Photonic zero-energy modes in a metal-based Lieb lattice

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Abstract

We design a photonic tight-binding system using the dispersive background, and observe the noncompact photonic zero-energy modes for both monopole and dipolar states in a finite Lieb lattice with flat truncations. In such a photonic Lieb system, the compact localization of s, p, and d flat bands is also checked. We show that this photonic zero-energy mode is provided by one dispersive band for singular touching, which has the same frequency with the flat band states. Specially, the zero-energy mode can be completely excited by merely one point source at the flat band frequency, covering all the minority sites and forming a noncompact state. This work may provide a deep understanding about the photonic zero-energy mode for higher order states in the Lieb or other flat band models.

1. Introduction

Photonic flat band has become a hot research topic in the field of photonic systems to analogize the flat-band properties in condensed matter systems, characterized by a dispersionless energy band existing in band structures [1, 2]. Due to their peculiar flat bands and compact localized eigenstates (CLS), many intriguing phenomena are discussed in the photonic lattices, e.g. diffraction-free image transmission [3–5], Aharonov–Bohm photonic cages [6–8], and edge states [9, 10]. Currently, there are three main ways to achieve the artificial photonic flat-band lattices, including exciton-polaritons in structured microcavities [11–16], waveguide arrays [9, 17–21], and metamaterials [22–24]. The first two ways obtain the energy bands through the tight-binding (TB) approximation, while the third way is based on a coupled oscillator model.

On the other hand, in the electronic models, the flat bands can have singular character due to the immovable discontinuity of Bloch wave functions, which result in the incompleteness of the CLSs and bring forth the new states at flat band energy. Several previous works have theoretically studied the additional noncompact states in the frustrated lattices and other singular flat band models [25–28]. In photonic systems, a missing line state has recently been experimentally found in a finite Lieb system under bearded truncations using the photonic writing waveguides [29]. However, there are another noncompact modes existing due to the triply degeneracy of Lieb lattice, which not yet to be observed in photonic system. Moreover, the noncompact flat band modes for higher order states (such as the dipolar state) have not been explored, which may contain more interesting mechanism in higher orbital dynamics [15, 30, 31].

In this Letter, we propose a special metal-based photonic system, and show that this model can obtain the quantitative band structure of photonic Lieb lattice. The additional noncompact zero-energy modes in Lieb lattice are observed in the designed photonic system for both the fundamental and dipolar states, which satisfy the condition for destructive interference. Such a photonic zero-energy mode can be excited by merely a point source and occupy all minority sites. Our study also provides a comprehensive understanding about the noncompact zero-energy mode for dipolar state in the Lieb lattice.

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2. Theoretical model and results

Let us start from a photonic system with material Ag (described by the Drude model \( 1 - \omega_p^2/(\omega_p^2 + i\omega\gamma) \), \( \omega_p \) is the plasma frequency and \( \gamma \) is the damping constant) as the background, in which air cavities are etched into photonic crystals of different structures [32]. For the dielectric and metallic regions, the Helmholtz equations can be written as follows (consider the TM modes with the electromagnetic field components \((H_x, H_y, E_z)\))

\[
\begin{align*}
-e^2\nabla^2E_z &= \omega^2E_z, & \text{for } \varepsilon &= 1 \\
(-e^2\nabla^2 + \omega_p^2)E_z &= \omega^2E_z, & \text{for real part } \varepsilon &= 1 - \frac{\omega_p^2}{\omega^2},
\end{align*}
\]

where \( \omega \) is the angular frequency of light, \( E_z \) is the electric field in \( z \)-direction, and \( e \) is the speed of light in vacuum. Comparing to the time independent Schrödinger equation, \( \omega_p^2 \) plays the role of potential energy \( V \), thus the particles are mainly bound to the air cavities, which can well mimic the TB approximation. As shown in figure 1(a) of a photonic Lieb lattice, the lattice spacing \( d \) relates to effective coupling strengths \( t = e^{-d/\kappa} \), where \( \kappa \) is a constant coefficient depending on concrete structures. Besides, these emulated meta-atoms can couple with each other through the evanescent waves, with the coupling strength \( t \) modulated by the lattice spacing \( d \) (or indirectly adjusted by the radius \( r \)). In the following, we will investigate the compact localized states and the noncompact photonic zero-energy mode occurred in such a system for both the monopolar (s) and dipolar (p) states. All the numerical results are simulated by the finite element commercial software COMSOL Multiphysics.

