Dark matter constraints on the parameter space and particle spectra in the nonminimal SUSY standard model

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Abstract

We investigate the dark matter constraints for the nonminimal SUSY standard model (NMSSM). The cosmologically restricted mass spectra of the NMSSM are compared to the minimal SUSY standard model (MSSM). The differences of the two models concerning the neutralino, sfermion and Higgs sector are discussed. The dark matter condition leads to cosmologically allowed mass ranges for the SUSY particles in the NMSSM: $m_{\tilde{\chi}^0_1} < 300 \text{GeV}$, $m_{\tilde{\epsilon}_R^e} < 300 \text{GeV}$, $300 \text{GeV} < m_{\tilde{u}_R} < 1900 \text{GeV}$, $200 \text{GeV} < m_{\tilde{t}_1} < 1500 \text{GeV}$, $350 \text{GeV} < m_{\tilde{g}} < 2100 \text{GeV}$ and for the mass of the lightest scalar Higgs $m_{S_1} < 140 \text{GeV}$. 
i) Introduction. Supersymmetry (SUSY) is suggested to solve the hierarchy problem of the standard model (SM), if it is embedded in a grand unified theory (GUT). The simplest SUSY extension of the SM is the minimal SUSY standard model (MSSM) [1, 2]. In the nonminimal SUSY standard model (NMSSM) [3] the Higgs sector of the MSSM is extended by a Higgs singlet N. By introducing a Higgs singlet N the $\mu$-parameter of the MSSM ($\mu$-problem) can be dynamically generated via $\mu = \lambda x$ with the vacuum expectation value (VEV) $\langle N \rangle = x$ and $\lambda$ being the Yukawa coupling of the Higgs fields. The phenomenology of the NMSSM has been analysed in several papers [4] - [13]. SUSY models with Higgs singlets can be derived from superstring inspired $E_6$ or $SU(5) \times U(1)$ GUT models and they offer the possibility of spontaneous breaking of the CP symmetry. The discrete $Z_3$ symmetry of the NMSSM causes a domain wall problem, because the $Z_3$ symmetry is spontaneously broken during the electroweak phase transition in the early universe. The domain wall problem of the NMSSM and possible solutions to it are discussed in [14].

In addition to the particle spectrum of the MSSM there are two extra Higgses and one extra neutralino in the NMSSM, which can mix with the other Higgses and neutralinos, and thus modify their properties. In many SUSY models the lightest SUSY particle (LSP) is a neutralino, which is a good candidate for cold dark matter. In the NMSSM the LSP neutralino can have a larger singlino portion. The cosmology of these LSP singlinos has been discussed in [15] at a given fixed low energy scale. The experimentally and cosmologically allowed parameter space of the NMSSM and the dark matter neutralinos have been investigated in [16] using RG evolutions.

In the present paper we show, how the dark matter condition restricts the mass spectra of the SUSY particles in the NMSSM. The cosmologically restricted mass spectra of the MSSM and NMSSM are compared and differences are discussed. In contrast to the previous investigation on NMSSM dark matter [16], we search for solutions which are the (global) minimum of the effective 1 loop Higgs potential and whose parameters are evolved with the SUSY RGEs from the GUT scale to the electroweak scale using the latest experimental constraints. Furthermore we consider all possible decay channels which occur in neutralino annihilation.
ii) The NMSSM. The NMSSM [3] is a supersymmetric $SU(3)_C \times SU(2)_I \times U(1)_Y$ gauge theory with two Higgs doublets $H_1$, $H_2$ and a Higgs singlet $N$. This model is defined by the superpotential (with only dimensionless couplings)

$$W = h_d H_1^T \epsilon \bar{Q} \bar{D} - h_u H_2^T \epsilon \bar{Q} \bar{U} + \lambda H_1^T \epsilon H_2 N - \frac{1}{3} k N^3$$  \hspace{1cm} (1)$$

