Flux flow resistivity in the two-gap superconductivity

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Abstract
We investigate the flux flow state in a two-gap superconductor in which two s-wave gaps with different amplitudes exist on two separate Fermi surfaces. The flux flow resistivity is obtained on the basis of the Bardeen-Stephen relation and the result agrees well with the anomalous field dependence of the flow resistivity recently observed in the two-gap superconductor MgB$_2$. Some typical properties of the vortex in this system are also discussed.

key words: MgB$_2$, two-gap structure, flux flow, extension of Bardeen-Stephen relation

The vortex lines in type II superconductors are subject to the Lorentz force under an external current and begin to flow perpendicular to the current and magnetic field when the Lorentz force exceeds the pinning force. This is the flux flow state and a finite resistivity arises\cite{1}. The basis of the flux flow resistivity is the presence of bound states inside the vortex core. The energy gaps of these states are so small that the conductivity in the core is practically normal. Such states were found by Caroli et al. by microscopic methods\cite{2}. In both dirty and moderately clean s-wave superconductors, the flux flow resistivity $\rho_f$ is proportional to the magnetic field $H\cite{1}$, namely,

$$\rho_f = \frac{H}{H_{c2}} \rho_n,$$

where $H_{c2}$ is the upper critical field and $\rho_n$ the normal-state resistivity. This is the so-called Bardeen-Stephen relation\cite{3}. In the low-temperature region, this relation holds well for almost the entire field range of the vortex state. \cite{4}

The superconductivity of MgB$_2$ has been investigated with keen interest since it has the highest transition temperature ($T_c \approx 39K$) among metallic compounds at present and a great number of investigations have been carried out\cite{5}. One of the most characteristic features of this superconductor is that two s-wave gaps with different amplitudes exist on the two separate Fermi surfaces having roughly equal densities of states (DOSs). The two-gap model was proposed on the basis of first-principle calculations,\cite{6, 7} and experimental results obtained by point-contact spectroscopy,\cite{8} specific heat measurements,\cite{9} and angle-resolved photoemission spectroscopy\cite{10} support the model.

Recently, the measurement of the flux flow resistivity of MgB$_2$ was reported by Shibata et al.\cite{11} Large deviation from the $H$-linear dependence for the flux flow resistivity has been observed, in spite of the s-wave pairing symmetry of MgB$_2$\cite{12, 13, 14}. Such anomalous behavior is expected to be related to the two-gap structure, but a clear explanation has not yet been proposed. In this study, we investigate the flux flow state in the two-gap system and propose a new scenario for the anomalous flux flow resistivity.

First, we investigate the vortex in the two-gap system, before discussing the flux flow state. Let $\Psi_L$ and $\Psi_S$ stand for the order parameter for the large energy gap and that for the small energy gap,
respectively. We use a Ginzburg-Landau (GL) free energy for the two-gap system in a weak coupling approach with a Josephson-type interaction [15],

\[
F = \int d^3r \left[ K_{L1} |D_\perp \Psi_L(r)|^2 + K_{S1} |D_\perp \Psi_S(r)|^2 \\
+ K_{Lz} |D_z \Psi_L(r)|^2 + K_{Sz} |D_z \Psi_S(r)|^2 \\
+ \alpha_L(T) |\Psi_L(r)|^2 + \alpha_S(T) |\Psi_S(r)|^2 \\
- \gamma \{ \Psi_L^*(r) \Psi_S(r) + \Psi_L(r) \Psi_S^*(r) \} \\
+ \frac{\beta_L}{2} |\Psi_L(r)|^4 + \frac{\beta_S}{2} |\Psi_S(r)|^4 + \frac{1}{8\pi} H(r)^2 \right],
\]

where \( D = \nabla - i \frac{2e}{\hbar c} A \) and index \( \perp = x, y \). We introduce a magnetic field along the \( z \)-axis. Let \( \theta_i(r) \) denote the phase of \( \Psi_i(r) \) (\( i = L, S \)). There are two types of vortex, one of which consists of the winding of \( \theta_L(r) \) and the other consists of that of \( \theta_S(r) \). The temperature dependences of the two gaps [8, 10] indicate that the mixing effect between the large gap and the small gap should play an important role in MgB2. This means that the term proportional to \( \gamma \) in the GL free energy, eq. (2), is not negligible. The term is rewritten as

\[
- \gamma \{ \Psi_L^*(r) \Psi_S(r) + \Psi_L(r) \Psi_S^*(r) \} = \\
- 2\gamma |\Psi_L(r)||\Psi_S(r)| \cos (\theta_L(r) - \theta_S(r)),
\]

and locks the relative phase as

\[
\theta_L(r) - \theta_S(r) = \begin{cases} 
0 & (\gamma > 0), \\
\pm \pi & (\gamma < 0).
\end{cases}
\]

Therefore, we may state that (i) the two types of vortex have the same \( H_{c1} \), and that (ii) it is favorable for their cores to be overlapped energetically, i.e., there is an attractive interaction between the two types of vortex. This phase-locking effect could be modified by some boundary effects [16] or in the thin film system and this possibility will be discussed elsewhere [17].

