Anharmonic mixing in a magnetic trap

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Abstract. We have experimentally observed re-equilibration of a magnetically trapped cloud of metastable neon atoms after it was put in a non-equilibrium state. Using numerical simulations we show that anharmonic mixing, equilibration due to the collisionless dynamics of atoms in a magnetic trap, is the dominant process in this equilibration. We determine the dependence of its time on trap parameters and atom temperature. Furthermore we observe in the simulations a resonant energy exchange between the radial and axial trap dimensions at a ratio of trap frequencies $\omega_r/\omega_z = 3/2$. This resonance is explained by a simple oscillator model.

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1. Introduction

Magnetic traps have become a standard and inexpensive tool in cold atom physics in recent years [1]. Together with the application of cooling techniques such as evaporative cooling and sympathetic cooling, they have enabled the formation of Bose-Einstein condensates [2] and, more recently, degenerate Fermi gases [3]. The most commonly used species for cold atom experiments (Rb, Cs, Na) are easily cooled to the micro-Kelvin regime, where the atomic dynamics in a magnetic trap is usually well described by regular harmonic motion. For other species such as the noble gases [4, 5] and the group II elements [6, 7] however, the lowest attainable temperatures can be in the milli-Kelvin regime. In this regime the atomic dynamics is more complicated, with coupling between the different dimensions of the trap becoming important.

Anharmonic mixing is a process that couples the motion of atoms in different dimensions of a magnetic trap. This coupling enables redistribution of energy over the coupled dimensions. Therefore, anharmonic mixing has to be considered when applying techniques that rely on the motion of atoms in a trap, such as evaporative cooling [8], Doppler cooling in a magnetic trap [9] and rethermalization experiments aimed at measuring a scattering length [10]. In order to investigate on what timescale
anharmonic mixing plays a role, we perform a Monte Carlo simulation of a cloud of atoms in a Ioffe-Quadrupole magnetic trap. We determine the dependence of this timescale on trap parameters and on atom temperature. Surkov et. al. [11] have investigated anharmonic mixing in the limit of low temperature where it can be treated as a perturbation of regular harmonic motion, but to the best of our knowledge this is the first paper discussing anharmonic mixing at higher temperatures. We will also present here an experiment showing that anharmonic mixing does play an important role in the atomic dynamics, and can even be the dominant mechanism of energy redistribution under certain conditions. To conclude we will look at possible applications of anharmonic mixing.

2. Monte Carlo calculations

A Ioffe-Quadrupole magnetic trap (MT) is one of the most common magnetic traps. Its potential is given by

\[ U(x, y, z) = \mu [B_0^2 + (\alpha^2 - B_0 \beta)(x^2 + y^2) + 2B_0 \beta z^2 \]

\[ + \frac{1}{4} \beta^2 (x^2 + y^2)^2 + \beta^2 z^4 + 2\alpha \beta (x^2 - y^2) z]^{1/2} - \mu B_0, \]

(1)

where \( x \) and \( y \) are the radial trap dimensions, \( z \) is the axial trap dimension, \( \mu \) is the magnetic moment of the atom, \( \alpha \) the gradient of the magnetic field, \( \beta \) the curvature of the magnetic field and \( B_0 \) the magnetic bias field. In this expression for the trap potential terms of order higher than four have been neglected. The MT has trap frequencies \( \omega_r \) and \( \omega_z \) in the limit \( 3k_BT \ll \mu B_0 \), where the trap shape is harmonic:

\[ U(r, z) = \frac{m}{2} (\omega_r^2 r^2 + \omega_z^2 z^2). \]

(2)

Here \( m \) is the atomic mass, \( r = (x^2 + y^2)^{1/2} \) the radial coordinate, \( \omega_r = \left[ \frac{\mu}{m} \left( \alpha^2 / B_0 - \beta \right) \right]^{1/2} \) the radial trap frequency, and \( \omega_z = \left( \mu \beta / m \right)^{1/2} \) the axial trap frequency. The higher order terms in (1) are in this limit negligible compared to the harmonic terms, the term \( 2\alpha \beta (x^2 - y^2) z \) that couples the motion in the axial and radial directions is therefore absent from (2).

