The lineshape of the compact fully heavy tetraquark

Zejian Zhuang,1, 2, * Ying Zhang,1, 2, † Yuanzhuo Ma,1, 2, ‡ and Qian Wang1, 2, 3, §

1Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China
2Guangdong–Hong Kong Joint Laboratory of Quantum Matter, Southern Nuclear Science Computing Center, South China Normal University, Guangzhou 510006, China
3Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
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Hadrons and their distributions are the most direct observables in experiment, which would shed light on the non-perturbative mystery of quantum chromodynamics (QCD). As the result, any new hadron will challenge our current knowledge on the one hand, and provide additional inputs on the other hand. The fully heavy $cc\bar{c}\bar{c}$ system observed by LHCb recently opens a new era for hadron physics. We first extract the internal structure of the fully heavy tetraquarks directly from the experimental data, within the compact tetraquark picture. By fitting to the di-$J/\psi$ lineshape, we find that the $X(6900)$ is only a cusp effect from the $J/\psi\psi(3770)$ channel. In addition, there is also a cusp slightly below 6.8 GeV stemming from the $J/\psi\psi'$ channel. The two $0^{++}$ tetraquarks behave as two resonances above the di-$J/\psi$ and di-$J/\psi'$ threshold, respectively. The $2^{++}$ state is a bound state below the di-$J/\psi'$ threshold. Furthermore, we find that the $X_{1^{++}}(6035)$ shows a significant structure in the di-$J/\psi$ lineshape even after the coupled channel effect. This is an unique feature which can distinguish compact $cc\bar{c}\bar{c}$ tetraquark from the loosely hadronic molecules.

INTRODUCTION

The formation of hadrons is from the non-perturbative mechanism of Quantum chromodynamics (QCD), the theory of strong interaction. As the result, the properties of hadrons are expected to shed light on the mystery of non-perturbative dynamics of QCD. The success of the conventional quark model attracts the attention of the community to search for the predicted missing particles for several decades. The situation breaks up since the observation of the $X(3872)$, as the first exotic candidate, in 2003. That challenges the conventional quark model and stimulates research enthusiasm on the so called exotic hadrons. Up to now, tens of exotic candidates have been observed by experimental collaborations (such as LHCb, BESIII, BelleII, JLab, CMS, ATLAS) and numerous studies [1–10] have been proposed for the understanding of their properties.

Most of them contain a pair of heavy quarks and locate at heavy quarkonium region. Two competitive scenarios, i.e. tetraquark and hadronic molecular pictures, are proposed for their nature based on two clusters, i.e. $cq-\bar{c}q'$ and $c\bar{q}-\bar{c}q'$ respectively. Although great efforts have been put forward to distinguish these two interpretations for a given particle, no definite conclusion about any particle yet in the community. One potential solution is pinning the hope on a fully heavy system, which is one goal of several experimental collaborations. e.g. the LHCb [11, 12] and CMS [13] collaborations. The importance of the fully heavy system is because that it cannot be classified to several clusters intuitively. One expects that a comparison of the fully heavy system with the $cc\bar{c}\bar{q}$ system would give some hints for the formation of hadrons. Luckily, the LHCb collaboration [12], recently, reports a narrow structure around 6.9 GeV and a broad structure within the range 6.2 GeV $\sim$ 6.8 GeV in the di-$J/\psi$ invariant mass distribution, using the data at 7 TeV, 8 TeV and 13 TeV center-of-mass energies. Because of the observed channel, the quark content of those structures is $cc\bar{c}\bar{c}$. The narrow structure around 6.9 GeV can be explained by a Breit-Wigner parametrization with and without the interference between the resonant contribution and the nonresonant contribution [12]. This structure is named as the $X(6900)$.

The study of the fully heavy system came back to 1970s which was motivated by the observation of the $\psi'$ [14] and the severe emerging structures in $e^+e^-$ annihilation [15]. However, the study becomes a dilemma because of no experimental data. The observation of the $X(6900)$ [12] breaks through the situation. Because of the equal masses of the components, the hadron with $cc\bar{c}\bar{c}$ quarks is more expected to be a compact one [16–47]. However, none of them fit the di-$J/\psi$ line shape within this scenario, analogous to that within the molecular picture [48–55]. As we know, the experimental events distribution is the most direct input for theoretical analysis and can provide the underlying structure of the interested hadrons, even for the compact object [5, 56].

