Reassessing the importance of the colour-singlet contributions to direct $J/\psi + W$ production at the LHC and the Tevatron

J.P. Lansberg\textsuperscript{a}, C. Lorce\textsuperscript{a,b}

\textsuperscript{a}IPNO, Université Paris-Sud, CNRS/IN2P3, F-91406, Orsay, France
\textsuperscript{b}LPT, Université Paris-Sud, CNRS, F-91405, Orsay France

Abstract

We show that the colour-singlet contributions to the hadroproduction of $J/\psi$ in association with a $W$ boson are sizable, if not dominant over the colour-octet contributions. They are of two kinds, $s g \to J/\psi + c + W$ at $\alpha_s^2\alpha$ and $q\bar{q}' \to \gamma^*/Z/W \to J/\psi W$ at order $\alpha_s$. These have not been considered in the literature until now. Our conclusion is that the hadroproduction of a $J/\psi$ in association with a $W$ boson cannot be claimed as a clean probe of the colour-octet mechanism. The rate are small even at the LHC and it will be very delicate to disentangle the colour-octet contributions from the sizable colour-singlet ones and from the possibly large double-parton-scattering contributions. During this analysis, we have also noted that, for reactions such as the production of a $J/\psi$ by light quark–antiquark fusion, the colour-singlet contribution via an off-shell photon is of the order of the expectation from the colour-octet contribution via an off-shell gluon. This is relevant for inclusive production at low energies close to the threshold. Such an observation also likely extends to other processes naturally involving light-quark annihilation.

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1. Introduction

Since the mid eighties, the field of quarkonium physics has faced a number of puzzles challenging our understanding of QCD at the interplay between its short- and long-distance domains. The puzzles related to the quarkonium production at the Tevatron have been attributed to the colour-octet mechanism (COM), i.e. the non-perturbative transition of heavy quark-antiquark pairs in colour-octet state into quarkonia (see [1, 2, 3] for reviews).

Since a few years, we know that $\alpha_s^2\alpha$ and $\alpha_s^2$ corrections to the colour-singlet mechanism (CSM) [4] are essential to try to explain the $P_T$ dependence of the $J/\psi$ and $\Upsilon$ cross sections observed at high-energy hadron collisions [5, 6, 7, 8, 9, 10]. As far as the $P_T$-integrated yield is concerned, colour-singlet $Q\bar{Q}$ configurations have been shown to be sufficient\textsuperscript{1} to account for the experimental data [14, 15].

In this Letter, we reassess the importance of the leading-$v^3$ contribution to $J/\psi + W^\pm$ – i.e. from the colour-singlet transitions. In the previous analyses of $J/\psi + W$ [18, 19, 20], these have been disregarded since formally appearing at higher orders in $\alpha_s$. In fact, they are not negligible at all. Quotes such as "$\psi + W$ offers a clean test of the colour-octet contributions" from [18] and "If the $J/\psi + W$ production is really detected, it would be a solid basis for testing the colour-octet mechanism of the NRQCD" from [20] are overstated if not misleading. This observable is not cleaner than the inclusive production for instance.

We have identified two classes of important colour-singlet contributions. The first comes from the strange-quark–gluon fusion which produces a $W + c$ pair where the charm quark fragments into a $J/\psi$ (see Fig. 1a). This is reminiscent of the leading-$P_T$ contribution to $J/\psi + c\bar{c}$ for instance [6]. In the past, $W + c$ has indeed been identified as a probe of the strange quark PDF [21]\textsuperscript{2}. The other class

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Representative diagrams contributing to $J/\psi + W^\pm$ hadroproduction in the CSM at orders $\alpha_s^2\alpha$ (a), $\alpha_s^3$ (b) and in the COM at orders $\alpha_s^2\alpha$ (c). The quark and antiquark attached to the ellipsis are taken as on-shell and their relative velocity $v$ is set to zero.}
\end{figure}

\textsuperscript{1}The CSM is nonetheless known to be plagued by infrared divergences in the case of $P$-wave decay at NLO, earlier regulated by an ad-hoc binding energy [16], which can however be rigorously cured [17] in the more general framework of NRQCD.

