Low-Density Instability of Multi-Component Matter with Trapped Neutrinos

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Abstract

The effect of neutrino trapping on the longitudinal dielectric function at low densities has been investigated by using different relativistic mean field models. Parameter sets G2 of Furnstahl-Serot-Tang and Z271 of Horowitz-Piekarewicz, along with the adjusted parameter sets of both models, have been used in this study. The role of the isovector adjustment and the effect of the Coulomb interaction have been also studied. The effect of the isovector adjustment is found to be more significant in the Horowitz-Piekarewicz model, not only in the neutrinoless matter, but also in the matter with neutrino trapping. Although almost independent to the variation of the leptonic fraction, the instability region of matter with neutrino trapping is found to be larger. The presence of more protons and electrons compared to the neutrinoless case is the reason behind this finding. For parameter sets with soft equation of states at low density, the appearance of a large and negative $\varepsilon_L(q,q_0 = 0)$ in some parts of the edge of the instability region in matter with neutrino trapping is understood as a consequence of the fact that the Coulomb interaction produced by electrons and protons interaction is larger than the repulsive isovector interaction created by the asymmetry between the proton and neutron numbers.

PACS numbers: 13.15.+g, 25.30.Pt, 97.60.Jd
I. INTRODUCTION

At low densities both the relativistic and the non-relativistic mean field models predict a liquid-gas phase transition region for nuclear matter leading, for dense star matter, to a non-homogeneous phase commonly called pasta phase, which is formed by a competition between the long range Coulomb repulsion and the short range nuclear attraction [1]. This transition has substantial consequences on the properties of stellar matter and neutrino transport [2]. Considerable efforts to comprehend the uniform ground state stability of multi-component systems consisting of electrons, neutrinos, protons, and neutrons as a good approximation of this transition have been recently devoted, not only in the zero temperature approximation, but also for finite temperature [3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. It is obvious that in order to understand the physics inside the non-homogeneous (unstable) regions like the mechanism of nuclear creation with slab-like or rod-like shape, we have to go beyond the mean field approximation. Attempts in this direction are discussed in Refs. [1, 13, 14, 15, 16]. Moreover, in the collapsing supernova core and at sub-nuclear densities, the transition of nuclear shape from sphere to other exotic shapes has significant effects to the neutrino mean free path. However, how these effects modify the neutrino mean free path is not fully understood yet [15, 16].

Another motivation to study this transition comes from the fact that a neutron star is expected to have a solid inner crust of nonuniform neutron-rich matter above its liquid mantle [5] and the mass of its crust depends sensitively on the density of its inner edge and on its equation of state (EOS) [4]. On the other hand, the critical density ($\rho_c$), a density at which the uniform liquid becomes unstable to a small density fluctuation, can be used as a good approximation of the edge density of the crust [3]. By generalizing the dynamical stability analysis of Ref. [17] in order to accommodate the various nonlinear terms in the relativistic mean field (RMF) model of Horowitz-Piekarewicz [6], Carriere et al. [5] found a strong correlation between $\rho_c$ in the neutron star and the density dependence of nuclear matter symmetry energy ($a_{sym}$). This leads to a suggestion that a measurement of the neutron radius in $^{208}$Pb will provide useful information on the $\rho_c$ [5, 6].

In our previous work [11], the critical densities of uniform matter with and without neutrino trapping have been calculated and analyzed by means of different RMF models. In this analysis it is shown that the interplay between the dominant contribution of the matter
composition and the effective masses of mesons and nucleons leads to higher critical densities for matter with neutrino trapping. Furthermore, it was also found that the predicted critical density is insensitive to the number of trapped neutrinos as well as to the RMF model used. However, the discussion about the reason behind these findings was not quite robust. On the other hand, as we mentioned above, the neutrino transport is very crucial in the dynamics of the core-collapsing supernova due to the fact that the neutrinos carry most of the energy away and will lose also their energies by exciting collective nuclear and plasmon modes. Similar situation can also be found in the neutron stars. Moreover, it was also shown in Ref. [8] that the behavior of electrons in matter depends strongly on the wavelength or momentum of the external perturbation \( q \). Note that this momentum is related to the energy transfer of the neutrinos that propagate in matter.

