A classical model for a photodetector in the presence of electromagnetic vacuum fluctuations

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Abstract

The main argument against the reality of the electromagnetic vacuum fluctuations is that they do not activate photon detectors. In order to meet this objection we propose a classical model of a photodetector which, in the simple case of a light signal with constant intensity, gives a counting rate which is a non-linear function of the intensity. For sufficiently large signal intensity, the counting rate is proportional to the intensity, in agreement with the standard quantum result, but there is a dark rate when the signal intensity is low.

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1 Introduction

The aim of this paper is to present a model of a photon counter resting upon classical electromagnetic (Maxwell’s) theory, but assuming that the whole space is filled with a random radiation field (zero-point field, ZPF). The ZPF is assumed to have an average energy $\frac{1}{2}h\nu$ per normal mode, $\nu$ being the frequency and $h$ Planck’s constant. Thus the ZPF corresponds precisely to the vacuum fluctuations of quantum electrodynamics, taken as a real field, and light signals would be radiation above that “sea” of ZPF. The reader may ask what is the use of the model which we propose because, according to the standard wisdom, classical Maxwell’s theory has been superseded by quantum electrodynamics and there exists already a well established quantum theory of photon detection (see, e.g. the book of Mandel[1]). In order to answer the question the present section will be attempt to explain the relevance of our model.

More than eighty years have elapsed since the discovery of quantum mechanics, but a vivid discussion still exists about its interpretation. The various interpretations may be roughly classified in two groups which could be called realistic and pragmatic. According to the pragmatic view, physics just consists of a set of rules for the prediction of the results of
experiments. It is assumed that the question whether these rules lead to a picture of the world does not belong to the domain of physics but rather to the realm of philosophy. This view was supported by Niels Bohr and gave rise to the “Copenhagen interpretation”, which has been accepted by the mainstream of the scientific community until recently. In contrast, the realistic view claims that the essential aim of natural sciences, physics in particular, is to provide knowledge about the material world, the possibility of predicting the results of experiments being a by-product. Of course it is an important by-product, because on it rests the criterion for the validity of the theory, that is the agreement of the predictions with the empirical results. The realistic view was maintained by several of the founding fathers, in particular Albert Einstein, and we strongly support it. Actually the realistic interpretation follows the tradition of all sciences, not only physics, until the advent of quantum mechanics. If the Copenhagen interpretation has had such a big success it is because people have failed to find a general and coherent realistic interpretation of quantum mechanics.

The difficulty for a realistic interpretation of quantum mechanics is that it seems incompatible with the quantum formalism, at least if realism is combined with the demand of locality, an incompatibility proved by John Bell more than forty years ago[2]. During the eighty years of life of quantum mechanics very many of its predictions have been tested and the agreement between theory and experiments has been spectacular, especially in quantum electrodynamics. Thus it is not strange that most people are so fond of quantum mechanics that Bell’s theorem has been taken as a proof of the impossibility of a local realistic theory of the world, even without the need of any additional empirical support. This prejudice has produced a bias in the analysis of performed experiments, leading to the current wisdom that local realism has been empirically refuted, modulo a few irrelevant loopholes[4]. However, the truth is that all experiments performed till now have had results compatible with local realism and the alleged empirical disproof rests upon some unjustified extrapolations[3]. Of course quantum mechanics has not been violated, so that local realism and quantum mechanics are compatible for all experiments performed till now. Therefore it is an open question whether they are compatible for all experiments which can be actually performed. If this is true, then some of the assumptions needed to prove Bell’s theorem are flawed. This means that either the quantum formalism or, most probably, the quantum measurement theory should be modified.

The goal of devising a modification or alternative to quantum mechanics compatible with local realism (and the experiments) is not easy. A possible procedure is to begin with some theory which: a) provides from the start a clear picture of the natural world, and b) allows an interpretation of the experiments within some restricted domain. The said theory should be generalized later in order to cover a wider domain. A theory of this type is stochastic electrodynamics (SED)[5] whose basic idea is that quantum behaviour is related to the existence of a real ZPF.

