How Robust are Randomized Smoothing based Defenses to Data Poisoning?

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Abstract

The prediction of certifiably robust classifiers remains constant around a neighborhood of a point, making them resilient to test-time attacks with a guarantee. In this work, we present a previously unrecognized threat to robust machine learning models that highlights the importance of training-data quality in achieving high certified robustness. Specifically, we propose a novel bilevel optimization based data poisoning attack that degrades the robustness guarantees of certifiably robust classifiers. Unlike other data poisoning attacks that reduce the accuracy of the poisoned models on a small set of target points, our attack reduces the average certified radius of an entire target class in the dataset. Moreover, our attack is effective even when the victim trains the models from scratch using state-of-the-art robust training methods such as Gaussian data augmentation[7], MACER[35], and SmoothAdv[28]. To make the attack harder to detect we use clean-label poisoning points with imperceptibly small distortions. The effectiveness of the proposed method is evaluated by poisoning MNIST and CIFAR10 datasets and training deep neural networks using the previously mentioned robust training methods and certifying their robustness using randomized smoothing. For the models trained with these robust training methods our attack points reduce the average certified radius of the target class by more than 30% and are transferable to models with different architectures and models trained with different robust training methods.

1. Introduction

Data poisoning [3, 16, 30, 31, 36] is a training-time attack in which the attacker is assumed to have access to the training data on which the victim will train the model. The attacker can modify the training data by either injecting poison samples or modifying the existing data in a manner that the model trained on this poisoned data performs as the attacker desires. Since modern machine learning methods require large amounts of data, data poisoning becomes easy as an attacker can place the poisoned data online and wait for it to be scraped by victims looking to increase the size for their dataset. Another easy target for data poisoning is data collection by crowd sourcing where malicious users can corrupt the data they contribute. In most cases an attacker can modify only certain parts of the training data such as change the features or labels for a specific class or modify a small subset of the data from all classes.

In this work, we assume the attacker wants to affect the performance of the victim’s models on a target class and thus modifies the features of the points belonging to that class (without affecting the labels). To evade detection, the attacker is also assumed to only add imperceptibly small perturbations to the points of the target class. Many previous works [25, 30, 15, 18, 36, 6, 17, 32] have shown the effectiveness of using data poisoning to affect the accuracy of models trained on poisoned data compared to the accuracy of the models achievable by training with clean data. In most of these works it is assumed that the victim will use standard training by minimizing the empirical loss on the poisoned data to train the models and thus the attack points are optimized to hurt the accuracy of standard training. However, in light of the recent research on test-time evasion attacks [5, 1, 33, 4] where it was shown that models trained with standard training are not robust to adversarial examples, the assumption that a victim will use standard training to train the models for deployment might not hold.

Thus, in a realistic scenario, where the aim of the attacker is to make the victim deploy the poisoned model, its better to assume that the victim will use a training procedure that can generate classifiers which are provably robust to test-time evasion attacks. Recently, researchers have proposed several methods for training certifiably robust models whose predictions are guaranteed to be constant in a neighbourhood of a point. However, many of these methods [27, 12, 14, 34] do not scale to deep neural networks or large datasets, due to their high complexity. Moreover, the effect of training data quality on the performance of these certified defenses at test time remains largely unexplored. Recently, randomized smoothing (RS) based certification methods [19, 20, 7] were shown to be scalable to deep neural networks and high dimensional datasets enabling researchers to propose training procedures [28, 35] that lead to models with high certified
Table 1. Failure of traditional data poisoning attacks optimized against standard training in affecting the test accuracy (of target class) of models trained with certifiably robust training procedures. Details of the experiment are present in Appendix D.1. Certifiably robust training methods [7, 28, 35] are trained with \( \sigma = 0.25 \) and accuracy of their base classifiers are reported.

| Training method          | Model trained on clean data | Model trained on poison data |
|--------------------------|-----------------------------|-----------------------------|
| **MNIST**                |                             |                             |
| Standard                 | 99.28±0.01                  | 60.08±12.6                  |
| Gaussian augmented [7]   | 98.99±0.14                  | 98.31±1.65                  |
| SmoothAdv [28]           | 99.18±0.23                  | 99.31±0.29                  |
| MACER [35]               | 99.21±0.56                  | 98.31±0.58                  |
| **CIFAR10**              |                             |                             |
| Standard                 | 92.71±1.31                  | 0.36±0.37                   |
| Gaussian augmented [7]   | 88.84±2.39                  | 88.38±2.13                  |
| SmoothAdv [28]           | 79.48±2.69                  | 74.95±3.45                  |
| MACER [35]               | 87.12±1.17                  | 88.54±4.52                  |

robustness. Thus, in this work we assume that a victim will rely on certification methods based on RS to measure the certified robustness of the model trained with a robust training procedure. The fact that victim can use a robust training method on poisoned data is an immediate challenge for current data poisoning attack methods which optimize the poison data to affect the accuracy of standard training. As demonstrated in Table 1, the poison optimized against standard training can significantly reduce the accuracy of the victim’s model (left to right) when the victim also uses the standard training (1st and 5th row). However, the table also shows that the poison fails to affect the accuracy when the victim uses a certifiably robust training method [7, 28, 35].

Are certified defenses robust to data poisoning? In this work we study this question and demonstrate that data poisoning is a serious concern even for certified defenses. We propose a novel data poisoning attack that can significantly compromise the certified robustness guarantees achievable from training with robust training procedures. We formulate the poisoning against certified defenses (PACD) attack as a constrained bilevel optimization problem and theoretically analyze its solution for the case when the victim uses linear classifiers. Our theoretical analysis and empirical results suggest that the decision boundary of the smoothed classifiers (used for RS) learned from the poisoned data is significantly different from the one learned from clean data there by causing a reduction in certified radius. Our bilevel optimization based attack formulation is general since it can generate poisoned data against a model trained with any certifiably robust training method (lower-level problem) and certified with any certification procedure (upper-level problem). Fig. 1 shows the overview of the proposed PACD attack.

Unlike previous data poisoning attacks which aim to reduce the accuracy of the models on a small subset of data, our attack can reduce the certified radius of an entire target class in the dataset. The poisoning points generated by our method have clean labels and imperceptible distortion making the attack difficult to detect. Our attack remains effective when the victim trains the models from scratch or uses data augmentation during training. Moreover, we observe that the attack points generated against a certified defense remain effective (are transferable) to models trained with other RS based certified defenses as well as to models with different architectures. Our work highlights that training data quality and curation are critical for obtaining any meaningful gains from certified defenses at test time, a factor not considered by current certified defense research.

