Probabilistic Approach to Soil Fertility Conservation by Mathematical Modeling of Technological Processes and Optimization of Resource Use

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Abstract. The well-being of the state directly depends on food security. Many factors and indicators affect the stability of food security in the country. It consists of the availability and sufficiency in quantity and quality of food to each resident of the country. Therefore, the main goals of the economic and agricultural policy pursued in Russia are to provide the best and balanced solutions for producing more agricultural products that meet all domestic and foreign quality requirements. In the article, the authors consider the operation mode of a rice irrigation system from a probabilistic point of view. A model of the process of setting the price of planned activities for step-by-step price changes has been compiled. Each period of new planned activities is called a phase. It is assumed that events form a Poisson flow of constant intensity. At the n-th phase, a satisfactory state of the system occurs. The optimization problem of optimal assignment of the events price at each phase is solved and a numerical iterative algorithm for its solution is obtained.

1. Introduction

The greatest anthropogenic load is experienced by the land of rice irrigation systems. This is due to the technology of rice cultivation, which implies either complete flooding of rice paddy fields using channels, or maintaining the maximum moisture capacity of the soil by irrigation during the growing season. In this period, the soil is subject to the maximum anthropogenic load, which consists in restoration processes in the soil, leaching of organic and mineral substances from the arable layer, processes of soil fusion and secondary salinization [1]. Taking into account the main factors that affect the potential soil fertility of rice irrigation systems in obtaining planned rice yields and rice crop rotation crops is an integrated approach to rice cultivation, namely, the development and ranking of criteria for assessing the state and quality of negative processes in rice soils, including their pollution. However, this task is significantly complicated by the fact that the above-described processes and various pollutants that accompany the rice cultivation technology can interact, turning into harmless compounds or, conversely, forming complexes, the destruction of which is much slower than for each of the components of such a complex separately.

Given that the “hazard index” of processes and pollutants in rice production is a function of the intensity of these processes and the concentration of pollutants [2], there is a problem of choosing adequate cost indicators for the process of rice cultivation. Determining the cost function $f(c)$ is an
intermediate step towards accounting for all the costs necessary to improve the agricultural resource state of the agricultural landscape, reduce the rate of negative processes and reduce the level of pollutants to the level of the required concentration of c. The means of preventing negative processes in rice soils and the accumulation of pollutants in them now include a wide range of technical and organizational measures. The optimization reserve is the search for the optimal distribution of costs of the planned activities [3].

At the first stage, it is advisable to solve such problems on a regional model of the subject, which is used to select typical conditions for setting local observations [4]. Typical conditions are identified through mapping ameliorative and hydrological conditions. This reflects not only typical hydrological conditions, but also such engineering and ameliorative elements as reservoirs, main and inter-farm channels, distribution network, irrigation conditions and characteristics of drainage systems [5].

The construction of regional models should reflect the specific features of each district that are primarily determined by its natural and climatic, reclamation conditions, as well as the resources available to an enterprise or a district (labor, energy, technical and technological, economic ones) [6].

2. Relevance, scientific significance

The current global issues of reducing fresh water and agricultural land volumes exacerbate the problem of implementing the goals under economic and agricultural policies. Scarce freshwater resources are mostly caused by poorly controlled, and sometimes not controlled at all, discharge of wastewater both to open sources and underground ones. This also includes the deterioration or complete absence of treatment facilities, which jointly leads to rapidly disappearing animal and plant life environments suitable for humans [7].

The problem of the reducing agricultural resource potential of soils should be considered as a separate problem. Its solutions should be divided into several clusters (reclamation, natural and climatic, technological, economic and environmental) [8]. The solution of such problems can be solved by creating mathematical models with a multi-level algorithm and using long-term continuous monitoring of all factors, attributes, indicators and criteria of such clusters [9]. Moreover, the adequacy of such models will depend both on the duration of the observations, but also on the quality and extensiveness of the list of observed factors, signs, indicators and criteria considered in the continuous monitoring [10].

Therefore, the program for maintaining food security is primarily a set of measures consisting of systematic, mathematically based models, and the target functions should take into account the largest number of factors, features, indicators and criteria that affect the stability of production.

The studies of the land reclamation status and technical condition of the land require obtaining and accumulating information on vast territories. Therefore, their successful implementation is impossible without identifying the typical conditions for the regime-balance observations [11].

