Higgs CP properties
using the tau decay modes at the ILC

Stefan Berge*1, Werner Bernreuther†2 and Hubert Spiesberger*3

* PRISMA Cluster of Excellence, Institut für Physik (WA THEP),
  Johannes Gutenberg-Universität, 55099 Mainz, Germany
† Institut für Theoretische Physik, RWTH Aachen University, 52056 Aachen, Germany

Abstract

We investigate the prospects of determining the CP nature of the 126 GeV neutral spin-0 (Higgs) boson $h$, discovered at the LHC, at a future linear $e^+e^-$ collider (ILC). We consider the production of $h$ by the Higgsstrahlung process $e^+e^- \rightarrow Z + h$ and its subsequent decays to $\tau$ leptons, $h \rightarrow \tau^- \tau^+$. We investigate how precisely a possible pseudoscalar component of $h$ can be detected by the measurement of a suitably defined angular distribution, if all major decay modes of the $\tau$ lepton are used. From our numerical simulations, we estimate the expected precision to the scalar-pseudoscalar mixing angle $\phi$, including estimates of the background and of measurement uncertainties, to be $\Delta \phi \simeq 2.8^\circ$ for Higgs-boson production at a center-of-mass energy of 250 GeV and for a collider with integrated luminosity of 1 ab$^{-1}$. 

PACS numbers: 11.30.Er, 12.60.Fr, 14.80.Bn, 14.80.Cp
Keywords: Linear collider physics, Higgs bosons, tau leptons, parity, CP violation

1berge@uni-mainz.de
2breuther@physik.rwth-aachen.de
3spiesber@uni-mainz.de
I. INTRODUCTION

The recent results [1–4] by CMS and ATLAS on the production, decays, and properties of the neutral boson $h$ of mass $\simeq 126$ GeV that was discovered last year by these experiments [5,6] at the LHC support the hypothesis that $h$ is the long-sought Standard Model (SM) Higgs boson. Nevertheless, much more detailed investigations will be necessary to firmly establish this expectation. In particular, the spin-parity analyses made in $h \to ZZ^* \to 4l$ [1,3] do not yet prove that $h$ is a pure scalar – they imply that $h$ cannot be a pure pseudoscalar, but do not rule out the possibility that it is a mixture of a scalar and a pseudoscalar state. It is expected that the profile of this resonance can be explored to a large extent at the LHC.

A high-energy linear $e^+e^-$ collider would be an ideal machine to investigate the properties of this spin-0 resonance in great detail, i.e., its decay modes, couplings, and CP parity (and, of course, also the properties of other, not too heavy resonances of similar type if they exist). For assessing the prospects of exploring this particle at a future linear collider, one may revert to the many existing phenomenological investigations, within the SM and many of its extensions, of Higgs-boson production and decay in $e^+e^-$ collisions. As to the prospects of exploring the spin and CP properties of a Higgs boson, there have been a number of proposals and studies, including [7–40], that are relevant for Higgs-boson production and decay at a linear collider.

In this paper, we apply a method [33,35] for the determination of the CP nature of a neutral spin-zero (Higgs) boson in its $\tau^+\tau^-$ decays to the production of $h$ at a future $e^+e^-$ linear collider (ILC). For definiteness, we consider $e^+e^- \to Zh$, but the analysis outlined below is applicable to any other $h$ production mode, where the $h$ production vertex can be determined. In our analysis, all major 1-prong and 3-prong $\tau$ decays are taken into account. We demonstrate that the CP nature of $h$ can be determined by this approach in a precise and unambiguous way.

II. HIGGS-BOSON PRODUCTION AND DECAY

The CMS and ATLAS results [2,4] on $h$ production and its couplings to the weak gauge bosons are consistent with expectations for the Standard Model Higgs boson. Therefore, the $e^+ e^-$ production of $h$ by the Higgsstrahlung process

\[ e^+ e^- \to Z + h \]  

has a cross section $\sim \sigma_{SM}(Zh)$. We are interested here in the decay mode $h \to \tau^- \tau^+$, with subsequent decays

\[ h \to \tau^- \tau^+ \to a^- a'^+ + X , \]  

where $a^\pm, a'^\pm \in \{ e^\pm, \mu^\pm, \pi^\pm, a_1^{L,T,\pm} \}$ and $X$ denotes neutrinos and $\pi^0$.  

2
The interaction of a Higgs boson $h$ of arbitrary $CP$ nature to $\tau$ leptons is described by the Yukawa Lagrangian

$$\mathcal{L}_Y = -\left(\sqrt{2}G_F\right)^{1/2}m_\tau (a_\tau \bar{\tau}\tau + b_\tau \bar{\tau}i\gamma_5\tau) h,$$

where $G_F$ denotes the Fermi constant and $a_\tau, b_\tau$ are the reduced dimensionless $\tau$ Yukawa coupling constants. In order to be able to compare with other studies in the literature, we use in the following sections, instead of (3), the equivalent parameterization

$$\mathcal{L}_Y = -g_\tau (\cos \phi \bar{\tau}\tau + \sin \phi \bar{\tau}i\gamma_5\tau) h,$$

where $g_\tau$ is the effective strength of the Yukawa interaction and $\phi$ describes the degree of mixing of the scalar and pseudoscalar component:

$$g_\tau = (\sqrt{2}G_F)^{1/2}m_\tau \sqrt{a_\tau^2 + b_\tau^2}, \quad \tan \phi = \frac{b_\tau}{a_\tau}.$$  

In the following sections, we take into account the main 1- and 3-charged prong $\tau$ decay modes:

\begin{align*}
\tau & \to l + \nu_l + \nu_\tau, \quad l = e, \mu, \quad (6) \\
\tau & \to \pi + \nu_\tau, \quad (7) \\
\tau & \to \rho + \nu_\tau \to \pi + \pi^0 + \nu_\tau, \quad (8) \\
\tau & \to a_1 + \nu_\tau \to \pi + 2\pi^0 + \nu_\tau, \quad (9) \\
\tau & \to a_1^{L,T} + \nu_\tau \to 2\pi^\pm + \pi^\mp + \nu_\tau. \quad (10)
\end{align*}

The decay mode (10), in fact a 3-prong $\tau$ decay with three charged pions, will also be called ‘1-prong’ because the track, i.e., the 4-momentum of the $a_1^\pm$ resonance can be obtained from the 4-momenta of the three charged pions. Moreover, by using known kinematic distributions, the longitudinal ($L$) and transverse ($T$) helicity states of the $a_1$ resonance can be separated \([41-44]\). Thus, the $\tau^-\tau^+$ decays that we analyze are of the form (2) with $a, a'$ as specified below (2).

The observables that we use to determine the $CP$ nature of $h$ in its $\tau$ decays are based on $\tau$-spin correlations \([9, 21, 32, 33, 35]\). The charged lepton $l = e, \mu$ in (6), the charged pion in (7) - (9), and the $a_1^{L,T}$ in (10) act as $\tau$-spin analyzers. The $\tau$-spin analyzing power is maximal for the direct decays to pions, $\tau^\pm \to \pi^\mp$, and for $\tau^\pm \to a_1^{L,T}$, For $a_1^T$ and $a_1^{T}$ it is +1 and −1, respectively. For the decays (6), (8), and (9), the $\tau$-spin analyzing power of $l^\pm$ and $\pi^\pm$ depends on the energy of these particles, cf. Appendix A. We will apply cuts on the respective energy to optimize the $\tau$-spin analyzing power.

