ENHANCED MOTOR IMAGERY-BASED EEG CLASSIFICATION USING A DISCRIMINATIVE GRAPH FOURIER SUBSPACE

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ABSTRACT
Dealing with irregular domains, graph signal processing (GSP) has attracted much attention especially in brain imaging analysis. Motor imagery tasks are extensively utilized in brain-computer interface (BCI) systems that perform classification using features extracted from Electroencephalogram signals. In this paper, a GSP-based approach is presented for two-class motor imagery tasks classification. The proposed method exploits simultaneous diagonalization of two matrices that quantify the covariance structure of graph spectral representation of data from each class, providing a discriminative subspace where distinctive features are extracted from the data. The performance of the proposed method was evaluated on Dataset IVa from BCI Competition III. Experimental results show that the proposed method outperforms two state-of-the-art alternative methods.

Index Terms— graph signal processing, EEG, simultaneous diagonalization, classification.

1. INTRODUCTION
Electroencephalogram (EEG) is a non-invasive, high temporal resolution brain imaging modality that captures functional and physiological changes within the brain [1, 2]. Motor imagery (MI) tasks are dynamic states during which neuronal activity in the primary sensorimotor areas modifies similar to a real executed movement [3]. MI tasks are extensively utilized in brain-computer interface (BCI) systems and can be classified by extracting features from EEG signals to identify a user’s mental state [4, 5]. Many methods have been proposed to classify MI tasks from EEG signals. Some approaches are based on extracting key information from the time and frequency domains [6-8]. Some other approaches of MI classification are focused on learning spatial filters from multichannel EEG signals to extract discriminative features from data [9, 10]. There are also many studies which have proposed applying mathematical transforms, such as wavelet transforms, to extract discriminative features via decomposition of EEG signals [11, 12].

The recently emerged field of graph signal processing (GSP) [13-15] has attracted great interest in different signal processing applications, in particular, signals defined on irregular domains such as the human brain [16-18]. In [19] the role of the GSP on the classification and dimensionality reduction of functional MRI (fMRI) data was evaluated. Promising results have also been presented that suggest the benefits of GSP in classification of EEG signals [20, 21].

In this work, using EEG data, we define a brain graph that characterizes the temporal correlation structure between the EEG electrodes. We then transform the EEG data into a spectral graph representation. The covariance structure of the resulting spectral representations is then computed, resulting in a matrix for each class of data. A classification framework is then proposed, in which simultaneous diagonalization of these two matrices provides the basis of a discriminative subspace that can be used to differentiate the two motor imagery tasks. An exploratory analysis is then performed to identify which spectral graph components from the data provide the most discriminative features. Results from the proposed method are also compared to two alternative methods that use simultaneous diagonalization.

The remainder of this paper is structured as follows. Section 2 gives an overview of the fundamental concepts and the proposed framework. Section 3 presents the experimental results and provides a discussion. Section 4 presents our concluding remarks.

2. MATERIALS AND METHODS

2.1. Graph signal processing fundamentals
Let $G = (V, E, A)$ denote an undirected, weighted graph, where $V = \{1, 2, \ldots, N\}$ denotes the graph’s finite set of $N$ vertices, $E$ denotes the graph’s edge set, i.e., pairs $(i, j)$ where $i, j \in V$, and $A$ is a symmetric matrix that denotes the graph’s weighted adjacency matrix. To exploit the spectral properties of the graph, the graph’s normalized Laplacian matrix is defined as $L = I - D^{-\frac{1}{2}}AD^{-\frac{1}{2}}$, where $D$ is the diagonal matrix of vertex degrees, i.e., $D_{ii} = \sum_j A_{ij}$ and $I$ is the identity matrix. Let $\ell^2(G)$ denote the Hilbert space of all square-integrable graph signals $f: V \rightarrow \mathbb{R}$ that are defined on $V$;
a graph signal \( f \in \ell^2(G) \) is an \( N \times 1 \) vector, whose \( n \)-th component represents the signal value at the \( n \)-th vertex of \( G \). Since \( \mathcal{L} \) is a real and positive semi-definite matrix, it can be diagonalized via eigenvalue decomposition as:

\[
\mathcal{L} = \mathbf{U} \Lambda \mathbf{U}^T, \tag{1}
\]

where \( \mathbf{U} = \{ \mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_N \} \) is a matrix of orthonormal eigenvectors of \( \mathcal{L} \) (an eigenvector in each column) with corresponding eigenvalues \( 0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N \leq 2 \) in the diagonal matrix \( \Lambda \). The eigenvalues define the graph Laplacian spectrum, and the corresponding eigenvectors form an orthonormal basis that spans the \( \ell^2(G) \) space. By using the Laplacian eigenvectors, a graph signal \( f \) can be transformed into a spectral representation, commonly denoted as the graph Fourier transform (GFT) of \( f \), obtained as \( \hat{f} = \mathbf{U} f \); the inverse GFT is obtained as \( f = \mathbf{U} \hat{f} \). Importantly, the GFT satisfies Parseval’s energy conservation relation [22], i.e., \( \| f \|_2^2 = \| \hat{f} \|_2^2 \). Graph Laplacian eigenvectors corresponding to larger eigenvalues entail a larger extent of variability, and as such, eigenvalues of the graph Laplacian matrix can be seen as an extension of frequency elements that define the Fourier domain in classical signal processing [15].

