Thermodynamics of parametric dark energy models

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A comparative study of a set of parametric dark energy models is performed by studying the evolution of dark energy both in the past and future epochs. In addition, the age of the universe and time till the distant future \((a = 1000)\) are estimated. The validity of generalized second law of thermodynamic in different parametric models is also ascertained.

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I. INTRODUCTION

In contemporary cosmology, the story of dark energy is quite checkered and enigmatic in nature. The fact that dark energy is a dominant component of cosmic constituents has been well established directly from the observation of acceleration in cosmic expansion as indicated by SN Ia data \([1, 2]\) and indirectly through different cosmological probes, namely CMB anisotropy measurements \([3, 4]\), BAO and \([5, 6]\) strong \([7]\) as well as weak \([8]\) gravitational lensing. As the acceleration of expansion due to repulsive dark energy can be provided by the cosmological constant \(\Lambda\), the standard cold dark matter \(\Lambda CDM\) model has consequently been replaced by the ΛCDM model. In spite of the fact that the ΛCDM model provides the best fit to all the available cosmological data, two well known theoretical issues, namely coincidence and fine tuning remain to be explained \([9]\).

In the absence of definitive answer regarding the source of dark energy, several alternative scenarios based on scalar field, string theory, modified gravity, Chaplygin gas, polytropic gas and interacting dark energy have been explored \([10–18]\). Within scalar field models, quintessence, phantom, k-essence, DBI-essence, H-essence, tachyon, dilaton, quintom and ghost condensate have been proposed. An exhaustive list of parametric models has been reported by Pacif \([19]\). In case of parametric models with varying \(w = w(z)\), different parametrizations of \(w(z)\), namely one index \([20]\), two index such as, linear parametrization \([21, 24]\), Chevallier-Polarski-Linder (CPL) \([25, 26]\), Jassal-Bagla-Padmanabhan (JBP) \([27]\), generalized CPL as well as JBP \([28]\), Feng-Shen-Li-Li (FSLL) \([29]\), Barboza-Alcaniz (BA) \([30]\), square-root \([31]\), Ma-Zhang (MZ) \([32]\), logarithmic \([32, 33]\) and oscillating \([37, 38]\), three index \([39, 40]\) and four index \([41, 42]\) parametrizations have been proposed.

Usually, the parameters of a specific model are marginalized in the red shift range \(z = 0 – 1090\) employing available cosmological data namely SN Ia, BAO, CMB, \(H(z)\) data. By adopting this procedure, it turns out that the ΛCDM model and parametric models exhibit similar dynamics of dark energy. The age of the universe is an integrated quantity. Hence, it can be used in the comparative study of different parametric models. As the fitting procedure cannot include future data, it will also be interesting to explore the future behavior of these parametric models. In Ref. \([43]\), two requirements of generalized second law (GSL) of thermodynamics as thermodynamic viability of dark energy models are given by \(S'(a) \geq 0\) and in the far future \(S''(a) \leq 0\). We calculate first derivative \(S'(a)\) and second derivative \(S''(a)\) of entropy \(S\) and thereby investigate the thermodynamic viability of parametric dark energy models.

II. THERMODYNAMICS OF DARK ENERGY MODELS

In a spatially flat \((k = 0)\) universe, the constituents of cosmic energy density are non-relativistic matter (baryonic and dark matter), neutrinos, CMB photons, and dark energy. In this work, contribution of neutrinos to the cosmic energy density is neglected. With the equation of state \(P = \omega \rho\), the Friedmann-Raychaudhury equations are written as

\[
H^2(a) = \frac{8 \pi G}{3} \sum_i \rho_i(a) \quad (1)
\]

\[
\frac{\ddot{a}}{a} = -\frac{4 \pi G}{3} \sum_i \rho_i(a) [1 + 3 \omega_i(a)] \quad (2)
\]

The equation of state parameters \(\omega_M = 0\) and \(\omega_R = 1/3\) for matter and radiation, respectively. In Section 3, the dark energy parameter \(\omega_X\) of different parametric models is discussed.

