Supersymmetric Brane-Antibrane Configurations

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We find a class of flat supersymmetric brane-antibrane configurations. They follow from ordinary brane-antibrane systems by turning on a specific worldvolume background electric field, which corresponds to dissolved fundamental strings. We have clarified in detail how they arise and identified their constituent charges as well as the corresponding supergravity solutions. Adopting the matrix theory description, we construct the worldvolume gauge theories and prove the absence of any tachyonic degrees. We also study supersymmetric solitons of the worldvolume theories.

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1 Introduction

Recently a new class of supersymmetric brane configurations has been discovered in string theory, the so called supertubes. They were originally constructed as 1/4 BPS solutions to the Born-Infeld action of a D2 brane with worldvolume gauge fields turned on \[1\]. Many supertubes and their worldvolume theory have been investigated using matrix theory in \[2\] and a corresponding supergravity solution was constructed in \[3\]. The supertube is a cylindrical D2 brane which is prevented from collapse by angular momentum. The configuration has zero D2 brane charge but carries a D2 dipole moment, as expected from a tube like configuration. The reason it can be BPS is that the worldvolume fields turned on, which carry the angular momentum, induce D0 and F1 brane charge, which ultimately become the charges appearing in the corresponding supersymmetry algebra. More recently, generalizations including junctions of tubes have been studied in Ref.\[4\].

In this paper we want to explore a much simpler system that exhibits a similar physics: a parallel configuration of D2 and anti-D2 brane in flat space can be made 1/4 BPS by turning on worldvolume B and E fields. This configuration can be obtained by deforming the circular tube into an elliptic tube and by taking the limit where the ellipse degenerates into two parallel lines. The separation between the branes is a free parameter. Both branes have the same E-field and opposite B-field. The fact that this deformation into an ellipse still preserves supersymmetry can easiest be seen in the matrix theory description of the tube, where the BPS equations for the matrices \(X\), \(Y\) and \(Z\) encoding the positions of the constituent D0 branes can be brought into the form \[2\]:

\[
[X,Y] = 0, \quad [X,[X,Z]] + [Y,[Y,Z]] = 0.
\]

This is solved by setting \([X,Z] = ilY\) and \([Z,Y] = ilX\). For the circular tube the same constant \(l\) has been chosen in both equations, but in order to solve the BPS equations one only needs that \([X,Z] \sim Y\) and \([Z,Y] \sim X\), so different constants can be chosen. This corresponds to the deformation into the ellipse.

At first it may sound counterintuitive that a brane and an antibrane can be BPS together. The way this works is that in the presence of both E and B-field, the supersymmetry preserved by a D2 brane is the same as the supersymmetry of an anti D2-brane, as long as the B-fields come with opposite signs. This will be shown in Section 2 by studying the kappa symmetry on the D2 worldvolume. Once we established that D2 and anti-D2 preserve the same SUSY it is clear that one can actually have an arbitrary number of D2 and anti-D2 branes at arbitrary positions in the transverse space with or without net D2 brane charge and still describe a stable 1/4 BPS configuration. Even the magnitude of the B-field can vary from brane to brane as long as we keep the sign choice correlated with brane or antibrane. In section 3 we write down the supergravity solution for these supersymmetric brane configurations and analyze T-dual setups.

Section 4 is devoted to a study of the D2 anti-D2 system using matrix theory. We once more demonstrate that the configuration preserves 1/4 of the supersymmetry. Analyzing small fluctuations we show explicitly that by turning on the electric field on the worldvolume the tachyon in the D2 anti-D2 system disappears. The worldvolume theory is non-commutative SYM. We again construct configurations corresponding to arbitrary superpositions of branes. We also exhibit solutions corresponding to branes at angles. In Section 5 we study the worldvolume theory using the open string metric. Possible decoupling limits are discussed. Section 6 is devoted to a study of worldvolume solitons.
2 Worldvolume action and kappa-symmetry

For a D2 brane to be supersymmetric, one has to find Killing spinors of the background geometry, which we take to be just flat space, that satisfy

$$\Gamma \epsilon = \pm \epsilon$$

(2)

where $\Gamma$ is the matrix appearing in the worldvolume kappa symmetry action that depends on the embedding, the type of brane and the worldvolume fields that are turned on. The upper or lower sign refers to brane and antibrane respectively. So it seems obvious that the spinors that satisfy the equation with the plus sign can’t satisfy the equation with the minus sign simultaneously. This conclusion can be avoided, if we turn on different worldvolume fields on D2 and anti-D2 respectively, so that the left side picks up an extra sign as well. What we like to show is that in the presence of an worldvolume $E$-field 1/4 of the supersymmetries are preserved as long as one turns on $B$-fields of opposite sign on D2 and anti-D2 branes. As we will see, the magnitude of $B$ actually doesn’t matter.

In the case of a D2 brane along $txz$ with $F_{xz} = B$ and $F_{zt} = E$ turned on, the matrix $\Gamma$ becomes [4]:

$$\Gamma = \frac{\sqrt{\det(g)}}{\sqrt{\det(g + 2\pi \alpha' F)}} (\gamma_{txz} + E \gamma_x \gamma_{11} + B \gamma_t \gamma_{11}).$$

(3)

In what follows we will take the background metric to be 10d flat space. For analyzing possible scaling limits we will later restore the constants $g_{tt}, g_{xx}$ and $g_{zz}$, but for now we just work with a metric $g_{\mu \nu} = \eta_{\mu \nu}$. Similarly, we will frequently set $2\pi \alpha' = 1$. The $\gamma$ matrices are the induced worldvolume Dirac matrices. Since we are dealing with a flat brane in flat space, they are just equal to their embedding space counterparts.

For $E = B = 0$, Eq. (3) reduces to

$$\gamma_{txz} \epsilon = \pm \epsilon$$

(4)

which is the usual condition for a D2 along $txz$. D2 and anti-D2 preserve opposite supersymmetries.

What we do corresponds to solving (3) by setting

$$\gamma_{txz} + E \gamma_x \gamma_{11} = 0$$

(5)

and

$$B \gamma_t \gamma_{11} = 0.$$ 

(6)

Notice that the sign that differs for D2 and anti D2 only makes it into (6), not into (5). As we will see momentarily, imposing those two conditions simultaneously preserves 1/4 of the supersymmetries.

