Decomposition of Harmonic and Jet Contributions to Particle-pair Correlations at Ultra-relativistic Energies

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Methodology is presented for analysis of two-particle azimuthal angle correlation functions obtained in collisions at ultra-relativistic energies. We show that harmonic and di-jet contributions to these correlation functions can be reliably decomposed by two techniques to give an accurate measurement of the jet-pair distribution. Results from detailed Monte Carlo simulations are used to demonstrate the efficacy of these techniques in the study of possible modifications to jet topologies in heavy ion reactions.

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A primary objective for studying ultra-relativistic collisions between complex nuclei is the creation and characterization of a new phase of hot and dense nuclear matter, where quarks and gluons are no longer confined to the interior of single hadrons. The existence of this Quark-Gluon Plasma (QGP), was conjectured based on the observation of asymptotic freedom [1, 2] and is now very strongly indicated by lattice QCD calculations [3, 4, 5, 6, 7]. The energy loss of hard scattered partons in this de-confined medium is predicted to be larger than that for cold hadronic matter [4, 5]. Such an energy loss can result in an apparent softening of the fragmentation function for jets, leading to suppression in the observed yield of high $p_T$ hadrons and to modifications of jet topologies. Thus, a careful study of jet properties and yields in heavy ion collisions provides a sensitive probe for possible QGP formation [6, 7] and possibly for QGP properties.

It is known that characteristic back-to-back jets of hadrons are copiously emitted in sufficiently energetic collisions. Studies of these jets provide information on related fragmentation functions in a “cold” nuclear medium [8, 9, 10, 11]. The more recent program of study at the Relativistic Heavy Ion Collider (RHIC) has involved p-p, d-Au, and Au-Au collisions. It is conjectured that the first produces jets in free space, the second produces jets that could be influenced by relatively modest initial state effects, and the third produces jets that could be strongly influenced by final-state effects such as those resulting from a reaction zone of very high energy-density nuclear matter. Hadron pair correlations provide a primary tool for the study of the yield and topology of these jets. Therefore, much excitement revolves around the observation, comparison, and characterization of such correlations in these three situations [12, 13, 14, 15, 16].

Pair correlation studies of d-Au and Au-Au collisions have to deal with a relatively large “underlying event”. For d-Au reactions, this underlying event is uncorrelated with the reaction plane (RP). By contrast, for Au-Au reactions there are rather strong correlations between the reaction plane and the azimuthal emission angle of hadrons [12, 13, 18, 14, 15, 16]. This “elliptic flow” leads to particle-plane-particle or “harmonic” pair correlations that must be taken into account as one tries to identify and characterize jet pair correlations. Flow correlations are presumed to result primarily from pressure gradients developed in the initial anisotropic (or almond shaped) overlap region of high energy-density nuclear matter [20, 21, 22, 23, 24]. The interactions within this overlap region may also alter jet properties, and one has to allow for this possibility in any analysis.

Recent experimental studies of jet properties have focused on the decomposition of $\Delta \phi$ distributions [14, 16, 25, 26]. Here, we present two methods which exploit the use of correlation functions to separate jet driven pair correlations from harmonic correlations. The use of correlation functions circumvents the need for full azimuthal correlation functions to allow for this possibility in any analysis.

Azimuthal angle correlation functions: To emphasize jet correlations, one often selects events with at least one high-$p_T$ particle. This “trigger particle” is then paired with other “associated” particles to obtain azimuthal angle pair correlations. Hereafter, we refer to the trigger and associated particles as A and B respectively. The AB correlations are termed “fixed correlations” if A and B are from the same $p_T$ bin or “assorted correlations” if A and B are from separate $p_T$ bins. AB pairs are used to construct a correlation function $C^{AB}(\Delta \phi)$ in azimuthal angle difference $\Delta \phi = (\phi_A - \phi_B)$; this function is constructed by dividing an area normalized (foreground) distribution of real particle pairs by an area normalized (background) distribution of mixed pairs obtained by pairing particles from different events.
The foreground distribution $F$ is defined in terms of the number of coincident (or real) pairs $N^{AB}$ per event:

$$F(\Delta \phi) = N^{AB}(\Delta \phi)/\Sigma N^{AB}(\Delta \phi),$$

where the sum is taken over the bins in $\Delta \phi$.

