Cosmology of non-minimal derivative coupling to gravity in Palatini formalism and its chaotic inflation

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We consider, in Palatini formalism, a modified gravity of which the scalar field derivative couples to Einstein tensor. In this scenario, Ricci scalar, Ricci tensor and Einstein tensor are functions of connection field. As a result, the connection field gives rise to relation, \( h_{\mu\nu} = fg_{\mu\nu} \) between effective metric, \( h_{\mu\nu} \) and the usual metric \( g_{\mu\nu} \) where \( f = 1 - \kappa \phi^2 \phi_{,\mu}/2 \). In FLRW universe, NMDC coupling constant is limited in a range of \(-2/\phi^2 < \kappa \leq \infty \) preserving Lorentz signature of the effective metric. Slowly-rolling regime provides \( \kappa < 0 \) forbidding graviton from travelling at superluminal speed. Effective gravitational coupling and entropy of blackhole’s apparent horizon are derived. In case of negative coupling, acceleration could happen even with \( w_{\text{eff}} > -1/3 \). Power-law potentials of chaotic inflation are considered. For \( V \propto \phi^2 \) and \( V \propto \phi^4 \), it is possible to obtain tensor-to-scalar ratio lower than that of GR so that it satisfies \( r < 0.12 \) as constrained by Planck 2015 [35]. The \( V \propto \phi^2 \) case yields acceptable range of spectrum index and \( r \) values. The quartic potential’s spectrum index is disfavored by the Planck results. Viable range of \( \kappa \) for \( V \propto \phi^2 \) case lies in positive region, resulting in less blackhole’s entropy, superluminal metric, more amount of inflation, avoidance of super-Planckian field initial value and stronger gravitational constant.

I. INTRODUCTION

Astrophysical observations strongly convinces us that the space is in the state of accelerating expansion. Results obtained from supernova type Ia (SNIa) [1–10], large-scale structure surveys [11, 12], cosmic microwave background (CMB) anisotropies [13–16] and X-ray luminosity from galaxy clusters [15, 17, 18] are examples of the evidence of the acceleration. If the expansion is to be accelerated, some unknown form of dark energy [19–22] is suggested as a driving force of the dynamics. Typically dark energy is in form of cosmological constant or scalar field [19–22] such as quintessence [23]-scalar with canonical kinetic term, and classes of k-essence type kinetic energy [24–26] which are hypothesized as dark energy. Alternative of Einstein gravity such as braneworlds, \( f(R) \) could as well result in present acceleration (see e.g. [27–29]). Other the other situation, inflationary expansion [30–34] in the early universe is also strongly supported by the recent CMB anisotropy observations [35]. Scalar field models or modified gravities should provide explanation to either or both inflationary acceleration and present acceleration.

A gravitational theory with non-minimal coupling (NMC) between scalar field’s derivative term and a gravity sector could as well give accelerating expansion. In metric formalism, of which the metric \( g_{\mu\nu} \) is a dynamical variable, the coupling function \( f(\dot{\phi},\phi,\phi_{,\mu},\phi_{,\mu\nu},\ldots) \) can be motivated by requirement of scalar quantum electrodynamics to preserve U(1) symmetry or by models with gravitational constant as a function of the mass density [36]. Non-minimal derivative coupling (NMDC) to \( R \) term can be found in lower energy limits of higher dimensional theories and in Weyl anomaly of \( N = 4 \) disformal supergravity [37, 38]. Without loss of generality, other possible coupling terms apart from \( R\phi_{,\mu}\phi^{,\mu} \) and \( R^{\mu\nu}\phi_{,\mu}\phi_{,\nu} \) are not necessary [39]. Hence gravitational theory with NMDC terms, \( R^{\mu\nu}\phi_{,\mu}\phi_{,\nu} \) and \( R\phi_{,\mu}\phi_{,\mu} \) with a free canonical kinetic term but without \( V(\phi) \) nor \( \Lambda \) term was studied and found cosmologically interesting, i.e. it gives de Sitter expansion [40]. Moreover types of NMDC models with two separated couplings have been investigated in various contexts and with further modifications [41–43].

Considering a special case of \( \kappa_1 R\phi_{,\mu}\phi^{,\mu} \) and \( \kappa_2 R^{\mu\nu}\phi_{,\mu}\phi_{,\nu} \) term, one can set \( \kappa_1 \equiv \kappa_2 = -2\kappa_1 \). As a result, the two NMDC terms combine into the Einstein tensor, \( G^{\mu\nu} \) coupling to scalar field’s kinetic part as \( \kappa G^{\mu\nu}\phi_{,\mu}\phi_{,\nu} \). The field equations contain terms with second-order derivative in \( g_{\mu\nu} \) and \( \phi \) at most order hence it is a good dynamical theory as Lagrangian contains only divergence-free tensors [44]. In flat FLRW universe, for \( \kappa > 0 \), there is quasi-de Sitter phase at very early stage and there is initial singularity at very early stage for \( \kappa < 0 \). The expansion is \( a \propto t^{1/3} \) at very late time for any sign of \( \kappa \) [44]. When adding \( V = \text{constant} \) and allowing phantom sign of the free kinetic term, it is possible to transit from de Sitter phase to other types of expansions [45]. If without free kinetic term, the model

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gives superluminal sound speed [46] and if having both free and coupling $\kappa G_{\mu\nu} \phi^\mu \phi^\nu$ term with $V(\phi) = 0$, the model can not have phantom crossing. Therefore potential is added into the theory and the acceptable action is

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{8\pi G_N} - (\varepsilon g_{\mu\nu} + \kappa G_{\mu\nu}) \phi^\mu \phi^\nu - 2V(\phi) \right] + S_m. \quad (1)$$

It is found that the potential must be less steep than quadratic potential in acquiring inflation [47]. With a constant potential and a matter term in the model, it is able to describe transition from inflation to matter domination epoch without reheating and this description includes transition to late de-Sitter epoch [48]. For positive potential, $\kappa > 0$ gives unbound $\phi$ value with restricted Hubble parameter [47]. For $V = $ constant and $\kappa > 0$, inflationary phase is always possible and the inflation depends solely on the value of coupling constant. Gravitational heavy particles are less produced during inflation when coupling to the inflaton field or to the particles gets stronger [49]. Perturbations and inflationary analysis of the model with a cosmological constant (or equivalently, the constant potential) was shown in [50] confronting observational data and in [51] as of exponential and monomial potentials. Other very interesting studies of slightly different versions of NMC model are reported, see such as [52–65]. The NMDC model considered here falls into a subclass of the Horndeski action (with $G_5 = \phi \kappa / 2$) which is generalized action that is Ostrogradski instability free [66].

