Vortex tubes of turbulent solar convection

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Abstract

The investigation of the turbulent properties of solar convection is important for understanding the multi-scale dynamics observed on the solar surface. In particular, recent high-resolution observations have revealed ubiquitous vortical structures, and numerical simulations have demonstrated links between vortex tube dynamics and the magnetic field organization. Simulations have shown the importance of vortex tube interactions in mechanisms of acoustic wave excitation on the Sun. In this paper, we investigate the mechanisms of formation of vortex tubes in highly turbulent convective flows near the solar surface by using realistic radiative hydrodynamic large-eddy simulations. Analysis of data from the simulations indicates two basic processes of vortex tube formation: (i) the development of small-scale convective instability inside convective granules and (ii) a Kelvin–Helmholtz-type instability of shearing flows in intergranular lanes. Our analysis shows that vortex stretching during these processes is a primary source of the generation of small-scale vorticity on the Sun.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Turbulent convection is a source of various phenomena observed on the Sun, such as coronal mass ejections, self-organized magnetic structures (that appear on the surface as sunspots and pores), and other nonlinear multi-scale dynamical structures and phenomena. Modern realistic numerical simulations of solar turbulent phenomena are based on physical first principles and take into account the real-gas equation of state, radiative transfer, chemical composition and the effects of magnetic fields. Realistic three-dimensional (3D) numerical simulations have reproduced and explained many observed effects in sunspots and magnetic active regions [1–4], as well as in the quiet Sun [5–7].

Vorticity is one of the basic properties of turbulent flows. Therefore, it is not surprising that swirling motions are found in observations of highly turbulent solar magnetoconvection. Large-scale vortical behavior, once evidenced by sunspot rotation, was first observed on the Sun by Secchi in 1875 [8]. Later, vortex flows in the photosphere (∼3 Mm in diameter) were detected by Brandt et al [9], and small-scale swirling flows (∼0.5 Mm) were observed by Wang et al [10].

Observations have shown that vortices are ubiquitous in non-magnetic quiet-Sun regions [11–13] and are associated with intergranular lanes. Such swirling motions correspond to vertically oriented vortex tube structures that have been found in numerical simulations [5, 7, 14]. The simulations have demonstrated important links between vortex tube dynamics and processes of magnetic self-organization [2] and also with acoustic wave excitation [7]. They have shown that turbulent vortex tubes play a fundamental role in solar magnetic flux dynamics. Also, small-scale horizontal vortex
tubes located along granule edges were found both in numerical simulations and in observations with the balloon observatory SUNRISE [6] and also with the New Solar Telescope (BBSO) [15]. However, even the highest-resolution observations are not capable of resolving the internal structure of the vortex tubes. Thus, it is important to investigate in detail the mechanism of vortex formation and dynamics using high-resolution numerical simulations.

Vortex tube formation can occur due to different processes. According to our numerical simulations, generation of vortex tubes on the Sun can be driven by the development of convective instabilities, accompanied by the emergence of small-scale plumes inside granules or granule splitting, and can also be generated by Kelvin–Helmholtz instability of shearing flows in the intergranular lanes. Both vertical and horizontal vortex tubes are generated, but the vertical tubes predominate, as the horizontal tubes are often dragged into intergranular lanes where they become vertical due to the surrounding downflows.

2. Computational setup

For this investigation of vortical structures in the turbulent convective near-surface boundary layer of the Sun, we use the 3D radiative MHD ‘SolarBox’ code developed at the NASA Ames Research Center and the Stanford Center for Turbulence Research by Alan Wray and his colleagues [16]. The code takes into account fluid flow compressibility in a highly stratified medium, the real-gas equation of state, the standard model of the solar interior [17] and the OPAL opacity tables. Radiative transfer between fluid elements is calculated using a 3D multi-spectral-bin method assuming local thermodynamic equilibrium. The physical description of the dynamical properties of solar convection was improved through the implementation of subgrid-scale turbulence models, which effectively increases the Reynolds number and provides a representation of small-scale motions closer to reality. This approach, based on large-eddy simulation (LES) models of turbulence, has demonstrated good agreement between numerical modeling and observations [16].

