Top quark pair production and decay at hadron colliders: Predictions at NLO QCD including spin correlations

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We present results at NLO QCD for hadronic production and decay of top quark pairs, taking into account \( \bar{t}t \) spin correlations.

The interactions of the top quark have not been precisely tested so far. The expected data samples of Run II of the Tevatron collider (eventually about \( 10^4 \bar{t}t \) pairs per annum) and the LHC (\( \sim 10^7 \bar{t}t \) pairs p.a.) will make possible detailed investigations of top quark production and decay. In particular, one may search for possible deviations from the QCD production mechanism and the V-A structure of the top decay by analysing energy and angular distributions of the \( \bar{t}t \) decay products. Such studies will be feasible since top quarks do not hadronize due to their very short lifetime, and perturbation theory gives a reliable description of top quark production and decay.

To search for new physics effects, precise theoretical predictions for top quark pair production and decay at hadron colliders within the Standard Model are needed. We consider here in particular the following processes:

\[
\begin{align*}
\text{h}_1\text{h}_2 & \to \bar{t}t + X \\
& \to \ell^+\ell^- + X \\
& \to \ell^+j_\ell + X \\
& \to \ell^-j_\ell + X \\
& \to j_\ell j_\ell + X,
\end{align*}
\]

where \( \text{h}_{1,2} = p, \bar{p}; \ell = e, \mu, \tau \), and \( j_\ell \) (\( j_{\ell'} \)) denote jets originating from hadronic \( t \) (\( \bar{t} \)) decays. An observable that is very sensitive to the dynamics of top quark production and decay in the above reactions is the double differential angular distribution of the top decay products, e.g. for the dilepton channel:

\[
\frac{1}{\sigma} \frac{d^2\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1}{4}(1 - C \cos\theta_+ \cos\theta_-). \tag{2}
\]

In (2), \( \theta_\pm \) are the angles between the \( \ell^\pm \) direction of flight in the \( t(\bar{t}) \) rest frame with respect to arbitrary axes \( \hat{a} (\hat{b}) \) which will be specified below. (Terms linear in \( \cos\theta_\pm \) are absent in (2) for our choices of \( \hat{a}, \hat{b} \) due to parity invariance of QCD.)

The calculation of the distribution (2) at next-to-leading order (NLO) QCD simplifies enormously in the leading pole approximation (LPA), which amounts to expanding the full amplitudes for (3) around the complex poles of the \( t \) and \( \bar{t} \) propagators. Only the leading pole terms are kept in this expansion, i.e. one neglects terms of order \( \Gamma_t/m_t \approx 1\% \). Within the LPA, the radiative corrections can be classified into factorizable and non-factorizable \( \bar{t}t \) contributions. Here we consider only the factorizable corrections. In this case the coefficient in the distribution (2) factorizes:

\[
C = \kappa_+ \kappa_- D. \tag{3}
\]

In (3), \( D \) is the \( \bar{t}t \) double spin asymmetry

\[
D = \frac{N(\uparrow\downarrow) + N(\downarrow\uparrow) - N(\uparrow\downarrow) - N(\downarrow\uparrow)}{N(\uparrow\uparrow) + N(\downarrow\downarrow) + N(\uparrow\downarrow) + N(\downarrow\uparrow)}, \tag{4}
\]
where \( N(\uparrow\uparrow) \) denotes the number of \( t\bar{t} \) pairs with \( t \) \( (\bar{t}) \) spin parallel to the reference axis \( \hat{a} \) \( (\hat{b}) \) etc. The directions \( \hat{a} \) and \( \hat{b} \) can thus be identified with the spin quantization axes for \( t \) and \( \bar{t} \), and \( D \) reflects the strength of the correlation between the \( t \) and \( \bar{t} \) spins for the chosen axes. Further, \( \kappa_{\pm} \) is the spin analysing power of the charged lepton in the decay \( t(\bar{t}) \to b(\bar{b})\ell^\pm \nu(\bar{\nu}) \) defined by the decay distribution

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \vartheta_{\pm}} = \frac{1 \pm \kappa_{\pm} \cos \vartheta_{\pm}}{2},
\]

where \( \vartheta_{\pm} \) is the angle between the \( t \) \( (\bar{t}) \) spin and the \( \ell^\pm \) direction of flight. From [2] we obtain

\[
\kappa_+ = \kappa_- = 1 - 0.015 \alpha_s,
\]

which means that the charged lepton serves as a perfect analyser of the top quark spin. For hadronic decays \( t \to bq\bar{q}' \) one has a decay distribution analogous to (4), and the spin analysing power of jets can be defined. To order \( \alpha_s \) \( (\text{full}) \) (and using \( \alpha_s(m_t) = 0.108 \)),

\[
\kappa_b = -0.408 \times (1 - 0.340 \alpha_s) = -0.393,
\]

\[
\kappa_j = +0.510 \times (1 - 0.654 \alpha_s) = +0.474,
\]

where \( \kappa_j \) is the analysing power of the least energetic non-\( b \)-quark jet. The results for \( C \) given below are for the dilepton channel. The corresponding results for the single lepton channel are obtained by replacing \( \kappa_+(or \ \kappa_-) \) in Eq. (4) by \( \kappa_b \) or \( \kappa_j \). The loss of analysing power using hadronic final states is overcompensated by the gain in statistics and in efficiency to reconstruct the \( t \) \( (\bar{t}) \) rest frames.

