Elastic Light Scattering by Semiconductor Quantum Dots

I.G.Lang, L.I.Korovin
A. F. Ioffe Physical-Technical Institute, Russian Academy of Sciences, 194021 St. Petersburg, Russia

S.T.Pavlov\dag\ddagger
\dag Facultad de Fisica de la UAZ, Apartado Postal C-580, 98060 Zacatecas, Zac., Mexico
\ddagger P. N. Lebedev Physical Institute, Russian Academy of Sciences, 119991 Moscow, Russia

Elastic light scattering by low-dimensional semiconductor objects is investigated theoretically. The differential cross section of resonant light scattering on excitons in quantum dots is calculated. The polarization and angular distribution of scattered light do not depend on the quantum-dot form, sizes and potential configuration if light wave lengths exceed considerably the quantum-dot size. In this case the magnitude of the total light scattering cross section does not depend on quantum-dot sizes. The resonant total light scattering cross section is about a square of light wave length if the exciton radiative broadening exceeds the nonradiative broadening. Radiative broadenings are calculated.

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I. INTRODUCTION

Measurement of elastic light scattering by size-quantized low-dimensional semiconductor objects (quantum wells, quantum wires and quantum dots) is a simple and convenient method to investigate excitonic excitations in these objects.

If exciton energy levels are discrete, light scattering becomes stronger resonantly at coincidence of the light frequency $\omega_l$ with the exciton frequency $\omega_0$. A resonant peak width is determined by the exciton damping $\Gamma$. The same concerns to the light absorption.

For the first time the role of so-called radiative damping $\gamma_r$ of excitons was discovered in light reflection of quantum wells. It was shown that the total damping $\Gamma$ consists of two parts: $\Gamma = \gamma_r + \gamma$, where $\gamma$ is the exciton nonradiative damping [1]. The same method was extended on light absorption by quantum wells [2] (see also review [3]). The light reflection by quantum wells, quantum wires and quantum dots was considered for the first time in [4].

There are two methods to investigate theoretically the light scattering by semiconductor objects. First of them we name as semiclassical method, since it is reduced to calculation of classic electric fields, whereas the description of electronic systems is quantum (it needs to be mentioned that a creation of an electron-hole pair is described by a non-diagonal matrix elements $p_{cv}$ of the momentum operator). The semiclassical method is described in [5]. It consists of calculation of averaged (on the ground state of a crystal) current and charge densities induced by a stimulating electric field and solution of the Maxwell equations inside and outside of the object with subsequent use of boundary conditions for electric and magnetic fields. Inelastic light scattering (for example, the Raman scattering) is caused by fluctuations of currents and charges. Let us emphasize that calculations of current and charge densities are made in view of exciton nonradiative broadenings $\gamma$ [5], that allows to calculate not only scattering, but the light absorption also (at $\gamma = 0$ the light absorption is absent). The quasi-classical method in calculations of reflection and absorption coefficients in quantum wells was used in [4,6-8]. In [9] the same method was applied to calculation of electric fields arising at resonant light scattering on excitons $\Gamma_0 \times \Gamma_7$ in spherical quantum dots, consisting of crystals of a class $T_d$ (for example, GaAs) and limited by an infinitely high potential barrier.

The second method is quantum one. The electric field is quantized and the quantum perturbation theory is used. We have checked up that both methods give identical results for dimensionless light reflection coefficients in quantum wells if the light-exciton interaction is taken into account in the lowest order. That is admitted at $\gamma_r \ll \gamma$.

Certainly, the quasi-classical method has a number of advantages before the quantum method. First, it allows to take into account precisely the light-electron interaction, i.e., all processes of light reradiation and reabsorption. The exact description is being achieved, if to substitute genuine electric fields in expressions for current and charge densities. Then we obtain automatically that the resonant contributions of excitons in dimensionless light reflection and absorption coefficients in quantum wells and in the light scattering cross sections in quantum wires and quantum dots contain factors $[(\omega_l - \omega_0)^2 + \Gamma^2/4]^{-1}$, where $\omega_0 = \omega_0 + \Delta \omega$ is the renormalized exciton frequency [6,8,9].

