DEUTERON POLARIZATION OBSERVABLES IN ELASTIC $\vec{dp}$ SCATTERING AT INTERMEDIATE ENERGIES

M.A. Shikhalev†

Veksler and Baldin Laboratory of High Energies, JINR, Dubna
† E-mail: shehalev@theor.jinr.ru

Abstract
A calculation of vector and tensor polarization observables of a deuteron in an optical potential framework for elastic proton-deuteron scattering at the incident deuteron energy $E_d = 880$ MeV is presented. The main aim is to point out the role of various effects in explaining the wide-range angle distribution of these observables. Under the investigation are a double-scattering mechanism in the optical potential, the influence of various $NN$-partial waves and as well a comparison of calculations with two different deuteron wave-functions derived from the Bonn-CD $NN$-potential model and dressed bag model of Moscow-Tuebingen group.

1 Introduction
The reaction of elastic nucleon-deuteron scattering are considered both by experimentalists and theoreticians as one of the clue problems in few-nucleon physics. For three decades it has been served as a hope to obtain more information about intermediate- and short-range $NN$ interaction and as a probe of the deuteron structure at small distances (for a review, see Ref. [1]). During the last decade it is also studied with the purpose of testing of various $3N$-forces and particularly their spin-dependance [2, 3]. It is also of interest as a basic reaction to establish a polarimetry for vector-tensor mixed polarized beams.

Now, there is a set of high quality nucleon-nucleon potentials that describe two-nucleon elastic scattering data perfectly below the pion production threshold. But they underestimate the three nucleon bound energy. The incorporation of a $3N$ force in the calculations improves the situation but some unsolved questions remained, especially it concerns its spin-dependant properties. The study of polarization observables in elastic $pd$ scattering, as expected, can provide an additional information about the properties of $3N$ force. To describe $pd$ elastic scattering below the pion production threshold different techniques have been applied. Among them the momentum space Faddeev equations can now be solved with high accuracy for the most modern two- and three-nucleon forces. The $3N$ forces in such a calculations are of Fujita-Miyazawa [4, 5] or Tucson-Melbourne [6] type. Also, in Ref. [7] a coupled-channel model with excitation of the stable $\Delta$-isobar was successfully applied to $pd$ scattering employing Chebyshev expansion of two-body amplitudes.

But calculations at incident proton energies greater than 200 MeV in lab encounter with some nontrivial difficulties. The first problem is that up to now there is no reliable quantitative model for $NN$-interaction above the inelastic threshold. All existing models that pretend to description of two-nucleon scattering up to 1 GeV do it only in
semi-quantitative manner [8]. The second problem is that it is no longer conventional three-body problem above the pion production threshold. The nucleon isobar and meson degrees of freedom start to play a significant role. And last but not least, there are purely computational difficulties in performing relativistic three-body calculations at higher energies [9]. Hence one must resort to some kind of simplifications and approximations in the study of this reaction.

In this work, the optical potential framework to calculate polarization observables of a deuteron is used. The incident deuteron energy is taken to be 880 MeV in lab. This energy is just below the region where the influence of ∆-isobar excitation becomes significant masking a large momentum component of the deuteron wave function [10]. Moreover, this energy lies in the interval of energies proposed for the measurements of dp elastic scattering analyzing powers at internal target station of the Nuclotron at JINR [11].

2 Theoretical framework

The basic equations for three-body problem in quantum physics are the Faddeev equations which can be written in operator form:

\[ U = PG_0^{-1} + PTG_0U, \]

where \( U = U_{\mu d\mu',\mu d\mu N}(\vec{q}',\vec{q}) \) is an amplitude of elastic dp-scattering, \( \mu_d, \mu_N \) are spin quantum numbers, \( T - NN \) scattering matrix, \( G_0 = (E - H_0 + i\epsilon)^{-1} \) is a free propagator of 3N system and \( P \equiv P_{12}P_{23} + P_{13}P_{23} \) stands for a permutation operator that takes into account the property of identity of three nucleons. Here and further in this work we have dropped out all 3N force contributions.

