FORMATION OF BLACK HOLE X-RAY BINARIES IN GLOBULAR CLUSTERS

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ABSTRACT

Inspired by the recent identification in extragalactic globular clusters of the first candidate black hole–white dwarf (BH–WD) X-ray binaries, where the compact accretors may be stellar-mass black holes (BHs), we explore how such binaries could be formed in a dynamical environment. We provide analyses of the formation rates via well-known formation channels like binary exchange and physical collisions and propose that the only possibility of forming BH–WD binaries is via coupling these usual formation channels with subsequent hardening and/or triple formation. In particular, we find that the most important mechanism for the creation of a BH–WD X-ray binary from an initially dynamically formed BH–WD binary is mass transfer induced in a triple system via the Kozai mechanism. Furthermore, we find that BH–WD binaries that evolve into X-ray sources can be formed by exchanges of a BH into a WD–WD binary or possibly by collisions of a BH and a giant star. If BHs undergo significant evaporation from the cluster or form a completely detached subcluster of BHs, then we cannot match the observationally inferred production rates even using the most optimistic estimates of formation rates. To explain the observations with stellar-mass BH–WD binaries, at least 1% of all formed BHs, or presumably 10% of the BHs present in the core now, must be involved in interactions with the rest of the core stellar population.

Key words: galaxies: star clusters: general -- stars: kinematics and dynamics -- X-rays: binaries

Online-only material: color figure

1. INTRODUCTION

The question of whether black holes (BHs) are present in globular clusters (GCs) has been discussed extensively in the literature (e.g., Kulkarni et al. 1993; Sigurdsson & Hernquist 1993; Portegies Zwart & McMillan 2000; Miller & Hamilton 2002b). The most obvious way to detect a BH in a GC is if the BH is in an X-ray binary. Kalogera et al. (2004) showed that, if a BH X-ray binary is formed dynamically by an exchange interaction in the core of a dense GC, it is very unlikely to be detected, as the duty cycle for such binaries is extremely low. Tidally captured BH binaries, in contrast, would be continuously luminous, and the lack of Galactic BH X-ray binaries implies that any such tidal captures destroy the potential companions. We note that only non-degenerate donors were considered by Kalogera et al. (2004). Indeed, in Galactic GCs, where the total number of low-mass X-ray binaries (LMXBs) is relatively small, no BH X-ray binary has been found so far (e.g., Verbunt & Lewin 2006). Among LMXBs in the GCs of early-type galaxies, however, likely BH X-ray binary candidates have been reported (e.g., Angelini et al. 2001; Di Stefano et al. 2002; Kundu et al. 2002; Sarazin et al. 2003; Kim et al. 2006; Brassington et al. 2010). The strongest candidates have X-ray luminosities $L_X > 10^{39}$ erg s$^{-1}$, well above the Eddington limit for a neutron star (NS) accreting helium, and are known as ultraluminous X-ray sources (ULXs).

A particularly interesting ULX has been identified in a GC in NGC 4472. It has $L_X \sim 4 \times 10^{39}$ erg s$^{-1}$ and has shown strong variability (likely due to variable absorption), indicating that it is a single source and thus likely a BH X-ray binary (Maccarone et al. 2007). Keck spectroscopy of the GC (RZ 2109) associated with this source identified strong, broad (2000 km s$^{-1}$) [O iii] emission lines, interpreted as an outflow from a stellar-mass BH accreting above or near its Eddington limit (Zepf et al. 2008). The low Hα/[O iii] ratio suggests a hydrogen-poor (white dwarf (WD)) donor, while the assumption that the breadth of the line is due to a wind driven from near-Eddington accretion implies a BH mass of $5 \sim 20 M_\odot$ (Gnedin et al. 2009).

The ULXs CXO J033831.8-352604 is associated with a GC in the Fornax elliptical galaxy NGC 1399, with $L_X \sim 2 \times 10^{39}$ erg s$^{-1}$ (Irwin et al. 2010). This source shows strong [O iii] emission lines (less broad; $\sigma_3 \sim 70$ km s$^{-1}$) and little or no hydrogen emission. It may be another BH–WD X-ray binary accreting near Eddington, although Irwin et al. (2010) suggest a tidal disruption of a white dwarf by an intermediate mass black hole (IMBH).

How common are such systems? Kim et al. (2006, K06) found eight such ULXs, with $L_X \gtrsim 10^{39}$ erg s$^{-1}$, in 6173 GCs, where these GCs have an average mass of $6 \times 10^5 M_\odot$. Humphrey & Buote (2008, HB08) found 2 such objects among 3782 globulars with total $L_V \sim 10^9 L_\odot$ (total mass $M \sim 3 \times 10^5 M_\odot$). These surveys suggest that GC ULXs are present at the rate of $2.0^{+1.5}_{-1.0} \times 10^{-9}$ and $7^{+15}_{-4}$ per $M_\odot$, respectively, using Gehrels’ statistics at 90% confidence (Gehrels 1986). From the overlap in these numbers, we estimate that there is $\sim 1$ such object per $1-2 \times 10^4 M_\odot$ in GCs.

These surveys were not completely independent, as several galaxies (including NGC 1399) were contained in both surveys. Some fraction of the observed globular ULXs may not be BH–WD systems, but accreting IMBHs. The similarities between the two well-observed ULXs in GCs, discussed in Irwin et al. (2010), suggest that they may both be BH–WD binaries or may both be accreting IMBHs. For this paper, we explore the consequences of the assumption that all GC ULXs are BH–WD.
Formations of black hole X-ray binaries in globular clusters

2. FATE OF A BH–WD BINARY

2.1. Characteristic Times

In a dense stellar system, once a BH–WD binary is formed through a dynamical encounter, its further binary evolution could be affected by subsequent encounters with other stars. The cross section for an encounter between two objects of total mass $m_{\text{tot}}$ with a distance of closest approach less than $r_{\text{max}}$ is computed as

$$\sigma = \pi r_{\text{max}}^2 \left(1 + \frac{v_p^2}{v_\infty^2}\right),$$

where the second term accounts for gravitational focusing, with $v_p^2 = 2Gm_{\text{tot}}/r_{\text{max}}$ and $v_\infty$ the relative velocity at infinity. For strong interactions, $r_{\text{max}}$ is usually on the order of the binary semimajor axis: $r_{\text{max}} = ka$, where $k$ is of order unity (Hut & Bahcall 1983). In the limit of strong gravitational focusing, $v_p^2 \gg v_\infty^2$ and

