Physical Origin of Chiral States and
Near-Threshold Resonances Observed at BES

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The purpose of my talk is to explain physical backgrounds of the chiral states, which are predicted to exist generally in the low mass regions of hadrons, containing light quarks, and to stimulate the experimental and phenomenological search for their candidates. The contents are organized as follows:

§1. Present Status of Hadron Spectroscopy
§2. Some Basic Consideration on Quark Confinement
§3. Essentials of Description of Composite Hadron in $\tilde{U}(12)$-Scheme
§4. Revisal of Meson and Baryon Wave Functions and Chiral States
§5. Multi-Quark Hadrons and Near-Threshold Resonances
§6. Concluding Remarks

§1. Present Status of Hadron Spectroscopy

Now hadron spectroscopy seems to meet a revolutionary stage. There exist (Two Contrasting Views for Classification of Hadrons),

| Non-Relativistic View | Relativistic View |
|-----------------------|------------------|
| Bases                 | Field-Theoretical Models |
| Framework             | Spont. Broken Chiral Symm. |
| Difficulty            | Unable to treat |
|                       | Internal Excitations |

which, on the one hand, have respective successes and, on the other hand, include their own difficulties as shown in Table I.

(Observation of New “Exotic” Hadrons)

Meanwhile, new hadrons, seemingly out of conventional classification scheme, have been reported\(^1\) to exist successively with the Features of new exotic hadrons;

(F1) Mass, close to thresholds of their observed or intermediate decay channels.

(F2) Decay width, unexpectedly narrow.
Several years ago, corresponding to this situation, we made a

(Proposal of Covariant $\tilde{U}(12)_{SF}$-Classification Scheme)$^{2),3)}$

which is an unification, keeping their successful points and getting rid of the difficulties
of the above two views.

**Framework :**

It has the following symmetry in the rest frame of hadrons, which is embedded
in the Lorentz-covariant space as,

\[
\text{Hadron} = \begin{cases} 
\text{at rest}(\mathbf{P} = 0), & \text{Static } U(12)_{SF} \otimes O(3)_{L} - \text{Symm.} \\
\text{moving}(\mathbf{P} \neq 0), & \text{Covariant } \tilde{U}(12)_{SF} \otimes O(3,1)_{\text{Lorentz-space}}
\end{cases}
\]

where the above groups contain their sub-groups as follows:

\[
\tilde{U}(12)_{SF} \supset SU(3)_{F} \otimes \tilde{U}(4)_{\text{Dirac Spinor}},
\]

\[
\tilde{U}(4)_{\text{D.S.}} \mathbf{P=0} U(4)_{S} \supset SU(2)_{\sigma} \otimes SU(2)_{\rho} \quad (\text{Dirac } \gamma \equiv \sigma \times \rho).
\]

The remarkable point in this scheme is that it contains a new approximate
symmetry$^{3)} SU(2)_{\rho}$ for “Confined-Light Quarks”, and the non-relativistic symmetry
$SU(6)_{SF} \supset SU(3)_{F} \otimes SU(2)_{\sigma}$ is extended into $SU(12)_{SF} \supset SU(3)_{F} \otimes SU(2)_{\sigma} \otimes SU(2)_{\rho}$.

The basic vectors for the new $SU(2)_{\rho}$ space consist of $\{\Phi_{+\alpha}(X), \Phi_{-\alpha}(X)\}$ $^{*}$, which have, respectively the quantum numbers and are called $^{**}$, as

\[
\begin{align*}
\Phi_{+\alpha}(X) : & \quad \text{Pauli Ur-(exciton) } \quad j^p = \frac{1}{2} \quad \rho_3 = +1 \\
\Phi_{-\alpha}(X) : & \quad \text{Chiral Ur-(exciton) } \quad j^p = \frac{1}{2} \quad \rho_3 = -1.
\end{align*}
\]

They are connected mutually by$^{5)}$

\[
\text{Chirality Transformation; } \Phi_{\pm \alpha}^{\pm}(X) = \Phi_{\mp \alpha}(X) = \left[ -\gamma_5 \Phi_{\pm}(X) \right].
\]

The chiral urciton $\Phi_{-\alpha}$ with exotic quantum number $(j^p = \frac{1}{2}^{-})$ leads to Chiral
States, a new-type of “Exotic Hadrons”.

§2. Some Basic Considerations on Quark-Confinement

First we recapitulate the definition of and related formulas to

$^{*}$ Here we concern only the transformation properties of hadron Wave Function(WF) related
with the C.M. coordinates and simply write as $\Phi(X, r, \cdots) \rightarrow \Phi(X)$, where $X/r$ is center of mass /relative coordinate of hadrons.

$^{**}$ The notion of urciton is introduced in Ref. 4). It is a dimensionless Dirac spinor simulating
confined quarks inside hadrons.
(Lorentz Covariance for Local Spinning Particle).

For Lorentz transformation of space-time coordinates of “local spinning hadron”

\[ X_{\mu} \rightarrow X'_{\mu} = \Lambda_{\mu\nu}X_{\nu} = (\delta_{\mu\nu} + \epsilon_{\mu\nu})X_{\nu} \]  \hspace{1cm} (2.1)

the hadron wave function generally of multi-component is transformed as

\[ \Phi(X) \rightarrow \Phi'(X') = S(A)\Phi(X), \quad S(A) \equiv (1 + \frac{i}{2}\epsilon_{\mu\nu}\Sigma_{\mu\nu}), \] \hspace{1cm} (2.2)

where the \( S(A) \) are relevant matrices to its spin and the generators \( \Sigma_{\mu\nu} \) are classified into those for two sub-groups as

for Rotation \( J_i \equiv \frac{1}{2}\epsilon_{ijk}\Sigma_{jk} \), for Boost \( K_i \equiv i\Sigma_{i4} \).

(unitary group) \hspace{1cm} (non-unitary group)

**Spin-1/2 case**  \hspace{1cm} The relevant concrete formulas in the case of Dirac field are given as

\[ J_i = \frac{1}{2}\sigma_i \otimes \rho_0, \quad K_i = \frac{i}{2}\rho_1 \otimes \sigma_i, \quad S(A) = R(\omega) = e^{-i\omega J}, \quad S(A) = B(\chi) = e^{-ib\cdot K}, \] \hspace{1cm} (b = \chi\hat{v}; \quad \tanh \chi = |v|, v = P/P_0)

where \( P_\mu \) is the four-momentum of Dirac particle. Here it should be stressed that the \( SU(2)_\rho \)-freedom is indispensable for Lorentz covariance.

**Higher spin case**  \hspace{1cm} The WF of particle with higher-spin is given as the tensor with multi-Dirac indices \( \Phi_{\alpha_1\alpha_2...\alpha_N}(X) \), of which generator is given as a sum of relevant ones for respective indices.

(Hole Theory for Negative Energy solution of Dirac Equation)

As is well known, the Lorentz covariant Dirac equation with spin one-half leads necessarily to the negative-energy solutions, which are generally interpreted, applying the Hole Theory, as representing the anti-particle of relevant particles. However, contrary to the free electrons, application of H.T. to the confined quarks inside hadrons produces the difficulty as follows:

Applying Hole Theory to respective confined quarks separately from the other constituents makes the change of quantum numbers explained in Fig.1, and thus violating the color-singlet constraint of Hadrons.