The energy densities \(\rho_i(a)\) and density parameters \(\Omega_i(a)\) are related by

\[
\rho_i(a) = \rho_0 f_i(a) \quad (3)
\]

\[
\Omega_i(a) = \frac{\rho_i(a)}{\rho_0} \quad (4)
\]

where the dark energy function \(f_i(a) = \rho_i(a)/\rho_0\) and

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$E(a)$ are defined by

$$f_i(a) = \exp \left[ -3 \int_1^a \frac{1 + w_i(a')}{a'} da' \right]$$ \hspace{1cm} (5)

$$E^2(a) = \frac{H^2(a)}{H_0^2} = \sum_i \Omega_0 f_i(a)$$ \hspace{1cm} (6)

Using Eq. (6), the second Friedmann-Raychaudhury equation can be written as

$$\frac{2q - 1}{3} = \frac{\Omega_R}{3} + w_X \Omega_x$$ \hspace{1cm} (7)

The age of the universe is calculated by using

$$t_0 = \int_0^1 \frac{da}{aH(a)} = H_0^{-1} \int_0^1 \frac{da}{a [\Omega_{0M} a^{-3} + \Omega_{0R} a^{-4} + \Omega_{0X} f_x(a)]^{1/2}}$$ \hspace{1cm} (8)

According to the GSL of thermodynamics, two requirements of thermodynamic viability of dark energy models are given by $S'(a) \geq 0$ and in the far future $S''(a) \leq 0$. The total entropy of the universe $S(a)$ is due to the entropy of horizon $S_H(a)$ as well as entropy of cosmic fluids within the horizon, namely matter $S_M(a)$, radiation $S_R(a)$ and dark energy $S_X(a)$. Explicitly, the entropy $S_H(a)$, $S_M(a)$, $S_R(a)$ and $S_X(a)$ are written as follows [43].

$$S_H(a) = \frac{k A_H}{4 l_p^2} = \frac{\pi k c^2}{4 H^2}$$ \hspace{1cm} (9)

$$S_M(a) = k n V_H$$

$$S_R(a) = 4 \pi k c^2 n_0$$

$$S_X(a) = 3 a^3 H^3$$

Entropy of radiation and dark energy are calculated using

$$T dS_i = d(\rho_i V_H) + \rho_i \omega_i d(V_H)$$ \hspace{1cm} (11)

In dimensionless form

$$\tilde{S}_\alpha(a) = \left( \frac{4GhH_0^3}{3kc^5} \right) S_\alpha$$ \hspace{1cm} (12)

where $\alpha = H, M, R$ and $X$ denote horizon, matter, radiation, and dark energy, respectively, the first derivative of total entropy of the universe $\tilde{S}'(a)$ is written as

$$\tilde{S}'(a) = \tilde{S}'_H(a) + \tilde{S}'_M(a) + \tilde{S}'_R(a) + \tilde{S}'_X(a)$$ \hspace{1cm} (13)

with

$$\tilde{S}'_H(a) = \frac{4\pi}{aE^2(a)} \left[ 1 + \sum_i w_i(a) \Omega_i(a) \right]$$ \hspace{1cm} (14)

$$\tilde{S}'_M(a) = \left( \frac{H_0 h}{k T_{0M}} \right) \frac{1}{a^4 E^3(a)} \left[ 1 + 3 \sum_i w_i(a) \Omega_i(a) \right]$$ \hspace{1cm} (15)

$$\tilde{S}'_R(a) = \left( \frac{H_0 h}{k T_i} \right) \frac{\Omega_i(a) (1 + w_i(a))}{a E(a)} \left[ 1 + 3 \sum_j w_j(a) \Omega_j(a) \right]$$ \hspace{1cm} (16)

Here, “i” denotes radiation/dark energy. Temperature of cosmic fluids are defined as

$$T_{0M} = \left( \frac{3 c^2 H_0^2}{8 \pi G k n_0} \right)$$ \hspace{1cm} (17)

$$T_i = T_{0M} a f_i(z)$$ \hspace{1cm} (18)
where \( n_0 \sim 10^{-6} \text{ m}^{-3} \), \( T_{0R} = 2.725 \text{ K} \) and
\[
T_{0X} = \tau_X \frac{H_0 h}{k}
\]
with an arbitrary parameter \( \tau_X \). Finally, the second derivative of total entropy \( S''(a) \) can be calculated from \( S'(a) \) in a straightforward manner.

