Longitudinal flight control laws for high aspect ratio light utility aircraft

S Bahri
National Institute of Aeronautics and Space (LAPAN), Bogor, Indonesia

Abstract. The development of Flight Control Laws (FCLs) for high aspect ratio light utility aircraft have been previously accomplished by considering each control module to be independent. For the longitudinal motion, the FCLs comprise mainly the airspeed control and the flight path angle control. In addition, the altitude control and the vertical speed control have been designed based on the design of flight path angle control. Further investigation is conducted to reveal possibility to design the longitudinal control, *i.e.* airspeed and flight path angle control by considering the coupling behavior between both controls but still utilizing the same control design strategy that is the advanced Proportional-Integral (PI) control technique. The paper shows the concept and architecture of coupled longitudinal flight control laws, the design strategy chosen, implementation to the aircraft model and investigation of control performance. The results achieved from this research are the architecture and procedure to design longitudinal FCLs. Additionally, knowledge of coupling control behavior between the airspeed and flight path angle via aircraft simulations is acquired.

1. Introduction
The usage of high aspect ratio light utility aircraft as a research aircraft is pursued by LAPAN. The aircraft is named LAPAN Surveillance Aircraft - 02 (LSA-02) whose purpose is to be an Unmanned Aerial Vehicle (UAV) technology demonstrator and the project has been conducted since 2014. The project goal is to develop Electronic Flight Control System (EFCS) which will be supplemented to the basic aircraft (see Figure 1) [4]. The ECFS consists of Basic EFCS (BEFCS) and Experimental System (XS). The development of FCLs for BEFCS has been accomplished in software level and FCLs comprise numerous control modules to perform modes such as speed mode, vertical mode and lateral mode [1].

The development of FCLs for BEFCS of LSA-02 follows the systematic and strict development process namely the V model which comes from recommended guidelines for development of civil aircraft and systems [10]. The FCLs design started from the requirements definition, continued by coding and ended by verification and validation. Some researchers have adopted this development process and successfully developed their FCLs to be implemented to the same type of aircraft [6][8]. The requirements of FCLs for BEFCS have been derived from several sources, *i.e.* the aircraft standard for flight control system, the specific aircraft specifications, the lessons learned from aircraft flight dynamic analysis and obviously the research project LSA-02 requirements [2][3][5][7][12].

The FCLs are activated interactively from the commands of Flight Control Panel (FCP) which are airspeed, altitude, vertical speed, flight path angle and heading commands. Here, the lateral motion command only comes from heading command while the others belong to the longitudinal motion commands. These infer that longitudinal motion covers majority of aircraft maneuver with respect of flight time. Furthermore, there are more control elements for longitudinal motion than lateral motion.
Additionally, in flight guidance loop, the airspeed and flight path angle influence each other. Hence, the complexity of longitudinal FCLs design is higher and thus challenging.

![Diagram of aircraft](image.png)

**Figure 1.** Aircraft STEMME ES15 which is modified with EFCS to become LSA-02.

For earlier versions, FCLs design simplification is desirable because the aim is to comply given requirements. Often design strategy chosen is the simplest strategy and requires short computation time to calculate control gains. The previous longitudinal FCLs design has been simplified by considering airspeed and flight path angle to be independent each other and excited by specific control variables, *i.e.* throttle substitute command and pitch angle command respectively. However, the real behavior is more complex as the both throttle substitute and pitch angle commands excite the airspeed and flight path angle at the same time. Hence, the former longitudinal FCLs design is refined by taking into account the coupling between airspeed and flight path angle.

The research objectives presented in this paper are to design longitudinal FCLs whose architecture considers coupling between airspeed and flight path angle by using the same design strategy that is the advanced PI control technique, to investigate the design performance with respect of requirements compliance and previous design comparison, and to propose design procedure for longitudinal FCLs.

2. **Longitudinal flight control laws design**

The FCLs for BEFCS adopts cascade structure and modularity concept [9], [11]. This concept proves to be advantageous during the project development phases as it provides clear flight control functions for each control loops, certain improvement can be executed locally to the specific control module, and it enables possibilities to add new flight control functions. The controller structure of FCLs for BEFCS is depicted in **Figure 2**.
**Figure 2.** The controller structure inside outer and inner loops for FCL of BEFCS [1].

