Research on rolling element bearing fault diagnosis based on genetic algorithm matching pursuit

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Abstract. In order to solve the problem of slow computation speed, matching pursuit algorithm is applied to rolling bearing fault diagnosis, and the improvement are conducted from two aspects that are the construction of dictionary and the way to search for atoms. To be specific, Gabor function which can reflect time-frequency localization characteristic well is used to construct the dictionary, and the genetic algorithm to improve the searching speed. A time-frequency analysis method based on genetic algorithm matching pursuit (GAMP) algorithm is proposed. The way to set property parameters for the improvement of the decomposition results is studied. Simulation and experimental results illustrate that the weak fault feature of rolling bearing can be extracted effectively by this proposed method, at the same time, the computation speed increases obviously.

1. Introduction
Rolling element bearings are one of the most extensively used elements in rotary machine. They also play a key role and are easily damaged components in the whole system. It may cause serious economic loss and safety accident when failures occur in rolling bearing. In order to ensure the safety of the machine and reduce the cost of production, it is meaningful to do research on rolling bearing fault diagnosis.

The most common rolling bearing fault detecting method is vibration monitoring method. Vibration signal generated by normal rolling bearing is constructed by the low frequency components which are caused by the rotation of shaft, the changing of the stiffness and the variation of loading. Once local fault occurred, the impulse components will be excited when rolling elements pass the fault position. These components are high frequency impulse components and these impulses lead to the vibration signal non-stable. However, the fault feature is weak to incipient defects and the mixed faults have a high proportion in practical engineering [1]. The key to the diagnosis of rolling bearing is to extract out the fault feature from vibration signals effectively by property methods.

Recently, a lot of meaningful researches have been done on the analysis and processing of non-stationary signal. Lots of relative theories and methods have proposed, such as, Fourier transform [2], wavelet transform [3], Wigner-Ville distribution [4], and so on. However, all the above methods have fixed basis functions which have great effect on the expression of vibration signals [5]. In order to enhance the flexibility of the expression of vibration signals and make the decomposition process self-adaptive, the idea that over-complete redundant atom dictionary can be used to obtain signal sparse decomposition results is proposed and the MP algorithm is introduced by Mallet [6]. MP algorithm
soon got widely used. MP algorithm is applied to analyzing bearing vibration signal by Liu, and the bearing faults are analyzed by time-frequency distribution [7]. Wang regards the feature signal waveform as the basis atom of dictionary, and successfully applied to the pattern recognition of bearing fault [8]. The MP algorithm is used in the complexity analysis of vibration signal by Tang, realized the feature extraction of rolling bearing [9]. However, because the dictionary is very enormous, the vast majority of computation time is spent on the search of optimum atoms, so the decomposition speed is very slow. Therefore, it is important to select property dictionary and improve the search method to increase the computation speed.

Because the process of searching for optimum atom is an optimization problem, so we can use genetic algorithm to solve the problem. In this paper, because the Gabor expression is constructed by the functions which can reflect the local time-frequency character well, the Gabor function is used to construct the dictionary [10]. Combined the genetic algorithm with MP algorithm, the decomposition speed increased obviously, a time-frequency analysis method based on GAMP is proposed. The effectiveness and practicable of this method is verified by simulation and experimental.

2. The basic principle and algorithm fundamental of Gabor dictionary

According to the signal processing theory, signal $f(t)$ can be represented as some basis functions [11]:

$$f(t) = \sum_{n=0}^{M-1} a_{\gamma n} g_{\gamma n}(t)$$  \hspace{1cm} (1)

where $M$ is the order of decomposition, $a_{\gamma n}$ are the Gabor coefficients, and $g_{\gamma n}(t)$ are the Gabor atoms, also called basis functions. A Gabor atom consists of a cosine-modulated Gaussian window function:

$$g_{\gamma n}(t) = \frac{1}{\sqrt{s}} g(t-u) \cos(\nu t + \omega)$$ \hspace{1cm} (2)

where $g(t) = e^{-\pi t^2}$ is the Gaussian window function and $\gamma = (s, u, \nu, \omega)$ are the time-frequency parameters. The function $g_{\gamma}(t)$ is centered at $u$ and its energy is mostly concentrated in a neighborhood of $u$ whose size is proportional to $s$. The space of time-frequency parameters can be discretized as $\gamma = (a', pa', ku', kv', k\nu, k\omega)$, with $a = 2$, $\Delta u = 1/2$, $\Delta \nu = \pi$, $\Delta \omega = \pi/6$, $0 < j \leq \log_2 N$, $0 \leq p < N 2^{-j+1}$, $0 \leq k < 2^{j+1}$, $0 \leq i \leq 12$ to form the so called Gabor dictionary [10]. Here $N$ is the number of samples.

