Quark Masses and Mixing Angles

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Abstract

Inspired by the vector-like phenomenon of chiral gauge theories at short distances, we postulate that the $W^\pm$-gauge boson has a vector-like gauge coupling in the high-energy region, beside its purely right-handed gauge coupling observed in the low-energy region. It is discussed that the top quark acquires its mass via spontaneous symmetry breaking, while the $u,d,c,s,b$ quarks acquire their masses via explicit symmetry breakings. We discover four relationships between four inter-generation mixing angles and six quark masses by examining Dyson equations for quark self-energy functions. A preliminary analysis shows that the hierarchical pattern of quark masses is closely related to the hierarchical pattern of inter-generation mixing angles. In particular, the CP-violating phase is predicted to be approximately $79^\circ$.

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1. Motivation

The vector-like phenomenon of chiral gauge theories (e.g., the Standard Model) at short distances, shown by the “No-Go” theorem of Nielsen and Ninomiya [1], is that the fermion spectrum and gauge coupling of regularized chiral gauge theories must be vector-like if one insists on preserving chiral gauge symmetries. This seems to run into a paradox that the successful Standard Model possesses, its very peculiar features of purely left-handed gauge coupling of $W^\pm$-boson and only left-handed neutrinos. However, this inconsistency may imply what are the Nature’s possible choices for the Standard Model at short distances [2, 3, 4].

There have been many attempts to find a resolution to this paradox. It is currently a very important issue for chiral gauge theories and the problem has not been completely resolved. We will not enter into the details of this issue, instead, inspired by this vector-like phenomenon, we postulate that the effective gauge coupling of $W^\pm$-bosons to quarks is vector-like in the high-energy region, beside its purely left-handed gauge coupling observed in the low-energy region:

$$\Gamma_{\mu ij}^w(q) = ig_w(q)\gamma_\mu V_{ij}(P_L + f(q))$$  \hspace{1cm} (1)
$$f(q) \neq 0, \quad q \sim \Lambda,$$  \hspace{1cm} (2)

where $g_w(q)$ is a renormalized coupling constant and $V_{ij}$ is the Cabibbo-Kobayashi-Maskawa (CKM) [5] representation of the mixing matrix that is parameterized by the following four inter-generation mixing angles [6],

$$\theta_{12}, \theta_{23}, \theta_{13}, \delta_{13},$$  \hspace{1cm} (3)

where $\theta_{12} = \theta_c$ is the Cabibbo angle and $\delta_{13}$ is the CP-violating phase. In eq.(1), the non-vanishing of the vertex function $f(q)$ in the high-energy region $q \sim \Lambda$, which may be the Planck scale, implies that the gauge coupling (1) is vector-like. Whereas, to coincide with the parity-violating gauge coupling observed in low-energy experiments, it is assumed there is an intermediate energy-threshold $\epsilon$ between the weak scale ($\sim 250\text{GeV}$) and the cutoff $\Lambda$,

$$250\text{GeV} < \epsilon < \Lambda,$$  \hspace{1cm} (4)

at which the vertex function $f(q)$ vanishes

$$f(q)|_{q \rightarrow \epsilon} = 0.$$  \hspace{1cm} (5)
The gauge coupling (1) seems to violate chiral gauge symmetries. However, it can be chiral gauge symmetric owing to the possible existence of right-handed partners that are three-fermion bound states with appropriate quantum numbers (7) in the high-energy region,

\[(\bar{t}_R \cdot t_L) t_R, \quad (\bar{b}_R \cdot b_L) b_R, \quad \cdots, \quad (\bar{u}_R \cdot u_L) u_R.\]  \hspace{1cm} (6)

These right-handed three-fermion bound states disappear at such a threshold \(\epsilon (4)\), corresponding to the vanishing of \(f(\epsilon) (5)\). We will discuss these three-fermion states (4), the intermediate energy-threshold (4) and the vertex function \(f(q)\) in eq.(1) somewhere else. Fortunately, we do not need the detailed information of the energy-threshold \(\epsilon\) and vertex function \(f(p)\) in the following relevant discussions and preliminary analysis. The vector-like feature of gauge coupling (1), the intermediate scale (4) and right-handed partners are reminiscent of “left-right” symmetric extensions of the Standard model [8].

