Enhancing the hybridization of plasmons in graphene with 2D superconductor collective modes

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Abstract
We explore ways in which the close proximity between graphene sheets and monolayers of 2D superconductors can lead to hybridization between their collective excitations. We consider heterostructures formed by combinations of graphene sheets and 2D superconductor monolayers. The broad range of energies in which the graphene plasmon can exist, together with its tunability, makes such heterostructures promising platforms for probing the many-body physics of superconductors. We show that the hybridization between the graphene plasmon and the Bardasis–Schrieffer mode of a 2D superconductor results in clear signatures on the near-field reflection coefficient of the heterostructure, which in principle can be observed in scanning near-field microscopy experiments.

Keywords: graphene, 2D superconductor, plasmons, collective modes

(Some figures may appear in colour only in the online journal)

1. Introduction

Within the broad class of quantum two-dimensional (2D) materials, superconductors are arguably the most challenging, both from the theoretical and the experimental perspectives. On the other hand, their complex behaviour is teeming with possibilities, from unveiling new physics to providing the basis for disruptive technologies.

As with any other material, elementary excitations can provide insight about the fundamental physics of 2D superconductors. The strongly correlated nature of superconductors endows them with several collective modes, each carrying complementary pieces of information about the superconducting state. For instance, the Higgs mode, associated with oscillations of the amplitude of the superconducting order parameter, has energy dispersion \[ \hbar \Omega_{\text{Higgs}} \approx \sqrt{\frac{4\Delta^2}{d} + \frac{(h v_F)^2}{d} q^2}, \] (1)
to leading order in the wave vector \( q \). Here, \( \Delta \) is the superconducting gap and \( v_F \) is the Fermi velocity and \( d \) is the dimensionality of the superconductor. Since its energy range lies close to and above the single particle excitation edge, the Higgs mode is damped. Moreover, its coupling to far-field electromagnetic radiation is rather weak \[ 2 \], being suppressed by the small factor \( \Delta / E_F \). Back in 1961, Bardasis and Schrieffer proposed \[ 3 \] the existence of exciton-like collective modes in superconductors. Their eponymous excitation is a bound quasiparticle pair, and has a dispersion relation very similar to that of the Higgs mode. Its exciton-like character, however, implies its energy lies deep within the superconducting gap, making them long-lived excitations. They arise whenever the effective attractive electron–electron interaction,
responsible for the pairing instability, has competing angular momentum components. It has been noted in the recent literature that characterization of the Bardasis–Schrieffer mode can help shed light on the nature of unconventional superconductivity, especially in Fe-based superconductors [4, 5].

From the recently synthesized 2D superconductors, one of the most promising and intriguing is FeSe, due to the record-high critical temperatures achieved for monolayers [6, 7]. Despite all the activity this system has attracted, the microscopic mechanisms by which \( T_c \) is dramatically enhanced from the modest bulk value of \( \sim 8 \) K to \( \sim 65 \) K or even \( \sim 109 \) K [8] remain largely unknown [9].

It has been shown recently that combining graphene with superconductors in heterostructures can lead to fruitful interplay between their collective modes [10–12]. For instance, hybridization with graphene plasmon polaritons (GPP) can enhance the visibility of the superconductor’s collective modes in optical experiments. By carefully designing the geometry of the heterostructure, several features of its electromagnetic response can be fine-tuned. Moreover, the incorporation of graphene provides a very convenient ‘handle’ to modify the behaviour of the heterostructure during operation, namely its doping level.

In this paper we study the near-field electromagnetic response of planar heterostructures combining monolayers of a 2D superconductor and graphene. We considered three kinds of heterostructures (shown schematically in figure 1): graphene–SC bilayers, graphene–SC–graphene sandwiches and SC–graphene–SC sandwiches. In all of them, a uniaxial dielectric (such as hexagonal boron nitride (hBN)) is assumed as spacer between graphene and superconductor monolayers or between two superconductor monolayers. We calculate the heterostructure’s reflection coefficient associated with the incidence of \( p \)-polarized waves by solving Maxwell’s equations subject to appropriate boundary conditions. The optical properties of hBN are incorporated into the calculations through its relative permittivity tensor, \( \epsilon_{\text{hBN}} = \text{diag}(\epsilon_x, \epsilon_y, \epsilon_z) \). Given the two-dimensional character of both graphene and the 2DSC, their properties only enter the calculations through BC. We model both graphene and the 2DSC by their non-local optical conductivity tensors [2, 13, 14]. For the superconductor we consider contributions coming from the Higgs mode and the Bardasis–Schrieffer mode [2]. As noted above, their features are directly tied to parameters that characterize the superconducting state, such as the superconducting gap and its symmetry, which makes them valuable probes into the nature of unconventional superconductivity. Moreover, their small dispersion in energy makes them perfect candidates for hybridizing with graphene plasmons [12]. The parameters that characterize FeSe were taken from reference [7]. There, through a multi-step annealing procedure, it was possible to change the carrier density from \( n \approx 0.07 \) to \( n \approx 0.12 \) electrons per Fe atom. They observe a pure superconducting phase for \( n \gtrsim 0.1 \) electrons per Fe atom.

