Symmetries and similarities for spin orientation parameters in $e^+e^- \rightarrow ZH, Z\gamma, ZZ$ at SM thresholds

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Abstract

We consider the spin orientation of the final $Z$ bosons for the processes in the Standard Model. We demonstrate that at the threshold energies of these processes the analytical expressions for the $Z$ boson polarization vectors and alignment tensors coincide ($e^+e^- \rightarrow ZH, Z\gamma$) or are very similar ($e^+e^- \rightarrow ZZ$). In addition, we present interesting symmetry properties for the spin orientation parameters.
1 Introduction

The Standard Model (SM) of elementary particle physics has been phenomenologically very successful. However, as a theory it is less impressive. There are many fundamental questions that remain unanswered by the SM. Due to this, there are strong reasons to expect that there is a great deal of (new) physics beyond the SM, more fundamental with a characteristic mass scale. The present good agreement between the accelerator-based experimental data and the SM predictions suggests that the energy scale associated with any new physics model should be much higher than the energy reach of the colliders up to now. It is believed that the new accelerator, the Large Hadron Collider (LHC), started in 2009, is able to produce new physics particles, first of all new on-shell resonances or a single heavy new particle in association with the SM ones [1]. However, such a scenario for visualizing new physics experimentally is obviously beyond the reach of future $e^+e^-$ colliders, like the planned International Linear Collider (ILC).

There are methods to probe new physics at energies below the new physics mass scale. These more indirect scenarios are based on observations of small deviations from the SM predictions for processes with SM particles where new physics effects can arise only from non-standard interactions. The price to pay for such a possibility is the need for higher sensitivity, both experimentally and theoretically. In the case of future ILC-type colliders, the achievable sensitivities are expected to be dramatically (i.e. up to orders of magnitude) improved as compared to the ones obtained up to now. Among the various ways to increase the sensitivity we emphasize two. First, there is a reason to expect that processes including heavy particles are more sensitive to new physics manifestations. Second, processes including polarization effects have higher sensitivity than unpolarized processes. The possibility at ILC to use both longitudinally and transversely polarized initial beams provide additional means to increase the sensitivity for new physics.

Aforementioned circumstances considerably increases the potential to probe anomalous
interactions via spin effects in $e^+e^-$ annihilation into heavy particles. Therefore, the processes $e^+e^- \rightarrow ZH$, $Z\gamma$, and $ZZ$ have been studied extensively. At the same time it is obvious that the search for new physics, even though performed at very sensitive future colliders, cannot be successful without knowing the SM predictions with appropriate precision. In Ref. [1] Lykken writes: “[…] to first approximation […] experimenters do not need to know anything about BSM [Beyond the SM] models in order to make discoveries – but they need to know a lot of the Standard Model physics!” In the context of searching for new physics in aforementioned processes via spin orientation, this means that the knowledge of all possible spin effects in these processes in the framework of the SM forms a good basis for rejecting or limiting anomalous couplings. In this paper we try to add our contribution to this basis.

In the framework of the SM we present and analyse analytical expressions for the $Z$ boson spin polarization vectors and alignment tensors for the processes $e^+e^- \rightarrow ZH$, $Z\gamma$, and $ZZ$ near the production threshold. For all these processes the initial beams are taken to have both longitudinal and transverse polarization components. We demonstrate that the expressions for the spin orientation parameters in these processes have similar shape and symmetry properties. Even more, the expressions for the $Z$ boson polarization vector and alignment tensor for the processes $e^+e^- \rightarrow Z\gamma$ and $ZH$ coincide.

The paper is organized as follows. In Sec. 2 we explain how spin orientation parameters can be described by density matrices. In addition we show how to obtain the actual spin polarization vector and alignment tensor of the final $Z$ boson. In Sec. 3 we present the analytical expressions for the spin orientation parameters of the three processes and dwell on similarities and interesting symmetry features. The results are checked with the help of positivity conditions. At the end of this paper we give our conclusions.


2 Description of spin orientation parameters

In physical processes the particles with non-zero spins are generally in a mixed spin state. Contrary to the pure state which can be described by a single wave function, the description of a mixed state deserves an incoherent superposition of $2s + 1$ orthogonal pure state wave functions. By definition this superposition means that in order to calculate the probability for finding a certain mixed state, one has to calculate the average of the probabilities of the pure states, assigning to each pure state an appropriate weight.