For the present study, the simulation results have been obtained for two different computational domains: 6.4 × 6.4 × 5.5 Mm^3 (with a 1 Mm layer of the atmosphere) and 3.2 × 3.2 × 7.5 Mm^3 (with a 2 Mm atmospheric layer), and various grid resolutions: 12.5 and 6.25 km in the horizontal direction and 10 and 6 km in the vertical direction. The lateral boundary conditions are periodic. The top boundary is open to mass, momentum and energy transfers and also to radiative flux. The bottom boundary is open only for radiation and simulates the energy input from the interior of the Sun. These high-resolution simulations reveal details of vortex tube formation and dynamics in the solar granular convection, as described in the following sections.

3. Vortex tube formation by convective granular instability

Subsurface layers of the Sun are highly turbulent and inhomogeneous. According to observations, the location of swirling motions (interpreted as vertical vortex tubes) is mostly associated with the intergranular lanes [11, 12]. A similar association was found in numerical simulations [5, 7, 18]. The numerical simulations presented in this paper show that vertical vortex tubes can also be formed inside granules because of a local instability caused by small-scale upflowing plumes. Also, splitting of granules due to convective instability can produce ‘cookie cutter’ structures at granular edges, which perturb flows at the granule top boundary and can produce vortices. In figure 1 we show a snapshot of the solar surface properties obtained in the simulations: vertical velocity (panel (a)), temperature (b), density (c) and enstrophy (panel (d)). The vortex tube structures are best visible in the density distribution (figure 1(c)) as low-density (dark) points in the intergranular lanes. For a detailed analysis we selected two small subregions, marked ‘A’ and ‘B’, to illustrate the typical vortex formation process.

3.1. Upflow plumes inside granular eddies

Vortex tube formation inside granules is a result of complicated interaction of turbulent flows. In the surface layer, the development of convective instability initially represents a localized upflow in a granule (red arrow in figure 2(c)), which is accompanied by a mixture of weak vortical motions on the surface (the vertical vorticity distribution in region ‘A’ is shown by contour lines in figures 2(a)–(c)). Overturning this upflow plume increases the instability region and destroys the granule (figures 2(d) and (e)). Finally, the overturning vortical motions form a compact and relatively stable vortex tube (figures 2(d)–(h)). The formation of the vortex tube and its development on the solar surface are indicated by the green arrow. However, the vortical flows observed on the surface are only a part of the complicated turbulent motions below the surface. In order to better understand the physics of the vortex development in convective granules, we investigate the flow behavior in subsurface layers.

The flow evolution of a typical granule revealed in our high-resolution simulations and illustrated in figures 2 and 3 is fairly complicated and can be described as follows. The diverging flows of a granule contain weak vortical motions along the granule edges (figure 3(a)). Enstrophy isosurfaces shown in figure 3(a) indicate that at this stage the lengths of two vortex tubes (marked as structures 1 and 2) are 0.5–1 Mm. During further evolution, surrounding flows compress the granule and transform the weakly helical structure 1 into a vortex sheet stretched by horizontal diverging flows (figures 3(b) and (c)). Magnified by compression, the granular upflows carry the vortex into the upper layers. Also, surrounding shearing flows compress the horizontal vortex tube 2 and can break it into small-scale vortical features (figures 3(c)–(f)).

In the subphotospheric layers the vortex sheet becomes unstable and splits into several segments (figures 3(c) and (d)). At the same time, the sheet-like part of structure 1 starts overturning (figures 3(c) and (d)) because of the vertical velocity gradient between the middle region of the granule and its edge. In this case, the process of sheet overturning is magnified by the second vortical structure as shown in
Figure 1. Snapshots of a surface layer illustrate distributions of (a) vertical velocity, (b) temperature, (c) density and (d) enstrophy. The rectangular regions indicate places of the formation of a vortex tube from the granular instabilities due to (i) a local upflow in region ‘A’ (solid rectangle; this region is shown in detail in figures 2 and 3); and (ii) splitting of granules, region ‘B’ (dashed rectangle; analyzed in detail in figures 5 and 6).

