New generalization of the simplest $\alpha$-attractor $T$ model

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Abstract: The simplest $\alpha$-attractor $T$ model is given by the potential $V = V_0 \tanh^2(\lambda \phi/M_p)$. However, its generalization to the class of models of the type $V = V_0 \tanh^p(\lambda \phi/M_p)$ is difficult to interpret as a model of inflation for most values of $p$. Keeping the basic model, we propose a new generalization, where the final potential is of the form $V = V_0(1 - \text{sech}^p(\lambda \phi/M_p))$, which does not present any of the problems that plague the original generalization, allowing a successful interpretation as a model of inflation for any value of $p$ and, at the same time, providing the potential with a region where reheating can occur for any $p$ (including odd and fractional values) without difficulty. In the cases $p = 1, 2, 4$ we obtain the solutions $r(n_s, N_{ke})$ where $r$ is the tensor-to-scalar ratio, $n_s$ the spectral index and $N_{ke}$ the number of $e$-folds during inflation. We also show how these solutions connect to the $\phi^2$ monomial.

I. INTRODUCTION

Inflation models of the type $\alpha$-attractors have captivated considerable attention in recent years because they cover a substantial part of the observationally favorable region reported mainly by the Planck Collaboration (see [2, 15] for most of the basic material and [13, 33] for a sample of subsequent work on the subject). These are well-motivated models and their origin is traced to conformal, superconformal, and supergravity theories all of which are well grounded mathematically. The simplest model of $\alpha$-attractors is given by a potential of the form

$$V = V_0 \tanh^2(\frac{\phi}{M_p}), \tag{1}$$

where $\lambda$ (with $\lambda = 1/\sqrt{6\alpha}$ in the original notation) is directly related to the curvature of the inflaton scalar manifold and $M_p$ is the reduced Planck mass $M_p = 2.44 \times 10^{18}$ GeV however, in the plots, we work in Planck units such that $M_p = 1$. This basic model is generalized to the class of models $V = V_0 \tanh^p(\lambda \phi/M_p)$ characterized by the parameter $p$ [7]. However, values of $p$ other than $p = 2$ present certain difficulties in being interpreted as inflation models.

In this article we have tried a different generalization from the previous one but at the same time keeping the basic structure that seems so promising. Therefore we generalize the basic potential $V = V_0 \tanh^2(\lambda \phi/M_p) = V_0(1 - \text{sech}^2(\lambda \phi/M_p))$ to the form $V = V_0(1 - \text{sech}^p(\lambda \phi/M_p))$. This small modification brings with it important changes in the class of resulting models. The models being well defined for every reasonable value of $p$, allowing a region where reheating can occur for any $p$, including odd and fractional values, and covering practically the entire phenomenologically acceptable region in the $n_s$ vs. $r$ plane.

The organization of the article is as follows: In Sec. II we discuss the new generalization of the basic model that we propose and show how the resulting potential is positive definite for every value of the power $p$. We also discuss the expansion of the model around its minimum. In particular, we see that the dependency on $p$ is weak, being only a multiplicative constant of the leading quadratic term, while the dependency of the previous generalized model is very strong, being a power of the inflaton. This has the consequence that the new generalization allows reheating for any $p$ (including odd and fractional values). In Sec. III, given the impossibility of carrying out an analytical study for arbitrary $p$, we consider several interesting examples with $p = 1, 2, 4$ and we write the potential in terms of the observables $n_s$ and $r$. This allows us to obtain the limit of the potential when $n_s \to 1 - r/4$, equivalently $\lambda \to 0$, showing that the potential is reduced to the quadratic potential $V = \frac{1}{2} m^2 \phi^2$, connecting the solution, in the $n_s$ vs $r$ plane, with the monomial $\phi^2$. We show figures for the number of $e$-folds during inflation $N_{ke}$, the tensor-to-scalar ratio $r$, and the inflation scale as functions of $p$ for various values of the parameter $\lambda$. Finally, Sec.IV contains our conclusions on the main points discussed in the article.

II. THE MODEL

Without going into the details of the construction of the $\alpha$-attractor models we begin by writing [17]

$$\frac{1}{\sqrt{-g}} \mathcal{L} = \frac{1}{2} M_p^2 R - \frac{1}{2} M_p^2 (\partial_\mu \Psi)^2 - V(\Psi), \tag{2}$$

as a phenomenological model and propose a function for $V(\Psi)$ where $\Psi \sim \tanh(\lambda \phi/M_p)$ makes $\phi$ a canonically normalized field identified with the inflaton. The simplest $\alpha$-attractor model is given by [7]

$$V(\Psi) \propto \Psi^2, \tag{3}$$

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As stated before, the $p = 2$ case gives the simplest $\alpha$-attractor model. However more general cases of the parameter $p$ are difficult to interpret as models of inflation giving rise to unattractive potentials (see Fig. [1]).

