We show that if the presently observed L/E-flatness of the electron-like event ratio in the Super-Kamiokande atmospheric neutrino data is confirmed then the indicated ratio must be unity. Further, it is found that once CP is violated the exact L/E flatness implies: (a) The CP-violating phase, in the standard parameterization, is narrowed down to two possibilities ±π/2, and (b) The mixing between the second and the third generations must be maximal. With these results at hand, we argue that a dedicated study of the L/E-flatness of the electron-like event ratio by Super-Kamiokande can serve as an initial investigatory probe of CP violation in the neutrino sector. The assumptions under which these results hold are explicitly stated.

1. Introduction

The Super-Kamiokande data on the atmospheric neutrinos have opened a new realm of physics research [1]. The simplest interpretation of these data is flavor oscillations arising from neutrino being linear superposition of some underlying mass eigenstates. This circumstance not only takes us into the physics beyond the standard model of the high energy physics, but it also allows to probe various aspects of quantum gravity [2,3,4,5,6]. As such much theoretical and experimental effort is being devoted to deciphering the nature of neutrino. Here, using a very specific aspect of the Super Kamiokande data, we shall analytically constrain the CP-violating neutrino oscillation mixing matrix. This would help the design of future experiments, allow for more analytically-oriented theoretical research, and provide a new direction of research at the existing experimental facilities.
This work joins the on-going research with the observation that as soon as the first results from the Super-Kamiokande on atmospheric neutrinos became available, one of us emphasized that the L/E flatness noted in the abstract places a set of constraints on the neutrino oscillation mixing matrix [7]. However, in that, and our subsequent work [8], CP violation has been neglected. Apart from reasons of simplicity, there is no a priori reason to assume the absence of CP violation in the neutrino sector. In addition, the observed cosmological baryonic asymmetry may be deeply connected with a CP violation in the leptonic sector [9]. This becomes particularly important, as we shall comment below, if the neutrino-sector CP violation is affected by gravity. As such, here, we present a non-trivial generalization of the constraints presented in the early work [7, 8] to obtain a CP-violating bimaximal matrix for neutrino oscillations.

2. Analytical constraints on the neutrino-oscillation mixing matrix

To generalize the discussion of Refs. [7, 8], we start from the probability formula of neutrino oscillations. As in the quark sector, when neutrinos have non-zero masses, their weak eigenstates may not coincide with the mass eigenstates, but may be linear superposition of the mass eigenstates. The latter choice is precisely what is suggested by the existing data [1, 10, 11, 12, 13]. As such, in a phenomenology of neutrino oscillations, a flavor eigenstate of a neutrino is postulated to be a linear superposition of some underlying mass eigenstates

\[ |\nu_\alpha\rangle = \sum_j U_{\alpha j} |\nu_j\rangle, \]

where \(U_{\alpha j}\) is an element of the mixing matrix with \(\alpha = e, \mu, \tau\) and \(j = 1, 2, 3\) in the framework of three generations. In the literature, \(U\) is usually taken as the standard parameterization matrix [14]

\[
V = \begin{pmatrix}
  c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\
  -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & -s_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} \\
  s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -s_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13}
\end{pmatrix}
\]

multiplied by a phase matrix

\[
P = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & e^{i\phi_2} & 0 \\
  0 & 0 & e^{i\phi_3 + \delta_{13}}
\end{pmatrix}
\]

if neutrinos are of the Majorana type. Here, \(c_{ij} = \cos \theta_{ij}\), \(s_{ij} = \sin \theta_{ij}\), and \(\phi_2\) and \(\phi_3\) are the additional phases for Majorana neutrinos. Due to the un-observable effect of \(P\) in flavor oscillation experiments, we will drop it in the discussion and simply equate the mixing matrix \(U\) to \(V\) in calculations that follow. Furthermore, \(\theta_{12}, \theta_{23},\) and \(\theta_{13}\) in \(U\) can all be made to lie in the first quadrant by an appropriate re-definition of the relevant fields.

