On a supersymmetric completion of the $R^4$ term in 

**type IIB supergravity**

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Abstract

We examine the question of the supersymmetric completion of the $R^4$ term in type IIB supergravity by using superfield methods. We show that while there is an obstruction to constructing the full action, a subset of the terms in the action can be consistently analyzed independent of the other terms, and these can be obtained from the superfield. We find the complete type IIB action involving the curvature and five-form field strengths.

1 Introduction

At low energies, string theory can be reduced to an effective field theory of the massless modes. All string theories have a massless graviton, and to leading order, the action for this field is the Einstein-Hilbert action. Supersymmetric type II strings have a large number of fields in addition to the graviton, and the complete two-derivative action for these fields is the $N = 2$ supergravity action in ten dimensions.

The effective action also contains an infinite series of higher derivative terms, suppressed by powers of the string scale $\alpha'$, and the complete action has the form

$$\kappa^2 S = S_2 + (\alpha')^4 S_8 + (\alpha')^5 S_{10} + \ldots$$

(1.1)

where $S_n$ contains terms with $n$ derivatives. $S_2$ is the supergravity action. The leading correction in type II theories is the eight-derivative action, which contains the famous $R^4$ term [1, 2]

$$S_{8;R^4} = \int d^{10}x \, t^8 t^8 R^4$$

(1.2)

This term occurs in all string theories and in the eleven-dimensional theory.

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There are also several other terms at the eight derivative level which involve the other fields of the theory (the Kalb-Ramond field, the dilaton and the Ramond-Ramond fields). These terms are believed to be related to the $R^4$ term by supersymmetry. Here we will discuss a procedure for finding (some of) these terms.

There are many reasons that one wishes to know the full action at the eight-derivative level.

At the basic level, knowledge of these terms will tell us a lot more about actions with maximal supersymmetry, which may lead to fundamental understandings like the off-shell nature of the theory.

From a phenomenological viewpoint, there has been a lot of interest in flux compactifications, where fluxes are turned on in the internal Calabi-Yau manifold (e.g. [3]). The potential for moduli in this background can be efficiently computed in the low energy effective theory, and can be used to gain information about stable compactifications at large radius. However, one needs to know the full action including the Ramond-Ramond (RR) field strengths.

Another place where the full effective action is required is for computing $\alpha'$ corrections in Anti-de-Sitter (AdS) backgrounds, for applications to the AdS/CFT correspondence. These can be applied to find corrections to black hole entropy, or to correlation functions.

Despite these motivations, it has not been possible so far to determine the complete eight derivative action. Several different approaches have been tried:

- The action can be computed by evaluating all the relevant string diagrams, and extracting the low energy action (see for example [1, 2, 4, 5]). The $R^4$ term can be found in this way. A related approach is to use sigma model techniques [6]. Another related approach was tried in [23].

- One can attempt to use dualities to generate terms involving RR fields, using the known terms involving NSNS fields.

- It is believed that the eight-derivative action is completely determined by supersymmetry alone. One can therefore attempt to construct the action by using the Noether method to generate terms step by step until supersymmetry is satisfied. This has been attempted for the heterotic string action [7, 8, 9, 10].

- If the complete superfield can be found, then the action can be written as an integral over one-half of superspace. This has also been attempted for the heterotic string in [11], and discussed for the maximally supersymmetric theories [12] (see also [13]).

Each of these approaches has serious difficulties.

- String diagrams contain much more information than just the eight-derivative terms. One needs an effective way of extracting the low energy limit without doing the entire computation. Furthermore, once we get to five-point amplitudes and beyond, we have to worry about extracting contributions to the amplitude involving the exchange of massless fields, for example those coming from a combination of the four-point
eight derivative amplitude and a tree level three-point interaction. Furthermore, the plethora of fields in the supergravities means that many amplitudes need to be computed. Sigma model techniques also require intense computational effort.

- Dualities do not determine the action. In general, dualities exchange an action at one value of a modulus with another action at another value of the modulus. While this constrains the way moduli can appear in any particular term in the action, it does not, in general, relate different terms in the action.

- The Neother method is completely general, but the vast number of fields and the plethora of possible terms make it impractical to use this method directly in ten dimensional supergravity.

- The superfield approach is by far the most promising. Unfortunately, there is a no-go result [14]: it is known that the eight-derivative action of type IIB supergravity cannot be written as the integral over one-half of superspace of a scalar superfield.

In this paper, we shall show that the superfield approach can be modified to obtain some information about the effective action.

Previous work on the eight derivative terms has produced many important results [15, 16]. Most importantly, several nonrenormalization theorems are known which strongly restrict the moduli dependence of the eight-derivative terms. We shall review some of these results further on.

Secondly, a lot of work has been done on the superfield approach to type IIB supergravity, starting with the seminal work of [17]. Particularly important for us will be the work of [14]. These authors applied the superfield approach to constructing the eight derivative terms in type IIB supergravity, and showed that there was an obstruction to constructing a viable supersymmetric action. We shall review their approach, and show how the obstruction they found can be (partially) avoided.