($\epsilon$ is the antisymmetric tensor with $\epsilon_{12} = 1$) and the SUSY soft breaking terms

$$L_{soft} = - m_{H_1}^2 |H_1|^2 - m_{H_2}^2 |H_2|^2 - m_N^2 |N|^2 - M_Q |\bar{Q}|^2 - M_U |\bar{U}|^2 - M_D |\bar{D}|^2$$

$$- h_d A_d H_1^T \epsilon \bar{Q} \bar{D} + h_u A_u H_2^T \epsilon \bar{Q} \bar{U}$$

$$+ \lambda A_\lambda H_1^T \epsilon H_2 N + \frac{1}{3} k A_k N^3$$

$$+ \frac{1}{2} M_3 \lambda_3 \lambda_3 + \frac{1}{2} M_2 \lambda_2^a \lambda_2^a + \frac{1}{2} M_1 \lambda_1 \lambda_1 + \text{h.c.}$$  \hspace{1cm} (2)$$

The effective 1 loop Higgs potential consists of the tree-level potential and the radiative corrections

$$V_{1\text{loop}} = V_{\text{tree}} + V_{\text{rad}}.$$  \hspace{1cm} (3)$$

The contributions of the top quark and stops $\tilde{t}_1,2$ are considered in the radiative corrections to the effective Higgs potential [3]

$$V_{\text{rad}} = \frac{1}{64 \pi^2} \sum_i C_i (-1)^{2S_i} (2S_i + 1) m_i^4 \ln \left( \frac{m_i^2}{Q^2} \right),$$  \hspace{1cm} (4)$$

where the sum is taken over all particles and antiparticles with field-dependent mass $m_i$, spin $S_i$ and color degrees of freedom $C_i$. The electroweak gauge-symmetry $SU(2)_I \times U(1)_Y$ is spontaneously broken to the electromagnetic gauge-symmetry $U(1)_{\text{em}}$ by the Higgs VEVs $\langle H_i^0 \rangle = v_i$ with $i = 1,2$ and $\langle N \rangle = x$. The three minimum conditions for the VEVs have the form:

$$\frac{1}{2} m_Z^2 = \frac{m_{H_1}^2 + \Sigma^1 - (m_{H_2}^2 + \Sigma^2) \tan^2 \beta}{\tan^2 \beta - 1} - \lambda^2 x^2$$  \hspace{1cm} (5)$$

$$\sin(2\beta) = \frac{2 x (A_\lambda + k x)}{m_{H_1}^2 + \Sigma^1 + m_{H_2}^2 + \Sigma^2 + 2 \lambda^2 x^2 + \lambda^2 v^2}$$  \hspace{1cm} (6)$$

$$(m_N^2 + \Sigma^3) x^2 - k A_k x^3 + 2 k^2 x^4 + \lambda^2 x^2 v^2 - \frac{1}{2} (A_\lambda + 2 k x) \lambda x v^2 \sin(2\beta) = 0$$  \hspace{1cm} (7)$$
where \( v = \sqrt{v_1^2 + v_2^2} = 174 \text{ GeV} \), \( \tan \beta = v_2/v_1 \) and \( \Sigma' = \partial V_{\text{rad}}/\partial v_i^2 \).

The mass of the neutralinos follows from the following part of the lagrangian
\[
L = -\frac{1}{2} \Psi^T M \Psi + h.c.
\]  

\[
\Psi^T = (-i \lambda_1, -i \lambda_2^3, \Psi^0_{H_1}, \Psi^0_{H_2}, \Psi_N).
\]  

The symmetric mass matrix \( M \) of the neutralinos in the basis given in (9) has the form:
\[
\begin{pmatrix}
M_1 & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta & 0 \\
0 & M_2 & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta & 0 \\
-m_Z \sin \theta_W \cos \beta & m_Z \cos \theta_W \cos \beta & 0 & \lambda x & \lambda v_2 \\
m_Z \sin \theta_W \sin \beta & -m_Z \cos \theta_W \sin \beta & \lambda x & 0 & \lambda v_1 \\
0 & 0 & \lambda v_2 & \lambda v_1 & -2 k x
\end{pmatrix}.
\]  

With \( \mu = \lambda x \) the first \( 4 \times 4 \) submatrix recovers the mass matrix of the MSSM \([2]\). The mass of the neutralinos is here obtained by diagonalising the mass matrix \( M \) with the orthogonal matrix \( N \). (Then some mass eigenvalues may be negative \([17]\).)

\[
L = -\frac{1}{2} m_i \tilde{\chi}_i^0 \tilde{\chi}_i^0
\]  

\[
\tilde{\chi}_i^0 = \left( \begin{array}{c}
\chi_i^0 \\
\tilde{x}_i^0
\end{array} \right) \text{ with } \chi_i^0 = N_{ij} \Psi_j \text{ and } M_{\text{diag}} = N M N^T.
\]