\textsuperscript{2}Pending the available statistics, $J/\psi + W + c$, as $W + c$, could in principle be discriminated experimentally owing to the presence of an additional charmed hadron in the final state.
is simply a contribution à la vector-meson dominance. The $3S_1$ quarkonium bound-state is simply produced by an off-shell photon (or $Z$) emitted by the quark which also radiates the $W$ boson (see Fig. 1b).

The latter contribution is clearly enhanced in $p\bar{p}$ collisions at the Tevatron owing to the presence of valence antiquarks in the antiiproton, whereas the former contribution is getting larger at LHC energies in $pp$ collisions thanks to the enhancement of the gluon PDF at lower $x$. In any case, these CSM processes are not at all negligible compared to the leading colour-octet contributions (see Fig. 1c). Interestingly, both these Born contributions possess a leading-$P_T$ contribution ($P_T^2$). Not only are they significant at low $P_T$, but they remain large at large $P_T$. This is at variance with the inclusive case where the Born contributions are not leading power in $P_T$.

In section 2, we briefly discuss how we have evaluated the cross sections of the different contributions. In section 3, we present and discuss our results. Section 4 gathers a detailed discussion of the relative size of the CSM contribution via an off-shell photon w.r.t. that of the COM via an off-shell gluon. We finally conclude in section 5.

2. Cross-section evaluation

In the CSM [4], the amplitude for the production of a $3S_1$ quarkonium $Q$ of a given momentum $P$ and of polarisation $\lambda$ accompanied by other partons, noted $j$, and a $W$ boson is written as the product of the amplitude to create the corresponding heavy-quark pair, a spin projector $N(\lambda|s_1, s_2)$ and $R(0)$, the radial wave function at the origin in the configuration space. Precisely, one has

$$\mathcal{M}(ab \to Q^j(P) + W + j) = \sum_{s_1, s_2, \lambda, \ell} \frac{N(\lambda|s_1, s_2)}{\sqrt{m_Q^2}} \frac{\delta^{\lambda\ell}}{\sqrt{N_\ell}} R(0) \times \mathcal{M}(ab \to Q_\ell^0 \bar{Q}_\ell^0(p = 0) + W + j),$$

(1)

where one defines $P = p_Q + p_\bar{Q}$, $p = (p_Q - p_\bar{Q})/2$, and where $s_1, s_2$ are the heavy-quark spin components and $\delta^{\lambda\ell}/\sqrt{N_\ell}$ is the projector onto a colour-singlet state. $N(\lambda|s_1, s_2)$ has a simple expression in the non-relativistic limit: $\frac{e_j^\ell}{2\sqrt{2m_0}} \Psi_s^\lambda(P, s)\gamma^\mu u(P, s_1)$ where $e_j^\ell$ is the quarkonium polarisation vector. Once one sums over the heavy-quark spin components, one obtains traces which can be evaluated in a standard way. In particular, for LO evaluations -without loops- one can simply use the framework described in [22] based on the tree-level matrix element generator MADONIA [23]. Another possibility would be to use HELAC-Onia [24].

For the cross-section evaluation, we have used the parameters $|R_{Q\bar{Q}}(0)|^2 = 1.01$ GeV$^3$ and $Br(J/\psi \to \ell^+\ell^-) = 0.0594$. Neglecting relativistic corrections, one has in the CSM, $M_{J/\psi} = 2m_c$. We have taken $m_W = 80.39$ GeV and $\sin^2(\theta_W) = 0.23116$. The uncertainty bands for the resulting predictions are obtained from the combined variations of the heavy-quark mass within the range $m_c = 1.5 \pm 0.1$ GeV, with the factorisation $\mu_F$ and the renormalisation $\mu_R$ scales chosen among the couples (0.75, 0.75); (0.75, 2); (1, 2); (1, 1); (2, 1); (2, 0.75); (2, 2) × $m_c$.

