Geometrothermodynamic model for the evolution of the Universe

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Abstract. Using the formalism of geometrothermodynamics to derive a fundamental thermodynamic equation, we construct a cosmological model in the framework of relativistic cosmology. In a first step, we describe a system without thermodynamic interaction, and show it to be equivalent to the standard ΛCDM paradigm. The second step includes thermodynamic interaction and produces a model consistent with the main features of inflation. With the proposed fundamental equation we are thus able to describe all the known epochs in the evolution of our Universe, starting from the inflationary phase.

Keywords: gravity, inflation, physics of the early universe

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1 Introduction

The standard cosmological model of general relativity states that the evolution of our Universe is described by the ΛCDM model, and postulates that there must have been an inflation era early in the history in which the vacuum energy dominated over other forms of energy density such as baryonic matter, dark matter or radiation [1]. All the physical phases contained in the ΛCDM scenario and inflation are characterized by particular equations of state which usually relate the corresponding energy density with the pressure. There are many ways to impose or derive a particular equation of state. Recently in [2], we proposed a method in which the equations of state are derived from a fundamental thermodynamic equation which, in turn, can be obtained by using the formalism of geometrothermodynamics (GTD).

One of the central ideas of GTD [3] is that any thermodynamic system is represented by its corresponding space of equilibrium states $E$. Moreover, the thermodynamic properties of the system can be represented in terms of the geometric properties of the equilibrium space — e.g., the curvature of the equilibrium space can be interpreted as a measure of the thermodynamic interaction and curvature singularities represent phase transitions. This has been shown to be true for a large number of thermodynamic systems (see [4] and references therein).

If the system contains $n$ different thermodynamic degrees of freedom (number of independent thermodynamic variables that are necessary for its description), the corresponding equilibrium space is $n$-dimensional whose geometric properties are completely determined by an $n$-dimensional metric $g$. In thermodynamic geometry, the components of the metric $g$ are usually identified with the second derivatives of a particular thermodynamic potential which can be chosen as the internal energy (Weinhold metric), the entropy (Ruppeiner metric), or in terms of a partition function (Fisher-Rao metric). These particular Hessian metrics have been extensively applied to explore the properties of a large number of thermodynamic systems and also in connection with information theory [5–7]. The approach of GTD is different. The metric $g$ is chosen such that it is invariant with respect to Legendre transformations. This is an additional condition which is necessary in GTD in order to incorporate the well-known property of the Legendre invariance of classical thermodynamics into the geometric description, i.e., the independence of the physical behavior of the...
system on the choice of thermodynamic potential. Furthermore, in order to consider a Legendre transformation as a coordinate transformation, in GTD it is necessary to introduce the \((2n + 1)\)-dimensional phase space \(\mathcal{T}\) which is endowed with a contact one-form \(\Theta\) and a metric \(G\). A Legendre transformation in \(\mathcal{T}\) can be expressed as a coordinate transformation \(Z^A \to \tilde{Z}^A\) \((A = 0, 1, \ldots, 2n)\) where the Jacobian is the unity matrix \([3, 8]\). The contact one-form can be written in a canonical manner as \(\Theta = d\Phi - \delta_{ab}I^a_dE^b\) \((a = 1, \ldots, n)\) with \(\delta_{ab} = \text{diag}(1, 1, \ldots)\), where the coordinates have been chosen as \(Z^A = \{\Phi, E^a, I^a\}\). Whereas the contact form is Legendre invariant in the sense that under a Legendre transformation \(Z^A \to \tilde{Z}^A\) it behaves as \(\Theta \to \tilde{\Theta} = d\tilde{\Phi} - \delta_{ab}\tilde{I}^a_dE^a\), the components of the metric \(G\) are in general not Legendre invariant. Nevertheless, it is possible to find particular solutions which preserve Legendre invariance. For instance, the metric (summation over all repeated indices implied)

\[
G = (d\Phi - I_adE^a)^2 + \Lambda(E_aI_a)^{2k+1}dE^a dI^a,
\]

where \(I_a = \delta_{ab}I^b, k\) is an integer, and \(\Lambda = \Lambda(Z^A)\) is an arbitrary non-zero Legendre-invariant function, is the most general metric found so far which is invariant with respect to partial and total Legendre transformations.

