Structural analysis of non-standard geometric variants of a shifted spur gear

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Abstract. Today's industrial practice shows that - in contrast to the past years, where gears were designed as standard as possible - machine parts are constructed on the basis of selecting non-standard parameters. This is because producers protect themselves against making additional parts from local suppliers for much less money by customers. The paper presents a geometric and strength analysis of an exemplary spur gear, which works as the second stage of a bevel-helical gear in the coiling mechanism of a multi-module machine producing bonell springs for mattresses. It was checked whether the transmission could be redesigned in such a way as to make its geometry as complicated as possible while maintaining the appropriate strength properties. Finally, there are graphs that can be helpful in this type of reconstruction process, and the subsequent stages of this work can be treated as a kind of algorithm for the discussed conversion of geometric parameters of the gear.

Keywords: spur gear, gear design, gear geometry, gear reverse engineering, modified gears, non-standard gear, bending stresses, maximum reduced stresses

1. Introduction

Of course, K. Ochęduszko [1] has already dealt with a very detailed study of gears [1], but his works contain nothing about the latest trends in gear design by manufacturers. It is about the habit of complicating the geometry in the direction of deviating from standard or generally accepted parameters in order to protect against spare parts on your own. In the available scientific databases, there are very few papers dealing with this type of phenomenon, and thus there is not enough knowledge to efficiently navigate this issue. And although a large part of the answers to some questions (such as what strength properties have such non-standard gears) can be roughly deduced on the basis of tedious analytical analyzes, there are no experimental validations and no numerical simulations that can diagnose the problem even more accurately.

The problem of the tooth root transition curve, which has already been mentioned [2], will have a very significant impact on the development of research, and it is also discussed in this paper, however, it requires extension to include also the juxtaposition of various non-standard geometric parameters.

The course of the first part of the article is similar to that of [3], however, the results of the analytical and numerical analysis for one set of standard geometric data of the gear were compared...
there. Meanwhile, here the authors investigate the influence of a slight modification of the geometry on the distribution of bending stresses towards the general complexity of the gears as part of a kind of reverse engineering in the form of redesigning an existing application.

The work focuses only on equivalent stresses, which mainly consist of bending stresses. However, scientists have already investigated the second state of gear loads, i.e. contact stresses [4]. There, the method based on the standard [5] (which is also used here) was compared with numerical calculations, but considering different ratios and different materials for the gear. The aim of this study, however, is to investigate the impact of a minor modification of the gear geometry within the same gear ratio and the same material, with priority being the complexity of parameters towards non-standard ones.

Another important aspect in this work will be the distribution of the shift factors between the small and large gear in the transmission. They dealt with it in great detail [6], even devising a new method of this decomposition. It is expected that in the future the results of these authors’ research will be used, but here it was necessary to approach the topic again, freshly, because - as already mentioned - the authors of the article depart from the standard parameters and it should be checked in the future whether the current calculation methods apply also to such an application.

Other papers, such as [7], [8], also deal with the verification of stresses and modified tooth profiles, but nowhere was the subject of departing from the standard geometric parameters.

2. Exposure of the analyzed example

Gears with non-standard geometric parameters, and hence - atypical design, are more and more often found in the industry. Apart from other important design reasons, it is important to prevent the making of spare parts by not using the original box service. An example of such a gear is shown in Figure 1. In order to produce such gears in at least as a batch production, a special tool must first be made – e.g. a Fellows cutter with such a non-standard module and other parameters. Of course, this involves a huge cost of producing the tool, but it is taken into account in the general settlements having regard to the original purpose - in this way causing that customers will be forced to order original spare parts. Often, entrepreneurs also have to face an attempt to recreate the geometry of the inch gear while switching to the metric system, in order to optimally carry out the reverse engineering process locally in the country. Unfortunately, in the imperial system there is no gear module, only the Diametral Pitch, i.e. how many teeth per inch of the pitch diameter. Converting to the metric system is almost always associated with the selection of very non-standard geometric parameters of the reproduced gear.

![Figure 1. An example of a custom gear for an aircraft engine. The basic parameters resulting from the drawing and calculations are: module $m = 4.98$, pitch diameter $d = 89.64$ mm](image-url)
pressure angle $\alpha = 26^\circ 19' 30''$, addendum modification coefficient $y = 0.795$, clearance factor $c^* = 0.383$, shift factor $x = +0.0695$.

