Stock Liquidity Risk Pricing Model Driven by Systematic and Unsystematic Risk

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In the stock pricing, liquidity risk has become one of the important factors that affect the stock realizable value. Systematic and unsystematic risk decided a stock’s liquidity risk. The author uses the stock price index growth rate and net outer disk ratio to describe a systematic and unsystematic risk faced by investors. With the help of correlation and regression analysis in SPSS software, the paper tries to establish the systematic and unsystematic risk-driven stock liquidity risk pricing model. Empirical study shows that systematic and unsystematic risk has significant influence on stock liquidity risk. The bigger circulation stock, the greater the systemic risk influence; the less the circulation stock, the larger the non-system risk influence. Calendar factor on stock returns ratio has no significant effect. Trading volume on the stock returns ratio of small companies had no significant effect. The model has important reference value for the measure of stock liquidity risk value loss.

Keywords: stock liquidity risk, systematic risk, unsystematic risk, calendar effect

Introduction

Liquidity decides asset liquidation cost. The higher liquidity of the assets is, the lower the liquidation cost is; the lower the liquidity of the assets is, the higher the liquidation cost is. Therefore, in order to asset pricing, liquidity becomes one of the important influential factors. Capital asset pricing model (Sharpe, 1964; Lintner, 1965) only considers the systemic risk influence on the stock returns ratio, without considering the effect of unsystematic risk to stock returns ratio. VAR model only points out that portfolio (or stock) market value is lower (or higher) than the threshold probability. The two models did not solve the stock liquidity risk value.

In option pricing models, Black and Scholes (1973) assumed that drift rate and diffusion rate formed stock logarithmic return. LUO (2003) believed that Chinese stock market risk factors were beta coefficient, circulation market value, the turnover rate, P/E etc.. Cetin, Jarrow, and Protter (2004) believed that the stock trading volume decided stock logarithmic returns ratio. LUO (2006) found the systematic risk of China stock market ratio in total risk was more than 50%, while the American stock market systemic risk was only 25%. GU (2006) thought that systematic risk of the Chinese stock market was gradually reduced from 1973 to 2005. HUANG and ZOU (2007) argued that trading volume had important influence on transaction cost. HUANG, HU, and LIAO (2010) thought that the stock liquidity and transaction date had the relations.

The daily stock trading volume is divided into inner disk and outer disk. The outer disk trading volume is to promote the stock price rises, inner disk trading volume is to promote decline in stock price. If the author does not
consider other factors, only when the outer disk is greater than the inner disk trading volume, stock prices will rise. Similarly, only when the inner disk is larger than the outer disk trading volume, stock prices will fall.

If the outer disk trading volume is greater than the inner disk trading volume, said active buying volume more than active selling volume, the stock price will go up. If the outer disk trading volume is smaller than the inner disk trading volume, said active buying volume less than active selling volume, stock prices will fall. If the outer disk trading volume is equal to the inner disk trading volume, said actively buying volume equal to active selling volume, there is no impact on the stock prices.

If investors want to put the stock immediately into cash, must sell the stock according to the buyer’s offer, so that the inner disk trading volume rise, prompted a decline in stock price. If investors want to put cash immediately into stock, must buy the stock according to the seller's offer, so that the outer disk trading volume increases, make the stock price rises.

In the process of stock trading, the stock price trends to be connected not only with the inner disk and the outer disk trading volume, but also with the Shanghai Stock Exchange Composite Index (SSECI). When the stock price index rises, stock prices also rise. Since the outer disk trading volume is greater than the inner disk trading volume, stock prices will also rises.

The author thinks that there are many stock liquidity risk factors, in which the systematic and unsystematic risks are the two most important factors. Systematic risk can use stock index returns ratio to describe, while unsystematic risk can use net outer disk ratio to describe. The second part of the thesis establishes the systematic and unsystematic risk-driven stock liquidity risk pricing model. The third part is the empirical study. The fourth part is the conclusion.