For the colour-octet contributions, the only relevant parameter is the NRQCD Long Distance Matrix Elements (LDME) $\langle Q_{J/\psi}(3S_1) |O_{R}(|s_0)^I |\rangle$. We have set it to $2.2 \times 10^{-3}$ GeV$^3$, i.e. the value obtained in the recent global NLO analysis of Butenschoen and Kniehl [25]. This value is also of the order of what was obtained in another recent NLO NRQCD fit [26].

There are of course drawbacks in using LDME obtained from NLO fits. First, NLO corrections to the hard part of color-octet processes to inclusive production show a $K$ factor higher than one which leads to a reduction of the CO LDME compared to those extracted from a LO fit. Yet, a comparison to the NLO results of [20] for CO channels indicate that our evaluation is reasonable. Moreover, various CO contributions can interfere and fits can yield negative values. For instance, a recent NLO fit has obtained such a negative result for this LDME [27]. It would not make much sense to use such a value in LO computations since the cross section would then be negative. It is therefore important to recall that our choice is also close to the LO analysis of [28] and from analyses which partially took into account QCD corrections [29, 30].

3. Results

Our leading-order results for the differential cross sections in $P_T$ are shown in Fig. 2 for the Tevatron $a)$, and for the LHC at 8 TeV $b)$ and 14 TeV $c)$.

At the Tevatron, the COM contribution3 (orange band) is significantly larger than that of the CSM via $sg$ fusion (dark green band). However, it is of similar size as the CSM contribution via $\gamma^\star$ (light blue band). Note that the light-blue band actually also contains other electroweak contributions appearing at the same order, i.e. via $Z^\star$, but the yield is strongly dominated by processes via $\gamma^\star$. At LHC energies, the three contributions are of the same order. The total CSM cross section is thus about twice as large as the COM one, probably a bit more at 14 TeV and at large $P_T$ (see Fig. 2c).

Such results clearly demonstrate that, contrary to earlier claims in the literature [18, 20], the yield for the production of $J/\psi$ in association with a $W$ boson cannot actually be used as a clean probe of the COM. This remains true over the whole range in $P_T$. The Born CSM contributions considered here are indeed leading $P_T$ at variance to the inclusive case where leading-$P_T$ contributions [6, 9] only appear at higher orders in $\alpha_s$.

In addition to the $P_T$ dependence, we present in Fig. 3 our CSM results for the differential cross sections in $y$ for

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3We note once again that our COM results are compatible with those of [19] and the LO of [20] once the differences in the choices of the scales, of the LDME and of the kinematical cuts are taken into account.
the LHC at 8 TeV (a) and 14 TeV (b). One observes that the CSM yields via $\gamma^*$ and via $sg$ fusion are of the same order at the LHC energies, with an increasing proportion of $sg$ fusion as the energy increases.

4. Singlet contributions via an off-shell photon vs. octet contributions via an off-shell gluon in processes involving quarks

As we have seen above, the contributions from the CSM via an off-shell photon and from the COM via an off-shell gluon –namely via a $^3S_1^{[8]}$ state– are similar, with nearly exactly the same $P_T$ dependence. We have found it instructive to investigate this.

To this end, we have evaluated the cross section for a $q\bar{q}$ annihilation into a $^3S_1$ quarkonium in both channels. Apart from the radiation of the $W$, this is the same process as discussed above.

The partonic cross section for the singlet contribution via an off-shell photon, $q(p_1)\bar{q}(p_2) \rightarrow \gamma^* \rightarrow Q(p_Q)$, is:

$$\hat{\sigma}^{[1]}_{\gamma^*} = \frac{4\pi\alpha_s^2 e_Q^2 e_q^2}{M_Q^2 s} \delta\left(x_1 x_2 - M_Q^2/s\right) |R(0)|^2,$$  \hspace{1cm} (2)

with $\delta = (p_1 + p_2)^2 = x_1 x_2$, $e_Q$ the heavy quark charge and $e_q$ the light quark charge.

