Discrete Symmetries, Strong CP Problem and Gravity *

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Abstract

Spontaneous breaking of parity or time reversal invariance offers a solution to the strong CP problem, the stability of which under quantum gravitational effects provides an upper limit on the scale of symmetry breaking. Even more important, these Planck scale effects may provide a simple and natural way out of the resulting domain wall problem.

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I. Introduction

One of the main characteristics of black holes is the fact that in their vicinity one loses any notion of global charge [1]. The trouble is that if you come close enough to learn about it, you yourself will be trapped and not able to send any information out. This of course is not true of local charges, since through the information carried by gauge fields one can know about them even far from the black hole. In other words, using Gauss theorem

$$\oint \vec{E} \cdot d\vec{S} = q$$

one can obtain the information about the charge $q$ of the black hole by measuring the electric field away from it. It is therefore highly suggestive that the non-gauged global symmetries, both continuous and discrete are violated in the presence of black holes [2]. If so, this should have a profound impact on anything which utilizes the idea of a global symmetry. Normally, even if the renormalizable part of the Lagrangian is invariant under such a symmetry, there is no reason to expect the same of higher degree effective operators. In other words, we would expect $d \geq 5$ operators to be cut off by $1/M_{Pl}$ to break the global symmetry [3]. So what, you will say, for any physical effect due to such terms would be of order $(E/M_{Pl})^n$ $(n \geq 1)$, where $E$ is the relevant energy in the physical process. For $E \approx M_W$ which is of our interest, this effect is less than $10^{-17}$! Can this have any impact on present day physics?

The answer is yes, as I discuss here. I will focus on the issue of discrete symmetries and the strong CP problem. The most important role that gravity may play is in providing a way out of a domain wall problem. Normally, a spontaneous breaking of a discrete symmetry causes the formation of domain walls, whose existence is a serious problem for the big-bang scenario [3] (unless the scale of symmetry breaking is below 10-100 MeV). However, an explicit breaking of such a symmetry induced by quantum gravitational effects (virtual black holes, wormholes), would provide a source of instability of domain walls allowing them to disappear before ever dominating the energy content of the universe. This is particularly important in the case of time reversal $T$ and parity invariance $P$, symmetries which cannot be gauged (at least not in four dimensions) [3]. It is aesthetically appealing to have these symmetries broken spontaneously, as suggested in the case of $T(CP)$ more than twenty years ago by Lee [3]. It would be only appropriate that gravity, a theory of space-time dynamics, may provide a way out of breaking of space-time symmetries such as $T$ and $P$. Now, there is more to this than just aesthetics. Spontaneously broken theories are in general softer, less divergent than the explicitly broken ones. A spontaneous breaking of either $P$ or $T$ symmetry may thus render the measure of CP breaking by QCD (through instanton effects) finite and calculable [7]. In other words, in such theories $\theta_{QCD} \leq 10^{-10}$ may be achieved naturally, without any fine-tuning. We will show here that the gravity effects (or better Planck scale effects) which explicitly break CP keep this picture valid as long as the scale of $T$ (or $P$) breaking $\Lambda_T(\Lambda_P)$ is limited from above [9]. Thus, these effects may provide a badly needed upper limit on $\Lambda_T(\Lambda_P)$, and offer hope of experimental verification of these ideas.

On the other hand, the most elegant and popular solution of the strong CP problem is based on the global Peccei-Quinn axial $U(1)_{PQ}$ symmetry [10], whose
breaking leads to the existence of an almost massless, pseudo Goldstone boson, the axion [11]. It is known that the scale of symmetry breaking $\Lambda_{PQ}$ must be extremely large, $\Lambda_{PQ} \geq 10^9$ GeV in order for axions not to be overproduced in stars [12]. The trouble then is that the Planck scale effects, if they do break global symmetries, would spoil this picture completely, leading in general to a heavy axion and a large $\theta_{QCD}$ [2].

Thus quantum gravity may turn out to be a curse for a Peccei-Quinn scenario and a blessing for the idea of $P$ or $T$ providing a solution to the strong CP problem.

