A Distributor-Retailer Inventory Model for Pharmaceutical Supply Chain With Expiry Cost

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Abstract. This paper presents an integrated inventory model involving a distributor and a retailer is proposed in the paper. The retailer manages inventories using periodic review policy and orders products from distributor to fulfill the customer’s demand. The distributor also manages its inventory periodically, orders products from supplier, and delivers products to fulfill the retailer’s demand. The model developed in this paper explores the problem of expiry cost. We assume that the demand is stochastic and follows a normal distribution. The model’s objective is to determine the optimum value of the review period, safety stock, and the number of deliveries from distributor to retailer which can maximize the joint total profit. The model is formulated using vendor-managed inventory (VMI) concept where the ordering cost and transportation cost handled by distributor. Furthermore, an algorithm is suggested to find the optimal solution and a numerical example is presented to illustrate the application of the proposed model.

Keywords: integrated inventory model, periodic review, expiry cost, vendor managed inventory

1. Introduction

The common problem faced by companies is the problem of inventory control. Inventory control must be handled properly because it is an important part in supporting the company's operations. In the absence of inventory, a company will be faced with the risk that at one time it will not be able to fulfill its customers’ demands optimally. Inventory control is also a very important managerial function, because its cost involves a large investment. If a company invests too much in inventory, this will also cause excessive storage costs.

The research concerning with an inventory lot-sizing in vendor-buyer system was introduced by Goyal [1]. Then, many scholars studied this issue under various conditions. For instance, Banerjee [2] developed vendor-buyer model with assuming that the replenishment rate is constant. Goyal [3] relaxed the lot-for-lot assumption used in the previous models and proposed that the vendor’s lot size should be determined as an integral multiple of the buyer’s order quantity. There’s still a shortcoming of Goyal [1],that is the demand is assumed to be deterministic. In reality, however, the demand may fluctuate over time, therefore it’s more suitable to be formulated as stochastic function rather than deterministic.
Some researchers relaxed the deterministic demand assumption and proposed that the demand is modeled under stochastic environment. Ouyang et al. [4] developed a mathematical model of vendor-buyer integrated inventory model which minimizes expected total cost and assumes that the shortage during lead time is allowed and the lead time can be reduced using an added cost. Later, Glock [5] developed a continuous review inventory policy between vendor-buyer and investigated the effect of lead time reduction on decision variables and expected total cost. Chan et al. [6] developed a periodic review inventory policy for a vendor and a buyer where buyer have access to choose two delivery modes in making order to vendor.

The management of pharmaceutical supplies is one of the most important managerial problems in the pharmaceutical industry. However, many pharmaceutical industries face difficulties in controlling their products [7]. Pharmaceutical Supply Chain (PSC) can be defined as the coordination of activities related to the flow of drug in multi parties system to minimize the operational cost and increase the competitive advantage [8].

Some researchers have conducted inventory research in area of pharmaceutical supply chain. Priyan and Uthayakumar [8] developed a continuous inventory model for pharmaceutical supply chain consisting of two participants, which are a hospital and a pharmaceutical company. Nematollahi et al. [9] developed a periodic review inventory model for a pharmaceutical supply chain involving two parties with economic and social collaborative decision making scenarios. The results of this study indicate that the number of shipments and service level are very influential on the optimal solutions of these two scenarios. Furthermore, Nematollahi et al. [10] examined the optimal distributor’s visit interval and safety stock in a two-level supply chain inventory model involving a distributor and a retailer.

This paper is structured as follows. Section 1 provides introduction and literature survey. Section 2 presents notations and assumption used to develop the model. Section 3 and section 4 describe model development and proposed procedure, respectively. Finally, section 5 illustrates numerical example and section 6 gives a brief conclusion.