Eq. (3) is solved by setting $E = -1$ and imposing

$$\gamma_{txz} \epsilon = -\gamma_{11} \epsilon.$$ 

(7)

In the absence of a magnetic field, $|E| = 1$ would be the critical value of the electric field, where the fundamental string becomes tensionless, $\sqrt{g + F}$ vanishes. Since we turned on a $B$ field in addition,  

*One can choose either signatures of $E$. But for the later comparison, we here choose the negative one.
$|E| = 1$ isn’t critical in the sense that the tension of strings goes to zero. For a single D2 brane, we could go to a frame where only an electric field or only a magnetic field is turned on depending on whether the Lorenz invariant quantity $E^2 - B^2$ is positive or negative. As the magnitude of $B$ is arbitrary, we can choose $E^2 - B^2$ to have either sign. But even if we choose it to be positive with a nonvanishing $B$ field in the original frame, in the new frame with $E$ field only, $|E_{\text{new}}| < 1$. Since we will soon be dealing with many branes, all with the same $E$, but with different $B$s, we will stay in the original frame where both are non-zero. What does happen at $|E| = 1$ is that

$$\sqrt{g + F} = \sqrt{1 + B^2 - E^2} = \sqrt{B^2}$$

which we will still loosely refer to as a “critical” electric field in what follows.

Eq. (8) is solved by

$$\gamma_t \gamma_{11} \epsilon = \pm \frac{\sqrt{B^2}}{B} \epsilon$$

where again the two different signs refer to D2 brane and anti-D2 brane respectively. Now it is obvious that as long as we choose the sign of $B$ to be positive for a D2 brane and negative for an anti-D2 brane, Eq. (8) in both cases reduces just to

$$\gamma_t \gamma_{11} \epsilon = \epsilon$$

This is just the supersymmetry preserved by a D0 brane. Similarly (9) is the supersymmetry preserved by a fundamental string along $t$ and $z$. We see that the supersymmetry of this configuration is only sensitive to the constituents, that is the lower brane charges induced by the background fields. For D0’s and F1’s we know that the conditions (8) and (9) are consistent with each other and preserve $1/4$ of the supersymmetries (the D0-F1 system is dual to D3-D5 or D1-F1).

But now since both D2 and anti-D2 with the appropriate sign of the $B$-field preserve the same supersymmetries (those associated with F1 and D0) it is clear that there is no force and we can have configurations with an arbitrary number of D2 and an arbitrary number of anti-D2’s at arbitrary positions in the transverse directions. Note that the magnitude of the $B$-field is allowed to differ from brane to brane, as long as the sign choice is correlated with the brane being D2 or anti-D2.

### 3 Supergravity solutions

The supergravity solution for a single D2 brane with the $E$ and $B$ field of the kind we discussed turned on was constructed in (3) as a limit of the supertube metric, zooming in to the region very close to the tube where the tube looks planar. The metric in this limit reads

$$ds^2 = -U^{-1}V^{-1/2}(dt - kdx)^2 + U^{-1}V^{1/2}dz^2 + V^{1/2}dx^2 + V^{1/2}(dr^2 + r^2d\Omega_6^2)$$

for $N$ coincident D2 branes along $t$, $x$ and $z$, and the harmonic functions $U$, $V$ and $k$ being given by the usual expressions for smeared F1, D0 and D2 brane charge along $t$, $x$ and $z$ localized at the origin $r = 0$ in the transverse seven space:

$$U = 1 + \frac{12\pi^3 g_s^2 \rho_n}{r^5}$$

$$V^{1/2} = \left(1 - \frac{k^2}{U^{1/2}}\right)^{-1/2}$$

$$k = \frac{12\pi^3 g_s^2 \rho_n}{U^{1/2}}$$

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\[
V = 1 + \frac{24\pi^4 g_s f_s^7 n_0}{r^5}
\]
\[
k = \frac{6\pi^2 g_s f_s^5 N}{r^5}
\]

where \(n_0\) and \(n_1\) are the D0 and F1 charge densities in string units respectively. The other nonzero SUGRA fields are given by

\[
B_{(2)} = -U^{-1}(dt - kdx) \wedge dz + dt \wedge dz
\]
\[
C_{(1)} = -V^{-1}(dt - kdx) + dt
\]
\[
C_{(3)} = -U^{-1}kdt \wedge dz \wedge dx
\]
\[
e^\phi = U^{-1/2}V^{3/4}
\]
giving rise to a 4-form field strength

\[
G_{(4)} = dC_{(3)} - dB_{(2)} \wedge C_{(1)} = U^{-1}V^{-1}dt \wedge dz \wedge dx \wedge dk
\]

appropriate for \(N\) D2 branes at \(r = 0\). Since this configuration is BPS, as shown in [3], one can construct many-centered brane solutions with branes at \(\vec{y}_a\) by just superposing harmonic functions with \(\frac{1}{r^n}\) replaced by \(\frac{1}{|\vec{y} - \vec{y}_a|^n}\). While only positive contributions can be added to \(U\) and \(V\), \(k\) can receive both positive and negative contributions corresponding to a D2 or an anti-D2 respectively located at \(\vec{y}_a\).

As in the case of the supertube we can find a bound on the number of D2 brane charge in the system. Basically every D2 brane and anti-D2 brane we introduce comes with a fixed amount of D0 and F1 charge. If for a given amount of D0 and F1 brane charge we want to maximize the D2 brane charge, we should have no antibranes and no extra unbounded D0’s or F1’s. Every other configuration has less D2 brane charge. This way one obtains an upper bound on \(N\) in terms of \(n_0\) and \(n_1\). As in the case of the tube in the gravity solution this bound can be reproduced by studying closed time like curves [3]. If \(g_{xx} < 0\) in addition to \(g_{tt} < 0\) we can find a continuous path \(x^\mu(s)\) for \(s \in [0, 1]\) such that

\[
x^\mu(s = 0) = x^\mu(s = 1)
\]

and

\[
g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} < 0
\]

for all \(s \in [0, 1]\). For this not to happen, we have to require

\[
-U^{-1}V^{-1/2}k^2 + V^{1/2} \geq 0
\]

which translates into

\[
N^2 \leq n_0 n_1 (2\pi)^3 g_s f_s^3.
\]

From this solution it is straightforward to T-dualize to higher dimensional branes, that is Dp and anti-Dp branes which are 1/4 BPS because they carry (smeared) D(p-2) and F1 charge. To do this one has to implement the following changes:

- the harmonic functions now only depend on the radial coordinate in the \(9 - p\) dimensional transverse space, their falloff being given by \(\frac{1}{r^{9-p}}\).
the formerly transverse directions that become worldvolume directions upon T-duality come with a $V^{-1/2}$ instead of the $V^{1/2}$. Otherwise the metric is the same as above.

- the R-R forms $C_{(3)}$ and $C_{(1)}$ get replaced with $C_{(p)}$ and $C_{(p-2)}$ by wedging the old solution with the new worldvolume directions.

