Recently, mirror symmetry, a phenomenon in superstring theory, has been used to give tentative calculations of several numbers in algebraic geometry. This yields predictions for the number of rational curves of any degree $d$ on general Calabi-Yau hypersurfaces in $\mathbb{P}^4$, $\mathbb{P}(2, 1^4)$, $\mathbb{P}(4, 1^4)$, and $\mathbb{P}(5, 2, 1^3)$ [4, 9, 11]. The techniques used in the calculation rely on manipulations of path integrals which have not yet been put on a rigorous mathematical footing. On the other hand, there is currently no prospect of calculating most of these numbers by algebraic geometry.

Until this point, three of these numbers have been verified, all for the quintic hypersurface in $\mathbb{P}^4$: the number of lines (2875) was known classically, the number of conics (609250) was calculated in [4], and the number of twisted cubics (317206375) was found recently by Ellingsrud and Strømme [3].

Even more recently [6], higher dimensional mirror symmetry has been used to predict the number of rational curves on Calabi-Yau hypersurfaces in higher dimensional projective spaces which meet 3 linear subspaces of certain dimensions. Again, there is no known way to calculate these using algebraic geometry.

The purpose of this paper is to verify some of these numbers in low degree, giving more evidence for the validity of mirror symmetry. In §1, the number of weighted lines in a weighted sextic in $\mathbb{P}(2, 1^4)$ is calculated,

\footnote{See the papers in [14] for general background on mirror symmetry.}
as well as the number of weighted lines in a weighted octic in \( P(4,1^4) \). In §2, the number of lines on Calabi-Yau hypersurfaces of dimension up to 10 which satisfy certain incidence properties is calculated. In §3, the number of conics on these same Calabi-Yau hypersurfaces satisfying the same incidence properties is calculated. These numbers are closely related to the Gromov-Witten invariants defined in [13, 12, 6]; and it is these numbers that are recorded here. In all instances, the calculations agree with those predicted by mirror symmetry. Thus the number of verified predictions has increased from 3 to 65.

There are two parts to all of these calculations. The first part is to express the desired numbers in terms of the standard constructs of intersection theory. The second part is to evaluate the number using the Maple package SCHUBERT \[8\] (although the number of weighted lines in a weighted octic in \( P(4,1^4) \) was first found via classical enumerative geometry, using a classical enumerative formula). The short SCHUBERT code is not included here, but is available upon request.

While it is checked that the data being enumerated is finite, no attempt has been made here to check that the multiplicities are 1. All enumeration takes multiplicities into consideration. This suffices for comparison to the numbers arising in physics, since the Feynman path integrals would take account of any multiplicities greater than 1 as well.

Some of the Gromov-Witten invariants were computed in [3] using an intriguing relation between the various invariants. These relations arise in conformal field theory. A mathematical proof of the relations for the invariants corresponding to lines is sketched here.

It is appropriate to point out the recent work of Libgober and Teitelbaum [11, 12], who have apparently correctly guessed the mirror manifold of complete intersection Calabi-Yau threefolds in an ordinary projective space. Their conjectured mirrors yield predictions for the numbers of rational curves. The predicted number of lines coincides with the results of a calculation done by Libgober 20 years ago, and the predicted number of conics coincides with the results of an unpublished calculation done by Strømme and Van Straten in 1990.

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1 Weighted projective spaces and their Grassmannians

Let $P(k, 1^n)$ denote an $n$-dimensional weighted projective space with first coordinate having weight $k > 1$, all other coordinates having weight 1. Thus $P(k, 1^n)$ consists of all non-zero $(n + 1)$-tuples $(x_0, \ldots, x_n)$, with $(x_0, \ldots, x_n)$ identified with $(\lambda^k x_0, \lambda x_1, \ldots, \lambda x_n)$ for any $\lambda \neq 0$. Note that $P(k, 1^n)$ is smooth outside the singular point $p = (1, 0, \ldots, 0)$. There is a natural rational projection map $\pi: P(k, 1^n) \longrightarrow P^{n-1}$ defined outside $p$ given by omitting the first coordinate. Let $X$ be a weighted hypersurface of weight $d$. Assume $p \notin X$ (this implies that $k|d$, and that the monomial $x_0^{d/k}$ occurs in an equation for $X$). It is further assumed that $X$ is smooth. The general weighted hypersurface whose weight is a multiple of $k$ is an example of such an $X$.

