A $C_2$-EQUIVARIANT ANALOG OF MAHOWALD’S THOM SPECTRUM THEOREM

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Abstract. We prove that the $C_2$-equivariant Eilenberg-MacLane spectrum associated with the constant Mackey functor $\mathbb{F}_2$ is equivalent to a Thom spectrum over $\Omega^\rho S^{\rho+1}$.

1. Introduction

Let $\mu$ be the Möbius bundle over $S^1$, regarded as a virtual bundle of dimension 0. The mod 2 Moore spectrum is the Thom spectrum

$$M(2) \simeq (S^1)^\mu.$$ 

The classifying map for $\mu$ extends to a double loop map

$$\tilde{\mu} : \Omega^2 S^3 \to BO.$$ 

Mahowald proved the following theorem [Mah77]:

Theorem 1.1 (Mahowald). There is an equivalence of spectra

$$(\Omega^2 S^3)^{\tilde{\mu}} \simeq H\mathbb{F}_2.$$ 

The bundle $\mu$ may also be regarded $C_2$-equivariant virtual bundle over $S^1$, by endowing both $S^1$ and the bundle with the trivial action. Since $BC_2 O$ is an equivariant infinite loop space [Ati68], the classifying map for $\mu$ extends to an $\Omega^\rho$-loop map

$$\tilde{\mu} : \Omega^\rho S^{\rho+1} \to BC_2 O.$$ 

Here, $\rho$ is the regular representation of $C_2$. The purpose of this paper is to prove the following.

Theorem 1.2. There is an equivalence of $C_2$-spectra

$$(\Omega^\rho S^{\rho+1})^{\tilde{\mu}} \simeq H\mathbb{F}_2.$$ 

(Here, $\mathbb{F}_2$ denotes the constant Mackey functor with value $\mathbb{F}_2$.)

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**Conventions.** Equivariant objects in this paper either live in $\text{Top}^{C_2}$, the category of $C_2$-spaces, or $\text{Sp}^{C_2}$, the category of genuine $C_2$-spectra. In both of these categories, the equivalences are those equivariant maps which induce equivalences on both $C_2$-fixed points and underlying fixed points. We let $H$ denote the Eilenberg-Maclane spectrum $H_{F_2}$, with underlying spectrum $H := HF_2$. We use $H_*$ and $\pi_{\sigma}^{C_2}$ to denote $RO(C_2)$-graded homology and homotopy groups (i.e. not the Mackey functors) of $C_2$-equivariant spaces and spectra, and $H_\ast$ and $\pi_\ast$ to denote the ordinary homology and homotopy groups of non-equivariant spaces and spectra. We let $\sigma$ denote the sign representation of $C_2$, and let $\rho = 1 + \sigma$ denote the regular representation. For a representation $V$, $S(V)$ denotes the unit sphere in $V$, and $S^V$ denotes its one point compactification, and $|V|$ denotes its dimension.

### 2. Equivariant preliminaries

**Euler class.** Let $a$ denote the Euler class in $\pi_{\sigma}^{C_2}S$, given geometrically by the inclusion

$$S^0 \hookrightarrow S^\sigma.$$ 

There is a cofiber sequence

$$C_2 \to S^0 \hookrightarrow S^\sigma$$

so the cofiber of $a$ is stably given by

$$Ca \simeq \Sigma^{1-\sigma} C_2.$$ 

The transfer induces a map

$$u : S^{1-\sigma} \xrightarrow{tr} \Sigma^{1-\sigma} C_2 \simeq Ca$$

which serves as a Thom class for the representation $\sigma$:

$$u : S^1 \to Ca \wedge S^\sigma.$$ 

For $X \in \text{Sp}^{C_2}$, we have

$$\pi_k^{C_2}(X) \cong \pi_k(X^{C_2}),$$

$$\pi_V^{C_2}(X \wedge Ca) \cong \pi_{|V|}(X^v).$$

Said differently,

$$\pi_\ast^{C_2} X \wedge Ca \cong \pi_\ast X^e[u^\pm].$$

**Tate square.** We will let

$$X^h := F(EC_2, X),$$

$$X^\Phi := X \wedge \widetilde{EC}_2$$

denote the homotopy completion and geometric localization of $X$, respectively. The fixed points of $X^h$ are the homotopy fixed points of $X$, and the fixed points of $X^\Phi$
are the geometric fixed points of $X$. $X$ is recovered from these approximations by the pullback (“Tate square”) \cite{GM95} 

$$
\begin{array}{ccc}
X & \longrightarrow & X^\Phi \\
\downarrow & \quad & \downarrow \\
X^h & \longrightarrow & X^t
\end{array}
$$

where the spectrum $X^t$ is the equivariant Tate spectrum 

$X^t := (X^h)^\Phi$.

Note that a generalization of the argument establishing (2.2) yields an equivalence 

$\Sigma^{k\sigma-1} C(a^k) \simeq S(k\sigma)_+$.

Taking a colimit, we see that we have 

$$
\text{hocolim}_k \Sigma^{k\sigma-1} C(a^k) \simeq EC_2^+,
\text{hocolim}_k S^{k\sigma} \simeq EC_2.
$$

It follows that homotopy completion and geometric localization can be reinterpreted as $a$-completion and $a$-localization:

$X^h \simeq X_\wedge^a,
X^\Phi \simeq X[a^{-1}]$.

In this manner, the Tate square is equivalent to the “$a$-arithmetic square”

$$
\begin{array}{ccc}
X & \longrightarrow & X[a^{-1}] \\
\downarrow & \quad & \downarrow \\
X_\wedge^a & \longrightarrow & X_\wedge^a [a^{-1}]
\end{array}
$$

Using (2.3), the $a$-Bockstein spectral sequence takes the form 

$\pi_* (X^\sigma)[u^\pm, a] \Rightarrow \pi_*^{C_2}(X^h)$.

The $a$-Bockstein spectral sequence can be regarded as an $RO(C_2)$-graded version of the homotopy fixed point spectral sequence (see \cite[Lem. 4.8]{HM17}).

**The mod 2 Eilenberg-MacLane spectrum.** We have \cite{HK01} 

$$
\pi_*^{C_2} H = \mathbb{F}_2[a, u] \oplus \mathbb{F}_2[a, u^\pm] \{\theta\}
$$

where 

$|u| = 1 - \sigma,$
$|\theta| = 2\sigma - 2.$

The $a$-u divisible factor in $\pi_* H$ is best understood from the Tate square, using 

$\pi_*^{C_2} H^h \simeq \mathbb{F}_2[a, u^\pm],
\pi_*^{C_2} H^\Phi \simeq \mathbb{F}_2[a^\pm, u]$. 


Actually, the second isomorphism lifts to an equivalence
\[ H^{\Phi C_2} \simeq H[a^{-1}] := \bigvee_{i \geq 0} \Sigma^i H \]
so we have
\[ H^{\Phi} X \cong H_a(X^{\Phi C_2})[a^\pm, u] \]
and, restricting the grading to trivial representations, we get
(2.4) \[ H^{\Phi} X \cong H_a(X^{\Phi C_2})[a^{-1}u]. \]
By applying \( \pi_V^{C_2} \) to the map
\[ H \wedge X \to H \wedge X \wedge Ca \]
we get a homomorphism
(2.5) \[ \Phi^e : H^V(X) \to H|V|^{|X^e|}. \]
Taking geometric fixed points of a map
\[ S^V \to H \wedge X \]
gives a map
\[ S^V \to H^{\Phi C_2} \wedge X^{\Phi C_2} \]
Using (2.4) and passing to the quotient by the ideal generated by \( a^{-1}u \), we get a homomorphism
(2.6) \[ \Phi^{C_2} : H^V(X) \to H|V|^{C_2}(X^{\Phi C_2}). \]

A useful lemma. Our main computational lemma is the following.