Figure 3(b), which is later destroyed by convection. During the overturning, the forming vortex tube is gradually moving into the intergranular lane, where surrounding downflows contribute to the overturning shearing motions and deform the sheet-like structure into an inclined vortex tube (figures 3(c) and (f)), which then becomes vertical.

The overturning motions significantly affect the flow dynamics and thermodynamic properties in a local region of the instability and can be observed as fluctuations of mean quantities in the region. The typical flow speed in a granule is \( \approx 1 \text{ km s}^{-1} \), but in the local upflows, the sizes of which are \( \sim 100 \text{ km} \), the flow speed can be \( \sim 2 \text{ km s}^{-1} \), and the temperature can be higher by \( \sim 400 \text{ K} \). These upflows are accompanied by weak helical motions in the subphotospheric layers (figure 4). Thus, in this case, vortex tube formation is caused by a granular instability, accompanied by high-speed small-scale upflows inside granules, which form a vortex sheet and drag it from below to the surface.

Figure 4 shows the temporal evolution of temperature, density, velocity and vorticity with time at different depths. The initially expanding and rising vortex sheet causes a local compression of the flow and a temperature increase (figures 4(a) and (d)). The behavior of the velocity field with time at different depths shows an increase of velocity during the compression of the granule by surrounding flows, followed by granule decay occurring during the overturning phase (figures 4(b) and (e)). The interplay of the horizontal and vertical vorticity components shown in figures 4(c) and (f) illustrates a partial conversion of the horizontal vorticity into vertical vorticity during overturning of the horizontally oriented vortex sheet. The temporal evolution of the vertical vorticity shows that the formation of a vortex tube starts in the subsurface layers. This is indicated by deep minima in the vertical vorticity at \( t \sim 3.2 \text{ min} \) in figure 4(f), which extend with time toward the surface. This trend is shown by a gray line in figure 4(f).

In addition to the convective instability resulting in a strong localized upflow motion inside granules, spitting of granules is also observed in our simulations. The results show that this process can also be accompanied by the formation of vortex tubes. In the following subsection, we consider this type of granular instability.
Figure 2. The formation of a vortex tube on the solar surface in region ‘A’ of figure 1. Grayscale background shows the distribution of the vertical velocity and contour lines show the vertical vorticity. This sequence illustrates the process of development of the vortical structure, which starts inside a granule. The red arrow indicates local upflow in the granule. Green arrows indicate the changes of the initial negative vorticity in a part of the vortex sheet ((c), (d)) and in the vertical vortex tube ((g), (h)).

Figure 3. 3D rendering of enstrophy in region ‘A’ of figure 1, illustrating the evolution of helical structures. The grayscale surfaces correspond to the enstrophy value: \( (\nabla \times \mathbf{u})^2 = 0.013 \text{ s}^{-2} \). Deformation of the vortical structure into a sheet-like belt by strong upflows and its stretching by horizontal flows cause overturning of the structure’s velocity gradients. Finally, this structure becomes more compact and evolves into a vertical vortex tube. The black streamlines illustrate the behavior of convective flows. Scales on the coordinate axes are in Mm.

3.2. Granule splitting

Splitting of granules has been observed previously with different instruments [19, 20] and also in numerical simulations [18]. Our numerical simulations reveal a process of formation of relatively short-lived vortices inside granules during the splitting process. Figure 5 shows a time sequence of the vertical velocity on the solar surface, where the splitting of a granule is accompanied by a new shearing flow, which produces vortices. Actually, the initial stage of vortex formation is similar to the process described in section 3.1. The main difference in the vortex formation process during granule splitting is that the vortices are
continuously formed inside granules, without a tendency to move into the intergranular lanes.

