Spontaneous Photoemission From Metamaterial Junction: A Conjecture

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The possibility of spontaneous photon pair emission from a normal material - metamaterial junction is investigated in a quantum field theory setting. We consider a pair of photons arising from vacuum fluctuations of the electromagnetic field close to the junction where one photon each comes from the normal and metamaterial sectors. Mixing between the positive and negative norm photon modes can give rise to spontaneous photoemission, the rate of which is calculated.

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In the present work we conjecture the possibility of spontaneous photon pair creation in normal material - metamaterial junction. Our computational framework is based on perturbative quantum field theory. In recent years, metamaterials, a novel form of artificially designed material with electric permittivity \(\varepsilon\) and magnetic permeability \(\mu\) both being negative, have created a lot of interest among experimentalists as well as theorists. It has many peculiar (with respect to conventional material) properties such as negative refractive index, negative phase velocity, among others that give rise to a plethora of observational consequences, such as reverse Doppler effect, reverse Cerenkov effect, perfect lensing, electromagnetic cloaking and many others.

It is quite natural that normal material - metamaterial interface will have many unique features, such as negative refraction, superlensing, anti-parallel directions of phase velocity and Poynting vector, unconventional surface waves, among others. However, most of the developments on the theoretical side have been in the classical electromagnetic wave (EMW) theory framework where consequences of negative \(\varepsilon\) and \(\mu\) are derived from Maxwell’s equations. Essentially these are refinements of the original results of Veselgo. But, a quantum mechanical or field theoretic study of metamaterials has not been attempted, to the best of our knowledge. In the present paper we show that the counterintuitive features of metamaterial, in particular negative phase velocity, can lead to a novel phenomenon - photon pair production in conventional positive (refractive) index material (PIM) - negative (refractive) index material (NIM) junction. We refer this proposed new effect as photoemission from metamaterial junction (PMJ).

Many of the observational consequences of NIM appear when dealing with PIM-NIM junction because of the boundary conditions involved, when electromagnetic wave crosses the boundary, in a purely classical setting. This feature has proved to be very worthwhile in Analog Gravity models, after the seminal work of Unruh. In the latter one tries to simulate (theoretically) gravitation like features in condensed matter or classical fluid systems and study theoretical predictions of gravitation in experiments that can be performed in the laboratory. In general, signatures of many of the interesting predictions of gravity, such as Hawking radiation from Black Holes, gravitational waves, etc., are extremely weak and their conclusive detection is very difficult. That is why one looks for similar effects in Analogue Gravity models. In a series of very interesting works, Smolyaninov and Narimanov have analyzed the possibility of simulating many exotic space-time behaviors, governed by gravitation, such as effective Hawking radiation, metric signature changing events, “two-time” physics and even “end of time” scenarios, in the framework PIM-NIM composites. The Analogue gravity concept comes in to play since it is well known that the dynamical equations governing the EMW in a curved space-time is similar to the EMW dynamics in PIM with inhomogeneous refractive index. There is a specific mapping between the metric coefficients on one hand and permittivity and permeability components on the other. Hence the possibility of EMW going from PIM to NIM through a junction can lead to metric signature change in the Analogue Gravity sector where this can amount to time and space components being interchanged, or even vanishing of conventional time, as one crosses the (PIM-NIM) interface. In the classical EMW theory, singularities in the electric field are encountered as one crosses the PIM-NIM junction, which have been observed experimentally. In the works of this type of phenomena have been identified as infinite number of particle (and energy) creation in signature changing events that have been predicted earlier in quantum field theory computations. However we feel that this identification is too naive and premature since the field singularities are purely classical in nature and interpreting this effect as particle (photon) creation, a totally quantum field theory (QFT) effect, requires further analysis. For this identification to hold forth rigorously one needs to study the (possibility of) photon creation in a quantum field theoretic setting. Precisely this has been attempted in the present paper.

A perturbative QFT scheme for photon pair production where \(\varepsilon(\vec{x}, t)\) undergoes a space and time varying...
perturbation has been provided by Schutzhold et al. [19]. It has been successfully applied in a specific problem by Belgiorno et al. where the ε perturbation is produced by a laser pulse inducing non-linear Kerr effect in the medium. We follow the notation of Belgiorno et al. [20]. The generic form of amplitude $A_{(\vec{k}, \mu; \vec{k}', \mu')}$ for a vacuum to a photon pair transition is,

$$A_{(\vec{k}, \mu; \vec{k}', \mu')} = \langle \left( \vec{k}, \mu; \vec{k}', \mu' \right) | S | 0 \rangle$$  \hspace{1cm} (1)

where the photon pair is labeled by momenta $\vec{k}, \vec{k}'$ and polarizations $\mu, \mu'$ and the $S$-matrix at first order of ε-perturbation is $S \sim 1 - i \int d^4x \mathcal{H}_I(x)$. The interaction Hamiltonian density is defined as $\mathcal{H}_I(x) = \xi \vec{D}(\vec{x}, t)^2$, where

$$\xi = \frac{1}{2} \left( \frac{1}{n^2(\vec{x})} - \frac{1}{(n_0)^2} \right).$$ \hspace{1cm} (2)

