Making waves on CMB power spectrum and inflaton dynamics

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Abstract

We discuss cosmic microwave background anisotropies in models with an unconventional primordial power spectrum. In particular, we consider an initial power spectrum with some “spiky” corrections. Interestingly, such a primordial power spectrum generates “wavy” structure in the CMB angular power spectrum.
Result of the first year Wilkinson microwave anisotropy probe (WMAP) [1] is almost consistent with the following assumption [2]: the present universe consists of 5% baryon, 23% cold dark matter (CDM) and 72% dark energy. The primordial power spectrum is almost scale invariant with adiabatic fluctuations. These assumptions form so-called the standard scenario of cosmology.

Although the fit to the WMAP data with the standard scenario seems to be excellent, there may be some deviations from the standard scenario in the angular power spectrum given by WMAP. One of them is that the angular power spectrum seems to be suppressed at low multipoles compared to the prediction of the standard cosmological scenario #1.

Since the cosmic variance error is very large at low multipole region, we may say that the suppression is just a statistical fluctuation. However, if we take this seriously, it may suggest some new physics. There are many works proposing mechanisms to explain this suppression [4].

The other one is that, if you look at the WMAP angular power spectrum carefully, there is “wavy” structure around multipole region from $l \sim \mathcal{O}(10)$ up to the first peak. In particular, there seems to be a dip around the first peak. Although this feature is not robust at this point, if it is an intrinsic one, it may suggest some unusual physics since this kind of structure cannot be explained by the conventional scenario. There have been some works inspired by this feature. In Ref. [5], using a cosmic inversion method, reconstruction of the primordial power spectrum which can accommodate this feature is discussed. The authors of Refs. [6, 7, 8] considered a primordial power spectrum with oscillatory corrections which can be generated from trans-Planckian physics. They showed that, with such an initial power spectrum, we can improve the fit to the WAMP data compared with the best-fit value of the standard scenario.

Although we may not be at the position where we should take this result seriously, it is interesting to consider some models which give the wavy structure on the CMB angular power spectrum. In this letter, we discuss a mechanism which generates such structure in the CMB power spectrum. Here we consider a scenario where the primordial fluctuation is modified from the standard form. If we assume an initial power spectrum with some “spiky” parts, wavy structure of the CMB power spectrum can appear. The initial power spectrum we assume is written as

$$P_s = A_s k^{n-1} \left(1 + \sum_i \alpha_i \exp \left[-\frac{(k - k_{ci})^2}{2\sigma_i^2}\right]\right) ,$$

where $A_s$ and $n$ are the amplitude of primordial fluctuation and the spectral index as usually assumed. In the spectrum, “spiky” corrections are added to the standard power spectrum which can be parameterized by $\alpha_i, k_{ci}$ and $\sigma_i$ for the $i$-th spike. We assume $\alpha_i \geq -1$ to keep the initial power spectrum $P_s$ positive. $k_{ci}$ and $\sigma_i$ represent the position of spike and its width respectively. These parameters can be determined by models of the mechanism generating the primordial fluctuation. Here we consider a situation where

#1In fact, this suppression was already observed by COBE and this issue was discussed in Refs. [3].
the width of the Gaussian function is very narrow. This kind of primordial spectrum may arise from a particular kind of inflaton dynamics, which will be discussed later.