$$\sigma = 2\pi Gkam_{\text{tot}}v_\infty^{-2}. \quad (2)$$

In our case, the first object is the BH–WD binary of mass $m_{\text{BH}} + m_{\text{WD}}$ while the second object is a core star of mass $m_\star$. In GCs, the close approaches that we are interested in have $v_p^2 \gg v_\infty^2$, and the BH mass is significantly more massive than both its WD companion and a typical core star. Thus, the rate at which a BH–WD binary undergoes a strong (binary-single) encounter is

$$\Gamma_{\text{BS}} = \sigma n_\star v_\infty$$

$$\approx 2\pi G km_{\text{BH}} n_\star a v_\infty^{-1}$$

$$\approx 0.1 k \frac{m_{\text{BH}}}{15 M_\odot} \frac{n_\star}{10^5 \text{pc}^{-3}} \frac{10 \text{km s}^{-1}}{v_\infty} \frac{a}{R_\odot} \text{Gyr}^{-1}$$

$$= 0.1 k \frac{a}{R_\odot} \text{Gyr}^{-1}. \quad (3)$$

The final equal sign in Equation (3) defines the dimensionless parameter $K$. The timescale for a BH–WD binary to experience a strong encounter can be calculated as $\tau_{\text{BS}} = 1/\Gamma_{\text{BS}}$. For example, in a typical GC, the timescale for a strong encounter is of order 10 K$^{-1} R_\odot/a$ Gyr: see Figure 1. Throughout this paper, we consider clusters with core number densities $n_\star$ near $10^5$ pc$^{-3}$, velocity dispersions of $\sim 10$ km s$^{-1}$, and BH masses of $\sim 15 M_\odot$; consequently, $K$ is of order unity.

Once formed, the orbit of a BH–WD binary starts to shrink due to gravitational radiation. The time $\tau_{\text{gw}}$ for the orbit to
Figure 1. Orbital parameters (semimajor axis $a$ and eccentricity $e$) of a 15 \( M_\odot \) BH and 0.6 \( M_\odot \) WD binary system. Lines of constant merger time $\tau_{gw}$ (dotted curves) and encounter time $\tau_{gw}$ with a single star (dashed lines) are shown at specified values. Also shown is the semimajor axis curve $a_{sep}$ for which $\tau_{gw} = \tau_{gw}$ (solid curve), assuming the dimensionless parameter $K$ defined by Equation (3) equals 1.

2.2. Encounters with Single Stars

Here, we consider the outcomes and their frequencies for encounters between a BH–WD and a single star which, as we found in Section 2.1, typically occurs only if $a \gtrsim a_{sep}(e)$. An encounter between a soft binary and a third star can lead to ionization if the third star approaches with sufficient speed; specifically, for ionization to be energetically possible, the relative velocity at infinity $v_\infty$ between the binary and the third star must exceed the binary’s critical velocity $v_c$ defined such that the total energy of the binary-single system is zero:

$$v_c^2 = \frac{m_{BHWD} + m_\ast \cdot m_{WD}}{m_\ast} \cdot \frac{Gm_{BH}}{a}.$$  \hspace{1cm} (4)

For $m_{BH} = 15 \ M_\odot$, $m_{WD} = 0.6 \ M_\odot$, and $m_\ast = 0.6 \ M_\odot$, the critical velocity $v_c$ is less than 10 km s$^{-1}$ only if the semimajor axis $a$ of the binary exceeds $\sim 3 \times 10^4 \ R_\odot$. Even for encounters between a BH–WD binary and a 15 \( M_\odot \) BH, the critical velocity $v_c$ is less than 10 km s$^{-1}$ only for $a \gtrsim 2000 \ R_\odot$. Consequently, in a typical cluster with velocity dispersion $\sim 10 \ km \ s^{-1}$, ionization by itself cannot destroy BH–WD binaries that are able to start MT within 10 Gyr, as all such binaries are easily of sufficiently small semimajor axis (see Figure 1). Essentially, a BH–WD binary with $a \lesssim 2000 \ R_\odot$ can be conservatively classified as a hard binary.

A strong encounter between a hard binary and a third star would ultimately lead to preservation of the BH–WD binary, a companion exchange, or a physical collision. Such outcomes, which we now consider, could potentially prevent a BH–WD binary from ever reaching the MT stage. We note that if an encounter occurs with another binary, a possible outcome is also a hierarchically stable triple system; we consider this option in the next subsection.

The collision cross section for binary-single interactions is approximately (for a non-eccentric binary):

$$\sigma_{coll} \sim \pi a^2 \left( \frac{V_\infty}{v_\infty} \right)^2 \left( \frac{215R}{a} \right)^{0.65},$$  \hspace{1cm} (5)

where $V_\infty^2 = Gm_{WD}/(2a)$ and the maximum radius $R$ among the encounter participants is presumed to be less than $a$. This approximation was derived using results from Fregeau et al. (2004). Our definition of $V_\infty$ follows that of Heggie et al. (1996). We note that $V_\infty = v_c$ in the case of three equal mass stars. We have verified with numerical experiments for many mass combinations that the use of $V_\infty^2$ yields the appropriate scaling of the cross section with mass (provided $v_\infty \ll V_\infty$).

Using Equations (2) and (5), we can now estimate the fraction of binary-single encounters that result in a collision:

$$\frac{\sigma_{coll}}{\sigma} \sim \frac{1}{4k} \left( \frac{215R}{a} \right)^{0.65},$$  \hspace{1cm} (6)

where $k$ is the ratio of the maximum pericenter considered as an encounter to the semimajor axis $a$ of the binary. Equation (6) implies that if a non-eccentric BH–WD has $a \lesssim 5 \ R_\odot$, then all encounters with $R = 0.6 \ R_\odot$ main-sequence (MS) star at $k < 2$ will result in a merger. Similarly, we can estimate that for a non-eccentric BH–WD binary with $a \lesssim 15 \ R_\odot$, a merger will result for all encounters with $k \lesssim 1$. Consequently, we expect that most BH–WD binaries that have $a > a_{sep}$ and are able to start MT within 10 Gyr (see Figure 1) will instead experience a merger if a binary-single encounter with an MS star occurs.

To examine the collision cross section more carefully, we perform scattering experiments as in Fregeau et al. (2004), but for our BH–WD binary with the separations from 1 to 100 \( R_\odot \) and fixed $v_\infty = 10 \ km \ s^{-1}$. We find the $k_{coll}$, defined by Equation (2) with $\sigma = \sigma_{coll}$ and $k = k_{coll}$, varies from $\sim 1$ for $a = 1 \ R_\odot$ to $\sim 0.1$ for $a = 100 \ R_\odot$. For example, $k_{coll} \approx 0.24$ for $a = 10 \ R_\odot$ and, accordingly, from Equation (3), the merger rate
for such BH–WD binaries that are able to start MT within 10 Gyr exceeds 0.2 per Gyr. We note that effectively this rate could be a factor of few smaller as a fraction of collisions will result in a merger between a BH and an MS star. We have verified with hydrodynamical simulations that at least some of the collisions where an MS star collided with the BH will not strongly affect the initial BH–WD binary. We assume the BH–WD binary does not survive if an MS collides with the WD.