For the octet contribution via a $^3S_1^{[8]}$ state, one can follow Petrelli et al. [31] and obtain

$$\hat{\sigma}^{[8]}_{\gamma^*} = \frac{4\pi\alpha_s^2 \rho^2}{27 M_Q^2 s} \delta\left(x_1 x_2 - M_Q^2/s\right) \langle Q_0(3^1S_1^{[8]}) \rangle.$$  \hspace{1cm} (3)

where $\langle Q_0(3^1S_1^{[8]}) \rangle$ is to be fitted to reproduce the $P_T$ spectrum of the data at the Tevatron and at the LHC and, in principle also, the yield polarisation. In the singlet case, one can connect the wave function at the origin to a similar matrix element: $\langle Q_0(3^1S_1^{[1]}) \rangle = 2N_c(2J+1)|\langle 0|\pi |0\rangle|^2/4\pi$. 

Figure 2: Differential cross section at LO for $J/\psi + W$ vs. $P_T$ for the Tevatron (a) and the LHC at 8 TeV (b) and 14 TeV (c). The orange band is for the COM while the light blue, dark green and blue bands are for the CSM via $\gamma^*$, via $sg$ fusion and total contributions, respectively.

Figure 3: Differential cross section at LO for $J/\psi + W$ vs. $y$ for the LHC at 8 TeV (a) and 14 TeV (b). The colour code is the same as in figure 2. Note that these results are obtained without cut on the $J/\psi P_T$. 


One can now make the ratio of the singlet to octet contribution:

$$\frac{\hat{\sigma}^{[1]}_{\text{vis} \gamma^*}}{\hat{\sigma}^{[8]}_{\text{vis} \gamma^*}} = \frac{6\alpha^2 e_q^2 e_\gamma^2 (O_2(3S_1^{[1]}))}{\alpha^2 (O_2(3S_1^{[8]}))}.$$  

(4)

The difference in the colour structure gives the relative factor $2\alpha$ between the octet and the singlet contributions. In the $J/\psi$ case, we have $(O_2(3S_1^{[1]})) = 1.45 \text{ GeV}^3$, $(O_2(3S_1^{[8]})) = 2.2 \times 10^{-3} \text{ GeV}^3$ as we used above and $\alpha_S(M_{J/\psi}) = 0.26$. The ratio is then about two thirds for $u\bar{u}$ fusion. In the case of $\Upsilon$ production $(O_2(3S_1^{[1]})) \approx 10 \text{ GeV}^3$, $(O_2(3S_1^{[8]})) = 0.4 \pm 3 \times 10^{-2} \text{ GeV}^3$ at LO [32] and $\alpha_S(M_{\Upsilon}) = 0.16$, the ratio is similar to that of $J/\psi$.

Along the same lines, if the quark line emits a $W$ boson, one expects the same ratio up to factors involving the quark electric charges $\alpha$.

4In fact, the ratio is expected to become more favourable to the CSM contributions by a factor of 5 for the $J/\psi$ and 2 for the $\Upsilon$, since the natural scale of the process would then be $m_W$ rather than $m_Q$; the strong coupling would then be smaller and the electroweak one larger.