Gauge coupling unification in non-supersymmetric GUTs is already excluded by the precision measurements at LEP \([18]\). In supersymmetric GUTs gauge coupling unification \( g_a(M_X) = g_5 \simeq 0.72 \) is possible at a scale \( M_X \simeq 1.6 \times 10^{16} \text{ GeV} \). For the SUSY soft breaking parameters we suppose universality at the GUT scale \( M_X \):

\[
m_i(M_X) = m_0 \\
M_a(M_X) = m_{1/2} \\
A_i(M_X) = A_0 \text{ (} A_\lambda(M_X) = -A_0 \).
\]

The Yukawa couplings at the GUT scale take the values \( \lambda(M_X) = \lambda_0, k(M_X) = k_0 \) and \( h_t(M_X) = h_{t0} \). With the SUSY RGEs \([19]\) the SUSY soft breaking parameters and the Yukawa couplings are evolved from the GUT scale to the electroweak scale \( M_{\text{weak}} \simeq 100 \text{ GeV} \). At \( M_{\text{weak}} \) we minimize the effective 1 loop Higgs potential. The 8 parameters of the NMSSM are \( m_0, m_{1/2}, A_0, \lambda_0, k_0, h_{t0}, \tan \beta \) and \( x \), with only 5 of them being independent, which in our procedure are taken as \( \lambda_0, k_0, h_{t0}, \tan \beta \) and \( x \). The remaining 3
parameters \((m_0, m_{1/2}, A_0)\) are numerically calculated from the three minimum conditions (5-7) following the method described in [6].

Since we take here \(\tan \beta\) and \(x\) as (randomly chosen) fixed input parameters, the three minimum conditions (5-7) depend only on the low energy parameters \(m_i, A_i, \lambda, k\) and \(h_t\) generalized as \(p_k(M_{\text{weak}})\):

\[
\frac{\partial V_{1\text{loop}}}{\partial v_i} = F_i(p_k) = 0 \quad (i = 1 - 3). \tag{13}
\]

The low energy parameters \(\lambda, k\) and \(h_t\) are fixed by the SUSY RGEs and the remaining (randomly chosen) fixed input parameters \(\lambda_0, k_0\) and \(h_{t0}\). With the SUSY RGEs the low energy parameters \(m_i\) and \(A_i\) are then only depending on the GUT scale parameters \(m_0, m_{1/2}\) and \(A_0\). Therefore all low energy parameters \(p_k(M_{\text{weak}})\) can be written as functions \(f_k\) (which can be constant) of the GUT scale parameters \(m_0, m_{1/2}\) and \(A_0\):

\[
p_k(M_{\text{weak}}) = f_k(m_0, m_{1/2}, A_0). \tag{14}
\]

The functions \(f_k\) are determined by numerically solving the SUSY RGEs [19]. In the three minimum conditions (13) we then substitute the low energy parameters \(p_k(M_{\text{weak}})\) by the functions \(f_k\) and obtain the equations

\[
F_i(f_k(m_0, m_{1/2}, A_0)) = G_i(m_0, m_{1/2}, A_0) = 0 \quad (i = 1 - 3). \tag{15}
\]

The SUSY breaking parameters \(m_0, m_{1/2}\) and \(A_0\) are then determined by numerically solving the equations \(G_i(m_0, m_{1/2}, A_0) = 0 \quad (i = 1 - 3)\).

Because we assume \(h_t \gg h_b, h_\tau\), we restrict \(|\tan \beta|\) to be smaller than 20 [6]. For the singlet VEV \(x\) we choose the range \(|x| < 80 \text{ TeV}\) [3, 4, 11].

The physical minimum of the Higgs potential should be the global minimum. We check, whether the physical minimum is lower than the unphysical minima of \(V_{1\text{loop}}(v_1, v_2, x)\) with at least one vanishing VEV [3].

