Conversed spin Hall conductance in two dimensional electron gas in a perpendicular magnetic field

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Using the microscopic theory of the conserved spin current [Phys. Rev. Lett. 96, 076604 (2006)], we investigate the spin Hall effect in the two dimensional electron gas system with a perpendicular magnetic field. The spin Hall conductance $\sigma_{\mu\nu}^{st}$ as a response to the electric field consists of two parts, i.e., the conventional part $\sigma_{\mu\nu}^{c0}$ and the spin torque dipole correction $\sigma_{\mu\nu}^{s\tau}$. It is shown that the spin-orbit coupling competes with Zeeman splitting by introducing additional degeneracies between different Landau levels at certain values of magnetic field. These degeneracies, if occurring at the Fermi level, turn to give rise to resonances in both $\sigma_{\mu\nu}^{c0}$ and $\sigma_{\mu\nu}^{s\tau}$ in spin Hall conductance. Remarkably, both of these two components have the same sign in the wide range of variation in the magnetic field, which result in an overall enhancement of the total spin Hall current. In particular, the magnitude of $\sigma_{\mu\nu}^{s\tau}$ is much larger than that of $\sigma_{\mu\nu}^{c0}$ around the resonance.

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Spintronics, which combines the basic quantum mechanics of coherent spin dynamics and technological applications in information processing and storage devices,\textsuperscript{4,5} has grown up to become a very active field in condensed matter physics. One central issue in spintronics is on how to generate and manipulate spin current as well as to exploit its various effects. In the ideal situation where spin is a good quantum number, spin current is simply defined as the difference between the currents of electron carried by spin-up and spin-down states. This concept of the spin current has served well in early studies of spin-dependent transport in metals and although the ubiquitous presence of spin-orbit coupling inevitably makes the spin non-conserved, this treatment on spin current sustains to be reasonable if the net effect of spin-orbit coupling is only considered as one source to spin relaxation. In recent years, however, it has been found that the extrinsic or intrinsic spin-orbit coupling can provide a route to generate transverse spin current in ferromagnetic metals\textsuperscript{6,7} or semiconductor paramagnets\textsuperscript{8,9} by the driving of an electric field. The fundamental question of how to define the spin current properly in the general situation then needs to be answered.\textsuperscript{10,11,12,13,14} In most of previous studies of bulk spin transport, it has been conventional to define the spin current simply as a combined thermodynamic and quantum-mechanical average over the symmetric product of spin and velocity operators. Unfortunately, the recent spin-accumulation experiments\textsuperscript{15,16,17} cannot directly verify it since there is no deterministic relation between this spin current and the boundary spin accumulation.

Recently, Shi et al.\textsuperscript{18} have established an alternative definition of spin current, which is given by the time derivative of the spin displacement (product of spin and position observable). This differs from the conventional definition by inclusion of one additional spin torque dipole term. As a result, the new spin transport coefficients $\sigma_{\mu\nu}^{st}$ have been shown to consist of two parts, i.e., the conventional part $\sigma_{\mu\nu}^{c0}$ and the spin torque dipole correction part $\sigma_{\mu\nu}^{s\tau}$. As one key consequence, the Onsager relation between $\sigma_{\mu\nu}^{st}$ and other kinds of force-driven transport coefficients has been shown\textsuperscript{19,18}. A general Kubo formula for the spin transport coefficients $\sigma_{\mu\nu}^{st}$ in terms of single-particle Bloch states has been given\textsuperscript{19,18} and applied to study the conserved spin Hall conductivity (SHC) in the two dimensional hole gas (2DHG). Also, the SHC based on the new spin current has been recently calculated\textsuperscript{20} for two dimensional electron gas (2DEG) with $k$-linear or $k$-cubic spin-orbit coupling and found to depend explicitly on the scattering potentials.