**Lemma 2.7.** Suppose that \( X \in \text{Sp}^C \) and suppose that \( \{b_i\} \) is a set of elements of \( H_*(X) \) such that

1. \( \{\Phi^e(b_i)\} \) is a basis of \( H_a(X^e) \), and
2. \( \{\Phi^{C_2}(b_i)\} \) is a basis of \( H_a(X^{\Phi C_2}) \).

Then \( H_*(X) \) is free over \( H_* \), and \( \{b_i\} \) is a basis.

**Proof.** The set \( \{b_i\} \) corresponds to a map
\[ H \wedge \bigvee S^{[b_i]} \to H \wedge X. \]
Assumption (1) implies this map is an equivalence upon applying \( \Phi^e \), while assumption (2) implies this map is an equivalence upon applying \( \Phi^{C_2} \). The result follows.

\[ \square \]

3. HOMOLOGY OF \( \rho \)-LOOP SPACES

We spell out some specific algebraic structure carried by the equivariant homology of a \( \rho \)-loop space. A more detailed and general study of this algebraic structure will appear in [Hil].
Products. Suppose $X = \Omega^p Y \in \text{Top}^{C_2}$ is a $\rho$-loop space. Then $X$ is in particular a 1-loop space, and is therefore an equivariant $H$-space with product
\[ m : X \times X \to X. \]
However, the $\sigma$-loop space structure also endows $X$ with a twisted product related to the transfer. Namely, let
\[ S^\sigma \to S^\sigma/S^0 \cong C_2^+ \wedge S^1 \]
be the pinch map. This gives rise to a twisted product
\[ \tilde{m} : N^\times \Omega Y \to \Omega^\rho Y \]
where
\[ N^\times Z := \text{Map}(C_2, Z) = Z \times Z \]
is the norm (with respect to Cartesian product). In particular, there is a map
\[ (3.1) \quad \tilde{m} : N^\times \Omega^2 Y \to X. \]
Upon applying fixed points to the map (3.1), we get an additive transfer
\[ (3.2) \quad t : X^e \to X^{C_2}. \]
In homology, the $H$-space structure give rise to a product
\[ m : H_Y X \otimes H_W X \to H_{Y+W} X. \]
Using the equivariant commutative ring spectrum structure of $H$ [Ull13], the twisted product $\tilde{m}$ gives rise to a “norm map” (see [BH15, Thm. 7.2])
\[ n : H_k X^e \to H_k \rho X. \]

Dyer-Lashof operations. $X$ has even more structure: $X$ is an $E_\rho$-algebra [GM17]. Specifically, regard $S(\rho)$ as a $C_2 \times \Sigma_2$-space where $C_2$ acts on $\rho$ and $\Sigma_2$ acts antipodally. Then the $E_\rho$-structure gives a map
\[ S(\rho) \times \Sigma_2 X^{\times 2} \to X. \]
Note that $H$ is itself an $E_\rho$-ring spectrum, because it is actually an equivariant commutative ring spectrum, so $H \wedge X_+$ is an $E_\rho$-ring in $H$-modules. Given $x \in H_Y(X)$, represented by a map
\[ x : S^V \to H \wedge X_+, \]
there is an induced composite
\[ H \wedge S(\rho)_+ \wedge \Sigma_2 S^{2V} \xrightarrow{1 \wedge 1 \wedge x \wedge x} H \wedge S(\rho)_+ \wedge \Sigma_2 (H \wedge X_+)^{\times 2} \]
\[ \to H \wedge H \wedge X_+ \]
\[ \to H \wedge X_+ \]
(where the unlabeled maps come from the $E_\rho$-ring and $H$-module structure of $H \wedge X_+$). Applying $\pi_{C_2}^*$, we get a total power operation
\[ P(x) : \tilde{H}_*(S(\rho)_+ \wedge \Sigma_2 S^{2V}) \to \tilde{H}_* X. \]
For the purposes of this paper we will be only concerned with the case of $V = k\rho - \sigma$.