Assuming the underlying mass eigenstates to be relativistic in the observer’s frame [15], the flavor-oscillation probability is given by [3, 16]

\[
P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{j<k} \text{Re}(U_{\alpha j} U_{\beta j}^* U_{\alpha k}^* U_{\beta k}) \sin^2(\phi_{jk})
\]

* To avoid confusion, we note in advance that in this paper we distinguish between fluxes and events. The former refers to the number of particles of a given species that pass a unit area in a unit time, while the latter depends on the detector sensitivity and the relevant cross sections.
\[ + 2 \sum_{j<k} \text{Im}(U_{\alpha j}U_{\beta j}^*U_{\alpha k}^*U_{\beta k}) \sin(2\varphi_{jk}). \quad (4) \]

where \( L \), measured in meters, refers to the source-detector distance, and the flavor-oscillation inducing kinematic phases \( \varphi_{ij} \), are defined as

\[ \varphi_{ij} = 1.27 \Delta m^2_{ij} \frac{L}{E}, \quad (5) \]

where \( E \) (MeV) refers to the "energy" of the flavor state, and, \( \Delta m^2_{ij} = m_i^2 - m_j^2 \), is the mass-squared difference of the underlying mass eigenstates and is measured in eV\(^2\).

For the CP conjugate channel, the CP-odd term, that is, the last term in Eq. (4), changes sign. So,

\[ P(\bar{\nu}_\alpha \xrightarrow{L} \bar{\nu}_\beta) = \delta_{\alpha\beta} - 4 \sum_{j<k} \text{Re}(U_{\alpha j}U_{\beta j}^*U_{\alpha k}^*U_{\beta k}) \sin^2(\varphi_{jk}) \]
\[ - 2 \sum_{j<k} \text{Im}(U_{\alpha j}U_{\beta j}^*U_{\alpha k}^*U_{\beta k}) \sin(2\varphi_{jk}). \quad (6) \]

Note that, all \( \text{Im}(U_{\alpha j}U_{\beta j}^*U_{\alpha k}^*U_{\beta k}) \) with \( \alpha \neq \beta \) and \( j \neq k \) take the same value

\[ J_{CP} = c_{12} s_{13} c_{23} s_{13} s_{13} (s_{\delta} = \sin\delta_{13}, \ c_{\delta} = \cos\delta_{13}), \]

which is the measure of CP violation.

The Super-Kamiokande measured ratio, \( R_e \), of the electron-like events is defined as

\[ R_e = \frac{N'_e + N'_\mu}{N_e + N'_e}, \quad (7) \]

where \( N_e \) and \( N'_e \) are the numbers of predicted \( \nu_e \) and \( \bar{\nu}_e \) events in the absence of neutrino oscillations, whereas the primed quantities are the corresponding numbers of observed events, allowing for the presence of neutrino oscillations.

If at the top of atmosphere, i.e. the "source," the number of \( \nu_e \) (\( \bar{\nu}_e \)) and \( \nu_\mu \) (\( \bar{\nu}_\mu \)) are \( N_{\nu_e} (N_{\bar{\nu}_e}) \) and \( N_{\nu_\mu} (N_{\bar{\nu}_\mu}) \) respectively, while the cross-sections for \( \nu_e \) and \( \bar{\nu}_e \) are \( \sigma_{\nu_e} \) and \( \sigma_{\bar{\nu}_e} \); then we obtain the following set of event predictions for the detector:

\[ N_e = N_{\nu_e} \sigma_{\nu_e} \quad (8) \]
\[ N_{\bar{\nu}_e} = N_{\bar{\nu}_e} \sigma_{\bar{\nu}_e} \quad (9) \]
\[ N'_e = N_{\nu_e} \frac{P(\nu_e \xrightarrow{L} \nu_e) \sigma_{\nu_e} + N_{\nu_\mu} P(\nu_\mu \xrightarrow{L} \nu_e) \sigma_{\nu_\mu}}{N_{\nu_e} \sigma_{\nu_e} + N_{\nu_\mu} \sigma_{\nu_\mu}} \quad (10) \]
\[ N'_{\bar{\nu}_e} = N_{\bar{\nu}_e} \frac{P(\bar{\nu}_e \xrightarrow{L} \bar{\nu}_e) \sigma_{\bar{\nu}_e} + N_{\bar{\nu}_\mu} P(\bar{\nu}_\mu \xrightarrow{L} \bar{\nu}_e) \sigma_{\bar{\nu}_\mu}}{N_{\nu_e} \sigma_{\nu_e} + N_{\nu_\mu} \sigma_{\nu_\mu}}. \quad (11) \]

The first two equations correspond to absence of flavor oscillations, and the last two equations incorporate effects of flavor oscillations of neutrinos.

Now, inserting Eqs. (10) and (11) into Eq. (7), and taking note of the fact that due to CPT symmetry,

\[ P(\nu_\mu \xrightarrow{L} \nu_e) = P(\bar{\nu}_\mu \xrightarrow{L} \bar{\nu}_e), \]

we arrive at

\[ R_e - P(\nu_e \xrightarrow{L} \nu_e) = \frac{N_{\nu_e} P(\nu_\mu \xrightarrow{L} \nu_e) \sigma_{\nu_e} + N_{\nu_\mu} P(\bar{\nu}_\mu \xrightarrow{L} \bar{\nu}_e) \sigma_{\bar{\nu}_e}}{N_{\nu_e} \sigma_{\nu_e} + N_{\nu_\mu} \sigma_{\nu_\mu}}. \quad (12) \]
Finally, on defining
\[
\frac{N_{\bar{\nu}_e}}{N_{\nu_e}} = x, \quad \frac{N_{\bar{\nu}_\mu}}{N_{\nu_\mu}} = y \quad \frac{\sigma_{\bar{\nu}_e}}{\sigma_{\nu_e}} = \lambda, \quad \frac{N_{\nu_e}}{N_{\nu_e}} = r,
\]
(13)
it is easy to show that
\[
R_e - P(\nu_e \xrightarrow{L} \nu_e) = \frac{r_1 + \lambda x}{1 + \lambda x} \left( P(\nu_\mu \xrightarrow{L} \nu_e) + \lambda y P(\bar{\nu}_\mu \xrightarrow{L} \bar{\nu}_e) \right).
\]
(14)

Now, substituting Eqs. (4,6) into the above equation, and after simplifying, we obtain
\[
\begin{align*}
\left\{ |U_{e1}|^2 |U_{e2}|^2 + r \frac{1 + \lambda y}{1 + \lambda x} \text{Re}(U_{\mu1}^* U_{e1}^* U_{\mu2}^* U_{e2}) \right\} \sin^2 (\varphi_{12}) \\
+ \left\{ |U_{e1}|^2 |U_{e3}|^2 + r \frac{1 + \lambda y}{1 + \lambda x} \text{Re}(U_{\mu1}^* U_{e1}^* U_{\mu3}^* U_{e3}) \right\} \sin^2 (\varphi_{13}) \\
+ \left\{ |U_{e2}|^2 |U_{e3}|^2 + r \frac{1 + \lambda y}{1 + \lambda x} \text{Re}(U_{\mu2}^* U_{e2}^* U_{\mu3}^* U_{e3}) \right\} \sin^2 (\varphi_{23}) \\
- \frac{r_1 - \lambda y}{2} J_{CP} [\sin (2 \varphi_{12}) + \sin (2 \varphi_{13}) + \sin (2 \varphi_{23})] \\
= \frac{1}{4} (1 - R_e).
\end{align*}
\]
(15)

It is worth noting that in case \(x = y\) and \(J_{CP} = 0\), i.e., if the ratio of the numbers of \(\bar{\nu}_e\) to \(\nu_e\) equals the ratio of the numbers of \(\bar{\nu}_\mu\) to \(\nu_\mu\) at the source, and if there is no CP violation in the neutrino sector, Eq. (15) looses dependence on the neutrino and anti-neutrino cross sections.

