Transient process modeling in micrologistic transport systems

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Abstract. Micrologistic transport systems are ports, transport hubs, railway terminals (railway stations and marshalling yards), and other micro-level transport objects. Such systems are dynamic ones, whose parameters are time-dependent. Consequently, transient processes appear. These processes last for some time and then subside. Then the system goes into a stationary mode of operation. To solve most engineering problems, it is sufficient to find the stationary characteristics of the system. However, some problems require knowledge of the properties of transients. In this paper, we use the previously presented technology for modeling the operation of micrologistic transport systems to study transients in various transport objects. We consider two types of micrologistic transport systems, such as passenger and cargo, and design their models in the form of two-phase and three-phase queuing systems with BMAP flows. For these queuing systems, we compile Kolmogorov systems of ordinary differential equations. Their analytical study is difficult; therefore, to solve such systems, we use numerical methods. Overall, we find the transition probabilities of system states. We compare the obtained probabilities with stationary characteristics, which allows us to draw conclusions about the properties of transients in the considered systems.

1. Introduction
Micrologistic transport systems (micro-LTS) provide the proceeding of the primary operations for the organization of transport flows. Examples of such systems are freight railway stations, ports, and transport hubs [1, 2]. The speed of passenger and cargo transportation, the stability of the transportation process, and the capacity of transport networks directly depend on the efficiency of micro-LTS operation. At the same time, on the one hand, these systems must have sufficient bandwidth and processing capacity; on the other hand, overstating their technical potential entails an unjustified increase in costs. In such conditions, the determination of the required technical and technological characteristics of micro-LTS is relevant. Since natural (full-scale) experiments, in this case, are problematic, the traditional research tools for micro-LTS are mathematical modeling and simulation.

When modeling the operation of micro-LTS, we should keep in mind the following facts. Firstly, we should consider several transport flows that have different parameters, such as the type of transport, the arrival intensity, the distribution of the size of the cargo or the number of passengers, etc. [2, 3]. Secondly, servicing can be multi-level, i.e., several consecutive devices are used [1]. These features can be taken into account by using the mathematical apparatus of queuing theory [4, 5].

Previously, we developed a technology based on the use of multiphase queueing systems (QS) with BMAP flows to simulate the operation of micro-LTS [6]. Such systems allow us to describe in detail
the incoming traffic flow and its route within an object having a hierarchical structure. Due to this, we can achieve acceptable accuracy of models for solving engineering problems, in particular, at the stages of planning and upgrading the system, as well as flexibility, i.e., the resulting models can be adapted to different types of micro-LTS in minimum time.

In previous studies, we considered only stationary states of systems, without transients. However, another important problem in the study of micro-LTS is that they are dynamic systems, whose parameters are time-dependent. First of all, these concerns the intensity of incoming traffic flows. Consequently, transients occur in micro-LTS. Accordingly, some questions require additional study. For example, how long will these processes last before the system reaches a steady (stationary) mode of operation? How much do the characteristics of transient operation differ from stationary ones? Is it possible to ignore the influence of transients on the output characteristics of the system, and can this reduce the accuracy of the forecast? This paper considers these issues.

2. Models and methods

2.1. Subject model of micrologistic transport system operation

Micro-LTS contains a large number of different parameters, such as elements, relationships between them, and their attributes. We will highlight the most significant ones that will be taken into account in the model.

The incoming traffic flow consists of several separate flows characterized by the type of transport arriving in the system (trains, cars, etc.), the direction of its movement, and the time of arrival in the system. The arrived transport unit should be considered as a group of requests, since cargo (passengers) are served independently and occupy a certain place in the system. The sizes of groups of requests and the time of their arrival in the micro-LTS are random variables because they are unknown in advance [3, 6]. Thus, to describe the incoming flow of requests, it is necessary to determine the number of transport sub-flows entering the system, the distribution of the size, and arriving time of groups of requests for each of them.

Micro-LTS structure. We briefly describe the components and parameters that are taken into account in the model. In the micro-LTS structure, we can specify several (usually three) independent subsystems with the same or similar functions. They are arranged sequentially, have feedbacks and individual technical characteristics, such as the capacity and number of servicing devices (channels) in each subsystem, the distribution of servicing time, and acceptable sizes of groups in channels.

