Quantum Processes, Space-time Representation and Brain Dynamics

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ABSTRACT

The recent controversy of applicability of quantum formalism to brain dynamics is critically analyzed. The prerequisites for applicability of any type of quantum formalism or quantum field theory is to investigate whether the anatomical structure of brain permits any kind of smooth geometric Hilbert structure or four dimensional Minkowskian structure. The present understanding of brain function clearly denies any kind of space-time representation in the Minkowskian sense. However, three dimensional space and one dimensional time can be assigned to the neuro-manifold and the concept of probabilistic geometry is shown to be an appropriate framework to understand the brain dynamics. The possibility of quantum structure for brain dynamics is also discussed in this framework.

Subject headings: Minkowski geometry, Hilbert space, quantum formalism, quantum field theory, functional geometry, probabilistic geometry

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1. Introduction

The applicability of quantum formalism to brain dynamics has raised lot of interest among the scientific community (Tegmark 2000; Hagan et al. 2002). Several authors (Stapp 1990; 1993; Hameroff et al. 1996; Riccardi et al. 1967; Umezawa 1993; Alfinito et al. 2000; Jibu et al. 1996) claimed that quantum processes and collapse of wave function in the brain are of importance to help us to understand information processing and higher order cognitive activities of brain. Even before, Pribram (1991) had proposed his holographic model to help in understanding the information processing in the brain. However, the most fundamental issue which should be solved before applying any quantum approach has not been addressed by any of the above authors. One of the prerequisites of applying any form of quantum mechanics to brain dynamics (in the non-relativistic domain) is to investigate whether the anatomical structure of brain permits assigning any kind of smooth geometric notion like distance function, or orthogonality relation between the vectors in the neuro-manifold. For applying quantum field theoretic model to memory function or spontaneous symmetry breaking, one needs to construct space-time geometry in Minkowskian sense over this neuro-manifold. Then it is necessary to look into the plausibility of indeterminacy relation with Planck’s constant $h$ or any other constant, say, a brain constant at any level of brain functions. We think one should address these issues before applying any kind of quantum formalism to understand the information processing and higher order cognitive activities (Roy et al. 2003).

The plan of this paper is as follows: At first we will analyze the anatomical structure of brain and its relation to Euclidean or non-Euclidean distance and then the possibility of assigning space-time (four dimensional) representation. Pellionisz and Llinas (1982; 1985) have shown that our present understanding of brain function does not permit to assign any space-time representation in the usual four dimensional form. They considered tensor network theory where they assigned a metric tensor $g_{ij}$ to the Central Nervous System.
(CNS). However, for global activities of the brain i.e., to define the metric tensor over the whole neuro-manifold, this raises a lot of difficulties. For example, some cortical areas are non-linear or rough, so the tensor network theory becomes very much complicated and almost intractable to solve the relevant mathematical equations. In one of our recent papers (Roy et al. 2002), we proposed that the statistical distance function may be considered over the entire neuro-manifold considering the selectivity properties of neurons (Hubel 1995). In this paper, we shall show that the statistical distance function and the statistical metric tensor ought to be considered, as they are expected to be very important concepts to understand the above mentioned issues. Then the possibility of Hilbert space structure and quantum processes are discussed with respect to structure of neuro-manifold.

2. Functional Geometry and Space-time Representation

The internalization of external geometries into the Central Nervous System (CNS) and the reciprocal issue have created lot of interest for the last two decades. The central tenet of the (Pellionisz and Llinas (1982) hypothesis is that brain is a tensorial system using vectorial language. This hypothesis is based on the consideration of covariant sensory and contravariant vectors representing motor behaviour. Here, CNS acts as the metric tensor and determines the relationship between the contravariant and covariant vectors. The contravariant observable theorem has been discussed in the context of Minkowskian geometry as well as in stochastic space-time and quantum theory. It can be stated that measurements of dynamical variables are contravariant components of tensors. This means that whenever a measurement can be reduced to a displacement in a particular coordinate system, it can be related to contravariant components of the coordinate system. To make an observation of a dynamical variable as position or momentum, the measurement is usually
done in the form a reading of a meter or something similar to that. Through a series of calculations one can reduce the datum to a displacement of a coordinate system.

