Research on Influence of Microburst on Ballistic Characteristics of Rockets

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Abstract. In order to study the influence of microburst on the ballistic characteristics of rockets, an engineering model was established by using the vortex ring method based on the basic principle of hydrodynamics and the theory of the exterior ballistic. The wind field model is combined with the six-degree-of-freedom rigid-body ballistic model of rockets. This study examined a kind of empennage-rocket whose flight process was influenced by a microburst. The simulation results show that the microburst model based on the vortex method is simple and the three-dimensional characteristics are good, which can describe the main wind field characteristics of the microburst in a certain extent. Meanwhile, the microburst affects the ballistic characteristics of rockets such as the trajectory, velocity and angular motion. The extent of the effect is concerned with the vertical wind velocity along the central axis and the vortex ring radius of the microburst model.

1. Introduction

Wind shear is the vector difference between the two points of the wind of space. It is known as low-level wind shear when occurred at 600m below. Low-level wind shear is an atmospheric phenomenon associated with strong convective weather, frontal weather, geography and environmental factors [1]. A large number of researches indicate that the microburst is one of the most dangerous and most probable type of wind shear. It is a small, strong, downward impulse that rushes to the ground to produce a horizontal airflow in all directions [2]. When the airplane and other aircrafts such as rockets moving through the microburst wind shear region, the rapid changes in wind speed will change the state of the aircraft to affect its flight stability and trajectory. In the initial stage of the launch, especially when the rocket is out of the deflector, the speed of the rocket is low and the anti-disturbance capability is weak. The change of the wind speed can cause a large distance and direction deviation on the trajectory of the rocket [3]. In the rocket range test and the actual battles, the complex and varied surrounding climate and the geographical environment are very favourable for the microburst wind shear to form. Therefore, how to consider and reflect the influence of the microburst on the rocket flight and the ballistic characteristics is extremely important to the launching and control of rockets.

The effect of the change in wind field on the process of the rocket flight has always been an important issue in the ballistic research. A simplified wind field model for the trajectory calculation was established according to the statistical characteristics of the wind field, and the wind shear was described by a linear function [4-5]. The model of the atmospheric wind field at 100 km below was developed, and the analysis for the movement of a long-range rocket under the influence of high-altitude winds was studied[6]. A numerical simulation was conducted to analyze the impact of different wind fields on the trajectory of the projectile based on the external ballistics and the meteorological theory [7]. Simplified models of two kinds of low-level wind shear were established...
and their effect on the fly-by-ground flight of the aircraft is studied [8]. The main structure and the evolution process of a microburst formed in Beijing area in 1997 was simulated by a non-hydrostatic and elastic mesoscale model [9]. A model of vortex ring induced microburst is developed [10]. On the basis of previous studies [10], a multi-vortex ring model and an inclined vortex ring model for microburst were designed and its influence on the take-off and the landing process of the aircraft was explored [11]. The multiple and inclined vortex ring model of the microburst was combined with the atmospheric turbulence velocity field [12]. It can be seen from the above research that the studies on the microburst wind shear is mainly focuses on the modeling method and the influence on the navigation or the flight of the airplane, but little is known about the research on the influence of the microburst on the flight and the ballistic characteristics of the projectile. This paper gives the engineering model of the microburst wind shear according to the flow features combined with the rocket's six-degree-of-freedom (6-DOF) trajectory model. We analyzed the ballistic characteristics of the rocket under the influence of the microburst, meanwhile, the influence of the characteristic parameters of the microburst on the ballistic characteristics of the rocket is studied. The results would provide some reference for the research of the ballistic trajectory and the launch control of the rocket.

2. Microburst model based on vortex ring method

Microburst is one of the most common convective storm wind shear, which behave as a localized subsided airflow. When the airflow reaches the ground, it collides with the ground and spreads to the surrounding. The formation process of a microburst is shown in Figure 1. There are several engineering modeling methods of a microburst, such as the dipole-based panel method, and the vortex ring method. The former method requires a large number of mathematical integral operations and the solving process are complex. This paper used the vortex ring method for the microburst wind field modeling.

