The Hysteretic Behaviour of Partial Slip Elastic Contacts Undergoing a Fretting Loop

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Abstract. Fretting defines an intrinsically multidisciplinary process, involving adhesion, oxidation, abrasion and pitting, in which mechanical contacts are subjected to alternating tangential displacements, small compared to dimensions of contact area. Assessment of hysteretic behaviour of stress and strain state in the contacting bodies is the main goal of this paper, which employs numerical analysis to extend the few existing analytical results to technologically important contact scenarios. An incremental iterative algorithm based on three levels of iterations, fully incorporating the interconnectivity between normal and tangential tractions, is reviewed in this work and used to simulate the hysteretic behaviour of partial slip elastic contacts undergoing fretting. Numerical simulations suggest that the fretting contact between dissimilarly elastic materials exhibits a unique path in the first two loading cycles, and then stabilizes with subsequent oscillating loading to a fixed trajectory, as in case of similarly elastic materials.

1. Introduction
Many industrial mechanisms involve mechanical contacts undergoing oscillating tangential displacements, small compared to dimensions of contact area. This type of loading conditions defines a fretting contact process. Related damages, i.e. fretting wear and fatigue, lead to a dramatic decrease of service lifetime of contacting machine elements. From a mechanical point of view, the problem is treated in the frame of Contact Mechanics, incorporating the Linear Theory of Elasticity in description of contacting bodies’ responses to loading. Achievement of a closed-form description of the fretting contact between dissimilarly elastic materials is mainly impeded by the coupling between normal and tangential effects, the latter related to the capacity of the contact interface to sustain friction leading to development of shear tractions. A numerical study is therefore expected to advance the understanding of fretting phenomena and to provide assistance to the design of contacts with improved load-carrying capacity.

The method employed in this paper, also referred to as semi-analytical (SAM) [1], is based on superposition of fundamental solutions (i.e. Green functions) for the elastic half-space, applied within an iterative contact solver. The strong point of this approach is that domain discretization is limited to a surface region of the half-space boundary, imposed by trial-and-error as to enclose the actual contact area, as opposed to finite element analysis (FEA), in which a volume mesh of the entire bulk is required. It is asserted in [1] that SAM can be employed to solve a three-dimensional contact scenario with the computational complexity needed for FEA to simulate a two-dimensional contact process.
Study of gross-slip fretting contact using SAM originated in the works of Gallego, Nélia and Jacq [2], who solved repeatedly the contact model in the normal direction while accounting for the change in conformity due to wear. Chen and Wang [3] advanced a numerical model for the partial slip non-conforming contact considering tangential tractions, and predicted the effect of a tangential force, i.e. fretting mode I, in the contact between dissimilarly elastic materials. However, the loading history, related to the irreversibility of friction, was not accounted for. Further extensions addressed the problems of a supplementary torsional moment [4], i.e. fretting mode III, as well as the heterogeneity of elastic layered half-spaces [5]. Gallego, Nélia and Deyber [6] firstly implemented the incremental load application in simulation of the fretting contact, but the coarse temporal discretization impeded achievement of well-converged solutions, as proven by the jagged shape of the shear tractions profiles. Consequently, the study of hysteretic behaviour of fretting contacts was not properly conducted.

The main goal of this paper is to assess numerically the memory effect in the fretting contact between dissimilarly elastic materials. To this end, existing fast-converging algorithms [7,8] for the steady-state slip-stick contact, based on the Conjugate Gradient Method (CGM), are adapted to allow for transient conditions, and combined in an incremental iterative process, aiming to accurately reproduce the loading history in the contact process. The use of state-of-the-art numerical tools for convolution computation in the frequency domain, as the Discrete Convolution Fast Fourier Transform (DCFFT) technique [9], allows for simulations to be conducted on a fine spatial and temporal mesh, leading to well-converged numerical solutions.

