Measuring the spectrum of primordial gravitational waves with CMB, PTA and Laser Interferometers

Paolo Campeti, a,b,c Eiichiro Komatsu, d,e Davide Poletti a,b,c and Carlo Baccigalupi a,b,c

a SISSA - Scuola Internazionale Superiore di Studi Avanzati, Via Bonomea 265, 34136, Trieste, Italy
b IFPU - Institute for Fundamental Physics of the Universe, Via Beirut 2, 34014, Trieste, Italy
c INFN - National Institute for Nuclear Physics, Sezione di Trieste, Via Valerio 2, 34127, Trieste, Italy

d Max Planck Institute for Astrophysics, Karl-Schwarzschild-Str.1, 85741 Garching, Germany
e Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU, WPI), UTIAS, The University of Tokyo, Chiba, 277-8583, Japan

E-mail: paolo.campeti@sissa.it, komatsu@mpa-garching.mpg.de, davide.poletti@sissa.it, carlo.baccigalupi@sissa.it

Abstract. We investigate the possibility of measuring the primordial gravitational wave (GW) signal across 23 decades in frequencies, using the cosmic microwave background (CMB), pulsar timing arrays (PTA), and direct detection with laser and atomic interferometers. For the CMB and PTA experiments we consider the LiteBIRD mission and the Square Kilometer Array (SKA), respectively. For the interferometers we consider space mission proposals including the Laser Interferometer Space Antenna (LISA), the Big Bang Observer (BBO), the Deci-hertz Interferometer Gravitational wave Observatory (DECIGO), the µAres experiment, the Decihertz Observatory (DO), and the Atomic Experiment for Dark Matter and Gravity Exploration in Space (AEDGE), as well as the ground-based Einstein Telescope (ET) and Cosmic Explorer (CE) proposals. We implement the mathematics needed to compute sensitivities for both CMB and interferometers, and derive the response functions for the latter from the first principles. We also evaluate the effect of the astrophysical foreground contamination in each experiment. We present binned sensitivity curves and error bars on the energy density parameter, \( \Omega_{GW} h^2 \), as a function of frequency for two representative classes of models for the stochastic background of primordial GW: the quantum vacuum fluctuation in the metric from single-field slow-roll inflation, and the source-induced tensor perturbation from the spectator axion-SU(2) inflation models. We find excellent prospects for joint measurements of the GW spectrum by CMB and space-borne direct detection mission proposals.

Corresponding author.
1 Introduction

The cosmic inflation paradigm [1–5] predicts the primordial Stochastic Background of Gravitational Waves (hereafter SGWB) [6, 7]. In the standard picture the scalar and tensor perturbations are generated by the quantum vacuum fluctuations during inflation [7–12]. The scalar modes are the seeds for the large-scale structure of the Universe and have been subject to meticulous measurements (see e.g. [13]), while the primordial tensor modes still remain undetected. The importance of their detection cannot be overstated, since the primordial SGWB would contain an unparalleled information on the very early Universe physics. If the single-field slow-roll inflationary scenario is confirmed, a detection of the tensor-to-scalar ratio \( r \), i.e., the ratio of the tensor and scalar power spectra, can be used to directly infer the energy scale of inflation, allowing us to probe the ultra-high energy scales not accessible by terrestrial particle colliders [14].

There are (at least) three ways to search for the SGWB at widely separated frequencies: the cosmic microwave background (CMB) at \( f \approx 10^{-20} \)–\( 10^{-16} \) Hz, pulsar timing arrays at \( f \approx 10^{-6} \)–\( 10^{-7} \) Hz, and direct detection with laser and atomic interferometers at \( f > 10^{-7} \) Hz (see [15–17] for reviews).
For CMB, the primordial SGWB would imprint its signature in the B-mode polarization \cite{18, 19}, which is currently the most promising channel for a near-future detection. Numerous ground-based experiments are currently scanning the microwave sky in search of the primordial B-mode, among them the Background Imaging of Cosmic Extragalactic Polarization 2 (BICEP2)/Keck Array \cite{20}, POLARBEAR/Simons Array \cite{21, 22}, the Atacama Cosmology Telescope (ACT) \cite{23}, the South Pole Telescope (SPT) \cite{24}, and the Cosmology Large Angular Scale Surveyor (CLASS) \cite{25}. Furthermore, the next decade will see a great increase in the efforts for detection with a new generation of experiments the Simons Observatory (SO) \cite{26}, the South Pole Observatory (SPO) and the Stage-IV network of ground-based observatories (CMB-S4) \cite{27–29}. As for space-borne experiments, the Japan Aerospace Exploration Agency has selected the LiteBIRD \cite{30} as the second Strategic Large-class mission.

For pulsar timing arrays (hereafter PTA), the current generation experiments such as the Nanohertz Observatory for Gravitational Waves (NANOGrav) \cite{31}, the European PTA \cite{32} and the Perkes PTA \cite{33} are placing limits on the SGWB. In future the Square Kilometre Array (SKA) \cite{34} will add to this international network of PTA.

For direct detection experiments, the current generation of ground-based laser interferometers (LIGO \cite{35}, VIRGO \cite{36}, KAGRA \cite{37}) will be succeeded by the Cosmic Explorer (CE) \cite{38} and Einstein Telescope (ET) \cite{39}, operating between a few Hertz and a few kilo-Hertz. The space-borne Laser Interferometer Space Antenna (LISA) \cite{40, 41} will be probing in the milli-Hertz band. In addition there are a host of proposals for future space missions including the \( \mu \)Ares \cite{42} in the micro-Hertz band; the Advanced Millihertz Gravitational-wave Observatory (AMIGO) \cite{43} in the milli-Hertz band; the Big Bang Observer (BBO) \cite{44, 45}, the Decihertz Interferometer Gravitational wave Observatory (DECIGO) \cite{46, 47}, the Decehertz Observatory (DO) \cite{48}, and the Atomic Experiment for Dark Matter and Gravity Exploration in Space (AEDGE) \cite{49} in the deci-Hertz bands.

Combining these experiments, we can measure the SGWB spectrum across 23 decades in frequency. If we include indirect probes using the Big Bang Nucleosynthesis (BBN) and the number of relativistic degrees of freedom, the range extends to 29 decades \cite{33}. This combination enables a detailed characterization of the SGWB that goes beyond the simple detection of \( r \), which will be of utmost importance to determine if the detected primordial SGWB was sourced by the quantum vacuum fluctuations in the metric tensor, as in the single-field slow-roll scenario, or from alternative scenarios that can also produce the SGWB. In this context, the possibility of SGWB production from gauge fields, both Abelian \cite{50–57} and non-Abelian \cite{58–68}, has been investigated in the literature.

These sourced gravitational waves come with distinct observational signatures: they can be non-scale-invariant, partially chiral (circularly polarized), and strongly non-Gaussian. In this paper, we focus on the first signature, i.e., the spectrum of the SGWB, which can be blue, red, or with a bump. See the above list of references for the other two signatures. Specifically, we seek to gather in one resource the expectations on the SGWB from the quantum vacuum fluctuations in the metric tensor, as in the single-field slow-roll scenario, or from alternative scenarios that can also produce the SGWB. In this context, the possibility of SGWB production from gauge fields, both Abelian \cite{50–57} and non-Abelian \cite{58–68}, has been investigated in the literature.

To this end, we try to use coherent assumptions for each experiment and, whenever possible, to derive the relevant quantities from the first principles using the latest available information in the literature. We provide therefore a quick reference for both communities of cosmologists and GW astronomers for the sensitivities of future experiments capable of...
detecting a SGWB, summarizing the mathematical tools needed to compute such sensitivities for both the CMB and the direct detection experiments. Finally, we show our results in a coherent manner by plotting error bars representing the uncertainty on the binned tensor power spectrum for each experiment. For example, we derive forecasts for the precision on the tensor-to-scalar ratio \( r \) and the tensor spectral index \( n_T \), for the combination of CMB B-modes experiments and laser interferometers (LiteBIRD+LISA and LiteBIRD+BBO), using a Monte Carlo Markov Chain exploration of the full cosmological parameters space.

We differentiate our work from the previous literature in three ways. First, we provide frequency-integrated error bars from the binned sensitivity curves for all the detectors. Second, we include astrophysical foregrounds for all experiments. Finally, we use the latest and realistic CMB sensitivity curves for the LiteBIRD mission, including state-of-the-art simulations for the CMB foregrounds.

The paper is organized as follows. In Section 2 we describe the two theoretical tensor power spectrum models for which we will provide forecasts in the subsequent sections: the single-field slow-roll model and the spectator axion-SU(2) model. In Section 3 we discuss the experimental setup for the CMB B-mode experiment LiteBIRD, including the instrumental noise, the lensing contribution and the astrophysical foregrounds contamination. In Section 4 we construct the instrumental sensitivity curves for the direct GW experiments and illustrate the effect of the astrophysical foregrounds on each direct GW experiment as well as on the PTA. Section 5 is dedicated to the discussion of our results concerning forecasts on the sensitivity of all the experiments for the spectator axion-SU(2) and single-field slow-roll models. We also present the updated forecasts on the tensor spectral index \( n_T \) exploiting the combination of CMB experiments and laser interferometers. We conclude in Section 6 with future perspectives.

2 Theoretical Models for the Primordial Tensor Power Spectrum

In this Section we review the theoretical models of the primordial tensor power spectrum for which we will provide forecasts in the rest of the paper. We consider two possibilities in this respect: one is the nearly scale-invariant tensor power spectrum predicted in the context of the single field-slow roll inflation, while the other is the one produced by the spectator axion-SU(2) model [65].

2.1 Single-Field Slow-Roll Model

In the single-field slow-roll inflationary scenario [3–5], cosmological scalar [8–11] and tensor [7, 12] perturbations are produced by the quantum vacuum fluctuations. The power spectrum for the scalar perturbations is parametrized by a power-law \( P_{\text{vac}}^R(k) = A_S (k/k_0)^{n_S-1} \), where \( A_S \) is the amplitude of the scalar perturbations, \( n_S \) the scalar spectral index, \( k \) the wavenumber of the perturbation, \( k_0 = 0.05 \text{ Mpc}^{-1} \) the pivot-scale and the superscript \( \text{vac} \) indicates that it is produced by the quantum vacuum fluctuations. The same applies to the tensor power spectrum

\[
P_{\text{vac}}^T(k) = A_T \left( \frac{k}{k_0} \right)^{n_T},
\]

where \( A_T \) is the amplitude of the tensor perturbations and \( n_T \) the tensor spectral index. We then define the tensor-to-scalar ratio \( r \) as \( r = A_T/A_S \). We also enforce the inflationary consistency relation in single-field slow-roll inflation [14], connecting the spectral index and the amplitude of the tensor spectrum as \( n_T = -r/8 \).
Currently no detection of $r$ exists and there are only upper limits available. The best limits come from CMB experiments, $r < 0.06$ at 95% CL, from the combination of the B-mode polarization data of the BICEP2/Keck [70] and Planck 2018 data [13].

2.2 Spectator Axion-SU(2) Model

Gauge fields are ubiquitous in physics and can affect the predictions of inflation (see [58] for a review). In this paper we consider the SGWB produced in the spectator axion-SU(2) model [65] based on the “chromo-natural” inflation model [71]. This model has the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{inflaton}} + \frac{1}{2} \left( \partial_\mu \chi \right)^2 - \mu^4 \left[ 1 + \cos \left( \frac{\chi}{f} \right) \right] - \frac{1}{4} F^{a\mu\nu} F_{a\mu\nu} + \frac{\lambda}{4 f} \chi F_{\mu\nu} \tilde{F}^{\mu\nu},$$

(2.2)

where $\mathcal{L}_{\text{inflaton}}$ represents a generic inflaton sector generating the quasi-de Sitter expanding background and the curvature perturbations in agreement with the current CMB observations, $\chi$ is a pseudo-scalar axion field with a cosine-type potential, $\mu$ and $f$ are dimensionful parameters and $\lambda$ is a dimensionless coupling constant for the axion and gauge fields. The field strength tensor of the SU(2) gauge field is given by $F^{a\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g e^{abc} A^b_\mu A^c_\nu$ with $g$ being the gauge field self-coupling constant, and $\tilde{F}^{\mu\nu}$ is its dual. We ignore the effect of the gravitational Chern-Simons term $R \tilde{R}$ because its effect on the SGWB is sub-dominant compared to the $F \tilde{F}$ term [72].

During inflation the SU(2) gauge field establishes a homogeneous and isotropic vacuum expectation value, $\bar{A}^b_i = a(t) Q(t) \delta^b_i$ [62, 63], which is an attractor solution [73–75]. The perturbation round this value contains scalar, vector, and tensor modes [62, 63], and the tensor mode linearly mixes with gravitons to produce the SGWB. In particular, the gauge field produces a chiral SGBW with either left- or right-handed circular polarization, depending on which circular polarization mode experiences a transient growth near horizon crossing [58–61].

Assuming that only left-handed polarized GWs are produced, we can write the sourced contribution to the tensor spectrum as [69]

$$P_{T,\text{Sourced}}^L(k) = r_s P_R(k) \exp \left[ \frac{1}{2 \sigma^2} \ln^2 \left( \frac{k}{k_p} \right) \right],$$

(2.3)

$$P_{T,\text{Sourced}}^R(k) \approx 0,$$  

(2.4)

where $P_R$ is the scalar curvature perturbation power spectrum, the parameter $r_s$, which is the tensor-to-scalar ratio at the peak scale $k = k_p$, controls the amplitude of the tensor power spectrum, and the parameter $\sigma$ controls the width of the Gaussian-shaped feature produced in the spectrum by this model. These parameters are related to the model parameters given in Eq. 2.2 (see below). This form of the tensor power spectrum is valid for the cosine potential given in Eq. 2.2 as well as for axion potentials with an inflection point [76].