The differential cross section of the production process (1) and subsequent decay (2) can be derived from Eq. (4) of \([35]\). For a Higgs boson of arbitrary $CP$ nature it is given by

$$d\hat{\sigma} = \frac{N_{\tau}|M_{a^-}\overline{a^+}|^2d\Omega_\tau d\Omega_{\chi} dE_{a^-} dE_{\chi} d\Omega_{a^-} d\Omega_{a^+}/(2\pi)}{(11) \times n (E_{a^-}) n (E_{a^+}) \{A - b (E_{a^-}) b (E_{a^+}) [c_1 \hat{q}^- \cdot \hat{q}^+ + c_2 \hat{k} \cdot \hat{q}^- \hat{q}^+ + c_3 \hat{k} \cdot (\hat{q}^- \times \hat{q}^+) \}},$$
where

\[ N_\tau = \frac{\sqrt{2} G_F m_\tau^2 \beta_\tau}{128 \pi^3 s} , \quad |M_{a^- d^+}|^2 = \sum |M(e^- e^+ \rightarrow Zh)|^2 |D^{-1}(h)|^2 B_{\tau^- a^-} B_{\tau^+ d^+} . \]

In Eq. (11), \( \hat{k} \) denotes the normalized \( \tau^- \) momentum in the Higgs-boson rest frame and \( \hat{q}^- \) (\( \hat{q}^+ \)) is the \( a^- (a^+) \) direction of flight in the \( \tau^- (\tau^+) \) rest frame. The functions \( n \) and \( b \) are defined in Appendix A, the coefficients \( A, c_i \) are given in Table I of [33], \( \beta_\tau \) is the \( \tau \) velocity, \( s = E_{cm}^2 \), and \( D^{-1} \) is the Higgs-boson propagator.

Choosing \( \hat{k} \) to be the \( z \) axis of a right-handed coordinate system and integrating Eq. (11) over the polar angles \( d \theta_{a^-} \) and \( d \theta_{a^+} \), we obtain:

\[ d\sigma = N_\tau |M_{a^- d^+}|^2 d\Omega_Z d\Omega_z dE_{a^-} dE_{d^+} d\phi \left[ v + u \cdot \cos(\varphi - 2\phi) \right] , \quad (12) \]

where

\[ \varphi = \delta_{a^-} - \delta_{a^+} , \quad 0 \leq \varphi \leq 2\pi , \]

is the difference of the azimuthal angles of \( a^- \) and \( a^+ \),

\[ u = -n(E_{a^-}) b(E_{a^-}) n(E_{d^+}) b(E_{d^+}) \frac{p^2_{\tau} m^2_H}{8 \sqrt{2} G_F m^2_\tau} , \quad v = 4n(E_{a^-}) n(E_{d^+}) A , \]

and \( \phi \) is the Higgs mixing angle defined in (5).

The distribution (12) holds also in the \( \tau\tau \) zero-momentum frame (ZMF). The angle \( \varphi \) is equal to the angle between the signed normal vectors of the \( \tau^- \to a^- \) and \( \tau^+ \to a^+ \) decay planes spanned by the unit vectors \( \hat{k}, \hat{q}^- \) and \( -\hat{k}, \hat{q}^+ \), respectively. Instead of determining \( \varphi \) in the \( \tau\tau \) ZMF one can measure this angle also in the zero-momentum frame of the charged prongs \( a^- \) and \( a^+ \) (cf. 33 and below). This has the advantage that the \( \tau^\pm \) momenta need not be reconstructed.

### III. METHOD AND OBSERVABLES

Our method to determine the \( CP \) nature of \( h \) requires, in the case of the 1-prong \( \tau^- \tau^+ \) decays (2), the measurement of the 4-momenta of the charged prongs \( a^- \), \( a^+ \) and their impact parameter vectors (unit vectors) \( \hat{n}_\tau \) in the laboratory frame. The 4-vectors \( n^\mu_\tau = (0, \hat{n}_\tau) \) are then boosted into the \( a^- d^+ \) zero-momentum frame (ZMF). The spatial parts of the resulting 4-vectors \( n^\mu_\tau \) are decomposed into their normalized components \( \hat{n}_\tau^+ \) and \( \hat{n}_\tau^- \) that are parallel and perpendicular to the respective \( a^- \) and \( a^+ \) 3-momentum.

With this prescription, one determines in the \( a^- d^+ \) ZMF the unsigned normal vectors \( \hat{n}_\tau^- \) and \( \hat{n}_\tau^+ \) of the \( \tau^- \to a^- \) and \( \tau^+ \to a^+ \) decay planes. The distribution of the angle between these two planes [33],

\[ \varphi^* = \arccos(\hat{n}_\tau^+ \cdot \hat{n}_\tau^-) , \quad (13) \]

where \( 0 \leq \varphi^* \leq \pi \), discriminates between \( CP = \pm 1 \) Higgs boson states. The simultaneous measurement of (13) and of the \( CP \)-odd and \( T \)-odd triple correlation

\[ O_{CP}^\alpha = \hat{q}_\tau^- \cdot (\hat{n}_\tau^+ \times \hat{n}_\tau^-) , \quad (14) \]

4
Figure 1: Left side: Normalized distribution of $\varphi^*_{\text{CP}}$ for $\tau^- \tau^+ \rightarrow \pi^+ \pi^- + 2\nu$. The red solid line, the black dotted line, and the black dashed line show the distributions for a CP-even Higgs boson ($\phi = 0$), for a CP-odd boson ($\phi = \pm \pi/2$), and for a CP mixture with $\phi = -\pi/4$, respectively. Right side: The normalized distribution of $\varphi^*_{\text{CP}}$ for $\tau^- \tau^+ \rightarrow \pi^- a^+_1 + 2\nu$ for a CP-even Higgs boson and a CP mixture with $\phi = -\pi/4$.

where $\hat{q}^-$ is the normalized $a^-$ momentum in the $a^- a^+ \quad$ ZMF, allows for an unambiguous determination of the CP nature of $h$ \cite{33}. If $h$ is a mixture of a CP-even and -odd state, the distribution of \cite{14} is asymmetric with respect to $\vartheta^*_{\text{CP}} = 0$. In order to determine the ratio $b_\tau/a_\tau$ of the reduced Yukawa couplings \cite{3} or, equivalently, the mixing angle $\phi$ defined in \cite{5}, one would fit theoretical predictions for $\sigma^{-1} d\sigma/d\varphi^*$ and $\sigma^{-1} d\sigma/d\vartheta^*_{\text{CP}}$ to the corresponding measured distributions. In addition, associated asymmetries can be measured. Some results of this approach, applied to the reactions \cite{1,2}, were presented in the workshop report \cite{38}.

Here we use a slight variation of our approach just outlined. Instead of using both the distribution of the ‘unsigned’ angle $\varphi^*$, Eq. \cite{13}, which is defined in the range $0 \leq \varphi^* \leq \pi$, and of $\vartheta^*_{\text{CP}}$, the same information is of course contained in the distribution of the ‘signed’ angle between the $\tau^- \rightarrow a^-$ and $\tau^+ \rightarrow a^+$ decay planes in the $a^- a^+ \quad$ ZMF. This angle which will be called $\varphi^*_{\text{CP}}$ in the following and that varies between 0 and $2\pi$ is obtained by the following prescription:

$$\varphi^*_{\text{CP}} = \begin{cases} \varphi^* & \text{if } \vartheta^*_{\text{CP}} \geq 0, \\ 2\pi - \varphi^* & \text{if } \vartheta^*_{\text{CP}} < 0. \end{cases} \quad (15)$$

In terms of this angle the triple correlation \cite{14} $\vartheta^*_{\text{CP}} = \sin \varphi^*_{\text{CP}}$. The distribution of \cite{15} is given by \cite{12} with $\varphi \rightarrow \varphi^*_{\text{CP}}$.

In order to illustrate the discriminating power of \cite{15}, we consider the $h \rightarrow \tau^- \tau^+ \rightarrow \pi^- \pi^+$ decay mode. The normalized distribution of $\varphi^*_{\text{CP}}$ for this decay channel is shown on the left side of Fig. \cite{1}. The red solid line shows the distribution for a pure CP-even Higgs boson ($\phi = \ldots$
the black dashed line results from the decay of an ideal CP mixture (with $\phi = -\pi/4$). The distance between the maxima of the distributions for the CP-even Higgs boson and the CP mixture is given by 2\(\phi\). For completeness, the distribution resulting from the decay of a CP-odd state ($\phi = \pm \pi/2$) is also shown.

