The effect of impact parameters on the formation of massive black hole binaries in galactic mergers

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Abstract

By employing N-body simulations, we investigate the formation of massive black hole binaries (MBHBs) through the sinking of two massive black holes (MBHs) during galaxy mergers. With different impact parameters and different central stellar density of the progenitor galaxies, we analyze the orbits of the MBHs from the beginning of the merger until the time when the bound MBHB forms. Contrary to the previous theory that the timing of the dual MBHs entering their dynamical radius is similar as the timing of the formation of the bound MBHB, we find that these two timings could deviate when the central stellar density of the progenitors galaxies are lower. On the other hand, when the central stellar density of the progenitor galaxies is higher and the mergers have small impact parameters, each MBHs would move directly into the core radius of the other progenitor galaxies, and therefore cause a variation in the timings of the MBHB formation.

Keywords

Methods: numerical · Galaxies: kinematics and dynamics · Galaxies: interactions · Black hole physics

1 Introduction

Massive black holes (MBHs), viewed as the central engines of active galactic nuclei (AGNs), are now generally considered a fundamental component of massive galaxies due to their ubiquitous detection in these galaxies. It is known that many AGNs are found at very high redshift. For example, a luminous quasar ULAS J112001.48+064124.3 was reported to host a MBH of $2 \times 10^9 \, M_\odot$ at redshift $z \sim 7.085$ (Mortlock et al. 2011), and a quasar ULAS J134208.10+092838.61 was found to host a MBH with a mass of $8 \times 10^8 \, M_\odot$ at redshift $z \sim 7.54$ (Bañados et al. 2018). These results imply that MBHs shall exist in the very early Universe.

In the hierarchical formation scenario of galaxies in the Local Group, such as the Milky Way and the Andromeda Galaxy, are expected to have accreted their mass through mergers of progenitor galaxies (Cole et al. 2000; Behroozi et al. 2013; Somerville and Davé 2015; Behroozi et al. 2019). Massive elliptical galaxies are also speculated to form through major mergers of galaxies (Bender 1996; De Lucia et al. 2006; Bournaud et al. 2007; Deeley et al. 2017). Because MBHs are ubiquitously found in these massive galaxies, which very likely have experienced mergers in the past, it is important to study the evolution of pair MBHs during merger, particularly the formation of massive black hole binaries (MBHBs).

The formation of MBHBs is also important in terms of the connection between the AGN periods and galaxy mergers. The first thorough discussion by Begelman et al. (1980) showed that the motion of black hole binaries in a newly merged galaxy may cause the bending or precession features in some AGN jets. However, due to observational limitations, such as the limitation of angular resolutions, the number of resolved MBHBs is still small (Maness et al. 2004; Burke-Spolaor 2011; Tremblay et al. 2016) and thus the evolution of MBHBs in the Universe still remains largely unclear.

The formation of MBHB was divided into four stages by Yu (2002), including the dynamical friction stage, the non-hard binary stage, the hard binary stage, and the gravitational radiation stage. In the dynamical friction stage, two MBHs that are originally located at the center of the progenitor galaxies lose energy due to dynamical friction and
therefore migrate towards the center of the merged system (Quinlan 1996; Yu 2002; Milosavljević and Merritt 2003; Bortolas et al. 2016). These two MBHs fall from a kpc-scale separation down to a pc-scale separation and eventually are bound by gravity. This marks the formation of the MBHB and the end of the dynamical friction stage. The timescale of this stage strongly depends on the geometry of the MBHs’ orbits and the stellar density surrounding the MBHs. In addition, this is also the stage at which dual or offset AGNs can appear (Liu et al. 2010; Comerford et al. 2015; Hou et al. 2019). Therefore, estimating the timescale of the dynamical friction stage is crucial in terms of predicting the lifetime of dual AGNs.

The non-hard binary stage is the stage where the MBHB keeps losing energy until its orbital speed becomes comparable to the velocity dispersion of the stars in the central galactic region (Begelman et al. 1980). In this stage, the MBHB loses its orbital energy through interactions with nearby stars, and therefore its semi-major axis decreases. This process is so-called dynamical hardening. During this process, dynamical friction gradually becomes less dominant because the velocity of the MBHs increase.

The third stage is the hard binary stage at which the orbital speed of the MBHB is comparable to the velocity dispersion of the central stars (Begelman et al. 1980), and thus the MBHB is considered a hard binary. In this stage, dynamical friction no longer plays an important role since only a small portion of stars can get sufficiently close to the hard MBHB. Moreover, the MBHB would clear off a so-called “loss cone” region of stars in the phase space (Quinlan 1996; Merritt and Poon 2004; Merritt 2006). The timescale of this stage depends on the re-population of stars into the loss cone.

Finally, in the gravitational radiation stage, when enough stars can diffuse into the loss cone, the MBHB continues to harden to the point where gravitational wave radiation takes over as the main mechanism subtracting the orbital energy. Eventually, the MBHB will coalesce (Quinlan 1996).

Numerical simulations are powerful tools to investigate the formation of MBHBs (Quinlan 1996; Berczik et al. 2006; Berentzen et al. 2009; Khan et al. 2012). However, due to the wide range in spatial resolution and the involvement of various physical mechanisms, it is very difficult to trace the evolution of the MBHs from the very beginning, when the MBHs were originally located at the centers of each progenitor galaxy, down to the scales where gravitational wave radiation drives the MBHB to coalescence, and leads to the formation of a more massive MBH (Khan et al. 2016). As the total number of particles \(N\) is one of the important parameters in numerical simulations, Makino (1997) performed N-body simulations with \(2 \times 10^3 \leq N \leq 2.56 \times 10^5\) to study the effects of \(N\) on the formation and evolution of MBHBs during galaxy mergers. They found that the evolution timescale of an MBHB is independent of \(N\) until the separation of the MBHB reaches a critical value. Berczik et al. (2006) further used \(2.5 \times 10^4 \leq N \leq 10^6\) to study the evolution of MBHBs and found that the hardening rate (the rate at which the semimajor axis of MBHB decreases) is independent of \(N\). Since hard binaries are formed at parsec separations (Quinlan 1996; Yu 2002), the above results suggest that N-body simulations with \(N \sim \text{tens-of-thousand particles}\) should be sufficient to study the orbital evolution of the MBHBs from kpc to pc scale.