The most natural way to describe mixed states is to use the spin density matrix formalism. Technically, if interested in the spin orientation of a certain particle in the process, one replaces the element $u \bar{u}$ (spin-1/2 case) or $\epsilon_\mu \epsilon^*_\nu$ (spin-1 case) in the squared amplitude of the process by the mixed state spin density matrix. For our calculations the relativistic spin-1/2 density matrices for the initial beams and the non-relativistic spin-1 density matrices for the final $Z$-bosons are needed. The relativistic spin-1/2 density matrices have the familiar shape

$$\rho_\pm = \frac{1}{2}(\not{k}_\pm \pm m)(1 + \not{\gamma}_\pm)$$

where the upper signs refer to the electron and the lower ones to the positron. Here and in the following we use Feynman’s slash notation $a = a_\mu \gamma^\mu$. The polarization four-vectors are given by

$$s_\pm = (s_0, \vec{s}_\pm) = \left(\frac{\vec{k}_\pm \cdot \vec{\xi}_\pm}{m} \vec{\xi}_\pm + \frac{(\vec{k}_\pm \cdot \vec{\xi}_\pm)\vec{k}_\pm}{m(k_0 - m)}\right)$$

where $\vec{\xi}_\pm$ are the polarization vectors in the resp. rest frame of the particle. For the absolute value $|\vec{\xi}_\pm|$ of the mixed state polarization vectors one can take any value between zero and one. If the electron and positron have both longitudinal polarization (LP) and transverse polarization (TP), it is useful to divide the polarization vector $s_\pm$ up into LP and TP parts.

The limit $m/k_0 \to 0$ which is used in our calculations can be conveniently taken by making use of the approximation

$$s_\mu^\pm \approx \frac{h_\pm k_\mu^\pm}{m} + \tau_\mu^\pm$$
and subsequently setting $m = 0$. $h_\mp$ is the measure of LP and $\tau_\mp = (0, \vec{r}_\mp)$ is the TP four-vector with $\vec{r}_\mp$ the transverse component of the polarization vector ($k_\mp \cdot \vec{r}_\mp = 0$).

After inserting Eq. (3) into Eq. (1) and performing the limit $m \to 0$, one obtains

$$\rho = \frac{1}{2} (1 \pm h_\mp \gamma_5 + \gamma_5 \vec{r}_\mp) \vec{k}_\mp$$

convenient for our calculations.

For describing mixed states by $3 \times 3$ non-relativistic hermitian spin-density matrixes, at most 8 real parameters are needed. For expanding such a matrix one can use a basis containing the unit matrix $I$, the three spin matrices $S_i$, and the six combinations of products of two spin matrices in the form

$$S_{ij} = \frac{3}{2} \left( S_i S_j + S_j S_i - \frac{4}{3} \delta_{ij} \right).$$

In such a basis the basic elements are hermitian zero-trace matrices which makes their usage convenient. The density matrix expanded in this basis reads

$$\rho = \frac{1}{3} \left( I + \frac{3}{2} t_i S_i + \frac{1}{3} t_{ij} S_{ij} \right)$$

where here and in the following we sum over all indices which occur twice (Einstein’s convention). Note that this basis is over-determined. Instead of 9 matrices needed to expand the density matrix it contains 10 matrices. However, not all the elements of the basis are linearly independent. The relation

$$S_{xx} + S_{yy} + S_{zz} = 0$$

holds for three of the basic elements. In order to reduce the number of parameters from 9 to 8 independent ones, it is necessary to claim

$$t_{xx} + t_{yy} + t_{zz} = 0.$$  

Due to this constraint, one obtains

$$\rho_{ij} = \frac{1}{3} \delta_{ij} + \frac{1}{2} t_k (S_k)_{ij} + \frac{1}{6} t_{kl} (S_k S_l + S_l S_k)_{ij}$$
where $t_k = \text{Tr}(\rho S_k)$ is the polarization vector and $t_{kl} = \text{Tr}(\rho S_{kl})$ is the orientation or alignment tensor describing the alignment of spins.

