Optimal forwarding ratio on dynamical networks with heterogeneous mobility

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Received 12 November 2012 / Received in final form 15 January 2013
Published online 7 May 2013 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2013

Abstract. Since the discovery of non-Poisson statistics of human mobility trajectories, more attention has been paid to understand the role of these patterns in different dynamics. In this study, we first introduce the heterogeneous mobility of mobile agents into dynamical networks, and then investigate packet forwarding strategy on the heterogeneous dynamical networks. We find that the faster speed and the higher proportion of high-speed agents can enhance the network throughput and reduce the mean traveling time in random forwarding. A hierarchical structure in the dependence of high-speed is observed: the network throughput remains unchanged at small and large high-speed value. It is also interesting to find that a slightly preferential forwarding to high-speed agents can maximize the network capacity. Through theoretical analysis and numerical simulations, we show that the optimal forwarding ratio stems from the local structural heterogeneity of low-speed agents.

1 Introduction

The notion of mobile agent is really important in the study of many dynamical systems, such as collective motion in biology [1], information transmission on wireless ad-hoc network [2], and traffic congestion in city transportation system [3]. In all of these dynamical systems, each mobile agent is free to move independently in any direction, and locally interacts with current nearby agents. Take wireless ad-hoc network for example [2]. The continuous movement of mobile devices will frequently change their links to other devices due to limited forwarding range. Thus, these dynamical systems can be treated primitively as dynamical networks, in which connections between nodes always change temporarily. Network theory provides, therefore, a natural framework to study the emergence of structural and dynamical properties of these systems [4–8].

Physicists recently became increasingly fascinated by the dynamics on dynamical networks, such as epidemic spreading of mobile individuals [9–13], synchronization of mobile oscillators [14–16], and consensus of communicating agents [17]. Especially, for packet forwarding of mobile devices, how to enhance transmission efficiency has become a crucial problem with the rapid development of wireless communication technology. Considering cost and technology limitation, more efforts should be made to improve the routing strategy [2,18–23]. Yang et al. first studied transportation dynamics on dynamical networks [24], and compared random routing with greedy routing [25].

However, previous researches have only focused on the case of identical mobile agents, which cannot reflect the diversity of mobile individuals. Since the discovery of non-Poisson statistics of human behaviors such as human interaction activities [26] and mobility trajectories [27,28], more and more scientists have been paying attention to the role of these patterns in different dynamics. Most recent research results show that both time and space activities have significant impacts on spreading dynamics [29–36]. In this paper, we introduce the heterogeneous mobility of mobile agents into dynamical networks, and then propose an optimal forwarding strategy on the dynamical networks with heterogeneous mobility.

2 Model

We consider \(N\) agents performing random walk on an area of \(L \times L\) with periodic boundary conditions. Initially, the agents are randomly put on the area. Let \(v_i(t)\) be the speed and \(\theta_i(t)\) be the angle with respect to the \(x\)-axis characterizing the direction of the velocity of agent \(i\), and \(x_i(t)\) and \(y_i(t)\) be the coordinates of the agent \(i\) at time \(t\). The coordinates and the velocity of agent \(i\) at time \(t + 1\) are

\[x_i(t + 1) = x_i(t) + v_i(t) \cos(\theta_i(t)), \quad y_i(t + 1) = y_i(t) + v_i(t) \sin(\theta_i(t)).\]
updated according to
\[ \begin{align*}
  x_i(t + 1) &= x_i(t) + v_i(t) \cos \theta_i(t), \\
  y_i(t + 1) &= y_i(t) + v_i(t) \sin \theta_i(t), \\
  v_i(t + 1) &= v_i, \\
  \theta_i(t + 1) &= \xi_i(t),
\end{align*} \]
where \( \xi_i(t) \) is a random variable chosen uniformly between the interval \([−\pi, \pi]\) after the coordinates are updated. To account for heterogeneous mobilities of the mobile agents [27,28], we assume that the agents only take on either a low speed \( v_l \) or a high speed \( v_h \). The fraction of high speed agents is \( f \), i.e., the system is characterized by a distribution of speeds \( D(v) \) among the agents represented by
\[ D(v) = (1 - f)\delta(v - v_l) + f\delta(v - v_h), \]
where \( \delta(x) \) is the Dirac \( \delta \)-function. The speed of an agent, once assigned initially, does not vary with time. Therefore, an agent moves at a constant speed but in random directions.

