Bacteria Inspired Multi-Flagella Propelled Soft Robot at Low Reynolds Number

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Abstract

The locomotion and mechanical efficiency of microorganisms, specifically micro-swimmers, have drawn interest in the fields of biology and fluid dynamics. A challenge in designing flagellated micro- and macro-scale robots is the geometrically nonlinear deformation of slender structures (e.g. rod-like flagella) ensuing from the interplay of elasticity and hydrodynamics. Certain types of bacteria such as *Escherichia coli* propel themselves by rotating multiple filamentary structures in low Reynolds flow. This multi-flagellated propulsive mechanism is qualitatively different from the single-flagellated mechanism exhibited by some other types of bacteria such as *Vibrio cholerae*. The differences include the flagella forming a bundle to increase directional stability for cell motility, offering redundancy for a cell to move, and offering the ability of flagella to be the delivery material itself. Above all, multi-flagellated biological system can inspire novel soft robots for application in drug transportation and delivery within the human body. We present a macroscopic soft robotic hardware platform and a computational framework for a physically plausible simulation model of the multi-flagellated robot. The fluid-structure interaction simulation couples the Discrete Elastic Rods algorithm with the method of Regularized Stokeslet Segments. Contact between two flagella is handled by a penalty-based method due to Spillmann and Teschner. We present comparison between our experimental and simulation results and verify that the simulation tool can capture the essential physics of this problem. The stability and efficiency of a multi-flagellated robot are compared with the single-flagellated counterpart.

Keywords: Discrete Elastic Rod, Soft Robotics, Bacteria-inspired Robot

1. Introduction

Locomotion of micro-swimmers has drawn significant attention in biology and fluid dynamics since the 1950s. A seminal paper by Purcell in 1977 introduced the counter intuitive behavior of cell locomotion in low Reynolds flow where viscosity dominates over inertial effects. In this regime, the so called scallop theorem elaborates reciprocal motions do not provide propulsive force. In order to overcome this constraint, several micro-swimmers in nature use elastic corkscrew like structures, or flagella, to propel themselves forward. It was also found that monotrichous bacteria, or single flagellated bacteria, such as *Vibrio cholerae* exploit buckling instability in the flagellum to make directional change in their motion. Consequently, numerous biological findings, mechanical experiments, hydrodynamic theories for low Reynolds flow, and medical microbots explored and exploited such a mechanism.

Multi-flagellated locomotion displays fundamentally different mechanism from monotrichous locomotion. There are two modes for the former type of locomotion: “run” and “tumble”. For example, *Escherichia coli*, flagella are formed as left-handed helices. Run is a period of near straight line motion that is caused by bundling of flagella. As multiple flagella turn in the same direction (counter clock wise (CCW) for *E. coli*),
they form a single or multiple bundles of helical shape. On the other hand, tumble is a period of random directional change which is caused by change in rotational direction of flagella i.e. if single or multiple flagella of *E. coli* rotates in clockwise (CW) direction. In essence, multi-flagellated mechanism is an intricate interplay between geometric nonlinearity, hydrodynamics, and contact.

Inspired by the complexity of mechanics behind simple locomotion, mechanical engineers also tried to formulate the motion of multi-flagellated bacteria. It was only until recently that the bundling behavior was shown to be a purely mechanical phenomenon that is due to the interaction of the soft helical structures and the viscous fluid. Besides its complexity in physics, the multi-flagellated mechanism is important from both robotic and biological perspectives due to the following features: (1) directional stability, (2) redundancy of actuation, (3) chemical secretion using flagella, (4) improved efficiency in swarm and propagation.

Prior works on soft robots actuated by flagella have considered both simulation and experiments. To solve this fluid-structure interaction problem, computational fluid dynamics model and slender body theory (SBT) were used to predict the motion of single flagellated small-scale robot with rigid flagellum ignoring the effect of flexibility of the flagella. With recent advancements in computational capability, the structural flexibility in a single-flagellated system can also be accounted for; the flagellum can be modeled as a linear elastic Kirchhoff rod. Multiple studies have demonstrated the modeling of multi-flagellated systems. However, the coupling between long-range hydrodynamics, geometrically nonlinear deformation, and contact, has not been accounted for until recently.

In the field of microbots, there are several studies that investigated the effect of multiflagellated mechanism. Due to limited modes of locomotion that a single flagellated mechanism provide, Beyrand et al. presented multiple flagella microswimmers that covers rolling, running, and tumbling motion. Ye et al. investigated the benefit of multiple flagellated locomotion and the advantages of sinusoidal 2D geometry. Even biohybrid microbots have been developed, which are created by assembly of biological flagellated organism with the man made magnetic structure, showed remarkable results of controlled locomotion using magnetic field. However, due to the nontrivial coupling between hydrodynamics, contact, and elasticity, researchers investigated lower order of coupling to solve the problem. e.g. (1) using rigid flagellum for flagellated robot under viscous fluid, (2) using a single elastic flagellum without modeling self-contact or (3) using ribbon-like multiple flagella without bundling.
behavior\textsuperscript{19,20} Overall, a comprehensive numerical model and physical prototype for flagellar bundling still needs further investigation.

In this paper, we present a macroscopic soft robotic platform based on propulsive mechanism of flagellated micro-organisms and a physics-based computational framework to simulate the robot. The computational tool uses the Discrete Elastic Rods (DER) algorithm for elastic rod dynamics, Regularized Stokeslet Segments (RSS) method for hydrodynamics including long-range interaction\textsuperscript{17} and Spillman and Teschner’s method of contact\textsuperscript{45}. We first verify the simulation against the experiments with qualitative and quantitative comparison. As shown in Figure 1, the experiment and simulation show good qualitative agreement; the simulation successfully captures the attraction between two flagella arising from hydrodynamic interaction. Following quantitative comparison between experiments and simulations, the shortcomings of the model are discussed and directions for future research are suggested. We find from experiments and simulations that the flagellar buckling\textsuperscript{46} does not occur for the multiflagellated mechanism, while a single flagellated system may undergo buckling due to excessive hydrodynamic loads. An efficiency comparison between a single- and a multi-flagellated system shows that the single-flagellated robot has a slight efficiency advantage over its multi-flagellated counterpart. This simulation and the observations on the propulsive mechanism will set the foundation for further developing the soft robotic prototype. Eventually, we expect that the robotic prototype can be miniaturized towards a microbot that harnesses structural flexibility and hydrodynamic interaction for functionality.