Margenau (1959) analyzed this issue and claimed that the above reduction can give rise to a measurement if it satisfies the two requirements. It must be repeatable with the same results and must yield a physically useful quantity. This can be easily shown in the context of Minkowski space. The motivation of Pellionisz and Llinas (1982) was to find a single underlying entity capable of representing any set of particular neuronal networks; i.e., data derived from particular neuronal networks, generalized to the set of neuronal networks (from a brain to the brain). In fact, if this is the case, it would imply that this is equivalent to consider the brain as a geometric object. However, they have shown that a space-time representation (in the sense of Minkowskian geometry) cannot be assigned to the internal representation. The arguments can briefly be described as follows:

CNS can be compared with a procedure such as taking the picture of a moving object not with an instantaneous flash, but replacing the light with a set of axons (each having a different conduction time). Now through differently delayed neuronal signals, the simultaneous external events will not be represented in the CNS as simultaneous. In other words, the simultaneous onset of firings of a group of neurons with different conduction times will not produce a set of simultaneous external events. Hence, the assignment of space-time geometry to the functional geometry of neurons is not possible, at least within the perview of our present understanding of brain dynamics. It appears that a three dimensional space and one time coordinate system can be assigned to the internal representation. But to assign this kind of space and time structure over the global activities of brain (to account for, i.e., the holonomic like information processing)(Pribram 1991), one needs to define a smooth metric tensor over the whole neuro-manifold. A family of neural networks forms a neuro-manifold. However, as some cortical areas of brain are more non-linear and rougher than others, it
seems to be very difficult to construct a smooth metric tensor over the entire neuromanifold. In fact, the mammalian cerebral cortex has the form of a layered thin shell of grey matter surrounding white matter. The cortical mantle is one of the most important features of the brain and it plays a tremendously important role in understanding brain functions. Although the cortical surface is an important feature of mammalian brain, the precise geometry and variability of the cortical surface are little understood. Attempts have been made to construct mathematical representation of a typical cortical surface using the data from the Van Essen Laboratory (Washington, Seattle, USA). This representation allows one to make statements about the geometry of the surface as well as its variability. Considering the surface as two-dimensional manifold in brain volume, it enables one to compute geometrical properties such as the Mean and the Gaussian curvature of the surface. However, much work is needed to understand the functional geometry of the brain.

Recently, Nakahara et al. (2002) used the concept of Information Geometry to understand the geometrical structure of a family of information systems. The family of information systems, stated here, are taken as consisting of a hierarchial structure of neuronal systems with feedback and feedforward connections. Amari introduced a duality structure in the Bayesian framework from the point of view of information geometry. However, he considered a manifold equipped with Riemannian metric formed by a family of distributions. In this framework, the four dimensionai metric in the Minkowskian sense is also not realizable due to the lack of existence of positive definite distribution functions for four dimensional space-time.

3. Quantum Formalism

Recent interests (Stapp 1993, 1990; Hameroff et al. 1996; Riccardi et al. 1997; Umezawa 1993; Alfinito et al. 2000; Beck 1996; Beck et al. 1998) on the applicability of quantum formalism
in understanding brain function leads one to consider several fundamental issues related to functional geometry of brain. In the quantum theory of mind-brain described by Stapp (1993;1990) there are two separate processes:

First, there is the unconscious mechanical brain process governed by the Schrödinger equation. It involves processing units that are represented by complex patterns of neural activity (or more generally, of brain activity) and subunits within these units that allow "association", i.e., each unit tends to be activated by the activation of several of its subunits. An appropriately described mechanical brain evolves by the dynamical interplay of these associative units. Each quasi-classical element of the ensemble that constitutes the brain creates, on the basis of clues, or cues, coming from various sources, a plan for a possible coherent course of action. Quantum uncertainties entail that a host of different possibilities will emerge. This mechanical phase of the processing already involves some selectivity, because the various input clues contribute either more or less to the emergent brain process according to the degree to which these inputs activate, via associations, the patterns that survive and turn into a plan of action.

Hameroff and Penrose (1996) discussed the issue of quantum coherence and consciousness in following way:

1. Quantum coherence and the wave function collapse are essential for consciousness and occur in cytoskeleton microtubules and other structures within each of the brain’s neurons.

2. Quantum coherence occurs among tubulins in micro-tubules, pumped by thermal and biochemical energies. Evidence for some kind of coherent excitation in proteins has recently been reported by Vos et al.(1993). The feasibility of quantum coherence in, seemingly, noisy, chaotic cell environment is supported by the observation that quantum spins from biochemical radical pairs which become separated, retain their
correlation in the cytoplasm.

3. For the Orchestrated Objective Reduction (OrchOR), as put forward by Penrose and Hameroff, each of superposed states have their own space-time geometries. When the degree of coherent mass-energy difference leads to sufficient separation of space-time geometry, the system must choose and decay to a single universe state. Thus the OrchOR model makes it necessary for inclusion of self selections in the fundamental space-time geometry.

4. It is known that a typical brain neuron has roughly $10^7$ tubulins. If the tubulins in each neuron are involved in quantum coherent state, then roughly $10^3$ neurons would be required to sustain coherence for 500 msec, by which time, according to Penrose and Hameroff, the quantum gravity threshold is reached and only then OOR occurs.