2. Notations and assumptions
2.1. Notations

- \(T\) review period (year) (decision variable)
- \(k\) safety factor (decision variable)
- \(n\) delivery frequency (decision variable)
- \(\mu\) demand rate (units/year)
- \(\sigma\) standard deviation of demand (units/year)
- \(p\) retailer’s product price ($/unit)
- \(w\) distributor’s product price ($/unit)
- \(c\) supplier’s product price ($/unit)
- \(v_r\) lost sale cost ($/unit)
- \(h_r\) retailer’s holding cost ($/unit/year)
- \(h_d\) distributor’s holding cost ($/unit/year)
- \(f_r\) fraction holding cost (%)
- \(A\) ordering cost ($/order)
- \(\xi\) transportation cost ($/shipment)
- \(C_d\) expiry cost ($/unit)
- \(d\) expiration rate (%)
- \(L\) lead time (year)
- \(G(k)\) unit normal loss

2.2. Assumptions

1. The demand is stochastic and follows a normal distribution.
2. Lost sale is allowed.
3. The finished product is always in good condition (not defective).
4. Lead time delivery product is constant.
5. Expiry cost depends on lead time.

3. Model development

In this paper, we assume that $DT$ units are ordered by retailer from distributor at certain period. Facing the retailer’s order, the distributor will fulfill the order by making a replenishment of $nDT$ units to supplier. Therefore, we can determine the length of ordering cycle, which is $T$.

3.1. Retailer’s annual total profit per unit time

Annual profit for retailer consists of revenue from selling the products and costs, which are holding cost, lost sale cost, and expiry cost. Revenue of retailer ($R_r$) per unit time is the gross profit obtained by retailer when selling product to end customers, which is

$$R_r = \mu p - w I = d$$  

(1)

Holding cost ($h_r$) per unit time is cost incurred by retailer for storing product from distributor in a certain period of time. The formulation of holding cost is given by equation (2)

$$C_{hr} = (f_r x p) \left( \frac{\mu T}{2} + \sigma G(k) \sqrt{T + L} + k \sigma \sqrt{T + L} \right)$$  

(2)

Lost sale cost ($v_r$) per unit is cost incurred by retailer if the retailer can’t fulfill the customer’s demand. The formulation of lost sale cost is given by

$$C_{v_r} = (p-w+v_r) \sigma G(k) \sqrt{T+L}$$  

(3)

Expiry cost of retailer ($C_d$) per unit is cost incurred by retailer to handle a number of product that have expired. Equation (4) shows the formulation of expiry cost

$$C_d = dL\mu C_{dr}$$  

(4)

Therefore, the retailer’s expected annual total profit per unit time can be computed by subtracting the total cost from the revenue. Mathematically, the retailer’s total profit is formulated by

$$\Pi_R = \mu (p-w) - \left( f_r x p \right) \left( \frac{\mu T}{2} + \sigma G(k) \sqrt{T + L} + k \sigma \sqrt{T + L} \right) - \frac{(p-w+v_r) \sigma G(k) \sqrt{T+L}}{T} - dL\mu C_{dr}$$  

(5)

where $\psi k = f sk - k[1 - Fs(k)]$.

3.2. Distributor’s annual total profit per unit time

As we assume in the previous section that we adopt vendor-managed inventory policy to formulate the model, then the distributor will incur ordering cost. To calculate the distributor’s annual profit we use a similar method as described in the above sub section. Revenue of distributor ($R_d$) per unit time is the gross profit obtained by distributor when sending a number of products to retailer. The formulation of revenue is given below

$$R_d = (w-c)(\mu - \frac{\sigma G(k) \sqrt{T+L}}{T})$$  

(6)

Ordering cost ($A_d$) per unit time is cost incurred by distributor when the retailer order a number of products. Equation (7) presents the calculation of ordering cost

$$C_{od} = \frac{A_d}{T}$$  

(7)

Holding cost ($h_d$) per unit time is cost incurred by distributor for storing product from supplier in a certain period of time. The formulation of holding cost is

$$C_{hd} = \frac{1}{2} \left( f_r x w \right) (\mu T - \sigma G(k) \sqrt{T + L})(n - 1)$$  

(8)

Transportation cost of distributor ($\zeta$) per unit time is cost incurred by distributor during delivery product to retailer. The formulation of transportation cost is given below

$$C_{\zeta} = \frac{\zeta}{T}$$  

(9)

