Results on $D^0 - \bar{D}^0$ Mixing and CP violation at B factories

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Abstract. We begin with a brief presentation of the phenomenology of $D^0 - \bar{D}^0$ mixing. This is followed by a summary of experimental results in various final states. In particular, we will first describe the discovery of $D^0 - \bar{D}^0$ mixing by BaBar and Belle using two-body decay modes, and follow this with more recent studies in three-body decay modes. Finally, we will mention semileptonic searches for $D^0 - \bar{D}^0$ mixing and end with a summary of all experimental results.

1. The physics of $D^0 - \bar{D}^0$ Mixing

Neutral D mesons can be viewed as a two-state system that mixes due to doubly-weak transitions that connect the two states. These two basis states are often taken to be either the flavor eigenstates $D^0$ and the $\bar{D}^0$ which evolve according to the equation

$$i \frac{\partial}{\partial t} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = \begin{pmatrix} M & \frac{i}{2} \Gamma \\ \frac{i}{2} \Gamma & -M \end{pmatrix} \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix},$$

(1)

(where $M$ and $\Gamma$ are the mass and decay matrices) or can be taken to be the mass eigenstates $D_1$ and $D_2$ defined as follows:

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle,$$

(2)

where $|p|^2 + |q|^2 = 1$.

For reference, let us also define the mean decay width parameter $\Gamma$ using

$$\Gamma \equiv \frac{\Gamma_1 + \Gamma_2}{2},$$

(3)

and the mixing parameters $x$ and $y$:

$$x \equiv \frac{m_1 - m_2}{\Gamma}, \quad \text{and} \quad y \equiv \frac{\Gamma_1 - \Gamma_2}{2\Gamma}.$$ 

(4)

These neutral $D$ mesons can decay to Cabibbo-favored (CF), typically called right-sign (RS) final states, or to double-Cabibbo-suppressed (DCS) or wrong-sign (WS) final states.

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2. Two-body final states

$D^0 - \bar{D}^0$ mixing was discovered in decays to DCS and singly-Cabibbo-suppressed (SCS) final states by the BaBar and Belle experiments [1, 2]. The chief results from the BaBar collaboration’s study of 384 fb$^{-1}$ of data in the WS $K^+\pi^-\pi^-$ mode included the measurements

$$x'^2 = (-0.22 \pm 0.30 \pm 0.21) \times 10^{-3}, \quad y' = (9.7 \pm 4.4 \pm 3.1) \times 10^{-3}$$

and

$$R_D' = (3.03 \times 0.16 \times 0.10) \times 10^{-3},$$

where $R_D$ is ratio of integrated DCS and CF decay rates, and $x'$ and $y'$ are the mixing parameters $x$ and $y$ defined above and rotated by a strong phase for $K^-\pi^+$ decays. In all cases the first error is statistical and the second systematic.

Belle’s initial observation of $D^0 - \bar{D}^0$ mixing came in comparisons of lifetimes measured in different two-body final states.

For neutral $D$ meson decays to CP eigenstates, we can further define two parameters related to lifetimes: $y_{CP}$ which measures the difference between the lifetime for a “CP-averaged” decay state such as $K^-\pi^+$ and a CP-eigenstate such as $K^+K^-$, and the parameter $\Delta Y$ which measures the lifetime difference in decays to CP-eigenstates when the neutral D meson is produced as a $D^0$ or as a $\bar{D}^0$; such initial states can be tagged when for instance, the neutral D meson arises from the decay of a charged $D^*$ meson:

$$y_{CP} = \frac{\tau_{hh}^+ - \tau_{hh}^-}{\left\langle \tau_{hh} \right\rangle}\ A_f,$$

$$\Delta Y = \frac{\left\langle \tau_{hh} \right\rangle}{\left\langle \tau_{hh} \right\rangle}\ A_f,$$

where $\left\langle \tau_{hh} \right\rangle = (\tau_{hh}^+ + \tau_{hh}^-)/2$ is the mean lifetime for neutral D mesons decaying into CP eigenstates and $A_f = (\tau_{hh}^+ - \tau_{hh}^-)/\left(\tau_{hh}^+ + \tau_{hh}^-\right)$ is the asymmetry in their lifetimes. Both $y_{CP}$ and $\Delta Y$ are zero if there is no $D^0 - \bar{D}^0$ mixing. In the limit of CP conservation the parameter $y_{CP}$ equals the parameter $y$ and the parameter $\Delta Y$ is zero.