When taking the spin-1 matrices in the representation $(S_k)_{ij} = -i\epsilon_{ijk}$ where $\epsilon_{ijk}$ is the totally antisymmetric tensor, one obtains

$$
(S_k S_l + S_l S_k)_{ij} = 2\delta_{kl}\delta_{ij} - \delta_{ki}\delta_{lj} - \delta_{kj}\delta_{li}.
$$

(10)

In this case, $\rho_{ij}$ from Eq. (9) can be written in the simple form

$$
\rho_{ij} = \frac{1}{3} \left( \delta_{ij} - \frac{3}{2} it_k \epsilon_{ijk} - t_{ij} \right).
$$

(11)

This form is used in our calculations.

### 2.1 Spin orientation of final particles

In our calculations it is supposed that the spin orientation of a single final $Z$ boson is observed. Accordingly, the squared amplitude of the process can be given in the form

$$
|M|^2 \sim S + V_i t_i + T_{ij} t_{ij}
$$

(12)

where $S$, $V_i$ and $T_{ij}$ are the scalar, vector and tensor built from the polarization parameters of the initial electron and positron, the kinematical parameters of all the particles participating in the process, and the coupling constants. Since the calculations are performed in the CM frame with vanishing electron mass at threshold energies, there are only two kinematical parameters – the electron three-momentum $\vec{k}$ and the mass $M_Z$ of the $Z$ boson.

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1 The spin orientation parameters $t_i$ and $t_{ij}$ determine the angular distribution of the decay products. Therefore, the $Z$ boson spin orientation parameters can be seen by analyzing the angular distribution of its decay products. According to the Particle Data Group [8], the $Z$ boson decays with 10% probability into lepton pairs ($e^+e^-$, $\mu^+\mu^-$, $\tau^+\tau^-$), and with 70% probability into hadron pairs. Considering only lepton pairs which can be clearly distinguished from the background, the analysis can be found in Sec. 4 of Ref. [3], giving the correlation between the spin orientation parameters and the angular distribution.
The squared amplitude determines the probability that the process produces a final $Z$ boson with spin orientation parameters $t_i$ and $t_{ij}$. On the other hand, the same probability can be expressed also as the traced product of two density matrices

$$\text{Tr}(\rho^r \rho) \sim 1 + \frac{3}{2} t^r_i t_i + \frac{1}{3} t^r_{ij} t_{ij}$$

where the real (actual) density matrices and their parameters are denoted by an upper index $r$. The symbols without this index are the density matrices and their parameters that are substituted into the squared amplitude instead of $\varepsilon^Z_{\mu} \varepsilon^{Z*}_{\nu}$. By comparing Eqs. (12) and (13) one can find the actual polarization vector and alignment tensor of the $Z$ boson,

$$t^r_i = \frac{2}{3S} V_i, \quad t^r_{ij} = \frac{3}{S} (T_{ij} + T\delta_{ij})$$

where the term $T$ incorporates the tracelessness (8).

### 2.2 Positivity conditions

The values of the parameters $t_i$ and $t_{ij}$ are restricted by their physical boundaries as extremal values. In this physical region parameters are linear independent. However, due to the positivity (non-negativity) condition of the density matrices they depend on each other non-linearly. It can be shown that the most restrictive requirements for the spin-1 density matrix can be given by the inequality

$$2 \text{Tr} \rho^3 - 3 \text{Tr} \rho^2 + 1 \geq 0.$$ (15)

If one expresses the density matrix through its parameters, one obtains

$$\frac{2}{9} - \frac{1}{2} t_i t_i + \frac{1}{2} t_i t_j t_{ij} - \frac{1}{9} t_{ij} t_{ij} - \frac{2}{27} t_{ij} t_{jk} t_{ki} \geq 0.$$ (16)

From this expression one can clearly see how the vector and tensor parameters depend non-linearly on each other. Eq. (16) is useful when analyzing the possibilities of tuning the final $Z$ boson spin orientation by varying the polarization of the initial beams.
3 Z boson spin orientation parameters at threshold

We are now ready to present the final $Z$ boson spin orientation parameters (polarization vectors and alignment tensors) at the threshold energies for the three processes $e^+e^- \rightarrow ZH, Z\gamma, ZZ$, calculated in the center-of-mass system. Note that the kinematics simplifies significantly when calculating at the threshold. On the other hand, the results obtained are also valid close to the threshold.