2.2. Data description

In order to evaluate the proposed method, we used EEG signals from the publicly available BCI Competition III-Dataset IVa [23]. The signals were recorded from five healthy subjects (labelled as \( aa, al, av, aw, \) and \( ay \)) using 118 electrodes arranged in the extended international 10/20-system at a sampling rate of 100 Hz. Subjects were presented with 280 3.5-second-long visual cues during which they were asked to perform right hand or right foot motor imagery; 140 trials were acquired for each class. According to the competition instructions, for each class the trials were divided into training and test sets, wherein the set sizes differed across the five subjects. The first two subjects have the most labelled trials (60% and 80%, respectively), while the other three have 30%, 20% and 10% labelled trials, respectively; as such, performance classification is more challenging on subjects \( av, aw, \) and \( ay \) due to their small training set size.

2.3. Graph-based representation of brain signals

We modeled the structure of the brain by a graph with vertices corresponding to the EEG electrodes and edges quantifying the degree of functional connectivity between the electrodes in each subject. Let \( f_{i,t} \) and \( f_{j,t} \) denote the time series of electrodes \( i \) and \( j \), respectively. The absolute value of the Pearson correlation between \( f_{i,t} \) and \( f_{j,t} \), providing an estimation of statistical dependency of the two temporal signals, was considered as the weight of the edge connecting vertices \( i \) and \( j \).

For each trial, we used the time points within the 0.5-2.5 second interval after the visual cue to construct graph signal; this 2-second interval has been previously proposed by the winner of BCI Competition III-Dataset IVa. Given that motor activity, be it real or imagined, causes modulations of the mu and beta rhythms [5], we filtered the extracted signal with a third-order Butterworth filter with a pass band of 8-30 Hz. Graph signals were then extracted from these filtered signals; in particular, we defined one graph signal per time instance, i.e., a signal representing EEG values across the 118 electrodes, which thus resulted in \( T=200 \) graph signals per trial. We then used the eigenvectors of the EEG graph normalized Laplacian matrix to compute the GFT of each signal. As such, we obtain a representation of brain signals that jointly encodes structural, functional, and temporal characteristics of the data.

2.4. Discriminative subspace through simultaneous diagonalization

Inspired by the methods presented in [18, 24], simultaneous diagonalization of two matrices was considered to provide a discriminative subspace for two-class (right hand and right foot) MI classification. For graph signal \( \mathbf{f} \) defined on \( G \), let \( \mathbf{f} \) denote the de-meaned and normalized version of \( \mathbf{f} \), obtained as [16]:

\[
\mathbf{\bar{f}} = \frac{(\mathbf{f} - \mu_f \mathbf{u}_1)}{\left\| (\mathbf{f} - \mu_f \mathbf{u}_1) \right\|_2}.
\]

More precisely, let \( \mathbf{F}_c \) denote an \( N \times T \) matrix with elements \( \{ f_{c,t} \} \), where \( c = 1, \ldots, N \) and \( t = 1, \ldots, T \), denote the \( k \)-th trial of the EEG time series, where \( c \) denotes electrode number; similarly, let \( \mathbf{\hat{F}}_k \) denote the GFT matrix of the \( k \)-th de-meaned and normalized trial. The goal is to determine a transform \( \mathbf{\hat{P}} \) that simultaneously diagonalizes the following two symmetric matrices that are computed for each class:

\[
\tilde{\Xi}_1 = \frac{1}{K_1} \sum_{t=1}^{K_1} \mathbf{\hat{F}}_t \mathbf{\hat{F}}_t^\top, \quad \tilde{\Xi}_2 = \frac{1}{K_2} \sum_{t=1}^{K_2} \mathbf{\hat{F}}_t \mathbf{\hat{F}}_t^\top,
\]

where \( T \) and \( \text{tr}(.) \) denote the transpose and the trace operator, respectively, and \( K_1 \) is the number of the trials in class \( i \). As a first step, we whiten \( \mathbf{\Xi} = \tilde{\Xi}_1 + \tilde{\Xi}_2 \) such that:
\[ P^T \bar{\Xi} P = P^T (\bar{\Xi}_1 + \bar{\Xi}_2) P = \bar{\Xi}_1 + \bar{\Xi}_2 = I. \]  

(4)

Due to positive definiteness of \( \bar{\Xi} \), whitening transform \( P \) can be derived via singular value decomposition as:

\[ \bar{\Xi} = \Phi \Theta \Phi^T; \quad P = \Phi \Theta^{-\frac{1}{2}}. \]  

(5)

