NUMERICAL INVESTIGATION OF Nusselt Number for Nanofluids Flow in an Inclined Cylinder

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ABSTRACT

Numerical investigation is performed for the determination of Nusselt number of ZnO, TiO2 and SiO2 nanoparticles dispersed in 60% ethylene glycol and 40% water inside inclined cylinder for adiabatic and isothermal process. The present study was conducted for both the constant heat flux (10,000 W/m²) and constant wall temperature (313.15 K) boundary conditions. At the inlet, the uniform axial velocity and initial temperature (293 K) were assumed. The results show the change of average Nusselt number at Reynolds number (400), Rayleigh number (10⁶) and volume fraction percentage (2%). From results for adiabatic process when increasing the slope up to (45°), the Nusselt number augments, while Nusselt number is reduced by further tube inclination. As it is clearly seen, the highest value of Nusselt number is corresponded to (45°) and for isothermal process it is shown that the value of Nusselt number in horizontal position is higher than the vertical position. The values of Nusselt number ratio were evaluated to be (45%, 31%, 25%) for the three Nano fluids (ZnO, TiO₂ and SiO₂) respectively, with the insulation (adiabatic) and (36%, 27%, 22%) without insulation (isothermal).

Keywords: concentration nanofluids, ethylene glycol, nanofluids, nanoparticles, thermal conductivity

1. INTRODUCTION

Nanofluids are suspensions of nanometer-size particles, generally smaller than 100 nm average particle size, suspended in base fluids such as water, ethylene glycol, propylene glycol and oil. With increasing energy demand of the modern society, nanofluid research has received great attention due to the thermal energy efficiency, resulting from the enhanced thermal characteristics of nanofluids with heat exchangers, heat sinks and solar collectors (Farhan et al., 2020) etc.

Nguyen and Menn (2009) had numerically studied the problem of a forced laminar flow and heat transfer of Al2O3-water nanofluid, with two different particle sizes, 36nm and 47 nm, inside a two-dimensional flat microchannel. Both the single-phase fluid and two-phase fluid models were used. Some significant results showing the effects due to the use of nanofluid on the heat transfer coefficient as well as the comparison of between the two models are presented and discussed. Shariat et al. (2011) laminar mixed convection Al2O3-water nanofluid flow in elliptic pipes with different aspect ratio (AR) had been investigated, employing the Brownian motions of nanoparticles for determining the thermal conductivity and dynamics viscosity of Al2O3-Water nanofluid, which depend on temperature. The axial velocity, secondary flow pattern, contours of temperature, distribution of nanoparticles, skin friction factor and Nusselt number profiles were presented and discussed. Sahoo et al. (2012) had focused on determining the thermal conductivity of 60:40 EG/W based SiO2 nanofluid. The 60:40 mass ratio is chosen because it guarantees no freezing, and subsequently no damage to the equipment, down to ~48.3 °C as per the ASHRAE data. The particle volumetric concentrations up to 10% had been experimentally investigated. This concentration may be too high requiring a large pumping power in applications as heat transfer fluid.

However, it gives us a broader database and establishes a general trend of the variation of thermal conductivity with concentration.

Ahmed et al. (2014) investigated the boundary layer flow and heat transfer characteristics due to stretching tube when the tube surface was permeable in the presence of heat source/sink utilizing nanofluids. The effects due to uncertainties of thermal conductivity and dynamic viscosity had been also investigated. Abdellateef (2016) investigated the peristaltic transport of Nano viscous incompressible Newtonian fluid in an inclined annulus tube under the assumptions of long wavelength and lower Reynolds number. Attention has been focused on the behaviors of Brownian motion parameter (Nb), thermophoresis parameter (Nt) and inclination of the annulus cylinder. The result indicated an appreciable increase in the temperature and Nanoparticles concentration with the increase in the strength of Brownian motion effects, and the inclination angle increases the pressure and all these also discussed through graphs. Hekmatpour et al. (2017) the mixed convective heat transfer and pressure drop characteristics of a buoyancy-aided nanofluid flow in a vertical tube was investigated experimentally. As such, this research was conducted to study the effect of using copper oxide nanoparticles on the heat transfer and pressure drop characteristics of the heat transfer oil flow. The tube wall temperature was constant and the flow rate was low enough to ensure that the flow regime was always laminar.

