Research article

Compressive sensing based maximum-minimum subband energy detection for cognitive radios

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ABSTRACT

To satisfy the growing spectrum demands of emerging wireless applications, cognitive radios have been considered as a viable option. It enables dynamic spectrum access opportunistically using wideband spectrum sensing (WSS) methods to discover the temporarily free frequency bands. WSS requires a high-speed analog-to-digital converter (ADC), which has high power consumption and hardware complexity. Improving the power consumption and hardware complexity of the ADC is one of the existing challenges in energy-constrained applications. To alleviate this problem, we propose compressive sensing (CS) in maximum-minimum subband energy detection method to sense the wideband spectrum by utilizing the sparse nature of spectrum occupancy with the minimal possible number of measurements. The CS method uses Fourier Transform and chaotic sequence in designing the measurement matrix to achieve both determinacy and randomness. The Bayesian method is used to reconstruct the signal from the available measurements. From the reconstructed signal, the maximum-minimum subband energy detection (ED) method is used to decide whether the primary user (PU) is absent or present in a particular frequency band. The simulation results show that the proposed CS-based maximum-minimum subband energy detection approach improves the probability of detection by 7.5% compared to the conventional maximum-minimum subband energy detection method of spectrum sensing. The proposed spectrum sensing method is simple and robust to noise uncertainty and signal strength variations.

1. Introduction

The present quick increment in applications, for example, high definition video streaming, interactive media applications, web browsing, broadband cell, social media, and so forth are highly data-intensive and resource-hungry, giving rise to a new sort of challenge called spectrum scarcity to provide high data rate services to a large number of users [1, 2]. The other contributing factor for the shortage of spectrum is the current inflexible frequency allocation systems with no dynamism in the manner and way regulatory agencies allocate spectrum to PUs [3]. Due to the fixed frequency allocation systems and exponential growth in the number of wireless devices, services, and applications, it is difficult to allocate spectrums for newly emerging technologies [4].

However, the trials by the Federal Communication Commission (FCC) demonstrate that at any given time and area, much (somewhere in the range of 80% to 90%) of the licensed spectrum is underutilized [4, 5, 6, 7, 8, 9]. Such temporarily unused spectrum slots are called spectrum holes, resulting in spectral wastefulness. Subsequently, the development of new wireless applications might be hindered, including broadband access, public safety, health care, and business [10]. So as to alleviate the impact of these two causative factors of spectrum scarcity, the cognitive radio (CR) system is recommended that allows utilization of primary (licensed) user’s radio spectrum again and again by secondary (unlicensed) users, as far as secondary users (SUs) do not cause any harmful interference to the primary users (PUs) in that frequency bands. This property of re-usability makes the radio spectrum a high-valued commodity with the capacity of permitting an enormous number of simultaneous users to benefit from it [3, 11].

CR is a smart wireless communication system. To accomplish the desired smartness, the principal challenge is spectrum sensing (SS) that the SUs achieve the awareness about the portion of a licensed spectrum that is not being utilized by PUs before opportunistically utilizing the temporally idle frequency [12]. Fundamentally, there are two types of SS depending on their bandwidth: narrowband SS and wideband SS [8]. Narrowband SS enables a SU to sense one narrowband channel just in one sensing period and decide in that specific slice of the spectrum whether the PU is available or absent [3, 13]. Since the next age of communication systems require high data rate and bandwidth, the con-
vctional narrowband SS methods are impractical. On the other hand, wideband SS scans multiple channels over a wide frequency range to discover more spectral opportunities to improve spectrum efficiency. However, the Shannon-Nyquist theorem states that to guarantee the recovery of the signals from a set of uniformly spaced samples, the signal must be sampled at a rate which is at least twice the bandwidth of the signal. As the spectrum bandwidth gets wider, the amount of data acquired by sensing system grows significantly, which needs a very high-speed analog-to-digital converter (ADC) that is excessively costly, requires complicated hardware implementation that is even physically impossible to build devices capable of acquiring samples at that necessary rate for power-constrained SUs [8, 14, 15, 16, 17, 18].

The narrowband spectrum sensing techniques cannot be straightforwardly utilized for performing wideband SS [8]. Several types of approaches have been proposed to perform wideband SS. The first method was using reconfigurable bandpass filters by dividing the wideband spectrum into several narrow bands and perform sensing sequentially. This method requires a considerable sensing time to cover a broad spectrum. Due to this, the sensing delay in the process will decrease the data transmission time and subsequently results in less throughput for SUs [8, 9, 11]. Another solution is to utilize a bank of parallel narrowband filters with different center frequencies to simultaneously sense multiple frequency bands and perform joint detection. This technique provides faster sensing as compared to a reconfigurable bandpass filter-based system. However, this method requires a large number of radio frequency (RF) front-end components, which increases the size of the receiver and the power consumption of the SU during SS [11, 12].

However, the samples obtained at or above the Nyquist rate are not fully needed to represent a signal in light of the fact that, from this tremendous acquired samples, a significant portion is immediately discarded and relatively a few important ones are stored at the compression stage. In order to eliminate the collection of unnecessary samples, the compressive sensing (CS) method is proposed that combines compression with signal acquisition step [17, 18, 19]. CS technique provides the capability to recover the signal from fewer samples or measurements than the conventional approach of reconstructing a signal from acquired data which is governed by the Shannon-Nyquist sampling theorem [16, 19, 20, 21]. Therefore, the primary challenge in CR is spectrum sensing, in which the SUs achieve the awareness about the portion of a licensed spectrum that is not being used by PUs.