II. Discrete Symmetries and a Domain Wall Problem: Gravity as a Way Out

In order to illustrate our ideas, let us imagine a simple case of a discrete symmetry, a real scalar field with no cubic self-interaction

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{\lambda}{2} (\phi^2 - v^2)^2$$

(2)

The discrete symmetry $\phi \to -\phi$ is broken spontaneously through $\langle \phi \rangle = \pm v$. It is easy to show that the theory allows for a static classical solution in say $x - y$ plane (the so-called kink)

$$\phi_{cl}(z) = v \tanh(vz \sqrt{\lambda})$$

(3)

which interpolates between the two physically distinct values $+v$ and $-v$, since $\phi_{cl}(z) \to \pm v$ as $z \to \pm \infty$. This is the infamous domain wall, which carries energy density per unit area

$$\rho_{cl}(z) = v^3 \left[ \coth(vz \sqrt{\lambda}) \right]^4$$

(4)

expected on physical grounds, since it costs energy to move from one vacuum to another. It differs from its vacuum values in a region of width $\delta \approx (v \sqrt{\lambda})^{-1}$.

In a classic paper Zeldovich et al. have pointed out a serious cosmological problem imposed by the existence of these classical solutions. In discussing this problem we follow here closely the discussion of Ref. [4], including also gravity as a potential cure of this problem.

Now, at high temperatures the potential $V(\phi)$ receives an additional contribution

$$\delta V = \frac{\lambda}{4} T^2 \phi^2$$

(5)

Since $\delta V$ is necessarily positive, for sufficiently high temperature $T > T_c \approx v$ the symmetry is restored [12]. In the standard big-bang cosmological scenario, the field $\phi$ is expected to undergo a phase transition as the universe cools down from $T > T_c$ to $T < T_c$. For separations larger than the correlation length or horizon size around the time of phase transition, the field $\phi$ will independently take either of its vacuum values giving rise to corresponding domains and domain walls. To understand the generic features of this system of domain walls one may consider the following idealised problem. Imagine splitting space into cubes of the size of
correlation length. And, say, the probability for the field to take a particular vacuum value in a given cube is $p$ ($= \frac{1}{N}$, where $N$ is the number of degenerate vacua). The nature of domain structure obtained, a domain being a set of connected cubes carrying same vacuum value of the field, is a basic question in percolation theory [13]. The main result we will need here is that if $p$ is greater than a certain critical value $p_c$, then apart from finite size domains there will be one and only one domain of “infinite” size formed. For $p$ less than $p_c$ there will not be any infinite size domain. Generically, i.e. considering even other lattices than just cubic, $p_c$ happens to be less than 0.5. In the example of real scalar field we have been considering, $p$ equals 0.5. Thus there would be an “infinite” domain corresponding to each vacuum and therefore infinite wall of a very complicated topology. Of course, there will also be a network of finite size walls. The question one is interested in is the energy density contribution of this domain wall system as it evolves. Following crude analysis addresses this question [14].

The dynamics of the wall is mainly decided by the force per unit area $f_T$ due to tension and frictional force $f_F$ with surrounding medium. Since tension in the wall is proportional to the energy per unit area $\sigma$, we get $f_T \sim \sigma R$ for radius of curvature scale $R$. Moreover, $f_F \sim s T^4$ where $s$ is the speed of the wall and $T$ the temperature of the system [15]. When the speed of the wall has stabilised we have

$$s T^4 = \frac{\sigma}{R}$$

Thus, the typical time $t_R \sim R/s$ taken by a wall portion of radius scale $R$ to straighten out would be

$$t_R \sim \frac{R^2 T^4}{\sigma} \approx \frac{R^2}{G \sigma t^2}$$

Making the plausible assumption that if $t_R < t$, the wall curvature on the scale $R$ would be smoothened out by time $t$, we get that the scale on which the wall is smooth grows as

$$R(t) \approx (G \sigma)^{\frac{1}{2}} t^{\frac{3}{2}}$$

Energy density contribution $\rho_W$ to the universe by walls goes as

$$\rho_W \sim \frac{\sigma R^2}{R^3} \sim \left(\frac{\sigma}{G t^3}\right)^{\frac{1}{2}}$$

Therefore $\rho_W$ becomes comparable to the energy density $\rho \sim \frac{1}{G t^2}$ of the universe in the radiation dominated era around $t_0 \sim \frac{1}{G \sigma}$. Thus domain walls would significantly alter the evolution of the universe after $t_0$.

