Solution to Chance Constrained Programming Problem in Swap Trailer Transport Organisation based on Improved Simulated Annealing Algorithm

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Abstract

Swap trailer transport organisation problem originates from the traditional vehicle routing problem (VRP). Most of the studies on the problems assume that the travelling times of vehicles are fixed values. In this paper, the uncertainties of driving times are considered and a chance constrained programming problem is proposed. An improved simulated annealing algorithm is used to solve the problem proposed. The model and algorithm described in this paper are studied through a case study, and the influence of uncertainty on the results is analysed. The conclusion of this study provides theoretical support for the practice of trailer pickup transport.

Keywords: road transport, swap trailer transport, tractor despatching, random driving time, simulated annealing algorithm.

AMS 2010 codes: 90C05

1 Introduction

Swap trailer transport, as an efficient freight organisation mode, is widely used in developed countries. There are many kinds of vehicle combination modes for swap trailer transport in foreign countries, such as truck and trailer; tractor, semi-trailer and tractor; semi-trailer and trailer and so on. Since the Ministry of Transport issued the notice on promoting the development of swap trailer transport in 2009, the swap trailer transport has received strong support and fewer institutional constraints. However, at the present stage, the mode of vehicle combination allowed by enterprises is still limited to tractor and semi-trailer. Therefore, this paper focuses on the tractor despatch, which is the most core issue in the organisation work of swap trailer transport in China.

Compared with trucks, swap trailer transport can achieve higher vehicle efficiency by free separation and combination of the power part and the cargo part [1, 2]. The academic community generally believes that the swap trailer routing problem is very complicated and it is an NP-hard problem. Some scholars think that the
traditional vehicle routing problem (VRP) is only a special case of swap trailer transport organisation problem [3]. At present, the research of the international academic community on such issues is mainly reflected in three categories. The first problem is truck and trailer routing problem (TTRP). Some customer points can be serviced by either a truck trailer or a separate truck, whereas other customers can provide only freight services by trucks. Set the number of trucks and trailers which are known, and the constraint of vehicle’s capacity. The goal is to find the lowest cost vehicle route set, if the route is closed, and each customer point is provided only one freight service. Representative study of TTRP can be found in Villegas et al. [4]. The second problem is roll-on–roll-off VRP (RRVRP) and it stems from urban garbage transport activities. The garbage truck transports the empty car to the garbage collection point. After the car is full of garbage, the garbage truck transports the heavy car to the garbage centralised processing station for unloading and subsequent processing. The operation mode of the garbage truck is similar to that of the tractor train and the semi-trailer. The research results of this problem are represented by Bodin et al. and Baldacci et al. [5, 6]. The third problem is tractor and semi-trailer routing problem (TSRP). So far, there were two main research backgrounds for the TSRP issue in the academic world: short-distance distribution and long-distance truck transportation. Short-distance transportation application background for TSRP problems includes in-plant transportation and local transportation. Liang studied TSRP for internal material transportation of large steel companies [7]. Fan established a mathematical model aiming at minimising the operating cost of collecting and despatching operations to study the scheduling problem of the towed and hoisted tractors [8]. Zhang used the LPG refinery as the tractor station and applied the tractor and semi-trailer mode to LPG transport [9]. In view of the economies of scale of the TSRP application scenario and its ability to solve the multi-to-multi relationship, this application scenario is more optimistic and the research in this paper is based on TSRP.

Most of the research on the swap trailer transport organisation problem assumes that the vehicle travel time is a certain value. However, in reality, unexpected factors such as traffic congestion, road maintenance and traffic restrictions and vehicle damage will lead to uncertainty in driving time. Therefore, it is more suitable for practical applications to consider the tractor optimisation problem with uncertain driving time. Some scholars have considered the random factors into the optimisation model of VRP and proposed the VRP with stochastic travel time (VRPSTT) [10, 11]. However, VRP is aimed at one-to-multi scenarios. This paper introduces the uncertainty of driving time into TSRP and proposes TSRP with stochastic travel time (TSRPSTT). In the multi-to-multi scenario, problems of tractor optimisation are studied, and the influence of uncertainty on the optimisation results is analysed. The TSRPTST model is discussed in Section 2, and its objective function and constraints are explained. The model solving algorithm is introduced in Section 3. In Section 4, a case study is analysed. Finally, in Section 5, the main conclusions of this paper are summarised.