The conditions [19], that the minimum of the scalar potential does not break the conservation of charge and colour, can be formulated as follows:

\[
A_{u_i}^2 \leq 3(m_{H_2}^2 + m_{Q_i}^2 + m_{U_i}^2) \quad \text{at scale } Q \sim A_{u_i}/h_{u_i}. \tag{16}
\]

\[
A_{d_i}^2 \leq 3(m_{H_1}^2 + m_{Q_i}^2 + m_{D_i}^2) \quad \text{at scale } Q \sim A_{d_i}/h_{d_i}. \tag{17}
\]

\[
A_{e_i}^2 \leq 3(m_{H_1}^2 + m_{L_i}^2 + m_{E_i}^2) \quad \text{at scale } Q \sim A_{e_i}/h_{e_i}. \tag{18}
\]
For the top quark pole mass we take the range $169 \, GeV < m_t < 181 \, GeV$ corresponding to the recent world average measurement \[20\]. The SUSY particles in the NMSSM have to fulfill the following experimental conditions: $m_{\tilde{\nu}} \geq 41.8 \, GeV$ \[21\], $m_{\tilde{\tilde{e}}} \geq 65 \, GeV$ \[22\], $m_{\tilde{\tilde{q}}} \geq 176 \, GeV$ (if $m_{\tilde{\tilde{g}}} < 300 \, GeV$) \[21\] and $m_{\tilde{\tilde{q}}} \geq 70 \, GeV$ (for all $m_{\tilde{\tilde{g}}}$) \[23\], $m_{\tilde{\tilde{t}}} \geq 58.3 \, GeV$ \[24\], $m_{\tilde{\tilde{g}}} \geq 173 \, GeV$ \[25\], $m_{\tilde{\tilde{q}}} \geq 176 \, GeV$ \[22\], $m_{H^+} \geq 43.5 \, GeV$ \[21\], $m_{S_1} \geq 70.7 \, GeV$ \[22\] or the coupling of the lightest scalar Higgs $S_1$ to the $Z$ boson is reduced compared to the SM \[22, 27\] assuming visible decays with branching ratios like the SM Higgs \[7\], and

$$\sum_{i,j} \Gamma(Z \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0) < 30 \, MeV$$ \[28\]
$$\Gamma(Z \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0) < 7 \, MeV$$ \[28\]
$$BR(Z \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0) < 10^{-5} \, , \, (i,j) \neq (1,1)$$ \[28\].

### iii) Dark matter constraints.
Inflationary cosmological models suggest a flat universe with $\Omega = \rho/\rho_{crit} = 1$. Big bang nucleosynthesis restricts the baryon density to $\Omega_{baryons} \leq 0.1$. The missing matter is called dark matter. One of the favoured theories to explain the structure formation of the universe is the cold + hot dark matter model (CHDM) \[23\]. In this model the dark matter consists of hot dark matter (massive neutrinos) and cold dark matter (neutralinos) with $\Omega_{hot} \simeq 0.3$ and $\Omega_{cold} \simeq 0.65$. With the Hubble constant $H_0 = 100 \, h_0 \, km \, s^{-1} \, Mpc^{-1}$ and the range $0.4 < h_0 < 1$, the condition for the cosmic density of the neutralinos in the CHDM model reads

$$0.1 \leq \Omega_{\chi} h_0^2 \leq 0.65.$$ \[19\]

With the annihilation cross section of the neutralinos $\sigma_{ann} v = a + b \, v^2$ ($v$ is the relative velocity of the neutralinos) the cosmic neutralino density \[30, 31, 32\] can be calculated

$$\Omega_{\chi} h_0^2 = \frac{1.07 \times 10^9 \, x_{fr}}{\sqrt{g_*}} \left( \frac{1}{M_P} \right) \left( \frac{a + \frac{6\, b}{x_{fr}}}{g_*} \right) \, GeV^{-1},$$ \[20\]

where $M_P = 1.221 \times 10^{19} \, GeV$ is the Planck mass and $g_* \approx 81$ is the effective number of degrees of freedom at $T_{fr}$. The freeze-out temperature of the neutralinos $T_{fr}$ follows from the equation \[30, 31, 32\]

$$x_{fr} = \ln \left( \frac{0.0764 \, |m_{\tilde{\chi}_1^0}| \, M_P \, a + \frac{6\, b}{x_{fr}}}{\sqrt{g_*} \, x_{fr} \, c} \right) \, (2 + c)$$ \[21\]

5
with \( x_{fr} = |m_{\tilde{\chi}_1^0}|/T_{fr} \) and \( c = 1/2 \).

