Regular Stringy Black Holes?

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We study the first-order α' corrections to the singular 4-dimensional massless stringy black holes studied in the nineties in the context of the Heterotic Superstring. We show that the α' corrections not only induce a non-vanishing mass and give rise to an event horizon, but also eliminate the singularity giving rise to a regular spacetime whose global structure includes further asymptotically flat regions in which the spacetime’s mass is positive or negative. We study the timelike and null geodesics and their effective potential, showing that the spacetime is geodesically complete. We discuss the validity of this solution, arguing that the very interesting and peculiar properties of the solution are associated to the negative energy contributions coming from the terms quadratic in the curvature. As a matter of fact, the 10-dimensional configuration is singular. We extract some general lessons on attempts to eliminate black-hole singularities by introducing terms of higher order in the curvature.

A very well-known class of 4-dimensional extremal stringy black holes is characterized by 4 real functions \(Z_0, Z_+, Z_-, H\) (which are harmonic in 3-dimensional Euclidean space \(\mathbb{E}^3\) at zeroth order in \(\alpha'\)) that occur in the metric and real scalar fields \(\phi, k, \ell\) as follows: [1]

\[
ds^2 = e^{2U} dt^2 - e^{-2U} d\vec{x}^2,
\]

\[
e^{-2U} = \sqrt{Z_0 Z_+ Z_- H}, \quad e^{2\phi} = \frac{Z_0}{Z_-},
\]

\[
\ell = \ell_{\infty} \left( \frac{Z_0 Z_+ Z_-}{H^3} \right)^{1/6}, \quad k = k_{\infty} \left( \frac{Z_+^2}{Z_+ Z_0} \right)^{1/4}.
\]

This 4-dimensional configuration can be obtained from a 10-dimensional (zeroth-order in \(\alpha'\)) solution of the Heterotic Superstring effective action of the form considered in Ref. [2]. They describe fundamental strings (associated to \(Z_+\), solitonic (NS) 5-branes (associated to \(Z_0\)) and waves traveling along the fundamental strings (associated to \(Z_-\)). These configurations are also solutions of the STU model that arises in the compactification on a geometrically complete. We investigate if these corrected and probably exact solutions satisfy some of the properties that are expected to occur in a UV complete theory, and in particular, the resolution of singularities.

As mentioned in Footnote [5], we have already studied how α' corrections can resolve the singular horizon of small black holes (with 2 or 3 charges) as in the classical example of Ref. [7], yielding a smooth horizon with non-vanishing area, though some divergencies persist in the KK scalars. In this paper we are going to study a particularly interesting set of singular solutions that have 4 non-vanishing charges: massless black holes [8, 9] (referred to as massless quadrupoles in Ref. [10]). Research on massless black holes was originally motivated by Strominger’s description of the conifold transition in Ref. [11]. Although his description was based on type II string theory and black holes with Ramond-Ramond charge, the solutions may be related by duality and the metrics are indeed identical.

The massless quadrupoles are a particular case of the solutions Eq. (1). They correspond to the choice

\[
g_0 = g_- = -q = -q_+ = Q \geq 0,
\]

which can be achieved if the string coupling constant \(g_s\) and the radii of the compactification circles at infinity satisfy

\[
g_s = \sqrt{N_{S5}} \frac{R_5}{\ell_s}, \quad \frac{R_5}{\ell_s} = \sqrt{-N_{W}} \frac{R_4}{\ell_s}, \quad \frac{R_4}{\ell_s} = \sqrt{-N_{S5}} \frac{R_{KK}}{N_{KK}}.
\]
Here $N_{SS}$, $N_{F1}$, $N_{W}$ and $N_{KK}$ are integer numbers associated to the stringy objects of the ten-dimensional configuration. The usual requirements $g_s \ll 1$, $R_{4.5} > \ell_s$ are satisfied if these numbers fulfill the hierarchy

$$|N_W| > N_{F1} >> N_{SS} > |N_{KK}|.$$  

(5)

In the absence of $\alpha'$ corrections, the metric of the massless quadruholes reads

$$ds^2 = \left(1 - \frac{Q^2}{r^2}\right)^{-1} dt^2 - \left(1 - \frac{Q^2}{r^2}\right) \left(dr^2 + r^2 d\Omega^2_{(2)}\right).$$  

(6)

This geometry clearly contains a naked singularity at $r = Q$, where the curvature as well as some scalars diverge. It has some interesting properties though, such as the fact that this solution is massless and that the dilaton takes a constant value $e^{2\phi} = e^{2\phi_{\infty}}$. The repulsive behavior noticed in Ref. [9] is characteristic of timelike singularities such as those of the Reissner-Nordstr"om or negative-mass Schwarzschild solutions.

