First Numerical Simulations of Anomalous Hydrodynamics

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Abstract

Anomalous hydrodynamics is a low-energy effective theory that captures effects of quantum anomalies. We develop a numerical code of anomalous hydrodynamics and apply it to dynamics of heavy-ion collisions, where anomalous transports are expected to occur. This is the first attempt to perform fully non-linear numerical simulations of anomalous hydrodynamics. We discuss implications of the simulations for possible experimental observations of anomalous transport effects. From analyses of the charge-dependent elliptic flow parameters ($v_2^\pm$) as a function of the net charge asymmetry $A_\pm$, we quantitatively verify that the linear dependence of $\Delta v_2 \equiv v_2^- - v_2^+$ on the net charge asymmetry $A_\pm$ cannot be regarded as a sensitive signal of anomalous transports, contrary to previous studies. We, however, find that the intercept $\Delta v_2 (A_\pm = 0)$ is sensitive to anomalous transport effects.

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**Introduction.**— Macroscopic transport phenomena induced by the triangle anomalies have recently attracted much attention. One such example is the chiral magnetic (separation) effect (CME/CSE), which states that a dissipationless vector (axial) current is generated along a magnetic field [1–4]. This kind of anomalous transport phenomena is expected to occur in a variety of materials such as the quark-gluon plasma (QGP), interiors of neutron stars [5], or Weyl semimetals [6, 7]. Theoretically, the anomalous transport effects have been derived in hydrodynamics [8], kinetic theory [9–11], or the linear response theory.

The quark-gluon plasma (QGP), which consists of deconfined quarks and gluons, is a place in which anomaly-induced transport would be taking place. The QGP is experimentally produced in heavy-ion collisions at Relativistic Heavy Ion Collider (RHIC) in BNL, and Large Hadron Collider (LHC) in CERN. Through the analysis of the collision experiments, it has been revealed that the QGP behaves like a perfect fluid [12–15]. Since two charged nuclei collide at very high energy, extremely strong magnetic fields are created in off-central collisions, along which vector and axial currents flow due to CME/CSE [1–4]. Recently, it has been theoretically discovered that interplay between CME and CSE results in a novel type of the gapless excitation, called the chiral magnetic wave (CMW) [17]. The CMW is a charge propagating wave along magnetic fields. It brings a charge quadrupole deformation in the QGP, which could be used as an experimental signal of anomalous transport effects [18, 19]. The results of linearized calculations in an uniform static temperature seem to be consistent with experimental data [20, 21]. However, linearization procedure is by no means justified since it lacks the dynamics of the QGP fluid which is crucially important in order to compare theoretical calculations with experimental data. Thus, in order to assess the contribution from the anomalous transport in the QGP, fully non-linear numerical simulations of anomalous hydrodynamics are indispensable, which have not been performed yet.

In this Letter, we perform the first numerical simulation of anomalous hydrodynamics in full three-dimensional space. This allows us to estimate the effect of anomalous transport quantitatively. We apply anomalous hydrodynamics to the space-time evolution of the QGP created in heavy-ion collisions and demonstrate how the chiral magnetic wave propagates in an expanding plasma. According to the simulations, the slope parameter \( r_e \), which is recently proposed as a signal of the CMW [18, 19], is not a sufficient signal of anomalous transport. Rather the charge-dependent elliptic flow \( v_2^± \) at zero charge asymmetry turns out
to be sensitive to the anomalous transport effects.

Hydrodynamics with the triangle anomaly. — Here we briefly discuss formulation of anomalous hydrodynamics \[8\]. We consider a system with one $U(1)_V$ vector current $j^\mu = \langle \bar{\Psi} \gamma^\mu \Psi \rangle$ and one $U(1)_A$ axial vector current $j^5_\mu = \langle \bar{\Psi} \gamma_5 \gamma^\mu \Psi \rangle$. The axial current is not conserved because of the axial anomaly. Hydrodynamic equations, therefore, represent (non-)conservation laws of energy, momentum, vector and axial currents:

\[
\partial_\mu T^{\mu\nu} = e F^{\nu\lambda} j_{\lambda}, \\
\partial_\mu j^\mu = 0, \\
\partial_\mu j^5_\mu = -CE^\mu B_\mu,
\]

where $T^{\mu\nu}$ is the energy-momentum tensor of the fluid, $e$ the elementary charge, $F^{\mu\nu}$ the electromagnetic field strength tensor, $E^\mu \equiv F^{\mu\nu} u_\nu$, $B^\mu \equiv \tilde{F}^{\mu\nu} u_\nu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}$ the electric and magnetic fields, and $C$ the anomaly coefficient. We here treat electromagnetic fields as external fields. The right hand sides of Eqs. (1) and (3) represent contributions of the Lorentz force and the triangle anomaly, respectively.

For a perfect fluid, we express the energy-momentum tensor using the four-velocity $u^\mu = \gamma(1, \vec{v})$ as

\[
T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu}.
\]

Here $\varepsilon$ is the energy density, $P$ the pressure, and $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ the metric tensor. Although vector and axial currents are non-dissipative in a perfect fluid, they should have space-like components $\nu^\mu$ and $\nu^5_\mu$ in the presence of the triangle anomaly:

\[
j^\mu = nu^\mu + \nu^\mu, \\
j^5_\mu = n_5 u^\mu + \nu^5_\mu,
\]

where $n = n_R + n_L$ denotes the charge density, $n_5 = n_R - n_L$ the axial charge density, and $n_{R,L}$ the charge density of particles with right/left-handed chirality.

Corrections from the triangle anomaly, $\nu^\mu$ and $\nu^5_\mu$, include novel transport phenomena known as the chiral magnetic effect, the chiral separation effect, and the chiral vortical effect. The constitutive equations in the presence of anomaly can be written in terms of the
magnetic field $B^\mu$ and the four-vorticity $\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\beta\gamma} u_\nu \partial_\beta u_\gamma$ as

$$\nu^\mu = \kappa_B B^\mu + \kappa_\omega \omega^\mu, \quad (7)$$

$$\nu_5^\mu = \xi_B B^\mu + \xi_\omega \omega^\mu, \quad (8)$$

where the transport coefficients are given by

$$e\kappa_B = C\mu_5 \left( 1 - \frac{\mu_5 n_5}{\varepsilon + p} \right), \quad e^2 \kappa_\omega = 2C\mu_5 \left( 1 - \frac{\mu n}{\varepsilon + p} \right), \quad (9)$$

$$e\xi_B = C\mu \left( 1 - \frac{\mu n}{\varepsilon + p} \right), \quad e^2 \xi_\omega = C\mu^2 \left( 1 - \frac{2\mu_5 n_5}{\varepsilon + p} \right). \quad (10)$$

Here, $\mu = \mu_R + \mu_L$ and $\mu_5 = \mu_R - \mu_L$ are the chemical potentials for the vector and axial charges, respectively.

In this Letter, we adopt the following constitutive equations,

$$j^\mu = nu^\mu + \kappa_B B^\mu, \quad (11)$$

$$j_5^\mu = n_5 u^\mu + \xi_B B^\mu, \quad (12)$$

where we omitted the vortical current due to technical difficulties. Thus, our calculations are applicable when vorticity (times chemical potential) is not large compared to the strength of the magnetic fields, which is the case in heavy-ion collisions considered in this work. In order to solve hydrodynamic equations, we also need to employ an appropriate equation of state, the pressure as a function of the energy and charge densities $P = P(\varepsilon, n, n_5)$. We discretize hydrodynamic equations above in the Cartesian coordinates $x^\mu = (t, x, y, z)$ and perform numerical simulations. This implementation allows us to study the dynamics of a fluid and anomalous transport phenomena simultaneously.

**Application to heavy-ion collisions.**— It has been argued that experimental signal of macroscopic anomalous transport effects can be obtained in heavy-ion collisions [18–21]. Strong magnetic fields are expected to be produced especially in off-central heavy-ion collisions, along which the current flows as a result of axial anomaly. However, the evidence of such transport effects has been elusive, mainly due to the lack of quantitative theoretical predictions. Numerical simulations of anomalous hydrodynamics allow us to gain an insight into the anomalous transport in the QGP.
For that purpose, we apply the anomalous hydrodynamic model introduced above to the space-time evolution of the QGP created in off-central heavy-ion collisions. Based on the results of the simulations, we discuss the implications in the experiments.

