Breached superfluidity of fermionic atoms in magnetic field.

G.M.Genkin∗.

Physics Department and Center for Polymer Studies, Boston University, Boston, MA 02215.

Abstract

We derived the energy gap of a breached pairing superfluidity phase of fermionic atoms in an external magnetic field in Feshbach resonance experiments which is determined by the magnetic-field detuning from the Feshbach resonance. We show that a BCS superfluid state exists only for the magnetic-field detuning smaller than one critical, and this critical magnetic-field detuning is determined by the equality of the Zeeman energy splitting for the magnetic-field detuning to the energy gap $\Delta_0$.

03.75.Ss, 03.75.Kk
The trapping and cooling of gases with Fermi statistics has become one of the central areas of research within the field of ultracold atomic gases. Much progress has been made in the achievement of degenerate regimes of trapped atomic Fermi gases \([1 - 5]\). The major goals of studies of these systems is to observe a transition to a paired - fermion superfluid state. There has been considerable interest in achieving superfluidity in an ultracold trapped Fermi gas in which a Feshbach resonance is used to tune the interatomic attraction by variation of a magnetic field. The interactions which drive the pairing in these gases can be controlled using a Feshbach resonance, in which a molecular level is Zeeman tuned through zero binding energy using an external magnetic field. Via a Feshbach resonance it is possible to tune the strength and the sign of the effective interaction between particles. In result, magnetic - field Feshbach resonances provide the means for controlling the strength of cold atom interactions, characterized \(s\) - wave scattering length \(a\), as well as whether they are effectively repulsive \((a > 0)\) or attractive \((a < 0)\). Therefore, the tunability of interactions in fermionic atoms provides a unique possibility to explore the Bose - Einstein condensate to Bardeen - Cooper -Schrieffer (BEC - BCS) crossover \([6 - 8]\), an intriguing interplay between the superfluidity of bosons and Cooper pairing of fermions. A Feshbach resonance offers the unique possibility to study the crossover between situations governed by Bose - Einstein and Fermi - Dirac statistics. When the scattering length \(a\) is positive the atoms pair in a bound molecular state and these bosonic dimers can form a Bose - Einstein condensate; when \(a\) is negative, one expects the well - known BCS model for superconductivity to be valid.

On the other hand, recently has been considerable interest in the study of asymmetric fermionic systems and the possibility of new form of superfluidity in this systems. An interesting new phase in Fermi matter, termed as breached pairing state, has been recently predicted by Liu and Wilczek \([9]\). This pairing phenomenon gives rise to a superfluidity having superfluid and normal Fermi liquid components simultaneously. The pairing between fermions with different Fermi momenta in asymmetric fermion systems produces a state with coexisting superfluid and normal fluids. Possible realization of this phase was
considered for different Fermi systems (an analogous state for superconductivity in conventional superconductors in a strong spin-exchange field was predicted many years ago by Sarma [10], in ultracold fermionic atom systems composed of two particle species with different densities and unequal masses [11], in quantum chromodynamics in the context of color superconductivity [12]).

We will consider the breached superfluidity of cold fermionic atoms in an external magnetic field under conditions of Feshbach resonance experiments. Usually, these experiments are initiated by preparing atoms in a mixture of states with different spin projections (for example, spin-up and spin-down) with equal populations. We show that the energy gap parameter of a breached pairing superfluidity state in Feshbach resonance experiments is determined by the magnetic-field detuning $\Delta B$ from the Feshbach resonance. We show that a BCS superfluid state (SF) exists only for the magnetic-field detuning smaller than one critical, and this critical magnetic-field detuning is determined by the equality of the Zeeman energy splitting for the magnetic-field detuning to the energy gap $\Delta_0$ of fermionic atoms without a magnetic field. A SF state is lost for $\Delta B$ larger than a critical. For example, for $^6$Li atoms this critical magnetic-field detuning $\Delta B_{cr}$ is order of $200mG$, and for $\Delta B > 200mG$ a SF state is lost. Note that the destroying a superfluid state for the magnetic-field detuning $\Delta B$ larger than $\Delta B_{cr}$ corresponds to a well-known dichotomy between superconductivity and ferromagnetism. The strong field destroys the superfluid state when the field is strong enough to break Cooper pairs, and an external magnetic field provides the pair-breaking mechanism.

