Erratum: Primordial black hole origin for thermal gamma-ray bursts

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The paper Primordial black hole origin for thermal gamma ray bursts was published in MNRAS, 506, 806–812 (2021). In the original article, an error in equation (14) was found. Once taken into account that the ratio \( R_c/r_0^3 \) is Lorentz-invariant (Misner 1973), the correct expression for the observed spectral radiance should be rewritten as

\[
R_v = \frac{2\pi h \nu^3}{c^2} \left( \exp \left( \frac{h \nu}{kT_p^{(G)}} \right) - 1 \right)^{-1},
\]

where \( T_p^{(G)} \) is given by equation (16) (with an appropriate modification of the emission angle, as described later). Accordingly, equations (15) and (19) should be modified by the following expressions

\[
\nu_{\text{max}} = \frac{2.821}{h} kT_p^{(G)}.
\]

and

\[
\nu_s = \frac{2\pi h \nu^4}{c^2} \left( \exp \left( \frac{h \nu}{kT_p^{(G)}} \right) - 1 \right)^{-1}.
\]

As a consequence, the original selection of the initial parameters resulted in an improbable observational scenario. Nevertheless, the fundamental theory and conclusions of the original article remain the same.

In Fig. 1, the emission angle as observed from the PBH frame \( \theta'_e \) is shown. Compared to previous set-up, no critical angle \( \theta'_c \) is considered in the model, since photons emitted at that angle will rarely reach the Earth. Moreover, under realistic initial parameters (described next), the measurable PBH Hawking radiation will be restricted to a region above the CBH photon sphere.

In relation to the PBH Hawking temperature for a distant observer \( T_p^{(G)} \), a recent publication by McMaken (2022) alleged a supposed inconsistency with equation (16) in the original paper (Barco 2021). Let us show that there is no discrepancy.

Following the formalism by Yoshino (2019) for a star under gravitational collapse (where the star emission is modelled as a blackbody emitter, receding away from the Earth), the redshift factor \( \alpha \) which relates the Planck temperature as measured by the emitter and observer (i.e., \( T_p = \alpha T_p' \) in our case) can be expressed as (equation (49) of this reference)

\[
\alpha = \sqrt{\frac{f(R)}{f(r_0)}} \left( 1 - \cos \theta'_c \right) \sqrt{1 - \frac{f(r_c)}{f(R)}}.
\]

where \( r_c (r_0) \) are the coordinate positions for the emitter (observer), \( M \) and \( R \) correspond to the mass and radius of the static star, respectively, and \( f(r) = 1 - 2M/r \).

Taking into account equation (39) of Yoshino (2019) for the velocity \( \beta \) of the star surface measured in the static frame (in our case, the velocity at which the PBH moves away from the Earth), the redshift factor \( \alpha \) can be rewritten as

\[
\alpha = \sqrt{\frac{f(R)}{f(r_0)}} \left( 1 - \beta \cos \theta'_c \right).
\]

After some simple algebra, we obtain for the first term on the right hand side of equation (5)

\[
\sqrt{\frac{f(R)}{f(r_0)}} = \sqrt{\frac{f(r_c)}{f(r_0)}} \sqrt{\frac{f(R)}{f(r_c)}} = \sqrt{\frac{f(r_c)}{f(r_0)}} \left( 1 - \frac{f(r_c)}{f(R)} \right)^{-1/2} = \sqrt{\frac{f(r_c)}{f(r_0)}} (1 - \beta^2)^{-1/2},
\]

where the first factor in equation (6) represents the gravitational redshift between the emitter and the observer \( \gamma_g \), and the second one is the kinematic factor \( \gamma = (1 - \beta^2)^{-1/2} \). So, the transformation relation can expressed as

\[
T_p^{(G)} = \alpha T_p' = \gamma_g \gamma (1 - \beta \cos \theta'_c) T_p'.
\]

Please note that the latter expression for \( T_p^{(G)} \) is equivalent to equation (16) in the original paper (Barco 2021), with the exception of the emission angle \( \theta'_c \) (in the corrected version, no critical angle \( \theta'_c \) is considered). Identical result can be easily deduced from equation (14) of McMaken (2022), simply by considering the emission angle transformation relation (equation (41) of Yoshino (2019))

\[
\cos \theta'_c = \frac{\cos \theta'_c - \beta}{1 - \beta \cos \theta'_c},
\]

and performing some elementary calculations.