So far the results given above are obtained in metric formalism. Considering Einstein-Hilbert action with matter term, the metric formalism gives equivalent field equation as that of the Palatini formalism. When GR is modified, the Palatini approach does not give the same field equations as those of the metric formalism as there is a non-minimal coupling between geometrical part and matter field and/or having some form of functions of the Ricci scalar. The affine connection and the metric are fundamentally independent concepts of geometrical entities [67]. Hence in the Palatini formalism, we consider metric tensor and connection field as independent dynamical fields. This independent connection field may look like disformal type [77] (generalization introduced by Bekenstien [78]). However here the situation is not about transformation between conformal or disformal frames. Not to mislead the readers, therefore we shall not refer to it as conformal nor disformal factor, but only a factor or a relation.

We shall investigate cosmological scenario of the model in Sec. III. whereas it is suggested that the field should be slowly-rolling and the coupling constant should be negative in order to preserve Lorentz invariance and to prevent the graviton from traveling faster than light. Cosmological field equations under slow-roll condition are stated in Sec. IV. Inflationary consideration is explored in Sec. V in which slow-roll parameters and spectral index were derived. We consider power-law potential of chaotic inflation in Sec. VI. We conclude our work and give comments in Sec. VII.

II. PALATINI NMDC GRAVITY

In the metric formalism, Sushkov’s NMDC action is [44]

$$S_g = \int d^4x \sqrt{-g} \left\{ R(g) - \left[ \varepsilon g_{\mu\nu} + \kappa \left( R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) \right) \right] \phi^\mu \phi^\nu - 2V(\phi) \right\} + S_m[g_{\mu\nu}, \Psi], \quad (2)$$

where the $\Psi$ represents matter fields and we set the unit $c = 1$ and $8\pi G_N = 1$. Canonical scalar field Lagrangian density is $L_\phi = -\varepsilon g_{\mu\nu} \phi^\mu \phi^\nu - 2V(\phi)$ where $\varepsilon = \pm 1$ is for canonical and the phantom cases respectively. The Einstein tensor is conventional, $G_{\mu\nu}(g) = R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g)$, as in Eq. (1). Differently, in Palatini formalism, the NMDC action is expressed as [76]

$$S_{\text{Palatini}} = \int d^4x \sqrt{-g} \left\{ \bar{R}(\Gamma) - \left[ \varepsilon g_{\mu\nu} + \kappa_1 g_{\mu\nu} \bar{R}(\Gamma) + \kappa_2 \bar{R}_{\mu\nu}(\Gamma) \right] \phi^\mu \phi^\nu - 2V(\phi) \right\} + S_m[g_{\mu\nu}, \Psi]. \quad (3)$$

Tilde symbol denotes variables that depend on the connection field. Following Sushkov [44], we set $\kappa = \kappa_2 = -2\kappa_1$ and define the Einstein tensor in Palatini formalism,

$$\bar{G}_{\mu\nu}(\Gamma) = \bar{R}_{\mu\nu}(\Gamma) - \frac{1}{2} g_{\mu\nu} \bar{R}(\Gamma). \quad (4)$$
Hence Eq. (3) is

$$S_{\text{Palatini}} = \int d^4x \sqrt{-g} \left\{ \tilde{R}(\Gamma) - \left[ \varepsilon g_{\mu\nu} + \kappa \tilde{G}_{\mu\nu}(\Gamma) \right] \phi^\mu \phi^\nu - 2V(\phi) \right\} + S_m[g_{\mu\nu}, \Psi]. \quad (5)$$

The Palatini Ricci tensor is defined by the independent dynamical connection,

$$\tilde{R}_{\mu\nu}(\Gamma) = \tilde{R}^\lambda_{\mu\nu}(\Gamma) = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\sigma \Gamma^\lambda_{\mu\sigma} + \Gamma^\lambda_{\sigma\lambda} \Gamma_{\mu\nu} - \Gamma^\lambda_{\sigma\nu} \Gamma_{\mu\lambda}, \quad (6)$$

and the Palatini Ricci scalar is $\tilde{R} = \tilde{R}(\Gamma) = g^{\mu\nu} \tilde{R}_{\mu\nu}(\Gamma)$. Varying the Palatini NMDC action in Eq. (5) with respect to the metric, we obtain the first field equation,

$$T_{\mu\nu} = \tilde{G}_{\mu\nu}(\Gamma) + \left[ \frac{\kappa}{2} \tilde{G}_{\mu\nu}(\Gamma) \phi^\lambda \phi^\lambda + \frac{\kappa}{2} \tilde{R}_{\alpha\beta}(\Gamma) g_{\mu\nu} \phi^\alpha \phi^\beta - \kappa \tilde{R}_{\mu\lambda}(\Gamma) \phi^\lambda \phi^\nu + \frac{\kappa}{2} \tilde{R}(\Gamma) \phi_{\mu} \phi_{\nu} \right.$$

$$
-2\kappa \tilde{R}_{\mu\lambda}(\Gamma) \phi^\nu \phi^\lambda + \varepsilon g_{\mu\nu} \phi^\alpha - \varepsilon \phi_{\mu} \phi_{\nu} + g_{\mu\nu} V(\phi)] ,
$$

whereas the matter energy-momentum tensor is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_m[g_{\mu\nu}, \Psi]}{\delta g^{\mu\nu}}. \quad (8)$$

The second Palatini NMDC field equation comes from the second degree of freedom, the independent connection field $\Gamma^\lambda_{\mu\nu}$ and it is

$$\nabla^\Gamma_{\lambda} \left\{ \sqrt{-g} \left[ g^{\mu\nu} \left( 1 - \frac{1}{2} \kappa \phi^\alpha \phi_{\alpha} \right) \right] \right\} = 0, \quad (9)$$

where $\nabla^\Gamma_{\lambda}$ is the covariant derivative with respect to the independent connection. This is written as

$$\nabla^\Gamma_{\lambda} \left( \sqrt{-g} g^{\mu\nu} f \right) = 0, \quad (10)$$

where

$$f = 1 - \frac{1}{2} \kappa \phi^\alpha \phi_{\alpha}. \quad (11)$$

Solving Eq. (10), the new metric or the effective metric $h_{\mu\nu}$ is related to the metric $g_{\mu\nu}$ via a transformation factor $f$ as