The present paper reports on the extension of our previous investigation [11] by calculating the longitudinal dielectric function of ERMF models and analyzing the relation between the obtained results and the isovector sector adjustment, the presence of the long-range Coulomb interaction, as well as the presence of electrons. The purpose of this work is to explain the reason behind the appearance of each point along the onset of the instability. To this end, we should emphasize here that we need to calculate the dielectric function in the edge of non-homogeneous regions because information from the critical density alone is insufficient. Furthermore, it is also important to emphasize that our definition of the instability is not the non-homogeneous area, but rather it is connected with the points where these non-homogeneities start to appear. As a consequence, the assumption of the uniform matter in the calculation is still valid.

This paper is organized as follows. The RMF models and some constraints used in the present analysis are briefly discussed in Sec. II. In Sec. III, a discussion of the longitudinal dielectric function is given. In Sec. IV we present the graphical results of the onset of instability along with the corresponding discussions. Finally, we give the conclusion in Sec. V.

II. RMF MODELS

To describe the multi-component matter, we use the Lagrangian density [11]

\[ \mathcal{L} = \mathcal{L}_N + \mathcal{L}_M + \mathcal{L}_{HP} + \mathcal{L}_L , \] (1)
where the first three terms describe the nucleons in the mean field level, while the last term indicates a free Lagrangian for leptons. The first term is the Lagrangian for nucleons interacting with each other via meson exchanges. The second term is the Lagrangian for mesons, containing also their nonlinear self coupling information. The third term is added to accommodate the Horowitz-Piekarewicz isovector nonlinear term [6].

In this study we investigate two RMF models, namely, the G2 parameter set of Furnstahl-Serot-Tang [19] (also known as the ERMF model) and the Z271 parameter set of Horowitz-Piekarewicz [6]. The effective Lagrangian density of Furnstahl-Serot-Tang model has been constructed to fulfill the symmetries of quantum chromodynamics and is expanded in powers of the fields and their derivatives up to order $\nu = 4$. Furthermore, the Lagrangian exploits the natural coupling constants and its application to study the properties of finite nuclei is quite successful. The inclusion of the higher order terms was found to be unimportant (and undetermined) for the nuclear observables of interest [19]. On the other hand, the Horowitz-Piekarewicz model is an extension of the standard RMF model with an additional isovector-vector nonlinear term.

Therefore, the two models can be considered as the generalization of the standard RMF models. Details of the individual terms and coupling constants of both models can be found in Ref. [11]. For each model we use two different parameter sets, i.e., the standard ones (G2 and Z271) and the adjusted ones which produce softer symmetry energy predictions at high densities (G2* and Z271*). Parameter set Z271* is obtained by adding an isovector-vector nonlinear term with the coupling constant $\Lambda_V$ in the Lagrangian density of Z271 parameter set and followed by an adjustment of the $g_\rho$ and $\Lambda_V$ [6]. On the other hand the G2* parameter set is obtained by using a similar procedure. However, since the ERMF model already contains an isovector-vector nonlinear term, the G2* parameter set is obtained by merely an adjustment of the $g_\rho$ and $\eta_\rho$ parameters, keeping the symmetry energy at the same value with the G2 parameter set, i.e., $E_{\text{sym}} = 24.1$ MeV at $k_F = 1.14$ fm$^{-1}$ [11].

By comparing the low density instability regions for matter with and without neutrino trapping obtained from both parameter sets we can investigate the role of isovector terms and the correlation between the instability regions and the symmetry energy. Furthermore, in this study we use the zero temperature approximation. We note that in the real situation the temperature of protoneutron stars is not equal to zero and a supernovae inner core can have a temperature around $T \approx (10–50)$ MeV. Indeed, the stability of uniform matter is
sensitive to temperature. This indicates that investigations at finite temperature will need to be addressed in the future.

In our approximation, the following constraints can be used to determine the fraction of every constituent in matter which are later used to calculate the Fermi momentum of every constituent involved:

- the balance equation for the chemical potentials

\[ \mu_n + \mu_{\nu_e} = \mu_p + \mu_e , \]  

(2)

- conservation of the charge neutrality

\[ \rho_e = \rho_p , \]  

(3)

- and fixed electronic-leptonic fraction

\[ Y_{le} = Y_e + Y_{\nu_e} , \]  

(4)

where the total baryon density is limited by

\[ \rho_B = \rho_n + \rho_p . \]  

(5)

Note that in the case of matter without neutrino trapping we have \( Y_{\nu_e} = 0 \) and the value of \( Y_{le} \) is not fixed.

