The Hubble constant and dark energy from cosmological distance measures

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Abstract. We study how the determination of the Hubble constant from cosmological distance measures is affected by models of dark energy and vice versa. For this purpose, constraints on the Hubble constant and dark energy are investigated using the cosmological observations of cosmic microwave background, baryon acoustic oscillations and type Ia supernovae. When one investigates dark energy, the Hubble constant is often a nuisance parameter; thus it is usually marginalized over. On the other hand, when one focuses on the Hubble constant, simple dark energy models such as a cosmological constant and a constant equation of state are usually assumed. Since we do not know the nature of dark energy yet, it is interesting to investigate the Hubble constant assuming some types of dark energy and see to what extent the constraint on the Hubble constant is affected by the assumption concerning dark energy. We show that the constraint on the Hubble constant is not affected much by the assumption for dark energy. We furthermore show that this holds true even if we remove the assumption that the universe is flat. We also discuss how the prior on the Hubble constant affects the constraints on dark energy and/or the curvature of the universe.

Keywords: dark energy theory, classical tests of cosmology

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1. Introduction

Recent precise cosmological observations provide us with a large amount of data to probe the evolution and the present state of the universe. We now customarily extract the information from them by constraining cosmological parameters. The Hubble constant, the expansion rate of the universe at present, is one of the most important cosmological parameters and which is measured in various ways.

In most studies which consider the constraint on the Hubble constant from cosmological observations, a cosmological constant is often assumed as dark energy. However, the nature of dark energy is not understood yet, which is one of the challenging problems in cosmology today. Thus the constraints on the Hubble constant should also be considered assuming some possible dark energy models other than a cosmological constant. Since the energy density of dark energy can change some distance measures which have been used to constrain the Hubble constant, the assumption concerning dark energy is expected to have an influence on the constraint on it. In some works, the constraint on the Hubble constant is obtained without assuming a cosmological constant (e.g. [1, 2]).
but a constant equation of state is still assumed. However, a lot of models of dark energy proposed so far have a time-varying equation of state and most of the recent works on dark energy accommodate its time dependence in some way. Thus, in light of these considerations and the importance of the Hubble constant, it is interesting to study constraints on the Hubble constant including time-evolving dark energy equations of state. This is one of the issues which we are going to investigate in this paper.

On the other hand, notice that the Hubble constant is often treated as a nuisance parameter over which we marginalize when we investigate dark energy since the parameters for dark energy themselves are those of interest in such a case. However, it should be mentioned that, when one studies the nature of dark energy, observations of distance measures such as the angular diameter distance which is relevant to the position of acoustic peaks in the cosmic microwave background (CMB) power spectrum and the scale of baryon acoustic oscillations (BAO) are often used. Since the Hubble constant affects such distance measures, it is expected that the prior on the Hubble constant can affect constraints on the nature of dark energy. Furthermore, the Hubble constant can also be determined with the cosmic distance ladder measurements independently from the cosmological distance measurements mentioned above. Hence, investigating dark energy with some priors on the Hubble constant is also an interesting subject, which is discussed in this paper too.

To study the issues mentioned above, we study the constraints on the Hubble constant and dark energy using the observations of CMB, BAO and type Ia supernovae (SN). For dark energy, we assume some types of time-varying equations of state as well as the case with a constant equation of state. By assuming several priors on the Hubble constant and dark energy equation of state, we can see how the determination of one of them can affect that of the other. In addition, we study them allowing a non-flat universe to make our analysis more general.

The structure of this paper is as follows. In the next section, we summarize our method for obtaining constraints from cosmological observations. Some parameterizations of dark energy adopted in this paper are also briefly explained. The data used for the analysis are mentioned there too. In section 3, we present our results and discuss some implications of our results for the study of the Hubble constant, dark energy and the curvature of the universe. In the final section, we summarize our results and give the conclusion. In the appendix, we give some quantitative explanations how the combinations of CMB, BAO and SN can break degeneracies among the Hubble constant, matter density, dark energy parameters and the curvature of the universe.

2. Dark energy parameterizations and method

In this section, we explain the method to constrain the cosmological parameters such as the Hubble constant and dark energy equation of state. We use the data from CMB, BAO and SN. Before we describe the method in detail, first we mention the dark energy parameterizations adopted in this paper, which accommodate the time variations of its equation of state. Among various possible parameterizations of the dark energy equation of state, we use the simple and often-used ones. The first parameterization adopted in the following analysis is [3, 4]

\[
w_X = w_0 + (1 - a)w_1 = w_0 + \frac{z}{1 + z}w_1,
\]  

(1)
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which varies in proportion to the scale factor $a$ (normalized as $a = 1$ at the present epoch). If we define dark energy as a fluid which can accelerate the universe, this can motivate the following prior:

$$w_X \leq -\frac{1}{3}. \quad (2)$$

For the particular parameterization equation (1), this is ensured by forcing $w_0$ and $w_1$ to satisfy

$$w_0 + w_1 \leq -\frac{1}{3}, \quad w_0 \leq -\frac{1}{3}. \quad (3)$$

In principle, a dark energy fluid can have its equation of state larger than $-1/3$. Thus a weaker prior $w_X \leq 0$ is sometimes adopted, requiring that dark energy does not dominate the universe at early time, say at the epoch of recombination, since such early domination of dark energy is apparently inconsistent with cosmological data. However, this prior allows the case with $w_X \sim 0$ where dark energy behaves in almost the same way as dark matter as far as the background evolution is concerned. In such a case, dark energy can be considered as a part of dark matter, which complicates the interpretation of the constraints on dark energy\(^3\). With the prior equation (2), the dark energy cannot be a dominant component of the universe at early times and even its fraction of the total energy density is negligible. Thus we can safely avoid the complication mentioned above by using the prior equation (2). We provide more detailed discussion on the effects of dark energy perturbation on the CMB power spectrum in appendix A.

We also adopt another type of dark energy parameterization:

$$w_X(z) = \begin{cases} \tilde{w}_0 + \frac{\tilde{w}_1 - \tilde{w}_0}{z_*} z & \text{for } z \leq z_* \\ \tilde{w}_1 & \text{for } z \geq z_* \end{cases}, \quad (4)$$

where we interpolate the value of $w_X$ linearly with respect to the redshift $z$ from the present epoch back to some transition redshift $z_*$. $w_X$ becomes $\tilde{w}_0$ at $z = 0$ and $\tilde{w}_1$ for $z \geq z_*$. The case with $\tilde{w}_0 = \tilde{w}_1 = -1$ corresponds to a cosmological constant. The prior equation (2) is satisfied by setting $\tilde{w}_0 \leq -1/3$ and $\tilde{w}_1 \leq -1/3$. This parameterization is essentially the same as the one which has a linear dependence on the redshift $z$ such as $w_X = w_0 + w_b z$. In fact, this form is adopted in much of the literature with a cutoff at some redshift to avoid large values of $w_X$ at $z \sim 1000$ which is relevant to the CMB constraint [5]–[7]. In the following, we consider several values of $z_*$ to see how the dark energy parameterization affects the constraints on the Hubble constant and vice versa. We note that this parameterization assumes a finite derivative of $w_X$ at the present epoch so we cannot probe the dark energy model which behaves like a cosmological constant (or a constant $w_X$) from some earlier epoch to the present but variable before that epoch. Also, we should note that, although this parameterization appears to have three parameters, we regard it as a family of two-parameter models labeled by $z_*$. Namely, we consider several two-parameter models each with a different value of $z_*$. Such a distinction among

\(^3\) This holds true when one considers the background evolution alone, which is the case of the present paper. If we include the information of perturbation of dark energy, the effect of dark energy can be distinguishable even if its equation of state is almost the same as that of dark matter. In that case, we need a full CMB angular power spectrum analysis to obtain the constraint from CMB but such an analysis is beyond the scope of this paper.
dark energy parameters may be artificial, but it is simpler and sufficient for our goal to show how priors for the Hubble constant and ways of parameterizing the dark energy equation of state (and the assumption for the curvature of the universe) could influence a conclusion drawn from the analysis of CMB, BAO and SN data.

Now we give detailed descriptions of the method how we make use of observational data. Incidentally, the model parameter fitting to the data is performed by $\chi^2$ minimization. We use the Brent method \[8\] extended to multi-parameters, as described in \[9\], to search a minimum efficiently.

2.1. CMB

To fit a model to the CMB data, we first note that here we do not use the whole information of the CMB power spectrum but use only the acoustic scale $\theta_A$ and the matter density $\omega_m$ as equation (18).\footnote{Recently, \[10,11\] noted that it is better to use $\ell_a = \pi/\theta_A$ and $R = \sqrt{\Omega_m H_0^2 r_\theta(z_{\text{rec}})}$ simultaneously to obtain the constraint from CMB. In this paper we use a rather conventional way of using $\theta_A$ and $\omega_m$ as in, for example, \[12,13\], but both ways make use of the same features of the CMB power spectrum, the peak position and height, and should give similar result. We checked that the constraints on the $\Omega_m - h$ plane from these two methods are almost the same for the case with a cosmological constant.}

This is because if we include the effects of perturbation of dark energy and perform a full CMB angular power spectrum analysis to obtain the constraint from CMB, it would be very time-consuming. One of the purposes of this paper is to investigate cosmological constraints assuming various dark energy parameterizations. Thus, in this case, a much faster method is preferable and it is known that, if dark energy dominates the universe only at late time, the constraint derived from the information on the background evolution captures well the nature of dark energy. This condition is satisfied by adopting the prior equation (2).

The acoustic scale $\theta_A$ which defines the characteristic angular scale of the acoustic oscillations is written as

$$\theta_A = \frac{r_s(z_{\text{rec}})}{r_\theta(z_{\text{rec}})}, \quad (5)$$

$\theta_A$ is given once we determine the comoving angular diameter distance to the last scattering surface $r_{\theta}(z_{\text{rec}})$ and the sound horizon at the recombination epoch $r_s(z_{\text{rec}})$, where $z_{\text{rec}}$ is the redshift of the epoch of recombination. The comoving angular diameter distance to the last scattering surface is given as

$$r_{\theta}(z_{\text{rec}}) = \frac{1}{H_0 \sqrt{|\Omega_k|}} S \left( \sqrt{|\Omega_k|} \int_0^{z_{\text{rec}}} \frac{dz'}{H(z')/H_0} \right), \quad (6)$$

where $S$ is defined as $S(x) = \sin(x)$ for a closed universe, $S(x) = \sinh(x)$ for an open universe and $S(x) = x$ with the factor $\sqrt{|\Omega_k|}$ being removed for a flat universe. $H_0$ is the Hubble constant and we sometimes denote it using a renormalized quantity $h$ defined as $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$. The expansion rate $H(z)$ is given as

$$H^2(z) = H_0^2 \left[ \Omega_r (1 + z)^4 + \Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_X \exp \left( 3 \int_0^z (1 + w_X(z)) \frac{dz}{1 + z} \right) \right], \quad (7)$$
where \( \Omega_r, \Omega_m, \Omega_k \) and \( \Omega_X \) represent the present values of the energy density of radiation, matter, curvature and dark energy normalized by the critical density. Note that \( \Omega_k = 1 - \Omega_r - \Omega_m - \Omega_X \). The sound horizon at recombination is

\[
r_s(z_{\text{rec}}) = \int_0^{a_{\text{rec}}} \frac{c_s}{a^2 H} \, da,
\]

where \( a_{\text{rec}} = 1/(1 + z_{\text{rec}}) \) and \( c_s \) is the sound speed of the photon–baryon fluid:

\[
c_s^2(a) = \frac{\dot{p}_\gamma}{\rho_\gamma + \rho_b} = \frac{1}{3(1 + R)},
\]

with \( R = 3\rho_b/4\rho_c \) being the scale factor normalized to 3/4 at the photon–baryon equality. The radiation-to-matter and baryon-to-photon ratios at the epoch of recombination, \( r_{\text{rec}} \) and \( R_{\text{rec}} \), are given as

\[
r_{\text{rec}} = 0.042 \omega_m^{-1} (z_{\text{rec}}/10^3), \quad \text{(11)}
\]
\[
R_{\text{rec}} = 30\omega_b (z_{\text{rec}}/10^3)^{-1}. \quad \text{(12)}
\]

In fact, the redshift at the epoch of recombination slightly depends on the energy density of baryon and matter. We include the dependence by adopting the fitting formula \( z_{\text{rec}} \) [16]:

\[
z_{\text{rec}} = 1048[1 + 0.00124 \left[ 1 + 0.23 \omega_b \right]\left[ 1 + g_1 \omega_m^2 \right]], \quad \text{(13)}
\]

where the functions \( g_1 \) and \( g_2 \) are given as

\[
g_1 = 0.0783 \omega_b^{-0.238} \left[ 1 + 39.5 \omega_b^{0.763} \right]^{-1}, \quad \text{(14)}
\]
\[
g_2 = 0.560 \left[ 1 + 21.1 \omega_b^{1.81} \right]^{-1}. \quad \text{(15)}
\]

We fix the value of \( \omega_b \) with the mean value from the WMAP3 analysis \( \omega_b = 0.02229 \) which is obtained for the \( \Lambda \)CDM model.

