A phenomenological bound on the viscosity of Hadron Resonance Gas

Snigdha Ghosh\textsuperscript{1,2}*, Sabyasachi Ghosh\textsuperscript{3}† and Sumana Bhattacharyya\textsuperscript{4}‡

\textsuperscript{1}Indian Institute of Technology Gandhinagar, Palaj, Gandhinagar 382355, Gujarat, India
\textsuperscript{2}Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700064, India
\textsuperscript{3}Indian Institute of Technology Bhilai, GEC Campus, Sebjahar, Raipur 492015, Chhattisgarh, India
\textsuperscript{4}Center for Astroparticle Physics and Space Science, Bose Institute, Block EN, Sector V, Salt Lake, Kolkata 700091, India

We have explored some phenomenological issues during calculations of transport coefficients for hadronic matter, produced in the experiments of heavy ion collisions. Here, we have used an ideal hadron resonance gas model to demonstrate the issues. On the basis of dissipation mechanism, the hadronic zoo is classified into resonance and non-resonance members, who participate in dissipation via strong decay and scattering channels respectively. Imposing our phenomenological restriction, we are able to provide a rough upper and lower bound estimations of transport coefficients. Interestingly, we find that our proposed lower limit estimation for shear viscosity to entropy density ratio is little larger than its quantum lower bound. By taking a simple example, we have demonstrated how our proposed restriction help to tune any estimation of transport coefficients within its numerical band, proposed by us.

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I. INTRODUCTION

Shear viscosity ($\eta$) to entropy density ($s$) ratio is the measure of fluidity of the medium. Being roughly proportional to the ratio of mean free path to de-Broglie wavelength of medium constituent, the $\eta/s$ of any fluid can never be vanished, because mean free path of any constituent can never be lower than its de-Broglie wavelength. It indicates that quantum fluctuations prevent the existence of perfect fluid in nature and $\eta/s$ of any fluid should have some lower bound, which is also claimed from the string theory calculation \cite{1}. Interestingly, a small value of $\eta/s$, close to this quantum lower bound, is observed in super hot medium, produced in Relativistic Heavy Ion Collider (RHIC) experiment as well as in some other many body systems like cold atoms \cite{2}, graphene \cite{3} and in low energy nuclear matter \cite{4}. This nearly perfect fluid behavior, at extreme conditions, has drawn immense attention from scientific communities working on the field of condense matter physics to nuclear physics to string theory.

In our present work, we emphasize on some phenomenological issues of $\eta/s$ for hadronic matter. We get a long list of Refs. \cite{5–25}, which had addressed different microscopic calculations of this $\eta/s$, based on different hadronic models \cite{5–14}, different effective QCD models \cite{15–21} and bulk simulations \cite{22–25}. The predicted values of $\eta/s$ from earlier estimations reside within a broad numerical band. Same is observed for bulk viscosity $\zeta$ \cite{1, 8, 13, 20, 24, 33}.

Here we have found a possibility of comparatively narrower band for $\eta/s$ of hadronic matter, when we put a restriction in the calculations of $\eta/s$, based on an ideal hadron resonance gas (HRG) model. The restriction is to consider the dissipation of hadrons within a finite size of RHIC or LHC matter. When we follow the expressions of different transport coefficients in the framework of relaxation time approximation (RTA), we assume that relaxation length (time) should be lower than the size (life time) of the system or medium. Owing to the fact, when we consider the hadronic matter, the resonances, whose mean life time are larger than the life time of the system, will not take part in dissipation process. So we have to eliminate them during the calculation of transport coefficients for finite size hadronic matter. On the other hand, the hadrons like pion, kaon, nucleon can have a momentum dependent relaxation length, whose high momentum component may become larger than the system size. Therefore, we have to eliminate the high-momentum part by imposing a upper momentum cut-off in the calculation. This fact of finite size dissipation is pointed out in the present work with the help of ideal HRG model. A generic qualitative message of the present study is that the theoretical tools should have to take care of this fact of finite size dissipation when we try to give the estimation of transport coefficients for RHIC or LHC matter.

