Ward Identities for $N=2$ Rigid and Local Supersymmetry in Euclidean Space

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Abstract

We construct and check by explicit Feynman diagram calculations the BRST Ward identities for $N=2$ rigid super Yang-Mills theory and $N=2$ extended supergravity in four-dimensional Euclidean space without auxiliary fields. We use the Batalin-Vilkovisky formalism. In the supergravity case we need one new contractible pair of complex spinor fields to obtain the usual gauge-fixing term and corresponding Nielsen-Kallosh ghosts.

1 Introduction

Many calculations these days of processes involving the AdS/CFT correspondence and its connection to ten-dimensional supergravity theories use Euclidean formulations of the field theories instead of Minkowskian formulations, see for example [1]. It seems useful to discuss the properties of these field theories at the quantum level, in particular to study the Ward identities. The present paper is intended as a pilot program; we shall consistently make simplifications when they reduce the amount of algebra and do not change the fundamental issues we study. For example, we shall evaluate the Ward identities for extended supergravity in flat space instead of anti-de Sitter space. Our final results are therefore easy to understand and check, but the more realistic cases will be more complicated.

One problem that arises with Euclidean $N=1$ supergravities in $D=10$ is that there do not exist Majorana-Weyl spinors in $D=10$ Euclidean space, while the Minkowskian theories use such Majorana-Weyl spinors. One solution to the problem is to view all fields, bosonic as well as fermionic, as complex in Euclidean space and to forget about such reality conditions as Majorana spinors, real self-dual antisymmetric tensors etc. Since all fields appear “holomorphically”, i.e., the (complex) fields but not their complex conjugates

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appear in the action and transformation rules, supersymmetry holds equally well in the complex theory in Euclidean space as in the real theory in Minkowski space. In particular, the symmetries of fermion bilinears and Fierz rearrangements of terms quartic in fermion fields needed in the proof of (rigid or local) supersymmetry only depend on properties of the charge conjugation matrix, but do not require real spinors [2]. In this sense Euclidean $N = 1$ supersymmetries and supergravities do exist.

Whether these $N = 1$ theories exist at the nonperturbative level, i.e., whether the path integral makes sense, is another question which we do not answer in this paper. For Euclidean path integrals, Gibbons, Hawking and Perry [3] have suggested to make one more Wick-like rotation on the conformal part of the metric, such that the action in the path integral becomes bounded from below. It may be that one can also make a corresponding Wick-like rotation on a conformal part of the gravitino, in such a way that the new action is still supersymmetric, but we have not studied this problem. We will study a Ward identity in Euclidean $N = 2$ supergravity, and find that with the standard (unmodified) Feynman rules this Ward identity in Euclidean space is satisfied.

In this article we consider $N = 2$ supersymmetric theories. For them, one can combine a pair of Majorana spinors into a Dirac spinor, and a complex Dirac spinor in Euclidean space can also be written as a pair of Majorana spinors, but with a modified (“symplectic”) Majorana condition. The real bosonic fields in Minkowski space are again real in Euclidean space. Hence, for $N = 2$ theories complexification is not needed.

However, there is a second problem with Euclidean theories, namely certain spinless fields acquire an extra minus sign in front of their kinetic action, and extra factors of $i$ in front of their Yukawa couplings [4]. The origin of these minus signs and factors of $i$ was pointed out in [5], where it was noted that a pseudoscalar in four dimensions can be represented in terms of four scalar fields $\phi_i$ as $\varepsilon^{\mu \nu \rho \sigma} \partial_\mu \phi_1 \partial_\nu \phi_2 \partial_\rho \phi_3 \partial_\sigma \phi_4$. Precisely one derivative is timelike and hence the Wick rotation produces a factor $i$. This mechanism explains why the kinetic action of the axion in $D = 10$ changes sign in Euclidean space.

In [5] the Wick rotation was extended to fermions. The basic idea was that the Wick rotation for a vector $(A_0, \vec{A}) \rightarrow (iA^E_0, \vec{A}^E)$ is really an induced representation $A_\mu(t, \vec{x}) \rightarrow O_\mu^\nu A^E_\nu(\tau, \vec{x})$ with $O_\mu^\nu = \text{diag}(i, 1, 1, 1)$ (we denote Minkowski time by $t$ and Euclidean time by $\tau$). It was then natural to assume that for fermions $\psi \rightarrow O\psi$, where $O$ is again a $4 \times 4$-matrix with spinor indices, but now the matrix will not be diagonal. Consistency arguments then show that $O$ is a complex Lorentz rotation in the $t-\tau$ plane, which is the spinor representation of the Wick rotation. With these matrices $O$ for (real or complex) bosons and complex fermions one can construct from every field theory in Minkowski spacetime a corresponding theory in Euclidean space. One finds then that certain internal compact rigid symmetries become noncompact after the Wick rotation, just like the noncompact spacetime symmetry $SO(3,1)$ becomes the compact $SO(4)$. This exchange of compactness and noncompactness between spacetime and internal symmetries is not surprising if we take the spacetime origin of internal symmetries provided by the Kaluza-Klein program into account. For example, the $SO(9,1)$ symmetry of $N = 1, D = (9,1)$ super Yang-Mills theory becomes an $SO(3,1) \times SO(6)$ symmetry if one compactifies on a 6-torus, but it becomes $SO(4) \times SO(5,1)$ with a noncompact internal $SO(5,1)$ if one compactifies on a torus $T(5,1)$ where the time axis has been compactified [5]. The same features arise in the compactification of $N = 1, D = (9,1)$ supergravity on
$S^3 \times \text{AdS}_3$ leading to $N = 4$, SU(2)×SU(1,1) supergravity in four-dimensional Euclidean space \([7]\).

Although some of the properties of Euclidean supersymmetric field theories have been explained in this way, the next question, dynamics and Ward identities in Euclidean space, has been less studied. One reason for this absence of such studies is that for super Yang-Mills theories in $x$-space the gauge fixing term and the ghost action break the rigid susy. One must then derive Ward identities for explicit symmetry breaking \([8]\). For $3 + 1$ dimensional susy gauge theories one can (at least for $N = 1$ models) go to superspace with (anti-) ghost superfields and preserve rigid susy. However, for $D > 4$ and $N > 1$ no full-fledged off-shell superspace formalism exists, whereas for $D = 4$, $N = 1$ one runs into the problems with Majorana spinors mentioned above.

Over a decade ago, it was noticed \([9]\) that one can treat rigid symmetries of Minkowskian field theories on the same footing as local gauge symmetries, and use a BRST formalism \([10]\) (more precisely a field-antifield formalism, also called a BV-formalism \([11, 12]\)) for all (rigid and local) symmetries simultaneously. This was applied to rigid supersymmetry in \([13, 14, 15]\). One of the purposes of this paper is to apply this formalism to supersymmetric field theories in Euclidean space and to derive Ward identities. In particular, we shall analyze in explicit examples whether the minus signs and factors of $i$ due to the Wick rotation of pseudoscalars are compatible with the BRST Ward identities in Euclidean space.

Of course, the BV formalism allows one to deal with locally or globally supersymmetric theories without auxiliary fields. This is in our case an enormous advantage, because the auxiliary fields for $N = 2$ supergravity are quite complicated \([16]\). For $N = 2$ super Yang-Mills theory \([17]\) they are not complicated and we could have incorporated them.

We shall study $N = 2$, $D = 4$ Euclidean super Yang-Mills theory. In this model the two Majorana spinors are written, in Minkowski spacetime and in Euclidean space, as one complex spinor. This model has one real scalar and one real pseudoscalar, so it is an ideal testing ground for our purposes. Applying the Wick rotation rules, one obtains a hermitean action in Euclidean space, which has again $N = 2$ supersymmetry, but with a kinetic action for the pseudoscalar with the wrong sign. We derive some Ward identities and explicitly verify them at the one-loop level. Our conclusion is that the Euclidean Ward identities are satisfied if one uses the minus sign and factors of $i$ for the pseudoscalars, but one can equally well redefine $\varphi \rightarrow i\varphi$ both in the action and in the Ward identities, thus obtaining the standard positive-definite kinetic action, at the expense of hermiticity of the action. Since unitarity is a concept in Minkowski space only, violation of hermiticity of a Euclidean action poses no problem.