2. Methods

2.1. Experimental setup

For experimental data collection, we used glycerin as the viscous medium for our robot. A cylindrical tank with diameter of 28 cm and height of 45 cm was used with glycerin filled up to a height of approximately 40 cm of the tank. The robot was initially placed at the center of the glycerin tank to remain approximately 10 cm apart from the sidewalls. For every experiment, temperature and viscosity were measured. Viscosity was measured using USS-DVT4 rotary viscometer and the viscosity measurement was in a good agreement with the nominal value of the glycerin; dynamic viscosity of $\mu = 0.956 \pm 0.2 \text{ Pa}\cdot\text{s}$. The temperature of glycerin was approximately 22°C throughout the experiment to minimize the effect of temperature on viscosity. The fluid was mixed thoroughly before the experiment to avoid variation in density inside the tank.

Figure 2. Experimental set up, robot schematic and symbol notation

The robot head contained Wemos D1 mini micro controller unit used for the motor control, two 3.7 V 500 mAh Lipo batteries, and two mini geared DC motors. The motor was calibrated for angular velocity using Cybertech DT6236B Tachometer. PWM-RPM was calibrated within $\pm 1(0.01\%)$ rpm at 3.8 V. The head is in cylindrical shape with radius of $3.1 \pm 0.01$ cm and height ($h = 8.2 \pm 0.01$ cm) and was built using fused deposition modeling (FDM) 3D printer with polylactic acid plastic (PLA) material ($\rho = 1.26 \text{ g/cm}^3$). The robot head was comprised of the casing and the main body. Urethane wax was applied inside of the casing, outside of the main body, and the motor chamber to further prevent the glycerin from penetrating into the robot. Ballast was placed near the robot centroid for neutral buoyancy and weight balance. Flagella were attached on the bottom plates that are connected to the motors.
Table 1. Table of geometric, physical and simulation parameters with symbol representations

| Parameter | Value(Exp/Sim) | Description                       |
|-----------|----------------|-----------------------------------|
| $r$       | 0.0064         | Radius of helix (m)               |
| $\lambda$| 0.0572         | Pitch of helix (m)                |
| $l$       | 0.0954         | Axial length of helix (m)         |
| $r_h$     | 0.031          | Radius of robot head (m)          |
| $r_0$     | 0.0016         | Radius of the rod (m)             |
| $E$       | $1.255 \times 10^6$ | Young’s Modulus (Pa)            |
| $\nu$     | 0.5            | Poisson’s Ratio                   |
| $\rho$    | 1260           | Density (kg/m$^3$)                |
| $\mu$     | $0.956 \pm 0.2 / 1.0$ | Viscosity (Pa·s)              |
| $\Delta t$| $1.0 \times 10^{-4}$ | Time step                        |
| $\epsilon$| $1.67 \times 10^{-4}$ | Regularization parameter        |
| $|r|$     | $5.0 \times 10^{-3}$ | Discretization length (m)        |
| $C_t$     | 4.8            | Translational drag coefficient    |
| $C_r$     | 0.36           | Rotational drag coefficient       |

Figure 3. Schematics of robot with relevant motion. $\omega_h$ and $\omega_t$ represents head and tail rotational velocity respectively.

For the flagella, VPS elastomeric material was used for fabrication with Young’s modulus $E = 1255 \pm 49$ kPa and Poisson’s ratio $\nu \approx 0.5$ (i.e. nearly incompressible). Catalyst and base – both liquid – were mixed with 1:1 volume ratio. Iron fillings were added to the liquid mixture to match the density of the glycerin ($\rho = 1.26$ g/cm$^3$). Left handed helix shaped molds with different pitch parameters ($\lambda = 3.18, 4.45, 5.72$ cm) were 3D printed. Hollow PVC tubes were placed inside the molds and the liquid mixture was injected inside the tubes. After waiting a few hours to cure, the PVC tubes were cut out and filamentary soft helical rods were obtained.

For data processing, we captured the video of the robot submerged inside the glycerin tank over 300 seconds. Out of the 300 seconds, we used the data between 30 seconds and 270 seconds to ignore the initial transience during the speed ramp-up from 0 rpm to a prescribed total rpm. Then, we converted the videos into jpg files with frame rate of 1 frame per second. The image files were then processed by ImageJ image processing tool using stacked image processing centroid calculation.

2.2. Physics based simulation of the soft robot

Figure 4 shows the discretized schematic of our robot. The rod is discretized into $N$ nodes: $x_k = [x_k, y_k, z_k]^T$ for $0 \leq k \leq N - 1$. and $N$-1 corresponding edges: $e_k = x_{k+1} - x_k$ for $0 \leq k \leq N - 2$. The degree of freedom (DOF) vector of discretized robot is defined as $q = [x_0^T, \theta_0, x_1^T, \theta_1, ..., x_{N-2}^T, \theta_{N-2}, x_{N-1}^T]^T$, where $\theta_k$ is the scalar twist angle at edge $e_k$. Therefore, the size of the DOF vector for $N$ nodes is $4N - 1$. Hereafter, subscripts are used for the node-based quantities and superscripts are used for the edge-based quantities.