The total tensor spectrum will be the sum of the sourced and the vacuum contributions

$$P_T(k; k_p, r_s, \sigma) = P_T^{\text{vac}}(k) + P_T^{\text{Sourced}}(k; k_p, r_s, \sigma),$$

(2.5)

$$P_T^{\text{Sourced}}(k; k_p, r_s, \sigma) = P_{T,\text{Sourced}}^L(k) + P_{T,\text{Sourced}}^R(k),$$

(2.6)

while the contribution of the axion and SU(2) gauge fields to $P_R$ is negligible with respect to the vacuum one for an appropriate choice of the model parameters$^1$, i.e., $m_Q \equiv g Q / H \geq \sqrt{2}$

$^1$There is a possibility of having a non-negligible contribution to the scalar sector for a very large $\sigma$ parameter choice, if the energy fraction of the axion grows after inflation and the axion decays faster than the inflaton [see 69, and references therein].
where $H$ is the Hubble expansion rate during inflation \cite{61, 65}; thus, $P_R(k) = P^{\nuac}_R(k)$. The parameters \{r*, k*, $\sigma$\} can be connected to the physical parameters in the model Lagrangian \{g, $\lambda$, $\mu$, $f$\} \cite{69, 76}. The peak wavenumber $k_p$ corresponds to the time $t_*$ at which $\chi$ is at the inflection point of the potential, $\chi(t_*) = \pi f/2$. The value of $m_Q$ is given by $m_* \equiv m_Q(t_*) = (g^2 \mu^4 / 3 H^4)^{1/3}$. The other relevant dimensionless variable is $\xi_* \equiv \lambda \chi(t_*)/(2fH) \approx m_* + m_*^{-1}$. With these variables, we can write $k/k_p = e^{H(t-t_*)}$, $\sigma^2 = (\lambda/2\xi_*^2)/(2G(m_*)^3)$, and $G(m_*) \approx 0.666 + 0.81 m_* - 0.0145 m_*^2 - 0.0064 m_*^3$. The effective tensor-to-scalar ratio at the peak scale $r_*$ can also be related to the model parameters, but in principle can assume any positive value, while the width of the Gaussian feature $\sigma$ is bounded by the peak scale choice $k_p$ because of the attractor behaviour of the background axion field coupled to the SU(2) gauge fields.

In the rest of this paper we will consider three sets of parameters:

\[ \{r*, k_p, \sigma\} = \{400, 10^{15} \text{ Mpc}^{-1}, 9.1\}, \quad \{0.15, 10^{11} \text{ Mpc}^{-1}, 8\}, \quad \{50, 10^6 \text{ Mpc}^{-1}, 4.8\}, \]

and we will refer to them as $AX1$, $AX2$ and $AX3$ models, respectively. For all cases we will assume the vacuum contribution to the tensor-to-scalar ratio of $r_{\nuac} = 10^{-5}$ \cite{69}, although this choice might be subject to backreaction of particle production of the gauge field \cite{77, 78}. To avoid this we can simply assume a larger value for $r_{\nuac}$, which would add the scale-invariant component to all the figures we show in this paper.

We chose the parameters given in Eq. 2.7 to provide representative examples for our analysis. The first set of parameters represents a tensor spectrum model that is simultaneously detectable by both CMB and laser interferometers, while still satisfying the upper bound provided by the BICEP2/Keck/Planck analysis (see the end of Section 2.1). The second set produces instead a spectrum that is just outside the reach of LiteBIRD and at the same time comfortably detectable by the advanced interferometers $\mu$Ares, DECIGO and BBO, thanks to the large bump feature produced at $k_p = 10^{11} \text{ Mpc}^{-1}$. The third parameter set produces a spectrum that peaks in the PTA experiments frequency range while still being compatible with the BICEP2/Keck/Planck upper limit in the CMB range. Due to the relationship between $\sigma$ and $k_p$, which tends to flatten out the spectrum, we could not get a SGWB detectable by SKA (Section 5).

In Figure 1 we show the tensor power spectra $P_T$ as a function of the wavenumber $k$ for the five cases considered in this paper. We have checked that all models are consistent with the current CMB shortwave and second-order back-reaction \cite{79}, indirect upper bounds \cite{80}, PTA limits \cite{81} and ground-based interferometers LIGO/Virgo \cite{82} limits.

2.3 Gravitational Wave Energy Density

A quantity commonly used in the literature to show the sensitivities of GW observatories is the fractional energy density in GWs at the present (conformal) time $\tau_0$ \cite{83}:

\[ \Omega_{GW}(k, \tau_0) = \frac{1}{\rho_0(\tau_0)} \frac{\partial \rho_{GW}(k, \tau_0)}{\partial \ln k} = \frac{P_T(k)}{12 H_0^2} \cdot \left[ T'(k, \tau_0) \right]^2 \]  

In the equation above $\rho_c$ is the critical energy density of the Universe and $\rho_{GW}$ the energy density of GWs, given by $\rho_{GW} = \langle h_{ab} h^{ab} \rangle / (32 \pi G)$, where the tensor $h_{ab}$ represents the GW metric perturbation and the $'$ indicates the conformal time derivative. The second equality in

\[ 2\text{Throughout this paper we adopt the notation } c = 1 \text{ unless stated otherwise.} \]
Eq. 2.8 can be obtained from the definition of the tensor power spectrum and by expressing the time evolution of the primordial GW amplitude – solution of the linearized Einstein equation – in terms of the GW transfer function $T(k, \tau)$ [see 84, and references therein]. In the rest of this paper, we will use approximate analytical expressions for $\Omega_{GW}$, derived in [84], and valid for two different regimes:

$$\Omega_{GW}(k, \tau_0) = \frac{P_T(k)}{12H_0^2}k^2 \cdot \begin{cases} \frac{\tau^2_{eq}}{\tau_0^2} \left[ A(k)j_2(k\tau_0) + B(k)y_2(k\tau_0) \right]^2, & \text{if } k > k_{eq}, \\ \left[ \frac{3j_2(k\tau_0)}{k\tau_0} \right]^2, & \text{if } k < k_{eq}, \end{cases}$$

(2.9)

where $\tau_{eq}$ is the conformal time at the epoch of the matter-radiation equality and $k_{eq} \approx 0.01038 \text{ Mpc}^{-1}$ is the comoving wavenumber of the modes that entered the horizon at that time, $j_n$ and $y_n$ with integer $n$ are the spherical Bessel functions of first and second kind, respectively, and the functions $A(k)$ and $B(k)$ are given by

$$A(k) = \frac{3}{2k\tau_{eq}} - \frac{\cos(2k\tau_{eq})}{2k\tau_{eq}} + \frac{\sin(2k\tau_{eq})}{(2k\tau_{eq})^2},$$

(2.10)

$$B(k) = -1 + \frac{1}{(k\tau_{eq})^2} - \frac{\cos(2k\tau_{eq})}{(k\tau_{eq})^2} - \frac{\sin(2k\tau_{eq})}{2k\tau_{eq}}.$$  

(2.11)

In the following we will often pass from the GW wavenumber $k$ to the frequency $f$ of the GW today, which are related to each other via $k = 2\pi f/c$ (here we reinstate the factor $c$),

$$\frac{k}{\text{Mpc}^{-1}} = 6.5 \times 10^{14} \frac{f}{\text{Hz}},$$

(2.12)
making explicit the units of measure.

3 CMB B-mode Experiments

CMB experiments are at the forefront of the search for a primordial SGWB. As we discussed in Section 2.1, the current best observational bounds on the SGWB come from the CMB. Furthermore, as it will be shown in Section 5, they represent our best opportunity to detect a SGWB if the correct model for its production is the single-field slow-roll inflation with \( r \lesssim 0.001 \).

The current generation of operating CMB experiments includes BICEP2/Keck, POLARBEAR, ACT, SPT and CLASS while the next generation of experiments, planned for this decade, will comprise the Simons Array, SO, SPO and CMB-S4 on the ground-based side, and the LiteBIRD mission observing from space. In this paper, we will focus on making forecasts for the LiteBIRD, which is expected to be – together with CMB-S4 – the most sensitive among the planned missions, capable of detecting a tensor-to-scalar ratio \( r \lesssim 0.001 \).

The signature of the primordial SGWB in the B-mode polarization has two main contributions: one at very large scales (around \( k \sim 6 \times 10^{-4} \text{Mpc}^{-1} \)) where the CMB photons are re-scattered by the free electrons made available by cosmic reionization [85], producing the so-called reionization bump, and the other at intermediate scales (\( k \sim 6 \times 10^{-3} \text{Mpc}^{-1} \)) corresponding to the recombination bump [86]. This primordial signal, however, is fainter than the contaminating signals of the secondary origin: smaller scales are dominated by the gravitational lensing due to the cosmological large-scale structure, which converts the E-mode polarization of the CMB into a secondary B-mode [87], while larger scales are contaminated by the presence of the diffuse Galactic foregrounds.

In this Section we first review the formalism of CMB power spectra (Section 3.1). We then describe the relevant noise sources for CMB experiments, including the instrumental, the lensing and the astrophysical foreground contributions (Section 3.2). Finally, we review the Fisher matrix approach for computing the binned uncertainties on the tensor power spectrum for a CMB experiment (Section 3.3).

3.1 CMB Angular Power Spectra

CMB experiments do not observe directly the scalar or tensor power spectra described in Sections 2, but rather their effects on the CMB temperature and polarization angular power spectra \( C_{\ell}^{XX'} \), defined by the correlation function \( \langle a_{\ell m}^{X} a_{\ell m'}^{X'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_{\ell}^{XX'} \), where the indices \( X, X' = \{ T, E, B \} \) label the total intensity (T), gradient (E) and curl (B) modes of the CMB polarization [18, 19] and the \( a_{\ell m}^{X} \) are the coefficients of the spherical harmonic expansion of the total intensity and polarization.

Assuming that vector modes get diluted by the expansion of the Universe, each angular power spectrum will have contributions only from scalar and tensor modes, so that \( C_{\ell}^{XX', \text{prim}} = C_{\ell}^{XX'} + c_{\ell}^{XX'} \). We can now connect the observable angular power spectra to the primordial scalar and tensor ones through the scalar or tensor transfer functions \( T_{\ell y}^{X} \)

\[
C_{\ell xx}^{X} = \frac{2\pi}{\ell (\ell + 1)} \int d\ln k \, \mathcal{P}_{y} (k) \, T_{\ell y}^{X} (k) T_{\ell y}^{X'} (k),
\]

with indices \( X, X' = \{ T, E \}, x = \{ s \} \) and \( y = \{ R \} \) for the scalar case and indices \( X, X' = \{ T, E, B \}, x = \{ t \} \) and \( y = \{ T \} \) for the tensor one. The transfer functions depend on
| Experiment | Frequency [GHz] | Sensitivity [µK-arcmin] | FWHM [arcmin] |
|------------|----------------|-------------------------|---------------|
| LiteBIRD   | 40             | 59.29                   | 60            |
|            | 50             | 32.78                   | 56            |
|            | 60             | 25.76                   | 48            |
|            | 68             | 15.91                   | 43            |
|            | 78             | 13.10                   | 39            |
|            | 89             | 11.25                   | 35            |
|            | 100            | 7.74                    | 29            |
|            | 119            | 5.37                    | 25            |
|            | 140            | 5.65                    | 23            |
|            | 166            | 5.81                    | 21            |
|            | 195            | 6.48                    | 20            |
|            | 235            | 15.16                   | 19            |
|            | 280            | 17.98                   | 24            |
|            | 337            | 24.99                   | 20            |
|            | 402            | 49.90                   | 17            |

Table 1: Instrumental specifications adopted for the LiteBIRD CMB experiment (LiteBIRD collaboration, private communication).

the cosmological parameters, for which we assume the Planck 2018 values [13], and can be computed from a Boltzmann solver such as CAMB [88] or CLASS [89].

To conclude this Section, we specialize Eq. 3.1 to the axion-SU(2) sourced contribution to the tensor spectrum, defined in Section 2.2

\[
C^{XX',Sourced}_{\ell,t} = \frac{2\pi}{\ell(\ell + 1)} \int d\ln k \left[ P_{T,Sourced}^L(k) + P_{T,Sourced}^R(k) \right] T_{XX'(k)}^{\ell,T}(k) T_{XX'(k)}^{\ell,T}(k),
\]

with \( XX' = \{TT, EE, TE, BB\} \). Note that the chiral tensor spectrum produced in the axion-SU(2) model also yields non-zero parity-odd cross-spectra such as \( TB \) and \( EB \) spectra, which could be used as an observational marker to distinguish it from the standard SGWB from the vacuum fluctuations [90–92]. However, these cross-power spectra are difficult to detect unless \( r \gtrsim 0.05 \) [69]; thus, in this paper we will be concerned only by the intensity of the SGWB rather than by its circular polarization, and consider only the \( BB \) spectrum in our analysis.

3.2 Noise and Foregrounds for CMB Experiments

In this paper we will consider the LiteBIRD satellite and its constraining power on the SGWB. For our purpose we can characterize this instrument using the following parameters: the polarization sensitivity (in \( \mu K\text{-arcmin} \) units) at each frequency channel, the Full Width at Half Maximum (FWHM) for the instrument beams, the observed sky fraction \( f_{sky} \) and the multipole range of the measurement. For LiteBIRD we adopt a multipole range from \( \ell_{min} = 2 \) to \( \ell_{max} = 200 \). We report all the other specifications in Table 1.

As we already mentioned above, there are three relevant noise sources which contribute to the total observed CMB B-mode spectrum \( C_{\ell}^{BB} \):

\[
C_{\ell}^{BB} = C_{\ell}^{BB,prim} + C_{\ell}^{BB,noise} + C_{\ell}^{BB,lens} + C_{\ell}^{BB,fgs},
\]
where $C^{BB,prim}_\ell$ is the primordial signal, $C^{BB,lens}_\ell$ is the gravitational lensing B-mode, $C^{BB,fgs}_\ell$ the residual contamination due to polarized diffuse foregrounds, and $C^{BB,noise}_\ell$ the post component separation noise. We model the instrumental noise [93] at each frequency channel $\nu$ as

$$N^{BB}_{\ell,\nu} = \left[w^{-1/2}_{B,\nu} \exp\left(\frac{\ell(\ell + 1)\theta_{FWHM,\nu}^2}{8 \ln 2}\right)\right],$$

(3.4)

where $w^{-1/2}_{B,\nu}$ is the white noise level (or sensitivity) in each frequency channel in $\mu$K-rad and $\theta_{FWHM,\nu}$ is the beam size in radians.

The lensing represents a contaminant of the unknown amplitude when searching for a primordial signal and affects especially the smaller angular scales of the CMB B-modes. We compute $C^{BB,lens}_\ell$ using the CAMB code. Note that for LiteBIRD we conservatively do not consider any cleaning from the lensing contamination, i.e., a procedure called “delensing” [94–97], but we stress that high resolution ground-based experiments such as CMB-S4 can be exploited to delens LiteBIRD data to enhance its capability in reconstructing the SGWB.