An example from industrial practice will be analyzed. Well, in the machine for coiling of bonell springs for mattresses, the so-called coiling mechanism. The wire passes through it, which is coiling by the gear with a specially shaped internal recess - this gear belongs to one of the body of the mechanism in question. All the above-described elements are shown below in Figure 2.

The engineering task was to recreate the geometry of, among others gears, preparation of their drawings and ordering parts to be made by local partners, because ordering original gears has already become unprofitable and dangerous for the company's future. An interesting design node of the coiling mechanism (from the research point of view) is, on the other hand, a small two-stage bevel-helical gear located in the main part of the mechanism body (also visible in Figure 2b). The gears belonging to the cylindrical mesh system (Figure 3) are shifted in order to obtain the appropriate axis distance, which was probably the superior parameter imposed from above in the design process. The thesis put forward in this paper is that with the use of an appropriate modification of the basic geometric parameters, but maintaining a constant gear ratio, constant axle distance and overall dimensions, the gear can obtain better strength properties.

Figure 2. Coiling machine: a) General view of the entire multi-module assembly (the red outline shows the coiling mechanism), b) Exploded view of the coiling mechanism (the blue outline shows the coiling gear), c) General view of the coiling gear (1 - body made of high-alloy steel for carburizing, 2 - induction brazed cemented carbide insert).

Figure 3. The second stage of the bevel-helical gear of the coiling mechanism: a) Gear No. 2-3 (double gear), b) Exploded view of the coiling mechanism (the orange outline shows a cylindrical gears), c) Gear No. 4.
In this work, the above example was analyzed. It was simulated through a series of calculations how such type of gear could perform in terms of strength with the use of non-standard geometric parameters. The authors hope that in the future, gear manufacturers' constructors will be able to use the simulation results to design an even more optimal variant while protecting themselves against the making of spare parts by customers. It can even be said that in the past, the aim was to standardize tools cutting teeth in order to reduce costs (this is why the module was introduced as the ratio of the pitch \( p \) and the number \( \pi \)), and now the aim is to possibly complicate the geometry of the same tools, while maintaining relatively constant expenses thanks to technical progress in the field of manufacturing cutting tools.

3. Geometric analysis
The analysis assumed constant parameters for the gears. These data are presented in Table 1.

| Number of teeth | Ratio | Center distance given in advance [mm] | Pressure angle [°] | Clearance factor | Addendum modification |
|-----------------|-------|---------------------------------------|-------------------|-----------------|----------------------|
| \( z_1 \) | \( z_2 \) | \( u \) | \( a_r \) | \( \alpha \) | \( c^* \) | \( y \) |
| 20 | 24 | 1.2 | 44 | 20 | 0.2 | 1.0 |

The individual gear variants differ slightly in modules. The pitch diameters of smaller gears and theoretical zero distances of the axes of the mating gears were calculated. The results in Table 2.

| Variant No. | I | II | III | IV | V |
|-------------|---|----|-----|----|---|
| Module \( m \) | 1.98 | 1.99 | 2.01 | 2.02 |
| Pitch diameter \( d \) [mm] | 39.60 | 39.80 | 40 | 40.20 | 40.40 |
| Standard center distance \( a_0 \) [mm] | 43.56 | 43.78 | 44 | 44.22 | 44.44 |

It was previously assumed that variant III would be a basic variant, because the normalized module was used in it and the standard center distance was equal to the center distance given in advance. Therefore, in this one case, profile shifting is unnecessary. For variants I-II a positive shifting is necessary, for variants IV-V a negative shifting is necessary. The sum of profile shift coefficient was calculated from the formula 1.

\[
\text{inv } \alpha_w = 2 \frac{x_1 + x_2}{z_1 + z_2} \tan \alpha + \text{inv } \alpha
\]  

Explanations:
\( \text{inv } \) – involute function
\( \alpha, \alpha_w \) – nominal and working pressure angle
\( z_1, z_2 \) – the number of teeth
\( x_1, x_2 \) – gear shift factors

The apparent center distance [1] distances of the gear axis were calculated from the formula 2.

\[
a_p = a_0 + (x_1 + x_2)m
\]
It was also checked whether the tip shortening factor is less than 0.1. If this relationship is maintained, then it is not necessary to tip shortening while maintaining sufficient apical clearance. This was calculated from formula 3.

\[ km = a_p - a_r \]  

(3)

Explanations:
- \( k \) – tip shortening factor
- \( a_p \) – the so-called apparent center distance [1]
- \( a_r \) – center distance given in advance

The results from formulas 1 to 3 are shown in Table 3.