As shown in Yu (2002), whether an MBHB could form within a Hubble time would depend on the mass ratio of two MBHs. Using N-body simulations, Khan et al. (2012) found that the timescale for forming a MBHB is well within a Hubble time when it forms by the merger of two equal-mass MBHs. However, for a pair of MBHs with a small mass ratio, the timescale is too large to form a MBHB within a Hubble time. In addition, it was demonstrated that the timescale of the dynamical friction stage is associated with the stellar density profile around MBHs. Using the analytical method and N-body integrations, Dosopoulou and Antonini (2017) found that the dynamical friction timescale can be long for shallower central stellar density cusps (\(\rho < r^{-1}\)), and minor mergers with mass ratio \(q < 10^{-3}\) could result in the less-massive MBH stalling at a distance of one-tenth the influence radius of the more-massive MBH.

To understand the effects of gas on the dynamical evolution of MBHBs, Pfister et al. (2017) used hydrodynamical simulations to evaluate the process of the MBHBs formation. They found that under a smooth stellar potential of the galactic nucleus, dynamical friction is the main mechanism driving the MBHs from kpc to pc scale and gas does not play an important role when the initial gas fraction is 30%. A higher gas fraction up to 50% was considered by Tamburello et al. (2017). They studied the evolution of an MBH pair (with a mass ratio of 0.2) in a clumpy gaseous disk, and found that the orbital decay of the less-massive MBH could be either accelerated or delayed depending on whether a positive or negative torque was exerted on the less-massive MBH by the surrounding massive clumps. In some cases, the time-delay of the orbital decay could result in the formation of MBHBs taking longer than 1 Gyr. Furthermore, on smaller scales (\(\lesssim 100\) pc), Fiacconi et al. (2013) studied the evolution of two unequal mass MBHs in a clumpy gaseous disk, and showed stochastic behavior of the MBH orbits in their simulations. The less-massive MBH was perturbed by massive clumps and then scattered out of the disk plane. Its orbital decay timescales is therefore longer compared to the case where MBHs are in a smooth gaseous disk, and ranges from \(~1\) to \(~50\) Myr.

In addition to examining the effects of gas, several studies have also been performed to investigate the effects of subgrid physics, such as gas cooling, star formation, supernova
feedback and feedback from black holes. For example, Lupi et al. (2015) considered mergers of two disk galaxies with a gas fraction of ∼30%, and found that the MBH orbits are not significantly affected by the recipes used to model the supernova feedback, but mainly affected by the gas clumpiness. In their simulations, the orbit decay timescale is ∼10 to 20 Myr. In addition, Souza Lima et al. (2017) investigated the effects of feedback from black holes (BHs). The timescale of the orbital decay of two BHs could be increased from ∼10 Myr to more than ∼300 Myr in their simulations when the feedback from black holes is considered. They suggested that the delay of the orbital decay is due to the wake evaporation effect, in which a hot bubble is created behind the less-massive BH and expels gas from its corresponding region. Therefore, the effect of the dynamical friction is reduced and the orbital decay is delayed.

In this work, we employed gravitational N-body simulations to study the formation of MBHBs over a range of different impact parameters and investigated the orbital evolution of the MBHB from 300 kpc to sub-kpc separations. In addition, we also built our progenitor galaxies with different core radii, so that we can study the effects of different impact parameters as well as core radii on the decay timescale of the MBHs orbit and the formation time of the MBHBs.

For simplicity, we only considered spherical progenitors without rotation and did not consider the hydrodynamical effects, i.e. we focused on a major merger between two gas-poor spherical galaxies, although it is clear that gas may play a role in the decay timescale of the MBHB orbit based on the discussions in the previous paragraphs. If gas were included, clumps could form during the merging process and exert dynamical frictions onto the MBHs. Thus, the orbital decay of MBHs would be enhanced (Pfister et al. 2017; Li et al. 2020). A qualitative study exploring the effects of gas in our simulations will be investigated in future work. This paper is organized as follows. The model is described in Sect. 2. The results are presented in Sect. 3. The discussions about physical implications are mentioned in Sect. 4, and the study is concluded in Sect. 5.

2 Numerical simulations

According to Milosavljević and Merritt (2001), the simulations of galactic mergers with MBHs can be done by tree codes. As shown in their Fig. 1, the time evolution of the stellar density and the stellar velocity dispersion near MBHs obtained by tree codes are completely consistent with those in direct N-body calculations. Only after the MBHB becomes a hard binary, the orbital evolution obtained from tree codes would start to differ from the results of direct N-body calculations.

In this work, since we only focus on the orbital evolution of the MBHs before a hard binary is formed, the publicly available tree-PM code GADGET-2 (Springel 2005) is adequate to perform simulations to achieve our goals. The initial galaxy models and the setup of mergers are presented below.

2.1 Progenitor models

Our progenitor galaxy consists of three components, including a MBH, a stellar component, and a dark halo. The MBH is represented by one massive particle at the center of the galaxy. The density profiles of the stellar component and the dark halo in the galaxy follow spherical symmetric models in Hernquist (1990) and Hernquist (1993). The density profile of the stellar component is

\[
\rho_s(r) = \frac{M_s}{2\pi r} \frac{1}{\left(r + r_s\right)^3},
\]

where \(M_s\) is the total stellar mass, and \(r_s\) is the characteristic radius, which determines the half-mass radius \(r_{1/2} = (1 + \sqrt{2})r_s\). The density profile of the dark halo follows the form:

\[
\rho_h(r) = \frac{M_h}{2\pi^{3/2} r_0} \frac{\alpha \exp(-r^2/r_h^2)}{r^2 + r_h^2},
\]

where \(M_h\) is the halo mass, \(r_0\) is the cutoff radius, and \(r_h\) is the characteristic radius of the profile. The normalization constant \(\alpha\) is defined as

\[
\alpha = \left\{ 1 - \sqrt{\pi q} \exp(q^2) \left[ 1 - \text{erf}(q) \right] \right\}^{-1},
\]

where \(q = r_h/r_0\). The initial velocities of particles were set to be in dynamical equilibrium, which can be achieved by calculating the distribution function over the spherical shells as described in Binney (1982).

