Large Solar Neutrino Mixing and Radiative Neutrino Mechanism

Teruyuki Kitabayashia * and Masaki Yasueb †

a Accelerator Engineering Center
Mitsubishi Electric System & Service Engineering Co.Ltd.
2-8-8 Umezono, Tsukuba, Ibaraki 305-0045, Japan

b Department of Natural Science
School of Marine Science and Technology, Tokai University
3-20-1 Orido, Shimizu, Shizuoka 424-8610, Japan

and

Department of Physics, Tokai University
1117 KitaKaname, Hiratsuka, Kanagawa 259-1292, Japan
(TOKAI-HEP/TH-0106, October, 2001)

We find that the presence of a global $L_e - L_\mu - L_\tau$ ($\equiv L'$) symmetry and an $S_2$ permutation symmetry for the $\mu$- and $\tau$-families supplemented by a discrete $Z_4$ symmetry naturally leads to almost maximal atmospheric neutrino mixing and large solar neutrino mixing, which arise, respectively, from type II seesaw mechanism initiated by a discrete $Z_4$-symmetric triplet Higgs scalar $s$ with $L' = 2$ and from radiative mechanism of the Zee type initiated by two singly charged scalars, an $S_2$-symmetric $h^+$ with $L' = 0$ and an $S_2$-antisymmetric $h'^+$ with $L' = 2$. The almost maximal mixing for atmospheric neutrinos is explained by the appearance of the democratic coupling of $s$ to neutrinos ensured by $S_2$ and $Z_4$ while the large mixing for solar neutrinos is explained by the similarity of $h^+$- and $h'^+$-couplings described by $f^h_+ \sim f^h_-$ and $f^{h'}_+ \sim f^{h'}_-$, where $f^h_+ (f^h_-)$ and $f^{h'}_+ (f^{h'}_-)$ stand for $h^+$- ($h'^+$)-couplings, respectively, to leptons and to Higgs scalars.

PACS: 12.60.-i, 13.15.+g, 14.60.Pq, 14.60.St

Keywords: neutrino mass, neutrino oscillation, radiative mechanism, triplet Higgs

Neutrino oscillations have been long recognized to occur if neutrinos have masses [1]. The experimental confirmation of such neutrino oscillations has been given by the Super-Kamiokande collaboration [2] for atmospheric neutrinos and the clear evidence of the solar neutrino oscillations has been released by the SNO collaboration [3]. These observed oscillation phenomena can be explained by the mixings between $\nu_e$ and $\nu_\mu$, with $\Delta m^2_{\odot} \lesssim 10^{-4}$ eV$^2$ for solar neutrinos and between $\nu_\mu$ and $\nu_\tau$ with $\Delta m^2_{\text{atm}} \sim 3 \times 10^{-3}$ eV$^2$ for atmospheric neutrinos [4]. Their masses are implied to be as small as $\mathcal{O}(10^{-2})$ eV and the smallness of neutrino masses can be explained by either the seesaw mechanism [5] or the radiative mechanism [6,7]. The mixing specific to atmospheric neutrinos is found to prefer maximal mixing [8]. It has also been suggested for solar neutrinos that solutions with large mixing angles are favored while solutions with small mixing angles are disfavored [9]. Therefore, both neutrino oscillations are characterized by large neutrino mixings.

One of the promising theoretical assumptions to account for the observed mixing pattern is to use the bimaximal mixing scheme [10,11]. The radiative mechanism of the Zee-type [6] provides the natural explanation on bimaximal neutrino mixing [12] when combined with a global $L_e - L_\mu - L_\tau$ ($\equiv L'$) symmetry [13,14] since the Zee model only supplies flavor-off-diagonal mass terms. However, the recent extensive analyses on solar neutrino oscillation data imply that the maximal solar neutrino mixing is not well compatible with the data, which prefer $\sin^2 \theta_{12} \sim 0.8$ for the large mixing angle (LMA) MSW solution [15]. If this observed tendency of solar neutrino oscillations with large mixing but not with maximal mixing is really confirmed, the bimaximal structure in the Zee model should be modified [16].

In this report, we discuss a possible modification of the Zee model with the $L'$ symmetry to accommodate the LMA solution without the maximal solar neutrino mixing [17]. The original Zee model requires the presence of a Higgs scalar of $\phi'$, the duplicate of the standard Higgs scalar $\phi$, which initiates radiative neutrino mechanism together with a singly charged scalar of $h^+$, and assumes $\phi'$ to couple to no leptons. One of the modifications is to relax this constraint such that $\phi'$ couples to leptons. By allowing $\phi'$ to generate lepton masses, the authors of Ref. [18] have found that solar neutrinos can exhibit $\sin^2 \theta_{12} \sim 0.8$ but their realization of the large solar neutrino mixing entails various fine-tunings, which seem unnatural. We, instead, rely upon a certain underlying symmetry to constrain the

*E-mail:teruyuki@post.kek.jp
†E-mail:yasue@keyaki.cc.u-tokai.ac.jp
interactions of \( \phi' \) with leptons and utilize an \( S_2 \) permutation symmetry for the \( \mu \)- and \( \tau \)-families, which is responsible for the appearance of the almost maximal atmospheric neutrino mixing [19]. Under \( S_2 \), \( \phi \) transforms as a symmetric state and \( \phi' \) transforms as an antisymmetric state.

In order to realize the large mixing, the natural resolution is to include flavor-diagonal mass terms because the main source of \( \sin^2 2\theta_{12} \approx 1 \) in the Zee model comes from the constraint on neutrino masses of \( m_{1,2,3} \) given by \( m_1 + m_2 + m_3 = 0 \) specific to flavor-off-diagonal mass terms. It is known that flavor-diagonal mass terms can be supplied by an \( SU(2)_L \)-triplet Higgs scalar \([20]\) denoted by

\[
s = \begin{pmatrix} s^+ & s^{++} & 0 \\ s^0 & -s^+ & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]

whose vacuum expectation value (VEV), \( \langle 0 | s^0 | 0 \rangle \), generates neutrino masses via interactions of \( \bar{\nu}_L^T s \nu_L \), where the subscript \( c \) denotes the charge conjugation including the \( G \)-parity of \( SU(2)_L \). The smallness of the neutrino masses can be ascribed to that of \( \langle 0 | s^0 | 0 \rangle \), which is given by \( \sim \mu \langle |\phi| |\phi| / m_s \rangle^2 \) produced by the combined effects of \( \mu |\phi| \phi^c \) and \( m_s^2 \text{Tr}(s^T s) \), where \( \mu \) and \( m_s \) are mass parameters. The type II seesaw mechanism \([21]\) can ensure tiny neutrino masses by the dynamical requirement of \( \langle 0 |\phi| 0 \rangle \ll m_s \) with \( \mu \sim m_s \).

