Variability of Magnetically Dominated Jets from Accreting Black Holes

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Abstract

Structured jets have recently been invited to explain the complex emission of gamma-ray bursts (GRBs), such as GW170817. Based on accretion simulations, the jets are expected to have a structure that is more complex than a simple top-hat structure. Also, the structure of the launch regions of blazar jets should influence their large-scale evolution. This was recently revealed by the interactions of jet components in TXS 0506+056, where the jet was observed at a viewing angle close to zero. Observational studies have also shown an anticorrelation between the jet variability, measured, e.g., by its minimum variability timescale, and the Lorentz factor, which spans several orders of magnitude and covers both blazars and GRBs samples. Motivated by those observational properties of black hole sources, we investigate the accretion inflow and outflow properties by means of numerical gamma-ray MHD simulations. We perform axisymmetric calculations of the structure and evolution of a central engine, composed of a magnetized torus around a Kerr black hole that is launching a nonuniform jet. We probe the jet energetics at different points along the line of sight, and we measure the jet-time variability as localized in these specific regions. We quantify our results by computing the minimum variability timescales and power density spectra. We reproduce the MTS–L correlation and we attribute it to the black hole’s spin as the main driving parameter of the engine. We also find that the power density spectral slope is not strongly affected by the black hole’s spin, while it differs for various viewing angles.

1. Introduction

Highly variable accretion flows are found in a number of different types of astrophysical black hole sources. At the largest scales, they are present in the cores of active galaxies. In radio-loud objects, such as blazars, the variability of the inflow can be transmitted to the outflow properties. In these sources, the relativistic jets point to our line of sight. In addition, many similarities are found between the jet physics in blazars and in gamma-ray bursts (GRBs). The latter are observed from extragalactic distances, but operate at smaller scales, within stellar-mass accreting black holes and in a collapsing star’s environment. Blazars and GRBs share the properties of their jets, despite different Lorentz factors and accreting black hole masses (Wu et al. 2016). Launching and collimation mechanisms are common: thick disk or corona, pressure gradient in a surrounding wall, external (matter-dominated) jet, or toroidal magnetic field. Acceleration of jets occurs due to both a magnetic field action field and accretion-disk rotation (see Fragile 2008 for a review). The blazar jets are Poynting-dominated, and powered by the Blandford–Znajek mechanism which can extract energy from a rotating black hole. This mechanism is now well known and tested for the purpose of a jet launch, but observations show variability in the jet emission. Multiple shocks that collide in the jet can lead to multiple emission episodes and can account for the fluctuating light curves of GRBs (Kobayashi et al. 1997). A reasonable interpretation of this effect is that the variability observed in the jets can directly reflect the central engine variability. The latter is tightly related to the action of magnetic fields in the center of the galaxy, or in the GRB central engine. Furthermore, the structure of a jet at its base is possibly much more complex than a simple top-hat structure and can be revealed by the afterglow observations and interactions of the large-scale jet with the surrounding medium, e.g., with the postmerger wind in GW170817 (Urrutia et al. 2021). Even though the observed light curves and spectra are primarily the result of the jet’s interaction with the circumburst medium, the initial structure of the jet at its base also affects the final emission. Also, interactions are possible between the different components of precessing blazar jets, such as in TXS 0506+056 (Britzen et al. 2019).

The variable energy output from the central engine implies the varying jet Lorentz factor, as shown, e.g., by Sapountzis & Janiuk (2019). This may lead to occurrence of internal shocks, and affect the observed variability of both GRBs and blazars (Begelman et al. 2008; Bromberg & Tchekhovskoy 2016). Unification of the models across the black hole mass scale, from GRBs to blazars, is not straightforward though. The most uncertain aspect is whether the magnetically arrested disk (MAD) state drives the jets in both types of source, or rather halts the GRB emission, as studied by Lloyd-Ronning et al. (2018). In the MAD mode, the flux accumulated at the black hole horizon, and the interchange instability rather than magnetorotational instability (MRI) governs the minimum timescale of variability (Tchekhovskoy et al. 2011, 2014). In contrast, in the standard and normal accretion evolution (SANE) mode, the MRI divies variability of the jets, as directly related to the accretion variability timescales (Penna et al. 2013; Porth et al. 2019). Finally, the blazar disks are subject to different physical conditions than the GRB disks, and in the latter, thermal instabilities of the neutrino-dominated accretion flows may play a role, also triggering the episodic jet ejections (Janiuk & Yuan 2010; Cao et al. 2014).

