Orientifolds of Non-Supersymmetric, 
Asymmetric Orbifolds

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\textbf{Abstract}
We consider certain four dimensional supersymmetric and non-supersymmetric asymmetric orbifolds with vanishing cosmological constant up to two loops and gauge the world sheet parity transformation. This leads to new string vacua, in which $D_p$ and $D(p-4)$ branes or $D_p$ and $\overline{D}(p-4)$ are identified. Moreover, it is shown that different degrees of supersymmetry can be realized in the bulk and on the brane. We show that for non-supersymmetric models the cosmological constant still vanishes at one loop order.

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1. Introduction

In the recent past we have seen some interesting developments in the field of non-supersymmetric string theory. Motivated by AdS-CFT duality, S. Kachru, J. Kumar and E. Silverstein (KKS) proposed a peculiar $\mathbb{Z}_2 \times \mathbb{Z}_2$ asymmetric orbifold of type IIB which breaks all supersymmetry but nevertheless features a vanishing cosmological constant both at one and two loops in string perturbation theory [1–2]. Strong-weak duality of such models suggests that such a behaviour may even persist at higher loops and non-perturbatively [3]. For a slightly different model, by using duality to heterotic strings it was shown that there exist non-zero non-perturbative contributions to the cosmological constant [4]. Generalisations of the original bosonic models to backgrounds described by free world sheet fermions were discussed in [5].

All the models studied so far contain at best abelian gauge symmetries in the tree level massless spectrum. In this paper we will pursue the question whether one can construct four dimensional non-supersymmetric string vacua with vanishing one loop cosmological constant and non-abelian gauge symmetries in the massless spectrum. It seems to be impossible to obtain perturbative heterotic models with the desired features. In a simple example, assume a $\mathbb{Z}_2$ symmetry $f$ breaks one half of the supersymmetry and another $\mathbb{Z}_2$ breaks the other half. Then, since all supersymmetry comes from the right moving sector in the heterotic string, the twisted sector $fg$ breaks already all supersymmetry, so that the argument of KKS fails. Another kind of models with non-abelian gauge symmetries are orientifolds which we will consider in this paper.

Naively, it seems to be nonsense to gauge the world sheet parity operation for an asymmetric orbifold. It seems simply not to be a symmetry. However, as also suggested in [6] one can arrange a situation where, for instance, the two asymmetric $\mathbb{Z}_2$ symmetries get exchanged by parity reversal. We will study examples of exactly this type, for which the entire discrete symmetry group is non-abelian. In the usual way tadpole cancellation requires the introduction of open strings in the theory which are allowed to end on certain D-branes. Depending on what exactly the two $\mathbb{Z}_2$ symmetries are, we are lead to introduce Dp and D(p-4) branes or Dp and D(p-4) in the theory. We consider examples with either $p = 9$ or $p = 8$. As already proposed in [7], the two types of branes are transformed into each other by the asymmetric $\mathbb{Z}_2$ actions in the open string sector. As a result one gets sort of a bound state between these two different kinds of branes.

By analysing the massless spectrum, we find that independent of the degree of supersymmetry in the bulk, one finds $N = 2$ supersymmetry in the open string sector. This is
similar to the supersymmetry breaking mechanism first introduced by Scherk and Schwarz and recently discussed in the string framework by various authors. This paper is organised as follows. In section 2 we provide some background material for the definition of the three types of models we will consider in the remaining part of the paper. In section 3 we compute all non-oriented one-loop diagrams, showing that they are still zero. Consequently one has the same number of bosonic and fermionic degrees of freedom at every mass level. In section 4 we compute the massless spectra in the open string sector and in section 5 we will end with some conclusions.

2. Asymmetric orbifolds

In this section we introduce six asymmetric orbifolds with either $N = 2$ or $N = 0$ supersymmetry in the bulk. However, let us start with some preliminaries. T-duality exchanges type II B (type II A) on a circle of radius $R$ with type II A (type II B) on a circle of radius $\alpha'/R$. If one considers an even number of circles at the self-dual radius $R = \sqrt{\alpha'}$, T-duality becomes a symmetry of the model and can be gauged. In all cases discussed in the following we will fix at least four circles at the self-dual radius. Thus we start with a compactification on the $SU(2)^4$ torus. Furthermore, from now on we set $\alpha' = 1/2$. It is known that T-duality can be understood as a left moving reflection and is therefore denoted as $(-1, 1)$. However, modding out $f = (-1, 1)^4$ alone is not possible, for it does not satisfy level matching. This can be seen by computing the partition function in the $f^1$ sector

