TRACTABILITY OF MULTI-PARAMETRIC EULER AND WIENER INTEGRATED PROCESSES

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Abstract: We study average case approximation of Euler and Wiener integrated processes of \( d \) variables which are almost surely \( r_k \)-times continuously differentiable with respect to the \( k \)-th variable and \( 0 \leq r_k \leq r_{k+1} \). Let \( n(\varepsilon, d) \) denote the minimal number of continuous linear functionals which is needed to find an algorithm that uses \( n \) such functionals and whose average case error improves the average case error of the zero algorithm by a factor \( \varepsilon \). Strong polynomial tractability means that there are nonnegative numbers \( C \) and \( p \) such that

\[
 n(\varepsilon, d) \leq C\varepsilon^{-p} \quad \text{for all } d \in \mathbb{N} = \{1, 2, \ldots\}, \text{ and } \varepsilon \in (0, 1). 
\]

We prove that the Wiener process is much more difficult to approximate than the Euler process. Namely, strong polynomial tractability holds for the Euler case iff

\[
 \liminf_{k \to \infty} \frac{r_k}{\ln k} > \frac{1}{2\ln 3},
\]

whereas it holds for the Wiener case iff

\[
 \liminf_{k \to \infty} \frac{r_k}{k^s} > 0 \quad \text{for some } s > \frac{1}{2}.
\]

Other types of tractability are also studied.

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