Table 3. Sum of coefficient of profile shift, apparent center distance and tip shortening factor

| Variant No. | I     | II    | III   | IV    | V     |
|-------------|-------|-------|-------|-------|-------|
| Suma \( x_1 + x_2 \) | 0.2304 | 0.1126 | 0     | -0.1074 | -0.2094 |
| Apparent center distance \( a_p \) [mm] | 44.016 | 44.004 | 44    | 44.004 | 44.017 |
| Tip shortening factor \( k \) | 0.008  | 0.002  | 0     | 0.002  | 0.008  |

As for the distribution of the sum of profile shift coefficient, three principles were initially followed:
- I – most of the positive shifting was transferred to the small gear to thereby increase the strength properties,
- II – the profile shifting could not exceed minimum value required to avoid undercutting teeth in the smaller gear
- III – the coefficient of profile shift could not exceed the maximum value to avoid excessive sharpening of the teeth at the tip in the smaller gear

The minimum number of teeth without undercut, was calculated from the formula 4.

\[ z_g = y \frac{2}{\sin^2 \alpha} = 1 \cdot \frac{2}{\sin^2 20^\circ} = 17.09 \rightarrow 17 \]  

(4)

In both considered gears, the number of teeth is greater than the minimum value, therefore in the basic variant (i.e. III) the undercut will not occur at all.

In the case of a non-uniform structure of the material (i.e. when carburizing), it is worth having the tooth thickness at the tip greater than the minimum value calculated by the formula 5.

\[ \bar{s}_{a_{\text{min}}} = 0.4 \cdot m \]  

(5)

A simulation was carried out in a spreadsheet, which allowed to generate the appropriate geometric parameters so that the tooth thickness at the tip in the smaller gear corresponded to at least the minimum value. For this purpose, the following parameters were used: tooth thickness on the pitch circle from formula 6, addendum from formula 7, tip diameter from formula 8, base diameter from formula 9, pressure angle at a tip diameter from formula 10, involute at a tip diameter from formula 11 and the tooth thickness at the tip from formula 12. The greatest negative value of the profile shift was also calculated from the formula 13, at which the undercut would certainly not occur. The results of the analysis based, among others, on in Formulas 5 to 13 are shown in Table 4.

\[ \bar{s}_{ck} = \left( \frac{\pi}{2} + 2 \cdot x \cdot \tan \alpha \right) \cdot m \]  

(6)

\[ h_{ak} = (y + x) \cdot m \]  

(7)
\[ d_{ak} = d + 2 \cdot h_a \quad (8) \]
\[ d_b = d \cdot \cos \alpha \quad (9) \]
\[ \alpha_a = \cos^{-1}\left(\frac{d_b}{d_a}\right) \quad (10) \]
\[ \text{inv} \\alpha_a = \tan \alpha_a - \alpha_a \quad (11) \]
\[ s_{ak} = d_{ak} \cdot \left(\frac{s_{ck}}{d} + \text{inv} \alpha - \text{inv} \alpha_a\right) \quad (12) \]
\[ X = x_{gr} \cdot m = y \cdot \frac{z_g - z}{z_g} m \quad (13) \]

**Table 4.** Results of simulation of parameters from formulas 5 to 13 for individual variants.

| Variant No. | I  | II | III | IV  | V  |
|-------------|----|----|-----|-----|----|
| Thickness \(s_{a \, \text{min}}\) | 0.792 | 0.796 | 0.8 | 0.804 | 0.808 |
| Thickness \(s_{ck}\) | 3.442 | 3.126 | 3.142 | 3.157 | 3.173 |
| Addendum \(h_{ak}\) | 2.436 | 1.990 | 2 | 2.010 | 2.020 |
| Tip diameter \(d_{ak}\) | 44.472 | 43.780 | 44 | 44.220 | 44.440 |
| Base diameter \(d_b\) | 37.212 | 37.400 | 37.588 | 37.776 | 37.964 |
| Pressure angle \(\alpha_a\) | 0.579494 | 0.546659 | 0.546659 | 0.546659 | 0.546659 |
| Involute of \(\alpha_a\) | 0.074951 | 0.061858 | 0.061858 | 0.061858 | 0.061858 |
| Thickness \(s_a\) | 1.195 | 1.383 | 1.39 | 1.397 | 1.404 |
| Profile shift \(X\) | -0.349 | -0.351 | -0.353 | -0.355 | -0.356 |
| Profile shift \(x_1\) | 0.230 | 0 | 0 | 0 | 0 |
| Profile shift \(x_2\) | 0 | 0.113 | 0 | -0.107 | -0.209 |

Although it was previously assumed that a larger (or the whole) part of the sum of the positive profile shift would be transferred to the smaller gear, this was done only in variant I. In variant II, this shifting was transferred entirely to the larger gear. The explanation of this issue can be found in the next chapter concerning the strength analysis.