The developed model is based on the analysis of large amounts of data about the rice irrigation system and the calculation of informative indicators for choosing the design of rice maps. Justification of optimal options requires new theoretical research and development of innovative projects that allow a comparison between the technical and economic indicators of options based on the results of mathematical modeling of the studied processes [12].

3. Problem statement

Consider the process of setting the price of measures when selecting the design and operational parameters of a rice irrigation system. The rice irrigation system is designed for irrigation of rice and associated crop rotation crops [13]. It consists of irrigation and drainage network channels, irrigation maps (divided into paddy fields by rollers), special facilities (water intakes, pumping stations, settling tanks). Water in rice paddy fields comes from cart sprinklers that is the last link of the irrigation network. An important factor in the impact on the environment are seepage losses from main and distribution canals. The magnitude of these losses largely determines the negative changes in the reclamation and hydrogeological situation (land flooding, groundwater pollution) [14].
When the process of crop formation is being managed, the entire growing season is divided into component interphase intervals. At a certain point in time, the control system input receives a set of parameter values characterized by the vector: \( (a_1, a_2, ..., a_n, b_1, b_2, ..., b_m, c_1, c_2, ..., c_l) \). Denote this set of situations \( S(t) \). \( S(t) \) contains all information about the control object at this time. The task of the control system is to exert control influence \( (u_1, u_2, ..., u_l) \) on the object based on knowledge about the object, that is, to obtain some technologically acceptable solutions, and if there are several acceptable solutions, to choose the best solution from the point of view of a certain control criterion. For the rice irrigation system, this is the flooding mode, fertilization, soil treatment, and protection from pests [15].

In the model of the rice irrigation system described in [16], the irrigation period of the rice growing season is divided into 10 stages, the irrigation norms for which are different. Each of the stages includes either one phase of rice development, or a part of it, for which the requirements for the flooding layer are the same. The duration of each stage depends on the combination of factors that affect the speed of vegetation phases and the selected irrigation regime for specific conditions [17]. So, the duration of the first stage depends on the water absorption capacity of the soil, the capacity of the sprinkler and the capacity of the channel network of the rice system itself. The timing of the other stages generally depends on the clogging of rice, the capacity of the discharge network, and so on.

4. Theory

A set of indicator parameters is selected for each stage. For example, the parameter can be a temperature factor that is one of the essential features of the rice development phases.

If the salinity of the water \( C_Q \) exceeds the maximum permissible salinity \( C_m \), the value of the irrigation norm, which would prevent secondary salinization, is determined by the formula:

\[
Q_c = \begin{cases} 
\frac{Q}{1 - \frac{C_Q}{C_E}} & \text{if } C_Q \leq C_M, \\
0 & \text{if } C_Q > C_M,
\end{cases}
\]

where \( Q \) is the calculated value of water supply in the rice irrigation system without taking into account the danger of salinization, \( C_i \) is the value of the salinity of water in the paddy fields and saturating the root layer.

For a rice irrigation system as an object of operational management, the relationship between output and input parameters is dynamic and cannot be adequately described by a static deterministic analytical model [20]. We will consider the planned activities as a Poisson flow of a certain intensity. Assume the price of the events varies in a stepwise manner. At the same time, various models of its change are possible. When the work has started, events are planned at a price \( S \), that is kept for a long time \( T_i \). If the desired result is not achieved during this time, the following events are held, the price of which is \( S \) kept for a long time \( T_i \). If the conditions do not become satisfactory during this time, the price is set \( S \), which is kept for a long time \( T_i \), and so on. Each such time interval is called a phase.

Here are some examples of the measures described above: discharge of irrigation water, part of precipitation, filtration water and soil drainage during the growing season, as well as the lowering ground water level and soil drainage during the inter-vegetation period, measures to combat erosion. Water erosion affects paddy fields: when water is discharged, as soon as its level in the discharge sprinkler falls below the surface of the check, the liquefied surface layer of soil is flushed into the discharge sprinkler. The intensity of erosion is so great that a 1.1–1.2 m deep irrigation ditch turns into a 0.3–0.4 m deep ditch in 1–2 years.

Consider the task of optimizing the price of the measures at each phase.

