Teacher’s strategies to promote student’s mathematical competencies in algebra: a case study

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Abstract. Mathematical competence is knowledge, skill, and ability to use mathematics for solving problems of both mathematical and non-mathematical context. The PISA and TIMSS reports indicate the low quality of mathematical competence of students in Indonesia. One of the most influential factors in the quality of learning is teachers. The paper aims to describe the teacher’s strategies to promote students' mathematical competence in algebra. The case study method was used to investigate the strategies used by mathematics teacher in conducting teaching activity on 7th grade. The data was collected through observation and interview. The observation was conducted three times. Field notes, observation guideline, and video recorder were used to collect the data of teacher's strategies in the classroom. Interview was conducted at the end of the observation activity. A semi-structured interview was done to collect further information regarding the teacher's strategies. The results show that the teacher used various strategies to promote students mathematical competence in understanding the mathematical concept and using reasoning. However, a lack of strategies to promote students in using the pattern as a conjecture to solve the problem and communicating the idea is identified.

1. Introduction
Mathematical competence is knowledge, skill, and ability to use math to solve problems of both mathematics and non-mathematical context [1]. This competence is very important to develop mathematical literacy [2] as a goal of mathematics learning [3] and the focus on PISA survey [4]–[6]. Moreover, it is a fundamental mathematical capability employed as a framework for assessing students’ performance in mathematics in the 2015 PISA test [1].

Several studies have constructed the components of mathematical competence ( [2], [5], [7]–[10]). Meanwhile, OECD ( [5], [7], [8]) identified seven components of mathematical competence including communication, mathematizing, representation, reasoning and argument, devising strategies for solving problems, using symbolic, formal and technical language and operations, and using mathematical tools. Turner [2] constructed the components of mathematical competencies, which are similar to OECD, without using mathematical tools. In addition, Niss [9] formulated the mathematical competence in a KOM project (in Denmark, KOM is an abbreviation of Competencies and the Learning of Mathematics) with eight components, namely thinking mathematically, posing and solving mathematical problem, modelling mathematically, reasoning mathematically, representing mathematical entities, handling
mathematical symbols and formalisms, communicating in, with, and about mathematics, and making use of aids and tools. NCTM [10], in process standards, identified the competencies that should be acquired by students in learning mathematics, involving problem-solving, reasoning and proof, communication, connections, and representation. Shortly, previous studies have discussed mathematical competence as a cognitive activity.

In [3], the Indonesian Ministry of Education and Culture states that the goal of learning mathematics in the 2013 curriculum (K-13) entails three aspects of: knowledge, skills, and attitude. The knowledge competency covers the understanding of the mathematical concept, using the pattern as a conjecture to solve the problem, reasoning, and communicating the idea. Furthermore, the skills competency comprises the using of mathematical aids and applying mathematical knowledge. The goal of the third aspect, attitude, relates to the usefulness of mathematics in life as well as attitudes and behaviours in accordance with the values in mathematics and learning. In the present study, the researcher limited the investigation on the strategies of the teacher to promote the knowledge aspect of learning mathematics. The Indonesian Ministry of Education and Culture [3] identifies the indicators of the knowledge aspect as presented in Table 1.

| No | Component                              | Indicator                                                                 |
|----|----------------------------------------|---------------------------------------------------------------------------|
| 1  | Understanding the mathematical concept | a. Restate the concept that has been learned.                              |
|    |                                        | b. Classify objects based on whether the requirements of a concept are met.|
|    |                                        | c. Identify the properties of operations or concepts.                      |
|    |                                        | d. Apply the concept logically.                                            |
|    |                                        | e. Provide examples or counter-examples of the concepts learned.           |
|    |                                        | f. Present concepts in various forms of mathematical representation (tables, graphs, diagrams, drawings, sketches, mathematical models, or other means). |
|    |                                        | g. Associate various concepts in mathematics and non-mathematics.          |
| 2  | Using the pattern as a conjecture to solve the problem | a. Conjecture.                                                             |
|    |                                        | b. Draw conclusions from a statement.                                      |
|    |                                        | c. Provide an alternative to an argument.                                  |
|    |                                        | d. Find patterns on a mathematical symptom.                                |
| 3  | Using reasoning                        | a. Understand the problem.                                                 |
|    |                                        | b. Organize and select relevant information in identifying problems.       |
|    |                                        | c. Presents a mathematical problem statement in various forms.             |
|    |                                        | d. Choose the right approach and strategy to solve the problem.            |
|    |                                        | e. Use or develop problem-solving strategies.                              |
|    |                                        | f. Interpret the results of answers obtained to solve problems.            |
|    |                                        | g. Solve the problem.                                                      |
| 4  | Communicating an idea                  | a. Provide reasons or evidence for the validity of a statement.             |
|    |                                        | b. Estimate and examine conjecture.                                        |
|    |                                        | c. Check the validity of a statement with inductive reasoning.             |
|    |                                        | d. Eliminate or prove a formula with deductive reasoning.                  |
|    |                                        | e. Estimate and examine conjecture.                                        |

