The role of electron-screening deformations in solar nuclear fusion reactions and the solar neutrino puzzle

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Abstract

Thermonuclear fusion reaction rates in the solar plasma are enhanced by the presence of the electron cloud that screens fusing nuclei. The present work studies the influence of electron screening deformations on solar reaction rates in the framework of the Debye-Hückel model. These electron-ion cloud deformations, assumed here to be static and axially symmetric, are shown to be able to considerably influence the solar neutrino fluxes of the \textit{pp} and the \textit{CNO} chains, with reasonable changes in the macroscopic parameters of the standard solar model (SSM). Various known deformation sources are discussed but none of them is found strong enough to have a significant impact on the SSM neutrino fluxes.

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I. INTRODUCTION

As the quest for the solution to the solar neutrino puzzle continues, all the parameters associated with the production and detection of solar neutrinos are exhaustively investigated. Naturally, stellar thermonuclear reactions have attracted a lot of attention in the nuclear physics community since not only do they govern the evolutionary stages of a star but also they can possibly hide the solution to the notorious discrepancy between theoretically and experimentally calculated solar neutrino fluxes.

The effects of the electron-ion screening on stellar fusion cross sections have drawn a lot of attention, especially in relation to the solar neutrino puzzle. Various screening prescriptions have been suggested, each of which has some inherent inadequacies. Salpeter’s weak screening formula can be safely used in the study of pp solar fusion reactions where the weak screening regime is obeyed, though a recent study showed that, for most practical purposes, it can reasonably be applied to other solar reactions as well. On the other hand, the formula given by Mitler is considered the most reliable as it describes fairly well all the screening regimes, weak screening (WES), intermediate (IS) and strong screening (SS), though the assumption of a constant electron density around the fusing nuclei is rather arbitrary.

As regards the solar neutrino puzzle, the prevalent belief is that, under the present standard solar model (SSM), the screening effect is under control. However, various non-standard solar models have been proposed, in an attempt to explain the discrepancy between the theoretical and experimental neutrino fluxes. Some of them are plausible, while others are exaggerated. Among those attempts, the existence of a primordial magnetic field in the solar interior has received relatively little attention, while its relation to the screened fusion reactions themselves has never been studied.

This lack of interest stems from the disheartening fact that if a large magnetic field (of the order of $\sim 10^9$ Gauss) is added to the equation of solar structure, the predicted event rate of the Cl$^{37}$ experiment is increased by a factor of two. As the magnetic field has the opposite effect from the one that is desired, no further investigation has been made by the same author. Admittedly, some other studies appeared which showed that a combination of differential rotation and strong magnetic fields in the sun can actually reduce the Cl$^{37}$ signal but none of them has been adapted as a component of the SSM. Nevertheless, as it has been recently shown, a superstrong magnetic field can accelerate hydrogen fusion reactions in stars and in the laboratory, an effect which could influence the solar neutrino fluxes.

This is due to the fact that strong magnetic fields ($B > 10^8G$) compress hydrogen atoms both perpendicular and parallel to the field direction. The magnetic field ties the electrons to the field lines so that their response to a Coulomb attraction is essentially restricted to a one-dimensional motion parallel to the field. It is therefore plausible to assume that the presence of such a field would modify the screening effect of the electron cloud just as it happens on the surface of neutron stars. Such screening deformations can be studied qualitatively by means of the Debye-Hückel model. Actually, in the presence of a large magnetic field the Debye–Hückel radius which represents a spherical distribution of the electron cloud around the nuclei will become orientation dependent. This effects causes,
inevitably, reaction rates and neutrino fluxes to be orientation dependent themselves.

The purpose of this paper is to investigate the dependence of thermonuclear fusion reaction rates on the deformations of the ionic-electron cloud that screens the reacting nuclei and its possible consequences on solar neutrino fluxes. The layout of the paper is as follows: In §II the advantages and disadvantages of the Debye-Hückel potential in the study of solar nuclear reactions are briefly investigated. In §III the formalism for an axially deformed screening cloud is established, underlining its effects on the \( pp \) reaction rates and the associated \( pp \) neutrino fluxes. In §IV there is given a measure of the uncertainty of the solar neutrino fluxes due to the presence of such screening deformations. Finally §V investigates some potential sources of deformation, while the main results of the present paper are summarized in §VI.

