Rich structure of the hidden-charm pentaquarks near threshold regions

Alessandro Giachino1,2, Atsushi Hosaka3,4, Elena Santopinto1,6
Sachiko Takeuchi1,2,5, Makoto Takizawa4,5,8, and Yasuhiro Yamaguchi5,9

1 Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Genova, via Dodecaneso 33, 16146 Genova, Italy
2 Institute of Nuclear Physics Polish Academy of Sciences Radziszewskiego 152, 31-342 Cracow, Poland
3 Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan
4 Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan
5 Meson Science Laboratory, Cluster for Pioneering Research, RIKEN, Hiyoshi, Wako, Saitama 351-0198, Japan
6 Department of Physics, Nagoya University, Nagoya 464-8602, Japan

The recent abundant observations of pentaquarks and tetraquarks by high-energy accelerator facilities indicate the realization of the conjecture by Gell-Mann and Zweig, and by De Rujula, Georgi and Glashow [1–3]. We construct a coupled-channel model for the hidden-charm pentaquarks with strangeness whose quark content is $udsc\bar{c}$, described as $\Lambda_c\bar{D}^{(*)}$, $\Xi_c^{(*)}\bar{D}^{(*)}$ molecules coupled to the five-quark states. These molecules are formed by the suitable cooperation of heavy quark and chiral symmetries. We reproduce the experimental mass and quantum numbers $J^P$ of $P_{cs}(4338)$ for which LHCb has just announced the discovery. We make other predictions for new $P_{cs}$ states as molecular states near threshold regions that can be studied by LHCb.

The past decade has witnessed tremendous progress in the experimental and theoretical explorations of the exotic hadrons. These are the strongly-interacting particles made up of quarks, but are considered to have more complicated structures than those of ordinary hadrons such as protons and neutrons, which were already mentioned in the early stages of the prediction of quarks and the discovery of the charm quark [1,3]. The saga of exotic hadrons dates back to 2003, when the Belle Collaboration discovered the first tetraquark candidate, $X(3872)$, with quark content $\bar{c}c\bar{u}u$ [4]. While further analyses are going on, the Large Hadron Collider beauty (LHCb) experiment revealed as many as 59 signals for new hadrons [5].

What should we learn from the observation? The question that should be clarified was nicely formulated in [6]: how are quarks organized inside these multiquark states – as compact objects with all quarks within one confinement volume; interacting via color forces, or as deuterons-like hadronic molecules, bound by light-meson exchange? Indeed, though the existence of these states has now been confirmed, their internal structure is still controversial.

A new phase of quest was triggered in 2015, when the LHCb collaboration reported the first discovery of two pentaquark states, which have been called $P_c^+(4380)$ and $P_c^+(4450)$ [7], in $\Lambda_b\rightarrow P_c^+K^-\rightarrow (J/\psi p)K^-$ decay channel. The quark content of these states is implied by the observed particles $J/\psi p \sim c\bar{c}uud$. Four years later, a new analysis [8] with nine times more statistics was performed; this revealed $P_c^+(4312)$, as well as the splitting of the $P_c^+(4450)$ into two narrow peaks, $P_c^+(4440)$ and $P_c^+(4457)$. Later on, evidence emerged for a new pentaquark state with mass $M \approx 4337$ MeV [9]. All of the above states appear near a two-hadron threshold, for instance $P_c^+(4312)$ near the threshold of the $D$ meson and $\Sigma_c$ baryon.