Our first new result will be the construction of the quartic terms in type IIB supergravity. This construction can be done using the linearized superfield, and has not appeared in the literature to our knowledge (although we believe that this construction is well known to the experts.) We shall also describe the moduli dependence of these terms.

We then go beyond the quartic level. We do this by the superfield construction. The supersymmetric measure does not exist in general, but we shall show that if we restrict ourselves to a well-defined subset of the terms, the measure can be constructed, and supersymmetric actions can be constructed. Our main result will be the complete type IIB action involving curvature and five-form field strengths alone.

We should emphasize that the specific form for this action has been found before in the literature [21]. These papers also used the integral of the superfield to find the action. Unfortunately, the analysis of [14] has shown that the full action cannot be written using superfields, and this a priori invalidates the results in these previous papers. The present paper shows that, in fact, the results obtained by these previous authors was correct,
despite the no-go theorem of [14]. This paper can hence be taken partly as a justification of the previous results in [21].

We will finally close with a discussion of our results and make a conjecture as to the extension of our results to find the other terms in the action.

2 Type IIB in components

We now discuss type IIB supergravity. This was first discussed in [18]; we will follow the conventions of [14].

The field content of Type IIB supergravity consists of the vielbein \( e_\mu^a \), a complex two-form field \( a_{\mu\nu} \), a real four form field \( a_{\mu\nu\rho\sigma} \), and a complex scalar \( a \). The fermionic fields are a graviton \( \psi_\mu \) and a dilatino \( \lambda \).

The scalar \( a \) transforms in a nonlinear representation under the \( SU(1,1) \). To represent the symmetry linearly, we introduce the new fields

\[
\mathcal{V} = \begin{pmatrix} u & v \\ v^* & u^* \end{pmatrix}
\]

with \( uu^* - vv^* = 1 \). The scalars \( u, v \) parametrize the \( SU(1,1)/U(1) \) coset manifold. \( SU(1,1) \) acts on the left, and \( U(1) \) acts from the right

\[
\mathcal{V}' = \begin{pmatrix} z & w \\ w^* & z^* \end{pmatrix} \begin{pmatrix} u & v \\ v^* & u^* \end{pmatrix} \begin{pmatrix} e^{-i\Sigma} & 0 \\ 0 & e^{i\Sigma} \end{pmatrix}.
\]

The physical field \( a \) is given by \( a = \frac{w}{w^*} \). It is invariant under \( U(1) \), and transforms under \( SU(1,1) \) as

\[
a' = \frac{za + w}{w^*a + z^*}.
\]

Furthermore, the axion \( C_0 \) and dilaton \( \phi \) of type IIB string theory can be defined by

\[
\tau = C_0 + i \exp(-\phi) = i \frac{1 - a}{1 + a}.
\]

To represent the scalar kinetic terms, we introduce the \( SU(1,1) \) invariant objects

\[
p = u^*dv - vdu^*, \quad q = \frac{1}{2i}(u^*du - vdv^*)
\]

\( q \) transforms as a \( U(1) \) connection.

The gauge field strengths are defined with weight \( n \) i.e.

\[
\mathcal{F}_{abc} = 3\partial_{[a}a_{bc]} \quad (2.8)
\]

\[
g_{abcde} = 5\partial_{[a}a_{bcde]} - 10i(a_{[ab}^*\mathcal{F}_{cde]} - a_{[ab}\mathcal{F}_{cde}^*)
\]

as is the gravitino field strength

\[
\psi_{ab} = 2\partial_{[a}\psi_{b]}.
\]

The five-form field strength \( g_5 \) is self-dual, and also invariant under \( SU(1,1) \). The field strengths \( \mathcal{F}_3 \) transform as a doublet under \( SU(1,1) \); we therefore define the \( SU(1,1) \) invariant 3-form field strengths

\[
(f_3^*, f_3) = (\mathcal{F}_3^*, \mathcal{F}_3)\mathcal{V}
\]
Under the local U(1), the complex field strength $f_3$ then has charge 1.

In terms of these fields the supersymmetry transformation laws are

$$\delta e_\mu^a = -i \left( (\zeta^* \gamma^a \psi_\mu) + (\zeta \gamma^a \psi_\mu^*) \right)$$  \hspace{1cm} (2.12)

$$\delta \psi_\mu = D_\mu \zeta - \frac{3}{16} \hat{f}_{abc} \gamma^{ab} \zeta^* + \frac{1}{48} \hat{f}_{abcd} \gamma_{abcd} \zeta^* - \frac{1}{192} i \hat{g}_{abcd} \gamma^{abcd} \zeta$$

$$+ \frac{1}{16} i \left[ -\frac{21}{2} (\lambda^* \gamma_\mu \lambda) + \frac{3}{2} (\lambda^* \gamma^a \lambda) \gamma_\mu a + \frac{5}{4} (\lambda^* \gamma_{\mu ab} \lambda) \gamma_{ab} - \frac{1}{4} (\lambda^* \gamma_{abc} \lambda) \gamma_{abc} \right] \zeta - (\zeta^* \gamma^a \psi_\mu^*) (\gamma_a \lambda) + (\psi_\mu^* \lambda) \zeta^* - (\zeta \lambda) \psi_\mu$$  \hspace{1cm} (2.13)