An important characteristic of the DER method is computation of the twisting of a rod simply by using a set of single scalar quantities $\theta_k$ embedded in the DOF vector. In this formulation, each edge has a reference frame (noted as $d_{i+1}^k$, $d_{i}^k$, $t^k$ in Figure 4) that is orthonormal and adapted (i.e. $t^k$ is the tangent along the $k$-th edge). The construction of reference frame is first initialized at the first edge ($k = 0$) at time $t = 0$ with an arbitrary set of orthonormal vectors (with the condition that the third vector $t^0$ is the tangent to the first edge). Then, the reference frame is parallel transported to the subsequent edges to form...
the reference frame on all the edges. After this initialization, reference frame can be updated at each time step by parallel transporting $d_1^k$, $d_2^k$, $t^k$ from the “old” configuration (DOF vector before the time step) to the “new” configuration (DOF vector after the time step). The material frame is also an adapted orthonormal frame (noted as $m_1^k$, $m_2^k$, $t^k$ in Figure 4) that is identical to the reference frame at $t=0$. Since both the frames share a common director ($t^k$), a single scalar quantity – the twist angle, $\theta^k$ – can be used to compute the material frame from the reference frame. Algorithmic representation of this update of frame is shown in Algorithm 1.

Based on the discretization, the elastic strains are required to calculate the energy and formulate the equations of motion (EOM) to march from time $t = t_i$ to time $t = t_{i+1} = t_i + \Delta t$, where $\Delta t$ is the time step size in the simulation outlined in Algorithm [1]. An elastic rod has three types of strains – bending, twisting, and stretching – associated with its deformation. Using these strains, we can calculate stretching, bending and twisting energy of our system; the sum of three energy is noted as the elastic energy and is represented as Eq[1] Details on calculation of the strains and energy term can be found on Supplementary methods section S1.

$$E_{\text{elastic}} = \sum_{k=0}^{N-2} E_s^k + \sum_{k=1}^{N-2} E_b^k + \sum_{k=1}^{N-2} E_t^k,$$

We can simply take the gradient of the energy terms with respect to the DOFs to get the elastic force at each DOF. The elastic force at the $k$-th DOF is $-\frac{\partial E_{\text{elastic}}}{\partial q_k}$. The simulation marches forward in time by updating the configuration, i.e. DOF vector, of the robot based on EOM. We can even impart artificial configuration updates in the simulation that are dynamic. In particular, for actuation of the robot we implement a time-dependent fixed rate natural twist on nodes $x_{m1}$, and $x_{m2}$ shown in Figure 4. In order to propagate in time, the equation of motion to be solved at the $k$-th node is

$$f_k \equiv m_k \frac{q_k(t_{i+1}) - q_k(t_i)}{\Delta t} - \dot{q}_k(t_i) + \frac{\partial E_{\text{elastic}}}{\partial q_k} - f_k^{\text{ext}} = 0, \quad (2)$$
where \(q_k(t_i)\) is the old position (and the \(k\)-th element of the vector \(q(t_i)\)), \(\dot{q}_k(t_i)\) is the old velocity, \(m_k\) is the lumped mass at the \(k\)-th DOF, and \(F^\text{ext}_k\) is the external force on the \(k\)-th DOF. Note that Eq. 2 is simply a statement of Newton’s second law. External forces may include gravity, contact, and hydrodynamics and the \((4N-1)\)-sized external force vector can be written as

\[
f^\text{ext} = f^h + f^\text{head} + f^c, \tag{3}
\]

where \(f^h\) is the hydrodynamic force vector on the flagella, \(f^\text{head}\) is the hydrodynamic force vector on the head, and \(f^c\) is the contact force vector.

(Supplementary methods section S2)

Now that the EOM is defined, we need to solve the system of \((4N-1)\) equations defined by Eq. 2 to compute the new position vector, \(q(t_{i+1})\). Newton-Raphson method can be used to solve the equations which requires the Jacobian of Eq. 2. The \((k,m)\)-th element of the square Jacobian matrix is

\[
J_{km} = \frac{\partial f_k}{\partial q_m} = J^\text{inertia}_{km} + J^\text{elastic}_{km} + J^\text{ext}_{km}, \tag{4}
\]

The expressions for the Jacobian terms associated with the elastic forces are available in the literature.\(^{19}\) The Jacobian terms associated with some external forces \((f^h, f^\text{head})\) cannot be analytically evaluated and those terms are simply set to zero. In other words, those forces are incorporated into the simulation in an Euler forward fashion.

After solving Eq. 2 to calculate the new position \(q_k(t_{i+1})\), new velocity can be trivially computed from \(\dot{q}_k(t_{i+1}) = (q_k(t_{i+1}) - q_k(t_i))/\Delta t\).

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**Algorithm 1 Multiflagella soft robot simulation**

