Aspects of SU(3) baryon extrapolation

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Abstract. We report on a recent chiral extrapolation, based on an SU(3) framework, of octet baryon masses calculated in 2+1-flavour lattice QCD. Here we further clarify the form of the extrapolation, the estimation of the infinite-volume limit, the extracted low-energy constants and the corrections in the strange-quark mass.

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In recent times, there has been significant advances in numerical studies of lattice-regularised QCD. In particular, the past year has seen several studies of baryon systems using 3 flavours of light dynamical quarks, including Refs. [1, 2, 3, 4, 5, 6]. While the quark masses are relatively light, one still requires an extrapolation to the physical point to make comparison with observables. Here we highlight some features of a recent SU(3) chiral extrapolation of octet baryon masses [7].

The chiral expansion of baryon masses have been studied extensively, eg. see [8, 9]. For our formulae, we most closely follow the work of Walker-Loud [10], writing

\[ M_B = M_B^{(0)} + \delta M_B^{(1)} + \delta M_B^{(3/2)} + \ldots \] (1)

The leading term, \( M_B^{(0)} \), denotes the degenerate mass of the baryon octet in the SU(3) chiral limit — where \( m_u = m_d = m_s = 0 \). The leading corrections, as one moves away from this limit, are linear in the quark mass. Assuming SU(2) symmetry in the light quark sector, with \( m_l = (m_u + m_d) / 2 \), these leading corrections can be written as

\[ \delta M_B^{(1)} = -C_B^{(1)} m_l - C_B^{(1)} m_s, \] (2)

with coefficients \( C_B^{(1)} \) listed in Table 1 [10]. We make a convenient substitution by reexpressing the expansion in terms of \( m_\pi \) and \( m_K \), with \( m_l \to m_\pi^2 / 2 \) and \( m_s \to (m_K^2 - m_\pi^2) / 2 \). The coefficients, \( \alpha_M \), \( \beta_M \) and \( \sigma_M \) are then each redefined by a dimensionful scale factor, as similarly done in Ref. [1]. To the order we work in this manuscript, the use of either quark masses or meson masses squared is equivalent.

Beyond this linear order come the leading loop corrections, which have long been known to generate large corrections (at the physical quark masses), up to \( O(100\%) \) of the leading terms [8, 9]. This problem in the SU(3) expansion has been overcome through introducing a finite scale into the regularization of these loop diagrams [9]. Concurrently, the introduction of a finite regularization scale was demonstrated to tame the chiral extrapolation problem for lattice QCD [11]. While these early studies required the regularization scale to be input from phenomenology, this study uses the lattice results themselves to pick this scale.
TABLE 1. Coefficients for the leading quark-mass expansion of the octet baryons about the SU(3) chiral limit.

|   | \( C_{B1}^{(1)} | \( C_{B5}^{(1)} |
|---|-----------------|------------------|
| \( B \) | \( 2\alpha_M + 2\beta_M + 4\sigma_M \) | \( 2\sigma_M \) |
| \( N \) | \( \alpha_M + 2\beta_M + 4\sigma_M \) | \( \alpha_M + 2\sigma_M \) |
| \( \Lambda \) | \( \frac{5}{2}\alpha_M + \frac{5}{2}\beta_M + 4\sigma_M \) | \( \frac{1}{2}\alpha_M + \frac{3}{2}\beta_M + 2\sigma_M \) |
| \( \Sigma \) | \( \frac{1}{2}\alpha_M + \frac{3}{2}\beta_M + 4\sigma_M \) | \( \frac{1}{2}\alpha_M + \frac{5}{2}\beta_M + 2\sigma_M \) |

TABLE 2. The relevant coefficients for the one-loop diagrams involving intermediate octet and decuplet baryons. These contributions are counted at the same chiral order in the present scheme.

|   | \( \pi \) | \( \chi_{B\phi} K \) | \( \eta \) | \( \pi \) | \( \chi_{T\phi} K \) | \( \eta \) |
|---|----------|-----------------|---|---|-----------------|---|
| \( N \) | \( \frac{3}{2}(D + F)^2 \) | \( \frac{1}{2}(5D^2 - 6DF + 9F^2) \) | \( \frac{1}{2}(D - 3F)^2 \) | \( \frac{1}{4}C^2 \) | \( \frac{1}{4}C^2 \) | \( 0 \) |
| \( \Lambda \) | \( 2D^2 \) | \( \frac{3}{2}(D^2 + 9F^2) \) | \( \frac{1}{2}D^2 \) | \( C^2 \) | \( \frac{3}{2}C^2 \) | \( 2C^2 \) |
| \( \Sigma \) | \( \frac{1}{2}(D^2 + 6F^2) \) | \( 2(D^2 + F^2) \) | \( \frac{3}{2}D^2 \) | \( \frac{2}{9}C^2 \) | \( \frac{10}{9}C^2 \) | \( \frac{1}{2}C^2 \) |
| \( \Xi \) | \( \frac{1}{2}(D - F)^2 \) | \( \frac{1}{2}(5D^2 + 6DF + 9F^2) \) | \( \frac{1}{2}(D + 3F)^2 \) | \( \frac{3}{2}C^2 \) | \( C^2 \) | \( \frac{1}{2}C^2 \) |