$$\delta u = 2(\zeta^* \lambda^*) v \hspace{1cm} \delta v = -2(\zeta \lambda) u$$  \hspace{1cm} (2.14)

$$\delta \lambda = \frac{1}{24} i \hat{f}_{abc} \gamma^{abc} \zeta + \frac{1}{2} i \hat{p}_a \gamma^a \zeta^*$$  \hspace{1cm} (2.15)

$$\delta (a^*_\mu, a_{\mu}) = - \left( (\zeta \gamma_{\mu \rho} \lambda^*) + 2i (\zeta^* \gamma_{\mu \rho} \lambda), -(\zeta^* \gamma_{\mu \rho} \lambda) + 2i (\zeta \gamma_{\mu \rho} \lambda_\mu) \right) \nu^{-1}$$  \hspace{1cm} (2.16)

$$\delta a_{\mu \rho \sigma} = -4(\zeta \gamma_{\mu \rho} \lambda^*_\sigma) + 4(\zeta^* \gamma_{\mu \rho} \lambda_\sigma) + 12i \left( a_{\mu \rho} \delta a^*_\sigma - a^*_{\rho \sigma} \delta a_{\mu \rho} \right)$$  \hspace{1cm} (2.17)

Here

$$D_\mu \epsilon = \partial_\mu \epsilon - \frac{1}{4} \omega_{\mu}^{bc} \gamma_{bc} \epsilon$$  \hspace{1cm} (2.18)

and the spin connection is defined as

$$2 \omega_{\mu}^{bc} = -e^{b\sigma} (\partial_\mu e^c_\sigma - \partial_\sigma e^c_\mu) + e^{c\sigma} (\partial_\mu e^b_\sigma - \partial_\sigma e^b_\mu) + e^{b\sigma} e^c_\mu (\partial_\sigma e_{\rho \sigma} - \partial_\rho e_{\sigma \sigma})$$  \hspace{1cm} (2.19)

We have also defined the supercovariant expressions

$$\hat{p}_a = p_a + 2(\psi_a \lambda)$$

$$\hat{f}_{abc} = f_{abc} - 3(\psi^*_a \gamma_{bc} \lambda) - 3i (\psi_{[a} \gamma_{bc]} \lambda)$$

$$\hat{f}_{abcde} = f_{abcde} + 20(\psi^*_a \gamma_{bcde} \lambda)$$  \hspace{1cm} (2.20)

The U(1) charges of every field are now fixed by the consistency of the supersymmetry transformations. We find that under the local U(1), the charges of the fields are

$$e_\mu^a, g_5 : 0 \hspace{1cm} \psi_\mu, \epsilon : \frac{1}{2} \hspace{1cm} f_3 : 1 \hspace{1cm} \lambda : \frac{3}{2} \hspace{1cm} p_a : 2$$  \hspace{1cm} (2.21)

3 The $R^4$ terms

The eight-derivative effective action $S_8$ is a sum of several terms, each of which has a moduli-dependent coefficient. For type IIB, the effective action has the generic form

$$S_8 = f_{R^4} (\tau, \tau^*) R^4 + f_{R^2 f^3_3} (\tau, \tau^*) R^2 f^3_3 + f_{R^2 \lambda \lambda} (\tau, \tau^*) R^2 \lambda \lambda + \ldots$$  \hspace{1cm} (3.22)

where we have shown a few of the many possible terms.
The structure of the $R^4$ terms can be made more explicit. Calculations in type IIB string theory \cite{19} show that the complete action involving the curvature alone is

$$S_{8,R^4} = f_{R^4}(\tau, \tau^*)(t^{abcd}f_{gh}e_{ijklmnop} + \epsilon^{ijklmnop}(e_{ijklmnop})R_{ab}^ijR_{cd}^{kl}R_{ef}^{mn}R_{gh}^{op}) \quad (3.23)$$

where $t^{abcd}f_{gh}$ is defined in Appendix 9.A of \cite{19}. The structure of the remaining terms have not been explored in as much detail.

The requirement of $U(1)$ invariance strongly constrains the moduli-dependent coefficients. For example, $R^4$ is invariant under the local $U(1)$ symmetry, and so $f_{R^4}(\tau, \tau^*)$ must also be invariant under the symmetry. Similarly, $R^2f_3^2$ has a charge 2, and so $f_{R^2f_3^2}(\tau, \tau^*)$ should have charge $-2$ under the symmetry.

