Free metal surface resonance under alternating magnetic field low-frequency oscillations.

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Abstract. The article describes the definition of relationship between parameters of induction crucible furnace and meniscus shape using a mathematical model. The data obtained has been analyzed and the mathematical model qualitatively assessed using the open non-profit software Elmer and OpenFOAM. The models derived have been checked out to study the occurrence of free-surface oscillation resonance when subjected to pulsed action of an alternating electromagnetic field. Transients in an induction furnace crucible have been analyzed. A mathematical model with the electromagnetic field pulsed action has been created and free surface resonance occurrence and stability examined.

1. Introduction
Molten metal in a pulsating magnetic field of an induction crucible furnace (ICF) affected by the radial component of the bulk Lorentz forces \( f_L \) starts moving and shaping a meniscus on the free melt surface (Figure 1). The natural double-streamline electrodynamic circulation of the melt being formed has both positive effects, i.e. composition temperature equalization, metal-slag reaction, and negative one, i.e. the oxide film breakthrough, inefficient slag use, relatively poor mixing of circulating streamlines, etc. \[1, 2, \text{and} \ 3\].

In magnetic hydrodynamics (MHD), the issue of primary melt circulation streamline mixing or transient mass transfer intensification is resolved in different ways \[4\]. One of them is a liquid resonance driven by the pulsed action of a single-phase alternating electromagnetic field \[5\]. This paper is devoted to the study of free melt surface resonance oscillations under low-frequency pulsed action of an alternating electromagnetic field.

We emphasize the already known relationship for the meniscus in the ICF: meniscus height increases as the specific power rises; at a constant power transmitted to the melt, the force impact thereon strengthens as frequency decreases \[2\]. It has also been established that the meniscus intensity will depend not only on electromagnetic (EM), but also on the geometric parameters of the EM system, namely coil end position relative to the melt surface, melt to coil distance \[1, 2\].

Both analytical and numerical models can be constructed to solve this issue. The major restriction for the analytical method is typically rough assumption in the form of relatively simple (one-dimensional) models needed to achieve the solution. Numerical special case-oriented methods (FDM, FEM, FVM) are used for more accurate calculations. There is both commercial (Ansys, COMSOL, etc.) and free software (OpenFOAM, Elmer, etc.) with various types of partial differential equations solvers for specific problems in applied physics.
Previous meniscus studies were carried out using analytical methods [1,2], one-dimensional numerical models ignoring the hydrodynamics [6], two-dimensional axisymmetric numerical models with account for the hydrodynamics [7], as well modern software improved to link EM and hydrodynamic problems: Ansys classic + CFX [8], Ansys classic + Ansys fluent [9], Elmer + OpenFOAM [10] and Comsol + Ansys fluent [11]. The free melt surface in an alternating electromagnetic field is an MHD problem falling into a category of complex physics problems. To solve this type of problem, an open EOF-Library [12] is proposed, which allows linking software products with open source code in order to solve hydrodynamics (OpenFOAM) and electromagnetism (Elmer) problems. The method chosen for the numerical solution of the MHD problem has been repeatedly verified against a problem with a levitating melt drop in a conical coil field, which is common for this category [13].

This paper is aimed to conduct a theoretic experiment to check the occurrence of a free surface oscillation resonance under pulsed action an alternating electromagnetic field (EMF).

![meniscus](image1)

**Fig. 1** - Meniscus formation in the induction furnace crucible

![axis of symmetry](image2)

**Fig. 2** – Computational region geometry

2. Materials and Methods

The ICF electromagnetic process studies consider the inductor download system. Based on the system’s energy parameters, the inductor may be a natural air-cooled multi-turn alternating current coil or a massive forced water-cooled tube. The charge, if a non-conductive crucible is used or a crucible EMF-transparent at these frequencies, is only the melt.

For the purpose of numerical solution, it has been assumed that the actual inductor’s turn are replaced by a single turn with the corresponding magnetomotive force (MF); non-conductive and minor geometric structural elements are neglected; the problem is solved in a two-dimensional axisymmetric arrangement; in the electromagnetic problem, the field changes harmonically; thermal processes and natural convective motion of the melt are neglected. Assumptions are standard for this class of problems and do not drive any significant increase in the result deviation, but allow for significant saving of computing resources. See Figure 1 for a scheme of computational region geometry indicating main dimensions, where 1 is the melt, 2 is the inductor. The dimensions were taken on the scale of an induction crucible furnace physical model to study MHD phenomena using a low-temperature eutectic alloy with metallic conductivity.