The coherence length of \( \Psi_i(r) \) is [1]

\[
\xi_i(T) = \left( \frac{K_i}{\alpha_i(T)} \right),
\]

and for \( |r| >> \xi_L, \xi_S \), the amplitudes of order parameters become constant and one obtains a London equation

\[
\nabla \times H + \left( \frac{1}{\lambda(L)^2} + \frac{1}{\lambda(S)^2} \right) \left( A - \frac{\Phi_0}{2\pi} \nabla \varphi \right) = 0,
\]

where \( \Phi_0 = \hbar c/2e \),

\[
\varphi(r) = \theta_L(r) = \begin{cases} 
\theta_S(r) & (\gamma > 0), \\
\theta_S(r) \pm \pi & (\gamma < 0),
\end{cases}
\]

\[
\lambda^{(i)}(T) = \left( \frac{32\pi e^2 K_{Lz}}{\hbar^2 c^2 |\Psi_i^{(0)}(T)|^2} \right)^{-1/2},
\]

and \( \Psi_i^{(0)}(T) \) is the stationary value of \( \Psi_i(r) \) in the homogeneous system at a temperature \( T \). One can see from eqs. (6) and (8) that the London penetration depth in this system is

\[
\tilde{\lambda} = \left( \frac{1}{\lambda(L)^2} + \frac{1}{\lambda(S)^2} \right)^{-1/2}.
\]
Let us now discuss flux flow resistivity. In the two-gap system, the applied current $J$ is divided between the band with the large gap (L-band) and that with the small gap (S-band):

$$J = J_L + J_S.$$  
(10)

The divided current $J_i$ provides the Lorentz driving force to the vortices in the two bands. We assume that the two types of vortex have the same velocity because of the presence of the attractive interaction, as we mentioned before. The ratio of the two distributed currents is determined to minimize the energy dissipation of flux flow, more precisely, the power loss density of the flux flow

$$W = W_L + W_S,$$  
(11)

where

$$W_i = \pi R^2 \rho_f^{(i)} J^2$$  
(12)

is the power loss per unit cell of the vortex lattice (with a lattice constant $R$) per unit length along the $z$-axis[1] and $\rho_f^{(i)}$ is the flux flow resistivity in the $i$-band. The field dependence of $\rho_f^{(i)}$ will be discussed later.

To minimize eq. (11) under the constraint of eq. (10), one obtains

$$J_L = \frac{\rho_f^{(S)}}{\rho_f^{(L)} + \rho_f^{(S)}} J,$$

$$J_S = \frac{\rho_f^{(L)}}{\rho_f^{(L)} + \rho_f^{(S)}} J.$$  
(13)

Then,

$$W = \pi R^2 \frac{1}{1/\rho_f^{(L)} + 1/\rho_f^{(S)}} J^2,$$  
(14)

and this equation indicates that the total flux flow resistivity in this system is

$$\rho_f^{\text{two}} = \frac{1}{1/\rho_f^{(L)} + 1/\rho_f^{(S)}},$$  
(15)

which is the parallel connection of the resistivity in the L-band and that in the S-band.

Let us discuss the field dependence of $\rho_f^{(i)}$. Since the two gaps have the $s$-wave pairing symmetry, these two resistivities are considered to obey the Bardeen-Stephen relation, i.e.,

$$\rho_f^{(i)} = \frac{H}{H_{c2}^{(i)}} \rho_n^{(i)}$$  
(16)

for $H_{c1} < H < H_{c2}^{(i)}$, where $\rho_n^{(i)}$ is the normal-state resistivity in the $i$-band, and

$$H_{c2}^{(i)} = \frac{\Phi_0}{2\pi \xi_i^2}.$$  
(17)

For MgB$_2$, the results of the point-contact spectroscopy experiment suggest that the small gap is suppressed quicker than the large gap.[8] This implies that $\xi_L < \xi_S$ and $H_{c2}^{(S)} < H_{c2}^{(L)}$. The value $H_{c2}^{(L)}$ coincides with the upper critical field in this system. The kink point that has been observed in the field dependence of the specific heat[9] corresponds approximately to $H_{c2}^{(S)}$, since the presence of the kink indicates that the small-gap pairing is almost suppressed.[18] Then, the S-band would quite
The robustness of the coincidence in the

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two bands is considered to obey the Bardeen-Stephen relation, since the two gaps have the
same velocity because of the presence of the attractive interaction. The flux flow resistivity in the
other. We have examined the flux flow resistivity in this system. The two types of vortex have the
for the small gap. These two types of vortex have the same lower critical field and attract each
consists of the phase winding of the order parameter for the large gap and the other consists of that
amplitudes that arise on these two Fermi surfaces. There are two types of vortex, one of which
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resistivity in the band with the large gap and that in the band with the small gap.