The first process $e^+e^- \rightarrow ZH$ could be important in clarifying the mechanisms of electroweak symmetry breaking proposed by various beyond-the-SM models (for an independent work on this subject see e.g. Refs. [5, 6]). In the SM at tree level this process is described by an $s$-channel Feynman diagram with a point-like $ZH$ vertex (Fig. 1). Actually, at lowest order there are three Feynman diagrams corresponding to the process.

![Figure 1: $s$-channel SM tree level diagram for $e^+e^- \rightarrow ZH$ with point-like $ZH$ vertex](image)

However, the two diagrams where the Higgs boson couples to the electron line can be ignored due to the smallness of the coupling which is proportional to the electron mass.

The two remaining processes $e^+e^- \rightarrow Z\gamma, ZZ$ have been most of all used to investigate possible new physics manifestations via anomalous gauge boson self-couplings $Z\gamma\gamma$, $Z\gamma Z$, and $ZZZ$. In the SM at tree level these processes are described by a $t$-channel and an $u$-channel diagram (Fig. 2).

The $Z$ boson spin polarization vectors and alignment tensors for the processes $e^+e^- \rightarrow$
\( e^- \rightarrow k_- \rightarrow p_1 \rightarrow Z, Z^\gamma, Z^H, Z^\gamma \)

\( e^+ \rightarrow k_+ \rightarrow p_2 \rightarrow Z, \gamma \)

Figure 2: (a) \( t \)-channel and (b) \( u \)-channel SM tree level diagram for \( e^+ e^- \rightarrow Z\gamma, ZZ \)

\( ZH, Z\gamma \) with vanishing electron mass have been calculated by us earlier [3, 7]. At the threshold energies the spin orientation parameters reduce to the same expressions,

\[
\bar{t}^Z_{\text{thres}} = \frac{f^{(2)\pm}}{f^+} \hat{k},
\]

\[
t^Z_{ij \text{thres}} = \frac{3}{2} (\hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij}) + \frac{3g_Lh_R}{f^+} \left[ (\bar{\tau}_- \cdot \bar{\tau}_+)(\hat{k}_i \hat{k}_j - \delta_{ij}) + \tau_- \tau_+ + \tau_- \tau_+ \right]
\]

where we used \( g_L = \frac{1}{2} (g_V + g_A) \), \( g_R = \frac{1}{2} (g_V - g_A) \) in order to write the expressions in a more symmetric form, and the short hand notation \( f^{(2)\pm} := g^2_L(1-h_-)(1+h_+) \pm g^2_R(1+h_-)(1-h_+) \).

\( h_+ \) and \( \bar{\tau}_\pm \) are the measures of the electron (positron) longitudinal polarizations and the electron (positron) transverse polarization vectors, respectively. \( \hat{k} \) indicates the unit vector along the electron momentum.

Even though somewhat peculiar, the coincidence of the \( Z \) boson spin orientation parameters of a \( s \)-channel diagram with the spin orientation parameters of a \( t \)-channel and an \( u \)-channel diagram at corresponding thresholds is expected. One can say that Eqs. (17) and (18) give the spin orientation parameters of the real \( Z \) boson in the process \( e^+ e^- \rightarrow Z \).

The Higgs boson is a spin-0 particle and, therefore, cannot affect the spin orientation of the \( Z \) boson at the threshold of \( e^+ e^- \rightarrow ZH \). In the same way, for \( p_1 \rightarrow 0 \) the photon in the process \( e^+ e^- \rightarrow Z\gamma \) can be considered as a radiative correction to the main process \( e^+ e^- \rightarrow Z \). The coincidence of the \( Z \) boson orientation parameters for the two different
processes, therefore, can be taken as a cross-check for the final expressions.

