Dealing with T and CPT violations in mixing as well as direct and indirect CP violations for neutral mesons decaying to two vectors

Anirban Karan

Indian Institute of Technology Hyderabad, Kandi, Sangareddy 502285, Telengana, India

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Abstract A large number of observables can be constructed from differential decay rate based on the polarization of final state while considering decay of a neutral meson ($P_0$ or $\bar{P}_0$) to two vector particles. But all of these observables are not independent to each other since there are only a few independent theoretical parameters controlling the whole dynamics and therefore various relations among observables emerge. In this paper, we have studied the behaviour of observables for neutral meson decaying to two vectors in presence of $T$ and $CPT$ violations in mixing accompanied by both direct and indirect $CP$ violations. We have expressed all of the fourteen unknown theoretical parameters for this scenario in terms observables only and constructed the complete set of thirty four relations among observables whose violation would signify the existence of some new Physics involving direct violation of $CPT$. In addition, using this formalism we have studied three special cases too: (a) SM scenario, (b) SM plus direct $CP$ violation (c) SM plus $T$ and $CPT$ violation in mixing.

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1 Introduction

$CPT$ invariance is one of the most fundamental principles in Physics. It is believed that any natural process must be described by a $CPT$ invariant Lagrangian. According to $CPT$ theorem, any quantum field theory involving point-particles in flat Minkowski space, delineated by Hermitian, local, Lorentz-invariant Lagrangian (or Hamiltonian), is certainly $CPT$ invariant [1,2]. The primary proof of this theorem was given by Lüders, Pauli and others [3–7] (an updated version of this approach can be found in Ref. [8]) depending on the formulation of Hamiltonian (or Lagrangian) for quantum field theory. Later on the theorem was proven rigorously by Jost and others [9–11] in the axiomatic formalism of quantum field theory based on the assumptions of Lorentz invariance, existence of unique vacuum state and weak local commutativity obeying ‘right’ statistics. The line of proof in this approach mainly depends on Wightman axioms, Wightman functions, the Wightman reconstruction theorem and Bargmann-Hall-Wightman theorem on complex Lorentz transformations [11–13].

Nonetheless, there are various models in literature that violates $CPT$ by evading some of the necessary conditions of $CPT$ theorem. Both the approaches, mentioned above, address $CPT$ theorem for fundamental particles only and they fail to handle the same with QCD bound states (although Ref. [14] that uses dynamical principle and variation of action methods of Schwinger [7] claims a third way to prove $CPT$ theorem incorporating bound states too). Open Bosonic strings, which are not point-like object, can go through spontaneous $CPT$ violation [15]. Again, violation of Lorentz symmetry may also lead to $CPT$ violation [16–22]. Non-trivial space-time topology could be a reason for $CPT$ viola-
tion, e.g. the vacuum state for a model with one of the three spatial dimensions compactified into a circle of cosmological size fails to be Lorentz invariant which in turn produce violation of \( CP T \) symmetry [23,24]. However, it should be kept in mind that the above condition is not a necessary one, for example QFT on non-commutative space-time can give rise to Lorentz invariance violating effects while conserving \( CP T \) [25–28]. A certain class of models can violate \( CP T \) through non-locality too while preserving Lorentz symmetry [29–31]. Non-point interactions, which are also a possible source of \( CP T \) violation, emerge in some models where particle and antiparticle are both contained in the same isospin multiplet [32–36]. \( CP T \) violation may arise from modifications of conventional quantum mechanics due to gravitational effects [37,38], specially near event horizon where inaccessibility of full information leads to non-unitarity of states. Quantum-mechanical decoherence in quantum gravity could also be responsible for \( CP T \) breaking [39,40]. It has been shown in Refs. [41–43] that Abelian Chern-Simons like terms in Lagrangian as well as fields with infinite components also violate \( CP T \).

Given its great importance to theoretical Physics, much attention has been devoted to scrutinize the plausibility of \( CP T \) symmetry experimentally. The observed equality between masses and life times of particle and antiparticle with striking precision [44], which is a consequence of \( CP T \) invariance, obligates us to believe that \( CP T \) is a good symmetry of nature. But, these quantities are mainly dominated by strong or electromagnetic interactions and therefore, the possibility for existence of tiny \( CP T \) violating effects mediated by weak interactions, which might remain undetected in direct measurements, cannot be ruled out. Apart from neutrino sector, mixing of neutral pseudoscalar meson \([K^0, D^0, B^0_s, B^0]\) with its own antiparticle, in this regard, is a promising place to search for \( CP T \) violating effects [45–49] as it is predominantly a second order electroweak phenomenon. However, in addition to \( CP T \) violating effects, since the most general mixing matrix involves \( T \) and \( CP \) violation as well, all those effects must be considered together.

In literature, there exist extensive studies on probing \( T \), \( CP \) and \( CP T \) violation using leptonic, semi-leptonic, two pseudoscalars and one pseudoscalar plus one vector decay modes of neutral pseudoscalar meson [50–67]. But the modes with neutral pseudoscalar mesons decaying to two vectors \((P^0 or \bar{P}^0 \to V_1V_2)\) are not very well assessed in light of \( CP T \) violation. Refs. [68–70] consider the SM scenario (i.e. \( CP \) violation in mixing only) and its extension to models with \( CP T \) conserving generic new physics effects only while probing two vectors decay modes of \( B \)-mesons. Howbeit, Ref. [71] has taken \( CP T \) violation into account for describing the mode \( B^0_s \to J/\psi \phi \) and Ref. [72] has discussed about triple products and angular observables for \( B \to V_1V_2 \) decays in context of \( CP T \) violation. Furthermore, two vectors decay modes of neutral mesons have been studied in Ref. [73] contemplating \( T \), \( CP \) and \( CP T \) violation in mixing only. In this paper, we have extended the idea of Ref. [73] to search for \( T \) and \( CP T \) violation in mixing through \( P^0 \to V_1V_2 \) decays using helicity-based analysis in presence of \( CP \) violation in decay as well as in mixing. Notwithstanding, the presence of \( CP \) violation in decay changes the scenario drastically and complicate all the equations compared to Ref. [73] and consequently recasting the whole approach for this analysis is essential. It should also be noted that we do not consider any specific model that might lead to \( CP T \) violation which implies that it’s a model-independent approach.

While dealing with oscillations of neutral pseudoscalar mesons \((P^0, \bar{P}^0)\), usually a common final state \( f \), to which both \( P^0 \) and \( \bar{P}^0 \) can decay, is considered. When \( f \) contains two vectors, there emerge three transversity amplitudes for each of the transitions \( P^0 \to f \) and \( \bar{P}^0 \to f \) depending on the orbital angular momentum of the final state. This fact helps us to construct a large number of observables from time-dependent differential decay rates of the two modes. But, all of the observables will not be independent to each other since independent theoretical parameters are lesser in number than the observables. Therefore, various relations among observables appear automatically. These relations have already been addressed in context SM scenario in Refs. [69,70,73], where first two references consider two vector decay modes of \( B^0_s \) only with vanishing width deference between the physical states. Moreover, Ref. [73] talks about these relations in the presence of \( T \), \( CP \) and \( CP T \) violation in mixing. In this paper we advance one step further by exploring these relations in the presence of \( CP \) violation in decay in addition to \( T \), \( CP \) and \( CP T \) violating effects in mixing. These new relations will break down only if any \( CP T \) violating effect is present in decay itself. Furthermore, we have used our formalism to study the relations among observables for SM scenario, SM plus direct \( CP \) violation case and SM plus indirect violation of \( T \) and \( CP T \) scenario, which are three special cases of our picture.

The paper is organized as follows. In the next section (Sect. 2), we briefly describe the theoretical formalism for \( CP T \) violation in \( P^0 - \bar{P}^0 \) mixing and express the time dependent differential decay rates of \( P^0 \) and \( \bar{P}^0 \) in terms of the mixing parameters. In Sect. 3, we construct helicity-dependent observables from the differential decay rates and express the “dummy observables” in terms of them as well as small \( T \) and \( CP T \) violating parameters. Section 4 deals with solving unknown theoretical parameters in terms of observables. The relations among observables in present scenario have been established in Sect. 5. In Sect. 6, we use this formalism to find the relations among observables for three special cases: (a) SM scenario (\( CP \) violation in mixing only), (b) SM plus direct \( CP \) violation, (c) SM plus \( T \) and \( CP T \) violation in
mixing. The phenomenological aspects have been discussed in Sect. 7 and finally, we summarize and conclude in Sect. 8.

2 Theoretical framework

Let us first briefly review the most general formalism incorporating CPT and T violation for $P^0 - \bar{P}^0$ mixing, which has already been discussed in Refs. [2,64,73]. In the flavour basis $(P^0, \bar{P}^0)$, the mixing Hamiltonian can be expressed in terms of two $2 \times 2$ Hermitian matrices, namely mass-matrix $M$ and decay-matrix $\Gamma$, as $M - (i/2)\Gamma$. Since three Pauli matrices $\sigma_j$ along with identity matrix $I$ constitute a complete set of bases spanning the whole vector-space of $2 \times 2$ matrices, one can write:

$$M - \frac{i}{2} \Gamma = E \sin \theta \cos \phi \sigma_1 + E \sin \theta \sin \phi \sigma_2 + E \cos \theta \sigma_3 - i D I$$

(2.1)

where $E$, $\theta$, $\phi$ and $D$ are complex entities in general. Comparing both sides of this equation, we obtain:

$$D = \frac{i}{2} (M_{11} + M_{22}) + \frac{1}{4} (\Gamma_{11} + \Gamma_{22}),$$

$$E \cos \theta = \frac{1}{2} (M_{11} - M_{22}) - \frac{i}{4} (\Gamma_{11} - \Gamma_{22}),$$

$$E \sin \theta \cos \phi = \text{Re} M_{12} - \frac{i}{2} \text{Re} \Gamma_{12},$$

$$E \sin \theta \sin \phi = - \text{Im} M_{12} + \frac{i}{2} \text{Im} \Gamma_{12}.$$  

(2.2)

where $M_{ij}$ and $\Gamma_{ij}$ are $(i, j)$-th elements of $M$ and $\Gamma$ matrices respectively.

The mass eigenstates or physical states $|P_L\rangle$ and $|P_H\rangle$ are the eigenvectors of the mixing Hamiltonian $M - (i/2)\Gamma$ and they can be expressed as linear combinations of the flavour eigenstates $(|P^0\rangle$ and $|P^0\rangle$) as follows:

$$|P_L\rangle = p_1|P^0\rangle + q_1|\bar{P}^0\rangle,$$

$$|P_H\rangle = p_2|P^0\rangle - q_2|\bar{P}^0\rangle.$$  

(2.3)

where $p_1 = N_1 \cos \frac{\theta}{2}$, $q_1 = N_1 e^{i\phi} \sin \frac{\theta}{2}$, $p_2 = N_2 \sin \frac{\theta}{2}$, $q_2 = N_2 e^{i\phi} \cos \frac{\theta}{2}$ with $N_1, N_2$ being two normalization factors and the $L, H$ tags indicating light and heavy physical states, respectively. Since, the physical states, as given by Eq. (2.3), depend only on the complex parameters $\theta$ and $\phi$, they are called the mixing parameters for $P^0 - \bar{P}^0$ system. It should be noted that the physical states are not orthogonal in general since the mixing matrix is non-Hermitian.

The time evolution of flavour states $(|P^0\rangle \equiv |P^0(t = 0)\rangle$ and $|\bar{P}^0\rangle \equiv |\bar{P}^0(t = 0)\rangle$) is given by:

$$|P^0(t)\rangle = h_+ |P^0\rangle + h_- \cos \theta |\bar{P}^0\rangle + h_- e^{i\phi} \sin \theta |\bar{P}^0\rangle,$$

$$|\bar{P}^0(t)\rangle = h_+ |\bar{P}^0\rangle - h_- \cos \theta |P^0\rangle + h_- e^{-i\phi} \sin \theta |P^0\rangle,$$

where, $h_\pm = e^{-i(M - i \frac{\theta}{2}) t} \pm e^{-i(M + i \frac{\theta}{2}) t}$.  

(2.4)

Here, $M = (M_H + M_L)/2$, $\Delta M = M_H - M_L$, $\Gamma = (\Gamma_H + \Gamma_L)/2$ and $\Delta \Gamma = \Gamma_H - \Gamma_L$ with $M_{HL}$ and $\Gamma_{HL}$ to be masses and decay widths of the light and heavy mass eigenstates respectively which can be found from the eigenvalues of the mixing Hamiltonian and measured directly in experiments.