*Electronic address: snigdha.physics@gmail.com
†Electronic address: sabyaphy@gmail.com
‡Electronic address: response2sumana91@gmail.com
The article is organized as follows. Next, in the formalism part [II], first [II A] we have addressed the standard expression of different transport coefficients and then [II B] we have provided a brief description of ideal HRG model, whose detail expressions are given in the Appendix [A]. After getting the expression of transport coefficients and thermodynamical quantity like entropy density, they have been folded by spectral function of hadrons. Its generic equation is written in the subsection [II C] of formalism part. Next we will come to the results section [III], where we have explored the issues of finite size dissipation, which can give us a rough numerical band in the values of different transport coefficients. Then, we have provided an example of microscopic calculation of transport coefficients, whose values don’t remain within our proposed band but after utilizing the appropriate restriction of finite size dissipation, we get their modified values, which ultimately remain within our proposed band. At last, we summarize our studies in section [IV].

II. FORMALISM

A. Transport Coefficients in Kubo Formalism

Our aim of this work is to calculate these transport coefficients with the help of HRG model, so we have to add the contributions of all mesons (M) and baryons (B) for getting total transport coefficients of hadronic matter. We know that the mathematical structure of transport coefficients, obtained from the one-loop diagram in quasi-particle Kubo approach and relaxation time approximation (RTA) in kinetic theory approach, are exactly same. Without going those background formalism part of transport coefficients like shear viscosity $\eta$ [8,9,43,44] and bulk viscosity $\zeta$ [8,18] therefore, let us start with their standard expressions:

$$\eta = \sum_{h \in \{\text{hadrons}\}} \frac{g_h}{15T} \int \frac{d^3k}{(2\pi)^3} \tau_h \left( \frac{k^2}{\omega_h} \right)^2 f_h (1 - a_h f_h),$$

$$\zeta = \sum_{h \in \{\text{hadrons}\}} \frac{g_h}{T} \int \frac{d^3k}{(2\pi)^3} \omega_h^2 \tau_h \left\{ \left( \frac{1}{3} - c_s^2 \right) k^2 - c_s^2 m_h^2 \right\}^2 f_h (1 - a_h f_h),$$

where $g_h$, $\omega_h = \sqrt{k^2 + m_h^2}$ and $f_h = \left[ e^{\omega_h/T} + a_h \right]^{-1}$ are respectively the degeneracy factor, energy and thermal distribution function (Fermi-Dirac or Bose-Einstein) of hadron $h$; $a_h = \pm 1$ if $h$ is a Fermion/Boson. In the above equation, $\tau_h$ is the relaxation time of $h$ which proportionally controls the numerical strength of the transport coefficients. Obviously, the thermal phase space factors, depend on the thermal distribution functions of different hadrons, are another controlling component for transport coefficients.

B. Thermodynamics from ideal HRG

As we are interested on the (nearly) perfect fluid nature of the medium, produced in HIC experiments, so we focus on the quantity - fluidity, which is quantified by the $\eta/s$, where $S$ is entropy density. To calculate $s$ of hadronic matter, we follow the standard procedure of ideal HRG model [15], where all thermodynamic quantities like energy density ($\epsilon$), pressure ($P$), entropy density ($s$), speed of sound ($c_s$) etc. are calculated from the partition function. The Grand Canonical partition function is given by,

$$\ln Z (T, V, \{\mu\}) = V \int \frac{d^3p}{(2\pi)^3} \sum_{h \in \{\text{hadrons}\}} g_h a_h \ln \left[ 1 + a_h \exp \left\{ -\beta \left( \omega_h - \sum_{\mu_k \in \{\mu\}} q_h^k \mu_k \right) \right\} \right]$$

where $\{\mu\} = \{\mu_B, \mu_Q, \mu_S, ...\}$ is the set of chemical potentials corresponding to the conserved quantities (like net baryon ($n_B$), net charge ($n_Q$), net strangeness ($n_S$) etc.) and $q_h^k$ is the corresponding quantum number of $h^{th}$ hadron.
From the partition function, all the thermodynamic quantities can be calculated:

\[ P = \left( \frac{T}{V} \right) \ln Z \]  

(5)

\[ \varepsilon = \left( \frac{T^2}{V} \right) \left( \frac{\partial}{\partial T} \right) \ln Z \]  

(6)

\[ n_k = \left( \frac{T}{V} \right) \left( \frac{\partial}{\partial \mu_k} \right) (\ln Z) \quad ; \quad k = B, Q, S, \ldots \]  

