Gravity, duality and conformal symmetry

Chris Hull

The Blackett Laboratory, Imperial College London, Prince Consort Road, London SW7 2AZ, UK

The (4, 0) supermultiplet in six dimensions contains a fourth-rank tensor gauge field with the symmetries of the Riemann tensor and is superconformal, with $32 + 32$ supersymmetries. Dimensional reduction on a circle gives the five dimensions $\mathcal{N} = 8$ supergravity multiplet, with the fourth-rank tensor reducing to the graviton. If there is an interacting (4, 0) theory, it should reduce to the full $\mathcal{N} = 8$ supergravity theory and so would give a conformal theory of gravity that would reduce to conventional gravity with the usual two-derivative action at low energies. This paper revisits the conjecture that a non-lagrangian interacting (4, 0) superconformal theory arises from a strong coupling limit of five-dimensional supergravity (suitably embedded in M-theory) describing M-theory at energies beyond the Planck scale. A key test for this is identified: M-theory toroidally compactified to five dimensions should have certain BPS states carrying a singlet central charge. These $1/2$ BPS states are not related to any of the standard BPS states by dualities and do not correspond to non-singular soliton solutions—they appear to correspond to singular solutions related to gravitational instantons. Such states are needed to provide the Kaluza–Klein modes for the compactified six-dimensional theory. If there are no such states, then the conjecture is false, while the presence of such states would be strong indication that the conjecture could be true.

1. Symmetry beyond the Planck scale

A lot of progress has been made in understanding quantum gravity at low energies and small curvatures, but what happens at the Planck scale and beyond remains something of a mystery. It is possible that a
radically different theory could emerge at the Planck scale and there have been many speculations as to what might happen there.

One possibility is that, as one tries to probe distance scales smaller than the Planck length, one could find that space–time is no longer a continuum at such scales and there is some discrete structure instead—perhaps a matrix model or some form of lattice theory.

Another speculation is that there could be a new highly symmetric phase emerging at the Planck scale. An attractive proposal is that the fundamental theory should be conformally invariant with no dimensionful parameters, with the Planck scale arising via symmetry breaking—see [1] for a recent discussion of this idea. A major obstacle to this is that the conformal theories of a graviton field that we know of are all higher derivative, so that although they have good ultraviolet properties, they have ghosts and it is difficult to recover Einstein’s theory from them. In string theory, it has been proposed that at trans-Planckian energies, there is a highly symmetric phase with an infinite number of symmetries associated with an infinite number of gauge fields becoming massless in some tensionless string limit. Evidence for such a picture was found in high-energy scattering amplitudes by Gross and Mende [2,3]. More recently, it has been shown that for string theory in anti-de Sitter space, tensionless strings arise when the curvature approaches a critical value at the string scale giving an infinite set of symmetries and signalling a transition to a new phase [4–8].

The standard picture of quantum gravity is that it arises from the quantization of a massless spin two field, and this field is then interpreted as the metric governing the Riemannian geometry of space–time. However, it is interesting to ask whether a massless spin two field provides the only way of formulating gravity. At low energies, we would require that the theory can be viewed as a theory of a graviton. However, this theory could have an ultraviolet origin that is a theory of another field, perhaps in higher dimensions.

This paper will revisit the conjecture of [9] in which it was proposed that gravity could be conformally invariant but formulated not in terms of the familiar spin-two graviton field but in terms of something more exotic. In the free limit, the theory of [9] is formulated in terms of a four index gauge field \( C_{MNJPQ} \) (with the symmetries of the Riemann tensor), but on compactification on a circle, a conventional ghost-free theory of a graviton emerges at scales that are small compared to the compactification scale, which also sets the Planck scale in the compactified theory. The outstanding question is whether there could be an interacting version of this theory that reduces to general relativity or supergravity.

This conjecture arose from supersymmetry. There is a remarkable supermultiplet in six dimensions with \((4, 0)\) supersymmetry which is fact a superconformal multiplet so that the theory has 32 normal supersymmetries plus 32 conformal supersymmetries. It has the gauge field \( C_{MNJPQ} \) together with 27 self-dual 2-forms and 42 scalars. Reducing on a circle gives five-dimensional \( N = 8 \) supergravity. It exists as a free theory, but again it is not known whether there is an interacting version. If there is, it would be expected to be a non-lagrangian theory of the kind arising in the \((2, 0)\) theory that arises as the world-volume theory of the M5-brane. An interacting theory would be very interesting, giving a conformal theory of gravity that reduces to a conventional theory of gravity at low energies. Little progress has been made on this proposal since it was made in 2000, partly because it was waiting for progress in understanding the \((2, 0)\) theory. Now much more is known about the \((2, 0)\) theory, it is perhaps time to revisit this conjecture. In this paper, the \((4, 0)\) theory will be reviewed following [9–12] and the conjectures concerning it [9] will be re-examined.

2. The six-dimensional \((2, 0)\) theory and five-dimensional Yang–Mills

There is considerable evidence that there exists an interacting \((2, 0)\) non-lagrangian superconformal field theory (SCFT) in six space–time dimensions [13–15]. Moreover, this provides a key to understanding strongly coupled maximally supersymmetric Yang-Mills (SYM) theory in \( D = 5 \) [16–19] and S-duality in \( D = 4 \) [16,20].
The free (2, 0) supersymmetric theory in six dimensions has a 2-form gauge field $B$ whose field strength $H = dB$ is self-dual, $H = *H$. The other fields are five scalars transforming as a 5 under the $R$-symmetry group $\text{Sp}(2)$ and four symplectic Majorana–Weyl fermions transforming as a 4 under $\text{Sp}(2)$.

