Model-based design of tube pumps with ultra-low flow rate pulsation

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ABSTRACT
In this paper, a model-based design problem of tube pumps are considered to achieve an ultra-low flow rate pulsation. Starting from the modelling of conventional tube pumps having two rollers, we clarify the fact that even the 1 degree of freedom (DoF) structure can realize a flat flow rate by adjusting the rotation speed. Then, it is shown that the 2 DoF setting reduces the amount of the required acceleration of the pushing roller. Equivalently, this can be achieved by a deceleration of the leaving roller. An alternative way to realize it with a constant rotation speed is to modify the wall curve profile at the leaving sector. Its modelling and an optimal design procedure based on this model are given. Finally, we propose a control strategy combining a quasi-optimal wall curve with slight speed adjustment at the bottom of the periodic operation cycle to achieve an almost pulsation-free flow rate.

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1. Introduction
A tube pump, sometimes called a peristaltic pump, is a fluid transfer mechanism that transports the fluid inside an elastic tube by squeezing the tube from the outside (Figure 1). For a concise review of this pump from an engineering viewpoint, the readers are referred to [1]. The largest advantage of the tube pump is that it causes no contamination of the fluid inside from its structure. This is promising for biological, medical, health-care or other application fields, e.g. [2–6]. Typically, tube pumps are driven with a constant rotation speed reference. Due to the periodic nature of the pump operation, its tracking problem is studied under the context of repetitive control, e.g. [7–9]. Further, even when the effect of the periodic disturbance on the rotation speed is completely removed, a plain implementation inevitably introduces a pulsation of the flow rate also from its structure. To remedy this, different types of tube pump or control strategies are investigated in the patents, e.g. [10, 11] and the literature, e.g. [12–15]. From their personal experiences in developing the pumps in [14, 15], the authors are strongly motivated to have a systematic understanding on the design issue, rather than isolated knowledge on particular cases. For this purpose, we study how pulsation is generated and review what we can do to prevent it based on mathematical models in this paper. Our model-based design concept is integrated step by step toward the final proposal.

This paper is organized as follows: In Section 2, we describe the structure of a typical tube pump and derive a mathematical model to quantify the flow rate change under a normal operation where it is driven with a constant rotation speed. Section 3 investigates the problem how to determine the rotation speed of the rollers to suppress the flow rate pulsation under 1 and 2 degree of freedom (DoF) settings. The latter corresponds to the pushing roller acceleration strategy. The effect of the leaving roller deceleration and its alternative modification of the inner wall profile is studied in Section 4. A geometric model to describe the wall with varying curvature is introduced. In Section 5, the optimal design problem of the wall curve is examined. The derived quasi-optimal profile is combined with a small speed compensation at the bottom of the periodic operation cycle. The concluding remarks are summarized in Section 6.

2. Description and modelling of tube pump
Figure 2 shows a schematic diagram of a typical tube pump with two rollers. It consists of a casing, a tube and a rotating component. The inner wall of the casing consists of a half circle and two straight lines. An elastic tube filled with fluid is placed along the wall. The rotor rotates around the axis at the centre of the half circle. Two rollers are attached to both ends of the rotor. One of them pushes the tube against the wall so as to...
The fluid is transported to the direction of the rotation. After the pushing period, the roller leaves from the tube not to prevent the flow generated by the following roller. The angular velocity of the rotation is usually kept constant. It gives a constant flow rate after the leaving of the front roller is completed. However, the leaving action of the roller causes a non-uniform temporal change of the closed volume inside the tube. It becomes the source of a flow rate pulsation during the leaving process. We investigate this phenomenon via a mathematical model and give a solution to obtain a pulsation-free operation.

