Generalized Cahn effect and parton 3D motion in covariant approach

Petr Zavada

Institute of Physics of the AS CR, Na Slovance 2, CZ-182 21 Prague 8, Czech Republic

The Cahn effect and the unintegrated unpolarized parton distribution function \( f_1^q(x, p_T) \) are studied in a covariant approach. Cahn effect is compared with some other effects due to the parton intrinsic motion. The comparison suggests that present understanding of the parton transverse momenta and the intrinsic motion in general is still rather incomplete. The new rule for \( f_1^q(x, p_T) \) is obtained in the framework of the covariant parton model from which a prediction for this distribution function follows.

PACS numbers: 12.39.-x 11.55.Hx 13.60.-r 13.88.+e

1. INTRODUCTION

Studies of the transverse momentum dependent (or 'unintegrated') parton distribution functions (TMDs) open the new way to better understanding of the partonic quark-gluon structure of the nucleon. At the same time it is evident, that explanation of some experimental observations could be hardly possible without more accurate and realistic 3D picture of the nucleon, which naturally includes transverse motion. The azimuthal asymmetry in distribution of hadrons produced in deep-inelastic lepton-nucleon scattering (DIS), known as the Cahn effect, is a classical example. The role of the quark (transversal) intrinsic motion is crucial for explanation of some spin effects, like the asymmetries in particle production related to the direction of proton polarization.

In our previous study we proposed the covariant parton model, in which the 3D picture of parton momenta with rotational symmetry in the nucleon rest frame represents a basic input. At the same time the model is based on the assumption, that for sufficiently great momentum transfer \( Q^2 \), the quarks can be considered as almost free due to the asymptotic freedom. It appears, that the main potential of this approach is implication of some old and new sum rules and relations among various parton distribution functions. The sum rules which relate structure functions \( g_1 \) and \( g_2 \) of the Wanzura-Wilczek type and some others are proved in [12]. Assuming the SU(6) symmetry (in addition to the covariance and rotational symmetry) we have proved relations between polarized and unpolarized structure functions [13], which agree very well with the experimental data. In [14] we studied transversity in the framework of this model and we proved the rules, which relate transversity and helicity. Recently, we generalized the model to include also the pretzelosity distribution [16] and proved the common relations for helicity, transversity and pretzelosity. Finally, with the same model we studied the TMDs and a set of relations among them [17]. Further, in framework of the model we demonstrated that the 3D picture of parton momenta inside the nucleon is a necessary input for consistent accounting for quark orbital angular momentum (OAM). Dominant role of the OAM for generating nucleon spin is a consequence of the quark relativistic motion inside the nucleon, i.e. when the quark mass \( \ll \) momentum in the nucleon rest frame. In this case only the total angular momentum \( J^2_q = L^2_q + S^2_q \) is good quantum number and we obtained mean values of the quark orbital and spin components: \( \langle L^2_q \rangle = 2 \langle S^2_q \rangle = \Delta \Sigma \) or \( \langle J^2_q \rangle = 3 \frac{2}{2} \Delta \Sigma \) [15].

Comparison of obtained relations and predictions with experimental data is very important and interesting from phenomenological point of view. It allows to judge to which extent the experimental observation can be interpreted in terms of simplified, intuitive notions. The obtained picture of the nucleon can be the useful complement to the exact but more complicated theory of the nucleon structure based on the QCD. For example, the covariant parton model can be useful tool for separating effects of the QCD from effects of relativistic kinematics.

In this work we study further aspects of intrinsic motion of partons. In Sec. 2 we analyze some general physical conditions, which induce the Cahn effect. In Sec. 3 we make a comparison of the data on average transverse momentum obtained by the method based on the Cahn effect and by some another methods, including the analysis of the structure function \( F_2 \) in the framework of covariant parton models. In Sec. 4 we analyze the unpolarized TMDs defined in [17]. As a result we obtain the rule from which a prediction for this distribution function follows. Finally in Sec. 5 we summarize obtained results.