It is known that there is a quantum interacting (2, 0) SCFT, but it is not clear how to formulate the interacting theory. It is clearly not a conventional theory of gauge fields—it is said to be non-lagrangian—but nonetheless it is known that its dimensional reduction to five or fewer dimensions does give a Yang–Mills gauge theory. Dimensional reduction of the free (2, 0) theory on a circle gives five-dimensional $N=4$ abelian gauge theory with $B$ giving rise to the five-dimensional vector potential $A$. The interacting (2, 0) SCFT is non-lagrangian but it reduces to a conventional field theory, five-dimensional $N=4$ SYM.

The (2, 0) theory arises as the strong coupling limit of five-dimensional SYM [16–19]. The interacting (2, 0) theory compactified on a circle of radius $R$ gives five-dimensional SYM with coupling constant $g_{\text{YM}}$ with $g_{\text{YM}}^2 = R$, so that as $g_{\text{YM}} \to \infty$ the radius $R \to \infty$ and at strong coupling the circle decompactifies to give a six-dimensional theory. The five-dimensional SYM theory is non-renormalizable and is known to have divergences [21] so here it is regarded as being embedded in some UV complete theory, such as string theory. The (2, 0) theory also arises as a decoupling limit of the world-volume theory of a stack of $M5$-branes and in IIB string theory compactified on K3.

The five-dimensional SYM theory has BPS 0-branes given by lifting self-dual Yang–Mills instantons in four Euclidean dimensions to soliton world-lines in $4 + 1$ dimensions, taking the product of the four-dimensional instanton solution with a timelike line. These solitons have mass $M \propto |n|/g_{\text{YM}}^2$ where $n$ is the instanton number, so that these become light as the dimensionful Yang–Mills coupling $g_{\text{YM}}$ becomes large. These solitons should be interpreted as Kaluza–Klein modes for a $D=6$ theory compactified on a circle of radius $R = g_{\text{YM}}^2$ so that in the strong coupling limit an extra dimension opens up to give a six-dimensional (2, 0) supersymmetric theory [16]. In the abelian theory, the five-dimensional vector gauge field is replaced by a six-dimensional 2-form gauge field $B_{MN}$ with self-dual field strength, and the five scalar fields are all promoted to scalar fields in six dimensions.

The $D=6$ theory is believed to be a non-trivial superconformally invariant quantum theory [14] and the $D=5$ gauge coupling arises as the radius of the compactification circle, $g_{\text{YM}}^2 = R$ [16]. The relationship between the $D=5$ and $D=6$ theories is straightforward to establish for the free case in which the Yang–Mills gauge group is abelian, but in the interacting theory the six-dimensional origin of the $D=5$ non-abelian interactions is mysterious; there are certainly no local covariant interactions that can be written down that give Yang–Mills interactions when dimensionally reduced [22]. Nonetheless, the fact that these $D=5$ and $D=6$ theories arise as the world volume theories of $D4$ and $M5$ branes, respectively, gives strong support for the existence of such a six-dimensional origin for the gauge interactions.

The $W$-bosons and magnetic strings in $D=5$ arise from self-dual strings in $D=6$. At the origin of moduli space, the $W$-bosons become massless and the tensions of the self-dual strings in $D=6$ must also become zero. The nature of the theory at such points is unclear. Nonetheless, given that mysterious interactions with no conventional field theory formulation arise in the $M5$-brane world-volume theory, it is natural to seek similar unconventional interactions elsewhere in $M$-theory.

### 3. The six-dimensional (4, 0) theory and five-dimensional supergravity

Given the close relationship between gauge theory and gravity, it is natural to ask whether there could be a story for gravity similar to that of the relation between the six-dimensional (2, 0) theory and five-dimensional SYM. Remarkably, there is a free SCFT with (4, 0) supersymmetry in six dimensions, and if this has an extension to an interacting theory, it would provide an exotic conformal theory giving supergravity in $D < 6$ and could arise as a strong coupling limit of five-dimensional supergravity.
Five-dimensional $N = 8$ supergravity (ungauged) has a global $E_6$ symmetry and a local $Sp(4) = USp(8)$-$R$-symmetry. It is non-renormalizable, and will be regarded as arising as a massless sector of some consistent theory, such as $M$-theory compactified on a six-torus, in which the global $E_6$ symmetry is broken to a discrete $U$-duality subgroup [23]. The massless bosonic fields consist of a graviton, 27 abelian vector fields and 42 scalars. The dimensional coupling constant is the five-dimensional Planck length $l$, so that strong gravitational coupling is the limit as $l \to \infty$. For such a limit, it is natural to look first for a theory with 32 supersymmetries and $Sp(4)$-$R$-symmetry and to expect that BPS states are protected and survive as the coupling $l$ is increased.

The multiplet has $(4, 0)$ supersymmetry in $D = 6$ and was studied in [9]. The particle content was identified in [24] and in [9] the covariant field content and gauge symmetry was found and further studied in [10–12]. Actions for the free theory have been considered in [25–27] and further discussion of the theory can be found in [28–32].