One is of course also interested in local supersymmetry, i.e. supergravity. We consider $N = 2$, $D = (3, 1)$ supergravity which was the first extended supergravity, constructed in 1976 \([18]\). It unifies electromagnetism and gravity, and does so by introducing two gravitinos. In this model, the two Majorana gravitinos combine in Minkowski spacetime into one complex gravitino $\psi^\mu_\alpha$. The Wick rotation now involves both a matrix $O_\mu^\nu$ for the vector index and a matrix $O^\alpha_\beta$ for the spinor index. The other fields, the real vielbein $e_\mu^\alpha$ and the real vector field $A_\mu$, are not complexified and are again real in Euclidean space. We construct $N = 2$, $D = 4$ Euclidean supergravity including the supersymmetric cosmological constant by using the Wick rotation of \([2]\) with the $O$-matrices. For example,
the Minkowski transformation rule for the gravitino $\psi_\mu$ with $\mu = 0$ reads

$$\delta \psi_0 = \ldots + \frac{1}{\sqrt{2}} g \kappa^{-2} \gamma_0 \epsilon, \quad \gamma_0 = e_0^a \gamma_a,$$

in anti-de Sitter supergravity \[^{[19]}\]. It becomes in Euclidean space \[^{[3]}\]

$$i O^{\alpha \beta} \delta \epsilon \psi_4^{\beta E} = \ldots + \frac{1}{\sqrt{2}} g \kappa^{-2} (i \gamma_4 \epsilon)^{\alpha},$$

and defining $\epsilon \equiv O \epsilon_E$, one obtains

$$\delta \epsilon \psi_4^{E} = \ldots + \frac{1}{\sqrt{2}} g \kappa^{-2} (O^{-1} \gamma_4 O) \epsilon_E.$$

One identifies then $O^{-1} \gamma_4 O$ as the Euclidean Dirac matrix $\gamma_4^E$, which, in fact, is equal to the usual (Minkowski) matrix $\gamma^5 \[^{[3]}\]$. Similarly, in Dirac actions we encounter the structure $\mathcal{L} = \psi^a \gamma_5^E \gamma_0 \partial_\mu \psi$, where $\mu$ runs from 1 to 4, because $\gamma_4^E$ in $\psi = \psi^a \gamma_4^E$ becomes $O \gamma^4 O = \gamma_4^E = -\gamma_5^E$. This action clearly preserves SO(4) symmetry with generators $\frac{1}{4} [\gamma^\mu_E, \gamma^\nu_E]$.

In the quantization of gravity theories with spinors in Minkowski space, there are not only Einstein ghosts $C^\mu$ and antighosts $\bar{C}^\mu$, but also Lorentz ghosts $C^{ab}$ and antighosts $\bar{C}^{ab}$. We can also first make a Wick rotation on the classical theory, and then quantize. One obtains then SO(4) (anti-) ghosts. The Lorentz ghosts are, as expected, related to the SO(4) ghosts by a Wick rotation. We can fix the Lorentz symmetry at the classical level by requiring the vielbein to be symmetric, but it is better to avoid such constraints on the vielbein at the quantum level and to accept the presence of the Lorentz (anti-) ghosts. If desired, one can eliminate them from the quantum theory by their algebraic field equations \[^{[20]}\]. (We recall that this does not mean that we put the theory on-shell, but, rather, that one performs a simple Gaussian integration in the path integral.) There is no problem with the negative signs due to the Lorentz metric for Berezin integration. We keep the Lorentz or SO(4) (anti-) ghosts at all stages.

Since the standard BV formalism deals with gauge theories and the presence of rigid symmetries requires some extra discussion, and also since for applications the supergravity part is more interesting than the super Yang-Mills part, we begin with the former.

## 2 Euclidean N=2 Supergravity

Pure $N = 2$ extended supergravity in four-dimensional Minkowski spacetime was formulated in \[^{[18]}\]. The field content consists of a vielbein $e^a_{\mu}$, a vector $A_\mu$ (the graviphoton) and two Majorana spinors $\chi^1_\mu, \chi^2_\mu$ (the gravitinos). In \[^{[19]}\] the model was generalized to include a minimal coupling of the graviphoton to the gravitinos, which requires the presence of a non-vanishing cosmological constant. For the continuation to Euclidean space it is convenient to combine the gravitinos into a complex Dirac spinor $\psi_\mu = (\chi^1_\mu + i \chi^2_\mu) / \sqrt{2}$. \[^{[3]}\] Use that if $t \to -i \tau$, the covariant index $\mu = 0$ of $e^{a}_\mu$ acquires a factor $+i$, but the contravariant index $a = 0$ a factor $-i$. Then use $-i \gamma_0 = \gamma_4$. Note that in flat space $\delta^{a}_\mu = \delta^{a}_\mu$ both in Minkowski and in Euclidean space.
The action then reads for the Minkowski theory\footnote{We use the following conventions in Minkowski spacetime: \( \{ \gamma^a, \gamma^b \} = 2 \eta^{ab}, \eta_{00} = -1, \gamma^{ab} = \gamma^a \gamma^b = \frac{1}{2}(\gamma^a \gamma^b - \gamma^b \gamma^a), \bar{\psi} = \psi^\dagger \gamma^0, \gamma^5 = \gamma^1 \gamma^2 \gamma^3 \gamma^4, \gamma^\mu = \gamma^\mu e^\mu_a, \epsilon^{0123} = 1, \epsilon = \text{det} e_\mu^a. \)}

\[
S_0 = \int d^4x \left[ \frac{e}{2\kappa^2} R(e, \omega) - i \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu \mathcal{D}_\rho \psi_\sigma - \frac{1}{4} e F^{\mu\nu} F_{\mu\nu} + \frac{6g^2}{\kappa^4} e \right.
- \frac{i\kappa}{2\sqrt{2}} \bar{\psi}_\mu \left[ e (F^{\mu\nu} + \tilde{F}^{\mu\nu}) + i\gamma_5(F^{\mu\nu} + \tilde{F}^{\mu\nu}) + \frac{4g}{\kappa^2} e \gamma^{\mu\nu} \psi_\nu \right],
\]

(1)

where a long bar denotes Dirac conjugation. (We shall use short bars as in (23) to denote antighosts.) The curvature scalar is given in terms of the inverse vielbein and the spin connection by

\[
R = e_a^\mu e_b^\nu R_{\mu\nu}^{ab}, \quad R_{\mu\nu}^{ab} = \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} + \omega_\mu^{ac} \omega_\nu^{cb} - \omega_\nu^{ac} \omega_\mu^{cb},
\]

(2)

while \( \mathcal{F}_{\mu\nu} \) and \( \tilde{\mathcal{F}}_{\mu\nu} \) denote the supercovariant field strength and its Hodge-dual respectively,

\[
\mathcal{F}_{\mu\nu} = F_{\mu\nu} + \frac{i\kappa}{\sqrt{2}} (\bar{\psi}_\mu \psi_\nu - \bar{\psi}_\nu \psi_\mu), \quad \tilde{\mathcal{F}}_{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\rho\sigma}.
\]

(3)

The operator \( \mathcal{D}_\mu \) entering the kinetic term of the gravitinos is the Lorentz- and gauge-covariant derivative,

\[
\mathcal{D}_\mu \psi_\nu = (D_\mu - igA_\mu) \psi_\nu, \quad D_\mu \psi_\nu = \partial_\mu \psi_\nu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \psi_\nu, \quad D_\mu e_\nu^a = \partial_\mu e_\nu^a - \omega_\mu^{ab} e_\nu^b.
\]

(4)

The spin connection \( \omega_\mu^{ab} \) is the function of the vielbein and gravitinos that satisfies its equation of motion \( \delta S_0 / \delta \omega_\mu^{ab} = 0 \) (1.5 order formalism \cite{21}),

\[
\omega_{\mu ab} = \omega_{\mu ab}(e) + \frac{\kappa^2}{4} (\bar{\psi}_a \gamma_\mu \gamma_b \psi_\nu + \bar{\psi}_a \gamma_b \psi_\mu - \bar{\psi}_\mu \gamma_b \psi_a - a \leftrightarrow b).
\]

(5)

The supersymmetry transformations with Dirac (i.e., complex) spinor parameter \( \epsilon(x) \) read

\[
\delta_\epsilon e_\mu^a = \frac{1}{2} \kappa (\bar{\epsilon} \gamma^a \psi_\mu - \bar{\psi}_\mu \gamma^a \epsilon), \quad \delta_\epsilon A_\mu = \frac{1}{\sqrt{2}} (\bar{\psi}_\mu \epsilon - \bar{\epsilon} \psi_\mu), \quad \delta_\epsilon \psi_\mu = \kappa^{-1} D_\mu \epsilon - \frac{i}{2\sqrt{2}} \gamma^\nu (\mathcal{F}_{\mu\nu} + i e \gamma_5 \tilde{\mathcal{F}}_{\mu\nu}) \epsilon + \frac{1}{\sqrt{2}} g \kappa^{-2} \gamma_\mu \epsilon.
\]

(6)

Using these rules one easily verifies that \( \mathcal{F}_{\mu\nu} \) in (3) and \( \omega_\mu^{ab} \) in (3) are supercovariant (in the variations the \( \partial_\mu \epsilon \) terms cancel).

