Radial quadrupole and scissors modes in trapped Fermi gases across the BCS phase transition

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The excitation spectra of the radial quadrupole and scissors modes of ultracold Fermi gases in elongated traps are studied across the BCS superfluid-normal phase transition in the framework of a transport theory for quasiparticles. In the limit of zero temperature, this theory reproduces the results of superfluid hydrodynamics, while in the opposite limit, above the critical temperature, it reduces to the collisionless Vlasov equation. In the intermediate temperature range, the excitation spectra have two or three broad peaks, respectively, which are roughly situated at hydrodynamic and collisionless frequencies, and whose strength is shifted from the hydrodynamic to the collisionless modes with increasing temperature. By fitting the time dependent quadrupole deformation with a damped oscillation of a single frequency, we can understand the “jump” of the frequency of the radial quadrupole mode as a function of interaction strength which has recently been reported by the Innsbruck group.

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I. INTRODUCTION

In the recent years, experiments with ultracold trapped fermionic atoms like $^6$Li or $^{40}$K have become a powerful tool for improving our understanding of strongly correlated Fermi systems. For example, the continuous tunability of the interatomic interaction via Feshbach resonances made it possible to study the crossover from a Bose-Einstein condensate (BEC) of diatomic molecules to a BCS-type superfluid. The creation of vortex lattices in rotating Fermi gases with attractive interaction (scattering length $a < 0$) showed unambiguously that the superfluid BCS phase was indeed reached $^{1}$. By studying the collective modes of these systems, one can obtain a lot of interesting information which cannot be obtained from static properties alone. Near the BCS phase transition, several collective modes of the trapped Fermi gases show interesting features like an extremely strong damping or a sudden change of the frequency $^{2,3,4,5}$. However, most of these experiments were done in the vicinity of the Feshbach resonance ($k_F|a| > 1$, where $k_F$ is the Fermi momentum in the center of the trap), and in this strongly interacting regime no theory is available so far which could quantitatively explain the observed damping effects. An obvious qualitative difference between the strongly and the weakly interacting regimes is that in the latter ($k_F|a| < 1$) the normal phase is in the collisionless regime, whereas in the former ($k_F|a| \gtrsim 1$) collisions can produce a hydrodynamic behavior of the normal phase $^{6}$.

The aim of the present paper is to discuss the properties of experimentally accessible collective modes, namely of the radial quadrupole and scissors modes, in the BCS regime ($k_F|a| < 1$). In particular, we want to see how the properties of the modes change across the superfluid-normal phase transition. At zero temperature, the collective modes can be described within the framework of superfluid hydrodynamics, but this is no longer true at non-zero temperature, where superfluid and normal components coexist. In Ref. $^{7}$ it was proposed to use Landau’s two-fluid hydrodynamics in this case. However, although this approach may be valid in the strongly interacting regime, it cannot be applied in the weakly interacting regime because of the lack of collisions. The theoretical framework which will be used in the present paper is the semiclassical transport theory developed for clean superconductors by Betbeder-Matibet and Nozières $^{8}$, which has recently been applied to the case of trapped Fermi gases $^{9,10}$. In the case of a quadrupole oscillation in a spherical trap, this theory predicts a strong damping of the hydrodynamic mode in the superfluid phase due to its coupling to the collisionless normal component of the system, made of thermally excited quasiparticles.

The article is organized as follows. Sec. II gives a brief summary of the theoretical method (for details, see Refs. $^{9}$ and $^{10}$). In Sec. III the results for the quadrupole mode are discussed, and it is shown that the frequency jump observed in a recent experiment at Innsbruck $^{5}$ can be understood. In Sec. IV results for the scissors mode are presented and Sec. V contains further discussions and conclusions.

II. THEORETICAL METHOD

The method used in the present work to describe collective modes in the BCS phase at finite temperature is based on the quasiparticle transport theory by Betbeder-Matibet and Nozières $^{8}$. The basic degrees of freedom are the phase $\phi(r,t)$ of the superfluid order parameter $\Delta(r,t)$, describing the collective flow of the Cooper pairs, and the quasiparticle distribution function $\nu(r,p,t)$, describing the normal-fluid component. In equilibrium, one can choose $\phi = 0$, and the quasiparticle distribution function $\nu$ reduces to the usual equilibrium Fermi function $f(E) = 1/(e^{E/T} + 1)$, where $E = \sqrt{p^2 + \Delta^2}$ with $h = p^2/2m + V(r) - \mu$, $V$ and $\mu$ being the trap po-
potential and chemical potential, respectively. In the non-equilibrium case, one writes $\Delta = |\Delta e^{-2i\phi}|$, such that the collective velocity of the Cooper pairs is given by $\mathbf{v}_s = -\hbar \nabla \phi/m$. In this case, $\nu$ is defined in the rest frame of the Cooper pairs, and the density $\rho$ and current $j$ are given by