Davarnejad and Hekmat (2018) a new correlation for the dynamic viscosity prediction was applied. The simulation was carried out in a fully developed turbulent regime. The conditions were exactly extracted from the experimental work of Heris (2006). The simulated data (obtained from two models: mixture and VOF model) were compared with each other and the experimental ones. Kandwal et al. (2019) investigations were concentrated on exploring the performance of nanofluid flow through different geometries. They utilized several latest methods to describe the prevailing equations with various conditions, but they
ignored the combined action of heat generation (or absorption) and viscous dissipation on inclined cylinder due to suction/injection and solid volume fraction of nanoparticles. Raei and Peyghambarzadeh (2019) an experimental study had been performed to measure local convective heat transfer coefficient and friction factor of stabilized \(\gamma\)-Al2O3/water nanofluid in a fully-developed turbulent flow regime in a double tube heat exchanger. Experiments performed at different nanofluid concentrations, operating temperatures, and nanofluid flow rates. Jalali et al. (2020) used nanofluids and downward flow to improve the convective heat transfer under constant wall temperature. There were a couple of numerical articles which were qualified to predict the experimental result. Sometimes, the numerical outcomes were against experimental results Hekmatipour et al. (2019), Hekmatipour and Jalali (2020). Therefore, numerical model was not displayed in this research. Therefore, the introduction of correlation in downward flow was valuable. The impact of using copper oxide-thermal oil nanofluid and inclination angle on the convective heat transfer and pressure drop in the slope circular tube was analyzed empirically. The boundary condition was really important in experimental analysis. Qu et al. (2020) provided a comprehensive state-of-the-art review of the thermo physical properties of nanoscale materials. The review was presented in two parts, the first part (Part I) mainly introduced the latest research progress on thermo physical properties of solid nanostructured materials, which included theoretical and experimental research advances in \(k\), \(c_p\), and thermal diffusivity \(\alpha\). The second part (Part II), mainly introduced the latest research progress on the thermo physical properties of nanofluids, including various mathematical models (from classical to advanced models), experimental measurement techniques, theory vs. experiments and the main factors affecting the \(k\), \(c_p\), \(\rho\), \(\mu\) and heat transfer rate of nanofluids. In addition, studied on molecular dynamic (MD) simulation that was used for modeling the properties at the nanoscale were reviewed.

In the present study, numerical investigation was carried out for the determination of Nusselt number of ZnO, TiO2 and SiO2 nanoparticles dispersed in 60% ethylene glycol and 40% water inside inclined cylinder for adiabatic and isothermal process.

2. GOVERNING EQUATION AND FORMULATION

Fig. 1 shows a schematic diagram of tube \((D=0.015\ \text{m and } Z=2\ \text{m})\) with uniform heat flux. The fluid in the tube is a water-based nanofluid containing different types of nanoparticles such as (TiO2, ZnO and SiO2). It is assumed that the base fluid (60% ethylene glycol and 40%water) and nanoparticles are in thermal equilibrium and no slip accurse between them. The thermo–physical properties of the nanofluid are assumed to be constant. The governing equations for the laminar, steady and single phase fluid (Homogenous model).

![Physical representation of the considered](image)

**Continuity Equation**

The equation of conservation of mass in the cylindrical coordinates is given as, (Shareef et al., 2010).

\[
\rho_{nf} \frac{\partial (ru)}{\partial r} + \frac{\rho_{nf}}{r} \frac{\partial (rv)}{\partial \theta} + \rho_{nf} \frac{\partial (rw)}{\partial z} = 0
\]

(1)

where:

\(\rho_{nf}\) - Density of the nanofluid, (kg/m3)
\(r\) - Inner radius of cylinder, (m)
\(u\) - Radial velocity component (r), (m/s)
\(v\) - Tangential velocity component (\(\theta\)), (m/s)
\(w\) - Axial velocity component (z), (m/s)
\(\theta\) - Tangential coordinate (Degree).

And, by using the second assumption (i.e., \(w = 0\)), equation (1) can be reduced to the following form:

\[
\rho_{nf} \frac{\partial (ru)}{\partial r} + \frac{\rho_{nf}}{r} \frac{\partial (rv)}{\partial \theta} = 0
\]

(2)

**Momentum Equations**

The equations of conservation of momentum in the cylindrical coordinates in the radial, tangential, and axial directions (\(r\), \(\theta\), \(z\), respectively) can be written, (Shareef et al., 2010).