In this paper we study the conserved spin Hall effect of 2DEG with Rashba spin-orbit coupling in a perpendicular magnetic field. This system with the same setup has been studied recently by Shen et al.\textsuperscript{21} within the conventional spin current framework. They have found that the conventional SHC can be made resonant or even divergent by tuning the sample parameters and/or magnetic field $B$. The behavior of the spin torque dipole contribution, and subsequently the total conserved SHC remains yet to be exploited. As we will show in this paper, the torque term in the SHC can also be made resonant or divergent by tuning the magnetic field or the Rashba spin splitting. Moreover, we find that the resonant amplitude of torque term $\sigma_{\mu\nu}^{s\tau}$ is even more prominent than that of the conventional term $\sigma_{\mu\nu}^{c0}$. The oscillations and resonances in both of these two terms stem from energy crossing of different Landau levels near the Fermi level due to the competition between Zeeman energy splitting and spin-orbit coupling. Another fact we find is that different from the case without magnetic field, the spin Hall currents contributed from the conventional part and the spin torque dipole part flow in the same direction, which means an overall enhancement of the total conserved spin Hall current compared to the conventional spin Hall current.

We consider a 2DEG with the Rashba coupling in the $x$-$y$ plane of an area $L \times L$ subject to a perpendicular magnetic filed $B = -B \hat{z}$. The electrons are confined
between \(-L/2\) and \(L/2\) in the \(y\) direction by an infinite potential wall, and its wave function is periodic along the \(x\) direction. We choose the Landau gauge \(A = xB\hat{y}\).

The Hamiltonian for a single electron of spin-1/2 with a Rashba coupling is given by

\[
H_0 = \frac{\Pi^2}{2m} + \frac{\lambda}{\hbar} (\Pi_x \sigma_y - \Pi_y \sigma_x) - \frac{1}{2} g_s \mu_B B \sigma_z ,
\]

where the confining potential is implied. \(m\), \((-e)\), and \(g_s\) are the electron’s effective mass, charge and effective magnetic factor respectively, \(\mu_B\) is the Bohr magneton, \(\Pi = \vec{p} + e\vec{A}/c\) is the kinetic operator, \(\lambda\) is the Rashba coupling, and \(\sigma\) are the Pauli matrices.

In the presence of the Rashba spin-orbit coupling, the electron momentum \(p_x = \hbar k\) along the \(x\)-direction remains to be a good quantum number, thus in the Hilbert subspace of given \(k\), the Hamiltonian can be written as

\[
H_0(k) = \hbar \omega \left[ a_k |a_k \rangle + \frac{1 - g \sigma_z}{2} + \sqrt{2} \eta (i a_k \sigma_- - a_k^\dagger \sigma_+) \right] ,
\]

where \(\omega = eB/mc\) is the cyclotron frequency, \(\eta = \lambda m l_B/\hbar^2\) is the effective Rashba coupling, and \(g\) is the mass of a free electron and \(l_B = \sqrt{\hbar c/eB}\) the magnetic length. \([a_k, a_k^\dagger] = 1, [\sigma_\pm, a_k] = (\sigma_\pm + i \sigma_y)/2\) are the Pauli matrices. The energy spectrum of \(H_0(k)\) is given by

\[
\epsilon_{ns} = \hbar \omega \left( n + \frac{s}{2} \sqrt{(1 - g)^2 + 8m^2} \right) + \frac{1}{2} g \mu_B B \sigma_z ,
\]

with \(s = \pm 1\), for \(n \geq 1\); and \(s = 1\) for \(n = 0\). The corresponding two-component eigenstates for \(\epsilon_{ns}\) are given by

\[
|nks\rangle = \left( \begin{array}{c} \cos \theta_{ns} |n\rangle_k \\ \i \sin \theta_{ns} |n - 1\rangle_k \end{array} \right) ,
\]

where \(|n\rangle_k\) is the eigenstate of the \(n\)th Landau level in the absence of the Rashba interaction. For \(n = 0\), \(\theta_{01} = 0\), otherwise for \(n \geq 1\), \(\tan \theta_{ns} = -u_n + s \sqrt{1 + u_n^2}\), with \(u_n = (1 - g)/\sqrt{8m}\). The eigenstate \(|n, k, s\rangle\) has a degeneracy \(N_\phi = \ell^2 \epsilon cB/\hbar c\), corresponding to \(N_\phi\) quantum values of \(k\).