Thus, we would like to keep the very nice features of the $\tanh^2$ potential while at the same time generalize the model to well defined and viable potentials. We could try a close expression to the one before by noticing that

\[ V_t = V_0 \tanh^p(\lambda \frac{\phi}{M_{pl}}). \]

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We study some properties of the model defined by the Eq. (8). In particular, we eliminate the model parameters $V_0$ and $\lambda$ in terms of the observables $n_s$ and $r$ which facilitate a better understanding of the model. Typically the global scale $V_0$ is of no interest because quantities like the number of e-folds during inflation $N_{ke}$ and the
observables $n_s$ and $r$ are related to the potential by ratios of the potential and its derivatives which eliminate $V_0$. For this model, however, it is not possible to solve the corresponding equations and to make some progress we are led to consider first the solution for the inflaton at horizon crossing $\phi_k$ by solving the equation for the amplitude of scalar perturbations $A_s$

$$A_s(k) = \frac{V}{24\pi^2 \epsilon M_{pl}^4},$$  \hfill (11)

which, however, involves the scale $V_0$. The solution is given by

$$\text{sech}(\lambda \phi_k / M_{pl}) = \left(1 - \frac{3A_s \pi^2 r}{2V_0 M_{pl}^4}\right)^{1/p}. \hfill (12)$$

From the equation $16\epsilon = r$ we get

$$\lambda = \left(\frac{r(1 - \text{sech}^p(\lambda \phi_k / M_{pl}))^2}{8p^2 \text{sech}^{2p}(\lambda \phi_k / M_{pl})(1 - \text{sech}^2(\lambda \phi_k / M_{pl}))}\right)^{1/2}, \hfill (13)$$

where the $\text{sech}(\lambda \phi_k / M_{pl})$ is given by Eq. (12) above. Unfortunately it is not possible to solve for $V_0$ for a general $p$ thus, in what follows, we discuss a few particular cases.

\textbf{A. The $p = 1$ case}

The $p = 1$ case corresponds to the model [35, 36] (see also [37, 38])

$$V = V_0 \left(1 - \text{sech}(\lambda \phi / M_{pl})\right). \hfill (14)$$

From the equation

$$n_s = 1 + 2\eta - 6\epsilon, \hfill (15)$$

written in the form $\delta_{n_s} + 2\eta - 6\epsilon = 0$, where $\delta_{n_s}$ is defined as $\delta_{n_s} = 1 - n_s$, we obtain

$$V_0 = \frac{3A_s \pi^2 r(24\delta_{n_s} - r + \sqrt{17r^2 + 16r\delta_{n_s} + 64\delta_{n_s}^2})}{16(4\delta_{n_s} - r)} M_{pl}^4, \hfill (16)$$

in this case the potential [14] can be written in terms of the observables $n_s$ and $r$ as follows

\begin{equation}
V = \frac{3A_s \pi^2 r(24\delta_{n_s} - r + R_1)}{16(4\delta_{n_s} - r)} \left(1 - \text{sech}\left(\frac{1}{8}\sqrt{45r - 16\delta_{n_s} - 11R_1 + \frac{8\delta_{n_s}}{r}(8\delta_{n_s} + R_1)} \frac{\phi}{M_{pl}}\right)\right) M_{pl}^4, \tag{17}
\end{equation}

where $R_1 = \sqrt{17r^2 + 16r\delta_{n_s} + 64\delta_{n_s}^2}$ in the limit $\delta_{n_s} \to r/4$ the potential becomes

$$V = \frac{3}{64} M_{pl}^2 A_s \pi^2 r^2 \phi^2. \hfill (18)$$

This potential is exactly the potential for the monomial $V = \frac{1}{2} m^2 \phi^2$ once the parameter $m^2$ is eliminated by means of Eq. (11) above. Thus, in the limit $\delta_{n_s} \to r/4$ (equivalently $\lambda \to 0$) the potential [14] transitions to the $\phi^2$.
monomial as shown in Fig. 3, panel p = 1. For r > 4δn, there is yet another transition to a sec(λϕ/Mpl) potential but we do not study it here because it is not phenomenologically acceptable with values for r beyond its upper bound. Plots for the number of e-folds Nke the tensor-to-scalar ratio r and the scale of inflation Δ ≡ V_k/\kappa for several values of the parameter λ are given as functions of p in Figs. 4 and 5, respectively. Also, a plot of r(n_s, Nke) in the n_s versus r plane for the number of e-folds Nke = 50, 60 is shown in Fig. 6, together with the p = 2 and p = 4 cases. From this last figure we see that the sech^p potential always ends in the φ^2 monomial as expected.