To consider the packet routing efficiency on a network of mobile agents, let \( R \) packets be generated at one time step. Each of these packets is generated randomly at some agent \( i \) and placed at the end of the queue of agent \( i \), with a destination that is also randomly chosen among the agents in the system. To forward the packet to its destination, each agent is capable of forwarding at most \( C \) packets in its queue to the selected neighbors at every time step. The delivery follows a First-In-First-Out policy. As the agents are mobile and in view of the connectivity of electronic devices, we regard the agents that are instantaneously within an area of radius \( r \) of an agent \( i \) to be the neighbors, i.e., the neighbors of agent \( i \) are those with \( d_{ij} < r \), where \( d_{ij} \) is the distance between agent \( i \) and agent \( j \). The radius \( r \) thus sets the range over which a packet could be forwarded in a time step. In this point of view, a mobile-agent network corresponds to a dynamical network, with the links constantly broken and established as the agents move. The algorithm for forwarding a packet goes as follows. If the destination of a packet is found among the neighbors, the packet will be delivered to its destination and removed immediately. If not, an agent decides whether to forward it to a high-speed neighbor with a probability \( p \) or to a low-speed neighbor with a probability \( 1 - p \). After making a decision, a neighbor of the chosen type is randomly picked and then the forwarded packet is put at the end of the queue of the selected neighbor. If there is no neighbor of the chosen type, the packet will not be delivered. The process is repeated for each packet to be forwarded and every agent carries out the algorithm in every time step. As there is a fraction \( f \) of high-speed agents in the system, the case of \( p = f \) is roughly equal to the case of randomly choosing a neighbor to forward a packet regardless of its type, and the case of \( p < f \) (\( p > f \)) corresponds roughly to forwarding a packet to low-speed (high-speed) neighbors preferentially.

Let \( S(t) \) be the total number of packets in the system at time \( t \). In the balanced phase, the number of delivered and removed packets balances that of packets generated. As the packet generation rate \( R \) increases, there will be a critical value of \( R_c \) that characterizes the traffic phase transition from free flow to congested state. In the congested phase, more and more packets will be accumulated in the system. These two phases can be characterized by [23]:
\[ \eta = \frac{C}{R} \lim_{t \to \infty} \frac{S(t + \Delta t) - S(t)}{\Delta t}, \]
which plays the role of the order parameter in that \( \eta = 0 \) in the balanced phase and \( \eta > 0 \) in the congested phase. For a given value of \( C \), the system is in the balanced (congested) phase for packet generation rates \( R < R_c \) (\( R > R_c \)). As a figure of merit for packet delivery, a higher \( R_c \) corresponds to a better algorithm.

### 3 Simulation results
First, we investigate how the high-speed agents influence the network throughput \( R_c \) in random forwarding, i.e., \( p = f \). Figure 1a depicts the phase transition (from free flow state to congestion state) for different \( v_h \) values when \( f = 0.4 \). \( \eta(R) \) is obtained by averaging over \( 10^3 \) time steps after disregarding \( 3.5 \times 10^3 \) initial steps as transients. (b) The network throughput \( R_c \) versus \( v_h \) when \( f = 0.2, 0.4, 0.8 \). The lines are the theoretical predictions from equation (10) where \( (T), k_{\nu, max}^l \) and \( k_{\nu, max}^h \) are obtained by numerical simulations. The parameters are chosen as \( N = 10^5, L = 10, r = 1, v_l = 0.001, C = 1, p = f \). The results are obtained by averaging over 10 independent realizations.
denotes more high-speed agents can enhance the network throughput. Then the question is: why do the speed $v_h$ and the proportion $f$ of high-speed agents influence the network throughput? We will discuss it later (see Eq. (16)).

Next, we consider the case of preferential forwarding, i.e., $p \neq f$. As shown in Figure 3a, it is very interesting to note that there is a maximum $R_m$ value when $p$ is slightly greater than $f$, e.g., $R_m \approx 15$ at $p_m \approx 0.48$ when $f = 0.4$ and $R_m \approx 22$ at $p_m \approx 0.88$ when $f = 0.8$. As $p_m > f$ corresponds to the preferential forwarding to high-speed agents in Figure 3b, this optimal phenomenon means that the slightly preferential forwarding can enhance network throughput to some extent. For example, $R_m \approx 22(p \approx 0.88) > R_c \approx 16(p = 0.8)$ when $f = 0.8$. Moreover, Figure 4 shows that the faster speed and the higher proportion of high-speed agents can reduce the mean traveling time $\langle T \rangle$ (defined as the average number of hops for all data packets from their sources to destinations) in the case of random forwarding.

**4 Theory**

In order to understand the above optimal phenomenon, here, we present a mean field analysis of network throughput. As all agents are randomly distributed on the planar space at any time $t$ due to the random walk of agents, the degree distribution will be approximated by [37]:

$$P(k) = \frac{e^{-\langle k \rangle} \langle k \rangle^k}{k!},$$

where $\langle k \rangle = N \pi r^2 / L^2$.

