Application of the Townsend-George wake theory to field measurements of wind turbine wakes

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Abstract. As wind turbines are usually clustered in wind farms, knowledge of the evolution of the wind turbine wakes is important because downstream turbines will be exposed to them, causing higher loads and maintenance times. For that reason, wind turbine wakes have been studied intensively and different engineering wake models were derived. However, none of them is constructed from a few basic and robust assumptions, while such a formalism already exists for the axisymmetric turbulent wake. Therefore, we will apply these models to data obtained in a wind farm using a scanning LiDAR. The wakes of two wind turbines are analyzed in four different wind directions chosen so that their wakes will have different degrees of interaction. The axisymmetric wake models are found to perform better than the Jensen wake model, and the main source of improvement is shown to be the presence of a virtual origin. Indeed, a virtual origin is shown to also improve the Jensen wake model significantly. Overall, our results indicate that a view from classical axisymmetric wake theory can help to improve the understanding of the evolution of wind turbine wakes in the atmospheric boundary layer.

1. Introduction

Wind turbines are exposed to turbulent structures both from the atmospheric wind and, within a wind farm, from the wakes of other wind turbines. As the loads experienced by a wind turbine are directly related to turbulence, e.g. [1], knowledge of the turbulence properties of a wind turbine wake is important. In addition, the turbines exposed to the wakes of other turbines yield less power due to the reduced velocity upstream the wake generator. A lot of effort has been put into the streamwise scaling of the averaged velocity deficit (e.g. [2, 3]), and those models are used for wind farm layout and control algorithms optimization. However, a physical model that describes the wake of a wind turbine and the turbulence within from a few basic and robust assumptions is still missing. Nevertheless, an analytical model exists for the axisymmetric turbulent far wake evolving e.g. downstream of a bluff body in uniform inflow that has been a subject of research for the past decades. Above all, Townsend [4] and George [5] developed the theories used to model these flows, that rely on the self-preservation of the averaged turbulence quantities and the assumption that the turbulent wake becomes axisymmetric far downstream of the wake generator. Here, self-preservation is defined as the self-similarity of flow structures that depends on the same scale for all quantities, and it is a common concept for the turbulent
decay of developing flows, for which the structure remains similar [6]. For self-similarity, we follow the explication given in [5]: “self-similarity is said to occur when the profiles of velocity (or any other quantity) can be brought into congruence by simple scale factors which depend on only one of the variables”. Axisymmetry can be understood as axisymmetry of the mean quantities, implying, in cylindrical coordinates, an independence of all averaged quantities on the polar angle theta [5].

The wake of a rotating wind turbine in the atmospheric boundary layer does undoubtedly exceed the complexity of the wake generated by a static body facing a steady laminar flow. Nevertheless, the question arises to what degree these self-preserving axisymmetric turbulent wake (SPATW) models can be applied to the wake of a wind turbine.

For the description of SPATWs, the two most prominent problems are the evolution of the mean centerline velocity deficit \( \Delta u \) and the wake width \( \delta \). Here, one fundamental difference between the engineering wind turbine wake models and SPATW models becomes obvious: In wind turbine wake models, the wake width is derived empirically while it is derived analytically in SPATW models.

The Townsend-George theory was developed to model boundary-free SPATWs in the far wake where the shear layers surrounding the wake have already met, and the model is based on two cornerstone assumptions. First, the averaged one-point statistical quantities have to evolve fulfilling self-preservation with increasing downstream distance \( x \). Second, the dissipation rate of the turbulent kinetic energy is assumed to scale as \( \varepsilon = C_\varepsilon K_c^{2/3}/L \) in the case of Richardson-Kolmogorov turbulence. Here, \( K_c \) denotes the centerline turbulent kinetic energy and \( L \) is an integral length scale. The assumption \( C_\varepsilon = \text{const.} \) for a given set of boundary conditions at high Reynolds numbers is crucial, and it is consequently used in the modelling of many turbulent flows (e.g. [7]).