![Figure 1. Forming process of microburst wind field.](image)

2.1. Modeling process

Since the velocity field induced by the vortex ring is very similar to the wind field of the microburst, the wind field model of the microburst can be constructed by using the vortex ring principle in fluid mechanics. The vortex ring model in the ground coordinates system (O-xyz) is shown in Figure 2, in which the center of the main vortex ring is \( O_x \) and the radius of the main vortex ring is \( R \).

![Figure 2. Schematic diagram of vortex ring model.](image)
The main vortex curvilinear equation can be written as
\[
(x_c - x_p)^2 + (y_c - y_p)^2 = R
\]
where \((x_c, y_c, z_c)\) are the coordinates of the points on the curve and \((x_p, y_p, z_p)\) is coordinate of \(O_p\).

The main vortex circulation equation is
\[
\psi_p = \frac{\Gamma}{2\pi} (r_{\text{max}} + r_{\text{min}})F(k)
\]
where \(r_{\text{max}}\) and \(r_{\text{min}}\) are the maximum distance and the minimum distance from any point in space \(O_\mu(x_M, y_M, z_M)\) to the vortex ring curve, \(F(k)\) is the elliptic integral function, and \(\Gamma\) is the vortex strength, which is determined by the vertical velocity in the vortex center \(V_{wz}(0)\) and the vortex ring radius \(R\).

\[
\Gamma = 2RV_{wz}(0)
\]
The variable is given by
\[
k = \frac{r_{\text{max}} - r_{\text{min}}}{r_{\text{max}} + r_{\text{min}}}
\]
If \(0 \leq k \leq 1\), \(F(k)\) can be approximated as
\[
F(k) \approx \frac{0.788k^2}{0.25 + 0.75\sqrt{1-k^2}}
\]
Based on Eq. (2), the radial (parallel to the horizontal plane) induced velocity and the axial (parallel to axis \(oz\)) induced velocity can be obtained by
\[
\begin{align*}
\nu_{r}^p &= \frac{1}{r_p} \frac{\partial \psi_p}{\partial z_R} \\
\nu_{z}^p &= -\frac{1}{r_p} \frac{\partial \psi_p}{\partial r_p}
\end{align*}
\]
where \(r_p\) is distance from \(O_\mu\) to the central axis of the vortex ring \((r_p = \sqrt{(x_M - x_p)^2 + (y_M - y_p)^2})\). The velocity components along axis \(ox\) and axis \(oz\) calculated from Eq. (6) are
\[
\begin{align*}
\nu_{x}^p &= \frac{x_M - x_p}{r_p} \nu_r^p \\
\nu_{z}^p &= \frac{y_M - y_p}{r_p} \nu_r^p
\end{align*}
\]
In order to make the vertical component of the wind vector on the ground zero, a mirror vortex ring should be configured at the symmetry point \(O_I(x_p, y_p, -z_p)\) of the center of the main vortex ring with respect to the horizontal plane. The mirror vortex circulation equation is \(\psi_{I} = -\psi_p\). So the wind vectors induced by the main vortex and the mirror vortex are equal in magnitude and symmetric with respect to the ground. Following the same steps with Eq. (6) and Eq. (7), the induced velocity \(\nu_{x}^I\), \(\nu_{y}^I\) and \(\nu_{z}^I\) of the mirror vortex ring at the point \(O_\mu\) can be obtained, and the resultant velocity of the two vortex rings at the point are expressed as
Thus, the streamline equation of point \( M \) is given by

\[
\psi = \psi_p + \psi_f = \frac{1}{2\pi} \left[ \frac{0.788 k^2 (r_{\text{max}}^2 + r_{\text{min}}^2)}{0.25 + 0.75 \sqrt{1 - k^2}} - \frac{0.788 k^2 (r_{\text{max}}^2 + r_{\text{min}}^2)}{0.25 + 0.75 \sqrt{1 - k^2}} \right]
\]

where \( \psi_f \) is the streamline of the mirror vortex ring, \( r_{\text{max}} \) and \( r_{\text{min}} \) are the maximum distance and the minimum distance from point \( O_M \) to the mirror vortex ring curve.