Taking into account the $\alpha'$ corrections given by the general formula Eq. (2), the metric function $e^{-2U}$ reads

$$e^{-2U} = \sqrt{\left(1 - \frac{Q^2}{r^2}\right)^2 + \frac{\alpha'}{2Q} \left(\frac{1}{r} + \frac{Q}{r^2} - \frac{Q^2}{r^2}\right)},$$  

(7)

and many interesting things start happening:

1. First of all, note that now this solution has a mass

$$M = \frac{\alpha'}{8QG_N^{(4)}}.$$  

(8)

2. The geometry [12] is now regular at $r = Q$. Indeed, for $Q^2 > \alpha'/8$, the solution can be extended up to $r = 0$, where a smooth $AdS_2 \times S^2$ near-horizon geometry arises. The area of the horizon is given by the $\alpha'$-independent expression [13]

$$A = 4\pi Q^2.$$  

(9)

This is the standard expression, in terms of the charges, for the entropy of an extremal 4-charge black hole up to $\alpha'$ corrections. However, the relation between the entropy and the mass is very unconventional: $A$ grows with $Q$ while $M$ goes to zero.

3. The most striking property of the metric above is that, if $Q^2 > \frac{\alpha'}{8}$, which corresponds to masses $M < \sqrt{\frac{\alpha'}{\pi}G_N^{(4)}}$, it does not contain any singularity behind the horizon. In order to extend the solution beyond $r = 0$, let us introduce the tortoise coordinate $r_*$ such that $dr_* = e^{-2U} dr$. We define the ingoing Eddington-Finkelstein coordinate

$$v \equiv t + r_*,$$  

(10)

which is constant along ingoing null radial geodesics. In terms of $v$, the metric reads

$$ds^2 = e^{2U} dv^2 - 2dvdr - e^{-2U} r^2 d\Omega^2_{(2)}.$$  

(11)

The metric is clearly regular at $r = 0$, and it can be extended to $r < 0$. A singularity would appear whenever $e^{-2U} = 0$, but looking at (7), we see that this function is strictly positive for all values of $r$ if $Q^2 > \alpha'/8$. Hence, this spacetime contains no singularity and we can extend it up to $r \rightarrow -\infty$, where it describes another asymptotically flat region.

4. Without loss of generality, we can consider the motion of a test particle in the equatorial plane $\theta = \pi/2$. Associated to the Killing vectors $\partial_t$ and $\partial_\varphi$, there are two constants of motion $\epsilon, L$ which are given by

$$\epsilon \equiv e^{2U} \dot{\varphi} - \dot{r},$$  

(12)

$$L \equiv \dot{r}e^{-2U} \dot{\varphi}.$$  

(13)

Then, we can write the mass-shell condition as

$$\dot{r}^2 + V_{\text{eff}}(r) = \epsilon^2.$$  

(14)

where the radial effective potential for massless and massive particles ($\kappa = 0, 1$ resp.) is given by

$$V_{\text{eff}}(r) = e^{2U} \left(\kappa + \frac{\epsilon^2L^2}{r^2}\right).$$  

(15)

The qualitative behavior of the geodesics can be found by studying this effective potential, which we have plotted for several values of $Q$ for timelike and null geodesics in Figure 1.

We see that the effective potential has a smoother form for larger values of $Q$ [14]. In all cases, it presents two hills and a valley separating them which contains the black hole horizon. Let us consider the geodesic of a massive particle. Coming from large positive values of $r$, the first hill represents a repulsive behavior which can be overcome if the particle has enough energy. If
FIG. 1. Effective potential for different types of geodesics and for several values of the charge $Q$. Top: Massive particle moving along radial geodesics ($L = 0$). Middle: Massive particle in a non-radial geodesic with $L = \alpha'$. Bottom: Massless particle in a general geodesic (for $L = 0$, $V_{\text{eff}} = 0$).

If the energy of the particle is higher than the summit

of the second, leftmost, hill of the effective potential, it will cross over it towards the $r \to -\infty$ of the type II region in the Penrose diagram, pushed by a repulsive force. This repulsive force is, now, associated to the negativity of the mass of the central object as seen from the type II regions. In some sense, it can be said that the mass of the $\alpha'$ corrected solutioin remains zero because it has opposite values in contiguous type I and type II regions of the Penrose diagram.

5. The effective potential for null geodesics, plotted in Figure 1, has two hills of the same height: if a light ray has small enough impact parameter $L/\epsilon$, then it always goes from type I to type II regions in the Penrose diagram. The maximums of the potential are exactly

$$V_{\text{eff}}^{\text{max}} = \frac{16L^2Q^2}{\alpha'(8Q^2 - \alpha')}.$$ 

6. In the central valleys of the effective potentials it is possible to have geodesics that never reach infinity and are confined between the hills. The particles cross the
horizons (future and past) of different regions an infinite number of times and for an infinite number of regions as shown in the Penrose diagram.

7. Finally, observe that, in the timelike case the effective potential has another minimum in the right-hand side of the diagram (type I region) that becomes shallower the larger $Q$ is. This happens even for $L = 0$ (radial motion), which means that there can be massive particles with purely radial motion confined between two radii.