*Electronic address: zavada@fzu.cz
FIG. 1: Interaction of lepton with quark defines two axes of different symmetry (a). Azimuthal asymmetry as a result of variable collision energy (b), see text.

2. CAHN EFFECT: MANIFESTATION OF THE INTRINSIC MOTION

The Cahn effect is due to the nonzero transverse momentum of quarks inside the nucleon. Probability \( W = |M_{fi}|^2 \) of the elementary lepton - quark scattering in one photon exchange approximation is given by the expression

\[
W(\hat{s}, \hat{u}) \propto \hat{s}^2 + \hat{u}^2,
\]

where Mandelstam variables depend on the azimuthal angle \( \phi \) (angle between leptonic and hadronic planes) as:

\[
\hat{s}^2 = \frac{Q^4}{y^2} \left[ 1 - 4 \frac{p_T}{Q} \sqrt{1 - y \cos \phi} \right] + \mathcal{O} \left( \frac{p_T^2}{Q^2} \right),
\]

\[
\hat{u}^2 = \frac{Q^4}{y^2} (1 - y)^2 \left[ 1 - 4 \frac{p_T}{Q} \cos \phi \sqrt{1 - y} \right] + \mathcal{O} \left( \frac{p_T^2}{Q^2} \right),
\]

where \( p_T \) is the quark momentum component transverse to the photon momentum \( q \), \( Q^2 = -q^2 \). Apparently, dependence on \( \phi \) disappear for \( p_T \to 0 \). Intrinsic motion of the constituents creating the composite target is a necessary condition for appearance of the effect. The Cahn effect is a kinematical effect accompanying the QED scattering of leptons on quarks inside the nucleon and its origin is different from that of the QCD higher-twist effects \[19\]. At the same time it is evident, that intrinsic quark motion in itself is due to unpertubative QCD. Mandelstam variables in terms of lepton and quark momenta \((l, p)\) read

\[
\hat{s} = (l + p)^2 = 2pl + m_l^2 + m_q^2,
\]

\[
\hat{u} = (p - l')^2 = -2pl + Q^2 + m_l^2 + m_q^2,
\]

where \( m_l, m_q \) are corresponding masses. Obviously, one can substitute the variables of the probability \[1\]:

\[
\hat{s}, \hat{u} \to pl, Q^2; \quad W(\hat{s}, \hat{u}) \to W(pl, Q^2).
\]

The probability \( W \) expressed in terms of the new variables clearly demonstrates azimuthal symmetry of \( p \) with respect to the lepton beam direction \( l \), which represents the axis of azimuthal symmetry. It follows, that the photon direction \( q \) being different from the direction \( l \), in general cannot be the second axis of azimuthal symmetry. In fact, this is essence of the Cahn effect, see Fig. \[1\]. Let us consider the two reference frames:

A. The nucleon rest frame, where the first axis is directed along \( q \) and projection of \( l \) on the plane transversal to \( q \) defines second axis. Azimuthal angle \( \phi \) and transverse momentum \( p_T \) are defined equally as above (\( p_T \) and \( \phi \) do not change under any Lorentz boost along \( q \)), so the quark momentum \( p_A \) in this frame has the components:

\[
p_A = (p_1, \quad p_T \cos \phi, \quad p_T \sin \phi).
\]

\[7\]
B. The nucleon rest frame, where the first axis is directed along \( \mathbf{l} \) and projection of \(- \mathbf{q}\) on the plane transversal to \( \mathbf{l} \) defines second axis. This reference frame is obtained by rotation of the frame A by angle \( \gamma \) around third axis, so the quark momentum has the new components:

\[
\mathbf{p}_B = (p_1 \cos \gamma - p_T \sin \gamma \cos \varphi, \quad p_T \cos \gamma \cos \varphi + p_1 \sin \gamma, \quad p_T \sin \varphi).
\]  

(8)

The angle \( \gamma \) is defined as

\[
\cos \gamma = \frac{q_L}{|\mathbf{q}|}, \quad \sin \gamma = \frac{q_T}{|\mathbf{q}|},
\]  