We summarize the loop contributions by

\[
\delta M_B^{(3/2)} = -\frac{1}{16\pi^2 f^2} \sum \phi \left[ \chi_{B\phi} I_R(m_\phi, 0, \Lambda) + \chi_{T\phi} I_R(m_\phi, \delta, \Lambda) \right],
\]

where the loop integrals, \( I \), and appropriate renormalisations, in a variety of regularisations, can be found in Ref. [12].

The relevant coefficients for these diagrams are displayed in Table 2 [10]. For this study, the baryon-baryon-meson coupling constants are fixed by phenomenology, with \( D + F = g_A = 1.27 \) and from SU(6) we use \( F = \frac{3}{2}D \) and \( C = -2D \). We note a similar value for \( C \) can also be inferred from the decay width of the \( \Delta \). We adopt a chiral perturbation theory estimate for the pion decay constant in the SU(3) chiral limit, \( f = 0.0871 \text{GeV} \) [13]. The final phenomenological input we use is the octet-decuplet splitting, where we take the physical \( N-\Delta \) splitting, \( \delta = 0.292 \text{GeV} \). All these inputs could potentially be constrained by actual lattice simulation results, or at least in the near future. The full analysis reported in [7] allows each of the chiral axial charges to vary from these estimates by 15%, and a 5% variation in \( f \) — precisions which should certainly be within reach of the current or next generation of simulations.

We apply our chiral expansion to two recent 2+1-flavour calculations of the octet baryon spectrum, those by LHPC [11] and PACS-CS [2]. To determine the absolute masses of the baryons, we need to rely on the lattice scale to be fixed from an alternate source. The MILC collaboration have gone to tremendous effort to tune the lattice spacing to reproduce hyperfine splittings in \( b \)-quark systems [14]. For the purposes of matching, it is convenient to relate to this scale determination through the Sommer scale, using \( r_0 = 0.465 \pm 0.012 \text{fm} \) [14].
The expansion described is formulated for the continuum and infinite-volume limit. While the approach to the continuum can be improved by choice of lattice action, the finite-volume effects are dominated by infrared chiral physics. This is the same chiral physics that contributes to the chiral expansion described above, where the leading finite-volume effects can be estimated by replacing the continuum loop integrals with discrete sums over lattice momenta \[15, 16, 17\]. In the left panel of Figure 1, we show an example of the EFT prediction of the approach to the infinite-volume limit — a prediction that is strongly supported by numerical studies in SU(2) simulations \[16\]. Our band shows potential differences in how the ultraviolet component of the meson loops are treated, using either a finite regulator \[15\] or a scale-free approach \[17\]. The uncertainty is added in quadrature to the statistical error in the estimate of the infinite-volume limit.

We choose to remain as close to the chiral regime as possible, and fit just those results for \(m_{\pi}^2 < 0.2\,\text{GeV}^2\). Independent fits to each of the LHPC and PACS-CS data sets yield excellent agreement in the low-energy constants of the fits, see Fig. 2. Further, the one additional fit parameter, given by the finite regularization scale is also in agreement between the two simulations.

This determination of the LECs and the relevant correlations lead to the accurate values for the baryon masses reported in \[7\]. Beyond the masses that have been extracted, the analysis demonstrates the ability to correct for the difference between the lattice and physical strange quark masses — in this case, the larger lattice strange quark masses are realised in the right panel of Fig. 1. The reliability of these corrections is demonstrated in \[7\], where the fit to the PACS-CS results are used to predict the baryon masses of the different strange quark mass ensemble.

Expressions for the baryon masses as a function of the meson masses (or quark masses...
by Gell-Mann–Oakes–Renner) have also enabled accurate determinations of the relevant sigma terms [7]. These sigma terms are of particular importance in the context of current dark matter searches [18]. Our extracted values are in satisfactory agreement with both the latest light-quark sigma term inferred from experiment [19], and other recent lattice determinations of the strange-quark sigma term [20, 21]. These “small” strangeness sigma terms are noted to be consistent with phenomenological expectations [22, 23].

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