Furthermore, in our galaxy model, the mass ratio between each component follows the statistical results in Kormendy and Ho (2013), Moster et al. (2010) and Larkin and McLaughlin (2016). The mass ratio between the MBH and the stellar part was obtained through the scaling relation derived by Kormendy and Ho (2013):

\[
\frac{M_\bullet}{10^7 M_\odot} = \left( 0.49 \pm 0.06 \right) \left( \frac{M_s}{10^{11} M_\odot} \right)^{1.16 \pm 0.08},
\]

where \(M_\bullet\) is the MBH mass, and \(M_s\) is the total stellar mass of the galaxy. The relation between the stellar and dark halo mass is

\[
\frac{M_s}{M_h} = 0.0564 \left\{ \left[ \frac{M_h}{7.66 \times 10^{11} M_\odot} \right]^{-1.06} + \left[ \frac{M_h}{7.66 \times 10^{11} M_\odot} \right]^{0.556} \right\}^{-1},
\]

where \(M_h\) is the dark halo mass (Moster et al. 2010; Larkin and McLaughlin 2016).
Fig. 1 An illustration of the merger simulation setup, where c.o.m. is the center of mass, $R$ is the orbital radius. The galaxies are assumed to have parabolic encounters, and the initial velocities are set to be anti-parallel. The impact parameter $b$ is set to be 0, 10, 20, 30, 40, 50, 60, 70, and 80 kpc.

Given that the MBH mass of $10^8 M_\odot$ is adopted in our galaxy model, the total stellar mass of the galaxy $M_\star$ is $2.5410 \times 10^{10} M_\odot$ and the dark halo mass $M_h$ is $8.7620 \times 10^{11} M_\odot$. In addition, we adopt $r_h = 10$ kpc and $r_0 = 25$ kpc in all our galaxy models. However, the $r_s$ is set to either 5 kpc or 8 kpc in two different galaxy models. With this setup, the stellar part is embedded within the dark halo. The number of particles representing the stellar component and dark halo in one galaxy are 127,050 and 730,164, respectively.

2.2 Merger setup

Fig. 1 shows a schematic diagram of the initial setup of our simulations. The center of mass of two MBHs is placed at the coordinate origin in all merger simulations. The radial distance from one MBH to this center is called orbital radius and denoted by $R$. It is set to be 150 kpc initially for both MBHs, so the initial separation between two MBHs is $2R = 300$ kpc. We assume the approach of the progenitor galaxies to be a parabolic encounter. Therefore, the initial velocity of each progenitor would be

$$v_0 = \sqrt{\frac{GM}{2R}},$$

where $M$ is the total mass of each progenitor. To investigate the effects of the impact parameter $b$, for each galaxy model, we perform nine simulations with impact parameters ranging from 0 to 80 kpc with a step of 10 kpc.

The gravitational softening lengths are set to be $10^{-5}$ kpc for the MBH particles, 0.03 kpc for the stellar particles, and 0.27 kpc for the dark halo particles. The rest-frame is set to be at the center of mass of the two MBHs, so that the pair of MBHs would meet near the coordinate origin. We run the simulations until the MBHs form a bound binary, or until the evolution reaches 13 Gyr, i.e. about one Hubble time.

3 Results

In this section, we present the evolution of MBH sinking process, the MBHB orbits, and the stellar structure of the merged core in two different models. Hereafter, we use “Model A” referring the mergers of the progenitor galaxies for which the characteristic radius $r_s$ is 5 kpc, and “Model B” for the mergers of the galaxies for which $r_s$ is 8 kpc. In addition, each model contains nine simulations with different impact parameters ranging from 0 to 80 kpc, as mentioned in Sect. 2.2. Then, we compared the results between Model A and Model B to see the effects of impact parameter and $r_s$ of the progenitors on the formation and evolution of MBHBs.

3.1 The MBH sinking

In addition to the orbital radius mentioned in Sect. 2, to better understand the orbital evolution of the two MBHs, we define two characteristic radii below, including the core radius $r_c$ and the dynamical radius $r_{\text{dyn}}$. The evolution of each radius and the timings of the MBHs reaching $r_c$ and $r_{\text{dyn}}$ will be presented in this section. Note that the orbital radius of the two MBHs are identical in our simulations because we perform axisymmetric mergers with two equal-mass galaxies. Therefore, in the following we only present one orbital radius $R$ for both MBHs.

$r_c$ and $r_{\text{dyn}}$ are defined in the same reference frame as the orbital radius mentioned in Sect. 2, that is the spherical coordinate system centered at the center of mass of the two MBHs. The core radius $r_c$ is useful to trace the evolution of the stellar component during the merger, and is defined as

$$r_c = \frac{r_e}{8},$$

where $r_e$ is the effective radius, which is the radius that encloses half of the total stellar mass of two galaxies. This definition is the same as in Jorgensen et al. (1995) and Ferrarese and Merritt (2000) for the purpose of the measurements of velocity dispersion. On the other hand, we define the dynamical radius $r_{\text{dyn}}$ as the radius that contains the stellar mass equaling to the MBHB mass $M_{\text{BB}}$ (Binney and Tremaine 1987):

$$M_\star(r_{\text{dyn}}) \equiv M_{\text{BB}} = 2M_\star.$$

$$
As the MBHs migrate inward during the merger process, the orbital radius $R$ could become comparable to $r_c$ and $r_{\text{dyn}}$. However, since the MBHs are not on circular orbits, their orbital radius $R$ may oscillate, which means that $R$ becomes smaller or larger than $r_c$ and $r_{\text{dyn}}$. This process can be repeated several times. The final timing of the orbital radius $R$ being the same as the core radius $r_c$ is defined as $t_c$, i.e.

$$R(t_c) = r_c.$$  \hfill (9)

Similarly, $t_{\text{dyn}}$ is defined as the final timing when $R$ is the same as $r_{\text{dyn}}$, i.e.

