Magnetized Particle Motion Around Black Hole in Braneworld

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We investigate the motion of a magnetized particle orbiting around a black hole in braneworld placed in asymptotically uniform magnetic field. The influence of brane parameter on effective potential of the radial motion of magnetized spinning particle around the braneworld black hole using Hamilton-Jacobi formalism is studied. It is found that circular orbits for photons and slowly moving particles may become stable near \( r = 3M \). It was argued that the radii of the innermost stable circular orbits are sensitive on the change of brane parameter. Similar discussion without Weil parameter has been considered by de Felice et all in Ref. 1, 2.

Keywords: braneworld models; magnetized particle’s motion

PACS Nos.: 04.50.-h, 04.40.Dg, 97.60.Gb.

1. Introduction

One of the exact solutions of gravitational field equations in the braneworld was obtained by Dadhich et al 3 (so called DMPR solution). This solution is the analog of solution of Reissner-Nordström, which describes the gravitational field of black hole with electric charge \( Q \).

Observational possibilities of testing the braneworld black hole models at an astrophysical scale have intensively discussed in the literature during the last years, for example through the gravitational lensing, 5 6 7 9 the motion of test particles 10 and the classical tests of general relativity (perihelion precession, deflection of light and the radar echo delay) in the Solar system 11. The motion of charged test particle around black hole in braneworld in the presence of external uniform magnetic field have been studied in Ref. 12.

A braneworld corrections to the charged rotating black holes and to the perturbations in the electromagnetic potential around black holes are studied in Ref. 13 14 15. In Ref. 16 authors considered the stellar magnetic field configurations of relativistic stars with brane charge.

Here we consider motion of magnetized particle around non-rotating DMPR black hole in braneworld and motion of magnetized particle in axial symmetric gravitational field. Innermost stable circular orbits 17 18, storage and release of energy of particle have been considered taking account of magnetic coupling parameter \( \beta \) in Schwarzschild spacetime (i.e. \( Q^* = 0 \)). Following the idea of the paper Ref. 1 19 we assume that the black hole is immersed in the external uniform magnetic
field. We plan to study the influence of brane parameter and magnetic field on stable circular orbits of magnetized particle around compact object in braneworld. We obtain the equation of motion of the particles using Hamilton-Jacobi formalism.

The paper is organized as follows. In section 2 we introduce the scenario for our model, specifying the spacetime metric and the external magnetic field. In section 3 we discuss the electromagnetic interaction of a magnetized particle in a curved spacetime. In section 4 we solve the radial equation, obtained by means of the Hamilton-Jacobi formalism, in order to find the admissible circular orbits. Section 5 is devoted to the study of the inner stable circular orbits, with some numerical estimates and possible astrophysical implications. Finally, we conclude our results in section 6.

We use in this paper a system of units in which \( c = 1 \), a space-like signature \((-+,+,+,+)\) and a spherical coordinate system \((t, r, \theta, \varphi)\). Greek indices are taken to run from 0 to 3, Latin indices from 1 to 3 and we adopt the standard convention for the summation over repeated indices. We will indicate vectors with bold symbols (e.g. \( B \)).

2. Electromagnetic Field Around Black Hole

Consider a black hole immersed in external asymptotically uniform magnetic field \( B \). A particle of mass \( m \), carrying a magnetic dipole moment \( \mu \), is assumed to move around the black hole, following a circular orbit at equatorial plane. The magnetic field is taken perpendicular to the orbital plane.

Spacetime metric of the DMPR black hole solution in braneworld takes form

\[
ds^2 = -A^2 dt^2 + H^2 dr^2 + r^2 d\theta^2 + r^2 \sin^2 \varphi,\]

where

\[
A^2 = H^{-2} = \left(1 - \frac{2M}{r} + \frac{Q^*}{r^2}\right),
\]

\( Q^* \) is the bulk tidal charge and \( M \) is the total mass of the central black hole. The polar axis was choose along the direction of \( B \). So we may assume, the particle motion was happening in the equatorial plane.