Now, the discrete symmetries relevant for particle physics typically tend to be broken at mass scales above the weak scale $M_W \approx 100 GeV$, giving $t_0 \leq 10^8$ sec.. This would be certainly true of P and T (CP), the examples we are most interested in. Hence from above considerations one would conclude that discrete symmetries cannot be broken spontaneously.

There are two possible ways out of this impasse. One possibility is that, even for low scales of symmetry breaking, the phase transition that would have restored the
symmetry does not take place, at least not until high enough temperatures to allow inflation to dilute the energy density in the domain walls. This in general requires a more complicated Higgs structure than the minimal one and realistic examples have been discussed in the literature [16].

Another way out [14], is the possibility that a spontaneously broken discrete symmetry is also explicitly broken by a small amount, which lifts the degeneracy of the two vacua \( +v \) and \( -v \). For instance, in our example we could imagine adding to the Lagrangian a small \( \phi^3 \) term which, obviously, breaks \( \phi \to -\phi \) symmetry. It should not come as a surprise that this effect may provide a mechanism for the decay of domain walls; after all now there is a unique vacuum. Crudely, the way it works is as follows [14]. Lifting of the degeneracy of the two vacua by an amount \( \epsilon \) gives a pressure difference of the same amount, between the two sides of the wall, with a tendency to push the wall into false vacuum region. Thus the dynamics of the wall is now going to be decided by combination of the pressure \( \epsilon \), forces \( f_T \) due to tension and \( f_F \) due to friction mentioned before. Clearly at some point the forces due to friction and tension become small, compared to pressure difference \( \epsilon \), because they are proportional to \( T^4 \sim \frac{1}{Gt^4} \) and \( \frac{\sigma}{R} \sim \left( \frac{\sigma}{Gt} \right)^{\frac{3}{2}} \) respectively. At that stage the pressure difference will dominate and cause shrinking of the false vacuum. Actually it is difficult to find out precisely when the false vacuum region, and hence the domain walls, disappear. However, it may be crudely estimated to be the time when the pressure \( \epsilon \) exceeds the force due to tension, or when it exceeds the force due to friction for a relativistically moving wall so as to dominate the dynamics. For either requirement to be satisfied before \( t_0 \sim \frac{1}{Gt} \), the time when wall contribution \( \rho_W \) would have become comparable to the energy density of the universe, one obtains

\[
\epsilon \geq G\sigma^2 \sim \frac{v^6}{M_{Pl}^2}
\]

(10)

Of course, it is not very appealing to introduce ad hoc the symmetry breaking terms just in order to eliminate the domain-wall problem. Ideally, we would prefer these effects to be a natural consequence of underlying theory. An interesting example recently discussed in the literature [17] is that of a discrete symmetry explicitly broken due to instanton induced effects.

**Role of Gravity**

In this paper we invoke the possibility that the needed mechanism for explicit breaking may be naturally provided by gravity. One expects that gravity, because of black-hole physics, may not respect global symmetries, both continuous and discrete ones. This expectation is motivated by two important points: firstly, the “no-hair” theorems of black-hole physics that state that stationary black-holes are completely characterised by quantum numbers associated with long-range gauge fields, and secondly, that the Hawking radiation in evaporation of black-hole is thermal [1]. Now, consider a process in which a certain amount of normal matter, which is in a state that is “odd” under the discrete symmetry in consideration, collapses under gravity to form a black-hole. Because of no hair being associated to the global discrete symmetry, any information regarding it is lost to observers outside the
black hole. Hawking radiation from the black hole being thermal in nature does not carry any information about internal states of the black-hole either. Of course, it is not certain what the properties of evaporation would be at late stages when semi-classical approximation breaks down. Unless for some reason the processes at late stages cause the final system to have same global discrete charges as those of the initial normal matter that collapsed, the symmetry would stand violated.