For each decay channel $aa'$ as specified above, the Higgs mixing angle $\phi$ can be obtained from the measured differential distributions by fitting the function $f = u\cos(\phi_{CP}^* - 2\phi) + v$ (subject to the constraint $\int_0^{2\pi} d\phi_{CP} f = 2\pi v = \sigma_{aa'}$) to the measured distribution of $\phi_{CP}^*$. For a fixed number of $aa'$ events the sensitivity to the mixing angle $\phi$ of this channel depends on the product of the $\tau$-spin analyzing powers of $a$ and $a'$. We find it convenient to define the following asymmetry that embodies this fact:

$$A_{aa'} = \frac{1}{\sigma_{aa'}} \left[ \sigma_{aa'} (u\cos(\phi_{CP}^* - 2\phi) > 0) - \sigma_{aa'} (u\cos(\phi_{CP}^* - 2\phi) < 0) \right] = \frac{-4u}{2\pi v}. \quad (16)$$

The value of $A_{aa'}$ does not depend on the mixing angle $\phi$, but it reflects the combined strengths of the $\tau$-spin analyzing powers of $a$ and $a'$. Thus, the larger $A_{aa'}$, the smaller the statistical error $\Delta \phi$ for a given number of events. The asymmetry is largest if both $\tau$ leptons decay either directly to $\pi^\pm$ or to three charged prongs, i.e., to $a_1^\pm$ ($\lambda = L, T$). For these decays $|A_{aa'}| = 39.3\%$.

In the next section, we determine the statistical uncertainty $\Delta \phi$ of the mixing angle in the following way [27]. We assume an integrated collider luminosity of 1 ab$^{-1}$. For some value of $\phi$, for instance $\phi = -\pi/8$, we generate for each decay channel the differential distribution of $\phi_{CP}^*$, using 20 bins between 0 and 2\(\pi\). Then we perform a fit to this distribution with the 3-parameter function $u\cos(\phi_{CP}^* - 2\phi) + v$. This yields a certain value of $\phi$. We repeat this procedure a 1000 times, then fit a Gaussian curve to the resulting distribution of $\phi$ and take the width of the Gaussian as the expected statistical uncertainty $\Delta \phi$.

As already mentioned above, the $\tau$-spin analyzing power is maximal for the direct decays $\tau^+ \to \pi^\mp$ and for $\tau^+ \to a_1^\mp T^\mp$. The $\tau$-spin analyzing power of the charged lepton in $\tau^+ \to l^\mp$ and of the charged pion from $\tau^+ \to \rho^\mp$ and $\tau^+ \to a_1^\mp$ can be enhanced by applying an appropriate cut on the energy of the lepton and the pion, respectively (see Figs. 1a and 4 in [35]). Ideally, this cut should be applied in the $\tau^+_{\text{rest}}$ rest frames. The reconstruction of these rest frames is, however, not possible for the leptonic $\tau$ decay modes.

For $\tau^- \tau^+ \to a^- a'^+ + \nu_\tau \bar{\nu}_\tau$, $a, a' = \rho, a_1$, the rest frames of $\tau^-$ and $\tau^+$ can be reconstructed by solving kinematic constraints [46], a technique that is well known in $\tau$ physics at $e^+ e^-$ colliders (cf., e.g., [44] for a review). Therefore, we apply for the decays $\tau \tau \to \rho\rho, a_1 a_1, \rho a_1$ an appropriate cut on the energy of the charged pion from $\rho$ and $a_1$ decay in the respective $\tau$ rest frame. Because the $\tau$-spin analyzing functions $b(E_{\pi}^{\tau-\text{rest}})$ are slowly varying functions of the energy $E_{\pi}^{\tau-\text{rest}}$, uncertainties in these reconstructed energies due to measurement uncertainties will not have a dramatic effect on our conclusions below.

This method does not work if one $\tau$, or both, decay to leptons. In this case, we use the fact that the 4-momentum of the $Z$ boson in the reaction [11] can be determined unambiguously...
from its visible decay modes. In this case the rest frame of the Higgs boson $h$ is also known. For the decays $\tau^- \tau^+ \rightarrow l^- l'^{\pm}, l^\mp \pi^\pm$, where the pion results from $\rho$ or $a_1$ decay, we apply cuts on the energies of the charged lepton and pion in the rest frame of $h$, in order to enhance their $\tau$-spin analyzing powers and in order to analyze whether or not such cuts increase the sensitivity of these decay modes to the mixing angle $\phi$ in a significant way.

At this point we briefly recall several other detailed studies on the determination of the $CP$ parity of a Higgs boson in its $\tau^+ \tau^-$ decays at a linear collider. In Refs. [25–27] the hadronic 1-prong decay $\tau \rightarrow \rho \nu$ was analyzed. The observable used there, namely the acoplanarity angle of the $\rho^+$ and $\rho^-$ decay planes, requires the reconstruction of the $\rho^+ \rho^-$ ZMF, i.e., the measurement of the $\pi^\pm$ and the $\pi^0$ momenta. In addition, the reconstruction of approximate $\tau^\pm$ rest frames is needed, because the method of [27] requires event selection that involves the knowledge of the $\pi^\pm$ and the $\pi^0$ energies in these frames. The authors of Ref. [27] conclude that at a linear collider with integrated luminosity of $1 \text{ ab}^{-1}$ the scalar-pseudoscalar mixing angle can be measured with an uncertainty $\Delta \phi = 6^\circ$. Background $\tau \tau$ studies were not made in this analysis.

In Ref. [34] the hadronic $\tau$-decay channels $\tau \rightarrow \pi, \rho, a_1$ were analyzed. (A similar analysis was made before by [28] for $\tau^\mp \rightarrow \pi^\mp \nu$.) The observable used in this study is the difference of the azimuthal angles of the polarimeter vectors of the decays $\tau^\mp \rightarrow \pi^\mp, \rho^\mp, a_1^\mp$. The measurement of this observable requires the reconstruction of the $\tau$ rest frames. The study includes a full detector simulation, takes into account the relevant background from ZZ production and determines appropriate kinematical cuts for suppressing this background. The resulting signal-to-background ratio is estimated to be $S/B = 4.5$. Assuming an integrated luminosity of 300 fb$^{-1}$, the authors of [34] conclude that a mixing angle of $\phi = -\pi/8$ can be excluded with 4.5$\sigma$ with respect to the $\phi = 0$ hypothesis.

The recent proposal [40] for determining the mixing angle $\phi$, both at the LHC and ILC, is based on the decay mode $\tau^\mp \rightarrow \rho^\mp \nu$ and relies on a complete reconstruction of the $\rho^\mp$ 4-momenta.

IV. NUMERICAL RESULTS

For our analysis we use a Higgs boson mass of $m_h = 126$ GeV. We assume that the strength of the ZZ$h$ vertex is as predicted by the SM. If $h$ has a pseudoscalar component, this component would not couple to ZZ at tree level. As expected from multi-Higgs extensions of the SM, it is likely that in this case the coupling of the scalar component of $h$ to ZZ is reduced with respect to the SM coupling. A coupling of the pseudoscalar component of $h$ to ZZ can be induced at the loop-level but, using the results of [49] and current LHC results, one concludes that such a coupling must be very small as compared to the respective coupling of the scalar component. The decay mode $h \rightarrow \tau \tau$ is, however, already affected at tree level if $h$ is a $CP$ mixture.
We assume in the following that the 126 GeV Higgs boson is SM-like, i.e., is predominantly a scalar. Therefore we use both for the cross section of the Higgsstrahlung process (1) and the branching ratio 
\[ B(h \rightarrow \tau\tau) \]
the respective SM predictions (c.f., for instance, [47]). Furthermore, we assume that the electron and positron beams have a longitudinal polarization of \(-0.8\) and \(0.3\), respectively. Then we obtain \( \sigma = 325 \text{ fb} \) at \( \sqrt{s} = 250 \text{ GeV} \) for the cross section of (1) and use this value to estimate the number of events in the various decay channels.