To see which masses of flavor neutrinos give contributions to yield \( \sin^2 2\theta_{12} \neq 1 \), we examine a possible neutrino mass texture that can be diagonalized by \( U_{\text{MNS}} \) with two mixing angles, \( \theta_{12} \) and \( \theta_{23} \), which, respectively, connect \( (\nu_1, \nu_2) \) with \( (\nu_e, \nu_\mu) \) and \( (\nu_2, \nu_1) \) with \( (\nu_\mu, \nu_\tau) \), where \( (\nu_1, \nu_2, \nu_3)^T = (|\nu_{\text{mass}}\rangle) \) with \( (m_1, m_2, m_3) \) and \( (\nu_e, \nu_\mu, \nu_\tau)^T = (|\nu_{\text{weak}}\rangle) \) are related by \( |\nu_{\text{weak}}\rangle = U_{\text{MNS}} |\nu_{\text{mass}}\rangle \). The resulting mass matrix denoted by \( M' \) takes the form of

\[
M' = \begin{pmatrix} a & b & c = -t_{23} b \\ b & d & e \\ c & e & f = (-d + (t_{23}^{-1} - t_{23}) e) \end{pmatrix},
\]

where the atmospheric neutrino mixing is specified by \( t_{23} = \sin \theta_{23} / \cos \theta_{23} \) [18,19,22]. The masses and the solar neutrino mixing angle of \( \theta_{12} \) are calculated to be:

\[
m_1 = a - \frac{1}{2} \sqrt{\frac{b^2 + c^2}{2}} (x + \eta \sqrt{x^2 + 8}), \quad m_2 = (\eta - \eta \text{ in } m_1),
\]

\[
m_3 = d + t_{23}^2 \left( d - a + x \sqrt{\frac{b^2 + c^2}{2}} \right),
\]

\[
\sin^2 2\theta_{12} = \frac{8}{8 + x^2} \text{ with } x = \frac{a - d + t_{23} e}{\sqrt{(b^2 + c^2)/2}},
\]

where \( c = -t_{23} b \) and \( |m_1| < |m_2| \) is always maintained by adjusting the sign of \( \eta (= \pm 1) \). The result shows that the significant deviation of \( \sin^2 2\theta_{12} \) from unity is only possible if \( (a - d + t_{23} e)^2 = \mathcal{O}(b^2 + c^2) \). In our subsequent discussions, we take the “ideal” solution [23] with \( t_{23} = \pm 1 \) \((\equiv \sigma)\) given by

\[
M'_{\text{ideal}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & d & \sigma d \\ 0 & \sigma d & d \end{pmatrix},
\]

which provides \( m_1 = m_2 = 0 \) and \( m_3 = 2d \). The deviation from this solution that yields \( b \neq 0 \) and \( c \neq 0 \) is caused by radiative effects, which also add additional contributions to \( d \) and \( e \), and \( (d - t_{23} e)^2 = \mathcal{O}(b^2 + c^2) \) is ensured by \( S_2 \). Then, the splitting between \( m_1 \) and \( m_2 \) is induced to yield the LMA solution with \( \sin^2 2\theta_{12} \sim 0.8 \).

To realize the “ideal” solution, we introduce a permutation symmetry, \( S_2 \), for the \( \mu \)- and \( \tau \)-families \([24]\) as have been announced, which is compatible with the requirement of \( L' \). For \( s \) with \( L' = 2 \), \( s \) only couple to the \( \mu \)- and \( \tau \)-families, which provide an \( S_2 \)-symmetric democratic mass texture \([25]\) for \( \nu_{\mu,\tau} \) with an additional \( Z_4 \) discrete symmetry to ensure the “ideal” structure of Eq.(5), leading to one massless neutrino \( \nu_2 \) and one massive neutrino \( \nu_3 \). These two neutrinos radiatively mix with \( \nu_e \) finally give observed neutrino mixings.

All interactions are taken to conserve \( L' \) and to be invariant under the transformation of \( S_2 \) as well as \( Z_4 \). The scalars of \( \phi, s \) and \( h^+ \) are assigned to \( S_2 \)-symmetric states. The other scalar of \( \phi' \) is assigned to an \( S_2 \)-antisymmetric state and we introduce an additional copy of \( \phi' \) and \( h^+ \) as \( S_2 \)-antisymmetric states denoted by \( \phi'' \) and \( h''^+ \). The inclusion of \( \phi'' \) and \( h''^+ \), respectively, allows us to meet the mass hierarchy of \( m_{\mu} \ll m_{\tau} \) and the large solar neutrino mixing satisfying \( (d - t_{23} e)^2 = \mathcal{O}(b^2 + c^2) \). To distinguish these copies from the original fields, it is sufficient to use a discrete symmetry of \( Z_4 \). The quantum numbers of the participating fields are tabulated in TABLE I, where \( \psi_{\pm L} \).
= (ψ_L^\pm \pm \psi_R^\mp)/\sqrt{2} and \ell_{\pm R} = (\tau_R \pm \mu_R)/\sqrt{2}.\) The assignment of the \(Z_2\)-charges of \(\psi_{\pm L}\) and \(s\) forbids the coexistence of \(\psi_{+L}s\psi_{+L}\) and \(\psi_{-L}s\psi_{-L}\), which disturbs the democratic structure of the “ideal” solution. The present assignment corresponds to \(\sigma = 1\) solution of Eq.(5). Since charged leptons simultaneously couple to the Higgs scalars of \(\phi, \phi'\) and \(\phi''\), flavor-changing interactions are induced by the exchanges of these Higgs scalars, which will be shown to give well-suppressed contributions at the phenomenologically consistent level. It is obvious that quarks that are \(S_2\)-symmetric can have couplings to \(\phi\) but not to \(\phi'\) and \(\phi''\); therefore, quarks do not have this type of dangerous flavor-changing interactions.

The Yukawa interactions for leptons are, now, given by

\[
-\mathcal{L}_Y = f_\phi \overline{\psi}_L \phi \rho_R + \overline{\psi}_{+L} (f_+ \phi_{+R} + f_- \phi_{-R}) + \overline{\psi}_{-L} (g_+ \phi_{-R} + g_- \phi'' \ell_{+R}) + f_{\phi'}^L (\overline{\psi}_L \phi' \rho_R + f_{\phi'}^L \overline{\psi}_{+L} \phi' \rho_R + f_{\phi'}^L \overline{\psi}_{-L} \phi' \rho_R + f_{\phi'}^L \overline{\psi}_{+L} \phi' \rho_R + (h.c.),\]

where \(f\)'s stand for coupling constants. Higgs interactions are described by usual Hermitian terms composed of \(\varphi^\dagger (\varphi = \phi, \phi', \phi'', h^+, h^{+\mp}, s)\) and by non-Hermitian terms in \(V_0 = (\lambda_1 \phi'' s \phi + \lambda_2 \phi^3 s \phi') \phi'^\dagger + (h.c.),\)

where \(\lambda_{1,2}\) are Higgs couplings, which conserves \(L\) and \(L'\). The soft breaking terms of \(L\) and \(L'\) can be chosen to be:

\[
V_1 = \mu_+ \phi'^\dagger \phi'' \phi'^\dagger + (h.c.), \quad V_2 = \mu_- \phi'^\dagger \phi'' \phi'^\dagger + (h.c.), \quad V_3 = \mu_1 s \phi^* + \mu_2 s \phi'^* + (h.c.),
\]