\[r - component\]

\[
\rho_{nf} \left( \frac{u \frac{\partial u}{\partial r}}{r} + \frac{v \frac{\partial u}{\partial \theta}}{r} + \frac{w \frac{\partial u}{\partial z}}{r} \right) = -\frac{\partial P_r}{\partial r} + \mu_{nf} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} - \frac{u}{r^2} \right)
\]

(3)

where:

\(P_r\) - Prandtl number
\(\mu_{nf}\) - Dynamic viscosity of the nanofluid (kg/m.s)
\(\alpha\) - Angle of inclination cylinder (Degree)

\[\theta - Component\]

\[
\rho_{nf} \left( \frac{u \frac{\partial v}{\partial r}}{r} + \frac{v \frac{\partial v}{\partial \theta}}{r} + \frac{w \frac{\partial v}{\partial z}}{r} \right) = -\frac{\partial P_{\theta}}{\partial \theta} + \mu_{nf} \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} - \frac{v}{r^2} \right)
\]

(4)

\[z - Component\]

\[
\rho_{nf} \left( \frac{u \frac{\partial w}{\partial r}}{r} + \frac{v \frac{\partial w}{\partial \theta}}{r} + \frac{w \frac{\partial w}{\partial z}}{r} \right) = -\frac{\partial P_z}{\partial z} + \mu_{nf} \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial z^2} \right) - \rho_{nf}(\sin \theta \cos \alpha)
\]

(5)

\(\rho_{nf} = (1 - \theta) \rho_f + \theta \rho_s\)

(6)

where:

\(\rho_f\) - Density of fluid (kg/m³)
\(\rho_s\) - Density of solid (kg/m³)

Inner wall temperature at each section is expressed by:

\[t_w = t_{wa} + \left( \frac{\partial t}{\partial z} \right)_{w} \]

(7)
Since \( \beta \) is constant along axis in the direction of the main flow.

**Energy Equation**

The energy equation in the cylindrical coordinates takes the following form, (Shareef et al., 2010).

\[
\rho c_p f \left( \frac{dt}{dr} + \frac{v}{r} \frac{dt}{d\theta} + \frac{w}{z} \frac{dt}{dz} \right) = \frac{k_f}{r} \left( \frac{1}{r} \frac{d}{dr} \left( r \frac{dt}{dr} \right) + \frac{1}{r^2} \frac{d^2t}{dz^2} \right)
\]

where:

- \( \rho \) - Density (kg/m³)
- \( c_p \) - Specific heat at constant pressure (kJ/kg·K)
- \( k_f \) - Thermal conductivity of the fluid (W/m·K)
- \( \nu \) - Kinematic viscosity (m²/s)
- \( \alpha_f \) - Thermal diffusivity of the fluid (m²/s)
- \( \tau \) - Time (s)

**Effective thermal conductivity**

Effective thermal conductivity calculates by the following (Azeez et al., 2020).

\[
k_{nf} = \frac{k_s + (n-1)k_f - (n-1)\Phi(k_f-k_s)}{k_s + (n-1)k_f + \Phi(k_f-k_s)} k_f
\]

where:

- \( n \) - Shape factor and equals to 3 for spherical nanoparticles
- \( k_s \) - Thermal conductivity of the solid (W/m·K)
- \( k_f \) - Thermal conductivity of the fluid (W/m·K)
- \( \Phi \) - Volume fraction (Vol. %)

**Thermal diffusivity**

Thermal diffusivity of nanofluids calculates from the following formula (Hussein et al., 2016).

\[
\alpha_{nf} = \frac{k_{nf}}{(1-\Phi)(\rho c_p)_s + \Phi(\rho c_p)_f}
\]

**Thermal expansion coefficient**

Thermal expansion of nanofluids evaluates by the following equation (Hussein et al., 2017).

\[
\beta_{nf} = \left[ \frac{1}{1+\Phi(\rho c_p)_f} \right] \beta_f + \left[ \frac{1}{1+\Phi(\rho c_p)_s} \right] \beta_s
\]

where:

- \( \beta_{nf} \) - Thermal expansion coefficient of the nanofluid (1/K)
- \( \beta_f \) - Thermal expansion coefficient of the fluid (1/K)
- \( \beta_s \) - Thermal expansion coefficient of the solid (1/K)

**Specific heat**

Following formula used to calculate specific heat of nanofluids (Hussein et al., 2017).

\[
c_{nf} = \frac{[1-\Phi(\rho c_p)_f + \Phi(\rho c_p)_s]}{(1-\Phi)\rho c_p_s + \Phi\rho c_p_f}
\]