Belle, using 540 fb$^{-1}$ of data, measured $y = (1.31 \pm 0.32 \pm 0.25)\%$ using the $K^+K^-$ and $\pi^+\pi^-$ final states, but found no evidence for CP violation in these decays since the measured value of $A_f$ was found to be $(0.01 \pm 0.30 \pm 0.15)\%$. More recently, the BaBar collaboration has reported measurements in these decay mods to be $y = (1.24 \pm 0.39 \pm 0.13)\%$ and $A_f = (-0.26 \pm 0.36 \pm 0.08)\%$ using 384 fb$^{-1}$ of data [3]. In all cases, the first error is statistical and the second systematic.

All these results are summarized in figure 1. It should be noted that an earlier search by the Belle collaboration for mixing in the $K^+\pi^+$ final state using 400 fb$^{-1}$ of data did not yield clear evidence for mixing [4].

Finally, for two-body decays it is noteworthy that the CLEO collaboration has determined the strong phase in the $K^-\pi^+$ decay mode, using external input for other mixing parameters, to be $(22^{+11}_{-12} \pm 9^0)$, where the first error is statistical and the second systematic [5].

3. Three-body final states

Next, let us consider observations of $D^0 - \bar{D}^0$ mixing in three-body (Dalitz) decays such as $D^0 \rightarrow K^-\pi^-\pi^0$. In such decays the decay rate is a function of both the Dalitz variables as well as the lifetime. For instance, in $D^0 \rightarrow K^-\pi^-\pi^0$ decays the WS decay rate (when $|x|,|y| \ll 1$) can be written as

$$\frac{dN_f(s_{12},s_{13},t)}{ds_{12}ds_{13}dt} = e^{-\Gamma t}\left\{ |A_f|^2 + |A_f||\bar{A}_f|\left[ y\cos \delta_f - x\sin \delta_f \right] (\Gamma t) + \frac{x^2 + y^2}{4} |\bar{A}_f|^2 (\Gamma t)^2 \right\}$$

(6)
Figure 1. In these plots we see fits to lifetime distributions from 384 fb$^{-1}$ of BaBar data. Shown are the $K^-\pi^+$ CF mode (upper left), with shaded backgrounds, tagged $K^+K^-$ data (middle row) and $\pi^+\pi^-$ data (bottom row). In each case events arise either from $D^{*+}$ decays (left) or $D^{*-}$ decays (right). The upper right plot shows the extracted mean lifetimes.
where the variables on the $x$ and $y$ axes of the Dalitz plot are $s_{12} = m_{K^+\pi^-}^2$ and $s_{13} = m_{K^+\pi^0}^2$, the DCS amplitude $A_{DF}(s_{12}, s_{13}) = \langle \mathcal{F}|\mathcal{H}|D^0 \rangle$, the CF amplitude $A_{CF}(s_{12}, s_{13}) = \langle \mathcal{F}|\mathcal{H}|D^0 \rangle$, the subscripts $D$ and $F$ refer to CF and DCS final states respectively and where $\delta_f(s_{12}, s_{13}) = \arg[A_{DF}(s_{12}, s_{13})/A_{CF}(s_{12}, s_{13})]$. In this equation the first term is due to DCS, the last due to mixing and the second term is due to interference between the two.

The BaBar collaboration determined the CF amplitude $A_f$ in a time-independent Dalitz plot analysis of the RS decay sample. This amplitude was then used in the analysis of the WS sample where the DCS amplitude $A_f$ is extracted along with the mixing parameters. Each of the amplitudes $A_f$ and $A_D$ are in turn described as a coherent sum of amplitudes, each of which describes a separate resonance (the usual isobar approach). Figure 2 shows the RS and WS Dalitz plots, as well as the $D^0$ mass and the $\Delta m$ distributions for the WS sample. The lifetime distributions for the RS and WS samples as well as projections on to the Dalitz plot $x$ and $y$ axes are shown in figure 3.

We may summarize the results of BaBar’s study of 384 fb$^{-1}$ of data in the $K^+\pi^+\pi^0$ mode as follows. The $x$ and $y$ parameters (rotated by the strong phase for this mode) were determined to be $2.61^{+0.53}_{-0.39}$% and $-0.06^{+0.55}_{-0.64}$ ± 0.34% respectively, where the first error is statistical and the second systematic. The probability for no mixing was determined to be 0.1%, or 3.2$. No evidence was found for CP violation [6].

Another 3-body decay mode which has been investigated and in which the parameters and technique are somewhat different is the $K_S^0\pi^+\pi^-$ final state. This technique was first used by CLEO [7] and has more recently been used by the Belle collaboration [8] to determine that, not allowing for CP violation, $x = (0.80 \pm 0.29^{+0.99}_{-0.10} + 0.10)$% and $y = (0.33 \pm 0.24^{+0.08}_{-0.12} + 0.08)$%.