The ability of Indonesian students in mathematical competence can be identified from the result of the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). OECD [1] reported that the competence of Indonesian Junior High School students in mathematics as indicated by the 2015 PISA assessment is relatively lower than the competence of students from other countries. The average math scores of Indonesian students was 386, while the average score at international scale was 490 [1]. The Indonesian students were only able to solve particular problems in the familiar context using a procedure of algorithm or simple formula.
In addition to PISA, the performance of the junior high school students in mathematics can also be identified from the 2011 TIMSS report since Indonesian students did not participate in the 2015 TIMSS. The average score of the Indonesian 8th grade students in TIMSS was 386. They were only able to solve mathematical problems using basic mathematical knowledge. The 2011 TIMSS also reported that the average score of the Indonesian 8th grade students in algebra is 392. The reports indicate the low level of mathematical competence of Indonesian students.

One of the most influential factors in the learning process is the teacher's competence [8]. Several studies have attempted to investigate the relationship between teacher’s strategy in teaching and student learning outcomes. Kunter et al. [11] showed the positive correlation between teacher’s instructional approaches and student learning outcomes. According to [11], the teacher's instruction or reaching strategy is influenced by pedagogical content knowledge, enthusiasm in teaching, and teacher’s self-regulated in teaching. Ellis et al. [12] also showed that the implementation of different instructions significantly improve the students learning outcome in mathematics. In general, previous studies indicate that the students’ performance can be improved with the support of innovative instructional strategies.

Study on the strategies of teacher in the teaching and learning process is useful particularly to promote students' mathematical competence. This study will be able to give an overview about the extent of teacher’s efforts to achieve the objectives of mathematics learning as stated in [3]. It will also give an overview on the implementation of new learning approaches in K-13. Therefore, the aim of the present study is to investigate the teacher’s strategies to promote mathematical competence in algebra. In this study, the knowledge aspect of learning mathematics goals developed by [3] was employed as the theoretical framework to investigate the competence.

2. Methods

The research used a qualitative approach with a case study method. The subject was a 7th grade mathematics teacher at a public junior high school in Sukoharjo, Central Java, Indonesia. The teacher had have more than 15-years teaching experience. The students of the 7th grade under observation had reported to have high mathematics ability. The teacher devised lesson plan without any intervention from the researcher. The data was collected through observation and interview. The teacher was observed and the three sessions of teaching learning activity on the topic of Algebra were recorded. During the observation, the researcher noted the important points carried out by the subject in promoting the students’ mathematical competence while recorded the teacher’s activities in the classroom. Additionally, the teacher was interviewed for further understanding of the strategies employed in the classroom. The data were validated using data triangulation method. Subsequently, the data were analyzed using Miles and Huberman’s method, which involve data reduction, data display, and conclusion drawing.

3. Results and Discussion

The subject, Mrs. Nita, is a mathematics teacher who teaches at school with high mathematical ability students. She has nearly 15 years of teaching experience. Her education background is a bachelor in mathematics education. She is a civil servant and has been equipped with professional educator certificate. The summary of the strategy used by the subject to promote mathematical competence as analyzed based on classroom observation is presented in Table 2.

| No | Components | Strategy |
|----|------------|----------|
| 1  | Understanding mathematical concept | a. Provide contextual problems related to the mathematical concepts.  
                        b. Identify the operations to solve the problem.  
                        c. Give examples of problem-solving using procedures.  
                        d. Use a graph to explain the solution set |
Using pattern as a conjecture to solve a problem

- Explain the direction of a number line graph to solve a linear inequality.