Once we give the Hubble constant, matter density, baryon density and dark energy parameters, \( \theta_A \) can be calculated. To constrain the cosmological parameters, we use the value of \( \theta_A \) reported by WMAP3 for \( \Lambda \)CDM

\[
\theta_{A, \text{obs}} = 0.5952^\circ \pm 0.0021^\circ. \quad \text{(16)}
\]

In addition to \( \theta_A \), we also use the prior for \( \omega_m \) which is given by WMAP3 for \( \Lambda \)CDM as

\[
\omega_{m, \text{obs}} = 0.1277 \pm 0.008. \quad \text{(17)}
\]

Notice that the values of \( \theta_{A, \text{obs}} \) and \( \omega_{m, \text{obs}} \) are almost unchanged even if we consider a model with a constant equation of state or a non-flat universe [1, 11].
Using $\theta_A$ and $\omega_m$, we calculate the $\chi^2$ from CMB as

$$\chi^2_{\text{CMB}} = (\theta_A - \theta_{A,\text{obs}})^2 + (\omega_m - \omega_{m,\text{obs}})^2 \sigma^2_{\theta_A} + \sigma^2_{\omega_m}. \quad (18)$$

Roughly speaking, in this way, although we are not using the whole information of the CMB power spectrum but using only the information of the first peak position and height (after we extracted $\omega_b$ from the second peak height to calculate $\theta_A$), since $\theta_A$ and $\omega_m$ are determined from such different, horizontal and vertical, features of the CMB power spectra, the $\chi^2$ for the parameters we concern in this paper can be constructed as above. This is explicitly justified in section 3 for the flat $\Lambda$CDM model. Furthermore, since the effects of the curvature of the universe and dark energy equation of state on the CMB power spectrum mainly appear as the change of $\theta_A$, we can use this $\chi^2_{\text{CMB}}$ for constraints involving these parameters.

### 2.2. BAO

We also use the baryon acoustic oscillation scale measured by SDSS [25]. To take the BAO data into account, we use the parameter $D_V$ which is defined as [25]

$$D_V(z) = \left[ r_\theta(z)^2 \frac{z}{H(z)} \right]^{1/3}, \quad (19)$$

where $r_\theta(z)$ is the comoving angular diameter distance defined in section 2.1. This quantity depends slightly on the scalar spectral index $n_s$ and $\omega_b$. We take $n_s = 0.958$ and $\omega_b = 0.02229$ which are the mean values for the $\Lambda$CDM model from WMAP3 data alone and calculate $D_V(0.35)$ following the procedure of [25] to find $D_V(0.35) = 1402$ Mpc. Then we use

$$D_V(0.35)_{\text{obs}} = 1402 \pm 64 \text{ Mpc}, \quad (20)$$

for the constraint from BAO. The value of $\chi^2$ is calculated as

$$\chi^2_{\text{BAO}} = \frac{(D_V(0.35) - D_V(0.35)_{\text{obs}})^2}{\sigma^2_{D_V(0.35)}}. \quad (21)$$

We note that the measurement of $D_V(0.35)$ assumes a flat $\Lambda$CDM model and there is no explicit check that the value does not change in a non-flat model and/or dark energy models with $w_X \neq -1$. However, since the effect of non-flatness or dark energy is considered to be only the geometrical one (their effects on the perturbation do not affect

5 The curvature of the universe and dark energy equation of state also affect the CMB power spectrum on large scales through the late-time integrated Sachs-Wolfe effect, but since the error from the cosmic variance dominates on large scales, such an effect can be neglected in most cases [17, 18]. On the other hand, it should be mentioned that such large scale fluctuation may be interesting when one considers scenarios such as dark energy isocurvature fluctuation [19, 20], the nature of dark energy perturbation [18], [21]–[24] and so on.

6 In most work on dark energy, the so-called $A$ parameter is used. However, we use $D_V(z)$ which explicitly depends on the Hubble constant for the purpose of this paper. The relation between $D_V(z)$ and the $A$ parameter is

$$D_V(0.35) = \frac{A \times 0.35}{\sqrt{\Omega_m H_0^2}} = 3.0 \times 10^3 \left( \frac{A \times 0.35}{\sqrt{\omega_m}} \right) \text{ Mpc.}$$
the peak of the galaxy correlation function which is located at smaller scales), we can use $D_V(0.35)$ to probe the background evolution just as we use $\theta_A$ of CMB if the cosmology is not radically different from a flat $\Lambda$CDM model. This condition is considered to be satisfied in our analysis because we place the prior equation (2) discussed in the previous section and our resulting constraints as shown later are not far away from a flat $\Lambda$CDM model when we combine with CMB and SN.

2.3. SN

As for SN data, we calculate the distance modulus

$$\mu = m - M = 5 \log d_L + 25,$$

where $m$ is apparent magnitude and $M$ is absolute magnitude. Here $d_L$ is the luminosity distance in units of Mpc which is written as

$$d_L(z) = \frac{1 + z}{H_0 \sqrt{|\Omega_k|}} S \left( \sqrt{|\Omega_k|} \int_0^z \frac{dz'}{H(z')/H_0} \right).$$

Although the luminosity distance depends on the Hubble constant explicitly, its dependence is indistinguishable from the uncertainty in $M$. Thus when we calculate $\chi^2$, the dependence on $h$ vanishes by marginalizing over $M$ as a nuisance parameter.

For the analysis, we use 182 SNe from the Gold dataset of Riess et al (Gold06) [26] or 192 SNe of Davis et al (Davis07) [26]–[28]. In this paper, we present our results for these two datasets separately (namely we do not combine these two datasets).

The main difference between these two datasets is that Davis07 uses the first data release of the ESSENCE supernova survey [27,29] at the intermediate redshift. In detail, Gold06 consists of 119 SNe (38 SNe are nearby, $z \lesssim 0.05$) from the previous Gold dataset [30], 16 SNe which are recently discovered by the Hubble Space Telescope (HST) [26] and 47 SNe from the first data release of the SNLS project [31]. Davis07 includes 60 SNe from ESSENCE, 57 SNe from SNLS, 16 SNe of [26], 14 SNe discovered by HST which are included in the previous Gold dataset and 45 nearby ($z \lesssim 0.05$) SNe. Thus, 30 SNe which are discovered by HST and have relatively high redshifts are common in both datasets. Also there are 25 nearby SNe which they have in common. The SNLS first data release [31] has 73 SNe but Gold06 and Davis07 place a different criterion for the selection. We may roughly say that Davis07 is constructed from Gold06 by replacing SNe data at the intermediate redshifts which are discovered in earlier times by the recent ESSENCE data.

2.4. Hubble constant

One of the purposes of this paper is to constrain the Hubble constant using the cosmological observations introduced above. However, as mentioned in the introduction, the Hubble constant can be measured with other methods such as using the cosmic distance ladder measurements. We summarize the recent values obtained with the distance
Table 1. The values of the Hubble constant in units of \( \text{km s}^{-1} \text{ Mpc}^{-1} \) determined from cosmic distance ladder measurements.

| Refs          | \( H_0 \pm \text{(statistic)} \pm \text{(systematic)} \) |
|---------------|----------------------------------------------------------|
| Freedman et al [32] | 72 ± 3 ± 7                                                  |
| Sandage et al [33]  | 62.3 ± 1.3 ± 5.0                                          |
| Macri et al [34]   | 74 ± 3 ± 6                                                  |

ladder in Table 1. For a recent review on the Hubble constant, see, e.g., [35]. We would like to compare such \( H_0 \) values measured in a relatively direct way with those obtained from the measurements of CMB, BAO and SN. Furthermore we also investigate constraints on dark energy, putting a prior on the Hubble constant to see how the determination of the Hubble parameter affects the constraints on dark energy. Thus we also use these values as priors when deriving constraints on dark energy parameters and/or the curvature of the universe. When we use the prior on the Hubble constant, we include them by calculating the \( \chi^2 \) which is given by

\[
\chi^2_{H_0} = \frac{(H_0 - H_{0,\text{obs}})^2}{\sigma_{H_0}^2}.
\]

For the error \( \sigma_{H_0} \), since we do not obtain a significant effect with those of the current measurements given in Table 1, as will be briefly discussed in the following section (section 3.2), we take a hypothetical value \( \sigma_{H_0} = 2 \text{ km s}^{-1} \text{ Mpc}^{-1} \). This value is motivated by the expected accuracy of 1% [34] which could be obtained through measuring the maser distance to a large number of galaxies in the Hubble flow by planned radio telescopes such as the square kilometer array (SKA) [38]. We adopt a slightly more conservative value than that.

3. Results

Now we discuss the constraints on the Hubble constant assuming some dark energy models including time-evolving dark energy equations of state described in section 2. We also investigate how the constraints on dark energy parameters are affected by the assumption of the Hubble constant. Effects of the curvature of the universe in constraining these parameters and the constraint on itself are also investigated.

3.1. Constraint on the Hubble constant

3.1.1. Case with a cosmological constant. First we show the constraints in the \( \Omega_m - h \) plane for the case with a cosmological constant in figure 1. In the figure, we show the constraints from several data sets, i.e. the cases with CMB alone, CMB + BAO, CMB + SN and

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7 In particular, interesting constraints on the Hubble constant are obtained via gravitational lens time delays and the Sunyaev–Zel’dovich effect, although the uncertainties are comparable to or slightly larger than those of the distance ladder measurements. See [36] and [37] for the recent measurements by these methods. Since these two methods in principle depend also on a dark energy model, their future improvements are expected to be useful for probing the cosmological constraints on the Hubble constant and dark energy parameters independently from CMB, BAO and SN.
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Figure 1. Contours of 1σ and 2σ allowed regions in the Ω_m–h plane for the case with a cosmological constant in a flat universe. (a) CMB alone, (b) CMB + BAO, (c) CMB + SN and (d) CMB + BAO + SN. In panels (c) and (d), we treat the SN data from Gold06 (red solid line) and Davis07 (blue dashed line) separately; thus two different constraints are shown.

CMB + BAO + SN. We do not show the constraint from SN data alone and BAO data alone here. Since, for SN data, the dependence of the Hubble constant on the luminosity distance is absorbed into the uncertainty in the absolute magnitude of SN, which is marginalized over, we cannot obtain the information on \( H_0 \) from SN alone. For the constraint from BAO, we use the parameter \( D_V \) for the analysis in this paper, which cannot constrain the Hubble constant much by itself from the currently available data. Thus we only show the constraints from CMB alone and the combinations of CMB and other datasets.