The relative fraction of WDs in the core, at the age of several to 14 Gyr, is \( f_{\text{WD}} \sim 0.2 \) of the total core population, and the encounter rate between a BH–WD binary and a WD is \( \Gamma_{\text{BS,WD}} = f_{\text{WD}} \Gamma_{\text{BS}} \). We also note that some simulations have shown that WDs can contribute up to 70% of the total core population, as was found in simulations performed for Fregeau et al. (2009b) without WD birth kicks. From Equation (6), we roughly estimate that a physical collision with an \( R = 0.01 R_\odot \) WD would occur only in binaries with \( a \lesssim 0.1 R_\odot \), which is well below \( a_{\text{sep}}(e) \). A more likely outcome in such cases would be preservation or a companion exchange. Exchanges preferentially occur if the incoming star is more massive than the pre-encounter companion and accordingly could happen if a BH–WD binary had a light WD companion. As a result of exchange, the post-encounter semimajor axis will be increased by the ratio of new companion mass to the old companion mass. We conclude that exchanges with WDs will not tend to make harder binaries.

To reiterate, we consider here only the cases when a BH–WD binary has \( a > a_{\text{sep}}(e) \) (because if \( a < a_{\text{sep}} \), a merger occurs before a strong encounter). Therefore, the resulting binary will most likely never be able to reach MT, as its binary separation will exceed the value necessary for MT to start within 10 Gyr. If a binary is preserved, though, it will most likely then experience a subsequent encounter with a single star. Even if WDs make up as much as 70% of the core population, this subsequent encounter has a high probability of having an MS star as a participant and therefore to result in a merger.

We have shown that BH–WD binaries that can evolve to an LMXB within 10 Gyr and yet are wide enough to have an encounter with a single star (so their \( a > a_{\text{sep}} \)) will either experience a physical collision with destruction of the BH–WD binary, or, after an exchange, will become too wide to reach MT. As such, we conclude that strong encounters with single stars will not create BH–WD binaries with \( a < a_{\text{sep}}(e) \) and therefore will not lead to the direct formation of BH–WD X-ray binaries.

### 2.3. Encounter with Binary Stars: Role of Triples

If a BH–WD has an encounter with another binary, a possible outcome is a hierarchically stable triple. For identical binaries, the maximum impact parameter for which strong encounters still could occur is a bit larger than for a binary-single encounter (Mikkola 1983):

\[
\sigma_{\text{BB}} \approx 4.6\pi (a_1 + a_2)^2 \frac{v_c^2}{v_\infty^2}. \tag{7}
\]

Here, \( v_c \) is the critical velocity of the two binaries with the masses \( m_1 \) and \( m_2 \):

\[
v_c^2 = \frac{G}{\mu} \left( \frac{m_1 m_2}{a_1} + \frac{m_1 m_2}{a_2} \right), \tag{8}
\]

where the reduced mass \( \mu = m_1 m_2 / (m_1 + m_2) \), where \( m_1, m_2, m_2 \), and \( m_2 \) are the masses of the binary components, and where \( a_1 \) and \( a_2 \) are the semimajor axes.

For hard binaries with equal masses and binary separations, \( \sim 25\% \) of all binary–binary encounters within maximum approach described by Equation (7) and \( v_c^2 / v_\infty^2 = 10 \) form a triple system (Mikkola 1983; Fregeau et al. 2004), with triple formation increasing with \( v_c^2 / v_\infty^2 \). For a larger mass ratio between binaries, or for a softer incoming binary of less mass, this fraction can be even higher.

We performed numerical experiments in which BH–WD binaries consisting of a 15 \( M_\odot \) BH and a 0.6 \( M_\odot \) WD encountered at \( v_\infty = 10 \text{ km s}^{-1} \) binaries consisting of 0.6 \( M_\odot \) companions. We record the fraction of encounters that result in a stable triple with an inner binary consisting of the initial BH and a companion 0.6 \( M_\odot \) star. We run the simulations for 3 values of BH–WD binary separation and 11–17 values of the incoming binary separations, performing 10,000 encounters for each. For the numerical integration, we use Fewbody (Fregeau et al. 2004), a small N-body integrator. The results demonstrate thatperiastron somewhat larger than those suggested by Equation(7) can result in a triple formation, and we choose the maximum possible periastron for the encounters to be fairly large, \( r_{\max,p} = (a_1 + a_2) \) (that is, \( k = 20 \)). The impact parameters for every encounter are distributed uniformly in area. The eccentricities are distributed thermonally. The results are shown in Figure 2.

We find that for almost all the encounters in which the incoming binary is wider than the BH–WD binary, the efficiency of triple formation is \( \sim 20\%–30\% \) with \( k = 20 \). This gives

\[
\sigma_{\text{triples}} \approx 120\pi (a_1 + a_2)^2 \left( 1 + \frac{Gm_{\text{BH}}}{10(a_1 + a_2)v_\infty^2} \right). \tag{9}
\]

The formation rate of triple systems, for hard binaries, compared to binary-single encounters rate (3) with \( k = 2 \), is then

\[
\frac{\Gamma_{\text{triples}}(a_1, a_2)}{\Gamma_{\text{BS}}(a_1)} \sim 3\sqrt{2} f_{\text{wb}} \left( 1 + \frac{a_2}{a_1} \right). \tag{10}
\]
Here, $f_{wb}$ is the fraction of stars that are hard binaries, but still wider than the BH–WD binary. In deriving the above equation, we have applied equipartition of energy, so that the $v_\infty$ of a typical binary in the core is $\sqrt{2}$ times less than that of a typical single star.

The overall binary fraction in observed dense, but not core-collapsed, clusters at the current epoch, is about a few percent (Albrow et al. 2001; Ivanova et al. 2005a; Milone et al. 2008). With $f_{wb} = 5\%$ and a flat distribution of binary separations between, e.g., 20 and 2000 $R_\odot$, a BH–WD binary that has $a = 20 R_\odot$ can form a triple about 30 times per Gyr. This exceeds the interaction rate with single stars.