Let us now consider a final state $f$ to which both $P^0$ and $\bar{P}^0$ can decay. Using Eq. (2.4), the time dependent decay amplitudes for the neutral mesons are given by:

$$A_{f} = \langle f|H_{\Delta F = 1}|P^0\rangle$$

(2.5)

and $\bar{A}_f = \langle f|H_{\Delta F = 1}|\bar{P}^0\rangle$ with $H_{\Delta F = 1}$ indicating the Hamiltonian related to the transition from flavour states to $f$. Therefore, incorporating the mixing, the time dependent decay rates $\Gamma(P^0(t) \rightarrow f)$ and $\Gamma(\bar{P}^0(t) \rightarrow f)$ can be expressed as:

$$\left[ \begin{array}{l}
\frac{\Gamma}{dt} (P^0(t) \rightarrow f) \\
\frac{\Gamma}{dt} (\bar{P}^0(t) \rightarrow f)
\end{array} \right] = \left[ \begin{array}{c}
\frac{1}{2} e^{-i\Gamma t} \left[ \sinh (\Delta \Gamma t/2) \right] \left[ 2 \text{Re} (\cos \theta |A_f|\bar{A}_f + e^{i\phi} \sin \theta |A_f|^2) \right] \\
\cos (\Delta \Gamma t/2) \left[ |A_f|^2 + |\cos \theta|^2 |\bar{A}_f|^2 + |e^{i\phi} \sin \theta|^2 |\bar{A}_f|^2 \right] + 2 \text{Re} (e^{i\phi} \cos \theta \sin \theta |A_f| \bar{A}_f) \\
\cos (\Delta \Gamma t) \left[ |A_f|^2 - |\cos \theta|^2 |\bar{A}_f|^2 - |e^{i\phi} \sin \theta|^2 |\bar{A}_f|^2 \right] - 2 \text{Re} (e^{i\phi} \cos \theta \sin \theta |A_f| \bar{A}_f) \\
\sin (\Delta \Gamma t) \left[ 2 \text{Im} \left( \cos \theta |A_f|^2 + e^{i\phi} \sin \theta |A_f|^2 \right) \right]
\end{array} \right].$$

(2.6)

where $A_f = \langle f|H_{\Delta F = 1}|P^0\rangle$
3 Observables

3.1 T and CPT violating parameters

The properties of $M$ and $\Gamma$ matrices in light of $T$ and CPT symmetries have been discussed in Ref. [75]. First, if CPT invariance holds, then, independently of $T$ symmetry [64, 73],

$$M_{11} = M_{22}, \quad \Gamma_{11} = \Gamma_{22} \quad \implies \theta = \frac{\pi}{2} \quad (\text{Using Eq. (2.2)}).$$

(3.1)

Secondly, if $T$ invariance holds, then, independently of $CPT$ symmetry [64, 73],

$$\frac{\Gamma_{12}^*}{\Gamma_{12}} = \frac{M_{12}^*}{M_{12}} \quad \implies \text{Im } \phi = 0 \quad (\text{Using Eq. (2.2)}).$$

(3.2)

Hence, incorporating $T$, $CP$ and $CPT$ violation in $P^0 - \bar{P}^0$ mixing, we parametrize $\theta$ and $\phi$ as [64,73]:

$$\theta = \frac{\pi}{2} + \epsilon_1 + i \epsilon_2 \quad \text{and} \quad \phi = -2\beta + i \epsilon_3$$

(3.3)

where $\beta$ is the $CP$ violating weak mixing phase, $\epsilon_1$ and $\epsilon_2$ are $CPT$ violating parameters and $\epsilon_3$ is $T$ violating parameter other than $CP$ violation. The notation of Belle, BaBar and LHCb collaborations [52–55] is a bit different from ours; however, the two notations are related to each other by the following transformation [64,73]:

$$\cos \theta \leftrightarrow -z, \quad \sin \theta \leftrightarrow \sqrt{1 - z^2}, \quad e^{i \phi} \leftrightarrow \frac{q}{p},$$

or, equivalently: $\epsilon_1 = \text{Re}(z), \quad \epsilon_2 = \text{Im}(z), \quad \epsilon_3 = 1 - \left| \frac{q}{p} \right|.$

(3.4)

3.2 Decay rates and observables

Any state consisting of two vectors can have three different values for orbital angular momentum quantum number $\{0, 1, 2\}$ which correspond to the polarization states $\{0, \perp, ||\}$, respectively. Since $CPT$ violation in decay has not been considered, the decay amplitudes for modes and conjugate modes can be expressed in terms of transversity amplitudes as [68–70,72,73]:

$$A_f(P^0 \rightarrow V_1 V_2) = A_0 g_0 + A_{\perp} g_{\perp} + i A_{||} g_{||},$$

$$\bar{A}_f(P^0 \rightarrow V_1 V_2) = \bar{A}_0 g_0 + \bar{A}_{\perp} g_{\perp} - i \bar{A}_{||} g_{||}.$$  (3.5)

where the factors $g_{\lambda}$ with $\lambda \in \{0, ||, \perp\}$ are the coefficients of transversity amplitudes ($A_{\lambda}$ or $\bar{A}_{\lambda}$) in linear polarization basis and depend only on the kinematic angles [74].

Now, using Eqs. (2.6)–(3.5), the time-dependent decay rates for $P^0(t) \rightarrow V_1 V_2$ and $\bar{P}^0(t) \rightarrow V_1 V_2$ modes can be written as [68–73]:

$$\frac{d\Gamma}{dt}(P^0(t) \rightarrow V_1 V_2) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left[ A_{\lambda,\sigma} \cosh \left( \frac{\Delta \Gamma t}{2} \right) + \eta_{\lambda,\sigma} \sinh \left( \frac{\Delta \Gamma t}{2} \right) \right] + \sum_{\lambda,\sigma} \cos(\Delta M t) - \rho_{\lambda,\sigma} \sin(\Delta M t) g_{\lambda,\sigma}.$$  (3.6)

$$\frac{d\Gamma}{dt}(\bar{P}^0(t) \rightarrow V_1 V_2) = e^{-\Gamma t} \sum_{\lambda \leq \sigma} \left[ \bar{A}_{\lambda,\sigma} \cosh \left( \frac{\Delta \Gamma t}{2} \right) + \bar{\eta}_{\lambda,\sigma} \sinh \left( \frac{\Delta \Gamma t}{2} \right) \right] + \bar{\Sigma}_{\lambda,\sigma} \cos(\Delta M t) + \bar{\rho}_{\lambda,\sigma} \sin(\Delta M t) g_{\lambda,\sigma}.$$  (3.7)

where both $\lambda$ and $\sigma$ take the value $\{0, ||, \perp\}$. It is important to note that in the entire paper we have considered a particular ordering for the combination $\lambda \sigma$ with $\lambda \neq \sigma$ and they are $\{ ||, \perp, || \}$. For defining the observables, we have taken the convention of Ref. [73]. On the other hand, Refs. [68–70] use a bit different notations involving some additional negative signs. Hence, several inferences of our paper may differ from their results by some signs only; however, all the outcomes of our paper are self-consistent.

Now, we see from Eq. (3.6) that for each of the helicity combinations, there are four types of observables ($\Lambda_{\lambda,\sigma}, \eta_{\lambda,\sigma}, \Sigma_{\lambda,\sigma}, \rho_{\lambda,\sigma}$) and six such helicity combinations are possible. Hence, we get total 24 observables for $P^0(t) \rightarrow V_1 V_2$ mode. Similarly, there will be 24 different observables ($\bar{\Lambda}_{\lambda,\sigma}, \bar{\eta}_{\lambda,\sigma}, \bar{\Sigma}_{\lambda,\sigma}, \bar{\rho}_{\lambda,\sigma}$) for $\bar{P}^0(t) \rightarrow V_1 V_2$ mode too, as shown in Eq. (3.7). These observables can be measured by performing a time dependent angular analysis of $P^0(t) \rightarrow V_1 V_2$ and $\bar{P}^0(t) \rightarrow V_1 V_2$ [68–70]. The procedure described in Ref. [72] can be helpful in this regard. On the other hand, probing polarizations of the final state particles may also aid in measurement of these observables. It should be noticed that Ref. [68–70] did not consider $\sinh \left( \frac{\Delta \Gamma t}{2} \right)$ terms in the decays of $B_d^0$ and $B_s^0$ since $\Delta \Gamma$ is consistent with zero [44]. In that case, $\eta_{\lambda,\sigma}$ and $\bar{\eta}_{\lambda,\sigma}$ remain undetermined and one should work with remaining $(18 + 18) = 36$ observables only for a mode and its conjugate mode. However, we have kept all the terms in our analysis since a general scenario has been considered here.

3.3 Parametric expansion of observables

Comparing Eqs. (2.6) and (2.7) to Eqs. (3.6) and (3.7) one can easily infer that all of the observables will be functions of the complex quantities $\theta$ and $\phi$. As $T$ and $CPT$ violations are expected to be very small [52–55,66,76,77], we can expand all the observables in terms of $\epsilon_j$ ($j \in \{1, 2, 3\}$) keeping up to the linear orders. However, following Refs. [69,70], if we divide all the transversity amplitudes into $CP$ conserving and $CP$ violating parts, it would become very complicated to handle all the unknown parameters. So, we implement a new method to reduce the complexity. After substitution of Eqs.
we define these twenty four combinations as follows:

\[
\begin{align*}
\Lambda^\prime_{i\ell} &= \frac{1}{2}(\Lambda_i^2 + \bar{A}_i^2), \\
\Sigma^\prime_{i\ell} &= \frac{1}{2}(\Sigma_i^2 + \bar{\Sigma}_i^2), \\
\Lambda^\prime_{ii} &= -\text{Im}(A_i A^*_i + \bar{A}_i \bar{A}^*_i), \\
\Sigma^\prime_{ii} &= -\text{Im}(A_i A^*_i + \bar{A}_i \bar{A}^*_i), \\
\Lambda^\prime_{i0} &= \text{Re}(A_i \bar{A}^*_0 + \bar{A}_i A^*_0), \\
\Sigma^\prime_{i0} &= \text{Re}(A_i \bar{A}^*_0 - \bar{A}_i A^*_0),
\end{align*}
\]  

(3.8)

where \(i \in \{0, ||\} \) and \(\lambda \in \{0, ||, \perp\} \). It should be noted that the quantities, mentioned above, which we name as “dummy-observables”, are not observables, in general; rather they are some theoretical tools for our convenience. Now, using Eqs. (3.6) and (3.7) one can express the actual observables in terms of the dummy-observables as well as the \(T\) and \(CPT\) violating parameters \(\epsilon_j\). For our purpose, we invert those equations and express dummy-observables as functions of the original ones keeping only the linear orders in \(\epsilon_j\) as follows:

\[
\begin{align*}
\Lambda^\prime_{ii} &= \epsilon_1 \eta_{ii} + (1 + \epsilon_3) \Lambda_{ii} + \epsilon_2 \rho_{ii} - \epsilon_3 \Sigma_{ii}, \\
\eta^\prime_{ii} &= (1 + \epsilon_3) \eta_{ii} + \epsilon_1 \Lambda_{ii} + \epsilon_3 \Sigma_{ii}, \\
\Lambda^\prime_{i\perp} &= \epsilon_1 \eta_{i\perp} + (1 + \epsilon_3) \Lambda_{i\perp} + \epsilon_2 \rho_{i\perp} - \epsilon_3 \Sigma_{i\perp}, \\
\eta^\prime_{i\perp} &= (1 + \epsilon_3) \eta_{i\perp} + \epsilon_1 \Lambda_{i\perp} + \epsilon_3 \Sigma_{i\perp}, \\
\Lambda^\prime_{\perp\perp} &= \epsilon_1 \eta_{\perp\perp} + (1 + \epsilon_3) \Lambda_{\perp\perp} + \epsilon_2 \rho_{\perp\perp} - \epsilon_3 \Sigma_{\perp\perp}, \\
\eta^\prime_{\perp\perp} &= (1 + \epsilon_3) \eta_{\perp\perp} + \epsilon_1 \Lambda_{\perp\perp} + \epsilon_3 \Sigma_{\perp\perp}, \\
\Lambda^\prime_{i0} &= \epsilon_1 \eta_{i0} + (1 + \epsilon_3) \Lambda_{i0} + \epsilon_2 \rho_{i0} - \epsilon_3 \Sigma_{i0}, \\
\eta^\prime_{i0} &= (1 + \epsilon_3) \eta_{i0} + \epsilon_1 \Lambda_{i0} + \epsilon_3 \Sigma_{i0}, \\
\Lambda^\prime_{\perp0} &= \epsilon_1 \eta_{\perp0} + (1 + \epsilon_3) \Lambda_{\perp0} + \epsilon_2 \rho_{\perp0} - \epsilon_3 \Sigma_{\perp0}, \\
\eta^\prime_{\perp0} &= (1 + \epsilon_3) \eta_{\perp0} + \epsilon_1 \Lambda_{\perp0} + \epsilon_3 \Sigma_{\perp0}, \\
\Lambda^\prime_{0\perp} &= \epsilon_1 \eta_{0\perp} + (1 + \epsilon_3) \Lambda_{0\perp} + \epsilon_2 \rho_{0\perp} - \epsilon_3 \Sigma_{0\perp}, \\
\eta^\prime_{0\perp} &= (1 + \epsilon_3) \eta_{0\perp} + \epsilon_1 \Lambda_{0\perp} + \epsilon_3 \Sigma_{0\perp}, \\
\Lambda^\prime_{00} &= \epsilon_1 \eta_{00} + (1 + \epsilon_3) \Lambda_{00} + \epsilon_2 \rho_{00} - \epsilon_3 \Sigma_{00}, \\
\eta^\prime_{00} &= (1 + \epsilon_3) \eta_{00} + \epsilon_1 \Lambda_{00} + \epsilon_3 \Sigma_{00},
\end{align*}
\]  