While \( C_\varepsilon = \text{const.} \) leads to the standard scaling of \( \Delta u \) and \( \delta \) of turbulent wakes, there is also experimental and numerical evidence that for some wakes, \( C_\varepsilon \) is not constant but scales according to \( C_\varepsilon \sim Re_{L}^{m}/Re_{\Lambda}^{n} \) (e.g. [8, 9, 10, 11]), which leads to different scalings of \( \Delta u \) and \( \delta \). \( Re_{L} = \delta u'/\nu \) denotes a local and streamwisely position-dependent Reynolds number depending on the wake width, the standard deviation of the streamwise velocity \( u' \) and the kinematic viscosity \( \nu \). \( Re_{\Lambda} = \sqrt{\Lambda u_{\infty}/\nu} \) is a global Reynolds number depending on the frontal area of the wake generator \( \Lambda \) and the inlet velocity \( u_{\infty} \). For fully turbulent flows, \( m = n = 1 \).

By means of the evolution of the mean velocity deficit, conclusions can thus be drawn on the turbulence within the wake, particularly on the nature of the energy cascade and its corresponding energy dissipation scalings. This is of interest as it allows a better description of the turbulence wake properties like the dissipation rate \( \varepsilon \).

While a lot of work has been put into the analysis and description of the evolution of the mean velocity deficit downstream of a wind turbine, only few works exist that connect the turbulence decay in the wake of a wind turbine with wake modeling. The standard Richardson-Kolmogorov scaling has been applied to experimental wind turbine wake data by [12], where it is found that this model fits the wake well. Also, [13] investigate the dissipation scaling within the wake of a model wind turbine exposed to an atmospheric boundary layer flow in a wind tunnel, applying the Richardson-Kolmogorov and the non-equilibrium scaling. They confirm the self-similar behavior of one-point statistical quantities, and they also find evidence of non-equilibrium behavior of the Reynolds stress tensor. [14] investigates the application of the Richardson-Kolmogorov and the non-equilibrium theories to wind turbine wakes and show that the assumptions hold for uniform laminar and wake inflow conditions and also to a certain degree in the wake of a turbine exposed to one half of the wake of an upstream wind turbine. They identify a wake core with a radius of approximately 0.2 turbine diameters where the turbulence properties are independent on the radial position. In this wake core, the models perform well even away from the centerline. Also, they show how the SPATW models outperform the Jensen
wake model and the Bastankhah-Porté-Agel wake model [15, 16] due to the presence of a virtual origin, and how the implementation of a virtual origin significantly improves the engineering wake models.

While the results from the studies above are very promising, the question remains to what degree they are valid also for wakes of full-scale wind turbines exposed to the instationary atmospheric inflow. Therefore, we will apply the Townsend-George SPATW models to data obtained during a seven months scanning LiDAR field campaign where the wakes of two wind turbines were investigated. Depending on the wind direction, one turbine is exposed to the wake of the other turbine. In addition, the Jensen wake model will be applied as a comparison.

In the following, the SPATW models and the Jensen model will be introduced and the main parameters of the database will be given in section 2. There, necessary modifications of the data will also be explained. The results are shown in section 3 and they are discussed in section 4.

2. Theory and Setup
In this section, a brief overview over the most important aspects of the SPATW models that will be used here will be given. For a more detailed explanation, the reader is referred to [9, 8, 14]. Afterwards, the Jensen wake model is briefly introduced, then, the setup is shown, and it is explained how the data was prepared to apply the different wake models.

In the following, \( x \) will refer to the direction downstream of the turbine, \( y \) is the horizontal cross-wise coordinate and \( z \) is the vertical coordinate. The origin is at the center of the turbine.

