Acoustic coherent perfect absorbers

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Received 15 December 2013, revised 10 February 2014
Accepted for publication 13 February 2014
Published 21 March 2014
New Journal of Physics 16 (2014) 033026
doi:10.1088/1367-2630/16/3/033026

Abstract
In this paper, we explore the possibility of achieving acoustic coherent perfect absorbers. Through numerical simulations in two dimensions, we demonstrate that the energy of coherent acoustic waves can be totally absorbed by a fluid absorber with specific complex mass density or bulk modulus. The robustness of such absorbing systems is investigated under small perturbations of the absorber parameters. We find that when the resonance order is the lowest and the size of the absorber is comparable to the wavelength in the background, the phenomenon of perfect absorption is most stable. When the wavelength inside both the background and the absorber is much larger than the size of the absorber, perfect absorption is possible when the mass density of the absorber approaches the negative value of the background mass density. Finally, we show that by using suitable dispersive acoustic metamaterials, broadband acoustic perfect absorption may be achieved.

Keywords: metamaterial, acoustic metamaterial, coherent perfect absorber, acoustic absorption

Recently, acoustic metamaterials [1–36], which can exhibit almost any value of mass density and modulus, have provided a new approach to manipulating acoustic waves. Similar to their electromagnetic counterparts, acoustic metamaterials can exhibit a rich variety of new material types, such as negative mass density [1, 2], negative modulus [3], double negativity [4–13], controlled anisotropy [14, 15], etc. The origins of such special properties are attributed to the
local resonance [1–10], or coiled space [11–13], or anisotropic design in metamaterials [14, 15]. Based on these unique parameters, unprecedented wave phenomena such as low-frequency sound blocking [1, 2], acoustic negative refraction and lensing [7–22], and acoustic cloaking [23–34] have been proposed. Recently, the concept of acoustic metamaterials has also been further applied to enhance the absorption of sound energy [35, 36]. In [35], a type of ‘dark’ acoustic metamaterial composed of a resonant membrane structure has been designed, which can absorb sound waves of wavelength much larger than its thickness. In [36], broadband large absorption has been achieved by designing porous lamella-crystals. It has been shown that acoustic metamaterials have great potential in the applications of sound absorption.

Recently, in the field of optics, a new concept named optical coherent perfect absorbers (CPAs) [37] which can achieve perfect absorption (100%) of light have attracted a lot of research interest [37–46]. Optical CPAs may be regarded as the reverse process of laser generation. Besides the function of achieving total absorption, they also demonstrate a unique way of controlling light absorption by interference effects. Optical CPA imposes strict conditions on the material and loss parameters of the absorber. For most optical materials, the permeability is unity. Therefore, both the real parts and the imaginary parts of the permittivity have to be adjusted to specific values to realize the phenomenon of optical CPA.

In this paper, we investigate the possibility and conditions of achieving acoustic CPAs, which can totally absorb sound energy instead of light. Unlike optical materials, both of the two parameters of the acoustic materials, i.e. the mass density and modulus, can exhibit many different values. In particular, acoustic metamaterials have provided an approach to engineering the effective mass density and modulus in an almost arbitrary manner. Here, through numerical analysis and simulations in two dimensions, we show that acoustic CPA can be achieved by designing the mass density or bulk modulus of the absorber to be specific complex values. CPAs can be classified by angular momentum $m$ and resonance order $n$. We also investigated the robustness of such absorbing systems under small perturbations of the absorber parameters. Among different resonance orders, we find that the CPA corresponding to the lowest resonance order is the most stable. When the size of the absorber is much larger or much smaller than the wavelength in the background, the CPA effect turns out to be quite sensitive to the ratio change of the real parts of the absorber parameters. If the wavelengths both outside and inside the absorber are much larger than the size of the absorber, coherent perfect absorption is possible when the effective mass density of the absorber approaches the negative value of the background mass density. Finally, we investigate the dependence of the CPA parameters as a function of frequency. By using suitable dispersive acoustic metamaterials that match the CPA parameter dispersion, broadband acoustic perfect absorption may be achieved.

