Gradient forces on double-negative particles in optical tweezers using Bessel beams in the ray optics regime

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Abstract: Gradient forces on double negative (DNG) spherical dielectric particles are theoretically evaluated for \( v \)-th Bessel beams supposing geometrical optics approximations based on momentum transfer. For the first time in the literature, comparisons between these forces for double positive (DPS) and DNG particles are reported. We conclude that, contrary to the conventional case of positive refractive index, the gradient forces acting on a DNG particle may not reverse sign when the relative refractive index \( n \) goes from \(|n| > 1\) to \(|n| < 1\), thus revealing new and interesting trapping properties.

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OCIS codes: (080.0080) Geometric Optics; (160.3918) Metamaterials; (350.4855) Optical tweezers or optical manipulation; (350.3618) Left-handed materials.

References and links

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1. Introduction

During the past two decades, optical tweezers have been extensively under investigation and, as far as ordinary particles are concerned — i.e., particles with positive refractive index — its theory is fully established and plenty of applications have been proposed and successfully experimented, especially in biomedical optics, including, for example, biological cells manipulation such as trapping of viruses and bacteria [1], induced cell fusion [2], studies of chromosome movement [3] and cellular microscopy [4].

The principle behind optical trapping with one single beam is usually explained by considering the momentum transfer from the photons to an isotropic, linear and homogeneous spherical particle and, in the ray optics regime, total gradient and scattering forces can be easily calculated by summing up the force contributions of all individual rays impinging the particle [5]. Alternatively, electromagnetic theory can be used as a general rule for calculating the optical forces for any optical regime using the Mie scattering and the beam shape coefficients, the latter being associated with the arbitrary incident beam [6].

Whatever the optical regime considered, if the relative refractive index \(n = n_p/n_m\) (\(n_p\) and \(n_m\) are, respectively, the refractive index of the particle and of the surrounding medium) is positive and higher than one, it is to be expected that the particle be directed towards high intensity regions of the beam, whereas if \(n\) is higher than zero but less than unity, the contrary take place: the particle is directed away from these regions. Finally, if \(n \gg 1\), scattering forces (parallel to the optical axis of the beam) can make optical trapping inefficient or even impossible to be achieved, and other schemes, such as two counter-propagating beams, must be used [7]. Still for this \(n \gg 1\) case, even gradient forces (perpendicular to the optical axis of the beam) can reverse sign, making that previous scheme useless [8].

But what would happen if the particle were of double negative (DNG) nature, i.e, \(n_p\) or, equivalently, \(n\), were negative? In a recent paper [9], gradient forces where calculated for a single incident ray in the ray optics regime, revealing new and interesting properties. It was shown, for example, that they present both repulsive and attractive values, depending on the incidence angle, regardless of any particular fixed \(|n|\), and a repulsive pattern for incident angles above some critical angle (see, e.g., Fig. 2 of ref [9]). From this, first results were obtained for a DNG spherical particle trapped by a focused Gaussian beam, being further extended to other regimes by adopting the so-called generalized Lorenz-Mie theory together with the integral localized approximation [10].

In this paper we calculate, for the first time in the literature, the total gradient forces exerted on a (simple, lossless and non-magnetic) DNG spherical particle by a \(v\)-th order Bessel beam \(J_v(.)\), using geometrical optics. Multi-ringed beams offers several advantages over focused beams, such as the simultaneous trapping of several biological particles of arbitrary shapes and sizes [11,12] including trapping in multiple planes [13]. Predicting how a DNG spherical particle would be trapped by these beams amounts to an increasing knowledge of what could be called “double-negative optical trapping”.

2. Theoretical analysis

Figure 1 shows the coordinate system considered in our simulations. The optical axis of the beam is parallel to \(+z\), meaning that all rays impinging the spherical particle comes from \(-z\). In the ray optics regime, we can assign a specific power \(P\) to each of these rays according to a Bessel beam profile, as follows:
(r, θ, φ) being spherical coordinates relative to the center of the particle, \( k_\rho \) the radial wave-number of the beam and \( \rho_0 \) and \( \phi_0 \) defined according to Fig. 1.