**Effective viscosity**

Following formula used to calculate effective viscosity of nanofluids (Abed et al., 2020).

\[
\mu_{nf} = [123\Phi^2 + 7.3\Phi + 1]
\]

2. Transformation of the Governing Equations into Non-Dimensional Form

The pressure term can be eliminated from the equations of momentum in directions \((r, \theta)\) by cross differentiating between two momentum components. Differentiation of eq. (3) with respect to \(r\) and \(\theta\) gives:

\[
\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} - \frac{1}{r} \frac{\partial v}{\partial \theta} - \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} = - \frac{1}{\rho_{nf}} \frac{\partial p}{\partial r} + \frac{\partial^2 v}{\partial r \partial \theta} + \frac{\partial^2 v}{\partial \theta \partial r} - \frac{\partial^2 u}{\partial \theta^2} - \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} - \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} - \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2}
\]

By subtracting eq. (15) from eq. (16), dividing by \(r\) and rearranging, the following equation can be obtained:

\[
\frac{\partial v}{\partial r} - \frac{\partial u}{\partial \theta} = - v \frac{\partial \Omega}{\partial r} + \nu \left( \frac{1}{r \partial^2 \Omega} - \frac{1}{r \partial^2 \alpha} - \frac{1}{r \partial^2 \nu} \right) - \frac{1}{r} \frac{\partial \nu}{\partial \theta} \left( \frac{1}{r \partial^2 \nu} \right) \cos \alpha + \frac{\partial \nu}{\partial r} \left( \frac{1}{r \partial^2 \nu} \right) \sin \alpha
\]

The stream function in the cylindrical coordinates is defined as:

\[
u = \frac{1}{\rho_{nf}} \frac{\partial \nu}{\partial r} , \text{ and } v = - \frac{\partial \nu}{\partial \theta}
\]

As a result, eq. (17) can be reduced into following form:

\[
\frac{1}{r} \frac{\partial^2 \Omega}{\partial r \partial \theta} + \frac{\partial^2 \Omega}{\partial r \partial \theta} = \nu_f \left( \frac{1}{r \partial^2 \Omega} + \frac{1}{r \partial^2 \alpha} + \frac{1}{r \partial^2 \nu} \right) + \nu \left( \frac{1}{r \partial^2 \Omega} + \frac{1}{r \partial^2 \alpha} + \frac{1}{r \partial^2 \nu} \right) \cos \alpha + \frac{\partial \nu}{\partial r} \left( \frac{1}{r \partial^2 \nu} \right) \sin \alpha
\]
The final form of non-dimensional and momentum equations in where:

\[ \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \left( \frac{\partial \theta}{\partial r} - \frac{\partial \psi}{\partial \theta} \right) = \frac{\nu_{nf}}{r} \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \theta}{\partial \theta^2} \right) + \frac{g \beta n_f}{r} \left( \frac{\cos \theta}{r} \frac{\partial \psi}{\partial \theta} + \sin \theta \frac{\partial^2 \psi}{\partial \theta^2} \right) \cos \alpha \]

(20)

where:

\( \nu_{nf} \) - Kinematic viscosity of nanofluids (m²/s)

\( \frac{\partial \psi}{\partial r} + \frac{1}{r} \left( \frac{\partial \psi}{\partial \theta} - \frac{\partial \psi}{\partial \theta} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \theta}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \theta}{\partial \theta^2} \)

(21)

The equation of vorticity in the cylindrical coordinates can be written as:

\[ \nabla^2 \psi = - \Omega \]

(22)

\[ \Omega = \frac{\partial \psi}{\partial r} + \frac{1}{r} \frac{\partial \psi}{\partial \theta} \]

(23)

By substituting the following non-dimensional parameters:

\[ \lambda = \frac{\alpha}{r^2}, \ W = \frac{w}{W}, \ \psi = \frac{\psi}{\eta_{nf}}, \ R = \frac{r}{R}, \ Z = \frac{z}{R}, \ P = \frac{Pr}{\rho W \eta_{nf}}, \ T = \frac{T - t_b}{2} \]

(31)

\[ \xi = \frac{R^2 \eta_{nf}}{\eta_{nf}}, \ G = \frac{gR^2}{\eta_{nf} W} \]

where:

\( \lambda \) - Dimensionless time
\( R \) - Dimensionless radius
\( \Psi \) - Dimensionless stream function
\( W \) - Dimensionless axial velocity in Z direction
\( W \) - Mean axial velocity (m/s)
\( P \) - Dimensionless pressures
\( T \) - Dimensionless temperature
\( t_b \) - Bulk temperature (°C)
\( q_w \) - Heat flux (W/m²)
\( G \) - Dimensionless gravity acceleration