where \(\mu\)'s represent the mass scales and \(V_1\) and \(V_3\) are, respectively, used to activate the radiative mechanism and the type II seesaw mechanism. Although \(L\) and \(L'\) are spontaneously broken by \(|0\rangle s|0\rangle\), \(L+L' (\propto L_\tau)\) is still conserved. In terms of the \(L_\tau\)-conservation, \(V_2\) and \(V_3\) are classified as \(L_\tau\)-conserving interactions and its explicit breaking is provided by \(V_1\). \(L_\tau\)-breaking interactions such as those causing \(\mu, \tau \to ee\) and \(\to e\gamma\) necessarily involve \(V_1\). All other interactions are forbidden by the conservation of \(L_\tau\) and \(Z_2\). Especially, \((h^+ h^{+\mp})^\dagger \phi\) could give a divergent term of \(\nu_\tau\nu_\tau\) at the two loop level as depicted in FIG.1 (a), which then would require a tree level mass term of the \(\nu_\tau\nu_\tau\)-term as a counter term. The appearance of this counter term is not consistent with the absence of the tree-level \(\nu_\tau\nu_\tau\)-term in Eq.(5). Since \(L'\) is explicitly broken, \(\nu_\tau\nu_\tau\) is induced by interactions shown in FIG.1 (b) with two \(V_1\) insertions. Fortunately, this diagram leads to the finite convergent term.

Charged lepton masses are generated via the Higgs couplings to leptons, which are specified by the following matrix of \(M_0^L(\phi, \phi', \phi''):\)

\[
M_0^L(\phi, \phi', \phi'') = \begin{pmatrix}
 f_\phi & 0 & 0 \\
 0 & \frac{1}{2} [(f_+ + g_+) \phi - (f_- \phi' + g_- \phi'')] & \frac{1}{2} [(f_+ - g_+) \phi + (f_- \phi' - g_- \phi'')] \\
 0 & \frac{1}{2} [(f_+ - g_+) \phi - (f_- \phi' - g_- \phi'')] & \frac{1}{2} [(f_+ + g_+) \phi + (f_- \phi' + g_- \phi'')]
\end{pmatrix}.
\]

The masses for leptons denoted by \(M_0^L\) are, thus, described by

\[
M_0^L = \langle 0 | M_0^L(\phi, \phi', \phi'') | 0 \rangle = \begin{pmatrix}
 m_e & 0 & 0 \\
 0 & m_{\mu \mu} & m_{\mu \tau} \\
 0 & m_{\tau \mu} & m_{\tau \tau}
\end{pmatrix}
\]

with

\[
m_e = f_\phi v, \quad m_{\mu \mu} = \frac{1}{2} [(f_+ + g_+) v - (f_- \phi' + g_- \phi'')] , \quad m_{\tau \tau} = \frac{1}{2} [(f_+ + g_+) v + (f_- \phi' + g_- \phi'')] ,
\]

\[
m_{\mu \tau} = \frac{1}{2} [(f_+ + g_+) v - (f_- \phi' + g_- \phi'')] , \quad m_{\tau \mu} = \frac{1}{2} [(f_+ + g_+) v - (f_- \phi' + g_- \phi'')],
\]

where \(v = \langle 0 | \phi^0 | 0 \rangle\), \(v' = \langle 0 | \phi'^0 | 0 \rangle\) and \(v'' = \langle 0 | \phi''^0 | 0 \rangle\). To be consistent with the pattern of the observed hierarchy of \(m_e \ll m_\mu \ll m_\tau\), we simply adopt the parameterization based on the “hierarchical” one [26].

It is straightforward to reach \(U_L(V_L)\) that links the original states of \(|\ell^0_L(R)\rangle\) to the states with the diagonal masses of \(|\ell^0_L(R)\rangle: |\ell^0_L\rangle = U_L|\ell_L\rangle\) and \(|\ell^0_R\rangle = V_L|\ell_R\rangle\). The original mass matrix \(M_0^L\) is transformed into \(M^L\) according to \(M^L = U_L^T M_0^L V_L = \text{diag.}(m_e, m_\mu, m_\tau):\)

\[
U_L = \begin{pmatrix}
 1 & 0 & 0 \\
 0 & c_\alpha & s_\alpha \\
 0 & -s_\alpha & c_\alpha
\end{pmatrix}, \quad V_L = \begin{pmatrix}
 1 & 0 & 0 \\
 0 & c_\beta & s_\beta \\
 0 & -s_\beta & c_\beta
\end{pmatrix},
\]

(12)
where \( c_\alpha = \cos \alpha \) etc. defined by

\[ c_\alpha = \sqrt{\frac{m_{\tau \tau}^2 + m_{\tau \mu}^2 - m_{\mu}^2}{m_{\tau}^2 - m_{\mu}^2}}, \quad s_\alpha = \sqrt{\frac{m_{\tau}^2 - m_{\tau \mu}^2 - m_{\mu}^2}{m_{\tau}^2 - m_{\mu}^2}}, \quad (13) \]

\[ c_\beta = \sqrt{\frac{m_{\tau \tau}^2 + m_{\tau \tau}^2 - m_{\mu}^2}{m_{\tau}^2 - m_{\mu}^2}}, \quad s_\beta = \sqrt{\frac{m_{\tau}^2 - m_{\tau \mu}^2 - m_{\mu}^2}{m_{\tau}^2 - m_{\mu}^2}}, \quad (14) \]

The \( \mu \) and \( \tau \) masses are calculated to be \( m_{\mu}^2 = \lambda_- \) and \( m_{\tau}^2 = \lambda_+ \) with \( \lambda_\pm \) given by

\[ \lambda_{\pm} = \frac{1}{2} (m_{\tau \tau}^2 + m_{\mu}^2 + m_{\mu}^2 \pm M^2), \quad (15) \]

where

\[ M^4 = (m_{\tau \tau}^2 - m_{\mu}^2)^2 + (m_{\tau \mu}^2 - m_{\mu}^2)^2 + 2 (m_{\tau \tau}^2 m_{\mu}^2 + m_{\mu}^2 m_{\mu}^2 + m_{\mu}^2 m_{\mu}^2) + 2 (m_{\tau \tau}^2 m_{\mu}^2 + m_{\mu}^2 m_{\mu}^2 m_{\mu}^2). \quad (16) \]

The hierarchical mass pattern of \( m_{\mu} \ll m_{\tau} \) can be realized by the hierarchical conditions of \( |s_\alpha|, |s_\beta| \ll 1 \). It is convenient for our later discussions to relate \( m_{\mu, \tau} \) with \( m_{\mu, \tau} \), which are described by

\[ m_{\mu}^2 = S^2 m_{\mu} + C^2 m_{\mu}, \quad m_{\tau}^2 = C^2 m_{\tau} + S^2 m_{\mu}, \]

\[ m_{\mu}^2 = \frac{1}{S_\alpha^2 - S_\alpha^2} \left[ (c_\alpha s_\alpha C^2 - c_\beta s_\beta S^2) m_{\tau} - (c_\beta s_\beta C^2 - c_\alpha s_\alpha S^2) m_{\mu} \right], \]

\[ m_{\tau}^2 = \frac{1}{S_\beta^2 - S_\beta^2} \left[ (c_\beta s_\beta C^2 - c_\alpha s_\alpha S^2) m_{\tau} - (c_\alpha s_\alpha C^2 - c_\beta s_\beta S^2) m_{\mu} \right], \quad (17) \]

where

\[ C^2 = \frac{c_\alpha^2 + c_\beta^2}{2}, \quad S^2 = \frac{s_\alpha^2 + s_\beta^2}{2}. \quad (18) \]

By combining Eqs.(11) and (17), we find that

\[ f_+ v \sim g_+ v \sim (m_{\tau} + m_{\mu}) / 2, \quad f_- v' \sim g_- v'' \sim (m_{\tau} - m_{\mu}) / 2, \quad (19) \]

should be satisfied for \( |s_\alpha|, |s_\beta| \ll 1 \).