**Stock Liquidity Risk Pricing Model**

In option pricing models, Black and Scholes (1973) assumed that the underlying asset was a function of time, denoted by \( S(t) \). In the stock option pricing when the author assumes stock logarithmic returns ratio which consists of two parts: one is determine \( \mu dt \); the other is random \( \sigma dW \):

\[
\ln\left(\frac{S(t+dt)}{S(t)}\right) = \mu dt + \sigma dW
\]  

(1)

where \( dt \) is time increment; \( \mu \) is stock logarithmic return ratio in unit of time; \( \sigma \) is the standard deviation of stock logarithmic return ratio; \( dW \) is Wiener process increment and obeys the normal distribution, \( dW \sim N(0, dt) \).

Actually, the stock logarithmic returns ratio is influenced by many factors, in which the systematic and unsystematic risks are the two most important factors. The stock logarithmic returns ratio has relationship not only with the exception of the time factor and random factors, but also with the stock price index increment \( dv \), the net outer disk increment \( du \) and trading volume increment \( dq \). That is to say, the stock returns ratio is the time, stock price index, net outer disk and trading volume function, denoted by \( S(t,v,u,q) \). Stock logarithmic returns ratio should be written in the following form:

\[
\ln\left(\frac{S(t+dt,v+dv,u+du,q+dq)}{S(t,v,u)}\right) = \mu dt + \alpha dv + \beta du + \gamma dq + \sigma dW
\]  

(2)

of which \( dv \) is stock price index increment; \( \alpha \) is effects of unit stock price index incremental changes to the stock logarithm return ratio; \( du \) is net outer disk increment; \( \beta \) is effects of unit net outer disk
incremental changes on the stock logarithm returns ratio; \(dq\) is trading volume increment; and \(\gamma\) is effects of unit trading volume increment on the stock logarithm returns ratio.

If the sampling interval is one day, with stock price index daily log returns rate \(x_i\) accounted for the stock price index increment \(dv\). Using the net outer disk trading volume ratio \(y_i\) instead of the net outer disk trading volume increment \(du\). The author use the trading volume ratio \(z_i\) instead of trading volume increment \(dq\). Considering the calendar effect on stock returns ratio, the author can put the equation (2) written in the following form:

\[
 r_i = \mu + \alpha x_i + \beta y_i + \gamma z_i + \sum_{k=1}^{5} \lambda_k w_k + \varepsilon_i
\]

where \(r_i\) is daily stock logarithmic returns ratio; \(\mu\) is mean of daily logarithmic returns ratio, when the net outer disk ratio, stock index logarithmic returns ratio and trading volume ratio are zeros, daily stock logarithmic returns ratio; \(x_i\) is stock price index daily logarithmic returns ratio; \(\alpha\) is contribution of stock price index daily logarithmic returns ratio on stock logarithmic returns ratio; \(y_i\) is net outer disk ratio, the ratio =[(outer-inner)/circulation stock]; \(\beta\) is contribution of unit net disk ratio on stock logarithmic returns ratio; \(\gamma\) is the effect of trading volume ratio \(z_i\) on stock returns ratio, trading volume ratio = [trading volume/circulation stock]; \(w_k\) is logical variable, \(k = 1, 2, \ldots, 5\), if it is Monday, \(w_1 = 1\), the other is equal to zero, and so on; \(\lambda_k\) is influence coefficient of the calendar on the stock daily logarithm returns ratio, there are five days in one week, \(k = 1, 2, \ldots, 5\); \(\varepsilon_i\) is error, other factors’ effects on the stock daily logarithmic returns ratio, \(\varepsilon_i \sim N(0, \sigma^2)\).

If investors to sell or buy of \(N\) stocks. If the investors to sell the \(n_i\) shares, where \(i = 1, 2, \ldots, N\), net outer disk ratio \(y_i\) is negative; if the investors to buy \(n_i\) shares, net outer disk ratio \(y_i\) is positive. Portfolio at time \(T\) market value for:

\[
 \sum_{i=1}^{N} [n_i S_i(T, x_i, y_i, z_i, w_k)] = \sum_{i=1}^{N} [n_i S_i(T, 0, 0, 0, 0)e^r]
\]

In which: \(n_i\) is the buy or sell stock quantity; \(r_i\) is logarithmic returns ratio of stock, showed with the equation (3); \(S_i(T, 0, 0, 0, 0)\) is the stock price per share before selling (or buying); \(S_i(T, x_i, y_i, z_i, w_k)\) is the stock price per share after selling (or buying).