Although these triples are stable in isolation, they can be destroyed during their next dynamical encounter. Before this next encounter occurs, the presence of the third component can significantly affect the eccentricity of the inner BH–WD binary via the Kozai mechanism (Kozai 1962), provided the next encounter occurs, the presence of the third component can bring the inner binary into MT via the Kozai mechanism: that can be achieved via the Kozai mechanism is (e.g., Innanen et al. 1997; Eggleton & Kiseleva-Eggleton 2001)

$$e_{\text{max}} \approx \sqrt{5/3} \sin^2(i_0) - 2/3.$$  \hspace{1cm} (11)

In dynamically formed triples, the inclinations are distributed almost uniformly, and so the number of Kozai triples is expected to be proportional to the solid angle (e.g., Ivanova 2008, where, depending on the sample of runs, the triples affected by the Kozai mechanism constituted from 30% to 40% of all formed triples, while a uniform distribution predicts 37%). Thus, the fraction of the dynamically formed triples with a post-encounter inclination greater than $i$ is $f_i = 1 - \sin i$.

The BH–WD binaries that could become MT systems are only those that have $a$ less than the maximum $a_{\text{sep}} (\lesssim 80 R_\odot$ for $e < 0.99$). To find the formation rate of such triples, we first determine the minimum value of $e_{\text{max}}$, that a triple with the specific binary semimajor axis $a$ must achieve: this is found by inverting the numerically obtained $a_{\text{sep}}(e)$ at $k = 2$. The obtained $e_{\text{max}}$ then provides the fraction of formed triples that can bring the inner binary into MT via the Kozai mechanism:

$$f_i = 1 - \sin i = 1 - \sqrt{3/5 \ e_{\text{max}}^2 + 2/5}.$$  \hspace{1cm} (12)

We define triple induced mass transfer (TIMT) systems to be those binaries that are brought into MT via the Kozai mechanism. We find that the fraction $f_i$ of TIMT systems ranges between approximately 0.07 (for $a = 15 R_\odot$) and 0.01 (for most semimajor axes, $a \gtrsim 40 R_\odot$). Accounting for the triple formation rate, we find that all BH–WD binaries with $15 R_\odot \lesssim a_1 \lesssim 80 R_\odot$ could make it to the MT through the triple formation mechanism. At a conservative level, assuming even that all the binaries that have a binding energy 10 times more than the kinetic energy of an average object in the core are destroyed, we find that a BH–WD binary with $a_1 \gtrsim 50 R_\odot$ has a 10%–40% chance to become a TIMT system within 1 Gyr (10% for $a_1 \lesssim 80 R_\odot$ and 40% for $a_1 \approx 50 R_\odot$); this chance is 100% for $a_1 \lesssim 35 R_\odot$. During several Gyr, all BH–WD binaries with $a_1 \lesssim 80 R_\odot$ can become a TIMT system at least once.

This TIMT system formation will be successful only if the time between subsequent encounters is longer than the time necessary to achieve $e_{\text{max}}$. The period of the cycle to achieve $e_{\text{max}}$ is (Innanen et al. 1997; Miller & Hamilton 2002a)

$$\tau_{\text{Koz}} \approx \frac{0.42 \ln(1/e_i)}{\sqrt{\sin^2(i_0) - 0.4}} \left( \frac{m_{12} + m_{12} b_1^3}{m_o a_1^3} \right)^{1/2} \left( \frac{b_1^3}{G m_o} \right)^{1/2},$$  \hspace{1cm} (13)

where $m_{12}$ and $m_{12}$ are the companion masses of the inner binary, $e_i$ is the initial eccentricity of the inner binary, $m_o$ is the mass of the outer star, $a_o$ is the initial semimajor axis for the outer orbit, and $b_1 = a_0 (1 - e_o^2)^{1/2}$ is the semiminor axis of the outer orbit. Roughly, $a_o$ will be of the order of magnitude of $a_2$. Considering the extreme case $a_2 \gg a_1$, for which $\tau_{\text{Koz}}$ would be maximum, we find:

$$\tau_{\text{Koz}} \approx 4 \times 10^{-14} f_b \frac{0.42 \ln(1/e_i)}{\sqrt{\sin^2(i_0) - 0.4}} \left( \frac{a_2^2}{a_1 R_\odot} \right)^{5/2} \times \left( \frac{m_{\text{BH}}}{15 M_\odot} \right)^{3/2} \left( \frac{M_\odot}{m_o} \right) \left( \frac{1 - e_o^2}{n_c / 10^5 \text{pc}^{-3}} \right) \frac{10 \text{ km s}^{-1}}{v_\infty},$$  \hspace{1cm} (14)

Here, $f_b$ is the binary fraction, $\tau_{\text{BB}} = \frac{1}{\Gamma_{\text{BB}}} = (\sigma_{\text{BB}} f_b n_c v_\infty)^{-1}$, and $\tau_{\text{BS}} = 1/\Gamma_{\text{BS}}$ has been evaluated at $k = 2$.

An average dynamically formed triple would have $e_o \approx 0.9$ (Ivanova 2008). For the purpose of the estimate, we adopt that a BH–WD binary would have its initial eccentricity distributed thermally, with an average $e_i \approx 2/3$, and adopt $i_0 = 54^\circ$ (this is the mean inclination of dynamically formed Kozai affected triples). From Equation (15), we have then that triples with $15 R_\odot \lesssim a_1 \lesssim 80 R_\odot$ and $a_2 \lesssim 60 R_\odot$ (where 4500 $R_\odot$ is the boundary between hard and soft binaries) would have their Kozai time significantly shorter than their collision time with either binary or single stars. We conclude that once a potential TIMT system is formed, it will succeed in bringing its inner BH–WD to MT before its next encounter. Accordingly, all BH–WD binaries with $a \lesssim 80 R_\odot$ will become X-ray binaries via the TIMT mechanism.