In the balanced phase, the number of delivered and removed packets balances that of packets generated. At each
time step, all packets on each node will be delivered because the packet number is smaller than the processing capacity $C$. According to the forwarding strategy, the evolution of the packet numbers of low-speed agent $l$ with $k_l(t)$ and high-speed agent $h$ with $k_h(t)$ can be written as:

$$
\frac{dn_l(t)}{dt} = -n_l(t) + \sum_{j=1}^{N} \frac{(1-p)A_{lj}n_j(t)}{1-fk_j(t)} , \\
\frac{dn_h(t)}{dt} = -n_h(t) + \sum_{j=1}^{N} \frac{pA_{hj}n_j(t)}{f k_j(t)} ,
$$

(5)

where the sum runs over all nodes of the network, and $A_{ij}$ is the element of the adjacency matrix, in which $A_{ij} = 1$ when $d_{ij} < r$, otherwise $A_{ij} = 0$. At large time $t$, all agents will reach the balance of traffic flow, and we have

$$
n_l(t) = \frac{\sum_{j=1}^{N} (1-p)A_{lj}n_j(t)}{1-fk_j(t)} ,
$$

$$
n_h(t) = \frac{\sum_{j=1}^{N} pA_{hj}n_j(t)}{f k_j(t)} ,
$$

(6)

Considering $N_f$ high-speed agents randomly distributed on the planar space, we know that an agent with $k_l(t)$ has $f(k_l(t))$ high-speed neighbors and $(1-f)k_l(t)$ low-speed neighbors. We suppose $n_l(t) = B_1 k_l(t)$ and $n_h(t) = B_2 k_h(t)$, and then equation (6) is transformed into

$$
n_l(t) = \frac{(1-p)k_l(t)}{1-f}[(1-f)B_1 + f B_2] ,
$$

$$
n_h(t) = \frac{pk_h(t)}{f}[(1-f)B_1 + f B_2] ,
$$

(7)

where the occupation packet number is independent of degree correlations [37]. From the above, we know that $B_1/B_2 = [(1-p)f]/[p(1-f)]$, and equation (7) is also written as:

$$
n_l(t) = B(1-p)k_l(t) \frac{1-f}{1-f} ,
$$

$$
n_h(t) = Bpk_h(t) \frac{1-f}{f} ,
$$

(8)

where $B$ is a constant. In the balanced phase, there are $S = \sum_l n_l(t) + \sum_h n_h(t) \approx R(T)$ packets on the network at each time step [38]. Submitting equations (4) and (8) into this self-consistent relationship, we obtain

$$
B = \frac{R(T)}{N(k)}
$$

(9)

At the critical point, the dynamic balance of traffic flow happens in a relatively long time with the same order as $O(T)$, which is different from the case in the balanced phase. The node possessing the maximum occupation packet number must meet the relationship

$$
\sum f^O(T) n_{max}(t) \approx CO(T) \text{ in } O(T) \text{ steps. According to equations (8) and (9), we have}
$$

$$
\frac{R(T)p k_{l,max}^c}{N(k)f} \leq C ,
$$

$$
\frac{R(T)(1-p)k_{h,max}^c}{N(k)(1-f)} \leq C ,
$$

(10)

where $k_{h,max}^c$ and $k_{l,max}^c$ are the maximum effective degree of high-speed agents and low-speed agents respectively, and the effective degree of node $i$ is defined as $k_i^c = \langle 1/p_i \rangle \sum_l k_l(t)$. Owing to the heterogeneous mobility, $k_{l,max}^c$ of low-speed agents is different from that of high-speed agents. All high-speed agents can span a broad area of planar space in $O(T)$ steps due to their fast moving. According to the law of large number, we have

$$
k_h^c = \lim_{O(T) \rightarrow \infty} \frac{1}{O(T)} \sum_t k_h(t) \simeq \langle k \rangle .
$$

(11)

The maximum effective degree of high-speed agents $k_{h,max}^c$ is approximatively equal to $\langle k \rangle$. On the other hand, low-speed agents are taken as motionless nodes in this stage because of its very slow speed. For this reason, the effective degree of a low-speed agent is divided into two parts: $f(k)$ high-speed neighbors and $k_l'$ settled low-speed neighbors. The number of settled low-speed neighbors follows a Poisson distribution

$$
P(k_l') = \frac{e^{-\langle k_l \rangle \langle k_l' \rangle k_l'^c}}{k_l'^c!} ,
$$

(12)

where $\langle k_l'^c \rangle = (1-f)N\pi r^2/L^2$. Thus, the distribution of $k_l^c$ follows

$$
P(k_l^c) = \frac{e^{-\langle k_l \rangle \langle k_l'^c \rangle - f(k)}\langle k_l^c \rangle}{(k_l^c - f(\langle k_l \rangle))!} .
$$

(13)

This feature is supported by the simulation results in Figure 5. The natural cut-off effective degree of low-speed agents can be calculated by [39]:

$$
N(1-f) \int_{k_{l,max}^c}^{\infty} P(k_l^c) dk_l^c \sim 1 .
$$

(14)

