Vacuum Alignment in SUSY $A_4$ Models

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Abstract

In this note we discuss the vacuum alignment in globally supersymmetric models with spontaneously broken flavour symmetries in the presence of generic soft supersymmetry (SUSY) breaking terms. We show that the inclusion of these soft SUSY breaking terms can give rise to non-vanishing vacuum expectation values (VEVs) for the auxiliary components of the flavon fields. These non-zero VEVs can have an important impact on the phenomenology of this class of models, since they can induce an additional flavour violating contribution to the sfermion soft mass matrix of right-left (RL) type. We carry out an explicit computation in a class of globally SUSY $A_4$ models predicting tri-bimaximal mixing in the lepton sector. The flavour symmetry breaking sector is described in terms of flavon and driving supermultiplets. We find non-vanishing VEVs for the auxiliary components of the flavon fields and for the scalar components of the driving fields which are of order $m_{SUSY} \times \langle \varphi \rangle$ and $m_{SUSY}$, respectively. Thereby, $m_{SUSY}$ is the generic soft SUSY breaking scale which is expected to be around 1 TeV and $\langle \varphi \rangle$ is the VEV of scalar components of the flavon fields. Another effect of these VEVs can be the generation of a $\mu$ term.

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1 Introduction

A well-known problem of SUSY extensions of the Standard Model (SM) with superparticles at the TeV scale is the presence of new sources of flavour violation, see e.g. [1]. They are due to the couplings between SUSY particles and ordinary particles that, for generic soft SUSY breaking terms, are incompatible with the present limits on rare flavour-changing transitions. It is reasonable to expect that in the presence of a flavour symmetry, helpful to reproduce the observed pattern of fermion masses and mixing angles, the soft SUSY breaking terms are more constrained and the above problem is alleviated. We consider a globally SUSY framework where the soft SUSY breaking terms are already present at the scale relevant to flavour dynamics, so that their boundary conditions at that scale are dictated by the flavour symmetry.

An important aspect of the problem is the relative alignment in flavour space of fermion and sfermion mass terms. Focusing on the lepton sector, in models invariant under a flavour symmetry group, the lepton masses are described by a superpotential of the type

\[ w = e^c_\alpha H_d Y_{\alpha\beta}(\varphi) l_\beta + w_d(\varphi) + ... \] (1)

where \( \varphi_i \) denotes the set of chiral superfields neutral under gauge interactions, the flavons, whose scalar components break the flavour symmetry through their VEVs. The analytic functions \( Y_{\alpha\beta}(\varphi) \) and \( w_d(\varphi) \) depend only on the supermultiplets \( \varphi_i \) and they generally admit an expansion in inverse powers of some ultraviolet cutoff \( \Lambda_f \) representing the flavour scale, such that \( \langle \varphi_i \rangle / \Lambda_f \) are small parameters and one can truncate the expansion after the first few terms. In the SUSY limit the term \( w_d(\varphi) \) determines the scalar potential of the flavons and is responsible for the breaking of the flavour symmetry. The lepton mass matrix is proportional to \( Y_{\alpha\beta}(\langle \varphi \rangle) \), while slepton masses receive contributions of different types. In this note we are interested in the contribution arising in the scalar potential after eliminating the auxiliary fields of the flavon supermultiplets. Assuming a canonical Kähler potential we get

\[ V = e^c_\alpha H_d \left\langle \frac{\partial Y_{\alpha\beta}}{\partial \varphi_i} \frac{\partial w_d}{\partial \varphi_i} \right\rangle \tilde{l}_\beta + \text{h.c.} + ... \] (2)

Dots stand for additional contributions, not relevant in this context. In the SUSY limit of course the auxiliary components of the flavons vanish at the minimum, that is \( \langle \partial w_d/\partial \varphi_i \rangle = 0 \) and the above contribution vanishes, but including SUSY breaking effects in general we expect \( \langle \partial w_d/\partial \varphi_i \rangle \neq 0 \) and of order \( m_{\text{SUSY}} \times \langle \varphi_i \rangle \). Thus it is important to establish whether the combinations

\[ \left\langle \frac{\partial Y_{\alpha\beta}}{\partial \varphi_i} \frac{\partial w_d}{\partial \varphi_i} \right\rangle \] (3)

and

\[ Y_{\alpha\beta}(\langle \varphi \rangle) \] (4)

can be simultaneously diagonalized (in flavour space) or not, since a misalignment would represent a source of flavour violation. If the expansion of \( Y_{\alpha\beta}(\varphi) \) is linear in \( \varphi_i \) at the
leading order (LO)
\[ Y_{\alpha\beta}(\varphi) = Y_{\alpha\beta}^{(1)i} \frac{\varphi_i}{\Lambda_f} + \ldots \]  
(5)

where \( Y_{\alpha\beta}^{(1)i} \) are constants, a necessary condition for the alignment of the above combinations is

\[ \langle \frac{\partial w_d}{\partial \varphi_i} \rangle = \alpha \langle \varphi_i \rangle \]  
(6)

where \( \alpha \) is a constant. Note that \( \alpha \) has to be the same for all (irreducible) flavon multiplets which couple to charged leptons at this order. Whether this condition is sufficient to eliminate all relevant sources of flavour violation associated to the contribution given in eq. (2), depends in general also on the structure of the subleading terms represented by dots in the expansion in eq. (5). Another possibility to render the effect of the contribution in eq. (2) phenomenologically harmless would be to suppress the size of \( \langle \partial w_d/\partial \varphi_i \rangle \) below its generic value of \( m_{SUSY} \times \langle \varphi_i \rangle \). As it has been shown in [2] this is indeed possible in the context of supergravity. In general, the VEVs of the auxiliary components of flavon fields are expected to be of order \( m_{3/2} \times \langle \varphi_i \rangle \) [3], \( m_{3/2} \) being the gravitino mass and \( m_{3/2} \sim m_{SUSY} \), due to the contribution to the \( F \)-term proportional to the superpotential \( w \) [1]. However, as discussed in [2] this contribution can be canceled against the global SUSY contribution to the \( F \)-terms so that the VEVs of the auxiliary components are \( \ll m_{3/2} \times \langle \varphi_i \rangle \). We will comment on this possibility in Section 4.

In this note we analyze \( \langle \partial w_d/\partial \varphi_i \rangle \) in the specific case of a globally SUSY model with \( A_4 \) flavour symmetry [5,6] in which tri-bimaximal mixing [7] can be successfully generated in the lepton sector. We show explicitly through minimization of the flavon potential including generic soft SUSY breaking terms that the auxiliary components of the flavons acquire in general non-vanishing VEVs. From the explicit expressions of these VEVs, we show that there is a special case in which they vanish, corresponding to universal soft SUSY breaking terms in the flavon potential. Furthermore, we show that the possibility of completely aligned VEVs of flavons and their auxiliary components, eq. (6), has to be considered as a fine-tuning. We comment on the impact of this effect on lepton flavour violating decays in the concluding section, but we leave a detailed discussion for a separate work [8]. Additionally, we confirm the result for the VEVs of the flavons at the LO and the next-to-leading order (NLO) of [5]. We note that we perform the calculation in the limit of canonical kinetic terms, although these are in general non-canonical. We will comment on this assumption below. Furthermore, we comment on the introduction of a \( \mu \) term and also on the mass spectrum in the flavour symmetry breaking sector at the LO in the SUSY limit, which indicates the presence of two real flat directions which give rise to the undetermined complex parameter in the flavon VEVs.

\[ ^1 \text{For further examples see [4].} \]
We consider a class of SUSY models invariant under a discrete flavour symmetry group, $A_4$. In the simplest case the gauge group is the SM one. Crucial ingredients of this type of models are the following: (a) additional degrees of freedom, flavons and driving fields, which are responsible for the breaking of the flavour symmetry and which do not transform under the gauge group and (b) additional symmetries apart from $A_4$ which are necessary for achieving the vacuum alignment. In this note we assume that $A_4$ is accompanied by a cyclic symmetry $Z_3$, necessary to separate the charged lepton and the neutrino sector, a continuous $R$ symmetry $U(1)_R$, simplifying the construction of the scalar potential and a Froggatt-Nielsen symmetry $U(1)_{FN}$ giving rise to the charged lepton mass hierarchy and not relevant for the present discussion. The following flavons and driving fields are assumed in this model: a triplet $\phi_T$ giving masses to charged leptons at the LO, two singlets $\xi$ and $\bar{\xi}$ and another triplet $\phi_S$ leading to neutrino masses and driving fields $\phi_T^0$, $\phi_S^0$ and $\xi_0$. These are collected in Table 1. Flavons have a vanishing charge under $U(1)_R$, whereas driving fields are assigned the charge +2. Through this the superpotential is linear in the driving fields. The model also includes two electroweak doublets, $H_{u,d}$, responsible for electroweak symmetry breaking. In the minimization of the scalar potential we work in the limit $\langle H_{u,d} \rangle = 0$.