We now employ the Wick rotation prescription for Dirac spinors presented in \cite{2},

\[
\psi \rightarrow O \psi_E, \quad \psi^\dagger \rightarrow \psi^\dagger_E O, \quad O = e^{\gamma^4 \gamma^5 \pi/4},
\]

(7)
along with the usual rules $t \to -i\tau$ and $A_\mu = (A_0, \vec{A})_\mu \to (iA_0^E, \vec{A}^E)_\mu$. The matrix $O$ is unitary and satisfies $O\gamma^4 = \gamma^4 O^{-1}$. The Euclidean $\gamma$-matrices are related to those in Minkowski spacetime by
\[
\gamma^a_E = O^{-1}\gamma^a O \quad \Rightarrow \quad \gamma^i_E = \gamma^i, \quad \gamma^4_E = \gamma^5, \quad \gamma^5_E = \gamma^1 E \gamma^2 \gamma^3 \gamma^4_E = -\gamma^4,
\]
where $a$ now runs from 1 to 4. In particular, one has $\bar{\psi} \to -\bar{\psi}_E \gamma^5 O^{-1}$. These rules yield the following Euclidean action, where we have included a factor $(-1)$ so that it enters the path integral via $\exp(-S_0)$ (from now on we drop all subscripts $E$ that denote Euclidean quantities)

\[
S_0 = \int d^4x \left[ -\frac{e}{2\kappa^2} R(e, \omega) - \varepsilon^{\mu\nu\rho\sigma} \psi^\dagger_\mu \gamma_\nu D_\rho \psi_\sigma + \frac{1}{4} e F^{\mu\nu} F_{\mu\nu} - \frac{6g^2}{\kappa^4} e \right.
\]
\[
\left. - \frac{i\kappa}{2\sqrt{2}} \psi^\dagger_\mu \gamma_5 \left[ e (F^{\mu\nu} + F^{\nu\mu}) + \gamma_5(F^{\mu\nu} + \tilde{F}^{\mu\nu}) + \frac{4i\bar{\epsilon}}{\kappa^2} e \gamma^{\mu\nu} \right] \psi_\nu \right].
\]

Note that $S_0$ is hermitean.

The supercovariant field strength and spin connection in Euclidean space read
\[
F_{\mu\nu} = F^{\mu\nu} - \frac{i\kappa}{\sqrt{2}} (\psi^\dagger_\mu \gamma_5 \psi_\nu - \psi^\dagger_\nu \gamma_5 \psi_\mu)
\]
\[
\omega_{\mu ab} = \omega_{\mu ab}(e) = \frac{\kappa^2}{4} (\psi^\dagger_\mu \gamma_5 \gamma_\mu \psi_b + \psi^\dagger_\mu \gamma_5 \gamma_b \psi_\mu - \psi^\dagger_\mu \gamma_5 \gamma_b \psi_a - a \leftrightarrow b).
\]

The former occurs in the action and the transformations only in the combination
\[
G_{\mu\nu} = F_{\mu\nu} + e \gamma_5 \tilde{F}_{\mu\nu}.
\]

The Wick rotation produces the following local supersymmetry transformation rules in Euclidean space
\[
\delta_\epsilon e_\mu^a = \frac{1}{2} \kappa (\psi^\dagger_\mu \gamma_5 \gamma^a e - e^\dagger \gamma_5 \gamma^a \psi_\mu)
\]
\[
\delta_\epsilon A_\mu = \frac{i}{\sqrt{2}} (e^\dagger \gamma_5 \psi_\mu - \psi^\dagger_\mu \gamma_5 \epsilon)
\]
\[
\delta_\epsilon \psi_\mu = \kappa^{-1} D_\mu \epsilon - \frac{i}{2\sqrt{2}} \gamma^\nu G_{\mu\nu} \epsilon + \frac{1}{\sqrt{2}} g\kappa^{-2} \gamma_\mu \epsilon.
\]

Using these rules one can again verify that $F_{\mu\nu}$ and $\omega_{\mu ab}$ are supercovariant, and that the action (12) is invariant.

$S_0$ is also invariant under the following BRST transformations of the supergravity multiple
\[
\begin{align*}
\delta_0 e_\mu^a &= C^\nu \partial_\nu e_\mu^a + \partial_\mu C^\nu e_\nu^a - C_b^a e_\mu^b + \frac{1}{2} \kappa (\psi^\dagger_\mu \gamma_5 \gamma^a \xi - \xi^\dagger \gamma_5 \gamma^a \psi_\mu), \\
\delta_0 A_\mu &= C^\nu \partial_\nu A_\mu + \partial_\mu C^\nu A_\nu + \partial_\mu C + \frac{1}{\sqrt{2}} (\xi^\dagger \gamma_5 \psi_\mu - \psi^\dagger_\mu \gamma_5 \xi), \\
\delta_0 \psi_\mu &= C^\nu \partial_\nu \psi_\mu + \partial_\mu C^\nu \psi_\nu + \frac{1}{4} C^{ab} \gamma_5 \psi_\mu - \kappa^{-1} D_\mu \xi + \frac{1}{2\sqrt{2}} \gamma^\mu G_{\mu\nu} \xi \\
&\quad - \frac{1}{\sqrt{2}} g\kappa^{-2} \gamma_\mu \xi + igC \psi_\mu.
\end{align*}
\]

\[5\text{Replace } \epsilon \text{ by } -\Lambda \xi \text{ with imaginary } \Lambda \text{ and remove } \Lambda \text{ from the left.} \]
\[
s_0 \psi_\mu^\dagger = C^\nu \partial_\nu \psi_\mu^\dagger + \partial_\mu C^\nu \psi_\nu^\dagger - \frac{i}{4} C^{ab} \psi_\mu^\dagger \gamma_{ab} + \kappa^{-1} D_\mu \xi^\dagger + \frac{i}{\sqrt{2}} \xi^\dagger G_{\mu \nu} \gamma^\nu \\
+ \frac{1}{\sqrt{2}} g \kappa^{-2} \xi^\dagger \gamma_\mu - i g C \psi_\mu^\dagger ,
\]
which include general coordinate transformations, local SO(4) rotations, U(1) gauge transformations and supersymmetry transformations with corresponding ghosts \( C^\mu \), \( C^{ab} \), \( C \) and \( \xi \), \( \xi^\dagger \) respectively. The latter are commuting Dirac spinors. The BRST transformations of the ghosts follow from the commutator algebra of the local symmetries; alternatively, they can be derived from imposing (on-shell) nilpotency of \( s_0 \) on the above fields. We obtain in either case

\[
\begin{align*}
    s_0 C &= C^\mu \partial_\mu C + \frac{i}{\sqrt{2}} \kappa^{-1} \xi^\dagger \gamma_5 \xi + \frac{i}{2} A_\mu \xi^\dagger \gamma_5 \gamma^\mu \xi \\
    s_0 C^\mu &= C^\nu \partial_\nu C^\mu - \frac{1}{2} \xi^\dagger \gamma_5 \gamma^\mu \xi \\
    s_0 C^{ab} &= C^\mu \partial_\mu C^{ab} + C^{ac} C^b - \frac{1}{2} \xi^\dagger \gamma_5 (\gamma^\mu \omega_\mu^{ab} + \frac{i}{\sqrt{2}} \kappa G^{ab} + \sqrt{2} g \kappa^{-1} \gamma^{ab}) \xi \\
    s_0 \xi &= C^\mu \partial_\mu \xi + \frac{i}{4} C^{ab} \gamma_{ab} \xi - \frac{i}{2} \kappa \psi_\mu (\xi^\dagger \gamma_5 \gamma^\mu \xi) + i g C \xi \\
    s_0 \xi^\dagger &= C^\mu \partial_\mu \xi^\dagger - \frac{1}{2} C^{ab} \xi^\dagger \gamma_{ab} + \frac{i}{2} \kappa (\xi^\dagger \gamma_5 \gamma^\mu \xi) \psi_\mu^\dagger - i g C \xi^\dagger .
\end{align*}
\]

The ghosts behave like tensors under coordinate transformations, except for the coordinate ghosts themselves, for which \( dC^\mu \) transform as coordinate vectors [22].

Since we did not include auxiliary fields in the supergravity multiplet, the commutator algebra closes only on-shell on the gravitinos, which implies that on these \( s_0 \) is not nilpotent but squares into a trivial gauge transformation (namely a field equation). Also one finds that nilpotency holds on the SO(4) rotation ghosts only modulo the field equations of the gravitinos,

\[
\begin{align*}
    s_0^2 \psi_\mu &= M_{\mu \nu} \frac{\delta S_0}{\delta \psi_\nu} + \frac{\delta S_0}{\delta \psi_\nu} N_{\nu \mu} , \\
    s_0^2 C^{ab} &= \Sigma_{\mu \nu}^{ab} \frac{\delta S_0}{\delta \psi_\mu} + \frac{\delta S_0}{\delta \psi_\mu} \Sigma_{\mu \nu}^{ab} , \\
    s_0^2 (\text{other fields}) &= 0 ,
\end{align*}
\]

where

\[
\begin{align*}
    M_{\mu \nu}^{\alpha \beta} &= -\frac{1}{8} e \left[ (\gamma_5 \gamma^\rho \xi)^\alpha (\xi^\dagger \gamma_5 \gamma_\nu \gamma_\mu)_\beta - (\gamma_5 \xi)^\alpha (\xi^\dagger \gamma_5 \gamma_\nu \gamma_\mu)_\beta - (\gamma_5 \gamma_\mu \xi)^\alpha (\xi^\dagger \gamma_5)_\beta \\
    &+ (\gamma_5 \gamma^\rho \xi)^\alpha (\xi^\dagger \gamma_5 \gamma_\rho \gamma_\nu \gamma_\mu)_\beta + 2 \xi^\dagger (\gamma_5 \gamma^\rho \xi)^\alpha \right] \\
    N_{\nu \mu}^{\alpha \beta} &= -\frac{1}{8} e \left[ (\gamma_5 \gamma_\mu \gamma_\nu \xi)^\beta (\gamma_5 \gamma^\rho \xi)^\alpha + (\gamma_5 \xi)^\beta (\gamma_5 \gamma_\mu \xi)^\alpha + (\gamma_\mu \gamma_\nu \xi)^\beta \xi^\alpha \right] \\
    \Sigma_{\mu \nu}^{\alpha \beta} &= \frac{\kappa}{8} e_\alpha^{\rho} e_\beta^{\sigma} (\gamma_5 \xi)^\alpha \gamma_5 \gamma^\nu \xi + (\gamma_\rho \gamma_\mu \gamma_5 \xi)^\alpha \xi^\dagger \xi \right] .
\end{align*}
\]

