Associated graphs of Certain Arithmetic IASI Graphs

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Abstract

An integer additive set-indexer is defined as an injective function $f : V(G) \to 2^{\mathbb{N}_0}$ such that the induced function $f^+ : E(G) \to 2^{\mathbb{N}_0}$ defined by $f^+(uv) = f(u) + f(v)$ is also injective. A graph $G$ which admits an IASI is called an IASI graph. An arithmetic integer additive set-indexer is an integer additive set-indexer $f$, under which the set-labels of all elements of a given graph $G$ are arithmetic progressions. In this paper, we discuss about admissibility of arithmetic integer additive set-indexers by certain associated graphs of the given graph $G$, like line graph, total graph, etc.

Key words: Integer additive set-indexers, arithmetic integer additive set-indexers, isoarithmetic integer additive set-indexers, biarithmetic integer additive set-indexer, semi-arithmetic set-indexer.

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1 Introduction

For all terms and definitions, not defined specifically in this paper, we refer to [6] and for more about graph labeling, we refer to [2]. Unless mentioned otherwise, all graphs considered here are simple, finite and have no isolated vertices. All sets mentioned in this paper are finite sets of non-negative integers. We denote the cardinality of a set $A$ by $|A|$.

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Definition 1.1. An integer additive set-indexer (IASI, in short) is defined as an injective function \( f : V(G) \to 2^{\mathbb{N}_0} \) such that the induced function \( f^+ : E(G) \to 2^{\mathbb{N}_0} \) defined by \( f^+(uv) = f(u) + f(v) \) is also injective. A graph \( G \) which admits an IASI is called an IASI graph.

Definition 1.2. The cardinality of the labeling set of an element (vertex or edge) of a graph \( G \) is called the set-indexing number of that element.

In [5], the vertex set \( V \) of a graph \( G \) is defined to be \( l \)-uniformly set-indexed, if all the vertices of \( G \) have the set-indexing number \( l \).

Definition 1.3. An IASI \( f \) is said to be a weak IASI if \( |f^+(uv)| = \max(|f(u)|, |f(v)|) \) for all \( u, v \in V(G) \). A graph which admits a weak IASI may be called a weak IASI graph. A weak IASI is said to be weakly uniform IASI if \( |f^+(uv)| = k \), for all \( u, v \in V(G) \) and for some positive integer \( k \).

Definition 1.4. An IASI \( f \) is said to be a strong IASI if \( |f^+(uv)| = |f(u)||f(v)| \) for all \( u, v \in V(G) \). A graph which admits a strong IASI may be called a strong IASI graph. A strong IASI is said to be strongly uniform IASI if \( |f^+(uv)| = k \), for all \( u, v \in V(G) \) and for some positive integer \( k \).

Definition 1.5. An arithmetic integer additive set-indexer is an integer additive set-indexer \( f \), under which the set-labels of all elements of a given graph \( G \) are the sets whose elements are in arithmetic progressions. A graph that admits an arithmetic IASI is called an arithmetic IASI graph.

If all vertices of \( G \) are labeled by the set consisting of arithmetic progressions, but the set-labels are not arithmetic progressions, then the corresponding IASI may be called semi-arithmetic IASI.

Theorem 1.6. A graph \( G \) admits an arithmetic IASI graph \( G \) if and only if all vertices of \( G \) have same deterministic index or for any two adjacent vertices in \( G \), the deterministic index of one vertex is a positive integral multiple of the deterministic index of the other vertex and this positive integer is less than or equal to the cardinality of the set-label of the latter vertex.

Definition 1.7. If all the set-labels of all elements of a graph \( G \) consist of arithmetic progressions with the same common difference \( d \), then the corresponding IASI is called isoarithmetic IASI.

Definition 1.8. Let \( f \) be an arithmetic IASI of a graph \( G \). For two vertices \( v_i \) and \( v_j \) of \( G \), let the common differences of \( f(v_i) \) and \( f(v_j) \) be \( d_i \) and \( d_j \) respectively. If either of \( d_i \) and \( d_j \), for the adjacent vertices \( v_i \) and \( v_j \), is a positive integral multiple of the other, then \( f \) is called a biarithmetic IASI. For \( k \in \mathbb{N}_0 \), if \( d_i = k \cdot d_j \) for all adjacent vertices \( v_i \) and \( v_j \) in \( G \), then \( f \) is called a biarithmetic IASI of \( G \).
By the term, an arithmetically progressive set, (AP-set, in short) with
difference $d$, we mean a set whose elements are in arithmetic progression
with difference $d$. In this paper, we investigate the admissibility of arithmetic
integer additive set-indexers by certain graphs that are associated to a given
graph $G$ and establish some results on arithmetic IASIs.