Even after the rotation that gives the diagonal mass matrix of \( U_{\ell}^\dagger (0| M_\ell^0 (\phi, \phi', \phi'') | 0) V_{\ell} \), our Yukawa interactions corresponding to \( U_{\ell}^\dagger (M_\ell^0 (\phi, \phi', \phi'') | V_{\ell} \) still contain flavor-off-diagonal couplings. In fact, Eq.(10) is transformed into \( M_{ij}^\ell (\phi, \phi', \phi'') \), whose elements denoted by \( M_{ij} \) are calculated to be:

\[ M_{11} = \alpha_{\phi} m_{\tau}, \quad M_{12} = M_{21} = M_{13} = M_{31} = 0, \]

\[ M_{22} = \frac{m_{\tau} + m_{\mu}}{2} \alpha_{\phi} + \frac{m_{\tau} - m_{\mu}}{4} (\alpha_{\phi'} + \alpha_{\phi''}) - (s_\alpha - s_\beta) \frac{m_{\tau}}{2} (\alpha_{\phi'} - \alpha_{\phi''}), \]

\[ M_{33} = \frac{m_{\tau} + m_{\mu}}{2} \alpha_{\phi} + \frac{m_{\tau} - m_{\mu}}{4} (\alpha_{\phi'} + \alpha_{\phi''}) + (s_\alpha - s_\beta) \frac{m_{\tau}}{2} (\alpha_{\phi'} - \alpha_{\phi''}), \quad (20) \]

up to \( O(s_\alpha, s_\beta) \), where \( \alpha_{\phi} = 0 / v, \alpha_{\phi'} = 0 / v' \) and \( \alpha_{\phi''} = 0 / v'' \). One can readily find that the identification of \( \alpha_{\phi'} \) and \( \alpha_{\phi''} \) with \( \alpha_{\phi} \) corresponding to the case of the standard model gives diagonal interactions, leading to the diagonal lepton masses. The flavor-changing interactions involving \( \tau \) and \( \mu \) such as \( \tau \rightarrow \mu \gamma \) and \( \tau \rightarrow \mu m_{\mu} \) are roughly controlled by the coupling of \( m_{\mu} / m_{\mu} \) (\( = \xi_i \)), where \( i = e, \tau \) and \( m_{\mu} \) is a mediating Higgs boson mass. We find constraints on \( \xi_{e, \tau} \) to suppress these interactions to the phenomenologically consistent level, which are given by examining the following typical processes:

1. for \( \tau^- \rightarrow \mu^- e^- e^- \) mediated by \( \phi, |s_\alpha, s_\beta, \xi_\epsilon / m_{H}^2| \leq 2.1 \times 10^{-7} \text{ GeV}^{-2} \) from \( B(\tau^- \rightarrow \mu^- e^- e^-) < 1.7 \times 10^{-6} \),
2. for $\tau^- \rightarrow \mu^+ \mu^- \mu^-$ mediated by $\phi'$ and $\phi''$, $|\xi_\tau/m_H|^2 < 2.2 \times 10^{-7}$ GeV$^{-2}$ from $B(\tau^- \rightarrow \mu^- \mu^+ \mu^-) < 1.9 \times 10^{-6}$,

3. for $\tau^- \rightarrow \mu^- \gamma$ mediated by $\phi'$ and $\phi''$, $|\xi_\tau/m_H|^2 < 4.2 \times 10^{-6}$ GeV$^{-2}$ from $B(\tau^- \rightarrow \mu^- \gamma) < 1.1 \times 10^{-6}$,

where the data are taken from Ref. [27]. Since $m_H > v_{\text{weak}}$ is anticipated, where $v_{\text{weak}} = (2\sqrt{2}G_F)^{-1/2} = 174$ GeV for the weak boson masses, $\xi_\tau \sim m_\tau/v_{\text{weak}}$ and $\xi_\tau \sim m_\ell/v_{\text{weak}}$ with $|\xi_{s,\beta}| \ll 1$ readily satisfy these constraints. As stated previously, there are no such Higgs interactions for quarks that only couple to $\phi''$ in the $P'$s and $P''$s are taken from Ref. [27].

The radiative neutrino masses, $\delta m_{ij}^{\text{rad}}$, are generated by interactions corresponding to FIG.2. Let us denote by $M_0^{\text{vertex}}$ the amplitude involving contributions from the vertices connected by the mediating Higgs scalar, either one of $\phi, \phi'$ or $\phi''$, and kinematical factors due to one-loop contributions denoted by $P$, $P'$ and $P''$:

$$M_0^{\text{vertex}} = \begin{pmatrix}
PU & 0 & 0 \\
0 & PV - P'V' - P''V'' & PW + P'V' - P''V'' \\
0 & PW - P'V' + P''V'' & PV + P'V' + P''V''
\end{pmatrix}$$  \hspace{1cm} (21)

with

$$U = f_{\phi\mu} - \nu'\bar{h}' \quad V = (f_+ + g_+) \mu - \nu'\bar{h}' \quad W = (f_+ - g_+) \mu - \nu'\bar{h}'$$

where $\bar{h}'$ and $\bar{h}'$ project out the contributions of $h^+$ and $h^+$ with $\bar{h}h = \bar{h}'h' = 1$ and $\bar{h}h' = 0$. The $U$-term arises from the interaction of $\mu - \phi^\dagger \phi''h'^{++}$ giving $\mu - \bar{h}'$ and $\langle 0|\phi'^\dagger \phi R \; (v')'' \rangle$ and of $f_{\phi\nu'\phi''} \phi R$ giving $f_{\phi}$ with the mediating $\phi$ and $h'^{++}$ involved in $P$ and similarly for other terms. The one-loop factors of $P$'s are defined by

$$P = \frac{1}{16\pi^2} \ln \frac{m_h^2 - m_{\phi}^2}{m_h^2 - m_{\phi}^2}.$$  \hspace{1cm} (23)

where $m$'s are masses of the relevant scalars and $m_h = m_{h^+}$ ($m_{\nu^+}$) if $P$'s accompany $\bar{h}$ ($\bar{h}'$) in Eq.(21) and similarly for $P'$ with $m_{\phi}^2 \rightarrow m_{\phi'}^2$, and $P''$ with $m_{\phi}^2 \rightarrow m_{\phi''}^2$.