Accordingly the solution of Dirac Eq. is expanded respectively as follows:

\[ \psi_{D,\alpha}(X) = \sum_{P, P_0 = |E|} (u_\alpha(P, +|E|)e^{iPX-iEt} + u_\alpha(-P, -|E|)e^{-iPX+iEt}), \]  \hspace{1cm} (2.3)

\( \Phi_\alpha = (\phi_1 \phi_2 \chi_1 \chi_2)^T \). The relevant Dirac \( \gamma \)-matrices are decomposed into \( \rho \) and \( \sigma \) matrices as \( \gamma(4 \text{ by } 4) \equiv \rho(2 \text{ by } 2) \otimes \sigma(2 \text{ by } 2) \), where the \( \sigma \) operating on \( SU(2) \)-space expanded by \( (\phi_1 \phi_2) \) on \( (\chi_1 \chi_2) \) and the \( \rho \) operating on \( SU(2) \)-space expanded by \( (\phi \chi) \), respectively.
Fig. 1. Applying Hole Theory for negative-energy solutions -Free electrons versus Confined-quarks-
Application of H.T. to confined quarks induces violation of the color-singlet condition of hadrons.

\[
u^{(e^-)}_\alpha(-P, -|E|) \Rightarrow v^{(e^+)}_\alpha(P, |E|) \quad \text{for electrons}
\]

\[
u^{(a)}_\alpha(-P, -|E|) \neq v^{(a^*)}_\alpha(P, |E|) \quad \text{for quarks}
\]

where the \((-\))\text{-frequency solution} \(u_\alpha(-P)\) is replaced, in the electron case, by the
\((+\))\text{-frequency positron spinor} \(v_\alpha(P)\); while this is not possible for the confined quark
as explained above. In the case of confined quarks both of \(u_+, \alpha(P) \equiv u_\alpha(P, |E|)\) and
\(u_-, \alpha(P) \equiv u_\alpha(-P, -|E|)\) are required for completeness, becoming basic vectors of
the \(SU(2)_\rho\) space.

(Overlooked Freedom of \(SU(2)_\rho\) for Confined Quarks and Chiral States)

In the \(\tilde{U}(12)_{SF}\)-classification scheme the spin WF of hadrons are supposed to
transform like tensors in \(\tilde{U}(4)_{D.S.}\)-space for the Lorentz transformation of CM coordinate
of hadrons as

\[
\Phi^{\beta_1 \cdots \beta_n}_{\alpha_1 \cdots \alpha_n}(X, r, \cdots) \propto \Phi_{\alpha_1}(X) \cdots \Phi_{\alpha_n}(X) \tilde{\Phi}^{\beta_1}(X) \cdots \tilde{\Phi}^{\beta_n}(X).
\]

(2.4)

Here \(\Phi_{\alpha}(X)\) is representing the respective tensor-component and simulating the
transformation property of confined quarks, and is called “Uricton”. In this con-
nexion, it is required only to satisfy the K.G. equation, because the observable is not
quarks but hadrons.

\[
\left( \frac{\partial^2}{\partial X^2} - M^2 \right) \Phi_\alpha(X, \cdots) = 0. \tag{2.5}
\]

\textit{SU}(2)_\rho\text{-freedom :} Since our master equation contains the second-order time derivative it contains\(^{\ast}\) the two-type of Dirac spinors, which constitute the \textit{SU}(2)_\rho space, as \(^{**}\)

Basic vectors; \( \Phi_\alpha(X) = \{ \Phi_{+,\alpha}, \Phi_{-,\alpha} \} \),

defined as solutions of

\[
(\gamma_\mu \partial_\mu \pm M) \Phi_\pm(X) \equiv 0. \tag{2.6}
\]

They are evidently related by the chirality transformation\(^{\ast\ast}\) \(^{\ast\ast}\)

\[
\Phi_\pm(X) = -\gamma_5 \Phi_\mp(X). \tag{2.7}
\]

We define \textit{Chiral states}/ \textit{Chiralons}, whose spin WF is tensors in \( \tilde{U}(4)_{D.S.} \) space with at least-one chiral urciton-spinor component, while \textit{Pauli states}/\textit{Paulons} with all Pauli-urciton components. Chiralons show anyhow some “Exotic” properties out of conventional framework, as was mentioned in §1.

The freedom for quarks, \textit{SU}(2)_\rho, had been overlooked for long time. Here we describe

\textit{(Physical Meaning of SU}(2)_{\rho}\text{-Freedom)}

For space-time reflection of CM-coordinates of hadron the WF is transformed, as

\[
X_\mu \to X'_\mu : \quad \Phi(X, r, \cdots) \to \Phi'(X', r, \cdots) = S^{(H)} \Phi(X, r, \cdots), \tag{2.8}
\]

where the relevant operator \( S^{(H)} \) is given through that for respective constituent-urcitons \( S \), defined by

\begin{align*}
\text{Space R.} & \quad X'_i = -X_i \quad S = \gamma_4 = \rho_3 \otimes \sigma_0 \quad \text{Intrinsic Parity} \\
\text{Time R.} & \quad X'_0 = -X_0 \quad S = \gamma_1 \gamma_2 \gamma_3 = i \rho_2 \otimes \sigma_0 \quad \text{Temporal Parity} \\
\text{Space-time R.} & \quad X'_\mu = -X_\mu \quad S = \gamma_1 \gamma_2 \gamma_3 \gamma_4 = -\rho_1 \otimes \sigma_0 \quad \text{Chirality}.
\end{align*}

Note that the chirality transformation is different from the conventional chiral transformation, as is compared in Table II. However, it is notable that the chirality transformation corresponds, mathematically, to the \( U(1) \)-chiral transformation \( S = \exp(i \omega \gamma_5) \) with an “angle” \( \omega = \frac{\pi}{2} \).

\(^{\ast}\) In this connection, note that the urciton field are described by totally 8 (= 4 \times 2)-components of complex field, while the conventional Dirac field by 4-components.

\(^{**}\) The (+, -) sign of \( \Phi_\pm(X) \) also corresponds to that of \( \rho_3 \)-eigen values of relevant spinors in the rest frame.

\(^{\ast\ast}\) Here it may be worthwhile to note that, as far as Eq.(2.6) concerned, the chirality transformation is equivalent to the mass reversal\(^{6}\).
Table II. Comparison between chirality and chiral transformation

| Chirality Transf. | Chiral Transf. |
|-------------------|----------------|
| discrete          | continuous     |
| for massive constituent-quarks | for massless current-quarks |

(A New Symmetry, SU(2), for Hadrons with Light-Quarks)

The SU(2)ρ freedom for confined quarks leads to a new symmetry in hadron physics as follows:

In QCD, a non-relativistic basis for hadron physics, is valid Chiral Symmetry on light quarks in the limit of no effects of vacuum condensation. Correspondingly, in hadron physics, simulating the constituent-quarks with the urcitons and substituting the transformation $\gamma_5 = -\rho_1$ on urcitons for the $\gamma_5$-transformation on quarks, leads to approximate Chiral Symmetry on hadrons with constituent light-quarks. This implies also validity of, in the case without direct effects of vacuum condensation,

A New SU(2)ρ-Symmetry for Hadrons concerning Light constituent (even massive) Quarks with respective flavors.

