Thermal Characteristics of Borehole Stability Drilling in Hot Dry Rock

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1. INTRODUCTION

Hot dry rock, an important reservoir of geothermal resources, is clean energy that attracts worldwide attention, and its development and utilization has become a research hotspot in recent years.1,2 The prerequisite for the exploitation and utilization of geothermal resources is to establish a channel—borehole. The drilling process in hot dry rock mainly faces several major problems. First, the low breaking rock efficiency is the prominent feature of the granite stratum that is one of the critical hot dry rock reservoirs for its high rock hardness, abrasive resistance, poor drillability, and huge uniaxial compressive strength (above 200 MPa). Then, lost circulation in the fault or natural fracture formation zones is the most common problem in drilling hot dry rock.4−7 Finally, borehole instability usually happens in drilling hot dry rock for the cracks enlarged by the huge thermal stress.8,9 There is higher temperature difference between drilling fluid and formation. The drilling fluid would cool the borehole wall rapidly, and the high temperature gradient in rock bringing about the thermal stress would make new cracks or aggravate the original crack propagation, which results in the rock strength reduction and the wall of the shaft peeling off and falling blocks. The above three problems are related to the cracks around the borehole. Therefore, it is very necessary to analyze the cause of the cracks in the borehole wall during drilling in the hot dry rock.

There are many theoretical research studies on heat transfer in borehole and formation. Most of researchers regarded the formation as a porous medium. Keller et al.10 ignored the axial heat conduction of drilling fluid, established a one-dimensional mathematical model of downhole circulating temperature in the well, two-dimensional in the formation, and solved it with the finite difference method. Thompson et al.11 proposed a one-dimensional unstable state model of drilling fluid heat transfer in the borehole. The influence of formation converted into the heat boundary changing over time. The characteristic line method accelerated the solving speed to the model. Wu and Pruess12 introduced Ramey’s hypothesis and provided analytical solutions for the borehole heat transfer in multilayer formations with different thermal physical properties. They found that Ramey’s method was suitable for long time but would produce large deviations for short time. Kabir and Hasan13−15 applied Fourier heat conduction law to describe...
the heat transfer of the second interface. They showed the
general analytic solution for the positive cycle and reverse
circulation fluid temperature. Romero\textsuperscript{16} proposed a temperature
prediction of deep-water well drilling and cementing
operation for the design and evaluation of the circulation
temperature numerical simulation program. Most of
the solution methods are suitable for the steady state and quasi-
transient state of heat transfer but not for the transient state of
heat transfer.

When the rock is heated or cooled sharply, a large amount of
heat exchange occurs inside the rock in a very short period and
the temperature changes dramatically. The unsteady tem-
perature gradient in the rock leads to the impact of thermal stress
that leads to the initiation, expansion, and coalition of cracks in
rock, which is a process from microscopic damage to
macroscopic damage. Many research studies mainly focused
on the rock mechanical properties and damage evolution law in the transient process of
rapid rock cooling.\textsuperscript{17–20} Shao et al.\textsuperscript{29,30} conducted experi-
mental studies and theoretical analysis on the mechanical
properties and fracture characteristics of granite under the
thermal stress caused by rapid cooling. Kumari et al.\textsuperscript{31}
conducted a series of studies on the thermal shock fracture
characteristics at different cooling rates and applied the results
of the research to improve the thermal recovery and yield
increase of hot dry rock. Xi and Zhao,\textsuperscript{34,35} Tang,\textsuperscript{16} and Xu\textsuperscript{37}
studied the mechanical properties and permeability of high-
temperature granites after water cooled sharply. Xu et al.\textsuperscript{38}
conducted a numerical simulation of the crack in the
surrounding rock of geothermal liquid petroleum gas storage,
simplifying the storage wall surface to an infinite plate, the
temperature range of which was $\pm 162$–$15^\circ$C. Tang et al.\textsuperscript{39}
used two-dimensional plates to establish a heat-conduction
differential equation to study rock fracture induced by the
formation temperature and established the fracture propaga-
tion theory of rock induced by low temperature. There were
few studies\textsuperscript{40–43} on heat transfer law, thermal conductivity
coefficient, temperature gradient, thermal impact velocity,
and damage of granite under thermal impact but more studies on
brittle materials such as ceramics and glass. Zhang,\textsuperscript{44} Yang,\textsuperscript{45}
and Zhang et al.\textsuperscript{46} studied the thermal stress distribution of
rock at the bottom of gas drilling in the spherical coordinates.

