Composition of the cores in massive neutron stars in the pseudoconformal model

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(Dated: June 26, 2020)

The composition of the cores in massive neutron stars is analyzed using the pseudoconformal model (PCM) proposed by us. We find that, in massive neutron stars there are cores made of matter in which the sound velocity approaches $v_s/c = 1/\sqrt{3}$ — the conformal limit — and the polytropic index is $\gamma \leq 1.75$, near 1, but the conformal symmetry is not restored and the chiral symmetry is in the Nambu-Golstone mode. The constituents are fractionally (baryon-)charged and not deconfined. With the help of the Cheshire-Cat principle, the matter in the cores can be interpreted as quasi-quarks resulting at $n \gtrsim 2.5n_0$ from quark-hadron duality.

Introduction.— The phase structure of the strong interactions at extreme conditions has been investigated for several decades but there are still many totally uncharted domains in it. The observation of massive neutron stars and detection of gravitational waves arising from neutron star merger provide indirect information of nuclear matter at low temperature and high density, say, up to ten times the saturation density $n_0$. So far, such phenomena can be accessed by neither terrestrial experiments nor lattice simulation.

The study of dense nuclear in the literature has largely relied on either phenomenological approaches anchored on density functionals or effective field theoretical models implemented with certain QCD symmetries, constructed in terms of set of relevant degrees of freedom appropriate for the cutoff chosen for the EFT, such as baryons, pions and vector mesons with $\rho$ or without $\omega$ hybridization with quarks. The ultimate aim is to understand what’s going on at high density relevant to compact stars.

Very recently, combining the astrophysical observations and theoretical ab initio calculations, Annala, et al. concluded that, inside the maximally massive stars there is a quark core consisting of deconfined quarks [4]. Their analysis is based on the observation that, in the cores of the maximally massive stars, the sound velocity approaching the conformal limit $v_s/c \rightarrow 1/\sqrt{3}$ and the polytropic index $\gamma < 1.75$ — the value close to the minimal one obtained in hadronic models.

In a series of publications, we proposed a pseudo-conformal model (PCM) for dense nuclear matter for compact stars [5, 6] in terms of a particular topological structure of baryonic matter embodying both nucleonic and quarkonic properties. This model with a few controllable parameters is found to account satisfactorily for all known properties of the normal nuclear matter and the astrophysical observations so far available, including the data from the gravitational detection emitted from the neutron star merger. For a comprehensive analysis, we refer to Ref. [7].

In this Letter, we show that this PCM makes a striking prediction that the core of massive stars is populated by topology (a.k.a. baryon)-number-1/2 objects. In this core, quarks are still confined with the scale symmetry spontaneously broken (or more appropriately hidden) but the sound velocity approaches the conformal limit $v_s/c \approx 1/\sqrt{3}$ and the polytropic index goes to $\gamma \leq 1.75$, approaching 1 as density exceeds the topology change density $n_{1/2} \sim 3n_0$. Given that the trace of energy-momentum tensor (TEMT) is not equal zero, it is not conformal, so we interpret it “pseudo-conformal,” signaling emerging hidden scale symmetry.

Pseudo-conformal model.— The PCM we proposed before has the following characteristic features (for details, see Refs. [5, 7]):

- The effective Lagrangian includes the lowest-lying vector mesons $\rho$ and $\omega$, the lightest scalar meson $f_0(500)$ and the nucleon, in addition to the Nambu-Goldstone boson pions. The vector mesons are the gauge fields of the hidden flavor symmetry (HLS) and $f_0(500)$ as the Goldstone boson of the hidden scale symmetry. We refer to the resulting Lagrangian as “GbsHLS”, standing for generalized scale-symmetric baryonic HLS.
- The density dependence comes from both the intrinsic density dependence (IDD) inherited from fundamental QCD and nuclear correlations in nuclear matter, such as the effect of three-body potential integrated out, denoted as $\text{DD}_{\text{induced}}$.
- The IDD is divided into two density regimes R-I and R-II delineated by the topology change density $n_{1/2}$. By the Cheshire Cat Principle expounded in [7], $n_{1/2}$ corresponding to the skyrmion-half-skyrmion transition density is found to be confined in the range $2 \lesssim n/n_0 < 4$ to capture the putative hadron-quark continuity. In R-I, the IDDs are
fixed by only one parameter that gives the density dependence of the in-medium pion decay constant determined from deeply bound pionic atoms, while in R-II, topology together with the assumed high density properties of hidden local symmetry and (hidden) scale symmetry determine how the EoS should vary with density. The pseudo-conformal structure results from this property in R-II.