$$h_{\mu\nu} = fg_{\mu\nu} = (1 - \frac{1}{2} \kappa \phi^\alpha \phi_{\alpha})g_{\mu\nu}, \quad (12)$$

of which $\sqrt{-h} = \sqrt{-g}f^2$ and its inverse is $h^{\mu\nu} = f^{-1}g^{\mu\nu}$. Conformal invariance and disformal invariance between dual conformal frames [77–81, 84] are not the case here since the $h_{\mu\nu}$ metric is the effect of the Palatini connection field, not mathematical transformation of the Lagrangian from one frame to another. The relation is in form of

$$h_{\mu\nu} \equiv \alpha(\phi, X)g_{\mu\nu} + \beta(\phi, X)\phi_{\mu} \phi_{\nu}, \quad (13)$$

where $\alpha(\phi, X)$ and $\beta(\phi, X)$ are generalized factors. In general, the factors depend on the field kinetic term, $X = g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$. The effective metric, $h_{\mu\nu}$ in Eq. (12) is hence related to $g_{\mu\nu}$ with $\beta(\phi, X) = 0$. Eq. (13) can be written as

$$h_{\mu\nu} = \alpha(\phi, X)g_{\mu\nu} = [\alpha_1(\phi) + \alpha_2(\phi)\phi^\sigma \phi_{\sigma}] g_{\mu\nu}, \quad (14)$$

such that $\alpha_1(\phi) = 1$ and $\alpha_2(\phi) = -\kappa/2$. The relation (12) allows us to write the action (5) as function of $h_{\mu\nu}$,

$$S_{\text{Palatini}} = \int d^4x \sqrt{-h} \left\{ \frac{\tilde{R}(h)}{f^2} - \left[ \varepsilon h_{\mu\nu} + \kappa \tilde{G}_{\mu\nu}(h) \right] \phi^\mu \phi^\nu - \frac{2}{f^2} V(\phi) \right\} + S_m \left( \frac{h_{\mu\nu}}{f}, \Psi \right). \quad (15)$$
III. COSMOLOGICAL SCENARIO

The factor (14) in flat FLRW geometry with homogenous scalar field is

$$f(\dot\phi) = 1 - \frac{\kappa}{2}g^{00}\frac{d\phi}{dt}\frac{d\dot\phi}{dt} = 1 + \frac{\kappa}{2}\dot\phi^2.$$  \hspace{1cm} (16)

Note that there is a factor $8\pi G_N \equiv 1$ multiplying with $\kappa$ in this equation. With Eq. (14), the new metric $h_{\mu\nu}$ preserves Lorentz signature (−,+,+,+) if $\alpha = -\kappa/2$ and $-2/\dot\phi^2 < \kappa$ due to positivity of the factor $f(\dot\phi)$. For fast-rolling field, the coupling is allowed in positive region or in very small negative-value region. Hence the coupling estimably ranges from 0 < $\kappa$. For slowly-rolling field, the coupling is permitted in vast negative region. The FLRW effective metric is therefore

$$h_{\mu\nu} = \begin{pmatrix} -1 - \frac{\kappa}{2}\dot\phi^2 & 0 & 0 & 0 \\ 0 & a^2(1 + \frac{\kappa}{2}\dot\phi^2) & 0 & 0 \\ 0 & 0 & a^2(1 + \frac{\kappa}{2}\dot\phi^2) & 0 \\ 0 & 0 & 0 & a^2(1 + \frac{\kappa}{2}\dot\phi^2) \end{pmatrix},$$ \hspace{1cm} (17)

of which graviton speed is modified with the Palatini NMDC effect. Slowly-rolling field allows negative $\kappa$ hence graviton travels under the speed of light. On the other hand, fast-rolling case could result in superluminal graviton. We hence restrict our consideration to the slowly-rolling case. The result above enables us to find that $\nabla^f_k(\sqrt{-h}g^\mu\nu) = \nabla^f_k(\sqrt{-g}g^\mu\nu f)$ and the independent connection $\Gamma^\lambda_{\mu\nu}(h)$ is constructed with the effective metric $h_{\mu\nu}$ as

$$\Gamma^\lambda_{\mu\nu}(h) = \frac{1}{2}h^\lambda\sigma(\partial_\mu h_{\sigma\nu} + \partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu}).$$ \hspace{1cm} (18)

Following e.g. [82, 83], the effective gravitational coupling of Palatini NMDC gravity is hence

$$G_{\text{eff}} = \frac{f^2}{8\pi} = \frac{1}{8\pi} \left(1 + \frac{\kappa}{2}\dot\phi^2\right)^2,$$ \hspace{1cm} (19)

leading to modification of the entropy of blackhole’s apparent horizon for this theory,

$$S_{\text{AH}} = \frac{A}{4(1 + \frac{\kappa}{2}\dot\phi^2)^2/8\pi}. \hspace{1cm} (20)$$

The effective gravitational coupling strength of the model could be tested by observing temporal variation of the effective gravitational coupling,

$$\frac{\dot{G}_{\text{eff}}}{G_{\text{eff}}} = \frac{2\kappa\ddot{\phi}\dot{\phi}}{\left(1 + \frac{\kappa}{2}\dot\phi^2\right)^2}.$$ \hspace{1cm} (21)

For fast-rolling field, $\dot{G}_{\text{eff}}/G_{\text{eff}} \approx 4\dot{\phi}/\dot{\phi}$ and on the other hand, $\dot{G}_{\text{eff}}/G_{\text{eff}} \approx 2\kappa\ddot{\phi}\dot{\phi}$ for slowly-rolling field. Since $\Gamma = \Gamma(h, \partial h)$ hence the field equation (7) is expressed as function of $h$ and $\partial h$, for example, $\tilde{R}_{\mu\nu}(\Gamma)$ is $\tilde{R}_{\mu\nu}(h, \partial h)$ (written $\tilde{R}_{\mu\nu}(h)$ for brevity). The energy-momentum tensor obeys the relation $T_{\mu\nu} = \tilde{f}^{-1}T_{\mu\nu}$ (see Appendix A). Considering time-indexing of the field equation,

$$T_{00} = \tilde{G}_{00}(h) - \frac{\kappa}{2}\tilde{G}_{00}(h)\dot{\phi}^2 + \frac{5\dot{\kappa}}{2}\tilde{R}_{00}(h)\dot{\phi}^2 + \frac{\kappa}{2}\tilde{R}(h)\dot{\phi}^2 - \left(\frac{\varepsilon}{2}\dot{\phi}^2 + V(\phi)\right).$$ \hspace{1cm} (22)