The effective action can therefore be separated into different pieces according to the $U(1)$ transformation law of the moduli-independent terms

$$S_8 = S_{8,0} + S_{8,1} + \ldots + S_{8,24} \quad (3.24)$$

where the $S_{8,i}$ have the schematic form

$$S_{8,0} = f^{(0,0)}(\tau, \tau^*) (R^4 + R^2\psi^\dagger \psi + (g_5)^8 + R^2f_3^2f_3^* + \ldots) \quad (3.25)$$

$$S_{8,1} = f^{(1, -1)}(\tau, \tau^*) (R^2\psi^\dagger \psi + \ldots) \quad (3.26)$$

$$S_{8,2} = f^{(1, -1)}(\tau, \tau^*) (R^2\psi^\dagger \lambda + R^2f_3^2f_3 + \ldots) \quad (3.27)$$

$$\ldots \quad (3.28)$$

$$S_{8,24} = f^{(12, -12)}(\tau, \tau^*) (\lambda^{16}) \quad (3.29)$$

The coefficients have been written in terms of the modular forms $f^{(w, -w)}(\tau, \tau^*)$. Also, we have made an assumption here that the $U(1)$ transformation uniquely determines each moduli-dependent coefficient up to a constant. This assumption was justified by the detailed analysis of \cite{15}.

Under a $SL(2,\mathbb{Z})$ transformation

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \quad (3.30)$$

these functions transform as

$$f^{(w, -w)}(\tau, \tau^*) \rightarrow f^{(w, -w)}(\tau, \tau^*)(c\tau + d)^w(c\tau^* + d)^{-w} \quad (3.31)$$

The explicit form of the modular forms can be determined by supersymmetry \cite{15}. We define the action of the modular covariant derivative on non-holomorphic forms $f^{(w, -w)}(\tau, \tau^*)$ as

$$D_w = i\left(\tau_2 \frac{\partial}{\partial \tau} - i\frac{w}{2}\right) \quad (3.32)$$

Then supersymmetry implies that

$$f^{(w+1, -w-1)}(\tau, \tau^*) = D_w f^{(w, -w)}(\tau, \tau^*) \quad (3.33)$$

Furthermore, we find that $f^{(w, -w)}(\tau, \tau^*)$ are eigenfunctions of the Laplacian on the $SL(2,\mathbb{Z})$ space. In particular, one finds

$$\nabla^2 f^{(0,0)}(\tau, \tau^*) = \frac{3}{4} f^{(0,0)}(\tau, \tau^*) \quad (3.34)$$
For weak coupling, \( f^{(0,0)}(\tau, \tau^*) \) should have a power law behavior. This fixes \( f^{(0,0)}(\tau, \tau^*) \) to be the Eisenstein series of order \( \frac{3}{2} \)

\[
f^{(0,0)}(\tau, \tau^*) = \sum_{m,n \neq (0,0)} \frac{\tau^{\frac{3}{2}}}{|m + n\tau|^3}
\]

(3.35)

The other \( f^{(w, -w)}(\tau, \tau^*) \) are then determined by (3.33).

4 Type IIB in superfields

We now turn to the superfield formulation of type IIB supergravity. This section follows the formulation of [14] very closely.

The superspace formulation of type IIB supergravity was first constructed in [17]. One introduces a Grassmann variable \( \theta^\alpha \) which is a 16-component Weyl spinor of \( SO(9,1) \). The superspace coordinates are then \( (x^\mu, \theta^\alpha, (\theta^\alpha)^\dagger) \). The covariant derivatives satisfy the algebra

\[
[D_A, D_B] = -T^C_{AB} D_C + \frac{1}{2} R_{ABC}^D L^C_D + 2i M_{AB} \kappa
\]

(4.36)

where \( T^C_{AB} \) is the torsion, \( R_{ABC}^D \) is the curvature, and \( M_{AB} \) is the U(1) curvature (the explicit values of these tensors can be found in [14]). The super-Jacobi identities then produce a large set of relations.

To obtain the field content of type IIB supergravity, we need to impose constraints on the superspace. After the imposition of the constraints, the relations obtained from the super-Jacobi identities become nontrivial and need to be solved. This was done in [17], where it was shown that the fields of type IIB supergravity could be obtained from a chiral superfield \( V \) satisfying

\[
D^*_\alpha V = 0
\]

(4.37)

The super-Jacobi identities provide several further constraints on the superfield. These constraints were summarized in [17, 14].

All components of the superfield \( V \) can be obtained by solving these constraints. The first few components are found to be [17, 14]

\[
V| = v
\]

\[
D_\alpha V| = -2u\lambda_\alpha
\]

(4.38)

\[
D_{[\alpha D_\beta]} V| = \frac{i}{12} u\gamma_{\alpha\beta} \hat{f}^{abc} \hat{f}_{abc}
\]

(4.39)

\[
D_{[\gamma D_\beta]D_\alpha]} V| = \frac{i}{12} u\gamma_{\alpha\beta} \left\{ -\frac{1}{32} \left( \gamma_{abcedf} \hat{f}^{abcedf} + 3\hat{f}_{[a}^{de} \gamma_{bc]d} + 52\hat{f}_{[ab} \gamma_{c]}d + 28\hat{f}_{abc}\right) \gamma^{\epsilon\lambda} + 3\hat{f}_{[a} \gamma_{bc]}\gamma^{\lambda} \hat{f}^{\epsilon}_{\gamma} \right\}.
\]