In conventional data acquisition system, all samples of the original spectrum are acquired. For the wideband spectrum, this number of spectrum samples can be in the order of torrent. This acquisition process is followed by compression, which exploits the redundancy in the signal to express in a space where most signal coefficients can be discarded with little or no loss in information. Notwithstanding, because of the practical limitations on the capabilities of SU’s receiver hardware components, for the most part, ADC, it is hard to implement a wideband spectrum sensing algorithm in practice. Hence, in the conventional acquisition system, we initially acquire a considerable amount of data in which a significant portion is immediately discarded at the compression stage. This creates significant wastefulness in SU during SS. Compressive sensing tends to reduce this wastefulness by successfully combining the acquisition and compression processes simultaneously. Therefore, in this paper, the SS is employed to sense the wideband spectrum in one sensing period in CR by using CS.

CR is capable of acquiring various RF parameters to become aware of its surrounding radio environment. This can be accomplished with SS. Several spectrum sensing algorithms have been proposed in the literature. However, none of them fully satisfy the requirements of all associated measurements, for example, efficiency, implementation complexity, reliability, and secondary system throughput degradation [22].

Cyclostationary detection is a powerful spectrum sensing technique which relies on the built-in periodicity of modulated signal since they are coupled with sine wave carriers, repeating spreading code sequence or cyclic prefixes where noise has no spectral correlation. By utilizing this property, PU signals are easily differentiated from noise [23, 24, 25, 26]. Hence, this method is robust to noise uncertainty (NU). It exhibits high computational complexity. Furthermore, cyclostationary-based SS is sensitive to cyclic frequency mismatch, and it has to know the PU transmission, pulse-shaping filter, and roll-off factor, which are not easy to know in real-world scenarios [2, 11, 22, 27].

Another spectrum sensing technique is matched filter detection which is obtained by correlating the received signal with priorly known spectral features of the primary signal. In this technique, the detection needs less time to accomplish high processing task because of the coherent nature of the matched filter processing. However, a matched filter detection requires the SUs to have the exact copy of PU signals that may not be available in a real-world situations. Moreover, for coherence detection, the SUs require a dedicated receiver for licensed users which increases its complexity and it is a significant challenge for an enormous number of PUs [23, 24, 25, 26].

Eigenvalue-based advanced spectrum detecting techniques were proposed to overcome the impact of NU on the performance of the sensing [23, 24]. However, because of the required calculation of the received signals’ covariance matrix and its eigenvalues, these techniques involve exceptionally high computational complexity [27].

The most widely recognized and a non-coherent way of SS is energy detection, that CR receiver does not need to know any information of the PU’s signals, and it has simple hardware realization. However, the detection threshold is highly sensitive to the NU, which degrades its performance under low SNR levels. Effective SS technique needs to identify weak primary signals in view of multipath fading and shadowing phenomena, which results in power fluctuation of received PU signals [2, 11, 13, 22, 23, 24, 26, 27]. An alternative method, maximum-minimum subband energy detection based spectrum sensing technique, was proposed which is efficient and robust. It has low computational complexity, which utilizes the subband decomposition of the received signal and the difference of maximum and minimum subband energy as a test statistic. This method gives an acceptable performance at the low SNR regime with NU by viewing the minimum subband energy as an estimation for a noise variance [22, 27].

In this paper, CS-based maximum-minimum subband energy detection is proposed to enable reliable and fast sensing of the wideband spectrum. The proposed spectrum sensing technique utilizes the sparse nature of wideband signals in the frequency domain on the ground that at any time and geographical area, only a few percentages of total available channels are occupied by PUs. In this manner, the CS-based spectrum sensing technique acquires wideband signals using sampling rates lower than the Nyquist rate and detects potentially vacant spectral bands using these compressed measurements. In other words, this method replaces multiple parallel narrowband ADC or reconfigurable bandpass filters by a single ADC based spectrum sensing technique.

So as to utilize the benefits of CR communication efficiency, a CR should have the capability to monitor the surrounding radio environment over a broad spectrum range. This environmental learning over a wideband range helps to apply a versatile resource allocation and spectrum utilization techniques for the powerful utilization of the underutilized radio spectrum. The main contributions of this paper are summarized as follows:

- The proposed spectrum sensing method achieves higher performance than the existing energy detection methods.
- The impact of different parameters, such as number of active PUs, NU factor, and number of measurements (compression ratio of CS), on maximum-minimum subband energy detection performance are thoroughly investigated both analytically and using simulations.
- The effect of decreasing the number of measurements on spectrum sensing time of the proposed technique is also studied. The proposed technique reduces the SS time.
- Finally, methods that can enhance the performance of maximum-minimum subband energy detection are recommended.
2. Compressive sensing process

Because of the unpredictability of PU, fast and immediate moving over the sensed empty channel is great importance and is a difficult task in cognitive radios. To mitigate this issue, a fast and efficient SS method is required. This quick sensing can be accomplished by acquiring a compressed version of the wideband spectrum and reconstructing that spectrum using a lower number of measurements than those required by the Shannon-Nyquist theorem. Therefore, CS speeds up the process of wideband SS in distinguishing the unoccupied spectrum. This compressive sensing based wideband SS technique permits overcoming the need of a very high-speed ADC to acquire and a high-speed digital signal processing (DSP) to process such a wideband signal, which needs to operate at or above the Shannon-Nyquist rate. There are three primary processes in compressive sensing: sparse representation, measurement, and sparse recovery [20].

2.1. Sparse representation

Sparse representation comprises finding a suitable transform basis to describe useful signals characteristic as linear combinations of a few elements. So, sparse signal has a few significant components that permit the discarding of coefficients with relatively small amplitude without introducing distortion in the reconstructed signals. This enables us to accomplish high rates of compression, and the sampling process can itself be designed to acquire only essential information [28]. Due to low spectrum utilization, the more significant part of the spectrum is idle, which means the wideband spectrum is inherently sparse in the frequency domain by its nature [11, 13].