$$f^1 \sim \left( \Theta_{0,1}(\tau) \right)^4,$$

where $\Theta_{m,k}$ denote the $SU(2)_k$ theta-functions. The $f^1$ sector is determined by a modular $S$-transformation and is proportional to $\left( \Theta_{0,1} + \Theta_{1,1} \right)^4$. Apparently this contains levels $\tilde{h} \in \mathbb{Z} + 1/4$ which violate the level matching condition. As was observed in [1] this can be repaired by equipping $f$ with further shifts. In the following we will work with the $\mathbb{Z}_2$ shifts $A_1, A_2, A_3$ introduced in [10]. Table 2 shows how the three kinds of shifts act on Kaluza-Klein (KK) and winding states $(m, n)$ and to what left-right shift they correspond to.
Note, that only $A_3$ is a left-right symmetric shift and thus can really be interpreted as an ordinary shift of the coordinate $X = X_L + X_R$. The existence of the asymmetric shift $A_1$ is a purely stringy effect and can be understood as a shift in momentum space. In particular, in the open string sector Wilson-lines get affected by these shifts. Nevertheless, the partition function obtained after modding out each one of the three shifts is left-right symmetric reflecting the transformation property $\Omega A_i \Omega^{-1} = A_i$. Under $T$-duality however, $A_1$ and $A_3$ get exchanged whereas $A_2$ is invariant, $TA_1T^{-1} = A_3$ and $TA_2T^{-1} = A_2$.

Due to this non-trivial transformation property, the discrete symmetries $f = (-1, 1; A_3)^4$ and $f = (1, -1; A_3)^4$ satisfy $f^2 = A_2^4$ and are no longer $Z_2$ transformations. In order to make them of order two, we first have to divide the $SU(2)^4$ torus by the asymmetric shift $A_2^4$. The resulting partition function is nothing else than the partition function for the $SO(8)$ torus. Now, we could follow two possible paths. First, we could realize the $SO(8)$ torus directly with some background metric $G_{ij}$ and some non-vanishing background two form $B_{ij}$. Second, we could continue to consider the $SO(8)$ torus as an orbifold of the $SU(2)^4$ torus. In the following, we will follow the second path, but keep in mind that our result has to be consistent with what one expects to get following the first path. In particular, in the open string sector it is known that the background $B_{ij}$ field reduces the rank of the gauge symmetries by a factor $2^b$, where $b = rk(B)$ [11].

The partition function in the $\widetilde{f}_1$ sector is

$$f_1\widetilde{f}_1 = f_1\widetilde{f}_1 + f_{A_2^2} A_2^2 + f_{A_2^4} A_2^4$$

$$\sim (\Theta_{0,4} - \Theta_{4,4})^4 + (\Theta_{-2,4} - \Theta_{-4,4})^4.$$  \hfill (2.2)

The second term vanishes, so that after a modular S-transformation on obtains

$$1\widetilde{f}_1 \sim (\Theta_{-3,4} + \Theta_{-1,4} + \Theta_{1,4} + \Theta_{3,4})^4,$$

$$\hfill (2.3)$$
which has conformal weight $\bar{h} = 1/4$ violating level matching. This can be repaired by compactifying on a further circle with shift $A_2$.

Having defined various symmetry action for toroidal compactifications, we can list the six models we will study in the course of this paper

|   | $f$                                      | $g$                                      | $\Omega'$ | susy |
|---|-----------------------------------------|-----------------------------------------|-----------|------|
| Ia| $(-1, 1; A_3)^4 A_2 A_3$                | $(1, -1; A_3)^4 A_2 A_3$                | $\Omega$  | N=2  |
| Ib| $(-1, 1; A_3)^4 A_2 A_3 (-1)^F$         | $(1, -1; A_3)^4 A_2 A_3 (-1)^F$        | $\Omega$  | N=0  |
| IIa| $(−1, 1; A_3)^4 A_2 A_3 (-1)^{FL}$    | $(1, -1; A_3)^4 A_2 A_3 (-1)^{FR}$     | $\Omega$  | N=2  |
| IIb| $(−1, 1; A_3)^4 A_2 A_3 (-1)^{FR}$    | $(1, -1; A_3)^4 A_2 A_3 (-1)^{FL}$     | $\Omega$  | N=0  |
| IIIa| $(−1, 1; A_3)^4 A_2 A_3 (-1)^{FL}$    | $(1, -1; A_3)^4 A_3 A_2 (-1)^{FR}$     | $\Omega P$| N=2  |
| IIIb| $(−1, 1; A_3)^4 A_2 A_3 (-1)^{FR}$    | $(1, -1; A_3)^4 A_3 A_2 (-1)^{FL}$     | $\Omega P$| N=0  |

Table 2: Definition of models

Apparently, for model III the world sheet parity reversal $\Omega$ is not a symmetry, but combining it with a permutation $P$ of the $X_5$ and $X_6$ directions one obtains a symmetry $\Omega' = \Omega P$. However, one has to be a bit more careful. For $\Omega$ to be a symmetry of the models I and II we had to start with type II B. Since the permutation $P$ changes a chiral ten dimensional spinor into an antichiral spinor, we should better start with type II A to guarantee that $\Omega P$ is indeed a symmetry.