In order to obtain the spin orientation parameters for one of the final $Z$ bosons in the process $e^+e^- \rightarrow ZZ$ at threshold, we again take the (massless) electron and positron beams to be polarized, having both longitudinal and transverse polarization components. In summing over the spin orientations of the second $Z$ boson (in Fig. 2 indicated with momentum $p_2$), we obtain for the squared amplitude of the process at threshold

$$|M_{ZZ\text{thres}}|^2 = \frac{32e^4}{3\sin^2(2\theta_W)} \left\{ f_+^{(4)} + 3\frac{3}{4}f_-^{(4)}\hat{k} \cdot \hat{t} - \frac{1}{4}f_+^{(4)}\hat{k}_i\hat{k}_j t_{ij} \right. \left. + \frac{g_L^2g_R^2}{4} \left[ (\bar{\tau}_- \cdot \bar{\tau}_+)\hat{k}_i\hat{k}_j + \tau_-\tau_{+j} + \tau_-\tau_{+i} \right] t_{ij} \right\}$$

(19)

where the short-hand notation $f_\pm^{(4)} := g_L^4(1 - h_-(1 + h_+) \pm g_R^4(1 + h_-)(1 - h_+)$ is used. According to the rules given in Sec. 2.1 for the spin orientation parameters one obtains

$$\hat{t}_{ZZ\text{thres}} = \frac{f_+^{(4)}}{2f_+^{(4)}}\hat{k},$$

(20)

$$t_{ij\text{thres}} = -\frac{3}{4}(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}) + \frac{3g_L^2g_R^2}{2f_+^{(4)}} \left[ (\bar{\tau}_- \cdot \bar{\tau}_+)\hat{k}_i\hat{k}_j - \delta_{ij} + \tau_-\tau_{+j} + \tau_-\tau_{+i} \right].$$

(21)

Note that the terms $\delta_{ij}$ are introduced into $t_{ij}$ in order to make it traceless. Obviously, the spin orientation parameters in Eqs. (20) and (21) do not coincide with those of the processes $e^+e^- \rightarrow ZH, Z\gamma$ (Eqs. (17) and (18)) but are similar to them. This is quite understandable. Due to the two $Z$ boson final state the constants $g_L$ and $g_R$ appear twice in the squared amplitude. Therefore, the squares in $f_\pm^{(2)}$ had to be replaced by fourth powers in $f_\pm^{(4)}$. On the other hand, only the direction $\hat{k}$ of the electron and the transverse polarization vectors $\bar{\tau}_\pm$ are available to build the spin orientation parameters. As a result, besides the change of the power of the coupling constants the expressions can differ only by the constants in front of the various three contributions. This is indeed the case in the expressions obtained.
3.1 Symmetry properties of the polarization vectors

In this section we demonstrate that the polarization vectors of the Z boson in the different processes under consideration feature interesting symmetries. Expressing the polarization vectors as a function of the effective polarization parameter

\[ \chi = \frac{h_+ - h_-}{1 - h_+ h_-} \]  

(22)

of the initial beams, for the processes \( e^+ e^- \rightarrow ZH, Z\gamma \) we obtain \( \vec{t}^{Z}_{\text{thres}} = t^Z(\chi)\hat{k} \) where

\[ t^Z(\chi) = \frac{\chi + a}{a\chi + 1}, \quad a = \frac{g^2_L - g^2_R}{g^2_L + g^2_R} = \frac{2g_V g_A}{g^2_V + g^2_A} \approx 0.147 \]  

(23)

where we used \( g_V = \frac{1}{2}(-1 + 2\sin^2 \theta_W) \approx -0.037, g_A = -\frac{1}{2} = -0.5, g_L = -\frac{1}{2} \cos^2 \theta_W, \) and \( g_R = \frac{1}{2} \sin^2 \theta_W \) (with \( \sin^2 \theta_W = 0.2315, \cos^2 \theta_W = 0.7685 \) [8]). For the process \( e^+ e^- \rightarrow ZZ \) one obtains \( \vec{t}^{ZZ}_{\text{thres}} = t^{ZZ}(\chi)\hat{k} \) where

\[ t^{ZZ}(\chi) = \frac{1}{2} \frac{\chi + b}{b\chi + 1}\hat{k}, \quad b = \frac{g^4_L - g^4_R}{g^4_L + g^4_R} = \frac{4g_V g_A (g^2_V + g^2_A)}{(g^2_V + g^2_A)^2 + 4g^2_V g^2_A} \approx 0.294. \]  

(24)

For longitudinally unpolarized initial beams (\( \chi = 0 \)) one obtains quite similar values \( t^Z(\chi = 0) = a \approx 0.147 \) and \( t^{ZZ}(\chi = 0) = \frac{1}{2}b \approx 0.144 \). The similarity is related to the smallness of \( g_V \). Exact coincidence would hold for \( g_V = 0 \). However, in this case one would obtain \( a = b = 0 \).