2.1. SPATW Models
Within classical SPATW research, similarly to wind turbine wake research, the prediction of the mean centerline velocity deficit \( \Delta u = u_\infty - u_c \) and the wake width \( \delta \) are of interest. \( u_\infty \) is the inflow velocity and \( u_c \) is the velocity at the centerline downstream of the turbine. The wake width is defined here as the integral wake width \( \delta^2(x) = \frac{1}{\Delta u} \int_{-\infty}^{\infty} (u_\infty - u) r dr \) that is integrated along the radial component \( r \). In the following, we will briefly introduce the revisited Townsend-George theory from [9]. The Richardson-Kolmogorov and the non-equilibrium predictions require an axisymmetric turbulent wake where the normalized mean velocity profile \( (u_\infty - u(x,y))/\Delta u \) (where \( u(x,y) \) is the streamwise mean velocity at hub height), the turbulent kinetic energy (TKE), the Reynolds shear stresses, and the turbulence dissipation evolve self-similarly. Both predictions are derived from the Reynolds-averaged streamwise momentum and TKE equations, however, the scaling behavior of \( \Delta u \) and \( \delta \) varies due to their dependence on \( C_\varepsilon \), as explained in the introduction.

In the case of Richardson-Kolmogorov turbulence, the predictions are

\[
\Delta u(x) \propto a_{RK} u_\infty (x - x_{0,RK})^{-2/3}
\]

\[
\delta(x) \propto b_{RK} (x - x_{0,RK})^{1/3}
\]

and in the case of non-equilibrium turbulence, the predictions are

\[
\Delta u(x) \propto a_{NEQ} u_\infty (x - x_{0,NEQ})^{-1}
\]

\[
\delta(x) \propto b_{NEQ} (x - x_{0,NEQ})^{1/2}
\]

\( a \) and \( b \) are dimensionless constants and \( x_0 \) is a virtual origin.
2.2. Jensen Wake Model

One of the first engineering wind turbine wake models has been derived by N. O. Jensen in 1983 [15]. It is still used as it is simple and computationally inexpensive. From the conservation of mass and the assumption of a top-hat shaped velocity deficit in the wake of a turbine, the evolution of the normalized velocity deficit \( \Delta u/u_\infty \) (where \( u_\infty \) denotes the uniform wake velocity of the top-hat shaped velocity deficit) with increasing downstream distance \( x \) is derived as

\[
\Delta u/u_\infty = (1 - \sqrt{1 - c_T}) \cdot \left(1 + \frac{2k_Jx}{D}\right)^{-2},
\]

where \( D \) is the turbine diameter, \( c_T \) its thrust coefficient and \( k_J \) the wake expansion that was suggested to be \( k_J \approx 0.1 \) by Jensen, however, values of \( k_J \approx 0.075 \) for onshore scenarios and \( k_J \approx 0.04 \) or \( k_J \approx 0.05 \) for offshore scenarios are suggested in the literature [2, 17]. We will use \( u_W = u_c \).

As one main difference between engineering wake models and SPATW models is the virtual origin, we will also apply a modified Jensen model with virtual origin (not predicted in the original theory) where \( x \) is substituted with \( x - x_0 \) to show possibilities that can arise from such a rather simple alteration. However, for now, the aim is mainly to demonstrate how the implementation of a virtual origin can help to improve wake models, and not to provide an improved wake model.

2.3. Data Base and further post-processing

![Figure 1. Setup: The data was collected in a wind farm in the north of France (a). The wind farm consists of seven wind turbines, marked as red dots, and the flow field that was scanned using the LiDAR is marked in blue; additionally, the four investigated wind directions 207°, 220°, 233° and 246° are indicated (b). Three elevation angles were scanned (c).](image)

The data base that we will use has been collected using a LiDAR at a wind farm in the north of France over the course of seven months from November 15th 2015 to May 31st 2016. The wind farm consists of seven Senvion MM82 wind turbines with a hub height of 80 m, a rotor
Figure 2. Illustration of the estimation of the inflow velocity in dependence of the height for case 1 (207°): (a) average wind field for elevation angle 2 (note that the coordinate system is turned so that the wakes downstream of turbine 6 and 5 are aligned with the x axis for this inflow case). The regions that are not influenced by the wind turbine wakes and that are used for the calculation of the wind profile in (b) are underlaid in grey.