We stress that our attitude in this paper is that the effective gauge coupling of the \(W^\pm\) bosons (1) is just an assumption. The theoretical problems relating to this effective gauge coupling (1) are intentionally not discussed here. Armed with this assumption and preliminary analysis, we show that the hierarchical pattern of inter-generation mixing angles

\[\theta_{12} \gg \theta_{23} \gg \theta_{13},\]  \hspace{1cm} (7)

is closely related to the hierarchical pattern of quark masses

\[m_u \ll \cdots \ll m_t.\]  \hspace{1cm} (8)

Finally, an approximate prediction of the CP-violating phase \((\delta_{13} \simeq 79^\circ)\) is given.

2. Spontaneous symmetry breaking for the top quark mass

As is known, the gauge interactions in the Standard Model cannot be the intrinsic dynamics for generating quark masses due to their perturbative feature. The six Dyson equations for quark self-energy functions \(\Sigma_i(p)\) (gap-equations) are self-consistent integral equations that can be, in general, written in the following form:

\[\Sigma_i(p) = \int p' V_i(p,p') \frac{\Sigma_i(p')}{{p'}^2 + \Sigma_i(p')} + m_0, \quad i = u, d, \cdots, t \quad \text{quarks},\]  \hspace{1cm} (9)
where the kernel $V_i(p,p')$ of the integral equations represents the contributions from all possible gauge interactions in the Standard Model, and $m_\circ$ is an explicit chiral-symmetry breaking, which is an inhomogeneous term of these Dyson equations. The notation of internal momentum integration up to the cutoff $\Lambda$ is defined as,

$$\int_{p'} = \int_\Lambda \frac{d^4p'}{(2\pi)^4}. \quad (10)$$

In these Dyson-equations, an appropriate gauge (Landau gauge) has been chosen so that the wave-renormalization $Z_2$ is zero \[9\] in the case of massless gauge bosons\[1\].

When $m_\circ = 0$, Dyson equations are homogeneous integral equations, and they have only trivial solution:

$$\Sigma_i(p) = 0, \quad i = u, d, \cdots, t \quad \text{quarks}, \quad (11)$$

for small gauge couplings \[9, 10\]. It is very important to point out that $W^\pm$ boson does not contribute to the kernel $V_i(p', p)$ of these Dyson equations \[4\] because of its purely chiral-gauge coupling in the Standard Model. These six Dyson equations \[9\] are decoupled from each other, namely, there is no mixing between these Dyson equations for different quarks.

Bardeen, Hill and Lindner proposed a phenomenal $t\bar{t}$-condensate model \[11\], where a four-fermion interaction of the Nambu-Jona Lasinio type \[12\] was introduced for the third quark generation ($t, b$) only,

$$g\bar{\psi}_i^L(x) \cdot t_R(x) \bar{t}_R(x) \cdot \psi_j^L(x), \quad (12)$$

with $\psi_i^L(x)$ being the left-handed doublet of the third generation. In this case, the kernel in eq.(9) for the top quark includes the contribution from the four-fermion coupling beside those from gauge couplings. When the four-fermion coupling is larger than a certain critical value, spontaneous symmetry breaking takes place and the Dyson equation for the top quark possesses a consistent massive solution even for small gauge couplings, which can be written as \[10\].

$$\Sigma_i(p) \sim m_i \left( \frac{p}{m_i} \right)^G, \quad m_i < p \ll \Lambda, \quad i = t, \quad (13)$$

\[1\] As for the case of the massive gauge boson $Z^\circ$, we consider this to be an approximation owing to the small weak-coupling.
where $G$ is related to the anomalous dimension of fermion mass operator, which is a function of perturbative gauge couplings. The top quark mass (relating to the weak-scale) is proportional to the condensate $\langle \bar{t}t \rangle$. The fine-tuning of the four-fermion coupling is necessary so as to have $m_t \ll \Lambda$. The $u, d, s, c$ and $b$ quarks remain massless, since the Dyson equations (9) for these quarks are homogeneous and possess only trivial solutions (11).