II. ADVANTAGES AND DISADVANTAGES OF THE DEBYE-HÜCKEL POTENTIAL

In the stellar plasma gravitational compression and quantum mechanical tunneling combine in order to achieve the classically impossible fusion between light nuclei. The electron gas that surrounds the nuclei acts as a catalyst in the reaction, by lowering the repulsive Coulomb barrier which prevents atomic nuclei from approaching each other.

In the framework of the Debye-Hückel model each nuclei is assumed to polarize its neighborhood creating a spherically symmetric but inhomogeneously charged ionic cloud around it. In this model, the potential \( V(r) \) of a given nucleus of charge \( Z_1 \) can be found by the equation of Poisson:

\[
\nabla^2 V(r) = r_D^{-2} V(r)
\]

with

\[
 r_D^{-2} = \frac{4\pi e^2}{kT} \left( \sum_i Z_i^2 n_i + n_e \theta_e \right)
\]

where \( r_D \) is the Debye-Hückel radius, \( \theta_e \) is the electron degeneracy factor, and \( n \) the number densities of ions \( (n_i) \) with atomic number \( Z_i \) and electrons \( (n_e) \), respectively. The solution of Eq. (1) is a Yukawa potential which acts in the vicinity of the ion \( Z_1 e \) and has the form:

\[
 V_D(r) = \frac{Z_1 e}{r} \exp\left( -\frac{r}{r_D} \right)
\]

A measure of the stellar plasma response to Coulomb interactions is the plasma coupling parameter, which is defined as:

\[
 \Gamma_{ij} = \frac{Z_i Z_j e^2}{akT}
\]

where \( a \) is the mean interionic distance, \( Z_{ij} \) the atomic numbers of the reactants and \( kT \) the thermal kinetic energy. Although the domains of the weak screening (WES), the
intermediate screening (IMS) and strong screening (SS) are not precisely defined, in an ion fluid one can define them as follows:

\[ WES : \Gamma_{ij} << 1, \quad IMS : \Gamma_{ij} \sim 1, \quad SS : \Gamma_{ij} >> 1 \] (5)

It is now easy to show that the assumption \( \Gamma_{ij} << 1 \) of the \( WES \) regime for the proton-atom reaction channel between the test nuclei \( Z_2 e \) and the generators of the above potential, i.e. nuclei \( Z_1 e \) plus ionic cloud, can be written:

\[
\frac{Z_1 Z_2 e^2}{r_D kT} << 1
\] (6)

The above condition has generated a lot of controversy in the study of solar fusion cross sections as it is violated for solar nuclear reactions with \( Z_1 Z_2 \) larger than unity.

Another interesting fact is that according to the above model for small \( r \) the ion density now vanishes while the electron density diverges. By noting this, Mitler [3] assumed the above model to be valid only beyond some radius \( r_1 \) while for shorter distances he assumed that the electron density around the nucleus is constant and equal to the mean electron density in the plasma, \( n_e (\infty) \).

However this is also an oversimplification, especially for the heavier nuclei of astrophysical interest where it underestimates the electron charge density around the ion. For instance near a \( Be^7 \) nucleus in central solar conditions we have a density, roughly \( 3.8 e n_e (\infty) \).

Moreover, the model in question is forced to undergo another compromise when calculating the thermalized cross section \( \langle \sigma v \rangle^{sc} \) which appears in the screened reaction rate between nuclei \( i \) and \( j \):

\[
r_{ij}^{sc} = (1 + \delta_{ij})^{-1} n_i n_j \langle \sigma v \rangle^{sc}
\] (7)

where \( n_i, n_j \) are the number densities of nuclei \( i \) and \( j \) respectively, and \( \delta_{ij} \) is the Kronecker delta. As the WKB integral involved cannot be found analytically, we inevitably resort to a linear approximation of potential [3]. However even if we don’t resort to that approximation and take into account non-linearities [9] of the Debye-Hückel potential this would add a negligible contribution to Salpeter’s \( WES \) prescription. Note that whenever the \( WES \) condition is challenged one should always investigate non-linear corrections as will be shown in the study that follows.

Finally, the spherical symmetry around the two reactants is taken for granted by the standard Debye-Huckel model. The fact that no deformations are assumed for the ionic-electron cloud around the point like nucleus can obviously be the source of uncertainties in the calculation of stellar reaction rates. It is the principle objective of this paper to investigate the effects of such deformations on reaction rates and the associated solar neutrino fluxes, without focusing in detail on the sources of deformations themselves.

