Probing Two types of Gluon Jets at LHC

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We propose a simple way to test the Abelian decomposition of QCD, the existence of two types of gluons, experimentally at LHC. The Abelian decomposition decomposes the gluons to the color neutral neurons and colored chromons gauge independently. This refines the Feynman diagram in a way that the color conservation is explicit, and generalizes the quark model to the quark and chromon model. We predict that the neuron jet has the color factor 3/4 and has a sharpest jet radius and smallest particle multiplicity, while the chromon jet with the color factor 9/4 remains the broadest jet. Moreover, the neuron jet has a distinct color flow which forms an ideal color dipole, while the quark and chromon jets have distorted dipole pattern.

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A common misunderstanding on QCD is that the non-Abelian gauge symmetry is so tight that it defines the theory almost uniquely, and thus does not allow any simplification. The Abelian decomposition of QCD tells that this popular wisdom is not true [1, 2]. It tells that QCD has two types of gluons, the color neutral neutrons and colored chromons, which play totally different roles. Moreover, it has a non-trivial core, the restricted QCD (RCD) which describes the Abelian sub-dynamics of QCD but has the full non-Abelian gauge symmetry, which we can separate from QCD gauge independently.

There are ample motivations for the Abelian decomposition. Consider the proton made of three quarks. Obviously we need the gluons to bind the quarks in the proton. However, the quark model tells that the proton has no valence gluon. If so, what is the binding gluon which bind the quarks in proton, and how do we distinguish it from the valence gluon?

Another motivation is the color confinement in QCD. Two popular proposals to resolve this problem are the monopole condensation [1–3] and the Abelian dominance [4, 5]. To prove the monopole condensation, we first have to separate the monopole potential gauge independently. Similarly, to prove the Abelian dominance we have to know what is the Abelian part and how to separate it. How can we do that?

The Abelian decomposition decomposes the non-Abelian gauge potential to two parts, the restricted Abelian part which has the full non-Abelian gauge symmetry and the gauge covariant non-Abelian part which describes the colored valence gluons (the chromons) [1, 2]. Moreover, it separates the restricted potential to the non-topological Maxwell part which describes the colorless binding gluons (the neutrons) and the topological Dirac part which describes the non-Abelian monopole [1, 2].

This has deep consequences. It tells that there is a simpler QCD called the restricted QCD (RCD) made of the restricted potential which describes the Abelian sub-dynamics of QCD, and that QCD can be viewed as RCD which has the chromons as the colored source. More importantly, it tells that QCD has two types of gluons which play totally different roles.

This allows us to prove the Abelian dominance, that RCD is responsible for the confinement [1, 5]. Since the colored chromons have to be confined, it can not play any role in the confinement. Moreover, this allows us to prove the monopole condensation. Integrating out the chromons under the monopole background, we can demonstrate that the true QCD vacuum is given by stable monopole condensation [6–8]. This makes the experimental verification of the Abelian decomposition, the confirmation of two types of gluons, an urgent issue. The purpose of this letter is to discuss how to do this at LHC.

When the Abelian decomposition was proposed first, there was no way to verify this experimentally. During the last twenty years, however, there has been huge progress on jet physics. Theoretically new features of the jet substructure have been known which can tag the different jets [9–13]. Moreover, ATLAS and CMS have succeeded to separate different types of jet experimentally [14–16]. We argue that these progresses could allow

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us to confirm the existence of two types of gluon jets experimentally at LHC.

Before we do this, we have to know how QCD has two types of gluons. To show this consider the SU(2) QCD first, and select the Abelian direction \( \hat{n} \). Impose the isometry to project out the restricted potential \( \hat{A}_\mu \). \[ \{1\} \] 

\[ D_\mu \hat{n} = (\partial_\mu + g\hat{A}_\mu \times)\hat{n} = 0, \]
\[ \hat{A}_\mu \rightarrow \hat{A}_\mu = A_\mu \hat{n} - \frac{1}{g} \hat{n} \times \partial_\mu \hat{n} = \hat{A}_\mu + \hat{C}_\mu, \]
\[ \hat{A}_\mu = A_\mu \hat{n}, \quad A_\mu = \hat{n} \cdot \hat{A}_\mu, \quad \hat{C}_\mu = -\frac{1}{g} \hat{n} \times \partial_\mu \hat{n}. \] (1)