**Require:** \(q(t_j), \dot{q}(t_j)\)

**Require:** \((d^k_1(t_j), d^k_2(t_j), t^k(t_j))\)

**Ensure:** \(q(t_{j+1}), \dot{q}(t_{j+1})\)

**Ensure:** \((d^k_1(t_{j+1}), d^k_2(t_{j+1}), t^k(t_{j+1}))\)

1: Guess : \(q^{(1)}(t_{j+1}) \leftarrow q(t_j)\)
2: \(n \leftarrow 1\)
3: Calculate \(f^h\) and \(f^\text{head}\) \(\triangleright \text{Supp. methods - sec. } S2\)
4: \(\triangleright \text{Eq. 2} 4\)
5: solved \(\leftarrow 0\)
6: while solved \(\neq 0\) do
7: \(\triangleright \text{while error > tolerance do}\)
8: Compute ref. frame using \(q^{(n)}(t_{j+1})\)
9: \((d^m_1(t_{j+1}), d^m_2(t_{j+1}), t^m(t_{j+1}))^{(n)}\)
10: Compute ref. twist \(\Delta m_{k,\text{ref}}^{(n)}\)
11: Compute material frame \((m_1^k(t_{j+1}), m_2^k(t_{j+1}), t^k(t_{j+1}))^{(n)}\)
12: Compute \(f\) and \(\partial J\) \(\triangleright \text{Eq. 2} 4\)
13: \(\Delta q \leftarrow J \setminus f\)
14: \(q^{(n+1)} \leftarrow q^{(n)} - \Delta q\)
15: error \(\leftarrow \text{sum (abs (f) )}\)
16: \(n \leftarrow n + 1\)
17: end while
18: solved\(\leftarrow 1\)
19: for \(l = 0\) to \(l = N - 2\) do
20: \(\triangleright \text{for } m = 0\) to \(m = N - 2\) do
21: Compute \(\delta_{l,m}^{(n)}\)
22: if \(\delta_{l,m}^{(n)} < 2r_0\) then
23: Compute \(f^c_l, f^c_{l+1}, f^c_m, f^c_{m+1}\)
24: \(\triangleright \text{Supp. methods - sec. } S3\)
25: solved \(\leftarrow 0\)
26: end if
27: end for
28: end for
29: end while
30: \(q(t_{j+1}) \leftarrow q^{(n)}(t_{j+1})\)
31: \(\dot{q}(t_{j+1}) \leftarrow q(t_{j+1}) - q(t_j)/\Delta t\)
32: \((d^k_1(t_{j+1}), d^k_2(t_{j+1}), t^k(t_{j+1}))^{(n)}\)
33: \((d^k_1(t_{j+1}), d^k_2(t_{j+1}), t^k(t_{j+1}))^{(n)}\)

3. Results and Discussion

3.1. Validation of Physics-based Simulation of Multi-flagellated robot

To demonstrate the validity of our simulation model, we investigated the locomotion of our multi-flagellated robot. Figure 5a shows snapshots from experiments and simulations at three
Figure 5. Comparison of data for experiment and simulation for different pitch to helix radius ratio of flagella ($\bar{\lambda} = 5, 7, 9$) : (a) Snapshot comparison between experiment and simulation at 60 rpm, scale bar: 1 cm ; (b) Comparison of experimental data with simulation at 30, 40, 50 rpm for $\bar{\lambda} = 7$. (b.1) shows comparison of normalized $x$ position and (b.2) shows comparison of normalized $y$ position against normalized time.

Different values of pitch: $\bar{\lambda} = \{5, 7, 9\}$. From Figure 5a, we first qualitatively analyze the match between the transitional motion of the flagella crossing and bundling behavior. We noted during experimental observations that $\bar{\lambda} = 5$ case formed a bundle throughout the entire range of angular velocity variation (30 to 70 rpm). On the other hand, $\bar{\lambda} = 7$ case formed a partial bundle at 50 and 60 rpm, and $\bar{\lambda} = 9$ did not bundle but had continuous contact between two flagella. In Figure 5b, we present quantitative comparison of experimental data with simulation at 30, 40, and 50 rpm for $\bar{\lambda} = 7$ and plot the position of the robot along $x$ and $y$ directions against normalized time. The data frequency of the experiment is 1 frame per second (fps) and the data frequency of simulation is 10 fps.

We obtain the values of the numerical prefactors $C_t$ and $C_r$ by data fitting using data obtained on experiment with $\bar{\lambda} = 7$. By comparing the mean total least squared error for the axial velocity $v$. rotational velocity, the prefactors that provided minimal error between experiment and simulation were used. Both prefactors are the coefficients to account for non-spherical shape of the robot body in translational, rotational hydrodynamic drag calculation based on sphere at Stokes flow.

The experiment and simulation results show agreement in the positional data for both $\bar{x}$ and $\bar{y}$ in Figure 5b. The normalized $x$ position, $\bar{x}$, in particular, shows a good match between the simulation and experiment (Figure 5b.1). From Figure 5b.2, we can observe that as the angular ve-
locity increases, the oscillation frequency in normalized \( y \) position increases accordingly. This represents an undulatory sideways motion of the robot as its is moving upward (along \( x \) direction). Even though both experiments and simulations show the same trend, there exist discrepancies in the higher rpm and lower normalized pitch cases. We attribute this to the friction between the flagellar surfaces. Unlike simulation where we resolve contact between the two flagella without consideration on friction between each flagellum, we observed in our experiment that once it partially or fully bundles, the bundled part had high frictional force that makes the flagella to be kept in bundled configuration when transitioning to partially bundled regime.

![Figure 6](image6.png)

**Figure 6.** Comparison of simulation and experiment for head and tail angular velocity.

Simple moment balance tells us that the cell body (i.e., the head of the robot) and the flagella have to rotate in opposite directions for net zero torque. It is known through previous works that the counter rotation of cell body in bacterial locomotion contributes to the trajectory and efficiency of the organism and could even contribute to the bundling of the flagella.\cite{20, 21} We use our robot to investigate the rotation of the head and, in Fig. 6, plot the angular velocity of the head as a function of the total angular velocity from both experiments and simulations at three different values of pitch. At lower values of total angular velocity (\( \bar{\omega}_T \lesssim 100 \)), there is little variation in the angular velocity of the head at different values of pitch. In both experiments and simulations, we see that the angular velocity of the head increases almost linearly with the total angular velocity. We exploit our simulations to probe the higher angular velocity regime and clearly see that the head angular velocity increases sublinearly with the total angular velocity. The variation in the head angular velocity as a function of the pitch of the flagella is worth mentioning. Among the three examined here (\( \lambda = \{5, 7, 9\} \)), the head angular velocity is the highest at \( \lambda = 7 \). This nonlinear dependence on the geometric parameter (pitch) of the flagella may be counter intuitive; however, it is a manifestation of the highly nonlinear and coupled nature of the problem. This type of nonlinearity with variation of geometry can be also be found in Ref.\cite{13} that used a single-flagellated system and analyzed the normalized force with respect to the normalized pitch.

![Figure 7](image7.png)

**Figure 7.** Non-linear relationship of angular velocity and robot \( x \)-velocity captured through simulation. The experimental data are in symbol with error bars.