Fundamental of MP algorithm

MP algorithm is a greedy algorithm that decomposes the signal according to atom dictionary. It spends the majority of time searching for optimum atom. The signal component which matches the optimum atom well is extracted from original signal when the optimum atom is selected. This procedure is repeated each time on the following signal component that is obtained. The procedure will not stop until the energy of the residual signal is less than given threshold value.

$D = \{ g_{\gamma}, \gamma = 1, 2, K, K \}$ is the dictionary. Let $H = \mathbb{R}^N$, for $K \leq N$, be a Hilbert space. The atoms of the dictionary are the unit vector of a Hilbert space. In order to select an optimum atom from $D$, the original signal $f$ is projected on a vector $g_{\gamma 0} \in D$. It must satisfy the following condition:

$$\left| \langle f, g_{\gamma 0} \rangle \right| = \sup | \langle f, g_{\gamma} \rangle |$$ \hspace{1cm} (3)

The original signal $f$ is decomposed into
\[ f = \langle f, g_{\gamma_0} \rangle g_{\gamma_0} + R'f \]  

(4)

where \( R'f \) is the residual vector after approximating \( f \) in the direction of \( g_{\gamma_0} \). Clearly \( g_{\gamma_0} \) is orthogonal to \( R'f \), hence

\[ \|f\|^2 = \left| \langle g_{\gamma_0}, f \rangle \right|^2 + \|R'f\|^2 \]  

(5)

To minimize \( \|Rf\| \), we must choose \( g_{\gamma_0} \in \mathbf{H} \) so that \( \langle f, g_{\gamma_0} \rangle \) is maximum. In infinite dimensional space, it is only possible to find a vector \( g_{\gamma_0} \) that is almost the best in the sense that

\[ \langle f, g_{\gamma_0} \rangle \geq \alpha \sup \left\langle f, g_{\gamma} \right\rangle \]  

(6)

where \( \alpha \) is an optimality factor that satisfies \( 0 < \alpha \leq 1 \). In finite dimensional space, \( \left\langle f, g_{\gamma} \right\rangle \) has maximum. In this case, we usually choose \( \alpha = 1 \).

MP algorithm is an iterative algorithm that decomposition the \( Rf \) by projecting it on a vector of \( D \) that matches \( R'f \) almost at best, as it was done for \( f \). After compute \( n+1 \)th iteration, the result is obtained as follow:

\[ R^n f = \left\langle R^n f, g_{\gamma_0} \right\rangle + R^{n+1} f \]  

(7)

Because \( R^n f \) is orthogonal to \( g_{\gamma_0} \), hence

\[ \left| \left\langle R^n f, g_{\gamma_0} \right\rangle \right| = \sup \left| \left\langle R^n f, g_{\gamma} \right\rangle \right| \]  

(8)

Let us carry this decomposition up to the order \( M \). \( f \) is decomposed into the concatenated sum

\[ f = \sum_{n=0}^{M-1} \left\langle R^n f, g_{\gamma_0} \right\rangle g_{\gamma_0} + R^M f \]  

(9)

Similarly, energy \( \|f\|^2 \) is decomposed into a concatenated sum

\[ \|f\|^2 = \sum_{n=0}^{M-1} \left| \left\langle R^n f, g_{\gamma_0} \right\rangle \right|^2 + \|R^M f\|^2 \]  

(10)

The \( M \) th estimate of vector \( Rf \) is obtained by the above equation, and the error is \( R^M f \).

- Time-frequency analysis method based on GAMP algorithm

The genetic algorithm is an iterated self-adaptive probability search method which is based on natural selection and natural genetic principle. It achieves the optimal goal according to Darwin’s theory of evolution and Mendel’s theory of gene. The process of searching for optimum atom when using MP algorithm is an optimized problem. Therefore, we can combine genetic algorithm with MP algorithm.