It is made of two parts, the non-topological (Maxwellian) \( \hat{A}_\mu \) which describes the color neutral Abelian gluon (the neutron) and the topological (Diracian) \( \hat{C}_\mu \) which describes the non-Abelian monopole \([17, 18]\). With this we have

\[ \hat{F}_{\mu\nu} = (F_{\mu\nu} + H_{\mu\nu}) \hat{n} = F'_{\mu\nu} \hat{n}, \]
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \]
\[ H_{\mu\nu} = -\frac{1}{g} \hat{n} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n}) = \partial_\mu C_\nu - \partial_\nu C_\mu, \]
\[ C_\mu = -\frac{1}{g} \hat{n}_1 \cdot \partial_\mu \hat{n}_2, \]
\[ F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu, \quad A'_\mu = A_\mu + C_\mu. \] (2)

Notice that \( \hat{F}_{\mu\nu} \) is Abelian but is made of two potentials, the non-topological \( A_\mu \) and topological \( C_\mu \). With \([1\] we can construct RCD which has the full non-Abelian gauge symmetry,

\[ \mathcal{L}_{RCD} = -\frac{1}{4} \hat{F}_{\mu\nu}^2 = -\frac{1}{4} F_{\mu\nu}^2 \]
\[ + \frac{1}{2g} F_{\mu\nu} n \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n}) - \frac{1}{4g^2} (\partial_\mu \hat{n} \times \partial_\nu \hat{n})^2, \] (3)

which describes the Abelian sub-dynamics of QCD.

We can express the full SU(2) potential adding the non-Abelian part \( \hat{X}_\mu \) to \( \hat{A}_\mu \). \[ \{1\} \] 

\[ \hat{A}_\mu = \hat{A}_\mu + \hat{X}_\mu, \quad \hat{n} \cdot \hat{X}_\mu = 0, \]

and show that \( \hat{A}_\mu \) has the full gauge degrees of freedom while \( \hat{X}_\mu \) transforms gauge covariantly. Moreover, with

\[ \hat{F}_{\mu\nu} = \hat{F}_{\mu\nu} + \hat{D}_\mu \hat{X}_\nu - \hat{D}_\nu \hat{X}_\mu + g \hat{X}_\mu \times \hat{X}_\nu, \] (5)

we obtain the extended QCD (ECD) to recover the full QCD

\[ \mathcal{L}_{ECD} = -\frac{1}{4} \hat{F}_{\mu\nu}^2 = -\frac{1}{4} \hat{F}_{\mu\nu}^2 - \frac{1}{4} (\hat{D}_\mu \hat{X}_\nu - \hat{D}_\nu \hat{X}_\mu)^2 \\
- \frac{g}{2} \hat{F}_{\mu\nu} \cdot (\hat{X}_\mu \times \hat{X}_\nu) - \frac{g^2}{4} (\hat{X}_\mu \times \hat{X}_\nu)^2, \] (6)

This shows that QCD can be viewed as RCD which has the chromon as the colored source \([2]\).

The Abelian decomposition of SU(3) QCD is similar \([6,8]\). Let \( \hat{n}_i \) \((i = 1, 2, \ldots, 8) \) be the orthonormal SU(3) basis and choose \( \hat{n}_3 = \hat{n} \) and \( \hat{n}_8 = \hat{n}' \) to be the Abelian directions, and impose the isometry

\[ D_\mu \hat{n} = 0, \quad D_\mu \hat{n}' = 0. \] (7)