In this work, we first fit the di-$J/\psi$ line shape with the compact tetraquark picture and extract the corresponding pole positions. The bare pole positions are extracted from a parameterization with both chromoelectro and chromomagnetic interactions. This study can tell to which extent the compact tetraquark picture can explain the lineshape directly.
FRAMEWORK

Hamiltonian

The interaction in the fully heavy tetraquark system can be described by both chromoelectro and chromomagnetic interaction among the constituent quarks, i.e.

\[
H = \sum_i (m_i + T_i) + \sum_{i<j} \left[ A_{ij} \lambda_i \cdot \lambda_j + \frac{B_{ij}}{m_i m_j} \lambda_i \cdot \lambda_j S_i \cdot S_j \right]
\]

where \( m_i, S_i = \frac{1}{2} \sigma_i, \lambda_i \) and \( T_i \) are mass, spin matrix, Gell-Mann matrix and kinematic energy for the \( i \)th quark, respectively. For antiquark, \( \lambda_i \) is replaced by \(-\lambda_i^*\). The expected values of \( A_{ij} \) and \( B_{ij} \) will be extracted from hadrons. Due to the non-relativistic property of the fully heavy system, we take the non-relativistic approximation here, i.e. neglecting the kinematic terms in Eq.(1).

Wave functions of heavy tetraquark system

Before proceeding to the solutions of the Hamiltonian of Eq.(1), one need to analyze the wave function of the fully heavy tetraquark system. The total wave function of a tetraquark system is constructed by space, flavor, spin and color wave functions individually, i.e. the total wave function

\[
|\psi\rangle = |\text{space}\rangle \otimes |\text{flavor}\rangle \otimes |\text{spin}\rangle \otimes |\text{color}\rangle.
\]

In this work, as we only focus on the ground \( S \)-wave fully heavy tetraquarks, the spatial wave function is symmetric and negligible. Thus, we first construct the spin-color wave function in the diquark-antidiquark configuration. Here, \(|Q_1 Q_2 S_{12} \rangle, (Q_3 Q_4 S_{34})_S \rangle \)

are the spin wave functions with subscripts \( S_{12}, S_{34} \) and \( S \) the total spins of the first two quarks, the latter two antiquarks and the sum of them, respectively. The spin wave functions for various \( J^{PC} \) are listed below

1) \( J^{PC} = 0^{++} \):

\[
|(Q_1 Q_2)_{0}, (Q_3 Q_4)_{0}\rangle_1, \|(Q_1 Q_2)_{1}, (Q_3 Q_4)_{1}\rangle_0 .
\]

2) \( J^{PC} = 1^{+-} \):

\[
\frac{1}{\sqrt{2}} \left[ |(Q_1 Q_2)_1, (Q_3 Q_4)_0\rangle_1 - |(Q_1 Q_2)_0, (Q_3 Q_4)_1\rangle_1 \right],
\]

3) \( J^{PC} = 1^{++} \):

\[
\frac{1}{\sqrt{2}} \left( |(Q_1 Q_2)_1, (Q_3 Q_4)_0\rangle_1 + |(Q_1 Q_2)_0, (Q_3 Q_4)_1\rangle_1 \right),
\]

4) \( J^{PC} = 2^{++} \):

\[
|(Q_1 Q_2)_1, (Q_3 Q_4)_1\rangle_2 .
\]

As we only consider \( S \)-wave fully tetraquark system in this work, the orbital angular momentum does not appear in the above equations. The color confinement tells us that all the observed hadrons are color singlet, which gives the potential color wave functions below

\[
|(Q_1 Q_2)_6, (Q_3 Q_4)_6\rangle^1 , \quad (Q_1 Q_2)_3, (Q_3 Q_4)_3\rangle^1 .
\]

Here the superscripts \( 6(\bar{6}) \) and \( 3(\bar{3}) \) denote the corresponding irreducible representations of color SU(3) group for diquark(antidiquark). Accordingly, 1 stands for color singlet. In total, the spatial and color wave functions of \( S \)-wave ground fully heavy systems are symmetric and antisymmetric, which leaves the product of spin and flavor wave functions symmetric due to Pauli principle. Flavor wave function is symmetric for full-charm(bottom) tetraquarks and could be symmetric or anti-symmetric for the \( b\bar{b}c\bar{c} \) tetraquarks. As the result, all the potential total wave functions for various \( J^{PC} \) are collected in Table 1.

Parameters

In this work, we do not aim at solving Schrödinger equation explicitly, but using a parametrization scheme to extract the mass spectra of the \( S \)-wave ground fully heavy tetraquarks. Thus to investigate the mass spectra, the expectation values of the parameters \( A_{ij} \) and \( B_{ij} \) for various systems should be extracted. As discussed in the above, the contributions from color and spin spaces have been factorized out by the \( \lambda \) and \( S \) matrixes, respectively. In addition, the contribution of flavor part can be obtained via the corresponding flavor wave functions in Table 1. The residue contribution is only from the spacial part via

\[
A_{ij} := \langle A_{ij} \rangle , \quad B_{ij} := \langle B_{ij} \rangle ,
\]

with \( \langle \cdots \rangle \) the expected values of the spacial wave functions. As we only consider \( S \)-wave ground tetraquarks, the expected values can be approximated constants and extracted from \( S \)-wave ground pseudoscalar and vector mesons. Due to the symmetry of color and spin spaces, these expected values have the relation

\[
A_{12} = A_{14} , \quad A_{13} = A_{24} = A_{14} = A_{23} ,
\]