The Lagrangian being quadratic in fermion fields is one of the prerequisites of the “No-Go” theorem for the non-existence of a consistent chiral gauge theory at short distances. This prerequisite strongly implies additional four-fermion interactions to the Standard Model at short distances. One can conceive that these four-fermion interactions are democratically shared by all quarks, for the reasons that all quarks can be considered to be massless at the cutoff and the underlying physics that induces effective four-fermion interactions could be flavour-blind. Thus, unlike the phenomenal $\bar{t}t$-condensate model (12), we consider that the dynamics of four-fermion interactions [12] should be applied to all quark flavours with a single four-fermion coupling $g$ at short distances,

$$g \sum_{f=1}^{3} \bar{\psi}^i_{L}(x) \cdot \psi^j_{R}(x) \bar{\psi}^j_{R}(x) \cdot \psi^i_{L}(x),$$

(14)

where the index “$f$” is for quark generations and

$$\psi^i_{L} = \begin{pmatrix} u \\ d \\ s \\ t \\ b \end{pmatrix}_L,$$

$$\psi^i_{R} = \begin{pmatrix} u_R \\ d_R \\ c_R \\ s_R \\ t_R \\ b_R \end{pmatrix}_R.$$  

(15)

All quark fields are eigenstates of the mass operator. The four-fermion interactions (14) should then be involved in the kernel $V_i(p, p')$ of the Dyson equations (9) for all quarks.

These four-fermion interactions are supposed to undergo the NJL spontaneous symmetry breaking and the Dyson equations (9) possess non-trivial solution (13) for the four-fermion coupling $g$ being larger than its critical

\footnote{That may be the Quantum Gravity.}
value \((g > g_c)\). One of non-trivial solutions can be

\[
\begin{pmatrix}
  m_u & 0 & 0 \\
  0 & m_c & 0 \\
  0 & 0 & m_t
\end{pmatrix}^{Q=\frac{2}{3}} \quad ; \quad \begin{pmatrix}
  m_d & 0 & 0 \\
  0 & m_s & 0 \\
  0 & 0 & m_b
\end{pmatrix}^{Q=-\frac{1}{3}},
\]

which we call the maximum non-trivial solution\(^{[1]}\). In addition, there are many possible solutions allowed by the Dyson equations \(^{[3]}\):

\[
\begin{pmatrix}
  0 & 0 & 0 \\
  0 & 0 & 0 \\
  0 & 0 & m_t
\end{pmatrix}^{Q=\frac{2}{3}} \quad ; \quad \begin{pmatrix}
  0 & 0 & 0 \\
  0 & 0 & 0 \\
  0 & 0 & 0
\end{pmatrix}^{Q=-\frac{1}{3}},
\]

\[
\begin{pmatrix}
  0 & 0 & 0 \\
  0 & m_c & 0 \\
  0 & 0 & m_t
\end{pmatrix}^{Q=\frac{2}{3}} \quad ; \quad \begin{pmatrix}
  0 & 0 & 0 \\
  0 & 0 & 0 \\
  0 & 0 & m_b
\end{pmatrix}^{Q=-\frac{1}{3}},
\]

etc., namely, Dyson equations of some quarks possess trivial solution \(^{[1]}\), which indicates the possibility of some quarks being massive and others being massless. We call eq.(18) the minimum non-trivial solution.

When these four-fermion interactions undergo the NJL spontaneous symmetry breaking, quarks acquire mass and the ground states of the whole system favourably gain negative energy. From this point of view, the maximum non-trivial solution \(^{[17]}\) seems to be the “real” solution. As a result, the Dyson equations \(^{[3]}\) engender all quarks to be approximately equally massive after fine tuning \(g \rightarrow g_c^+\),

\[
m_u \simeq m_d \simeq m_s \simeq m_c \simeq m_b \simeq m_t \ll \Lambda,
\]

since gauge couplings are perturbatively small, and corresponding contributions to eqs.\(^{[3]}\) are negligible. Obviously, this spectrum \(^{[20]}\) of equal quark masses is not in agreement with what is observed.

However, on the other hand, corresponding numbers of Goldstone and Higgs modes are produced \(^{[13]}\) when spontaneous symmetry breaking takes place. These modes carry positive energy in these ground states, and the system gains positive energy. The more quarks acquire mass, the more such scalar and pseudoscalar modes are produced. As a consequence, the system

\(^{[3]}\)All condensate violating electric charge conservation must be zero.
should be stabilized by balancing these two opposite contributions to obtain
an energetically favourable solution. In a lattice model ref.[14], it is shown
that the ground states of the system should be realized by the minimum
non-trivial solution (18) with only one massive quark in the generation that
is called \((t,b)\) and three Goldstone modes,

\[
m_t \neq 0
\]
\[
m_u, m_c, m_d, m_s, m_b = 0,
\]
which is actually very close to the real pattern of quark masses. Those Gold-
stone modes are to become longitudinal modes of the intermediate gauge
bosons \(W^\pm\) and \(Z^0\). There are no extra Goldstone modes. This is just the
phenomenal \(\bar{t}t\)-condensate model proposed in ref.[11]. These ground states
that are non-perturbatively built by the phenomenon of spontaneous sym-
metry breaking should be considered as approximate ground states, where
the approximate pattern (21) is realized. These arguments based on the en-
ergetically favourable realization of the ground states should be applicable
to any models of dynamical symmetry breaking.