After implementing the corrections, a possible astrophysical scenario includes an asteroid-mass PBH (Wang 2021; Coogan 2021) with \( M_P \approx 5 \times 10^{-18} M_\odot \) describing a free fall from rest at infinity towards a 10 \( M_\odot \) CBH at 48.6 kpc. In Fig. 2, the vertical axis now
represents the emission angle $\theta_e'$ (provided that, in the current situation, the PBH specific energy $E_s = 1$ with specific angular momentum $L_s = 3.3 \times 10^{-4}$ m, both in geometrized units).

It can be observed that, for a given emission angle $\theta_e'$, the PBH Hawking temperature diminishes as the light BH approaches its heavier companion. This cooling behaviour can be easily understood after inspection of equation (7): as the PBH gains velocity during its free fall, the term $1 - \beta \cos \theta_e'$ tends progressively to zero when the emission angle $\theta_e' = \theta_e = 0$ rad (i.e., along the Earth’s line of sight). As a numerical example, when the asteroid-mass PBH is located at $10^5$ m above the CBH horizon, $\beta = 0.48c$, $\gamma = 1.14$ and $\gamma_G = 0.88$ with an observed Hawking temperature of 552.5 keV. At $10^5$ m above the horizon, $\beta = 0.98c$, $\gamma = 5.53$ and $\gamma_G = 0.18$, where now $T_p^{(G)} = 17.5$ keV. It should be underlined that the emitted PBH Hawking radiation is not Lorentz-boosted (as incorrectly commented in the original version).

The numerical study of the PBH fluence spectrum $\nu S_\nu$ (via equation (3) of this revised version) and the associated flux density $S_\nu = \Omega R_\nu$, for different horizon separations $D_H$ is represented in Fig. 3. These parameters strongly depend on the solid angle $\Omega$ subtended by the PBH (considered as a point source) and obey the well-known inverse square law. So, it is assumed that the beginning of the GRB occurs when the fluence spectrum $\nu S_\nu$ is sufficiently high to be detected by ground-based or space observatories (that is, when the PBH is close enough to Earth). In our updated model, this happens when $D_H = 10^5$ m (please, see again Fig. 3(a)) while the final stage of the burst occurs at about 3 km above the CBH horizon, when $\nu S_\nu$ drops below the sensitivity of the Joint European X-Ray Monitor (JEM-X).

The astrophysical scenario is the same as the previously described in Fig. 2, with an emission angle $\theta_e' = \theta_e = 0$. A decreasing $\nu S_\nu$ fluence spectrum is also observed as the horizon separation $D_H$ decreases, with measurable time intervals $\Delta t$ between different PBH approximations. Unlike the original version, the closest approach to the CBH event horizon contributing to the observable GRB is now about 3 km. It should be also noted that the coordinate time interval is clearly reduced as the PBH gains speed during its free fall, achieving a velocity of 0.82c at the CBH photon sphere.

In connection with the numerical results depicted in Fig. 3, the cooling behaviour of the PBH Hawking temperature $T_p^{(G)}$ is described in Fig. 4. For a better visualization, the horizontal axis now represents the coordinate time $t$ once the estimated GRB duration ($t_{\text{GRB}} = 13.2$ s) is subtracted. In this revised version, the broken power-law parameters correspond to $a = 0.0$, $b = -12.5$, $t_0 = 9 \times 10^{-3}$ s, $\delta = 1.0$ s, $t_n = 1.5 \times 10^{-3}$ s and $T_n = 36$ keV. It is also worth mentioning that the slope index $b$ (which takes into account the cooling process at the final stage of the GRB) is steeper than in the original version. This
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... is due to the above-mentioned shortening of the time intervals as the PBH is more accelerated.