Secondly, the quasi-classical method allows to calculate precisely the light absorption by low-dimensional objects. For quantum wells this task is solved in [6-8]. In [9] $\gamma = 0$, and the light absorption by a quantum dot is absent.

Thirdly, the quasi-classical method allows to go from a monochromatic irradiation to a pulse irradiation and to obtain new interesting results [10-14].

Finally, the quasi-classical method allows to take into account a permittivity difference of objects and environment [6,9,12].
However, the quantum method has one decisive advantage. It is much easier especially for quantum wires and QDs, that is shown below.

II. THE QUANTUM THEORY

Let us calculate a probability of a stimulating photon absorption and a scattered photon creation. According to the perturbation theory

\[ W_i = \frac{2\pi}{\hbar} \sum_f |M_{fi}|^2 \delta(E_f - E_i), \]  

(1)

where \( E_i(E_f) \) is the initial (final) state energy,

\[ M_{fi} = \sum_m \frac{\langle f | V | m \rangle \langle m | V | i \rangle}{E_i - E_m + i\delta\hbar} \]  

(2)

is the compound matrix element, \( E_m \) is the intermediate state energy, \( \delta \to +0 \).

The charge - electric field interaction is as follows

\[ V = -\int d^3r \mathbf{d}(r) \mathbf{E}(r), \]  

(3)

where the polarization density

\[ \mathbf{d}(r) = \sum_i r_i \rho_i(r), \quad \rho_i(r) = e\delta(r - r_i) \]  

(4)

is introduced. We use the field \( \mathbf{E}(r) \) in the secondary quantization representation [15, p. 579]

\[ \mathbf{E}(r) = \frac{i}{\nu} \sqrt{\frac{2\pi\hbar}{V_0}} \times \sum_{k,\mu} \sqrt{\omega_k} (c_{\mu k} e_{\mu k} e^{i\mathbf{k}\cdot\mathbf{r}} - c_{\mu k}^* e^{i\mathbf{k}\cdot\mathbf{r}}), \]  

(5)

where \( V_0 \) is the normalization volume, \( \omega = ck/\nu \) is the frequency, \( \mathbf{k} \) is the light wave vector, \( \nu = \sqrt{\varepsilon} \) is the light refraction coefficient, which is identical inside and outside of the semiconductor volume, \( e_{\mu k} \) is the polarization vector, \( \mu \) is the polarization index, \( c_{\mu k}^*(c_{\mu k}) \) is the photon creation (annihilation) operator. In (5), the approximation \( u_{\mu k} = c/\nu \), where \( u_{\mu k} \) is the light group velocity. The field (5) is normalized so that the energy in the volume \( V_0 \) equals \( \sum_{k,\mu} \hbar \omega_k (N_{\mu k} + 1/2) \).

In the effective mass approximation [5],

\[ \mathbf{d}(r) = \mathbf{d}^{nd}(r) \]  

\[ = \sum_\eta [\mathbf{d}_{\text{exc}} F_\eta^*(\mathbf{r}) a_\eta^* + \mathbf{d}_{\text{exc}}^* F_\eta(\mathbf{r}) a_\eta], \]  

(6)

where superscript \( nd \) means a non-diagonal part of the operator (having only non-diagonal matrix elements), \( a_\eta^*(a_\eta) \) is the exciton creation (annihilation) operator with an index \( \eta \), \( F_\eta(\mathbf{r}) \) is the exciton envelope wave function at \( \mathbf{r}_e = \mathbf{r}_h = \mathbf{r}, \mathbf{r}_h(\mathbf{r}_h) \) is the electron (hole) radius -

vector, \( \mathbf{d}_{\text{exc}} = -(ie/m_0\omega_g)\mathbf{p}_{\text{exc}}, m_0 \) is the free electron mass, \( \hbar\omega_g \) is the energy gap, \( \mathbf{p}_{\text{exc}} \) is the momentum interband matrix element. Using (5) and (6) in (3), we obtain

\[ V_1 = -\frac{e}{m_0\omega_g \nu} \left( \frac{2\pi\hbar}{V_0} \right)^{1/2} \sum_\eta \sum_{k,\mu} a_\eta^* c_{\mu k} \omega_k^{1/2} \times (\mathbf{p}_{\text{exc}} e_{\mu k}) P_\eta^*(\mathbf{k}), \]  