One can rewrite (1) in the form that is more appropriate for calculations at higher energies where an impulse approximation is justified:

\[ U = V_{opt} + V_{opt}G_dU. \]

This is the optical potential framework and \( V_{opt} \) is a nucleon-deuteron optical potential:

\[ V_{opt} = PG_0^{-1} + PTcG_0V_{opt}. \]

Here \( T_c \) is a NN T-matrix with subtracted deuteron pole term and \( G_d \) is a deuteron contribution in the spectral decomposition of the two-body Hamiltonian.

Although, the Eq. (2) seems quite simple and represents a common two-body scattering equation, the derivation of the optical potential by solving Eq. (3) is as difficult as solving Faddeev equations themselves. However, as was mentioned in Introduction, at energies higher than the pion production threshold, new degrees of freedom, other than nucleons, come into the game. But this new physics is not taken into account in (2). Also, these are the energies at which the impulse approximation is a good starting point, so one can make a series expansion of \( V_{opt} \) in (3), keeping a possibility to include in \( V_{opt} \) some additional terms:

\[ V_{opt} = PG_0^{-1} + PTcP + PTcG_0PTcP + \ldots \]
Pictorially it can be described as the sum of nine graphs (up to the second order in series expansion), see Fig. 1. The first diagram is a usual one-nucleon exchange mechanism, the next three diagrams are so called triangle diagrams and represent the single-scattering process and the last five diagrams take into account the inelastic break up channel.

The one-nucleon exchange contribution is written in a usual way:

\[
2(31) \langle f | G_0^{-1} | i \rangle = - \left[ \sqrt{q^2 + M_d^2} - \sqrt{q^2 + M_N^2} - \sqrt{4q^2 \cos^2 \frac{\theta}{2} + M_N^2} \right] \Psi_{13} \left( \vec{q'} + \frac{1}{2} \vec{q} \right) \Psi_{23} \left( \vec{q'} + \frac{1}{2} \vec{q} \right) .
\]

Here, we omit a complicated notation related to a summation over spin and orbital quantum numbers. \( \Psi_{23} \) is a wave function of the deuteron composed of nucleons 2 and 3.

To evaluate a single scattering diagram, one must implement an integration over the internal momentum of nucleons in the deuteron. To do this, a knowledge about the fully off-shell behavior of \( NN \) \( T \)-matrix is required:

\[
1(23) \langle f | T_c | i \rangle = \int \frac{d^3 p}{(2\pi)^3} \Psi_{23} \left( \vec{p'} + \frac{1}{4} \vec{k} \right) T_c \left( \vec{q'}, \vec{p} - \frac{3}{4} \vec{q'} + \frac{1}{4} \vec{q}; \vec{q}, \vec{p} - \frac{3}{4} \vec{q} + \frac{1}{4} \vec{q'} \right) \Psi_{23} \left( \vec{p} - \frac{1}{4} \vec{k} \right) ,
\]

here \( k = \vec{q} - \vec{q'} \) is a transferred momentum.

However, the absence of high quality \( NN \) interaction model above the inelastic threshold at the present moment forces anyone to make some approximate evaluation of the integral, proceeding from the assumption of either an off-shell behavior of somehow parameterized \( T \)-matrix or taking only its on-shell value. In this work, the \( T \)-matrix is put on-shell in the way that minimizes the off-shell corrections. This is an optimal impulse approximation \[12, 13]:
Then $T$-matrix is evaluated in the so called NN Breit frame, where an effective scattering energy is

$$E_{\text{eff}} = \frac{s}{2M_N} - 2M_N,$$

$$s = \left( \sqrt{q^2 + M_N^2} + \sqrt{\frac{q^2}{8}(5 - 3\cos\theta) + M_N^2} \right)^2 - \frac{q^2}{4}\cos^2\theta.$$

For example, at deuteron laboratory energy $T_{\text{lab}} = 880$ MeV and scattering angle $\theta = 10^\circ$ the effective scattering energy of two nucleons is $E_{\text{eff}} = 443$ MeV, whereas at the angle $\theta = 170^\circ - E_{\text{eff}} = 796$ MeV. The numerical values of the $T$-matrix are obtained from partial wave analysis SAID [14] that provides the $NN$ phase shifts up to 3 GeV and the total angular momentum $J_{NN} = 7$.