When paddy fields covering a bigger area are developed, deep cuts and large backfills of the fertile arable layer are necessary [21]. \( K_n \) is losses to the farm if a satisfactory state of the rice irrigation
system is achieved in the \( n \)-th phase. Obtain the average household income using the formula for mathematical expectation

\[
\Phi = \sum_{n=1}^{\infty} (S_n - K_n)(1 - P_n) \prod_{i=1}^{n-1} P_i ,
\]

where \( n \) is the number of events, \( S_n \) is the price of the event, and \( P_i \) the probability that a satisfactory state is not reached in the \( i \)-th time interval (\( i \)-th phase).

Consider the problem of software optimization \( \Phi \{ S_n \} \). Finding the partial derivative of the function \( f \) in \( S_m \), we obtain the equation

\[
\frac{\partial \Phi}{\partial S_m} = 0, \quad m = 1, \infty.
\]

Take a closer look at composing an explicit expression for a partial derivative. The value \( S_m \) occurs in the summand

\[
(S_m - K_m)(1 - e^{-\lambda R(S_m)T_m}) \prod_{i=1}^{n-1} P_i ,
\]

The derivative of this term is equal to \( S_m \)

\[
\left[1 - e^{-\lambda R(S_m)T_m} + (S_m - K_m)\lambda R'(S_m)T_m e^{-\lambda R(S_m)T_m} \prod_{i=1}^{n-1} P_i - \lambda R'(S_m)T_m \cdot \sum_{n=m+1}^{\infty} (S_n - K_n) \prod_{i=1}^{n-1} P_i (1 - P_n) .
\]

In terms of \( C \), \( n > m \) the value \( S_m \) is present only in the cofactor \( P_m = \exp(-\lambda R(S_m)T_m) \), the derivative of which is equal to

\[
P'_m = -\lambda R'(S_m)T_m \cdot P_m .
\]

Now we can write

\[
\frac{\partial \Phi}{\partial S_m} = \left[1 - e^{-\lambda R(S_m)T_m} + (S_m - K_m)\lambda R'(S_m)T_m e^{-\lambda R(S_m)T_m} \prod_{i=1}^{n-1} P_i - \lambda R'(S_m)T_m \cdot \sum_{n=m+1}^{\infty} (S_n - K_n) \prod_{i=1}^{n-1} P_i (1 - P_n) .
\]

Equating the composed expression to zero

\[
\left[1 - e^{-\lambda R(S_m)T_m} + (S_m - K_m)\lambda R'(S_m)T_m e^{-\lambda R(S_m)T_m} - \lambda R'(S_m)T_m \cdot F_m = 0 ,
\]

where

\[
F_m = \sum_{n=m+1}^{\infty} (S_n - K_n) \prod_{i=1}^{n-1} P_i (1 - P_n) .
\]

Preparing a recurrence relation for \( F_m \). For the first term:

\[
F_m = (S_{m+1} - K_{m+1}) P_{m+1} (1 - P_{m+1}) + P_m \sum_{n=m+2}^{\infty} (S_n - K_n) (1 - P_n) \prod_{i=1}^{n-1} P_i ,
\]

As a result, we get

\[
F_m = (S_{m+1} - K_{m+1}) P_{m+1} (1 - P_{m+1}) + P_m F_{m+1} .
\]

Next, derive the recurrence relation for \( S_m \). Write expression (5) as

\[
1 - e^{-\lambda R(S_m)T_m} + (S_m - K_m)\lambda R'(S_m)T_m e^{-\lambda R(S_m)T_m} = F_m .
\]

By the same reasoning

\[
1 - e^{-\lambda R(S_{m+1})T_{m+1}} + (S_{m+1} - K_{m+1})\lambda R'(S_{m+1})T_{m+1} e^{-\lambda R(S_{m+1})T_{m+1}} = F_{m+1} .
\]
\[
\frac{1 - e^{-\lambda R(S_m)T_m}}{\lambda R'(S_m)T_m^2} + (S_m - K_m)\lambda R'(S_m)T_m e^{-\lambda R(S_m)T_m} = (S_{m+1} - K_{m+1})e^{-\lambda R(S_{m+1})T_{m+1}} \left(1 - e^{-\lambda R(S_{m+1})T_{m+1}}\right) + \\
e^{-\lambda R(S_m)T_m} \left[ \frac{1 - e^{-\lambda R(S_{m+1})T_{m+1}}}{\lambda R'(S_{m+1})T_{m+1}^2} + (S_{m+1} - K_{m+1})\lambda R'(S_{m+1})T_{m+1} e^{-\lambda R(S_{m+1})T_{m+1}} \right].
\]

