Regression Analysis of the Heat Balance Thickness of a Blast Furnace Hearth Lining

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Abstract. The reasonable structure and operation of a hearth can reduce the probability of serious depression and erosion of the lining of a blast furnace (BF). The temperature of the hearth lining is high inside and low outside. Theoretically, there is a heat balance state in the hearth. A three-dimensional computational fluid dynamics (CFD) calculation model that can simulate the theoretical heat balance state of the lining for different hearth design structures and operating parameters was established in this paper. Based on the calculation model, the response surface method was used to study the regression relationship between different factors and the thickness of the lining. In addition, the sensitivity of each factor to the thickness in a state of heat balance was evaluated. The analysis results showed that the blast furnace volume utilization factor and tapping temperature had a greater impact on the theoretical heat balance thickness of the hearth lining. These two factors had an obvious interaction. With the increase in the blast furnace volume utilization factor, the sensitivity of the heat balance thickness to the tapping temperature gradually decreased. The cooling water flow rate had a small effect on the heat balance thickness. The theoretical basis for the optimization of the hearth structure and the improvement of the operation method was provided in this paper.

Keywords: Blast furnace hearth, heat balance thickness, response surface method, computational fluid dynamics, solidification and melting model.

1. Introduction
The internal state of a BF hearth cannot be directly explored and is often regarded as a "black box". The flow and heat transfer state of hot iron in the hearth directly affect the safety and life of the BF [1]. For many years, scholars have been working to model the ironmaking process in BFs. The research methods include industrial tests, laboratory tests and numerical simulations [2-4]. The implementation of industrial tests is difficult and expensive. At present, it is common to disassemble and study BFs after the end of service [5, 6]. Laboratory tests can provide information about some phenomena inside BFs, but these tests are often too simple to ensure the accuracy of the results. With the development of computer hardware and numerical calculations, computer simulations have become a feasible alternative method. Scholars have carried out many related studies and proposed different models for
numerical calculations.

Dash et al. simulated the shear stress caused by the flow of molten iron on the inner surface of a hearth by solving the Navier-Stokes and Darcy equations [7]. The study showed that as the length of the taphole increases, the peak stress gradually decreases. They also found that 15° is the optimum tap angle, and the shear stress of the corresponding tap wall is the smallest. Cheng et al. solved the three-dimensional turbulent Navier-Stokes equation and energy transport equation with FLUENT software, simulated the flow and heat transfer process of hot iron in a BF hearth, and evaluated the accuracy of the model with No. BHP 5 BF [8]. Guo et al. established a three-dimensional CFD model of coupled heat transfer that considers the flow of molten iron [9]. The corrosion morphology and refractory properties of the hearth lining have a great influence on the temperature distribution of the hearth.

The above research studied the flow and heat transfer state of molten iron given the boundary of the hearth lining. This paper focuses on the heat balance state of the hearth lining, that is, the critical state of slagging on the thermal surface of the lining. In the initial stage of hearth service, the inner lining mainly relies upon its own resistance to erosion by molten iron and erodes gradually. With increasing amount of erosion, the working thickness of a hearth decreases. The inner lining is determined by its ability to protect itself from slag and its resistance to erosion after the hot metal condenses due to external cooling. Therefore, it is of theoretical and technical significance to study the critical morphology and thickness of slagging on the hot surface of the lining. In this paper, a reverse solidification method of the theoretical heat balance state of the hearth lining is proposed. Based on the solidification and melting model, a heat balance state calculation model of the hearth lining containing hot iron flow is established, and the model is solved to find the heat balance state of the hearth lining for different hearth structures and operating parameters. Response surface methodology was used to study the regression relationship between various factors and the theoretical heat balance thickness of the hearth lining and to evaluate the sensitivity of various factors to the theoretical heat balance thickness of the hearth lining.

