TDA 3D Data Model and Feature Simplification Analysis

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ABSTRACT. Topology data analysis (TDA) is a combination of topology and data science. The analysis process will not cause information loss. It has been widely used in various industries in recent years. However, how to display TDA data with simpler data structure is still under exploration. This paper designs and simulates a method of TDA point cloud data segmentation and classification data topology information simplification. Firstly, the point cloud data is segmented based on the region growing method. Secondly, the idea of "topological compression" is used to express the richer data information of each segmentation part with a simpler data structure; in the experimental model, the experiment is completed on the platform of MATLAB 2016 (b), and the simple and complex representation of segmented vehicle targets completely includes all the pixel positions and gray 3D data information of the vehicle.

1. Introduction
In recent years, with the rapid development of 3D imaging technology, low-cost and miniaturized 3D sensors such as Microsoft Kinect, Intel's realsense and Google's Tango can capture the 3D information of the scene, help intelligent devices better perceive and understand the world. At the same time, it greatly reduces the threshold for people to obtain the real-world information in a three-dimensional way. On the other hand, with the iterative updating of GPU computing power and the emergence of large-scale 3D model data, the deep learning idea of big data processing gradually occupies an absolute dominant position in the tasks of 3D model classification and retrieval. This makes efficient, accurate and direct processing of three-dimensional data technology become a wide range of needs, and become the key to the development of automatic driving, virtual reality and remote sensing surveying and mapping. Among them, topological data analysis is a common image processing method (TDA).

Topological data analysis (TDA) is a combination of topology and data science. By definition, TDA is a method to analyze topological features of data based on persistent homology. The general purpose of TDA is to extract effective information from high-dimensional data, which belongs to unsupervised learning and representational learning from the viewpoint of machine learning. The analysis process does not cause information loss and is considered to be stable for missing and noisy samples. Because the analysis object of TDA is metric independent topological features (generally expressed as "abstract shape" and "relationship between points"), TDA can integrate and analyze data sets under different metrics (coordinates). In recent years, TDA is mainly used in computational geometry, especially in the analysis, segmentation, reconstruction and visualization of point cloud data from 3D scanners.

TDA 3D data segmentation is to divide the disorderly 3D model point cloud data obtained by specific equipment into several disjoint subsets, and the data in each subset have basically the same
attribute characteristics or certain semantic information. In this way, in the process of scene understanding or virtual reconstruction, the point cloud data can be regarded as the data of an independent object, so the shape, size and other attributes of the target can be easily determined.

At present, due to the technical limitations of acquisition equipment, the sampling density of point cloud data obtained by various methods is uneven, usually disordered and sparse, and there are a large number of noise points and abnormal points. In addition, the surface shape and distribution of point cloud data can be any form of physical characteristics, without fixed or distinct statistical distribution characteristics. At the same time, point cloud data has high redundancy, uneven sampling density and lack of clear structural features. Due to the characteristics of the above point cloud data, it is very difficult to realize the point TDA 3D model segmentation technology, so it has become a research hotspot and difficulty.

In order to solve the data processing difficulties caused by disordered, sparse, noise points and outliers in the current TDA data analysis, reduce the waste of computing resources caused by the huge scale and information characteristics of TDA data, and increase the processing efficiency. In this paper, a method of TDA point cloud data segmentation and classification data topology information simplification is designed and simulated. Firstly, the point cloud data is segmented based on region growing method. Secondly, the idea of "topological compression" is used to express more abundant data information with simpler data structure; in the experimental model, the simple complex representation of segmented vehicle targets completely contains all the pixel positions and gray 3D data information of the vehicle.

2. The theory and method of TDA data segmentation

2.1 Mathematical principle of TDA data segmentation

Topology studies some special geometric properties, which can remain unchanged after the shape of a graph changes continuously, which is called "topological property". In the complex high-dimensional data, there are similar structural properties, which we can call the shape (feature) of data vividly. Compared with the pairwise relationship usually studied, the shape of this relationship may have great research value. To understand the shape of data, we must resort to topology. What TDA does is extract this shape and analyze it.

TDA segmentation is to divide the disorderly point cloud data obtained by specific equipment into several disjoint subsets by certain methods. The data in each subset have basically the same attribute characteristics or certain semantic information. In this way, these point cloud data can be treated as data on an independent object during scene understanding or virtual reconstruction, it is convenient to determine the shape, size and other attributes of the target.