$$R(t_{\text{dyn}}) = r_{\text{dyn}}.$$  \hfill (10)

Fig. 2 presents the time evolution of the core radius $r_c$ (blue line), the dynamical radius $r_{\text{dyn}}$ (red line), and the orbital radius $R$ (black line) of Model A. In Fig. 2, nine panels beginning from top-left to bottom-right are for different impact parameters $b$. In general, the core radius $r_c$ decreases from an initial value of $\sim 19$ kpc, and settles to $\sim 2$ kpc. The dynamical radius $r_{\text{dyn}}$ begins from around 100 kpc, and settles to $\sim 1$ kpc. Since the definitions of these radii are based on the cumulative mass profile of the merger remnant, the point where both radius become stationary shows that the merger remnant has reached a stabilized mass profile. The orbital radius $R$ decreases from an initial value of 150 kpc, and exhibits several damping oscillations. Finally it reaches saturation at $\sim 10^{-2}$ kpc. As the orbital radius decreases, $t_c$ and $t_{\text{dyn}}$ can then be determined, as indicated by the circle and the triangle in Fig. 2, respectively. It is clear that, for all impact parameters, the timings $t_c$ and $t_{\text{dyn}}$ are all located near $t = 2.0$ Gyrs, and have no clear dependence on $b$.

Fig. 3 shows the similar analysis for Model B. The time evolution of $r_c$ and $r_{\text{dyn}}$ are basically the same in both Model A and Model B, except that the final values in Model B are slightly larger than those in Model A. Table 1 summarizes the final values of $r_c$ and $r_{\text{dyn}}$ for both models. Because the mass of the MBHB is the same ($M_{\text{BB}} = 1.6 \times 10^9 M_{\odot}$), a greater final value of $r_{\text{dyn}}$ in Model B means a lower mean density within $r_{\text{dyn}}$. In addition, we found that the final values of $r_c$ in Model B are systematically greater than those of Model A by $\sim 0.5$ kpc. Since the total mass in both models are the same and $r_c$ is derived from $r_e$ (see Equation(7)), the larger values of $r_c$ in Model B imply that the values of $r_e$ in Model B are systematically larger and the core stellar
density slope of the remnants in Model B are shallower than those in Model A.

The evolution of $R$, however, shows a completely different feature in Model A and B. Comparing Fig. 2 and Fig. 3, we notice that the orbital radii $R$ decrease much slower in Model B than those in Model A, and the dual MBHs remain for a longer period between $t_c$ and $t_{\text{dyn}}$ in Model B. Except for the mergers with the impact parameter $b = 0$ and $b = 10$, all other dual MBHs in Model B seem to slow down their orbital decay rate at a separation of $\sim 1$ kpc after $t_{\text{dyn}}$. This is probably because the stellar particles originally surrounding the MBHs are scattered away during the merging process, therefore the efficiency of the dynamical friction decreases. The case of $b = 80$ even has $t_{\text{dyn}} > 6$ Gyr.

After the MBHs reach and remain within $r_{\text{dyn}}$, the dual MBHs continued to dissipate their orbital energy to the surrounding particles via dynamical friction, to eventually form a bounded MBHB. However, during the evolution, the dual MBHs may oscillate between bound and unbound state due to the interaction with the stellar and dark halo particles.

The MBHB’s orbital energy $E_{\text{BB}}$ is defined as

$$E_{\text{BB}}(t) = \frac{2(K + U)}{M_\bullet},$$

where $K$ is the MBHB’s kinetic energy, and $U$ is the potential energy. Considering the MBHB as an isolated system, we derive $K$ by removing the center of mass velocity of the MBHB, and neglect the potential of other particles. The final timing when $E_{\text{BB}} = 0$ is defined as the bound time, $t_b$.

Fig. 4 shows the time evolution of the MBHB orbital energy for both Model A and Model B. The black and red squares indicate the final timings when $E_{\text{BB}}$ drops to zero, i.e. the bound time $t_b$, in Model A and Model B, respectively. In Model A, the orbital energy goes up and down and finally becomes negative. Clearly, $t_b$ is right after $t = 2$ Gyr for all mergers with nine different impact parameters. This result, together with Fig. 2, shows that this timing $t_b$ always follows after $t_{\text{dyn}}$, in agreement with the previous studies (Merritt 2006; Bortolas et al. 2018). However, for Model B, not all dual MBHs could form a bound binary within 13 Gyr. Only the head-on merger shows similar evolution as in Model A, but the other cases have dual MBHs alternating between bound and unbound for very long periods.

The fact of having a large time difference between $t_b$ and $t_{\text{dyn}}$ in Model B is contrary to the previous studies that show $t_b \approx t_{\text{dyn}}$ (Yu 2002; Merritt 2006), and has significant implications. First, it implies the fundamental difference between
Table 1 Final values of the defined radii $r_c$ and $r_{dyn}$

| $b$  | 0   | 10  | 20  | 30  | 40  | 50  | 60  | 70  | 80  |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $r_{c,A}$ | 2.0293 | 2.0275 | 2.0034 | 2.0339 | 2.0396 | 2.0166 | 2.0630 | 2.0486 | 2.0914 |
| $r_{c,B}$ | 2.0425 | 2.5199 | 2.5285 | 2.5324 | 2.5048 | 2.5150 | 2.5536 | 2.5915 | 2.6448 |
| $r_{dyn,A}$ | 0.5799 | 0.6220 | 0.4693 | 0.5668 | 0.5394 | 0.5520 | 0.6243 | 0.6087 | 0.5088 |
| $r_{dyn,B}$ | 0.71499 | 1.3629 | 1.3815 | 1.3760 | 1.4238 | 1.3992 | 1.4312 | 1.4096 | 1.4510 |

Table of the final core radii and dynamical radii of the merger remnant. $r_{c,A}$ and $r_{c,B}$ are the core radii of Model A and Model B, respectively. Similarly, $r_{dyn,A}$ and $r_{dyn,B}$ are the dynamical radii of the two models.