- the dilaton changes under T-duality, and the new dependence is

$$e^{\phi} = U^{-1/2}V^{\frac{3-(p-2)}{4}}$$

(19)

It is a little more difficult to understand what happens if we T-dualize along one of the worldvolume directions of the two branes. In the case of the supertube this T-duality leads to a superhelix configuration, see [6, 7]. One thing we can do is to go back to the worldvolume action and choose a gauge, where the worldvolume gauge field is

$$A_z = Bx - Et$$

(20)

and all other components zero. Under T-duality a worldvolume gauge field of the D2 translates into the position in the transverse direction of the T-dual D1. We see that our D2 brane T-dualizes into a D1 string whose $z$ coordinate is given by (20), that is it is moving with a speed $E$ in the $z$ directions and rotated by $B$. Since D2 and anti-D2 branes have opposite signs of $B$ they are rotated away from each other.

4 Matrix theory

So far we have discussed the supersymmetric D2 and anti-D2 systems from the viewpoints of the supergravity or the Born-Infeld descriptions. In this section, we shall exploit their properties employing the matrix theory description. As mentioned earlier, the supersymmetric D2 and anti-D2 system may be obtained from the noncommutative supersymmetric tubes [2, 4] as a limiting case of the elliptic deformation. We will first describe the details of such deformation within the matrix theory description. A constituent D2 brane obtained this way preserves only a quarter of the total 32 supersymmetries due to the presence of the worldvolume gauge field. We shall provide a detailed study of the worldvolume gauge theory that is noncommutative. Unlike the ordinary D2-D2 which is tachyonic, there should not be any tachyonic degrees in our case. This will be proved using the noncommutative worldvolume gauge theory. We then provide more general supersymmetric brane configurations like branes at angles.

Let us begin with the matrix model Lagrangian [8]

$$L = \frac{1}{2R} \text{tr} \left( \sum_I (D_0 X_I)^2 + \frac{1}{(2\pi \alpha')^2} \sum_{I \neq J} [X_I, X_J]^2 + \text{fermionic part} \right)$$

(21)

where $I, J = 1, 2, \cdots 9$, $R = g_s l_s$ is the radius of the tenth spatial direction, and $\alpha'(\equiv l_s^2)$ is related to the eleven dimensional Planck length by $l_{11} = (R\alpha')^{\frac{1}{3}}$. The scale $R$ (together with $2\pi \alpha'$) will be frequently omitted below by setting them unity. One could introduce a target space metric $g_{\mu \nu}$ into the action, which may be a convenient way to get the decoupling limit related to the
noncommutative field theory. However, for simplicity, we shall not go into this complication in this section. Related to the descriptions below, one thing we like to emphasize is that the above model is valid for any finite $R$ and $\alpha'$. One does not need any further decoupling limit for the validity of the description\[9].

For the supersymmetric tubes or D2-$\overline{\text{D}2}$ systems, we shall turn on the first three components, $X$, $Y$ and $Z$. Using the Gauss law, the Hamiltonian can be written in a complete square form plus commutators terms as

$$H = \frac{1}{2} \text{tr} \left( (D_0 X + i[Z,X])^2 + (D_0 Y + i[Z,Y])^2 + (D_0 Z)^2 + ||X,Y||^2 + 2C_J \right) \geq \text{tr} C_J \quad (22)$$

where the central charge $\text{tr} C_J$ is defined by

$$\text{tr} C_J = i \text{tr} \left( [X,Z(D_0 X)] + [Y,Z(D_0 Y)] \right). \quad (23)$$

Note that the central charge here is a trace of commutator terms. Hence for any finite dimensional representations, the central charge vanishes. As will be shown later on, this central charge is related to the stretched strings in the $z$-direction\[10]. The relevant BPS equations were identified in Refs.\[3,4]; in the gauge $A_0 = \frac{1}{2\pi\alpha'} Z$, they are given by Eq.(1) with all the components static, i.e. $\partial_0 X = \partial_0 Y = \partial_0 Z = 0$. Notice that the supersymmetric tube solutions in Refs.\[3,4] satisfy the algebra, $[Z,X] = ilY$, $[Y,Z] = ilX$ and $[X,Y] = 0$ where $l$ is an arbitrary parameter related to the noncommutativity scale. The representations of the algebra describe tubes extended in $z$-direction. In particular, $\rho^2 \equiv X^2 + Y^2$ is a Casimir of the algebra and proportional to the identity for any irreducible representations; the solutions describe circular shaped tubes. There seem many variations of the algebra that lead to the solutions of the BPS equations. One simple deformation of interest is given by

$$[Z,X] = iaY, \quad [Y,Z] = ibX, \quad [X,Y] = 0, \quad (24)$$

with $a$ and $b$ arbitrary. One can easily show that the corresponding configuration describes an elliptic tube and the ellipse is described by a Casimir, $\rho^2 = \frac{1}{a} X^2 + \frac{1}{b} Y^2$. We then take the degenerate limit where the scale $a$ becomes large while keeping $b$ and $\rho$ fixed. The resulting configuration is describing two separated planar D2 branes. Since the tube carries no D2-brane charges, it is clear that the resulting two brane configuration corresponds to a brane-antibrane system. As will be shown shortly, the limit is in fact described by the reduced BPS equations

$$[X,Y] = [Z,Y] = 0 \quad [X,[X,Z]] = 0. \quad (25)$$

Any nontrivial solutions of these equations will be again 1/4 BPS. This one may see as follows. Note that the supersymmetric variation of the fermionic coordinates $\psi$ in the matrix theory is

$$\delta \psi = \left( D_0 X^I \gamma_I + \frac{i}{2} [X^I,X^J] \gamma_{IJ} \right) \epsilon' + \bar{\epsilon}, \quad (26)$$

where $\epsilon'$ and $\bar{\epsilon}$ are real spinors of 16 components parameterizing total 32 supersymmetries. These are related to the 11 dimensional 32 component real spinor $\epsilon$ by

$$\epsilon' = \gamma_{11} \Omega_{+}^{11} \epsilon, \quad \bar{\epsilon} = \Omega_{-}^{11} \epsilon \quad (27)$$
with the projection operators $\Omega_{\pm}^{11} = \frac{1}{2}(1 \pm \gamma_t \gamma_{11})$. Using the BPS equations in (25), the invariance condition becomes

$$D_0 X(\gamma_x + \gamma_{xz})\epsilon' + \tilde{\epsilon} = 0,$$

(28)

For the nontrivial configurations, this implies that

$$\Omega_{\pm}\epsilon' = 0, \quad \tilde{\epsilon} = 0,$$

(29)

where $\Omega_{\pm} \equiv \frac{1}{2}(1 \pm \gamma_z)$ are projection operators. These two conditions agree respectively with (7) and (10) of the kappa symmetry consideration. The kinematical supersymmetries parametrized by $\tilde{\epsilon}$ are completely broken while only half of remaining dynamical supersymmetries are left unbroken. Hence in total the configuration preserves a quarter of the 32 supersymmetries of the matrix model and the unbroken supersymmetries are the same as those of the supersymmetric tubes.