The exact solution for black hole in braneworld immersed in external uniform magnetic field is considered in Ref. [12]. The potential of the electromagnetic field around black hole in braneworld has the form

\[
A^\mu = \hat{A}^\mu + a^\mu,
\]

where \( a^\mu \) proportional to the angular momentum of black hole \( a \) as

\[
a^\mu = \frac{\pi a B}{6M^2} (1, 0, 0, 0).
\]

Since we consider the nonrotating black hole in braneworld, the second term of the expression vanishes. In this paper we will consider only first term of the
potential of the electromagnetic field around black hole in braneworld, which has
the following form
\[ A_\mu = \frac{1}{2} \delta^{\mu}_\varphi B r^2 \sin^2 \theta. \] (4)

Nonvanishing components of the Faraday electromagnetic tensor
\[ F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \] reads:
\[ F_{r \varphi} = B_0 r \sin^2 \theta, \]
\[ F_{\theta \varphi} = B_0 r^2 \sin \theta \cos \theta. \] (5)

Expressions (5) is the tensors of electromagnetic field. They are identical with the
tensors of electromagnetical field in Schwarzschild metric.

3. The equation of motion

The Hamilton-Jacobi equation
\[ g^{\mu \nu} \left( \frac{\partial S}{\partial x^\mu} - qA_\mu \right) \left( \frac{\partial S}{\partial x^\nu} - qA_\nu \right) = -m^2 + mD^{\mu \nu} F_{\mu \nu} , \] (6)

for motion of the charged test particles with mass \( m \) is applicable as a useful com-
putational tool only when separation of variables can be effected. Equation (6) can
be considered as the Hamilton-Jacobi equation for the particle, interacting with
external electromagnetic field. \( D^{\mu \nu} \) is polarization tensor and taken to be pro-
portional to the particle spin,
\[ D^{\mu \nu} = \frac{q}{m} S^{\mu \nu}, \] (7)

where \( S^{\mu \nu} \) is the antisymmetric spin tensor.

Consider now particle with zero charge \( (q = 0) \) preserving a magnetic dipole
moment \( \vec{\mu} \), due to some inertial electrodynamical structure.

Then the Hamilton-Jacobi equation (6) can be read as:
\[ H^2 \mathcal{L}^2 + \frac{1}{r^2 \sin^2 \theta} \mathcal{L}^2 + A^2 \left( \frac{\partial S_\varphi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial S_\varphi}{\partial \theta} = -m^2 + 2m \mu B_0 K[\lambda]. \] (8)

The magnetized particle interacts with the exterior magnetic field \( B \) via the
\( D \cdot F \) term in (8). In the present case \( D^{\mu \nu} \) will be proportional only to \( \vec{\mu} \). So, using
the similarity with the spin theory one can define define
\[ D^{\mu \nu} = \eta^{\mu \nu \rho \lambda} u_\rho u_\lambda, \] (9)

where \( u^\mu \) is the 4-velocity and \( \mu^\lambda \) is the particle magnetic moment 4-vector. In our
coordinate \( D^{\mu \nu} \) is not constant. Such an equation has to be found and solved along
with (8). An alternative approach is to exploit the fact that the interaction term
\( D \cdot F \) is a scalar.

From equation (9) one can rewrite in the rest frame of a fiducial comoving
observer \( u_\nu D^{\mu \nu} \) is totally transverse, namely
\[ D^{\mu \nu} u_\nu = 0. \] (10)
4. The circular orbits

Our goal in this section is to explore orbital motion around the compact object in braneworld, that is influences brane parameter to the radii of stable orbits using equation (8). For an equatorial orbit one can write $\theta = \pi/2$ and consequently we have $p_\theta = 0$. The spacetime symmetries are preserved by the axisymmetric configuration of the magnetic field and therefore they still allow for two conserved quantities: $p_\phi = L$ and $p_t = -E$. These are energy and angular momentum of particle respectively.

Equations of motion (8) take the following form:

$$2M \frac{dt}{d\sigma} = \left(1 - \frac{1}{\rho} + \frac{\hat{Q}^*}{\rho^2}\right)^{-1} e,$$

$$4M^2 \left(\frac{d\rho}{d\sigma}\right)^2 = e^2 - V(\rho, \lambda, e, \hat{Q}^*),$$

$$2M \frac{d\phi}{d\sigma} = \frac{\lambda}{\rho^2 \sin^2 \theta},$$

here $\hat{Q}^* = Q^*/4M^2$ and $V(\rho, \lambda, e, \hat{Q}^*)$ is the effective potential and it was described in the following form:

$$V(\rho, \lambda, e, \hat{Q}^*) = \left(1 - \frac{1}{\rho} + \frac{\hat{Q}^*}{\rho^2}\right) \left(1 + \frac{\lambda^2}{\rho^2} - \beta K[\lambda_\alpha]\right).$$