All units are in kpc

Fig. 4 Time evolution of the MBHB orbital energy $E_{BB}$. The impact parameter $b$ is shown in the upper-right corner of each panel. The black dotted line represents $E_{BB}$ for Model A, and the red dotted line is for Model B. The black and red squares indicate $t_b$, i.e. the final timings when $E_{BB} = 0$, in Model A and Model B, respectively. Note that the mergers with the impact parameter $b = 50$ and $b = 80$ could not form a bound binary within 13 Gyr.

The definitions of $E_{BB}$ and $r_{dyn}$. $E_{BB}$ describes a dynamical state of the dual MBH system, and does not depend on the surrounding environment directly. However $r_{dyn}$ is defined through the enclosed stellar mass compared with the MBHB mass only, which has nothing to do with the dynamics of the dual MBHs. Secondly, our simulations focus on different combinations of density structure of the progenitors and merger geometries. Thus, the discrepancy between $t_b$ and $t_{dyn}$ reflects the fact that the evolution of the MBHB is a complicated process affected by many factors. In Model A, a denser stellar core surrounding the initial MBH enables the core to maintain its original structure for a longer period during the merging process. Therefore a stronger dynamical friction could drive the dual MBHs to a closer distance, and eventually form a bound MBHB at the radius $\sim r_{dyn}$. While in Model B, a larger stellar core with a shallower central stellar density is adopted. Since the merger remnant in Model B has a more diffuse central stellar density, a longer time is re-
Fig. 5 The important timings over different impact parameters. Please see the main text for the definitions of \( t_{10}, t_c, t_{\text{dyn}}, \) and \( t_b \). The black markers are for Model A, and the red markers are for Model B. Note that the \( b = 50 \) and the \( b = 80 \) cases both have \( t_b > 13 \) Gyr, therefore are not shown in the figure.

required for dynamical frictions to diminish the energy of the dual MBHs until forming a bound pair. Even after the MBHs reach the radius \( r_{\text{dyn}} \) of \( 1.36 \sim 1.45 \) kpc, their orbital radius continue reducing but the two MBHs remain unbound.

The above results show that the effects of the stellar density structure of the progenitor and the merger geometry on the evolutionary timescale of the MBHB formation are actually a coupled situation, in a way that both factors should be considered simultaneously. To provide complete information about the MBH sinking and the MBHB formation in our simulations, all timings defined in this subsection are summarized in Fig. 5. Furthermore, in order to have more idea about the early stage of MBH sinking, the timing when the MBH orbital radius equals 10 kpc, i.e. \( t_{10} \), is also determined and plotted in Fig. 5.

In general, for Model A, the timings have no significant dependence on impact parameters. Only the merger with \( b = 10 \) forms an MBHB slightly later than the other cases and has the relatively longer duration between \( t_{\text{dyn}} \) and \( t_b \), up to a few hundreds of Myr. To investigate this, we have further tested several more simulations with \( b \) near 10 kpc, from \( b = 6 \) to \( b = 18 \) kpc. The results of the characteristic timings are shown in Fig. 6. It can be seen that the deviation of \( t_b \) from \( t_{\text{dyn}} \) is clear for most mergers with impact parameters between \( b = 8 \) kpc and \( b = 18 \) kpc. It is also shown that the deviation exhibits significant variations.

As for Model B, although all the \( t_{10} \) coincides with those of Model A, the \( t_c \) differs from that of Model A significantly. There is a slight trend for \( t_c \) to increase with \( b \) when \( b > 40 \) kpc, but the most prominent signature of Model B is a significantly longer dynamical friction timescale for the dual MBHs to evolve from \( t_{\text{dyn}} \) to \( t_b \).
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3.2 The MBHB orbital properties

To further investigate the MBHB orbital properties, the MBHB angular momentum is discussed here. Although the MBHB forms after $t_b$, the angular momentum is still calculated and presented from $t = 0$ as it could give hints about the MBH orbits during the MBHB formation process.

To give an example of MBH orbits, Fig. 7 presents the trajectories of two MBHs for the case of $b = 50$ kpc in Model A. From $t = 0.95$ Gyr (top-left) to $t = 1.07$ Gyr (bottom-left), it can be seen that the MBHB orbit encounters a turn-over, such that the moving directions of the MBHs change drastically. The orbital direction of the MBHs first switch from clockwise to counter-clockwise, and then back to clockwise.

The MBHB angular momentum is defined as

$$ L_{BB} \equiv M_{BB} \sum_{i=1,2} r_i \times v_i, \quad (12) $$

where $r_i, v_i$ are the position and velocity vectors of two MBHs relative to their barycenter. Fig. 8 shows the time evolution of the MBHB angular momentum. The MBHB orbital plane lies on the $xy$-plane, and hence only the $z$-component of the $L_{BB}$ is shown in the figure. Since the initial velocity as well as the initial separation of two MBHs are fixed in all mergers, the total energy of two MBHs are the same.
initially in all simulations. However, due to the difference in impact parameters, the initial angular momentum $L_{BB}$ are not the same. Fig. 8 shows that, for the case of $b = 0$, the angular momentum $L_{BB}$ is zero at the beginning of the simulation and then oscillates quickly between positive and negative values. For the other cases of Model A, the angular momentum is negative initially. After two major sign-switchings at $t = 1 - 2$ Gyr, $L_{BB}$ decreases steadily and converges to zero. Furthermore, together with Fig. 7, we can see that for the merger with $b = 50$ kpc, the first major sign-switching of $L_{BB}$ happens after the first encounter of two MBHs at $t = 0.95 - 1.03$ Gyr, as shown in the top two panels of Fig. 7. In addition, the second major sign-switching happens around $t = 1.43$ Gyr, as shown in the bottom right panel of Fig. 7. After that, both MBHs sink into the center of the merged system.

As for Model B, the evolution of $L_{BB}$ basically follows that of the trend in Model A, of the same $b$. However, except for the merger with $b = 0$, all other cases in Model B show significant oscillations after two major sign-switchings. It means that dynamical friction from the stellar background is not efficient at removing angular momentum from $L_{BB}$, and therefore the dual MBHs may stay unbound for a much longer period.