## 2 Isoarithmetic IASIs of Associated Graphs

In the following discussions, we study admissibility of isoarithmetic IASIs
and biarithmetic IASIs by certain graphs associated to a given arithmetic
IASI graph.

Throughout this section, we denote the set-label of a vertex $v_i$ of a given
graph $G$ by $A_i$, which is a set of non-negative integers. All sets we consider
in this section have at least three elements which are in ascending order.

**Proposition 2.1.** Let $G$ be an isoarithmetic IASI graph. Then, any subgraph
of $G$ is also an isoarithmetic IASI Graph.

By *edge contraction operation* in $G$, we mean an edge, say $e$, is removed
and its two incident vertices, $u$ and $v$, are merged into a new vertex $w$, where
the edges incident to $w$ each correspond to an edge incident to either $u$ or $v$.
We establish the following theorem for the graphs obtained by contracting
the edges of a given graph $G$. The following theorem verifies the admissibility
of the graphs obtained by contracting the edges of a given isoarithmetic IASI
graph $G$.

**Theorem 2.2.** Let $G$ be an isoarithmetic IASI graph and let $e$ be an edge of
$G$. Then, $G \circ e$ admits an isoarithmetic IASI IASI.

**Proof.** Let $G$ admits a weak IASI. Let $e$ be an edge in $E(G)$. Since $G$ is
isoarithmetic IASI graph, the set set-label of each edge $e$ of $G$ is also an
AP-set with difference $d$. $G \circ e$ is the graph obtained from $G$ by deleting $e$ of
$G$ and identifying the end vertices of $e$. Label the new vertex thus obtained,
say $w$, by the set-label of the deleted edge. Then, each edge incident upon
$w$ has a set-label which is also an AP-set with difference $d$. Hence, $G \circ e$ is
an isoarithmetic IASI graph.

**Definition 2.3.** Let $G$ be a connected graph and let $v$ be a vertex of $G$
with $d(v) = 2$. Then, $v$ is adjacent to two vertices $u$ and $w$ in $G$. If $u$ and
$v$ are non-adjacent vertices in $G$, then delete $v$ from $G$ and add the edge $uw$
to $G - \{v\}$. This operation is known as an *elementary topological reduction*
on $G$.

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**Theorem 2.4.** Let $G$ be a graph which admits an isoarithmetic IASI. Then any graph $G'$, obtained by applying finite number of elementary topological reductions on $G$, also admits an isoarithmetic IASI.

**Proof.** Let $G$ be a graph which admits an isoarithmetic IASI, say $f$. Then, all the elements of $G$ are labeled by AP-sets with the same difference, say $d$. Let $v$ be a vertex of $G$ with $d(v) = 2$. Then $v$ is adjacent two non adjacent vertices $u$ and $w$ in $G$. Now remove the vertex $v$ from $G$ and introduce the edge $uw$ to $G - v$. Let $G' = (G - v) \cup \{uw\}$. Now $V(G') = V(G)$. Let $f' : V(G') \rightarrow 2^{\mathbb{N}_0}$ such that $f'(v) = f(v) \forall v \in V(G') (or V(G))$ and the associated function $f'^+ : E(G') \rightarrow 2^{\mathbb{N}_0}$ and defined by

$$f'^+(e) = \begin{cases} f^+(e) & \text{if } e \neq uw \\ f(u) + f(w) & \text{if } e = uw \end{cases}$$

Hence, $f'$ is an isoarithmetic IASI of $G'$.

Another associated graph of a given graph $G$ is its graph subdivision. The notion of graph subdivision is given below and its admissibility of arithmetic IASI are established in the following theorem.

**Definition 2.5.** [11] A subdivision of a graph $G$ is the graph obtained by adding vertices of degree two into its edges.