Using reasoning

- Determine the solution of linear inequalities with graphs of number lines.
- Give examples of solving contextual problems.

Communicating an idea

- Instruct students to present their works.

**Table 2** shows the strategies used by the subject to promote students' mathematical competence, especially in the knowledge aspect. The discussion of each component of the strategies is presented in subsequent section.

### 3.1 The understanding of mathematical concept

We identified four strategies implemented to achieve the first goal of the knowledge aspect of mathematical competence. The activities of each strategy are clarified as follows:

**First**, the teacher posed a contextual problem related to mathematical concepts that had been learned to the student as presented below.

| Problem: |
| --- |
| A farmer has a rectangular plot of land. The width of the land is 6 meters shorter than its length. If the land circumference is 60 meters, determine the area of the farmer's land! |

The problem can be found in the real-world and can be associated with at least two mathematical concepts, namely rectangular (plane geometry) and linear equations of one variable (algebra). Therefore, the strategy is relevant with the concept, which is to achieve the indicator associated with various concepts in both mathematics and non-mathematics. In addition, the students also need to transform the problem into mathematical form to solve the problem. It means that they should represent the word problem into mathematical form. So, the students will be able to achieve the indicator of present concepts in various forms of mathematical representation. Representation skills are needed to understand the mathematical ideas as stated by [10]. Algebraic expression, equations, and graphs are several forms of mathematical representations. Moreover, students also need to recall their knowledge about circumference and area of rectangular to solve the problem. It is relevant with the indicator of to restate the concept that has been learned. Therefore, it can be claimed that the strategy to provide a contextual problem is able to promote at least three indicators of mathematical competence.

**Second**, the teacher encouraged the students to identify the proper operations to solve the problem by posing a question. As an example, the teacher posed a problem as follows:

\[
\frac{7}{x - 8} + \frac{5}{x - 3} = ?
\]

Subsequently, the teacher questioned the students, "what are the steps to solve the problem? What should we do first?". To solve the question, the students should recall their knowledge of procedure in solving fractions number that equates the denominator. So, it is in accordance with the indicator of to restate the concept that has been learned. It is also relevant to the indicator of to identify the properties of operations or concepts. Therefore, the second strategy is able to promote at least two indicators of mathematical competence.

**Third**, the teacher provided examples to solve the algebra problems using a systematic procedure. The example of the problem is as follows:

Solve the following inequalities.
Subsequently, the teacher explicated the procedure to solve the problem as follows:

\[
2x - 4 > 3x + 9
\]

2x - 3x > 9 + 4  => moving segment, grouping the similar terms.

\[-x > 13\]

\[\Rightarrow\] because the left side is negative then each segment is multiplied by -1

\[x < 13\]

\[\Rightarrow\] the final result, then drawn a number line

![Figure 1. Number line graph of example problem](image)

The procedure to solve the example problem requires students to understand the properties of operations of inequalities. For example, in grouping terms, the sign (\(+/\)) of each term will be changed into (\(+/-\)) while moving from the left side to the right or vice versa. Multiplying each term by (\(-\)) will also change the inequality sign from "\(>\)" to the "\(<\)", or vice versa. So, the strategy meets the indicator to identify the properties of operations or concepts. The problem-solving also involves the knowledge of line number. The students need to recall their knowledge about the properties of the line number. Therefore, this strategy is relevant to the indicator of to restate the concept that has been learned. The graph of line number to solve the problem also demonstrates the indicator of to present concepts in various forms of mathematical representation. It can be claimed that the third strategy at least promotes three indicators of mathematical competence.

**Fourth**, the teacher used the graph of the number line to determine the solution for linear inequalities problem. In the previous strategy, the teacher posed the inequality problem of \(2x - 4 > 3x + 9\) to the students. Subsequently, the students were guided to solve the problem with algebraic manipulation until obtaining the algebraic solution that of \(x < 13\). To obtain the solution set, the number line graph as presented in Figure 1 was employed. This strategy is in line with the indicator of to present concepts in various forms of mathematical representation. In this case, the teacher uses a number line graph to present the solution of linear inequalities.