When CP violation is permitted in the fit, they find that $x = (0.80 \pm 0.29^{+0.08}_{-0.07} + 0.10)$%, $y = (0.33 \pm 0.24^{+0.06}_{-0.12} + 0.06)$%, $|q/p| = 0.86^{+0.30}_{-0.29} + 0.06 \pm 0.08$ and $\arg\{\lambda_{K^0\pi^+\pi^-}\} = (-14^{+16}_{-18} + 5_{-3} + 2)$°.

Using yet another technique in the $K^0_S K^+ K^-$ final state, where the $\phi K^0_S$ region and its sidebands are examined to extract CP-odd and CP-even amplitudes, the Belle collaboration has determined from 673 fb$^{-1}$ that $y_{CP} = (0.21 \pm 0.63(stat.) \pm 0.78(sys.st.) \pm 0.01(model))$% [9]. This is done by measuring the mean lifetime $\tau'$ in the $\phi K^0_S$ region and the mean lifetime $\tau''$ in the sidebands, along with the corresponding fractions $f_1'$ and $f_1''$ of CP-even events in these regions. The lifetime asymmetry in the regions can then be related to $y_{CP}$ from which relation the latter is then determined:

$$\frac{\tau' - \tau''}{\tau' + \tau''} = \frac{y_{CP}(f_1'' - f_1')} {1 + y_{CP}(1 - f_1'' - f_1')} \approx y_{CP}(f_1'' - f_1').$$

where $\tau$ is the mean $D^0$ lifetime and $f_1 = \int |A_1|^2/\int (|A_1|^2 + |A_2|^2)$ is the aforementioned CP-even fraction.

4. Results from semileptonic modes

Early results on $D^0 - \bar{D}^0$ mixing using semileptonic decay modes include those from E791 [10] and CLEO [11]. In principle, these decay modes are cleaner because of the absence of a DCS amplitude. However, the problems caused by the missing neutrino more than compensate for this advantage, and the upper limits at the 90% CL from E791 and CLEO were 0.50% and 0.78%, respectively. More recently, BaBar [12] has placed a 90% CL limit of 0.12% on $r_M$ and Belle [13] has placed an even better upper limit of 0.06%, also at the 90% CL.
Figure 2. BaBar Dalitz plots for the (a) RS and (b) WS $D^0 \to K^{\mp} \pi^\pm \pi^0$ samples. Also shown are (c) the reconstructed $D^0$ mass and (d) $\Delta m$ distributions for the WS sample. The fit results used to extract the yields are shown as superimposed curves. The light histogram represents the mistag background, while the dark histogram shows the combinatoric background.

5. HFAG combination of all the results

There is now a wide variety of results in the field of $D^0 - \bar{D}^0$ mixing; the majority of the significant new ones having been described above. The HFAG group has combined these results with and without the assumption of CP conservation [14]. Figures 4 and 5 show the results. Allowing for the possibility of CP violation, HFAG determine the following parameter values: $x = (1.00^{+0.24}_{-0.25})\%$, $y = (0.76^{+0.17}_{-0.18})\%$, $|q/p| = 0.86^{+0.17}_{-0.15}$, and $\arg(\lambda_{K^{S+\pi-}}) = (8.8^{+7.6}_{-7.2})^\circ$.

6. Conclusions

After 30 years of searches, evidence for $D^0 - \bar{D}^0$ mixing is now compelling. It is clear that the world averages of the mixing parameters exclude the no $D^0 - \bar{D}^0$ mixing scenario at the 10$\sigma$ level even though no single measurement exceeds 5$\sigma$. At this point, future measurements must focus on determining the mixing parameters with precision and searching for CP violation in
Figure 3. Proper time distribution for (a) RS and (b) WS BaBar $D^0 \to K^{\mp} \pi^\pm \pi^0$ events with the fit result superimposed. Background event distributions are shown by the shaded histograms. Projections of the Dalitz distributions are shown for the $m_{K^\mp \pi^0}^2$ axis (c) and the $m_{K^0 \pi^0}^2$ axis (d). For these the light histogram represents the mistag background, while the dark histogram shows the combinatoric background.

$D^0 - \bar{D}^0$ mixing.

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Figure 4. Two-dimensional contours for the mixing parameters \((x, y)\) determined by the HFAG group using all available data for the case of no allowed CP violation. The no-mixing point \((0, 0)\) is clearly excluded with more than 5\(\sigma\) significance.

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Figure 5. Two-dimensional contours for the mixing parameters \((x, y)\) (top) and \((|q/p|, \phi)\) (bottom) determined by the HFAG group using all available data allowing for the possibility of CP violation. In the top plot the no-mixing point \((0, 0)\) is clearly excluded with more than \(5\sigma\) significance. In the bottom plot we see that the no-CPV point \((1, 0)\) is very much allowed.