the perturbation breaks down because the coefficients of the higher order terms $\alpha$ or $s_{13}$ become large. Therefore, it is needed to involve the higher order terms of $\alpha$ and $s_{13}$, in order to make a good approximate formula. Our method partially realizes this request.

At the end of this section, let us give a brief comment. In ref. [20, 21], the formula calculated by single expansion on $\alpha$ is also given and this includes all order terms of $s_{13}$. So, this approximate formula gives a good approximation in the high energy MSW resonance region compared with the formula (22)-(24), while the difference between the single expansion formula and numerical calculation becomes large in the low energy region as commented also in ref. [21].

4 Earth Matter Effect for $A$, $B$ and $C$

In this section, we perform numerical calculations of $A$, $B$ and $C$ by using the PREM. We give a qualitative understanding of the behavior of the coefficients $A$, $B$ and $C$ by matter effects of the mantle and the core.

4.1 Numerical Calculation of $A$, $B$ and $C$

In the previous section, the order of the reduced amplitudes are estimated as $S'_{\mu e} = O(\alpha)$ and $S'_{\tau e} = O(s_{13})$ in the case that we takes only the first order term of $\alpha$ and $s_{13}$ in (24)-(26). The order of coefficients are also obtained as $A = O(s_{13} \alpha)$, $B = O(s_{13} \alpha)$ and $C = O(s_{13}^2) + O(\alpha^2)$ by substituting $S'_{\mu e} = O(\alpha)$ and $S'_{\tau e} = O(s_{13})$ into (14)-(13). In the case of $\alpha < s_{13}$, the magnitude of ratios is give by $A/C = O(\alpha/s_{13})$ and $B/C = O(\alpha/s_{13})$. Therefore, it is expected from the perturbative point of view that the CP violating effect due to $A$ and $B$ becomes large and can reach a few ten % of $C$. However, because non-perturbative
effect becomes important in MSW region as shown in fig. 3, the CP violating effect should be investigated more carefully.

At first, let us numerically calculate how the coefficients $A$, $B$ and $C$ are enhanced by the earth matter effect, in the case of the baseline length $L = 6000\,\text{km}$ and $12000\,\text{km}$ for $\sin^2 2\theta_{13} = 0.10$, and $0.04$, respectively. These values of $\sin^2 2\theta_{13}$ correspond to the values within the upper bound of the CHOOZ experiments. The PREM is used as the earth matter density and the same mass squared differences and the mixing angles given in sec. 3 are also used.

![Graphs showing energy dependence of $A$, $B$ and $C$](image)

**Fig. 4.** Energy dependence of $A$, $B$ and $C$ by numerical calculation. We use the PREM as earth matter density model with two baseline length 6000 km and 12000 km, and we choose the 1-3 mixing angle $\sin^2 2\theta_{13} = 0.10$ and 0.04 as representative values.

In the case of $L = 6000\,\text{km}$, the behavior can be understood by using the formulation (19)-(21) in
constant matter. The value of $C$ becomes large around $E = 5$ GeV, which comes from the enhancement of the effective mixing angle $\sin \theta_h$ for $a \simeq \Delta h$, and then oscillates depending on the factor $\sin \phi_h$. The values of $A$ and $B$ become small compared with that of $C$ in high energy region, as the suppression factor $S^\ell_{\mu e} \propto 1/E$. See details in refs. \cite{13, 20, 23} for example of the constant matter density.

In the case of $L = 12000$ km, three main peaks appear in $C$. On the other hand, the pattern of the enhancement for $A$ and $B$ seems to become more complicated than that of $L = 6000$ km. In the next subsection, we give a qualitative understanding of the above results by using our approximate formula for $A$, $B$ and $C$.

We also represent the values of $A$, $B$ and $C$ around the energy of the three peaks of $C$ in Table 1. These values are computed by the numerical calculation using the PREM.

| $L$ (km) | $\sin^2 2\theta_{13}$ | $E$ (GeV) | $A$ ($|A/C|$) | $B$ ($|B/C|$) | $C$ | peak type |
|---------|-----------------|----------|-------------|-------------|-----|----------|
| 6000    | 0.10            | 4.9      | -0.025 (8.3%) | -0.003 (1.0%) | 0.301 | MSW(mantle) |
| 6000    | 0.04            | 4.8      | -0.017 (12.0%) | -0.001 (0.7%) | 0.142 | MSW(mantle) |
| 12000   | 0.10            | 2.1      | -0.061 (17.9%) | 0.066 (19.4%) | 0.340 | MSW(core)  |
| 12000   | 0.10            | 2.8      | 0.017 (6.3%)  | -0.028 (10.4%) | 0.269 | mantle-core |
| 12000   | 0.10            | 5.2      | -0.026 (7.4%)  | 0.013 (3.7%)  | 0.352 | MSW(mantle) |
| 12000   | 0.04            | 2.0      | -0.068 (31.6%) | 0.038 (17.7%) | 0.215 | MSW(core)  |
| 12000   | 0.04            | 3.0      | -0.030 (6.0%)  | -0.037 (7.4%) | 0.500 | mantle-core |
| 12000   | 0.04            | 5.4      | -0.014 (8.9%)  | 0.015 (9.6%)  | 0.157 | MSW(mantle) |