The acoustic wave equation in fluids is written as:

$$\nabla \cdot \left( \frac{1}{\rho} \nabla p \right) + \frac{\omega^2 \rho}{\kappa} = 0,$$

where $p$ is the pressure field, $\rho$ is the mass density, $\kappa$ is the bulk modulus. The time average of the divergence of energy flux $\vec{T} = \rho \vec{v}$, where $\vec{v}$ is velocity, gives

$$\int_0^T \nabla \cdot \vec{T} \, dt = \frac{1}{2} \left( \text{Re} \left( -i \omega \rho^* |\vec{v}|^2 \right) + \text{Re} \left( \frac{-i \omega}{\kappa^*} |p|^2 \right) \right).$$

If the medium is absorptive, then
\[ \int_0^T \nabla \cdot \mathbf{I} dt < 0. \] Therefore, it is easy to see that \( \text{Im}(\rho) > 0 \) and \( \text{Im}(\kappa) < 0 \) indicate energy absorption. Therefore, the absorption of sound energy shall be attributed to the imaginary parts of mass density and bulk modulus in theory.

We consider the case of cylindrical absorbers in two dimensions. A cylindrical absorber with radius \( R \), mass density \( \rho_1 \) and bulk modulus \( \kappa_1 \) is embedded in a background fluid of mass density \( \rho_0 \) and bulk modulus \( \kappa_0 \). In two dimensions, the pressure wave can be expanded into Hankel and Bessel functions as:

\[
\begin{align*}
    p_{\text{inside}} &= \sum_{m=-\infty}^{+\infty} c_m J_m(k_1 r) e^{im\varphi}, \\
    p_{\text{outside}} &= \sum_{m=-\infty}^{+\infty} [\alpha_m H_m^{(2)}(k_0 r) + \beta_m H_m^{(1)}(k_0 r)] e^{im\varphi},
\end{align*}
\]

in which \( \alpha_m, \beta_m \) and \( c_m \) are the coefficients of Hankel and Bessel functions for angular momentum quantum number \( m \). The wave vectors in the cylinders and background are \( k_1 = \omega/c_1 = \omega \sqrt{\rho_1/\kappa_1} \) and \( k_0 = \omega/c_0 = \omega \sqrt{\rho_0/\kappa_0} \), respectively. By substituting (2) into the boundary conditions, i.e. \( p_{\text{inside}} \big|_{r=R} = p_{\text{outside}} \big|_{r=R} \) and \( \left( \nabla_{\text{inside}} \right) \big|_{r=R} = \left( \nabla_{\text{outside}} \right) \big|_{r=R} \), we obtain:

\[
\begin{align*}
    c_m J_m(k_1 R) &= \left[ \alpha_m H_m^{(2)}(k_0 R) + \beta_m H_m^{(1)}(k_0 R) \right], \\
    c_m \rho_1 J_m'(k_1 R) &= \rho_0 \left[ \alpha_m k_0 H_m^{(2)}(k_0 R) + \beta_m k_0 H_m^{(1)}(k_0 R) \right],
\end{align*}
\]

and the scattering coefficient \( s_m \) is obtained as

\[
    s_m = \frac{\beta_m}{\alpha_m} = \frac{c_0 \rho_1 J_m(k_1 R) H_m^{(2)}(k_0 R) - c_0 \rho_0 J_m'(k_1 R) H_m^{(2)}(k_0 R)}{c_0 \rho_0 J_m'(k_1 R) H_m^{(1)}(k_0 R) - c_0 \rho_1 J_m(k_1 R) H_m^{(1)}(k_0 R)}. \tag{4}
\]

When the energy of the incident acoustic waves with angular momentum \( m \) is perfectly absorbed, the scattered waves should vanish, i.e. \( s_m = 0 \). This is the condition for acoustic coherent perfect absorbers. However, \( s_m = 0 \) is difficult to solve analytically. In this paper, we mainly rely on numerical analysis and simulations to solve the problem.

Comparing with most of the previous optical CPAs in which the loss parameter is the imaginary part of the permittivity, here, the loss parameter can be the imaginary parts of either the mass density or bulk modulus or both. Here we consider the following simple cases: (1) \( \text{Im}(\rho) > 0 \) and \( \text{Im}(\kappa) = 0 \); (2) \( \text{Im}(\rho) = 0 \) and \( \text{Im}(\kappa) < 0 \). For acoustic metamaterials, these cases correspond to absorptive meta-atoms with dipolar or monopolar resonances, respectively.