(7)

\[ V_2 = -\frac{e}{m_0\omega_g \nu} \left( \frac{2\pi\hbar}{V_0} \right)^{1/2} \sum_\eta \sum_{k,\mu} a_\eta^* c_{\mu k}^{*\prime} \omega_k^{1/2} \times (\mathbf{p}_{\text{exc}} e_{\mu k})^* P_\eta(-\mathbf{k}), \]  

\[ P_\eta(k) = \int d^3r e^{-i\mathbf{k}\cdot\mathbf{r}} F_\eta(\mathbf{r}). \]  

(8)

The compound matrix element (2) consists of two parts

\[ M_{fi} = M^{(1)}_{fi} + M^{(2)}_{fi}, \]  

(9)

where

\[ M^{(1)}_{fi} = \sum_m \frac{\langle f | V_{1(2)}^* | m \rangle \langle m | V_{1(2)}^* | i \rangle}{E_i - E_m + i\delta\hbar}. \]  

In the initial state \( |i\rangle \), there are the semiconductor ground state and \( N_l \) photons with a wave vector \( \mathbf{k}_l \) and polarization \( e_l \), \( N_l \gg 1 \). In the final state \( |f\rangle \), there are \( N_f - 1 \) photons of exciting light and one photon of scattered light with a wave vector \( \mathbf{k}_s \) and polarization \( e_s \).

In an intermediate state (process 1), there are \( N_l - 1 \) photons of exciting light and an exciton \( \eta \); in the intermediate state (process 2), there are \( N_f \) photons of exciting light, one photon of scattered light and an exciton \( \eta \).

For probability \( W_l \) we obtain the result

\[ W_l = \frac{(2\pi)^3}{V_0^2\hbar^2} \left( \frac{e^2}{m_0^2\omega_g^2} \frac{N_l \omega_g}{\nu^4} \right)^2 \sum_{k,\mu} \left| \tilde{A}_\eta \right|^2 \delta(\omega_l - \omega_s), \]  

(10)

where the designation

\[ \tilde{A}_\eta = \frac{(\mathbf{p}_{\text{exc}} e_l)(\mathbf{p}_{\text{exc}} e_s)^* P_\eta(\mathbf{k}_l) P_\eta^*(\mathbf{k}_s)}{\omega_l - \omega_s + i\delta} - \frac{(\mathbf{p}_{\text{exc}} e_l)(\mathbf{p}_{\text{exc}} e_s)^* P_\eta(-\mathbf{k}_l) P_\eta^*(-\mathbf{k}_s)}{\omega_l + \omega_s + i\delta} \]  

(11)

is introduced. The summation in the RHS of (10) is carried out on wave vectors \( \mathbf{k}_s \) and polarizations \( \mu \) of scattered light.

Let us consider the resonant scattering, when energies \( \hbar\omega_l \) and \( \hbar\omega_s \) exceed slightly the energy gap. Then the "non-resonant" second term in the RHS of (11) must be omitted.
Substituting the summation $\mathbf{k}_s$ by the integration and integration on the module $k_s$ by the integration on frequency $\omega_s = ck_s/\nu$, we obtain

$$W_i = \left( \frac{e^2}{m_0^2 \omega^2} \right)^2 \frac{N_l \omega^4}{V_0 \hbar^2 c^3 \nu} \times \sum_{\mu} \int d\omega_s \left| \sum_{\eta} A_{\eta} \right|^2,$$

where $A_{\eta}$ is the resonant term from (11). The expression (12) is universal one: It is applicable to any low-dimensional semiconductor objects, including the presence of quantizing magnetic field.