**Definition** A weighted $r$-plane in $P(k, 1^n)$ is the image of a section of $\pi$ over an $r$-plane in $P^{n-1}$.

Note that weighted $r$-planes do not contain $p$.

Let $P$ be a weighted $r$-plane, with $L$ its image in $P^{n-1}$. Let $(q_0, \ldots, q_r)$ be any homogeneous coordinates on $L \simeq P^r$. Then $P$ may be thought of as the image of $L$ via the mapping $x_0 = f_k(q_0, \ldots, q_r)$, $x_i = l_i(q_0, \ldots, q_r)$, where $f_k$ is a form of degree $k$, and the $l_i$ are all linear. Once $L$ is fixed, we may fix in mind a choice of the $q_i$ and $l_i$.

The moduli space of weighted $r$-planes can be represented (and compactified) as follows. Conventions have been chosen to be consistent with those in Schubert [8]. Let $G = G(r+1, n)$ be the Grassmannian of $r$-dimensional linear subspaces $L$ of $P^{n-1}$. Let $V$ be the $n$-dimensional vector space of linear forms on $P^{n-1}$. This identifies $P^{n-1}$ with $P(V) = Proj(S^*V)$. $G$ is then the space of $r + 1$ dimensional quotients of $V$ (since the space of linear forms restricted to $L$ is an $r + 1$-dimensional quotient of $V$). Let $Q$ be the universal rank $r + 1$ quotient bundle on $G$.

The equation $x_0 - f_k(q_0, \ldots, q_r) = 0$ which describes a section of $\pi$ over an $r$-plane may be identified with a section $s$ of the bundle $\mathbf{C} \oplus S^k(Q)$, where $\mathbf{C}$ denotes the trivial bundle. A scalar multiple of this section would correspond to the equation $ax_0 - af_k(q_0, \ldots, q_r) = 0$, which defines the same weighted $r$-plane. Note that (up to scalar) $s$ does not depend on any of the choices.
which have been made. So $M = \mathbf{P}(\mathbf{C} \oplus S^k(Q)^*)$ gives a compactification of the space of weighted $r$-planes. Here, $\mathbf{P}(E)$ denotes the space of rank 1 quotients of the fibers of the bundle $E$; hence the need for dualizing in defining $M$.

In the sequel, we will also refer to $M^o \subset M$, the open subset which corresponds to the actual weighted $\mathbf{P}^1$’s, in other words, $M^o = M - \mathbf{P}(S^kQ^*)$, where $\mathbf{P}(S^kQ^*)$ is included in $M$ via the map induced by the natural projection $\mathbf{C} \oplus S^kQ^* \to S^kQ^*$.

2 Lines on weighted hypersurface Calabi-Yau threefolds

Now let $k > 1$, and let $X \subset \mathbf{P}(k,1^4)$ be a smooth weighted hypersurface of weight $k + 4$ with $(1,0,0,0,0) \notin X$. As has been noted in the previous section, this implies that $k|k + 4$, which in turn implies that $k = 2$ or $k = 4$. The weight $k + 4$ has been chosen to ensure that $X$ is Calabi-Yau, i.e. that $X$ has trivial canonical bundle.

The rational projection map $\pi$ restricts to a morphism $\pi : X \to \mathbf{P}^3$. This is a 3-1 cover for $k = 2$, and a 2-1 cover for $k = 4$. The goal of this section is to enumerate the weighted $\mathbf{P}^1$’s contained in $X$.