2. Heat balance model of the hearth lining

2.1. Calculation principle of the theoretical heat balance state of the hearth lining

BF ironmaking is a continuous process in which hot iron produced in the BF continuously converges into the hearth and is discharged from the taphole. The process follows the laws of mass conservation, momentum conservation and energy conservation. Under forced cooling conditions outside the hearth, the temperature of the molten iron near the lining decreases, and if it is below 1150 °C, it condenses and adheres to the hot surface of the lining. In this paper, the solidification phenomenon of hot iron in the hearth is simulated by using the FLUENT solidification melting model. If hot iron is an incompressible viscous fluid, then the calculation is performed using the k-epsilon turbulence model. Since the radiated heat proportion is less than 5%, it is not considered in the calculation model. The initial properties of all materials in the calculation model are set as those of hot iron, and the properties of hot iron (thermal conductivity) in the solidified state are changed to the properties of the original refractory material corresponding to the position during the iterative process. The position of the 1150 °C isothermal surface does not change after repeated iterations of the calculation, which is the theoretical heat balance position of the hearth lining, and the distance from this position to the cold surface of the hearth lining is the theoretical heat balance thickness of the hearth. Based on this model, a calculation is constructed to simulate the theoretical heat balance state of the hearth lining.

2.2. Physical model

Taking the hearth of No. 5 BF as the research object, according to the calculation principle, the initial properties of all refractory materials are set to those of hot iron. To improve the calculation efficiency, the bottom graphite brick layer, microporous carbon brick with a thickness of 200 mm around the hearth, and the sidewall carbon brick are reserved in the calculation model, as shown in Fig. 1. The taphole mud is solid while retaining the adjacent carbon brick. To achieve a fully developed state of
hot iron as it flows through the original inlet, the entire inlet of the model is extended by 1000 mm. The physical properties of the hot iron used in the calculation model and the thermal conductivity of each refractory material are shown in Table 1 and Table 2, where \( T \) is the temperature, indicating that the property value changes with the temperature.

**Table 1.** Thermal conductivity of the solid materials (W/(m·K))

| Material                        | Thermal conductivity (W/(m·K)) |
|--------------------------------|--------------------------------|
| Graphite brick                 | 46.61-0.01342\( T \)          |
| Microporous carbon brick       | 8.88+0.00440\( T \)           |
| Carbon composite brick         | 17.58-0.00499\( T \)          |
| Taphole mud                    | 2                              |
| Solidified iron                | 3                              |
| Packing layer                  | 12                             |
| Cooling wall                   | 35                             |

**Table 2.** Physical properties of hot-iron

| Property               | Density \(/	ext{kg} \cdot \text{m}^{-3}\) | Specific heat capacity \(/	ext{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}\) | Heat conductivity \(/	ext{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}\) | Viscosity \(/	ext{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}\) | Melting heat \(/	ext{J} \cdot \text{kg}^{-1}\) | Solidification temperature \(/	ext{°C}\) | Melting temperature \(/	ext{°C}\) |
|------------------------|---------------------------------------------|-------------------------------------------------|-------------------------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
|                        | 6700                                        | 756                                             | 0.0158T                                         | 0.007                           | 103343                          | 1150                            | 1300                            |

The initial state of zones A–D in Fig. 1 is hot iron, where zone A is a deadman region with a porosity of 0.3; zone B is a hot iron coke-free zone; and zone C is the carbon brick zone of the original sidewall. The physical parameters of the solidification part of zone B and zone C are changed to the physical parameters of the carbon brick of the sidewall of the hearth. Zone D is the original microporous carbon brick zone, and the physical property parameters of the solidified part in the zone are changed to those of microporous carbon brick. The above parameters are set and changed using the FLUENT UDF function.

**Figure 1.** Schematic diagram of the calculation model.

2.3. **Physical model**

The velocity inlet condition is adopted at the inlet of the calculation model, and its value is related to the BF volume utilization coefficient. The calculation formula is:

\[
v_{\text{inlet}} = \frac{m}{\rho_{\text{iron}} S_{\text{inlet}}}
\]

where \( m \) is the mass flow rate of the hot iron inlet; \( \rho_{\text{iron}} \) is the density of hot iron; and \( S_{\text{inlet}} \) is the entrance area. The inlet temperature of the model is calculated according to the tapping temperature, the outlet is selected as the pressure outlet, and the pressure value is the standard atmospheric pressure. The sidewall and bottom of the heart are the convection heat transfer boundaries, and the equivalent
convection heat transfer coefficients of the sidewall and bottom are \( h_c \) and \( h_d \), respectively. The equivalent convection heat transfer coefficient of the sidewall of the hearth is related to the cooling specific surface area, cooling water flow rate and packing thickness. Formula (2) is an empirical formula for calculating the convection heat transfer coefficient on the surface of a water pipe with a cooling wall. Formulas (3) – (5) are the calculation formulas of the equivalent convection heat transfer coefficient on the central surface of the cooling wall water pipe, the equivalent convection heat transfer coefficient on the cold surface of the packing layer and the equivalent convection heat transfer coefficient on the cold surface of the sidewall, respectively. \( h_d \) is taken as 40 W/(m\(^2\)·K), the cooling water temperature is taken as 25 °C~40 °C, and the other boundary is set as an adiabatic boundary.