(7)

\[ c_s^2 = \left( \frac{\partial p}{\partial \varepsilon} \right) = \left( \frac{\partial p}{\partial T} \right) / \left( \frac{\partial \varepsilon}{\partial T} \right) + \sum_{\mu_k \in \{\mu\}} \left( \frac{\partial p}{\partial \mu_k} \right) / \left( \frac{\partial \varepsilon}{\partial \mu_k} \right). \]  

(8)

The entropy density \( s \) can be obtained from

\[ s = \frac{(\varepsilon + P)}{T} - \frac{1}{T} \sum_{\mu_k \in \{\mu\}} n_k \mu_k. \]  

(9)

The momentum integration in Eq. (4) can be analytically performed in terms of modified Bessel function details of which are provided in Appendix A. In this work we have taken \( \mu_B = \mu_Q = \mu_S = \ldots = 0 \) which implies that \( \{\mu\} \) is a null set.

C. Spectral Folding

The transport coefficients as well as the thermodynamic quantities as given in Eqs (1)-(2), (5)-(9), depend on the masses of all the hadrons \( \{m_h\} \). To take into account the finite widths of the unstable hadrons, the transport coefficients and the thermodynamic quantities are folded with the corresponding hadronic spectral functions \( \rho^{m,b}_h(M) \).

Let \( \Phi \) denotes any of the transport coefficients (such as \( \eta, \zeta \)) or thermodynamic quantities (such as \( \varepsilon, P, s \) etc.). In this work, the spectral foldings are done through,

\[ \Phi_{\text{folded}} = \sum_{h \in \{\text{mesons}\}} \frac{1}{N^m_h} \int_{(m_h-2\Gamma_h)^2}^{(m_h+2\Gamma_h)^2} dM^2 \rho^{m}_h(M) \Phi (m_h = M) \]

\[ + \sum_{h \in \{\text{baryons}\}} \frac{1}{N^b_h} \int_{m_h-2\Gamma_h}^{m_h+2\Gamma_h} dM \rho^{b}_h(M) \Phi (m_h = M) \]  

(10)

with \( N^m_h = \int_{(m_h-2\Gamma_h)^2}^{(m_h+2\Gamma_h)^2} dM^2 \rho^{m}_h(M) \) and \( N^b_h = \int_{m_h-2\Gamma_h}^{m_h+2\Gamma_h} dM \rho^{b}_h(M) \). In the above equation, the mesonic and baryonic spectral functions are respectively,

\[ \rho^{m}_h(M) = \frac{1}{\pi} \text{Im} \left[ \frac{1}{M^2 - m_h^2 + iM \Gamma_h} \right] \]  

(11)

\[ \rho^{b}_h(M) = \frac{1}{\pi} \text{Im} \left[ \frac{1}{M - m_h + i\frac{\Gamma_h}{2}} \right]. \]  

(12)

III. NUMERICAL RESULTS

Let us begin this section by showing numerical results for the thermodynamic quantities obtained from ideal HRG model in Figs 1 and 2. From Eqs. (7) and (5), one can obtain \( \varepsilon \) and \( P \), which are shown by blue dash line in Fig. 1(a), (b). Using the folding technique, as given in Eq. (10), the values of \( \varepsilon \) and \( P \) are little bit enhanced as shown by red line in Fig. 1(a), (b). Similar kind of results for trace anomaly \( (\varepsilon - 3P)/T^4 \) is shown in Fig. 1(c). One of the success of HRG model is that its estimated values of different thermodynamical quantities are quite close to the results obtained by Lattice Quantum Chromodynamics (LQCD). We have added two set of lattice QCD data from Ref. [46] (cyan band) and [47] (green dash dot line), which are well agreement with the HRG results of present work. In same pattern, results of \( C_S^2 \) and entropy density \( s \) are also plotted in Fig. 2(a) and (b).
FIG. 1: (Color online) (a) Energy density (b) Pressure and (c) Trace anomaly scaled with fourth power of inverse temperature as a function of temperature compared among results from ideal HRG and two lattice QCD data from Ref. [46] and [47] abbreviated as Lattice I and Lattice II respectively. Set-1 and Set-3 (see Table III) corresponds to results from ideal HRG without and with spectral folding.

FIG. 2: (Color online) (a) Speed of sound (b) Entropy density scaled with third power of inverse temperature as a function of temperature compared among results from ideal HRG and two lattice QCD data from Ref. [46] and [47] abbreviated as Lattice I and Lattice II respectively. Set-1 and Set-3 (see Table III) corresponds to results from ideal HRG without and with spectral folding.