Let us first consider the case $k = 4$. Then an equation for $X$ has the form

$$F = ax_0^2 + g_4(x_1, \ldots, x_4)x_0 + g_8(x_1, \ldots, x_4) = 0,$$

where $a \in \mathbf{C}$ and $g_i$ has degree $i$ for $i = 4$ or 8. Such an equation naturally induces a section $s$ of the bundle $\mathbf{C} \oplus S^4Q \oplus S^8Q$. Consider a point $C \in M^o$. We abuse notation by allowing $C$ to also denote the corresponding curve. Let $(q_0, q_1)$ be homogeneous coordinates on $\mathbf{P}^3$. Identifying $C$ with $\mathbf{P}^1$, we may describe $C$ by equations of the form

$$x_0 = f_4(q_0, q_1), \quad x_i = l_i(q_0, q_1).$$

The equation $x_0 - f_4(q_0, q_1) = 0$ and its multiples for varying $C$ form the tautological subbundle $\mathcal{O}_\mathbf{P}(-1) \subset \mathbf{C} \oplus S^4Q$ on $\mathbf{P} = \mathbf{P}(\mathbf{C} \oplus S^4Q^*)$. $C$ is contained in $X$ if and only if an equation for $X$, when pulled back to $\mathbf{P}^1$ via a parametrization of $C$, vanishes. Substituting from the second of equations (2) into (1), it is seen that this happens if and only if $ax_0^2 +
Table 1: The number of lines.

| Ambient space | Weighted degree | Number of lines |
|---------------|-----------------|-----------------|
| P(1^5)       | 5               | 2875            |
| P(2,1^4)     | 6               | 7884            |
| P(4,1^4)     | 8               | 29504           |

Putting all this together, we see that \( C \subset X \) if and only the section \( \bar{s} \) of
\[ B = (C \oplus S^4Q) \oplus O_P(-1) \rightarrow C \oplus S^4Q \oplus S^8Q. \]
induced by \( s \) vanishes at \( C \). Note that if \( C \in M - M^0 \), then \( C \) corresponds to
a curve defined by equations of the form \( f_4(q_0, q_1) = 0, \ x_i = l_i(q_0, q_1) \). Since
such a curve would contain \( p \), it follows that \( C \) is not in the zero locus of \( \bar{s} \).
Also note that \( \dim(M) = \text{rank}(B) = 9 \); so one expects finitely many zeros
of such a section; hence finitely many weighted \( P^1 \)'s. It is easy to prove that
this is indeed the case for general \( X \). The actual number is the degree of
\( c_9(B) \). This may be calculated by standard techniques in intersection theory and the calculation may be implemented via SCHUBERT.

The case \( k = 2 \) is similar. Changing the meaning of the notation in the
obvious manner, one must consider \( M = P(C \oplus S^2Q^*) \), and calculate the
degree of \( c_7(B') \), where
\[ B' = (C \oplus S^2Q \oplus S^4Q \oplus S^6Q)/((C \oplus S^2Q \oplus S^4Q) \otimes O_P(-1)). \]

Combining these with the well-known number of lines on a quintic three-fold, the calculation of some examples considered in via mirror
symmetry may be verified. The results are displayed in table 1.

**Problem:** Verify the predictions of mirror symmetry for weighted \( P^1 \)'s in a
weight 10 hypersurface in \( P(5,2,1^3) \). Also, verify the predictions of mirror
symmetry for weighted conics on the weighted hypersurfaces considered in this section.

Remark: The family of weighted conics on the general weighted octic in \( \mathbb{P}(4,4^4) \) is positive dimensional (independently observed by Kollár); hence part of the problem in this case is to systematically assign numbers to positive dimensional families. This can be defined as the number of such curves that remain almost holomorphic under a general almost complex deformation; but it is desirable to give a purely algebraic description.