The connection between the phase space \(\mathcal{T}\) and the equilibrium space \(\mathcal{E}\) is determined through an embedding map \(\varphi : \mathcal{E} \to \mathcal{T}\) such that its pullback \(\varphi^*\) satisfies the condition \(\varphi^*(\Theta) = 0\). In particular, we can choose \(\{E^a\}\) as the coordinates of \(\mathcal{E}\), in which case the pullback condition reads

\[
d\Phi = I_adE^a.
\]

This is the general form of the first law of thermodynamics where \(\Phi\) is the thermodynamic potential, the \(E^a\) are the extensive variables and the \(I_a\) the intensive variables dual to \(E^a\), i.e., \(I_a = \Phi, a \equiv \frac{\partial\Phi}{\partial E^a}\). Furthermore, the pullback induces a canonical metric \(g\) on the equilibrium space \(\mathcal{E}\),

\[
g = \Lambda(E_a\Phi, a)^{2k+1}\delta^{ab}\Phi, bc dE^a dE^c.
\]

The function \(\Phi = \Phi(E^a)\) represents the fundamental equation of classical thermodynamics \([9]\). Each thermodynamic system is given by a particular fundamental equation which, in turn, determines a particular metric \(g\) for \(\mathcal{E}\). In this manner, GTD provides a particular equilibrium space for each thermodynamic system. In classical thermodynamics, fundamental equations are usually obtained in the laboratory by applying empirical methods \([9]\). In GTD, we can use a theoretical approach to obtain new fundamental equations \([10]\). If we suppose that the equilibrium space \(\mathcal{E}\) is an extremal subspace of \(\mathcal{T}\), i.e., the action \(I = \int \sqrt{|\text{det}(g)|} d^nE\) satisfies the variational principle \(\delta I = 0\), we obtain the Nambu-Goto differential equations

\[
\frac{1}{\sqrt{|\text{det}(g)|}} (\sqrt{|\text{det}(g)|} g^{ab} Z^A_{,a} )_b + \Gamma^A_{BC} Z^B_{,h} Z^C_{,i} g^{bc} = 0,
\]

where \(\Gamma^A_{BC}\) are the Christoffel symbols associated with the metric \(G\), which is a system of differential equations for \(Z^A(E^a)\) and in particular for the fundamental equation \(\Phi(E^a)\). Consider the special case \(\Phi = S\) and \(E^a = (U, V)\), where \(S\) is the entropy, \(U\) the internal energy and \(V\) the volume. Then, the resulting differential equations contain the free function \(\Lambda\) and the constant \(k\) from the metric \((1.3)\). It is then possible to show \([10]\) that for particular choices of \(\Lambda\) and \(k\) the fundamental equation (up to an arbitrary additive constant) reads

\[
S = c_1 \ln \left( U + \frac{\alpha}{V} \right) + c_2 \ln(V - \beta),
\]
where $c_1$, $c_2$, $\alpha$ and $\beta$ are real constants. If we choose $c_1 = 2/3$, $c_2 = 1$, $\alpha > 0$ and $\beta > 0$, we obtain the fundamental equation for the van der Waals gas. In general, however, the set of constants entering the fundamental equation (1.5) are not fixed by the Nambu-Goto equations. The main object of the present work is to investigate the physical consequences of the above fundamental equation in the context of relativistic cosmology.

We note that from the point of view of GTD, in general the equilibrium space of the fundamental equation (1.5) is curved. The presence of curvature in the equilibrium space is interpreted as representing the interaction between the constituents of the thermodynamic system. In the limiting case $\alpha = \beta = 0$, the thermodynamic curvature vanishes, indicating the absence of thermodynamic interaction. For particular values of $c_1$ and $c_2$ this limiting fundamental equation represents an ideal gas which, from the point of view of statistical physics, is a system without thermodynamic interaction. However, the arbitrary constants $c_1$ and $c_2$ do not affect this property. As we will see, this simple generalization of the ideal gas plays an important role in relativistic cosmology.

In this work, we consider the consequences of the presence of thermodynamic interaction in cosmology. We will show that with nonzero $\alpha$ and $\beta$, and a particular choice of $c_2/c_1$, it is possible to describe an inflationary period in the early Universe.

This work is organized as follows. In section 2, we analyze the simple case of a fundamental equation without interactions, and show that such systems can be used to describe the standard cosmological evolution. In section 3, we consider an interacting fluid and determine the parameters of the model necessary to achieve an inflationary period. In section 4, we briefly comment on some of the thermodynamic properties of the fluid, before we conclude our work in section 5.

Throughout the paper we use geometric units with $G = c = \hbar = k_B = 1$.