Therefore, the annual total profit for distributor which is composed of revenue, ordering cost, holding cost, and transportation cost, is expressed by the following equation
3.3. The joint expected annual total profit per unit time

As we assume in the previous section that both parties make a decision related to inventory jointly to maximize their profit. Hence, they share the information, such as costs, demand and lead time to each other. Therefore, the joint total profit per unit time for a pharmaceutical supply chain is given by

\[ \Pi_T = \mu (p - w) + (w - c) \left( \mu - \frac{\sigma G(k) \sqrt{T+L}}{T} \right) - \frac{1}{2} \left( f_r x w \right) \left( \mu T - \sigma G(k) \sqrt{T+L} \right) \left( n - 1 \right) - \frac{\xi}{T} \]  

(10)

4. Solution methodology

In this section, we construct an iterative procedure to determine the optimal values of \( T, k, n \) and \( \xi \) in order to maximize the value of expected total profit. Here, basic procedure of Jauhari [11] is used to develop this procedure. The proposed algorithm to find the solutions is listed below

1. Set \( n = 1 \) and \( \Pi_T (T_{n-1}, k_{n-1}, n-1) = 0 \)
2. Start with review period \( T = \frac{A_d + \xi_d}{\mu f_r w (n - 1) + \frac{\xi}{2} f_r p} \)
3. Substitute \( T \) into Equation (15) to find \( k \).
4. Compute \( T \) using Equation (14).
5. Repeat steps 3-4 until no change occurs in the values of \( T \) and \( k \).
6. Set $T^* = T$ and $k^* = k$, compute $\Pi_T (T_{n-1}, k_{n-1}, n-1)$ using Equation (11).
7. If $\Pi_T (T^*_n, k^*_n, n^*_n) \geq \Pi_T (T_{n-1}, k_{n-1}, n-1)$ repeat steps 2 – 6 with $n = n+1$, otherwise go to step 8.
8. Compute $\Pi_T (T^*_n, k^*_n, n) = \Pi_T (T_{n-1}, k_{n-1}, n-1)$, then $T^*_n, k^*_n, n$ are the solutions of the model.

This algorithm is proposed to find the solutions that were described in Section 3. We use $n = 1$ as initial values of order interval of finished product. Then, step 3 until step 5 are developed to find the convergence values of $T$ and $k$. After finding the convergence values of $T$ and $k$, we determine the optimal values of $T$ and $k$. The computation of the total profit is shown in step 7. If total profit is higher than the previous one, then we must increase the value of $n$ by one. Finally the algorithm is stop when we can’t find the higher value of total profit again.

5. Numerical example
In this section, a numerical example is presented to illustrate is the feasibility of the above solution procedure. Let us consider an inventory problem with the following data: $\mu = 1,800$ units/year, $\sigma = 15$ units/year, $p = $ 45/unit, $w = $ 33/unit, $c = $ 20/unit, $vr = $ 3/unit, $fr = 10\%$ (0.1), $A = $ 150/order, $\xi = $ 50/shipment, $Cd = $ 1/unit, $d = 5\%$ (0.05), $L = 0.02$ year. By using the proposed procedure explained above, the solutions can be obtained easily.
Therefore, the optimal values of $T^*, k^*$, and $n^*$, satisfying the proposed solution procedure, are 0.178 years, 2,253, and 12 deliveries which can maximize the value of $JTP$ at $44,362.78.