**Theorem 2.6.** The graph subdivision $G^*$ of a given isoarithmetic IASI graph $G$ also admits isoarithmetic IASI.

**Proof.** Let $u$ and $v$ be two adjacent vertices in $G$. Since $G$ admits an isoarithmetic IASI, the set-labels of $u$, $v$ and the edge $uv$ are AP-sets with difference, say $d$. Introduce a new vertex $w$ to the edge $uv$. Now, we have two new edges $uw$ and $vw$ in place of $uv$. Extend the set-labeling of $G$ by labeling the vertex $w$ by the same set-label of the edge $uv$. Then, both the edges $uw$ and $vw$ have the set-labels which are AP-sets with the same difference $d$. Hence, $G^*$ admits an isoarithmetic IASI.

Recall the following definition of line graph of a graph.

**Definition 2.7.** [12] For a given graph $G$, its line graph $L(G)$ is a graph such that each vertex of $L(G)$ represents an edge of $G$ and two vertices of $L(G)$ are adjacent if and only if their corresponding edges in $G$ incident on a common vertex in $G$.

An interesting question we need to address here is whether the line graph an isoarithmetic IASI graph admits an isoarithmetic IASI. The following theorem answers this question.
Theorem 2.8. If $G$ is an isoarithmetic IASI graph, then its line graph $L(G)$ is also an isoarithmetic IASI graph.

Proof. Since $G$ is an isoarithmetic IASI graph, the elements of $G$ have the set-labels whose elements are in arithmetic progression with the same common difference, say $d$. Label each vertex of $L(G)$ by the same set-label of the corresponding edge in $G$. Hence, all the vertices $u_r$ in $L(G)$ have the set-labels consisting of elements that are in arithmetic progressions with the same common difference $d$. Therefore, all the edges of $L(G)$ have the set-labels are also arithmetic progressions with the same common difference $d$. Hence, $L(G)$ is also an isoarithmetic graph.

Definition 2.9. The total graph of a graph $G$ is the graph, denoted by $T(G)$, is the graph having the property that a one-to-one correspondence can be defined between its points and the elements (vertices and edges) of $G$ such that two points of $T(G)$ are adjacent if and only if the corresponding elements of $G$ are adjacent (either if both elements are edges or if both elements are vertices) or they are incident (if one element is an edge and the other is a vertex).

Theorem 2.10. If $G$ is an isoarithmetic IASI graph, then its total graph $T(G)$ is also an isoarithmetic IASI graph.

Proof. Since $G$ admits an isoarithmetic IASI, say $f$, by the definition of an IASI $f(v), \forall v \in V(G)$ and $f^+(e), \forall e \in E(G)$ are AP-sets of non-negative integers with the same difference, say $d$. Define a map $f^*: V(T(G)) \rightarrow 2^{N_0}$ which assigns the same set-labels of the corresponding elements in $G$ under $f$ to the vertices of $T(G)$. Clearly, $f^*$ is injective and each $f^*(u_i), u_i \in V(T(G))$ is an AP-set with difference $d$. Now, define the associated function $f^+: E(T(G)) \rightarrow 2^{N_0}$ defined by $f^+(u_i, u_j) = f^*(u_i) + f^*(u_j), u_i, u_j \in V(T(G))$. Then, $f^+$ is injective and each $f^+(u_i, u_j)$ is also an AP-set with the difference $d$. Therefore, $f^*$ is an isoarithmetic IASI of $T(G)$. This completes the proof.

3 Biarithmetic IASI of Associated Graphs

Definition 3.1. Let $f$ be an arithmetic IASI of a graph $G$. For two vertices $v_i$ and $v_j$ of $G$, let the differences of $f(v_i)$ and $f(v_j)$ be $d_i$ and $d_j$ respectively. If either of $d_i$ and $d_j$, for the adjacent vertices $v_i$ and $v_j$, is a positive integral multiple of the other, then $f$ is called a biarithmetic IASI. For $k \in N_0$, if $d_i = k.d_j$ for all adjacent vertices $v_i$ and $v_j$ in $G$, then $f$ is called a biarithmetic IASI of $G$. 
In this section, we discuss the admissibility of biarithmetic IASIs by the associated graphs of a given biarithmetic IASI graph.