**Empirical Study**

Industrial and Commercial Bank of China Limited (ICBC), Beijing Tongrentang Co., Ltd. (BTCL) and Beijing Beilu Pharmaceutical Co., Ltd (BPCL) three listed companies as the research object, and from April 1, 2010 to March 31, 2011 data were done by the empirical study. The relevant statistical parameters of the three companies are listed in Table 1, Table 2, and Table 3.

ICBC, BTCL, and BPCL Corporation’s sample size respectively are 232, 241, and 237. For company ICBC, there are three indicators of the returns ratio of stock, net outer disk ratio, and trading volume ratio, kurtosis is greater than 3. For BTCL and BPCL companies, the kurtosis of the net outer disk ratio is greater than 3, indicating that they do not follow a normal distribution. The distribution of these indicators is very concentrated. The kurtosis of three companies’ other variables is less than 3, their distribution is scattered.

In the three listed companies, the kurtosis of ICBC Company’s trading volume ratio is the biggest, showed that the company’s daily trading volume is very orderly. The BPCL Company’s trading volume ratio kurtosis is
the smallest, showed that the daily trading volume of the corporation most does not have the rule.

The use of SPSS software, the author analyzed the three company’s data in detail in Table 4. The results show that the ICBC stock returns ratio is significant correlation with the index returns ratio and trading volume ratio. In addition, ICBC stock daily returns ratio and net outer disk ratio are not significantly correlated with each other at the 0.05 level (2-tailed). The correlation of BTCL stock returns ratio with the index returns ratio, net outer disk ratio and trading volume ratio are significant at the 0.01 level (2-tailed). The correlation is significant between BPCL stock returns ratio and the index returns ratio, net outer disk ratio, volume ratio at the 0.01 level (2-tailed).

Table 1
ICBC Statistical Parameters

| Parameters       | Returns ratio | Index returns ratio | Net outer disk ratio | Trading volume ratio |
|------------------|---------------|---------------------|----------------------|----------------------|
| Sample size      | 232           | 232                 | 232                  | 232                  |
| Mean             | -0.0096       | 0.0118              | -0.0020              | 0.0392               |
| Std. deviation   | 1.5248        | 1.3982              | 0.0064               | 0.0332               |
| Variance         | 2.3250        | 1.9550              | 0.0000               | 0.0010               |
| Skewness         | -0.3900       | -0.5970             | -0.8650              | 4.7500               |
| Kurtosis         | 10.4330       | 1.7350              | 7.8490               | 34.3850              |
| Minimum          | -9.9800       | -5.1600             | -0.0400              | 0.0100               |
| Maximum          | 6.8400        | 3.4800              | 0.0200               | 0.3500               |

Note. Source: The author.

Table 2
BTCL Statistical Parameters

| Parameters       | Returns ratio | Index returns ratio | Net outer disk ratio | Trading volume ratio |
|------------------|---------------|---------------------|----------------------|----------------------|
| Sample size      | 241           | 241                 | 241                  | 241                  |
| Mean             | 0.2147        | -0.0165             | 0.0038               | 1.1967               |
| Std. deviation   | 2.8682        | 1.4037              | 0.1992               | 0.7293               |
| Variance         | 8.2260        | 1.9700              | 0.0400               | 0.5320               |
| Skewness         | 0.2740        | -0.5730             | 2.5710               | 1.4700               |
| Kurtosis         | 1.2990        | 1.5700              | 16.6400              | 2.3450               |
| Minimum          | -9.4000       | -5.1500             | -0.4700              | 0.1700               |
| Maximum          | 10.0200       | 3.4800              | 1.5600               | 4.1800               |

Note. Source: The author.

