Unique identification of stiffness parameters for nonlinear, anisotropic textile fabrics based on full-field measurements on a single experiment

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This work discusses a hybrid experimental-numerical framework by which the stiffness parameters for nonlinear anisotropic textile fabrics can be uniquely identified. In the experimental part, a non-standard setup has been designed to activate the deformation modes in a free energy description describing the mechanical material behavior. As a result, one inhomogeneous deformation field is captured by means of measuring full-field kinematics, which includes sufficient information regarding the nonlinear response for varying deformation modes and intensities. In the numerical part, the measured full-field data are inserted into a suitable discretized model of the specimen. Reformulating the free energy function to be linear in material parameters is considered in order to enable the definition of a quadratic objective function in terms of differences of discrete internal and external forces and hence to assure the uniqueness of the parameter identification. Finally, the sensitivity of the method to noisy data is studied.

1 Identification framework

1.1 Material model reformulation and classical parameters adjustments

Coated textile fabrics are thin structures that consist of two interlaced orthogonal fiber families: warp and fill. A suitable strain energy function describing the nonlinear anisotropic behavior of the textile membrane is illustrated in [1] and given as:

\[ \psi := \psi_w + \psi_f + \psi_{\text{orth}} + \psi_{\text{orth}} \]

where the terms \( \psi_w \) and \( \psi_f \) are contributions coming from each fiber family and from the interaction between the fibers, respectively. By analogy with [2], the free energy function is reformulated to be linear in material-stiffness related parameters as follows:

\[ \psi := \alpha_w \Psi_w + \alpha_f \Psi_f + \alpha_{\text{orth}} \Phi_{\text{orth}} + \alpha_{\text{orth}} \Phi_{\text{orth}} \]

with \( \Phi_i \) and \( \Phi_{\text{orth}} \) being polyconvex functions defined as [3, 4]:

\[ \Phi_i = (J_{4i} - 1)^{\beta_i}, \quad \Phi_{\text{orth}} = \frac{(\tilde{J}^{(\beta_2 + 1)}_{4i} - 1) (\beta_2 + 1) g_i^{\beta_2 i}}{(\beta_2 + 1) g_i^{\beta_2 i} - \ln(I^i_3)} \]

where \( i \) indicates either warp or fill. The mixed invariant of the right Cauchy-Green tensor \( C \) and the metric tensor \( G \) are:

\[ J_{4i} = \text{tr}[CG_i] \quad J_3 = \text{tr}[CG_i] \quad I_3 = \det C \]

The exponents \( \beta_1, \beta_2, \) and \( g_i \) are model parameters.

According to the Japanese guideline MSAJ/M-02-1995, the loading profile for the characterization of textile membranes consists of three biaxial stress ratios warp:fill 1:1, 2:1 and 1:2, and two uniaxial stress ratios 1:0 and 0:1. The model parameters appearing in the functions \( \Phi_i \) and \( \Phi_{\text{orth}} \) have been attained in a classical fashion by fitting the five stress-strain data resulting from the standard tests simultaneously as shown in Fig. 1 (a).

1.2 The equilibrium gap method

The objective of our strategy is to identify uniquely and efficiently the material parameters \( \alpha_i \) linearly appearing in the constitutive equations. Therefore, we make use of the Equilibrium Gap Method [5], where the displacement field experimentally measured is directly inserted into a suitable FE discretization. As a result, the system does not have to be solved but rather the internal forces need to be evaluated for certain material parameters. The set of the constitutive parameters \( \alpha \) minimizes the discrepancy between the experimental applied forces \( R_{\text{ext}} \) and the internal computed forces \( R_{\text{int}} \) so that the equilibrium equation represented by the discrete residual vector \( R = R_{\text{int}}(\alpha, D) - R_{\text{ext}} \) is satisfied as best as possible in terms of chosen material model and the measurement accuracy. Thus, the norm of residual vector is considered as the basis of the objective function to be minimized \( g(\alpha, D) = R(\alpha, D)^T R(\alpha, D) \), where \( D \) is the discrete matrix including the nodal displacements caused by \( R_{\text{ext}} \).