### 3.2 Using pattern as a conjecture to solve the problem

Based on the observation, we identified a strategy to promote mathematical competence in comply with this component. Fig. 2 demonstrated the session when teacher explained to the students of how to draw the number line graph to represent the solution of linear inequality. The subject gave an example of how to draw the solution of the inequality \(x > 0\) in number line. Firstly, she drew an open circle at 0 which represents that 0 is not a member of the solution. Then, she asked the students, "where is the direction of the arrow if the inequality is more than (\(>\))?". Almost all of the students answered "the right side of the number line". In addition, the teacher also gave another example of inequality \(p \geq 0\). She drew the closed circle at 0 which represents that 0 is a member of the solution. Then, the graph solution is similar to the previous example. By using this strategy, the teacher used the pattern of the symbols "\(>\)" or "\(\geq\) " to determine the form of a circle at the number line, the open or closed circle. The symbols of inequality also determine the pattern of the direction of the arrow line at the number line. This strategy is relevant to the indicator of to find patterns on a mathematical problem.
3.3 Using reasoning

Based on the observation in the classroom, we identified two strategies were in line with the indicators in the component of using reasoning. The sample of each strategy is discussed as follows.

First, the teacher used the graph of the number line to determine the solution of the linear inequality problem. This strategy was similar to the fourth strategy in the first component of the knowledge aspect of mathematics learning (Table 1), which is by using a graph to explain the solution set. In this component, the teacher’s strategy is relevant with at least three indicators of to understand the problem, present a mathematical problem statement in various forms, and solve the problem. The students would be able to use the graph of the number line to determine the solution of linear inequality if they are able to solve the problem in the algebraic form correctly. It requires students to understand the problem, which is the first indicator of this component. The students are also able to present a mathematics statement in various forms. In this case, they will be able to present the solution of linear inequality from the algebraic form into the graph of number line. This strategy is a part of the steps in solving the linear inequality problem. The sample of student’s answer in solving the inequality problem is presented in Fig. 3.

Second, the teacher provided an example in solving contextual problems. This strategy was similar to the previous strategy in the first component, which is by posing a contextual problem related mathematical concepts that have been learned to the students. To solve the contextual problem, the students must understand the problem, organize or select the relevant information, present a problem in mathematical form, choose the right strategy to solve the problem and interpret the solution. In solving the contextual problem, the teacher instructed the students to read and understand the problem, which is consistent with the first indicator of to understand the problem. Furthermore, the teacher guided the
students to identify the relevant data or information in the problem. Subsequently, the information in the problem was formulated as follows:

- **The length of the land** \( l \) = \( x \)
- **The width of the land** \( w \) = \( x - 6 \)
- **The land circumference** \( K \) = 60 m
- **Area of the land** \( L \) = \( l \times w = x(x - 6) \)

These steps are able to promote the students to achieve two indicators of organizing and selecting the relevant information, and to present a problem in mathematical form.

Furthermore, the teacher also guided the students to determine the solution of the problem with algebraic manipulation expressed as follows.

\[
K = 2(l + w) \\
\Rightarrow 60 = 2(x + x - 6) \\
\Rightarrow 60 = 2(2x - 6) \\
\Rightarrow 60 = 4x - 12 \\
\Rightarrow 72 = 4x \\
\Rightarrow x = 18
\]

Then, substitute the value of \( x = 18 \) into the formula of the area and the area of the land is as follows:

\[
Area\ of\ the\ land = x(x - 6) \\
= 18(18 - 6) \\
= 18(12) = 216
\]

Shortly, the area of the land is 216 m². To solve the contextual problem, the ability to choose the right strategy in order to obtain the correct solution is required. This strategy encourages students to choose the most suitable strategy to solve the problem. In this case, the teacher guides students to find the value of \( x \) using the circumference formula and then substitute the value in the formula of the area. The solution also showed the interpretation of the solution, which is the value of the area of land. So, it can be claimed that this strategy is able to promote at least five indicators of using reasoning, which are to understand the problem, organize or select the relevant information, present a problem in mathematical form, choose the right strategy to solve the problem and interpret the solution.

### 3.4 Communicating an idea

Based on observation, we identified a strategy to promote the component of communicating idea, which is to ask students to explain their ideas. In each session, after presenting an example, the teacher gave problems as an exercise for students, both individually or in groups. After finishing the exercise, the teacher instructed the students to present their works to others (Fig. 4). During the presentation, the students were required to demonstrate the reason for each step of their solution. This strategy is in line with the indicator of to provide reasons or evidence for the accuracy of an argument.

