Design of Terminal Sliding Mode Controller Based on Nonlinear Dynamic Inversion

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Abstract: In this paper, the extended state observer was applied to satisfy the high accuracy demand of the nonlinear dynamic inversion through compensating the uncertainty and disturbance of the nonlinear dynamic system as a total disturbance. The terminal sliding mode controller was adopted to assure the system converging fast and accurately. The effectivity and quickly converging of the proposal method was demonstrated by the simulation of the attitude control problem of the upper stage, with the comparison of other methods.

1. Introduction

Nonlinear dynamic inversion (NDI) is one of the basic control methods of nonlinear system. The main idea of NDI is transforming the complex nonlinear systems into linear ones with the form of integral chain through the conversion of coordinates, and then adopting linear control methods to design the controller \cite{1}. However, the demand of the high accuracy for the system limits the implement of the controller, which could only working well just around the characteristic points\cite{2}. The controller designed based on the NDI directly for nonlinear system with external disturbance and uncertainty will produce the large deviation, and even result in the divergence of the state and the failure of the controller \cite{3}.

In this paper, the extended state observer which could estimate the total disturbance (including external disturbance and the uncertainty of the system’s parameters) was introduced to compensate for the NDI for the shortage of the system with uncertainty and external disturbance\cite{4}, combining with the terminal sliding mode control(TSMC) which could make the state convergence in finite time\cite{5}, a controller of nonlinear system was designed. The implement of the attitude control system for the upper stage the demonstrated the effectivity and quick convergence of the proposed method.

The outline of this paper is organized as follows. In the upcoming section, the method proposed will presented detailly. Next, the proposed method is applied in the control problem of the attitude system for the upper stage. Finally, conclusions are drawn in the last section.
2. Method of designing the controller

2.1. State feedback linearization

Considering the following multi-output nonlinear system (1)
\[
\begin{align*}
\dot{x} &= f(x) + g(x)u \\
y &= h(x,u)
\end{align*}
\]  

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^m$ represent the state, the input and the output of the system respectively, and $f = [f_1, f_2, \ldots, f_n]^T \in \mathbb{R}^n$, $g = [g_1, g_2, \ldots, g_m]^T \in \mathbb{R}^m$, $h = [h_1, h_2, \ldots, h_m]^T \in \mathbb{R}^m$ are smooth functions. To simplify the form, the Lie derivatives of $f, g, h$ can be rewrite as
\[
L_f h = \frac{\partial h}{\partial x^T} f \\
L_f^j h = L_f L_f^{j-1} h \\
L_g^i h = \frac{\partial L_f^i h}{\partial x^T} g_i
\]

**Remark 1:** For the multi-output nonlinear system (1), if the following two conditions are satisfied, the relative degree of the system will be the vector $\{r_1, \ldots, r_m\}$ around $x_0$.

I. For $1 \leq i \leq m$, $k < r_i - 1$, and $1 \leq i \leq m$, all state $x$ in the neighborhood of $x_0$ could produce
\[
L_{g_i} L_f^j h_i = 0
\]

And the sum of the relative degrees is $n$, namely $r_1 + r_2 + \ldots + r_m = n$.

II. The $m \times m$ order matrix $G(x)$ is nonsingular when $x = x^0$
\[
G(x) = \begin{bmatrix}
L_{g_1} L_f^{r_1-1} h_1 & \cdots & L_{g_1} L_f^{r_1-1} h_1 \\
\vdots & \ddots & \vdots \\
L_{g_m} L_f^{r_m-1} h_m & \cdots & L_{g_m} L_f^{r_m-1} h_m
\end{bmatrix}
\]

Then the system could be linearized exactly.

It can be seen from the Remark 1 that if the multi-input multi-output nonlinear system (1) met the linearization conditions, the $i$th output would be rewritten as Eq. (5)
\[
\begin{align*}
\dot{y}_i &= L_f h_i \\
\dot{y}_i &= L_f^2 h_i \\
\vdots \\
\dot{y}_i^{(r_i)} &= L_f^{r_i} h_i + L_g L_f^{r_i-1} h_i u
\end{align*}
\]

Take all outputs of the system into consideration, all the highest derivate of the output can be written as
\[
\begin{bmatrix}
\dot{y}_1^{(r_1)} \\
\vdots \\
\dot{y}_m^{(r_m)}
\end{bmatrix} = F(x) + G(x) u
\]

where $F(x) = [L_f^1 h_1, \ldots, L_f^r h_m]^T$.