III. ELECTRON SCREENING DEFORMATIONS

In the adiabatic model the target and the projectile nuclei are assumed to be surrounded by a static, spherical electron cloud, whose electron charge density falls off exponentially
with respect to the distance from the center of the cloud which is the nucleus itself. In central solar conditions the mean ion velocity $\langle u_i \rangle = (8kT/\pi \mu)^{1/2}$ is roughly fifty times smaller than the mean electron velocity $p_e^2/(2m_e) \simeq (3/2)kT$ thus justifying the fact that as the nucleus moves the electron cloud has enough time to re-arrange itself so that it practically screens the nucleus at all times. However oscillations of the ionic cloud are inevitable due to their speed which is much lower than that of the electrons. For the main sequence stars Mitler showed that in the framework of the standard solar model (SSM) the distortion of the common charge clouds has only a small effect on the screening calculations ($\sim 2\%$).

Moreover, the possible presence of a strong magnetic field is bound to cause substantial deformations of the electron cloud by compressing it both parallel and perpendicular to the field direction. On the other hand the existence of other, as yet unspecified, sources of deformations cannot be ruled out.

Therefore treating the screening cloud as a rigid (albeit inhomogeneous) sphere is an assumption which must by further investigated, especially when primordial magnetic fields are considered.

Studies of heavy nuclei fusion reactions have shown that theoretical predictions of cross section can be greatly improved by assuming rotations and deformations of the fusing nuclei. It is therefore plausible to consider similar effects in the study of screened thermonuclear reactions where the electron cloud is assumed to be deformed. In fact this deformation can be parametrized in the framework of the liquid-drop model so that the Debye-Hückel radius is considered a measure of the electron cloud. The deformed DH radius is now:

$$r_D(\theta, \phi) = r_D^{(0)} \left[ 1 + \sum \beta_m Y_m^2(\theta, \phi) \right]$$

where $r_D^{(0)}$ takes care of volume conservation and $Y_m^2$ is the usual spherical harmonic function. For simplicity and reasons that will soon become clear only quadrupole deformations will be considered. Moreover, we assume that the deformation is axially symmetric and take the $z$ axis along the axis of symmetry. Disregarding the rotational degree of freedom we obtain the surface shape

$$r_D(\theta) = r_D^{(0)} \left[ 1 + \beta Y_2^0(\cos \theta) \right]$$

where the angle $\theta$ is measured from the axis of symmetry i.e. the $z$ axis. Note that $r_D^{(0)}(\beta) \simeq r_D^{(0)}(-\beta)$.

For $\beta > 0$ the single axis is larger than the double axis and the cloud is a prolate spheroid, that is cigar-shaped. For $\beta < 0$ the single axis is smaller than the double axis and the cloud is an oblate spheroid i.e. disk-shaped.

Note that the weak screening approximation restricts the possible values of $\beta$. A reasonable assumption which stems from Eq. is that in the WES regime the following relation must be fulfilled:

$$\frac{Z_1Z_2e^2}{r_D(\theta)kT} \leq 0.1$$

\[10\]
In solar conditions for the $pp$ reaction where the use of the $WES$ formalism is incontrovertible we have

$$\frac{e^2}{r_D kT} \simeq 0.05$$  \hspace{1cm} (11)

Therefore, for all orientations, the following inequality must hold:

$$-0.8 \leq \beta \leq 0.8$$  \hspace{1cm} (12)

where we have disregarded the contribution of volume conservation which is always less than 5%. The deformation parameter can now take all the above values without violating the $WES$ condition (6), thus rendering the use of the deformed screening formalism legitimate.