The starting point of a simulation is an atom cloud in equilibrium in a harmonic trap. The initial positions and velocities of the atoms are determined by a Monte Carlo method. For the properties of the atom we use the values for metastable neon in the \(^3P_2\) state, as this is the atom we use in our experiment. Two kinds of clouds have been used, namely thermal clouds and clouds in which all the atoms have the same energy. The former is used to compare the results of the simulations with experiments while the latter is used to gain a better understanding of the dynamics of atoms at a certain energy. During the simulation the position and velocity of each atom is determined as a function of time by integrating the equations of motion. At the start of the simulation the magnetic bias field \( B_0 \) is adiabatically ramped down, thereby increasing the energy of the atoms in the radial direction and changing the effective shape of the potential from harmonic to almost linear. The ramping is adiabatic if the condition \( \frac{1}{2} \frac{d\omega_r}{dt} < 1 \)
Figure 1. Monte Carlo calculation of the radial temperature as a function of time for a thermal cloud with an initial temperature of 1 mK and $10^4$ atoms (gray). The exponential fit is also shown (black); the temperature during the ramping of the bias field ($t = 0 – 50$ ms) is left out.

is fulfilled [11]. The simulation yields the kinetic and potential energies of the cloud in every direction as a function of time. From the kinetic energies in different directions the transfer of energy from one direction to the other can be determined, and a comparison with the experiment can be made.

We choose the trap parameters at the start of the simulation as $\alpha = 1 \cdot 10^4$ G/m, $\beta = 47.5 \cdot 10^4$ G/m$^2$ and $B_0 = 99.6$ G. The bias field is then ramped down to 1.5 G. These parameters are chosen because they are easily accessible in our experiment. A typical calculated result of the temperature evolution, or the average kinetic energy, in the radial direction of a thermal cloud of 1 mK after ramping the bias field is shown in Figure 1. The first few tens of milliseconds show a linear decrease of temperature, after that the decrease is approximately exponential with a mixing time of 112 ms for this particular cloud with a temperature of 1 mK and a relative change in temperature of 8%. The linear decrease of temperature in the first tens of ms is caused by high-energy atoms that mix on a timescale of $\frac{1}{2}\omega_z^{-1} = 11$ ms. This is the average time an atom needs to reach a singular point in the potential [11], where almost instantaneous mixing can occur.

The dependence of the behavior of a thermal cloud on initial trap parameters and cloud temperature is relatively weak. When $\alpha$ is varied over the range $0.9 – 1.2 \cdot 10^4$ G/m and $B_{0, end}$ over the range $1.5 – 20$ G, the mixing time stays within the range $100 – 130$ ms. Only when $\beta$ is decreased, a significant change in mixing time occurs: $\tau_{mix} \approx 400$ ms when $\beta = 37.5 \cdot 10^4$ G/m$^2$. The reason for this weak dependence on trap parameters is that a significant part of the energy of the cloud is carried by atoms that are in a regime where mixing does not occur or where it occurs on a timescale of $\frac{1}{2}\omega_z^{-1}$.

To show that indeed the timescale of energy transfer between dimensions of an atom depends strongly on the energy of that atom, we now take clouds of atoms that all have the same energy $E_{atom}$ in order to determine the mixing timescale as a function
of energy. This gives us the average behavior of an atom with a certain energy. The result is shown in Figure 2. Below $E_{\text{atom}}/3k_B = 375 \mu K$ no mixing occurs on the longest timescale we considered, i.e. the lifetime of the metastable state $\text{Ne}^*({^3}P_2)$ of 14.7 s [12]. For temperatures above 1.5 mK the mixing time is determined by $\frac{1}{2}\omega_z^{-1}$ and becomes independent of energy.