2.2. Mathematical model

To describe mathematically the operation of micro-LTS, we apply multiphase QS with BMAP flows. They allow us to describe a set of incoming threads with different characteristics and a single structure. First, we represent a description of BMAP flows, then multiphase QS.

BMAP (Batch Markovian Arrival Process) is set by process control that is a Markov chain \(v_i\) having continuous time and finite state space \(\{0,1,\ldots,N\}\) [8]. The residence time in each state \(v\) is exponentially distributed with parameter \(\lambda_v\), \(v=0,N\). With probability \(p_v^{(k)}\), \(k \geq 0\) the process can go to state \(r, r=0,N\). This generates a group of random size \(k \geq 0\). Probabilities \(p_v^{(k)}\) satisfy the normalization condition \(\sum_{k=0}^{\infty} \sum_{r=0}^{N} p_v^{(k)} = 1, \quad i=0,N\).

The parameters of the Markov chain can be written as matrices \(D_v, i=0,k (k \geq 1)\) of size \((N+1)\times(N+1)\), with the elements \(d_v^{(k)} = \lambda_v p_v^{(k)}, k \geq 1; \quad d_v^{(0)} = -\lambda_v; \quad d_v^{(0)} = \lambda_v p_v^{(0)}, v \neq r\), where \(\lambda_v\) is the intensity of the flow \(v\). Notably, in the case when \(D_v\) \(k \geq 2\) are zero matrices, the flow is not a group one.

Multiphase QS is a complex of sequentially connected queueing systems (phases) [5]. We describe each micro-LTS subsystem as a separate phase. Further, we use only open multiphase QS with an exponential
distribution of service time and finite queues at each phase. If there are no available places for accepting requests from the previous phase in the queues of phases 2 or 3, then the channels of phases 1 or 2, respectively, are temporarily blocked [9]. This procedure means feedback. In terms of queuing theory, the multiphase QS can be written as

\[ A_1 / B_1 / m_1 / n_1 \rightarrow A_2 / B_2 / m_2 / n_2 \rightarrow \ldots \rightarrow A_s / B_s / m_s / n_s. \] (1)

System (1) has \( s \) phases. Positions \( A_i \) and \( B_i \) have letters or letter combinations. \( A_i \) denotes an incoming flow of requests; \( B_i \) means the distribution of service time in channels of phase \( i \). Positions \( m_i \) and \( n_i \) are integers, which are the number of serving channels and the queue length in the corresponding phase. The symbol \( \rightarrow \) denotes the arrival of a request to phase \( u+1 \) when processing is completed in phase \( u \) [4, 5]. If the parameters of the incoming flow are unknown, then we put “*” in the first position.

Thus, to identify the model, we need to determine the number of phases, the parameters of the input BMAP, and for each phase, set the maximum queue length, the number of channels, the size of the served group of requests, and the distribution of servicing time (required), as well as the type of the incoming flow of applications (if possible).

2.3. Kolmogorov system of differential equations

To study transient processes in the model, we compose and solve Kolmogorov system of differential equations (SDE). It is a system of ordinary differential equations, in which the unknown functions are the probabilities of the system states \( p_i(t), i=1,\ldots, K \), where \( K \) is a maximum number of requests in the QS, and time is independent variable [4, 5]. Initial conditions are the state of the system at an initial time.

In the case of QS with BMAP, the pair \((v_i; i)\) serves as the state of the system, where \( v_i \) is the state of the Markov chain, and \( i \) is the number of requests in the system at time \( t \). Let \( p_i^{(v)}(t), v=0,N, i=0,\ldots, K \) denote the probability of the system being in state \((v; i)\) at time \( t \). Let \( \overline{P}_i(t) \) be a vector with components \( p_i^{(0)}(t), p_i^{(1)}(t), \ldots, p_i^{(N)}(t) \). To find the probability of the system state \( i \) without taking into account the state of the Markov chain, it is necessary to sum up all the components of \( \overline{P}_i(t) \), \( p_i(t) = \sum_{v=0}^{N} p_i^{(v)}(t) \).