Mean axial velocity evaluate from the following equation (Holman, 2010).

\[ \bar{W} = \frac{2}{n_{nf}} \int_0^\pi \int_0^R W \rho \nu W \rho dr dr \theta \]

(24)

Bulk temperature evaluate from the following equation (Kaviani, 2012).

\[ t_b = \frac{2}{m_{nf}} \int_0^\pi \int_0^R \rho_p \nu W \rho dr dr \theta \]

(25)

Where:

\( m_{nf} \) - Mass flow rate of nanofluids (kg/s)

From equations (19) and (23) give:

\[ \frac{\partial W}{\partial \theta} + \frac{1}{R} \left( \frac{\partial W}{\partial \theta} - \frac{\partial W}{\partial \theta} \right) = - \frac{\partial P}{\partial \theta} + Pr_{nf} \nabla^2 W - G \sin \alpha \]

(26)

where:

\( Pr_{nf} \) - Prandtl number for nanofluid

\[ \frac{\partial \theta}{\partial \theta} + \frac{1}{R} \left( \frac{\partial \theta}{\partial \theta} - \frac{\partial \theta}{\partial \theta} \right) = \frac{z_{nf}}{\alpha} \nabla^2 \xi + \frac{g \beta n_f R}{k_{nf} \nu_{nf}} \left( \frac{\cos \theta}{r} \frac{\partial \psi}{\partial \theta} + \sin \theta \frac{\partial^2 \psi}{\partial \theta^2} \right) \cos \alpha \]

(27)

The final form of non-dimensional and momentum equations in (r, θ)

and z directions will be:

\[ \frac{\partial W}{\partial \theta} + \frac{1}{R} \left( \frac{\partial W}{\partial \theta} - \frac{\partial W}{\partial \theta} \right) = Pr_{nf} \nabla^2 \xi + Pr_{nf} \alpha R \left( \frac{\cos \theta}{r} \frac{\partial \psi}{\partial \theta} + \sin \theta \frac{\partial^2 \psi}{\partial \theta^2} \right) \cos \alpha \]

(28)

where:

\( R \) - Rayleigh number for nanofluid

Since the pressure gradient along the pipe axis is constant, then the

axial density gradient can be neglected.

\[ W = \bar{W} \]

(29)

Where:

\( \bar{W} \) - Dimensionless velocity in z direction

By dividing eq. (26) by \( \frac{\partial W}{\partial \theta} \) and substituting eq. (29) in itself, the following equation can be obtained:

\[ \frac{\partial \bar{W}}{\partial \theta} = - \frac{1}{R} \left( \frac{\partial \bar{W}}{\partial \theta} - \frac{\partial \bar{W}}{\partial \theta} \right) - 1 + Pr_{nf} \nabla^2 \bar{W} - \frac{G}{\bar{W}} \sin \alpha \]

(30)

Since the average velocity remains constant, the pressure gradient \( \frac{\partial P}{\partial \theta} \) takes the following integral form:

\[ \frac{\partial P}{\partial \theta} = \frac{\pi}{R} \int_0^R \int_0^\pi W_r dr d\theta \]

(31)

Applying Simpson’s rule to the double integral which includes the definition of a new function, (F) taking the following form:

\[ F = \bar{W}(R, 0), R \]

(32)

The velocity at each node will be multiplied by the radius of this node, and eq. (29) becomes as follows:

\[ \frac{\partial \bar{W}}{\partial \theta} = \frac{\pi}{R} \int_0^R \int_0^\pi W_r dr d\theta \]

(33)

The equation of energy, eq. (21) can be simplified as follows:

\[ \frac{\partial \bar{W}}{\partial \theta} + \frac{1}{R} \left( \frac{\partial \bar{W}}{\partial \theta} - \frac{\partial \bar{W}}{\partial \theta} \right) = \alpha_{nf} \nabla^2 \bar{\theta} - \bar{W} \frac{\partial \bar{W}}{\partial \theta} \]

(34)

Substituting the non-dimensional parameters in the energy equation gives:

\[ \frac{\partial \bar{\theta}}{\partial \theta} + \frac{1}{R} \left( \frac{\partial \bar{\theta}}{\partial \theta} - \frac{\partial \bar{\theta}}{\partial \theta} \right) = \nabla^2 \bar{\theta} - \bar{W} \frac{\partial \bar{W}}{\partial \theta} \]