\[
B_{12} = B_{14} , \quad B_{13} = B_{24} = B_{14} = B_{23} .
\]
tracting the parameters. The values of the first four mesons in the constituent quark model \[60\]. In the end, the mass quark masses \(m\) are fixed to the values \(m_c = 1.5 \text{ GeV}, \ m_b = 5 \text{ GeV}\), \(\text{m}_{c}\) in the constituent quark model \[60\]. In the end, the mass

\[
\begin{array}{ccc}
\text{Wave function} \\
\{cc\}_i^6 \{cc\}_i^6 \{cc\}_i^6 \{cc\}_i^6 & \{cc\}_i^6 \{cc\}_i^6 \{cc\}_i^6 \{cc\}_i^6 & \{cc\}_i^6 \{cc\}_i^6 \{cc\}_i^6 \{cc\}_i^6 \\
\{bb\}_i^6 \{bb\}_i^6 \{bb\}_i^6 \{bb\}_i^6 & \{bb\}_i^6 \{bb\}_i^6 \{bb\}_i^6 \{bb\}_i^6 & \{bb\}_i^6 \{bb\}_i^6 \{bb\}_i^6 \{bb\}_i^6 \\
\{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 & \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 & \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \\
\{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 & \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 & \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \\
\{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 & \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 & \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \{bc\}_i^3 \\
\end{array}
\]

The parameters \(A_{bc}\) and \(B_{bc}\), the masses of either \(cb\) or \(bc\) mesons are as inputs. However, only one \(cb\) state, i.e., \(B_c\) is observed in experiments \[57, 58\]. Alternatively, the theoretical results of Ref. \[59\] is used as an input. All the input masses are collected in Table II. Here the heavy quark masses \(m_i\) are fixed to the values

\[
m_c = 1.5 \text{ GeV}, \ m_b = 5 \text{ GeV}, \ \text{m}_{c}\)

in the constituent quark model \[60\]. In the end, the mass

\[
\begin{array}{cccc}
\text{Mass (MeV)} & \eta_c & J/\psi & \eta_b \\
\hline
m & 2983.9 & 3096.9 & 9398.7 & 9460.3 & 6276 & 6331 \\
\Delta m & 0.4 & 0.006 & 2 & 0.26 & 7 & 7
\end{array}
\]

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\begin{array}{cccc}
\text{Mass (MeV)} & \eta_c & J/\psi & \eta_b \\
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m & 2983.9 & 3096.9 & 9398.7 & 9460.3 & 6276 & 6331 \\
\Delta m & 0.4 & 0.006 & 2 & 0.26 & 7 & 7
\end{array}
\]
and $\Omega_{bcb}$ and compare with the predicted results by lattice QCD simulation [62–64] as shown in Fig. 1. From Fig. 1, one can see that the masses of triply heavy baryons are very close to results given by lattice QCD, which indicates the applicability of our framework.

**FIG. 1.** The comparison of normal heavy hadrons with the predicted values from our framework. The red dotted lines are results from either PDG or lattice QCD group. The green dashed and blue solid lines are our inputs and predictions, respectively. The values of $J/\psi$, $\eta_c$, $\Upsilon$ and $\Upsilon_1$ masses are from PDG[61]. Those of $B_{c}^{(*)}$, $\Omega_{bc}^{(*)}$ and $\Omega_{cc}^{(*)}$ are from Ref. [64]. The masses of $\Omega_{bbb}$ and $\Omega_{ccb}^{*}$ are from Ref. [62] and Ref. [63], respectively.

**NUMERICAL RESULTS AND DISCUSSIONS**

Mass spectra of fully heavy tetraquark system with the non-relativistic parametrization

With all the parameters extracted from the $S$-wave ground pseudoscalar and vector heavy mesons, one can obtain the matrix elements of Hamiltonian, i.e., Eq. (1) in the bases listed in Table I. The numerical matrixes of Hamiltonian for various systems can be found in the appendix. After diagonalizing Hamiltonian, one can obtain the mass spectra and eigenvectors which can be found in the appendix. To get an intuitive impression of the bare tetraquarks, we also plot the mass spectra of $ccc\bar{c}$, $b\bar{b}b\bar{b}$, $bcb\bar{c}$ and their potential hidden charm decay channels in Fig. 2. One can see that most of them are above the lowest allowed decay channels and are expected to illustrate themselves as broader structures. However, this expectation might be invaded due to the couplings of the decay channels as discussed below.

**FIG. 2.** Mass spectra (blue solid boxes) of $ccc\bar{c}$, $b\bar{b}b\bar{b}$ and $bcb\bar{c}$ tetraquarks for different $J^{PC}$'s and their potential hidden charm decay channels (red dashed boxes). The bands are uncertainties either from the framework or the experimental data. The green and yellow points are the poles after the coupled channel effect, which will be discussed in the next sections. The errors inherit from the experimental data.