Our main contributions are

- We study the problem of using data poisoning attacks to affect the robustness guarantees of classifiers trained using certified defense methods. To the best of our knowledge, this is the first clean label poisoning attack that significantly reduces the certified robustness guarantees of the models trained on the poisoned dataset.
- We propose a bilevel optimization based attack which can generate poison data against several robust training and certification methods. We specifically use the attack to highlight the vulnerability of randomized smoothing based certified defenses to data poisoning.
- We demonstrate the effectiveness of our attack in reducing the certifiable robustness obtained using randomized smoothing on models trained with state-of-the-art certified defenses [7, 28, 35]. For these training methods our attack reduces the average certified radius of the target class by more than 30%.

2. Background and related work

Randomized smoothing: The RS procedure [7] uses a smoothed version of the original classifier \( f : \mathbb{R}^d \rightarrow Y \) and certifies the adversarial robustness of the new classifier. The smoothed classifier \( g(x) = \max_{\eta} \mathbb{P}_{\eta \sim N(0, \sigma^2 I)}(f(x + \eta) = c) \), meaning the label for a point \( x \) under \( g \) is the class whose decision region \( \{ x' \in \mathbb{R}^d : f(x') = c \} \) has the largest measure under the distribution \( N(x, \sigma^2 I) \), where \( \sigma \) is used for smoothing. Suppose that while classifying a point \( N(x, \sigma^2 I) \), the original classifier \( f \) returns the class \( c_A \) with probability \( p_A = \mathbb{P}(f(x + \eta) = c_A) \), and the “runner-up” class \( c_B \) is returned with probability \( p_B = \max_{c \neq c_A} \mathbb{P}(f(x + \eta) = c) \). Suppose that \( x \) is an adversarial example with \( g(x) = c \). Then, the prediction of mistake attack hypothesis \( g(x; \sigma) = \Phi^{-1}(p_A) - \Phi^{-1}(p_B) \), where \( \Phi^{-1} \) is the inverse CDF of the standard Normal distribution. In practice, Monte Carlo sampling is used to estimate a lower bound on \( p_A \) and an upper bound on \( p_B \) as it difficult to estimate the actual values for \( p_A \) and \( p_B \). Since standard training of the base classifier does not achieve high robustness guarantees, [7] proposed to use Gaussian data augmentation based training in which the base classifier is trained on
Gaussian noise corruptions of the clean data. Recent works [35, 28] showed that the certified robustness guarantees of RS can be boosted using different training procedures. In particular, [28] proposed to train the base classifier using adversarial training where the adversarial examples are generated against the smoothed classifier. Although effective at increasing the certified radius, their approach requires generation of adversarial examples against the smoothed classifier and is therefore slow to train. Another recent work [35] proposed a different training procedure which is significantly faster to train and relies on directly maximizing the certified radius for achieving high robustness guarantees.

Due to the effectiveness of these approaches in improving the certified robustness guarantees of the machine learning models, we craft poison data against these approaches. A recent attack method [10] showed that it is possible to fool a robust classifier to mislabel an input and give an incorrect certificate using perturbation large in $\ell_p$ norm at test-time. Our work is different from theirs since we focus on train-time attacks against certified defenses using imperceptibly small perturbations to the poison data.

**Bilevel optimization:** A bilevel optimization problem is of the form $\min_{u \in U} \xi(u, v^*)$ s.t. $v^* = \arg \min_{v \in V(u)} \xi(u, v)$, where the upper-level problem is a minimization problem with $v$ constrained to be the optimal solution to the lower-level problem (see [2]). Our data poisoning attack is formulated as a constrained bilevel optimization problem. Although general bilevel problems are difficult to solve, under some simplifying assumptions their solution can be obtained using gradient based methods. Several methods for solving bilevel problems in machine learning have been proposed previously [8, 26, 9, 21, 29, 23]. We review these in Appendix B and use the method based of approximating the hypergradient by approximately solving a linear system (ApproxGrad Alg. 2 in Appendix B) in this work. Previous works [24, 25, 23, 15] have shown the effectiveness of solving bilevel optimization problem for data poisoning to affect the accuracy of models trained with standard training. Our work on the other hand proposes a bilevel optimization based formulation to generate a data poisoning attack against RS based certified defenses and shows its effectiveness against state-of-the-art robust training methods.

### 3. Poisoning against certified defenses

In this section we present the bilevel optimization formulation of our PACD attack for generating poisoned data to compromise the certified robustness guarantees of the models trained using certified defenses. Specifically, we discuss how to use our attack to generate poison data against Gaussian data augmentation [7], SmoothAdv [28] and MACER [35] to degrade the certified robustness obtained with RS.

#### 3.1. General attack formulation

Let $D^{\text{clean}} = \{(x_i^{\text{clean}}, y_i^{\text{clean}})\}_{i=1}^{N_{\text{clean}}}$ be the clean, unalterable portion of the training set. Let $u = \{u_1, ..., u_n\}$ denote the attacker’s poisoning data which is added to the clean data. For clean-label attack, we require that each poison example $u_i$ has a limited perturbation, for example, $\|u_i - x_i^{\text{base}}\| = \|\delta_i\| \leq \epsilon$ from the base data $x_i^{\text{base}}$ and has the same label $y_i^{\text{base}}$, for $i = 1, ..., n$. Thus $D^{\text{poison}} = \{(u_i, y_i^{\text{base}})\}_{i=1}^{N_{\text{poison}}}$. The goal of the attacker is to find $u$ such that when the victim uses $D^{\text{clean}} \cup D^{\text{poison}}$ to train a classifier, the certified robustness guarantees of the model on the target class ($D^{\text{val}} = \{(x_i^{\text{val}}, y_i^{\text{val}})\}_{i=1}^{N_{\text{val}}}$) are significantly diminished compared to a classifier trained on clean data. The attack can be formulated as follows:

$$\min_{u \in U} \mathcal{R}(D^{\text{val}}, \theta^*)$$

s.t. $\theta^* = \arg \min_{\theta} \mathcal{L}_{\text{robust}}(D^{\text{clean}} \cup D^{\text{poison}}; \theta)$.  

(1)

The upper-level cost $\mathcal{R}$ denotes a certified robustness metric such as the certified radius from RS. The goal of the upper-level problem is to compromise the certified robustness guarantees of the model on clean validation data $D^{\text{val}}$. The solution to the lower-level problem $\theta^*$ are the parameters of

Figure 1. Overview of our poisoning against certified defenses (PACD) attack which generates poisoned data to reduce the certified robustness of the victim’s model trained with methods such as Gaussian data augmentation [7], SmoothAdv [28] and MACER [35] on a target class.
the machine learning model learned from \( \mathcal{D}^{clean} \cup \mathcal{D}^{poison} \) using a robust training method with loss function \( \mathcal{L}_{robust} \). The fact that any robust training method and any certification procedure can be incorporated into this formulation by changing the lower- and upper-level problems, respectively, makes the attack formulation broadly applicable. Recent works [7, 28, 35] have shown RS based methods to be effective at certifying and producing robust classifiers. The scalability of these methods to large datasets and deep models makes them useful for real-world applications. Thus, we focus on using our poisoning attack against these methods.

