Theory of High $T_c$ Ferrimagnetism in a Multi-orbital Mott Insulator

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We propose a model for the multi-orbital material Sr$_2$CrOsO$_6$ (SCOO), an insulator with remarkable magnetic properties and the highest $T_c \approx 725$ K among all perovskites with a net moment. We derive a new criterion for the Mott transition $(\tilde{U}_1 \tilde{U}_2)^{1/2} > 2.5W$ using slave rotor mean field theory, where $W$ is the bandwidth and $\tilde{U}_1(2)$ are the effective Coulomb interactions on Cr(Os) including Hund’s coupling. We show that SCOO is a Mott insulator, where the large Cr $\tilde{U}_1$ compensates for the small Os $\tilde{U}_2$. The spin sector is described by a frustrated antiferromagnetic Heisenberg model that naturally explains the net moment arising from canting and also the observed non-monotonic magnetization $M(T)$. We predict characteristic magnetic structure factor peaks that can be probed by neutron experiments.

Transition metal oxides [1] with partially filled 3d and 4d shells have dominated materials research in past decades, leading to such spectacular phenomena as high $T_c$ superconductivity and colossal magnetoresistance. We expect the next revolution to involve 5d oxides where the interplay of spin-orbit coupling (SOC) and strong correlations can lead to novel phases of matter. Recent experimental efforts have focused on iridium oxides [2][5], driven by the possibility of exotic phases [6][9]. Here we explore other equally promising experimental systems, the 5d oxides containing osmium that have received far less attention and provide a rich area for investigation.

We focus on the double perovskite (DP) family of oxides with the formula $A_2B'B'O_6$. A is an alkali or alkaline earth metal while B and B' are two transition metals (TM) arranged on a 3D checkerboard lattice. With two TM ions, the range of properties span metals to insulators, ranging from ferromagnets, antiferromagnets, ferroelectrics, multiferroics, to spin liquids [10][13].

The most studied DP is Sr$_2$FeMoO$_6$, a half metallic ferrimagnet with $T_c \approx 420K$ [11][14]. It is now well-understood that a generalized double exchange mechanism can explain ferromagnetism with the scale of $T_c$ set by the kinetic energy of itinerant electrons [15]. From this perspective, the observation of an even higher $T_c \approx 725K$, the highest $T_c$ amongst all perovskites with a net moment, in an insulator Sr$_2$CrOsO$_6$ (SCOO) [17] is rather puzzling.

There have been several important density functional theory (DFT) calculations of SCOO [17][25][27], nevertheless many puzzling questions remain open. (i) Why is SCOO an insulator [28]? It is not a band insulator, since the bands are partially filled. It is not obviously a Slater insulator, given the large moment observed on Cr. It is not a priori clear how it can be a Mott insulator either, given the weak correlations on Os relative to the large bandwidth of 5d orbitals. (ii) Why is there a net moment, given that both Cr and Os are in $d^5$ configurations? What is the role of SOC on Os? DFT calculations [25][27] find a net moment only if SOC is included, but XMCD experiments [17] show insignificant Os orbital moment. (iii) What sets the scale for the high magnetic $T_c \approx 725K$? DFT does not give a clear answer [29]. (iv) Why is the magnetization $M(T)$ a non-monotonic function of $T$? Finite temperature properties are, of course, very hard for DFT to address.

In the Letter we gain insight into all these questions within a theoretical framework that describes the hierarchy of energy scales in the charge and spin sectors. (1) We use a slave-rotor mean field theory to analyze the charge sector of SCOO and show that it is a multi-orbital Mott insulator. We derive a new criterion for the Mott transition for this system: $(\tilde{U}_1 \tilde{U}_2)^{1/2} > 2.5W$ where W is the bandwidth and $\tilde{U}_1(2)$ the effective Coulomb repulsion on Cr(Os), which includes the important effects of Hund’s coupling in addition to Hubbard $U$. Thus a small $\tilde{U}_2$ on Os is compensated by a strong $\tilde{U}_1$ on Cr and drives the system into a Mott insulating state.