The hydrodynamic problem was solved in the melt region only. The expected Reynolds number in the melt is $Re \approx 38 \, 000$, which is much more than the critical value, so turbulent flows should be expected. These were taken into account using the k-ε turbulence model. Under alternating magnetic
field conditions, the hydrodynamic problem took into account the force action through an additional source of motion. A free surface is simulated using the volume of fluid method (VOF), where \( \alpha \) characterizes the volume fraction of fluid in the mesh cell.

The electromagnetic problem was solved in a quasi-stationary arrangement, indicating the equivalent current density. The computational region, in addition to the inductor and the charge, is filled with air with boundaries in the electromagnetic problem distanced from the inductor to reduce the impact of boundary conditions.

During the calculation of the MHD problem, the EM part recalculation would be initiated based on the change in the free surface position determined during the solution of the hydrodynamic part. This approach reduces the number of inner solutions of the electromagnetic part greatly declining the total amount of calculations.

The pulsed Lorentz force in the melt is created by interrupting current generation in the inductor with a frequency of \( f_{puls} = 1/T_{FS} \). The ratio of the pause time when the melt moves driven by inertia to the period of time during which the Lorentz force affected the melt is called relative pulse duration and is always equal to \( \Psi = 1 \) in this study. The created impulse action at the mathematical model level is implemented by adding the Lorentz force matrix multiplier in the EOF-library \( f_{puls}(t) = f_L \cdot k_{10}(t) \). Pulse ratio used \( k_{10} \):

\[
k_{10} = \frac{1}{2} \left( 1 + \frac{\sin(2\pi f_{puls} \cdot t)}{\sin(2\pi f_{puls} \cdot t)} \right)
\]

where \( f_{puls} \) is resonant frequency, \( t \) is process time.

Hereinafter in the text and in the graphs, the total current \( I, A \cdot turn \) is used to indicate the MF value. All study results related to the media interface will be shown for \( \alpha = 0.5 \). Properties of materials used necessary to solve the MHD problem are shown in Table 1.

**Table 1. - Physical properties of materials**

| Item no. | Parameter                     | Symbol | Units of measure | Value      |
|----------|-------------------------------|--------|------------------|------------|
| 1        | Density                       | \( \rho \) | kg/m\(^3\)      | 6080       |
| 2        | Conductivity                  | \( \sigma \) | S/m             | 3.85\times10\(^6\) |
| 3        | Kinematic viscosity           | \( \nu \)   | m\(^2\)/s       | 2.6315\times10\(^{-7}\) |
| 4        | Relative magnetic permeability | \( \mu \)   | rel. units      | 1          |
| 5        | Conductivity                  | \( \sigma \) | S/m             | 5.96\times10\(^7\) |
| 6        | Density                       | \( \rho \)   | kg/m\(^3\)      | 1          |
| 7        | Kinematic viscosity           | \( \nu \)   | m\(^2\)/s       | 1.48\times10\(^{-5}\) |
| 8        | Melt-air surface tension      | \( \gamma \) | N/m             | 0.72       |
3. Results and discussion

3.1. The study of meniscus shape as a function of energy and geometric parameters

The determination of relationships connecting ICF parameters and the meniscus shape has required the introduction of the following evaluative attributes: total meniscus height \( \Delta h = h_{\text{max}} - h_{\text{min}} \), where \( h_{\text{max}} \), \( h_{\text{min}} \) are meniscus’ maximum and minimum axial position; meniscus’ deviation from equilibrium \( \Delta h_0 = h_{\text{max}} - h_0 \), where \( h_0 \) is initial melt level (Figure 3); \( U_{\text{ave}} \), is the average melt circulation rate.

These parameters have allowed to characterize the observed processes in the melt in terms of hydrodynamics. In course of the numerical experiment, two EM parameters of the system have been varied – inductor power frequency \( f_0 \) and full current \( I \), as well as the fullness of the crucible with the melt.

We consider the dependence of evaluative parameters on the total current, as shown in Figure 4.

Figure 4 shows a quadratic dependence of the meniscus height (\( \Delta h \)) on the current and, accordingly, a linear dependence on the system capacity. The average speed (\( U_{\text{ave}} \)) is in turn linearly related to the current magnitude.