By considering the rotation effects due to $U_0$ that transforms the original states of $|\nu_0\rangle$ into $|\nu_{\text{weak}}\rangle$: $|\nu_{\text{weak}}\rangle = U_0^\dagger |\nu_0\rangle$, we find that $\delta m_{ij}^{\text{rad}}$ can be parameterized by

$$\delta m_{ij}^{\text{rad}} = \left(U_0^\dagger U_0 M_0^{\text{vertex}} U_0\right)_{ij} = \left(U_0^\dagger M^2 M_0^{\text{vertex}} U_0\right)_{ij}$$  \hspace{1cm} (24)

for $|\nu_{\text{weak}}\rangle$, where $f_{ij} = f_{[ij]}$ with $f_{[\nu]\mu} = f_{[\nu\tau]} = f_{[\nu]^h} = \frac{1}{\sqrt{2}} f_{[\nu]}$ and $f_{[\nu\tau]} = f_{[\nu]^h} = \frac{1}{\sqrt{2}} f_{[\nu]}$ and $M^{\text{vertex}} = U_0^\dagger M_0^{\text{vertex}} U_0$. The radiative neutrino masses given by $\delta m_{\nu}^{\text{rad}} = 2\delta m_{\nu}^{\text{rad}}$ and $\delta m_{\nu}^{\text{rad}} = \delta m_{\nu}^{\text{rad}} + \delta m_{\nu}^{\text{rad}} (i \neq j)$ are calculated to be:

$$\delta m_{\nu}^{\text{rad}} = \begin{pmatrix}
0 & \delta m_{\nu}^{\text{rad}} & \delta m_{\nu}^{\text{rad}} \\
\delta m_{\nu}^{\text{rad}} & 0 & \delta m_{\nu}^{\text{rad}} \\
\delta m_{\nu}^{\text{rad}} & \delta m_{\nu}^{\text{rad}} & 0
\end{pmatrix}$$

$$= \begin{pmatrix}
\frac{1}{2} f_{[\nu]}^2 h^{\dagger} (r^{-1} - P - P'^{-1}) m_\nu + m_\nu^2 & \frac{1}{2} f_{[\nu]}^2 (r^{-1} - P - P'^{-1}) m_\nu + m_\nu^2 \\
\frac{1}{2} f_{[\nu]}^2 (r^{-1} - P - P'^{-1}) m_\nu + m_\nu^2 & 0 & \frac{1}{2} f_{[\nu]}^2 (r^{-1} - P - P'^{-1}) m_\nu + m_\nu^2 \\
\frac{1}{2} f_{[\nu]}^2 (r^{-1} - P - P'^{-1}) m_\nu + m_\nu^2 & \frac{1}{2} f_{[\nu]}^2 (r^{-1} - P - P'^{-1}) m_\nu + m_\nu^2 & 0
\end{pmatrix}$$

where $r = v'/v''$ and $r'' = v''/v$ and we have neglected the non-leading contributions of $O(s_{\alpha,\beta})$ and $O(m_{\mu,\tau}/m_{\tau})$. The tree level masses, $m_\nu^{\text{ij}} (i, j = \mu, \tau)$, are given by the type II seesaw mechanism to be:

$$m_\nu^{\text{ij}} = A_{ij} f_{[\nu]}^2 v_s \approx A_{ij} f_{[\nu]}^2 \mu v' v''$$

for $m_s \sim \mu \sim \mu' \gg v', v'', v''$, where $v_s = \langle 0|s_s^0|0\rangle$, $A_{\mu\mu} = (c_\alpha - s_\alpha)^2 (\sim 1 - 2s_\alpha)$, $A_{\tau\tau} = (s_\alpha + c_\alpha)^2 (\sim 1 + 2s_\alpha)$ and $A_{\mu\tau} = A_{\tau\mu} = c_\alpha - s_\alpha^2 (\sim 1)$. Our mass matrix of Eq.(2) has the following mass parameters:

$$a = 0, \quad b = \delta m_{\nu}^{\text{rad}}, \quad c = \delta m_{\nu}^{\text{rad}},$$

$$d = (c_\alpha - s_\alpha)^2 m_\nu + \delta m_{\nu}^{\text{rad}}, \quad e = (s_\alpha - c_\alpha)^2 m_\nu + \delta m_{\nu}^{\text{rad}}$$

$$f = (c_\alpha + s_\alpha)^2 m_\nu + \delta m_{\nu}^{\text{rad}}.$$  \hspace{1cm} (27)
where we have used the exact expressions for $d$, $e$ and $f$ as far as the tree-level contributions are concerned. The possible contribution to the mass parameter of $a$ from the two-loop convergent diagram of FIG.1 (b) is well suppressed by $m_s$, arising from the propagator of $s$ and does not jeopardize $a=0$.

We are now in a position to estimating various neutrino oscillation parameters. In the course of calculations, we assume for the simplicity that $m_{2^+}^2 = m_{2^+}^2, m_{2^+}^2 = m_{2^+}^2, m_{2^+}^2 = m_{2^+}^2$, leading to $P = P' = P''$. The mixing angle $t_{23}$ for the atmospheric neutrino oscillations is computed to be:

$$t_{23} = \frac{1 + r^2}{1 - r^2}, \tag{28}$$

from $t_{23} = -c/b(= -\delta m_{ee}^e/\delta m_{\mu\mu}^e)$. In the limit of $\delta m_{ee}^e = 0, t_{23}$ is also given by $t_{23} = (1 - c)/c$ given from $f = d = (t_{23} + t_{23})c$, which ensures the almost maximal atmospheric neutrino mixing characterized by $t_{23} \approx 1$ because $|s_\alpha| \ll 1$ for the hierarchical $\mu$ and $\tau$ mass texture. To be consistent, we require that $r^2 = -t_\alpha$, thereby, $v'' \gg v'$ for $r^2 \ll 1$ leading to $g_1^2/f_1^2 \ll 1$ from Eq.(19). By including $\delta m_{\mu\mu}^e$, we find that $r^2 = -t_\alpha$ is modified into

$$r^2 = -s_\alpha - \frac{1}{2 (1 + 2r^2)} \frac{\delta m_{\mu\mu}^e}{m^e}, \tag{29}$$

up to $O(s_\alpha)$, where we have replaced $f_{\mu}^h P_{\mu-\nu} m_{\nu}^2$ by $\delta m_{\mu\nu}^e$ defined in Eq.(25). The mixing angle $\sin^2 2\theta_{12}$ for the solar neutrino oscillations is given by $\sin^2 2\theta_{12} = 8/(\alpha^2 + x^2)$ of Eq.(4) with $x$ calculated to be:

$$x = \frac{2m^e}{\sqrt{(b^2 + c^2)/2}} \frac{s_\alpha - r^2}{1 - r^2} - 2\sqrt{2r} + r^2 - 1 \frac{f_{\mu-\nu}^h f_{\mu+}^h}{(1 - r^2)^2} \frac{f_{\mu-\nu}^h f_{\mu-\nu}^h}{(1 + 2r^2)} \frac{\delta m_{\mu\mu}^e}{\delta m_{\mu\mu}^e}, \tag{30}$$

up to $r^2$ and $|s_\alpha|$, where we have used the relation of

$$\frac{f_{\mu-\nu}^h f_{\mu-\nu}^h}{f_{\mu+}^h f_{\mu+}^h} = -\frac{1}{\sqrt{2r}} \frac{1 - r^2}{2r + r^2 - 1} \frac{\delta m_{\mu\mu}^e}{\delta m_{\mu\mu}^e} \tag{31}$$

supplied by Eq.(25). The tree-level contributions to $x$ involving $s_\alpha$ vanish in Eq.(30) because of the use of Eq.(29).