2.2. Measurements

The second process is a measurement or experiment to gather information about the spectrum occupancy. In CS, the measurement is required to obtain the compressed version of signals directly by taking only a few measurements from the sparse signals. This dimensional reduction in CS is done by linear projection or multiplying the sparse signal by a set of carefully chosen measurement matrix [20, 29].

\[ Y + \eta = \Phi BX = AX \]  

where \( Y \in \mathbb{R}^M \) measurements, \( A = \Phi B \in \mathbb{R}^{N \times M} \) is measurement matrix, \( X \in \mathbb{R}^N \) sparse signals, \( \eta \) is measurement noise, \( B \) is sparsity bases in our case Fourier transform and \( \Phi \) is dimensional reduction matrix.

The matrix \( A \) represents a dimensional reduction by mapping \( R^N \), where \( N \) is generally large, into \( R^M \), where \( M \) is much smaller than \( N \). Thus, significant difficulties in CS is to develop a useful measurement matrix in which the compression must preserve the information in the sparse signal and easy hardware implementation. To guarantee that, the measurement matrix is required to satisfy certain desirable properties that can ensure a certain recovery quality [30].

The vital property for the measurement matrix that is used in CS is a spark. The spark of the matrix is the smallest number of columns that are directly dependant or the largest number of columns from the matrix that are independent. If two columns of the measurement matrix are linearly dependent, it is difficult to distinguish whether the signal's energy comes from one column or another column. However, the spark of a matrix is challenging to compute, which requires a combinational search over possible subsets of columns from the measurement matrix [31, 32]. The other significant property of the measurement matrix is to have uncorrelated columns in the matrix with sparse bases in order to recover the signal accurately. The mutual coherence of measurement matrix \( A \) is the level of closeness between the sparsifying bases \( (B) \) and measurement matrix \( (\Phi) \) [18, 28, 33]. Hence, a large value of spark and low coherence between measurement matrix and sparsifying bases are required to reduce the number of measurements for the recovery of a signal. The minimum the coherence is an assurance that every measurement carries out its responsibility of conveying valuable data in the sensed signal, the less the required number of measurements, and the higher the probability of reconstruction [18]. Therefore, an effective technique of fulfilling optimal incoherence is required for the design of the measurement matrix.

Random measurement matrix is usually used in CS since such a matrix is universal that a measurement matrix will be uncorrelated with high probability regardless of the choice of a sparsifying basis matrix [17, 29]. However, the huge memory requirement for storing the entries of random sensing matrix, its high computational complexity because of unstructured nature, and high processing time makes its hardware implementation expensive. So as to solve the problem of random measurement matrix, deterministically designing a good measurement matrix was proposed by authors of [29, 30].

The deterministic measurement matrix is simple to implement, generate, and reproduce. Its deterministic constructions, simplicity in sampling and recovery process, and small storage requirement makes it a preferable choice. However, these deterministic measurement matrix elements result in a high possibility of correlating with sparsifying bases, which implies that the incoherence criterion is hardly met, and the reconstruction accuracy of the deterministic measurement matrix is inadequate [29]. In order to eliminate the contradiction among determinacy and randomness in deterministic measurement matrix, chaotic system is proposed. A chaotic system is generated by chaotic maps, which is used in the measurement matrix to achieve both determinacy and randomness properties. In this paper, the logistic chaotic sequence generated by the logistic map is used because of its exceptional characteristics such as sensitive dependence on the initial condition, non-periodicity, and pseudo-random property that introduce the idea of the incoherence factor in measurement matrix [29, 30, 34]:

\[ x_{n+1} = \mu(1 - x_n) \]  

(2)

Where \( \mu \in (0, 4) \), \( x_n \in (0, 1) \). Moreover, in this paper, Toeplitz operation has been used to remove the high correlation between measurement matrix and sparsifying bases.

2.3. Sparse recovery

The final and the challenging task in CS is to accurately recover the sparse signals from the available noisy measurement which has a reduced size of elements. The sparse signal recovery problem should also consider measurement noise and additional noise, as mathematically expressed in [20].

\[ Y + \eta = AX \]  

(3)

Where A is the designed measurement matrix, Y is observed measurement vector, X is a sparse signal to be recovered, and \( \eta \) is measurement noise containing measurement errors, quantization error and model imperfection which is generally modeled as an independent zero-mean Gaussian distributed with a variance \( \sigma^2 \).

Generally, from linear algebra, when the number of unknown variables (dimensionality of sparse signal X), exceeds the number of observations (dimensionality of measured signal Y), the system is an underdetermined system of linear equations which have infinitely many solutions. This implies that there exists an infinite possible number of signal X that may explain the observation vector Y. The problem of reconstructing X from Y appears, therefore, somewhat infeasible. In order to narrow the choice to one well-known solution, it is necessary to specify additional criteria [31, 35].

The knowledge of signal X only holds a few non-zero entries relative to the signal dimension allows us to recover the signal uniquely within high probability. Therefore, we need to introduce an objective function \( Q(X) \) to evaluate the desirability of an answer X and the additional property of signal, which is sparsity. This objective function
significantly reduces the set of candidates for X and sufficiently clarifies the observation without unnecessary complexity [33]. In general, signal recovery from noisy measurement is formulated as a constrained optimization problem.

\[
\text{minimize } O(X) \quad \text{Subject to } \|Y - AX\|_2 < \eta
\]

The desired characteristic of a sparse signal with the sparsest solution can be characterized as the solution with the smallest number of nonzero entries. The objective function is \(l_1\)-norm that uniquely combines both sparsity and convexity simultaneously. Therefore the \(l_1\) minimization is given by

\[
\min \|X\|_1 \quad \text{Subject to } \|Y - AX\|_2 < \eta
\]

Various methods have been developed to recover sparse signal X from its noisy compressed measurement. The principal signal recovery algorithms are convex and relaxation that employs a set of techniques that depend on \(l_1\)-norm minimization using a convex optimization technique. Convex optimization-based techniques change equation (5) to linear programming [31, 35]. These algorithms generally possess good recovery performance in solving a convex minimization problem. However, these techniques are difficult to apply in practice when N is large due to high computational cost.