We can see that \( \rho^{\text{two}} \) is linear with \( H \) in the low-field regime, curves convexly in the high field
regime and is continuously connected to the normal-state resistivity in the two-band system \((1/\rho_n^{(L)} + 1/\rho_n^{(S)})^{-1}\).

Let us compare eq. (19) to the experimental results of the flux flow resistivity in MgB\(_2\)[11]. The measurements were performed with the magnetic field parallel to the \( c \)-axis of the crystal and also with the field in the \( ab \)-plane[11]. The kink point that has been observed in the field dependence of the specific heat[9] suggests that \( H_{c2}^{(S)} / H_{c2}^{(L)} = 0.02 \) for the \( H \parallel ab \) case and \( H_{c2}^{(S)} / H_{c2}^{(L)} = 0.1 \) for the
\( H \parallel c \) case[21]. Unfortunately, there is no definite information for the ratio \( \rho_n^{(S)} / \rho_n^{(L)} \) at present, and we use it as a fitting parameter and choose \( \rho_n^{(S)} / \rho_n^{(L)} = 0.26 \) for the \( H \parallel ab \) case and \( \rho_n^{(S)} / \rho_n^{(L)} = 0.5 \) for the \( H \parallel c \) case. As is shown in Fig. 1, it should be emphasized that both cases of the experimental
results\[11\] are consistent with the convex-type behavior of eq. (19), particularly for the \( H \parallel c \) case. The robustness of the coincidence in the \( H \parallel c \) case is supported by the fact that the 20% change in the
parameter \( \rho_n^{(S)} / \rho_n^{(L)} \) around the value 0.5 causes at most a 10% change in \( \rho_j^{\text{two}} \). The difference between our calculated results and the experimental results in the high-field region of the \( H \parallel ab \) case is considered to be related to complications of the analysis of experimental data, since the vortices driven by the microwave field[11] move in the \( ac \)-plane and the anisotropy of the crystal causes the current-direction dependence of the flux flow resistivity. The experimental data for the \( H \parallel ab \) case correspond to the averaged value of this anisotropy.

In summary, we have considered a two-gap superconductor such as MgB\(_2\).[5] In this system, there are two bands that cross the Fermi level in the metallic phase and two \( s \)-wave gaps with different
amplitudes that arise on these two Fermi surfaces. There are two types of vortex, one of which consists of the phase winding of the order parameter for the large gap and the other consists of that for the small gap. These two types of vortex have the same lower critical field and attract each other. We have examined the flux flow resistivity in this system. The two types of vortex have the same velocity because of the presence of the attractive interaction. The flux flow resistivity in the two bands is considered to obey the Bardeen-Stephen relation, since the two gaps have the \( s \)-wave pairing symmetry. We have obtained that, to minimize the power loss (energy dissipation per unit time) caused by the flux flow, the total flux flow resistivity is expressed as a parallel connection of the resistivity in the band with the large gap and that in the band with the small gap. It should be noted that the amplitude of the small gap is suppressed faster than that of the large gap by the magnetic field in MgB\(_2\)[8], and the resistivity in the band with the small gap is approximately equal to its normal state value in the field region where the small gap is almost entirely suppressed. Taking this into account, our formula agrees quite well with the anomalous field dependence of the flux flow resistivity as observed experimentally\[11\].

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Figure 1: We compare the experimental data of flux flow resistivity in MgB$_2$ given in ref. 11 and our results calculated with eq. (19). Black dots and triangles denote experimental results for the $H \parallel ab$ case and $H \parallel c$ case, respectively. The green line denotes eq. (19) with parameters $H_{c2}^{(S)}/H_{c2}^{(L)} = 0.02$ and $\rho_n^{(S)}/\rho_n^{(L)} = 0.26$, and the red line denotes eq. (19) with $H_{c2}^{(S)}/H_{c2}^{(L)} = 0.1$ and $\rho_n^{(S)}/\rho_n^{(L)} = 0.5$. The red line agrees very well with the data for $H \parallel c$.

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