Summarizing, we have found that $\alpha'$ corrections transform the singular massless black holes (6) into a geodesically complete spacetime which represents a regular black hole with no singularity. We focused on the particular example of the massless black holes for convenience, but the same result is found for more general values of the charges, provided that their signs are chosen as in (3).

Being extremal and supersymmetric, these solutions might evade some of the stability issues related to regular black holes, as the ones reported in [15]. In particular, we note that our black holes have a different structure as compared to the models analyzed there: they do not contain a de Sitter core, and the instability associated to it does not directly apply.

Although we have argued that the $\alpha'$-corrected solutions may receive no further corrections, it is not clear to us how seriously they should be taken from the string theory point of view, [16] because they are singular in $d = 10$ dimensions. The singularity is to be expected since some compactification circles diverge (shrink to zero radii in the dual theory) at given values of $r$. This pathology, on the other hand, may be interpreted as a sign of the relation between these solutions and the massless black holes of Ref. [11].

A feature of these solutions that may also be considered as another sign of this relation is the two-sided structure of the solution, which exhibits masses of opposite signs in contiguous type I and type II regions of the Penrose diagram. Some of the states that become massless in the conifold transition could be 2-particle states and, therefore, they should have opposite masses.

Despite the pathological character of the ten-dimensional solution, it should be noted that the cancellation of the black hole singularity in $d = 4$ is a highly non-trivial effect related to the precise form of the corrections in $Z_+\alpha$ given in (2). This function diverges with the right degree precisely at the points where the functions $\mathcal{H}$, $Z_0$ and $Z_-$ vanish, and this is the only way in which the singularity could be removed. Even though the compactification is singular, everything conspires to produce a regular four-dimensional geometry.

Finally, since these solutions are also solutions of General Relativity with complicated couplings to matter, an explanation for their completely out of the ordinary features must be proposed. As we mentioned, the mass of the solution has opposite sign in type I and type II regions. The presence of negative masses is usually associated to that of naked singularities and the absence of the latter can only be attributed to the lack of positivity of the energy in the theory that we are considering. The terms of higher order in the curvature associated to the $\alpha'$ corrections typically have the wrong sign compared with terms quadratic in Yang-Mills curvatures [17]. Repulsive gravitational behavior associated to these corrections associated to these terms has been observed, for instance, in Ref. [18].

There have been many attempts in the literature to get rid of the singularities at the core of black holes, though the analysis is usually restricted to finding appropriate regular black hole models [19, 20]. The theories that achieve that goal usually introduce higher-derivative terms in the curvature [21–24] or in the matter fields [25, 26] which may (or may not) be associated to quantum corrections of a theory of quantum gravity such as string theory. Effectively, many of those terms may introduce negative energy in the theory in a more or less consistent way (nobody really knows) that it is ultimately responsible for the removal or softening of the singularities. We believe that this aspect of the higher-derivative terms deserves to be understood in depth if these theories are to be considered internally consistent.

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[1] For the sake of simplicity, we omit the 4 non-vanishing vector fields associated to the 4 harmonic functions. They can be derived from the general 10-dimensional solution given in Ref. [2].

[2] S. Chimento, P. Meessen, T. Ortin, P. F. Ramírez, and A. Ruiperez, (2018), arXiv:1803.04463 [hep-th].
The scalars $B_{451}$ and $B_{372}$ are non-vanishing even for trivial $Z_8$ and $H$, in which case the solutions would be special cases of the chiral null model discussed in Refs. [8, 27]. In those references it was argued that those solutions describing fundamental strings and wave traveling along them receive no $\alpha'$ corrections and are exact to all orders in $\alpha'$ in some renormalization scheme which is natural for this model. In the context of the Heterotic Superstring and in the scheme associated to quartic action given in Ref. [28] that we are using, though, these corrections are expected on physical grounds [29]. Furthermore, they are expected to play an important role: they can resolve the singular horizon of small black holes with 2 charges in $d = 5$ dimensions [6] and with 2 or 3 in $d = 4$, as we will show in a forthcoming paper in which we will study the $\alpha'$ corrections of general 4-dimensional black holes [30]. It should also be mentioned that, being associated to the $uu$ component of the Einstein equation ($\alpha$ being a null coordinate), these corrections cannot be seen by merely observing the curvature invariants.

The scalars $\ell$ and $k$ diverge there. This is a common feature of small black hole solutions and is associated to a problematic (singular) compactification from 10 (actually, from 6) to 4 dimensions. In the original picture of Ref. [11], there is a cycle around which a D-brane is wrapped whose volume shrinks to zero. The regularity of the geometry at that point in spite of the singularity of the scalars suggests that something strange is happening, as we will discuss later.

A calculation of the $\alpha'$ corrections to the area that give the entropy using Wald’s formula [31, 32] will be given in Ref. [30]. A naked singularity arises for $Q^2 = \alpha'/8$. In the timelike case, the height of the peaks grows as $Q/\sqrt{\alpha'/2}$ when $Q$ is large, so the potential is actually smoother for intermediate values of $Q$.

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