(9)

where \( q_L \) and \( q_T \) are longitudinal and transversal components of the photon momentum in the frame B, \( q_B = (q_L, q_T, 0) \). For lepton energy \( l_0 \) (lepton mass will be neglected in the next) one can obtain [11]:

\[
\frac{|q_L|}{\nu} = 1 + \frac{M}{l_0} x_B, \quad \frac{|\mathbf{q}|}{\nu} = \sqrt{1 + \frac{4M^2}{Q^2} x_B^2},
\]  

(10)

and

\[
\frac{q_T}{\nu} = \sqrt{\left(\frac{4M^2}{Q^2} - \frac{M^2}{l_0^2}\right) x_B^2 - \frac{2M}{l_0} x_B},
\]  

(11)

where the standard notation is used:

\[
x_B = \frac{Q^2}{2M\nu}, \quad \nu = l_0 - l'_0.
\]  

(12)

Now the variable \( p_l \) can be expressed as

\[
p_l = (p_0 - p_1 \cos \gamma - p_T \sin \gamma \cos \varphi) l_0.
\]  

(13)

This variable, after inserting to relations (4), (5) allows to exactly calculate azimuthal dependence of the probability [1].

If one assumes

\[
Q^2 \gg 4M^2 x_B^2, \quad l_0 \gg M x_B,
\]  

(14)

then the relations (9) and (10) give

\[
|\mathbf{q}| \approx |q_L| \approx \nu, \quad \cos \gamma \approx 1.
\]  

(15)

Now, since

\[
p_1 = \frac{\mathbf{pq}}{|\mathbf{q}|} = \frac{p_0 \nu - pq}{|\mathbf{q}|},
\]  

(16)

the relation (13) is modified as

\[
p_l \approx \left( \frac{\mathbf{pq}}{\nu} - \frac{p_T q_T}{\nu} \cos \varphi \right) l_0.
\]  

(17)

Further, Eq. (11) is rearranged as

\[
\frac{q_T}{\nu} = \frac{2M x_B}{Q} \sqrt{1 - \frac{\nu}{l_0} - \frac{Q^2}{4l_0^2}}.
\]  

(18)

Since the complete expression for the probability \( W(\hat{s}, \hat{u}) \) involves the \( \delta - \) function term

\[
\delta \left( (p + q)^2 - m_q^2 \right) = \delta \left( 2pq + q^2 \right) = \frac{1}{2Pq} \delta \left( \frac{pq}{Pq} - x_B \right),
\]  

(19)
where $P$ is nucleon momentum, one can replace product $pq$ in (17) by $Mx_{B\nu}$. Then, assuming $4l_0^2 \gg Q^2$, after inserting (18) into (17) one gets

$$pl \approx \frac{Q^2}{2y} \left(1 - \frac{2p_T \sqrt{1 - y}}{Q} \cos \varphi \right); \quad y = \frac{\nu}{l_0} = \frac{Pq}{Pl}, \quad \frac{Q^2}{2y} = x_B Pl.$$  \hspace{1cm} (20)

Now, the term

$$\lambda = \frac{2p_T \sqrt{1 - y}}{Q} \cos \varphi$$  \hspace{1cm} (21)

represents a "small" correction and one can check, that Mandelstam variables (4),(5) in which the term $pl$ is replaced by the expression (20) and quark masses are neglected, give the relations (2),(3).

Now the probability $W(pl,Q^2)$ can be expanded as

$$W(pl,Q^2) = W(pl,Q^2)|_{\lambda=0} - \frac{\partial W(pl,Q^2)}{\partial (pl)}|_{\lambda=0} \lambda + ...$$  \hspace{1cm} (22)

$$\approx W(pl,Q^2)|_{\lambda=0} \left(1 - \frac{\partial \ln W(pl,Q^2)}{\partial \ln (pl)}|_{\lambda=0} \lambda \right).$$

Let us make some remarks on this relation:

i) The relation implies, that azimuthal asymmetry of the recoiled quark is described by the distribution

$$P(\varphi) = (1 - a \cos \varphi),$$  \hspace{1cm} (23)

where

$$a = \frac{2\sqrt{1 - y}}{Q} \cdot \left[\frac{\partial \ln W(pl,Q^2)}{\partial \ln (pl)}\right]_{\lambda=0} \langle p_T \rangle.$$  \hspace{1cm} (24)