Here we explore the scenario of magnetically driven accretion and jet variability related to the MRI timescale. We confirm the existence of the correlations between the inferred minimum variability timescale and magnetic field strength as...
well as the black hole spin, as expected for the Blandford–Znajek-driven jets. We compare the resulting timescales with observed ones, taken from the sample by Wu et al. (2016), and conclude that they represent both classes of sources, namely blazars and GRBs, well.

2. Model

We present here the two-dimensional magnetohydrodynamical models computed in full general relativity (general relativistic magnetohydrodynamics, GR MHD). The numerical scheme is our implementation of the code HARM (Gammie et al. 2003; Janiuk et al. 2013; Sapountzis & Janiuk 2019). Our initial condition assumes the existence of a pressure equilibrium torus, embedded in the poloidal magnetic field (Figure 2, top). Such 2D studies of a compact magnetized tori around black holes have already been performed by different groups (McKinney et al. 2012; Fernández & Metzger 2013; Sadowski et al. 2015; Qian et al. 2017). Our approach is based on similar methodology, while the focus of the present study is given to measuring the variability of the jet. The novel aspect of our analysis is that we consider a structured jet morphology and we attempt to compare our results with some observables.

The jet launched from the central engine is powered by a rotating black hole and mediated by magnetic fields. The Kerr black hole accretes matter from the torus, and its rotation affects the magnetic field evolution. The models are parameterized with the black hole’s spin and the initial magnetization of the matter. Code works in a GR framework, so dimensionless units are adopted, with \( G = c = M = 1 \). Hence, geometrical time is given as \( t = GM/c^3 \), where \( M \) is the black hole’s mass. In this way, we are able to model the launch and variability of jets in both a supermassive black hole environment and in GRBs.

The chosen configuration of the torus structure is that of Chakrabarti (1985). Here, the angular momentum distribution has a power-law (PL) relation with the von Zeipel parameter \( \lambda = (l/\Omega)^{1/2} \), where \( l \) denotes the specific angular momentum and \( \Omega \) denotes the angular velocity. The size of the torus is fixed in geometrical units, and its inner radius is located at \( r_{\text{in}} = 6\,r_g \), its density maximum at \( r_{\text{max}} = 16.5\,r_g \), and the outer edge at about \( r_{\text{out}} = 40 \).

We embed the initial torus in a poloidal magnetic field that was proven to drive the bipolar jets after the initial configuration has been relaxed (see, e.g., Liska et al. 2020, however, for recent results with toroidal field initial configurations). We chose the magnetic field configuration produced by a circular current, the same as in Sapountzis & Janiuk (2019). The only nonvanishing component of the vector potential is given by:

\[
A_\phi(r, \theta) = A_0 \frac{(2 - k^2)K(k^2) - 2E(k^2)}{k\sqrt{4Rr\sin \theta}}
\]

where \( E, K \) are the complete elliptic functions and \( A_0 \) is used to scale the magnetic field and the initial gas to the magnetic-pressure ratio, \( \beta = p_{\text{gas}}/p_{\text{mag}} \), across the torus.

We define the family of models with varying magnitudes of \( \beta \), normalizing them to the maximum value within the torus (the point where \( \beta \) reaches its maximum also depends on the black hole spin parameter, \( a \), because of the properties of torus models is depicted in Figure 1. In all our simulations we used the resolution of \( 768 \times 512 \) grid points in \((r, \theta)\) directions. This allowed us to keep a proper MRI resolution, defined as the minimum number of cells per MRI wavelength (Siegel & Metzger 2018).

3. Results

The initial configuration of an equilibrium torus as given by the solution of Chakrabarti (1985) is depicted in Figure 2. The solution for the gas-pressure distribution). Our family of models is depicted in Figure 1.
the model with the parameter represents the true inner edge of the torus and is kept within the region of about 40$r_g$. The geometrical thickness of the structure is less than $H/r = 0.2$. The solution shown in the plot is parameterized by the black hole spin $a = 0.99.$