We first consider case 1 of \( \text{Im}(\rho) > 0 \) and \( \text{Im}(\kappa) = 0 \). The background medium is set to be water \( (\kappa_0 = 2.24 \times 10^9 \text{ Pa}, \rho_0 = 998 \text{ kg m}^{-3}) \) and the cylinder radius \( R = 0.037 \text{ m} \). The bulk modulus of the cylinder \( \kappa_1 \) is set to be \( 3 \times 10^9 \text{ Pa} \). The frequency is chosen to be 2000 Hz and the wavelength in the background is \( \lambda_0 = 0.75 \text{ m} \gg R \). Then, we can numerically obtain the complex value of \( \rho_1 \) according to \( s_m = 0 \) for different angular momentum \( m \). In figure 1, we show the field maps of four cases of coherent perfect absorption corresponding to \( m = 0, 1, 2, 3 \), respectively. For \( m = 1, 2, 3 \), a spiral wave front is produced due to the lack of reflection from
the absorber placed at the center. The corresponding $\rho_1$ are found to be $77\,830.2 + 2\,728.28i\text{kg m}^{-3}$, $-1\,133.23 + 170.56i\text{kg m}^{-3}$, $-1\,030.92 + 1.91i\text{kg m}^{-3}$, $-1\,009.38 + 0.008i\text{kg m}^{-3}$, respectively. In this case, one interesting observation is that for $\text{Re}(\rho_1) > 0$ for $m=0$ and $\text{Re}(\rho_1) < 0$ for $m=1, 2, 3$. The negative real parts of $\rho_1$ for $m \neq 0$ indicate that absorption is actually induced by surface-plasmon-like surface acoustic waves [20]. It can be realized by using acoustic metamaterials with negative mass density [1, 2].

Next, we consider case 2 of $\text{Im}(\rho) = 0$ and $\text{Im}(\kappa) < 0$. The background medium is still water but the cylinder radius $R$ is increased to be 0.371 m. The mass density of the cylinder $\rho_1$ is set to be 1300 kg m$^{-3}$. The frequency is still 2000 Hz. Thus, the wavelength in the background $\lambda_0 = 0.75m \sim 2R$. Then, we obtain the complex value of $\kappa_1$ according to $\kappa_m = 0$. In figure 2, we show the field maps of four cases of coherent perfect absorption for $m=1$. It can be seen that they correspond to different resonance orders $n = 1, 2, 3, 4$, which may be defined as the number of amplitude peaks from the center to the surface of the cylinder. The corresponding $\kappa_1$ are

**Figure 1.** (a)–(d) The field maps of total coherent perfect absorption for cases of $m = 0$, 1, 2, 3. The background is water and the incident wave frequency is 2000 Hz. The radius and the bulk modulus of the absorber are 0.037 m and $3 \times 10^9$ Pa, respectively. The complex mass densities of the absorber are obtained as (a) $77\,830.2 + 2\,728.28i\text{kg m}^{-3}$, (b) $-1\,133.23 + 170.56i\text{kg m}^{-3}$, (c) $-1\,030.92 + 1.91i\text{kg m}^{-3}$, (d) $-1\,009.38 + 0.008i\text{kg m}^{-3}$.
found to be $2.1 - 1.2i \times 10^9$ Pa, $0.82 - 0.24i \times 10^9$ Pa, $0.37 - 0.042i \times 10^9$ Pa, and $0.2 - 0.012i \times 10^9$ Pa, respectively. We note that the imaginary parts are all negative, indicating absorption. Similarly, for angular momentum $m = 2$, we also observe coherent perfect absorption corresponding to different resonance orders, which are shown in figure 3.

Figures 1, 2 and 3 show that acoustic CPA can be achieved under different angular momentum $m$ and resonance order $n$ by using absorbers with specific complex $\rho_1$ or complex $\kappa_1$. Such complex $\rho_1$ or complex $\kappa_1$ may be realized by using composite acoustic materials or acoustic metamaterials. However, in practice, there are always perturbations and the absorber parameters may deviate from the required precise value. Therefore, an important question is: how robust is the coherent perfect absorption system? To answer this question, we calculate the variation of $|s_m|$ within a small perturbation range of the specific complex $\rho_1$ or complex $\kappa_1$ of CPA ($s_m = 0$) and show it in figure 4. Generally, it is seen that $|s_m|$ is more sensitive to perturbations in the real parts than in the imaginary parts. In figures 4(e), (f), the sensitivity of

Figure 2. (a)–(d) The field maps of total coherent perfect absorption for cases of resonance order $n = 1, 2, 3, 4$. Angular momentum quantum number $m = 1$. The background is water and the incident wave frequency is 2000 Hz. The radius and the mass density of the absorber are $R = 0.371$ m and $1300$ kg m$^{-3}$, respectively. The complex bulk moduli of the absorber are obtained as (a) $2.1 - 1.2i \times 10^9$ Pa, (b) $0.82 - 0.24i \times 10^9$ Pa, (c) $0.37 - 0.042i \times 10^9$ Pa, (d) $0.2 - 0.012i \times 10^9$ Pa.
on \( \kappa_1 \) is observed to increase when the resonance order \( n \) increases. Therefore, although there are infinite resonance orders that can achieve CPA, the lowest one, i.e. \( n = 1 \), is the most stable one.