Now, we construct formulas that allow us to compose the Kolmogorov SDE for three-phase and two-phase QS with BMAP flows. To simplify the formulas, we introduce the following notation: let \( V \) be the maximum size of the incoming request group; \( I \) – a unit matrix of size \((N+1)\times(N+1)\); \( K_s = m_s + n_s \) – a maximum number of requests in phase \( s \), and \( \mu_s \) – request service intensity in the phase \( s \).

For three-phase QS, let \( \overline{P}_{i,j,z}(t) \) be vectors of size \( N+1 \), whose elements are the probabilities of the system being in state \((v; i,j,z)\), \( v=0,N \), at time \( t \), where \( i \), \( j \), and \( z \) are the numbers of requests in phases 1, 2, and 3, respectively; \( \overline{P}_{i,K2+1,z}(t) \) be vectors \( \overline{P}_{i;,(K)(t)} \) for the case when the system has \( i \), \( K2 \), and \( z \) requests in phases 1, 2, and 3, and the phase 1 channels are blocked; \( \overline{P}_{i,j,K3+1}(t) \) be vectors \( \overline{P}_{i;,(K)(t)} \) for the case when the system has \( i \), \( j \) and \( K3 \) requests in phases 1, 2, and 3, and the phase 2 channels are blocked; \( \overline{P}_{i,K2,K3+1}(t) \) be vectors \( \overline{P}_{i;,(K)(t)} \) for the case when the system has \( i \), \( K2 \) and \( K3 \) requests in phases 1, 2, and 3, and the phase 1 channels, as well as the phase 2 ones, are blocked. \( \overline{P}_{i,j}(t) \) is a null vector if at least one of the conditions \( i < 0; i > K1; j < 0; j > K2+1; z < 0; z > K3+1; j = K2+1 \) and \( z = K3+1 \) holds. Kolmogorov SDE for three-phase QS has the form...
$$\bar{P}_{ij}(t) = D_0 + \sum_{k=1}^{N} D_k - \min(i,m_i)\mu_1 - \min(j,m_j)\mu_2 - \min(z,m_z)\mu_3 \bar{P}_{ij}(t) + \sum_{k=1}^{N} D_k \bar{P}_{i-k,j,z}(t) +$$
$$+ \min(i+1,m_i)\mu_i \bar{P}_{i+1,j-z}(t) + \min(j+1,m_j)\mu_j \bar{P}_{i,j+1}(t) + \min(z+1,m_z)\mu_z \bar{P}_{i,j+z}(t), \quad i=0, K1, \quad j=0, K2+1, \quad z=0, K3+1.$$  

For two-phase QS, let $X$ be the size of the served request group in phase 2; let $\bar{P}_{ij}(t)$ be vectors of size $N+1$, whose elements are the probabilities of being the system in state $(i, j)$, $\nu=0, N$, at time $t$, where $i$ and $j$ are the number of requests in phases 1 and 2, respectively; $\bar{P}_{i,K2+1}(t)$ be vectors $\bar{P}_{ij}(t)$ for the case when the system has $i$ and $K2$ requests in phases 1 and 2, and the phase 1 channels are blocked. $\bar{P}_{ij}(t)$ is a null vector if at least one of the conditions $i < 0; i > K1; j < 0; j > K2+1; i = K1$ and $j = K2+1$ holds. Kolmogorov SDE for two-phase QS with group service in phase 2 has the form

$$\bar{P}_{ij}(t) = D_0 + \sum_{k=1}^{N} D_k - \min(i,m_i)\mu_1 - \min(j,m_j)\mu_2 \bar{P}_{ij}(t) + \sum_{k=1}^{N} D_k \bar{P}_{i-k,j}(t) +$$
$$+ \min(i+1,m_i)\mu_i \bar{P}_{i+1,j}(t) + F(i,j), \quad i=0, K1, \quad j=0, K2+1,$$

$F(i,j) = \begin{cases} 
\sum_{k=1}^{X} \min(j+k,m_j)\mu_j \bar{P}_{i,j+k}(t), & i=0, \\
\min(j+X,m_j)\mu_j \bar{P}_{i,j+X}(t), & i > 0.
\end{cases}$  

Formulas (2) and (3) are applicable to QS with an exponential distribution of request service time at each phase. Construction of the Kolmogorov SDE by these formulas should start with $p_{000}(t)$ and $p^{00}_{00}(t)$, respectively.