This was not the end of story. In 2020, the first evidence of a pentaquark with strangeness, $P_{cs}(4459)$, was reported in the $\Xi_b^0\rightarrow P_{cs}(4459)K^-\rightarrow (J/\psi\Lambda)K^-$ decay channel with statistical significance of $3.1\sigma$ [10]. It is worth noting that this resonance can be equally well described by a two-peak structure, with the two peaks split by 13 MeV: $P_{cs}(4455)$ and $P_{cs}(4468)$ [10,11]. The experimental masses of $P_{cs}(4455)$ and $P_{cs}(4468)$ are $M = 4549.9 \pm 2.7$ MeV and $M = 4667.8 \pm 3.7$ MeV. According to LHCb, the two-peak structure hypothesis has the same statistical significance as the single-peak hypothesis [10]. Unfortunately, owing to limited signal yield, the $J^P$ of the $P_{cs}(4555)$ and $P_{cs}(4468)$ states could not be determined in this analysis [10]. $P_{cs}(4555)$ and $P_{cs}(4468)$ lie below the $\Sigma_cD^*$ threshold and so this situation is similar to what happens in the non-strange sector to the two states $P_c(4440)$ and $P_c(4457)$, which are just below the $\Sigma_cD^*$ threshold.

Very recently LHCb has announced the discovery of a new state with mass $M = 4338.2 \pm 0.7$ MeV and width $\Gamma = 7.0 \pm 1.2$ MeV with statistical significance $> 10\sigma$ in $B^-\rightarrow P_{cs}\bar{p}\rightarrow (J/\psi\Lambda)\bar{p}$: thus, $P_{cs}(4338)$ [5]. The amplitude analysis performed by LHCb favors spin and parity $J^P = \frac{1}{2}^+$ [5]. Again, these states appear very close to the two-hadron threshold. Indeed, this applies not only to the pentaquarks but also to tetraquarks, well-known candidates for which are $X(3872)$ and $T_{cs}(3875)$ [12,13].

In Ref. [14] several hidden charm pentaquarks with strangeness have been predicted by means of a $SU(4)$ extension of the Local Hidden Gauge approach to the charm sector and, in particular, a $J^P = \frac{1}{2}^-$ state with

...
mass 4277 MeV; in Ref. [15] a pentaquark state with \( J^P = \frac{1}{2}^- \) and mass 4319.4\(^{+2.4}_{-3.0} \) MeV has been predicted within chiral effective field theory with only leading-order contact interactions; in Ref. [16] a pentaquark state with \( J^P = \frac{1}{2}^- \) and mass 4290\(^{+180}_{-120} \) MeV has been predicted with QCD sum rule; in Ref. [17] a pentaquark state with \( J^P = \frac{1}{2}^- \) and mass 4292 MeV has been predicted in the compact diquark model; in Ref. [18] a pentaquark state with \( J^P = \frac{1}{2}^- \) and mass 4485 MeV has been predicted within the hadrocharmonium model. Finally, in Ref. [19] a coupled-channel calculation limited to \( \Xi_c^+ \bar{D}^- \) channels has been studied as a function of the cut-off parameter. Just after the LHCb collaboration announcement of 5th of June [5] a quark model interpretation appeared [11], and in Ref. [20] a molecular interpretation within a coupled-channel model limited to \( \Xi_c^0 \bar{D}^+ \) channels, while in Ref. [21] within a coupled-channel model using only contact range interactions. Finally, in [22] a triangle singularity interpretation is proposed.

The new \( P_{cs}(4338) \) state is very intriguing because, as discussed by LHCb, its mass is very close to the \( \Xi_c^+ \bar{D}^- \) meson-baryon threshold, which lies at 2467.7 + 1869.7 = 4337.4 MeV, and indeed its favorable quantum numbers, \( J^P = 1/2^- \), are just what one expects for the \( \Xi_c^+ \bar{D}^- \) meson-baryon system in an S-wave. The most natural decay channel for such a state should be the \( J/\Psi \Lambda \) channel, whose threshold is located 126 MeV below the \( P_{cs}(4338) \) mass. The 7 MeV width \( P_{cs}(4338) \) is unnaturally small for such a large phase space so there must be some decay-suppressing mechanism at work [11]. The small experimental decay width in the \( J/\Psi \Lambda \) channel can be understood only if the \( P_{cs}(4338) \) is a very shallow resonant \( \Xi_c^+ \bar{D}^- \) meson-baryon system, in which the formation of a \( c\bar{c} \) pair is suppressed by the long distance between the \( \bar{D}^- \) meson and the \( \Xi_c^+ \) baryon [11].