Before delving into the case of the supersymmetric D2-\overline{D2}, we will first construct a supersymmetric D2 with electric flux from the BPS equations in (25) and study its charge and worldvolume dynamics. A D2 with electric flux is described by the Heisenberg algebra

$$[x, z] = i\theta, \quad y = 0$$

(30)

where $\theta$ is the noncommutativity parameter. As usual one may find the representation of this algebra introducing the annihilation and creation operators, $c$ and $c^\dagger$, by

$$c = \frac{1}{\sqrt{2\theta}}(x + iz), \quad c^\dagger = \frac{1}{\sqrt{2\theta}}(x - iz)$$

(31)

with $[c, c^\dagger] = 1$. The minimal irreducible representation of the algebra will then be

$$x + iz = \sqrt{2\theta} c = \sqrt{2\theta} \sum_{n=0}^{\infty} \sqrt{n + 1}|n\rangle\langle n + 1|.$$  

(32)

This background describes D0’s distributed uniformly in the $x$-$z$ plane. The description respects the rotational symmetry around the origin. The operator $r^2 \equiv x^2 + z^2$ is diagonalized by the states $|n\rangle$ with eigenvalues $\theta(n + 1/2)$.

For the charges, we will use the nonabelian Chern-Simons couplings of D-particles to the R-R gauge fields\cite{11},

$$S_{CS} = \mu_0 \int dt \text{tr} \left( C_{t}^{(1)} + C_{t}^{(1)} D_t \phi^I + \frac{i\lambda}{2} C_{t I J}^{(3)} [\phi^I, \phi^J] + \frac{i\lambda^2}{3} \phi^I \phi^J \phi^K F_{t I J K} + \text{h.o.t.} \right),$$

(33)

where $\mu_p^{-1} = (2\pi)^p g_s l_s^{p+1}$, $\lambda = 2\pi \alpha'$, $X^I = 2\pi \alpha' \phi^I$ and $F^{(p+1)}$ is the field strength corresponding to the R-R p-form potential, $C^{(p)}$. The first term implies that the charges of D0 is counted by $\text{tr} I$. The third term implies there is now net D2-brane charge. The last term vanishes and there is no dipole moment that couples to the R-R four form field strength, partly because the D2 is located at the origin ($\vec{y} = 0$) in the transverse space.

Noting

$$S_{CS}^{D2} = \frac{i\mu_0 \lambda}{2} \int dt \text{tr} [\phi^I, \phi^J] C_{t I J}^{(3)} = \frac{1}{(2\pi)^2 g_s l_s^3} \int dtdxdz C_{txx}^{(3)},$$

(34)
we see that the D2 charge density is given by \( \mu_2 \) as expected. For the density of D0’s on the D2 brane, we use the relation \( \int dx dz = 2\pi \theta I \) where \( \theta I \) corresponds to the number of D0’s as said above. Thus the number density \( n_0 \equiv \theta I/ \int dx dz \) is

\[
n_0 = \frac{1}{2\pi \theta}.
\] (35)

The fundamental strings are stretched in the z directions producing a worldvolume electric field. To evaluate the corresponding number density \( n_1 \equiv N_s/ \int dx \), we note that the central charge\(^{[10, 4]} \) is related to the number of strings \( N_s \) by

\[
\int dt \theta I \rho = \frac{1}{2\alpha'} N_s \int dt dz.
\] (36)

Hence one finds that

\[
n_1 = \frac{\theta}{(2\pi)^2 g_s l_s^3}.
\] (37)

The densities \( n_0 \) and \( n_1 \) obtained so far are for just one D2 brane. For \( N \) D2 branes with the same noncommutativity on each brane, which will be constructed below, a similar computation leads to

\[
n_0 = \frac{N}{2\pi \theta}, \quad n_1 = \frac{N\theta}{(2\pi)^2 g_s l_s^3}.
\] (38)

Thus we obtain relations

\[
N^2 = n_0 n_1 (2\pi)^3 g_s l_s^3, \quad \theta^2 = \frac{n_1}{n_0} 2\pi g_s l_s^3.
\] (39)

These are for the case where all the strings and D0’s are used up constructing the D2 branes without any extra D0’s or strings. The first equation in (39) agrees with the saturated supergravity bound\(^{[38]} \). The second equation can be reproduced by looking at \( B_{(2)} \) in the supergravity solution. One finds that \( B_{yz}^{(2)} = (2\pi l_s^2) B = \frac{k}{l^2} \) and looking at the near horizon region of \( r \ll l_s (g_s n_1 R)^{1/3} \) for large \( g_s R n_1 \), the second relation of (38) follows. Combining this with the relation between \( N \) and \( n_1, n_0 \) in the saturated case, one gets perfect agreement with the second equation in (38).

For the noncommutative Yang-Mills theory description of the worldvolume dynamics, we introduce gauge fields by

\[
X = x + \theta A_z, \quad Z = z - \theta A_x.
\] (40)

Using \([x,] = i\theta \partial_z\) and \([z,] = -i\theta \partial_x\), one gets

\[
[X, Z] = i\theta^2 \left( \frac{1}{\theta} + F_{xz} \right).
\] (41)

Thus it is clear that the worldvolume background magnetic field is

\[
B = \frac{1}{\theta}.
\] (42)

In order to evaluate the background electric field, we shall use the relation \( D_0 X = -\theta E_z \) and \( D_0 Z = \theta E_x \). Evaluated on the explicit solution, one finds that

\[
E_x = 0, \quad E_z = -\frac{1}{2\pi \alpha'}.
\] (43)
From this, we conclude that the background electric field in the z-direction is “critical”.

The analysis of worldvolume gauge theory is straightforward and simpler than that of the supersymmetric tubes. To organize the worldvolume theory, one may use either a standard noncommutative gauge theory with the background electric field or a deformed noncommutative gauge theory without any background. Here we shall take the latter approach, which turns out to be more convenient for a D2 brane with electric flux. First we define the worldvolume gauge field by $A_t = A_0 - Z$ and $X = x + \theta A_x$, $Z = z - \theta A_x$ and $Y = \theta \varphi$. Inserting this into the original matrix model and ignoring total derivative terms, one is led to

$$L = \frac{\theta^2}{2} \text{tr} \left( \mathcal{F}_{tx}^2 + (\mathcal{F}_{tx} - \theta \mathcal{F}_{xz})^2 - \theta^2 \mathcal{F}_{xz}^2 + (\nabla_t \varphi - \theta \nabla_x \varphi)^2 - \theta^2 \left( (\nabla_x \varphi)^2 + (\nabla_z \varphi)^2 \right) \right),$$

where

$$\nabla_\mu = \partial_\mu - i[A_\mu, \cdot] , \quad \mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

In this worldvolume action, the metric takes a rather unusual form, which has nonvanishing off diagonal elements. However as we will verify in the next section, the metric appearing in the action is the natural open string metric induced from the matrix theory description.