### III. LIGHT SCATTERING ON QUANTUM DOTS.

A quantum dot may be of any form (sphere, cube, disk) and be limited by any energy potential (parabolic, rectangular) of any height. All the features of quantum-dot structure influence only on $P_\eta (k)$ for an exciton $\eta$. The angular distribution of scattered light depends also on the structure of vectors $\mathbf{p}_{cv\eta}$, which, generally speaking, are complex. For cubic crystals (class $T_d$) these vectors are different for excitons, containing heavy or light holes from the valence band, chipped off by the spin-orbital interaction [16,17]. Under an exciton we consider any state of an electron-hole pair with a discrete energy level.

In a case of a quantum dot, it is naturally to introduce the light scattering cross section. According to Eq. (12), the scattered energy flux in an solid angle interval $d\omega_s$ in time unit equals

$$\hbar \omega_i dW_i = \left( \frac{e^2}{m_0^2 \omega^2} \right)^2 \frac{N_l \omega^4}{V_0 \hbar^2 c^3 \nu} \times \sum_{\mu} \left| \sum_{\eta} A_{\eta} \right|^2 d\omega_s.$$  

The energy flux of stimulating light on the unit area in time unit equals

$$S_i = \frac{N_l \hbar \omega_i c}{V_0 \nu}.$$  

Eq. (13) divided by (14) is the light scattering differential cross section

$$d\sigma = \left( \frac{e^2}{\hbar c} \right)^2 \frac{\omega^4}{c^2 \omega^2 m_0^2} \sum_{\mu} \left| \sum_{\eta} A_{\eta} \right|^2 d\omega_s.$$  

Eq. (15) describes an angular dependence and (without sum on $\mu$) polarization of scattered radiation. If $\omega_i$ is close to $\omega_\eta$ of some excitonic state, the resonant amplification of scattered light is observed.

If the excitonic state (see section VI below) is degenerated (i.e., the same energy $\omega_\eta = \omega_\nu$ corresponds to some set of indexes $\eta$, and the function $P_\eta (k) = P(k)$ and only vectors $\mathbf{p}_{cv\eta}$ depend on $\eta$) the contribution of this state into the light scattering section equals

$$\frac{d\sigma_0}{d\omega_s} = \sum_{\mu} \frac{d\sigma_\mu}{d\omega_s},$$  

$$\frac{d\sigma_\mu}{d\omega_s} = \left( \frac{e^2}{\hbar c} \right)^2 \omega^4 \left| \frac{\Im (e_i, e_s)}{\omega_\nu} \right|^2 \times \frac{|P(k)|^2 |P(k_s)|^2}{(\omega_\nu - \omega_\eta)^2 + \delta^2},$$  

where

$$\Im (e_i, e_s) = \sum_{\eta} (\mathbf{p}_{cv\eta} e_i)(\mathbf{p}_{cv\eta} e_s)^*.$$  

Let us designate by $R$ the greatest quantum-dot size and consider a case, when a light wave length is much greater than $R$, i.e., $kR \ll 1$. Then the value $P(k)$ is $P(0) = \int d^3 r F_\eta (r)$ does not depend on a wave vector $k$, and the resonant contribution to the cross section is described by the formula (17), in which it is necessary to make the replacement $|P(k)|^2 |P(k_s)|^2 \approx |P(0)|^4$.

It is possible to make the following conclusions. At $kR \ll 1$ the polarization and angular distribution of scattered light are determined only by the factor $|\Im (e_i, e_s)|^2$, containing the vector $\mathbf{p}_{cv\eta}$, i.e., it does not depend on the quantum-dot form and on the exciton wave function. The section magnitude does not depend on the quantum-dot sizes. Certainly, the energy level $E_\eta$ position depends on the quantum-dot form and sizes.

### IV. THE RADIATIVE BROADENING OF THE EXCITON LEVEL

It is well known (see, for example, [1-8]) that the exact account of the electron-electromagnetic field interaction and account of the exciton nonradiative broadening $\gamma_\eta$ results in the replacement of the factor $(\omega_i - \omega_\eta + i \delta)^{-1}$ by $[\omega_i - \omega_\eta + i (\gamma_\eta + \gamma_n)/2]^{-1}$ in (11), where $\gamma_n$ is the radiative broadening, $\hbar \omega_\eta$ is the renormalized exciton energy, $\omega_\eta = \omega_\eta + \Delta \omega_\eta$.