Submitting equation (11) into equation (10), the network throughput is given by:

$$
R_c = \operatorname{Min} \{ R_b^c , R_l^c \} ,
$$

(15)

and

$$
R_b^c = \frac{NFc}{(T)p} ,
$$

$$
R_l^c = \frac{N\langle k \rangle C(1-f)}{(T)(1-p)k_{l,max}^c} ,
$$

(16)

where $\operatorname{Min}\{}$ represents the less of the two. If $p = f$, equation (16) will go back to the case of random forwarding.
Fig. 5. Effective degree distribution for two kinds of agents when $f = 0.2$ (a) and $f = 0.8$ (b). The parameters are chosen as $N = 10^5, L = 10, r = 1, v_h = 0.001, v_h = 2, \mathcal{O}(\langle T \rangle) = 50$. The results are obtained by averaging over $10^6$ independent realizations. The lines are the theoretical predictions from equation (13).

When $v_h$ is much smaller than $r$, the dynamic network is approximatively taken as a fixed structure in a comparatively long time, so the network throughput changes very little. As $v_h$ increases to the value with the same order as $r$, the shorter mean traveling time, which is due to the longer forwarding distance in each hop, results in the increase of network throughput. For a large value of $v_h$, the network throughput remains unchanged because of the fixed mean traveling time and maximum degree [24]. For this reason, a hierarchical structure in the dependence of high-speed is observed in Figure 1b. In this case, $\langle T \rangle$ (due to more high-speed agents) and $k_{e, \text{max}}^f$ (from Eq. (14)) decrease with $f$. Thus, $R_e = R_e^0$ increases with $f$, which is validated by the results in Figure 2b. From equation (10), the theoretical predictions are basically consistent with the numerical results in Figures 1b and 2b. When $v_h$ is not very large (e.g., $v_h = 1$), there is a difference between the theoretical predictions and the numerical results due to the invalidation of effective degree hypothesis. An interesting work for future work is to present an analytical approach with the purpose of filling this gap.

When $f$ is fixed, $\langle T \rangle$ decreases with $p$ because the preferential forwarding to high-speed agents can increase the distance which packets are forwarded in one time step (see numerical results in Fig. 4). From equation (16), we know that $R_e^0$ decreases with $p$ because of the increase of $\langle T \rangle p$ (confirmed by numerical results in the inset of Fig. 4), while $R_e^0$ increases with $p$ because of the decrease of $\langle T \rangle (1 - p)$. There exists a maximum network throughput $R_{\text{max}}$ in Figure 3a. A difference between the theoretical predictions and the numerical results is originated from the real mean traveling time $\langle T \rangle_{\text{real}} > \langle T \rangle (C = \infty)$ at the critical point. As the dynamic balance of traffic flow exists in a relatively long time at the critical point, some packets may stay more than one time step on those nodes with more packets due to the fluctuation of traffic flow. When $R_e^0 = R_e^h$, the corresponding optimal $p_m$ is thus obtained by:

$$p_m = \frac{f k_{e, \text{max}}^f}{(1 - f)(k) + f k_{e, \text{max}}^f}. \quad (17)$$

Interestingly, $p_m$ only depends on the structure of the dynamical network. Owing to the Poisson characteristic of effective degree of low-speed agents, i.e., $k_{e, \text{max}}^f > \langle k \rangle$, $p_m$ will be slightly greater than $f$, which means that the optimal forwarding rate is the slightly preferential forwarding to high-speed agents. As shown in Figure 3b, the theoretical predictions are in good agreement with the simulation results.

5 Conclusion and discussion

In conclusion, we have investigated the forwarding strategy on the dynamical networks with heterogeneous mobility. First, we have shown the faster speed and the higher proportion of high-speed agents can enhance the network throughput and reduce the mean traveling time in random forwarding. A hierarchical structure in the dependence of high-speed is observed; the network throughput remains unchanged at small and large high-speed value. Second, it would be interesting to know that the slightly preferential forwarding to high-speed agents can maximize the network capacity. By combining theoretical analysis with numerical simulations, we have obtained the optimal forwarding rate, and found this optimal phenomenon stems from the local structural heterogeneity of low-speed agents. This work provides us further understanding and new perspective in the effect of human dynamics on the forwarding strategy on dynamical networks, and thus may help to optimize the delivering strategy on heterogeneous dynamical networks.

Ming Tang would like to thank Pakming Hui for stimulating discussions. This work is supported by the NNSF of China (Grant No. 11105025), China Postdoctoral Science Foundation (Grant No. 20110491705), the Specialized Research Fund for the Doctoral Program of Higher Education (Grant No. 20110185120021), China Postdoctoral Science Special Foundation (Grant No. 2012T50711), and the Fundamental Research Funds for the Central Universities (Grant No. ZYGX2011J656).

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