As has been reported in the previous work [11], besides the EOS, at low density regimes the asymmetry between the proton and neutron number \( (\alpha=Y_N - Y_P) \) in matter with and without neutrino trapping (NT) is also different, i.e., \( \alpha^{\text{without NT}} \) is closer to the asymmetry of the pure neutron matter (PNM), whereas \( \alpha^{\text{with NT}} \) is closer to the symmetric nuclear matter (SNM). Thus, \( \alpha^{\text{without NT}} \) has a strong correlation with \( a_{\text{sym}} \), in contrast to the \( \alpha^{\text{with NT}} \). This behavior is also found in the properties of Fermi momentum of each constituent in matter. Another different phenomenon is that in matter with NT we have \( k_F^e = k_F^p \sim k_F^n \), while matter without NT has \( k_F^e = k_F^P \ll k_F^n \). The latter indicates that the role of isovector contribution is more significant in matter without NT than in matter with NT, while the Coulomb interaction has a more significant effect in matter with NT due to the presence of more protons and electrons. Note that Fermi momentum of every constituent is one of the required information, besides the nucleon effective mass, to calculate the polarizations.
FIG. 1: (Color online) Proton (electron) Fermi momentum as a function of the ratio between baryon and nuclear saturation densities for the G2, G2*, Z271, and Z271* parameter sets in the neutrinoless matter and in matter with neutrino trapping with $Y_e = 0.3$ and 0.4.

in the longitudinal dielectric function. In Fig. 1 the effects of the neutrinos in matter on the electron or proton Fermi momentum at low density regimes are shown.

The symmetry energies ($a_{\text{sym}}$) of the corresponding parameter sets are shown in Fig. 2. It is clearly seen that, different from the Horowitz-Piekarewicz model, the high density adjustment in the isovector-vector channel of the Furnstahl-Serot-Tang model does not significantly affect its $a_{\text{sym}}$ at low density. On the other hand, parameter set with softer symmetry energy at high density of Horowitz-Piekarewicz model becomes stiffer at low density regimes. It means that, in contrast to the Furnstahl-Serot-Tang model, the high density isovector adjustment in the Horowitz-Piekarewicz model leads to a more repulsive isovector interaction than the standard one at low density regimes. The different forms of the nonlinear terms in both models are responsible for these different behaviors.

From both figures, it can be explicitly seen that the presence of neutrinos in matter leads to higher Fermi momenta of protons and electrons compared to the case of matter without NT, while in the latter Fermi momentum of every constituent, which is represented by protons and electrons, is independent to the model used. Furthermore, in matter without neutrino trapping, due to its proton-neutron asymmetry that is closer to PNM, the Fermi momentum of each constituent can be correlated to the $a_{\text{sym}}$. 

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FIG. 2: (Color online) Symmetry energy as a function of the ratio between baryon and nuclear saturation densities for the G2, G2*, Z271, and Z271* parameter sets.

III. LONGITUDINAL DIELECTRIC FUNCTION

The longitudinal dielectric function can be written as

$$\varepsilon_L(q, q_0) = \det \left[ 1 - D_L(q, q_0)\Pi_L(q, q_0) \right].$$

(6)

The uniform ground state system becomes unstable to small-amplitude density fluctuations with perturbation momentum $q$ when $\varepsilon_L(q, q_0 = 0) = \varepsilon_L \leq 0$. Note that in Eq. (6) $q_0$ is the time-component of the four-momentum $q^\mu = (q_0, \vec{q})$ and $q = |\vec{q}|$. The critical density $\rho_c$ is the largest density for which the above condition has a solution. For matter consisting of protons, neutrons, and electrons, the longitudinal meson propagator is given by

$$D_L = \begin{pmatrix} d_g & 0 & -d_g & 0 \\ 0 & -d_s & d_{sv\rho}^+ & d_{sv\rho}^- \\ -d_g & d_{sv\rho}^+ & d_{33} & d_{vp}^- \\ 0 & d_{sv\rho}^- & d_{vp}^- & d_{44} \end{pmatrix},$$