The inner edge of the torus is $r_{in} = 6r_g$ and the position of maximum pressure in the torus is $r_{max} = 16.5r_g$. All the simulations start from an initial configuration and evolve afterwards. The value of $\lambda$ ranges between 0.04–10.21 $M$ within the torus. The outer radius of the torus is affected by change in Kerr parameter $a$ and $r_{in}^{(von Zeipel)}$. The surfaces of constant specific angular momentum $l$ and angular velocity $\Omega$ are called the von Zeipel cylinders. With the choice of Chakrabarti’s solution for the torus structure, $l = constant$ surfaces become the von Zeipel cylinders. The $r_{in}^{(von Zeipel)}$ parameter represents the true inner edge of the torus and is kept at $8r_g$ for all the models in the simulation to constrain the outer radius of the torus near to $40r_g$. We note that using bigger values for $r_{in}^{(von Zeipel)}$ would reduce the size of the torus. By changing the Kerr parameter from $a = 0.60$ to 0.99, we can change the outer radius of torus from 42.5 to 37$r_g$. The average value of $\beta$ within the torus is calculated from the inner point in the torus where the value rises above $10^{-3}$ to the outer point in the torus where the value falls below $10^{-3}$.

The magnetic field imposed on top of the stationary torus configuration has an electric wire shape, with circular loops concentrated on the pressure maximum radius. This radius was chosen as $16.5r_g$ for all models. The magnetic field allows material to start accrete onto the black hole. The evolved structure of the flow, which has already relaxed from its initial configuration, is depicted in the bottom panel of Figure 2.

In our simulation the accretion flow has an axisymmetric configuration. To assess the MADness of the dynamical solution, we would need to cover the nonaxisymmetric modes. However, even in our setup we can evaluate the ratio between the magnetic flux and mass-accretion rate on the black hole’s horizon. In fact, the magnetically arrested state appears in the most-magnetized models, i.e., those with $\beta_{max} = 60$. At the beginning of the simulation, before time $t = 2000 \, M$, the parameter $\Phi_{BH} = \frac{1}{2M} \int |B|^2 (r_H) \, dA_{H0}$ is greater than 10 (note that in our code we use the Gaussian units, so the factor $4\pi$ is not incorporated in the magnetic flux).

The jet energetics determine the Lorentz factor at infinity, and as was shown by Vlahakis & Königl (2003) and Sapountzis & Janiuk (2019) it is given by the $\mu$ parameter. It is defined as

$$\mu = -\frac{\varepsilon}{\rho u'}$$  \hspace{1cm} (2)

where $\varepsilon$ is the energy component of the energy-momentum tensor, which consists of gas and magnetic parts, $\rho$ is the gas density, and $u'$ is the radial velocity, i.e., the total plasma energy flux normalized to the mass flux. It is therefore given by the sum of the inertial–thermal energy of the plasma and its Poynting flux, which can be transferred to the bulk kinetic energy of the jets at large distances.

The distribution of the jet-energetics parameter in an evolved state of the simulation is shown in Figure 3. The snapshots compare two values of magnetic field normalizations, $\beta = 600$.
and $\beta = 60$, in the top and bottom rows, respectively. We show three different values of black hole spin, $a = 0.6, 0.8,$ and $0.95$. We note highly inhomogeneous outflows, where larger values of $\mu$ are reached at the edges of the jets rather than at the $z$ polar axis. From the color scales of the distribution it can already be seen that more energetic jets are produced from rapidly spinning black holes, which confirms our intuitions. The relation with magnetization $\beta$ is not that clear though, and it seems to be affected by the black hole’s spin value. The details of the simulation results are therefore summarized quantitatively in Table 1.

The table shows the minimum variability timescale and Lorentz factor values with the changing black hole spin value. Three models with different magnetic field normalizations are shown here. For our calculations the Lorentz factor is taken as the average of $\mu$ in time. The averages were calculated from $t = 600$ to $t = 3100r_g$. The minimum variability timescale is calculated as the average of peak widths at their half maximum on the $\mu$ variability plot.

In Figure 4 we show the time variability of jet energetics (i.e., the $\mu$ parameter) for the models with a magnetic field normalization $\beta_{\text{max}} = 300$ and different values of black hole spin. The variability is measured here at a chosen specific point, $r = 150r_g$ and $\theta = 5^\circ$. Here we show the values of $\mu$, from $t = 1000r_g$ to $t = 3000r_g$. For each simulation, the parameter $\mu$ is computed at two different points located at $r = 150r_g$, $\theta = 5^\circ$ and $\theta = 10^\circ$.

