Spin number dependent dissipative coupling strength

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ABSTRACT
A system consisting of a yttrium iron garnet (YIG) sphere coupled to a 1D circular-rectangular cavity is tuned between level repulsion and attraction by rotating the angular position of the YIG sphere within the cavity. The dominance of coherent or dissipative coupling mechanisms was determined, and the coupling strength was deduced by fitting the transmission spectra. By changing the diameter of the YIG sphere from 0.5 mm to 0.3 mm and 1.0 mm, we confirm that the $\sqrt{N}$ scaling of coherent coupling strength is also applicable in dissipative coupling. A large YIG sphere leads to an enhanced coupling strength that is useful for information processing. Alternatively, a small YIG sphere results in a narrow transition regime, which may be helpful for identifying the dissipative coupling dominated regime and providing insight into the physical origin of dissipative coupling.

Strong light-matter interaction is an essential component of research into quantum information exchange between two subsystems and achieving the ambitious goal of workable quantum technologies.1–4 During these interactions, atoms, ions, photons, spins, etc., could be coupled to harness their unique characteristics and produce distinct features.5–10 In the past decade, hybrid systems based on magnetic materials enclosed in a microwave cavity have been found to provide an elegant way to control the light-matter interaction.11 This form of system merges the properties of quantum electrodynamics and magnetism12–14 and serves as an interesting platform to demonstrate phenomena, including spin pumping manipulation,15–19 magnon quintuplet states,20 the exceptional point,21 magnon-polariton bistability,22 and the quantum nature of magnon-qubit coupling.23,24 All these effects are based on coherent magnon-photon coupling. The resulting mode hybridization has a characteristic anticrossing between the two eigenmodes, representing the oscillatory exchange of energy quanta.25 Here, the anticrossing gap and coupling strength are proportional to the square root of the spin number $N$, according to the Clebsch-Gordan coefficient.26

In addition to level repulsion dominated by the coherent coupling mechanism, considering the dissipation of the microwave cavity and magnetic material, very recently level attraction dominated by the dissipative coupling mechanism has been realized in coupled magnon-photon systems.24–27 Here, the two hybridized modes converge toward each other, instead of the avoided-crossing seen in level repulsion. This dissipative coupling can be described using both classical and quantum models27–29 and displays effects such as nonreciprocity and unidirectional invisibility,30,31 distant magnetic moments,32 anti-Parity-Time symmetry,32 and two-tone driven Cavity-Magnon-Polariton.33–35 Due to these unique features of dissipative coupling, the dependence of coupling strength on the spin number, which is widely acknowledged in coherent coupling, needs to be investigated in dissipative coupling. This dependence is essential for research related to tunable coupling and may give insight into developing future magnon-based quantum systems.

In this work, we couple a yttrium iron garnet (YIG) sphere36,37 to a 1D circular-rectangular cavity. By rotating the YIG sphere’s angular position within the cavity, both level repulsion and attraction can be achieved.38 Then we change the diameter of the YIG
sphere from 0.5 mm to 3.0 mm and 1.0 mm, and the $\sqrt{N}$ scaling of coupling strength seen in coherent coupling is confirmed to be applicable in dissipative coupling. Increasing the YIG size leads to an enhanced coupling strength that is useful for information processing. On the other hand, the transition regime also changes with the size of the YIG sphere. Shrinking the YIG size results in a weak average effect of local field and a narrow transition regime, which may be helpful for identifying the dissipative coupling dominated regime and providing insight into the physical origin of dissipative coupling.

Our setup is shown in Fig. 1(a). A 1D Fabry-Perot-like cavity is formed by connecting a Ku-band circular waveguide to circular-rectangular transitions. The inner diameter of the circular waveguide is 16.1 mm. The length of the cavity is aligned to the $x$-direction, while an external magnetic field $H_z$ is applied in the $z$-direction. At the middle plane of the cavity, the dominant term of the cavity mode $TE_{11}$'s microwave magnetic field is along the $x$-direction and its amplitude $h_x$ is calculated as a color map in Fig. 1(b). Positions A and B denote the antinode and node of the cavity. The angular position $\theta$ of the YIG sphere is defined with respect to $\theta = 0$ (position A). This rotation process thus changes the local microwave magnetic field experienced by the YIG sphere and alters the magnon-photon coupling phenomenon. Using a vector network analyzer (VNA), we then measure the microwave transmission $S_{21}$.