The calculation of radiative broadening is made according to Eq. (1). The matrix element $M_{fi} = \langle f | V | i \rangle$ corresponds to a direct transition from an initial state (in which there is the exciton $\eta$, but the photons are absent) in final state, containing the ground state of a crystal and a photon with a wave vector $k$ and polarization $\mu$. Using Eqs. (3) - (6), we obtain

$$\gamma_\eta = \frac{4 \pi^2 \epsilon^2}{\hbar} \frac{m_0^2 \omega_\eta^2 r^2 V_0}{\omega_\eta} \sum_{k, \mu} \omega_k |\mathbf{p}_{cv\eta} \mathbf{e}_{\mu k}|^2 \times |P_\eta (k)|^2 \delta (\omega_\eta - \omega_k).$$
Having replaced summation on \( k \) by integration, we obtain the result
\[
\gamma_{\eta} = \frac{e^2 \omega_n^3 \nu}{2 \pi \hbar m_0^2 \omega_d^2 c^3} \times \sum_{\mu} \int dk_{\eta} |p_{cv} e_{\mu k_{\eta}}|^2 |P_{\eta}(k_{\eta})|^2,
\]
(20)
where \( k_{\eta} \) is the vector with module \( k_{\eta} = \omega_n \nu / c \). The formula (20) is applicable to any quantum wells, quantum wires and quantum dots at any magnitudes of the parameter \( k_{\eta} d \), where \( d \) is the characteristic size.

Under condition \( k_{\eta} R \ll 1 \), we obtain for a quantum dot
\[
\gamma_{\eta} = \frac{e^2 \omega_n^3 \nu |P_{\eta}(0)|^2}{2 \pi \hbar m_0^2 \omega_d^2 c^3} \sum_{\mu} \int dk_{\eta} |p_{cv} e_{\mu k_{\eta}}|^2,
\]
whence it follows that the radiative broadening does not depend on quantum-dot sizes.

\section{V. Estimation of the Magnitude of the Light Scattering Section in the Resonance}

Taking into account the exciton energy corrections and both radiative and nonradiative broadenings, with the help of (16) and (17) we obtain the formula for the total light scattering cross section for any quantum dot near the resonance \( \omega_l = \tilde{\omega}_0 \):
\[
\sigma_0 = \left( \frac{e^2}{\hbar c} \right)^2 \frac{\omega_d^4}{\omega_n^2 m_0^2 c^2} \times \frac{|P(k_l)|^2}{(\omega_l - \omega_0)^2 + (\gamma_r + \gamma)^2 / 4} \times \int d\omega \sum_{\mu} |\Xi_{\mu}(e_l, e_s)|^2.
\]
(22)
For an estimation of the magnitude of the radiative broadening, we use Eq. (20). Let us assume that \( \gamma \ll \gamma_r \). Then, we obtain
\[
\sigma_0(\omega_l = \tilde{\omega}_0) = \frac{e^2}{\omega_l \nu^2 \omega_d^2 c^2} = \frac{x}{k_l^2},
\]
(23)
where \( x \) is a number about unit.

It follows from (23) that the elastic light scattering cross section in the resonance is about a square of wave length of the exciting light in the case, when the radiative broadening exceeds nonradiative one. It can be carried out in perfect quantum dots. This result is true at \( k_l R \ll 1 \), and at \( k_l R \gg 1 \) for any quantum dot, in which exciton energy levels exist. Otherwise at \( \gamma \gg \gamma_r \) the magnitude of the cross section in resonance decreases in \( (\gamma / \gamma_r)^2 \) times.

\section{VI. Excitons \( \Gamma_6 \times \Gamma_7 \) in Cubic Crystals of the Class \( T_d \)}

As an example, we consider an exciton formed by an electron from the twice degenerated conduction band \( \Gamma_6 \) and by a hole from the twice degenerated valence band \( \Gamma_7 \), chipped off by the spin-orbital interaction. Such exciton is considered in [9].