There are two special cases of the above-mentioned vortex ring model:

- The induced velocity along the central axis of the vortex ring (\( r_p = 0 \)) calculated from Eq.(6) is close to infinity. The potential function of the vortex ring can be introduced here to complete the derivation of the induced velocity along the central axis.

\[
\begin{align*}
V_{wx} &= V_{wy} = 0 \\
V_{wz} &= \frac{\Gamma}{2R} \frac{1}{(1 + (z_M - z_p)^2)^{1.5}}
\end{align*}
\]

- The induced velocity at any points along the vortex filament (\( r_p = R, z_M = z_p \)) calculated from Eq.(6) is also close to infinity. Thus, a method based on Rankine compound vortex is used here [10]. It takes the vortex core as an annular cylinder with a radius of \( r \) as shown in Figure 3. The vorticity inside the vortex core is evenly distributed. The velocity along the vortex filament is kept at 0, while the flow outside the vortex core is still subject to the streamline equation \( \psi_f \). From the vortex core to the vortex radius, the induced velocity is linearly distributed.

According to the above-mentioned derivation, the wind speed vector at any point in the ground coordinate system space can be calculated.
2.2. Numerical simulation verification

In order to verify the rationality and effectiveness of the microburst model based on the vortex ring method, the numerical simulation method is used here to obtain the wind field model. The simulation results are compared with the measured wind field distribution from the relevant data. According to the statistical data of the intensity and the spatial scale of the microburst wind field, it usually occurs below 600m and the horizontal outflow range is within 4km. For the moderate intensity microburst, the vertical velocity of the central axis is about 10m/s. According to the above information, the basic parameters of the vortex ring model in the ground coordinate system are shown in Table 1. Based on these parameters, the wind vector diagram of vertical section of vortex ring is shown in Figure 4(a) and the wind vector diagram of horizontal section(y=100m) is shown in Figure 4(b).

| Parameters                                      | Value  |
|------------------------------------------------|--------|
| Coordinate of center of main vortex ring (m)   | 23.56  |
| Radius of vortex ring (m)                      | 34.64  |
| Radius of vortex core (m)                      | 23.76  |
| Vertical flow velocity of the vortex center (m/s) | 27.9   |

Figure 4(a) shows that the vertical velocity near the central axis of vortex ring is higher, and along the horizontal direction, the farther away from the central axis of the vortex, the smaller the wind speed is. Figure 4(b) shows that the maximum wind velocity appears at the center of the vortex ring in the horizontal section, and the wind velocity is distributed around the center of the vortex ring and gradually decreases.

![Figure 4(a). Wind vector diagram of vertical section](image1)

![Figure 4(b). Wind vector diagram of horizontal section(y=100m)](image2)
Figure 5(a). JAWS 1982 wind-shear data showing a downburst flow pattern

Figure 5(b). Wind field of 1988 microburst event of DEN by NASA

From the comparison of the simulation results in Figure 4(a), Figure 4(b) and the measured wind field data in Figure 5(a), Figure 5(b), it can be seen that the wind field model based on the vortex ring method is very similar to the actual microburst wind distribution, and it also has a good three-dimensional characteristic.

3. 6-DOF rigid-body trajectory model for rockets

Rocket is regarded as a free rigid body. The movement of the rocket is composed of centroid movement and the movement encircling the centroid of rockets. Based on the theory of the external ballistics[14], we select a reasonable reference coordinate system for rocket force analysis. When we set up the rocket equation of motion, the rocket is treated as a constant mass object. Meanwhile, we ignore the gas inertia force and its torque, and take no account of the influence of gunpowder combustion caused by the acceleration of mass center of gravity and the rate of change in inertia.

