Evidence of radiation-driven Landau states in 2D electron systems: Magnetoresistance oscillations phase shift

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Abstract – We provide the ultimate explanation of one of the core features of microwave-induced magnetoresistance oscillations in high-mobility two-dimensional electron systems: the 1/4-cycle phase shift of minima. We start with the radiation-driven electron orbits model with the novel concept of scattering flight-time between Landau states. We calculate the extrema and nodes positions obtaining an exact coincidence with the experimental ones. The main finding is that the physical origin of the phase shift is a delay of $\pi/2$ of the radiation-driven Landau guiding center with respect to radiation, demonstrating the oscillating nature of the irradiated Landau states. We analyze the dependence of this minima on radiation frequency and power and its possible shift with the quality of the sample.

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Introduction. – Microwave-induced magnetoresistance ($R_{xx}$) oscillations (MIRO) [1,2] show up in high-mobility two-dimensional electron systems (2DES) when they are irradiated with microwaves (MW) at low temperature ($T \sim 1$ K) and under low magnetic fields ($B$) perpendicular to the 2DES. At high enough MW power ($P$) maxima and minima oscillations increase but the latter evolve into zero-resistance states (ZRS) [1,2]. Both effects were totally unexpected when they were first obtained revealing some type of new radiation-matter interaction or coupling assisting electron magnetotransport [3–5].

Despite that over the last few years important experimental [6–25] and theoretical efforts [26–42] have been made on MIRO and ZRS, their physical origin still remains controversial and far from reaching a definite consensus among the scientists devoted to this field. For instance, the two, in principle, accepted theoretical models explaining MIRO (displacement [31] and inelastic [35] models) are under question in regard to recent (and even older) experimental results [43,44] that they are not able to explain. In fact, experimentalists on MIRO are calling for other theoretical approaches that might be more successful offering solid arguments on MIRO and ZRS physical origin [26,27,32].

Among the different features defining MIRO we can underline some that can be considered as fundamental, turning up in most experiments irrespectively of the semiconductor platform and carrier (holes or electrons). Thus, we can highlight three of them: MIRO are periodic in $B^{-1}$, MIRO dependence with $P$ follows a sublinear law whose exponent is around 0.5 [22,24] and finally they present a 1/4-cycle phase shift in the minima position [8,11,45].

In this letter, we present, based on the radiation-driven electron orbits model [26–28], a theoretical analysis of MIRO where we explain the 1/4-cycle shift of minima and the peculiar position of the extrema and nodes. According to this model, when a Hall bar is illuminated, the guiding centers of the Landau states perform a classical trajectory consisting in a harmonic motion along the direction of the current. Thus, the electron orbits move in phase and harmonically with each other at the radiation frequency, altering dramatically the scattering conditions and giving rise eventually to MIRO and, at higher $P$, ZRS.

In this letter, we present, based on the radiation-driven electron orbits model [26–28], a theoretical analysis of MIRO where we explain the 1/4-cycle shift of minima and the peculiar position of the extrema and nodes. According to this model, when a Hall bar is illuminated, the guiding centers of the Landau states perform a classical trajectory consisting in a harmonic motion along the direction of the current. Thus, the electron orbits move in phase and harmonically with each other at the radiation frequency, altering dramatically the scattering conditions and giving rise eventually to MIRO and, at higher $P$, ZRS.

Now, our approach consists on two main physical effects. The first is that the Landau orbit guiding center displacement lags behind the driving force (radiation) by a definite phase constant of $\pi/2$. And secondly, in a scenario of remote charged impurity scattering, we introduced the concept of scattering flight time as the time it takes the electron to
jump from one Landau orbit to another in a scattering process. The core finding is that this time is equal to the cyclotron period. Thus, during the flight time the electrons complete one loop in their cyclotron orbits. For the second effect to occur it is essential that magnetotransport is dominated by the guiding center position of the Landau orbit.

In our calculations we recover the experimental expressions for the extrema and nodes positions showing that they only depend on \( w \) and \( w_c \). We finally discuss and predict the situation of a possible change in the minima shift for ultrahigh-mobility samples when ramping up the magnetic field. The reason is that the phase constant delay of the MW-driven guiding center with respect to radiation, depends on the damping that the Landau orbits suffer along the swinging motion. This damping drops in samples with extreme high mobility (much lower disorder), affecting the phase constant (delay) and eventually the extrema positions and minima shift.