$\vec{D}(\vec{x}, t)$ is the displacement vector (or equivalently the canonical momentum [19, 20]) and $\epsilon(\vec{x}) = n^2(\vec{x})$ and $\epsilon_0 = n_0^2$ are the variable and constant background refractive indices, respectively. Indeed in these applications only PIMs are considered. For a non-zero amplitude to exist, it is essential for the perturbation to be explicitly time-dependent [19, 20]. This follows simply from the principle of energy conservation which for a static perturbation will appear as $\delta(\omega_k + \omega_{k'})$ in a generic transition amplitude $A_{(\vec{k}, \mu; \vec{k}', \mu')}$ can and never be satisfied for +ve $\omega$ for PIM. For this reason a time-dependent $\epsilon(\vec{x}, t)$ perturbation is necessary for a non-vanishing amplitude since it introduces additional terms in the energy conservation $\delta$-function that helps to saturate the $\delta$-function (for details see [20]).

But for a NIM such as metamaterials things are radically different. Inside a PIM medium $\omega_k = ck/n_0, k = |\vec{k}|$ and the right (left) moving plane waves (in one spatial dimension) are represented by $\sim \exp(ikx - i\epsilon t)$ ($\sim \exp(-ikx - i\epsilon t)$). In NIM one can simulate the negative phase velocity by replacing $n_0$ by $-n_0$ which converts the above waves to $\sim \exp(ikx + i\epsilon t)$ and $\sim \exp(-ikx + i\epsilon t)$. Notice that the NIM states turn out to be complex conjugates of the PIM states and hence are negative norm states whereas the PIM states are positive norm states. This mode-mixing phenomenon at the PIM-NIM junction can give rise to our conjectured PMJ effect. It should be stressed that negative phase velocity of NIM modes have striking physical consequences such as negative refraction, reverse Cerenkov effect, reverse Doppler effect, etc., that are already established.

We propose the following scenario: in a vacuum quantum fluctuation a photon pair is produced in a PIM-NIM junction such that one photon is in the PIM sector and the other is in the NIM sector. Then, even for a time independent perturbation, the $\delta$-function will appear as $\delta(\omega_k^{\text{PIM}} - \omega_k^{\text{NIM}})$ that can be saturated yielding a non-zero vacuum to photon pair transition amplitude. Still a question remains as to from where will the energy come for the photon pair production. Note that throughout our analysis we have ignored dispersion effects and loss indicating that we are restricted to a narrow frequency band. Without dispersion the NIM is unstable with negative energy density and losslessness leads to violation of Kramers-Kronig relation. Hence energy from outside is needed to stabilize the NIM and this energy comes out in the PMJ. A somewhat similar idea was suggested in [12] that energy needs to be supplied from outside to compensate for the dissipation in NIM, which we have not considered. The main point is that, in a dispersionless and lossless model (as considered in many previous works), PMJ can occur in a static PIM-NIM junction with no time-dependent [19] (eg. laser induced [20]) $\epsilon$-perturbation in the process. Indeed it is imperative to study effects of dispersion and loss on the PMJ.

We notice a qualitative resemblance between PMJ and the celebrated Hawking-Unruh effect of Black Hole evaporation [22, 23] or more closely with Hawking-Unruh radiation in Analogue Gravity scenario [9]. In the semi-classical explanation of Hawking effect, out of the photon pair (or any other particle pair) created close to the event horizon, the negative energy particle is trapped inside the Black Hole (thereby reducing the Black Hole’s mass) whereas its positive energy partner escapes. The latter constitutes the Hawking radiation. Energy conservation is taken care of since the Black Hole mass reduces. In the Analogue Gravity scenario this effect is captured by mode-mixing between positive and negative norm states [21]. The major differences between Hawking effect and PMJ are that (i) in PMJ, both photons of the correlated pair should be observable; (ii) the energy needed in PMJ has to be provided from outside to preserve the NIM and (iii) only photon pairs are involved in PMJ whereas Hawking-Unruh radiation, in principle, can constitute of all types of elementary particles. In Analogue Gravity scenario, Belgiorno et al. [24] have claimed to provide evidence of analogue Hawking radiation in a controllable moving refractive index perturbation using ultrashort Laser pulse filamentation [25]. However the above claim [24] has been debated in [26].

We now attempt to provide a quantitative estimate of
PMJ. In general it is tricky to consider a perturbation small if it incorporates a change of sign in the physical parameter in question, as is true for the present case. We have considered the composite material such that the negative $x$ (positive $x$) consists of NIM (PIM) of constant background values $-n_0^2$ ($n_0^2$) and the perturbation smooths out the crossover at the junction. Hence the overall refractive index is given by,

$$n^2(\vec{x}) = n_0^2 \text{sign}(x) - 2n_0\eta \tanh(x/l) e^{-R^2/2\sigma^2},$$

where, $\text{sign}(x) = (+)1$ for $x > (<)0$, $\eta$ is a small parameter and $l, \sigma$ describe the crossover at the junction and $R^2 = x^2 + y^2 + z^2$. At large distance from the junction at

$$x = 0$$

the left and right side materials are NIM and PIM, respectively. This is depicted in Fig. 1 for two values of $\sigma$, where the red and blue lines correspond to the exact and approximated expressions given in Eq. 3, respectively.

$$x = 0$$

The energy contributions in the $\delta$-function appear with opposite signature which, as explained before, is the highlight of our conjecture. Rest of the calculation is straight-

where we plot the exact function (red curve) given in the equality Eq. 4 and its approximate version (blue curve) that we actually use given in the third equality in Eq. 4. The results are presented for two different values of $\sigma$, for the sake of simplicity.