From the analysis of experimental data one can obtain the parameter $a$. Obviously for obtaining $\langle p_T \rangle$ one has to know also the term involving differentiation of $W$. This term can be estimated either from the model (Eq. (1)) or from the experiment, if the data for a few lepton energies are available.

ii) Azimuthal asymmetry generated by the probability $W(pl,Q^2)$ has simple geometrical interpretation. In figure Fig. 1b. two momenta $p_1, p_2$ with opposite transverse components $p_{T1}, p_{T2}$ correspond to different collision energies $\hat{s}_1, \hat{s}_2$ since

$$\hat{s} = 2lp = 2 \left(\hat{s}_0 p_0 - \hat{s}_0 \right) \hat{p} \cos \beta,$$  \hspace{1cm} (25)

where $\beta$ is angle between the lepton and quark momenta. Obviously $\hat{s}_1 < \hat{s}_2$ in this figure and because $W$ depends on $\hat{s}$, then two corresponding momenta $p_1, p_2$ give different probabilities. In this way the asymmetry is generated. The figure reflects necessary conditions for the asymmetry:

$$\sin \gamma > 0, \quad \frac{\partial W}{\partial \hat{s}} > 0, \quad \langle p_T \rangle > 0,$$  \hspace{1cm} (26)

which correspond to the three factors in asymmetry parameter (24).

iii) In fact we have shown, that this asymmetry can be expected in any process $l + p \rightarrow l' + p'$, which is defined only by the lepton vector $l$, parton vector $p$ (with some distribution of $p_T$) and momentum transfer $q$, and which is described by the probability $W(\hat{s}, \hat{u}).$

### 2.1. Cross section and intrinsic motion of the constituents

The probability $W$ is related to the individual lepton - quark scattering. The experimentally accessible quantity is the corresponding cross section integrated over the distribution of quarks inside the nucleon. The cross section for the single lepton - quark scattering reads:

$$d\sigma \propto \frac{W(pl,Q^2)}{f}; \quad f = 4pl$$  \hspace{1cm} (27)
where $f$ is the corresponding flux factor. If the set of quarks in the nucleon is represented by the 3D distribution function $G(p)d^3p$, then the resulting probability of the lepton scattering is given [3] by the integral

$$W_{\text{nuc}} \propto \int \frac{P_0}{p_0} G(p) W(p, Q^2) d^3p,$$

where $P_0$ is the nucleon energy, $P_0 = M$ in the nucleon rest frame. The corresponding total cross section for lepton - nucleon scattering reads:

$$d\sigma_{\text{nuc}} \propto \frac{W_{\text{nuc}}}{f}; \quad f = 4l_0M.$$

Alternatively, the same result can be obtained directly by integrating of the cross section [27], but with the weight, which reflects effectively different amounts of quarks upstream and downstream the lepton beam in the nucleon volume [11]. For calculation of differential cross section the integration region in [28] is modified accordingly, for example the azimuthal dependence reads:

$$\frac{d\sigma_{\text{nuc}}}{d\varphi} \bigg|_{\varphi = \varphi_0} \propto \frac{M}{f_2} \int G(p) W(p, Q^2) \delta(\varphi - \varphi_0) \frac{d^3p}{p_0}.$$

The single lepton – parton scattering is only one stage of the Cahn effect. For complete phenomenology of the effect in lepton – nucleon DIS one needs further inputs:

a) 3D distribution $G(p)d^3p$ of parton momenta in the nucleon. The covariant approach will be studied in Sec. 4.

b) Fragmentation of recoiled quark and transfer of azimuthal asymmetry to hadrons. It is a complex stage comprising of both perturbative and unpertubative QCD aspects, currently applied parameterization related to this process is defined in [18].