*Figure 2* shows that the throttle substitute $\sigma_F$ is used individually to generate airspeed $V$ while the pitch angle $\Theta$ is utilized to excite the flight path angle $\gamma$. The other variables which are altitude $h$ and vertical speed $\dot{h}$ are derived from the design of flight path angle control. The airspeed control module is firstly designed and later followed by the design of flight path angle control module. The block diagram of airspeed and flight path angle control modules are displayed in *Figure 3* and *Figure 4*. The airspeed and flight path angle control modules originally consist of proportional and integral gains which are obtained from gain calculation. Later, through some tests and tunings, it is decided to change the proportional gain $K_{\Theta Y, P}$ from flight path angle control module to be the state feedback gain $k_{\Theta Y}$ (see *Figure 4*).

The new architecture is then proposed to combine the airspeed and flight path angle control modules. The throttle substitute $\sigma_F$ and the pitch angle $\Theta$ are simultaneously used to generate both airspeed $V$ and flight path angle $\gamma$. The altitude and vertical speed control modules are not modified because they are located outside the loop of flight path angle control module. By combining the airspeed and flight path angle control modules, the gains calculation is done at one time but later they are placed to their respective control module. The block diagram of the proposed longitudinal control is given in *Figure 5.*

**Figure 3.** Block diagram of airspeed control module [1].
Figure 4. Block diagram of flight path angle control module [1].

Figure 5 shows that the airspeed and flight path angle control modules are expanded so that each control module consists of 2 PI gains. Note that inner loops and aircraft block diagrams are not shown in that figure. The first gains, i.e. $K_{\sigma V}$ and $K_{\Theta V}$ are considered as the main gains as what previous design has shown (see Figure 3 and Figure 4.). The second gains, i.e. $K_{\Theta \sigma}$ and $K_{\sigma \Theta}$ are the coupling gains which are expected to improve the FCLs performance. The outputs and initial conditions are summed outside the control modules.

Figure 5. Block diagram of longitudinal control consisted of airspeed and flight path angle control modules.
The advanced PI control technique starts by creating extended system in form of state space equation. The state variables vector consists of the airspeed \( V \), flight path angle \( \gamma \), airspeed error \( e_V \) and flight path angle error \( e_\gamma \). The control variables vector consists of the pitch angle command \( \Theta_c \) and throttle substitute command \( \sigma_{P,c} \). By considering that there are no change of commands, \( i.e. \), \( \delta V_c = \delta \gamma_c = 0 \), then the control modules shown in Figure 5 can be considered to consist of state feedback gains. Subsequently, the Riccati design method can be used to solve the state feedback gains which later will be returned as PI gains. However, executing the advanced PI control technique with Multiple Input Multiple Output (MIMO) extended system will create difficulty during gains solving which finally may produce incorrect signs for the gains. These errors originated from natural behavior of airspeed \( V \) and flight path angle \( \gamma \) from pitch angle \( \Theta \) excitation. Positive changes of pitch angle \( \Theta \) will produce positive changes of flight path angle \( \gamma \) but negative changes of airspeed \( V \). Therefore, applying the advanced PI control technique with Single Input Single Output (SISO) extended system is most favorable and able to completely obtain gains with concise procedure.

The longitudinal FCLs design is based from 9 linear aircraft models whose trim points are listed in Table 1. The airspeed \( V \) and flight path angle \( \gamma \) responses are investigated by exciting pitch angle command \( \Theta_c \) and throttle substitute command \( \sigma_{P,c} \) consecutively. The simulation example is displayed in Figure 6. Here, red line is reference and trim point number is 1 (see Table 1). Subsequently, the airspeed \( V \) and flight path angle \( \gamma \) responses are approached with PT1 function and put in the same plot. The PT1 function in Laplace domain is written to be

\[
F = \frac{K}{T_1 s + 1}.
\]

Hence, the approximation is given to be

\[
d\dot{x} = F_{xu} \delta u.
\]

Where state variables \( x \) are airspeed \( V \) and flight path angle \( \gamma \), while the control variables \( u \) are pitch angle command \( \Theta_c \) and throttle substitute command \( \sigma_{P,c} \).