To compute \( D \) at NLO QCD, the differential cross sections for the following parton processes are needed to order \( \alpha_s^3 \), where the full information on the \( t \) and \( \bar{t} \) spins has to be kept:

\[
q\bar{q} \to t\bar{t}, \; t\bar{t}g; \; gg \to t\bar{t}, \; t\bar{t}g; \; q(\bar{q})g \to t\bar{t}q(\bar{q}).
\]

Results at NLO QCD for the \( \overline{\text{MS}} \) subtracted parton cross sections \( \hat{\sigma} \) for the above processes with \( t\bar{t} \) spins summed over have been obtained in [8], [9], while

\[
\hat{\sigma} D = \hat{\sigma}(\uparrow\uparrow) + \hat{\sigma}(\downarrow\downarrow) - \hat{\sigma}(\uparrow\downarrow) - \hat{\sigma}(\downarrow\uparrow)
\]

has been computed for different spin quantization axes in [8], [9]. This combination of spin-dependent cross sections can be written as follows

\[
\hat{\sigma} D = \frac{\alpha_s^2}{m_t^2} \left[ g^{(0)}(\eta) + 4\pi \alpha_s G^{(1)} \right],
\]

with

\[
G^{(1)} = g^{(1)}(\eta) + \tilde{g}^{(1)}(\eta) \ln \left( \frac{\mu^2}{m_t^2} \right),
\]

where \( \eta = \hat{s}/(4m_t^2) - 1 \), and we use a common scale \( \mu \) for the renormalization and factorization scale. As an example, Fig. 1 shows, for the parton process \( gg \to t\bar{t}(g) \), the three functions that determine \( \hat{\sigma} D \) for the helicity basis, i.e. \( \hat{a} \) \( (\hat{b}) \) is chosen to be the \( t \) \( (\bar{t}) \) direction of flight. Apart from the helicity basis we also consider the beam basis, where \( \hat{a} \) and \( \hat{b} \) are equal to the direction of one of the hadron beams in the laboratory frame, and the so-called off-diagonal basis [9], which is defined by the requirement that \( \hat{\sigma}(\uparrow \downarrow) = \hat{\sigma}(\downarrow \uparrow) = 0 \) for the process \( q\bar{q} \to t\bar{t} \) at tree level.

Table 1 lists our results [9] for the coefficient \( C \) in the double differential distribution [8] for
the contributions from PDFs is largely due to the
dependence on the PDFs rather than the scale uncertainty. This rather strong
spread of results for different PDFs is larger
than the scale uncertainty. This rather strong
dependence on the PDFs is largely due to the
fact that the contributions from $q\bar{q}$ and $gg$ initial
states contribute to $C$ with opposite signs. This
may offer the possibility to constrain the quark
and gluon content of the proton by a precise
measurement of the double angular distribution $\alpha$.

In all results above we used $m_t = 175$ GeV. A
variation of $m_t$ from 170 to 180 GeV changes the
results for the Tevatron, again for $\mu = m_t$ and
PDFs of GRV98 as follows: $C_{\text{hel}}$ varies from $-0.378$
and $C_{\text{off}}$ from $0.790$ to $0.817$, and $C_{\text{beam}}$
from $0.797$ to $0.822$. For the LHC, $C_{\text{hel}}$ changes
by less than a percent.

The results above have been obtained without
imposing any kinematic cuts. Standard cuts on
the top quark transverse momentum and rapidity
only have a small effect on $C$: For the Tevatron,
demanding $|k_T^{T,i}| > 15$ GeV and $|\alpha| < 2$ leads to
the following results: $C_{\text{hel}} = -0.386$, $C_{\text{beam}}$
= 0.815, $C_{\text{off}} = 0.823$. For the LHC, when impos-
ing the cuts $|k_T^{T,i}| > 20$ GeV and $|\alpha| < 3$, we
find $C_{\text{hel}} = 0.295$.

In summary, $t\bar{t}$ spin correlations are large ef-
fects that can be studied at the Tevatron and
the LHC by measuring double angular distribu-
tions both in the dilepton and single lepton decay
channels. The QCD corrections to these distribu-
tions have been computed and are under control.
Spin correlations are suited to analyse in detail
top quark interactions, search for new effects, and
may help to constrain the parton content of the
proton.

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### Table 1

| $\mu$          | LO | NLO | LO | NLO |
|----------------|-----|-----|----|-----|
| $C_{\text{hel}}$ | 0.456 | 0.389 | 0.305 | 0.311 |
| $C_{\text{beam}}$ | 0.910 | 0.806 | -0.005 | -0.072 |
| $C_{\text{off}}$ | 0.918 | 0.813 | -0.027 | -0.089 |

### Table 2

Upper part: Dependence on $\mu$ of the correlation
coefficients $C$, computed at NLO with the PDFs
of ref. [10]. Lower part: Correlation coefficients
$C$ at NLO for $\mu = m_t$ and different sets of parton
distribution functions: GRV98 [11], CTEQ5 [10],
and MRST98 (c-g) [12].

| $C_{\text{hel}}$ | $C_{\text{beam}}$ | $C_{\text{off}}$ | $C_{\text{hel}}$ |
|----------------|----------------|---------------|-------------|
| Tevatron       | LHC            | LHC           | LHC         |
| $m_t/2$        | -0.364 | 0.774 | 0.779 | 0.278 |
| $m_t$          | -0.389 | 0.806 | 0.813 | 0.311 |
| $2m_t$         | -0.407 | 0.829 | 0.836 | 0.331 |
| PDF            | $C_{\text{hel}}$ | $C_{\text{beam}}$ | $C_{\text{off}}$ | $C_{\text{hel}}$ |
| GRV98          | -0.325 | 0.734 | 0.739 | 0.332 |
| CTEQ5          | -0.389 | 0.806 | 0.813 | 0.311 |
| MRST98         | -0.417 | 0.838 | 0.846 | 0.315 |
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