### 3. WHAT DO WE KNOW ABOUT INTRINSIC MOTION?

In the lepton - quark scattering the distribution $G(p)$ controls the distributions of momenta of the scattered lepton and the recoiled quark. And vice versa, from the knowledge of the distributions of the scattered leptons or quarks (in real analysis hadrons from the quark fragmentation), one can obtain information about the initial distribution by the two independent methods. Comparison of the results can serve as a consistency check. Actually we can analyze two sets of data:

**A. Leptonic data**

The nucleon structure function $F_2(x, Q^2)$ is obtained by analysis of lepton data from the DIS experiments.

i) Interpretation of this function in framework of the usual, non-covariant parton model suggests, that the quarks (valence+sea) carry approximately only 50% of the nucleon energy-momentum. It follows that one valence quark can carry less than roughly 15%. This estimation follows from approach in the nucleon infinite momentum frame, where the transversal momentum of the quarks is neglected.

ii) The analysis of the function $F_2(x, Q^2)$ in the framework of the covariant parton models gives the following results. The model [20] gives the prediction for the dependence $\langle p_T^2/M^2 \rangle$ on $x$: the ratio vanishes at $x = 0$ and $x = 1$ and reaches the peak value $0.04 - 0.05$ at $x \approx 0.5$. This means, that $p_T/M \approx 0.2$ at the peak, so the mean value over $x$ must be smaller. These results are quite consistent with those obtained in the covariant model in which we obtained for massless quarks the relation [11]

$$p_T^2 \leq M^2 x (1 - x) \equiv p_{T\text{max}}(x)$$

and for average momentum of the valence quarks in the nucleon rest frame we get [15]

$$\langle |p_{\text{val}}| \rangle \approx 0.1 GeV; \quad \langle p_{T\text{val}} \rangle = \frac{3}{4} \langle |p_{\text{val}}| \rangle.$$  \hspace{1cm} (32)

iii) The statistical model [21] of the nucleon gives very good description of the unpolarized $(F_{2n})$ and polarized $(g_{1n})$ structure functions in a broad kinematical region. The temperature, one of free parameters of the model, is fixed to the value $T \approx 0.06 GeV$. Similar estimations follow also from the other versions of statistical model [22] [23]. Let us remark, that the QCD calculations on the lattice suggest, that the temperature corresponding to the transition of the nuclear matter to the quark-gluon plasma is around $T \approx 0.175 GeV$ [24].
Despite of variance of applied models, analysis of the structure functions gives the compatible results on the measure of intrinsic motion of quarks inside the nucleon. Roughly speaking, average momentum of the quark, if “measured” by the scattered lepton should not exceed \( \approx 0.15 \text{GeV} \) in the nucleon rest frame, or \( \approx 15\% \) of the nucleon energy-momentum regardless of the reference frame. One can add, that the leptonic information is straightforward, since after interaction with a quark, the lepton state is not affected by other processes (final state interaction).

B. Hadronic data (quark line)

The Cahn effect represents method of measuring transversal momenta of quarks by means of produced hadrons. The related process has two stages:

1. Lepton - quark interaction generates azimuthal asymmetry on the level of recoiled quarks, which is defined by the relations \( f_1^q(x, p_T) \) and by the distribution of their transverse momentum.

2. Fragmentation of the recoiled quark - in this stage the asymmetry is partially smeared. Inclusion of this effect requires additional free parameters, so this method of evaluation of the quark intrinsic motion is less direct.

The \( p_T \) dependence of the quark distribution function is usually parameterized as

\[
f_1^q(x, p_T) = f_1^q(x) \frac{1}{\pi \langle p_T^2 \rangle} \exp \left( - \frac{p_T^2}{\langle p_T^2 \rangle} \right),
\]

where

\[
\int \frac{1}{\pi \langle p_T^2 \rangle} \exp \left( - \frac{p_T^2}{\langle p_T^2 \rangle} \right) d^2p_T = 1.
\]

