Analysis of Time Scale Algorithm in Satellite Autonomous Navigation Mode

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Abstract. Using satellite clock to establish and maintain constellation reference time datum is the basis of satellite autonomous navigation, reference time Datum algorithm is the core of realization, the paper first elaborated the commonly used time scale algorithm: ALGOS algorithm and Kalman algorithm, ALGOS is the most commonly used reference time Datum algorithm in the world, The Kalman algorithm can assign weights dynamically, but there are some disadvantages of slow convergence speed. Based on the analysis of the computational complexity, traffic requirements and algorithm convergence of the reference time Datum algorithm in the space satellite environment, the experimental results show that it takes 3 minutes for the ALGOS algorithm to complete 1 reference time datum calculation, the communication volume is about 398kb, and the algorithm converges about 6-9 seconds. The Power acquisition strategy is the most core part of the reference time algorithm, the paper finally gives the experimental analysis to the different power acquisition strategy, the results show that, from the stability point of view, the short stability of the 12-hour power calculation is better than the short stability of the calculation of 5 days of right, but its instability is slightly lower than that of 1 days and 5 days.

1. FRONTIER CONSTELLATION

Autonomous navigation refers to the satellite in the absence of ground control system support, the independent completion of all navigation services, including satellite orbit determination, satellite calendar determination, satellite signal broadcasting, system reference time datum determination and so on. The system reference time Datum realizes the foundation and key of Constellation Autonomous navigation, and how to establish and maintain the system reference time datum is the core content of realizing satellite autonomous navigation. With the support of the ground control system, the satellite navigation system is supported by the ground support system using ground and satellite clock resources to establish the system reference time datum, the support of the ground control system is lost under the satellite autonomous navigation navigation mode, and the system reference time datum is established and maintained by using the satellite's own satellite clock resources.

The process involves the selection of the Computing node satellite, the realization of the integrated atomic time algorithm, the transmission of interstellar data, the evaluation of the computing power on the star, etc., which will affect the establishment and maintenance of the reference time datum to a certain extent. In this paper, we simulate the new generation navigation system constellation in China, discuss the establishment and maintenance of the system reference time datum under the satellite autonomous navigation mode under the composition of the constellation, and analyze the influence of various factors as far as possible.
It is assumed that the number of Constellation satellites is 30, the main atomic time algorithm is compared and analyzed, the complexity of the algorithm, communication requirements, computational complexity and so on are analyzed in detail, and finally, through the experimental analysis, some conclusions are given to establish and maintain the reference time datum of the system under the satellite autonomous navigation mode.

2. INTEGRATED ATOMIC TIME ALGORITHM

2.1 ALGOS algorithm
The ALGOS algorithm can obtain the Atomic time (TA) when the paper atom is obtained. The algorithm is briefly described below, see formula (1), (2), (3):

\[ TA(t) = \sum_{i=1}^{N} w_i(t)\{h_i(t) + h'_i(t)\}, \quad \sum_{i=1}^{N} w_i(t) = 1 \]  
\[ x_i(t) = TA(t) - h_i(t) \]  
\[ h'_i = x_i(t_0) + \dot{x}_i(t)(t-t_0) \]  

\( h_i(t) \) (i=1,2,⋯N) is the atomic clock reading, \( h'_i(t) \) is the correction value of the clock \( i \) prediction at the \( t \) moment, and \( w_i(t) \) represents the weight of the atomic clock \( i \).

The equations used in the actual calculation are shown in (4):

\[
\begin{cases}
X_0(t) = x_i(t) - x_j(t), & i = 1,2,\ldots,N, i \neq j \\
x_j(t) = \sum_{i=1}^{N} w_i(t)\{h_i(t) - X'_i(t)\}
\end{cases}
\]  

\( X'_i(t) \) is the clock difference. Formula (4) The atomic time scale obtained can be obtained by solving the equation group.

2.2 KALMAN algorithm
The basic models of atomic clocks are shown in formulas (5), (6), (7):

\[ x(t) = x_i(t) + n_i(t) \]  
\[ \dot{x}_i(t) = x_2(t) + n_2(t) \]  
\[ \ddot{x}_2(t) = x_3(t) + n_3(t) \]  
\[ \dddot{x}_3(t) = 0 \]  

Among them, \( x_i(t), x_2(t) \) and \( x_3(t) \) are 3 states of the clock: phase, frequency and frequency change rate, \( x(t) \) is the measurement value of \( x_i(t) \), \( \dot{x}(t) \) represents the derivative, \( n_i(t), n_2(t), n_3(t) \) is independent of each other white noise, respectively, represents the phase noise in the phase white noise \( h_2 \), frequency white noise \( h'_0 \) and frequency random walk noise \( h_2 \). If the \( n_i(t), n_2(t), n_3(t) \) variance is \( \sigma^2, \sigma_2^2, \sigma_3^2 \) and the sampling interval is \( \tau \), the discrete state transfer model and observation model are shown separately (9), (10):

\[
\begin{bmatrix}
x_i(k+1) \\
x_2(k+1) \\
x_3(k+1)
\end{bmatrix} =
\begin{bmatrix}
1 & \frac{1}{2} \tau^2 & \frac{1}{2} \tau \\
0 & 1 & \tau \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_i(k) \\
x_2(k) \\
x_3(k)
\end{bmatrix}
+ 
\begin{bmatrix}
u_i(k) \\
u_2(k) \\
u_3(k)
\end{bmatrix}
\]  