This cancellation is realized by the “ideal” structure of the tree-level mass terms thanks to the presence of $S_2$ and $Z_4$ and can be traced back to the fact that the tree-level contributions alone give $x=0$ since the relations of $t_{23} = (1 - t_\alpha)/(1 + t_\alpha)$ and $x = -(c_\alpha - s_\alpha)^2 + t_{23}(c_\alpha^2 - s_\alpha^2)$ yields $x = 0$. The masses of neutrinos that of course depend on $x$ satisfy the “normal” mass hierarchy of $|m_1| < |m_2| \ll m_3$ determined by Eq.(3) to be:

$$m_1 \approx -\frac{1}{2} \delta m_{rad} (\sqrt{x^2 + 8 - |x|}) , \quad m_2 \approx \frac{1}{2} \delta m_{rad} (\sqrt{x^2 + 8 + |x|}), \quad m_3 \approx 2(1 - 2s_\alpha)m^e + \delta m_{\mu\mu}^e + x\delta m_{\mu\mu}^e \tag{32}$$

with $\delta m_{rad} = \sqrt{\delta m_{\mu\mu}^e + \delta m_{\tau\tau}^e}$, where $\eta$ is chosen such that $\eta x = |x|$. Then, $\Delta m_{atm,\odot}^2$ are calculated to be:

$$\Delta m_{atm,\odot}^2 = m_3^2 - m_2^2 \approx 4m^e + 4m^e (\delta m_{\mu\mu}^e + x\delta m_{\mu\mu}^e) = 4s_\alpha m^e, \quad \Delta m_{\odot}^2 \approx m_3^2 - m_2^2 \approx |x| \sqrt{8 + 2x^2}\delta m_{rad}^2. \tag{33}$$

To get numerical estimations, let us fix $|x| = \sqrt{2}$ corresponding to $\sin^2 2\theta_{12} = 0.8$ and also fix $r = 1 (v = v''')$ and $r = 1/9 (v' = v''/3)$ corresponding to $\sin^2 \theta_{23} = 0.95$, where $v (=v'' > v') \approx \epsilon_{weak}/\sqrt{2}$ to satisfy $v^2 + v'^2 + v''^2 = v_{weak}^2$. In the end, we derive $\sin^2 2\theta_{12} = 0.78$ from reasonable assumptions on the couplings. The conditions of Eq.(19) in turn require

$$|f_+ / g_-| \sim 3 \tag{34}$$

to be consistent with $r^2 = 1/9$ and give the estimation of the Yukawa couplings to be:

$$f_+ \sim g_+ \sim g_- \sim f_- / 3 \sim 0.005. \tag{35}$$

The tree level mass of $m^e$ is estimated to be $\sim 0.03$ eV for $\Delta m_{atm}^2 = 3 \times 10^{-3}$ eV$^2$ and $\delta m_{rad}^e = 3.2 \times 10^{-3}$ eV$^2$ is obtained for $\Delta m_{\odot}^2 = 4.5 \times 10^{-3}$ eV$^2$. Since $r^2$ in Eq.(29) is almost saturated by $-s_\alpha$ for these values of $m^e$ and $\delta m_{rad}$, we observe that $s_\alpha \sim -1/9$. The type II seesaw mechanism for $m^e$ yields an estimate of the mass of $s$: $m_s$
\[ (= \mu ) = 1.7 \times 10^{14} \times (|f_x^\phi|/e) \text{ GeV} \], where \( e \) is the electromagnetic coupling. From the expression of \( \delta m_{\mu}^\nu \) in Eq.(25), we find that the estimation of \( \delta m_{rad}^\nu \) yields
\[ f_+^h \sim 2.3 \times 10^{-7}, \quad (36) \]
where \( \mu_+ = m_\phi = v_{\text{weak}} \) and \( m_{\phi}^2 \) are used to compute the loop-factor of \( P \). From Eqs.(30) and (31), we also find that \( x = 26 \delta m_{\mu}\nu/27 \delta m_{rad}^\nu \), which yields \( |x| = \sqrt{2} \) for \( \delta m_{\mu}\nu/\delta m_{rad}^\nu = 1.47 \), leading to
\[ f_+^h/\mu_+ = -9 \sqrt{2} (1-r^2) x / 52r = \pm 0.92, \quad (37) \]
from Eq.(31) with \( |\delta m_{\mu}\nu| \approx |\delta m_{rad}^\nu| \). Finally, the masses of \( m_{1,2,3} \) are predicted to be:
\[ |m_1| = 2.8 \times 10^{-3} \text{ eV}, \quad |m_2| = 7.3 \times 10^{-3} \text{ eV}, \quad m_3 = 5.5 \times 10^{-2} \text{ eV}. \quad (38) \]

It is remarkable to note that the result of these numerical estimates is consistent with the reasonable expectation of \( |f_+^h| = \mathcal{O}(f_h^h) \) and \( \mu_+ = \mathcal{O}(\mu_-) \), but with neither \( |f_+^h| \gg |f_h^h| \) nor \( |f_h^h| \ll |f_0^h| \), to yield the large solar neutrino mixing. This result should be contrasted with the requirement of “inverse” hierarchy for the original Zee model [29]. If the relation of \( f_+^h \sim f_h^h \) and \( \mu_+ \sim \mu_- \) is assumed, one finds that \( |x| \sim 1.53 \) for \( r^2 = 1/9 \) leading to \( \sin^2 2\theta_{12} \sim 0.78 \) in good agreement with the observed data.