$$\rho = \int \frac{d^3p}{(2\pi\hbar)^3} \left( \frac{1}{2} - \frac{\hbar_+}{2E_+}(1 - 2\nu_+) \right),$$

$$j = \rho \mathbf{v}_s + \int \frac{d^3p}{(2\pi\hbar)^3} \frac{P}{m} \nu_-.$$  

The subscripts $+$ and $-$ denote the time-even and time-odd parts of a given function (e.g., $\nu_\pm(\mathbf{r}, \mathbf{p}, t) = (\nu(\mathbf{r}, \mathbf{p}, t) \pm \nu(\mathbf{r}, -\mathbf{p}, t))/2$),

$$\hbar = \frac{(\mathbf{p} - i\hbar \nabla \phi)^2}{2m} + V - \hbar \dot{\phi} - \mu$$

is the hamiltonian and

$$E = \sqrt{\hbar_+^2 + |\Delta|^2 + \hbar_-}$$

the quasiparticle energy in the rest frame of the Cooper pairs.

The equation of motion of $\nu$ can be written in the compact form

$$\dot{\nu} = \{E, \nu\},$$

where $\{E, \nu\}$ denotes the Poisson bracket of $E$ and $\nu$, similar to the usual Vlasov equation, which can be written as $f = \{h, f\}$, where $f$ denotes the usual distribution function. The equation for $\phi$ can be obtained from the continuity equation

$$\dot{\rho} + \nabla \cdot j = 0,$$

which becomes a second-order differential equation for $\phi$ if one inserts the explicit expressions for $\rho$ and $j$.

In order to describe collective modes, one writes $V = V_0 + V_1$, $\nu = \nu_0 + \nu_1$, and $\phi = \phi_0 + \phi_1$, with $\nu_0 = f(E_0)$ and $\phi_0 = 0$, the subscripts 0 and 1 referring to equilibrium quantities and deviations from equilibrium, respectively. Assuming that the perturbation $V_1$ is weak, one keeps only terms which are linear in the deviations from equilibrium. The explicit equations of motion for $\phi_1$ and $\nu_1$ are quite cumbersome and can be found in Refs. 11 and 10.

Note that, contrary to Ref. 10, the mean-field (Hartree) potential $g\rho$ ($g$ being the coupling constant) has been neglected in the definitions of $h$ and $\hbar$. If one wants to approach the crossover region ($kF|a| \sim 1$), the expression $g\rho$ for the mean-field potential is not applicable. One possible solution of this problem is to replace $g\rho$ by the real part of the single-particle self-energy in ladder approximation 11. Anyway, previous experimental and theoretical studies showed that the mean-field shift does not modify the qualitative behaviour of the modes and leads only to minor changes of the mode frequencies. For example, in the recent Innsbruck experiment on the radial quadrupole mode 3, the frequency shift in the collisionless normal phase from $2\omega_r$ ($\omega_r$ being the radial trap frequency) to $\approx 2.1\omega_r$ was attributed to the mean field. However, the mean field can considerably change the equilibrium density profile, thereby modifying the gap, the critical temperature, etc. For a quantitative study, the inclusion of the mean field is therefore desirable, but beyond the scope of the present paper.

Except in very simple cases (e.g., a sound wave propagating in a uniform system 3), the equations of motion for $\nu_1$ and $\phi_1$ have to be solved numerically. In Ref. 10, a test-particle method was proposed which is similar to the one often used for solving the Boltzmann equation. In the context of the linearized theory, it is useful to write $\nu_1$ in the form

$$\nu_1(\mathbf{r}, \mathbf{p}, t) = \frac{df(E_0)}{dE_0} y(\mathbf{r}, \mathbf{p}, t).$$

