We present a \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold compactification of the \( E_8 \times E_8 \) heterotic string which gives rise to the exact chiral MSSM spectrum. The GUT breaking \( SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \) is realized by modding out a freely acting symmetry. This ensures precision gauge coupling unification. Further, it allows us to break the GUT group without switching on flux in hypercharge direction, such that the standard model gauge bosons can remain massless when the orbifold singularities are blown up. The model has vacuum configurations with matter parity, a large top Yukawa coupling and other phenomenologically appealing features.

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geometrical interpretation in terms of CY manifolds, and the computation of the chiral spectra based on index theorems is well under
close. On the other hand, the validity of the supergravity description requires moderately large radii, which is sometimes problematic
as this easily leads to too small gauge couplings. Moreover, since the underlying CYs are complicated spaces, the calculation of couplings,
needed to make detailed predictions for phenomenology, is still far from straightforward.

As is well known, orbifolds and CYs are not unrelated; rather, in many cases orbifold singularities can be resolved, thus reproducing
compactifications based on smooth manifolds (see e.g. [14–19]). The transition from an orbifold to a smooth compactification is achieved
by giving VEVs to twisted states, stringy degrees of freedom residing at the orbifold singularities. The reverse of the “blow-up” process,
in which a compact hyper surface (i.e. exceptional divisor) shrinks down to zero size, is commonly referred to as a “blow-down”:\footnote{Since the VEV and the corresponding volume are schematically related by VEV \( \sim \exp(\text{volume}) \), the naive definition of the volume has to go to \(-\infty\) in order to arrive at the orbifold point [19,20]. Hence, the blow-down in the supergravity sense does not describe the orbifold point.} A setting in
which one has both an exact orbifold CFT picture as well as a smooth CY description, would be quite powerful, because one can combine
the calculability of the orbifold with the generic features of CY compactifications.

So far no phenomenologically appealing model has been obtained that allows for an orbifold as well as a CY description. It is unknown
whether a complete blow-down of the potentially realistic smooth compactifications, obtained so far [7,8,21], to an exact (free orbifold)
CFT description exists. On the other hand, many phenomenologically attractive orbifold models [10–12] cannot be completely blown up
without destroying the phenomenological viability of these settings, as the hypercharge or another part of the standard model gauge
group gets broken in the complete blow-up [19].

Our aim is to describe in detail how to construct (phenomenologically attractive) orbifolds which allow for complete blow-ups without
breaking the standard model gauge group \( G_{\text{SM}} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \). We base our discussion on an explicit model that can be seen
as a specific realization of a proposal made by Witten [22] in that the GUT breaking \( \text{SU}(5) \to G_{\text{SM}} \) is achieved by dividing out a freely
acting symmetry. (The idea of associating a Wilson line with an involution of the underlying CY manifold in concrete model building has
particularly well to the paradigm of MSSM precision gauge coupling unification [24,25].)

This Letter is organized as follows: Section 2 is devoted to the construction of a concrete orbifold model that allows for a freely acting
involutions. In addition, such settings allow us to avoid GUT scale threshold corrections to the gauge couplings and therefore fit
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corresponding to the six torus directions $e_\alpha$. These shifts and Wilson lines satisfy the modular invariance conditions

$$2\left[(k_1 v_1 + k_2 v_2 + n_0 W_\alpha)^2 - (k_1 v_1 + k_2 v_2)^2\right] = 0 \mod 2 \quad \forall k_1, n_0 \in \{0, 1\},$$

and, furthermore, fulfill the consistency requirements of Ref. [30], Eq. (3) is obtained by noticing that the theta-functions inside the corresponding partition function are periodic under the change of the modular parameter ($\rho \rightarrow \rho + 2$ for an order two element) up to some phase factors that needs to be cancelled.

