Extending Class Activation Mapping Using Gaussian Receptive Field

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Abstract

This paper addresses the visualization task of deep learning models. To improve Class Activation Mapping (CAM) based visualization method, we offer two options. First, we propose Gaussian upsampling, an improved upsampling method that can reflect the characteristics of deep learning models. Second, we identify and modify unnatural terms in the mathematical derivation of the existing CAM studies. Based on two options, we propose Extended-CAM, an advanced CAM-based visualization method, which exhibits improved theoretical properties. Experimental results show that Extended-CAM provides more accurate visualization than the existing methods.

1 Introduction

Deep Convolutional Neural Networks (DCNNs) have exhibited remarkable accuracy in various image processing fields such as object detection [Redmon et al., 2016; Girshick, 2015], semantic segmentation [Long et al., 2015; Chen et al., 2017], and monocular depth estimation [Godard et al., 2017; Fu et al., 2018]. DCNNs have been confirmed in various studies to possess strong modeling abilities in general machine learning problems. DCNN, however, is a black-box model and does not provide the cause or process for the results it outputs. As practical applications require stability and understanding, researches have been conducted to visualize the inner behavior of DCNN.

The visualization methods of DCNN are largely divided into two popular branches. The first is the gradient-based methods [Zeiler and Fergus, 2014; Springenberg et al., 2014]. They produce activation by flowing the gradient of DCNN to the learned weights. The gradient-based methods mainly pay attention to detail. The second is the Class Activation Mapping (CAM) methods [Zhou et al., 2016; Selvaraju et al., 2017; Chattopadhay et al., 2018]. CAM methods operate on feature maps and represent rough shapes. Each algorithm is developed independently, but recently, [Selvaraju et al., 2017] mixed the two methods to complement their advantages and disadvantages.

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In this paper, several problems in existing CAM studies are identified. We propose Extended-CAM, a new visualization method that improves on these problems. The main contributions of this paper are summarized as follows:

1. We propose an improved upsampling method, Gaussian upsampling. The Gaussian upsampling offers a natural way for upsampling CAM and reflects the characteristics of DCNN. We argue that bilinear upsampling is an inappropriate upsampling method for CAM.

2. Problematic mathematical derivation on existing CAM studies are identified. We discuss the correct mathematical derivation for DCNN that exhibits non-linearities.

3. Combining the two options, we propose Extended-CAM, a new CAM-based visualization method. We verified that our Extended-CAM provides more accurate visualization than the existing methods.

This paper is organized as follows. Section 2 reviews the literature on CAM. Section 3 describes the two components that consist of Extended-CAM, Gaussian upsampling and modified mathematical derivation. We verify the accuracy of the visualization mask with experiments in Section 4 and discuss underlying meaning in Section 5.
2 Related Work

This section reviews existing CAM studies. While the CAM approach is general for DCNNs, we mainly focus on DCNN for classification problems of 2D images such as the VGG models. DCNN consists of a feature extractor and classifier. The feature extractor in front of DCNN consists of convolutional layers and pooling layers, obtaining feature maps from the input image. The final output of the feature extractor is the last feature map $A_{ijk}$, where $i, j$ are spatial indices and $k$ is channel index. The last feature map $A_{ijk}$ passes through the Fully-Connected layers, which forms classifier at the back, to obtain a score $y_c$. Finally, $y_c$ is passed through the softmax layer to obtain a probability for classification.

The goal of the CAM methods is to obtain $L_{ij}^c$ which satisfies $y_c = \sum_{i,j} L_{ij}^c$. Here, $i, j$ are the indices of $A_{ijk}$, the last feature map of DCNN. In other words, the CAM aims to calculate the contribution to prediction $y_c$ in units of $i, j$. Original CAM [Zhou et al., 2016] obtains $L_{ij}^c$ by replacing the classifier architecture of DCNN after feature extractor with linear layers. The linear layers consist of Global Average Pooling (GAP) and Fully-Connected layer (FC). At this time, the last feature map $A_{ijk}$ and prediction $y_c$ are represented in a linear relationship. Specifically, GAP outputs, $F_k = \sum_{i,j} A_{ijk}$, and FC that holds weight $w_k^c$ outputs $y_c = \sum_k w_k^c F_k$, so we derive,

\[
y_c = \sum_k w_k^c F_k = \sum_k w_k^c \sum_{i,j} A_{ijk} = \sum_{i,j} \sum_k w_k^c A_{ijk}. \tag{1}
\]

As the goal is to find $L_{ij}^c$ that satisfies $y_c = \sum_{i,j} L_{ij}^c$, we obtain $L_{ij}^c = \sum_k w_k^c A_{ijk}$.