### Table 1. Trim points in different center of gravity locations \( x_{CG} \) and flight path speed \( V_K \).

| Trim Point | \( h[m] \) | \( m[kg] \) | \( x_{CG}[m] \) | \( V_K[km/h] \) |
|------------|------------|------------|----------------|----------------|
| 1          | 1000       | 1000       | -2.71          | 130            |
| 2          | 1000       | 1000       | -2.71          | 160            |
| 3          | 1000       | 1000       | -2.71          | 190            |
| 4          | 1000       | 1000       | -2.62          | 130            |
| 5          | 1000       | 1000       | -2.62          | 160            |
| 6          | 1000       | 1000       | -2.62          | 190            |
| 7          | 1000       | 1000       | -2.51          | 130            |
| 8          | 1000       | 1000       | -2.51          | 160            |
| 9          | 1000       | 1000       | -2.51          | 190            |

By deriving equation (1), relating with block diagram in Figure 5 and assuming that there is no command. The extended system in time domain can be written as

\[
\begin{bmatrix}
\delta \dot{x} \\
\delta \dot{e}
\end{bmatrix} = \begin{bmatrix}
-1/T_1 & 0 \\
-1 & 0
\end{bmatrix} \begin{bmatrix}
\delta x \\
\delta e
\end{bmatrix} + \begin{bmatrix}
K/T_1 \\
0
\end{bmatrix} \delta u.
\]

(3)
Figure 6. Example of airspeed and flight path angle responses for $1^\circ$ pitch angle excitation (left column) and 10% throttle substitute excitation (right column).

Equation (3) corresponds to ordinary differential state space equation

$$\delta \dot{x}_{ext} = A_{ext} \delta x_{ext} + b_{ext} \delta u_{ext}. \quad (4)$$

Block diagram in Figure 5 can now be seen as a state feedback system whose control variable $\delta u_{ext}$ can be solved to be

$$\delta u_{ext} = -k^T \delta x_{ext}, \quad (5)$$

$$\delta u_{ext} = -[k_1 \quad k_2] \begin{bmatrix} \delta x \end{bmatrix}, \quad (6)$$

$$\delta u_{ext} = -[K_p \quad -K_1] \begin{bmatrix} \delta x \end{bmatrix}. \quad (7)$$

The controllability check is done to determine if the control variable from the extended system is able to change state variables. The controllability matrix $Q_c$ is constructed and to fulfill the controllability, the rank of controllability matrix $Q_c$ must equal to the length of dynamic matrix $A_{ext}$. The definition of controllability matrix and the condition of controllability are written respectively as following

$$Q_c = [b_{ext} \quad A_{ext} b_{ext}]. \quad (8)$$

$$\text{rank}(Q_c) = \text{length}(A_{ext}). \quad (9)$$

Once the controllability is fulfilled then the FCLs calculation can be proceeded. The Riccati design method also known as optimal control linear quadratic is utilized to calculate longitudinal FCLs gains. It firstly needs to define the weighting matrices $Q$ and $R$ which are used in quadratic function $J = x^T Q x + u^T R u$. The matrix elements are chosen from the control variable excitation and state variable.
response which are used in PT1 approximation (see Figure 6). The weighting matrices $Q_n$ and $R_n$ for each trim point $n$ can be calculated using following relations

$$Q_n = \begin{bmatrix} 1/\delta x^2 & 0 \\ 0 & 1/\delta x^2 \end{bmatrix},$$

$$R_n = r_n = 1/\delta u^2.$$  

(10)

(11)

Riccati matrix $P$ is subsequently solved from Riccati equation $-\dot{P} = P A + A^T P - P B R^{-1} B^T P + Q = 0$. The extended state feedback gains can be computed to be

$$k_n^T = r_n^{-1} b_{ext}^T P_n.$$  

(12)

Extracting the PI gains from equation (12), the longitudinal FCL can be written to be

$$\begin{bmatrix} \Theta_c \\ \sigma_{F,c} \end{bmatrix} = \begin{bmatrix} K_{\theta v,P} + \frac{K_{\theta v,I}}{S} & K_{\theta y,P} + \frac{K_{\theta y,I}}{S} \\ K_{\sigma v v,P} + \frac{K_{\sigma v v,I}}{S} & K_{\sigma y v,P} + \frac{K_{\sigma y v,I}}{S} \end{bmatrix} \begin{bmatrix} V_c - V \\ \gamma_c - \gamma \end{bmatrix} + \begin{bmatrix} \Theta_{init} \\ \sigma_{F,init} \end{bmatrix}.$$  

(13)