(35)

But (Holman, 2010).

\[ \frac{\partial \bar{\theta}}{\partial \theta} = z_{nf} \bar{W} \theta \]

(36)

Then:

\[ \frac{\partial \bar{\theta}}{\partial \theta} + \frac{1}{R} \left( \frac{\partial \bar{\theta}}{\partial \theta} - \frac{\partial \bar{\theta}}{\partial \theta} \right) = \nabla^2 \bar{\theta} - 2 \bar{W} \]

(37)

The equation of vorticity in terms of stream function is an elliptic partial differential equation and can be represented in the following non-dimensional form (Kaviani, 2012).

\[ \nabla^2 \psi = - \xi \]

(38)

Where:

\[ \nabla^2 \psi = \frac{\partial^2 \psi}{\partial \theta^2} + \frac{1}{R} \frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \theta} \]

(39)

3. Boundary Conditions

Table 1 represents the boundary conditions of this study.

| Surface | Stream line \( \Psi \) | Axial velocity \( \bar{W} \) | Temperature \( T \) | Vorticity \( \xi \) |
|---------|----------------|----------------|----------------|----------------|
| Symmetry wall | \( \Psi(R,0,Z) = 0 \) | \( \bar{W}(R,0,Z) = 0 \) | \( T_0(R,0,Z) = 0 \) | \( \xi(R,0,Z) = 0 \) |
| \( \Psi(R,\pi,Z) = 0 \) | \( \bar{W}(R,\pi,Z) = 0 \) | \( T_0(R,\pi,Z) = 0 \) | \( \xi(R,\pi,Z) = 0 \) |

(40)
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5. Numerical implementation

The governing equation in the cylindrical coordinates (equations 1, 2, 3, 4, and 5) as well as boundary conditions were discretized by finite difference method. In this study the finite difference equation were derived by using central difference approximation for the partial derivatives except the convective terms for which upwind difference formula was employed. Derivative at the boundary were approximated by three point forward backward difference. The explicit method was chose for the solution of flow and energy fields, while relaxation method was chose for stream function calculation. A time increment \( \Delta t = 10^{-5} \) has been used for \( Ra=10^6 \). In order to evaluate how the presence of the Nanofluids affect the heat transfer rate around the perimeter of pipe according to the parameters Rayleigh number, Nano particles volume fraction, and theta it is necessary to observe the variation of the Local Nusselt number on the perimeter of pipe. In generalized coordinate the local and average Nusslet number defined as:

\[
\text{Nu}_{\text{local}} = \frac{\text{Nu}_{\text{local}}}{\gamma} \left[ 3T_{\text{int},n}^k - 4T_{\text{int},n-1}^k + T_{\text{int},n-2}^k \right] 
\]

(40)

The mean Nusselt number around the perimeter of pipe at location \((k+1)\) is deduced by integrating local Nusselt number as follows:

\[
\text{Nu}_{\text{mean}}^{k+1} = \frac{1}{2\pi} \int_0^{2\pi} \text{Nu}_{\text{local}}^{k+1} d\theta 
\]

(41)

The Nusselt number is used to calculate the surface temperature at the location \((k+1)\), but it is found that the boundary conditions causes unstable state in the solution at the value of relaxation factor \((S=0)\). Therefore; relaxation factor \((S=0.8)\) is used for stability considerations (Thomas et al., 2003). The above integral was calculated using Simpson’s rule 1/3 method. To show the effect of the nanofluids on heat transfer rate, we introduce a variable called Nusselt number ratio (NUR) with its definition given as:

\[
\text{NUR} = \frac{\text{Nu}_{\text{mean}}^{\text{Nanofluid}}}{\text{Nu}_{\text{mean}}^{\text{Pure Fluid}}} 
\]

If the value of NUR greater than 1 indicated that the heat transfer rate is enhanced on that fluid, whereas reduction of heat transfer is indicated when NUR is less than 1 (Fadhil et al., 2019).

