Abstraction ability in number patterns problems

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Abstract. This study aims to describe number patterns problems that are designed to measure the abstraction ability of junior high school students. This study was design research which consists: preliminary design, focus group discussion, trial, interview, and retrospective analysis. The instruments used in this study were test and interview to measure abstraction ability that has 5 indicators, which are: specialization, observation of pattern, generalization, conjecturing and testing conjecture. The subjects of this study were 3 students in grade 8 and the data analysis used in this study was qualitative. As a result of this study, the problem can be used to measure students’ abstraction ability.

1. Introduction

There are so many mathematical abilities, for instance, reasoning, abstracting, conjecturing, representing and switching between different representations, visualizing, deducing, inducing, analyzing, synthesizing, connecting, generalizing and proving [1 - 3]. Abstraction is one of the mathematical abilities. Abstraction is a fundamental process in mathematics because abstraction ability allows students to construct a mathematics concept in their mind by using their initial knowledge [4, 5]. That is why the abstraction process needs high cognitive-mentally work [6]. As we know, mathematics has no concrete object, that is why the mathematics concept should be constructed in the mind [5 - 7]. These are the reason why abstraction ability becomes an urgent thing for students.

Abstraction ability appears by these 5 indicators, which are specialization when students able to find a relationship between informations that consist on the problem in order to find a solution for the problem [8 - 10], observation of pattern when students able to find a pattern for the problem [9 - 12], after analyze the pattern students will make a conjecture in order to solve the problem [1, 9, 10] generalization when students able to change a pattern into a general form [8, 11, 13]. A conjecture that students already made might be correct or wrong, that is why students have to check or test the conjecture [6]. These 5 indicators of abstraction ability are in accordance with the objectives of learning mathematics [14].

From the previous study of number patterns, students have difficulty in developing the principle of number patterns, which means that students have difficulties in generalizing the correct principles and modifying principles. Generalizing and modifying patterns is a characteristic of abstraction ability. It can be concluded that these problems in number patterns are also problems in student’s abstraction ability [7].

Based on the explanation above, this study aims to design number patterns problems that are expected to be able to measure students’ abstraction ability. Furthermore, the problems that are designed are problem-solving questions so that students are trained to use their skills and knowledge to deal with problems [9, 15].
2. Method
This is a design research study that aims to describe number patterns problems that are designed to measure abstraction ability of junior high school students. In this study, there are 5 stages, which are: preliminary design, focus group discussion, trial, interview, and retrospective analysis. The instrument used in this study were two questions to measure students’ abstraction ability. The subjects of this study were three students in grade 8.

3. Result and Discussion
This study was conducted based on the stages of design research. As for the following details are the stages of this study.

3.1. Preliminary Design
In this stage, the literature study was conducted to the related to focus of this study which are theories about abstraction, abstraction ability, specialization, observation of pattern, conjecturing, generalization, and testing conjecture. Based on several references, there are several problems that can be used to measure junior high school student’s abstraction ability. Some of these problems were adopted from several books and articles [14]. Here are 3 question designs that have been prepared.

| Problem Number | Problems                                                                                   |
|----------------|-------------------------------------------------------------------------------------------|
| 1              | What is the quotient of 1 divided by 500,000,000,000?                                       |
| 2              | Determine the results of \(\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \cdots + \frac{1}{9900}\) and show that your answer is correct! |
| 3              | Consider the following sequence, \(a, b, c, c, a, b, b, c, c, c, a, b, c, c, \cdots\) What letter in the 100th place? |

3.2. Focus Group Discussion
The three problems that had been designed were discussed with 2 Mathematics Education Lecturers. The following table summarizes the result of the discussion.

| Lecturers | Comments and Suggestions                                                                 |
|-----------|------------------------------------------------------------------------------------------|
| 1         | Pay attention to the order in which the questions are placed. It should be sorted by difficulty level. |
| 2         | Consider the difficulty level of the problems.                                             |

Based on the Table 2, revision were made to the designed problems that had been prepared. There are rearrange the order of the placement of the questions, then delete questions number 2 because they were considered too difficult. Here are two questions as the result of the discussion.

| Question Number | Questions                                                                                   |
|----------------|-------------------------------------------------------------------------------------------|
| 1              | What is the quotient of 1 divided by 500,000,000,000?                                       |
| 2              | Consider the following sequence, \(a, b, b, c, c, a, b, b, c, c, c, a, b, b, c, c, \cdots\) What letter in the 100th place? |

3.3. Trial
At this stage, 2 questions that were designed and discussed with the lecturer were tested on 3 subjects. The subjects did the test in one hour. The result of this trial shows that the questions that had been designed can be used to measure student’s abstraction ability through 5 indicators which are specialization, observation of pattern, conjecturing, generalization, and testing conjecture.

3.4. Interview
The interview was immediately carried out with three subjects in the trial stage. It was conducted in order to clarify which indicator of abstraction ability that appears. Furthermore, the collected data from this stage is used to the next stage, retrospective analysis.

3.5. Retrospective analysis
The analysis of subject’s answer in problem number 1. The figure below is abstraction ability from the subjects' answers on problem number 1.