**Theoretical model.** – The radiation-driven electron orbits model, was proposed to study the magnetoresistance of a 2DES subjected to MW at low B and temperature \( T \) [26,27,46–48]. The total electronic Hamiltonian \( H \) can be exactly solved and the solution for the total wave function of \( H \) [26,36,46–48] reads \( \Psi_n(x,t) \propto \phi_n(x-X_0-x_{cl}(t),t) \), where \( \phi_n \) is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator. Thus, the obtained wave function (Landau state or Landau orbit) is the same as the one of the standard quantum harmonic oscillator where the guiding center of the Landau state, \( X_0 \), without radiation, is displaced by \( x_{cl}(t) \). \( x_{cl}(t) \) is the classical solution of a negatively charged, forced and damped, harmonic oscillator [49,50]:

\[
x_{cl}(t) = \frac{-eE_0}{m^* \sqrt{(w_0^2 - w^2)^2 + \gamma^4}} \cos(wt - \beta) = -A \cos(wt - \beta),
\]

where \( E_0 \) is the amplitude of the MW electric field and \( \beta \) is a phase constant. \( \beta \) is the phase difference between the radiation-driven guiding center and the driving radiation; its expression reads \( \tan \beta = \frac{\gamma^2}{w^2 - w_0^2} \). Thus, the guiding center lags behind the time-dependent driving force, (radiation), a phase constant of \( \beta \). In the above calculations radiation has been expressed as \( E(t) = E_0 \cos wt \). \( \gamma \) is a phenomenologically introduced damping factor for the interaction of electrons with the lattice ions giving rise to the emission of acoustic phonons. When the damping parameter \( \gamma \) is important, \( \gamma > w \Rightarrow \gamma^2 \gg w^2 \), then \( \tan \beta \to \infty \) and \( \beta \to \frac{\pi}{2} \). Now, the time-dependent guiding center is \( X(t) = X_0 + x_{cl}(t) = X_0 - A \sin wt \). This physically implies that the orbit guiding centers oscillate harmonically at the MW frequency, but radiation leads the guiding center displacement in \( \frac{\pi}{2} \). This expression automatically fulfills the initial condition of \( X(t=0) = X_0 \) and then we do not need to add any initial phase.

This radiation-driven behavior of the orbit guiding centers is supposed to affect the electron-charged impurity scattering and eventually \( R_{xx} \) [47,51,52]. If one electron, without radiation, is scattered from the Landau state \( \Psi_n \) located at \( X_{0,n} \) to the final state \( \Psi_m \) at \( X_{0,m} \) in a time \( \tau \), the average advanced distance, is given by \( \Delta X_0 = X_{0,m} - X_{0,n} \). This flight time, \( \tau \), is the time it takes the electron to "fly" from one orbit to another due to scattering. This time is part of the quantum scattering time (quantum lifetime), \( \tau_q \), that it is normally defined as the average time between scattering events or collisions, considering that all of them are equally weighted. In high-mobility 2DES the electron magnetotransport is ruled or dominated by the guiding center position of the Landau orbit and not by the electron position itself. In a semiclassical approach, this implies that the electron scattering begins and ends in the same relative position inside the initial and final cyclotron orbits. In other words, during the scattering jump from one orbit to another, in a time \( \tau \), the electrons in their orbits complete one full loop. Then, \( \tau \) must be equal to \( T_c \): \( \tau = \frac{2\pi}{eB} = T_c \) where \( T_c \) is the cyclotron period. Interestingly, applying the time-energy uncertainty relation [53] \( \Delta t \cdot \Delta E \geq h \), it turns out that, being \( \Delta t = \tau \), then, \( \tau \times \Delta E = \frac{2\pi}{eB} \times \Delta E \geq h \Rightarrow \) the uncertainty of energy is \( \Delta E \approx \hbar w_c \). Then the scattered electron ends up, most likely, in the next Landau level: \( m = n + 1 \).

When we turn on the radiation, the guiding centers begin to harmonically oscillate according to the expression of \( X(t) \). Later, in a time \( t_f \), scattering starts and the electron jumps from \( \Psi_n(t) \) at \( X_n(t) \) to \( \Psi_m(t) \) at \( X_m(t) \) in a total time \( t_f = t_i + \tau \). The average advanced distance will change accordingly: \( \Delta X(t) = X_m(t_f) - X_n(t_i) = \Delta X_0 - A \sin w(t_i + \tau) + A \sin wt_i \). The time \( t_i \) is on average the time it takes the electron to experience a scattering process, i.e., the scattering time \( \tau_q \), that for constant \( B \) it is constant too. Therefore, \( t_i \simeq \tau_q \) and \( \Delta X(t) = \Delta X_0 - A \sin (\tau_q + \phi) + A \sin \phi \) being \( \phi = wt_q \). Now shifting the time origin so that \( \phi = 0 \), i.e., to when the electron scattering begins, we obtain the final expression \( \Delta X(t) = \Delta X_0 - A \sin wt \).