The states in the spectrum originate from different sectors: the untwisted sectors $U_i$ (with $i = 1, 2, 3$ corresponding to the 4th plane, spanned by $e_2, e_3$ and $e_0$) and the twisted sectors $T_{k_1 k_2}$ (corresponding to the orbifold twist $k v_1 + v_2$). In total, the spectrum contains $6 \times \text{Tor} + 15 \times 5 + 9 \times 5$ of $SU(5)$, 52 non-Abelian singlets and some representations with respect to a hidden sector gauge group $SU(4)^2$. Of the nine vector-like pairs of $5/\overline{5}$-plets in the untwisted sectors $U_i$, $i = 1, 2, 3$, originating from the 10D bulk; the remainder of the $SU(5)$-charged spectrum resides in the various twisted sectors. In particular, the six generations of $SU(5)$ are all twisted states.

### 2.2. Modding out a freely acting $\mathbb{Z}_2$ involution

Next, we divide out the $\mathbb{Z}_2$ symmetry corresponding to

$$\tau = \frac{1}{2}(e_2 + e_4 + e_6)$$

with a gauge embedding denoted by $W$. Since $\tau$ acts freely, i.e. it does not produce fixed points, we refer to $W$ as freely acting Wilson line. This is a slight abuse of terminology, since (field-theoretic) Wilson lines are always non-local. However, in the context of orbifold model building discrete Wilson lines usually denote the differences between local shifts, i.e. they are Wilson lines on the underlying torus but not on the orbifold (see e.g. [10,31]). By contrast, $W$ is a Wilson line also on the orbifold.

The strict identification of $W_2$, $W_4$ and $W_6$ in Eq. (2e) allows us to mod out $\tau$. Further, from its definition (4) it follows that $W$ is an element of order four,

$$W = \frac{1}{2}(W_2 + W_4 + W_6) = \frac{3}{2}W_2,$$

as $W_2$ is of order two.

Modular invariance of the resulting partition function for this order four element amends the conditions (3) by

$$4(n_0 W_\alpha + n_0 W)^2 = 0 \mod 2 \quad \forall n_0, n_\alpha \in \{0, 1\}.$$ \hspace{1cm} (6)

In particular, we have chosen the Wilson line $W_2$ in equation (2b) such that $W$ satisfies all the conditions (6) and breaks the $SU(5)$ GUT group down to $G_{SM}$.

By contrast, the Wilson line associated with an involution employed on smooth CY are taken to be perpendicular to the gauge bundle [7,8]. This is not the case in our construction; precisely for that reason our Wilson line $W$ is an order 4 element instead of order 2.

### 2.3. Massless spectrum

After modding out $\tau$, the 4D gauge group is $G_{SM}$ times eight $U(1)$ factors and a non-Abelian hidden sector $SU(3) \times SU(2) \times SU(2)$. One combination of the $U(1)$ factors with generator $t_{\alpha\text{anom}}$ denotes the anomalous $U(1)$. Furthermore, the standard hypercharge generator $t_Y$ from $SU(5)$ can be identified and turns out to be orthogonal to the anomalous direction,

$$t_{\alpha\text{anom}} = (-2, -1, 2, 1, 1, 1, 1, 1)(-\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0, 0, 0),$$

$$t_Y = \left(0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right),$$

$$\alpha \text{anom}_\text{f} = \left(0, 0, 0, 0, 0, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right).$$

The model has a local $SU(5)$ GUT structure (for the discussion of the concept of local GUTs see [32] and cf. the related earlier discussion in [26,33]).

Dividing out the freely acting symmetry $\tau$ reduces the number of fixed points from 48 to 24 and breaks the symmetry from $SU(5)$ to $G_{SM}$. It further splits the untwisted $5$- and $\overline{5}$-plets in the $(e_1, e_2)$-plane to a pair of Higgs candidates, denoted by $h_1$ and $h_\text{t}$, removing the
triplets. In the other two planes it removes the doublets, leaving two pairs of triplets/anti-triplets $\delta_i / \bar{\delta}_i$ ($i = 1, 2$) massless. A compact summary of the spectrum is given in Table 1; more complete details have been listed in Table 2 in Appendix A.