On the other hand, the dominant source of noise on large scale B-mode polarization is the diffuse Galactic foregrounds [see, e.g., 98, and references therein]. In particular, in this paper we will consider the two main sources of foregrounds for B-mode experiments: the thermal emission of dust grains and the synchrotron radiation emitted by cosmic-ray electrons spiraling in the Galactic magnetic field [see 99, and references therein]. We generate simulated sky maps of the polarized Galactic foreground emission using the “d1s1” sky model in the Python Sky Model (PySM) code [100], and degrade them to a HEALPIX [101] resolution $N_{side} = 128$. We add to the simulated maps an instrumental white noise realization generated by the model in Eq. 3.4. We perform component separation for three possible spectral energy distributions (SEDs): the CMB SED, for which we assume no free parameters; the thermal dust SED, for which we take the one-component modified black-body

$$A_{dust}(\nu) = \left(\frac{\nu}{\nu_d}\right)^{\beta_d+1} \frac{h\nu_d}{e^{h\nu_d/kT_d} - 1},$$

(3.5)

with the spectral index $\beta_d$ and the temperature $T_d$ as free parameters and the reference frequency $\nu_d$ fixed to 353 GHz ; and the synchrotron SED, for which we assume the curved power-law

$$A_{sync}(\nu) = \left(\frac{\nu}{\nu_s}\right)^{\beta_s + C_s \ln(\nu/\nu_s)},$$

(3.6)

with the spectral index $\beta_s$ and the curvature $C_s$ as free parameters and $\nu_s = 70$ GHz.

We compute the contributions of residual foregrounds $C^{BB,fgs}_\ell$ and post component separation noise $C^{BB,noise}_\ell$ to the observed spectrum using the parametric maximum likelihood approach [93, 102–104] implemented in the publicly available ForeGroundBuster (FGBuster) code. This code allows for several different choices of cleaning techniques, among which we choose the Multi-Resolution procedure, an evolution of the Multi-patch technique presented in [105]. While in the Multi-Patch approach we fit all the spectral parameters over independent sky patches equal to HEALPIX pixels with the same resolution parameter $N_{side}$, in the Multi-resolution approach, each of the free spectral parameters is fitted on a different HEALPIX grid

---

3See https://github.com/fgbuster/fgbuster and reference therein.
with different resolution. The patches resolution for each parameter are gathered in the Multi-resolution vector $N_{\text{side}}$, for which we adopt the choice $N_{\text{side}} = [\beta_d, T_d, \beta_s, C_s] = [64, 8, 8, 0]$, obtained by prioritizing the characterization of dust SED over synchrotron SED and by requiring that systematic residuals are much smaller than the statistical ones (J. Errard 2019, private communication). This selection of parameters provides appropriate residuals for the current foreground modeling in LiteBIRD.

We average the resulting residual foregrounds plus post-component separation noise spectra over 100 noise realizations, obtaining the final spectrum in Figure 2 (red curve). This spectrum is roughly composed by two parts. In the angular domain, the diffuse Galactic foregrounds are usually characterized by a decaying power law with the angular multipole. Therefore, at high $\ell$, the foreground contamination is less relevant, and the component separation noise is the co-addition of sensitivity in multi-frequency channels corresponding to the CMB solution. On the other hand, at low and intermediate multipoles, the structure is dominated by the component separation residuals from the large scale pattern of foregrounds.

### 3.3 Fisher Matrix for the Tensor Power Spectrum

To compute the binned uncertainties on the tensor power spectrum for LiteBIRD, we use a Fisher matrix approach similar to the one described in Refs. [86, 106]. We report here the
main ingredients of the method. The tensor power spectrum $\mathcal{P}_T$ can be discretized as

$$
\mathcal{P}_T(k) = A_S \sum_i p_i W_i(\ln k),
$$

(3.7)

where $W_i$ is the discretization window function, which we choose to be equal to 1 inside the $i$-th of the $N$ power spectrum bins and 0 outside

$$
W_i(\ln k) = \begin{cases} 
1 & \text{for } k_{i-1} \leq k < k_i \text{ with } 1 \leq i \leq N, \\
0 & \text{for } k < k_{i-1} \text{ and } k > k_i,
\end{cases}
$$

(3.8)

and $\Delta \ln k = (\ln k_{i+1} - \ln k_i)$ is the width of the $i$-th bin. The discretization process allows to write the derivative of the $C_{\ell}^{XX'}$ with respect to the power spectrum parameters $p_i$ in a simple way:

$$
D_{\ell i}^{BB} = \left. \frac{\partial C_{\ell}^{BB}}{\partial p_i} \right|_{\text{fid}} = \frac{2\pi}{\ell(\ell+1)} A_S \int d\ln k T_{\ell}^B(k) T_{\ell'}^B(k) W_i(\ln k).
$$

(3.9)

We choose the $k$ range to be $10^{-4} \text{Mpc}^{-1} < k < 1 \text{Mpc}^{-1}$ such that it contains the whole sensitivity curve of the LiteBIRD experiment. To obtain the error bar on each power spectrum wavenumber bin, we first compute the Fisher information matrix [see, e.g., 107]

$$
F_{ij} = f_{sky} \sum_{\ell=2}^{\ell_{\text{max}}} 2\ell + 1 \frac{1}{2} \text{Tr} \left[ D_{\ell i}^{BB} (C_{\ell}^{BB})^{-1} D_{j\ell}^{BB} (C_{\ell}^{BB})^{-1} \right],
$$

(3.10)

where the factor $f_{sky}$ takes into account the loss of modes by a partial sky coverage. We then take the diagonal of its inverse to obtain

$$
\sigma^2_{PS}(k_i) = (F^{-1})_{ii}.
$$

(3.11)

The desired binned uncertainty on $\Omega_{GW}$ is then easily obtained from Eq. 2.8

$$
\sigma_{\Omega_{GW}}(k_i) = \sigma_{PS}(k_i) \cdot \frac{A_S}{12H_0^2} \left[ T'(k, \tau_0) \right]^2.
$$

(3.12)

4 Direct Detection Experiments and PTA

The landscape of the current and future GWs direct detection experiments is vast, characterized by their complementarity in probing the GW spectrum across a wide range of frequencies. The frequency window between $\sim 10^{-7}$ and $\sim 10$ Hz is expected to be observed from space through a host of funded and proposed laser interferometers, ranging from $\mu$Ares [42] in the micro-Hertz band, to LISA [41] and AMIGO [43] in the milli-Hertz band, to BBO [44, 45], DECIGO [46] and DO [48] in the deci-Hertz bands. In this work we also include the recently proposed space-based atom interferometer AEDGE [49], which will observe the deci-Hertz band as well. Going higher in the GW frequency ($\sim 10 - 10^3$ Hz), the next-generation ground-based detectors (CE [38] and ET [39]), also exploiting laser interferometry, will complement the previous observations in the high-frequency part of the GW spectrum.

We summarize in Table 2 the main instrumental characteristics and capabilities of GW observatories treated in this paper, including the experiment type, the arm length ($L$) for traditional interferometers, the total observation length ($T_{\text{obs}}$), the observational efficiency
Table 2: Summary of the instrumental specifications for the direct GW experiments and PTA considered in this work. “M.I.” stands for Michelson Interferometer, “F.P.I.” stands for Fabry-Pérot Interferometer and “A.I.” stands for Atomic Interferometer. The binning used to compute the values of $\Omega^{\text{min}}_{GW}$ is $\Delta \ln k = 1.2$.

| Experiment | Type    | L [m]   | $T_{\text{obs}}$ [yr] | $\epsilon$ | Freq. [Hz] | $\Omega^{\text{min}}_{GW}$ w/o Fgs | $\Omega^{\text{min}}_{GW}$ w/ Fgs | Ref.   |
|------------|---------|---------|------------------------|------------|-----------|------------------------------------|----------------------------------|--------|
| LISA       | Space M.I. | $2.5 \times 10^9$ | 4 | 75% | $10^{-4} - 10^{-1}$ | $5.9 \times 10^{-14}$ | $1.0 \times 10^{-13}$ | [41]   |
| DO Cons.   | Space M.I. | $10^8$ | 4 | 75% | $10^{-3} - 10^1$ | $3.7 \times 10^{-15}$ | $6.2 \times 10^{-15}$ | [48]   |
| DO Opt.    | Space M.I. | $10^8$ | 4 | 75% | $10^{-3} - 10^1$ | $7.1 \times 10^{-16}$ | $2.5 \times 10^{-15}$ | [48]   |
| $\mu$Ares  | Space M.I. | $430 \times 10^9$ | 10 | 100% | $10^{-6} - 10^{-2}$ | $4.7 \times 10^{-18}$ | $1.2 \times 10^{-15}$ | [42]   |
| DECIGO     | Space F.P.I. | $10^6$ | 10 | 100% | $10^{-4} - 10^1$ | $5.9 \times 10^{-18}$ | $9.7 \times 10^{-18}$ | [108]  |
| BBO        | Space M.I. | $5 \times 10^7$ | 10 | 100% | $10^{-4} - 10^1$ | $1.9 \times 10^{-18}$ | $1.9 \times 10^{-18}$ | [109]  |
| AEDGE      | Space A.I. | - | 5 | 60% | $10^{-2} - 1$ | $4.2 \times 10^{-16}$ | $2.4 \times 10^{-15}$ | [49]   |
| CE         | Ground M.I. | $4 \times 10^4$ | 1 | 100% | $1 - 10^4$ | $1.1 \times 10^{-13}$ | $2.1 \times 10^{-12}$ | [38, 110] |
| ET         | Ground M.I. | $1 \times 10^4$ | 1 | 100% | $1 - 10^4$ | $4.5 \times 10^{-14}$ | $5.5 \times 10^{-13}$ | [39, 110] |
| SKA        | PTA | - | 15 | 100% | $10^{-9} - 10^{-7}$ | $3.3 \times 10^{-13}$ | $1.2 \times 10^{-10}$ | [34] (Sec.4.3) |

$\epsilon$ to compute the actual observation time $T_{\text{eff}} = \epsilon T_{\text{obs}}$, the frequency range at which the experiment is operating, and the minimum of the binned sensitivity curve with and without foregrounds.

Going lower in the frequency, PTAs will probe GWs in the $\sim 10^{-9} - 10^{-7}$ Hz region. There are several planned and ongoing PTA experiments (NANOGrav [81], EPTA [111], PPTA [112, 113], IPTA [114]). In this paper we show the expected constraints for the most ambitious experiment of this kind, i.e., the SKA [34].

All of the experiments listed above will target several GW sources, both stochastic and deterministic, but in the following we will be interested only in the stochastic ones, and in particular in the possibility of detecting a SGWB of the primordial origin. Therefore, we will consider other SGWB sources, such as unresolved Galactic and extra-Galactic compact binaries for instance, as a foreground or confusion noise to our sought-after primordial signal.

In this Section, we first describe the formalism required to compute the sensitivity curves for the direct detection experiments (Section 4.1). We then describe in detail our choices con-
cerning the astrophysical foreground contamination (Section 4.2) and how it affects the sensitivity curve for each experiment. In Section 4.3 we describe how we calculate the sensitivity curve for the SKA. To supplement these sections, in Appendix we describe the construction of the interferometers response functions (Appendix A) and the noise properties of each direct experiment (Appendix B).

4.1 Instrumental Sensitivity Curves

In this Section we derive the equation for the sensitivity curve of a GW laser interferometer to an homogeneous and isotropic SGWB. Three of the experiments considered in this work (µAres, DECIGO, BBO) are designed as two independent triangular interferometers, with consequently uncorrelated instrumental noises. The target of these experiments is to measure the cross-correlation of the outputs of the two independent detectors. Therefore, in the following we will provide formulae for both the sensitivity of a single detector (suited for LISA, DO, ET and CE) and for the cross-correlation of two independent detectors. Our discussion follows Refs. [45, 115, 116], and we refer to those papers for a more complete and detailed derivation. For a derivation of the sensitivity curve of a PTA experiment, which will not be reproduced here, we refer the reader to Refs. [115, 117].

A SGWB can be expanded in plane waves as

\[ h_{ab}(t, \vec{x}) = \int_{-\infty}^{+\infty} df \int d^2\hat{n} \sum_{P=+,\times} \tilde{h}_P(f, \hat{n}) e^{P}_{ab}(\hat{n}) e^{2\pi f(t-\hat{n} \cdot \vec{x})}, \]  \hspace{1cm} (4.1)

where \( \tilde{h}_P \) is the amplitude of a sinusoidal plane GW, \( P = +, \times \) is the linear polarization state of GW, \( \hat{n} \) the GW propagation direction and \( e^P_{ab} \) the polarization tensor. In time domain, the data \( d_I \) of a detector \( I \) can be written as the sum of the signal \( s_I \) and noise \( n_I \)

\[ d_I(t) = s_I(t) + n_I(t). \]  \hspace{1cm} (4.2)

Moving to Fourier space, the noise spectrum for a single detector is determined by

\[ \langle \tilde{n}_I(f) \tilde{n}_I^*(f') \rangle = \frac{1}{2} \delta(f - f') S_n^I(f). \]  \hspace{1cm} (4.3)

Similarly, we define the GW signal strain power spectrum \( S_s \) through the correlation of the GW Fourier modes defined in Eq. 4.1:

\[ \langle \tilde{h}_P(f, \hat{n}) \tilde{h}_{P'}^*(f', \hat{n}') \rangle = \frac{1}{2} \delta(f - f') \frac{\delta^{(2)}(\hat{n} - \hat{n}')}{{4\pi}} \delta^{PP'} S_s(f). \]  \hspace{1cm} (4.5)

We can now introduce the response function \( T_I^P(f, \hat{n}) \) to describe the signal response of a detector \( I \) to a sinusoidal plane GW, which will be computed in Appendix A for several

---

\^ More generally, the covariance matrix of \( \tilde{h}_P \) can be written in terms of the “GW Stokes parameters” in analogy to the electromagnetic waves [118]

\[ \langle \tilde{h}_P(f, \hat{n}) \tilde{h}_{P'}^*(f', \hat{n}') \rangle = \frac{1}{2} \delta(f - f') \frac{\delta^{(2)}(\hat{n} - \hat{n}')}{{4\pi}} \left( \frac{I + Q}{U + iV} \frac{U - iV}{I - Q} \right). \]  \hspace{1cm} (4.4)

Here, \( I \) is the Stokes I and should not be confused with the index for the detector used in the main text. Circular polarization from chiral GW due to the SU(2) gauge field would appear as the Stokes V [69]. In this paper we are concerned only with the total intensity of the SGWB and ignore \( Q, U, \) or \( V, \) hence \( \delta^{PP'} \) in Eq. 4.5.
different detector configurations. Using this we write the signal response \( \tilde{s}_I(f) \) of a detector \( I \) in Fourier space as

\[
\tilde{s}_I(f) = \int d^2 \hat{n} \sum_{P=+,-} T_P^I(f, \hat{n}) h_P(f, \hat{n}),
\]

(4.6)

with \( T_P^I(f, \hat{n}) = T^{ab}_I(f, \hat{n}) c_{ab}^P(\hat{n}) e^{-i2\pi f \hat{n} \cdot \hat{x}} \).