### 3.3 The triaxiality

To investigate the effect of impact parameters on the structures of merged cores, the triaxiality of the stellar core is determined by the method of the moment-of-inertia tensor (Dubinski and Carlberg 1991; Berczik et al. 2006). For the region within the core radius, an ellipsoidal density distribution is set as

$$\rho \equiv \rho(\zeta), \ \text{with} \ \zeta = \left(\frac{x_1^2}{\eta^2} + \frac{x_2^2}{\eta^2} + \frac{x_3^2}{\xi^2}\right)^{\frac{1}{2}},$$

where $\zeta$ is the elliptical radius, and $\eta$ and $\xi$ are the axial ratios with $\xi \leq \eta \leq 1$. The $\eta$ and $\xi$ can be determined through the calculations of the moment-of-inertia tensor and then rotate the coordinates of the stellar particles in the core (Within $r_c$). We compute the inertia tensor iteratively until both differences between the input and output of $\eta$ and $\xi$ are smaller than $10^{-5}$. After $\eta$ and $\xi$ are determined, the triaxiality in-

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**Fig. 8** Time evolution of the MBHB angular momentum $L_{BB}$. Since the initial MBHB orbital plane lies on the $xy$-plane, $L_{BB}$ is along the $z$-axis. The impact parameter $b$ is shown in the upper-right corner of each panel. The values of Model A are shown in black, and the values of Model B are shown in red.
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Fig. 9 Time evolution of the axial ratio $\eta$ and $\xi$ for Model A. The impact parameter $b$ is shown in the upper-right corner of each panel. The $\eta$ is shown in black, and $\xi$ in red.

Index $T$ is calculated as (Merritt 2006; Bortolas et al. 2018)

$$T \equiv \frac{1 - \eta^2}{1 - \xi^2}.$$  \hspace{1cm} (14)

They are only evaluated after $t_c$ when the MBHs are in the core of the merged system.

Fig. 9 and Fig. 10 present the time evolution of the axial ratios $\eta$ and $\xi$ for different impact parameters of Model A and Model B, respectively. The most prominent difference between Model A and Model B is that $\eta - \xi$ is significantly greater in Model A. This implies that the core structure is close to the shape of an oblate ellipsoid in Model A. Since the progenitor in Model A has a more concentrated stellar center, the oblateness of the remnant core can be expected if the original stellar density structure surrounding the original MBH is better preserved during the merging process. On the other hand, the smaller $\eta - \xi$ in Model B suggests that the remnant stellar core is closer to a spheroid. Secondly, $\eta$ and $\xi$ are systematically closer to unity in Model B than in Model A. Note that $\eta = \xi = 1$ gives a spherical symmetric core, thus having $\eta$ and $\xi$ closer to unity suggests that the remnant core structures in Model B are closer to spherical.

In terms of impact parameters, $b$ mainly governs the dynamical properties of the evolving core, such that mergers with the same $b$ would result in their remnant cores having similar motion. For the case of $b = 0$, both $\eta$ and $\xi$ oscillate violently all the time. As for $b = 10$, there still many peaks of oscillations are observed, but the difference between $\eta$ and $\xi$ becomes clearer. For all the remaining cases with larger impact parameters, the axial ratios vary gently.

The time evolution of triaxiality index $T$ is shown in Fig. 11. The stellar core is considered to be oblate if $0 \leq T < 0.5$ and prolate if $0.5 < T \leq 1$, while $T = 0.5$ indicates the maximum triaxiality. It can be seen that the evolution of $T$ follows the same trend for Model A and Model B, i.e. the black lines and the red lines basically overlap with each other in Fig. 11. However, for different impact parameters, similar to the axial ratios in Fig. 9 and Fig. 10, the triaxiality indexes in the mergers with $b = 0$ and $b = 10$ oscillate violently. In addition, the merger with $b = 10$ has its triaxiality index at $T \approx 0.5$ for the longest duration.

To have an overview of the above evolutions, Fig. 12 shows the mean values of $\eta$, $\xi$, and $T$ as a function of the impact parameter. The black markers are for Model A, and the red markers are for Model B. Panel (a) shows that rem-
nants in Model B generally have a higher value of axial ratios $\eta$ and $\xi$ when compared with Model A, except for $b = 0$. However, such dependencies are not found on the mean values of $T$.

In general, the periodic variations of $\eta$, $\xi$, and $T$ are caused by the rotation of the remnant core. Since the evolution of $T$ in Model A and Model B clearly overlaps, this variation in $T$ only depends on the impact parameter.

4 Discussions

Based on the results presented in Sect. 3, we discuss the relations between different impact parameters and the half-mass radii of the progenitor galaxies here. Note that the half-mass radius $r_{1/2}$ of the progenitor galaxies in Model A and Model B are 12.07 kpc and 19.31 kpc, respectively. Because all progenitors have the same mass, a greater value of $r_{1/2}$ means a lower average stellar density within $r_{1/2}$.

4.1 Head-on mergers

During a head-on merger ($b = 0$), two MBHs move toward each other and enter the density peak of the merged system directly. They approach each other nearly along a straight line and thus the initial angular momentum $L_{BB}$ is close to zero. However, due to the strong interaction between the MBHs and other particles, as shown in Fig. 8, the angular momentum $L_{BB}$ keeps alternating between positive and negative values, which would be impossible if both MBHs were only governed by their two-body gravitational force. This strong interaction between the MBHs and other particles also makes the triaxiality index $T$ varying rapidly all the time, as can be seen from Fig. 11. Nevertheless, because the initial angular momentum $L_{BB}$ is zero, with enough dynamical friction from other particles, MBHs sink toward the center of the merging system smoothly as presented in Fig. 2 and Fig. 3. Furthermore, the evolution of MBHs during the head-on merger shows little dependence on the models of the progenitor galaxies. The evolution of the MBHs in Model A and Model B are basically the same although the timings $t_c$, $t_{\text{dyn}}$ and $t_b$ in Model B are slightly longer than in Model A. This is reasonable since the progenitors in Model B have a lower stellar density at their central region and therefore the dynamical friction is not as efficient as in Model A.
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4.2 Mergers with non-zero impact parameters

When $b > 0$, because the initial angular momenta of the progenitors are not zero, the merged remnant core becomes more triaxial, and the discrepancies between $\eta$ and $\xi$ are generally larger than those in a head-on merger.