The first four models in Table 2 are similar to those considered by J. Harvey in [4], whereas the last two ones are similar to the original one of KKS 1[1]. Before gauging $\Omega'$ all six models have 32 massless bosonic degrees of freedom and 32 massless fermionic degrees of freedom in the untwisted sector. Models I and II also contain 64 bosonic and fermionic degrees of freedom in the $R$ twisted sector. Since in model III $R = f g A_2^4$ still contains a shift in the $X_5$ and $X_6$ directions, there do not appear additional massless states in the $R$ twisted sector. Furthermore, the complete perturbative massive spectrum is also bose-fermi degenerated, leading to a vanishing cosmological constant at one loop. Now, we would like to gauge also the parity reversal $\Omega'$. Considering only the massless spectrum one finds that the degrees of freedom arising in the closed string sector are exactly halved by

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1 As in reference [4] we consider $f$ and $g$ as commuting generators in the point group with the sector $\frac{f}{g} = 0$. This implies $Z = \frac{1}{2}(Z_f + Z_g + Z_{fg} - Z_1)$ where every term preserves some supersymmetry.
this projection. However, one expects to find massless tadpoles arising in the Klein bottle amplitude, which makes it necessary to introduce an open string sector in the theory. One might argue that the open string only sees the left-right symmetric part of the orbifold and thus should live in a world with $N = 2$ supersymmetry independently of the supersymmetry in the bulk.

3. Tadpole cancellation

Computationally, the new aspect we are facing is to gauge a non-abelian discrete symmetry group containing $\Omega'$. Using the relation $\Omega' f \Omega' = g$ and that $f$ and $g$ commutes one can easily show that the two symmetries

$$\theta = \Omega' f \quad \text{and} \quad r = f$$

generate the non-abelian group $D_4$. Let us give a formal argument how the partition function looks like. Modding out a closed string by $D_4$ one knows that, since one has to sum only over commuting twists along the two fundamental cycles the partition function can be written as a sum over abelian orbifolds

$$Z = \frac{1}{2} (Z_\theta + Z_r + Z_{r\theta} - Z_{\theta^2}).$$

$Z_\theta$ is a $\mathbb{Z}_4$ orbifold with in our case elements $(1, \Omega' f, R, \Omega' g)$, $Z_r$ is a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with elements $(1, f, g, R)$, $Z_{r\theta}$ is also a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with elements $(1, R, \Omega', \Omega' R)$ and finally $Z_{\theta^2}$ is a $\mathbb{Z}_2$ orbifold with elements $(1, R)$. Here, we have formally identified sectors twisted by $\Omega'$ as open string sectors. Note, that $Z_\theta$ and $Z_{\theta^2}$ do only contain closed string sectors. Moreover, due to the shifts in $f$ one convinces oneself that the orientifold $Z_\theta$ does not lead to any massless tadpole in the tree channel Klein bottle amplitude. Therefore, there is no need to introduce open strings here. In other words, would we not had included the twists in $f$ and $g$, we would now had been in trouble with explaining what a sector twisted by $\Omega' f$ should be.

Summarising, the only open string sector appears in the abelian orientifold $\frac{1}{2} Z_{r\theta}$, which is obviously related to the kind of orientifold firstly studied by Gimon and Polchinski (GP)

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1 The non-abelian discrete symmetry group $D_4$ has eight elements and contains two generators $r$ and $\theta$ satisfying the relations $r^2 = 1$, $\theta^4 = 1$ and $r \theta = \theta^3 r$. Moreover, $D_4$ has 5 irreducible representations, four of them are one dimensional and one is two dimensional.
This simple argument shows that in the open string sector the degrees of freedom in our non-abelian orientifold model are expected to be half the number of degrees of freedom in the related GP model. This immediately implies that the one-loop cosmological constant in the open string sector also vanishes. In the following we will see, that due to the shifts in the 5-6 directions the mass levels of the 95 and 59 strings get shifted against the mass levels of the 99 and 55 open strings. But still the one-loop cosmological constant vanishes.

The GP model contains 9-branes and 5-branes which are now further related by the action of $f$. $f$ contains a T-duality transformation in four directions, thus simply exchanging the two kinds of branes. Furthermore, positions of 5-branes in the transverse directions get mapped onto Wilson lines of 9-branes around those four circles. Moreover, due to the $A_2$ shift in the $x_5$ direction, the Wilson lines inside the 9 and 5-branes have to be shifted against each other. In a T-dual version the $x_5$ positions of the 8-branes and 4-branes would have to be on opposite sides of the circle. This immediately implies that there are no massless modes from open strings stretched between two such branes. In the next part we will compute in detail the remaining three different one loop contributions to the cosmological constant, namely the Klein bottle, cylinder and Möbius strip amplitude.