This gives the probability distribution for detecting an AB pair in a bin of $\Delta \phi$. The background distribution is defined in the same way, but by selecting A and B particles from separate events to form pairs:

$$B(\Delta \phi) = [N^{AB}(\Delta \phi)/\Sigma N^{AB}(\Delta \phi)]_{\text{mix}}.$$  

This gives the detection efficiency for an AB pair in the bin of $\Delta \phi$. The ratio of these two distributions defines the correlation function

$$C(\Delta \phi) = F(\Delta \phi)/B(\Delta \phi).$$

This correlation function gives the efficiency corrected probability distribution for real coincident pairs.

The filled circles in Fig. 1 show a typical correlation function generated from simulated data tuned to recent experimental data from RHIC [17, 27]. There is a relatively narrow peak at $0^\circ$, a broader peak at $180^\circ$, and a minimum at $\sim 80^\circ$. Our task is to devise an analysis method that allows one to retrieve the input jet-pair distribution used to generate the correlation function.

Two source model: For Au-Au reactions at RHIC it has been shown that a dominant aspect of hadron pair correlations is collective flow of second order, i.e. elliptic flow $v_2$. This gives a harmonic distribution for particle type A (or B) with respect to the angle of the reaction plane $\psi_R$.

$$N^A(\phi_A - \psi_R) \propto [1 + 2v_2^A \cos 2(\phi_A - \psi_R)]$$

and a corresponding harmonic correlation function between AB pairs [23, 24]

$$C^{AB}_H(\Delta \phi) = [1 + 2v_2 \cos 2(\Delta \phi)]; \quad v_2 = (v_2^A \times v_2^B).$$

In addition to these harmonic correlations one expects characteristic contributions from jets. To simulate both source contributions we include both harmonic and di-jet particle emission patterns into a Monte Carlo code. The simulations use exponential $p_T$ distributions and Poisson sampling for the number of jets per event, the number of particles per jet and the number of harmonically flowing particles per event. The average number of near and away-side jet particles were set to be equal, and the near and away-side jets were generated with an effective $j_T$ and $k_T$ respectively. Here, $j_T$ reflects the average transverse momentum projection of hadrons perpendicular to the near-side jet axis ($j_T = p_T^{\text{hadron}} \sin (\phi_{\text{jet}} - \phi_{\text{hadron}})$) and $k_T$ reflects the acoplanarity of the near- and away-side jets, ($k_T = (p_T^\text{jet}/\sqrt{2}) \sin (\Delta \phi_{\text{jet}} - \phi_{\text{jet}})$). Input parameters were adjusted to yield correlation functions in rough agreement with the measured ones. An absorption factor was introduced to study the effects of possible suppression of the away-side jet ($J_n$). The axis of the nearside jet ($J_n$) was also made to correlate with varying strength to the RP via a harmonic function (as in Eq. 4) or left uncorrelated.

It can be shown [30] that the pair correlations from such flow and jet sources are given by

$$C^{AB}(\Delta \phi) = a_o[C^{AB}_H(\Delta \phi) + J(\Delta \phi)],$$

where $C^{AB}_H(\Delta \phi)$ is a harmonic function of effective amplitude $v_2$, Eq. 5, and $J(\Delta \phi)$ is a di-jet function. No explicit or implicit assumption is made for the functional form of $J$. In Fig. 1, the dot-dashed and solid curves represent the input harmonic and jet functions respectively.