The gradient force exerted on a DNG spherical particle by a single ray is

\[
P(r, \theta, \phi) = J_1 \left( k_\rho \sqrt{r^2 \sin^2 \theta + \rho_0^2 - 2r \rho_0 \sin \theta \cos (\phi - \phi_0)} \right)
\]

(1)

where \( R \) and \( T \) are the Fresnel coefficients of reflection and transmission, respectively, \( \theta_i \) the incident angle of the incident ray and \( \theta_t \) the angle of the transmitted ray [9]. The difference between (2) and the gradient force in the DPS case lies solely on the positive sign in the argument \( 2\theta_i + 2\theta_t \), because all infinite reflected/transmitted rays follow the inverted Snell’s law, due to \( n_p \) being negative [14]. Total gradient forces are found by numerically integrating (2) over the impinged hemisphere of the DNG particle, in a process analogous to that of a focused Gaussian beam [9]. A right-hand circularly polarized beam is assumed.

\[
F_x = \frac{n_m}{c} P(r, \theta, \phi) \left\{ R \sin 2\theta_i - \frac{T^2 \left[ \sin (2\theta_i + 2\theta_t) + R \sin 2\theta_t \right]}{1 + R^2 + 2R \cos 2\theta_t} \right\}
\]

(2)

Suppose a Bessel beam with a wavelength \( \lambda = 1064 \) nm, an axicon angle \( \alpha = 0.0141 \) rad or, equivalently, a spot of \( \Delta \rho = 28.89 \) μm and a radial wavenumber \( k_\rho = 83261.3 \) m\(^{-1}\). A dielectric, homogeneous, isotropic and linear spherical particle with radius \( a = 10\lambda \) is assumed throughout in all simulations. These parameters were chosen in order to ensure the ray optics requirements and that the scalar power formula approximation, given by (1), is completely within its range of validity.

For these parameters, Fig. 2 shows the total gradient force profile expected when the particle is DPS for \( \phi_0 = -\pi (F_x = F_z) \), \( \nu = 0 \), \( n_m = 1.33 \) and several values of \( n = n_p/n_m \). Because of \( \phi_0 \), negative \( F_x \) means an attractive force. The intensity profile of the beam is also plotted as a solid line. Note the common repulsive/attractive pattern according to the intensity of the beam and its reversion when \( n = n_p/n_m \) goes from \( n > 1 \) to \( 0 < n < 1 \). For \( n > 1 \), points of stable equilibrium occurs at \( \rho_0 \approx 0, 46, 84 \) μm and all subsequent high intensity regions of the beam (not seen in the picture), whereas for \( 0 < n < 1 \), those points are \( \rho_0 \approx 30, 67 \) μm and so forth. It is obvious that \( F_x \) would be zero for all \( \rho_0 \) whenever \( n = 1 \) (matched case).

Due to the inversion of Snell’s law, however, total gradient force presents an unusual trapping profile, as can be appreciated in Fig. 3 for the same parameters of Fig. 2. Note that, because \( F_x \neq 0 \) for \( n = -1 \), this curve is also presented for completeness. In Figs. 4 and 5, re-
results for $F_x$ over a DPS and a DNG particle are shown for $n = 3$, respectively. In all cases, positive values of $F_x$ means that this force is repulsive, and vice-versa. To ensure that our theoretical proposal is adequate, all simulations were compared with those obtained by means of the generalized Lorenz-Mie theory (GLMT) with the integral localized approximation [15,16]. Good agreement was achieved.