3.1. Klein bottle

In the loop channel Klein bottle amplitude one has to sum over all sectors

$$K(t)_{g,h} = \text{Tr}_g \left( \Omega \ h \ e^{-2\pi t (L_0 + \overline{L}_0)} \ P \ S \right), \quad (3.3)$$

for which $g$ and $\Omega h$ commute. This is in agreement with the consistency conditions derived in [12] for the tree channel Klein bottle amplitude. $P$ denotes the GSO projection and $S = \frac{1}{2}(1 + A_4^2)$ denotes the projection onto states invariant under the asymmetric shift $A_4^2$. Applied to our case (3.3) leads to the following contribution to the cosmological constant

$$\Lambda_{KB} \sim 4 \int_0^\infty \frac{dt}{t^3} \frac{1}{4} \text{Tr}_{1,R} \left( \Omega \ (1 + f + g + R) e^{-2\pi t (L_0 + \overline{L}_0)} \ P \ S \right), \quad (3.4)$$

where one only has to take the trace over the untwisted and $R = fgA_4^2$ twisted sectors. Moreover, since $\Omega$ exchanges left and right movers the trace needs only to be taken over the NS-NS and R-R sectors. Therefore the space time fermion number operator $(-1)^F$ appearing in the definition of $f$ and $g$ does not matter in the Klein bottle amplitude. The effect of the $(-1)^{F_L,F_R}$ insertions in $f$ and $g$ is also marginal, for it only changes some
of the signs in front of terms containing contributions from the spin structure which is zero anyway. Thus, also for the non-supersymmetric cases Ib and IIb the Klein bottle amplitude vanishes. Moreover, the shifts contained in \( f \) and \( g \) permute the 16 different fixed points of \( R \) and therefore the contributions \( K(t)_{R,f} \) and \( K(t)_{R,g} \) are identical to zero.

Summarising, the Klein bottle amplitude for model I and II can be written as

\[
\Lambda_{KB}^{I,II} \sim (1 - 1/4) \frac{4}{t^3} \left\{ \frac{f_3(e^{-2\pi t})}{f_8(e^{-2\pi t})} \left( \sum_{m \in \mathbb{Z}} e^{-\pi t m^2/\rho} \right)^2 - \frac{1}{2} \left[ \left( \sum_{m \in \mathbb{Z}} e^{-\pi t m^2/\rho} \right)^4 + \right. \right.
\]

\[
\left. \left. \left( \sum_{m \in \mathbb{Z}} (-1)^m e^{-\pi t m^2/\rho} \right)^4 + \left( \sum_{n \in \mathbb{Z}} e^{-\pi t n^2/\rho} \right)^4 + \left( \sum_{n \in \mathbb{Z}} (-1)^n e^{-\pi t n^2/\rho} \right)^4 \right) \right\} +
\]

\[
8 \frac{f_4(e^{-2\pi t})}{f_7(e^{-2\pi t}) f_2(e^{-2\pi t})} \left( \sum_{m \in \mathbb{Z}} (-1)^m e^{-\pi t m^2/\rho} \right)^2 \right\}.
\]

(3.5)

Here we have defined \( \rho = r^2/\alpha' \) which is actually one at the self-dual radius. However, we keep it in the formulae in order to follow the different volume factors in the amplitudes. We have also set the radii of the 5-6 circles to the self-dual radius. In order to detect massless tadpoles one has to transform the amplitude into the tree channel by a modular transformation \( t = \frac{1}{4t} \). Since the Poisson resummation formula implies

\[
\sum_{m} (-1)^m e^{-\pi m^2/\rho} \frac{1}{4} = \sum_{m} e^{-4\pi \rho (m + \frac{1}{2})^2}
\]

(3.6)

there are no new tadpoles arising from the second term, the \( K(t)_{1,f} \) and \( K(t)_{1,g} \) sectors, in (3.5). The only tadpoles have their origin in the first term of (3.5), which up to factor \( \frac{1}{2} \) yields exactly the same tadpole as in the associated GP model.

The computation for model III is slightly different. Let us, for instance, determine how \( \Omega P \) acts on the KK and winding states in the 5-6 directions:

\[
\Omega P |m_5, n_5; m_6, n_6\rangle = |m_6, -n_6; m_5, -n_5\rangle,
\]

(3.7)

so that only states with \( m_5 = m_6 \) and \( n_5 = n_6 \) contribute in the trace. The complete
calculation yields the following amplitude

\[
A_{KB}^{III} \sim (1 - 1) 4 \int_0^\infty \frac{dt}{t^3} \left\{ \frac{f_5^2(e^{-2\pi t})}{f_1^2(e^{-2\pi t})} \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right) \left( \sum_{n \in \mathbb{Z}} e^{-2\pi t \rho n^2} \right) + \frac{1}{2} \left[ \left( \sum_{m \in \mathbb{Z}} e^{-\pi t \frac{m^2}{\rho}} \right)^4 + \left( \sum_{m \in \mathbb{Z}} (-1)^m e^{-\pi t \frac{m^2}{\rho}} \right)^4 + \left( \sum_{n \in \mathbb{Z}} e^{-\pi t \rho n^2} \right)^4 + \left( \sum_{n \in \mathbb{Z}} (-1)^n e^{-\pi t \rho n^2} \right)^4 \right] + 8 \frac{f_4^4(e^{-2\pi t}) f_1^4(e^{-2\pi t})}{f_1^4(e^{-2\pi t}) f_2^4(e^{-2\pi t})} \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right) \left( \sum_{n \in \mathbb{Z}} (-1)^n e^{-2\pi t \rho n^2} \right) \right\} + \text{(3.8)}
\]

The second term in (3.8) does not generate a tadpole either and the two tadpoles from the first term scale with volume factors \(V_9\) and \(V_5\), pointing already to compensating brane contributions from D8 and D4 branes. The next step is to introduce an open string sector, on which one first has to determine the action of the asymmetric generators \(f\) and \(g\).