The motion vector equation of rocket mass center is decomposed into the ballistic coordinate system ignoring the minor factors such as the earth rotation, and the equations of the scalar form are obtained by projecting the motion vector equation around the center of mass to the projectile axis coordinate system [14]. The motion of the center of mass of rocket relative to the inertial coordinate system follows the theorem of center of mass motion

\[ m \frac{dv}{dt} = F \]  

(13)

Where \( t \) is the flight time, \( m \) is the mass of rocket, \( v \) is the velocity vector on the mass center of rocket. By projecting Eq.(13) onto the ballistic coordinate system, the components of the net force \( F \) on the rocket in the ballistic coordinate system are \( F_{x_1}, F_{y_2} \) and \( F_{z_2} \) are obtained. Thus, the scalar equation of mass center of the rocket in the ballistic coordinate system is given by

\[
\begin{align*}
\frac{dv}{dt} &= \frac{1}{m} F_{x_1} \\
\frac{d\theta_1}{dr} &= \frac{1}{mv \cos \psi_2} F_{y_2} \\
\frac{d\psi_2}{dr} &= \frac{1}{mv} F_{z_2}
\end{align*}
\]

(14)

Where \( \theta_1 \) is the velocity pitch angle and \( \psi_2 \) is the velocity yaw angle. Eq. (14) determines the relationship between the center-of-mass velocity of the rocket and the direction change and the force acting on the rocket, and the equation that describing the change of the coordinates of the mass center of the rocket is as follows
\[ \frac{\text{d}x}{\text{d}t} = v \cos \psi_2 \cos \theta_1 \]
\[ \frac{\text{d}y}{\text{d}t} = v \cos \psi_2 \sin \theta_1 \]
\[ \frac{\text{d}z}{\text{d}t} = v \sin \psi_2 \]

(15)

Where \( x \), \( y \) and \( z \) are the coordinates of the mass center of the rocket in the ground coordinate system, and the motion of the rocket around the mass center depends on the theorem of moment of momentum.

\[ \frac{\text{d}G}{\text{d}t} = M \]

(16)

Where \( G \) is the moment of momentum acting on the mass center of rocket and \( M \) is the external torque acting on the mass center of rocket. Projecting Eq.(15) to the projectile axis coordinate system and omit some minor factors, the dynamic equation and kinematics equation for the motion around the mass center of rocket are obtained as

\[ \frac{\text{d}\omega_z}{\text{d}t} = \frac{1}{C} M_z \]
\[ \frac{\text{d}\omega_\eta}{\text{d}t} = \frac{1}{A} M_\eta - \frac{C}{A} \omega_z \omega_\eta + \omega_\eta \tan \varphi_2 \]
\[ \frac{\text{d}\omega_\zeta}{\text{d}t} = \frac{1}{A} M_\zeta + \frac{C}{A} \omega_z \omega_\eta - \omega_z \omega_\eta \tan \varphi_2 \]

(17)

\[ \frac{\text{d}\varphi_a}{\text{d}t} = \frac{1}{\cos \varphi_2} \omega_z \]
\[ \frac{\text{d}\varphi_2}{\text{d}t} = -\omega_\eta \]
\[ \frac{\text{d}\gamma}{\text{d}t} = \omega_z - \omega_\eta \tan \varphi_2 \]

(18)

Where \( \omega_z \), \( \omega_\eta \) and \( \omega_\zeta \) are the components of the rocket rotational angular velocity in the projectile axis coordinate system, \( C \) and \( A \) are the polar moment of the inertia and the equator moment of the inertia of the rocket respectively, \( M_z \), \( M_\eta \) and \( M_\zeta \) are the components of the external torque acting on the rocket in the projectile axis coordinate system, \( \varphi_a \) and \( \varphi_2 \) are the pitch angle and the yaw angle of rocket, \( \gamma \) is the angle of the rotation of the rocket.

