Numerical Method to Calculate Low-Frequency Sound Field in Non-anechoic Pools

Rui TANG\textsuperscript{1,2,3,*}, He TIAN\textsuperscript{1,2,3}, Qi LI\textsuperscript{1,2,3} and Yi-ming ZHANG\textsuperscript{1,2,3}

\textsuperscript{1}Acoustic Science and Technology Laboratory, Harbin Engineering University, Harbin 150001, China
\textsuperscript{2}Key Laboratory of Marine Information Acquisition and Security (Harbin Engineering University), Ministry of Industry and Information Technology; Harbin 150001, China
\textsuperscript{3}College of Underwater Acoustic Engineering, Harbin Engineering University, Harbin 150001, China

*Corresponding author

Keywords: Low frequency, Non-anechoic pools, Impedance boundary, Numerical calculation.

Abstract. A numerical method of low-frequency sound field calculation in non-anechoic pools with impedance boundary based on finite element software ACTRAN is proposed in this paper. A virtual Kundt’s tube is established in ACTRAN to obtain the normal incidence surface impedance of non-anechoic pools’ walls. The normal incidence surface impedance is then added to the surface of fluid domain to simulate the sound field boundary of non-anechoic pools. The numerical results are obtained by computing the model established in ACTRAN. The sound field of a spherical sound source has been tested in a glass tank to verify the accuracy of the proposed numerical method. As a comparison, the sound field in the glass tank with absolutely soft boundary is also calculated. It is concluded that the proposed method can effectively predict the low-frequency sound field in non-anechoic pools with arbitrary material impedance boundary and will provide a reference analysis method for the study of the low-frequency sound field characteristic in non-anechoic pools with complex impedance boundary.

Introduction

The characteristic of sound field in a rectangular enclosed space is an important and classical problem in acoustics. Considering the acoustic absorption ability of anechoic pools will be limited at low frequency, it is of great significance to extend the low-frequency application of non-anechoic pools. The accuracy of sound field prediction in non-anechoic pools depends on whether the sound field boundary is set correctly. Since there is no ideal boundary in reality, it is more appropriate to consider the boundary of non-anechoic pools as impedance boundary.

In order to solve the acoustic problem of enclosure cavity with impedance boundary, a large amount of effort has been devoted. Morse [1] first developed a nonlinear transcendental characteristic equation through combining the assumed sound pressure modes with complex impedance boundary conditions on the walls. In order to obtain the solutions of these highly nonlinear transcendental characteristic equations, numerical root searching algorithms need to be utilized. Bistafa [2] compared two different numerical procedures: Newton’s method and homotopic continuation procedure technique, and the latter procedure is faster in finding all the possible roots. Naka et al. [3] utilized an interval Newton/generalized bisection method to find the roots of the nonlinear characteristic equation within any given interval for the modal analysis of a rectangular room with arbitrary wall impedances. Recently, Du et al. [4] proposed a modified Fourier series method for the acoustic analysis of a rectangular cavity with impedance boundary conditions arbitrarily specified on any of the walls. Jeong et al. [5] proposed phased geometrical acoustics methods to evaluate various approximate boundary conditions for a phased beam tracing model. Jin et al. [6] proposed a general Chebyshev-Lagrangian method to obtain the analytical solution for a rectangular acoustic cavity with arbitrary impedance boundary conditions. Xie et al. [7] proposed a weak variational principle based
approach to study the sound field inside the acoustic enclosures with walls in arbitrary inclination and impedance conditions.

It is necessary to learn sound field characteristics before underwater experiments such as measuring the low-frequency sound power of an underwater source in a non-anechoic pool [8]. In the previous study, several methods for predicting the sound field in a non-anechoic tank with elastic boundary is investigated [9]. In the present paper, a low-frequency sound field prediction method in non-anechoic pools based on finite element software ACTRAN is proposed. In order to obtain the normal incidence surface impedance of non-anechoic pools’ walls, a virtual Kundt’s tube is established in ACTRAN. The normal incidence surface impedance of non-anechoic pools’ walls is then used to establish the model of the sound field in non-anechoic pools with impedance boundary and the numerical results are obtained by computing the model established in ACTRAN. The originality of this paper is the successful attempt of applying virtual Kundt’s tube to numerically calculate the normal incidence surface impedance of non-anechoic pools’ walls and then using the normal incidence surface impedance to numerically calculate the sound field of the non-anechoic pools’ walls. The numerical method proposed in the paper is applicable to predict the low-frequency sound field in enclosed space with arbitrary material and shaped boundary.

**Theory and Method**

**The Sound Field in Non-anechoic Pools with Absolutely Soft Boundary**

The boundary of non-anechoic pools is generally been considered as ideal boundary. For small-scale water tanks which are placed in the air, the boundary is considered as absolutely soft. A rectangular non-anechoic pool of dimensions \( L_x \times L_y \times L_z \) and the associated coordinate system are sketched in Figure 1. The boundary of the pool is absolutely soft boundary.

![Figure 1. Coordinate system of a rectangular non-anechoic pool.](image)

It is assumed that the sound pressure satisfies the wave equation

\[
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0,
\]

where \( p \) is the sound pressure, \( c \) is the sound speed in water, and \( t \) is the time of sound propagation. The boundary conditions can be described as

\[
p|_{\Sigma=0} = 0,
\]

where \( \Sigma \) denotes the six faces of the tank. The analytical solution of the sound pressure can be expressed as

\[
p = \sum_{n_x} \sum_{n_y} \sum_{n_z} A_{n_x,n_y,n_z} \sin \frac{n_x \pi}{L_x} x \sin \frac{n_y \pi}{L_y} y \sin \frac{n_z \pi}{L_z} z \cdot e^{i \omega t}, n_x, n_y, n_z = 1, 2, 3...
\]
The natural frequencies of the wave equation are then obtained as

\[
f_{n_x, n_y, n_z} = \frac{c}{2} \sqrt{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2}
\]  

(4)

**Numerical Method**

Absolutely soft boundary is an ideal sound field boundary condition and does not exist in reality. Thus, a low-frequency sound field prediction method in non-anechoic pool based on ACTRAN finite element software is proposed in this paper. The sound field of a glass tank is calculated in this section.