This permits the $T$-matrix to be shifted outside the integral and the remaining integration produces the deuteron form factor:

$$1(23) \langle f | T_0 | i \rangle = T_c \left( \vec{q}^\prime, -\frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}, \vec{q}^\prime, -\frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}; E_{\text{eff}} \right) S(\vec{k}).$$

For the double scattering term one has two integrals – one over the internal momentum in the deuteron and another over the intermediate momentum of the scattered nucleon:

$$1(23) \langle f | T_2 G_0 T_1 | i \rangle = \int \frac{d^3p}{(2\pi)^3} \frac{d^3p^\prime}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \delta^3 \left( \vec{p}^\prime + \vec{p} - \frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q} \right) \times \Psi_{23} \left( \vec{q}^\prime - \frac{3}{8} \vec{q} - \frac{1}{2} \vec{q}^\prime \right) T_2 \left( \vec{q}^\prime, \vec{p}^\prime - \frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}; \vec{q}^\prime, \vec{p}^\prime - \frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}^\prime \right) G_0$$
$$\times T_1 \left( \vec{q}, \vec{p} - \frac{3}{4} \vec{q}, \frac{1}{4} \vec{q}; \vec{q}, \vec{p} - \frac{3}{4} \vec{q} + \frac{1}{4} \vec{q} \right) \Psi_{23} \left( \vec{q} - \frac{3}{2} \vec{q} + \frac{3}{4} \vec{q} - \frac{1}{2} \vec{q}^\prime \right),$$

Taking into account in $G_0$ only its pole part and employing the impulse approximation

$$T_2 \left( \vec{q}^\prime, \vec{p}^\prime - \frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}; \vec{q}^\prime, \vec{p}^\prime - \frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}^\prime \right) G_0 T_1 \left( \vec{q}^\prime, \vec{p}^\prime - \frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}; \vec{q}^\prime, \vec{p}^\prime - \frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}^\prime \right)$$
$$\longrightarrow -i\pi \frac{2M_N}{3q} \delta(q'' - q) T_2 \left( \vec{q}^\prime, -\frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}; \vec{q}^\prime, -\frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}^\prime; E_{\text{eff}2} \right) T_1 \left( \vec{q}^\prime, -\frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}; \vec{q}, -\frac{3}{4} \vec{q} + \frac{1}{4} \vec{q}^\prime; E_{\text{eff}1} \right),$$

finally one gets

$$1(23) \langle f | T_2 G_0 T_1 | i \rangle = -\frac{2i}{3\pi} q M_N \int \frac{d^2q}{(2\pi)^3} T_2 \left( \vec{q}^\prime, -\frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}; \vec{q}^\prime, -\frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}^\prime; E_{\text{eff}2} \right)$$
$$\times T_1 \left( \vec{q}^\prime, -\frac{3}{4} \vec{q}^\prime + \frac{1}{4} \vec{q}; \vec{q}, -\frac{3}{4} \vec{q} + \frac{1}{4} \vec{q}^\prime; E_{\text{eff}1} \right) \delta \left( q'' - q \right) S \left( q'' - \frac{1}{2} (\vec{q} + \vec{q}^\prime) \right).$$