When drilling in the hot dry rock, borehole wall instability is
one of the focused issues. The heat transfer process between
the drilling fluid and the wall is an unstable heat-transfer state.
The temperature gradient in a shallow area around the
borehole wall similar to thermal shock or cold shock is bigger.
The research about the mechanical properties of rock in a
relatively short time is few. The paper mainly studies four
aspects: first, to analyze the borehole fluid flow process and
borehole heat transfer process of the surrounding rock; then,
to study the transient heat transfer in the stratum and
macroscopic thermodynamic parameters of rock with tempera-
ture change effect on the heat transfer rate; next, to determine
the heat transfer rate around the borehole, the temperature
gradient, and thermal stress distribution inside the rock; and
finally, to analyze the crack change rule according to the stress
intensity factor of fracture mechanics. Therefore, the influence
law of thermal stress on the stability of surrounding rock
during drilling in hot dry rock formation is clearly
demonstrated.

![Figure 1. Wellbore and drill string were considered as concentric cylinders, with the inner pipe fluid flowing downward and annular fluid flowing upward. Transversely, heat is exchanged between the fluid and the solid side wall.](image)

2. MATHEMATICAL MODEL

2.1. Temperature Distribution Model of the Drilling Fluid in the Borehole. Taking the well head as the coordinate origin, the $z$-axis is along the borehole axis, and downward is positive. The unit at any well section $z$ is shown in Figure 1. The drilling fluid flows downward in the drill string and upward in the annulus. Heat exchange occurs due to the temperature difference between the drilling fluid in the drill string and the annular drilling fluid. The heat entering the unit is equal to the energy increment in the unit according to the first law of thermodynamics. The temperature equations of the drilling fluid in the drill string and annulus were established. Assuming

- The drill string and borehole are concentric.
- Heat transfer in the drill string fluid and annulus fluid is by axial convection and by radial conduction.
- Heat transfer in the wall of the drill string and casing is by radial conduction. Heat transfer in the formation is by radial conduction.
- The formation is isotropic and impermeable.
- The heat transfer direction is axisymmetric.
- The drilling fluid is incompressible and circulates at a constant rate.

2.1.1. Temperature Model of the Drilling Fluid in the Drill String. In the process of downward flow of the drilling fluid in the drill string, the heat entering the unit body of length $dz$ in $dt$ period includes the quantity of heat flowing into the unit body $Q_p(z) - Q_p(z + dz)$; the quantity of heat entering the drill string from the annulus through the drill string wall is $Q_{ap}$.  

$$Q_p(z) - Q_p(z + dz) = -q_m \frac{\partial (C_p T_p)}{\partial z} dz dt$$  \hspace{1cm} (1)

$$Q_{ap} = U \pi d_p \left(T_a - T_p(z)\right) dt$$  \hspace{1cm} (2)

According to the principle of conservation of energy, the heat entering the unit after $dt$ eventually causes the heat of the unit to increase. Therefore, the conservation of energy is expressed by eqs 3a and 3b.
In the upward temperature, \( \frac{\partial (C_T T_p)}{\partial z} \) dz dt

\[
U d_{fl} (T_e - T_f) dt - \frac{\partial (C_T T_e)}{\partial z} dz dt
\]

\[
= \frac{\pi d_{fl}^2}{4} \frac{\partial (C_T T_f)}{\partial z} dz dt
\]

The total heat transfer coefficient \( U \) from the annulus to drill string is expressed as eq 4.

\[
1 = \frac{1}{h_{po} d_{po}} + \frac{1}{h_{po} d_{po}} + \ln \frac{d_{po}/d_{fl}}{2 \lambda_{dp}}
\]

where, \( q_m \) is the mass flow rate of the drilling fluid, kg/s; \( C_i \) is the specific heat of the drilling fluid in the drill string, J/(kg °C); \( T_p \) is the temperature of the drilling fluid in the drill string, °C; \( U \) is the total heat transfer coefficient from the annulus to drill string, W/(m² °C); \( d_{fl} \) is the inner diameter of the drill string, m; \( d_{po} \) is the outer diameter of the drill string, m; \( T_s \) is the temperature of the drilling fluid in the annulus, °C; \( \rho_f \) is the density of the drilling fluid in the drill string, kg/m³; \( h_{po} \) is the convective heat-transfer coefficient of the drill string inner wall, W/(m² °C); \( h_{po} \) is the convective heat transfer coefficient of the drill string outer wall, W/(m² °C); \( \lambda_{dp} \) is the thermal conductivity of the drill string wall, W/(m² °C).