2. Numerical modelling of fretting contact

The set of equations and inequalities describing the elastic contact are overviewed in this section, based on the works developed in [3,6,7,10]. The numerical formulation assumes all problem parameters are piece-wise constant on a \( N_1 \times N_2 \) uniformly spaced rectangular grid, also referred to as the computational domain \( A_p \), established in the common plane of contact. The set \( A_p \) is expected to enclose the contact area \( A_C \) at any point on the loading curve. Problem spatial discretization is performed along directions \( \bar{x}_1 \) and \( \bar{x}_2 \) of a Cartesian coordinate system having its axes aligned with the grid sides. In this manner, analytical integration of arbitrary functions over arbitrary domains is circumvented and a numerical estimate is achieved through DCFFT assisted multi-summation.

Although a purely elastic analysis is intended in this paper, due to the irreversibility of friction, the state achieved at any point on the loading curve is expected to depend on all previous states. Consequently, discretization of the loading path, i.e. in the temporal domain, is required as well, implying that load should be applied in small increments, and the contribution of each increment should be superimposed to previous levels in order to acquire a good assessment of the current state. However, no rate-dependence is considered in this model, meaning no impact study is conducted.

In the numerical formulation, it is convenient to denote all contact parameters as functions of integers indexing the grid cells, \( i=1,N_1 \), \( j=1,N_2 \), as well as of the number of imposed loading steps, \( k=1,N_3 \), e.g. \( p^{(k)}(i,j) \) denotes the nodal pressure at the intersection of the line \( i \) with the column \( j \) of the rectangular grid, achieved after application of \( k \) loading increments. With this notation convention, the well-known equations governing the elastic contact in the normal direction yield:

\[
W^{(k)} = \Delta \sum_{(i,j) \in A_p} p^{(k)}(i,j) ;
\]

\[
h^{(k)}(i,j) = h(i,j) + u_x(i,j,k) - \omega^{(k)}(i,j), \quad (i,j) \in A_p ;
\]

\[
p^{(k)}(i,j) > 0 \quad \text{and} \quad h^{(k)}(i,j) = 0, \quad (i,j) \in A_C^{(k)} ;
\]

\[
p^{(k)}(i,j) = 0 \quad \text{and} \quad h^{(k)}(i,j) > 0, \quad (i,j) \in A_p - A_C^{(k)} .
\]
Equation (1) expresses the static force equilibrium, relating the normal force $W$ to the induced pressure distribution $p$. Digitization of geometrical condition of deformation yields the interference equation (2), in which the gap between deformed surfaces $h$ is expressed in terms of initial gap $(\text{or contact geometry}) \; h_i$, of surface normal displacement $u_i$, and of rigid-body translation $\omega_i$. The contact or non-contact status of every cell in the $A_p$ is assessed by the complementarity contact conditions in equations (3) and (4). An analogous model can be established for the slip-stick elastic contact in the tangential direction, provided a torsional moment is not present:

$$T^{(k)} = \Delta \sum_{(i,j) \subset A^{(k)}} q^{(k)}(i,j), \; n = 1,2;$$

$$\begin{bmatrix} s_1^{(k)}(i,j) - s_1^{(k-1)}(i,j) \\ s_2^{(k)}(i,j) - s_2^{(k-1)}(i,j) \end{bmatrix} = \begin{bmatrix} u_1^{(k)}(i,j) - u_1^{(k-1)}(i,j) \\ u_2^{(k)}(i,j) - u_2^{(k-1)}(i,j) \end{bmatrix} - \begin{bmatrix} \omega_1^{(k)} - \omega_1^{(k-1)} \\ \omega_2^{(k)} - \omega_2^{(k-1)} \end{bmatrix}, \; (i,j) \in A^{(k)};$$

$$\begin{cases} \|q^{(k)}(i,j)\| \leq \mu p^{(k)}(i,j) & \text{and} \; \|s^{(k)}(i,j) - s^{(k-1)}(i,j)\| = 0, \; (i,j) \in A^{(k)} \\ \|s^{(k)}(i,j) - s^{(k-1)}(i,j)\| > 0, \; (i,j) \in A^{(k)} - A^{(k-1)} \end{cases}$$