We wish to note that from very different viewpoints there have been discussions in the literature regarding possibility of CP or T violation in context of gravity. Ashtekar et. al. have discussed [18] CP "problem" in the framework of canonical quantisation of gravity. In Ashtekar variables reformulation of general relativity, the canonical variables of the theory resemble those of Yang-Mills theory. This allows for discussion of $\theta$ sectors in canonical quantisation framework for Yang-Mills to be taken over to the gravity case. Moreover, an analogue of $\theta FF^d$ term in the action can also be given.

Another set of observations that interest us particularly were made by Penrose about T-asymmetry [19]. He contends, based on arguments related to Bekenstein-Hawking formula, that there must be some as yet unknown theory of quantum gravity that is time-asymmetric. We recall here only an easy to state, interesting point from his discussion. Corresponding to a solution of Einstein’s equation describing collapse of normal matter to form a black-hole that stays for ever (classically) , there would be a time-reversed solution, white-hole, describing explosion of a singularity into normal matter. Now, according to Bekenstein-Hawking formula the surface area $A$, of a black hole’s horizon is proportional to its intrinsic entropy, $S$

$$S = \frac{k c^3}{4\hbar G} A$$

(11)

In classical processes area is non-decreasing with time and hence so is the entropy. If an intrinsic entropy is associated with a white-hole, it is again expected to be proportional to the area of its horizon. Time reverse of area principle would give that this area, and hence the corresponding entropy can never increase, an anti-thermodynamic behaviour. Especially it would be a strongly anti-thermodynamic behaviour by the white hole when it ejects substantial amount of matter. This is among the reasons that lead Penrose to consider the possibility that there may be a general principle that rules out the existence of white holes and would therefore be time-asymmetric.

With the premise, in view of preceding discussion, that gravity may violate a global discrete symmetry we wish to explore its consequences for the domain-wall problem.

The crucial issue one faces in implementing this kind of approach is determination of the precise form of these symmetry-breaking terms. At the present day understanding of gravity it does not seem possible to give a satisfactory answer to this question. The strategy followed in the literature [2], which we also adopt here, in analogous discussions has been to write all the higher dimensional effective operators allowed by gauge invariance of an underlying theory. Of course, one could take a point of view that the dimension four and lower terms may also break the discrete symmetry. We take no stand on this point. In any case, even if this hap-
pens it can only help in destabilising the domain walls due to increased symmetry breaking. Our point is that even the tiny higher dimensional symmetry breaking terms, cut-off by powers of Planck-mass, may be sufficient in solving the domain wall problem.

To illustrate how this works, we turn again to our simple example of a real scalar field. The effective higher dimensional operators would take the form

\[ V_{\text{eff}} = \frac{C_5}{M_{Pl}} \phi^5 + \frac{C_6}{M_{Pl}^2} \phi^6 + \ldots \]  

(12)

Obviously, all the terms with odd powers of \( \phi \) break the discrete symmetry \( \phi \to -\phi \). [We should mention that while discussing difficulties with a certain compactification scheme in superstring theory, it was remarked by Ellis et. al. [20] that a specific discrete symmetry in their model may be broken by terms inversely proportional to \( M_{Pl} \). But they note further that massless modes of string theory would not induce such terms and that the massive modes, could be the only possible source of such effects. However, as we have been pursuing here, the non-perturbative effects are expected to be a natural source of breaking of global discrete symmetries, independent of whether string theory turns out to be a correct theory of gravity. Furthermore, as we have emphasised before, we feel that these effects should be taken seriously as a possible solution to the domain-wall problem associated with the fundamental discrete symmetries of nature such as Parity or Time-reversal invariance.]