B. The p = 2 case

We solve again the equation Δ_n + 2η - 6ε = 0 with the result

\[ V_0 = \frac{6A_s \pi^2 r \delta_n}{4\delta_n - r} M_{pl}^4, \]

in this case the potential is given by

\[ V = \frac{6A_s \pi^2 r \delta_n}{4\delta_n - r} \left( 1 - \text{sech}^2 \left( \frac{\sqrt{6(4\delta_n - r)} \ \phi}{2\sqrt{r}} \frac{\phi}{M_{pl}} \right) \right) M_{pl}^4, \]

and in the limit δ_n → r/4 the potential again transitions to

\[ V = \frac{3}{64} M_{pl}^2 A_s \pi^2 r^2 \phi^2. \]
In the $p = 2$ case we can find a simple expression for $r$ as a function of $n_s$ and $N_{ke}$: from the equation
\[ \delta_{n_s} + 2n - 6c = 0 \]
we get
\[ \cosh^2 \left( \frac{\phi_k}{M_{pl}} \right) = \frac{\delta_{n_s} + 8\lambda^2 + \sqrt{\delta_{n_s}^2 + 16\lambda^2\delta_{n_s} + 64\lambda^4}}{2\delta_{n_s}}, \]
while the solution to $\epsilon = 1$ gives the end of inflation
\[ \cosh^2 \left( \frac{\phi_c}{M_{pl}} \right) = \frac{1}{2} \left( 1 + \sqrt{1 + 8\lambda^2} \right). \]
The number of e-folds $N_{ke} = -\frac{1}{M_{pl}^2} \int_{\phi_k}^{\phi_c} \sqrt{V} d\phi$ is
\[ N_{ke} = \frac{1}{4\lambda^2} \left( \cosh^2 \left( \frac{\phi_k}{M_{pl}} \right) - \cosh^2 \left( \frac{\phi_c}{M_{pl}} \right) \right), \]
or
\[ N_{ke} = \frac{8\delta_{n_s} - r - \sqrt{r^2 + r\delta_{n_s}(4\delta_{n_s} - r)}}{\delta_{n_s}(4\delta_{n_s} - r)}. \]
Solving for $r$
\[ r = \frac{4(N_{ke}\delta_{n_s} - 2)^2}{1 + N_{ke}(N_{ke}\delta_{n_s} - 2)}, \]
This solution should be supplemented with conditions which guarantee that $r$ is a well-defined real positive number. To have a positive $r$ the condition on the denominator $1 + N_{ke}(N_{ke}\delta_{n_s} - 2) > 0$ has to be satisfied. The numerator implies that $r = 0$ if and only if $N_{ke}\delta_{n_s} - 2 = 0$ (small dots as reference points in the upper panel of Fig. 7). For $N_{ke}\delta_{n_s} - 2 < 0$ we get the almost vertical rhs branch of the solution with $n_s$ barely increasing from the dot. For $N_{ke}\delta_{n_s} - 2 > 0$ we get the lhs branch with $n_s$ decreasing from the dot. If we substitute $r$ as given by Eq. (26) back in the rhs of Eq. (25) we will find that to get $N_{ke}$ as we should) $N_{ke}\delta_{n_s} - 2$ has to be negative within Planck’s ranges for $n_s$ and $r$ [1]. Thus, the lhs branch is unphysical. Also, if we combine these two conditions ($N_{ke}\delta_{n_s} - 2 < 0$ and $1 + N_{ke}(N_{ke}\delta_{n_s} - 2) > 0$), it is easy to show that they are equivalent to the following conditions on $n_s$
\[ 1 - \frac{2}{N_{ke}} \leq n_s < 1 - \frac{2}{N_{ke}^2} + \frac{1}{N_{ke}^4}. \]
Thus, the solution given by Eq.(26) should be supplemented with the conditions (27) above. The solution (26) is plotted for various values of $N_{ke}$ in the lower
FIG. 6: Plot of the tensor-to-scalar ratio $r$ as a function of the spectral index $n_s$ for the models defined by $p = 1, 2, 4$. This is shown for a number of $e$-folds during inflation $N_{ke} = 50$ and $N_{ke} = 60$. The region inside the “bags” gets filled as $p$ continuously sweeps from $p = 1$ to $p = 4$ covering most of the interesting zone bounded by the results presented by Planck’s article [1].