For a network of detectors \( I, J = 1, 2, ... \), we write

\[
\langle \tilde{s}_I(f) \tilde{s}_J^*(f') \rangle = \frac{1}{2} \delta(f - f') \tilde{C}_{IJ}(f) = \frac{1}{2} \delta(f - f') \mathcal{R}_{IJ}(f) S(f),
\]

(4.7)

where \( \tilde{C}_{IJ} \) is the covariance matrix of the signal response defined by

\[
\tilde{C}_{IJ} = \langle \tilde{s}_I \tilde{s}_J \rangle - \langle \tilde{s}_I \rangle \langle \tilde{s}_J \rangle,
\]

(4.8)

and \( \mathcal{R}_{IJ}(f) \) is the so-called overlap reduction function for the detector pair \( IJ \) [119] (see also discussion in Appendix A)

\[
\mathcal{R}_{IJ}(f) = \int d^2 \hat{n} \sum_{P=+,-} T_P^I(f, \hat{n}) T_P^J(f, \hat{n}).
\]

(4.9)

It can be shown that the optimal signal-to-noise ratio (hereafter SNR) for a cross-correlation measurement of a SGWB using a network of detectors \( I, J = 1, 2, ... \), takes the form [115, 116]

\[
\text{SNR} = \left[ \frac{3H_0^2}{4\pi^2 f^3} \Omega_{GW}(f) \right]^{1/2},
\]

(4.10)

where \( n_{\text{det}} \) is the number of detectors in the network, \( T \) is the mission observation time and \( [f_{\text{min}}, f_{\text{max}}] \) is the detector pair bandwidth.

Since the GW strain power spectrum density can be related to the fractional energy density spectrum in GW as [115]

\[
S_s(f) = \frac{3H_0^2}{4\pi^2 f^3} \Omega_{GW}(f),
\]

(4.11)

we can write the sensitivity curve in terms of the minimum detectable gravitational wave energy density \( \Omega_{GW} \) with the desired SNR in a frequency bin \( \Delta f \) as [45]

\[
\Omega_{GW}^{\text{min}}(f_i) = \left[ n_{\text{det}} T \int_{f_i-\Delta f/2}^{f_i+\Delta f/2} \left( \frac{3H_0^2}{4\pi^2} \right)^2 \sum_{J>I} \mathcal{R}_{IJ}^2(f) \frac{S_{n}^I(f)S_{n}^J(f)}{S_{n}^I(f)S_{n}^J(f)} \right]^{-1/2}.
\]

(4.12)

Another useful quantity, which is common in the literature, is the strain spectral sensitivity \( S_h \) for the detector network, defined as

\[
S_h = \left( \sum_{J>I} \frac{\mathcal{R}_{IJ}^2(f)}{S_{n}^I(f)S_{n}^J(f)} \right)^{-1/2}.
\]

(4.13)

In Appendices A and B, we give details on our computations for the overlap reduction function \( \mathcal{R}_{IJ}(f) \) and the noise spectrum \( S_n(f) \) for each experiment, respectively. In Figure 3 we show the strain sensitivity curves for all the direct detection experiments considered in this paper.

– 14 –
Figure 3: Strain sensitivity curves (without the contribution of astrophysical foregrounds discussed in Section 4.2) for all the direct detection and PTA experiments considered in this paper. We also plot for reference the strain curve for the Advanced LIGO (aLIGO) experiment.

4.2 Astrophysical Foregrounds for GW Direct Detection Experiments

The main sources of astrophysical foregrounds for direct detection experiments are Galactic compact binaries, mainly White Dwarf (hereafter WD) binaries, extra-Galactic WD binaries and Neutron Star (NS) binaries. Moreover, at frequencies below $\sim 10^{-5}$ Hz, which are probed by the $\mu$Ares and SKA experiments, we cannot neglect the contribution due to the coalescence of Massive Black Hole Binaries (MBHB). Therefore, the total strain sensitivity for the SGWB should be given by the sum of the instrumental sensitivity $S_h$, computed in Section 4.1, and the strain noise associated to each of the foreground components

$$S_{\text{tot}} = S_h + S_{\text{gal}} + S_{\text{exgal}} + R_{\text{NS}} S_{h}^{\text{NS}} + S_{h}^{\text{MBHB}},$$

where $S_{\text{gal}}$ represents the contribution of the Galactic WD binaries, $S_{\text{exgal}}$ the extra-Galactic WD, $S_{h}^{\text{NS}}$ the NS binaries, and $S_{h}^{\text{MBHB}}$ the MBHB. We consider also a suppression factor $R_{\text{NS}}$ multiplying $S_{h}^{\text{NS}}$ to quantify the subtraction of individually resolved NS binaries, according to the experiment specifications [120].

We now describe the model adopted for each of the foreground contributions mentioned above, starting with the Galactic binaries confusion noise. Following Refs. [121, 122], we parametrize it as

$$S_{h}^{\text{gal}}(f) = A f^{-7/3} e^{-f^\alpha + \beta f} \sin \kappa f [1 + \tanh(\gamma (f_k - f))] \text{Hz}^{-1},$$

where $A = 9 \times 10^{-45}$ and the parameters $\alpha$, $\beta$, $\kappa$, $\gamma$ and $f_k$ are reported in Table 1 of Ref. [122]. These parameters vary according to the total mission observation time, hence the amount of cleaning that is possible to perform on the data. On the other hand, the contribution of the extra-Galactic WD binaries remaining after the subtraction of the individually resolvable
sources can be analytically approximated as [120]

\[ S_{h}^{\text{exgal}} = 4.2 \times 10^{-47} \left( \frac{f}{1 \text{ Hz}} \right)^{-7/3} \exp \left( -2 \left( \frac{f}{5 \times 10^{-2} \text{ Hz}} \right)^{2} \right) \text{Hz}^{-1}. \]  

(4.16)

Finally, we model the NS binaries’ strain spectrum as [120]

\[ S_{h}^{\text{NS}} = 1.55 \times 10^{-47} h^{-3} \left( \frac{M_{c}}{1.22 M_{\odot}} \right)^{5/3} \left( \frac{f}{1 \text{ Hz}} \right)^{-7/3} \left( \frac{\dot{n}_{0}}{10^{-6} \text{ Mpc}^{-3} \text{ yr}^{-1}} \right), \]  

(4.17)

where \( M_{c} = 1.22 M_{\odot} \) and \( \dot{n}_{0} = 10^{-6} \text{ Mpc}^{-3} \text{ yr}^{-1} \) is the binary NS merger rate at the present time. As for the unresolved MBHB foreground, we use the analytical model given in [123]

\[ S_{h}^{\text{MBHB}} = h_{0}^{2} \left( \frac{f}{f_{0}} \right)^{-4/3} \left( 1 + \frac{f}{f_{0}} \right)^{2\gamma}, \]  

(4.18)

where the parameters \( h_{0}, f_{0} \) and \( \gamma \) are determined by the particular astrophysical model assumed for the MBHB system. The shape and amplitude of the MBHB foreground can vary greatly according to the theoretical model considered and, in particular, to the eccentricity of the binary system. However, just for the purpose of showing an indicative level for this foreground, we adopt the VHMhopk model [124], with parameters \( h_{0} = 0.69 \times 10^{-15}, f_{0} = 4.27 \times 10^{-8} \text{ Hz} \) and \( \gamma = -1.08 \), which are consistent with the current upper limits from the 11-year NANOGrav data set [81].

We discuss now, on a case-by-case basis, the effect of the contamination of the astrophysical foregrounds on the detection of a SGWB by direct detection experiments.

Let us start with LISA. As evident from the left panel of Figure 4, the WD binaries constitute the relevant confusion noise source in the LISA band. This foreground can be progressively reduced as the observation time is accumulated during the mission, allowing to resolve more sources and iteratively subtract them from the data. Therefore, the parameters for the model in Eq. 4.15 are determined by the effective duration of the LISA mission (Table 2). The NS and extra-Galactic WD foregrounds, also shown in the same figure, are well below the LISA instrumental noise, and therefore can be neglected in this case.

The DO interferometer, in its ‘‘Optimal’’ incarnation, suffers mainly from the presence of the NS binaries, while its less sensitive ‘‘Conservative’’ design is almost unaffected (right panel of Figure 4). The contribution from WD, both Galactic and extra-Galactic, is almost irrelevant in both designs after the subtraction procedure. The same holds for the AEDGE experiment (left panel of Figure 5), since it has similar sensitivity and frequency range to DO ‘‘Optimal’’ (Figure 3).

The low-frequency part of the \( \mu \)Ares sensitivity curve appears to be strongly affected by the Galactic WD foreground, which dominates over the confusion noise from the extra-Galactic WD and NS binaries by an order of magnitude in almost all the experiment bandwidths (right panel of Figure 5). In addition to the usual three foreground components we described above, we must also take into account the non-negligible contribution of the coalescence of MBHB (dot-dashed purple curve) between \( 10^{-7} \) to \( 10^{-5} \text{ Hz} \).

---

5 We report here two recent papers on the subject of foreground cleaning for LISA. In [125], the authors propose to perform foreground cleaning for LISA using observations in other frequency bands, e.g., the ones typical of ground-based detectors. In [126], instead, a Principal Component Analysis technique is used to reduce the degradation due to the black hole and neutron star binaries.
Figure 4: Characteristic strain curves with (solid lines) and without foregrounds (dashed lines) for LISA (left panel) and DO Optimal/Conservative (right panel). Also the foregrounds due to the Galactic WD after removal of resolved sources (dot-dashed blue) and the NS (dot-dashed orange) and extra-Galactic WD binaries (dot-dashed red) are shown for each experiment.

Figure 5: Same as Figure 4 but for AEDGE (left panel) and µAres (right panel). The relevant foregrounds are the NS (dot-dashed orange) and extra-Galactic WD binaries (dot-dashed red) for AEDGE and additionally the unresolved MBHB (dot-dashed purple) and Galactic WD binaries (dot-dashed blue) for µAres.

As for DECIGO and BBO, we adopt the procedure outlined in Ref. [120]. As can be seen from the left panel of Figure 6, DECIGO will be affected mainly by the extra-Galactic WD confusion noise at frequencies $10^{-3} - 10^{-1}$ Hz, while the NS binaries contribution can be almost completely cleaned out, reaching a fractional residual $R_{NS} = 4.62 \times 10^{-3}$ over an observation time of 10 yr. The situation of BBO is similar (right panel of Figure 6), but its deeper sensitivity allows to fully subtract the contamination of the NS binaries ($R_{NS} = 0$).

For the two ground based detectors ET and CE, probing similar frequency bands with similar sensitivities, we use the residual astrophysical foregrounds computed in Ref. [110] assuming a network of 5 detectors: one detector with the sensitivity of ET located at the current Virgo location and four with the sensitivity of CE at the location of LIGO Hanford, LIGO Livingston, LIGO India and KAGRA [see 110, and references therein]. The main source of the confusion noise in this frequency band is the NS binaries, while the contribution from black hole binaries has been shown to be completely removable through the cleaning procedure. The residual foregrounds contribution is so overwhelming with respect to the
Figure 6: Same as Figure 4 but for DECIGO (left panel) and BBO (right panel). The relevant foregrounds are the extra-Galactic WD (dot-dashed red) and the NS binaries (dot-dashed orange) before cleaning ($R_{NS} = 1$). For DECIGO we also show the NS binaries after cleaning with $R_{NS} = 4.62 \times 10^{-3}$ (dot-dashed green), while BBO is supposed to subtract all the contamination of the NS binaries ($R_{NS} = 0$), hence not shown in the right panel.

instrumental sensitivity of the ground-based network (see Figure 8) that we sum it to the single detector sensitivity of either ET or CE rather than to the network one, obtaining rather similar results.

4.3 SKA

For SKA we optimistically include only the white noise component in the noise budget; however, we note that the so-called “red noise” component due to pulsar timing noise [117] could be present in the data, raising considerably the noise level in the lower frequency part of the PTA sensitivity curves. We use the the codes hasasia\textsuperscript{6} [117] and gwent\textsuperscript{7} to compute the sensitivity to the SGWB, choosing for the pulsars an rms timing residual of $\sigma_t = 10$ ns, an observing time $T = 15$ yr, a number of pulsars $N_p = 200$ and an average observation cadence of 1 per week. Moreover, we choose the “Realistic Noise” model for SKA described in the gwent documentation, which uses the NANOGrav 11-yr data for 34 pulsars for the individual pulsar noises. Since we simulate 200 pulsars, the hasasia code draws the remaining pulsars from distributions based on the NANOGrav pulsar noise.

As for the foreground, Figure 7 shows that the most sensitive part of the SKA bandwidth will be limited by the presence of the MBHB astrophysical foreground.

5 Results

In this Section we present the forecasts for CMB, PTA, and direct detection experiments, described respectively in Sections 3 and 4. We will first show the binned sensitivity curves obtained for LiteBIRD, SKA, and all the direct detection experiments (Section 5.1). Then, in Section 5.2 we will proceed to show the error bars for each experiment and each of the five example tensor power spectrum models described in Section 2.

\textsuperscript{6}https://hasasia.readthedocs.io/en/latest/index.html
\textsuperscript{7}https://gwent.readthedocs.io/en/latest/index.html
Figure 7: Characteristic strain curves with (solid line) and without foregrounds (dashed line) for SKA. The relevant foreground for this experiment, namely the unresolved MBHB noise, is shown in the dot-dashed purple line.

Figure 8: Sensitivity curves on the energy density of gravitational waves $\Omega_{GW}$ with (solid lines) and without (dashed lines) the contribution of the astrophysical foregrounds for all the experiments considered in this work, obtained with a logarithmic binning in wavenumber with $\Delta \ln k = 1.2$. We also plot for reference the sensitivity curve for the aLIGO experiment without the astrophysical foregrounds.

5.1 Binned $\Omega_{GW}$ Sensitivity Curves

We calculate the binned sensitivity curves to the gravitational wave energy density $\Omega_{GW}$ using Eq. 3.12 for the LiteBIRD CMB experiment, Eq. 4.12 for all the direct detection experiments, and the procedure described in Section 4.3 for the SKA. We plot them in Figure 8, choosing $\Delta \ln k = 1.2$ as the power spectrum discretization scale. The solid and dashed lines show the sensitivities obtained with and without the foregrounds, respectively. The sensitivity of a CMB experiment to $\Omega_{GW}$, as computed in Section 3.3, depends on the fiducial tensor power spectrum used to compute the $C_l$ in the Fisher matrix given in Eq. 3.10; thus, the sensitivity of LiteBIRD is computed for $r = 0.01$ (in green) and for $r = 0.001$ (in red).
We find that the best sensitivity of LiteBIRD (including foregrounds) at frequencies $f \sim 10^{-17}$ Hz is similar to those of the most advanced among the interferometers, namely DECIGO and BBO at $f \sim 10^{-1}$ Hz. However, when plotting error bars on the model predictions in the next sub-Section, we find that the shape of the GW spectrum is very different for CMB and interferometer frequencies. It has a rising spectrum towards the CMB frequency after the transition between the matter and radiation dominated eras, while for the single-field slow-roll model it rapidly flattens out at higher frequencies, making a detection challenging for interferometers. The situation changes dramatically for some parameter choices of the axion-SU(2) model, which can produce a strongly blue-tilted signal easily detectable at interferometer frequencies [69].