§3. Essentials of Description\(^2\) of Composite Hadrons in \(\tilde{U}(12)\)-Scheme

In this section we give a brief summary of the relevant manifestly-covariant scheme. Note first that, we are not treating a dynamical composite problem, but giving a kinematical framework.

3.1. General Framework

Hadron Wave Function should represent all attributes of hadrons, such as “definite-mass,\(^a\)JP, Lorentz Transformation Property, and -quark structure”.

Concerning the definite-Lorentz T.P. and -quark structure, we set up the WF $\Phi$ and its Pauli-conjugate $\bar{\Phi}$, as follows.

$$
\Phi_{A_1\cdots B_1\cdots}(x_1,\cdots,y_1,\cdots) \approx \psi_{A_1}(x_1)\cdots \bar{\psi}_{B_1}(y_1)\cdots, \quad (3.1)
$$

$$
\bar{\Phi}^{A_1\cdots B_1\cdots}(x_1,\cdots,y_1,\cdots) \approx \bar{\psi}^{A_1}(x_1)\cdots \psi_{B_1}(y_1)\cdots, \quad (3.2)
$$

where $x_i$ and $y_i$ denote the space-time coordinates of constituent quarks and antiquarks; $A = (\alpha,a)$ and $B = (\beta,b)$ do their Dirac and flavor indices. The WF transform like tensors in $\tilde{U}(12)_{SF} \otimes O(3,1)_{\text{Lorentz}}$ space.

Basic Equation is, concerning the definite-mass, given\(^7\) by Yukawa-type Klein-Gordon\(^*)\) Equation:

$$
[(\frac{\partial}{\partial X_\mu})^2 - \mathcal{M}^2(r_\mu, \partial/\partial r_\mu)]\Phi(X,r,\cdots) = 0. \quad (3.3)
$$

\(^*)\) Note that hadron is observable but quark is not, as was mentioned in §2.
Then WF $\Phi$ and its Pauli conjugate $\bar{\Phi}$ are expanded by the four-dimensional Fourier amplitudes:

\[
\Phi(X,r,\cdots) = \sum_{N,P,N,\mu(P_{N,0}>0)} \left\{ e^{iP_{N}\cdot X} \Psi^+(H)(P_{N},r,\cdots) + e^{-iP_{N}\cdot X} \Psi^-(H)(P_{N},r,\cdots) \right\}
\]

\[
\bar{\Phi}(X,r,\cdots) = \sum_{N,P,N,\mu(P_{N,0}>0)} \left\{ e^{-iP_{N}\cdot X} \bar{\Psi}^-(H)(P_{N},r,\cdots) + e^{+iP_{N}\cdot X} \bar{\Psi}^+(H)(P_{N},r,\cdots) \right\},
\]

(3.4)

where *)

\[
\bar{\Phi}(X,r,\cdots) \equiv \gamma_{4} \cdots \Phi^\dagger(X,r,\cdots) \gamma_{4} \cdots,
\]

The description of internal excitation becomes possible due to the squared-mass operator on relative coordinates as

\[
M^2_r(\partial/\partial r_{\mu}) \Psi^\pm(N)(P_{N},r,\cdots) = M^2_N \Psi^\pm(N)(P_{N},r,\cdots),
\]

(3.5)

where $M^2_r$ is $U(12)_{SF}$ symmetric and $\delta M^2$ represents effects of vacuum condensation and of perturbative QCD.

**Second Quantization** The positive / negative frequency parts of WF $\Phi(X,r,\cdots)$ and $\bar{\Phi}(X,r,\cdots)$ are supposed to become, as a result of the second-quantization,

\[
\Psi^+(H)(P_{\cdots})/\Psi^-(H)(P_{\cdots}); \text{annihilation-/creation-operator of the relevant hadrons}
\]

\[
\bar{\Psi}^+(H)(P_{\cdots})/\bar{\Psi}^-(H)(P_{\cdots}); \text{creation-/annihilation-operator of their charge-conjugates}
\]

(3.6)

where $\bar{\Psi}^\pm(P) \equiv \gamma_4 \cdots [\Psi^\mp(P)]^\dagger \gamma_4 \cdots$ is the Pauli adjoint both on each lower and upper sufix of $\Psi(P_{\cdots})_{\alpha_1,..\beta_1,..}$. 

This leads to the crossing relation of relevant hadrons, which is another attribute of hadrons.

**Composition of WF with definite $J = L + S$** Concerning the attribute of definite $J^P$-property; the Fourier amplitudes of WF $\Phi(X,r \cdots)$ is composed, as

\[
\Psi_{J,\alpha_\cdots}^{(\pm)\beta_\cdots}(P,r \cdots) = \sum_{i,j} c^{(\pm)}_{ij} \underbrace{W^{(\pm)}_{\alpha_\cdots}^{(i)\beta_\cdots}(P)}_{\text{Spin WF}} \times \underbrace{O^{(j)}(P,r \cdots)}_{\text{Space–time WF}}
\]

(3.7)

*) Note that this implies that $\bar{\Psi}^\pm(P_{N},r,\cdots) \equiv \gamma_{4} \cdots [\Psi^\mp(P_{N},r,\cdots)]^\dagger \gamma_{4} \cdots$. 


from complete sets of eigen functions in respective spaces, of the $\tilde{U}(4)_{DS}$ and of the relative space-time coordinates.

In Eq.(3.7) the spin WF are tensors in $\tilde{U}(4)_{DS}$ and defined by the basic vectors\textsuperscript{a) $W_\alpha$ and $\tilde{W}_\beta$ as,

$$
W^{(\pm)}_{\alpha_1 \cdots \alpha_n}(P) = u_{\alpha_1}(P) \cdots u_{\alpha_n}(P) \tilde{w}^{\beta_1}(P) \cdots \tilde{w}^{\beta_m}(P) \quad (P_0 > 0) \\
W^{(-)}_{\alpha_1 \cdots \alpha_n}(P) = v_{\alpha_1}(P) \cdots v_{\alpha_n}(P) \tilde{u}^{\beta_1}(P) \cdots \tilde{u}^{\beta_m}(P) \quad (P_0 > 0),
$$

(3.8)

where $W^{(\pm)}_{\alpha_1 \cdots \alpha_n}(P)$ are Bargmann-Wigner spinors, and $u_{\alpha}(P)/v_{\alpha}(P)$ are Dirac spinors for $q/\bar{q}$.

The spin WF of the Pauli-conjugate $\bar{\Phi}(X,r \cdots)$ are given as $\tilde{W}^{(\pm)}_{\beta_1 \cdots \beta_m}(P) = W^{(\mp)}_{\alpha_1 \cdots \alpha_n}(P)$.