To understand the family structure note that, due to the absence of the Wilson line in the $e_1$ direction ($W_1 = 0$), states in the $T_{(1,1)}$ and $T_{(1,0)}$ sectors form doublets under a discrete group $D_4$, which is unaffected by modding out the freely acting symmetry $\tau$. As two families reside in the $T_{(1,1)}$ sector, the two light families transform as a doublet under this $D_4$ flavor symmetry. The third family comes from $T_{(1,0)}$ sector and hence is a $D_4$ singlet. Such a $D_4$ symmetry is known to be phenomenologically attractive as it can ameliorate supersymmetric flavor problems [34]. In this respect the structure of the model is very similar to the $\mathbb{Z}_6$-II models discussed in [10,12,35].

2.4. Gauge coupling unification

As explained in Section 2.2, the GUT symmetry breaking is accomplished by the action of a freely acting symmetry $\tau$, leading to a completely non-local breaking. This mechanism was introduced originally in the context of smooth manifold compactifications [22]. Later it was considered as an alternative to the standard (localized) breaking in orbifold constructions [36].

In order to discuss the virtues of non-local breaking, let us briefly recall the usual obstructions in embedding the beautiful picture of MSSM gauge coupling unification in the heterotic string. There are three main issues:

1. huge, “power-like” threshold corrections around the string scale;
2. the MSSM unification scale, $M_{\text{GUT}} = \text{few} \cdot 10^{16}$ GeV, is by an order 10 factor below the heterotic string scale;
3. the appearance of split multiplets at the high scale generically leads to logarithmic thresholds.

The first problem is absent in the scheme of ‘local grand unification’ as the bulk gauge group in extra dimensions contains $G_{\text{SM}}$ such that power-like corrections are universal. This may be overcome by considering anisotropic compactifications [37,24]. As described in detail in [24], by using a discrete (rather than continuous) Wilson line associated with the involution to break the GUT symmetry, the breaking scale is related to the length of the corresponding Wilson line cycle. This length can be of order $M_{\text{GUT}}^2$ with the volume of compact space being so small that a description in terms of the perturbative heterotic string is still justifiable. This mechanism also ameliorates the third problem. In fact, the remaining logarithmic corrections may even mitigate the discrepancy between string and GUT scales [25]. In this respect our model is “cleaner” than the MSSMs based on $\mathbb{Z}_6$-II, where various logarithmic corrections to gauge unification from localized states and vector-like exotics coming in incomplete GUT multiplets are expected (cf. the discussion in [38]). Hence, the mechanism of non-local GUT breaking provides us with one of the most compelling realizations of precision gauge coupling unification. The implications of precision gauge unification for the MSSM superpartner spectrum have been discussed very recently in [39].

3. Semi-realistic VEV configuration

In order to obtain the MSSM, the unwanted $U(1)$ gauge group factors have to be broken. This can be accomplished by switching on VEVs of standard model singlet fields consistently with vanishing $F$- and $D$-terms (cf. the discussion in [10,12]). In addition, these VEVs give rise to effective Yukawa couplings for quarks and leptons, they serve as effective mass terms decoupling the exotics and may generate supersymmetric flavor problems [34]. In this respect the structure of the model is very similar to the $\mathbb{Z}_6$-II models discussed in [10,12,35].

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\( \mathcal{M}_{ij}^{\delta} \sim \begin{pmatrix} \phi^3 & s_1 & \phi^3 & \phi^3 \\ s_2 & \phi^3 & \phi^5 & s_{16} \\ s_{28} & \phi^3 & s_{19} & s_{10} \\ s_{33} & \phi^3 & s_{23} & \phi^3 \end{pmatrix} \),

where here and in the following \( \phi^n \) denotes a sum of known monomials in the VEVs of the fields of (9) with \( n \) being its lowest degree. Obviously, due to the \( \mathbb{Z}_2^8 \) matter parity, there is no mixing between \( \bar{d} \) quarks and the quark-like exotics \( \delta \). Switching on the VEVs of the untwisted states \( s_1 \) and \( s_2 \) is sufficient to decouple the untwisted triplets \( \delta_i, \bar{\delta}_i, i = 1, 2 \). Even more, as can be seen from (10), all triplets decouple at linear order in the \( \phi^{10} \) fields. Similar features have been reported in the context of free fermionic model building (see e.g. [27]).