For the analysis with CMB data alone, our method gives consistent results with those obtained by the WMAP team who uses information of the entire CMB power spectrum. For the marginalized 1σ values of \( h \) and \( \Omega_m \), theirs are \( h = 0.732^{+0.031}_{-0.032} \) and \( \Omega_m = 0.241 \pm 0.034 \), which are satisfactorily close to our values \( h = 0.729 \pm 0.035 \) and \( \Omega_m = 0.244 \pm 0.038 \). Closer comparison reveals that their two-dimensional contours in this plane (figure 10 in [1]) are somewhat larger than ours (figure 1(a)) but the difference scarcely affects the estimation of \( h \) and \( \Omega_m \) as mentioned above. This small difference arises most likely because we fix \( \omega_b \) to the WMAP central value and our method cannot incorporate the uncertainty due to the value of \( n_s \). As for the SN datasets, we show our results with Gold06 and Davis07 separately. In table 2, we report the central values and 1σ errors from various combinations of datasets for the case with a cosmological constant. The values of the minimum \( \chi^2 \) for each case are also shown. When we combine SN with CMB, the central value of \( h \) becomes smaller since the constraint on \( \Omega_m \) and \( h \) lies along
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Figure 2. $2\sigma$ constraints are shown for the case with a constant equation of state for dark energy. We use the data from CMB alone (red solid line), CMB + BAO (green dashed line), CMB + SN (blue dotted line) and all combined (purple shaded region). As for the treatment of SN data, we use those from Gold06 (left panel) and Davis07 (right panel). The value of $w_0$ is marginalized over $-3 \leq w_0 \leq -1/3$.

Table 2. The central values and $1\sigma$ errors from several combinations of datasets for the case with a cosmological constant. The values of the minimum $\chi^2$ and the numbers of data are also shown.

| Dataset                  | $h$ ($1\sigma$) | $\Omega_m$ ($1\sigma$) | $\chi^2_{\text{min}}$ | # of data |
|--------------------------|------------------|-------------------------|-------------------------|-----------|
| CMB alone                | 0.729 ± 0.035    | 0.244 ± 0.038           | 0.00                    | 2         |
| CMB + BAO                | 0.704 ± 0.027    | 0.270 ± 0.033           | 1.41                    | 3         |
| CMB + SN (Gold06)        | 0.682 ± 0.021    | 0.299 ± 0.028           | 162.0                   | 184       |
| CMB + SN (Davis07)       | 0.714 ± 0.022    | 0.259 ± 0.025           | 195.9                   | 194       |
| CMB + BAO + SN (Gold06)  | 0.678 ± 0.019    | 0.304 ± 0.026           | 162.2                   | 185       |
| CMB + BAO + SN (Davis07) | 0.704 ± 0.019    | 0.269 ± 0.023           | 197.0                   | 195       |

the region around $\Omega_m h^2$ being constant (for the case with WMAP3, $\Omega_m h^2 = 0.1277$) and SN data favor somewhat larger $\Omega_m$ than CMB. This is more conspicuous for Gold06 than Davis07. Another point is that the BAO data also favor a slightly smaller value of $h$. Hence when we use the dataset of CMB + BAO + SN (Gold06), the favored value of $h$ becomes considerably smaller.

3.1.2. Case with a constant equation of state. Next we show the constraints for the case with a constant equation of state for dark energy (which corresponds to the cases with $w_1 = 0$ in equation (1)) and with $\tilde{w}_0 = \tilde{w}_1$ in equation (4). In figure 2, the constraints in the $\Omega_m - h$ plane are shown after marginalizing over $w_0$. In the figure, $2\sigma$ constraints from CMB alone (red solid line), CMB + BAO (green dashed line), CMB + SN (blue dotted line) and all combined (purple shaded region) are shown. We treat the SN data from Gold06 and Davis07 separately; thus we show the constraints using them in the left and right panels in figure 2, respectively.

Since the dark energy equation of state directly affects the angular diameter distance, the allowed region from CMB alone becomes significantly larger compared to the case
with a cosmological constant, which means that there is a strong degeneracy among $\Omega_m$, $w_0$ and $h$. However, if we combine other datasets such as BAO and SN with CMB data, the allowed region becomes similar to that for the case with a cosmological constant (the region becomes larger but only a little). Although the preferred values of $w_0$ which is marginalized over in the figure are in general different for each observation for fixed $h$ and $\Omega_m$, only around the allowed region in the figure, those values from different observations happen to coincide to be $w_0 \sim -1$ (i.e. a cosmological constant). Thus we obtain almost the same result as the cosmological constant case even for the case where we marginalize over a constant equation of state. We see that the combination of the data from CMB + SN seems to be enough to constrain $h$ and $\Omega_m$ whereas the CMB + BAO constraint is much weaker. This is because the degeneracy curves in the $\Omega_m$–$w_0$ plane extend to almost the same direction for CMB and BAO, but those of CMB and SN are complementary (see figure 6 in [39]). Thus, as far as the degeneracy between $\Omega_m$ and $w_0$ is concerned, the combination of CMB + SN gives a severe constraint.

3.1.3. Case with time-evolving equations of state. In figure 3, we show the results for the case with the time-evolving dark energy equation of state parameterized as equation (1). In the figure, the values of $w_0$ and $w_1$ are marginalized over. As the case with a constant equation of state, although the allowed region from CMB data alone is significantly larger compared to that for a cosmological constant case, when other data are combined, the allowed region lies around the same region as the $\Lambda$CDM case and also it is not so significantly larger compared to that for the $\Lambda$CDM model. Notice that the combination of CMB + SN again already gives severe constraint in the $\Omega_m$–$h$ plane. This result shows that, even for a time-evolving equation of state, the SN data works well to break the degeneracy among $\Omega_m$ and dark energy parameters for the parameterization equation (1).

We also performed the same analysis using the parameterization given in equation (4) with $z_s = 0.5$ and the results are shown in figure 4. As seen from the figure, the
Figure 4. Same as figure 2 except we assume the dark energy equation of state as equation (4) with \(z^* = 0.5\) and marginalize over the values of \(\tilde{w}_0\) and \(\tilde{w}_1\) in the ranges \(-3 \leq \tilde{w}_0 \leq -1/3\) and \(-3 \leq \tilde{w}_1 \leq -1/3\).

Constraints on \(\Omega_m\) and \(h\) are almost the same even if we assume this type of dark energy parameterization. Furthermore it should be noticed that here again the combination of CMB + SN gives a severe constraint.

Although we have adopted just two types of parameterization of the dark energy equation of state, as far as we consider some typical types of dark energy evolution, it seems that dark energy models do not affect much the determination of the Hubble constant using the combined data of CMB, BAO and SN. As already mentioned and seen from figures 2, 3 and 4, CMB + SN gives sufficiently tight constraints in the \(\Omega_m - h\) plane. Meanwhile, the allowed region from CMB + BAO is rather broad and BAO does not seem to have much constraining power. (However, when we allow a non-flat universe, BAO becomes important, which we are going to discuss in section 3.1.4.) Thus we focus on the constraints from CMB and SN for a while and explain how the constraints can be obtained.

Notice that, since \(h\) and \(\Omega_m\) are degenerate with respect to \(\theta_A\), to break the degeneracy we assume the prior on \(\omega_m\) determined from the height of the CMB power spectrum. Thus the fact that \(h\) is well determined even if we allow some variations of dark energy models eventually indicates that the determination of \(\Omega_m\) is not affected much by the assumption for dark energy. (Also notice that, as already mentioned, the SN data cannot determine the Hubble constant since the dependence on \(h\) in the luminosity distance is totally indistinguishable from the uncertainty in the absolute magnitude of SN.) We remind ourselves that the SN data can give information of the background evolution from the present up to \(z_{SN}\) which represents the redshift of the furthest SN currently available (in observations used in the analysis, \(z_{SN} \approx 1.8\)). In this period, the background evolution is determined by dark energy and matter. Since the distance measure such as the luminosity distance involves some integration with respect to redshift, it is well known that there is a severe degeneracy among \(\Omega_m\) and dark energy parameters. However, we can break the degeneracy by combining the constraint from \(\theta_A\) which can also probe the background evolution earlier than \(z_{SN}\). In the epoch earlier than \(z_{SN}\), we can approximate that the universe is dominated by the matter component. Thus, knowing the distance between the present epoch and \(z_{SN}\) by the SN data, the matter density is
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Table 3. The central values and 1σ errors, along with the minimum value of $\chi^2$ from CMB + BAO + SN for the cases with various dark energy models. Here the dark energy parameters are marginalized over. The cases where the equation of state is always larger than −1 are also presented. A flat universe is assumed. The best fit values of dark energy parameters are also shown.

| Model                | $h$ (1σ)   | $\Omega_m$ (1σ) | $\chi^2_{\text{min}}$ | $w_0$ (\$\tilde{w}_0\$) | $w_1$ (\$\tilde{w}_1\$) |
|----------------------|------------|-----------------|------------------------|---------------------------|---------------------------|
| Cosmological constant| 0.678 ± 0.019 | 0.304 ± 0.026 | 162.2                  | —                         | —                         |
| $w_0$ ($w_1 = 0$)    | 0.667 ± 0.020 | 0.295 ± 0.027 | 160.2                  | -0.87                     | —                         |
| $w_0$ and $w_1$      | 0.656 ± 0.020 | 0.301 ± 0.026 | 158.3                  | -1.06                     | 0.72                      |
| $w_0$ and $w_1$ ($w ≥ -1$) | 0.656 ± 0.020 | 0.299 ± 0.027 | 158.5                  | -1.00                     | 0.60                      |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 0.1$) | 0.678 ± 0.023 | 0.279 ± 0.027 | 157.9                  | -2.49                     | -0.71                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 0.2$) | 0.666 ± 0.021 | 0.289 ± 0.026 | 157.5                  | -1.63                     | -0.68                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 0.5$) | 0.655 ± 0.022 | 0.299 ± 0.027 | 157.5                  | -1.21                     | -0.61                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 1.0$) | 0.649 ± 0.023 | 0.306 ± 0.029 | 158.1                  | -1.09                     | -0.52                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 2.0$) | 0.651 ± 0.022 | 0.307 ± 0.029 | 158.6                  | -1.02                     | -0.37                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 0.5$) ($w ≥ -1$) | 0.658 ± 0.022 | 0.294 ± 0.027 | 158.3                  | -1.00                     | -0.71                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 1.0$) ($w ≥ -1$) | 0.652 ± 0.023 | 0.301 ± 0.027 | 158.3                  | -1.00                     | -0.60                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 2.0$) ($w ≥ -1$) | 0.651 ± 0.023 | 0.305 ± 0.028 | 158.6                  | -1.00                     | -0.41                     |

| Model                | $h$ (1σ)   | $\Omega_m$ (1σ) | $\chi^2_{\text{min}}$ | $w_0$ (\$\tilde{w}_0\$) | $w_1$ (\$\tilde{w}_1\$) |
|----------------------|------------|-----------------|------------------------|---------------------------|---------------------------|
| Cosmological constant| 0.704 ± 0.019 | 0.269 ± 0.023 | 197.0                  | —                         | —                         |
| $w_0$ ($w_1 = 0$)    | 0.703 ± 0.020 | 0.266 ± 0.024 | 196.9                  | -0.98                     | —                         |
| $w_0$ and $w_1$      | 0.689 ± 0.025 | 0.272 ± 0.024 | 195.5                  | -1.16                     | 0.83                      |
| $w_0$ and $w_1$ ($w ≥ -1$) | 0.695 ± 0.022 | 0.265 ± 0.024 | 196.4                  | -1.00                     | 0.27                      |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 0.1$) | 0.707 ± 0.021 | 0.259 ± 0.024 | 195.5                  | -2.03                     | -0.85                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 0.2$) | 0.700 ± 0.020 | 0.264 ± 0.024 | 195.9                  | -1.40                     | -0.84                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 0.5$) | 0.693 ± 0.023 | 0.268 ± 0.024 | 195.9                  | -1.18                     | -0.77                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 1.0$) | 0.684 ± 0.025 | 0.274 ± 0.025 | 195.5                  | -1.15                     | -0.60                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 2.0$) | 0.682 ± 0.024 | 0.277 ± 0.026 | 195.4                  | -1.12                     | -0.38                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 0.5$) ($w ≥ -1$) | 0.698 ± 0.020 | 0.264 ± 0.024 | 196.6                  | -1.00                     | -0.90                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 1.0$) ($w ≥ -1$) | 0.693 ± 0.023 | 0.265 ± 0.023 | 196.3                  | -1.00                     | -0.80                     |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z = 2.0$) ($w ≥ -1$) | 0.689 ± 0.025 | 0.267 ± 0.023 | 196.2                  | -1.00                     | -0.63                     |

determined accurately from the distance measurement between $z_{\text{SN}}$ and $z_{\text{rec}}$. The point is that the distance between $z = 0$ and $z_{\text{SN}}$, where both of the matter and dark energy contribute, can be inferred by the SN data independently from the dark energy model. This is why the combination of SN and CMB can determine $\Omega_m$ in a dark energy model-independent manner. We will provide a more quantitative demonstration of this point in appendix B.1.