On the other hand, the polarizations vanish for \( \chi = -a \) and \( \chi = -b \), respectively, i.e. \( t^Z(\chi = -a) = 0 \) and \( t^{ZZ}(\chi = -b) = 0 \). This is because the reciprocal functions read

\[ \chi^Z(t) = \frac{-t + a}{at - 1}, \quad \chi^{ZZ}(t) = \frac{-2t + b}{2bt - 1}. \]  

(25)

Note that for the process \( e^+ e^- \rightarrow t\bar{t} \) at threshold the SM result for the final top quark polarization can be expressed as \( \xi = \xi(\chi)\hat{k} \) where the function \( \xi(\chi) = -(\chi + c)/(c\chi + 1) \) has similar symmetry properties with \( \xi(\chi = 0) = \chi(\xi = 0) = -c \approx -0.408 \) [9].
3.2 Positivity condition for longitudinally polarized initial beams

In Fig. 3 the dependence of $t^Z(\chi)$ and $t^{ZZ}(\chi)$ as functions of the effective polarization parameter $\chi$ is presented. Note that $\chi$ can obtain values between $-1$ and $+1$ only if the initial beams have no transverse polarization components. While $t^Z$ takes all values between $-1$ and $+1$, the range for $t^{ZZ}$ is restricted to $|t^{ZZ}| \leq \frac{1}{2}$. This is mirrored also by the positivity condition in Sec. 2.2, giving a possibility to check our result in an independent way.

As mentioned before, the polarization vectors and the alignment tensors can depend only on the direction $\hat{k}$ of the electron beam and the transverse polarization vectors of the initial beams. Considering first longitudinally polarized beams ($\vec{\tau}_\pm = 0$) and choosing the $z$-axis to be the direction of the electron beam, one obtains the non-vanishing components

$$
t^Z_z = \frac{\chi + a}{a\chi + 1} \hat{k}_z = \frac{\chi + a}{a\chi + 1},
$$

$$
t^Z_{zz} = 1, \quad t^Z_{xx} = t^Z_{yy} = -\frac{1}{2}.
$$

(26)

Inserting the $t_{ij}$ into the positivity condition (16) it turns out that the condition is satisfied.
independently of the value of $t^Z$ and, therefore, also independently of the value of $\chi$. Hence, by varying the effective polarization parameter $\chi$ one can theoretically force the $Z$ boson polarization vector to take any value between $-1$ and $+1$. Practically achievable values are not far from the theoretical ones. When using the values $h_- = \pm 0.8$, $h_+ = \pm 0.6$, planned to be achieved at the ILC, one can reach value $\chi = \pm 0.95$ for the effective polarization parameter, leading to similar values for the $Z$ polarization (cf. Fig. 3(a)).

When doing the same with the components of the spin orientation parameters for the process $e^+e^- \rightarrow ZZ$, one obtains

\begin{align*}
t^Z_{zz} &= \frac{1}{2} \frac{\chi + b}{b\chi + 1} k_z = \frac{1}{2} \frac{\chi + b}{b\chi + 1}, \\
t^Z_{xz} &= -\frac{1}{2}, \quad t^Z_{xx} = t^Z_{yy} = \frac{1}{4}.
\end{align*}

(27)

Inserting the $t_{ij}$ into the positivity condition (16), one obtains $|t^Z_{z}| \leq 1/2$ which is consistent with Fig. 3(b).

### 3.3 Positivity condition as a test for the result

Also in the general case, i.e. with transversely and/or longitudinally polarized initial beams, the positivity condition (16) is in harmony with the spin orientation parameters $t_i$ and $t_{ij}$ of the processes $e^+e^- \rightarrow ZH, Z\gamma, ZZ$. Our investigation checks the obtained expressions for the spin orientation parameters and shows how much one can tune these parameters when varying the initial beam polarization parameters $h_{\mp}$ and $\vec{\tau}_{\mp}$.