2 Problem Formulation

2.1 Problem Description

The problem scenario can be described as follows. The transport network node set $V$ consists of a central station $v_0$, $n$ semi-trailer distribution points $v_i (i = 1, 2, ..., n)$ and several sides $A = \{(i, j)|i \neq j, i, j = 0, 1, 2, ..., n\}$. It is assumed that all tractors are stored at the central station $v_0$, and the freight demands come from both central station $v_0$ and semi-trailer distribution points $v_i (i = 1, 2, ..., n)$. The freight demand between each two nodes is recorded as matrix $B$.

$$B = \begin{pmatrix}
0 & r_{01} & 0 & \cdots & r_{0n} \\
r_{10} & 0 & r_{13} & \cdots & r_{1n} \\
r_{20} & r_{21} & 0 & \cdots & r_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
r_{n0} & r_{n2} & \cdots & r_{n(n-1)} & 0
\end{pmatrix}$$ (1)
where \( r_{ij}(i,j = 0, 1, ..., n) \) represents the freight demand between two nodes, and the value of demand is reflected by the number of full-loaded semi-trailers.

Traditional studies on VRP, TTRP or TSRP mostly focus on the minimum driving distance, fleet size or other traditional parameters. Scholars are increasingly inclined to incorporate environmental factors into organisational optimisation because of environmental problems. Moreover, one of the important indicators for evaluating the pilot scheme of enterprises’ swap trailer transport is CO\(_2\) emission in China. Therefore, this paper takes CO\(_2\) emission per ton kilometre as the objective function. The lower the index, the higher the efficiency of road freight transportation, and the smaller the impact on the environment.

Due to the increasingly stringent requirements of customers for the delivery time of goods, swap trailer transport organisations must consider how to complete the freight organisation within the specified time, named as time window. Therefore, it is assumed that there is a fixed time window \([e_i, l_i](i = 1, 2, ..., n)\) at each semi-trailer distribution point, where \( e_i \) and \( l_i \) represent the earliest and the latest service time at the \( i \)-th semi-trailer distribution point, respectively. If vehicle \( k \) arrives at node \( i \) earlier than \( e_i \), additional waiting time will be generated which is recorded as \( w_{ik} \). If the time is within the time window when the vehicle arrives at the node, then \( w = 0 \). In addition, the planning period of central station is represented by \([e_0, l_0]\), while \( e_0 \) and \( l_0 \) represent the start and end time of the scheme, respectively.

In practice, traffic congestion, road maintenance and traffic restriction, vehicle damage and other accidental factors will lead to the uncertainty of driving time. Vehicle travel time \( t_{ij}(i, j \in V) \) is assumed to obey the orthodox distribution in this paper, that is \( t_{ij} \sim N(\mu_{ij}, \sigma^2_{ij}) \), where \( \mu_{ij} \) represents the expected travel time between node \( i \) and node \( j \), and \( \sigma^2_{ij} \) is variance, indicating the degree of uncertainty in driving time. We assume that \( \mu_{ij} \) and \( \sigma^2_{ij} \) are independent of vehicle load.

### 2.2 Modelling

Since the scenarios in this paper involve random variables and belong to stochastic programming problem, Chance Constrained Programming (CCP) model is selected to solve the problem [12]. The CCP model is commonly used to analyse the stochastic decision-making system, and its constraints feature is that the chance-constrained conditions are established at least with a certain probability \( \alpha \) that represents the confidence level. In TSRPSTT, we consider two kinds of chance constraints: one is time window constraints, that is, the probability of vehicle arriving at semi-trailer distribution points or central station in time window is higher than the confidence level \( \alpha \). The time window chance constraints are as follows:

\[
\begin{align*}
\{ P\{e_i \leq s_i \leq l_i\} \geq \alpha \} & \quad i = 1, 2, ..., n \\
\{ P\{s_0 \leq l_0\} = 1 
\end{align*}
\]