3 Lines on higher dimensional varieties

In this section and the next, we consider rational curves on the generic Calabi-Yau hypersurface \( X \) in \( \mathbb{P}^{k+1} \). This is a hypersurface of dimension \( k \) and degree \( k+2 \). For \( k > 3 \), there will be infinitely many lines and conics contained in \( X \). But there will only be finitely many lines or conics which satisfy certain incidence properties with fixed linear subspaces.

Since the normal bundle \( N \) of \( C \) in \( \mathbb{P}^{k+1} \) has degree \(-2\), one expects that for general \( X \) and any \( C \subset X \), \( N \cong \mathcal{O} \oplus \ldots \oplus \mathcal{O} \oplus \mathcal{O}(-1) \oplus \mathcal{O}(-1) \) (with \( k-3 \) \( \mathcal{O} \)'s). Since \( h^0(N) = k-3 \) and \( h^1(N) = 0 \) in this case, the scheme of rational curves on \( X \) is expected to have dimension \( k-3 \).

For each \( i \), let \( L_i \subset \mathbb{P}^{k+1} \) denote a general linear subspace of codimension \( i \). Pick positive integers \( a, b, c \) such that \( a+b+c = k \). Following [13, 12, 1], define an invariant \( n^a_b(d) \) of \( X \) as the number of holomorphic immersions \( f : \mathbb{P}^1 \to X \) with \( f(\mathbb{P}^1) \) of degree \( d \) such that \( f(0) \in L_a \), \( f(1) \in L_b \), \( f(\infty) \in L_c \). These numbers, called “Gromov-Witten invariants” in [12], are expected to be finite. Note that the value of \( c \) is implicit in the notation \( n^a_b(d) \) by virtue of the equation \( a+b+c = k \).

These invariants are essentially the same as the number of reduced, irreducible rational curves of degree \( d \) in \( X \) which meet each of \( L_a, L_b, \) and \( L_c \). The \( n^a_b(d) \) differ from the corresponding numbers of curves by one factor of \( d \) for each of the indices \( a, b, \) or \( c \) equal to 1 (since \( C \) meets a general \( L_1 \) \( d \) times). There is no difference for lines; and for conics, we will see that in the calculation of the number of conics satisfying the required incidence properties, the Gromov-Witten invariants arise naturally. So the Gromov-Witten invariants will be calculated and tabulated, while the numbers of
rational curves follow immediately by division by the appropriate power of $d$, if necessary.

In the remainder of this section, we specialize to $d = 1$, i.e. lines. A theorem of Barth-van de Ven [3] states that the Fano variety of lines on a degree $l$ hypersurface $X \subset \mathbb{P}^n$ is smooth of dimension $2n - l - 3$ for generic $X$ when $l + 3 \leq 2n$. Applied in the present context of $X_{k+2} \subset \mathbb{P}^{k+1}$, we find that the variety of lines must be smooth of dimension $k - 3$ whenever $k \geq 3$. From this, standard techniques show that a general $X$ contains finitely many lines which meet each of $L_a$, $L_b$, and $L_c$. So we can calculate the Gromov-Witten invariants by using the Schubert calculus. The lines are parametrized by the Grassmannian $G(2, k+2)$. The class of lines meeting $L_a$ is the Schubert cycle $\sigma_{a-1}$; similarly for $L_b$ and $L_c$. Let $Q$ be the rank 2 universal quotient bundle on $G$. Since the class of the variety of lines on $X$ is represented by $c_k S^{k+2}Q$ and dimensions work out correctly, the answer is the degree of $c_{k+3} S^{k+2}Q \cdot \sigma_{a-1} \cdot \sigma_{b-1} \cdot \sigma_{c-1}$. These may be easily worked out as integers using Schubert. The answers obtained are displayed in table 2.