In Eq.(3.7) the space-time WF, $O(P,r)$, satisfies the subsidiary constraint\textsuperscript{b) on relative-time, as

$$
\langle P_\mu r_\mu \rangle = \langle P_\mu p_\mu \rangle = 0 \quad (P=0) \Rightarrow \langle t \rangle = 0.
$$

(3.9)

Then the 4-dimensional oscillator function becomes three-dimensional, effectively, in the hadron rest frame as

$$
O(P_\mu r_\mu) \quad \Rightarrow \quad O(M_\mu, r).
$$

tensors in $O(3,1)_{\text{Lorentz}}$
tensors in $O(3)_{\text{L}}$

3.2. Formulas concerning basic Vectors in $\tilde{U}(4)_{DS}$-Tensor Space

As was mentioned in §2, our urciton spinors, simulating the Lorentz transformation property of confined quarks, are required only to satisfy Klein-Goldon equation with second-order time-derivative. This leads to existence of two kinds of Dirac spinors as its solution. In the following we collect mathematical formulas related to the $\tilde{U}(4)_{DS}$-space.

$$
\text{ucritons} \quad \Phi_{\cdots \cdots}(X; r, \cdots) \propto \Phi_\alpha(X) \quad (\alpha : \text{Dirac spinor index}),
$$

(3.10)

\text{ucritons} \quad \Phi_{\cdots \cdots}(X; r, \cdots) \propto \Phi_\alpha(X) \quad (\alpha : \text{Dirac spinor index}),

(3.10)

Where basic vectors in $\tilde{U}(4)_{DS}$ tensor space are given by $W_\alpha(P) = \{u_\alpha(v), \bar{v}_\alpha(v)\}$, and $\tilde{W}_\beta(P) = \{\tilde{u}^\beta(v), \tilde{v}^\beta(v)\}$ which are urciton-Dirac spinors represented by the four-velocity of relevant hadrons, $v_\mu \equiv P_\mu/M (M > 0)$. Here note the restriction, $P_0 > 0$, in Eq.(3.8). This, in addition to the mass-shell condition $P_\mu^2 = -M^2$, implies the zero-th component $v_0(\equiv -iv_4)$ becomes $v_0 = 1$ in the rest frame.

\textsuperscript{a) Basic vectors in $\tilde{U}(4)_{DS}$ tensor space are given by $W_\alpha(P) = \{u_\alpha(v), \bar{v}_\alpha(v)\}$, and $\tilde{W}_\beta(P) = \{\tilde{u}^\beta(v), \tilde{v}^\beta(v)\}$ which are urciton-Dirac spinors represented by the four-velocity of relevant hadrons, $v_\mu \equiv P_\mu/M (M > 0)$. Here note the restriction, $P_0 > 0$, in Eq.(3.8). This, in addition to the mass-shell condition $P_\mu^2 = -M^2$, implies the zero-th component $v_0(\equiv -iv_4)$ becomes $v_0 = 1$ in the rest frame.
\( K.G.Eq. \)
\[
(\Box - M^2)\Phi_\alpha(X) = [(\gamma_\mu \partial_\mu + M)(\gamma_\lambda \partial_\lambda - M)\Phi]_\alpha = 0
\]
\[
\Phi_\alpha(X) = \Phi_{+,\alpha}(X) + \Phi_{-,\alpha}(X)
\]
\[
(\gamma_\mu \partial_\mu \pm M)\Phi_\pm(X) \equiv 0 \quad (M > 0).
\]

**Basic Vectors for SU(2)_\rho:** \( \Phi_r(X) = \{\Phi_+(X), \Phi_-(X)\} \).

**Fourier Expansion of urciton spinor**
\[
\Phi_\alpha(X) = \sum_{\mathbf{P}(P_0 \equiv E_\mathbf{P} > 0), r, s, \bar{r}, \bar{s} = \pm 1} \left( b_{r, s, \mathbf{P}} u_{r, s, \alpha}(\mathbf{P}) e^{i\mathbf{P}X} + d_{\bar{r}, \bar{s}, \mathbf{P}} \bar{u}_{\bar{r}, \bar{s}, \alpha}(\mathbf{P}) e^{-i\mathbf{P}X} \right)
\]
\[
\bar{\Phi}^\alpha(X) = \sum_{\mathbf{P}(P_0 \equiv E_\mathbf{P} > 0), r, s, \bar{r}, \bar{s} = \pm 1} \left( b_{r, s, \mathbf{P}} \bar{u}_{r, s, \alpha}(\mathbf{P}) e^{-i\mathbf{P}X} + d_{\bar{r}, \bar{s}, \mathbf{P}} \bar{v}_{\bar{r}, \bar{s}, \alpha}(\mathbf{P}) e^{+i\mathbf{P}X} \right). \tag{3.12}
\]

Here it should be noted that a summation on the new freedom, \( SU(2)_\rho \), denoted as \( r = \pm \) (the eigen value of \( \rho_3 \)-spin), is appearing in addition to the conventional one \( s = \pm \) on \( SU(2)_\sigma \).

**Fourier Conjugate-Basic Vectors in \( \tilde{U}(4)_{DS}-Tensor Space**
\[
W_\alpha(v) = \{u_\alpha(v), v_\alpha(v)\},
\]
\[
\bar{W}^\beta(v) = \{\bar{u}^\beta(v), \bar{v}^\beta(v)\}. \quad (v_\mu \equiv \frac{P_\mu}{M}; \quad v_0 > 0) \tag{3.13}
\]

**Complete set of basic vectors in \( SU(2)_\rho \)- space**
\[
u_r = \{u_+, u_-\}, \quad v_r = \{v_+, v_-\}
\]
\[
\bar{\nu}_r = \{\bar{u}+, \bar{u}-\}, \quad \bar{v}_r = \{\bar{v}+, \bar{v}-\}. \tag{3.14}
\]

**Chirality Partners**
\[
W_\pm(v) = -\gamma_5 W_{\mp}(v) ; \quad \bar{W}_\pm = \bar{W}_{\mp}(v)\gamma_5
\]
\[
\begin{cases}
  r = +; & \text{Pauli- (urciton) spinor} \\
  r = -; & \text{Chiral- (urciton) spinor} \quad \text{opposite relative-parity}
\end{cases} \tag{3.15}
\]

### 3.3. Static-\( U(4)_S \)-embedded spinor WF

In order to embed\(^*\) the static \( U(4)_S \)-symmetry in the covariant \( \tilde{U}(4)_{DS} \)-space, it is necessary to replace the basic vectors \( W^\beta(v) \) by \( W^\beta_U(v) \) and to make corresponding modification on the spinor WF of general hadrons, as follows:

\(^*\) As for details, see Ref. 3)
The $F_{U}(v)$, unitarizer, becomes $\gamma_{4}$ in the rest frame of relevant hadrons, while Pauli-adjoint does Hermite conjugates. Accordingly the new scalar product of basic vectors becomes equal to the unitary-invariant product in the rest frame of relevant hadrons, as

$$\langle \bar{W}_{U}(P)_{\alpha}W_{U}(P)_{\alpha} \rangle \stackrel{(P=0)}{=} \langle W^{\dagger}(P)_{\alpha}W(P)_{\alpha} \rangle_{P=0}. \quad (3.17)$$