The advantage of this rewriting is that $y$ is a smooth function in phase space, contrary to $\nu_1$ [this is analogous to writing $f_1 = f_0(1 - f_0)\Phi$ in the context of the linearized Boltzmann equation, which is a rather common trick]. The test-particle method consists in replacing the continuous function $\nu_1(\mathbf{r}, \mathbf{p}, t)$ by a sum of $\delta$ functions,

$$\nu_1(\mathbf{r}, \mathbf{p}, t) \rightarrow \sum_{i=1}^{N_v} y_i(t) \delta(\mathbf{r} - \mathbf{R}_i(t)) \delta(\mathbf{p} - \mathbf{P}_i(t))$$

where $y_i(t) = y(\mathbf{R}_i(t), \mathbf{P}_i(t), t)$. The trajectories of the test particles in phase space, $\mathbf{R}_i(t)$ and $\mathbf{P}_i(t)$, satisfy the following equations of motion:

$$\dot{\mathbf{R}} = \frac{\partial E_0}{\partial \mathbf{P}}, \quad \dot{\mathbf{P}} = -\frac{\partial E_0}{\partial \mathbf{R}}.$$  

These equations of motion do not conserve the energy $E_0$, but the quasiparticle energy $E_0 = \sqrt{\hbar_0^2 + \Delta_0^2}$. This can lead to surprising effects which make the numerical solution more involved than that of usual Newtonian equations of motion. A well-known example is Andreev reflection: If a particle ($h > 0$) arrives at a point with $\Delta = E$, it is reflected as a hole ($h < 0$), and vice versa.

In the present calculations, the number of test particles is set to $N_v = 10^5$ as in Ref. 10. However, practically identical results are already obtained with $N_v = 10^4$. In fact, the large number of test particles was only needed in Ref. 10 in order to obtain a stable mean field, while here, where the mean field is neglected, a much smaller number of test particles is sufficient.

The time dependence of the coefficients $y_i(t)$ can rather easily be solved. For the time dependence of the phase, $\phi$ is expanded in a basis and the time dependence of the coefficients is determined by minimizing the violation of the continuity equation. In the case of the modes under consideration, the velocity field and therefore the form of $\phi$ is well known. In this case, one or two basis functions are completely sufficient 10.
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The radial quadrupole mode is excited at time \( t = 0 \) by applying a perturbation of the form \( V_1(\mathbf{r}, t) = V_1(\mathbf{r}) \delta(t) \) with \( V_1(\mathbf{r}) \propto x^2 - y^2 \). The equations of motion are then solved for \( t > 0 \). The excitation spectrum is obtained as the Fourier transform of the time evolution of the quadrupole moment \( Q = (x^2 - y^2) \).

We choose here parameters similar to the recent Innsbruck experiments: \( N = 4 \times 10^5 \) atoms, trap frequencies \( \omega_r = 2\pi \times 570 \) Hz, \( \omega_z = 2\pi \times 22 \) Hz. Since on the one hand our theory is only valid in the BCS limit, but on the other hand the scattering length of \( ^{6}\text{Li} \) saturates at quite a large value at high magnetic fields, we choose as a compromise \( 1/k_F a = -1.5 \). For these parameters, the critical temperature within BCS theory is \( T_C \approx 0.058T_F \approx 43 \) nK (where \( T_F = e_F/k_B \) is the Fermi temperature). However, one should remember that BCS theory cannot be trusted quantitatively. To give an example, the inclusion of the correlated density according to Nozières and Schmitt-Rink [12], keeping the number of atoms \( N = 4 \times 10^5 \) and the parameter \( 1/k_F a = -1.5 \) fixed, reduces the critical temperature by almost 20\% to \( T_C \approx 0.049T_F \) (\( k_F \) and \( T_F \) being defined as usual by the density of a non-interacting system with the same number of atoms).

The resulting excitation spectra for different temperatures \( T \) are shown in Fig. 1. At \( T = 0.15T_C \), the spectrum is dominated by a peak situated at the hydrodynamic frequency, \( \sqrt{2}\omega_r \). There is, however, a considerable broadening of the peak, i.e., damping, due to the coupling of the hydrodynamic mode to the thermally excited quasiparticles. At higher temperatures, the peak is further broadened and slightly shifted towards lower frequencies. At the same time, a second peak appears, which is situated at \( 2\omega_r \), the frequency of the radial quadrupole mode in an ideal (collisionless normal-fluid) Fermi gas. Close to \( T_C \), this mode is practically undamped (the width of the peak at \( 2\omega_r \) in the uppermost spectrum in Fig. 1 is due to an artificial broadening which has been introduced in order to improve the visibility of the peaks). As one can clearly see, the peak position does not move from \( \sqrt{2}\omega_r \) to \( 2\omega_r \) with increasing temperature, but the strength is shifted from the lower to the upper peak. There seems to be no simple explanation for the downshift of the hydrodynamic mode, but it can partially be understood as a consequence of the usual repulsion between two coupled modes. The general behavior is very similar to that found in previous work on the quadrupole mode in spherical traps [10-12].