(7)

where $d_{sv\rho}^+ = -(d_{sv} + d_{sp})$, $d_{sv\rho}^- = -(d_{sv} - d_{sp})$, $d_{vp} = d_v - d_\rho$, $d_{33} = d_g + d_v + d_\rho + 2d_{vp}$ and $d_{44} = d_v + d_\rho - 2d_{vp}$. In this form, mixing propagators between isoscalar-scalar and isoscalar-vector ($d_{sv}$), isoscalar-vector and isovector-vector ($d_{vp}$), isoscalar-scalar and isovector-vector ($d_{sp}$) are present due to the nonlinear mixing terms in the model, in addition to the standard photon, omega, sigma and rho propagators ($d_g$, $d_v$, $d_s$ and $d_\rho$). These propagators are
determined from the quadratic fluctuations around the static solutions which are generated by the second derivatives of energy density \( \left( \partial^2 \epsilon / \partial \phi_i \partial \phi_j \right) \), where \( \phi_i \) and \( \phi_j \) are the involved meson fields. The explicit forms of the \( \sigma, \omega \), and \( \rho \) propagators are

\[
d_s = \frac{g_s^2 (q^2 + m_\omega^2)(q^2 + m_\sigma^2)}{(q^2 + m_\omega^2)(q^2 + m_\rho^2)(q^2 + m_\sigma^2) + (\Pi^0_{\sigma\omega})^2(q^2 + m_\sigma^2) + (\Pi^0_{\sigma\rho})^2(q^2 + m_\rho^2)} ,
\]

\[
d_v = \frac{g_s^2 (q^2 + m_\omega^2)(q^2 + m_\sigma^2)}{(q^2 + m_\omega^2)(q^2 + m_\rho^2)(q^2 + m_\sigma^2) + (\Pi^0_{\sigma\omega})^2(q^2 + m_\sigma^2) - (\Pi^0_{\omega\rho})^2(q^2 + m_\rho^2)} ,
\]

\[
d_\rho = \frac{1/4 g_s^2 (q^2 + m_\sigma^2)}{(q^2 + m_\omega^2)(q^2 + m_\rho^2)(q^2 + m_\sigma^2) + (\Pi^0_{\sigma\rho})^2(q^2 + m_\rho^2) - (\Pi^0_{\omega\rho})^2(q^2 + m_\omega^2)} ,
\]

and in the mixing propagators,

\[
d_{sv} = \frac{g_{sv} \Pi^0_{\sigma\omega}(q^2 + m_\sigma^2)}{H(q, q_0 = 0)} ,
\]

\[
d_{sp} = \frac{1/2 g_{sp} \Pi^0_{\sigma\rho}(q^2 + m_\sigma^2)}{H(q, q_0 = 0)} ,
\]

\[
d_{vp} = \frac{1/2 g_{vp} \Pi^0_{\omega\rho}(q^2 + m_\sigma^2)}{H(q, q_0 = 0)} ,
\]

with

\[
H(q, q_0 = 0) = (q^2 + m_\omega^2)(q^2 + m_\rho^2)(q^2 + m_\sigma^2) + (\Pi^0_{\sigma\omega})^2(q^2 + m_\sigma^2) + (\Pi^0_{\sigma\rho})^2(q^2 + m_\rho^2)
\]

\[
+ (\Pi^0_{\sigma\rho})^2(q^2 + m_\sigma^2) - (\Pi^0_{\omega\rho})^2(q^2 + m_\omega^2) ,
\]

where the effective mass of each meson is

\[
m_{\sigma}^2 = \frac{\partial^2 \epsilon}{\partial \sigma^2} = m_\sigma^2 + 2b_2 \sigma + 3b_3 \sigma^2 - d_3 V_0^2 - \tilde{\Lambda}_b b_0^2 ,
\]

\[
m_{\omega}^2 = -\frac{\partial^2 \epsilon}{\partial V_0^2} = m_{\omega}^2 + 2d_2 \sigma + d_3 \sigma^2 + 3c_3 V_0^2 + \tilde{\Lambda}_v b_0^2 ,
\]