According to these authors, one possible scenario for emergence of quantum coherence leading to OrchOR and conscious events is cellular vision. Albrecht-Buehler (1992) has observed that single cells utilize their cytoskeletons in cellular vision-detection, orientation and directional response to beams of red/infrared light. Jibu et al. (1996) argued that this process requires quantum coherence in microtubules and ordered water.

Pribram (1991) developed the first holographic model of the brain. According to Pribram, the brain encodes information on a three dimensional energy field that enfolds time and space, yet allows to recall or reconstruct specific images from the countless millions of images, stored in a space slightly smaller than a melon. Further, he and his collaborators studied the implications of Gabor’s quanta of information (1946) on brain function and their relation to Shannon’s measure (1948) for the amount of information related to the data, as obtained in their investigations. Furthermore, Pribram along with Jibu and Yasue (1996) investigated how the quantum mechanical processes can operate at the synaptodendritic level. According to their view, something like the phenomenon of superconductivity can
occur by virtue of boson condensation over short ranges when the water molecules adjacent to the internal and external hydrophilic layers of the dendritic membrane become aligned by the passive conduction of post synaptic excitatory and inhibitory potential changes, initiated at synapses (Jibu et al. 1996). It is generally argued that the brain is warm and wet. It is interesting to note that recent theoretical and experimental papers support the prevailing opinion (Tegmark 2000) that such large warm systems will rapidly lose quantum coherence and classical properties will emerge. In fact, the rapid loss of coherence would naturally be expected to block any crucial role for quantum theory in explaining the interaction between our conscious experiences and the physical activities of our brains. However, very recently, Hagan et al.(2002) claimed that at certain level the quantum coherence can be retained.

It is clear from the above discussions that it is necessary to use either non-relativistic quantum mechanics or quantum field theory for the description of activities of brain. For any kind of quantum field theoretic approach one needs to define the field functions over relativistic space-time as a prerequisite. So far, experimental evidence regarding the anatomy of brain does not permit the space-time description (in a Minkowskian sense), and consequently, any type of quantum field theoretic model is hard to be conceivable, at least, at the present state of understanding in brain dynamics. However, in order to apply any kind of non-relativistic quantum mechanics, the basic prerequisites are Hilbert space structure and the indeterminacy relation. Before going into the context of indeterminacy relation in brain dynamics, let us try to analyze the micro- and macro- structures in the brain.
3.1. Brain activity at various scales

The different scales of activities of brain can be classified in the following manner (Freeman 1999):

1. Pulses of single neurons, microtubules in milliseconds and microns may be considered as the part of microstructures.

2. Domains of high metabolic demand managed in seconds and centimetres (for measuring the spatial patterns of cerebral blood flow). This can be designated as the macrostructure.

3. Millimeters and tenths of a second are the patterns of the massed dendritic potentials in EEG recordings from the waking and sleeping brains. This can be considered as an area where there might be a level of mesostructure.

Freeman (1991) suggested that the perception cannot be understood solely by examining properties of individual neurons, i.e., only by using the microscopic approach (dominant approach in neuroscience). Perception depends on the simultaneous activity of millions of neurons spread throughout the cortex. Such global activity has to be of the macroscopic approach.

Micro- or macrostructures in the brain are distinguished by the scale of time or energy. The macrostructure can be characterized by the fact that brain lives in hot and wet surroundings with $T = 300K$. This should be discussed in the context of quantum coherence versus thermal fluctuations. The physiological temperature $T = 300K$ corresponds to an energy $E \sim 1.3 \times 10^{-2} \text{eV}$. Let us now define a signal time $\tau = 2\pi\omega$ and $\hbar\omega = E$. As $\hbar\omega = E$, we can get

$$\omega = 2 \times 10^{13}\text{s}^{-1} \quad \text{or} \quad \tau = .03\text{ps}$$
This indicates that the physiological temperatures correspond to frequencies smaller than the picosecond scale. They correspond to the time scale involving electronic transitions like electron transfer or changes in molecular bonds. In cellular dynamics, the relevant time scale is of the order of $\tau > 0.4$ ns, where, $E_{\text{cell}} \sim 10^{-5}$eV (Beck 1996). To allow comparison with quantum scales, let us distinguish the two scales as follows:

1. The macroscopic or cellular dynamics level with time scales in the milli, down to the nanosecond range.

2. The microscopic or quantal dynamics level with time scales in the pico, down to the femtosecond range.

The large difference between the two time scales indicates quantum processes might be involved in the individual microsites and decoupled from the neural networks. Recently, using dimensional analysis, Bernroider (2003) made an analysis by applying the concept of Lagrangian action to brain processes at different scales of resolutions in order to clarify the current dispute whether classical neurophysics or quantum physics are relevant to brain function.