In equations (14), (15) and (16) $\sigma$ is the proper time of the particle. In these expressions we introduced the following notations:

$$\rho = \frac{r}{2M}, \quad \frac{e}{m} = \frac{E}{m}, \quad \frac{\lambda}{2M m} = \frac{L}{2M m}, \quad \beta = \frac{2\mu B_0}{m},$$

and

$$K[\lambda_\alpha] = h^\psi \left(1 - \frac{1}{\rho} + \frac{\hat{Q}^*}{\rho^2}\right) = h^\psi \frac{\Delta}{\rho^2},$$

$$4 \mu B_\alpha B^\alpha,$$
here $h^\psi = (1 - 1/\rho + \dot{Q}^*/\rho^2 - 4M^2\Omega^2\rho^2)^{-\frac{1}{2}}$.

The parameter $\beta$ is responsible for the intensity of the magnetic interaction. In this paper we suppose $\beta > 0$. And now the circular orbits are obtained from

$$\frac{d\rho}{d\tau} = 0, \quad \text{and} \quad \frac{\partial V(\rho, e, \ell, \dot{Q}^*)}{\partial \rho} = 0. \quad (18)$$

Using these equations we get

$$\beta(\rho, \lambda, e, \dot{Q}^*) = \frac{1}{K[\lambda_\alpha]} \left[ 1 - \frac{\lambda^2}{\rho^2} - \frac{e^2\rho^2}{\rho^2 - \rho + \dot{Q}^*} \right]. \quad (19)$$

Moreover, using equation (15) and (18) one can write for the first derivative of potential:

$$\frac{\partial V}{\partial \rho} = \left( 1 - \frac{1}{\rho} + \frac{\dot{Q}^*}{\rho^2} \right) K[\lambda_\alpha] \frac{\partial \beta}{\partial \rho}. \quad (20)$$

One can write (19) and (20) in the following form:

$$\beta(\rho, \lambda, e, \dot{Q}^*) = \left( 1 - \frac{\lambda^2}{\rho^2} - 4M^2\Omega^2\rho^2 \right)^{\frac{1}{2}} \left( 1 + \frac{\lambda^2}{\rho^2} - \frac{e^2\rho^2}{\rho^2 - \rho + \dot{Q}^*} \right), \quad (21)$$

and

$$\frac{\partial V}{\partial \rho} = \left( 1 - \frac{1}{\rho} + \frac{\dot{Q}^*}{\rho^2} \right)^2 \left( 1 - \frac{1}{\rho} + \frac{\dot{Q}^*}{\rho^2} - 4M^2\Omega^2\rho^2 \right)^{-\frac{1}{2}} \frac{\partial \beta}{\partial \rho}. \quad (22)$$

using the equation (17) for the angular velocity of particles together with two constant of motion $E$ and $L$,

$$\Omega = \frac{\lambda}{2Me\rho^3}. \quad (23)$$

one can rewrite (21) in following form:

$$\beta(\rho, \lambda, e, \dot{Q}^*) = \left( \frac{\rho^2}{\rho^2 - \rho + \dot{Q}^*} - \frac{\lambda^2}{e^2\rho^2} \right)^{\frac{1}{2}} \left( 1 + \frac{\lambda^2}{\rho^2} - \frac{e^2\rho^2}{\rho^2 - \rho + \dot{Q}^*} \right). \quad (24)$$

Since $h^\psi \geq 0$, $\partial V/\partial \rho$ and $\partial \beta/\partial \rho$ have the same sign, thus, for a given value of the parameter $\beta$, the determination of circular orbits as well as the analysis of their stability reduces to the solution of the following set of equations:

$$\beta = \beta(\rho, e, \ell, \dot{Q}^*), \quad \frac{\partial \beta(\rho, e, \ell, \dot{Q}^*)}{\partial \rho} = 0. \quad (25)$$

This is a system of two equations with five unknowns $\beta$, $\rho$, $e$, $\ell$, and well parameter $\dot{Q}^*$, so its solutions can be parametrized in terms of any two of the five independent variables. We will use as free parameters the magnetic coupling, and the orbital radius $\rho$. Our aim is then to find the angular momentum $\lambda$ and the energy $e$ of the particle as functions of $\beta$ and $\rho$ for the innermost stable circular orbits, i.e. stable orbits near the critical radius $\rho_{\text{crit}} = 3/2$ (orbit of photon).
5. Towards inner circular orbits