(2) We find that the orbital angular momentum on Os is quenched, and the Os moment is purely of spin origin, in agreement with XMCD measurements [17].

(3) We show that the effective spin Hamiltonian for SCOO is a frustrated Heisenberg model with antiferromagnetic exchange between nearest neighbor Cr-Os as well as next-nearest neighbor Os-Os spins. We find that the net magnetic moment $M(0)$ arises, not because of SOC, but rather because of a canted magnetic ground state, found using both Monte Carlo (MC) simulations and a variational approach.

(4) The scale for the high $T_c$ is set by a $J_1 = 35$ meV superexchange between Cr-Os near neighbors.

(5) We find that we can obtain an unusual non-monotonic $M(T)$, in agreement with magnetization and neutron experiments [17], due to a subtle frustration effect between Cr-Os $J_1$ and Os-Os $J_2$.

(6) Finally, we predict distinct magnetic structure factor peaks for our canted ground state. These can be measured by neutrons and provide a “smoking gun” test of our theory.

Hamiltonian: The cubic crystal field splits $d$-orbitals and
only the $t_{2g}$ orbitals are occupied, with both Cr and Os in $d^{3}$ configurations. To investigate the metal-insulator transition, we consider

$$H = -t \sum_{\langle ij \rangle, \alpha} (d_{i \alpha}^{\dagger} c_{j \alpha} + h.c) - t' \sum_{\langle ij \rangle, \alpha} (c_{i \alpha}^{\dagger} c_{j \alpha} + h.c) + \hat{U}_{2} \sum_{i} \left[ \sum_{\alpha} n_{1i \alpha} \right]^{2} + \hat{U}_{2} \sum_{i} \left[ \sum_{\alpha} n_{2i \alpha} \right]^{2} - \mu \sum_{i, \alpha} (n_{1i \alpha} + n_{2i \alpha}) - \Delta \sum_{i, \alpha} (n_{1i \alpha} - n_{2i \alpha}). \quad (1)$$

This has orbitally-symmetric onsite Coulomb interactions that make the problem analytically tractable and provide insight, while capturing the essence of strong correlations in a multi-band system. Hund’s coupling will be discussed below. Here $d_{i \alpha}^{\dagger}$ ($d_{i \alpha}$) and $c_{i \alpha}^{\dagger}$ ($c_{i \alpha}$) are the creation (annihilation) operators on the $i^{th}$ Cr and Os sites respectively. $n_{1i \alpha}$ ($n_{2i \alpha}$) are the number operators on the $i^{th}$ Cr (Os) site. Note that the label $\alpha$ includes both spin and $t_{2g}$ orbital index. $t$ and $t'$ are the hopping amplitude between neighboring Cr-Os and Os-Os sites. The effective on-site Coulomb interaction $\hat{U}_{1(2)}$ is related to the atomic charge gap on Cr(Os), as discussed in detail below. The chemical potential is $\mu$ and $\Delta$, the charge transfer gap between Cr and Os.

**Multi-band Mott criterion:** We investigate the charge sector of (1) using the slave-rotor mean field (MF) theory [30]. This method captures the qualitative physics of dynamical mean field theory, and is well suited to studying the multi-band Mott transition; it has recently been used for various other multi-band systems including pyrolores and iron pnictides [8, 31, 32]. We decompose the physical electron into an O(2) rotor that carries the charge and a spinon that carries the spin and orbital degrees of freedom. We write $d_{i \alpha}^{\dagger} = f_{i \alpha}^{\dagger} e^{i \phi_{i}}$ and $c_{i \alpha}^{\dagger} = g_{i \alpha}^{\dagger} e^{i \phi_{i}}$ where $f_{i \alpha}^{\dagger}$ and $g_{i \alpha}^{\dagger}$ are the fermionic spinon creation (annihilation) operators on the $i^{th}$ Cr and Os sites respectively. The bosonic rotor operator $\phi_{i}$ is conjugate to the charge on Cr(Os) at site $i$. Substituting into (1) and performing a MF decomposition of the hopping terms, we obtain spinon and rotor Hamiltonians that we solve self-consistently.