As we have just argued, the Hubble constant derived from CMB, BAO and SN is not affected much by the assumption for a dark energy model. Notice that, however, when we impose a prior on the Hubble constant by using a value, for instance, of the cosmic distance ladder measurement mentioned in section 2.4, the constraints on dark energy parameters can be affected by the prior on $H_0$. Naturally, its effect depends on the accuracy of the determination of the Hubble constant. We will return to this issue in section 3.2.
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Figure 5. 2σ constraints in the $\Omega_m$–$h$ plane without assuming a flat universe. The curvature is marginalized over in the range $-0.5 \leq \Omega_k \leq 0.5$. A cosmological constant is assumed for dark energy. Constraints from the data from CMB alone (red solid line), CMB + BAO (green dashed line), CMB + SN (blue dotted line) and all combined (purple shaded region) are shown. As for the treatment of the SN data, we use those from Gold06 (left panel) and Davis07 (right panel).

We summarize our results on the constraints on $h$ and $\Omega_m$ for time-evolving dark energy equations of state in table 3 along with the best fit values of the dark energy parameters. As discussed above, even if we assume a different dark energy parameterization, the constraints on $h$ and $\Omega_m$ are almost unchanged. Furthermore, even if we restrict ourselves to the dark energy model with its equation of state being larger than $-1$ in the course of the history of the universe, the results are not affected by this assumption as seen from table 3. It should be mentioned that two SN datasets (i.e., Gold06 and Davis07) give somewhat different values of the Hubble constant as discussed above. The difference due to the SN dataset is more significant than the one caused by the dark energy parameterization.

3.1.4. Case with a non-flat universe. Here we discuss the constraint on the Hubble constant without assuming a flat universe. When one considers the constraint on the Hubble constant from cosmological observations, a flat universe is often assumed. However, to make our analysis general, here we allow a non-flat universe to obtain the constraint on the Hubble constant. The constraint on the curvature of the universe itself with some priors on the Hubble constant will be discussed in section 3.3.

In figure 5, we show the constraints in the $\Omega_m$–$h$ plane for the case with a cosmological constant while marginalizing over $\Omega_k$ in the range $-0.5 \leq \Omega_k \leq 0.5$. We show the constraints from CMB alone (red solid line), CMB + BAO (green dashed line), CMB + SN (blue dotted line) and all combined (purple shaded region) separately. The case with marginalizing over a constant equation of state in addition is presented in figure 6. When we allow a non-flat universe, there is a strong degeneracy in the distance measures as in the case we assume a dynamical dark energy in a flat universe which can be seen in figures 2, 3 and 4. However, when all the data are combined, there is little difference between the constraints for the cases with a non-flat universe being allowed and a cosmological constant with a flat universe. This is because the...
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Figure 6. Same as figure 5 (i.e. the curvature is marginalized over) except that a constant equation of state is assumed and marginalized over in the range $-3 \leq w_0 \leq -1/3$.

combination of all three cosmological datasets works well to remove the degeneracy even if we consider the possibilities of a non-flat universe. On the other hand, when one focuses on the combination of two datasets such as CMB + BAO and CMB + SN, we can see some differences between a flat case and a non-flat case. Recall that when one assumes a flat universe, the combination of CMB and SN already gives a relatively severe constraint even for the case with time-varying equations of state but that of CMB + BAO is not so severe. For the case with a non-flat universe, in contrast to the flat universe case, the combination of CMB + SN does not give a severe constraint. When one allows the nonzero curvature of the universe, the degeneracy in CMB (here this means $\theta_A$) becomes worse: it involves $\Omega_m$, $\Omega_k$ and dark energy parameters. Thus just adding SN data is not enough to break this degeneracy. BAO is necessary to remove such degeneracy. In appendix B, we discuss this point in a quantitative way.

We have seen that the distance measures suffer from the degeneracies among parameters describing the dark energy equation of state and the curvature of the universe. Due to those degeneracies, if we use CMB alone, CMB + BAO and CMB + SN, we get different constraints, as clearly seen in figures 5 and 6. It seems to be necessary to combine all three observations to remove the degeneracies. Once that is done, the constraints are almost the same as the case with the flat $\Lambda$CDM model. In table 4, the central values and 1σ errors of $h$ and $\Omega_m$ are summarized for the cases with a cosmological constant, a constant equation of state and dark energy parameterizations given in equations (1) and (4) when a non-flat universe is allowed in the analysis. The table shows that the constraint on $h$ is almost unchanged even if we allow a non-flat universe under some types of dark energy parameterization.

3.1.5. Summary of constraints on the Hubble constant. In figure 7, we summarize some of the results for the constraints on $h$ and $\Omega_m$ presented in this section. See also tables 3 and 4 for more detailed constraints on $h$ and $\Omega_m$ and the best fit values of dark energy parameters and/or the curvature. As discussed above, even if we assume different types of dark energy parameterization, there is no considerable change in the
constraints on $h$ and $\Omega_m$, although the allowed region for the case with a cosmological constant and a flat universe is slightly smaller compared to the other cases. It is interesting to compare this result with the distance ladder measurements of $h$ (section 2.4). In particular, when all the data are combined, the central values of $h$ cannot be as low as Sandage’s central value of 0.62 and cannot be as high as Macri’s central value of 0.74 even if we relax the assumptions of a cosmological constant as dark energy and/or the flatness of the universe. Of course, since the current measurements of $h$ by the distance ladder have somewhat large systematic errors (see table 1), this must be taken only as a quick comparison. The most seemingly problematic case arises when we compare CMB + BAO + Davis07 and Sandage’s $h$, but their 1σ errors overlap. Our analysis shows that the allowed region of the Hubble constant from CMB, BAO and SN can be different from the WMAP flat ΛCDM value of $h = 0.73 \pm 0.03$ by more than 1σ, depending on the assumption for dark energy and the cosmic curvature, but $h < 0.59$ or $h > 0.76$ are not allowed at 2σ level which can be read off from tables 3 and 4. This conclusion is obtained for rather limited types of dark energy parameterization but the interpretation presented in the appendix leads us to speculate that this holds true for any dark energy parameterization.

It should also be noticed that the choice of the SN dataset makes a larger difference than that of the assumptions on dark energy and the curvature of the universe. The Gold06 data tend to give lower $h$ and higher $\Omega_m$ than the Davis07 data. Thus, there is much room for improvement in SN data as well as the distance ladder measurement of $h$.

| CMB + BAO + Gold06 | $h$ (1σ) | $\Omega_m$ (1σ) | $\chi^2_{\text{min}}$ | $\Omega_k$ | $w_0$ (\tilde{w}_0) | $w_1$ (\tilde{w}_1) |
|---------------------|---------|----------------|---------------------|----------|----------------|----------------|
| Cosmological constant | 0.644 ± 0.027 | 0.319 ± 0.029 | 159.8 | -0.014 | — | — |
| $w_0$ ($w_1 = 0$) | 0.649 ± 0.029 | 0.310 ± 0.032 | 159.5 | -0.010 | -0.93 | — |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 0.1$) | 0.656 ± 0.029 | 0.301 ± 0.031 | 158.3 | -0.000 | -1.05 | 0.72 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 0.2$) | 0.685 ± 0.041 | 0.276 ± 0.037 | 157.8 | 0.003 | -2.62 | -0.69 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 0.5$) | 0.680 ± 0.037 | 0.279 ± 0.034 | 157.3 | 0.009 | -1.80 | -0.59 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 1.0$) | 0.676 ± 0.032 | 0.279 ± 0.034 | 157.3 | 0.009 | -1.80 | -0.59 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 2.0$) | 0.654 ± 0.029 | 0.303 ± 0.032 | 158.5 | 0.006 | -1.02 | -0.33 |
| CMB + BAO + Davis07 | $h$ (1σ) | $\Omega_m$ (1σ) | $\chi^2_{\text{min}}$ | $\Omega_k$ | $w_0$ (\tilde{w}_0) | $w_1$ (\tilde{w}_1) |
| Cosmological constant | 0.677 ± 0.028 | 0.278 ± 0.025 | 195.6 | -0.011 | — | — |
| $w_0$ ($w_1 = 0$) | 0.674 ± 0.029 | 0.284 ± 0.029 | 195.5 | -0.012 | -1.04 | — |
| $w_0$ and $w_1$ | 0.677 ± 0.034 | 0.281 ± 0.033 | 195.4 | -0.008 | -1.10 | 0.41 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 0.1$) | 0.688 ± 0.036 | 0.272 ± 0.033 | 195.1 | -0.008 | -1.69 | -0.93 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 0.2$) | 0.680 ± 0.034 | 0.279 ± 0.033 | 195.4 | -0.010 | -1.20 | -0.97 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 0.5$) | 0.676 ± 0.034 | 0.282 ± 0.033 | 195.5 | -0.011 | -1.07 | -0.98 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 1.0$) | 0.678 ± 0.034 | 0.281 ± 0.033 | 195.4 | -0.008 | -1.09 | -0.83 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_s = 2.0$) | 0.677 ± 0.032 | 0.282 ± 0.032 | 195.4 | -0.005 | -1.10 | -0.53 |
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Figure 7. Contours 1σ and 2σ allowed regions in the $\Omega_m$–$h$ plane for the cases with: (a) a cosmological constant (same as figure 1(d)), (b) a constant equation of state $w_X$, marginalized over $w_X$, (c) the parameterization defined by equation (1), marginalized over $w_0$ and $w_1$, and (d) the parameterization defined by equation (4) with $z_*=0.5$, marginalized over $\tilde{w}_0$ and $\tilde{w}_1$. Constraints using the SN data from Gold06 (red solid line) and Davis07 (blue dashed line) are shown separately. In panels (e)–(h), we marginalize over $\Omega_k$ in addition to panels (a)–(d), respectively.

We need more precise measurement from both fields of observation to tell whether there is a discrepancy or not.

The errors on the Hubble constant (and on $\Omega_m$) are slightly larger for the cases with a non-flat universe than those with a flat universe which can be seen from tables 3 and 4. For a flat case, 1σ errors on $h$ are about 0.02 and, for a non-flat case, it is around 0.03. Therefore, when we use the current observations of CMB, BAO and SN,
we can expect that this level of accuracy is needed for the distance ladder measurement of the Hubble constant to give a meaningful external prior on \( h \) in constraining the dark energy parameters and the curvature of the universe. Assuming that such sensitivity on \( h \) is achieved in the future, we investigate its effects on the determination of the dark energy equation of state and the curvature of the universe in the following sections 3.2 and 3.3.

### 3.2. Constraint on dark energy equations of state

Here we discuss the constraints on parameters of dark energy which describe the time dependence of its equation of state. Our special emphasis is on investigating how they are affected by several external priors on the Hubble constant and/or by the assumption of the flat universe. In most analyses on this issue so far, a flat universe is usually assumed. Once one invokes the inflationary paradigm, the flatness assumption seems to be natural. However, the flatness itself should be tested including the uncertainties of dark energy. In this respect, when we investigate the equation of state for dark energy, we also remove the assumption of a flat universe to obtain a more conservative constraint. In particular, in the following, we discuss implications of the results taking a cosmological constant as a reference point. Since a cosmological constant is the simplest and most conventional model for dark energy (and it can fit the observations satisfactorily as shown below), it is useful for illustrating the roles of the priors.