Figure 8 also highlights the fact that the frequency window between $\sim 10^{-16} - 10^{-9}$ Hz is devoid of any experiment. The constraints on the SGWB intensity in this range come only from indirect limits, such as the BBN, second-order back-reaction and CMB shortwave calculations [79]. Also the range $\sim 10^{-9} - 10^{-7}$ Hz, belonging to the PTA experiments, calls for new methods to improve upon the sensitivity to a SGWB. PTA experiments may also be more useful when considering different SGWB models, see for instance [127].

Concerning the effect of the foregrounds, we find significant impacts in the frequency range $10^{-9} - 10^{-3}$ Hz. The SKA and the $\mu$Ares experiments are particularly affected and require more efficient foreground cleaning techniques to reach interesting levels of sensitivity to an SGWB from single-field slow-roll inflation models, while the situation changes for the axion-SU(2) models, as described in the next sub-Section. Note that the LiteBIRD sensitivity always includes the foregrounds.

5.2 Error bars for the spectator Axion-SU(2) models

Next, we calculate the $1\sigma$ error bars on $\Omega_{GW}$ for five models of the primordial tensor spectrum. Of these, three are the AX1, AX2 and AX3 models defined in Section 2.2 (see Eq. 2.7), while two are single-field slow-roll ones with $r = 0.01, 0.001$ and $n_T = -r/8$. In this Section we discuss the results for the former models, while in the next Section 5.3 we discuss the latter.

In Figure 9 (Figure 10), we show the results for LiteBIRD, SKA, LISA and ET (CE). The light and dark shaded areas show the error bars for the AX1 model with and without the astrophysical foregrounds included in our calculation. We always take the foregrounds into account for the LiteBIRD CMB satellite, as explained in Section 3.2.

For what concerns the AX1 model, we tuned its parameter set to have simultaneous detections in both the CMB and the interferometers ranges, while still being consistent with the BICEP2/Keck/Planck upper bound at CMB scales (the dashed pink curve in Figure 9). As can be seen from the plots, this model cannot be detected in the PTA range, even neglecting the foreground contamination. By observing closely the CMB part of the spectrum, the LiteBIRD error bars clearly show two peaks of sensitivity corresponding to the reionization bump (second bin from the left) and the recombination bump (fourth and fifth bins from the left), as we anticipated in Section 3.

The ground-based ET and CE (Figures 9 and 10) show similar sensitivities in roughly the same frequency range, and both of them have detections only in the absence of the foreground. We have tried to tune the axion-SU(2) parameter set to have detections from ground-based interferometers in the presence of the foreground, but were not successful due to the attractor behaviour of the theory and the CMB upper bounds, as explained in Section 2.2.

In Figures 11–13, we show the expected error bars for the AX1 model for the other direct detection experiments. We show the error bars only for the experiments that can give
Figure 9: Expected 1σ error bars on $\Omega_{GW}$ for the AX1 model (the solid blue line) for the LiteBIRD (green), SKA (orange), LISA (blue), and ET (purple). We show the constraints with and without the astrophysical foregrounds in the light and dark shared areas, respectively. We use the logarithmic binning in wavenumber with $\Delta \ln k = 1.2$. We also show for comparison the other tensor spectrum models adopted in this paper (dashed lines), including the BICEP2/Keck/Planck upper bound $r = 0.06$.

Figure 10: Same as Figure 9 but with the CE (cyan).

... a detection (without the foregrounds contamination) in at least one bin. Therefore, we show DO Conservative, DO optimal and AEDGE only for the AX1 model, which has the strongest
signal in the frequency range favorable to them. We do not show them for the other models because they would not be able to have a detection in at least one bin. However, we make an exception for DECIGO and BBO and do not show their error bars for the AX1 and AX3 models despite excellent prospects for the detection, since these experiments are so sensitive that the error bars would be invisible.

Figures 11 and 12 show that the error bars for the DO Conservative and Optimal designs are similar for this particular model, with the less-sensitive Conservative setup having two

---

**Figure 11**: Same as Figure 9 but for the DO Conservative.

**Figure 12**: Same as Figure 9 but for the DO Optimal.
Figure 13: Same as Figure 9 but for the AEDGE. Note a different scale for the vertical axis.

detections missing with respect to the Optimal case in the last two bins. In both cases, the foreground contamination appears to be negligible, apart from the first bin. Figure 13 shows the error bars for the AEDGE atomic interferometer: this detector shows a similar sensitivity to the DO Optimal design, with the latter being slightly less sensitive while covering a wider frequency range.

The error bars on the AX1 model for the μAres mission are shown in Figure 14. The foreground contamination plays a minor role in this very high SNR case, and the μAres is capable of detecting this model across an impressive range of frequencies $\sim 10^{-6} - 10^{-2}$ Hz.

Next, we show the error bars for the AX2 model. This set was specifically tuned to show the capability of the axion-SU(2) to produce a signal out of the reach of LiteBIRD while being detectable in the interferometer bands. For this case we use a larger bin size, $\Delta \ln k = 2.0$. The situation is now different for the μAres experiment (Figure 15): the effect of the foregrounds contamination is dramatic in this case and we shift from highly significant detections in three bins to no detection at all. In Figures 16 and 17, we show that DECIGO and BBO can detect at high significance the AX2 model in two bins if we account for foregrounds, and in three bins if we ignore them.

We have explored the possibility of having an axion-SU(2) tensor spectrum peak in the PTA frequency range with the AX3 model, but we could not succeed in obtaining a signal detectable by SKA (even without the foreground contamination) while still complying with the BICEP2/Keck/Planck upper bound on CMB scales (Figure 18). This is due to the attractor nature of the axion-SU(2) model, which poses a minimum value for the Gaussian width of the spectrum bump $\sigma$ for a given peak scale $k_p$ (see Section 2.2).

### 5.3 Error bars on single-field slow-roll models and combined constraints on $n_T$

We consider now the two single-field slow-roll models with $r = 0.01$ and 0.001, both with $n_T = -r/8$. We choose a bin size of $\Delta \ln k = 2.0$. The model with the larger $r = 0.01$ is easily detected by LiteBIRD in multiple bins. On interferometric scales, it can be detected in
Figure 14: Same as Figure 9 but for the $\mu$Ares. Note a different scale for the vertical axis.

Figure 15: Expected $1\sigma$ error bars on $\Omega_{GW}$ for the AX2 model (the solid orange line) for the LiteBIRD (green) and $\mu$Ares (orange). We show the constraints with and without the astrophysical foregrounds in the light and dark shared areas, respectively. We use the logarithmic binning in wavenumber with $\Delta \ln k = 2.0$. We also show for comparison the other tensor spectrum models adopted in this paper (dashed lines).

one bin by DECIGO (Figure 19) and in two bins by BBO (Figure 20), with negligible effects of the foreground contamination. On the other hand, the effect of the foregrounds is severe for the $\mu$Ares, as evident from Figure 21.
To have a detection in at least one bin for the lower $r = 0.001$, we increase the binning scale to $\Delta \ln k = 4.0$. This model is detected by LiteBIRD on the CMB side, while only BBO, the most sensitive among the considered direct detection experiments, can detect it on the side of interferometers (Figure 22). Notably, BBO is able to detect this signal even when the foregrounds are taken into account.

With the extremely high sensitivity of BBO at frequencies $\sim 16$ orders of magnitude larger than the CMB, ones creates a significant lever-arm, providing interesting constraints.
Figure 18: Same as Figure 10 but for the AX3 model, with a logarithmic binning of $\Delta \ln k = 2.0$.

Figure 19: Expected 1σ error bars on $\Omega_{GW}$ for the single-field slow-roll model with $r = 0.01$ and $n_T = -r/8$ (the solid red line) for the LiteBIRD (green) and DECIGO (blue). We show the constraints with and without the astrophysical foregrounds in the light and dark shared areas, respectively. We use the logarithmic binning in wavenumber with $\Delta \ln k = 2.0$. We also show for comparison the other tensor spectrum models adopted in this paper (dashed lines).
on the spectrum tilt $n_T$. The path of multi-frequency measurements of the primordial tensor spectrum has been explored in the past, in the context of forecasts [see for instance 128, for the combination of Planck/CMBPol and DECIGO/BBO], as well as of the analysis of available datasets, combining for instance the Planck, BICEP2/Keck, PPTA and LIGO data [129], and adding SPTPol [130] or COrE and other indirect constraints [80] to the previous datasets.

Here, we update the forecasts on the tensor power spectrum amplitude $r$ and the tilt

Figure 20: Same as Figure 19 but for the BBO.

Figure 21: Same as Figure 19 but for the µAres.
Figure 22: Same as Figure 20 but for $r = 0.001$ with a logarithmic binning of $\Delta \ln k = 4.0$.

... from the combination of CMB and laser interferometers, considering in particular the two configurations LiteBIRD+LISA and LiteBIRD+BBO. We take into account foregrounds for all experiments. We bin the LISA and BBO sensitivity curves with $\Delta \ln k = 2.0$ and 4.0, respectively. We explore the full cosmological parameters space including $\{A_S, n_S, \tau, \Omega_b h^2, \Omega_c h^2, H_0, r, n_T\}$ via the Monte Carlo Markov Chain (MCMC). We modify the MontePython MCMC package [131, 132] by adding a Gaussian likelihood for the interferometers [133]

\[ L(\hat{\Omega}_i, \sigma_i|\vec{\theta}) \propto \exp \left[ \frac{1}{2} \sum_i \frac{(\hat{\Omega}_i - \Omega_M(f_i; \vec{\theta}))^2}{\sigma^2_i} \right], \quad (5.1) \]

where $\Omega_M(f|\vec{\theta})$ is the proposed model as a function of frequency $f$ and model parameters $\vec{\theta}$, $\hat{\Omega}_i$ the fiducial model in the frequency bin $f_i$ and $\sigma^2_i$ its variance in the same bin. For the CMB, we adopt instead the standard Gaussian likelihood [134], with noise and foregrounds $C_\ell$ spectra determined from the LiteBIRD specifications (see Section 3.2).

We adopt for the fiducial model $r = 0.01$ and $n_T = -r/8$ given by the inflationary consistency relation, while the values for the other cosmological parameters are taken from Ref. [13]. We show in Figure 23 the 1D and 2D marginal distributions of the $n_S$, $r$ and $n_T$ parameters for four possible observational configurations: (i) constraints from LiteBIRD alone (red contours); (ii) constraints from LiteBIRD and LISA (grey contours), (iii) constraints from LiteBIRD and BBO (blue contours); and (iv) constraints from LiteBIRD and BBO assuming the fiducial signal in the LiteBIRD range but no signal in the BBO range, that is $\hat{\Omega}_i = 0$ in every bin $f_i$ (orange contours). This configuration is chosen to quantify possible deviations from the consistency relation in the eventuality of a detection at $\sim 5\sigma$ by LiteBIRD, but no detection in BBO.

For (i) we recover the following best-fitting parameters with 1σ uncertainties: $n_S = 0.9665^{+0.017}_{-0.018}$, $r = 0.013^{+0.005}_{-0.006}$ and $n_T = 0.09^{+0.18}_{-0.20}$; thus, a test of the consistency relation is
out of discussion using the CMB alone: only extreme deviations from the consistency relation (e.g., axion-SU(2) models) can be detected in this case.

For (ii) the addition of LISA impacts mainly the error on $n_T$ by limiting the range of allowed blue-tilted models, but this is still not enough to distinguish the consistency relation from the scale-invariant case. In this case the recovered parameters are $n_S = 0.9665^{+0.017}_{-0.017}$, $r = 0.0108^{+0.003}_{-0.004}$ and $n_T = 0.017^{+0.18}_{-0.10}$. The further inclusion of the ground-based interferometers ET or CE jointly with LISA does not improve significantly the constraints with respect to LISA alone because of the large foreground contamination affecting both experiments.

For (iii) the effect of adding BBO is evident in Figure 23: the constraints on $r$ and $n_T$ become significantly tighter and also the maxima of the marginal distributions for the recovered parameters are very close to their fiducial values. Using the LiteBIRD+BBO configuration, we recover the following parameters: $n_S = 0.9649 \pm 0.017$, $r = 0.0100 \pm 0.0011$ and $n_T = -0.00125 \pm 0.0045$. Also in this case, however, the error on the tensor spectral index, although remarkably smaller than the LiteBIRD only case, does not allow to distinguish the
consistency relation from a scale-invariant case.

For (iv) we recover $n_S = 0.970 \pm 0.017$, $r = 0.0073^{+0.0016}_{-0.0015}$ and $n_T = -0.16^{+0.11}_{-0.04}$. As it can be argued from Figure 23, the recovered tensor-to-scalar ratio shows a bias: this is because, to have an undetectable signal at interferometers scales, the spectrum must have a large red tilt, so large that it affects also the CMB scales. Therefore, even in the absence of a consistency relation detection, if we do not detect a signal in BBO, the red tilt in the power-law model of tensor power spectrum has to be so large that we can detect its departure from the single-field slow-roll consistency relation.

6 Conclusions and prospects

We have calculated the sensitivities of CMB, PTA, and direct detection experiments for SGWB from the primordial GW across 23 decades in frequency. Not only do we provide the sensitivity curves for the GW energy density parameter $\Omega_{GW}$ (Figure 8) as commonly done in the literature, but also we provide the binned 1σ error bars on the model predictions for $\Omega_{GW}$ from two representative classes of sources of the primordial SGWB: the quantum vacuum fluctuation in the metric tensor (i.e., the homogeneous solution of Einstein’s equation) from single-field slow-roll inflation models with $r = 0.01$ and 0.001 and the tensor tilt given by the consistency relation $n_T = -r/8$, and the source-induced primordial GW from the spectator axion-SU(2) model (i.e., from the stress energy tensor in the right hand side of Einstein’s equation).

For CMB and PTA we considered the most ambitious future experiments LiteBIRD and SKA, respectively, while for direct detection experiments we considered a host of funded and proposed space (LISA, $\mu$Ares, DO, AEDGE, DECIGO, BBO) and ground-based (ET, CE) GW observatories covering a wide range of frequencies from $10^{-7}$ to $10^3$ Hz. We took into account the instrumental noise, the response functions, and most importantly the contamination of the astrophysical foregrounds in the forecasts. We have presented all the details in our computation with homogeneous assumptions for all experiments in one place, which should provide convenient resources for the experiments in search of the primordial SGWB.

