Non-contact heat transfer models identification for laser hyperthermia of semi-transparent materials

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Abstract. The identification of mathematical models of heat transfer traditionally involves the installation of temperature sensors inside the sample under study and registration of the response to external thermal effects. In cases where the use of contact methods for measuring temperature is impossible, it is necessary to develop new approaches to determining the unknown thermophysical and radiation-optical characteristics. Laser hyperthermia of superficial tissues is one such case. The paper proposes a method for identifying a model of one-dimensional unsteady heating of a semitransparent sample using non-contact thermometry. A feature of the physical process under consideration is the possibility of its discretization. Due to this, a two-stage iterative procedure for solving the inverse heat transfer problem was formulated. The implementation of the proposed algorithm using software made it possible to carry out a computational experiment. The results showed the effectiveness of this approach. The presented method can be used in the development of means for monitoring and regulating the laser hyperthermia procedure.

1. Introduction

The use of hyperthermia in medicine has become widespread both in the form of heating the whole body and locally. This method is used in the treatment of cancer, cardiac arrhythmias, Parkinson's disease, joint hypermobility, hyperopia, hyperplasia, etc., and the case of tissue ablation (heating over 50°C) [1].

To prevent the occurrence of health risks, as well as to achieve the required therapeutic effect, it is necessary to ensure that the temperature field of the tissue changes within the limits specified by the requirements of the method. Prediction and regulation of these processes require mathematical modeling, which is impossible without an accurate assessment of the thermophysical and radiation-optical characteristics of the tissue [2,3].

This study is devoted to the development of a method for radiation-conductive heat transfer mathematical models identifying as applied to laser hyperthermia of human surface tissues. At the previous stage, a computational and experimental method was proposed for determining the parameters of the simplest one-dimensional model of heating an opaque material [4]. However, the iterative approach applied to the complication of the model to improve the accuracy of planning the physical process requires modernization and identification procedures. Therefore, the purpose of this work was to develop a method for identifying a mathematical model of laser hyperthermia of the surface semitransparent human tissue.
2. Materials and methods

2.1. Formulation of the problem

The process under consideration can be described as follows. A homogeneous plate of thickness \( d \) on one side (left) is acted upon by a pulsed laser heat flux \( q_l(\tau) \) and heat exchange with the environment occurs. The switching times of the laser facility are specified by an array \((\tau^l_i)\) so that the piecewise-specified function \( q_l(\tau) \) is determined by the system (1).

\[
\begin{aligned}
q_l(\tau) &= q_l, \quad 0 \leq \tau < \tau^l_1 \\
q_l(\tau) &= 0, \quad \tau^l_1 \leq \tau < \tau^l_2 \\
q_l(\tau) &= q_l, \quad \tau^l_{n-1} \leq \tau < \tau^l_n
\end{aligned}
\] (1)

On the other side (right), there is an exchange of temperature with deeper tissues of a conventionally constant temperature, characterized by a heat flow \( q_2 \). Before the start of the procedure, the temperature distribution in the sample is considered linear, the temperature of the left (surface) border is determined by the ambient temperature, and the right one is constant and equal to 36.6°C. Then, turning on the laser device, the surface temperature of the irradiated tissue rises to 42°C and is maintained in the range of 40–42°C by pulsed laser on/off. The temperature, in this case, is recorded at the left border using a thermal imaging camera.

The mathematical model of one-dimensional heat transfer in such a situation can be formulated as follows:

\[
\begin{aligned}
C \frac{\partial T}{\partial \tau} &= \lambda \frac{\partial^2 T}{\partial x^2} + Aq_l(\tau)e^{-rx} \\
T(x, 0) &= T_0, \quad 0 < x < d \\
-\lambda \frac{\partial T}{\partial x}(0, \tau) &= Aq_l(\tau) + \alpha (T(0, \tau) - T_e(\tau)), \quad 0 \leq \tau \leq \tau_{\text{max}} \\
-\lambda \frac{\partial T}{\partial x}(d, \tau) &= q_2(\tau), \quad 0 \leq \tau \leq \tau_{\text{max}}
\end{aligned}
\] (2)

where \( C \) is the heat capacity; \( \lambda \) is the coefficient of thermal conductivity; \( T \) is the temperature; \( d \) is the thickness of the sample; \( \tau \) is the time; \( \alpha \) is the heat transfer coefficient; \( r \) is the linear absorption coefficient; \( A \) is the absorption capacity.

In this case, the translucency of the material is taken into account by the additional term \( Aq_l(\tau)e^{-rx} \) in the heat transfer equation [5].