To understand the behaviour of the flow rate pulsation in detail, let us construct a mathematical model approximating the actual phenomenon. We expect that this model captures the essential properties of the physical system. The fluid is assumed to be incompressible. For simplicity, we employ a two-dimensional (2D) model by ignoring the effect of the depth direction, i.e. instead of the displacement volume, the corresponding area is considered for the flow rate computation. Also the effect of the tail shape of the elastic deformation of the tube (shown in Figure 2) and the thickness of the tube are ignored. An important assumption is that the state of the fluid inside the tube is uniform, i.e. no fluid dynamical effect (spatial change of the pressure and velocity distribution) is taken into account (cf. [16]). Also an elastic vibration of the tube is not focused. These might be reasonable under a relatively slow operation of the pump with the fluid of low viscosity. As illustrated in Figure 3, the centre of the rotor and the (leaving) roller are denoted by $O_1$ and $O_2$, respectively. Let $R$, $r$, and $d$ denote the distance between $O_1$ and $O_2$, the radius of the roller and the tube diameter, respectively. We assume that $r \geq d$. The angles of the leaving and pushing roller measured from the horizontal position in clockwise are denoted by $\theta_l$ and $\theta_p$, respectively. Let $h$ be the distance between the leaving roller and the wall. It is given by

$$h = \min\{R(1 - \cos \theta_l), d\}. \quad (1)$$

Then, the displaced fluid area $S$ (shaded part) is calculated as

$$S(h) \simeq r^2 \cos^{-1}\left(\frac{\ell(h)}{r}\right) - \ell(h) \sqrt{r^2 - \ell^2(h)}, \quad (2)$$

where $\ell(h) := r - d + h$, via elementary geometry. We consider the change of the total volume (area) $V$ between the pushing roller and a fixed point somewhere downstream (for an experimental setup, the point where the flow meter is placed). Let $V_0$ be the initial volume of the closed area inside the tube between the pushing roller and the flow meter when the leaving roller is completely disengaged. The displaced volumes caused by the engagement of the leaving roller and the forward movement of the pushing roller should be subtracted to obtain an actual volume. Then, by assuming that the shape of the deformation caused by the pushing roller is time-invariant, $V$ is given by

$$V \simeq V_0 - d\theta_p \ell(R) - S. \quad (3)$$

Now, the flow rate denoted by $F$ is computed as

$$F = -\frac{dV}{dt}. \quad (4)$$

By using this model, let us verify the flow pulsation behaviour under the rotation with a constant angular velocity. This is Case 1. Note that $dS/dt$ is given by a simple form as

$$\frac{dS}{dt} = -2 \frac{dh}{dt} \sqrt{r^2 - \ell^2(h)}. \quad (5)$$

In this case, 1 DoF operation is assumed, i.e. $\theta_l(t) - \theta_p(t) = \phi$ (constant). The pump is driven by $\theta_p(t) = \omega_0 t - \phi$ (to make $\theta_l(0) = 0$). The parameters
are chosen as \( R = 5 \times 10^{-2} \) [m], \( r = 1 \times 10^{-2} \) [m] and \( d = 1 \times 10^{-2} \) [m]. To achieve the target flow rate \( \alpha = 3 \times 10^{-4} \) [m\(^2\)/s], the angular velocity \( \omega_0 \) is set to \( \alpha/(d\ell(R)) \). The simulation result is shown in Figure 4 with the solid lines.\(^3\) As seen in the second plot, it takes about 2.6 [s] for the \( \pi/2 \) [rad] rotation of the roller. For the time being, we do not pay attention to the periodicity (when the next cycle begins). During the time interval \([0,1.07]\), \( h \) increases from 0 to \( d \) as shown in the third plot. This implies that the front roller is leaving at this time interval. As shown in the bottom plot, the dip of the flow rate caused by the roller leaving is up to 75% of the target value. This might be an intrinsic drawback of the plain tube pump.

3. Suppression of flow rate pulsation

We would like to reduce the flow rate pulsation mentioned above. First of all, one must note that any kind of feedback control using a flow metre is not preferable to preserve the contamination-free characteristic of the tube pump, cf. [12, 13]. This could be a partial reason that the rotation speed regulation (instead of the flow rate regulation) via repetitive control is focused in [8, 9]. Thus, to suppress the flow rate pulsation, various mechanisms have been proposed. The basic idea is to accelerate the pushing roller to compensate for the dip caused by the leaving roller. To this end, two rollers should be driven independently and hence, the driving mechanism must have 2 degrees of freedom. The work [14] is a direct implementation of this idea and the proposals in [10, 15] are its variant to achieve 2 DoF-like motions with only one motor using a cum or non-circular gear mechanism. In this section, we review this idea using our model.