2.1.2. Temperature Model of the Drilling Fluid in the Annulus. In the upward flow of the drilling fluid in the annulus, the quantity of heat entering the unit in \( dt \) period includes the quantity of heat flowing into the unit \( Q(x + dz) - Q(x) \). The quantity of heat entering the drill string from the annulus through the drill string wall is \( Q_{ap} \). The quantity of heat in the formation is transferred from the borehole wall to the annular fluid through convection heat transfer \( Q_{fa} \).

\[
Q_{ap} = \frac{\pi d_{fl}^2}{4} \frac{\partial (C_T T_f)}{\partial z} dz dt
\]

\[
Q_{fa} = h_{pi} d_{wi} (T_i - T_f) dt
\]

According to the principle of conservation of energy, the heat that enters the body of the unit after \( dt \) time eventually causes the heat of the body of the unit to increase. Therefore, the conservation of energy is expressed by eqs 8a and 8b.

\[
h_{pi} d_{wi} (T_i - T_f) dt - U d_{fl} (T_e - T_f) dt + q_m \frac{\partial (C_T T_e)}{\partial z} dz dt
\]

\[
= \frac{\pi (d_{wi}^2 - d_{po}^2)}{4} \frac{\partial (C_T T_f)}{\partial z} dz dt
\]

where, \( C_i \) is the specific heat of the drilling fluid in the annulus, J/(kg °C); \( d_{wi} \) is the well diameter, m; \( T_i \) is the formation temperature, °C; \( \rho_i \) is the density of drilling fluid in annulus, kg/m³; \( h_i \) is the convective heat transfer coefficient of borehole wall, W/(m² °C); \( h_{po} \) is the convective heat transfer coefficient of drill string outer wall, W/(m² °C).

2.1.3. Determine Solution Conditions.

\[
T_p = T_s = T_i
\]

\[
T_p(z = 0) = \text{const}, \quad T_p(z = H) = T_i(z = H),
\]

\[
T_i(r = \infty) = T_i(r = \infty, t = 0)
\]

where, \( H \) is the well depth in meters; \( r \) is the distance from the formation to borehole axis in meters; \( t \) is the time in seconds.

2.2. Temperature Field Model of the Rock around the Borehole. Longitudinally, a \( dz \) length borehole cross section is shown in Figure 2. The borehole radius is \( R \), and there is a crack on the surface of the borehole with a depth of \( a \). The fluid temperature at the borehole wall is \( T_p \) and the original formation temperature is \( T_0 \). For any time \( t \), the temperature at radius \( r \) is \( T(z, r, t) \). The heat transfer coefficient of rock is \( \lambda \), density is \( \rho \), and specific heat capacity is \( C \).

Assuming

- The rock formation is isotropic and homogeneous.
- Heat transfer in the annulus fluid on the surface of the borehole is by axial convection.
- Heat transfer in the rock formation is transient by radial conduction.
- Heat transfer direction is axisymmetric.

According to the differential equation of heat conduction

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T(z, r, t)}{\partial r} \right) = \frac{\rho C \partial T(z, r, t)}{\partial t},
\]

\[
R \leq r \leq \infty, \quad t > 0
\]

Initial conditions

\[
T(z, r, 0) = T_0
\]

The boundary conditions are:

According to Fourier’s law, the second kind of boundary conditions can be given as follows

\[
-2 \pi \lambda R \frac{\partial T(z, r = R, t)}{\partial r} = \frac{dQ_{fa}}{dz}
\]