6. Thermo physical Properties

| Base Liquid Type | Density (kg/m³) | Specific Heat (J/kg.K) | Viscosity (kg/m.s) | Thermal Conductivity (W/m.K) | Pr |
|------------------|----------------|------------------------|-------------------|-----------------------------|----|
| Water            | 997            | 4170                   | 1.00*10⁻³          | 0.606                       | 6.96 |
| Ethylene Glycol (EG) | 1111          | 2415                   | 1.57*10⁻²          | 0.252                       | 150.4 |

Table 2 Thermo physical properties of base liquid types.
The present study was conducted for both the constant heat flux ($q_{\text{w}} = 10,000 \text{ W/m}^2$) and constant wall temperature ($T_{\text{w}} = 313.15 \text{ K}$) boundary conditions. At the inlet, the uniform axial velocity $u_o$ and initial temperature ($T_o = 293 \text{ K}$) were assumed. The thermo physical properties of the base fluids (60% ethylene glycol and 40% water) and nanoparticles (TiO$_2$ (30 nm), SiO$_2$ (30 nm), ZnO (30 nm)) are specified in tables 2 and 3, respectively. These properties were adopted from Al-damook et al. (2020), Al-damook et al. (2020) and Abed and Afgan (2020).

**Table 3 Thermo physical properties of different nanoparticles types.**

| Nanoparticles Type | Density (kg/m$^3$) | Specific Heat (J/kg.K) | Thermal Conductivity (W/m.K) |
|--------------------|--------------------|------------------------|-----------------------------|
| TiO$_2$            | 3900               | 692                    | 8.40                        |
| SiO$_2$            | 2200               | 745                    | 1.40                        |
| ZnO                | 5600               | 495                    | 13                          |

7. Results and Discussion:

Fig. 4 represents the relation between the variations of Nusselt number (Nu) with angle of inclination cylinder ($\alpha$) for adiabatic process of ZnO nanoparticles at Reynolds number ($Re = 400$), Rayleigh number ($Ra = 10^6$) and volume fraction percentage (vol. = 2%). In general, increasing the slope up to $\alpha=45^0$, the Nusselt number augments, while Nusselt number is reduced by further tube inclination. As it is clearly seen, the highest value of Nusselt number is corresponded to $\alpha=45^0$.

![Fig. 4 Variation of Nu with $\alpha$ for adiabatic of ZnO at Re = 400, Ra = 10$^6$ and $\Phi = 2$ vol. %](image)

Fig. 5 shows the relation between the variations of Nusselt number (Nu) with angle of inclination cylinder ($\alpha$) for adiabatic process of TiO$_2$ nanoparticles at Reynolds number ($Re = 400$), Rayleigh number ($Ra = 10^6$) and volume fraction percentage (vol. = 2%). In general, increasing the slope up to $\alpha=45^0$, the Nusselt number augments, while Nusselt number is reduced by further tube inclination. As it is clearly seen, the highest value of Nusselt number is corresponded to $\alpha=45^0$.

![Fig. 5 Variation of Nu with $\alpha$ for adiabatic of TiO$_2$ at Re = 400, Ra = 10$^6$ and $\Phi = 2$ vol. %](image)

Fig. 6 represents the relation between the variations of Nusselt number (Nu) with angle of inclination cylinder ($\alpha$) for adiabatic process of SiO$_2$ nanoparticles at Reynolds number ($Re = 400$), Rayleigh number ($Ra = 10^6$) and volume fraction percentage (vol. = 2%). In general, increasing the slope up to $\alpha=45^0$, the Nusselt number augments, while Nusselt number is reduced by further tube inclination. As it is clearly seen, the highest value of Nusselt number is corresponded to $\alpha=45^0$.

![Fig. 6 Variation of Nu with $\alpha$ for adiabatic of SiO$_2$ at Re = 400, Ra = 10$^6$ and $\Phi = 2$ vol. %](image)

Fig. 7 represents the relation between the variations of Nusselt number (Nu) with angle of inclination cylinder ($\alpha$) for isothermal process of ZnO nanoparticles at Reynolds number ($Re = 400$), Rayleigh number ($Ra = 10^6$) and volume fraction percentage (vol. = 2%). It is clearly shown that increasing the Nusselt number in horizontal position of the isothermal case is higher than the vertical position.

![Fig. 7 Variation of Nu with $\alpha$ for isothermal of ZnO at Re = 400, Ra = 10$^6$ and $\Phi = 2$ vol. %](image)

Fig. 8 shows the relation between the variations of Nusselt number (Nu) with angle of inclination cylinder ($\alpha$) for isothermal process of TiO$_2$ nanoparticles at Reynolds number ($Re = 400$), Rayleigh number ($Ra = 10^6$) and volume fraction percentage (vol. = 2%). It is clearly shown that increasing the Nusselt number in horizontal position of the isothermal case is higher than the vertical position.