In the following, we investigate how the complex \( \rho_1 \) or complex \( \kappa_1 \) changes as a function of frequency or the other absorber parameter. The frequency dependence is significantly important because if we can engineer the frequency dispersion of the absorber material to exhibit the CPA dispersion, then we can realize broadband CPA in principle. From (4), we can see that \( s_m \) is dependent on \( k_0 R \) instead of \( \omega \) alone. In figures 5(a)–(c), we demonstrate the dependence of complex \( \kappa_1 \) as a function of \( k_0 R \) for \( m = 0, 1, 2 \), respectively. The background medium is water. The mass density of the absorber is set as 1300 kg m\(^{-3}\). The resonance order \( n \) is chosen to be 1. It is clearly seen that when \( k_0 R \) increases, bulk modulus \( \kappa_1 \) also increases in both the real and imaginary parts (negatively). This is reasonable because the wave vector inside the absorber...
Figure 4. (a)–(d) and (e)–(h) show, respectively, the absolute value of the scattering coefficient, i.e. \( |s_m| \), as a function of the real and imaginary parts of complex parameters near coherent perfect absorption for the cases shown in figures 1 and 2. Generally, \( |s_m| \) is found to be more sensitive to ratio changes in the real parts than in the imaginary parts. For different \( n \), the case of \( n = 1 \) is the most stable.
$k_iR = (k_i/k_0)k_0R = \sqrt{\kappa_i\rho_i/k_0\rho_0} \cdot k_0R$ should not change much so as to maintain resonance order $n = 1$. In figures 5(d), we show the dependence of complex $\kappa_1$ as a function of $\rho_1$ when $k_0R = \pi$ and $m = 1$. It is seen that $\kappa_1$ increases when $\rho_1$ increases. $k_iR = (k_i/k_0)k_0R = \sqrt{\kappa_i\rho_i/k_0\rho_0} \cdot k_0R$ is also changed less to maintain $n = 1$.

In figure 6(a)–(c), we demonstrate the dependence of complex $\rho_1$ as a function of $k_0R$ for $m = 0, 1, 2$, respectively. The background medium is water. The bulk modulus of the absorber is set as $3 \times 10^9$ Pa. The resonance order $n$ is chosen to be 1. For $m = 0$, it is clearly seen that when $k_0R$ increases, mass density $\rho_1$ decreases, in both the real and imaginary parts. This is reasonable because the wave vector inside the absorber $k_iR = (k_i/k_0)k_0R = \sqrt{\kappa_i\rho_i/k_0\rho_0} \cdot k_0R$ is almost unchanged so as to maintain resonance order $n = 1$. However, for $m > 0$, when $k_0R$ is large, $\rho_1$ also decreases when $k_iR$ increases; but when $k_iR$ is small, Re ($\rho_1$) turns negative and approaches $-\rho_0$, and Im ($\rho_1$) decreases to zero when $k_iR$ goes to zero. Such behavior is totally different from the cases in figures 5(a)–(c) and 6(a). The physical origin of this change in $k_iR$ dependence lies in the different behaviors of the resonant mode. It is known that when Re ($\rho_1$) is negative and $m > 0$, there exists surface-plasmon-like surface acoustic waves [20]. Such a resonant mode can induce a strong field on the surface of the absorber. Therefore, even when the wavelengths inside and outside the cylinder are much larger than the size of the cylinder, CPA can still be
achieved. In this case, for \( m > 0 \), we can expand the Bessel functions in (4) as
\[
J_m(x) \approx \frac{x^m}{2^m \cdot m!}
\]
and obtain
\[
\rho_1 \approx \rho_0 \frac{\pi (k_i R)^{2m} + i 2^{2m} (m - 1) ! m!}{\pi (k_i R)^{2m} - i 2^{2m} (m - 1) ! m!}.
\]
Obviously when \( k_0 \rightarrow 0 \), \( \rho_1 \rightarrow - \rho_0 \). Figure 6(d) shows the dependence of complex \( \rho_1 \) as a function of the bulk modulus for \( n=1, m=1 \), and \( k_i R = \pi \).