In the SUSY limit the $F$-terms of all fields are required to vanish and from the conditions
\[ \left\langle \frac{\partial w}{\partial \varphi_T} \right\rangle = 0, \quad \left\langle \frac{\partial w}{\partial \varphi_S} \right\rangle = 0 \quad \text{and} \quad \left\langle \frac{\partial w}{\partial \xi} \right\rangle = 0 \] (7)
we can derive
\[ \frac{\langle \varphi_T \rangle}{\Lambda_f} = (\epsilon, 0, 0) + (c' \epsilon^2, c \epsilon^2, c \epsilon^2), \quad \frac{\langle \varphi_S \rangle}{\Lambda_f} = c_0 (\epsilon, \epsilon, \epsilon) + (c_1 \epsilon^2, c_2 \epsilon^2, c_3 \epsilon^2), \]
\[ \frac{\langle \xi \rangle}{\Lambda_f} = c_a \epsilon \quad \text{and} \quad \frac{\langle \bar{\xi} \rangle}{\Lambda_f} = c_c \epsilon^2 \] (8)
where $c'$, $c$, $c_{a,b,c}$ and $c_i$ are complex numbers with absolute value of order one, depending on one undetermined parameter. The undetermined parameter indicates that there is a flat direction in the subspace $(\varphi_S, \xi)$. The parameter $\epsilon$ is defined as the VEV of the first component of the triplet $\varphi_T$, in units of the cutoff scale $\Lambda_f$. In this model $\epsilon$ is a small parameter, which controls the expansion in inverse powers of $\Lambda_f$. Its typical size is $0.007 \lesssim \epsilon \lesssim 0.05$. We have displayed the result at the NLO, up to and including $\epsilon^2$ terms. At the same time we deduce from the vanishing of the $F$-terms associated to the flavons,
\[ \left\langle \frac{\partial w}{\partial \varphi_T} \right\rangle = 0, \quad \left\langle \frac{\partial w}{\partial \varphi_S} \right\rangle = 0, \quad \left\langle \frac{\partial w}{\partial \xi} \right\rangle = 0 \quad \text{and} \quad \left\langle \frac{\partial w}{\partial \bar{\xi}} \right\rangle = 0, \] (9)
that the VEVs of the driving fields vanish in the SUSY limit, to all orders in the expansion parameter $\epsilon$. This result has been discussed in detail in [5] and we recover it as a byproduct.
of our computation. We just recall that the special pattern of VEVs in eq. (8) is the crucial ingredient to reproduce the tri-bimaximal mixing in this class of models. The explicit expressions of these VEVs are shown in the Appendix.

| Field | $\phi_T$ | $\phi_S$ | $\xi$ | $\xi$ | $\phi_0^T$ | $\phi_0^S$ | $\xi_0$ | $H_{u,d}$ |
|-------|----------|----------|-------|-------|------------|------------|--------|-----------|
| $A_4$ | 3        | 3        | 1     | 1     | 3          | 3          | 1      | 1         |
| $Z_3$ | 1        | $\omega$| $\omega$| $\omega$| 1         | $\omega$  | $\omega$| 1         |
| $U(1)_R$ | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 0 |

Table 1: Supermultiplets of the model given in [5], relevant for the breaking of the flavour symmetry $A_4 \times Z_3$, the electroweak doublets $H_u, H_d$ and their transformation properties under the symmetries $A_4 \times Z_3 \times U(1)_R$.

The presence of an undetermined VEV in the flavon sector is associated to the existence of a complex flat direction in the flavon potential. This can be checked by analyzing the mass spectrum of flavons and driving fields in the SUSY limit. In doing so we find that the massive modes have masses either proportional to the mass parameter of the superpotential $w_d$ or to the undetermined VEV. This mass spectrum receives corrections of several types: from the NLO terms in the scalar potential, from deviations of the Kähler potential from the assumed canonical form, from SUSY breaking effects and from radiative corrections. The two last classes of corrections can be important to give mass to the modes associated to the flat direction as well as to stabilize the undetermined VEV around a value of order $\epsilon \Lambda_f$.

### 3 Generic soft SUSY breaking terms

We proceed to include generic soft SUSY breaking terms, which originate from another sector of the theory, completely neutral under the action of the gauge group and under $A_4 \times Z_3 \times U(1)_{FN}$. However, without referring to a specific SUSY breaking mechanism this sector remains undetermined and thus also no further information of the form of the soft SUSY breaking terms can be used. The search for field configurations that minimize the energy density by simultaneously varying both the field content of the SUSY breaking sector and the flavon fields is beyond the scope of this work, but we present a schematic discussion of this point in Section 4, where we embed our model into a supergravity framework. In our model we mimic the new sector through a set of generic soft SUSY breaking terms obtained by promoting the coupling constants of the theory to constant superfields with non-vanishing auxiliary components [10]. For instance a superpotential coupling constant $g$ is expanded as $g + g m_{SUSY} \theta^2$. In our analysis we work under the assumption that the dynamics of the flavon sector do not appreciably affect that of the SUSY breaking sector, so that at the scale $\Lambda_f$ and below we can separately discuss the minimization of the scalar
potential with respect to the flavons and the driving fields, in the presence of fixed soft SUSY breaking terms for them.

We discuss the Kähler potential of the flavon and driving fields. We make the assumption that it is canonical even though the symmetries of the model would allow subleading corrections to the canonical form. We show that, even in the absence of these corrections, in general the alignment in eq. (6) is not realized. We do not expect that the introduction of a new set of parameters related to the non-canonical part of the Kähler potential could restore the alignment, unless a fine-tuning is enforced. The Kähler potential represents also a possible source of soft SUSY breaking terms for the flavons and the driving fields, once we promote its parameters to constant superfields. In the $D$-term for the flavons and the driving fields, collectively denoted as $\varphi_i$

$$
\int d^2\theta d^2\bar{\theta} \varphi_i Z_{ij} \varphi_j
$$

we regard the matrix $Z_{ij}$ as a constant superfield

$$
Z_{ij} = \delta_{ij} + \Gamma_{ij} m_{SUSY} \theta^2 + \Gamma_{ji} m_{SUSY} \bar{\theta}^2 + C_{ij} m_{SUSY}^2 \theta^2 \bar{\theta}^2
$$

where the first term gives rise to the kinetic terms, while the remaining ones generate soft SUSY breaking terms. It is not restrictive to set the matrix $\Gamma_{ij}$ to zero in the above decomposition. Indeed, as can be shown, after eliminating the auxiliary fields the effect of a non-vanishing $\Gamma_{ij}$ can be absorbed by redefining the matrix $C_{ij}$ and the parameters describing the soft SUSY breaking terms originating from the superpotential $w_d(\varphi)$. By setting $\Gamma_{ij}$ to zero we are left with the contribution of $C_{ij}$ which is proportional to $m_{SUSY}^2$. When calculating the VEVs of flavons and driving fields including soft SUSY breaking terms into the flavon potential, we perform a double expansion in the small symmetry breaking parameter $\epsilon$ and in the soft SUSY breaking mass $m_{SUSY}$ which is assumed to be much smaller than the cutoff scale of the theory $\Lambda_f$ and the VEVs of the flavons. As we see below, the quantities $\langle \partial w / \partial \varphi_i \rangle$ we are interested in are proportional to $m_{SUSY} \times \epsilon \Lambda_f$, at the LO in our expansion. One can show that the SUSY breaking terms coming from the Kähler potential contribute to $\langle \partial w / \partial \varphi_i \rangle$ with terms at the order $m_{SUSY}^2 \ll m_{SUSY} \times \epsilon \Lambda_f$ and therefore they can be safely neglected in our analysis. Thus we take $Z_{ij} = \delta_{ij}$ in the following.

Under the above assumptions, the relevant object of our computation is the superpotential of the theory and its dependence upon the flavons and the driving fields. From this we subsequently derive the soft SUSY breaking terms by promoting the coupling constants in the superpotential to superfields with constant $\theta^2$ components following [10]. The superpotential $w_d$ of flavons and driving fields at the NLO level, generated by including terms which are suppressed by one power of the cutoff scale $\Lambda_f$, has already been calculated and discussed in detail [5]. In order to establish our notation we repeat the results found in [5]. The superpotential can be expanded in the parameter $\epsilon$

$$
w_d = w_d^{(0)} + w_d^{(1)} + ...$$

5
At the LO the superpotential $w_d$ is of the form \(^2\)

\[
  w_d^{(0)} = M(\varphi^T_0 \varphi_T) + g_0(\varphi^T_0 \varphi_T \varphi_T) \\
  + g_1(\varphi^S_0 \varphi^S \varphi_S) + g_2(\varphi^S_0 \varphi_S) + g_3(\varphi_0 \varphi^S_S) + g_4\xi_0 \xi^2 + g_5\xi_0 \xi \xi + g_6\xi_0 \xi^2 
\]  

(13)

where the mass parameter $M$ and the coupling constants $g_i$ are expanded as superfields

\[
  M = M + a_m M_{SUSY} \theta^2 \quad \text{and} \quad g_i = g_i + g_{i} M_{SUSY} \theta^2 \quad (i = 0, ..., 6) 
\]  