C. The $p = 4$ case

From the equation $\delta_{n_s} + 2\eta - 6\epsilon = 0$ we obtain the result

$$V_0 = \frac{3A_s \pi^2 r}{32(8\delta_{n_s} - 3r)(4\delta_{n_s} - r)} \left( \frac{512\delta_{n_s}^2 - 192r\delta_{n_s} + 9r^2 - r\sqrt{r(256\delta_{n_s} - 15r)}}{32(8\delta_{n_s} - 3r)(4\delta_{n_s} - r)} \right) M_{pl}^4, \quad (28)$$

in this case the potential is given by

$$V = \frac{3A_s \pi^2 r R_2}{32(8\delta_{n_s} - 3r)(4\delta_{n_s} - r)} \left( 1 - \text{sech}^4 \left( \frac{\sqrt{2r(8\delta_{n_s} - 3r)(4\delta_{n_s} - r)} \phi}{\sqrt{R_3 \left( 1 - \sqrt{R_3/R_2} \right) M_{pl}}} \right) \right) M_{pl}^4, \quad (29)$$

where $R_2 = 512\delta_{n_s}^2 - 192r\delta_{n_s} + 9r^2 - r\sqrt{r(256\delta_{n_s} - 15r)}$ and $R_3 = 128r\delta_{n_s} - 39r^2 - r\sqrt{r(256\delta_{n_s} - 15r)}$. In the limit $\delta_{n_s} \to r/4$ the potential becomes

$$V = \frac{3}{64} M_{pl}^2 A_s \pi^2 r^2 \phi^2, \quad \text{(30)}$$

as in the previous two cases. We expect this to be a general result: from Eq. (13) we can express $r$ in terms of $\lambda$. A small $\lambda$ expansion gives

$$r = \frac{32M_{pl}^2}{\phi^2} - \frac{16}{3} (2 + 3p)\lambda^2 + \cdots, \quad \text{(31)}$$

Thus, in the limit of vanishing $\lambda$ and using Eq. (11), we get

$$V = \frac{r\phi^2}{32M_{pl}^2} V = \frac{r\phi^2}{32} M_{pl}^2 \frac{3}{2} A_s \pi^2 r = \frac{3}{64} M_{pl}^2 A_s \pi^2 r^2 \phi^2. \quad \text{(33)}$$

for any value of $p$. 
FIG. 7: The upper figure shows $r(n_s)$ for $N_{ke} = 55, 60$ according to Eq. (26) with only the condition on the denominator $1+N_{ke}(N_{ke}\delta_n - 2) > 0$ to guarantee a positive $r$. The dots occur when $N_{ke}\delta_n - 2 = 0$ (thus, $r = 0$) and are there for reference only. For $N_{ke}\delta_n - 2 < 0$ we get the almost vertical rhs branch of the solution with $n_s$ barely increasing from the dot. For $N_{ke}\delta_n - 2 > 0$ we get the lhs branch with $n_s$ decreasing from the dot. Substituting $r$, as given by Eq. (26), back into Eq. (25) we find that to get $N_{ke}, N_{ke}\delta_n - 2$ has to be negative within Planck’s ranges for $n_s$ and $r$ [1]. From here we conclude that the lhs branch is unphysical. In the lower figure we plot $r(n_s)$ supplemented with the conditions [27] which guarantee a consistent solution for $r$ for (from left to right) $N_{ke} = 50, 52, 54, 56, 58, 60$.

IV. CONCLUSIONS

Starting from the simplest monomial function for $\alpha$-attractors we have proposed a new generalization of the $T$ models leading to the potential $V = V_0(1 - \text{sech}^p(\lambda\phi/M_{pl}))$, that does not present the difficulties of interpretation of the original generalization given by $V = V_0 \tanh^p(\lambda\phi/M_{pl})$. The resulting class of potentials have also the particularity that they are quadratic around the minimum for all values of the power $p$ giving rise to viable inflation models while at the same time presenting a region where reheating can occur without difficulty for any reasonable value of $p$, including odd and fractional values. We have also shown how the generalized models transition to $\phi^2$ monomials when the tensor-to-scalar ratio $r$ approaches the value $4(1 - n_s)$, equivalently $\lambda \to 0$, where $n_s$ is the spectral index. The resulting models are phenomenologically viable, covering most of the area preferred by the observations reported by the Planck 2018 collaboration article [1].

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