In order to obtain a nilpotent BRST operator and to quantize the model, we employ the Batalin-Vilkovisky approach [12, 11]. There exist off-shell formulations for \( N = 2 \) extended supergravity [18], but these are quite complicated, and working with the BV formalism without auxiliary fields is much simpler for our aim of checking Ward identities. With auxiliary fields the local gauge algebra of general coordinate transformations, local Lorentz (SO(4) in the Euclidean case) transformations, local supersymmetry transformations and U(1) gauge transformations closes off-shell. In particular the commutator of two
local susy transformations produces a local Lorentz transformation with a parameter that depends on the auxiliary fields. This explains why \( s_b C^{ab} \) is nonvanishing when these are absent, because the auxiliary fields transform under local supersymmetry transformations into gravitino field equations.

The BV formalism involves the construction of an extension \( S[\Phi, \Phi^*] \) of the action \( S_0[\Phi] \) that satisfies the master equation (the superscripts \( L \) and \( R \) denote left and right differentiation respectively, where we frequently omit the \( L \) for left-derivatives)

\[
(S, S) = -2 \int d^4x \frac{\delta^R S}{\delta \Phi^*_A} \frac{\delta^L S}{\delta \Phi^A} = 0 \tag{18}
\]

Here the \( \Phi^A \) collectively denote the fields in the supergravity multiplet and the ghosts, while the \( \Phi^*_A \) are the antifields conjugate to \( \Phi^A \) with respect to the antibracket, i.e.

\[
(\Phi^A(x), \Phi^*_B(y)) = \delta^A_B \delta(x - y).
\]

One assigns antifield numbers \((af)\) and ghost numbers \((gh)\) to the antifields according to the relation \( af \Phi^*_A = -gh \Phi^*_A = 1 + gh \Phi^A \); for the fields \( af \Phi^A = 0 \). (For example, \( af A^*\mu = 1 \) and \( af C^* = 2 \).) Given a solution to the master equation, the corresponding BRST operator

\[
s = (S, \cdot) = \int d^4x \left( \frac{\delta^R S}{\delta \Phi^*_A} \frac{\delta}{\delta \Phi^*_A} - \frac{\delta^R S}{\delta \Phi^A} \frac{\delta}{\delta \Phi^A} \right)
\]

is automatically nilpotent. It decomposes into pieces \( s_k \) of definite antifield number \( k \), \( s = \sum_{k \geq -1} s_k \), with \( s_{-1} \Phi^A = 0 \) and \( s_0 \Phi^A \) as in (14) and (15).

The minimal solution to (18) contains terms bilinear in the antifields and is given by

\[
S_{\text{min}} = S_0 - \int d^4x \left( s_0 e^\mu_a e^a_{\mu} + s_0 A_\mu A^*\mu + \psi^{*\mu} s_0 \psi^\mu + s_0 \psi^\mu \psi^{*\mu} + s_0 C \psi + s_0 C^{*} C + s_0 C^{ab} C_{ab} - \xi^\mu s_0 \xi_{\mu} + s_0 \xi^\mu \xi^{*\mu}
+ \psi^{*\mu} M_{\mu\nu} \psi^{*\mu} + \frac{1}{2} \psi^{*\mu} N_{\mu\nu} \psi^{*\mu} + \frac{1}{2} \psi^{*\mu} N_{\mu\nu} \psi^{*\mu}
+ \psi^{*\mu} \Sigma^{ab} C_{ab} + \Sigma^{ab} \psi^{*\mu} C_{ab} \right). \tag{20}
\]

The terms in the first line have antifield number 1 (except \( S_0 \), which has vanishing antifield number), those in the second and third line have \( af = 2 \) and the last line has \( af = 3 \). This extended Euclidean action is hermitean if we define \((\Phi_A^*)^\dagger = -(\Phi_A)^{*\dagger}\) as usual.

From \( S_{\text{min}} \) we obtain the nilpotent BRST transformations of \( \psi^\mu \) and \( C^{ab} \)

\[
s \psi^\mu = (S_{\text{min}}, \psi^\mu) = \frac{\delta S_{\text{min}}}{\delta \psi^\mu} = s_0 \psi^\mu + M_{\mu\nu} \psi^{*\nu} + \psi^{*\nu} N_{\nu\mu} + \Sigma^{ab} C_{ab}^\mu
\]

\[
s C^{ab} = (S_{\text{min}}, C^{ab}) = -\frac{\delta S_{\text{min}}}{\delta C_{ab}^\mu} = s_0 C^{ab} + \psi^{*\mu} \Sigma^{ab} + \Sigma^{ab} \psi^{*\mu}. \tag{21}
\]

On all other fields we have \( s \Phi^A = s_b \Phi^A \) exactly.

In order to gauge-fix the various local symmetries of the action, we introduce as usual a nonminimal sector, consisting of Nakanishi-Lautrup auxiliary fields \( b_\mu, b_{ab}, b, \beta, \beta^\dagger \), the antighosts \( C_\mu, C_{ab}, \bar{C}, \bar{\xi}, \bar{\xi}^\dagger \) (short bars denote antighosts), and the corresponding antifields. To fix supersymmetry, we employ in addition a second pair of Dirac spinors
\[ η, η^\dagger \] and \( \varrho, \varrho^\dagger \), whose role will be explained below. \( \bar{C}, \bar{C}_\mu, \bar{C}_{ab}, \beta, \beta^\dagger, η \) and \( η^\dagger \) are anticommuting, the other fields are commuting.

The action for the nonminimal sector is

\[
S_{\text{non}} = \int d^4x \left( -b_\mu \bar{C}^{*\mu} - b_{ab} \bar{C}^{*ab} - b \bar{C}^{*} + \bar{\xi}^* \beta - \beta^\dagger \bar{\xi}^{\dagger*} + \varrho \eta - η^\dagger \varrho^{\dagger*} \right) .
\]

(22)

Because the fields and antifields in \( S_{\text{non}} \) are different from the fields and antifields in \( S_{\text{min}} \), \( S = S_{\text{min}} + S_{\text{non}} \) satisfies the master equation (18). The BRST transformations for the nonminimal sector are obtained by taking the antibracket with \( S \),

\[
\begin{align*}
    s\bar{C}_\mu &= b_\mu , \\
    sb_\mu &= 0 \\
    s\bar{C}_{ab} &= b_{ab} , \\
    sb_{ab} &= 0 \\
    s\bar{C} &= b , \\
    sb &= 0 \\
    s\bar{\xi} &= \beta , \\
    s\beta &= 0 , \\
    s\varrho &= \eta , \\
    s\eta &= 0 \\
    s\bar{\xi}^{\dagger} &= \beta^\dagger , \\
    s\beta^\dagger &= 0 , \\
    s\varrho^{\dagger} &= \eta^\dagger , \\
    s\eta^{\dagger} &= 0 .
\end{align*}
\]

(23)

The reason for the last two definitions will become clear below.

We now replace the antifields by

\[
\Phi^{* \text{A}} = -\frac{δ}{δ\Phi^{* \text{A}}} \Psi[Φ] ,
\]

(24)

where \( \text{A} \) refers to all fields including the nonminimal sector and \( \Psi \) is a suitably chosen gauge-fixing fermion of ghost number \(-1\). Upon elimination of the \( \Phi^{* \text{A}} \) in \( S = S_{\text{min}} + S_{\text{non}} \) we obtain

\[
S_\Psi[Φ] = S[Φ, Φ^{* \text{A}} = -δΨ/δΦ] = S_0[Φ] + s_0Ψ + \ldots .
\]

(25)

The ellipses denote four-ghost terms that arise from the antifield bilinears in \( S \) and will be given explicitly below. \( S_Ψ \) is invariant under the gauge-fixed BRST transformations

\[
sΨ \Phi^{* \text{A}} = sΨ \Phi^{* \text{A}}|_{Φ^{* \text{A}} = -δΨ/δΦ} .
\]

(26)

At this point we have specified the quantum action for any choice of \( Ψ \).