If two (or more) particles interact suitably with each other they form a weakly bound or resonant state, just as atoms form molecules. \( P_{cs} \)s are the molecules of \( \Lambda_c^- \bar{D}^+ \) and \( \Xi_c^0 \bar{D}^+ \). The molecule’s constituent hadrons, such as \( \Xi_c^0 \) and \( \bar{D}^+ \), are colorless clusters of quarks that are strongly bound by colored force of Quantum Chromodynamics (QCD) mediated by gluons. Otherwise, the constituent hadrons interact weakly via a colorless force mediated by mesons. Hence, the molecular states are realized by finely-tuned conditions of the formation of the constituent hadrons as quark clusters with suitable interaction strength and masses. While the masses of constituent hadrons are well known, not much is known about their interaction. Recently, LHC has started to look for correlation functions in high-energy hadron-hadron collisions [23] and lattice QCD calculations [24] are ongoing. But these have not yet been fully achieved.

Hence, we attempt a model construction on the basis of the knowledge of the strong interaction that has been accumulated so far. The best established model is the one-pion and kaon exchange force between the constituent hadrons via meson coupling to light \( u, d, s \) quarks. The interactions between the heavy baryons and heavy mesons via pion and kaon exchange are derived from Lagrangians that satisfy the heavy quark and chiral symmetries [25]. This is illustrated on the left in Fig. 1.

![FIG. 1: Pictorial representation of the pentaquarks described as five-quark core + meson-baryon molecular components interacting via \( \pi \) and \( K \) exchange-mediated potential.](Image)

If we consider that the hadron size is in the order of half fm and the distance between hadrons in a hadronic molecule (molecular size) is one fm or larger, then the hadrons may overlap for quite some time while forming the molecule. Unlike ordinary molecules, in which constituent atoms repel each other at short distances, constituent hadrons may not. In such a situation, there should exist a transition between hadronic molecular components and compact five-quark components. Refs. [26]-[27] studied the possible configurations of the compact five-quark components and their energies. Thus, we arrive at a coupled-channel model of hadronic molecules and five-quark states, as shown in Fig. 1 [28]-[29] (see also the supplementary material). The coupling structure of the molecules and five-quarks is dictated by the so-called spectroscopic factor, which is familiar in the discussion of cluster structure of atomic nuclei. The spectroscopic factor corresponds to the probability of finding a molecular component in a five-quark state. Thus, there is only one parameter for the overall coupling strength which we have denoted as \( f/f_0 \equiv F \).

Having constructed the model, we show the results for \( P_{cs} \) in Fig. 2 in comparison with the existing experimental data. The figure is made for \( F = 27 \) which forms a weakly bound state of \( \Xi_c \bar{D} \) with binding energy \( E_B \sim 0.1 \) MeV for \( P_{cs}(4338) \). The very shallow bound state has a large spatial size \( \sim 10 \) fm, according to the formula for the root mean square radius of a weakly bound state, \( \langle r^2 \rangle^{1/2} = 1/(2\sqrt{\mu E_B}) \) where \( \mu \) is a reduced mass of the two-body system. Such a large state has only a small overlap with the interaction region \( \sim 1 \) fm\(^3 \) affected by the \( P_{cs}(4338) \to J/\Psi \Lambda \) decay. This explains the small width of \( P_{cs}(4338) \). Therefore, \( P_{cs}(4338) \) is very likely to be a weakly bound molecular state of \( \Xi_c \bar{D} \) in an S-wave, with \( J^P = 1/2^- \).
In Fig. 2 in addition to that state, we find six more states with various spin and parity $J^P = 1/2^−, 3/2^−, 5/2^−$. All of these are molecular states of the near-threshold particles. By increasing the $F$ parameter, it is possible to lower the two predicted states located only slightly below the $Ξ_c \bar{D}^*$ threshold and a better agreement with $P_{cs}(4468)$ and $P_{cs}(4468)$ experimental masses is achieved. Numerical values of the obtained masses and widths for the above two cases are summarized in Table 2 in the supplementary material. What is important is that our model predicts two states as $J^P = 1/2^−$ and $3/2^−$ molecules of $Ξ_c(J = 1/2)$ and $\bar{D}^*(J = 1)$ in the $S$-wave, supporting the two-peak interpretation of the experimental analysis by LHCb [10]. We suggest conducting a higher statistical data analysis in order to improve the statistical significance of those two states.