A computer analysis of equation (25) gives us

\[ e_{\text{min}}(\rho, \lambda, \dot{Q}^*) = \frac{\sqrt{2}\lambda(\rho^2 - \rho + \dot{Q}^*)}{\sqrt{\rho^2 - 2\dot{Q}^*\rho^4}}. \]  

Putting \( e_{\text{min}} \) into (24), we obtain

\[ \beta_{\text{min}}(\rho, \lambda, \dot{Q}^*) = \sqrt{\frac{\rho^2(2\rho^2 - 3\rho + 4\dot{Q}^*)}{(\rho^2 - \rho + \dot{Q}^*)^2}} \left( \frac{\sqrt{2}}{2} - \frac{\lambda^2(2\rho^2 - 3\rho + 4\dot{Q}^*)}{\sqrt{2}\rho^2(\rho^2 - 2\dot{Q}^*)} \right). \]  

\( \beta_{\text{min}} \) yields the value of the magnetic coupling \( \beta \) for a circular stable orbit having radius \( \rho \) with parameter \( \lambda \) and \( e_{\text{min}} \).

The locus \( \beta_{\text{extr}}(\rho, \dot{Q}^*) \) of the maxima of \( \beta_{\text{min}} \) is obtained by solving \( \partial \beta_{\text{min}} / \partial \rho = 0 \) with respect to \( \lambda \), and inserting the solution in \( \beta_{\text{min}} \) again. This gives

\[ \beta_{\text{extr}}(\rho, \lambda, \dot{Q}^*) = \sqrt{\frac{\rho^2(2\rho^2 - 3\rho + 4\dot{Q}^*)}{(\rho^2 - \rho + \dot{Q}^*)^2}} \left( \frac{\sqrt{2}}{2} + \frac{2\rho^2 - 3\rho + 4\dot{Q}^*}{\sqrt{2}(\rho^2 - 4\dot{Q}^* \rho + \dot{Q}^*)} \right). \]  

In equation (28) we neglect infinitely small expressions (i.e. \( \dot{Q}^* \cdot \beta^2 \)) and obtain maximum value \( \rho_+ \) of \( \rho \) for circular orbits

\[ \rho_+ = \frac{4\beta^2 - 27 - 3\sqrt{81 - 8\beta^2 - 288\dot{Q}^*}}{4(\beta^2 - 9)}. \]  

For any given value of \( \beta \) we also obtain value \( \rho_- \) from \( \beta = \beta_{\text{min}} \mid_{\lambda=0} \). Actually, \( \rho_- \) is the minimum value of \( \rho \) for admissible circular stable orbits,

\[ \rho_- = \frac{2\beta^2 - 3 - \sqrt{9 - 12\beta^2 - 32\dot{Q}^*}}{2(\beta^2 - 2)}. \]  

Thus for a given values of parameters \( \beta \) and \( \dot{Q}^* \), stable circular orbits near \( \rho_{\text{crit}} \) are confined in the range

\[ \rho_-(\beta, \dot{Q}^*) < \rho < \rho_+(\beta). \]  

Obviously \( \beta \ll 1 \) and we may neglect \( \beta^4 \) and \( \beta^2 \dot{Q}^* \), and obtain

\[ \Delta \rho(\beta, \dot{Q}^*) = \rho_+ - \rho_- = \frac{\beta^2(4\beta^2 - 29 + 56\dot{Q}^*)}{6(18 - 11\beta^2)}. \]  

Then make transformation \( \rho = \frac{x}{m_{\text{p}}^2} \) and \( \beta = \frac{2\mu B_0}{m_{\text{p}}} \) and take into consideration the particle with \( \mu \simeq \mu_{\text{Bohr}} = 5.27 \times 10^{-21} \text{ erg} G^{-1} \) (the Bohr magneton), \( m_{\text{p}} = 1.67 \times 10^{-24} \text{ g} \) (the proton rest mass) and magnetic field \( B_0 \simeq 10^{12} \text{ G} \) we obtain \( \beta \simeq 7 \times 10^{-6} \). Assumption \( M \simeq 10^3 M_{\odot} \), we adduce this results in table1:

| \( \rho_+ \) (\text{m}) | \( \rho_- \) (\text{m}) | \( \Delta \rho \) (\text{m}) |
|----------------|----------------|----------------|
| 3.47 | 0.00 | 3.47 |