We consider an uniform gas of Fermi atoms with two hyperfine (spin-up $\uparrow$ and spin-down $\downarrow$) states in an external magnetic field. Our starting model can be described by a Hamiltonian

$$H - \mu_\uparrow N_\uparrow - \mu_\downarrow N_\downarrow = \sum_p [(\varepsilon_p - \mu_\uparrow - \mu_{mag}B) a_{p\uparrow}^+ a_{p\uparrow} + (\varepsilon_p - \mu_\downarrow + \mu_{mag}B) a_{p\downarrow}^+ a_{p\downarrow}] - \frac{g}{V} \sum_{p,q} a_{p\uparrow}^+ a_{-p\downarrow}^+ a_{-q\downarrow} a_{q\uparrow},$$

with the coupling constant $g = \frac{4\pi\hbar^2|a|}{m}$, volume $V$. Here a coupling constant corresponds
the attractive ( \( a < 0 \) ) pairing interaction; \( a_{p\uparrow}a_{p\uparrow}^\dagger \) represent the annihilation (creation) operators of a Fermi atom with the kinetic energy \( \varepsilon_p = \frac{p^2}{2m} \), and \( \mu \) is the chemical potential. The Zeeman energy in an external magnetic field \( \mathbf{B} \) is \( -\beta \sigma \mathbf{B} \), and corresponding terms for spins \( \uparrow \) and \( \downarrow \) are \( \pm \mu_{\text{mag}} B \) where \( \mu_{\text{mag}} \) is the atomic magnetic moment. In general, \( \mu_\uparrow \) and \( \mu_\downarrow \) are not equal. Usually, in Feshbach resonance experiments the chemical potentials of the states are determined by experimental densities. In these experiments there is using the radiofrequency driving the Zeeman transition \( \Delta_{Zee}^{(0)} \) between spin states \( \uparrow \) and \( \downarrow \), and these states are preparing with equal populations. Here \( \Delta_{Zee}^{(0)} \) is the Zeeman energy splitting for the magnetic field \( B_0 \), i.e. \( \Delta_{Zee}^{(0)} = 2\mu_{\text{mag}} B_0 \), and \( B_0 \) is the magnetic field in the vicinity of the Feshbach resonance. Due to equal populations of spin states \( \uparrow \) and \( \downarrow \) we will assume
\[
\mu_\downarrow = \mu_\uparrow + \Delta_{Zee}^{(0)}.
\] (2)
It is correspond to well-known statement [13] that for a spin in a magnetic field we may assume that the chemical potential is equal to \( \mu_\uparrow + \mu_{\text{mag}} B \) for atoms with the spin projection \( \uparrow \) along the field and to \( \mu_\downarrow - \mu_{\text{mag}} B \) for atoms with the spin projection \( \downarrow \) opposite to the field. In the Hamiltonian (Eq. (1)) we introduce the standard canonical transformation to the Bogolyubov quasiparticles
\[
a_{p\uparrow} = u_p b_{p\uparrow} + v_p b_{-p\downarrow}^\dagger,
\]
\[
a_{p\downarrow} = u_p b_{p\downarrow} - v_p b_{-p\uparrow}^\dagger,
\] (3)
where the coefficients \( u_p \) and \( v_p \) are real, depend only on \(|p|\). They are chosen from the condition [14] that the energy \( E \) of the system has a minimum for a given entropy. We have
\[
E = \sum_p (\varepsilon_p - \mu_\uparrow - \mu_{\text{mag}} B)[u_p^2 n_{p\uparrow} + v_p^2 (1 - n_{p\downarrow})] + \varepsilon_p - \mu_\downarrow + \mu_{\text{mag}} B)[u_p^2 n_{p\downarrow} + v_p^2 (1 - n_{p\uparrow})] - \frac{g}{V} \sum_p u_p v_p (1 - n_{p\uparrow} - n_{p\downarrow})^2.
\] (4)
Varying this expression with respect to the parameter \( u_p \) and using the relation \( u_p^2 + v_p^2 = 1 \), which the transformation coefficients must be satisfy, the condition for a minimum is \( \frac{\delta E}{\delta u_p} = 0 \).
Using this condition for Eq.(4) we have the standard [14] equation for the energy gap

\[ \frac{g}{2V} \sum_p \frac{(1 - n_{p\uparrow} - n_{p\downarrow})}{\sqrt{\Delta^2 + \eta_p^2}} = 1, \]  

(5)

where the energy gap

\[ \Delta = \frac{g}{V} \sum_p u_p v_p (1 - n_{p\uparrow} - n_{p\downarrow}) \]  

(6a)

and

\[ u_p^2 = \frac{1}{2} \left( 1 + \frac{\varepsilon_p^+}{\sqrt{\Delta^2 + \varepsilon_p^+}} \right), \quad v_p^2 = \frac{1}{2} \left( 1 - \frac{\varepsilon_p^+}{\sqrt{\Delta^2 + \varepsilon_p^+}} \right), \]  