The original predictions for the numbers found in [6] resulted from a two-step process arising from mirror symmetry and conformal field theory. First, the $n_{a,b}^l(d)$ are found, followed by what amounts to an expression for any $n_{i,j}^l(d)$ in terms of the various $n_{a,b}^l(d')$ for $d' \leq d$. Most of these expressions remain a mathematical mystery at present. However, the case $d = 1$ can be established mathematically as follows.

**Theorem** Let $X$ be any Calabi-Yau manifold of dimension $k$ in any projective space. Define $n_{a,b}^0(1)$ as above. Assume that there are finitely many lines in $X$ satisfying each of the respective incidence conditions needed to define the $n_{a,b}^0(1)$. Then

$$n_{j}^l(1) = \sum_{i=0}^{j-l} n_{i+l}(1) - \sum_{l=1}^{j-1} n_{i}^l(1).$$

**Proof (sketch).** Follows immediately by intersecting the cycle class (in the appropriate Grassmannian) of the scheme of lines in $X$ with the identity

$$\sigma_{i-1} \sigma_{j-1} \sigma_{k-i-j-1} = \sum_{l=0}^{j-1} \sigma_{i+l-1} \sigma_{k-i-l-2} - \sum_{l=1}^{j-1} \sigma_{l-1} \sigma_{k-l-2},$$

an identity which can be proven by a few applications of Pieri’s formula.
| $k$ | $n^k_1(1)$ | $n^k_2(1)$ | $n^k_3(1)$ |
|-----|-------------|-------------|------------|
| 3   | $n^3_1(1) = 2875$ |             |            |
| 4   | $n^4_1(1) = 60480$ |             |            |
| 5   | $n^5_1(1) = 1009792$, $n^5_2(1) = 1707797$ | |            |
| 6   | $n^6_1(1) = 15984640$, $n^6_2(1) = 37502976$, $n^6_3(1) = 59021312$ | | |
| 7   | $n^7_1(1) = 253490796$, $n^7_2(1) = 763954092$, $n^7_3(1) = 1069047153$ | $n^7_2(1) = 1579510449$ | |
| 8   | $n^8_1(1) = 4120776000$, $n^8_2(1) = 15274952000$, $n^8_3(1) = 27768048000$ | $n^8_2(1) = 38922224000$, $n^8_3(1) = 51415320000$ | |
| 9   | $n^9_1(1) = 69407571816$, $n^9_2(1) = 307393401172$, $n^9_3(1) = 695221679878$ | $n^9_2(1) = 905702054829$, $n^9_3(1) = 933207509234$, $n^9_3(1) = 1531516162891$ | $n^9_3(1) = 1919344441597$ |
| 10  | $n^{10}_1(1) = 1217507106816$, $n^{10}_2(1) = 6306655500288$ | $n^{10}_3(1) = 17225362851840$, $n^{10}_4(1) = 28015971489792$ | $n^{10}_2(1) = 22314511245312$, $n^{10}_3(1) = 44023827234816$ |
|     | $n^{10}_1(1) = 54814435872768$, $n^{10}_3(1) = 65733143224320$ | $n^{10}_3(1) = 54814435872768$, $n^{10}_3(1) = 65733143224320$ | |

Table 2: Gromov-Witten invariants for lines.
Table 3: Gromov-Witten invariants for conics.