Since \( \psi \) is linear in the material parameters, the vector \( R_{\text{int}} \) is linear as well. Therefore, the objective function \( g \) is quadratic. Moreover, for a known displacement field \( D \), the internal forces become a function only of \( \alpha \). Based thereon, the set of material parameters \( \alpha = \arg \min_\alpha g(\alpha) \) is unique and can be more or less directly computed without any sophisticated global minimization procedure.

1.3 Non-standard experimental setup

Standard identification techniques are based on performing several tests resulting in several homogeneous stress/strain fields. Consequently, for more parameters governing the constitutive model, more tests are required, especially, when nonlinear

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anisotropic materials, such as textile membranes, are under investigation. Unlike this classical approach, a non-standard experimental setup is proposed here by which one inhomogeneous strain field is obtained on purpose, whose most important feature is the ability to excite all the deformation modes in the free energy function. The experimental setup with the corresponding dimensions is illustrated in Fig. 1 (b).

![Experimental Setup Illustration](image)

Fig. 1: (a) The experimental stress-strain data of a textile membrane specimen under uniaxial and biaxial tension tests with the fitted response. (b) Schematic illustration of the disassembled experimental setup that consists of a steel frame fixed in space clamping the specimen and a circular disk by which load/moment is applied.

## 2 Results and discussion

Two sets of examples have been analyzed. The first set considers the displacements obtained from performing a FE simulation representing the experimental setup as the full-field data. This set of examples serves as a feasibility study in order to verify the identification approach. Two in-silico loading programs have been considered; 1) vertical displacements Fig. 2 (a), and 2) vertical displacements combined with torsion Fig. 2 (b). The relative error measure, defined as \( \eta = \frac{\alpha_{\text{ide}} - \alpha_{\text{pre}}}{\alpha_{\text{pre}}} \), is used to assess the quality of the identified parameters, where \( \alpha_{\text{ide}} \) and \( \alpha_{\text{pre}} \) are the identified and the predefined parameters respectively.

The second set of tests is concerned with noisy displacements. In the real case scenario, the displacements are not measured exactly but rather subject to an error. Therefore, the robustness of the method to noisy displacements should be addressed. For this purpose, white noise is added to the computed displacements as follows: \( D^\text{noise} = D + (1 + \Theta_j)D_j \), where \( D \) and \( D^\text{noise} \) are the vectors containing the nodal displacements and their noisy counterpart respectively, and \( \Theta \) is the vector of random Gaussian distributed variables in the range \( \pm 10^{-4} \) with confidence interval 97.5%.

From the results presented in Table 1, it can be noted that for the case without noise the identified parameters are identical up to computer precision when comparing to the prescribed ones. Nevertheless, for noisy displacements resulting from the first loading program, the relative error for the parameter associated with the transversely isotropic term in fill direction is more than 100%. This implies that the non linearity of the fill direction is still not activated and hence a more suitable loading program is needed. For the second loading program the quality of the results associated with the fill direction improves significantly and the relative error does not exceed a few percent.

![Deformed Geometry](image)

Fig. 2: Deformed geometry

### Table 1: The relative error for the identified parameters.

| Relative error | Reference values | \( \alpha_{\text{ide}} \) | \( \alpha_{\text{pre}} \) |
|----------------|-----------------|-----------------|-----------------|
| \( \eta \) (no noise) | 3039400 | 47259 | 1.17 | 2.02 |
| \( \eta \) (noise range ±0.1%, LP 1) | 10^{-15} | 10^{-15} | 10^{-15} | 10^{-15} |
| \( \eta \) (noise range ±0.1%, LP 2) | 0.12 | > 1 | 0.12 | 0.14 |
| \( \eta \) (noise range ±0.1%, LP 2) | 0.14 | 0.07 | 0.12 | 0.06 |

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