First we assume the dark energy parameterization of equation (1). In figure 8, the constraints in the \( w_0-w_1 \) plane are shown for several cases. In panels (a)–(c), we marginalized over the values of \( \Omega_m \) and \( h \). In panels (d)–(f), we repeat the same analysis except that we allow a non-flat universe and marginalize over \( \Omega_k \). The best fit values of \( w_0 \) and \( w_1 \) and the minimum \( \chi^2 \) are summarized in tables 5 and 6. The best fit values of \( h \) and \( \Omega_m \) (and \( \Omega_k \) for the case of a non-flat universe in table 6) are also shown. In panels (b), (c), (e) and (f), we assume some priors on the Hubble constant to see how it can affect the determination of dark energy parameters. We impose Gaussian priors as \( h = 0.72 \pm 0.02 \) (panels (b) and (e)) and \( h = 0.62 \pm 0.02 \) (panels (c) and (f)) where we use Freedman’s and Sandage’s (see table 1) as the central values and hypothetical errors of 0.02.

The hypothetical errors adopted here are motivated from the results we obtained in section 3.1. As summarized in section 3.1.5, the uncertainties of \( h \) from the cosmological
Figure 8. 1σ and 2σ constraints from CMB + BAO + SN in the $w_0$–$w_1$ plane marginalizing over $\Omega_m$ and $h$ are shown for the cases with (a) no prior on the Hubble constant, (b) assuming a Gaussian prior on the Hubble constant $h = 0.72 \pm 0.02$ and (c) $h = 0.62 \pm 0.02$. In panels (d)–(f), we allow a non-flat universe and marginalize over $\Omega_k$ in addition to $\Omega_m$ and $h$. The black dashed lines show the boundary of the prior equation (3). The constraints using the SN datasets from Gold06 (red solid line) and Davis07 (blue dashed line) are shown separately.

Table 6. The best fit values for $\Omega_m$, $h$, $\Omega_k$, $w_0$ and $w_1$ for the analysis presented in panels (d)–(f) of figure 8. The minimum values of $\chi^2$ are also shown.

| CMB + BAO + Gold06 | $\chi^2_{\text{min}}$ | $\Omega_m$ | $h$ | $\Omega_k$ | $w_0$ | $w_1$ |
|-------------------|----------------------|-----------|-----|-----------|-------|-------|
| No prior          | 158.3                | 0.301     | 0.655 | 0.000     | -1.06 | 0.72  |
| Prior $h = 0.72 \pm 0.02$ | 161.3                | 0.263     | 0.702 | 0.014     | -1.02 | 0.69  |
| Prior $h = 0.62 \pm 0.02$ | 159.2                | 0.320     | 0.634 | -0.007    | -1.06 | 0.73  |

| CMB + BAO + Davis07 | $\chi^2_{\text{min}}$ | $\Omega_m$ | $h$ | $\Omega_k$ | $w_0$ | $w_1$ |
|-------------------|----------------------|-----------|-----|-----------|-------|-------|
| No prior          | 195.4                | 0.279     | 0.676 | -0.009    | -1.10 | 0.39  |
| Prior $h = 0.72 \pm 0.02$ | 196.6                | 0.254     | 0.709 | 0.006     | -1.14 | 0.80  |
| Prior $h = 0.62 \pm 0.02$ | 197.3                | 0.315     | 0.638 | -0.026    | -0.93 | -0.92 |

observations are 0.02–0.03. Thus we can expect that this level of accuracy on the Hubble constant prior is required to have an influence on constraining the dark energy sector. In fact, we have also made the analysis adopting a Gaussian prior with the current errors as shown in table 1 but we cannot see a noticeable difference compared with the case of no prior on the Hubble constant. It should be noted that, as mentioned in section 2.4, this level of improvement on the Hubble constant determination is not unimaginable in the near future [34]. Note that we can expect that other observations (CMB, BAO and SN) would be more precise at the time when this level of accuracy in the Hubble constant determination is realized. Thus, our combining the hypothetical Hubble priors and the current observational data is not a forecast of future status in a strict sense but should
rather be regarded as an illustration of how the Hubble external priors affect conclusions on properties of dark energy and the curvature of the universe.

Let us start by looking at the constraints from the present cosmological observations assuming no Hubble prior. When we assume a flat universe, although the best fit value is slightly away from a cosmological constant (especially in the positive direction of \( w_1 \); see table 5), a cosmological constant is within the 2\( \sigma \) allowed regions for both SN datasets as shown in figure 8(a). This situation holds when we allow a non-flat universe and marginalize over \( \Omega_k \) as shown in figure 8(d). We can understand this by noticing that the region closely around a flat universe is favored in this parameterization with no Hubble prior as seen in table 6 or in the analysis presented in the following section 3.3.

We now turn to the constraints when we assume the external priors on the Hubble constant. The constraints in the \( w_0-w_1 \) plane for this case are shown in figures 8(b), (c), (e) and (f). Probably the most interesting feature is seen in panel (c): in a flat universe, if Sandage’s value is confirmed at this level, a cosmological constant (\( w_0 = -1 \) and \( w_1 = 0 \) in the figure) would be rejected at nearly the 2\( \sigma \) level in this parameterization. However, as shown in panel (f), if we allow a non-flat universe and marginalize over \( \Omega_k \), a cosmological constant is well within the allowed regions. This is an example which shows the importance of the priors on the Hubble constant and the curvature of the universe when we probe the nature of dark energy.

Next we show the constraints on the dark energy parameters for the parameterization equation (4) in figures 9 and 10. Although this parameterization includes three parameters, we present our result in the \( \tilde{w}_0-\tilde{w}_1 \) plane fixing \( z_* \) to several values: 0.1, 0.2, 0.5, 1.0 and 2.0. Namely we regard \( z_* \) as labeling the model expressed as equation (4) which has two parameters \( \tilde{w}_0 \) and \( \tilde{w}_1 \), and do not marginalize over \( z_* \). In figure 9, a flat universe is assumed and in figure 10 we allow a non-flat universe and marginalize over the curvature. The best fit parameter values are shown respectively in tables 7 and 8. We also impose some priors on the Hubble constant as is done for the analysis of the dark energy parameterization of equation (1).

For the case with no Hubble prior, as the panels in the left column in figures 9 and 10 indicate, a cosmological constant (\( \tilde{w}_0 = -1 \) and \( \tilde{w}_1 = -1 \)) is in the allowed regions with or without the assumption of a flat universe. This is because, as is the case with the parameterization equation (1) discussed above, the region around the flat universe is more or less favored in this parameterization with no Hubble prior as seen in tables 7 and 8.

However, if we impose some external priors on the Hubble constant, the constraints in the \( \tilde{w}_0-\tilde{w}_1 \) plane receive some interesting effects of the external priors on the Hubble constant and the assumption for the curvature of the universe. We can see them from the panels in the middle and right columns in figures 9 and 10.

For example, when we impose \( h = 0.62 \pm 0.02 \) for the \( z_* = 1.0 \) model assuming a flat universe, a cosmological constant is not within the 2\( \sigma \) allowed region for both SN datasets (see figure 9). However, if we allow a non-flat universe and marginalize over \( \Omega_k \), a cosmological constant is allowed with the 2\( \sigma \) level (see figure 10). This is similar to the tendency which is seen in the case of the parameterization equation (1) discussed above. Another interesting point can be observed in this parameterization, which is actually an opposite tendency to the one just mentioned. When we impose \( h = 0.72 \pm 0.02 \) for the \( z_* = 0.5 \) model assuming a flat universe, a cosmological constant is well within the allowed regions for both SN datasets (see figure 9). However, when we use the Gold06 dataset,
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Figure 9. 1σ and 2σ constraints from CMB + BAO + SN in the $\tilde{w}_0$–$\tilde{w}_1$ plane marginalizing over $h$ and $\Omega_m$ are shown for several values of $z_\star$. For each $z_\star$, we show the cases with no prior on the Hubble constant and Gaussian priors $h = 0.72 \pm 0.02$ and $h = 0.62 \pm 0.02$. A flat universe is assumed. The constraints using the SN datasets from Gold06 (red solid line) and Davis07 (blue dashed line) are shown separately.

if we allow a non-flat universe and marginalize over $\Omega_k$, a cosmological constant is out of the 2σ allowed region (see figure 10). This is considered to be an infrequent case in which relatively large $\Omega_k$ gives a significantly better fit than a flat universe ($\chi^2_{\text{min}}$ decreases by 4.3 in this case) with the equation of state well away from a cosmological constant, but we should bear in mind that such a case could happen.

In summary, we have seen that the prior on the Hubble constant, the way to parameterize the equation of state and the assumption for the curvature of the universe are all very important to probe the nature of dark energy. These would be true for data from future experiments of CMB, BAO and SN. In particular, since there is a well-known severe degeneracy among $w_X$ and $\Omega_X$, the independent determination of the Hubble constant will still affect the constraint on dark energy parameters by pinning down the value of $\Omega_m$. 

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Figure 10. 1σ and 2σ constraints from CMB + BAO + SN in the \( \tilde{w}_0 - \tilde{w}_1 \) plane marginalizing over \( h, \Omega_m \) and \( \Omega_k \) are shown for several values of \( z_\ast \) for the cases with no Hubble prior and assuming Gaussian priors \( h = 0.72 \pm 0.02 \) and \( h = 0.62 \pm 0.02 \).

more precisely. The answer to the simplest question of ‘a cosmological constant or not’ can be altered by changing some of these assumptions.

3.3. Constraint on the curvature of the universe

Here we discuss the constraints on the curvature of the universe assuming some priors on the Hubble constant including some types of dark energy model in addition to a cosmological constant and a constant equation of state using the parameterizations equations (1) and (4). As already mentioned, a flat universe is assumed in most cosmological parameter estimations in the literature because the inflationary paradigm strongly suggests it. However, the paradigm should be tested through the test of the flatness and it should be done including the uncertainty in the dark energy sector since its nature is not understood yet. In this respect, the investigation of the curvature of the universe in connection with dark energy models has been done in [4], [39]–[55]. In [39]–[41],
it was shown that an open universe can be largely allowed for some particular dark energy parameterizations. However, in the previous works, the Hubble constant was treated as a nuisance parameter to be marginalized over. Here we analyze how the constraint on the curvature is correlated with the Hubble constant and how the priors on it affect the constraint. We also investigate whether the constraint varies according to the choice of dark energy parameterization.