According to the definition of the relative order, the coefficient matrix $G(x)$ is nonsingular, and then the control law can be designed based on NDI as Eq. (7).
\[
u = G^{-1}(x)[y - F(x)]
\]

Thus, the linear system (8) will be available by combining (6) and (7).
where, \( v_1, v_2, \ldots v_m \) are the designed linear control law which will be studied in section 2.3.

It can be known from the above deduce that the all-order deriva tes of the outputs should be available accurately when the NDI is applied to achieve the accurate linearization of a nonlinear system, and if there is a nonnegligible uncertainty of the system or an external disturbance of the input, the accuracy of each derivate model will be declining with the increase of the order of the derivate, which eventually may lead to the accuracy of the dynamic inverse model falling sharply. Thus, outputs of the system based on NDI would be tracked badly, and then the system would become instable. It is very important to study the method to improve the accuracy of dynamic inverse system when NDI is adopted in nonlinear system control. The extended state observer (ESO) will be introduced in Section 2.2 to estimate the uncertainty of the system and the external disturbance of the input, which will improve the stability of the NDI substantially.

2.2. the Extended State Observer

The ESO is an important part of the active disturbance rejection controller (ADRC) proposed by professor Han in the 1980s [7]. Its main idea is regarding the uncertainties of system model parameter, the unknown and coupling items and other unknown factors as the total disturbance of the system, which is extended to a new state of the system, and then the state of the extended system can be estimated with the help of the state observer, and compensating the original system for the total disturbance in its input finally.

Consider the following continuous integral system (9)

\[
\begin{bmatrix}
1_{11}^{(t)} \\
\vdots \\
1_{m}^{(t)}
\end{bmatrix} = \begin{bmatrix}
y_1 \\
\vdots \\
y_m
\end{bmatrix}
\] (8)

where, \( v_1, v_2, \ldots v_m \) are the designed linear control law which will be studied in section 2.3.

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Consider the following continuous integral system (9)

\[
\begin{bmatrix}
x_1 \\
\vdots \\
x_n \\
\hat{x}_1 = x_2 \\
\vdots \\
\hat{x}_n = f_0 + g_0 u + d_L
\end{bmatrix}
\] (9)

where \( x = [x_1, \ldots, x_n]^T \) is the system state, \( f_0 \) and \( g_0 \) are the nonlinear known functions of the system, and \( d_L \) is the total disturbance which includes the uncertainties of system model parameter, the unknown and coupling items and other unknown factors, which satisfies the conditions of Remark 2.

**Remark 2**: The total disturbance \( d_L \) or its derivatives is bounded.

The total disturbance \( d_L \) of the system is regarded as the new state of the system, namely \( x_{n+1} \), and then the state observer is designed as Eq. (10).

\[
\begin{bmatrix}
e_1 = z - x_1 \\
\hat{e}_1 = z_2 - \beta_1 e_1 \\
\vdots \\
\hat{e}_n = f_0 + g_0 u + z_{n+1} - \beta_n e_1 \\
\hat{e}_{n+1} = -\beta_{n+1} e_1
\end{bmatrix}
\] (10)

where \( z = [z_1, \ldots, z_{n+1}]^T \) is the estimation of the system corresponding to the extended state \( \hat{x} = [x_1, \ldots, x_n, x_{n+1}]^T \), and \( \beta_1, \ldots, \beta_{n+1} \) are the coefficient of the observation that need to be designed.

To demonstrate the stability of ESO and the accuracy of the estimated total disturbance, let \( e = [e_1, \ldots, e_n]^T \) to be the difference of the estimation of the extended state, then Eq. (11) can be deduced by combining (9) and (10).

\[
\dot{e} = A_e e - B_e \dot{d}_L
\] (11)

where
If the matrix $A_e$ is Hurwitz, and at least one of the following two conditions is true, $e$ will converge to zero.

1. For all time $t$, there will be a constant $M_1$ to satisfy $\|d_1\| \leq M_1$.
2. For all time $t$, there will be a constant $M_2$ to satisfy $\|\dot{d}_1\| \leq M_2$.