Solving this we have the SU(3) Abelian projection,

\[ \hat{A}_\mu \rightarrow \hat{A}_\mu = A_\mu \hat{n} + A'_\mu \hat{n}' - \frac{1}{g} \hat{n} \times \partial_\mu \hat{n} - \frac{1}{g} \hat{n}' \times \partial_\mu \hat{n}' = \sum_{p=1}^{3} \frac{2}{3} \hat{A}_p^p + \hat{W}_p^p, \]

\[ (p = 1, 2, 3), \]
\[ \hat{A}_\mu = A_\mu^p \hat{W}_p^p - \frac{1}{g} \hat{W}_p^p \times \partial_\mu \hat{W}_p^p = A_\mu \hat{W}_p^p + C_\mu \]
\[ A_\mu^1 = A_\mu, \quad A_\mu^2 = -\frac{1}{2} A_\mu + \sqrt{3} A'_\mu, \]
\[ A_\mu^3 = -\frac{1}{2} A_\mu - \sqrt{3} A'_\mu, \quad \hat{n}_1 = \hat{n}, \]
\[ \hat{n}_2 = -\frac{1}{2} \hat{n} + \frac{\sqrt{3}}{2} \hat{n}', \quad \hat{n}_3 = -\frac{1}{2} \hat{n} - \frac{\sqrt{3}}{2} \hat{n}', \]

where \( A_\mu \) and \( A'_\mu \) are two SU(3) neurons, and the sum is the sum of the three Abelian directions \((\hat{n}_1, \hat{n}_2, \hat{n}_3)\) of three SU(2) subgroups. Notice that \( \hat{A}_\mu \) is expressed by three SU(2) restricted potential \( \hat{A}_\mu^p \) \((i = 1, 2, 3) \) in Weyl symmetric way. From this we have the SU(3) RCD

\[ \mathcal{L}_{RCD} = -\frac{1}{4} \hat{F}_{\mu\nu}^2 = \sum_{p=1}^{3} \frac{1}{6} \big( \hat{F}_{\mu\nu}^p \big)^2, \] (9)

which has the full SU(3) gauge symmetry.

Adding the valence part \( \hat{X}_\mu \) to \( \hat{A}_\mu \) we have the SU(3) Abelian decomposition,

\[ \hat{A}_\mu = \hat{A}_\mu + \hat{X}_\mu = \sum_{p} \frac{2}{3} \hat{A}_\mu^p + \hat{W}_\mu^p, \]
\[ \hat{W}_\mu^1 = X_\mu^1 \hat{n}_1 + X_\mu^2 \hat{n}_2, \quad \hat{W}_\mu^2 = X_\mu^6 \hat{n}_6 + X_\mu^7 \hat{n}_7, \]
\[ \hat{W}_\mu^3 = X_\mu^4 \hat{n}_4 + X_\mu^5 \hat{n}_5. \] (10)

Notice that \( \hat{X}_\mu \) can be decomposed to three (red, blue, and green) SU(2) chromons \((\hat{W}_\mu^1, \hat{W}_\mu^2, \hat{W}_\mu^3)\).

From this we have

\[ \hat{F}_{\mu\nu} = \hat{F}_{\mu\nu} + \hat{D}_\mu \hat{X}_\nu - \hat{D}_\nu \hat{X}_\mu + g \hat{X}_\mu \times \hat{X}_\nu = \sum_{p} \bigg[ \frac{2}{3} \hat{F}_{\mu\nu}^p + (\hat{D}_\mu \hat{W}_\mu^p - \hat{D}_\nu \hat{W}_\mu^p) \bigg] + \sum_{p,q} \hat{W}_\mu^p \times \hat{W}_\mu^q, \]
\[ D_\mu = \partial_\mu + g \hat{A}_\mu \times, \] (11)
The essential characteristics of QCD \cite{17,18}.