In order that Eq. (15) holds for all values of \(L/E\) we must impose the constraints:
\[
\frac{r_1 - \lambda y}{2} J_{CP} = 0
\]
(16)

and
\[
\begin{align*}
|U_{e1}|^2 |U_{e2}|^2 + r \frac{1 + \lambda y}{1 + \lambda x} \text{Re}(U_{\mu1}^* U_{e1}^* U_{\mu2}^* U_{e2}) &= 0 \\
|U_{e1}|^2 |U_{e3}|^2 + r \frac{1 + \lambda y}{1 + \lambda x} \text{Re}(U_{\mu1}^* U_{e1}^* U_{\mu3}^* U_{e3}) &= 0 \\
|U_{e2}|^2 |U_{e3}|^2 + r \frac{1 + \lambda y}{1 + \lambda x} \text{Re}(U_{\mu2}^* U_{e2}^* U_{\mu3}^* U_{e3}) &= 0.
\end{align*}
\]
(17)

3. The constrained CP-violating matrix

\[\text{†}\]The Super-Kamiokande data spans roughly five orders of magnitude in \(L/E\). However, as a mathematical theorem, it can be shown that if \(R_e\) carries an \(L/E\) independence over a finite range of \(L/E\) then it must be so over the entire range of \(L/E\).
Since Eq. (15) holds for any value of $L/E$, we are also free to set $L/E = 0$. This yields:

$$R_e = 1.$$  (20)

Although we invoke the Super-Kamiokande observed flatness for $R_e$ from the beginning, we did not refer to a specific value of $R_e$. The present analysis predicts $R_e$ to be unity. This circumstance is in sharp contrast to the framework of references [7, 8] where one assumes both the indicated flatness and the value unity for $R_e$.

Furthermore, Eq. (16) requires that $J_{CP} = 0$ and/or $\lambda y = 1$. We consider each of these cases in turn.

3.1. $J_{CP} = 0$ Case:

The constraints (16-19), after some algebraic manipulations, reduce to:

$$c_{12}s_{12}c_{13}^2 + \frac{1 + \lambda y}{1 + \lambda x}(c_{12}s_{12}(s_{23}^2s_{13}^2 - c_{23}^2) + (s_{12}^2 - c_{12}^2)c_{23}s_{23}s_{13}) = 0$$  (21)

$$c_{12}s_{13} - r\frac{1 + \lambda y}{1 + \lambda x}s_{23}(c_{12}s_{23}s_{13} + s_{12}c_{23}) = 0$$  (22)

$$s_{12}s_{13} - r\frac{1 + \lambda y}{1 + \lambda x}s_{23}(s_{12}s_{23}s_{13} - c_{12}c_{23}) = 0.$$  (23)

We find no non-trivial solution that satisfies this set of equations. However, a limit of the second case to be considered next does yield a non CP violating mixing matrix and reproduces the results given in Ref. [8].

3.2. $\lambda y = 1$ Case:

According to the definition, $\lambda y = 1$ indicates that, if the ratio of the numbers of $\nu_\mu$ to $\bar{\nu}_\mu$ is close to the ratio of the cross-sections of $\bar{\nu}_e$ to $\nu_e$, then this circumstance allows to ignore the last term on the left hand side of Eq. (15). From Table 1 of Ref. [18] we estimate $y \approx 2.06 \pm 0.31$, while from Ref. [19] we infer $\lambda \approx 1/2.4$. Thus, the required condition is fulfilled on “accidental” grounds. Further justification for ignoring the indicated term lies in the fact that $J_{CP}$ is significantly suppressed by data-indicated $U_{e3} \ll 1$. In any case $E$-dependent deviations from $\lambda y = 1$ would contribute to departures from the exact $L/E$ flatness of the e-like event ratio. Similarly, we point out that in certain range of $L/E$ the matter effects may become operative, and these too would contribute to the indicated departure.

