Compensation of errors due to incorrect model geometry in electrical impedance tomography

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Abstract. Electrical impedance tomography (EIT) is a highly unstable problem with respect to measurement and modeling errors. This instability is especially severe when absolute imaging is considered. With clinical measurements, accurate knowledge about the body shape is usually not available, and therefore an approximate model domain has to be used in the computational model. It has earlier been shown that large reconstruction artefacts result if the geometry of the model domain is incorrect. In this paper, we adapt the so-called approximation error approach to compensate for the modeling errors caused by inaccurately known body shape. This approach has previously been shown to be applicable to a variety of modeling errors, such as coarse discretization in the numerical approximation of the forward model and domain truncation. We evaluate the approach with experimental data from a thorax phantom, with absolute imaging considered. We show that the related modeling errors can be efficiently compensated for by the approximation error approach. We also show that recovery from simultaneous discretization related errors is feasible, allowing the use of computationally efficient reduced order models.

1. Introduction

The reconstruction of the conductivity in electrical impedance tomography (EIT) problem is a non-linear inverse boundary value problem which is highly unstable with respect to measurement and modeling errors. The effect of the measurement errors can be reduced by using an accurate measurement system and by careful modeling of the statistics of the measurement error.

Modeling errors, on the other hand, are often related, for example, to discretization of the forward model, truncation of the computational domain and unknown boundary data. Furthermore, a common modeling error in medical EIT is related to inaccurate knowledge on the shape of the target body. Most of the available reconstruction methods assume that the boundary of the target body is known. In principle, the shape of the patient’s thorax could be obtained from other imaging modalities. However, such information is often not available and therefore the reconstruction has to be computed using an approximate model domain.

The traditional way to circumvent the problem of inaccurately known body shape has been to use difference imaging. In spite of the difference imaging modality being able to suppress some of the effects of model uncertainties, it has been shown that the breathing artefacts are still present in the reconstructions.
In this paper, we propose the reduction of the reconstruction errors caused by inaccurately known body shape in EIT by using the Bayesian approximation error approach, which was originally proposed in [1]. The key idea in the approximation error approach is, loosely speaking, to represent not only the measurement error, but also the effects of the computational model errors and uncertainties as an auxiliary additive noise process in the observation model. The realization of the modeling error is obviously unknown since its value depends on the actual unknown conductivity and body shape. However, the statistics of the related approximation error can be estimated using the prior distribution models of the conductivity and parameterization of the body shape. The approximation error statistics are then used in the reconstruction process for compensating the inaccurately known body shape. In this paper, the proposed approach is evaluated with a 3D example with experimental data from a thorax shaped measurement tank.

2. Methods

2.1. Forward model and notation

We model the measurements with the complete electrode model. The numerical approximation of the forward model is usually based on the finite element (FEM) approximation. In the following, we use notation $U_h(\sigma, \gamma) \in \mathbb{R}^m$ for the FEM based forward solution. The dependence of the forward model on the domain $\Omega$ is expressed by the parameterization $\gamma$ of the boundary $\partial \Omega$ and the subindex $h$ denotes the discretization level parameter.

The measurement noise in EIT experiments is commonly modeled as Gaussian additive noise which is mutually independent with the unknown conductivity. This leads to measurement model

$$V = U_h(\sigma, \gamma) + e, \quad e \sim \mathcal{N}(e_s, \Gamma_e)$$

where $V \in \mathbb{R}^m$ is the vector of the measured voltages, $\sigma \in \mathbb{R}^N$ is the conductivity vector, and $e \in \mathbb{R}^m$ is a Gaussian distributed measurement noise with mean $e_s \in \mathbb{R}^m$ and covariance matrix $\Gamma_e$.

Note that if the boundary $\partial \Omega$ of the target body is not known accurately ($\gamma$ is incorrect) or the FEM discretization is too coarse ($h$ is too large), the error in the FEM approximation $U_h(\sigma, \gamma)$ may become significant compared to the measurement error $e$.

2.2. Approximation error approach

In this section, we explain shortly how the modelling error caused by using the model domain $\bar{\Omega}$ instead of the actual (unknown) domain $\Omega$ in the computational (forward) model can be embedded in the likelihood model. For more detailed discussion, see [2].

Let $U_\delta(\bar{\sigma}, \gamma)$ denote a (sufficiently) accurate forward model. Here the parameters $\gamma$ of the boundary $\partial \bar{\Omega}$ and discretization level parameter $\delta$ are such that the error in the FEM approximation is smaller than the measurement error. Let $U_h(\sigma, \tilde{\gamma})$ denote a approximate forward model where the discretization level parameter $h > \delta$ and $\tilde{\gamma}$ are the parameters of the boundary $\partial \Omega$ of the model domain.

The relation of the representation of the conductivities in models $U_\delta(\bar{\sigma}, \gamma)$ and $U_h(\sigma, \tilde{\gamma})$ is of the form

$$P : \Omega \rightarrow \bar{\Omega}, \quad P\bar{\sigma} = \sigma,$$

where $P$ is a mapping that describes the deformation of the conductivity from the domain $\Omega$ into $\bar{\Omega}$. In the numerical examples considered in this study, $P$ is a linear deformation mapping that preserves angle and relative distance from the center of the domain.