3. Explicit symmetry breaking for \(u,d,s,c,b\) quark masses

When \(m_\circ \neq 0\) in the Dyson equations (9), there is an explicit chiral
symmetry breaking, these Dyson equations turn out to be inhomogeneous.
For small gauge couplings, inhomogeneous Dyson equations for fermion self-
energy functions have non-trivial solutions \(\Sigma_i(p)\) stemming from the inhom-
ogeneous term \([9,10]\)

\[
\Sigma_i(p) \sim m_i\left(\frac{p}{m_i}\right)^{G'}, \quad i = u, d, s, c, b,
\]
where \(G'\) is also related to the anomalous dimension of the fermion mass
operator, and the infrared mass scale \(m_i\) is proportional to inhomogeneous
terms, i.e., explicit chiral symmetry breaking \(m_\circ\).

Taking into account the vector-like feature of \(W^\pm\) boson coupling (11) in
the high-energy region, we find that this vector-like coupling of \(W^\pm\) boson
does contribute to the Dyson equations in the high-energy region:

\[
\Sigma_i(p) = \int_{p'} V_{\frac{2}{3}}(p,p') \frac{\Sigma_i(p')}{p'^2 + \Sigma_i(p')}
\]
\[
\begin{align*}
\Sigma_j(p) &= \int_{p'} W_{\frac{1}{3}}(p, p') \frac{\Sigma_j(p')}{p'^2 + \Sigma_j(p')} \\
+ |V_{ij}|^2 &\int_{|p'| \geq \epsilon} W_{\frac{1}{3}}(p, p') \frac{\Sigma_j(p')}{p'^2 + \Sigma_j(p')}, \quad i = u, c, t \text{ quarks,} \\
+ |V_{ij}|^2 &\int_{|p'| \geq \epsilon} W_{-\frac{1}{3}}(p, p') \frac{\Sigma_i(p')}{p'^2 + \Sigma_i(p')}, \quad j = d, s, b \text{ quarks,} 
\end{align*}
\]

where the six Dyson equations (1) are classified into two sets of equations for quarks with charge \( Q = \frac{2}{3} \) and quarks with charge \( Q = -\frac{1}{3} \) due to the fact that they have different gauge interactions. In eqs. (23) and (24), the kernel \( W(p, p') \) is proportional to the vector-like vertex function \( f(q) \) and \( q = p' - p \sim p' \). The integration of the internal momentum \( p' \) starts from the intermediate threshold \( \epsilon \) to the cut-off \( \Lambda \). One can see that eqs. (23) and (24) are mixed between different generations and charge sectors of quarks via the vector-like vertex function \( f(q) \) and the CKM matrix \( V_{ij} \). Thus, these self-consistent Dyson equations for the six quark self-energy functions \( \Sigma_i(p) \) are coupled together.

If the top quark mass is generated by spontaneous symmetry breaking, as discussed in previous section, the Dyson equations for \( u, d, s, c, b \) quarks acquire inhomogeneous terms that are the last terms in eqs. (23) and (24). With respect to the Dyson equations of \( u, d, s, c, b \) quarks, these inhomogeneous terms are explicit chiral symmetry breakings, which are analogous to \( m_\sigma \) in eq. (3). Thus, the Dyson equations of \( u, d, s, c, b \) quarks possess non-trivial solutions of the type (22) and \( u, d, s, c, b \) quarks are massive. Because these masses are generated by explicit symmetry breakings, no extra Goldstone bosons are produced. It is worth noting that possible global symmetries associated with quark generations are explicitly broken by inhomogeneous terms that contain the CKM matrix, and there are no Goldstone bosons associated with these global symmetries.