The longitudinal conductivity \( \sigma_{xx} \) is given by [54] \( \sigma_{xx} \propto \int dE \frac{\Delta X_{MW}^4}{\tau} \) being \( E \) the energy and \( \frac{1}{\tau} \) the remote charged impurity scattering rate [26]. To obtain \( R_{xx} \) we use the common tensorial relation \( R_{xx} = \sigma_{xx} - \sigma_{xz} \sigma_{zy}^* \sigma_{yx}^* \), where \( \sigma_{xy} \approx \frac{ne}{m^*} \), \( n_i \) being the electrons density, and \( \sigma_{xz} \ll \sigma_{xy} \). Thus [52], \( R_{xx} \propto -A \sin wt \).

Importantly, according to the last expression, the MIRO minima positions are given by \( \frac{2\pi}{\tau} + 2\pi j \Rightarrow w = \frac{2\pi}{\tau} (j + 1/2) \). And the left ones, known as integer fixed points (ifp), \( w = \frac{2\pi}{\tau} j \). If we compare the calculated
expression with the ones previously obtained in experiments by Mani et al. [8,11],

\[
\min \to w = \frac{2\pi}{\tau} \left( \frac{1}{4} + j \right) \quad \Leftrightarrow \quad \frac{w}{w_e} = \left( \frac{1}{4} + j \right),
\]

\[
\max \to w = \frac{2\pi}{\tau} \left( \frac{3}{4} + j \right) \quad \Leftrightarrow \quad \frac{w}{w_e} = \left( \frac{3}{4} + j \right),
\]

\[
\text{hifp} \to w = \frac{2\pi}{\tau} \left( j + \frac{1}{2} \right) \quad \Leftrightarrow \quad \frac{w}{w_e} = \left( j + \frac{1}{2} \right),
\]

\[
\text{ifp} \to w = \frac{2\pi}{\tau} j \quad \Leftrightarrow \quad \frac{w}{w_e} = j \quad (2)
\]

and thus, \( \tau = \frac{2\pi}{w_0} = T_e \). This interesting result confirms our previous approach about the physical insight of the flight time. Now, the 1/4-cycle phase shift of the MIRO minima can be traced back to a phase constant of \( \frac{\pi}{4} \) that is the delay that presents the driven guiding center with respect to radiation in the whole range of \( B \). This delay is revealed by the interplay between the swinging nature of the Landau states under radiation and the remote charged impurity scattering. Then, we can state that the 1/4-cycle phase shift in MIRO minima is an evidence of the fact that the Landau states are not fixed but they oscillate driven by MW. The final expression of the irradiated magnetoresistance turns out to be

\[
R_{xx} \propto -\frac{eE_0}{m^*\sqrt{(w_e^2 - w^2)^2 + \gamma^4}} \sin \left( 2\pi \frac{w}{w_e} \right). \quad (3)
\]

**Results.** – In fig. 1 we exhibit the dependence of MIRO extrema and nodes with respect to \( w \). In fig. 1(a), we show calculated irradiated \( R_{xx} \) vs. \( B \) for an intermediate range of \( w \). We observe, as expected according to the set of equations (3), that extrema and nodes displace to higher \( B \) as \( w \) increases. In fig. 1(b), we plot \( w_e \) vs. \( w = 2\pi f \), where \( f \) is the radiation frequency in GHz, for the extrema and nodes labelled in fig. 1(a). For the hifp, we have used the square symbol, the index \( j = 1 \) and then from eq. (3) we obtain, \( w_e = w \times \frac{1}{2} \). For the maxima, the black dot symbol, the index \( j = 1 \) and then \( w_e = w \times \frac{1}{2} \). For the ifp, the triangle symbol, \( j = 2 \) and \( w_e = w \times \frac{1}{2} \). And finally, for the minima, black down triangle, \( j = 2 \) and \( w_e = w \times \frac{1}{2} \). These calculated results demonstrate and explain that the corresponding shifts for extrema and nodes are independent of radiation frequency. They only depend on the phase difference between radiation and the harmonic displacement of the Landau orbit guiding center. In the lower panel we present the fits corresponding to the four sets of data resulting, as expected, in straight lines where the slopes are given according to the extrema and nodes positions in the upper panel.