**Theorem 3.2.** A biarithmetic IASI of a graph $G$ is a $l$-uniform IASI if and only if $G$ has $p$ bipartite components, where $p$ is the number of distinct pair $(m_i, n_j)$ of positive integers such that $m_i$ and $n_j$ are the set-indexing numbers of adjacent vertices in $G$ and $l = m_i + n_j - 1$.

What are the characteristics of the line graph of a biarithmetic graph? The following results provide a solution to the problem.

**Theorem 3.3.** For $k > 1$, the line graph of a biarithmetic IASI graph admits an isoarithmetic IASI if and only if $G$ is bipartite.

**Proof.** Let $G$ be bipartite graph which admits a biarithmetic IASI, with the bipartition $(X, Y)$. Label the vertices of $X$ by distinct sets of non-negative integers consisting of arithmetic progressions with common difference $d$ and label the vertices of $Y$ by distinct sets of non-negative integers consisting of arithmetic progressions with common difference $kd$. Then, every edge of $G$ has the set-label consisting of arithmetic progressions with the common difference $d$. Therefore, all vertices in $L(G)$ has the set-labels which are arithmetic progressions with the same common difference $d$ and hence every edge of $L(G)$ is also set-labeled by arithmetic progressions with the same common difference $d$. Hence, $L(G)$ admits an isoarithmetic IASI.

Conversely, let $L(G)$ is an isoarithmetic IASI graph. Hence, every element of $L(G)$ must be labeled by arithmetic progressions with common difference $d$. Therefore, the all the edges in $G$ must have set labels which are arithmetic progressions with common difference $d$. Since, $G$ admits a k-augmented IASI, one end vertex of every edge must have one end vertex labeled by arithmetic progressions with common difference $d$ and other end vertex labeled by arithmetic progressions with the common difference $kd$. Let $X$ be the set of all vertices of $G$ which are labeled by arithmetic progressions with common difference $d$ and $Y$ be the set of all vertices of $G$ which are labeled by arithmetic progressions with common difference $kd$. Since $k > 1$, no two vertices in $X$ and no two vertices in $Y$ can be adjacent. $(X, Y)$ is a bipartition of $G$. Hence $G$ is bipartite. This completes the proof.

**Theorem 3.4.** For $1 < k \leq |f(v_i)|_{\min};$ $v_i \in V(G), if the line graph of a biarithmetic IASI graph admits a biarithmetic IASI, then $G$ is acyclic.

**Proof.** First assume that $L(G)$ is a biarithmetic IASI graph. If possible, let $G$ contains a cycle $C_n = v_1v_2v_3…v_nv_1$. Let $e_i = v_iv_{i+1}, 1 \leq i \leq n$
and let \( u_i \) be the vertex in \( L(G) \) corresponding to the edge \( e_i \) in \( G \). Label each vertex \( v_i \) of \( G \) by the set whose elements are arithmetic progression with common difference \( d_i \) where \( d_{i+1} = k.d_i \); \( k \geq |f(v_i)|_{\text{min}} \). Without loss of generality, let \( f(v_1) \) has the minimum cardinality. Since \( L(G) \) admits a biarithmetic IASI, adjacent vertices \( u_i \) and \( u_{i+1} \) in \( L(G) \) are labeled by the sets whose elements are in arithmetic progressions whose common differences are \( d_i \) and \( d_{i+1} = k.d_i \) respectively. Therefore, the corresponding edges \( e_i \) and \( e_{i+1} \) of \( G \) must also have the same set-labeling. Hence, alternate vertices of \( G \) can not have the set-labels with the same common difference. Then, \( d_i = k^i.d_1, 1 < k \leq |f(v_1)| \). Here, we notice that the set-label of one end vertex \( v_n \) of the edge \( v_nv_1 \) in the cycle \( C_n \) has the common difference \( k^n.d_1 \) and the set-label of other end vertex \( v_1 \) has the common difference \( d_1 \), which is a contradiction to the fact that \( G \) is biarithmetic IASI graph. Therefore, \( G \) is acyclic.

Remark 3.5. The converse of the theorem need not be true. For example, the graph \( K_{1,3} \) admits a biarithmetic IASI and is acyclic, but its line graph does not admit a biarithmetic IASI.