where $T(T_1,T_2)$ is the tangential force, $q(q_1,q_2)$ the vectors of shear tractions, $s(s_1,s_2)$ the relative slip distances, $\mu$ the frictional coefficient, and $A^s$ the stick area. The torsional moment, which adds a supplementary unknown, namely the rotation angle along direction of $\hat{x}_2$, but also an additional moment equation in the static equilibrium, should always be considered in simulation of fretting mode III. However, this analysis is restricted for brevity to fretting modes I and II. The model (5) – (8) also assumes the tangential force is not large enough to induce gross-slip, which is guaranteed at any point on the loading curve if $T^{(k)}$ is smaller than the instantaneous limiting friction force $\mu W^{(k)}$. On the other hand, both Cattaneo [11] and Mindlin [12] proved independently that, even when the contacting bodies are globally sticking, a peripheral region of slip is required to relinquish the infinite shear tractions at the boundary of the contact area, or else the no-slip contact scenario is not compatible with either Linear Theory of Elasticity or Coulomb’s law of friction.

Providing a subsequent tangential load increment is applied in a slip-stick contact, the stick or slip status of specific points on the contact area is expected to change, as predicted by the complementarity conditions in equations (7) and (8). Points included in the stick zone conserve their status if and only if no relative motion occurs between matching particles on the two contacting surfaces, i.e. the increment of relative slip distance in the considered timeframe is nil. Points for which this condition is not met pass into the slip zone. As pointed out by Hills, Newell, and Sackfield [13], the existence of slip is intrinsically conditioned by an increase or decrease in the level of tangential load, and therefore a purely static model is not appropriate. Moreover, when the tangential load increment changes sign, the rate of change of shear tractions and of relative tangential displacements also changes sign. As shear tractions no longer oppose relative motion between the contacting bodies, instantaneous stick occurs, and further loading leads to partial slip in the reversed direction. Subsequent state can be computed starting from a fully-stuck one, i.e. with vanishing relative slip distances, but with an initial shear tractions distribution. The model advanced herein is consistent with this formulation, provided the point on the loading curve when the tangential load trend is reversed is accurately reproduced.

Considering the similarity between the set of equations (1) – (4) and (5) – (8), the same type of algorithm could be used to solve either model considered independently. The main difficulty consists in the system being intrinsically non-linear due to presence of rigid-body translations. Also, its size is not known in advance, as neither contact nor stick areas are known a priori. In order to use the CGM, which is proven to converge only for systems with symmetric and positive definite matrix, the static
equilibrium is imposed in a correction of the system solution outside the CGM core, as depicted in figure 1, and estimates for the rigid-body translations are computed numerically from the interference equation, thus linearizing the system. This scheme aims to preserve the influence coefficients matrix, which is intrinsically symmetric and positive definite, as the system matrix. Detailed description of the algorithms can be found in [7] for the set of equations (1) – (4), and in [8] for the set (5) – (8).

Moreover, when materials of contacting bodies are dissimilarly elastic, the two set of equations are coupled, i.e. resolution of each requires solution of the other, as elastic displacement result by superposing contributions of all contact tractions, both normal and shear. To surmount this mutual determination, an iterative approach, fully described in [14], is employed, allowing simulation of the elastic three-dimensional fretting contact between dissimilarly elastic materials with the additional cost of a supplementary outer loop.

3. Results and discussions
The spherical contact between similarly or dissimilarly elastic materials, subjected to fretting, is simulated using the proposed computer program. A ball of radius $R = 18 \text{ mm}$ is pressed against an elastic half-space following the loading path depicted in figure 2. The maximum level of $W$ is fixed in all simulations, while the maximum tangential force $T_{\text{lim}}$ is chosen in relation to the imposed frictional coefficient, i.e. $T_{\text{lim}} = 0.9 \mu W$, assuring a gross-slip regime is not reached. The load is imposed in a variable number of increments, and the effect upon solution convergence is discussed.