The Ricci tensor for the effective metric $h_{\mu\nu}$ in $n$ dimensions is related to the usual Ricci tensor by the following formula (see e.g. [85, 86]),

$$\tilde{R}_{\mu\nu}(h) = R_{\mu\nu}(g) - \frac{(n-2)\delta^\mu_\sigma\delta^\nu_\rho + g_{\mu\nu}g^{\sigma\rho}}{\sqrt{f}} \frac{1}{\sqrt{f}}(\nabla^\alpha_\mu \nabla^\beta_\nu \sqrt{f}) + \frac{2(n-2)\delta^\mu_\sigma\delta^\nu_\rho - (n-3)g_{\mu\nu}g^{\sigma\rho}}{\sqrt{f}} \frac{1}{\sqrt{f}}(\nabla^\alpha_\mu \sqrt{f})(\nabla^\beta_\nu \sqrt{f}),$$ \hspace{1cm} (23)

where $\nabla^\alpha_\mu$ is the usual covariant derivative constructed from $g_{\mu\nu}$. Ricci tensor and Ricci scalar for the metric $g_{\mu\nu}$ in flat FLRW universe are,

$$R_{00}(g) = -3(\dot{H} + H^2), \hspace{0.5cm} R_{ii}(g) = a^2(\dot{H} + 3H^2), \hspace{0.5cm} R(g) = 6(\dot{H} + 2H^2),$$ \hspace{1cm} (24)

where $H = \dot{a}/a$. In case of the metric $h_{\mu\nu}$

$$
\tilde{R}_{00}(h) = -3(\dot{H} + H^2) - \frac{3}{2} \left( \frac{\ddot{f}}{f} - \frac{2\dot{f}^2}{f^2} \right), \quad \tilde{R}_{ii}(h) = R_{ii}(g) + \frac{\ddot{a}^2}{2f}.
$$

(25)

First and second-order time derivative of the factor are expressed in term of $\dot{\phi}$, $\ddot{\phi}$ and $\ddot{\phi}$, i.e.

$$
\dot{f} = \kappa \dot{\phi}, \quad \ddot{f} = \kappa \left( \ddot{\phi}^2 + \dot{\phi} \ddot{\phi} \right).
$$

(26)

The Ricci scalar is (see [86])

$$
\tilde{R}(h) = f^{-1}R(g) - 2(n - 1)g^{\alpha\beta}f^{-3/2} \left( \nabla_\alpha \nabla_\beta \sqrt{f} \right) - (n - 1)(n - 4)g^{\alpha\beta}f^{-2} \left( \nabla_\alpha \sqrt{f} \right) \left( \nabla_\beta \sqrt{f} \right).
$$

(27)

In four dimensions, this is

$$
\tilde{R}(h) = \frac{1}{f}R(g) + 3 \left( \frac{\ddot{f}}{f^2} - \frac{\dot{f}^2}{2f^3} \right) = \frac{6}{f} \left( \dot{H} + 2H^2 \right) + 3 \left( \frac{\ddot{f}}{f^2} - \frac{\dot{f}^2}{2f^3} \right).
$$

(28)

Using $\tilde{T}_{\mu\nu} = f^{-1}T_{\mu\nu}$, the $T_{00}$ component of NMDC-Palatini field equation is the matter density,

$$
\rho_m = \dot{H} \left[ 12f + \frac{6}{f} - 18 \right] + H^2 \left[ 12f + \frac{12}{f} - 21 \right] - \frac{3}{2} (1 - f) \left( \frac{4\ddot{f}}{f} - \frac{8\dot{f}^2}{f^2} \right) - \frac{3\ddot{f}}{2f} + \frac{3\dot{f}}{f} \left( \frac{3\ddot{f}}{2f} + \frac{3\dot{f}}{f} \right) - \frac{3\ddot{f}}{2f^3} - \rho_\phi,
$$

(29)

where $\rho_{\text{tot}} \equiv \rho_m + \rho_\phi$ and $\rho_\phi = \varepsilon\dot{\phi}^2/2 + V(\phi)$. The matter pressure found from the $T_{ii}$ components is

$$
p_m = \dot{H} (4f - 6) + H^2 (6f - 9) - \frac{3}{2} (1 - f) \left( \frac{\ddot{f}}{f} - \frac{2\dot{f}^2}{f^2} \right) + \frac{\ddot{f}}{f} - \frac{3\ddot{f}}{2f} + \frac{3\ddot{f}}{4f^2} - p_\phi,
$$

(30)

where $p_{\text{tot}} \equiv p_m + p_\phi$. The pressure of scalar field is $p_\phi = \varepsilon\dot{\phi}^2/2 - V(\phi)$. For brevity, we define

$$
A \equiv 4f - 6, \quad B \equiv 6f - 9, \quad C \equiv -\frac{3}{2} (1 - f) \left( \frac{\ddot{f}}{f} - \frac{2\dot{f}^2}{f^2} \right) + \frac{\ddot{f}}{f} - \frac{3\ddot{f}}{2f} + \frac{3\ddot{f}}{4f^2},
$$

(31) \hspace{2cm} (32) \hspace{2cm} (33)

$$
D \equiv 12f + \frac{6}{f} - 18, \quad E \equiv 12f + \frac{12}{f} - 21,
$$

(34) \hspace{2cm} (35)

$$
F \equiv \frac{3}{2} (1 - f) \left( \frac{4\ddot{f}}{f} - \frac{8\dot{f}^2}{f^2} \right) - \frac{3\ddot{f}}{2f} + \frac{3\ddot{f}}{f} \left( \frac{3\ddot{f}}{2f} + \frac{3\dot{f}}{f} \right) - \frac{3\ddot{f}}{2f^3}.
$$

(36)

The effective equation of state parameter is hence

$$
w_{\text{eff}} \equiv \frac{p_{\text{tot}}}{\rho_{\text{tot}}} = \frac{AH + BH^2 + C}{DH + EH^2 + F}.
$$

(37)

The modified $\dot{H}$ and modified Friedmann equations are found as (the Friedmann equation in the $h_{\mu\nu}$ metric is shown in the Appendix B.)

$$
\dot{H} = \frac{[(B - Ew_{\text{eff}})H^2 - Fw_{\text{eff}} + C]}{Dw_{\text{eff}} - A}, \quad H^2 = \rho_{\text{tot}} \left[ \frac{1 - \frac{3(C - Dw_{\text{eff}})}{Dw_{\text{eff}} - A} - \frac{F}{D}}{[\frac{B - Ew_{\text{eff}}}{Dw_{\text{eff}} - A}] - \frac{3}{3}} \right].
$$