FIG. 2: Comparison between experimental masses of $P_{cs}$ and theoretical predictions of our model when $F = 27$ is employed. The correspondence between the theoretical predictions and experimental data is denoted with arrows.

The nature of these states that appear near threshold regions depends considerably on the attraction strength. They may be either weakly bound or virtual states. Mathematically, the difference lies in the location of their poles; bound states are on the first Riemann sheet, while virtual states are on the second Riemann sheet. Whichever the case, the production rates of these states are amplified near the thresholds; from the experimental point of view this near-threshold amplification is a physically important feature.

In addition to the above comparison with data, the present model contains important physics. (1) The coupling to the compact five-quark components is effectively expressed as a short-range attraction in the hadronic molecules. It is noticeable that such an interaction plays a dominant role in generating bound states. (2) The tensor force of the pion exchange causes $SD$-wave channel-couplings, which provides additional attraction. More interestingly, it controls decay widths, the inverse of the life time. Without the tensor force, the decay width of, for instance, $Ξ_c^0 \bar{D}^*$ (3/2−) and $Ξ_c^+ \bar{D}^*$ (5/2−) molecules becomes smaller by one order of magnitude.

In hadronic systems, the above features are characteristic of those containing both heavy and light quarks, and hence are a result of the cooperation of chiral and heavy quark symmetries with colorful and colorless forces of the strong interaction, QCD. These conditions have confirmed the conjecture regarding the rich structure of hadronic molecules near the threshold, which was made almost half century ago [11][13].

The molecular structure near threshold region is a universal phenomenon of quantum systems that may appear in various matter hierarchies; quarks, hadrons (nuclei), atoms and molecules. Therefore, we expect to see interdisciplinary opportunities for various research activities to implement and discuss.

SUPPLEMENTARY MATERIAL

The coupled-channel Hamiltonian for meson-baryon and five-quark channels is written in the form of block matrix as $[28]$ $[29]$ $H = \begin{pmatrix} H^{MB} & V \\ V^\dagger & H^{5q} \end{pmatrix}$

\[ \begin{pmatrix} K_1 + V_{11}^m & V_{12}^m & \cdots \\ V_{21}^m & K_2 + V_{22}^m & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \]  

\[ \begin{pmatrix} M_1 & 0 & \cdots \\ 0 & M_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \]  \hspace{1cm} \text{(2)}

and

\[ V_{i\alpha} = f(i|\alpha) = \begin{pmatrix} V_{11} & V_{12} & \cdots \\ V_{21} & V_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \]  \hspace{1cm} \text{(3)}

In these equations, the label $m$ indicates the kind of mesons (either pion or kaon) exchanged between a meson and a baryon, $K_i$ the kinetic energy of the $i$-th meson-baryon pair and $M_\alpha$ the masses of the $\alpha$-th five-quark channel. The couplings of the meson-baryon and five-quark channels $V_{i\alpha}$ are expressed by the products of
the overlap $\langle i|\alpha \rangle$ (spectroscopic factor) and the overall strength $f$. The overlap is computed when the channel’s meson and baryon are in a region of interaction with the five-quark state. This is a good working hypothesis, known in the study of cluster dynamics. The spectroscopic factor is obtained as the overlap of the color-spin-flavor wave functions of the meson-baryon and five-quark states, $\langle i|\alpha \rangle = \langle \phi_{MB}^{\alpha}(CSF)|\phi_{5q}^{\alpha}(CSF) \rangle$, as discussed in Ref. [28].

