Multi-period Decision-making and Governing of Carbon Emissions

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Abstract: The carbon emission governing has been more and more widely applied around the world and its establishment has brought opportunities and challenges to the management of microenterprises or governments. In order to solve the problem of multi-period decision-making or governing in carbon emission, it studies the problem of dynamic carbon emission control, constructs a multi-period cost-minimizing model, and designs a dynamic programming algorithm to solve the problem based on the optimal solution properties. In addition, the model is extended to the case where the governance capability has upper limit.

1. Introduction

Global warming and a series of other environmental issues have led to human concern for carbon emissions. As a solution to carbon emissions policy tools, dynamic governing of the carbon emissions has been applied in more and more countries. The full implementation of carbon emission governing brings opportunities and challenges to the management of the microenterprises or governments, and it has also become an important factor in the production and operation of enterprises or governments.[1]

At present, most of the microenterprise or governments decision-making research under the constraint of carbon emission governing focus on the issue of mass production. Hua Guowei et al. (2011) studied the influence of economic order quantity, total emissions, and total cost of carbon emission trading quotas and carbon emission quota enterprises under the quota-trading mechanism.[2] Benjaafar et al. (2013) studied the problem of multi-period production inventory decision-making in individual firms and supply chains under different carbon policies, such as carbon tax, quota-trade, quota-offset. In a dynamic mass production model, they also analyzed the impact of carbon emission rights quotas, carbon policies, energy-saving emission reduction technologies, and supply chain joint reductions on total emissions and total costs through numerical experiments.[3] Absi et al. (2013) studied the time complexity of the dynamic batch model under the constraints of periodic unit production carbon emission constraint, accumulation cycle unit production carbon emission constraint, rolling cycle unit production carbon emission constraint, and full cycle unit production carbon emission constraint. [4] Other papers approach the carbon emission problem from a similar point of view, such as Fang et al. (2011), Chaabane et al. (2012), Chen et al. (2013) and Chen et al. (2016).[5-8]

However, few scholars have paid much attention to the dynamic governing of carbon emission.

The rest of the paper is organized as follows. Section 2 formulates the problem and presents the
model. Section 3 develops the dynamic programming algorithm to solve the problem. Section 4 extends the model with capacity constraints. Section 5 concludes the paper.

2. Model formulation.

The following symbols need to be defined in the model construction.

- $T$: Time period;
- $P(t)$: t-periodic problem, $t = 1, 2, ..., T$;
- $d_t$: Emissions of pollutants in the period of $t$;
- $x_t$: The amount of pollutants removed(controlled) in the period of $t$;
- $I_t$: Stock of pollutants in the period of $t$;
- $h_t$: Hazard costs resulting from the stock of unit pollutants in the period of $t$;
- $\sigma_t$: The fixed cost of pollution control in the period of $t$;
- $c_t$: Unit cost of pollution control in the period of $t$;
- $\delta_t = \begin{cases} 1 & \text{when } x_t > 0 \\ 0 & \text{when } x_t = 0 \end{cases}$, $t = 1, 2, ..., T$

Based on the symbols and assumptions defined above. The objective function is expressed as:

$$\min \sum_{t=1}^{T} [\sigma_t \delta(x_t) + c_t x_t + h_t I_t]$$

Subject to:

1. $I_0 = I_T = 0$ (1)
2. $I_t = I_{t-1} - x_t + d_t$, $t = 1, 2, ..., T$ (2)
3. $x_t \geq 0$, $t = 1, 2, ..., T$ (3)

Constraint (1) indicates that the stocks of pollutants in the initial and end periods are 0. Constraint (2) indicates the relationship between the stock of pollutants, the control amount and the emission in the period of $t$ and the stock of pollutants in the previous period.

Constraint (3) is that the amount of pollutant control is non-negative.

**Definition 1.** In an optimal solution, period $t$ is defined as a transaction point (periods) if $x_t > 0$, $1 \leq t \leq T$.

3. Properties and dynamic programming algorithm

In this section, we will research two properties and develop a dynamic programming algorithm to solve our problem.

