EFFECTS OF RESISTIVITY ON MAGNETIZED CORE-COLLAPSE SUPERNOVAE

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Received 2012 June 4; accepted 2012 November 26; published 2013 January 21

ABSTRACT

We studied the role of turbulent resistivity in the core-collapse of a strongly magnetized massive star, carrying out two-dimensional resistive-MHD simulations. Three cases with different initial strengths of magnetic field and rotation are investigated: (1) a strongly magnetized rotating core, (2) a moderately magnetized rotating core, and (3) a very strongly magnetized non-rotating core. In each case, one ideal-MHD model and two resistive-MHD models are computed. As a result of these computations, each model shows an eruption of matter assisted by magnetic acceleration (and also by centrifugal acceleration in the rotating cases). We found that resistivity attenuates the explosion in cases 1 and 2, while it enhances the explosion in case 3. We also found that in the rotating cases, the main mechanisms for the amplification of a magnetic field in the post-bounce phase are an outward advection of the magnetic field and a twisting of poloidal magnetic field lines by differential rotation, which are somewhat dampened down with the presence of resistivity. Although magnetorotational instability seems to occur in the rotating models, it plays only a minor role in magnetic field amplification. Another impact of resistivity is that on the aspect ratio. In the rotating cases, a large aspect ratio of the ejected matter, \( \geq 2.5 \), attained in an ideal-MHD model is reduced to some extent in a resistive model. These results indicate that resistivity possibly plays an important role in the dynamics of strongly magnetized supernovae.

Key words: magnetohydrodynamics (MHD) – methods: numerical – stars: magnetars – supernovae: general

Online-only material: color figures

1. INTRODUCTION

Studies on magnetized core-collapse supernovae (CCSNe) have gathered steam for the past several years. Numerical simulations so far have shown that the presence of a strong magnetic field together with rapid rotation results in a vigorous eruption of matter accompanied by bipolar jets with an explosion energy of \( \sim 10^{51} \) erg (magnetorotational explosion: LeBlanc & Wilson 1970; Symbalisty 1984; Yamada & Sawai 2004). One of the main driving forces is a toroidal pressure amplified by differential rotation that becomes intense after the collapse in the vicinity of the proto-neutron star surface. This mechanism requires a magnetic field strength of \( \geq 10^{15} \) G after collapse, which is comparable to an inferred surface magnetic field of magnetar candidates, soft-gamma repeaters (SGRs), and anomalous X-ray pulsars (AXPs). Magnetically driven explosions may be related to such supernovae which produce magnetars.

Most numerical simulations of magnetized core-collapse have so far been performed in the regime of ideal magnetohydrodynamics (MHD; e.g., Kotake et al. 2004; Sawai et al. 2005; Moiseenko et al. 2006; Burrows et al. 2007; Sawai et al. 2008; Takiwaki et al. 2009; Obergaulinger & Janka 2011; Endeve et al. 2012). The only exception is the numerical study by Guilet et al. (2011), in which resistivity is introduced to one-dimensional MHD simulations investigating the dynamics of an Alfvén surface in the context of CCSNe. Although a numerical computation with a finite differential scheme inevitably involves numerical diffusion, the effects of electric resistivity on the dynamics have not been investigated systematically. The reason why resistivity has been neglected is that it would be infinitesimally small assuming that the source is the Coulomb scattering (Spitzer resistivity; Spitzer 1956). The magnetic Reynolds number, the ratio of the resistive timescale to the dynamical timescale, estimated with typical parameters of the proto-neutron star surface is

\[
R_m \sim 8 \times 10^{15} \left( \frac{Z}{26} \right)^{-1} \left( \frac{T}{5 \times 10^{10} \text{K}} \right)^{3/2} \left( \frac{L}{4 \times 10^5 \text{cm}} \right) \left( \frac{v}{2 \times 10^8 \text{cm s}^{-1}} \right),
\]

where \( Z, T, L, \) and \( v \) are, respectively, an atomic charge number, temperature, scale length of the magnetic field, and flow velocity. Since the magnetic Reynolds number is much larger than unity, resistivity seems unimportant for the dynamics. However, it is uncertain whether Coulomb scattering is the unique origin of resistivity in a supernova core, and other sources may exist that give rise to a dynamically important value of resistivity. One such candidate is turbulence. In the collapsed core of a massive star, convection, which occurs due to a negative gradient of entropy and/or the electron fraction, may play a key role in producing a turbulent state and turbulent resistivity (and viscosity) along with that. Thompson et al. (2005) roughly estimated the amplitude of turbulent viscosity arising from convective motions in a supernova core by the product of the correlation of length and convective velocity, \( \xi_{\text{con}} \sim l v_{\text{con}}/3 \). They found that \( \xi_{\text{con}} \) is around \( 10^{13} \)–\( 10^{14} \) cm² s⁻¹. As far as this level of estimation is concerned, the amplitude of turbulent resistivity may be evaluated by the same formula and may be comparable to turbulent viscosity, since the magnetic field would have a similar timescale and length scale to those of the velocity in the present situation. Yoshizawa (1990) showed in the framework of the so-called two-scale direct-interaction approximation that the magnitude of turbulent viscosity and turbulent resistivity are of the same order, albeit in the context of...
an incompressible MHD turbulence. With the above amplitude of the turbulent resistivity, the magnetic Reynolds number becomes $\sim 1$–$10$ around the surface of a proto-neutron star, and then resistivity is possibly important for the dynamics.