Theoretically, the transmission of our coupled magnon-photon system can be written as:

$$S_{21} = 1 + \frac{\gamma_c}{i(\omega - \omega_c) - (\gamma_c + \gamma_m) + \gamma_c^2 G + \pi \gamma_c^2 \gamma_m^2},$$

where $\omega_c$ and $\omega_m$ are the resonant frequencies of the cavity and magnon modes, respectively. $\gamma_c$ is the intrinsic damping of the cavity, while $\gamma_m$ is the extrinsic damping coming from the cavity input/output connections and sample loading. $\gamma_m$ is the damping of the magnon. The total coupling strength $G = g \sqrt{\omega_c \omega_m}$ has an absolute value $g > 0$ and phase $\Phi$. For pure level repulsion, $G = g$ and $\Phi = 0$, and for pure level attraction, $G = ig$ and $\Phi = \pi$.

The eigenfrequencies of the hybridized magnon-photon modes are

$$\omega_k = \frac{1}{2} [\omega_c + \omega_m \pm \sqrt{(\omega_c - \omega_m)^2 + 4G^2}],$$

with $\omega_c = \omega_c - i\gamma_c$ and $\omega_m = \omega_m - i\gamma_m$.

By sweeping the external field $H_z$ to tune the frequency of the magnon mode with respect to the cavity mode, amplitude mappings of the microwave transmission spectra $S_{21}$, measured with the YIG sphere at positions A and B, are shown in Figs. 1(c) and 1(d) as a function of the frequency and field detunings, $\Delta = \omega - \omega_c$ and $\Delta_H = \omega_m(H) - \omega_c$. Here, the intrinsic damping and extrinsic damping of the cavity mode are $\gamma_c/2\pi = 4$ MHz and $\gamma_m/2\pi = 100$ MHz, respectively. The uncoupled magnon mode has a damping of $\gamma_m/2\pi = 1$ MHz and follows the Kittel dispersion $\omega_m(H) = \gamma(H + H_A)$, where $\gamma = 2\pi \times 28.6$ GHz/T is the gyromagnetic ratio and $\mu_0 H_A = 7.5 \pm 2$ mT is the magneto-crystalline anisotropy field.

At position A ($\theta = 0^\circ$), we observe level repulsion in the mapping. The two hybridized modes are repelled by each other and open a Rabi-like gap. To change the coupling behavior, we move the YIG sphere to position B by rotating it to $\theta = 90^\circ$. Level attraction is now observed with the hybridized modes bending and coalescing toward each other.

For the field detuning $\Delta_H = 0$, the amplitude spectra $|S_{21}|$ (green dots) at positions A and B are plotted as a function of $\Delta_c$ in Figs. 1(e) and 1(f). Black curves are fittings from Eq. (1). At position A, $g = 14.8$ MHz and $\Phi = 0$, and at position B, $g = 5.4$ MHz and $\Phi = \pi$. Therefore, through rotation of the YIG sphere, both conventional level repulsion and newly reported level attraction can be observed in this system, consistent with the previous result in Ref. 24 for a 1.0-mm YIG sphere.

![FIG. 1.](image-url)

(a) Schematic picture of the 1D circular-rectangular cavity. The external magnetic field $H_z$ is applied in the $z$-direction. (b) The amplitude $h_x$ of the dominant microwave magnetic field at the middle plane of the cavity. A 0.5 mm YIG sphere (black dot) is placed approximately 3.0 mm from the inner edge at this middle plane and could be rotated clockwise (white arrow) relative to the cavity. The angular position $\theta$ of the YIG sphere is defined with respect to $\theta = 0$ (position A). A vector network analyzer (VNA) is employed to measure the microwave transmission $S_{21}$. When the YIG is placed in position A, the $|S_{21}|$ amplitude mapping (c) and amplitude spectrum (e) at zero field detuning show the level repulsion phenomenon. When the YIG is placed in position B, the $|S_{21}|$ amplitude mapping (d) and spectrum (f) show the level attraction phenomenon. Black curves in (e) and (f) are fittings using Eq. (1).
We then replace the 0.5-mm YIG sphere with a 0.3-mm YIG sphere and a 1.0-mm YIG sphere to compare the measured coupling strengths. By fitting the transmission spectra, the 0.3-mm YIG sphere shows level repulsion with \( g = 6.8 \) MHz and \( \Phi = 0 \) at position A and level attraction with \( g = 2.7 \) MHz and \( \Phi = \pi \) at position B, and the 1.0-mm YIG sphere shows level repulsion with \( g = 39.7 \) MHz and \( \Phi = 0 \) at position A and level attraction with \( g = 15.0 \) MHz and \( \Phi = \pi \) at position B. The measured frequency dispersions of the 0.3-mm (orange dot), 0.5-mm (green dot), and 1.0-mm (blue dot) spheres are shown in Fig. 2. Calculations from Eq. (2) are added as black curves. The 2g Rabi-like gap at \( \Delta_H = 0 \) in level repulsion and the 4g convergence of eigenfrequencies at \( \Delta_H = \pm 2g \) in level attraction are indicated. 

