Evaporative Cooling of a Photon Fluid to Quantum Degeneracy

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We demonstrate that the process of evaporative cooling, as associated with the cooling of atomic gases, can also be employed to condense a system of photons giving rise to coherent properties of the light. The system we study consists of photons in a high-quality Fabry-Perot cavity with photon interactions mediated by a nonlinear atomic medium. We predict a macroscopic occupation of the lowest energy mode and evaluate the conditions for realizing a narrow spectral width indicative of a long coherence time for the field.

The phenomenon of coherence has played a crucial role in many areas of physics. An extraordinarily long coherence time is the fundamental property that distinguishes laser light from ordinary light [1]. The extensive coherence properties of matter have recently been investigated in trapped ultracold atoms by studying de Broglie matter waves in Bose-Einstein condensates [2, 3, 4]. Coherence is a ubiquitous phenomenon in many other systems, such as a collection of polaritons [5], also show similar coherent attributes.

In the case of atoms, coherence presents itself when the atomic gas has been condensed to quantum degeneracy. Condensation is often achieved using the technique of evaporative cooling in which the highest energy atoms are forced to escape the trap, and those that remain rethermalize, thereby reducing the temperature. In a nonequilibrium system, continuous evaporative cooling may generate a coherent atom laser [6]. In this article, we demonstrate how evaporative cooling and Bose-stimulated emission can be used to condense a photon fluid into a quantum degenerate superfluid.

In a general sense, a photon fluid is a collection of spatially localized photons which, through their interactions, exhibit hydrodynamic or similar fluidic behavior. A condensed photon fluid should possess superfluid properties such as coherence, phase rigidity, quantized vortices, a critical Landau velocity, and the Bogoliubov dispersion [2]. This is distinct from a normal laser, where the effective photon interactions are so weak that they are unimportant. Another significant difference with usual lasers is that a population inversion of an internal atomic state is not necessary to generate the coherent light. Instead the phase-space compression to produce a macroscopically occupied mode relies on an inversion of photon cavity mode populations with respect to the Planck distribution. Since the relevant modes of the system are determined by the structural properties of the cavity and not atomic quantities, the frequency of the coherent field could be highly tunable.

Two high-reflectivity mirrors placed close together form a Fabry-Perot cavity and support many photon modes. The closeness of the mirrors creates a cavity mode volume which has a high aspect ratio, with large energy gaps between modes which have adjacent longitudinal quantum numbers and small energy gaps between modes which have adjacent transverse quantum numbers. We utilize this to pump the cavity such that only one longitudinal quantum number is relevant, and the modes considered are distinguished only by their transverse degrees of freedom. Photons in vacuum interact weakly, therefore, it is necessary to incorporate a nonlinear medium, such as an atomic Rydberg gas or nonlinear crystal, into the cavity to allow for atom-mediated photon-photon interactions. Since the longitudinal degree of freedom is frozen out due to a small cavity length, an intriguing link exists between this system and condensation in two dimensions with the well-known physics of the Berezinskii-Kosterlitz-Thouless transition, which has recently been investigated in atomic gases [3, 4].

The photon fluid in a Fabry-Perot cavity is governed by the Hamiltonian,

\[ \hat{H} = \sum_i \hbar \omega_i \hat{a}^\dagger_i \hat{a}_i + \sum_{ijkl} \hbar \Gamma_{ijkl} \hat{a}^\dagger_i \hat{a}^\dagger_j \hat{a}_k \hat{a}_l, \]

where \( \hat{a}_i \) is the annihilation operator for a photon in mode \( i \) with energy \( \hbar \omega_i \), and \( \Gamma_{ijkl} \) is the scattering or collision (C) rate for photons from modes \( k \) and \( l \) into modes \( i \) and \( j \). There are two kinds of interactions, as illustrated in Fig. 1. Terms which do not change populations give rise to a self-interaction energy and leave the photons in the same mode they entered in, e.g. \( \hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_1 \hat{a}_0 \). Population transfer is due to recombination terms, such as \( \hat{a}_0^\dagger \hat{a}_2 \hat{a}_2^\dagger \hat{a}_1 \).

Photons can also enter and leave the cavity through irreversible pumping and decay, which is not described by the Hamiltonian in Eq. 1. Pumping and decay can be incorporated into the system dynamics through the quantum master-equation formalism,

\[ \frac{\partial \hat{\rho}}{\partial t} = -i \hbar [\hat{H}, \hat{\rho}] + \sum_i \Gamma_P^i \hat{L}[\hat{a}_i^\dagger] + \sum_i \Gamma_D^i \hat{L}[\hat{a}_i], \]

where \( \hat{\rho} \) is the density matrix operator of the system, \( \Gamma_P^i \) is the pumping rate (P) and \( \Gamma_D^i \) is the decay rate (D) for mode \( i \), and the Lindblad superoperator is

\[ \hat{L}[\hat{O}] \equiv 2\hat{O} \hat{\rho} \hat{O}^\dagger - \hat{O}^\dagger \hat{O} \hat{\rho} - \hat{\rho} \hat{O}^\dagger \hat{O}. \]
FIG. 1: (Color online) The five primary processes that occur in the Fabry-Perot cavity are presented. Photons enter the system from a pumping source. While in the cavity, the photons recombine and self-interact. Finally, the photons can leave the cavity through evaporation or output coupling.