Instead of a graviton, it has an exotic fourth-rank tensor gauge field $C_{MNPQ}$ with the algebraic properties of the Riemann tensor

$$C_{MNPQ} = -C_{NMPQ} = -C_{MNQP} = C_{PNMQ}$$

and the gauge symmetry

$$\delta C_{MNPQ} = \partial_M \chi_{NPQ} + \partial_P \chi_{QM} - 2\partial_Q \chi_{MPN}$$

with parameter $\chi_{MNPQ} = -\chi_{MPNQ}$. The invariant field strength is

$$G_{MNPQRS} = \frac{1}{36} (\partial_M \partial_S C_{NPQR} + \cdots) = \partial_M C_{NP][QRS]$$

and in the $(4, 0)$ multiplet it satisfies the self-duality constraint

$$G_{MNPQRS} = \frac{1}{6} \epsilon_{MNPQRSTUV} C_{TUVQRST}$$

or $G = *G$. In addition, there are 27 2-form gauge-fields with self-dual 3-form field strengths, 42 scalars, 48 symplectic Majorana–Weyl fermions and, instead of gravitini, eight spinor-valued 2-forms $\psi^a_{MN}$ which satisfy a symplectic Majorana–Weyl constraint and have self-dual field strengths. The fermionic gauge symmetry is of the form

$$\delta \psi^a_{MN} = \partial_M \epsilon_N^a$$

with parameter a spinor-vector $\epsilon_N^a$. The free theory based on this multiplet is a superconformally invariant theory, with conformal supergroup $OSp^*(8/8)$ [9,33]. This has bosonic subgroup $USp(8) \times SO^*(8) = Sp(4) \times SO(6,2)$ and 64 fermionic generators, consisting of the 32 supersymmetries of the $(4, 0)$ superalgebra and 32 conformal supersymmetries.

It is remarkable that, in going from $D = 5$ to $D = 6$ in the free theory, the vector gauge fields $A_{\mu}$ are lifted to 2-forms $B_{MN}$, the gravitini $\psi_{\mu}$ are lifted to spinor-valued 2-forms $\psi_{MN}$ and the graviton $h_{\mu \nu}$ is lifted to the gauge field $C_{MNPQ}$, with these $D = 6$ gauge-fields all satisfying self-duality constraints. Electrically charged 0-branes and magnetic strings in $D = 5$ lift to BPS self-dual strings in $D = 6$. In [9], it was shown that the dimensional reduction of the free $(4, 0)$ theory on a circle indeed gives the linearized $D = 5, N = 8$ supergravity theory, with gravitational coupling (Planck length) given by the circle radius $l = R$. However, there are no covariant local interactions in $D = 6$ for this multiplet that could give rise to the $D = 5$ supergravity interactions.

There is then a close analogy between the $D = 5, N = 4$ Yang–Mills theory and the $D = 5, N = 8$ supergravity. The linearized versions of these theories both arise from the dimensional reduction of a free SCFT in $D = 6$, with the dimensional $D = 5$ coupling constant arising from the radius of compactification, so that the strong coupling limit of the $D = 5$ free theories is a decompactification to $D = 6$. For the interacting $D = 5$ Yang–Mills theory, there are a number of arguments to support the conjecture that its strong coupling limit should be an interacting superconformal theory in $D = 6$ with $(2,0)$ supersymmetry, even though such a theory has not been constructed directly and indeed cannot have a conventional field theory formulation. This led to the conjecture of [9] that
the situation for $D = 5$ supergravity is similar to that of $D = 5$ super-Yang–Mills, and that a certain strong coupling limit of the interacting supergravity theory should give an interacting theory whose free limit is the $(4,0)$ theory in $D = 6$. In this case, there is no analogue of the M5-brane argument to support this, although the M5-brane case does set a suggestive precedent.

The limiting theory should have some novel form of interactions which give the non-polynomial supergravity interactions on reduction. It could be that there are some non-local or non-covariant self-interactions of the $(4,0)$ multiplet, or it could be that other degrees of freedom might be needed; one candidate might be some form of string field theory. However, if a strong coupling limit of the theory does exist that meets the requirements assumed here, then the limit must be a $(4,0)$ theory in six dimensions, and this would predict the existence of interactions arising from the strong coupling limit of the supergravity interactions. Although it has not been possible to prove the existence of such a limit, it is remarkable that there is such a simple candidate theory for the limit with so many properties in common with the $(2,0)$ limit of the $D = 5$ gauge theory. Conversely, if there is a six-dimensional phase of $M$-theory which has $(4,0)$ supersymmetry, then its circle reduction to $D = 5$ must give an $N = 8$ supersymmetric theory and the scenario described here should apply.

In the context in which the $D = 5$ supergravity is the massless sector of $M$-theory on $T^6$, then the $D = 6$ superconformal theory would be a field theory sector of a new six-dimensional superconformal phase of $M$-theory. This could tell us a great deal about $M$-theory: it would be perhaps the most symmetric phase of $M$-theory so far found, with a huge amount of unbroken gauge symmetry, and would be a phase that is not well described by a conventional field theory at low energies, so that it could give new insights into the degrees of freedom of $M$-theory.