Figure 5 shows the relationship between evaluative parameters and the relative penetration depth \( \Delta' = 1/(R \sqrt{\pi f_0 \sigma \mu_0}) \), where \( R \) is the radius of the melt, \( \mu_0 \) is the magnetic constant, \( \sigma \) is the melt conductivity, \( f_0 \) is the inductor power frequency.
Increasing inductor power frequency (decreasing $\Delta'$) results in a steady increase in the integral value of the Lorentz forces $F_L$, which is shown by an increase in all the evaluative parameters. When the penetration depth approaches the workpiece radius (for the sizes used in the calculation of about 500 Hz), an increase in electromagnetic pressure induces an increase in hydrostatic pressure to balance the system, as can be seen from the meniscus deviation from the equilibrium position ($\Delta h_0$). Melt speeds ($U_{ave}$) depend on the value $F_L$, while free surface parameters and Lorentz forces are of a more complex relationship. The meniscus shape depends on the size, however, the local nature of $F_L$ force distribution, namely the axial distribution in the meniscus zone due to the boundary effect has a greater effect on the free surface, which is characterized by different values $\Delta'$ of the specified parameter maxima. A further increase in the frequency results in a decreased integral value, but an increased density of Lorentz forces, which can explain differences in the behavior of parameters $\Delta h_0$ and $\Delta h$ within this frequency range. This meniscus height ($\Delta h$) increasing behavior is explained by the fact that the meniscus point’s $h_{min}$ decline is due to increased EM pressure in the boundary layer exerted on the melt surface up to the complete separation of the melt from the crucible walls.

To indicate the fullness of the crucible with the melt relative to the inductor, $k_m = h_0/h_{ind}$ parameter is introduced, where $h_{ind}$ is the inductor height; its effect on meniscus characteristics is analyzed (Figure 6).

![Fig. 6 - Evaluative parameters vs. crucible fullness with melt](image)

When emptying the crucible $k_m < 1$ (Figure 6, a) the meniscus height will increase, this is explained by an increase in the axial component of the Lorentz force due to edge effects. The additional created pressure exerted on the free surface near the contact point of the three media (air-melt-crucible) intensifies the upper circulation streamline and increases the meniscus height. Further emptying of the crucible leads to a slight decrease in the meniscus height due to a limited total melt volume for closing hydrodynamic flows. When the crucible is overfilled $k_m > 1$ (Figure 6, b) the axial component of the electromagnetic forces disappears along with the meniscus. From [1] it is also known that this region can form secondary flows over the inductor.

3.2. Description of free surface-related transients.
A numerical experiment was carried out with the MHD system configured to achieve an apparent meniscus height of 3 mm. These system parameters were selected with the intention to continue the research on the physical model. To make it practicable, industrial power frequency $f = 50$ Hz, total current allowing to construct the unit with no considerable financial costs for additional power supplies, $I = 40 \text{ kA} \cdot \text{turn}$ as well as a small volume of liquid metal $\sim 0.02 \text{ m}^3$ were used.
Figure 7 shows the results of a non-steady MHD problem calculation with several states. The bottom graph is given to indicate the corresponding process control behavior, namely, the dynamics of instantaneous currents and effective electromagnetic forces.

Figure 8 shows the dynamics of the MHD processes at characteristic points of time indicated in Figure 7. The left side shows the bulk Lorentz force vector field modulus for the melt $f_L = Re\{\delta \times \bar{B}\}$ where $\delta$ is the complex current density vector, $\bar{B}$ is the complex conjugate magnetic induction vector. The right side shows the velocity modulus distribution.

In the initial period of time (before I), the equilibrium state of the melt unaffected by Lorentz force is shown. At time point I, the coil is energized and the current is created in it. The force effect of induced fields interacts with the melt and a velocity transient begins with 2 distinguishable phases: (I-II). Instantaneous reaction of the melt to EM pressure, characterized by meniscus extrusion with cylindrical damped waves formed on the surface, which cause a complex nature of meniscus pulses due to interaction with their part reflected from the crucible;
(I-III). The hydrodynamic effect has its transient rate up to \( \approx 3 \) sec during which the average melt velocity rises and the average vibrational position increases, higher than in phase (I-II), to a maximum value.

Then, within the interval (III-IV), the hydrodynamic and EM pressures inside the melt begin to equalize and a steady state with a “permanent” meniscus (IV-V) sets in. At time point V, the ICF turns off and an inverse transient occurs with damped cylindrical vibrations, after which the free melt surface reaches a calm state (VI) again, while velocities inside the melt tend towards zero.

Based on observations of transients in such systems, it was hypothesized that the free surface resonance is possible [14], and the possibility to maintain the resonance state by a pulsed action of EMF forces with a frequency equal to that of free surface vibrations was assumed.