### 2.4. Multiple encounters: hardening

It is well known that through multiple fly-by encounters hard binaries get harder (e.g., Hut 1983). The number of encounters necessary to change the binary energy, e.g., by a factor of 4, can be estimated as (Heggie & Hut 2003)

$$N_{\text{hard}} \approx \log(4) / \log(1 + \delta),$$  \hspace{1cm} (16)

where $\delta$ is the relative change in the binary’s binding energy caused by an encounter. This change, for an eccentric binary with a large mass ratio of one star to a companion and an encountering $m_3$ star, if occurred at the distance $q$ ($q \gg a$), can be roughly estimated as (e.g., Heggie 1975; Roy & Haddow 2003)

$$\delta \approx \frac{m_3}{m_2} \left( \frac{a}{q} \right)^{3/4} e^{-(q/a)^{3/4}}.$$  \hspace{1cm} (17)
Assuming that all hardening happened with the maximum δ \( \sim m_3/m_{\text{bh}} \sim 0.04 \), it can therefore be estimated that, the hardening of a 1000 \( R_\odot \) BH–WD binary to, e.g., 35 \( R_\odot \) should take about 100 encounters. As the collision time for a 1000 \( R_\odot \) BH–WD binary is only about \( 10^7 \) yr, and is still only about \( 10^9 \) yr for a 35 \( R_\odot \) BH–WD binary, it is plausible therefore that all 1000 \( R_\odot \) BH–WD binary can be hardened to 35 \( R_\odot \) within a few Gyr. We choose here 35 \( R_\odot \) as the critical separation at which triple formation starts to have 100% efficiency in making this binary an MT binary (see Section 2.3).

However, this is an idealistic picture. There are plenty of encounters that this binary would have undergone in order to reach 35 \( R_\odot \), and certainly not all of them will be simple fly-by hardening encounters. Some of the encounters would result in exchanges (where the lighter companion is often exchanged with a more massive incoming star, and the binary gets wider), mergers, or even binary ionizations, therefore reducing the chance for a binary to harden to the separation we are interested in.

We set up a numerical experiment, by taking a BH–WD binary with the specific initial binary semimajor axis \( a_i \). At the collision rate predicted by the current binary semimajor axis and the adopted \( n_c = 10^5 \) pc\(^{-3}\), we bombard the binary with single stars drawn from the simplified core’s population, randomizing impact parameters and using Fewbody. Initial eccentricities of BH–WD binaries for this experiment are adopted to have a thermal distribution. A simplified core population is taken from Monte Carlo runs (Ivanova et al. 2008) and has only four different groups for the non-BH population: stars with masses 0.22, 0.506, 0.75, and 1.13 \( M_\odot \), with, respectively, 33%, 38%, 25%, and 4% contributions to the non-BH population. Then we vary the BH component, starting from the highest possible, where the number of BHs is 0.4% of all other stars in the core (this is about 100% of the initially formed BH population, see the discussion in Section 1, or \( f_{\text{BH},0.1} = 10 \), 0.04% (this corresponds to the case of \( f_{\text{BH},0.1} = 1 \), 0.004%, and no contribution at all (this corresponds to the case when only one BH, which is in the considered BH–WD binary, is left). Here, one BH corresponds to 0.001% of the core population.

We consider the fraction of BH–WD binaries that would harden to 35 \( R_\odot \) within 10 Gyr as well as the average time it will take them. We also separately consider cases where we insist that the original BH–WD binary makes it to 35 \( R_\odot \), or simply any binary containing an initial BH would make it (allowing multiple exchanges). The latter case could correspond to the case when almost all the stellar non-BH population in the core are WDs (e.g., in some runs performed in Fregueau et al. 2009b; WDs fraction in the core reached 70%). The results are shown in the Table 1.

There are several trends in the results, all of them well expected theoretically. Indeed, as expected, the fraction of survived and hardened BH–WD binaries \( f_{\text{hard}} \) is well below 1. The fraction \( f_{\text{hard}} \) decreases with increasing \( a_i \) and increases when the BH population in the core drops, as other BHs contribute significantly into exchange reactions or ionization. The average time that it takes to harden a binary agrees with the analytic estimate above and is about 1–2 Gyr. Even though \( f_{\text{hard}} \) increases when the BH population drops, it does not increase by a factor comparable to the relative drop in the BH population. Accordingly, since the resulting formation rate of hardened BH–WD binaries (their production from initially wider binaries) \( f_{\text{hard}} \propto f_{\text{BH},0.1} f_{\text{hard}} \), the highest \( f_{\text{hard}} \) occurs in an unlikely case when most of initially formed BHs are retained (\( f_{\text{BH},0.1} = 10 \)). The fraction \( f_{\text{hard}} \) for \( f_{\text{BH},0.1} \lesssim 1 \) does not vary much with \( f_{\text{BH},0.1} \).

We also study what fractions can be hardened to \( a = 80 R_\odot \), for our optimistic scenario when all BH–WD binaries with \( a = 80 R_\odot \) make it to the MT via TIMT. In this case, significant fractions of wide binaries can be successfully hardened.

### Table 1

| Hardening | Black hole fraction in the total core population (%) | Corresponding \( f_{\text{BH},0.1} \) |
|-----------|---------------------------------------------------|----------------------------------|
|          | 0.4 0.04 0.004 0 0.4 0.04 0.004 0               |                                  |
| \( a_i \) | 10 1 0.1 0 10 1 0.1 0                            |                                  |
| Hardened fraction (%) | Average time (Gyr) |                           |
| 100 \( R_\odot \) | 4.6 12.9 14.8 14.9 0.86 1.38 1.48 1.52       |                                  |
| 250 \( R_\odot \) | 0.53 2.5 3.4 3.4 1.07 1.9 1.9 1.9             |                                  |
| 500 \( R_\odot \) | 0.0 0.8 1.0 1.3 ... 1.9 2.1 2.3               |                                  |
| Exchanges are allowed |                                  |                                  |
| 100 \( R_\odot \) | 9.4 30.4 38.1 38.9 1.06 1.50 1.58 1.60       |                                  |
| 250 \( R_\odot \) | 1.3 9.8 13.9 13.3 1.17 1.85 2.00 2.07       |                                  |
| 500 \( R_\odot \) | 0.0 3.2 4.8 5.9 ... 1.78 2.15 2.09       |                                  |
| Hardening to 80 \( R_\odot \) |                                  |                                  |
| Original binary survival only |                |                                  |
| 100 \( R_\odot \) | 37.9 56.0 58.2 60.6 0.22 0.29 0.30 0.31       |                                  |
| 150 \( R_\odot \) | 12.7 27.5 30.0 30.9 0.35 0.52 0.57 0.57       |                                  |
| 200 \( R_\odot \) | 6.5 16.9 19.1 20.7 0.41 0.64 0.68 0.70       |                                  |
| 250 \( R_\odot \) | 3.5 11.3 14.4 14.4 0.46 0.73 0.77 0.77       |                                  |
| 500 \( R_\odot \) | 1.1 3.1 3.8 3.8 0.37 0.90 0.98 0.98       |                                  |

**Notes.** The table shows what fraction of binaries, with a 15 \( M_\odot \) BH and a 0.6 \( M_\odot \) WD and with an initial binary semimajor axis \( a_i \), can be hardened to 35 \( R_\odot \) and 80 \( R_\odot \), as a function of the relative BH content in the core: 0.4%, 0.04%, 0.004%, and 0% of the population. The corresponding values of \( f_{\text{BH},0.1} \) are the fraction of BHs that are available for interactions with normal stellar population: see Section 1. The average time corresponds to the time that it takes to harden those binaries that successfully get harder. The number of encounters used for the statistics are 10,000 for 100 \( R_\odot \) and 80 \( R_\odot \) and 5000 for 100 \( R_\odot \) and 1000 for 500 \( R_\odot \).