In designations of [17], electron wave functions have the structure
\[
\Psi_{e1} = i S \uparrow, \quad \Psi_{e1} = i S \downarrow,
\]
(24)
and hole wave functions are
\[
\Psi_{h1} = \frac{1}{\sqrt{3}} (X - i Y) \uparrow - \frac{1}{\sqrt{3}} Z \downarrow,
\]
\[
\Psi_{h2} = \frac{1}{\sqrt{3}} (X + i Y) \downarrow + \frac{1}{\sqrt{3}} Z \uparrow.
\]
(25)
Combining functions (24) and (25) in pairs, we obtain the four times degenerated excitonic states, for which the vector \( p_{cv} \) equals
\[
p_{cv1} = \frac{p_{cv}}{\sqrt{3}} (e_x - i e_y),
\]
\[
p_{cv2} = \frac{p_{cv}}{\sqrt{3}} (e_x + i e_y),
\]
\[
p_{cv3} = \frac{p_{cv}}{\sqrt{3}} e_z,
\]
\[
p_{cv4} = \frac{p_{cv}}{\sqrt{3}} e_z,
\]
(26)
where we have introduced a scalar \( p_{cv} = i \langle S | \hat{p}_x | X \rangle \). \( e_x, e_y, e_z \) are unite vectors along crystallographic axes.

Let us consider a circular polarization of exciting and scattered light
\[
e^{\pm}_{l} = \frac{1}{\sqrt{2}} (e_{xl} \pm i e_{yl}),
\]
\[
e^{\pm}_{s} = \frac{1}{\sqrt{2}} (e_{xs} \pm i e_{ys}),
\]
(27)
where unite vectors \( e_{xl} \) and \( e_{yl} \) are perpendicular to the axis \( z_l \) directed along the vector \( k_l \). Unite vectors \( e_{xs} \) and \( e_{ys} \) are perpendicular to the axis \( z_s \) directed along the vector \( k_s \).

Direction of \( k_l \) concerning crystallographic axes is arbitrarily. It is described by angles \( \theta_l, \varphi_l \), where \( \theta_l \) is the angle between chosen crystallographic axis \( z \) and \( k_l \). The direction of \( k_s \) is described by angles \( \theta_s, \varphi_s \). Direct calculation of \( \Xi(e_l, e_s) \) (18) results in
\[
\Xi(e^+_l, e^+_s) = \Xi^*(e^-_l, e^-_s)
\]
\[
= \frac{p_{cv}^2}{3} \{ (1 + \cos \theta_l \cos \theta_s) \cos(\varphi_s - \varphi_l) + \sin \theta_l \sin \theta_s + i (\cos \theta_l - \cos \theta_s) \sin(\varphi_s - \varphi_l) \},
\]
(28)
\[ \Xi(e_{i+}^+, e_{i-}^-) = \Xi^*(e_{i-}^-, e_{i+}^+) = \]
\[ = \frac{p_i^2}{3} ((1 - \cos \vartheta_i \cos \vartheta_s) \cos(\varphi_s - \varphi_i) \]
\[ - \sin \vartheta_i \sin \vartheta_s \]
\[ + i(\cos \vartheta_i - \cos \vartheta_s) \sin(\varphi_s - \varphi_i)). \]

Squaring (28) and (29) on the module, we obtain
\[ |\Xi(e_{i+}^+, e_{i+}^-)|^2 = |\Xi(e_{i-}^-, e_{i-}^-)|^2 = \frac{p_i^4}{9} (1 + \cos \theta)^2, \]
\[ \frac{\sigma^{++}}{\sigma_0} = \frac{\sigma^{--}}{\sigma_0} = \frac{\sigma^{+-}}{\sigma_0} = \frac{\sigma^{-+}}{\sigma_0} = \frac{\sigma_0 (1 + \cos \theta)^2}{9}, \]
\[ \sigma^{+}\sigma^{-}\]

where \( \theta \) is the angle between \( k_i \) and \( k_s \).