4. The superalgebra and BPS states

The five-dimensional $N = 2n$ superalgebra with automorphism group $Sp(n)$ is

$$\left\{ Q^a_{\alpha}, Q^b_{\beta} \right\} = \Omega^{ab}_{\alpha\beta}(\Gamma^{\mu}_{\alpha}\omega P_{\mu} + \Omega^{ab}_{\alpha\beta}C_{\alpha\beta}K + (\Gamma^{\mu}_{\alpha}\omega Z_{ab}^{\mu} + C_{\alpha\beta}Z_{\mu}^{ab} + \frac{1}{2}(\Gamma^{\mu\nu}\omega)_{\alpha\beta}Z_{\mu\nu}^{ab}) \right\} \tag{4.1}$$

where $\mu, \nu, \ldots, 4$ are space–time indices, $\alpha = 1, \ldots, 4$ are spinor indices, $a = 1, \ldots, N$ are $Sp(n)$ indices, $C_{\alpha\beta}$ is the charge conjugation matrix and $\Omega^{ab}_{\alpha\beta}$ is the symplectic invariant of $Sp(n)$. The supercharges $Q^a_{\alpha}$ are symplectic Majorana spinors satisfying

$$\langle Q^a_{\alpha} | = C_{\alpha\beta} \Omega_{ab}Q^b_{\beta} \rangle.$$

For the $D = 5$, $N = 4$ super-Yang–Mills theory, the charges can be identified as follows [18]. The five central charges $Z^{ab} = -Z^{ba}$ with $\Omega^{ab}_{\alpha\beta}Z^{ab} = 0$ are proportional to the five electric charges $\phi^j \propto \int tr(\ast F \phi^j)$ and are carried by massive vector multiplets. There is no vector field coupling to the singlet central charge $K$, which is a topological charge carried by the instantonic 0-branes and is proportional to the instanton number. There is a corresponding conserved topological current

$$j = \ast tr(F \wedge F). \tag{4.2}$$

The spatial components $Z^{ab}_{ij}$ $(i,j = 1, \ldots, 4)$ of $Z^{ab}_{i}$ are the magnetic charges carried by the magnetically charged strings. The other charges on the right-hand-side of (4.1) are 2, 3 and 4-brane charges. For example, a vortex solution in $2 + 1$ dimensions lifts to a 2-brane in $4 + 1$ dimensions with charge $Z^4_{ij}$.

The $N = 8$ superalgebra has automorphism group $Sp(4)$ and is given by (4.1) with $a, b = 1, \ldots, 8$. The algebra with scalar central charges only is

$$\left\{ Q^a_{\alpha}, Q^b_{\beta} \right\} = \Omega^{ab}_{\alpha\beta}(\Gamma^{\mu}_{\alpha}\omega P_{\mu} + C_{\alpha\beta}(Z^{ab}_{\mu} + \Omega^{ab}_{\alpha\beta}K) \right\}. \tag{4.3}$$

The 27 central charges $Z^{ab}_{} = -Z^{ba}_{}, \Omega^{ab}_{\alpha\beta}Z^{ab}_{\mu} = 0$, and are the electric charges for the 27 vector fields (dressed with scalars). There are 28 central charges but only 27 vector fields, and $K$
is not the conserved charge for any gauge field. However, on dimensional reduction from five to four dimensions, $K$ becomes one of the 28 magnetic charges of $D = 4, N = 8$ supergravity, and is the one coupling to the gravi-photon field (i.e. the electromagnetic field from the dimensional reduction of the metric) so that it is the Kaluza–Klein monopole charge. $U$-duality requires that $1/2$-supersymmetric states with $M = |K|$ occur in the $D = 4$ BPS spectrum, and $K$ is quantized. A key question is whether there are $1/2$-supersymmetric states in $D = 5$ carrying the central charge $K$ with $M = |K|$. Presumably $K$ should be quantized, so that $K \propto n/l$ where $n$ is an integer. In any case, there is a conserved current analogous to (4.2) given by
\[ j = \ast \text{tr}(R \wedge R). \] (4.4)

The $U(1)$ theory has 27 self-dual 2-forms and these couple to 27 self-dual BPS strings, or more precisely, to strings whose charges take values in a 27-dimensional lattice. In addition, there are 42 scalars and these can couple to BPS 3-branes in six dimensions.

The $U(1)$ superalgebra in six dimensions with central charges is
\[ \{ Q^a_\alpha, Q^b_\beta \} = \Omega_{ab} \left( \Pi_+ G^MC_{a\alpha} + \frac{1}{6} \left( \Pi_+ G^{MNP} C_{a\alpha} \right) Z_{MNP}^{ab} \right), \] (4.5)
where $\Pi_\pm$ is the chiral projector
\[ \Pi_\pm = \frac{1}{2} (1 \pm \Gamma_7). \]
The 27 one-form charges satisfy $Z_{MNP}^{ab} = -Z_{MNP}^{ba}$, while $Z_{ab}^{MNP} = Z_{ab}^{MNP}$ and is a self-dual 3-form,
\[ Z_{MNP}^{ab} = \frac{1}{6} \epsilon_{MNPQRS} Z_{QR}^{ab}. \] (4.6)
The 27 charges $Z_{ij}^{ab}$ with spatial indices $i, j = 1, \ldots, 5$ are the string charges, and $Z_{ijk}^{ab}$ are the 36 3-brane charges, while $Z_{0}^{ab}$ are the charges for space-filling 5-branes. (Note that the self-duality condition (4.6) implies that $Z_{ij}^{ab}$ are not independent charges.)

