Abstract. We report on work done by the Virgo consortium, an international collaboration set up in order to study the formation and evolution of Large Scale Structure using N–body simulations on the latest generation of parallel supercomputers. We show results of $256^3$ particle simulations of the formation of clusters in four Dark Matter models with different cosmological parameters. Normalizing the models such that one obtains the correct abundance of rich clusters yields an interesting result: The peculiar velocities of the clusters are almost independent of $\Omega$, and depend only weakly on $\Gamma$, the shape parameter of the power spectrum. Thus, it is nearly impossible to distinguish between high and low $\Omega$ models on the basis of the peculiar velocities.
1. Introduction

Clusters of galaxies are the largest objects one finds today in the Universe. They are well studied from both observational and theoretical viewpoints (for a recent review c.f. Bahcall (1996a)).

It may be possibile to distinguish between high and low Ω Universes by using the peculiar velocities of the clusters. On this basis Bahcall et al. (1996b) concluded that the observed clusters velocity function "is most consistent with a low mass-density (Ω ∼ 0.3) CDM model". However, we show in section 4 that this is dubious when models are normalized such that they all give the correct abundance of rich clusters. We present results from the simulations that support our theoretical considerations in section 5.

Before we talk about the details of the analysis of clusters from the simulations we give in section 2 a brief description of the numerical code. In section 3 we discuss the models used.

2. The Code

The Virgo consortium was formed in order to study the evolution of both Dark Matter and gas in the expanding Universe using the latest generation of parallel supercomputers.

The code used is called "Hydra" which was developed by Couchman et al. (1995) and parallelized by Pearce et al. (1995). Gravity is treated using an adaptive P^3M technique. This places mesh refinements on the regions of strongest clustering. Large refinements are done in parallel across the processors, smaller ones are farmed out to single processors.

The simulations were run on the Cray T3D supercomputers at the computer center of the Max Planck Society in Garching and at the EPCC in Edinburgh.

3. The Dark Matter Simulations

We have carried out a set of very large N–body simulations of CDM universes with four different choices of parameters. The models chosen were Standard CDM (SCDM), a high Ω model with an additional radiative component (τCDM), an Open CDM model (OCDM) and a flat low Ω model with a Cosmological Constant (ΛCDM). With the exception of the Open Model, which only had 200^3 particles, each simulation followed the evolution of 256^3 particles in a box of 239.5 h^{-1} Mpc on a side.

\(^1\)As usual we express the Hubble constant as \(H_0 = 100 \, h \, \text{km/(Mpc sec)}\).
TABLE 1. The Virgo models

| Model  | Ω   | Λ   | h   | σ₈  | Γ   |
|--------|-----|-----|-----|-----|-----|
| SCDM   | 1.0 | 0.0 | 0.5 | 0.51| 0.50|
| τCDM   | 1.0 | 0.0 | 0.5 | 0.51| 0.21|
| ΛCDM   | 0.3 | 0.7 | 0.7 | 0.90| 0.21|
| OCDM   | 0.3 | 0.0 | 0.7 | 0.85| 0.21|

In all models, the initial fluctuation amplitude was set by requiring that the models should reproduce the observed abundance of rich clusters (for details see section 4). Table 1 gives the parameters of the models.

Figure 3 shows the evolution of the same cluster$^2$ in the τCDM and the ΛCDM model for the redshifts 2, 1, and 0. Structure forms earlier in the low Ω universe. But already at a redshift of 2 a very large filamentary structure can be seen in the high Ω universe, too$^3$.

4. Peculiar Velocities of Galaxy Clusters from Linear Theory

According to linear theory, the mean–square peculiar velocity of galaxy clusters is

$$
\langle v_{3D}^2 \rangle = \frac{1}{2\pi^2} \frac{H_0^2}{3} \Omega^{1.2} \int_0^{\infty} P(k, \Gamma) W^2(k, R) dk,
$$

where $W^2(k, R) = \exp[-k^2 R^2]$ is a window function of radius $R$ (see, e.g., Croft & Efstathiou (1994)). The power spectrum $P(k)$ is taken in its parametric form introduced by Bond & Efstathiou (1984),

$$
P(k) = \frac{Bk}{(1 + [a k + (b k)^{3/2} + (c k)^2]^\nu)^{2/\nu}},
$$

where $a = (6.4/\Gamma) h^{-1}$ Mpc, $b = (3.0/\Gamma) h^{-1}$ Mpc, $c = (1.7/\Gamma) h^{-1}$ Mpc and $\nu = 1.13$. The quantity $\Gamma$ is given by

$$
\Gamma = \begin{cases} 
\Omega_0 h & \text{models with } \Omega_0 + \Lambda_0 = 1 \\
\Omega_0 h/[0.861 + 3.8(m_{10} \tau_d)^{2/3}]^{1/2} & \text{models with decaying } \nu
\end{cases}
$$

where $m_{10}$ is the neutrino mass in units of 10 keV and $\tau_d$ is its lifetime in years (Bond & Efstathiou (1991)). Equation (3) is valid for any model in a spatially flat universe.