For the Lieb geometry in figure 1(a), the effective tight-binding Hamiltonian can be written as (we only consider the A–B and A–C nearest neighbors (NN), and hopping constant \( t \) in \( x \) and \( y \) direction is the same)

\[
H_{\text{TB}} = \sum_{n,m} [-t[a^+_n(b_{m} + b_{n-1,m}) + a^+_m(c_{m} + c_{n,m-1})] + \text{h.c.}],
\]

where \( a^+_n \) and \( b_m \) (\( c_m \)) are the creation and annihilation operators in the \((n, m)\)th unit cell on A, B (C) sites, respectively. By Fourier transforming \( H_{\text{TB}} \), we can obtain the eigenvalues expressed as three branches: \( E = \pm t\sqrt{4 + 2 \cos(k_x) + 2 \cos(k_y)} \), \( E_{\text{FB}} = 0 \). The dispersion relations are displayed in figure 1(c) using red dashed curves, including a zero energy band and two conically intersecting bands, and these three bands all intersect at the M points. Meanwhile, the dispersion spectrum is also symmetric around zero energy, which is determined by its eigenvalues. Next, we turn to our photonic Lieb structure (PLS) to study the character of its band structure. In figure 1(a), the lattice constant \( a \) is set as 700 nm, the radius of air cavities \( r \) is 140 nm (the lattice spacing \( d \) is 70 nm), while the parameters of Drude model of metal Ag are taken as \( \omega_p = 1.37 \times 10^{16} \text{s}^{-1} \) and \( \gamma = 2.37 \times 10^{13} \text{s}^{-1} \). The simulated real parts of band structures are shown in figure 1(b), which display a perfect Dirac cone at M point. Next, we extract the frequencies along the boundaries of irreducible BZ and plot the band structure in figure 1(c), as the black lines shown. This photonic band structure perfectly coincides with the results from TB model, verifying that our model can well mimic the photonic TB model and have only NN couplings due to the evanescent wave coupling of short distance. For the imaginary bands, the magnitudes are three orders smaller than those of the real bands, indicating that the metal loss has no great influence on the real parts of band structures in such a Lieb lattice (though it might cause large effects on other structures), as shown in the part I of supplementary materials is available online at stacks.iop.org/NJP/21/113046/mmmedia.

Therefore, in the following we will only display the real parts of band structures to analyze the phenomena in such system.
To study whether the CLS of the flat band photons also exists in our Lieb structure, we investigate the band structure including s, p, and d states (see figure 2(a)). The radii of air cavities are increased to 160 nm to enhance the evanescent coupling, while keeping $a = 700$ nm unchanged ($d = 30$ nm). The band structure in figure 2(a) shows the distinct isolated s, p, and d modes, each with the flat bands inserting in the middle, including one s-flat band (yellow solid line), two degenerate p-flat bands (magenta solid line), and one d-flat band (green solid line). This photonic band structure is highly consistent with that of the TB approach under NN couplings. In figures 2(b1)–(b4), we show the distributions of electric fields $E_z$ for these flat bands at $\Gamma$ point, which clearly illustrate that the electric fields are all absent from the A site and highly localized on the B and C sites for these flat bands. These characteristics are caused by the destructive wave interference of this Lieb geometry, thus leading to these flat bands under NN couplings. Intriguingly, the s and d flat bands exhibit the opposite phases on the B and C sites (see figures 2(b1), (b4)), while for the p flat band, $p_s$ orbitals dominate on the C site and $p_d$ orbitals for the B site (see figures 2(b2), (b3)). In addition, we also explore the situation for $\varepsilon_B = \varepsilon_C = \varepsilon_A + \Delta\varepsilon$ (e.g. we randomly set $\varepsilon_A = 1$, $\Delta\varepsilon = 0.1$), and other parameters are the same to those in figure 2(a). It is shown that a gap is opened at M point, with the flat band keeping touching to the lower dispersive band, as shown in figure 2(c). Therefore, such a PLS can perfectly recur all the features of flat bands or CLS, building up the foundation for further study of noncompact photonic zero-energy modes.

Having investigated some inherent characteristics in an infinite Lieb lattice (equally, a unit cell with periodic boundary conditions), we now give the quantitative analysis in a semi-infinite and finite Lieb system. As depicted in figure 3(a), a semi-infinite PLS is constructed by cutting the air cavities on both sides into flat truncations, while keeping the y direction infinite. We next calculate the projected band structure for this system by taking the dotted rectangle in figure 3(a) as unit cell, and setting $a = 700$ nm, $r = 140$ nm. As shown in figure 3(b), there are eight flat band states (the overlapped yellow lines) degenerating at the flat band frequency, which is determined by the number difference between majority sites (B, C) and minority site (A) [26]. In addition, two symmetrical dispersive bands touch these flat bands at $k_y = \pi / a$ due to additional degeneracies. We remark that the incomplete degeneracy between the lower dispersive band and flat bands is due to the approximation of photonic TB model. In exact TB approach, the degeneracy is yet to be complete, as shown in the part II of supplementary materials. The band characteristics are almost the same, and the flat bands in photonic system correspond exactly to the zero-energy bands in TB model. Therefore, ten states are degenerated with the same flat band frequency, and the field patterns $E_z$ of them all satisfy the conditions for destructive interference, as shown in the part III of supplementary materials. Specially, we find that a noncompact state occupies all the A sites (the inset in figure 3(b)) and cannot be produced by the superposition of the above CLSs, representing an independently state. We call this noncompact state as photonic zero-energy mode, which is due to the dependence of CLSs in Lieb lattice. Considering a realistic (finite) PLS with $8 \times 8$ unit cells (see the inset in figure 3(c)), we study the eigenmodes of it for the fundamental (s) state, as shown in figure 3(c). Many degenerate flat band modes exist in the middle gapped region, indicating the CLSs with their eigenfields distributing in the B and C sites. Moreover, we also observe the noncompact photonic zero-energy mode in this finite lattice, as denoted at the red point of figure 3(c). The field distributions can be found in figure 3(d), which extends and locates at all the A sites, with neighbor sites having different phases, i.e. 0 and $\pi$. Different from previous line...
states found in the Lieb lattice under bearded truncations [30], here we observe this additional noncompact state in the photonic Lieb lattice under flat truncations, which is compatible with that in finite systems. They all belong to the missing states that result from the singular touching at \( k_y = \pi / a \), and can supplement the flat band complete.