By redefining $x \to x + \theta t$ and $A_t \to A_t - \theta A_x$, the Lagrangian becomes

$$L = \frac{\theta^2}{2} \text{tr} \left( (\mathcal{F}_{tx})^2 + (\mathcal{F}_{tx})^2 - \theta^2 \mathcal{F}_{xz}^2 + (\nabla_t \varphi)^2 - \theta^2 \left( (\nabla_x \varphi)^2 + (\nabla_z \varphi)^2 \right) \right).$$

One could also add the contributions of the remaining 6 transverse scalars. Rescaling the time coordinate $t \to \frac{\theta}{2\pi \alpha'} t$, one can make the worldvolume theory to be the noncommutative Yang-Mills theory with a standard flat metric $\eta_{\mu\nu}$. Or one may rescale the spatial coordinate to get the noncommutative Yang-Mills theory with a standard flat metric $\eta_{\mu\nu}$.

We now move to the case of D2-\(\overline{D2}\). The solution is

$$X + iZ = \sqrt{2\theta} \sum_{n=0}^{\infty} \sqrt{n+1} \left( |2n\rangle \langle 2n + 2| + |2n + 3\rangle \langle 2n + 1| \right),$$

$$Y = \frac{\xi}{2} \sum_{n=0}^{\infty} \left( |2n\rangle \langle 2n| - |2n + 1\rangle \langle 2n + 1| \right).$$

The basis labelled by the even numbers describes a D2 brane located at $y = \xi/2$. This brane is extended in the $xz$ directions and its worldvolume background fields are $B = 1/\theta$ and $E_z = -1/(2\pi \alpha')$. On the other hand, the odd-number basis is for an anti-D2 brane extended again $xz$ directions but located at $y = -\xi/2$. The worldvolume background fields of the anti-D2 brane are $B = -1/\theta$ and $E_z = -1/(2\pi \alpha')$. Thus the branes are separated by $\xi$ in the $y$ direction. This is the configuration of D2-\(\overline{D2}\) system obtained by taking the degenerate planar limit of the supersymmetric tube.

To determine the related charges, we use (13). From the first term, the number of D0’s are again given by $\text{tr} I$. The third term vanishes and the net D2 charge is zero. The moment is now nonvanishing and evaluated by

$$S_{CS}^{\text{dipole}} = \frac{i\mu_0 \alpha^2}{3} \int dt \text{tr} \phi^J \phi^J \phi^K F_{tJK}^{(4)} = \frac{1}{3} \frac{\xi}{(2\pi)^2 g_s l_s^3} \int dtdxdz F_{txyz}^{(4)}.$$
Here we have used $2\pi\theta tr' = \int dxdz$ where $tr'$ is the trace operation over the matrices with basis $|n\rangle'\langle m\rangle'$. The dipole moment density is then
\[ d_2 = \frac{1}{3} \left( \frac{\xi}{(2\pi)^2 g_s l_s^3} \right) = \frac{1}{3} \mu_2 \xi . \] (49)

The dipole moment is proportional to the transverse separation of the D2 and $\overline{D2}$ and agrees with that of the supertube[4].

For the worldvolume description, the language of U(2) gauge theory is more appropriate. The U(2) basis can be constructed by writing $|2n - 1 + a\rangle\langle 2m - 1 + b| = |n\rangle'\langle m\rangle'|_{\tau_{ab}} (a, b = 1, 2)$. Here $|n\rangle'$ is interpreted as a new basis for the space while $\tau_{ab}$ generates an U(2) algebra. We further introduce the notation $C \equiv \frac{1}{\sqrt{2}} (X + iZ)$ with
\[ C = \left( \frac{U^T V^T}{T^V} \right) = \tau_{11} U + \tau_{22} W + \tau_{21} T^\dagger + \tau_{22} V^\dagger \] (50)
where U, V, T, and W are $\infty \times \infty$ matrices of basis $|n\rangle'\langle m\rangle'$. The D2-$\overline{D2}$ background is described by
\[ \bar{C} = \left( \frac{\sqrt{\theta} c}{0} \frac{0}{\sqrt{\theta} c^\dagger} \right) = \tau_{11} \sqrt{\theta} c + \tau_{22} \sqrt{\theta} c^\dagger , \] (51)
and the U(2)-valued background field strength reads
\[ [\bar{C}, \bar{C}^\dagger] = \theta (\tau_{11} - \tau_{22}). \] (52)

As we are dealing with brane-antibrane system, the background magnetic field on each brane has opposite signature. For the worldvolume description, we organize the fluctuation as follows;
\[ C = \sqrt{\theta} \left( \begin{array}{cc} c & 0 \\ 0 & c^\dagger \end{array} \right) + \sqrt{\theta} \left( \begin{array}{cc} 0 & 0 \\ 0 & c^\dagger - c \end{array} \right) - \frac{i\theta}{\sqrt{2}} (A_x + iA_z) \] (53)
The first term will generate the spatial derivatives and the second term is for the background gauge field,
\[ A_x^{\text{back}} = \frac{2z}{\theta} \tau_{22}, \quad A_z^{\text{back}} = 0 \] (54)
which is nonvanishing only for the anti-D2 brane. The presence of such nontrivial background magnetic field is due to the fact that we describe anti D2-brane from the view point of D2-brane. In the ordinary D2-$\overline{D2}$, this background magnetic field makes the strings connecting D2 to $\overline{D2}$ be tachyonic. To see this explicitly, let us turn on the $T$ and $W$ that describes D2-$\overline{D2}$ strings and compute the potential
\[ \text{tr} [C, C^\dagger]^2 = tr' \left( (\theta + W W^\dagger - TT^\dagger)^2 + (\theta + W^\dagger W - T^T)^2 + 2\theta |cT - Tc^\dagger + Wc - c^\dagger W|^2 \right) . \] (55)
As is well known, certain components of $T$ have a negative mass squared and become tachyonic in case of the ordinary D2-$\overline{D2}$ system[12, 13, 14]. For example, the mode $T = u|0\rangle'\langle 0\rangle'$ has a quadratic potential term $-4\theta u^2$, so it is tachyonic. In the ordinary D2-$\overline{D2}$ system, the vortex-antivortex
annihilation is argued to be an important process for the decay of the tachyons\cite{12, 13, 14}. In our case, the vortices (D0's) are also stable objects as we will see later on.