In the approximation error approach, the accurate measurement model is written in the form

$$V = U_h(\sigma, \tilde{\gamma}) + (U_\delta(\bar{\sigma}, \gamma) - U_h(\sigma, \tilde{\gamma})) + e$$

$$= U_h(\sigma, \tilde{\gamma}) + e(\bar{\sigma}, \gamma) + e = U_h(\sigma, \tilde{\gamma}) + \eta,$$

(3)
where $\varepsilon(\bar{\sigma}, \gamma)$ represents the modelling error due to the discretization and incorrect boundary, and we denote $\eta = \varepsilon + \epsilon$. The objective in the modelling error approach is to estimate the statistics of the error $\eta$ and construct the likelihood model using this statistics instead of using only the statistics of the measurement error.

To obtain a computationally feasible and efficient approximation, we make the Gaussian approximation for the joint distribution $\pi(\sigma, \eta)$. This is the core of the most common implementation of the approximation error approach, in particular when computational efficiency is sought. To simplify the analysis further, we make an approximation where the mutual dependence of parameters $\sigma$ and $\eta$ is ignored. This approximation is referred to as an enhanced error model. In problems such as EIT that involve nonlinear forward models, the statistics of the approximation error have to be estimated by stochastic simulation.

For the Monte Carlo simulation, we generate a set of $N_s = 150$ draws from the prior models $\pi(\gamma)$ and $\pi(\bar{\sigma})$. For the draws for the domains and the boundary shapes, $N_s$ CT images of different individuals were segmented. This resulted in the ensemble of domains $\{\Omega^{(l)}, \ell = 1, 2, \ldots, N_s\}$ and the corresponding parametric representations for the boundaries $\{\gamma^{(l)}\}$. The samples of the conductivity $\bar{\sigma}^{(l)}$ were drawn from a proper Gaussian smoothness prior [1]: $\bar{\sigma} \sim \mathcal{N}(\bar{\sigma}, \Gamma_{\bar{\sigma}})$. Given the accurate and target forward solutions, the samples $\varepsilon^{(l)}$ of the approximation error were obtained as $\varepsilon^{(l)} = U_h(\bar{\sigma}^{(l)}, \gamma^{(l)}) - U_h(\sigma^{(l)}, \hat{\gamma})$. The mean $\varepsilon_s$ and covariance $\Gamma_{\varepsilon}$ of the approximation error were estimated as a sample average.

When the likelihood model is based on the estimated statistics of the error $\eta$ and Gaussian prior is used, the computation of the MAP estimate amounts to solving the minimization problem

$$
\sigma_{\text{MAP}} = \arg \min_{\sigma \geq 0} \{ \| L_\eta(V - U_h(\sigma, \hat{\gamma}) - \varepsilon_s - \varepsilon) \|^2 + \| L_\sigma(\sigma - \sigma_s) \|^2 \},
$$

where Cholesky factors are $L_\eta^T L_\eta = (\Gamma_{\varepsilon} + \Gamma_{\varepsilon})^{-1}$ and $L_\sigma^T L_\sigma = \Gamma_{\sigma}^{-1}$. We refer to the MAP estimate (4) as MAP with the approximation error model (MAP-AEM).

In the conventional approach, approximation errors are ignored and the mean $\varepsilon_s$ and covariance $\Gamma_{\varepsilon}$ in (4) are assumed to be zero. This approach is referred to MAP with conventional error model (MAP-CEM).

3. Results

The experimental data was measured from a vertically symmetric measurement tank $\Omega$, see Fig. 1. The height of the tank $\Omega$ and the model domain $\tilde{\Omega}$ was 5cm. Thus, no model error due to truncation of the computational domain in the vertical direction was present. To construct the phantom, heart and lung shaped inclusions were made of agar and placed in the measurement tank filled with saline of conductivity 3.0 mS cm$^{-1}$. The conductivity of the lung and heart targets were 0.73 mS cm$^{-1}$ and 5.8 mS cm$^{-1}$, respectively.

The reconstructions are shown in Fig. 1, together with a photograph of the measurement phantom. The images that are shown, are the central horizontal cross sections of the reconstructed 3D conductivity, that is, 2.5 cm from both the top and bottom layers. The MAP-CEM estimate with the accurate forward model $U_h(\sigma, \gamma)$ can be considered as a reference estimate since no discretization errors or geometrical modeling errors are present. As can be seen, the MAP-CEM estimate with the reduced forward model $U_h(\sigma, \hat{\gamma})$ show intolerable errors. In contrast, the MAP-AEM estimate with reduced model $U_h(\sigma, \hat{\gamma})$ is able to capture the geometry of the organs well. Also, the actual values of the conductivity distribution match well.

Although it is clear that the MAP-CEM estimate with the actual geometry is a better estimate than the MAP-AEM with $U_h(\sigma, \hat{\gamma})$, the computational time with the latter is only about 21% of the former. Of course, the central issue is that the actual domain does not need to be known.
### 4. Conclusions

In this paper, we considered electrical impedance tomography and the recovery from incorrectly modelled boundary shape. An incorrectly modelled boundary has been known to induce significant errors to the estimates, typically making absolute imaging impossible.

In this paper, we applied the recently proposed approximation error approach for the modelling of the errors induced by the unknown boundary shape. We considered a real tank measurement case, corresponding to a (absolute) thorax imaging problem. For the construction of the prior distribution for the thorax shape, CT images were used. The results show that the approximation error approach is well suited for the recovery from simultaneous uncertainty in boundary shape and the errors caused by using highly approximative forward solvers. The results of this paper, together with earlier results concerning the handling of the truncation of the computational domain, suggest that absolute EIT imaging might be a clinically relevant possibility.

### References

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