In this paper, we investigate how (turbulent) resistivity alters the dynamics of a magnetized core-collapse, paying particular attention to the explosion energy, the magnetic field amplification, and the aspect ratio of ejected matters. To this end, we carried out axisymmetric two-dimensional resistive-MHD simulations of the core-collapse of a massive star, assuming a strong magnetic field and large resistivity. Constant resistivities of $10^{13}$ and $10^{14}$ cm$^2$ s$^{-1}$ are used based on the above discussion. Both rapidly rotating and non-rotating cores are studied. In the computations, we omitted any treatments of neutrinos and adopted a nuclear equation of state (EOS) produced by Shen et al. (1998a, 1998b).

Before proceeding to the next section, we go into a little more detail on convection as a source of turbulent resistivity and also mention our position in choosing the initial strength of a magnetic field and a rotation.

In a collapsed stellar core, there are mainly two convectively unstable regions (see, e.g., Herant et al. 1994). One is a region behind the shock surface, where a negative entropy gradient is created as the shock surface propagates with its amplitude decreasing, and is maintained by a neutrino heating. The other is a region around the proto-neutron star surface, where a negative gradient of lepton fraction is created because it is easier for a neutrino to escape from the core with a larger radius. Since we do not deal with neutrinos, convections related to them are not captured in the computations. The only convection that seems to appear in our computation is one due to the shock propagation with a decreasing amplitude. Nevertheless, we assume all of the above convections as sources of turbulent resistivity adopted in this study, since they will occur in nature. What we have done here is to effectively introduce an impact of these convections on the simulations. In doing so, it does not seem quite important whether they are properly captured in the simulations. Note that we do not consider that the standing accretion shock instability, which is not present in our computations, is one of the origins of turbulent resistivity, because all of our models explode before this instability can develop (several 100 ms after bounce), owing to strong magnetic fields initially assumed.

Although turbulent resistivity will appear only around convectively unstable regions, in our computations constant resistivity is assumed everywhere except in the vicinity of the center (see Section 3). Also, we should note that the estimation made by Thompson et al. (2005) is very uncertain. Moreover, strong magnetic fields initially assumed may decrease the strength of a turbulent resistivity. Therefore, the strengths of the adopted turbulent resistivity are perhaps too large to be realistic. However, at present, a probable value of turbulent resistivity in a collapsed-stellar core is very unclear. Under such a circumstance, it is meaningful to parametrically study its effect with some possible values and to grasp dynamical trends. We consider that the adopted strengths of turbulent resistivity may be maximum possible values.

A strongly magnetized core prior to collapse, assumed in the present study, is based on the so-called fossil-field hypothesis, which supposes that the progenitor of a magnetar already has a magnetar-class magnetic flux during the main-sequence stage. Assuming this hypothesis, Ferrario & Wickramasinghe (2006) have done population synthesis calculations from main-sequence stars to neutron stars, to fit observational data of radio pulsars. Their calculation produces a number of magnetars consistent with those observed in the Galaxy, where both the age and rotational period of magnetars are taken into account. The fossil field hypothesis is also supported by observations. There are several O-type stars whose surface magnetic flux is inferred to be a magnetar class, e.g., HD148937 (Wade et al. 2012) and HD19612 (Donati et al. 2006). Aurèilhac et al. (2010) measured surface magnetic fields of Betelgeuse, a red supergiant star, to be $\sim 1$ G, which indicates a magnetar-class magnetic flux. Note, however, that the origin of strong magnetic fields in magnetars is still controversial. Alternatively, they may be produced during the core-collapse by a dynamo mechanism (Thompson & Duncan 2003).

Heger et al. (2005) found that the so-called Tayler–Spruit dynamo (Spruit 2002) drastically slows down the rotation of a star especially during the early phase of a red supergiant, where an angular momentum of the rapidly rotating helium core is transported into the slowly rotating hydrogen envelope. According to their computation, an inferred rotational period of a pulsar is $\sim 10$ ms for a 15 $M_\odot$ progenitor, in which the available rotational energy is insufficient for the explosion. On the other hand, Woosley & Heger (2006) carried out stellar evolution computations of inherently rapid rotators and showed that the rotation of a pre-supernova core could be fast. Due to the fast rotation, matter is almost completely mixed, and instead of forming a red supergiant it becomes a compact helium core star, where magnetic torque works less efficiently. One of their 16 $M_\odot$ magnetic star models with solar metallicity results in the expected pulsar rotation period of 2.3 ms, sufficient for a magnetorotational explosion. Note that both works involve uncertainties about mass-loss rate and multi-dimensional effects. At present, it is unclear whether either slow or fast rotation is appropriate for the progenitor of a magnetar. Hence, in this study both rapidly rotating and non-rotating models are investigated, where the latter, in effect, correspond to a slow rotation case.