3.3. Determining the natural surface oscillation period

Determining a more accurate value of the natural surface oscillation period \( T_{FS} \) for the crucible, has required the solution of an additional hydrodynamic problem with no effect of Lorentz forces. In this case, the initial conditions in the free surface problem, are represented by an arbitrary meniscus height (Figure 9, a).

The graph shown in Figure 9 demonstrates the dynamics of damped oscillations \( h_{\text{max}} \). The bottom of Figure 9 (a - e) shows the distribution \( \alpha \) at time points indicated on the graph. The adjusted free surface oscillation period was equal to \( T_{FS} = 0.211 \) sec. Therefore, Lorentz force pulsed action frequency \( f_{puls} = 1/T_{FS} \approx 4.7 \) Hz.
3.4. *Analysis of non-steady processes with pulse behavior of EM forces.*

Figure 10 shows the nature of process development within a single oscillation period.

At time point (a) the meniscus moves up driven by Lorentz forces, which coincides in phase with the free surface oscillation period, thereby "neutralizing" the attenuation of oscillation. Upon reaching a peak in the meniscus height (c) the coil is deenergized, which allows the meniscus to move freely downward under the action of the gravity field. When the minimum height is reached, the coil is energized again and the cycle described continues.

For ease of comparison, Figure 11, similarly to Figure 7, shows transient dynamics for main parameters when affected by pulsed Lorentz forces.
The hydrodynamic transient begins at time point I and, as in the constant exposure case $F_L$ (Figure 7), consists of 2 phases. Under pulsed action during the first phase of the transient, free surface oscillation amplitude intensifies sharply reaching the maximum value at point II, which indicates the onset of the free surface oscillation resonance. It is clearly seen that when the maximum melt (III) velocity is reached, the resonance starts decaying, which suggests the dependence of the melt surface resonant frequency on the melt circulation rate. In addition, given the studies of relationship of the fullness of the crucible with the melt, described in the first part, it is now interesting to study the local effect on the free surface by the pulsed method to minimize the velocity field by reducing the radial component $F_L$. At time point IV, a quasistationary state of a hydrodynamic system with a constant pulsation amplitude $U_{ave}$ and $h_{max}$ sets in. In this case, the velocity fluctuations are saw-toothed. Drastic free surface changes noticeably affect $F_L$ value, which is indicated by uneven pulse maxima in the lower graph of Figure 11.

To study the effect of the velocity field on the resonance frequency, a series of numerical experiments was carried out to determine the dependence of the free surface oscillation amplitude in the steady state on the force pulsed action frequency. Resonance frequency $f_{pls}$ has been varied within a small range from the onset of steady-state condition under the resonant pulsed action of Lorentz force (Figure 11, IV).

Figure 12 shows an example with a pronounced increase in the free surface oscillation amplitude after a slight decrease $f_{pls}$, which confirms the assumed velocity field effect. Obviously, when using an automated electromagnetic force fluctuation meniscus position feedback control system for an ICF, the system becomes self-oscillating i.e. resonates automatically.

Figure 13 shows the dependence of the meniscus height and average melt circulation rate on frequency $f_{pls}$.
As can be seen on the graphs, the velocity field shifts the resonant frequency towards the decrease. It is also seen that with the free surface resonance, the meniscus’ equilibrium state deviation ($\Delta h_0$) is almost twice as high as with constant exposure to EMF, despite the fact that the average value $F_L$, and hence the power is actually half as great. On the other hand, apart from the obvious general decrease in the average velocity, the resonant frequency demonstrates its additional deviation indicating an increase in the kinetic energy dissipation under these conditions.

4. Conclusions
1. The study of the basic laws of meniscus formation in a steady-state electromagnetic field has allowed to assess the conformity of the models created to the verified results of authoritative papers in these areas of study and to checkout them. The results obtained have allowed the selection of basic values of main MHD system parameters for further solution of the unsteady problem aimed to study oscillatory processes.
2. The presence of the free surface resonance with negative and positive properties has been confirmed.
3. The creation of self-oscillations using an automated control system is proposed, which can be verified by reworking the numerical model or creating a physical one.
4. A relationship has been found between the melt flow velocity and the frequency of the Lorentz force's pulsed action on the melt, which is of interest for further studies in various conditions and system configurations.
5. Also, this method has been assumed as a separate exciter of local EM pressure to create a mixing streamline due to free surface oscillations in an inert medium.

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