By rearrangement of Eq. 6 one obtains

$$J(\Delta \phi) = [C^{AB}(\Delta \phi) - a_oC^{AB}_H(\Delta \phi)]/a_o, \quad (7)$$

Knowledge of $a_o$ is required to evaluate $J$. Next we discuss an ansatz for assigning the value of $a_o$.

From measurements of the di-jet functions in d+Au reactions, one finds that they are very similar to the ones obtained in p+p reactions and can be reasonably described by a double Gaussian function with relatively narrow near- and away-side widths ($\sigma_2$ and $\sigma_2$, respectively) [15, 31]. For similar $p_T$ selection, Au+Au reactions are found to give near-side jet widths which are similar to those obtained in d+Au. By contrast, the away-side jet seems to broaden and to diminish in amplitude as the centrality is increased [36]. Guided by these observations it is reasonable to assume, as a starting point, that the di-jet function in Au+Au reactions has negligible intensity at the minimum $\Delta \phi_{\text{min}}$, in the jet function (cf. Fig. 1), i.e.

$$a_oC^{AB}_H(\Delta \phi_{\text{min}}) = C^{AB}(\Delta \phi_{\text{min}}), \quad (8)$$

which can be solved to obtain $a_o$. Thus the condition, zero yield at minimum (ZYAM), serves to fix the normalization constant $a_o$.

From the two source model described above, one obtains a single particle distribution, with respect to the reaction plane $\psi_R$ which can be fit with a harmonic. Its amplitude $v_2$, reflects an average over the particles from both sources. For example, if one simulates a flow source (90%) with $v_2 = 0.2$ and an uncorrelated jet source (10%), then the observed value of $v_2$ will be $0.9 \times 0.2 + 0.1 \times 0$ or $0.18$. A finite resolution in the determination of $\psi_R$ requires a correction factor in the evaluation of $v_2$ as described in Refs. [32, 33, 34, 35]. Knowledge of $v_2$ determines $C^{AB}_H$ in Eq. 6. The determination of the effective $v_2$ utilizing the reaction plane as discussed above, requires that the reaction plane is itself determined by a procedure free of non-flow effects. One such example is a measurement which demands a large (pseudo)rapidity gap ($\sim 3 - 4$ units) between the reaction plane and the particles correlated with it [27, 36]. It is expected that the $v_2$ values so obtained are much less affected by jet contributions [23, 34]. Earlier work has shown that $v_2$ varies with both centrality and $p_T$. 

$$\frac{\Delta \phi}{\Delta \phi_{\text{min}}} = \frac{1}{1 + \left[1 + 2v_2 \cos 2(\Delta \phi_{\text{min}}/4) \right]} \sin (\Delta \phi_{\text{jet}} - \phi_{\text{jet}}).$$

Input parameters were adjusted to yield correlation functions in rough agreement with the measured ones. An absorption factor was introduced to study the effects of possible suppression of the away-side jet ($J_n$). The axis of the nearside jet ($J_n$) was also made to correlate with varying strength to the RP via a harmonic function (as in Eq. 4) or left uncorrelated.

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Therefore, to avoid biases, one must measure \( v_A^2 \) and \( v_B^2 \) for exactly the same data set to be analyzed for its jet function. To accomplish this, one also uses the reaction planes and their resolution corrections for exactly the same events of interest and determines \( v_A^2 \) and \( v_B^2 \) from Eq. 4 for the particle of interest. From Eq. 5 one then obtains the effective amplitude \( v_e \).

With the ZYAM condition and the measured values of \( v_e \), one can decompose the observed correlation function into jet and harmonic components. This is illustrated in Fig. 1. The solid circles represent the simulated correlation data. The dot-dashed curve represents the effective harmonic component about \( a_0 \). The filled triangles show the difference with an \( a_0 \) offset, which can be compared to the input jet function (also with an \( a_0 \) offset) represented by the solid line. Note that the values of \( v_A^2 \) and \( v_B^2 \) are obtained from fits to the azimuthal distributions with respect to the reaction plane.