After multiplying by \(e^{\lambda R(S_m)T_m}\), we get
\[
S_m - K_m + \frac{e^{\lambda R(S_m)T_m} - 1}{\lambda R'(S_m)T_m^2} = S_{m+1} - K_{m+1} + \frac{1 - e^{-\lambda R(S_{m+1})T_{m+1}}}{\lambda R'(S_{m+1})T_{m+1}^2}.
\]

Relation (11) connects \(S_m\) and \(S_{m+1}\). We will use it as follows: given \(S_0\), we find numerically \(S_1\), then, knowing \(S_1\), we find \(S_2\), and so on. After that, you can calculate the value \(\Phi\), which will now depend only on \(S_0\). Next, we find \(\max_{S_0} \Phi\) using the algorithm for finding the extremum of a function of one variable.

5. Practical significance, proposals and implementation results

Based on the proposed model, the following is justified:

1) types of rice systems that provide guaranteed and highly efficient combined irrigation (a combination of irrigation by sprinkling and flooding) of rice crop rotation without deterioration of the land.

2) organization and implementation of efforts to repair the network and its facilities, paddy fields planning.

3) environmental and reclamation crop rotations. When switching to the herbicide-free technology of rice cultivation in crop rotations, it is necessary to increase the share of legumes to 50-75 %, and also apply a moisture-saving technology of soil treatment. At the same time, gross production will remain at the same level, and the cost of cultivation will be reduced by five or more times.

4) water use regulations. Due to the shortage of water resources in recent years, a regime of variable flooding has been developed: the environmental and economic regime of irrigation (Figure 1).

![Figure 1](image)

Figure 1. Model of eco-economical rice irrigation regime: 1-traditional irrigation mode; 2-variable flooding mode.

5) method of flooding the rice fields. The Krasnodar reservoir is completely emptied to the level of dead volume by August 5-10, which is why rice crops in the critical phase of their development (maturation and grain filling) are not provided with water – the yield decreases from 15 to 45 %. One
of the methods of significant water saving and rational use of water resources during this period is the transition from constant flooding to variable flooding by supplying water in the form of a triangular damped symmetrical pulse.

The regulations include a science-based water-saving regime for rice irrigation and program water distribution both for drawing up dispatching schedules for the operation of reservoirs and regulatory waterworks, as well as in-system water use plans:

- the irrigation season starts on April 20 and ends on September 5;
- the initial flooding of paddy fields is carried out programmatically, setting a two-stroke water cycle between the paddy fields on the map and a three-stroke one on the area distributor between the maps;
- technological discharges of water from the paddy fields are not allowed. If it is necessary (expedient) to lower the water layer, stop its supply from the sprinkler in advance, so that due to evapotranspiration and inevitable filtration, the layer drops to the set level by the scheduled date.

6. Conclusions

The stability of agricultural production directly depends not only on the degree of use of advanced technologies by the region or/and the economy (optimal crop rotations together with eco-adaptive technology for processing crop fields, selection achievements, the use of modern mineral and organic fertilizers, the use of advanced herbicides, pesticides and other agrochemicals), technical equipment with modern domestic and/or foreign agricultural equipment, but also on the degree of optimization of the use of available resources in farms.

The article considers the production of agricultural products in the economy as a multi-level system. At each level there are certain resources ranked by the mathematical model according to the importance of their use and by time and place of their use. At the same time, the existing resources (energy, labor, economic, technical and technological) are redistributed in order to obtain programmed yields.

The compiled mathematical model reflects the conditions of its operation and will be expanded when considering all possible factors that influence the assessment of the operational condition. The study includes the environmental factor, which is an effective method for solving environmental problems.

The probabilistic model of the reducing the price of planned measures allows approaching the issue of operation of rice irrigation systems in a different way, since it becomes possible to form a data bank that determines a particular state of technology, choosing an algorithm for preventing risks and damages.

7. References

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