Summarizing our discussions, in the radiative mechanism based on the conservation of \( S_2 \) and the invariance under the \( S_2 \)-transformation as well as under the discrete \( Z_4 \)-transformation, we have demonstrated that the almost maximal atmospheric neutrino mixing is guaranteed by the \( S_2 \)-symmetric coupling of \( \nu \) to neutrinos and the large solar neutrino mixing is derived by the radiative effects only, where the tree-level contributions from \( s \) vanish owing to the presence of \( S_2 \). Our model spontaneously breaks \( L \) and \( L' \) but preserves \( L + L' \), namely \( L_\tau \). This remaining \( L_\tau \)-conservation is used to select the Higgs interactions that include the key interactions for type II seesaw mechanism and radiative mechanism. The massless Nambu-Goldstone boson associated with the spontaneous breakdown of \( L - L' \), namely \( L_\tau \), can be removed by introducing a soft breaking such as \( \phi^\nu \phi^\nu h^{+-} \). The model seems to suffer from the emergence of the dangerous flavor changing interactions that disturbs the well-established low-energy phenomenology of leptons because there are three Higgs scalars of \( \phi, \phi' \) and \( \phi'' \). However, the explicit calculations show that the lepton sector has couplings to those Higgs scalars at most of order \( m_\tau/v_{\text{weak}} \), which are shown to be sufficiently small to suppress these interactions to the phenomenologically consistent level.

In our scenario, properties of neutrino masses are summarized as follows:

1. The smallness of neutrino masses is ensured by type II seesaw mechanism for atmospheric neutrinos and by radiative mechanism for solar neutrino neutrinos.

2. The observed hierarchy of \( |\Delta m_{\alpha\beta}^2| \gg |\Delta m_{\gamma\alpha}^2| \) is reproduced by the huge mass scale of \( s \) of \( \mathcal{O}(10^{14}) \) GeV, which determines the democratic neutrino mass to be around 0.03 eV, and by the feeble couplings of \( h^+ (h^{+-}) \) to neutrinos of \( \mathcal{O}(10^{-7}) \), which determine the radiative neutrino mass to be around 0.003 eV.

and neutrino mixings are explained as follows:

1. The mixing angle of \( \theta_{23} \) for atmospheric neutrinos is determined to be \( t_{23} = (1-t_\alpha)/(1+t_\alpha) \) by \( S_2 \)- and \( Z_4 \)-symmetric tree-level mass terms without radiative effects ensuring \( t_{23} \approx 1 \) because \( |\sin \alpha| \ll 1 \) for the hierarchical \( \mu \) and \( \tau \) mass texture. Radiative effects in turn give \( t_{23} = -\Delta m_{\mu\tau}^\nu/\Delta m_{\mu\mu}^\nu = (1+r^2)/(1-r^2) \) with \( r=\langle 0|\phi^0|0 \rangle / \langle 0|\phi^0|0 \rangle \) fixing \( f_-/f_+ \sim r \) from Eq.(19), which becomes consistent if \( r^2 \sim -s_\alpha \) as in Eq.(29).

2. The mixing angle of \( \theta_{12} \) for solar neutrinos is determined to be \( \sin^2 2\theta_{12} = 8/(8+x^2) \) by radiative mass terms, where \( x = \delta m_{\mu\mu}^\nu/\delta m_{\mu\tau}^\nu \), which is subject to the cancellation of the tree-level contributions in \( x \) ensured by \( S_2 \). The ratio \( |x| \) of \( \mathcal{O}(1) \) needed for the explanation of the large solar neutrino mixing can be realized by the requirement of \( f_+^h \sim f_-^h \) and \( \mu_+ \sim \mu_- \), where \( f_+^h (f_-^h) \) and \( \mu_+ (\mu_-) \) stand for \( h^+ (h^{+-}) \)-couplings, respectively, to leptons and to Higgs scalars.

It should be noted that the hierarchical mass texture for \( \mu \) and \( \tau \) characterized by the finite mixing angle with \( \sin^2 \alpha \) \( \sim \sin^2 \beta \) \( \ll 1 \) is inevitable to be consistent with the almost atmospheric neutrino mixing.

We have also estimated various couplings by assuming the reasonable parameter setting based on \( \langle 0|\phi^0|0 \rangle \sim \langle 0|\phi^0|0 \rangle \) and \( \langle 0|\phi^0|0 \rangle \sim \langle 0|\phi^0|0 \rangle / 3 \) or equivalently \( s_\alpha \sim -1/9 \), leading to \( \sin^2 2\theta_{23} \sim 0.95 \). The solar neutrino mixing parameter of \( x \) is estimated to be \( |x| = 26 \delta m_{\mu\mu}^\nu/27 \delta m_{\mu\tau}^\nu \approx 1.3 f_+^h \mu_-/6 \sqrt{2} f_+^h \mu_+ \). This estimation yields \( |\delta m_{\mu\mu}^\nu/\delta m_{rad}^\nu = 1.47 \) with \( f_+^h \mu_-/f_+^h \mu_+ = \pm 0.92 \) for \( |x| = \sqrt{2} \) corresponding to \( \sin^2 2\theta_{12} = 0.8 \) and implies the acceptable assumption of \( f_+^h \sim f_-^h \) and \( \mu_+ \sim \mu_- \) giving the large solar neutrino mixing of \( \sin^2 2\theta_{12} \sim 0.78 \). In this
respect, our mechanism provides a natural solution to the large solar neutrino mixing. The presence of the permutation symmetry of $S_3$ for the $\mu$- and $\tau$-families in radiative neutrino mechanism enhances the significant deviation of $\sin^2 2\theta_{12}$ from unity as suggested by the latest data provided that the hierarchical mass texture is realized for $\mu$ and $\tau$.

ACKNOWLEDGMENTS

The authors are grateful to Y. Koide for useful discussions on the status of the Zee model. One of the authors (M.Y.) also thanks the organizers and participants in the Summer Institute 2001 at FujiYoshida, Yamanashi, Japan, especially M. Bando and M. Tanimoto, for useful suggestions at the preliminary stage of this work. The work of M.Y. is supported by the Grants-in-Aid for Scientific Research on Priority Areas A: “Neutrino Oscillations and Their Origin” (No 12047223) from the Ministry of Education, Culture, Sports, Science, and Technology, Japan.

[1] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870. See also B. Pontecorvo, JETP (USSR) 34 (1958) 247; B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53 (1967) 1717; V. Gribov and B. Pontecorvo, Phys. Lett. 28B (1969) 493.

[2] Super-Kamiokande Collaboration, Y. Fukuda, et al., Phys. Rev. Lett. 81 (1998) 1562; Phys. Lett. B 433 (1998) 9; Phys. Lett. B 436 (1998) 33; N. Fornengo, M.C. Gonzalez-Garcia and J.W.F. Valle, Nucl. Phys. B 580 (2000) 58; T. Kajita and Y. Totsuka, Rev. Mod. Phys. 73 (2001) 85; K. Nishikawa, Talk given at the Third International Workshop on Neutrino Factories based on Muon Storage Rings (NuFACT’01), 24-30 May, Tsukuba, Japan (http://psux1.kek.jp/~nufact01/Docs/programs.html); C. Walter, Talk given at the Third International Workshop on Neutrino Factories based on Muon Storage Rings (NuFACT’01), 24-30 May, Tsukuba, Japan (http://psux1.kek.jp/~nufact01/Docs/agenda_wg1.html).

[3] SNO Collaboration, Q.R. Ahmad, et. al., Phys. Rev. Lett. 87 (2001) 07301.

[4] K2K Collaboration, J.E. Hill, hep-ex/010034.