(38)

The acceleration equation can be found from the $\ddot{a}/a = \dot{H} + H^2$. In the GR limit, $f = 1$ making $A = -2$, $B = -3$, $E = 3$ and $C = D = F = 0$ hence $p_{\text{tot}}$ and $w_{\text{eff}}$ reduce to the usual $p_{\text{tot}} = -2\dot{H} - 3H^2$ and $w_{\text{eff}}$ =
\[ -1 - 2\dot{H}/3H^2 \]. The Friedmann and acceleration equations can as well reduce to standard GR case, \( H^2 = \rho_{\text{tot}}/3 \), and \( \ddot{a}/a = -(1/6) (\rho_{\text{tot}} + 3\rho_{\text{tot}}) \). Modified Klein-Gordon equation can be obtained from the Euler-Lagrange equation for scalar field (see e.g. [86] for standard result),

\[
\ddot{\phi} - \kappa \dot{\phi} \left[ \hat{R}(h) - \tilde{R}_{00}(h) \right] - \kappa \dot{\phi} \nabla^h \tilde{R}_{00}(h) + \frac{\kappa}{2} \dot{\phi} \nabla^h \hat{R}(h) - 3\varepsilon H \dot{\phi} - V' = 0,
\]

where \( V' = dV(\phi)/d\phi \) and \( \nabla^h \dot{\phi} = \nabla^h \phi = \partial_t \phi \), and

\[
\nabla^h \nabla^h \phi = \nabla^\mu \nabla^\nu \phi - \left( \delta^\mu_\nu \delta^\beta_\alpha + \delta^\beta_\nu \delta^\alpha_\mu - g_{\mu\nu} g^{\alpha\beta} \right) \frac{1}{\sqrt{g}} \left( \nabla^\alpha \sqrt{f} \right) \left( \nabla^\beta \phi \right).
\]

The time component of the equation (40) reads, \( \nabla^0 \nabla^h \phi = \ddot{\phi} = \ddot{\phi}/f \). The modified Klein-Gordon equation hence reads

\[
\ddot{\phi} - \frac{\kappa}{2} \left[ \frac{6\dot{H} + 12H^2}{f} + 3 \left( \frac{\dot{f}}{f^2} - \frac{\dot{f}^2}{2f^3} \right) - 3 \left( H + H^2 \right) - \frac{3}{2} \left( \frac{\ddot{f}}{f} - \frac{2\dot{f}^2}{f^2} \right) \right] \ddot{\phi} + \frac{\kappa}{2} \nabla^0 \left[ \frac{6\dot{H} + 12H^2}{f} + 3 \left( \frac{\dot{f}}{f^2} - \frac{\dot{f}^2}{2f^3} \right) \right] - 3\varepsilon H \dot{\phi} - V' = 0.
\]

which recovers the usual Klein-Gordon equation, \( \varepsilon \ddot{\phi} + 3\varepsilon H \dot{\phi} + V' = 0 \) in the GR limit.

**IV. SLOW-ROLL REGIME**

Slowing-rolling field obeys the condition \( 0 < |\phi| \ll 1 \) and we approximate further that \( |\ddot{\phi}| \ll |\dot{\phi}| \ll |\phi| \), i.e. \( 0 \sim |\ddot{\phi}| \ll |\dot{\phi}| \ll |\phi| \). This enables us to neglect \( \dot{\phi}, \ddot{\phi}, \dddot{\phi} \). The modified Klein-Gordon equation (41) in the slow-roll regime is

\[
\ddot{\phi} \left[ \varepsilon - \frac{9\kappa}{2} \dot{H} \left( 1 - \kappa \dot{\phi}^2 \right) - \frac{3\kappa}{2} \dot{H}^2 \left( 5 - 6\kappa \dot{\phi}^2 \right) \right] + 3\dot{H} \phi \left[ \varepsilon - \left( \frac{\dot{H}}{H} + 4\dot{H} \right) \kappa \left( 1 - \frac{\dot{\phi}^2}{2} \right) \right] + V' \approx 0.
\]

**V. SLOW-ROLL INFLATION**

Considering the early universe when the scalar field dynamically drives the inflation. Scalar field density dominates the universe and slow-roll regime is plausible during the era. The Friedmann equation (44) reads

\[
H^2 \approx \frac{\rho_{\phi}}{3M_P^2} \left[ 1 + \frac{3}{2} \kappa \dot{\phi}^2 \left( 1 + \frac{\rho_{\phi}}{\rho_{\phi}} \right) \right] \approx \frac{1}{3} \frac{V(\phi)}{M_P^2}.
\]
as the condition $\dot{\phi}^2 \ll V(\phi)$ is assumed. Here we restore $8\pi G_N = M_P^{-2}$ to the equation. The coupling constant can be considered in mass$^{-2}$ dimension, $\kappa = M^{-2} \ll M_P^{-2}$. The other useful relation is

$$\dot{H} \simeq \frac{V'\dot{\phi}}{6HM_P^2} \simeq \frac{\sqrt{3V'\dot{\phi}}}{6\sqrt{V}M_P}$$

(47)

as the slow-roll approximated Friedmann equation is used. Further slow-roll approximation, $\ddot{\phi} \phi^2 \simeq 0$, $\phi^3 \simeq 0$ and $|\dot{H}| \ll |H\ddot{H}| < |H^3|$ give the Eq. (45) as

$$\ddot{\phi} \left( \varepsilon - \frac{9\kappa H}{2} - \frac{15\kappa}{2} H^2 \right) + 3H\dot{\phi} \left( \varepsilon - 4H\kappa \right) + V' \simeq 0. $$

(48)

If we let $\ddot{\phi} \simeq 0$, this gives

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H(\varepsilon - 4\kappa H)}. $$

(49)

Using this in (47) hence,

$$\dot{H} \simeq -\frac{(V'(\phi))^2}{18H^2M_P^2(\varepsilon - 4\kappa H)}. $$

(50)

Deriving slow-roll parameters, the first one is

$$\epsilon_v \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_P^2}{2(\varepsilon - 4\kappa H)} \left( \frac{V'}{V} \right)^2, $$

(51)

with a slow-roll condition, $\epsilon_v \ll 1$. The second parameter

$$\delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}} \simeq -\frac{V''(\phi)}{3H^2(\varepsilon - 4\kappa H)} + \frac{V'(\phi)H}{3H^3(\varepsilon - 4\kappa H)} - \frac{4\kappa \ddot{H}V'(\phi)}{3M_P^2H^2(\varepsilon - 4\kappa H)^2}\phi. $$