We calculate $\langle \cos \theta \rangle$ and $\langle \cos \phi \rangle$ which are used as diagnostics of the metal-insulator transition. When both quantities vanish, the system is a Mott insulator with suppressed charge fluctuations, while non-zero values lead to a metal. For simplicity, we first work at the particle-hole (p-h) symmetric point: $\mu = \mu_{0}$; see text. Combination of small $\hat{U}_{2}$ on Os and large $\hat{U}_{1}$ on Cr can drive the system Mott insulating. Effect of (c) Os-Os hopping $t'$ and (d) Cr-Os charge-transfer energy $\Delta$.

FIG. 1: Phase diagram obtained from slave-rotor mean field theory. (a) Schematic of Cr and Os orbitals and relevant energies. (b) Phase boundary of eq. (2) for the particle-hole symmetric case $t' = 0$, $\Delta = \Delta_{0}$, and $\mu = \mu_{0}$; see text. Combination of small $\hat{U}_{2}$ on Os and large $\hat{U}_{1}$ can drive the system Mott insulating. Effect of (c) Os-Os hopping $t'$ and (d) Cr-Os charge-transfer energy $\Delta$. 

where $W = 8t$ is the $t_{2g}$ bandwidth. This new Mott criterion generalizes the well-known result $U \gtrsim W$.

Deviations from the p-h symmetric point, relevant for real materials, require numerical solution of the slave rotor mean field equations. In general, the right-hand side of (2) is replaced by the the average kinetic energy, which depends on the filling and band structure [33].

The resulting phase diagram is shown in Fig. 1. We see from Fig. 1(c) that increasing $t'$ favors the metallicity by introducing an additional route for gaining kinetic energy. Fig. 1(d) shows that increasing the deviation from the symmetric point $\Delta = \Delta_{0}$ for a fixed $\mu = \mu_{0}$ and $t' = 0$, also favors metallicity as it increases charge fluctuations within a unit cell [33].

**Determining the effective $U$:** In order to apply our results to SCO0, we must relate the effective interaction $\hat{U}_{1(2)}$ to material parameters: the Hubbard $U$, Hund’s coupling $J_{H}$ and SOC $\xi$. This is done by matching the excitation energy in the atomic limit of (1) to that of the more general atomic Hamiltonian

$$H_{a}^{tt} = (U_{a} - 3J_{Ha}) \frac{\hat{N}_{a}(\hat{N}_{a} - 1)}{2} - 2J_{Ha} (\hat{S}_{a})^{2} + \frac{1}{2} J_{Ha} (\hat{T}_{a})^{2} - \lambda_{a} \hat{T}_{a} \cdot \hat{S}_{a} - \mu_{a} \hat{N}_{a}. \quad (3)$$

Here the label $a = 1, 2$ indicates Cr and Os sites respectively. $\hat{N}$ is the total number operator, $\hat{S}$ the total spin.
operator, and $T$ the total orbital angular momentum operator in the $t_{2g}$ manifold. The effective SOC $\lambda$ in a given $TS$ manifold is a function of $T$, $S$ and $\xi$ [33]. We work with a $d^{3}$ atomic ground state.

For the Cr site, we ignore SOC and find the atomic excitation gap to obtain $\tilde{U}_1 = U_1 + 2J_{H1}$ [33]. The effective repulsion is enhanced by Hund’s coupling, well known [34, 35] for half-filled orbitals. On the Os site, we include $\lambda_2$ and find $\tilde{U}_2 = U_2 + 2J_{H2} - 3\xi_{2}/2$ [33], so that SOC reduces the atomic charge gap.