The relative pumping rates as a function of the frequency of the system modes are dependent on the pumping source, which, for simplicity, we take here to be constant across all relevant modes, $\Gamma_i^P \rightarrow \Gamma^P$.

In contrast, evaporative cooling requires a decay rate from the cavity which is strongly dependent on the energy. A mirror with a frequency dependent reflectivity, which allows modes to decay at different rates, could be utilized. A reflectivity which drops for large frequencies can create conditions where high energy photons leave the cavity faster than low energy photons. Since a high energy photon carries away more than the average photon energy, subsequent rethermalization through photon interactions should reduce the system temperature, in much the same way as in the evaporative cooling of atoms.

We implement energy dependent reflectivity by dividing the decay rates into two classes. High energy photons ($H$) are described by a decay rate of $\Gamma^H$ and low energy photons ($L$) by a decay rate of $\Gamma^L$, with $\Gamma^H \gg \Gamma^L$. The large decay rate of the high energy modes means they can be eliminated in the following way. Consider an interaction in which photons from modes $i$ and $j$ create a photon of low energy in mode $k$ and a photon of high energy in mode $l$. The rapid decay of mode $l$ causes this process to be effectively irreversible. This is described most simply by extracting such processes which involve a high energy mode from the reversible $\hat{H}$ term in Eq. 2 and adding them back to the density matrix evolution as an irreversible term of superoperator form,

$$\Gamma^E \hat{L}[\hat{a}_i \hat{a}_j],$$

where $\Gamma^E \equiv (\Gamma^C)^2 / \Gamma^H$ is the evaporation rate ($E$), taken to be constant. The system Hilbert space is now reduced and the high energy modes no longer appear in the theory explicitly.

Even with these simplifications, the problem is still intractable in general. The dimensionality of the Hilbert space grows exponentially with the number of photons in the lower energy modes. Most of this complexity is uninteresting from the point of view of evaporative cooling of photons since it arises from all possible pathways to redistribute and entangle photons amongst the lowest energy modes. Consequently, we divide the system into what we will refer to as plaquettes, each plaquette consisting of a pair of modes $i$ and $j$, which may interact to produce a photon in the lowest energy mode $k = 0$ and a photon in a high energy mode $l$ which is lost through the mirrors rapidly. Adding the contributions from $D$ distinct plaquettes as an incoherent sum of pumping rates into the ground state, the density matrix equation of motion in the interaction picture is,

$$\frac{\partial \hat{\rho}}{\partial t} = +\Gamma^P \sum_i \hat{L}[\hat{a}_i] + \Gamma^L \sum_i \hat{L}[\hat{a}_i]$$

$$-i\Gamma^C \sum_i \{[\hat{a}^\dagger_i \hat{a}_i \hat{a}_0, \hat{\rho}] + \Gamma^E \sum_{(ij)} \hat{L}[\hat{a}^\dagger_i \hat{a}_i \hat{a}_j]\},$$

where the sum over $i,j$ implies modes $i$ and $j$ are from the same plaquette.

Amongst the populations, the evolution time scales are not all equivalent. If only one or no atoms are in a plaquette, a slow evolution takes place on time scales governed by $\Gamma^P$ and $\Gamma^L$. If two atoms are in a plaquette, however, and providing $\Gamma^E \gg \Gamma^P, \Gamma^L$, they will rapidly collide and form a ground state photon, and a high energy photon which will evaporate away. Consistent with this is that we may neglect the possibility of three or more photons in the plaquette.

The basis states needed then describe the number of photons in each plaquette and the number of photons in the lowest energy mode. The precise occupation of the modes is not important, only how many plaquettes have a given number of photons. For instance, an arbitrary basis state $|\Psi\rangle$ can be uniquely identified with a reduced computational state described by,

$$|\Psi\rangle \rightarrow |n,abcd\rangle \equiv |n,(00)^a(01)^b(11)^c(02)^d\rangle,$$

where $n$ is the number of ground state photons, $a$ is the number of plaquettes with no photons, $b$ is the number of plaquettes with one photon, and $c$ and $d$ are the number of plaquettes with two photons in different modes or the same mode, respectively.

The photon number distribution is an important quantity to characterize the photon state since a coherent state has a Poissonian distribution while a thermal state has an exponentially decaying distribution. The populations of all states $|\Psi\rangle$ that can be expressed as $|n,abcd\rangle$ are given by

$$P_{n,abcd} \equiv \langle \Psi | \hat{\rho} | \Psi \rangle.$$

We can now take advantage of the natural separation of time scales by adiabatically eliminating the fast pro-
The number distribution of photons in the low energy mode during steady state operation for systems with 20 photons in the low energy mode. With a large effective temperature, as denoted in the figure by $k_B T = 21h\omega$, the number distribution is essentially what would be expected for a thermal source. As the effective temperature decreases, by decreasing the pumping strength and increasing the number of pumped plaquettes, the effects of bosonic amplification dramatically increase. This leads to a nearly Poissonian number distribution associated with zero temperature condensates. The key to converting the incoherent thermal pump into a coherent system is that the occupation of the lowest energy mode comes from stimulated emission from higher energy modes.