\[
h_w = 208.8 + 47.5 \nu_w
\]

\[
h_1 = \gamma h_w
\]

\[
h_2 = \frac{r_2}{r_1} \frac{h_2 \lambda_1}{\lambda_1 + h_1 r_1 \ln \left(\frac{r_1}{r_2}\right)}
\]

\[
h_2 = \frac{r_2}{r_3} \frac{h_2 \lambda_2}{\lambda_2 + h_2 r_2 \ln \left(\frac{r_2}{r_3}\right)}
\]

where \( \nu_w \) is the cooling water flow rate; \( \gamma \) is the cooling specific surface area; \( r_2 \), \( r_3 \) are the radii of the outer and inner walls of the packing layer; \( \lambda_1 \) is the heat conduction system of the cooling stave; and \( \lambda_2 \) is the thermal conductivity of the packing layer.

3. Multifactor Test Design of the Heat Balance Thickness of the Furnace

3.1. Value range and code of each factor

In this paper, seven factors related to the theoretical heat balance thickness of the hearth lining are selected: specific cooling surface area (\( A \)), cooling water flow rate (\( B \)), cooling water temperature (\( C \)), packing layer thickness between the cooling stave and lining (\( D \)), BF volume utilization coefficient (\( E \)), tapping temperature (\( F \)), and thermal conductivity coefficient of the carbon brick sidewall of the hearth (\( G \)). The relationship between the above seven factors and the theoretical heat balance thickness of the hearth lining was studied by the BBD method. Because the value range and dimension of each factor are different, to facilitate data processing, the high value of each factor is coded as 1, the low value is coded as -1, and the intermediate value is coded as 0. The values of each factor are shown in Table 3.

| Random parameters \( \times 10^2 \) | Coded value |
|-----------------------------------|------------|
|                                    | -1        | 0        | 1        |
| \( A \) (\( \times 10^2 \))        | 0.6       | 1.0      | 1.4      |
| \( B/(\text{m} \cdot \text{s}^{-1}) \) | 1.50      | 2.75     | 4.00     |
| \( C/°C \)                         | 25.0      | 32.5     | 40.0     |
| \( D/\text{mm} \)                  | 0         | 50       | 100      |
| \( E/(\text{t} \cdot \text{m}^3 \cdot \text{d}^{-1}) \) | 2.50      | 3.00     | 3.50     |
| \( F/°C \)                         | 1480      | 1500     | 1520     |
| \( G/(\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}) \) | 10        | 20       | 30       |
3.2. Calculation of the heat balance state of the hearth
First, the simulation calculation is carried out for the case in which all parameters are coded 0. To better track the position of the theoretical heat balance surface and improve the calculation accuracy and model convergence, the hot iron region (fluid zone) and the region around the taphole are discretized by using a dense polyhedral mesh. After a grid independence analysis, there are 2553980 computational meshes, including 1719577 meshes in the fluid zone and 834403 meshes in the solid zone. The local temperature distribution of the hearth is shown in Fig. 2. Under the action of externally forced cooling, the overall temperature of the hearth has a distribution pattern of internal high and external low values. The theoretical heat balance surface of the hearth lining is shown in Fig. 3, and the overall shape of the heat balance surface is that of a bowl.

![Figure 2. Temperature distribution of the hearth.](image)

3.3. Response index setting of the regression model
The intersection line between the horizontal plane at the height of the taphole and the heat balance surface of the hearth is taken as the evaluation line, as shown in Fig. 3. The average distance from the evaluation line to the cold surface of the hearth lining is the theoretical heat balance thickness, expressed in $Y$, and used as the response index. To eliminate the influence of the taphole on the calculation result, the data in the direction of the taphole along the Z-axis, plus or minus 45 degrees, are not included in the result. Fifty-seven test protocol samples were designed by the BBD method with the use of Design-Expert software. The FLUENT UDF function is used to adjust the values of the factors according to the parameters of the samples.

