In [1], the authors show numerically that spanning and percolation probabilities in two-dimensional systems with different aspect ratios obey a form of “superscaling”. This scaling form makes it possible to relate the percolation properties of a system with one set of parameters \((L, \epsilon, R)\), to a system with another set of parameters.

In this comment, we would like to point out some difficulties with their proposed scaling ansatz and suggest why this remained undetected in their numerical analysis. Starting from the central result for the existence probability Eq. (8), one observes that it cannot account for their proposed scaling ansatz and suggest some inconsistencies did not show up in the numerical analysis because of the way in which the authors “correct for finite-size effects”. After introducing this correction in Eq. (9), their data analysis is based on an \(\epsilon = \rho - \rho_c\) in the scaling function shifted by a small amount to \(\epsilon' = \rho - \rho_c'(L, R)\) where \(\rho_c'(L, R)\) is defined implicitly by \(E_p(L, \rho_c'(L, R) - \rho, R) = E_p(L, 0, 1)\). However, this shift is not a finite size correction because, regardless of the size, it is a necessary adjustment in order to produce the intended superscaling behaviour.

Although the shift vanishes in the thermodynamic limit, \(\lim_{L \to \infty} \rho'_c - \rho_c = 0\), it remains crucial: the difference between the values of the observable with and without shift does not vanish in the thermodynamic limit, but converges to a finite constant, for example

\[
\lim_{L \to \infty} E_p(L, \rho'_c - \rho_c, R) - E_p(L, 0, R) \neq 0 \quad \text{if } R \neq 1.
\]

This is possible because \(\lim_{L \to \infty} E_p(L, \rho - \rho_c, R)\) is discontinuous at \(\rho = \rho_c\).

Allowing for such a shift undermines the notion of universality: In fact, there is a \(\rho_c'(L, R)\) that deviates from \(\rho_c\) only by \(O(L^{-y_t})\) such that \(E_p(L, \rho'_c - \rho_c, R)\) or \(P(L, \rho'_c - \rho_c, R)\) is equal to any arbitrary constant \(0 < c < 1\).

What the authors actually show is that, for example, \(E_p(L, \rho_c, R) \propto F((\epsilon + \Delta\rho(L, R))L^{y_t}R^b)\) where \(\Delta\rho = \rho'_c - \rho_c\), which is in fact a function of system size and aspect ratio. This limitation of their result becomes even clearer in the other scaling form they suggest which takes “into account higher order corrections to the scaling”, \(E_p(L, \rho, R) \propto F(c_0(L, R) + \rho c_1(L, R) + \rho^2 c_2(L, R))\). This form, while interesting in itself, is not a form of scaling and therefore should not be called “superscaling”.

GP would like to thank the Alexander von Humboldt foundation as well as the NSF (DMR-0308548/0414122) for support.
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