The 6D rigid-body trajectory differential equations of the rocket can be obtained by associating Eq.(14), Eq.(15) and Eq.(17), Eq.(18). When the structural parameters and aerodynamic parameters of the rocket are given and the firing conditions, weather conditions and other starting conditions are determined, the trajectory differential can be solved by using the ballistic trajectory integral method.

4. Simulation and analysis

4.1. Simulation conditions

As an example, a kind of 122mmempennage-rocket is studied, and the basic parameters and initial launching conditions of projectile are given by table.2 and table.3 respectively. During the boost phase of the rocket projectile, the mass of the rocket is reduced by a uniform rate, and the moment of inertia and the position of the center of mass change with time.
Table 2. Basic parameters of vortex ring.

| Parameters                                    | Value  |
|-----------------------------------------------|--------|
| Diameter of projectile (m)                   | 0.122  |
| Length of projectile (m)                     | 2.9    |
| Specific impulse (s)                         | 250    |
| Engine’s working time (s)                    | 3.0    |
| Initial mass of rocket in boost phase (kg)   | 70.01  |
| Initial equator moment of inertia (kgm²)     | 40.05  |
| Initial polar moment of inertia (kgm²)       | 0.147  |
| Initial position of center of mass (m)       | 1.533  |
| Mass change rate of projectile (kg/s)        | -8.0   |

Table 3. Initial launching condition of rocket.

| Parameters                      | Value   |
|---------------------------------|---------|
| Initial velocity, m/s           | 40      |
| Elevation angle, deg            | 50      |
| Direction angle, deg            | 0       |
| Rotational angular velocity, rad/s | 0       |
| Launching position in           | (0,0,0) |
| ground coordinate system        |         |

4.2. Influences on trajectory and impact point

Figure 6 shows the trajectory of the rocket under the influence of the microburst wind field with a central vertical induced velocity of $V_0 = 10\text{ m/s , } V_0 = 15\text{ m/s and } V_0 = 20\text{ m/s respectively.}$ figure 7 shows the trajectory of the rocket under a vortex ring radius of $R = 800\text{ m , } R = 900\text{ m , } R = 1000\text{ m and } R = 1100\text{ m respectively.}$ In addition, the trajectory under windless condition is also shown in figure 6 and figure 7 for comparison. Table 4 summarizes the data of ballistic performance under different central vertical induced velocity and vortex ring radius.

![Figure 6. Trajectory of rocket under different central vertical wind velocity](image-url)
As shown in Figure 6 and the data in group 1 and group 2 of Table 4, due to the impact of the vertical airflow in the boost phase of the rocket, the lower dynamic pressure caused a corresponding decrease in lift and a static stabilization moment acting on the rocket. The curvature of the trajectory is reduced because of the decrease of the trajectory inclination angle. So the maximum trajectory height, the longitudinal range and transverse range of the projectile are less than that under the non-wind condition. It can be seen from the data in group 2, 5, 7 of Table 4 and figure 6 that the relative velocity between the projectile and the air decreases with the decrease of the aerodynamic force in the pitch direction due to the increase of the vertical induced velocity of the microburst center. Thus, the maximum trajectory height and the projectile's longitudinal range and transverse range are reduced correspondingly. The flight time becomes shorter.

Table 4. Ballistic performance under different model conditions.

| Group number | Vertical wind velocity(m/s) | Vortex ring radius(m) | Flight time(s) | Maximum trajectory height(m) | Impact point velocity(m/s) | Longitudinal range(m) | Transverse range(m) |
|--------------|-----------------------------|-----------------------|----------------|-----------------------------|---------------------------|----------------------|---------------------|
| 1            | 0                           | 1000                  | 105.8          | 13126.3                     | 367.6                     | 34478.8              | -9.3                |
| 2            | 10                          | 1000                  | 99.2           | 11590.8                     | 357.4                     | 33505.2              | -8.5                |
| 3            | 15                          | 800                   | 99.3           | 11605.7                     | 357.5                     | 33514.4              | -8.6                |
| 4            | 15                          | 900                   | 97.3           | 11164.2                     | 354.5                     | 33185.2              | -8.4                |
| 5            | 15                          | 1000                  | 95.9           | 10851.4                     | 352.6                     | 32949.9              | -8.2                |
| 6            | 15                          | 1100                  | 95.1           | 10690.4                     | 351.6                     | 32826.5              | -8.1                |
| 7            | 20                          | 1000                  | 92.5           | 10133.1                     | 348.3                     | 32369.8              | -8.0                |