2.1. Advantages of TDA model segmentation

1) Strong versatility
The input of TDA can be a distance matrix representing the distance between any two data points. It studies coordinate independent shapes and is not limited by coordinates. This also means that the construction of topological shape depends on the definition of distance function or similarity concept. The coordinate independent feature allows TDA to integrate data from different platforms, although the structure of the data is not the same, you only need to give a reasonable distance function. This is an advantage of TDA, versatility.

For example, TDA's success in cancer analysis is an important reason for its versatility. Because different cancer data sets have different indicators and structures, TDA can be easily integrated.
2) **Strong robustness**
The shape of data studied by TDA can tolerate a small range of data deformation and distortion. Imagine writing a letter "a" on a piece of rubber. You squeeze and pull the rubber. Although the letter "a" is a little distorted, the basic feature of "one triangle with two feet" still exists.

3) **The expression is simple**
If we want to sketch the outline of a lake, the simplest is to use a polygon. Topology deals with abstract shapes. The most typical example is to use hexagon to represent a circle, which only needs 6 points and 6 edges. TDA uses this form of data compression, using limited points and edges to represent a large number of data, and retains the important characteristics of the data.

2.2. **Common methods of TDA data segmentation**
The point cloud segmentation algorithm based on region growth is to combine the points with the same attributes to form an isolated region in the neighborhood, while ensuring the maximum difference of the rest of the surrounding areas. Compared with the edge-based segmentation algorithm, the region growth-based algorithm has strong anti-noise ability, but it is easy to produce over segmentation or insufficient segmentation results because it cannot get the determined segmentation edge.

Attribute based segmentation algorithm is a relatively stable segmentation algorithm, which is implemented in the feature space and is not affected by the spatial relationship of point clouds. The selection of feature space and clustering method largely determines the performance of the algorithm, and the change of point cloud density has a great impact on the algorithm. When processing large-scale complex point cloud data, the time complexity is also large.

3. **Classification Algorithm Design**

3.1. **Data segmentation**
After preprocessing the video frame image, the gray information of the image is extracted by using the gray image, and the image is transformed into 3D point cloud data, which contains the target shape and gray information in the image.

Point cloud segmentation technology divides the point cloud data into several disjoint subsets according to the spatial position, geometry and texture features, so that the data in each subset have the same or similar attributes. Point cloud segmentation is the premise of subsequent feature extraction, target classification and recognition at present, there are three kinds of segmentation methods for 3D point cloud, which are based on feature clustering, model fitting and region growing. Euclidean distance clustering and random sampling consensus (RANSAC) are typical algorithms of the first two categories. Region growing segmentation is based on normal, which mainly compares seed points and neighbors If the normal angle between domain points is less than the smoothing threshold (the angle between normals), it is regarded as a part of the same smooth surface.

The segmentation method based on region growth realizes clustering according to the similarity of pixels in the region of the same object. Starting from the initial position, adjacent pixels or small regions with the same or similar attributes are merged into the current region to realize gradual growth until all adjacent pixels or small regions are traversed.

Accoring to the characteristics of vehicle target, a region growing segmentation algorithm based on normal direction consistency is proposed. The angle between seed point and neighborhood point in normal direction is calculated. If the angle between the two points is smaller than the threshold value, it is regarded as belonging to the same smooth surface, otherwise it is regarded as different planes to be segmented.

The algorithm steps are as follows:
Using the point cloud, a point \( y \) and its \( K \) nearest neighbors \( y_1, y_2 \), the covariance matrix \( C_{3 \times 3} \) is calculated by the following formula, and its eigenvalues are \( k_0, k_1, k_2 \). Where \( \bar{y} \) represents the three-dimensional centroid of the nearest neighbor element.

\[
\begin{bmatrix}
    y_1 - \bar{y} \\
    \vdots \\
    y_K - \bar{y}
\end{bmatrix} \begin{bmatrix}
    y_1 - \bar{y} \\
    \vdots \\
    y_K - \bar{y}
\end{bmatrix}^{-1} \begin{bmatrix}
    y_1 - \bar{y} \\
    \vdots \\
    y_K - \bar{y}
\end{bmatrix}
\]

(1)

Then the curvature \( K_P \) of \( Y \) point can be estimated by the following formula:

\[
k_p = \frac{k_0}{k_0 + k_1 + k_2}
\]

(2)

The curvature \( k_p \) of each point is calculated and compared, and the input point cloud is sorted according to the curvature value of the point cloud. The point with the smallest curvature is selected as the initial seed point, and the region where the initial seed point is located is the flattest region. Growing from the smoothest region can reduce the total number of segments and improve the efficiency.