Substituting the relevant elements of $U$ into Eqs. (17-19), similarly, we obtain

$$c_{12}s_{12}c_{13}^2 + \frac{2r}{1 + \lambda x}(c_{12}s_{12}(s_{23}^2s_{13}^2 - c_{23}^2) + (s_{12}^2 - c_{12}^2)c_{23}s_{23}s_{13}) = 0$$  (24)

$$c_{12}s_{13} - \frac{2r}{1 + \lambda x}s_{23}(c_{12}s_{23}s_{13} + s_{12}c_{23}) = 0$$  (25)

$$s_{12}s_{13} - \frac{2r}{1 + \lambda x}s_{23}(s_{12}s_{23}s_{13} - c_{12}c_{23}) = 0.$$  (26)

From Eqs. (25,26) we infer,

$$s_{23}^2 = \frac{1 + \lambda x}{2r}$$  (27)

‡It being the value associated with the lowest atmospheric density in the experiment, identified here as “the top of the atmosphere.”
and

\[ c_\delta = 0. \] (28)

So, the CP phase is \( \pi/2 \) or \(-\pi/2\). Inserting Eqs. (27, 28) into Eq. (24), we have

\[ c_{23}^2 = \frac{1 + \lambda x}{2r} \] (29)

Finally, combining Eq. (27) and Eq. (29), we achieve the results:

\[ \theta_{23} = \pi/4, \quad r = 1 + \lambda x, \] (30)

That is, the mixing between the second and the third generations is maximal, and that the ratio of the numbers of \( \nu_\mu \) to \( \nu_e \) equals to one plus the ratio of the numbers of \( \bar{\nu}_e \) to \( \nu_e \) events in case of no oscillations.

As a result, the indicated L/E flatness in the the Super-Kamiokande data on the atmospheric neutrinos implies CP-violating maximal mixing matrix:

\[
U^\pm = \begin{pmatrix}
-\frac{1}{\sqrt{2}} (s_{12} \pm i c_{12} s_{13}) & \frac{1}{\sqrt{2}} (c_{12} \mp i s_{12} s_{13}) & \mp i s_{13} \\
\frac{1}{\sqrt{2}} (s_{12} \mp i c_{12} s_{13}) & -\frac{1}{\sqrt{2}} (c_{12} \pm i s_{12} s_{13}) & c_{13} / \sqrt{2} \\
\frac{1}{\sqrt{2}} (s_{12} \pm i c_{12} s_{13}) & \frac{1}{\sqrt{2}} (c_{12} \mp i s_{12} s_{13}) & -c_{13} / \sqrt{2}
\end{pmatrix}
\] (31)

where \( U^+ \) corresponds to \( \delta_{13} = \pi/2 \), and \( U^- \) arises from \( \delta_{13} = -\pi/2 \).

4. Concluding Remarks

Corresponding to the two general forms for \( U \), we obtain the following two measures of CP violation:

\[
J_{CP}^\pm = \pm \frac{1}{2} c_{12} s_{12} c_{13}^2 s_{13} = \pm \frac{1}{8} \sin (2\theta_{12}) \sin (2\theta_{13}) \cos (\theta_{13})
\] (32)