We will need the following lemma.
Lemma 3.3. We have the following identification of the $C_2$-fixed point space of the extended power:

$$(S(\rho)_+ \wedge_{\Sigma_2} S^{2(kp-\sigma)})_{C_2} \cong S^{2k-1} \lor S^{2k}.$$ 

Proof. The extended power can be identified with the Thom complex of the equivariant vector bundle

$$S(\rho) \times_{\Sigma_2} \mathbb{R}^{2(kp-\sigma)} \to S(\rho)/\Sigma_2.$$ 

The fixed points is the Thom complex of the fixed point bundle. Thinking of $S(\rho)$ as the unit circle in $\mathbb{C}$, with $C_2$ acting by conjugation, the fixed points of the base are given by

$$[S(\rho)/\Sigma_2]_{C_2} = \{[1], [i]\}.$$ 

The bundle has fiber $\mathbb{R}^{2(kp-\sigma)}$ over $[1]$, and because $\Sigma_2$ acts with the antipodal action mixed with the interchange action, the fiber over $[i]$ is given by

$$\mathbb{R}^{p(kp-\sigma)} = \mathbb{R}^{(2k-1)p}.$$ 

The result follows. \hfill \Box

Proposition 3.4. We have

$$\tilde{H}_* S(\rho)_+ \wedge_{\Sigma_2} S^{2(kp-\sigma)} \cong \tilde{H}_* \{e_{2kp-\sigma-1}, e_{2kp-\sigma}\}.$$ 

Proof. Theorem 2.15 of [Wil17] implies there is a cofiber sequence

$$S^{2kp-2\sigma} \to S(\rho)_+ \wedge_{\Sigma_2} S^{2(kp-\sigma)} \to S^{2kp-1}.$$ 

There are two possibilities for the long exact sequence in $\tilde{H}_*$: either (a) the connecting homomorphism sends $i_{2kp-1}$ to zero, or (b) the connecting homomorphism sends it to $\theta_{2kp-2\sigma}$. Only possibility (b) is compatible with Lemma 3.3 from geometric fixed point considerations. The result follows. \hfill \Box

Thus we get a pair of Dyer-Lashof operations

$$Q^{kp} : \tilde{H}_{kp-\sigma} X \to \tilde{H}_{2kp-\sigma} X,$$

$$Q^{kp-1} : \tilde{H}_{kp-\sigma} X \to \tilde{H}_{2kp-\sigma-1} X$$

given by the formulas

$$Q^{kp}(x) := \mathcal{P}(x)(e_{2kp-\sigma}),$$

$$Q^{kp-1}(x) := \mathcal{P}(x)(e_{2kp-\sigma-1}).$$

Remark 3.5. If $X$ is actually an equivariant infinite loop space, then $\tilde{H}_* X$ has an action by equivariant Dyer-Lashof operations [Wil17], and these operations agree with those defined in that paper.
Compatibility with fixed points. The compatibility of all this structure with the maps $\Phi^e$ and $\Phi^{C_2}$ of (2.5) and (2.6) is summarized as follows.

**Products:** Note that $X^e$ is an $E_2$-algebra, and $X^{C_2}$ is an $E_1$-algebra. The maps $\Phi^e$ and $\Phi^{C_2}$ are algebra homomorphisms.

**Norms:** The following diagram commutes:

$$
\begin{array}{ccc}
H_k X^e & \xrightarrow{t} & H_k X^e \\
\downarrow n & & \downarrow \text{Sq} \\
H_k X^{C_2} & \xleftarrow{\Phi^{C_2}} & H_k X^e \\
& \xleftarrow{\Phi^e} & H_{2k} X^e
\end{array}
$$

Here $t$ is the transfer (3.2) and Sq is the squaring map.