(3.9)

4 Solutions of the theoretical parameters

In this section, we discuss how to solve for the unknown theoretical quantities in terms of observables. These theoretical parameters are six helicity amplitudes \((A_\lambda, \bar{A}_\lambda\) with \(\lambda \in \{0, ||, \perp\}\), which are complex entities and three parameters \(\epsilon_j\) \((j \in \{1, 2, 3\})\) related to \(T\) and \(CPT\) violation in mixing. It should be noted that the \(CPT\) violating weak phase \(\beta\) cannot be probed directly in the presence of direct \(CPT\) violation; it can only be measured if there is no \(CPT\) violation in decay itself. Now, it is impossible to measure the absolute phases for all the helicity amplitudes; rather relative phases can be estimated. Hence, we define the following quantities that indicate the relative phases of five transversity amplitudes with respect to \(A_\perp\):

\[
\Omega_i = \text{Arg}[A_i] - \text{Arg}[A_\perp], \quad \tilde{\Omega}_i = \text{Arg}[\bar{A}_i] - \text{Arg}[A_\perp]
\]

(4.1)

where ‘Arg’ implies argument of a complex number, \(i \in \{0, ||\}\) and \(\lambda \in \{0, ||, \perp\}\). Thus we have to solve for fourteen unknown parameters (three of \([A_\lambda]\), three of \([\bar{A}_\lambda]\), three of \(\epsilon_j\), two of \(\Omega_i\) and three of \(\tilde{\Omega}_i\)).

For convenience, we define nine angular quantities as follows:

\[
\omega_{i\lambda} = \text{Arg}[A_{i\lambda}] - \text{Arg}[A_{i\perp}], \quad \tilde{\omega}_{i\lambda} = \text{Arg}[\bar{A}_{i\lambda}] - \text{Arg}[\bar{A}_{i\perp}], \\
\phi_{i\lambda}^{\text{max}} = -2\beta + \text{Arg}[\bar{A}_{i\lambda}] - \text{Arg}[A_{i\perp}].
\]

(4.2)
where \((\lambda, \sigma) \in \{0, \parallel, \perp\}\). As mentioned earlier, we will consider the combination \(\lambda \sigma\) to be one of \(\{\perp \lambda, \lambda \parallel, \parallel \lambda\}\) only; one should not be bothered about reverse ordering. Now, instead of the five relative phases of the transversity amplitudes we use five of the above-defined angular entities (three of \(\phi^{\text{meas}}_\lambda\) and two of \(\omega_{\lambda\perp}\)) as our unknown parameters to solve for. The rest four angular quantities in Eq. (4.2) will be used later in order to find relations among various observables. The relative phases of helicity amplitudes can easily be expressed in terms of the five angular entities mentioned above in the following way:

\[
\Omega_i = -\omega_{\lambda \perp}, \quad \tilde{\Omega}_i = \phi^{\text{meas}}_\lambda + 2\beta - \omega_{\lambda \perp}, \quad \tilde{\Omega}_\perp = \phi^{\text{meas}}_\perp + 2\beta, \quad (4.3)
\]

where \(i \in \{0, \parallel\}\). Thus the fourteen theoretical parameters that we are going to solve are three of \(|A_i|\), three of \(|A_\lambda|\), three of \(\varepsilon_j\), two of \(\omega_{\lambda \perp}\) and three of \(\phi^{\text{meas}}_\perp\).

The modulus of helicity amplitudes are given by:

\[
|A_\lambda| = \sqrt{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}} \quad \text{and} \quad |\bar{A}_\lambda| = \sqrt{\bar{\Lambda}_{\lambda\lambda} + \bar{\Sigma}_{\lambda\lambda}}. \quad (4.4)
\]

The value of sin \(\Phi^\text{meas}_\lambda\) can be found by solving the following cubic equation:

\[
sin^3 \Phi^\text{meas}_\lambda = -\frac{\rho^\prime_{\lambda\perp} + \rho^\prime_{\lambda}}{\sqrt{1 - C^2}} \sin^2 \Phi^\text{meas}_\lambda + \left(\frac{C^2 - \Sigma_{\lambda\perp} - \bar{\Sigma}_{\lambda\perp}}{1 - C^2}\right) \sin \Phi^\text{meas}_\lambda
\]

\[+ \frac{C_i}{\sqrt{1 - C^2}} \left[\frac{\rho^\prime_{\lambda\perp}}{1 + C_\lambda} - \frac{\rho^\prime_{\lambda}}{1 - C_\lambda}\right] = 0, \quad (4.5)
\]

where \(\Phi^\text{meas}_\lambda = \phi^\text{meas}_\lambda\) and \(\Phi^\text{meas}_\perp = -\phi^\text{meas}_\perp\) in the above expression.

The quantities with superscript ‘r’ and \(C_\lambda\) are defined as:

\[
C_\lambda = \frac{(\Lambda_{\lambda\lambda} - \bar{A}_{\lambda\lambda} + \Sigma_{\lambda\lambda} - \bar{\Sigma}_{\lambda\lambda})}{(\Lambda_{\lambda\lambda} + \bar{A}_{\lambda\lambda} + \Sigma_{\lambda\lambda} + \bar{\Sigma}_{\lambda\lambda})} \quad \text{and} \quad Y_{\lambda\lambda} = \frac{Y_{\lambda\lambda}^r}{(\Lambda_{\lambda\lambda} + \bar{A}_{\lambda\lambda} + \Sigma_{\lambda\lambda} + \bar{\Sigma}_{\lambda\lambda})}, \quad (4.6)
\]

where \((Y \equiv \rho, \bar{\rho}, \Sigma, \bar{\Sigma})\) with \(\lambda \in \{0, \parallel, \perp\}\). Knowing \(\phi^\text{meas}_\lambda\) form the above equations, the \(T\) and \(CPT\) violating parameters in mixing can be obtained from the following equations:

\[
\varepsilon_1 = -\left[\frac{2}{\sin 2\phi^\text{meas}_\lambda}\right] \left[\frac{\rho^\prime_{\lambda\perp}}{1 - C_\lambda} - \frac{\rho^\prime_{\lambda}}{1 + C_\lambda} - \frac{\Sigma_{\lambda\perp} - \bar{\Sigma}_{\lambda\perp}}{2} \sin \Phi^\text{meas}_\lambda\right],
\]

\[
\varepsilon_2 = -\left(\rho^\prime_{\lambda\perp} + \rho^\prime_{\lambda}\right) + \sqrt{1 - C^2} \sin \Phi^\text{meas}_\lambda, \quad (4.7)
\]

\[
\varepsilon_3 = -C_\lambda - \csc \phi^\text{meas}_\lambda \left[\frac{\rho^\prime_{\lambda\perp}}{\sqrt{1 + C_\lambda}} - \frac{\rho^\prime_{\lambda}}{\sqrt{1 - C_\lambda}}\right].
\]

Thus we solve for twelve of the fourteen unknown parameters. To obtain the solutions we have inverted the expressions for \(\Lambda_{\lambda\lambda}, \Sigma_{\lambda\lambda}\) and \(\rho^\prime_{\lambda\perp}\) in Eqs. (3.9) and (3.10) and then use the definitions of those dummy-observables from Eq. (3.8). It should be noticed that each of \(\varepsilon_j (j \in \{1, 2, 3\})\) can obtained in three ways since \(\lambda \in \{0, \parallel, \perp\}\). This fact will be used later to find some relations among observables. Now, the remaining two angular quantities \(\omega_{\lambda \perp} (i \in \{0, \parallel\})\) are calculated in the following way:

\[
\Lambda_{\lambda \perp} + \Sigma_{\lambda \perp} = \Lambda_{\lambda \perp} + \Sigma^r_{\lambda \perp} = -2 \Im (A_{\lambda \perp}^\ast)
\]

(Using Eqs. (3.8) and (3.9))

\[
\Rightarrow \omega_{\lambda \perp} = \text{Arg}[A_{\lambda \perp}] - \text{Arg}[A_{\lambda \perp}^\ast] \quad (4.8)
\]

\[= \sin^{-1} \left(-\frac{\Lambda_{\lambda \perp} + \Sigma_{\lambda \perp}}{2\sqrt{(\Lambda_{\lambda \perp} + \Sigma_{\lambda \perp})(\Lambda_{\lambda \perp} + \Sigma_{\lambda \perp})}}\right).
\]

5 Relations among observables

In this section we are going to derive complete set of relations among observables for the scenario with \(T\) and \(CPT\) violation in mixing along with \(CP\) violation in both mixing and decay. As discussed before, we have forty eight observables combining mode and conjugate mode, but the number of unknown theoretical parameters are fourteen. Therefore, we must have forty eight minus fourteen equals to thirty four relations among observables. If we simply substitute the solutions of unknown parameters into Eqs. (3.8)–(3.10), we would overcount the number of independent relations among observables.

Firstly, it is evident from Eq. (4.7) that each of the \(\varepsilon_j (j \in \{1, 2, 3\})\) can express in three ways depending on the value of \(\lambda\). Hence, one will give the solution for \(\varepsilon_j\) while the rest two can be recast as relations among observables and this happens for each \(\varepsilon_j\). Thus we have two times three equals to six relations among observables which are the following:

\[
\left[\frac{\sin 2\phi^\text{meas}_\lambda}{\sin 2\phi^\text{meas}_\perp}\right] \left[\frac{\rho^\prime_{\lambda\perp}}{1 - C_\perp} - \frac{\rho^\prime_{\lambda}}{1 + C_\perp} - \frac{\Sigma^r_{\lambda\perp} - \bar{\Sigma^r}_{\lambda\perp}}{2} \sin \Phi^\text{meas}_\perp\right]
\]

\[= \left[\frac{\rho^\prime_{\lambda\perp}}{1 - C_\perp} - \frac{\rho^\prime_{\lambda}}{1 + C_\perp} - \frac{\Sigma^r_{\lambda\perp} - \bar{\Sigma^r}_{\lambda\perp}}{2} \sin \Phi^\text{meas}_\perp\right] \sin \Phi^\text{meas}_\perp \quad (5.1)
\]

\[\sqrt{1 - C^2_\perp} \sin \Phi^\text{meas}_\perp - \sqrt{1 - C^2_\perp} \sin \Phi^\text{meas}_\perp = (\rho^\prime_{\lambda\perp} + \rho^\prime_{\lambda}) - (\rho^\prime_{\lambda\perp} + \rho^\prime_{\lambda}) \quad (5.2)
\]

\[(C_\perp - C_\perp) + \csc \Phi^\text{meas}_\perp \left[\frac{\rho^\prime_{\lambda\perp}}{1 - C_\perp} - \frac{\rho^\prime_{\lambda}}{1 + C_\perp} \left(1 - C^2_\perp\right)\right] = \csc \Phi^\text{meas}_\perp \left[\frac{\rho^\prime_{\lambda\perp}}{1 - C_\perp} - \frac{\rho^\prime_{\lambda}}{1 + C_\perp} \left(1 - C^2_\perp\right)\right] \quad (5.3)
\]

where \(i \in \{0, \parallel\}\). The above six relations can be interpreted from a different perspective too. Looking at Eqs. (4.4)–(4.7), it can be realized that eighteen observables (three for each of \(\Lambda_{\lambda\lambda}, \Sigma_{\lambda\lambda}, \rho_{\lambda\lambda}, \bar{\Lambda}_{\lambda\lambda}, \bar{\Sigma}_{\lambda\lambda}, \bar{\rho}_{\lambda\lambda}\) have been used to solve for twelve different quantities (three for each \(|A_i|, |A_\lambda|, \varepsilon_j\) and \(\sin \phi^\text{meas}_\lambda\)). Hence, eliminating the unknown quantities,
one should get eighteen minus twelve equals to six relations among observables which are given by Eqs. (5.1)–(5.3).