The spectrum of the photons is also an important quantity to examine as it provides information about the coherence time and spectral width. Since the mode separation is greater than the collisional interaction energy and the system is in steady state, the fluctuation spectrum is determined by,

$$S(\nu) = \int_{-\infty}^{\infty} d\tau e^{-i\nu\tau} \frac{\langle \hat{a}_0^\dagger(\tau)\hat{a}_0(0) \rangle}{\langle \hat{a}_0^\dagger(0)\hat{a}_0(0) \rangle},$$

which is the Fourier transform of a two-time correlation function. The correlation function can be expressed as,

$$\langle \hat{a}_0^\dagger(t)\hat{a}_0(0) \rangle = \sum_{n,\{i\}} \sqrt{n}C_{n,\{i\}}(t),$$

with,

$$C_{n,\{i\}}(t) \equiv Tr[\hat{\rho}|n, \{i\}\rangle\langle n - 1, \{i\}|(t)\hat{a}_0(0)|0\rangle,$$

where $n$ is the number of ground state photons and $\{i\}$ represents the configuration of the remaining modes. From the quantum regression theorem, the equation of
motion for the quantity $C_{n,\{i\}}$ is given by,
\[
\frac{\partial C_{n,\{i\}}}{\partial t} = \sum_{m,\{j\}} M_{n,\{i\}m,\{j\}} C_{m,\{j\}},
\]  
(15)
where the matrix elements are determined from the off-diagonal density matrix equation of motion,
\[
\frac{\partial \rho_{n,\{i\},m,\{j\}\{i\}}}{\partial t} = \sum_{m,\{j\}} M_{n,\{i\}m,\{j\}} \rho_{(m-1),\{j\},m,\{j\}}.
\]  
(16)
as given by Eq. 5. As this is a reduced set of the full density matrix equations of motion, it is merely necessary to replace these matrix elements into Eq. (15) with the initial condition $C_{n,\{i\}}(0) = \sqrt{p_n} \langle n, \{i\} \rangle$, where $|n, \{i\}\rangle$ corresponds to the reduced basis state $|n, abc\rangle$.

There are two driving forces of decoherence, incoherent transfer through the mirrors and phase dispersion due to collisions, which both limit the coherence time. The decoherence due to transfer through the mirrors can be examined by setting a vanishing collision rate. Due to bosonic amplification, the photons are able to support a state with a spectral width much narrower than that given by the linewidth of the photon decay.

Figure 3 presents the spectrum for vanishing collision rates for systems with 20 pumped plaquettes and different average photon numbers. As the photon number is increased due to a higher pumping rate, the spectral width decreases due to Bosonic amplification. The spectrum can be much narrower than the width associated with the decay rate through the mirrors. As shown in the inset of Fig. 3, the spectral width, defined as half width at half maximum, is inversely proportional to the average photon number as expected by the Schawlow-Townes linewidth [1].

The effects of non-vanishing collision rates are of course an important aspect of the system. Figure 4 presents the fluctuation spectrum for a system with an average of 100 photons as the collision rate is increased. There is a mean-field interaction shift of the frequency since the energy of the photons is no longer just the bare mode energy. The spectral width rapidly grows as the collision rate becomes larger. One possible way to remedy this problem is to create a system that has a small collision rate for the low energy photons but still possesses a large recombination rate for the high energy photons. Such a system can be manufactured by creating a spatially dependent nonlinear medium. The low energy photons are more localized to the center of the cavity than the high energy photons. Therefore, if the medium is less dense in the center of the trap than toward the edges, the cavity will allow for fast recombinations of high energy photons to promote rapid evaporative cooling but small collision rates for low energy photons to promote coherent evolution.

The condensation of a photon fluid using evaporative cooling provides a new mechanism for creating a high-intensity narrow-spectrum coherent beam of light. Unlike a laser, the photon fluid, due to the presence of interactions, is expected to be superfluid and should, for example, exhibit signature properties such as phase rigidity of the order parameter, and the ability to support quantized vortices.

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[1] A. L. Schawlow and C. H. Townes, Phys. Rev. 112, 1940 (1958).
[2] M. H. Anderson et al., Science 269, 198 (1995).
[3] K. B. Davis et al., Phys. Rev. Lett. 75, 3969 (1995).
[4] C. C. Bradley, C. A. Scakett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. 75, 1687 (1995).
[5] R. Balili et al., Science 326, 1007 (2007).
[6] M. Holland et al., Phys. Rev. A 54, R1757 (1996).
[7] R. Y. Chiao and J. Boyce, Phys. Rev. A 60, 4114 (1999).
[8] Z. Hadzibabic et al., Nature 441, 1118 (2006).
[9] V. Schweikhard, S. Tung, and E. A. Cornell, Phys. Rev. Lett. 99, 030401 (2007).