Table 1. Resonance values of $A$, $B$ and $C$ calculated numerically by using the PREM with baseline length $6000$ km and $12000$ km, $\sin^2 2\theta_{13} = 0.10$ and 0.04.

Table 1 shows that the coefficients $A$ and $B$ can be rather large at the three main peaks of $C$. The absolute values of $A$ and $B$ become about 0.06 at the peak of the core. Furthermore, the ratios $|A/C|$ and $|B/C|$ also become a few ten % even for the case of including non-perturbative effect. These energy regions $E = 2 \sim 6$ GeV are explored by the atmospheric neutrino and the long baseline experiments. In actual experiments averaging of various parameters are necessary for example, energy, zenith-angle distribution, the sum of particle and anti-particle, and so on. Therefore, the CP phase effect may be weakened to some extent, but we consider that the CP phase effect should be estimated precisely in order to determine the value of $\theta_{13}$ in future experiments.

### 4.2 Approximate Formula in Matter with Three Layers

In order to give a qualitative understanding of the results obtained in the previous subsection, let us approximate the earth matter density with baseline $L = 12000$ km as three constant layers such that the first and the third layers have the same density and length.

At first, we calculate the amplitude $S^\ell_{\mu e}$ from the low energy Hamiltonian $H_\ell$. We use the superscript $m$ and $c$ for representing the amplitude in the first and third layer (mantle), and the second layer (core). Taking the limit $s_{13} \to 0$, the amplitudes $S^{m}_{\tau \mu}$, $S^{m}_{\tau e}$ and so on vanish. Only four terms contribute to the amplitude in three layers $S^\ell_{\mu e}$ as

$$S^\ell_{\mu e} = S^{m}_{\mu e} S^{c}_{e e} S^{m}_{e e} + S^{m}_{\mu \mu} S^{c}_{e e} S^{m}_{e e} + S^{m}_{\mu e} S^{c}_{e \mu} S^{m}_{e \mu} + S^{m}_{\mu \mu} S^{c}_{e \mu} S^{m}_{e \mu}.$$  \hspace{1cm} (34)

Substituting \cite{14} and

$$S^{m}_{e e} = \exp(-i H^m_L)_{e e} = (\cos \phi^m_e + i \cos 2\theta^m_e \sin \phi^m_e) \exp \left(-i \frac{\Delta_{21} + a^m L^m}{2} \right),$$  \hspace{1cm} (35)
\[
S_{\mu\mu}^m = \left[\exp(-iH_\ell^m L)\right]_{\mu\mu} = (\cos \phi_\ell^m - i \cos 2\theta_\ell^m \sin \phi_\ell^m) \exp \left(-i\frac{\Delta_{21} + a \cdot L^m}{2}\right) \quad (36)
\]

into (34), we obtain

\[
S_\mu^\ell = -i \exp \left(-i\frac{\Delta_{21} + 2a \cdot L^m + a' \cdot L^c}{2}\right) F(\phi_\ell^m, \phi_c^m, \theta_\ell^m, \theta_c^m), \quad (37)
\]

where the function \( F \) is defined by

\[
F(\phi^m, \phi^c; \theta^m, \theta^c) = \sin 2\phi^m \cos \phi^c \sin 2\theta^m + \cos^2 \phi^m \sin \phi^c \sin 2\theta^c + \sin^2 \phi^m \sin \phi^c \sin(2\theta^c - 4\theta^m). \quad (38)
\]

We can easily extract the physical meaning from this expression, although this function \( F \) becomes the same one as given in refs. 12, 13 after a short calculation. We describe the meaning of each term later.