There are four Higgs pair candidates with mass matrix, defined by the superpotential terms \( h_i \mathcal{M}_{ij} \mathcal{F}_{ij} \),

\( \mathcal{M}_i^h \sim \begin{pmatrix} \phi^3 & s_3 & \phi^3 & \phi^3 \\ s_{15} & \phi^5 & s_{19} & s_{23} \\ s_{26} & \phi^3 & s_{10} & \phi^3 \\ \phi^3 & s_{31} & \phi^3 & s_{10} \end{pmatrix} \)

of maximal rank. Thus, this VEV configuration suffers under the stringy version of the \( \mu \) problem. The Yukawa couplings of the quarks \( \langle q_i \mathcal{M}_i^q \mathcal{F}_j \rangle \) and of the charged leptons \( \langle \ell_i \mathcal{M}_i^\ell \mathcal{F}_j \rangle \) are of the form

\( \mathcal{M}_i^q \sim \begin{pmatrix} \tilde{h}_1 \phi^4 & \tilde{h}_1 \phi^4 & 0 \\ \tilde{h}_1 \phi^4 & \tilde{h}_1 \phi^4 & 0 \\ 0 & 0 & \tilde{h}_4 \end{pmatrix} \) and \( \mathcal{M}_i^\ell \sim \mathcal{M}_i^2 \sim \begin{pmatrix} 0 & 0 & h_3 \\ 0 & 0 & h_4 \\ h_3 & h_4 & 0 \end{pmatrix} \).

Generically, they depend on the VEVs of all four Higgs-pairs and, due to \( \mathbb{Z}_2^8 \), there is no mixing between the lepton-doublets \( \ell \) and the Higgses \( h \). The top-quark couples to \( \tilde{h}_1 \) already at order \( \phi^5 \), hence this coupling is not suppressed compared to the first and second generations. Moreover, as the Higgs \( \tilde{h}_1 \) is part of the untwisted sector, i.e. an internal part of the 10D gauge field, it couples with a strength proportional to the gauge coupling, realizing a gauge-top unification [41].

In general, couplings between localized states exhibit \( \text{SU}(5) \) relations, as they are not subject to the non-local symmetry breakdown due to \( W \). Furthermore, as a consequence of our choice of \( U(1)_{B-L} \), the three generations of quarks and leptons originate only from the twisted sectors, hence their couplings originate from \( \text{SU}(5) \). This explains why the charged lepton mass matrix \( \mathcal{M}^\ell \) and the \( d \)-type mass matrix \( \mathcal{M}^d \) are identical in Eq. (12), a feature that is actually only desirable for the third generation.

### 4. Interpretation as a complete blow-up

The second objective of our work is to show that our orbifold model may be related to a smooth Calabi–Yau compactification. The \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold has 48 \( \mathbb{Z}_2 \) fixed tori that constitute codimension four singularities, which are identified pairwise by the freely acting symmetry. To obtain a smooth space all these singularities have to be polished out. Below we will explain why the configuration discussed in the previous section defines a complete blow-up within the effective 4D theory.

From the perspective of the orbifold model smoothing out singularities corresponds to non-vanishing VEVs of twisted states. To smooth out all singularities at least one twisted state per fixed torus needs to acquire a VEV. Such a VEV can either lead to a blow-up or to a deformation of these singularities [42]. In the former case the cycle hidden inside the singularity, called the exceptional divisor, acquires a finite volume w.r.t. the Kahler form of the geometry. When the singularity is deformed, i.e. the complex structure is modified, its volume remains zero, and is in this sense still singular. As all twisted states of the \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold are six dimensional, they form hyper multiplets of \( \mathcal{N} = 1 \) in 6D. Which of the two chiral multiplets within these hyper multiplets takes a VEV decides whether one has a blow-up or a deformation.