The diameter of $D = 82$ m and a nominal power of 2050 kW at 14.5 ms$^{-1}$. The turbine positions are illustrated in figure 1. The main wind direction is from the south-west, and the LiDAR, a Windcube 200S by Leosphere, has been used to scan the wakes of turbine 5 and 6. These turbines are 3.7D apart from each other, and they are aligned at a 207° reference line. As shown in figure 1(b) and (c), the wind field was scanned using plan position indicator scans covering azimuth angles between 243° and 273° at three different elevation angles $\alpha_1 = 2.5°$, $\alpha_2 = 3.8°$, and $\alpha_3 = 5.2°$. The line of sight (LOS) velocity was measured sweeping the azimuth angle at a fixed elevation angle with a spatial resolution of 1° and a temporal resolution of 2°/s. Starting from 150 m away from the LiDAR, the LOS resolution is 25 m, and 115 positions are measured along each LOS.

In this work, we use the data processed and discussed in [18] to test the wake models. There, the streamwise velocity was already derived from the LOS velocity scans by fitting a sinus to the LOS velocity at a fixed height. Measurements at an inflow velocity of approximately 13 ms$^{-1}$ at 50 m and a turbulence intensity of 15% in a neutral atmosphere were selected for four different wind directions. Different interactions between the wakes from the two turbines were chosen:

- case 1 - 207°: turbines 5 and 6 are aligned and turbine 5 operates in the wake of turbine 6,
- case 2 - 220°: turbine 5 operates partially in the wake of turbine 6,
- case 3 - 233°: turbine 5 is exposed to a fraction of the wake downstream of turbine 6 but the wakes interact,
- case 4 - 246°: turbine 5 and 6 are both exposed to free inflow but the wakes might interact in the far field.

The wind directions are also indicated in figure 1(b). Similarly to [18], we analyze the mean flow fields that are averaged over at least 200 scans.

Here, we propose to analyze the evolution of the mean centerline velocity deficit. However, the
Table 1. Fit parameters for atmospheric wind profile $u^*(z) = d \cdot z^n$ and the resulting velocities at hub height.

|       | case 1 | case 2 | case 3 | case 4 |
|-------|--------|--------|--------|--------|
| $d$   | 3.79   | 4.52   | 4.11   | 4.45   |
| $n$   | 0.27   | 0.22   | 0.22   | 0.20   |
| $u_\infty = u^*(z = 80m)/ms^{-1}$ | 12.18  | 11.91  | 10.96  | 10.92  |

height where the wind speed is measured with the LiDAR varies along the LOS, and therefore, a centerline measurement (that implies $z = \text{const}$) is not available downstream of the two turbines. To correct the measured velocity, we therefore use the areas of the three scans at the different elevation angles that are unaffected by the wind turbine wakes to obtain an approximated profile of the wind velocity with increasing height $z$. This is illustrated exemplarily for case 1 in figure 2. In (a), the complete flow field is plotted for $\alpha_2$, and the wakes downstream of turbine 5, 6 and 7 are visible. The unaffected areas that were used to obtain the profile in (b) are underlaid in grey in the scan in (a). Then, a fit according to $u^*(z) = d \cdot z^n$ was applied. The fit parameters can be found in table 1 together with the velocities at hub height. Note that the exponents differ from those found in [18] as a different procedure was used to choose the data for the fits.

For the data evaluation, we will assume $u_\infty = u^*(z = 80m)$.

In the next step, the streamwise velocities measured along the LOS at different heights, $u_m(z)$, will be corrected by balancing the difference between the calculated velocity at hub height, $u^*(z = 80m)$, and the calculated velocity at the respective measurement height $z_m$, $\Delta u_{cor}(z_m) = u^*(z_m) - u^*(z = 80m)$, so that $u_c \approx u_m(z_m) - \Delta u_{cor}(z_m)$. The results are shown in figure 3. Here, the data at $y = 0$ was obtained by interpolating the measured data to a Cartesian grid. In the left column, the height of the three scans at different elevation angles is plotted over the downstream distance for the four cases, and in the right column, the normalized velocity deficit is plotted. For both turbines, the downstream distance is given with respect to their rotor plane except in case 1 where the position of turbine 5 is indicated. In the plots that indicate the height, the part of the respective scan that is closest to the centerline is indicated in red. For case 1 and 2, figures 3 (a) and (c), the scans with elevation angle $\alpha_2$ are closest to the centerline for the whole measurement region. We will therefore use the velocity obtained at the elevation angle $\alpha_2$ and correct it according to the vertical distance of the scan from the hub height. The results are shown in figures 3 (b) and (d) for case 1 and 2, respectively. Empty symbols show the original data and filled symbols the corrected curves. For case 3 and 4, cf. figures 3 (e) and (h), however, first, the scans with elevation angle $\alpha_2$ are closest to the hub height but in the far field, the scans with elevation angle $\alpha_3$ are closest. We will therefore correct the velocities according to the hub height and switch from elevation angle $\alpha_2$ to elevation angle $\alpha_3$, which is indicated in figures 3 (f) and (g) for case 3 and (i) and (j) for case 4 with the red line. In case 3, we see that the switch from one elevation angle to the other is discontinuous. Even though the post-processed data has this discontinuity, applying a fit to it will give a better indication of the velocity at the centerline than only taking into account one elevation angle and fitting the velocity data away from the centerline.