- The equation of state (EoS) has been calculated in the $V_{lowk}$ renormalization-group ($V_{lowk}$-RG) approach [8] with the GsHLS Lagrangian with the parameters in R-I and R-II of the Lagrangian suitably matched as stated above. For computational ease, it turns out be convenient to map the $V_{lowk}$-RG results to simple analytic functions for both R-I and R-II. This can be done for a given $n_{1/2}$ in the range admitted. The parameterized function of the EoS takes in R-I the form

$$E/A = A^\alpha \left( \frac{n}{n_0} \right) + B^\alpha \left( \frac{n}{n_0} \right)^{D^\alpha}$$

(1)

and in R-II the form

$$E/A = -m_N + X^\alpha \left( \frac{n}{n_0} \right)^{1/3} + Y^\alpha \left( \frac{n}{n_0} \right)^{-1}$$

(2)

with $\alpha = (N - Z)/(N + Z)$. The pressure and the chemical potential of the R-I and R-II regions are of course matched at $n_{1/2}$.

**Equation of state and star properties.**— In the PCM, the only parameter that cannot be constrained by the theory is the location of the topology change $n_{1/2}$. As has been extensively discussed in Ref. [7], the range can be determined by the constraints of the star properties and it comes to $2.0 \leq n_{1/2}/n_0 < 4.0$. For the present purpose, we take a typical value $n_{1/2} = 2.5n_0$. There is very little difference for a different value within the range (see Ref. [7] for a comprehensive analysis).

We list in Tables I and II the parameters that accurately reproduce the $V_{lowk}$-RG results. Now using these parameters, one can easily obtain the nuclear matter properties before the topology change as given in Table II. From this table, one clearly sees that the nuclear matter properties from PCM agree with the constraints found from analyses in the literature.

**TABLE I.** Fitting parameters (in of MeV) in R-I for symmetric nuclear matter ($\alpha = 0$) and neutron matter ($\alpha = 1$).

| Parameter | Prediction | Empirical |
|-----------|------------|-----------|
| $n_0$     | 0.161      | 0.16 ± 0.01 |
| B.E.      | 16.7       | 16.0 ± 1.0 |
| $E_{sym}(n_0)$ | 30.2       | 31.7 ± 3.2 |
| $E_{sym}(2n_0)$ | 56.4       | 46.9 ± 10.1 |
| $L(n_0)$  | 67.8       | 58.9 ± 16 |
| $K_0$     | 250.0      | 230 ± 20   |

We first show in Fig. 1 the TEMT $\langle \theta_{\mu}^\alpha \rangle$ predicted by the PCM,

$$\langle \theta_{\mu}^\alpha \rangle = \langle \theta^{00} \rangle - \sum_i \langle \theta^{ii} \rangle = \epsilon - 3P$$

(3)

which is the order parameter for the scale symmetry, as function of density. What is noteworthy in this figure is that after the topology change, the trace of the energy momentum tensor $\langle \theta_{\mu}^\alpha \rangle$ does not scale with density but stays non-zero , $\langle \theta_{\mu}^\alpha \rangle \neq 0$, within the range of density relevant to massive compact stars. Thus the scale symmetry is not restored. How this comes about is intrinsically connected with the precursor to the emergent scale symmetry and the approach to the “vector manifestation fixed point” at which the vector meson mass goes massless – as explained in detail with necessary references in [5].

**TABLE II.** Fitting parameters (in unit MeV) in R-II for symmetric nuclear matter ($\alpha = 0$) and neutron matter ($\alpha = 1$).

| Parameter | $X^\alpha$ | $Y^\alpha$ |
|-----------|------------|------------|
| $\alpha = 0$ | 571.45     | 678.92     |
| $\alpha = 1$ | 433.46     | 259.21     |

**FIG. 1.** Density dependence of the TEMT in neutron matter.

Now, let’s look at the sound velocity defined as

$$c_s^2 = \frac{\partial P(n)}{\partial n} / \frac{\partial s(n)}{\partial n}$$

(4)
which can be easily computed from the EoS. The result is given in Fig. 2. Remarkably the sound velocity, sharply peaking below the density \( n_{1/2} \) up to \( v_s/c \sim 0.7 \), comes down below \( v_s/c = \sqrt{1/3} \) at slightly above \( n_{1/2} \) and then rapidly converges and stays at \( \sqrt{1/3} \). This striking behavior can be easily understood by taking the derivative with respect to density of \( \langle \theta_\mu^\mu \rangle \)

\[
\frac{\partial}{\partial n} \langle \theta_\mu^\mu \rangle = \frac{\partial \epsilon(n)}{\partial n} (1 - 3v_s^2). \tag{5}
\]

As plotted in Fig. 1 after the topology change, \( \langle \theta_\mu^\mu \rangle \) stays (density-independent constant). Therefore the left-hand side of (5) vanishes. Since \( \partial \epsilon(n)/\partial n \neq 0 \), i.e., no Lee-Wick-type states, it follows that \( v_s/c - 1/\sqrt{3} = 0 \).

FIG. 2. Density dependence of the sound velocity in neutron matter.