The conventional description of the coherent coupling between spins and a microwave field with Rabi-like splitting is based on the Tavis-Cummings model, with the coupling strength \( g \) as a function of the angular position \( \theta \) of the YIG sphere.

\[
g_N = \frac{\eta}{2} \sqrt{\frac{\hbar g_0 \mu_0}{V_c}} \sqrt{2N_s} \tag{3}
\]

where \( \eta \) is the overlapping coefficient that relates to local magnetic field at the YIG position, \( g_0 \) is the vacuum permeability, \( V_c \) is the mode volume of the cavity, \( N \) is the total number of polarized spins, and \( s = 5/2 \) is the spin number of the ground state Fe\(^{3+} \) ion.

Considering \( g_0 = \sqrt{\frac{\pi \mu_0}{V_c}} \) as the coupling rate of a single spin to the microwave field, the collective coupling strength \( g_N \) of an \( N \)-spin ensemble is enhanced by a factor of \( \sqrt{N} \). The total spin number \( N = \rho V \), where \( \rho \) is the spin density and \( V \) is the spin volume. The enhancement factor of coherent coupling strength is thus the square root of the spin number (volume) ratio. As shown in Figs. 2(g) and 2(h), the coupling strengths of both level repulsion at position A and level attraction at position B increase linearly with \( \sqrt{V} \); the black dashed line is the fitting result.

To investigate this enhancement factor of both coherent coupling and dissipative coupling in detail in this system, we precisely rotate the YIG spheres (0.3-mm, 0.5-mm, or 1.0-mm) from \( 0^\circ \) to \( 180^\circ \). A panoramic view of the evolution between level repulsion and attraction is observed. By using Eq. (1) to fit the measured transmission spectra, we determine the total coupling strength \( G = g e^{i \Phi/2} \) as a function of the angular position \( \theta \) of the YIG sphere. The absolute coupling strength \( g \) is shown in polar coordinates in Fig. 3(a), and the phase \( \Phi \) is in Fig. 3(b). The gray shade is used to indicate the level attraction regime where \( \Phi = \pi \), and the yellow shade indicates the transition regime where \( 0 < \Phi < \pi \).

As we rotate the YIG sphere away from \( \theta = 0 \) (position A) clockwise, the system initially has \( \Phi = 0 \) and is dominated mainly by the coherent coupling mechanism. As \( \theta \) increases, the Rabi-like gap gradually closes. When \( g \) becomes small enough, the system enters a transition regime where \( \Phi \) increases from 0 to \( \pi \), and both coupling mechanisms play a non-negligible role. Inside this transition regime, \( \theta = 67^\circ \) with a minimum \( g \) marks the matching condition of coherent and dissipative coupling mechanisms. After the transition regime, \( \Phi = \pi \), and the system is mainly dominated by the dissipative coupling mechanism. The absolute \( g \) increases again until the strongest level attraction is observed at \( \theta = 90^\circ \) (position B). As the YIG rotates further beyond this magnetic field node, a symmetric change appears again. The absolute \( g \) diminishes until reaching another transition regime where \( \Phi \) drops from \( \pi \) to 0, and the matching condition is \( \theta = 110^\circ \) with a minimum \( g \). Beyond this transition regime, the system shows level repulsion again and \( g \) increases until a maximum at \( \theta = 180^\circ \).