![Image](image-url)
The description of the findings of this research can be summarized as presented in Table 3 as follows.

### Table 3. The summary of teacher’s strategies and mathematical competence indicators

| Strategy | Mathematical competence indicator |
|----------|----------------------------------|
| 1. Provide contextual problems related to the mathematical concepts | a. Link various concepts in both mathematics and non-mathematics.  
| | b. Present concepts in various forms of mathematical representation.  
| | c. Restate the concept that has been learned.  |
| 2. Identify the operations to solve the problem | a. Restate the concept that has been learned.  
| | b. Identify the properties of operations or concepts.  |
| 3. Give examples of problem-solving using procedures | a. Identify the properties of operations or concepts.  
| | b. Restate the concept that has been learned.  
| | c. Present concepts in various forms of mathematical representation.  |
| 4. Use a graph to explain the solution set | a. Present concepts in various forms of mathematical representation.  |
| 5. Explain the direction of a number line graph to solve a linear inequality | a. Find patterns on a mathematical symptom.  |
| 6. Determine the solution of linear inequalities with graphs of number lines | a. Understand the problem.  
| | b. Presents a problem statement mathematically in various forms.  
| | c. Solve the problem.  |
| 7. Give examples of solving contextual problems | a. Understand the problem.  
| | b. Organize or select the relevant information.  
| | c. Present a problem in the mathematical form.  
| | d. Choose the right strategy to solve the problem.  
| | e. Interpret the results of answers to solve problems.  |
| 8. Instruct students to explain their works | a. Provide reasons or evidence for the accuracy of an argument  |

Effective teaching should motivate the students to engage in mathematical thinking and reasoning, and to provide opportunities that challenge students of understanding mathematics [10]. However, the teacher must conduct a complex effort to carry out effective teaching. Table 3 shows the teacher's strategy is associated with the knowledge aspect of the objectives of learning mathematics as presented in Table 1. The first four strategies are corresponding with the strategy to achieve an understanding of the mathematical concept. Based on Table 3, the strategies devised by the teacher in providing opportunities for students to restate the concept that has been learned, present concepts in various forms of mathematical representation, link various concepts in mathematics and non-mathematics, and identify the properties of operations or concepts. In other words, teacher’s strategies are able to promote the mathematical competence.

The fifth strategy is corresponding with the effort made by teacher in using the pattern as a conjecture to solve the problem. Problem-solving is an important goal of learning mathematics [10]. The students should have the opportunities as many as possible to formulate and solve the problems as reflected from their thinking ability. In solving problems, devising strategies is one of the steps to solve the problem systematically as formulated by [13]. In this step, the students should select a strategy or devise a plan to solve problems. The understanding pattern is one of the strategies to solve the mathematical problem effectively. Based on Table 3, the teacher provides opportunities for students to find patterns on a
The sixth and seventh strategies in Table 3 are associated with the teacher's strategy to embody one of the objectives of learning mathematics, which is to use reasoning. The development of reasoning ability and stating a mathematical argument is one of the important abilities to be achieved in learning mathematics. Learning mathematics with understanding can develop mathematical reasoning skills [14]. Table 3 shows that the teacher's strategies provide opportunities for the students in promoting reasoning skills. The strategies are able to encourage the students to understand the problem, organize or select the relevant information, present a problem in mathematical forms, choose the right strategy to solve the problem, and interpret the results of answers to solve problems.

The last strategy in Table 3 is related to the fourth objectives of mathematics learning, which is communicating an idea. Communication skills are essential for students in learning mathematics [10]. The ability to explain or present ideas both orally and in writing shows the students' understanding of the subject-matter or topic that have been learned. It entails the ability to understand written or oral statements from others. The teacher's strategy to promote communication skills by instructing students to present their problem-solving. This strategy gives the students opportunities to provide the reason or evidence of the truth of their answers. Nevertheless, the teacher had not explored further the students' communication skills. When the students present their works, they were only required to write answers on the whiteboard without any verbal explanation. So, the students' way of thinking in solving mathematical problems cannot be explored more deeply. The teacher had not explored other strategies to develop student communication skills, such as group discussions and other active learning models. Based on the interview, the subject said that learning with group discussions and active learning models is indeed good and interesting for students. However, it is not easy to carry out these instructional models because they require a longer time while the material that should be learned by students is very huge.