### 3.2. Cylinder and Möbius amplitude

First, we would like to discuss the examples I and II. Since it is essentially the GP computation we have to introduce D9-branes and D5(D5)-branes in the theory in order to cancel the tadpoles from the Klein bottle amplitude. We have to arrange the branes in such a way that they respect the symmetries \(A_4^2, f, g\) we would like to mod out. For instance, since the shift \(A_4^2\) acts on the Wilson lines of the D9-branes and the positions of the D5(D5)-branes, respectively, we can not choose them equal for all D9- and D5(D5)-branes. At best, we can have half of the D9-branes with Wilson lines \(\Theta^i\) and the other half with Wilson-lines \(\Theta^i + \frac{1}{2\pi}\), so that they get mapped to each other by \(A_4^2\). Analogously, we can place half the D5(D5)-branes at a fixed point \(X_i\) and the other half at the shifted fixed point \(X_i + \frac{R}{2}\). We will see, that this has exactly the effect of reducing the rank of the gauge group in the expected way. In order not to confuse the reader too much, we restrict ourselves to this case of maximal gauge symmetry.

Moreover, under T-duality \((-1, 1)^4\) the D9- and D5- branes get exchanged, where positions \((X^i_\mu)_{i=1, \ldots, N_9}\) of D5(D5)-branes in the transverse space are mapped to Wilson lines \((\Theta^i_\mu)_{i=1, \ldots, N_9}\) of D9-branes around those four cycles. Since we would like to gauge \(f\) we should guarantee that it is indeed a symmetry in the open string sector, as well. To this end, one needs the same number \(N\) of D9- and D5(D5) branes just from the very beginning. Furthermore, the branes need to carry Wilson lines or need to have transverse positions, respectively, in just the right way to get mapped onto each other by \(f\). We also restrict
ourselves to the case where all D9-brane Wilson lines around cycles in the 5-6 torus are equal. Of course, the D5(D5)-brane Wilson lines are shifted by one half but are also equal among themselves. In the original GP model, D9 brane Wilson line moduli and D5(D5)-brane position moduli were completely independent, whereas now they become related. It is in this sense that we speak of a bound state of D9 and D5(D5) branes. Figure 1 shows that, when $f$ maps Wilson lines of an open string ($\Theta^1, \Theta^2$) to positions ($X^1, X^2$) then $g$ maps it to the inverse positions ($-X^1, -X^2$).

As visualised in figure 2, taking into account $\Omega \delta p \Omega = \delta p$ and $\Omega \delta x \Omega = -\delta x$, it can be shown that this is compatible with the relation $\Omega f \Omega = g$ now realized in the open string sector.

Depending on the boundary conditions, the action of the various shifts in the open string sector is presented in Table 3.
The asymmetric fermion number operator \((-1)^{FL}\) flips the charges of all R-R fields, thus transforming a D-brane into its charge conjugate anti-brane \([13]\). Thus, the orientifolds in class I contain D9 and D5 branes whereas the orientifolds in class II must have D9 and \(\overline{D5}\) branes. Note, that contrary to the examples of non-supersymmetric Dp-\(\overline{Dp}\) systems discussed recently in a series of papers, a pair of flat D9 and \(\overline{D5}\) branes does not break all the supersymmetry. We conclude that all the asymmetric operations contained in \(f\) and \(g\) have a well defined action in the open string sector.

To describe the action of \(f\) on the Chan-Paton factors we introduce a matrix \(\gamma_f\) which must be of the form

\[
\gamma_f = \begin{pmatrix} 0 & f_{95} \\ f^{-1}_{95} & 0 \end{pmatrix},
\]

where \(f_{95}\) is an \(N \times N\) matrix. Remember, that the actions of all the other symmetries like \(\Omega\) and \(R\) were block diagonal in the GP model. Given the choice made in GP

\[
\gamma_\Omega = \begin{pmatrix} 1 & 0 \\ 0 & M \end{pmatrix}, \quad \gamma_R = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} \quad \text{with} \quad M = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix},
\]

consistent with all conditions for the \(\gamma\) matrices we can choose \(f_{95} = 1\).