If we take into account the nuclear potential $V_N (r)$ then the total potential of the reaction is

$$V (r; \theta; \beta) = V_N (r) + V_D (r; \theta; \beta) + \frac{\hbar^2}{2\mu r^2} l (l + 1)$$  \hspace{1cm} (13)

where the centrifugal term is assumed to be independent of the orientation. This assumption is immaterial here as we will only consider very low-energy reactions where $s$-interactions dominate. In that case the orientation dependent cross section of the nuclear fusion reaction is given by:

$$\sigma (E; \theta; \beta) = \frac{S (E)}{E} P (E; \theta; \beta)$$  \hspace{1cm} (14)

where

$$P (E; \theta; \beta) = \exp \left[ -\frac{2\sqrt{2\mu}}{\hbar} \int_{R}^{r_c (\theta; \beta)} \sqrt{V_D (r; \theta; \beta) - E} dr \right]$$  \hspace{1cm} (15)

and $r_c (\theta; \beta)$ is the classical turning point given by

$$V_D (r_c; \theta; \beta) = E$$  \hspace{1cm} (16)

The thermonuclear reaction which can be studied safely by means of the above formalism is the one that dominates the solar neutrino production namely: $H^1 (p, e^+ \nu_e) H^2$. For that reaction, in the undeformed weak screening case, it turns out that in the region of the maximum energy production $R = 0.09 R_\odot$ the Gamow peak is $E_0 = 5.599 keV$, the classical turning point is roughly $r_c = 0.01 r_D$, while the $WES$ enhancement factor is to good approximation: $f_{\text{WES}}^0 = 1.049$.

Actually, non-linear screening corrections [9] can be safely neglected. Along the whole profile of orientations and $\beta$ parameters the contribution of higher order terms to the shifts of the screening corrections and the Gamow peak has been found negligible. Hence, screening deformation effects can be simply represented by Salpeter’s $WES$ formula

$$f_D (\theta; \beta) = \exp \left( \frac{e^2}{k T r_D (\theta)} \right)$$  \hspace{1cm} (17)
where the DH radius $r_D(\theta)$ is now orientation dependent. Finally we obtain:

$$f_D(\theta; \beta) = (f_{\text{WES}}^{\text{we)}})^{-1}(\theta; \beta)$$

(18)

where $g(\theta; \beta)$ is the ratio $r_D(\theta; \beta)/r_D$:

$$g(\theta; \beta) = \left\{ \frac{1}{2} \int_{-1}^{1} \left[ 1 + \beta \sqrt{\frac{5}{16\pi}} (3u^2 - 1) \right]^3 du \right\}^{-1/3} \left[ 1 + \beta \sqrt{\frac{5}{16\pi}} (3\cos^2 \theta - 1) \right]$$

(19)

In Fig. 1 the orientation dependent DH radius $r_D(\theta; \beta)$, measured in units of $r_D$, is plotted in polar coordinates with respect to the azimuthal angle $\theta$ and the deformation parameter $\beta$. It is obvious that along the axis of symmetry of the cloud, $z$-axis ($\theta = 0$), a positive $\beta$ parameter "stretches out" the ionic cloud (a prolate spheroid shape) while a negative $\beta$ parameter "sucks in" the cloud (an oblate spheroid shape). As one would expect, for both positive and negative parameters the larger the absolute value of $\beta$ the more pronounced the deformation.

Note that Fig. 1 actually represents the deformation factor $g(\theta; \beta)$ while the corresponding classical turning point is $r_c(\theta; \beta) = 261g(\theta; \beta)\text{fm}$. For example for a $\beta = 0.8$ deformation a proton cruising along the $z$-axis in the plasma with an energy equal to the Gamow peak $E_0 = 5.56\text{keV}$ will come up against the potential wall at a distance roughly 1.43 times further than in the undeformed case ($r_c = 261\text{fm}$), that is $r_c(\theta = 0, \beta = 0.8) = 373\text{fm}$. On the other hand for a $\beta = -0.8$ the classical turning point is reduced to $r_c(\theta = 0, \beta = -0.8) = 123\text{fm}$.

The screened Coulomb potential $V(r; \theta; \beta)$ can be visualized by means of Fig. 2 where we have plotted in polar coordinates the deformed shapes of the potential at a distance $r = r_D$ from the origin. At that distance the potential contours $V_D(r_D; \theta; \beta)$ of Fig. 2 are only a function of the orientation. Hence, as a proton enters the ionic cloud of the Hydrogen atom on its way to fusion, according to the angle at which it enters the cloud it will experience a different (thicker or thinner) potential wall. The thicker the wall, the most improbable the reaction and of course the smaller the reaction rate.

The orientation dependent acceleration of the reaction is displayed in Fig. 3 where the screening factor is plotted with respect to the azimuthal angle and the deformation parameter for the $pp$ reaction at $R = 0.09R_\odot$. The reaction rate can be 1.1 times faster if the proton enters a disk-shaped cloud ($\beta = -0.8$) at zero angle. On the other hand a much slower reaction is obtained for a cigar-shaped ionic cloud (almost no enhancement at all for $\beta = 0.8$).