3. Results

In the following, we will apply the Richardson-Kolmogorov and the non-equilibrium predictions to the four different cases and the wakes of the two turbines 5 and 6. The results will be compared to the Jensen model.
Figure 3. Illustration of the data correction: in the left column, the height of the scan at the centerline downstream of the respective turbine is plotted for the four cases. In red, the part of the scan that is used is highlighted. The blue horizontal lines indicate the hub height and the rotor radius. In the right column, the measured wind speed downstream of the respective turbine is plotted as empty symbol and the corrected wind speed is plotted as filled symbol. In addition, in plots (f), (g), (i) and (j), the red line shows the curves that are corrected and closest to the hub height.
Before applying the SPATW predictions, it has to be checked first that the theory can be applied. As the small scales of turbulence are not resolved, data obtained using LiDAR has the drawback that turbulence quantities are unknown, and we can only verify the self-similar behavior of the mean velocity deficit. As the self-similar behavior of other one-point statistical quantities has been verified by [13] for a wake exposed to an atmospheric boundary layer, and the applicability of the SPATW theory to wind turbine wakes was verified in [14], we will assume that the remaining requirements are fulfilled sufficiently.

To look for self-similar behavior, the normalized wake profiles \( \frac{u_\infty - u(y/\delta)}{u_\infty - u_c} \) are plotted over the normalized span-wise coordinate \( y/\delta \) in figure 4 for the four cases and the two turbines. In the case of self-similarity, the curves should collapse. Note that the velocity profiles were corrected according to the measurement height. As can be seen, the curves indeed collapse in case 1 for both turbine wakes, cf. figure 4 (a) and (b). In case 2, figure 4 (c), the curves collapse close to turbine 6, but the wake is deflected towards turbine 5 and becomes asymmetrical in the far wake. Downstream of turbine 5, figure 4 (d), the wake profiles are a bit skewed but collapse overall quite well. In case 3, figure 4 (e) and (f), we find that the mean velocity deficit behaves self-similarly close to the respective turbine. In the far wake, a clear wake profile is not identifiable anymore. The same is true for case 4, figure 4 (g) and (h). Here, it can also be seen, that the wake of turbine 6 is again deflected towards turbine 5.

Overall, this investigation shows that self-similarity of the mean velocity deficit is sufficiently fulfilled to apply the Townsend-George phenomenology.