Setting the full-component wave function as $\psi = (\psi_{MB}^{i}, \psi_{5q}^{\alpha})$, we obtain the coupled-channel equation,

$$
H_{MB}^{i} \psi_{MB}^{i} + V \psi_{5q}^{\alpha} = E \psi_{MB}^{i},
$$

$$
V^\dagger \psi_{MB}^{i} + H_{5q}^{\alpha} \psi_{5q}^{\alpha} = E \psi_{5q}^{\alpha}.
$$

(4)

By eliminating the five-quark channels (Feshbach’s method [30, 31]), we find the equation for the meson-baryon channels

$$
(K_{MB}^{i} + U) \psi_{MB}^{i} = E \psi_{MB}^{i},
$$

$$
U = V_{m}^{i} + V \frac{1}{E - H_{5q}^{\alpha}} V^\dagger.
$$

(5)

The second term of the effective potential $U$ provides a short-range interaction that is induced by a mixture of hadronic molecules and compact five-quark states. This effective potential consists of the meson-baryon One Meson Exchange Potential (OMEP), $V_{m}^{i} \equiv V_{m}^{i\alpha}$, with $m = \pi$ or $K$ meson, and the coupling between the five-quark core configurations, $\alpha$, and the meson-baryon channels, $i$, $V \equiv V_{i\alpha}$. The explicit expressions of the OMEP and the meson-baryon coupling values are reported in Appendix. In general, the effective potential $U$ is given as a non-local form with an energy dependence. As discussed in Refs. [28, 29], we reduce the complicated term to an energy-independent contact potential approximately. This reduction is reasonable, if masses of the compact five-quark states are sufficiently larger than the threshold energies in which we are interested.

**Methods and numerical results**

We can solve the coupled-channel Schrödinger equation for meson-baryon states by means of the Gaussian expansion method [32] with the complex scaled coordinates [33], thereby finding their poles on the complex energy plane for the masses and decay widths for the pentaquarks.

Expecting the lower partial-wave dominance for states near thresholds, we consider $S$-waves and $D, G$-waves that are coupled by the tensor force of the meson-exchange force.

The numbers of channels depend on the quantum numbers, and are summarized in Table I. Masses and decay widths of the pentaquarks predicted in this study are summarized in Table II for two coupling strengths, $F = 27$ and 51. The former corresponds to Fig. 2 in the main text, while the latter, with the larger strength, shows the rich structure of hadronic molecules of $P_{cs}$.

**Explicit form of the meson-baryon potential and couplings**

The OMEP for pion and kaon exchange used in this work are the following:
TABLE I: Meson-baryon channels coupled to the hidden-charm strange pentaquarks $P_{cs}$ of $J^P$ with $I = 0$. The symbol $^2S^1L$ in the parentheses indicates possible spin ($S$) and orbital angular momentum ($L$) of each meson-baryon channels.

| $J^P$ | Channels |
|-------|----------|
| 1/2   | $\Lambda_c D_s(^2S), \Xi_c D(^2S, D), \Xi_c D(^4S), \Xi_c D(^4D)$, $\Xi_c D(^6D), \Xi_c D^*(^2S, D)$, $\Xi_c D^*(^4S, D)$, $\Xi_c D^*(^6S, D)$ |
| 3/2   | $\Lambda_c D_s(^2S), \Xi_c D(^2S), \Xi_c D(^4S), \Xi_c D^*(^4S, D)$, $\Xi_c D^*(^6S, D)$, $\Xi_c D^*(^2S, D)$, $\Xi_c D^*(^4S, D)$, $\Xi_c D^*(^6S, D)$ |
| 5/2   | $\Lambda_c D_s(^2S), \Xi_c D(^2S), \Xi_c D(^4S), \Xi_c D^*(^4S, D)$, $\Xi_c D^*(^6S, D)$, $\Xi_c D^*(^2S, D)$, $\Xi_c D^*(^4S, D)$, $\Xi_c D^*(^6S, D)$ |