The formation temperature field is not disturbed at an infinite distance, and its boundary condition is as follows

\[
\frac{\partial T(z, r = \infty, t)}{\partial r} = 0
\]
The dimensionless radius and dimensionless time are defined as follows

\[ r_t = \frac{r}{R}, \quad t_t = \frac{\lambda t}{\rho R} \]

Equations 9–12 are replaced by the dimensionless radius and the dimensionless time

\[ \frac{\partial^2 T(z, r, t)}{\partial r_t^2} + \frac{1}{r_t} \frac{\partial T(z, r, t)}{\partial r_t} = \frac{\partial T(z, r, t)}{\partial t_t}, \]

\[ 1 \leq r_t \leq \infty, \quad t_t > 0 \]  \hspace{1cm} (15)

\[ T(z, r_t, 0) = T_0 \] \hspace{1cm} (16)

\[ \frac{\partial T(z, r_t = 1, t_t)}{\partial r_t} = -\frac{1}{2\pi \lambda} \frac{dQ_{tb}}{dz} \] \hspace{1cm} (17)

\[ \frac{\partial T(z, r_t = \infty, t_t)}{\partial r_t} = 0 \] \hspace{1cm} (18)

Analytical solutions of model eqs 13–16 are obtained by Laplace transformation

\[ T(z, r_t, t_t) = T_0 + \frac{dQ_{tb}}{2\pi \lambda} \int_0^\infty \frac{1 - e^{-\varepsilon t_t} Y_0(\varepsilon r_t) - J_1(\varepsilon r_t) Y_0(\varepsilon r_t)}{\varepsilon^2 J_1^2(\varepsilon) + Y_1^2(\varepsilon)} \, d\varepsilon \] \hspace{1cm} (19)

When \( r_t = 1 \), the temperature \( T_b \) at the borehole wall can be obtained as

\[ T_b = T_0 + \frac{dQ_{tb}}{\pi \lambda} \int_0^\infty \frac{1 - e^{-\varepsilon t_t} Y_0(\varepsilon) J_0(\varepsilon r_t) - J_1(\varepsilon) Y_0(\varepsilon r_t)}{\varepsilon^2 J_1^2(\varepsilon) + Y_1^2(\varepsilon)} \, d\varepsilon \] \hspace{1cm} (20)

Let the dimensionless temperature \( T_{bd} \) of the borehole wall be

\[ T_{bd} = \frac{2\pi \lambda}{dQ_{tb}} (T_b - T_0) \] \hspace{1cm} (21)

According to formula (20), we can get

\[ T_{bd} = \frac{2}{\pi} \int_0^\infty \frac{1 - e^{-\varepsilon t_t} Y_0(\varepsilon) J_0(\varepsilon r_t) - J_1(\varepsilon) Y_0(\varepsilon r_t)}{\varepsilon^2 J_1^2(\varepsilon) + Y_1^2(\varepsilon)} \, d\varepsilon \] \hspace{1cm} (22)

The dimensionless temperature \( T_{bd} \) is just a function of the dimensionless time \( t_t \), but in the process of solving the problem, it is necessary to carry out infinite integration of the Bessel function, which makes the calculation more complicated. According to the calculation formula of dimensionless temperature of borehole wall given by Hassan,\textsuperscript{13} the actual calculation is greatly simplified. Its calculation formula is as follows

\[ T_{bd} = \begin{cases} 1.1281 \sqrt{t_t} (1 - 0.3 \sqrt{t_t}), & t_t \leq 1.5 \\ (0.04063 + 0.5 \ln t_t)(1 + 0.6/t_t), & t_t > 1.5 \end{cases} \] \hspace{1cm} (23)

Transforming eq 21 into

\[ \frac{dQ_{tb}}{dz} = \frac{2\pi \lambda}{T_{bd}} (T_b - T_0) \] \hspace{1cm} (24)

Substituting eq 24 into eq 19, the temperature distribution in the rock around the well is obtained

\[ T(z, r_t, t_t) = T_0 + \frac{2(T_b - T_0)}{\pi T_{bd}} \int_0^\infty \frac{1 - e^{-\varepsilon t_t} Y_0(\varepsilon) J_0(\varepsilon r_t) - J_1(\varepsilon) Y_0(\varepsilon r_t)}{\varepsilon^2 J_1^2(\varepsilon) + Y_1^2(\varepsilon)} \, d\varepsilon \] \hspace{1cm} (25)