We showed that it is possible to tune the axion-SU(2) model parameters to have detections with high significance in multiple frequency bins in both the CMB and space interferometers frequency ranges, even when accounting for the foreground contamination (Figures 9-14), while remaining consistent with all current upper limits. We also showed that the parameters of the axion-SU(2) model can be chosen in such a way that the signal is out of reach for CMB experiements, while being detectable by the most sensitive space interferometers, i.e., $\mu$Ares (but only without the foregrounds) and DECIGO and BBO (even with the foregrounds; Figures 15-17).

On the other hand, the situation is different for future ground-based interferometers, for which the current estimates for the foreground contamination prevent detections of the axion-SU(2) model. It is also difficult to obtain a tensor spectrum detectable by the SKA experiment on PTA scales (even in the absence of the foreground), while still complying with the BICEP2/Keck/Planck upper bound on CMB scales (Figure 18). This is due to the attractor behaviour of the axion-SU(2) model, posing an upper limit on the width of the spectrum bump for a given peak scale $k_p$.

For what concerns the single-field slow-roll power spectrum, we showed that the $r = 0.01$ model can be detected comfortably and simultaneously by LiteBIRD, DECIGO and BBO, while $\mu$Ares is able to measure it only if we do not account for the foregrounds (Figures
We also found that the lower tensor-to-scalar ratio $r = 0.001$ can be detected only by LiteBIRD and BBO (Figure 22).

Finally, we presented updated constraints on $r$ and $n_T$ combining LiteBIRD with LISA and LiteBIRD with BBO, to leverage on the scale dependence of the tensor spectrum. We conclude that distinguishing the single-field slow-roll consistency relation from the scale-invariant case remains out of reach even for LiteBIRD+BBO. However, if we detect tensors in the CMB but not in BBO, we would detect a significant deviation from the consistency relation in the context of the power-law primordial spectrum.

If the primordial SGWB is discovered during the next decade by ground-based CMB observatories or LiteBIRD, characterizing the power spectrum beyond the value of $r$ and testing chirality and Gaussianity would be of utmost importance for deciphering of the origin of the SGWB. If the discovered SGWB were found to be nearly scale-invariant, parity even and Gaussian, it would set a target for the DECIGO and BBO to test the prediction of single-field slow-roll inflation models. On the other hand, if the SGWB were found to be blue-tilted, chiral or non-Gaussian, it would give excellent prospects for direct detection by LISA in the 2030s as well as by other proposed post-LISA direct detection experiments at any frequencies, opening up a new window to particle physics during inflation.

Acknowledgments

We thank C. Berry, A. Sesana, and the AEDGE collaboration for providing us with the noise power spectra of D0, µAres, and AEDGE, respectively. We also thank J. Errard for sharing the multi-resolution analysis of the foreground removal and Joint Study Group of the LiteBIRD collaboration for useful discussions and the instrument specification given in Table 1. PC thanks Lumen Boco and Nicoletta Krachmalnicoff for useful discussions. EK thanks SISSA and IFPU for hospitality, where this work was initiated, and A. Buonanno for useful discussion on the future direct detection mission proposals. This work has been supported by the network COSMOS by the Italian Space Agency (cosmosnet.net) and by the INDARK specific initiative of the National Institute of Nuclear Physics. The work of EK was supported in part by the Excellence Cluster ORIGINS which is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy – EXC - 2094 – 390783311. We acknowledge the NERSC super-computing center in Berkeley and the Ulysses super-computer at SISSA for supporting numerical analyses in this work.

A Interferometers Designs and Response Functions

A necessary ingredient to compute the sensitivity curve of a GW direct SGWB experiment (Eq. 4.12) is the overlap reduction function $R_{IJ}(f)$ of the detector pair $IJ$ (Eq. 4.9) [119], which is computed from the response function $T_I(f, \hat{n})$ of each of the detector involved in the cross-correlation (Eq. 4.6). We summarize here the formalism necessary to compute it, following Ref. [45] to which we refer the reader for further details.

The overlap reduction function depends on the design of the detector and the combination of laser signals from the interferometer arms that we choose to form at the detector output. The response of space interferometers can also depend on time because of the orbital motion of the spacecrafts composing the detector; however, for simplicity we ignore this dependence.
Let us start by considering the response of a single arm of the interferometer, from which we build the response of the full detector. The physical principle behind the detection of GWs in a laser interferometer is simple: the passage of GWs changes the proper distance between two freely moving test-masses at the opposite ends of an interferometer arm, causing phase-shifts in the laser beams which are traveling back-and-forth in each arm. It can be shown [45] that the phase change due to light traveling from the test-mass \( i \) to the test-mass \( j \) along a single interferometer arm is

\[
\Delta \varphi_{ij}(t) = \int_{-\infty}^{+\infty} df \int d^2 \hat{n} \sum_{P=-,+,0} \bar{h}_P(f, \hat{n}) e^{i2\pi f t_i} e^{i\pi L (\hat{l}_{ij} \cdot \hat{n}, f)} T_{ij}^{ab}(\hat{l}_{ij} \cdot \hat{n}, f),
\]

(A.1)

where \( L \) is the arm length, the test-masses \( i \) and \( j \) are located at \( \vec{x}_i \) and \( \vec{x}_j + L \hat{l}_{ij} \), respectively, \( t_i \) is the time at which light left the mass \( i \), \( t \) is the time of arrival at the mass \( j \) and \( T^{ab} \) is the single-arm response function given by

\[
T^{ab}(\hat{l} \cdot \hat{n}, f) = \hat{\delta}^{ab} \mathcal{T}(\hat{l} \cdot \hat{n}, f) e^{-i2\pi \hat{n} \cdot \vec{x}_i},
\]

(A.2)

\[
\mathcal{T}(\hat{l} \cdot \hat{n}, f) = \frac{1}{2} \text{sinc} \left[ \frac{f}{2f^*} (1 - \hat{l} \cdot \hat{n}) \right] e^{i\pi f^* (1 - 2\hat{l} \cdot \hat{n})},
\]

(A.3)

where \( f^* = 1/(2\pi L) \). To measure the SGWB it is necessary to correlate the phase differences from different arms or paths around the interferometer. For example, we write the correlation between the \( i \to j \) and the \( k \to l \) paths as

\[
\langle \Delta \varphi_{ij}(f) \Delta \varphi_{kl}(f^*) \rangle = \frac{1}{2} \delta(f - f^*) \mathcal{R}_{ij,kl}(f) \mathcal{S}_s(f),
\]

(A.4)

where \( \mathcal{R}_{ij,kl} \) is the overlap reduction function defined in Eq. 4.9, which we rewrite in this case as

\[
\mathcal{R}_{ij,kl}(f) = \int \frac{d^2 \hat{n}}{4\pi} T^{ab}(\hat{l}_{ij} \cdot \hat{n}, f) T^{ac}(\hat{l}_{kl} \cdot \hat{n}, f).
\]

(A.5)

To build the detector responses for the experiments we consider in this paper, we start from the simplest design adopted for the LISA mission. The current proposal for LISA showcases three spacecrafts, each occupying a vertex from the simplest design adopted for the LISA mission. The current proposal for LISA triangle \( ABC \) response function at the detector vertex \( T_{ABC}(\hat{n}, f) \).

\[
T_{ABC}(\hat{n}, f) = \frac{1}{2} e^{-i2\pi f \hat{n} \cdot \vec{x}_A} \left[ \hat{l}_{AB} \otimes \hat{l}_{AB} \mathcal{T}(\hat{l}_{AB} \cdot \hat{n}, f) - (\hat{l}_{AC} \otimes \hat{l}_{AC}) \mathcal{T}(\hat{l}_{AC} \cdot \hat{n}, f) \right],
\]

(A.6)

\[
\mathcal{T}(\hat{l} \cdot \hat{n}, f) = \frac{1}{2} W(f, f^*) \left( \text{sinc} \left[ \frac{f}{2f^*} (1 - \hat{l} \cdot \hat{n}) \right] e^{-i\pi f^* (3 + \hat{l} \cdot \hat{n})} \right.
\]

\[
+ \left. \text{sinc} \left[ \frac{f}{2f^*} (1 + \hat{l} \cdot \hat{n}) \right] e^{-i\pi f^* (1 + \hat{l} \cdot \hat{n})} \right),
\]

(A.7)

where \( W(f, f^*) = 1 \) for the Michelson signals and \( W(f, f^*) = 1 - e^{-2i(f/f^*)} \) for the TDI signals we are interested in. Specifically, the TDI \( A \) and \( E \) modes overlap reduction function\(^8\) for

\(^8\)The three TDI signals are constructed by diagonalizing the signal covariance matrix and are named the \( A \), \( E \) and \( T \) modes. Note that Eq. A.8 is valid only for the \( A \) and \( E \) TDI modes, which happen to be the most sensitive to the SGWB, while the \( T \) mode is much less sensitive and is used instead to remove noise from the \( A \) and \( E \) modes [41].
LISA (the blue curve in Figure 24) will be
\[ \mathcal{R}_{A,E} = \mathcal{R}_{ABC,ABC} - \mathcal{R}_{ABC,BCA}, \tag{A.8} \]
where \( \mathcal{R}_{ABC,ABC} \) is the response for the auto-correlation at the vertex \( A \) and \( \mathcal{R}_{ABC,BCA} \) is the one for the cross-correlation between the signals at the vertices \( A \) and \( B \) \[41\].

In addition to TDI, another useful signal combination that we can form from the Michelson signals \( s_{mich,A}(t) \) and \( s_{mich,C}(t) \) at the vertices \( A \) and \( C \), respectively, is \[45\]
\[ s_X(t) = s_{mich,A}(t) + 2s_{mich,C}(t). \tag{A.9} \]
As shown in \[45\], it is convenient then to correlate the Michelson signal \( s_{mich,A}(t) \) with the signal combination \( s_X(t) \), because the total noises for these two signals will be uncorrelated over the frequencies at which space-based interferometers are typically most sensitive. In this case, the detector response function for the Michelson signal \( s_{mich,A}(t) \) at the vertex \( A \) takes the form in Eq. A.6, while the one for the \( s_X(t) \) signal combination is given by
\[ T_{ab}^{ab}(\hat{n}, f) = T^{ab}_{ABC}(\hat{n}, f) + 2T^{ab}_{CAB}(\hat{n}, f), \tag{A.10} \]
and for both responses the transfer function \( T(\hat{l} \cdot \hat{n}, f) \) is given by Eq. A.7 with \( W(f, f^*) = 1 \). The final overlap reduction function for this signal combination \[45\] will be
\[ \mathcal{R}_X = \mathcal{R}_{ABC,ABC} + \mathcal{R}_{X,X} + 2\mathcal{R}_{ABC,X}. \tag{A.11} \]
We use this particular combination of signals to compute the overlap reduction function for DO (green curve in Figure 24), which has been proposed as a LISA-like interferometer with shorter arms of length \( L = 10^8 \) m.

Differently from the LISA and DO detectors, BBO will feature six spacecrafts forming two independent triangular LISA-like interferometers \( ABC \) and \( A'B'C' \) with sides \( L = 5 \times 10^7 \) m. The two interferometers will be co-planar with one being rotated by 180 degrees with respect to the other, creating the so-called “hexagram” configuration. To compute the overlap reduction function for such a configuration, we cross-correlate the Michelson signal \( s_{mich,A}(t) \) at the vertex \( A \) on the interferometer \( ABC \) and the combination \( s_X(t) = s_{mich,A}(t)+2s_{mich,C}(t) \) on the other interferometer \( A'B'C' \) \[45\] (the black curve in Figure 24). The DECIGO design is similar to the BBO, with two independent triangular interferometers with arms \( L = 10^6 \) m disposed in the hexagram configuration. Unlike BBO, however, the current DECIGO design envisages Fabry-Pérot (hereafter FP) interferometers; the response function at the vertex \( A \) \[135\] becomes therefore
\[ T_{FP}^{ab}(\hat{n}, f) = \frac{1}{2} e^{-i2\pi f\hat{n} \cdot \hat{x}_A} \left[ (\hat{i}_{AB} \otimes \hat{i}_{AB}) - (\hat{i}_{AC} \otimes \hat{i}_{AC}) \right], \tag{A.12} \]
and – similarly to what we do for BBO – we cross-correlate it with the response at the vertex \( A' \) on the second interferometer, obtaining the overlap reduction function depicted in the orange curve in Figure 24.

The \( \mu \)Ares experiment will be composed, similarly to DECIGO and BBO, by two identical triangular LISA-like constellations with arms \( L = 430 \times 10^9 \) m. However, in this case one of the two triangular interferometers would be trailing Mars orbit within the ecliptic plane while the other would be in the same orbit but 90 degrees tilted with respect to the ecliptic plane \[42\]. In order to compute the overlap reduction function for \( \mu \)Ares, we adopt again the
Figure 24: Absolute value of the overlap reduction functions $|R_{IJ}|$ normalized to 1, computed for the interferometers LISA, DECIGO, DO, BBO, $\mu$Ares and CE.

We consider now CE, an L-shaped next-generation ground-based detector with arms $L = 4 \times 10^4$ m. We compute the overlap reduction function for the CE using the analytical fit shown in Eq. (A.33) of [116] (the red curve in Figure 24). Finally, we take into consideration the ET ground-based experiment. The current proposal consists of a network of three interferometers with arm opening of 60 degrees, arranged in a such a way to form an equilateral triangle. For the ET experiment there is no need to compute the overlap reduction function, since the strain sensitivity curves (as defined in Eq. 4.13) are publicly available\(^9\).