Based on the transient probabilities, we determine the efficiency indicators, such as the loss probability ($P_{loss}$), absolute bandwidth ($A_{QS}$), the average number of working channels ($\bar{K}$) and queue length ($L$) in phase $s$, the probability of blocking ($P_{block}$), and others (depending on the simulation goals).

3. Numerical experiment

To test the presented approach, we perform calculations for two micro-LTS. The first is a freight railway station and the second is a transport hub.

3.1. Research of cargo micro-LTS

Consider a model freight railway station, whose characteristics correspond to Sukhovskaya station located on the East Siberian railway, Russia. The structure of Sukhovskaya station includes a receiving yard (RY), a sorting bowl (SB), a cargo yard (CY), and a departure yard (DY). The purpose of the study is to determine the traffic capacity of the CY. Therefore, we exclude the DY from consideration, since it does not affect the functioning of the CY.

The characteristics of the model station are as follows. The station receives three trains per day shift and two trains per night shift on average. The average length of each train is 85 cars. All trains arrive only at the RY. It has a low-capacity hump (1 channel) with a processing capacity of approximately 820 cars per day and 5 railway tracks with an average capacity of 87 conventional cars (conv. cars). Cars are delivered from the RY to the SB. The sorting bowl has two locomotives (2 channels) and contains 10 tracks with an average capacity of 73 conv. cars. From the SB, locomotives move the train to the CY, which has ten service devices. Each of them has a queue with a capacity of 30 conv. cars.

If each car is considered as a single request for service, then in terms of queue theory we have the following model of the station $BMAP/M/1/435 \rightarrow */M/2/730 \rightarrow */M/10/300$. For this three-phase QS, the Kolmogorov SDE consists of approximately 100 million equations.
To reduce the dimension of the Kolmogorov SDE, we assume that the arriving train is one service request. Then we obtain the following parameters of the model station. The station receives an average of three daytime and two night shift trains, i.e.$\lambda_{11} = 0.25$ and $\lambda_{12} = 0.17$ trains per hour. The hump processes up to 9.64 train per day or 0.4 train per hour. The RY capacity is five trains. The SB can accommodate nine trains and operates two diesel locomotives (two channels), which move trains to the CY with an average intensity of 0.222 trains per hour. For the CY, there is the following assumption. We consider the three service devices and the tracks in front of them as one channel. This allows us to accept the entire train. The processing capacity of CY is four trains per day. Then each channel serves 1.333 trains per day (0.056 trains per hour). Thus, in terms of the queuing theory, we obtain a three-phase QS $BMAP/M/1/5 \rightarrow */M/2/8 \rightarrow */M/3/0$. $BMAP$ matrices taking the form

$$D_0 = \begin{pmatrix} -0.25 & 0 \\ 0 & -0.17 \end{pmatrix}, \quad D_i = \begin{pmatrix} 0.125 & 0.125 \\ 0.085 & 0.085 \end{pmatrix}.$$ 

Kolmogorov SDE constructed according to formulas (2) includes $(K1+1) \cdot (K2+2) \cdot (K3+3) - K2=408$ equations. We do not show here a specific view of the resulting system because of its large size.

To construct the solution, we use the fourth-order Runge-Kutta method [10] with the 0.05 step of the grid and the parameters of QS, $\mu_1 = 0.4$, $\mu_2 = 0.222$, $\mu_3 = 0.056$, and the initial probability distribution that defines the Cauchy conditions: $p_{000}(0) = 1$, $p_{ij}(0) = 0$, $i = 0, 6$, $j = 0,11$, $z = 0,4$. We found the transition probabilities of the states of the considered QS. Table 1 and figure 1 show efficiency indexes based on the transition probabilities.