Secondly, as mentioned in the last section, we have used only five angular quantities so far (three of Φ, and two of ω⊥⊥). The rest four (ω∥ and three of ϕσ,ω), as defined in Eq. (4.2), will now be used to find four relations among observables. Let us first express these angles in terms of observables as follows:

\[
\lambda_{ij0} + \Sigma_{ij0} = \lambda'_{ij0} + \Sigma'_{ij0} = 2 \text{Re} (\Lambda_{ij0} A_{i0}) \quad \text{(Using Eqs. (3.8) and (3.9))}
\]
\[
\Rightarrow \omega_{ij0} = \text{Arg} [\lambda_{ij0}] - \text{Arg} [A_{i0}]
\]
\[
= \cos^{-1} \left( \frac{\lambda_{ij0} + \Sigma_{ij0}}{2 \sqrt{(\lambda_{i}\| \Lambda_{ij0} + \Sigma_{i}\|)}} \right)
\]
\[
\Rightarrow \omega_{ij0} = \text{Arg} [\lambda_{ij0}] - \text{Arg} [A_{i0}]
\]
\[
= \cos^{-1} \left( \frac{\lambda_{ij0} + \Sigma_{ij0}}{2 \sqrt{(\lambda_{i}\| \Lambda_{ij0} + \Sigma_{i}\|)}} \right)
\]
\[
\Rightarrow \omega_{ij0} = \text{Arg} [\lambda_{ij0}] - \text{Arg} [A_{i0}]
\]
\[
= \sin^{-1} \left( \frac{\lambda_{ij0} + \Sigma_{ij0}}{2 \sqrt{(\lambda_{i}\| \Lambda_{ij0} + \Sigma_{i}\|)}} \right)
\]

where \( i \in \{0, \|, \perp \} \). Now, from the definitions of these angles, as shown in Eq. (4.2), it easy to establish the following relations:

\[
\tilde{\omega}_{i\lambda \sigma} = \phi_{\lambda \sigma}^{\text{meas}} - \phi_{\sigma}^{\text{meas}} + \omega_{\lambda \sigma} \quad \text{and} \quad \omega_{i0} = \omega_{\perp0} = \omega_{\perp \perp}
\]

(5.7)

Fourthly, all the expressions related to \( \rho'_{\lambda \sigma} \) have already been used in solving the unknown parameters and deducing the first six relations among observables. But none of the expressions involving \( \rho'_{\lambda \sigma} \) with \( \lambda \neq \sigma \) has been used yet. Let us first rewrite \( \rho'_{\lambda \sigma} \) in terms of observables and the measured angles (\( \phi_{\lambda \sigma}^{\text{meas}} \) and \( \omega_{\lambda \sigma} \)) as follows:

\[
\rho'_{ij0} = -\text{Re} \left[ \xi (A_i^* \tilde{A}_{i0} + A_{i0}^* \tilde{A}_{ij0}) \right]
\]
\[
= -\sqrt{(\lambda_{i\perp\perp} + \Sigma_{i\perp\perp})(\tilde{A}_{i0} + \tilde{\Sigma}_{i0})} \cos (\phi_{\lambda \sigma}^{\text{meas}} - \omega_{ij0})
\]
\[
-\sqrt{(\lambda_i + \Sigma_i)(\tilde{A}_{i\perp\perp} + \tilde{\Sigma}_{i\perp\perp})} \cos (\phi_{\sigma}^{\text{meas}} + \omega_{ij0}),
\]
\[
\rho'_{i00} = \text{Im} \left[ \xi (A_i^* \tilde{A}_{i0} + A_{i0}^* \tilde{A}_{i0}) \right]
\]
\[
= \sqrt{(\lambda_{i\| \perp\perp} + \Sigma_{i\| \perp\perp})(\tilde{A}_{i0} + \tilde{\Sigma}_{i0})} \sin (\phi_{\lambda \sigma}^{\text{meas}} - \omega_{i00})
\]
\[
+ \sqrt{(\lambda_{i\| \|\perp\perp} + \Sigma_{i\| \|\perp\perp})(\tilde{A}_{i0} + \tilde{\Sigma}_{i0})} \sin (\phi_{\sigma}^{\text{meas}} + \omega_{i00}).
\]

(5.9)

where \( i \in \{0, \| \} \). Now using the six equations involving \( \rho'_{\lambda \sigma} \) with \( \lambda \neq \sigma \) from Eqs. (3.9) and (3.10) six more relations among observables can be obtained with the help of Eq. (4.7).

Lastly, the expressions for \( \eta'_{\lambda \sigma} \forall (\lambda, \sigma) \in \{0, \|, \perp \} \) have not been utilized so far since \( \eta_{\lambda \sigma} \) and \( \eta_{\lambda \sigma} \) become non-measurable in the systems with vanishing \( \Delta \Gamma \) (like \( B_\theta^0 \)). That is why we have tried to eliminate them from most of our solutions and relations (although Eq. (5.8) contains them). Nonetheless, one can overcome the problem for systems with vanishing \( \Delta \Gamma \) as well as find the rest of the relations in case of general \( P^0 - B_\theta^0 \) systems. We have to express \( \eta'_{\lambda \sigma} \forall (\lambda, \sigma) \in \{0, \|, \perp \} \) in terms of observables and the measured angles (\( \phi_{\lambda \sigma}^{\text{meas}} \) and \( \omega_{\lambda \sigma} \)) first (like \( \rho' \) in the last paragraph) as follows:

\[
\eta'_{i0} = -\text{Re} \left[ \xi (A_i^* \tilde{A}_{i0} + A_{i0}^* \tilde{A}_{i0}) \right]
\]
\[
= \sqrt{(\lambda_{i\perp\perp} + \Sigma_{i\perp\perp})(\tilde{A}_{i0} + \tilde{\Sigma}_{i0})} \cos (\phi_{\lambda \sigma}^{\text{meas}} - \omega_{i00})
\]
\[
- \sqrt{(\lambda_i + \Sigma_i)(\tilde{A}_{i\perp\perp} + \tilde{\Sigma}_{i\perp\perp})} \cos (\phi_{\sigma}^{\text{meas}} + \omega_{i00}),
\]
\[
\eta'_{i0} = -\text{Re} \left[ \xi (A_i^* \tilde{A}_{i0} + A_{i0}^* \tilde{A}_{i0}) \right]
\]
\[
= \sqrt{(\lambda_{i\perp\perp} + \Sigma_{i\perp\perp})(\tilde{A}_{i0} + \tilde{\Sigma}_{i0})} \cos (\phi_{\lambda \sigma}^{\text{meas}} - \omega_{i00})
\]
\[
+ \sqrt{(\lambda_i + \Sigma_i)(\tilde{A}_{i\perp\perp} + \tilde{\Sigma}_{i\perp\perp})} \cos (\phi_{\sigma}^{\text{meas}} + \omega_{i00}),
\]

(5.10)

where \( i \in \{0, \| \} \). Now using the six equations involving \( \rho'_{\lambda \sigma} \) with \( \lambda \neq \sigma \) from Eqs. (3.9) and (3.10) six more relations among observables can be obtained with the help of Eq. (4.7).

Thirdly, using \( \Delta'_{\lambda \sigma} \) and \( \Sigma'_{\lambda \sigma} \) for \( \lambda \neq \sigma \), we have solved for two independent angular quantities \( \omega_{\perp \lambda} \), as given by Eq. (4.8), and established four independent relations among observables, given by Eq. (5.7). However, we see from Eqs. (3.9) and (3.10) that there are total twelve equations involving \( \Delta'_{\lambda \sigma} \) and \( \Sigma'_{\lambda \sigma} \) with \( \lambda \neq \sigma \) combining the mode and conjugate mode. Therefore, we must use six more relations involving them. Actually, so far we have used \( (\Delta'_{\lambda \sigma} + \Sigma'_{\lambda \sigma}) \) for mode and \( (\Delta'_{\lambda \sigma} - \Sigma'_{\lambda \sigma}) \) for conjugate mode separately. Now, equating the expressions for \( (\Delta'_{\lambda \sigma} + \Sigma'_{\lambda \sigma}) \) from both mode and conjugate mode and repeating it for \( (\Delta'_{\lambda \sigma} - \Sigma'_{\lambda \sigma}) \) too we derive the rest six relations as:

\[
\lambda_{i\perp\perp} + \Sigma_{i\perp\perp} = (1 - 2 \epsilon_3) (\tilde{\lambda}_{\lambda \sigma} - \tilde{\Sigma}_{\lambda \sigma}) - 2 \epsilon_1 \eta_{\lambda \sigma} + 2 \epsilon_2 \rho_{\lambda \sigma},
\]
\[
\lambda_{i\perp\perp} + \Sigma_{i\perp\perp} = (1 + 2 \epsilon_3) (\tilde{\lambda}_{\lambda \sigma} - \tilde{\Sigma}_{\lambda \sigma}) + 2 \epsilon_1 \eta_{\lambda \sigma} + 2 \epsilon_2 \rho_{\lambda \sigma},
\]

(5.8)

where \( (\lambda, \sigma) \in \{0, \|, \perp \} \) and \( \sigma \neq \lambda \). Here, one has to use the solutions for \( \epsilon_3 \) from Eq. (4.7). It should be noticed that each of two equations in Eq. (5.8) contains three relations for three different combination of \( \lambda \) and \( \sigma \) with \( \sigma \neq \lambda \).
observables can be established. For vanishing $\Delta \Gamma$, those twelve relations can be used for theoretical estimation of $\eta_{\lambda\sigma}$ and $\bar{\eta}_{\lambda\sigma}$ which can used in Eq. (5.8) to verify those observable relations. Thus excluding the last twelve relations, we have total twenty two observable relations in vanishing $\Delta \Gamma$ scenario, whereas in general cases we have total thirty four relations among observables. Some of these relations will get violated only if there exists direct violation of $CPT$ (i.e. violation in the decay itself.)

6 Special cases

In this section, we will study following three special cases using our formalism.

6.1 SM scenario

In SM scenario, there is no violation of $T$ (apart from $CP$ violating effects) and $CPT$ in mixing. Hence, $\epsilon_j = 0 \forall j \in \{1, 2, 3\}$ which readily infer from Eqs. (3.9) and (3.10) that

$$\Lambda_{\lambda\sigma} = \bar{\Lambda}_{\lambda\sigma}, \quad \eta_{\lambda\sigma} = \bar{\eta}_{\lambda\sigma}, \quad \Sigma_{\lambda\sigma} = -\bar{\Sigma}_{\lambda\sigma}, \quad \rho_{\lambda\sigma} = \bar{\rho}_{\lambda\sigma},$$

(6.1)

where $(\lambda, \sigma) \in \{0, \|, \bot\}$. It should be kept in mind that the forty eight equations in Eqs. (3.9) and (3.10) have been recast as solutions of fourteen parameteral, as given in Sect. 4, and thirty four relations among observables, as described in Sect. 5. Therefore, the twenty four relations in Eq. (6.1) are also embedded in the solutions or relations among observables. But it would take a bit more algebraic complexity to dig them out from there and so we simply derive them from Eqs. (3.9) and (3.10).