(14)

and $a_m$, $g_i$ and $g_{i}$ are complex numbers with absolute value of order one. $(\ldots)$ denotes in eq. (13) the contraction to an $A_4$ invariant. From eq. (13) one can derive the superpotential given in [5] at the LO. At the NLO in the expansion parameter $\epsilon$ all non-renormalizable terms which are suppressed by one power of the cutoff scale $\Lambda_f$ and respect all symmetries of the model are included (called $\Delta w_d$ in [5])

\[
  w_d^{(1)} = \frac{1}{\Lambda_f} \left( \sum_{k=3}^{13} t_k I_k^T + \sum_{k=1}^{12} s_k I_k^S + \sum_{k=1}^{3} x_k I_k^X \right) 
\]  

(15)

where $\{I_k^T, I_k^S, I_k^X\}$ represent a basis of independent quartic invariants

\[
  I_3^T = (\varphi^T_0 \varphi_T)(\varphi_T \varphi_T) \\
  I_4^T = (\varphi^T_0 \varphi_T)'(\varphi_T \varphi_T)' \\
  I_5^T = (\varphi^T_0 \varphi_T)''(\varphi_T \varphi_T)'' \\
  I_6^T = (\varphi^T_0 \varphi_T)'(\varphi_T \varphi_T)' \\
  I_7^T = (\varphi^T_0 \varphi_T)'(\varphi_T \varphi_T)' \\
  I_8^T = (\varphi^T_0 \varphi_T)'(\varphi_T \varphi_T)' \\
  I_9^T = (\varphi^T_0 \varphi_T)(\varphi_T \varphi_T) \\
  I_{10}^T = (\varphi^T_0 \varphi_T)(\varphi_T \varphi_T) \\
  I_{11}^T = (\varphi^T_0 \varphi_T)(\varphi_T \varphi_T) \\
  I_{12}^T = (\varphi^T_0 \varphi_T)(\varphi_T \varphi_T) \\
  I_{13}^T = (\varphi^T_0 \varphi_T)(\varphi_T \varphi_T) \\
  I_1^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_2^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_3^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_4^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_5^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_6^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_7^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_8^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_9^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_{10}^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_{11}^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_{12}^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_{13}^S = (\varphi^S_0 \varphi^S_T)(\varphi^S_T \varphi^S_T) \\
  I_1^X = \xi_0 (\varphi_T \varphi^S_T \varphi^S_T) \\
  I_2^X = \xi_0 (\varphi_T \varphi^S_T \varphi^S_T) \\
  I_3^X = \xi_0 (\varphi_T \varphi^S_T \varphi^S_T) \\
\]  

(16)

(17)

(18)

and the parameters $t_k$, $s_k$, and $x_k$ are expanded in terms of superfields with constant $\theta^2$ component

\[
  t_k = t_k + t_k M_{SUSY} \theta^2, \quad s_k = s_k + s_k M_{SUSY} \theta^2 \quad \text{and} \quad x_k = x_k + x_k M_{SUSY} \theta^2 . 
\]  

(19)

By $(\ldots)'$ and $(\ldots)''$ the contraction to a non-trivial $A_4$ singlet $1'$ and $1''$ is denoted, respectively. For the product of two triplets $(\ldots)_{A, S}$ denotes the (anti-) symmetric triplet in this product. We agree on the result found in [5] (up to misprints).

\(^2\)We change the notation slightly compared to the original version found in [5] by renaming $g$ into $g_0.$
With the information given above we find for the potential \( V \) of flavons and driving fields that it is of the following form

\[
V = V_{\text{SUSY}} + V_{\text{soft}}
\]

with

\[
V_{\text{SUSY}} = \sum_i \left| \frac{\partial w}{\partial \varphi_i} \right|^2
\]

with \( \varphi_i \) being in the list \( \{ \varphi_T, \varphi_S, \xi, \tilde{\xi}, \varphi_0^T, \varphi_S^0, \xi^0 \} \) of all flavons and driving fields. The relevant part of \( V_{\text{soft}} \) is given by

\[
V_{\text{soft}} = \sum_{k=3}^{13} b_k I_k^T + \sum_{k=1}^{12} g_k I_k^S + \sum_{k=1}^{3} x_k I_k^X + \text{h.c.}
\]

In the course of calculation it is convenient to make the following re-definition of the couplings \( g_3, g_4 \) and \( g_3, g_4 \) in the potential [5]

\[
g_3 \equiv 3 \tilde{g}_3^2, \quad g_4 \equiv -\tilde{g}_4^2, \quad g_3 \equiv 3 \tilde{g}_3^2 \quad \text{and} \quad g_4 \equiv -\tilde{g}_4^2.
\]

We calculated all contributions to the flavon VEVs and VEVs of the driving fields up to the NLO, that is \( O(\epsilon^2) \) for the flavons and \( O(\epsilon) \) for the driving fields. Concerning the expansion in \( m_{\text{SUSY}} \), all flavon VEVs are given at order \( O(m_{\text{SUSY}}^0) \) and all VEVs of driving fields are at the LO \( O(m_{\text{SUSY}}) \). For the VEVs of the flavons we confirm the result given in [5], which we list in the Appendix for completeness. This result can be cast into the form as given in eq. [8] by properly defining the order one parameters. Apart from the specific structure which is revealed in the LO result, the NLO results only have as special property that the shifted vacua of \( \langle \varphi_T^2 \rangle \) and \( \langle \varphi_T^3 \rangle \) coincide, as already noted in [5]. As we can see from the explicit expression given in the Appendix, the VEV of one combination of the flavons \( \varphi_S^0 \) and \( \xi \) is undetermined, indicating the existence of a flat direction in the potential. Furthermore, note that the vacua of \( \varphi_T^2, \varphi_T^3 \) and \( \tilde{\xi} \) only arise at the NLO level.

Similarly, we find for the vacua of the driving fields

\[
\begin{align*}
\frac{\langle \varphi_0^T \rangle}{m_{\text{SUSY}}} &= c_0(1, 0, 0) + (c_0^0 \epsilon, c_0^0 \epsilon, c_0^0 \epsilon), \\
\frac{\langle \varphi_0^S \rangle}{m_{\text{SUSY}}} &= c_0^0(1, 1, 1) + (c_1^0 \epsilon, c_2^0 \epsilon, c_3^0 \epsilon), \\
\frac{\langle \xi_0 \rangle}{m_{\text{SUSY}}} &= c_0^a + c_4^0 \epsilon.
\end{align*}
\]

The explicit expressions of the coefficients \( c_0^0, c_0^0 \) and \( c_0^a \) can be deduced from the
explicit expression of the VEVs. For instance, the VEV of $\varphi_{01}^T$ is given by

$$\langle \varphi_{01}^T \rangle = m_{SUSY} \left[ Y + \left( \frac{\tilde{g}_4}{2g_0g_3} \left( 3t_{11} + \frac{\tilde{g}_3^2}{g_3^2} (t_6 + t_7 + t_8) \right) Y + \frac{3}{2g_0} \left( 3\frac{\varphi_0}{g_0} t_{11} - \xi_{24} \right) \right) + \frac{3}{8g_6g_3^2g_4} (t_{11}(3\tilde{g}_3^2 - 2\tilde{g}_1^2) + 3\tilde{g}_4^2(t_6 + t_7 + t_8) Z) \left( \frac{u^3}{v_T^2 \Lambda_f} \right) - \frac{9}{2g_0} (t_3 Y + \frac{1}{2g_0})(\xi_{3} - \frac{\varphi_0}{g_0} t_{3}) \left( \frac{v_T}{\Lambda_f} \right) \right]$$

where

$$Y = \frac{3}{2g_0^2} (a_{in}g_0 - g_0)$$

and

$$Z = \frac{a_3^2 g_2^2 - a_5^2 g_4^2}{3g_3^2 + g_4^2}.$$

The full expressions of the VEVs of all driving fields are listed in the Appendix. Given this result one can eventually calculate the vacuum of the auxiliary components of the flavon supermultiplets and can find the following structure

$$\frac{1}{\Lambda_f} \left< \frac{\partial w}{\partial \varphi_T} \right> = \zeta_T m_{SUSY} \{ (\epsilon, 0, 0) + (c_F^T \epsilon^2, c_F^T \epsilon^2, c_F^T \epsilon^2) \}$$

$$\frac{1}{\Lambda_f} \left< \frac{\partial w}{\partial \varphi_S} \right> = \zeta_S m_{SUSY} \{ (\epsilon, \epsilon, 0) + (c_F^S \epsilon^2, c_F^S \epsilon^2, c_F^S \epsilon^2) \}$$

$$\frac{1}{\Lambda_f} \left< \frac{\partial w}{\partial \xi} \right> = \zeta_T m_{SUSY} \{ \epsilon + \epsilon^2 \}$$

$$\frac{1}{\Lambda_f} \left< \frac{\partial w}{\partial \xi} \right> = \zeta_T m_{SUSY} \{ \epsilon \}$$

where the LO is proportional to $m_{SUSY} \times \epsilon \Lambda_f$. The coefficients $\zeta_T, \zeta_S, \zeta_T, c_F^T, c_F^T, c_F^S$ and $c_T$ can be computed from eqs. (8) and (24) and the content of the Appendix.

