PARITY-VIOLATING LONGITUDINAL RESPONSE

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The parity-violating quasielastic electron scattering response is explored within the context of a model that builds antisymmetrized random phase approximation and Hartree-Fock correlations on a relativistic Fermi gas basis. Particular emphasis is put on the weak-neutral longitudinal response function, since this observable displays a strong sensitivity to isospin correlations: specifically, it is shown how, through a diagrammatic cancellation/filtration mechanism, this response acts as a magnifier of pionic correlations in the nuclear medium. The parity-violating longitudinal response function also displays appreciable sensitivity to the electric strangeness content of the nucleon, thus making quasielastic electron scattering a possible candidate to measure the nucleon electric strange form factor at relatively high momentum transfers. Finally, we discuss how observables, related to the asymmetry, can be constructed to disentangle the nuclear and nucleonic effects.

1 Introduction

Parity-violating (pv) electron scattering is a promising tool to study the structure of the nucleon, a notable issue being, for instance, the strangeness content (for a general review of pv studies see Ref. [1]). The proton alone, however, is not enough to extract sufficiently constrained information, because of the large number of poorly known form factors entering the game. One is thus led to consider the neutron as well, which in turn implies, generally, the use of nuclei as target. A natural first choice is then elastic scattering off very light nuclei, in order to minimize the nuclear structure uncertainties and to have a nuclear form factor that falls off not too rapidly with the momentum transfer.

At intermediate energies also quasielastic scattering shows promising features, such as large cross sections (being proportional to the number of nucleons) and nuclear form factors slowly decreasing with the momentum transfer. The issue here is, of course, whether one is able to control the nuclear dynamics sufficiently well to extract accurate information on the single nucleon form factors. This issue has been discussed at length in Ref. [3], where it has been shown that observables can be constructed, which are selectively sensitive to the nuclear or nucleonic physics content (more on this in the last Section).

In the following we focus on the pv (or weak-neutral) longitudinal response,
which had been shown to be extraordinarily sensitive to nuclear isospin correlations; this response function also displays an appreciable sensitivity to the nucleon electric strange form factor and, as mentioned above, in the last Section we discuss a possible way to disentangle the two dependences.

Let us start by introducing the asymmetry $A$, defined in terms of the double-differential cross sections, $d^2\sigma^{\pm}/d\Omega' d\Omega$, for longitudinally polarized electrons with helicity $\pm 1$ as:

$$A = \frac{d^2\sigma^+ - d^2\sigma^-}{d^2\sigma^+ + d^2\sigma^-} = \frac{v_L R^L_{AV}(q,\omega) + v_T R^T_{AV}(q,\omega) + v_T' R^T'_{AV}(q,\omega)}{v_L R^L(q,\omega) + v_T R^T(q,\omega)},$$

where $v_L$, $v_T$ and $v_T'$ are kinematical factors and

$$A_0 = \frac{G |Q^2|}{2\pi\alpha \sqrt{2}} \simeq 3.1 \times 10^{-4} \tau$$

sets the scale of the asymmetry; here $\tau = |Q^2|/(4m_N^2) - Q^2 = \omega^2 - q^2$ being the four-momentum transfer, $-\alpha$ is the electromagnetic and $G$ the Fermi coupling constant.

In the denominator of Eq. (1) the standard electromagnetic (em) response functions appear, while the numerator contains the new weak-neutral longitudinal, transverse and axial responses, respectively.

2 Weak-neutral longitudinal response

The peculiar role played by $R^L_{AV}$ can be understood by decomposing the response functions into their isospin components. The em charge response then reads

$$R^L(q,\omega) = R^L(\tau = 0) + R^L(\tau = 1),$$

with the isoscalar and isovector pieces entering with the same sign and the same norm in an independent-particle model (apart from effects due to the nucleon form factors). The weak-neutral longitudinal response, on the other hand, is given by

$$R^L_{AV}(q,\omega) = - \left[ \beta^{(0)}_V R^L(\tau = 0) + \beta^{(1)}_V R^L(\tau = 1) \right],$$

where

$$\beta^{(0)}_V = 1 - 2 \sin^2 \theta_W$$

$$\beta^{(1)}_V = -2 \sin^2 \theta_W.$$
Figure 1: Feynman diagrams representing the free particle-hole polarization propagator for the em (a) and pv (b) longitudinal response. The excitation of proton (p) and neutron (n) particle-hole pairs is shown separately. The labels strong and weak refer to the strength of the nucleon coupling to the photon $\gamma$ or to the vector boson $Z^0$.