Before the issue above, a fundamental and interesting question worth asking is “whether 1 DoF is really not enough.” It seems that the suppression by 1 DoF has been thought to be impossible because the acceleration of the rollers increases the pushing flow rate and the leaving speed at the same time. We can answer this question also using our dynamical model.

Let us denote this 1 DoF variable speed case as Case 2. In this case, \( \theta_l(t) \) is chosen to make

\[
\frac{d\ell}{dt} = \frac{\alpha}{d\ell(R) - 2R\sin\theta_l\sqrt{r^2 - \ell^2(h)}}.
\]

(7)

Recall that the right-hand side including \( \ell(h) \) is a function of \( \theta_l(t) \). Thus, if the ordinary differential Equation (7) in terms of \( \theta_l(t) \) has a solution, the pulsation can be suppressed by only 1 DoF. It really does have a solution under the initial condition \( \theta_l(0) = 0 \) and the result is shown in Figure 4 with the dashed lines. A flat
flow rate is achieved as expected. The reason for the success is that although the acceleration increases both the pushing flow rate and the leaving speed in a qualitative sense, the way they really work is different in a quantitative sense or dynamical sense. However, it is done at the expense of a rapid change of the angular velocity around \( t = 0.5 \text{ [s]} \). The maximum value is more than 4 times larger than the steady state one. It might be difficult to realize it in practice.

Case 3 investigates 2 DoF mechanisms. While the leaving roller is driven in a constant angular velocity as \( \theta_l(t) = \omega_0 t \), the pushing roller whose angle is denoted by \( \theta_p \) is manipulated to achieve a constant flow rate. By replacing \( \theta_l \) in (6) by \( \theta_p \) and using \( dh/dt = R \omega_0 \sin \omega_0 t \), one can obtain the following equation:

\[
\frac{d\theta_p}{dt} = \alpha + 2R \omega_0 \sin \omega_0 t \sqrt{f(t)}/d\ell(R),
\]

where

\[
f(t) = r^2 - (r - d + R(1 - \cos \omega_0 t))^2.
\]

Since the right-hand side is an explicit function of \( t \), one can obtain the desired \( \theta_p(t) \) by simply integrating (8) over time from the initial condition \( \theta_p(0) = -\phi \). The corresponding time response is plotted in Figure 4 with the dash-dot line. The suppression is also perfect and the acceleration is limited to 1.6 times of the steady-state. This seems to be easier to realize than Case 2, but 2 DoF mechanism becomes much more complex than 1 DoF case as we experienced in [14,15].

4. Effect of inner wall profile at leaving

The basic idea of Case 3 is to speed up the pushing roller compared to the leaving roller. Let us consider an opposite situation, namely, to slow down \( dh/dt \) compared to \( d\theta_p/dt \). It is easy to verify how this idea works via numerical simulations. First, suppose that the leaving speed is 1/3 of the constant pushing speed. The result for \( \theta_l(t) = (\omega_0/3)t \), \( \theta_p(t) = \omega_0 t - \phi \) (Case 4) is shown in Figure 5 with the solid line. Since the leaving speed is 1/3 of the nominal case (Case 1), the dip of the flow rate is reduced to about 26% of the steady-state.

Next, we attempt to suppress the pulsation by shaping \( \theta_p() \). As in Case 2, a flat flow rate can be achieved by solving

\[
h = \min\{R(1 - \cos(\theta_p/3)), d\},
\]

\[
\ell(h) = r - d + h,
\]

\[
\frac{d\theta_p}{dr} = \frac{3\alpha}{3d\ell(R) - 2R \sin(\theta_p/3) \sqrt{r^2 - \ell^2(h)}},
\]