![Fig. 8 Variation of Nu with $\alpha$ for isothermal of TiO$_2$ at Re = 400, Ra = 10$^6$ and $\Phi = 2$ vol. %](image)

Fig. 9 represents the relation between the variations of Nusselt number (Nu) with angle of inclination cylinder ($\alpha$) for isothermal process of SiO$_2$ nanoparticles at Reynolds number ($Re = 400$), Rayleigh number ($Ra = 10^6$) and volume fraction percentage (vol. = 2%). It is clearly shown that increasing the Nusselt number in horizontal position of the isothermal case is higher than the vertical position.

![Fig. 9 Variation of Nu with $\alpha$ for isothermal of SiO$_2$ at Re = 400, Ra = 10$^6$ and $\Phi = 2$ vol. %](image)
8. Conclusions

The present study was carried out to investigate about effect nanofluids flow of nanoparticles of ZnO, TiO$_2$ and SiO$_2$ base fluids (60% ethylene glycol and 40% water) (through horizontal and inclined cylinder on the Nusselt number for adiabatic and isothermal process, the flow at Reynolds number (Re = 400), Rayleigh number (Ra = 10$^6$) and volume fraction percentage (vol. = 2%), from results we can conclude the following:

1. For adiabatic process when increasing the slop up to $\alpha=45^\circ$, the Nusselt number augments, while Nusselt number is reduced by further tube inclination. As it is clearly seen, the highest value of Nusselt number is corresponded to $\alpha=45^\circ$. Nusselt number of ZnO is greater than of TiO$_2$ and SiO$_2$, because thermal conductivity of ZnO is higher than of TiO$_2$ and SiO$_2$.

2. For isothermal process Nusselt number in horizontal position of the isothermal case is higher than at the vertical position. For this process Nusselt number of ZnO is greater than of TiO$_2$ and SiO$_2$ because thermal conductivity of ZnO is higher than of TiO$_2$ and SiO$_2$.

NOMENCLATURE

c$_{np}$ specific heat of nanofluid at constant pressure (kJ/kg.K) 
g gravity acceleration (m/s$^2$) 
k$_{np}$ thermal conductivity of the nanofluid (W/m.K) 
r$_h$ Mass flow rate (kg/s) 
N$_U$ Nusselt number 
p pressure (N/m$^2$) 
P dimensionless pressures 
P$_{Pr}$ Prandtl number of nanofluid 
$\bar{q}$$_f$ Heat flux (W/m$^2$) 
R dimensionless radius 
Ra$_{nf}$ Rayleigh number of nanofluid 
$(R, \theta)$ Radial and tangential direction 
$(R, \theta, Z)$ Dimensionless cylindrical coordinates 
r tube radius (m) 
S relaxation factor 
T dimensionless temperature 
t temperature (°C) 
t$_w$ wall temperature (°C) 
t$_b$ Bulk temperature (°C) 
u radial velocity component (r) (m/s) 
v tangential velocity component (θ) (m/s) 
W dimensionless axial velocity ($\Phi$) direction 
w axial velocity component (z) (m/s) 
$\bar{W}$ Dimensionless velocity in the Z direction 
$\bar{W}$ Mean axial velocity (m/s) 
Z dimensionless axial coordinate 
z axial coordinate (m)

GREEK SYMBOLS

$\alpha$ angle of inclination of the tube (degree) 
$\alpha_{nf}$ Thermal diffusivity of the nanofluid (m$^2$/s) 
$\beta_{nf}$ The thermal expansion coefficient of the nanofluid (1/K) 
$\lambda$ dimensionless time 
$\mu_{nf}$ dynamic viscosity of the nanofluid (kg/m.s) 
$\nu_{nf}$ kinematic viscosity of the nanofluid (m$^2$/s) 
$\xi$ Dimensionless vorticity 
$\rho_{nf}$ Density of the nanofluid (kg/m$^3$) 
$\tau$ Time (s) 
$\phi$ volume fraction (Vol %) 
$\psi$ Stream function (m^2/s) 
$\Psi$ Dimensionless stream function 
$\Omega$ vorticity (1/s) 
$\emptyset$ Tangential coordinate

SUBSCRIPTS

b bulk 
b$_f$ base fluid 
f fluid 
m mean 
mt number of radial points in the numerical mesh network 
nt number of tangential points in the numerical mesh network 
s solid 
w wall

SUPERSCRIPTS

k time step 
k+1 forward time step

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