\[
m_{\rho}^2 = -\frac{\partial^2 \epsilon}{\partial b_0^2} = m_{\rho}^2 + 2f_2 \sigma + \tilde{\Lambda}_v \sigma^2 + \tilde{\Lambda}_v V_0^2 ,
\]

and the effective mixing masses read

\[
\Pi^0_{\sigma\omega} = -\frac{\partial^2 \epsilon}{\partial \sigma \partial V_0} = 2d_2 V_0 + 2d_3 \sigma V_0 ,
\]

\[
\Pi^0_{\sigma\rho} = -\frac{\partial^2 \epsilon}{\partial \sigma \partial b_0} = 2f_2 b_0 + 2\tilde{\Lambda}_v \sigma b_0 ,
\]

\[
\Pi^0_{\omega\rho} = \frac{\partial^2 \epsilon}{\partial V_0 \partial b_0} = -2\tilde{\Lambda}_v V_0 b_0 ,
\]

whereas the propagator of photon (Coulomb) is

\[
d_g = \frac{e^2}{q^2} .
\]
The longitudinal polarization matrix given in Eq. (6) reads

\[
\Pi_L = \begin{pmatrix}
\Pi_{00}^e & 0 & 0 & 0 \\
0 & \Pi_s & \Pi_m^p & \Pi_m^n \\
0 & \Pi_m^p & \Pi_{00}^p & 0 \\
0 & \Pi_m^n & 0 & \Pi_{00}^n \\
\end{pmatrix}
\]  

(22)

The formulas for polarization elements in \( \Pi_L \) are given in, e.g., Ref. [11]. Note that for the Horowitz-Piekarewicz model \( \Pi^0_{0\sigma\omega} \) and \( \Pi^0_{0\sigma\rho} \) equal to zero. On the other hand, for the Furnstahl-Serot-Tang model, \( \Pi^0_{\omega\rho} \) and \( \tilde{\Lambda}_s \) in \( \Pi^0_{\sigma\rho} \) are zero. In Figs. 3 and 4, we show some plots as examples of the polarizations and propagators at \( q = 0.5 \text{ fm}^{-1} \). It is clearly seen that all polarizations depend strongly on whether the neutrinos are trapped or not in matter. In matter with NT, at a density close to 0.6 \( \rho_B \), they have only a weak dependence on the model used due to the increasing role of the nucleon effective mass. For matter without NT, a similar model dependence on its Fermi momentum \( k_F \) shows up (see Fig. 11). This indicates that the polarizations can be correlated with \( a_{\text{sym}} \). Note that the nucleon effective mass has no correlation with the symmetry energy \( a_{\text{sym}} \) [11]. This demonstrates that, instead of the effective mass of nucleons, Fermi momentum of each constituent controls the behavior of each polarization.

Figure 4 shows the behavior of the propagators as a function of the ratio between baryon and nuclear saturation densities. The scalar \( (d_s) \), vector \( (d_v) \) and rho \( (d_\rho) \) propagators are clearly model dependent and they are insensitive to the presence of the neutrino (NT) in matter. Only \( d_\rho \) depends on the isovector adjustment. The presence of neutrinos has a significant effect only in the mixing propagators \( (d_{\sigma\rho} \) and \( d_{\omega\rho} \). However, their contributions are smaller compared to the rest. We also note that the propagator \( d_{sv} \), which appears in the Furnstahl-Serot-Tang model, has an order of magnitude that equals to the propagators \( d_{\sigma\rho} \) and \( d_{\omega\rho} \), and, furthermore, it does not depend on the adjustment of the isovector sector. These properties are a manifestation of the interplay among \( \sigma, \omega \) and \( \rho \) meson effective masses as well as mixing effective masses \( \Pi^0_{0\sigma\omega}, \Pi^0_{0\sigma\rho} \) and \( \Pi^0_{0\rho\omega} \).

To simplify our discussion, let us neglect the minor contributions of the mixing propagators. In this picture we obtain \( d^{\text{with NT}} \simeq d^{\text{without NT}} \). Together with the fact that \( d_\rho \) depends on the adjustment of isovector sector (model dependent), this indicates that \( d_\rho \) can be related with \( a_{\text{sym}} \).
FIG. 3: (Color online) The proton-scalar, -longitudinal, -mixing, and electron-longitudinal polarizations as a function of the ratio between baryon and nuclear saturation densities for the G2, G2*, Z271, and Z271* parameter sets in neutrinoless matter and matter with neutrino trapping with $Y_{\ell_e} = 0.4$. All curves have been obtained by using $q = 0.5$ fm$^{-1}$.