under the initial condition \( \theta_p(0) = -\phi \). In this case, Case 5, the required increment of the angular velocity is limited to 1/3 of the steady-state (Figure 6). This is much smaller than Case 2 and hence, it can be said that the reduction of the leaving speed is quite effective from a practical viewpoint. It is verified that the reverse strategy works either. However, a significant difference from the case of the pushing roller acceleration is that 2 DoF is not necessarily required to realize the converse operation, at least approximately. Figure 7 illustrates a solution where the straight line of the inner wall is replaced by a curve whose curvature is increased gradually. By this change, the leaving speed can be reduced in an arbitrary manner. Unfortunately, it was found that this is not our own invention and the same idea had been already reported in [11]. If we confine ourselves to the case with constant rotational speed for simplicity, then it might be possible to design a curve giving
desired leaving behaviour by adjusting the curvature in principle.

Let us extend the model to deal with the curved wall profiles mathematically. Suppose that the curve is described in a polar coordinate system shown in Figure 8 where the distance between \( O_1 \) and a point \((x, y)\) on the curve having the angle \( \theta \) from the \( X \)-axis is denoted by \( D(\theta) \). Since

\[
(x, y) = D(\theta)(\cos \theta, -\sin \theta),
\]

the tangent of the curve \(\frac{dy}{dx}\) at the point is given as

\[
\frac{dy}{dx} = \frac{dy}{d\theta} \frac{d\theta}{dx} = \frac{-D'(\theta) \sin \theta - D(\theta) \cos \theta}{D'(\theta) \cos \theta - D(\theta) \sin \theta} = t(\theta).
\]

One must note that the way to calculate the distance \( h \) should be modified from the straight wall case. While the distance is measured along a horizontal line crossing \( O_2 \) in the straight wall case, it should be measured, in the curved wall case, along a normal line of the curve crossing \( O_2 \) (Figure 9). Let \( Q \) be the crossing point of the curve and this normal line and denote the angle of \( O_1Q \) by \( \theta_q \). Then from (13), the equation of the normal line \( \ell \) is given as

\[
\ell : y + D(\theta_q) \sin \theta_q = -\frac{1}{t(\theta_q)}(x - D(\theta_q) \cos \theta_q).
\]

The condition that \( \ell \) goes through \( O_2 \) gives an implicit relationship between \( \theta_l \) and \( \theta_q \). It is finally reduced to

\[
f(\theta_l, \theta_q) := \frac{D(\theta_q)D'(\theta_q)}{R} - D(\theta_q) \sin \delta
- D'(\theta_q) \cos \delta = 0,
\]

where \( \delta = \theta_l - \theta_q \). Let \( h_c \) be the distance between the roller at angle \( \theta_l \) and the wall curve. Given \( \theta_l, \theta_q \) is determined from (15) and \( h_c \) is given as \( h_c = \min(h_1, d) \) where

\[
h_1 = \text{dist}(O_2, Q) - r
= \sqrt{R^2 + D^2(\theta_q) - 2RD(\theta_q) \cos(\theta_l - \theta_q) - r}.
\]

The flow rate is computable from \( S(h_c) \) as in the previous cases.

Before considering the wall curve design problem, we verify the effect of \( D(\theta) \) by using a simple function.
Let $D(\theta)$ be given by a linearly increasing function in terms of $\theta$, i.e.

$$D(\theta) = R + r + \beta \theta, \quad \beta > 0.$$  

(17)