**Dyer-Lashof operations:** The following diagrams commute, where $\epsilon = 0, 1$:

$$
\begin{array}{ccc}
H_{kp-\sigma} X & \xrightarrow{\Phi^e} & H_{2k-1} X^e \\
\downarrow Q^{kp-\epsilon} & & \downarrow Q^{2k-\epsilon} \\
H_{2kp-\sigma-\epsilon} X & \xrightarrow{\Phi^e} & H_{4k-1-\epsilon} X^e
\end{array}
$$

$$
\begin{array}{ccc}
H_{kp-\sigma} X & \xrightarrow{\Phi^{C_2}} & H_k X^{C_2} \\
\downarrow Q^{k+\epsilon} & & \downarrow \text{Sq} \\
H_{2kp-\sigma} X & \xleftarrow{\Phi^{C_2}} & H_{2k} X^{C_2}
\end{array}
$$

4. **Homology of $\Omega^p S^{p+1}$**

**Theorem 4.1.** There is an additive isomorphism (of $H_\ast$-modules)

$$H_\ast \Omega^p S^{p+1} \cong H_\ast \otimes E[t_0, t_1, \ldots] \otimes P[e_1, e_2, \ldots]$$

with

$$|t_i| = 2^i \rho - \sigma,$$

$$|e_i| = (2^i - 1) \rho.$$

**Proof.** Note that we have

$$H_\ast \Omega^2 S^3 = \mathbb{F}_2[x_1, x_2, \ldots]$$

with

$$|x_i| = 2^i - 1.$$

Here $x_1$ is the fundamental class $t_1$, and

$$x_i := Q^{2^i} Q^{2i-1} \cdots Q^2 x_1.$$
Define $t_0 \in H_\rho \Omega^\rho S^{\rho+1}$ to be the fundamental class, and define the other "generators" $e_i$ and $t_i$ by

$$e_i := n(x_i),$$
$$t_i := Q^2\rho Q^{2i-1}\rho \cdots Q^\rho t_0.$$ 

Consider the product

$$t^e_k := t_0^e t_1^e \cdots e_1^{k_1} e_2^{k_2} \cdots \in H_\rho(\Omega^\rho S^{\rho+1})$$

with $e_i \in \{0,1\}$ and $k_i \geq 0$. We compute

$$\Phi^e(t^e_k) = x_1^{2k_1 + e_1} x_2^{2k_2 + e_1} \cdots .$$

Mapping out of the cofiber sequence (2.1) gives a fiber sequence

$$\Omega N^\times \Omega S^{\rho+1} \to \Omega^\rho S^{\rho+1} \to \Omega S^{\rho+1} \xrightarrow{\Delta} N^\times \Omega S^{\rho+1}.$$ 

Upon taking fixed points we get a fiber sequence

$$\Omega^2 S^3 \xrightarrow{t} (\Omega^\rho S^{\rho+1})^{C_2} \to \Omega S^2 \xrightarrow{\text{null}} \Omega S^3$$

In particular there is an equivalence

$$(\Omega^\rho S^{\rho+1})^{C_2} \simeq \Omega S^2 \times \Omega^2 S^3.$$

and we have

$$H_\rho(\Omega^\rho S^{\rho+1})^{C_2} \cong P[y] \oplus P[t(x_1), t(x_2), \ldots]$$

where $y$ is the image of the fundamental class under the map

$$S^1 \to (\Omega^\rho S^{\rho+1})^{C_2}.$$ 

It follows that

$$\Phi^{C_2}(t^e_k) = y^{e_0 + 2e_1 + 4e_2 + \cdots} t(x_1)^{k_1} t(x_2)^{k_2} \cdots .$$