(Spinor WF of Hadron)

$$W_{\alpha_{1}...\alpha_{n}}^{\beta_{1}...\beta_{m}}(P) = W_{\alpha_{1}}(P) \cdots W_{\alpha_{n}}(P)\bar{W}^{\beta_{1}}(P) \cdots \bar{W}^{\beta_{m}}(P)$$

$$W_{U,\alpha_{1}...\alpha_{n}}^{\beta_{1}...\beta_{m}}(P) = W_{U,\alpha_{1}}(P) \cdots W_{U,\alpha_{n}}(P)\bar{W}^{\beta_{1}}(P) \cdots \bar{W}^{\beta_{m}}(P) \stackrel{(P=0)}{=} W_{\alpha_{1}}(P = 0) \cdots W_{\alpha_{n}}(P = 0)W^{\dagger,\beta_{1}}(P = 0) \cdots W^{\dagger,\beta_{m}}(P = 0). \quad (3.18)$$

Accordingly the scalar product of the $U(4)$-embedded spin WF becomes equal to the unitary-invariant product in the rest frame, as

$$\langle \bar{W}_{U}(P)_{\alpha_{1}...\alpha_{n}}W_{U}(P)_{\beta_{1}...\beta_{m}} \rangle \stackrel{(P=0)}{=} \prod_{i=1}^{n}\langle W^{\dagger,\alpha_{i}}(P)W_{\alpha_{i}}(P) \rangle_{P=0}\prod_{j=1}^{m}\langle W^{\dagger,\beta_{j}}(P)W_{\beta_{j}}(P) \rangle_{P=0}. \quad (3.19)$$

§4. Revival of Meson and Baryon Wave Functions and Chiral States

In this section we give a brief review on the new treatment of the conventional $(q\bar{q})$-mesons and $(qqq)$-baryons in the $\tilde{U}(12)$-scheme, and describe the level structures, referring to the chiral states.

4.1. Spinor WF of $(q\bar{q})$-Mesons and $(qqq)$-Baryons

In this sub-section we give the concrete form of spinor WF of $(q\bar{q})$ mesons and $(qqq)$ baryons.

**Dirac Spinor**

$$\psi_{D}(X)_{\alpha} = \Phi_{+\alpha}(X) = \sum_{P(P_{0} > 0)}(W_{q_{1}+\beta}(P)e^{iPX} + W_{q_{1}+\alpha}(P)e^{-iPX}). \quad (4.1)$$
Urciton Spinor \[ \Phi_\alpha(X) = \Phi_{+,\alpha}(X) + \Phi_{-,\alpha}(X) \]

\[ \Phi_\alpha(X) = \sum_{P(P_0>0), r, \bar{r}=\pm} (W_{q,r,\alpha}(P)e^{iPX} + W_{q,\bar{r},\alpha}(P)e^{-iPX}) \]

\[ \bar{\Phi}^\beta(X) = \sum_{P(P_0>0), r, \bar{r}=\pm} (\bar{W}_{q,r}^\beta(P)e^{iPX} + \bar{W}_{q,\bar{r}}^\beta(P)e^{-iPX}) \] \hspace{1cm} (4.2)

Complete set of Basic Vectors in SU(2)_ρ space

\[ W_\alpha(P) = \{W_{q,r,\alpha}(P), W_{q,\bar{r},\alpha}(P)\}, r, \bar{r} = \pm \]

\[ \bar{W}^\beta(P) = \{\bar{W}_{q,r}^\beta(P), \bar{W}_{q,\bar{r}}^\beta(P)\}, r, \bar{r} = \pm \] \hspace{1cm} (4.3)

Meson Spinor bi-Dirac Spinor

\[ W_{\alpha\beta}(P) = W_{q,r,\alpha}(P)\bar{W}_{q,\bar{r}}^\beta(P) \] \hspace{1cm} (4.4)

\[ (r, \bar{r}) = (+,+) \text{ boosted Pauli States} \]
\[ (+, -), (-, +) \}
\[ “Chiral States” \]

Baryon Spinor tri-Dirac Spinor

\[ W^{(B)}_{\alpha\beta\gamma}(P) = W_{q,r_1,\alpha}(P)W_{q,r_2,\beta}(P)W_{q,r_3,\gamma}(P) : \text{for Baryons} \]

\[ (r_1, r_2, r_3) = (+, +, +) \text{ boosted Pauli States} \]
\[ (+, +, -), (+, - , -) \}
\[ “Chiral States” \]

\[ W^{(\bar{B})}_{\alpha\beta\gamma}(P) = W^{(B)}\{W_{q,r_i,\alpha} \Rightarrow W_{q,\bar{r}_i,\alpha}\} : \text{for anti-Baryons} \]

4.2. Level Structure and Excitation Trajectories of Mesons

(Ground Sates) in \( \tilde{U}(12)_{SF} \)-scheme are classified, respectively, into the multiplets as follows:

Meson: \( (12 \times 12^*) = 144 \)

\[ J^{PC} \begin{array}{c|c|c|c|c|c|c} p_{E}^{(N)} & v_{E}^{(N)} & (\text{Chiral States}) \\ \hline 0^{+} & 1^{--} & P_{E}^{(E)} & v_{E}^{(E)} & S^{(N)} & A_{E}^{(N)} & S^{(E)} & A_{E}^{(E)} \\ \hline 0^{-} & 1^{--} & 0^{++} & 0^{++} & 1^{++} & 0^{-} & 1^{+-} \end{array} \]

Baryon: \( (12 \times 12 \times 12)_{\text{Symm}} = 364 = 182_\beta \oplus 182_{\bar{\beta}} \)
(Excited States) in $\bar{U}(12)_{SF} \otimes O(3)_L$-scheme show\(^{18}\), respectively, the excitation trajectories, as

\[
(mass)^2 \quad M_N^2 = M_0^2 + N\Omega
\]  

\[
\text{meson: } M_M^{(N)} = m_q^{(N)} + m_{\bar{q}}^{(N)}
\]

\[
\text{baryon: } M_B^{(N)} = m_{q_1}^{(N)} + m_{q_2}^{(N)} + m_{q_3}^{(N)},
\]

where \(m_q^{(N)}\) being the mass of quarks in N-th excited states. In Eq.(4.6) the mass of ground states \(M^{(G)} = M_0\) is given. In the ideal case, as a sum of that of respective constituent-quarks \(m_q\), taking \(m_q^{(0)} = m_q\), while the mass of N-th excited states \(M^{(N)}\) are given as a sum of those of respective excited-constituent quarks \(m_q^{(N)}\), being given by \(m_q^{(N)} = \gamma_N m_q\) with \(\gamma_N \equiv M^{(N)}/M^{(G)}\).

(Effective Chiral Symmetry) We suppose a phenomenological rule for approximate chiral symmetry;

\[
m_q^{(N)2} \ll \Lambda_{conf}^2 \approx 1\text{GeV}^2.
\]  

This leads to a condition on number of excitation quanta as

| (m$\bar{n}$)Meson | (n$c$)Meson | (n$b$)Meson |
|-----------------|-------------|-------------|
| \(N \leq 1\) or 2 | \(N \leq 0\) or 1 | \(N \leq 0\) or 1 |

Accordingly, for the Lower-mass states with these \(N\) values we expect\(^2)\(^{,9}\) the Existence of Chiral States. The situations on the meson excitation-trajectory are shown in Fig.2.