This finding suggests the following interpretation of the results of the Innsbruck experiment [2], where a sudden “jump” of the frequency as a function of the interaction strength was found. In the analysis of the experiment, the deformation of the atom cloud was fitted with an ansatz function containing only a single frequency and damping rate. Let us try what happens if we apply the same analysis to our theoretical results.

In order to follow as closely as possible the experiment, we will do the calculation as a function of the coupling strength for fixed temperature, which we choose rather arbitrarily as \( T = 0.046T_F \). We also simulate the experimental initial conditions, which are different from the \( \delta \)-like perturbation used above. In the experiment, the potential is first slowly deformed from \( V_0(\mathbf{r}) \) to \( V_0(\mathbf{r}) + V_1(\mathbf{r}) \), with \( V_1(\mathbf{r}) \propto x^2 - y^2 \), giving enough time to the collisions to keep the momentum distribution spherical, and then the deformation \( V_1(\mathbf{r}) \) is suddenly switched off at \( t = 0 \). Unfortunately there is one step in the experiment which cannot be simulated within the present linearized theory, namely the expansion of the cloud before the picture is taken. However, if one assumes that the picture taken after the expansion reflects in some way the state of the system at the moment when the trap was switched off, it seems plausible to assume that the expansion is not important for the observed frequency and damping, but only for the observed amplitude and phase of the oscillation.

In Fig. 1 the obtained time dependence of the quadrupole moment \( Q(t) \) is shown for different values of \( 1/k_F a \). The solid lines are the calculations, while the dashed lines correspond to fits with functions of the form

\[
Q_{\text{fit}}(t) = Ae^{-\kappa t} \cos(\omega t + \phi) + Ce^{-\xi t}.
\]

The parameters \( A, \kappa, \omega, \phi, C, \) and \( \xi \) are determined by minimizing the error

\[
\langle (\Delta Q)^2 \rangle = \frac{1}{t_{\text{max}}} \int_0^{t_{\text{max}}} dt \left[ Q(t) - Q_{\text{fit}}(t) \right]^2,
\]

which is plotted as a function of \( 1/k_F a \). The excitation spectra for different temperatures and for different coupling strengths are shown in Figs. 2 and 3, respectively. The parameters \( A, \kappa, \omega, \phi, C, \) and \( \xi \) are determined by minimizing the error

\[
\langle (\Delta Q)^2 \rangle = \frac{1}{t_{\text{max}}} \int_0^{t_{\text{max}}} dt \left[ Q(t) - Q_{\text{fit}}(t) \right]^2,
\]

which is plotted as a function of \( 1/k_F a \). The excitation spectra for different temperatures and for different coupling strengths are shown in Figs. 2 and 3, respectively.
where \( t_{\text{max}} = 14 \text{ ms} \) is chosen as in Ref. [5]. It can be seen that for \( 1/k_Fa = -1.1 \) and for \( 1/k_Fa = -1.5 \) the fit reproduces almost exactly the full calculation, while for the intermediate couplings, in particular for \( 1/k_Fa = -1.3 \), the fit is quite bad.

In Fig. 3 the fitted frequency \( \omega \) and damping rate \( \kappa \) are shown as functions of the parameter \( 1/k_Fa \). The third panel shows the corresponding rms error of the fit, defined as \( \sqrt{\langle (Q(t) - Q(t=0))^2 / Q(t=0) \rangle} \). There is a couple of striking similarities with the experimental results: At strong coupling, where \( T_C \) is large and hence \( T/T_C \) is small, the fit gives the hydrodynamic frequency. With decreasing coupling, i.e., increasing \( T/T_C \), the damping increases and the frequency of the hydrodynamic mode is shifted downwards. The quality of the fit becomes quite bad (this corresponds probably to the shaded area in Ref. [5]), and at some critical coupling strength the fit jumps to the collisionless (normal-fluid) mode. As seen in the experiment, the damping rate of the collisionless mode is weaker than that of the hydrodynamic mode before the jump.