Taking both transverse polarization vectors $\vec{\tau}_{-}$ and $\vec{\tau}_{+}$ to be directed along the $x$-axis of the coordinate system, for the processes $e^+e^- \rightarrow ZH, Z\gamma$ one obtains

\begin{align*}
t^Z_{z} &= \frac{f^{(2)}_{+}}{f^{(2)}_{+}}, \\
t^Z_{xx} &= -\frac{1}{2} + \frac{3g_{L}g_{R}}{f^{(2)}_{+}} r_{-x} r_{+x}, \\
t^Z_{yy} &= -\frac{1}{2} - \frac{3g_{L}g_{R}}{f^{(2)}_{+}} r_{-x} r_{+},
\end{align*}
Inserting the $t_{ij}$ into the positivity condition (16), one finds that the condition takes the form $0 = 0$. This means that the positivity condition is satisfied in any case, i.e. for any values $h_{\pm}$ and $\tau_{\mp}$. As a consequence, by varying the LP parameters $h_{\pm}$ (or $\chi$) one can tune the $Z$ boson polarization over the whole range $[-1,1]$ if $\bar{\tau}_-$ or $\bar{\tau}_+$ or both of them are equal to zero. In the same way, by varying $\bar{\tau}_{\pm}$ one can tune the alignment tensor for $h_-=h_+=0$.

Under the same conditions, for the process $e^+e^- \rightarrow ZZ$ one obtains

\begin{align*}
  t_{zz}^Z &= 1. \quad (28) \\
  t_{zz}^{ZZ} &= \frac{1}{2} \left\{ 1 - \frac{g_L^4}{f_+^{(4)}}(\tau_{-x}\tau_{+x})^2 \right\}. \\
  t_{xx}^{ZZ} &= \frac{1}{4} \left( g_L^2 g_R^2 \right)^2 \frac{1}{2} \left\{ 1 - \frac{g_L^4}{f_+^{(4)}}(\tau_{-x}\tau_{+x})^2 \right\}, \\
  t_{yy}^{ZZ} &= \frac{1}{4} \left( g_L^2 g_R^2 \right)^2 \frac{1}{2} \left\{ 1 - \frac{g_L^4}{f_+^{(4)}}(\tau_{-x}\tau_{+x})^2 \right\}, \\
  t_{zz}^{ZZ} &= -\frac{1}{2}. \quad (29)
\end{align*}

After inserting $t_{ij}$ into the positivity condition (16), one obtains the restriction

\begin{equation}
  (t_{ZZ}^Z)^2 \leq \frac{1}{4} - \frac{g_L^4 g_R^4}{f_+^{(4)}}(\tau_{-x}\tau_{+x})^2. \tag{30}
\end{equation}

In absence of the term depending on $\tau_{-x}\tau_{+x}$ this result reproduces the previous result, i.e. that by varying the parameters $h_{\pm}$ (or $\chi$) one can theoretically tune the $Z$ boson polarization from zero to 1/2 ($t_{zz}^{ZZ} = 0$ for $\chi = -b = -0.288$ and $t_{zz}^{ZZ} = 1/2$ for $\chi = 1$).

Finally, if we insert also the value $t_{zz}^{ZZ}$ into the positivity condition (16), we obtain a condition of the form

\begin{equation}
  (\tau_{-x}\tau_{+x})^2 \leq (1 - h_2^2)(1 - h_2^2). \tag{31}
\end{equation}

For $h_-=h_+=0$ the condition $(\tau_{-x}\tau_{+x})^2 \leq 1$ puts no constraints on the transverse polarization of the initial beams. On the other hand, if one of the initial beams is longitudinally polarized, restrictions on the transverse polarizations of the initial beams are imposed.
4 Conclusions

We have presented the spin orientation parameters of the final $Z$ boson for the three processes $e^+e^- \rightarrow ZH$, $Z\gamma$ and $ZZ$ in the Standard Model at the threshold energies and have found their properties. Note that a part of the properties found are valid also at higher energies. Therefore, the processes depend on the longitudinal polarization of the initial beams via the factors $f^{(2)}_{\pm} = g_L^2(1-h_-(1+h_+)+g_R^2(1+h_-)(1-h_+)$ or $f^{(4)}_{\pm} = g_L^4(1-h_-)(1+h_+)+g_R^4(1+h_-)(1-h_+)$ not only at the thresholds of the processes but also in the general case. Such properties could be helpful in separating SM contributions from new physics ones.

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