(52)

with a slow-roll condition, $|\delta| \ll 1$. The first term on the right-hand side of Eq. (52) is, in fact, the other slow-roll parameter, $\eta_v$, manifesting another NMDC-Palatini effect,

$$\eta_v \equiv \frac{4\kappa \ddot{H}}{HM_P^2(\varepsilon - 4\kappa H)} \simeq -\frac{4\kappa \ddot{H}V'(\phi)}{3H^2M_P^2(\varepsilon - 4\kappa H)^2}\phi \simeq \frac{4\kappa}{M_P^2(\varepsilon - 4\kappa H)^3} \left( \frac{V''(V')^2}{18V} - \frac{V'^4}{36V^2} \right). $$

(53)

Therefore $\delta = -\eta_v + \epsilon_v + \eta_v$. The spectral index of the model can be derived from $n_s - 1 = -4\epsilon_v - 2\delta$ to obtain [87],

$$n_s - 1 = -6\epsilon_v + 2\eta_v - 2\eta_v = -6\epsilon_v - 2\delta$$

(55)

or written in full form of $V(\phi)$ and its field derivatives,

$$n_s - 1 = -\frac{3M_P^2}{(\varepsilon - 4\kappa H)} \left( \frac{V'}{V} \right)^2 + \frac{2M_P^2}{(\varepsilon - 4\kappa H)} \frac{V''}{V} - \frac{8\kappa}{M_P^2(\varepsilon - 4\kappa H)^3} \left[ \frac{V''(V')^2}{18V} - \frac{(V')^4}{36V^2} \right]. $$

(56)

The $e$-folding number during the inflationary epoch can be found from $N = \ln(a_f/a_i) = \int_{t_i}^{t_f} H \, dt$, where $i$ and $f$ denotes the beginning and the end of inflationary phase. From Eqs. (46) and (49), $Hdt = [-V(\varepsilon - 4\kappa H)/V'(M_P^2)]d\phi$. During inflation, $H$ is almost constant due to slowly-rolling field as seen in Eq. (47), the number of e-folds is hence approximately

$$N \simeq \frac{(\varepsilon - 4\kappa H)}{M_P^2} \int_{\phi_i}^{\phi_f} \frac{V(\phi)}{V'(\phi)} \, d\phi = (\varepsilon - 4\kappa H) \int_{\phi_i}^{\phi_f} \frac{1}{\sqrt{2\epsilon_v GR M_P}} \, d\phi, $$

(57)

where here $\epsilon_v, GR \equiv (M_P^2/2)(V'/V)^2$. A known from of the scalar field potential is necessary in order to evaluate $n_s$ and $\epsilon_v$. From now on, we consider only the non-phantom field, i.e. $\varepsilon = 1$. For slow-roll scalar field dynamics, $\dot{H} < 0$, if the NMDC-Palatini effect, $\kappa < 0$, it reduces the amount of inflation from that of the GR case. On the other hand, for $\kappa > 0$, the effect is opposite, that is to enhance the amount of inflation.
VI. CHAOTIC INFLATION POTENTIALS

Consider single monomial potential (chaotic inflation [88]) in form of

\[ V(\phi) = V_0 \phi^n, \]

(58)

where \( V_0 \equiv \lambda (M_P^4/M_P^n) \). With \( V'/V = n/\phi \) and \( V''/V = n(n-1)/\phi^2 \), the slow-roll parameters and the spectral index can be found,

\[ \epsilon_{\nu} = \frac{n^2 M_P^2}{2(1-4\kappa H)} \phi^2, \quad \eta_{\nu} = \frac{n(n-1)}{2N_{\text{GR}}} \left( \frac{1}{1-4\kappa H} \right), \quad \eta_{\kappa} = \frac{n^3(n-2)}{9(1-4\kappa H)^3} \frac{\kappa V_0^2 \phi^{2n-4}}{M_P^2}. \]

(59)

In GR, scalar field rolling in power-law potential (with \( n > \sqrt{2} \)) is super-Planckian in order to satisfy the slow-roll condition. Considering \( \epsilon_{\nu} \ll 1 \), the GR slow-roll condition is modified with the NMDC-Palatini effect, \( |n|/(\sqrt{2} \sqrt{1-4\kappa H})M_P < \phi \). For \( \kappa > 0 \) case, the slowly-rolling scalar field can avoid the super-Planckian regime if the coupling takes the value in a range, \( \kappa < -(n^2 - 2)/(8\dot{H}) \), (note that \( \dot{H} < 0 \)). That is for \( V \propto \phi^{-2} \) or \( V \propto \phi^2 \), it is \( \kappa < 1/(4\dot{H}) \) and for \( V \propto \phi^4 \), it is \( \kappa < 7/(4\dot{H}) \). Oppositely if \( \kappa < 0 \), the field takes more super-Planckian value in order to slowly roll. For simplification, we let \( \phi_i = 0 \) and the integral (57) is \( N \simeq (1-4\kappa H)\phi_i^2/(2nM_P^2) \) so that we define

\[ \phi^2 = \phi_i^2(n, N, \dot{H}) \simeq \frac{2nM_P^2}{1-4\kappa H} N. \]

(61)

The e-folding number of the GR case is \( N_{\text{GR}} = \phi^2/(2nM_P^2) \), therefore \( N = N_{\text{GR}}(1-4\kappa H) \) and \( N > N_{\text{GR}} \) for \( \kappa > 0 \). The slow-roll parameters can be expressed in term of \( N_{\text{GR}} \) as

\[ \epsilon_{\nu} = \frac{n}{4N_{\text{GR}}} \left( \frac{1}{1-4\kappa H} \right), \quad \eta_{\nu} = \frac{n-1}{2N_{\text{GR}}} \left( \frac{1}{1-4\kappa H} \right), \quad \eta_{\kappa} = \frac{\kappa V_0^2 2^{n-2} M_P^{2n-6}}{9(1-4\kappa H)^3} (n-2)n^{n+1}N_{\text{GR}}^{n-2}, \]

(62)

where the \( \epsilon_{\nu, \text{GR}} \equiv n/(4N_{\text{GR}}) \) and \( \eta_{\nu, \text{GR}} \equiv (n-1)/(2N_{\text{GR}}) \). The spectral index is hence

\[ n_s \simeq 1 - \frac{n+2}{2N_{\text{GR}}} \left( \frac{1}{1-4\kappa H} \right) - 2^{n-1} \frac{\kappa V_0^2 M_P^{2n-6}}{9(1-4\kappa H)^3} (n-2)n^{n+1}N_{\text{GR}}^{n-2}. \]