In the limit \( \theta_{13} \) vanishes the \( U^\pm \) reduces to the result contained in Eq. (26) of Ref. [8], as it should. Preliminary indications that the \( U \) matrix carries a similar form as given in Eq. (31) can also be deciphered from a recent work of Barger, Geer, Raja, and Whisnant [20]. Furthermore, for \( \theta_{12} = \pi/4 \), \( U^+ \) reads

\[
U^+ = \begin{pmatrix}
c_{13} / \sqrt{2} & c_{13} / \sqrt{2} & -i s_{13} \\
-(1 + i s_{13}) / 2 & (1 - i s_{13}) / 2 & c_{13} / \sqrt{2} \\
(1 - i s_{13}) / 2 & -(1 + i s_{13}) / 2 & c_{13} / \sqrt{2}
\end{pmatrix}
\] (33)

which coincides with the Xing postulate [21]. The latter, originally invoked to simultaneously allow for the a neutrino-oscillation explanation of the atmospheric and solar neutrino data, turns out to be dictated upon us by the indicated L/E flatness.

Since the CHOOZ experiment [22] constraints, for large-\( \delta m^2 \), \( \sin^2 (2\theta_{13}) \) to be about 0.1, even the large value of \( \delta_{13} = \pm \pi/2 \) implied by the present analysis, does not result in a maximal CP-violating difference:

\[
P(\nu_\alpha \xrightarrow{L} \nu_\beta) - P(\bar{\nu}_\alpha \xrightarrow{L} \bar{\nu}_\beta) = 4 J_{CP} \sum_{j<k} \sin (2 \varphi_{jk})
\] (34)

However, we note that Eqs. (4, 6) define a set of flavor-oscillation clocks, and these clocks must red-shift when introduced in a gravitational environment. If this environment is characterized by a dimensionless gravitational potential, \( \Phi_{grav} \), then in
order that the flavor-oscillations suffer a gravitationally-induced red-shift we must replace, in Eq. (34), $\varphi_{jk}$ by $(1 + \Phi_{\text{grav}}) \varphi_{jk}$. For other quantum-gravity effects on neutrino oscillations we refer the reader to Ref. [19]. Such gravitationally-induced modifications to a neutrino-sector CP violation may carry significant physical implications.

5. Summary

In summary, firstly, our discussion extended in this work seems to obligate us to accept a CP violated neutrino sector. And secondly, once CP is violated in neutrino system, the exact $L/E$ flatness of $R_e$ implies that: (i) The mixing between the second and the third generations must be maximal, (ii) The ratio $R_e$ must be unity, (iii) The CP-violating phase in the standard parameterization matrix is $\pi/2$ up to a sign ambiguity, (iv) $N_{\nu_e} \sigma_{\nu_e} = N_{\bar{\nu}_e} \sigma_{\bar{\nu}_e}$, and finally that (v) $N_{\nu_e}/N_{\nu_e} = 1 + N_{\bar{\nu}_e} \sigma_{\bar{\nu}_e}/N_{\nu_e} \sigma_{\nu_e}$.

Therefore, a dedicated study of the ratio $R_e$ in terms of its precise value, and its $L/E$ dependence, can become a powerful probe to study CP violation in the neutrino sector. Within the framework of this study, if the future data confirms $R_e$ to be unity for all zenith angles, then we must conclude that either there is no CP violation in the neutrino sector, or it is of the form predicted by equation (32). This precise result, in conjunction with knowledge of $\theta_{12}$, $\theta_{13}$, and the associated mass-squared differences, up to a sign ambiguity, completely determines the expectations for CP violation in all neutrino-oscillation channels.

However, the assumptions made in arriving the above results may be violated to some extent, and we once again point out that the $E$-dependent deviations from $\lambda_y = 1$ would contribute to departures from the exact $L/E$ flatness of the e-like event ratio. Similarly, we note that in certain range of $L/E$ the matter effects may become operative, and these too would contribute to the indicated departure. Once deviations from $\lambda_y = 1$ are fully incorporated, the study of the $L/E$ flatness of the e-like event ratio at Super-Kamiokande probes not only CP violation in the neutrino sector, but it also explores absence/presence of matter effects in atmospheric neutrino oscillations. At present the available data contains significant systematic and statistical errors, and, for that reason, these higher order corrections are left to a future investigation.

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