| $k$ | $n_k^2(2)$ |
|-----|------------|
| 3   | $n_1^2(2) = 4874000$ |
| 4   | $n_1^2(2) = 1763536320$ |
| 5   | $n_1^2(2) = 488959144352, n_2^2(2) = 1021575491286$ |
| 6   | $n_1^2(2) = 133588638826496, n_2^2(2) = 448681408315392, n_3^2(2) = 821654025830400$ |
| 7   | $n_1^2(2) = 39031273362637440, n_2^2(2) = 187554590257349088, n_3^2(2) = 506855012110118424$ |
| 8   | $n_1^2(2) = 1260796543571824000, n_2^2(2) = 80684596772238448000, n_3^2(2) = 295035175517918176000$ |
| 9   | $n_1^2(2) = 4565325719860021608624, n_2^2(2) = 3700500182380218657624, n_3^2(2) = 444475303469701680000$ |
| 10  | $n_1^2(2) = 186179182239762093573744, n_2^2(2) = 18415607624138339954786304, n_3^2(2) = 4987056763838232368404990$ |
|     | $n_1^2(2) = 83885220561474498867757056, n_2^2(2) = 1799828409247495843588669444, n_3^2(2) = 1072278991421919158312960$ |
|     | $n_2^2(2) = 1072278991421919158312960, n_3^2(2) = 297755098999730079369412608$ |
|     | $n_3^2(2) = 41795036446757098415214592, n_4^2(2) = 527556832251612742800359424$ |

4 Conics on higher dimensional varieties

It can easily be shown that if $X$ is a general hypersurface of degree $k + 2$ in $\mathbb{P}^{k+1}$, then the variety of conics on $X$ has the expected dimension $k - 3$. Standard techniques show that given positive integers $a, b, c$ with $a + b + c = k$, there will be a finite number of conics in $X$ which meet each of $L_a, L_b,$ and $L_c$. Thus the Gromov-Witten invariants are finite. They will be calculated here; the answers obtained are displayed in table 3.

We start with the well known description of the moduli space of conics in $\mathbb{P}^{k+1} = \mathbb{P}(V)$, where $V$ is a $k + 2$-dimensional vector space. To describe a
conic, we first describe the 2 plane it spans, and then choose a quadric in that 2-plane (up to scalar). So let $G = G(3, V)$ be the Grassmannian of 2-planes in $\mathbb{P}(V)$ (that is, of rank 3 quotients of $V$), and let $Q$ be the universal rank 3 quotient bundle of linear forms on the varying subspace. Then the moduli space of conics is $M = \mathbb{P}(S^2Q^*)$. Following the reasoning in section 2 (or [7]), the scheme of conics on $X$ is given by the locus over which a certain section of $F = S^{k+2}Q/(S^k Q \otimes \mathcal{O}_{\mathbb{P}}(-1))$ vanishes. Here $\mathcal{O}_{\mathbb{P}}(1)$ is the tautological sheaf on $\mathbb{P}(S^2Q^*)$. Since $F$ has rank $2k+5$, the conics on $X$ are represented by $c_{2k+5}(F)$.

It remains to find the condition that a conic $C$ meets $L_a$. One way to find this is to consider the moduli space $\mathcal{M}$ of pointed conics, i.e. pairs $(p, C)$, with $C$ a conic, $p \in C$. This may easily be constructed as a bundle over $\mathbb{P}^{k+1}$, with fiber over $p \in \mathbb{P}^{k+1}$ being the set of conics containing $p$. We start by constructing the moduli space of pointed 2-planes as follows. Consider the tautological exact sequence on $\mathbb{P}^{k+1}$:

$$0 \to K \to V_{\mathbb{P}^{k+1}} \to \mathcal{O}(1) \to 0,$$

where $V_{\mathbb{P}^{k+1}}$ is a trivial bundle of rank $k+2$ on $\mathbb{P}^{k+1}$ (more generally, $E_Y$ will stand for the pullback of $E$ to $Y$, the morphism used for the pullback assumed to be clear in context). Let $H = G(2, K)$ be the Grassmannian of rank 2 quotients of $K$, $Q$ its universal rank 2 quotient, and $S \subset K_H$ the universal subbundle. These fit into the exact sequence

$$0 \to S \to K_H \to Q \to 0$$

of sheaves on $H$. The natural quotient $V_H \to V_H/S$ induces a map $H \to G$ by the universal property of the Grassmannian; since $V_H \to \mathcal{O}(1)_H$ clearly factors through $V_H/S$, it is easy to see that $H$ may be identified with the space of pointed 2-planes in $\mathbb{P}(V)$. Here $\mathcal{O}(1)$ denotes the tautological sheaf on $\mathbb{P}(V)$ as before.