Our results show a strong hybridization between the Bardasis–Schrieffer mode and the graphene plasmon, specially in the Gr–SC–Gr geometry. All geometries allow for tuning the optical response by changing either the heterostructure geometry (the thickness of the hBN spacer layers) or graphene’s doping level. The SC–Gr–SC geometry displays long-lived hybrid modes, which can be relevant for future applications. Moreover, we show that the hybridized modes impart their signature to the Purcell factor, which can be probed in scanning near-field optical microscopy experiments [15].

2. Calculation of the reflection coefficient

In order to study the near-field electrodynamicsof heterostructures formed by graphene monolayers and 2D superconductors we calculate the reflection coefficient of the structure. We assume \( p \) polarization of the electromagnetic field within and near the heterostructure. We adopt the coordinate system depicted in figure 1, where the \( z \) direction is perpendicular to the graphene sheet(s) and to the 2D superconductor(s). Thus, the electric field in the spacer dielectric and in the vacuum surrounding the heterostructure has the general form

\[
\vec{E}(\vec{r}, t) = \begin{cases} 
E_x^e & \text{for } \vec{r} \parallel \hat{z} \\
E_y^e & \text{for } \vec{r} \parallel \hat{x} \\
E_z^e & \text{for } \vec{r} \parallel \hat{y} 
\end{cases} \quad \vec{e}^{\vec{k} \rightarrow \vec{r} - \vec{t}}
\]

where we have assumed, without loss of generality, that the projection of the wave vector on the plane of the layers, \( k_|| \), is parallel to \( x \). The corresponding magnetic field is parallel to \( \hat{y} \),

\[
\vec{H}(\vec{r}, t) = (H_x e^{-ik_\parallel x} + H_y e^{ik_\parallel y}) \vec{e}^{\vec{k} \rightarrow \vec{r} - \vec{t}}
\]

We model the spacer dielectric by a relative permittivity tensor,

\[
\epsilon_{\parallel} = \begin{pmatrix}
\epsilon_{\parallel} & 0 & 0 \\
0 & \epsilon_{\parallel} & 0 \\
0 & 0 & \epsilon_{\parallel}
\end{pmatrix}
\]

Thus, within the dielectric regions we have

\[
k_{\parallel} = \sqrt{\epsilon_{\parallel} \left( \frac{\omega^2}{c^2} - \frac{1}{\epsilon_{\parallel}} k^2 \right)}
\]

The graphene monolayers and the 2D superconductors are modelled by their respective optical conductivities. For graphene we employ the non-local conductivity described in detail in [13]. We assume the optical conductivity of the 2D superconductor is given by the sum of two contributions,
each associated with the excitation of either the Higgs mode or the Bardasis–Schrieffer mode, as described in [2]. The contribution from the Higgs mode reads

\[ \sigma_{\text{Higgs}}(\omega, q) = \frac{n_i e^2 i}{m} \frac{1}{\omega} \frac{1}{1 + \frac{v_F^2}{2\omega^2} \left( \frac{\omega}{\omega_{\text{Higgs}}} \right)^2}, \]

with

\[ \omega_{\text{Higgs}} = \sqrt{4\Delta^2 + \frac{v_F^2 q^2}{\omega}}, \]

and

\[ \kappa_{\text{Higgs}} = \frac{\lambda}{E_F} \sqrt{2 \sinh^{-1} \left( \frac{\omega_D}{\Delta} \right)}. \]