The method can be extended to obtain information on the correlation of the jet function with respect to the reaction plane \( \Delta \phi \). Two new correlation functions are constructed as follows: (1) the trigger particle \( A \) is constrained to have its \( \phi \) angle within a certain cut-angle \( \Delta \phi_e \), about the reaction plane, and the associated particle \( B \) is unconstrained. The mixed-event background \( B(\Delta \phi) \) is constructed as before in Eq. 6.

\[
C_{in}^{\Delta \phi}(\Delta \phi) = F_{in}^{\Delta \phi}(\Delta \phi) / B(\Delta \phi).
\]  

(9)

(2) The trigger particle is constrained to have its \( \phi \) angle within \( \Delta \phi_e \) of the normal to the reaction plane

\[
C_{out}^{\Delta \phi}(\Delta \phi) = F_{out}^{\Delta \phi}(\Delta \phi) / B(\Delta \phi).
\]  

(10)

Figure 2 shows the effect of these constraints on simulated data for pure flow with no di-jet contribution. Despite a sizable dispersion of the reaction plane \( \Delta \psi_R = 40^\circ \), Fig. 2 shows that the effects of these constraints are strong and are in opposing directions for \( C_{in}^{\Delta \phi}(\Delta \phi) \) versus \( C_{out}^{\Delta \phi}(\Delta \phi) \). The smooth curves in the figure show results derived from the analytical formulas developed in Ref. 37 for the relationship between \( (v_A^2)^{in}(\Delta \phi) \) and \( (v_A^2)^{out}(\Delta \phi) \) for given values of \( v_A^2 \), the reaction plane resolution \( \Delta \psi_R \), and the angular selection \( \Delta \phi_e \).

\[
(v_A^2)^{out} = \frac{2v_A^4 (\Delta \phi_e) - \sin (2 \Delta \phi_e) (\cos (2 \Delta \psi_R)) + \frac{v_A^4}{2} \sin (4 \Delta \phi_e) (\cos (4 \Delta \psi_R))}{2 (\Delta \phi_e) - 2v_A^2 \sin (2 \Delta \phi_e) (\cos (2 \Delta \psi_R))}
\]  

(11)

\[
(v_A^2)^{in} = \frac{2v_A^4 (\Delta \phi_e) + \sin (2 \Delta \phi_e) (\cos (2 \Delta \psi_R)) + \frac{v_A^4}{2} \sin (4 \Delta \phi_e) (\cos (4 \Delta \psi_R))}{2 (\Delta \phi_e) + 2v_A^2 \sin (2 \Delta \phi_e) (\cos (2 \Delta \psi_R))}
\]  

(12)

The harmonic amplitude \( v_2 \) in Eq. 5 is replaced by \( (v_A^2)^{in}(v_B^2) \) and by \( (v_A^2)^{out}(v_B^2) \). In the presence of jets, one can also use the ZYAM condition to decompose in-plane or out-of-plane correlations. This is illustrated in Fig. 3 where results are generated that include harmonic emissions of 92% of the particles and di-jet emissions of 8%. Panel (a) shows simulated data points compared to a fit by Eqn. 4 for the particle distribution with respect to the reaction plane. This fit gives the effective value of \( v_2 \) to be used in the analysis of the two-particle correlation functions. Panels (b), (c), and (d) show simulated data for an inclusive, in-plane and out-of-plane correlation function (filled circles) respectively. The harmonic correlations in each figure (dashed-dot curve) are calculated analytically from Eqn. 4 using \( v_2 \) from the fit in panel (a) along with \( (v_A^2)^{out} \) and \( (v_A^2)^{in} \) from Eqs. 11 and 12. The ZYAM condition is used to fix the constant \( a_0 \) (in each case) as described above. Then the extracted points for jet-pair distributions are determined by difference via Eq. 4. Comparison to the input jet-pair distribution (solid curves) shows an excellent agreement, thereby giving confidence in the decomposition procedure. It is noteworthy that a similar analysis performed for the case in which di-jets were made to correlate with the reaction plane, gave equally good results.

