High Energy Neutrinos from Novae in Symbiotic Binaries: The Case of V407 Cygni

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Detection of high-energy ($\gtrsim 100$ MeV) $\gamma$ rays by the Fermi Large Area Telescope (LAT) from a nova in the symbiotic binary system V407 Cygni has opened possibility of high-energy neutrino detection from this type of sources. Thermonuclear explosion on the white dwarf surface sets off a nova shell in motion that expands and slows down in a dense surrounding medium provided by the red giant companion. Particles are accelerated in the shocks of the shell, and interact with surrounding medium to produce observed $\gamma$ rays. We show that proton-proton interaction, which is most likely responsible for producing $\gamma$ rays via neutral pion decay, produces $\gtrsim 0.1$ GeV neutrinos that can be detected by the current and future experiments at $\gtrsim 10$ GeV.

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I. INTRODUCTION

High-energy ($\gtrsim 1$ GeV) neutrinos are produced dominantly via decays of charged pions and kaons created by proton-proton ($pp$) and proton-photon ($p\gamma$) interactions. Powerful astrophysical sources of $\gamma$-rays, long duration ($\gtrsim 2$ s) gamma ray bursts (GRBs); short duration ($\lesssim 2$ s) GRBs and active galactic nuclei (AGNs), as well as sources such as core-collapse supernovae, supernova remnants and microquasars have been proposed as the sources of high-energy $\nu$'s (see e.g. [1]). Protons and ions accelerated by a Fermi mechanism in the shocks of these sources interact with ambient particles and/or soft photons to produce $\gamma$ rays, via neutral pion decay, and $\nu$'s. Modeling $\gamma$ ray emission from an astrophysical source by a $\pi^0$ model thus inevitably predicts high-energy $\nu$ flux from the same source (see e.g. [2]). Detection of these $\nu$'s can provide a conclusive proof of the $\pi^0$ model and discriminate against a lepton model for observed $\gamma$ rays.

Detection of $\gtrsim 100$ MeV $\gamma$ rays by the Fermi Large Area Telescope (LAT) from the Nova 2010 on March 10 in the symbiotic binary V407 Cygni is the first from any nova [3]. The binary system consists of a Mira-type pulsating star, a red giant (RG) which may be swollen to a radius $\sim 500R_\odot$, and a white dwarf (WD) with mass $M_{WD} \gtrsim 1M_\odot$ [4]. The WD accretes material from the stellar wind of the RG and forms an envelope on its surface. As the mass of the envelope increases, an increasing WD surface temperature ignites a thermonuclear runaway (see e.g. [5]). A shell of mass $M_{ej}$ can be ejected from the WD surface when the proper pressure at the core-envelope interface exceeds $2 \cdot 10^{20}(M_{ej}/10^{-6}M_\odot)(M_{WD}/M_\odot)/(R_{WD}/10^8\text{ cm})^{-4}$ dyn cm$^{-2}$. Note, however, that the WD mass and radius are related roughly as $R_{WD} \propto M_{WD}^{-1/3}$. Thus a smaller envelope mass and corresponding $M_{ej}$ is required to produce a thermonuclear explosion on the WD surface as $M_{WD}$ increases.

An optical nova is detected when the initially hot shell expands to a large radius from the WD, thus cooled to a temperature $\lesssim 10^4$ K, and when the opacity dropped due to recombination of ionized hydrogen gas [6]. Non-thermal $\gamma$ rays detected by LAT for 15 days following the optical detection requires particle acceleration and their interactions in the nova shell and in the surrounding medium of the RG. Particle acceleration in the shock of the nova shell has been perceived for the 2006 nova outburst in another symbiotic binary RS Ophiuchi [7], but LAT detection of $\gtrsim 100$ MeV $\gamma$ rays from V407 Cygni prove existence of these high-energy particles in the nova shock and their interactions.

In this brief report we calculate the expected $\nu$ flux from the nova 2010 in the symbiotic binary V407 Cygni following the $\pi^0$ decay model of $\gamma$ ray emission from $pp$ interactions reported in Ref. [3]. We also estimate expected $\nu$ events in current and future detectors and discuss detection prospects of these type of novae.