### B Interferometers Noise Models

To compute the sensitivity curve in Eq. 4.12 we need not only the overlap reduction function, but also the noise power spectral density $S_n(f)$ for each detector (Eq. 4.3). Let us start from the LISA mission. Following Ref. [41], we use the noise models reported in the LISA Science Requirements Document\(^10\): the two main noise sources are acceleration noise and optical metrology noise, with spectra

\[
S^{LISA}_{acc}(f) = \left(\frac{\sqrt{\langle \delta a \rangle^2}}{L}\right)^2 \left(1 + \left(\frac{f_1}{f}\right)^2\right) \text{Hz}^{-1},
\]

\[
S^{LISA}_{opt}(f) = \left(\frac{\sqrt{\langle \delta x \rangle^2}}{L}\right)^2 \text{Hz}^{-1},
\]

where $\sqrt{\langle \delta a \rangle^2} = 3 \times 10^{-15}$ m/s$^2$ and $\sqrt{\langle \delta x \rangle^2} = 1.5 \times 10^{-11}$ m are the rms amplitudes for acceleration and optical metrology noise, respectively, and $f_1 = 0.4$ mHz. The noise spectra for the TDI $A$ and $E$ signals that we used to compute the response function for LISA in

\(^9\)http://www.et-gw.eu/index.php/etsensitivities

\(^10\)https://www.cosmos.esa.int/web/lisa/lisa-documents
Appendix A are
\[
S_{n}^{A,E}(f) = |W(f, f^*)|^2 \left[ (4 + 2 \cos(f/f^*)) S_{\text{opt}}^{\text{LISA}} + 8(1 + \cos(f/f^*)) + \cos^2(f/f^*) S_{\text{acc}}^{\text{LISA}}(f) \right].
\] (B.3)

Combining the A and E modes, we reduce the noise power by a factor \(\sqrt{2}\) to obtain \([41]\)
\[
S_h^{\text{LISA}} = \left[ \left( \frac{R_A}{S_A} \right)^2 + \left( \frac{R_E}{S_E} \right)^2 \right]^{-1/2}.
\] (B.4)

For BBO \([109]\) we use
\[
S_{\text{acc}}^{\text{BBO}}(f) = 2.3 \times 10^{-52} (1 \text{ Hz}/f)^4 \text{ Hz}^{-1},
\] (B.5)
\[
S_{\text{opt}}^{\text{BBO}} = 8 \times 10^{-50} \text{ Hz}^{-1},
\] (B.6)

and the noise model for one of the two identical triangular interferometers proposed in \([45]\)
\[
S_n^{\text{BBO}} = \frac{5}{2} \left[ S_{\text{opt}}^{\text{BBO}}(f) + 2S_{\text{acc}}^{\text{BBO}}(f)(1 + \cos^2(f/f^*)) \right].
\] (B.7)

For DECIGO we use the noise model \([108]\):
\[
S_n^{\text{DECIGO}} = S_{\text{opt}}^{\text{DECIGO}}(f) + S_{\text{rad}}^{\text{DECIGO}}(f) + S_{\text{acc}}^{\text{DECIGO}}(f),
\] (B.8)

with shot noise, radiation pressure noise and acceleration noise given by
\[
S_{\text{shot}}^{\text{DECIGO}}(f) = \frac{\hbar \pi \lambda}{P_{\text{eff}}} \left( \frac{1}{4fL} \right)^2 \left[ 1 + \left( \frac{f}{f^*} \right)^2 \right],
\] (B.9)
\[
S_{\text{rad}}^{\text{DECIGO}}(f) = \frac{hP}{\pi \lambda} \left( \frac{16F}{ML} \right)^2 \left( \frac{1}{2 \pi f} \right)^4 \left[ 1 + \left( \frac{f}{f^*} \right)^2 \right]^{-1},
\] (B.10)
\[
S_{\text{acc}}^{\text{DECIGO}}(f) = \frac{hP}{\pi \lambda} \left( \frac{16F}{3ML} \right)^2 \left( \frac{1}{2 \pi f} \right)^4,
\] (B.11)

where \(P = 10 \text{ W}\) is the laser output power, \(\lambda = 532 \text{ nm}\) is the laser wavelength, \(M = 100 \text{ kg}\) is the mirror mass, \(R = 0.5 \text{ m}\) is the mirror radius, \(F = 10.18\) is the FP cavity finesse and \(P_{\text{eff}} = 6.68 \text{ W}\) is the effective laser output power.

For DO we use the noise curves shown in Ref. \([48]\) and kindly provided by Christopher Berry. Also for \(\mu\)Ares we use the noise curves kindly provided by Alberto Sesana, as shown in Ref. \([42]\). For AEDGE we use the strain sensitivity curve shown in Ref. \([49]\) and kindly provided by the AEDGE collaboration. For ET we use the strain sensitivity curve available from Ref. \([39]\) (see also website in footnote 9). The sensitivity curve for CE is also publicly available and can be downloaded from the LIGO website\(^{11}\).

References

[1] A. H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, Phys. Rev. D23 (1981) 347.

\(^{11}\)https://dcc.ligo.org/LIGO-T1500293-v11/public
[2] K. Sato, *First Order Phase Transition of a Vacuum and Expansion of the Universe*, *Mon. Not. Roy. Astron. Soc.* **195** (1981) 467.

[3] A. D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, *Phys. Lett.* **108B** (1982) 389.

[4] A. Albrecht and P. J. Steinhardt, *Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking*, *Phys. Rev. Lett.* **48** (1982) 1220.

[5] A. A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, *Phys. Lett.* **91B** (1980) 99.

[6] L. P. Grishchuk, *Amplification of gravitational waves in an isotropic universe*, *Sov. Phys. JETP* **40** (1975) 409.

[7] A. A. Starobinsky, *Spectrum of relict gravitational radiation and the early state of the universe*, *JETP Lett.* **30** (1979) 682.

[8] V. F. Mukhanov and G. V. Chibisov, *Quantum Fluctuations and a Nonsingular Universe*, *JETP Lett.* **33** (1981) 532.

[9] S. W. Hawking, *The Development of Irregularities in a Single Bubble Inflationary Universe*, *Phys. Lett.* **115B** (1982) 295.

[10] A. H. Guth and S. Y. Pi, *Fluctuations in the New Inflationary Universe*, *Phys. Rev. Lett.* **49** (1982) 1110.

[11] A. A. Starobinsky, *Dynamics of Phase Transition in the New Inflationary Universe Scenario and Generation of Perturbations*, *Phys. Lett.* **117B** (1982) 175.

[12] L. F. Abbott and M. B. Wise, *Constraints on Generalized Inflationary Cosmologies*, *Nucl. Phys.* **B244** (1984) 541.

[13] PLANCK collaboration, *Planck 2018 results. VI. Cosmological parameters*, **1807.06209**.

[14] D. H. Lyth and A. Riotto, *Particle physics models of inflation and the cosmological density perturbation*, *Phys. Rept.* **314** (1999) 1 [hep-ph/9807278].

[15] M. Kamionkowski and E. D. Kovetz, *The Quest for B Modes from Inflationary Gravitational Waves*, *Ann. Rev. Astron. Astrophys.* **54** (2016) 227 [1510.06042].

[16] M. Kramer and D. J. Champion, *The European Pulsar Timing Array and the Large European Array for Pulsars*, *Class. Quant. Grav.* **30** (2013) 224009.

[17] N. Bartolo et al., *Science with the space-based interferometer LISA. IV: Probing inflation with gravitational waves*, *JCAP* **12** (2016) 026 [1610.06481].

[18] M. Kamionkowski, A. Kosowsky and A. Stebbins, *Statistics of cosmic microwave background polarization*, *Phys. Rev.* **D55** (1997) 7368 [astro-ph/9611125].

[19] U. Seljak and M. Zaldarriaga, *Signature of gravity waves in polarization of the microwave background*, *Phys. Rev. Lett.* **78** (1997) 2054 [astro-ph/9609169].

[20] The BICEP/Keck Collaboration, : : P. A. R. Ade et al., *Measurements of Degree-Scale B-mode Polarization with the BICEP/Keck Experiments at South Pole*, arXiv e-prints (2018) [1807.02199].

[21] POLARBEAR Collaboration, P. A. R. Ade et al., *A Measurement of the Cosmic Microwave Background B-mode Polarization Power Spectrum at Subdegree Scales from Two Years of polarbear Data*, *Astrophys. J.* **848** (2017) 121 [1705.02907].

[22] POLARBEAR collaboration, *The POLARBEAR-2 and the Simons Array Experiment*, *J. Low. Temp. Phys.* **184** (2016) 805 [1512.07299].
[23] T. Louis, E. Grace, M. Hasselfield, M. Lungu, L. Maurin, G. E. Addison et al., The Atacama Cosmology Telescope: two-season ACTPol spectra and parameters, JCAP 6 (2017) 031 [1610.02360].

[24] SPTPol collaboration, Detection of B-mode Polarization in the Cosmic Microwave Background with Data from the South Pole Telescope, Phys. Rev. Lett. 111 (2013) 141301 [1307.5830].

[25] S. Dahal et al., The CLASS 150/220 GHz Polarimeter Array: Design, Assembly, and Characterization, J. Low. Temp. Phys. 199 (2020) 289 [1908.00480].

[26] The Simons Observatory Collaboration, P. Ade, J. Aguirre, Z. Ahmed, S. Aiola, A. Ali et al., The Simons Observatory: Science goals and forecasts, arXiv e-prints (2018) [1808.07445].

[27] K. N. Abazajian, P. Adshead, Z. Ahmed, S. W. Allen, D. Alonso, K. S. Arnold et al., CMB-S4 Science Book, First Edition, arXiv e-prints (2016) [1610.02743].

[28] K. Abazajian et al., CMB-S4 Science Case, Reference Design, and Project Plan, 1907.04473.

[29] K. Abazajian et al., CMB-S4 Decadal Survey APC White Paper, Bull. Am. Astron. Soc. 51 (2019) 209 [1908.01062].

[30] M. Hazumi et al., LiteBIRD: A Satellite for the Studies of B-Mode Polarization and Inflation from Cosmic Background Radiation Detection, J. Low. Temp. Phys. 194 (2019) 433.

[31] NANOGrav collaboration, The NANOGrav Nine-year Data Set: Limits on the Isotropic Stochastic Gravitational Wave Background, Astrophys. J. 821 (2016) 13 [1508.03024].

[32] L. Lentati et al., European Pulsar Timing Array Limits On An Isotropic Stochastic Gravitational-Wave Background, Mon. Not. Roy. Astron. Soc. 453 (2015) 2576 [1504.03692].

[33] P. D. Lasky et al., Gravitational-wave cosmology across 29 decades in frequency, Phys. Rev. X 6 (2016) 011035 [1511.05994].

[34] A. Weltman et al., Fundamental Physics with the Square Kilometre Array, Publ. Astron. Soc. Austral. 37 (2020) e002 [1810.02680].

[35] LIGO Scientific collaboration, Advanced LIGO: The next generation of gravitational wave detectors, Class. Quant. Grav. 27 (2010) 084006.

[36] VIRGO collaboration, Advanced Virgo: a second-generation interferometric gravitational wave detector, Class. Quant. Grav. 32 (2015) 024001 [1408.3978].

[37] KAGRA collaboration, Detector configuration of KAGRA: The Japanese cryogenic gravitational-wave detector, Class. Quant. Grav. 29 (2012) 124007 [1111.7185].

[38] D. Reitze et al., Cosmic Explorer: The U.S. Contribution to Gravitational-Wave Astronomy beyond LIGO, Bull. Am. Astron. Soc. 51 (2019) 035 [1907.04833].

[39] S. Hild et al., Sensitivity Studies for Third-Generation Gravitational Wave Observatories, Class. Quant. Grav. 28 (2011) 094013 [1012.0908].

[40] J. Baker, J. Bellovary, P. L. Bender, E. Berti, R. Caldwell, J. Camp et al., The Laser Interferometer Space Antenna: Unveiling the Millihertz Gravitational Wave Sky, arXiv e-prints (2019) arXiv:1907.06482 [1907.06482].

[41] T. L. Smith and R. Caldwell, LISA for Cosmologists: Calculating the Signal-to-Noise Ratio for Stochastic and Deterministic Sources, Phys. Rev. D100 (2019) 104055 [1908.00546].

[42] A. Sesana et al., Unveiling the Gravitational Universe at µ-Hz Frequencies, 1908.11391.

[43] V. Baibhav et al., Probing the Nature of Black Holes: Deep in the mHz Gravitational-Wave Sky, 1908.11390.

[44] J. Crowder and N. J. Cornish, Beyond LISA: Exploring future gravitational wave missions, Phys. Rev. D 72 (2005) 083005 [gr-qc/0506015].
T. L. Smith and R. Caldwell, Sensitivity to a Frequency-Dependent Circular Polarization in an Isotropic Stochastic Gravitational Wave Background, Phys. Rev. D95 (2017) 044036 [1609.05901].

N. Seto, S. Kawamura and T. Nakamura, Possibility of direct measurement of the acceleration of the universe using 0.1-Hz band laser interferometer gravitational wave antenna in space, Phys. Rev. Lett. 87 (2001) 221103 [astro-ph/0108011].

S. Kawamura, M. Ando, N. Seto, S. Sato, M. Musha, I. Kawano et al., Current status of space gravitational wave antenna DECIGO and B-DECIGO, arXiv e-prints (2020) arXiv:2006.13545 [2006.13545].

M. A. Sedda et al., The Missing Link in Gravitational-Wave Astronomy: Discoveries Waiting in the Decihertz Range, 1908.11375.

Y. A. El-Neaj et al., AEDGE: Atomic Experiment for Dark Matter and Gravity Exploration in Space, 1908.00802.

L. Sorbo, Parity violation in the Cosmic Microwave Background from a pseudoscalar inflaton, JCAP 1106 (2011) 003 [1101.1525].

N. Barnaby and M. Peloso, Large Nongaussianity in Axion Inflation, Phys. Rev. Lett. 106 (2011) 181301 [1011.1500].

N. Barnaby, J. Moxon, R. Namba, M. Peloso, G. Shiu and P. Zhou, Gravity waves and non-Gaussian features from particle production in a sector gravitationally coupled to the inflaton, Phys. Rev. D86 (2012) 103508 [1206.6117].

J. L. Cook and L. Sorbo, Particle production during inflation and gravitational waves detectable by ground-based interferometers, Phys. Rev. D85 (2012) 023534 [1109.0022].

J. L. Cook and L. Sorbo, An inflationary model with small scalar and large tensor nongaussianities, JCAP 1311 (2013) 047 [1307.7077].

R. Namba, M. Peloso, M. Shiraishi, L. Sorbo and C. Unal, Scale-dependent gravitational waves from a rolling axion, JCAP 1601 (2016) 041 [1509.07521].

M. Shiraishi, C. Hikage, R. Namba, T. Namikawa and M. Hazumi, Testing statistics of the CMB B-mode polarization toward unambiguously establishing quantum fluctuation of the vacuum, Phys. Rev. D94 (2016) 043506 [1606.06082].

V. Domeke, M. Pieroni and P. BinÃľtruy, Primordial gravitational waves for universality classes of pseudoscalar inflation, JCAP 06 (2016) 031 [1603.01287].

A. Maleknejad, M. M. Sheikh-Jabbari and J. Soda, Gauge Fields and Inflation, Phys. Rept. 528 (2013) 161 [1212.2921].

P. Adshead, E. Martinec and M. Wyman, Gauge fields and inflation: Chiral gravitational waves, fluctuations, and the Lyth bound, Phys. Rev. D88 (2013) 021302 [1301.2598].

P. Adshead, E. Martinec and M. Wyman, Perturbations in Chromo-Natural Inflation, JHEP 09 (2013) 087 [1305.2930].

E. Dimastrogiovanni and M. Peloso, Stability analysis of chromo-natural inflation and possible evasion of Lyth’s bound, Phys. Rev. D87 (2013) 103501 [1212.5184].