As indicated earlier, this method of decomposition makes no assumption about the functional form of the jet function. From the shape of the jet functions observed in p+p and d+Au reactions, one is led to expect a double Gaussian jet function peaking at 0 and 180 deg. However, if the reaction creates a strongly interacting medium, then that medium could distort the jets, especially on the away-side. Such distortions have been recently attributed to the influence of collective flow and the generation of shock waves around partons propagating through the medium. We have performed several simulations in which strongly distorted away-side jets were introduced. In each case the decomposition method retrieves the input jet function in detail, confirming that the decomposition procedure is robust even for unusual di-jet distributions.

**Extinction of Harmonic Correlations:** Figure 3 shows pair correlations with strong harmonic components \( C_H^{AB}(\Delta \phi) \) and relatively weak jet components \( J(\Delta \phi) \). Only after subtraction of \( C_H^{AB}(\Delta \phi) \) is the true shape revealed for \( J(\Delta \phi) \). It would be desirable to select a data
set that contained only those correlations due to the jets; Fig. 2 and Eqs. 4 and 11 point the way. As discussed above for Fig. 2, the out-of-plane constraint on particle A changes the phase of the harmonic correlation to favor pairs at $\Delta \phi = 90^\circ$. By using Eq. 11 one can select the particular cut angle $\Delta \phi_c = \phi_{xt}$ such that $(v_2^A)^{out} = 0$. For this case the value of $v_2$ is driven to zero in Eq. 4, and the harmonic correlations are extinguished.

In Fig. 4 we demonstrate the extinguishing of harmonic correlations. The filled circles show the inclusive $\Delta \phi$ distribution obtained for a simulation with an input $v_2 \sim 0.16$. The filled squares show the out-of-plane $\Delta \phi$ distribution after the cut angle $\phi_{xt}$ is set to the extinction value, i.e. $\phi_c = \phi_{xt}$. The latter distribution is flat and demonstrates that this technique gives a good method to determine the jet correlations directly from a data set, without the blurring effect from harmonic correlations mediated by the reaction plane. The latter technique provides clear advantages for future jet studies at RHIC.

Jet pair fraction and conditional yield: The sum of the jet-pair function $J$ over $\Delta \phi$ can be related to the average fraction of jet-correlated particle pairs per event, hereafter termed jet pair fraction JPF, as follows,

$$JPF = \sum a_0 J(\Delta \phi)/\sum C(\Delta \phi).$$

The hatched area in Fig. 11 illustrates this sum. This value of JPF represents the fraction of all mid-rapidity AB pairs that are correlated by jets i.e. those correlated by effects over and above those mediated by the reaction plane.

One may ask what is the event-averaged number of jet-associated particles per trigger particle. This quantity is commonly referred to in jet-analyses as per-trigger or conditional yield. The conditional yield can be calculated from the jet pair fraction as follows.

For a given data set, one records the detected number of AB pairs per event $n_{AB}^p$, the detected number of A particles per event $n_A^p$, and the detected number of B particles per event $n_B^p$. The ratio $R_p$ of the pair rates to the product of the single particle rates

$$R_p = n_{AB}^p/(n_A^p \times n_B^p)$$

(14)

carries information about pair-production over and above the combinatorial background. These detected rates should be corrected for efficiencies to get the true rates for particle-pairs and singles at all $\phi$ but with the relevant $\eta$ acceptance. Assuming that the pair-efficiency approximately factorizes into the single particle efficiencies, we can denote the true values per event by:

$$n_A = (n_A^p)(Ef^A)$$

(15)

$$n_B = (n_B^p)(Ef^B)$$

(16)

$$n_{AB} \approx (n_{AB}^p)(Ef^A)(Ef^B)$$

(17)

i.e. $n_{AB}^p/n_A^p n_B^p = n_{AB}/n^A n^B$.