Furthermore, in Model A, we notice that the difference between $t_b$ and $t_{\text{dyn}}$ in the merger with $b = 10$ kpc is larger than that in the mergers with larger $b$, as shown in Fig. 5. We believe the cause of this is the combination of a small half-mass radius of the progenitor galaxies and a small impact parameter $b$. In Model A, since the progenitors have a smaller half-mass radius, their stellar particles are more concentrated around the central MBHs. When the impact parameters are small, the two progenitors will have their half-mass radii overlapping with each other and therefore result in strong interaction between the stellar particles during the merging process. Meanwhile, each MBH moves directly into the high-density region of the other progenitor, and suffers strong perturbations from the stellar particles. Hence, $t_b$ and the difference between $t_b$ and $t_{\text{dyn}}$ increase, as seen in the merger with $b = 10$ kpc. However, if the impact parameters are significantly larger than the half-mass radius of the progenitors, the stellar particles will remain near each MBH for a longer period, and continue to subtract the orbital energy of the MBHs. We will discuss the variation of $t_b$ in Model A in more details in Sect. 4.3.

On the other hand, as shown in Fig. 5, the timing $t_b$ is much later than $t_{\text{dyn}}$ in Model B, comparing with in Model A. In the cases $b = 50$ and $b = 80$ kpc in Model B, their $t_b$ are even longer than our simulation time. In Model B, the progenitors have a larger half-mass radius and a lower density stellar core. Although the half-mass radii of the progenitors also overlap, dynamical frictions are less efficient in subtracting the orbital energy of the MBHs because the two progenitors merge into a core with lower stellar density. Meanwhile, since there are less stellar particles near each MBH, the original stellar particles surrounding the MBH get scattered away more easily, leaving the two MBHs “naked”. The orbital radii of the MBHs will then continue to shrink by interacting with the background stars at a much lower rate, as shown in Fig. 3, and therefore the timings $t_b$ becomes much later than $t_{\text{dyn}}$. In addition, the timings $t_b$ and $t_{\text{dyn}}$ in Model B show similar variations as in Fig. 6. The MBHs’ orbits are perturbed by the direct collision between the pro-
genitor cores, and less stellar particles are around the MBHs, it is thus even harder for the MBHs to form a bound binary.

It can also be seen from Fig. 12 that mergers in Model B leave rounder stellar cores in the orbital plane when compared with those in Model A, such that \( \eta \) and \( \xi \) are systematically closer to unity.

In general, by comparing Fig. 2 with Fig. 3, it is clear that the orbital evolutions between Model A and Model B are quite different for non-zero impact parameters. In Model A, the orbital radii are able to shrink smoothly after \( t_{\text{dyn}} \), and reaching saturation at values of a few times 0.01 kpc. While in Model B, the respective final values are generally close to 0.1 kpc. This suggests that the dynamical friction timescale could be much longer in mergers of progenitors with low central density, and that MBHs could stay at hundreds-of-parsec separations much longer in these mergers. However, simulations with a larger number of stellar particles are required to resolve the three-body interactions between the stellar particles and the MBHs, which are beyond the scope of this work.

### 4.3 Model a with impact parameters near 10 kpc

Lastly, it is worth noticing that in Model A, \( t_b \) fluctuated near \( b \approx 10 \) kpc as shown in Fig. 6. A possible cause is that, when the two progenitors approach each other under small impact parameters, the MBH goes into the high-density part of another galaxy directly, which leads to strong interactions between the MBHs and the surrounding stellar particles, causing more perturbed MBH orbits than the other cases. Note that the perturbations might be numerical, since there is no general trend in the distribution of \( t_b \) for \( b \) between 10 kpc and 20 kpc. To estimate the scale of the perturbations, we run the \( b = 10 \) kpc case in Model A for three times using the same identical setup, i.e. we build progenitors using the same density profile and performed merger simulations with \( b = 10 \) kpc. We obtained the standard deviations for \( t_c \), \( t_{\text{dyn}} \), and \( t_b \) to be 1.57, 7.49, and 3.02 Myrs, respectively. The scale of perturbations are much smaller than the variations observed in the distribution of \( t_b \) in Fig. 6. Future refined simulations, such as increasing the number of particles or using more accurate force integration techniques, are required to confirm the MBHB orbital evolutions in these cases.

### 5 Conclusions

N-body simulations with more than 1.6 million particles are employed to investigate the MBHB formation during a merger of two galaxies. With different impact parameters and different half-mass radii of the progenitor galaxies, we...
study the orbital evolution of the MBHs from an initial separation of 300 kpc until the formation of a bound MBHB. We categorize the evolution of the MBHs into different stages according to the important timings $t_\text{c}$, $t_\text{dyn}$ and $t_\text{b}$, and also present the time evolution of the MBH orbital radius, the MBHB orbital angular momentum, and the triaxiality of the merged stellar core.

Our results show that, for mergers of progenitor galaxies with a denser stellar core, the timings $t_\text{c}$, $t_\text{dyn}$ and $t_\text{b}$ are earlier than those of progenitors with a less-denser stellar core, which is expected because dynamical friction is stronger when the MBHs are moving within a denser stellar environment, therefore the orbital radius of the two MBHs shrinks faster. In addition, we see that the timing $t_\text{dyn}$ could deviate from $t_\text{b}$ greatly, especially when the progenitors have less-denser stellar core. The deviation between $t_\text{dyn}$ and $t_\text{b}$ are large in mergers of progenitors with less-denser stellar core. Furthermore, the final values of the MBHB orbital radii are also different, such that mergers with less-denser stellar cores generally have greater final values than those with denser ones by an order.

Another important finding in this work is that, when the impact parameters are small in Model A, the MBHs would collide directly within the core radius of the progenitor galaxies, and the dual MBHs would suffer strong perturbations from the stellar particles, which leads to variations in the timings $t_\text{b}$ for impact parameters between 10 kpc and 20 kpc. This suggests that resolving the later stages of the MBHB under higher stellar density in the progenitors is more difficult if the MBHs would collide directly into the central region with nonzero initial angular momentum.