(6b)

where

\[ 2\varepsilon_p^+ = (\varepsilon_p - \mu\uparrow - \mu_{mag} B) + (\varepsilon_p - \mu\downarrow + \mu_{mag} B) = 2\varepsilon_p - (\mu\uparrow + \mu\downarrow), \]  

(6c)

and the quasiparticle occupation numbers \( n_{p\alpha} = b_{p\alpha}^0 \). Note that due to the standard canonical transformation ( Eq.(3)) the Cooper pair has the zero summary momentum, therefore, we have an uniform energy gap \( \Delta \) ( Eq.(5)). The energy of the elementary excitations \( E_\uparrow(p), E_\downarrow(p) \) can be find [14] from the change of the energy \( E \) of the system when the quasiparticle occupation numbers are changing, i.e. by varying \( E \) with respect to \( n_{p\uparrow} \) and \( n_{p\downarrow} \). Therefore, start from the equation

\[ \delta E = \sum_p [E_\uparrow(p) \delta n_{p\uparrow} + E_\downarrow(p) \delta n_{p\downarrow}], \]

we have two branches of quasiparticle excitations with the spectra

\[ E_\uparrow(p) = \frac{\delta E}{\delta n_{p\uparrow}} = \sqrt{\varepsilon_p^+ + \Delta^2} - \frac{1}{2} \delta \Delta_{Zee}, \]

\[ E_\downarrow(p) = \frac{\delta E}{\delta n_{p\downarrow}} = \sqrt{\varepsilon_p^+ + \Delta^2} + \frac{1}{2} \delta \Delta_{Zee}, \]

\[ \delta \Delta_{Zee} = \Delta_{Zee} - \Delta_{Zee}^{(0)} = 2\mu_{mag}(B - B_0). \]

(7)

The quasiparticle occupation numbers satisfy Fermi - Dirac statistics.

For \( |\delta \Delta_{Zee}| < 2\Delta \) we have a BCS superfluidity with both gapped excitations ( Eqs.(7)). Meanwhile, for \( |\delta \Delta_{Zee}| > 2\Delta \) one branch of quasiparticle excitations ( for \( \Delta B = B - B_0 > 0 \)
the branch \( E_\uparrow(p) \), for \( \Delta B < 0 \) the branch \( E_\downarrow(p) \) may be negative for momenta \( p_1 \leq p \leq p_2 \) where

\[
\frac{p_{1,2}^2}{2m} = \frac{\mu_\uparrow + \mu_\downarrow}{2} \pm \sqrt{\frac{1}{4}\delta^2_{\text{Zee}} - \Delta^2},
\]

and we have a breached pairing superfluidity with one gapped excitation branch and one gapless. In this case for finding the energy gap \( \Delta \) we will follow the method of Ref.\[13\], Chapter 21.2. In the BCS gap equation (Eq. (5)) using cutting off \[14\] the logarithmic integral at same \( \eta = \bar{\epsilon} \) (\( \bar{\epsilon} \) is the ultraviolet cutoff) we have the well-known result for the energy gap \( \Delta_0(g) \) for \( T = 0 \) and \( B = 0 \)

\[
\ln \frac{\bar{\epsilon}}{\Delta_0(g)} = \frac{2\pi^2 h^3}{gmp_F}.
\]

For \( B \neq 0 \), in general, the energy gap depends on the magnetic field \( \Delta(g, B) \). If \( \Delta(g, B) < \frac{1}{2}|\delta \Delta_{\text{Zee}}B| \) there is a nonzero minimal value in the integral (Eq. (5)) \( \bar{\epsilon}_{\mu(\text{min})} = \sqrt{\frac{1}{4}\delta^2_{\text{Zee}} - \Delta^2} > 0 \), and using the table integral \( \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln|x + \sqrt{x^2 + a^2}| \) for \( \Delta < \mu_{\text{mag}}|\Delta B| \) we have in this case

\[
\ln \frac{\bar{\epsilon}}{\Delta_0(g)} + \sqrt{\frac{1}{4}\delta^2_{\text{Zee}} - \Delta^2(g, B)} = \frac{2\pi^2 h^3}{gmp_F}.
\]