The other constrain in SM is that each of helicity amplitudes for mode and the conjugate mode are equal to each other (i.e. $A_{\lambda} = A_{\lambda}$). Equating the modulus of helicity amplitudes one gets the following three relations from Eqs. (4.4) and (6.1):

$$\Sigma_{\lambda\lambda} = 0 \quad \forall \lambda \in \{0, \|, \bot\},$$

(6.2)

and hence, $|A_{\lambda}| = \sqrt{\Lambda_{\lambda\lambda}} \quad \forall \lambda \in \{0, \|, \bot\}$.  

(6.3)

On the other hand, equating the phases one would get the following three relations from Eq. (4.2):

$$\omega_{\lambda\sigma} = \bar{\omega}_{\lambda\sigma} \quad \forall (\lambda, \sigma) \in \{0, \|, \bot\} \implies \Sigma_{\lambda0} = 0 \quad \text{and} \quad \Lambda_{\perp i} = 0 \quad \text{with} \; i \in \{0, \|\}.$$  

(6.4)

Here, we have used the Eq. (4.8) and Eqs. (5.4)–(5.6) for the expressions of $\omega_{\lambda\sigma}$ and $\bar{\omega}_{\lambda\sigma}$. The expressions for $\omega_{\perp i}$ and $\omega_{\perp i0}$ in this scenario turn out to be following which will be used later:

$$\omega_{\perp i} = \sin^{-1} \left( -\frac{\Sigma_{\lambda \perp i}}{2\sqrt{\Lambda_{\perp \perp} \Lambda_{\perp i}}} \right) \quad \text{and} \quad \omega_{\perp i0} = \cos^{-1} \left( \frac{\Lambda_{\perp i0}}{2\sqrt{\Lambda_{\perp \perp} \Lambda_{00}}} \right).$$

(6.5)

From Eq. (4.2), we also get that $\varphi_{\lambda0}^{\text{meas}} = -2\beta$. Combining this information with Eqs. (4.5), (4.6), (6.1) and (6.2) results in following two relations:

$$\frac{\rho_{0i}}{\Lambda_{0i}} = -\frac{\rho_{0\perp}}{\Lambda_{\perp \perp}} \quad \text{with} \; i \in \{0, \|\}.$$  

(6.6)

along with the expression of $\sin 2\beta$ as:

$$\sin 2\beta = -\sin \varphi_{\lambda0}^{\text{meas}} = -\left( \frac{\rho_{00}}{\Lambda_{00}} \right).$$

(6.7)

Thus first part of Eq. (5.7) (i.e. $\omega_{\lambda0} = \omega_{\perp i0} - \omega_{\perp i}$) gets satisfied automatically. After a couple of discussions we will come back to the second part of the equation.

Now, substituting the Eqs. (5.9) and (5.10) into Eq. (3.9) and using the expressions of angular quantities $\omega_{\perp i}$, $\omega_{\perp i0}$ and $\varphi_{\lambda0}^{\text{meas}}$ from Eqs. (6.5) and (6.7) along with Eqs. (6.1) and (6.2), one arrives at the following three relations:

$$\frac{\rho_{0\perp}}{\Lambda_{0\perp}} = \frac{\rho_{00}}{\Lambda_{00}}.$$  

(6.8)

$$\frac{\rho_{0\perp}}{\Lambda_{0\perp}} = \frac{\rho_{00}}{\Lambda_{00}}.$$  

(6.9)

In the same way, using the expressions for $\eta_{\lambda0}^{\prime}$ in Eqs. (5.11)–(5.14), the following six relations for $i \in \{0, \|\}$ can be achieved with the help of a bit of algebraic and trigonometric operations:

$$\eta_{ii} = \eta_{i0} = -\eta_{\perp \perp} = \Lambda_{\perp \perp},$$

(6.10)

$$\frac{\eta_{\perp \perp}}{\rho_{0\perp}} + \frac{\eta_{00}}{\rho_{00}} = 0,$$  

(6.11)

$$\eta_{00}^2 + \rho_{0\perp}^2 = \Lambda_{\perp \perp}^2.$$  

(6.12)

Finally, we use the last part of Eq. (5.7) (i.e. $\omega_{\perp i0} = \omega_{\perp i0} - \omega_{\perp i}$) to reach the last relation:

$$\Lambda_{i0} = \frac{1}{2\Lambda_{\perp \perp}} \left[ \Sigma_{0\perp} \Sigma_{\perp \perp} + \rho_{\perp 0} \rho_{\perp \perp} \left( \frac{\Lambda_{\perp0}^2 - \rho_{0\perp}^2}{\Lambda_{00}^2 - \rho_{00}^2} \right) \right].$$

(6.13)

Thus we have six unknown parameters (three of $|A_{\lambda}|$, two of $\omega_{\perp i}$, and one $\beta$) in this case to solve for which are given by Eqs. (6.3), (6.5) (first part) and (6.7). Therefore, one should get a complete set of forty two independent relations among observables which consists of twenty four in Eq. (6.1), three in each of Eqs. (6.2), (6.4) and (6.10), two in each of Eqs. (6.6), (6.8), (6.11) and one in each of Eqs. (6.9), (6.12) and (6.13) respectively. All the other expressions in Sect. 4 and 5.
satisfy automatically. Except the twenty four relations in Eq. (6.1), the other eighteen relation have already been discussed in Ref. [73]. These relations will get violated by the presence of direct \( CP \) violation or some \( CPT \) non-conserving new Physics effects.

6.2 SM plus direct \( CP \) violation

In this case also \( \epsilon_j = 0 \forall j \in \{1, 2, 3\} \) which immediately imply from Eqs. (3.9) and (3.10) that

\[
\Lambda_{\lambda,\lambda} = \sqrt{\Sigma_{\lambda,\lambda}^2 + \rho_{\lambda,\lambda}^2 + \eta_{\lambda,\lambda}^2},
\]

\[
\left[ \frac{\rho_{\lambda,\lambda}}{\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda}} + \left( \frac{\rho_{\lambda,\lambda}}{\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda}} \right)^2 - 2 \left( \frac{\eta_{\lambda,\lambda}}{\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda}} \right) - 2 \left( \frac{\eta_{\lambda,\lambda}}{\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda}} \right)^2 \right] = - \left( \frac{\eta_{\lambda,\lambda}}{\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda}} \right)^2 - \left( \frac{\eta_{\lambda,\lambda}}{\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda}} \right)^2,
\]

\[
\frac{4 \left( \Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda} \right)}{(\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda})^2} - \left( \frac{\eta_{\lambda,\lambda} \rho_{\lambda,\lambda} - \eta_{\lambda,\lambda} \rho_{\lambda,\lambda}}{\Lambda_{\lambda,\lambda} - \Sigma_{\lambda,\lambda}} \right)^2 - \left( \frac{\eta_{\lambda,\lambda} \rho_{\lambda,\lambda} - \eta_{\lambda,\lambda} \rho_{\lambda,\lambda}}{\Lambda_{\lambda,\lambda} - \Sigma_{\lambda,\lambda}} \right)^2 = 1,
\]

\[
4 \left[ \frac{(\Lambda_{\lambda,\lambda} - \Sigma_{\lambda,\lambda}) (\Lambda_{\lambda,\sigma} - \Sigma_{\lambda,\sigma})}{(\Lambda_{\lambda,\sigma} - \Sigma_{\lambda,\sigma})^2} \right] = 1,
\]

\[
4 \left[ \frac{(\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda}) (\Lambda_{\lambda,\sigma} + \Sigma_{\lambda,\sigma})}{(\Lambda_{\lambda,\sigma} + \Sigma_{\lambda,\sigma})^2} \right] = 4 \left[ \frac{(\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda}) (\Lambda_{\lambda,\sigma} + \Sigma_{\lambda,\sigma})}{(\Lambda_{\lambda,\sigma} + \Sigma_{\lambda,\sigma})^2} \right].
\]

\[
\Lambda_{\lambda,\sigma} = \tilde{\Lambda}_{\lambda,\sigma}, \quad \eta_{\lambda,\sigma} = \tilde{\eta}_{\lambda,\sigma}, \quad \Sigma_{\lambda,\sigma} = -\tilde{\Sigma}_{\lambda,\sigma}, \quad \rho_{\lambda,\sigma} = \tilde{\rho}_{\lambda,\sigma},
\]

where \( (\lambda, \sigma) \in \{0, \|, \perp\} \) like the SM scenario. However, the presence of direct \( CP \) violation in this case makes the helicity amplitudes of the mode to be different from that of the conjugate mode. Thus, here we have eleven theoretical parameters (three for each of \( |A_{\lambda,\lambda}|, |\tilde{A}_{\lambda,\lambda}| \) and \( \phi_{\lambda}^{\text{meas}} \) respectively and two of \( \omega_{\perp\perp}, \) which follows from Sect. 4 and Eq. (6.14) as:

\[
|A_{\lambda,\lambda}| = \sqrt{\Lambda_{\lambda,\lambda} + \Sigma_{\lambda,\lambda}}, \quad |\tilde{A}_{\lambda,\lambda}| = \sqrt{\tilde{\Lambda}_{\lambda,\lambda} - \Sigma_{\lambda,\lambda}},
\]

\[
\sin \phi_{\lambda}^{\text{meas}} = \frac{\rho_{\lambda,\lambda}}{\sqrt{\Lambda_{\lambda,\lambda}^2 - \Sigma_{\lambda,\lambda}^2}},
\]

\[
\omega_{\perp\perp} = \sin^{-1} \left( -\frac{\Lambda_{\perp\perp}}{2 \sqrt{(\Lambda_{\perp\perp})^2 + (\Sigma_{\perp\perp})^2} \langle A_{\perp\perp} \rangle} \right).
\]

It should be noticed that the other two solutions for \( \sin \phi_{\lambda}^{\text{meas}} \) from the cubic equation in Eq. (4.5) are imaginary in general since \( \Lambda_{\lambda,\lambda} > \Sigma_{\lambda,\lambda} \) is required for positive definiteness of \( |A_{\lambda,\lambda}| \). With the help of Eqs. (6.14) and (6.15), the expressions for \( \epsilon_j \) in Eq. (4.7) vanish automatically.

In this case, number of independent relations among observables is forty eight minus eleven equals to thirty seven. Among them twenty four are listed in Eq. (6.14). The remaining thirteen can be found in the following way: (1) four relations can be found from Eq. (5.7), (2) three can be established by substituting Eqs. (5.9) and (5.10) in to Eq. (3.9), (3) the last six can be obtained by replacing Eqs. (5.11)–(5.14) into Eq. (3.9). After a bit of mathematical juggling these thirteen relations can be described in the following form:

\[
\text{where} \quad (\lambda, \sigma) \in \{0, \|, \perp\} \quad \text{and} \quad \lambda \neq \sigma. \quad \text{It should be noticed that each of the four equations from Eqs. (6.16) to (6.19) contains three relations for different values of } \lambda \text{ and } \sigma \quad \text{with } \lambda \neq \sigma. \quad \text{And Eq. (6.20) contains only one.} \quad \text{Violation of these relations would definitely imply existence of } CPT \text{ violating new Physics phenomenon.}
\]

6.3 SM plus T and CPT violation in mixing

In this case, one can follow the entire procedure described in Sects. 4 and 5 to get the solutions of the theoretical parameters and find the relations among observables. But to reach the expressions in the form of Ref. [73], that already discusses this scenario, one has to encounter various algebraic complexities.

As there is no direct \( CP \) violation in this case, the helicity amplitudes for mode and conjugate mode will be equal to each other (like SM). All of the three \( \phi_{\lambda}^{\text{meas}} \) become \( -2\beta \). Therefore, there will be total nine unknown parameters (three of \( |A_{\lambda,\lambda}| \), two of \( \omega_{\perp\perp} \), one \( \beta \) and three of \( \epsilon_j \)). It implies that the total number of independent relations among observables is
forty eight minus nine equal to thirty nine. It should be noticed from Sect. 6.1 that Σ_{λλ}, Σ_{00}, Λ_{∥∥}, (ρ_{00}^i + ρ_{00}^{∥i}), (ρ_{00}^i + η_{00}^{∥i}), (ρ_{00}^i - ρ_{00}^{∥i}), etc. were zero in SM case. However, inverting Eq. (3.9) one can find that they are O(ε) in the present scenario. Since we are keeping track up to the linear order terms in ε, any quadratic term involving the above expressions will be neglected. The same rule applies for the observables of conjugate mode too.