The disadvantages of original CAM lie in the linear layers. If linear layers replace the classifier architecture, retraining of DCNN is required and non-linearity of classifier vanishes. To enable the direct use of the CAM approach in DCNN without any modification of the architecture, Grad-CAM/Grad-CAM++ [Selvaraju et al., 2017; Chattopadhay et al., 2018] have been proposed. As in the original CAM, they also frame the form $L_{ij}^c = \sum_k w_k^c A_{ijk}$ but use mathematical derivation to obtain the coefficient $w_k^c$ under the assumption of the non-linear classifier. The resulting Grad-CAM equation is:

\[
w_k^c = \frac{1}{Z} \sum_{i,j} \frac{\partial y_c}{\partial A_{ijk}}, \tag{2}
\]

where $Z$ is the area of the last feature map. Grad-CAM++ adds a term $\alpha_{ijk}$ to generalize Grad-CAM and uses,

\[
w_k^c = \sum_{i,j} \alpha_{ijk} ReLU\left( \frac{\partial y_c}{\partial A_{ijk}} \right), \tag{3}
\]

where $\alpha_{ijk} = \frac{\partial^2 y_c}{\partial x \partial y A_{ijk}}$. By combining $w_k^c$ and $A_{ijk}$, we obtain $L_{ij}^c = \sum_k w_k^c A_{ijk}$.

3 Method

3.1 Gaussian Upsampling

In DCNN, downsampled feature maps appear due to the pooling layers or the strided convolution layers. Accordingly, the units of last feature map $i, j$ are different from pixel-level units. Assume a $224 \times 224$ image $I$ is given to a DCNN. Using a CAM method, we obtain a grid-level CAM like $14 \times 14 L_{ij}^c$. To obtain pixel-level CAM $224 \times 224 L_{xy}^c$, where $x, y$ are units of pixel-level, an upsampling method is required.

Here, bilinear upsampling method is used in CAM, Grad-CAM, and Grad-CAM++. Nevertheless, bilinear upsampling is a simple method, which linearly interpolates only through adjacent values. The fatal problem of bilinear upsampling is that it fails to link the physical meaning between the grid-level CAM $14 \times 14 L_{ij}^c$ and the pixel-level CAM $224 \times 224 L_{xy}^c$. We want to revise the upsampling method to reflect how the grid and pixel link, i.e., how many pixels a grid-level cell covers.

Such a link can be investigated from the nature of the convolutional operation. Indeed, a cell value of the feature map is determined only by a local region of the pixel-level, which is known as a receptive field. Meanwhile, [Luo et al., 2016] found effective receptive field, which is different from the previously known square type receptive field. By checking the pixel that is actually activated, the shape of the effective receptive field is shown to be a 2D Gaussian.

Thus, a pixel-level region of effective receptive field determines the value of a grid-level cell as it passes through a feature extractor of the DCNN. Conversely, the pixel-level region covered by a grid-level cell would appear as the 2D Gaussian of the effective receptive field (Fig. 2). We describe this as spraying 2D Gaussian on the pixel-level. Therefore, to transform a grid-level CAM to hold meaning at the pixel-level, all cell values in the $14 \times 14$ grid must spray 2D Gaussians on the pixel-level.

Now, we investigate the CAM value of a pixel. The CAM value of a pixel is determined by the combination of Gaussian sprays of all the cells in the grid-level CAM. Due to the characteristics of Gaussian spray, a pixel is sprayed much from
the close cell, and less from the far cell. We express this as a combination of 2D Gaussian, as follows.

\[
L_{xy}^c = \sum_{i,j} L_{ij}^c \exp \left( -\frac{(x - w)^2}{2\sigma_x^2} - \frac{(y - h)^2}{2\sigma_y^2} \right),
\]

where \(w, h = 14, w, h = 224, \) and \(\sigma_x, \) and \(\sigma_y\) are the standard deviations for 2D Gaussian of the effective receptive field. We call this improved upsampling method that links physical meanings of grid and pixel, Gaussian upsampling.