(4.40)

The bosonic part of the fourth component is given by

\[
D_{[\beta D_\gamma} D_\beta D_\alpha]} V| = u\gamma_{\beta\alpha} \hat{f}^{de} \hat{f}_{de}
\]

(4.41)

(4.42)
\begin{align*}
R_{abcdef} &= \frac{1}{16} \left( g_{ad} c_{bce} f - \frac{i}{6} D_b g_{acdef} \right) - \frac{1}{1536} \left( 3 g_{bfmn} g_{ced}^m n - g_{abcmn} g_{def}^m n \right) + f_3^* f_3 \text{ terms}.
\end{align*}

where \( c_{bce} \) is the Weyl tensor.

Finally, the supersymmetry variations of the component fields can be obtained by using the equation for the variation of the superfield under supersymmetry transformations

\[ \delta \xi V = \xi^* D^*_\alpha V + \xi D_\alpha V \]

The superfield \( U^* \) is also chiral; it is however not independent since it satisfies

\[ UU^* - VV^* = 1 \]

Any function of \( V,U^* \) is also chiral.

5 From superfields to the action

Now that we have constructed superfields, one can attempt to use them to construct supersymmetric actions. To get an eight derivative action, we integrate over one-half of superspace.

5.1 The linear case

We will first try this with the linearized superfield. We first gauge fix the \( U(1) \) symmetry by assuming that the vacuum solution is given by \( u = 1, v = 0 \), i.e.

\[ V_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

Linearized perturbations about this background value will be represented by

\[ V_{lin} = \begin{pmatrix} 1 & v \\ v^* & 1 \end{pmatrix} \]

At the linearized level, we can set \( v_{lin} = a_{lin} \).

The first few components of the linearized superfield can now be immediately obtained from the above formulae

\begin{align*}
|V_{lin}| &= v \quad (5.48) \\
D_\alpha |V_{lin}| &= -2 \lambda_\alpha \quad (5.49) \\
D_{[\alpha} D_{\beta]} |V_{lin}| &= \frac{i}{12} \gamma_{\alpha \beta \gamma} f_{abc} \quad (5.50) \\
D_{[\gamma} D_{\beta]} D_{\alpha]} |V_{lin}| &= -\frac{1}{4} \gamma_{\alpha \beta \gamma} \gamma_\epsilon \gamma_\psi f_{bc} \quad (5.51)
\end{align*}

while the bosonic part of the fourth component is

\[ D_\beta D_\gamma D_\beta D_\alpha |V_{lin}| = \frac{1}{16} \gamma^{abc \gamma def} \left( g_{ad} R_{bce} - \frac{i}{6} D_b g_{acdef} \right) \]

5}
To get the $R^4$ action, we integrate $V^4$ over one half of superspace to get [14]

$$\int d^{16}\theta (V_{\text{lin}})^4 = (t^{abcdefgh}t^{ijklmnop} + \epsilon^{IJabcdefgh}\epsilon_{IJijklmnop}) R^{ij}_{ab} R^{kl}_{cd} R^{mn}_{ef} R^{op}_{gh} + \ldots \tag{5.53}$$

This is in agreement with the string calculation, which is an indication that we may be on the right track.

To fill out the rest of the factors in (3.25), we construct the action

$$S_{8,\text{lin}} = \int d^{10}x \sqrt{g} f^{(0,0)}(\tau, \tau^*) \int d^{16}\theta (V_{\text{lin}})^4 \tag{5.54}$$

This expression is not completely supersymmetric; however all variations containing at most four fields cancel.

The other terms from the expansion of the action will then produce the quartic action of type IIB supergravity.

### 5.2 The nonlinear case and a failure

We now review the results obtained by [14] for the nonlinear action.

When we try to go beyond the quartic action, we will need the full nonlinear superfield. In addition we need a supersymmetric measure; the supersymmetric analogue to the $\sqrt{g}$ factor. The suggested form of the eight-derivative action is then

$$S_8 = \int d^{10}x \int d^{16}\theta \Delta W[V, U^*] \tag{5.55}$$

where $\Delta$ is by definition a superfield whose lowest component is

$$\Delta|_{\theta=0} = \sqrt{g} \tag{5.56}$$

$\Delta$ is to be constructed order by order by requiring that the action be supersymmetric. (When we were considering the linear action, we did not need to worry about constructing $\Delta$, since it only contributes to higher order terms.)