At first, using the equality of helicity amplitudes for mode and conjugate mode, one achieve the following six relations from Eqs. (4.4), (4.8), (5.4)–(5.7) (first part):

\[ Λ_{λλ} + Σ_{λλ} = \tilde{Λ}_{λλ} + \tilde{Σ}_{λλ}, \quad Λ_{∥i} + Σ_{∥i} = -(\tilde{Λ}_{∥i} + \tilde{Σ}_{∥i}), \]

\[ Λ_{00} + Σ_{00} = \tilde{Λ}_{00} + \tilde{Σ}_{00} + \bar{Ω}. \]  

(6.21)

where λ ∈ {0, ||, ⊥} and i ∈ {0, ||}. Along with the above six relations we also get the expressions for five unknown parameters as:

\[ |A_{λi}| = \sqrt{Λ_{λλ} + Σ_{λλ}} \quad \text{and} \quad \omega_{∥i} = \sin^{-1} \left( -\frac{Λ_{∥i} + Σ_{∥i}}{2\sqrt{(Λ_{∥∥} + Σ_{∥∥})(Λ_{∥i} + Σ_{∥i})}} \right). \]  

(6.22)

At second, we use the three expressions for ψ_{λ}^{meas} (≡ −2β) from Eq. (4.5) and six relations in Eqs. (5.1)–(5.3) that equate the different expressions for ε_j (j ∈ {1, 2, 3}). These nine expressions can be paraphrased as one equation for sin 2β and eight relations among observables as follows:

\[ \sin 2β = -\frac{1}{2} \left( \frac{ρ_{00}^0 - ρ_{00}^i}{Λ_{00} - Σ_{00}} \right). \]  

(6.23)

\[ \begin{align*}
\rho_{00}^0 - ρ_{00}^i &= \frac{2ρ_{00}^i - \tilde{ρ}_{00}^i}{Λ_{00} - Σ_{00}}, \\
\rho_{00}^0 + ρ_{00}^i &= \frac{2ρ_{00}^i + \tilde{ρ}_{00}^i}{Λ_{00} + Σ_{00}} \\
ρ_{00}^i + Π &= \frac{2ρ_{00}^i + Π}{Λ_{00} + Σ_{00}} \\
0 &= \left( \begin{array}{ccc}
\rho_{00}^0 + Σ_{00} & Π & \rho_{00}^i + Σ_{00} \\
\rho_{00}^0 + Σ_{00} & Π & \rho_{00}^i + Σ_{00} \\
\rho_{00}^0 + Σ_{00} & Π & \rho_{00}^i + Σ_{00}
\end{array} \right) \sin 2β = 0.
\end{align*} \]  

(6.24)

(6.25)

(6.26)

(6.27)

With the help of the above expressions the values of ε_2 and ε_3 in Eq. (4.7) can be written as:

\[ ε_2 = -\frac{1}{2} \left( \frac{ρ_{00}^0 + Σ_{00} + Π}{Λ_{00} + Σ_{00} + Π} \right), \quad ε_3 = \frac{1}{2} \left( \frac{Σ_{00} + Σ_{00} + Π}{Λ_{00} + Σ_{00} + Π} \right). \]  

(6.28)

Thirdly, substituting the expressions for η_{λλ} for λ ∈ {0, ||, ⊥} from Eqs. (5.11) and (5.12) into Eqs. (3.9) and (3.10) one would end up with following six relations:

\[ \begin{align*}
\eta_{00} &= \eta_{∥∥}, \quad \tilde{η}_{00} = \tilde{η}_{∥∥}, \quad \eta_{00} = \tilde{η}_{∥∥}, \\
(ρ_{00} - ρ_{00}^{∥})^2 + (ρ_{00} - η_{00}^{∥})^2 &= 4, \\
(η_{00} - η_{00}^{∥})^2 &= 4, \\
ρ_{00}^2 + η_{00}^2 &= η_{∥∥}^2 + η_{∥∥}^2.
\end{align*} \]  

(6.29)

(6.30)

(6.31)

(6.32)

It should be noticed that some of the above relations could be further simplified as: Λ_{λλ} = ϵ_{λλ}^2 and Λ_{∥∥} = ϵ_{∥∥}^2. Nonetheless, to reproduce the relations in Ref. [73], we stick to the former ones only. Using the above relations, the expression for ε_j from Eq. (4.7) can be interpreted as following:

\[ \epsilon_j = -\frac{1}{2} \left( \frac{η_{jj} + Π_{jj}}{Λ_{jj} + Σ_{jj} + Π_{jj}} \right). \]  

(6.33)

With the help of above relations involving η_{λλ}, one can also abandon Eq. (6.27) and recast it as:

\[ \left( η_{jj} - Π_{jj} \right) + \left( Π_{jj} + Π_{jj} \right) = 0. \]  

(6.34)

Fourthly, we use the Eq. (5.8) to arrive at the expressions for cos ω_{σσ} and cos Ω_{σσ} that can be rewritten as the following six relations:

\[ \begin{align*}
\left( Λ_{||} + Σ_{||} \right) &+ 4X^2 Λ_{||} Σ_{||} \left( Λ_{||} + Σ_{||} + 2Λ_{||} Λ_{||} \right) = 4. \\
\left( Λ_{||} + Σ_{||} \right) &+ 4X^2 Λ_{||} Σ_{||} \left( Λ_{||} + Σ_{||} + 2Λ_{||} Λ_{||} \right) = 4, \\
\left( Σ_{00} + Σ_{00} \right) &+ \bar{Σ}_{00} = \frac{Σ_{00}}{Λ_{00}}. \end{align*} \]  

(6.35)

(6.36)

(6.37)

where

\[ X_i = \left[ \frac{1}{Λ_{||} Λ_{||} Λ_{||} + Σ_{||} Σ_{||} Σ_{||}} \right] \times \left[ \left( Λ_{||} Λ_{||} Λ_{||} + Σ_{||} Σ_{||} Σ_{||} \right) + 2(Λ_{||} Λ_{||} Λ_{||} - Σ_{||} Σ_{||} Σ_{||}) \right]. \]  

and,

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\[ \tilde{X}_i = \left[ \frac{1}{(\tilde{\eta}_{i\perp} - \tilde{\eta}_{i\parallel}) (\Lambda_{i\perp} + \Sigma_{i\perp}) (\Lambda_{i\parallel} + \Sigma_{i\parallel})} \right] \times \left[ (\tilde{\Lambda}_{i\perp} - \tilde{\Sigma}_{i\perp}) (\tilde{\Lambda}_{i\perp} - \tilde{\Sigma}_{i\perp}) \right], \\
+ 2(\tilde{\Lambda}_{i\parallel} \tilde{\Lambda}_{i\perp} - \tilde{\Sigma}_{i\parallel} \tilde{\Sigma}_{i\perp}). \]

with \( i \in \{0, \parallel\} \). It is very important to note that to get a correct \( X_i \) or \( \tilde{X}_i \) up to \( \mathcal{O}(\epsilon_j) \) one should keep the quadratic terms of \( \epsilon_j \) in the numerator and denominator separately while defining \( X_i \) or \( \tilde{X}_i \), since the leading order terms in both of the numerator and denominator are \( \mathcal{O}(\epsilon_j) \). Notwithstanding, the relations in Eqs. (6.35) and (6.36) can be untangle a bit by using \( \cos \omega_{i\perp} = \frac{1}{2} X_i \sqrt{\Lambda_{i\perp} \Lambda_{i\parallel}} \) and \( \cos \tilde{\omega}_{i\perp} = \frac{1}{2} \tilde{X}_i \sqrt{\Lambda_{i\perp} \Lambda_{i\parallel}} \).

Fifthly, by replacing the expressions for \( \rho^*_\sigma \) with \( \lambda \neq \sigma \) from Eqs. (5.9) and (5.10) into Eqs. (3.9) and (3.10), as described in Sec. 5, one would end up with following six relations:

\[ \rho_{i\perp} = \frac{1}{2} \left[ \frac{\Sigma_{i\perp}}{\Lambda_{i\perp} \Lambda_{i\parallel}} \left( \rho_{i\perp} (\Lambda_{i\perp} + \Sigma_{i\perp}) + \rho_{i\parallel} (\Lambda_{i\parallel} + \Sigma_{i\parallel}) \right) \right] \]

\[ - X_i \left[ (\Lambda_{i\perp} \eta_{i\parallel} - \Lambda_{i\parallel} \eta_{i\perp}) \right], \]  
(6.38)

\[ \tilde{\rho}_{i\perp} = \frac{1}{2} \left[ \frac{\tilde{\Sigma}_{i\perp}}{\Lambda_{i\perp} \Lambda_{i\parallel}} \left( \tilde{\rho}_{i\perp} (\tilde{\Lambda}_{i\perp} + \tilde{\Sigma}_{i\perp}) + \tilde{\rho}_{i\parallel} (\tilde{\Lambda}_{i\parallel} + \tilde{\Sigma}_{i\parallel}) \right) \right] \]

\[ - \tilde{X}_i \left[ (\tilde{\Lambda}_{i\perp} \tilde{\eta}_{i\parallel} - \tilde{\Lambda}_{i\parallel} \tilde{\eta}_{i\perp}) \right], \]  
(6.39)

\[ \frac{\rho_{0\parallel}}{\Lambda_{0\parallel}} = \frac{\rho_{0\perp}}{\Lambda_{0\perp}} \] and \[ \frac{\tilde{\rho}_{0\parallel}}{\Lambda_{0\parallel}} = \frac{\tilde{\rho}_{0\perp}}{\Lambda_{0\perp}}, \]  
(6.40)

where \( i \in \{0, \parallel\} \). Similarly, substituting the expressions for \( \eta^*_\sigma \) with \( \lambda \neq \sigma \) from Eqs. (5.13) and (5.14) into Eqs. (3.9) and (3.10), would lead to the following six relations:

\[ \eta_{i\perp} = \frac{1}{2} \left[ \frac{\Sigma_{i\perp}}{\Lambda_{i\perp} \Lambda_{i\parallel}} \left( \eta_{i\perp} (\Lambda_{i\perp} + \Sigma_{i\perp}) + \eta_{i\parallel} (\Lambda_{i\parallel} + \Sigma_{i\parallel}) \right) \right] \]

\[ + X_i \left[ (\Lambda_{i\perp} \rho_{i\parallel} - \Lambda_{i\parallel} \rho_{i\perp}) \right], \]  
(6.41)

\[ \tilde{\eta}_{i\perp} = \frac{1}{2} \left[ \frac{\tilde{\Sigma}_{i\perp}}{\Lambda_{i\perp} \Lambda_{i\parallel}} \left( \tilde{\eta}_{i\perp} (\tilde{\Lambda}_{i\perp} + \tilde{\Sigma}_{i\perp}) + \tilde{\eta}_{i\parallel} (\tilde{\Lambda}_{i\parallel} + \tilde{\Sigma}_{i\parallel}) \right) \right] \]

\[ + \tilde{X}_i \left[ (\tilde{\Lambda}_{i\perp} \tilde{\rho}_{i\parallel} - \tilde{\Lambda}_{i\parallel} \tilde{\rho}_{i\perp}) \right], \]  
(6.42)

\[ \frac{\eta_{0\parallel}}{\Lambda_{0\parallel}} = \frac{\eta_{0\perp}}{\Lambda_{0\perp}} \] and \[ \frac{\tilde{\eta}_{0\parallel}}{\Lambda_{0\parallel}} = \frac{\tilde{\eta}_{0\perp}}{\Lambda_{0\perp}} \]  
(6.43)

Finally, second part of Eq. (5.7) \( (\omega_{0\parallel} = \omega_{0\perp} - \omega_{\perp}) \) indicates the last independent relation as:

\[ \left( \Lambda_{0\parallel} + \Sigma_{0\parallel} \right) - \frac{1}{2} \left[ \frac{(\Lambda_{0\perp} + \Sigma_{0\perp}) (\Lambda_{0\parallel} + \Sigma_{0\parallel})}{(\Lambda_{i\parallel} + \Sigma_{i\parallel})} \right] \]

\[ = \left[ \frac{2 X_0 X_1 \Lambda_{0\parallel}}{\Lambda_{i\parallel} \Sigma_{0\parallel} + \Lambda_{0\parallel} \Sigma_{i\parallel} + 2 \Lambda_{i\parallel} \Lambda_{0\parallel}} \left( \Lambda_{0\parallel} \right) \right] \]

\[ \times \left[ \frac{(\Lambda_{0\parallel} + \Sigma_{0\parallel}) (\Lambda_{0\parallel} + \Sigma_{i\parallel}) (\Lambda_{0\parallel} + \Sigma_{0\parallel})}{(\Lambda_{i\parallel} \Sigma_{0\parallel} + \Lambda_{0\parallel} \Sigma_{i\parallel} + 2 \Lambda_{i\parallel} \Lambda_{0\parallel})} \right] \].  
(6.44)

Thus, the expressions for nine theoretical parameters (three of \( |A_{ij}| \), two of \( \omega_{i\perp} \), one \( \beta \) and three of \( \epsilon_j \) in this scenario are given by five equations in Eq. (6.22), two expressions in Eq. (6.28) and one in each of Eqs. (6.23) and (6.33). On the other hand, the thirty nine relation among observables are presented as: (a) six relations in Eq. (6.21), (b) seven equations from Eqs. (6.24)–(6.26) (we have not counted Eq. (6.27) since it has been recast as Eq. (6.34)), (c) six expressions from Eqs. (6.29)–(6.32) and (d) twenty relations from Eqs. (6.34)–(6.44). If some of these relations do not hold true, that will indicate the presence of \( CPT \) or \( CP \) violation in decay itself.