We now turn to a suitable choice for \( Ψ \). If one considers a complex gravitino in a background gravitational field, a gauge-fixing term for local supersymmetry that manifestly preserves the Euclidean space symmetries is given by \( e \psi^{\dagger} \cdot γ_5 \bar{\varrho} \gamma \cdot \psi \). This gauge-fixing term is obtained from the usual one in Minkowski spacetime [21] by the Wick rotation discussed above. It is well-known that such a gauge-fixing term which depends on the vielbein field leads to additional Nielsen-Kallosh (NK) ghosts [23]. Since we are considering a dynamical gravitational field and want to avoid spurious vertices, we prefer a vielbein-independent gauge-fixing term \( ψ^{\dagger} \cdot \hat{γ}_5 \varrho \hat{\gamma} \cdot ψ \) with \( \hat{γ}_5 \equiv γ_5 \delta_\mu \) and \( \varrho \equiv \hat{γ}_\mu \partial_\mu \). Both terms give the same propagators. However, we find it worthwhile first to consider the more general case \( e \psi^{\dagger} \cdot γ_5 \varrho \gamma \cdot ψ \) and afterwards to take the flat space limit.

In a path integral approach we could start from

\[
\Delta_{FP} δ(γ \cdot ψ - χ) δ(\psi^{\dagger} \cdot γ - χ^\dagger) \int dx \int d\chi \exp \left( -\frac{1}{2k} \int d^4x \ e \chi^{\dagger} γ_5 \varrho \chi \right) (\det γ_5 \varrho)^{-1} .
\]

(27)
The term \((\det \gamma_5 \mathcal{D})^{-1}\) is a normalization factor of the path integral over \(\chi\) and \(\chi^\dagger\) and leads to a complex commuting NK ghost. (For \(N = 1\) theories one has Majorana spinors \(\chi\) and a factor \((\det \gamma_5 \mathcal{D})^{-1/2}\). One replaces this by \((\det \gamma_5 \mathcal{D})^{1/2} (\det \gamma_5 \mathcal{D})^{-1}\) and obtains then one real anticommuting NK ghost and one pair of commuting NK ghosts. In Euclidean space one can take complex holomorphic NK ghosts, similar to the approach for Majorana spinors in Euclidean space discussed in the introduction. For \(N = 2\) theories we can directly take one complex commuting ghost because the Dirac action for a complex commuting spinor is nonvanishing.) The factor \(\Delta_{\text{FP}}\) yields the susy ghost action of the form \(\kappa^{-1} e \xi^\dagger \gamma_5 \mathcal{D} \xi + \text{h.c.}\) (obtained from the usual ghost action in Minkowski spacetime by the Wick rotation).

We must now find the corresponding expressions in the BV formalism. This is surprisingly complicated. The most straightforward way to proceed would be to start from the following nonlocal gauge-fixing fermion \(\Psi\),

\[
\Psi = \int d^4 x \ e \, \xi^\dagger \gamma_5 (\gamma \cdot \psi - k \, \mathcal{D}^{-1} \beta) - \text{h.c.}
\]

\[
s_0 \Psi = \int d^4 x \ e \left[\beta^\dagger \gamma_5 (\gamma \cdot \psi - k \, \mathcal{D}^{-1} \beta) + \xi^\dagger \gamma_5 s_0 (\gamma \cdot \psi)\right] + \text{h.c.}
\]

where \(k\) is the gauge-fixing parameter. (We are considering a background gravitational field and a gravitino-independent spin connection for the time being, so the vielbein fields in the gauge-fixing fermion are not varied.) Eliminating \(\beta\) we would indeed find the desired gauge-fixing term \(e \psi^\dagger \gamma_5 \mathcal{D} \gamma \cdot \psi\), and the complex NK ghost from the path integral over \(\beta\) and \(\beta^\dagger\). However, the main virtue of the BV formalism is that it tries to avoid problems connected to the path integral measure, and therefore it is preferable to extend the above approach in such a way that all nonlocal terms in the action cancel. This is possible for \(N = 1\) theories if one introduces two contractible pairs \(\eta_1, \varrho_1\) and \(\eta_2, \varrho_2\) (all of which are Majorana spinors) with BRST transformations

\[
s \eta_1 = \varrho_1 \ , \ s \varrho_1 = 0 \ , \ s \varrho_2 = \eta_2 \ , \ s \eta_2 = 0 \ .
\]

The \(\eta_i\) are anticommuting, while the \(\varrho_i\) are commuting. One may add to the action the BRST exact term

\[
s (\eta_1^i \mathcal{C} \varrho_2) = \varrho_1^i \mathcal{C} \varrho_2 - \eta_1^i \mathcal{C} \varrho_2 = \eta_1^i \mathcal{C} \varrho_2 - \frac{1}{2} \lambda^i \mathcal{C} \varrho \lambda + \frac{1}{2} \lambda^i \mathcal{C} \varrho \lambda - \frac{1}{2} D_\mu (\eta_1^i \mathcal{C} \gamma^\mu \eta_2) \ ,
\]

where \(\mathcal{C}\) is the Euclidean charge conjugation matrix and \(\lambda = (\eta_1 + \eta_2)/\sqrt{2}\), \(\chi = (\eta_1 - \eta_2)/\sqrt{2}\). Shifting \(\chi \to \chi + \mathcal{D}^{-1} \beta\), we find the anticommuting NK ghost \(\lambda\), the two commuting NK ghosts \(\rho_i\), and further a term \(\beta^i \mathcal{C} \varrho \lambda\) which cancels the nonlocal contribution from the BRST variation of the gauge-fixing fermion. One is left with terms of the form \(\beta^i \mathcal{C} (\gamma \cdot \psi + 2k \chi) + \chi^i \mathcal{C} \varrho \chi\), and integration over \(\beta\) and \(\chi\) yields the desired gauge-fixing term \((\gamma \cdot \psi)^i \mathcal{C} \varrho \chi\). In this way we end up with the same result as from the path integral.

---

\(^6\) The Euclidean \(\mathcal{C}\) is obtained from the Minkowski matrix \(\mathcal{C}_M\) via \(\mathcal{C} = O^i \mathcal{C}_M O\) with \(O\) as in \(\footnote{I}\) and satisfies the same relations in Euclidean space as in Minkowski spacetime, namely \(\mathcal{C}^i = -\mathcal{C}\) and \(\mathcal{C} \gamma_\mu = -\gamma_\mu \mathcal{C}\), c.f. \(\footnote{II}\).
We now generalize these results to the case $N = 2$, but at the same time we aim for $\psi^\dagger \gamma_5 \tilde{\eta} \gamma \cdot \psi$ as the supersymmetry fixing term. We have been able to achieve this with only one new complex contractible pair of fields $\eta$ and $\varrho$. Consider the following gauge-fixing fermion

$$
\Psi = \int d^4 x \left[ C(\partial_\mu A_\mu - k_1 b) + \bar{C}^\mu (\partial_\nu (eg^{\mu\nu}) - k_2 b^\nu) + \bar{C}^{ab} \left( (\delta_\mu e_{ab} - \delta_\nu e_{ma}) - k_3 b_{ab} \right) + k_4 \eta^\dagger \gamma_5 \tilde{\eta} - k_4 \eta^\dagger \gamma_5 \bar{\eta} \right] 
$$

We chose to contract world indices with $\delta_{\mu\nu}$ rather than the metric in order to have a minimal number of vertices to deal with when we check the Ward identities. Upon elimination of the $\Phi^\dagger$ by using (24) in $S = S_{\text{min}} + S_{\text{non}}$, we obtain

$$
S_\Psi = S_0 + s_0 \Psi + \int d^4 x \left( \xi^\dagger \gamma_5 \gamma^\mu M_{\mu\nu} \gamma_5 \xi - \frac{1}{2} \xi^\dagger \gamma_5 \gamma^\mu N_{\mu\nu} (\xi^\dagger \gamma_5 \gamma^\nu) \right) 
$$

where we have integrated by parts. The nonlocal term $\beta^\dagger \gamma_5 \tilde{\eta}^{-1} \beta$ can be removed by means of a shift $\eta \rightarrow \eta - \tilde{\eta}^{-1} \beta$, after which the third line in the above equation reads

$$
-\beta^\dagger \gamma_5 (\gamma \cdot \psi - 2k_4 \eta^\dagger \gamma_5 \bar{\eta}) - (\psi^\dagger \gamma - 2k_4 \eta^\dagger \gamma_5 \bar{\eta}) 
$$

We can now eliminate the auxiliary fields by their algebraic equations of motion, which results in

$$
s_0 \Psi = \int d^4 x \left[ \frac{1}{4k_1} (\partial_\mu A_\mu)^2 + \frac{1}{4k_2} (\partial_\mu (eg^{\mu\nu}))^2 + \frac{1}{4k_3} (e_{ab} - e_{ba})^2 + \partial_\mu \bar{C} s_0 A_\mu + \partial_\mu \bar{C}^\nu s_0 (eg^{\mu\nu}) - \bar{C}^{ab} s_0 (e_{ab} - e_{ba}) 
$$

and the BRST transformations of the antighosts turn into

$$
s_\Psi \bar{C} = \frac{1}{2k_1} \partial_\mu A_\mu, \quad s_\Psi \bar{C}^\mu = \frac{1}{2k_2} \partial_\nu (eg^{\mu\nu})$$
\[ s_\Psi \bar{C}^{ab} = \frac{1}{2k_3} (e^{ab} - e^{ba}), \quad s_\Psi \bar{\xi} = \frac{1}{2k_4} \partial(\hat{\gamma} \cdot \psi). \] (36)

At this point the complete quantum action is explicitly known: it is given by (32) and (35).