Our method [33] requires that the Higgs production vertex is known. It can be determined from the charged tracks of the Z-boson decay products. In the case of leptonic \( \tau \) decay modes the application of energy cuts to enhance the \( \tau \)-spin analyzing power requires also the knowledge of the Higgs-boson 4-momentum (see below). Therefore, only the visible Z boson decay modes are taken into account. In fact, we only consider
\[ Z \rightarrow e^-e^+, \mu^-\mu^+, u\bar{u}, d\bar{d}, s\bar{s}. \] (17)

The other decays \( (Z \rightarrow \tau^-\tau^+, c\bar{c}, b\bar{b}) \) would require a more detailed study. For instance, \( Zh \) production with \( Z \rightarrow b\bar{b}, h \rightarrow \tau\tau \) has to be disentangled from \( Zh, Z \rightarrow \tau\tau, h \rightarrow b\bar{b} \). The branching ratio of the decays (17) is \( B(Z \rightarrow \text{light}) = 0.5 \) [48].

We analyze the reactions (1), (2) in the narrow width approximation both for the Higgs boson and the \( \tau \) leptons. In this section we do not apply cuts for the selection of the signal events. An estimate of the effect of such cuts will be done in Sec. VI. The branching fractions of the \( \tau^-\tau^+ \) decays that we use in the following sections are summarized in Table I [48].

The cross sections for the various signal reactions that we use, e.g., \( Zh \) with \( Z \rightarrow \text{light} \) and \( h \rightarrow \tau\tau \rightarrow ll' \), are denoted by \( \sigma_{0}^{ll'} \), etc., in the Tables below. For \( \tau \rightarrow l, \rho, a_1 \), we apply energy cuts in order to enhance the \( \tau \)-spin analyzing power. The resulting reduced cross section ratios \( \sigma/\sigma_{0} \) are also given in Tables II – V below.

For each decay channel \( aa' \), the value of the asymmetry (16) is also given in these Tables. Note, however, that we use this asymmetry only as a qualitative measure for the sensitivity to the scalar-pseudoscalar mixing angle \( \phi \). The estimates of the statistical uncertainty \( \Delta \phi \) were made for the various channels as described above (see text after Eq. (16)). The estimates given in this section are based on an integrated luminosity of 1 ab\(^{-1}\).

A. \( \tau^-\tau^+ \rightarrow \{\pi^-, a_1^{-\lambda}\} \{\pi^+, a_1^{+\lambda'}\} \)

First we consider the case where both \( \tau \) leptons decay either directly to a charged pion or to \( a_1 \) with a subsequent 3-prong decay, and we assume that the polarization state of the \( a_1 \), i.e., \( a_1^{\pm\lambda} (\lambda = L, T) \) can be reconstructed efficiently at a linear collider [41–44]. As a measure of the expected efficiency one may consider the sensitivity of the decay distributions of \( \tau \rightarrow \pi, \rho, a_1 \) to the \( \tau \) polarization. If one uses all the information about the respective hadronic final state, the sensitivity computed from the theoretical distributions is the same for the three
| Branching ratios [%] | $l^-$ | $\pi^-$ | $\rho^- \to \pi^- + \pi^0$ | $a_1^- \to \pi^- + 2\pi^0$ | $a_1^{T-} \to 2\pi^- + \pi^+$ |
|---------------------|-------|---------|------------------------|------------------|-----------------|
| $l^+$               | 12.4  | 7.7     | 18.0                   | 6.6              | 6.3             |
| $\pi^+$             | 1.2   | 5.6     | 2.0                    | 2.0              |                 |
| $\rho^+ \to \pi^+ + \pi^0$ | 6.5   | 4.7     |                        | 4.6              |                 |
| $a_1^+ \to \pi^+ + 2\pi^0$ |       | 0.9     |                        | 1.7              |                 |
| $a_1^{T+} \to \pi^- + 2\pi^+$ |       |         |                        | 0.8              |                 |

Table I: Branching fractions of the different decay modes [48]. The off-diagonal elements are the sum of the branching fractions of the respective decay mode and the charge-conjugated mode, for instance, $B_{l^+\pi^+} + B_{l^+\pi^-}$.

decay modes ($S = 0.58$) [42]. For $\tau$ pair production at LEP and subsequent semi-hadronic decays of both $\tau$ leptons, it was shown by the ALEPH collaboration [45] that experimentally $S \simeq 0.48$ can be achieved, which amounts to a degradation of only about 17%. It seems not too optimistic to assume that such an efficiency can also be achieved – and probably improved – at a future $e^+e^-$ collider. Therefore we assume that in the case of both $\tau$ leptons decaying semi-hadronically, the spin-analyzing power of $a_1$ is maximal and the asymmetry (16) takes its maximal value $|A^{a\alpha}| = 39.3\%$ for all combinations of $a, a' = \pi, a_1^\lambda$. The resulting distributions of $\phi_{CP}$ for $a a' = \pi \pi$ are shown in Fig. 1 (left part). Identical distributions are obtained for $a a' = a_1^\lambda a_1^{\lambda'}$, $\pi a_1^T$, and $a_1^T a_1^{T'}$. For $a a' = \pi a_1^T, a_1^T a_1^{\lambda'}$ the corresponding distributions are shifted by $\phi_{CP}^* = \phi_{CP} + \pi$ (see Fig. 1 right part), because the $\tau^\mp$-spin analyzing power of $a_1^{\mpT}$ is $\mp 1$, i.e., opposite to the analyzing powers of $\pi^\pm$ and $a_1^{\mpT}$.

To minimize $\Delta \phi$ it is therefore essential to disentangle the polarization states $a_1^\lambda$ in an efficient way. The cross sections of the $\pi \pi, a_1^\lambda a_1^{\lambda'},$ and $\pi a_1^T$ channels are $\sigma^{\pi \pi}_0 = 0.12$ fb, $\sigma^{a_1^\lambda a_1^{\lambda'}}_0 = 0.08$ fb, and $\sigma^{\pi a_1^T}_0 = 0.20$ fb, respectively. Adding the three channels yields 400 events for a luminosity of 1 ab$^{-1}$. Using the procedure described above we estimate the statistical error on the mixing angle from fitting the $\phi_{CP}^*$ distribution of these decay modes to be $\Delta \phi = 3.3^\circ$.

B. $\tau^-\tau^+ \to l^-l'^+$

The $\tau^-\tau^+ \to l^-l'^+$ decay mode has a rather large branching fraction of 12.4%. However, the spectral function $b(E_l)$ changes sign at $E_l = m_\tau/4$ (see Fig. 1b of Ref. [35]). The inclusive $\tau$-spin analyzing power of $l = e, \mu$, that is, the spin-analyzer quality that results from integrating over the lepton energy $E_l$ in the $\tau$ rest frame, is therefore small. Thus the $\phi_{CP}^*$ distribution of the $l^-l'^+$ channel is not very sensitive to the mixing angle $\phi$ if no energy cuts are made.