The blow-up described within the effective 4D theory takes into account only those twisted states as blow-up-modes that are massless in 4D. Due to the presence of Wilson lines it may happen that some orbifold singularities do not provide 4D massless states. In fact, a quick glance over Table 2 in Appendix A reveals that three fixed tori of the orbifold model defined in Section 2 do not support 4D zero modes. So they might remain singular in a complete blow-up within the effective 4D theory. On the other hand, each fixed torus supports 6D massless twisted states, which may provide non-trivial profiles over the internal tori that might remove the singularities. However, a detailed discussion of these issues is beyond the scope of the present Letter.

In general, VEV configurations correspond to complicated gauge bundles on the Calabi–Yau space that need to fulfill the integrated Bianchi identities

\[ \int_S (\text{tr} \mathcal{R}^2 - \text{tr} \mathcal{F}^2) = 0 \]

for all four-cycles \( S \). These consistency equations can be viewed as the smooth analog of the modular invariance conditions, equation (3), for the shifts \( V_1, V_2 \) and the Wilson lines \( W_a \). However, there seems to be no condition(s) on the Wilson line \( W \) associated with the involution \( \tau \) for our blow-up or for other smooth CY constructions. By contrast, on the orbifold we encounter the additional requirements (6). Their derivation necessarily involves winding modes. The fact that in the supergravity approximation they are usually ignored,
might explain why the modular invariance conditions of the freely acting Wilson line \( W \) do not have a smooth counterpart. Nevertheless, such conditions might be essential to ensure that a given smooth CY compactification of supergravity has a full string lift.

The VEV configuration (9) has been chosen such that the standard model group and matter parity (in particular also the hypercharge) remain unbroken. Moreover, according to Table 2 in Appendix A all non-empty fixed tori support twisted states to the 4D theory with VEVs switched on. Therefore, this corresponds to a complete blow-up in the effective 4D theory. Hence, it shows that the obstructions to a full blow-up encountered in the \( \mathbb{Z}_6 \)-II mini-landscape models can be overcome in settings with non-local GUT breaking.

To summarize, we have shown that the \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold model with a freely acting \( \mathbb{Z}_2 \) involution allows for VEV configurations where 4D zero modes originating from all non-empty fixed tori are switched on without breaking the standard model gauge group. The construction and interpretation of such configuration from the point of view of smooth compactifications will be discussed elsewhere [43].

5. Conclusions

We have presented a \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold compactification of the heterotic string exhibiting the exact chiral MSSM spectrum and gauge group as well as matter parity. The starting point of this model is the \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold with SU(5) gauge group. The SU(5) GUT symmetry is non-locally broken by modding out a freely acting symmetry. This ensures that there is no flux in hypercharge direction such that there is no obstruction to a complete blow-up. Further, Wilson line breaking is known to avoid large thresholds to the gauge coupling such that our construction complies with the beautiful picture of MSSM gauge coupling unification.

Accompanying an inversion of the geometry with a Wilson line has been considered previously in smooth compactifications leading to the MSSM [7,8]. However, in our approach we encounter novel modular invariance conditions on this freely acting Wilson line that seem to have no analog in smooth CY compactifications in the supergravity approximation. This might suggest that some of the smooth CYs with involutions dressed with Wilson lines may exist only as supergravity models, but do not have a lift to consistent string theory constructions.

The model has the chiral MSSM spectrum and many other phenomenologically appealing features, like non-trivial Yukawa couplings and admits vacua with matter parity. On the other hand, we cannot claim that the configuration presented here is fully realistic. In more detail, due to the presence of a \( D \)-term for an anomalous U(1) some states need to acquire VEVs. We identified and discussed a specific VEV configuration with an exact matter parity (hence proton decay is avoided at the dimension four level), where all unwanted U(1) gauge group factors are broken, and all exotics decouple. Unfortunately, also the Higgs fields generically attain large masses. This unpleasant feature is shared with smooth CY MSSM models [7,21], where generically the \( \mu \)-term is of the order of the fundamental scale [21,44] (while in orbifold models there are symmetries that allow us to relate the size of \( \mu \) to the scale of supersymmetry breakdown [12,45, 46]). Our model also avoids the problem of an additional U(1)\(_{\mu-\tau} \) symmetry that cannot be broken without breaking supersymmetry. In summary, we have presented an explicit orbifold compactification satisfying all stringy consistency conditions. We identified vacua which correspond to resolutions of the orbifold fixed points, have properties very similar to those of the most promising smooth heterotic compactifications known so far, and are, in addition, endowed with an exact matter parity.