The existence of vacuum fluctuations is a straightforward consequence of field quantization[6]. In addition, quantum vacuum fluctuations have consequences which have been tested empirically. For instance, the vac-
uum fluctuations of the electromagnetic field give rise to the main part of the Lamb shift \[7\] and to the Casimir effect \[8\]. Thus Planck’s radiation law should be written in the form

$$\rho(\omega, T) = \frac{\omega^2}{\pi^2 c^3} \left( \frac{\hbar \omega}{\exp (\hbar \omega / k_B T) - 1} + \frac{1}{2} \hbar \omega \right) = \frac{\hbar \omega^2}{\pi^2 c^3} \coth \frac{\hbar \omega}{k_B T},$$

(1)

where the second term represents the ZPF. That the thermal spectrum contains an $\omega^3$ term has been proved by experiments measuring current fluctuations in circuits with inductance at low temperature \[9\]. Of course, the term is ultraviolet divergent so that some cutoff should be assumed, possibly at about the Compton wavelength, where the fluctuations of the Dirac electron-positron sea become important. The ZPF of SED is identical with these quantum vacuum fluctuations, taken as a real stochastic field.

The standard wisdom is that the vacuum fluctuations cannot be interpreted as a real random electromagnetic field because they do not activate photodetectors in the absence of signals. There is also a gravitational problem because, if the quantum vacuum fluctuations are at the origin of the cosmological constant, as is usually assumed, that constant should be many orders of magnitude larger than the observed value, but we shall not be concerned with gravitational effects in this paper. A common explanation of the fact that the zeropoint field (ZPF) does not activate photodetectors is to say that the ZPF is not real, but virtual. In our opinion replacing a word, real, by another one, virtual, with a less clear meaning is not a good solution. In the present article we shall prove that the behaviour of photodetectors can be explained without renouncing the reality of the ZPF. The proof goes via constructing an explicit model of detector, producing a counting rate proportional to the intensity of the signal, that is also able to subtract efficiently the ZPF. The mere existence of the model proves that the said objection to the ZPF reality is untenable. Such a proof is the first purpose of the present paper.

In addition we think that a classical model of detection may add to understand the origin of an open problem lasting for more than 30 years, namely the existence of a detection loophole. In fact, as is well known all experiments aimed at a discrimination between quantum mechanics and local realism, via the test of Bell’s inequalities using optical photons, suffer from a loophole due to the lack of efficient detectors. The standard wisdom is that the difficulty in manufacturing efficient photon counters is a minor technical problem. However the persistence of the difficulty for so many years may indicate that the problem is a fundamental one. The comparison of the classical model that we propose with the quantum detection theory may throw light on the problem, and this is another purpose of the present paper.

2 Stochastic properties of the zeropoint field

Now we shall derive some relevant properties of the ZPF in free space, that is far from any material body. The ZPF is characterized by the electric
field $\mathbf{E}(r, t)$, and the magnetic field $\mathbf{B}(r, t)$. The stochastic properties of the field may be summarized saying that it is Gaussian, so that the mean and the correlation functions are sufficient to characterize all stochastic properties. The mean is zero and the correlation functions of the field components are \cite{10, 11}

\begin{align}
\langle E_i(r, t)E_j(r', t') \rangle &= \int d^3k \left( \delta_{ij} \frac{k_i k_j}{k^2} \right) \frac{n\omega}{4\pi^2} \cos \left[ k \cdot (r - r') - \omega (t - t') \right], \\
\langle B_i(r, t)B_j(r', t') \rangle &= \langle E_i(r, t)E_j(r', t') \rangle, \\
\langle E_i(r, t)B_j(r', t') \rangle &= \int d^3k \varepsilon_{ijl} \frac{n\omega}{k^2} \cos \left[ k \cdot (r - r') - \omega (t - t') \right],
\end{align}

where $k = |k| = \omega/c$. The integrals in these expressions do not converge but a natural cut-off in $\omega$ appears as explained in the following. Our aim is to make a photodetection model and we may assume that any detection event takes place essentially at a single atom. Thus the quantity of interest for us will be the time autocorrelation of the averages of the fields over the volume of the atom. We define the averaged quantities

\begin{equation}
E_j(t) \equiv \int d^3r E_j(r, t) \rho(r), \quad B_j(t) \equiv \int d^3r B_j(r, t) \rho(r),
\end{equation}

where no confusion should arise by the use of the same label for the field at a point and the averaged quantity. The function $\rho(r)$ represents a normalized effective electron density in the atom, which I assume spherically symmetrical. From eqs. \eqref{2} it is straightforward to get the time correlations of the averaged quantities and we get

\begin{equation}
\langle E_j(t)E_k(t') \rangle = \langle B_j(t)B_k(t') \rangle = \delta_{jk} F(t - t'), \quad \langle E_j(t)B_k(t') \rangle = 0,
\end{equation}

where

\begin{equation}
F(\tau) = \frac{32\pi n}{3c} \int_0^\infty \omega \cos (\omega \tau) d\omega \left[ \int_0^\infty \sin (\omega r/c) \rho(r) r dr \right]^2.
\end{equation}

For any reasonable density, $\rho(r)$, the function $F(\tau) = F(-\tau)$ decreases slowly for $|\tau| \lesssim a/c$ and rapidly for higher $\tau$, becoming negligible for $|\tau| >> a/c$, $c$ being the velocity of light and $a$ the atomic radius.