**Partial decay width of bare fully heavy tetraquarks**

As discussed in the above section, most of the bare fully heavy tetraquarks are above their lowest allowed hidden charm/bottom decay channels [16–29, 65–69]. As the result, we will discuss their partial widths of hidden charm/bottom channels. As our framework considers all the possible two-body interactions among the four constituents, the $Q_1Q_2 \otimes Q_3Q_4$ base is equal to the $Q_1Q_3 \otimes Q_2Q_4$ (or $Q_1Q_4 \otimes Q_2Q_3$) base. These two bases can be transformed to each other by Fierz rearrange-
TABLE III. The values and errors of the parameters $A_{ij}$ and $B_{ij}$ in this work.

| Parameters | $A_{QQ'}[\text{GeV}]^3$ | $B_{QQ'}[\text{GeV}]^3$ |
|------------|--------------------------|--------------------------|
| $A_{cc}$   | -0.01287                 | -0.04767                 |
| $A_{bh}$   | 0.10408                  | 0.03427                  |
| $A_{cb}$   | -0.04767                 | -0.28785                 |
| $B_{cc}$   | 1.04 x 10^{-3}           | 9.45 x 10^{-3}           |
| $B_{bb}$   | 2.1 x 10^{-3}            | 1.392 x 10^{-2}          |
| $B_{bb}$   | 9.45 x 10^{-3}           | 1.392 x 10^{-2}          |

Here we use the $c\bar{c}c\bar{c}$ system as an example and the transformation for other systems are analogous. Because the observed hadrons are color singlet, only the $\{|c\bar{c}\rangle,\{|c\bar{c}\rangle\}$ components on the right hand side of the above equations contribute to the hidden charm decay channels. Here $\langle c\bar{c} \rangle$ means either $\{c\bar{c}\}$ or $\{c\bar{c}\}$ which depends on the spin of the $c\bar{c}$ pair. The coefficient of the $\langle|c\bar{c}\rangle_0,\langle c\bar{c} \rangle_0 \rangle_{PC}$ component is denoted as $\alpha_{i}^0$. Here $i$ indicates the allowed two-body charm collision decay channels. $s_{i1}$ and $s_{i2}$ are the spins of the two $c\bar{c}$ pairs in the $i$-th channel. The subscript $J^{PC}$ means the $n$-th $J^{PC}$ base. The two physical $0^{++}$ tetraquarks are a combination of the first two bases with the mixing coefficients listed in the last column Table VII of the appendix. Collect these coefficients into a matrix as

$$\beta = \begin{pmatrix} 0.58 & 0.81 \\ -0.81 & 0.58 \end{pmatrix}. \quad (19)$$

With all the pieces ready, the transition rates of physical tetraquarks to two hidden charm collision channels can be read through the coefficient matrices

$$\mu(X_{0^{++}}(6035)) = \left( \sqrt{\frac{1}{6}} \beta_{11} + \frac{1}{2} \beta_{12} \right) \sqrt{\frac{1}{2}} \beta_{11} - \frac{1}{2 \sqrt{3}} \beta_{12} \right), \quad (20)$$

$$\mu(X_{0^{++}}(6254)) = \left( \sqrt{\frac{1}{6}} \beta_{21} + \frac{1}{2} \beta_{22} \right) \sqrt{\frac{1}{2}} \beta_{21} - \frac{1}{2 \sqrt{3}} \beta_{22} \right), \quad (21)$$

$$\mu(X_{1^{++}}(6137)) = \sqrt{\frac{1}{6}} \beta_{11} \beta_{12} + \sqrt{\frac{1}{6}} \beta_{21} \beta_{22} \right), \quad (22)$$

$$\mu(X_{2^{++}}(6194)) = \sqrt{\frac{1}{3}}, \quad (23)$$

The definitons

$$g_{d1-J/\psi} := \langle J/\psi | J/\psi | \hat{H}_{\text{strong}} | \{c\bar{c}\} \{c\bar{c}\} \rangle, \quad (24)$$

$$g_{d1-n_c} := \langle n_c | n_c | \hat{H}_{\text{strong}} | \{c\bar{c}\} \{c\bar{c}\} \rangle \right), \quad (25)$$

indicating the hadronization process, one can obtain the
relative transition rate to di-J/ψ is

\[ |\mathcal{M}(X_{2^{++}J/\psi}(6194))|^2 : |\mathcal{M}(X_{0^{++}J/\psi}(6254))|^2 : |\mathcal{M}(X_{0^{++}J/\psi}(6035))|^2 \]

\[ = \frac{1}{3} g_{J/\psi-J/\psi}^2 : \left( \sqrt{\frac{1}{2} \beta_{21} - \frac{1}{2 \sqrt{3}} \beta_{22}} \right)^2 g_{J/\psi-J/\psi}^2 : \left( \sqrt{\frac{1}{2} \beta_{11} + \frac{1}{2} \beta_{12}} \right)^2 g_{J/\psi-J/\psi}^2 \sim 33 : 55 : 3 \] (26)

and that to di-ηc is

\[ |\mathcal{M}(X_{0^{++}\eta_c}(6254))|^2 : |\mathcal{M}(X_{0^{++}\eta_c}(6035))|^2 \]

\[ = \left( \sqrt{\frac{1}{6} \beta_{21} + \frac{1}{2} \beta_{22}} \right)^2 g_{\eta_c-\eta_c}^2 : \left( \sqrt{\frac{1}{6} \beta_{11} + \frac{1}{2} \beta_{12}} \right)^2 g_{\eta_c-\eta_c}^2 \sim 0 : 41. \] (27)