In figure 5, the corrected mean centerline velocity deficit is plotted over the increasing downstream distance for the four cases and the two turbines. We apply the Richardson-Kolmogorov scaling (equation 1; dark blue), the non-equilibrium scaling (equation 3; petrol, dashed), the Jensen model (equation 5; dark green, dotted), and the Jensen model modified with a virtual origin (green, dot-dashed) to the region where the velocity recovers (we chose \( \approx 5D \) downstream of the maximum velocity deficit as starting point). The thrust coefficient \( c_T \) used to fit the Jensen model is shown in table 2 and was taken from [19]. There, it was calculated from the average streamwise wind speed as measured by the turbines’ Supervisory Control and Data Acquisition (SCADA) systems and using the power curve provided by the manufacturer. The wake growth rate was treated as a fit parameter. The fit parameters are given in table 3 together with the root-mean-square error (RMSE) of the respective fit. Overall, we can see that both the Richardson-Kolmogorov model and the non-equilibrium model fit the data well from \( x/D \approx 2 \) with the exception of case 4, turbine 6, in figure 3 (f). This is also mirrored in the RMSE values. The reason for this may be that we are looking at the centerline data downstream of the turbine while the wake is deflected towards turbine 5, as we saw in figure 4 (g). This means that we look at data rather far away from the wake center, and even leave the wake core where the wake models were shown to perform well by [14] despite a deviation from the centerline. The original Jensen model does not perform well in any case as it under-predicts the velocity deficit up to approximately \( 4D \) but over-predicts it farther downstream. The results can be improved significantly when implementing a virtual origin. The modified Jensen model does perform significantly better than the original model. It performs as well as the Richardson-Kolmogorov and the non-equilibrium wake models, which is also indicated by the RMSE values.

It has to be noted that this is a demonstration of the possibilities coming with this additional parameter that is commonly used in SPAWT models, and that the virtual origin is treated as a fit parameter in this work.

When looking at the fit parameters, we see that they are not correlated with the thrust coefficients of the turbines. However, some conclusions can be drawn regarding the dependence of the fit parameters on the inflow. For example, in case 1, the coefficients of all models are different from all other cases because turbine 5 is fully immersed in the wake of turbine 6. In contrast, the coefficients are similar for turbine 5 in case 2 and 3 where the turbine
Figure 4. Investigation of the self-similarity in the wakes: the normalized velocity deficit \((u_\infty - u(x, y/\delta))/(u_\infty - u_c)\) is plotted over the normalized span-wise position \(y/\delta\) for the four cases and the two turbines. The downstream position is indicated in color.
Table 2. Thrust coefficients of turbine 5 and 6 for the four cases (taken from [19]).

|      | case 1     | case 2     | case 3     | case 4     |
|------|------------|------------|------------|------------|
| $c_{T,T_6}$ | 0.5345     | 0.5342     | 0.5291     | 0.5765     |
| $c_{T,T_5}$ | 0.7419     | 0.7094     | 0.5811     | 0.6111     |

Table 3. Fit coefficients for the two turbines 5 and 6 and the four models (Jensen [15], Jensen VO [14], RK and NEQ [5, 8, 9, 10]).

| model       | case 1 | case 2 | case 3 | case 4 |
|-------------|--------|--------|--------|--------|
|             | T5     | T6     | T5     | T6     | T5     | T5     |
| RK          |        |        |        |        |
| $a_{RK}$    | 0.930  | 1.026  | 0.863  | 0.367  | 0.789  | 0.330  | 0.384  |
| $x_{0,RK}$  | 3.467  | -1.949 | -0.559 | 1.540  | -0.961 | 0.911  | 0.932  |
| RMSE        | 0.013  | 0.020  | 0.008  | 0.016  | 0.013  | 0.047  | 0.009  |
| NEQ         |        |        |        |        |
| $a_{NEQ}$   | 1.923  | 2.818  | 1.961  | 0.708  | 1.986  | 0.562  | 0.775  |
| $x_{0,NEQ}$ | 2.197  | -4.930 | -2.248 | 0.588  | -3.261 | 0.318  | -0.165 |
| RMSE        | 0.010  | 0.021  | 0.008  | 0.014  | 0.014  | 0.036  | 0.006  |
| Jensen      |        |        |        |        |
| $k_J$       | 0.003  | 0.005  | 0.022  | 0.034  | 0.018  | 0.058  | 0.051  |
| RMSE        | 0.095  | 0.055  | 0.045  | 0.060  | 0.034  | 0.078  | 0.034  |
| Jensen VO   |        |        |        |        |
| $k_{JVO}$   | 0.063  | 0.027  | 0.053  | 0.090  | 0.039  | 0.130  | 0.088  |
| $x_{0,JVO}$ | 6.147  | 4.034  | 1.998  | 2.848  | 2.336  | 2.027  | 1.737  |
| RMSE        | 0.010  | 0.022  | 0.009  | 0.020  | 0.015  | 0.023  | 0.010  |