### 3.2. Poison randomized smoothing based defenses

For an input at test time RS produces a prediction from the smoothed classifier \( g \) and a radius in which this prediction remains constant. Since the certified radius of a “hard” smooth classifier \( g \) is non-differentiable, it cannot be directly incorporated in the upper-level of the attack formulation Eq. (1).

To overcome this challenge, we use the “soft” smooth classifier \( \tilde{g} \) as an approximation. Similar technique has been used in [28, 35]. Let \( z_0 : X \to \mathcal{P}(\mathcal{K}) \) be a classifier whose last layer is softmax with parameters \( \theta \) and \( \sigma > 0 \) is the noise used for smoothing, then soft smoothed classifier \( \tilde{g}_0 \) of \( z_0 \) is 
\[
\tilde{g}_0(x) = \arg \max_{y \in \mathcal{Y}} \mathcal{E}_{y \sim \mathcal{N}(0, \sigma^2 I)}[\gamma \hat{z}_0^\sigma(x + \eta)]
\]

It was shown in [35] that if the ground truth of an input \( x \) is \( y \) and \( \tilde{g}_0 \) classifies \( x \) correctly then \( \tilde{g}_0 \) is provably robust at \( x \), with the certified radius \( \tilde{R} \).

To generate poison data that reduces the robustness guarantee obtained by RS for a classifier trained with Gaussian data augmentation is as follows.

\[
\min \tilde{R}(\tilde{g}_0; \mathcal{D}^{val}, \sigma)
\]
s.t. \( \| \delta \|_\infty \leq \epsilon \), \( i = 1, \ldots, n \), and

\[
\theta^* = \arg \min_{\theta} \mathcal{L}_{GaussAug}(\mathcal{D}^{clean} \cup \mathcal{D}^{poison}, \theta, \sigma).
\]

### Poisoning against MACER [35]

Another recent work proposed a method for robust training by maximizing the certified radius (MACER). Their approach uses a loss function which is a combination of the classification loss and the robustness loss of the soft smoothed classifier \( \tilde{g}_0 \). In particular, the loss of the smoothed classifier on a point \((x, y)\) is given by

\[
\ell_{macer}(\tilde{g}_0; x, y) = -\log \tilde{z}_0^\sigma(x) + \alpha \max \{ \gamma - \tilde{z}_0^\sigma(x), 0 \} \cdot \delta_1(\tilde{g}_0(x) = y).
\]

where \( \eta_1, \ldots, \eta_k \) are i.i.d. samples from \( \mathcal{N}(0, \sigma^2 I) \), \( \tilde{z}_0^\sigma(x) = \frac{1}{k} \sum_{j=1}^k \tilde{z}_0^\sigma(x + \eta_j) \) is the empirical expectation of \( z_0(x + \eta) \), \( \tilde{z}_0^\sigma(x, y) = \Phi^{-1}(\tilde{z}_0^\sigma(x)) - \Phi^{-1}(\max_{y \neq y} \tilde{z}_0^\sigma(x, y)) \), \( \gamma \) is the hinge factor, and \( \lambda \) balances the accuracy and robustness trade-off. Using this we can define \( \ell_{macer}(\mathcal{D}; \theta, \sigma) = \frac{1}{|\mathcal{D}|} \sum_{(x, y) \in \mathcal{D}} \ell_{macer}(\tilde{g}_0; x, y) \). To generate poison data that reduces the robustness guarantees of classifier trained with MACER we can use the loss \( \ell_{macer}(\mathcal{D}; \theta, \sigma) \) in the lower-level problem in Eq. (2).
Poisoning against SmoothAdv [28]. In this work it was shown that the certified robustness guarantees obtained from RS can be improved by training the classifiers using adversarial training with adversarial examples generated against the smooth classifier. In particular the classifier trained with SmoothAdv optimizes the following objective for a point \( (x, y) \). \[ \min_{\theta} \max_{\|x' - x\|_2 \leq \alpha} - \log \frac{\sum_{j=1}^{k} \eta_j}{\sum_{j=1}^{k} \eta_j} (x' + \eta_j) \] where \( \eta_1, \ldots, \eta_k \) are \( k \) i.i.d. samples from \( \mathcal{N}(0, \sigma^2 I) \) and \( \alpha \) is the permissible \( \ell_2 \) distortion to \( x \). To generate poisoning data against SmoothAdv we must use this objective as the lower-level problem in Eq. (2). To make it easier for bilevel solvers to solve this problem we use an approximation to the min-max problem. For doing that we first compute the adversarial example \( x' = \arg \max_{\|x' - x\|_2 \leq \alpha} - \log \frac{\sum_{j=1}^{k} \eta_j}{\sum_{j=1}^{k} \eta_j} (x' + \eta_j) \) using PGD attack on the points in \( D_{\text{clean}} \cup D_{\text{poison}} \) and then use these examples as our new dataset to train the model parameters in the lower-level as in Eq. (2).

Specifically, the lower-level problem in Eq. (2) becomes \[ \arg \min_{\theta} \mathcal{L}_{\text{GaussAug}}(D_{\text{adv}} \cup D_{\text{adv}}; \theta, \sigma) \] where \( D_{\text{adv}} \) denotes the adversarial examples generated against \( \tilde{g}_{\theta} \). We update \( D_{\text{adv}} \) in every step of the bilevel optimization.

### 3.3. Generation and evaluation of poisoning attack

In this work, we focus on creating a poisoned set to compromise the certified adversarial robustness guarantees of all points in a target class. We initialize the poisoning data with clean data from the target class (i.e., base data) and optimize the perturbation to be added to each point by solving the bilevel problem in Eq. (2) for attack against Gaussian data augmentation based training. We use a small value of \( \epsilon \) to ensure the perturbations added are imperceptible and the poison points have clean labels when inspected visually (See Fig. 5 in the Appendix). The bilevel optimization is solved using the ApproxGrad algorithm described in Alg. 2 in Appendix B. The full attack algorithm for generating poisoning data against the Gaussian data augmentation based defense of [7] is shown Alg. 1. Attack against other methods are generated similarly by replacing the lower-level objective \( \zeta \) in Alg. 1 with the appropriate loss function for MACER[35] and SmoothAdv[28]. We evaluate the effect of poisoning, by training the models from scratch using Gaussian data augmentation approach of [7], MACER [35] and SmoothAdv [28] on their respective poisoned sets and report the average certified radius and approximate certified accuracy on the clean test points from the target class.

