Field-induced axion emission via process
\[ e^+e^- \to a \] in plasma

N.V. Mikheev, A.Ya. Parkhomenko and L.A. Vassilevskaya
Yaroslavl State (Demidov) University,
Sovietskaya 14, Yaroslavl 150000, Russia

Abstract
The annihilation into axion \( e^+e^- \to a \) is investigated in a plasma and an external magnetic field. This process via a plasmon intermediate state has a resonant character at a particular energy of the emitted axion. The emissivity by \( e^+e^- \to a \) is compared with the axion cyclotron emissivity.

1 Introduction
The axion [1, 2], the pseudo-Goldstone boson arising as a result of the spontaneous breakdown of the global Peccei-Quinn (PQ) symmetry \( U_{PQ}(1) \), is one of the well-motivated candidates for the cold dark matter (see, for example, Refs. [3, 4, 5] and references therein). Although the original axion, associated with the PQ symmetry breakdown at the weak scale \( f_w \), is excluded experimentally, many variants of the PQ models and their accompanying axions are of great interest. If the breaking scale of the PQ symmetry \( f_a \) is much larger than the electroweak scale \( f_a \gg f_w \) (the latest astrophysical data yield \( f_a \gtrsim 10^{10} \) GeV), the resulting “invisible axion” [6, 7] is very light \( (m_a \sim f_a^{-1}) \) and very weakly coupled (coupling \( \sim f_a^{-1} \)).

In view of the smallness of the coupling constant, axion effects could be noticeable under astrophysical conditions – high matter densities, high temperature, and strong magnetic fields. So, it is important to take into account the influence of a plasma and magnetic fields in studies of axion processes in stars. One of the most physically realistic situations presented in many astrophysical objects is that when from both these components of the active medium the plasma dominates:
\[ eB \ll \mu^2, T^2, \] (1)
where \( \mu \) and \( T \) are the electron chemical potential and temperature, respectively. While this situation corresponds to a relatively weak magnetic field, it can still be strong \( B \gg B_e \)
in comparison with the electron’s Schwinger value $B_e = m_e^2/e \simeq 4.41 \times 10^{13}$ G. At present an existence of magnetic fields up to $B \sim 10^{15} - 10^{17}$ G is not exotic in astrophysics and cosmology. A possible manifestation of axions’ effects under such conditions is of great interest.

In the recent paper the axion cyclotron emission of the plasma $e^- \rightarrow e^- a$ was studied as a possible source of energy losses by astrophysical objects. In this paper we study the annihilation process $e^+e^- \rightarrow a$ in medium as an additional channel of energy losses and compare the emissivity by this process with the axion cyclotron emissivity under the condition of a nondegenerate hot plasma.

2 S-matrix element

The process $e^+e^- \rightarrow a$ is described by two diagrams in Fig. where solid double lines imply the influence of medium in the electron and positron wave functions and the photon propagator. Figure (a) describes the axion annihilation of $e^+e^-$-pair due to a direct axion-fermion coupling:

$$\mathcal{L}_{af} = -ig_{af} \left( \bar{f} \gamma_5 f \right) a, \quad (2)$$

where $g_{af} = C_f m_f / f_a$ is a dimensionless Yukawa coupling constant; $C_f$ is a model-dependent factor, $m_f$ is the fermion mass (the electron mass in our case); $f$ and $a$ are the fermion and axion fields, respectively.

Figure (b) describes the electron-positron annihilation via the photon intermediate state. This channel becomes possible due to an effective axion-photon interaction with the Lagrangian:

$$\mathcal{L}_{a\gamma} = \bar{g}_{a\gamma} \left( \partial_\mu A_\nu \right) \tilde{F}^{\mu\nu} a, \quad (3)$$

where $A_\mu$ is the four-potential of the quantized electromagnetic field; $\tilde{F}^{\mu\nu}$ is the dual tensor of the external field; $g_{a\gamma}$ is an effective coupling in the presence of the magnetic field with the dimension $(energy)^{-1}$:

$$\bar{g}_{a\gamma} = g_{a\gamma} + \Delta g_{a\gamma}. \quad (4)$$

![Figure 1: The annihilation process.](image-url)
Figure 2: Axion-photon coupling in an external electromagnetic field.