[5] T. Yanagida, in Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe edited by A. Sawada and A. Sugamoto (KEK Report No.79-18, Tsukuba, 1979), p.95; Prog. Theor. Phys. 64 (1980) 1103; M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity edited by P. van Nieuwenhuizen and D.Z. Freedmann (North-Holland, Amsterdam 1979), p.315; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

[6] A. Zee, Phys. Lett. 93B (1980) 389; Phys. Lett. 161B (1985) 141; L. Wolfenstein, Nucl. Phys. B 175 (1980) 93; S.T. Petcov, Phys. Lett. 115B (1982) 401.

[7] A. Zee, Nucl. Phys. 264B (1986) 99; K. S. Babu, Phys. Lett. B 203 (1988) 132; D. Chang, W-Y.Keung and P.B. Pal, Phys. Rev. Lett. 61 (1988) 2420; J. Schechter and J.W.F. Valle, Phys. Lett. B 286 (1992) 321.

[8] See for example, J.W.F. Valle, hep-ph/0104085; E. Lisi, Talk given at the Third International Workshop on Neutrino Factories based on Muon Storage Rings (NuFACT’01), 24-30 May, Tsukuba, Japan (http://psux1.kek.jp/~nufact01/Docs/programs.html).
Bahcall, M.C. Gonzalez-Garcia and C. Peña-Garay, JHEP 08 (2001) 014; V. Barger, D. Marfatia and K. Whisnant, hep-ph/0106207; P.I. Krastev and A.Yu. Smirnov, hep-ph/0108177.

[16] For the explicit demonstration, see P.H. Frampton and S.L. Glashow, in Ref. [12]; Y. Koide, Phys. Rev. D 64 (2001) 077301.

[17] T. Kitabayashi and M. Yasue, Talk given at Summer Institute 2001 (SI2001), 13-20 August, FujiYoshida, Yamanashi, Japan, to appear in the Proceedings.

[18] K.R.S. Balaji, W. Grimus and T. Schwetz, Phys. Lett. B 508 (2001) 301.

[19] W. Grimus and L. Lavoura, hep-ph/0105212 and 0110041.

[20] E. Ma and U. Sarkar, Phys. Rev. Lett. 80 (1998) 5716.

[21] E. Ma, M. Raidal and U. Sarkar, Phys. Rev. Lett. 85 (2000) 3769; S.K. Kang and C.S. Kim, Phys. Rev. D 63 (2001) 113010; C.S. Lam, Phys. Lett. B 507 (2001) 214. See also, I. Dorsner and S.M. Barr, hep-ph/0108168.

[22] E. Ma, M. Raidal and U. Sarkar, Phys. Rev. Lett. 85 (2000) 3769; S.K. Kang and C.S. Kim, Phys. Rev. D 63 (2001) 113010; C.S. Lam, Phys. Lett. B 507 (2001) 214. See also, I. Dorsner and S.M. Barr, hep-ph/0108168.

[23] For leptons with \( S_3 \), see for example, M. Fukugita, M. Tanimoto and T. Yanagida, in Ref. [11]; M. Tanimoto, in Ref. [11]; R.N. Mohapatra, A. Perez-Lorenzana and C.A. de S. Pires, in Ref. [10].

[24] For quarks with \( S_3 \), see H. Harari, H. Haut and J. Weyers, Phys. Lett. 78B (1978) 459.

[25] See for example, H. Harari, H. Haut and J. Weyers, in Ref. [24]; Y. Koide, Phys. Rev. D 28 (1983) 252; Phys. Rev. D 39 (1989) 1391; P. Kaus and S. Meskov, Mod. Phys. Lett. A 3 (1988) 1251; M. Tanimoto, Phys. Rev. D 41 (1990) 1589; G.C. Branco, J.I. Silva-Marcos and M.N. Rebelo, Phys. Lett. B 237 (1990) 451.

[26] See for example, H. Harari, H. Haut and J. Weyers, Phys. Lett. 78B (1978) 436; Phys. Lett. 73B (1978) 317; S. Weinberg, Trans. New York Acad. Sci. 38 (1977) 185; F. Wilczek and A. Zee, Phys. Lett. 70B (1977) 418.

[27] Particle Data Group, D.E. Groom et al., Euro. Phys. J. C 15 (2000) 1.

[28] See for example, E. Mitsuda and K. Sasaki, Phys. Lett. B 516 (2001) 47; A. Ghosal, Y. Koide and H. Fusaoka, Phys. Rev. D 64 (2001) 053012.

[29] C. Jarlskog, M. Matsuda, S. Skadhauge and M. Tanimoto, in Ref. [12]. See also, Y. Koide and A. Ghosal, Phys. Rev. D 63 (2001) 037301.

TABLES

TABLE I. The lepton number \( (L) \), \( L' \) and \( S_2 \) and \( Z_4 \) for leptons and Higgs scalars, where \( S_2 = (+) \) denotes symmetric (antisymmetric) states.

| \( \psi_eL \) | \( e_R \) | \( \psi_eL' \) | \( e_R' \) | \( L+R \) | \( L-R \) | \( \phi \) | \( \phi' \) | \( \phi'' \) | \( h^+ \) | \( h^{++} \) | \( s \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| \( L \) | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | -2 | -2 | -2 |
| \( L' \) | 1 | 1 | -1 | -1 | -1 | 0 | 0 | 0 | 2 | 2 | 2 |
| \( S_2 \) | + | + | + | + | + | i | i | i | + | -i | + |
| \( Z_4 \) | + | + | + | i | + | i | i | + | -i | + |

TABLE II. \( L \) and \( L' \) for Higgs interactions with \( S_2=Z_4=+ \).

| \( \phi'' \phi^{++} \) | \( \phi'^{+} \phi'^{+} \) | \( \phi'' \phi^+ \) | \( \phi' \phi'^+ \) | \( \phi'' \phi^{++} \) | \( \phi'^+ \phi'' \) | \( \phi'' \phi'' \) | \( \phi'^+ \phi' \) | \( \phi'' \phi'' \) | \( \phi'^+ \phi'' \) |
|---|---|---|---|---|---|---|---|---|---|
| \( L \) | 2 | 2 | 0 | 0 | 0 | 0 | -2 | 2 | 2 |
| \( L' \) | 0 | -2 | 4 | 0 | 2 | 2 | 2 | 2 | 2 |

FIGURES

FIG. 1. (a) Divergent two-loop diagram for Majorana mass terms of \( \nu_{eL} \nu_{eL} \). (b) The same as (a) but for the finite diagram.

FIG. 2. One-loop diagrams for Majorana mass terms, where \( i,j = e, \mu, \tau \) with \( m,n = \mu, \tau \) and \( M_{\ell} (\phi, \phi', \phi'') \) is defined by Eq.(9).
FIGS.

FIG.1: (a) Divergent two-loop diagram for $\nu_L \nu_L$; (b) The same as (a) but for the finite diagram.

FIG.2: One-loop diagrams for Majorana mass terms, where $i, j = e, \mu, \tau$ with $m, n = \mu, \tau$ and $M_0^i (\phi, \phi', \phi'')$ is defined by Eq.(9).