Generally, the cylinder amplitude is defined by

\[
\Lambda_C \sim \int_0^\infty \frac{dt}{t^3} \frac{1}{4} \text{Tr}_{99,55,95,59} \left((1 + f + g + R) e^{-2\pi t L_0} P S (-1)^F\right). \quad (3.11)
\]

Since \(f\) and \(g\) change the type of brane, the trace with \(f\) and \(g\) insertions is trivially zero. We obtain for the complete cylinder amplitude

\[
\Lambda_C^{I,II} \sim \int_0^\infty \frac{dt}{t^3} (Z_{99} + Z_{55} + Z_{95} + Z_{59}) \quad (3.12)
\]
with the contribution from the 99 open strings being

\[
Z_{99} = \frac{f_3^8(e^{-\pi t}) - f_2^8(e^{-\pi t}) - f_4^8(e^{-\pi t})}{f_1^8(e^{-\pi t})} \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right)^2
\]

\[
\frac{1}{2} \left[ \sum_{i,j=9} \left( \gamma_{1,9} \right)_{ii} \left( \gamma_{1,9} \right)_{jj} \left\{ \prod_{k=1}^{4} \sum_{m \in \mathbb{Z}} e^{-2\pi t \left( \frac{m}{\rho} + \Theta_k' - \Theta_k' \right)^2 / \alpha'} + \right. \\
\prod_{k=1}^{4} \sum_{m \in \mathbb{Z}} (-1)^m e^{-2\pi t \left( \frac{m}{\rho} + \Theta_k' - \Theta_k' \right)^2 / \alpha'} \right\} \\
4 \left( \frac{f_4^4 f_4^4 - f_2^4 f_2^4 + f_4^4 f_4^4 - f_2^4 f_2^4}{f_1^4 f_2^4} \right) \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right)^2 \sum_{j=1}^{16} (Tr(\gamma_{R,j}))^2.
\]

For open strings stretched between two D5-branes we obtain

\[
Z_{55} = \frac{f_3^8(e^{-\pi t}) - f_2^8(e^{-\pi t}) - f_4^8(e^{-\pi t})}{f_1^8(e^{-\pi t})} \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right)^2
\]

\[
\frac{1}{2} \left[ \sum_{i,j=5} \left( \gamma_{1,5} \right)_{ii} \left( \gamma_{1,5} \right)_{jj} \left\{ \prod_{k=1}^{4} \sum_{n \in \mathbb{Z}} e^{-2\pi t \left( \frac{n}{\rho} + X_k' - X_k' \right)^2 / \alpha'} + \right. \\
\prod_{k=1}^{4} \sum_{n \in \mathbb{Z}} (-1)^n e^{-2\pi t \left( \frac{n}{\rho} + X_k' - X_k' \right)^2 / \alpha'} \right\} \\
4 \left( \frac{f_4^4 f_4^4 - f_2^4 f_2^4 + f_4^4 f_4^4 - f_2^4 f_2^4}{f_1^4 f_2^4} \right) \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right)^2 \sum_{l=1}^{16} (Tr(\gamma_{R,l}))^2.
\]

Finally, the 95 and 59 sector together contribute

\[
Z_{95} = 2 \left( \frac{f_4^4 f_2^4 + f_2^4 f_2^4 + f_4^4 f_4^4}{f_1^4 f_4^4} \right) \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right) \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{(m+\frac{1}{2})^2}{\rho}} \right) Tr(\gamma_{1,5}) Tr(\gamma_{1,9}) + \\
2 \left( \frac{f_4^4 f_0^4 + f_2^4 f_2^4 + f_4^4 f_4^4}{f_1^4 f_4^4} \right) \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right) \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{(m+\frac{1}{2})^2}{\rho}} \right) Tr(\gamma_{R,5}) Tr(\gamma_{R,9}),
\]

where \( f_0 = 0 \) denotes the trace for the spin structure \(+\square\). The upper sign in the above equations belongs to model I and the lower one to model II. We have deliberately written the complete partition function, for we would like to discuss the physical relevance of the different signs. For model I everything is like in the GP case, we have computed the one loop graph for the interaction of two D-branes, which of course vanishes due to supersymmetry.
or bose-fermi degeneracy, respectively. For model II two extra signs occur. One is due to the \((-1)^F\) action in \(R\) and the other is due to the fact that we have to consider a \(\overline{D}5\) brane instead of a \(D5\) brane. This is achieved in the loop channel by changing the GSO projection from \(\frac{1}{2}(1 + (-1)^f)\) to \(\frac{1}{2}(1 - (-1)^f)\) implying that the tree level exchange of \(R-R\) fields contribute with the other sign. This nicely confirms that we indeed need \(D5\) branes for model II, since otherwise the brane would be attracted to a \(D9\)-brane and we would not get a stable background.