TABLE II: Comparison between the experimental masses and decay widths with our numerical results for isospin $I = 0$ in units of MeV.

| Threshold | EXP $|^5{[53]}$ | Our results for $F = 27$ | Our results for $F = 51$ |
|-----------|----------------|---------------------------|---------------------------|
|           | EXP | State | Mass | Width | $J^P$ | Mass | Width | $J^P$ | Mass | Width | $J^P$ | Mass | Width |
| $\Lambda_c D_s$ | - | - | - | - | 1/2 | 4252.65 | - | - | - | 1/2 | 4292.11 | 1.54 |
| $\Xi_c D$ | $P_{cs}$ (4338) | 4338.2 | 7.0 | - | 1/2 | 4363.34 | 7.20 x 10^{-2} | - | 1/2 | 4394.97 | 7.31 x 10^{-4} |
| $\Lambda_c D_s^*$ | - | - | - | - | - | - | - | - | - | 3/2 | 4395.76 | 8.78 x 10^{-4} |
| $\Xi_c D$ | - | - | - | - | 1/2 | 4455.21 | 0.341 | - | 1/2 | 4436.24 | 2.12 |
| $\Xi_c D^*$ | $P_{cs}$ (4455) | 4454.9 | 7.5 | - | 3/2 | 4476.92 | 0.559 | - | 3/2 | 4465.24 | 1.08 |
| $P_{cs}$ (4468) | - | - | - | - | 1/2 | 4477.81 | 0.210 | - | 1/2 | 4469.24 | 2.31 |
| $\Xi_c D$ | - | - | - | - | 3/2 | 4511.98 | 1.74 | - | 3/2 | 4502.91 | 4.09 |
| $\Xi_c D^*$ | - | - | - | - | 3/2 | 4583.29 | 8.30 | - | 3/2 | 4567.12 | 9.95 |
| $\Xi_c D^*$ | - | - | - | - | - | - | - | - | 1/2 | 4587.53 | 1.25 |
| $\Xi_c D^*$ | - | - | - | - | 5/2 | 4649.04 | 12.4 | - | 5/2 | 4629.81 | 14.7 |
| - | - | - | - | - | 3/2 | 4653.02 | 5.52 |
\[ f_\pi \] is the pion decay constant given by \( f_\pi = 92.3 \text{ MeV} \). The coupling constant \( g_\pi = 0.59 \) is determined by the strong decay of \( D^* \to D\pi \) \cite{33}. \( g_1 = 1 \) is estimated by the quark model \cite{36}. \( \vec{\varepsilon} \) and \( \vec{\sigma} \) are the polarization vector and the Pauli matrices, respectively. The spin matrices \( \vec{\Sigma} \) are given by \cite{28}

\[
\vec{\Sigma}^\mu = \begin{pmatrix}
\varepsilon^\mu(+) & \sqrt{2/3} \varepsilon^\mu(0) & \sqrt{1/3} \varepsilon^\mu(-) & 0 \\
0 & \sqrt{1/3} \varepsilon^\mu(+) & \sqrt{2/3} \varepsilon^\mu(0) & \varepsilon^\mu(-)
\end{pmatrix}.
\] (25)