2.3. Thermal Stress Model of the Rock around the Borehole. The dry-hot rock reservoir is mainly granite formation, and the formation temperature is generally 200–300 °C or higher. The temperature of the borehole wall will drop rapidly when it contacts the circulating drilling fluid. The temperature difference between the borehole wall and the

Figure 3. By comparing the heat transfer time of 2, 10, and 60 s with the temperature of Zhang’s model at different locations of the borehole wall, it is found that the calculated results of the two models are very close to each other, thus verifying the reliability of the model.
surrounding rock produces huge thermal stress in a certain range near the borehole wall. The thermal stress is similar to the impact thermal stress obtained by Lidman and Bobrowsky in brittle materials according to the experimental data in the literature. In the I scheme, all known parameters are taken as the values when the maximum temperature is 300 °C; in schemes II-1 to II-5, linear interpolation is carried out for one of the parameters with temperature as the variable, and the values correspond to the minimum and maximum temperature. Scheme II-6 involves linear interpolation of thermal conductivity and specific heat at the same time.

### 3. RESULTS AND DISCUSSION

Mechanical rock breaking is generally adopted in drilling engineering. It is necessary to keep the well wall smooth and stable as far as possible, but there are inevitably more or less microcracks and microdefects on the well wall. Biot,51,52 took into account the microcracks surrounding the rock produced by stress and proposed the classical theory of porous elastic media to analyze the borehole stability. Murphy’s study showed that the thermal stress in the rock has a great influence on the initiation and development of secondary cracks. The influence of thermal stress in the rock on crack propagation in the rock surrounding the borehole is inescapable. Stephens calculated the thermal stress generated when the hot fracturing fluid was injected into the cold target layer in the process of hydraulic fracturing, obtaining the correction of the thermal stress on the original site stress of the rock.

#### 3.1. Variation Law of Rock Properties with Temperature

A large number of experimental results show that the thermodynamic properties of rocks change with the temperature. Xi34,35 tested and inverted the elastic modulus, Poisson’s ratio, and thermal expansion coefficient inversion of the granite with a hole in the center at high temperature and high pressure. The result showed that the elastic modulus of the rock had a trend of decrease with the increase of temperature. The parameters are the elastic modulus of 68 GPa that dropped to 51.42 GPa, Poisson’s ratio from 0.28 to 0.41, and the thermal expansion coefficient at 4000 m depth range is from 1.68 × 10⁻⁶ to 10.5 × 10⁻⁶ °C⁻¹ when the temperature from 100 to 300 °C. Tang found in the granite experiment that the temperature increased from 60 to 300 °C, the specific heat capacity increased gradually from 970 to 1850 J/(kg·°C), and the thermal conductivity from 2.34 to 4.03 W/(m·°C). Xu’s experimental results show that the temperature has no obvious effect on the density of granite before 1000 °C. Therefore, with the decrease of temperature, the elastic modulus of granite increases, the Poisson’s ratio, thermal expansion coefficient, thermal conductivity coefficient, and specific heat capacity decrease, while the density remains const. According to the change of thermodynamic properties of granite within 300 °C, the analysis scheme is formulated, as shown in Table 1.

#### 3.2. Influence of Temperature-Dependent Rock Properties on Crack Propagation

When the temperature of the drilling fluid in the borehole is 200 °C and the original formation temperature is 300 °C, the temperature field,

| scheme | elasticity modulus E/GPa | Poisson’s ratio $\nu$ | thermal expansion coefficient $\alpha/10^{-6}$ °C⁻¹ | thermal conductivity $\lambda$/W m⁻¹ °C⁻¹ | specific heat capacity $c_p$/J kg⁻¹ °C⁻¹ | density $\rho$/kg m⁻³ |
|--------|--------------------------|-----------------------|-----------------------------------------------|----------------------------------------|----------------------------------------|-------------------------|
| I      | 68                        | 0.41                  | 10.5                                          | 4.03                                   | 1850                                   | 2640                    |
| II-1   | 51.42                     | 0.41                  | 10.5                                          | 4.03                                   | 1850                                   | 2640                    |
| II-2   | 51.42                     | 0.28                  | 10.5                                          | 4.03                                   | 1850                                   | 2640                    |
| II-3   | 51.42                     | 0.41                  | 8.68                                          | 4.03                                   | 1850                                   | 2640                    |
| II-4   | 51.42                     | 0.41                  | 10.5                                          | 4.03                                   | 1850                                   | 2640                    |
| II-5   | 51.42                     | 0.41                  | 8.68                                          | 4.03                                   | 1850                                   | 2640                    |
| II-6   | 51.42                     | 0.41                  | 10.5                                          | 2.34                                   | 970                                    | 1850                    |