By comparing Eqs. (9) the energy gap \( \Delta(g, B) \) is determined by the equation

\[
\frac{|\delta \Delta_{\text{Zee}}|}{2} + \sqrt{\frac{1}{4}\delta^2_{\text{Zee}} - \Delta^2(g, B)} = \Delta_0(g),
\]

or

\[
\Delta(g, B) = \sqrt{\Delta_0(g)(|\delta \Delta_{\text{Zee}}| - \Delta_0(g))},
\]

\[
\delta \Delta_{\text{Zee}} = 2\mu_{\text{mag}}|\Delta B|, \Delta B = B - B_0.
\]

We derived the energy gap \( \Delta(g, B) \) of the breached pairing superfluidity phase for fermions in an external magnetic field under actual conditions of Feshbach resonance experiments. This breached pairing superfluidity phase exists in the range \( 0 \leq \Delta(g, B) \leq \Delta_0(g) \) for \( \Delta_0(g) \leq \delta \Delta_{\text{Zee}} \leq 2\Delta_0(g) \). The equation (10a) has a threshold at the value \( |\delta \Delta^\text{cr}_{\text{Zee}}| = \Delta_0(g) \) for which this equation first has solutions for \( |\Delta B| \geq \Delta B^\text{cr} \), and, accordingly, the energy gap.
\( \Delta(g, B) \) has a solution for \( |\Delta B| \geq \Delta B_{cr} \). In result, we have a breached pairing superfluidity phase with the critical magnetic - field detuning from the Feshbach resonance \( \Delta B_{cr} \).

Note that for fermions in an external magnetic field for \( \mu_\uparrow = \mu_\downarrow \) ( without a radiofrequency driving ) the energy gap parameter of the breached pairing superfluidity phase is determined by \( \Delta_{zee} = 2\mu_{mag}B \) instead \( \delta\Delta_{zee} \) ( Eq.(10b)), and in this case for the conventional superconductors in a strong exchange field \( B_{ex} \) there is a Sarma unstable phase [10] with \( \Delta_{zee}^{ex} = 2\mu_{mag}B_{ex} \) in Eq.(10b).

We consider the thermodynamic properties of Fermi atoms with two hyperfine states in the breached pairing superfluidity phase (for the magnetic - field detuning \( |\Delta B| \geq \Delta B_{cr} \)). In calculations it is convenient to start from the thermodynamic potential \( \Omega \), and the condensation energy ( the difference between the thermodynamic potential \( \Omega_s \) in the superfluid state and the value in the normal state \( \Omega_n \) at the same temperature [14]) is given by ( at \( T = 0 \))

\[
\Omega_s - \Omega_n = - \int_0^g \frac{\Delta^2(g_1)}{g_1} dg_1. \tag{11}
\]

Changing in Eq.(11) from integration over \( dg_1 \) to that over \( d\Delta \), and using Eq.(10b) we obtain the difference between the ground - state energies of the superfluid \( E_s \) and normal \( E_n \) states for magnetic - field detuning \( 2\mu_{mag}|\Delta B| \geq \Delta_0 \), or \( |\delta\Delta_{zee}| \geq \Delta_0 \)

\[
E_s - E_n = \frac{mp_F}{4\pi^2\hbar^3}(\delta\Delta_{zee} - \Delta_0)^2. \tag{12a}
\]

The positive sign of this difference \( E_s - E_n > 0 \) indicates that for magnetic - field detuning \( |\delta\Delta_{zee}| > \Delta_0 \) the normal state has a smaller energy. The condensation energy is positive, indicating that this breached superfluidity phase is unstable, and the normal state is energetically favored. For magnetic - field detuning \( |\delta\Delta_{zee}| < \Delta_0 \) we have a BCS state. In result, our system passes from a BCS state to an unstable breached superfluidity state as the magnetic - field detuning from the Feshbach resonance \( |\Delta B| = |B - B_0| \) increases to a critical value

\[
\Delta B_{cr} = \frac{\Delta_0}{2\mu_{mag}}, \tag{13a}
\]
i.e. a critical value of the Zeeman splitting for the magnetic-field detuning $\delta \Delta_{Zee}^\text{cr} = 2\mu_{\text{mag}}\Delta B_{\text{cr}}$ is

$$\delta \Delta_{Zee}^\text{cr} = \Delta_0.$$ \hspace{1cm} (13b)

By means of Eq.(13) we have

$$\Delta B_{\text{cr}} = 2\frac{\Delta_0}{\Delta_{Zee}^\text{(0)}} B_0.$$ \hspace{1cm} (14)

For example, for $^6\text{Li}$ atoms a Feshbach resonance is located [15] at $B_0 \simeq 850G$, the Fermi energy [15] $\varepsilon_F \simeq 1.1\mu K$, and in experiments there is driving the Zeeman transition $\Delta_{Zee}^\text{(0)}$ between the $|1/2, 1/2>$ and $|1/2, -1/2>$ states with [16] $\Delta_{Zee}^\text{(0)} = 76MHz$ rf field. In the present experiments $\Delta_0/\varepsilon_F = 0.2 - 0.4$. For $\Delta_0/\varepsilon_F \simeq 0.4$ we have $\Delta B_{\text{cr}} \simeq 200mG$ for $^6\text{Li}$ atoms, and for $\Delta B > 200mG$ a SF state is lost.