The central issue of his analysis is to estimate the order of action from dimensional analysis, relevant to brain physiology and its close proximity to quantum action i.e., $\hbar$ (Planck’s constant). For example, spiking action at the single cell level is found to involve $1.8 \times 10^{-15}$ Lagrange (using mechanical units) down to $2.1 \times 10^{-16}$. This lies between $10^{18}$ and $10^{19}$ times Planck’s constant. This is in good agreement with the time scale difference and spiking time estimated by Tegmark (2000). However, he pointed out that the action behind the selective ion permeation and channel gating might be of interest at the molecular level. Considering $10^5$ ions permeating per msec and employing a saturating barrier model with one or two ions crossing during that time, the action turns out to be of the order of $0.48 \times 10^{34}$ Lagrange, which is in the range of the quantum action $\hbar$ ($1.05459 \times 10^{-34}$ MKS
units). It implies that brain functioning at a certain level might be a proper arena to apply the quantum formalism.

3.2. Indeterminacy Relations

In communication theory, Gabor (1946) considered an uncertainty relation between frequency($\omega$) and time($t$) as

$$\delta \omega \delta t = \frac{1}{2}$$

This is similar to the Heisenberg energy($E$)-time($t$) uncertainty relation:

$$\delta E \delta t = \hbar \quad [\hbar = \frac{\hbar}{2\pi}]$$

where $\hbar$ is Planck’s constant. Now if the quantum formalism is valid (even in its non-relativistic form) in brain dynamics, there should exist a similar type of uncertainty relation between frequency/energy and time, i.e.,

$$\delta \omega \delta t = b$$

where $b$ may be termed here as the “brain constant”. Even if there exists such a constant in brain dynamics, one needs to relate it to action quanta like Planck’s constant. *Future research in brain function can shed light on this important issue.*

As far as the existence of Hilbert space structure is concerned, one needs to define a smooth distance function over the cortical surface of the brain. It should be mentioned that Joliot et al. (1994) found a minimum interval in sensory discrimination. Considering this aspect, they claimed that consciousness is a non continuous event determined by the activity in the thalamocortical system. Now, if this is the case, then one needs to introduce discrete time or some form of granularity in space and time geometry.
4. Probabilistic Geometry and the Neuromanifold

Let us now describe first the geometroneuro-dynamics as proposed by Roy and Kafatos (2002) considering the neurophysiological point of view. Then we shall generalize the approach and investigate the relevant issues from a more generalized perspectives.

4.1. Orientation Selectivity of Neurons and Statistical Distance

Recent research on Planck scale physics (Roy 2003a) sheds new light on the possible geometrical structure for discrete and continuum levels. The idea of probability in geometric structure as proposed and developed by Karl Menger around 1940, (1942;1949) seems to be a very useful tool in defining distance function over the cortical areas of brain. There is a large variety as well as number of neurons in the brain. Collective effects which can only be accounted for in terms of statistical considerations, are clearly important in such case. Experimental evidences point to more than 100 different type of neurons in the brain, although the exact number is not yet decided. It is found that no two neurons are identical, and it becomes very difficult to say whether any particular difference represents more than the other i.e., between individuals or between different classes. Neurons are often organized in clusters containing the same type of cell. The brain contains thousands of clusters of cell structures which may take the form of irregular clusters or of layered plates. One such example is the cerebral cortex which forms a plate of cells with a thickness of a few millimeters.

In the visual cortex itself (Hubel 1995), certain clear, unambiguous patterns in the arrangement of cells with particular responses have been found. Even though our approach could apply to non-visual neurons, here we limit our study to the neurons in the visual cortex as the visual cortex is smoother preventing non-linear effects. For example, as the measurement electrode is moving at right angles to the surface through the grey matter,
cells encountered one after the other have the same orientation as their receptive field axis. It is also found that if the electrode penetrates at an angle, the axis of the receptive field orientation would slowly change as the tip of the electrode is moved through the cortex. From a large series of experiments in cats and monkeys it was found:


*Neurons with similar receptive field axis orientation are located on top of each other in discrete columns, while we have a continuous change of the receptive field axis orientation as we move into adjacent columns.*

The visual cortex can be divided into several areas. The most important areas in the visual cortex are V1, V2, V3, V4 and MT (V5). The primary visual cortex area V1 is important for vision. It is the principal entry point for visual input to the cerebral cortex. From this area, relays pass through a series of visual association areas in parietal and temporal regions and finally to the prefrontal cortex where substrates for decision making on the basis of visual cues are found. The main issues related to visual cortex are linked to intrinsic and extrinsic relays of each cortical region, geometrically ordered microcircuitry within appropriate areas etc. Because of the stripy appearance of area V1, this area is also known as the striate cortex. Other areas of visual cortex are known as the extrastriate (nonstriate) visual cortex.