Because the $\tau$ rest frames cannot be reconstructed for these channels, we try to enhance the $\tau$-spin analyzing powers by applying the cuts $E_{l^-} = E_{l^+} > E_{cut}$ on the lepton energies in
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$E_{l^-} = E_{l^+}$ & $A^{ll'}$ [%] & $\sigma/\sigma_0$ & $\Delta\phi$ [°] & $E_{l}$ in Higgs & $A^{l\pi(l\lambda)}$ [%] & $\sigma/\sigma_0$ & $\Delta\phi$ [°] \\
rest frame & & $\sigma_0^{ll'} = 1.25 \text{ fb}$ & & rest frame & & $\sigma_0^{l\pi} = 0.78 \text{ fb}$ & & \\
\hline
$> 0 \text{ GeV}$ & 4.4 & 1 & 20.4 & $> 0 \text{ GeV}$ & $-13.1 \ (-13.1)$ & 1 & 5.2 \\
$> 10 \text{ GeV}$ & 6.5 & 0.54 & 19.6 & $> 10 \text{ GeV}$ & $-16.0 \ (-16.0)$ & 0.74 (0.74) & 5.0 \\
$> 15 \text{ GeV}$ & 7.6 & 0.37 & 19.6 & $> 15 \text{ GeV}$ & $-17.2 \ (-17.2)$ & 0.61 (0.61) & 5.5 \\
$> 20 \text{ GeV}$ & 8.6 & 0.24 & 22.1 & $> 20 \text{ GeV}$ & $-18.3 \ (-18.3)$ & 0.5 (0.5) & 5.4 \\
\hline
\end{tabular}
\caption{Asymmetries, cross sections, and uncertainties $\Delta\phi$ for the decay modes $\tau\tau \rightarrow ll'$ (left part) and $\tau\tau \rightarrow l\pi, l\lambda_1$, ($\lambda = L, T$) (right part) for several cuts on the energy of the lepton(s) in the Higgs-boson rest frame.}
\end{table}

The asymmetry is increased slightly by these cuts – however, this growth is outweighed by the decrease of the cross section. We conclude that the statistical error $\Delta\phi$ cannot be reduced in a significant way by applying a cut on the energy of the lepton(s) in the Higgs-boson rest frame. The sensitivity to $\phi$ of these channels is rather poor.

**C. $\tau^-\tau^+ \rightarrow l^\pm\{\pi^\pm, a_1^{z\lambda}\}$**

The results given in the right part of Table II show whether or not a cut on the energy of the charged lepton in the Higgs-boson rest frame leads to a higher sensitivity to the mixing angle $\phi$ for these decay modes. A cut of $E_{l^-} = E_{l^+} > 10 \text{ GeV}$ reduces the uncertainty $\Delta\phi$. For larger energy cuts, the increase in spin-analyzing power is completely offset by the decrease of the cross section. In any case, the sensitivity to $\phi$ of these decay modes is higher than the sensitivity of the $\tau^-\tau^+ \rightarrow l^-l'^+ \text{ decay channels}$.

**D. $\tau^-\tau^+ \rightarrow l^\pm\{\rho^\pm, a_1^{z}\}$**

The inclusive $\tau$-spin analyzing power of the charged pion from $\rho$ decays and from the 1-prong $a_1$ decays is very small. As in the case of the lepton this is due to the fact that the corresponding spectral functions $b(E_{\pi^\pm})$ change sign within the physical region, see Fig. 4 of Ref. [35]. For these decay modes, where the decay of $a_1$ involves only one charged pion and the final state contains three neutrinos, the reconstruction of the $\tau^\pm$ rest frames is not possible. Therefore, we investigated the effect of cuts on the energies of the charged lepton and pion in the Higgs-boson rest frame. The resulting sensitivities to the mixing angle are given in Table II. We observe that for the $l^\pm\rho^\mp$ decay channel, a cut on the charged pion energy is essential to obtain a reasonable sensitivity to $\phi$. E.g., with a minimum cut of $E_{\pi^\pm} > 25 \text{ GeV}$ an asymmetry of $-10.2\%$ is obtained. Similarly, the asymmetry of the $l^\pm a_1^\mp$ decay channel...
Table III: Asymmetries, cross sections, and uncertainties $\Delta \phi$ for the decays $\tau \tau \rightarrow l \rho$ (left part) and $\tau \tau \rightarrow l a_{1} \rightarrow l + \pi + 2 \pi^{0}$ (right part). $E_{\pi}$ and $E_{l}$ denote the energy of the charged pion and lepton in the Higgs-boson rest frame. The cut $E_{l} \geq 10 \text{ GeV}$ has been applied. The values given for $\sigma_{l}^{\rho}$ and $\sigma_{l}^{a_{1}}$ were computed without this cut.

Table IV: Asymmetry, cross section, and uncertainty $\Delta \phi$ for the decay $\tau \tau \rightarrow \rho \rho$ (left part). $E_{\pi}$ denotes the energy of the charged pion in the respective $\tau$ rest frame. The right part of the Table contains the asymmetry and cross section for $\tau \tau \rightarrow a_{1} a_{1}$. The sensitivity of this decay mode to $\phi$ is very low; therefore $\Delta \phi$ is not given.

can be increased by a small amount. However, even with such cuts the overall sensitivity to $\phi$ of these decay modes is low.

\section*{E. $\tau^{-} \tau^{+} \rightarrow \rho^{-} \rho^{+}, a_{1}^{-} a_{1}^{+}$}

For the double-hadronic decay channels, we assume that the $\tau$ rest frames can be reconstructed \cite{46}. The ALEPH experiment at LEP showed \cite{45} that this is possible with an efficiency of 80% for this type of decay modes. Therefore, a cut on the energy of the charged pion from the decay of $\rho$ or $a_{1}$ can be applied in the respective $\tau$ rest frame.

The left part of Table IV contains our results for $\tau \tau \rightarrow \rho \rho$. Without a cut on the energies of $\pi^{\pm}$ from the decays of $\rho^{\pm}$, this decay mode is useless for the determination of $\phi$, as indicated by the small value $A^{\rho \rho} = 0.1\%$. Judiciously chosen cuts on $E_{\pi^{-}}$ and $E_{\pi^{-}}$, however,
Table V: Asymmetry, cross section, and uncertainty $\Delta \phi$ for the channel $\tau \tau \rightarrow \rho a_{1}$.

| $E_\pi$ in $\tau$ rest frame in GeV | $A^{\rho a_1}$ [%] | $\sigma/\sigma_0$ | $\Delta \phi$ [°] |
|-------------------------------------|------------------|-----------------|-----------------|
| $0.38$                              | $1$              | $0.38$          | $32.0$          |
| $E_\pi(\rho) > 0.6, E_\pi(a_1) > 0.53$ | $11.5$           | $0.14$          | $32.0$          |
| $E_\pi(\rho) < 0.6, E_\pi(a_1) < 0.53$ | $12.6$           | $0.38$          | $18.6$          |
| $E_\pi(\rho) > 0.6, E_\pi(a_1) < 0.53$ | $-13.3$          | $0.31$          | $19.1$          |
| $E_\pi(\rho) < 0.6, E_\pi(a_1) > 0.53$ | $-11.0$          | $0.17$          | $30.0$          |
| combined                            | $12.4$           | $1$             | $10.4$          |

lead to a dramatic increase in sensitivity to $\phi$. As the three sets of cuts given in Table IV do not intersect, one can compute their combined effect. We find that the mixing angle can be determined with an uncertainty $\Delta \phi \simeq 4°$ in this decay mode.

The right part of Table IV contains the asymmetry and cross section for $\tau \tau \rightarrow a_{1}a_{1}$ for three sets of cuts on the energies $E_{\pi^-}, E_{\pi^+}$ of the charged pions from the 1-prong $a_{1}$ decays. The increase in $\tau$-spin analyzing power by these cuts is offset by the decrease of the cross section which is small already without cuts. The sensitivity of this decay mode to $\phi$ is very low; therefore $\Delta \phi$ is not given.

**F. $\tau^-\tau^+ \rightarrow \rho^+a^+_1$**

Table V contains our results for the ‘non-diagonal’ 1-prong hadronic decay mode, $\tau \tau \rightarrow \rho a_{1} \rightarrow \pi^+\pi^-$. Various sets of cuts were applied to the energies of the charged pions from the $\rho$ and $a_{1}$ decay in the respective $\tau$ rest frames. As the four sets of cuts given in Table V do not intersect, we can compute the resulting combined statistical uncertainty $\Delta \phi$, and we obtain $\Delta \phi \simeq 10°$.