III. PARAMETRIC MODELS

In addition to \( \Lambda \)CDM, quintessence and phantom models, a set of two index parametric models in which \( w(z) \) is bounded in both distant past \((a \rightarrow 0)\) and far future \((a \rightarrow \infty)\) are considered. Although, the choice of parametrizations is arbitrary, they are representative of different scenarios. With \( w = \) constant, there are three possible scenarios, namely \( \Lambda \)CDM, quintessence and phantom models. In general, the dark energy function \( f_X(a) \) is given by
\[
f_X(a) = a^{-3(1+w)}
\]

I. \( \Lambda \)CDM model

In \( \Lambda \)CDM model, \( w(a) = -1 \) and dark energy function is given by
\[
f(a) = 1
\]

II. Quintessence model

In quintessence models, scalar fields with constant \( w > -1 \) are considered. The dark energy function \( f_X(a) \) is same as Eq. (21) and \( w = -0.80 \) with \( \Omega_{0M} = 0.30 \).

III. Phantom model

Scalar fields with \( w < -1 \) are called phantom fields and in general, the dark energy function \( f_X(a) \) given by
\[
f_X(a) = \frac{3w_1(1 - a)}{a}
\]

By \( \chi^2 \) minimization of SN Ia ‘gold set’, galaxy power spectrum and CMB power spectrum data, the 1\( \sigma \) and 2\( \sigma \) constraints on dark energy parameters are \( w_0 = -1.38^{+0.08}_{-0.05} \), \( w_1 = 1.27^{+0.20}_{-0.16} \), and \( \Omega_{0M} = 0.31 \). With these parameters, the dark energy has phantom origin.

V. Feng-Shen-Li-Li (FSLL(I)) parametrization

Using the parametrization given by Feng et al. [29]
\[
w(a) = \frac{w_0 + \frac{w_1 a(1 - a)}{1 - 2a + 2a^2}}{1 - 2a + 2a^2}
\]

the dark energy function \( f_X(a) \) is written as
\[
f_X(a) = a^{-3(1+w_0)}(1 - 2a + 2a^2)^{3w_1/4} \exp \left[ \frac{3w_1}{2} \tan^{-1} \left( \frac{1 - a}{a} \right) \right]
\]
energy, the 1σ and 2σ constraints on dark energy parameters are $w_0 = -1.0148^{+0.5183}_{-0.5183} + 0.6482^{+0.9834}_{-0.9834}$, $w_1 = -0.0155^{+2.1257}_{-0.0155} + 2.9673^{+0.9834}_{-0.9834}$, $\Omega_{DM} h^2 = 0.1094^{+0.0126}_{-0.0133} - 0.0163$ and $\Omega_b h^2 = 0.0223^{+0.0016}_{-0.0014} - 0.0021$.

VI. Feng-Shen-Li-Li (FSLL(II)) parametrization

Second parametrization Feng et al. [29] given by

$$w(a) = w_0 + \frac{w_1(1-a)^2}{1-2a+2a^2}$$  \hspace{1cm} (27)

results in the dimensionless dark energy function $f_X(a)$ given by

The 1σ (best fit) values of parameters for fitting SN Ia Hubble diagram by JLA sample, BAO (6dFGS), SDSS DR7, CMB (Plank13) and $H(a)$ data are $w_0 = -0.947^{+0.539}_{-1.010}$, $w_1 = -0.013^{+1.143}_{-0.143}$ (-0.076), $\Omega_{OM} = 0.256^{+0.052}_{-0.087}$ (0.290) and $h = 0.707^{+0.097}_{-0.060}$ (0.699).

VII. Barboza-Alcaniz (BA) parametrization

Barboza and Alcaniz have parametrized $w(a)$ [31] as

$$w(a) = w_0 + w_1 \frac{(1-a)}{1-2a+2a^2}$$  \hspace{1cm} (29)

and $f_X(a)$ is given by

$$f_X(a) = a^{-3(1+w_0+w_1)}(1-2a+2a^2)^{3w_1/4} \exp \left[ \frac{-3w_1}{2} \tan^{-1} \left( \frac{1-a}{a} \right) \right]$$  \hspace{1cm} (28)

By the same procedure as FSLL (I), the 1σ and 2σ constraints on dark energy parameters are $w_0 = -1.0214^{+0.2846}_{-0.3373}$, $w_1 = -0.0113^{+2.5469}_{-3.9002}$, $\Omega_{DM} h^2 = 0.1107^{+0.0118}_{-0.0141} + 0.0161$ and $\Omega_b h^2 = 0.0225^{+0.0015+0.0018}_{-0.0017-0.0021}$.