Table 3
BPCL Statistical Parameters

| Parameters       | Returns ratio | Index returns ratio | Net outer disk ratio | Trading volume ratio |
|------------------|---------------|---------------------|----------------------|----------------------|
| Sample size      | 237           | 237                 | 237                  | 237                  |
| Mean             | -0.0715       | -0.0114             | -0.0856              | 2.3449               |
| Std. deviation   | 2.8541        | 1.4105              | 0.4271               | 1.6481               |
| Variance         | 8.1460        | 1.9890              | 0.1820               | 2.7160               |
| Skewness         | -0.2930       | -0.5900             | 1.1770               | 1.5350               |
| Kurtosis         | 0.9260        | 1.5580              | 6.2690               | 2.1610               |
| Minimum          | -8.9800       | -5.1600             | -1.6600              | 0.4300               |
| Maximum          | 10.0100       | 3.4800              | 2.3100               | 8.4800               |

Note. Source: The author.
Using multiple regression analysis, in three equations between dependent variable and independent variable significantly associated in Table 5 and Table 6. The correlation coefficients respectively are 0.655, 0.711, and 0.801. The less the circulation stock is, the more the correlation coefficient is.

Table 4

|                      | Index returns ratio | Net outer disk ratio | Trading volume ratio |
|----------------------|---------------------|----------------------|---------------------|
| ICBC returns ratio   | 0.596** (0.000)     | -0.111 (0.091)       | 0.256** (0.000)     |
| BTCL returns ratio   | 0.368** (0.000)     | 0.661** (0.000)      | 0.181** (0.005)     |
| BPCL returns ratio   | 0.506** (0.000)     | 0.751** (0.000)      | 0.133* (0.041)      |

*Correlation is significant at the 0.05 level (2-tailed); **Correlation is significant at the 0.01 level (2-tailed). Source: The author.

Table 5

| Company   | Company code | Circulation stock (hand) | Sample size | Correlation coefficient | F     | Sig.  |
|-----------|--------------|--------------------------|-------------|-------------------------|-------|-------|
| ICBC      | 601398       | 2,622,245,012.77         | 232         | 0.655                   | 57.048| 0.000 |
| BTCL      | 600085       | 5,208,262.78             | 241         | 0.711                   | 80.785| 0.000 |
| BPCL      | 300016       | 537,985.26               | 237         | 0.801                   | 139.355| 0.000 |

Note: Source: The author.

Table 6

| Company | μ         | t        | Sig. | α         | t        | Sig. | β         | t        | Sig. | γ         | t        | Sig. |
|---------|-----------|----------|------|-----------|----------|------|-----------|----------|------|-----------|----------|------|
| ICBC    | -0.412    | -3.482   | 0.001| 0.668     | 11.983   | 0.000| -42.158   | -3.378   | 0.001| 7.873     | 3.350    | 0.001|
| BTCL    | -0.090    | -0.347   | 0.729| 0.552     | 5.740    | 0.000| 8.617     | 12.377   | 0.000| 0.236     | 1.246    | 0.214|
| BPCL    | 0.216     | 1.051    | 0.295| 0.602     | 7.093    | 0.000| 4.346     | 15.212   | 0.000| 0.039     | 0.556    | 0.579|

Note: Source: The author.

Three listed companies stock daily returns ratio calendar effect is not apparent, that is to say, Monday to Friday stock return rate has no obvious difference. Investors trading in the stock market from Monday to Friday, daily returns ratio is the same.

At 10% significant level, the correlation of BTCL and BPCL stock returns ratio with time trends is not significant. In other words, in one year the stock price of BTCL and BPCL, there is no obvious time trend. The ICBC company stock prices have the quite obvious time tendency at 1% significant level.

In addition the ICBC companies, the BTCL and BPCL stock returns ratio has nothing to do with trading volume ratio at 10% significant level. For medium-sized and small companies, trading volume and returns ratio has nothing to do.

In addition, the three companies stock returns ratio and SSE composite index rate of return is related to at 1% significant level. This shows that the SSE composite index rises, stock prices also rises. SSE composite index is a main power of stock prices.

ICBC stock price returns ratio and net outer disk ratio negative correlation. When ICBC stock prices rises, inner disk trading volume is greater than the outer trading volume, and the active selling of stocks is more than the active buying, which is the main reason of the big companies to support stock price index. ICBC stock price rise was mainly driven by the SSE Composite index.
BTCL and BPCL stock returns ratio and net outer disk ratio are strongly correlated. The net outer disk ratio of the two companies is one of the main engines of stock prices. The less the circulation stock is, the bigger the influence of net outer disk ratio is.