In estimating precisely the amount of symmetry-breaking we would need to know the values of coefficients \( C_n \). Barring some unexpected conspiracy, in the following we will assume that \( C_n \) will be \( O(1) \) as they are dimensionless. Moreover, it is understood that the scale of spontaneous symmetry-breaking \( v \) lies below the Planck scale. With all this in mind, the energy-density split between the two vacua would be \( \approx \frac{C_5}{M_{Pl}} v^5 \). This is obviously much bigger than the amount \( v^6/M_{Pl}^2 \) needed to make the domain walls disappear. This holds true as long as \( C_5 \gg v/M_{Pl} \), which for lower scales of symmetry breaking gets to be more and more plausible. For example, if \( v = M_{\text{GUT}} \approx 10^{15} \text{ Gev} \), we need \( C_5 > 10^{-4} \), whereas for \( v = M_w \approx 100 \text{ Gev} \), we only need \( C_5 > 10^{-17} \) ! In general, if a leading operator in eq.(10) is of dimension \( n \), the condition for disappearance of domain walls is \( C_n > (M_{Pl}/v)^{n-6} \). Clearly \( n = 6 \) is the critical value, since for \( n > 6 \) \( C_n \) would have to be unreasonably large whereas for \( n = 6 \) \( C_6 \sim O(1) \) may suffice.

III. T and P and a Strong CP Problem

As we mentioned in the Introduction, the most popular solution to the strong CP problem is based on the idea of \( U(1)_{PQ} \) chiral global symmetry. Recall that the strong CP problem is the fact that the \( d = 4 \) renormalizable part of QCD Lagrangian in general contains a term

\[ \Delta \mathcal{L}_{QCD} = \theta_{QCD} g^2/32\pi^2 \epsilon_{\mu\alpha\beta} \tilde{F}^{\mu\nu} F^{\alpha\beta} \]  

(13)

Although this is a total derivative, due to instanton effects it does contribute
to the action. Now in QCD alone we could always choose $\theta_{QCD} = 0$, but in the presence of CP violating weak interactions the effective $\theta$ parameter becomes

$$\bar{\theta} = \theta_{QCD} + \theta_{QFD}$$

where $\theta_{QFD} = \arg \det M_q$ and $M_q$ is the quark mass matrix. Since $\theta_{QFD}$ is perturbatively renormalized, setting $\bar{\theta} = 0$ is plagued by infinities. In the Peccei-Quinn scenario, $\bar{\theta}$ is promoted to a dynamical variable, an axion field $a$, i.e. $\bar{\theta} = \langle a \rangle$. Due to $U(1)_{PQ}$ breaking by instantons, $a$ is not a purely massless Goldstone boson, but gets a mass through a potential $\Delta V \simeq \Lambda_{QCD}^4 (1 - \cos a / \Lambda_{PQ})$ In other words, $\bar{\theta} = \langle a / \Lambda_{PQ} \rangle = 0$, but

$$m_a \simeq \frac{\Lambda_{QCD}^2}{\Lambda_{PQ}}$$

This is what makes Peccei-Quinn solution to the strong CP problem so appealing, the fact that $\bar{\theta} = 0$ is a dynamical phenomenon.

What about Planck scale effect? After all $U(1)_{PQ}$ is a global symmetry and so we expect the quantum gravitational effects to break it, i.e. there should be Planck scale corrections to $\Delta V$

$$\Delta V_{Pl} \simeq \frac{\Lambda_{Pl}^5}{M_{Pl}} \cos(a / \Lambda_{PQ} + \delta)$$

where $\delta$ is some arbitrary phase.

Since $\Lambda_{PQ} \geq 10^9$ GeV, the above form dominates the instanton contribution by at least 26 orders of magnitude, implying $\bar{\theta} = -\delta$. In other words, Peccei-Quinn scenario is rendered completely inoperative (notice that for $\Lambda_{PQ} \leq M_W$ it would still work).

On the other hand, there is another possible strategy that could provide $\bar{\theta}$ calculable and small in the perturbation theory, i.e. $T$ or $P$ symmetry. Simply $T$ or (and) $P$ symmetry allows us to set $\bar{\theta} = 0$. To see how it works, let us take an example of parity $P$, which can be used to set $\theta_{QCD} = 0$. Furthermore, $P$ becomes L-R symmetry in the fermionic sector, i.e. $q_L \overset{P}{\longrightarrow} q_R$ and so the mass matrix $M_q$ defined through $\bar{q}_L M_q q_R$ becomes hermitian: $M_q = M_q^\dagger$. This in turn implies $\theta_{QFD} = \arg \det M_q = 0$, or in other words $\bar{\theta} = 0$.