**The Virtual Kundt’s Tube.** As shown in Figure 2, a virtual Kundt’s tube is established in ACTRAN to calculate the normal incidence surface impedance. The cut-off frequency of the virtual Kundt’s tube is

\[
f_{10} = \frac{1.84 \cdot c}{\pi D},
\]  

(5)

where \(c\) is the sound speed of the fluid in the Kundt’s tube and \(D\) is the diameter of the tube.

![Figure 2. The virtual Kundt’s tube in ACTRAN.](image)

The virtual Kundt’s tube consists of a finite length tube. The velocity source is placed at one end of the tube and the material to be tested is placed at the other end. The tube wall is set as rigid boundary. A field point is placed at the interface of two medium. The sound pressure and the complex particle velocity at the field point are extracted by the post-processing part of ACTRAN. The normal incidence surface impedance of material can be calculated by Eq.(6)

\[
Z = \frac{P}{u},
\]  

(6)

where \(P\) is sound pressure and \(u\) is the complex particle velocity at the field point.

**Numerical Calculation of the Sound Field.** The model of a glass tank, of which the demission is 1.5m × 0.9m × 0.6m, with impedance boundary is shown in Figure 3(a). The water surface is set as absolutely soft boundary and the other surfaces are set as impedance boundary. As a comparison, the model of the glass tank with absolutely soft boundary is also established, as shown in Figure 3(b), in which all the surfaces are absolutely soft boundary. A point source is placed at (1.3m, 0.15m, 0.3m).

![Figure 3. The model of the glass tank with different boundary.](image)
Results and Discussions

The numerical results of the sound field in the glass tank with impedance boundary (at the left side) and absolutely soft boundary (at the right side) at the same depth are shown in Figure 4. As shown in Figure 4, as the frequency increases, the difference of the numerical results with different boundary conditions becomes more obvious.

![Figure 4. The numerical results of the sound field in glass tank.](image)

In order to verify the numerical method, the verification experiment was carried out in a glass tank with the same demission. A hydrophone (B&K 8103) was used to measure the sound field of the glass tank. The results of the experiment and the numerical results of the sound field in the glass tank with impedance boundary are shown in Figure 5. As a comparison, the numerical results of the sound field in the glass tank with absolutely soft boundary are also shown in Figure 5. It is observed that the results of the experiment and numerical results with impedance boundary are in good agreement. The numerical results with impedance boundary are more accurate than that with absolutely soft boundary. Thus, the effectiveness of proposed method is verified.

![Figure 5. The comparison results along the length direction of the glass tank.](image)

Conclusions

In this paper, the virtual Kundt’s tube is established in ACTRAN to numerically calculate the normal incidence surface impedance of non-anechoic pools’ walls at different frequency. Then the model of the non-anechoic pool with impedance boundary is established based on the calculated normal incidence surface impedance. The results shown that the method proposed in this paper can produce more accurate results than that with ideal boundary condition. The numerical method proposed in this paper is applicable to predict the low-frequency sound field in enclosed space with arbitrary material and shaped boundary.
Acknowledgement
This research was financially supported by the National Science Foundation of China under grant numbers 11504065.

References
[1] P.M. Morse, R.H. Bolt, Sound waves in rooms, Rev. Modern Phys. 16 (1944) 69-150.
[2] S.R. Bistafa, J.W. Morrissey, Numerical solutions of the acoustic eigenvalue equation in the rectangular room with arbitrary (uniform) wall impedances, J. Sound Vib. 263 (2003) 205-218.
[3] Y. Naka, A.A. Oberai, B.G. Shinn-Cunningham, Acoustic eigenvalues of rectangular rooms with arbitrary wall impedances using the interval Newton/generalized bisection method, J. Acoust. Soc. Am. 118 (2005) 3662-3671.
[4] J.T. Du, W.L. Li, Z.G. Liu, Z.L. Ji, Acoustic analysis of a rectangular cavity with general impedance boundary conditions, J. Acoust. Soc. Am. 130.2 (2011) 807-17.
[5] C.H. Jeong, Absorption and impedance boundary conditions for phased geometrical- acoustics methods, J. Acoust. Soc. Am. 132.4 (2012) 2347.
[6] G.Y. Jin, Y.H. Chen, Z.G. Liu, A Chebyshev-Lagrangian method for acoustic analysis of a rectangular cavity with arbitrary impedance walls, Appl. Acoust. 78.4 (2014) 33-42.
[7] X. Xie, H. Yang, H. Zheng, A weak formulation for interior acoustic analysis of enclosures with inclined walls and impedance boundary, Wave Motion, 65 (2016) 175-186.
[8] Y. Zhang, R. Tang, Q. Li, D. Shang, The low-frequency sound power measuring technique for an underwater source in a non-anechoic tank, Meas. Sci. Technol. 29.3 (2018).
[9] R. Tang, Y. Zhang, Q. Li, D. Shang, The investigation of the methods for predicting the sound field in a non-anechoic tank with elastic boundary, 2016 IEEE/OES China Ocean Acoustics Symposium, COA 2016.