**G. $\tau^-\tau^+ \rightarrow \{\rho^\pm,a_1^\pm\}\{\pi^\pm,a_1^{\lambda}\}$**

Finally, we consider the double-hadronic decay channels where one of the $\tau$ leptons decays either directly to a pion or to three charged prongs via $a_1^\lambda$ ($\lambda = L, T$). We assume that the polarization states $a_1^\lambda$ can be reconstructed also in these channels. Notice that the sum of the branching ratios of the $\rho\pi$ and $\rho a_1^{T,L}$ decay modes is almost twice as large as that of the $\rho \rho$ mode analyzed in Sect. IV E. In addition, we recall that the $\tau$-spin analyzing power is maximal for $\tau \rightarrow \pi/a_1^{L,T}$. Therefore we expect that the sensitivity to the mixing angle $\phi$ of these two channels, $\rho\pi$ and $\rho a_1^{T,L}$, will be better than for the $\rho \rho$ channel given in Sect. IV E if an appropriate cut on the energy of the charged pion from the $\rho$ decay in the respective $\tau$ rest frame is applied.
| $E_{\pi(\rho)}$ in GeV | $A^{\rho\pi}$ ($A^{\rho a_1^L}$) [%] | $\sigma/\sigma_0$ | $\Delta\phi$ [°] | $E_{\pi(a_1)}$ in GeV | $A^{a_1\pi}$ ($A^{a_1 a_1^L}$) [%] | $\sigma/\sigma_0$ | $\Delta\phi$ [°] |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\tau$ rest frame in GeV | | | | | | | |
| $< 0.5$ | $-31.9$ | $0.48$ | $3.7$ | $<0.5$ | $20.6$ | $0.19$ | $18.5$ |
| | $(-31.9)$ | | | | $(-20.5)$ | | |
| $> 0.7$ | $36.0$ | $0.36$ | $3.7$ | $<0.4$ | $-22.3$ | $0.41$ | $10.1$ |
| | $(35.9)$ | | | | $(22.4)$ | | |
| combined | $31.0$ | $1$ | $2.6$ | combined | $15.7$ | $1$ | $9.1$ |
| | $(31.0)$ | | | | | | |
| $> 0.6$ | $31.9$ | $0.45$ | $3.8$ | $>0.53$ | $14.2$ | $0.31$ | $19.8$ |
| | $(31.8)$ | $(0.45)$ | | | $(14.2)$ | $(0.31)$ | |
| $< 0.6$ | $-30.3$ | $0.55$ | $3.7$ | $<0.53$ | $-16.3$ | $0.69$ | $11.1$ |
| | $(30.4)$ | $(0.55)$ | | | $(16.4)$ | $(0.69)$ | |
| combined | $31.0$ | $1$ | $2.6$ | combined | $15.7$ | $1$ | $9.1$ |
| | $(31.0)$ | | | | | | |
| $< 0.5$ | $-2.1$ | $1$ | $6.9$ | combined | $1$ | $24.2$ |
| $(-2.1)$ | | | | | | |

Table VI: Left part: Asymmetries, cross sections, and uncertainties $\Delta\phi$ for the decays $\tau\tau \rightarrow \rho\pi$, $\rho a_1^L$ ($\lambda = L, T$). The energy of the charged pion from a $\rho$ decay in the respective $\tau$ rest frame is denoted by $E_{\pi(\rho)}$. Right part: Asymmetries, cross sections, and uncertainties $\Delta\phi$ for the decays $\tau\tau \rightarrow a_1\pi$, $a_1 a_1^L$ ($\lambda = L, T$).

Our results for the $\rho\pi$ and $\rho a_1^L$ decay channels are given in the left part of Table VI. Because the pion-energy cuts of the third and fourth row of this Table do not intersect, we can compute their combined effect. We find that with these decay channels the mixing angle can be determined with a statistical uncertainty $\Delta\phi \simeq 3^\circ$.

The right part of Table VI contains our results for the $a_1\pi$ and $a_1 a_1^L$ decay modes. Because both the asymmetries $A^{a_1\pi}$, $A^{a_1 a_1^L}$ and the cross sections are smaller than in the case where one $\tau$ decays to a $\rho$, one can achieve only a moderate sensitivity to $\phi$ with these decay channels.

V. UNCERTAINTIES

There are a number of uncertainties that will affect the measurement of the distributions $\phi_{CP}^*$ in the various decay channels. One source of uncertainty results from beamstrahlung and initial state radiation (ISR) which change the total cross section. While beamstrahlung effects on the Higssstrahlung process [1] are small (for $m_H = 126$ GeV and at $\sqrt{s} = 250$ GeV), ISR reduces the cross section by a few percent [11].

The normalized $\phi_{CP}^*$ distribution is not directly affected by beamstrahlung and ISR, be-
cause the quality of the distribution depends, first of all, on the precision with which the normalized impact parameter vectors $\mathbf{\hat{n}}_{\pm}$ can be measured. For the lepton-lepton and lepton-hadron decay channels, we found the effect of ISR on the normalized $\varphi_{CP}$ distribution to be negligibly small, even if cuts on the final lepton or pion energies in the reconstructed Higgs-boson rest frame are applied. (The Higgs-boson rest frame is reconstructed using $\mathbf{p}_h = -\mathbf{p}_Z$.) On the other hand, as discussed above, for some of the double hadronic decay modes, it is necessary to reconstruct the $\tau$ rest frames for applying energy cuts that enhance the sensitivity to the $CP$ mixing angle $\phi$. Beamstrahlung and ISR will affect the quality of this reconstruction and will therefore reduce the sensitivity to $\phi$ to some extent [34].

The largest uncertainty involves the measurement uncertainty of the impact parameter vectors $\mathbf{\hat{n}}_{\pm}$. In order to study this issue we have performed a Monte Carlo simulation taking into account the expected measurement uncertainties by smearing the impact parameter direction. Measurement errors were assumed to be described by a Gaussian with a $1\sigma$ uncertainty of $\sigma_{\text{impact}} = 25^\circ$, as suggested in Ref. [27]. The resulting effect on the $\varphi_{CP}$ distributions is shown in Fig. 2 for a $CP = +1$ state and for the decay modes $h \rightarrow \tau^- \tau^+ \rightarrow \pi^- \pi^+ + \nu_\tau + \bar{\nu}_\tau$ and $h \rightarrow \tau^- \tau^+ \rightarrow l^- \pi^+ + \nu_\tau + \bar{\nu}_\tau + \bar{\nu}_l$. The effect turns out to reduce the asymmetry by a factor of $\approx 0.9$. We expect that a more realistic simulation would not drastically change our conclusions.