### 3.4. Analysis of poisoning with linear classifiers

To gain a deeper insight into the effect of poisoning, we analyze the analytical solution of our bilevel problem for the case of linear classifiers trained with Gaussian data augmentation. Suppose we have a one-dimensional two-class problem and the attacker’s goal is to poison the distribution of the negative class \( P^- \) so that the average certified radius \( \bar{R} \) of the poisoned model on the test points of the negative class is reduced. Let the maximum permissible perturbation to the points of the class \( P^- \) be bounded by \( |u_i - x_i^-| \leq \epsilon \), for \( i = 1, ..., n \). We do not assume any specific distributions for \( P^+ \) and \( P^- \) here, but only that \( \sum_{i} x_i^+ < \sum_{i} x_i^- \) without loss of generality. Here \( x_i^+ \) and \( x_i^- \) refer to the training points of the positive and the negative class, respectively. A linear classifier in one-dimension is either \( f(x) = 1 \) iff \( x \geq t \) or \( f(x) = 1 \) iff \( x \leq t \) parameterized by the threshold \( t \). For linear classifiers, it is known [7] that for any value of \( \sigma \) used for smoothing, the smoothed classifier \( g \) is the same as the unsmoothed classifier \( f \) and the certified radius for a point is the distance to the decision boundary. To make the problem analytically tractable, we use the squared-loss for the linear classifier at the lower-level i.e., \( f(x) = wx + b \) and \( l(x, y; w, b) = (wx + b - y)^2 \).

The bilevel formulation for the poisoning problem is
\[
\min_{u} \mathbb{E}_{P_-} \left[ \max \{ \text{sign}(w^*) (-b^*/w^* - x), 0 \} \right] \\
\text{s.t.} \quad -\epsilon \leq u_i - x_i^- \leq \epsilon, \quad \text{for} \quad i = 1, ..., n \\
w^*, b^* = \arg \min_{w, b} \frac{1}{2n} \sum_{i=1}^{n} l(x_i^+, 1) + \sum_{i=1}^{n} l(u_i, -1) \tag{3}
\]

**Theorem 1.** If the perturbation is large enough, i.e., \( \epsilon \geq \sum_{i} x_i^+ - \sum_{i} x_i^- \) then there are two locally optimal solutions to (3) which are \( u_i = x_i^- - \epsilon \) (Case 1) and \( u_i = x_i^- + \epsilon \) (Case 2) for \( i = 1, ..., n \). Otherwise, there is a unique globally optimal solution which is \( u_i = x_i^- - \epsilon \) (Case 1) for \( i = 1, ..., n \).

The theorem states that the optimal poisoning is achieved by shifting all points of the negative class either towards left or right by the maximum amount \( \epsilon \) (See Fig. 2 and Appendix D.2). For the nonlinear case, direct analysis is intractable, but we empirically observe that poisoning on neural networks also moves the decision boundary closer to the points in the target class as measured by the mean dis-
Table 2. Decrease in certified radius and certified accuracy of models trained with Gaussian augmentation [7] on poison data compared to those of models trained on clean and watermarked data.

| σ  | Data          | Certified radius of target class ACR | ACA(%)  |
|----|---------------|-------------------------------------|---------|
|    | Clean         | 0.986±0.01                          | 98.92±0.32 |
| 0.25| Watermarked   | 0.908±0.01                          | 99.24±0.29 |
|     | Poisoned      | 0.325±0.10                          | 71.96±8.28 |
| 0.5 | Clean         | 1.481±0.02                          | 99.16±0.34 |
|     | Watermarked   | 1.514±0.06                          | 99.12±0.47 |
|     | Poisoned      | 0.733±0.10                          | 90.68±3.37 |
| 0.75| Clean         | 1.549±0.11                          | 98.48±0.35 |
|     | Watermarked   | 1.566±0.06                          | 98.36±0.39 |
|     | Poisoned      | 0.698±0.13                          | 84.92±5.14 |

Table 3. Decrease in certified radius and certified accuracy of models trained with MACER [35] on poison data compared to those of models trained on clean and watermarked data.

| σ  | Data          | Certified radius of target class ACR | ACA(%)  |
|----|---------------|-------------------------------------|---------|
|    | Clean         | 0.915±0.01                          | 99.64±0.21 |
| 0.25| Watermarked   | 0.894±0.01                          | 98.84±0.53 |
|     | Poisoned      | 0.431±0.13                          | 79.81±9.26 |
| 0.5 | Clean         | 1.484±0.11                          | 98.56±0.41 |
|     | Watermarked   | 1.475±0.08                          | 98.68±0.39 |
|     | Poisoned      | 0.685±0.16                          | 84.36±6.17 |
| 0.75| Clean         | 1.353±0.13                          | 93.81±2.08 |
|     | Watermarked   | 1.415±0.11                          | 94.52±1.58 |
|     | Poisoned      | 1.008±0.19                          | 88.41±4.64 |

This bottleneck we leave the problem of poisoning ImageNet for future work. For the experiments with MNIST we randomly selected digit 8 and for CIFAR10 the class “Ship” as the target class for the attacker. The attack results for other target classes are similar and are presented in the Appendix D. To ensure that the attack points satisfy the clean label constraint, the maximum permissible $\epsilon_\infty$ distortion is bounded by $\epsilon = 0.1$ for MNIST and $\epsilon = 0.03$ for CIFAR10 which is similar to the value used to generate imperceptible adversarial examples in previous works [22, 11]. We used convolutional neural networks for our experiments with MNIST and the Resnet-20 model for our CIFAR10 experiments. Model architectures, hyperparameters, generated attack examples (Fig. 5 in Appendix), and additional results on transferability of our poisoned samples to different architectures are presented in Appendix D. In this work we present poisoning attack with models trained using robust training procedures since standard training does not achieve high certified robustness with randomized smoothing, due to its use of a smooth classifier for certification. For comparison, ACR on the target class “Ship” with Resnet-20 trained with standard training on clean CIFAR10 dataset is close to zero whereas for the same model trained with Gaussian data augmentation ($\sigma = 0.25$) ACR is close to 0.5.

4.1. Poisoning Gaussian data augmentation [7]

Here we discuss the effectiveness of our data poisoning attack to compromise the certified robustness guarantees as reported by RS on a model trained using the Gaussian data augmentation approach. The results of the attack are present in Table 2 which show a significant decrease in the average certified radius and the certified accuracy of the target classifier (See Sec. 4.6).

4. Experiments

In this section we present the results of our PACD$^1$ attack on poisoning deep neural networks trained using various certifiably robust training methods. All the results presented here are averaged over models trained with five random initialization. For all experiments we report the average certified radius (ACR) as the average of the certified radius obtained from the RS based certification procedure of [7] for correctly classified points. Certified radius is zero for misclassified and abstained points. The approximate certified accuracy (ACA) is the fraction of points correctly classified by the smoothed classifier. All results are reported over 500 randomly sampled images from the target classes. We use the same value of $\sigma$ for smoothing during attack, retraining and evaluation. We compare our results to watermarking [30] which has been used previously for clean label attacks (opacity 0.1 followed by clipping to make $\ell_\infty$ distortion equal to $\epsilon$), and show that the solution to the bilevel optimization is significantly better at reducing the certified radius.