II. BINARY SYSTEM AND PARTICLE ACCELERATION IN NOVA SHELL

Modeling of $\gamma$-ray light curve from the nova 2010 in V407 Cygni system suggests a binary separation of $a \approx 10^{14} \pm 14$ cm, a mass-loss rate of $\dot{M}_w \approx 3 \cdot 10^{-7} A \cdot M_\odot \text{ yr}^{-1}$ from the RG that blows a wind with a measured velocity of $v_w \approx 10 v_{w,6}$ km s$^{-1}$ and a shell kinetic energy of $E_k \approx 10^{44}$ erg with a measured velocity $v_{ej} = 3200 \pm 345$ km s$^{-1}$ [8]. Close proximity between the WD and RG allows the parts of the nova shell that expand toward the RG to decelerate and enter the Sedov-Taylor phase rapidly, resulting in efficient conversion of shell kinetic energy into shock-accelerated particle energy.

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For a distance $R$ and polar angle $\theta$ from the WD center towards the RG, the density of particles in the RG wind can be calculated as $n(R, \theta) = (M_w/v_w \bar{m}) (R^2 + a^2 - 2aR \cos \theta)^{-1}$ using an inverse square-law density profile of the RG wind. The magnetic field in the forward shock that propagates into the surrounding medium with a velocity $v_{sh} \sim v_{ej} \approx 3.2 \cdot 10^8 \text{cm s}^{-1}$, before significant deceleration, can be calculated from the thermal energy density in the RG wind with temperature $T_w \approx 700 T_{w,2.8} \text{ K}$ [3] as

$$B(R, \theta) = \alpha_B \sqrt{\frac{8kT_w M_w/v_w \bar{m}}{R^2 + a^2 - 2aR \cos \theta}} \approx 0.04 \alpha_B T_{w,2.8}^{1/2} \frac{A^{1/2} v_{th,6}^{-1/2}}{a_{14} - v_{sh,8.5d}} \text{ GeV}; \ \theta = 0, \ (1)$$

in $t = t_d$ day time scale. Here $\bar{m} = 10^{-24}$ g is the mean particle mass; $\alpha_B$ is a magnetic field amplification factor, which may arise from shock modification, and we assumed a factor 4 increase in particle density expected to arise in strong shocks. The maximum proton energy, accelerated by a Fermi mechanism, in this magnetic field can be estimated by assuming that the acceleration time scale is equal to the Larmor time scale as

$$E_{p, \text{max}} = \frac{eBt_{sh}^2}{\varphi c} \approx 160 \frac{\alpha_B T_{w,2.8}^{1/2} A^{1/2} v_{th,6}^{-1/2} t_d v_{sh,8.5}^{1/2}}{\varphi_{1.3}(a_{14} - v_{sh,8.5d})} \text{ GeV}; \ \theta = 0. \ (2)$$

Here we assumed that $\varphi = 20 \varphi_{1.3}$ is the number of $e^-$ folding required for a thermal proton with $\sim 1$ keV kinetic energy to reach $E_{p, \text{max}}$. For $\theta = \pi$, i.e. in the part of the shell opposite to the RG, $E_{p, \text{max}} \approx 90$ GeV at $t = t_d$ day.

Evolution of the shock velocity and radius in the deceleration phase at later time ($\gtrsim 3$ day for $\theta \sim 0$), after the shell accumulates swept up RG wind material of mass equal to its own in a given solid angle, depends on the detail calculation for different $\theta$, and is beyond the scope of this paper. Here we simply assume that protons can be accelerated to an energy up to the value in Eq. (2) throughout the 15 day $\gamma$ ray emission episode, as required by the $\pi^0$ model.

III. NEUTRINO FLUXES ON EARTH

The observed $\gamma$ ray flux, averaged over 15 day outburst, from the nova in V407 Cygni at a distance $D$ is fitted with a $\pi^0 \rightarrow 2 \gamma$ decay model [8] as

$$\Phi_\gamma(E_\gamma) \approx \frac{\langle n_H \rangle}{4\pi D^2 cL} \int_{E_{p,th}}^\infty Q(E_p, E_\gamma) N_p(E_p) dE_p. \ (3)$$