Table 1. Results of numerical example

| n  | k     | T      | TPR   | TPD   | JTP   |
|----|-------|--------|------|------|-------|
| 1  | 1.758 | 0.337  | $20,240.28$ | $22,786.69$ | $43,226.97$ |
| 2  | 1.891 | 0.280  | $20,466.13$ | $22,232.74$ | $43,198.87$ |
| 3  | 1.973 | 0.250  | $20,581.33$ | $21,553.23$ | $43,284.56$ |
| 4  | 2.031 | 0.232  | $20,654.17$ | $20,957.24$ | $43,461.41$ |
| 5  | 2.076 | 0.219  | $20,705.52$ | $20,409.89$ | $43,565.41$ |
| 6  | 2.113 | 0.209  | $20,744.24$ | $19,894.76$ | $43,589.00$ |
| 7  | 2.145 | 0.202  | $20,774.77$ | $19,402.77$ | $43,727.54$ |
| 8  | 2.172 | 0.195  | $20,799.66$ | $18,928.31$ | $43,877.96$ |
| 9  | 2.196 | 0.190  | $20,820.44$ | $18,467.66$ | $43,938.10$ |
| 10 | 2.217 | 0.186  | $20,838.13$ | $18,018.24$ | $44,106.37$ |
| 11 | 2.236 | 0.182  | $20,853.44$ | $17,578.14$ | $44,281.58$ |
| 12 | 2.253 | 0.178  | $20,866.85$ | $17,145.93$ | $44,362.78$ |
| 13 | 2.269 | 0.175  | $20,878.73$ | $16,720.51$ | $43,949.24$ |

Sensitivity analysis is carried out on four parameters, which are expiration rate, lead time, distributor’s holding cost, and transportation cost. The results of the analysis are depicted in figure 1, figure 2, figure 3, and figure 4. Figure 1 shows the correlation of expiration rate to decision variables and joint profit. The expiration rate indicate how many products have expired from the total inventory. The value of review period is significantly affected by the change of expiration rate. Safety factor is constant due to the change of expiration rate. While joint profit slightly increases as there is increase in expiration rate.
Figure 1. Correlation between expiration rate to decision variable and joint profit

Figure 2 shows the influence of lead time on decision variables and joint profit. When the value of lead time decreases 40%, the value of period review decreases 4% and the value of safety factor and total joint profit seem unchanged. The value of period review increases 4% due to the increase in expiration rate.

Figure 2. Correlation between lead time to decision variable and joint profit

Figure 3. Correlation between holding cost of distributor to decision variable
From figure 3, it can be seen that the decision variables change as there is variation in distributor’s holding cost. The value of safety factor increases 3.8%, the value of period review decreases 9%, and the value of joint total profit decreases 5% when the holding cost of distributor is reduced 40%. The value of safety factor increases 1%, the value of period review increases 3%, and the value of joint total profit increases 4% when the holding cost of distributor is increased 40%. While the value of the number of deliveries from distributor to retailer is constant in 12 deliveries. Figure 4 shows the influence of transportation cost on decision variables and joint profit. When the transportation cost decreases 40%, the joint profit decreases 5%. Later, the safety factor seems unchange due to the change of transportation cost. However, there is a change in the value of review period (3%) The value of safety factor increases 1%, the value of period review increases 4%, and the value of joint total profit increases 5% when the transportation cost of distributor is increased 40%. While the value of number deliveries product from distributor to retailer is constant in 12 deliveries.

The proposed model can be used as a guidance for the manager working in a pharmaceutical supply chain consisting of single distributor and single retailer. The proposed model can help the practitioners to keep inventories efficiently by controlling the optimum level of inventory such as, review period and safety factor, so the minimum cost can be achieved. Our focus is to consider expiration rate which is the most important issue in pharmaceutical supply chain. As our model is developed in distributor-retailer system, the expiry cost will be incurred by the distributor. However, if the system is extended to more complex system such as a three echelon system, the model may not suitable to be applied. In a tree echelon perspective, the expiry cost should be charged by the manufacturer. Furthermore, the proposed model may not appropriate for managing the pharmaceutical products which are classified in class A (tight class). For products categorized in class A is better to use a tight policy, namely base stock system or continuous review policy.

6. Conclusion

This paper proposes a vendor-buyer integrated model for a distributor-retailer system with expiry cost and periodic review policy. The model is proposed to determine the optimal values of delivery frequency, safety factor and review period. A simple procedure is suggested to determine the model’s optimal solutions. The numerical example results show that the proposed algorithm can be used to determine optimal solution which maximizes the joint total profit. This model can help managers to manage the inventory of pharmaceutical supply chain efficiently.

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