Analysis of Unsteady Magneto Hydro Dynamic (MHD) Nano Fluid Flow Past A Sliced Sphere

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Abstract. A mixture of among solid small size nanoparticles with liquid fluid as the basic fluid, called nano fluid. Size of nano fluid partical 1-100 nm. Magnetohydrodynamic (MHD) is the movement of a fluid flow which is affected by a magnetic field, has high efficiency. The research discussed, analysis unsteady magnetohydrodynamics (MHD) at nano fluid boundary layer that past sliced sphere. The research will search how to form Dimensional governing equations. Nano fluid flow is influenced by the magnetic field cause the boundary layer until formed Non-similar equations boundary layer and then solved numerically using finite difference method with Keller-Box scheme. In this research will show to influence the variations in non dimensional nano fluid density parameters ($\rho_{ND}$), nano fluid heat capacity non dimensional variations ($\rho C_{ND}$), and nano particle volume fraction variations ($\chi$) to velocity profile ($f'$) and temperature profile ($s$). The results of the research, if the variation of non dimensional nano fluid density $\rho_{ND}$ is greater then the velocity profile increases but the temperature profile decreases, the greater variation in value of non-dimensional nano fluid heat capacity $\rho C_{ND}$ the velocity profile will increase constantly but the temperature profile increases significantly, the greater temperature the velocity profile variation of nanoparticle volume fraction $\chi$ increases when the value is $0 \leq \chi \leq 0.5$ and decreases at the value is $0.6 \leq \chi \leq 0.9$, the greater nanoparticle volume fraction $\chi$ variation the temperature profile increases.

Keyword: magnetohydrodynamic, unsteady, nano fluids, sliced sphere.

1. Introduction
At this time there has been a lot of research on nanotechnology and has evolved towards fluids called nano fluids. Nano fluid is a new fluid that has its own uniqueness. The nano fluid component is a mixture of liquid fluid with nanoparticles that have sizes between 1-100 nm. Such as the properties of nanoscale fluids namely the density of nano fluid, viscosity, specific heat of nano fluids, and thermal conductivity. Magnetohydrodynamic (MHD) is the fluid flow which is affected by a magnetic field, has high efficiency. In a recent study it has had a lot of research on the development of the flow of magnetohydrodynamic (MHD). There are two types of nano fluid that was recently studied, namely Newtonian and Non-Newtonian nano fluid. Newtonian fluid is a fluid that does not change viscosity when there are forces acting on the fluid and non-Newtonian fluid is a fluid that changes viscosity when there are forces acting on the fluid. In this research using Newtonian nano fluid type. The boundary layer is a thin layer on a solid surface confined to a very narrow area close to the contour surface which affected by the presence of viscosity and the inertial force of the object. Many cases of
magnetohydrodynamics are usually in a steady state even though at the boundary layer time changing is very important. Meanwhile, speaking of fluid flow through a solid sphere, which involves a pulling force (the force that makes a moving object) arising around it, this event understood through the identification of flow separation and this occurs for an unstable flow. (Widodo et al, 2015)

In this research discusses about analysis of the unsteady flow of nano fluid magnetohydrodynamic (MHD) passed the sliced sphere in the boundary layer, at the point of stagnation. There are governing equations like continuity equation, momentum equation, and energy equation. Dimensional equations are transformed into a non-dimensional equation. Then change the form of non-dimensional similarity to the equation. So we get the boundary layer of similar equations that can be solved numerically using finite difference schemes Keller-Box. After that it was simulated to determine the effect of velocity profiles and temperature profiles on non dimensional nano fluid density variations ($\rho_{ND}$), nano fluid heat capacity non dimensional variations ($\rho C_{ND}$), and nano particle volume fraction variations ($\chi$).

2. Mathematical modelling

2.1 3D to 2D Shapes

![Figure 1. Physical Model and Coordinate System](image)

2.2 Construction of mathematical models in the form of dimensional mathematical

2.2.1. Continuity Equation

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = 0$$

(1)

2.2.2. Momentum Equation Axis - x

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial P}{\partial x} + \nu_{n} \left( \frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} \right) - \frac{\sigma B_0^2 \bar{u}}{\rho_{nf}}$$

(2)

2.2.3. Momentum Equation Axis - y

$$\frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial P}{\partial y} + \nu_{n} \left( \frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 \bar{v}}{\partial y^2} \right) - \frac{\sigma B_0^2 \bar{v}}{\rho_{nf}}$$

(3)

2.2.4. Energy Equation

$$\frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} = \alpha_{n} \left( \frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} \right)$$

(4)