For our detection model the most relevant quantity is the radiation intensity represented by the Poynting vector,

\begin{equation}
\mathbf{S}(t) = \frac{c}{4\pi} \mathbf{E}(t) \times \mathbf{B}(t),
\end{equation}

where $\mathbf{E}(t)$ and $\mathbf{B}(t)$ are the averages over the volume of the atom defined in \eqref{3}. The mean of the Poynting vector is zero and the correlation functions of its components may be derived from \eqref{4} to be

\begin{equation}
\langle S_j(t)S_k(t') \rangle = \frac{c^2}{8\pi^2} \delta_{jk} F \left( t' - t \right)^2 \simeq \sigma^2 \delta_{jk} \delta \left( t' - t \right),
\end{equation}

where $\delta_{jk}$ is Kronecker’s delta, $\delta \left( t' - t \right)$ is Dirac’s delta and $\sigma$ is a constant of the order of $10^{-3} e^{3/2} a^{-7/2}$. The second equality involves approximating every component of the Poynting vector by a white noise, which
is plausible provided we study detection of light signals whose coherence time, $\tau$, is much larger than the time interval where $F(\tau)^2$ is not negligible, that is $\tau >> a/c$. These properties characterize the components of the Poynting vector as three stationary stochastic processes, statistically independent and having the autocorrelation of a white noise.

Actually we are interested in the ZPF within a photon detector and the question is whether we may use there the statistical properties (6) of free space. Our answer is in the affirmative because we should assume that the ZPF permeates everything. Indeed in SED it is supposed that the vacuum fluctuations of the electromagnetic field - and possibly other fields also - is the cause of the random position of the electron inside the atom. In the standard interpretation of quantum mechanics it is also assumed that the vacuum fields fluctuate even inside material bodies, with an average energy $\frac{1}{2}h\nu$ per normal mode, although the modes are modified by the presence of matter (this gives rise, for instance, to the Casimir effect and the Lamb shift as mentioned above). However the radiation modes which are most modified by the presence of matter are those of low frequency, much lower than the ones most strongly involved in $F(\tau)$, eq.(5). Thus we may use the free space statistical properties (6) in our model.

We shall study the situation where we have a determinate light signal superimposed on the ZPF. The statistical independence of signal and random fields leads to (compare with (6))

$$\langle S_j(t) \rangle = \delta_{j3} I_s, \; \langle S_j(t)S_k(t') \rangle = \delta_{j3}\delta_{k3} I_s^2 + \sigma^2 \delta_{jk} \delta(t'-t), \quad (7)$$

where $S_3$ is the component of the Poynting vector in the direction of the beam and $I_s$ the signal intensity. Eq.(7) will be the basis of our subsequent study but we should bear in mind that the approximations involved may be too crude for some applications. Possible improvements will be considered elsewhere.

### 3 Detection model

Several models of photon counter have been proposed in the context of SED, resting upon the idea that there exists a “detection time”, $T$, independent of the light intensity and such that the probability of a count depends on the radiation, including the ZPF, which enters the detector during the time $T$[12]. Those models, however, are not compatible with empirical evidence[13]. Instead of fixing the detection time $T$, here we shall assume that every atom accumulates the energy and momentum of the radiation arriving at it and a count is produced whenever that energy surpasses some threshold. If $\mathbf{S}(t)$ is the Poynting vector of the radiation arriving at the detecting atom at time $t$, the accumulated energy at time $T$ will be

$$E(T) = A \int_0^T |\mathbf{S}(t)| dt,$$

where $A$ is the effective cross section of the atom. In the following we shall put $A = 1$ in calculating the detection rate and multiply times $A$ at the end.
The essential assumption of our model is that a detection event is produced at a time $T$, after the previous count, when $T$ is such that

$$E(T) = |E(T)| = E_m, \ E(t) \equiv \int_0^t S(t')dt'$$

and $E$ is a parameter characteristic of the detector.