We see that flavon VEVs and the VEVs of the corresponding auxiliary components have a similar structure. Indeed, at the LO in the $\epsilon$ expansion, each flavon VEV has exactly the same orientation in flavon space as the VEV of the corresponding auxiliary component. In the specific model, we consider, only the flavons $\varphi_T$ contribute to the charged lepton mass matrix at the LO as well as the NLO, for details see [5]. Thus, the condition given in eq. (6) applies and the RL slepton mass terms of eq. (2) are diagonal in the basis in which the charged lepton mass matrix is diagonal. At the NLO, however, this does not hold anymore. At the NLO and for the $A_4$ triplets the condition in eq.(6) would require a special relation between the two sets of coefficients $(c_F^T, c_F^T, c_F^S)$ and $(c, c', c_1/c_6)$. Such a relation is not natural in our model, since the coefficients $(c, c', c_1/c_6)$ only depend on the superpotential parameters that remain in the SUSY limit, whereas $(c_F^T, c_F^T, c_F^S)$ depend on the full set of parameters, including those that describe the soft SUSY breaking terms. In the case studied here realizing such a relation would be even sufficient in order to eliminate all relevant sources of flavour violation, associated to this type of contribution, because the
subleading contributions in the Yukawa couplings do not induce further sources of flavour violation. We found an interesting case where the condition in eq. (6) is trivially realized, i.e. with \( \alpha = 0 \). This is the limit of universal soft SUSY breaking parameters, which is defined as

\[
\begin{align*}
\alpha_m &= \beta , \quad g_i = \beta g_i , \\
\kappa_k &= \beta t_k , \quad s_k &= \beta s_k \quad \text{and} \quad x_k = \beta x_k .
\end{align*}
\]

(28)

The two re-defined parameters, see eq. (23), are in the universal limit

\[
\tilde{g}_3 = \sqrt{\beta} \tilde{g}_3 \quad \text{and} \quad \tilde{g}_4 = \sqrt{\beta} \tilde{g}_4 .
\]

(29)

\( \beta \) is a complex number with absolute value of order one. We see from eq. (26) that the parameters \( Y \) and \( Z \) vanish in this limit, as well as all other terms on the right-hand side of eq. (25). Thus, \( \langle \varphi^T_{01} \rangle \) becomes zero. From the expressions of the VEVs in the Appendix we can see that also all other VEVs of the driving fields are zero and as a consequence also the VEVs of the auxiliary components of the flavons vanish. The case of universal soft SUSY breaking parameters is simple and reduces the number of parameters but, without further specification of the mechanism of SUSY breaking, it is not a natural result in our model.

We notice that non-vanishing VEVs for the driving fields contribute to the \( \mu \) term of the two Higgs doublets \( H_u \) and \( H_d \). The lowest order operator of this type in the superpotential \( w \), allowed by all symmetries of the model, is

\[
(\varphi^T_{01}\varphi^T) H_u H_d / \Lambda_f .
\]

(30)

It generates a contribution to the \( \mu \) term of the order of \( m_{\text{SUSY}} \times \epsilon \). The size of such a term is expected to be \( \lesssim 50 \text{ GeV} \) for \( m_{\text{SUSY}} \sim \mathcal{O}(1 \text{ TeV}) \).

4 Relation to supergravity

In this section we briefly discuss the constraints that can arise by embedding our setup into the supergravity formalism. In supergravity SUSY is realized as local symmetry and its breaking is always spontaneous. The consequences for the physics at low energies depend on the specific mechanism of SUSY breaking. To make contact with our previous discussion, where soft breaking terms are already present at the scale relevant to flavour dynamics, we assume that SUSY is broken at a large scale in a hidden sector and its breaking is transmitted to the observable one via gravitational effects.

The hidden sector of the theory contains a gauge singlet chiral supermultiplet \( h \) (there can be more, but this does not change our discussion) and the observable sector describes
both, flavons, $\varphi_i$, and matter, $y_a = (t, e^c, H_d, ...)$, supermultiplets that we collectively denote by $z_I$. The superfield $h$ can develop a VEV of the order of the Planck scale $M_{Pl}$, whereas the supermultiplets $z_I$ develop VEVs much smaller than $M_{Pl}$, or no VEV at all. The interactions between hidden and observable sectors take place via the dimensionless combinations $h/M_{Pl}$. We are interested in the flat limit of the supergravity formalism, in which $M_{Pl}$ is taken to infinity and the gravitino mass $m_{3/2}$ is kept fixed. For a compact explicit discussion we restrict ourselves to the case of canonical Kähler potential

$$K = |h|^2 + |z_I|^2$$

(31)

and we parametrize the superpotential $\hat{w}$ as

$$\hat{w} = \hat{w}_h(h) + \hat{w}_d(h, \varphi) + \hat{w}_m(h, \varphi, y)$$

(32)

where $\hat{w}_m$ is a polynomial of third degree in the matter fields $y_a$, as the first term on the right-hand side of eq. (1). Neglecting gauge interactions, the scalar potential of the theory is given by

$$V = e^{K/M_{Pl}^2} \left[ |\hat{F}_h|^2 + |\hat{F}_{z_I}|^2 - 3 |\hat{w}|^2/M_{Pl}^2 \right]$$

(33)

where

$$\hat{F}_h = \partial \hat{w}/\partial h + h \hat{w} M_{Pl}^2, \quad \hat{F}_{z_I} = \partial \hat{w}/\partial z_I + z_I \hat{w} M_{Pl}^2$$

(34)

In supergravity SUSY is broken by the non-vanishing VEV of some auxiliary field, $F_h$, $F_{z_I}$. Assuming a vanishing cosmological constant, when SUSY is broken the gravitino acquires a mass

$$m_{3/2} = |\langle \hat{w} \rangle| M_{Pl}^2$$

(35)

where we have defined a rescaled superpotential

$$w = \langle e^{K/2M_{Pl}^2} \rangle \hat{w}$$

(36)

We will also make use of the rescaled quantities

$$F_h = \langle e^{K/2M_{Pl}^2} \rangle \hat{F}_h, \quad F_{\varphi_i} = \langle e^{K/2M_{Pl}^2} \rangle \hat{F}_{\varphi_i}$$

(37)

Assuming an appropriate asymptotic behaviour of $\hat{w}$, it is possible to take the flat limit of $V$ in eq. (33) and derive the soft SUSY breaking terms for the matter fields $y$ [11]. These include

---

4We have conventionally chosen to call the flavons $\varphi$ observable fields, but this does not exclude the possibility that their $F$-terms develop sizable VEVs. This is precisely the point that we would like to discuss in this section. Since $\varphi$ are gauge singlets that couple to the matter fields $y$ (except for right-handed neutrinos) only through non-renormalizable interaction terms and develop VEVs not much smaller than the cutoff scale of the theory, we could have included them into the hidden sector as well.
• Universal soft scalar masses
  \[ m_{3/2}^2 |y_a|^2 \].

• Additional bilinear and trilinear soft terms of three different types
  \[ m_{3/2} \, w_m \ , \ m_{3/2} \frac{\partial w_m}{\partial y_a} \ , \ m_{3/2} \frac{\partial w_m}{\partial h} \].

• A contribution from the $F$-terms of the flavon supermultiplets
  \[ \langle F_{\varphi_i} \rangle \frac{\partial w_m}{\partial \varphi_i} = \left( \frac{\partial w_m}{\partial \varphi_i} \right) \frac{\partial w_m}{\partial h} + m_{3/2} \langle \varphi_i \rangle \frac{\partial w_m}{\partial \varphi_i} \].

In this paper we have analyzed the first contribution on the right-hand side of the previous equation, in the context of global SUSY. In supergravity it is more natural to discuss the sum of the two contributions as a whole, since they both arise from the VEV of the $F$-terms of the flavons, $F_{\varphi_i}$. Moreover, it has been observed that the two contributions can approximately cancel against each other in a class of supergravity models and then the VEVs of $F_{\varphi_i}$ scale as $m_{3/2}^2 (p \geq 2)$ instead of being proportional to $m_{3/2}$ [2].