So as to develop computationally cheap while keeping acceptable reconstruction accuracy using \(l_1\) minimization, different greedy algorithms were proposed. Greedy algorithms iteratively attempt to find the signal support (its nonzero indices) by correlating the measured vector Y with the columns of measurement matrix A. The support (non-zero) indices of the sparse signals are expected to have relatively large magnitudes of correlation. A number of highest magnitude components of correlated values are chosen in every iteration, and their indices are added to a set of identified supports [19, 32, 37]. Generally, greedy algorithms exhibit good performance when the prior information on the sparsity of signal (number of the non-zero components) is known. However, in SS, sparsity means the number of active PU at the time of sensing which is impossible to know in reality.

At the point when the sparsity level of the wideband spectrum (number of active PUs) is unknown, the preferable signal reconstruction method is the Bayesian method, which utilizes a probabilistic approach by assigning a prior distribution for a known parameter to calculate the posterior distribution of the unknown parameters. In SS, we have specified the prior knowledge of the reality that X has a few nonzero rows, which are produced by a sparse coefficient expansion to estimate noise variance \(\sigma^2\), the signal mean \(\mu_s\), and signal variance \(\sigma^2_s\) [36, 38, 39, 40].

2.4. Minimum number of measurements in compressive sensing

The most widely recognized \(l_1\) norm based optimization solution that produces a minimal squared error of the reconstructed signal is euclidian norm \(\|X\|_2^2\). The minimal squared error implies that the minimization of the difference between the available measurements and their values produced by reconstructed coefficients [36]. Mathematically, the squared error is defined as follows.

\[
e^2 = \|Y - AX\|_2^2 = (Y - AX)^H (Y - AX)
\]

We take the derivative of \(e^2\) with respect to X and equate it to zero to get minimum error of the reconstruction algorithm.

\[
\frac{\partial e^2}{\partial X} = -2A^HY + 2A^HA X = 0
\]

\[
X = \frac{2A^HY}{2A^HA} = (A^HA)^{-1} A^HY
\]

It is recalled in equation (1) that the available samples are corrupted by additive noise. The signal reconstruction in CS using a small set of samples which are corrupted by a noise causes miss-detection in sparse signal components detection procedure and using least square minimization, the arrangement can be given as:

\[
X = (A^HA)^{-1} A^HY + (A^HA)^{-1} A^Hg = X_s + X_n
\]

where \(A^H\) is conjugate transpose and \(X_s\) is the noise impact on the reconstruction coefficients. There are two possible cause of noise in CS, the first is noise appearing as an outcome of missing samples while the other is noise that appear due to an external source.

The noise variance that appears as an outcome of missing samples can be determined as follows. The \(n\)th DFT coefficient of length N sequence X(n) is defined as follows:

\[
X(k) = \sum_{n=0}^{N-1} X(n) e^{-\frac{2\pi i kn}{N}}, k = 0, 1, ..., N - 1
\]

and its inverse transform (IDFT) as

\[
X(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{2\pi i kn}{N}}, n = 0, 1, ..., N - 1
\]

From equation (9) and (10)

\[
X(k) = \sum_{n=0}^{N-1} X(n) e^{-\frac{2\pi i kn}{N}} = \sum_{n=0}^{N-1} X(n) e^{-\frac{2\pi i kn}{N}} e^{-\frac{2\pi i k(L-n)}{N}}
\]

\[
= \sum_{n=0}^{N-1} X(n) e^{\frac{2\pi i k(L-n)}{N}} (e^{\frac{2\pi i kn}{N}})^{-iX(n)}
\]

\[
= \sum_{n=0}^{N-1} \Lambda_n e^{-\frac{2\pi i kX(n)}{N}}
\]

In CS, we reduce measurement \(Y(m), m = 0, 1, 2, ..., M - 1\), where M is the number of measurements and \(M < N\) for reduced measurements. The missing samples can be represented as noise in the transform domain, destroying the signal sparsity. The reconstruction conditions with the goal that the recovery algorithm performs spectrum reconstruction from the largest K coefficients. The rest of the N-K spectrum component will produce noise in these K largest components. Therefore, discarding samples from the transform domain signal can be considered as a complete set of samples affected by additive noise introduced due to missing samples and mathematically expressed in [41, 42].

\[
\sum_{n=1}^{M} Y(n) = \sum_{n=1}^{N} X(n) + \eta(n)
\]

where

\[
\eta(n) = \begin{cases} -\Lambda_n e^{-\frac{2\pi i kX(n)}{N}} & \text{For missed signal samples} \\ 0 & \text{For available samples} \end{cases}
\]

Due to the missing samples, the variance of noise \(\sigma^2_{M,S}\) is defined as follows [41, 42]:

\[
\sigma^2_{M,S} = M \frac{N-M}{N-1} \sum_{n=1}^{K} \Lambda_n^2
\]

As we can see in (14), the noise appears due to missing sample relies upon the signal amplitude, the number of missing sample and the total number of samples and the variance of signal component \(\sigma^2_{S,i}\) is given by:

\[
\sigma^2_{S,i} = M \frac{N-M}{N-1} \sum_{i=1}^{K} \Lambda_i^2 = M \frac{N-M}{N-1} \left( \sum_{i=1}^{K} \Lambda_i^2 - \sum_{j=1}^{K} \Lambda_j^2 \right)
\]
In addition to noise due to missed samples, the measurement is corrupted by Gaussian noise with variance $\sigma_e^2$ and the total expected error energy ($\sigma^2$) caused by both the measurement noise and noise due to the missing samples is given below [42].

$$\sigma^2 = \sigma_{M+N}^2 = M \frac{N-M}{N-1} \sum_{i=0}^{K} \lambda_i^2 + M \sigma_e^2$$  (16)