Figure 2. The loading path imposed in the fretting contact simulation.
When the contacting materials are assumed similarly elastic (Young modulus $E = 210 \text{ GPa}$, Poisson’s ratio $\nu = 0.3$), the Hertz frictionless theory predicts a central pressure $p_H = 1.996 \text{ GPa}$ and a contact radius $a_H = 0.489 \text{ mm}$, which are used as normalizers in figures 3-4, together with the rigid-body tangential displacement $\omega_{\text{lim}}$ corresponding to $T_{\text{lim}}$. Due to decoupling between normal and tangential effects, any state on the loading trajectory $OA$ can be obtained in one step, by running one solver iteration consisting in resolution of contact problem in the normal direction. As no shear force is explicitly applied, the friction related memory effect is not present. When an oscillating tangential force is subsequently applied (point $A$ in figure 2), shear tractions profiles on the loading path $ABCD$, corresponding to equal loading levels $T_i = 0$ and $T_i = T_{\text{lim}}/2$ but laying on different trajectories, prove that the current state depends not only on the achieved loading level, but also on the loading history, as depicted in figure 3. However, provided the states when the rate of change in the tangential load changes sign are accurately captured by a suitable choice of the loading increments, no additional temporal discretization is required. For example, any state on the loading trajectory $BC$ can be accurately predicted in only two loading steps, the first imposing the load corresponding to point $B$, and the second the currently achieved loading level.

The numerical predictions for this contact scenario allow validating the computer program against closed-form solutions reviewed by Johnson [10]. The numerical data, depicted using discrete symbols in figure 3, match well its analytical counterpart, traced as continuous line. As depicted in figure 4, the force $-\text{rigid-body displacement}$ curve exhibits a hysteretic behaviour, also referred to as a fretting loop. The closed-form description of this contact scenario predicts that the loop closes perfectly, implying states corresponding to points $B$ and $D$ on the loading curve overlap, which is also obtained through numerical simulation within an imposed precision. A unique path is thus established from the simulation of trajectory $BCD$. In simulation of this path, one can take advantage of the feature of symmetry, i.e. states on the $CD$ trajectory are a complete reversal of those on $BC$.

A different behaviour is predicted numerically when the normal and tangential effects are coupled. A dissimilarly elastic contact scenario, for which, to our best knowledge, no analytical solution has been advanced, is subsequently simulated using the advanced algorithm. To this end, the ball in the previous contact scenario is considered rigid. The Hertz frictionless framework predicts that $p_H = 3.168 \text{ GPa}$ and $a_H = 0.388 \text{ mm}$, which are used as normalizers in figures 5-7.
On the OA loading trajectory, although no tangential force is explicitly applied, divergent pressure induced tangential displacements of initially matching surface points arise due to dissimilarity in elastic responses of contacting materials. The resulting relative tangential motion is resisted by friction, leading to development of shear tractions. It should be noted that tangential displacements due to pressure also arise in the contact between similarly elastic materials, but of the same magnitude and in the same direction, thus leading to no relative motion and consequently to vanishing shear tractions.

The frictional contact process which is established in case of dissimilar contacting materials is found to be very sensitive to the temporal discretization. As also stated in [6], large loading increments predict jagged shear tractions profiles, due to important contractions / dilatations of the stick zone following application of subsequent loading increments. In this simulation, the loading process was divided into 900 increments (200 load steps for the OA trajectory, and 700 for remaining of the loading path in figure 3), leading to well-converged solutions.