2 Cosmology without thermodynamic interaction

Let us consider the Friedmann-Lemaître-Robertson-Walker (FLRW) metric,

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2),$$

(2.1)

which, in connection with a perfect fluid energy-momentum tensor, yields the Friedmann equations

$$\frac{\dot{a}}{a^2} = \frac{8\pi}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3P),$$

(2.2)

where we assume zero spatial curvature in accordance with observations. To integrate the above differential equations we need an additional equation, which we take to be the non-interacting limit of the fundamental equation (1.5), i.e.,

$$S = c_1 \ln U + c_2 \ln V,$$

(2.3)

following from GTD. For any fundamental equation to be physically relevant, it must satisfy the first law of thermodynamics, which in this case reads $TdS = dU + PdV$. This leads to the equations of state,

$$\frac{1}{T} = \frac{\partial S}{\partial U} \quad \text{and} \quad P = \frac{\partial S}{\partial V}.$$ 

(2.4)

A straightforward calculation shows that

$$P = \frac{c_2}{c_1} \frac{U}{V} = \frac{c_2}{c_1} \rho,$$

(2.5)
i.e., a barotropic equation of state. Different choices of the ratio $c_2/c_1$ correspond to different eras of the evolution of the Universe. For instance, $c_2/c_1 = 1/3$ describes a radiation-dominated Universe, $c_2 = 0$ describes a dust-dominated Universe, and $c_2/c_1 = -1$ corresponds to dark energy. We see that the fundamental equation contains all the epochs of the standard cosmological model as particular cases for specific choices of $c_2/c_1$. In all cases, eq. (2.3) represents the fundamental equation of the standard cosmological model. This simple fact allows us to interpret the Universe as a thermodynamic system whose properties are entirely described by a single fundamental equation.

Other generalizations of this fundamental equation have been used to describe dark fluids [2] representing both matter and dark energy simultaneously. In a different context [11], this way of deriving cosmological models has been found useful as well and the systems described by GTD fundamental equations were called GTD fluids.

3 Cosmology with thermodynamic interaction

From the fundamental equation for the interacting fluid (1.5), the corresponding equations of state can be calculated, giving the temperature of the fluid as

$$T = \frac{U}{c_1} + \frac{\alpha}{c_1V},$$

(3.1)

whereas the pressure reads

$$P = \frac{c_2UV^2 + \alpha[\beta c_1 + (c_2-c_1)V]}{c_1V^2(V-\beta)}.$$

(3.2)

We now introduce an energy density as $\rho = U/V$ and parametrize the volume as a function of the scale factor as $V = V_0 a^3$ so that the pressure becomes a function of $\rho$ and $a$. This parametrization uses the standard convention of cosmology that the scale factor at the current time $t_0$ is $a(t_0) = 1$, and thus $V_0$ is the volume of the Universe at current time. Furthermore, the continuity equation,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0,$$

(3.3)

becomes a differential equation for $\rho$ and $a$ which can be solved explicitly to give

$$\rho = K \left( a^3 V_0 - \beta \right)^{\frac{-c_2}{c_1}} - \frac{\alpha}{a^6 V_0^2},$$

(3.4)

where $K$ is an integration constant. It is always possible to choose the values of the constants $K$ and $\alpha$ such that the energy density is positive. Moreover, by fixing $c_2/c_1$, we can obtain in principle any polynomial dependence of the density of the kind

$$\rho_{\text{inf}} \sim \frac{1}{a^m}.$$  

(3.5)

Combinations of terms with different powers are also possible. By choosing $c_2/c_1$ appropriately, we can thus obtain a large number of models with inflationary behavior, for instance, a cosmological constant-type term. However, it is also possible to achieve a period of strong expansion with an appropriate number of e-folds that is not an exact inflationary term, i.e. not a constant. This can be done with the choice $c_2/c_1 = -8/9$, under the assumption that
the constant $\beta$ is small. Indeed, using this value in (3.4) and expanding the first term for small values of $\beta$, we obtain the density

$$\rho \simeq \frac{KV_0^{8/9}}{a^{1/3}} - \frac{8\beta K}{9V_0^{1/9}a^{10/3}} - \frac{\alpha}{a^{6V_0^2}}. \tag{3.6}$$