VIII. Ma-Zhang (MZ) parametrization

Ma and Zhang have parametrized $w(a)$ [32] as

$$w(a) = w_0 + w_1 \left[ a \ln \left( \frac{1+a}{a} \right) - \ln 2 \right]$$  \hspace{1cm} (31)

and the resulting dark energy evolution function $f_X(a)$ is written as

$$f_X(a) = a^{-3(1+w_0-w_1 \ln 2)} 2^{6w_1} (1+a)^{-3w_1} \left( \frac{1+a}{a} \right)^{-3w_1 a}$$  \hspace{1cm} (32)

IX. Pan-Saridakis-Yang (PSY1) parametrization

Using the second parametrization of $w(a)$ given by Pan et al. [33],

$$w(a) = w_0 + b \left[ 1 - \cos(\ln a) \right]$$  \hspace{1cm} (33)

$f_X(a)$ is written as

$$f_X(a) = a^{-3(1+w_0+b)} \exp \left[ 3b \sin(\ln a) \right]$$  \hspace{1cm} (34)

Using the same procedure as in PSY1 parametrizations, the marginalized dark energy parameters with mean $\pm 1\sigma \pm 2\sigma \pm 3\sigma$ (best fit values) are $w_0 = -1.0078^{+0.0231+0.068+0.094}_{-0.032-0.059-0.080}$ (-1.0031), $b = -0.1468^{+0.275+0.431}_{-0.142-0.550-0.803}$ (-0.1127), $\Omega_0^{\text{M}} = 0.306^{+0.008+0.017+0.025}_{-0.009-0.017-0.020}$ (0.308), and $H_0 = 68.05^{+1.20+1.77+2.25}_{-0.90-2.02-2.68}$ (67.84).

X. Pan-Saridakis-Yang (PSY2) parametrization

Using the parametrized $w(a)$ given by Pan et al. [33],

$$w(a) = w_0 - b \sin(\ln a)$$  \hspace{1cm} (35)

doing the dark energy function $f_X(a)$ is found to be

$$f_X(a) = a^{-3(1+w_0)} \exp \left[ 3b (1 - \cos(\ln a)) \right]$$  \hspace{1cm} (36)

The dark energy parameters have been marginalized by using SNIa (JSL sample), BAO, CMB, red shift space distortions, weak gravitational lensing, and Hubble parameters. Constrained parameters with mean $\pm 1\sigma \pm 2\sigma \pm 3\sigma$
(best fit values) are \( w_0 = -0.9817^{+0.0535+0.0938+0.1175} \) \(-0.0144\), \( b = -0.0114^{+0.0378+0.0739+0.1001} \) \(-0.0039\), \( \Omega_{0M} = 0.311^{+0.011+0.019+0.023} \) \(0.300\) and \( H_0 = 67.32\) \(-1.39\) \(-1.95\) \(-2.37\) (68.74).

IV. RESULTS AND DISCUSSIONS

In order to ascertain the similarities and differences between \( \Lambda \)CDM and various parametric models, we compare the evolution of \( \Omega_X \) due to different parametrizations both in the past and future epochs. In the past epoch, the dark energy density parameter \( \Omega_X \) is calculated at red-shifts of a few astrophysical epochs, namely \( 1 \leq z \leq 3 \) (galaxy formation era), \( z = 1090 \) (last scattering surface), and \( z = 10^{10} \) (BBN). Constraints on dark energy density parameters during galaxy formation era, LSS and BBN are \( \Omega_X \leq 0.5 \) \(\Omega_X^{\text{LSS}} < 0.1 \) \(\Omega_X^{\text{BBN}} < 0.14 \) \((\Omega_X^{\text{BBN}} < 0.21)\) respectively. These constraints have been obtained such that during the galaxy formation era, \( \Omega_X < \Omega_M \) and the observed Hubble parameter in the universe should not be disturbed due to the presence of dark energy until BBN epoch. The evolution of \( \Omega_X \) in the future epoch is reported at a few arbitrary redshifts corresponding to \( a = 0.25, 0.50 \) and 1.00.

Parameters of the above mentioned models are fitted to different sets of the then available cosmological data. In most of the \( \chi^2 \) minimization but for Ref. [42], contribution of radiation has not been taken into account. The inclusion of radiation may play significant role while fitting data of early universe, such as BBN. In the present analysis, we include the contribution of radiation with \( \Omega_{0R} = 2.475 \times 10^{-5}/h^2 \). Although, the Hubble constant has been fitted with a few parametrizations, the results of the present work are almost independent of the magnitude of \( h \) and hence, \( h = 0.699 \) is used for convenience.