Although we use the term “upsampling”, Gaussian upsampling differs from the conventional upsampling method. Gaussian upsampling results in 16 × upsampling, but it aims to naturally link the physical meaning of the pixel and grid. Gaussian upsampling does not aim to increase resolution by making interpolation more accurate. According to Gaussian upsampling, the CAM value of a pixel is determined by a Gaussian combination of the entire grid. Even if a grid cell is far away, a pixel may be influenced. Thus, conventional upsampling methods that interpolate only through adjacent values are completely different from Gaussian upsampling. Conversely, such a conventional upsampling method is unsuitable for the upsampling of CAM because distant, non-adjacent grid cell values cannot influence the corresponding pixel.

### 3.2 Modified Mathematical Derivation

CAM replaced DCNN’s classifier architecture with linear layers of Global Average Pooling (GAP) and a Fully-Connected (FC) layer. However, the neural network is essentially a non-linear model. To apply the CAM approach and allow non-linearity of DCNN, Grad-CAM/Grad-CAM++ have been proposed.

However, Grad-CAM/Grad-CAM++ hold two common problems. First, even Grad-CAM/Grad-CAM++ assume linear layers of GAP and FC in mathematical derivation. This is unsuitable for assuming non-linear layers. Second, Grad-CAM/Grad-CAM++ use \(w^c\) as the coefficient of \(A_{ijk}\), and it is assumed to be independent of \(i, j\). We will demonstrate that this is also an unnecessary assumption and the use of \(w^c\), which include dependent information on \(i, j\), is more natural.

Following mathematical derivation we provide improves on these two problems. In DCNN, the last feature map \(A_{ijk}\) passes through Fully-Connected layers, which outputs the prediction score \(y^c\). We represent the Fully-Connected layers by a non-linear function \(f\).

\[
y^c = f(A_{111}, A_{112}, ...)
\]

\[
y^c = \sum_{k} \frac{\partial y^c}{\partial A_{ijk}} A_{ijk}.
\]

The goal of the CAM approach is to obtain \(L_{ij}^c\) such that \(y^c = \sum_{i,j} L_{ij}^c\). So,

\[
L_{ij}^c = \sum_{k} \frac{\partial y^c}{\partial A_{ijk}} A_{ijk}.
\]

In the context of existing CAM studies which use \(w^c\), Equation 8 can be interpreted as using the coefficient \(w^c = \frac{\partial y^c}{\partial A_{ijk}}\), which includes \(i, j\) units. We can understand that the use of \(w^c\), which consider \(i, j\) units, is mathematically natural. It is unnatural to additionally assume that the coefficient is independent of \(i, j\) and to use \(w^c\), which is averaged over \(i, j\), like Grad-CAM. The meaning of \(w^c\) will be discussed again in the discussion section.

On the other hand, additional post-processing is applied in the existing CAM studies. Grad-CAM applies ReLU on \(L_{ij}^c\) and clips the negative value to 0 to ignore the negative contribution. Grad-CAM++ also takes ReLU on the gradient term to ignore the negative gradient. These manipulations using ReLU lead to several problems: the information of negative value is lost, an unnatural formula is created, and the design choice of where to assign ReLU is required. However, our method uses Gaussian upsampling, so even if a negative value appears in \(L_{ij}^c\), it is smoothed by blending with the surrounding value. Thus, the information of negative \(L_{ij}^c\) can be naturally reflected in the surrounding value. Experimenting with various design choices that apply ReLU on \(L_{ij}^c\) and the gradient term, we confirmed that the original Equation 8 without any ReLU yielded the best results.

### 4 Experiments

#### 4.1 Estimation of Effective Receptive Field

To use Gaussian upsampling, we need to estimate \(\hat{\sigma}_x, \hat{\sigma}_y\), the standard deviations of 2D Gaussian of the effective receptive field. First, we obtained the effective receptive field of the last feature map of the VGG-16 model [Simonyan and Zisserman, 2014]. This can be achieved by obtaining the gradient of an image for the last feature map and averaging it over the various images (Fig. 3).