Setting to zero from the beginning the matter fields $y_a$, the minima of $V$ with vanishing cosmological constant for the remaining fields should obey \[ \langle V \rangle = 0, \langle \partial V / \partial h \rangle = 0 \text{ and } \langle \partial V / \partial \varphi_i \rangle = 0. \] These are equivalent to

\[ |F_h|^2 + |F_{\varphi_i}|^2 - 3m_{3/2}^2 M_{Pl}^2 = 0 \],

\[ \left( \frac{\partial^2 w}{\partial h^2} + \frac{\bar{h}}{M_{Pl}^2} \frac{\partial w}{\partial h} \right) F_h + \left( F_h - 3 \frac{\partial w}{\partial h} \right) m_{3/2} + \left( \frac{\partial^2 w}{\partial h \partial \varphi_i} + \frac{\bar{\varphi}_i}{M_{Pl}^2} \frac{\partial w}{\partial \varphi_i} \right) F_{\varphi_i} = 0 \],

\[ \left( \frac{\partial^2 w}{\partial \varphi_i \partial h} + \frac{\bar{h}}{M_{Pl}^2} \frac{\partial w}{\partial \varphi_i} \right) F_h + \left( F_{\varphi_i} - 3 \frac{\partial w}{\partial \varphi_i} \right) m_{3/2} + \left( \frac{\partial^2 w}{\partial \varphi_i \partial \varphi_j} + \frac{\bar{\varphi}_j}{M_{Pl}^2} \frac{\partial w}{\partial \varphi_i} \right) F_{\varphi_j} = 0 \],

where we have used the rescaled functions of eqs. (36) and (37). Our aim is to analyze the behaviour of the previous equations in the limit of small $m_{3/2}$, by performing a series expansion in powers of $m_{3/2}$. From eq. (41) we see that the $F$-terms should vanish when we take $m_{3/2}$ to zero and some of them should scale as $m_{3/2}$ at the minimum: we assume that $F_h \propto m_{3/2}$. We also assume that at the minimum of $V$, $h$ as well as some of the fields $\varphi_i$ tend to some non-vanishing constant in the limit of vanishing $m_{3/2}$. In realistic models $h$ is of order $M_{Pl}$ at the minimum, and the flavon fields have a VEV much larger than $m_{3/2}$, though smaller than $M_{Pl}$. We now impose that also the VEV of $F_{\varphi_i}$ scales as $m_{3/2}$ in the limit of small gravitino mass

\[ F_{\varphi_i} \propto m_{3/2} \].

We analyze the conditions under which such behaviour is consistent with eqs. (42, 43). From eq. (44) we see that we also have $\partial w / \partial h \propto m_{3/2}$ and $\partial w / \partial \varphi_i \propto m_{3/2}$ at maximum. Then it is easy to see that all the terms of eqs. (42, 43) not containing second derivatives of $w$ scale at least as $m_{3/2}^2$. The equations can be satisfied if either of the following two cases occurs.
I) In the first case we have

\[ \frac{\partial^2 w}{\partial \varphi_i \partial h} \propto m_{3/2}^p \quad p \geq 1 \ , \tag{45} \]

(including the case \( \frac{\partial^2 w}{\partial \varphi_i \partial h} = 0 \)). Then eq. (42) can be satisfied, for instance, by \( \frac{\partial^2 w}{\partial h^2} = 0 \) as in the Polonyi model and eq. (43) requires

\[ \frac{\partial^2 w}{\partial \varphi_i \partial \varphi_j} \propto m_{3/2}^p \quad p \geq 1 \ . \tag{46} \]

At first sight it may seem surprising or unnatural that at the minimum \( \frac{\partial^2 w}{\partial \varphi_i \partial \varphi_j} \) vanish in the limit of zero \( m_{3/2} \), since this matrix controls the flavon masses and its dynamics is expected to be related to a much higher scale. This tuning is however no worse than that occurring in the Polonyi model, where \( F_h \) is of order \( m_{3/2} M_{Pl} \) (instead of \( M_{Pl}^2 \)) despite the VEV for \( h \) of order \( M_{Pl} \). In the Polonyi model this is obtained by tuning by hand the overall scale of the superpotential, \( \hat{w}_h = m M_{Pl} (h + b M_{Pl}) \) with \( m \) of order \( m_{3/2} \). Therefore a minimum with \( F_{\varphi_i} \) proportional to \( m_{3/2} \) can occur, if the superpotential is of the type

\[ \hat{w} = m \left( M_{Pl} h + b M_{Pl}^2 + a_{ij} \varphi_i \varphi_j + \frac{g_{ijk}}{M_{Pl}} \varphi_i \varphi_j \varphi_k \right) \tag{47} \]

and we have checked in an explicit example that \( \langle F_{\varphi} \rangle \) can be of the order \( m_{3/2} \times \langle \varphi_i \rangle \).

Notice that this case also includes the possibility where \( \frac{\partial^2 w}{\partial \varphi_i \partial h} \) and \( \frac{\partial^2 w}{\partial \varphi_i \partial \varphi_j} \) vanish at the minimum, which can occur if the superpotential does not depend on the flavon fields \( \varphi_i \), or depends on them in combination with fields having vanishing VEVs. Eq. (42) can then be satisfied by a superpotential linear in the hidden sector field, as in the Polonyi model.

II) In the second case

\[ \frac{\partial^2 w}{\partial \varphi_i \partial h} \neq 0 \quad \text{for vanishing } m_{3/2} \ . \tag{48} \]

Barring cancellations, in this case also the remaining second derivatives, \( \frac{\partial^2 w}{\partial h^2} \) and \( \frac{\partial^2 w}{\partial \varphi_i \partial \varphi_j} \), should be non-vanishing when \( m_{3/2} \) tends to zero and the terms containing second derivatives should cancel against each other in the equations. When this happens, the couplings between the supermultiplets \( h \) and \( \varphi_i \) are large and it would be more appropriate to include \( \varphi_i \) into the hidden sector. We do not know examples of this type among the most common superpotentials considered in supergravity, but we think that it is not possible to discard a priori this possibility.

We also observe that, if there is no coupling between the hidden sector and the flavon fields, \( \frac{\partial^2 w}{\partial \varphi_i \partial h} = 0 \), and if \( \frac{\partial^2 w}{\partial \varphi_i \partial \varphi_j} \) is non-vanishing in the limit of zero \( m_{3/2} \), then eq. (43) can only be solved if \( F_{\varphi_i} \propto m_{3/2}^p \), with \( p \geq 2 \), i.e. the two contributions to \( F_{\varphi_i} \)
have to cancel up to terms of order $m_{3/2}^p$ \(^5\). In this case, the contribution shown in eq. (\(\text{eq. (40)}\)), which is the subject of the present work, is harmless, as has been noticed in ref. [2]. It is quite interesting that in this class of models the suppression of this contribution occurs dynamically through the minimization of the scalar potential of the underlying supergravity theory.

The framework considered in this section is not the most general one. We could also allow for a non-canonical Kähler potential, a possibility that leads to soft SUSY breaking terms and minimum conditions more general than those analyzed here. In the setup of global SUSY which we have analyzed in the previous sections, we wished to contemplate the most general possibility, without making any assumption about the origin and the specific pattern of the SUSY breaking terms. Even if cancellations in the VEVs of the $F$-terms for the flavon fields can occur and do occur in specific cases, as we have seen these cancellations are not model-independent features of the underlying supergravity theory and the general parametrization of our global framework is more appropriate to cover the most general possibility.

### 5 Summary and Conclusions

In this note we have studied the effect of generic soft SUSY breaking terms on the vacuum alignment in a globally SUSY model invariant under the flavour symmetry $A_4 \times Z_3 \times U(1)_{FN} \times U(1)_R$. In such a model, lepton masses and mixing angles directly depend on how the flavour symmetry is broken by the flavon fields. In the SUSY limit the minimization of the scalar potential of the theory leads to a special pattern of flavon VEVs $\langle \phi \rangle$, that reproduces the nearly tri-bimaximal mixing observed in neutrino oscillations. The question addressed here is how this vacuum structure is modified when generic soft SUSY breaking terms are added to a globally SUSY theory. At first sight the impact of such terms would seem negligible, due to the large separation between the flavour symmetry breaking scale, $\langle \varphi \rangle \approx 10^{14}$ GeV and $m_{\text{SUSY}} \approx 1$ TeV. Indeed the corrections to the VEVs of the flavon fields induced by the soft SUSY breaking terms are of order $m_{\text{SUSY}}$ and thus completely irrelevant as far as lepton masses and mixing angles are concerned.

Even if lepton masses and mixings are unaffected by the soft SUSY breaking terms, there are important corrections to the VEVs of the auxiliary components of the flavon supermultiplets. These are zero in the SUSY limit, and become non-vanishing when soft SUSY breaking terms are included. By an explicit computation we find that $\langle \partial w/\partial \varphi \rangle$ are of order $m_{\text{SUSY}} \times \langle \varphi \rangle$. These VEVs give rise to a contribution to the RL slepton masses of order $m_{i} m_{\text{SUSY}}$, $m_i$, $i = e, \mu, \tau$, denoting the lepton masses. The important feature of this contribution is its orientation in flavour space, which is completely determined at the LO

\(^5\) Notice that this case is different from the one included under condition I) above, where the vanishing of $\partial^2 w/\partial \varphi_i \partial h$ is considered as possible solution, but $\partial^2 w/\partial \varphi_i \partial \varphi_j$ is assumed to fulfill $\partial^2 w/\partial \varphi_i \partial \varphi_j \propto m_{3/2}^p (p \geq 1)$.
by the relative orientation of $\langle \partial w/\partial \phi \rangle$ with respect to $\langle \phi \rangle$. A misalignment is a source of lepton flavour violation, since it gives rise to non-diagonal terms in the RL slepton mass matrix in the basis in which the charged lepton mass matrix is diagonal. In our model we can compute both $\langle \partial w/\partial \phi \rangle$ and $\langle \phi \rangle$ in a systematic expansion in the parameter $\epsilon$, the scale at which the flavour symmetry is broken measured in units of the cutoff scale $\Lambda f$. At the LO in this expansion, for each irreducible multiplet $\phi$ of the flavour symmetry we find $\langle \partial w/\partial \phi \rangle \propto \langle \phi \rangle$. At the NLO however, by including terms of $O(\epsilon^2 \Lambda f)$ in $\langle \phi \rangle$ and terms of $O(\epsilon^2 m_{SUSY} \Lambda f)$ in $\langle \partial w/\partial \phi \rangle$, such a proportionality does not hold any longer. The VEVs of the flavons and of their auxiliary components are misaligned and there are non-diagonal contributions to the RL slepton mass matrix in the flavour basis.