In the monkey striate cortex, about 70% to 80% of cells have the property of orientation specificity. In a cat, all cortical cells seem to be orientation selective, even those with direct geniculate input (Hubel, 1995). Hubel and Wiesel found a striking difference among orientation-specific cells, not just in the optimum stimulus orientation or in the position of the receptive field on the retina, but also in the way cells behave. The most useful distinction is between two classes of cells: simple and complex. These two types differ in the complexity of their behavior and one can make the reasonable assumption that the cells with the simpler behavior are closer in the circuit to the input of the cortex.

The first oriented cell recorded by Hubel and Wiesel (1995) which responded to the edge
of the glass slide was a complex cell. The complex cells seem to have larger receptive fields than simple cells, although the size varies. Both type of cells do respond to the orientation specificity. There are certain other cells which respond not only to the orientation and to the direction of movement of the stimulus but also to the particular features such as length, width, angles etc. Hubel and Weisel originally characterized these as hypercomplex cells but it is not clear whether they constitute a separate class or, represent a spectrum of more or less complicated receptive fields. We now ask how the computational structure or filters can manifest as orientation detectors?

Pribram (1981) discussed this question, whether single neurons serve as feature or channel detectors. In fact, Pribram and his collaborators (1981 & references therein) made various attempts to classify "cells" in the visual cortex. This proved to be impossible because each cortical cell responded to several features of the input such as orientation, velocity, the spatial and temporal frequency of the drifted gratings. Furthermore, cells and cell groups displayed different conjunctions of selectivities.

From these findings and analysis, he concluded that cells are not detectors, that their receptive field properties could be specified but that the cells are multidimensional in their characteristics (Pribram 1991). Thus, the pattern generated by an ensemble of neurons is required to encode any specific feature, as found by Vernon Mountcastle's work on the parietal cortex and Georgopoulos's data (Pribram 1998) on the motor cortex.

It is worth mentioning that Freeman and his collaborators (1991) suggested that perception cannot be understood solely by examining properties of individual neurons i.e. by using the microscopic approach that currently dominates neuroscience research. They claimed that perception depends on the simultaneous, cooperative activity of millions of neurons spread throughout expanses of the cortex. Such global activity can be identified, measured and explained only if one adopts a macroscopic view alongside the microscopic one.
4.2. Statistical Distance

We can define the notion of distance between the “filters” or the orientation selective neurons. This distance is similar to the statistical distance between quantum preparations as introduced by Wotters (1981). The statistical distance is most easily understood in terms of photons and polarizing filters:

Let us consider a beam of photons prepared by a polarizing filter and analyzed by a nicol prism. Let $\theta \in [0, \pi]$ be the angle by which the filter has been rotated around the axis of the beam, starting from a standard position ($\theta = 0$) referring to the filter’s preferred axis as being vertical. Each photon, when it encounters the nicol prism, has exactly two options: to pass straight through the prism (with “yes” outcome) or to be deflected in a specific direction characteristic of the prism ( “no” outcome). Let us assume that the orientation of the nicol prism is fixed once and for all in such a way that vertically polarized photons always pass straight through. By counting how many photons yield each of the two possible outcomes, an experimenter can learn something about the value of $\theta$ via the formula $p = \cos^2 \theta$, where $p$ is the probability of “yes” (Wotters 1981), as given by quantum theory.

If we follow this analogy in the case of oriented neurons in the brain i.e. as if the filters are oriented in different directions like oriented analyzers, we can proceed to define the statistical distance.

4.3. Statistical Distance and Hilbert Space

It can be shown (Wotters 1981) that the statistical distance between two preparations is equal to the angle in Hilbert space between the corresponding rays. The main idea is as follows:

Let us imagine the following experimental set up. There are two preparing devices, one of which prepares the system in a specific state, say $\psi_1$, and the other prepares in
Here, the statistical distance between these two states can be thought as the measure of the number of distinguishable preparations between $\psi_1$ & $\psi_2$. However, in treating quantum systems, new features should be observed as opposed to rolling the dice. For dice, there is only one possible experiment to perform, i.e., rolling the dice, whereas for a quantum system, there are many, one for each different analyzing device. Furthermore, two preparations may be more easily distinguished with one analyzing device than with another. For example, the vertical and horizontal polarizations of photons can easily be distinguished with an appropriately oriented nicol prism, but can not be distinguished at all with a device whose eigenstates are the right and left handed circular polarizations. Due to this reason, one can speak of the statistical distance between two preparations $\psi_1$ & $\psi_2$ as related to a particular measuring device which means the statistical distance is device dependent. The absolute statistical distance between $\psi_1$ & $\psi_2$ is then defined as the largest possible such distance, i.e., statistical distance between $\psi_1$ & $\psi_2$ when the states are analyzed by the most appropriate or discriminating apparatus. We can illustrate this point in the following way:

Let $\phi_1$, $\phi_2$ .... $\phi_N$ be the eigenstates of a measuring device $A$, by which $\psi_1$ & $\psi_2$ are to be distinguished. It is assumed that these eigenstates are non-degenerate so that there are $N$-distinct outcomes of each measurement. The probabilities of various outcomes are $|\langle \phi_i, \psi_1 \rangle|^2$ if the apparatus is described by $\psi_1$ and $|\langle \phi_i, \psi_1 \rangle|^2$ if the apparatus is described by $\psi_2$. Then the statistical distance between $\psi_1$ & $\psi_2$ with respect to the analyzing device $A$ is

$$d_A(\psi_1, \psi_2) = \cos^{-1}[\Sigma_{i=1}^N |\langle \phi_i, \psi_1 \rangle||\langle \phi_i, \psi_2 \rangle|]$$ (1)

This quantity attains its maximum value if it assumes one of the eigenstates of $A$, (say, $\phi_1$). In that case, we get the statistical distance as

$$d(\phi_1, \phi_2) = \cos^{-1}|\phi_1, \phi_2|$$ (2)
This clearly indicates that the statistical distance between two preparations is equal to the angle in Hilbert space between the corresponding rays. The equivalence between the statistical distance and the Hilbert space distance might be very surprising at first. It gives rise to the interesting possibility that statistical fluctuations in the outcome of measurements might be partly responsible for Hilbert space structure of quantum mechanics. These statistical fluctuations are as basic as the fact that quantum measurements are probabilistic in their nature.

However, it should be mentioned that although representation of orientation of objects in the visual cortex is fairly fine-scaled, visual information regarding the nonstriate visual processing and in superior colliculus is very rough and varies in a non-linear way from that in striate cortex. This type of nonlinearity is neglected here as we have considered statistical considerations which average out this type of nonlinearity. Instead, we considered here the distance between the different clusters of neurons or between the ensemble of neurons.

4.4. Perception and Relational Aspects in Probabilistic Geometry

The issue of continuum and discreteness remains a long standing problem over the last few centuries. In mathematics, if the quantity $A$ is equal to the quantity $B$ and $B$ is equal to $C$, then $A$ is equal to $C$, i.e, mathematical equality is a transitive relation. In the observable continuum “equal” means indistinguishable. In psychology, following Weber and Fechner (1860), we can say that $A$ may lie within the threshold of $B$ and $B$ within the threshold of $C$. Poincaré (1905) suggested

“For the raw result of experience, $A = B, B = C$; $A \leq C$, which may be regarded as the formula for the physical continuum”

that is to say that physical continuum is not a transitive relation. Menger (1949) tried to solve this problem from the positivist point of view. Following his words:
“Instead of distinguishing between a transitive mathematical and intransitive physical relation, it thus seems more hopeful to retain the transitive relation in mathematics and to introduce for the distinction of physical and psychological quantities a probability, that is, a number lying between 0 and 1”.

He considered the role of probability in geometry and introduced the concept of probabilistic metric. He introduced the concept of a set of hazy lumps instead of considering set of points. Then the problem turns out to be similar to finding a probability of the overlapping lumps. For more intuitive understanding, the lumps were considered as the “seat” of elementary particles like electrons, protons etc. These lumps are taken as not to be reducible to any other structures. In other words, they are the ultimate building blocks of space and time. Therefore, a kind of granularity is introduced here at the very basic level. Mathematically speaking, it can be stated as:

“for each pair of elements A and B of probabilistic geometry, it is possible to associate a distribution function $F_{AB}(z)$ which can be interpreted as the probability that the distance between the points is less than $z$”.

Essentially the relational aspect of geometry has been proposed and elaborated by Menger. In our approach, towards geometroneurodynamics (Roy and Kafatos 2002), we have considered the same relational aspect of geometry by considering the orientation selectivity of neurons. Recently, Mogi (1997) tried to reinterpret Mach’s principle in the context of the response selectivity of neurons. He proposed that in perception, the significance of firing of a neuron is determined by the relation of the firing to other neurons at that very psychological moment. He called it Mach’s principle in perception. According to his proposal it is not meaningful to talk about the firing of a single neuron in isolation and its significance in perception.

Menger replaced the usual metric function by a distribution function and showed that this distribution function satisfies all the axioms of the metric. Hence it is known as
probabilistic metric space. There are various types of probabilistic metric spaces used in different branches of physical science (Schewizer and Sklar 1983; Roy 1998). On a large scale, taking averages over these distributions, one can get the usual metric structure.

