One-Loop $\beta$ Functions for Yukawa Couplings in the Electroweak-Scale Right-Handed Neutrino Model

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Abstract. Fermions in the model of electroweak-scale right-handed neutrinos (EWRH) with masses of the order of 300 GeV or more could result in dynamical electroweak symmetry breaking by forming condensates through the exchange of a fundamental Higgs scalar doublet or triplet. These condensates are dynamically studied within the framework of the Schwinger-Dyson equation. With the electroweak symmetry broken by condensates, the fully worked-out model of EWRH in which there are two doublets and two triplets, one of which is composite and the others being the original fundamental scalar doublet and triplet could be suitable for recent LHC discovery of the 125 GeV scalar particle.

1. Introduction

The SM has been established for a long time and has shown its abilities to address many problems in High Energy Physics. Nevertheless, some problems can not be explained by SM. One of which is SM Higgs mechanism where spontaneous symmetry breaking (SSB) is put in by hand. With Higgs potential $V(\phi) = \mu^2\phi^+\phi + \lambda(\phi^+\phi)^2$, this mechanism does not explain why $\mu^2$ is negative when it could, a priori, have been positive. $\mu^2$ and hence $m_H^2 = -2\mu^2 = 2\lambda v^2$ are unstable to large radiative corrections from much higher energy scales (gauge hierarchy problem, fine-tuning needed to keep Higgs light, etc.). Moreover, the SM Yukawa mechanism for generating fermion masses, with $m_f = y_f v/\sqrt{2}$, does not give insight into these masses, since it just puts in the value of Yukawa couplings $y_f$ by hand, and some $y_f$’s range down to $10^{-5}$ with no explanation. Might it be a possible way to give a dynamical reason for fermion masses? Consider two major previous cases of superconductivity and Gell-Mann Levy $\sigma$ model for spontaneous chiral symmetry breaking ($S\chi$SB) where the fundamental scalar fields were used to model SSB. For these cases, the actual underlying physics did not involve fundamental scalar fields but instead a bilinear fermion condensate. For instance, the physical properties of superconductor materials can be explained by dynamically forming a condensate state of Cooper pairs (ee) in BCS theory; since these are charged, they give mass to photon, resulting in Meissener effect. In the second case, the actual origin of $S\chi$SB in QCD is the dynamical formation of a $\{\bar{q}q\}$ condensate. These previous models of symmetry breaking, therefore, bring about the underlying physics responsible for this symmetry breaking may well be a dynamically induced fermion condensate.

The condition for condensation mentioned in [1] is that the Yukawa couplings are larger than critical Yukawa couplings $\alpha^c$. For instance, when the energy scale is of the order of $O(1\text{TeV})$, the Yukawa coupling of a heavy fourth generation in [2] is large enough, exceeding $\alpha^c$, to form...
condensates through the exchange of a fundamental Higgs scalar doublet. In this paper, a similar mechanism has been discussed. We have also investigated the one-loop $\beta$ functions in the model of [3] to find the energy scale forming condensate states of fermions. For the fact that condensates of mirror fermions can be a part of DEWSB, the fully worked-out model of EW\(\nu_R\) in which there are one singlet, two doublets and two triplets could be compatible with recent LHC discovery of Higgs particle [4], [5], while the SM with fourth generations has been ruled out.

The plan of the paper is as follows. Firstly, we summarize the essential elements of the EW\(\nu_R\) model of [3]. Secondly, the analytical formula and numerical calculations related to the one-loop beta functions of Yukawa couplings are presented. We conclude with some remarks concerning the DEWSB in the model of [3].

2. The EW\(\nu_R\) model

The essential elements of the model presented in [3] can be reviewed as below

The \(SU(2)_L \times U(1)_Y\) fermion content of the EW\(\nu_R\) model of [3]
- \(SU(2)_L\) lepton doublets:

\[
SM : l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix};
\]

\[
Mirror : l^M_R = \begin{pmatrix} \nu_R \\ e^M_R \end{pmatrix};
\]

for the SM left-handed lepton doublet and for the right-handed mirror lepton doublet respectively.
- \(SU(2)_L\) lepton singlets:

\[
SM : e_R;
\]

\[
Mirror : e^M_L
\]

for the SM right-handed lepton singlet left-handed mirror lepton singlet respectively.

Similarly, for the quarks, we have
- \(SU(2)_L\) quark doublets:

\[
SM : q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix};
\]

\[
Mirror : q^M_R = \begin{pmatrix} u_R \\ d^M_R \end{pmatrix},
\]

for the SM left-handed quark doublet and for the right-handed mirror quark doublet respectively.
- \(SU(2)_L\) quark singlets:

\[
SM : u_R, d_R; Mirror : u^M_L, d^M_L,
\]

for the SM right-handed lepton singlet left-handed mirror lepton singlet respectively.

Higgs content in the model of EW\(\nu_R\)
- Higgs doublets

\[
\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = (1, 2, Y/2 = 1/2).
\]

- Higgs triplets

\[
\tilde{\chi} = \frac{1}{\sqrt{3}} \sigma^i \cdot \chi^i = \begin{pmatrix} \frac{1}{\sqrt{2}} \chi^+ \\ \chi^0 \\ -\frac{1}{\sqrt{2}} \chi^+ \end{pmatrix} = (1, 3, Y/2 = 1),
\]
\[ \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix} = (1, 3, Y/2 = 0). \quad (8) \]

-Higgs singlet

\[ \phi_S = (1, 1, Y/2 = 0). \quad (9) \]