§5. Multi-Quark System and Near-Threshold Resonances

An overview of general multi-quark hadrons, obtained by applying the Joined Spring Quark Model\(^{10}\), in the $\bar{U}(12)_{SF}$-classification scheme is given in Fig.3. The JSQM had been proposed so as to lead to the triality-zero and color-singlet multi-quark hadrons. Here the model applied in the $\bar{U}(12)$-scheme is manifestly covariant, and gives chirality($\gamma_5$)-symmetric (concerning the light quarks) mass spectra in the ideal limit. First in this section I shall give an interpretation of basic properties of new “exotic” hadrons in the $\bar{U}(12)$ scheme. Then I give some comments in identifying the near-threshold resonances observed at BES.
5.1. Properties of Tetra-quarks - an Example of $X(3872)$-Family

(Experimental Candidates of Tetra-quarks)

The three resonances $X(3872)$, $Y(3940)$, and $Y(4260)$ observed recently in Belle and BaBar experiments, may be promising candidates of the tetra-quark system. The properties of three resonances are collected in Table III.

| Decay Channel | $J^P_C$ | $I$ | $I$ (MeV) |
|---------------|---------|-----|----------|
| $\omega(\rho) + J/\psi$ | $1^{++}$ | $0$ | $< 2.3$ |
| $\omega + J/\psi$ | $2^{++}$ | $0$ | $\approx 87$ |
| $S(f_0(\rightarrow \pi^+\pi^-)).J/\psi$ | $1^-$ | $0$ | $50 - 90$ |
Fig. 3. Overview of Hadrons in $\tilde{U}(12)$-Classification Scheme. The view is obtained applying the JSQM in our covariant classification scheme, where the $q_+/q_-$ denotes Pauli/chiral urciton-spinor, respectively. The hadrons described with $q_-$ or $\bar{q}_-$ show anyhow some exotic-properties.

**Level Structure**

The $SU(2)_T$-WF of tetra-quark system in JSQM is classified, depending upon the (chiral or Pauli type) properties of constituent diquark/anti-diquark, into the three types (given in Fig. 4), where $T^{\chi\chi}$ etc. denotes tetra-quark system with the constituent diquark and anti-diquark as follows:

\[
T^{\chi\chi} \equiv [d^\chi(cq_-) \cdot d^\chi(\bar{c}\bar{q}_-) ], \quad T^{\chi P} \equiv [d^\chi(cq_-) \cdot d^P(\bar{c}\bar{q}_+) ] / [d^P(cq_+) \cdot d^\chi(\bar{c}\bar{q}_-) ], \\
T^{PP} \equiv [d^P(cq_+) \cdot d^P(\bar{c}\bar{q}_+) ]. \quad (5.1)
\]

The possible Spin-Parity values $J^P$ for constituent di-quarks and tetra-quarks are, respectively, as

Diquarks; \quad $d^P(0^+,1^+)$, \quad $d^\chi(0^-,1^-) \quad (5.2)$

Tetraquarks; \quad $T^{\chi\chi}[(0,1,2)^+]$, \quad $T^{\chi P}[(0,1,2)^-]$, \quad $T^{PP}[(0,1,2)^+]$. \quad (5.3)
Fig. 4. Classification of Tetra-quarks. The SU(2)_{\rho} WF is classified into the three groups.

(Mass Relation)

The mass of tetra-quarks are given by

\[ M_T = M_T^{(0)} + (\delta^X M_T + \delta^J M_T), \]

(5.4)

where \( M_T^{(0)} \) is \( U(12) \) symmetric; \( \delta^X M \) represents the effect of vacuum condensation \( \langle q\bar{q} \rangle_{VEV} \); and \( \delta^J M \) does the one due to hyperfine spin-spin interaction

\[ H_J \approx (\bar{c}\sigma_{\mu\nu}c)(\bar{q}U\sigma_{\mu\nu}q) \propto \langle \sigma^{(c)} \rangle \langle \sigma^{(q)} \rangle. \]

(5.5)

The \( M_T^{(0)} \) is given by, and satisfies the following equations

\[ M_T^{(0)} = \sum_{i\text{ (constit.)}} m_i = 2(m_c + m_q), \]

(5.6)

\[ M_T^{(0)} = M_d^{(0)}(cq) + M_d^{(0)}(c\bar{q}) = M_{\psi}^{(0)}(c\bar{c}) + M_M^{(0)}(q\bar{q}). \]

(5.7)

Both the \( \delta M_T^X \) and \( \delta M_T^J \) are supposed to be given as a sum of those of constituent diquarks. Then their numerical values are able to estimate by using the following formulas and the knowledge obtained from the analyses of \( D_s(c\bar{s}) \) mesons.

On \( \delta^X M \);

\[ \delta^X M(\equiv M^X - M^P) = \delta^X M_d(cq) + \delta^X M_d(c\bar{q}); \]

(5.8)

\[ \delta^X M_d(cq) = -\delta^X M_D(c\bar{q}), \]

(5.9)

\[ \delta^X M_D(c\bar{n}) = 242\text{MeV}, \]

\[ \delta^X M_D(c\bar{s}) = 348\text{MeV}. \]

(5.10)

On \( \delta^J M \);

\[ \delta^J M(\equiv M(J = 1) - M(J = 0), \]

(5.11)

\[ \delta^J M_d(cq) = \frac{1}{2} \delta^J M_D(c\bar{q}) = 71\text{MeV}. \]

(5.12)
Here I add several remarks on the equations given above:

The relation (5.7) reflects the static $U(12)$-symmetry in the ideal limit, a basic assumption in our classification scheme, and explains one of the features F1 of new exotic-hadrons mentioned in §1.

The relation (5.9) between chiral splittings $\delta^X M$ of $(cq)$ diquarks and of $D(c\bar{q})$ mesons are derived, considering the vacuum condensation effects through their constituent light-quark. It is notable that the opposite sign between the splittings reflects the charge conjugation property of the $c$-number scalar bilinear $(\bar{\Phi}1\Phi)$ of urciton spinors.

The relation (5.12) between hyperfine splittings in the two systems is derived, considering the color-gauge invariant and static $U(4)$-invariant interaction between the heavy $c$-quark and the light $q$-quark, Eq.(5.5). The same sign between the splittings reflects the properties of $c$-number tensor bilinear $(\bar{\Phi}\sigma_{\mu\nu}\Phi)$ (similarly as in Eq.(5.9)), and the factor $1/2$ represents the difference between color-space WF of the two systems.

The numerical values in (5.10) are obtained, respectively, on $\delta^X M_D(c\bar{s})$ from the experimental value, and on $\delta^X M_D(c\bar{n}) = (a/b)\delta^X M_D(c\bar{s})$ from the experimental value $a/b = 1/1.44$, where $a \propto \langle n\bar{n}\rangle_{\text{VEV}}$ and $b \propto \langle s\bar{s}\rangle_{\text{VEV}}$.

(Mass Spectra)

The level structure and mass spectra of ground state $X(3872)$ meson families, thus determined, are shown in Fig.5; where the $J^{PC}$-structure of $T^{PP}$ group is identical to that of $T^{\chi\chi}$, while the $T^{X\chi}$ group has the same $J$-structure and opposite Parity as the other groups. The mass values of corresponding members of these groups are heavier by twice of $\delta^X M_D = 242\text{MeV}$ in order of $M(\chi\chi) < M(P\chi) < M(PP)$. However, the $T^{PP}$ members are expected having too large width to be observed and not shown in Figure, see the next sub-sections.