(2)

where \( P \) is a probability measure; \( s_i \) represents the time when the vehicle arrives at the \( i \)-th node; \( e_i \) and \( l_i \) represent the earliest and the latest service time at the \( i \)-th semi-trailer distribution point and \( e_0 \) and \( l_0 \) represent the start and end time of the scheme, respectively. Since the travel time is a random variable, \( s_i \) is also a random variable. \( P\{s_0 \leq l_0\} = 1 \) means that all vehicles must return to the central station no later than the end of the plan. Confidence level \( \alpha \) controls the problem tightness and solving difficulty. The larger the value of \( \alpha \), the smaller the range of feasible solutions. The other is the travel time constraint, that is, the probability that the vehicle’s travel time cannot exceed a threshold \( B \) is higher than the confidence level \( \beta \). Its expression is as follows:

\[
P\left\{ \sum_{i,j \in V} (X^l_{ijk}t_{ij} + X^l_{ijk}t_{ij} + w_{ik}) \leq B \right\} \leq \beta, \quad \forall k
\]

(3)

where \( P \) is a probability measure; \( X^l_{ijk} \) and \( X^l_{ijk} \) are decision variables representing the number of times a tractor numbered \( k \) travels alone and with trailer between node \( i \) and node \( j \); \( t_{ij} \) represents the travel time between node \( i \) and node \( j \), that is, a random variable and \( w_{ik} \) represents the waiting time of the vehicle \( k \) at the node \( i \), which is also a random variable. Considering the CO\(_2\) emission per ton kilometre as the objective, the optimisation
model is established as follows:

\[
\gamma \left( \sum_i \sum_j \sum_k \left( X'_{ijk} c' d_{ij} + X^l_{ijk} c^l d_{ij} \right) \right) \min \frac{W \sum_i \sum_j \sum_k X^l_{ijk} d_{ij}}
\]

(4)

where \( \gamma \) is CO\(_2\) emission coefficient; \( c' \) and \( c^l \) indicate the fuel consumption coefficients of tractor travel alone and with trailer, respectively; \( W \) is the trailer load and \( d_{ij} \) denotes the distance between node \( i \) and node \( j \).

Subject to

\[
\sum_i X'_{ijk} + \sum_i X^l_{ijk} \geq 1, \forall j, k
\]

(5)

\[
\sum_i X'_{ijk} + \sum_i X^l_{ijk} \geq 1, \forall j, k
\]

(6)

\[
\sum_i X'_{ijk} + \sum_i X^l_{ijk} = \sum_i X'_{ijk} + \sum_i X^l_{ijk}, \forall j, k
\]

(7)

\[
\begin{cases}
P\{e_i \leq s_i \leq l_i \} \geq \alpha, i = 1, 2, ..., n
P\{s_0 \leq l_0 \} = 1
\end{cases}
\]

(8)

\[
P \left\{ \sum_{i,j \in V} (X'_{ijk} t_{ij} + X^l_{ijk} t_{ij} + w_{ik}) \leq B \right\} \leq \beta, \ \forall k
\]

(9)

where equations (5)–(7) ensure route closure and balance, and equations (8) and (9) are the opportunity constraints of time window and travel time, respectively.

3 An Improved Simulated Annealing Algorithm

Drawing on the research experience of VRP, TTRP, TSRP and VRPSTT algorithm, a simulated annealing (SA) algorithm is used to solve TSRPSTT. The specific process is as follows:

Step 1: Setting the initial number of tractors \( m = D/a \), where \( D \) is the total freight demand and \( a \) is an empirical parameter.

Step 2: A feasible tractor line set \( V \) is constructed according to the time window and the driving time constraint of the tractor, and \( m \) tractor routes are randomly selected as the initial solution \( X \).

Step 3: Setting initial temperature \( T = T_0 \). Let the outer loop count variable be \( \omega = 0 \) and the inner loop count variable be \( q = 0 \). The freight satisfaction rate is set to understand solution \( X \)'s freight demand satisfaction rate \( R_{best} = R(X) \).

Step 4: Outer loop count variable \( \omega = \omega + 1 \).

Step 5: Inner loop count variable \( q = q + 1 \).

Step 6: A tractor route is selected from \( V \) replacing a random one from \( X \) to generate a new solution \( Z \).

Step 7: Metropolis criterion judgement. According to the first Metropolis criterion, the freight demand satisfaction rates of route schemes \( X \) and \( Z \) are compared. If \( \Delta = R(Z) - R(X) \geq 0 \), \( X \) is replaced by \( Z \), that is \( X = Z \). If \( \Delta = R(Z) - R(X) < 0 \), enter the second Metropolis criterion. Let random number \( r \in (0, 1) \). If \( e^{(\Delta \lambda)/T} > \lambda \) (\( \lambda \) is the step size parameter), \( X \) is replaced by \( Z \), that is \( X = Z \).