We use our attack to poison MNIST and CIFAR10 dataset and use ApproxGrad to solve the bilevel optimization. The time complexity for ApproxGrad is $O(VT)$ where $V$ are the number of parameters in the machine learning model and $T$ is the number of lower-level updates. For datasets like Imagenet where the optimization must be performed over a very large number of batches, obtaining the solution to bilevel problems becomes computationally hard. Due to

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$^1$The code is available at https://github.com/akashyehra24/poisoning_certified_defenses
Table 4. Decrease in certified radius and certified accuracy of models trained with SmoothAdv \cite{28} on poison data compared to those of models trained on clean and watermarked data.

| σ  | Data          | Certified robustness of target class | AC(%)
|----|---------------|-------------------------------------|------
|    |               | ACR                                 | ACA  |
| 0.25 | Clean         | 0.896±0.01                          | 99.16±0.45 |
|      | Watermarked   | 0.906±0.01                          | 99.28±0.16 |
|      | Poisoned      | 0.672±0.04                          | 93.21±1.92 |
| 0.5  | Clean         | 1.408±0.05                          | 99.21±0.25 |
|      | Watermarked   | 1.401±0.02                          | 98.01±0.18 |
|      | Poisoned      | 1.037±0.06                          | 93.81±1.31 |
| 0.75 | Clean         | 1.262±0.05                          | 95.68±0.47 |
|      | Watermarked   | 1.433±0.03                          | 97.21±0.13 |
|      | Poisoned      | 0.924±0.06                          | 88.88±0.18 |

Table 4. Decrease in certified radius and certified accuracy of models trained with SmoothAdv \cite{28} on poison data compared to those of models trained on clean and watermarked data.

| σ  | Data          | Certified robustness of target class | AC(%) |
|----|---------------|-------------------------------------|-------|
|    |               | ACR                                 | ACA   |
| 0.25 | Clean         | 0.504±0.02                          | 78.76±0.81 |
|      | Watermarked   | 0.441±0.02                          | 70.16±2.12 |
|      | Poisoned      | 0.271±0.02                          | 55.78±0.96 |
| 0.5  | Clean         | 0.479±0.07                          | 65.84±4.81 |
|      | Watermarked   | 0.473±0.02                          | 62.51±2.12 |
|      | Poisoned      | 0.277±0.02                          | 49.11±3.19 |

class of the model trained on poisoned data compared to the model trained on clean and watermarked data. Since the certified radius and certified accuracy are correlated, our poisoning attack which targets the reduction in certified radius (upper-level problem in Eq. (2)) also causes a decrease in the certified accuracy. Significant degradation of the certified radius from 0.52 to 0.06 on CIFAR10 with imperceptible distortion to poisoning points shows the extreme vulnerability of this defense to data poisoning attacks.

4.2. Poisoning MACER \cite{35}

Here we use the bilevel formulation in Eq. (2) with $\mathcal{L}_\text{macer}$ as the loss in the lower-level and generate poison data to reduce the certification guarantees of models trained with MACER. The poison data is generated with $k = 2$, where $k$ are the number of noisy images used per training point to ease the bilevel optimization. However, during retraining $k = 16$ is used, which is similar to the one used in the original work. The average certified radius obtained using MACER is higher than that achievable using Gaussian augmentation based training consistent with the original work. However, our data poisoning attack is still able to reduce the certified radius of the method by more than 30% (Table 3) even though the attack is evaluated against a much stronger defense ($k = 16$ for retraining compared to $k = 2$ for poisoning) than the poison data was optimized for. This shows that using larger number of noise samples $k$ cannot eliminate the effect of the attack emphasising the importance of the threat posed by data poisoning.

4.3. Poisoning SmoothAdv \cite{28}

Here we present the results of our attack on SmoothAdv. Since this method of robust training requires the model parameters to be optimized using adversarial training, we used 2 step PGD attack with $k = 1$ to obtain adversarial example of each point in the batch. We used a single noisy instance of the adversarial example while doing adversarial training. Although using larger $k$ makes the defense better, we used $k = 1$ to save computational time needed for adversarial training. For bilevel training we followed the similar process to generate adversarial examples for clean and poison data. The adversarial examples are used as the data for the lower-level problem of Eq. (2) (Gaussian data augmentation training) to optimize the network parameters. The batch of adversarial examples are recomputed against the updated model after each step of bilevel training. Note that this is an approximation to the actual solution of the mini-max problem to be solved in the lower-level for generating poison data against SmoothAdv. However, the results of the attack present in Table 4 suggests that our approximation works well in practice and certified robustness guarantees achieved from SmoothAdv can be degraded by poisoning.

4.4. Effect of the imperceptibility constraint

Here we evaluate the effect of using different values of the perturbation strength $\varepsilon$ which controls the maximum permissible distortion to the poisoned data in Eq. (2). We use $\sigma = 0.25$ for smoothing and Gaussian data augmentation based training to generate and evaluate the attack. The results are summarized in Fig. 3, which show that the average certified radius of the target class decreases as $\varepsilon$ increases rendering certification guarantees useless. This is expected since larger $\varepsilon$ creates a larger distribution shift among the target class data in the training and the test sets. However, due to larger permissible distortion the attack is much more visible and is easily detectable by human inspection, which may not be desirable in certain applications.
4.5. Transferability of poisoned data

Here we report the performance of the models trained on the poison data using a different robust training procedure than the one attacker used to optimize the poison data. We used \( k = 1 \) and 2 steps of PGD attack to generate adversarial examples for all SmoothAdv experiments and \( k = 16 \) for all MACER experiments. The dataset generated against MACER was optimized with \( k = 2 \) which is a weaker poisoning attack for a victim using larger \( k \). However, this poison dataset is still able to cause sufficient decrease in the robustness guarantees of all methods. The results are summarized in Fig. 4, which show that poisoned data optimized against any robust training procedure causes significant reduction in the certified robustness of models trained with a different training method. Interestingly, poisoned data optimized against Gaussian data augmentation is extremely effective against other methods. This shows the ease of creating a poisoned set affecting victim’s models trained with RS based certified defense methods as its sufficient to create a single poisoned set for transferable attacks.