Now, the remaining geometrical parameters of the smaller gear have been precisely calculated, i.e. addendum using formula 14, dedendum using formula 15, the tip diameter using formula 16 and the root diameter using formula 17. The calculation results are presented in Table 5.

\[ h_{ak} = (y + x) \cdot m \quad (14) \]
\[ h_{fk} = (y - x) \cdot m + c \quad (15) \]
\[ d_{ak} = d + 2 \cdot h_{ak} \quad (16) \]
\[ d_{fk} = d - 2 \cdot h_f \quad (17) \]

**Table 5.** Calculation results from formulas 14 to 17 for individual variants.

Values expressed in [mm]. The counted values are for the smaller gear.
The same was also calculated for the larger wheel, and the results are presented in Table 6.

### Table 6. Calculation results from formulas 14 to 17 for individual variants.

Values expressed in [mm]. The counted values are for the larger gear.

| Variant No. | I    | II   | III  | IV   | V    |
|-------------|------|------|------|------|------|
| Addendum    | 2.436| 1.990| 2    | 2.010| 2.020|
| Dedendum    | 1.920| 2.388| 2.4  | 2.412| 2.424|
| Tip diameter| 44.472| 43.780| 44   | 44.220| 44.440|
| Root diameter| 35.760| 35.024| 35.2 | 35.376| 35.552|

From Formula 18, the contact ratio is calculated. This formula takes into account the general case, i.e. for uncut and shifted teeth, where the working circle does not coincide with the pitch circle. Due to the smooth running of the gears, what is desired is $\varepsilon_{\alpha_{\text{min}}}=1.4$. Of course, for variant III, where the working circle coincides with the pitch circle, formula 18 implicitly simplifies. Formula 18 consists of three members which are commonly described in the literature as $C_1$, $C_2$, and $C_3$ (respectively: the first gear term, the second gear term, and the closing member). The results are presented in Table 7.

### Table 7. Contact ratio for individual variants.

| Variant No. | I    | II   | III  | IV   | V    |
|-------------|------|------|------|------|------|
| Contact ratio $\varepsilon_{\alpha}$ | 1.51 | 1.55 | 1.58 | 1.62 | 1.66 |

4. **Strength analysis**

Data on the characteristics of work and load as well as parameters are presented in Table 8.

### Table 8. Load parameters of the analyzed gear.

| Power [kW] | Input rotational speed [min⁻¹] | Character of work | Drive engine | Power receiver | Working hours per day | The nature of the load |
|------------|---------------------------------|-------------------|--------------|----------------|-----------------------|------------------------|
| 5          | 500                             | Regular           | Electric     | Zapętlarka    | 24                    | With moderate fluctuations |
The overload coefficient can be selected according to the results of Dudley, Niemann or Henriot. The value is $C_p = 1.45$.

From formula 19, the tangential velocity of a small gear was calculated.

$$v = \frac{\pi \cdot d \cdot n}{1000 \cdot 60} \left[ \frac{m}{s} \right]$$

(19)

Assuming the 2nd class of accuracy of the gear, the dynamic load factor was calculated from the general formula 20. The constant $A$, selected from the tables, depends on the accuracy group and the value of the tangential velocity itself.

$$C_d = 1 + \frac{\sqrt{v}}{A}$$

(20)

The tangential force was calculated from formula 21, taking into account the power $N$ and the speed $n$ from Table 7.

$$P_{stat} = \frac{1000 \cdot N}{v} = \frac{2.955000 \cdot N}{d \cdot n} \left[ N \right]$$

(21)

The equivalent force was calculated from the formula 22.

$$P_{eq} = C_p C_d P_{stat} \left[ N \right]$$

(22)

In this strength analysis, a load case has been assumed that takes into account bending and compression, the sum of which makes up the maximum - equivalent loads. This is shown in Figure 4.
Figure 4. Stress distribution at the base of the tooth from the forces acting on the tooth at the tip when bending and compression are taken into account; $P_z$ – force acting along the pressure line (mesh), $P$ – circumferential force (acting on the pitch circle), $\sigma_b$ – bending stresses, $\sigma_c$ – compressive stresses, $\sigma_{eq}$ ($\sigma_{max}$) – equivalent bending stress (maximum) [1].