In the expressions above, \( \Delta \) is the superconducting gap at zero temperature, \( E_F, v_F \) are the Fermi energy and velocity of the SC in the normal state, \( n_i \) is the superfluid density, \( e \) is the quantum of electric charge, \( m \) is the electronic effective mass in the normal state and \( \omega_D \) is the Debye frequency of the SC. The parameter \( \lambda \) characterizes the extent to which particle-hole symmetry is broken around the Fermi level in the normal state. It is usually small in all known superconductor. We follow the authors of reference [2] and adopt \( \kappa_{\text{Higgs}} = 0.2 \). The contribution from the Bardasis–Schrieffer mode reads,

\[ \sigma_{\text{BaSh}}(\omega, q) = \frac{n_i e^2 i}{m} \frac{1}{\omega} \frac{1}{1 + \frac{v_F^2 q^2}{2\omega^2 - \omega_{\text{BaSh}}^2}}, \]

where

\[ \kappa_{\text{BaSh}} = \frac{\pi v_F^2}{2m \omega_D} \approx 1. \]

The frequency of the Bardasis–Schrieffer mode at zero wavevector, \( \omega_{\text{BaSh}}(0) \) is a fraction of \( 2\Delta \) that depends on the strength of the coupling constant in the \( d \)-wave channel compared to the strength of the coupling constant in the dominant pairing channel. We adopt \( \omega_{\text{BaSh}}(0) = 0.7 \), following reference [2]. These contributions to the optical conductivity have been derived assuming a temperature much smaller than the critical temperature of the superconductor. Moreover, at nonzero temperatures a contribution from the normal electrons should be added. Since this would not change our results qualitatively, we assume \( T = 0 \).

The conductivities of graphene and the 2D superconductor enter the calculation of the reflection coefficient exclusively through boundary conditions. The building block for the calculation of the reflection coefficient of the structure is the relationship between the electromagnetic fields across a 2D conductor parallel to the xy plane, placed at \( z_0 \), characterized by the surface conductivity \( \sigma(k_{\parallel}, \omega) \). It is simpler to consider the equations for the magnetic field. The electric field can be easily derived from the magnetic field once the whole system is solved. To simplify the notation we will assume the dielectric surrounding the 2D conductor is the same at both sides. Thus,

\[ H_L^i e^{-ik_{\parallel}z_0} + H_R^i e^{ik_{\parallel}z_0} = \begin{bmatrix} 1 + \frac{\mu e^2 k_{\perp}}{e^2 \omega} \sigma(\omega, k_{\parallel}) \end{bmatrix} H_R^o e^{ik_{\parallel}z_0} + \left[ 1 - \frac{\mu e^2 k_{\perp}}{e^2 \omega} \sigma(k_{\parallel}, \omega) \right] H_R^o e^{-ik_{\parallel}z_0}, \]

\[ \times H_L^o e^{ik_{\parallel}z_0} - H_R^o e^{-ik_{\parallel}z_0}. \]

Here, the superscripts \( i, o \) in the field amplitudes mean incoming and outgoing waves, respectively. The subscripts \( L, R \)
mean left and right relative to the 2D conductor (or any other interface between two media). It is convenient to cast the above equations into matrix form,

\[
\begin{bmatrix}
H^i_L \\
H^o_R
\end{bmatrix}
= \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
H^o_L \\
H^o_R
\end{bmatrix}.
\] (12)

With this relationship in hand it is straightforward to combine the scattering processes at several sequential interfaces into an effective scattering matrix for the whole structure. Once this is done, the reflection coefficient of the structure can be found by setting the incoming amplitude at the rightmost end of the structure to zero, leading to

\[
r \equiv \frac{H^o_L}{H^i_L} = \frac{S_{21}}{S_{11}}.
\] (13)

Surface plasmons occur whenever \(k^2_\perp < 0\); thus the electromagnetic field acquires an exponentially decaying envelope in the direction perpendicular to the surface. As a consequence, the reflection coefficient \(r\), defined above, acquires poles that appear as peaks in its imaginary part. In the next sections we will analyse the behaviour of \(\Im r(k_\parallel, \omega)\), and extract information about the dispersion relation of the hybrid modes from the positions of its peaks.