Taking into account these considerations, three resonances $X(3872)$, $Y(3940)$, and $Y(4260)$ may be assigned as shown in Fig.5.

(Decay Mechanism)

![Fig. 5. Mass Spectra and Level Structure of Ground State X(3872) Meson Family. The mass of X(3872) is used as input, and a tentative assignment of the Y(3940) and Y(4260) is also made.](image-url)
Fig. 6. Decay of $T[(cq)(\bar{c}\bar{q})]$ into $J/\psi + M(q\bar{q})$, $D(c\bar{q}) + \bar{D}(\bar{c}q)$. In the first decay channel the two cases of $M(q\bar{q})$ are observed.

In the relevant close-to-threshold decay, the dominant amplitude is considered to come from rearrangement of constituent quarks. This process, shown in Fig.6, contains overlapping of initial and final WF.

(Overlapping of quark spinor WF and $\rho_3$-linerule)

The overlapping contains the factor concerning the light-quark line, which has the properties, as

1. at threshold $(\bar{q}U_\pm(v), q_\pm(v)) = 1$, $(\bar{q}U_\pm(v), q_\mp(v)) = 0$

2. near threshold $(\bar{q}U_\pm(v_F), q_\mp(v_F = 0)) = \frac{|P_F|}{2M_F} \approx \epsilon|P_F| \ll 1$

These two relations reflect the basic contents of static $U(4)$ symmetry, as orthonormality of spinor WF in static $U(4)_S$ space and $\rho_3$ (third component of $\rho$-spin) conservation at threshold. We called them the $\rho_3$-line rule.

(Decay Width of Tetra-quarks with ideal $SU(2)_\rho$ WF)

In Table IV we have collected the qualitative decay properties of T-mesons with the typical $SU(2)_\rho$ WF, derived from the $\rho_3$-line rule, are given.

| Structure of WF | allowed/forbidden | Structure of Decay Channel | Wirth $\gamma$ |
|-----------------|-------------------|-----------------------------|---------------|
| $T[(cq_+)(\bar{c}\bar{q}_+)]$ | $\Rightarrow$ (allowed) | $D(cq_+) + \bar{D}(\bar{c}\bar{q}_+)$ | $\Gamma^0$ a few GeV |
| $T[(cq_\pm)(\bar{c}\bar{q}_\mp)]$ | $\Rightarrow$ (first forbidden) | $\psi(c\bar{c}) + M(q_\pm\bar{q}_\mp)$ | $\epsilon(P)^2 \Gamma^0 < \Gamma^0$ |
| $T[(cq_-)(\bar{c}\bar{q}_-)]$ | $\Rightarrow$ (second forbidden) | $D(cq_-) + \bar{D}(\bar{c}\bar{q}_-)$ | $\epsilon(P)^4 \Gamma^0 \ll \Gamma^0$ |

The inspection of this qualitative decay properties, in addition to the other results in this section, led us to the assignments of the $X(3872)$ meson families shown in Fig.5. Thus the $\rho_3$-rule seems to be able to explain the feature F2 of the new “exotic” hadrons, mentioned in §1.
5.2. Near-Threshold Resonances observed at BES

(Experimental Data)

Two Resonances\(^{12}\) with the following properties have been observed in \(J/\psi \rightarrow \gamma + X;\)
X[\(\omega \cdot \phi\)] \(\rightarrow \omega + \pi;\) \(M = 1810\text{MeV} \rightarrow \text{close to } M_{\text{th}}(\omega + \pi) = 1802\text{MeV},\)
\(\Gamma = 105\text{MeV}, (I, J^{PC}) = (0, 0^{++}).\)

\(X[p \cdot \bar{p}] \rightarrow p + \bar{p}; M = 1859\text{MeV} \rightarrow \text{close to } M_{\text{th}}(p + \bar{p}) = 1876\text{MeV},\)
\(\Gamma < 28\text{MeV}, J^{PC} = 0^{--}.\)

(Possible Structure in JSQM)

They seem to be identified as the four-quark (\(qq\bar{q}\)) and six-quark (\(qqq\bar{q}\bar{q}\)) systems, respectively with the structure shown in Fig. 7 as follows:

![Diagram](image.png)

**Fig. 7.** Possible Structure of \(X[\omega \cdot \phi]\) and \(X[p \cdot \bar{p}]\)

1. \(X[\omega \cdot \phi]\)
   - \(J^{PC}: 1^- \otimes 1^- = 0^{++}, 1^{+-}, 2^{++}\)
   - Mass: \(M_{X}^{(0)}[\omega \cdot \phi] = M_{\phi}^{(0)}(ss) + M_{\omega}^{(0)}(n\bar{n})\)
   - Decay: \(X[\omega \cdot \phi] \rightarrow \phi(s_+\bar{s}_+) + \omega(n_+\bar{n}_+)\); doubly forbidden by the \(\rho_3\)-line rule. Accordingly decay width to be \(\Gamma = |\epsilon(P)|^4 \Gamma^{(0)} < \Gamma^{(0)}.\)

2. \(X[p \cdot \bar{p}]\)
   - Mass: \(M_{X}^{(0)}[p \cdot \bar{p}] = M_{p}^{(0)}(nnn) + M_{\bar{p}}^{(0)}(n\bar{n}\bar{n})\)
   - Decay: \(X[p \cdot \bar{p}] \rightarrow N(n_+n_+n_+) + \bar{N}(n_+\bar{n}_+\bar{n}_+)\); forbidden by the \(\rho_3\)-line rule in the fourth. Accordingly \(\Gamma = |\epsilon(P)|^8 \Gamma^{(0)} \ll \Gamma^{(0)}.\)

These tentative identification has been made, considering only the features F1 and F2 of exotic hadrons (§1). Detailed theoretical investigations are necessary to ascertain it.
§6. Concluding Remarks

We have reviewed the essentials of covariant framework in the $\tilde{U}(12)$ classification scheme, where is predicted the existence of chiral states, new type of hadrons out of the conventional scheme. Hadron spectroscopy seems to be presently in serious difficulty, observing a new type of exotic hadrons successively. In going way out of this difficulty the notion of chiral states, closely connected to quark-confinement, is believed to play an important role. BES experiment is considered to be BEST place to seek for them. The serious correspondence to this situation be important and urgent!

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Appendix A

Present Picture of Hadrons and a History of Level-Classification

A seminar talk, of which theme is closely related to that of this report, was given at the Peking University on February 24, 2006 by the present author. The title and contents were as follows:

On the $\tilde{U}(12)_{SF}$ classification Scheme and a $U(4)$ Symmetry for the Confined Light-Quarks; §0 Preliminaries, §1 Present Status of Hadron Spectroscopy, §2 Some Basic Consideration on Quark Confinement, §3 Essentials of Description of Composite Hadrons in $\tilde{U}(12)$-Scheme, §4 Concluding Remarks.

The contents of §1, §2 and §3, are almost the same as those in the present talk. Here I give a summary of the other parts not given here.

(Proposal of $\tilde{U}(12)_{SF}$-Level Classification Scheme)

The $\tilde{U}(12)_{SF}$-classification scheme had been proposed\(^2\) by us several years ago as a kinematical framework so as to correspond to the relativistic properties of composite hadrons. In this scheme we have seriously regarded the two general principles to be satisfied by any physics theory:

P-E. Lorentz Covariance in describing the space-time structure of composite hadrons.