Next, we investigate the dependence of $\tau_{\text{mix}}$ on the trap parameters $\alpha$ and $\beta$ while keeping the energy constant at $E_{\text{atom}}/3k_B = 700 \mu K$. Figure 3 shows that $\tau_{\text{mix}}$ increases with $\alpha$ and decreases with $\beta$. This behavior can be understood qualitatively by comparing the strength of the harmonic ($\sim \alpha^2 - \beta$) and coupling ($\sim \alpha \beta$) terms in (1).

Furthermore we determined the mixing time as a function of the value of the bias field at the end of the ramp. When the final value of the bias field is higher, less energy is added to the atoms and the final temperature of the cloud is lower. Therefore we expect that the mixing time becomes larger for higher final values of the bias field. As can be seen from the result in Figure 4 however, there is a broad resonance (meaning small mixing time) in the mixing time around 14 G. This occurs exactly at a ratio of radial and axial trap frequencies of $\omega_r/\omega_z = 3/2$. The resonance is so broad because the radial trap frequency changes only very slowly with bias field. The reason for this slow change is that at higher bias field the final energy of the cloud is lower, resulting in a higher oscillation frequency in the MT because the trap is not harmonic, but at the same time the trap is less tightly confining, resulting in a lower oscillation frequency. These two effects almost completely cancel.

3. Resonance model

In order to explain this resonance the dynamics of an atom in the MT is now described as two coupled one-dimensional harmonic oscillators, whose energies represent the radial
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Figure 3. a) Calculated mixing time as a function of magnetic gradient in radial direction $\alpha$. b) Mixing time as a function of magnetic curvature in axial direction $\beta$. Atomic energy is $E_{\text{atom}}/3k_B = 700$ $\mu$K.

Figure 4. Calculated mixing time as a function of bias field after ramping. Atomic energy is $E_{\text{atom}}/3k_B = 700$ $\mu$K.

and axial energies of the atom. The coupling term is chosen to be $W = x^2z$, as shown in Figure 5. Now we examine the forces $F_z$ and $F_x$ that the two oscillators exert on one another, assuming they oscillate as $x \sim \cos(\omega_x t)$ and $z \sim \cos(\omega_z t)$:

\[
F_z = -\frac{\partial}{\partial z} W = x^2 \sim \cos^2(\omega_x t) \sim \cos(2\omega_x t) + 1,
\]

\[
F_x = -\frac{\partial}{\partial x} W = xz \sim \cos(\omega_x t) \cos(\omega_z t) \sim \cos((\omega_x + \omega_z)t) + \cos((\omega_x - \omega_z)t). \tag{3}
\]

A resonance occurs if an oscillator is driven by a periodic force with its
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$\text{Figure 5. Schematic picture of the coupled oscillator model. Two one-dimensional oscillators with frequencies } \omega_x \text{ and } \omega_z \text{ can exchange energy through a coupling.}$

eigenfrequency, $F_i \sim \cos(\omega_i t)$. From (3) it can be seen that this happens if $\omega_z = 2 \omega_x$. In general, a coupling term $x^a z^b$ yields a resonance at $a \omega_x = b \omega_z$. When potential (1) is linearized a term $x^2 z^3$ appears, explaining the resonance in Figure 4. A more formal treatment explaining the resonance is possible by transforming the Hamiltonian of an atom in the potential to Birkhoff-Gustavson normal form [13, 14], but that is beyond the scope of this paper.

4. Experiment

To check our simulation results, we now compare with experiment. The only difference between simulation and experiment is that in the experiment the atoms can also collide, which we did not include in the simulation. Therefore it is necessary to consider both anharmonic mixing and collisional equilibration of energy. Our setup has been described previously in [3, 15]. Briefly, a discharge source is used to create metastable $^3P_2$ neon atoms, and after passing through several laser-cooling sections the atomic beam flux is $6 \cdot 10^{10}$ s$^{-1}$ [3]. From this atomic beam about $10^9$ atoms are loaded into a magneto-optical trap (MOT) at a temperature of 1 mK. These atoms can be transferred to a Ioffe-Quadrupole magnetic trap with almost unity efficiency. To match the size and temperature of our MOT the trap parameters are chosen as $\alpha = 1 \cdot 10^4$ G/m, $\beta = 47.5 \cdot 10^4$ G/m$^2$ and $B_0 = 99.6$ G. The MT has trap frequencies of $\omega_r = (2\pi)376$ Hz and $\omega_z = (2\pi)44.7$ Hz.