In the MSSM the formulas for the annihilation cross section of the neutralinos into the different decay products \( X \) and \( Y \) can be found in [30]

\[
\sigma_{ann}(\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow X Y)v = \frac{1}{48\pi sS} \left[ |A(1S_0)|^2 + \frac{1}{3} |A(3P_0)|^2 + |A(3P_1)|^2 + |A(3P_2)|^2 \right]
\] (22)

\[
\beta_f = \sqrt{1 - \frac{2(m_X^2 + m_Y^2)}{s} + \frac{(m_X^2 - m_Y^2)^2}{s^2}}
\] (23)

\( s \approx 4m_{\tilde{\chi}}^2 \) is the center of mass energy squared. \( S \) is a symmetry factor, which is 2 if \( X = Y \). The partial wave amplitude \( A \) describes annihilation from an initial state \( ^{2S+1}L_J \) with spin \( S \), orbital angular momentum \( L \) and total angular momentum \( J \).

In the NMSSM the annihilation cross section of the neutralinos changes because of more particles in this model, which can mix, and modified vertices. In our numerical analysis we consider all possible decay channels. The relevant formulas for the NMSSM are obtained by substituting the MSSM couplings in [30] by the corresponding NMSSM couplings and by slightly modifying the partial wave amplitudes in [30] and will be given in [33]. Here we only give the NMSSM couplings for the dominant decay channel

\[
\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow f_a \bar{f}_a.
\]

The fermions are produced by Z boson, scalar Higgs \( S_i \) \((i = 1 - 3)\) and pseudoscalar Higgs \( P_{\alpha} \) \((\alpha = 1, 2)\) exchange in the s-channel and sfermion \( \tilde{f}_{1,2} \) exchange in the t- and u-channel. When the LSP neutralino, which is denoted with index 0 in [30] instead of 1, is predominantly a bino \((N_{01}^2 \approx 1 \) in eq. (11)), the annihilation cross section is dominated by sfermion exchange. The couplings of the unmixed sfermion \( \tilde{f}_{L,R} \) to the fermion \( f_a \) and LSP neutralino, are given by

\[
X_{a0} = -\sqrt{2}g_2 [T_{3a} N_{02} - \tan \theta_W (T_{3a} - e_{f_a}) N_{01}]
\] (24)

\[
Y_{a0} = \sqrt{2}g_2 \tan \theta_W e_{f_a} N_{01}
\] (25)

\[
Z_{a0} = -\frac{g_2 m_u}{\sqrt{2} \sin \beta m_W} N_{04}
\] (26)
\[ Z_{d0} = - \frac{g_2 m_d}{\sqrt{2} \cos \beta m_W} N_{03} \]  

and refer to eqs. (A32a)-(A32c) in [30]. These couplings have the same form in the MSSM and NMSSM. The only difference follows from the matrix \( N \), which diagonalizes the mass matrix of the neutralinos. The couplings \( X'_{a0}, W'_{a0}, Z'_{a0} \) and \( Y'_{a0} \) of the mixed sfermion \( \tilde{f}_{1,2} \) to the fermion \( f_a \) and LSP neutralino are easily given in terms of the unmixed case as described by eq. (A31) in [30]. When the Higgsino portion of the LSP neutralino is larger (larger \( N_{03}^2, N_{04}^2 \) in eq. (11)), \( Z \) boson exchange becomes important for the annihilation cross section. The coupling of the \( Z \) boson to the LSP neutralinos \( O''_{00} \) (eq. (A5e) in [30]) has the same form in the MSSM and NMSSM, but a different matrix \( N \) enters:

\[ O''_{00} = - \frac{1}{2} N_{03}^2 + \frac{1}{2} N_{04}^2. \]  

In the NMSSM the Higgs-couplings to the fermions have to be modified compared to the MSSM,

\[
\begin{align*}
    h_{P,a} &= \frac{g_2 m_a U_{a2}}{2 \sin \beta m_w} \\
    h_{P,d} &= -\frac{g_2 m_d U_{a1}}{2 \cos \beta m_w} \\
    h_{S,u} &= -\frac{g_2 m_u U_{i2}}{2 \sin \beta m_w} \\
    h_{S,d} &= -\frac{g_2 m_d U_{i1}}{2 \cos \beta m_w}.
\end{align*}
\]  