The calculations resting upon the model (9) are involved due to the fluctuations of the ZPF and the signal. Indeed constructing a detailed detection model on the basis of that equation would require using the theory of “first passage time” for the (vector) stochastic process $E(t)$, which has a finite, nonzero, correlation time. The problem is dramatically simplified if we assume that every cartesian component of $E(t)$ is a white noise (having a null correlation time) superimposed on a deterministic signal with constant intensity $I_s$, as in eq.(7). More specifically, we assume that every component of the stochastic process $E(t)$ is a Wiener (Brownian motion) process. The calculation of the first-passage time is now straightforward. We must begin solving the diffusion equation

$$\frac{\partial \rho}{\partial t} = \frac{1}{2} \sigma^2 \left(\frac{\partial^2 \rho}{\partial E_x^2} + \frac{\partial^2 \rho}{\partial E_y^2} + \frac{\partial^2 \rho}{\partial E_z^2}\right) - I_s \frac{\partial \rho}{\partial E_z}, \ \rho = \rho(E,t), \quad (10)$$

with the initial condition

$$\rho(E,0) = \delta^3(E) \equiv \delta(E_x) \delta(E_y) \delta(E_z), \quad (11)$$

and an absorbing barrier at $|E| = E_m$, that is the boundary condition

$$\rho(E,t) = 0 \ \text{for} \ |E| = E_m. \quad (12)$$

The solution of eq.(10) with these conditions is cumbersome due to the fact that the symmetry properties of the boundary condition, eq.(12), are different from those of the diffusion eq.(10) itself. In order to simplify the calculations, without changing qualitatively the model, we substitute for eq.(12) the following

$$\rho(E_x, E_y, E_z, t) = 0 \ \text{for either} \ E_x = \pm E_m \ or \ E_y = \pm E_m \ or \ E_z = \pm E_m. \quad (13)$$

This means replacing a sphere in the 3D space of the vectors $E$ by a cube. Thus our problem may be reduced to solving 3 one-dimensional equations by introducing the functions $f_x, f_y, f_z$ such that

$$\rho(E_x, E_y, E_z, t) = f_x(E_x, t) f_y(E_y, t) f_z(E_z, t).$$

Indeed the solution of eq.(10) with conditions eqs.(11) and (13) may be obtained from the solution of the three ordinary differential equations

$$\frac{\partial f_x}{\partial t} = \frac{1}{2} \sigma^2 \frac{\partial^2 f_x}{\partial E_x^2}, \ \frac{\partial f_y}{\partial t} = \frac{1}{2} \sigma^2 \frac{\partial^2 f_y}{\partial E_y^2}, \ \frac{\partial f_z}{\partial t} = \frac{1}{2} \sigma^2 \frac{\partial^2 f_z}{\partial E_z^2} - I_s \frac{\partial f_z}{\partial E_z}, \quad (14)$$

with initial and boundary conditions, respectively,

$$f_x(E_x, 0) = \delta(E_x), \ f_x(E_m, t) = f_x(-E_m, t) = 0,$$
and similar for $f_y$ and $f_z$. The appropriate solutions of eqs.(14) may be obtained by the method of the images and we get (we shall put $\sigma^2 = 2$ in the calculation and restore $\sigma$ in the final result)

$$f_z(E_z,t) = \frac{1}{\sqrt{4\pi t}} \exp \left[ -\frac{1}{2} E_z I_z \right] \sum_{n=-\infty}^{\infty} (-1)^n \exp \left[ -\frac{(E_z + 2nE_m)^2}{4t} \right]$$

and similar expressions, except replacing $I_s$ by 0, for $f_x (E_x, t)$ and $f_y (E_y, t)$.

Now our model predicts that, if we have a detection event at time $t = 0$, the probability that the next detection event takes place before time $t$ is

$$P(t) = 1 - \int_{-E_m}^{E_m} f_x (E_x, t) dE_x \int_{-E_m}^{E_m} f_y (E_y, t) dE_y \int_{-E_m}^{E_m} f_z (E_z, t) dE_z.$$  

(15)

Our aim is to calculate the detection rate, which is the inverse of the mean first passage time, that is

$$<t> = \int_0^\infty t \frac{dP(t)}{dt} dt.$$  

(17)

The proof that this average gives the inverse of the detection rate is as follows. We consider that the detector is active during a very long time interval. Within it we will have a large number of detection events on each atom. Let us assume, for the sake of clarity, that the time intervals between two detection events form a discrete sequence $t_1, t_2, ..., t_j, ...$. If we have $N_j$ time intervals of duration $t_j$ then the detection rate will be

$$R = \frac{\sum N_j}{\sum N_j t_j} = \frac{1}{\sum P_j t_j} = \frac{1}{<t>}.$$  

(18)

where $P_j$ is the probability that a time interval between two detection events has duration $t_j$. If we pass to the continuous, we shall replace the summation by an integral, giving a rate $R$ equal to the inverse of $<t>$, which completes the proof. The counting rate in a macroscopic detector consisting of many atoms may be obtained from the predicted rate in one atom but this will not be made in the present article.