5.1. Outlook

The main achievement in this Letter was to show how to construct a concrete orbifold compactification of the heterotic string in which the breaking SU(5) \( \to \) G\(_{SM} \) is non-local, i.e. due to a Wilson line. We have argued that this may allow us to obtain a potentially realistic model with an explicit orbifold limit and a clear interpretation in terms of smooth geometry. Our analysis is incomplete in three main respects. First, the phenomenological viability of the model has to be studied in more detail. The configuration discussed in this Letter suffers from the problem that the Higgses generically get ultra-heavy. Possible solutions to the \( \mu \)-problem will be discussed in a forthcoming publication [47].

Secondly, the configuration with VEVs discussed in this work seems to indicate that a complete blow-up within the effective 4D theory is possible. However to really show that this configuration corresponds to a smooth compactification, one has to construct the gauge bundle on the resolution of the compact orbifold explicitly and check that it fulfills all Bianchi identities for consistency. Work in this direction is in progress [43].

Finally, we have seen that some twisted sectors are empty in 4D. This seems to indicate that the corresponding orbifold singularity remain unresolved. Therefore, they correspond to partial (rather than full) blow-ups of the geometry. A geometric interpretation of such settings still needs to be obtained.

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Appendix A. Full spectrum

In Table 2, we present in detail the spectrum of the model discussed in Section 2. We decompose the states in untwisted (\( U_i \) with \( i = 1, 2, 3 \) corresponding to the \( i \)-th plane) and twisted (\( T_i^{(n_1, n_2, \ldots, n_6)} \)) sectors, where \( (k, \ell) \) and \( (n_1, n_2, \ldots, n_6) \) indicate the corresponding “constructing elements” (of twisted strings with boundary conditions \( X(\tau, \sigma + 2\pi) = e^{i\theta} e^{i\omega} X(\tau, \sigma) + n_6 e_{\sigma} \) with \( \theta \) and \( \omega \) denoting the rotations corresponding to \( v_1 \) and \( v_2 \), respectively). As all twisted states live on two-tori in six dimensions, we indicate the directions \( n_i \) where these tori lie by \( * \). The states that acquire a VEV in the configuration discussed in Section 3 are indicated with angular brackets (\( \langle \rangle \)).
### Table 2
Spectrum of the model at the orbifold point.