The impact of screening deformation on the neutrino fluxes will be discussed in a more quantitative way in the section that follows.

IV. SOLAR NEUTRINO FLUXES.

In the solar region of maximum energy production ($R/R_\odot = 0.09$) the thermal kinetic energy is $kT = 1.161\text{keV}$ while for the $pp$ reaction the standard (undeformed) weak-screening factor is $f_0^{\text{WES}} = 1.049$. On the other hand for a $\beta = -0.4$ deformation and
an angle of impact of \( \theta = 0 \) the screening factor is \( f_D = 1.067 \). This corresponds to an acceleration of the (undeformed) weakly screened \( pp \) reaction by roughly 1.7% which in turn reflects on the cross-section factor given by \( S_D = f_D S \). As the principal source of energy in the Sun is the \( pp \) reaction this acceleration would influence both the solar structure and the neutrino fluxes by reducing the central temperature and density in order to conserve luminosity. (An account of what happens in the sun if the cross-section factor \( S_{pp} \) increases can be found in Ref. [11]).

In most solar evolution codes the \( pp \) screening factor is evaluated by means of Salpeter’s formula which has been proved to be valid and accurate in standard conditions. In the deformed case the quantity \( f_D \) should be used, instead. We can obtain an approximation of the uncertainties introduced due to the presence of such deformations by using the proportionality formulae [7] [12] which relate neutrino fluxes to screening factors. In order to isolate the \( pp \) uncertainty, we will assume that except for the \( pp \) reaction all the other neutrino-producing reactions remain unaffected by the deformations, thus obtaining a minimum of the total associated uncertainty.

For various solar fusion reactions the ratios of the deformed neutrino fluxes \( \Phi^D \) to the ones obtained in the \( WES \) regime \( \Phi^{WES} \), are as follows:

\[
H^1 (p, e^+ \nu_e) H^2: \quad \left( \frac{\Phi^D_{pp}}{\Phi^{WES}_{pp}} \right)_{pp} = \left( \frac{f_D}{f_0^{WES}} \right)^{0.14}
\]

\[
H^1 (pe^-, \nu_e) H^2: \quad \left( \frac{\Phi^D_{hep}}{\Phi^{WES}_{hep}} \right)_{pp} = \left( \frac{f_D}{f_0^{WES}} \right)^{-0.08}
\]

\[
Be^7 (e^-, \nu_e) Li^7: \quad \left( \frac{\Phi^D_{Be^7}}{\Phi^{WES}_{Be^7}} \right)_{pp} = \left( \frac{f_D}{f_0^{WES}} \right)^{-0.97}
\]

\[
Be^7 (p, \gamma) B^8 (e^+, \nu_e) B^{8*}: \quad \Phi^D_B = \left( \frac{f_D}{f_0} \right)^{-2.6}
\]

\[
N^{13} (e^+ \nu_e) C^{13} \quad \text{and} \quad O^{15} (e^+, \nu_e) N^{15}: \quad \Phi^D_{N,O} = \left( \frac{f_D}{f_0^{WES}} \right)^{-22/8}
\]

According to the above formulae, the presence of a screening deformation of \( \beta = -0.4 \) and an angle of impact of \( \theta = 0 \) can perturb the solar neutrino fluxes calculated in the SSM by at least 4.5% for the \( N,O \) and \( B^8 \) neutrinos and by less than 1% for the less
sensitive $pp, hep,$ and $B^7$ neutrinos. Such a source of deformation would also perturb the electron cloud around the heavier nuclei involved in neutrino production, thus modifying their screening factors which we arbitrarily considered constant here.

It is very tempting to investigate what happens if the deformations are stronger. For a collision along the $z$-axis with $\beta = -0.8$ we obtain $f_D = 1.16$ and the corresponding uncertainties are at least $23\%$ for the $N, O$ and $B^8$ neutrinos, $9.2\%$ for the $Be^7$ neutrinos and less than $2\%$ for the $pp, hep$ ones. Adding the effects of the screening factors of the other reactions, which have been disregarded so far, the uncertainties can be dramatic. It seems therefore that the presence of screening deformations in the sun can tune the predicted neutrino fluxes in order to reduce the observed deficit.