(63)

For the \( V = V_0 \phi^2 \) case, we have

\[ \epsilon_{\nu} = \eta_{\nu} = \frac{1}{2N_{\text{GR}}} \left( \frac{1}{1-4\kappa H} \right), \quad \eta_{\kappa} = 0 \quad \text{and} \quad n_s = 1 - \frac{2}{N_{\text{GR}}(1-4\kappa H)}, \]

(64)

and for the \( V = V_0 \phi^4 \) case, they are

\[ \epsilon_{\nu} = \frac{3}{2N_{\text{GR}}} \left( \frac{1}{1-4\kappa H} \right), \quad \eta_{\nu} = \frac{3}{2N_{\text{GR}}} \left( \frac{1}{1-4\kappa H} \right), \quad \eta_{\kappa} = \frac{8192 \kappa V_0^2 M_P^2 N_{\text{GR}}^2}{9 (1-4\kappa H)^3}. \]

(65)

and

\[ n_s = 1 - \frac{3}{N_{\text{GR}}(1-4\kappa H)} - \frac{16384}{9} \frac{\kappa V_0^2 M_P^2 N_{\text{GR}}^2}{(1-4\kappa H)^3} N_{\text{GR}}^2. \]

(66)

The GR predictions for the \( V \propto \phi^n \) models, given \( N_{\text{GR}} = 60 \) and \( n = 2 \), are the tensor-to-scalar ratio \( r \simeq 16n_{\epsilon, \text{GR}} \simeq 0.13 \) and \( n_s \simeq 0.967 \). The \( n = 4 \) case has \( r \simeq 0.27 \) and \( n_s \simeq 0.95 \). These are disfavored by Planck 2015’s results [35] which are \( r < 0.12 \) at 95% CL (B-mode polarization constraint from the BICEP2/Keck Array/Planck joint analysis) and \( n_s = 0.968 \pm 0.006 \) (temperature and large angular scale polarization data). Eq. (47) for the power-law potential case is

\[ \dot{H} \simeq \frac{\sqrt{3} V_0}{6} n \phi(n-2)/2 \phi^2 M_P. \]

(67)
For $V = V_0 \phi^2$, it is found from Eq. (64) that the range, $\kappa \dot{H} \lesssim -0.027$ can satisfy the upper bound of $r < 0.12$, hence $\kappa > 0$ is favored. The Planck 2015 bound of $n_s = 0.968 \pm 0.006$ corresponds to the range $-0.071 \lesssim \kappa \dot{H} \lesssim 0.031$, hence the combined bound is

$$0.071 \gtrsim |\kappa| \dot{H} \gtrsim 0.027.$$  \hfill (68)

Using $\dot{H} \simeq \sqrt{V_0/3}(\phi/M_p)$ (the $n = 2$ case of Eq. (67)), the combined condition (68) is satisfied when

$$\frac{0.174}{|\phi| (m/M_p)} \gtrsim \kappa \gtrsim \frac{0.066}{|\phi| (m/M_p)},$$

where $V_0 \equiv (1/2)m^2 = \lambda M_p^2$ and hence $\lambda = (1/2)(m/M_p)^2$. The small value of $\dot{\phi}$ and the mass $m > M_p$ counterbalance the value of $\kappa$. For $V = V_0 \phi^4$, using Eq. (65), the range $\kappa \dot{H} \lesssim -0.306$ satisfies the bound of $r < 0.12$. Using $\dot{H} \simeq 2\sqrt{V_0/3}(\phi/M_p)$ (the $n = 4$ case of Eq. (67)), the condition $r < 0.12$ is satisfied as $\kappa \lesssim 0.264/|\sqrt{\phi}|(\phi/M_p)$ where in this case $V_0 = \lambda$. As seen in Eq. (66), $n_s$ of the $V = V_0 \phi^4$ case needs to be much fine-tuned due to the NMDC-Palatini effect of the $\eta_0$ term. It is noticed that $\kappa > 0$ results in superluminal nature of the metric, less blackhole’s entropy and stronger gravitational constant.

VII. CONCLUSIONS

We investigate cosmology of a non-minimal derivative coupling (NMDC) to gravity model in Palatini formalism imposing non-minimal constant coupling between the Einstein tensor and the scalar field derivative term. The Lagrangian contains also a free scalar field derivative term and a scalar potential as proposed in [44]. In Palatini formalism, the connection field is a dynamical variable hence Ricci scalar and Ricci tensor are also functions of connection field. As a result, the Einstein tensor is a function of the connection, $G_{\mu\nu} (\Gamma)$. Variation of the NMDC action with respect to the independent connection gives the factor, $f = 1 - \kappa \phi^\alpha \phi_{,\alpha}/2$. In FLRW spacetime the factor takes the form, $f(\phi) = 1 + \kappa \phi^2/2$. The NMDC coupling constant is enforced to be in a range of $-2/\phi^2 < \kappa \lesssim \infty$ in order to preserve the Lorentz signature of the metric. The coupling needs to be negative in order to prevent graviton traveling with superluminal speed. The effective gravitational coupling of the theory is $G_{\text{eff}} = (8\pi)^{-1}(1 + \kappa \phi^2/2)^2$ (in the unit of $8\pi G_N = c = 1$) which reduces to standard GR case when there is no NMDC coupling. The NMDC-Palatini effective gravitational constant leads to modification of the entropy of blackhole’s apparent horizon to $S_{\text{eff}} = A/[4(1 + \kappa \phi^2)^2/8\pi]$. The cosmological field equations found can reduce to standard form in the GR limit. Field equations are approximated in the slow-roll regime. We see that the acceleration condition is modified to $w_{\text{eff}} \lesssim -(1/3)(1 + 2\kappa \phi^2)$. The NMDC-Palatini effect of the $2\kappa \phi^2$ term, with $\kappa > 0$, results in acceleration to occur at $w_{\phi}$ value less than $-1/3$. The $\kappa > 0$ case can also enhance the amount of inflation. The NMDC-Palatini effect results in an extra term $\eta_0$ in the slow-roll parameter $\delta = -\eta_0 + \epsilon_\phi + \eta_\phi$ so that it affects the spectrum index. In case of $V \propto \phi^2$, the $\eta_0$ term vanishes in the expression of $n_s$, however in the case of $V \propto \phi^4$, it contributes to enormous value of $n_s$. In this model, the quartic potential is not likely to be viable compared to the Planck 2015’s predicted range of spectrum index [35]. The $\kappa > 0$ case can help avoiding super-Planckian region so that it can achieve slow-roll. In the $V \propto \phi^2$ case, the $\kappa > 0$ coupling could help resolving the Planck 2015’s tensor-to-scalar ratio constraint, $r < 0.12$. In the GR case, for $N_{\text{GR}} = 60$, the $V \propto \phi^2$ potential gives $r \simeq 0.13$ and $n_s \simeq 0.967$ which are disfavored by Planck 2015. The NMDC-Palatini model, the range $0.071 \gtrsim |\kappa| |\dot{H}| \gtrsim 0.027$ can satisfy Planck 2015 tensor-to-scalar ratio upper bound ($r < 0.12$) and the constraint $n_s = 0.968 \pm 0.006$ at the same time. The viable range corresponds to $\frac{0.174}{|\phi| (m/M_p)} \gtrsim \kappa \gtrsim \frac{0.066}{|\phi| (m/M_p)}$ of which the chaotic inflation $V \propto \phi^2$ could be viable in the range. It should be noted that the positive $\kappa$ would allow superluminal nature of the metric, less blackhole’s entropy and stronger gravitational constant. The other types of inflationary potential such as exponential potential and cosmological perturbations should be investigated in future works. Moreover, analysis on inflationary exit and possibility that the model could give eternal inflation are await to be done.