The rest of this paper is organized as follows. We describe the governing equations and essentials of our resistive-MHD code in Section 2, and computational set-ups in Section 3. The results are presented in Section 4. A discussion and our conclusion are given in Section 5.

2. GOVERNING EQUATIONS AND NUMERICAL SCHEMES

In order to follow the dynamics of magnetized core-collapses with resistivity, the resistive-MHD equations below are solved:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot \left( \rho \mathbf{v} \mathbf{v} - \frac{B \mathbf{B}}{4\pi} \right) = -\nabla \left( p + \frac{B^2}{8\pi} \right) - \rho \nabla \Phi, \quad (2)$$

$$\frac{\partial \left( e + \frac{\rho v^2}{2} + \frac{B^2}{8\pi} \right)}{\partial t} + \nabla \cdot \left( e + p + \frac{\rho v^2}{2} + \frac{B^2}{4\pi} \right) \mathbf{v} - \frac{(\mathbf{v} \cdot \mathbf{B}) \mathbf{B}}{4\pi} + \frac{\eta}{c} \mathbf{j} \times \mathbf{B} = -\rho \nabla \Phi \cdot \mathbf{v}, \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \times \mathbf{E} = 0, \quad (4)$$

\footnote{In each computation, we found that the square of Brunt–Väisälä frequency is negative due to a negative entropy gradient in some locations behind the shock surface and that a relatively large vorticity develops around there.}
in which the notations of the physical variables are customary.

To solve the above equations we have developed a two-dimensional resistive-MHD code called Yamazakura. This is a time-explicit, Eulerian code based on the high-resolution central scheme formulated by Kurganov & Tadmor (2000, hereafter the KT scheme). Below we briefly describe the features of Yamazakura. For the sake of simplicity, we deal in the case where the equations are written in Cartesian coordinates with plane symmetry in the z-direction.

The KT scheme adopts a finite volume method to solve conservation equations. Although the induction Equations (4) apparently seem to be written in non-conservation forms, they are rewritten into conservation forms (Ziegler 2004):

\[
\frac{\partial B^x}{\partial t} + c \nabla \cdot (0, E^x) = 0, \\
\frac{\partial B^y}{\partial t} + c \nabla \cdot (-E^y, 0) = 0, \\
\frac{\partial B^z}{\partial t} + c \nabla \cdot (E^z, -E^z) = 0.
\]

Then the evolutionary Equations (1)–(3) and (7) are all written in conservation forms with source terms,

\[
\frac{\partial u}{\partial t} + \nabla \cdot (f^x(u), f^y(u)) \\
+ \nabla \cdot (g^x(u, u_x, u_y), g^y(u, u_x, u_y)) = s(u),
\]

where an expression such as \( \nabla \cdot (f^x, f^y) \) means \( \frac{\partial f^x}{\partial x} + \frac{\partial f^y}{\partial y} \). The vectors in Equation (8), each of which has eight components, are given as follows:

\[
\begin{pmatrix}
\rho \\
\rho v^x \\
\rho v^y \\
\rho v^z \\
e + \rho v^2/2 + B^2/8\pi \\
B^x \\
B^y \\
B^z
\end{pmatrix}
\]

\[
\begin{pmatrix}
\rho v^x \\
\rho v^y \\
\rho v^z \\
e + \rho v^2/2 + B^2/4\pi \\
B^x \\
B^y \\
B^z
\end{pmatrix}
\]

\[
\begin{pmatrix}
\rho v^x (\rho v^x B^x - B^x B^y + B^y B^z) + p B^x/4\pi \\
\rho v^y B^y - B^y B^x + B^y B^z \\
\rho v^z B^z - B^z B^x + B^z B^y \\
e + \rho v^2/2 + B^2/4\pi \\
B^x \\
B^y \\
B^z
\end{pmatrix}
\]

\[
\begin{pmatrix}
\rho v^x \\
\rho v^y \\
\rho v^z \\
e + \rho v^2/2 + B^2/4\pi \\
B^x \\
B^y \\
B^z
\end{pmatrix}
\]

\[
\begin{pmatrix}
\rho v^x (\rho v^x B^x - B^x B^y + B^y B^z) + p B^x/4\pi \\
\rho v^y B^y - B^y B^x + B^y B^z \\
\rho v^z B^z - B^z B^x + B^z B^y \\
e + \rho v^2/2 + B^2/4\pi \\
B^x \\
B^y \\
B^z
\end{pmatrix}
\]

\[
\begin{pmatrix}
\rho v^x \\
\rho v^y \\
\rho v^z \\
e + \rho v^2/2 + B^2/4\pi \\
B^x \\
B^y \\
B^z
\end{pmatrix}
\]

\[
\begin{pmatrix}
\rho v^x (\rho v^x B^x - B^x B^y + B^y B^z) + p B^x/4\pi \\
\rho v^y B^y - B^y B^x + B^y B^z \\
\rho v^z B^z - B^z B^x + B^z B^y \\
e + \rho v^2/2 + B^2/4\pi \\
B^x \\
B^y \\
B^z
\end{pmatrix}
\]

\[
\begin{pmatrix}
\rho v^x \\
\rho v^y \\
\rho v^z \\
e + \rho v^2/2 + B^2/4\pi \\
B^x \\
B^y \\
B^z
\end{pmatrix}
\]