Although the hadronization parameters does not play a role here, it will when the coupled channel effect is included. Considering the S-wave phase space

\[ P.S. = \frac{1}{8\pi} \frac{|p|}{M^2} \] (28)

with |p| the three momentum of the final particle in the rest frame of the decaying tetraquark, the ratio to di-ηc becomes 1 : 127. It means that the lower X_{0^{++}}(6035) should be more significant in di-ηc channel, which is also the case after the coupled channel effect. After inclusion of the S-wave phase space, only the X_{0^{++}}(6254) is allowed to decay into di-J/ψ and would be expected to be significant in the di-J/ψ line shape. The values for other systems can be found in the appendix.

**Coupled channel effect to two hidden charmonium channels**

From Eqs. (20),(21),(22),(23) and Eqs. (24),(25) one also can obtain the potentials between two hidden charmonium channels via bare tetraquark poles in the above subsection. As the allowed C-parity of di-J/ψ system is positive, only the 0^{++} and 2^{++} quantum numbers could explain the structure in experiment [12]. For the 0^{++} channel, we consider the \eta_c, J/ψ, J/ψ', J/ψ(3770) channels. The inclusion of the latter two is because of the significant changes in the line shape. The corresponding 0^{++} potential reads as

\[ V_{iJ}^{0^{++}}(E) = \sum_{n=1,2} \sum_{\alpha=1,2} \frac{\mu_{n}^{\alpha} \mu_{m}^{\beta} \mu_{n}^{\beta} \mu_{m}^{\beta}}{E - E_{n0}} \] (29)

with \(E_{10}\) and \(E_{20}\) the masses of bare compact fully heavy tetraquarks \(X_{0^{++}}(6035)\) and \(X_{0^{++}}(6254)\), respectively.

\(\mu_{n}^{\alpha}\) is the coefficient of the \(\{|cc\}_i^{(1)}|\langle cc\rangle_0\rangle \) components of the n-th compact tetraquark. \(g_{n}^{\alpha}\) represents the \(\alpha\)-th component hadronization to the i-th channel (analogous to Eqs. (24),(25)). For the 2^{++} channel, we have \(J/\psi, J/\psi', J/\psi(3770)\) channels and the corresponding potential can be obtained analogous to Eq. (29).

**TABLE IV.** The values of the parameters extracted from the fit. The errors are from the experimental uncertainties. The subscripts are for the corresponding channels.

| parameters | 0^{++} | 2^{++} |
|------------|--------|--------|
| \(U_{J/\psi \eta_c}^{PC}\) | -572.92 ± 912.33 | - |
| \(U_{J/\psi \eta_c, J/\psi J/\psi'}^{PC}\) | 7.53 ± 3.87 | 30.67 ± 1.93 |
| \(U_{J/\psi J/\psi'}^{PC}\) | 34447.71 ± 4145.37 | 39111.96 ± 6605.72 |
| \(\alpha, \beta, \gamma, \delta, \epsilon, \zeta\) | -37513.64 ± 4035.80 | -51446.38 ± 7192.27 |
| \(g_{J/\psi \eta_c}\) | 0.99 ± 0.03 | |
| \(g_{\eta_c-\eta_c}\) | 0.924 ± 0.03 | |
| \(g_{J/\psi J/\psi'}\) | 0.177 ± 0.02 | |
| \(g_{J/\psi J/\psi'}\) | 0.134 ± 0.01 | |

The two-body propagator for the i-th channel is

\[ G_{i}(E) = \frac{1}{16\pi^{2}} \left\{ a(\mu) + \log \frac{m_{1}^{2}}{\mu^{2}} + \log \frac{m_{2}^{2} - m_{1}^{2} + s}{2s} \log \frac{m_{2}^{2}}{m_{1}^{2}} \right. \]

\[ + \frac{k}{E} \left[ \log (2k_{i}E + s + \Delta_{i}) + \log (2k_{i}E + s - \Delta_{i}) \right. \]

\[ - \log (2k_{i}E - s + \Delta_{i}) - \log (2k_{i}E - s - \Delta_{i}) \right\} \] (30)

where \(s = E^{2}\), \(m_{1}\) and \(m_{2}\) are the particle masses in the i-th channel, \(\Delta_{i} = m_{1}^{2} - m_{2}^{2}\), \(k_{i} = \lambda^{1/2}(E^{2}, m_{1}^{2}, m_{2}^{2})/2E\). Here the dimensional regularization is used to regularize the ultra-divergent and we take \(a(\mu) = -3, \mu = 1\text{GeV}\). The physical scattering T
The pole positions have also been extracted in Refs. [48–50] based on the S-wave phase space factor
\[ \rho(E) = \frac{|k|}{8\pi E}. \] (33)

with |k| the three momentum of the particle in the center-of-rest frame. similar to Refs [48]. The final fit function is

\[ \left( |P_2^{0+}|^2 + |P_2^{2+}|^2 \right) \rho(E). \] (34)