An empty seed point sequence and an empty clustering area were set up at first. After selecting the initial seed point, it was added to the seed point sequence. Then the neighborhood points were searched. For each neighborhood point, the angle between the normal of the neighborhood point and the current seed point should be compared. Besides, the current point to the current region should be added, if it is less than the smoothing threshold.

For each seed point, repeat step 3), and finally output a set of classes. Each class's points are considered as part of the same smooth surface.

When using the above method, \( k = 150 \), normal angle threshold value is 5° and curvature threshold value is 0.2. Fig. 1 (left) is the original image. After the original image is transformed into a gray image, the point cloud data of the original gray image is generated. After the point cloud data is segmented by region growth, the segmentation result is shown in Fig. 1 (right). It can be seen from the results that there is a big difference between the normal angle features of point clouds in the vehicle target area and the point clouds in the background area. Selecting region growth segmentation is more conducive to separating the point cloud data belonging to the target vehicle area from the background point cloud data.

![Figure 1. Segmentation effect of point cloud data.](image)

3.2. Topology simplification

Topological data analysis is a method based on topology. Distance function usually encodes the similarity of concepts in classification methods, but large-scale distance is meaningless in data set. Topological data analysis method not only retains the concept of similarity, but also distorts large-scale distance and ignores large-scale distance. The idea of mapper can simplify the structure of the original data set substantially and generate a simple complex invariant topology. It provides a method of comparison and matching in the field of shape. Its core idea is to produce a simple complex which is not sensitive to measurement. There is a persistent homology between the structure and the original data. When the data are expressed by simple complex structure, a large number of similar
homologous features are preserved. The topological data analysis of foreground target samples can use less

In order to find the simple topological structure expression of vehicle target point cloud, i.e., generate simple complex g, set the input point cloud set as y, convert the point cloud data in Y into parameter space by filtering function, and get the parameter space Z, i.e., F: y → Z, where f is the eccentric filter function, then the i-th of N data points is obtained the eccentricity of the data points is (where the exponent is set to 1).

\[
\text{eccentricity}(i) = \left( \frac{1}{N} \sum_{j=1}^{N} d(y_i, y_j)^{\text{exponent}} \right)^{1/\text{exponent}}
\]  

(3)

The filtered results are shown in Fig. 2.

Figure 2. Filtering of target sample point cloud data.

The interval length L and the interval coverage percentage P of the filter component are defined. The unjoin clustering method in hierarchical clustering is used to cluster each interval U:

\[
D_H(Z_i, Z_j) = \min_{x_j \in Z_i, y_j \in Z_j} \text{distance}(y_i, y_j)
\]  

(4)

The vertex cluster set C of simple complex G is obtained by the above formula, in which each cluster contains several points in the original segmented dataset y. In order to represent the association between vertex sets, the adjacency matrix \( D_H \) is defined:

\[
D_H(C_i, C_j) = \max \left( \frac{\sum_{x_j \in C_i} \min_{y_j \in C_j} d(x, y)}{N_i}, \frac{\sum_{y_j \in C_j} \min_{x_j \in C_i} d(x, y)}{N_j} \right)
\]  

(5)

\( D_H \) is defined in the smooth form of Hausdorff distance of two vertex sets, where n is the number of vertices. After visualizing the segmented data set processed by topological data analysis algorithm, the simplex complex g of different vehicle target samples can be seen. They have similar data structure. Fig. 3 shows Fig. 2 In order to quantify the output parameters of the simplex complex, the weights of each vertex are defined as follows:

\[
\mu_i = \frac{N_i}{\sum N_j}
\]  

(6)

The output of a simple complex can be expressed as \{\( (C, D_H, \mu C) \)\}
4. Experimental process

4.1. Experimental environment
This experiment is completed on MATLAB 2016 (b) platform, and the hardware configuration is Intel corei5 7200u CPU (@ 2.5 GHz) ram 4 GB. The detection speed of the proposed algorithm is 17.6 FPS (frames per second), which is higher than that of 3d-cnns algorithm and convnet algorithm, which are 9.2 FPS and 10.5 FPS, respectively the detection speed of FPS, self-coding network based on changing cost function and self-coding network based on target background separation are 15.2 FPS and 12.9 FPS respectively. It can be seen that compared with the existing deep learning algorithm, the proposed algorithm has better noise resistance and higher detection rate in snow video detection environment and to a certain extent, it improves the efficiency of deep learning algorithm for video sequence detection.