The KT scheme is written as follows:

\[
\frac{du_{i,j}(t)}{dt} = - \frac{F^x_{i+1/2,j}(t) - F^x_{i-1/2,j}(t)}{\Delta x_i} \\
- \frac{F^y_{i+1/2,j}(t) - F^y_{i-1/2,j}(t)}{\Delta y_j} \\
- \frac{G^x_{i+1/2,j}(t) - G^x_{i-1/2,j}(t)}{\Delta x_i} \\
- \frac{G^y_{i+1/2,j}(t) - G^y_{i-1/2,j}(t)}{\Delta y_j} + s_{i,j}(t),
\]

where \( \Delta x_i = x_{i+1/2} - x_{i-1/2} \), \( \Delta y_j = y_{j+1/2} - y_{j-1/2} \), and an integer and half-integer subscript, respectively, means that a variable is evaluated at the numerical cell center and interface. The numerical fluxes \( F \) and \( G \) are for the non-resistive and resistive terms, respectively. The numerical fluxes of non-resistive terms are given by

\[
F^x_{i+1/2,j}(t) \equiv \frac{f^x(u^+_{i+1/2,j}(t)) + f^x(u^+_{i+1/2,j}(t))}{2} \\
- \frac{a_{i+1/2,j}(t)}{2} [u^+_{i+1/2,j}(t) - u^+_{i+1/2,j}(t)],
\]

\[
F^y_{i+1/2,j}(t) \equiv \frac{f^y(u^+_{i+1/2,j}(t)) + f^y(u^+_{i+1/2,j}(t))}{2} \\
- \frac{a_{i+1/2,j}(t)}{2} [u^+_{i+1/2,j}(t) - u^+_{i+1/2,j}(t)],
\]

The numerical fluxes of the resistive terms, \( G^x \) and \( G^y \), are to be appear later.

In the original KT scheme, the interface values in Equation (10), which have a superscript “+” or “−,” are
We employ a minmod-like limiter suggested by Kurganov & Tadmor (2000). With this and the spatial interpolations in Equation (12), the original KT scheme is third order in time and second order in space, even though we adopt a non-uniform cell distribution and the spatial interpolations of the primitive variables (Equations (10)), e.g.,

\[
u^{+}_{i+1/2,j}(t) = u_{i+1,j}(t) - \frac{\Delta x_{i+1}}{2}(u_{x})_{i+1,j}(t),
\]

\[
u^{-}_{i+1/2,j}(t) = u_{i,j}(t) + \frac{\Delta x_{i}}{2}(u_{x})_{i,j}(t),
\]

\[
u^{+}_{i,j+1/2}(t) = u_{i,j+1}(t) - \frac{\Delta y_{j+1}}{2}(u_{y})_{i,j+1}(t),
\]

\[
u^{-}_{i,j+1/2}(t) = u_{i,j}(t) + \frac{\Delta y_{j}}{2}(u_{y})_{i,j}(t).
\]

Alternatively, we may be able to use interpolated interface values of the primitive variables \(q = (\rho, v_x, v_y, v_z, e, B_x, B_y, B_z)\),

\[
q^{+}_{i+1/2,j}(t) = q_{i+1,j}(t) - \frac{\Delta x_{i+1}}{2}(q_{x})_{i+1,j}(t),
\]

\[
q^{-}_{i+1/2,j}(t) = q_{i,j}(t) + \frac{\Delta x_{i}}{2}(q_{x})_{i,j}(t),
\]

\[
q^{+}_{i,j+1/2}(t) = q_{i,j+1}(t) - \frac{\Delta y_{j+1}}{2}(q_{y})_{i,j+1}(t),
\]

\[
q^{-}_{i,j+1/2}(t) = q_{i,j}(t) + \frac{\Delta y_{j}}{2}(q_{y})_{i,j}(t),
\]

In the KT scheme, we only need to know these maximum characteristic speeds instead of carrying out a complicated characteristic decomposition for wave propagations.

The original KT scheme is formulated for uniform spatial cells. We have followed the procedure for deducing the KT scheme described in Kurganov & Tadmor (2000) with non-uniform cells and found that the final semi-discrete form (9)–(11) is unchanged, except that the subscripts to \(\Delta t\) and \(\Delta y\) appear.