The fitted di-J/ψ invariant mass distribution comparing to the experimental data is shown in Fig. 3, with the fitted parameters in Table IV. From the figure, one can see a significant structure around 6.25 GeV which might stem from the shifted X_{0++}(6254). This structure could be seen when the experimental statistic increases and can be viewed as a strong evidence of the compact fully heavy tetraquarks. The structure around 6.9 GeV demonstrates itself as a cusp effect from the J/ψψ(3770) channel. As stated in above section, the bare X_{0++}(6035) strongly couples to the di-ηc channel and will demonstrate itself in this channel. In addition, even after the coupled channel effect, the shift of this bare state is marginal. That means the results for the bare pole almost survive and the X_{0++}(6035) will also show a significant peak structure in the di-ηc spectrum, which is a key physical observable for the nature of the fully heavy tetraquark. For the further measurement in experiment, the di-ηc line shape is presented in Fig. 5 without any background.

The fitted result (red solid curve) comparing to the experimental data [12]. The band is the error with 70% confidence level. The two orange vertical lines are the J/ψψ' and the J/ψψ(3770) thresholds.

To further check this understanding, the pole positions of the corresponding S-matrix are extracted and shown in Table V comparing to the bare pole masses. One can see that the shift of the pole positions to the bare masses are tiny which indicates that the physical states are dominated by compact tetraquarks. We do not find a pole around 6.9 GeV which indicates the structure is only the cusp effect from the J/ψψ(3770) channel. One can also see a dip structure in Fig. 3 around the J/ψψ'.
threshold. The pole positions with the parameters within 70\% confidence level are shown in Fig. 4. The two $0^{++}$ tetraquarks behave as two resonances above the di-$\eta_c$ and di-$J/\psi$ threshold, respectively. The $2^{++}$ state is a bound state below the di-$J/\psi$ threshold. The positions of these three states are listed in Table V comparing with the bare pole masses. In addition, the nature of full-charm tetraquark states can also be obtained by estimating their compositeness $X_A = 1 - Z_A$ [48, 56] with $Z_A = 1$ for molecules and $Z_A = 0$ for compact states, respectively. This idea was proposed by Weinberge in 1963 for bound states [73] and extended to virtual states and resonances [56, 74] recently. Practically, the di-$J/\psi$ to di-$J/\psi$ scattering amplitude can be parametrized in the effective range expansion

$$T(k) = -8\pi \sqrt{s} \left[ \frac{1}{a_0} + \frac{1}{2} r_0 k^2 - i k + \mathcal{O}(k^4) \right]^{-1}, \quad (35)$$

with $a_0$, $r_0$ the scattering length and effective range, respectively. $k$ is the three momentum of the $J/\psi$ in the di-$J/\psi$ center-of-mass frame. By comparing with the physical scattering amplitude, one can extract the S-wave scattering length $a_0$ and the effective range $r_0$. Furthermore, one can obtain the compositeness [56]

$$X_A = \frac{1}{\sqrt{1 + 2|r_0/a_0|}}. \quad (36)$$

The results are collected in Table VI. The absolute values of effective range are much larger than those of the scattering lengths for both $0^{++}$ and $2^{++}$ channels. That indicates that compact tetraquarks are the dominant contributions which can also be understood by the large values of $Z_A$ in Table VI. This behavior has also been explained by the frame with the so-called Castillejo-Dalitz-Dyson (CDD) pole [50, 75, 76], which state that the scattering length $a_0$ and effective range $r_0$ should be linearly and quadratically inversely proportional to the distance between the CDD pole $M_{\text{CDD}}$ and the relevant threshold $m_{\text{thr.}}$, respectively, i.e.

$$a_0 \propto M_{\text{CDD}} - m_{\text{thr.}}, \quad r_0 \propto (M_{\text{CDD}} - m_{\text{thr.}})^{-2}. \quad (37)$$

![FIG. 4. Pole positions of the $0^{++}$ (red triangles and blue boxes for the lower and higher ones, respectively) and $2^{++}$ (green circles) channels with parameters in 70\% confidence level. The yellow dashed vertical lines are the di-$\eta_c$ and di-$J/\psi$ channels, respectively.](image)

![FIG. 5. Prediction of the di-$\eta_c$ lineshape.](image)

**TABLE VI.** The scattering length $a_0$, effective range $r_0$ in the di-$J/\psi$ channel as well as the corresponding compositeness $X_A$ and wave function renormalization constants $Z_A$ for the $0^{++}$ and $2^{++}$ channels. The errors are from the uncertainties of the experimental data.