On dimensional reduction to five dimensions, the charges decompose as [10]
\[ \begin{align*}
Z_{ab}^{M} &\rightarrow (Z_{ab}^{M}, Z_{5}^{ab} = Z_{ab}), \\
P^{M} &\rightarrow (P^{\mu}, P^{5} = K), \\
Z_{MNP}^{ab} &\rightarrow Z_{\mu\nu5}^{ab} = Z_{\mu\nu}^{ab}.
\end{align*} \] (4.7)
The extra component of momentum becomes the $K$-charge, so that the five-dimensional instantonic 0-brane is, from the $D = 6$ viewpoint, a wave carrying momentum $P^{5}$ in the extra circular dimension. The electric 0-branes in $D = 5$ arise from strings winding around the 6th dimension while the magnetic strings arise from unwrapped $D = 6$ strings. The 2-branes and 3-branes in $D = 5$ arise from 3-branes in $D = 6$, while the space-filling $D = 5$ 4-branes come from wrapped $D = 6$ 5-branes.

5. Strong coupling limits and solitons

The five-dimensional SYM theory has BPS solitons carrying the central charge $K$ in the superalgebra (4.1). These arise from self-dual or anti-self-dual YM instantons in 4 Euclidean dimensions lifted to 0-branes in five dimensions by taking a product of the four-dimensional instanton with time. They have mass
\[ M \propto \frac{|n|}{g_{\text{SYM}}^{2}}, \] (5.1)
where $n$ is the instanton number, so that these become light as the dimensionful Yang–Mills coupling $g_{\text{SYM}}$ becomes large. These are interpreted as Kaluza–Klein modes for a $D = 6$ theory compactified on a circle of radius $R = g_{\text{YM}}^{-2}$, so that the strong coupling limit is a decompactification limit $R \rightarrow \infty$. The current $j = \ast \text{tr}(F \wedge F)$ represents the density of these solitons. The BPS states
carrying the charge $K$ fit into massive five-dimensional supermultiplets with precisely the right structure to be the KK modes for a $(2, 0)$ theory [34]. In particular, they have massive self-dual 2-form fields $B$ with mass $m$ satisfying

$$dB = \pm m \ast B,$$  

(5.2)

together with massive fermions and scalars.

The $D = 5$ SYM is non-renormalizable but a UV completion can be defined within string theory, e.g. the $D4$-brane theory. The strong coupling limit is then defined within string theory, e.g. the strong coupling limit of multiple $D4$ branes gives multiple $M5$ branes.

As the coupling $g_{YM}$ is dimensionful, the limit is better expressed as a high energy limit, going to energies $E$ large compared to the YM scale,

$$E \gg \frac{1}{g_{YM}},$$  

(5.3)

so that at ultra-high energies the extra circle dimension becomes observable.

In the free case, the dimensional reduction can be done explicitly and the free $(2, 0)$ theory reduced on a circle gives abelian SYM in five dimensions. However, in the abelian case, there are no non-singular instantons and so no solitonic 0-branes, but the Kaluza Klein modes can be associated with singular zero-size instantons. A similar issue arises for the non-abelian SYM with the gauge symmetry spontaneously broken to an abelian subgroup, with the instantonic solitons shrinking to zero size as the Higgs expectation value is turned on. However, in this case, the soliton can be stabilized by turning on an electric charge to give a regular BPS soliton solution [35], so that there should be BPS 0-branes in the five-dimensional theory even when the symmetry becomes abelian.

The free $(4, 0)$ theory compactified on a circle of radius $R$ gives five-dimensional linearized supergravity with five-dimensional Planck length $l = R$ plus Kaluza–Klein modes with mass

$$M \sim \frac{|n|}{R} = \frac{|n|}{l},$$  

(5.4)

for integers $n$.

For the interacting five-dimensional $N = 8$ supergravity theory (embedded in string theory to give a UV completion, as in $M$-theory compactified on a 6-torus), a necessary condition for it to decompactify to a six-dimensional $(4, 0)$ theory is that it should have 1/2-BPS states carrying the charge $K$ with a spectrum of the form (5.4). From [34], the 1/2-BPS states carrying $K$ fit into short five-dimensional multiplets whose highest spin field is a massive self-dual field $C_{\mu
u\rho\sigma}$ satisfying

$$\partial_{[\mu}C_{\nu\rho]\sigma\tau} = \pm \frac{1}{2}me^{ab}_{\mu\nu}C_{a\beta\sigma},$$  

(5.5)

as required for the KK modes of $C_{MNPO}$.

The key question is then whether there are BPS states carrying the charge $K$, and if so, what is their spectrum. This then provides a crucial test for the conjecture that there is a six dimensions $(4, 0)$ theory arising as a limit of $M$-theory. $M$-theory toroidally compactified to five dimensions should have certain BPS states carrying the singlet central charge $K$. If there are no such states, then the conjecture is false, while the presence of such states would be strong indication that the conjecture could be true.

Independent of this conjecture, it is interesting in any case to ask whether there are BPS states carrying the charge $K$. All other central charges in the five-dimensional superalgebra are carried by BPS states and so it would be strange if there were none carrying $K$. The charge $K$ is a singlet under the $E_6(\mathbb{Z})$ $U$-duality and the $Sp(4)$ $R$-symmetry so such BPS states would not be related to any of the standard BPS branes by duality.