In Table 1, transition redshift \( z_T \), dark energy density parameter at transition \( \Omega_X(T) \), \( \Omega_X(a = 0.5 \rightarrow 0.25) \) during galaxy formation era, and \( \Omega_X(a = 0.0009) \) at LSS \( (z = 1090) \) are displayed. Also, the age of the universe \( (\text{in dimensionless units}) \), \( tH_0 \) is given in the last column of the same Table. It is noticed that in comparison to \( \Lambda \)CDM model, the transition redshift \( z_T \) in all models but for UIS and BA differ by less than 5%. In fact, UIS parameters have been marginalized with earlier cosmological data set [24]. However, variation in \( \Omega_X(T) \) due to different parametrizations is within 5% except for quintessence, phantom, and UIS parametrizations. During galaxy formation epoch, the values of \( \Omega_X \) show the same trend albeit the constraint on \( \Omega_X < 0.5 \) corresponding to \( a = 0.5 \rightarrow 0.25 \) is satisfied by all the parametric models. Further, all the parametric models are consistent with LSS and BBN constraints as \( \Omega_X \rightarrow 0 \).

Due to tension in present values of \( H_0 \) \(\Omega_X \) \(-10.32 \) \(-0.1390 \), the age of the universe \( tH_0 \) is presented in the last column of Table 1. As \( tH_0 \) is an integrated observable, the dynamical evolution of dark energy vis-a-vis the signature and the functional form of \( w(z) \) leaves an indelible imprint on the age of the universe. The age \( tH_0 \) is close to 1 with BA parametrizations. In quintessence model, the value of \( tH_0 \) is maximally off by 7%.

In Table 2, we display the future evolution of dark energy parameter for \( a = 2, 4 \) and 1000. It is noticed that in comparison to \( \Lambda \)CDM, the variation in \( \Omega_X \) due to all the considered parametrizations at \( a = 2 \) and 4 lies within 4%. The dark energy parameter \( \Omega_X = 1 \) at \( a = 1000 \) in all the parametric models. In fact, \( \Omega_X \) is equal to unity as early as \( a = 10 \). However, a large variation is observed in \( t_{1000}H_0 \). The smallest and largest values of \( t_{1000}H_0 \) are 0.901 and 27.556 with UIS and quintessence parametrizations, respectively.

| TABLE I: Evolution of dark energy parameter \( \Omega_X \) in the past epoch. |
|----------------------|------------------|------------------|---------|
| Models               | \( z_T \)         | \( \Omega_X(T) \) | \( \Omega_X \) | \( t_{1000}H_0 \) |
| ACDM                 | 0.671            | 0.333            | 0.226–0.035 | 0.000 | 0.964 |
| QUINT                | 0.638            | 0.417            | 0.307–0.077 | 0.000 | 0.931 |
| Phantom              | 0.650            | 0.278            | 0.161–0.016 | 0.000 | 0.991 |
| UIS                  | 0.441            | 0.392            | 0.276–0.208 | 0.011 | 0.934 |
| FSLL1                | 0.699            | 0.326            | 0.227–0.034 | 0.000 | 0.976 |
| FSLL2                | 0.698            | 0.325            | 0.226–0.033 | 0.000 | 0.977 |
| BA                   | 0.810            | 0.348            | 0.286–0.051 | 0.000 | 0.998 |
| PSY1                 | 0.665            | 0.325            | 0.214–0.028 | 0.000 | 0.961 |
| PSY2                 | 0.644            | 0.338            | 0.222–0.035 | 0.000 | 0.952 |

| TABLE II: Evolution of dark energy parameter \( \Omega_X \) in the future epoch. |
|----------------------|------------------|------------------|---------|
| Model                | \( \Omega_X \)  | \( t_{1000}H_0 \) |
| ACDM                 | 0.949            | 1.000            | 8.182   |
| QUINT                | 0.925            | 0.985            | 1.000   | 27.56  |
| Phantom              | 0.966            | 0.997            | 1.000   | 3.426  |
| UIS                  | 0.987            | 1.000            | 1.000   | 0.901  |
| FSLL1                | 0.952            | 0.994            | 1.000   | 7.799  |
| FSLL2                | 0.954            | 0.994            | 1.000   | 7.140  |
| BA                   | 0.954            | 0.993            | 1.000   | 10.67  |
| PSY1                 | 0.950            | 0.994            | 1.000   | 2.930  |
| PSY2                 | 0.944            | 0.992            | 1.000   | 4.312  |