To explore the excitation ways of the photonic zero-energy mode and the effect of loss on the excitation efficiency, we numerically establish the \( 8 \times 8 \) Lieb system (the number of lattices can be arbitrary) without or with loss in the Ag background. First, we discuss the ideal system with \( \gamma = 0 \), as shown in figure 4(a). By setting a line current with frequency \( f_D = 7.07 \times 10^{14} \) Hz at the red circle, all the sites A are lighten up and this noncompact photonic zero-energy mode is perfectly excited. Besides, the patterns around the excitation cite (in site B and site C) are also observed, which are the additionally excited CLSs and localize in the NN (the strength is

**Figure 3.** (a) The diagrams of semi-infinite PLS with flat truncations. (b) Projected band structure for the semi-infinite PLS with \( a = 700 \) nm, \( r = 140 \) nm and \( \omega / \omega_p = 0.32 \) at the real frequency of flat band. The inset shows the noncompact state at the red point \( f_D \). (c) Eigenfrequencies of \( s \) mode for a finite Lieb lattice with \( 8 \times 8 \) unit-cells, as shown in the inset. (d) Eigenmodes for the \( s \)-photonic zero-energy mode in Lieb lattice.

**Figure 4.** The distributions of \( |E|^2 \) when excited by the source frequency \( f_D = 7.07 \times 10^{14} \) Hz. (a), (b) One point source at site A with the damping constants (a) \( \gamma = 0 \) and (b) \( 6 \times 10^{12} s^{-1} \). (c) One point source at site B, (d) five out-of-phase sources at site A. The red circles indicate sources with +1 A, while the yellow circles indicate sources with −1 A.
much weak). When it comes to the lossy system with the damping constant $\gamma = 6 \times 10^{12}$ s$^{-1}$, the results are illustrated in figure 4(b). We can see that this state can still be found though the intensity of the optical signal decays away from the point source. In this situation, the strength of CLS is stronger than that of the photonic zero-energy mode. In figures 4(c) and (d), we excite this state using other two ways, i.e. one point source at site B and five out-of-phase sources at site A. It can be seen that only the CLSs are excited in the NN in figure 4(c), while the photonic zero-energy mode is suppressed (the same results for C site excitation). For the out-of-phase excitation in figure 4(d), this state is also excited, and the CLSs are offset in neighbors. Compared with the excitation way in the waveguide system [9, 29], our theoretical photonic model is simpler to be excited and not required the phase matching with input source.

Now we extend our results to the dipolar ($p$) state and give a similar analysis of its eigenmodes in an $8 \times 8$ Lieb lattice, as shown in figure 5(a). The number of $p$ eigenmodes is twice that of the $s$ eigenmodes due to the existence of $p_x$ and $p_y$ states. Similar to the $s$ state, we can see that there are also many degenerate modes existing in the middle flat band energy, indicating the CLSs of dipolar ($p$) state. Moreover, at the red points of figure 5(a), two degenerate photonic zero-energy modes for dipolar state still can be observed as illustrated in figures 5(b) and (c), i.e. $p_x$-photonic zero-energy mode, and $p_y$-photonic zero-energy mode. They have opposite phases at the neighbor A sites both along the $x$ and $y$ directions, thus fulfilling the conditions for destructive interference. The two noncompact $p_x$ and $p_y$ zero-energy modes have the same causes as that of the $s$ state, and enrich our understanding of the singular band touching of $s$ and $p$ flat bands.

3. Conclusion and outlook

In summary, we offer a simple metal-based photonic model to realize the tight-binding approximation with nearest-neighbor couplings. The photonic zero-energy mode for both the fundamental $s$ state and higher order $p$ state are observed in a finite Lieb lattice, presenting intrinsic destructive interference in all minority sites. We find that this state could be perfectly excited by merely one point source, even with attenuation of strength in a lossy system. Our work supplements the research on photonic zero-energy mode in a Lieb structure. Moreover, this photonic model may realize the pseudopotential by gradient refractive index in cavities, thus can analogize the wave function behavior of electrons authentically and relate to other topics, such as topological insulators and molecular hybridization.

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