These inhomogeneous terms, i.e., explicit chiral symmetry breakings are quite small, since they are proportional to the off-diagonal elements of the CKM matrix. One can conceive that these small explicit symmetry breakings are perturbative on the approximate ground states, where the pattern (21) is realized by the spontaneous symmetry breaking. In other words, when the gauge couplings and the CKM mixing angles are perturbatively turned on, spontaneous-symmetry-breaking generated Vacuum alignment must be re-arranged to the real ground states, where the real pattern is realized. This
real pattern should deviate slightly from the approximate pattern (21), due to the fact that gauge couplings are perturbatively small and the observed CKM mixing angles are small deviations from triviality.

This new alignment of vacua (the ground states) certainly minimizes the energy in such a way that all quark self-functions $\Sigma_i(p)$ satisfy the six self-consistently coupled Dyson equations (23) and (24). These equations (23) and (24) show that six quark masses and four CKM mixing angles are closely related. This means the six quark masses and four CKM mixing angles are no longer completely free and independent parameters. Nevertheless, we are still far from entirely determining the quark masses and CKM mixing angles, since the Dyson equations (23) and (24) are not complete equations for the dynamics of the whole system. The best we can achieve is to find the explicit relationships between the six quark masses and four CKM mixing angles.

4. Preliminary analysis of the Dyson equations

The Dyson equations (23) and (24) in general are very complicated. It is hard to see any explicit relationships between the six quark masses and four CKM mixing angles. In order to find the relationships between quark masses and mixing angles, we have to figure out the most important mass-dependence in these Dyson equations and make a reasonable approximation. Based on the arguments of all quarks being massive, which we discussed in previous sections, we adopt the non-trivial solution (13,22) as an ansatz for each quark self-energy function. Substituting these ansatze into eqs.(23) and (24), we obtain approximately

$$Q^i_+ (p, \{m\}) m^i_+ = \sum_j K^j_+ (p, \{m\}) |V_{ij}|^2 m^j_+, \quad i = u, c, t$$

$$Q^j_- (p, \{m\}) m^j_- = \sum_i K^i_- (p, \{m\}) |V_{ji}|^2 m^i_-, \quad j = d, s, b$$

where the equations are factorized with

$$m^i_+ = m_u, m_c, m_t$$

$$m^j_- = m_d, m_s, m_b$$

being the quark masses for the $Q = \frac{2}{3}$ and $Q = -\frac{1}{3}$ sectors respectively. The factors $Q^i_+ (p, \{m\})$, $Q^j_- (p, \{m\})$, $K^j_+ (p, \{m\})$ and $K^j_- (p, \{m\})$ in (25,26)
are still quite complicated,

\begin{align}
Q_i^j(p, \{m\}) & = \left( \frac{p}{m_i} \right)^G - \int_{p'} V_i^j(p, p') \frac{(\frac{p'}{m_i})^G}{p'^2 + m_i(\frac{p'}{m_i})^G}, \quad i = u, c, t, \quad (28) \\
Q_{-\frac{1}{3}}^j(p, \{m\}) & = \left( \frac{p}{m_j} \right)^G - \int_{p'} V_{-\frac{1}{3}}^j(p, p') \frac{(\frac{p'}{m_j})^G}{p'^2 + m_j(\frac{p'}{m_j})^G}, \quad j = d, s, b \quad (29)
\end{align}

and

\begin{align}
K_i^j(p, \{m\}) & = \int_{|p'| \geq \epsilon} W_i^j(p, p') \frac{(\frac{p'}{m_i})^G}{p'^2 + m_i(\frac{p'}{m_i})^G}, \quad i = u, c, t, \quad (30) \\
K_{-\frac{1}{3}}^j(p, \{m\}) & = \int_{|p'| \geq \epsilon} W_{-\frac{1}{3}}^j(p, p') \frac{(\frac{p'}{m_j})^G}{p'^2 + m_j(\frac{p'}{m_j})^G}, \quad j = d, s, b \quad (31)
\end{align}

where

\[ \{m\} = m_u, m_d, m_s, m_c, m_b, m_t. \quad (32) \]

Obviously, these functions \( Q \) and \( K \) depend on all quark masses (32) through renormalizations. However, these mass-dependences are logarithmically weak \((\ell n \frac{\Lambda}{m_i})\) for renormalizable gauge interactions. These can be seen by dimensional counting of integrations in eqs.\( (28)-(31) \), in which functions \( V^i(p', p) \) and \( W(p', p) \) contain propagators of gauge bosons and behave as \( O(\frac{1}{p'^2}) \) in the high-energy region \((p' \rightarrow \Lambda)\). The contributions of NJL four-fermion interactions to functions \( Q(p, \{m\}) \) are

\[
1 - \frac{8\pi^2}{N_c g_c \Lambda^2} - \left( \frac{m_i}{\Lambda} \right)^2 \ell n \frac{\Lambda}{m_i} + O \left( (\ell n \frac{\Lambda}{m_i})^2 \right) , \quad (33)
\]

which is also logarithmically dependent on quark masses \( \{m\} \) after fine-tuning \( g \rightarrow g^+ \) to cancel quadratic divergent terms.