P-H. Observable Notion in connection to the confinement of constituent quarks.

(Non-Relativistic Picture of Hadrons and Facing Problem)
The hadron is conventionally regarded as the color-singlet bound-states of quarks and anti-quarks, where the constituent quarks seem to obey the Non-Relativistic quantum mechanics and have approximately the $SU(6)_{SF} \otimes O(3)_L$ symmetry. On the contrary, the composite hadrons are surely relativistic entities, since the pion, for example, has the properties of Nambu-Goldstone boson in the case of spontaneously broken-chiral symmetry. Accordingly we are facing the problem of extending the above NR picture relativistically.

(Difficulty of Relativistic Extension)

One the most natural covariant-extension of $SU(6)_{SF} \otimes O(3)_L$ scheme is thought to be the $\tilde{U}(12)_{SF} \otimes O(3,1)_{Lorentz}$, treating separately the external (center of mass) and the internal (relative) coordinates of composite hadrons. However, this way of extension seems to be almost closed:

On the one hand, the original theory\textsuperscript{13} of $\tilde{U}(12)_{SF}$ symmetry, which was the first attempt of the extension of $SU(6)_{SF}$, had been shown to have the serious difficulty as follows.

First the physical state condition (restricting within the boosted-Pauli spinors) reduces to the violation of unitarity\textsuperscript{14}, and secondly the “no-go-theorem\textsuperscript{15}” states that a relativistic extension of the $SU(6)_{SF}$ symmetry is impossible.

On the other hand, in the framework of bilocal-field theory by Yukawa (which is regarded as the first attempt considering covariantly the spatial-extension of composite system, ) it had been shown that the framework leads to the violation of general principle such as causality, unitarity and even Lorentz-covariance. This implies also another “no-go-theorem” against the above way of extension.

Here we remind ourselves (consciously) that there exists, presently, no consistent non-local (to represent the internal spatial-extension), quantum field theory. As a matter of fact that various attempts for relativistic generalization of the above hadron picture ever-appeared have anyhow some unsatisfactory points as follows:

The Bethe-Salpeter equation for relativistic bound-state problem starts from the asymptotically free quark states as the S-matrix bases, being against the principle P-H.

The heavy quark effective theory is still a static theory, being against the principal P-E, because the heavy-quark stays at rest.

The method of effective Lagrangian is, able to treat the chiral symmetry, but the framework is a local field theory, unable to treat the spatial extension of hadrons.

(Exciton Picture of Quarks and Static $U(12)_{SF}$ Symmetry in the $\tilde{U}(12)$-scheme)

In the new $\tilde{U}(12)_{SF}$-classification scheme the constituent quark is regarded as, not an elementary entity of bound state hadrons but the exciton\textsuperscript{4}, which is a mathematical entity simulating the center of mass motion of confined quarks inside relevant hadrons.

The $U(12)_{SF}(\supset U(4)_S \otimes SU(3)_F)$ symmetry is embedded \textsuperscript{4} in the covariant $\tilde{U}(12)_{SF} \otimes$

\textsuperscript{4} As an origin of this thought I should like to refer to the $SU(6)_{SF}$ symmetry as a “rest condition”\textsuperscript{16}.
Table V. A Histirical Road led to $\tilde{U}(12)$-Classification Scheme

| Year | Name | Description |
|------|------|-------------|
| 1950 | H. Yukawa | Bi-local Field Theory; To avoid divergence. Wave Function $\Phi(x_{\mu}, r_{\mu})$ violates, in higher orders, covariance, causality and unitarity(C. Hayashi). |
| 1953 | H. Yukawa | Covar. Oscillator WF; Freedom on $r_{\mu}$ gives mass spectra of mesons. $\to$ Oscillator Model $\sim$1970 (Sogami, Feynman, Namiki, COQM). |
| 1956 | S. Sakata | Composite Model; Origin of flavor $[q_{a} : a = (1,2,3)]$ $\to$ $SU(3)_{F}$ symmetry $\sim$ 1959 (Ikeda-Ogawa-Ohnuki, Yamaguchi). |
| 1964 | M. Gell-Mann & G. Zweig | Quark Model; Addition of $SU(6)_{SF}$ symmetry $\sim$ 1964 (F. Gursey and L.A.Radicati, B. Sakita). |
| 1965 | A. Salam et al. & B. Sakita-K.C.Wali | “Original” $\tilde{U}(12)$-symmetry; Attempt for relativ. ext. of $SU(6)_{SF}$ symmetry $[q_{a,\alpha} : \alpha = (1 \sim 4)]$. $\tilde{U}(12)_{SF} \supset SU(3)_{F} \otimes SU(2)_{\sigma}$ with physical state condition. Physical states be Boosted-Pauli spinors $\to$ “Violation of unitarity.” |
| 1967 | S. Coleman & J. Mandula | “No-Go Theorem”; Lorentz covariant symm. including $SU(6)_{SF} \supset SU(3)_{F} \otimes SU(2)_{\sigma}$, not existing. |
| 1968 | S. Ishida-P. Roman | $SU(6)_{SF}$ Symm.” as “Rest Condition”. |
| 1970 | S. Ishida, M. Oda, K. Yamada | Urcton Scheme; urciton, simulating confined quarks, WF $\Phi_{A}^{\lambda}(X_{\mu}, r_{\mu}) \ A = (a, \alpha) :$ Tensors in $\tilde{U}(12)_{SF} \otimes O(3,1)$ space with $SU(6)_{SF} \otimes O(3)_{L}$ symm. “at Rest”. $\sim$ The kinematical framework (still keeping P.S. condition and suffering from the “violation of unitarity.”), COQM, had been applied to Born term of hadron-reactions. |
| 2000 | S.Ishida-M.Ishida & T.Maeda | New $\tilde{U}(12)$ level classification scheme ; Static $U(12)_{SF} \otimes O(3)_{L}$ symm. embedded in $\tilde{U}(12)_{SF} \otimes O(3,1)$ space. |
| 2002 | S.Ishida-M.Ishida | Free from “No-Go Theorem” and “Violation of Unitarity” |

$O(3,1)_{L}$ Lorentz space in the hadron rest-frame. Accordingly our new scheme is out of the above mentioned “no-go-theorem”.

Furthermore, it is also free from the trouble related to unitarity, since there is now no need for the physical-state condition. The “unphysical” states (at that time), described with chiral(urciton)-spinors, are now considered to be an origin of promising candidates of new “exotic hadrons” out of the conventional NR-classification scheme.

In the Table V a brief history of the hadron symmetry and classification, in relation to the $\tilde{U}(12)$-classification scheme, is given.

(Implication of Possible Existence of Chiral States)

In the $\tilde{U}(12)$-classification scheme the existence of chiral states, which are out of the conventional NR-classification scheme, is predicted.

Their confirmation would imply the realization in nature of the lowest and the higher-number-index solutions of the original Dirac Equation as representing the leptons and the hadrons, respectively. The equation had been proposed long ago, as is well known, in order to derive the Lorentz-covariant (first-quantized) wave equation without negative-energy solutions.
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