\( \vec{S} \) and \( \vec{\Xi} \) are obtained by \( \vec{S} = i\varepsilon \times \varepsilon^\dagger \) and \( \vec{\Xi} = (3/2)i\vec{\Sigma} \times \varepsilon^\dagger \), respectively. The tensor operator \( S_{\Omega_{1}\Omega_{2}}(\hat{r}) \) is defined by \( S_{\Omega_{1}\Omega_{2}}(\hat{r}) = 3\hat{O}_{1} \cdot \hat{r}\hat{O}_{2} \cdot \hat{r} - \hat{O}_{1} \cdot \hat{O}_{2} \). The functions \( \hat{O}_{1} \) and \( \hat{O}_{2} \) are given by \( \hat{O}_{1} = \hat{O}_{2} = \frac{1}{2}(\hat{r} \cdot \hat{\sigma}) \). The functions \( \hat{O}_{1} \) and \( \hat{O}_{2} \) are the operators that correspond to the products of the polarization vectors and the Pauli matrices. The tensor operators are defined in terms of these functions.
$C(r, m)$ and $T(r, m)$ are defined by

$$C(r, m) = \int \frac{d^3q}{(2\pi)^3} \frac{m^2}{\bar{q}^2 + m^2} F_i(r, m, \bar{q}) \times F_M(L, m, \bar{q}) F_B(L, m, \bar{q}), \quad (26)$$

$$S_{\alpha_1, \alpha_2}(r) T(r, m) = \int \frac{d^3q}{(2\pi)^3} \frac{-\bar{q}^2}{\bar{q}^2 + m^2} S_{\alpha_1, \alpha_2}(q) e^{i\vec{q} \cdot \vec{r}} \times F_M(L, m, \bar{q}) F_B(L, m, \bar{q}), \quad (27)$$

where $F_i(L, m, \bar{q})$ ($i = M, B$) is the form factor introduced at each vertex:

$$F_i(L, m, \bar{q}) = \frac{\Lambda_i^2 - m^2}{\Lambda_i^2 + \bar{q}^2}. \quad (28)$$

As discussed in [28], the cutoff parameters $\Lambda_i$ are determined by the size ratio between the heavy meson and nucleon, $\Lambda_N/\Lambda_i = r_i/r_N$. With the nucleon cutoff $\Lambda_N = 837$ MeV, we obtain $\Lambda_M = 1.35\Lambda_N$ for the heavy mesons, and $\Lambda_B = \Lambda_N$ for the heavy baryons.