Assume that $A_e$ is Hurwitz, then it can ensure the difference $e$ bounded and especially when the derivate of $d_1$ is zero, Eq. (11) will be asymptotically stable, which can result in the difference converging to zero. Therefore, it is very significant to set the reasonable coefficient to make sure $A_e$ be a Hurwitz matrix.

To ensure that $A_e$ is Hurwitz, the coefficient of ESO can be set as Eq. (12)

$$
\beta_k = C_{n+1}^k \omega_0^k, \quad k = 1, \ldots, n + 1
$$

The $\omega_0 > 0$ can be regarded as the bandwidth of the ESO which can simplify the design, due to there is only one coefficient need to be set. Furthermore, all the roots of the characteristic polynomial (13) of $A_e$ locate in the left half plane, ensuring the stability of ESO and the accuracy of the estimated total disturbance.

$$
p(s) = s^{n+1} + \beta_1 s^n + \cdots + \beta_{n+1} = (s + \omega_0)^{n+1}
$$

2.3. Design of terminal sliding mode controller

Considering the single-input and single-output linear system in the $i$th channel of system (8), let $x_{i1} = y_i$, $x_{i2} = \dot{y}_i$, $\cdots$, $x_{in} = y_i^{(n-1)}$, and to make the definition have universality, regardless of the serial number in the meantime, then the linear system will be written as Eq. (14), in which $r$ is the relative degree and $v$ is the designed control law by TSMC.

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\vdots \\
\dot{x}_r &= v
\end{align*}
$$

The state $X = [x_1, x_2, \ldots, x_r]^T = [x_1, \dot{x}_1, \ldots, x_1^{(r-1)}]^T$ is expected to track the desired state in a finite time by designing the control law.

Define the error vector to be the difference between the state and the desired one.

$$
E = X - X_d = [e, \dot{e}, \cdots, e^{(r-1)}]^T
$$

The sliding mode function can be designed as Eq. (16)

$$
\sigma(X, t) = CE - CP(t)
$$

where $C = [c_1, c_2, \ldots, c_r]$ is the coefficient that makes the polynomial (17) Hurwitz stable, which could ensure the sliding mode to converge to zero.

$$
s' + c_1 s^{-1} + \cdots + c_r s + c_r = 0
$$

Thus, $C$ can be set by the same method that the coefficient of the ESO is designed, just like Eq. (18), in which $\Omega_e$ is the bandwidth of controller.
In addition, $P(t) = [p(t), \ldots, p^{(r-1)}(t)]^T$ will be designed according to the following **Remark 3** to make sure that the difference converge to zero in the finite time when the sliding mode is zero.

**Remark 3:** when $p(t): \mathbb{R}^r \to \mathbb{R}^r$, and $p(t)$ is the differentiable continuous functions of the order $r$ and all the derivatives of $p(t)$ belong to space of $L^\infty$. For a desired finite time $T > 0$, $p(t)$ is bounded over the time $[0, T]$, and it also satisfies the conditions that $p(0) = e(0)$, $\dot{p}(0) = \dot{e}(0)$, $\ldots$, $p^{(r-1)}(0) = e^{(r-1)}$ where $e(0), \dot{e}(0), \ldots, e^{(r-1)}$ are the value of each element of $E$ at initial time. One of the methods is provided by the reference [5] to set $p(t)$, shown in Eq. (19)

\[
p(t) = \left\{ \sum_{k=0}^{r-1} \frac{1}{k!} e(0)^k t^k + \sum_{j=0}^{r-1} \sum_{i=0}^{r-1} \frac{a_{ij}}{T^j} e(0)^j \dot{e}(0)^i \right\} t^{i+j} 0 \leq t \leq T
\]

\[
t > T
\]

It can be known from **Remark 3** that the sliding mode is zero at initial time which can avoid sliding mode too large at initial time. The parameters $a_{ij}$ can be obtained by **Remark 3** at the time $T$, for example when the relative degree $r = 2$, $p(t)$ can be set as Eq. (20)

\[
p(t) = e(0) + \dot{e}(0) t + \frac{1}{2} \ddot{e}(0) t^2 + \left[\frac{a_{00}}{T} e(0) + \frac{a_{01}}{T} \dot{e}(0) + \frac{a_{02}}{T^2} \ddot{e}(0)\right] t^3
\]

\[
+ \left[\frac{a_{10}}{T^2} e(0) + \frac{a_{11}}{T^3} \dot{e}(0) + \frac{a_{12}}{T^4} \ddot{e}(0)\right] t^4 + \left[\frac{a_{20}}{T^3} e(0) + \frac{a_{21}}{T^4} \dot{e}(0) + \frac{a_{22}}{T^5} \ddot{e}(0)\right] t^5
\]