The dependence of the values of \( e_{\text{geod}} \) and \( e_{\text{min}} \) from brane parameter. Evidently from this table, the value of \( e_{\text{geod}} \) is increasing with increase the of module of brane parameter, but the value of \( e_{\text{min}} \) inversely proportional to the brane parameter.
Table 1. The dependence of the $\rho_{\text{prop}}$ of brane parameter $Q$. $Q = 0$ corresponds to Schwarzschild black hole, which was considered by de Felice at all.

| $Q$     | $3 \times 10^{-5}$ | $1 \times 10^{-4}$ | $1 \times 10^{-3}$ | $3 \times 10^{-3}$ | $1 \times 10^{-2}$ | $3 \times 10^{-2}$ |
|---------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $r_{\text{prop}}(m)$ | 100          | 150          | 151          | 152          | 153          | 155          | 160          |

Let us now choose a circular stable orbit with a value of $\rho$ in the allowed range. We evaluate for such an orbit the corresponding $e$ and $\lambda$. From $\beta = \beta_{\text{min}}(\rho, \lambda, \hat{Q}^*)$ we obtain

$$\lambda_{\text{min}}(\rho, \beta, \hat{Q}^*) = \sqrt{\frac{2\beta\rho^4(2\hat{Q}^* - \rho)(\rho^2 - \rho + \hat{Q}^*) - \sqrt{2\rho^2(2\rho^2 - 3\rho + 4\hat{Q}^*)}}{2(\rho^2 - 3\rho + 4\hat{Q}^*)^{\frac{3}{2}}}}.$$  

(33)

Also, from $e(\rho, \lambda, \hat{Q}^*)$ and using (33) we obtain energy

$$e_{\text{min}}(\rho, \beta, \hat{Q}^*) = 2\rho^2 - 3\rho + 4\hat{Q}^* - 2\hat{Q}^*\beta(\rho^2 - \rho + \hat{Q}^*)^2.$$  

(34)

Equations (33) and (34) yield the required values for given $\beta$, $\rho$ and $\hat{Q}^*$.

In table 1 we provide the numerical values for shrink of stable orbits for photonic particle for the typical values of brane parameter. Apparently, the shrink of stable orbits depends on the module of brane parameter. Decreasing of brane parameter leads to increasing of radius of stable orbits. In the case when Weil parameter is equal to zero we get the results obtained in the Schwarzschild metric.

6. Storage and release of energy

The most interesting result which stems from the previous analysis is that the influence of brane tension parameter to the magnetic interaction, where provides a very efficient mechanism of particle confinement. As far as we know the combined effect of curvature and the pronounced relativistic character of the particle motion in the vicinity of the circular photon orbit, causes an amplification of the magnetic field which, in turn, provides not only the binding force required to sustain the orbital motion but also a large amount of magnetic interaction energy which adds to the gravitational one. So we end with a tiny ring of stable orbits with a high content of negative binding energy (magnetic and gravitational) which is subtracted from the positive (kinetic and rest) energy of the relativistic particle to give $e_{\text{min}}(\rho, \beta, \hat{Q}^*)$. Following to [1] it is instructive to compare $e_{\text{min}}(\rho, \beta, \hat{Q}^*)$ with the corresponding energy $e_{\text{geod}}(\rho, \hat{Q}^*)$ of a geodesic particle (no magnetic interaction: $\beta = 0$) moving on the same orbit:

$$e_{\text{geod}}(\rho, \hat{Q}^*) = \frac{\rho^2 - \rho + \hat{Q}^*}{\hat{Q}^*} \sqrt{\frac{2}{2\rho^2 - 3\rho + 4\hat{Q}^*}}.$$  