3. FORMATION OF A BH–WD BINARY

In the previous section, we considered what can happen to a BH–WD binary in a dense stellar system. Here, we analyze which kind of a BH–WD binary could be formed and, accordingly, which formation mechanism dominates in the formation of potential BH–WD X-ray binaries. We neglect the possibility that an unperturbed primordial binary with a BH could be formed and survive in a dense stellar system: Downing et al. (2009), for example, have shown that the fraction of primordial BH binaries is well below 1% compared to all BH binaries. It has been discussed that there are two processes through which a BH could acquire a non-degenerate stellar companion: exchange interactions with (primordial) binaries and tidal captures (for a thorough discussion of the latter mechanism with a BH, see Kalogera et al. 2004, and references therein). Here, we do not consider tidal captures, as they are inefficient for interactions between a BH and a WD; it was suggested that these encounters would result rather in a tidal disruption of a WD or its nuclear ignition (Rosswog et al. 2009). We consider exchange interactions, when an acquired
companion is a WD, as well as two other possible mechanisms: physical collision with red giants (RGs) and three-body binary formation. The latter mechanism is usually neglected from consideration, as for all non-degenerate stars it is assumed that such encounters would lead to a merger rather than a formation of a hard binary.

3.1. Exchange Encounters

In a binary that has been newly formed via exchange, the post-encounter semimajor axis will be \( a_{\text{post}} \sim a_{\text{pre}} m_{\text{BH}}/m_{\text{c}} \), where \( m_{\text{c}} \) is the lower mass companion replaced by a BH. The post-encounter eccentricities are distributed thermally with the mean \( e \sim 2/3 \). The mass \( m_{\text{c}} \) would normally not exceed that of an average star in the core, and accordingly the post-encounter semimajor axis will be at least 25 times larger than pre-encounter semimajor axis.

Thus, in Figure 1, we see that only those binaries with pre-encounter semimajor axes \( a_{\text{pre}} \lesssim 7 R_{\odot}/25 \approx 0.3 R_{\odot} \) could create binaries that will evolve to MT directly (though up to \( a_{\text{pre}} \approx 80 R_{\odot}/25 \approx 3 R_{\odot} \) if the post-encounter eccentricity is as large as 0.99). For an MT binary formed via a triple, \( a_{\text{pre}} \lesssim 35 R_{\odot}/25 \approx 1.4 R_{\odot} \). In both cases, \( a_{\text{pre}} \) is small enough to expect a merger if at least one companion in the pre-encounter binary was an MS star.

Therefore, to create a BH–WD binary with \( a_{\text{post}} \lesssim 35 R_{\odot} \), only encounters with WD–WD binaries are relevant. The fraction of WD–WD binaries is just a few percent of the total binary population. From the simulations set in Ivanova et al. (2008, 2006), we find the fraction of the short-period WD–WD binaries at 10 Gyr in their “standard” GC (corresponds to our typical dense GC) to be \( f_{\text{b}} \approx 0.8\% \) (as the fraction of the all objects in the core). Adopting \( a_{\text{pre}} \approx 1 R_{\odot} \), we find that the formation rate of potential BH–WD X-ray binaries through exchanges coupled with TIMT is \( \Gamma_{\text{exch},1} \approx 2 \times 10^{-3} \) per Gyr per BH.

Relatively wide hard BH–WD binaries (\( a \approx 80–1000 R_{\odot} \)) can be formed through encounters with both MS–WD and WD–WD binaries, as pre-encounter binaries can be up to 20 \( R_{\odot} \) (here, MS–WD binaries could be only those that have \( a \gtrsim 15 R_{\odot} \)). Again, using the same set of simulations, we find that such binaries have a relative \( f_{\text{b}} \approx 1.4\% \) and an average \( a_{\text{pre}} \approx 22 R_{\odot} \). The resulting formation rate is fairly similar to the formation via encounters with WD–WD binaries, \( \Gamma_{\text{exch,2}} \approx 0.06 f_{\text{hard}} \approx 2 \times 10^{-3} \) per Gyr per BH.

In our “optimistic” case, TIMT is successful in all binaries with \( a_{\text{post}} \lesssim 80 R_{\odot} \). Then, the fraction of binaries playing a role in direct exchanges (with semimajor axes \( a_{\text{pre}} \lesssim 80 R_{\odot}/25 = 3.2 R_{\odot} \)) is \( f_{\text{b}} \approx 2\% \). When averaged over all binaries in several numerical simulations, \( f_{\text{hard}} \approx 25\% \) (for \( f_{\text{BH},0.1} = 1 \). We find then \( \Gamma_{\text{exch,1}} \approx 6 \times 10^{-3} \) and \( \Gamma_{\text{exch,2}} = 1.5 \times 10^{-2} \) per Gyr per BH.

3.2. Physical Collisions

Physical collisions between compact stars (NSs) and low-mass RGs can lead to a formation of NS–WD X-ray binaries (Ivanova et al. 2005b), which, during their persistent phase, are also known as ultracompact X-ray binaries (UCXBs). From observations of UCXBs in the Milky Way’s galactic GCs, we know that UCXBs are formed at a high rate. The fraction of UCXBs among all LMXBs is much larger in GCs than in the field (Deutsch et al. 2000).

When natal kicks from Hobbs et al. (2005) are adopted for NSs formed via core collapse and reduced kicks are adopted for NS formed via electron capture supernova, the theoretically predicted UCXB formation rate through physical collisions is consistent with the observations of GCs both in the Milky Way and in external galaxies (Ivanova et al. 2005b).