4.3. Influence on Velocity Characteristics
The most direct effect of the microburst wind field on a rocket is the change of its velocity. Figure 8 shows the curve of the rocket’s velocity during the flight varying with time. Under action of the thrust of the engine, the rocket reaches a speed of nearly 1000m/s in a short time. The wind speed of the microburst wind field is smaller than that of the high speed of the projectile so that the speed change is not obvious when the rocket crosses the area due to the long flight time of the passive free flight, the velocity and acceleration of the projectile are gradually accumulated by the influence of the wind speed, resulting in the decrease of the impact point velocity and longitudinal range compared to that of the windless condition. In addition, the higher the vertical induced wind speed, the smaller the projectile’s impact point velocity. From Figure.8(b), it can be seen that the increase of the vortex ring radius also decreases the impact point velocity of the projectile, but the velocity decrease is not linearly related to the increase of vortex ring radius.

![Figure 8(a). Velocity curve under different vertical wind velocity](image1)

![Figure 8(b). Velocity curve under different vortex ring radius](image2)

Due to the long flight time of the passive free flight, the velocity and acceleration of the projectile are gradually accumulated by the influence of the wind speed, resulting in the decrease of the impact point velocity and longitudinal range compared to that of the windless condition. In addition, the higher the vertical induced wind speed, the smaller the projectile’s impact point velocity. From Figure.8(b), it can be seen that the increase of the vortex ring radius also decreases the impact point velocity of the projectile, but the velocity decrease is not linearly related to the increase of vortex ring radius.

4.4. Influences on angular motion

The Figure 9 shows the curve of the attack angle of the rocket under the microburst with different wind field characteristics parameters.

It can be seen from Figure 9(a) and Figure 9(b) that the lift force acting on the rocket is obviously reduced, as well as the pitch moment compared to the windless condition. The axis of the projectile moves along with direction of the airflow of microburst and the pitch attack angle is changed from positive to negative. The amplitude of attack angle increases with the vertical induced wind speed. However, due to the stabilizing moment of the tail-fin projectile, the rocket gradually returns to steady state, and the attack angle amplitude decreases. Compared to the windless condition, the recovery time of the attack angle of rocket increases. The lateral wind velocity component is small because the central axis of the microburst is in the launching plane of the rocket. Therefore, the microburst wind shear has little influence on the yaw attack angle and its restore time of returning to steady.

Similar conclusions can be obtained from Figure.9(c) and Figure.9(d) that, the influence of the microburst on the pitch attack angle of the projectile is greater than that on the yaw attack angle. The larger the radius of the vortex ring, the greater the impact of the microburst on the attack angle of the rocket.
5. Conclusions

Footnotes should be avoided whenever possible. If required they should be used only for brief notes that do not There are three main conclusions of this research:

- The microburst wind field model based on the vortex ring method is simple and has good three-dimensional characteristics. It can reflect the wind field characteristics of the microburst to a certain extent;
- The microburst has certain influence on the trajectory parameters such as the flight time, the range, the lateral deviation, the maximum trajectory height and the impact point velocity of the rocket. The greater the vertical induced wind velocity or the radius of vortex ring, the higher the degree of the influence of the microburst;
- Under the influence of the microburst, the pitch attack angle of the rocket is reduced to a negative value and the time of the rocket returning to steady state increases. When the central axis of the microburst is in the vertical launching plane of the projectile, the effect of the wind field on the pitch attack angle is greater than that of the yaw attack angle of rockets;

6. References

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