According to eqs.(16) and (17) the mean “first passage time” is given by

$$<t> = \int_0^\infty dt L K^2,$$  

(19)

where $L \equiv \int_{-E_m}^{E_m} f_x (E_x, t) dE_x$ and $K \equiv \int_{-E_m}^{E_m} f_y (E_y, t) dE_y$.

Hence the calculation is straightforward putting eqs.(14) in eq.(19) and performing the integrals. But before proceeding at this calculation we will work a one-dimensional model consisting of the replacement of $K(t)$ by unity, which corresponds to substituting a layer for the atom. As we shall see that model gives predictions not too different from the three-dimensional one and it has the advantage of leading to a simple analytical...
expression for the detection rate. From eqs. (15) and (19) we obtain, making the time integral first,

\[ < t > = \int_0^\infty dtL = \int_{-\infty}^{\infty} dE \frac{1}{\sqrt{4\pi}} \exp \left[ \frac{1}{2} EI_s \right] \sum_{n=-\infty}^{\infty} (-1)^n J_n, \]  

(20)

\[ J_n = \int_0^\infty dt \frac{1}{\sqrt{t}} \exp \left[ -\frac{(E + 2nE_m)^2}{4t} - \frac{I_s^2 t}{4} \right] = \sqrt{\frac{1}{4\pi I_s}} \exp \left[ -\frac{1}{2} I_s |E + 2nE_m| \right], \]

where the integral has been taken from Gradshteyn[15]. In order to perform the sum in \( n \) it is convenient to separate terms with \( n = 0 \), \( n > 0 \) and \( n < 0 \), giving

\[ \text{Sum} = \exp \left[ -\frac{1}{2} I_s |E| \right] - 2 \exp[I_s E_m]^{-1} \cosh \left[ \frac{1}{2} I_s E \right]. \]

(21)

The integral in \( E \) is now easy and we obtain

\[ < t > = \frac{E_m}{I_s} \tanh \left( \frac{1}{2} I_s E_m \right) \]

(22)

and, restoring the parameter \( \sigma \),

\[ R = \frac{1}{< t >} = \frac{I_s E_m}{\sigma \cosh \left( \frac{I_s E_m}{\sigma^2} \right)} \]

(23)

which has the limits

\[ R \approx \frac{I_s E_m}{\sigma^2} \left( 1 + 2 \exp \left( -\frac{2I_s E_m}{\sigma^2} \right) \right) \text{ for } I_s \gg \frac{\sigma^2}{E_m}, \]

\[ R \approx \frac{\sigma^2}{E_m^2} \text{ for } I_s \ll \frac{\sigma^2}{E_m}. \]

(24)

In order to solve the 3D model we begin by calculating

\[ K(t) = \frac{1}{\sqrt{4\pi t}} \int_{-\infty}^{E_m} dE_x \sum_{n=-\infty}^{\infty} (-1)^n \exp \left[ -\frac{(E_x + 2nE_m)^2}{4t} \right] \]

\[ = \frac{1}{\sqrt{4\pi t}} \sum_{n=-\infty}^{\infty} \int_{(2n-1)E_m}^{(2n+1)E_m} (-1)^n \exp \left( -\frac{s^2}{4t} \right) ds, \]

(25)

where the last expression is obtained by means of a trivial change of variable. We see that \( K(t) \) is the integral of a Gaussian times a function, \( F(s) \), taking alternately the values +1 and -1 in intervals of length \( 2E_m \).