On the basis of the factors \( Q_i^j(p, \{m\}), \quad Q_{-\frac{1}{3}}^j(p, \{m\}), \quad K_i^j(p, \{m\}) \) and \( K_{-\frac{1}{3}}^j(p, \{m\}) \) logarithmically depending on quark masses, which are very weakly dependent in comparison with the linear mass-dependence in eqs.\( (25) \) and \( (26) \), we can make the completely reasonable approximations:

\begin{align}
Q_u^i & \simeq Q_c^i \simeq Q_t^i; \quad Q_{-\frac{1}{3}}^i \simeq Q_s^i \simeq Q_b^i, \\
K_u^i & \simeq K_c^i \simeq K_t^i; \quad K_{-\frac{1}{3}}^i \simeq K_s^i \simeq K_b^i, \quad (34)
\end{align}
in eqs. (25) and (26). This means that the variations of the functions $Q$ and $K$ in terms of different quark masses are negligible in these equations. It is important to note that the factors $Q^i_2(p, \{m\})$ and $Q^j_{-\frac{1}{3}}(p, \{m\})$ are different due to different gauge interactions.

Taking the ratios from the six Dyson equations (25, 26), the functions $Q$ and $K$ are approximately cancelled and we obtain the following equations:

\[
\begin{align*}
\frac{m_u}{m_c} &= \frac{|V_{ud}|^2m_d + |V_{us}|^2m_s + |V_{ub}|^2m_b}{|V_{cd}|^2m_d + |V_{cs}|^2m_s + |V_{cb}|^2m_b}, \\
\frac{m_u}{m_t} &= \frac{|V_{ud}|^2m_d + |V_{us}|^2m_s + |V_{ub}|^2m_b}{|V_{td}|^2m_d + |V_{ts}|^2m_s + |V_{tb}|^2m_b}, \\
\frac{m_c}{m_t} &= \frac{|V_{cd}|^2m_d + |V_{cs}|^2m_s + |V_{cb}|^2m_b}{|V_{td}|^2m_d + |V_{ts}|^2m_s + |V_{tb}|^2m_b}, \\
\end{align*}
\]

(35)

(36)

(37)

and

\[
\begin{align*}
\frac{m_d}{m_u} &= \frac{|V_{du}|^2m_u + |V_{dc}|^2m_c + |V_{dt}|^2m_t}{|V_{su}|^2m_u + |V_{sc}|^2m_c + |V_{st}|^2m_t}, \\
\frac{m_d}{m_s} &= \frac{|V_{du}|^2m_u + |V_{dc}|^2m_c + |V_{dt}|^2m_t}{|V_{bu}|^2m_u + |V_{bc}|^2m_c + |V_{bt}|^2m_t}, \\
\frac{m_b}{m_s} &= \frac{|V_{bu}|^2m_u + |V_{bc}|^2m_c + |V_{bt}|^2m_t}{|V_{bu}|^2m_u + |V_{bc}|^2m_c + |V_{bt}|^2m_t}.
\end{align*}
\]

(38)

(39)

(40)

There are only four independent equations that completely determine the four CKM mixing angles in terms of six quark masses.

Using the standard parameterization of the CKM matrix [6] with four mixing angles (3), one can obtain definite, though rather complicated relationships between such mixing angles and the pattern of quark masses. Defining, for convenience

\[
\begin{align*}
x &= \tan^2 \theta_c; & y &= \sin^2 \theta_{23} \\
z &= \cot^2 \theta_{13}; & w &= \cos \delta_{13},
\end{align*}
\]

we cast eqs.(33, 36, 38, 39) into,

\[
\frac{m_u}{m_c} = \frac{z(m_d + zm_s) + (1 + x)m_b}{(zm_d + zm_s)(1 - y)(1 + z) + y(m_d + zm_s) + (1 + x)yzm_b + (m_d - m_s)wc}.
\]