**4. Jet Properties and Central Engine**

Here, we investigate the influence of the central engine properties, as scaled by its magnetization, and the Kerr parameter $a$ of the black hole, on the variability and energetics of the jet. The total energetics are described by the parameter $\mu$, which represents the total, thermal, and Poynting energy in the jet. Note that this parameter is dimensionless, as it is given by the ratio of the $r$ component of the linear momentum, to the mass flux across the radial surface (see Equation (2)). Therefore, it can be related to the maximum achievable Lorentz factor, reached at “infinity”, and available under the “infinite” efficiency of conversion to the bulk kinetic energy of particles injected to the jet. We identify therefore the time-averaged value of $\mu$ as the proxy of the jet Lorentz factor, $\Gamma$. The variability is also measured by $\mu$ changes with time, at a given point. We propose that the frequency of these changes, measured in the base of the jet, is related to the frequency of collisions between the shells transported downstream by the jet and is the source of observable gamma-ray pulses, produced in the internal-shock scenario (Kobayashi et al. 1997).

The jet structure is clearly nonuniform, and more energetic blobs are always located in the outer regions, while less energetic ones travel close to the axis. This is revealed by the systematic differences between $\Gamma$ measured at point $p_1$, which ranges between $\sim 200$ and $350$, and those measured at point $p_2$, which is ranges from $\sim 300$ up to $1000$ (see Table 1 and the bottom panel in Figure 6). The jet bulk velocity, and hence its power, increases with the black hole’s spin, and reaches remarkably average values if the black hole rotates close to the Kerr limit. This is expected to be a result of Blandford–Znajek-driven process. The dependence on the magnetic pressure in the disk, and the $\beta$ parameter, is not linear, however. Only in the case of the most spinning black hole, $a = 0.99$, and the most-magnetized disk, with average $\beta$ in the torus on the order of $15$ ($\beta_{\text{max}} = 10^{-3}$ and $\beta_{\text{max}} = 60$), is the jet power the largest, compared to more thermally dominated tori. If the black hole does not rotate at close to the maximum Kerr limit, then the more thermally dominated tori, with an average $\beta_{\text{max}}$ of 300, or

| $\beta_{\text{max}}$ in Torus | Average $\beta$ | Spin ($a$) | MTS | Lorentz Factor $\Gamma$ |
|-----------------------------|---------------|-----------|-----|------------------------|
|                             | Point 1       | Point 2   | Average |
| $\beta_{\text{max}} = 600$ | 145.54        | 0.99      | 19.73 | 347.26 602.29 474.77   |
| 145.86                      | 26.40         | 280.66    | 594.80 | 437.73   |
| 146.87                      | 25.34         | 265.89    | 505.47 | 385.68   |
| 148.43                      | 36.88         | 268.89    | 435.86 | 352.38   |
| 149.70                      | 35.56         | 256.27    | 404.21 | 330.12   |
| 151.01                      | 39.66         | 223.22    | 343.72 | 283.47   |
| $\beta_{\text{max}} = 300$ | 73.35         | 0.99      | 24.20 | 337.80 651.93 494.87   |
| 74.07                       | 26.55         | 301.88    | 584.01 | 442.94   |
| 73.99                       | 28.31         | 281.92    | 476.51 | 379.21   |
| 74.76                       | 31.98         | 257.70    | 491.84 | 374.77   |
| 74.85                       | 31.61         | 248.67    | 428.22 | 338.44   |
| 75.50                       | 45.65         | 228.90    | 332.53 | 280.71   |
| $\beta_{\text{max}} = 60$  | 14.91         | 0.99      | 22.91 | 359.43 1000.72 680.08  |
| 14.93                       | 23.15         | 234.38    | 607.69 | 421.03   |
| 14.91                       | 26.73         | 230.75    | 479.02 | 354.89   |
| 15.06                       | 34.75         | 225.11    | 377.68 | 301.40   |
| 15.41                       | 31.15         | 204.59    | 314.61 | 259.60   |
| 15.42                       | 43.14         | 192.39    | 309.97 | 251.18   |