The Fourier transform turns out to be

$$\xi = \frac{n_0^2}{\eta} \int \frac{d^3k}{2\pi^3} \frac{k_x}{V} e^{-R^2/\sigma^2}$$

$$= \frac{8\pi^2\eta}{n_0^2} \int \frac{d^3k}{2\pi^3} \frac{\delta(k_x - k'_x)}{e^{-|k_x + k'_x|^2/\sigma^2}}.$$ (5)

FIG. 2: (Color online). The variation of $\xi$ as a function of $x$, where (a) $\sigma = 1.5$ and (b) $\sigma = 2.5$. The red and blue lines correspond to the exact and approximated expressions given in Eq. 4 respectively.

$$N_{k,\mu} = \frac{\sqrt{V}}{2\pi} \int d^3k \frac{\omega k^2}{k} \frac{\left[\hat{\xi}(k + k')^2 - 1 - (\hat{e}_k \hat{e}_k')^2\right]}{\left[1 - (\hat{e}_k \hat{e}_k')^2\right]}$$

$$= \frac{16\pi^2\eta^2\sigma^6 k}{V n_0^2} \int d^3k \frac{\delta(k - k')\sigma^2(k_x + k'_x)^2}{\sqrt{V}} \frac{1}{e^{-|k_x + k'_x|^2/\sigma^2}}.$$ (6)

FIG. 3: (Color online). The rate of photoemission $dN/d\Omega$ computed from Eq. 5 for different values of $\alpha$ and $\lambda$ when we set $\sigma = 1 \mu m$ and $n_0 = 2.5$.

$$N_{k,\mu} = \frac{16\pi^2\eta^2\sigma^8 k}{V n_0^2} \int \frac{d^3r}{2\pi} \frac{r^2}{f^2 + r^2}$$

$$\times \left[(k_x + f)^2 + k_{\perp} r \cos \theta\right]^2$$

$$\times e^{-\sigma^2|k_x + f + k_{\perp} r \cos \theta|^2/k^2}$$

$$\times e^{-|k_{\perp} + r + 2k_{\perp} r \cos \theta|^2/(2\sigma^2)}.$$ (7)

Forward. Defining $(\vec{k}')^2 = (k_x')^2 + r^2$, the result becomes

$$N_{k,\mu} = \frac{16\pi^2\eta^2\sigma^8 k}{V n_0^2} \int \frac{d^3r}{2\pi} \frac{r^2}{f^2 + r^2}$$

$$\times (k_x + f)^2 + k_{\perp} r \cos \theta\right]^2$$

$$\times e^{-\sigma^2|k_x + f + k_{\perp} r \cos \theta|^2/k^2}$$

$$\times e^{-|k_{\perp} + r + 2k_{\perp} r \cos \theta|^2/(2\sigma^2)}.$$ (7)
given by,
\[
\frac{dN}{d\Omega} = \frac{64\pi^3 L n^2 \alpha^8}{n_0^2} \int_0^\infty \int_0^{2\pi} \int_0^{\pi} (k_x + f)^2 (k_x f + k_\perp r \cos \theta) e^{-\sigma^2[(k_x + f)^2 + k_\perp^2 + r^2 + 2k_\perp r \cos \theta]} \, dkdkdr \, d\theta.
\]

This constitutes our main result. Figure 3 describes a numerical evaluation of our result (Eq. 8) for \( n_0 = 2.5 \), \( \eta = 0.01 \), and \( \sigma = 1 \mu m \). The photoemission number density is plotted as functions wavelength \( \lambda \) and emission angle \( \alpha \). We notice that the process is fairly localized in \( \alpha \) and \( \lambda \) with a maximum emission at around \( \lambda = 2 \mu m \). The maximum count rate is \( \sim 200 \) but indeed the number should not be taken too seriously at our present level of analysis.

We conclude that the possibility of a completely new form of spontaneous photoemission is suggested from a normal material - metamaterial interface. The junction acts as a “horizon” that separates two types of excitations with opposite phase velocities. This makes spontaneous emission of a photon pair feasible due to mixing between positive and negative norm modes, provided one should not be taken too seriously at our present level of analysis.

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[28] An exponential damping term in the Fourier integral of \( x\)-sector is dropped so that the Fourier transform is computed analytically. Note that part of the contribution of the exponential term in \( x \) appears in \( \exp[-(\sigma^2 k_x^2)/2] \) in Eq. 5. The above simplification can affect our numerical estimates of photon emission number.