We used the LmFit library [Newville et al., 2016] to fit the effective receptive field into 2D Gaussian. The unused pixels which appear as microscopic holes in the effective receptive field have negative values. We observed that ignoring them does not affect the fitting results.

The fitting results showed that the \(R^2\) value is quite close to 1, indicating that the effective receptive field matches well to the shape of the 2D Gaussian (Table 1). The resulting \(\hat{\sigma}_x, \hat{\sigma}_y\) are used in the Gaussian upsampling.
Figure 3: The effective receptive field of the VGG-16 model. The bright areas are highly activated areas due to large gradients, and the dark areas are areas that are not.

Table 1: 2D Gaussian fitting result of the effective receptive field. The $R^2$ value which is close to 1 indicates that the effective receptive field well matches to the 2D Gaussian shape. The resulting $\hat{\sigma}_X$, $\hat{\sigma}_Y$ are used for Gaussian upsampling.

| Item | Value |
|------|-------|
| $R^2$ | 0.9905 |
| $\hat{\sigma}_X$ | 31.7797 |
| $\hat{\sigma}_Y$ | 33.3606 |

4.2 Visualization with Extended-CAM

We implemented Extended-CAM, which uses Gaussian upsampling and the modified mathematical equation using $w_{ijk}$. We obtained several examples of visualization from Extended-CAM and existing CAM methods (Fig. 4). This visualization represents important parts for the VGG-16 model to classify a given image. The visualization results of Grad-CAM and existing CAM methods (Fig. 4). This visualization mask suggested by Extended-CAM. In other words, the visualization of Extended-CAM highlights more important parts of the image than the existing CAM methods.

Meanwhile, the visualizations of Extended-CAM tend to be wider than Grad-CAM and Grad-CAM++ (Fig. 4). The area of the CAM mask can be an important factor. Depending on the distribution of $L_{xy}$, the original image $I$ may be masked much or less masked. Thus, indicators such as Average Drop % or % Increase may be influenced. In the visualization task, the key is to find relatively important parts in the image, regardless of the absolute area of the mask. In consideration of this, we designed additional experiments with relative masking.

We designed the following experiments with relative masking, which masks with a fixed area. Based on the value of $L_{xy}$, we left only the top 50 % pixels in the image and masked out the remaining 50 % of pixels. Using masked images from relative masking, Average Drop % and % Increase are measured again (Table 3). Extended-CAM also exhibits better results than existing CAM methods in relative masking. This means that Extended-CAM finds important parts in the image well regardless of the distribution of $L_{xy}$.

5 Discussion

We proposed two options to improve the CAM method: 1) bilinear upsampling or Gaussian upsampling in the upsampling process, and 2) the existing formula using $w_i$ or the generalized one using $w_{ijk}$ in the $L_{ij}$ calculation. Apparently, the two options seem to be independent. However, in this section, we will discuss in detail that the two options are not independent and determine a factor, smoothness.

First, Gaussian upsampling has the effect of making $L_{ij}$ smoother. In terms of signal processing, Equation 4 is equivalent to applying zero-insertion and filtering. The latter is commonly known as Gaussian smoothing. In another aspect, as mentioned in Section 3, Gaussian upsampling differs from conventional upsampling methods and more naturally represents grid-pixel relationships. If we use Gaussian upsampling, a pixel-level CAM value is also influenced by distant grid cells. That is, Gaussian upsampling combines $L_{ij}$ through a wide area. Thus, when bilinear upsampling is replaced with Gaussian upsampling, $L_{xy}$ becomes smooth overall. We call this additional smoothing explicit smoothing.

Second, consider the setting of $w$. We illustrate an intuitive comparison of the use of $w_i$ and $w_{ijk}$ (Fig. 5). $w_i$ averages the gradient in the $i, j$ direction and ignores the $i, j$ units. Because $L_{ij}$ is calculated from $w_i$ and $A_{ijk}$, the averaged $w_i$ creates a smoothing effect on $L_{ij}$. We call this effect implicit smoothing. In contrast, if $w_{ijk}$ is used, implicit smoothing as it removes unnecessary parts in the image. To compare the confidences, we used two indicators, Average Drop % and % Increase. We measured them on average using the PASCAL VOC 2007 validation dataset [Everingham et al., 2007].