4.6. Empirical robustness of poisoned classifiers

We report the empirical robustness of the smoothed classifier where the base classifier is trained on clean and poisoned data using Gaussian data augmentation. The poisoned data is generated with Gaussian augmentation based training in the lower-level as in Eq. (2). We report the mean \( \ell_2 \)-distortion required to generate an adversarial example using the PGD attack [28] against the smoothed classifier using 200 and 100 randomly sampled test points of the target class from MNIST and CIFAR10, respectively, in Table 5. We observe that our poisoning leads to a decrease in the empirical robustness of the smoothed classifier on clean test data. This backs up our hypothesis that the decision boundary of the smooth classifier must be changed to reduce the certified radius in nonlinear classifiers, similar to linear classifiers (Fig. 2), where we proved that the decision boundary must move closer to the clean test data.

|     | \( \sigma \) | Clean data | Poisoned data |
|-----|-------------|------------|---------------|
| MNIST | 0.25        | 3.271±0.10 | 1.339±0.16    |
|      | 0.5         | 3.637±0.15 | 2.170±0.09    |
|      | 0.75        | 3.961±0.18 | 2.213±0.31    |
| CIFAR10 | 0.25      | 1.754±0.17 | 0.132±0.04    |
|      | 0.5         | 1.996±0.09 | 0.367±0.06    |

5. Conclusion

Certified robustness has emerged as a gold standard to gauge with certainty the susceptibility of machine learning models to test-time attacks. In this work, we showed that these guarantees can be rendered ineffective by our bilevel optimization based poisoning attack which adds imperceptible perturbations to the points of the target class. Unlike
previous poisoning attacks, our attack can reduce the average
certified radius of an entire class and is even effective against
models trained using state-of-the-art certifiably robust train-
ing methods. Our results suggests that data quality is a cru-
cial factor in achieving high certified robustness guarantees
but is overlooked by current certified defense methods.

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Appendix

We present the proof of Theorem 1 in App. A followed by a review of bilevel optimization (App. B) and our attack algorithm (App. C). In App. D, we present results of poisoning different classes in the dataset along with successful transferability of our poisoning attack to deeper models. We conclude in App. E by providing experimental details such as hyperparameters of the algorithms and models architectures.

A. Proofs

Theorem 1. If the perturbation is large enough, i.e., \( \epsilon \geq \sum_i x_i^+ - \sum_i x_i^- \) then there are two locally optimal solutions to (3) which are \( u_i = x_i^+ - \epsilon \) (Case 1) and \( u_i = x_i^- + \epsilon \) (Case 2) for \( i = 1, \ldots, n \). Otherwise, there is a unique globally optimal solution which is \( u_i = x_i^+ - \epsilon \) (Case 1) for \( i = 1, \ldots, n \).

Proof. Let \( t = -\frac{b}{w} \) be the threshold of the linear classifier. Also let \( \Phi(t) := \int_{-\infty}^{t} P_-(x) \, dx \) and \( \Psi(t) := \int_{t}^{\infty} x P_-(x) \, dx \). There are two cases to consider.

Case 1 (\( w > 0 \)): The upper-level cost function is

\[
f(t) = \int_{-\infty}^{t} (-t + x) P_-(x) \, dx = -t(1 - \Phi(t)) + (1 - \Psi(t)),
\]

which is non-increasing (i.e., \( f'(t) = -(1 - \Phi(t)) + tP_- - tP_\leq 0 \)) and has the constraints:

\[
-\frac{\epsilon}{2} + \frac{\sum_i x_i^+ + \sum_i x_i^-}{2n} \leq t \leq \frac{\epsilon}{2} + \frac{\sum_i x_i^+ + \sum_i x_i^-}{2n}.
\]

For the solution to be feasible, it is required that \( \frac{\sum_i x_i^+}{n} \leq \frac{\epsilon}{2} + \frac{\sum_i x_i^+ + \sum_i x_i^-}{2n} \) (remember the assumption \( \sum_i x_i^+ \leq \sum_i x_i^- \)). Therefore if the perturbation is large enough, i.e., \( \epsilon \geq \frac{\sum_i x_i^+ - \sum_i x_i^-}{2n} \) holds, then the minimum is achieved at the right-most boundary \( t = \frac{\epsilon}{2} + \frac{\sum_i x_i^+ + \sum_i x_i^-}{2n} \) which corresponds to \( u_i = x_i^- - \epsilon, \; i = 1, \ldots, n \).

Case 2 (\( w < 0 \)): The upper-level cost function is now

\[
f(t) = \int_{t}^{\infty} (-t + x) P_-(x) \, dx = -t(1 - \Phi(t)) + (1 - \Psi(t)),
\]

and

\[
t = \frac{\sum_i x_i^+ + \sum_i x_i^-}{2n}.
\]

B. Review of bilevel optimization

A bilevel optimization problem is of the form \( \min_{\xi \in \mathbb{R}} \xi(u, v^*) \) s.t. \( v^* = \arg \min_{v \in V(u)} \Phi(u, v) \), where the upper-level problem is a minimization problem with \( v \) constrained to be the optimal solution to the lower-level problem. General bilevel problems are difficult to solve but if the solution to the lower-level problem can be computed in closed form then we can replace the lower-level problem with its solution, reducing the bilevel problem into a single level problem. We can then use the gradient-based methods to solve the single level problem. The total derivative \( \frac{d}{du}(\xi(u, v^*)(u)) \) (hypergradient) using the chain rule is

\[
\frac{d\xi}{du} = \nabla_u \xi + \frac{dv}{du} \cdot \nabla_v \xi.
\]

Since \( \nabla_v \xi = 0 \) at \( v = v^*(u) \) and assuming \( \nabla_v^2 \xi \) is invertible we can compute \( \frac{dv}{du} \) using the implicit function theorem (this can be done even if the solution to lower-level problem can’t be found in closed form) which gives

\[
\frac{dv}{du} = -\nabla_v^2 \xi \left( \nabla_v^2 \xi \right)^{-1}.
\]

Thus the hypergradient is

\[
\frac{d\xi}{du} = \nabla_u \xi - \nabla_{uv}^2 \xi \left( \nabla_v^2 \xi \right)^{-1} \nabla_v \xi \text{ at } (u, v^*(u)).
\]

Since computation of \( \left( \nabla_v^2 \xi \right)^{-1} \) is difficult, [8, 26] proposed to approximate the solution to \( q = \left( \nabla_v^2 \xi \right)^{-1} \nabla_v \xi \) by solving the linear system of equations \( \nabla_v q = \nabla_v \xi \) by minimizing
\[ \| \nabla^2_{uv} \xi \cdot q - \nabla_v \xi \| \] using any iterative solver. Other methods for solving the bilevel optimization problems include using forward/reverse mode differentiation \cite{9, 21, 29} to approximate the inverse and penalty method \cite{23} to solve the single level problem as a constrained minimization problem.