This is a gross misunderstanding. It represents the gauge and obtain the SU(3) ECD \cite{6–8}

In (A) it is decomposed to the restricted potential (kinked line) and the chromon (straight line). In (B) the restricted potential is further decomposed to the neuron (wiggly line) and the monopole (spiked line).

and obtain the SU(3) ECD \cite{6} \cite{8}

\[
\mathcal{L}_{ECD} = -\frac{1}{4} F_{\mu\nu}^2 = \sum_p \left\{ -\frac{1}{6} (\mathcal{F}_{\mu\nu\rho})^2 \right. \\
- \frac{1}{4} (\bar{W}_\mu^p W_\nu^\mu - \bar{D}_\mu^p W_\nu^\mu)^2 - \frac{g}{2} \bar{F}_{\mu\nu\rho} \cdot (\bar{W}_\mu^p \times \bar{W}_\nu^p) \bigg\} \\
- \frac{g^2}{4} \left( \bar{W}_\nu^p \times \bar{W}_\mu^p \right)^2 \\
\left. - \sum_{p,q,r} \left[ (\bar{W}_\nu^p \times \bar{W}_\mu^q) \cdot (\bar{W}_\mu^q \times \bar{W}_\nu^r) \right] + (\bar{W}_\mu^p \times \bar{W}_\nu^p) \cdot (\bar{W}_\mu^q \times \bar{W}_\nu^q) \right]. \tag{12}
\]

The Abelian decomposition is known as the Cho decomposition, Cho-Duan-Ge (CDG) decomposition, or Cho-Faddeev-Niemi (CFN) decomposition \cite{19} \cite{23}.

We can add quarks in the Abelian decomposition,

\[
\mathcal{L}_q = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi = \bar{\Psi}(i\gamma^\mu \bar{D}_\mu - m)\Psi + \frac{g}{2} \bar{\lambda}_\mu \cdot \bar{\Psi}(\gamma^\mu \bar{\lambda})\Psi \\
= \sum_p \left[ \bar{\Psi}^p(i\gamma^\mu \bar{D}_\mu - m)\Psi^p + \frac{g}{2} \bar{\Psi}^p \cdot \bar{\Psi}(\gamma^\mu \bar{\sigma})\Psi^p \right], \\
\bar{D}_\mu = \partial_\mu + \frac{g}{2\ell} \bar{\lambda} \cdot \bar{A}_\mu, \quad \bar{D}_\mu = \partial_\mu + \frac{g}{2\ell} \bar{\sigma} \cdot \bar{A}_\mu, \tag{13}
\]

where \( m \) is the mass, \( p \) denotes the color of the quarks, and \( \Psi^p \) represents the three SU(2) quark doublets \((r,b), (b,g), \) and \((g,r)\). Notice that the Lagrangian is also Weyl-symmetric.

There has been an assertion that the introduction of the Abelian direction \( n \) adds a new dynamical degree \cite{19}. This is a gross misunderstanding. It represents the gauge degree, not a dynamical degree \cite{28}. Nevertheless \( n \) plays important roles representing the topological structure of QCD, the monopole topology \( \pi_3(S^2) \) as well as the vacuum topology \( \pi_3(S^2) \simeq \pi_3(S^2) \), both of which are the essential characteristics of QCD \cite{17} \cite{18}.

The Abelian decomposition is expressed graphically in Fig. 1. Although the decomposition does not change QCD, it reveals the important hidden structures of QCD. In particular, it shows the existence of two types of gluons, the neuron and chromon, which play totally different roles. This is evident in \cite{6}, \cite{12}, and \cite{13}. The neurons, just like the photons in QED, provide the binding. But the chromons, just like the quarks, become the colored source.

This has deep implications. In the perturbative regime this tells that the Feynman diagram can be decomposed in such a way that the color conservation is explicit. This is graphically shown in Fig. 2. Notice that the monopole does not appear in the Feynman diagram, because it does not represent a dynamical degree.

To see the decomposition is non-trivial, consider the Feynman diagrams of two neurons, two chromons, and quark-antiquark pair shown in Fig. 3. Clearly the neutron binding looks very much like two photon binding in QED, while the chromon binding looks just like the quark-antiquark binding in QCD. This is because the three point vertex made of two or three neurons are forbidden. This strongly implies that the quarks can hardly make a bound state, and may not be viewed as the constituent of hadrons. However, the chromon binding strongly implies that they, just like the quarks, become the constituent of hadrons and form hadronic bound states. This generalizes the quark model to the quark and chromon model, which provides a new picture of hadrons \cite{24} \cite{25}.