Figure 2: Feynman diagrams corresponding to the correlated pv longitudinal response. The meaning of the labels is as in Fig. 1. In (a) one has the HF case, where the bubbles represent the nucleon self-energy; in (b) and (c) the first-order exchange diagrams induced by the exchange of isoscalar and isovector mesons, respectively; in (d) a first-order ring diagram involving the exchange of neutral mesons.
Since \( \sin^2 \theta_W = 0.227 \) (\( \theta_W \) being the Weinberg angle), one has \( \beta_{V}^{(0)} \approx -\beta_{V}^{(1)} \) and one gets a combination of the isospin components that is nearly orthogonal to the one entering Eq. (3); in particular, in an independent-particle model \( R_{AV}^L \) nearly vanishes. It is however clear that any nuclear correlations altering the isoscalar-isovector balance will markedly affect \( R_{AV}^L \).

A more direct picture of why \( R_{AV}^L \) is so small in the independent-particle model and why isospin correlations have dramatic effects (as we shall see quantitatively later on) can be gained looking at the representation of the response function in terms of many-body Feynman diagrams. Indeed, by inspecting Fig. 1 one can easily understand why the uncorrelated em response is expected to be larger than its pv counterpart, since the corresponding polarization propagator can involve two strong \( \gamma p \) vertices (here strong and weak refer to the relative strength of the couplings), while in the other case there must be a weak \( \gamma n \) or \( Z_0^0 p \) vertex (note that the \( Z_0 \) couples strongly to the neutron and weakly to the proton).

Introducing correlations, one sees (Fig. 2) that there are classes of diagrams that cannot change this picture, such as Hartree-Fock (HF) insertions (Fig. 2a) or exchange contributions involving neutral mesons (Fig. 2b). On the other hand, exchange diagrams involving charged mesons (Fig. 2c) or ring diagrams with neutral mesons (unless forbidden by spin selection rules, as in the case of \( \pi^0 \) and \( \rho^0 \) (Fig. 2d) can turn a proton into a neutron: because of the two strong vertices, these contributions will now dominate over the free ones.

### 2.1 Nuclear correlations

The calculations displayed in the following span a rather wide range of transferred momenta. Hence, one has to cope with relativity and we have resorted to the relativistic Fermi gas (RFG) model, with inclusion of fully antisymmetrized random phase approximation (RPA) correlations built on a HF basis (from the discussion above, it should be clear that antisymmetrizing RPA – i. e. including contributions such as those of Fig. 3c – is mandatory). As the input nucleon-nucleon interaction, we have chosen a version of the Bonn potential, which accounts for \( \pi, \rho, \sigma \) and \( \omega \) exchange. We refer the reader to Ref. 6 for a thorough discussion of the model, together with its application to the calculation of the em charge response.

A technical issue one should note here concerns the choice of the Fermi momentum \( k_F \), the only free parameter of the model. In the free Fermi gas, \( k_F \) is usually chosen to reproduce the experimental width of the inclusive \((e, e')\) cross section; when correlations are introduced, the width of the response is modified and one should then change \( k_F \) accordingly. As shown in Ref. 6, one
Figure 3: The pv longitudinal response $R^L_{AV}$ is shown as a function of $\omega$ at $q = 300$, 500, 800 and 1000 MeV/c. The dotted curves correspond to the free RFG calculation with $k_F = 225$ MeV/c, the dashed curves to the HF-RPA calculation with $k_F = 225$ MeV/c, the solid curves to HF-RPA with $k_F = 200$ MeV/c.

