Propagation stability of a chirped soliton in birefringent fibers

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Abstract. The propagation stability of a chirped soliton at anomalous dispersion region in birefringent fibers is numerically studied by using the split-step Fourier-method. It is found that initial linear chirp can change obviously the threshold value \( A_{th} \) above which soliton stably propagates in birefringent fibers, the \( A_{th} \) increases with the decrease of the polarization angle \( |\alpha| \). The positive chirp makes obviously the \( A_{th} \) smaller for group velocity mismatch parameter \( \gamma > 0.5 \), the negative one makes the \( A_{th} \) larger for \( \gamma < 0.5 \). The effect of initial positive chirp on the \( A_{th} \) is greater than that of negative chirp for high birefringent fibers, is less than that of negative chirp for low birefringent fibers.

1. Introduction
The soliton propagation in birefringent fibers has been investigated extensively for its importance in science and practical applications. It is governed by a set of coupled nonlinear Schrödinger (CNLS) equations which have been receiving a great deal of attention in recent years [1-9]. The previous works on soliton propagation in birefringent fibers have been studied in the case of unchirped pulse [3-9]. However, optical pulses generally have frequency chirps which have great effects on characteristics of the pulses [10-16]. The frequency chirp can be controlled by changing the input current of laser or changing the length of input fiber, and so on. In this paper, propagation stability of chirped soliton at anomalous dispersion region in birefringent fibers is investigated numerically by using the split-step Fourier-method (SSFM), is compared with that of unchirped pulses.

2. Theoretical model of soliton propagation in birefringent fibers
The soliton propagation in birefringent fibers is described by the normalized coupled nonlinear Schrödinger equations (CNLS) [1, 2]

\[
\begin{align*}
&i\left( \frac{\partial u}{\partial \xi} + \beta \frac{\partial u}{\partial \tau} \right) + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + \left( |u|^2 + B |v|^2 \right) u = 0, \tag{1a} \\
&i\left( \frac{\partial v}{\partial \xi} - \beta \frac{\partial v}{\partial \tau} \right) + \frac{1}{2} \frac{\partial^2 v}{\partial \tau^2} + \left( |v|^2 + B |u|^2 \right) v = 0, \tag{1b}
\end{align*}
\]

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where $u$ and $v$ are the normalized complex amplitudes of the two orthogonal polarization components in CNLS system, $\xi$ is the propagation distance which normalized to the soliton period, $\tau$ is the time which normalized to $T_0$ (half-width of the input pulse at 1/e intensity), $\delta$ is the group velocity mismatch parameter and $B$ is the cross-phase modulation (CPM) coefficient where $0 \leq B \leq 1$. $u$ is slow component and $v$ is fast one for $\delta > 0$. The fiber loss is neglected because the propagation distance is short for all-optical switch application. Eqs.(1) reduce to an uncoupled NLS system for $B=0$ \cite{1}, reduce to the Manakov equations and have analytical solutions for $B=1$ \cite{1,5-9}. However, Eqs.(1) are unintegrable for $B=2/3$ in linear birefringent fibers \cite{1-4}. We numerically study the propagation characteristics of the chirped soliton by using the SSFM according to Eqs.(1) for $B=2/3$. The two orthogonal polarization components of an input soliton with initial linear chirp are, respectively,

$$u(\xi = 0, \tau) = A \cos \alpha \sec h(\tau) \exp\left(-\frac{i C \tau^2}{2}\right),$$  \hspace{1cm} (2a)

and

$$v(\xi = 0, \tau) = A \sin \alpha \sec h(\tau) \exp\left(-\frac{i C \tau^2}{2}\right),$$  \hspace{1cm} (2b)

where $A$ is the amplitude, $C$ is the linear chirp parameter, $\alpha$ determines the relative strengths of input soliton in each of the two polarizations. The dispersion and the nonlinearity effects can be taken into account respectively in propagating form $\xi$ to $\xi + h$ (small distance $h$) according to the SSFM algorithm. It becomes simple to deal with the pulse propagation by using the SSFM. To deal with the edge effects we enlarge the computational region to $\tau = (-80, 80)$, set the number of sampling $2^{13}$ and use the edge damping method in the Ref.\cite{4}. The initial amplitudes of input soliton in each polarizations are equal ($= /4$) in the section 3, are unequal ($\neq /4$) in section 4.