Now let us come to the results of transport coefficients. In the expression of $\eta$, $\zeta$, given in Eqs. (1), (2), we see that the thermodynamical phase space parts of different hadrons are known components from the HRG model but their relaxation times are unknown components, which we should have to include in the model from outside, based on our phenomenological understanding. Owing to this phenomenological picture of relaxation of different hadrons, we have classified them into two categories - non-resonance (NR) and resonance (R) components. Let us call pseudo-scalar meson nonet and baryon octet as NR members as these long lived particles can’t decay inside the fireball, produced in HIC experiments. Among them, pion, kaon and nucleon are most abundant constituents in the medium, hence we consider only them as NR members for simplicity. Their strong interaction elastic scattering will provide their relaxation times, which are expected to be important in dissipation within the life time of fireball. The hadrons, other than pseudo-scalar meson nonet and baryon octet, are considered as R members as maximum of them follow strong decays. Their mean life times which are the inverse of their strong decay widths, are comparable with the life time of fireball. So these hadrons also live in the fireball along with the NR members but they exist in resonance states. Maximum of them decays within the medium during its life time and therefore, their strong decays contributes to the total dissipation of the medium. So, by ignoring other interactions (weak and electromagnetic decays and scattering channels) [15], the strong interaction scale is our matter of interest for calculating dissipation in the medium, which survives in that scale. So we see that R and NR members participate in dissipation by their strong decay and scattering processes respectively.

To explore our phenomenological studies on the dissipation process, we have plotted mean life times (red circles) of different hadrons upto $M = 2.5$ GeV in Fig. 3, which is basically covering the strong interaction spectra of hadronic zoo at a glance. The horizontal blue dash line indicates the life time of fireball, which is approximately taken as 10 fm. Hence, the hadrons, whose mean life times are less than the life time of medium, decay inside the medium and they will only participate in the dissipation. Considering only those hadrons and using their mean life times as the relaxation times $\tau$ in Eq. (11), we will get shear viscosity of R component. One has to always consider this amount
FIG. 3: (Color online) The values of mean life times (red points) for different hadron resonances up to 2.5 GeV masses. The horizontal line indicates an approximated life time of the hadronic medium, produced in heavy ion experiments. Blue, pink and green bars are denoting the ranges of collisional time or relaxation time for non-resonance members π, K and N respectively.

FIG. 4: (Color online) (a) Shear viscosity (η) (b) shear viscosity to entropy density ratio (η/s) as a function of temperature for Set-1, Set-2 and Set-3, as given in Table III. An approximate numerical band of both are shown by cyan color and the dash-dot-dot line indicates the KSS limit for η/s.

of shear viscosity for hadronic matter, which may be considered as a lower estimation of η in HRG even when we don’t take any NR contribution. Now, we focus on the NR component, whose in-medium scattering contribution will not be a fixed value like R component. In different hadronic model calculation [5–11] we notice different numerical strength of η from this NR component, although some refs. [7–10] are concentrated only in pion medium. Let us take Compton lengths (1/mπ,K,N) of NR particles (π, K, N) as their minimum scattering lengths in Eq. (1) and then add this contribution with R component to get an approximate lowest estimation of total η. On the other hand, life time (maximum size) of fireball can be considered as upper limit of relaxation times (relaxation lengths) of NR particles and after adding this contribution with R part contribution, we get an upper limit estimation of η. These ranges of relaxation times for π, K and N are shown by blue, pink and green bars in Fig. 3 and using these ranges, we get a numerical band of η, which is shown by cayan color in Fig. 4(a).

Normalizing this η by that entropy density s, we get a similar numerical band for η/s ratio, as shown in Fig. 4(b). In Fig. 4(b), we show that lowest possible value of η/s is little greater than its quantum lower bound (η = 1/4π ≈ 0.08). At T = 0.160 GeV, our proposed band provides an approximate inequality 0.3 < η/s < 0.85. Now, analyzing the earlier estimations of η/s for hadronic matter [5, 8, 11, 14, 23, 25], we see that η/s(T = 0.160GeV) ≈ 0.8 [6, 0.45 [12], 0.32 [8, 0.3 [14], 0.3 [11] remain within the inequality, except η/s(T = 0.160GeV) ≈ 1 [22, 0.2 [13] and 0.13 [11]]]. Whereas, at freeze out temperature T = 0.100 GeV (say), their η/s (≈ 2 [12], 1.2 [8, 1 [14, 22, 25], 0.9 [8, 0.45 [11], 0.4 [14]) are not at all located within our proposed inequality 0.007 < η/s < 0.4. Absence of R members in some formalism [5, 8, 11] and absence of considering dissipation of hadrons within the finite size hadronic matter in Refs. [12, 14, 22, 25] may be possible reason for being outside of our proposed band.