(35)
We emphasize that $e_{\text{geod}}$ is the energy of a geodesic unstable orbit, while $e_{\text{min}}$ is the energy of a magnetized stable one. Then the quantity
\[ \Delta e(\rho, \beta, \hat{Q}^*) = e_{\text{min}}(\tilde{\rho}, \beta, \hat{Q}^*) - e_{\text{geod}}(\tilde{\rho}, \hat{Q}^*) = -e_{\text{geod}}(\tilde{\rho}, \hat{Q}^*) \left[ 1 - \sqrt{1 - \beta e_{\text{geod}}(\tilde{\rho}, \hat{Q}^*)} \right], \] (36)
corresponds to the (negative) magnetic binding energy, stored in that orbit. This would be just the maximum energy released in the case of an abrupt disappearance of the magnetic coupling $\beta$. In table 2 we expressed the dependence of brane parameter $\hat{Q}^*$ from $e_{\text{geod}}(\tilde{\rho}, \hat{Q}^*)$ and $e_{\text{min}}(\tilde{\rho}, \beta, \hat{Q}^*)$.

Making the numerical calculation one can write some values for $e_{\text{geod}}(\tilde{\rho}, \hat{Q}^*)$ in the following table, $\tilde{\rho} = \frac{1}{2} (\rho_+ + \rho_-) = \rho_{\text{crit}} + \frac{13}{48} \beta^2 - \hat{Q}^*$. $e_{\text{geod}} = e_{\text{geod}}(\tilde{\rho}, \hat{Q}^*)$, $e_{\text{min}} = e_{\text{min}}(\tilde{\rho}, \beta, \hat{Q}^*)$.

Fig. 1. Radial dependence of effective potential $\beta$ for constant values of $e$ and $\lambda$. The value of $\hat{Q}^*$ is different for each graph.

Table 2. The dependence of the values of $e_{\text{geod}}$ and $e_{\text{min}}$ from brane parameter. Evidently from this table, the value of $e_{\text{geod}}$ is increasing with increase of module of brane parameter, but the value of $e_{\text{min}}$ inversely proportional to the brane parameter.

| $\hat{Q}^*$ | 0   | $1 \cdot 10^{-13}$ | $3 \cdot 10^{-13}$ | $1 \cdot 10^{-12}$ | $3 \cdot 10^{-12}$ | $5 \cdot 10^{-12}$ | $7 \cdot 10^{-12}$ |
|-------------|-----|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| $e_{\text{geod}}$ | $11.2 \cdot 10^4$ | $11.3 \cdot 10^4$ | $11.5 \cdot 10^4$ | $11.9 \cdot 10^4$ | $13.6 \cdot 10^4$ | $16.2 \cdot 10^4$ | $21.3 \cdot 10^4$ |
| $e_{\text{min}}$ | $5.2 \cdot 10^4$ | $5.1 \cdot 10^4$ | $5.06 \cdot 10^4$ | $5.01 \cdot 10^4$ | $4.9 \cdot 10^4$ | $4.7 \cdot 10^4$ | $4.4 \cdot 10^4$ |

In table 2 we provide the value of energy of geodesic particle (where $\beta = 0$) and minimum energy of particle ($\beta = \beta_{\text{min}}$). Decreasing of the value of brane parameter
$Q$, leads to increasing of $e_{geod}$ and decreasing of $e_{min}$.

7. Conclusion

Using the Hamilton-Jacobi formalism, we have analytically solved the radial equation for the motion of a magnetized particle orbiting braneworld black hole surrounded by a strong magnetic field. The case of the Schwarzschild black hole was extensively discussed in [1] for example magnetic coupling parameter $\beta$, which is function of $\rho, \lambda$ and $e$. Consequently, in Schwarzschild case, a sudden change in the value of the parameter $\beta$ could cause an abrupt release of stored energy, perhaps providing a novel mechanism for jets or bursts.

In this article we perform the similar calculations in metric of DMPR [3] (Dadhich N et al). In limiting case (i.e $Q = 0$) our expressions match the expressions, which were calculated in the Schwarzschild spacetime. We also recall that the particle magnetic moment $\mu$ has been taken parallel to the external magnetic field.

Obviously, from figure 1 one can see the existence region of stable circular orbits shifts to about $2M$, as far as we know the circular orbits around the Schwarzschild black hole appear to be in $3/2M$. This implies, the region of existence of stable orbits shifts sideways to observer, i.e. moves away off the black hole.

Acknowledgements

The author thanks Bobomurat Ahmedov and Ahmadjon Abdujabbarov for their help, useful advices, editing the text and making important corrections and comments. This research is supported in part by the UzFFR (projects 1-10 and 11-10) and projects FA-F2-F079 and FA-F2-F061 of the UzAS.

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