First we discuss effects of the spiky corrections in the initial power spectrum. In Fig. 1 we show the resultant CMB angular power spectrum from the initial power spectrum given by Eq. (1). In the upper panel of the figure, we assume a single “dip” which is parameterized as $\alpha = -0.9, k_c = 2.2 \times 10^{-2} h \text{ Mpc}^{-1}$, and $\sigma = 8.7 \times 10^{-5} h \text{ Mpc}^{-1}$. In the lower panel of the figure, we assumed a single spike and a single dip in the initial power spectrum with the parameters $\alpha_1 = 3.0, k_{c1} = 4.3 \times 10^{-3} h \text{ Mpc}^{-1}, \sigma_1 = 7.6 \times 10^{-5} h \text{ Mpc}^{-1}$ and $\alpha_2 = -0.9, k_{c2} = 2.5 \times 10^{-3} h \text{ Mpc}^{-1}, \sigma_2 = 1.4 \times 10^{-4} h \text{ Mpc}^{-1}$. The cosmological parameters are chosen to be $\Omega_{m} h^2 = 0.145, \Omega_{b} h^2 = 0.023, h = 0.69, n_s = 0.97$ and $\tau = 0.116$ which give the minimum value of $\chi^2$ in the power-law $\Lambda$CDM model. Here a flat universe and no tensor mode are assumed. Very interestingly, with the parameter in the upper panel, you can see a dip around the first peak, while wavy structure around $l \sim 50$ appears in the lower panel. Accordingly, the fit to the WMAP data becomes better. We calculated $\chi^2$ using the code provided by WMAP [10] and obtained smaller $\chi^2$ by $\Delta \chi^2 \sim -10$ compared to the value for the best-fit parameter of the power-law $\Lambda$CDM model in both panels in Fig. 1.

The reason why such structure arises is simple. The angular power spectrum can be written as

$$C_l = \frac{2}{\pi} \int_0^\infty dk \ k^2 P_s(k)|\Theta_l(k, \eta_0)|^2$$

where $\Theta_l(k, \eta)$ is the transfer function and $P_s$ is the primordial power spectrum. $\eta$ is the conformal time and the subscript 0 represents a quantity defined at the present time. The transfer function is given by

$$\Theta_l(k, \eta_0) = \int_0^{\eta_0} d\eta \ g(\eta)(\Theta_0(k, \eta) + \Psi(k, \eta))j_l(k(\eta_0 - \eta))$$

$$- \int_0^{\eta_0} d\eta \ g(\eta) \frac{iv_b(k, \eta)}{k} \frac{d}{d\eta} j_l(k(\eta_0 - \eta))$$

$$+ \int_0^{\eta_0} d\eta \ e^{-\tau} \left[ \Phi(k, \eta) - \dot{\Psi}(k, \eta) \right] j_l(k(\eta_0 - \eta))$$

where $j_l$ is a spherical Bessel function of degree $l$, and $v_b$ is the velocity of baryon. $\Psi$ and $\Phi$ are the gravitational potential and the curvature perturbation, respectively. $g(\eta)$ is the visibility function which has a peak around $\eta_*$ where $*$ represents the epoch of the recombination. Since we are considering the case where $\sigma_i$ is very small (i.e., the width is very narrow), we can replace the second term of Eq. (1) with delta functions $\sum_i \delta(k - k_{ci})$.

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#2 Although these cosmological parameters are a bit different from those given by WMAP [2], this choice of cosmological parameters gives a better fit to the data with $\chi^2 = 1428.6$ [9].

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#3 For the case with the upper panel of the figure, assuming a dip introduces three additional parameters thus the degrees of freedom increases by 3. Similarly, in the case with the lower panel of the figure, the degrees of freedom increases by 6 since we assume a dip and a spike.
Figure 1: The CMB angular power spectrum with the primordial spectrum Eq. (11). The model parameters and cosmological parameters are given in the text. The binned WMAP data is also plotted.
Approximating with the delta function in the initial power spectrum Eq. (1), we can write \( C_l \) as

\[
C_l \simeq C_l^{(\text{standard})} + \sum_i \alpha_i \left| F_l^{(1)}(k_{ci}, \eta_0, \eta_*) j_l(k_{ci}(\eta_* - \eta_0)) + F_l^{(2)}(k_{ci}, \eta_0, \eta_*) \frac{d}{d\eta} j_l(k_{ci}(\eta_* - \eta_0)) \right|^2,
\]