\[
x(k) = x_i(k) + n_i(k)
\]
clock group consisting of M clock, whose state vectors are made up, adding two elements of each clock to a column vector of 2M in length. State transfer matrix $\Phi$ is a $2M \times 2M$ -square, the main diagonal is $2 \times 2$ of the block, the rest of the elements are 0. Similarly, the covariance matrix is also a $2M \times 2M$ -square, the main diagonal is $M \times M$ block, the rest is 0. The main clock in the clock group is the reference clock, each measurement of the remaining clocks relative to the main clock of the clock difference, a total of M-1 measurements, the measurement matrix is $(M-1) \times 2M$ -dimensional. The time Kalman model of the clock group is shown in (11) to (14):

$$
\tilde{X}(k + 1) = \Phi \tilde{X}(k) + \bar{\mu}(k)
$$  \hspace{1cm} (11)

$$
\ddot{Z}(k) = H\dot{X}(k)
$$  \hspace{1cm} (12)

$$
\tilde{X}(k) =
\begin{bmatrix}
x_1(k) \\
y_1(k) \\
x_2(k) \\
y_2(k) \\
x_3(k) \\
y_3(k) \\
\vdots
\end{bmatrix}, \ \ \ \Phi =
\begin{bmatrix}
1 & \tau & & & \\
0 & 1 & \ddots & \\
& \ddots & \ddots & \ddots \\
0 & \cdots & 1 & \tau
\end{bmatrix}
$$  \hspace{1cm} (13)

$$
Q =
\begin{bmatrix}
\sigma_{\varepsilon_1}^2 & 0 & \cdots & 0 \\
0 & \sigma_{\varepsilon_1}^2 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & \sigma_{\varepsilon_2}^2 & 0 \\
0 & \cdots & 0 & \sigma_{\varepsilon_3}^2
\end{bmatrix}
$$  \hspace{1cm} (14)

$\tau$ is the time interval, $\sigma_{\varepsilon}^2$ and $\sigma_{\eta}^2$ respectively for the frequency white noise and frequency random walk noise produced by the Allan variance. The correction amount of the time scale is the Kalman estimator corresponding to the reference time element in the state vector.

### 3. ALGORITHM COMPLEXITY ANALYSIS

Space satellite environment and ground environment are different, the star load load, traffic volume, convergence speed and so on have special requirements, here based on the ALGOS algorithm, from the computational complexity, traffic, convergence time, algorithm accuracy and other aspects, the use of centralized calculation model complexity analysis.

#### 3.1 Calculation analysis

Assume that the number of satellite clocks N is the 30, Allan variance for the calculated sampling point of 60, the least squares fitting, sampling every 5 minutes, and the sampling point per hour is 12 points. It can be estimated that the multiplication complexity of the centralized punctuality algorithm is $O(10^7)$, and the addition complexity $O(10^8)$.

At present, the main frequency of domestic on-board computer is 80MHz, processor instruction multi-emission, addition and multiplication parallel processing, multiplication parts using pipeline structure, on-board computer to complete a centralized punctuality algorithm to calculate the time required for at least 3 seconds.
3.2 Traffic Analysis
Assuming that the number of satellite clocks is N (24MEO+3GEO+3IGSO), the calculated sampling point for Allan variance is 60, the least squares are timed, and the sampling is sampled every 5 minutes. Assuming that each satellite carries a time difference measurement ratio with the surrounding 8 satellites, the total amount of data traffic per calculation for punctuality is: \( \text{7992} \times (\text{20} + \text{31}) = \text{407592} \) bit, approximately 398 KB.

3.3 Algorithm Convergence time analysis
When calculating the weight of each clock, the relative clock difference correction number needs to be obtained when calculating the relative clock velocity correction number, and the weight of the clock and the relative clock speed correction number need to be used to calculate the relative clock difference correction number. Therefore, this is a circular iterative process. Generally overlapping 2 to 3 times can converge. According to the computational analysis of the algorithm, it takes 3 seconds per calculation, and 6-9 seconds to complete the convergence analysis of the centralized punctuality algorithm.

4. IMPACT OF THE PRINCIPLE OF RIGHT OF ACCESS
In the process of calculating atomic time, it is necessary to calculate the weight of atomic clock, the determination of weight is determined by calculating stability, and different power-taking strategies will have different effects on the calculation results of integrated atoms, and the influence of the time scale on the synthesis of atoms is analyzed by the method of hourly stability, daily stabilization and 5-day stabilization.

In the test, the clock group selected 6 hydrogen atomic clocks and 24 rubidium atomic clocks with a fixed data length of 20 days, a data sampling rate of 30 minutes, and a data calculation frequency of 1 hours, respectively, for 12 hours, 1 days and 5 days of data calculation Allan variance extraction.

![Fig. 1](image)

As can be seen from Figure 1, the 1-day and 5-day pick-up has little effect on the results of atomic calculation, and the short stability of 12-hour power calculation is better than that of 5 days of right calculation, but its stability is slightly lower than that of 1 days and 5 days to obtain the right calculation results. The stability of the three methods is basically equal, because the index is similar in the clock simulation, and it is not possible to fully determine the effect of different extraction principles on the calculation results of atoms.

5. CONCLUSION
Through the above analysis, some conclusions can be drawn:
1) ALGOS algorithm and Kalman algorithm are relatively classical comprehensive atomic time algorithm, the difference lies in the power method. Kalman algorithm is to filter the original timing data without smoothing to improve the stability of the time scale. ALGOS algorithm is more focused on the long-term stability of time scale, Kalman algorithm has advantages in short-term stability prediction, but the abnormal data will cause the Kalman filter divergence, and eventually lead to the increase of error.

2) It is suggested that the ALGOS weighted average algorithm should be used for the calculation of atomic time, and the weight of each clock should be controlled adaptively according to the actual operation performance and ratio of the clock, and the Allan variance of a given time should be used as the basis for the calculation of the weight in the actual calculation. According to the result of weight calculation, the atomic clock data of the free operation of the clock group are processed, and the system time is obtained by frequency control when the free synthetic atoms are treated.

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