3. Bilayers

Here we consider the simplest heterostructure containing one graphene sheet and one monolayer of FeSe, separated by a thin layer of the insulator hBN of thickness \(d\). We calculate the frequency and wave vector dependent reflection coefficient to look for signatures of the FeSe collective modes. In figure 2 we show the spectrum of electromagnetic waves as revealed by the imaginary part of the reflection coefficient for \(p\)-polarized waves, \(\Im r_p\). By zooming into the spectral region where the SC collective modes live we can get a clear picture of their hybridization with the graphene plasmon. It is known that the coupling of the Bardasis–Schrieffer mode with the electromagnetic field is stronger than the corresponding Higgs’ coupling by a factor of \(E_F/\Delta \gg 1\) [2]. This is reflected by the size of the splitting between branches around the respective anticrossings, seen in figures 2(b) and (c). In the case of the Higgs mode, the tiny splitting between branches (\(\sim 85\mu eV\)) makes it harder to observe it than the Bardasis–Schrieffer mode.
4. Planar cavities

Here we consider two kinds of planar cavities: (i) FeSe sandwiched between two graphene sheets, (ii) or a graphene sheet is sandwiched between two monolayers of FeSe. Again, the separation between graphene and FeSe is achieved by a thin layer of hBN.

4.1. Gr–FeSe–Gr

When compared with the results for the bilayer geometry, we see an increase in the splitting between the branches around the anticrossing, both for the Bardasis–Schrieffer and for the Higgs mode (figure 3). Also noticeable is a reduction of the linewidth of both branches (better seen in an energy cut along a fixed wave vector, as in figure 4). This means that the sandwich geometry produces longer-lived hybrid modes than the simpler bilayer geometry, which could be important for applications.

This can be understood in terms of the amplitude of the electric field across the structure. In figure 5 we show the absolute value of the parallel component of the electric field between the graphene layers in a Gr–FeSe–Gr heterostructure, for $q_{||} = 44 \mu m^{-1}$, and $\hbar \omega = 17.5$ meV (blue line) and $\hbar \omega = 18.6$ meV (orange line). The FeSe layer is at $z = 0$, graphene layers are at $z = \pm 2$ nm. $E_{Gr}^{FS} = 400$ meV, $\gamma_{Gr} = 1$ meV. Superconducting gap $\Delta = 13$ meV, $\hbar \omega_{BaSh}(0) = 18.2$ meV.

In order to enhance the visibility of the features associated with the hybridized modes, we have adopted a fairly small relaxation rate for graphene ($\gamma_{Gr} = 1$ meV). Nevertheless, the hybrid modes can still be clearly seen at higher relaxation rates, as shown in figure 7. In fact, those linewidths are considerably smaller than $\gamma_{Gr}$. For $\gamma_{Gr} = 1$ meV the linewidths are $\sim 0.05$ meV, and for $\gamma_{Gr} = 5$ meV we find linewidths $\sim 0.2$ meV. This means that observation of the hybrid graphene plasmon—Bardasis–Schrieffer mode is possible with existing graphene preparation techniques (figure 8).
Figure 6. Effect of changing the Gr–SC distance in the sandwich geometry (Gr–SC–Gr). Left: \( d = 4 \) nm, right: \( d = 2 \) nm. Top: zoom at the region of the crossing between the Bardasis–Schrieffer mode and the GPP. Bottom: zoom at the region of the crossing between the Higgs mode and the GPP. \( E_{Gr}^F = 400 \) meV, \( \gamma_{Gr} = 1 \) meV. Superconducting gap \( \Delta = 13 \) meV, \( \hbar \omega_{BaSh}(0) = 18.2 \) meV.

Figure 7. Effect of increasing the graphene relaxation rate \( \gamma_{Gr} \) on the visibility of the hybrid graphene plasmon—Bardasis–Schrieffer mode. We plot the imaginary part of the reflection coefficient as a function of energy \( \hbar \omega \) at a fixed wave vector \( q_\parallel = 20 \) \( \mu \text{m} \). The graphene–SC distance is 10 nm and the graphene doping level is \( E_{Gr}^F = 500 \) meV.

We note that, in this sandwich geometry, only the anti-symmetric graphene plasmon (the lower energy branch) couples to the superconductor. This is related to the fact that the electric field at the position of the SC sheet (right in the middle of the sandwich) is zero for the symmetric mode. To allow both branches to couple to the SC we must break the symmetry, either by placing the SC sheet off centre, or by applying different gate voltages to the two graphene sheets. In figure 9 we show the results for the latter case. It is interesting to note that Fermi energy imbalance between the graphene sheets must be relatively large to make the coupling of the high energy mode to the SC visible.