Next, we calculate the amplitude \( S_{\tau\tau}^h \) taking the limit \( \alpha \to 0 \). In this limit, \( S_{\mu\mu}^m, S_{\mu\mu}^c \) and so on vanish, so the amplitude \( S_{\tau\tau}^h \) in three layer is calculated as

\[
S_{\tau\tau}^h = S_{\tau\tau}^m S_{\tau\tau}^c S_{\tau\tau}^m + S_{\tau\tau}^m S_{\tau\tau}^c S_{\tau\tau}^m + S_{\tau\tau}^m S_{\tau\tau}^c S_{\tau\tau}^m + S_{\tau\tau}^m S_{\tau\tau}^c S_{\tau\tau}^m. \quad (39)
\]

Substituting (10) and

\[
S_{ee}^m = \left[\exp(-iH_\ell^m L)\right]_{ee} = (\cos \phi_h^m + i \cos 2\theta_h^m \sin \phi_h^m) \exp \left(-i\frac{\Delta_{31} + a \cdot L^m}{2}\right), \quad (40)
\]

\[
S_{\tau\tau}^m = \left[\exp(-iH_\ell^m L)\right]_{\tau\tau} = (\cos \phi_h^m - i \cos 2\theta_h^m \sin \phi_h^m) \exp \left(-i\frac{\Delta_{31} + a \cdot L^m}{2}\right) \quad (41)
\]

into (39), we obtain

\[
S_{\tau\tau}^h = -i \exp \left(-i\frac{\Delta_{31} + 2a \cdot L^m + a' \cdot L^c}{2}\right) F(\phi_h^m, \phi_c^m, \theta_h^m, \theta_c^m), \quad (42)
\]

which corresponds to equation 34 by replacing the subscript and superscript as \((\ell) \to (h)\).

Substituting 34 and 42 into 10 - 14, the coefficients \( A, B \) and \( C \) in three layers are given by

\[
A \simeq \sin 2\theta_{23} \cos \left(\frac{\Delta_{32} L}{2}\right) F(\phi_\ell^m, \phi_c^m; \theta_\ell^m, \theta_c^m) F(\phi_h^m, \phi_c^m; \theta_h^m, \theta_c^m), \quad (43)
\]

\[
B \simeq \sin 2\theta_{23} \sin \left(\frac{\Delta_{32} L}{2}\right) F(\phi_\ell^m, \phi_c^m; \theta_\ell^m, \theta_c^m) F(\phi_h^m, \phi_c^m; \theta_h^m, \theta_c^m), \quad (44)
\]

\[
C \simeq \left(c_{23}^2 F(\phi_\ell^m, \phi_c^m; \theta_\ell^m, \theta_c^m)^2 + s_{23}^2 F(\phi_h^m, \phi_c^m; \theta_h^m, \theta_c^m)^2\right). \quad (45)
\]

Thus, we can calculate the coefficients \( A \) and \( B \), which are related to the magnitude of the CP effect, by using our approximate formula. We can see the following from the expressions of \( A, B \) and \( C \). The expression of \( C \) is given as the sum of \( F_h \) and \( F_\ell \), where we use the abbreviation \( F_h \) and \( F_\ell \) as \( F(\phi_h^m, \phi_c^m; \theta_h^m, \theta_c^m) \) and \( F(\phi_\ell^m, \phi_c^m; \theta_\ell^m, \theta_c^m) \). On the other hand, the expressions \( A \) and \( B \) are both given as the product of \( F_h \) and \( F_\ell \) and furthermore multiplied by the oscillating factor related to \( \Delta_{32} \). This is the main difference between \( A, B \) and \( C \). However, all the coefficients depend on the function \( F \). In the following, we study the behavior of this function \( F \).
At first, we divide $F$ given in (38) into three parts as

$$F(\phi^m, \phi^c; \theta^m, \theta^c) = F_1 + F_2 + F_3,$$  \hspace{1cm} (46)

$$F_1 = \sin 2\phi^m \cos \phi^c \sin 2\theta^m, \hspace{1cm} (47)$$

$$F_2 = \cos^2 \phi^m \sin \phi^c \sin 2\theta^c, \hspace{1cm} (48)$$

$$F_3 = \sin^2 \phi^m \sin \phi^c \sin(2\theta^c - 4\theta^m). \hspace{1cm} (49)$$

This separation of the function $F$ is useful to understand, which contribution becomes large in the amplitude because $F_1$, $F_2$ and $F_3$ correspond to the MSW effect in the mantle, and in the core, and the mantle-core effect, respectively. By using the above expressions, the following interpretation in refs. [12, 13] can be understood more clearly.

1. $\cos 2\phi^m = 0$ and $\sin \phi^c = 0$

Only $F_1$ remains and the function takes the form $F = \pm \sin 2\theta^m$ because $F_2 = F_3 = 0$ due to $\sin \phi^c = 0$. In the case that the above conditions are approximately satisfied around the MSW resonance region of the mantle, namely around the energy determined by $\sin 2\theta^m = \pm 1$, the function $F$ is enhanced.