The explicit form for $K$ can be found from the superalgebra and was given in [36]. If the five-dimensional theory is compactified on a circle to four dimensions, then there are BPS states carrying $K$ which are the KK monopoles, with $K$ related to the NUT charge and hence the four-dimensional magnetic charge [36].
Just as for five-dimensional SYM, for supergravity taking the product of a self-dual gravitational instanton in four Euclidean dimensions with a timelike line gives a 1/2-supersymmetric configuration associated with the charge $K$. It would be desirable to have such a solution which is asymptotic to five-dimensional Minkowski space but unfortunately there are no such non-trivial non-singular solutions. There are no asymptotically Euclidean gravitational instantons but there are ALE ones which are asymptotic to a discrete quotient of four-dimensional Euclidean space (see e.g. [37]). Thus there are no regular solitons with the desired asymptotics. However, this does not preclude the existence of BPS states of this kind: they could be associated with singular solutions or zero-size gravitational instantons. The role of gravitational instantons is further suggested by the topological symmetry associated with the current $j = \ast \text{tr}(R \wedge R)$ associated with the gravitational instanton density. It is interesting that there remains a possibility of further BPS states in $M$-theory that are not dual to any of the known branes and which correspond to singular supergravity solutions.

6. Gravity and geometry

Particularly intriguing are the consequences for gravity. In $D=5$, gravity is described geometrically in terms of a metric $g_{\mu\nu}$, but at strong coupling it is described instead in terms of the gauge field $C_{MN\tilde{P}Q}$ in $D=6$ (at least in the free case). This suggests the possibility of some new structure which reduces to Riemannian geometry but which is more general and is the appropriate language for describing gravity beyond the Planck scale. For example, while $g_{\mu\nu}$ provides a norm for vectors and a notion of length, $C_{MN\tilde{P}Q}$ could provide a norm $C_{MN\tilde{P}Q}\omega^{M\tilde{P}}\omega^{N\tilde{Q}}$ for 2-forms $\omega^{MN}$ and hence gives a notion of area that is not derived from a concept of length [11]. It may be that the interacting theory is not described in conventional $D=6$ space–time at all, but in some other arena.

There seem to be three main possibilities. The first is that there is no interacting version of the $(4,0)$ theory, that it only exists as a free theory, and that the limit proposed in [9] only exists for the free $D=5$ theory. The second is that an interacting form of the theory does exist in six space–time dimensions, with $D=6$ diffeomorphism symmetry. The absence of a space–time metric means that such a generally covariant theory would be of an unusual kind. The third and perhaps the most interesting possibility is that the theory that reduces to the interacting supergravity in $D=5$ is not a diffeomorphism-invariant theory in six space–time dimensions, but is something more exotic.

For gravity in any dimension, the full nonlinear gauge symmetry is

$$\delta g_{\mu\nu} = 2\nabla(\xi \delta) .$$

(6.1)

If the metric is written as

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu},$$

(6.2)

in terms of a fluctuation $h_{\mu\nu}$ about some background metric $\tilde{g}_{\mu\nu}$ (e.g. a flat background metric) then two main types of symmetry emerge. The first consists of ‘background reparameterizations’

$$\delta \tilde{g}_{\mu\nu} = 2\nabla(\xi \delta) \quad \text{and} \quad \delta h_{\mu\nu} = \mathcal{L}_\xi h_{\mu\nu},$$

(6.3)

where $\nabla$ is the background covariant derivative with connection constructed from $\tilde{g}_{\mu\nu}$, while $h_{\mu\nu}$ transforms as a tensor ($\mathcal{L}_\xi$ is the Lie derivative with respect to the vector field $\xi$), as do all other covariant fields. The second is the ‘gauge symmetry’ of the form

$$\delta \tilde{g}_{\mu\nu} = 0 \quad \text{and} \quad \delta h_{\mu\nu} = 2\nabla(\xi \delta)$$

(6.4)

in which $h_{\mu\nu}$ transforms as a gauge field and the background is invariant. There is in addition the standard shift symmetry under which

$$\delta \tilde{g}_{\mu\nu} = \alpha_{\mu\nu} \quad \text{and} \quad \delta h_{\mu\nu} = -\alpha_{\mu\nu}.$$
In terms of the full metric $g_{\mu\nu}$, there is no shift symmetry and a unique gauge symmetry (6.1); the various types of symmetry (6.3)–(6.5) are an artifice of the background split. The shift symmetry is a signal of background independence and plays an important role in the interacting theory.

The linearized $D = 5$ supergravity theory has both background reparameterization and gauge invariances given by the linearized forms of (6.3) and (6.4), respectively, and both of these have origins in $D = 6$ symmetries of the free $(4,0)$ theory. The background reparameterization invariance lifts to the linearized $D = 6$ background reparameterization invariance

$$\delta \bar{g}_{MN} = 2\partial(M\bar{\xi}_N) \quad \text{and} \quad \delta C_{MNPQ} = \mathcal{L}_\xi C_{MNPQ}$$

with the transformations leaving the flat background metric $\bar{g}_{MN}$ invariant forming the $D = 6$ Poincaré group. The $D = 5$ gauge symmetry given by the linearized form of (6.4) arises from the $D = 6$ gauge symmetry (3.2) with $\delta \bar{g}_{MN} = 0$ and the parameters related by $\zeta^\mu = \chi^{55\mu}$. The $D = 6$ theory has no analogue of the shift symmetry, and the emergence of that symmetry on reduction to $D = 5$ and dualizing to formulate the theory in terms of a graviton $h_{\mu\nu}$ comes as a surprise from this viewpoint.

The gravitational interactions of the full supergravity theory in $D = 5$ are best expressed geometrically in terms of the total metric $g_{\mu\nu}$. If an interacting form of the $(4,0)$ theory exists that reduces to the $D = 5$ supergravity, it must be of an unusual kind. One possibility is that there is no background metric of any kind in $D = 6$, and the full theory is formulated in terms of a total field corresponding to $C$, with a space–time metric emerging only in a particular background $C$ field and a particular limit corresponding to the free theory limit in $D = 5$.