In figure 11, the constraints in the $\Omega_k - h$ plane are shown for the cases with a cosmological constant, a constant equation of state and time-varying equations of state parameterized as equations (1) and (4) with $z_a = 0.5$. For each model, we impose no prior and two types of prior on the Hubble constant as in section 3.2.
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Table 8. The best fit values for \( \Omega_m, h, \Omega_k, \tilde{w}_0 \) and \( \tilde{w}_1 \) for the analysis presented in figure 10 in which a non-flat universe is allowed. The minimum values of \( \chi^2 \) are also shown.

| CMB + BAO + Gold06 | \( \chi^2_{\text{min}} \) | \( \Omega_m \) | \( h \) | \( \Omega_k \) | \( \tilde{w}_0 \) | \( \tilde{w}_1 \) |
|---------------------|-----------------|-------------|--------|-------------|-------------|-------------|
| \( z_s = 0.1 \) (No prior) | 157.8 | 0.273 | 0.684 | 0.003 | -2.66 | -0.68 |
| Prior \( h = 0.72 \pm 0.02 \) | 158.4 | 0.251 | 0.714 | 0.013 | -3.20 | -0.60 |
| Prior \( h = 0.62 \pm 0.02 \) | 159.7 | 0.318 | 0.636 | -0.013 | -1.58 | -0.85 |
| \( z_s = 0.2 \) (No prior) | 157.3 | 0.278 | 0.678 | 0.009 | -1.80 | -0.59 |
| Prior \( h = 0.72 \pm 0.02 \) | 158.3 | 0.253 | 0.711 | 0.025 | -2.06 | -0.49 |
| Prior \( h = 0.62 \pm 0.02 \) | 159.2 | 0.316 | 0.637 | -0.010 | -1.44 | -0.75 |
| \( z_s = 0.5 \) (No prior) | 156.6 | 0.279 | 0.676 | 0.032 | -1.39 | -0.33 |
| Prior \( h = 0.72 \pm 0.02 \) | 157.9 | 0.255 | 0.709 | 0.042 | -1.38 | -0.33 |
| Prior \( h = 0.62 \pm 0.02 \) | 158.6 | 0.314 | 0.638 | 0.002 | -1.25 | -0.51 |
| \( z_s = 1.0 \) (No prior) | 157.5 | 0.294 | 0.661 | 0.018 | -1.12 | -0.33 |
| Prior \( h = 0.72 \pm 0.02 \) | 160.0 | 0.260 | 0.704 | 0.032 | -1.09 | -0.33 |
| Prior \( h = 0.62 \pm 0.02 \) | 158.7 | 0.318 | 0.635 | 0.009 | -1.14 | -0.33 |
| \( z_s = 2.0 \) (No prior) | 158.5 | 0.302 | 0.653 | 0.005 | -1.01 | -0.33 |
| Prior \( h = 0.72 \pm 0.02 \) | 161.7 | 0.263 | 0.702 | 0.020 | -0.98 | -0.33 |
| Prior \( h = 0.62 \pm 0.02 \) | 159.3 | 0.321 | 0.633 | -0.003 | -1.03 | -0.33 |

| CMB + BAO + Davis07 | \( \chi^2_{\text{min}} \) | \( \Omega_m \) | \( h \) | \( \Omega_k \) | \( \tilde{w}_0 \) | \( \tilde{w}_1 \) |
|---------------------|-----------------|-------------|--------|-------------|-------------|-------------|
| \( z_s = 0.1 \) (No prior) | 195.1 | 0.271 | 0.686 | -0.008 | -1.70 | -0.93 |
| Prior \( h = 0.72 \pm 0.02 \) | 195.7 | 0.251 | 0.713 | -0.001 | -2.04 | -0.86 |
| Prior \( h = 0.62 \pm 0.02 \) | 197.5 | 0.312 | 0.639 | -0.023 | -0.94 | -1.09 |
| \( z_s = 0.2 \) (No prior) | 195.4 | 0.278 | 0.678 | -0.010 | -1.20 | -0.97 |
| Prior \( h = 0.72 \pm 0.02 \) | 196.4 | 0.253 | 0.711 | 0.000 | -1.38 | -0.86 |
| Prior \( h = 0.62 \pm 0.02 \) | 197.4 | 0.313 | 0.638 | -0.023 | -0.94 | -1.14 |
| \( z_s = 0.5 \) (No prior) | 195.5 | 0.281 | 0.675 | -0.011 | -1.07 | -0.98 |
| Prior \( h = 0.72 \pm 0.02 \) | 196.7 | 0.254 | 0.710 | 0.002 | -1.16 | -0.79 |
| Prior \( h = 0.62 \pm 0.02 \) | 197.3 | 0.315 | 0.637 | -0.026 | -0.93 | -1.29 |
| \( z_s = 1.0 \) (No prior) | 195.4 | 0.279 | 0.677 | -0.008 | -1.09 | -0.84 |
| Prior \( h = 0.72 \pm 0.02 \) | 196.5 | 0.253 | 0.710 | 0.013 | -1.17 | -0.47 |
| Prior \( h = 0.62 \pm 0.02 \) | 197.3 | 0.315 | 0.637 | -0.026 | -0.96 | -1.52 |
| \( z_s = 2.0 \) (No prior) | 195.4 | 0.278 | 0.677 | -0.005 | -1.10 | -0.53 |
| Prior \( h = 0.72 \pm 0.02 \) | 196.6 | 0.255 | 0.709 | 0.011 | -1.10 | -0.33 |
| Prior \( h = 0.62 \pm 0.02 \) | 197.3 | 0.315 | 0.637 | -0.027 | -0.96 | -2.08 |

From the current data of CMB, BAO and SN without an external Hubble prior, we may conclude that the universe is constrained to be around flat. In almost all of the cases, the curvature is limited as |\( \Omega_k | < 0.05 \) at 2\( \sigma \) level (the only exception is that case where the Gold06 set is used for the parameterization equation (4) with \( z_s = 0.5 \). The 2\( \sigma \) boundary extends to \( \Omega_k \sim 0.07 \). However, when we look at them more closely, we notice some dependence on dark energy parameterization. For the cases with a cosmological constant and a constant equation of state, the allowed region extends into a closed universe, which is a rather widely known result (e.g. figure 17 of [1]). In contrast, for the case with a time-varying equation of state equation (4) with \( z_s = 0.5 \), the allowed region has a much wider area in an open universe as has been found in [40].
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Table 9. Best fit values of $\Omega_k$ and $h$ for various assumptions on dark energy. Here we also show the best fit values for marginalized parameters such as $\Omega_m$, $w_0(\tilde{w}_0)$ and $w_1(\tilde{w}_1)$.

| CMB + BAO + Gold06 | $\chi^2_{\text{min}}$ | $\Omega_k$ | $h$ | $\Omega_m$ | $w_0(\tilde{w}_0)$ | $w_1(\tilde{w}_1)$ |
|--------------------|----------------------|----------|------|-----------|----------------|-----------------|
| Cosmological constant | 159.8 | -0.014 | 0.644 | 0.318 | — | — |
| Prior $h = 0.72 \pm 0.02$ | 164.5 | -0.001 | 0.696 | 0.277 | — | — |
| Prior $h = 0.62 \pm 0.02$ | 160.2 | -0.018 | 0.630 | 0.331 | — | — |
| $w_0$ (0.10) | 159.5 | -0.010 | 0.648 | 0.308 | -0.092 | — |
| Prior $h = 0.72 \pm 0.02$ | 163.3 | 0.005 | 0.700 | 0.265 | -0.088 | — |
| Prior $h = 0.62 \pm 0.02$ | 160.0 | -0.015 | 0.632 | 0.323 | -0.094 | — |
| $w_0$ and $w_1$ | 158.3 | 0.000 | 0.654 | 0.301 | -1.06 | 0.72 |
| Prior $h = 0.72 \pm 0.02$ | 161.3 | 0.015 | 0.702 | 0.262 | -1.01 | 0.68 |
| Prior $h = 0.62 \pm 0.02$ | 159.2 | -0.007 | 0.634 | 0.320 | -1.07 | 0.74 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_*=0.5$) | 156.6 | 0.032 | 0.676 | 0.280 | -1.38 | -0.33 |
| Prior $h = 0.72 \pm 0.02$ | 157.9 | 0.042 | 0.708 | 0.256 | -1.38 | -0.33 |
| Prior $h = 0.62 \pm 0.02$ | 158.6 | 0.002 | 0.638 | 0.313 | -1.25 | -0.52 |

| CMB + BAO + Davis07 | $\chi^2_{\text{min}}$ | $\Omega_k$ | $h$ | $\Omega_m$ | $w_0(\tilde{w}_0)$ | $w_1(\tilde{w}_1)$ |
|--------------------|----------------------|----------|------|-----------|----------------|-----------------|
| Cosmological constant | 195.6 | -0.011 | 0.674 | 0.278 | — | — |
| Prior $h = 0.72 \pm 0.02$ | 197.1 | -0.003 | 0.706 | 0.258 | — | — |
| Prior $h = 0.62 \pm 0.02$ | 198.0 | -0.020 | 0.642 | 0.302 | — | — |
| $w_0$ (0.10) | 195.5 | -0.012 | 0.674 | 0.282 | -1.04 | — |
| Prior $h = 0.72 \pm 0.02$ | 197.1 | -0.004 | 0.706 | 0.258 | -1.01 | — |
| Prior $h = 0.62 \pm 0.02$ | 197.6 | -0.022 | 0.640 | 0.311 | -1.07 | — |
| $w_0$ and $w_1$ | 195.4 | -0.009 | 0.676 | 0.280 | -1.10 | 0.37 |
| Prior $h = 0.72 \pm 0.02$ | 196.6 | 0.006 | 0.710 | 0.254 | -1.15 | 0.81 |
| Prior $h = 0.62 \pm 0.02$ | 197.3 | -0.026 | 0.638 | 0.314 | -0.93 | -0.93 |
| $\tilde{w}_0$ and $\tilde{w}_1$ ($z_*=0.5$) | 195.5 | -0.010 | 0.676 | 0.280 | -1.07 | -0.99 |
| Prior $h = 0.72 \pm 0.02$ | 196.7 | 0.002 | 0.710 | 0.254 | -1.15 | -0.80 |
| Prior $h = 0.62 \pm 0.02$ | 197.3 | -0.025 | 0.638 | 0.315 | -0.94 | -1.28 |

Figure 11 also shows the known tendency that the lower Hubble constant is favored in a closed universe (a positive correlation between $h$ and $\Omega_k$). Due to this correlation, when we assume the prior of $h = 0.62 \pm 0.02$ for the Hubble constant, the region with a closed universe occupies a larger space in the allowed region. On the other hand, when the prior $h = 0.72 \pm 0.02$ is assumed, a wider region of an open universe is allowed compared to a closed universe. For example, a flat universe is rejected at the 2σ level and a closed universe is favored for a cosmological constant with the prior $h = 0.62 \pm 0.02$. Meanwhile, the prior $h = 0.72 \pm 0.02$ enhances the preference of an open universe to the 2σ level for the parameterization equation (4) with $z_* = 0.5$.

The final remark is that, in the previous works [40, 41], it has been shown that, when we parameterize the dark energy equation of state as equation (4) and take $z_* \sim 0.5$, the region of an open universe as large as $\Omega_k \sim 0.2$ is allowed. However, for the result presented in figure 11, this is not the case even though we use the same type of parameterization. This is because in this paper we use a strong prior $w_X \leq -1/3$ on the equation of state given in equation (2) but in the previous works [40, 41], a weak prior $w_X \leq 0$ has been
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Figure 11. 1σ and 2σ constraints from CMB + BAO + SN in the Ω_k–h plane marginalizing over Ω_m and dark energy parameters. The best fit values are summarized in table 9. The dark energy parameterization adopted in the analysis are shown on the right side of the panels. We show the cases with no prior on the Hubble constant and Gaussian priors $h = 0.72 \pm 0.02$ and $h = 0.62 \pm 0.02$. The constraints using the SN datasets from Gold06 (red solid line) and Davis07 (blue dashed line) are shown separately.

As is already noticed in [40, 41], an open universe tends to be preferred for dark energy whose equation of state approaches zero at earlier time. Since we remove this possibility by adopting a strong prior, we do not have a large allowed region with an open universe. In other words, our result here confirmed explicitly that the dark energy whose equation of state is close to zero at earlier time, which is sometimes called early dark energy, can help to allow an open universe at least as long as the background evolution is concerned.

4. Conclusions and discussion

We studied the constraints on the Hubble constant from CMB, BAO and SN assuming several types of dark energy parameterization. Although the Hubble constant and dark energy parameterization adopted9. As is already noticed in [40, 41], an open universe tends to be preferred for dark energy whose equation of state approaches zero at earlier time. Since we remove this possibility by adopting a strong prior, we do not have a large allowed region with an open universe. In other words, our result here confirmed explicitly that the dark energy whose equation of state is close to zero at earlier time, which is sometimes called early dark energy, can help to allow an open universe at least as long as the background evolution is concerned.