Thus, the identification problem in this formulation is reduced to solving the inverse problem of heat transfer, that is, determining the vector of unknown characteristics of the tissue \( \vec{u} = \{\alpha, \lambda, A, r\} \).

2.2. Identification Problem Solution

In the case when the external thermal effect on the object under study is caused by pulsed radiation (as in laser hyperthermia of surface tissues), discretization of the calculation is expedient to optimize the computational process.

Since the implementation of the hyperthermia procedure in clinical practice is an alternating sequence of heating and cooling, this paper proposes the following algorithm for identifying the mathematical model.

At the first (preparatory) stage, the surface of the sample is heated by a laser to a temperature of 36.6°C and maintained at that temperature for a sufficiently long time until a uniform distribution of the thermal field is achieved throughout the volume. Then the laser system is turned off and the
surface temperature is recorded during cooling. In this case, the heat transfer equation and the left boundary condition have the form:

$$ \frac{C}{\tau} \frac{\partial T}{\partial \tau} = \lambda \frac{\partial^2 T}{\partial x^2} $$

$$ -\lambda \frac{\partial T}{\partial x}(0, \tau) = \alpha(T(0, \tau) - T_e(\tau)), \quad 0 \leq \tau \leq \tau_{\text{max}} $$

The readings of the thermal imager are used as input data for the calculation by the iterative regularization method according to the proposed at the first step of the study algorithm [4]. Then, assuming the heat capacity of the sample to be known, the inverse heat transfer problem is solved and the initial approximations of the coefficients of thermal conductivity $\lambda$ and heat transfer $\alpha$ are determined.

After the completion of the preparatory stage, the standard hyperthermia procedure begins. The laser device is switched on, the sample surface is heated to a temperature of 42°C, and is maintained in a predetermined range by periodically switching on and off the heat flux. Using the same approach [4], and assuming the previously determined values of $\lambda$ and $\alpha$ are known, the values of $A$ and $r$ are determined.

The exit from the iterative process is performed by the value of the residual functional of the calculated and actual temperatures.

To implement the described algorithm, special software was developed.

2.3. Computational experiment.

To test the proposed algorithm, a computational experiment was carried out. Based on the experience of conducting thermal tests in this direction, the values of the model coefficients were selected, comparable with the real ones: $C = 1200$ kJ/kg*K, $\alpha = 3.5$ W/m²*K, $\lambda = 0.3$ W/m*K, $r = 5000$ cm⁻¹, $A_{q1} = 600$ W/m². The ambient temperature was 24.6°C. The solution to the direct problem of heat transfer in this formulation was "experimental data".

In order to identify the mathematical model, considering its parameters to be unknown, zero values were set: $\alpha = 15$ W/m²*K, $\lambda = 0.05$ W/m*K, $r = 10$ cm⁻¹, $A_{q1} = 1000$ W/m². Then, using the experimental data generated in the previous step, the algorithm described in paragraph 2.2 of this work was reproduced.

3. Results

The result of this stage of the study is the obtained algorithm for the identification of mathematical models of radiation-conductive heat transfer in semitransparent materials without using contact temperature measurements.

The performed computational experiment has shown the effectiveness of the algorithm for solving the problem. The desired characteristics of the model were restored in 8-10 iterations. The graphs of characteristic values from the experiment are shown in Fig. 1.
Figure 1. The results of the computational experiment: values of desired characteristics at iteration $i$: a – heat transfer coefficient $\alpha$, b – coefficient of thermal conductivity $\lambda$, c – linear absorption coefficient $r$, d – absorbed heat flux $Aq_l$.

4. Discussion
The development of a set of methodological tools for the identification of radiation-conductive heat transfer mathematical models based on the apparatus of inverse heat transfer problems is an urgent task. Besides the laser hyperthermia, the results of the project can be effectively used in the creation of promising structures for aviation and rocket and space technology, medicine, thermal and nuclear power engineering, cryogenic technology.

In this work, a fairly simple and at the same time quite effective version of the iterative regularization method was developed. When identifying heat transfer processes, it becomes necessary to determine the entire set of coefficients appearing in the analyzed mathematical model. In such a situation, it seems appropriate to use a sequence of experimental studies. This approach makes it possible to take into account all the errors in determining the characteristics, admitted at the previous stages when identifying effective thermophysical and radiation-optical characteristics. Simultaneously with the implementation of this approach, it is necessary to create effective software to increase the number of identifiable characteristics.

The next stages of the study are experimental development of the method, development of a feedback module for regularization of heating based on the results of calculations, and testing on biological materials (in vitro).

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