Table 1. Main Thermodynamic Parameters of Rock Vary with Temperature$^a$

$^a$Seven kinds of calculation schemes are designed. The values of each parameter are obtained according to the thermophysical parameters tested experimentally in the literature. In the I scheme, all known parameters are taken as the values when the maximum temperature is 300 °C. In schemes II-1 to II-5, linear interpolation is carried out for one of the parameters with temperature as the variable, and the values correspond to the minimum and maximum temperature. Scheme II-6 involves linear interpolation of thermal conductivity and specific heat at the same time.
thermal stress field, and stress intensity factor at the crack tip of the surrounding rock in the borehole are simulated.

3.2.1. Temperature Field of the Rock around the Borehole When Changing with Temperature. The distances to the surface borehole wall are 1, 2, and 10 mm, respectively, in Figure 2. The temperature change of each distance cylinder is calculated according to the scheme I, II-4, II-5, and II-6. The thermal parameters adopt the method of linear interpolation to get the values via the temperature $T$. The specific calculation is shown in eq 29.

$$Y(T) = Y(T_i) + \frac{Y(T_2) - Y(T_1)}{T_2 - T_1} (T - T_i)$$

The calculation results are shown in Figure 3. Compared with constant specific heat and thermal conductivity coefficient, the rock temperature influenced by the thermal conductivity coefficient is higher than the specific heat. However, when both of them change with temperature, the heat transfer inside the rock is almost the same as that without considering the change with temperature. Therefore, the influence of the specific heat and thermal conductivity of the rock with temperature on heat transfer is ignored while the heat transfer inside the rock is almost the same as that without considering the change with temperature. Therefore, the influence of the specific heat and thermal conductivity of the rock with temperature on heat transfer is ignored while

Figure 4. Temperature changes with time at the positions 1, 2, and 10 mm away from the borehole wall were calculated, respectively. The farther the distance from the wellbore wall, the smaller the range of temperature variation and the smaller the influence of the wellbore fluid temperature. For a certain location, the effect of thermal conductivity on temperature is lower than that of specific heat, and the effect of thermal conductivity and specific heat as variables is basically the same as that of the constant.

Figure 5. Temperature gradient of the crack surface at 1 mm at crack tip. (a) Temperature gradient absolute value increases rapidly, from 0, jumping to around 9000 °C/m. Then, the temperature gradient turns, gradually flattens, and tends to be stable. (b) Temperature gradient changes sharply in a short period of time (around 20 s).
studying the heat transfer inside the rock. At the same time, Figure 4 can also show the temperature change speed of a point inside the rock around the well. The period of intense temperature change is short and then tends to be stable soon.

3.2.2. Temperature Gradient of the Rock around the Borehole. The crack tip stress field tends to be infinite from a mathematical perspective, and the damage behavior describes at the tip of crack. Therefore, the temperature gradient at the crack surface 1 mm away from the crack tip is as the observation site when the crack depth is 5 mm. The results are shown in Figure 5. Its temperature gradient changes sharply in a short period of time (about 20 s), and its absolute value increases rapidly, from 0, jumping to around 9000 °C/m. Then, the temperature gradient turns, gradually flattens, and tends to be stable.

3.2.3. Thermal Stress Distribution of Rocks around the Borehole. The thermal stress is affected by the temperature gradient, thermal conductivity coefficient, elastic modulus, and linear expansion coefficient. The separation of variables for thermodynamic parameters was used to calculate the thermal stress around the borehole with time, as shown in Figure 6. In terms of the overall change trend, it takes a short time for the thermal stress to reach the maximum value, which is completed within dozens of seconds. The peak value of thermal stress can almost reach the intensity of 2500 MPa instantaneously, and then, the thermal stress gradually decreases.