![Figure 2: The normalized $\varphi_{CP}$ distributions for the decays of a $CP$-even Higgs boson $h \rightarrow \tau \tau \rightarrow \pi \pi$ and $h \rightarrow \tau \tau \rightarrow l \pi$ if the impact parameter direction is smeared by a Gaussian with $\sigma_{\text{impact}} = 25^\circ$. The solid line shows the distributions without smearing.](image)

**VI. ESTIMATE OF $\Delta \phi$**

In this section we estimate, for the different $\tau$ decay channels, the statistical uncertainty $\Delta \phi$ by taking into account estimates of measurement uncertainties and background. For the hadron-hadron decay channels, a judicious choice of cuts can raise the signal-to-background
Table VII: Estimate of the asymmetries, the number of events, and the precision to the $CP$ mixing angle $\phi$ after taking into account measurement uncertainties as described in the text; $\sqrt{s} = 250$ GeV.

| $\tau\tau$-decay channel | $A$ [%] | # of events for $\mathcal{L} = 1 \text{ab}^{-1}$ | $\Delta\phi$ [$^\circ$] for $\mathcal{L} = 1 \text{ab}^{-1}$ | $\Delta\phi$ [$^\circ$] for $\mathcal{L} = 500 \text{fb}^{-1}$ | $\Delta\phi$ [$^\circ$] for $\mathcal{L} = 300 \text{fb}^{-1}$ |
|---------------------------|---------|---------------------------------|------------------------|------------------------|------------------------|
| $\pi + a_1^\ell \pi + a_1^\ell$ | 28.9 | 269 | 5.5 | 7.9 | 10 |
| $\pi + a_1^\ell$ | 3.4 | 292 | 7.7 | 11 | 15 |
| $\rho + a_1^\ell$ | 18.0 | 443 | 5.5 | 7.9 | 10 |
| $\rho \rho$ | 22.8 | 686 | 4.4 | 6.3 | 8.2 |
| $l l$ | 4.8 | 454 | 30 | 36 | 39 |
| $l \pi + a_1^\ell$ | 6.0 | 723 | 19 | 27 | 31 |
| $l a_1$ | 11.8 | 706 | 8.7 | 13 | 18 |
| $\tau\tau$-decay channel | $\Delta\phi$ [$^\circ$] for $\mathcal{L} = 1 \text{ab}^{-1}$ | $\Delta\phi$ [$^\circ$] for $\mathcal{L} = 500 \text{fb}^{-1}$ | $\Delta\phi$ [$^\circ$] for $\mathcal{L} = 300 \text{fb}^{-1}$ |
|---------------------------|------------------------|------------------------|------------------------|
| $\tau\tau$-decay channel | $\Delta\phi$ [$^\circ$] for $\mathcal{L} = 1 \text{ab}^{-1}$ | $\Delta\phi$ [$^\circ$] for $\mathcal{L} = 500 \text{fb}^{-1}$ | $\Delta\phi$ [$^\circ$] for $\mathcal{L} = 300 \text{fb}^{-1}$ |
| all had-had: | 3.0 | 4.3 | 5.5 |
| all lep-had: | 2.8 | 4.0 | 5.1 |

Table VII contains our results after inclusion of these effects. The asymmetries (second column) are reduced due to the additional flat background and due to the uncertainty of the impact parameter measurement. The number of events (for $1 \text{ab}^{-1}$, third column) is affected by two opposing effects: the limited efficiency leads to a reduction of signal events while the $ZZ$ background increases the event numbers. The last three columns show the resulting uncertainty $\Delta\phi$, for Higgs-boson production at $\sqrt{s} = 250$ GeV and for luminosities of $1 \text{ab}^{-1}$, $500 \text{fb}^{-1}$ and $300 \text{fb}^{-1}$. In each case, cuts on the lepton or pion energies as given in Tables II to VI have been chosen such that the uncertainty $\Delta\phi$ is minimized.

The results of Table VII show that the most precise measurement of the $CP$ mixing angle can be made using the hadron-hadron decay modes. The decays $\tau^+ \rightarrow \pi^+ \pi^\ell$, $\rho^+ \pi^\ell$, $a_1^+ \pi^\ell$ have the largest impact. While the lepton-lepton decay modes can be neglected in the determination of

---

4 We have checked for $e^+ e^- \rightarrow Z \rightarrow \tau\tau \rightarrow \pi\pi\nu\nu$, using the formulae for the matrix element given e.g. in [50], that this is indeed the case. This indicates that the $\phi^CP$ distribution is flat, too, for the background reaction $e^+ e^- \rightarrow ZZ$ where one $Z$ decays to $\tau\tau$. 

---
\( \phi \), including the lepton-hadron decays in the fit can lead to an improvement of the precision to \( \phi \) by about 8\% as compared to a fit based on hadron decays only. We finally estimate the precision of a \( \phi \) measurement to be \( \Delta \phi = 2.8^\circ \) for a luminosity of 1\,ab\(^{-1}\), \( \Delta \phi = 4.0^\circ \) for 500\,fb\(^{-1}\), \( \Delta \phi = 5.1^\circ \) for 300\,fb\(^{-1}\), and \( \Delta \phi = 5.9^\circ \) for 250\,fb\(^{-1}\).

If one considers Higgs-boson production at higher collider energies and assumes that efficiencies and measurement uncertainties do not change significantly then one obtains the following estimates: \( \Delta \phi = 6.9^\circ \) for \( \sqrt{s} = 350 \) GeV and 350\,fb\(^{-1}\), \( \Delta \phi = 8.8^\circ \) for \( \sqrt{s} = 500 \) GeV and 500\,fb\(^{-1}\), and \( \Delta \phi = 14^\circ \) for \( \sqrt{s} = 1 \) TeV and 1\,ab\(^{-1}\). The increase of \( \Delta \phi \) with increasing center-of-mass energy is due to the rapid decrease of the cross section \( \sigma(e^+e^- \to Zh) \).

VII. CONCLUSIONS

We have studied how precisely the \( CP \) nature of the 126 GeV spin-zero Higgs resonance \( h \), discovered last year at the LHC, can be determined at a future high-luminosity linear \( e^+e^- \) collider in its decays \( h \to \tau^-\tau^+ \) with subsequent decays of the \( \tau \) leptons to 1 or 3 charged prongs. The \( CP \) nature of \( h \) is reflected in the shape of the distribution of an angle \( \phi_{CP}^* \) which is defined in the zero-momentum frame of the charged prongs that result from \( \tau^\pm \) decay. Its measurement does not require the reconstruction of the \( \tau \) rest frames or the reconstruction of the momentum of the \( \rho \) meson or of the \( a_1 \) in its 1-prong decay mode. This may increase the precision with which the \( CP \) mixing angle \( \phi \) can be determined.

We have analyzed all major 1- and 3-prong decays of the \( \tau \) leptons. The most precise measurement of the mixing angle \( \phi \) that parameterizes the \( CP \) nature of \( h \) can be made using the hadron-hadron decay modes. The decays \( \tau^\pm \to \pi^\mp, \rho^\mp, a_1^{\mp,\lambda} (\lambda = L, T) \) have the highest sensitivity to \( \phi \). Moreover, we find that taking into account also the lepton-hadron decays of \( \tau^-\tau^+ \) leads to an improvement of the precision to \( \phi \) by about 8\%, while the sensitivity to \( \phi \) of the modes \( \tau^-\tau^+ \to l^-l'^+ \) is rather low. Assuming an integrated luminosity of 1\,ab\(^{-1}\) and Higgs-boson production at \( \sqrt{s} = 250 \) GeV we estimate, including background and measurement uncertainties, that the mixing angle \( \phi \) can be determined with a statistical uncertainty of \( \Delta \phi = 2.8^\circ \). We recall, however, that in the analysis presented here, several reconstruction efficiencies were assumed. Therefore, the achievable experimental precision on the \( CP \) mixing angle might be somewhat different than this number.

Acknowledgments

We wish to thank H. Videau for a correspondence and for providing us with a copy of [34]. The work of S. B. is supported by the Initiative and Networking Fund of the Helmholtz Association, contract HA-101 (‘Physics at the Terascale’) and by the Research Center ‘Elementary Forces and Mathematical Foundations’ of the Johannes-Gutenberg-Universität Mainz. The work of W. B. is supported by BMBF.
Appendix A

The normalized distributions of polarized $\tau$ decays to a charged lepton $l = e, \mu$, and to a charged pion via $\rho$ and $a_1$ decay have, in the $\tau$ rest frame, the form

$$\frac{1}{\Gamma(\tau^\pm \to a^\pm + X)} \frac{d\Gamma(\tau^\pm(\hat{s}^\pm) \to a^\pm(q^\pm) + X)}{dE_{a^\pm}d\Omega_{a^\pm}/(4\pi)} = n(E_{a^\pm}) \left(1 \pm b(E_{a^\pm}) \hat{s}^\pm \cdot \hat{q}^\pm\right). \quad (18)$$

Here, $\hat{s}^\pm$ denote the normalized spin vectors of the $\tau^\pm$ and $\hat{q}^\pm$ is the direction of flight of $a^\pm = l^\pm, \pi^\pm$ in the respective $\tau$ rest frame. The spectral functions $n$ and $b$ are shown in Figs. 1a and 4 of Ref. [35]. The function $b(E_a)$ encodes the $\tau$-spin analyzing power of particle $a^\pm = l^\pm, \pi^\pm$. The results described in [35] show that for all three decays, the function $b(E_a)$ changes sign in the allowed kinematic range.