| sector | irrep   | $R_1$, $R_2$, $R_3$ | $q_{atom}$ | $q_1$ | $q_2$ | $q_3$ | $q_4$ | $q_5$ | $q_6$ | $q_7$ | $q_8$ | label(s) |
|--------|---------|----------------------|------------|-------|-------|-------|-------|-------|-------|-------|-------|-----------|
| $U_1$  | (1, 1, 1, 1) | $-1, 0, 0$ | 2 | 0 | 2 | 4 | 20 | 7 | $-37$ | 0 | 0 | $\{s_1\}$ |
|        | (1, 2, 1, 1) | $-1, 0, 0$ | 2 | $-\frac{1}{2}$ | 2 | 4 | $-9$ | 6 | 6 | $-4$ | 0 | $h_1$ |
|        | (1, 2, 1, 1) | $-1, 0, 0$ | 2 | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-4$ | 9 | $-6$ | 6 | 4 | 0 | $\tilde{h}_1$ |
|        | (1, 1, 1, 1) | $-1, 0, 0$ | 2 | 0 | $-\frac{1}{2}$ | $-4$ | $-20$ | $-7$ | 37 | 0 | $0$ | $\{s_2\}$ |
| $U_2$  | (1, 1, 1, 1) | 0 | $-1, -\frac{1}{2}$ | 2 | $\frac{1}{2}$ | 2 | 4 | $-4$ | 7 | $\frac{13}{4}$ | $\frac{5}{2}$ | 0 | $\tilde{h}_2$ |
|        | (1, 1, 1, 1) | 0 | $-1, -\frac{1}{2}$ | 2 | $-\frac{1}{2}$ | 2 | 4 | $-4$ | $\frac{13}{4}$ | $\frac{5}{2}$ | 0 | $h_3$ |
|        | (1, 1, 1, 1) | 0 | $-1, -\frac{1}{2}$ | 2 | $-\frac{1}{2}$ | $-4$ | $-8$ | 6 | $\frac{21}{4}$ | $\frac{13}{2}$ | $\frac{5}{2}$ | 0 | $\{s_3\}$ |
|        | (3, 2, 1, 1) | 0 | $-1, -\frac{1}{2}$ | 1 | $\frac{1}{2}$ | 1 | 2 | $-2$ | 3 | 3 | $-2$ | 0 | $\tilde{p}_1$ |
| $U_3$  | (3, 1, 1, 1) | 0 | $-1, -\frac{1}{2}$ | 1 | $-\frac{1}{2}$ | 1 | 2 | $-2$ | 3 | 3 | $-2$ | 0 | $\tilde{p}_1$ |
|        | (3, 1, 1, 1) | 0 | $-1, -\frac{1}{2}$ | 1 | $-\frac{1}{2}$ | 1 | 2 | $-2$ | 3 | 3 | $-2$ | 0 | $\tilde{p}_1$ |

For the other entries, the table continues with similar entries.
Table 2 (continued)

| sector | irrep | $R_1$, $R_2$, $R_3$ | $q_{\text{assign}}$ | $q_1$ | $q_2$ | $q_1$ | $q_4$ | $q_5$ | $q_6$ | $q_7$ | $q_8$ | label(s) |
|--------|-------|-----------------|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| $T^{(0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)}_{(1, 1)}$ | (1, 1, 1, 1, 1, 1, 1, 1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\langle y_4, y_6 \rangle$ |

Appendix B. Selection rules

The usual string selection rules are modified due to the freely acting symmetry which we mod out. Starting from the general rules [48,49] we find that a superpotential term $\prod_i \Phi^{(i)}$ between the superfields $\Phi^{(i)}$ is allowed if the following conditions are met:

\begin{align}
\text{gauge invariance:} & \quad \sum_i p^{(i)}_{\text{th}} = 0, \\
\text{R-invariance:} & \quad \sum_i R^{(i)} = (-1, -1, -1) \mod (2, 2, 2), \\
\text{point group rule:} & \quad \sum_i k^{(i)} = 0 \mod 2, \\
\text{space group rule:} & \quad \sum_i n^{(i)} = 0 \mod 2, \\
\text{space group rule:} & \quad \sum_i n^{(i)} = 0 \mod 2, \\
\text{space group rule:} & \quad \sum_i (n^{(i)} + n^{(i)} + n^{(i)}) = 0 \mod 2.
\end{align}

Here $p^{(i)}_{\text{th}}$ denote the shifted $E_8 \times E_8$ momenta, the discrete $R$ charges $R^{(i)}$ are computed from the (shifted) SO(8) momenta and oscillator quantum numbers, $R^{(i)} = n^{(i)} - \tilde{N}^{(i)} + \tilde{N}^{(i)}$, $k^{(i)}$, $e^{(i)}$ and $n^{(i)}$ specify the constructing element $(\theta^{(i)} \varphi^{(i)}, n^{(i)} \epsilon^{(i)})$ of the corresponding state. In contrast to the space group selection rule of the standard $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, where similar conditions to the rules (14e)-(14g) also apply for $n^{(i)}$, we find the single condition (14h) in the freely acting case.

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