Therefore, in the case of matter with NT, we can understand that the critical density is higher and the instability region is larger than those in the case without NT. Furthermore, the fact that both of them are insensitive to the value of $Y_{\ell_e}$ and they are not influenced by the isovector treatment appears as a consequence of the small proton-neutron asymmetry $\alpha$, which makes the role of the polarizations more dominant compared to the role of the
FIG. 4: (Color online) The scalar, vector, rho and mixing scalar-rho or vector-rho propagators as a function of the ratio between baryon and nuclear saturation densities for the G2, G2*, Z271, and Z271* parameter sets in neutrinoless matter and matter with neutrino trapping with $Y_e = 0.4$ and $q = 0.5$ fm$^{-1}$. Note that for the Horowitz-Piekarewicz model $d_{s\rho} = 0$, whereas for the Furnstahl-Serot-Tang model $d_{\nu\rho} = 0$.

The correlation between critical density, as well as the onset of the instability, and $a_{sym}$ in matter without NT are caused by two sources: the dependence of the polarization on the $a_{sym}$ as well as on the isovector propagators ($d_\rho, d_{s\rho}$ and $d_{\nu\rho}$).
IV. THE ONSET OF INSTABILITY

As mentioned in the Introduction, here we intend to investigate every point in the onset of the instability region in a great detail. To this end, we plot the projection of the longitudinal dielectric function given in Eq. (6) on the $\rho/\rho_0 - q$ plane in the case that $\varepsilon_L = 0$. To obtain more information on the role of the Coulomb interaction in the limit of $q \approx 0$ (almost zero perturbation), we also present the longitudinal dielectric function at $q$ close to zero, i.e., $\varepsilon_L(q = 0.01 \text{ fm}^{-1}, q_0 = 0)$, as a function of the $\rho/\rho_0$.

The effects of the isovector-vector channel adjustment in both models are exhibited in Fig. 5. Obviously, from the size and the position of the boundary of the instability region, the adjustment in the isovector-vector sector has a more significant effect in the Horowitz-Piekarewicz model compared with the Furnstahl-Serot-Tang one. It is also clear from the figure that this adjustment leads to a higher critical density in both models. This result is certainly consistent with our previous study (cf. Fig. 8 of Ref. [11] in the case of $Y_e = Y_e$).

Another important finding obtained from these figures is that in both cases the ERMF model of Furnstahl-Serot-Tang yields a smaller onset of instability. This is due to the fact that the parameter sets $Z_{271}^*$ and $G_2^*$ have larger symmetry energies compared to the $Z_{271}$ and $G_2$. Therefore, beta-equilibrium of these parameter sets is attained with larger proton fractions.
FIG. 6: (Color online) Onset of the instability of the neutrinoless matter obtained by using the Furnstahl-Serot-Tang model with and without (symbolized with $-$) Coulomb interaction and electrons contributions. Note that the solid (red) and dash-dotted (blue) curves [as well as the dashed (green) and dotted (black) ones] are coincident.

and a smaller contribution from the repulsive isovector channel. Thus, as we expected, Fig. 5 shows explicitly the correlation between the onset of the instability region with $a_{\text{sym}}$. The reason of this fact has been explained in the previous section.
FIG. 8: (Color online) Onset of the instability for matter with NT as functions of the ratio between baryon and nuclear saturation densities and the perturbation momentum $q$. All curves have been obtained by using $Y_e = 0.3$. The results are obtained by using the Horowitz-Piekarewicz (Z271 and Z271*) and Furnstahl-Serot-Tang (G2 and G2*) models.

In Fig. 6, we show the effect of the electron absence (indicated by “$-e$” in the figure) on the onset of instability by switching off their contribution in the case of matter without NT using G2 parameter set, with and without Coulomb contribution. Note that the latter is indicated with “$-\text{Coulomb}$” in the figure. The effect of the electron presence on the size of the instability region is found to be negligible in both cases. The reason is that the number of the electrons is too small in matter without NT, and therefore, the effect of the attractive Coulomb interaction generated by electrons and protons is too weak to produce a visible impact on every point in the onset of instability. Such behavior is observed even in the limit of $q$ close to zero (see Fig. 7).