Next, in Figure 7, the velocity of the bacterium robot along the \( x \)-axis is shown as a function of the normalized total angular velocity, \( \bar{\omega}_T \). Interestingly, the translational velocity increases superlinearly with the total angular velocity in the regime explored here. The nonlinear dependence of translational velocity on the pitch of the flagella is also obvious. The case with \( \lambda = 7 \) results in lower propulsive speed compared with the \( \lambda = 5 \) and \( \lambda = 9 \) cases. In Figure 7, the simulation results overestimate the velocity of the experimental robot.

This can be attributed to the friction between the two flagella that was observed in experiments. Since the amount of contact is more dominant at lower values of pitch, the experiments and the simulations differ further for the case with \( \bar{\lambda} = 5 \).
compared with \( \bar{\lambda} = 7 \) and \( \bar{\lambda} = 9 \) cases. The simulation tool, at this stage of our research, enforces non-penetration condition but does not incorporate friction. This implies that one flagellum can smoothly slide past another flagellum without any resistance from friction. However, that is not the case in the real world. In experiments, the flagella form a tighter bundle compared with simulations which leads to a lower net propulsive force forward. Incorporating physically-accurate friction inside low Reynolds environment is a direction of future research. The simulation tool presented in this paper, that models the entire system as a single rod for computational efficiency, can be used to explore various models of friction and eventually formulate an accurate model that matches experiments. A few recent works have explored friction among rods in simpler settings\cite{51,52} and our simulations can be augmented to include such friction models.

### 3.2. Comparison with Single-Flagellated Robot

In this section, we take one step towards a mechanistic understanding of the difference between these two modes of locomotion – single-flagellated and multi-flagellated. Locomotion of a robot (or bacterium) with single flagellum was recently investigated using a DER-based numerical framework\cite{34} A single-flagellated robot cannot exhibit bundling; however, it can undergo buckling instability beyond a critical value of the total angular velocity when the resulting external hydrodynamic force is too large. In Figure 8, we utilize our same simulation tool to model a single-flagellated robot.

We assumed that all the parameters are the same between the single-flagellated robot and the multi-flagellated case discussed above. The only difference is the number of flagella. Figure 8.a shows the Euclidean distance, \( L' \), between the head node and the tail node (the node at the free tip of the flagellum) as a function of the total angular velocity, \( \omega_T \), obtained from simulations. This apparent length, \( L' \), has been normalized by its value at \( \omega_T = 0 \) so that all the curves for three different values of pitch start at \((0, 1)\). The star symbols represent the angular velocity beyond which the apparent length, \( L' \), of the robot abruptly drops and the flagellum buckles. Two snapshots – one of an unbuckled configuration and one of a buckled shape – are also shown on Figure 8.a. For \( \bar{\lambda} = 5 \), the flagella buckle at \( \bar{\omega}_T \approx 230 \) and the \( \bar{\lambda} = 7 \) case buckles at \( \bar{\omega}_T \approx 363 \). The case for \( \bar{\lambda} = 9 \) does not buckle in the regime explored in this figure. The findings on single-flagellated robot are similar to the study in Huang et al.\cite{34}

![Figure 8](image)

**Figure 8.** Comparison between single-flagellated robot and multi-flagellated robot simulation: (a) Plot of normalized tail to head distance with respect to normalized angular velocity. Star symbols represent the critical angular velocity where the flagellum buckles. Rendered image of unbuckled (left), and buckled (right) state of the robot shown within the graph. (b) Figure of normalized propulsive force with respect to normalized angular velocity. Star symbols represent the buckling. The multi flagellated case does not exhibit buckling behavior.

Next, we employ the simulation tool to comparatively explore the propulsive forces of the two types of systems – single-flagellated and multi-flagellated. Figure 8.b shows the non-dimensionalized propulsive force as a function of total angular velocity at three different values of pitch for both the single-flagellated and multi-flagellated cases. Propulsive force is de-
fined as the \( x \)-component (direction of motion of the robot) of the force exerted on the head (Eq. 15, Supplementary methods section S2). The propulsive force for the multi-flagellated robot was divided by the number of flagella. The propulsive force generated by the robot is small (on the order of \( 10^{-3} \) N), which makes it difficult to experimentally measure. Our robotic platform does not have a force sensor and, therefore, we use simulations to analyze the propulsive force and efficiency of the soft robot. The trend is qualitatively different between the two cases. For single-flagellated robot buckles at a critical angular velocity (indicated by star symbols) and its propulsive force dramatically drops at that point. Multi-flagellated robot does not exhibit buckling behavior even at larger angular velocities; the flagella bundle instead. A point of note is the relatively larger propulsive force per flagellum in the single-flagellated robot prior to buckling than the multi-flagellated robot. The propulsive force depends on the deformed shape of the flagella and this shape differs between the single-flagellated case (no bundling, only buckling) and multi-flagellated case (prominent bundling). However, beyond the critical threshold for buckling in a single-flagellated robot, the propulsive force is larger in the multi-flagellated system. In short, single-flagellated robot generates larger propulsive force per flagellum; however, its propulsion is limited by an instability. The non-monotonic dependence of propulsion force on the geometry of the flagella (pitch) is observed in the multi-flagellated case. The robot with \( \lambda = 5 \) generates the largest force and \( \lambda = 7 \) generates the least; \( \lambda = 9 \) falls in between. In contrast, a single-flagellated robot generates more propulsion as the pitch decreases. However, this observation is true only for the range of parameters explored in this study. Prior works\(^{14}\) show non-monotonic dependence of propulsion on the pitch of the flagellum; however, the flagellum was assumed to be rigid.

The observations on propulsive force lead us to address the efficiency of the flagellated robots using numerical simulations. Figure 9 shows the variation of efficiency with the total angular velocity at three values of pitch in both the cases. Efficiency is defined as \( \eta = \frac{f_{\text{head}}}{T_h} \), where \( f_{\text{head}} \) is the hydrodynamic force on the head, and \( T_h \) is the torque due to rotation of the head. Qualitatively, efficiency is a measure of the ratio of the propulsive force and the torque exerted by the motor. At smaller values of angular velocity, multi-flagellated robot decreases in efficiency. This decrease is particularly prominent at small values of pitch and, thus, high interaction between flagella. Efficiency is the highest when \( \lambda = 7 \) and lowest at \( \lambda = 5 \), which further signifies the non-linear nature of the problem. If the robot has a single flagellum, there is no bundling and the shape of the flagellum remains almost helical until the threshold for buckling. Therefore, the efficiency remains almost constant as a function of angular velocity before buckling. The efficiency drops to almost zero post buckling.