Let us notice that at a direction of light along the crystallographic axis \( z \), i.e., \( k_i \) along \( z \), at polarization \( e_i^+ \)
only the exciton with \( \eta = 1 \) and \( p_{c_{ei}} = (p_{c_{ei}}/\sqrt{3})(e_x - ie_y) \) is excited, and at polarization \( e_i^- \)
only exciton with \( \eta = 2 \) and \( p_{c_{ei}} = (p_{c_{ei}}/\sqrt{3})(e_x + ie_y) \) is excited. However, at any direction of light concerning crystallographic axes, all four excitons are excited.

Substituting (30) and (31) in (17), we obtain the differential light scattering cross sections
\[ \frac{d\sigma^{++}}{d\sigma_s} = \frac{d\sigma^{--}}{d\sigma_s} = \frac{d\sigma^{+-}}{d\sigma_s} = \frac{d\sigma^{-+}}{d\sigma_s} = \frac{\sigma_0 (1 + \cos \theta)^2}{9}, \]

where superscript \( + \) \( + \) means polarization of exciting (scattered) light \( e_i^+(e_i^-) \) etc.,
\[ \sigma_0 = \left( \frac{e^2}{4\pi \hbar c} \right)^2 \frac{\omega_0^4 p_{cv}^4 |P(k_i)|^2 |P(k_s)|^2}{\omega_0^4 4 \epsilon_0^2 ((\omega_1 - \omega_0)^2 + \delta^2)} \quad (34) \]

Summarizing on polarizations of scattered light, we obtain
\[ \frac{d\sigma^+}{d\sigma_s} = \frac{d\sigma^-}{d\sigma_s} = \frac{2}{9} \frac{\sigma_0 (1 + \cos \theta)^2}{9}, \]

where superscript \( + \) \( - \) means polarization of exciting light \( e_i^+(e_i^-) \).

Finally, the total light scattering cross section \( \sigma^+ = \sigma^- \) turns out as a result of integration on angles determining a direction of \( k_s \). The factor \( |P(k_i)|^2 \) can cause dependence of the cross section on a direction of exciting light, for example, if a quantum dot is a disk.

If the magnitude \( P(k_i) \) depends only on the module \( k \)
\[ P(k_i) = P(k_s) = P(k_1 R), \]

the magnitude of \( \Sigma_0 \) does not depend on a direction of \( k_i \) and \( k_s \). But in this case (as it follows from (34) and (35))

the light scattering is not isotropic. Under condition (36), integration on angles, determining a direction of \( k_s \), is carried out easily, and we obtain the result for the total section
\[ \sigma^+ = \sigma^- = \frac{2\pi}{\gamma (2\pi)} \left( \frac{e^2}{4\pi \hbar c} \right)^2 \frac{\omega_0^4 p_{cv}^4}{\omega_0^4 4 \epsilon_0^2 ((\omega_1 - \omega_0)^2 + \delta^2)}, \]

For example, using the envelope wave function
\[ F(r) = \frac{1}{2\pi R} \sin^2(\pi r/R) \theta(R - r), \]

we obtain
\[ \sigma^+ = \sigma^- = \frac{2}{\gamma (2\pi)} \int_0^\pi dx \sin^2(\pi r/R) \theta(R - r), \]

Under condition \( k_1 R \ll 1 \) in Eqs. (34) and (37), we believe \( P(k) \simeq P(0) \) and obtain results applied in case of small quantum dots of any form, sizes and potential configurations for light scattering in cubic crystals of a class \( T_d \), having holes from the chipped off valence band. In cases of heavy or light holes in the exciton structure, we obtain other results caused by other structure of vectors \( p_{cv} \).

At \( k_1 R \geq 1 \) it may be essentially to take into account a difference of permittivities inside and outside of the quantum dot.