To further prove the need for incremental load application in simulation of frictional contact processes when dissimilarly elastic materials are involved, a closed-form relation derived by Spence [15] based on the condition that shear stresses should be continuous over the boundary between the slip and the stick regions is used as benchmark:

$$\frac{\mu}{\beta} = \ln \frac{1 + \bar{a}_s}{1 - \bar{a}_s} - \frac{2\bar{a}_s K}{\sqrt{1 - \bar{a}_s^2}}$$

(9)

where $a_s$ is the stick radius, $\bar{a}_s = a_s/a_H$, $K$ the complete elliptic integral of the first kind, and $\beta$ the Dundurs constant [16], assessing the dissimilarity in elastic properties of contacting materials ($\beta = 0.286$ in the considered contact scenario). The stick radii predicted numerically with or without simulation of the loading history (i.e. when load is applied incrementally or in one step, respectively) are compared in figure 5 to the analytical curve (9). Although Spence’s result is derived under the Goodman approximation [17], the agreement is considered satisfactory giving the model complexity. The gap to the analytical curve when $\bar{a}_s \rightarrow 1$ proves the inability of the numerical approach to handle asymptotic behaviours. Indeed, in the discretized formulation, the stick region can only vary in increments equal to the area $\Delta$ of the elementary patch of the imposed grid. Consequently, the finer the grid, the smaller the slip zone that can be predicted, but also the greater the computational effort required per iteration. When the never vanishing slip region decreases toward zero, extents of the stick area may therefore be underestimated in the numerical solution due to inherent discretization errors. However, predictions obtained without properly simulating the loading history may be unreliable, as depicted in figure 5, and should be considered with great care for designing purposes.

Figure 5. Comparison of existing closed-form expression with numerical predictions obtained with or without proper simulation of loading history in the indentation (fretting mode II) involving dissimilarly elastic materials.
The load-displacement curves for various frictional coefficients depicted in figure 6 predict that the fretting loop does not close perfectly, as in case of similarly elastic materials, but a stabilized regime emerges past point E on the loading curve, when the tangential force reaches the value $-T_{lim}$ for the second time. This assertion is also supported by the shear tractions profiles depicted in figure 7 for $\mu = 0.6$, although the same tendency was also predicted for all simulated frictional coefficients values ranging between 0.1 and 0.6. However, the shear tractions profiles were found to vary from one case to another, making the achievement of a global description improbable. While states corresponding to points B and D on the loading curve differ significantly, as depicted in figure 7, those matching points C and E overlap within an imposed precision. It was verified by continuing the simulation on an extended loading path that further periodical loading leads to states superimposing the loop established in the contact process between points C and E. A similar periodic stability was numerically found in [5] for the fretting contact of layered half-spaces.

Figure 6. The fretting loop during the first two loading cycles in the contact between dissimilarly elastic materials subjected to oscillating tangential displacement (fretting mode I).

Figure 7. Achievement of a periodic stability proven by matching shear tractions profiles in the first two loading cycles of the fretting contact between dissimilarly elastic materials, $\mu = 0.6$. 


4. Conclusions
A numerical solution for the three-dimensional slip-stick elastic contact between dissimilarly elastic materials is employed in this paper to predict the hysteretic behaviour of the fretting contact, related to the irreversibility of friction.

Simulation of fretting contact between similarly elastic materials allows for program validation, and proves that any state on the fretting loop, which is established as a fixed path from the first loading cycle, can be predicted with a minimum number of increments, related to the number of changes in the monotonicity of the tangential load.

Further validation is achieved in case of indentation involving dissimilarly elastic materials (fretting mode II), when shear stresses arise due to relative motion induced by the dissimilarity in elastic response of contacting materials. To obtain well-converged numerical solutions, a fine discretization in the temporal domain is critical.

When an oscillating tangential load is subsequently applied (fretting mode I), the contact of dissimilarly elastic materials exhibits a unique trajectory in the first two loading cycles, then stabilizes to a steady path, as in case of similarly elastic materials. The knowledge of this path and of corresponding contact tractions and stresses is essential in prediction of yield inception or crack nucleation in the mechanical contact, providing grounds for a better understanding of premature contact failure due to fretting wear or fretting fatigue.

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