The steps of the time-frequency analysis method based on GAMP are similar to MP algorithm. The absolute value of the inner product of atom and signal is regarded as the fitness function of genetic algorithm, and the parameters of the atom is regarded as the chromosome. Thereby, the optimum atom is found out from dictionary by genetic algorithm. The process is shown in figure 1.
Figure 1. The process of the time-frequency analysis method based on GAMP.

Genetic algorithm possesses favorable whole situation optimum performance. The translation parameter, scale parameter and modulating parameter are the parameters waiting for optimization. The fitness function is established by the inner product of atom and signal. Therefore, the model of the time-frequency analysis method based on GAMP is established.

- **Time-frequency energy distribution**

Wigner distribution possesses well time-frequency resolution. However, it exists cross terms when applied to complicated signal. It is hard to reflect the time-frequency character well.

\[
Wf(t, \omega) = \sum_{n=0}^{\infty} \left( \mathbf{R}^{n} f, g_{yn} \right)^{2} W_{g_{yn}}(t, \omega) + \sum_{n=0}^{\infty} \sum_{m=0, m \neq n} \left( \mathbf{R}^{n} f, g_{yn} \right) \mathbf{g} \left( \mathbf{R}^{n} f, g_{ym} \right) W[g_{yn}, g_{ym}](t, \omega)
\] (11)

The second term in equation (11) is the cross term. We can separately get the Wigner distribution of the optimum atoms, then we can overlap the result. In this way, the effect of cross terms is eliminated, and the energy distribution with well time-frequency resolution is obtained.

3. **Simulation studies**

In order to validate the correctness and validity of the rolling bearing fault signal extraction method based on GAMP algorithm, according to the fact that inner race prone to failure, the simulation signal of rolling bearing surface fault was studied and analyzed [12].

3.1. **Simulated signal**

The vibration signal model [13] for rolling bearing inner ring single damage is shown in equation (12).

\[
x(t) = \sum_{i=1}^{N} A_{i} s(t - iT - \tau_{i}) + B(t) + n(t)
\]

\[
A_{i} = A_{0} \cos(2\pi f_{r} t + \phi_{A}) + C_{A}
\]

\[
B(t) = B_{0} \cos(2\pi f_{m} t + \phi_{B})
\]

\[
s(t) = e^{-\xi t} \sin(2\pi f_{n} t + \phi_{n})
\]

(12)

Where \( A_{i} \) is the amplitude modulation. \( f_{r} \) is rotation frequency of the shaft. \( B(t) \) is the background
harmonic component. $f_m$ is the frequency of background signal. $s(t)$ is exponential decay pulse. $T$ is the interval of two adjacent shock. $\tau_i$ is cycle delay of the $i$th impulse due to slip. $n(t)$ is white noise. $A_0, B_0, C_A, A > A_0$. $R$ is the attenuation coefficient determined by the system. $f_a$ is the natural frequency of the system.

3.2. Verification and analysis

The sampling frequency is 16384 Hz, $f_r$ is 52 Hz, fault frequency is 180 Hz, and the duration of the signal is 62.5 ms. In early stage of bearing failures, the feature information is usually buried by strong background noise coming from gear meshing vibration, mass unbalance, misalignment, etc. The hybrid signal waveform obtained from equation (12) is shown in figure 2(a), the time-domain waveform contains significant harmonic components, with the influence of noise, the fault impact cannot be seen clearly. The time-frequency distribution of the hybrid signal is shown in figure 2(b), the energy distribution of each frequency component can’t be distinguished obviously.

Figure 2. Simulation signal: (a) time-domain waveform, (b) Wigner distribution.

Figure 3. Comparison of the decomposition speed.

The GAMP algorithm and MP algorithm are applied to vibration signal separately. We find that both of them possess similar reconstruction property, but there is a big difference in decomposition speed. The comparison of two methods in decomposition speed is shown in figure 3. The value of vertical axis means the time lag of two methods. It is obvious that the calculating speed of GAMP is many times quicker than MP method, and the advantage of GAMP method reflects more and more
obviously when it regards to the longer time signal.