Figures 5 and 6 show that in order to have broadband acoustic CPA, the material of the absorber must be dispersive. Such materials may be constructed using acoustic metamaterials. It is well known that locally resonant metamaterials (see, e.g. [1]) are inherently dispersive. Here, we use the Lorentz model to describe the dispersion of the complex mass density and bulk modulus as
\[
\rho_1(f) = \rho_{1, \text{inf}} - \frac{f_p^2}{f^2 - f_0^2 + i f T_0} \quad \text{and} \quad \kappa_1^{-1}(f) = \kappa_{1, \text{inf}} - \frac{f_p^2}{f^2 - f_0^2 + i f T_0},
\]
and try to fit...
the dispersions of CPA parameters shown in figures 6(b) and 5(b), respectively, where \( f_0 \) is the resonant frequency, \( \Gamma_0 \) is the damping constant, and \( f_p \) is the plasma frequency. In figure 7, we can see that a good match is achieved between 4000 Hz and 6000 Hz by fitting the complex mass density in figure 7(a), and achieved between 3500 Hz and 4500 Hz by fitting the complex bulk modulus in figure 7(b). Acoustic metamaterials with such Lorentz responses should be capable of extending the phenomenon of acoustic CPA from a single frequency to a broadband regime.

Finally, we discuss the robustness dependence of acoustic CPAs for cases under various frequency regimes. In figure 8, we show the sensitivity of \( |s_m| \) on the complex \( \rho_1 \) or complex \( \kappa_1 \) for cases of \( m = 1 \), \( n = 1 \), but with different frequencies of \( k_0 R = 10 \times 2\pi \), \( k_0 R = 2\pi \), \( k_0 R = 2\pi/10 \), respectively. The real \( \kappa_1 \) and real \( \rho_1 \) has been chosen as \( 3 \times 10^9 \text{Pa} \) and 1300 kg m\(^{-3}\), respectively. It is seen that \( |s_m| \) becomes much more sensitive on the real parts of the complex \( \rho_1 \) or \( \kappa_1 \) in the low and high frequencies than in the middle frequency. In other words, the CPA effect is most stable when the background wavelength is comparable to the size of the absorber. Interestingly, the dependence of \( |s_m| \) on the imaginary parts of the complex \( \rho_1 \) or \( \kappa_1 \) is much more robust. Even when the imaginary parts change 20% from the CPA value, very large absorption \( (|s_m| < 0.1) \) is still observed.

We note that the condition of CPA can actually be regarded as the issue of impedance matching of the boundary fields of the dissipative resonant mode inside the absorber and the incident waves in the background. Here we have discussed the characteristics of some typical resonant solutions that demonstrate acoustic CPA, but not all the solutions. For instance, under
In summary, we have shown that acoustic CPAs can be achieved by using absorbers with specific mass density and bulk modulus. CPAs may be classified by angular momentum $m$ and resonance order $n$. Under a small disturbance of the absorber parameters, this effect is most robust when the resonance order is the lowest and the size of the absorber is comparable to the wavelength in the background. When both the wavelengths inside and outside the absorber are much larger than the size of the absorber, perfect absorption requires that the mass density of the absorber be the negative value of the background mass density. The dependence of the CPA parameters as a function of frequency is also investigated. By using suitable dispersive acoustic metamaterials that match the CPA parameter dispersion, broadband acoustic perfect absorption may be achieved.

Some extreme conditions such as the long wavelength limit, other interesting solutions of CPA may exist, as has been shown in [35].

\textbf{Figure 8.} (a–c) and (d–f) show the absolute value of the scattering coefficient, i.e. $|\varepsilon_m|$, as a function of the real and imaginary parts of complex mass density ($\text{Re}(\rho)$ and $\text{Im}(\rho)$) or complex bulk moduli ($\text{Re}(\kappa)$ and $\text{Im}(\kappa)$) for the coherent perfect absorption cases all corresponding to $m=1$, $n=1$, but at different frequencies of $k_{\omega} R = 10 \times 2\pi$, $k_{\omega} R = 2\pi$, $k_{\omega} R = 2\pi/10$, respectively. $|\varepsilon_m|$ turns more sensitive on the real parts of the complex $\rho$ or $\kappa$ in both low and high frequency regimes, but its dependence on the imaginary parts of the complex $\rho$ or $\kappa$ is changed much less with frequency.
Acknowledgments

We thank H Y Chen and J Luo for helpful discussions. This work is supported by the State Key Program for Basic Research of China (no. 2012CB921501), the National Natural Science Foundation of China (no. 11374224, 11104196, 11304215), the Natural Science Foundation of Jiangsu Province (no. BK2011277, BK20130281), the Program for New Century Excellent Talents in University (NCET), and a project funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD).

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