Now one writes for the true efficiency corrected conditional yield TCY

$$TCY = JPF(n_{t}^{AB}/(n_t^A \times n_t^B)) \times n_t^B$$

(18)

$$= JPF \times R_p \times n_t^B$$

(19)

From the procedure outlined above one gets JPF, and one has $R_p$ from the detected values $n_{AB}^p$, $n_A^p$ and $n_B^p$ for each data set. From the literature (see for example Refs. 41, 42) one has previously published $p_T$ spectra for many particles corrected for azimuthal efficiency. These can be integrated over the $p_T$ intervals for particle B to get $n_B^p$.

The evaluation of systematic errors in this procedure will center on the determination of the jet pair fraction (shaded area in Fig. 11 for example). This is because the jet-pair distribution could have a finite (albeit small) intensity at $\Delta \phi_{min}$, in certain situations. If this is indeed the case, the extracted jet shape is essentially unaffected, but the yields extracted under the ZYAM assumption would constitute lower limit yields. On the other hand, one can adopt an appropriate functional form for the shape of the jet function to make an estimate of the possible jet yield at $\Delta \phi_{min}$, and hence, assign a systematic error via iteration.

In the discussion of jet yields we used Eq. (17)

$$n_{AB} = n_{t}^{AB}(Ef^A)(Ef^B).$$

In fact, we note that AB pairs that are correlated in $\Delta \phi$ may well have a somewhat modified efficiency, designated below by $M_t$.

$$n_{AB} = n_{t}^{AB}(Ef^A Ef^B(M_t))$$

(20)

After determination of the harmonic amplitude one may get the $M_t$ factor by Monte Carlo simulation techniques. Similarly one can get modification factors $M_{n}^{aj}$ and $M_{aj}^{n}$ for the near-side jets and the away-side jets respectively.

Correlations between the multiplicities $n_A^p$ and $n_B^p$ make the mean number of combinatoric pairs $n_{com}^{AB}$ greater than the product $n_A^p n_B^p$. One can write

$$n_{com}^{AB}/n_A^p n_B^p \xi = 1, \; \xi \neq 1.$$

(21)

However the quantity $\xi$ cancels out in the product (JPF)$n_{AB}^p$ in the present analysis.

Summary: A Monte Carlo reaction code has been used to generate simulated data sets for hadron pair correlations tuned to the general behavior from Au + Au reactions at RHIC. These data sets contain pair correlations from both elliptic flow and from jets. A method is presented for the analysis of such data to retrieve the jet pair fractions, conditional yields, and widths for both near and away side jets. A novel technique is also presented for direct quenching of harmonic correlations to reveal jet correlations.
FIG. 1: Correlation function from simulated data (filled circles). The dotted curve shows the input harmonic component. The solid curve and full squares show input and output jet pair distributions referenced to an offset.

FIG. 2: Simulated data, closed and open circles, for inplane $C^\text{in}$ and out of plane $C^\text{out}$ correlation functions as described in the text with $\Delta \phi_\ell = 45^\circ$ and $\Delta \psi_R = 40^\circ$. Only harmonic correlations are present as shown by the dotted curve with amplitude $v_A^2 = v_B^2$.

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FIG. 3: (a) Simulated data points and smooth curve for the correlation with respect to the reaction plane. (b) Simulated data (filled circles) for an inclusive correlation function including harmonic and jet correlations. The dotted curve indicates the harmonic term. The solid curve and full squares show input and output jet distributions referenced to $\alpha_0$. (c) Same as (b) but for an in-plane correlation function Eq.(9). (d) Same as (b) but for an out-of-plane correlation function Eq.(10).

FIG. 4: Simulated correlation function for unconstrained particles (filled circles) and for a trigger particle constrained within the cut angle $\phi_c = \phi_{c1}$ perpendicular to the reaction plane (filled squares), see text. The results are for a pure harmonic simulation with $v_2 \sim 0.16$. 

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