Now let us consider a uniform electric field \(E\) applied in the \(y\)-direction. The total Hamiltonian in this case is

\[
H = H_0 + eEy \text{ in which the term } eEy \text{ is usually treated as a small perturbation. Within the conserved spin current formalism, the spin current is defined as a time-derivative of the spin displacement operator, i.e., } J_\sigma = \frac{d(\hat{\mathbf{s}})}{dt} ,
\]

where \(\hat{\mathbf{s}}\) is the spin operator for a particular component (\(z\) here, to be specific), and \(\hat{\mathbf{r}}\) is the electron position operator. Compared to the conventional spin current operator, there has an extra term, \(\hat{\mathbf{r}}(d\hat{\mathbf{s}}_z/dt)\) in \(J_\sigma\), which accounts the contribution from the spin torque dipole. As a consequence, the conserved SHC \(\sigma^a_{xy}\) as a linear response to the external electric field defined by \(J_{s,xx} = \sigma^a_{xy} E\), includes two components,

\[
\sigma^a_{xy} = \sigma^{a0}_{xy} + \sigma^a_{xy} ,
\]

and for the conventional SHC, which is ready to be rewritten in the Landau spinor space (consisting of states \(|nks\rangle\)) as

\[
\sigma^{a0}_{xy} = -\frac{e \hbar \omega}{V} \sum_{\{n, s\} \neq \{n', s'\}, k} |f(\epsilon_{ns}) - f(\epsilon_{n's'})| \times \langle \epsilon_{nks} - \epsilon_{n'k's'} \rangle^2 + \delta^2 ,
\]

where the velocities are given by

\[
v_x = \frac{\partial H_0(k)}{\partial p_x} = \frac{p_x}{m} + \lambda \sigma_y = \frac{\omega \hbar}{\sqrt{2}} (a_k^\dagger + a_k) + \lambda \hbar \sigma_y ,
\]

\[
v_y = \frac{\partial H_0(k)}{\partial p_y} = \frac{p_y}{m} - \frac{\lambda \hbar}{\sqrt{2}} \sigma_x = \frac{\omega \hbar}{\sqrt{2}} (a_k^\dagger - a_k) - \lambda \hbar \sigma_x ,
\]

and \(f(\epsilon_{ns})\) is the equilibrium fermi function. The limit of \(\delta \to 0\) in Eq. (6) should be taken at the last step of calculation, and as a result, there is no intra-band \((\{n, s\} = \{n', s'\})\) contribution. The Kubo formula (9) for the conventional SHC has been used by most of previous investigations. The second component \(\sigma^a_{xy}\) in the conserved SHC (5) comes from the contribution of the spin torque dipole term. In the present context, its expression reads

\[
\sigma^a_{xy} = -\frac{e \hbar \omega}{V} \lim_{q \to 0} \frac{1}{q} \sum_{\{n, s\} \neq \{n',s'\}, k} |f(\epsilon_{nks}) - f(\epsilon_{n'k'q})| \times \langle \epsilon_{nks} - \epsilon_{n'k'q} \rangle^2 + \delta^2 ,
\]

where \(\tau_{n, q} = \frac{1}{2} \{\tau_n(k + q), v_y(k + q)\}\), \(v_y(k, q) = \frac{1}{2} [v_y(k) + v_y(k + q)], \) with \(\tau_n = \frac{1}{\hbar}[s_z, H_0].\) Note that to properly calculate \(\sigma^a_{xy}\) in practice, all the terms in Eq. (8) with the subscript \(k + q\) should be expanded at \(k\) to first order in \(q\).

By substitution of the expressions for single-particle velocities \(v_{x,y}\) and eigenstates \(|nks\rangle\), and after a tedious but straightforward derivation, we obtain the conventional part of conserved SHC as follows

\[
\sigma^{a0}_{xy} = -\frac{e}{4\pi} \sum_{n, s, s'} \frac{f(\epsilon_{ns}) - f(\epsilon_{n+1s'})}{(\epsilon_{ns} - \epsilon_{n+1s'})/\hbar \omega}[A(n, s, s')] ,
\]
where

$$A(n, s, s') = (n + 1) \cos^2 \theta_{ns} \cos^2 \theta_{n+1s'}$$

$$- n \sin^2 \theta_{ns} \sin^2 \theta_{n+1s'}$$

$$+ \frac{n}{\sqrt{2}} (\sqrt{n + 1} \cos^2 \theta_{ns} \sin 2\theta_{n+1s'}$$

$$- \sqrt{n} \sin^2 \theta_{n+1s'} \sin 2\theta_{ns}).$$

The other component contributed from the spin torque dipole term turns out to be given by