3. Aircraft simulation

The design of longitudinal FCLs is implemented to the FCLs model which is done by means of Simulink/MATLAB (see Figure 7). The 00_Flight_Control_Panel block consists of FCP commands which are airspeed command $V_c$, altitude command $h_c$, vertical speed command $\dot{h}_c$, flight path angle command $\gamma_c$ and heading command $\Psi_c$. The 01_Flight_Control_Laws_Parameter block consists of center of gravity locations $x_{CG}$ parameter which is inputted prior simulation. The 02_Flight_Control_Laws consists of outer and inner loops blocks where certain control modules are located to command the aircraft (see Figure 2). The 03_Aircraft block consists of linear aircraft models which are also supplemented with outputs calculation.

![Figure 7. Simulink model for FCLs of BEFCS.](image)

The longitudinal FCLs design which is proposed like Figure 5 is implemented in the FCLs Simulink model. Numbers of test are prepared to verify the performance of the longitudinal FCLs. The tests include airspeed command $V_c$, flight path angle command $\gamma_c$ and altitude command $h_c$ in all trim points. The simulation is then compared between the new design and the previous design.
The simulation example in Figure 8 shows that airspeed acquire from coupled design (new longitudinal FCLs design) generates very low overshoot compared to the independent design (previous FCLs design). Here, red line is the reference and trim point number is 1 (see Table 1). The pitch angle $\Theta$ looks to correct the airspeed $V$ to reduce the overshoot. On the other hand, the flight path angle hold from coupled design produces higher deviation although the maximum deviation is less than 1°. Overall, the altitude and vertical load factor from both designs are considered undisturbed from the airspeed command.

The flight path angle command simulation example is given in Figure 9. Here, red line is the reference and trim point number is 1 (see Table 1). The coupled design has slower rise time but has relative faster settling time for the flight path angle compared to the independent design. The different performance does not affect the altitude as both designs has relatively at the same altitude changes. The coupled design produces low deviation of airspeed and settles faster compared to the independent design. The coupled design generates lower vertical load factor which is less than 0.1g. The simulation shows that the coupling commands produces better airspeed hold.

Figure 10 shows the simulation example for the altitude command. Both coupled and independent designs generate relatively the same altitude changes. Here, red line is the reference and trim point number is 1 (see Table 1). The differences in the flight path angle between coupled and independent design do not affect the altitude performance. The coupled design produces lower deviation of airspeed compared to the independent design. Additionally, the vertical load factor from the coupled design is lower compared than the independent design.
Figure 9. Example of flight path angle command simulation.

Figure 10. Example of altitude command simulation.
From three types of command test, i.e. airspeed, flight path angle and altitude command tests, it can be shown that the new longitudinal control design (coupled design) produces relatively different response for the respective commands including the vertical load factor. The throttle substitute $\sigma_f$ (in the chart it is always represented as throttle $\eta_f$ as they are the same) as the main input for the airspeed $V$ also acts to generate the flight path angle $\gamma$.

The pitch angle $\Theta$ which is the main input to generate flight path angle $\gamma$ also reduces the airspeed $V$. The coupled inputs from the longitudinal FCLs design reduces work of the main inputs. This affects the FCLs performance especially better performance in airspeed acquire and hold and the vertical load factor. The flight path angle acquire and hold are slightly lower but it does not relatively affect the altitude performance as both designs produce the relatively the same performance for altitude acquire and hold.

4. Conclusions
Some conclusions which can be drawn from the research work are as following:
1. The development of longitudinal flight control laws which considers coupling behavior between airspeed and flight path angle has been accomplished. The pitch angle and throttle substitute are the main inputs for flight path angle and airspeed respectively and are coupled inputs for airspeed and flight path angle respectively.
2. The procedure of developing longitudinal control has been described. The advanced proportional integral control technique is chosen with utilizing the Riccati design method as gains solver. The extended system is built by creating 1st order function approximation which is a benefit using single input single output approach.
3. In comparison with previous design, the simulation shows that the coupled longitudinal flight control laws slightly reduces performance of flight path angle acquire and hold, relatively maintains the performance of altitude acquire and hold, but significantly increases performance of airspeed acquire and hold and greatly reduces vertical load factor. By enhancing capability in airspeed control and vertical load factor reduction, the coupled design enables harder requirements to be defined for future flight control laws design.

Acknowledgement
The work of this research is realized through LSA-02 project funded by LAPAN.

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