In the experiment, 80000-pixel level snow video images are selected for vehicle target detection. The size of each frame in the video is 240*320. The experimental data in this paper are collected from the real-time urban traffic monitoring video under snow weather environment, including video images taken under various road conditions, and the sample size is 70*100 The vehicle image with 10 000 pixels includes 10 000 samples, 80% of which are used as training samples and 20% as test samples. When there is more than 90% overlap between the detection frame and the vehicle target, the target detection is regarded as successful. The ROC curve is used as the performance evaluation index of the target detection method.

4.2. Results evaluation method
In the field of image segmentation, how to evaluate the segmentation quality effectively is a very important research problem. In the current evaluation system, it is mainly divided into subjective evaluation analysis, objective analysis and the time consumed to evaluate the segmentation algorithm and segmentation results. Subjective evaluation analysis is mainly to make a certain evaluation by comparing the segmentation results with the expected segmentation objects. In the evaluation, the following points are mainly used for analysis:
• Observe the integrity of the segmentation target: compare the image segmentation results with the original image to see whether the target object is completely segmented from the image and whether there is over segmentation phenomenon;
• Observe the under-segmentation results: on the basis of the integrity of the segmentation target, check whether there are other regions in the segmentation results except the target region. The smaller the proportion of other regions in the segmentation results, the better the segmentation results are;
• Observe the edge of the segmentation object: observe whether the edge of the target object is smooth; whether there is an area of under segmentation; the segmentation result around the boundary of the object is smooth; there is no under segmentation condition in the segmentation result, which proves that the segmentation result is better;

The main reason is that in the field of image segmentation, different researchers or image segmentation users are segmenting personal images, and there is no fixed and recognized image segmentation result in this part of the image. Results evaluation can only be based on personal standards to evaluate the quality of image segmentation results.

At present, some researchers have designed a series of subjective evaluation methods. For example, the scores can be made according to the above subjective evaluation points, and the image segmentation results can be scored by people in different professional fields and different age groups. The researchers make statistical score results to evaluate the segmentation quality as accurately as possible. As far as possible to achieve the effectiveness and accuracy of the subjective evaluation method, but this method is still not scientific to a certain extent.

In order to accurately compare and judge the quality of image segmentation and the advantages and disadvantages of image segmentation algorithm, researchers label the image pixels one by one, and divide the image pixels into target pixels and background pixels, and form a series of public data sets, such as the standard data set msra1000 and the standard data set bsds300. Through the use of a unified image, researchers in related fields can obtain the relevant formula to judge the quality of image segmentation and carry out quantitative analysis. This method is called objective analysis method. In the objective analysis method, recall, precision, F-measure and OCR curve are all good indicators to objectively evaluate the segmentation results of the model.

4.3. Classifier training and experimental steps

It is one of the common algorithms in deep learning to capture the important factors that can represent the input by reproducing the output. It is composed of input layer, hidden layer and output layer. The input layer and the hidden layer encode, and a decoder is formed between the hidden layer and the output layer to transform the coding into the output signal. For a group of unlabelled training samples, the self-encoder learns the input data to make its reconstructed output as close to the input data as possible, so as to obtain an efficient and robust feature representation.

Firstly, the quantitative output \{\{C, DH, \mu \ C\}\} of input point cloud y obtained by TDA is used as the input of deep self-coding network to train the network:

\[ x(m+h) = (x(0), x(1), \ldots, x(m), x(0), x(1), \ldots, x(h)) \]  

Where \(x(I) \in \text{RM}\) is the element in D, \(X(I) \in \text{RH}\) is the element in \(\mu\), the encoding and decoding of data are as follows:

\[
\begin{align*}
\{h' &= f(Wx' + p) \\
\hat{x'} &= g(W h' + q)
\end{align*}
\]  

Among them, W and W are transposed to each other, P and Q are encoding function and decoding function respectively, the offset vector of function, F and G are sigmoid functions.

In order to make the output layer restore the state of the input layer to the maximum extent, the objective function is as follows:
The gradient descent method is used to calculate the parameters \( w, P \) and \( Q \). In the gradient descent method, the parameters \( w, P \) and \( Q \) are updated in each iteration. The features learned from the previous layer of automatic encoder are used as the input of the later layer. By stacking the multilayer self-encoder, a deep neural network structure can be obtained. By multi-layer reconstruction of the input data, the robust and stable multi-layer feature expression can be obtained.