**Property 1.** There exists an optimal solution to problem $P(t)$ such that: if $x_t > 0$, then $I_t = 0$.

**Proof.** Suppose there are two consecutive transaction periods $t < t'$. Assume that there exists an optimal solution with $I_t = \epsilon > 0$ and $x_t = \delta > 0$, if $c_t + \sum_{i=t}^{t-1} h_i \geq c_{t'}$, then reduce $I_t$ from $\epsilon$ to 0 and increase $x_t$ to $x_t + \epsilon$, this alternation does save the cost $(c_t + \sum_{i=t}^{t-1} h_i - c_{t'}).$ If $c_t + \sum_{i=t}^{t-1} h_i < c_{t'}$, then reduce $x_t$ from $\delta$ to 0 and increase $I_t$ to $I_t + \epsilon$, it is that increase $x_t$ to $x_t + \epsilon$, this
alteration saves the cost \( (c_i - c_i - \sum_{t=1}^{i-1} h_t)\delta + \sigma_i \). Hence, if \( x_i > 0 \), then \( I_i = 0 \).

Property 1 shows that if governance measures are taken during the period, all the stock pollutants will be controlled.

From Property 1, we can derive the following corollary easily.

**Corollary 1.** In the optimal solution of \( P(t) \), \( x_i = 0 \) or \( x_i = \sum_{t=1}^{i} d_t \).

Now, we show how to solve the problem based on Property 1 and Corollary 1. First, we define \( F(t) \) and \( F(i, t) \) before we show the dynamic programming algorithm.

\( F(t) \) is the minimum costs for \( P(t) \);

\( F(i, t) \) is the minimum costs for \( P(t) \) in which the final setup performs in period \( t \) and the next-to-last setup performs in period \( i - 1 \);

The dynamic programming algorithm is as follows:

\[
F(t) = \min \{ F(i, t) \} = \min \{ F(i-1) + \sigma_i + c_i \sum_{u=i}^{i-1} d_u + \sum_{u=1}^{i} \sum_{v=1}^{u} h_v d_v \}
\]

which satisfies the following conditions:

1) \( 1 \leq i \leq t \leq T \); (2) \( F(0) = 0 \)

We will illustrate how this recursion formulation solves \( P(t) \) with a toy example. Assume \( T = 5 \), we start with solving \( P(1) \) with an optimal value \( F(1) \). From the recurrence formula,

\[
F(1) = F(1,1) = F(0) + \sigma_1 + c_1 d_1
\]

We then proceed to solve \( P(2) \).

\[
F(2) = \min \{ F(1,2); F(2,2) \} \quad \text{where}
F(1,2) = F(0) + \sigma_2 + c_2 (d_1 + d_2) + h_1 d_1
F(2,2) = F(1) + \sigma_2 + c_2 d_2
\]

\( P(3) \) is solved next.

\[
F(3) = \min \{ F(1,3); F(2,3); F(3,3) \} \quad \text{where}
F(1,3) = F(0) + \sigma_3 + c_3 (d_1 + d_2 + d_3) + h_1 d_1 + h_2 (d_1 + d_2)
F(2,3) = F(1) + \sigma_3 + c_3 (d_2 + d_3) + h_2 d_2
F(3,3) = F(2) + \sigma_3 + c_3 d_3
\]

\( F(4) = \min \{ F(1,4); F(2,4); F(3,4); F(4,4) \} \quad \text{where}
F(1,4) = F(0) + \sigma_4 + c_4 (d_1 + d_2 + d_3 + d_4) + h_1 d_1 + h_2 (d_1 + d_2) + h_3 (d_1 + d_2 + d_3)
F(2,4) = F(1) + \sigma_4 + c_4 (d_2 + d_3 + d_4) + h_2 d_2 + h_3 (d_2 + d_3)
F(3,4) = F(2) + \sigma_4 + c_4 (d_3 + d_4) + h_3 d_3
F(4,4) = F(3) + \sigma_4 + c_4 d_4
\]

Finally, we proceed to solve \( P(5) \).