In particular, this gives us a new picture of glueball different from all existing glueball models. In existing glueball models all gluons are treated equally, and the color singlet combinations of the gluon octet make the glueballs. But in the quark and chromon model only the chromons become the constituent of glueballs, and the neurons provide the chromon binding \cite{1} \cite{2}. Moreover, this picture describes the glueball-quarkonium mixing successfully. The numerical analysis of the mixing in this picture below 2 GeV shows that \( f_0(1500), f_2(1950), \eta(1405), \) and \( \eta(1475) \) in \( 0^{++}, 2^{++}, \) and \( 0^{++} \) sectors can be identified as predominantly the glueball states \cite{24} \cite{25}.

In the non-perturbative regime, the Abelian decom-
position proves the Abelian dominance. Implementing the Abelian decomposition on lattice we can calculate the Wilson loop integral with the full potential, the restricted potential, and the monopole potential separately, and show that all three potentials produce exactly the same linear confining force \[22,23\]. This proves not only the Abelian dominance but also the monopole dominance.

As importantly the Abelian decomposition puts QCD to the background field formalism, because we can treat the restricted part as the classical background and the valence part as the fast moving quantum field \[27,28\]. This makes ECD an ideal platform for us to calculate the QCD effective potential and prove the monopole condensation. Indeed, choosing the monopole potential as the background and integrating out the chromons gauge invariantly, we can prove that the true QCD vacuum is given by the gauge invariant monopole condensation \[6,8\].

The prediction and subsequent confirmation of the gluon jet was a great success of QCD \[29,31\]. To promote QCD further, what we badly need is the experimental confirmation of the Abelian decomposition (the existence of two types of gluons). This could be done at LHC. Obviously two types of gluons mean two types of gluon jets, the neuron jet and chromon jet, and the recent progress on jet physics could allow us to separate the neutron jet from the chromon jet. This could be a difficult task, but ATLAS and CMS have already separated the gluon jets from the quark jets successfully \[14,16\]. In doing so they have developed powerful techniques to tag different jets. This could allow them to identify the neutron jet.

To show this might be possible, we have to know what are the expected differences of the neutron jet from other jets. The gluons and quarks emitted in the p-p collisions evolve into hadron jets in two steps, the parton shower described by the perturbative process and the hadronization described by the non-perturbative process. The neutron behaves differently in the first step. This is shown in Fig. 4. Clearly the neutron jet shown in (A) has no parton shower made of neuron emission which exists in the chromon jet (B) and the quark jet (C), but the chromon and quark jets look almost the same. This is because the three-point vertex made of three neurons is forbidden. This strongly suggests that the neuron jet must have different jet shape, sharp with relatively small radius compared to the chromon and quark jets.

There are other differences. The neuron jet should have different (charged) particle multiplicity, considerably smaller than that of the quark and/or chromon jets. This again must be clear from Fig. 4 which shows that chromon and quark couple to neurons which make secondary showers but the neutron jet is not allowed to have such interaction.

To quantify the differences, we have to know the color factor of the neuron and chromon jets, a most important quantity which determines the characters of the jet. Although the neurons are color neutral, they have a finite color factor. But we can not find it from the SU(3) Casimir invariant, because the color gauge symmetry is replaced to the 24-element color reflection symmetry after the Abelian decomposition \[1,2\]. There are various ways to find the neuron color factor, but from the fact that the adjoint representation has the color factor 3, we can deduce the neuron color factor to be 3/4. This comes from the simple democracy of the gauge interaction. Since the neurons constitute one quarter of total gluons their color factor becomes 3/4 and that of of chromons becomes 9/4, while that of of quarks remains the same. This endorses the above predictions.

Another important feature of the neuron jet is that it has different color flow. Clearly the chromons and quarks carry color charge, but the neurons are color neutral. So the neutron jet must have different color flow. In fact, Fig. 4 tells that the color flow of the neuron jet generates an ideal color dipole pattern, but the other two jets have distorted dipole pattern. We could check this
prediction using Pythia and FastJet. This means that the neuron jet must be quantitatively different from the chromon and quark jets. To tell more detailed differences, of course, we need more serious theoretical calculations. But the above differences give us enough tools to identify the neuron jet.