We will perform the integral using the Fourier expansion of this function, that is

\[ F(s) = \sum_{k=0}^{\infty} \frac{4}{(2k+1)^2} \cos \left( \frac{(2k+1)\pi s}{2E_m} \right), \]

whence we get

\[ K(t) = \frac{4}{\pi} \sum_{k=0}^{\infty} (-1)^k \int_{-\infty}^{\infty} \cos \left( \frac{(2k+1)\pi s}{2E_m} \right) \exp \left( -\frac{s^2}{4t} \right) ds = \sum_{k=0}^{\infty} K_k(t), \]

\[ K_k(t) = \frac{4}{\pi} \frac{(-1)^k}{2k+1} \exp \left( -\frac{(2k+1)^2 \pi^2 t}{4E_m^2} \right). \]

(26)
After that we have, from eqs. (26) and (19),
\[
< t > = \int_0^\infty \frac{dt}{\sqrt{4\pi t}} K(t)^2 \int_{-E_m}^{E_m} dE \exp \left[ \frac{1}{2} EI_s \right] \sum_{n=\infty}^{\infty} \frac{(-1)^n}{\sigma} \exp \left[ -\frac{(E + 2nE_m)^2}{4t} - \frac{I_k^2 t}{4} \right].
\]

(27)

Making the time integral first we get (compare with eq. (20))
\[
< t > = \frac{16}{\pi^2} \sum_k \sum_{l=0}^{\infty} \int_{-E_m}^{E_m} dE \frac{1}{\sqrt{4\pi}} \exp \left[ \frac{1}{2} EI_s \right] \sum_{n=\infty}^{\infty} \frac{(-1)^{n+k+l}}{2(2k+1)(2l+1)} J_{nk1},
\]

(28)

where
\[
J_{nk1} = \int_0^\infty dt \frac{1}{\sqrt{t}} \exp \left[ -\frac{(E + 2nE_m)^2}{4t} - \frac{I_k^2 t}{4} \right] \exp \left[ \frac{1}{2} I_k |E + 2nE_m| \right].
\]

(29)

The sum in \( n \) is similar to that in (21) with \( I_k \) substituted for \( I_s \), the integrals in \( E \) are straightforward and we obtain
\[
< t > = \frac{64}{\pi^2} \sum_k \sum_{l=0}^{\infty} \frac{(-1)^{k+l}}{(2k+1)(2l+1) \cosh \left( \frac{1}{2} I_k E_m \right)} \left[ \frac{1}{\sigma} \frac{\cosh \left( \frac{1}{2} I_k E_m \right)}{\cosh \left( \frac{1}{2} I_k E_m \right)} \right]
\]

\[
= \frac{128 F^2}{\pi^2 \sigma^2} F \left( \frac{I_k E_m}{\sigma^2} \right),
\]

\[
F(x) = \sum_k \sum_{l=0}^{\infty} \frac{(-1)^{k+l}}{(2k+1)^2 + (2l+1)^2} G(x),
\]

\[
G(x) = 1 - \frac{\cosh x}{\cosh \sqrt{x^2 + \frac{1}{4} \pi^2 [(2k+1)^2 + (2l+1)^2]}},
\]

(30)

where we have restored the parameter \( \sigma \) (see eq. (15)). The function \( F(x) \) is defined by an infinite series which should be calculated numerically. Here we shall make only the calculation for large and small values of \( x \).

For \( x \gg 1 \) we may write (using the approximation \( \cosh \alpha \approx \frac{1}{2} \exp \alpha \), valid for \( \alpha \gg 1 \))
\[
G(x) \approx 1 - \exp \left\{ x - \sqrt{x^2 + \frac{1}{4} \pi^2 [(2k+1)^2 + (2l+1)^2]} \right\}
\]

\[
\approx 1 - \exp \left\{ -\frac{\pi^2 [(2k+1)^2 + (2l+1)^2]}{8x} + O \left( \frac{1}{x^3} \right) \right\}. \quad (31)
\]

Hence it is easy to get \( F(x) \) for large \( x \), that is
\[
F(x) \sim \frac{\pi^2}{8x} \sum_k \sum_{l=0}^{\infty} \frac{(-1)^{k+l}}{(2k+1)(2l+1)} = \frac{\pi^2}{8x} \left( \sum_k \frac{(-1)^k}{(2k+1)} \right)^2 = \frac{\pi^4}{128x}.
\]

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whence we obtain

\[ < t > = \frac{E_m}{I_s} + O \left( \frac{\sigma^2}{I_s^2} \right) \rightarrow R \simeq \frac{I_s}{E_m}, \]

which is similar to the result of the one-dimensional model but the deviation from the standard quantum result decreases as \( 1/I_s^2 \) instead of exponentially (see eq. (24)).