The probability that at least one of N-K noisy DFT values are higher than one of K signal DFT components follow Rayleigh probability distribution function and mathematically given by [41, 42]:

$$P_k = 1 - \left[1 - \exp\left(-\frac{(M \lambda_i^2 - A_{\min})^2}{\sigma^2}\right)\right]^N$$  (17)

where $P_k$ is probability of error and $i = 1, 2, 3, ..., K$. In the case of a reduced number of measurements, some of the components are masked by noise. Therefore, the number of signal components with the expected value of the minimal component amplitude that is above the noise level can be detected. From equation (15), the expected value of the minimal signal component amplitude is given by $A_{\min} = M \frac{N-M}{N-1} \left(\sum_{i=1}^{K} \lambda_i^2 - \Lambda_{\min}\right)$.

$$P(M) = 1 - \left[1 - \exp\left(-\frac{(M \lambda_{\min}^2 - A_{\min})^2}{\sigma^2}\right)\right]^N$$  (18)

Where $P(M)$ helps determine the minimum number of measurements M required for proper signal reconstruction in the presence of noise, ensuring no false component detection and $\sigma^2$ is given in (16). This minimum number of measurements consequently eliminates the misdetection of sparse signal components due to the missing samples and external noise.

2.5. Effect of compressive sensing on SNR

The SNR of the recovered spectrum without applying CS is given by:

$$SNR_0 = 10 \log\left(\frac{\sum_{n=0}^{N-1} X(n)^2}{\sigma_e^2}\right)$$  (19)

The energy of the spectrum is concentrated in a small number of Fourier coefficients due to the sparse nature of its utilization. Hence, the spectrum can be sparsely represented in the frequency domain, whereas noise is not sparse in a frequency domain. For efficient signal reconstruction, it is important to separate those (N-K) coefficients belonging to the noise component from signal components. To separate signals from noise, we need to define the probability (P($T$)) that noise samples in the frequency domain are below a threshold T. The probability that all (N-K) samples corresponding to noise value, which are below T, is given in [43].

$$P(T) = \left[1 - \exp\left(-\frac{T^2}{\sigma_e^2}\right)\right]^{N-K} \approx \left[1 - \exp\left(-\frac{T^2}{\sigma^2}\right)\right]^N$$  (20)

A predefined probability of error ensures that the signal components can be separated from noise and for the desired value of P(T), the threshold value of detection (T) is given follows.

$$T = \sqrt{-\sigma_e^2 \log\left(1 - P(T)^\frac{1}{N}\right)}$$  (21)

Hence, it is possible to reduce the influence of noise on signal component detection by keeping only the large Fourier transform coefficients using appropriately chosen threshold level and throw away other insignificant components which are below the threshold value.

In the case of a reduced set of measurements (from N measurements to M measurements), only M available measurements, the observed noise energy ($\sigma_{R,n}^2$) is reduced [36, 43].

$$\sigma_{R,n}^2 = \frac{N^2}{M^2} \sigma_e^2$$  (22)

Since just K out of N coefficients are used in the reconstruction, the energy of the reconstructed samples is reduced by the factor of $\frac{K}{N}$

$$\sigma_{R,k}^2 = \frac{K N^2}{M^2} \sigma_e^2 = \frac{K M}{M^2} \sigma_e^2$$  (23)

Using the reality that the variances in measurements are the same $\frac{1}{N} \sigma_e^2 = \frac{1}{N} \sigma^2$, we can write

$$\sigma_{R_k}^2 = \frac{K}{M} \sigma^2$$  (24)

substituting equation (24) in to (19) and we get the SNR due to reduced measurement

$$SNR = 10 \log\left(\frac{\sum_{n=0}^{N-1} X(n)^2}{\sigma_{R,k}^2}\right) = 10 \log\left(\frac{\sum_{n=0}^{N-1} X(n)^2}{\frac{K}{M} \sigma_e^2}\right)$$

$$= SNR_0 - 10 \log\left(\frac{K}{M}\right)$$  (25)

where $K$ is the number of active PU, $M$ is the number of measurements, $SNR$ is the signal to noise ratio of the recovered spectrum and $SNR_0$ is the signal to noise ratio when all signal sample used (without CS).

From equation (25), the SNR is significantly improved by $10 \log\left(\frac{K}{M}\right)$ dB because of the application of CS. The SNR of the spectrum of the samples depends on the number of active PUs (K) and the number of available measurements (M).

3. Compressive sensing based maximum-minimum subband energy detection

In this section, we analyze the impact of applying CS on the performance of maximum-minimum subband energy detection. The performance of the spectrum detection algorithm is influenced by noise variance uncertainty due to multipath fading, shadowing, and varying channel conditions. Hence, the noise uncertainty (NU) impacts are considered in spectrum occupancy decision making process.

3.1. Problem formulation in spectrum detection

There are two hypotheses in SS problem. The null hypothesis defines no signal transmission of PUs which is represented as $H_0$, and the other hypothesis, $H_1$, indicates that the channel is occupied by PUs signals along with noise. Mathematically, these hypotheses are defined as [4, 11, 23].

$$Y[n] = \begin{cases} \eta[n], & H_0; \text{Signal is absent} \\ \frac{X[n]}{S[n] \otimes h[n]} + \eta[n], & H_1; \text{Signal is present} \end{cases}$$  (26)

where $Y[n]$ is the recovered signal, $\eta[n]$ denotes the measurement noise, $h[n]$ denotes the channel impulse response and $S[n]$ denotes the signal which is transmitted by PU.