The action can be written

$$S_8 = \int d^{10}x \epsilon^{\alpha_1\ldots\alpha_{16}} \sum_{n=0}^{16} \frac{1}{n!(16 - n)!} D_{\alpha_1}\ldots D_{\alpha_n} \Delta | D_{\alpha_{n+1}}\ldots D_{\alpha_{16}} W|$$

$$= \int d^{10}x \sum_{n=0}^{16} \frac{1}{n!} D_{\alpha_1}\ldots D_{\alpha_n} \Delta | D^{16-n,\alpha_1\ldots\alpha_n} W| \tag{5.57}$$

where we have introduced the notation

$$D^{16-n,\alpha_1\ldots\alpha_n} W = \frac{1}{(16 - n)!} \epsilon^{\alpha_1\ldots\alpha_{16}} D_{\alpha_{n+1}}\ldots D_{\alpha_{16}} W . \tag{5.58}$$

Invariance of the action under supersymmetry requires

$$\delta S = \int d^{10}x \sum_{n=0}^{16} \frac{1}{n!} \left( \delta D_{\alpha_1}\ldots D_{\alpha_n} \Delta | D^{16-n,\alpha_1\ldots\alpha_n} W| + D_{\alpha_1}\ldots D_{\alpha_n} \Delta | \delta D^{16-n,\alpha_1\ldots\alpha_n} W| \right) = 0. \tag{5.59}$$
The first projection of $\Delta$ is determined by equation (5.56). The next projection can be determined by requiring the cancellation of variations containing $D^{16}W$ and $\zeta$. This yields [14]

$$D_\alpha \Delta |_{\theta=0} = -ie\epsilon^{c}_{\alpha;\beta} \psi^{*\beta}$$

Similarly, the cancellation of terms proportional to $D^{15}W$ determines

$$[D_\alpha, D_\beta] \Delta |_{\theta=0} = \frac{1}{12} ie\epsilon^{abc}_{\alpha;\beta} f^{*}_{abc} + O[\psi^{*}\psi^{*}, \lambda^{*}\psi]$$

At the same time, the terms proportional to $\zeta^{*}$ need to cancel as well, and this has to happen automatically for this construction to work. As it turns out, the terms containing $D^{16}W$ and $\zeta^{*}$ do cancel, but at next order the cancellation does not work [22, 14]. The uncANCELLED term is

$$\delta S = \frac{1}{2} \int d^{10} x e^{*} T_{\alpha;\beta} \tau^{\delta} T_{\gamma;\delta} D^{15}W$$

This then implies that the eight-derivative action cannot be written as an integral over one-half of superspace.

6 A second attempt at an action

This analysis suggests that we should give up the idea of reproducing the complete action from this integral, which as we have seen is a hopeless task. Instead we can try to use the superfield to reproduce a subset of the terms in the action. In fact, we will now show that the superfield construction can be used to find the complete effective action containing just the curvature and five-form field strength $g_{5}$.

First, we must show that there is a consistent way to restrict ourselves to this subset of terms. The $U(1)$ symmetry will help us here. The curvature and five-form field strength $g_{5}$ are both uncharged under the $U(1)$ symmetry. Hence all the terms containing only these two objects are in $S_{8;0}$, and will occur multiplied by $f^{(0,0)}(\tau, \tau^{*})$. The bosonic terms containing only $R, g_{5}$ then have the schematic form

$$S_{8;0;R,g_{5}} = \int d^{10} x f^{(0,0)}(\tau, \tau^{*})(R^{4} + g_{5}^{8} + \ldots)$$

Under a supersymmetry variation, these terms produce a huge set of variations that need to be cancelled. We will consider the subset of variations which have at most one fermion field, and where we set $\partial \tau^{*} = \partial \tau = \lambda = a_{2} = 0$. We can then ignore the variation of $f^{(0,0)}(\tau, \tau^{*})$.

The remaining variations are of the schematic form

$$\delta S_{8;0;R,g_{5}} = \int d^{10} x f^{(0,0)}(\tau, \tau^{*})(R^{3} \psi^{*} \epsilon + (g_{5})^{7} \psi^{*} \epsilon + \ldots)$$

These can be cancelled by variations coming from terms containing fermion bilinears which are of the schematic form

$$S_{8;0;R,g_{5},\psi^{*}\psi} = \int d^{10} x f^{(0,0)}(\tau, \tau^{*})(R^{2} \psi^{*} \psi + (g_{5})^{7} \psi^{*} \psi + \ldots)$$
We note that it is essential for the gravitino terms to have a $\psi^*$ as well as $\psi$ in order to cancel the $U(1)$ charge.

We now argue that we can consistently restrict ourselves to this subset of terms in the action viz. $S_{8;0;R,g_5}$ and $S_{8;0;R,g_5,\psi^*,\psi}$, as long as we only look for the cancellation of the subset of the variations $\delta S_{8;0;R,g_5}$ discussed above.

The reason this is not obvious is that the variations of a term linear in a field (say $a_2$) includes terms independent of $a_2$. So one might expect that in general, a cancellation of the supersymmetry variations $\delta S_{8;0;R,g_5}$ which satisfy $\partial \tau^* = \partial \tau = \lambda = a_2 = 0$ will require us to consider terms in the action which are linear in these fields.