The 3 \( \times \) 3 mass matrix of the scalar Higgses \( S_i \) in the basis \( (H^0_{1R}, H^0_{2R}, N_R) \) with \( H^0_{iR} = \sqrt{2} Re\{H_i^0\} \) (i = 1-3) is diagonalised by the matrix \( U^S \). The 3 \( \times \) 3 mass matrix of the pseudoscalar Higgses \( P_a \) and the Goldstone boson \( S \) in the basis \( (H^0_{1I}, H^0_{2I}, N_I) \) with \( H^0_{iI} = \sqrt{2} Im\{H_i^0\} \) (i = 1-3) is here diagonalised by the 3 \( \times \) 3 matrix \( U^P \). These NMSSM couplings have to be substituted for the MSSM couplings given in eqs. (A33a)-(A33c) of [30]. The coupling of the scalar Higgs \( S_a \) to the neutralinos is different in the NMSSM and the MSSM. In the NMSSM this coupling can be written as

\[
T_{S,a ij} = -U^S_{a1} Q''_{ij} + U^S_{a2} S''_{ij} + U^S_{a3} Z''_{ij}
\]  

\[
Q''_{ij} = \frac{1}{2 g_2} \left[ N_{i3} \left( g_2 N_{j2} - g_y N_{j1} \right) + \sqrt{2} \lambda N_{i4} N_{j5} + (i \leftrightarrow j) \right]
\]

\[
S''_{ij} = \frac{1}{2 g_2} \left[ N_{i4} \left( g_2 N_{j2} - g_y N_{j1} \right) - \sqrt{2} \lambda N_{i3} N_{j5} + (i \leftrightarrow j) \right]
\]
\[ Z''_{ij} = \frac{1}{2} g_2 \left[ -\sqrt{2} \lambda N_{i3} N_{j4} + \sqrt{2} k N_{i5} N_{j5} + (i \leftrightarrow j) \right], \] (34)

to be substituted for the MSSM couplings given in eqs. (A9b,A9c) of [30]. The coupling of the pseudoscalar Higgs \( P_\alpha \) to the neutralinos also changes. In the NMSSM this coupling has the following form

\[ T_{P_\alpha}^\alpha_{ij} = -U_{P_\alpha 1}^1 Q''_{ij} + U_{P_\alpha 2}^2 S''_{ij} - U_{P_\alpha 3}^3 Z''_{ij} \] (35)

\[ Q''_{ij} = \frac{1}{2} g_2 \left[ N_{i3} \left( g_2 N_{j2} - g_y N_{j1} \right) + \sqrt{2} \lambda N_{i4} N_{j5} + (i \leftrightarrow j) \right] \] (36)

\[ S''_{ij} = \frac{1}{2} g_2 \left[ N_{i4} \left( g_2 N_{j2} - g_y N_{j1} \right) + \sqrt{2} \lambda N_{i3} N_{j5} + (i \leftrightarrow j) \right] \] (37)

\[ Z''_{ij} = Z''_{ij}, \] (38)

which has to be used instead of the MSSM coupling in eq. (A9a) of [30].

**iv) Particle spectra.** To obtain the mass spectra of the SUSY particles in the NMSSM, we randomly generate \( \sim 5.5 \times 10^8 \) points in the 5 dimensional parameter space of the input parameters \( \lambda_0, k_0, h_{t0}, \tan \beta \) and \( x \). The SUSY breaking parameters \( m_0, m_{1/2} \) and \( A_0 \) are then determined by numerically solving the three minimum equations (15) as described there. Having found a solution to the SUSY RGEs, whose effective 1 loop Higgs potential has a local minimum for the input parameters \( \tan \beta \) and \( x \), we calculate the masses of all SUSY particles [3, 34] and Higgses [3, 5, 10, 11] and impose the additional theoretical and experimental constraints described above in ii). With all these requirements, about 4900 solutions remain, from which about 2000 solutions are cosmologically acceptable. For the constrained MSSM in [35] a scatter-plot of solutions in the \((m_{1/2}, m_0)\) plane is shown. Upper bounds of 1.1 \( TeV \) for \( m_{1/2} \) and 600 \( GeV \) for \( m_0 \) are found. For \( m_{1/2} \approx 100 \ GeV \) solutions with much larger \( m_0 \) are allowed due to Z-pole enhanced neutralino pair annihilation \((m_{\tilde{\chi}_1^0} \approx m_Z/2)\). The comparable scatter-plot for the NMSSM (more details in [33]) shows only a few cosmologically allowed solutions with \( m_0 \gg m_{1/2} \). The reason, why the case \( m_0 \gg m_{1/2} \) is somewhat suppressed in the NMSSM, is discussed in [3]. It follows from the condition, that the minimum of the scalar potential does not break
the conservation of charge and colour, and from the condition that the physical minimum is lower than the symmetric vacuum, \( A^2_0 > 9m^2_0 \).