4. Conclusion

In conclusion, we presented a multi-flagellated soft robotic platform and a numerical simulation method. These tools were used to explore the relationship between the motion of the robot and the geometry of the flagella. Both the experiments and simulations could capture the
bundling behavior of the elastic flagella caused by long range hydrodynamic interaction. Bundling is only possible if the flagella are flexible and there is long range interaction by flows induced by distant parts of the flagella. Prior works often neglected the flexibility of the flagella or ignored the long range hydrodynamics in favor of simplified (but computationally cheap) resistive force theory-based hydrodynamic model. Our study emphasizes the need to accurately capture these two ingredients – flexibility and long range hydrodynamics – in modelling bacteria and robots inspired by them. The simulation tool can successfully capture both the ingredients.

The accuracy of the simulation when compared to the experiment was reasonable, where our metrics for comparison were the translational velocity of the robot and the angular velocity of the head. Lack of an accurate friction model was identified as the main reason behind any mismatch between experiments and simulations. This simulation and the experimental platform gives a basis for the future development of multiple flagella based robots. The results were presented in non-dimensional format and, as long as the dimensionless groups are the same (e.g., low Reynolds number is low), they apply to robots and organisms of any size.

The robotic platform can be easily repurposed to explore tumbling – change in direction of swimming – when one flagellum rotates in a direction opposite to the other flagellum. In addition, the robot can be improved by integrating sensors and inertial measurement unit (IMU) to achieve 3D trajectory control of the robot.

Directions for future work include (1) analysis of the tumbling behavior, (2) incorporation of physically accurate friction model, and (3) formulation of control policy along with hardware improvement for an autonomous robot. Despite these limitations, we are close to realizing palm-sized flagellated robots that are simple in design (few moving part) and control (angular velocity is the only control input) yet functional (i.e., capable of following 3D trajectory by bundling and tumbling). With the ongoing advancements in material science and nano fabrication in creating tiny rotational actuators, we envision that our research, in the long term, can lead to small, simple, cheap but functional microbots for environmental monitoring, drug delivery, and inspection of engineering assets in hazardous and hard-to-reach locations.

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Author Disclosure Statement

Authors declare that they have no conflict of interest.

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Supplementary Information for

Bacteria Inspired Multi-Flagella Propelled Soft Robot at Low Reynolds Number

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This PDF file includes:
- Supplementary Methods
- Figs. S1 to S2
- Legend for Movie S1
- SI References

Other supplementary materials for this manuscript include the following:
- Movie S1
Supplementary Methods

Full details of physics-based simulation for multi-flagellated soft robot.

S1. Elastic Strains, Forces and Jacobians. In order to understand the Discrete Elastic Rod algorithm (DER), it is essential to understand the elastic strains, associated energy, gradient and hessian of the energy term. As described in the main text, there are three major elastic strains that a Kirchhoff rod-based formulation take into account: bending, twisting, and stretching. Bending and twisting strains are node-based quantities while stretching strain is an edge-based quantity. Bending is computed using the curvature binormal vector at each node:

$$ (\kappa_b)_k = \frac{2t_k e^{k-1} \cdot t_k}{1 + e^{2k-1} \cdot t_k^2} $$

[1] The magnitude of this vector is $2 \tan(\phi_k/2)$ where $\phi_k$ is the turning angle shown in Figure 4. The curvature vector (i.e. bending strain) at the $k$-th node is then

$$ \kappa_k = ( (\kappa_b)_k \cdot d_2^k, -(\kappa_b)_k \cdot d_1^k ) $$

[2] The twist at each node is

$$ \tau_k = \theta_k - \theta_{k-1} + m_{k,ref}, $$

[3] where $m_{k,ref}$ represents the reference twist (i.e. twist associated with the reference frame) and can be calculated from the reference frames (1). In order to account for the rotation of the motor, we included a “natural” twist angle, $\tau_{motor} = \omega_T \cdot t$ (where $t$ represents time), to the expression for twist at the two nodes representing the motors ($x_m1$, and $x_m2$) shown in Figure 4. Then the new equation for integrated twist at $x_m1$ and $x_m2$ becomes

$$ \tau_k = \theta_k - \theta_{k-1} + m_{k,ref} - \tau_{motor}, $$

[4] where $\tau_{motor}$ is zero everywhere except at $m1$ and $m2$. Axial stretching ($e^s$) is an edge-based quantity, which can be represented as

$$ e^s = \frac{|x_k+1 - x_k|}{|e^s|} - 1, $$

[5] where $|e^s|$ denotes undeformed magnitude of the $k$-th edge. The energy term associated with the elastic strains can be calculated to be

$$ E^t_k = \frac{1}{2} E A (e^t)^2 / |e^t|, $$

[6] $$ E^b_k = \frac{1}{2} E I (|\kappa_k - \kappa^0_k|)^2 / l_k, $$

[7] $$ E^s_k = \frac{1}{2} G J \tau_k^2 / l_k, $$

[8] where $EA = E \pi r^2$ is the stretching stiffness, $EI = E \pi r^3 / 4$ is the bending stiffness, $GJ = G \pi r^4 / 2$ is the twisting stiffness, $G = E / (2(1 + \nu))$ is the shearing modulus, and $l_k$ is the reference Voronoi length. $\delta_k$ is the Kronecker delta function ($\delta_{km} = 1$ if $k = m$; otherwise $\delta_{km} = 0$). The expressions for the Jacobian terms associated with the elastic forces are available in the literature (1).