On the other hand, the repulsive Coulomb interaction due to the presence of protons, even in a very small number, enlarges moderately the stability region of this matter in the range of $0.05 \leq \rho / \rho_0 \leq 0.4$, which is clearly shown in Fig. 6. Furthermore, Fig. 7 emphasizes and shows the important role of Coulomb interaction to stabilize matter without NT for almost zero perturbation.

When the neutrino contribution is taken into account, the situation dramatically changes. This is shown in Fig. 8 for $Y_e = 0.3$ and in Fig. 9 for $Y_e = 0.4$. For all parameter sets the instability boundaries expand, for which no substantial difference appears in the onset of
the instability due to the isovector adjustment, except for the region with $q$ close to zero. In this region, the variation of the neutrino fraction in matter also yields an insignificant effect in the onset of instability, as we have expected.

If we observe the longitudinal dielectric function at $q$ close to zero ($q = 0.01 \text{ fm}^{-1}$), as shown in Fig. 10, then we can clearly see that the Z271* parameter set shows a quite different behavior compared to the other parameter sets, i.e., the Z271* parameter always yields $\varepsilon_L > 0$ in this limit. This fact indicates that the transition to more stable region at the points with small perturbation $q < 0.1 \text{ fm}^{-1}$ around $0.1 \leq \rho / \rho_0 \leq 0.3$ of the Z271* is driven by a different mechanism compared to other parameter sets, i.e., it has a larger $a_{\text{sym}}$ that leads to larger proton and electron fractions (and less neutrinos for a fixed lepton fraction). As a consequence, a larger repulsion effect is produced compared to other parameter sets.

From the fact that the adjusted parameter sets (indicated with *) have narrower instability region compared to their counterparts (indicated without *) for both models, then we can conclude that in this limit the onset of the instability is strongly related to the adjustment of the isovector sector.

To dig up more information behind this fact, we show in Figs. 11 and 12 the effects of the electrons absence (indicated by “$-e$” in the figure) and the absence of the Coulomb interaction (indicated by “$-\text{Coulomb}$” on the onset of the instability and on the instability in the limit of $q$ close to zero, by switching off their contributions in the case of matter with NT. It can be seen from Fig. 13 that the Coulomb interaction plays an important role in
FIG. 10: (Color online) Longitudinal dielectric function for matter with NT as a function of the ratio between baryon and nuclear saturation densities. All curves have been obtained by using the perturbation momentum \( q = 0.01 \text{ fm}^{-1} \) and \( Y_{\text{le}} = 0.4 \). The results are obtained by using the Horowitz-Piekarewicz (Z271 and Z271*) and Furnstahl-Serot-Tang (G2 and G2*) models.

FIG. 11: (Color online) As in Fig. 6 but calculated for the case of neutrino trapping with \( Y_{\text{le}} = 0.4 \).

stabilizing the region with \( q < 0.6 \text{ fm}^{-1} \), in the range of \( 0.2 \leq \rho/\rho_0 \leq 0.6 \). Furthermore, from Fig. 12 we can see that if the electrons contribution were turned off, then the instability at \( q \) close to zero would disappear. Thus, for matter with NT the appearance of the large and negative \( \varepsilon_L \) for the G2, G2* and Z271 parameter sets is caused by the fact that the repulsive interaction induced by the proton-neutron asymmetry (isovector) is unable to cancel the strong-attractive Coulomb interaction created by the presence of a substantially
large number of electrons in matter.

Figure 12 also shows that if the electrons are present but their Coulomb interactions were turned off (electrons behave as free particles) then the longitudinal dielectric function $\varepsilon_L$ became larger and closer to $\varepsilon_L = 0$ but the instability boundary enlarges. This result can be used to emphasize the important role of the Coulomb interaction to stabilize the matter with NT in the limit of $q$ close to zero.