Now we like to prove that there are no tachyons in our D2-D2̅ system. For this, we proceed as follows. First using \( A_t = A_0 - Z \), rewrite the original matrix Lagrangian by

\[
L = \frac{1}{2} \text{tr} \left( (\nabla_0 X)^2 + (\nabla_0 Y)^2 + (\nabla_0 Z)^2 + [X, Y]^2 + 2i\nabla_0 X [X, Z] + 2i\nabla_0 Y [Y, Z] \right)
\]

(56)

where \( \nabla_0 = \partial_0 - i[A_0, ] \) and we suppress the contributions of the remaining scalars. Now we compute the corresponding Hamiltonian by the Legendre transform; the resulting expression reads

\[
\hat{E} = \frac{1}{2} \text{tr} \left( (\nabla_0 X)^2 + (\nabla_0 Y)^2 + (\nabla_0 Z)^2 + ||X, Y||^2 \right)
\]

(57)

which is in fact \( H - \text{tr} C_J \) with \( \text{tr} C_J \) being the central charge in (23). This Hamiltonian is dynamically equivalent to the original Hamiltonian and equally well describes the dynamics of the matrix model. We now evaluate contributions of any fluctuations around the BPS background using this new Hamiltonian. It is obvious to see that the leading contributions are quadratic in fluctuations and positive definite. The energy in total is positive definite, so any fluctuations cost energy. In conclusion there are no tachyons, as it should be since we are considering fluctuations around the BPS background.

The more general BPS solutions for the collection of parallel D2's and D̅2's are

\[
X = \frac{1}{\sqrt{2}} \sum_{a=0}^{N-1} u_a \sum_{n=0}^{\infty} \sqrt{n+1} \left( |N(n+1)+a\rangle\langle Nn+a| + |Nn+a\rangle\langle N(n+1)+a| \right),
\]

\[
Z = \frac{1}{\sqrt{2}} \sum_{a=0}^{N-1} v_a \sum_{n=0}^{\infty} i\sqrt{n+1} \left( |N(n+1)+a\rangle\langle Nn+a| - |Nn+a\rangle\langle N(n+1)+a| \right),
\]

\[
Y = \sum_{a=0}^{N-1} y_a \sum_{n=0}^{\infty} |Nn+a\rangle\langle Nn+a|,
\]

(58)

where the integer \( N \) is the total number of D2 and D̅2 and \( u_a, v_a \) and \( y_a \) are all real parameters. The parameter \( y_a \) represents the transverse location of the \( a \)-th D2-brane. \( |u_a v_a| \) (no sum) is the noncommutative parameters on \( a \)-th brane while the signature of \( u_a v_a \) indicates whether the \( a \)-th brane is D2 or D̅2. From our convention, the positive signature represents D2.

Here is an example of more nontrivial configuration;

\[
X = \frac{1}{\sqrt{2}} \sum_{n=0}^{\infty} \left[ u_0 \sqrt{n+1} \left( |2n+2\rangle\langle 2n| + |2n\rangle\langle 2n+2| \right)
\]

\[
+ u_1 \cos \chi \sqrt{n+1} \left( |2n+3\rangle\langle 2n+1| + |2n+1\rangle\langle 2n+3| \right) \right],
\]

\[
Y = \frac{u_1 \sin \chi}{\sqrt{2}} \sum_{n=0}^{\infty} \sqrt{n+1} \left( |2n+3\rangle\langle 2n+1| + |2n+1\rangle\langle 2n+3| \right),
\]

\[
Z = \frac{1}{\sqrt{2}} \sum_{a=0}^{1} v_a \sum_{n=0}^{\infty} i\sqrt{n+1} \left( |2(n+1)+a\rangle\langle 2n+a| - |2n+a\rangle\langle 2(n+1)+a| \right). \quad (59)
\]
This solution describes a configuration in which the 0th and 1st branes are extended respectively in the directions $txz$ and $tx'z$ with $x' = \cos \chi x + \sin \chi y$. Namely the two D2-branes make an arbitrary angle $\chi$ in the $xy$ plane with $z$ as a common direction. The solution is again 1/4 BPS but (25) is not an appropriate BPS equation for them. Rather they satisfy the original BPS equations in (1).

There is yet another intriguing class of BPS solutions for Eq. (25); an example is

$$X = \frac{\rho}{2} \sum_{n=-\infty}^{\infty} (|n+1\rangle \langle n| + |n\rangle \langle n+1|),$$

$$Z = l \sum_{n=-\infty}^{\infty} n|n\rangle \langle n|,$$

with $Y = 0$. This solution is obtained from the supersymmetric tube solution by setting $Y = 0$. There are no net D2-brane charges nor the dipole moment carried by this object. How to interpret this solution seems not clear. Perhaps a simple minded interpretation will be an elliptic tube with the limit where the length of minor axis becomes zero. Indeed when we diagonalize $X$, the magnitude of its eigenvalues are bounded by $\rho$. This indicates that we are dealing with a strip having a width $\rho$ in the $x$-direction, extended infinitely to the $z$-direction. The detailed worldvolume description of this kind of object is not known though the fluctuation analysis of the matrix model can in principle provide it.

5 Open string metric

Here we like to compute the metric for the noncommutative worldvolume theory on the D2. For this, we shall follow the procedure described in Ref.[1,17]. We begin with the closed string metric of the diagonal form $g_{\mu\nu} = \text{diag}(-|g_{tt}|, g_{xx}, g_{zz})$ and $B_{\mu\nu}$ of the form,

$$B_{\mu\nu} = \begin{pmatrix} 0 & 0 & E \\ 0 & 0 & B \\ -E & -B & 0 \end{pmatrix}$$

which is nothing but the $B_{\mu\nu}$ for the D2-brane. With the metric and after restoring $2\pi\alpha'$, the conditions for supersymmetry, (8) and (11) together with (8) read

$$|g_{tt}|g_{xx}g_{zz} = g_{xx}(2\pi\alpha'E)^2,$$

and, hence, $\lambda E = \pm \sqrt{|g_{tt}|g_{zz}}$. The open string metric and $\theta$ can be identified using the following relation,[15]:

$$\frac{1}{g + \lambda B} = \frac{\theta}{\lambda} + \frac{1}{G + \lambda \Phi},$$

where $G$ and $\theta^\mu\nu$ are respectively the open string metric and the noncommutativity of the worldvolume theory. The two form $\Phi$ is free to choose but there is natural one for the matrix theory description[15]. With the value $E$ in (62), note

$$\frac{1}{g + \lambda B} = \frac{1}{|g_{tt}|(\lambda B)^2} \begin{pmatrix} g_{xx}g_{zz} + (\lambda B)^2 & -\lambda B\lambda E & -g_{xx}\lambda E \\ -\lambda B\lambda E & 0 & |g_{tt}|\lambda B \\ g_{xx}\lambda E & -|g_{tt}|\lambda B & -|g_{tt}|g_{xx} \end{pmatrix}$$