Like in the case of UCXBs formation, it is plausible that BHs, if retained and not detached in a BHs subcluster from the rest of the stellar population, will experience physical collisions with RGs. The rate of such collisions, per BH, can be estimated as in Ivanova et al. (2005b):

\[
\Gamma_{\text{BH}RG} \approx 2 \frac{\Gamma_{\text{FG}} f_{\text{RG}} m_{\text{BH}} n_{\text{c}}}{15 M_{\odot}} \frac{10^5 \text{ pc}^{-3}}{R_{\odot}} \frac{R_{\text{RG}} 10 \text{ km/s}}{v_{\infty}} \text{per Gyr},
\]

where \( R_{\text{RG}} \) is the average radius of the RGs, \( f_{\text{RG}} = r_{\text{RG}}/R_{\text{RG}} \) describes how close has to be an encounter in order to result in the formation of a binary compact enough to start the MT. For NSs, \( f_{\text{RG}} \approx 1.3 \) (Lombardi et al. 2006). However, for BHs the maximum periastron that leads to Roche lobe overflow of the RG at the closest approach is larger, \( f_{\text{RG}} \approx 5 \). The fraction \( f_{\text{RG}} \) of stars at the RG stage is typically \( \sim 0.4\% \) of non-degenerate single stars at the age 10–12 Gyr, with the minimum stellar mass in the population being \( 0.08 M_{\odot} \) and with the initial mass function as in Kroupa (2002). Mass segregation of stars in the core results in a higher \( f_{\text{RG}} \). Analyzing the set of simulations from Ivanova et al. (2008), we find that this fraction is about 0.8% of the overall core population and \( R_{\text{RG}} = 3.7 \) at 10 Gyr; in the simulations presented in Fereguie et al. (2009a), the post-processed \( f_{\text{RG}} \approx 0.8\% \) as well. In our relatively dense stellar cluster, with \( n_{\text{c}} = 10^5 \text{ pc}^{-3} \) and velocity dispersion of about 10 km s\(^{-1} \), a BH of 15 \( M_{\odot} \) therefore could have \( \sim 3 \times 10^{-3} f_{\text{RG}} \) chance of a physical collisions during 1 Gyr. Therefore, physical collisions can provide a significant contribution to overall BH–WD X-ray binary formation only if all the collisions leading to the formation of bound binaries, with all of the periastra up to 5 \( R_{\text{RG}} \), lead to X-ray binary formation.

Let us estimate what fraction of the formed bound binaries can successfully become BH–WD X-ray binaries. A simple estimate for the outcome of a BH–RG collision can be done using an energy argument, as in the standard common envelope consideration, using the \( a_{\text{CE}} \lambda \) formalism (Webbink 1984). Specifically, the final semimajor axis \( a_{\text{f}} \) is given by

\[
a_{\text{f}} = R_{\text{RG}} \frac{a_{\text{CE}} \lambda}{2} \frac{m_{\text{BH}} m_{\text{RG,core}}}{m_{\text{RG}} m_{\text{RG,env}}}.
\]

where \( m_{\text{RG,core}} \) and \( m_{\text{RG,env}} \) are, respectively, the core and envelope masses of the RG, and \( R_{\text{RG}} \) is the RG radius. The parameter \( \lambda \) introduced to characterize the donor envelope central concentration can be found directly from stellar models and is about 1 for low-mass RGs. The parameter \( a_{\text{CE}} \) is introduced as a measure of the energy transfer efficiency from the orbital energy into envelope expansion, and is bound to be below 1. The energy balance in Equation (19) assumes that the amount of the energy that is transferred from the orbital motion to the envelope expansion does not exceed the energy that is

\[\footnote{We will show in the paper in preparation, where BH–RG encounters are modeled using the SPH code described in Lombardi et al. (2006), that this is indeed the case, and the collisions with \( f_{\text{RG}} \lesssim 5 \) lead to the formation of bound binary systems.}\]
required to eject the envelope to infinity. The kinetic energy that the ejected gas has at infinity is neglected, as well as that two stars had initially at the infinity, as it is very small compared to the binding energy of the envelope.

It can be seen, if $a_{C E} = 1$, that the final binary separation for a case of a BH collision with a low-mass RG will result in the formation of a binary with $a_e > 1.3R_G$. (This minimum value is for a case of an early subgiant, when the core mass is minimum: e.g., the core mass is only $\sim 0.12M_\odot$ for a $0.9M_\odot$ RG of radius $\sim 2.5R_\odot$ and metallicity $Z = 0.005$.) Comparison with the parameter space on Figure 1 shows that, if the post-collision eccentricity is small, only encounters with a subgiant satisfying $R_{RG}/R_{RG,core}/m_{RG,env} < 0.66R_\odot$ could lead directly to a BH–WD X-ray binary. This corresponds to a subgiant with $m_{RG,core} \approx 0.17M_\odot$ (so that for a $0.9M_\odot$ RG, the radius is then still only 2.75 $R_\odot$). The time this RG spends as a subgiant before it reaches $m_{RG,core} \approx 0.17 M_\odot$ is only 135 Myr, or $\sim 25\%$ of its RG lifetime. Assuming that the final separation after a physical collision does not exceed that predicted by simple energy conservation, the formation rates are then $\sim 2 \times 10^{-3} f_p$ per BH per Gyr. This rate is an upper limit, as it also is being due to finite size effects. Collisions or strong tidal interactions between the objects could lead directly to the formation of binaries with unequal masses and different energies; however, the treatment is applicable only when the resulting binaries are near the hard–soft boundary or a bit harder.

In their derivation of the formation rates of binaries with different energies, Ivanova et al. (2005a) approximate that the semimajor axis in the formed binary is the same as the size of the vicinity $a_v$, where the three objects meet. As shown by numerical experiments in Aarseth & Heggie (1976), this assumption is approximately satisfied at least for equal masses, where $a(1 + e)/a_v \approx 1$.

In our study, we are most interested in the formation rates where objects have a fairly large mass ratio and have high energies ($\eta \gtrsim 100$). Compared to $\eta = 1$ binaries, the decrease in the hard binary formation rate due to physical collisions and tidal effects could be significant. We therefore limit ourselves to the consideration of only degenerate objects meeting each other. On the other hand, the limitation of the considered cases to only very hard binaries eliminates the necessity to consider the second condition: Aarseth & Heggie (1976) showed that the probability of forming a binary strongly increases as viscosity decreases, $P \propto \eta^{-2}$, and, even in the case of $\eta = 2$, $77\%$ of three-body encounters resulted in binary formation. They have also shown that the average eccentricity of formed binaries is a bit higher than in the thermal distribution: $\langle e \rangle = 0.77$ for $\eta = 2$ cases, though this was the largest energy considered and the average values were slowly decreasing with hardness. We may expect that the average eccentricities could go as low as that of the thermal distribution.