Our aim is to explicitly check the infinite and finite parts of Ward identities. To avoid the complications of propagators in anti-de Sitter space, we set the cosmological constant \( g = 0 \) (the present paper is a pilot program for more complicated studies with \( g \neq 0 \)). Hence we expand about flat space. A convenient choice for the gauge-fixing parameters is \( k_1 = \frac{1}{2}, \ k_2 = \kappa^2, \ k_3 = 0, \ k_4 = 1 \), which yields the following propagators for the supergravity multiplet

\[ \langle A_\mu A_\nu \rangle_0 = \frac{1}{p^2} \delta_{\mu\nu}, \quad \langle c_{\mu\nu} c_{\rho\sigma} \rangle_0 = \frac{1}{2p^2} (\delta_{\mu\rho} \delta_{\nu\sigma} + \delta_{\mu\sigma} \delta_{\nu\rho} - \delta_{\mu\nu} \delta_{\rho\sigma}) \]
\[ \langle \bar{\psi}_\mu \psi_\nu \rangle_0 = \frac{i}{2p^2} \gamma_5 \hat{\gamma}_\mu \hat{\gamma}_\nu \]

where \( e_\mu^a = \delta_\mu^a + \kappa c_\mu^a \). The ghost propagators read

\[ \langle C \bar{C} \rangle_0 = \frac{1}{p^2}, \quad \langle C^\mu \bar{C}^\nu \rangle_0 = -\frac{1}{p^2} \delta^{\mu\nu} \]
\[ \langle \bar{C}_{ab} \bar{C}^{cd} \rangle_0 = \frac{1}{2} \delta^{[a}_b \delta^{d]}_{c}, \quad \langle \bar{\xi} \bar{\xi} \rangle_0 = \kappa \frac{i}{p^2} \gamma_5 \]

where we have shifted \( \hat{C}_{ab} = C_{ab} + \delta_{\mu[a} \partial_{b]} C^\mu \) in order to diagonalize the kinetic terms of the ghosts [21]. Note the occurrence of \( \gamma_5 \) in the propagators of the spinors.

We are now ready to compute Ward identities. These are derived by first making the substitution \( \Phi^*_A = K_A - \delta \Psi / \delta \Phi^A \) instead of (24), where the \( K_A \) are a set of external sources for the gauge-fixed BRST transformations introduced in [11]. Since this substitution is a canonical transformation, \( S_\Psi[\Phi, K] \) satisfies a master equation similar to (18)

\[ \int d^4x \frac{\delta R S_\Psi}{\delta K_A} \frac{\delta L S_\Psi}{\delta \Phi^A} = 0. \] (39)

Now consider the generating functional

\[ Z[J, K] = \int [d\Phi] \exp \left( -S_\Psi[\Phi, K] + \int d^4x \Phi^A J_A \right), \] (40)

and perform an infinitesimal change of integration variables \( \Phi^A \rightarrow \Phi^A + \delta R S_\Psi / \delta K_A \). Assuming the path integral measure to be invariant, one obtains

\[ \int d^4x \frac{\delta R Z}{\delta K_A} J_A = 0. \] (41)

The same relation holds for the generating functional \( W = \ln Z \) of connected Green functions. (Right-) differentiation with respect to the sources \( J_A \) and setting \( J_A = K_A = 0 \) afterwards then yields the Ward identities for connected graphs

\[ s_\Psi \langle \Phi^{A_1}(x_1) \ldots \Phi^{A_n}(x_n) \rangle = 0, \] (42)

\footnote{One has \( S_\Psi[\Phi, K] = S_0 + s_0 \Psi - \int s_\Psi \Phi^A K_A + \ldots \).}
expressing the BRST invariance of the Green functions.

We consider the identity for the two-point function \( \langle A_\mu(x) C(y) \rangle \). The Fourier transformed identity reads

\[
0 = -ip_\nu \langle A_\mu(p) A_\nu(-p) \rangle + ip_\mu \langle C(p) \bar{C}(-p) \rangle + \langle C^{\nu*} \partial_\nu A_\mu(p) \bar{C}(-p) \rangle + \langle \partial_\mu C^{\nu*} A_\nu(p) \bar{C}(-p) \rangle + \frac{1}{\sqrt{2}} \langle \xi_5 \gamma_5 \gamma_\mu \psi_\mu(p) \bar{C}(-p) \rangle - \frac{1}{\sqrt{2}} \langle \bar{\psi}_\mu \gamma_5 \xi(p) \bar{C}(-p) \rangle .
\]

It obviously holds at tree level. For \( g = 0 \) the relation is easily verified also at the one-loop level. From (52) and (53) we infer that there are no vertices involving the ghost \( C \), so only the Green function \( \langle A_\mu A_\nu \rangle \) receives any loop contributions at all and the Ward identity expresses the transversality of the sum of these graphs. The one-loop diagrams consist of a gravitino loop, a vector-graviton loop and various tadpoles (i.e., loops with one internal line). The latter vanish in dimensional regularization since all fields are massless. Transversality of \( \langle A_\mu A_\nu \rangle \) at one loop now follows from the fact that \( A_\mu \) enters the relevant vertices only via its field strength, which implies that the contraction with \( p_\nu \) vanishes. Of course, this argument hinges on a suitable regularization that can deal with the presence of \( \gamma_5 \) and the Levi-Civita tensor in the vector-gravitino vertices.

This Ward identity is admittedly very simple. Other identities are quite complicated due to the gravitational interactions. Our aim was to develop the formalism to the point of explicit Feynman diagrams, and this we have achieved.

### 3 Euclidean N=2 Super Yang-Mills

As explained in the introduction, the continuation to Euclidean space of theories which contain pseudoscalars gives rise to certain unusual signs and factors of \( i \) in the action and transformation rules that originate from the Wick rotation \( \varphi \to i \varphi_E \) of pseudoscalars \( \varphi \). For an ordinary scalar \( \phi \) one has of course just \( \phi \to \phi_E \). In particular, the kinetic terms of the pseudoscalars enter the Euclidean action with the “wrong” sign, which leads to propagators with a sign opposite to that of the scalar propagators. At first sight this seems to spoil the Ward identities, since diagrams containing internal pseudoscalar lines acquire a factor \( (-1) \) from each of the corresponding propagators. However, the vertices also acquire a factor \( i \) for each pseudoscalar, leading to an additional factor \( (-1) \) from the two endpoints of every internal pseudoscalar line. These restore the signs of the propagators, such that there is no violation of the Ward identities coming from the Wick rotation of the pseudoscalars.

We study the Ward identities in some detail in the example of \( N = 2 \) supersymmetric Yang-Mills theory. The on-shell field content consists of one vector \( A^I_\mu \), one scalar \( \phi^I \), one pseudoscalar \( \varphi^I \), and one Dirac spinor \( \lambda^I \) for each generator \( T_I \) of the gauge group. The classical action in Euclidean space has been given first by Zumino in \([4]\) and coincides with the one derived from the Wick rotation rules of \([5]\). It reads (dropping again all subscripts \( E \))

\[
S_0 = \int d^4x \left[ \frac{1}{4} F^{\mu\nu I} F_{\mu\nu}^I + \frac{1}{2} D^\mu \phi^I D_\mu \phi^I - \frac{1}{2} D^\mu \varphi^I D_\mu \varphi^I - \lambda^I \gamma^\mu D_\mu \lambda^I 
- ig f^{IJK} \lambda^K (\gamma_5 \phi^J + \varphi^J) \lambda^K - \frac{1}{2} (gf^{IJK} \phi^J \varphi^K)^2 \right],
\]

\[
(44)
\]
with covariant derivative $D_{\mu} \phi^I = \partial_{\mu} \phi^I + g f^{IJK} A^I_{\mu} \phi^K$, etc. Note the signs of the kinetic term of $\phi^I$ and of the scalar potential; clearly, the Euclidean action is not bounded from below. For this model a set of three real scalar auxiliary fields is known, but we shall not use them for reasons explained in the introduction.