In this case, the shearing flows and later a deformed vortex sheet appear on the surface as a wavy pattern (or ‘cake cutter’ structure, figure 5). Each ‘wave’ represents a horizontal cut through a vortex tube at the solar surface. In this case, three vortex tubes can be identified. In figure 5, we marked each vortex in the order of their formation. Figure 6 illustrates the evolution of enstrophy (shown by isosurfaces) below and above the solar surface layer (horizontal plane). The process of rising and overturning of the vortex sheet (figure 6(a)) produces the ‘wavy’ pattern in figure 5(f). During expansion and splitting, the vortex sheet initially forms vortex 1, then vortex tube 2 (figures 5(g) and 6(b)) and, finally, vortex 3 (figures 5(i) and 6(c)).

The first vortex has a loop-like (or ‘hairpin’) structure. The lifetime of the ‘hairpin’ vortex is very short, ∼5 min, and depends mainly on the magnitude of the diverging flows. The simulations show that such loop-like vortical structures are sensitive to surrounding flows and during the evolution they are stretched, split by diverging flows and diffuse. The vortex 2 is almost vertically oriented and has a very compact circular structure below and above the surface with a diameter of about 100 km. Because of diverging flows this vortex tube expands in diameter, up to ∼200 km where the diverging flows are strongest (figures 6(b) and (c)). In our example, the expanded part of vortex tube 2 transforms into a ring-like substructure, which is separated from the initial vortex and finally destroyed by the diverging flows. This vortex tube 2 is rather stable, with a lifetime up to 10 min, and almost all the time connected to the ‘parent’ vortex sheet. Vortex 3 has a lifetime of a few minutes and is formed during the decay of the ‘parent’ vortex sheet (figures 5(j) and 6(c)).

According to the numerical simulation, most of the vortex tubes are located in the intergranular lanes. Vortex tube formation caused by granular instabilities can be understood as a result of the overturning of a vortex-sheet structure in upflowing plumes inside granules. This mechanism can be realized by small-scale and high-speed local upflows and during granule splitting. In these cases when the formation of vortices is initialized by local upflows inside granules, diverging granular flows move these vortices into intergranular lanes where they form stable and relatively long-lived vortex tubes. These series of vortex tubes, formed during the splitting of granules from a ‘cake cutter’ structure, are usually short-lived for ∼10 min or less. Therefore, the mechanism of vortex formation by local upflows seems to produce more stable and longer-living vortex tubes than granular splitting. However, our simulations show that such instabilities are not sufficiently frequent to explain the ubiquitous distribution of vortex tubes in the intergranular lanes. In the next section we consider the vortex formation caused by shearing intergranular flows, which is probably the main mechanism of vortex tube formation.

4. The Kelvin–Helmholtz instability in intergranular lanes

The formation of vortex tubes is most common in intergranular lanes, where strong downflows and horizontal shearing flows along the edges of granules are present. In this case, the formation of one or several vortex tubes by shearing flows is due to the Kelvin–Helmholtz instability. Figure 7(a) shows the vertical velocity distribution on the solar surface obtained from our high-resolution simulations with a 6.25 km grid interval. An example of an area where the Kelvin–Helmholtz instability develops is indicated by the rectangle. A similar flow instability can often be observed in the intergranular lanes on smaller scales.
Figure 5. The formation of a series of small-scale vortex tubes on the surface of a granule during splitting in region ‘B’ of figure 1. Grayscale images show the vertical velocity distribution on the surface.

Figure 6. Snapshots of the 3D structure of enstrophy (isosurfaces) illustrate different stages of vortex formation in region ‘B’ of figure 1: 0.008 s⁻² in (a) and 0.014 s⁻² in (b) and (c). The horizontal plane shows the vertical velocity distribution on the surface; isocontours indicate the magnitude of the velocity.