2.2.5. Initial and Boundary Conditions
\( \tilde{t} < 0; \tilde{u} = \tilde{v} = 0, \) for \( x \cdot y \)

\( \tilde{t} \geq 0; \tilde{u} = \tilde{v} = 0, T \to T_\infty, \) for \( \tilde{y} = b - a \)

\( \tilde{u} = \tilde{u}_e(x), T \to T_\infty, \) for \( y \to \infty \)

when \( \tilde{u}_e = \frac{3}{2} U_\infty \sin \left( \frac{x}{b} \right) \) (5)

The dimensional construct equation is used as Non-Dimensional equation, then similarity and finally at the lowest stagnation point which becomes the ordinary differential equation (Ordinary PD), then decreasing in the initial condition is obtained by the equation function

\( f; f'; f'' \) and \( s; s' \)

\[
f = \frac{1}{\sqrt{D}} \left( 2 \left( \frac{1}{2} \text{erf} \left( \frac{\eta}{\sqrt{D}} \right) \eta \left( \frac{1}{D} + e^{-\frac{\eta^2}{4D}} \right) \right) - 2 \sqrt{\frac{D}{\pi}} \right) \]

(6)

\[
f' = \text{erf} \left( \frac{\eta}{\sqrt{D}} \right) \]

(7)

\[
f'' = e^{-\frac{\eta^2}{D} + c_1} \]

(8)

\[
s' = e^{-\frac{c_1}{D} + \frac{\rho R_n^2}{G}} \]

(9)

\[
s = -\text{erf} \left( \frac{\eta}{\sqrt{G}} \right) + 1 \]

(10)

With boundary conditions

\[ f(0,t) = f'(0,t) = 0, s(0,t) = 1 \]

\[ f' = 1, s = 0 \] untuk \( \eta \to \infty \) (11)

3. Numerical method

After getting a mathematical model analysis of the unsteady flow of magnetohidrodynamic (MHD) nano fluid passing a sliced sphere, it can be done numerically using the Keller-Box method. The initial step in applying the Keller-Box method by discretizing the model that has been obtained using different methods, then doing linearization by using the Newton method and in a vector matrix, equipped with a block matrix elimination method, and simulating it in the program (MATLAB).

4. Result and discussion

4.1 Non Dimensional Nano Fluid Density (\( \rho_{ND} \))

At Figure 2. When numerical simulations are performed using (\( Pr = 0.7, \theta_s = 53^\circ, \chi = 0.2, M = 1, \rho C_{ND} = 0.7 \)) the result the greater the density value of non-dimensional nano fluid will be causing the speed profile that passes through the sliced ball to increase. In the simulation is given a value of 0, 4, 7, 10, 20 with each value given a different color.

In the Figure 3, it can be seen that the greater density value of the non dimensional nano fluid (\( \rho_{ND} \)) is given, the temperature profile that passes through the sliced ball becomes increasingly decreasing. Given the input value of the parameters is the same as simulating the velocity profile (\( Pr = 0.7, \theta_s = 53^\circ, \chi = 0.2, M = 1, \rho C_{ND} = 0.7 \))
Figure 2. Velocity Profile on Non Dimensional Nano Fluid Density Variations ($\rho_{ND}$).

Figure 3. Temperature Profile of Non Dimensional Nano Fluid Density Variations ($\rho_{ND}$).

4.2 Non Dimensional Nano Fluid Heat Capacity ($\rho C_{ND}$)

From Figure 4 the numerical simulation above it is known the value $\rho C_{ND} = 0; 0.7; 1.4; 2$ then the velocity profile increases constantly. As in the momentum there is no heat transfer or convection that occurs, the heat capacity is very small. So it can be proven using Table 1 as follows.

Based on Figure 5. It can be seen that the temperature profile (s) increases when the variation in the $\rho C_{ND}$ is greater. The parameters in this simulation are $\rho C_{ND} = 0; 0.7; 1.4; 2$ with ($Pr = 0.7, \theta_s = 53^\circ, M = 1, \rho_{ND} = 4$). Inside the nano fluid there is an energy that is produced to affect the size of the nanoparticles. The heat capacity itself is the energy added to increase the object temperature by 1 °C. It can be proved by using the following Table 2.
Table 1. Large Value \( f' \) when value \( \eta \) is \( 0 \leq \eta \leq 7 \)

| \( \rho C_{ND} \) | 0   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.00             | 0.935304 | 0.999105 | 0.999998 | 1 | 1 | 1 | 1 |
| 0.70             | 0.935304 | 0.999105 | 0.999998 | 1 | 1 | 1 | 1 |
| 1.00             | 0.935304 | 0.999105 | 0.999998 | 1 | 1 | 1 | 1 |
| 1.40             | 0.935304 | 0.999105 | 0.999998 | 1 | 1 | 1 | 1 |
| 2.00             | 0.935304 | 0.999105 | 0.999998 | 1 | 1 | 1 | 1 |