As a few representative examples for the importance of the dark matter constraints we show in the following some scatter-plots for the mass of SUSY particles. In the upper left picture of fig. 1 the mass of the LSP neutralino versus the mass of the lighter selectron is shown for the MSSM. In the MSSM the LSP neutralino is mostly a bino. In the upper right picture the dark matter condition is imposed and gives an upper bound of 280 GeV for the LSP neutralino and a cosmologically allowed mass range of 100 GeV < \( m_{\tilde{e}_R} < 300 \) GeV for the lighter selectron. These bounds for the LSP neutralino and lighter selectron can be understood from the fact [30], that for a bino-like LSP neutralino the annihilation cross section is dominated by sfermion exchange. Heavier selectrons are cosmologically allowed, if Z-pole or Higgs-pole enhanced neutralino pair annihilation [35] occurs.

For the NMSSM the lower left picture of fig. 1 shows the mass of the LSP neutralino versus the mass of the lighter selectron. Compared to the MSSM there are not so many solutions in the NMSSM with small LSP neutralino mass and large selectron mass, which follows from the suppression of \( m_0 \gg m_{1/2} \) in the NMSSM [8]. In the NMSSM some LSP neutralinos can have a larger singlino portion. Most of these singlinos are decoupled pure singlinos [8, 11], but a smaller fraction of the singlinos can be mixed states ([15], more details in [33]). The LSP neutralinos with mass below 40 GeV and \( m_{\tilde{e}_R} \approx 70 \) GeV in the lower left picture of fig. 1 are lighter singlinos.

The lower right picture of fig. 1 shows the NMSSM solutions, which survive the dark matter condition. The cosmologically allowed solutions in the MSSM and NMSSM look quite similar except that heavier selectrons are allowed in the former. With the dark matter condition nearly all decoupled pure LSP singlinos can be excluded (more details in [33]), what can be seen for the lighter singlinos \( m_{\tilde{\chi}_1^0} < 40 \) GeV with \( m_{\tilde{e}_R} \approx 70 \) GeV) by comparing the lower pictures of fig. 1.

Another difference between the MSSM and NMSSM is the enhanced lower bound of 50 GeV for most of the LSP neutralinos in the NMSSM. The dark matter condition in the NMSSM in fig. 1 excludes nearly all LSP neutralinos with mass \( m_{\tilde{\chi}_1^0} \approx m_Z/2 \), even if
they are not singlinos. The reason for this difference is the larger coupling $O^{UL}_{00}$ in eq. (28) of the Z boson to the LSP neutralinos in the NMSSM corresponding to neutralinos with larger Higgsino portion. In the MSSM the coupling $O^{UL}_{00}$ can be very small for nearly pure bino-like LSP neutralinos, so that the neutralino pair annihilation becomes not too large from Z exchange near the Z-pole. The difference follows from the circumstance, that the NMSSM can be obtained from the MSSM in the limit $\lambda, k \rightarrow 0, |x| \rightarrow \infty$ with $\lambda x$ and $k x$ fixed. In this limit the first two minimum conditions (5),(6) of the NMSSM correspond to the two minimum conditions of the MSSM, whereas the third minimum condition (7) of the NMSSM would be an extra constraint for the MSSM [6, 11]. The third minimum condition restricts the parameter region of the NMSSM compared to the MSSM in such a way, that the coupling $O^{UL}_{00}$ of the Z boson to the LSP neutralinos is larger in the NMSSM for $m_{\tilde{\chi}_1^0} \approx m_Z/2$.