|         | $0^{++}$          | $2^{++}$          |
|---------|-------------------|-------------------|
| $a_0$ (fm) | $0.012^{+0.129}_{-0.142}$ | $-0.280^{+0.444}_{-0.397}$ |
| $r_0$ (fm) | $-37.966^{+1.010}_{-1.082}$ | $-60.803^{+3.592}_{-3.586}$ |
| $X_A$      | $0.013^{+0.024}_{-0.003}$ | $0.048^{+0.059}_{-0.042}$ |
| $Z_A$      | $0.987^{+0.048}_{-0.244}$ | $0.952^{+0.042}_{-0.095}$ |

To get a whole picture of the fully heavy tetraquark system, we compare our results with those presented in Refs [16, 26–29, 46, 66, 69, 77] in the compact tetraquark picture. Fig. 6 are the comparison of the spectra from these works. For the full-charm tetraquark system $cc\bar{c}\bar{c}$, the spectra has been obtained with parameterization scheme in Refs. [27, 28, 66] and our work, Gaussian expansion method (GEM) [78] in Refs [16, 26, 29, 46] and QCD sum rules in Refs. [69, 77]. The masses of fully charm tetraquarks mostly fall in the interval [6.2,6.8]GeV, which are in a good agreement with the range of mass for broad structure investigated by LHCb [12]. In Ref. [28], the mass splitting is dominated only by spin interaction. Refs. [6, 20, 66] further include the chromoelectro and chromomagnetic interaction and find that these two kinds of interaction cannot be neglected to obtain the correct spectrum. Among them, our method is similar to that in Ref. [27] and our results should be consistent with each other. However, one can see a deviation from Fig. 6, which could stem from the different methods of extracting the parameters. Our framework is under non-relativistic approxi-
FIG. 6. Comparison of the fully heavy tetraquark spectra (in units of GeV) with other works [16, 26–29, 32, 35, 36, 38, 41, 46, 66, 69, 77]. The black circles are for the compact fully heavy tetraquarks. The yellow circles in the $cc\bar{c}\bar{c}$ spectra are the pole positions. The gray dashed lines are for the thresholds of potential two-charmonium decay channels.

In this work, we first extract the internal structure of the fully heavy tetraquarks directly from the experimental data, within the compact tetraquark picture. The bare pole masses are obtained from the parametrization with both chromoelectro and chromomagnetic interaction and the kinematic terms have been ignored comparing with the heavy quark mass terms and chromoelectro/ chromomagnetic interaction. For Refs. [27, 79] both the light meson and heavy meson masses are used to extract the parameters, which is the reason for the deviation. The spectra can also be obtained by solving Schrödinger equation numerically with the variational method [16, 26, 29, 46], which can only give the upper limits of the system. It is the reason why the masses with the variational method is much larger than those with the parameterization ones [27, 79] and ours. QCD sum rule can also be used to obtain the spectra [69, 77] of fully heavy system. Ref [69] concludes that the broad structure around 6.2-6.8GeV and $X(6900)$ is an $S$-wave and a $P$-wave full-charm tetraquark states, respectively. On the contrary, Ref. [77] shows that the broad structure is the first radial excited state of $cc\bar{c}\bar{c}$ tetraquark and the $X(6900)$ is the second radial excited state of $cc\bar{c}\bar{c}$ tetraquark.

SUMMARY

In this work, we first extract the internal structure of the fully heavy tetraquarks directly from the experimental data, within the compact tetraquark picture. The bare pole masses are obtained from the parametrization with both chromoelectro and chromomagnetic interac-
tions. For the $S$-wave ground states, the spacial wave function is trivial and their overlapping can be neglected. The rest parameters are extracted from the masses of the $S$-wave ground heavy mesons, i.e. the $J/\psi$, $\eta_c$, $\Upsilon(1S)$, $\eta_b(1S)$, $B_c$ and $B_c^\ast$. Most of the bare masses are above the lowest allowed two-heavy-quarkonium decay channels. However, it does not mean that all of them could exhibit themselves as broader structures in the lineshape. For an illustration, although the $X_{0^{++}}(6035)$ mass is smaller than that of the $X_{0^{++}}(6254)$, its transition to the $\eta_c$ channel is much larger than that of the $X_{0^{++}}(6254)$. This makes the $X_{0^{++}}(6035)$ more significant in the $\eta_c$ lineshape even after the coupled channel effect. This is an unique feature which can distinguish compact $cc\bar{c}\bar{c}$ tetraquark from the loosely hadronic molecule. After fit to the di-$J/\psi$ lineshape, we find that the $X(6900)$ reported by LHCb is only a cusp effect from the $J/\psi\psi(3770)$ channel. In addition, there is also a cusp effect slightly below 6.8 GeV stemming from the $J/\psi\psi'$ channel. The two $0^{++}$ and $2^{++}$ di-tetraquarks behave as two resonances above the di-$\eta_c$ and di-$J/\psi$ threshold, respectively. The $2^{++}$ state is bound state below the di-$J/\psi$ threshold. Studying the lineshape from the compact tetraquark picture and comparing with those from the molecular picture can tell how much we have learnt from the experimental data and where we should go.