3.2. Conventional energy detection method

The principle of conventional energy detector is finding the energy of the received signal and comparing that with an appropriately selected threshold to determine the presence or the absence of the primary signal. The test for energy detector is given by [4, 27]:

$$\text{Energy of received signal} = E_s = \frac{1}{N} \sum_{n=0}^{N-1} |Y[n]|^2$$  (27)

This energy $E_s$ is then compared to a pre-defined threshold $T$; to obtain the sensing decision as follows:
The performance of spectrum sensing algorithm is evaluated using the probability of false rejection and probability of false alarm. The probability of false alarm of $H_0$ describes the probability of deciding the PU is present while the PU is absent.

$$P_{FA} = P\{E_y > T | H_0\} = Q\left(\frac{T - \sigma_y^2}{\sqrt{\frac{\sigma_y^2}{N}}}ight)$$ (29)

where $T$ is the detection threshold, $\sigma_y^2$ is the variance of the noise and $Q$ is standard Gaussian complementary cumulative distribution function (Q-Function). With regards to CR networks, a false alarm yields undetected spectrum holes. So a large $P_{FA}$ contributes to poor spectrum usage by SUs. While the probability of false rejection in $H_1$ is called Probability of missed detection ($P_M$) which refers to the probability of deciding the PU is absent while actually, the PU is present.

$$P_M = P\{E_y < T | H_1\}$$ (30)

which is equivalent to identifying a spectrum hole where there is none. Consequently, large $P_M$ introduces unexpected interference to PUs. Detection probability ($P_D = 1 - P_M$) is the probability of deciding the signal is present when actually the PU is present and identifying the channel as not busy, when the channel is truly not busy.

$$P_D = P\{E_y > T | H_1\} = P\{E_y < T | H_0\}$$ (31)

$$P_D = P\{E_y > T | H_1\} = Q\left(\frac{T - \sigma_y^2(1 + \gamma)}{\sqrt{\frac{\sigma_y^2(1 + \gamma)}{N}}}ight)$$ (32)

Where $\sigma_y^2$ is the power of the primary signal and $\gamma = \frac{\sigma_y^2}{\sigma_i^2}$ is the average signal-to-noise ratio.

Because the effect of applying CS in equation (25), the SNR in dB is given as

$$SNR = SNR_0 - 10\log\left(\frac{K}{M}\right)$$ (33)

where $\gamma$ is SNR when we apply CS, $\gamma$ is $SNR_0$ without applying CS, $K$ is number of active PUs, and $M$ is number of measurements.

Therefore, when we apply CS, $P_D$ is given as

$$P_D = Q\left(\frac{T - \sigma_y^2(1 + \gamma)}{\sqrt{\frac{\sigma_y^2(1 + \gamma)}{N}}}ight)$$ (34)

The variance of the PU information signal is practically unknown to the SU and thus the design of a test for $H_0$ versus $H_1$ involves a trade-off between the $P_{FA}$ and the $P_D$ since a reduction on the $P_{FA}$ will decrease the $P_D$, and an increment on the $P_D$ will increase the $P_{FA}$. The Neyman-Pearson criterion for making this trade-off is to place a bound on the $P_{FA}$ and then to maximize the detection probability within this constraint thus the value of $T$ is commonly calculated by the assumed noise variance and desired $P_{FA}$ as:

$$T = \sigma_y^2\left(1 + \frac{Q^{-1}(P_{FA})}{\sqrt{\frac{\sigma_y^2}{N}}}ight)$$ (35)

The possibility for the estimation of exact noise variance value is technically non-existing. Because of energy detection just observes the energy of the PU signal, determining the $P_{FA}$ and $P_D$ results are highly dependent upon the accuracy of the noise variance estimate. Therefore, the significant loss in the detection performance occurs because of a small change in noise power changeability. Practically, vulnerability of noise can be expected to lie within the range $\sigma_y^2 \in \left[\frac{1}{2} \sigma_i^2, \rho \sigma_i^2\right]$ [44], where $\sigma_i^2$ is the nominal noise power and $\rho$ is a parameter that measures the size of vulnerability. The worst case $P_{FA}$ and $P_D$ can be explicitly expressed as follows:

$$P_{FA} = \max_{\sigma_y^2} Q\left(\frac{T - \sigma_y^2}{\sqrt{\frac{\sigma_y^2}{N}}}ight) = Q\left(\frac{T - \sigma_i^2}{\sqrt{\frac{\sigma_i^2}{N}}}(1 + 3)\right)$$ (36)

$$P_D = \min_{\sigma_y^2} Q\left(\frac{T - \sigma_y^2(1 + \gamma)}{\sqrt{\frac{\sigma_y^2(1 + \gamma)}{N}}}ight) = Q\left(\frac{T - \sigma_i^2(1 + 3)\sqrt{\frac{\rho}{N}(1 + 3)}}{\sqrt{\frac{\rho \sigma_i^2}{N}(1 + 3)}}\right)$$ (37)

3.3. Maximum-minimum subband energy detection based spectrum sensing

Wideband energy detection is performed at subband level as the output of an Fast Fourier Transform (FFT). $Y_m[k]$ is the subband output signal, where $m$ is the subband sample index and where $k = 0, \ldots, K - 1$ is the subband index. With regard to SS, the subband signals can be expressed as follows [27]:

$$Y_k[m] = \begin{cases} 0 & H_0 \\ S_m[k] + \eta_m[k] & H_1 \end{cases}$$ (38)

Where $H_k, S_m[k]$ denotes the PU signal at $mth$ FFT output sample in subband $k$ under $H_k$ hypothesis, $H_1$ is complex channel frequency response gain of subband $k$, and $\eta_m[k]$ is the corresponding noise sample. Additionally, the noise $\eta_m[k]$ can be modeled as a zero-mean Gaussian random variable with variance as $N\left(0, \sigma_k^2\right)$ and $x_m[k] ~ N\left(0, \sigma_k^2\right)$ denoting the PU signal variance in subband $k$. For a uniform FFT, the subband noise variances $(\sigma_{n,k}^2)$ are assumed to be equal, such that

$$\sigma_{n,k}^2 = \frac{\sigma_n^2}{N}$$ (39)

The overall subband ED process can be summarized as

$$E_k = \frac{1}{L_k} \sum_{m=1}^{N - N_{FFT}} |Y_m[k]|^2$$ (40)

Where $L_k = \frac{N - N_{FFT}}{N_{FFT}}$, $N_{FFT}$ is N point fast Fourier transform and by using central limit theorem the expected value of energy $E_k$ is given by [27]

$$E_k = \begin{cases} N \left(\frac{\sigma_k^2}{2} \frac{N}{L_k} \right) & H_0 \\ N \left(\frac{K}{L_k} \frac{N}{2} \right) & H_1 \end{cases}$$ (41)

Where $K = \sigma_k^2 / (H_k^2 \gamma_m + 1)$ and $\gamma_m = \frac{\sigma_k^2}{\sigma_n^2}$ is signal to noise ratio.