Similar kind of numerical band can be obtained from standard RTA expressions of bulk viscosity (ζ) as given in Eq. (2). These bands are shown by cyan color in Fig. 5(a).
After getting an approximate numerical bands of transport coefficients of hadronic matter, now let us focus on absolute estimation. If we collect estimated values of transport coefficients for hadronic matter by earlier studies, then we will get a broad numerical band within which those estimations are located. In this regards, the present investigation provide a little narrow band and we are expecting that the values of transport coefficients for hadronic matter should be located within our proposed band, when one properly take care about the finite size dissipation phenomena. By taking an example, let us demonstrate how to consider the dissipation of hadrons within the finite size hadronic matter and how it will help to reshape the values of transport coefficients within our proposed band. Let us calculate the relaxation times of NR particles from the experimentally available data of their scattering lengths. Here, we are considering scattering lengths $R_{ab}^I$ for different isospin ($I$) states of $\pi\pi$, $\pi N$, $NN$, $KN$ interactions from Refs. [48–50] and $\pi K$ interaction from Refs. [51, 52] and then using these values, we have calculated isospin average cross sections

$$\sigma_{ab} = \sum (2I+1) \frac{4\pi |a_{ab}^I|^2}{\sum (2I+1)}.$$  \(13\)

These input details are displayed in Table I. Now, using these isospin average cross sections, we can calculate the relaxation time $\tau_a$ ($a=\pi$, $K$ and $N$) from the relation,

$$\frac{1}{\tau_a(k_a)} = \sum_{b\in\{\pi,K,N\}} \int \frac{d^3k_b}{(2\pi)^3} \langle \sigma_{ab} v_{ab} n_b \rangle,$$  \(14\)

For the calculation of relaxation times, we have used the input details displayed in Table I.
where \( n_b \) is BE/FD distribution function of meson/baryon:

\[
v_{ab} = \left( \frac{1}{2\omega_a \omega_b} \right) \sqrt{\left\{ s - (m_a + m_b)^2 \right\} \left\{ s - (m_a - m_b)^2 \right\}}
\]

is the relative velocity with \( \omega_{a,b} = \sqrt{k_{a,b}^2 + m_{a,b}^2} \) and \( s = (\omega_a + \omega_b) \). With help of the relaxation time \( \tau_a \) of \( \pi \), \( K \) and \( N \), one can calculate their relaxation length \( \lambda_a = \vec{k}_a \tau_a / \omega_a \), as shown in Fig. (6). Here we see that relaxation lengths for \( \pi \) and \( K \) exceed the dimension of fireball in the high momentum domain; although the nucleon relaxation length always remains lower than the dimension of fireball in the entire momentum range. Now, let us use these entire momentum distribution of \( \pi \), \( K \) and \( N \) in Eqs. (1), (2) to get NR contribution of \( \eta \) and \( \zeta \). Then after adding the contribution of R component, we will get the total as shown by green dash dot line in Figs. 4(a) and 5(a). Their dimensionless, normalized values, quantified as \( \eta/s \) and \( \zeta/s \) respectively, are shown by green dash dot line in Figs. 4(b) and 5(b). All curves are going beyond the upper bound of our proposed numerical band. The reason is that we are considering high momentum \( \pi \) and \( K \), which are not at all participants in dissipation process as their relaxation lengths exceed the system size. This can be well visualized from Figs. 6.