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**Several huge tables**

Several huge tables are collected in this appendix.

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* zejian.zhuang@m.scnu.edu.cn
* zhangying@m.scnu.edu.cn
* yuanzhuoma@m.scnu.edu.cn
* qianwang@m.scnu.edu.cn, Corresponding author

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TABLE VII. The Hamiltonian (the third column) in the bases listed in Table I, predicted mass spectra (the forth and fifth columns) for the $ccc\bar{c}, bb\bar{b}$ and $bc\bar{c}$ tetraquarks with various $J^{PC}$'s as well as their corresponding eigenvectors (the last column).

| $J^{PC}$ | Tetraquark | $H$ [MeV] | Mass [MeV] | Error [MeV] | Eigenvector |
|----------|------------|-----------|------------|-------------|-------------|
| 0++      | $ccc\bar{c}$ | -103.80 -103.80 | 6034.72 | 0.52 | 0.58 0.81 |
|          | $bb\bar{b}$  | -103.80 6109.05 | 6254.00 | 0.57 | -0.81 0.58 |
|          | $bc\bar{c}$  | -56.58 18874.41 | 18833.90 | 2.12 | 0.58 0.81 |

TABLE VIII. The overlaps between a tetraquark state and its possible decay channels

| $J^{PC}$ | Tetraquark | $\eta_c \otimes \bar{c}\bar{c}$ | $\bar{b}\otimes \bar{b}$ |
|----------|------------|-------------------------------|------------------------|
| 0++      | $X^{0+}_{cc\bar{c}}$ (6035) | 0.644 | -0.743 |
|          | $X^{0+}_{cc\bar{c}}$ (6254) | 0.041 | -0.743 |
|          | $X^{0+}_{cc\bar{c}}$ (18834) | 0.644 | -0.743 |
|          | $X^{0+}_{bbb}$ (18953) | 0.041 | -0.743 |
| 1++      | $X^{1+}_{cc\bar{c}}$ (6137) | 0.408 | -0.408 |
| 2++      | $X^{2+}_{cc\bar{c}}$ (6194) | 0.577 | -0.577 |

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| $J^{PC}$ | Tetraquark | $\eta_c,\eta_c$ | $J/\psiJ/\psi$ | $\eta_cJ/\psi$ | Tetraquark | $\eta_c,\eta_c$ | $YY$ | $\eta_cYY$ |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $0^{++}$ | $X_{bc}^{0++}$ (6035) | $2.03 \times 10^{-4}$ | $-$ | $-$ | $X_{bb\bar{b}\bar{b}}^{0++}$ (18834) | $2.726 \times 10^{-5}$ | $-$ | $-$ |
| | $X_{bc}^{0++}$ (6254) | $1.599 \times 10^{-6}$ | $2.431 \times 10^{-4}$ | $-$ | $X_{bb\bar{b}\bar{b}}^{0++}$ (18953) | $2.59 \times 10^{-5}$ | $3.49 \times 10^{-5}$ | $-$ |
| $1^{++}$ | $X_{bc,bc}^{1++}$ (6137) | $-$ | $2.435 \times 10^{-5}$ | $X_{bb\bar{b}\bar{b}}^{1++}$ (18990) | $-$ | $3.336 \times 10^{-6}$ | $-$ | $-$ |
| $2^{++}$ | $X_{bc,bc}^{2++}$ (6194) | $-$ | $-$ | $X_{bb\bar{b}\bar{b}}^{2++}$ (18921) | $7.198 \times 10^{-8}$ | $-$ | $-$ | $-$ |

| $\eta_c,\eta_c$ | $\eta_cJ/\psi$ | $YY$ | $\eta_cYY$ |
| --- | --- | --- | --- |
| $X_{bc,bc}^{0++}$ (12516) | $2.049 \times 10^{-3}$ | $-$ | $-$ |
| $X_{bc,bc}^{1++}$ (12597) | $5.47 \times 10^{-6}$ | $6.647 \times 10^{-6}$ | $-$ |
| $X_{bc,bc}^{2++}$ (12639) | $-$ | $-$ | $-$ |
| $X_{bc,bc}^{3++}$ (12573) | $8.401 \times 10^{-6}$ | $1.073 \times 10^{-5}$ | $-$ |

| $\eta_c,\eta_c$ | $\eta_cJ/\psi$ | $YY$ | $\eta_cYY$ |
| --- | --- | --- | --- |
| $X_{bc,bc}^{0++}$ (12538) | $-$ | $-$ | $-$ |
| $X_{bc,bc}^{1++}$ (12618) | $-$ | $-$ | $-$ |
| $X_{bc,bc}^{2++}$ (12597) | $1.436 \times 10^{-5}$ | $-$ | $-$ |
| $X_{bc,bc}^{3++}$ (12613) | $5.813 \times 10^{-6}$ | $-$ | $-$ |

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