The instructional strategies implemented by the teacher to promote students mathematical competence have been discussed by previous researchers [15], [16]. They used the tasks and questions for developing the students' mathematical thinking competence in a simple and routine way. [15] revealed several strategies, namely: using questions and task, providing activities focused on student thinking, using a representation with picture or table, and providing an opportunity to think and respond to promoting students' thinking about mathematics. Meanwhile, [16] emphasized on the use of interactive discourse to enhance students mathematical thinking. Similarly, the results of the present research show that the teacher has used various strategies to promote students mathematical competence. The teacher provides the contextual problem, use questions to identify the procedure or concepts, use procedure and pattern to solve the problem, use a graph to represent the solution, and instruct students to present their works. Although these strategies have not fully reached the mathematics learning indicators as stated by [3] yet, they have provided opportunities for students to achieve mathematical competence.

4 Conclusions
The results of the present study indicate that the teacher has used various strategies to promote students' mathematical competencies. The strategies to promote profound understanding in mathematical concepts include using contextual problems in learning, using graphics to represent solutions, using questions to identify the procedure or concepts, and using the procedure to solve the problem. Moreover, a strategy that explains the direction of a number line graph to solve a linear inequality is implemented to promote the pattern to solve the problem. In addition, the teacher also uses graphs to represent solutions and solve the contextual problem to promote reasoning competence. Finally, the teacher instructs students to present their works as a strategy to promote students communication skills. Even though these strategies can promote students' mathematical competencies, the development of effective learning strategies is still a difficult task for teachers in general. Moreover, the challenge ahead is more complicated, particularly in approaching the 4.0 industrial revolution era which requires teachers to be familiar with the digital world. However, teachers must prepare themselves to face that era.
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References
[1] OECD, “PISA 2015 Results (Volume I): Excellence and Equity in Education,” Paris, 2016.
[2] R. Turner, “Mathematical Competencies,” Res. Dev., vol. 24, no. 5, pp. 2–7, 2011.
[3] Kemendikbud, “Lampiran III Permendikbud RI Nomor 58 Tahun 2014,” 2014.
[4] OECD, “Assessing Scientific, Reading and Mathematical Literacy: A framework for PISA 2006,” Program. Int. Student Assess., p. 192, 2006.
[5] OECD, “Programme for international student assessment 2009 assessment framework: Key competencies in reading, mathematics and science.,” OECD Publ., p. 292, 2010.
[6] OECD, PISA 2012 results: What students know and can do-Student Performance in Mathematics, Reading and Science, vol. I, no. Volume I. 2014.
[7] OECD, PISA 2012 in Focus: What 15-years-old know and what they can do with what they know. 2014.
[8] OECD, “Teachers Matter: Attracting, Developing and Retaining Effective Teachers (Overview),” 2005.
[9] K. Stacey and R. Turner, “Assessing mathematical literacy: The PISA experience,” Assess. Math. Lit. PISA Exp., pp. 1–321, 2015.
[10] NCTM, Principles and Standards for School Mathematics. Reston, VA: The National Council of Teachers of Mathematics, 2000.
[11] M. Kunter, U. Klusmann, J. Baumert, D. Richter, T. Voss, and A. Hachfeld, “Professional Competence of Teachers: Effects on Instructional Quality and Student Development.,” J. Educ. Psychol., vol. 105, no. 3, pp. 805–820, 2013.
[12] D. K. Ellis, K. A. Ellis, L. J. Huemann, and E. A. Stolarik, “Improving Mathematics Skills Using Differentiated Instruction with Primary and High School Students,” Chicago, Illinois, 2007.
[13] G. Polya, How to Solve It: A New Aspect of Mathematical Method. Princeton, New Jersey: Princeton University Press, 1973.
[14] J. A. Van De Walle, K. S. Karp, and J. M. Bay-Williams, Elementary and Middle School Mathematics Teaching Developmentally, 7th Edition. Boston. MA: Pearson Education, 2010.
[15] S. An, G. Kulm, and Z. Wu, “The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S.,” J. Math. Teach. Educ., vol. 7, pp. 145–172, 2004.
[16] M. Walshaw and G. Anthony, “The Teacher’s Role in Classroom Discourse: A Review of Recent Research Into Mathematics Classrooms,” Rev. Educ. Res., vol. 78, no. 3, pp. 516–551, 2008.