For fermions in an external magnetic field with $\mu_\uparrow = \mu_\downarrow$ a BCS superfluid state exists only for the magnetic field $B < B_{\text{cr}}$, and this critical magnetic field $B_{\text{cr}}$ which destroys the SF state is determined from the condition that the Zeeman energy splitting $\Delta_{Zee}^\text{(0)}$ equal to the energy gap $\Delta_0$ without a magnetic field. Note that, although, an external magnetic field does not penetrate a bulk conventional superconductor, the strength of the field required to overcome the energy gap, i.e. to destroy superconductivity in conventional superconductors, is much more than for atomic Fermi gases because the energy gap $\Delta_{\text{sup}}^\text{0}$ of the conventional superconductors is much more than $\Delta_0$ for atomic Fermi gases, usually, $\Delta_{\text{sup}}^\text{0}/\Delta_0 > 10^6$.

In summary, we show that for fermionic atoms in an external magnetic field in Feshbach resonance experiments there is a breached pairing superfluidity for the magnetic-field detuning from the Feshbach resonance larger than one critical. We derived the energy gap parameter of this phase which is determined by the magnetic-field detuning; this unstable phase exists in the certain range of the Zeeman energy splitting determinable by the energy gap $\Delta_0$. We show that a BCS superfluid state exists only for the magnetic-field detuning from the Feshbach resonance smaller than the critical value, and this critical magnetic-field detuning is determined by the equality of the Zeeman energy splitting for the magnetic-field detuning to the energy gap $\Delta_0$. 

8
I thank H. E. Stanley for encouragement, J. Borreguero for assistance.
REFERENCES

* Electronic address: ggenkin@argento.bu.edu.

[1] B. DeMarco and D. S. Jin, Science 285, 1703 (1999).

[2] A. G. Truscott, K. E. Strecker, W. I. Mc Alexander, G. Partridge, and R. G. Hulet, Science 291, 2570 (2001).

[3] F. Schreck, L. Khaykovich, K. L. Corwin, G. Ferrari, T. Bourdel, J. Cubizolles, and C. Salomon, Phys. Rev. Lett. 87, 080403 (2001).

[4] S. R. Granade, M. E. Gehm, K. M. O’ Hara, and J. E. Thomas, Phys. Rev. Lett. 88, 120405 (2002).

[5] Z. Hadzibabic, S. Gupta, C. A. Stan, C. H. Schunck, M. W. Zwierlein, K. Dieckmann, and W. Ketterle, Phys. Rev. Lett. 91, 160401 (2003).

[6] E. Timmermans, K. Furuga, P.M. Milonni, and A. K. Kerman, Phys. Lett. A285, 228 (2001).

[7] M. Holland, S. M. F. Kokkelmans, M. L. Chiofalo, and R. Walser, Phys. Rev. Lett. 87, 120406 (2001).

[8] Y. Ohashi and A. Griffin, Phys. Rev. Lett. 89, 130402 (2002).

[9] W. V. Liu and F. Wilczek, Phys. Rev. Lett. 90, 047002 (2003).

[10] G. Sarma, J. Phys. Chem. Solids 24, 1029 (1963).

[11] P. F. Bedaque, H. Caldas, and G. Rupak, Phys. Rev. Lett. 91, 247002 (2003).

[12] E. Gubankova, W. V. Liu, and F. Wilczek, Phys. Rev. Lett. 91, 032001 (2003).

[13] See, for example, A. A. Abrikosov, *Fundamentals of the theory of metals* (North-Holland, New York, 1988), Chapter 10.1.

[14] See, for example, E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics*, Part 2 (
[15] M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, C. Chin, J. Hecker Denschlay, and R. Grimm, Phys. Rev. Lett. **92**, 120401 (2004).

[16] T. Bourdel, L. Khaykovich, J. Cubizolles, J. Zhang, F. Chevy, M. Teichman, L. Tarruell, S. M. F. Kokkelmans, and C. Salomon, Phys. Rev. Lett. **93**, 050401 (2004).