The first term has the exponent $m = 1/3$, and can produce the appropriate amount of e-foldings, if it is the dominating contribution to the density during the inflationary regime. Indeed, neglecting the last two terms in (3.6) for the duration of inflation, we can calculate the number of e-foldings from the inflationary density

$$\rho(a) \simeq \frac{KV_0^{8/9}}{a^{1/3}} \equiv \rho_{\text{inf}}(a) \tag{3.7}$$

as follows. The integration of the first Friedmann equation yields the scale factor and the Hubble parameter

$$a = \left(\frac{2\pi}{27KV_0^{8/9}}\right)^{1/6} t^6, \quad H = \frac{6}{t}. \tag{3.8}$$

The number of e-foldings then can be calculated as

$$N = \int H dt = 6 \ln \left(\frac{t_f}{t_i}\right), \tag{3.9}$$

where $t_i$ and $t_f$ are the times of the beginning and end of inflation, usually estimated to be $t_i = 10^{-36}$ s and $t_f = 10^{-32}$ s; in this case, we obtain $N = 6(-32 + 36) \ln 10 \simeq 55$, which is an appropriate amount of e-foldings [1, 12].

The assumption that the inflationary density is dominated by the first term given in (3.6) puts constraints on the possible values of the constants $\alpha$ and $\beta$, for instance, by requiring that the absolute value of each of the two additional terms in (3.6) is much smaller than the absolute value of the inflationary term, i.e.

$$|\alpha| \ll KV_0^{26/9}a_i^{17/3} =: \alpha_c, \tag{3.10}$$

$$|\beta| \ll \frac{9}{8} V_0 a_i^3 =: \beta_c, \tag{3.11}$$

where $a_i = a(t_i)$. Here the signs of $\alpha$ and $\beta$ do not matter, since only their absolute values have to be negligible. From the fundamental equation of GTD (1.5), it follows that the constant $\beta$ should be related with some characteristic volume. Therefore we assume that $\beta$ be positive. On the other hand, $\alpha$ usually is the interaction constant between the gas constituents, and could in principle be both positive and negative. However, if we choose $\alpha$ to be negative, and let the two terms proportional to $\alpha$ and $\beta$ cancel each other exactly at the beginning of inflation, we end up with the condition

$$|\alpha| = \frac{8}{9} \beta KV_0^{17/9}a_i^{8/3} \ll \alpha_c, \tag{3.12}$$

which, with the condition on $\beta$, leads to the same condition on $\alpha$ as in the previous case. This would mean that inflation starts off very cleanly, since the density at the beginning of inflation is exactly composed by the inflationary term only, and the contribution of the other two densities will first slightly grow, before the terms dilute away faster than the inflationary
term in the course of inflation due to their dependence on the scale factor. All dynamics and physical properties will thus be determined in terms of a small parameter

$$
\epsilon = \frac{\beta}{\beta_c} = -\frac{|\alpha|}{\alpha_c}.
$$

(3.13)

Moreover, the present inflationary model contains two additional parameters, namely, $V_0$ and $K$.

To get a quantitative grip on the involved constants, we consult what is considered the standard picture of cosmology. Inflation in our model lasts for about 55 e-foldings, i.e. during inflation the Universe expands for a factor of roughly $e^{55} \simeq 7 \cdot 10^{23}$ times. After that, the Universe grows for further $10^{30}$ times during the periods of radiation and matter dominance [12]. We further know that the current observable Universe has a diameter of about $l_0 \simeq 10^{26}$ m, i.e. a volume of about $V_0 \simeq 10^{78}$ m$^3$. The diameter of the Universe at the beginning of inflation was thus $l_i = l_0/(10^{30} e^{55}) \simeq 10^{-28}$ m. Using $V = V_0 a^3$ and the convention $a(t_0) = 1$, we can thus determine the scale factor at the beginning of inflation as $a_i = l_i/l_0 \simeq 10^{-54}$. Combining these numbers, we obtain

$$
\beta_c = \frac{9}{8} V_0 a_i^3 \simeq 10^{-84} \text{ m}^3,
$$

(3.14)
i.e. $\beta_c$ equals about the volume of the Universe at the onset of inflation — a small number, but still much larger than the Planck volume, $l_p^3 \simeq 10^{-105}$ m$^3$. For $\alpha_c$, we have to estimate the value of $K$, which can be fixed requiring that the energy scale at the onset of inflation was of the order of the GUT scale of about $10^{16}$ GeV,

$$
\rho_{\text{inf}}(t_i) = K V_0^{8/9} a_i^{1/3} = \frac{10^{16} \text{ GeV}}{l_i^3} \simeq \frac{10^6 \text{ J}}{10^{-84} \text{ m}^3} = 10^{89} \text{ J m}^3.
$$

(3.15)