To resolve it, we have first track numerically the upper momentum threshold or cut-off at different temperatures for \( \pi \) and \( K \), within which their relaxation lengths don’t exceed the fireball dimension (10 fm). From Fig. 6 one can visualize this fact graphically and then we have plotted the the upper momentum thresholds \( \vec{k}_{th} \) for \( \pi \) and \( K \) as a function of temperature, which is shown in Fig. 7. Now when we put those \( T \) dependent momentum thresholds as upper limit in integration of Eq. 1 and use the modified results of \( \pi \) and \( K \), the total values of \( \eta \), \( \eta/s \), \( \zeta \) and \( \zeta/s \) will politely remain within the numerical band. It is shown by blue dash lines in Figs. 4 and 5. Hence, our investigation states that the values of transport coefficients for hadronic matter will be within our proposed numerical band, when one will properly impose the finite size dissipation of NR and R components during the calculation. We are first time addressing this realistic or phenomenological issue, which should be considered for transport coefficients calculations of hadronic matter, which is not an infinite in size.

We may further extend our estimations by adopting folding technique, described by Eq. 10. Putting \( \eta \), \( \zeta \) from Eqs. 1, 2 in \( \Phi \), we will get their modified results, as shown by red solid lines in Fig. 4(a) and 5(a). We notice
that the values of $\eta$ and $\zeta$ becomes lower due to folding effect. For convenient of reader, the Table III is showing our different set of input choices, which we have considered.

|               | Transport coefficients | Entropy density |
|---------------|------------------------|-----------------|
| Lower bound   | NR ($\tau_{\text{low}}$) + R ($\tau < 10\text{fm}$) | -               |
| Upper bound   | NR ($\tau = 10\text{ fm}$) + R ($\tau < 10\text{fm}$) | -               |
| Set-1         | NR + R ($\tau < 10\text{ fm}$) | NR + R          |
| Set-2         | NR ($\tau(k) < 10\text{ fm}$) + R ($\tau < 10\text{ fm}$) | -               |
| Set-3         | Set-2 with folding      | Set-1 with folding |

TABLE II: Different set of inputs for transport coefficients ($\eta, \zeta$) and entropy density (or other thermodynamical quantities like pressure, speed of sound)

IV. SUMMARY AND DISCUSSIONS

In summary, we have pointed out a phenomenological issue of hadron resonance gas model, which should be seriously considered during the calculation of shear viscosity for RHIC or LHC matter and the facts are as follows. At first, on the basis of the dissipation process, we have classified our HRG members into two categories - Non-resonance members ($\pi$, $K$ and $N$) and Resonance members (hadrons other than pseudo-scalar meson nonet and baryon octet). Former members participate in dissipation via strong interaction scattering processes, where as the contribution from latter members is coming from their strong decay processes. We consider only strong interaction processes as other interaction (weak or electromagnetic) processes are meaningless for this scale of the (hadronic) system [15]. Beyond this normal filtering, we have chosen only those strong decays, whose mean life times are not exceeding the life time of the hadronic medium, which is roughly chosen as 10 fm. Now selecting those resonances and using their mean life times as relaxation times in the expression of shear viscosity, we get some non-zero value of $\eta$, which always has to be considered as a background value due to resonances in HRG model. Taking Compton lengths ($1/m_{\pi,K,N}$) as minimum scattering lengths of NR particles ($\pi, K, N$), we get a lower limit estimation for NR component, which has to be added with resonance component. We notice that the lower limit of total $\eta$, normalized by entropy density $s$ is greater than its quantum lower bound at high temperature range. Owing to this fact, we may conclude that $\eta/s$ in HRG model never reach its quantum lower bound near the transition temperature $T_c$ because of unavoidable resonance contribution.

Similar to lower limit, the upper limit of shear viscosity can be tuned by equating the relaxation length of NR particles to the dimension of medium, produced in heavy ion experiments. The contribution from resonance component is very definite or known since it is determined from the experimental values of mean life time of their strong decays, documented in PDG [53]. Hence, only adjustable quantities are relaxation lengths of NR particles, whose lower and upper possible values basically give a narrow numerical band in shear viscosity and other transport coefficients of hadronic matter.

Next, we have taken a particular example, where absolute values of transport coefficients are obtained. Here, we have estimated absolute values of relaxation lengths for non-resonance particles from their scattering length data. From the momentum distribution of their relaxation lengths, we have found that pion and kaon relaxation lengths exceed from the fireball dimension beyond some upper values of momentum, which is again different for different temperature. Now when we take this temperature dependent momentum cut-off as an upper limit of integration, then the values of transport coefficients remain within our proposed numerical band. However, when we take entire momentum distribution, those values don’t remain within the band. Through this example, we want to emphasize the point - the values of transport coefficients for hadronic matter will be remain within our proposed numerical band, if we impose the idea of finite size dissipation.