By inspecting the explicit expressions of $\langle \partial w/\partial \phi \rangle$, we have found a special case in which they vanish. This occurs when the soft SUSY breaking terms of the flavons are universal, that is they have the same form, up to an overall proportionality constant, as the superpotential terms. In our model however this special case is not a generic result and thus has to be considered as fine-tuning, as long as the mechanism of SUSY breaking is not specified. As has been discussed in [3] in the context of supergravity, the VEVs of the $F$-terms of the flavons are also generically expected to be of order $m_{3/2} \times \langle \phi_i \rangle \sim m_{SUSY} \times \langle \phi_i \rangle$ and thus give a relevant contribution to the sfermion soft masses of RL type. However, these terms can be suppressed [2] so that they are $\ll m_{3/2} \times \langle \phi_i \rangle$ through a dynamical mechanism in which the generic supergravity contribution is canceled against the globally SUSY one. In this work we have recovered in a model-independent way the conditions under which such a dynamical suppression occurs by performing a series expansion of the relevant minimum conditions in powers of $m_{3/2}$; for an explicit example see [2].

One relevant consequence of our result concerns processes in which lepton flavour is violated, especially $\mu \rightarrow e\gamma$, whose branching ratio is severely constrained. Actually it is possible to show that the amplitude for $l_i \rightarrow l_j\gamma$ is dominated regarding the expansion in the symmetry breaking parameter $\epsilon$ by the above mechanism, through a one-loop diagram with the insertion of the element $ij$ of the RL block of the slepton mass matrix. In particular the normalized branching ratios $R_{ij}$ for the lepton flavour violating transitions $l_i \rightarrow l_j\gamma$

$$R_{ij} = \frac{BR(l_i \rightarrow l_j\gamma)}{BR(l_i \rightarrow l_j\nu_i\bar{\nu}_j)} \quad ,$$

have the following asymptotic behaviour for small $\epsilon$

$$R_{ij} = \frac{48\pi^3 \alpha_{em}}{G_f^2 M_{new}^4} |w_{ij}|^2 \epsilon^2$$

where $\alpha_{em}$ is the fine structure constant, $G_f$ is the Fermi constant, $w_{ij}$ are dimensionless parameters of order one and $M_{new} = (4\pi/g)m_{SUSY}$ with $g$ being the $SU(2)_L$ gauge coupling constant. When the contribution to the RL slepton masses from the $F$-term of the flavon

\[ \text{Subleading terms present in the Yukawa couplings might be an additional source of flavour violation associated to this type of contribution to the RL slepton masses.} \]
multiplets is absent or negligible, a cancellation takes place in the amplitudes for lepton flavour violating transitions and $R_{ij}$ scale as

$$R_{ij} = \frac{48\pi^3\alpha_{em}}{G_F^2M_{new}^4} \left[ |w_{ij}^{(1)}\epsilon|^2 + \frac{m_j^2}{m_i^2}|w_{ij}^{(2)}\epsilon|^2 \right]$$  \hspace{1cm} (51)

where $w_{ij}^{(1,2)}$ are dimensionless quantities of order one. Given the smallness of the symmetry breaking parameter $\epsilon$, the branching ratios in eq. (51) are clearly much more suppressed than those in eq. (50). This shows the potential relevance of the effect analyzed in this paper. A detailed calculation of the branching ratios for radiative charged lepton decays will be presented elsewhere [8].

In this note we discussed a model in the framework of global SUSY and we carried out an explicit computation of the effect up to the NLO in the parameter $\epsilon$ showing, without specifying the SUSY breaking mechanism, the relevance of the $F$-terms of the flavon fields in model building. In our specific framework, a contribution to the $\mu$ term is generated as well due to the VEVs of the driving fields which are of the size of the generic soft SUSY breaking scale $m_{SUSY}$.

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A NLO vacua for flavons and driving fields

Here we list the explicit expressions of the VEVs of flavon and driving fields at the NLO in the expansion parameter $\epsilon$. Concerning the expansion in the soft SUSY breaking parameter $m_{\text{SUSY}}$, the results are given at the LO, namely at $O(m_{\text{SUSY}}^0)$ for the flavons and at $O(m_{\text{SUSY}})$ for the driving fields. For the flavons we find

$$\langle \varphi_{T1} \rangle = -\frac{3M}{2g_9} - \left[ \frac{3t_3}{2g_9} \left( v_T^2 \Lambda_f^2 \right) + \frac{\tilde{g}_4}{2g_9 g_3} \left( t_{11} + \frac{\tilde{g}_4^2}{3g_3^2} (t_6 + t_7 + t_8) \right) \left( \frac{u^3}{v_T \Lambda_f^2} \right) \right]$$

$$\langle \varphi_{T2} \rangle = -\frac{g_4}{4g_9 g_3} \left[ t_{11} + \frac{g_4^2}{3g_3^2} (t_6 + t_7 + t_8) \right] \left( \frac{u^3}{v_T \Lambda_f} \right)$$

$$\langle \varphi_{T3} \rangle = -\frac{g_4}{4g_9 g_3} \left[ t_{11} + \frac{g_4^2}{3g_3^2} (t_6 + t_7 + t_8) \right] \left( \frac{u^3}{v_T \Lambda_f} \right)$$

$$\langle \varphi_{S1} \rangle = -\frac{\tilde{g}_4 u}{3g_3} + \left[ \frac{1}{g_4} \left( \frac{g_4}{g_1} + \frac{g_5}{6g_2 g_4} \right) s_{10} - \frac{g_4}{6g_1 g_3} \right] (2s_3 - s_4 - s_5)$$

$$+ \frac{g_1}{18g_2 g_3^2} (s_3 + s_4 + s_5) - \frac{s_6}{3g_4} - \frac{x_2}{18g_3^2} \left( \frac{uv_T}{\Lambda_f} \right)$$

$$\langle \varphi_{S2} \rangle = -\frac{\tilde{g}_4 u}{3g_3} + \left[ \frac{1}{g_4} \left( \frac{g_3}{g_1} + \frac{g_5}{6g_2 g_4} \right) s_{10} - \frac{g_4}{6g_1 g_3} \right] (2s_4 - s_3 - s_5)$$

$$+ \frac{g_5}{18g_2 g_3^2} (s_3 + s_4 + s_5) + \frac{s_6}{6g_1} + \frac{s_8}{4g_1} - \frac{x_2}{18g_3^2} \left( \frac{uv_T}{\Lambda_f} \right)$$

$$\langle \varphi_{S3} \rangle = -\frac{\tilde{g}_4 u}{3g_3} + \left[ \frac{1}{g_4} \left( \frac{g_3}{g_1} + \frac{g_5}{6g_2 g_4} \right) s_{10} - \frac{g_4}{6g_1 g_3} \right] (2s_5 - s_3 - s_4)$$

$$+ \frac{g_5}{18g_2 g_3^2} (s_3 + s_4 + s_5) + \frac{s_6}{6g_1} - \frac{s_8}{4g_1} - \frac{x_2}{18g_3^2} \left( \frac{uv_T}{\Lambda_f} \right)$$

$$\langle \xi \rangle = u$$

$$\langle \tilde{\xi} \rangle = -\left[ \frac{\tilde{g}_3 s_{10}}{g_2 g_4} + \frac{\tilde{g}_4}{3g_3 g_3} (s_3 + s_4 + s_5) \right] \left( \frac{uv_T}{\Lambda_f} \right)$$