is partially exposed to the wake of turbine 6 and, thus, to higher turbulence levels (with $a_{RK} \approx 0.83 \pm 0.04$ and $x_{0,RK} \approx -0.75 \pm 0.20$ in the case of the RK model, $a_{NEQ} \approx 1.97 \pm 0.02$ and $x_{0,NEQ} \approx -2.75 \pm 0.50$ in the case of the NEQ model, $k_J \approx 0.20 \pm 0.02$ in the case of the Jensen model, and $k_{JVO} \approx 0.045 \pm 0.010$ and $x_{0,JVO} \approx 2.15 \pm 0.15$ in the case of the modified Jensen model). Also, the coefficients are comparable for turbine 6, case 3, and turbine 5 and 6, case 4 with $a_{RK} \approx 0.36 \pm 0.03$ and $x_{0,RK} \approx 1.25 \pm 0.30$ in the case of the RK model and $a_{NEQ} \approx 0.66 \pm 0.1$, $x_{0,NEQ} \approx -0.25 \pm 0.35$ in the case of the NEQ model, $k_J \approx 0.45 \pm 0.15$ in the case of the Jensen model, and $k_{JVO} \approx 0.11 \pm 0.02$ and $x_{0,JVO} \approx 2.3 \pm 0.55$ in the case of the modified Jensen model. This indicates that the wakes behave similarly in free inflow conditions despite the different thrust coefficients. While turbine 6 is also exposed to free inflow in case 2, the coefficients do not match with case 3 and 4. A reason for this may be that the wake is deflected towards turbine 5. In addition, the far wake of turbine 7 could interact with the far wake of turbine 6.

Overall, the results show that the Richardson-Kolmogorov model, the non-equilibrium model and the modified Jensen model that includes the virtual origin predict the mean centerline velocity deficit well. The fit coefficients appear to be independent of the thrust coefficient, but it can be differed between wake inflow and free inflow as long as the wakes do not interact too strongly.
4. Discussion and Conclusion
In this paper, we applied the Townsend-George SPATW theory to field measurements of wind turbine wakes. Data obtained with a scanning LiDAR was used with the objectives to test the applicability of these models and to test the accuracy of them.

First, we investigated the self-similar behavior of the normalized profiles of the velocity deficit at hub height for four different inflow directions downstream of two turbines. In the far wake, clear profiles could not be identified, and also, in some cases, the wake center was deflected, but overall, the wake profiles can be considered self-similar especially close to the turbines.

Next, the wake models were applied to the corrected mean centerline velocity deficit. Since
the wake models were derived for the far wake, they are only applicable to the far wake of the turbines that starts between $x/D \approx 1$ and $x/D \approx 2$. Similarly to [14], we could show that the SPATW models outperform the original Jensen wake model, but we also showed that the implementation of a virtual origin improves the Jensen model significantly. Overall, the results show that a power law fits the evolution of the mean centerline velocity deficit well as long as the wake center is sufficiently aligned with the centerline downstream of the turbine, i.e. as long as the wake core is investigated. Given the small streamwise range pertinent for wind energy applications, the exponent of the power law is secondary to the virtual origin that is used in many classical turbulence descriptions. While different exponents produce good fits of data, the latter proves useful to improve engineering wake models as well, as it is an additional parameter that increases the adaptability of a model to the ambient turbulence conditions. This field study gives a first idea of the possibilities coming with the implementation of a virtual origin, and the correlation with the ambient turbulence needs to be determined for each model. Finally, the fit coefficients show no clear dependence on the thrust coefficient of the respective turbine. On the contrary, they seem to be related to the inflow turbulence in the case that the wakes do not interact too strongly. These results show that classical SPATW models derived from a few robust turbulence assumptions can help to better understand the evolution of wind turbine wakes and to improve engineering wake models. The next step will be to identify the determining inflow parameters for the prediction of the fit coefficients, which are the inflow velocity and the atmospheric turbulence or, in the case of a downstream turbine, the wake turbulence of an upstream turbine. Finally, the question arises to what degree it is possible to identify parameters when wakes are interacting.

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