**Algorithm 2** Algorithm for ApproxGrad

**Input:** \( \xi, \zeta, M, T_1, T_2, \epsilon, u_{base}, \{ \tau_m = 0.1 \}, \{ \rho_m, t_2 = 0.001 \}, \{ \beta_m, t_2 = 0.001 \} \)

**Output:** \( (u_k) \)

Initialize \( u_0, v_0 \) randomly

Begin

\begin{enumerate}
  \item \# Approximately solve the lower-level problem
    \begin{enumerate}
      \item for \( m = 0, \cdots, M-1 \) do
        \begin{enumerate}
          \item \# Approximately solve the linear system
            \begin{enumerate}
              \item for \( t = 0, \cdots, T_2-1 \) do
                \begin{enumerate}
                  \item \( q_{t+1} \leftarrow q_t - \beta_m, t_2 \nabla \left( \| \nabla^2_{uv} \xi \cdot q_m - \nabla_v \xi \| \right) \)
                \end{enumerate}
            \end{enumerate}
          \end{enumerate}
        \end{enumerate}
      \end{enumerate}
    \end{enumerate}

  \item \# Compute the approximate Hypergradient
    \begin{align*}
      p_m &= \nabla_u \xi - \nabla^2_{uv} \cdot q_{T_2} \\
      \# \text{ Update } u_m \text{ and use projection for the} \\
      \# \text{ upper-level constraint}
    \end{align*}

    \begin{align*}
      u_{m+1} &= P(u_m - \tau_m p_m, \epsilon, u_{base})
    \end{align*}

\end{enumerate}

**C. Attack algorithm**

Alg. 1 shows the complete algorithm used to generate the poisoning attack when RS is used for certification and models are trained using Gaussian data augmentation. The algorithm relies on ApproxGrad (Alg. 2) to solve the bilevel optimization problem. The upper-level cost is a differentiable function that approximates the certified radius of the hard smooth classifier using a soft smooth classifier. The hyperparameter \( \alpha \) is the inverse temperature parameter of softmax. As \( \alpha \to \infty \), softmax converges to argmax almost everywhere. As a result \( g_\theta \) converges to \( g_\theta \) almost everywhere and thus soft randomized smoothing converges to hard randomized smoothing almost everywhere. Although in this paper we have RS as the procedure for certification (due to its scalability to large models and datasets), any other certification procedure can be used as the upper-level cost as long as its differentiable. Moreover, Alg. 1 uses \( \mathcal{L}_{GaussAug} \) in the lower-level to train the model, but like the case with upper-level cost any other loss function can be used to obtain the model parameters. This flexibility of our method allows us to generate poisoned data against MACER and SmoothAdv using their loss functions in the lower-level.

**C.1. ApproxGrad**

For an unconstrained bilevel problem of the form \( \min_u \xi(u, v^*) \text{ s.t. } v^* = \arg \min_v \zeta(u, v) \) if \( \zeta(u, v) \) is strongly convex then we can replace the lower-level problem with its necessary condition for optimality and write the bilevel problem as the following single level problem \( \min_u \xi(u, v^*) \text{ s.t. } \nabla_v \zeta(u, v) = 0 \). Assuming \( \nabla^2_{vv} \zeta \) is invertible everywhere we can compute the hypergradient at the point \((u, v^*(u))\) as \( \frac{d\zeta}{du} = \nabla_u \zeta - \nabla^2_{uu} \zeta (\nabla^2_{vv} \zeta)^{-1} \nabla_v \zeta \).

The ApproxGrad algorithm approximates the Hessian-inverse vector product by approximately solving a system of linear equation using an iterative solver such as gradient descent or conjugate gradient method. In this work we use Adam optimizer to solve this system. Since our problem for data poisoning in Eq. (2) involves a constraint in the upper-level we use projection to enforce the constraint. The full algorithm for solving the unconstrained bilevel optimization problem using ApproxGrad is present in Alg. 2. For our attack the lower-level problem involves a deep neural network, which can have multiple local minima and thus optimizing against a single local minima in the bilevel problem is not ideal. To overcome this problem we reinitialize the lower-level variable \( v \) after few upper-level iterations to prevent the poisoning points from overfitting to a particular local minima. Empirically, this helps us find poisoning points that remain effective even after the model is retrained from scratch making them generalize to different initialization of the neural network.

**D. Additional experiments**

**D.1. Comparison with standard data poisoning**

The standard data poisoning attack creates a poison data so that the accuracy of the victim’s model trained on it is significantly lower than the accuracy attainable with training on clean data. The bilevel optimization problem for this attack is as follows.

\[
\min_u \mathcal{L}_{standard}(D^{val}) \\
\text{s.t. } \| \delta_i \|_\infty \leq \epsilon, \ i = 1, \ldots, n, \ \text{and} \\
\theta^* = \arg \min_\theta \mathcal{L}_{standard}(D^{clean} \cup \bigcup D^{poison}, \theta, \sigma).
\] (4)

Here \( \mathcal{L}_{standard}(D; \theta) = \frac{1}{|D|} \sum_{(x_i, y_i) \in D} l_{ce}(x_i, y_i; \theta) \), where \( l_{ce} \) is the cross entropy loss. We used this formulation to generate the poisoned dataset for reporting the results in Table 1 with \( \epsilon = 0.1 \) for MNIST and \( \epsilon = 0.03 \) for
CIFAR10. The attack modifies all the points in the target classes. Specifically, our attack targets misclassification of the digit 8 in MNIST and class “Ship” in CIFAR10. The poisoned dataset obtained after solving the bilevel optimization was then used to train five models starting from random initializations with different training procedures. The results of which are reported in Table 1. As expected the models trained with standard training on the poisoned data perform the worst against poisoning since the attack was optimized against standard training. However, the generated poison data has little to no effect when a robust training procedure is used. This shows that the effect of standard data poisoning can easily be nullified if a victim trains the model with a robust training procedure. This could give a false sense of security of models trained with robust training procedures to data poisoning attacks. Thus, in this work we study the effect of poisoning on robust training procedures and show that they are also vulnerable to data poisoning.
D.2. Isotropic Gaussians

Here we validate the solution found by solving the bilevel optimization against the analytical solution of a toy problem. Consider a two-dimensional dataset comprising of points drawn from two isotropic Gaussian distributions. Let \( P(x|y = -1) = \mathcal{N}(\mu_1, \sigma^2 I) \) and \( P(x|y = 1) = \mathcal{N}(\mu_2, \sigma^2 I) \) and equal prior \( P(y = 1) = P(y = -1) \). For a point \( x \), the Bayes optimal classifier predicts \( y = 1 \) if \( P(y = 1|x) \geq P(y = -1|x) \) and predicts \( y = -1 \) otherwise. The decision boundary of the Bayes optimal classifier is given by \( (x - \mu_1)^T(x - \mu_1) = (x - \mu_2)^T(x - \mu_2) \). This is also the decision boundary of the smoothed classifier. Assuming the attacker is poisoning the class with label \(-1\) and maximum permissible distortion is \( \epsilon \), our analysis showed that maximum reduction in radius occurs if the entire distribution shifts by \( \epsilon \) i.e. the new mean of the class with label \(-1\) is \( \mu_1 - \epsilon \) and the decision boundary is \( (x - (\mu_1 - \epsilon))^T(x - (\mu_1 - \epsilon)) = (x - \mu_2)^T(x - \mu_2) \). Since the test distribution is unchanged, the average certified radius for the test points with labels \(-1\) is reduced by \( \frac{\epsilon}{\sqrt{2}} \). Using \( \mu_1 = 0.2, \mu_2 = 0.8, \sigma_1 = \sigma_2 = 0.3, \epsilon = 0.1 \) and using logistic regression in the lower-level, analytically computed certified radius must decrease from 0.4243 to 0.3546. The solution by solving the bilevel optimization numerically (Table 6) matches the analytic solution.