Table 2. Large Value \( s \) when value \( \eta \) is \( 0 \leq \eta \leq 7 \)

| \( \rho C_{ND} \) | 0   | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| 0.00             | 0.47534 | 0.13637 | 0.021693 | 0.002052 | 0.000364 | 0.0000291 | 0.000029 |
| 0.70             | 0.482006 | 0.142722 | 0.023969 | 0.002462 | 0.000463 | 0.000368 | 0.000366 |
| 1.00             | 0.48492 | 0.145551 | 0.02502 | 0.002663 | 0.000513 | 0.000407 | 0.000404 |
| 1.40             | 0.488862 | 0.149429 | 0.0265 | 0.002957 | 0.000589 | 0.000465 | 0.000462 |
| 2.00             | 0.494899 | 0.155481 | 0.028898 | 0.003461 | 0.000725 | 0.000569 | 0.000564 |

4.3 Nanoparticle Volume Fraction \( (\chi) \)

When the simulation is performed at Figure 6, the parameter value \( (\chi) \) is 0; 0.1; 0.3; 0.6; 0; 8 then the results obtained cause the nanoparticle velocity volume fraction \( (\chi) \) profile to increase but at a certain point the velocity profile graph decreases. After observing it turns out that the value of \( \chi \) is \( 0.6 \leq \chi \leq 0.9 \) the velocity profile increases and when the value of \( \chi \) is \( 0 \leq \chi \leq 0.5 \), the velocity profile decreases. This is due to the influence of the level of nano fluid viscosity and nano fluid density. If the composition of the volume of the nano fluid fraction increases, the fluid will become more concentrated. To clarify the results, the following Table 3. is given.

From the simulation results, given a value \( \chi \) of equal to 0; 0.1; 0.3; 0.6; 0.8 and \( (Pr = 0.7, \theta_s = 53^\circ, M = 1) \) the results of the temperature profile graph are increasing. Then it can be ascertained that the greater value \( \chi \) given, more temperature profile in the boundary layer increases. This is due to higher concentrated particle content in the fluid, resulting in friction between particles that can cause heat. Can be shown in the following Table 4.
Table 3. Large Value $f'$ when value $\eta$ is $0 \leq \eta \leq 7$

| $\chi$  | 0   | 0.00  | 0.10  | 0.30  | 0.60  | 0.80  |
|--------|-----|-------|-------|-------|-------|-------|
| $\eta$ | 0.00| 0.944724 | 0.999447 | 0.999999 | 1.000000 | 1.000000 |
| 1.00  | 0.901095 | 0.996844 | 0.999975 | 1.000000 | 1.000000 | 1.000000 |
| 1.10  | 0.752633 | 0.960668 | 0.998280 | 0.999994 | 1.000000 | 1.000000 |
| 1.20  | 0.394825 | 0.656055 | 0.817257 | 0.909825 | 0.959209 | 0.983708 |
| 1.40  | 0.172917 | 0.329642 | 0.471120 | 0.598985 | 0.714923 | 0.820673 |

Table 4. Large Value $s$ when value $\eta$ is $0 \leq \eta \leq 7$

| $\chi$  | 0   | 0.00  | 0.10  | 0.30  | 0.60  | 0.80  |
|--------|-----|-------|-------|-------|-------|-------|
| $\eta$ | 0.00| 1.000000 | 0.366620 | 0.056446 | 0.003066 | 5.67E-06 |
| 1.00  | 1.000000 | 0.439228 | 0.102921 | 0.011275 | 0.000582 | 5.81E-05 |
| 1.10  | 1.000000 | 0.557039 | 0.217621 | 0.054817 | 0.009208 | 0.001861 |
| 1.20  | 1.000000 | 0.689859 | 0.407837 | 0.197229 | 0.077175 | 0.027176 |
| 1.40  | 1.000000 | 0.752032 | 0.518066 | 0.318787 | 0.171890 | 0.081581 |

5. Conclusion

The results of numerical simulation using some variation of parameters such as non dimensional nano fluid density parameters ($\rho_{ND}$), non dimensional nano fluid heat capacity ($\rho C_{ND}$), and nano particle volume fraction ($\chi$) showed that:

- Variations of non dimensional nano fluid density ($\rho_{ND}$) is greater, the velocity profile increases but the temperature profile decreases.
- The greater variation in the value of non-dimensional nano fluid heat capacity ($\rho C_{ND}$) the velocity profile will increase constantly as the value $\eta$ is greater but the temperature profile increases significantly.
- The greater value of the nanoparticle volume fraction ($\chi$) causes the velocity profile to increase, at a certain point the velocity profile chart is getting lower but the temperature profile is decreasing.

6. References

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