![Fig. 2. A $n = 0$ Bessel beam (solid line) and $F_x$ for several values of $n (n_0)$. The surrounding media has $n_m = 1.33$. Note that, as expected, in trapping DPS spherical particles, $F_x$ inverts sign when $n$ goes from $n < 1$ to $n > 1.$](image)

To explain these results in view of the geometrical optics, one must look at the graphical representation of (2) for both $|n| < 1$ and $|n| \geq 1$ and compare it with the DPS case, as we have done before for a focused Gaussian beam (Fig. 2 of ref [9]). Depending on the incident angle of the rays that composes the beam, the gradient force exerted on the particle by each ray can be either attractive or repulsive. Consider the optical axis of the Bessel beam in Fig. 1 as before and, furthermore, the DNG particle at an arbitrary $\rho_0 < \Delta \rho$ (although we have assumed $\phi_0 = -\pi$, due to symmetry and the optical regime adopted, the choice of $\phi_0$ is irrelevant). If the overall presence of rays with an attractive nature relative to this axis, i.e., towards the optical...
axis, overcomes the repulsive effect (away from the optical axis), then the DNG particle will be directed towards lower values of $\rho_0$. The same qualitative analysis is valid for $\rho_0 > \Delta \rho$.

Fig. 4. A $v = 3$ Bessel beam (solid line) and $F_x$ for four values of $n$. Again, $n_n = 1.33$. Negative values of $F_x$ means a gradient force pulling the particle towards the optical axis of the beam. For a DPS particle, the points of stable equilibrium depends upon $n$ being higher or less (but higher than zero) than one.

Fig. 5. A $v = 3$ Bessel beam (solid line) and $F_x$ for four values of $n$. Again, $n_n = 1.33$. For this DNG particle, points of stable equilibrium are seen at $\rho_0 \approx 0, 76, 117 \mu m$ regardless of $|n|$ being higher or less than one.

This is exactly what is seen in Fig. 3 for $\rho_0 < \Delta \rho$ and all subsequent regions where the total gradient force is positive: for a zero-order Bessel beam and the parameters chosen, the DNG particle will always be directed towards low intensity regions of the beam (nulls of intensity) and there it will remain trapped, regardless of $|n|$ being higher or lower than one. Analogous considerations can be made for Fig. 5, where $v = 3$. Differently from what was observed for a Gaussian beam [12], when a DNG particle was shifted towards the optical axis, here the total gradient forces now seems to push the DNG particle away from high intensity regions of the beam. This conclusion, however, cannot be extended to any relative refractive index. A full tridimensional plot of $F_x$ as a function of both $\rho_0$ and $n$ for the zero-order Bessel beam used before, as shown in Fig. 6, reveals that it is also possible to have attractive forces towards...
high intensity regions (bright annular disks) of this multi-ringed beam even for DNG particles.

As Snell’s law is inverted, momentum transfer from the rays to a DNG particle leads to gradients forces whose repulsive/attractive profile is not seen for DPS particles. Depending upon the spatial distribution of power, $P(r, \theta, \phi)$ and the radius $a$ of the DNG particle, it can be pushed against or toward high intensity regions of the incident beam, regardless of $|n|$ being higher or less than one.

In this paper we proposed a new application for double negative metamaterials in biomedical optics researches, using Bessel beams of arbitrary order. A full understanding of this new optical force profile can be obtained by extending the results of this work to include other regimes such as the Mie regime, making use of the generalized Lorenz-Mie theory for evaluating and studying the scattered fields. Obviously, scattering optical forces along the optical axis of the beam must also be revised, and other characteristics such as torque could also be investigated. This is currently in progress.

Fig. 6. (a) $F_x$ as a function of the displacement $\rho_0$ and $n$ for a particle of radius $a = 10 \lambda$, where $\lambda = 1064$ nm is the wavelength of our zero-order Bessel beam with an axicon angle $\theta = 0.0141$ rad (spot of $\Delta \rho = 28.89 \mu m$). (b) A contour plot of (a).

Although micro-spheres made of negative refractive index are not physically available yet, their technical feasibility may be just a matter of time. We believe that the manipulation of this kind of metamaterial particles using optical tweezers can be useful in biological applications, including new cancer treatments. Further studies may confirm this possibility.

Acknowledgements

The authors wish to thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) under contracts 2009/54494-9 (L. A. Ambrosio’s post doctorate grant) and 2005/51689-2 (CePOF, Optics and Photonics Research Center), for supporting this work.