To calculate the tooth strength at the root, you need to know the point where the computational force is applied. This is point $A$ at the tip from Figure 4. Other parameters that should therefore be taken into account in this analysis are: bending stresses from formula 23, the resistance moment for the rectangular section of the tooth at the base from formula 24, tooth compressive stresses from formula 25, the force along the action line in formula 26 and the computational circumferential force in formula 27.
\[ \sigma_b = \frac{P_z \sin \vartheta h_0}{W_b} \quad (23) \]
\[ W_b = \frac{bs_f^2}{6} \quad (24) \]
\[ \sigma_c = \frac{P_z \cos \vartheta}{bs_f} \quad (25) \]
\[ P_z = \frac{P_{cal}}{\cos \alpha} \quad (26) \]
\[ P_{cal} = \frac{P_{eq}}{\epsilon \alpha} \quad (27) \]

Explanations:
\[ \vartheta \] – the angle between the action line and the symmetrical tooth
\[ h_0 \] – arm of the bending moment from point B (figure 3)
\[ h \] – moment arm from point A (figure 3)
\[ b \] – face width (tooth length)
\[ s_f \] – tooth thickness at the base
\[ \alpha \] – operating pressure angle (on the pitch circle)

Both the moment arm \( h \), jak i the tooth thickness \( s_f \) at the base depend on the nominal module \( m \), the number of teeth \( z \) in the gear, addendum modification \( y \) and nominal pressure angle \( \alpha \) (formulas 28 and 29).
\[ h_0 = f(z, m, \alpha, y) \quad (28) \]
\[ s_f = f(z, m, \alpha, y) \quad (29) \]

But for the same number of teeth \( z \) and the same angle \( \alpha \) and modification \( y \) these parameters are quantities dependent only on the magnitude of the nominal module \( m \), therefore the equations from formulas 30 and 31 apply.
\[ h_0 = \varphi m \quad (30) \]
\[ s_f = \psi m \quad (31) \]

Explanations:
\[ \varphi \] – bending moment arm factor
\[ \psi \] – tooth thickness coefficient at the base

Ultimately it is possible to calculate the equivalent bending stress on the compression side from formula 32
\[ \sigma_{eq} = \sigma_b + \sigma_c = \frac{P_{cal}}{bm} \left( \frac{6\varphi \sin \vartheta h_0}{\psi^2 \cos \alpha} + \frac{\cos \vartheta}{\psi \cos \alpha} \right) \left[ \frac{N}{mm^2} \right] \quad (32) \]
or exactly from formula 33, not taking into account the replacement by the coefficients \( \varphi \) and \( \psi \).
\[ \sigma_{eq} = \sigma_b + \sigma_c = \frac{P_{cal}}{b} \left( \frac{6h_0 \sin \vartheta}{s_f^2 \cos \alpha} + \frac{\cos \vartheta}{s_f \cos \alpha} \right) \left[ \frac{N}{mm^2} \right] \quad (33) \]
Often in the literature, the expression in the brackets in formula 3 is bending stress factor (formula 34) and it is selected from tables (e.g. according to L. D. Czasownikow) or graphs. However, some of the data presented in the literature available to the authors are not sufficient to accurately determine the values of the factors belonging to the geometric variants from this analysis, which differ so subtly in the values of the geometric parameters.

\[
\frac{6\varphi \sin \vartheta}{\psi^2 \cos \alpha} + \frac{\cos \vartheta}{\psi \cos \alpha} = q = \frac{1}{\lambda}
\]  

(34)