![FIG. 2. Density dependence of the sound velocity in neutron matter.](image)

It should be stressed that the precocious convergence \( (v_s/c)^2 \rightarrow 1/3 \) at \( \sim 3n_0 \) is most likely an oversimplification. Among others it ignores a possibly important effect of the anomalous dimension of the gluonic energy-momentum tensor \( \beta' \) figuring in the trace anomaly of QCD, which may move the onset density above \( 3n_0 \) and/or make the sound speed deviate from the conformal. In fact there is an indication of such an effect in the long-standing puzzle of quenched QCD simulation. In this Letter, we show that all the star properties can be consistently explained, with the striking prediction that massive stars of \( \sim 2M_\odot \) could support a core with fractionally (baryon-)charged objects with \( v_s/c \approx 1/\sqrt{3}, \gamma < 1.75 \) but \( \langle \theta_\mu^\mu \rangle \neq 0 \). These objects are not the usual hadrons figuring in the standard nuclear effective field theories nor are they 1/3-baryon-charged quarks. So what are they? This question is addressed below.

FIG. 3. Density dependence of the polytropic index \( \gamma \) in neutron matter.

![FIG. 3. Density dependence of the polytropic index \( \gamma \) in neutron matter.](image)

The polytropic index \( \gamma \) given by

\[
\gamma = \frac{d \ln P}{d \ln \epsilon}, \tag{6}
\]

can also be straightforwardly computed. The result is plotted in Fig. 3. We find that, after the topology change, \( \gamma \) is reduced to the region \( \gamma < 1.75 \) — the value close to the minimal value obtained in hadronic models — and, when the density increases, \( \gamma \) approaches to the conformal limit \( \gamma \rightarrow 1 \) from above.

We finally plot in Fig. 4 the neutron star mass as a function of the central density \( n_{\text{cent}} \). We find that for a star with mass \( \geq 1.55M_\odot \), the central density is \( n_{\text{cent}} \geq n_{1/2} = 2.5n_0 \). Therefore, for a star with mass \( \geq 1.55M_\odot \), the core of the star is in the pseudo-conformal phase with \( v_s/c \approx 1/\sqrt{3}, \gamma < 1.75 \) but \( \langle \theta_\mu^\mu \rangle \neq 0 \).

In summary, the analysis above tells us that, in the PCM constructed in terms of the hidden local and scalar field degrees of freedom, all the star properties can be consistently explained, with the striking prediction that massive stars of \( \sim 2M_\odot \) could support a core with fractionally (baryon-)charged objects with \( v_s/c \approx 1/\sqrt{3}, \gamma < 1.75 \) but \( \langle \theta_\mu^\mu \rangle \neq 0 \). These objects are not the usual hadrons figuring in the standard nuclear effective field theories nor are they 1/3-baryon-charged quarks. So what are they? This question is addressed below.

**Conclusion and remarks.**— The composition of the matter inside the compact stars is extremely interesting as it can help us to reveal the phase structure of QCD at high density (and low temperature) which can be accessed neither by terrestrial experiments nor by lattice QCD simulation. In this Letter, we show that all the experimental data at low density and astrophysical observations at high density can be calculated in a unified way using our model in terms of only hadronic variables without explicit QCD variables.
In contrast to the scenario of Ref. [4] where “deconfined” quarks figure crucially, the sound velocity of our model contains no explicit quark degrees of freedom and furthermore predicts the sound velocity $v_s$ very close to the conformal value $1/\sqrt{3}$ even though the conformal symmetry is not restored. In addition, we found that the polytropic index $\gamma \equiv d(\ln p)/d(\ln \epsilon) < 1.75$, approaching 1 as density goes beyond $n_{1/2}$. So the question is what are these “objects” masquerading quarks? We have no simple convincing answer but conjecture that they are both “quasi-quarks” and “quasi-baryons” in the sense of hadron-quark continuity. One can show that $N_c$ light quarks in a confinement bag can be traded, continuously, in for a soliton [15]. For instance a $N_f = 1$ baryon can be shown to be equivalent to $N_c$ fractionalized quarks within a fractional quantum Hall pancake or pita [16]. This suggests that fractionalized baryons can be equivalent to highly distorted or dressed quarks. It seems plausible that what we are seeing in the core of the star are indeed these objects. In fact there is an indication of such objects in the skyrmion matter simulated on crystal which behave like scale-invariant objects in the density regime $n \gtrsim n_{1/2}$ [3]. Given that in the large $N_c$ limit, the constituent-quark structure is the same as skyrmionic, the structure of the PCMs could be likened to the quarkyonic structure on the Fermi sphere where constituent quarks with hard-core interactions [2] behave similarly to fractionalized skyrmions with pseudoconformal symmetry.

Finally one can ask where the true conformal sound speed ($\mathcal{C}_s = 0$) would show up. In the line of reasoning we are adopting, this must take place at a density much higher than that relevant to compact stars, say, $\gtrsim 50n_0$ at which scale invariance and vector manifestation with both the scalar and vector mesons going massless are expected to figure. That’s the density at which the hidden local symmetry in the Higgs mode transits to a topological phase conjectured in duality theories with the explicit QCD degrees intervening [17]. Such density must lie much higher than what’s relevant in compact stars being studied.

Acknowledgments.— We are grateful for helpful correspondence from Jim Lattimer and Aleksi Vuorinen on their recent publications. The work of YLM was supported in part by the National Science Foundation of China (NSFC) under Grant No. 11875147 and 11475071.

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