The maximum-minimum ED method subdivide and estimate the energies over various subbands of the total frequency band of interest.
and evaluates the maximum and minimum subband energy levels or energy differentials. The difference of maximum and minimum energy levels are used as decision measurement and compared with the predefined energy threshold as shown in Fig. 1.

### 3.4. Decision device and threshold calculation

In the decision stage, the maximum and minimum of the subband energies are determined. The corresponding difference between the maximum and minimum values is compared with a predetermined threshold. The desired $P_{FA}$ with the help of Gumbel distribution is used to obtain the predefined threshold. As per this test when

$$E_{max-min} = E_{max} - E_{min} > T$$  \hspace{1cm} (42)

the PU is thought to be available; else it is expected that only noise is available in the band of interest.

The decision statistics depends on the maximum and minimum value of $E_k$. Probabilistic extreme value theory deals with its stochastic behavior. Extreme value distributions arise as limiting distributions for maximums or minimums (extreme values) of a sample of independent, identically distributed random variables. Gumbel distribution arises due to the limit of the maximum or minimum of N independent random variables, each with the standard exponential distribution. Based on Gumbel distribution, the probability of false alarm and probability of detection are given as [27]

$$P_{FA} = 1 - e^{-e}$$

and

$$P_D = 1 - e^{-e}$$

Where $C = 0.577215665$ is the Euler’s constant [27]. In realistic situations, NU must be considered in the expressions of $P_{FA}$ and $P_D$. The noise distribution can be summarized to be in the range $\sigma_n^2 \in \left[ \left( \frac{1}{\rho^2} \right) \sigma_n^2, \rho^2 \sigma_n^2 \right]$, where $\rho$ presents the corresponding NU parameter. As a result, the worst-case probability of false alarm and detection probability can be expressed as follows:

$$P_{FA} = 1 - \max_{\sigma_n^2 \in \left[ \left( \frac{1}{\rho^2} \right) \sigma_n^2, \rho^2 \sigma_n^2 \right]} (e^{-e^{2\sqrt{\frac{c}{\sigma_n^2}}}})$$

$$P_D = 1 - e^{-e}$$

Table 1. List of parameter values used in the performance analysis of the proposed spectrum sensing method.

| No. | Parameters | Values used |
|-----|------------|-------------|
| 1   | Probability of detection | ≥90%         |
| 2   | Probability of false detection | ≤10%         |
| 3   | Probability of missed detection | ≤10%         |
| 4   | Length of FFT | 8, 32       |
| 5   | Noise variance uncertainty | (0-1) dB     |
| 6   | Time record length | 10240 samples |
| 7   | Average signal to noise ratio | −12 dB      |
| 8   | Nominal noise power ($\sigma^2$) | 0.5 to 1 dB |

The energy threshold can be determined by

$$T = \frac{\rho \sigma_n^2}{2} + C \sqrt{\frac{6}{L_t} \frac{\rho \sigma_n^2}{\pi \sigma_n^2}}$$

$$+ \left( \frac{6}{L_t} \right)^{\frac{1}{6}} \sqrt{\frac{\pi}{6}} \sigma_n^2 \ln \left( \frac{1}{1 - P_{FA}} \right)$$

The worst case probability of detection with NU can be expressed as:

$$P_D = \min_{\sigma_n^2 \in \left[ \left( \frac{1}{\rho^2} \right) \sigma_n^2, \rho^2 \sigma_n^2 \right]} (1 - e^{-e^{2\sqrt{\frac{c}{\sigma_n^2}}}})$$

$$= 1 - e^{-e}$$

where $K' = \frac{\sigma_n^2}{\rho} (|H_1|^2 3_k + 1)$.

### 4. Simulation results and discussions

The performance of SS is characterized using the probability of detection ($P_D$). In cases where the $P_D$ characteristics are below the target minimum SNR, the spectrum sensing system will have high missed detections that cause interference to PU operations. In this paper, $P_D = 0.9$ is considered as a coarse lower limit. Moreover, for efficient CR operation, adequately low $P_{FA}$ needs to be reached in a reliable way. Therefore, $P_{FA} = 0.1$ is considered as a coarse upper limit [10]. Table 1 shows the parameter values used to evaluate the performance of the proposed spectrum sensing algorithm using simulations.

#### 4.1. Performance of the sparsity basis

The performance of the transform basis depends on sparsity, which is the number of non-zero elements in that domain. The number of non-zero elements is required to be much smaller than the total number of signal samples for the signal representation to be considered as sparse.

The simulation results in Fig. 2 show that for a signal with a length of 10240, there are only eight non-zero coefficients representing its spectrum, which is obtained by applying Discrete Fourier Transform (DFT).
However, there are about 5127 and 10240 non-zero DCT coefficients and DWT Coefficients, respectively, for the same signal. Hence, the DFT is an efficient sparsity basis for signals consisting of sinusoidal components.

Since most of the spectrum is idle at a given time in a given geographical area, it is conceivable to describe the spectrum characteristic using a few non-zero Fourier coefficient that allows discarding of numerous coefficients without introducing significant distortion in the reconstructed signal.

4.2. Signal recovery

Fig. 2 shows that since the highest frequency in the spectrum is 5,000 Hz, we need to recover the signal from the samples according to the Shannon-Nyquist theory. However, from simulation results shown in Fig. 3, only eight sinusoids are enough to describe the characteristic spectrum. The simulation result shows that the recovered spectrum from 200 samples without introducing significant distortion in the recovered signal spectrum.