$$\sigma_{xy}^{\tau} = \frac{2e\eta}{\pi} \sum_{n, s, s'} \left( \frac{f(\epsilon_{ns}) - f(\epsilon_{n+1s'})}{(\epsilon_{ns} - \epsilon_{n+1s'})/\hbar\omega} \right) B(n, s, s')$$

$$+ \frac{2e\eta}{\pi} \sum_{n, s \neq s', k} \left( \frac{f(\epsilon_{ns}) - f(\epsilon_{ns'})}{(\epsilon_{ns} - \epsilon_{ns'})/\hbar\omega} \right) C(n, s, s'),$$

where

$$B(n, s, s') = [n \cos \theta_{ns} \sin \theta_{n+1s'}$$

$$- (n + 1) \sin \theta_{ns} \cos \theta_{n+1s'}]$$

$$\times \left( \sqrt{n(n+1)} \cos \theta_{ns} \cos \theta_{n+1s'}$$

$$+ n \sin \theta_{ns} \sin \theta_{n+1s'}$$

$$+ \sqrt{2n\eta} \cos \theta_{ns} \sin \theta_{n+1s'} \right)$$

and

$$C(n, s, s') = \sin (\theta_{ns} - \theta_{ns'}) \times \left( n^2 \cos \theta_{ns} \cos \theta_{ns'}$$

$$- (n - 1) \sqrt{n(n-1)} \sin \theta_{ns} \sin \theta_{ns'}$$

$$+ n\sqrt{2n\eta} \cos \theta_{ns} \sin \theta_{ns'} \right).$$

A similar expression for the conventional component $\sigma_{xy}^{0}$ has also been obtained by Shen et al.\cite{Shen}, while the spin torque dipole term $\sigma_{xy}^{\tau}$ is derived for the first time in this paper. The difference between $\sigma_{xy}^{0}$ and $\sigma_{xy}^{\tau}$ is obvious. Note that the first line in Eq. (11) is obtained by expanding $|n'k + qs'|$ in Eq. (8) at k to the first order in q while remaining other quantities to be their values at $q = 0$. Whereas the second line in Eq. (11) is obtained by a linear expansion of $|n'k + qs'|$ with respect to $q$. The other terms occurring in Eq. (8) turn out to take no contributions to $\sigma_{xy}^{\tau}$ for the present model.

From Eqs. (9) and (11), specially at zero temperature, one can see that if $|n, s|$ and $|n+1, s'|$ are both occupied or empty, then the contribution from these two states to the SHC is zero. Thus only the states near the Fermi level are important to the SHC; this happens in the situation that the energy-lower state $|n+1|$ is fully occupied while the upper state $|n+1,1⟩$ is partially occupied or fully empty. Due to the denominator in Eqs. (9) and (11), one can find that there will occur contributions at the degeneracies between the Landau levels $\epsilon_{n,1}$ and $\epsilon_{n+1,1}$. To show this, we first plot in Fig. 2 the Landau levels as a function of the effective Rashba coefficient $\eta$ for experimentally accessible In$_{0.53}$GaAs/In$_{0.52}$AlAs system\cite{Fujisawa}. Here the effective magnetic factor is taken to be $g = g_s m/2m_c = 0.1$. One can see that with increasing the amplitude of the effective Rashba coefficient $\eta$, the pair of states $|n, s = 1⟩$ and $|n+1, s' = −1⟩$ approach to close, implying resonances in conserved SHC at these level crossings. From expressions for Landau levels $\epsilon_{n,s}$, one can see that the resonant condition for conserved SHC is given by