We here consider a plasma composed of massless gluons and single-flavor quarks with unit electric charges. Although it is simplified compared to the actual quark-gluon plasma, essential features are still captured. As an equation of state, we employ that of the ideal gas EOS with the one-component massless fermion and gluons, namely \( P(\varepsilon, n, n_5) = \varepsilon/3 \).

Let us specify the initial values and time evolutions of electromagnetic fields appropriate for the heavy-ion collisions. A typical configuration of the electromagnetic field in off-central heavy-ion collisions is illustrated in Fig. 1. The magnetic field has a special direction perpendicular to the reaction plane (RP) and the electric field tends to direct out-of-plane. We have defined the origin of the spatial coordinate as the middle of the centers of the colliding nuclei and \( x \) and \( y \) axes as in Fig. 1. We parametrize the magnitudes, spatial distributions, and duration times of the electromagnetic fields as

\[
e B_y(\vec{r}, t) = eB_0 \exp \left( -\frac{r^2_{EB}}{2\sigma_{EB}^2} - \frac{t}{\tau_B} \right),
\]

\[
e E_y(\vec{r}, t) = eE_0 \times y \exp \left( -\frac{r^2_{EB}}{2\sigma_{EB}^2} - \frac{t}{\tau_E} \right),
\]

where \( \sigma_{EB} \) and \( \tau_B \) are spatial distribution widths and time durations of the magnetic field, and \( \sigma_{EB} \) and \( \tau_E \) are spatial distribution widths and time durations of the electric field.
where initial values of the fields are taken to be $eB_0 = 0.08 \text{ GeV}^2 \simeq (2m_\pi)^2$ and $eE_0 = 0.02 \text{ GeV}^2 \simeq m_\pi^2$, which are estimated in Refs. 22, 23. The initial time $t = 0$ is defined as the time at which two colliding nuclei are completely overlapped. The parameter $r_z = \sqrt{x^2 + y^2 + (2z)^2}$ denotes the effect of Lorentz contraction of the fields, and $\sigma_{EB} = 4.0 \text{ fm}$. The duration times [24, 25] of electric and magnetic fields are controlled by the parameters $\tau_B$ and $\tau_E$. Other components of the electromagnetic fields are set to zero. The quantity $\vec{E} \cdot \vec{B}$ is positive(negative) above(below) the reaction plane (see also Fig. 1), which leads to the generation of axial charges through the source in Eq. (3).

As an initial condition for the fluid, we take $n_5(\vec{r}, t = 0) = 0$ and temperature and vector charge chemical potential distributions are chosen as

$$T(\vec{r}, t = 0) = T_0 \exp\left(-\frac{r_y^2}{2\sigma_T^2}\right),$$

$$\mu(\vec{r}, t = 0) = \mu_0 \exp\left(-\frac{r_y^2}{2\sigma_\mu^2}\right),$$

where $\sigma_T = 5.0 \text{ fm}$, $\sigma_\mu = 5.0 \text{ fm}$, and $T_0 = 0.3 \text{ GeV}$, which is the typical initial temperature achieved at RHIC and LHC. We take $\mu_0$ from 0.0 to 0.01 GeV, which represent the initial fluctuation of the charge distribution. The anisotropic parametrization $r_y = \sqrt{x^2 + (y/2)^2 + z^2}$ represents the initial almond-like shape of the QGP created in non-central collisions (see Fig. 1).