It should be noticed that with the present choice of parameters, the jump happens at \( 1/k_Fa \approx -1.32 \), corresponding to \( T/T_C \approx 0.6 \). This means that in the center of the trap the system is still superfluid, but since this region contributes only very little to the quadrupole moment, the quadrupole mode behaves almost as if the system was in the collisionless normal-fluid phase.

In spite of the surprising qualitative agreement with the findings of Ref. [5], there is no exact agreement on a quantitative level. First of all, in the experiment the sudden transition from hydrodynamic to collisionless normal-fluid behavior happens already at a stronger coupling than in the calculation. The position of the jump depends on the temperature, and if the temperature in the experiment had been higher than ours (0.0467\( T_F \)), this could explain the difference. In addition, one should re-member that the critical temperature predicted by BCS theory is too high. The second problem is the damping rate: The experimental damping rates are larger than the theoretical ones by roughly a factor of 2 near the jump, and even worse in the case of the collisionless normal-fluid mode far away from the transition. In fact, the experiment is not in the collisionless limit and collisions still play a role. In order to improve the theory in this respect, one would have to include a collision term similar to that of the Boltzmann equation into the quasiparticle transport equation. The small anharmonicity of the experimental trap potential could be an additional source of damping. However, a calculation where the radial harmonic potential, \( 4\pi m\omega_r^2(x^2+y^2) \), was replaced by a Gaussian one, \( -U_0 e^{-(x^2+y^2)/d^2} \), with a realistic width \( d = 0.05 \text{ mm} \), showed that this effect can explain only \( \sim 10\% \) of the experimentally observed damping in the normal phase. The third point concerns the frequencies: On the one hand, although the theory gives a down-shift of the hydrodynamic mode before the transition, it is not as strong as the down-shift observed in the experiment. On the other hand, the theoretical frequency of the collisionless normal-fluid mode above the transition is too low, because the mean-field has been neglected.

### IV. SCISSORS MODE

In the case of the radial quadrupole mode the velocity field is always irrotational and it does not change significantly from the superfluid to the normal-fluid phase. The different frequencies are a consequence of the Fermi sur-
Excitation spectra of the scissors mode as functions of the excitation frequency $\omega$ for different temperatures ranging from $0.3T_C$ to $0.9T_C$ (from bottom to top). The spectra have been folded with a Lorentzian of width $\pi \times 10$ Hz. The arrows indicate the predicted frequencies in the hydrodynamic $(\sqrt{\omega_x^2 + \omega_y^2})$ and collisionless limit $(\omega_x \pm \omega_y)$, respectively.

The excitation of the scissors mode consists in turning the gas back and forth around the $z$ axis [velocity field $v(r) \propto e_z \times r$], but also by an irrotational motion [velocity field $v(r) \propto \nabla(xy)$]. In an ideal gas, these two kinds of motion correspond to two distinct modes with frequencies $\omega_-$ and $\omega_+$, respectively, given by $\omega_{\pm} = \omega_x \pm \omega_y$. In the superfluid phase, of course, the rotational mode is suppressed and only the irrotational one exists. In the zero-temperature limit, its frequency approaches the hydrodynamic value $\omega_{sc} = \sqrt{\omega_x^2 + \omega_y^2}$.

The excitation of the scissors mode consists in turning the trap potential around the $z$ axis by a small angle. In the small-amplitude limit, this corresponds to an excitation operator $V_1(r) \propto xy$. The excitation spectra of the scissors mode for different temperatures, which are obtained analogously to those of the radial quadrupole mode (i.e. Fourier transform of $xy$ after a $\delta$-like perturbation), are shown in Fig. 4 for the case of $N = 3 \times 10^5$ atoms in a trap with $\omega_x = 2\pi \times 580$ Hz, $\omega_y = 2\pi \times 270$ Hz, $\omega_z = 2\pi \times 22$ Hz, and $1/k_Fa = -1.5$. At $T = 0.3T_C$, the spectrum has a single broad peak at the hydrodynamic frequency. The damping rate (width) is comparable with that of the hydrodynamic radial quadrupole mode at $T = 0.3T_C$. With increasing temperature, the two collisionless normal-fluid modes appear at the predicted frequencies, while the strength of the hydrodynamic mode decreases and its frequency is slightly shifted downwards. At $T = 0.6T_C$ and $0.75T_C$, the scissors-mode spectrum has three pronounced peaks. There is a striking analogy between these results and the experimental observations concerning the scissors mode in a BEC with a thermal cloud, where the BEC oscillates with $\omega_{sc}$ which is slightly shifted downwards while the thermal cloud oscillates with $\omega_z$.