The question is what happens next when parity is broken. Clearly, $\bar{\theta}$ cannot remain zero, but is necessarily finite and calculable in perturbation theory. It’s value becomes model dependant and by now various models have been offered based either on $P$ or $T$ symmetry (notice that $T$ can be as effective as $P$) which keep $\bar{\theta}$ small, less than it’s limit of $10^{-10}$ \cite{8}. To see what happens once gravity effects are switched on we must turn to a specific model. For simplicity, we discuss first a model of Barr et al. (called BCS hereafter) \cite{22} which is based on a generalized notion of parity, one that transforms $SU(2)_L \times U(1)$ into its mirror world $SU(2)^\wedge_L \times U(1)$. Imagine thus a gauge group

$$G = SU(2)_L \times U(1) \times SU(2)^\wedge_L \times U(1)$$

with the fermionic assignment $q, \ell$ and $\hat{q}, \hat{\ell}$ where $q$ and $\ell$ are the ordinary quarks and leptons, are $\hat{q}$ and $\hat{\ell}$ their parity counterparts. The Higgs sector is the minimal
one, i.e. the usual $SU(2)_L \times U(1)$ doublet $\phi$ and its parity analog $\hat{\phi}$. Due to L-R symmetry $\bar{\theta} = \theta_q + \hat{\theta}_q$ is forced to vanish. When $P$ is broken through $\langle \hat{\phi} \rangle \gg \langle \phi \rangle$, $\bar{\theta}$ is turned on, but due to the low energy theory being precisely the standard model, its value must be as small as the finite value $\bar{\theta}$ in the standard model (keep in mind that both $\langle \phi \rangle$ and $\langle \hat{\phi} \rangle$ are real)

$$\bar{\theta} \leq 10^{-19}$$

In order to avoid problems with fractionally charged bound states of ordinary and mirror quarks, one imagines $U(1)_L \times U(1)_\hat{\phi}$ symmetry being broken to a single $U(1)_L$ at some high enough energy (at the same this provides a necessary source needed to split the $\langle \phi \rangle$ and $\langle \hat{\phi} \rangle$).

Notice the end result of this scenario. Since $M_{\hat{q}} = \frac{\langle \hat{\phi} \rangle}{\langle \phi \rangle} M_{q}$, all the mirror fermion masses are scaled by the same factor

$$R \equiv \frac{\langle \hat{\phi} \rangle}{\langle \phi \rangle}$$

compared to the ordinary ones. In other words,

$$m_{\hat{u}} = R m_u, \ m_{\hat{e}} = R m_e, \text{etc.}$$

**IV. Gravity Effects**

Let us now switch on higher dimensional Planck scale effects. Since the model under discussion is not automatically invariant under $P$, i.e. since $P$ does not follow from gauge invariance, one could argue perhaps that even the renormalizable terms in the low energy theory should be allowed to break $P$. In the absence of any detailed calculations of the non-perturbative gravitational effects, one is not able to settle this issue definitely. In what follows (as in Ref. [8]), we make an assumption that only Planck scale induced non-renormalizable terms are allowed, terms which have a desirable property of vanishing in the $M_{Pl} \to \infty$ limit. This would be certainly true in any theory with an automatic $P$ symmetry, a situation we would like to achieve eventually.

The leading $P$ violating operators are then of dimension six

$$\mathcal{L}_{Pl} = \alpha/M_{Pl}^2 \bar{q}_L \phi q_R \hat{\phi}^+ \hat{\phi} + \text{other terms}$$

where other terms are assumed to break L-R symmetry and $\alpha$ is in general a complex matrix. Clearly these terms do not restrict $\langle \hat{\phi} \rangle$ severely, due to the suppression $\langle \hat{\phi} \rangle^2/M_{Pl}^2$. Furthermore, being of order 6, they are on the borderline of being large enough to destabilize the domain walls produced due to the spontaneous breaking of parity.