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Appendix A: Proof of relation between $\tilde{T}_{\mu\nu}$ and $T_{\mu\nu}$

\[
\tilde{T}_{\mu\nu} = -\frac{2}{\sqrt{-h}} \frac{\delta L_m(g_{\kappa\lambda}, \Psi)}{\delta h^{\mu\nu}} = -\frac{2}{f^2 \sqrt{-g}} \frac{\delta L_m(g_{\kappa\lambda}, \Psi)}{\delta (f^{-1}g^{\mu\nu})} = -\frac{2}{f \sqrt{-g}} \frac{\delta L_m(g_{\kappa\lambda}, \Psi)}{\delta g^{\mu\nu}} = f^{-1} T_{\mu\nu}.
\]

(A1)

Appendix B: Hubble parameter and the Friedmann equation derived with the metric $h_{\mu\nu}$

As in Eq. (17), the line element in the new metric $h_{\mu\nu}$ form is

\[
ds^2 = h_{\mu\nu} dx^\mu dx^\nu = -f dt^2 + f a^2 dx^2.
\]

Defining $d\tilde{t} = \sqrt{f} dt$, $\tilde{a} = \sqrt{f} a$ and $\tilde{H} = \tilde{a}^{-1} d\tilde{a}/d\tilde{t}$, hence $ds^2 = -d\tilde{t}^2 + \tilde{a}^2 dx^2$. Therefore

\[
H = \tilde{H} \sqrt{f} - \frac{1}{2} \tilde{f},
\]

(B1)

and the Friedmann equation of the $h_{\mu\nu}$ is

\[
\tilde{H}^2 = \frac{H^2}{(1 + \kappa \dot{\phi}^2/2)} + \frac{\kappa \dot{\phi} \dot{H}}{(1 + \kappa \dot{\phi}^2/2)^2} + \frac{\kappa^2 \dot{\phi}^2 \ddot{\phi}^2}{4(1 + \kappa \dot{\phi}^2/2)^3}.
\]

(B2)

In the slow-roll regime, $\tilde{H}^2 \simeq H^2/(1 + \kappa \dot{\phi}^2/2)$.

Appendix C: Conclusion of results in comparison to the metric NMDC gravity

Here we compare major results of the metric and Palatini approaches of the NMDC models. Please note that our coupling $\kappa$ is defined as $M^{-2}$ in some other Refs. such as in [49, 52, 53, 84].

- **Friedmann equation**
  - Metric approach [44]

\[
H^2 = \frac{1}{3 M_p^2} \left( \rho_{tot} - \frac{9 \kappa H^2 \dot{\phi}^2}{2} \right).
\]

(C1)

Palatini approach

From Eq. (44),

\[
H^2 \simeq \frac{\rho_{tot}}{3 M_p^2} \left( 1 + \frac{3 \kappa \dot{\phi}^2}{2 M_p^2} (1 + w_{eff}) \right),
\]

(C2)

where $\rho_{tot} = \varepsilon \dot{\phi}^2/2 + V(\phi) + \rho_m$. Of the metric approach, there is a NMDC coupling term $\kappa H^2 \dot{\phi}^2$ in the Friedmann equation. This is coupled to kinematic part, the Hubble function. Unlike the metric case, in the NMDC Palatini Friedmann equation, the NMDC coupling to the kinematic part is not via the $H$ or $\dot{H}$ terms but via the effective EoS, $w_{eff}$ which is either written as function of density or function of the $H$ or $\dot{H}$ terms.

- **Klein-Gordon equation**
  - Metric approach (See, e.g. [61, 63])

\[
\ddot{\phi} (\varepsilon - 3 \kappa H^2) + 3 H \dot{\phi} (\varepsilon - 3 \kappa H^2 - 2 \kappa \dot{H}) + V' = 0.
\]

(C3)

Palatini approach

Eq. (48) reads,

\[
\ddot{\phi} \left( \varepsilon - (9/2) \kappa \dot{H} - (15/2) \kappa H^2 \right) + 3 H \dot{\phi} \left( \varepsilon - 4 \kappa \dot{H} \right) + V' \simeq 0.
\]

(C4)
• Spectral index

Metric approach

During slow-roll, $H^2 \gg \dot{H}$ and in high friction limit ($-\kappa H^2 \gg 1$) [51, 54],

$$1 - n_s \simeq \frac{8\epsilon_{c, GR}}{(\varepsilon - 3\kappa H^2)} - \frac{2\eta_{c, GR}}{(\varepsilon - 3\kappa H^2)}. \tag{C5}$$

Palatini approach

From Eq. (56), in high friction limit, $-\kappa \dot{H} \gg 1$,

$$1 - n_s \simeq -\frac{6\epsilon_{c, GR}}{(\varepsilon - 4\kappa H)} + \frac{2\eta_{c, GR}}{(\varepsilon - 4\kappa H)}. \tag{C6}$$

where $\epsilon_{c, GR} \equiv (M^2_0/2)(V'/V)^2$ and $\eta_{c, GR} \equiv M^2_0(V''/V)$.
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