### 7 Phenomenology

Various experiments have been performed so far in order to probe \( CPT \) violation in neutral meson mixing. Although these experiments measure tiny non-zero values for \( CPT \) violating parameters, they become consistent to zero within 2\( \sigma \) due to the presence of experimental error bars with comparable size. In case of kaon system, \( CPT \) asymmetry is measured from the semileptonic \((\pi^+)^-\tilde{\nu}_1, \pi^- + \nu_1\) decay modes of \( K^0 \) and \( \tilde{K}^0 \) to estimate the \( CPT \)-violating complex parameter \( \delta \) whose real and imaginary parts are directly proportional to \( \epsilon_1 \) and \( \epsilon_2 \) respectively in our notation. From the data of KTeV collaboration [76], the real and imaginary parts of this parameter are estimated to be: \( \text{Re}(\delta) = (2.51 \pm 2.25) \times 10^{-4} \) and \( \text{Im}(\delta) = (-1.5 \pm 1.6) \times 10^{-5} \) which agree with \( CPT \) conservation. In case of \( D^0 \rightarrow \bar{D}^0 \) system, \( CPT \) asymmetry, which is constructed by comparing the time dependent decay probabilities of the modes \( D^0 \rightarrow K^- \pi^+ \) and \( \bar{D}^0 \rightarrow K^+ \pi^- \), has been measured by FOCUS collaboration [77]. This measurement leads to the estimation of \( CPT \)-violating complex parameter \( \xi \), whose real and imaginary parts are proportional to \( \epsilon_1 \) and \( \epsilon_2 \) respectively, to be: \( \text{Re}(\xi) \propto \frac{\text{Im}(\xi)}{\sqrt{x+\frac{\Delta M}{2}}} \) where, \( x = \frac{1}{2} \) and \( y = \frac{1}{2} \). The first measurement of \( CPT \) violation in \( B^0 \rightarrow \bar{B}^0 \) system was performed by BaBar collaboration [52,53]. The last update on it has been carried out by Belle collaboration [54]. For this purpose, they have fitted the time dependent decay rate of the charm: \( \Upsilon(4S) \rightarrow B^0_d \bar{B}_d^0 \rightarrow f_{rec} f_{tag} \), where one of the \( B \)-meson decays to reconstructed final state \( f_{rec} \) at time \( t_{rec} \) and the other one decays at time \( t_{tag} \) to a final state \( f_{tag} \), that distinguishes between \( B^0 \) and \( \bar{B}^0 \). Several hadronic and
semileptonic decay modes of $B_d^0 (J/\psi K_S, J/\psi K_L, D^- \pi^+, D^+ \pi^-, D^+ \rho^+)$ and $D^+ \pi^+$ have been used in this case to find the experimental value for the CPT-violating complex parameter $z$ as: $\text{Re}(z) = (1.9 \pm 3.7 \pm 3.3) \times 10^{-2}$ and $\text{Im}(z) = (-5.7 \pm 3.3 \pm 3.3) \times 10^{-3}$ which are consistent with zero. Similarly, using the time dependent decay rate to a CP eigenstate for the mode $B_s^0 \to J/\psi K^+ K^-$, the CPT violation in $B_s^0$ system has been measured by LHCb collaboration [55] as: $\text{Re}(z) = -0.022 \pm 0.033 \pm 0.005$ and $\text{Im}(z) = 0.004 \pm 0.011 \pm 0.002$ that also agrees with conservation of CPT. The dependence of the parameter $z$ on $\epsilon_1$ and $\epsilon_2$ is already given by Eq. (3.4). On the other hand, direct and indirect CP violations as well as $T$ violation in neutral meson mixing have also been measured at different occasions [44]. However, during all these measurements, CPT has always been assumed to be preserved which makes $T$ and CP violations equivalent to each other.

Due to unavailability of enough phase space, the kaon system cannot decay to any vector meson. Nevertheless, for $D^0$, $B_d^0$ and $B_s^0$ systems, several decay modes with two vectors in final state, to which both the pseudoscalar meson and its antiparticle can be accessible. As this paper addresses direct CP violation with the decays $P^0 \to f$ and $\bar{P}^0 \to f$, we should focus on the final states $f$ which are CP eigenstates. The modes that can be considered for $D^0$ system are $K^+ \bar{K}^*$, $\rho^0 \phi$, $K^{*+} K^{*-}$, $\rho^0 \rho^0$, $\rho^+ \rho^-$, etc. Similarly, for $B_d^0$ systems, modes like $\phi \rho_0 \rho_0$, $\rho^+ \rho^-$, $K^* \bar{K}^*$, $K^{*+} K^{*-}$, $D_s^{*+} D_s^{*-}$, etc. can be used. For $B_s^0$ system also, the modes like $K^\ast \rho_0 \rho_0$, $\rho^+ \rho^-$, $K^* \bar{K}^*$, $K^{*+} K^{*-}$, $D_s^{*+} D_s^{*-}$, $J/\psi \phi$, etc. should be used. Many of these modes have already been studied by several experimental groups [44]. But time dependence of the decay rates for these modes has not been used so far in connection with probing CPT violation.

Having said that, let us now consider the final states consisting of two vectors with no definite CP. The general analysis, described up to Sect. 5, does not require the final state $f$ to be a CP eigenstate. So, one can use it for any final state $f$ to which both $P^0$ and $\bar{P}^0$ can decay. In that case, the decay rates for the channels $P^0 \to f$ and $\bar{P}^0 \to f$ can be very different from each other and the SM behaviour would exactly be described by the scenario depicted in Sect. 6.2. But the equations of Sects. 6.1 and 6.3 must not be used for this particular scenario since these two sections inevitably presume the final state $f$ to be a CP eigenstate while imposing the condition $A_{\lambda} = A_{\bar{\lambda}}$ for CP being conserved. Generally, if $f$ does not represent a CP eigenstate, only one of the channels between $P^0 \to f$ and $\bar{P}^0 \to f$ becomes dominant due to additional CKM suppression or presence of extra $W$-boson propagator in the leading order Feynman diagram of the other process. For example, this happens in the decay channels like $K^\ast \rho_0$ or $K^{*+} \rho^-$ for $D^0$ system, $D^{*+} K^{*-}$, $D^{*+} \rho^-$, $\phi K^*$, $J/\psi K^*$, etc., modes in case of $B_d^0$ systems and $D_s^{*+} \rho^-$, $\phi K^*$, $J/\psi K^*$, etc., channels for $B_s^0$ systems. Nevertheless, it does not mean that all the observables for the disfavoured modes will be small enough in magnitude. To elaborate, let us consider a SM scenario (i.e. $\epsilon_j = 0$) with $|\bar{A}/A| \leq 1$. Then, Eqs. (3.8)–(3.10) readily imply that $(\rho_{\lambda}/A_{\lambda}) \approx (\bar{\rho}_{\bar{\lambda}}/A_{\bar{\lambda}}) \ll 1$, $(\eta_{\lambda}/A_{\lambda}) \approx (\bar{\eta}_{\bar{\lambda}}/A_{\bar{\lambda}}) \ll 1$ and $|\lambda_{\lambda} | \approx |\bar{\lambda}_{\bar{\lambda}} | \approx |\sigma | \approx |\bar{\sigma} |$. So, if we neglect the terms $\eta_{\lambda}$ and $\rho_{\lambda}$ with respect to $\lambda_{\lambda}$ and $\bar{\lambda}_{\bar{\lambda}}$, then we integrate the time dependent decay rates (given by Eqs. (3.6) and (3.7)) over time, we find the branching fractions for $P^0 \to f$ and $\bar{P}^0 \to f$ to be:

$$\text{Br}(P^0 \to f) \approx \left( \frac{1}{\Gamma^2 - \Delta \Gamma^2/4} + \frac{1}{\Gamma^2 + \Delta M^2} \right) \sum_{\lambda_{\lambda}} \Lambda_{\lambda \lambda} g_{\lambda \lambda} \sigma$$

$$\text{Br}(\bar{P}^0 \to f) \approx \left( \frac{1}{\Gamma^2 - \Delta \Gamma^2/4} + \frac{1}{\Gamma^2 + \Delta M^2} \right) \sum_{\lambda_{\lambda}} \Lambda_{\lambda \lambda} g_{\lambda \lambda} \sigma.$$  

This clearly shows that $\lambda_{\lambda}$ might not be small in this special case but still branching fraction for the disfavoured transition could be small due to cancellation between two large contributions coming from integration of $e^{-i\Gamma} \cosh(\Delta \Gamma t/2)$ and $e^{-i\Gamma} \cos(\Delta M t)$ terms depending on the values of $\Delta \Gamma$ and $\Delta M$ whereas for the dominant mode these contributions add up to a larger value. However, due to presence of large contributions from $\lambda_{\lambda}$ and $\bar{\lambda}_{\bar{\lambda}}$, precise measurement for $\eta_{\lambda}$ and $\rho_{\lambda}$ might be problematic in these kind of channels.

Now, by examining the decay products of the produced vectors, the helicity component for the two vector final state can be obtained. Then studying angular analysis and time dependence of the decay rates for $P^0 \to f$ and $\bar{P}^0 \to f$, all the observables can be measured [68]. However, due to existence of several relations among observables, the $T$, CP and CPT violating parameters can be expressed in various ways involving different observables. But in experiment, all these observables can be measured separately and hence several measurements for same theoretical parameter can be performed simultaneously which would help to reduce the error bar allowing a more precise measurement for the unknown parameter. Thus better results on CPT violation should be expected from future runs of LHCb and Belle II.

8 Conclusion

In conclusion, we have studied the behaviour of observables for neutral meson decaying to two vectors in the presence of $T$, CP and CPT violation in mixing as well as CP violation in decay. Polarizations of final state with two vectors provide us a large number of observables in these modes. The final state should be chosen in such a way that both $P^0$ and $\bar{P}^0$ can decay to it. We extract all of the fourteen unknown theoretical parameters in terms of the observables and then discuss the procedure to establish the complete set of independent relations among observables containing thirty four equations. These relations can be used as the smoking gun
signal to prove the existence of direct violation of CPT (if any) since those effects only can lead to non-obedience of them. Additionally, we explore three special cases e.g. SM case, SM plus direct CP violation scenario and SM plus CP violation in mixing case. Using our new formalism, we derive the expressions for unknown theoretical parameters and construct the complete set of independent relations among observables too in each special case. Experimental verification for each of the sets will signify the existence of some particular type of Physics. For example, the set of relations in SM plus CP violation in mixing scenario can be applied to probe direct violation of CPT or CP, the set of relations in SM plus CP case can be implemented to confirm the existence of any CPT violating new Physics (direct or indirect), whereas the set of observable relations in SM scenario should be used to detect direct CP violation or CPT non-conserving new Physics.