V. CONCLUSION

We have studied how the instability region starts to appear in low-density matter described by the Horowitz-Piekarewicz and Furnstahl-Serot-Tang models. To this end we have utilized the longitudinal dielectric function at $q_0 = 0$. The importance of the electron and Coulomb terms in matter with neutrino trapping has been investigated. It is found that the adjustment of the isovector terms has a more significant effect in the Horowitz-Piekarewicz model, i.e., producing a stronger repulsive isovector contribution which leads to a stronger correlation between its low density instability region and the $a_{sym}$ compared to the model of Furnstahl-Serot-Tang for matter without neutrino trapping. In the case of matter with neutrino trapping, the parameter sets with stiff EOS at low density lead to a large and positive $\varepsilon_L(q, q_0 = 0)$. This demonstrates that, although the onsets of the instability of parameter sets with stiff and soft EOS at low densities are similar, the driving mechanisms are
different. This fact might have an effect to the neutrino transport in matter. In both models the effect of the variation of the leptonic fraction is negligible, but the effect of the neutrino trapping on the onset of the instability region is significant. The presence of more protons and electrons in matter with neutrino trapping is the reason behind this phenomenon. The Coulomb term is found to be decisive in enlarging the stability of matter in this density region. The presence of the large and negative $\varepsilon_L(q, q_0 = 0)$ in some parts of the instability region of matter with neutrino trapping originates from the fact that the isovector term is insufficient to cancel the attractive Coulomb interaction contributions generated by the presence of electrons (and protons).

ACKNOWLEDGMENT

Support from the University of Indonesia is gratefully acknowledged.

[1] L. Brito, Ph. Chomaz, D. P. Menezes, and C. Providência, Phys. Rev. C 76, 044316 (2007).
[2] C. Ducoin, K. H. O. Hasnaoui, P. Napolitani, Ph. Chomaz, and F. Gulminelli, Phys. Rev. C 75, 065805 (2006).
[3] C. J. Pethick, D. G. Ravenhall, and C. P. Lorenz, Nucl. Phys. A 584, 675 (1995).
[4] F. Douchin and P. Haensel, Phys. Lett. B 485, 107 (2001).
[5] J. Carriere, C. J. Horowitz, and J. Piekarewicz, Astrophys. J. 593, 463 (2003).
[6] C. J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. 86, 5647 (2001).
[7] S. S. Avancini, L. Brito, D. P. Menezes, and C. Providência, Phys. Rev. C 71, 044323 (2005).
[8] C. Providência, L. Brito, S. S. Avancini, D. P. Menezes, and Ph. Chomaz, Phys. Rev. C 73, 025805 (2006).
[9] C. Providência, L. Brito, A. M. Santos, D. P. Menezes, and S. S. Avancini, Phys. Rev. C 74, 045802 (2006).
[10] L. Brito, C. Providência, A. M. Santos, S. S. Avancini, D. P. Menezes, and Ph. Chomaz, Phys. Rev. C 74, 045801 (2006).
[11] A. Sulaksono and T. Mart, Phys. Rev. C 74, 045806 (2006).
[12] H. Muller and B. D. Serot, Phys. Rev. C 52, 2072 (1995).
[13] P. Napolitani, Ph. Chomaz, F. Gulminelli, and K. H. O. Hasnaoui, Phys. Rev. Lett. 98, 131102 (2007).
[14] G. Watanabe, K. Sato, K. Yasuoka, and T. Ebisuzaki, Phys. Rev. C 69, 055805 (2004).
[15] C. J. Horowitz, M. A. Perez-Garcia, and J. Piekarewicz, Phys. Rev. C 69, 045804 (2004).
[16] H. Sonoda, G. Watanabe, K. Sato, T. Takiwaki, K. Yasuoka, and T. Ebisuzaki, Phys. Rev. C 75, 042801(R) (2007).
[17] C. J. Horowitz and K. Wehberger, Nucl. Phys. A 531, 665 (1991); ibid. Phys. Lett. B 266, 236 (1991).
[18] Guo Hua, Chen Yanjun, Liu Bo, Zhao Qi, and Liu Yuxin, Phys. Rev. C 68, 035803 (2003).
[19] R. J. Furnstahl, B. D Serot, and H. B. Tang, Nucl. Phys. A 598, 539 (1996); Nucl. Phys. A 615, 441 (1997).