(64)
The Seiberg-Witten limit \([16]\) corresponds to
\[
\lambda \sim \sqrt{\varepsilon}, \quad g_{xx} \sim g_{zz} \sim \varepsilon \tag{65}
\]
keeping \(B, E\) and \(g_{tt}\) fixed. In this limit, one finds that
\[
\theta = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & -1/B \\
0 & 1/B & 0
\end{pmatrix}, \tag{66}
\]
for any fixed \(\Phi\). One can go forward to compute the corresponding \(G_{\mu\nu}\) or the inverse \(G^{\mu\nu}\); the result agrees with the expression below.

Alternatively, from the matrix theory description we know that
\[
\theta = -\Phi^{-1} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & -1/B \\
0 & 1/B & 0
\end{pmatrix}. \tag{67}
\]
Those are the noncommutativity and \(\Phi\) used for the worldvolume theory. From this, one may compute \(G_{\mu\nu}\) without taking the Seiberg-Witten limit. From (63), they are
\[
G_{\mu\nu} = \left( g + \lambda B \right)^{-1} - \frac{\theta}{\lambda}^{-1} - \lambda \Phi. \tag{68}
\]
The straightforward evaluation gives
\[
G_{\mu\nu} = \begin{pmatrix}
0 & \pm \lambda B \sqrt{\frac{|g_{tt}|}{g_{zz}}} & 0 \\
\pm \lambda B \sqrt{\frac{|g_{tt}|}{g_{zz}}} & \frac{(\lambda B)^2}{g_{zz}} & 0 \\
0 & 0 & \frac{(\lambda B)^2}{g_{xx}}
\end{pmatrix}, \tag{69}
\]
and
\[
G^{\mu\nu} = \begin{pmatrix}
\frac{-1}{|g_{tt}|} & \pm \frac{\sqrt{g_{zz}}}{\lambda B \sqrt{|g_{tt}|}} & 0 \\
\pm \frac{\sqrt{g_{xx}}}{\lambda B \sqrt{|g_{tt}|}} & 0 & 0 \\
0 & 0 & \frac{g_{zz}}{(\lambda B)^2}
\end{pmatrix}. \tag{70}
\]
This \(G^{\mu\nu}\) is precisely the metric appearing in the action \([44]\). Here we do not have to take the Seiberg-Witten limit, but the same metric follows from the limit as mentioned before.

6 D0-D2 solutions

The supergravity solutions implies that one could have extra D0’s which are not used up to form D2 branes. The configurations are again 1/4 BPS. In this section, we shall briefly discuss such solutions. Since generalization to the case of N D2-brane is straightforward, we shall restrict our
discussion to the case of one D2 brane. As done for the case supersymmetric tubes, we introduce a shift operator defined by

\[ S = \sum_{n=0}^{\infty} |n + m\rangle \langle n|, \] (71)

It satisfies the relations

\[ SS^\dagger = I - P, \quad S^\dagger S = I, \] (72)

where the projection operator \( P \) is defined by

\[ P = \sum_{a=0}^{m-1} |a\rangle \langle a|. \] Then general soliton solutions including the moduli parameters are given by [17, 18]

\[ X_i = S x_i S^\dagger + \sum_{a=0}^{m-1} \xi_a^i |a\rangle \langle a|, \quad X_s = \sum_{a=0}^{m-1} \varphi_a^s |a\rangle \langle a|, \] (73)

where \( i = 1, 2 \) are respectively for \( x \) and \( z \), i.e. \( x = x_1, z = x_2 \) and the index \( s \) refers to the remaining seven transverse scalars. This certainly satisfies the BPS equations. Hence the solution is again 1/4 BPS and the presence of solitons does not break any further supersymmetries. The solutions describes \( m \) extra D0 branes with there positions \((\xi_a^i, \varphi_a^s)\) in the nine dimensional target space. Using the field defined in (46), the solution becomes

\[ A_i = -\frac{1}{\theta} \epsilon_{ij} \left( S x_j S^\dagger - x_j + \sum_{a=0}^{m-1} \left( \xi_a^j + \frac{\theta t}{2 \pi \alpha'} \delta_{j1} \right) |a\rangle \langle a| \right), \]

\[ \varphi_s = \frac{1}{\theta} \sum_{a=0}^{m-1} \varphi_a^s |a\rangle \langle a|, \] (74)

in an appropriate gauge. The field contents of the solution are identified as

\[ F_{tx} = \frac{1}{2 \pi \alpha'} P, \quad F_{xz} = -\frac{1}{\theta} P. \] (75)

This may be compared with the moving solitons found in [18]. It is then clear that this corresponds to moving D0’s with velocity \( v_x = \frac{2 \alpha}{m \alpha'} \). In fact the bosonic content of the Lagrangian in (46) is exactly the same as the noncommutative Yang-Mills theory describing D2 brane worldvolume in the decoupling limit. We know that the D0’s on D2 with the noncommutativity turned on are tachyonic, which was explicitly verified in [13, 20]. Hence we are lead to a seemingly contradictory result since the above solitons should be stable as the remaining supersymmetries dictate. However there is one way to avoid this conclusion; since in our case the D0’s are moving in a specific velocity, this specific motion can make the tachyonic spectrum disappear. Indeed, one can prove that \( \text{tr} \ F_{tx} \) is conserved in general using the equations of motion. Hence it is not possible to dynamically reduce the velocity to make them unstable, which confirms that there is no contradiction.

One may ask what happens to the worldvolume solitons for the 1/4 BPS N Dp or Dp anti-Dp systems that can be obtained by the T-dualization along the transverse directions. Our analysis above can be trivially extended to the case of noncommutative solitons describing branes of codimension two. It is, however, not so straightforward if one considers worldvolume solitons of Dp’ with \( p' < p - 2 \). For example, how magnetic monopoles corresponding to D-strings connecting D3-branes get affected due to the change of the worldvolume theory, seems quite interesting. Or one could study the possible deformation of instantons. Though we believe these are interesting issues, we like to leave them for the future work, partly due to the vastness of the subject itself.
7 Conclusion

In this note, we have obtained the supersymmetric brane-antibrane configurations. The configurations may be obtained by taking a degenerate limit of the elliptic deformation of the supersymmetric tubes. We have verified that the systems preserve 8 supersymmetries via the analysis of the worldvolume kappa-symmetry or the matrix theory and constructed corresponding supergravity solutions. The specific worldvolume background electric field, which is induced by the dissolved fundamental strings on the branes, makes the would-be tachyonic degrees disappear. We have shown that the worldvolume dynamics may be described by gauge theories with the spatial noncommutativity. Finally we study D0-solitons on the D2 brane, which is supersymmetric and stable.