![Figure 3. Shape of heat balance surface.](image)

4. Response surface analysis of the heat balance thickness of the hearth

4.1. Principles of the response surface methodology regression analysis
The method of determining the quantitative relationship between samples and their corresponding response values is called regression analysis. The response surface method (RSM) is a common regression analysis method that can be used to directly describe the relationship between samples and
response values. If a sample contains \( n \) factors, then the independent variable \( y = y(x) \) to be evaluated can be denoted as an \( n \)-dimensional vector. For a sample \( x^{(i)} \), its corresponding response value \( y^{(i)} \) can be obtained through experimental or numerical tests. After completing a reasonable test design, and if the number of tests is sufficient, then the approximate function of the function \( y = y(x) \) can be obtained by using the undetermined coefficient method, as shown in formula (6). The regression result is related to the fitting degree of the actual values and the form of the approximate function. In general, first-order or second-order fitting can meet engineering requirements. The specific form is shown in formula (7).

\[
\tilde{y} = f(x) = \sum_{j=0}^{n} \alpha_j x_j \tag{6}
\]

\[
\tilde{y} = \alpha_0 + \sum_{j=1}^{n} \alpha_j x_j + \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} x_i x_j \tag{7}
\]

where \( \alpha_0 \) is a constant coefficient to be determined; \( \alpha_j \) is the coefficient of the primary term to be determined; and \( \alpha_{ij} \) is the coefficient of the quadratic term to be determined.

### 4.2. Response surface model of the heat balance thickness of the hearth

The significance of the regression model and its terms can be determined by the F value and the p value; the larger the F value, the smaller the p value, and the more significant the corresponding term. When the p value of the model term is less than 0.05, the model term is significant and should be retained. When the p value of the model term is greater than 0.10, the model term is not significant and can be simplified. The calculation results are analyzed by the analysis software Design Expert. The results show that the primary terms \( D, E, F, G \), the cross-term \( EF \) and the quadratic term \( E^2 \) of the model have a significant impact on \( Y \). The influence of the primary terms \( A \) and \( C \) and the quadratic terms \( D^2, F^2 \) and \( G^2 \) on \( Y \) is significant. The effect of all the terms containing factor \( B \) is not significant, and the effect of the cooling water flow rate on the theoretical heat balance thickness is small when the cooling water flow rate is within the 1.5 m/s~4.0 m/s range.

The variance analysis results of the theoretical heat balance thickness regression model of the hearth lining are shown in Table 4. The F value of the regression model is 280.35, and the p value is less than 0.0001, indicating that the regression model is very significant. According to the value of F, the sensitivity of each factor to the heat balance thickness of the hearth lining is: \( E > F > G > D > C > A \).

| Project | Sum of square | Degree of freedom | Mean square | F value | p value |
|---------|--------------|-------------------|-------------|---------|---------|
| Model   | 1.13×10^5    | 11                | 10278.79    | 280.35  | <0.0001 |
| A       | 260.04       | 1                 | 260.04      | 7.09    | 0.0107  |
| C       | 376.04       | 1                 | 376.04      | 10.26   | 0.0025  |
| D       | 1908.17      | 1                 | 1908.17     | 52.04   | <0.0001 |
| E       | 55200.04     | 1                 | 55200.04    | 1505.54 | <0.0001 |
| F       | 42336.00     | 1                 | 42336.00    | 1154.68 | <0.0001 |
| G       | 3174.00      | 1                 | 3174.00     | 86.57   | <0.0001 |
| EF      | 2380.50      | 1                 | 2380.50     | 64.93   | <0.0001 |
| D^2     | 357.57       | 1                 | 357.57      | 9.75    | 0.0031  |
| E^2     | 5805.01      | 1                 | 5805.01     | 158.33  | <0.0001 |
| F^2     | 357.57       | 1                 | 357.57      | 9.75    | 0.0031  |
| G^2     | 436.81       | 1                 | 436.81      | 11.91   | 0.0012  |
| Residual| 1649.91      | 45                | 36.66       | -       | -       |
| Sum     | 1.14×10^5    | 56                | -           | -       | -       |
The p value of the cross-term $EF$ is less than 0.0001, indicating that the interaction between the BF volume utilization factor $E$ and tapping temperature $F$ is significant. The influence of the interaction of $E$ and $F$ on the heat balance thickness is shown in Fig 4. As shown in Fig. 4(a), the BF volume utilization coefficient and the tapping temperature are negatively correlated with the heat balance thickness within the range of test data. As shown in Fig. 4(b), when the BF volume utilization coefficient is 2.50 t/m$^3$·d, the tapping temperature increases from 1480 °C to 1520 °C, and the heat balance thickness decreases from 777 mm to 656 mm. When the BF volume utilization coefficient is 3.50 t/m$^3$·d, the tapping temperature increases from 1480 °C to 1520 °C, and the heat balance thickness decreases from 645 mm to 595 mm. When the BF volume utilization coefficient and the tapping temperature decrease, the total heat generated in the hearth is reduced, the hot iron flow rate is reduced, the heat balance surface moves toward the center of the hearth, and the heat balance thickness increases. As shown in Fig. 4(b), when the BF volume utilization coefficient decreases, the variation amplitude of the heat balance thickness caused by the change of the tapping temperature further increases, which shows that simultaneously reducing the BF volume utilization coefficient and tapping temperature has a very significant effect on increasing the heat balance thickness. The volume utilization coefficient of the BF is related to the economic benefit of the enterprise, and the tapping temperature is related to the activity of the BF hearth and the subsequent transport of hot iron. Therefore, in actual BF production operations, these problems should be fully considered to ensure that the BF is active and find the balance between economic benefits and BF working life.