9 The method of this paper to obtain a constraint from cosmological data is also slightly different from that used in [39]–[41]. However, the difference in the method is irrelevant to the conclusion on the constraint on the curvature of the universe.
Energy are both important issues in cosmology today, these two subjects have not been investigated much simultaneously. First we investigated the constraints in the $\Omega_m$–$h$ plane assuming several dark energy parameterizations. The constraints on the Hubble constant from the combination of CMB, BAO and SN observations obtained under different dark energy models and priors are summarized in table 3 when a flat universe is assumed and in table 4 when the flatness assumption is dropped, respectively. It is noticed that the constraints are not affected drastically by the dark energy model assumed and/or the assumption of the flatness of the universe. It is rather more affected by the choice of the SN dataset: the Gold06 set gives a slightly lower value of $h$ than the Davis07 set. We can conservatively conclude that $H_0 < 59$ and $H_0 > 76$ are highly unlikely from these cosmological observations: these parameter regions are not allowed at the $2\sigma$ level for any dark energy parameterization even if we do not restrict ourselves to a flat universe. Since the distance ladder estimations of $H_0$ have somewhat large systematic errors at present, we are not at the stage of arguing any possible discrepancies among these measurements now. Nevertheless, it is worth mentioning in passing that Sandage’s central value $H_0 = 62$ [33] and Macri’s value $H_0 = 74$ [34] are fairly close to the limit we have obtained here.

We have also investigated the constraints on some dark energy parameters assuming several priors on the Hubble constant. The constraints are derived with and without the assumption of the flatness of the universe. Using the present cosmological observations assuming no Hubble prior, we have found that a cosmological constant and a flat universe can fit all the data satisfactorily. When we impose a prior on the Hubble constant, we have adopted Gaussian priors $h = 0.72 \pm 0.02$ and $h = 0.62 \pm 0.02$. The central values are those of Freedman [32] and Sandage [33] but the error is taken to be a hypothetical value to give a meaningful effect on the dark energy parameter estimation. We have found that, even with some limited options discussed in this paper for the assumptions with respect to the prior on the Hubble constant, a parameterization of the dark energy equation of state and the curvature of the universe, they can have a great influence in determining the nature of dark energy. It should also be mentioned that the choice of the SN dataset affects the allowed region. We demonstrate these points by taking a cosmological constant as a reference dark energy model because it is the simplest and most conventional model which can fit the observational datasets. For example, when we adopt the parameterization equation (1) in a flat universe, the prior $h = 0.62 \pm 0.02$ reject a cosmological constant at the $2\sigma$ level for the Gold06 dataset, but the allowed region broadens to allow a cosmological constant if we do not adopt the flatness assumption or if we instead adopt the prior $h = 0.72 \pm 0.02$. Moreover, if we adopt the parameterization equation (4) with $z_* = 0.5$ and the prior $h = 0.72 \pm 0.02$, the Gold06 dataset is in good agreement with a cosmological constant in a flat universe, but if we drop the flatness condition and marginalize over the curvature, a cosmological constant is disfavored at the $2\sigma$ level. These examples imply that our understanding of the nature of dark energy can be varied by the assumptions on the Hubble constant, a parameterization of the dark energy equation of state and the curvature of the universe.

Finally, we have investigated the constraints on the curvature of the universe assuming several types of dark energy and the priors on the Hubble constant. In contrast to the constraints on the Hubble constant, we see the result depends on the dark energy parameterization we adopt. For the cases with a cosmological constant and a constant equation of state, the allowed region occupies a larger area in a closed universe, whereas
for the case with a time-varying equation of state parameterized as equation (4) with $z_* = 0.5$, the allowed region extends into a region of an open universe. Since there is an obvious positive correlation between $h$ and $\Omega_k$, the prior $h = 0.62 \pm 0.02$ exaggerates the preference for a closed universe and the prior $h = 0.72 \pm 0.02$ for an open universe. This is what we see in figure 11. We also reconfirmed that the preference of an open universe is caused by the equation of state for dark energy which is close to 0 at earlier time. This is because we have found that $\Omega_k$ as large as 0.2 is allowed in our previous papers [40, 41] under the weak prior $w_X \leq 0$, whereas $\Omega_k$ is found to be well below 0.1 in this paper under the stronger prior $w_X \leq -1/3$.

Since dark energy is one of the most important problems in science today, a large number of works are focusing on dark energy itself. However, when one tries to probe the nature of dark energy, other cosmological parameters such as the Hubble constant, which was discussed in this paper, should necessarily be involved in various manners. In light of precise measurements of cosmology that we are performing now, the works from this kind of viewpoint should be done to check our understanding of cosmology and may also give insights to probe the present state and the evolution of the universe.

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Appendix A. Effects of dark energy perturbation on CMB

In the analysis for the constraints, we make use of the quantities which can be thoroughly determined by the background evolution. Thus we only take into account the modification to the background evolution by dark energy to obtain constraints on some parameters although the dark energy component can fluctuate in general to affect the cosmic density fluctuation such as CMB anisotropies. As mentioned in the text, when dark energy becomes the dominant component at late time, the effect of fluctuation is not significant except on large scales and that from the modification to the background is enough to extract the cosmological constraints. However, when the equation of state for dark energy approaches zero at earlier time, which means that the energy density of dark energy behaves $\rho_X \propto a^{-3}$ and can be comparable to that of matter, fluctuation of dark energy becomes important to affect the structure of acoustic peaks. In such a case, fluctuation of dark energy should be properly taken into account in the analysis. Thus, as for the CMB, the constraint from the distance measure such as the acoustic scale which is given by the information on the background evolution alone becomes invalid. This is one of the reasons why we assume the prior of equation (2) to avoid such a case where dark energy has a significant fraction at earlier time. In addition, when fluctuation of dark energy becomes important, the other nature of dark energy such as the effective sound speed, which we denote here as $c_X^2$, can also modify the density fluctuation. In this sense, the equation of state is not enough to consider the effect of dark energy.

To see how the nature of dark energy can affect CMB, we show the CMB power spectra in figure A.1 for several cases of the equation of state and the effective sound
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Figure A.1. Left panel: CMB power spectra for a dark energy model with the parameterization of equation (1). Here we fix the value of $w_0 = -1$ and take several values for $w_1$ as $w_1 = 1$ (red solid line), 0.6 (green dashed line) and 0 (blue dotted line). In this panel, we assume the effective sound speed as $c_X^2 = 1$.

Right panel: CMB power spectra for the equation of state $w_0 = -1 + (1 - a)$ (i.e. we take $(w_0, w_1) = (-1, 1)$ for the parameterization of equation (1).) The cases with $c_X^2 = 1$ (red solid line), 0.1 (green dashed line) and 0 (blue dotted line) are shown.

speed. Here we assume the parameterization of equation (1) for dark energy. In the left panel, we fix the value of $w_0$ as $w_0 = -1$ and vary $w_1$ as $w_1 = 1$ (red solid line), 0.8 (green dashed line) and 0 (blue dotted line). For other cosmological parameters, we assume the mean values of a power law ΛCDM model from WMAP3 alone analysis as $ω_m = 0.1277$, $ω_b = 0.0229$, $τ = 0.089$, $h = 0.732$ and $n_s = 0.958$. As seen from the figure, when we compare the cases with $w_1 = 0$ and 0.6, the power spectra is just shifted to smaller $l$ but the structure of acoustic peaks is unchanged. This shift is caused by the change of the angular diameter distance to the last scattering surface due to the modification to the background evolution. However, when we take $w_1 = 1$ in which the energy density of dark energy is not negligible compared to that of matter at earlier time, the structure of acoustic peaks is significantly modified because fluctuation of dark energy can affect it in addition to the shift of the peaks in this case. Furthermore, this change depends on the nature of dark energy fluctuation. In the right panel of figure A.1, we plot the cases with $c_X^2 = 1$ (red solid line), 0.1 (green dashed line) and 0 (blue dotted line) for a parameter set $(w_0, w_1) = (-1, 1)$. Even though the equation of state is the same (namely, the background evolution is the same), the CMB power spectra are different when one assumes different sound speed. In this kind of case, since the effective sound speed (and/or possibly another property of dark energy fluctuation) can affect the CMB, one should take into account the whole information of CMB power spectrum and constraints from the background evolution become invalid.

We made another plot, figure A.2, to see when neglecting the fluctuation of dark energy component is valid. In the left panel, we plot the height of the CMB first peak as a function of $w_1$ for the parameterization equation (1) with fixing $w_0 = -1$. We can see that it is unchanged for $w_1 \lesssim 0.7$, which indicates that the fluctuation of dark energy does

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10 Other parameterizations which make the dark energy density non-negligible at the epoch of recombination are adopted in [56,57] and constraints on the dark energy parameters are investigated.
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Figure A.2. Left panel: the height of the first peak as a function of $w_1$ of the parameterization equation (1) for the cases with $c_X^2 = 1$ (red solid line) and 0 (green dashed line). The value of $w_0$ is fixed as $w_0 = -1$. Right panel: the ratio of the energy density of dark energy to that of matter at $z = 1089$ as a function of $w_1$.

not affect the structure of the acoustic peaks. In the right panel, as a function of $w_1$, we plot the ratio of energy densities of dark energy and matter at the recombination epoch, $z_{\text{rec}} = 1089$. Notice that, when $w_1 \lesssim 0.7$, the energy density of dark energy is negligible compared to that of matter at the recombination epoch. In other words, the effect of fluctuation of dark energy can be neglected when the energy density of dark energy is small enough at earlier time. In such a case, since the information on the background evolution alone captures well the effect of dark energy, we can constrain dark energy parameters by only studying the shift of acoustic peaks. Thus, our method to obtain constraints from observations of CMB is justified when the equation of state is in a range where the energy density of dark energy is negligible at earlier time. The prior we take in this paper, equation (2) $w_X \leq -1/3$, can satisfy this requirement. The final comment is that, as can be inferred from figure A.2, a slightly looser prior like $w_X \leq -0.3$ may make dark energy subdominant at the epoch of recombination and justify our analysis (and would not change our results much). However, since this condition depends on other cosmological parameters such as $\Omega_X$ and $\Omega_m$, we adopt the conservative prior of $w_X \leq -1/3$.

Appendix B. How $\Omega_m$ and $\Omega_k$ are determined from CMB, BAO and SN

In section 3.1, we found that $h$ is constrained rather tightly regardless of the dark energy model and/or the assumption of the flatness of the universe if all the CMB, BAO and SN data are combined. We also argued that this is equivalent to the determination of $\Omega_m$ from the acoustic scales, $\theta_A$ and $D_V(0.35)$, and the luminosity distance $d_L$ since $\omega_m = \Omega_m h^2$ is given by the height of the CMB acoustic peak. Thus, in this appendix, we explain how these three types of cosmological datasets can determine $\Omega_m$ without referring to any particular dark energy model and without assuming the flat universe.

B.1. What SN and CMB determine

In this section, we focus on SN and CMB observations. We start with defining

$$I_{\text{SN}} \equiv \int_0^{\mu_{\text{SN}}} \frac{dz'}{H(z')/H_0},$$

(B.1)
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Figure B.1. Contours of \( r_\theta(z_{\text{rec}}) = 14.3 \) Gpc for several values of \( \Omega_k \) (black solid lines) and \( I_{\text{SN}} \) derived from observations of Gold06 (red dotted line) and Davis07 (blue dotted-dashed line).

where \( z_{\text{SN}} \) denotes the highest redshift to which SN data can probe. For the present data, we take \( z_{\text{SN}} = 1.8 \). Using this integral, the comoving angular distance to the last scattering surface equation (6) is given by

\[
\begin{align*}
    r_\theta(z_{\text{rec}}) &= \frac{1}{H_0 \sqrt{|\Omega_k|}} S \left( \sqrt{|\Omega_k|} \left\{ I_{\text{SN}} + \int_{z_{\text{SN}}}^{z_{\text{rec}}} \frac{dz'}{H(z')/H_0} \right\} \right) \\
    &\approx 3.0 \times 10^3 \text{ Mpc} \sqrt{\omega_m} \sqrt{|\Omega_k|} \left( \sqrt{|\Omega_k|} \left\{ I_{\text{SN}} + 2 \Omega_m^{-1/2}(1 + z_{\text{SN}})^{-1/2} \right\} \right).
\end{align*}
\]

In the second line, we analytically performed the integration by approximating the universe to be matter-dominated for \( z_{\text{SN}} < z < z_{\text{rec}} \) and neglected the term proportional to \((1 + z_{\text{rec}})^{-1/2}\) since \( z_{\text{SN}} \ll z_{\text{rec}} \). Now, if \( I_{\text{SN}} \) is known from SN observation, since CMB observation gives \( \omega_m \) and \( r_\theta(z_{\text{rec}}) \), we obtain the degeneracy relation between \( \Omega_m \) and \( \Omega_k \). This is drawn in figure B.1 as a contour of \( r_\theta(z_{\text{rec}}) = 14.3 \) Gpc (this is derived from equation (16) and \( r_\theta(z_{\text{rec}}) = 149 \) Mpc, which is in turn from equation (10) using \( \omega_m = 0.1277 \) and \( \omega_b = 0.02229 \) in the \( \Omega_m - I_{\text{SN}} \) plane for several values of \( \Omega_k \). Specifically, if we impose a flat universe prior, \( \Omega_k = 0 \), \( \Omega_m \) is determined uniquely. In [45], essentially the same argument is used to investigate the prospect of measuring \( \Omega_k \) very precisely.