What is the required condition for an acyclic graph to have a biarithmetic IASI? The following theorem establishes the necessary and condition for a biarithmetic IASI graph to have its line graph, a biarithmetic IASI graph.

**Theorem 3.6.** For \( 1 < k \leq |f(v_i)|_{\text{min}}; \ v_i \in V(G), \) if the line graph of a biarithmetic IASI graph admits a biarithmetic IASI, then \( G \) is a path.

**Proof.** The necessary part of the theorem follows from Theorem 3.4. Conversely, assume that \( G \) is a path. Let \( G = v_1v_2v_3 \ldots v_n \). Label the vertex \( v_i \) by an AP-set of difference \( d_i \), where \( k \leq |f(v_i)|_{\text{min}} \). Without loss of generality, let \( f(v_1) \) has the minimum cardinality. Then, \( d_i = k^i.d_1, 1 < k \leq |f(v_1)| \). Then, each edge \( e_i \) of \( G \) has the AP-set-label with difference \( d_i = k.d_{i-1} \). Hence, the each vertex \( u_i \) in \( L(G) \) corresponding to the edge \( e_i \) has the AP-set-label with difference \( d_i = k.d_{i-1} = k^{i-1}.d_1 \). Hence, \( L(G) \) admits a biarithmetic IASI. This completes the proof.

In the above theorems, the value of \( k \) should be within \( 1 \) and \( |f(v_i)|_{\text{min}} \), the minimum among the set-indexing numbers of the vertices of \( G \). We note that if \( k > |f(v_i)|_{\text{min}} \), then the set-label of the edge \( v_iv_{i+1} \) is not an AP-set. That is, \( f \) is a semi-arithmetic IASI. Therefore, the vertices of its line graph are not labeled by AP-sets. Hence, we have the following theorem.

**Theorem 3.7.** For \( k > |f(v_i)|_{\text{min}}; \ v_i \in V(G), \) if the line graph \( L(G) \) of a biarithmetic IASI graph does not admit an arithmetic IASI.
**Theorem 3.8.** The total graph of an identical biarithmetic IASI graph is an arithmetic IASI graph.

*Proof.* The vertices of $T(G)$ corresponding to the vertices of $G$ have the same set-labels and the edges in $T(G)$ connecting these vertices also preserve the same set-labels of the corresponding edges of $G$. The vertices of $T(G)$ corresponding to the edges of $G$ are given the same set-labels of the corresponding set-labels of the edges of $G$. Hence, all these vertices in $T(G)$ have the same deterministic index, say $d$, and hence the edges in $T(G)$ connecting these vertices also have the same deterministic index $d$. As the deterministic indices of an edge and one of its end vertex are the same and the deterministic index of the other end vertex is a positive integral multiple of the deterministic index of the edge, where this integer is less than the cardinality of the set-label of the other end vertex, the edges corresponding to the incidence relations in $G$ also have the deterministic index $d$. Hence, $T(G)$ admits an arithmetic IASI. \[\square\]

**Theorem 3.9.** The total graph of a biarithmetic IASI graph is an arithmetic IASI graph.

*Proof.* The vertices of $T(G)$ corresponding to the vertices of $G$ have the same set-labels and the vertices of $T(G)$ corresponding to the edges of $G$ are given the same set-labels of the corresponding set-labels of the edges of $G$. Also, the deterministic indices of an edge and one of its end vertex are the same and the deterministic index of the other end vertex is a positive integral multiple of the deterministic index of the other end vertex, where this integer is less than the cardinality of the set-label of the other end vertex. Hence, for every two adjacent vertices in $T(G)$, the deterministic index of one is a positive integral multiple of the deterministic index of the other, where this integer is less than or equal to the set-indexing number of the latter. Therefore, by Theorem 1.6, $T(G)$ is an arithmetic IASI graph. \[\square\]

The following theorem checks whether the total graph corresponding to a biarithmetic IASI graph $G$ admits a biarithmetic IASI.