The GAMP algorithm is applied to simulation signal. Setting the parameters according to table 1, then the time-frequency distribution by decomposing 60 times is obtained as shown in figure 4(a). There exists some small-scale atoms, which are regularly spaced in frequency above 3 KHz, and the time interval between the atoms equals to the inner fault period of rolling bearing (about 5.6 ms). These are the impact components of simulation signal. In the extracted signal, the fault impact and amplitude modulation can be seen obviously. According to the above results, the inner fault can be judged easily. Because the resonance components of vibration signal usually appear in high frequency with short time period, we can improve the ability to detect the fault feature by the small scale atoms of high frequency components. According to the fact that the frequency of small scale atoms is greater than 3 KHz, we can set the scale parameter \( j = 4 \) to utilize the high frequency atoms. The extracted signal is shown in figure 4(b). In the extracted signal, the fault impact related to the bearing fault can be seen clearly, and the time interval between the adjoin impact equals to inner fault period of bearing (about 5.6 ms). At the same time, the amplitude modulation can be seen obviously. According to the above analysis, the rolling bearing fault feature can be extracted effectively by GAMP method when the parameters are set properly.

| Table 1. The related parameters of genetic algorithm. |
|------------------------------------------|
| Population size | Maximum generation | Hybrid method | Hybrid probability | Mutation probability |
|-----------------|---------------------|---------------|-------------------|---------------------|
| 80              | 200                 | Single hybridization | 0.9               | 0.1                 |

![Figure 4. Extracted signal: (a) time-frequency distribution, (b) time-domain waveform.](image)

4. Experimental studies

4.1. Experimental apparatus

The rolling bearing vibration signals are measured from rolling bearing fault simulation platform in laboratory. The type of rolling bearing is 6010. The structure parameters are as follows: the bearing pitch diameter is \( D = 65 \) mm, the rolling element diameter is \( d = 9 \) mm, the number of rolling element is \( Z = 13 \), the contact angle is \( \alpha = 0^\circ \). The rolling bearing faults include inner fault, outer fault, rolling element fault and cage fault. However, the inner fault is the most hard to be detected, and we analyze the vibration signal with inner fault.

The rev of the shaft is 2947 \( r/min \). The sampling frequency is 65536 Hz. According to the geometric parameters of bearing and the relationship between character frequency and rotation frequency, we can obtain the inner fault frequency, which is 363 Hz.
4.2. Verification and analysis
The time-domain waveform of rolling bearing vibration signal is shown in figure 5(a). Due to the effect of attenuation and machine’s vibration, it is hard to resolve the fault period impact. The time-frequency distribution is shown in figure 5(b). We cannot resolve the energy distribution at each frequency component of measured signal.

![Figure 5. Measured signal: (a) time-domain waveform, (b) Wigner distribution.](image)

The GAMP algorithm is applied to the measured signal. The time-frequency distribution by decompose 60 times is obtained as shown in figure 6(a). There are some small scale atoms with regularly and same spaced in time between 4 KHz and 7 KHz, and the time interval between the atoms equals to the inner fault period of rolling bearing (about 5.6 ms). These are the impact components of measured signal. In the extracted signal, the fault impact and amplitude modulation can be seen obviously. According to the above results, the inner fault can be judged easily. We can set the scale parameter $j = 5$ to utilize the high frequency atoms. The extracted signal is shown in figure 6(b). In the extracted signal, the fault impact related to the bearing fault can be seen clearly, and the time interval between the adjoin impact equals to inner fault period of bearing (about 2.75 ms). At the same time, the amplitude modulation can be seen obviously. According to the above analysis, the rolling bearing fault feature can be extracted effectively by GAMP method when the parameters are set properly.

![Figure 6. Reconstructed signal: (a) time-frequency distribution, (b) time-domain waveform.](image)

5. Conclusions
In this paper, based on the structure principle of Gabor dictionary and the principle of MP algorithm,
searching the optimum atoms from redundancy dictionary is the reason why the decomposition speed is slowly pointed out, at the same time, the searching process which is an optimization process is illustrated. Then the principle why GAMP algorithm can improve the calculate speed is analyzed. According to time-frequency energy distribution, setting the decomposition parameters, the fault feature can be extracted out effectively. Simulation and experimental results illustrate that the weak fault signal of rolling bearing can be extracted effectively by this proposed method, at the same time, the computation speed increases obviously.

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