The conics containing $p$ globalize to a rank 5 bundle $W$ on $H$. This bundle is in fact the kernel of the natural map $S^2(V_H/S) \to \mathcal{O}(2)_H$. Then the moduli space $M'$ of pointed conics may be seen to be $\mathbb{P}(W^*)$. Let $\mathcal{O}_W(1)$ be its tautological bundle.

Let $h = c_1(\mathcal{O}(1)_{M'})$. Consider the natural morphism $f : M' \to M$. The variety of conics meeting $L^a$ is represented by the class $f_*(h^a)$ for $a > 1$. Note that for $a = 1$, $f_*(h) = 2$. This factor exactly gives the factor
needed to give the Gromov-Witten invariants rather than the number of conics meeting three linear subspaces. So the Gromov-Witten invariants are given by the formula \[ n_a^b(2) = \int_{M} c_{2k+5}(Q) f_*(h^a) f_*(h^b) f_*(h^c), \]
which is valid since the dimensions work out correctly.

To compute these as numbers using \textsc{schubert}, everything is clear, except the description of the morphism \( f \). But this may be described merely by knowing the pullbacks \( f^*(Q) \) and \( f^*(\mathcal{O}_{\mathbb{P}(1)}) \). However, from the above description and the universal properties, this is just \( V_H/S \) and \( \mathcal{O}_W(1) \). \textsc{schubert} takes care of the rest.

\textbf{References}

[1] W. Barth and A. van de Ven. Fano-varieties of lines on hypersurfaces. \textit{Arch. Math.}, 31(1):96–104, 1978.

[2] P. Candelas, X.C. de la Ossa, P.S. Green, and L. Parkes. A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory. \textit{Nucl. Phys. B}, 359:21–74, 1991.

[3] G. Ellingsrud and S. A. Strømme. The number of twisted cubic curves on the general quintic threefold. Univ. of Bergen Preprint 63-7-2-1992.

[4] A. Font. Periods and duality symmetries in Calabi-Yau compactifications. Universidad Central de Venezuela preprint UCVFC/DF-1-92, 1992.

[5] William Fulton. \textit{Intersection Theory}. Springer-Verlag, Berlin Heidelberg New York Tokyo, 1984.

[6] B. R. Greene, D. R. Morrison, and M. R. Plesser. Mirror manifolds in higher dimension. In preparation.

[7] Sheldon Katz. On the finiteness of rational curves on quintic threefolds. \textit{Comp. Math.}, 60:151–162, 1986.

[8] Sheldon Katz and Stein Arild Strømme. \textsc{schubert}: a \textsc{maple} package for intersection theory. Available by anonymous ftp from ftp.math.okstate.edu or linus.mi.uib.no, cd pub/schubert.
[9] A. Klemm and S. Thiesen. Considerations of one-modulus Calabi-Yau compactifications: Picard-Fuchs equations, Kähler potentials and mirror maps. Universität Karlsruhe and Technische Universität München preprint KA-THEP-03/92, TUM-TH-143-92, 1992.

[10] A. Libgober and J. Teitelbaum. Lines on Calabi Yau complete intersections, mirror symmetry, and Picard Fuchs equations. Preprint, University of Illinois at Chicago.

[11] D. R. Morrison. Picard-Fuchs equations and mirror maps for hypersurfaces. In S.-T. Yau, editor, Essays on Mirror Manifolds, pages 241–264. International Press, Hong Kong, 1992.

[12] D. R. Morrison. Hodge-theoretic aspects of mirror symmetry. In preparation.

[13] E. Witten. Topological sigma models. Commun. Math. Phys., 118:411–449, 1988.

[14] S.-T. Yau, editor. Essays on Mirror Manifolds, Hong Kong, 1992. International Press.