Here $g_{a\gamma}$ corresponds to the well-known $a\gamma\gamma$ coupling in vacuum (Fig. 2a) with the constant $g_{a\gamma} = \alpha \xi / 2\pi f_a$ \cite{12}, where $\xi$ is the model-dependent parameter. The second term in Eq. (4) is the field-induced contribution to the effective axion-photon coupling $\bar{g}_{a\gamma}$ which comes from Fig. 2b:

$$
\Delta g_{a\gamma} = \frac{\alpha}{\pi} \sum_f \frac{Q_f^2 g_{af}}{m_f} (1 - J),
$$

$$
J = \left( \frac{4}{\chi_f} \right)^{2/3} \pi^{1/2} \int_0^{\pi/2} f(\eta) \sin^{-1/3} \phi \, d\phi,
$$

where the Hardy-Stokes function $f(\eta)$ of the argument $\eta = (4/\chi_f \sin^2 \phi)^{2/3}$ is defined as:

$$
f(\eta) = i \int_0^\infty du \exp \left\{ -i \left( \eta u + \frac{u^3}{3} \right) \right\}.
$$

Further, the dynamic parameter is $\chi_f^2 = e_f^2 (qFq)/m_f^5$, where $e_f = e Q_f$, $e > 0$ is the elementary charge, $Q_f$ is a relative electric charge of a loop fermion, $(qFq) = q_\mu F^{\mu\nu} q^\nu$, $q_\mu = (E_\alpha, q)$ is the four-momentum of the final axion, and $F_{\mu\nu}$ is the external electromagnetic field tensor. The field-induced coupling $\Delta g_{a\gamma}$ is a step function of the magnetic field strength because of the hierarchy of the fermion mass spectrum.

The case of ultrarelativistic electrons and the relatively weak external magnetic field, when a large number of the Landau levels is excited, is well described by a crossed field limit ($E \perp B$, $E = B$), and the dynamic parameter $\chi_f$ is the only field invariant.
The matrix element of the process $e^-(p) + e^+(p') \rightarrow a(q)$ shown in Fig. 1 is the sum:

$$S = S^{(a)} + S^{(b)}, \quad (6)$$

where $S^{(a)}$ corresponds to Fig. 1a:

$$S^{(a)} = \frac{g_{ae}}{\sqrt{2}E_a V} \int d^4x \bar{\psi}(-p', x) \gamma_5 \psi(p, x) e^{iqx},$$

and $S^{(b)}$ describes the contribution from Fig. 1b:

$$S^{(b)} = \frac{\bar{g}_{ae}}{\sqrt{2}E_a V} \int d^4x \bar{\psi}(-p', x) (\gamma h) \psi(p, x) e^{iqx},$$

$$h_\alpha = -ie(\gamma h)_{\alpha} = -ieq_{\mu} \bar{F}^{\mu\nu}G_L^\nu_{\alpha}(q).$$

Here, $\psi(p, x)$ is the exact solution of the Dirac equation in the external crossed field [13]; $p_\mu = (E, p)$ and $p'_\mu = (E', p')$ are the four-momenta of the initial electron and positron ($p^2 = p'^2 = m_e^2$); $(\gamma h) = \gamma_\mu h^\mu$, $\gamma_\mu$ are the Dirac $\gamma$-matrices; $V$ is the three-dimensional volume.

Note that from both components of an active medium, the plasma determines basically the properties of the photon (plasmon) propagator. The contribution to the amplitude from the transverse intermediate plasmons is negligible small in the ultrarelativistic limit. Finally, $G_L^{\alpha\beta}$ is the longitudinal plasmon propagator:

$$G_L^{\alpha\beta} = i \frac{\ell_\alpha \ell_\beta}{q^2 - \Pi^L}.$$

Here, $\Pi^L$ is the eigenvalue of the polarization operator of the longitudinal plasmon with the eigenvector:

$$\ell_\alpha = \sqrt{\frac{q^2}{(uq)^2 - q^2}(u_\alpha - \frac{uq}{q^2} q_\alpha)}, \quad (8)$$

where $u_\alpha$ is the four-velocity of medium.

By integrating over the variable $x$, Eq. (4) can be presented in the ultrarelativistic limit of the form:

$$S = \frac{(2\pi)^4 \delta^{(2)}(Q_+) \delta(kQ)}{\sqrt{2}E_a V \cdot 2EV \cdot 2EV \cdot \pi \alpha z} \Phi(\eta) \Psi(\eta) \bar{U}(p) \left[ g_{ae} \gamma_5 \left( 1 - \frac{iez^2}{m_e^2} (\gamma k)(\gamma a) \frac{\Phi'(\eta)}{\Phi(\eta)} \right) + \bar{g}_{ae} (\gamma h) \right] U(p),$$

$$\alpha^2 = -\frac{e^2 a^2}{m_e^2}, \quad z = \left( \frac{\chi_0}{2\chi} \right)^{1/3},$$