Note. Three families of models differ with respect to the magnetic field normalization, which is scaled with the maximum value of the gas to magnetic-pressure ratio within the torus (note that $\beta(\text{max})$, the value at the radius of pressure maximum, can be as small as $10^{-3}$–$10^{-2}$, see Figure 1, so that all our models are essentially representing strongly magnetized tori). We also give the value of the average average of $\beta$ in the second column. The third column gives the value of the black hole Kerr parameter, $a$, for each model. The resulting variability timescale and Lorentz factor measured as the averaged energetics parameter at two chosen points in the jet are given in the last two columns.
600 (average \( \beta \)) in the torus is 75 or 150, respectively, give more power to the jet. This result can be understood in the frame of magnetically driven transfer of accretion-disk energy to the jet when there is less Poynting flux available in the funnel for less spinning black holes, and while the thermal energy can still be transported through the horizon with enough efficiency. Notably, there is almost no difference between the jet power and angle-averaged Lorentz factors, in the \( \beta_{\text{max}} = 300 \) and 600 cases. For spins 0.7 \( \leq a \leq 0.9 \) the point \( p_2 \) meets more energetic blobs for smaller \( \beta \), while blobs at point \( p_1 \) are found to be more energetic for larger \( \beta \).

The variability of the jet, studied in terms of the duration of the pulses, is driven by the MRI in the disk. Here, however, some numerical constraints of our simulation, namely spatial resolution and the axisymmetric setup of the models, may also be of some importance. The MRI is resolved in terms of the minimum number of cells per wavelength, as shown in Figure 5. Nevertheless, the duration of the pulses duration only roughly correlates with \( t_{\text{MRI}} \) (Figure 4), and only the widest pulses clearly show this effect. The narrower pulses, which also contribute to our minimum variability timescale (MTS) and MTS estimate, behave more erratically. Therefore, as displayed in Figure 6 (upper panel), the MTS has a general trend of decreasing with the black hole’s spin, but it can either decrease or increase with \( \beta \), depending on the \( a \) value. In particular, we can note that the thermal pulses are shortest for \( a = 0.95 \) while they are longest for \( a = 0.8 \).

We note that important information about the jet engine and jet collimation comes from the angular jet structure. Our jet is not uniform and has a distribution of energy content that is both time and angle dependent. Here, we probed how the jet distributes its power and we plot \( \Gamma \) as function of polar angle. We calculated the time-averaged jet profile at a radius of 2000 \( r_g \), so at a large distance from the black hole. It is presented in Figure 7. The profile shows that most energetic part of the jet is located inside a narrow region at \( \theta < 15^\circ \) which is qualitatively very similar to the profiles found in recent 3D black hole jet studies (Kathirgamaraju et al. 2019, see also Nathanail et al. 2021). Compared with those results, our jets are accelerated to a larger \( \Gamma \), for the same black hole spin. This is due to our magnetization profile and initial different \( \beta \) distribution in the torus, which in those works has been adopted as uniform and larger on average (Fernández et al. 2019).

Finally, we verified whether our results depended on the adopted value of the density floor, i.e., the numerical floor in our simulation. The minimum density in our runs is forced to not drop below \( \rho_{\text{min}} = 10^{-7} \). As shown already in Sapountzis & Janiuk (2019), the time-averaged value of the energetics \( \mu \) parameter converged for various adopted density floors (see...
Similarly, even though we are using different initial conditions and magnetization in the current models, the density floor value does not significantly affect the time-averaged results, provided it is sufficiently low. We show our testing results in Table 2, where we compare the time-averaged Lorentz factors at two distinct points in the jet. The testing model was used with parameters $a = 0.9$ and $b_{\text{max}} = 60$. We also checked that the variability MTS, calculated at different locations in the jet, depended somewhat on the density floor value, however the results do not follow any specific trend. In general, MTS values at point p2 are always smaller than at point p1, and their ratio is about $2/3$ (with an exception of the floor $1 \times 10^{-9}$, where the ratio is almost $1/2$).

In order to better understand the jet variability in our models, and also to be able to compare it to observed light curves originating from gamma-ray emission at large distances, we performed a time-series analysis of our modeled sequences.

### 4.1. Time-series Analysis

We consider the time series of the $\mu$ parameter (defined in Equation (2)) in order to carry out a Fourier and power density spectral (PDS) analysis of it. We further impose logarithmic binning to this time series and we plot the averaged values over the bins. Figure 8 shows our simulated data, in a logarithmic scale, corresponding to the model with $\beta_{\text{max}} = 60$ and spin $0.9$.

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Figure 5. Color contour map of $Q_{\text{MRI}}$, defined as the number of grid cells per MRI wavelength, shown in the logarithmic scale at time $t = 0$ and at time $t = 2000 M$. The plots are made for the model with $\beta_{\text{max}} = 300$ and a Kerr parameter $a = 0.9$.