The worldvolume background gauge fields involve the magnetic as well as electric components. From this one may naively expect that the natural description would be spacetime noncommutative. However as we have constructed in detail, the study of the matrix model leads to the worldvolume gauge theory with only spatial noncommutativity. The metric appearing in the gauge theory is shown to agree precisely with the expected open string metric of the string theory. Note, however, that the only invariant combination $\lambda^2 F^2 = g^{tt} g^{zz} (\lambda E_z)^2 + g^{xx} g^{zz} (\lambda B)^2$ may flip the signature depending on the magnitude of $\theta = 1/B$. With the specific value of $E_z$ in (62) required by the supersymmetry, the invariant combination becomes $\lambda^2 F^2 = -1 + g^{xx} g^{zz} (\lambda/\theta)^2$. Depending on the value of the noncommutativity scale $\theta$, $F^2$ may be spacelike, lightlike or timelike. Yet in the worldvolume theory, there appears no apparent signals indicating possible breakdown or transitions of the theories. Is the worldvolume noncommutative theory we obtained somehow related to the lightlike noncommutative theory in Refs.[21, 22] when $F^2 = 0$? Currently, the issues here are not fully resolved. One thing clear is that our description of the worldvolume theory can be trusted if one takes the Seiberg-Witten decoupling limit [16]. In this limit, $F^2$ is dominated by the $B^2$ term and thus spacelike always.

Finally, we like to comment upon the nature of the worldvolume theory when we have many parallel D2-branes with varying noncommutative scales. This is contrasted to the case of 1/2 BPS branes, where the BPS condition dictates all the noncommutativity scale the same. The original matrix model has a $U(N)$ noncommutative gauge symmetry, $X_I \rightarrow U^\dagger X_I U$, if one considers N such branes. Since the noncommutativity scale controls the natural open string metric and the Yang-Mills coupling constant on each brane, this gauge symmetry is broken by the presence of such branes with varying noncommutativity scales. Of course one could separate the branes in the transverse directions, which leads to the conventional way of spontaneous breaking of the nonabelian gauge symmetry. Being independent of the transverse separation, the breaking induced by the varying noncommutativity scale makes the open string geometries and the couplings vary from one to another branes. Further investigation is necessary on the interplay between the geometries and the nonabelian symmetry.

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Nonetheless, one could think of varying noncommutativity scale in the transverse directions which leads to the mixing of the nonabelian symmetry and geometry[23].
References

[1] D. Mateos and P. K. Townsend, “Supertubes,” Phys. Rev. Lett. 87 (2001) 011602, hep-th/0103030.

[2] D. Bak and K. Lee, “Noncommutative supersymmetric tubes,” Phys. Lett. B509 (2001) 168–174, hep-th/0103148.

[3] R. Emparan, D. Mateos, and P. K. Townsend, “Supergravity supertubes,” JHEP 07 (2001) 011, hep-th/0106012.

[4] D. Bak and S.-W. Kim, “Junctions of supersymmetric tubes,” hep-th/0108207.

[5] E. Bergshoeff and P. K. Townsend, “Super D-branes,” Nucl. Phys. B490 (1997) 145–162, hep-th/9611173.

[6] J.-H. Cho and P. Oh, “Super D-helix,” hep-th/0105095.

[7] O. Lunin and S. D. Mathur, “Metric of the multiply wound rotating string,” Nucl. Phys. B610 (2001) 49–76, hep-th/0105136.

[8] T. Banks, W. Fischler, S. H. Shenker, and L. Susskind, “M theory as a matrix model: A conjecture,” Phys. Rev. D55 (1997) 5112–5128, hep-th/9610043.

[9] N. Seiberg, “Why is the matrix model correct?,” Phys. Rev. Lett. 79 (1997) 3577–3580, hep-th/9710009.

[10] T. Banks, N. Seiberg, and S. H. Shenker, “Branes from matrices,” Nucl. Phys. B490 (1997) 91–106, hep-th/9612157.

[11] R. C. Myers, “Dielectric-branes,” JHEP 12 (1999) 022, hep-th/9910053.

[12] P. Kraus, A. Rajaraman, and S. H. Shenker, “Tachyon condensation in noncommutative gauge theory,” Nucl. Phys. B598 (2001) 169–188, hep-th/0010016.

[13] M. Li, “Note on noncommutative tachyon in matrix models,” Nucl. Phys. B602 (2001) 201–212, hep-th/0010058.

[14] G. Mandal and S. R. Wadia, “Matrix model, noncommutative gauge theory and the tachyon potential,” Nucl. Phys. B599 (2001) 137–157, hep-th/0011094.

[15] N. Seiberg, “A note on background independence in noncommutative gauge theories, matrix model and tachyon condensation,” JHEP 09 (2000) 003, hep-th/0008013.

[16] N. Seiberg and E. Witten, “String theory and noncommutative geometry,” JHEP 09 (1999) 032, hep-th/9908142.

[17] D. Bak, “Exact multi-vortex solutions in noncommutative Abelian- Higgs theory,” Phys. Lett. B495 (2000) 251–255, hep-th/0008204.

[18] D. Bak, K. Lee, and J.-H. Park, “Noncommutative vortex solitons,” Phys. Rev. D63 (2001) 125010, hep-th/0011099.
[19] M. Aganagic, R. Gopakumar, S. Minwalla, and A. Strominger, “Unstable solitons in noncommutative gauge theory,” *JHEP* **04** (2001) 001, hep-th/0009142.

[20] A. Fujii, Y. Imaizumi, and N. Ohta, “Supersymmetry, spectrum and fate of D0-Dp systems with B-field,” hep-th/0105079.

[21] J. Gomis and T. Mehen, “Space-time noncommutative field theories and unitarity,” *Nucl. Phys. B* **591** (2000) 265–276, hep-th/0005129.

[22] O. Aharony, J. Gomis, and T. Mehen, “On theories with light-like noncommutativity,” *JHEP* **09** (2000) 023, hep-th/0006236.

[23] K. Dasgupta and Z. Yin, “Non-Abelian geometry,” hep-th/0011034.