This is Case 6. Although smaller $\beta$ is preferable for a slower leaving, structural limitations exist for a continuous periodic operation of the pump. Consider the 1 DoF case. For a simple periodic operation, a full rotation $2\pi$ should be divided into the intervals with equal length. Each interval is assigned its own role. One of them is used for pushing and another one is used for leaving. Let $n$ be the number of intervals. Then, the central angle $\psi$ of the sector corresponding to one interval is given by $2\pi/n$. For a complete disengagement at the end, $D(\psi) - D(0) = \beta \psi \geq d$ must hold. Thus, a larger $\psi$ is desired for a small $\beta$. The case $n = 1$ is not acceptable because it requires a complex structure where the tube has to be stacked and coiled. Due to possible interference between incoming and outgoing ports of the tube, $n = 2$ and 3 are also not desirable. Based on these observations, we select $n = 4$ and $\psi = \pi/2$. In addition, due to the lag $\delta$ between $\theta_l$ and $\theta_q$, $\beta \theta$ must reach $d$ slightly earlier than $\pi/2$. Thus, $\beta$ is chosen to be $2.1d/\pi$. When we drive the pump with the curve (17) at the constant angular velocity $\omega_0$, the result in Figure 10 is obtained. Around $t = 2.619$ [s], $\theta_l$ reaches $\pi/2$ and at this moment, the simulation is terminated since the next cycle will start for an actual pump. After $t = 2.514$ [s], $h_c$ coincides with $d$. This implies that a complete disengagement has already happened. Even with this $D(\theta)$ without any tuning, the flow rate change is moderate. Figure 11 depicts the sampled positions of the leaving roller and the point $Q$. One can observe the relative distance between the wall (outer tube side wall) and the leaving roller increases as $\theta_l$ approaches to $\pi/2$.

5. Optimal design of wall curve

In this section, the optimal design procedure for the wall curve $D(\theta)$ to minimize the flow rate pulsation is investigated. The primitive idea is to derive an ordinary differential equation in terms of $D(\theta)$ assuming the rotation with a constant angular velocity $d\theta_l/dt$. 

![Figure 9: Distance between wall and roller.](image)

![Figure 10: Time responses of Case 6.](image)

![Figure 11: Position of roller and point Q.](image)
This ODE-based approach is similar to the previous design of $\theta(t)$ to suppress the pulsation. But this time, we consider a function of the angle $\theta$, not the time $t$. By regarding $\theta(t) = \theta_q(t)$, the requirement is written as

$$\frac{dS}{d\theta_i} = -\gamma = \frac{dS}{dh_1} \frac{dh_1}{d\theta},$$

where

$$\frac{dS}{dh_1} = -2\sqrt{r^2 - (r - d + h_1)^2}.$$ 

Further calculation yields

$$\frac{dh_1}{d\theta} = \frac{\partial h_1}{\partial \theta} + \frac{\partial h_1}{\partial \theta_q} \frac{d\theta_q}{d\theta},$$

$$\frac{\partial h_1}{\partial \theta} = \frac{2RD_q \sin \delta}{2(h_1 + r)},$$

$$\frac{\partial h_1}{\partial \theta_q} = \frac{2D_qD_q' - 2RD_q \cos \delta - 2RD_q \sin \delta}{2(h_1 + r)},$$

$$\frac{d\theta_q}{d\theta} = \frac{RD_q \sin \delta - RD_q \cos \delta}{(D_q')^2 + D_qD_q' - RD_q \sin \delta - RD_q' \cos \delta}.$$ 

In the equations above, the following shorthand notations are used for a compact description: $D_q = D(\theta_q)$, $D_q' = D'(\theta_q)$, $D_q'' = D''(\theta_q)$. This will result in a differential equation of the form

$$\frac{d^2}{d\theta^2} D(\theta - \delta(\theta)) = H \left( \frac{d}{d\theta} D(\theta - \delta(\theta)), D(\theta - \delta(\theta)), \delta(\theta), \theta \right),$$

(18)

This system is left for future work and to circumvent the difficulty, we introduce a further approximation. Suppose that $|D'| \ll 1$. Then $t(\theta)$ is simply given by $\tan \theta$ from (13) and, correspondingly, $\delta$ is always zero (no lag) from (15). This greatly simplifies the process. Since the height $h$ is simply given by

$$h \simeq D(\theta_i) - R - r,$$

the derivative of $S$ becomes

$$\frac{dS}{d\theta_i} = -2D'(\theta_i)\sqrt{r^2 - (r - d + h_i)^2}.$$ 

To achieve a constant decreasing rate $-\gamma$, $D(\theta)$ must be designed by solving ODE:

$$D'(\theta) = \frac{\gamma}{2\sqrt{r^2 - (D(\theta) - R - d)}},$$

(20)

under the initial condition $D(0) = R + r$. The parameter $\gamma$ determines the way $D(\theta)$ increase. We select $\gamma = 1.05 \times 10^{-4}$ under the current setting so that a complete disengagement ($D = R + r + d = 0.07$) is achieved just before $\theta = \pi/2$. In an ideal situation,