\[
F(5) = \min \{ F(1,5); F(2,5); F(3,5); F(4,5); F(5,5) \} \quad \text{where}
F(1,5) = F(0) + \sigma_5 + c_5 (d_1 + d_2 + d_3 + d_4 + d_5) + h_1 d_1 + h_2 (d_1 + d_2) + h_3 (d_1 + d_2 + d_3) + h_4 (d_1 + d_2 + d_3 + d_4)
F(2,5) = F(1) + \sigma_5 + c_5 (d_2 + d_3 + d_4 + d_5) + h_2 d_2 + h_3 (d_2 + d_3) + h_4 (d_2 + d_3 + d_4)
F(3,5) = F(2) + \sigma_5 + c_5 (d_3 + d_4 + d_5) + h_3 d_3 + h_4 (d_3 + d_4)
F(4,5) = F(3) + \sigma_5 + c_5 (d_4 + d_5) + h_4 d_4
F(5,5) = F(4) + \sigma_5 + c_5 d_5
\]
\[ F(4, 5) = F(3) + \sigma_5 + c_5(d_4 + d_5) + h_5d_4 \]
\[ F(5, 5) = F(4) + \sigma_5 + c_5d_5 \]

4. Model extension.

In reality, due to the constraints of funds or the shortage of equipment and personnel, there are often cases where the governance capacity has an upper limit. In the case where the governance capacity has an upper limit, the property in theorem 1 is not valid. Under the condition that the unit governance cost is stable \( c_t = c \), \( t = 1, 2, ..., T \), this section studies the situation where there is a limit of the governance capacity so as to expand the properties in theorem 1.

\[ C \]: The maximum value of pollutant control in each period.

Based on the symbols and assumptions defined above, the objective function is expressed as:

\[
\min \sum_{t=1}^{T} \left[ \sigma_t \delta(x_t) + cx_t + h_tI_t \right]
\]

Subject to:

\[ I_0 = 0 \] (1)
\[ I_t = I_{t-1} - x_t + d_t \quad t = 1, 2, ..., T \] (2)
\[ 0 \leq x_t \leq C \quad t = 1, 2, ..., T \] (3)

Constraint (3) stands for the limit to ability of governing in every period.

For the capacitated problem, the following property holds, which can be proved by contradiction, hence its proof is omitted.

**Property 2.** There exists an optimal solution to problem \( P(t) \) such that:

\[ x_t(C - x_t)I_t = 0 . \]

Property 2 can make us to limit consideration to governing quantities with certain simplifying features. In particular, if the rest of emission in period \( I_t \) is positive, then the governing quantities are either at governing capacity or zero. On the other hand, if the governing quantities are positive at a level less than governing capacity, then the rest of emission in period \( I_t \) is zero.

**Corollary 2.** In the optimal solution of \( P(t) \), \( x_t = 0 \) \( x_t = C \) or \( x_t = I_{t-1} + \sum_{u=1}^{t} d_u \)

Based on Property 2 and Corollary 2, we also develop the dynamic programming algorithm to solve the problem.

If \( I_{t-1} + \sum_{u=1}^{t} d_u < C \), then

\[
F(t) = \min F(i, t) = \min \{ F(i-1) + \sigma_i + c(I_{i-1} + \sum_{u=1}^{i-1} d_u) + h_i(I_{i-1} + d_i) \\
+ h_{i+1}(I_{i-1} + d_i + d_{i+1}) + ... + h_{i+1}(I_{i-1} + d_i + d_{i+1} + ... + d_{i-1}) \}
\]

If \( I_{t-1} + \sum_{u=1}^{t} d_u \geq C \), then

\[
F(t) = \min F(i, t) = \min \{ F(i-1) + \sigma_i + cC + h_i(I_{i-1} + d_i) \\
+ h_{i+1}(I_{i-1} + d_i + d_{i+1}) + ... + h_{i+1}(I_{i-1} + d_i + d_{i+1} + ... + d_{i-1}) \}
\]

5. Conclusion

This paper studies the problem of dynamic carbon emission control, constructs a multi-period
cost-minimizing model, and designs a dynamic programming algorithm to solve the problem based on the optimal solution properties. In addition, the model is extended to the case where the governance capability has upper limit.

This paper can also be expanded from the following aspects: Firstly, it assumes that the amount of carbon emission in each period is determined, and an important expansion is to study the case where the amount of carbon emission in each period is random. Secondly, the case, where the governance capability has an upper limit under the condition that the governance variable cost is fixed and stable, is studied. Therefore, it is a well-worthy direction to study the problem of capacity constraints in the case of unstable governance cost. Thirdly, the research on synergistic control of waste gas, waste water and solid waste is a meaningful extension.

Acknowledgement:
The work was supported by the National Social and Scientific Fund of China (16AGL002), Major Program of the Ministry of Education (16JZD014), Innovation Team Training Plan of Tianjin University (TD13-5012).

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