3. Effects of initial chirp on soliton propagation for equal amplitudes

3.1 Variation of the interval of the two components with propagation distance

The interval of the two polarization components is defined as

$$\Delta \tau(\xi) = \tau_{\text{max}}^u(\xi) - \tau_{\text{max}}^v(\xi),$$  \hspace{1cm} (3)

where $\tau_{\text{max}}^u(\xi)$ and $\tau_{\text{max}}^v(\xi)$ are the positions above which $|u(\xi, \tau)|$ and $|v(\xi, \tau)|$ reach maximum value, respectively. We can obtain $\tau_{\text{max}}^u(\xi) = -\tau_{\text{max}}^v(\xi)$ from Eqs.(1) and (2). The slow component is before the fast one when $\Delta \tau(\xi) < 0$. The two components will separate when $\Delta \tau(\xi)$ always increases with the increase of propagation distance. If $\Delta \tau(\xi)$ varies slower and slower with propagation distance and becomes finally unchanged at $A=A_{\text{th}}$, $A_{\text{th}}$ is called the threshold value above which soliton stably propagates in birefringent fibers. The two polarization components can be mutually bound together for $A > A_{\text{th}}$, separate each other for $A < A_{\text{th}}$. The threshold value $A_{\text{th}}$ can be determined by variation of the interval with propagation distance. We firstly assume $\delta = 0.5$. Fig.1 shows that the intervals of two polarization components vary with propagation distance for $\delta = 0.5$. The dashed curve is for $A=0.845$ and $C=0.5$, solid curve is for $A=0.996$ and $C=0$, dot-dashed curve is for $A=1.02$ and $C=-0.5$. It is found from Fig.1 that the intervals of two components for different cases are almost unchanged when $\xi > 16$. Therefore, $A_{\text{th}}=0.996$ which is more accurate than that in the Ref.\cite{4} for $C=0$, $A_{\text{th}}=0.845$ for $C=0.5$ and $A_{\text{th}}=1.02$ for $C=-0.5$. It shows that initial chirp changes the threshold value $A_{\text{th}}$ of soliton bound state in birefringent fibers. The positive chirp enhances the nonlinear effect and makes the threshold value $A_{\text{th}}$ smaller, the negative one makes the threshold value $A_{\text{th}}$ larger. It is important to the applications in optical communication and its device.
Fig. 2 shows that the intervals vary with propagation distance for $A=1.1$ and $\delta = 0.5$. The curves (1-5) are for $C=-0.7$, -0.5, 0, 0.5 and 0.7, respectively. It is found that the intervals oscillate with the increase of propagation distance for $A>A_t$. It shows that the two polarization components are mutually bound together and stably propagates as a single unit. For a given magnitude of $A$, the larger $C$ (positive chirp) is, the earlier two components are bound; the larger $|C|$ (negative chirp) is, the later two components are bound.

![Figure 1](image1.png)  
**Figure 1.** Variation of the intervals with propagation distance for $\delta = 0.5$. The dashed curve is for $A=0.845$ and $C=0.5$, solid curve is for $A=0.996$ and $C=0$, dot-dashed curve is for $A=1.02$ and $C=-0.5$.  

![Figure 2](image2.png)  
**Figure 2.** Variation of the intervals with propagation distance for $A=1.1$ and $\delta = 0.5$. The curves (1-5) are for $C=-0.7$, -0.5, 0, 0.5 and 0.7, respectively.

### 3.2 Effect of linear chirp on the relationship between $A_t$ and $\delta$

Fig. 3 shows the effect of linear chirp on the relationship between $A_t$ and $\delta$. The solid curve is for $C=0$, dashed curve is for $C=0.5$, dotted curve is for $C=-0.5$. It is found from Fig. 3 that the initial chirp changes the threshold value $A_n$ which increases with the increase of $\delta$. For example, the $A_n$ is 1.608 at $\delta = 0.8$ for negative chirp, is 1.58 for unchirp case, is 1.34 for positive chirp; the $A_n$ is 0.88 at $\delta = 0.3$ for negative chirp, is 0.76 for unchirp case, is 0.743 for positive chirp. The threshold value $A_n$ of positive chirp is less than that of unchirped case for $\delta \cdot 0.3$, is almost consistent with that of unchirped case for $0.15 \cdot \delta < 0.3$; the difference of $A_n$ between positive chirp and unchirped case increases with increase of $\delta$ for $0.3 \cdot \delta < 0.6$, remains about 0.24 for $0.6 \cdot \delta \cdot 1$. The $A_n$ of negative chirp is slightly larger than that of unchirped case for $\delta \cdot 0.5$, is much larger than that of unchirped case for $\delta < 0.5$; the difference of $A_n$ between negative chirp and unchirped case decreases with the increase of $\delta$ for $0.3 < \delta < 0.5$, remains about 0.13 for $0.15 \cdot \delta \cdot 0.3$.