We show the numerical results of three-dimensional anomalous hydrodynamic simulations. We start with the initial condition in which there is no vector or axial charge at initial time ($\mu_0 = 0$). Figure 2 shows the vector and axial charge density distributions at $t = 6 \text{ fm}$ in a plane at $z = 0$, which is the plane parallel to the external magnetic field. The vector charges show quadrupole distributions (left), while the axial charges show dipole-like distributions (right). This can be understood as follows. First, the axial charge is generated through an anomaly term $\vec{E} \cdot \vec{B}$ in Eq. (3). This results in the dipole-like distributions of axial charge since the sign of $\vec{E} \cdot \vec{B}$ changes between above and below the reaction plane (Fig. 1). Then, the vector current flows in the presence of axial charge densities, which makes the quadrupole distribution of vector charges [26]. This is the CMW. At the same time, the large pressure gradients make the QGP expand radially. Thus, a propagating CMW in an expanding plasma is successfully demonstrated in the numerical simulations. Appearance of the charge quadrupole deformation due to the CMW is proposed as a signal
FIG. 2: (Color online) Vector (left) and axial (right) charge density distributions at \( t = 6 \) fm in the \( z = 0 \) plane, which is the plane parallel to the external magnetic field. Because of the anomaly source and CMW evolution, vector (axial) charge has quadrupole (dipole) distribution even if there is no initial vector charge.

Implications on experimental observables.— Here we discuss implications of the anomalous hydrodynamics for the results in heavy-ion collisions experiments. Since the initial shape of the QGP produced in off-central collisions is like an almond (see Fig. 1), the pressure gradients lead to the anisotropic momentum distribution of emitted particles. The elliptic flow \( v_2 \) is an experimental signal of this anisotropic momentum distribution [27]. If the CMW brings the charge quadrupole deformation in the QGP, charge-dependent particle distributions in the final state should also be affected. As a result, the elliptic flow of negatively-charged particles are larger than that of positively-charged particles: \( v_2^- > v_2^+ \). Charge differences of elliptic flow \( \Delta v_2^\pm = v_2^- - v_2^+ \) therefore have information about charge distributions.

In order to get the momentum distribution of particles, we employ the Cooper-Frye formula [28] using hydrodynamic valuables such as temperature \( T(\vec{r}) \), and chemical potential \( \mu(\vec{r}) \) at \( t = t_f \)

\[
E \frac{dN}{dp^3} = \frac{d}{(2\pi)^3} \int \frac{EdV}{\exp[(p^\mu u_\mu - \mu)/T] \mp 1}.
\] (17)

Here \( d \) is the degree of freedom and \( \mp \) corresponds to Bose/Fermi distribution for the particles considered. We assume that the freeze-out occurs at \( t = t_f \). We take \( t_f = 6 \) fm,
which is typical duration time of the QGP. From Eq. (17), we obtain charge-dependent azimuthal particle distributions,

\[
\frac{dN_\pm}{d\phi}(\phi) = \bar{N}_\pm \left( 1 + 2 \sum_n v_n^\pm \cos n\phi \right),
\]

where the azimuthal angle \( \phi \) is measured from the \( x \)-axis and \( \bar{N}_\pm \) is defined as the angle average of the number distribution,

\[
\bar{N}_\pm \equiv \int \frac{d\phi}{2\pi} \frac{dN_\pm}{d\phi}.
\]

Now we obtain the charge-dependent elliptic flow parameter \( v_2^\pm = \langle \cos(2\phi) \rangle_\pm \), where the bracket \( \langle \ldots \rangle_\pm \) denotes averages over the azimuthal particle distributions for positively (negatively) charged particles.

We calculate \( \Delta v_2^\pm \) as a function of \( A_\pm \) through different initial charge distributions by changing \( \mu_0 \) as a parameter, which controls the initial charge fluctuation. The function \( \Delta v_2^\pm(A_\pm) \) is a key quantity to exhibit the signal of anomalous transports. Recently, the characteristic linear dependence of \( \Delta v_2^\pm \) on \( A_\pm \), \( \Delta v_2^\pm = 2r_e A_\pm + O(A_\pm^2) \), is argued to be one of the possible signals of the anomalous transport [18, 19]. The experimental data also show the same tendency [20, 21]. However, according to the simulation of anomalous hydrodynamics, we conclude that the linear tendency cannot be regarded as a direct consequence of anomalous transports since it is present even when we switch off the anomaly constant [33].