Figure 6. Correlations between (a) the Kerr parameter $a$ and MTS and (b) the Kerr parameter $a$ and Lorentz factor. The upper panel shows results for three families of models, differing with average (and maximum) $\beta$ parameter: $\beta_{\text{max}} = 60$ (green), 300 (orange) and 600 (blue). The MTS is computed as the average duration of the pulses in the $\mu$ time series. The values given are averages from the two points p1 and p2. The bottom panel shows results for the $\Gamma$ factor, defined as the average energetic parameter $\mu$, measured from time 600 until 3000 $\tau_g$. Long-dashed lines represent measurements at point p1 in the jet, short-dashed lines are for point p2, and solid lines are the average between two points.

Figure 7. The time-averaged jet Lorentz factor as measured at a distance of 2000 $r_g$ in the function of polar angle $\theta$. The plot shows the results for four chosen models, differing with the maximum $\beta$ parameter: $\beta_{\text{max}} = 60$ (red and green), 300 (orange), and 600 (blue). The black hole spin was either $a = 0.8$ or $a = 0.9$, as marked in the plot.
\( a = 0.99 \). The jet variability is extracted at an inclination angle of \( \theta = 5^\circ \). The plot shows binned data with a PL fitting. The error bars can be seen to be very large at low frequencies in the middle panel of Figure 8 as the data are largely spread around the mean value, but gradually they decrease, giving perfectly binned data. We further fit the binned data with a PL function of \( \gamma(x) = Ax^\alpha \). The bottom panel of Figure 8 shows the residuals in the frequency function and it can be seen that the fitting is better in the low-frequency range. Out of our 18 models, we choose the PDS plot for this model (Figure 8) because its chi-square value is the lowest among all other models (reduced \( \chi^2 = 14.21 \)). We note that in fact the PL model might not be the best way to reproduce the jet variability in this model. On the other hand, there are no significant peaks at specific frequencies, which would be found in the PDS analysis. Also, our aim is to show the general trends and correlations between the central engine properties and variability probes in the modeled jets. Therefore, we limit our study below to the relations between black hole spin and PDS slope.

### 4.2. Relation between the Central Engine and Jet Variability

In Figure 9 we show the relation between the slope of the PL function and the black hole spin in the jet engine. The time variability of jet energetics (the \( \mu \) parameter) is measured at two different inclination angles, \( \theta = 5^\circ \) and \( \theta = 10^\circ \). We show three families of models here, with different magnetization of the torus. The values of the PL slope fitted for all models with different spins are listed in Table 3.

It can be inferred from our analysis that model with the lowest torus magnetization, i.e \( \beta_{\text{max}} = 600 \), has the steepest PDS, and the highest slope of the PL function is found at spin \( a = 0.7 \). This is measured at both inclinations chosen for observing the jet variability. The model with the higher magnetization, \( \beta_{\text{max}} = 60 \), is found to have steeper PDS slopes at a higher spin, \( a = 0.95 \), when measured at an inclination of \( \theta = 5^\circ \). As the inclination increases to \( \theta = 10^\circ \), the PDS is steeper at spin \( a = 0.7 \). For an intermediate model with

#### Table 2

| Density Floor | Time-averaged \( \Gamma \) |
|---------------|---------------------------|
| \( 1 \times 10^{-17} \) | 258.28, 469.20 |
| \( 1 \times 10^{-15} \) | 267.20, 577.17 |
| \( 1 \times 10^{-12} \) | 250.00, 421.46 |
| \( 1 \times 10^{-9} \) | 237.84, 472.90 |
| \( 1 \times 10^{-7} \) (original simulation) | 230.75, 479.02 |
| \( 1 \times 10^{-5} \) | 113.15, 239.35 |

Figure 8. The top panel shows the simulated time series along with the binned data and the fitted PL. The middle panel shows the error bar in the binned data and the bottom panel is the residual plot for the binned data and fitted PL. The plots in this figure correspond to the model with \( \beta_{\text{max}} = 60 \) and spin \( a = 0.99 \) at an inclination of \( \theta = 5^\circ \).

Figure 9. Relation between the black hole spin and the PL slope of the fitted PDS for all models, \( \beta_{\text{max}} = 600 \) (blue lines), \( \beta_{\text{max}} = 300 \) (red lines), and \( \beta_{\text{max}} = 60 \) (green lines). The first panel is chosen at an inclination of \( 5^\circ \) and the second one is at \( 10^\circ \).
the detected gamma-ray function is again steeper. In some parts of the jet, while the slope of its variability PL differs.