Step 8: Computing \( R_{best} \) under current solution \( X_{best} \). \( X_{best} = X \), \( R_{best} = R(X) \).

Step 9: Judgement of inner loop termination. If \( q \) reaches the upper limit of the number of inner loop \( N_e \) at isothermal condition, that is, \( q = N_e \), enter the next outer loop and set \( T = T/k \), \( q = 0 \). Then return to Step 5.

Step 10: Judgement of outer loop termination. If the outer loop reaches the termination temperature \( T_F \), that is, \( T \leq T_F \), the outer loop terminate, others, return to Step 4.
Table 1  Freight volume between urban nodes (unit: veh)

| Node number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|------------|---|---|---|---|---|---|---|---|---|----|----|
| 1          | 0 | 1 | 2 | 1 | 3 | 1 | 1 | 0 | 0 | 0  |    |
| 2          | 1 | 0 | 6 | 0 | 4 | 7 | 18| 0 | 1 | 3  | 3  |
| 3          | 1 | 4 | 0 | 1 | 2 | 3 | 1 | 1 | 1 | 1  |    |
| 4          | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0  | 0  |
| 5          | 2 | 3 | 2 | 0 | 1 | 1 | 0 | 1 | 0 | 0  |    |
| 6          | 1 | 6 | 2 | 0 | 2 | 0 | 5 | 0 | 0 | 1  | 1  |
| 7          | 3 | 15| 3 | 0 | 1 | 4 | 0 | 1 | 2 | 1  | 1  |
| 8          | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0  |    |
| 9          | 0 | 1 | 1 | 1 | 1 | 0 | 2 | 0 | 0 | 0  |    |
| 10         | 0 | 2 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0  |    |
| 11         | 0 | 3 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0  | 0  |

Step 11: Solution of the optimal number of tractors. If $R_{\text{best}} < 1$, let $m = m + 1$, return to Step 2. If $R_{\text{best}} = 1$, the optimal number of tractors is obtained, that is, $m_{\text{best}} = m$.

Step 12: Solution of tractor route scheme. Bring the optimal number of tractors $m_{\text{best}}$ into Step 1 and reuse SA algorithm by adding a judgement in Step 10, which is if $R(X) > 1$, return to Step 4, otherwise, terminate the outer loop.

Step 13: According to the tractor route scheme, the total fuel consumption $C$, cargo turnover $TK$, CO$_2$ emission per ton kilometre $E$ and the ratio of tractor to trailer $RT$ are calculated.

4 Case Study

In case study, 11 cities in a certain area of China are selected as the swap trailer transport nodes, of which the city node 1 is the centre station and the remaining 10 cities are the semi-trailer distribution points. The optimal scheme of CO$_2$ emission per ton kilometre, the number of tractors, total fuel consumption, cargo turnover and the ratio of tractor to trailer are calculated. According to the relevant information published by the Ministry of Transport on the Model Table of Fuel Consumption of Road Transportation Vehicles, the expected driving speed of tractor is 50 km/h, the fuel consumption of tractor with trailer is 32 L/(100 km) and the fuel consumption of tractor driving alone is 18 L/(100 km). The maximum driving time $B$ of the tractor is 8 h, and the confidence level is 95%. Consider the semi-trailer load $W = 30$ t. For convenience of calculation, the number of semi-trailers is used to represent the freight demand between nodes as shown in Table 1. In addition, the distance between nodes is shown in Table 2, and the time window of each node is shown in Table 3.

Assuming that the travelling speed of the tractor obeys the orthogonal distribution $N(\mu, \sigma^2)$, where $\mu = 50$ km/h, and let the value of variance $\sigma^2$ be 0, 1, 4, 9 and 16, respectively. It is easy to see that when $\sigma^2 = 0$ the travel time is a fixed value, and the larger the variance value, the greater the uncertainty of travel time. The solution results with different variance values are shown in Table 4.