We now can use our Mott criterion for SCOO. For Cr, we use $U_1 \approx 5$ eV and $J_{H1} \approx 1$ eV [1]. For Os, we use $U_2 \approx 2$ eV, $J_{H2} \approx 0.35$ eV and $\xi_2 \approx 0.3$ eV [36]. Thus $(\tilde{U}_1\tilde{U}_2)^{1/2} \approx 4$ eV which is larger than the critical value $2.5W \approx 3.75$ eV using the band theory estimate $W \approx 1.5$ eV [27]. We thus believe that SCOO is a Mott insulator. We emphasize that (2) is a conservative estimate [35, 36] for half-filled orbitals. On the Os site, we find

$$\tilde{U}_2\approx U_2 - 3\xi_{2}/2 J_{H2} + \lambda_2 \approx 5 eV,$$

and $W \approx 3 eV$, $(\tilde{U}_1\tilde{U}_2)^{1/2} \approx 1$ eV and $\tilde{U}_2\approx U_2 - 3\xi_{2}/2 J_{H2} + \lambda_2 \approx 5 eV$ and

$$\tilde{U}_2\approx U_2 - 3\xi_{2}/2 J_{H2} + \lambda_2 \approx 5 eV.$$

Among the assumptions, there is the possibility that SCCOO is likely an intermediate coupling material (see Fig. 2(b)).

Given the general rules of superexchange, while $J_2$ frustrates this ordering. Note that $J_2$ by itself is also frustrated since Os spins are on an FCC lattice. Given the complexity of estimating superexchange microscopically [35], we treat them as parameters and study the magnetic properties as a function of $J_2/J_1$.

**Effective Spin Hamiltonian:** Having shown that charge is localized, we turn to the spin sector. For simplicity, we take a strong coupling approach, although in reality SCOO is likely an intermediate coupling material (see Fig. 2(b)).

Given the large $J_H$ on Cr, the three spins align to yield $S=3/2$. We might expect that a large SOC $\lambda E \cdot S$ on Os might split the $5d_{3/2}$ into $J = 3/2$ and $J = 1/2$ manifolds. However, in the special case of a $d^{3}$ configuration, the orbital angular momentum is quenched. Thus we also get a $S=3/2$ spin on Os. While the absence of an orbital moment on Os is consistent with XMCD data [17], we must now explain the net moment, since SOC cannot be responsible for this.

The effective spin Hamiltonian is given by

$$H_{\text{eff}} = J_1 \sum_{<ij>} S_i^1 \cdot S_j^1 + J_2 \sum_{<ijj>} S_i^2 \cdot S_j^2,$$

Here $S_i^{1(2)}$ is a classical $S=3/2$ spin on the $i^{th}$ Cr(Os) site; see Fig 2(b). Given the general rules of superexchange for half-filled orbitals, $J_1$ between Cr-Os and $J_2$ between Os-Os are both antiferromagnetic. We have retained next nearest neighbor Os-Os couplings because $5d$ orbitals are much more extended than $3d$ orbitals. $J_1$ tends to order the spins into a Néel antiferromagnet, while $J_2$ frustrates this ordering. Note that $J_2$ by itself is also frustrated since Os spins are on an FCC lattice.

**Magnetic Structure:** In Fig. 2(a), we show the magnetic ground state obtained from MC simulation of $H_{\text{eff}}$ for $J_2/J_1 = 0.4$. The magnetic structure factor has two large peaks at $q_0 = (\pi, \pi, \pi)/a$ and $q_1 = (\pi, \pi, 0)/a$, where $a$ is the nearest neighbor Cr-Os separation. We gain insight into this ground state using a variational analysis. Fourier transforming (4) we get $H_{\text{eff}} = \sum q \phi^\dagger(q) \cdot H(q) \cdot \phi(q)$ where $H_{11} = 0$, $H_{12} = H_{21} = J_1 (\cos(q_x a) + \cos(q_y a) + \cos(q_z a))$, $H_{22} = J_2 (\cos(q_x - q_y a) + \cos(q_y - q_z a) + \cos(q_z - q_x a))$ and $\phi^\dagger = (S^1(q))^\dagger, S^2(q)^\dagger)$. For each $q$ we diagonalize $H(q)$, and find that $q_0 = (\pi, \pi, \pi)/a$, the usual antiferromagnetic Néel ordering of Cr and Os spins, minimizes the energy. While it satisfies $|S_1|^2 + |S_2|^2 = 2S^2$, it does not satisfy the constraints $|S_1|^2 = S^2$ and $|S_2|^2 = S^2$ for $J_2/J_1 \neq 0$.  