2. $\sin \phi^m = 0$ and $\cos \phi^c = 0$

Only $F_2$ remains and the function takes the form $F = \pm \sin 2\theta^c$ because $F_1 = F_3 = 0$ due to $\sin \phi^m = 0$ and $\cos \phi^c = 0$. In the case that the above conditions are approximately satisfied around the MSW resonance region of the core, namely around the energy determined by $\sin 2\theta^c = \pm 1$, the function $F$ is enhanced.

3. $\cos \phi^m = 0$ and $\cos \phi^c = 0$

Only $F_3$ remains and the function takes the form $F = \pm \sin(2\theta^c - 4\theta^m)$ because $F_1 = F_2 = 0$ due to $\cos \phi^m = 0$ and $\cos \phi^c = 0$. Around the energy determined by $\sin(2\theta^c - 4\theta^m) = \pm 1$, the function $F$ is enhanced. This can be large, even if both effective mixing angles in the mantle and in the core, $\theta^m$ and $\theta^c$, are small. It is considered as the mantle-core effect. It is realized in the case that $\Delta_{31}$ takes the intermediate value of the matter potentials $a^m$ and $a^c$, respectively, for the mantle and the core.

### 4.3 Interpretation of Numerical Results

In this subsection, the numerical result for $L = 12000$km can be understood, by using the analytical expression derived in the previous subsection. All the coefficients $A$, $B$ and $C$ are determined by the functions $F_\ell$ and $F_h$. Here, we study the behavior of $F_\ell$ and $F_h$ in the energy region larger than $E > 1$ GeV. We can approximate $F_\ell$ by using the fact $\Delta_{21} \ll a$ at $E > 1$ GeV. That is, the oscillation part and the mixing angle are approximated by

$$\phi_\ell = \frac{\Delta_{21}L}{2} \simeq \frac{aL}{2} \sim \text{const.},$$  \hspace{1cm} (50)

$$\sin 2\theta_\ell = \frac{\Delta_{21} \sin 2\theta_{12}}{\sqrt{(\Delta_{21} \cos 2\theta_{12} - a)^2 + \Delta_{21}^2 \sin^2 2\theta_{12}}} \simeq \frac{\Delta_{21} \sin 2\theta_{12}}{a} \propto \frac{1}{E},$$  \hspace{1cm} (51)

from (15). As a result, we can also approximate $F_\ell$ from (46-49) as

$$F_\ell \propto \frac{1}{E}. \hspace{1cm} (52)$$

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Thus, the value of $F_h$ decreases proportional to the inverse of the neutrino energy.

On the other hand, some of the peaks appear in $F_h$ corresponding to $F_{h1}$, $F_{h2}$ and $F_{h3}$, since $F_h$ includes the 1-3 MSW effect in the considered energy range. Fig. 5 shows the component of $F_h$ by using our analytical expression (40), where we use the matter densities in the mantle and the core as $\rho^m = 4.7g/cm^3$ and $\rho^c = 11.0g/cm^3$, the electron fraction as $Y^m_e = 0.494$ and $Y^c_e = 0.466$, calculated by the PREM in the case of the baseline $L = 12000$ km.

Table 2. Components of $F_h$ calculated from our analytical formula with $\sin^2 2\theta_{13} = 0.10$ and 0.04 from left to right. Solid, dashed, dash-dotted and dotted lines correspond to $F_{h1}^2$, $F_{h2}^2$, $F_{h3}^2$, respectively. Furthermore, we represent the values of each component $F_{h1}$, $F_{h2}$ and $F_{h3}$ at three peaks in Table 2.

| $\sin^2 2\theta_{13}$ | $E$ (GeV) | $F_h$ | $F_{h1}$ | $F_{h2}$ | $F_{h3}$ |
|------------------|---------|-------|----------|----------|----------|
| 0.10             | 2.1     | 0.821 | 0.278    | **0.509** | 0.034    |
| 0.10             | 3.0     | 0.726 | -0.029   | 0.011    | **0.744** |
| 0.10             | 5.4     | -0.810| -**0.821** | 0.015    | -0.004   |
| 0.04             | 2.0     | 0.648 | 0.093    | **0.532** | 0.023    |
| 0.04             | 3.1     | 0.999 | 0.051    | 0.055    | **0.893** |
| 0.04             | 5.7     | -0.535| -**0.545** | 0.013    | -0.003   |

Table 2. Components of $F_h$ calculated from our analytical formula with $\sin^2 2\theta_{13} = 0.10$, and 0.04.