Therefore, either the arm \( h_0 \) and the thickness \( s_f \) should be generated from the geometrical parameters or the \( \varphi \) and \( \psi \) coefficients should be determined very precisely, e.g. by linear interpolation or extrapolation to the available tables and graphs in the literature. In order to most accurately estimate these values, one should assume the type of fillet curve (single-element or multi-element with protuberance) and count, among others, radius of the fillet curve, which is a tedious process. Due to the volume of this work and its nature (introduction to further considerations), for the time being, for calculation purposes, the final values of the strength factor \( \lambda \), similar to those in the tables in the literature, were finally adopted. For this reason, for variant I, the sum of the shift factors was entirely transferred to the smaller gear, because the appropriate strength factor can be found in the table for this value. For variant II, such a factor is not available in the table, therefore the sum of the shift factors was transferred to the larger gear, and the small gear was left without shifting. This has very interesting consequences, which are presented in the next chapter on the graphical interpretation of the results. Finally, formula 35 will allow the calculation of equivalent stresses.

\[
\sigma_{eq} = \sigma_b + \sigma_c = \frac{P_{cal}}{bm\lambda} \left[ \frac{N}{mm^2} \right]
\]  

(35)

The face width was assumed to be constant for all variants: \( b = 10 \) mm. Additionally, according to G. Henriot, for the case of the least favorable load, there is an approximate relationship according to formula 36.

\[
\sigma_c = 0.119 \cdot \sigma_b
\]  

(36)

The results of the strength analysis are presented in Table 9.

| Variant nr | I     | II    | III   | IV    | V    |
|------------|-------|-------|-------|-------|------|
| Tangential velocity \( v \) [m/s] | 1.04  | 1.04  | 1.05  | 1.05  | 1.06 |
| Tangential force \( P_{stat} \) [N] | 4823  | 4799  | 4775  | 4751  | 4728 |
| Equivalent load \( P_{eq} \) [N] | 9367  | 9326  | 9286  | 9245  | 9205 |
| Computational force \( P_{cal} \) [N] | 6190  | 6026  | 5879  | 5721  | 5553 |
| Strength factor \( \lambda \) | 0.452 | 0.395 | 0.395 | 0.395 | 0.395 |
| Equivalent stresses \( \sigma_{eq} \) [MPa] | 692   | 767   | 744   | 721   | 696  |
| Bending stresses \( \sigma_b \) [MPa] | 618   | 685   | 665   | 644   | 622  |
| Compressive stresses \( \sigma_c \) [MPa] | 74    | 82    | 79    | 77    | 74   |
5. Interpretation of the analysis results

Figure 5 shows the graph from which the conclusions can be drawn. **A:** The standard center distance $a_0$ is directly proportional to the modulus $m$ – if it increases, the standard distance also increases. And conversely. **B:** Although with positive shifting the gears seem to "move away" from each other, and with negative shifting the gears "move closer" to each other, the difference of modules $m$ in individual variants is important here, which directly affects the pitch diameter $d$, and thus also the standard center distance $a_0$. Taking this into account in formula 2, it comes out that with the given output parameters, the apparent center distance $a_p$ will always increase in relation to the center distance given in advance $a_r$ regardless of whether the total sum of shift coefficient $(x_1 + x_2)$ is positive or negative.

![Figure 5. Standard-, apparent center distance $a_0$, $a_p$ and center distance given in advance $a_r$ for I-V.](image)

Figure 6 shows the graph from which the conclusions can be drawn. **A:** For variants II to V, the tooth thickness $s_{ck}$ on the pitch diameter $d$ of the gear 1 and addendum $h_{ak1}$ increases with the increase of the modulus $m$ with the constant number of teeth $z$ and the value of the shift factor $x$ (in this case $x = 0$). This is a linear increase, as shown by the dashed trend lines. **B:** For variant I, in which the total positive sum of the shift factors $x_1 + x_2$ was transferred to the smaller gear, a relatively significant increase $s_{ck}$ i $h_{ak1}$. This is because with positive shifting the cutting tool moves away from the pitch circle $d = 39.60$ mm, thus creating a working diameter $d_w = 40.00$ mm. As a result, the tooth thickness increases towards the base (but the tip is sharpened more, as shown in Table 4) and the addendum increases (relative to the dedendum, which decreases).

![Figure 6. Tooth thickness $s_{ck}$ on the pitch diameter and addendum $h_{ak1}$ for I-V.](image)
Figure 7 shows the graph from which the conclusions can be drawn. **A:** For variants II-V element \( C_1 \) is constant and equal to 1.94, because there is no shifting in gear 1, and therefore the term \( \frac{2h_{a1}}{d_1} \) in the formula 18 takes the same value. However, for variant I, this term has a greater value, because gear 1 has a positive shifting, which increases the addendum, and thus also the contact surface of the teeth of the gears. **B:** For all variants, a decrease in the \( C_2 \) term was observed with an increase in modulus \( m \). However, the interpretation of the results for the mating gear 2 is left to the next work. A declining linear trend alone with this set of output data does not necessarily indicate that the trend will always be proportional. **C:** Although the positive shifting has a positive effect on \( C_1 \) for variant I, the contact ratio grows relatively for individual variants without much revelation for case No. I.