![Fig. 2](image)


Upon generalizing the solution to include two different q’s that minimize the energy and also satisfy the constraints, we find that $q_0 = (\pi, \pi, \pi)/a$ and $q_1 = (\pi, \pi, 0)/a$ in agreement with the MC results. Our prediction of the structure factor peaks can be checked by neutron scattering, and should prove to be an important test of our theory.

**Magnetization $M(T)$:** We calculate $M(T)$ in 3D using finite temperature classical MC simulations. Fig. 2(c) shows that (i) $M(T = 0)$ is finite, (ii) $M(T)$ has an unusual non-monotonic function of temperature, (iii) a large value of $T_c$ for $J_1 = 35$ meV and $J_2/J_1 = 0.4$. Remarkably, all of these results compare very well with experimental data shown in the inset of Fig. 2(c) [17].

The non-zero value of $M(0)$ arises from the canted nature of the ground state discussed above; see Fig. 2(a). There are some quantitative differences with experiment, e.g., $M(T = 0) \approx 0.75\mu_B$ and it peaks at $\approx 0.9\mu_B$ while the simulation gives $M(T = 0) \approx 0.5\mu_B$, which could be due to any one of several effects ignored in our minimal model, like higher order effects of SOC or distortion from the cubic lattice.

The non-monotonicity of the net $M(T)$ arises from a competition between the magnetizations on the two sublattices: for essentially antiparallel moments $M = M_{Cr} - M_{Os}$. At low $T$, the Os moments are stuck in the canted state and $M_{Os}$ does not change significantly with temperature. However $M_{Cr}$ drops faster leading to a decrease in $M$. This trend continues till about 150K. At intermediate temperatures, Os spins have a smaller spin stiffness compared to Cr spins because $J_2$ acts only on Os sublattice. Thus Os spins become more “floppy” and depolarize faster leading to an increase in $M$ with temperature. Close to $T_c$, both Cr and Os moments drop very rapidly and the net moment goes to zero at $T_c$. This provides a qualitative understanding of the non-monotonic behavior. The behavior shown in Fig. 2(c) depends sensitively on the choice of $J_2/J_1$, and $M(T)$ for different $J_2/J_1$ is discussed in ref. [33].

**Discussion:** For weakly correlated materials, the magnetism is of the Slater/spin-density wave form where the state above $T_c$ is a metal and the charge gap is determined by magnetic ordering. In a local-moment Mott insulator, the charge gap scale is set by the Coulomb interaction and is much larger than $T_c$, set by superexchange. SCOO is in effect just on the Mott side of this crossover between Slater and Mott magnetism. By combining a weakly interacting 5d Os with a strongly interacting 3d Cr, we are able to tune the interaction strength to an intermediate regime and, therefore, maximize the $T_c$. We already see some evidence for the emergence of such a crossover behavior: NaOsO$_3$ is a Slater insulator with a $T_N \approx 410$ K, while LaCrO$_3$ a strong coupling Mott insulator with a 3.5 eV charge gap and $T_N \approx 320$ K. SCOO is an intermediate coupling on the Mott side, and this generates a very high ordering temperature.

Moreover, the insulating and magnetic mechanisms that we have unravelled in SCOO are an important step for understanding the trends, such as metal to insulator transitions, in materials with fixed Os and varying 3d ion (Sc-Cr-Fe-Co), as well as fixed Cr and varying 5d ion (W-Re-Os) [see Table 1].

**Conclusion:** We have established a general theoretical framework for understanding Mott insulators in DP’s with half-filled bands. It can be easily generalized to other materials like Sr$_2$MnOsO$_6$, Sr$_2$FeOsO$_6$ [29]. An important open question is the effect of doping these multiband Mott insulators. Away from half-filling, SOC will become important, and the interplay of strong SOC and interactions may produce exciting new physics.

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