It is not even clear that the interacting theory should be formulated in a $D = 6$ space–time. In $D = 5$, the diffeomorphisms act on the coordinates as

$$\delta x^\mu = \xi^\mu.$$  

In the $(4,0)$ theory, the parameter $\xi^\mu$ lifts to a parameter $\chi^{MNP}$. If the coordinate transformations were to lift, it could be to something like a manifold with coordinates $X^{MNP}$ transforming through reparameterizations

$$\delta X^{MNP} = \chi^{MNP},$$

with the $D = 5$ space–time arising as a submanifold with $x^\mu = X^{55\mu}$. Another possibility is as follows. The diffeomorphism $\delta x^\mu = \xi^\mu$ gives a gauge transformation for the graviton

$$\delta g_{\mu\nu} = \partial(\mu \xi^\nu) + \cdots$$

with the index on the parameter lowered with the metric

$$\xi_\mu = \xi^\nu g_{\mu\nu}.$$  

The parameter of the gauge transformation

$$\delta C_{MNPQ} = \partial(MX_{NPQ} + \partial[P X_{QMN} - 2\partial[M X_{NPQ}]}$$

could be related to that of a six-dimensional diffeomorphism

$$\delta X^M = \xi^M$$

by

$$\chi^{MNP} = \xi^Q C_{MNPQ}$$

so that the gauge symmetry could after all be that of conventional six-dimensional diffeomorphisms.\(^1\)

Similar considerations apply to the local supersymmetry transformations. In $D = 5$, the local supersymmetry transformations in a supergravity background give rise to ‘background supersymmetry transformations’ with symplectic Majorana spinor parameters $\epsilon^{\alpha a}$ (where $\alpha$ is a $D = 5$ spinor index and $a = 1,\ldots, 8$ labels the eight supersymmetries) in which the gravitino

\(^1\)I would like to thank Paul de Medeiros for discussions about this suggestion.
fluctuation $\psi^a_\mu$ transforms without a derivative of $\varepsilon^a$, and ‘gauge supersymmetries’ with spinor parameter $\varepsilon^{a\mu}$ under which

$$\delta\psi^a_\mu = \partial_\mu \varepsilon^a + \cdots$$

The background symmetries preserving a flat space background form the $D=5$ super-Poincaré group. In the free theory, the $D=5$ super-Poincaré symmetry lifts to part of a $D=6$ super-Poincaré symmetry with $D=5$ translation parameters $\xi^\mu$ lifting to $D=6$ ones $\xi^M$ and supersymmetry parameters $\varepsilon$ lifting to $D=6$ spinor parameters $\hat{\varepsilon}$. The corresponding $D=6$ supersymmetry charges $Q$ and momenta $P$ are generators of the $(4,0)$ super-Poincaré algebra with

$$\left\{ Q^a_\alpha, Q^b_\beta \right\} = \Omega^{ab} \left( \Pi_+ \Gamma^M C \right)_{a\beta} P_M$$

where $\Pi_\pm$ are the chiral projectors

$$\Pi_\pm = \frac{1}{2} (1 \pm \Gamma^7)$$

$\alpha, \beta$ are $D=6$ spinor indices and $a, b = 1, \ldots, 8$ are USp(8) indices, with $\Omega^{ab}$ the USp(8)-invariant anti-symmetric tensor. This is in turn part of the $D=6$ superconformal group OSp*(8/8).

The $D=5$ gauge symmetries including those with parameters $\xi^\mu, \varepsilon^a$ satisfy a local algebra whose global limit is the $D=5$ Poincaré algebra, but the $D=6$ origin of this (at least in the free theory) is an algebra including the generators $Q^a_\alpha$ of the fermionic symmetries with parameter $\varepsilon^a_N$ and the generators $P^{5M}$ of the bosonic symmetries with parameter $\chi^{MN}$. The global algebra is of the form

$$\left\{ Q^a_\alpha, Q^b_\beta \right\} = \Omega^{ab} \left( \Pi_+ \Gamma^M C \right)_{a\beta} P_{MN}$$

In the dimensional reduction, the $D=5$ superalgebra has charges $Q^a_\alpha = Q^a_{\alpha 5}, P_\mu = P_{5\mu}$.

Supersymmetry provides a further argument against the possibility of a background metric playing any role in an interacting $(4,0)$ theory in $D=6$. The $D=5$ supergravity can be formulated in an arbitrary supergravity background, but these cannot be lifted to $D=6$ $(4,0)$ backgrounds involving a background metric as there is no $(4,0)$ multiplet including a metric or graviton. The absence of a $(4,0)$ supergravity multiplet makes problematic the possibility of a background metric and the standard supersymmetry playing any role in the $D=6$ theory. Indeed, the interacting theory (if it exists) should perhaps be a theory based on something like the algebra (6.15) rather than the super-Poincaré algebra (6.14).

7. Double copy

It is intriguing that there is a sense in which gravity can be regarded as a ‘square’ of Yang–Mills theory and supergravity can be regarded as a ‘square’ of super-Yang–Mills theory. At the level of free theories, there is a direct construction of supergravity as a square of SYM, with the supergravity supermultiplet arising from the tensor product of two SYM multiplets [38–40]. In the same way, the free $(4,0)$ theory is a square of the free $(2,0)$ theory [33,46].