In relation to the capture rate of a PBH by a more massive BH, McMaken (2022) recently claimed an implausible occurrence based on Kouvaris (2008) and Capela (2013) research. Let us analyze in detail if McMaken’s arguments and conclusions (concerning such capture rate) can be properly applied to the binary black hole model here presented.

According to McMaken (2022), the capture rate $\mathcal{F}$ is estimated by integrating a Maxwellian dark matter distribution, with velocity dispersion $\bar{v}$, over the $E_s$ and $L_s$ space with well-defined integration limits (in accordance with equation (3) of Kouvaris (2008), within the context of weakly interacting massive particles (WIMPs) captured by a compact star). It must be borne in mind that, within Kouvaris and Capella’s formalism, the capture rate $\mathcal{F}$ is derived for a WIMP (or PBH, in Capela’s paper) which loses its initial energy due to the accretion of star’s material and dynamical friction, so it becomes gravitationally bound (please, see Section IIa of Capela (2013)).

Consequently, as stated by Kouvaris (2008), the capture rate $\mathcal{F}$ should be calculated in two steps: first, it must be determined what part of the $E_s$ and $L_s$ space can give orbits for the WIMPs that intersect with the neutron star (this is what McMaken is supposed to calculate). A second step involves a derivation of the fraction of such particles that lose enough energy to be trapped inside the star.

In that sense, equation (16) of McMaken (2022) (the alleged expression for calculating the capture rate of a PBH by a heavier black hole) is exactly the same as equation (13) of Capela (2013) in the context of a neutron star of radius $R$ (which, in turn, is also equivalent to equation (11) of Kouvaris (2008), once the integration of the Maxwellian distribution is carried out). Please note that Capela’s equation comes from a particular regime, where the PBH energy loss $E_{\text{loss}}$ is particularly high (i.e., when $E_{\text{loss}} \gg m_{\text{PBH}}c^2/3$, as well explained by Capela (2013) in Section IIb). How can Capella’s approach be valid for our binary BH scenario?

Even assuming that equation (16) of McMaken (2022) could properly be applied to our model, let us recalculate the supposed PBH capture rate $\mathcal{F}$ with the same parameters as McMaken’s paper. In this reference, all dark matter is composed of PBHs (that is, $\Omega_{\text{PBH}} = \Omega_{\text{DM}}$) with a current observed DM density of $\rho_{\text{DM}} = 0.5 \text{ GeV cm}^{-3} = 8 \times 10^{-3} \text{J m}^{-3}$. The velocity dispersion parameter is $\bar{v} = 7000 \text{ m s}^{-1}$ and a selected radius of capture $R = 1 \text{ kpc}$ from Earth.

Considering an all-DM primordial black hole of $5 \times 10^{17} \text{ kg}$ (Inomata 2017; Bartolo 2019; deLuca 2021) and a 10 $M_{\odot}$ central black hole with an Schwarzschild radius of $2.9 \times 10^4 \text{ m}$ (this last parameter can be easily replaced by other massive CBHs with identical consequences), the resulting capture rate is $\mathcal{F} = 8.6 \times 10^{-2} \text{s} = 2.7 \times 10^6 \text{yr}^{-1}$ (according to equation (16) of McMaken (2022)). This result is 15 orders of magnitude greater than the reported by McMaken, proving explicitly the inconsistency of such capture rate calculations.

As a final remark, it is worth noting that the astrophysical scenario described by Barco (2021) does not intend to reinterpret the thermal GRBs described satisfactorily by the fireball model (Ghirlanda 2003), but provide an alternative explanation of such astrophysical events based on PBH origin.
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