Higgs triplets such as in (7) were considered earlier in various contexts [6],[7],[8]. In the search for the beta functions discussed in this work, it is important to know Yukawa Lagrangian. The Yukawa couplings can be found in [9]. In the case of SM fermions, we have the usual Yukawa interactions

\[ \mathcal{L}_{SM} = -g_{SM} \bar{\Psi}_L \Phi \Psi_R + h.c. \quad (10) \]

For mirror fermions we consider the terms

\[ \mathcal{L}_{e^M} = -g_{eR}^M \bar{e}_L^M \Phi e_R^M + h.c., \quad (11) \]
\[ \mathcal{L}_{d^M} = -g_{dR}^M d_R^M - g_{dR}^M \bar{d}_L^M \Phi d_R^M + h.c., \quad (12) \]
\[ \mathcal{L}_{\nu_R} = g_{\nu_R}^{M,T} \sigma_2 \tilde{\nu}_L^M, \quad (13) \]
\[ \mathcal{L}_S = -g_{S\nu_R} \bar{L}_L^M \Phi_S + h.c. \quad (14) \]

As energy increases, the Yukawa interactions become strong, bound states of fermions can be formed. The top quark, however, can not form a bound state by exchanging the Higgs boson [2] because of a severely fine-tuned picture of DEWSB. Moreover, the vacuum expectation values of Higgs doublets and Higgs triplets break the global \( SU(2)_L \times SU(2)_R \) down to the custodial \( SU(2)_D \). Thus, we are interested in the running of Yukawa couplings: \( g_{eR}^M, g_{dR}^M \) and \( g_{\nu_R}^M \).

3. One-loop beta functions for Yukawa couplings in the EW\( \nu_R \) model

3.1. Analytical formula

In the EW\( \nu_R \) model, right-handed neutrino interacts with Higgs triplet \( \tilde{\chi} \). Hence, the running of Yukawa coupling \( g_M \) is firstly investigated in this section. The one loop contributions to the Yukawa coupling renormalization constant are composed of four terms: vertex corrections, fermion self-energy, scalar self-energy and gauge interactions. At high energy scale, gauge couplings, however, become too small compared to Yukawa couplings then gauge contributions to the one-loop \( \beta \) functions can be neglected. For this reason the first three terms are computed. Consider the vertex corrections which contribution diagrams modifying the proper one-loop \( \beta \) function for \( g_M \) are shown in fig. 1

\[ \Gamma_1 = -(i)^3 g_M^3 \left( 1 + \frac{1}{2} \right) \int \frac{d^4 k}{(2\pi)^4} \left( \frac{i}{\gamma^\mu p_\mu - \gamma^\nu k_\nu} \right)^2 \frac{i}{k^2}. \quad (15) \]

The contribution coming from equation (15) is computed by using dimensional regularization. We have

\[ \beta_1 = 3 \times \frac{g_M^3}{16\pi^2}. \quad (16) \]
The fermion self-energy term, which diagrams shown in fig. 2, is given by

\[ \Sigma_f(p) = \left[ 2(-ig_M)^2 + 2 \left( -\frac{i}{\sqrt{2}}g_M \right)^2 + (-ig_\gamma^M)^2 \right] \int \frac{d^4k}{(2\pi)^4} \frac{i(\gamma^\mu p_\mu \gamma^\nu k_\nu)}{(p-k)^2}, \]  

(17)
and its contribution to the beta function is
\[ \beta_2 = \left( \frac{3}{2}g_M^2 + \frac{1}{2}g_l^M \right) \frac{g_M}{16\pi^2}. \] (18)

Finally, the Feynman diagram for the scalar self-energy is shown in fig. 3 and
\[ \Sigma_s(p) = (-ig_M)^2 \int \frac{d^4k}{(2\pi)^4} Tr \left( \frac{i}{\gamma^\mu k_\mu} \frac{i}{\gamma^\nu p_\nu - \gamma^\mu k_\mu} \right), \] (19)
\[ \beta_3 = 2 \times \frac{g_M^3}{16\pi^2}. \] (20)

The total contribution to the beta function is then
\[ \beta = \beta_1 + \beta_2 + \beta_3 = \frac{(13g_M^3 + g_L^M g_M^2)}{32\pi^2}. \] (21)

3.2. Numerical calculations
As shown in fig.4, with initial values of \( g_M, g_l^M, \) and \( g_d^M \) being 2.50, 2.46 and 2.73 respectively or the corresponding induced masses being about 300 GeV, the Yukawa couplings of mirror fermions increase dramatically as energy increases and Landau pole singularities appear at the order of \( O(1\text{TeV}) \). Moreover, by using the Schwinger-Dyson equation in the rainbow approximation to compute the self-energy of fermions and subsequently their condensates, we found the critical Yukawa couplings of \( g_M, g_l^M, \) and \( g_d^M \) are \( \pi, \pi \) and \( \pi/2 \) respectively [10]. Above which condensates carrying the electroweak quantum numbers can be formed which could spontaneously break electroweak symmetry. It is amazing that from fig.4 both mirror fermions form condensates at the order of \( O(1\text{TeV}) \) while top quark which initial Yukawa coupling is approximately 1 hardly to do so. For this reason, mirror fermions are responsible for DEWSB in the EW\(\nu_R \) model.
4. Conclusion
The central results of our paper, on one hand, is eqs. (21), (22) and (23) which give beta functions for the Yukawa couplings in the EWνR model. For suitable initial values of Yukawa couplings, the Landau singularities appear at the order of O(1TeV). As critical Yukawa couplings are known, electroweak symmetry can be broken dynamically by condensates of mirror quarks and right-handed neutrinos. On the other hand, because DEWSB scale is of the order of O(1TeV) with forbidden case that mirror leptons get condensate states later than the others do (badly symmetry broken), the mass distribution of fermions in the EWνR model and constraints on form factors by using conditions assuring proper alignment of the two VEVs can be investigated in near future.

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