An unexpected consequence of this in the present study is that the (rare) 3-body decay $^3 W \rightarrow J/\psi + \mu + \nu_\mu$ cannot be disentangled from genuine $J/\psi + W \rightarrow J/\psi + \mu + \nu_\mu$ events. In fact, its contribution is not negligible with the typical cuts used at the LHC. We have indeed found that with the cuts used by ATLAS [34] ($E_T^{\text{miss}} > 20 \text{ GeV}, P_T^\nu > 25 \text{ GeV}, |\eta^\nu| < 2.4$, $m_W^4 = \sqrt{2P_T^\nu E_T^{\text{miss}}[1 - \cos(\phi^\nu - \phi^\gamma)] > 40 \text{ GeV}}$, the process $q\bar{q}^\gamma \rightarrow W \rightarrow J/\psi + \mu + \nu_\mu$ contributes nearly equally to that of $q\bar{q}^\gamma \rightarrow J/\psi + W \rightarrow J/\psi + \mu + \nu_\mu$, where $\text{Br}(W \rightarrow \mu + \nu_\mu) = 11\%$.

5.2. Double-parton-scattering contributions

Second, ATLAS has evaluated [34], using the pocket formula $\sigma_{\text{DPS}}^{J/\psi+W} = \sigma_{J/\psi}^W \sigma_{W}^W / \sigma_{\text{eff}}$, that a significant Double-Parton-Scattering (DPS) contribution – as high as 40% – is to be expected provided that this formula makes sense and that one can use the effective cross section $\sigma_{\text{eff}}$ as extracted from the $W + 2$ jets analysis [35].

In principle, the DPS signal is reducible since the $J/\psi$ and the $W$ should completely be uncorrelated in $\phi$ and $P_T$. In practice, since one expects only a handful of events per fb$^{-1}$, it will be very complicated to subtract with a good accuracy the DPS signal by looking at the $\Delta\phi$ or $\Delta P_T$ distributions.

5.3. $\chi_c$ feed-down

Third, as for most quarkonium-production observable, feed-down from excited-quarkonium states can be important and proceeds from different partonic reactions. We have indeed computed that the cross section for $\chi_c + W$ times the branching $\chi_c \rightarrow J/\psi + \gamma$ is about 6 times larger than the direct cross section for $J/\psi + W$. In short, the feed-down from $\chi_c$ is expected to be larger than in the inclusive case and cannot be disregarded. This simply comes from the possibility of a fragmentation contribution at $\alpha_S^2$.

Summing the direct contribution to the feed-down from $\chi_c$ and $\psi(2S)$, we find a total cross-section of $\sigma(\gamma|< 2.4) = 4.5 \pm 2.3 \text{ fb}$ at 7 TeV, comparable to the cross-section $\sigma(\gamma|< 2.4) = 15 \pm 10 \text{ fb}$ for DPS-subtracted prompt $J/\psi + W$ recently obtained by the ATLAS collaboration [34].

6. Conclusions

We have shown that the LO CSM contributions to direct $J/\psi + W^\pm$ are not negligible compared to the contribution arising from CO transitions which were previously thought to be dominant. These CSM contributions arise from two sub-processes: a) the fusion of a gluon and a strange quark which turns into a charm quark by the emission of the $W$, the charm quark subsequently fragments into a $J/\psi + c$ pair; b) the annihilation of a quark $q$ and an antiquark $\bar{q}$ into an off-shell photon, $\gamma^\ast$, and a $W$, the $\gamma^\ast$ subsequently fluctuates into a $J/\psi$. The former process appears at $\alpha_S^2$ and the

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[1] A similar decay channel of the $W$, $W \rightarrow \Upsilon + \mu + \nu_\mu$ has previously been considered in [33].
latter at $\alpha^2$ compared to $\alpha_3^2$ for the COM process which is however suppressed in the $v$ expansion of NRQCD.

We have also noted that, for any $^3S_1$ quarkonium-production process involving quark–antiquark annihilation, the CSM process via an off-shell photon numerically competes with the COM one via an off-shell gluon through a $^3S_1$ octet.

Finally, owing to the uncertainties on the CO LDME, the small rate for this process at the LHC and the possibility for large DPS contributions, our conclusion is that the study of direct $J/\psi + W$ yields cannot serve as a clean probe of the colour-octet mechanism, as previously stated in the literature.

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