$$\frac{dS}{d\theta_i} = -\gamma, \quad \theta_i = \omega_0 t, \quad \omega_0 = \alpha/(d\ell(R)),$$

we have $dS/d\theta = -\gamma \omega_0$ and hence

$$F = -dV/dt = d\ell(R)\omega_0 + dS/d\theta = (1 - k)\alpha,$$

where $k = \gamma \omega_0 / \alpha$ denotes the ratio of the resulting bias from the desired flow rate. It is easy to compensate for this bias of the flow rate by increasing $\omega_0$. The result of the numerical computation is shown in Figure 12. From (20), $D'(\theta)$ diverges when $D = 0.07$. To obtain a continuous curve over this singular point $\theta$, the derivative just before $\theta$ is fixed for $\theta > \theta_i$. This is Case 7. Figure 13 shows the change of the wall shape. The vertical solid black line represents the wall of a conventional pump, while the curve in red describes the position of the designed wall.\textsuperscript{7}

Next, we investigate a detailed pump behaviour taking the lag $\delta$ into consideration when this designed curve is used. Recall that the functions $D(\theta)$ and $D'(\theta)$ are necessary for this computation instead of the values $D(\theta_i)$ at the sampling points $\{\theta_i\}$. Spline interpolations and numerical differentiation are used for the purpose of this approximate reconstruction. The result is summarized in Figure 14. A complete disengagement

![Figure 12. Designed wall curve $D(\theta)$.](image-url)
occurs just before $\theta_l$ reaches $\pi/2$ and a flat flow rate is obtained except at the end of one cycle in which $\theta_l$ travels from 0 to $\pi/2$. The steady-state flow rate is $(1 - k)\alpha$ where $k = 0.21$. As explained before, this flow rate ripple at the bottom of a cycle is inevitable for a constant $\frac{d\theta_l}{dt}$.

We employ the 1 DoF compensation to remove the ripple. Remember that the flow rate is computed as

$$F = d\ell(R)\frac{d\theta_l}{dt} + \frac{dS}{dt}.$$

Even when the behaviour of $\frac{dS}{dt}$ is (slightly) different from $-\gamma\omega_0$, $F = (1 - k)\alpha$ holds true if we set

$$\frac{d\theta_l}{dt} = \frac{1}{d\ell(R)} \left( (1 - k)\alpha - \frac{dS}{dt} \right).$$

This implies that a flat flow rate is achieved by adjusting the angular velocity of the leaving roller according to $\frac{dS}{dt}$ (Case 8). The result of this compensated operation is presented in Figure 15. To prevent the flow rate ripple existing in the constant speed drive case, the angular velocity is gradually reduced to 78% of the initial setting as the end of a cycle approaches. The velocity change is relatively gentle compared to Cases 2 or 3. The flow rate is kept almost constant, being 79% of $\alpha$ as intended and the ripple becomes so small that it is hard to recognize in the current scale of the figure.

6. Conclusions

In this paper, a model-based design process of tube pumps using a simplified 2D mathematical model is considered. The goal is to analyse the generation mechanism of the flow rate pulsation inherent in usual tube pumps driven at a constant angular velocity of rotation and reduce it. After verifying the amount of pulsation for a typical situation, it is shown that 1 DoF speed control can remove the pulsation completely, seemingly contrary to our common sense. A practical drawback
is that rapid acceleration is required compared to the 2 DoF case considered in our previous research, in which the pushing roller is accelerated to cancel the effect of the leaving roller. To avoid a structural complexity caused by the 2 DoF configuration, we try an opposite strategy, i.e. to decelerate the leaving speed. The remarkable point is that this can be achieved by 1 DoF configuration by shaping the wall curve at the leaving sector. To examine it, we extend our model so that the curved wall is taken into consideration. A crucial point of the modelling is the existence of the phase lag between the leaving roller angle and that of the nearest point on the wall from the roller. By using this model, we further discuss the effect of the wall curve and the way to design a (quasi) optimal curve profile. Finally, it is demonstrated that the combination of optimal wall curve and slight speed adjustment at the bottom of the cycle can achieve an ultra-low flow rate pulsation. We think that this is the best way to produce and operate a tube pump having very small flow rate pulsation from a practical viewpoint.