![Figure 4](image)

**Figure 4.** Influence of the interaction of $F$ and $E$ on $Y$.

Fig. 5 shows the relationship between other significant factors ($p<0.05$) and the heat balance thickness of the hearth lining. As shown in Fig. 5(a), when the thermal conductivity of the carbon brick increases from 10 W/m·K to 30 W/m·K, the thermal balance thickness increases from 625 mm to 648 mm. The heat balance thickness increases with increasing thermal conductivity of the carbon brick on the sidewall of the hearth. After the thermal conductivity of the carbon brick increases, the cooling strength of the hearth increases, and the heat exchange between the hot iron and the hearth increases. In addition, when the thermal conductivity of the carbon brick increases to a certain extent, the rate of increase of the heat balance thickness gradually slows, which indicates that decreasing sensitivity of the heat balance thickness to the variation of the thermal conductivity of the carbon brick. Continuing to improve the thermal conductivity of carbon bricks has an increasingly smaller effect on increasing the thickness of the heat balance.

As seen from Fig. 5(b), after the thickness of the packing layer is increased from 0 mm to 100 mm, the heat balance thickness decreases from 656 mm to 638 mm, and the heat balance thickness is negatively correlated with the thickness of the packing layer. Since the thermal conductivity of the packing layer is relatively small, the convection heat transfer of the sidewall of the hearth decreases when the thickness of the packing layer increases, and the cooling strength of the hearth is reduced.

As seen from Figs 5(c) and (d), the heat balance thickness is inversely proportional to the cooling water temperature and directly proportional to the specific cooling surface area. When the cooling
water temperature rises from 25 °C to 40 °C, the heat balance thickness decreases from 646 mm to 638 mm. As the cooling specific surface area increases from 0.6 to 1.4, the heat balance thickness increases from 638 mm to 645 mm. The specific cooling surface area is related to the cooling stave structure, and the cooling water temperature is greatly influenced by the ambient temperature.

Figure 5. Relationship between single factor and heat balance thickness.

5. Conclusions
On the basis of understanding the heat balance state of the hearth lining and the theoretical and technical significance of the critical slagging surface, calculations of the flow and a model of the heat balance state of the hearth lining are established based on the solidification and melting principle. The simulation results for the hearth of the No. 5 BF show that the heat balance of the critical slagging surface has the shape of a bowl. The critical thickness of the sidewall lining of the hearth is 581 mm ~ 802 mm.

Response surface methodology was used to process the data. The results show that the degree of influence of various factors on the theoretical heat balance thickness of the hearth lining, in decreasing order, is: BF volume utilization coefficient, tapping temperature, thermal conductivity of carbon bricks on the sidewall of the hearth, thickness of the packing layer, cooling water temperature, and specific cooling surface area. The interaction between the BF volume utilization coefficient and tapping temperature is obvious. With increasing BF volume utilization coefficient, the theoretical heat balance thickness is less sensitive to the tapping temperature. The flow rate of cooling water has little effect on the thickness of the heat balance.

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