Figure 4: The pv longitudinal response versus $\omega$ at $q = 300$, 500, 800 and 1000 MeV/c. The dotted lines correspond to the free RFG case ($k_F = 225$ MeV/c), the solid lines to the HF-RPA calculation ($k_F = 200$ MeV/c); the dashed lines represent the pure ring approximation, whereas the dot-dashed ones the pure exchange contribution.
Figure 5: The pv longitudinal response is shown as a function of $\omega$ at $q = 500$ MeV/c. The dotted curve corresponds to the free RFG case, the solid curve to the HF-RPA approximation with the full Bonn potential, the dashed curve is obtained in the HF-RPA approximation with a pure pion-exchange interaction; $k_F = 225$ MeV/c for the free RFG and $k_F = 200$ MeV/c for the correlated response.

should allow for a moderate reduction of $k_F$: in $^{12}$C, for instance, one has $k_F = 225$ MeV/c in the free model and $k_F \approx 200$ MeV/c in the correlated one. These are the values employed in the following, together with $k_F = 225$ MeV/c in the correlated model, which should be adequate for medium-heavy nuclei and has the purpose of illustrating the dependence on $k_F$.

The results for $R_{L}^{A\nu}$ are displayed in Fig. 3 at various momentum transfers. At moderate momenta one can see the huge effect induced by correlations, which stays sizable even at 1 GeV/c. Note the oscillating behaviour at $q = 500$ MeV/c. As discussed above, not all the correlations contribute equally to this outcome. HF correlations (Fig. 2a) are essentially filtered out, apart from some hardening of the response already observed in $R_{L}^{A\nu}$, and most of the effect enters through RPA.

It is interesting to see how different mesons contribute to RPA: in Fig. 4 the pure ring (direct) and the pure exchange approximations for $R_{L}^{A\nu}$ are compared to the full HF-RPA calculation. Both the ring diagrams — due solely to the $\sigma$ and $\omega$, — and the exchange ones — dominated by the pion (the $\rho$ giving a small contribution), — have a strong impact on $R_{L}^{A\nu}$: however, the full calculation turns out to be rather close to the approximation including only exchange terms. What happens is that the interference of the pion with the $\sigma$ and $\omega$ is washing out their direct contribution; hence, the final result is close to what one should get in a pure pionic model. This outcome is illustrated in Fig. 5, where $R_{L}^{A\nu}$ is displayed at $q = 500$ MeV/c in the full and pion-only
Figure 6: The pv longitudinal response is displayed versus $\omega$ at $q = 300, 500, 800$ and $1000$ MeV/c. The dotted curves refer to the free RFG case ($k_F = 225$ MeV/c), the solid, dot-dashed and dashed curves represent the HF-RPA results ($k_F = 200$ MeV/c) with $G_E(s)$ given by the three models of Eq. (7), respectively.

models.

Note that in Ref. it had been shown that calculations in the pionic model can be adequately performed, over a wide range of momenta, in first order perturbation theory. It is rather amusing that, after the smoke has cleared, all the complexities introduced in the nuclear dynamics conspire to yield a pv longitudinal response described by just one exchange diagram.

2.2 Strangeness

$R_{AV}^{L}$ is, roughly speaking, proportional to the weak-neutral electric form factor $\tilde{G}_E$. The latter, in turn, can be expressed as a combination of the standard proton and neutron electric form factors, with weights dictated by the standard model, plus a possible electric strange form factor, $G_E^{(s)}$, induced by a nonzero strangeness content of the nucleon.

In order to test the sensitivity of $R_{AV}^{L}$ to variations in $G_E^{(s)}$, one has to
choose a parametrization for the latter. We have taken a Galster-like form:

$$G_{E}^{(s)}(\tau) = \rho_{s} \frac{\tau}{[1 + \lambda_{D}^{V}\tau]^{2}} \frac{1}{[1 + \lambda_{E}^{(s)}\tau]}$$

(6)

with $\lambda_{D}^{V} = 4.97$. Three “reasonable” choices for the parameters $\rho_{s}$ and $\lambda_{E}^{(s)}$ have been considered in past work, namely

$$\begin{align*}
\rho_{s} & = 0 \\
\rho_{s} & = -3, \quad \lambda_{E}^{(s)} = 5.6 \\
\rho_{s} & = -3, \quad \lambda_{E}^{(s)} = 0.
\end{align*}$$

(7)

They correspond, respectively, to the absence of strangeness, to the same momentum dependence as the electric neutron form factor and to a pure dipole form factor.

In Fig. 6, the results for $R_{AV}^{L}$ with these three choices for $G_{E}^{(s)}$ are presented. It is apparent that at large momenta, where $G_{E}^{(s)}$ is not suppressed by the factor of $\tau$ in the numerator of (6) (which is dictated by the fact that the total strangeness of the nucleon is zero), $R_{AV}^{L}$ is significantly sensitive to the strength of $G_{E}^{(s)}$.