S2. Hydrodynamics model. We used the Regularized Stokeslet Segments method for the hydrodynamic force on the flagella and Stokes law for the hydrodynamic force on the robot head. Built on the method of regularized Stokeslets, RSS method is beneficial to reduce the sensitivity of the velocity field to the regularization parameter due to its numerical treatment of a weakly singular integral. Cortez presented this method with the assumption that the force field along a filament is piece-wise linear and suggested the possibility of application to piece-wise quadratic or higher degree polynomial (2). Importantly, RSS method accounts for the long range hydrodynamic interaction among flows induced by different nodes on the flagellum. This interaction is ignored by widely used simplified methods, also known as Resistive Force Theory (3). Referring to Figure S1, RSS provides a relationship between the velocity at a point ($\mathbf{v}(\mathbf{x})$ in Figure S1) and the forces applied by each node on the fluid such that
\begin{equation}
8\pi \mu v(\hat{x}) = \sum_{k=0}^{N-2} (M_k^h f_k^h + M_2^k f_{k+1}^h),
\end{equation}

where $f_k^h$ is the force vector of size 3 that represents the force applied by the $k$-th node onto the fluid. This is equal and opposite to the hydrodynamic force onto the $k$-th node. The hydrodynamic force on the $k$-th node is the $4k$, $(4k+1)$, and $(4k+2)$-th elements of the hydrodynamic force vector $f_k^h$ of size $(4N-1)$ in equation 3 in the main text. The matrices (size $3 \times 3$) $M_k^h$ and $M_2^h$ are

\begin{equation}
M_2^k = |s_k|[(T_{k+1}^{h,k+1} + \epsilon^2 T_{k-1}^{h,k-1}) + T_{k+1}^{h,k+1} (r_k r_k^T) + ] T_{k+1}^{h,k+1} (r_k s_k^T + s_k r_k^T) + T_{k+1}^{h,k+1} (s_k s_k^T)],
\end{equation}

\begin{equation}
M_1^k = |s_k|[(T_{0,1}^{h,k+1} + \epsilon^2 T_{0,1}^{h,k-1}) + T_{0,1}^{h,k+1} (r_k r_k^T) + ] T_{0,1}^{h,k+1} (r_k s_k^T + s_k r_k^T) + T_{0,1}^{h,k+1} (s_k s_k^T)],
\end{equation}

where, as shown in Figure S1, \( x \) is the point of evaluation, \( \epsilon \) is regularization parameter, \( r_k = x - x_k \), \( s_k = x_k - x_{k+1} \), \( n \) is 3-by-3 identity matrix, and the scalar quantities denoted by \( T \) (e.g. \( T_{1,1}^{h,k+1} \)) are described next.

\begin{equation}
T_{0,1}^{h,k+1} = \frac{1}{|s_k|} \log([|s_k| R + (x_0 \cdot s_k)])
\end{equation}

\begin{equation}
T_{1,1}^{h,k+1} = \frac{1}{R(|s_k| R + (x_0 \cdot s_k))^2}
\end{equation}

\begin{equation}
T_{2,1}^{h,k+1} = \frac{1}{R(|s_k| R + (x_0 \cdot s_k))^2}
\end{equation}

\begin{equation}
T_{3,1}^{h,k+1} = \frac{1}{R(|s_k| R + (x_0 \cdot s_k))^2}
\end{equation}

where $x_\alpha = x_k - \alpha s_k$, and $R = \sqrt{|x_\alpha|^2 + \epsilon^2}$. Equation 12 can be used to formulate 3N equations (3 per node) that relate the velocities at each node with the forces applied by all the other nodes. Knowing the velocity of each node at the beginning of

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the time step in DER, this linear system of equations can be solved to obtain the forces and compute the hydrodynamic force vector $F$. Since the gradient of the forces with respect to the DOF vector is not available, this force is treated explicitly (Euler forward) in the simulation scheme. Complete details are found in Refs. (2) and (4).

Previously, MRS method prevailed for the analysis in low Reynolds hydrodynamics using Stokeslet methods, however, due to its dependence on the distance between contiguous cutoff functions, the choice of regularization parameter $\epsilon$ limited the accuracy of calculation. However, RSS method accounts for a continuum of regularized forces therefore decouples the necessity between discretization and the regularization. The regularization parameter used in RSS method $\epsilon$ for flagella could be interpreted as the radius of the slender filaments. Based on the analysis shown from Cortez (2) we used the regularization value of $\epsilon = 1.031 \cdot r_0 = 0.00165$ (m).

We now turn to the computation of the hydrodynamic forces on the head ($F_{\text{head}}$ in equation 3 in the main text). The middle node along the entire structure ($x_h$ in Figure 4) represents the head. As the head is translating (quantified by the velocity of $x_h$), the viscous medium exerts a drag force onto it. The head is also rotating (quantified by the angular velocity of the head, $\omega_h$) and the viscous fluid applies a torque to resist that rotation. We applied Stokes’ law to model the hydrodynamic drag. Since Stokes’ law is meant for spherical bodies and the robotic head is cylindrical, we used two numerical prefactors as fitting parameters as discussed next.

The hydrodynamic force on the head ($x_h$) at time $t = t_{i+1}$ is

$$F_{\text{head}}^h = -C_t \cdot 6\pi \mu r_h \left[ \frac{x_h(t_{i+1}) - x_h(t_i)}{\Delta t} \right], \quad [15]$$

where $C_t$ is a numerical prefactor to account for the non-spherical shape of the head, $r_h$ is the radius of the head (see Table 1), and $x_h$ is the position of the head node that can be extracted from the DOF vector, $q$. If the head is the $h$-th node in the structure, the vector $F_{\text{head}}^h$ of size 3 represents $(4h - 3), (4h - 2)$, and $(4h - 1)$-th elements of the vector $F_{\text{head}}$ in Eq.3.