Thermodynamic viability of GSL is given by \( S'(a) \geq 0 \) and in the far future \( S'(a) \leq 0 \). Hence, it is required that \( S'(a) \geq 0 \) through out the dynamical evolution of the universe \( \text{i.e.} \ a = 0 \rightarrow \infty \). As \( \Omega_X \rightarrow 0 \) for \( a < 0.0009 \) (BBN) and \( \Omega_X = 1 \) for \( a = 10 \), the behavior of \( S'(a) \) is investigated within range \( a = 0.001–1000 \) corresponding to \( z_{CMB} = 1090 \) and redshift at far future \( z = -0.999 \) as asymptotic limit \( a \rightarrow \infty \). In general, the contribution of \( S_R(a) \) and \( S_H(a) \) to total \( S'(a) \) is negligible. The evolution of \( S'(a) \) is governed by the relative contribution of \( S_X(a) \). In \( \Lambda \)CDM model, the \( S_X(a) = 0 \)
due to the $(1 + w_X)$ term in Eq. (13) albeit the dark energy parameter $\Omega_X$ contributes to the horizon entropy $S_H(a)$. In the vicinity of $a = 0.001$, $S'_H(a) \sim 0$ as $\Omega_X \sim 0$. As $a \to 1$, there is an increase in $S'_H(a)$. Further, $S'_H(a) \to 0$ as $\Omega_X \sim 1$ around $a = 10$. The second condition $S''(a)|_{a=1000} \leq 0$ is also satisfied as $S'_H(a)$ decreases with the increase of $a \to 1000$.

With other parametrizations, relative contributions of horizon and dark energy entropy decide the evolution of total entropy. The signs of $S'_H(a)$ and $S'_X(a)$ are decided by the terms $(1 + w_X(a)\Omega_X(a))$ and $(1 + 3w_X(a)\Omega_X(a))$ in Eq. (14) and Eq. (16), respectively. Further, the magnitude of $S'_X(a)$ and hence $S'(a)$ is modulated by the parameter $\tau_X$ as shown in Ref. [13]. Presently, $\tau_X = 1$ is used. In Figs. 1 and 2, variation of $S'(a)$ vs. $a$ due to different parametrizations is plotted within range $a = 0.001–1000$. In all cases, the peak in variation of $S'(a)$ is observed to be at the transition epoch corresponding to $\tau_T$. This is expected as the sign of $\Omega_X(a)$ changes at $\tau_T$. It is also observed that models presented in Fig. 1, namely $\Lambda$CDM, quintessence, BA, PSY1 and PSY2 have $S''(a) < 0$ and $S'(a)$ due to phantom, UIS, FSSL1, FSSL2 and MZ parametrizations displayed in Fig. 2 have $S''(a) > 0$. However, thermodynamic viability of parametric models cannot be decided due to the unknown value of arbitrary parameter $\tau_X$.

V. CONCLUSIONS

A comparative study of a set of parametric dark energy models has been performed by calculating the dynamical evolution of dark energy parameter $\Omega_X$ in the past and future epochs. In comparison to $\Lambda$CDM model, the variation in $\Omega_X$ due to different parametrization is about 5% in the expansion parameter range $a = 10^{-14} – 1000$. The maximum variation in the age of the universe $t_\text{bH}$ is about 7%. However, $t_{1000} \mathcal{H}$ (time till $a = 1000$) varies by a factor of 30.

Thermodynamic viability of dark energy models has been investigated by calculating the two requirements of GSL of thermodynamics i.e. $S'(a)$ and $S''(a)$. It has been observed that with reasonable assumptions, GSL of thermodynamics is satisfied by $\Lambda$CDM, quintessence, BA, PSY1 and PSY2 parametrizations. Parametrizations having phantom behavior are not suitable from thermodynamic perspective. Thermodynamic viability of parametric models cannot be conclusively ascertained to the unknown value of $\tau_X$.

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[1] S. Perlmutter et al., Nature (London) 391, 51 (1998); Bull. Am. Astron. Soc. 29, 1351 (1997); Astrophys. J. 517, 565 (1999).
[2] A.G. Riess et al., Astron. J. 116, 1009 (1998); P. Gar-