In this case, the $U(1)$ symmetry helps us out. $U(1)$ invariance of the bosonic terms implies that the terms multiplied by $f^{(0,0)}(\tau, \tau^*)$ cannot be linear in the fields $\partial \tau^*, \partial \tau, \lambda, a_2$, which are all charged under the $U(1)$. It is possible to have terms linear in these fields if they are multiplied by a compensating factor $f^{(w,-w)}(\tau, \tau^*)$ with the appropriate $U(1)$ charge, but these terms cannot contribute to the cancellation of the variations in $\delta S_{8;0;R,g_5}$.

We can have terms containing fermion bilinears which are linear in $a_2$, where the $U(1)$ charge is cancelled by having $\psi \psi$ instead of $\psi^* \psi$. However the variation of $a_2$ will then produce variations which are trilinear in fermion fields, and which again do not contribute to the cancellation of $\delta S_{8;0;R,g_5}$.

Accordingly, we can set $\partial \tau = \lambda = a_2 = 0$ for all terms in $S_{8;0;R,g_5}$ and $S_{8;0;R,g_5,\psi^*,\psi}$ contributing to cancel $\delta S_{8;0;R,g_5}$. In other words, we can restrict ourselves to the subset of terms $S_{8;0;R,g_5}$ and $S_{8;0;R,g_5,\psi^*,\psi}$.

We can now attempt to find the precise form of the action by requiring that the supersymmetry variations coming from $S_{8;0;R,g_5}$ and $S_{8;0;R,g_5,\psi^*,\psi}$ cancel. This would be an extremely tedious procedure. Instead we shall see that the superfield offers a quick way to reproduce these terms in the action.

As we have seen, we can set $\partial \tau = \lambda = a_2 = 0$. The $U(1)$ structure then ensures that the terms we are looking for all occur in the superfield only in the third, fourth and fifth components i.e. with a factor of $\theta^3, \theta^4$ or $\theta^5$. In particular the bosonic terms are all found in the $\theta^4$ component.

To get a 8-derivative term, we should consider the action

$$S_8 = \int d^{10}x g(\tau, \tau^*) \int d^{16} \theta \Delta V^4$$

(6.66)

which is the natural extension of the linearized action (5.54).

To obtain bosonic terms, we should look at the $\theta^4$ component in $V$. The 16 $\theta$ are then already saturated from the $V^4$ term. For the terms bilinear in fermions, at least 15 $\theta$ must be taken from the $V^4$ term.

Hence to construct the action, we only need the first two components of $\Delta$, i.e. $\Delta|_{\theta=0} \equiv \sqrt{g}$ and $D_\alpha \Delta|_{\theta=0}$. We do not need the other components of the measure, as long as we are restricting ourselves to this particular subclass of terms. That is, we may truncate the action to

$$S = \int d^{10}x \frac{1}{16!} g(\tau, \tau^*) \epsilon^{\alpha_1..\alpha_{16}} (\sqrt{g} D_{\alpha_1}...D_{\alpha_{16}} W| + 16 D_{\alpha_1} \Delta| D_{\alpha_2}...D_{\alpha_{16}} W|)$$

(6.67)
where we have set $W \equiv V^4$. We can now consider the variations of this action.

We are setting $\partial \tau = \lambda = a_2 = 0$. Furthermore, we are considering variations with at most one fermion field. In this case, we can set $D^nW = 0$ in the supersymmetry variations for all $n \leq 14$. We then only need to cancel the variations proportional to $D^{16}W$ and $D^{15}W$.

Now, as we discussed above, the components of $\Delta$ have already been computed in [14], yielding

$$\Delta|_{\theta=0} = \sqrt{g}$$

$D_\alpha \Delta|_{\theta=0} = -i e^{c A}_\alpha \psi^\beta$ (6.69)

Furthermore, it was shown that the variations proportional to $D^{16}W$ and $D^{15}W$ do cancel up to the obstruction shown in equation (5.62). However, the torsion factor is

$$T_{\gamma\beta} = (\gamma^a)_{\gamma\beta} (\gamma^\alpha)_{\delta\tau} \lambda^* - 2\delta_{\gamma}^\delta \lambda^*$$

which vanishes if we set $\partial \tau = \lambda = a_2 = 0$ in the variations. This means that the obstruction vanishes, and the variations $\delta S_{8;0;R,g}$ indeed cancel in the action (6.67).

Finally, we need to fix the moduli-dependent function $g(\tau, \tau^*)$. To do this, we require that the action reproduce the known $R^4$ term. Noting that the bosonic part of the fourth component is given by

$$D_{[\delta}D_{\gamma}D_{\beta}D_{\alpha]}V = \frac{1}{16} u^{abc} d^{def} (g_{\alpha\beta} R_{bcdf} + \ldots)$$

we find that up to an overall constant

$$g(\tau, \tau^*) = \frac{1}{16} f^{(0,0)}(\tau, \tau^*)$$

This reproduces the $R^4$ term. The supersymmetric completion including $g_5$ terms is then given by (6.67).