Here, we shall consider the stochastic space as proposed by Frederick (1976). In this model, the actual points of the space are stochastic in nature. These points can not be used as either a basis for a coordinate system or to define a derivative. However, the space of common experience at large scales or in the laboratory frame is nonstochastic. Therefore, we can start from large scale nonstochastic space and continue mathematically to stochastic space i.e. towards small scales. This stochasticity is considered to be manifested in a stochastic metric $g_{ij}$ and the corresponding mass distribution determines not only the space geometry but also the space stochasticity. However, as more and more mass is confined in a region of space, the less stochastic will be that space. The relation between covariant and contravariant quantity can be written as

$$x^i = g^{ij}x_j$$

As $g_{ij}$ is stochastic, one obtains a distribution of the contravariant quantity $x^i$ instead of a fixed quantity. This may play a significant role in accounting for the tremor in motor behaviour in neurophysiological experiments which will be considered in future publications.

Now let us assume that a Lagrangian can be defined taking this stochastic metric. Then we can define a pair of conjugate variables, one covariant and another contravariant, as

$$P_j = \frac{\partial L}{\partial \dot{q}^j}$$

where $P_j$ is a covariant quantity. However, $P_j$ is not observable in the laboratory as it is covariant. The observable quantity is the covariant one, i.e.,

$$P^j = g^{j\nu}P_\nu$$

As the metric is stochastic in nature, $P^j$ is a random variable. So, if one member of the conjugate pair is well defined, the other member will be random.
4.5. Neurophysiological Basis for Stochasticity in Metric

Let us now look into the origin of stochasticity in neuromanifold. The neurophysiological evidences shows that most neurons are spontaneously active, spiking at random intervals in the absence of input. Different neuron types have different characteristic spontaneous rates, ranging from a few spikes per second to about 50 pikes per second. The mechanism of regular activity is well studied whereas the mechanism of random spontaneous activity is now not well understood. Several possibilities are discussed by Lindhal and Arhem (1994).

One is the well known ion-channel hypothesis. According to this, the nerve impulses are triggered by the opening of single ion channels where the ion channel gating is random. Ion channels are membrane proteins through which the current causing the nerve impulse passes. Donald (1990) considered that the randomness may be related to quantum fluctuations. Lindhal et al. (1994) suggested that single channels may cause spontaneous activity in areas of the brain with consciousness. However, the detailed mechanism of ion-channel gating is still not well understood. Grandpierre (1999) made an attempt to study the effect of the fluctuation of the zero point field (ZPF) in the activity of brain. As such, the future investigations on the effect of ZPF on the neurons may shed new insights not only for the spontaneous activity of neurons but also on the actual process of consciousness. We like to emphasize that in our picture, the fluctuation associated with this kind of spontaneous activity of neurons is the physical cause behind the stochasticity of metric tensor. To start with, let us take Frederick’s (1976) version of stochasticity in geometry. He made several interesting postulates as follows:

1. **The metric probability postulate**: $P(x, t) = A\sqrt{(−g)}$, where for a one-particle system $P(x, t)$ is the particle probability distribution, $A$ is a real valued function, and $g$ is the determinant of the metric.
2. **The metric supersposition postulate**: If at a position of a particle the metric due to a specific physical situation is \( g_{ij}^1 \) and the metric due to a different physical situation is \( g_{ij}^2 \), then the metric at the position of the particle due to the presence of both of the physical situations is \( g_{ij}^3 \) can be written as \( g_{ij}^3 = \frac{1}{2}[g_{ij}^1 + g_{ij}^2] \)

3. **The metric \( \psi \) postulate**: There exists a local complex diagonal coordinate system in which a component of the metric is at the location of the particle described by the wave function \( \psi \).

We have started with Frederick’s approach not only for the attractive mathematical framework for neuromanifold but also for the use of Mach’s principle as the guiding rule for stochastic geometry. It then becomes possible to derive quantum mechanics by adopting a strong version of Mach’s principle such that in the absence of mass, space becomes non-flat and stochastic in nature. At this stage, the stochastic metric assumption is sufficient to generate the spread of wave function in empty space. Thus, following this framework, one obtains an uncertainty product for contravariant position vector \((q^1)\) and contravariant momentum vector \((P^1)\) as

\[
\Delta q^1 \Delta P^1 = \Delta q^1 \Delta (P_\nu g^{\nu 1})
\]

where \( P^\nu = g^{\nu \nu} P_\nu \). Now the question is what is the minimum value of this product. It can be shown that

\[
\Delta q^1 \Delta (P^\nu_{\min} g^{\nu 1}) = \Delta q^1 \Delta P^1 > k_{\min}
\]

which is nothing but the uncertainty principle with \( k \) as the action quantity similar to \( h \), i.e., Planck’s constant. Moreover, using the superposition postulate of metric tensor, it is also possible to account for the interference phenomena.
4.6. Penrose and Hameroff Approach : OrchOR Model