$$\chi^2 = \frac{e^2(p FF p)}{m_e^2}, \quad \chi_0 = \frac{e^2(q FF q)}{m_e^2}, \quad x'(p \rightarrow p').$$
Here, $F_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu$ ($k^2 = (ka) = 0$) is the external crossed field tensor; $Q = q - p - p'$, $Q_\perp$ is the perpendicular to $k$ component ($Q_\perp k = 0$). The bispinor $U(p)$, which is normalized by the condition $\bar{U}U = 2m_e$, satisfies the Dirac equation for the free electron $((\gamma p) - m_e)U(p) = 0$. Finally, $\Phi(\eta)$ is the Airy function:

$$\Phi(\eta) = \int_0^\infty dt \cos \left( \eta t + \frac{t^3}{3} \right),$$

$$\eta = z^2 (1 + \tau^2), \quad \tau = -\frac{e(p\bar{F}q)}{m_e^4 \chi},$$

and $\Phi'(\eta) = d\Phi(\eta)/d\eta$.

### 3 Axion emissivity

The plasma’s axion emissivity due to the process $e^+e^- \rightarrow a$ can be written as:

$$Q_a = \frac{1}{VT_0} \int dn_{e^-} \int dn_{e^+} \int \frac{V d^3 q}{(2\pi)^3} E_a \sum_{s,s'} |S|^2,$$

where $T_0$ is a time interval. Taking into account the condition (1), the numbers of plasma electrons and positrons can be estimated as the numbers without the field:

$$dn_{e^-} \simeq \frac{V d^3 p}{(2\pi)^3} f(E), \quad dn_{e^+} \simeq \frac{V d^3 p'}{(2\pi)^3} \bar{f}(E'),$$

where $f(E) = (e^{(E-\mu)/T} + 1)^{-1}$ and $\bar{f}(E') = (e^{(E'+\mu)/T} + 1)^{-1}$ are the electron’s and positron’s Fermi-Dirac distribution functions at the temperature $T$ and the chemical potential $\mu$, respectively. Carrying out the integration in Eq. (10), we obtain the expression for the axion emissivity in the form:

$$\frac{dQ_a}{dE_a} \simeq \frac{g_{ae}^2 (eB)^{2/3}}{40\pi^{5/2}3^{1/3} \Gamma(5/6)} \frac{E_a^{7/3}}{E_a^{1/3}} I_{-\frac{1}{3},-\frac{1}{3}}(E_a)$$

$$+ \frac{g_{ae}^2 (eB)^2}{36\pi^3} \frac{E_a^3}{(E^2 - E_a^2)^2 + \gamma^2 E^4} I_{1,1}(E_a),$$

where

$$I_{k,n}(E_a) = \int_{E_a}^{\infty} dE E^k (E_a - E)^n f(E) \bar{f}(E_a - E).$$

The second term in Eq. (12) describing the contribution of the longitudinal plasmon intermediate state has a resonant character at a particular energy of the emitted axion $E_a \sim E$. This is due to the fact that the axion and the longitudinal plasmon dispersion relations cross for a certain wave number $k = E$ as was shown in Fig. 3. The dimensionless
Figure 3: Dispersion relations \( \omega^2 = \omega^2_L(k) \) for longitudinal plasmons (solid line), axions \( E^2_a = k^2 + m^2_a \) (short dashes), and vacuum photons \( \omega = k \) (long dashes).

The resonance width \( \gamma \) in Eq. (12) is

\[
\gamma = \frac{\mathcal{E} \Gamma_L(\mathcal{E})}{q^2} \left( 1 - \frac{\partial \Pi^{(L)}}{\partial q^2} \right). \tag{13}
\]

Here, \( \Gamma_L(\mathcal{E}) \) is the total width of the longitudinal plasmon in the presence of the magnetic field, determined mainly by the absorption \( \gamma_{Le^- \to e^-} \) [11]. The expression in brackets in Eq. (13) comes from the renormalization of the longitudinal plasmon wave function. The second term in Eq. (12) defines the main contribution to the plasma’s axion emissivity by the annihilation \( e^+e^- \to a \) in the resonant region. So, below we neglect the nonresonant direct axion-electron term. The resonant behaviour of the axion spectrum is shown in Fig. 4. The expressions for \( \mathcal{E}, \gamma \) and the axion emissivity in the resonant point are:

i) degenerate plasma \((\mu \gg T)\)