At this point one might ask what are the gluon jets identified by ATLAS and CMS? Probably they are the chromon jets, because the chromon jet has the characteristics of the known gluon jet. This is evident from Fig. [4] Perhaps a more interesting question is why they have not found the neuron jet? There could be two explanations. They have not searched for the neuron jet yet, because they had no motivation to do that. Or they might have misidentified some of the neuron jets as the quark jet. This is because the color factor of neuron and chromon jets are not much different. This tells that we need a more careful analysis of quark and gluon jets.

If this is true, the recent experiments which separated the quark jet from the gluon jet based on the color factor ratio $C_A/C_F = 9/4$ need to be completely re-analysed [13]. According to the above reasoning we should look for three (neuron, quark, and chromon) jets which have the color factor ratio $3/4 : 4/3 : 9/4 \approx 0.56 : 1 : 1.69$. In this respect we notice two interesting reports which might indicate that the observed gluon jets are indeed the chromon jets. A re-analysis of DELPHI $e^+e^-$ three jet data at LEP strongly suggests that actual $C_A/C_F$ could be around 1.74, much less than the popular value 2.25 but close to our prediction 1.69 [32, 33]. Moreover, the $p\bar{p}$ DO jets experiment at Fermilab Tevatron shows that the quark to gluon jets particle multiplicity ratio is around 1.84, again close to our prediction [34]. They seem to imply that the above interpretation might be correct.

One advantage in searching for the neuron jet is that we do not need any new collider or detector. LHC produces billions of hadron jets in a second, and ATLAS and CMS have already filed up huge data on jets. So all that we have to do is to re-analyse these data. Actually we could even go back to the three jet events (the gluon jets) at DESY, LEP, and Tevatron, and search for the neuron jets [38, 39, 40]. Here again the simple number counting strongly suggests that one-quarter of the gluon jets coming from the three jets events could actually be the neuron jets. It would be very interesting to re-analyse the existing data and confirm the existence of the neuron jet.

The confirmation of the gluon jet justified the asymptotic freedom and extended our understanding of QCD very much [35, 36]. Clearly the experimental confirmation of two types of gluon jets would be at least as important. It will shed a new light on QCD, revealing the hidden structures of QCD. In particular it will endorse the decomposition of Feynman diagram and justify the quark and chromon model [24, 25]. The detailed discussion of the neuron jet and its color flow will be published separately [37].