The next issue is how tightly \( I_{\text{SN}} \) is constrained by the data. The SN data such as Gold06 or Davis07 used in this paper consist of the \( z-\mu \) relation with an error on \( \mu \) where \( \mu \) is given by equation (22). The integral \( I_{\text{SN}} \) is directly obtained from \( \mu(z_{\text{SN}}) \) if \( H_0 \) and \( M \) are known. However, the uncertainties in them cannot be resolved by SN data alone. Thus, to cancel these uncertainties, we use \( \mu \) at another redshift \( z_1 \) which is close to zero. \((z_1 \ll 1)\) is required not to make \( \mu(z_1) \) dependent on cosmology. Notice that, as we will
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adopt below, approximately we can use the relation \( d_L(z) \simeq z/H_0 \) for small \( z \). To cancel the constants, we take a difference of \( \mu(z) \) as

\[
\mu(z_{SN}) - \mu(z_1) = 5 \log \frac{d_L(z_{SN})}{d_L(z_1)}. \tag{B.4}
\]

Since

\[
d_L(z_{SN}) = \frac{1 + z_{SN}}{H_0 \sqrt{|\Omega_k|}} S \left( \sqrt{|\Omega_k|} I_{SN} \right), \tag{B.5}
\]

and, \( d_L(z_1) \approx z_1/H_0 \) for \( z_1 \ll 1 \), we obtain

\[
I_{SN} = \frac{1}{\sqrt{|\Omega_k|}} S^{-1} \left( \frac{z_1 \sqrt{|\Omega_k|}}{1 + z_{SN}} \right)^{10(\mu(z_{SN})-\mu(z_1))/5}. \tag{B.6}
\]

We derive \( \mu(z_{SN}) \) and \( \mu(z_1) \) by fitting the SN data to a power-law function \( \mu(z) = az^p \). For the Gold06 set,

\[
a_G = 44.288 \pm 0.022, \tag{B.7}
\]
\[
p_G = (6.048 \pm 0.033) \times 10^{-2}, \tag{B.8}
\]

and for the Davis07 set,

\[
a_D = 44.278 \pm 0.023, \tag{B.9}
\]
\[
p_D = (6.123 \pm 0.030) \times 10^{-2}. \tag{B.10}
\]

Note that it does not refer to a specific parameterization for dark energy. The \( \chi^2 \)’s are 158.9 for the Gold06 data and 199.0 for the Davis07 data which are comparable to the values by the usual approach [41,58] of assuming some dark energy models. \( I_{SN} \) has an uncertainty as regards the value of \( z_1 \). However, a numerical experiment reveals that \( I_{SN} \) barely depends on the value of \( z_1 \) for \( 0.02 \lesssim z_1 \lesssim 0.05 \) \((z \sim 0.02\) is the lowest redshift for the SN data we use here). Thus, we take \( z_1 = 0.05 \) hereafter. Then in a flat universe,

\[
I_{SN,G} = 1.10, \tag{B.11}
\]
\[
I_{SN,D} = 1.15, \tag{B.12}
\]

respectively for Gold06 and Davis07. Notice that they do not depend much on the curvature. In fact, they change less than 1% for \(|\Omega_k| < 0.2\). These values are plotted as horizontal lines in figure B.1. They cross the contour which satisfies the CMB observation for a flat universe (denoted by the thick black solid line) at around \( \Omega_m = 0.25 \). This is consistent with the full analysis result as presented in section 3.1. Moreover, we can see from the figure that, since \( I_{SN,G} < I_{SN,D} \) (which in turn comes from the difference in the power-law slope \( p_G < p_D \)), Gold06 favors larger \( \Omega_m \) than Davis07 does. This is another point which we have noted in section 3.1. We believe that these explanations are independent of the dark energy model we adopt because we use the SN data without referring to dark energy.

However, figure B.1 also shows that, if we abandon the flatness assumption and shift \( \Omega_k \) from zero, a different value of \( \Omega_m \) is favored. This is why we cannot constrain \( \Omega_m \) much from CMB + SN when we marginalize over \( \Omega_k \).
B.2. What BAO adds

The BAO data give $D_V(z_{\text{BAO}})$ defined by equation (19) where $z_{\text{BAO}} = 0.35$. This is written as

$$D_V(z_{\text{BAO}})^2 = r_\theta(z_{\text{BAO}})^2 H_0^{-1} \frac{z_{\text{BAO}}}{\sqrt{\Omega_m (1 + z_{\text{BAO}})^3 + \Omega_k (1 + z_{\text{BAO}})^2 + (1 - \Omega_m - \Omega_k) f(z_{\text{BAO}})^2}},$$

(B.13)

where $f(z)$ is a function which expresses the evolution of the dark energy density. Here, the dark energy dependence enters in two places, $r_\theta(z_{\text{BAO}})$ and $f(z_{\text{BAO}})$. Since $z_{\text{BAO}}$ is relatively small, we may neglect the dark energy evolution (namely, we approximate dark energy as a cosmological constant) to approximate as $f(z_{\text{BAO}}) \approx 1$. For $r_\theta(z_{\text{BAO}})$, since this is written as

$$r_\theta(z_{\text{BAO}}) = \frac{1}{H_0 \sqrt{|\Omega_k|}} S \left( \sqrt{|\Omega_k|} I_{\text{BAO}} \right),$$

(B.14)

where $I_{\text{BAO}}$ is the integral similar to equation (B.1) with $z_{\text{SN}}$ replaced by $z_{\text{BAO}}$, it can be calculated without referring to a dark energy model provided that we combine with the SN data. This is because we can infer $I_{\text{BAO}}$ by using the fit to the SN data in the same manner to derive $I_{\text{SN}}$ in appendix B.1. Similarly to $I_{\text{SN}}$, $I_{\text{BAO}}$ depends slightly on the SN data (but not on $z_1$ or the curvature). Gold06 and Davis07 give respectively

$$I_{\text{BAO},G} = 0.31,$$

(B.15)

$$I_{\text{BAO},D} = 0.32.$$  

(B.16)

Now, if we fix $\omega_m$ using the CMB value, we have a relation between $\Omega_m$ and $\Omega_k$ determined from SN and BAO data for a measured $D_V(0.35)$.

Thus, here and in appendix B.1, we have replaced the dark energy dependent part of CMB and BAO observables ($r_\theta(z_{\text{rec}})$ and $D_V(0.35)$) by the empirical values ($I_{\text{SN}}$ and $I_{\text{BAO}}$) inferred from the SN data. Namely, we can now draw contours of $r_\theta(z_{\text{rec}})$ and $D_V(0.35)$ in the $\Omega_m - \Omega_k$ plane without referring to any dark energy model. In figure B.2, we draw the contours $r_\theta(z_{\text{rec}}) = 14.3$ Gpc (see appendix B.1) by the black solid line and $D_V(0.35) = 1402$ Mpc (see section 2.2) by the black dotted line. We have fixed $\omega_m$ to the CMB value of 0.1277. Since the two SN datasets give slightly different values of $I_{\text{SN}}$ and $I_{\text{BAO}}$ as mentioned above, we show the results in two panels separately (the Gold06 data are used in the left panel and the Davis07 data in the right panel). The $r_\theta(z_{\text{rec}})$ contours run in a somewhat diagonal direction, showing the degeneracy between $\Omega_m$ and $\Omega_k$ for the CMB + SN combination as mentioned in appendix B.1. In contrast, the $D_V(0.35)$ contours run almost horizontally, showing that the BAO + SN combination is insensitive to $\Omega_k$ and can determine $\Omega_m$ regardless of the assumption on the curvature of the universe. This is reasonable because BAO measures the distance to relatively low redshift ($z = 0.35$). Reference [25] has derived a linearized relation from BAO measurement: $\Omega_m = 0.273 + 0.1375\Omega_k$ (equation (6) in [25]), which is quite similar to our slope for the $D_V(0.35)$ contours. But also note that this relation has been derived only for a cosmological constant. If we consider the case with a constant equation of state or a time-varying equation of state, $\Omega_m$ cannot be determined from BAO alone. Our point is that, when BAO and SN are combined, $\Omega_m$...
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Figure B.2. Contours of $r_\theta(z_{\text{rec}}) = 14.308.5$ Mpc (equation (B.3)) are drawn in black solid lines and those of $D_V(0.35) = 1402$ Mpc are drawn in black dotted lines. Here, $r_\theta(z_{\text{rec}})$ and $D_V(0.35)$ are approximate expressions in which the dependence on the dark energy model is dropped by using empirical values from SN data as discussed in appendices B.1 and B.2. Also shown are 2σ allowed regions derived by the full analysis assuming some dark energy models and marginalizing over two dark energy parameters and $h$. The models are: (i) the parameterization equation (1) and (ii) the parameterization equation (4) with $z_*=0.5$. The bands running diagonally are from SN, $\theta_A$ and $\omega_m$ (blue dotted–long-dashed line for model (i) and red long-dashed line for model (ii)), and the horizontal bands are from SN, $D_V(0.35)$ and $\omega_m$ (blue dotted–short-dashed line for model (i) and red short-dashed line for model (ii)). The shaded regions are from all data combined, SN, $\theta_A$, $D_V(0.35)$ and $\omega_m$ (the regions are bounded by blue dotted—dashed line for model (i) and red dashed line for model (ii)). We can see that the contours of the approximate expressions well indicate the degeneracy directions of the corresponding data combinations. We use the Gold06 SN dataset in the left panel and the Davis07 dataset in the right panel.

can be determined regardless of the assumption on the curvature of the universe and dark energy model.

In order to check the validity of these approximations, we plot the allowed regions by the full analysis such as explained in section 2. The $r_\theta(z_{\text{rec}})$ contours are compared with the analysis using $\chi^2$ constructed from $\theta_A$, $\omega_m$ and the SN dataset. The $D_V(0.35)$ contours are compared with the one using $\chi^2$ from $D_V(0.35)$, $\omega_m$ and SN. For completeness, we also performed the all-combined analysis using $\theta_A$, $D_V(0.35)$, $\omega_m$ and SN, which can be compared with the analyses of $\Omega_m$ and $\Omega_k$ done in sections 3.1 and 3.3. The $\chi^2$’s are minimized over $h$ and dark energy parameters. We analyzed several dark energy models parameterized as equations (1) and (4) but the results do not show much difference. In figure B.2, we show 2σ allowed regions for the cases with equation (1) (blue dotted–dashed lines) and equation (4) with $z_*=0.5$ (red dashed lines) as examples. Note that the allowed regions for the two models look alike. We can also see that the regions basically extend in the directions of the approximate $r_\theta(z_{\text{rec}})$ and $D_V(0.35)$ contours. Therefore, it is considered to be appropriate to drop the dark energy dependence by using the SN data as we have done here.
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