Here $Q(E_p, E_\gamma)$ is the $\gamma$-ray production rate per unit density of H atoms [5], $c_{\text{em}} = 1.84$ is the nuclear enhancement factor to take into account the contribution of other atoms [5], $N_p(E_p) = N_{p,0} E_p^{-s} \exp((m_p c^2 - E_p)/E_{p,0})$ is the shock-accelerated proton spectrum and $\langle n_H \rangle$ is the average number density of target particles. The best-fit parameter values are $s = 2.15_{-0.28}^{+0.45}$, $E_{p,0} = 32_{-8}^{+85}$ GeV with a total energy in protons $\approx 6.93_{-2.3}^{+3.6} \times 10^{42} (\langle n_H \rangle / 4.10^{48} \text{ cm}^{-3})^{-1} (D/2.7 \text{ kpc})^2 \text{ erg}$ in steady state [3]. Note that a delta-function approximation [9] as $\Phi_s(E_\gamma) \propto (c\langle n_H \rangle/4\pi D^2 K_\parallel) \int_{E_{p,th}}^\infty dE_p (E_\gamma^2 - m_p^2 c^4)^{-1/2} \sigma_{pp}(m_p c^2 + E_\gamma/K_\parallel) N_p(m_p c^2 + E_\gamma/K_\parallel)$ also produces reasonable fit to the observed $\gamma$-ray flux with the same spectral parameters $s$ and $E_{p,0}$. Here $E_{p,th} = E_p + m_p^2 c^4 / 4E_\gamma$ is the threshold pion energy and $K_\parallel \approx 0.17$ is the mean fraction of the proton kinetic energy converting to $\pi^0$.

Neutrino fluxes from $\pi^\pm$ decays, which are created in the $pp$ interactions along with $\pi^0$, can be estimated from the observed $\gamma$-ray source flux as [10][12]

$$\Phi_{\nu_s}(E) = \Phi_{\pi^0}(E) = 0.50\Phi_\gamma(E); \ \Phi_{\nu_e}(E) = 0.30\Phi_\gamma(E); \ \Phi_{\nu_x}(E) = 0.22\Phi_\gamma(E), \ (4)$$

for $s = 2.0$. For softer indices, $s > 2$, of the proton spectrum the $\nu$ fluxes decrease compared to the $\gamma$-ray flux, following the hadronic cascade theory [10]. Contributions from kaons to the neutrino fluxes are at the level of 10% or less and we ignore those.

Figure 1 shows the $\nu$ fluxes (thick curves) calculated using Eq. (4), and the $\pi^0$ decay model (thin solid curve) for the $\gamma$ ray spectrum from Ref. [3]. The data points and 2-$\sigma$ upper limits are also taken from Ref. [3]. Note that the source fluxes will be modified as they are measured by any Earth-based detector due to flavor oscillation in vacuum and in matter (MSW effect) inside the Earth, which is particularly important below $\lesssim 50$ GeV.
High-energy ν’s, created with definite flavors (να with α = e, μ, τ), arrive as coherent mass eigenstates (νi with i = 1, 2, 3) from V407 Cygni on the Earth’s surface. This is because the separation between the wave packets of different mass eigenstates

\[ d_s = L\Delta m^2/2E^2 \approx 2.4 \times 10^{-4}(L/2.7 \text{kpc})(\Delta m^2/2.4 \cdot 10^{-3} \text{eV}^2)/(E_\nu/10 \text{GeV})^{-2} \text{ cm,} \]

arising from propagation, is much smaller than the typical size of the ν wave packets from π± and μ± decays. However, the ν detectors do not have necessary energy resolution, ΔE/E, affected by matter in case of normal ν oscillations. The anti-neutrino conversion probabilities are not δ-violating phase respectively. The PMNS mixing matrix for the flavors and mass eigenstates are denoted with \( P_{\nu_\alpha \rightarrow \nu_\beta} \) and \( A^\oplus \) is the transition amplitude after propagation inside the Earth. The conversion probability at the source is neglected in the second equality, which is valid because of a negligible matter potential at the ν production site. (However an interesting situation may arise if ν’s pass through the RG, which will modify the source probability.)

For the position of V407 Cygni, Right Ascension (RA) = 315.55° and Declination (DEC) = 45.74°, ν’s travel a path length \( L = 2R_\oplus \cos \theta_n \), inside the Earth, with radius \( R_\oplus \), and a nadir angle \( \theta_n = 90° - \text{DEC} \).