Thus the set

$$\{t^e_k\} \subset H_\rho X$$

satisfies the hypotheses of Lemma 2.7, and the result follows. \qed

5. The equivariant Mahowald theorem

In order to prove Theorem 1.2 we will need to establish a Thom isomorphism

$$H_\rho(\Omega^\rho S^{\rho+1}) \cong H_\rho \Omega^\rho S^{\rho+1}.$$ 

We will do so in two steps. Recall that an $E_\rho$-algebra is just a spectrum $X$ equipped with a map $S^0 \to X$. Let $\text{Free}^\rho_{E_\rho}: \text{Alg}_{E_\rho}(Sp^{C_2}) \to \text{Alg}_{E_\rho}(Sp^{C_2})$ denote a homotopical left adjoint to the forgetful functor. An explicit model for this functor is the homotopy pushout of $E_\rho$-algebras:

\[
\begin{array}{ccc}
\text{Free}^\rho_{E_\rho}(S^0) & \longrightarrow & \text{Free}^\rho_{E_\rho}(X) \\
\downarrow & & \downarrow \\
S^0 & \longrightarrow & \text{Free}^\rho_{E_\rho}(X)
\end{array}
\]
We will need the following theorem.

**Theorem 5.1.** Let \( f : X \to BC_2O \) classify a virtual bundle of dimension zero and denote by \( \tilde{f} : \Omega^p \Sigma^p X \to BC_2O \) the associated \( \Omega^p \)-map. Then there is a canonical equivalence of \( E_\rho \)-algebras in \( Sp \):

\[
\text{Free}_{E_\rho}(X) \cong (\Omega^p \Sigma^p X) \tilde{f}.
\]

*Proof.* Combine the equivariant approximation theorem \([GM17, RS00]\) with Theorem IX.7.1 and Remark X.6.4 of [LMSM86]. \(\square\)

**Remark 5.2.** The non-equivariant version of Theorem 5.1 was first observed by Mark Mahowald, and then proven by Lewis. A nice modern account in the non-equivariant setting via universal properties can be found in [AB14].

**Proposition 5.3.** There is a Thom isomorphism

\[
H_* (\Omega^p S^{p+1}) \cong H_* \Omega^p S^{p+1}.
\]

*Proof.* Let \( \text{Free}_{E_\rho, H}^* : \text{Alg}_{E_\rho} (\text{Mod}_H) \to \text{Alg}_{E_\rho} (\text{Mod}_H) \) denote a homotopical left adjoint to the forgetful functor. Along with the previous theorem, we will need two facts:

1. \( H \wedge (-) : \text{Sp} \to \text{Mod}_H \) is symmetric monoidal.
2. There is a Thom isomorphism \( H \wedge (S^1)^\mu \cong H \wedge S^1_+ \).

The proposition is now proved by the following string of equivalences:

\[
\begin{align*}
H \wedge (\Omega^p \Sigma^p S^1) & \cong H \wedge \text{Free}_{E_\rho}^* ((S^1)^\mu) \quad \text{by Theorem 5.1} \\
& \cong \text{Free}_{E_\rho, H}^* (H \wedge (S^1)^\mu) \quad \text{by (1)} \\
& \cong \text{Free}_{E_\rho, H}^* (H \wedge S^1_+) \quad \text{by (2)} \\
& \cong H \wedge \text{Free}_{E_\rho}^* (S^1_+) \quad \text{by (1)} \\
& \cong H \wedge \Omega^p \Sigma^p S^1_+.
\end{align*}
\]

\(\square\)

**Proof of Theorem 1.2.** The Thom class is represented by a map

\[
(\Omega^p S^{p+1})^\mu \to H.
\]

We wish to show this map is an isomorphism on \( H_* \). The homology of \( H \) is the \( C_2 \)-equivariant Steenrod algebra, computed in [HK01] to be

\[
H_* H = H_* [\tau_0, \tau_1, \ldots, \xi_1, \xi_2, \ldots]/(\tau_i^2 = (u + a\tau_0)\xi_{i+1} + a\tau_{i+1})
\]

with

\[
|\tau_i| = 2^i \rho - \sigma,
\]

\[
|\xi_i| = (2^i - 1) \rho.
\]

It suffices to show it is surjective, since the two homologies are abstractly isomorphic and of finite type. Observe that the composite

\[
M(2) \cong (S^1)^\mu \to (\Omega^p S^{p+1})^\mu \to H
\]
hits $\tau_0$. Everything is hit then, by [Wil17, Thm. 5.4]. □

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