And the derivative and the second derivative can are Eq. (21) and Eq. (22) respectively.

\[
\dot{p}(t) = \dot{e}(0) + \ddot{e}(0) t + 3 \left[\frac{a_{00}}{T} e(0) + \frac{a_{01}}{T} \dot{e}(0) + \frac{a_{02}}{T^2} \ddot{e}(0)\right] t^2
\]

\[
+ 4 \left[\frac{a_{10}}{T^2} e(0) + \frac{a_{11}}{T^3} \dot{e}(0) + \frac{a_{12}}{T^4} \ddot{e}(0)\right] t^3 + 5 \left[\frac{a_{20}}{T^3} e(0) + \frac{a_{21}}{T^4} \dot{e}(0) + \frac{a_{22}}{T^5} \ddot{e}(0)\right] t^4
\]

\[
\ddot{p}(t) = \dddot{e}(0) + 6 \left[\frac{a_{00}}{T} e(0) + \frac{a_{01}}{T} \dot{e}(0) + \frac{a_{02}}{T^2} \ddot{e}(0)\right] t
\]

\[
+ 12 \left[\frac{a_{10}}{T^2} e(0) + \frac{a_{11}}{T^3} \dot{e}(0) + \frac{a_{12}}{T^4} \ddot{e}(0)\right] t^2 + 20 \left[\frac{a_{20}}{T^3} e(0) + \frac{a_{21}}{T^4} \dot{e}(0) + \frac{a_{22}}{T^5} \ddot{e}(0)\right] t^3
\]

Let $p(t) = 0$ at the time $T$, then it will be Eq. (23)

\[
p(T) = (1 + a_{00} + a_{01} + a_{02}) e(0) + T \left(1 + a_{01} + a_{11} + a_{21}\right) \dot{e}(0) + T^2 \left(\frac{1}{2} + a_{02} + a_{12} + a_{22}\right) \dddot{e}(0) = 0
\]

The equation set (24) can be deduced from (23).

\[
\begin{cases}
1 + a_{00} + a_{01} + a_{02} = 0 \\
1 + a_{01} + a_{11} + a_{21} = 0 \\
0.5 + a_{02} + a_{12} + a_{22} = 0
\end{cases}
\]

(24)

By the same way, Eq. (25) and (26) can be received when $\dot{p}(T) = 0$, $\ddot{p}(T) = 0$
Finally, the parameters (27) can be obtained by combining Eq. (24), (25) and (26).

$$\begin{align*}
6a_{00} + 12a_{01} + 20a_{20} &= 0 \\
6a_{01} + 12a_{11} + 20a_{21} &= 0 \\
1 + 6a_{02} + 12a_{12} + 20a_{22} &= 0
\end{align*}$$

Finally, the parameters (27) can be obtained by combining Eq. (24), (25) and (26).

$$\begin{align*}
a_{00} &= -10 & a_{01} &= -6 & a_{02} &= -1.5 \\
a_{10} &= 15 & a_{11} &= 8 & a_{12} &= 1.5 \\
a_{20} &= -6 & a_{21} &= -3 & a_{22} &= -0.5
\end{align*}$$

3. The simulation example

3.1. modeling

In this section, the proposed method is applied to the attitude control problem of the upper of the vehicle, in which the three attitudes of the system are coupled, and the unknown factors such as the density of the atmosphere leads to the system uncertainty as well as the volatility of the upper itself. And the actuator is assembled according to reference [8], in which the rolling angle will be controlled by a pair of rotating wheels and the other two angles will be changed through the swing of the nozzle of the engine.