One can calculate

\[
\langle p_T \rangle = \int \frac{p_T}{\pi \langle p_T^2 \rangle} \exp \left( - \frac{p_T^2}{\langle p_T^2 \rangle} \right) d^2p_T = \frac{\sqrt{\pi \langle p_T^2 \rangle}}{2}
\]

and from the transverse momentum estimate total momentum in the nucleon rest frame as

\[
\langle |\mathbf{p}| \rangle = \sqrt{\frac{3 \langle p_T^2 \rangle}{2}} = \sqrt{\frac{6}{\pi} \langle p_T \rangle}.
\]

The analysis of the experimental data on the azimuthal asymmetry suggests the following. In the paper \( [18] \) the value \( \langle p_T^2 \rangle \approx 0.25 \text{GeV}^2 \) (i.e. \( \langle p_T \rangle \approx 0.44 \text{GeV} \)) is obtained (note different notation). This result is close to the estimation \( \langle p_T \rangle \approx 0.5 - 0.6 \text{GeV} \) following from the analyses \( [20], [20] \). These numbers suggest, that the corresponding average energy-momentum of a quark in the nucleon rest frame amounts \( \approx 0.6 - 0.8 \text{GeV} \), i.e. \( \approx 64 - 85\% \) of the nucleon mass.

Obviously, two questions arise:

\( a) \) Why do the results related to the intrinsic quark momentum obtained by the methods \( A \) and \( B \), differ by a factor greater than four?

\( b) \) Why does the method \( B \) lead to a paradox, that total energy of quarks in the nucleon rest frame can considerably exceed the nucleon mass?

We do not know the answer, but we realize, that contradiction appears in the framework of the parton model, which has its limits of validity. Nevertheless, the questions are legitimate and require further discussion. In fact, the inconsistency can originate in arbitrary stage of the process. For example, approximation of the probability \( W \) only by the one photon exchange \( (1) \) can be insufficient without further QCD corrections. Or, another function \( W \) can generate different degree of azimuthal asymmetry in the general expression \( [24] \), which means, that fitting the data with the false \( W \) can give false \( \langle p_T \rangle \) even though the corresponding \( \chi^2 \) is good. Further, the quark fragmentation into hadrons is a complex stage comprising of both perturbative and unperturbative QCD aspects. So the present estimations of its impact on the smearing of primordial quark azimuthal asymmetry can be also rather approximate.

4. INTRINSIC 3D MOTION IN COVARIANT PARTON MODEL

This section follows from our previous study \( [15] \) and \( [17] \). In the present paper we again assume mass of quark \( m \to 0 \). This assumption substantially simplifies calculation and seems be in a good agreement with experimental data – in all model relations and rules, where such comparison can be done. But in principle, more complicated calculation with \( m > 0 \) is possible \( [13] \). After fixing the quark mass there are no free parameters and construction of the model is based only on the two symmetry requirements: covariance and rotational symmetry. Formulation of
the model in terms of the light–cone formalism is suggested in [17] and allows to define the unpolarized leading-twist TMDs $f_1$ and $f_{1T}$ by means of the light–front correlators $\phi(x, p_T)$ as:

$$\frac{1}{2} \text{tr} \left[ \gamma^+ \phi(x, p_T) \right] = f_1(x, p_T) - \frac{\varepsilon^{ik} p_T^i S_k}{M_N} f_{1T}(x, p_T).$$

(37)

Corresponding expressions for integrated or unintegrated distributions $f_1$ are given by Eqs. 5 and 25 in the cited paper and can be equivalently rewritten as:

$$f_1^q(x) = Mx \int G_q(p_0) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{dp_1 d^2p_T}{p_0},$$

(38)

$$f_1^q(x, p_T) = Mx \int G_q(p_0) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{dp_1}{p_0}.$$ 

(39)

Now we shall study these relations in more detail. Due to rotational symmetry in the nucleon rest frame the distribution $G_q$ depends on one variable $p_0$, in manifestly covariant representation the $p_0$ is replaced by the variable $p^p/M$. In this way the relation (38) defines transformation

$$G_q \rightarrow f_1^q,$$ 

(40)

where both functions depend on one variable. In [15] we showed, that the integral (38) can be inverted

$$G_q(p) = - \frac{1}{\pi M^3} \left( \frac{f_1^q(x)}{x} \right) \left( \frac{2p}{M} \right), \quad x = \frac{2p}{M}, \quad p \equiv p_0 = \sqrt{p_0^2 + p_T^2}.$$ 

(41)

and in this way the distributions $G_q$ can be obtained from distributions $f_1^q$, which are extracted from the structure functions by the global analysis. Apparently, there is one-to-one mapping

$$G_q(p) \leftrightarrow f_1^q(x)$$ 

(42)

so both distributions represent equivalent descriptions.