In the left picture of fig.2 we show the mass of the lighter u-squark and the lighter stop versus the mass of the gluino for the NMSSM. In contrast to the MSSM [36, 33], the squark masses in the NMSSM are nearly proportional to the gluino mass. The already mentioned reason is, that in the NMSSM solutions with $m_0 \gg m_{1/2}$ are somewhat disfavoured. In the right picture of fig.2 the cosmologically allowed lighter u-squark and lighter stop masses are shown. In contrast to the MSSM, the dark matter condition improves the lower bound for the gluino to 350 GeV in the NMSSM. The reason is the enhanced lower bound of 50 GeV for most of the LSP neutralinos in fig.1. Furthermore, the Higgsino or singlino portion of the lightest cosmologically allowed LSP neutralinos is larger in the NMSSM than in the MSSM as already discussed. For these LSP neutralinos with larger Higgsino or singlino portion, the gaugino mass $M_1$ is larger than their mass $m_{\tilde{\chi}_1^0}$. A larger gaugino mass $M_1$ leads to a larger gluino mass $M_3$ in the NMSSM. Because the mass of the squarks are nearly proportional to the gluino mass in the NMSSM, the lower bound for the lighter u-squark improves to 300 GeV and the lower bound for the lighter stop improves to 200 GeV. This is in contrast to the MSSM, where the dark matter condition does not change the lower bounds. The cosmologically upper bound of 1900 GeV for the lighter u-squark, 1500 GeV for the lighter stop and 2100 GeV for the gluino in the NMSSM are similar to the corresponding upper bounds in the MSSM and are connected with the
cosmologically upper bound of 300 GeV for the lighter selectron and the bino-like LSP neutralino.

In the left picture of fig. 3 the mass of the lightest scalar Higgs $S_1$ versus the mass of the gluino is shown for the NMSSM. The lightest scalar Higgs can be lighter than the MSSM Higgs bound of 62.5 GeV [22]. In this case the lightest scalar Higgs is predominantly a Higgs singlet $N$ [7, 9, 10]. The right picture of fig. 3 shows only solutions, which fulfill the dark matter condition. For bino-like LSP neutralinos the cosmologically allowed mass range for the lightest scalar Higgs $S_1$ is $40 GeV < m_{S_1} < 140 GeV$, which follows from the cosmologically allowed mass range for the lighter selectron of $100 GeV < m_{\tilde{e}_R} < 300 GeV$. Very light scalar Higgses $S_1$ can be cosmologically allowed, if the LSP neutralino is a mixed singlino [15] or a nearly pure singlino with mass $m_{\tilde{\chi}_1^0} = m_Z/2$, so that Z-pole enhanced neutralino pair annihilation occurs. The corresponding plots in the MSSM [37, 33] look very similar with the exception, that there are no Higgses below the bound of 62.5 GeV [22].

v) Conclusions. We have investigated the particle spectra of the MSSM and NMSSM without and with the dark matter condition. In contrast to the MSSM, the squark masses in the NMSSM are nearly proportional to the gluino mass. We found, that pure LSP singlinos and most of the light scalar Higgs singlets are cosmologically excluded. With the dark matter condition the following cosmologically allowed mass ranges for the SUSY particles in the NMSSM can be given: $m_{\tilde{\chi}_1^0} < 300 GeV$, $m_{\tilde{e}_R} < 300 GeV$, $300 GeV < m_{\tilde{\nu}_R} < 1900 GeV$, $200 GeV < m_{\tilde{t}_1} < 1500 GeV$, $350 GeV < m_{\tilde{g}} < 2100 GeV$, $m_{S_1} < 140 GeV$. The upper bounds for the mass of the SUSY particles in the NMSSM are comparable to the MSSM bounds. The larger Higgsino or singlino portion of the lightest LSP neutralinos in the NMSSM leads to improved lower bounds for the gluinos and squarks compared to the MSSM.
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Figure Captions

**Fig 1** The mass of the LSP neutralino versus the lighter selectron mass. The upper pictures are for the MSSM, the lower pictures are for the NMSSM. In the right pictures the dark matter condition is imposed.

**Fig 2** The mass of the lighter u-squark and the lighter stop versus the gluino mass in the NMSSM. In the right picture the dark matter condition is imposed.

**Fig 3** The mass of the lightest Higgs $S_1$ versus the gluino mass in the NMSSM. In the right picture the dark matter condition is imposed.
Fig. 1
Fig. 2
Fig. 3