The scattering equation (2) is solved in helicity basis employing $K$-matrix approximation, i.e. only the pole part of the two-body propagator $G_d$ is remained thus all terms in the equation contain only on-shell information about the optical potential and scattering amplitude:

$$\langle \lambda_d^J \lambda_d^J | U^J (q', q) | \lambda_d \lambda_N \rangle = \langle \lambda_d^J \lambda_d^J | V_{\text{opt}}^J (q', q) | \lambda_d \lambda_N \rangle - i \frac{A(q, q)}{q} \langle \lambda_d^J \lambda_d^J | V_{\text{opt}}^J (q', q) U^J (q, q) | \lambda_d \lambda_N \rangle +$$
$$+ \frac{2}{\pi} \int \frac{dq''}{q^2 - q''^2} \left( \langle \lambda_d^J \lambda_d^J | V_{\text{opt}}^J (q', q'') U^J (q'', q) | \lambda_d \lambda_N \rangle A(q'', q) - \langle \lambda_d^J \lambda_d^J | V_{\text{opt}}^J (q', q) U^J (q, q) | \lambda_d \lambda_N \rangle A(q, q) \right),$$

4
where the kinematical factor is
\[ A(q'', q) = q''^2 \frac{(E_q + E_{q''})((E_q^2 + E_{q''}^2)/2 - q^2 - q''^2 - M_N^2 - M_d^2)}{E_q^2 + E_{q''}^2}. \]

Then for the scattering amplitude in spin space one has (the incident particle is going along the z-axis):
\[
\langle \mu_1 \mu_N' | U(q, \theta) | \mu_2 \mu_N \rangle = 4\pi \sum J \sum_{\lambda_1} \left( -1 \right)^{\lambda_1 - \mu_d} (2J + 1) d^{1/2}_{\mu_d, \lambda_1} (\theta) d^{1/2}_{\lambda_1, \mu_1} (\theta) \\
\times d^J_{\lambda_1 - \lambda_N, \mu_N + \mu_d} (\theta) \langle \lambda_1 | U^J (q, q) | \lambda_N \rangle - \mu_d \mu_N. \]

It was found that at \( T_{lab} = 880 \text{ MeV} \) a convergent result is obtained at \( J = 37/2 \).

3 Results

The calculation of the polarization observables is performed with to different deuteron wave functions. The first one is a wave function derived in the meson-exchange Bonn-CD model [15]. The general trait of wave functions of this kind, derived from the most modern \( NN \) potentials, is their depletion at small distances. The other possible choice is the wave function with a nodal behavior. This node corresponds to a so-called forbidden state in \( NN \) system as a consequence of the six-quark dynamics and the fact that the mostly symmetric six-quark state \( |s^6\rangle \) has a small \( NN \) component [16]. As one of the representatives of such a wave functions with a nodal behavior can serve the wave function of dressed bag (DB) model of Moscow-Tuebingen group [17]. In Fig. 2 a comparison of two calculations with these different wave function is presented. As one can see, the remarkable difference in the polarization observables occurs at large scattering angles at which the high momentum component of the wave functions is probed. It should be noted however, that even if we can discriminate this two choices of the wave functions by experimental data, to obtain an ultimate conclusion, which of the models are preferable, one has to include in calculations a corresponding \( 3N \) force. And the \( 3N \) forces are different in these two concepts.

In Fig. 3 the influence of rescattering and break up channel on the polarization observables at the deuteron energy 880 MeV is shown. It is evident that taking into account the break up process or double-scattering in the optical potential is inevitable to obtain quantitative predictions at these energies. The rescattering terms which are imbedded in the \( pd \) scattering equation is not so pronounced and much less important, however not negligible.

Finally, the contribution from different partial waves in \( NN \) T-matrix is examined, see Fig. 4. The full calculation takes into account all \( NN \) partial waves that parameterized in SAID partial wave analysis [14], i.e. \( J_{NN} = 7 \). The other two calculations are done with total angular momentum up to \( J_{NN} = 5 \) - dashed curve and \( J_{NN} = 3 \) - dotted curve. Although at some scattering angles the differences between the observables are very small, the convergence is only partial. But it seems that \( J_{NN} = 7 \) is sufficient angular momentum at this energy range.
4 Summary and Conclusions