The regions of development of the Kelvin–Helmholtz instability can be characterized by the distribution of the Richardson number, \( Ri = N^2/(\frac{du}{dz})^2 \), where \( N \) is the Brunt–Väisälä frequency and \( u_h \) is the horizontal velocity. Usually, estimates of the Richardson number can indicate the transition from laminar to turbulent flow and vice versa. In the solar convection case characterized by very high Reynolds number (~10¹²), the Richardson number indicates a relative level of turbulence. Thus, the distribution of the Richardson number on the solar surface (figure 7(b)) qualitatively indicates, where it is small, the predominant regions of the formation of vortices.

In particular, the apparent decrease of Richardson number in the intergranular lanes corresponds to the transition to a more turbulent regime, where horizontal shearing flows of different scales can initialize vortices (figure 7(c)). Such shearing flows often have a complicated structure containing various flow streams propagating in different directions, strong density stratification and gradients in the downflow speed.

5. Enstrophy balance and sources of vorticity

To investigate the process of vortex formation in terms of enstrophy evolution, we compare the contributions of various vorticity sources, such as baroclinicity (\( B \)), compression (\( C \)) and stretching (\( S \)). The evolution of enstrophy, \( \omega^2 \), is given by the following equation [21]:

\[
\frac{d\omega^2}{dt} = B + S + C,
\] (1)
\[ B = 2 \tilde{\omega} \cdot \nabla P \times \nabla \frac{1}{\rho}, \quad S = 2 \tilde{\omega} \cdot \tilde{\omega} \cdot \nabla \tilde{u}, \quad C = -2\omega^2 \nabla \cdot \tilde{u}, \]

where \( \tilde{\omega} \) is the vorticity, \( P \) is the gas pressure, \( \rho \) is the density and \( \tilde{u} \) is the flow velocity. \( B \), \( S \) and \( C \) represent the vorticity sources corresponding to baroclinicity, vorticity stretching and compression, respectively.

A comparison of their contributions in the surface layer for different moments of time (figure 8) shows a dominant role of the vortex stretching effect. It is surprising that the baroclinicity effect does not play a significant role at this stage, but its contribution starts increasing when the vortex tube is almost formed. The variation of the stretching source with depth is significant only at a very early stage of vortex formation (figure 9(a)). During the vortex overturning stage the contribution of stretching to enstrophy production is similar at different depths (figures 9(b) and (c)) because of the strong turbulent mixing. The subsequent expansion of the mixing region increases the influence of stretching, but a very complicated flow topology in this region makes the enstrophy dependence on the individual sources unclear. During the final stage of vortex formation the stretching is very consistent (in both magnitude and sign) at all layers, from the surface to 100 km below the surface, because the formed vertical vortex tube has almost the same flow topology at all these depths (figure 9(d)).

The statistical distribution of the forming vortices is shown in figure 10(a), where black and gray curves correspond to the relative density of vorticity in intergranular lanes and in granules. The histograms are normalized by the total number of pixels of the developing vortices and were calculated for a near-surface 12.8 Mm\(^2\) slice, with a 12.5 km per step resolution (or 1024\(^2\) grid points), using a 15 min long data set. Vertical vorticity of a relatively weak magnitude is dominant in granules and is mostly related to unstable small-scale fluctuations as discussed in section 3. This also explains the dramatic decrease of the vorticity density in granules with increasing magnitude of vorticity (figure 10(a)). In the intergranular lanes, the decrease of the vorticity density with the magnitude is less steep for two reasons: (i) the stable vortex tubes formed inside granules migrate into the intergranular lanes and (ii) strong vortices are formed there by shearing flows. The dependence of the ratio...
between the vertical, $\Omega_z$, and horizontal vorticity, $\Omega_h$, magnitudes on the local vertical velocity, $V_z$, shows concentration of vertical vortex tubes (where $\Omega_z/\Omega_h \gg 1$) in the intergranular lanes with vertical downward velocities of about $4 \text{ km s}^{-1}$, and a weaker increase of the number of vertical vortices in granules, associated with strong upflows (figure 10(b)).