where \( F_l^{(1,2)}(k_{ci}, \eta_0, \eta_*) \) are functions which vary slowly with \( l \) and \( C_l^{(\text{standard})} \) is a CMB power spectrum calculated by using the standard initial power spectrum. As you can see from Eq. (4), when the spiky correction term in the initial power spectrum can give sizable contribution to \( C_l \), the form of \( C_l \) exhibits the oscillatory structure of \( j_l \). This is the reason why the wavy structure of \( C_l \) can be realized assuming the initial power spectrum given by Eq. (1). To see the effects of a spherical Bessel function, we plot \( \Delta C_l/C_l^{(\text{standard})} = (C_l^{(\text{standard})} - C_l)/C_l^{(\text{standard})} \) in Fig. 2 with the same model and cosmological parameters in the upper panel of Fig. 1. When \( \alpha_i \) is negative, the second term in Eq. (4) gives a negative contribution to \( C_l \), thus, in this case, a dip can appear as in the upper panel of Fig. 1. For a positive \( \alpha_i \) case, the second term is added to give oscillatory power spectrum. Without a spiky part in the initial power spectrum as it is usually assumed, oscillatory structure of the spherical Bessel function is smeared out by contributions from superposition of \( k \)-modes around the corresponding multipole region. Hence we do not usually see such structure. Notice also that if the width of the Gaussian part in the initial power spectrum of Eq. (1) is not narrow, the oscillatory structure of a spherical Bessel function is also smeared out, thus we cannot see the wavy structure on the CMB power spectrum. Then how narrow should the width of the spiky structure be to give a distinctive feature to \( C_l \)? Noting that the spherical Bessel function \( j_l(z) \) has the highest peak with the width \( \Delta z \sim l^{1/3} \) around \( z_* \sim l \), the following inequality needs to be satisfied in order to make wavy structure around multipole \( l \):

\[
\frac{\sigma_i}{k_{ci}} < \frac{\Delta z}{z_*} \sim l^{-2/3}.
\]

This inequality is satisfied for the model parameters we have adopted in Fig. 1.

Now we discuss possible models which generate the initial power spectrum of the form of Eq. (1). Usually, the primordial density fluctuation is represented by the amplitude of the density perturbation at the horizon crossing \( \delta_H(k) \) which can be approximately written in the spatially flat gauge as [12],

\[
\delta_H(k) \sim \frac{H^2}{5\pi\dot{\phi}},
\]

where \( H \) is the Hubble parameter during inflation and \( \phi \) is an inflaton field. The dot represents derivatives with respect to time. Using \( \delta_H \), the initial power spectrum can be written as \( P_s(k) \equiv \delta_H^2(k) \). From Eq. (6), we can see that, if the inflaton field almost
stops instantaneously, $\delta_H$ gets enhanced at corresponding scale $k_c$, which may make some spiky structure in the power spectrum #4. Inversely, if the inflaton field moves very fast instantaneously, $\delta_H$ gets suppressed at corresponding scale $k_c$. In this case, this makes some dips in $\delta_H(k)$. Thus the primordial power spectrum in the form of Eq. (1) may be generated from an unusual inflaton dynamics. However, this simple argument is not enough to evaluate the primordial power spectrum. For more detailed investigation of this issue, see Ref. [14].

In summary, we considered the primordial power spectrum with some spikes and/or spiky dips. Assuming this kind of initial power spectra, wavy structure appears in the CMB angular power spectrum, which may be inferred from the first year WMAP observation. Such wavy structure may originate to the oscillatory structure of a spherical Bessel function. When there is a spike in the initial power spectrum and its width is very narrow, for a fixed $l$, only the small number of $k$-modes is picked up to give contributions to $C_l$ with oscillatory structure of the spherical Bessel function. If this mode can give sizable contribution to $C_l$, the oscillatory behavior of the spherical Bessel function directly affects the form of $C_l$. When the width of the spike is not so narrow, several modes can contribute to $C_l$. Thus oscillatory structure is smeared out by the projection effect as the conventional case.

#4 Note that the power spectrum does not diverge even in the limit of $\dot{\phi} \rightarrow 0$ [13].
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