The BRST transformations in Minkowski space in absence of auxiliary fields in the supersymmetry multiplet have been worked out in [14]. Using the Wick rotation rules of [5] one finds for the antifield-independent parts of the Euclidean BRST transformation rules

\[
\begin{align*}
  s_0 A^I_{\mu} &= D^I_{\mu} + i \xi^I \gamma_{\mu} \lambda^I + i \lambda^I \gamma_{\mu} \xi + C^I \partial_{\mu} A^I_
u \\
  s_0 \phi^I &= -g f^{IJK} C^J \phi^K - \xi^I \gamma_{\mu} \lambda^I - \lambda^I \gamma_{\mu} \xi + C^I \partial_{\mu} \phi^I \\
  s_0 \varphi^I &= -g f^{IJK} C^J \varphi^K + \xi^I \lambda^I + \lambda^I \xi + C^I \partial_{\mu} \varphi^I \\
  s_0 \lambda^I &= -g f^{IJK} C^J \lambda^K - D_{\mu}(\phi^I - \varphi^I \gamma_{\mu}) \gamma_{\mu} \xi + \frac{i}{2} F_{\mu \nu} \gamma_{\mu \nu} \xi \\
  &\quad + ig f^{IJK} \phi^J \varphi^K \xi \gamma_{\mu} + C^I \partial_{\mu} \lambda^I \\
  s_0 \lambda^{\dagger I} &= -g f^{IJK} C^J \lambda^K + \xi^I \gamma^I + \frac{1}{2} \xi^I \gamma^I \gamma^I + C^I \partial_{\mu} \lambda^I \\
  &\quad + ig f^{IJK} \phi^J \varphi^K \xi \gamma_{\mu} + C^I \partial_{\mu} \lambda^{\dagger I} .
\end{align*}
\]

Here the $C^I$ are the ghosts of gauge transformations. Further, $\xi$, $\xi^I$ and $C^I$ denote again the supersymmetry and translation ghosts respectively, only now these are constant since we are considering rigid supersymmetry and translations.

The BRST transformations of the ghosts read

\[
\begin{align*}
  s_0 C^I &= -\frac{1}{2} g f^{IJK} C^J C^K + 2 i \xi^I (\gamma_{\mu} \phi^I + \varphi^I) \xi + 2 \xi^I \gamma^I \gamma^I + C^I \partial_{\mu} C^I \\
  s_0 C^I &= -2 \xi^I \gamma^I \gamma^I \\
  s_0 \xi &= 0 .
\end{align*}
\]

As in the supergravity case, the BRST operator $s_0$ is not nilpotent. We have

\[
\begin{equation}
\begin{align*}
  s_0^2 \lambda^I &= M \frac{\delta S_0}{\delta \lambda^I} + \frac{\delta S_0}{\delta \lambda^I} N , \\
  s_0^2 \text{ (other fields)} &= 0 ,
\end{align*}
\end{equation}
\]

where

\[
M^{\alpha}_{\beta} = 2(\xi^\alpha \delta^\beta_{\gamma}) - (\gamma^\alpha \delta^\beta_{\xi}) (\xi^\gamma \gamma^\beta) - (\gamma^\alpha \gamma^\beta) (\xi^\gamma \gamma^\beta) , \\
N^{\alpha}_{\beta} = -\xi^\alpha \xi^\beta .
\]

A nilpotent BRST operator $s$ can be obtained from a solution $S$ of an extended master equation [4, 24]

\[
(S, S) = -2 \int d^4 x \left( \frac{\delta^R S}{\delta \hat{\Phi}^A} \frac{\delta^L S}{\delta \hat{\Phi}^A} \right) - 2 \frac{\partial^R S}{\partial \hat{\Phi}^A} \frac{\partial^L S}{\partial \hat{C}^A} = 0 .
\]

Here the fields $\hat{\Phi}^A$ consist of the supersymmetry multiplet and the ghosts $C^I$ (and later also the BRST auxiliary fields and antighosts), while the $\hat{C}^A$ denote the constant ghosts $C^\mu$, $\xi$, $\xi^I$ of the rigid symmetries. The minimal solution is given by

\[
S_{\text{min}} = S_0 - \int d^4 x \left( s_0 A^I_{\mu} A^\mu I + s_0 \phi^I \phi^I + s_0 \varphi^I \varphi^I + \lambda^I s_0 \lambda^I + s_0 \lambda^I \lambda^{\dagger I} \right.
\]

\[
\left. + s_0 C^I C^I + \lambda^{\dagger I} M \lambda^I + \frac{1}{2}(\lambda^I \xi^2) - \frac{1}{2}(\xi^{\dagger I} \xi)^2 \right) + 2 \xi^I \gamma_{\mu} \gamma^I \xi C^\mu .
\]
This action is hermitean because \( (\Phi^*_A)^\dagger = -(\Phi^A)^\dagger \). The only constant term in this action is the last one. It follows that \( s = (S_{\text{min}}, \cdot) = s_0 \) on all fields \( \Phi^A \) and constant ghosts \( C^r \) except for the gauginos \( \lambda^I \), for which we find

\[
s\lambda^I = s_0 \lambda^I + 2(\xi^\dagger \xi) \lambda^{*I} - (\xi^\dagger \gamma_5 \lambda^{*I}) \gamma_5 \xi + (\xi^\dagger \gamma_5 \gamma_\mu \lambda^{*I}) \gamma_\mu \gamma^\mu \xi - (\lambda^{*I} \xi) \xi . \tag{51}
\]

In the gauge-fixing procedure we follow \([14]\) and include also a translation in the BRST transformations of the antighosts \( \bar{C}^I \). (In \((45)\) and \((46)\) there are already translations of the classical fields and internal symmetry ghosts.) The master equation then implies that the auxiliary fields are not BRST-invariant, for the solution for the nonminimal sector now reads

\[
S_{\text{non}} = -\int d^4x \left( s\bar{C}^I \bar{C}^{*I} + sb^I b^{*I} \right) , \tag{52}
\]

where

\[
s\bar{C}^I = b^I + C^\mu \partial_\mu \bar{C}^I , \quad sb^I = 2 \xi^\dagger \gamma_5 \gamma^\mu \xi \partial_\mu b^I + C^\mu \partial_\mu b^I . \tag{53}
\]

The term \( s\bar{C}^I = b^I \) is standard, but adding the translation term \( C^\mu \partial_\mu \bar{C}^I \) BRST nilpotency leads to a nonvanishing transformation of \( b^I \). As gauge-fixing fermion for the local symmetry we choose

\[
\Psi = \int d^4x \bar{C}^I \left( \partial_\mu A^{\mu I} - k b^I \right) , \tag{54}
\]

where the real constant \( k \) denotes again the gauge-fixing parameter. Upon replacing \( \Phi^*_A = -\delta \Psi / \delta \Phi^A \) in \( S = S_{\text{min}} + S_{\text{non}} \) the terms with two antifields vanish, whereas the terms with one antifield yield the BRST variation of \( \Psi \)

\[
s_0 \Psi = s\Psi = \int d^4x \left[ b^I (\partial_\mu A^{\mu I} - k b^I) + \partial_\mu \bar{C}^I (D^\mu C^I - 2k \xi^\dagger \gamma_5 \gamma^\mu \xi \bar{C}^I + i \lambda^{*I} \gamma_5 \gamma^\mu \xi) \right] . \tag{55}
\]

The final quantum action is given by \( S_\Psi[\Phi, \xi, \xi^\dagger] = S_0[\Phi] + s_0 \Psi \). The result for the gauge-fixing and ghost action is independent of the translation ghosts because we included translations in \((53)\) (as a result, \( \Psi \) in \((54)\) is translation invariant). After elimination of the \( b^I \) the transformations of the antighosts read

\[
s_\Psi \bar{C}^I = \frac{1}{2k} \partial_\mu A^{\mu I} + C^\mu \partial_\mu \bar{C}^I . \tag{56}
\]

The final BRST transformation rules \( s_\Psi \Phi^A \) are then given by \((13)\), \((18)\) and \((56)\).