A. Maleknejad and M. Sheikh-Jabbari, Non-Abelian Gauge Field Inflation, Phys. Rev. D 84 (2011) 043515 [1102.1932].

A. Maleknejad and M. M. Sheikh-Jabbari, Gauge-flation: Inflation From Non-Abelian Gauge Fields, Phys. Lett. B723 (2013) 224 [1102.1513].

A. Maleknejad, Axion Inflation with an SU(2) Gauge Field: Detectable Chiral Gravity Waves, JHEP 07 (2016) 104 [1604.03327].
[65] E. Dimastrogiovanni, M. Fasiello and T. Fujita, Primordial Gravitational Waves from Axion-Gauge Fields Dynamics, *JCAP* **01** (2017) 019 [1608.04216].

[66] I. Obata and J. Soda, Chiral primordial gravitational waves from dilaton induced delayed chromonatural inflation, *Phys. Rev.* **D93** (2016) 123502 [1602.06024].

[67] A. Agrawal, T. Fujita and E. Komatsu, Large tensor non-Gaussianity from axion-gauge field dynamics, *Phys. Rev.* **D97** (2018) 103526 [1707.03240].

[68] A. Agrawal, T. Fujita and E. Komatsu, Tensor Non-Gaussianity from Axion-Gauge-Fields Dynamics : Parameter Search, *JCAP* **05** (2018) 027 [1802.09284].

[69] B. Thorne, T. Fujita, M. Hazumi, N. Katayama, E. Komatsu and M. Shiraishi, Finding the chiral gravitational wave background of an axion-SU(2) inflationary model using CMB observations and laser interferometers, *Phys. Rev. D* **97** (2018) 043506 [1707.03240].

[70] BICEP2, Keck Array collaboration, BICEP2 / Keck Array x: Constraints on Primordial Gravitational Waves using Planck, WMAP, and New BICEP2/Keck Observations through the 2015 Season, *Phys. Rev. Lett.* **121** (2018) 221301 [1810.05216].

[71] P. Adshead and M. Wyman, Chromo-Natural Inflation: Natural inflation on a steep potential with classical non-Abelian gauge fields, *Phys. Rev. Lett.* **108** (2012) 261302 [1202.2366].

[72] L. Mirzagholi, E. Komatsu, K. D. Lozanov and Y. Watanabe, Effects of Gravitational Chern-Simons during Axion-SU(2) Inflation, *JCAP* **06** (2020) 024 [2003.05931].

[73] A. Maleknejad and E. Erfani, Chromo-Natural Model in Anisotropic Background, *JCAP* **03** (2014) 016 [1311.3361].

[74] V. Domecke, B. Mares, F. Muia and M. Pieroni, Emerging chromo-natural inflation, *JCAP* **04** (2019) 034 [1807.03358].

[75] I. Wolfson, A. Maleknejad and E. Komatsu, How attractive is the isotropic attractor solution of axion-SU(2) inflation?, 2003.01617.

[76] T. Fujita, E. I. Sfakianakis and M. Shiraishi, Tensor Spectra Templates for Axion-Gauge Fields Dynamics during Inflation, *JCAP* **05** (2019) 057 [1812.03667].

[77] A. Maleknejad and E. Komatsu, Production and Backreaction of Spin-2 Particles of SU(2) Gauge Field during Inflation, *JHEP* **05** (2019) 174 [1808.09076].

[78] A. Papageorgiou, M. Pelosi and C. Unal, Nonlinear perturbations from axion-gauge fields dynamics during inflation, *JCAP* **07** (2019) 004 [1904.01488].

[79] T. J. Clarke, E. J. Copeland and A. Moss, Constraints on primordial gravitational waves from the Cosmic Microwave Background, 2004.11396.

[80] G. Cabass, L. Pagano, L. Salvati, M. Gerbino, E. Giusarma and A. Melchiorri, Updated Constraints and Forecasts on Primordial Tensor Modes, *Phys. Rev. D* **93** (2016) 063508 [1511.05146].

[81] NANOGrav collaboration, The NANOGrav 11-year Data Set: Pulsar-timing Constraints On The Stochastic Gravitational-wave Background, *Astrophys. J.* **859** (2018) 47 [1801.02617].

[82] LIGO Scientific, Virgo collaboration, Search for the isotropic stochastic background using data from Advanced LIGO’s second observing run, *Phys. Rev. D* **100** (2019) 061101 [1903.02866].

[83] E. W. Kolb and M. S. Turner, *The Early Universe*. Addison-Wesley, 1990.

[84] Y. Watanabe and E. Komatsu, Improved Calculation of the Primordial Gravitational Wave Spectrum in the Standard Model, *Phys. Rev. D* **73** (2006) 123515 [astro-ph/0604176].

[85] M. Zaldarriaga, Polarization of the microwave background in resonized models, *Phys. Rev. D* **55** (1997) 1822 [astro-ph/9608050].
[86] T. Hiramatsu, E. Komatsu, M. Hazumi and M. Sasaki, Reconstruction of primordial tensor power spectra from B-mode polarization of the cosmic microwave background, Phys. Rev. D97 (2018) 123511 [1803.00176].

[87] M. Zaldarriaga and U. Seljak, Gravitational lensing effect on cosmic microwave background polarization, Phys. Rev. D 58 (1998) 023003 [astro-ph/9803150].

[88] A. Lewis, A. Challinor and A. Lasenby, Efficient Computation of Cosmic Microwave Background Anisotropies in Closed Friedmann-Robertson-Walker Models, Astrophys. J. 538 (2000) 473 [astro-ph/991177].

[89] D. Blas, J. Lesgourgues and T. Tram, The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes, JCAP 07 (2011) 034 [1104.2933].

[90] A. Lue, L.-M. Wang and M. Kamionkowski, Cosmological signature of new parity violating interactions, Phys. Rev. Lett. 83 (1999) 1506 [astro-ph/9812088].

[91] S. Saito, K. Ichiki and A. Taruya, Probing polarization states of primordial gravitational waves with cosmic microwave background anisotropies, JCAP 2007 (2007) 002 [0705.3701].

[92] C. R. Contaldi, J. Magueijo and L. Smolin, Anomalous CMB polarization and gravitational chirality, Phys. Rev. Lett. 101 (2008) 141101 [0806.3082].

[93] R. Stompor, J. Errard and D. Poletti, Forecasting performance of CMB experiments in the presence of complex foreground contaminations, Phys. Rev. D94 (2016) 083526 [1609.03807].

[94] L. Knox and Y.-S. Song, A Limit on the detectability of the energy scale of inflation, Phys. Rev. Lett. 89 (2002) 011303 [astro-ph/0202286].

[95] M. Kesden, A. Cooray and M. Kamionkowski, Separation of gravitational wave and cosmic shear contributions to cosmic microwave background polarization, Phys. Rev. Lett. 89 (2002) 011304 [astro-ph/0202434].

[96] W. Hu and T. Okamoto, Mass Reconstruction with Cosmic Microwave Background Polarization, Astrophys. J. 574 (2002) 566 [astro-ph/0111606].

[97] K. M. Smith, D. Hanson, M. LoVerde, C. M. Hirata and O. Zahn, Delensing CMB polarization with external datasets, JCAP 6 (2012) 014 [1010.0048].

[98] PLANCK collaboration, Planck 2018 results. IV. Diffuse component separation, 1807.06208.

[99] C. Dickinson, CMB foregrounds - A brief review, in Proceedings, 51st Rencontres de Moriond, Cosmology session: La Thuile, Italy, March 19-26, 2016, pp. 53–62, 2016, 1606.03606.

[100] B. Thorne, J. Dunkley, D. Alonso and S. Naess, The Python Sky Model: software for simulating the Galactic microwave sky, Mon. Not. Roy. Astron. Soc. 469 (2017) 2821 [1608.02841].

[101] K. M. Górski, E. Hivon, A. J. Banday, B. D. Wandelt, F. K. Hansen, M. Reinecke et al., HEALPix: A Framework for High-Resolution Discretization and Fast Analysis of Data Distributed on the Sphere, Astrophys. J. 622 (2005) 759 [astro-ph/0409513].

[102] J. Errard, F. Stivoli and R. Stompor, Framework for performance forecasting and optimization of CMB B-mode observations in presence of astrophysical foregrounds, Phys. Rev. D84 (2011) 063005 [1105.3859].

[103] J. Errard, S. M. Feeney, H. V. Peiris and A. H. Jaffe, Robust forecasts on fundamental physics from the foreground-obscured, gravitationally-lensed CMB polarization, JCAP 3 (2016) 052 [1509.06770].

[104] R. Stompor, S. M. Leach, F. Stivoli and C. Baccigalupi, Maximum Likelihood algorithm for parametric component separation in CMB experiments, Mon. Not. Roy. Astron. Soc. 392 (2009) 216 [0804.2645].
[105] J. Errard and R. Stompor, "Characterizing bias on large scale CMB B-modes after galactic foregrounds cleaning," arXiv e-prints (2018) [1811.00479].

[106] P. Campeti, D. Poletti and C. Baccigalupi, "Principal component analysis of the primordial tensor power spectrum," JCAP 1909 (2019) 055 [1905.08200].

[107] M. Tegmark, "How to measure CMB power spectra without losing information," Phys. Rev. D55 (1997) 5895 [astro-ph/9611174].

[108] S. Kuroyanagi, K. Nakayama and J. Yokoyama, "Prospects of determination of reheating temperature after inflation by DECIGO," PTEP 2015 (2015) 013E02 [1410.6618].

[109] J. Crowder and N. J. Cornish, "Beyond LISA: Exploring future gravitational wave missions," Phys. Rev. D 72 (2005) 083005 [gr-qc/0506015].

[110] S. Sachdev, T. Regimbau and B. S. Sathyaprakash, "Subtracting compact binary foreground sources to reveal primordial gravitational-wave backgrounds," 2002.05365.

[111] L. Lentati et al., "European Pulsar Timing Array Limits On An Isotropic Stochastic Gravitational-Wave Background," Mon. Not. Roy. Astron. Soc. 453 (2015) 2576 [1504.03692].

[112] M. Kerr et al., "The Parkes Pulsar Timing Array Project: Second data release," 2003.09780.

[113] G. Hobbs, "The Parkes Pulsar Timing Array," Class. Quant. Grav. 30 (2013) 224007 [1307.2629].

[114] B. Perera et al., "The International Pulsar Timing Array: Second data release," Mon. Not. Roy. Astron. Soc. 490 (2019) 4666 [1909.04534].

[115] J. D. Romano and N. J. Cornish, "Detection methods for stochastic gravitational-wave backgrounds: a unified treatment," Living Rev. Rel. 20 (2017) 2 [1608.06889].

[116] K. Schmitz, "New Sensitivity Curves for Gravitational-Wave Experiments," 2002.04615.

[117] J. S. Hazboun, J. D. Romano and T. L. Smith, "Realistic sensitivity curves for pulsar timing arrays," Phys. Rev. D 100 (2019) 104028 [1907.04341].

[118] N. Seto, "Prospects for direct detection of circular polarization of gravitational-wave background," Phys. Rev. Lett. 97 (2006) 151101 [astro-ph/0609504].

[119] E. E. Flanagan, "The Sensitivity of the laser interferometer gravitational wave observatory (LIGO) to a stochastic background, and its dependence on the detector orientations," Phys. Rev. D 48 (1993) 2389 [astro-ph/9305029].

[120] A. Nishizawa, K. Yagi, A. Taruya and T. Tanaka, "Cosmology with space-based gravitational-wave detectors — dark energy and primordial gravitational waves —," Phys. Rev. D85 (2012) 044047 [1110.2885].

[121] N. Cornish and T. Robson, "Galactic binary science with the new LISA design," J. Phys. Conf. Ser. 840 (2017) 012024 [1703.09858].

[122] T. Robson, N. J. Cornish and C. Liu, "The construction and use of LISA sensitivity curves," Class. Quant. Grav. 36 (2019) 105011 [1803.01944].

[123] A. Sesana, A. Vecchio and C. N. Colacino, "The stochastic gravitational-wave background from massive black hole binary systems: implications for observations with Pulsar Timing Arrays," Mon. Not. Roy. Astron. Soc. 390 (2008) 192 [0804.4476].

[124] G. Lodato and P. Natarajan, "Supermassive black hole formation during the assembly of pre-galactic discs," Mon. Not. Roy. Astron. Soc. 371 (2006) 1813 [astro-ph/0606159].

[125] Z. Pan and H. Yang, "Probing Primordial Stochastic Gravitational Wave Background with Multi-band Astrophysical Foreground Cleaning," 1910.09637.

[126] M. Pieroni and E. Barausse, "Foreground cleaning and template-free stochastic background extraction for LISA," 2004.01135.
[127] J. García-Bellido, M. Peloso and C. Unal, *Gravitational waves at interferometer scales and primordial black holes in axion inflation*, JCAP 12 (2016) 031 [1610.03763].

[128] T. L. Smith, H. V. Peiris and A. Cooray, *Deciphering inflation with gravitational waves: cosmic microwave background polarization vs. direct detection with laser interferometers*, Phys. Rev. D 73 (2006) 123503 [astro-ph/0602137].

[129] P. D. Meerburg, R. Hložek, B. Hadzhiyska and J. Meyers, *Multiwavelength constraints on the inflationary consistency relation*, Phys. Rev. D 91 (2015) 103505 [1502.00302].

[130] P. D. Lasky et al., *Gravitational-wave cosmology across 29 decades in frequency*, Phys. Rev. X 6 (2016) 011035 [1511.05994].

[131] B. Audren, J. Lesgourgues, K. Benabed and S. Prunet, *Conservative Constraints on Early Cosmology: an illustration of the Monte Python cosmological parameter inference code*, JCAP 1302 (2013) 001 [1210.7183].

[132] T. Brinckmann and J. Lesgourgues, *MontePython 3: boosted MCMC sampler and other features*, 1804.07261.

[133] V. Mandic, E. Thrane, S. Giampanis and T. Regimbau, *Parameter Estimation in Searches for the Stochastic Gravitational-Wave Background*, Phys. Rev. Lett. 109 (2012) 171102 [1209.3847].

[134] L. Perotto, J. Lesgourgues, S. Hannestad, H. Tu and Y. Y. Wong, *Probing cosmological parameters with the CMB: Forecasts from full Monte Carlo simulations*, JCAP 10 (2006) 013 [astro-ph/0606227].

[135] H. Kudoh, A. Taruya, T. Hiramatsu and Y. Himemoto, *Detecting a gravitational-wave background with next-generation space interferometers*, Phys. Rev. D73 (2006) 064006 [gr-qc/0511145].