In order to obtain the time evolution of conservative variables \(u_{i,j}(t)\), the semi-discrete Equation (9) is time integrated utilizing a third-order Runge–Kutta method according to Kurganov & Tadmor (2000). With this and spatial interpolations in Equations (11), the original KT scheme is third order in time and second order in space. In the Appendix (“Linear Wave Propagation”), it is shown that Yamazakura performs, approximately, at least second order in time and second order in space, even though we adopt a non-uniform cell distribution and the spatial interpolations of the primitive variables (Equations (12)).

In solving MHD equations, it is necessary to satisfy the divergence-free constraint of the magnetic field. To accomplish this, we apply a constraint transport (CT) method to the KT scheme based on Ziegler (2004), extending it to a resistive-MHD case. In a three-dimensional CT method, a magnetic field vector is placed at the center of a cubic cell interface while a numerical flux (or an electric field vector) is at a cell edge so that \(\nabla \cdot \mathbf{B}\) does not evolve (Evans & Hawley 1998). The positional relation between them in a two-dimensional case is shown in Figure 1. Due to this placement, the semi-discrete equations and the numerical fluxes of the induction equations should be different from Equations (9) and (10). With numerical fluxes, \(\vec{E} = \vec{F} + \vec{G}\), the semi-discrete form of the induction equations is written as

\[
\frac{d}{dt} B_{i+1/2,j}^{x} = -\frac{\vec{E}_{i+1/2,j+1/2}^{z} - \vec{E}_{i+1/2,j-1/2}^{z}}{\Delta y_{j}},
\]

\[
\frac{d}{dt} B_{i,j+1/2}^{y} = \frac{\vec{E}_{i+1/2,j+1/2}^{z} - \vec{E}_{i-1/2,j+1/2}^{z}}{\Delta x_{i}},
\]

\[
\frac{d}{dt} B_{i,j}^{z} = -\frac{\vec{E}_{i+1/2,j+1/2}^{y} - \vec{E}_{i-1/2,j+1/2}^{y}}{\Delta x_{i}} + \frac{\vec{E}_{i+1/2,j-1/2}^{y} - \vec{E}_{i-1/2,j-1/2}^{y}}{\Delta y_{j}}.
\]

The numerical fluxes of the non-resistive terms are

\[
\vec{F}_{i,j+1/2}^{x} = - F_{i,j+1/2}^{x},
\]

\[
\vec{F}_{i+1/2,j}^{y} = F_{i+1/2,j}^{y},
\]

\[
\vec{F}_{i+1/2,j+1/2}^{z} = \frac{1}{4} \left( - F_{i+1/2,j}^{z} - F_{i+1/2,j+1}^{z} + F_{i+1,j+1/2}^{z} + F_{i+1,j+1/2}^{z} \right),
\]

where \(F_{i,j}^{m}(x,y)\) denotes the \(m\)th component of the vector \(F_{i,j}^{m}(x,y)\).

The interpolations for the \(x\)-component and \(y\)-component of a magnetic field along the \(x\)-direction are given by

\[
B_{i+1/2,j}^{x} = B_{i+1/2,j}^{x},
\]

\[
B_{i+1/2,j}^{y} = B_{i+1/2,j}^{y},
\]

\[
B_{i+1/2,j}^{z} = \frac{1}{2} \left( B_{i+1,j+1/2}^{z} + B_{i+1,j-1/2}^{z} - \frac{\Delta x_{i+1}}{2} \left( B_{i+1,j+1/2}^{z} + B_{i+1,j-1/2}^{z} \right) \right).
\]
\[ B_{i+1/2,j} = \frac{1}{2} \left[ B_{i,j+1/2}^x + B_{i,j-1/2}^x + \frac{\Delta x_j}{2} \left( \left( B_{i+1,j+1/2}^x \right) + \left( B_{i+1,j-1/2}^x \right) \right) \right]. \]

Those along the y-direction are given in a similar way as the above. Note again that interpolations of \( B_z \) are the same as Equation (12), since in two dimensions it is defined at a cell center.

In order to obtain the numerical fluxes of the resistive terms that appear in the energy equation (3) and the induction equations (7), an evaluation of current density is required, which we simply give by

\[
\begin{align*}
J_{i+1/2,j}^x &= \frac{c}{4\pi} \frac{B_{i,j+1}^z - B_{i,j}^z}{\Delta y_{i+1/2}}, \\
J_{i+1/2,j}^y &= -\frac{c}{4\pi} \frac{B_{i+1,j}^z - B_{i,j}^z}{\Delta x_{i+1/2}}, \\
J_{i+1/2,j}^z &= \frac{c}{4\pi} \left[ \frac{B_{i+1,j+1/2}^z - B_{i,j+1/2}^z}{\Delta y_{i+1/2}} - \frac{B_{i+1/2,j+1}^z - B_{i+1/2,j}^z}{\Delta y_{i+1/2}} \right].
\end{align*}
\]