Figure 3 shows the \( \Delta v_2^\pm \) as a function of the net charge asymmetry \( A_\pm \). Different lines correspond to different conditions. Solid line is the result in the absence of the anomaly: The anomaly constant \( C \) is set to zero. Dashed and dotted lines correspond to the cases with anomaly, where the parameters \( (\tau_B, \tau_E) \) are set to \( (3.0 \text{ fm}, 1.0 \text{ fm}) \) and \( (6.0 \text{ fm}, 1.0 \text{ fm}) \), respectively. We find a linear relationship between \( \Delta v_2^\pm \) and \( A_\pm \) for all lines. This is the case even in the absence of the anomaly. The slopes \( r_e \) of the all lines are approximately the same, \( r_e \sim 0.01 \), which is of the same order of magnitude with the experimental data [20, 21]. This definitely tells us that the linear relation itself cannot be regarded as an indicator of the anomalous transport. The origin of the linear dependence in this simulation can be understood as follows. In the case of finite initial chemical potential, the central region tends to have a positive vector chemical potential, while the values of the potential are smaller in the outer region. Thus, the central region tends to emit more positively-charged particles than negatively-charged ones due to a positive vector chemical potential.
The central region does not make a large contribution to elliptic flow compared to the outer region. Therefore, \( v_2^+ \) becomes smaller than \( v_2^- \) at finite \( \mu \), which makes the quantity \( \Delta v_2^\pm \) an increasing function of \( A_\pm \). The linear behavior is the leading contribution at small \( A_\pm \).

However, the intercepts of these lines, \( \Delta v_2(A_\pm = 0) \), turn out to be sensitive to anomalous effects. While \( \Delta v_2^\pm(0) = 0 \) in the absence of anomaly \( (C = 0) \), it is finite in the presence of anomaly as shown in Fig. 3. The magnitude of intercepts depends on the duration time of magnetic fields: The larger duration time results in larger \( \Delta v_2(0) \). This can also be utilized to estimate the dynamics of electromagnetic fields.

Let us comment on the uncertainties in the calculations. Firstly, we have neglected the conductive current \([23, 26, 32]\) in this study. It also makes a contribution to charge quadrupole deformation in the QGP (see Fig. 1). Secondly, we have also neglected the back reaction of matter to electromagnetic fields. The back reactions would affect the duration time of the magnetic field, which is treated as parameters in this study. In order to quantify this effect, we have to calculate the time evolutions of dynamical electromagnetic fields simultaneously. Thirdly, the effects of the QCD sphalerons can be also important since it reduces the axial charge. Consideration of these effects is left as future work.

**Summary.** — In this Letter, we have performed the first numerical simulation of anomalous hydrodynamics. We have developed a numerical code to study anomalous transport effects quantitatively and applied the code to heavy-ion collision experiments. We have demonstrated the chiral magnetic wave in the expanding quark-gluon plasma and found that the charge quadrupole deformation is produced even if there is no initial vector charge. We have also shown that the slope parameter \( r_e \) is not a proper signal for the CMW since the slope is also present in the absence of anomaly. The results of simulations indicate that the intercepts, \( \Delta v_2(A_\pm = 0) \), are sensitive to the anomalous effects.

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FIG. 3: (Color online) $\Delta v_2 \equiv v_2^- - v_2^+$ is shown as a function of the net charge asymmetry $A_\pm$. Solid line is the result in the absence of the anomaly $C = 0$. Dashed and dotted lines correspond to the cases with anomaly, where the parameters are set to $(\tau_B, \tau_E) = (3.0 \text{ fm}, 1.0 \text{ fm})$ and $(\tau_B, \tau_E) = (6.0 \text{ fm}, 1.0 \text{ fm})$, respectively. Each dot corresponds to the different initial chemical potential $\mu_0$ from 0.0 GeV to 0.01 GeV. All lines show the linear relation between $\Delta v_2^\pm$ and $A_\pm$ with close to the same slopes, while the values of $\Delta v_2^\pm (A_\pm = 0)$ are different.

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The effects of finite baryon potentials on $\Delta v_2(p\bar{p}) \equiv v_2(\bar{p}) - v_2(p)$ is discussed in Refs. [29, 30].