$$\sqrt{(1 - g)^2 + 8\eta n^2} + \sqrt{(1 - g)^2 + 8(n + 1)^2} = 2,$$

where $2n \leq \nu < 2n+1$, $n = 0, 1, 2, \cdots$, and $\nu$ is the filling factor given by the ratio of the total electron number $N_e = \sum_{k \in \mathbb{K}} f(\epsilon_{nks})$ to the Landau level degeneracy factor $N_\nu$, i.e., $\nu = N_\nu / N_e$. Eq. (11) ensures that the state $|n,1⟩$ is fully occupied, while the state $|n+1,1⟩$ is not fully filled. Based on Eqs. (8)- (13) we have systematically calculated the SHC in a wide range of system parameters. Figure 3 plots $\sigma_{xy}^{s}$ and its two components as a function of inverse magnetic field (or the effective Rashba coupling coefficient $\eta$). One can see that there is a pronounced resonance at filling $\nu = 12.6$. At this filling the 13th Landau level (state $|n = 7, -1⟩$) is partially occupied, while the 12th Landau level (state $|n = 6, 1⟩$) is fully

FIG. 2: Electron Landau levels as a function of effective Rashba coupling $\eta = \lambda m_b / h^2$ for $g = 0.1$.

FIG. 3: (Color online). Conserved SHC and its two components versus $1/B$ at $T = 0$. The parameters are $\lambda = 9$ meV nm, $n_c = 1.9 \times 10^{-2} / \text{nm}^2$, $g_s = 4$, and $m = 0.05 m_c$, taken for the inversion heterostructure In$_{0.53}$GaAs/In$_{0.52}$Al$_{0.48}$As.
occupied. At the same time these two levels cross at \( \nu = 12.6 \), as shown in Fig. 2, therefore resulting in a giant resonance revealed in Fig. 3. Note that the resonant conditions are the same for the conventional term \( \sigma_{xy}^{0} \) and the spin torque dipole term \( \sigma_{xy}^{\tau} \), so one can find in Fig. 3 that the resonant peaks in \( \sigma_{xy}^{0} \) and \( \sigma_{xy}^{\tau} \) have the same location (at \( B = 6 \) T). Furthermore, it reveals in Fig. 3 that these two conductance components have the same sign in a wide range of magnetic field, and the resonant amplitude of \( \sigma_{xy}^{\tau} \) is even more larger than that of \( \sigma_{xy}^{0} \). As a result the total conserved SHC is prominently enhanced by the inclusion of the spin torque dipole term in. This result is different from that in the absence of the magnetic field, wherein it has been verified that the two components \( \sigma_{xy}^{0} \) and \( \sigma_{xy}^{\tau} \) always compete each other, giving the opposite contributions to the total spin Hall current\(^{32}\). Note that the small side peaks occurred in Fig. 3 reflect the variations in filling factor \( \nu \) as a function of the magnetic field.

We also calculate the conserved SHC by only varying the Rashba coefficient \( \lambda \) while the magnetic field remains unchanged. The result is shown in Fig. 4 for \( B = 6 \) T. The other parameters are set to be the same as used in Fig. 3. One can see that at \( \lambda = 9 \) meV nm, there occurs a resonance in both \( \sigma_{xy}^{0} \) and \( \sigma_{xy}^{\tau} \) with different peak amplitudes and line widths, the resonance behavior is even more prominent for \( \sigma_{xy}^{\tau} \). As a consequence, the resonant features of total conserved SHC is dominated by its spin torque dipole term. The resonance shown in Fig. 4 corresponds to the same case as Fig. 3 does, i.e., the 12th Landau level is fully occupied while the 13th Landau level is not fully filled. The difference is that in Fig. 4 the filling factor \( \nu \) remains unchanged when varying the Rashba coefficient. As a consequence, unlike what is shown Fig. 3, it reveals in Fig. 4 that there are no side peaks occurred.

In summary, we have studied the spin Hall effect in 2DEG system in a perpendicular magnetic field by using the conserved definition of spin current, which includes both the conventional part \( \sigma_{\mu
u}^{0} \) and the spin torque dipole correction term \( \sigma_{\mu
u}^{\tau} \). We have shown that the conserved SHC can be featured by giant resonances by tuning the amplitude of system parameters. Furthermore, it has been found that compared to the previous result of \( \sigma_{xy}^{0,21} \), the resonance features, including the height and line-width of the resonant peak, are even more prominent for \( \sigma_{xy}^{\tau} \). It is expected that the present results could have helpful implications on other aspects involving spin transport and spin accumulation in the presence of magnetic field.

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