The dynamic model is referred to reference [9] and to facilitate the subsequent presentation, the following matrixes and vectors are defined.

$$\begin{align*}
F_w &= \begin{bmatrix}
\cos \gamma / \cos \psi & \sin \gamma / \cos \psi & 0 \\
-\sin \gamma & \cos \gamma & 0 \\
\cos \gamma \tan \psi & \sin \gamma \tan \psi & 1
\end{bmatrix} \\
I &= \begin{bmatrix}
I_{z1} & 0 \\
0 & I_{y1} \\
0 & 0 & I_{x1}
\end{bmatrix} \\
F_a &= \begin{bmatrix}
-(I_{y1} - I_{z1}) \omega_z \omega_y \\
-(I_{x1} - I_{z1}) \omega_y \omega_z \\
-(I_{z1} - I_{x1}) \omega_y \omega_z
\end{bmatrix} \\
\Theta &= F_a \omega \\
I \dot{\omega} &= F_w + T + d_w \\
y &= \Theta
\end{align*}$$

where $\Theta = [\phi, \psi, \gamma]^T$ is the vector formed by the pitching, yawing and rolling angle,
\( \omega = [\omega_x, \omega_y, \omega_z]^T \) is the rotation vector around the center of mass, \( I \) is the rotational inertia of the upper, \( T = [T_{x1}, T_{y1}, T_{z1}]^T \) is the torque of actuator in the three channels, and \( d = [d_{x1}, d_{y1}, d_{z1}]^T \) is the disturbance of torque in the three channels.

Due to the complexity of thrust vector nozzle swinging motion process, and the nozzle swing process modeling is difficult to measure or estimate, and the influence of the flow field to the thrust also is unknown, the thrust will be regarded as a constant which can’t be influenced and the swing motion of the nozzle will be an ideal one.

The components of the thrust \( P \) in coordinate system of the upper \( o_1-x_1y_1z_1 \) are presented in the Fig. 2, in which \( P \) is norm of the thrust, \( \delta_x, \delta_y \) are the components of the swing angle in \( o_1-x_1y_1z_1 \), and the angle shown in the Fig. 2 is positive \([10]\).

\[
\begin{align*}
P_{x1} &= P \cos \delta_y \cos \delta_x \\
P_{y1} &= -P \sin \delta_y \\
P_{z1} &= -P \cos \delta_y \sin \delta_x
\end{align*}
\tag{32}
\]

Let \( l \) to be the distance between the outlet of the nozzle and the center of mass, then the torque will be received in Eq. (33)

\[
\begin{align*}
T_{x1} &= -Pl \cos \delta_y \sin \delta_x \\
T_{y1} &= Pl \sin \delta_y
\end{align*}
\tag{33}
\]

3.2. The simulation parameters

The process of simulation is based on MATLAB/SIMULINK, and the runge-kutta method is selected and the integral step is 1ms. Initials of angle and angular velocity are \( \Theta = [0, 0, 0]^T \) and \( \omega = [0, 0, 0]^T \) respectively, the other parameters of the system model are as follows: \( I = \text{diag}(1.6, 1.6, 1.6) \text{kgm}^2 \), \( P = 300 \text{N} \), and the maximum of the swing angle of the nozzle is 7.5 deg. The desired state is made up by step and sine signals shown in equation (34), of which the unit is deg.

\[
\begin{align*}
\varphi^* &= \begin{cases} 
10, & 0 < t < 2s \\
0, & 2 \leq t < 4s \\
10 \sin(\pi t / 2), & 4 \leq t \leq 10s 
\end{cases} \\
\psi^* &= \begin{cases} 
12, & 0 < t < 2s \\
0, & 2 \leq t < 4s \\
12 \sin(\pi t / 2), & 4 \leq t \leq 10s 
\end{cases} \\
\gamma^* &= \begin{cases} 
5, & 0 < t < 2s \\
0, & 2 \leq t < 4s \\
5 \sin(\pi t / 2), & 4 \leq t \leq 10s 
\end{cases}
\tag{34}
\]

3.3. Controller parameters

According to the above process the controller is designed, the bandwidth of the observer and controller is selected as \( \omega_o = \pi \text{ diag}(6.0, 6.0, 6.0) \) and \( \omega_c = \pi \text{ diag}(2.6, 2.6, 2.6) \) respectively when the order of system is second. The limited time is set \( T = 0.8s \), then the function \( p(t) \) shown in Eq. (35) can be calculated on the base of Eq. (27)

\[
\begin{align*}
p(t) &= e_0 - \frac{10}{T^3} e_0 t^3 + \frac{15}{T^4} e_0 t^4 - \frac{6}{T^5} e_0 t^5 \\
\dot{p}(t) &= \frac{30}{T^2} e_0 t^2 + 60 \frac{e_0}{T^3} t^3 - \frac{30}{T^4} e_0 t^4 \\
\ddot{p}(t) &= 60 \frac{e_0}{T^3} t^2 + 180 \frac{e_0}{T^4} t^3 - 120 \frac{e_0}{T^5} t^4
\end{align*}
\tag{35}
\]
At the same time, to demonstrate the advantage of the proposed method, and the multivariable decoupling control based on ADRC method (ADRC-DCP) and the conventional sliding mode controller based on NDI (NDI-SMC) [11] are introduced to compare.