The torque due to rotation of the head is $T_h = -C_r \cdot 8\pi \mu r_h^2 \omega_h$, where $C_r$ is a numerical prefactor due to the non-spherical shape of the head and $\omega_h$ is the angular velocity of the head. The angular velocity at $t = t_{i+1}$ can be computed from the velocities of the two neighboring nodes $x_{h-1}$, and $x_{h+1}$ (see Figure 4) such that

$$\omega_h = \frac{1}{|x_{h+1}(t_{i+1}) - x_{h-1}(t_{i+1})|^2} \left( |x_{h+1}(t_{i+1}) - x_{h-1}(t_{i+1})| \times [x_{h+1} - x_{h-1}] \right), \quad [16]$$

where $x_{h+1} = x_{h+1}(t_{i+1}) - x_{h+1}(t_i)$ is the velocity of the $(h+1)$-th node, $x_{h-1} = x_{h-1}(t_{i+1}) - x_{h-1}(t_i)$ is the velocity of the $(h-1)$-th node, and $\times$ represents vector cross product. The hydrodynamic torque is implemented in the simulation as a force-couple, i.e. a force on the node $x_{h-1}$ and an equal but opposite force on the node $x_{h+1}$. It turns out that, in case of the specific problem studied in this paper, the magnitude of each force in the force-couple can be approximated to a very good degree as $T_h/|x_{h+1}(t_{i+1}) - x_{h-1}(t_{i+1})|$ and the angle between two vectors in the right-side of equation 16 is 90°. The direction of the force can be approximated to be equal to $[x_{h+1} - x_{h-1}]$. The reason behind these approximations is they result in a simplified expression for the forces and allow us to take the gradient with respect to the DOFs (so that the forces are incorporated into the simulation implicitly). The resulting force on the $(h+1)$-th node is

$$F_{\text{head}}^{h+1} = -C_r \cdot 8\pi \mu r_h^2 \left[ \frac{r_h^2}{|x_{h+1}(t_{i+1}) - x_{h-1}(t_{i+1})|^2} \right] [x_{h+1} - x_{h-1}]. \quad [17]$$

The force on $x_{h-1}$ is

$$F_{\text{head}}^{h-1} = -F_{\text{head}}^{h+1}. \quad [18]$$

Equations 15, 17, and 18 are used to calculate the hydrodynamic forces on the head and populate the $(4N - 1)$-sized $F_{\text{head}}$ vector. Note that this vector has only 9 non-zero elements (resulting from the forces on 3 nodes).

We fitted the parameters $C_t$ and $C_r$ with the experiment velocity along the $x$-axis which is defined in Figure 5 and head rotation speed for $\lambda = 7$ case. The values for $C_t$ varied from 4.9 − 5.1 and $C_r$ varied from 0.6 ∼ 1.5 for the simulation sample data. The mean total least squared error from the experimental values were the smallest for the head angular velocity when $C_t = 9$, and $C_r = 4.1$ for the $x$-velocity. We used these values for the head hydrodynamics for analysis of other geometric cases for flagella as well ($\lambda = 5, \lambda = 9$).

**S3. Contact model.** In this section, the constraint-based contact forces are explained based on the non-penetrative condition between the two edges; we refer the reader to Ref. (5) for complete details. Figure S2 shows two edge segments undergoing collision. We denote the edge segments as $S_k = (x_k, x_{k+1})$ and $S_m = (x_m, x_{m+1})$, where $x_k, x_{k+1}, x_m,$ and $x_{m+1}$ can be extracted from our DOF vector $q$. In order to formulate a non-penetrative condition, penetration depth ($\epsilon_{k,m}$) between $S_m$ and $S_k$ and sum of the radii of the segment $S_k$ and $S_m$:

$$\epsilon_{k,m} = 2r_0 - \delta_{k,m}^{\text{penetration}}. \quad [19]$$

In case, the radius of all the segments are the same and therefore the sum of the radii is always $2r_0$. 

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A contact is detected when $\epsilon_{k,m} > 0$. The schematic shown in Figure S2 visualizes the contact condition. After detecting contact, iterations to find interference-free configuration needs to be executed. Using the detected points, the collision displacements are weighted with the barycentric coordinates of the collision to account for the conservation of mass such that

$$
\Delta x_k = -\frac{1}{2} n_{km} w_k
$$

$$
\Delta x_{k+1} = -\frac{1}{2} n_{km}(1 - w_k)
$$

$$
\Delta x_m = \frac{1}{2} n_{km} w_m
$$

$$
\Delta x_{m+1} = \frac{1}{2} n_{km}(1 - w_m).
$$

Here, the value of $\frac{1}{2}$ represents the barycentric ratio of masses with all points having same masses, $n_{km}$ represents the minimum distance vector between $S_k$ and $S_m$, $w_k$ represents the barycentric coordinate of the contact point on the segment. Iterative process over all the detected contact positions and summing the displacement points for each $x_k$, the displacements with mass conservation could be obtained. We use the penetration depth and compare it with the error tolerance until $\epsilon_{k,m} < \text{tolerance}$ for all the detected contact points. The contact force applied at $k$-th node due to collision between the contact segments $S_k$ and $S_m$, can be evaluated using the $\Delta x_k$ values and is represented as

$$
f^c_k = \frac{1}{\Delta t^2} \Delta x_k m_k,
$$

where $\Delta x_k$ represents weighted collision displacements with mass conservation consideration, $\Delta t$ represents the time discretization, $m_k$ represents the mass of the point (5).

While formulating the DER with contact, we found out that the level of discretization is limited by contact function. Contact method by Spillman and Teschner (5) considers the diameter of the rod so that when two nodes are within the boundary of its diameter, contact could be detected. Therefore, contact limits our length of discretized segment $|e|$ to be always greater than the diameter of the rod $2r_0$.

While the limitation in level of discretization could be a limitation, we used RSS method with our particular choice of the regularization parameter that decouples the viscous force along a line segment from discretization to overcome the limitation. It is shown by Cortez that the level of discretization has insignificant effect on the swimming speed and the trajectory waveform (2).

Movie S1. Comparison video of multi-flagellated robotic platform and simulation with flagella geometry variation

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