**Table 1. Efficiency indexes of system $BMAP/M/1/5 \rightarrow */M/2/8 \rightarrow */M/3/0$ for $T = 200$.**

| $P_{loss}$ | $A_{QS}$ | $\bar{k}$ | $L$ | $P_{lock}$ | $P_{lock}$ (Phase 1 and 2) |
|------------|----------|-----------|-----|------------|--------------------------|
| 0.242      | 0.318    | 0.878     | 2.355 | 0.243      |                          |
| Phase 1    |          |           |       |            |                          |
| Phase 2    |          | 1.986     | 7.093 | 0.109      | 0.393                    |
| Phase 3    |          | 2.848     | 0     | -          |                          |

The figure below shows graphs of changes in the loss probability for $t = [0.200]$ depending on the initial state of the system: 1) $p_{000}(t) = 1$, 2) $p_{2,5,2}(t) = 1$, 3) $p_{6,11,4}(t) = 1$, 4) $p_{3,5,0}(t) = 1$, 5) $p_{6,10,0}(t) = 1$.

**Figure 1.** Loss probability depending on the initial state of three-phase QS.

According to the results of calculations (table 1), we can see that the considered micro-LTS cannot cope with the arriving train flow. First, the loss probability deviates significantly from zero (0.242).
Second, there is a high probability of blocking channels of phases 1 and 2, which appears due to the low capacity of phase 3. Thus, the “bottleneck” in the structure of the station is phase 3 (cargo yard). To increase the system capacity, it is advisable to increase the number of channels or the intensity of service requests in phase 3.

In the analysis of transient processes (see figure 1), it was found that their behavior and the rate of convergence to stationary parameters significantly depend on the initial state of the system. The minimum time required for the transition probabilities converge to the stationary ones with a deviation of ± 0.05% is achieved at the average system load, and the maximum time – at the lowest or the highest. For example, for the initial state, \( p_{0,10,0}(t) \), the convergence time \( t^* = 54 \), i.e. a quarter of the total simulation time, and for \( p_{oo0}(t) \), \( t^* = 200 \). Additionally, when the system is loaded at an average speed, the transients are not monotonous, and local extremes appear (see figure 1, lines 2, 3, and 5).

### 3.2. Simulation of a passenger micro-LTS with a railway station

We consider the Kutuzovo transport hub (TH) located in Moscow. It includes a metro station (M), a Moscow Central Circle station (MCC city train), and bus stops (BS). Thus, it connects three passenger flows: No1 M and BS → MCC, No2 M and MCC → BS, No3 MCC and BS → M.

The aim of the study is to analyze the loading of the terminal and the efficiency of the MCC station at the Kutuzovo TH with new technical parameters. Therefore, we are only interested in passenger traffic No3 (to the MCC). Other passenger flows are omitted.

According to the aim of the study, we distinguish two subsystems in the Kutuzovo TH. The first one is a terminal with a capacity of 1000 people and 14 turnstiles. The service intensity of one turnstile is 18 people per minute. The second subsystem is the MCC station, which has two directions of train traffic and a platform that can accommodate up to 1000 people. With such parameters, we have the Kolmogorov SDE with more than 105 thousand equations. To simplify the problem, we make the following assumptions. First, let a group of 50 people be a single request. Secondly, to serve it in one minute, 2.8 turnstiles are required. Therefore, in the first subsystem, the number of channels is 14 / 2.8 = 5 with \( \mu_1 \) = 1 requests per minute. Third, the second subsystem has a single channel, but with doubled service intensity of \( \mu_2 = 0.5 \) requests per minute. The MCC station at Kutuzovo TH is an intermediate one. According to full-scale observations, the maximum size of the group of passengers that the train can take during rush hour is 250 people. Therefore, the channel in phase 2 can serve request groups with a maximum size of five people.

Under these assumptions, the general incoming request flow consists of two sub-flows with the following characteristics. The intensity of arriving applications from the metro station is \( \lambda_{21} = 0.8 \) groups per minute, and their size is distributed according to the binomial law with parameters \( n = 5 \) (number of independent experiments) and \( p = 0.45 \) (probability of success). From the bus stop, requests arrive one at a time (the flow is ordinary) with intensity \( \lambda_{22} = 1 \) per minute.