The only outlier from this trend is the jet is very strong, the Lorentz factor even reaches a magnetized torus around the fastest spinning black hole. Here with an increasing jet speed. The only outlier from this trend is 

\[ \alpha = 0.99 \pm 0.471 \] 

\[ \alpha = 0.95 \pm 0.476 \] 

\[ \alpha = 0.90 \pm 0.478 \] 

\[ \alpha = 0.80 \pm 0.472 \] 

\[ \alpha = 0.70 \pm 0.475 \] 

\[ \alpha = 0.60 \pm 0.470 \] 

Furthermore, several correlations between the variability properties and GRB energetics have been detected. The peak energy is anticorrelated with the PDS index (Dichiara et al. 2016). The general idea behind correlations of this kind invokes the jet Lorentz factor, \( \Gamma \), being the main driver responsible for relations between both peak energy and luminosity, and GRB duration and its luminosity (Dainotti & Del Vecchio 2017). The duration of the burst, \( T_{\text{M}} \), was also found to be related to the MTS. In the sample of long- and short-duration GRBs detected by Fermi, the statistical significance for the bimodal distribution of the events is higher when the MTS is taken into account (Tarnopolski 2015).

In our simulations, the variability in the jet is related to the action of the central engine and the timescale of the MRI. It can be seen that the duration of the pulses in the jet, which reveals the size and speed of blobs containing high thermal and Poynting energy, corresponds to the timescale of the fastest-growing mode of MRI (see Figure 4). Furthermore, we use the average duration of the pulses, measured at their half width, as

\[ \beta_{\text{max}} = 300, \text{ the PDS is always steeper at lower spins, } a = 0.7 \text{ and } a = 0.6, \text{ for inclinations of } \theta = 5^\circ \text{ and } \theta = 10^\circ, \text{ respectively.} \]

We also investigated how the slopes of the PDS behave for different Lorentz factors of the jets in different models (see Figure 10). For particular magnetization values of \( \beta_{\text{max}} \), we do not see any particular pattern. The PDS are steep, with slopes (\( \alpha \geq 0.55 \) for the model with lower magnetization, \( \beta_{\text{max}} = 600 \), only (see Tables 1 and 3). Other models have quite flat PDS spectra, with slopes (\( \alpha \leq 0.55 \)). The model with \( \beta_{\text{max}} = 300 \) presents the most varying relation between the slope versus Lorentz factor, as measured at inclination \( \theta = 10^\circ \).

On the other hand, a general anticorrelation between the jet Lorentz factor and PDS slope of the PL fit is seen when we abandon the dependence on the central engine magnetization. In other words, if the particular GRBs are treated individually, then most of them follow the trend of a decreasing PL slope with an increasing jet speed. The only outlier from this trend is the GRB which represents the model of most highly magnetized torus around the fastest spinning black hole. Here the jet is very strong, the Lorentz factor even reaches \( \Gamma = 1000 \) in some parts of the jet, while the slope of its variability PL function is again steeper.

### 5. Discussion and Conclusions

The variability of emission observed in the GRBs is a complex phenomenon. From the observational point of view, the detected gamma-ray flux exhibits a large variety of patterns that reflect complicated processes governing the high-energy radiation (Fishman et al. 1994). The flux varies on multiple timescales, and power spectral density of the light curves is frequently fitted with the PL function (\( P(f) \sim f^{-\alpha} \)). The values of the slope fitted to individual PDS spectra have a wide range. For the stochastic process driven by internal turbulence in the jet interior, a slope of \( \alpha = 5/3 \) is theoretically expected within the internal-shock scenario (Beloborodov et al. 2000). Also, \n
\[ \beta_{\text{max}} = 300 \]