Finally, we compute the M"obius amplitude

\[
\Lambda_M \sim \int_0^\infty \frac{dt}{t^3} \frac{1}{4} \text{Tr}_{99,55} \left( (1 + f + g + R) e^{-2\pi t L_0} \ P \ S \ (-1)^F \right) \tag{3.16}
\]

Again, since \(f\) and \(g\) change the type of brane the traces with \(f, g\) insertions vanish.

\[
\Lambda_{M,II} \sim (1 - 1) \int_0^\infty \frac{dt}{t^3} - f_j^2 (i e^{-\pi i t}) \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right)^2 \left[ \frac{1}{2} Tr(\gamma^{-1} \Omega,9 \gamma^T \Omega,9) \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right)^4 + Tr(\gamma^{-1} \Omega,9 \gamma^T \Omega,9) \left( \sum_{m \in \mathbb{Z}} (-1)^m e^{-2\pi t \frac{m^2}{\rho}} \right)^4 + Tr(\gamma^{-1} \Omega,5 \gamma^T \Omega,5) \left( \sum_{n \in \mathbb{Z}} e^{-2\pi t \rho n^2} \right)^4 + Tr(\gamma^{-1} \Omega,5 \gamma^T \Omega,5) \left( \sum_{n \in \mathbb{Z}} (-1)^n e^{-2\pi t \rho n^2} \right)^4 \right]. \tag{3.17}
\]

Here, the extra \((-1)^F\) factor in \(R\) in model II means that the orientifold plane carries the opposite charge, so that we have to put a \(\overline{D}5\)-brane in front of an \(O5\)-plane in order to get zero force. Working out the tadpole cancellation conditions one finds that they cancel exactly for 32 \(D9\)-branes and 32 \(D5\)(\(\overline{D}5\))-branes.

Before we discuss the massless spectra in the open string sector, we would like to briefly describe what happens for model III. Apparently, since we are in type IIA we should better introduce odd dimensional branes. Indeed, the tree channel Klein bottle amplitude contains an exchange of a \(R-R\) 9-form potential showing that we should have some \(D8\) branes and hence \(D4\)-branes involved. Denote by \(a\) and \(b\) the two fundamental cycles of the torus spanning the 5-6 directions. Both the \(D8\) and the \(D4\)-brane must wrap around the cycle \(a + b\) which is invariant under \(P\) and must be localised on the antisymmetric cycle \(a - b\).

This guarantees that on the oscillator modes of \(X^5 + X^6\) and \(X^5 - X^6\) the operation \(\Omega P\) acts in the same way. Open strings can wind around the \(a - b\) cycle with energies
proportional to $2\rho n^2$ and naively one might expect to find KK states with momentum in the $a + b$ direction with energy $m^2/2\rho$. This is however not correct, since a torus with cycles $a + b$ and $a - b$ is a double cover of the original torus with cycles $a$ and $b$. Said differently, moving the $a + b$ cycle along the $a - b$ cycle, we arrive at the original cycle already after half of the $a - b$ cycle. Thus, the relevant radius for the KK modes is not $\sqrt{2}\rho$ but $\rho/\sqrt{2}$. Then the cylinder and Möbius strip amplitude for model III can be obtained simply by substituting

$$
\left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right)^2 \rightarrow \left( \sum_{m \in \mathbb{Z}} e^{-4\pi t \frac{m^2}{\rho}} \right) \left( \sum_{n \in \mathbb{Z}} e^{-4\pi t \rho n^2} \right) \tag{3.18}
$$

and

$$
\left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{m^2}{\rho}} \right) \left( \sum_{m \in \mathbb{Z}} e^{-2\pi t \frac{(m+\frac{1}{2})^2}{\rho}} \right) \rightarrow \left( \sum_{m \in \mathbb{Z}} e^{-4\pi t \frac{(m+\frac{1}{2})^2}{\rho}} \right) \left( \sum_{n \in \mathbb{Z}} e^{-4\pi t \rho n^2} \right) \tag{3.19}
$$

in (3.13) and (3.17). The tadpole conditions get not affected by this substitution and therefore model III also contains 32 D8 and D4 branes. In the following section we discuss the massless spectrum in the open string sector.

4. Massless spectrum in the open string sector

Roughly speaking the open string spectrum in our models is one half of the massless spectrum in the associated GP model. Let us first discuss model I, with 16 D9 and D5 branes located at the same fixed point of $R$ and due to the $A_2^4$ shift with the other 16 D9
and D5 branes located at the shifted fixed point of \( R \). Then GP obtained a vector multiplet in the gauge group \( U(8)^{(1)}_{99} \times U(8)^{(2)}_{99} \times U(8)^{(1)}_{55} \times U(8)^{(2)}_{55} \) and the following massless hypermultiplets

\[
\begin{array}{cccc}
U(8)^{(1)}_{99} & U(8)^{(2)}_{99} & U(8)^{(1)}_{55} & U(8)^{(2)}_{55} \\
2 \times 28 & 1 & 1 & 1 \\
1 & 2 \times 28 & 1 & 1 \\
1 & 1 & 2 \times 28 & 1 \\
1 & 1 & 1 & 2 \times 28 \\
8 & 1 & 8 & 1 \\
1 & 8 & 1 & 8 \\
8 & 1 & 1 & 8 \\
1 & 8 & 8 & 1 \\
\end{array}
\]