4.3. Comparison of conventional and maximum-minimum subband ED

Fig. 4 shows the simulation of the receiver operating characteristic (ROC) curve under noise variance uncertainty. The simulation results show that the maximum-minimum subband ED can effectively reduce the NU effect on the performance. The detection performance degrades significantly with a small change in noise variance uncertainty in conventional ED. Whereas the detection performance reduction is negligible in the maximum-minimum subband ED compared to conventional ED as shown in Table 2.

4.4. Performance of CS in maximum-minimum subband ED

Fig. 5 shows the ROC curve, which is obtained by applying CS on maximum-minimum subband ED. The simulation result shows the performance of the proposed method is better than the conventional maximum-minimum subband ED method, which does not employ compressive sensing.

Fig. 6 depicts the simulation result of the probability of missed detection versus SNR. As we can see, CS-based maximum-minimum subband ED has better performance than maximum-minimum subband ED, governed by the Shannon-Nyquist rule in the sampling process. The result proves that the detection performance of maximum-minimum subband ED can be significantly improved by applying CS.

This improvement is obtained because of randomly positioned unavailable samples of a signal. Therefore, removing the samples corrupted by noise and computation of the Fourier transform using the remaining incomplete set of samples helps improve the SNR of the signal. This is the main reason that introducing CS in spectrum sensing that reconstructs the signal from a few available samples involves eliminating noisy samples. The removal of noisy samples can significantly improve the performance of the spectrum sensing algorithm.
4.5. Effect of number of measurements and number of active PUs on detection performance

Fig. 7 and 8 show the effect of increasing the number of active PUs and the number of measurements versus probability of detection. The simulation result shows that the number of active primary users is inversely proportional to the probability of detection i.e., as the number of active PUs increases; the probability of detection decreases significantly. Contrarily, the number of measurements and the probability of detection have direct relationship, i.e. as the number of measurements increases, the probability of detection also increases. Fig. 9 shows the number of required measurements for the successful reconstruction of the spectrum, which affects signal reconstruction performance through the phenomenon of induced noise. The simulation results show that the spectrum with high SNR is detected with a minimal number of random measurements. Additionally, the number of required measurements increases as the strength of the signal decreases to resolve the possibility of reliable signal components detection and reconstruction in noise.

From the above discussion, one of the reasons behind applying CS to spectrum sensing is to reduce the sampling time. The time consumption in CS is the time required to sense both spectrum value and the location of the sample spectrum point needed for the SU in reconstruction. When the number of measurements for spectrum sensing is reduced, the time required for SU in the sampling process is also reduced significantly. Therefore, the time consumed by SU during spectrum sensing increases along with the increasing number of measurements [45]. Additionally, a shorter sensing time during SS has a longer data transmission duration that increases the throughput of the SUs.

The time of acquiring and sending both the spectrum values and their locations information for 5000 points is near the time of acquiring every one of the 10000 spectrum values using the traditional sampling method governed by the Shannon-Nyquist sampling theorem [45]. Therefore, beginning from 5000 measurements, every reduction of 2000 measurement will lead to the 40% reduction of the original sampling time. From the result demonstrated in Fig. 9, the number of required measurements depends on the signal strength,
which means the time consumption improvement depends on the signal strength. The strongest signal requires 100 measurements that have around 98% reduction in sensing time.

Generally, a number of measurements required to ensure the expected minimal signal components amplitude above the noise level should be detected. Therefore in our case, the number of measurements required in Fig. 9 to detect minimal signal components is 1000 measurements, which decrease SS duration by 80% by applying CS.

### 4.6. Spectrum sensing time

Fig. 10 shows the probability of detection versus sensing time at a fixed stationary level of probability of false alarm. The simulation results show that increasing the sensing time in SS leads to an increased probability of detection and decreasing the probability of missed detection. Furthermore, for a given sensing time, applying CS on maximum-minimum subband ED gives an improved probability of detection and probability of missed detection. In general, to obtain better detection performance, longer sensing time is required. During the sensing period, the transmission of SUs is not allowed. From the SU point of view, these longer time in the sensing process will decrease the data transmission time for SUs and subsequently results in less throughput for SUs.

The comparison of various energy detection methods of spectrum sensing in cognitive radios is given in Table 3. The proposed spectrum sensing method is computationally simple and eliminates the effect of noise uncertainty leading to a good probability of detection, which makes it suitable for energy-constrained cognitive radio applications.

### 5. Conclusion

A spectrum sensing method based on maximum-minimum subband energy detection using compressive sensing for cognitive radios has been analyzed analytically and using simulations. The compressive sensing method uses a Fourier basis as sparsifying basis, and l1-norm minimization technique is used as a signal recovery mechanism. By introducing the compressive sensing in spectrum sensing methods, the signal to noise ratio of the signal improves, and the probability of detection increases. The application of compressive sensing improves the probability of detection of conventional maximum-minimum subband energy detection method by 7.5%.

The effects of the number of measurements, and the number of active users on the probability of detection for the proposed method have been investigated. The signal to noise ratio, which affects the probability of detection, decreases with an increasing number of active users in a given band of interest and increases with the increasing number of measurements. The proposed spectrum sensing method is both computationally inexpensive and has a good probability of detection, which makes it suitable for energy-constrained CR applications.

In the future, to improve probability of detection using cooperative sensing is recommended in which measurements from multiple SUs are melded together to make the final sensing decision. Accumulation of more measurements would lead to unnecessary samples. This can be avoided by making the measurement matrix adaptable to the number of active PUs in the spectrum using a machine learning algorithm. Additionally, in this paper, the spectrum is sparse in the frequency domain due to its lower utilization. This consideration might be applicable to the present situation; however, the circumstance may change in the future due to the radio spectrum’s improved usage.

### Declarations

**Author contribution statement**

Desalegn T. Dagne: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Kinde A. Fante: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

Getachew A. Desta: Analyzed and interpreted the data; Wrote the paper.
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Additional information

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