The results of Fig. 4 are quite different from those found in the experiment [6]. However, this is not surprising, since the experiment was not done in the BCS limit but for $1/k_Fa = -0.45$, where in particular the collisions are very important. As one can see in Fig. 2 of Ref. [6], the phase transition in the experiment does not lead directly from the superfluid to the collisionless regime, but the system enters first a collisionally hydrodynamic regime (i.e., the mode frequency does not change) and reaches the collisionless phase only at much higher temperature. As shown in Ref. [17], the transition from collisional hydrodynamic to the collisionless regime leads to a continuous increase of the upper (irrotational) branch of the scissors mode. This explains why in the experiment no sign of a sudden jump of the scissors mode frequency has been observed. It would be interesting to see if in the BCS regime, e.g. at $1/k_Fa = -1.5$, the experiment would observe a similar jump as in the case of the quadrupole mode.

V. CONCLUSIONS

In the present paper, the behavior of the radial quadrupole and of the scissors mode across the BCS phase transition was theoretically studied in the framework of a semiclassical theory, in which superfluid hydrodynamics is coupled to a quasiparticle transport theory for the normal component. The theory proved to be applicable to realistic situations (large numbers of atoms, arbitrary trap geometry), and it provides an explanation of the surprising behavior of the radial quadrupole mode as a function of $1/k_Fa$ as observed by the Innsbruck group [6]: The damping near the BCS phase transition is due to the coupling between the hydrodynamic mode and the thermally excited quasiparticles. In addition to the damping, this coupling leads to a two-peak spectrum, the two peaks belonging, respectively, to the hydrodynamic mode of the superfluid component and the collisionless mode of the normal-fluid component. The sudden jump from the hydrodynamic to the collisionless frequency reported in Ref. [6] is probably a consequence of the fit with a single frequency in the analysis of the experiment.

It should be mentioned that an alternative explanation of the damping was suggested in Ref. [18]. The frequency shift and damping are not necessarily a signal for a superfluid-normal phase transition. They could also be related to the breakdown of superfluid hydrodynamics when $h\omega$ is of the same order of magnitude as the pairing gap $\Delta$. In Ref. [14], effects of this kind...
were quantitatively studied by using a fully quantum mechanical formalism \cite{10}, (QRPA), corresponding to the time-dependent Bogoliubov-de Gennes equations in the case of small deviations from equilibrium, and it was shown that indeed, even at zero temperature, considerable damping and deviations from the hydrodynamic frequency can occur if $\hbar \omega / \Delta$ is not small. Although such effects may also play a role in the recent experiments (maybe they can give rise to an additional downshift of the hydrodynamic quadrupole mode, which would help to quantitatively explain the data), they are probably less important than the coupling between the hydrodynamic mode and the thermally excited quasiparticles considered here: In Ref. \cite{10} it was shown that for a ratio of mode frequency to zero-temperature gap of $\hbar \omega / \Delta (T = 0) \sim 0.3$ (which is similar to the most unfavorable ratio of all the cases considered in the present paper), the results of the semiclassical theory and of the quantum mechanical QRPA calculation for the quadrupole mode in a spherical trap are in good agreement, even if the finite-temperature gap is smaller.

Possible extensions of this theory which are needed for giving more quantitative results at the limit between the BCS and the crossover regime ($k_F |a| \sim 1$) have been mentioned. These are, on the one hand, the inclusion of quasi-particle collisions in order to reproduce the observed damping, and on the other hand the inclusion of the single-particle self-energy in order to account for the “mean field” energy shift. However, these extensions may not be enough to give a complete description of the BCS-BEC crossover ($k_F |a| \gtrsim 1$), including the unitary limit ($k_F |a| \to \infty$). In this regime, the normal-fluid component of the system is not only made of thermally excited quasiparticles, which are included in the present theory, but also of pair fluctuations \cite{19}, which persist even above the critical temperature $T_C$ up to the “pair breaking temperature” $T^*$ and which cannot easily be accounted for in the present theoretical framework. 

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