4.2. FeSe–Gr–FeSe

Superconducting waveguides have been discussed in the literature [16] as promising building blocks for future plasmonic technologies, due to the intrinsic low-loss associated with the dynamics of the Cooper-pair condensate. In the original proposal [16], the cavity geometry serves the purpose of overcoming the problem of weak confinement of the plasmon to the surface of bulk superconductors. In the context of 2D superconductors, the same geometry can be exploited to promote the coupling of the (small dispersion) Bardasis–Schrieffer and Higgs modes to the anti-symmetric Cooper-pair plasmon, as shown in figure 10. Of course, the anti-symmetric mode itself can be used as a resource for plasmonic technologies, as suggested in the literature [16]. The features of the anti-symmetric mode can be tuned in the manufacturing process, by adjusting the distance between the SC monolayers and/or changing the insulating spacer material. It may be desirable, however, to have the ability to control those features on-the-fly. Incorporating a graphene sheet into the device may provide such a control. Due to the anti-symmetric nature of the mode, however, it couples weakly to objects that are placed close to the middle point between the two FeSe monolayers. Thus, to obtain control over the dispersion relation of the anti-symmetric mode, the graphene sheet must be placed much closer to one of the FeSe monolayers than to the other.

For the sake of comparison, we start by showing the results for a cavity formed by two FeSe monolayers separated by a 12 nm thick slab of hBN. The anti-crossings between the Ba–Sh anti-symmetric plasmon mode is clearly visible. To notice the anti-crossing in the case of the Higgs mode it is necessary to zoom in, as shown in figure 10.
Figure 8. Effect of changing the SC carrier density (Gr–SC–Gr). Left: $n = 0.74$ nm$^{-2}$, $\Delta = 13$ meV, $\hbar \omega_{BaSh}(0) = 18.2$ meV. Right: $n = 0.81$ nm$^{-2}$, $\Delta = 17$ meV, $\hbar \omega_{BaSh}(0) = 23.8$ meV. Top: zoom at the region of the crossing between the Bardasis–Schrieffer mode and the GPP. Bottom: zoom at the region of the crossing between the Higgs mode and the GPP. $E_G^F = 400$ meV, $\gamma_G = 1$ meV.

Figure 9. Gr–SC–Gr with different Fermi energies for the two graphene sheets. One of the sheets is kept at $E_F = 500$ meV, while the $E_F$ of the other is changed from 50 meV (top left), to 100 meV (top right) to 200 meV (bottom).

By sandwiching one graphene sheet between the two FeSe monolayers, we add the possibility to tune certain features of the excitation spectrum of the heterostructure. For instance, by changing the graphene sheet’s doping level it is possible to control the dispersion relation of the anti-symmetric plasmon branch associated with the Cooper-pairs, as seen in figure 11.
Figure 10. Imaginary part of the reflection coefficient for a superconducting cavity formed by two sheets of FeSe separated by 12 nm of hBN. Superconducting gap $\Delta = 13$ meV, $\hbar \omega_{\text{Bardasis}}(0) = 18.2$ meV. The right panel shows a zoom at the region of the crossing between the Higgs mode and the 2D SC plasmon.

Figure 11. Superconducting FeSe cavity modified by a single graphene sheet placed (asymmetrically) inside the cavity. By changing the doping of graphene it is possible to tune the slope of the anti-symmetric Cooper-pair plasmon. Left: $E_{\text{Gr}}^\text{F} = 0.1$ eV; right: $E_{\text{Gr}}^\text{F} = 0.6$ eV. The distances between the graphene sheet and the left and right FeSe monolayers are 2 nm and 10 nm respectively. Superconducting gap $\Delta = 13$ meV, $\hbar \omega_{\text{Bardasis}}(0) = 18.2$ meV.

5. Purcell factor

The reflection coefficient is a directly measurable quantity in far-field optical experiments, for which the kinematic constraint $\omega^2 = c^2(k_\parallel^2 + k_\perp^2)$ applies. Surface plasmons exist at values of $k_\parallel$ that render $k_\perp$ imaginary, thus implying exponential confinement of the field amplitudes along the direction perpendicular to the surface. In this regime, the coefficient $r$ defined in equation (13) represents a so-called loss function, whose poles indicate the existence of confined modes. Thus, to excite surface plasmons one needs do overcome the kinematic constraint. This can be done, for instance, by patterning a diffraction grating on top of graphene. Another method that allows to partially circumvent this difficulty is to consider the effects of the hybrid modes on a nearby quantum emitter [13, 17–19]. Its rate of spontaneous emission is modified by the presence of the heterostructure, and this modification carries information about the reflection coefficients of the heterostructure. This is encoded in the ratio between the electromagnetic local density of states (LDOS) in the presence of the heterostructure and the LDOS of free space, given by [13]

$$\frac{\rho(\varepsilon, \omega)}{\rho_0(\omega)} = 1 + \frac{1}{2} \int_0^\infty \mathrm{d}s \Re \left[ \left( \frac{s^3}{s^2 - s \varepsilon} \right) e^{3\pi^2 s^2 r_p(s, \omega)} \right] ,$$

where $s_\varepsilon = \sqrt{1 - s^2}$, with $s = q/c/\omega$ and $\varepsilon$ is the distance between the emitter and the topmost hBN layer. This ratio is known as the Purcell factor [20–22], and can be very large when the emitter is placed close to surfaces that support localized electromagnetic modes.