Consequently, eigenvalue decomposition of \( \bar{\Xi}_1 \) gives:

\[ \bar{\Xi}_1 = \Psi \Theta_1 \Psi^T \rightarrow \bar{\Xi}_2 = \Psi (I - \Theta_1) \Psi^T. \]  

(6)

In particular, \( \bar{\Xi}_1 \) and \( \bar{\Xi}_2 \) share the same eigenvectors but their eigenvalues are complementary; that is, the eigenvector associated with the largest eigenvalue of \( \bar{\Xi}_1 \) corresponds to the smallest eigenvalue of \( \bar{\Xi}_2 \). Therefore, a small combination of the first and last eigenvectors of \( \Psi \) induces a suitable discriminatory transform for differentiating the two classes. Finally, the overall transformation matrix can be obtained as:

\[ \hat{P} = \Psi^T P. \]  

(7)

This matrix was used to project the GFT coefficients of the de-meaned and normalized graph signals to a discriminative feature space; these features were then used for classification.

### 3. RESULTS AND DISCUSSION

In our experiments, the algorithms were trained using the training set data available for each subject, and consequently, the test set data available for each subject were used to evaluate the performance of the methods by assigning a label to each trial. The variance of the projected GFT coefficients on the first and the last rows of \( \hat{P} \) were used to train an SVM classifier with a linear kernel. This projection maximizes the variance of the signals from one class while minimizing it for the signals of the other class [24].

In the first experiment, the GFT coefficients were used in four different settings. In the first setting, we used the entire set of GFT coefficients, i.e., all frequencies (AF), whereas in the second to fourth settings we only used a subset of the coefficients by equally dividing them into three sub-bands, low (LF), medium (MF) and high (HF) frequencies, respectively; division of the spectrum into 3 sub-bands was inspired by prior work on application of GFT on brain imaging data [25, 26]. These four sets of GFT coefficients were then used to derive the discriminative matrix \( \hat{P} \), and consequently, features for classification were extracted by projecting them on \( \hat{P} \). Table 1 shows the classification accuracies for each individual subject and also on average across subjects.

| Cut-off frequency bands | Mean ± std | (aa) | (al) | (av) | (aw) | (ay) |
|------------------------|-----------|------|------|------|------|------|
| SS-LF                  | 87.50±11.96 | 100  | 70.41| 93.3 | 88.09| 87.50±11.01 |
| O1-LF (23)             | 87.5      | 98.21| 59.18| 85.27| 86.11| 83.25±14.43 |
| O2-LF (30)            | 89.25     | 80.37| 93.3 | 91.27| 94.24| 88.44±11.93 |
| O3-LF (32)            | 91.96     | 100  | 68.88| 94.2 | 92.46| 89.5±11.96  |
| O4-LF (35)            | 89.28     | 100  | 68.88| 93.3 | 92.06| 88.71±11.76 |
| SRCSSP [9]            | 72.32     | 96.43| 60.2 | 77.68| 86.51| 78.63±13.78 |
| RCSSP [10]           | 82.14     | 96.42| 68.87| 98.21| 88.88| 86.91±11.94 |
Given that using SS cut-off frequencies did not result in better performance, we attempted to find an optimal cut-off that can be used across subjects. This was done by performing 10-fold CV on the training sets of each subject for different cut-off frequencies. Fig. 1 shows the resulting classification accuracies across subjects. Overall, on average across subjects, best performance was obtained at a cutoff frequency corresponding to spectral elements within the range \([20,40]\), after which point the performance almost saturated in all five subjects. Four cut-off values within the optimal window were then selected (see vertical dashed lines), the results for which are reported in Table 2, denoted by \(O_x\)-LF, where \(x = 1,...,4\).

![Classification accuracies for 10-fold CV on the training sets for different indices of cut-off frequencies (\(\lambda\)).](image)

In future work, the proposed method will be focused on investigating alternative methods for deriving brain graphs from EEG data using graph learning techniques [28].

### 4. CONCLUSIONS

In this paper, a GSP framework was presented for classification of MI tasks from multi-electrode EEG data. Experimental results suggested that decomposition of brain signals on a discriminative subspace of the graph Fourier domain outperforms two related conventional methods for classification of EEG signals. Graph frequency analysis of EEG data on brain functional connectivity graphs reveals the spatially smooth nature of motor activity signals, making it possible to classify these signals by exploiting the representation of the data on the low frequency range of the graph spectrum. In future work, the proposed method will be validated on a dataset with a greater number of subjects. Moreover, our future work will be focused on investigating the intrinsic properties of representing EEG data in the graph Fourier domain, with the goal of finding a generalizable approach to extract discriminative subspaces for other classes of EEG signals. We will also explore alternative methods for deriving brain graphs from EEG data using graph learning techniques [28].

### 5. COMPLIANCE WITH ETHICAL STANDARDS

The present research study was conducted retrospectively using human subject data made available in open access by provided by the Berlin BCI group [23]. Ethical approval for
an analyzing the openly available data is not required according to our local ethics committee.

6. ACKNOWLEDGMENTS

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