The influence of YIG’s size on coupling is observed over the whole rotation range. As shown in Fig. 3(a), to make the 0.3-mm (orange dots) and 0.5-mm data (green dots) comparable to the 1-mm-data (blue dots), they are multiplied by the enhancement factor \( \left( V_{1,0.3 \text{mm}} / V_{0.5 \text{mm}} \right)^{1/2} = (1/0.3)^{1/2} = 6.086 \) and \( \left( V_{1,0.3 \text{mm}} / V_{0.5 \text{mm}} \right)^{1/2} = (1/0.5)^{1/2} = 2.828 \) using Eq. (3). The good agreement confirms the \( \sqrt{N} \) scaling of the coupling strength for both coherent and dissipative couplings. The minor deviation may
FIG. 3. Complex coupling strength $G = g e^{i \Phi/2}$ measured when the cavity is coupled to a 0.3-mm, 0.5-mm, or 1.0-mm YIG sphere. (a) Absolute coupling strength $g$ is plotted as the radial axis of polar coordinates, while the YIG’s position $\theta$ is the angular axis. For a clear comparison, data for the 0.3-mm YIG sphere (orange dots) are enlarged by a factor of 6.086, and data for the 0.5-mm YIG sphere (green dots) are enlarged by a factor of 2.828. The gray shade indicates the level attraction regime. (b) Coupling phase $\Phi$ vs YIG’s position $\theta$. The yellow shade indicates the transition regime.

come from the averaged effect of overlapping coefficient, which will be talked about later.

For the coupling strength phase $\Phi$ in Fig. 3(b), it is noticeable that the transition regime changes with the size of the YIG sphere. This reflects the spatial overlapping between the YIG sphere and the local microwave magnetic field distribution of the cavity mode. For a large YIG sphere, the overlapping coefficient and the coupling are averaged over the YIG size, resulting in the minor deviation in coupling strength $g$ and a large transition regime. For a small YIG sphere with decreased spatial overlap, the weak average effect of the local field and overlapping coefficient results in a narrow transition regime. This narrow transition might be helpful for identifying the dissipative coupling dominated regime and providing insight into the physical origin of dissipative coupling.

To carefully check the narrow transition regime and matching condition of the 0.5-mm YIG sphere, several measurements extracted from the transition regime shown in Fig. 3 are analyzed. As shown in the top and bottom plots in Fig. 4, $\theta = 104^\circ$ shows weak level attraction and is dominated by dissipative coupling, and $\theta = 116^\circ$ shows weak level repulsion and is dominated by coherent coupling. Comparing these two coupling mechanisms, in addition to features shown in amplitude spectra and mappings which have been discussed above, a significant difference also appears in the phase spectra. Coherent coupling displays two $\pi$-phase drops and one opposite $\pi$-phase jump, corresponding to the two hybridized antiresonance modes and a resonance between them, while dissipative coupling displays a single $2\pi$ jump at zero field detuning, consistent with the coalescence of hybridized modes. The existence of this $2\pi$ phase jump offers an effective way to determine the dominant coupling mechanism of the system.

By precisely rotating the YIG sphere between $\theta = 104^\circ$ and $\theta = 116^\circ$, we could find the exact matching condition that separates the coherent coupling dominated regime and the dissipative coupling dominated regime. At $\theta = 109^\circ$, $110^\circ$, and $111^\circ$, due to the small coupling strength, amplitude mappings display a level crossing with suppressed transmission (bright yellow color) near zero field detuning, while amplitude spectra show a dip at zero frequency detuning in Fig. 4(a). To distinguish their difference, the corresponding phase spectra are considered. As shown in Fig. 4(b), when $\theta$ increases from $109^\circ$ to $111^\circ$, the $2\pi$-jump gradually closes and the phase becomes continuous; thus, the system evolves from the dissipative coupling dominated regime to the coherent coupling dominated regime within just $3^\circ$. The matching condition of the two coupling regimes is found to occur at $110^\circ$. Fittings of spectrum using Eq. (1) are added as black curves.

In conclusion, we couple a YIG sphere to a 1D cavity to study the magnon-photon coupling. By rotating the YIG sphere’s angular position within the cavity, both level repulsion and attraction can be
achieved. The dominance of coherent or dissipative coupling mechanisms was determined, and the coupling strength was deduced by fitting the transmission spectra. Then we change the diameter of the YIG sphere from 0.5 mm to 0.3 mm and 1.0 mm to investigate the dependence of coupling strength on the YIG’s size. By comparing the measured coupling strengths, we confirm that a $\sqrt{N}$ scaling of the coupling strength seen in coherent coupling is also applicable in dissipative coupling as a key feature. The transition regime changes with the size of YIG sphere as well. A small YIG sphere overlaps less with the local microwave magnetic field, and the weak average effect of the overlapping coefficient and coupling results in a narrow transition regime. This may be helpful for identifying the dissipative coupling dominated regime and providing insight into the physical origin of dissipative coupling.

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