Figure 6. Variation of thermal stress with time. (a) Influence of each thermodynamic parameter on the thermal stress has the same trend. Compared with the thermal conductivity coefficient and elastic modulus, the linear expansion coefficient has a stronger effect on the peak value of thermal stress, and the peak value of thermal stress is the highest. The degree of thermal stress decrease is also the largest with the increase of time. (b) It takes a short time for the thermal stress to reach the maximum value, which is completed within dozens of seconds. The peak value of thermal stress can almost reach the intensity of 2500 MPa instantaneously, and then, the thermal stress gradually decreases.

Figure 7. Stress intensity factor at the crack tip with a depth of 5 mm. The peak stress intensity factor reached above 200 MPa m^{0.5}, far more than the rock fracture toughness value.
3.2.4. Stress Intensity Factor at the Crack Tip. The crack tip stress field will be infinite for the material containing crack from a mathematical perspective. The damage behavior of materials cannot describe the crack tip stress field. The stress intensity factor is an important appraisal item for the mechanic parameters of the crack tip. The stress intensity factor contains the stress field and displacement field near the crack tip. The critical value of the stress intensity factor is defined as fracture toughness, which can be considered as the material constant and the main indicator to identify fracture stability under a certain temperature and loading rate. Zuo et al.\textsuperscript{21} pointed out that the granite fracture toughness is nearly unchanged through the experimental test within 25\text{\textdegree}C to 100\text{\textdegree}C. All 18 specimens of granite had the average fracture toughness around 0.67 MPa m\textsuperscript{0.5}. The higher is the temperature, the lower is the average fracture toughness. Above 100 \textdegree C, the fracture toughness of granite will be lower than 0.67 MPa m\textsuperscript{0.5}. The stress intensity factor at the crack tip with a depth of 5 mm is shown in Figure 7. The peak stress intensity factor reached above 200 MPa m\textsuperscript{0.5}, far more than the rock fracture toughness value. In this circumstance, the rock crack extension forward is almost inevitable.

3.2.5. Crack Width Changes with Temperature. Based on the calculation results of thermal stress at the point 1 mm away from the crack tip, the crack width at this point was calculated according to the schemes I, II-1, II-2, and II-3, and the results are shown in Figure 8. The crack width is affected by the linear expansion coefficient, elastic modulus, and Poisson's ratio. Single factor analysis shows that the peak value of the crack width was affected least by elastic modulus with temperature, the linear expansion coefficient and Poisson's ratio with the temperature is the same as without them.

![Figure 8](image_url)

Figure 8. (a) Crack width is affected by the linear expansion coefficient, elastic modulus, and Poisson's ratio. The crack width tends to be the same value. (b) Peak value of the crack width affected least by elastic modulus with temperature, and the comprehensive influence from the linear expansion coefficient and Poisson's ratio with the temperature is the same as without them.

![Figure 9](image_url)

Figure 9. Temperature distribution in the borehole and formation when drilling to 5000, 5500, and 6000 m.
Table 2. Temperature and Stress State at the Crack Tip at the Drilling Depth$^a$

| drilling depth m | annulus temperature °C | formation temperature °C | time of conduction s | thermal stress MPa | stress intensity factor MPa m$^{0.5}$ | crack width mm |
|------------------|------------------------|--------------------------|----------------------|-------------------|--------------------------------------|---------------|
| 5000             | 124                    | 208                      | 21.7                 | 2230              | 195.8                                | 1.276         |
| 5500             | 146                    | 245                      | 22.2                 | 2617              | 229.7                                | 1.539         |
| 6000             | 175                    | 281                      | 22.6                 | 2743              | 240.7                                | 1.664         |

$^a$Annular drilling fluid temperature and formation temperature at each drilling depth are the first three columns. The time of conduction, thermal stress, stress intensity factor, and crack width are results from the calculation based on the model.

The temperature and stress near the crack tip with a depth of 5 mm on the borehole wall at different drilling depths are similar. The deeper the drilling depth, the smaller the temperature drop.