Now, we will calculate the TMD integral (39). First we calculate roots of the expression in the $\delta-$ function for the variable $p_1$:

$$\frac{p_0 + p_1}{M} - x = 0,$$ 

(43)

there is just one root

$$\tilde{p}_1 = \frac{Mx}{2} \left( 1 - \left( \frac{p_T}{Mx} \right)^2 \right).$$ 

(44)

At the same time the corresponding variable $p_0$ reads:

$$\tilde{p}_0 = \frac{Mx}{2} \left( 1 + \left( \frac{p_T}{Mx} \right)^2 \right).$$ 

(45)

The $\delta-$function term can be modified as

$$\delta \left( \frac{p_0 + p_1}{M} - x \right) dp_1 = \frac{\delta (p_1 - \tilde{p}_1) dp_1}{d \tilde{p}_1} \left( \frac{p_0 + p_1}{M} - x \right)_{p_1=\tilde{p}_1} = \frac{\delta (p_1 - \tilde{p}_1) dp_1}{x/p_0},$$ 

(46)

then after inserting to Eq. (39) one gets:

$$f_1^q(x, p_T) = M \int G_q(p_0) \delta (p_1 - \tilde{p}_1) dp_1 = MG_q(\tilde{p}_0).$$ 

(47)

One can observe, that $f_1^q(x, p_T)$ depends on $x, p_T$ via one variable $\tilde{p}_0$ defined by Eq. (40). It is due to fact, that this variable in $G_q(\tilde{p}_0)$ reflects rotational symmetry in the rest frame. Obviously $x, p_T$ are not independent variables at fixed $p_0$ or $p_1$. Also in Eq. (47) both functions represent equivalent description.
Further, if we define

$$\xi = x \left(1 + \left(\frac{p_T}{Mx}\right)^2\right),$$

(48)

then

$$f^q_1(x, p_T) = MG_q \left(\frac{M}{2} \xi\right).$$

(49)

Since Eq. (41) implies

$$G_q \left(\frac{M}{2} \xi\right) = -\frac{1}{\pi M^3} \left(\frac{f^q_1(\xi)}{\xi}\right),$$

(50)

after inserting to Eq. (49) we get the result

$$f^q_1(x, p_T) = -\frac{1}{\pi M^3} \left(\frac{f^q_1(\xi)}{\xi}\right)'.$$

(51)

This relation represents new rule, which connects integrated and unintegrated unpolarized distribution functions. Before further discussion one can verify compatibility with Eqs. (38) and (39):

$$f^q_1(x) = \int f^q_1(x, p_T) d^2 p_T.$$

(52)

Eq. (51) implies

$$\int f^q_1(x, p_T) d^2 p_T = -\frac{2}{M^2} \int_0^{p_T\text{ max}(x)} \left(\frac{f^q_1(\xi)}{\xi}\right)' p_T dp_T,$$

(53)

where we replaced $d^2 p_T = 2\pi p_T dp_T$. From Eq. (48) we have

$$d\xi = \frac{2p_T dp_T}{M^2 x}.$$

(54)

and Eqs. (31) and (48) imply

$$x \leq \xi \leq 1.$$ 

(55)

Now the Eq. (53) can be modified as

$$\int f^q_1(x, p_T) d^2 p_T = -x \int_x^1 \left(\frac{f^q_1(\xi)}{\xi}\right)' d\xi,$$

(56)

from which now Eq. (52) follows easily.