**Table 4:** GP massless spectrum

The action of the shift \( A^4_2 \) relates the gauge group \( U(8)^{(1)}_{99} \times U(8)^{(1)}_{55} \) to the gauge group \( U(8)^{(2)}_{99} \times U(8)^{(2)}_{55} \) and the matter multiplets in subsequent rows 1 and 2, 3 and 4 and so on. Summarising, the massless spectrum after modding out the shift \( A^4_2 \) is presented in Table 5

\[
\begin{array}{cc}
U(8)_{99} & U(8)_{55} \\
2 \times 28 & 1 \\
1 & 2 \times 28 \\
8 & 8 \\
8 & 8 \\
\end{array}
\]

**Table 5:** massless spectrum after \( A^4_2 \) orbifold

Note, that this is exactly the massless spectrum of the orientifold of type II B on a \( Z_2 \) orbifold of the \( SO(8) \) torus [11]. Both the reduction of the rank of the gauge group and the doubling of the states in the 95 sector is as expected from computations using the background \( B_{ij} \) field explicitly from the very beginning.

The action of \( f \) relates the two gauge multiplets arising from the D9 and D5-branes to each other, finally leading to the \( N = 2 \) supersymmetric massless spectrum of one vector multiplet of \( U(8) \) and 2 hypermultiplets in the \( 28 \) representation. Due to the shifts in the 5-6 directions, the 95 sector is massive containing one hypermultiplet in the \( 8 \otimes 8 = 28 + 36 \)
representation. As in the GP model one can move the D9- and D5- branes away from the fixed point of $R$ leading to more general gauge groups

$$\prod_{I=1}^{8} U(m_I) \times \prod_{J=1}^{8} USp(n_J)$$

(4.1)

with the constraint $\sum m_I + \sum n_J = 8$. As in the GP model one expects abelian gauge anomalies, which should be cancelled by the Green-Schwarz mechanism [14]. Except some slightly different projections of the oscillator states, the computation of the massless spectrum in the open string sector is absolutely identical for the models II and III. Independently of the degree of supersymmetry in the bulk, one always gets $N = 2$ supersymmetry on the brane. Thus, we have a situation which is very similar to the supersymmetry breaking scenario proposed by Scherk and Schwarz [8]. Quantum effects of course will mediate supersymmetry breaking from the bulk to the brane.

One can obtain more interesting spectra by putting more general Wilson lines around the 5-6 direction. For instance, one can have two sets of 16 9-branes with 5-6 Wilson-lines $\Theta_1$ and $\Theta_2$ respectively. Then we also need two sets of 16 5-branes with the shifted Wilson lines $\Theta_1 + \frac{1}{2R}$ and $\Theta_2 + \frac{1}{2R}$. If in the other four compact dimensions we have no further Wilson lines, we get $U(4) \times U(4)$ gauge symmetry and matter multiplets $2(6,1) + 2(1,6)$ arising in the 99 and 55 open string sectors. If we choose $\Theta_2 = \Theta_1 + \frac{1}{2R}$, we can also get massless modes in the 95 sector. In our case we obtain two hypermultiplets in the bi-fundamental $(4,4)$ representation, so part of the $N = 2$ massive spectrum becomes massless.

5. Conclusions

Technically, in this paper we have studied new kinds of orientifolds, namely we have arranged asymmetric $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds of type II, such that the two $\mathbb{Z}_2$ symmetries get exchanged by the action of world sheet parity reversal. It was then possible to gauge the parity reversal operation thereby introducing an open string sector in the theory. The action of the former $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry in the open string sectors related $Dp$ and $D(p-4)$ branes to each other. Unaffected by the supersymmetry in the closed string sector, in all cases the supersymmetry in the open string sector was $N=2$. We computed the remaining three one-loop contributions to the cosmological constant and found that they were all vanishing. Since there is very little known about higher genus non-oriented string loop
diagrams, we have nothing to say about whether one might hope that the cosmological constant vanishes to higher loop order, as well.

From a phenomenological point of view these models may be interesting, because they combine a (possibly) vanishing cosmological constant with non-abelian gauge symmetries living on the branes. I would be interesting to study whether one can construct chiral models with only $N = 1$ supersymmetry on the brane in this set up.

Unfortunately, the recently discussed brane-world scenario [6--15], where one can have a string scale as low as 1TeV is not directly applicable. The essential condition there is, that one can grow a large radius transverse to the brane on which the gauge degrees of freedom live. In our case all radii orthogonal to the D(p-4)-branes are fixed at the self-dual radius and related to that the gauge degrees of freedom live not only on D(p-4)-branes but also on Dp-branes.

It is known that the GP model has an F-theory dual description [16], so it would be interesting to determine to which symmetry of F-theory $f$ and $g$ correspond to. Finally, let us make clear that our results are not in contradiction to the duality of Type I and heterotic string theory. In the introduction we claimed that it is impossible for perturbative heterotic strings to be non-supersymmetric and have vanishing cosmological constant. The orientifold models discussed so far would however be at best dual to non-perturbative heterotic backgrounds with also the solitonic heterotic 5-brane involved.

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