Let us calculate the radiative broadening for excitons with vectors \( p_{cv} \) from (26). The factors \( S_{\eta \eta} = |p_{cv} e_{\mu}\rangle |p_{cv} e_{\eta}\rangle |^2 \) enter in Eq. (21). Using (26), we obtain the following results
\[ S_{1+} = S_{2-} = \frac{p_{cv}^2}{6} (1 + \cos \vartheta)^2, \]
\[ S_{1-} = S_{2+} = \frac{p_{cv}^2}{6} (1 - \cos \vartheta)^2, \]
\[ S_{3+} = S_{3-} = S_{4+} = S_{4-} = \frac{p_{cv}^2}{6} \sin^2 \vartheta, \]

where subscripts 1 - 4 correspond to excitonic states (26). Subscripts + and - describe the circular polarization \( e_{\mu} = (e_{\nu} \pm i e_{\nu'})/\sqrt{2} \), if the vector \( k \) is directed along \( z' \), \( \vartheta \) is the angle between the crystallographic axis \( z \) and \( k \). Summarizing \( S_{\eta \mu} \) on polarizations, we obtain
\[ S_1 = S_2 = \frac{p_{cv}^2}{3} (1 + \cos \vartheta)^2, \]
\[ S_3 = S_4 = \frac{p_{cv}^2}{3} \sin^2 \vartheta, \]

where \( S_\eta = \sum_{\mu} |p_{cv} e_{\mu}\rangle |p_{cv} e_{\eta}\rangle |^2 \). Substituting (41) in (21), we obtain
\[ \gamma_1 = \gamma_2 = \frac{e^2 \omega_0^4 p_{cv}^4}{6 \pi \hbar n_0^2 \omega_0^2 c^2} \]
\[ \times \int d\mathbf{k}(1 + \cos^2 \vartheta)|P(\mathbf{k})|^2, \]
\[ \gamma_3 = \gamma_4 = \gamma_1/2, \quad k = \omega_0 \nu/c. \]  

(42)

In a case \( (36) \), integrating on angles determining a direction of \( \mathbf{k} \), we have
\[ \gamma_1 = \gamma_2 = \frac{8 e^2 \omega_0^2 \nu^2 \nu}{9 \hbar m_0^2 \omega^4 c^4}|P(kR)|^2, \]
\[ \gamma_3 = \gamma_4 = \gamma_1/2. \]  

(44)

In a case \( kR \ll 1 \), for radiative broadenings of any excitons \( \Gamma_0 \times \Gamma_7 \), we obtain Eq. (43), in which the factor \( |P(kR)|^2 \) is replaced by \( |P(0)|^2 \). At \( P(0) = 1 \), (43) coincides with Eq. (6) of [9], if in the last to put \( \varepsilon_1 = \varepsilon_2 = \nu^2 \).

**VII. RESULTS**

With the help of the quantum perturbation theory in the lowest order on the light-electrons interaction, the probability of the elastic light scattering on low-dimensional size-quantized semiconductor heterostructures (Eq. (12)) is calculated. For a quantum dot, the obtained expression allows to calculate dimensionless light reflectivity coefficient, in a case quantum wires the light scattering section on a length unit. For a quantum dot the dimension of cross section is \( cm^2 \).

The differential cross section of resonant light scattering on any exciton in a quantum dot of any form, potential configuration and sizes (17) is obtained.

Under condition \( kR \ll 1 \), the polarization and angular distribution of scattered light do not depend on the quantum dot form and on the exciton envelope wave function, but depend only on vectors \( \mathbf{p}_{cv} \), which are non-diagonal matrix elements of the momentum operator of excitonic states. The magnitude of the scattering cross section does not depend on quantum-dot sizes.

With the help of the quantum perturbation theory, the formula (20) for the radiative broadening of the exciton \( \eta \) is obtained. It is applicable to any exciton in any quantum well, quantum wire and quantum dot at any magnitudes of \( k_\eta R \), where \( R \) is the quantum-well width, quantum-wire diameter or quantum-dot size, \( k_\eta = \omega_\eta \nu/c, \hbar \omega_\eta \) is the exciton energy, \( \nu \) is the light refraction factor, which is considered identical inside and outside of the object. For a quantum dot at \( k_\eta R \ll 1 \), we find that the radiative broadening does not depend on the quantum-dot sizes (21).

The estimation of the magnitude of the total light scattering cross section in the resonance \( \omega_l = \omega_0 \) shows that at \( \gamma \ll \gamma_r \), the section is about \( \left( \lambda_1/2\pi \right)^2 x \), where \( x \) is some number.

The example of the exciton \( \Gamma_0 \times \Gamma_7 \) in cubic crystals \( T_d \) is considered.

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