**Theorem 3.10.** The total graph of a biarithmetic IASI graph is not a biarithmetic IASI graph.

*Proof.* We observe that every edge in $G$ corresponds to a triangle $K_3$ in its total graph. Since $K_3$ can not admit a biarithmetic IASI, $T(G)$ is not a biarithmetic IASI graph. \[\square\]
If $G$ is a biarithmetic IASI graph, will $G \circ e$, $e \in E(G)$ be an IASI graph? We observe that the cycle $C_4$ is a biarithmetic graph, but for any edge $e$ of $C_4$, $C_4 \circ e = C_3$, which does not admit a biarithmetic IASI. Hence, we have the following observation.

**Proposition 3.11.** A graph obtained from a biarithmetic IASI graph by contracting an edge of it, is not a biarithmetic IASI graph.

We also prove a similar for the graphs obtained from a biarithmetic IASI graph by a finite number of topological reductions.

**Proposition 3.12.** Let $H$ be a graph obtained by finite number of topological reduction on a biarithmetic IASI graph $G$. Then, $H$ is not a biarithmetic IASI graph.

*Proof.* Let $v$ be a vertex of $G$ with degree 2. Without loss of generality, let the set-label of $v$ be an AP-set with difference $d$. Let $u$ and $w$ be the adjacent vertices of $v$ which are not adjacent to each other. Since $G$ is a biarithmetic graph, both $u$ and $w$ must be labeled by distinct AP-sets with difference $k.d$. Now delete the vertex $v$ and join $u$ and $w$. Let $H = (G - \{v\}) \cup \{uw\}$. Then, both the end vertices of the edge $vw$ has the set labels which are AP-sets of the same difference $k.d$. Hence, $H$ does not admit a biarithmetic IASI. \(\square\)

**Theorem 3.13.** The graph subdivision $G^*$ of a given biarithmetic IASI graph $G$ does not admit a biarithmetic IASI.

*Proof.* Let $u$ and $v$ be two adjacent vertices in $G$ whose set-labels are AP sets with differences $d$ and $k.d$ respectively. Since $G$ admits a biarithmetic IASI, the set-label of the edge $uv$ is AP-set with different difference $d$. If we introduce a new vertex $w$ to the edge $uv$ and extend the set-labeling of $G$ by labeling the vertex $w$ by the same set-label of the edge $uv$, then, the set-labels of both $u$ and $w$ (or $v$ and $w$) are AP-sets with the same difference $d$. Hence, $G^*$ does not admit a biarithmetic IASI. \(\square\)

## 4 Further Points of Discussions

In this section we make some remarks on semi-arithmetic IASI graphs and their associated graphs. We observe that if the set labels of all vertices of $G$ are AP-sets with distinct differences, then the set-labels of edges will not be AP-sets. Hence, We have the following observations.

**Proposition 4.1.** The line graph $L(G)$ of a semi-arithmetic IASI graph $G$ does not admit an arithmetic IASI (or a semi-arithmetic IASI).
Proposition 4.2. The Total graph $T(G)$ of a semi-arithmetic IASI graph $G$ does not admit an arithmetic IASI (or a semi-arithmetic IASI).

From the fact that a graph $G$, its subdivision graph, the graph obtained by contracting an edge and the graph obtained by elementary topological reductions have some common edges, we observe the following results.

Proposition 4.3. The graph $G \circ e$, obtained by contracting an edge $e$ of a semi-arithmetic IASI graph $G$, does not admit an arithmetic IASI (or a semi-arithmetic IASI).

Proposition 4.4. The subdivision graph $G^*$ of a semi-arithmetic IASI graph does not admit an arithmetic IASI (or a semi-arithmetic IASI).

Proposition 4.5. The graph $G'$, obtained by applying elementary topological reduction on a semi-arithmetic IASI graph $G$, does not admit an arithmetic IASI (or a semi-arithmetic IASI).

5 Conclusion

In this paper, we have discussed some characteristics of certain graphs associated a given graph which admits an arithmetic IASI. We have formulated some conditions for those graph classes to admit arithmetic IASIs. Here, we have discussed about isoarithmetic IASI graphs and biarithmetic IASI graphs only. The existence of similar results for arbitrarily arithmetic IASI graphs and biarithmetic IASI graphs are yet to be studied. The IASIs under which the vertices of a given graph are labeled by different standard sequences of non negative integers, are also worth studying. The problems of establishing the necessary and sufficient conditions for various graphs and graph classes to have certain IASIs still remain unsettled. All these facts highlight a wide scope for further studies in this area.

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