---

1. alessandro.giachino@ifj.edu.pl
2. hosaka@rcnp.osaka-u.ac.jp
3. Corresponding author: elena.santopinto@ge.infn.it
4. takizawa@ac.shoyaku.ac.jp
5. yamaguchi@hken.phys.nagoya-u.ac.jp
6. M. Gell-Mann, Phys. Lett. 8, 214 (1964).
7. G. Zweig, An SU(3) model for strong interaction symmetry and its breaking. Version 1, CERN-TH-401 (1964).
8. A. De Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 37, 785 (1976).
9. S. K. Choi et al. (Belle), Phys. Rev. Lett. 91, 262001 (2003) arXiv:hep-ex/0309032.
10. E. Spadaro Norella and C. Chen (LHCb), Particle Zoo 2.0: New Tetra- and Pentaquarks at LHCb, CERN Seminar, 5th July (2022).
11. N. Brambilla et al. (2022), arXiv:2203.16583 [hep-ph].
12. R. Aaij et al. (LHCb), Phys. Rev. Lett. 115, 072001 (2015) arXiv:1507.03414 [hep-ex].
13. R. Aaij et al. (LHCb), Phys. Rev. Lett. 122, 222001 (2019) arXiv:1904.03947 [hep-ex].
14. R. Aaij et al. (LHCb), Phys. Rev. Lett. 128, 062001 (2022) arXiv:2108.04720 [hep-ex].
15. R. Aaij et al. (LHCb), Sci. Bull. 66, 1278 (2021) arXiv:2012.10380 [hep-ex].
16. M. Karliner and J. R. Rosner, (2022) arXiv:2207.07581 [hep-ph].
17. R. Aaij et al. (LHCb), Nature Phys. 18, 751 (2022) arXiv:2109.01038 [hep-ex].
18. R. Aaij et al. (LHCb). [Nature Commun. 13, 3351 (2022)] arXiv:2109.01056 [hep-ex].
19. C. W. Xiao, J. Nieves, and E. Oset, Phys. Lett. B 799, 135051 (2019) arXiv:1906.09010 [hep-ph].
20. B. Wang, L. Meng, and S.-L. Zhu, Phys. Rev. D 101, 034018 (2020) arXiv:1912.12592 [hep-ph].
21. H.-X. Chen, W. Chen, X. Liu, and X.-H. Liu, Eur. Phys. J. C 81, 409 (2021) arXiv:2011.01079 [hep-ph].
22. A. Ali, I. Ahmed, M. J. Aslam, A. Y. Parkhomenko, and A. Rehman, JHEP 10, 256 (2019) arXiv:1907.06507 [hep-ph].
23. J. Ferretti and E. Santopinto, Sci. Bull. 67, 1209 (2022) arXiv:2111.08650 [hep-ph].
24. R. Chen and X. Liu, Phys. Rev. D 105, 014029 (2022) arXiv:2201.07603 [hep-ph].
25. F.-L. Wang and X. Liu, (2022) arXiv:2207.10493 [hep-ph].
26. M.-J. Yan, F.-Z. Peng, M. Sánchez Sánchez, and M. Pavon Valderrama, (2022) arXiv:2207.11144 [hep-ph].
27. T. J. Burns and E. S. Swanson, (2022) arXiv:2208.05106 [hep-ph].
28. A. Collaboration et al. (ALICE), Nature 588, 232 (2020), Erratum: Nature 590, E13 (2021), arXiv:2005.11495 [nucl-ex].
29. T. Hatsuda, Front. Phys. (Beijing) 13, 132105 (2018).
30. Y. Yamaguchi, A. Hosaka, S. Takeuchi, and M. Takizawa, J. Phys. G 47, 053001 (2020) arXiv:1908.08790 [hep-ph].
31. E. Santopinto and A. Giachino, Phys. Rev. D 96, 014014 (2017) arXiv:1604.03769 [hep-ph].
32. S. Takeuchi and M. Takizawa, Phys. Lett. B 764, 254 (2017) arXiv:1608.05475 [hep-ph].
33. Y. Yamaguchi, A. Giachino, A. Hosaka, E. Santopinto, S. Takeuchi, and M. Takizawa, Phys. Rev. D 96, 114031 (2017) arXiv:1709.00819 [hep-ph].
34. Y. Yamaguchi, H. García-Tecocoatzi, A. Giachino, A. Hosaka, E. Santopinto, S. Takeuchi, and M. Takizawa, Phys. Rev. D 101, 091502 (2020) arXiv:1907.04684 [hep-ph].
35. H. Feshbach, Annals Phys. 5, 357 (1968).
36. H. Feshbach, Annals Phys. 19, 287 (1962).
37. E. Hiyama, Y. Kino, and M. Kamimura, Prog. Part. Nucl. Phys. 51, 223 (2003).
38. S. Aoyama, T. Miyo, K. Katō, and K. Ikeda, Progress of Theoretical Physics 116, 1 (2006) https://academic.oup.com/ptp/article-pdf/116/1/1/19571980/116-1.pdf.
39. R. Aaij et al. (LHCb), Sci. Bull. 66, 1278 (2021) arXiv:2012.10380 [hep-ex].
40. A. V. Manohar and M. B. Wise, Heavy quark physics, Vol. 10 (2000).
41. Y.-R. Liu and M. Oka, Phys. Rev. D 85, 014015 (2012) arXiv:1103.4624 [hep-ph].