![Figure 7](image)
Figure 7. Contact ratio \( \varepsilon_a \) and its components \( C_1, C_2, C_3 \) for I-V.

Figure 8 shows the graph from which the conclusions can be drawn. **A:** For variant I, where there is a shift factor \( x_1 = +0.230 \), a different strength factor \( \lambda = 0.452 \) is selected compared to the other variants, where \( \lambda = \text{const.} = 0.395 \) (in the available tables it was possible to read the relatively exact value of the coefficient \( \lambda \) only for the shifting case \( x_1 = 0 \) or \( x_1 = +0.2 \)). **B:** The coefficient \( \lambda \) is important because a slight change in its value is enough and the stresses at the base of the tooth vary drastically. However, \( \lambda \) is closely related to the arm \( h_0 \) on which the bending moment acts, and this in turn with the fillet curve, which was already described earlier - an issue for future works.

![Figure 8](image)
Figure 8. Shift factor \( x_1 \) and strength factor \( \lambda \) for I-V.
Figure 9 shows the graph from which the conclusions can be drawn. **A:** The value of each type of load decreases with the increase of the $m$ module and the other selected or calculated parameters. This is an approximately linear decline as shown by the dashed trend lines. **B:** However, these differences are minor and result from slightly different degrees of contact ratio $\epsilon$, tangential velocity $v$, constant power $N$ and rotational speed $n$. What is more important is the reception of this load by a suitably modified geometry, i.e. the stress distribution in non-standard variants - it is shown only in Figure 10.

![Figure 9](image)

**Figure 9.** Tangential force $P_{stat}$, equivalent force $P_{zast}$ and computational force $P_{obl}$ for I-V.

Figure 10 shows the graph from which the conclusions can be drawn. **A:** It can be seen that the mere increase of the modulus $m$ e.g. to the value of 2.04 in the variant V in relation to the basic variant III (which in itself is beneficial for the stress distribution) does not give such a revelation as variant I, in which the modulus $m$ is the smallest, but appropriate the shift factor $x_1$ and strength factor $\lambda$ reduce the stresses at the base of the tooth. **B:** You can observe a relative decrease in the maximum stresses $\sigma_{eq}$ by the following percentages: 6.5 % decrease in $\sigma_{eq}$ in variant V with respect to basic III, 7.0 % decrease in $\sigma_{eq}$ in variant I with respect to basic III and 0.6 % decrease in $\sigma_{eq}$ in variant I with respect to V.

![Figure 10](image)

**Figure 10.** Equivalent stresses $\sigma_{eq}$, bending stresses $\sigma_b$ and compressive stresses $\sigma_c$ for I-V.
Figure 11 shows a graphical comparison of the contours of the analyzed tooth geometries, from which the conclusions can be drawn. **A:** You can see relatively slight sharpening of the tip for variant I. Although this variant fared most optimally on the plane of bending stresses, possible tooth chipping should be verified here in the future. **B:** With the increase in the value of the module $m$ in variants II-V, the tooth thickness increases on the pitch diameter. But the thickest tooth is for variant I due to the appropriate correction. **C:** The working diameter is common to all I-V variants.

### 5. Summary

Based on the structural analysis of a spur gear, it can be concluded that in order to reduce the equivalent stresses $\sigma_{zast}$ (resulting mainly from the bending of the tooth at the base) and at the same time to maintain a constant center distance given in advance $a_r$ with the possible complexity of the nominal module $m$ for the purposes described in chapter 2 (mainly protection against making spare parts on your own), this module should be reduced first of all. Then, thanks to the generally positive shifting $x_1+x_2$ thus created, transferred the entire shifting to the smaller gear. In this way, the thickness of the tooth at the base will increase, and making the gear by customers will become very difficult due to the need to use a non-standard cutting tool.

In the future, further analytical analysis is foreseen in order to determine the cases that will definitely be taken into account in computer simulations using the finite element method. The authors will then make several key gears for experimental validation.

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