We calculate \( P_{\nu_\alpha \rightarrow \nu_\beta} \) with a numerical code [14] that uses the Preliminary Reference Earth Model [15] for the density profile inside the Earth, and ν mixing parameters: \( \Delta m^2_{31} = 2.4 \cdot 10^{-3} \text{ eV}^2 \), \( \Delta m^2_{21} = 8 \cdot 10^{-5} \text{ eV}^2 \), sin²θ12 = 0.31, sin²θ23 = π/4, sin²θ13 = 0.02 and the CP violating phase \( \delta = 0 \). We consider normal ν mass hierarchy only. The total conversion probabilities are plotted in Fig. 2 for neutrinos. The upper and lower panels correspond to \( \nu_e \rightarrow \nu_\alpha \) and \( \nu_\mu \rightarrow \nu_\alpha \) conversions, respectively. The anti-neutrino conversion probabilities are not affected by matter in case of normal ν mass hierarchy and vacuum conversion formalism apply.

The most prominent features in the probability curves, the dip in the \( P_{\nu_\mu \rightarrow \nu_\tau} \) (and associated peaks/dips for other ν’s) at \( \sim 6 \text{ GeV} \), can be understood analytically from 2-flavor ν oscillation framework with 1-3 mixing [16]. For \( \theta_n \gtrsim 34° \), which is the case for V407 Cygni, ν’s do not pass through the Earth’s core. Conversions mostly take place in the mantle with an average density of \( \rho \sim 5 \text{ g cm}^{-3} \). For this density, the low MSW resonance energy is \( E_L = \Delta m^2_{12} \cos 2\theta_{12}/[2\sqrt{2}G_F \langle \rho \rangle] \approx 0.08 \text{ GeV} \),

\[ \phi_{\nu_\mu} = \phi_{\nu_\mu}^{\text{src}} \left( 1 - \frac{dL_{\nu_\mu \rightarrow \nu_\alpha}}{dE_\nu} \right) \text{ GeV}^{-1} \]

1-2 mixing is strongly suppressed for our energy range of interest, \( E_\nu \gtrsim 1 \text{ GeV} \). The dip at \( \sim 6 \text{ GeV} \) for \( P_{\nu_\mu \rightarrow \nu_\tau} \) in Fig. 2 (upper panel) corresponds to the 1-3 or high MSW resonance energy \( E_H = \Delta m^2_{31} \cos 2\theta_{13}/[2\sqrt{2}G_F \langle \rho \rangle] \approx 6 \text{ GeV} \). The width of the dip is \( 2\tan^2 2\theta_{12} E_H \approx 3.6 \text{ GeV} \). At energies \( \gtrsim E_H \), the conversion probabilities are dominated by vacuum oscillation.

The ν fluxes at a detector buried under the Earth’s surface are calculated from Eqs. (1) and (5) as

\[ \phi_{\nu_\alpha} = \phi_{\nu_\alpha}^{\text{src}} \left( 1 - \frac{dL_{\nu_\mu \rightarrow \nu_\alpha}}{dE_\nu} \right) \text{ GeV}^{-1} \]

Event rates in a ν detector depend on these fluxes and we calculate the expected number of events from a γ-ray nova outburst such as Nova 2010 in V407 Cygni next.

IV. HIGH ENERGY NEUTRINO DETECTION

The total number of neutrino-nucleon charge current (cc) interactions by να, which produce secondary leptons of type α that may be detectable, can be calculated as

\[ N_{\nu_\alpha} = \frac{N_T}{V_{\text{det}}} \int dE_\nu \int d\Omega \; V_{\text{eff}}(E_\nu, \Omega) \times \left[ \sigma_{\nu_e}(E_\nu)\phi_{\nu_\mu}^{\text{det}}(E_\nu, \Omega) + \sigma_{\nu_\mu}(E_\nu)\phi_{\nu_\tau}^{\text{det}}(E_\nu, \Omega) \right], \]

below the energy (\( \sim 100 \text{ TeV} \)) at which ν absorption inside the Earth is negligible (see e.g. [14]). Here \( t \) is the
duration of the $\nu$ outburst, $V_{\text{det}}$ is the instrumented detector volume and $N_T$ is the total number of nucleons in that volume. The effective detector volume $V_{\text{eff}} \approx V_{\text{det}}$ for contained events at low energies. At very high energies $V_{\text{eff}} > V_{\text{det}}$, specially for $\nu_\mu$ because secondary $\mu$'s that are produced far from the physical detector can propagate inside due to their long range. We parameterize the $cc$ cross-sections in the $\sim 10 - 100$ GeV range as

$$
\sigma_{\nu}^{cc} = 7.3 \cdot 10^{-39} (E_\nu/\text{GeV}) \, \text{cm}^2
$$

$$
\sigma_{\bar{\nu}}^{cc} = 3.8 \cdot 10^{-39} (E_\nu/\text{GeV}) \, \text{cm}^2.
$$