There are other physical processes at play that could be important. Our work provides a bigger picture of the foundations of the later stages of the MBHB orbital evolutions. Future works using a larger number of particles and more refined integration accuracy will be important in resolving the later stages of the MBHB evolution. Moreover, further relevant processes such as multiple mergers, gas accretion, heating or scouring effect by the MBHB formation should also be considered.

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References

Bañados, E., Venemans, B.P., Mazzucchelli, C., et al.: Nature 553, 473 (2018)
Begelman, M.C., Blandford, R.D., Rees, M.J.: Nature 287, 307 (1980)
Behroozi, P.S., Wechsler, R.H., Conroy, C.: Astrophys. J. 770, 57 (2013)
Behroozi, P., Wechsler, R.H., Hearin, A.P., et al.: Mon. Not. R. Astron. Soc. 488, 3143 (2019)
Bender, R.: New Light on Galaxy Evolution, vol. 171, p. 181 (1996)
Berczik, P., Merritt, D., Spurzem, R., Bischof, H.-P.: Astrophys. J. Lett. 642, L21 (2006)
Berentzen, I., Preto, M., Berczik, P., Merritt, D., Spurzem, R.: Astrophys. J. 695, 455 (2009)
Binney, J.: Mon. Not. R. Astron. Soc. 200, 951 (1982)
Binney, J., Tremaine, S.: Galactic Dynamics, p. 747. Princeton University Press, Princeton (1987)
Bortolas, E., Gualandris, A., Dotti, M., Spera, M., Mapelli, M.: Mon. Not. R. Astron. Soc. 461, 1023 (2016)
Bortolas, E., Gualandris, A., Dotti, M., Read, J.I.: Mon. Not. R. Astron. Soc. 477, 2310 (2018)
Bournaud, F., Jog, C.J., Combes, F.: Astron. Astrophys. 476, 1179 (2007)
Burke-Spolaor, S.: Mon. Not. R. Astron. Soc. 410, 2113 (2011)
Cole, S., Lacey, C.G., Baugh, C.M., et al.: Mon. Not. R. Astron. Soc. 319, 168 (2000)
Comerford, J.M., Pooley, D., Barrows, R.S., et al.: Astrophys. J. 806, 219 (2015)
De Lucia, G., Springel, V., White, S.D.M., et al.: Mon. Not. R. Astron. Soc. 366, 499 (2006)
Deeleey, S., Drinkwater, M.J., Cunnamma, D., et al.: Mon. Not. R. Astron. Soc. 467, 3934 (2017)
Dospopoulou, F., Antonini, F.: Astrophys. J. 840, 31 (2017)
Dubinski, J., Carlberg, R.G.: Astrophys. J. 378, 496 (1991)
Ferrarese, L., Merritt, D.: Astrophys. J. Lett. 539, L9 (2000)
Fiaccioni, D., Mayer, L., Roskar, R., et al.: Astrophys. J. Lett. 777(2006), L14 (2013)
Hernquist, L.: Astrophys. J. 356, 359 (1990)
Hernquist, L.: Astrophys. J. Suppl. Ser. 86, 389 (1993)
Hou, M., Liu, X., Guo, H., et al.: Astrophys. J. 882, 41 (2019)
Jorgensen, I., Franx, M., Kaergaard, P.: Mon. Not. R. Astron. Soc. 276, 1341 (1995)
Khan, F.M., Preto, M., Berczik, P., et al.: Astrophys. J. 749, 147 (2012)
Khan, F.M., Fiaccioni, D., Mayer, L., Berczik, P., Just, A.: Astrophys. J. 828, 73 (2016)
Kormendy, J., Ho, L.C.: Annu. Rev. Astron. Astrophys. 51, 511 (2013)
Larkin, A.C., McLaughlin, D.E.: Mon. Not. R. Astron. Soc. 462, 1864 (2016)
Li, K., Bogdanović, T., Ballantyne, D.R.: Astrophys. J. 896, 113 (2020)
Liu, X., Greene, J.E., Shen, Y., et al.: Astrophys. J. Lett. 715, L30 (2010)
Lupi, A., Haardt, F., Dotti, M., et al.: Mon. Not. R. Astron. Soc. 453, 3437 (2015)
Makino, J.: Astrophys. J. 478, 58 (1997)
Maness, H.L., Taylor, G.B., Zavala, R.T., et al.: Astrophys. J. 602, 123 (2004)
Merritt, D.: Rep. Prog. Phys. 69, 2513 (2006)
Merritt, D., Poon, M.Y.: Astrophys. J. 606, 788 (2004)
Milosavljević, M., Merritt, D.: Astrophys. J. 563, 34 (2001)
Milosavljević, M., Merritt, D.: The Astrophysics of Gravitational Wave Sources, vol. 686, p. 201 (2003)
Mörtlock, D.J., Warren, S.J., Venemans, B.P., et al.: Nature 474, 616 (2011)
Moster, B.P., Somerville, R.S., Maulbetsch, C., et al.: Astrophys. J. 710, 903 (2010)
Pfister, H., Lupi, A., Capelo, P.R., et al.: Mon. Not. R. Astron. Soc. 471, 3646 (2017)
Quinnan, G.D.: New Astron. 1, 35 (1996)
Somerville, R.S., Davé, R.: Annu. Rev. Astron. Astrophys. 53, 51 (2015). https://doi.org/10.1146/annurev-astro-082812-140951
Souza Lima, R., Mayer, L., Capelo, P.R., et al.: Astrophys. J. 838, 13 (2017). https://doi.org/10.3847/1538-4357/aa5d19
Springel, V.: Mon. Not. R. Astron. Soc. **364**, 1105 (2005)
Tamburello, V., Capelo, P.R., Mayer, L., et al.: Mon. Not. R. Astron. Soc. **464**, 2952 (2017)
Tremblay, S.E., Taylor, G.B., Ortiz, A.A., et al.: Mon. Not. R. Astron. Soc. **459**, 820 (2016)

Yu, Q.: Mon. Not. R. Astron. Soc. **331**, 935 (2002)

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