It is to be noted that the above metric supersposition postulate can be shown to be valid under the weak approximation of the general theory of relativity. If there is more non-linearity in the cortical surface, the superposition may actually break down. In Penrose and Hameroff (Hameroff 1996) model, they considered a kind of superposition of space and time geometries in order to relate it with the superposition of wave functions and the decoherence due to variation of mass distribution and hence due to gravity effects. In the above framework, it is possible to relate the superposition of wave functions using the superposition postulate of metrics. Because of the existence of different curvatures at different points (due to different mass distributions) in the framework of statistical geometry, one can write the superposition of metrics or geometries. But it is necessary to investigate the real neurophysiological conditions under which the superposition of metrics would be a valid approximation. It raises a new possibility of constructing a Hilbert structure over the neuromanifold within the framework of statistical geometry. The construction of Hilbert structure over the neuromanifold is one of the prerequisites for applying any kind of quantum mechanical process to the brain.

5. Information Processing in the Brain

The information generated by integrated neural processes and its measurement has created a lot of interest among the scientific community for the last few years. The measure of information essentially depends on the basis of statistical foundation of information theory (Shannon 1948). One of the intriguing question arises is how far the statistical aspects of information theory can help one to assign a measure to differentiate the informative character of the neural processes without any reference to an external observer. The issue of the external observer has been debated in various branches of science and philosophy over
the last century since the birth of quantum mechanics. In fact, the issue of measurement procedure in the history of science has been reanalyzed and still under active consideration after the mathematical formulation of Von Neumann using the statistical concept of entropy. In the standard approach, one generally assigns a number to measure the information and probability of the states of the system that are distinguishable from the point of view of an external observer. But the brain not only processes the information but also interprets the pattern of activities (Pribram 1991). Therefore, one must avoid the concept of a privileged viewpoint of an external observer to understand the information processing in the neural processes in the brain.

In our approach, we have developed a framework (Roy et al. 2003b) where it is possible to avoid the concept of an external observer by reanalyzing the very basis of measurement procedure as well as the neurophysiological evidences in the standard paradigm. Edelman et al. (2000) discussed this problem in the context of neurophysiology and consciousness. The main problem is how to measure the differences within a system like the brain? He defined a concept of mutual information for this purpose. Here, they considered the entropic measure to define the information as considered in Shannons framework.

The principal idea that lies behind our approach can be summarized as follows: The concept of invariance plays a crucial role to understand the information processing and measurement issues in the brain. In the brain, a matching occurs between an input pattern and a pattern inherent in the synaptodentritic network by virtue of generic or learning experience. In the Holonomic theory, both the input and output patterns provide the initial conditions. The match between them is considered to be probabilistic in nature (Pribram 1991). We have introduced here a kind of invariance assisted by the context (as described by the inherent patterns in the dendrites). This is quite similar to environment-assisted invariance in Quantum Mechanics (Zurek 2002). It is one of the fundamental principles of quantum mechanics known as quantum determinism where it is possible to show that an
entangled state is formed between the input pattern and the output pattern. This state can be written in terms of its basis vectors. Now picking up the specific term from the expansion is generally known as selection. Redhead (1989) emphasized that the selection of the parts is related to the attention to particular subensemble of the whole. This means selection is not part of quantum physics.

In physicist’s language, the selection signifies measurement that marks the end of quantum physics. In contrast, the “yes-no”- experiment puts the selection process at the beginning and makes the involvement of brain dynamics (or the selection that underlies the pattern recognition in the brain) into the primitive of quantum mechanics. It may be mentioned that “yes-no”-experiment depends primarily on the act of cognition. In this framework it has been shown how the above kind of analysis and the concept of invariance will help us to understand the nature of ignorance (for example to understand the probabilistic nature of matching) and hence the origin of probability in the context of brain function, similar to quantum physics, without using concepts like collapse or measurement as commonly used in quantum mechanics. It is curious to note that Edelman et al.(2002) pointed out that selection is biologically the more fundamental process. He conjectured that there exists two fundamental ways of patterning thought: selectionism and logic. We think that the selectionism play very significant role in understanding information processing in brain.

6. Discussion

The above analysis clearly indicates that it is not understandable how the anatomy of brain can permit the joint space-time representation in the sense of special theory of relativity. So the applicability of any kind of quantum field theoretic approach is not realizable, at least, at the present stage of understanding of brain function. However, it may be possible
to define a smooth distance function and metric tensor in the probabilistic sense. Therefore, the probabilistic geometry seems to play a significant role in understanding Hilbert space structure and its connection to non-relativistic quantum mechanics. This approach sheds new light to understand the information processing and measurement procedure related to the brain. The implication of stochastic geometry in the inner world might have significant effects in the external world too, which will be considered in subsequent publications.

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