For \( x = 0 \) we get, from eqs. (30) and (29),

\[ F(0) = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{(-1)^{k+l} \left( 1 - 1/\cosh \left( \sqrt{\frac{(2k+1)^2 + (2l+1)^2}{2}} \right) \right)}{(2k+1)(2l+1)(2k+1)^2 + (2l+1)^2}. \]

The series converges quickly and a numerical calculation gives

\[ < t > \simeq 0.49 \frac{E^2}{\sigma^2} \rightarrow R \simeq 2.0 \frac{\sigma^2}{E_m}, \]

that is a rate at zero intensity about twice that in the one-dimensional model, eq. (24).

It is remarkable that we obtain a detection rate proportional to the signal intensity, that is a perfect subtraction of the ZPF, when the intensity of the signal is big, a result in agreement with the quantum mechanical prediction. However for low, or nil, intensity signals there is some counting rate, which we should interpret as a “fundamental dark rate” of the detector.

The result may be generalized to the case where the signal intensity is not a constant, but a known function of time. It would be enough to substitute \( \int_{t_0}^{t} I_s(t') dt' \) for \( I_s(t) \) in the above equations, although the solution of the differential equation (10) would be more involved. More difficult would be to treat the case where the signal itself fluctuates (with a correlation time of the order of the inverse of the frequency bandwidth). We shall not study these problems here.

We might also analyze coincidence counts in two detectors when the incoming beams, with intensities \( I_1(t) \) and \( I_2(t) \) above the ZPF, are correlated. The calculation would be straightforward, although lengthy. We may assume that eq. (24) is still valid for each detector and the coincidence rate, with a time delay \( \tau \), will be

\[ R_{12} \propto \langle I_1(t)I_2(t + \tau) \rangle, \]

again in agreement with the quantum prediction. However the current situation may not be that. In practice the signals may be stochastic and not independent of the ZPF. The calculation in these conditions would be rather involved.

4 Discussion

Our analysis shows that quantum vacuum fluctuations of the electromagnetic field (or ZPF) may be efficiently subtracted by a model which assumes that the radiation is a classical (Maxwell) field including a fluctuating ZPF, provided that the fluctuations of the signal have a large enough
correlation time in comparison with the effective correlation time of the
ZPF. This is usually the case in astronomical observations. In contrast, in
standard quantum optical experiments the fluctuations of the signal may
have a rather short correlation time. If the correlation time of the signal
does not fulfil the assumptions of the previous section, the presence of the
ZPF will probably give rise to departures from the standard quantum
predictions eqs. (23) and (32), that is they will produce some nonidealities
in the behaviour of optical photon counters. This is specially important
when it is necessary to measure coincidence counting rates with short time
windows, as is frequent in quantum optical experiments (e.g. optical tests
of Bell’s inequality). If this is the case, our approach may provide an
explanation for the difficulties of performing loophole-free tests of Bell’s
inequality using optical photons.

I emphasize that, although our model is semiclassical, probably its
main properties might be reproduced by a rigorous quantum treatment.
Furthermore the difficulties for reaching an intuitive picture of how detec-
tors subtract the ZPF probably do not derive from quantum theory itself,
but from the use of approximations like first-order perturbation theory or
taking the limit of time $t \to \infty$ in calculating the probability of photon
absorption per unit time. Indeed I conjecture that excessive idealizations
might be at the origin of the difficulties for understanding intuitively the
paradoxical aspects of quantum physics. Although simplifications are ex-
tremely useful for calculations, they tend to obscure the physics.

5 Conclusion

We want to compare our rate prediction

$$R_m = \frac{1}{< t >} = \frac{I_s}{E_m} \coth \left( \frac{I_s E_m}{\sigma^2} \right),$$

with the quantum prediction

$$R_q = \eta K I_s,$$

where $\eta$ is the quantum efficiency and $K$ is a dimensional constant taking
into account the energy of one “photon”, the effective cross section of an
atom and the number of atoms in the detector. The two predictions are
identical for high intensity provided we identify

$$E_m = \frac{1}{K\eta},$$

But for low intensity and/or high efficiency there is a departure. For
instance the departure becomes more than 10% (that is the “fundamental
dark rate” is more than 10% of the total rate) when $I_s E_m/\sigma^2 < 1.5$. Using
(35) this gives

$$I_s < 1.5 K \sigma^2 \eta,$$

that is when the intensity is too low and/or the quantum efficiency too
high. As the tests of Bell’s inequalities requires both high efficiency and
low intensity our model may explain the difficulty, or maybe impossibility
of such tests. A more quantitative result would require an estimate of $K$,
which is not easy.
6 Appendix

6.1 Evaluation of $F(\tau)$ with $\rho(r) = (8\pi a^3)^{-1} \exp(-r/a)$

The density $\rho(r)$ is normalized in the sense $\int_0^\infty \rho(r) 4\pi r^2 dr = 1$.