Table 7

Two Variable Parameters and Equations Test of Significance

| Company | \( \alpha \) | \( t \) | Sig. | \( \beta \) | \( t \) | Sig. | Correlation coefficient | \( F \) | Sig. |
|---------|-------------|--------|------|-------------|--------|------|------------------------|------|------|
| BTCL    | 0.530       | 5.570  | 0.000| 8.882       | 13.254 | 0.000| 0.708                  | 119.873 | 0.000 |
| BPCL    | 0.608       | 7.182  | 0.000| 4.231       | 15.436 | 0.000| 0.794                  | 200.475 | 0.000 |

Note. Source: The author.

Two variable parameters and equations test of significance are listed in Table 7. If we do not include a constant term, correlation coefficients of two equations are respectively 0.708 and 0.794. For BTCL and BPCL companies, the index returns ratio and net outer disk ratio on stock returns ratio have significant influence, but the influence of net outer disk ratio to the stock returns ratio is greater than to the index returns ratio.

Table 8

Single Variable Parameter and Equation Significance Test

| Company | \( \beta \) | \( t \) | Sig. | Correlation coefficient | \( F \) | Sig. |
|---------|-------------|--------|------|------------------------|------|------|
| BTCL    | 9.533       | 13.620 | 0.000| 0.660                  | 185.512 | 0.000 |
| BPCL    | 4.851       | 16.962 | 0.000| 0.741                  | 287.706 | 0.000 |

Note. Source: The author.

When trading stocks, people do not know the next moment index return ratio and only know the stock trading quantity. As pricing liquidity risk for small and medium-sized companies, people can only consider the net outer disk ratio on the impact of liquidity risk.

In Table 8 the author list single variable parameter and equation significance test. Two equation correlation coefficients are respectively 0.660 and 0.741, and net outer disk ratio on stock returns ratio has significant influence. When trading the stocks of small and medium-sized companies, a single variable linear regression equation can be used for stock liquidity risk pricing. If people consider only the net outer disk ratio influence the stock returns ratio, equation (4) can be written in equation (5):

\[
\sum_{i=1}^{N} [n_iS_i(T, y_i)] = \sum_{i=1}^{N} [n_iS_i(T, 0)e^r] \tag{5}
\]

If an investor holds BTCL company stock 3,000,000 shares and BPCL stock 500,000 shares, and listed in Table 9. Before selling the stock, BTCL and BPCL companies’ stock prices are respectively 14.51 and 9.33 RMB/share. After sells the stock in one day, BTCL and BPCL companies’ stock price respectively drop to 13.73 and 8.92 RMB/share. Stock returns ratios respectively are -5.38% and -4.39%, which is the liquidity cost.

Table 9

Sell the Stock Price Changes

| Company | Number of shares (share) | Before selling price (RMB/share) | After selling price (RMB/share) | Rate of price change (%) |
|---------|--------------------------|---------------------------------|--------------------------------|--------------------------|
| BTCL    | 3,000,000                | 14.51                           | 13.73                           | -5.38                    |
| BPCL    | 500,000                  | 9.33                            | 8.92                            | -4.39                    |

Note. Source: The author.
Conclusion

The calendar has not affected the stock returns ratio. In other words, whether Monday or Friday trading shares has no effect on stock returns ratio.

For small and medium-sized companies stock, trading volume has no effect on stock returns ratio. For large companies, trading volume affects stock returns ratio, but has little effect.

The stock returns ratio with net outer disk ratio and the SSE composite index returns ratio significant correlation. For small and medium-sized companies, stock returns ratio is determined by the SSE composite index and net outer disk ratio. In other words, systematic and unsystematic risks are the important impact factors of the stock returns ratio.

For large companies, the stock returns ratio is positively correlated with the SSE composite index and trading volume, negatively correlated with the net outer disk ratio. That is the greater the index return ratio and trading volume, the greater the stock return ratio of large companies; the greater the net outer disk ratio, the smaller the stock returns ratio.

The stock prices of large companies have a significant impact on the index. Large company stocks when prices rise, index also rose. When the market is overvalued, the weight stock to sell shares, when the market is undervalued, the weight stock to buy the shares.

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