On both Average Drop % and % Increase, Extended-CAM outperforms the existing CAM methods (Table 2). The result indicates that confidence drops less and the tendency to increase is larger. These confidence changes mean that important parts of the image remain well when applying the visualization mask suggested by Extended-CAM. In other words, the visualization of Extended-CAM highlights more important parts of the image than the existing CAM methods.
Figure 4: Visualization examples of existing CAM methods and Extended-CAM. We confirm that the visualization results of Extended-CAM represent more important areas.

| Method          | Grad-CAM | Grad-CAM++ | Extended-CAM (Ours) |
|-----------------|----------|------------|---------------------|
| Average Drop %  | 46.56    | 36.84      | 21.89               |
| % Increase      | 13.42    | 17.05      | 28.72               |

Table 2: Comparison of visualization performance of CAM-based methods.

| Method          | Grad-CAM | Grad-CAM++ | Extended-CAM (Ours) |
|-----------------|----------|------------|---------------------|
| Average Drop %  | 47.94    | 45.60      | 39.86               |
| % Increase      | 16.39    | 15.10      | 20.72               |

Table 3: Comparison of visualization performance of CAM-based methods with relative masking.

does not appear because $L_{ij}$ is calculated using the gradient that is not averaged in the $i,j$ direction.

In summary, the existing CAM method of using bilinear upsampling and $w_{i,j}^c$ causes implicit smoothing. Our method of using Gaussian upsampling and $w_{i,j,k}^c$ leads to explicit smoothing. Thus, both the upsampling method and the $w$ setting option determine smoothness. Indeed, Gaussian upsampling can mimic the effects of the use of $w_{i,j}^c$. We found that if we apply explicit smoothing using certain smoothness $\hat{\sigma}_X = \hat{\sigma}_Y = 20$ rather than the standard deviations of the effective receptive field, the results become quite similar to those of Grad-CAM where implicit smoothing occurs.

So, how should smoothing be applied? Note that implicit smoothing has a smoothing effect on $L_{ij}$ through averaging $w_{i,j}^c$. Here, smoothness is arbitrarily determined. If we use implicit smoothing, we won’t be able to control the smoothness as desired, even if we know the reasonable smoothness. Explicit smoothing, in contrast, allows us to set the smoothness directly. Note that Gaussian upsampling naturally links the grid-pixel relationship and it means smoothing. This implies that the appropriate amount of smoothness exists and is determined by explicit smoothing. Therefore, it is preferable to apply explicit smoothing by calculating the appropriate smoothness rather than arbitrarily determining the smoothness through implicit smoothing.

Indeed, the amount of smoothing strongly relates to the accuracy of the visualization mask. We believe that the reason Extended-CAM provides a more accurate visualization mask lies in the valid smoothness that explicit smoothing determines. We present the observations that other incorrect smoothnesses degrade performance (Table 4. We found that applying Gaussian upsampling to Grad-CAM/Grad-CAM++ results in worse visualization than Extended-CAM. This means that using both implicit smoothing and explicit
smoothing at the same time applies the smoothing twice and determines the wrong smoothness. Also, when bilinear upsampling and $w_{ijk}^c$ are used instead of Gaussian upsampling and $w_{ijk}^c$, the performance is degraded. We believe these options also determine wrong smoothness because both implicit and explicit smoothings do not appear. These observations agree with our analysis that applying explicit smoothing alone determines valid smoothness, leading to improved performance.

### 6 Conclusion and Future Work

This paper proposes Extended-CAM, a new CAM-based visualization method of DCNN. First, the limitations of bilinear upsampling are identified in the pixel-level upsampling. We discussed the validity of Gaussian upsampling using an effective receptive field. Second, the problems of the mathematical derivation presented in the previous papers are identified and corrected. We found that these two options are factors that determine smoothness and can be interpreted as explicit smoothing and implicit smoothing, respectively. Experimental results showed that Extended-CAM represents a more accurate visualization than previous studies.

There are still tasks left. All CAM approaches have a limitation in that the last feature map is highly coarse ($14 \times 14$). In this study, we focused on the last feature map. But in the future, we expect the finer structure in the image to be investigated by applying a CAM that sets another feature map in the front as a target layer. Also, while we used DCNN for 2D images, we expect the potentials of Extended-CAM for other general-purpose DCNNs such as speech recognition.

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