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References

1. R. Lehnert, CPT symmetry and its violation. Symmetry 8(11), 114 (2016)
2. I.I. Bigi, A.I. Sanda, CP Violation, 2nd edn. (Cambridge University Press, Cambridge, 2009)
3. G. Lüders, On the equivalence of invariance under time-reversal and under particle-antiparticle conjugation for relativistic field theories. Kong. Dan. Vid. Sel. Mat. Fys. Med. 26(5), 1–17 (1954)
4. G. Lüders, Proof of CPT theorem. Ann. Phys. 2, 1–15 (1957)
5. W. Pauli, Niels Bohr and the Development of Physics (McGraw-Hill, Hoboken, 1955), p. 30
6. J.S. Bell, Time reversal in field theory. Proc. R. Soc. Lond. 231, 479–495 (1955)
7. J.S. Schwinger, The theory of quantized fields. I. Phys. Rev. 82, 914–927 (1951)
8. S. Weinberg, The Quantum Theory of Fields, vol. 1 (Cambridge University Press, Cambridge, 1995)
9. R. Jost, A remark on the C.T.P. theorem. Helv. Phys. Acta 30, 409 (1957)
10. N.N. Bogoliubov, A.A. Logunov, I.T. Todorov, Introduction to Axiomatic Quantum Field Theory (Benjamin, Boston, 1975)
11. R.F. Streater, A.S. Wightman, PCT, Spin & Statistics, and All That (Princeton University Press, Princeton, 2000)
12. A.S. Wightman, Quantum field theory in terms of vacuum expectation values. Phys. Rev. 101, 860 (1956)
13. D.W. Hall, A.S. Wightman, A theorem on invariant analytic functions with applications to relativistic quantum field theory. Mat. Fys. Medd. Danske Vid. Selsk. 31, 5 (1957)
14. S. Mark, E.C.G. Sudarshan, Derivation of the TCP theorem using action principles. arXiv:1308.5110 [hep-th] (2019)
15. V.A. Kostelecký, R. Potting, CPT and strings. Nucl. Phys. B 359, 545–570 (1991)
16. V.A. Kostelecký, S. Samuel, Spontaneous breaking of Lorentz symmetry in string theory. Phys. Rev. D 39, 683 (1989)
17. D. Colladay, V.A. Kostelecký, CPT violation and the standard model. Phys. Rev. D 55, 6760 (1997)
18. D. Colladay, V.A. Kostelecký, Lorentz-violating extension of the standard model. Phys. Rev. D 58, 116002 (1998)
19. V.A. Kostelecký, Gravity, lorentz violation, and the standard model. Phys. Rev. D 69, 105009 (2004)
20. V.A. Kostelecký, N. Russell, Data tables for Lorentz and CPT violation. Rev. Mod. Phys. 83, 11 (2011)
21. V.A. Kostelecký, R. Lehnert, Stability, causality, and Lorentz and CPT violation. Phys. Rev. D 63, 065008 (2001)
22. O.W. Greenberg, CPT violation implies violation of Lorentz invariance. Phys. Rev. Lett. 89, 231602 (2002)
23. F.R. Klinkhamer, A CPT anomaly. Nucl. Phys. B 578, 277–289 (2000)
24. F.R. Klinkhamer, C. Rupp, Space-time foam, CPT anomaly, and photon propagation. Phys. Rev. D 70, 045020 (2004)
25. M. Chaichian, K. Nishijima, A. Tureanu, Spin statistics and CPT theorems in noncommutative field theory. Phys. Lett. B 568, 146 (2003)
26. M. Chaichian, M.N. Mnatsakanova, K. Nishijima, A. Tureanu, YuS Vernov, Towards an axiomatic formulation of noncommutative quantum field theory. J. Math. Phys. 52, 032303 (2011)
27. M.M. Sheikh-Jabbari, C. P and T invariance of noncommutative gauge theories. Phys. Rev. Lett. 84, 5265–5268 (2000)
28. L. Alvarez-Gaumé, M. A. Vazquez-Mozo, General properties of noncommutative field theories. Nucl. Phys. B 668, 293 (2003)
29. M. Chaichian, A.D. Dolgov, V.A. Novikov, A. Tureanu, CPT violation does not lead to violation of Lorentz invariance and vice versa. Phys. Lett. B 699, 177–180 (2011)
30. M. Chaichian, K. Fujikawa, A. Tureanu, Lorentz invariant CPT violation: particle and antiparticle mass splitting. Phys. Lett. B 712, 115 (2012)
31. M. Chaichian, K. Fujikawa, A. Tureanu, Lorentz invariant CPT violation. Eur. Phys. J. C 73, 2349 (2013)
32. P. Carruthers, Locality and the isospin of self-conjugate bosons. Phys. Rev. Lett. 18, 353 (1967)
33. P. Carruthers, Isospin symmetry, TCP, and local field theory. Phys. Lett. B 26, 158–160 (1968)
34. P. Carruthers, Local field theory and isospin invariance. I. Free-field theory of spinless bosons. J. Math. Phys. 9, 928–945 (1968)
35. P. Carruthers, Local field theory and isospin invariance. II. Free field theory of arbitrary spin particles. J. Math. Phys. 9, 1835–1845 (1968)
36. P. Carruthers, Local field theory and isospin invariance. III. Interactions of self-conjugate isofermion fields. Phys. Rev. 172, 1406 (1968)
37. S.W. Hawking, Breakdown of predictability in gravitational collapse. Phys. Rev. D 4, 2460 (1976)
38. S.W. Hawking, The unpredictability of quantum gravity. Commun. Math. Phys. 87, 395–415 (1982)
39. N.E. Mavromatos, CPT violation and decoherence in quantum gravity. Lect. Notes Phys. 669, 245–320 (2005)
40. N.E. Mavromatos, Models and (some) searches for CPT violation: from early universe to present era. J. Phys. Conf. Ser. 873, 012006 (2017)
41. C. Adam, F.R. Klinkhamer, Causality and CPT violation from an Abelian Chern-Simons like term. Nucl. Phys. B 607, 247–267 (2001)
42. E. Abers, I.T. Grodsky, R.E. Norton, Diseases of infinite-component field theories. Phys. Rev. D 159, 1222 (1967)
43. A.I. Oksak, I.T. Todorov, Invalidity of TCP-theorem for infinite-component fields. Commun. Math. Phys. 11, 125–130 (1968)
44. P.A. Zyla et al., (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)
45. L. Lavoura, J.P. Silva, Disentangling violations of CPT from other new physics effects. Phys. Rev. D 60, 056003 (1999)
46. M. Kobayashi, A.I. Sanda, On testing CPT symmetry in B decays. Phys. Rev. Lett. B 795, 620–626 (2019)
47. J.R. Ellis, N.E. Mavromatos, Comments on CP, T and CPT violation in neutral kaon decays. Phys. Rept. 320, 341–354 (1999)
48. B.R. Edwards, A. Kostelecky, searching for CPT violation with neutral meson oscillations. Phys. Lett. B 795, 012007 (2019)
49. A. Roberts, Testing CPT violation with correlated neutral mesons. Phys. Rev. D 96, 116015 (2017)
50. M.C. Bañuls, J. Bernabéu, Studying indirect violation of CP, T and CPT in a B factory. Nucl. Phys. B 590, 19 (2000)
51. E. Alvarez, J. Bernabéu, Correlated neutral B meson decays into CP eigenstates. Phys. Lett. B 579, 79 (2004)
52. B. Aubert et al., (BaBar Collaboration), Limits on the decay-rate difference of neutral B mesons and on CP, T, and CPT violation in $B^0\bar{B}^0$ oscillations. Phys. Rev. Lett. 92, 181801 (2004)
53. B. Aubert et al., (BaBar Collaboration), Limits on the decay rate difference of neutral–B mesons and on CP, T, and CPT violation in $B^0\bar{B}^0$ oscillations. Phys. Rev. D 70, 012007 (2004)
54. T. Higuchi et al., (Belle Collaboration), Search for time-dependent CPT violation in hadronic and semileptonic B decays. Phys. Rev. D 85, 071105 (2012)
55. R. Aaij et al., (LHCb Collaboration), Search for violations of Lorentz invariance and CPT symmetry in $B^0\bar{B}^0$ mixing. Phys. Rev. Lett. 116, 241601 (2016)
56. E. Alvarez, J. Bernabéu, N.E. Mavromatos, M. Nebot, J. Papavassiliou, CPT violation in entangled $B^0\bar{B}^0$ states and the demise of flavor tagging. Phys. Lett. B 607, 197 (2005)
57. E. Alvarez, A. Szykman, Direct test of time reversal invariance violation in B mesons. Mod. Phys. Lett. A 23, 2085 (2008)
58. J. Bernabéu, F. Martinez-Vidal, P. Villanueva-Perez, Time Reversal Violation from the entangled $B^0,\bar{B}^0$ system. JHEP 1208, 064 (2012)
59. E. Applebaum, A. Efратi, Y. Grossman, Y. Nir, Y. Soreq, Subtleties in the BABAR measurement of time-reversal violation. Phys. Rev. D 89(7), 076011 (2014)
60. Anirban Kundu, Soumira Nandi, Sunando Kumar Patra, Amarjit Soni, $B_s \rightarrow D \bar{D}$ as a probe of CPT violation. Phys. Rev. D 87(1), 016005 (2013)
61. A. Kundu, S. Nandi, S.K. Patra, Probing CPT violation in B systems. Phys. Rev. D 81, 076010 (2010)
62. J. van Tilburg, M. van Veghel, Status and prospects for CPT and Lorentz invariance violation searches in neutral meson mixing. Phys. Lett. B 742, 236 (2015)
63. J. Bernabéu, F.J. Botella, M. Nebot, Genuine $T$, $CP$, CPT asymmetry parameters for the entangled $B_d$ system. JHEP 1606, 100 (2016)
64. A. Karan, A.K. Nayak, R. Sinha, D. London, Using time-dependent indirect CP asymmetries to measure $T$ and CPT violation $B_s^0 - \bar{B}_s^0$ mixing. Phys. Lett. B 781, 459–463 (2018)
65. F.J. Botella, M. Nebot, CPT Violation in $B_s^0 - \bar{B}_s^0$ mixing and the measurement of CP Violation in $B_s \rightarrow K^+K^-$. arXiv:1903.04542 [hep-ph] (2019)
66. Y. Amhis et al., Heavy Flavor Averaging Group (HFAG) Collaboration, Averages of b-hadron, c-hadron, and $\tau$-lepton properties as of summer 2016, arXiv:1612.07233 [hep-ex], and online updates at http://www.slac.stanford.edu/xorg/hfag (2016)
67. J.H.A. Nogueria et al., Suppressed $B \rightarrow PV$ CP asymmetry: CPT constraints. Phys. Rev. D 94, 054028 (2016)
68. D. London, N. Sinha, R. Sinha, Extracting weak phase information from $B \rightarrow V(1)\bar{V}(2)$ decays. Phys. Rev. Lett. 85, 1807–1810 (2000)
69. D. London, N. Sinha, R. Sinha, Bounds on new physics from $B \rightarrow V(1)\bar{V}(2)$ decays. Phys. Rev. D 69, 114013 (2004)
70. D. London, N. Sinha, R. Sinha, Searching for new physics via an angular analysis of $B \rightarrow V_1\bar{V}_2$ decays. Europhys. Lett. 67, 579 (2004)
71. A. Dighe, D. Ghosh, A. Kundu, S.K. Patra, Reconciling anomalous measurements in $B_s^0 - \bar{B}_s^0$ mixing: The role of CPT-conserving and CPT-violating new physics. Phys. Rev. D 84, 056008 (2011)
72. S.K. Patra, A. Kundu, CPT violation and triple-product correlations in B decays. Phys. Rev. D 87, 116005 (2013)
73. A. Karan, A.K. Nayak, Behavior of observables for neutral meson decaying to two vectors in the presence of $T$, CPT and CPT violation in mixing only. Phys. Rev. D 101, 015027 (2020)
74. N. Sinha, R. Sinha, Determination of the angle gamma using $B \rightarrow D^*V$ modes. Phys. Rev. Lett. 80, 3706 (1998)
75. T.D. Lee, Particle physics and introduction to field theory. Contemp. Concepts Phys. 1, 1 (1981)
76. E. Abouzaid et al., (KTeV Collaboration), Precise measurement of direct CP violation, CPT symmetry and other parameters in neutral kaon system. Phys. Rev. D 83, 092001 (2011)
77. J.M. Link et al., (FOCUS Collaborations), Charm system tests of CPT and Lorentz invariance with FOCUS. Phys. Lett. B 556, 7–13 (2003)