Effects of numerical resolution on the distribution of vortices is significant for small-scale structures. Figure 11 shows a histogram of the vortex tube distribution per Mm$^2$.
Figure 10. Histograms of the distribution of vertical vorticity in intergranular lanes (black line) and granules (gray line), normalized by the total number of pixels for each region (a). The ratio of vertical and horizontal vorticity magnitudes versus the vertical velocity component (b).

Figure 11. Density distribution histogram of the vertical vorticity for the simulations with different spatial resolutions: from 50 km (thick line) to 12.5 km (thin line).

for the simulations with different grid spacings: 50, 25 and 12.5 km, calculated from 1h data sets. For the vortex identification we consider two criteria: (i) local extrema of the vertical vorticity, \( W_z \) (with a constraint \( W_z \geq 0.03 \, \text{s}^{-1} \), to filter out local, very short-lived perturbations of small scales); (ii) local minima of density perturbation. The histogram shows that the distribution of strong vortices with the vorticity value \( > 0.27 \, \text{s}^{-1} \) is similar for different grid resolutions. The low-resolution simulation, with 50 km spacing, shows a lower number of vortices than the higher-resolution simulations. This can be explained by the underresolving of weak vortices, which often have a tendency to cluster.

Vortex tube formation caused by vortex sheet overturning inside granules is a common phenomenon in our simulations. However, the distribution of vortices shows a clear tendency for accumulation in intergranular lanes. This can be explained by two effects: (i) the process of formation of vortex tubes is often accompanied by the transport of new vortices into the intergranular lanes by diverging flows formed inside granules; and (ii) preferential formation of vortices in the intergranular regions due to the Kelvin–Helmholtz instability in strong shearing flows.

6. Discussion and conclusion

Observations of turbulent solar convection have shown the existence of vortical motions of different scales. The small-scale vortices, detected on the solar surface with large ground-based telescopes and balloon observations (SUNRISE, NST/BBSO), are an important part of the convective dynamics of the granulation layer. Our simulations show that the topology of the turbulent flows can be very complicated and is often accompanied by arc-like vortical structures above the surface (figure 6(c)), which link the convective subsurface layers with the atmosphere. Therefore, the division of vortical structures into different types, e.g. horizontal and vertical vortex tubes, vortex sheets, etc, is only a simplification of the real picture of the turbulent flow.

For this initial investigation we concentrated on purely hydrodynamic turbulent effects, without magnetic fields, although these undoubtedly play a very important role in vortical motions on the Sun [22]. The simulation results have shown that small-scale vortex tubes are mostly concentrated in the intergranular lanes, but they are also formed inside convective granules (figure 1). The vortex tube formation can be initiated by two basic processes: (i) small-scale convective instability leading to localized upflows inside granules and (ii) the Kelvin–Helmholtz instability of shearing flows in the intergranular lanes.

The convective granular instability usually results in small-scale local upflows inside granules, which initially form vortex sheets. Flow overturning in these sheets and their simultaneous advection into intergranular lanes produce vertically oriented vortex tubes (figures 3 and 6). It is interesting that a similar process occurring during splitting of granules can be a source of a series of different types of vortices (figure 5). The mechanism of vortex tube formation due to the Kelvin–Helmholtz instability (figure 7(a)) works mostly in the intergranular lanes, where horizontal shearing flows are the strongest. However, these mechanisms often work together. For instance, the process of convective
overturning may be accompanied by shearing flows. The vortices interact with each other and can merge or quickly decay due to interactions. Some of them live only for a few minutes, but strong vortex tubes can be stable for a long time (>40 min). The 3D topological structure of shearing flows determines the dynamical properties of vortices and their extension into the interior. The simulations reveal that vortex formation is a very common process on small scales not yet resolved in solar observations.

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