The output features of the last two hidden layers of the trained trestle self-coding network are taken as the input training samples of the softmax classifier, and the supervised fine-tuning of the network is carried out by introducing tag variables. Finally, a complete deep self-coding classification network structure is obtained. Finally, the trained network classifies the input samples of vehicle data to achieve the purpose of detection.

The experimental steps are summarized as follows:
- Transforms the image of time \( t \) into 3D point cloud data, which contains the position and gray information of each point in the image.
- The transformed point cloud data are segmented into regions to obtain point cloud data sets \( Y_1, Y_2 \) Where \( n \) is the number of regions forming point clouds in different regions after segmentation.
- To point cloud dataset \( Y_1, Y_2 \) the topological data of \( Y_1, Y_2 \) the data are quantized as \( \{(C, DH, \mu_C)\}^n \)
- Input the quantitative data in step 3 into the sample vehicle extension.
- After analyzing the quantized data, the deep self-coding network is used for binary classification, and the vehicle video target detection at time \( t \) is completed.
- Cycle the video image at \( t + 1 \) time step 1.

5. Experimental results and analysis

In order to verify the correctness and effectiveness of the method, self-built 3D images are selected for testing. Visual effect is regarded as the evaluation standard of subjective evaluation. The experimental results are shown in Fig. 4. Fig. 4 shows the contrast detection results in the real-time monitoring scene of snowy days. It can be seen from the figure that there are a lot of noises that interfere with target recognition caused by random falling snowflakes in the video under snow conditions. When the noise interference or partial occlusion occurs, the proposed algorithm can still recognize the target better.
Through the left original image and the right segmentation results, it can be seen that the segmentation results in this chapter have good visual effect.

In order to objectively evaluate the segmentation effect of the algorithm, and further verify the effectiveness of the proposed method, the image segmentation quality is quantitatively analyzed.

5.1. FPPI
Fig. 5 is the ROC curve of the algorithm, in which abscissa is the false positive per image (FPPI). From Figure 5, we can see that when FPPI is 0.2, the detection rate of the method in this paper is at a very high level in snowy environment.

![Figure 5. ROC graph of different algorithms in real-time scenarios.](image)

5.2. Recall, Precision, F-measure
By comparing the recall and precision of image segmentation results and F-measure as the evaluation index of image segmentation.

| Number of iterations | Recall | Precision | F-measure |
|----------------------|--------|-----------|-----------|
| 2                    | 0.95   | 0.97      | 0.98      |
| 3                    | 0.96   | 0.9        | 0.97      |
| 4                    | 0.92   | 0.96       | 0.98      |
| 5                    | 0.94   | 0.97       | 0.9       |
| 6                    | 0.92   | 0.93       | 0.99      |
| 7                    | 0.96   | 0.98       | 0.93      |
| 8                    | 0.99   | 0.98       | 0.96      |
| 9                    | 0.91   | 0.9        | 0.95      |
| 10                   | 0.9    | 0.96       | 0.92      |
| 11                   | 0.91   | 0.9        | 0.95      |
| 12                   | 0.9    | 0.94       | 0.95      |

As can be seen from table 1, the recall rate, accuracy rate and F-measure of the segmentation results of our algorithm are very high, and the three parameters are all greater than 90%.

5.3. Experimental Experiment time and speed

| Serial number | Time | Number of iterations |
|---------------|------|----------------------|
| 1             | 3979 | 4                    |
| 2             | 6122 | 3                    |

Table 2. Experimental Experiment time and speed.
As can be seen from table 1, this algorithm has fewer iterations and shorter time.

6. Conclusion

In this paper, TDA algorithm is combined with topological feature simplification algorithm to detect video vehicles. Different from the traditional image as input data, the data obtained by TDA algorithm analysis and simplification is used as the input of self-encoder.

The experimental results are as follows: in the traffic scene under the snow environment, the vehicle detection is often affected by the random falling snowflakes, which makes the classification algorithm unable to detect normally under the interference of snowflake noise, and the feature extraction method cannot play its maximum benefit. In this complex environment, the vehicle itself may also have the influence of rotation. Topological data analysis is an algorithm of data simplification and dimensionality reduction. Topology data analysis algorithm itself has good robustness to noise. Without losing useful information of the original data, the simplified features of the original data are retained to the maximum extent. Combined with the advantages of self-coding network which can learn features automatically, it can be more satisfied at the same time, this algorithm improves the detection efficiency of TDA learning algorithm.

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