Remarkably, for the interacting theories four-dimensional and five-dimensional supergravity scattering amplitudes can be constructed from ‘squaring’ four-dimensional and five-dimensional SYM scattering amplitudes in the double copy construction [42]. This suggests another approach to seeking an interacting $(4,0)$ theory. Given amplitudes for the $(2,0)$ theory, one could seek to apply the double copy construction to obtain $(4,0)$ amplitudes. After all, on circle compactification, the $(2,0)$ amplitudes should reduce to five-dimensional SYM amplitudes and the $(4,0)$ ones should reduce to five-dimensional supergravity ones, which should in turn be the double copy of the five-dimensional SYM amplitudes. This approach has been attempted in [43–45].

A major problem with this approach is that we do not know the amplitudes for the $(2,0)$ theory even at tree level. In [43–45], formal expressions were derived that had many of the properties that would be required of $(2,0)$ amplitudes and which reduced to five-dimensional SYM amplitudes, but they had a number of problems. One was that they could only be derived for even numbers
of external states. Another was that the poles had problematic non-local residues. As such, the interpretation of these expressions is unclear. Nonetheless, the double copy can be applied to these to give formal expressions for the (4, 0) theory, but again it is unclear as to how they might be interpreted and whether they can be thought of as scattering amplitudes.

However, as the (2, 0) theory exists then it should have well-defined correlation functions and associated amplitudes, even though we do not yet understand what these might be. Whatever they are, they should then give (4, 0) amplitudes. As these theories do not have conventional local covariant interactions, it is to be expected that the corresponding amplitudes should also involve non-standard features. It will be interesting to see whether seeking amplitudes could be a useful way to unravel the mysteries of the interacting theories.

8. All four nothing?

There is a free (4, 0) superconformal theory in six dimensions which provides a conformal theory of an exotic ‘graviton’ but which nevertheless reduces to linearized five-dimensional $N = 8$ supergravity on circle reduction, with a standard graviton. It then gives a conformal theory of gravity with only two derivatives. Compactifying on a 2-torus to four dimensions gives an $SL(2, \mathbb{Z})$ symmetry acting through duality transformations, giving a gravitational duality for the free theory [9,10,12]. The usual gauge symmetry of gravity is replaced by one with a parameter which is a 3-tensor instead of a vector field.

The big question is whether there is an interacting version of this theory. It is of course possible that there is no interacting theory, but it would be disappointing if $M$-theory didn’t avail itself of the existence of such an interesting multiplet.

If an interacting theory exists, then it cannot be a conventional theory but should be some ‘non-lagrangian’ theory, presumably of a similar kind to the (2, 0) theory. If there is an interacting (4, 0) theory, then the next question is whether it arises in $M$-theory. It was argued in [9] that it should arise as a strong coupling limit of $M$-theory toroidally compactified to five dimensions, understood as a limit to energies much greater than the Planck scale.

A key question to ask here is whether $M$-theory compactified to five dimensions has BPS states carrying the singlet central charge $K$ and, if so, what the spectrum of these is. The conjecture that the (4, 0) theory arises as a strong coupling limit requires the five-dimensional theory to have BPS states carrying $K$ with the right spectrum to be Kaluza–Klein modes. This then provides an important test for the conjecture that there could be a six-dimensional (4, 0) theory arising as a limit of $M$-theory: if $M$-theory toroidally compactified to five dimensions does not have BPS states carrying $K$, then the conjecture is false. On the other hand, the presence of $K$-charged states with a spectrum consistent with a KK tower would be strong evidence that the conjecture could be true. Importantly, this is a test that can be examined within $M$-theory and would deepen our understanding of $M$-theory.

A promising approach to the (4, 0) theory is to first understand better amplitudes and correlation functions for the (2, 0) theory and then to use these to construct (4, 0) amplitudes via the double copy mechanism. Such (4, 0) tree amplitudes would then define the classical theory.

There is a further exotic multiplet in six dimensions with (3, 1) supersymmetry [9]. This also gives five dimensions $N = 8$ supergravity on circle reduction and involves a third-rank tensor gauge field with different symmetry properties to that arising in the (4, 0) theory, and the supermultiplet has both vector fields and self-dual 2-form gauge fields. This is not a superconformal supermultiplet and so requires a dimensionful coupling constant in six dimensions. Again it is interesting to ask whether an interacting theory exists and whether it has a role to play in $M$-theory. While the (4, 0) multiplet arises from the product of two (2, 0) ones, the (3, 1) multiplet arises from the product of a (2, 0) multiplet with a (1, 1) multiplet. Moreover, (3, 1) amplitudes can be obtained from combining (2, 0) with (1, 1) amplitudes and in [43–45] formal (3, 1) amplitude-like expressions were found using their (2, 0) expressions for ‘amplitudes’. As in the (4, 0) case, these have many of the properties one might expect from an amplitude but have non-local poles and other problematic features.
Whether or not the (4, 0) theory turns out to have a role to play in M-theory, it raises some interesting issues. Studying the multiplet led to the duality between the graviton and dual graviton in linearized gravity [9,10,12] and to a gravitational $S$-duality in four dimensions [10]. It raises the possibility that gravity could be fundamentally conformal and that at high energies it could be described by some field other than the usual symmetric 2-tensor graviton. More generally, it is interesting to ask how M-theory behaves at trans-Planckian energies, especially given the interesting structures found in string theory at ultra-high energies or string-scale curvatures.

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