In fig. 2, we present the \( P \)-dependence of the extrema and nodes positions of MIRO. In fig. 2(a) we plot \( R_{xx} \) irradiated with MW of 85 GHz vs. \( B \) for different values of \( P \), from 15 mW to dark and \( T = 1 \) K. We observe how MIRO decrease and tend to vanish as \( P \) drops. In fig. 2(b), we exhibit the same results as in fig. 2(a) but this time vs. \( w/w_e \). In both panels we mark the values defining the 1/4-cycle phase shift. The main finding is that the positions of extrema and nodes turn out to be immune with respect to \( P \). Now, we can theoretically explain these results according to our model. In eq. (3) \( P \) only shows up in the numerator of the amplitude as \( \sqrt{P} \propto E_0 \), and not in the phase of the sine function. Thus, \( P \) does not affect the phase of MIRO and the latter remains constant. The outcome is that extrema and node positions in MIRO turn out to be immune to \( P \).

Finally it is interesting to consider the case in which the damping parameter \( \gamma \) is small compared to \( w \). Under this condition we will come across with regimes where the 1/4-cycle phase shift will not be conserved. This can be found in ultrahigh-mobility samples where the damping parameter is expected to become smaller due to a much smaller disorder. Thus, \( \gamma < w \Rightarrow \gamma^2 \ll w^2 \), then the obtained values for \( \beta \) will be different from before when ramping up \( B \). Thus, if, along with this condition, \( w_e \simeq 0 \) (low values of \( B \)), then \( \tan \beta \to 0 \) and \( \beta \to \pi \Rightarrow X(t) = X_0 - A\cos(wt - \pi) \). And we need to add an initial
phase constant of $-\frac{\pi}{2}$ to fulfill $X(t = 0) = X_0$. Finally $X(t) = X_0 - A \cos(\omega t + \frac{\pi}{2}) = X_0 + A \sin \omega t$. Similarly as before, this result leads to $\Delta X(t) = \Delta X_0 + A \sin \omega \tau$. According to this we would expect different positions for extrema and nodes. For instance, now for maxima: $\omega \tau = \frac{\pi}{2} + 2\pi j \Rightarrow \omega = \frac{\Delta X_0}{(\frac{\pi}{2} + j)}$. And for minima: $\omega \tau = \frac{\Delta X_0}{\frac{\pi}{2} + 2\pi j} \Rightarrow \omega = \frac{\Delta X_0}{(\frac{\pi}{2} + j)}$. These are opposite results to the ones obtained in the previous scenario of $\gamma > \omega$. If now $B$ and $w_c$ increase, $\tan\beta$ increases too, but in negative and tends to $-\infty$ at resonance, where $\beta \approx \frac{\pi}{2}$. Accordingly, $X(t) = X_0 - A \cos(\omega t - \frac{\pi}{2} - \frac{\pi}{2}) = X_0 + A \cos \omega t$, and now maxima positions are given by $\omega \tau = 2\pi j$. If $w_c$ keeps increasing, $\tan\beta$ $\rightarrow 0$ but now $\beta \rightarrow 0$. Thus, as $B$ is ramping up from $B = 0$, $\beta$ evolves from $\pi$ to $\frac{\pi}{2}$ at resonance and, for higher $B$, ends up being equal to 0. Something similar to the above has been experimentally obtained previously by Dai et al. [55] but it has been overlooked by the physicists, experimentalists and theorists, devoted to MIRO; they obtain a clear shift of extrema and nodes positions as $B$ rises for ultrahigh-mobility samples.

In fig. 3 we present the calculated results of irradiated $R_{xx}$ and $\beta$ vs. $w/w_c$ for a regime of $\gamma < \omega$ and a frequency of 143 GHz. As in Y. Dai experiments, this simulation has been run considering that the resonance takes place in the second harmonic, i.e., $2w_c = \omega$. This permits in the simulation to see more clearly the transition of $\beta$ from $\gamma$ to $\omega$. In the upper panel we plot two curves, one for $\gamma < \omega$ (black curve) and the other for $\gamma > \omega$ (red curve) to compare both regimes in terms of extrema and nodes positions as $B$ rises. We observe that the black curve no longer shows the 1/4-cycle phase shift for minima. Yet it presents this shift but for the maxima and changes to $\pi$ to $\frac{\pi}{2}$. This work is supported by the MINECO (Spain) under grant MAT2014-58241-P and ITN Grant 234970 (EU).
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