![Diagram showing specialization, observation of pattern, conjecture, and generalization]

Figure 1. Subject 1’s answer.

Figure 1 shows how the abstraction ability of subject 1 in answering problem number 1. After the interview, it can be obtained that subject 1 got the answer by looking at the existing patterns. In subject 1, specialization indicator can be seen by subject 1 tries to experiment to solve the problem. In this case, subject 1 experimented at 1:5. The observation of pattern indicator was seen when subject 1 continued the problem-solving experiment to find the pattern and analyze the pattern that has been obtained. It can be seen that student 1 continued to divide 1:50 and 1:500. Subject 1 then begins to analyze the results of the division will form a pattern, the generalization indicator appears when subject 1 was able to form a general pattern which is how many 0 after commas will depend on how many 0 after 5.

Then, subject 1 proposes the answer that they have so that the conjecturing indicator appears. During the interview process, subject 1 had been asked how did he make sure his answer and subject 1 answer was the answer that he got was following the existing pattern, so that is why subject 1 also showed the appearance of testing conjecture indicator.
From Figure 2, the indicator of abstraction ability of subject 2 that appeared was specialization. The appearance of this indicator is marked by subject 2 understood the problem that must be solved and subject 2 has tried to solve the problem. Subject 2 used a manual divisor as an alternative solution. Based on the results of the interview, subject 2 chooses to use this alternative because it was the first that came to his mind.

The calculation process by subject 2 was correct, it was just that subject 2 having a mistake at understanding the problem. The problem was 1:500,000,000,000 but subject 2 wrote 1:500,000,000, so the answer obtained by subject 2 did not fulfill the question request. However, after the interview process, it can be concluded that subject 2 just misread the problem.

Last, from Figure 3, subject 3 first tried to find the result of 1:5 manually, which showed the appearance of indicators specialization in student 3. Based on the result of the interview, subject 3 immediately concludes that the result of the division would be related to the number of 0 behind 5. Subject 3 make a mistake by doing this without observing the pattern first.

3.5.1. The analysis of subject’s answer in question number 2.
Figure 3 shows abstraction ability from the subjects' answers on problem number 2.
Figure 4. Subject 1 and subject 2’s answer.

Figure 4 is the answer from subject 1 and subject 2. Student 1’s answer is above and subject 2’s is below. Based on subject 1 and subject 2 answers, there is the appearance of specialization indicator. It showed when subject 1 and subject 2 were trying to solve the problem. In this case, subject 1 and subject 2 divide 100 by 6 because there are 6 letters at the problem that always repeat. It can be concluded that there are the appearance of indicators specialization and conjecturing. The remainder of the division will show what letter will appear. Because the remainder of the 100:6 is 4, then the letters that appear in 100th place are letters in 4th place of the order, which is c. It was showing the appearance of observation of pattern, conjecturing and generalization indicators. Also, from the interview, subject 1 and subject 2 were showing the appearance of testing conjecture indicators because the answer that they had obtained was corresponding to the patterns that they have determined.

Figure 5. Subject 3’s answer.

Figure 5 was subject 3’s answer. Subject 3 acknowledges that he does not know how to answer this kind of problem. Therefore, subject 3 tried to look for a possible pattern. In this regard, the appearance of the indicator was specialization because subject 3 tried to solve the problem through special cases. Furthermore, subject 3 saw that each sequence which is a multiple of 18 so letter that appears would be c. It shows the appearance of observation of pattern, conjecturing and generalization indicators because subject 3 was able to analyze the unique rules that exist and can conclude it. After knowing that the letters that appear in the 90th place were c, subject 3 manually searches for the letters that appear in the 100th place. Subject 3 also believes that the answer is correct because the letter c that appears on every multiple of 18 is indeed in the pattern. However, subject 3 determines the letter that appears in the 100th manually.
The three subjects have shown abstraction ability that can be seen when they do problem-solving planning [9]. This starts with finding the relationship between the information contained in the problem to try a special case that might help in solving the problem so that the specialization indicator appears [9, 21]. Furthermore, the results of this specialization are observed and analyzed to make conjectures or conjectures that are generally accepted so that indicators emerge to observe patterns and make conjectures at this stage [9, 21]. However, the conjecture obtained is not necessarily appropriate for general application. Therefore, an effort is needed to prove the truth of the conjecture or the allegations that have been submitted so that the indicators appear to examine the conjecture [9]. Moreover, students decide how general rules apply based on the truth of the conjecture that has been proposed to form a generalization [9, 10, 21].

Based on the results of the data obtained from the subjects, the problems that have been designed can be used to measure abstraction ability. Although not all indicators can be seen on the subjects [3], this is not caused by those problems but caused by other factors, such as the difference in the cognitive abilities of each student [16, 17]. It is following [6] that abstraction ability requires high cognitive mental work. That is why students who are not up to the process of thinking in the direction of abstraction choose to solve problems using ordinary computing. This happens because the student's way of thinking is dominant in algorithmic thinking, which means focusing on the steps of the problem-solving procedure without knowing why the step is used [18, 19, 20].

4. Conclusion
Based on the data analysis above, it can be concluded that the 2 questions that have been designed can be used to measure student’s abstraction ability.

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6. References
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