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[1] Y. M. Cho, Phys. Rev. D21, 1080 (1980); Y. S. Duan and M. L. Ge, Sci. Sinica 11, 1072 (1979).
[2] Y. M. Cho, Phys. Rev. Lett. 46, 302 (1981); Phys. Rev. D23, 2415 (1981).
[3] Y. Nambu, Phys. Rev. D10, 4262 (1974); S. Mandelstam, Phys. Rep. 23C, 245 (1976); A. Polyakov, Nucl. Phys. B120, 429 (1977).
[4] G. ’t Hooft, Nucl. Phys. B190, 455 (1981).
[5] Y. M. Cho, Phys. Rev. D62, 074009 (2000).
[6] Y. M. Cho, Franklin H. Cho, and J. H. Yoon, Phys. Rev. D87, 085025 (2013).
[7] Y. M. Cho, Int. J. Mod. Phys. A29, 1450013 (2014); Y. M. Cho, Euro Phys. J. WcC, 182, 02029 (2018).
[8] Y. M. Cho, and Franklin H. Cho, submitted to Phys. Rev. D.
[9] H. Nilles and K. streng, Phys. Rev. D23, 1944 (1981);L. Jones, Phys. Rev. D39, 2550 (1989).
[10] Z. Fodor, Phys. Rev. D41,1726 (1990); L. Jones, Phys. Rev. D42,811 (1990); L. Lomblad, C. Peterson, and T. Rognvaldsson, Nucl. Phys. B349, 675 (1991); J. Pumplin, Phys. Rev. D44, 2925 (1991).
[11] J. Gallicchio and M. Schwartz, Phys. Rev. Lett. 107, 172001 (2011); A. Larkoski, G. Salam, and J. Thaler, JHEP, 06, 108 (2013).
[12] B. Bhattacharjee, S. Mukhopadhyay, M. Nojiri, Y. Sakaki, and B. Webber, JHEP. 04, 131 (2015); D. de Lima, P. Petrov, D. Soper, and M. Spannowsky, Phys. Rev. D95, 034001 (2017).
[13] J. Davighi and P. Harris, Eur. Phys. J. C78, 334 (2018); E. Metodiev and J. Taler, Phys. Rev. Lett. 120, 241602 (2018).
[14] ATLAS Collaboration, Eur. Phys. J. C73, 2676 (2013); C74, 3023 (2014); C75, 17 (2015).
[15] CMS Collaboration, Eur. Phys. J. C75, 66 (2015); Phys. Rev. D92, 032008 (2015).
[16] ATLAS Collaboration, Eur. Phys. J. C76, 322 (2016); Phys. Rev. d96, 072002 (2017).
[17] Y. M. Cho, Phys. Rev. Lett. 44, 1115 (1980).
[18] Y. M. Cho, Phys. Lett. B115, 125 (1982).
[19] L. Faddeev and A. Niemi, Phys. Rev. Lett. 82, 1624 (1999); Phys. Lett. B449, 214 (1999).
[20] S. Shabanov, Phys. Lett. B458, 322 (1999); B463, 263 (1999); H. Gies, Phys. Rev. D63, 125023 (2001).
[21] R. Zucchini, Int. J. Geom. Meth. Mod. Phys. 1, 813.
[22] S. Kato, K. Kondo, T. Murakami, A. Shibata, T. Shinohara, and S. Ito, Phys. Lett. B632, 326 (2006); B645, 67 (2007); B653, 101 (2007); B669, 107 (2008).
[23] K. Kondo, S. Kato, A. Shibata, and T. Shinohara, Phys. Rep. 579, 1 (2015).
[24] Y. M. Cho, X. Y. Pham, Pengming Zhang, Ju-Jun Xie, and Li-Ping Zou, Phys. Rev. D91, 114020 (2015); Y. M. Cho, Euro Phys. J. WoC, 182, 02031 (2018).
[25] Pengming Zhang, Li-Ping Zou, and Y. M. Cho, Phys. Rev. D98, 096015 (2018).
[26] N. Cundy, Y. M. Cho, W. Lee, and J. Leem, Phys. Lett. B729, 192 (2014); Nucl. Phys. B895, 64 (2015).
[27] B. de Witt, Phys. Rev. 162, 1195 (1967); 1239 (1967).
[28] W. S. Bae, Y. M. Cho, and S. W. Kim, Phys. Rev. D65, 025005 (2001).
[29] J. Ellis, M.K. Gaillard, and G.G. Ross, Nucl. Phys. B111, 253 (1976).
[30] R. Brandelik et al. (TASSO Collaboration), Phys. Lett. B86, 243 (1979); D.P. Barber et al. (MARK-J Collaboration), Phys. Rev. Lett. 43, 830 (1979).
[31] Ch. Berger et al. (PLUTO Collaboration), Phys. Lett. B86, 418 (1979); W. Bartel et al. (JADE Collaboration), Phys. Lett. B91, 142 (1980).
[32] J. Gary, Phys. Rev. D61, 114007 (2000).
[33] P. Abreu et al. (DELPHI Collaboration), Phys. Lett. B449, 383 (1999).
[34] V. Avazov et al. (DØ Collaboration), Phys. Rev. D65, 052008 (2002).
[35] G. Abbiendi et al. (OPAL Collaboration), Euro.Phys. J. C11, 217 (1999).
[36] D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. Politzer, Phys. Rev. Lett. 30, 1346 (1973).
[37] Y. M. Cho, Xiaohui Liu, and Pengming Zhang, to be published.