Currently, we are working on the design, implementation and experimental verification of a new type tube pump realizing the concept obtained from the investigation here. As mentioned earlier, due to the complexity of the mechanical structure and control, the experimental result of the 2 DoF attempt in [15] is not fully satisfactory and still reveals a flow rate pulsation about 30% of the steady state value at best. Our tentative goal is to make it within 10%. Since an accurate angular velocity setting is crucial in the development, a precision servo control and the rejection of the periodic disturbance via, for example, repetitive control is indispensable.

Obviously, the removal of the ripple at the bottom of a cycle shown in Section 5 is entirely dependent on the pump model (purely feedforward method). Thus how to determine the way of the speed compensation is a big problem in the practical implementation stage. The introduction of an experimental method such as iterative learning control [17] should be considered.

Notes

1. Actually, the design and control of the pumps in [14, 15] were successful under the 2D-based model with these assumptions. For verification, we attempt a reproduction of the flow rate measurement of a commercial tube pump shown in [15] by the current model. See Appendix 1.

2. When $\theta_1 \geq 0$, cutting by a straight line is an approximation, but it is reasonable as long as $R$ is not so large compared to $R_o$.

3. For the sake of the space limitation, the units of the $y$-axis variables are omitted in the figure. They are [rad/s], [rad], [m], [m$^2$] and [m$^2$/s] from the top to the bottom, respectively. The second plot shows the increment from the initial position $\Delta \theta_2(t) = \theta_2(t) - \theta_2(0)$. These are the same for other figures showing the time responses.

4. This is a potential motivation of the current research.

5. Recall that the movement of each roller is not independent. Actually, they are constrained by $\dot{\theta}_2(t) = (1/3) \dot{\theta}_p(t)/dt$.

6. This fact is related to the following fundamental limitation. Our displaced area consists of a circular arch and a chord. When the roller approaches to the complete disengagement point, the length of the chord tends to be zero. Thus, if we require a constant rate of change of the displacement area until the very end of the leaving process, the vertical leaving speed of the roller must be infinitely large at the endpoint. Obviously, this is impossible and, in practice, the flow rate returns to $\alpha$ at the bottom of the cycle. This is inevitable under the situation of a constant angular velocity driving no matter how the curve is designed as a continuous function of $\theta$.

7. Note that the basic configuration of the pump shown in Figures 1 and 2 can be invariant in spite of this large change of the outlook in Figure 13. It is achieved by employing the structure with four rollers with $\phi = \pi/2$ and a $-\pi/2$ rotation of the casing (The left sector of the semicircle in Figure 2 is not necessary when $\phi = \pi/2$).

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Appendix. Verification of proposed model

Let us try to reproduce the flow rate measurement of a commercial tube pump WP200 (WELCO Co., Ltd) by the current model. The experimental data are taken from Figure 2 of [15]. The model parameters $R = 2.16 \times 10^{-2}$ [m], $r = 1.14 \times 10^{-2}$ [m] and $d = 8 \times 10^{-3}$ [m] are determined from the dimension of the actual pump. The driving angular velocity, the phase and the depth factor to convert 2D volume to 3D are adjusted to fit to the actual steady-state flow rate. The computational result is overwritten to the original plot (Figure A1).

Figure A1. Comparison with actual pump flow rate.

The actual data shows an extremely large dip (including partial backflow) of the flow rate. Due to a limitation of the flow metre, the measurement is 0 for negative values. One can verify that the current model describes the dip very well. A major difference is the existence of a damped oscillation in the actual data. This is caused by the inertia of the fluid or local variation of the velocity and pressure triggered by the large dip. This is nothing but a fluid dynamical effect we ignored in our modelling. However, one must note that if we can prevent the large dip well predicted by the model through the design and control, the following oscillation is supposed to be not triggered. This is why we believe the proposed model works for our control objective.