The constant $K$ then is

$$
K = \frac{\rho_{\text{inf}}(a_i) a_i^{1/3}}{V_0^{8/9}} \simeq 2 \cdot 10^2 \text{ J m}^{-17/3}.
$$

(3.16)

The peculiar unit of $K$ is owed to the requirement that the inflationary density has the unit of energy density, and leads to

$$
\alpha_c \simeq 10^{-78} \text{ J m}^3.
$$

(3.17)

Ultimately, it is instructive to see how fine-tuned the value of the equation of state parameter $c_2/c_1$ has to be in order to achieve an appropriate amount of e-foldings. Already early on we have chosen a specific value for $c_2/c_1$, but we have to ask ourselves whether this is the only possible choice, or whether there is some freedom in fixing its value. In order to investigate this question, we use the expression for the density (3.4), without specifying $c_2/c_1$, and expand it for small $\beta$. Neglecting the $\alpha$- and $\beta$-terms, the inflationary density in the general case results in

$$
\rho_{\text{inf}} \simeq \frac{k}{V_0^{c_2/c_1} a^m}, \quad m = 3 + 3 \frac{c_2}{c_1}.
$$

(3.18)

Using (3.9) for the number of e-foldings, we can calculate the values of $m$ and $N$ for a range of choices of $c_2/c_1$. Fixing $c_2/c_1|_{1} = -0.912$ results in a power-law dependence of the
density of $m_1 \simeq 0.263$, slightly smaller than our previous choice, and thus leading to stronger expansion with a number of e-foldings of $N_1 \simeq 70$. In contrast, choosing $c_2/c_1|_2 = -0.898$ leads to the power $m_2 \simeq 0.307$ and the corresponding inflationary expansion of $N_2 \simeq 60$ e-foldings. Ultimately, defining $c_2/c_1|_3 = -0.877$ yields the value of $m_3 \simeq 0.368$, and thus a number of e-foldings of $N_3 \simeq 50$.

Our previously chosen value of $c_2/c_1 = -8/9$ corresponding to $m = 1/3$ leads to $N \simeq 55$ and lies somewhere between $N_2$ and $N_3$. We thus see that, although the equation of state parameter does not exactly have to be $c_2/c_1 = -8/9$, there is also not too much room for variation. Small alterations of $O(10^{-2})$ in the value of $c_2/c_1$ lead to sizable fluctuations of $O(10)$ in the number of e-foldings $N$, a crucial parameter in the description of inflation. The equation of state parameter is thus constrained in the regime of about $c_2/c_1 \in [-0.912, -0.877]$ in order for the model to work. As for the consequences on the dynamics, we don’t expect qualitative changes when varying the equation of state parameter in this regime. Naturally the values of the constants of the model, such as $\alpha$, $\beta$, or $K$, will slightly change accordingly, as well as the evolution of the thermodynamic variables, since they depend on the choice of $c_2/c_1 \in [-0.912, -0.877]$, but the qualitative features are preserved.

4 Thermodynamic behavior

Using the expressions for the volume $V = V_0 a^3$ and the density (3.4) for $c_2/c_1 = -8/9$, and carrying out an expansion for small $\beta$, the temperature (3.1) becomes

$$T(a) \simeq \frac{K}{c_1} a^{8/3} \left( \frac{V_0^{17/9}}{\beta} - \frac{8\beta V_0^{8/9}}{9} \frac{1}{a^3} \right).$$

(4.1)

Using the definition (3.13) for $\epsilon$, and rewriting the scale factor as multiples of its value at the onset of inflation,

$$a(t) = a_i x,$$

(4.2)

the temperature can be reexpressed as

$$T(x) \simeq \frac{K}{c_1} V_0^{17/9} a_i^{8/3} x^{8/3} (1 - \epsilon x^{-3}).$$

(4.3)

Without loss of generality we can assume that $c_1$ is positive. Then, the temperature is positive as long as the expression in the bracket is positive, i.e. as long as $x$ keeps growing and $\epsilon$ is small, which is one of the conditions which already ensures the dominance of the inflationary term, (3.11). Consequently, the same condition that must be satisfied in order to have clean inflation also guarantees that the temperature is positive at all times during inflation.