The opposite sign of the kinetic terms of the pseudoscalars \( \varphi^I \) implies an opposite sign in the propagators. They read for the bosonic fields (taking \( k = \frac{1}{2} \))

\[
\langle A^J_\mu A^{\mu J}_\nu \rangle_0 = \frac{1}{p^2} \delta_{\mu\nu} \delta^{IJ} , \quad \langle \phi^I \phi^J \rangle_0 = \frac{1}{p^2} \delta^{IJ} , \quad \langle \varphi^I \varphi^J \rangle_0 = -\frac{1}{p^2} \delta^{IJ} . \tag{57}
\]
The fields $\Phi^A$ we couple to external sources as usual, but the constant ghosts $C^r$ are treated themselves as external sources (so the generating functional $Z$ depends on $C^r$.) We have then a nondiagonal kinetic matrix due to the mixed terms in (55)

$$
\int d^4x \left[ - \lambda^I \gamma_5 \phi \lambda^I + \partial_\mu C^I (\partial^\mu C^I - 2k \xi^I \gamma_5 \gamma^\mu \xi \bar{C}^I + i \xi^I \gamma_5 \gamma^\mu \lambda^I + i \lambda^I \gamma_5 \gamma^\mu \xi) \right] =
$$

$$
= \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \left( - \bar{C}^I(p), C^I(p), \sqrt{2} \lambda^I(p) \right) \mathcal{M}^{IJ} \left( \begin{array}{c} \bar{C}^J(-p) \\ C^J(-p) \end{array} \right),
$$

(58)

where $\mathcal{M}^{IJ}$ is given by

$$
\mathcal{M}^{IJ} = \delta^{IJ} \left( \begin{array}{ccc} 4ik \xi^I \gamma_5 \phi \xi & -p^2 & \sqrt{2} \xi^I \gamma_5 \phi \\ -p^2 & 0 & 0 \\ -\sqrt{2} \gamma_5 \phi \xi & 0 & i \gamma_5 \phi \end{array} \right).
$$

(59)

This matrix is easily inverted. The result is particularly simple in the Feynman gauge $k = \frac{1}{2}$, which we had adopted already above,

$$
(\mathcal{M}^{-1})^{IJ} = \frac{i}{p^2} \delta^{IJ} \left( \begin{array}{ccc} 0 & i & 0 \\ i & (2 - 4k) \xi^I \gamma_5 \phi \xi / p^2 & -\sqrt{2} \xi^I \\ 0 & \sqrt{2} \xi & \gamma_5 \phi \end{array} \right).
$$

(60)

With $k = \frac{1}{2}$ we find the following nonvanishing propagators

$$
\langle \lambda^I \lambda^J \rangle_0 = -\frac{i}{p^2} \gamma_5 \phi \delta^{IJ}, \quad \langle C^I \bar{C}^J \rangle_0 = \frac{1}{p^2} \delta^{IJ}, \quad \langle \lambda^I C^J \rangle_0 = \frac{i}{p^2} \xi \delta^{IJ}.
$$

(61)

Note that it was the inclusion of translations in the BRST transformations of the anti-ghosts that led to the $4ik \xi^I \gamma_5 \phi \xi$ term in (59) and that enabled us to make the $\langle C^I C^J \rangle_0$ propagator vanish by a suitable choice of the gauge-fixing parameter. Note also that one may diagonalize the propagators in the $\lambda^I C^J$ sector by means of a field redefinition $\lambda^I \rightarrow \lambda^I - i \xi \bar{C}^I$. However, this would result in additional vertices and more complicated BRST transformations, which we prefer to avoid.

The nonvanishing $\langle \lambda^I C^J \rangle_0$ propagator signals the intertwining of gauge and supersymmetry. Rigid supersymmetry is broken as usual by the gauge-fixing terms, but it fuses with the usual BRST symmetry for the internal local symmetries into an extended BRST symmetry which connects the local and the rigid symmetries. This fusion can also be observed in the Ward identities. These are derived similarly to the previous case of supergravity, only we set $C^r_* = 0$ in addition to the substitution $\Phi^*_A = K^*_A - \delta \Psi / \delta \Phi^A$ in $S = S_\text{min} + S_\text{non}$. (We could also have introduced an external source $K^*_\mu$ for $C^\mu$ to remove the last term in (52), but we prefer to treat the constant ghosts as external fields.) The action $S_\Psi[\Phi, K, \bar{C}] = S_0 + s_0 \Psi - \int s_\Psi \Phi^A K^*_A + \ldots$ thus obtained then satisfies the master equation

$$
\int d^4x \left( \frac{\delta R S_\Psi}{\delta K^*_A} \frac{\delta L S_\Psi}{\delta \Phi^A} \right) + 2 \xi^I \gamma_5 \gamma^\mu \xi \frac{\partial S_\Psi}{\partial C^\mu} = 0,
$$

(62)
which implies the following Ward identity for the generating functional $Z[J,K,C]$

$$\int dx \left( \frac{\delta R Z}{\delta K_A} J_A \right) + 2 \xi^I \gamma^5 \gamma^\mu \xi \frac{\partial Z}{\partial C^\mu} = 0 \ .$$

We are now interested in the Ward identities [12], which are obtained by differentiating the above equation with respect to the sources $J_A$ and setting $J_A = K_A = 0$. The second term in (63) does not contribute to (12) since $C^\mu$ enters $S_\psi$ only via $\int C^\mu \partial_\mu \Phi^A K_A$, which when differentiated with respect to $C^\mu$ vanishes after setting $K_A = 0$.

The $\langle \lambda^I C^J \rangle_0$ propagator is crucial already at tree level. Consider as an example the Ward identity

$$s_\psi \langle \lambda^I (x) A^J_\mu (y) \rangle = 0 \ .$$

At tree level the composite operators in the BRST transformations do not contribute and the Fourier transformed relation reads, writing only the nonvanishing two-point functions,

$$\langle A^I_\mu A^J_{\mu 10} \gamma^\mu \xi p_\nu - i \langle \lambda^I \lambda^J \rangle_0 \gamma_5 \gamma_\mu \xi + ip_\mu \langle \lambda^I C^J \rangle_0 = 0 \ .$$

The first two terms come from the supersymmetry transformations of $\lambda^I$ and $A^J_\mu$ respectively, while the last one originates from the gauge transformation of $A^J_\mu$. The contributions from the translation ghosts in $s_\psi \lambda^I$ and $s_\psi A^J_\mu$ cancel due to translational invariance. Plugging in the propagators, we find

$$\frac{1}{p^2} \delta^{IJ} (\gamma_{\mu\nu} - \gamma_5 \gamma_\mu \gamma_5 \gamma_\nu - \delta_{\mu\nu}) \xi p^\nu = 0 \ ,$$

which is indeed satisfied. This checks the $\langle \lambda^I C^J \rangle_0$ propagator in (61).

In order to investigate the effects of the sign of the pseudoscalar propagator, we shall consider the following Ward identity for connected Green functions

$$\left. \frac{\partial}{\partial \xi} s_\psi \langle \lambda^I (x) \varphi^J (y) \rangle \right|_{\xi = \xi_1 = 0} = 0 \ .$$

The Fourier transformed identity reads

$$0 = \langle \varphi^I (p) \lambda^J (-p) \rangle - i \gamma_5 \Phi \langle \varphi^I (p) \varphi^J (-p) \rangle - g f^{IKL} \gamma_5 \gamma^\mu \langle A^K_\mu \star \varphi^L (p) \varphi^J (-p) \rangle$$

$$+ g \frac{\partial}{\partial \xi} [f^{IKL} \langle C^K_\mu \star \lambda^L (p) \varphi^J (-p) \rangle - f^{IKL} \langle \lambda^I (p) C^K_\mu \star \varphi^L (-p) \rangle] |_{\xi = \xi_1 = 0} \ ,$$

where we have written only terms that contribute up to one-loop order. Using (54) and (51) this relation is satisfied at tree level thanks to the nonstandard sign of the pseudoscalar propagator. At the one-loop level, the gaugino self-energy receives contributions from virtual $A^I_\mu$, $\phi^I$ and $\varphi^I$, while the pseudoscalar self-energy gets contributions from a $\lambda^I$ loop and a virtual $A^I_\mu$. We regularize the divergent integrals by dimensional reduction 26, where only coordinates and momenta are continued to dimensions $d < 4$, while the spinors and vectors remain four-dimensional. Furthermore, we use $\text{tr}(\gamma_\mu \gamma_\nu) = 4 \delta_{\mu\nu}$ and $\{\gamma_\mu, \gamma_5\} = 0$ for all $\mu$, i.e., algebraic manipulations involving $\gamma$-matrices are performed in four dimensions 26. We then find for the one-loop Green functions

$$\langle \varphi^I (p) \varphi^J (-p) \rangle = 0$$

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\[
\langle \lambda^I(p) \lambda^J(-p) \rangle = 2 i \gamma_5 \slashed{p} g^2 f^{IKL} f^{JKL} I(p^2; d)
\]

\[
\langle A^K \varphi^L(p) \varphi^J(-p) \rangle = \frac{3}{2} i p \mu g f^{JKL} I(p^2; d)
\]

\[
\langle C^K \lambda^L(p) \varphi^J(-p) \rangle = \frac{1}{2} i \gamma_5 \slashed{p} \xi g f^{JKL} I(p^2; d)
\]

\[
\langle \lambda^I(p) C^K \varphi^L(-p) \rangle = i \gamma_5 \slashed{p} \xi g f^{IKL} I(p^2; d),
\]

(69)

where

\[
I(p^2; d) = \frac{1}{p^2} \frac{1}{(4\pi)^2} \Gamma(2 - d/2) \int_0^1 d\alpha \left( \frac{4\pi}{\alpha(1 - \alpha)p^2} \right)^{2 - d/2}.
\]

When plugged into (68), the poles and finite terms cancel and the Ward identity is verified.

The examples we have investigated show that at the perturbative level the Ward identities are satisfied if one uses the straightforward propagators (with extra minus signs for the pseudoscalars). One can therefore calculate Feynman graphs in Euclidean supersymmetric theories as easily as in Minkowskian theories.

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