A calculation of the deuteron polarization observables $A_{yy}^d$, $A_{xx}$ and $A_{xz}$ through an optical potential model for elastic proton-deuteron scattering at the incident deuteron energy $E_d = 880$ MeV was presented. Under investigation were the double-scattering mechanism in the optical potential, the effect from precise treatment of two-body unitarity in proton-deuteron Lippman–Schwinger equation, the influence of various $NN$-partial waves and as well a comparison of calculations with two different deuteron wave-functions derived from the Bonn-CD $NN$-potential model \cite{15} and DB model of Moscow-Tuebingen group \cite{17}. For the $NN$ input, the model independent approach in which nucleon-nucleon $T$-matrix is taken to be on-shell and evaluated in the so-called Breit frame, was used. The calculation was carried out in the helicity partial wave decomposition and total angular momenta up to $J = 37/2$ in proton-deuteron system seem to be sufficient to obtain a convergent result. As for $NN$-partial waves, we took all SAID partial waves up to $J_{NN} = 7$. The large effect from double-scattering mechanism at 880 MeV as well as the large dependency on the short-range behavior of the deuteron wave function were observed.

This work is partly supported by the Russian Foundation for Basic Research, grant 04-02-17107a.

References

[1] W. Glöckle, H. Witala, D. Hüber, H. Kamada, J. Golak, Phys. Rep. 274, 107 (1996).
[2] K. Sekiguchi et al., Phys. Rev. C 70, 014001 (2004).
[3] Yu.N. Uzikov., JETP Lett. 75, 5 (2002).
[4] J. Fujita and H. Miyazawa, Prog. Theor. Phys. 17, 360 (1957).
[5] B.S. Pudliner, V.R. Pandharipande, J. Carlson, S.C. Pieper, and R.B. Wiringa, Phys. Rev. C 56, 1720 (1997).
[6] S.A. Coon M. T. Peña, Phys. Rev. C 48, 2559 (1993).
[7] A. Deltuva, R. Machleidt, and P.U. Sauer, Phys. Rev. C 68, 024005 (2003).
[8] K.O. Eyser, R. Machleidt, W. Scobel, and the EDDA Collaboration, Eur. Phys. J. A 22, 105 (2004).
[9] H. Liu, Ch. Elster, and W. Glöckle, Phys. Rev. C 72, 054003 (2005).
[10] Yu. Uzikov., PEPAN 29, 1405 (1998).
[11] T. Uesaka, V.P. Ladygin, L.S. Azhgirey, et al., PEPAN, Letters 3, 57 (2006).
[12] S.A. Gurvitz, J.-P. Dedonder, and R.D. Amado, Phys. Rev. C 19, 142 (1979).
[13] J.A. McNeil, L. Ray, S.J. Wallace, Phys. Rev. C 27, 2123 (1983).
[14] R.A. Arndt, I.I. Strakovsky, R.L. Workman, Phys. Rev. C62, 034005 (2000).

[15] R. Machleidt, Phys. Rev. C63, 024001 (2001).

[16] A.M. Kusainov, V.G. Neudatchin, and I.T. Obukhovsky, Phys. Rev. C44, 2343 (1991).

[17] A. Faessler, V.I. Kukulin, M.A. Shikhalev, Ann. Phys. 320, 71 (2005).
Figure 2: Deuteron vector and tensor polarization observables at the energy $E_d = 880$ MeV in lab. The solid and dashed curves are calculations with the deuteron wave function derived in the Bonn-CD model and DB model of NN interaction respectively.
Figure 3: The effect of a break up channel in the optical potential - dashed curve, and when all rescattering mechanisms are omitted - dotted curve. The solid curve is a full calculation with Bonn-CD deuteron wave function.
Figure 4: The results of calculations that take into account different amount of partial waves in \( NN \) \( T \)-matrix. The solid curve is a result when all \( J_{NN} = 7 \) SAID partial waves are included in calculation, whereas dashed and dotted curves are the calculations with only \( J_{NN} = 5 \) and \( J_{NN} = 3 \) respectively.