It is crucial to study what deviations from the SSM parameters such deformations can induce. As was previously noted conservation of luminosity implies a reduction in the central temperature $T_c$ as a result of an increase in $S_{pp}$, since $L_\odot \sim S_{pp}T^8_c$. This implies:

$$\frac{T^D_c}{T^{WES}_c} = \left( \frac{f_D}{f^{WES}_0} \right)^{-\frac{1}{8}}$$

Therefore, for the weaker (stronger) of the above considered deformations the ($WES$) central temperature of the sun would have to decrease by $0.2\%$ ($1.2\%$). On the other hand, as the ratio $\rho/T^3$ is approximately constant along the solar profile, the new central density would now be given by:

$$\frac{\rho^D_c}{\rho^{WES}_c} = \left( \frac{f_D}{f^{WES}_0} \right)^{-\frac{3}{8}}$$

that is roughly decreased by $0.6\%$ ($3.7\%$) with respect to the ($WES$) assumption. Both values represent reasonable deviations from the ($WES$) SSM considering that the ($WES$) assumption itself causes a $0.6\%$ deviation from the ($NOS$) central temperature and another $1.8\%$ deviation from the ($NOS$) central density.

V. SCREENING DEFORMATION SOURCES

Having established the general formalism for the investigation of screening deformations, a discussion of potential sources of such deformations is imperative. In a study of the effects of superstrong magnetic fields in the stellar plasma there was shown clearly that even at zero energies the screening electron cloud is deformed, in the sense that it becomes compressed perpendicular and parallel to the field direction. This deformation can dramatically accelerate hydrogen fusion reactions not only in neutron stars [13] but also in the laboratory as was shown recently [14], [15]. It is very plausible therefore to assume that the presence of a superstrong magnetic field in the solar interior would cause similar effects.

From the present work it is obvious that if a substantial correction to the solar neutrino fluxes is to be made by means of screening deformations those have to be larger than $\beta = -4$. To gain an idea of what kind of solar magnetic field would cause such deformation
in the center of the sun we can use the results of Ref. \[15\] where it shown clearly that any magnetic field weaker than the Intense Magnetic Field Regime:

\[ B_{IMF} = 4.7 \times 10^9 G \]

would have no effect on hydrogen fusion reactions. Hence, in order to decrease the solar neutrino fluxes by means of magnetically catalyzed thermonuclear fusion, the magnetic field required must be stronger than $10^{10} G$.

Admittedly such a superstrong field cannot be easily justified. After the disheartening result \[6\] that the presence of a strong magnetic ($\sim 10^9 G$) in the solar interior increases the predicted neutrino fluxes (doubles the $Cl^{37}$ signal by increasing the pressure gradient in the sun) few investigators have looked into the matter. This is also due to some additional arguments which indicate the implausibility of a solar magnetic field larger than $10^9 G$. Such arguments include the limiting strength set by Chandrasekhar and Fermi \[17\], stability reasons \[15\] and magnetic buoyancy \[18\], though there is a very interesting work \[8\] which argues that a combination of a differential rotation and magnetic field can reduce the $Cl^{37}$ signal opposing the results of Ref. \[6\]. Note that Helioseismology is another concern when considering such a strong magnetic field in the sun. Nevertheless, such a field, despite stability and buoyancy counter-arguments, can have been formed by the interstellar magnetic field which was frozen into the matter out of which the sun was formed, or there may be an unspecified mechanism of continuous generation.

It is now obvious that, regarding the solar neutrino puzzle, even if we accept the presence of a superstrong magnetic field the corrections induced are much smaller than the ones required to reconcile theory and experiment. Therefore, taking also into account other counter-arguments, it seems that magnetically induced screening deformations (magnetic catalysis) cannot possibly be the answer to the neutrino puzzle.