| \( \beta_{\text{max}} \) in Torus | Spin \( a \) | Slope | Point 1, \( \theta = 5^\circ \) | Point 2, \( \theta = 10^\circ \) |
|-------------------------------|-------------|------|-----------------|-----------------|
| \( \beta_{\text{max}} = 600 \) | 0.99        | 0.321 ± 0.053 | 0.298 ± 0.052 |
|                               | 0.95        | 0.384 ± 0.068 | 0.347 ± 0.056 |
|                               | 0.90        | 0.359 ± 0.081 | 0.328 ± 0.053 |
|                               | 0.80        | 0.278 ± 0.066 | 0.435 ± 0.091 |
|                               | 0.70        | 0.308 ± 0.063 | 0.495 ± 0.051 |
|                               | 0.60        | 0.327 ± 0.078 | 0.504 ± 0.052 |
| \( \beta_{\text{max}} = 300 \) | 0.99        | 0.282 ± 0.068 | 0.259 ± 0.038 |
|                               | 0.95        | 0.354 ± 0.067 | 0.386 ± 0.053 |
|                               | 0.90        | 0.298 ± 0.051 | 0.234 ± 0.071 |
|                               | 0.80        | 0.378 ± 0.063 | 0.278 ± 0.077 |
|                               | 0.70        | 0.423 ± 0.092 | 0.401 ± 0.064 |
|                               | 0.60        | 0.358 ± 0.063 | 0.542 ± 0.162 |
| \( \beta_{\text{max}} = 60 \)  | 0.99        | 0.476 ± 0.073 | 0.478 ± 0.067 |
|                               | 0.95        | 0.471 ± 0.095 | 0.515 ± 0.047 |
|                               | 0.90        | 0.468 ± 0.101 | 0.454 ± 0.067 |
|                               | 0.80        | 0.432 ± 0.049 | 0.508 ± 0.049 |
|                               | 0.70        | 0.535 ± 0.074 | 0.680 ± 0.123 |
|                               | 0.60        | 0.374 ± 0.094 | 0.462 ± 0.121 |

Zhang & Zhang (2014) proposed a turbulence scenario with magnetic reconnections in the ejected shells to explain a PL PDS shape for the Swift GRBs.

In some GRBs the quasiperiodic oscillations have been tentatively detected with a periodicity between 2–8 s for long events and a few milliseconds for short events. These oscillations can be attributed to the nonsteady accretion in the central engine of a collapsing star (Masada et al. 2007) or to the modulation caused by the spin misalignment of a black hole after the merger with a neutron star (Stone et al. 2013).
a proxy for the MTS. There is an anticorrelation found between this MTS proxy and the black hole spin parameter of the central engine. The latter is directly responsible for the jets launching via the Blandford–Znajek process, so that the jet Lorentz factor will increase with the black hole’s spin, while the MTS decreases with it. Thus, the observed anticorrelation between the MTS and $\Gamma$ is reproduced by our model (see Wu et al. 2016).

In addition, the MTS–$T_{90}$ correlation should be naturally reproduced. However, this is mainly due to the fact that the calculations are done in a dimensionless unit system. Therefore, the simulations we run in dimensionless time units, $t_G = GM_{BH}/c^3$, should be converted to physical timescales, assuming a fixed black hole mass. The time unit for a black hole of $10^9 M_\odot$ will be equal to $4.96 \times 10^{-5}$ s. The MTSs for this conversion unit are between 1–2 ms, while the timescale of operation of the engine, which we cover in our simulation, is of the order of 0.15 s (it has to be noted that we are not running the models for a longer time because of magnetic field decay and inefficient MRI turbulence at late times, which limits the effective accretion period, while the massive torus is still present and does not replenish, so the engine operation could last $\sim 100$ times longer). Therefore, adopting a range of black hole masses driving the central engine of a GRB, from $\sim 3$ to $\sim 30 M_\odot$, we will automatically be able to cover the range of $T_{90}$ duration times and MTSs in a correlated way. The scatter in this relation will be imposed by the additional factors, such as the mass of the disk available for accretion and its magnetization, hence the accretion rate. Furthermore, we can speculate that the relation between $\Gamma$ and MTS, which spans $\sim 10$ orders of magnitude in the observations presented by Wu et al. (2016), can also reach the blazar sample. The black hole mass in our simulations scales the MTS via the gravitational timescale, up to $\log(T) \sim 5$ for a black hole mass of $10^9 M_\odot$. The smaller values of Lorentz factor should be related mainly with a smaller black hole spin parameter.

We notice that our MTS (measured on average within the jet) is affected by the magnetic field strength, but assuming a given black hole spin we can have either the shortest timescales for the most-magnetized tori (i.e., $a = 0.99$, see Figure 6), or the opposite ($a = 0.7–0.8$). Therefore, we conclude that it is the total efficiency of the Blandford–Znajek process, rather than single parameter of the engine, which drives the jet variability timescales. Its observed value is further regulated by the factors describing the conversion efficiency of the jet bulk kinetic energy into radiation (Granot et al. 2015), which is beyond the scope of our present simulations.

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