3 Disentangling nuclear and nucleonic physics

In view of the results discussed in Sec. 2, one may obviously wonder whether it is possible to disentangle the sensitivities to the nuclear dynamics and to the nucleonic structure. This is, of course, a general problem and concerns all the response functions measurable in polarized electron scattering. Furthermore, since, given the present experimental capabilities, there is no way to separate all the responses, one has to tackle this problem through the asymmetry (1). A general procedure, within the RFG scheme, has been devised in Ref. 3 and there applied to the pure pionic model. Calculations in the full meson-exchange model are still in progress; however, pionic dynamics has been shown in Sec. 2 to be adequate for the pv longitudinal channel, whereas correlations in the other channels do not have the same disruptive effects and, moreover, act in the same way in the response functions entering the numerator and the denominator of Eq. (1), thus making the asymmetry an observable much less sensitive to transverse and axial correlations than the responses themselves. For these reasons, the results based on the pionic model should be adequate for the purpose of illustrating the procedure.
Figure 7: (I) The quantity $\Delta A$ and the asymmetry $A$ at $q = 500$ MeV/c; the solid, dashed and dash-dotted lines refer, respectively, to the three models for the electric strangeness of Eq. (7), for the free (RFG) and the pion-correlated ($\pi$) relativistic Fermi gas. (II) The quantity $R$ versus $\theta$ at $q = 300$, 500 and 1000 MeV/c; the curves are labeled as in (I).
To enhance the effect of nuclear correlations, one can introduce the following integrated (over the transferred energy) observable:

$$
\Delta A(q, \theta) = \frac{1}{\Delta \omega} \left[ \int_{\omega_{\min}}^{\omega_{\text{QEP}}} d\omega A(\theta; q, \omega) - \int_{\omega_{\text{QEP}}}^{\omega_{\max}} d\omega A(\theta; q, \omega) \right],
$$

where $\omega_{\min}$ and $\omega_{\max}$ represent the boundaries of the response region and $\omega_{\text{QEP}}$ the quasielastic peak (QEP) energy. Since a sizable fraction of the contribution induced by correlations is antisymmetric around the QEP, whereas variations of the nucleon form factors generate a uniform shift of the asymmetry, $\Delta A$ turns out to be rather unsensitive to modifications of the nucleon structure, as one can see in Fig. 3I, where, for the sake of illustration, the sensitivity of $\Delta A$ to the electric strangeness content of the nucleon is displayed.

Constructing an observable desensitized to the nuclear dynamics is slightly more involved and requires sum rule considerations. In fact, in a relativistic description of nuclear dynamics the nucleon form factors cannot be simply factored out; however, in the RFG one can define reduced response functions — for which the factorization is approximately true — satisfying the sum rule:

$$
S^\alpha(q, \omega) = v_\alpha R^\alpha(q, \omega)/X'_\alpha,
$$

with the dividing factors $X'_\alpha$ given in Ref. 3. One can then define the following observable:

$$
R(q, \theta) = \frac{\int_{\omega_{\min}}^{\omega_{\max}} d\omega W^{PV}(q, \omega)/\tilde{X}'_T}{\int_{\omega_{\min}}^{\omega_{\max}} d\omega W^{EM}(q, \omega)/\tilde{X}'_T},
$$

with $W^{PV} = v_L R^L_{AV} + v_T R^T_{AV} + v_T R^T_{V_A}$ and $W^{EM} = v_L R^L + v_T R^T$. $R$ is displayed in Fig. 3I, with $G_E^{(s)}$ given by the three models of Eq. (7). At high momenta, where the strange form factor is not suppressed by its momentum dependence, $R$ shows a remarkable sensitivity to $G_E^{(s)}$, while being practically independent of nuclear correlations.

Of course, a large amount of work has to be done to sophisticate the nuclear model (accounting, e.g., for meson-exchange currents, short range correlations, 2p-2h, ...) and to test the model dependence of the observables discussed above (on the input nucleon-nucleon interaction, on finite size effects, ...). These preliminary results look, however, promising enough to let one hope that quasielastic scattering may be included in the landscape of future $pv$ experiments.
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