Fig. 5 and Table 2 show that the peak in the right hand side is dominated by $F_{h1}$ and mainly depends on the MSW effect in the mantle. The MSW resonance in the mantle is realized at the condition $a^m = \Delta_{31} \cos 2\theta_{13}$. The energy determined by this condition is $E \sim \frac{\Delta_{31} \cos 2\theta_{13}}{2\sqrt{2} G F_N} \sim 5.7$ GeV. The peak in the left hand side is dominated by $F_{h2}$ and mainly depends on the MSW effect in the core. The MSW resonance in the core is realized at the condition $a^c = \Delta_{31} \cos 2\theta_{13}$. Noticing the relation $a^c \simeq 2.5 \times a^m$, the peak energy is given by around $E \sim 5.7/2.5 \sim 2.3$ GeV. The energy of these peaks do not largely depend on the value $\theta_{13}$ in the case of $\sin^2 2\theta_{13} \ll 1$. Furthermore, it is shown that the mantle-core effect mainly contributes to the peak at the center, when $F_{h3}$ becomes large. The energy determined by the condition $\sin(2\theta^c - 4\theta^m) \sim 1$ is about $E = 3.4$ GeV for $\sin^2 2\theta_{13} = 0.04$. In the case of $\sin^2 2\theta_{13} = 0.10$, this condition cannot be satisfied in any energy region and as a result the enhancement is weakened. This
phenomena is interesting because the value of $F_h$ for $\sin^2 2\theta_{13} = 0.04$ (small mixing) is larger than that for $\sin^2 2\theta_{13} = 0.10$ (large mixing). It is interpreted as the total neutrino conversion pointed out by Petcov et al. [12].

Next, let us study how we can understand the behavior of $A$, $B$ and $C$. From [14], $C$ is approximated by

$$ C = \frac{1}{2}(F_\ell^2 + F_h^2) \simeq \frac{1}{2}F_h^2, \quad (53) $$

where we neglect $F_\ell$, because of its smallness compared to $F_h$ as shown in fig. 5. Actually, the $C$-function has almost half of the size of the $F_h^2$-function. Therefore, $C$ has three peaks as $F_h^2$. $P(\nu_e \to \nu_\mu)$ in ref. [14] corresponds to $C$ in this paper. It means that the terms $A$ and $B$, related to the CP phase, were not considered in previous papers.

Next, we obtain the expressions for $A$ and $B$ from [17] and [18] as

$$ A \simeq \cos \left( \frac{\Delta_{32} L}{2} \right) F_\ell F_h \propto \frac{1}{E} \cos \left( \frac{\Delta_{32} L}{2} \right) F_h, \quad (54) $$

$$ B \simeq \sin \left( \frac{\Delta_{32} L}{2} \right) F_\ell F_h \propto \frac{1}{E} \sin \left( \frac{\Delta_{32} L}{2} \right) F_h. \quad (55) $$

From these expressions, we can see the following. First, the mantle-core effect, which is different from the usual MSW effect, appears not only in $C$ but also in $A$ and $B$ because of the multiplication of $F_h$. Second, $A$ and $B$ are suppressed compared with $C$ in the energy range $E > 1$ GeV because of $F_\ell \propto 1/E$. Third, $A$ and $B$ depend on the oscillation part $\Delta_{32} L/E$ additionally to $L/E$ dependence included in $F$. Because of this factor, the oscillation phases of $A$ and $B$ have a difference of about a quarter of the wavelength.

### 5 Summary

In this paper, we investigate the matter effect included in the terms related to the CP phase, particularly in the case that neutrinos pass through the earth core. The results are summarized as follows.

1. Our approximate formulas [21]-[41] include non-perturbative effect of the small parameters $\alpha = \Delta_{21}/\Delta_{31}$ and $s_{13}$. As a result, the precision of the formula is rather improved compared to the formula which includes up to second order of $\alpha$ and $s_{13}$ around the MSW resonance regions.

2. We numerically calculate the coefficients $A$, $B$ and $C$ for the baseline length $L = 6000$ km and 12000 km by using the PREM as the earth matter density. As a result, the magnitude of $A$ and $B$ can reach a few ten % of $C$ around the three main peaks of $C$ even for the case of including non-perturbative effect.

3. We give the qualitative understanding of the behavior for $A$, $B$ and $C$ by using our approximate formula. The mantle-core effect, which is different from the usual MSW effect, appears not only in $C$ but also in $A$ and $B$, although the effect is weakened.

From the results of this paper, it has been found that the effects of the leptonic CP phase can be comparatively large in the oscillation probability, when neutrinos pass through the earth. We should consider the CP phase effects more seriously in order to extract the information on $\theta_{13}$ and the sign of $\Delta m^2_{31}$ in future experiments.
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