$^2$The simulations were run with the same phases.

$^3$The clusters were taken from an additional set of runs with the same parameters as above but box sizes of $85 h^{-1}$ Mpc and $141 h^{-1}$ Mpc for the high and low Ω models, respectively.
Figure 1. The evolution of the same cluster in the $\tau$CDM (leftmost pictures) and in the $\Lambda$CDM model (rightmost pictures), for redshifts of 2 (top), 1 (middle), and 0 (bottom). The sizes of the regions shown are $21 \times 21 \times 8\ h^{-1}\ Mpc^3$ and $35 \times 35 \times 13\ h^{-1}\ Mpc^3$ for $\tau$CDM and $\Lambda$CDM, respectively.
The normalisation $B$ of the power spectrum can be obtained in two different ways: Either by using a relationship between $B$ and the COBE measurements (as shown in Efstathiou et al. (1992)) or by relating $B$ to the rms linear fluctuation in the mass distribution on scales of $8\,h^{-1}\text{Mpc}$, $\sigma_8$, which is defined by

$$\sigma_8^2 \equiv \frac{1}{(2\pi)^3} \int_0^{\infty} P(k, \Gamma) \left( \frac{3}{kR_8} j_1(kR_8) \right)^2 d^3k,$$

where $R_8 \equiv 8\,h^{-1}\text{Mpc}$, and $j_1$ is a spherical Bessel function.

Values for $\sigma_8$ were obtained by either using the mass or the X–ray temperature functions of rich clusters (White et al. (1993) (WEF), Viana & Liddle (1996), Eke et al. (1996) (ECF)). WEF obtain $\sigma_8 \approx 0.57\,\Omega_0^{-0.56}$, ECF get

$$\sigma_8 = \begin{cases} 
(0.52 \pm 0.04) \Omega_0^{0.46+0.10} & \text{for } \Lambda_0 = 0 \\
(0.52 \pm 0.04) \Omega_0^{-0.52+0.13} & \text{for } \Omega_0 + \Lambda_0 = 1
\end{cases}$$

with the quoted statistical uncertainties obtained using bootstrap methods.

Inserting eq. (4) into eq. (1) yields

$$\langle v_{3D}^2 \rangle = 10^4 \left( \Omega_0^{0.6} \sigma_8 \right)^2 f(\Gamma, R) \, [\text{km/sec}],$$

with the abbreviation

$$f(\Gamma, R) \equiv 4\pi h^2 \frac{\int P(k) W^2(k, R) dk}{\int P(k) \left( \frac{3}{kR_8} j_1(kR_8) \right)^2 d^3k}$$

Comparing eqs. (5) and (6) one sees that $(\Omega_0^{0.6} \sigma_8)^2$ only very weakly depends on $\Omega$. Thus, the peculiar velocities of galaxy clusters are nearly a function of $\Gamma$ alone.

Taking $R = 1.5\,h^{-1}\text{Mpc}$ one obtains

$$f(\Gamma, R = 1.5\,h^{-1}\text{Mpc}) = 12.5 \Gamma^{-1.08} + 49.4$$

5. Peculiar Velocities of Galaxy Clusters from the Simulations

When comparing N–body simulations with real data one has to find a way to select the clusters in the simulation. WEF find clusters by locating high–density regions with a friends–of–friends group finder, and then getting masses from the particle count within spheres of comoving radius $r = 1.5\,h^{-1}\text{Mpc}$. We use the same algorithm and treat objects with a mass larger than $5.5 \times 10^{14}h^{-1}\text{M}_\odot$ as galaxy clusters.
In table 2 we give the peculiar velocities for the four models from linear theory (first column, using eq.s (6) and (8)), from the simulations (second column), and the number of clusters found in the simulations (third column). Linear theory clearly predicts the simulation results to within their uncertainties.

From the numbers it is obvious that the peculiar velocities are not a good way to discriminate between the models if these are normalized such that they reproduce the correct abundance of rich clusters. In particular, there is no difference between high and low $\Omega$ models.

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