For this experiment the MOT was operated at a detuning of $-2\Gamma$ and at an intensity of 0.5 mW/cm$^2$ for the three MOT beams together. After turning off the MOT, a 50 $\mu$s long $\sigma^+$ spin polarization pulse of 2 mW/cm$^2$ was given to put all the atoms in the $|m_J = +2>$ state, and then the MT was turned on. The atom cloud now has a temperature of approximately 1 mK and contains approximately $1 \cdot 10^9$ atoms. After turning on the MT the atoms are held there long enough to allow the atom cloud to reach its equilibrium state. Then the bias field is ramped from 99.6 G to 1.5 G, adding
energy to the atoms in the radial direction and compressing them spatially. The cloud radius in the radial direction as a function of time after the ramping is determined by absorption imaging. The radius is directly proportional to the potential energy and therefore the temperature because the trap shape is linear. A typical result of a series of measurements is shown in Figure 6.

The exponential fit gives a characteristic equilibration time of 130 ± 25 ms and a relative change in width of 5 ± 3%. This result agrees with the timescale of the simulation as found from the data in Figure 1. If the equilibration would be due solely to collisions, this would give a collisional cross-section of \( \sigma = 8\pi \cdot (200a_0)^2 \), with \( a_0 \) the Bohr radius. However, the cross-section at 1 mK can not be larger than the unitary limit \( \sigma = 8\pi \cdot (\lambda_{dB}/2\pi)^2 = 8\pi \cdot (80a_0)^2 \). Here \( \lambda_{dB} \) is the thermal de Broglie wavelength of the atom. Also, we did not observe a dependence of the equilibration time on the atomic density, indicating that anharmonic mixing is the dominant mechanism here.

5. Applications

An interesting application for anharmonic mixing is to use it to enhance the efficiency of Doppler cooling in a magnetic trap. This cooling technique is one-dimensional because only along the axial direction of an MT all atoms are polarized the same way. Cooling in the radial directions can be achieved by reabsorption of scattered photons if the cloud is optically dense. If that is not the case anharmonic mixing can cool the radial directions, as shown in an early experiment by Helmerson et al. By making the mixing time as short as possible the cooling can be fast and the atom losses as a result of the cooling can be limited.

Another technique that could benefit from anharmonic mixing is evaporative cooling. One of the problems that can occur when an atom cloud is cooled evaporatively...
is gravitational sag, in which gravity shifts the equipotential surfaces of atoms in a trap in such a way that they do not coincide with the surfaces of constant magnetic field. This causes the cooling process to become one or two-dimensional, reducing its efficiency \[17\]. If the dimensions are coupled by anharmonic mixing the evaporation remains three-dimensional even when gravity plays a role.

A situation where anharmonic mixing is undesirable and needs to be suppressed is in rethermalization experiments to measure the scattering length \[10\], \[18\]. The scattering length is one of the properties of an atom that determine the feasibility of evaporative cooling and reaching the transition point of Bose-Einstein condensation for a given number of atoms at a certain density and temperature \[8\]. It can be measured by observing collisional equilibration after ramping the bias field as we did in our simulations and experiment. We can conclude that this method will only yield reliable results if the timescale on which anharmonic mixing occurs is long compared to the collision time.

6. Conclusion

We determined the timescale on which anharmonic mixing occurs and the dependence of that timescale on trap parameters and atom temperature. We observed a resonance in the mixing time as a function of the end bias field, and explained this with a simple oscillator model. We verified experimentally that anharmonic mixing does indeed occur, and that its timescale can be short compared to the timescale needed for collisional equilibration. The application of anharmonic mixing to improve Doppler cooling and lower-dimensional evaporative cooling is possible.

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