(8)

For a detector such as the IceCube Deep Core [18], with a 10 Mt fiducial volume, the total number of nucleons inside $V_{\text{det}}$ is $N_T = 10^{13} N_A$, where $N_A$ is the Avogadro's number. All astrophysical sources are essentially point sources in $\nu$'s and one needs to carry out integration over $E_\nu$ only in Eq. (7). The lower range $E_{\nu, \text{th}}$ depends on the energy of the secondary lepton at the detection threshold. The average muon energies are $\approx 0.52 E_\nu$ for neutrinos and $\approx 0.66 E_\nu$ for anti-neutrinos in the range $E_\nu \sim 10 - 100$ GeV. For a rough estimate of $N_{\nu_\mu}$ we assume a common $E_{\nu, \text{th}}$ for both $\nu$'s and $\bar{\nu}$'s for the same lepton energy. Here we consider $\nu_\mu$'s and $\bar{\nu}_\mu$'s only, as the detection prospect of $\sim 5$ GeV muon tracks is better than the showers created by other flavors. For $t = 15$ day, the duration of the $\gamma$ ray outburst from the nova in V407 Cygni, and for $V_{\text{eff}} = V_{\text{det}}$ we calculate

$$
N_{\nu_\mu} \approx 0.5^{+4.4}_{-0.4}, \quad N_{\bar{\nu}_\mu} \approx 0.3^{+2.3}_{-0.2}, \quad 10 \leq E_\nu/\text{GeV} \leq 100,
$$

using the $\pi^0$ model for $\gamma$ rays with the best-fit parameters, and their $\pm 1\sigma$ variation.

V. DISCUSSION AND IMPLICATIONS

We have calculated $\sim 0.1$-100 GeV $\nu$ fluxes from the $\gamma$ ray Nova 2010 in the symbiotic binary V407 Cygni, from $\pi^\pm$ decays while assuming that the $\gamma$ rays detected by the Fermi LAT are from $\pi^0$ decays. Detection of these $\nu$'s in the $\sim 10$-100 GeV range depends critically on the detector’s angular resolution and uncertainty in the atmospheric $\nu$ background. For the IceCube Deep Core the atmospheric $\nu_\mu + \bar{\nu}_\mu$ events during 15 days of the $\gamma$-ray nova is $\sim 60$ for $\Delta \theta = 10^\circ$ following Ref. [14]. Future large scale detectors with a better angular resolution, such as $\sim 3^\circ$ in the Super-Kamiokande [19], can reduce the background roughly by an order of magnitude to improve the signal-to-noise ratio, as well as measure background more precisely to reduce uncertainty. Moreover, a similar $\gamma$-ray nova in the nearest symbiotic nova system AG Peg ($D \sim 0.6$ kpc), among a dozen known, would improve the signal by a factor of $\sim 20$. More energetic $\gamma$ ray nova is also possible. The kinetic energy release in 2006 nova outburst in a nearby ($D \sim 1.4$ kpc) symbiotic system RS Ophiuchi is inferred to be an order of magnitude higher than in V407 Cygni 2010 nova [3].

The rate of classical novae in our galaxy is $\sim 30$ year$^{-1}$ in all binary systems [21]. The rate of novae among $\sim 100$ known symbiotic binary systems [22], which seem to require for particle acceleration and $\gamma$ ray production, is more uncertain, once every few years. Following up of future novae with $\nu$ detectors may establish a rate independent of the $\gamma$ ray instruments and answer the question of a hadronic or leptonic origin of the $\gamma$ rays from them. Non-detection of $\nu$'s from $\gamma$ ray nova can be used to constrain the emission models and system parameters as well.

While astrophysical $\gamma$ ray sources have been hypothesized as potential TeV - PeV $\nu$ sources, mostly from $\pi$'s and $K$'s created in $p\gamma$ interactions, relatively low-energy ($\sim 10$ GeV) $\nu$ sources which are simultaneous $\gamma$ ray sources were thought to be rare. Surprising discovery of the nova in V407 Cygni by the Fermi LAT adds a new $\nu$ source candidate class where, like supernova remnants, $pp$ interactions may produce both $\gamma$ rays and $\nu$'s.

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