The temperature and stress near the crack tip with a depth of 5 mm on the borehole wall were analyzed, and the results are shown in Figures 10–13. The peak time of thermal stress, stress intensity factor, and crack width are between 21 and 22 s. The stress intensity factor is around 200 MPa m$^{0.5}$, which is much higher than the fracture toughness of granite (0.67 MPa m$^{0.5}$). Figure 10 shows the temperature that nears the fracture tip changes in the same way as time with the increase of the drilling depth, and the initial temperature varies at a higher level for the higher original formation temperature. Figures 11–13 show that thermal stress, stress intensity factor, and crack width have the same trend.

4. CONCLUSIONS

(1) The transient heat conduction in the rock is affected by the specific heat and the heat conduction coefficient change with temperature. Their influence on the heat conduction velocity in rock is not significant. The influence of specific heat and thermal conductivity of rock changing with temperature on heat transfer inside the rock can be ignored. The temperature changes of a certain point inside the rock around the well has a short period when the temperature change is more intense, and it then tends to be stable soon.

(2) Compared with the thermal conductivity and elastic modulus, the linear expansion coefficient has a stronger effect on the peak value of thermal stress, and the peak value of thermal stress is the highest (above 2500 MPa),
and the degree of its decrease is the largest with the growth of time.

(3) The peak value of the crack width is least affected by the elastic modulus with temperature, and the effect of the linear expansion coefficient and Poisson’s ratio with temperature is basically equivalent to that of the linear expansion coefficient and Poisson’s ratio without temperature. However, the deeper the crack around the borehole, the weaker the influence of drilling fluid temperature on it, and the temperature gradient changes gently in space.

(4) The stress intensity factor at the crack tip around the borehole reaches around 200 MPa m\(^{0.5}\), which is much higher than the fracture toughness of granite (<1 MPa), and the crack opening width at 1 mm away from the crack tip is more than 1 mm. Therefore, the rock around the well will almost inevitably crack and expand under the action of thermal stress.

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■ NOMENCLATURE

\(q_{ \text{fl}}\), mass flow rate of the drilling fluid, kg/s; \(C_w\), \(C_p\), specific heat of the drilling fluid in the drill string, annulus, J/(kg °C); \(T_w\), \(T_p\), temperature of the drilling fluid in the drill string and annulus, °C; \(U\), total heat transfer coefficient from the annulus to drill string, W/(m\(^2\) °C); \(d_{i0}\), \(d_{fo}\) inner and outer diameter of the drill string, m; \(\rho\), \(\rho_w\), density of the drilling fluid in the drill string and annulus, kg/m\(^3\); \(h_w\), \(h_w^{\text{fl}}\) convective heat transfer coefficient of the drill string inner and outer walls, W/(m\(^2\) °C); \(\lambda_{d}\), thermal conductivity of the drill string wall, W/(m\(^2\) °C); \(d_s\), well diameter, m; \(T_0\), formation temperature, °C; \(h_f\), \(h_f^{\text{fl}}\) convective heat transfer coefficient of the borehole wall and drill string outer wall, W/(m\(^2\) °C); \(H\), well depth, m; \(r\), distance from formation to the borehole axis, m; \(t\), time, s; \(R\), borehole radius, m; \(a\), depth of crack on the surface of the borehole, m; \(T_p\), fluid temperature at the borehole wall, °C; \(T_0\), original formation temperature, °C; \(\lambda\), heat transfer coefficient of the rock, W/(m\(^2\) °C); \(\rho\), density of the rock kg/m\(^3\); \(C\), specific heat capacity of the rock, J/(kg °C); \(\sigma\), thermal stress of the rock, MPa; \(\alpha\), linear expansion coefficient of the rock, 1/°C; \(E\), elastic modulus of the rock, MPa; \(\nu\), Poisson’s ratio of the rock; \(K_s\), stress intensity factor at the crack tip, MPa m\(^{0.5}\); \(w_c\), crack opening displacement, m; \(T_{\text{dip}}\), dimensionless temperature; \(t_p\), dimensionless time; \(r_{\text{dip}}\), dimensionless radius; \(Q_{\text{hp}}\), quantity of heat flowing in the drill string, J/s; \(Q_{\text{fl}}\), quantity of heat flowing in the annulus, J/s; \(Q_{\text{hp}}\), quantity of heat transferred from the borehole wall to the annular fluid, J/s.

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