\[
\mathcal{E}^2 \simeq \frac{4\alpha}{\pi} \mu^2 \left( \ln \frac{2\mu}{m_e} - 1 \right), \quad \gamma \simeq \frac{2\alpha}{3} \frac{\mu^2}{\mathcal{E}^2}; \tag{14}
\]

\[
Q_{a\text{res}}^\gamma \simeq \frac{g_a^2 \gamma (eB)^2}{48\pi^2\alpha} \frac{\mathcal{E}^2 T^2}{\mu^2} e^{-\mu/T},
\]

ii) nondegenerate hot plasma \((T \gg \mu)\)

\[
\mathcal{E}^2 \simeq \frac{4\pi\alpha}{3} T^2 \left( \ln \frac{4T}{m_e} - 0.647 \right), \quad \gamma \simeq \frac{2\pi^2\alpha}{9} \frac{T^2}{\mathcal{E}^2}; \tag{15}
\]
Figure 4: The resonant behaviour of the axion emissivity spectrum where \( Q_{a\gamma} = (\bar{g}_{a\gamma}^2/36\pi^3)(eB)^2\mu^2E \). Line 1 corresponds to \( \mu = 100\, m_e, \, T = 10\, m_e, \, E = 20\, m_e \), while line 2 corresponds to \( \mu = 500\, m_e, \, T = 70\, m_e, \, E = 120\, m_e \).

\[
Q_{a}^{\text{res}} \approx \frac{\bar{g}_{a\gamma}^2 (eB)^2 E^5}{384\pi^3 \alpha T^2}.
\]

The conditions of the nondegenerate hot plasma are the most favourable for a possible manifestation of the annihilation \( e^+e^- \rightarrow a \). Below we compare the axion emissivity by the process \( e^+e^- \rightarrow a \) with the one by the axion cyclotron emission \([11]\) under these conditions:

\[
\frac{Q(e^+e^- \rightarrow a)}{Q(e^- \rightarrow e^-a)} \simeq \frac{1}{8\pi^2} \frac{E^3}{T^3} \sim \left[ \frac{\alpha}{3\pi^{1/3}} \left( \ln \frac{4T}{m_e} - 0.647 \right) \right]^{3/2}.
\]

It is seen from Eq. (16) that the axion emissivity by the annihilation is noticeably smaller than that by the cyclotron emission.

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References

[1] R.D. Peccei and H.R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977); *Phys. Rev.* **D16**, 1791 (1977).

[2] S. Weinberg, *Phys. Rev. Lett.* **40**, 223 (1978);
   F. Wilczek, *ibid.* **40**, 271 (1978).

[3] G.G. Raffelt, *Stars as Laboratories for Fundamental Physics* (University of Chicago Press, 1996).

[4] G.G. Raffelt, in *Proc. of Beyond the Desert*, eds. H.V. Klapder-Kleingrothaus and H. Paes (IOP, 1998), p. 808; astro-ph/9707268.

[5] G.G. Raffelt, in *Proc. of 1997 European School of High-Energy Physics*, eds. N. Ellis and M. Naubert (CERN, 1998), p. 235; hep-ph/9712538.

[6] J. Kim, *Phys. Rev. Lett.* **43**, 103 (1979);
   M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B166**, 493 (1980).

[7] M. Dine, W. Fischler and M. Srednicki, *Phys. Lett.* **B104**, 199 (1981);
   A.R. Zhitnitsky, *Sov. J. Nucl. Phys.* **31**, 260 (1980).

[8] R.C. Duncan and C. Thompson, *Astrophys. J.* **392**, L9 (1992);
   C. Thompson and R.C. Duncan, *ibid.* **194**, 408 (1993);
   M. Bocquet at al., *Astron. and Astrophys.* **301**, 757 (1995).

[9] G.S. Bisnovatyi-Kogan and S.G. Moiseenko, *Astron. Zh.* **69**, 563 (1992) [Sov. Astron. **36**, 285 (1992)];
   G.S. Bisnovatyi-Kogan, *Astron. Astrophys. Trans.* **3**, 287 (1993);
   G.J. Mathews et al., astro-ph/9710229.

[10] D. Lemoine, *Phys. Rev.* **D51**, 2677 (1995);
    T. Tajima et al., *Astrophys. J.* **390**, 309 (1992).

[11] N.V. Mikheev, G.G. Raffelt and L.A. Vassilevskaya, *Phys. Rev.* **D58**, 055008 (1998); hep-ph/9803486.

[12] G.G. Raffelt, *Phys. Rept.* **198**, 1 (1990).

[13] V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii, *Quantum Electrodynamics* (Pergamon Press, 1982), 2nd edition.