Now we can make two remarks about obtained results:

i) We should comment the rule (51), which allows to obtain the unintegrated distribution $f^q_1(x, p_T)$ from the integrated $f^q_1(x)$. In fact, due to rotational symmetry and relativistic covariance, all the following distributions involve equivalent information

$$f^q_1(x, p_T) \leftrightarrow G_q(p) \leftrightarrow G_q(p_0) \leftrightarrow G_q\left(\frac{p_T}{M}\right) \leftrightarrow f^q_1(x)$$

(57)

and also the sets of variables are equivalent:

$$p \leftrightarrow x, p_T, \quad d^3 p = \frac{p_0}{x} dx d^2 p_T.$$ 

(58)

Actually the rotational symmetry and covariance are conditions, which the rule (51) follows from.

ii) All the functions (57) depend also on $Q^2$. Due to this equivalence, the evolution of $f^q_1(x, p_T, Q^2)$ can be obtained by evolution $f^q_1(x, Q^2)$ and similarly for other distributions.

At the end, we can complete the obtained relations by corresponding figures. Transverse momentum distribution functions $f^q_1(x, p_T)$ are calculated from Eq. (51), for input distributions $f^q_1(x)$ we used the standard parameterization [27] (LO at the scale $4 GeV^2$). In Fig. 2 we have results for $u$ and $d$–quarks. Left part demonstrates, that $x$ and $p_T$ are not independent variables. In accordance with relation (31), in the sample of partons with fixed $p_T$ the region of low $x$ is effectively suppressed. For increasing $p_T$ the effect is getting more pronounced. The right part of the figure confirms, that typical value of $p_T$ in this approach corresponds to the estimates based on the leptonic data in Sec. 3.
FIG. 2: Transverse momentum dependent unpolarized distribution functions for $u$ (upper figures) and $d$–quarks (lower figures). **Left part**: dependence on $x$ for $p_T/M = 0.10, 0.13, 0.20$ is indicated by dash, dotted and dash-dot curves; solid curve corresponds to the integrated distribution $f_q(x)$. **Right part**: dependence on $p_T/M$ for $x = 0.15, 0.18, 0.22, 0.30$ is indicated by solid, dash, dotted and dash-dot curves.

5. SUMMARY AND CONCLUSION

We studied some questions related to the distribution of quark transverse momenta in the framework of covariant approach. From this point of view, the distribution is a projection of more general 3D motion of quarks inside the nucleon to the plane transverse to the momentum of probing particle. Due to general arguments, 3D motion of quarks in the nucleon rest frame has rotational symmetry. It follows, that both the pictures - 2D and 3D momenta distributions involve equivalent information. The main results obtained in this paper can be summarized as follows.

i) We analyzed physical conditions generating the Cahn effect, which represents important tool for measuring of the quark transverse motion. We suggested, that the effect has a more general origin than it is currently considered. At the same time we presented arguments, why the analysis of data on azimuthal asymmetry due to Cahn effect requires caution.

ii) We have done a comparison, which suggests that the data on transverse motion based on Cahn effect disagree with the data based on analysis of the structure functions ($F_2$) in the framework of various models. Both methods differ in estimation of $\langle p_T \rangle$ by factor $\approx 4$.

iii) We studied unpolarized parton distribution functions $f_1^u(x, p_T)$ in the framework of the 3D covariant parton model. We obtained a new rule, which relates this TMD to its integrated counterpart $f_1^q(x)$. Using this rule with the input on integrated distribution obtained from the global analysis, we calculated $f_1^u(x, p_T)$ also numerically.

We confirmed, that symmetry requirement of relativistic covariance combined with the nucleon rotational symmetry represent a powerful tool for revealing new rules connecting various parton distribution functions, including relations between the integrated (1D) and their unintegrated (3D) counterparts.

Acknowledgments

This work was supported by the project AV0Z10100502 of the Academy of Sciences of the Czech Republic. I am grateful to Anatoli Efremov, Peter Schweitzer and Oleg Teryaev for many useful discussions and valuable comments. I would like to thank also to Jacques Soffer and Claude Bourrely for helpful comments on earlier version of the manuscript.

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