[1] S. Chatrchyan et al. [CMS Collaboration], Phys. Rev. Lett. 110 (2013) 081803 [arXiv:1212.6639 [hep-ex]].
[2] S. Chatrchyan et al. [CMS Collaboration], JHEP 06 (2013) 081 [arXiv:1303.4571 [hep-ex]].
[3] G. Aad et al. [ATLAS Collaboration], arXiv:1307.1432 [hep-ex].
[4] G. Aad et al. [ATLAS Collaboration], arXiv:1307.1427 [hep-ex].
[5] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1 [arXiv:1207.7214 [hep-ex]].
[6] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716 (2012) 30 [arXiv:1207.7235 [hep-ex]].
[7] J. R. Dell’Aquila and C. A. Nelson, Phys. Rev. D 33 (1986) 93.
[8] J. R. Dell’Aquila and C. A. Nelson, Nucl. Phys. B 320 (1989) 61.
[9] W. Bernreuther and A. Brandenburg, Phys. Lett. B 314 (1993) 104.
[10] A. Soni and R. M. Xu, Phys. Rev. D 48 (1993) 5259 [hep-ph/9301225].
[11] V. D. Barger, K. Cheung, A. Djouadi, B. A. Kniehl and P. M. Zerwas, Phys. Rev. D 49 (1994) 79 [hep-ph/9306270].
[12] K. Hagiwara and M. L. Stong, Z. Phys. C 62 (1994) 99 [hep-ph/9309248].
[13] M. Krämer, H. Kühn, M. L. Stong and P. M. Zerwas, Z. Phys. C 64 (1994) 21 [hep-ph/9404280].
[14] T. Arens, U. D. J. Gieseler and L. M. Sehgal, Phys. Lett. B 339 (1994) 127 [hep-ph/9408316].
[15] A. Skjold and P. Osland, Phys. Lett. B 329 (1994) 305 [hep-ph/9402358].
[16] T. Arens and L. M. Sehgal, Z. Phys. C 66 (1995) 89 [hep-ph/9409396].
[17] A. Skjold and P. Osland, Nucl. Phys. B 453 (1995) 3 [hep-ph/9502283].
[18] S. Bar-Shalom, D. Atwood, G. Eilam, R. R. Mendel and A. Soni, Phys. Rev. D 53 (1996) 1162 [hep-ph/9508314].
[19] B. Grzadkowski and J. F. Gunion, Phys. Lett. B 350 (1995) 218 [hep-ph/9501339].
[20] J. F. Gunion, B. Grzadkowski and X. -G. He, Phys. Rev. Lett. 77 (1996) 5172 [hep-ph/9605326].
[21] W. Bernreuther, A. Brandenburg and M. Flesch, Phys. Rev. D 56 (1997) 90 [hep-ph/9701347].
[22] S. Bar-Shalom, D. Atwood and A. Soni, Phys. Lett. B 419 (1998) 340 [hep-ph/9707284].
[23] B. Grzadkowski, J. F. Gunion and J. Kalinowski, Phys. Rev. D 60 (1999) 075011 [hep-ph/9902308].
[24] S. Y. Choi, D. J. Miller, M. M. Mühlleitner and P. M. Zerwas, Phys. Lett. B 553 (2003) 61 [hep-ph/0210077].
[25] G. R. Bower, T. Pierzchala, Z. Was and M. Worek, Phys. Lett. B 543 (2002) 227 [hep-ph/0204292].
[26] K. Desch, Z. Was and M. Worek, Eur. Phys. J. C 29 (2003) 491 [hep-ph/0302046].
[27] K. Desch, A. Imhof, Z. Was and M. Worek, Phys. Lett. B 579 (2004) 157 [hep-ph/0307331].
[28] A. Rouge, Phys. Lett. B 619 (2005) 43 [hep-ex/0505014].
[29] J. R. Ellis, J. S. Lee and A. Pilaftsis, Phys. Rev. D 72 (2005) 095006 [hep-ph/0507046].
[30] E. Accomando et al., hep-ph/0608079.
[31] P. S. Bhupal Dev, A. Djouadi, R. M. Godbole, M. M. Mühlleitner and S. D. Rindani, Phys. Rev. Lett. 100 (2008) 051801 [arXiv:0707.2878 [hep-ph]].
[32] S. Berge, W. Bernreuther and J. Ziethe, Phys. Rev. Lett. 100 (2008) 171605 [arXiv:0801.2297 [hep-ph]].
[33] S. Berge and W. Bernreuther, Phys. Lett. B 671 (2009) 470 [arXiv:0812.1910 [hep-ph]].
[34] M. Reinhard, *CP violation in the Higgs sector with a next-generation detector at the ILC*, PhD thesis, LLR - École Polytechnique, November 2009; H. Videau, *Measuring the CP state of the Higgs through its decay H to tau tau*, IWLC2010 (International Workshop on Linear Colliders 2010) at CERN, https://espace.cern.ch/LC2010/default.aspx
[35] S. Berge, W. Bernreuther, B. Niepelt and H. Spiesberger, Phys. Rev. D 84 (2011) 116003 [arXiv:1108.0670 [hep-ph]].
[36] R. M. Godbole, C. Hangst, M. Mühlleitner, S. D. Rindani and P. Sharma, Eur. Phys. J. C 71 (2011) 1681 [arXiv:1103.5404 [hep-ph]].
[37] J. Ellis, D. S. Hwang, V. Sanz and T. You, JHEP 1211 (2012) 134 [arXiv:1208.6002 [hep-ph]].
[38] S. Berge, W. Bernreuther and H. Spiesberger, arXiv:1208.1507 [hep-ph].
[39] B. Ananthanarayan, S. K. Garg, J. Lahiri and P. Poulou, Phys. Rev. D 87 (2013) 114002 [arXiv:1304.4414 [hep-ph]].
[40] R. Harnik, A. Martin, T. Okui, R. Primulando and F. Yu, arXiv:1308.1094 [hep-ph].
[41] A. Rougé, Z. Phys. C 48 (1990) 75.
[42] M. Davier, L. Duflot, F. Le Diberder and A. Rougé, Phys. Lett. B 306 (1993) 411.
[43] J. H. Kühn, Phys. Rev. D 52 (1995) 3128.
[44] A. Stahl, Springer Tracts Mod. Phys. 160 (2000) 1.
[45] A. Heister et al. [ALEPH Collaboration], Eur. Phys. J. C 20 (2001) 401 [hep-ex/0104038].
[46] Y. -S. Tsai and A. C. Hearn, Phys. Rev. 140 (1965) B721.
[47] S. Dittmaier et al. [LHC Higgs Cross Section Working Group Collaboration], arXiv:1101.0593.
[hep-ph].

[48] J. Beringer et al. [Particle Data Group Collaboration], Phys. Rev. D 86 (2012) 010001.

[49] W. Bernreuther, P. Gonzalez and M. Wiebusch, Eur. Phys. J. C 69 (2010) 31 [arXiv:1003.5585 [hep-ph]].

[50] W. Bernreuther, G. W. Botz, O. Nachtmann and P. Overmann, Z. Phys. C 52 (1991) 567.