By virtue of these definitions, the divergence-free condition of a current density is automatically satisfied through a similar logic as the CT scheme:

\[ \frac{J_{i+1/2,j}^x - J_{i,j+1/2}^x}{\Delta x_{i+1/2}} + \frac{J_{i+1/2,j}^y - J_{i+1/2,j}^y}{\Delta y_{i+1/2}} = 0. \]

The representations for the numerical fluxes of the resistive terms in the induction equations are straightforward, since a current density is defined at the same grid position as a numerical flux:

\[
\begin{align*}
\tilde{G}_{i+1/2,j}^x &= \frac{4\pi \eta}{c^2} J_{i+1/2,j}, \\
\tilde{G}_{i+1/2,j}^y &= \frac{4\pi \eta}{c^2} J_{i+1/2,j}, \\
\tilde{G}_{i+1/2,j}^z &= \frac{4\pi \eta}{c^2} J_{i+1/2,j+1/2}.
\end{align*}
\]

The numerical fluxes of the resistive terms in the energy equation are given as\(^5\)

\[
\begin{align*}
G_{i+1/2,j}^{(5)}(t) &= \frac{\eta}{c} \left[ J_{i+1/2,j}^y \frac{B_{i+1/2,j}^x - B_{i+1/2,j}^x}{2} - \frac{J_{i+1/2,j}^y}{2} + \frac{J_{i+1/2,j}^y}{2} + \frac{B_{i+1/2,j}^x + B_{i+1/2,j}^x}{2} \right], \\
G_{i+1/2,j}^{(5)} &= \frac{\eta}{c} \left[ J_{i+1/2,j}^y \frac{B_{i+1/2,j}^x - B_{i+1/2,j}^x}{2} - \frac{J_{i+1/2,j}^y}{2} + \frac{J_{i+1/2,j}^y}{2} + \frac{B_{i+1/2,j}^x + B_{i+1/2,j}^x}{2} \right].
\end{align*}
\]

\(^5\) Numerical fluxes given here are a little different from those found in the original KT scheme. In the original scheme the average of cell center values, \( B_i^x \) and \( B_{i+1}^x, \) is used, while we employ the average of the left- and right-interpolated values, \( B_{i+1/2}^x \) and \( B_{i+1/2}^x. \)

In order to close the equation system (1)–(6), we further need to know a gravitational potential and the relation between pressure and other thermodynamic quantities. The former is done by solving the Poisson equation: \( \Delta \Phi = 4\pi G \rho. \) In *Yamazakura*, this is numerically solved by the modified incomplete Cholesky decomposition conjugate gradient method (Gustafsson 1983). For the latter, we adopt a tabulated nuclear EOS produced by Shen et al. (1998a, 1998b), which is commonly used in recent core-collapse simulations, e.g., Sumiyoshi et al. (2005), Murphy & Burrows (2008), Marek et al. (2009), and Iwakami et al. (2009). To derive a pressure from the EOS table, three thermodynamic quantities should be specified: in our case, density, specific internal energy, and electron fraction are chosen. We do not solve the evolution of an electron fraction. Instead it is assumed to be a function of density according to Liebendörfer (2005).\(^6\) A neutrino transport that may be important in the dynamics of a core-collapse is not dealt with in the present simulations. Since a neutrino cooling is not considered, only a photodissociation of heavy nuclei takes energy away from shocked matter. Nevertheless, our core-collapse simulation without a magnetic field and rotation still indicates that no explosion occurs (see Section 4). Although neutrino heating is also omitted, it may not be very important since the present computations are run until at most \( \sim 100 \) ms after bounce, a factor of several shorter than the heating timescale.

Although we adopt Liebendörfer’s prescription for electron fraction through the whole evolution, it is only valid until bounce. For example, that prescription does not properly reproduce a decrease in electron fraction around the neutrino sphere due to the neutrino burst. A numerical simulation performed by Sumiyoshi et al. (2007), which deals with sophisticated neutrino physics, shows that electron fraction around the neutrino sphere at 100 ms after bounce is \( \sim 0.1, \) while it is \( \sim 0.3 \) in our simulations. In the simulation without a magnetic field and rotation at 100 ms after bounce, we have tested how much pressure around the neutrino sphere varies when the electron fraction is changed from Liebendörfer’s value to 0.1, and found that the difference is at most only 20%. Note that in the present simulations, electron fraction may influence dynamics only through pressure and sound speed, where the latter is just related to the strength of numerical diffusion.

*Yamazakura* has passed several numerical test problems, which are demonstrated in the Appendix.