![Graph](image)

Figure 6. Successful poisoning attack against all classes in MNIST and CIFAR10 dataset.

| \( \sigma \) | Certified Robustness on clean data | Certified Robustness on poisoned data |
|---|---|---|
| | ACR | ACA(%) | ACR | ACA(%) |
| 0.25 | 0.4047 | 90.00 | 0.3585 | 88.00 |
| 0.50 | 0.4139 | 90.00 | 0.3587 | 87.60 |
| 0.75 | 0.4123 | 90.00 | 0.3544 | 87.60 |

D.3. Targeting other classes

In this section we present the results of our poisoning attack on different target classes where the models are trained using Gaussian data augmentation during poison generation and evaluation. Since MNIST and CIFAR10 both have 10 classes we create 10 poisoning sets each targeting a particular class. The results of retraining models from five random initializations on each of the 10 poisoning sets are summarized in Fig. 6. Reduction in certified radius for all classes shows that an attacker can generate poison data to affect any class in the dataset.

D.4. Transferability to different architectures

Here we present the results of transferability of the poisoned data generated against Resnet-20 targeting the class
“Ship” to bigger models. In particular we present the results on Resnet-56 and Resnet-110 [13] models in Fig. 7. As seen from the results the poisoned data generated against Resnet-20 is successful in reducing the certified radius of the target class even if the victim uses a larger model. We report the results of training the models on clean and poisoned data starting from three random initializations and certify using 500 randomly sampled points of the target class from the clean test set. Our results suggest that the poisoned data generated using our procedure are agnostic to the training procedure (Fig. 4), model (Fig. 7) and metric (RS or empirical robustness) used by the victim during evaluation highlighting the threat of data poisoning.

E. Details of the experiments

All codes are written in Python using Tensorflow/Keras, and were run on Intel Xeon(R) W-2123 CPU with 64 GB of RAM and dual NVIDIA TITAN RTX. Implementation and hyperparameters are described below.

E.1. Data splits

For MNIST, we use 55000 points as the training data and 5000 points for validation data. We have roughly 500 points belonging to the target class in the validation set which is used in the upper-level problem of the bilevel optimization presented in Eq. (1). For CIFAR10, we use 45000 points as the training data and 5000 points for validation data. Similar to MNIST we have roughly 500 points belonging to the target class in the validation set. The test sets of both the datasets comprises of 10000 points. We use 500 randomly sampled points of the target class from the test set to report the results of certified and empirical robustness of the models trained on clean and poisoned data.

E.2. Model Architecture

For the experiments on the MNIST dataset, our network consists of a convolution layer with kernel size of 5x5, 20 filters and ReLU activation, followed by a max pooling layer of size 2x2. This is followed by another convolution layer with 5x5 kernel, 50 filters and ReLU activation followed by similar max pooling and dropout layers. Then we have a fully connected layers with ReLU activation of size 500. Lastly, we have a softmax layer with 10 classes. The accuracy of the model on clean data when optimized with the Adam optimizer using a learning rate of 0.001 for 100 epochs with batch size of 200 is 99.3% (without Gaussian data augmentation). For the experiments on the CIFAR10 dataset, we use the Resnet-20 model. The accuracy of the model on clean data when optimized with the Adam optimizer using a learning rate of 0.001 for 100 epochs with batch size of 200 is 85% (without Gaussian data augmentation). For all CIFAR10 experiments except for the experiments with SmoothAdv, we trained the models using data augmentation (random flipping and random cropping). We used the same parameters for training the models with different robust training procedures on clean and poisoned data.

E.3. Hyperparameters

For experiments with MNIST we used $\epsilon = 0.1, K = 20, \alpha = 16$. The batch size used for lower-level training was 1000, of which 100 points belonged to the poisoned set (target class). The batch size for validation set was 100 which only consisted of points from the target class. The lower-level was trained using different training procedures on clean and poisoned data. For experiments with CIFAR10 we used $\epsilon = 0.03, \lambda = 0.06, M = 20, \alpha = 16$. The batch size used for lower-level training was 200, of which 20 points
belonged to the poisoned set (target class). The batch size for validation set was 20 which only consisted of points from the target class. For training with Gaussian data augmentation the lower-level is trained with a single noisy image of the clean and poisoned dataset. The same setting is used while retraining. For generating poison data against MACER the lower-level is trained with $K = 2, \lambda = 1, \gamma = 8$. During retraining, $K = 16, \lambda = 16, \gamma = 8$ are used for MNIST. For CIFAR10, we use $K = 16, \gamma = 8$ and $\lambda = 12$ for $\sigma = 0.25$ and $\lambda = 4$ for $\sigma = 0.5$. The hyperparameters during retraining are similar to the ones used in the original work. For SmoothAdv, $k = 1$ and 2-step PGD attack are used to generate adversarial examples of the smooth classifier. These adversarial examples along with Gaussian data augmentation are used to do adversarial training during poison generation and retraining.

In our experiments with generating poisoned data against Gaussian data augmentation and MACER we used $P = 50, T_1 = T_2 = 10, \tau = 0.1, \rho = 0.001, \beta = 0.01$ for ApproxGrad. We used all the same hyperparameters for SmoothAdv except $T_1 = 1$. For certification we used the CERTIFY procedure of [7], with $n_0 = 100, n = 100000, \alpha = 0.001$. For measuring empirical robustness of the smoothed classifier, we used the mean $\ell_2$ distortion required by PGD attack to generate an adversarial example as done in [28]. The attack is optimized for 100 iterations for different values of $\ell_2$ distortion between (0.01, 10). We used 20 augmentations for each test point of MNIST and 10 for CIFAR10. To report the results for empirical robustness we record the minimum distortion for a successful attack for each test point.

For the watermarking baseline, we randomly selected an image (other) from the classes other than the target class and over-layed them on top of the target class images (base) with an opacity of $\gamma = 0.1$ i.e. $(\text{poison\_image} = \gamma \cdot \text{other} + (1 - \gamma) \cdot \text{base})$. We then clip the images to have $\ell_\infty$ distortion of $\epsilon$ to make our bilevel attack comparable in terms of maximum distortion.