The bosonic part of the action can be immediately written down; up to an overall constant

$$S_{8;0;R,g_5} = \int d^{10} x f^{(0,0)}(\tau, \tau^*) \times$$

$$\epsilon^{\alpha_1 \ldots \alpha_{16}} (\gamma^\alpha_{\alpha_1 \alpha_2 \alpha_3} h_{b_1 b_2 b_3} (\gamma^c_{\alpha_4 \alpha_5 \alpha_6} \gamma^d_{\alpha_7 \alpha_8} \ldots \gamma^g_{\alpha_{13} \alpha_{14} \alpha_{15} \alpha_{16}}) \times$$

$$R_{a_{1 \ldots 3} b_1 b_2 b_3} R_{c_{1 \ldots 3} d_1 d_2 d_3} \ldots g_{1g_{2g_{3h_{1h_{2h_{3}}}}}} \times$$

where we have defined

$$R_{abdef} = \frac{1}{16} (g_{\alpha d c} c_{be f} - \frac{i}{6} D_{kb} g_{a cdef}) - \frac{1}{1536} (3 g_{bafmn} g_{ced}^{mn} - g_{abcmn} g_{def}^{mn})$$

The fermionic terms can be written down similarly.

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2In a previous draft, we had used the Riemann tensor instead of the Weyl tensor. We would like to thank the authors of [14] for pointing out this error.
This procedure can be generalized by considering any action of the form (6.67), with $W$ being replaced by any function of $V$, and with the first two components of $\Delta$ given in equation (6.68). Then our analysis proceeds unchanged, and we can shown that all variations with one fermion field and $\partial \tau = \lambda = a_2 = 0$ cancel. This allows us to generate a large set of actions involving $R, g_5$ and $\psi$ which have at least a partial cancellation of the supersymmetry variations.

7 Discussion and a conjecture

Let us now summarize our results.

We have found the quartic action of type IIB including the moduli dependence in equation (5.54). Supersymmetry variations of this action will produce a large number of variations; the terms which are quartic in the fields will cancel.

We then extended the analysis beyond the quartic level. We restricted ourselves to terms involving the curvature $R$ and five-form field strength $g_5$. To cancel the variations coming from these terms we also considered terms containing the curvature, the five form field strength and in addition, two gravitino fields.

These terms were found using the superfield construction of [17, 14]. In terms of this superfield, the terms discussed in the paragraph above can all be generated by the action (6.67) where the components of the measure are given in equation (6.68). In particular, all bosonic terms involving $R$ and $g_5$ are given by the action (6.73). This confirms the results of [21].

In hindsight, it is clear that the superfield construction cannot give us the full action. The action (5.55) depends on an arbitrary function $W$, while the real action is completely determined by supersymmetry up to an overall factor. In other words, we normally use superfields in order to be able to construct the most general supersymmetric action. But the action up to eight derivatives is unique, so the superfield construction (5.55) will not work. (It may be possible to use a non-scalar superfield, in which case the action would not be of the form (5.55). We would like to thank the authors of [14] for bringing this possibility to our attention.)

Here we argued that despite this failure, the superfield indeed reproduces a subset of terms; in particular all terms with only $R$ and $g_5$ can be reproduced. It is now natural to ask if we can extend this approach to obtain the remaining terms in the action.

First, we note that the vanishing of the obstruction (5.62) does not require us to impose $a_2 = 0$. This suggests the

Conjecture: A superfield $\Delta$ can be constructed such the subset of the supersymmetry variations of the action (5.55) which have one fermion field and which satisfy $\partial \tau^* = \partial \tau = \lambda = 0$ all vanish.
Proving this conjecture would require us to continue generating further projections of $\Delta$ by the procedure outlined in [14], and reviewed above. We will leave the proof or disproof of this conjecture to future work.

It does not seem possible to extend the scope of the superfield further than this conjectured extension. The apparently insuperable obstruction is the fact the moduli-dependent coefficients are non-holomorphic. It is hard to see how any integral of holomorphic field can produce such coefficients. The integral over superspace will generically produce moduli dependent coefficients $W[v, u^*]$, which cannot correspond to the required coefficients $f^{(w, -w)}(\tau, \tau^*)$.

This suggests that we will have to give up the idea of obtaining the coefficients from a superfield construction and instead consider an action of the form

$$S_8 = \int d^{10}x f^{(w, -w)}(\tau, \tau^*) \int d^{16}\theta \Delta W[V, U^*]$$  (7.75)

The cancellation of terms coming from the variations of the moduli-dependent coefficients will not occur, and accordingly, the supersymmetry variations of this action will have uncancelled terms containing either a derivative acting on a scalar ($\partial \tau^*$ or $\partial \tau$) or a dilatino factor. Hence it would seem impossible that the entire action can be constructed using superfields.

It is possible that to find the full action, one will have to modify the superfield approach drastically. In particular, when we constructed the superfield, we imposed certain constraints on the torsion and curvature. These constraints are required in order to reduce the large number of fields in the superfield. Now it is very possible that the $\alpha'$ corrections modify these constraints as well. The corrected constraints will produce a superfield different from the one we considered, and may lead to a supersymmetric action [20]. Unfortunately, it is a very difficult task to find the correct constraints with no additional help. The construction given in this paper may be of help in finding these constraints.

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