(41)
\[
\frac{m_u}{m_t} = \frac{z(m_d + xm_s) + (1 + x)m_b}{y(1+z)(xm_d + m_s) + (1+y)(m_d + xm_s) + (1+z)m_b + (m_s - m_d)wc},
\]
\[
\frac{m_d}{m_s} = \frac{zm_u + [(1+z)x(1-y) + y]m_c + [xy(1+z) + (1-y)m_t + (m_c - m_t)wc]}{xzm_u + [(1-y)(1+z) + xy]m_c + [y(1+z) + x(1-y)]m_t + (m_t - m_c)wc},
\]
\[
\frac{m_d}{m_b} = \frac{zm_u + [(1+z)x(1-y) + y]m_c + [xy(1+z) + (1-y)m_t + (m_c - m_t)wc]}{(1+y)[m_u + z (ym_c + (1-y)m_t)]},
\]

where

\[c = 2\sqrt{xy(1-y)(1+z)}.\]

These equations still look very complicated.

The relationships (35), (36), (38) and (39) still cannot tell us the observed hierarchical pattern of quark masses. However, we should emphasize at this point that in the Standard Model, the CKM mixing angles are totally extrinsic elements, qualifying the observed pattern as real as compared (and opposed) to any other possible pattern, where the CKM mixing angles can be either trivial (the approximate pattern (21)) or anything one wishes. We consider the observed real pattern so close to the approximate one (21), this implies in turn that we do live in a world where the CKM matrix is almost trivial.

The equations (41)-(44) can be drastically simplified if one takes into account the observed hierarchical pattern of quark masses, i.e., the fact that the top quark is much heavier than the others. One can then approximately solve the above four independent equations (41)-(44) and get

\[
\tan^2 \theta_c \simeq -\frac{m_d}{m_s}; \quad \sin^2 \theta_{23} \simeq -\frac{m_c}{2m_t},
\]
\[
\tan^2 \theta_{13} \simeq \frac{m_u}{m_t}; \quad \cos \delta_{13} \simeq \sqrt{\frac{m_s m_u}{2m_cm_d}},
\]

where the strange and ominous looking minus signs, can all be eliminated by chirally rotating the fields of the (c, s) generation so as to transform

\[-m_s, -m_c \rightarrow m_s, m_c.\]

We note that

\[\tan \theta_c \simeq \frac{m_d}{m_s}.\]
in eq. (46) has been around in completely different contexts for a quarter of a century [15] now, while the others[4] – to our knowledge – appear to be genuine consequences of the peculiar chiral symmetry breaking of both the spontaneousness and explicitness, which we have fully discussed in sections 3 and 4. The most interesting aspect of eqs. (46) and (47) is their tying the observed hierarchical pattern of the inter-generation mixing angles (7) to the observed hierarchy of mass ratios

$$\frac{m_s}{m_d} \gg \frac{m_c}{2m_t} \gg \frac{m_u}{m_t}. \quad (50)$$

Finally, as for the CP-violating phase $\delta_{13}$, eqs. (35), (36), (38) and (39) can be cast in the form

$$\cos \delta_{13} \simeq \frac{1}{2} \frac{\sin \theta_{13} \cos \theta_{23} \cos \theta_c}{\sin \theta_{23} \sin \theta_c}, \quad (51)$$

which by using the experimental determinations of the mixing angles (central values $\sin \theta_c = 0.221; \sin \theta_{23} = 0.040; \sin \theta_{13} = 0.0035$ [17]) yields the prediction

$$\delta_{13} \simeq 79^\circ. \quad (52)$$

In summary, starting with the postulation of the vector-like feature of the $W^\pm$ boson gauge coupling in the high-energy region, we study the Dyson equations for the six quark self-energy functions. We discuss the possible pattern of quark masses on the basis that the top quark mass is generated by NJL spontaneous symmetry breaking and $(u, d, s, c, b)$ masses are generated by explicit chiral symmetry breaking. The four relationships between the six quark masses and four CKM mixing angles are approximately derived by the preliminary analysis of the Dyson equations. These four relationships, which relate the hierarchical pattern of the four CKM mixing angles to the hierarchical pattern of the six quark masses, are in good agreement with observations. Of particular interest is the prediction (52) of the CP-violating phase. We have to confess that the practical analysis presented in this article is very preliminary and more precise analysis needs pursuing further.

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