The situation becomes more interesting in the case of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry, as in the work of Babu and Mohapatra[23] (which preceded the BCS work). The quark mass matrix now takes the form

$$q_L \begin{pmatrix} q_R & Q_R \\ 0 & \Gamma_{UL} \end{pmatrix} q_R\left( \begin{array}{c} q_L \\ Q_L \end{array} \right) \begin{pmatrix} \Gamma_{V_R} & M \end{pmatrix}$$
where $q_L$ and $q_R$ are $SU(2)_L$ and $SU(L)_R$ doublets respectively; $Q_{L,R}$ are singlets, $v_L$ and $v_R$ are vev's of $\phi$ and $\hat{\phi}$ respectively (called $\phi_L$ and $\phi_R$) and $\Gamma$ are Yukawa couplings. Since $v_L, v_R \in R$, once again $\det M_q = \det \Gamma \Gamma v_L v_R$ and thus $\bar{\theta} = 0$ at the tree level. Babu and Mohapatra proceed to show that $\bar{\theta}$ is induced at the two-loop level and it turns out to be small: $\bar{\theta} \leq 10^{-12}$.

In this case the $P$ breaking terms induced by gravity are of dimension five

$$\mathcal{L}_{P_L} = \beta/M_{Pl} \bar{q}_L \phi_L \phi_R \bar{q}_R + \text{h.c.}$$

(23)

where $\beta$ is a complex matrix. This means that the zero in the quark mass matrix (22) is now induced to be of order of $\beta v_L v_R / M_{Pl}$. It can be shown that the effective $\bar{\theta}$ induced through this explicit breaking of $P$ is

$$\bar{\theta} \simeq \frac{\beta v_L v_R}{m_u M_{Pl}}$$

(24)

where the leading contribution comes from the up quark $u$. In order to comply with $\bar{\theta} \leq 10^{-19}$ one obtains an upper limit on $v_R, v_R \leq 10^6$ GeV.

Furthermore, now we can argue that there is no domain wall problem, since $\mathcal{L}_{P_L}$ is of dimension five. Thus, a reasonably low lying left-right symmetry, with $M_R \leq 10^6$ GeV may account naturally for the smallness of $\bar{\theta}$ without running into conflict with either experiment or cosmological considerations.

Alternatively, one can utilize CP instead of parity. A nice example is provided in [24], which is based on the horizontal family symmetry. Similarly, as before, when the gravity effects are switched on one obtains an upper limit on the horizontal symmetry breaking scale of approximately $10^6$ GeV.

V. Summary and Discussion

Understanding quantum gravity remains one of the central issues of today's theoretical physics and before it is achieved in depth, it is important to look for the probes of its potential low energy effects. One promising direction lies in searching for the possible violations of global symmetries, expected through the experience with black holes. The notion of global symmetries, if valid at all, may be subject to breaking terms in the effective theory at low energies, cut-off by the Planck scale. On the other hand, the solutions to the strong CP problem which try to explain naturally the smallness of strong CP parameter $\bar{\theta} \leq 10^{-9}$ are in general based on global symmetries. In particular, Peccei-Quinn mechanism is based on the $U(1)_{PQ}$ global symmetry and seems to be unable to survive the gravitational effects. On the other hand, the approach which utilizes discrete symmetries, such as parity or time-reversal invariance tend to benefit from such effects, since they can provide an upper limit on the scale of breaking of $P$ or $T$. All this assuming that the Planck scale phenomena vanish in the limit $M_{Pl} \to \infty$.

Finally, in my opinion the most interesting aspect of these Planck scale effects lies in the possibility of resolving the domain wall problem. The symmetry breaking of the order of $\langle \phi \rangle / M_{Pl}$ (or even $\langle \phi \rangle^2 / M_{Pl}$) is enough to cause the dissappearance of domain walls before they dominate the energy density of the universe. In other
words, the idea of spontaneous breaking of discrete symmetries may be free from cosmological difficulties and moreover it may play (through the explicit breaking by gravity) a role in baryogenesis.