3. COMPUTATIONAL SET-UPS

We follow the collapse of the central 4000 km core of a 15 \( M_\odot \) star provided by S. E. Woosley (1995, private communication). To construct the initial condition of the core, the density and temperature distributions are taken from the stellar data. Note that the initial profile of electron fraction is determined using the prescription by Liebendörfer (2005) as mentioned above. In order for the collapse to proceed in the presence of a strong magnetic field and rapid rotation, the temperature of the initial core is reduced as

\[ T(r) = T_{\text{ref}} \left(1 - \frac{r^2}{r_T^2 + r^2}\right), \]

\(^6\) In Liebendörfer (2005), prescriptions not only for electron fraction, but also for neutrino stress and entropy change are suggested. In the present simulations, we only adopt the prescription for electron fraction.
where $r$ is the distance from the center of the core, $T_{\text{org}}(r)$ is the original temperature of the core, and $r_T$ is taken 1000 km.\(^7\) The initial internal energy distribution is obtained by the EOS table, using density, electron fraction, and temperature as the three parameters. Radial velocities are initially assumed to be zero. The magnetic field and rotation of the core are initially input by hand. We assume that the initial magnetic field is purely dipole like, and the core is either rotating or non-rotating.

The dipole-like magnetic field is produced by a toroidal electric current of a two-dimensional Gaussian-like distribution centered at $(\sigma, z) = (\sigma_0, 0)$,

$$
j_\phi(\sigma, z) = j_0 e^{-\sigma^2/2\sigma^2} \left( \frac{\sigma_0 \sigma}{\sigma_0^2 + \sigma^2} \right), \tag{22}
$$

where $\tilde{r} = \sqrt{(\sigma - \sigma_0)^2 + z^2}$. The last factor is multiplied to impose $j_\phi = 0$ along the pole. Width $\sigma$ is a function of $\theta \equiv \arccos(z/\tilde{r})$, defined by

$$
\sigma(\theta) = \frac{\tilde{r}_{\text{dec}}}{\sqrt{1 - e^2 \cos \theta}}, \tag{23}
$$

which traces a prolate ellipse centered at $(\sigma, z) = (\sigma_0, 0)$ with eccentricity $e$ and major radius $\tilde{r}_{\text{dec}}$. Parameters in Equations (22) and (23) are set as $\sigma_0 = 1000$ km, $\tilde{r}_{\text{dec}} = 710$ km, and $e = 0.5$ in every computation. The parameter $j_0$, which determines the field strength, is given later. From the above electric current distribution, the vector potential is calculated by

$$
A_\phi(\sigma, z) = \frac{1}{c} \int_0^{\sigma_{\text{core}}} \int_0^{2\pi} \int_0^{\sigma_{\text{core}}} \frac{j_\phi(\sigma_c, \phi_c, z_c) \cos \phi_c}{R(\sigma_c, \sigma_c, \phi_c, z_c)} \sigma_c d\sigma_c d\phi_c d\tilde{z}, \tag{24}
$$

where $R(\sigma, z, \sigma_c, \phi_c, z_c)$ is the distance between $(\sigma, 0, z)$ and $(\sigma_c, \phi_c, z_c)$, and $\sigma_{\text{core}} = z_{\text{core}} = 4000$ km. The magnetic field is obtained via $\mathbf{B} = \nabla \times \mathbf{A}$. Note that, evaluating $A_\phi$ at a cell corner, the initial magnetic field automatically satisfies the divergence-free condition in the same way as in the CT scheme. The initial magnetic field configuration and the distribution of magnetic energy per unit mass are shown in Figure 2.

In each rotating model, the initial angular velocity is given by

$$
\Omega(r) = \Omega_0 \frac{r_0^2}{r_0^2 + r^2}, \tag{25}
$$

where $r_0 = 1000$ km and $\Omega_0 = 3.9$ rad s\(^{-1}\).

Employing the above magnetic field and rotation, we study three different cases, namely, a strong magnetic field and rapid rotation (model series Bs-$\Omega$), a moderate magnetic field and rapid rotation (model series Bm-$\Omega$), and very strong magnetic field and no rotation (model series Bss-$\bar{\Omega}$). Parameters for each model series are given in Table 1.

In order to study the effects of resistivity on the dynamics, one ideal model and two resistive models are run in each model series. For the resistive models, we examine two different values of resistivity, $\eta = 10^{13}$ and $10^{14}$ cm\(^2\) s\(^{-1}\), as discussed in Section 1. Resistivity is uniform in space and time except that it is set to zero inside a radius of 10 km to save computational time. There, a magnetic field and thus resistivity are expected to be unimportant due to high density. For descriptive convenience, we use abbreviations $\eta_{-\infty}$, $\eta_{13}$, and $\eta_{14}$, respectively, for models with $\eta = 0$, $10^{13}$, and $10^{14}$ cm\(^2\) s\(^{-1}\), after the name of a model series. For example, a model with a strong magnetic field, rapid rotation, and $\eta = 10^{14}$ cm\(^2\) s\(^{-1}\) is referred to as model Bs-$\Omega$-$\eta_{14}$.