The rate at which three objects meet in the same vicinity $a_v$ is the product of the rate $\Gamma_2(a_v, m_1, m_2)$ for two objects to meet in this neighborhood and the probability $P_3$ that during this event a third object will be in the vicinity. As previously done, the two-body encounter rate $\Gamma_2(a_v, m_1, m_2)$ is standardly derived as a combination of geometric cross section and gravitational focusing:

$$\Gamma_2(a_v, m_1, m_2) = \pi n_\infty v_\infty a_v^2 \left(1 + \frac{v_p^2}{v_\infty^2}\right) \quad (20)$$

where $v_p^2 = 2G(m_1 + m_2)/a_v$. Two objects spend in this vicinity about a time $\tau_v = 2a_v/v_p$. The probability $P_3$ that a third object will be within the same vicinity is then usually found assuming that a third object will geometrically sweep only a certain volume during $\tau_v$ (Binney & Tremaine 1987; Ivanova et al. 2005a). A “geometric” cross section however is good only for cases of $a_v$ large enough so $v_p \lesssim v_\infty$. In the case, when we are interested in the encounters occurring within a small vicinity only, gravitational focusing must be taken into account. In some sense, this implies that we are looking for a probability that, at the same time, two objects will be gravitationally focused by a BH. The probability that a third object will pass within $a_v$ during $\tau_v$ is then

$$P_3 \simeq 2\pi n_3 a_v^3 \frac{v_p}{v_\infty}. \quad (21)$$

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7 Details of the hydrodynamical simulations of collisions between BH and RG will be presented in the paper in preparation.
X-ray binaries, how these binaries can be formed and at what served rates of BH–WD X-ray binary formation. Smaller), three-body binary formation cannot explain the observed formation rate of BH–WD X-ray binaries. {\textit{Conservative}} is the case when we take into account only BH–WDs with \( a \lesssim 35 R_\odot \), which are guaranteed to create a Kozai triple that results in TIMT within 1 Gyr. \{\textit{Optimistic}\} is the case when all BH–WD binaries with \( a \lesssim 80 R_\odot \) have a chance to be in such a Kozai triple that leads to TIMT, a process that takes several Gyr.

The three-body formation rate then is
\[
\Gamma_3(a_\nu) \simeq 2\pi^2 n_\nu a_\nu^5 v_\nu^3 \simeq 2^{5/2} \pi^2 G^{3/2} n_\nu a_\nu^3 v_\nu^{-2} m_{\text{BH}}^{3/2}.
\] (22)

For very hard binaries, this rate is significantly larger than the corresponding rate derived in Ivanova et al. (2005a) and can be written as
\[
\Gamma_3(a_\nu) \simeq 1.6 \times 10^{-16} \text{ per Gyr}
\times \left( \frac{n_\nu}{10^5 \text{ pc}^{-3}} \right)^2 \left( \frac{a_\nu}{10^8 R_\odot} \right)^{3.5} \left( \frac{10 \text{ km/s}}{v_\nu} \right)^2 \left( \frac{m_{\text{BH}}}{15 M_\odot} \right)^{3/2}.
\] (23)

Therefore, even in a relatively dense stellar cluster, a BH of \( 15 M_\odot \) will have no less than \( 2 \times 10^{-9} \) three-body encounters within its 100 \( R_\odot \) vicinity during 1 Gyr. The formation rate necessary to explain BH–WD X-ray binaries formation is consistent only with encounters within \( \sim 5000 R_\odot \), or \( \eta = 10 \). We also note that in this case \( v_\nu \gg v_\infty \), the assumption used for the derivation, is no longer valid. Even in a denser cluster \( (n_\nu \sim 10^6 \text{ pc}) \) with a somewhat smaller velocity dispersion, the rate given by the above estimate is still about an order of magnitude less than the observed formation rate.

We conclude that unless a significant fraction of small-vicinity three-body encounters lead to a formation of binaries much smaller than the vicinity where they met (10–50 times smaller), three-body binary formation cannot explain the observed rates of BH–WD X-ray binary formation.

4. CONCLUSIONS

We have discussed which BH–WD binaries can become X-ray binaries, how these binaries can be formed and at what rate. In Section 2, we find that the most important mechanism for making a BH–WD LMXB is when a third star brings the components of the binary sufficiently close via the Kozai mechanism that MT is induced: so-called TIMT. The second most important mechanism is hardening with subsequent TIMT (see Table 2).

| Channel          | Conservative | Optimistic |
|------------------|--------------|------------|
| EX + TIMT        | \( 2 \times 10^{-3} \) | \( 6 \times 10^{-3} \) |
| EX + HARD + TIMT | \( 2 \times 10^{-3} \) | \( 1.5 \times 10^{-2} \) |
| PC               | \( 4 \times 10^{-4} \) | \( 1.5 \times 10^{-2} \) |

Notes. The table shows the formation rates of BH–WD MT binaries, per BH per Gyr. Here, “EX + TIMT” is formation via an exchange encounter with the subsequent TIMT (triple induced mass transfer); “EX + HARD + TIMT” is an exchange with a wide binary formation and subsequent hardening and TIMT; “PC” is a physical collision directly leading to MT, and “PC + TIMT” is the formation through a physical collisions of a bound but wide BH–WD binary that is brought to MT by TIMT. “Conservative” is the case when we take into account only BH–WDs with \( a \lesssim 35 R_\odot \), which are guaranteed to create a Kozai triple that results in TIMT within 1 Gyr. “Optimistic” is the case when all BH–WD binaries with \( a \lesssim 80 R_\odot \) have a chance to be in such a Kozai triple that leads to TIMT, a process that takes several Gyr.

In Section 3, we consider several channels for dynamically forming the initial BH–WD binary: exchange encounters, physical collisions with giants, and three-body binary formation. All of these channels require that at least a fraction of BHs interacts strongly with other stars in the cluster. With our conservative estimate of the formation rates, we find that we can explain the formation rate inferred from observations, \( \sim 4 \times 10^{-3}/\text{BH}\text{-}\text{BH}_{0.1} \), but only if \( f_{\text{BH,0.1}} \approx 1 \). This, by our definition of \( f_{\text{BH,0.1}} \), corresponds to the case when 10% of all formed BHs remain in the cluster and interact with the core’s stellar populations.

With the optimistic scenario, we find that all the channels have comparable formation rates and each of the channels can explain the observed formation rate of BH–WD X-ray binaries. The combined rate of all the channels is \( \sim 4 \times 10^{-2} \) per BH per Gyr. Comparing this with the formation rates inferred from the observations, we find that we can explain the observations only with \( f_{\text{BH,0.1}} \gtrsim 0.1 \). In other words, even in the most optimistic case, we require that at least 1% of all initially formed BHs (\( f_{\text{BH,0.1}} = 0.1 \)) should be retained in the cluster and continuing to interact with the core’s stellar population. Future simulations could address this.
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