References

[1] See, e.g. “General Relativity” by R.M. Wald (1984).

[2] This idea has a long history. For early work, see e.g. R. Barbieri, J. Ellis and M.K. Gaillard, Phys. Lett. B90 (1980) 249. Recently, there has been a revival of interest in the subject. See R. Holman et al. Phys Lett. 282B (1992) 132; M. Kamionkowski and J. March-Russell, ibid. 137; S. Barr and D. Seckel, Phys. Rev. D46 (1992) 539; S. Ghigna, M. Lusignoli and M. Roncadelli, Phys. Lett. 282B (1992) 1978.

[3] Ya. B. Zeldovich, I. Yu. Kobzarev and L.B. Okun, JETP 40 (1974) 1.

[4] B. Rai and G. Senjanović, ICTP preprint IC/92/414. Recently, I discovered that almost ten years ago the same idea was discussed by B. Holdom in his work on domain walls and baryon number generation, Phys. Rev. D28 (1983) 1419. For further application of the decay of the domain walls to baryogenesis, see H. Lew and A. Riotto, SISSA preprint 93/49-A.

[5] K. Choi, D.B. Kaplan and A.E. Nelson, preprint UCSD/PTH 92-11; A. Strominger and A. Witten, Comm. Math. Phys. 101 (1985) 341; C. Wetterich, Nucl. Phys. B234 (1984) 413; C.S. Lim, Phys. Lett. 256B (1991) 233.

[6] T.D. Lee, Phys. Rev. D8 (1973) 1226.

[7] H. Georgi, Hadronic J. 1 (1978) 155; M.A.B. Beg and H.S. Tsao, Phys. Rev. Lett. 41 (1978) 278; R.N. Mohapatra and G. Senjanović, Phys. Lett. 79B (1978) 28.

[8] An interesting scheme was used by A. Nelson, Phys. Lett. 136B (1983) 387 and S.M. Barr, Phys. Rev. Lett. 53 (1984) 329; Phys. Rev. D30 (1984) 1805, who use CP and extra heavy fermions.

[9] Z. Berezhiani, R.N. Mohapatra and G. Senjanović, Phys. Rev. D47 (1993) 5565.

[10] R. Peccei and H. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16 (1977) 1791.

[11] S. Weinberg, Phys. Rev. Lett. 40 (1978) 223; F. Wilczek, ibid. 279.

[12] For a review and references, see J.E. Kim, Phys. Reports 150 (1987) 1.

[13] D.A. Kirzhnitz and A. Linde, Phys. lett. 72B (1972) 471; L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320; S. Weinberg, ibid. 3357.
[14] D. Stauffer, Phys. Rep. 54 (1979) 1.

[15] For a review and references, see A. Vilenkin, Phys. Rep. 121 (1985) 263.

[16] A.E. Everett, Phys. Rev. D10 (1974) 316.

[17] R.N. Mohapatra and G. Senjanović, Phys. Lett. 89B (1979) 57; Phys. Rev. D20 (1979) 3390. The fact that symmetries may remain broken at high $T$ was known to Weinberg, Ref. [12].

[18] J. Preskill, S.P. Trivedi, F. Wilczek and M. Wise, Nucl. Phys. B363 (1991) 207.

[19] A. Ashtekar, A.P. Balachandran and S. Jo, Int. J. Mod. Phys. A4, 6 (1989) 1993.

[20] R. Penrose, in “General Relativity: An Einstein Centenary Survey”, Ed. S.W. Hawking and W. Israel (Cambridge Univ. Press, 1979), p. 581.

[21] J. Ellis, K. Enqvist, D.V. Nanopoulos, K. Olive, M. Quiros and F. Zwirner, Phys. Lett. 1786B (1986) 403.

[22] S. Barr, D. Chang and G. Senjanović, Phys. Rev. Lett. 67 (1991) 2765.

[23] K.S. Babu and R.N. Mohapatra, Phys. Rev. D41 (1990) 1286.

[24] Z.G. Berezhiani, Mod. Phys. Lett. A6 (1991) 2437.