It is easy to see that when the travel time is fixed, $\sigma^2 = 0$, the optimal number of tractors required is the least, the total fuel consumption and CO$_2$ emission per ton kilometre are the smallest and the ratio of tractor to trailer is the largest. When the uncertainty of road travel time increases, the optimal number of tractors will increase, and the range will become larger and larger. This is because when the travel time is uncertain, the probability of the tractor arriving at the designated place within the specified time will decrease. In order to keep the confidence level unchanged, it is necessary to increase the number of tractors and reduce the travel distance of a single tractor to ensure the transportation timeliness requirement. Accordingly, the total fuel consumption and CO$_2$ emission per ton kilometre will increase, while the ratio of tractor to trailer will decrease. The turnover of goods does not change with the increase of road travel time uncertainty. This is because uncertainty of time
Table 2  Distance between urban nodes (unit: km)

| Node number | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1           | 0   | 60  | 40  | 100 | 80  | 55  | 90  | 75  | 25  | 105 | 110 |
| 2           | 60  | 0   | 80  | 155 | 100 | 70  | 130 | 115 | 75  | 160 | 130 |
| 3           | 40  | 80  | 0   | 120 | 45  | 30  | 50  | 105 | 55  | 100 | 150 |
| 4           | 100 | 155 | 120 | 0   | 160 | 145 | 135 | 50  | 80  | 50  | 85  |
| 5           | 80  | 100 | 45  | 160 | 0   | 30  | 45  | 145 | 100 | 130 | 185 |
| 6           | 55  | 70  | 30  | 145 | 30  | 0   | 65  | 130 | 80  | 130 | 165 |
| 7           | 90  | 130 | 50  | 135 | 45  | 65  | 0   | 135 | 100 | 100 | 185 |
| 8           | 75  | 115 | 105 | 50  | 145 | 130 | 135 | 50  | 85  | 50  | 90  |
| 9           | 25  | 75  | 55  | 80  | 100 | 80  | 100 | 50  | 0   | 85  | 90  |
| 10          | 105 | 160 | 100 | 50  | 130 | 128 | 100 | 85  | 85  | 0   | 130 |
| 11          | 110 | 130 | 150 | 85  | 185 | 165 | 185 | 50  | 90  | 130 | 0   |

Table 3  Time windows of semi-trailer distribution points in each urban node

| Node number | Time window       | Node number | Time window       | Node number | Time window       |
|-------------|-------------------|-------------|-------------------|-------------|-------------------|
| 1           | 7:00–19:00        | 5           | 7:30–13:00        | 9           | 9:30–16:00        |
| 2           | 9:00–15:00        | 6           | 9:30–14:00        | 10          | 10:00–18:00       |
| 3           | 8:30–17:00        | 7           | 8:00–17:00        | 11          | 8:30–15:30        |
| 4           | 8:00–17:30        | 8           | 9:00–16:30        |             |                   |

Table 4  Solution results of tractor despatching schemes with different variances

| The value of $\sigma^2_{ij}$ | 0   | 1   | 4   | 9   | 16  |
|-----------------------------|-----|-----|-----|-----|-----|
| Number of tractors          | 47  | 49  | 58  | 75  | 103 |
| $C$ (unit: L)               | 14,123 | 14,264 | 15,405 | 17,716 | 22,300 |
| $TK$ (unit: t·km)           | 567  | 567  | 567  | 567  | 567  |
| $RT$                        | 238  | 238  | 238  | 238  | 238  |
| $E$ (unit: g)               | 67.97 | 68.65 | 74.14 | 85.26 | 108.29 |
increases the number of tractors, which mainly increases the empty distance of tractors but it does not affect the distance of trailers. Therefore, the turnover of goods will not be affected.

5 Conclusion

In this paper, the uncertainties of travel time are introduced into TSRP, and the TSRPSTT is proposed. The model proposed in this paper considers the travel time as a random variable, which describes the unexpected situation that the vehicle may encounter in the process of travel. The modified model can better reflect the actual scene and easier be applied in practice. In the scenario of multi-to-multi, the tractor despatching problem is studied, and the influence of uncertainty on the optimisation results is analysed. The conclusions are as follows. When the travel time is a fixed value, the optimal number of tractors is the least, the ratio of tractor to trailer is the highest and the emission of CO$_2$ per ton kilometre is the smallest. With the increase of uncertainties in travel time, the number of tractors required increases, the ratio of tractor to trailer decreases, the CO$_2$ emission per ton kilometre increases and the range becomes larger and larger. The freight turnover of the system changes little with the increase of travel time uncertainty. In addition, this paper makes a preliminary discussion on the organisation of swap trailer transport under the condition of random travel time. However, assuming that the speed obeys the orthogonal distribution is too idealised, the model can be further validated by the actual speed probability distribution according to the actual investigation in the future, so as to enhance the practicability of the model.

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