Another plausible source of screening deformations is the fact that in the solar center the average interionic spacing is $a \approx 2.8 \times 10^{-9} cm$, which is similar to the Debye radius. Therefore the spherical electron cloud assumption is not well justified. The cloud is more likely to assume an ellipsoidal distribution around the two reactants, like a fissioning nucleus \[4\]. However, the argument that the incoming fusing nucleus will be carrying its own cloud, suggested in Ref. \[20\], and Ref. \[4\], thus increasing the deformations, doesn’t seem to have a significant effect as in that case the total of the nucleus-nucleus, nucleus-cloud, and cloud-cloud interactions would be \[20\]:

\[
V(r) = \frac{Z_1 Z_2 e^2}{r} - \frac{3}{2} \frac{Z_1 Z_2 e^2}{r_D}
\]

where the cloud-cloud interaction, in analogy to the recent results for the laboratory \[17\], would be much smaller than the screening shift itself. Therefore the screening shift would have been at least 50% larger than the (WES) one used in standard DH theory thus accelerating for example the hydrogen fusion reaction by roughly 2.5% with respect to the WES regime. This would result in an increase of the (NOS) $pp$ neutrino fluxes of the order of 1% and a similar decrease in the (NOS) central solar temperature. It is therefore obvious that any screening deformations due to SSM cloud-cloud interactions in the solar plasma are negligible.
Finally a non-Maxwellian distribution of velocities, such as the flat Maxwellian \([21]\), where the relative particle motion is frozen to 0 K along a specific direction, will establish a preferential direction of motion for ions thus inducing electron cloud deformations. In fact any kind of deviation from statistical equilibrium is a source of screening deformations. For example it has been shown \([22]\) that a progressive depletion of the Maxwellian tail can yield a neutrino counting rate below 1 SNU but a physical cause for that distribution has not been found. Taking into account other counter-arguments \([7]\) to the existence of a non-Maxwellian distribution in the sun this non-standard source of deformation has to be deferred to a forthcoming article where the issue will be studied in detail.

VI. CONCLUSIONS

In this work we investigate the response of the proton-proton reaction to electron-ion screening deformations in the solar plasma. Those deformations are studied in the framework of the Debye-Hückel model and the results show that they can induce an orientation-dependent thermalized cross section which causes the solar neutrino fluxes to be orientation-dependent themselves. Therefore, in principle, screening deformations can influence the solar neutrino fluxes with reasonable deviations from the macroscopic values of the SSM.

Various potential deformation sources are discussed but none of them is found capable of inducing deformations strong enough to have a significant impact on the SSM neutrino fluxes. However, the existence of other, as yet unspecified, deformation sources cannot be ruled out. It seems therefore necessary to further investigate the possible presence of such sources which could cause a substantial degree of uncertainty to the solar neutrino fluxes.

Regarding the novelties of the present paper they can be summarized as follows: The effects of deformations have received a minimal attention by only two authors (Ref. \([3]\) and Ref. \([4]\) ). They both concluded that the effect is small but none of them studied deformations in a non-standard solar model as they assumed SSM conditions from the very beginning of their calculations. The present paper studies screening deformations in a way completely independent of the source and solar model conditions. From now on, each time a non-standard source of deformation appears, it can possibly be connected to the deformation parameter \(\beta\) introduced here, just as it happens in heavy ion fusion reactions. However, that task is admittedly not a trivial one. Moreover, the effects of magnetically catalyzed fusion on the SSM have been discussed here for the first time and eventually overruled as a solution to the solar neutrino puzzle. Finally, here for the first time there is shown that, if sufficiently strong, screening deformation can actually perturb considerably the SSM neutrino fluxes with reasonable changes in the central solar temperature and density. That result warrants further research into the origin and the effects of such screening deformation sources in the solar interior.

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FIGURE CAPTIONS

Figure 1.
The variation of the orientation dependent DH radius $r_D(\theta; \beta)$, measured in units of $r_D$, with respect to the azimuthal angle $\theta$ and the deformation parameter $\beta$ in polar coordinates.

Figure 2.
The orientation dependent DH potential $V_D(r; \theta; \beta)$, measured in units of the screening term of Eq.(11), at a distance $r = r_D$ with respect to the azimuthal angle $\theta$ and the deformation parameter $\beta$ in polar coordinates. The effect is calculated for the $pp$ reaction in the region of the maximum energy production $R = 0.09R_\odot$ where $r_D = 25719\, fm$ and $\frac{e^2}{r_D kT} \simeq 0.056\, keV$.

Figure 3.
The variation of the orientation dependent screening factor $f_D(\theta; \beta)$ with respect to the azimuthal angle $\theta$ and the deformation parameter $\beta$ in polar coordinates. The effect is calculated for the $pp$ reaction in the region of the maximum energy production $R = 0.09R_\odot$ where $r_D = 25719\, fm$. 
\[ V_{D}(r_{D}, \theta, \beta)/V_e \]