From the general expression for the pressure (3.2) and the density (3.4) for the special inflationary case $c_2/c_1 = -8/9$ and small values of $\beta$, we obtain

$$P(a) \simeq -\frac{\alpha}{a^6 V_0^2} - \frac{8K}{9} V_0^{8/9} a^{-1/3} - \frac{8\beta K}{81} V_0^{-1/9} a^{-10/3}.$$  

(4.4)

Using (3.13) and (4.2), we get

$$P_{\text{inf}}(x) = \frac{P_{\text{inf}}(a_i)}{x^{1/3}} \left[ -\frac{8}{9} - \frac{\epsilon}{9} x^{-3} + \epsilon x^{-17/3} \right].$$

(4.5)
We see that for small values of $\epsilon$, the pressure is negative and nearly that of a cosmological constant with barotropic factor $\omega_{inf} = -8/9$. During inflation, $x$ will increase, which implies that the terms proportional to epsilon will dilute away much faster than the leading dependence on $x$. The pressure is thus always negative during inflation.

5 Summary

In this work, we have investigated the physical properties of a particular thermodynamic fundamental equation that is obtained from the GTD formalism, in the framework of relativistic cosmology. It represents the entropy of a thermodynamic system that explicitly depends on the internal energy and volume. In addition, it contains four real parameters $c_1, c_2, \alpha$ and $\beta$ which enter the corresponding equation of state. For a particular choice of these parameters, the fundamental equation corresponds to that of a van der Waals gas.

Furthermore, if we assume that this equation of state can be applied to the entire Universe, we construct a cosmological model that describes the evolution of the Universe. Indeed, based on the GTD interpretation of thermodynamic interaction as the curvature of the equilibrium space, we consider two different cosmological models, with and without interaction. The cosmological model without thermodynamic interaction, which corresponds to the limiting case $\alpha = \beta = 0$, turns out to be equivalent to the standard $\Lambda$CDM model. On the other hand, the resulting cosmology in the presence of thermodynamic interaction has been shown to reproduce the main features of inflation, namely, the number of e-foldings ($N \approx 55$), and is consistent with commonly assumed parameters as the initial time ($t_i \approx 10^{-36} s$) and the final time ($t_f \approx 10^{-32} s$). This inflationary model, however, is valid only for the particular value $c_2/c_1 = -8/9$ and under the condition that the parameters $\alpha$ and $\beta$ are small. Evaluating these parameters shows that $\beta$ corresponds to the volume of the Universe at the beginning of inflation and turns out to be $\approx 10^{-90} m^3$. On the other hand, the ratio $\alpha/\beta$ determines the internal energy of the Universe at the beginning of inflation. The interaction constant $\alpha$ turns out to be small and $\approx 10^{-78} J m^3$. These properties can be considered predictions of our cosmological model.

By relating $\alpha$ and $\beta$, we have developed a model for inflation depending on only one semi-free parameter, i.e. $\epsilon$ (semi-free because it has to obey certain limits), after fixing the remaining constants $V_0$ and $K$ to physically reasonable values expected from an inflationary model, and choosing the value of the parameter $c_2/c_1$ determining the equation of state of the fluid. The inflationary model derived in this work is thus unique in the sense that all the parameters entering the fundamental equation must be fixed or limited by critical values in order to reproduce the main features of inflation. A fine-tuning, however, is not necessary, as we have seen. In particular, the value of $c_2/c_1$ is free to vary in a regime of about $c_2/c_1 \in [-0.912, -0.877]$ around our chosen value, while still fulfilling the criteria for inflation. This variation of the equation of state parameter leads to a slight modification of the values of the constants of the model as well as the evolution of the thermodynamic state functions, but preserves their qualitative properties. Apart from this small uncertainty in $c_2/c_1$ however, strictly speaking, the GTD inflationary model presented in this work is not an element of a family of models in which many possible scenarios are consistent with observations, but instead predicts a specific scenario which can be compared with observations to determine its validity.

One way to test the model are cosmological perturbations, a very important aspect of any inflationary model. A detailed analysis of this issue is, however, beyond the scope of this work.
of the present work. Preliminary analysis have shown that a nearly scale-invariant power spectrum of primordial perturbations can be achieved within the framework of our proposal. However, since the model is constructed not from the dynamics of a scalar field but from the thermodynamic behavior of an interacting system, the conventional calculation of a primordial power spectrum from the quantum fluctuations of the scalar field does not apply here. Instead, one has to consider perturbations in the fluid limit of the Einstein equations, or some alternative thermodynamic description of perturbations in the fluid. A detailed analysis of the perturbation spectrum will be presented in a more extensive complementary publication, along with a thorough examination of the properties of the inflationary fluid, considering thermodynamic response functions, possible phase transitions and critical points.

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