It shows the effect of initial positive chirp on the threshold value $A_n$ is greater than that of negative chirp for high birefringent fiber (about $\delta > 0.5$), is less than that of negative chirp for low birefringent fiber (about $\delta < 0.5$). One should consider the effect of initial chirp on the threshold value $A_n$ changing in the practical applications.
Figure 3. Effect of linear chirp on the relationship between $A_{th}$ and $\delta$. The solid curve is for $C=0$, dashed curve is for $C=0.5$, dotted curve is for $C=-0.5$.

Figure 4. Variation of amplitude threshold value $A_{th}$ with the angle $\alpha$ for $\gamma=0.5$. The solid curve is for $C=0$, dashed curve is for $C=0.5$, dotted curve is for $C=0.5$, dotted curve is for $C=-0.5$.

4 Effects of initial chirp on soliton propagation for unequal amplitudes

4.1 Variation of the threshold value $A_{th}$ with the polarization angle

It is found from numerical results that the amplitude threshold value $A_{th}$ increases with the decrease of the polarization angle $|\Delta_{p}|$. Fig.4 shows that the $A_{th}$ varies with the angle $\gamma$ ($0< \gamma < \pi/2$) in the case of $\gamma=0.5$. The solid curve is for $C=0$, dashed curve is for $C=0.5$, dotted curve is for $C=-0.5$. The threshold value $A_{th}$ increases obviously with the decrease of the angle $|\Delta_{p}|$ and $C$, the maximum threshold value $A_{th}$ is at $\gamma=\pi/4$ (0.7854). The difference of $A_{th}$ between positive chirp and unchirped case is larger than that between negative chirp and unchirped case. It shows that effect of positive chirp on variation of $A_{th}$ with angle $|\Delta_{p}|$ is greater than that of negative chirp.

4.2 Variation of the threshold value $A_{th}$ with parameter

We assume $\gamma=\pi/6$ (0.5236). Fig.5 shows that the amplitude threshold value $A_{th}$ varies with group velocity mismatch parameter $\delta$ in the case of $\gamma=\pi/6$. The solid curve is for $C=0$, dashed curve is for $C=0.5$, dotted curve is for $C=-0.5$. It is found from Fig.5 that the initial chirp changes the threshold value $A_{th}$ which increases with the increase of $\delta$. The $A_{th}$ of positive chirp is larger than that of unchirped case for $0.15<\delta<0.4$; is much less than that of unchirped case for $0.4<\delta<1$. The $A_{th}$ of negative chirp is larger than that of unchirped case for $0.15<\delta<0.75$, is slightly less than that of unchirped case for $0.75<\delta<1$: the difference of $A_{th}$ between negative chirp and unchirped case decreases with the increase of $\delta$ for $0.15<\delta<0.75$, remains about 0.05 for $0.75<\delta<1$. For a given magnitude of $\delta$ and $C$, the threshold value for $\gamma=\pi/6$ is less than that for $\gamma=\pi/4$. Comparing the difference of $A_{th}$ between positive chirp and unchirped case with that between negative chirp and unchirped case, it shows that effect of initial positive chirp on the threshold value $A_{th}$ is still greater than that of negative chirp for high birefringent fiber (about $\delta>0.5$), is still less than that of negative chirp for low birefringent fiber (about $\delta<0.5$).
5 Conclusion

It is found that initial chirp can change obviously the threshold value $A_{th}$ which increases with the decrease of the polarization angle $|\theta/4-\theta|$. The positive chirp obviously makes the $A_{th}$ smaller for $>0.5$, the negative one makes the $A_{th}$ larger for $<0.5$. The effect of initial positive chirp on the threshold value $A_{th}$ is greater than that of negative chirp for high birefringent fiber, is less than that of negative chirp for low birefringent fiber. The threshold value $A_{th}$ increases with the decrease of the angle $|\theta/4-\theta|$ and $C$. The effect of positive chirp on variation of $A_{th}$ with angle $|\theta/4-\theta|$ is greater than that of negative chirp.

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