3.4. Results of Simulation.

(a) comparison for the curve of the roll angle
(b) comparison for the curve of the yaw angle
(c) comparison for the curve of the pitch angle
(d) comparison for the curve of $T_{x1}$
(e) comparison for the curve of $T_{y1}$
(f) Comparison for the curve of $T_{z1}$

Fig.3 Comparison for the result of the three different controllers

The results of simulation are shown in the Fig.3. As can be seen from the Fig. 3 (a)–(c), in the case of without external disturbance, all the three controllers can keep tracking the sine signal well with the
compensation of ESO, which indicates the three controllers are valid to deal with control problem of nonlinear system. But when the period 0 ~ 2s is taken into account in Fig. 3 (a)–(c), it can be known that the pitch angle and yaw angle can reach the step signal in 0.8s when NDI-TSMC is adopted, while it takes more than 1s to arrive at the step for the other two methods and there even exists an overshoot in the ADRC-DCP, which proves that the system will converge to the desired state faster when NDI-TSMC is applied than other two.

The control torques of the three methods are presented in Fig.3 (d) ~ (f), it is obvious that the control torques in the three channels of ADRC-DCP change more severely than the other two because of the nonlinear combination of the difference between the actual and the desired state; and when comparing the NDI-TSMC with NDI-SMC, although the change of control torques are similar to each other, the large value in the simulation at initial time of NDI-SMC can’t be ignored, which is difficult to achieve for the actual system, and as a result of NDI-TSMC which take the initial difference and its derivative into consideration, it can avoid the disadvantages.

4. Conclusion
The terminal sliding mode controller based on the nonlinear dynamic inversion is designed and proposed in this paper to address the nonlinear system with external disturbance and the uncertainty. The extended state observer was applied to compensate the total interference when the nonlinear dynamic inversion requests the high accuracy. The terminal sliding mode controller was adopted to assure the system converging fast and accurately. The effectiveness and quickly converging of the proposal method was demonstrated by the simulation of the compare with other method in the attitude control problem of the upper stage.

References
[1] Fang C Y, Lu G X. Nonlinear System Theory[M]. Beijing: Tsinghua university press, 2009.
[2] I Hameduddin, Bajodah A H. Generalized Dynamic Inversion for Multiaxial Nonlinear Flight Control[C]. American Control Conference, 2011. 250–255.
[3] Wang Q, Stengel R F. Robust Nonlinear Control of a Hypersonic Aircraft[J]. Journal of Guidance, Control, and Dynamics, 2000, 23(4): 577–585.
[4] Han J Q. From PID to Active Disturbance Rejection Control[J]. IEEE Transactions on Industrial Electronics, 2009, 56(3): 900–906.
[5] Liu J K. MATALB Simulation of Sliding Mode Control[M]. Beijing: Tsinghua university press, 2005.
[6] Hassan K K. Nonlinear System, Third Edition[M]. Beijing: Electronic industry press, 2012.
[7] Han J Q. Active Disturbance Rejection Control——Compensation for Estimated the Uncertainty[M]. Beijing: National defense industry press, 2009
[8] Kehl F, Mehta A, Pister K. An Attitude Controller for Small Scale Rockets[J], 2013.
[9] Jia P R, Chen K J and He L. Ballistics of Long-Range Rocket[M]. Changsha: University of defense science and technology press, 1993.
[10] Dong C Y, Jing S G and Wang Q. Neural Network Control for Air-to-Air Missiles with Thrust Vectoring[J]. Journal of System Simulation, 2001, 13(5): 585–588.
[11] Xia Y, Zhu Z, Fu M. Back-Stepping Sliding Mode Control for Missile Systems Based on an Extended State Observer[J]. IET Control Theory & Applications, 2011, 5(1): 93–102.