$v_T$ is defined as

$$v_T = -\frac{3M}{2g_9},$$

equal to the VEV of the first component of the flavon field $\varphi_T$ at the LO. Note that the parameter $u$ is undetermined in the VEVs.
With the parameters $Y$ and $Z$, as given in eq.\((20)\), we get for the VEVs of the driving fields

\[
\langle \varphi_{01}^T \rangle = m_{\text{SUSY}} \left[ Y + \left( \frac{\bar{g}_4}{2g_0 g_3} \left\{ \left( 3t_{11} + \frac{\bar{g}_4}{g_3} (t_6 + t_7 + t_8) \right) Y + \frac{1}{2g_0} \left( 3 \frac{g_6}{\bar{g}_4} t_{11} - \bar{t}_{31} \right) \right. \right. \right.
\]
\[
+ \frac{\bar{g}_4}{g_3} \frac{g_6}{g_0} (t_6 + t_7 + t_8) \left( \bar{t}_6 + \bar{t}_7 + \bar{t}_8 \right) \left) \right\} + \frac{3}{8g_0^2 g_3^2 g_4^2} (t_{11} (3g_3^2 - 2g_4^2) + \right. \right. \right.
\]
\[
+ 3g_4^2 (t_6 + t_7 + t_8) \right) Z \left( \frac{u^3}{v_T \Lambda_f} \right) - \frac{9}{2g_0} \left( t_3 Y + \frac{1}{2g_0} \left( \bar{t}_3 - \frac{g_6}{g_0} \right) \right) \left( \frac{v_T}{\Lambda_f} \right)
\]
\[
- \frac{g_5}{8g_0 g_2 g_3 g_4^3} \left( 3s_{10} + \frac{\bar{g}_4}{g_3} (s_3 + s_4 + s_5 - \frac{g_2}{g_5} x_2) \right) \left( \frac{u^2}{v_T \Lambda_f} \right) \right. \right. \right.
\]
\[
\langle \varphi_{02}^T \rangle = m_{\text{SUSY}} \left[ \left( \frac{\bar{g}_4}{16g_0 g_3} \left( 3 \frac{g_6}{g_0} t_{11} - \bar{t}_{31} \right) + \frac{\bar{g}_4}{g_3} \frac{g_6}{g_0} (t_6 + t_7 + t_8) - (\bar{t}_6 + \bar{t}_7 + \bar{t}_8) \right) \right.
\]
\[
+ \frac{3}{32g_0^2 g_3^2 g_4^2} (t_{11} (3g_3^2 - 2g_4^2) + \right. \right. \right.
\]
\[
+ 3g_4^2 (t_6 + t_7 + t_8) \right) Z \left( \frac{u^3}{v_T \Lambda_f} \right)
\]
\[
+ \frac{g_5}{16g_0 g_2 g_3 g_4^3} \left( 3s_{10} + \frac{\bar{g}_4}{g_3} (s_3 + s_4 + s_5 - \frac{g_2}{g_5} x_2) \right) \left( \frac{u^2}{v_T \Lambda_f} \right) \right. \right. \right.
\]
\[
\langle \varphi_{03}^T \rangle = m_{\text{SUSY}} \left[ \left( \frac{\bar{g}_4}{16g_0 g_3} \left( 3 \frac{g_6}{g_0} t_{11} - \bar{t}_{31} \right) + \frac{\bar{g}_4}{g_3} \frac{g_6}{g_0} (t_6 + t_7 + t_8) - (\bar{t}_6 + \bar{t}_7 + \bar{t}_8) \right) \right.
\]
\[
+ \frac{3}{32g_0^2 g_3^2 g_4^2} (t_{11} (3g_3^2 - 2g_4^2) + \right. \right. \right.
\]
\[
+ 3g_4^2 (t_6 + t_7 + t_8) \right) Z \left( \frac{u^3}{v_T \Lambda_f} \right)
\]
\[
+ \frac{g_5}{16g_0 g_2 g_3 g_4^3} \left( 3s_{10} + \frac{\bar{g}_4}{g_3} (s_3 + s_4 + s_5 - \frac{g_2}{g_5} x_2) \right) \left( \frac{u^2}{v_T \Lambda_f} \right) \right. \right. \right.
\]
\[
\langle \xi_0 \rangle = m_{\text{SUSY}} \left[ \left( \frac{\bar{g}_4}{4g_3 g_4} \right) \left( \frac{1}{6g_3 g_4 (3g_3^2 + g_4^2)} \left( 3g_3^2 - 2g_4^2 \right) t_{11} + 3g_4^2 (t_6 + t_7 + t_8) \right) \right.
\]
\[
- \frac{g_0 g_5}{3g_2} \left( 3 \frac{g_3^2}{g_4} s_{10} + s_3 + s_4 + s_5 - \frac{g_2}{g_5} x_2 \right) \left( \frac{v_T}{u} \right) \right. \right. \right.
\]
\[
\left. + \frac{1}{12g_0^2 g_3 g_4 (3g_3^2 + g_4^2)} \left( \frac{3g_3^2}{g_4} s_{10} + s_3 + s_4 + s_5 \right) \left( g_2 g_5 - g_2 g_5 \right) \left( \frac{v_T}{\Lambda_f} \right) \right. \right. \right.
\]
\[
+ g_2 g_5 \left( \frac{s_3 + s_4 + s_5 - \frac{g_2}{g_5} x_2}{3g_3^2 + g_4^2} - \frac{3g_3^2 + g_4^2}{3g_3^2 + g_4^2} \right) \left( s_3 + s_4 + s_5 - \frac{g_2}{g_5} x_2 \right) \left( \frac{v_T}{\Lambda_f} \right) \right. \right. \right.
\]
\[
+ \frac{g_3 g_5}{4g_2 g_3 g_4 (3g_3^2 + g_4^2)} \left( s_{10} - \frac{g_4}{g_3^2} s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) + \left( \frac{3g_3^2 + 2g_4^2}{4g_2 g_3 g_4 (3g_3^2 + g_4^2)} s_{10} \left( \frac{v_T}{\Lambda_f} \right) \right) \right. \right. \right.
\]
\[ \langle \varphi^S \rangle = m_{SUSY} \left[ \frac{g_5}{4g_1g_2g_4^2}Z + \left[ \frac{g_4}{6g_1g_3} \left( -2t_6 + t_7 + t_8 - \frac{4\tilde{g}_3}{\tilde{g}_4} t_{19} - \frac{6\tilde{g}_2^2}{\tilde{g}_4^2} t_{11} - \frac{2g_1\tilde{g}_3}{g_4^2} t_{12} \right) \left( \frac{u}{\Lambda_f} \right) Y + \frac{g_5}{2g_2(3\tilde{g}_3 + \tilde{g}_4^2)} \left( t_6 + t_7 + t_8 + \left( \frac{3\tilde{g}_3^2 - 2\tilde{g}_4^2}{3\tilde{g}_4^2} \right) t_{11} \right) \left( \frac{u}{\Lambda_f} \right) Y - \frac{g_0\tilde{g}_3}{3g_4^2 g_4} s_6 \left( \frac{v_T}{\Lambda_f} \right) Y \right) + \frac{g_0}{18g_4^2 g_4^2} \left( -2(2g_1^2 + 3g_2^2)s_3 - (4g_1^2 - 3g_2^2)(s_4 + s_5) - \frac{6g_2^2}{g_4^2} (2g_1^2 + 3g_2^2)s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) Y \right] + \frac{g_0 g_5^2}{18g_4^2 g_4^2 (3g_3^2 + \tilde{g}_4^2)} \left( -s_3 - s_4 - s_5 + \frac{g_2}{g_5} x_2 - \frac{3g_3^2}{g_4^2} s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) Y \right] + \frac{1}{6g_4^2 g_4^2} \left( \frac{1}{g_1 g_2} (3g_3 g_2^2 + 2g_1 g_2) s_3 - (2g_1^2 + 3g_2^2) s_3 \right) \left( \frac{v_T}{\Lambda_f} \right) Y \right] + \frac{1}{12g_1 g_2} \left( \frac{1}{g_1 g_2} (4g_1 g_2 - 3g_3 g_2^3) s_4 - (4g_1^2 - 3g_2^2) s_4 \right) \left( \frac{v_T}{\Lambda_f} \right) Y \right] + \frac{1}{12g_1^2 g_2^2} \left( \frac{1}{g_1 g_2} (4g_1 g_2 - 3g_3 g_2^3) s_5 - (4g_1^2 - 3g_2^2) s_5 \right) \left( \frac{v_T}{\Lambda_f} \right) Y \right] + \frac{g_0 g_5^2}{2g_2^2 g_4^2 g_4^2} \left( \frac{1}{g_1 g_2} (3g_3 g_2^2 + 2g_1 g_2) s_{10} - (2g_1^2 + 3g_2^2) s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) Y \right] + \frac{g_0}{4g_2 g_4^2 (3g_3^2 + g_4^2)} \left( g_2 g_5 - g_2 g_5 \right) s_{10} \left( \frac{v_T}{\Lambda_f} \right) + \frac{1}{4g_2 g_4^2 (3g_3^2 + g_4^2)} \left( \frac{g_4^2}{g_4^2} s_{10} - s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) \right] + \frac{g_0 g_5^2}{12g_1 g_2^2 g_4^2 (3g_3^2 + g_4^2)} \left( (3g_3^2 + g_4^2) s_3 - s_3 \right) \left( \frac{v_T}{\Lambda_f} \right) \right] + \frac{g_0 g_5^2}{12g_1 g_2^2 g_4^2 (3g_3^2 + g_4^2)} \left( \frac{1}{2} \left( \frac{g_4^2}{g_4^2} + \frac{g_4^2}{g_4^2} \right) s_5 - \frac{g_2}{g_5} x_2 - \frac{g_2}{g_5} x_2 \right) \left( \frac{v_T}{\Lambda_f} \right) \right] + \frac{g_5}{12g_1^2 g_2^2 (3g_3^2 + g_4^2)} \left( \frac{1}{g_1 g_2} (3g_3 g_2^2 + 2g_1 g_2) s_{10} - (2g_1^2 + 3g_2^2) s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) Y \right] - \frac{g_5}{6g_1 g_2 g_4^2} s_{11} \left( \frac{v_T}{\Lambda_f} \right) Z - \frac{g_5}{4g_1^2 g_4^2} s_{11} \left( \frac{v_T}{\Lambda_f} \right) Z \right] + \frac{1}{6g_2 g_4^2 g_4^2} x_1 \left( \frac{v_T}{\Lambda_f} \right) Z + \frac{1}{12g_2 g_4^2 g_4^2} x_3 \left( \frac{v_T}{\Lambda_f} \right) Z \right] + \frac{1}{4g_2 g_4^2 g_4^2} \left( -(3g_3^2 + g_4^2) s_6 + g_1 x_2 \right) \left( \frac{v_T}{\Lambda_f} \right) Z + \frac{1}{2g_1 g_2^2} \left( s_3 + s_4 + s_5 - s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) Z \right] + \frac{g_5}{8g_1 g_2 g_4^2 g_4^2} \left( 2s_3 - s_4 - s_5 + \frac{6g_2^2}{g_4^2} s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) Z \right] + \frac{g_5}{24g_2 g_4^2 g_4^2} \left( -s_3 - s_4 - s_5 + \frac{g_2}{g_5} x_2 \right) \left( \frac{v_T}{\Lambda_f} \right) Z \right] + \frac{3g_5^2}{8g_2 g_4^2} \left( s_{10} \left( \frac{v_T}{\Lambda_f} \right) Z - \frac{3g_5^2}{8g_2 g_4^2} \left( 3g_3^2 + g_4^2 \right) \left( -s_5 + 2s_{10} + \frac{g_2}{g_5} x_2 \right) \left( \frac{v_T}{\Lambda_f} \right) Z \right) + \frac{g_6}{6g_2 g_4^2 g_4^2} \left( -3g_3^2 s_{10} - s_3 - s_4 - s_5 \right) \left( \frac{v_T}{\Lambda_f} \right) Z - \frac{(g_1^2 + 3g_2^2)}{2g_1^2 g_2^2 g_4^2} \left( 3g_3^2 + g_4^2 \right) s_{10} \left( \frac{v_T}{\Lambda_f} \right) Z \right] \right] \]
\[
\langle \varphi_{02}^S \rangle = m_{SU3Y} \left[ \frac{g_5}{4g_2g_3g_4^2}Z + \left( \frac{\tilde{g}_3}{6g_1g_3} - \frac{2g_3}{g_4}t_9 + \frac{3\tilde{g}_3^2}{g_4^2}t_{11} - \frac{2g_1g_3}{g_2g_4}t_{12} \right) \left( \frac{u}{\Lambda_f} \right) Y \right] + \frac{g_5}{2g_3(3g_3^2 + \tilde{g}_3^2)}(t_6 + t_7 + t_8 + \frac{(3\tilde{g}_3^2 - 2g_1^2)}{3g_4^2}t_{11}) \left( \frac{u}{\Lambda_f} \right) Y + \frac{g_5g_3}{6g_1g_4}s_6 \left( \frac{v_T}{\Lambda_f} \right) Y + g_5g_3s_8 \left( \frac{v_T}{\Lambda_f} \right) Y + \frac{g_5}{18g_1g_2^2} \left( (-4g_1^2 + 3g_2^2)(s_3 + s_5) - 2(2g_1^2 + 3g_2^2)s_4 - 3\tilde{g}_3^2/g_4^2(4g_1^2 - 3g_2^2)s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) Y + \frac{g_5g_3}{18g_1g_2^2(3g_3^2 + \tilde{g}_3^2)}(-s_3 - s_4 - s_5 + g_2/g_5x_2 - 3\tilde{g}_3^2/g_4^2s_{10}) \left( \frac{v_T}{\Lambda_f} \right) Y + \frac{1}{12g_1g_2} \left( -3g_3g_2^2 + 4g_1^2g_3 \right) s_3 - (3g_2^2 + 4g_1^2)s_3 \left( \frac{v_T}{\Lambda_f} \right) Y + \frac{1}{12g_1g_2} \left( 4g_1^2g_2 - 3g_3g_2^2 \right)s_5 - (4g_1^2 - 3g_2^2)s_5 \left( \frac{v_T}{\Lambda_f} \right) Y - \frac{g_5}{4g_1g_2(3g_3^2 + \tilde{g}_3^2)}(3g_2^2 - 4g_1^2g_2)s_{10} - (4g_1^2 + 3g_2^2)s_{10} \left( \frac{v_T}{\Lambda_f} \right) Y
\]
\[
\langle \varphi_{03}^S \rangle = m_{\text{SUSY}} \left[ \frac{g_3}{4g_2g_3g_4^2} Z + \frac{\tilde{g}_4}{6g_1g_3} \left( t_6 + t_7 - 2t_8 + \frac{2g_3}{g_4} t_9 + \frac{3g_3^2}{g_4^2} t_{11} - \frac{2g_1g_3}{g_2g_4} t_{12} \right) \left( \frac{u}{\Lambda_f} \right) Y + \frac{g_5}{2g_4(3g_3^2 + \tilde{g}_4^2)} \left( t_6 + t_7 + t_8 + \frac{(3g_3^2 - 2g_4^2)}{3g_3^2} t_{11} \right) \left( \frac{u}{\Lambda_f} \right) Y + \frac{g_9 g_3}{6g_1^2 g_4 s_6} \left( \frac{v_T}{\Lambda_f} \right) Y - \frac{g_9 g_3}{4g_1^2 g_4 s_8} \left( \frac{v_T}{\Lambda_f} \right) Y + \frac{g_9}{18g_1^2 g_2^3} \left( - (4g_1^2 - 3g_3^2)(s_3 + s_4) - 2(2g_1^2 + 3g_3^2)s_5 - \frac{3g_3^2}{g_1^2}(4g_1^2 - 3g_3^2)s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) Y + \frac{g_9 g_3^2}{18g_1^2 g_2^3 (3g_3^2 + \tilde{g}_4^2)} \left( -s_3 - s_4 - s_5 + \frac{g_2}{g_5} x_2 - \frac{3g_3^2}{g_4^2} s_{10} \right) \left( \frac{v_T}{\Lambda_f} \right) Y + \frac{1}{12g_1 g_2} \left( \frac{1}{g_1 g_2} \left( -3g_3 g_2^2 + 4g_1^2 g_3 \right) s_3 - (4g_1^2 - 3g_3^2) s_3 \right) \left( \frac{v_T}{\Lambda_f} \right) + \frac{1}{12g_1 g_2} \left( \frac{1}{g_1 g_2} \left( 4g_1^2 g_2^3 - 3g_3 g_2^3 \right) s_4 - (4g_1^2 - 3g_3^2) s_4 \right) \left( \frac{v_T}{\Lambda_f} \right) + \frac{1}{6g_1^2 g_4^2} \left( \frac{1}{g_1 g_2} \left( 2g_1^2 g_2^3 + 3g_3 g_2^3 \right) s_5 - (2g_1^2 + 3g_3^2) s_5 \right) \left( \frac{v_T}{\Lambda_f} \right) + \frac{g_3^2}{4g_1^2 g_2^4 g_3 g_4} \left( \frac{1}{g_1 g_2} \left( -3g_3 g_2^3 + 4g_1^2 g_3^2 \right) s_{10} - (4g_1^2 - 3g_3^2) s_{30} \right) \left( \frac{v_T}{\Lambda_f} \right) + \frac{g_3^2 g_5}{4g_2 g_3^2 g_4^2 (3g_3^2 + \tilde{g}_4^2)} \left( g_2 g_5 - 2g_5 s_5 \right) \left( s_3 + s_4 + s_5 \right) \left( \frac{v_T}{\Lambda_f} \right) + \frac{g_5}{g_4 g_1 g_2 g_3 g_4^2} \left( \frac{g_3}{s_6 - s_6} \right) \left( \frac{v_T}{\Lambda_f} \right) - \frac{g_3}{8g_1^2 g_4} \left( s_6 - s_6 \right) \left( \frac{v_T}{\Lambda_f} \right) + \frac{g_5}{12g_1 g_2 g_3 g_4^2} \left( s_11 \right) \left( \frac{v_T}{\Lambda_f} \right) Z - \frac{g_5}{4g_2 g_4} \left( s_11 \right) \left( \frac{v_T}{\Lambda_f} \right) Z \right]
\]
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