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Appendix A: HRG

The momentum integral in Eq. (4) can be performed analytically and be expressed in terms of modified Bessel functions \( K_n(x) \). Below we summarize the final forms of all the thermodynamic quantities:

\[
\ln Z = \frac{VT^3}{2\pi^2} \sum_{h \in \{\text{hadrons}\}} \sum_{n=1}^{\infty} g_h \frac{(a_h)^{n+1}}{n^2} \left( \frac{m_h}{T} \right)^2 K_2 \left( \frac{nm_h}{T} \right) \exp \left( \frac{n \mu_h}{T} \right) \tag{A1}
\]

\[
P = \frac{T^4}{2\pi^2} \sum_{h \in \{\text{hadrons}\}} \sum_{n=1}^{\infty} g_h \frac{(a_h)^{n+1}}{n^2} \left( \frac{m_h}{T} \right)^2 K_2 \left( \frac{nm_h}{T} \right) \exp \left( \frac{n \mu_h}{T} \right) \tag{A2}
\]

\[
\varepsilon = \frac{T^4}{2\pi^2} \sum_{h \in \{\text{hadrons}\}} \sum_{n=1}^{\infty} g_h \frac{(a_h)^{n+1}}{n^2} \left( \frac{m_h}{T} \right)^2 \exp \left( \frac{n \mu_h}{T} \right) \left[ \left\{ 1 - \left( \frac{nm_h}{T} \right) \right\} K_2 \left( \frac{nm_h}{T} \right) + \frac{1}{2} \left( \frac{nm_h}{T} \right) \right] \tag{A3}
\]

\[
n_k = \frac{T^3}{2\pi^2} \sum_{h \in \{\text{hadrons}\}} \sum_{n=1}^{\infty} g_h \frac{(a_h)^{n+1}}{n^2} \left( \frac{m_h}{T} \right)^2 K_2 \left( \frac{nm_h}{T} \right) \exp \left( \frac{n \mu_h}{T} \right) ; \ k = B, Q, S, ... \tag{A4}
\]

\[
\left( \frac{\partial P}{\partial T} \right) = \frac{T^3}{2\pi^2} \sum_{h \in \{\text{hadrons}\}} \sum_{n=1}^{\infty} g_h \frac{(a_h)^{n+1}}{n^2} \left( \frac{m_h}{T} \right)^2 \exp \left( \frac{n \mu_h}{T} \right) \left[ \left\{ 2 - \left( \frac{nm_h}{T} \right) \right\} K_2 \left( \frac{nm_h}{T} \right) + \frac{1}{2} \left( \frac{nm_h}{T} \right) \right] \tag{A5}
\]

\[
\left( \frac{\partial \varepsilon}{\partial T} \right) = \frac{T^3}{2\pi^2} \sum_{h \in \{\text{hadrons}\}} \sum_{n=1}^{\infty} g_h \frac{(a_h)^{n+1}}{n^2} \left( \frac{m_h}{T} \right)^2 \exp \left( \frac{n \mu_h}{T} \right) \left[ \left\{ \frac{3}{4} \left( \frac{nm_h}{T} \right)^2 K_0 \left( \frac{nm_h}{T} \right) \right\} + \frac{1}{2} \left( \frac{nm_h}{T} \right) \right] \tag{A6}
\]

\[
\left( \frac{\partial P}{\partial \mu_h} \right) = n_k ; \ k = B, Q, S, ... \tag{A7}
\]

\[
\left( \frac{\partial \varepsilon}{\partial \mu_h} \right) = \frac{T^3}{2\pi^2} \sum_{h \in \{\text{hadrons}\}} \sum_{n=1}^{\infty} g_h \frac{(a_h)^{n+1}}{n^2} \left( \frac{m_h}{T} \right)^2 \exp \left( \frac{n \mu_h}{T} \right) \left[ \left\{ \frac{1}{2} \left( \frac{nm_h}{T} \right) \right\} K_1 \left( \frac{nm_h}{T} \right) \right] + \left\{ 2 - \left( \frac{nm_h}{T} \right) \right\} \left\{ K_2 \left( \frac{nm_h}{T} \right) + \frac{1}{2} \right\} \tag{A8}
\]

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