We start calculating

$$\int_0^\infty \sin(\omega r/c) \rho(r) r dr = \frac{c^3 \omega}{4\pi (\omega^2 a^2 + c^2)^2} = \frac{1}{4\pi a} \frac{x}{(x^2 + 1)^\tau} \equiv I(x),$$

where we have introduced the dimensionless variable $x \equiv \omega a/c$.

Thus we get

$$F(t - t') = \frac{32\pi hc}{3a^2} \int_0^\infty x \cos(\tau x) [I(x)]^2 dx,$$

Introducing the dimensionless parameter $\tau \equiv |t - t'| c/a$ and the new function

$$G(\tau) \equiv G(|t - t'| c/a) = F(t - t')$$

we get

$$G(\tau) = \frac{2\hbar c}{3\pi a^4} \int_0^\infty \frac{x^3 \cos(\tau x) dx}{(x^2 + 1)^4},$$

(36)

It is possible to get an analytical expression for $G(\tau)$ (an integration by parts may transform it in the integral no. 3.773/1 of Gradshteyn[15]) but we shall just obtain the behaviour for large and small $\tau$.

For $\tau << 1$, that is $|t - t'| << a/c$, we may approximate

$$\cos(\tau x) \simeq 1 - \frac{1}{2} \tau^2 x^2,$$

and we obtain, to order $\tau^2$, (with the change $x^2 = z$)

$$G(\tau) \simeq \frac{\hbar c}{3\pi a^4} \int_0^\infty \frac{z (1 - \frac{1}{2} \tau^2 z) dz}{(z + 1)^4} = \frac{hc}{18\pi a^4} (1 - \tau^2).$$

For $\tau >> 1$ the main contribution to the integral will come from values of $x$ close to the minimum of the quantity $y \equiv x^{-3} (x^2 + 1)^4$, that is $x = \sqrt{3/5}$. Making the change of variable $x = u + \sqrt{3/5}$ we may approximate, neglecting terms of order $u^4$ and higher,

$$y(u) \simeq \frac{8}{3.5^3} \sqrt{\frac{5}{3}} \left( 1 + \frac{25}{8} u^2 \right).$$

Thus we get, extending the integral in $u$ to $-\infty$ because only values of $x$ close to $\sqrt{3/5}$ will contribute,

$$G(\tau) \simeq \frac{5}{256} \sqrt{\frac{3}{5}} \frac{\hbar c}{5 \pi a^4} \int_{-\infty}^\infty \cos \left( \frac{u + \sqrt{3/5}}{\sqrt{5}} \tau \right) du$$

$$= \frac{5}{256} \sqrt{\frac{3}{5}} \frac{\hbar c}{5 \pi a^4} \cos \left( \sqrt{\frac{3}{5}} \tau \right) \int_{-\infty}^\infty \cos(\tau u) du$$

$$= \frac{25}{512} \sqrt{\frac{3}{10}} \frac{\hbar c}{10 a^4} \exp \left( -\frac{2\sqrt{\frac{3}{5}} \tau}{\sqrt{\frac{3}{5}}} \right) \cos \left( \sqrt{\frac{3}{5}} \tau \right).$$
6.2 Evaluation of $\sigma$

From eq. (6) it is natural to assume

$$\sigma^2 = \frac{c^2}{8\pi^2} \int_{-\infty}^{\infty} F(t)^2 dt.$$ 

The integral may be obtained from eq. (36) performing the $\tau$-integration first. Taking into account that

$$\int_{-\infty}^{\infty} \cos(x\tau) \cos(x'\tau) dt = \pi \left[ \delta(x-x') + \delta(x+x') \right],$$

we get

$$\sigma^2 = \frac{126\pi \hbar e^3}{9a^3} \int_0^{\infty} x^2 I(x)^4 dx = \frac{\hbar e^3}{18\pi a^2} \int_0^{\infty} \frac{x^5}{(x^2 + 1)^3} dx,$$

giving

$$\sigma = \frac{2\sqrt{3\pi} \hbar c^{3/2}}{3(4\pi a)^{7/2}} \simeq 2.12 \times 10^{-4} \frac{\hbar c^{3/2}}{a^{7/2}}.$$ 

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