In figure 12 we show the Purcell factor for a Gr–FeSe–Gr heterostructure as a function of the emitter frequency. The emitter has been placed at a distance $\varepsilon = 50$ nm from the surface, corresponding to an effective wavevector of $\sim 20 \mu\text{m}^{-1}$. In the limit of vanishing graphene doping ($E_{\text{Gr}}^\text{F} = 0.1$ meV, left panel) the LDOS displays a clear peak at the frequency of the Bardasis–Schrieffer mode. It also has a much smaller peak at the frequency of the Higgs mode, as expected due to the weakness of the coupling between the Higgs mode and the electromagnetic field [2]. In the right panel we show the Purcell factor for $E_{\text{Gr}}^\text{F} = 100$ meV. Both the lineshape and the size of the features (relative to the background) are strongly influenced by the charge density in the graphene sheets. These changes reflect the fact that the hybridization between the SC modes and the graphene plasmon transfers spectral weight from a very narrow frequency range to the hybrid excitations, both of which have significant dispersion.

The Purcell factor for the SC–Gr–SC heterostructure displays a much sharper feature when compared to the Gr–SC–Gr structure, as seen in figure 13. The Purcell factor in this case is dominated by the very sharp peak in $\Im r_p$ at small wavevector, associated with hybridization between the anti-symmetric Cooper pair plasmon and the Bardasis–Schrieffer mode (see
The left panel shows the limit of vanishing graphene doping placed at a distance of 50 nm from the surface of the heterostructure. SC is 10 nm. The Purcell factor has been calculated for an emitter placed at a distance of 50 nm from the surface of the heterostructure. Figure 12. Purcell factor for the Gr–SC–Gr heterostructure. The left panel shows the limit of vanishing graphene doping ($E^G_{Gr} = 0.1$ meV). For the right panel we have chosen $E^G_{Gr} = 100$ meV. The distance between the graphene sheets and the SC is 10 nm. The Purcell factor has been calculated for an emitter placed at a distance of 50 nm from the surface of the heterostructure.

This function has a maximum close to $q_{||} = 1/z$, but the width of this peak depends strongly on both $z$ and $\omega$. To illustrate the filtering property of the kernel we fix $\hbar \omega = 18$ meV, which is close to the energy of the Bardasis–Schrieffer mode, and plot

$$K(s, z, \omega) \equiv \frac{1}{N} \left( \frac{s^2}{\sqrt{s^2-1}} + s \sqrt{s^2-1} \right) e^{-2\sqrt{s^2-1}z}$$

as a function of $q_{||}$ for a few values of $z$. The normalization factor $N$

$$N \equiv \int_{1+}^{\infty} \left( \frac{s^2}{\sqrt{s^2-1}} + s \sqrt{s^2-1} \right) e^{-2\sqrt{s^2-1}z} ds$$

is introduced to facilitate the visual comparison between the curves with different values of $z$. The results are shown in figure 14. It is readily noticeable that the filtering function is much broader than the typical features of the reflection coefficient. This constrains the degree of detail with which features in $\Im r_p$ can be resolved, depending on the wave vector around which they appear. This is important to keep in mind when choosing the placement of the LDOS probe relative to the system.

6. Concluding remarks

We have shown that heterostructures of simple geometry, formed by graphene and 2D superconductors separated by a few nanometers, display clear signatures of hybridization between their respective collective modes. The planar cavity geometry Gr–SC–Gr promotes a strong enhancement of the hybridization. It also provides tunability, either through adjustment of geometric parameters or on-the-fly modification of graphene’s doping levels. On the other hand, in the SC–Gr–SC cavity the hybridization is typically weak, but still the graphene doping level can be used to control the features of the SC collective modes. Our results show that graphene–2D superconductor heterostructures are promising platforms...
for probing the fundamental properties of 2D superconductors and for future applications.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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