To describe the flow mathematically, we use the BMAP model. Matrices \( D_0 \) have the form

\[
D_0 = \begin{pmatrix}
-0.8 & 0 \\
0 & 1 \\
\end{pmatrix}, \\
D_1 = \begin{pmatrix}
0.055 & 0.07 \\
0.44 & 0.56 \\
\end{pmatrix}, \\
D_2 = \begin{pmatrix}
0.01155 & 0.147 \\
0 & 0 \\
\end{pmatrix}, \\
D_3 = \begin{pmatrix}
0.01155 & 0.147 \\
0 & 0 \\
\end{pmatrix}, \\
D_4 = \begin{pmatrix}
0.055 & 0.07 \\
0 & 0 \\
\end{pmatrix}, \\
D_5 = \begin{pmatrix}
0.011 & 0.144 \\
0.011 & 0.144 \\
\end{pmatrix}.
\]

Thus, in terms of the queueing theory, we obtain a two-phase QS: BMAP/M5/20 → */M5/1/20. Kolmogorov SDE for this QS can be obtained by formulas (3) and contains \((K1+1) \cdot (K2+2) - 1 = 597\) equations.

To construct the solution, we use the fourth-order Runge-Kutta method with the 0.05 step of the grid. The parameters of two-phase QS are \( \mu_1 = 1, \mu_2 = 0.5 \) and the initial probability distribution that defines the Cauchy conditions, \( p_{ci0}(0) = 1, p_{cj0}(0) = 0, i = 0.25, j = 0.22 \). We found the transition probabilities of the states of the considered QS. Table 2 shows efficiency indexes based on the transition probabilities.
Table 2. Efficiency indexes for $BMAP/M/5/20 \rightarrow * / M^1 / 1/20$ with $T = 75$

| $P_{loss}$ | $A_{QS}$ | $\bar{k}$ | $L$ | $P_{lock}$ |
|-----------|---------|---------|-----|---------|
| 0.001     | 1.798   | Phase 1 | 1.656 | 0.267   | 0.031 |
|           |         | Phase 2 | 0.818 | 4.834   | -     |

Figure 2 shows the loss probability depending on the initial state of the system: 1) $p_{00}(t) = 1$, 2) $p_{12,22}(t) = 1$, 3) $p_{13,11}(t) = 1$, and 4) $p_{14,05}(t) = 1$.

Based on the results of the simulation (table 2), we can conclude that the capacity of the terminal and the MCC station at the Kutuzovo TH is sufficient to serve incoming passenger traffic. The average number of busy channels and the average queue length in each phase are significantly lower than their maximum permissible values. The loss probability is negligible (0.001).

The minimum time required the transition probabilities converge to the stationary ones with a deviation of ± 0.05% is achieved for the initial state $p_{14,5}(t)$, the convergence time $t^* = 39$, i.e. more than half of the total simulation time. The maximum time $t^* = 56$ is required for the initial state $p_{24,22}(t)$. As in the previous experiment, transients are not monotonous at the average system load (see figure 2, lines 2, 3, and 4).

4. Results and discussion

The paper analyzes transients for two types of micro-LTS: cargo and passenger. We found that the behavior and rate of convergence to stationary parameters significantly depends on the initial state of the system. Initial states can be divided into the following groups: 1) the system is almost empty or, on the contrary, the maximum loaded; 2) the system has an average load. For the first group, the transients converge monotonically to the stationary characteristics. The releasing time of these processes is the longest. For the second one, transients have one or more local extremes, and their releasing time is the shortest. The presence of extrema is explained by the fact that the considered systems have a multiphase structure with feedback (channel blocking), and the overflow of one of the phases leads to a decrease in the bandwidth of all previous ones.
5. Conclusion
The study of transients in some micro-LTS is necessary for their comprehensive investigation. Freight railway stations and transport hubs characterized by significant daily fluctuations in traffic flow are an example. Currently, cities in various regions of Russia are actively developing urban transport infrastructure by transport hub designing. For example, it is planned to build four transport hubs in Irkutsk, two of which will include railway stations (On approval of the program for the integrated development of the transport infrastructure of the city of Irkutsk for 2016-2025 Duma of the city of Irkutsk, the decision of September 30, 2016, No 006-20-250396/6 (in Russian)). Thus, further research in this direction may be related to the adaptation of the proposed approach to transport hub modeling for Russian cities.

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