Each computation is performed in cylindrical coordinates. Assuming axisymmetry and equatorial symmetry, we take the numerical domain as $(\sigma, z) \in [0 \text{ km}, 4000 \text{ km}] \times [0 \text{ km}, 4000 \text{ km}]$. Until the central density reaches $10^{12}$ g cm\(^{-3}\), the number of numerical cells is $N_{\sigma} \times N_z = 320 \times 320$. After that, the number of cells is changed to $N_{\sigma} \times N_z = 720 \times 720$. There, the spatial width of a cell increases outward in each direction with a constant ratio of 1.0051 and 1.0056, before and after the re-gridding, respectively. Both the width $\Delta \sigma$ of the innermost cells of the $\sigma$-coordinate and the width $\Delta z$ of the innermost cells of the $z$-coordinate are 5 km and 400 m, before and after the re-gridding, respectively. When the numerical cells are redistributed, physical variables are linearly interpolated from the coarse into the fine cells. In this procedure, the divergence-free constraint of the magnetic field is usually violated, which stems from the poloidal components. To avoid this, we calculate $A_\phi$ from Equation (24) using the distribution of $j_\phi$ right after the cell redistribution, and then obtain a divergence-free poloidal magnetic field.

Each simulation is run until the shock front reaches radii of 2100, 3000, and 2300 km in model series Bs-$\Omega$, Bm-$\Omega$, and Bss-$\bar{\Omega}$, respectively.

---

\(^7\) In what follows we denote spatial points in the polar and cylindrical coordinates with $(r, \theta, \phi)$ and $(\sigma, \phi, z)$, respectively.
of the total energy of fluid elements that fulfill the criterion

\[ E_{\text{total}} = E_{\text{gravitational}} + E_{\text{internal}} + E_{\text{kinetic}} + E_{\text{magnetic}} + E_{\text{potential}}. \]

Here, the terms “positive kinetic energy” and “negative kinetic energy” mean the kinetic energy associated with fluid elements with a positive and a negative radial velocity, respectively.

4. RESULTS

In this section, we will present results of our computations for each model series separately, seeing how resistivity affects the dynamics of a magnetized supernova. Particular attention is paid to the explosion energy, magnetic field amplification, and the aspect ratio of the ejecta.

Before proceeding to the main results, here we describe the dynamical evolution in the simulation without a magnetic field and rotation. Soon after the start of computation, the core has a negative radial velocity everywhere and collapses toward the center. A bounce occurs at \( t = 133 \) ms due to nuclear force, and a shock wave is generated. The shock wave propagating outward first stalls around \( r \sim 200 \) km, but it starts to gradually expand around \( 165 \) ms. Afterward, the shock surface alternately expands and shrinks. We followed the evolution until \( t = 350 \) ms (217 ms after bounce) during which the maximum shock position is \( \sim 800 \) km.

In this way, our model without a magnetic field and rotation does not result in a stalled shock as seen in recent core-collapse simulations (see, e.g., Figure 2 of Nordhaus et al. 2010), which may be because we only consider photodissociation of heavy nuclei but no neutrino cooling as cooling processes. Figure 3 shows the evolutions of total, internal, gravitational, positive kinetic energy, and negative kinetic energy in the simulation without a magnetic field. This indicates that part of the fluid has a positive radial velocity. Nonetheless, we found that the estimated explosion energy is very small: less than \( \sim 10^{48} \) erg and sometimes exactly zero. We assume that a fluid element is exploding if the total fluid energy plus the gravitational potential energy at its position and the radial velocity are both positive, i.e., \( e + \rho v^2 / 2 + B^2 / 8 \pi + \rho \Phi > 0 \) and \( v_r > 0 \). Then the explosion energy is obtained by the sum of the total energy of fluid elements that fulfill the criterion plus the gravitational potential energy for the exploding fluid. We calculate the latter by \( E_{\text{exp},\text{grav}} = \int_{V_{\text{exp}}} [\rho \Phi_{\text{exp}} / 2 + \rho \Phi] dV \), where \( \Phi_{\text{exp}} \) is the gravitational potential due to the exploding fluid and \( \Phi \) is that due to the other fluid, and \( \Phi = \Phi_{\text{exp}} + \Phi_{\text{non-exp}} \).

Gravitational potentials \( \Phi_{\text{exp}} \) and \( \Phi \) are obtained by solving a Poisson equation with only the mass of the exploding fluid and non-exploding fluid, respectively. The fact that the explosion energy is quite small, as mentioned above, implies that most or all fluid elements including those with a positive radial velocity do not fulfill the criterion. It seems reasonable to assume that the model without a magnetic field and rotation does not explode.

The error in the total energy conservation of the system is 51% at the end of the simulation. We found that the error in the total energy conservation is 24%–33% at the end of all simulations involving the magnetic field, except that it is 14% in a different resolution run for model B-\( \Omega_{-14} \) described in Section 4.1.5. In Section 4.1.5, we will discuss whether these errors are problematic for results presented in this paper.

4.1. Strong Magnetic Field and Rapid Rotation—Model Series B-\( \Omega_{-14} \)

We start from briefly describing the dynamical evolution in model B-\( \Omega_{-14} \). In this model, a rotation hampers the collapse and a bounce occurs at \( t = 143 \) ms, 10 ms later than in the case without a magnetic field and rotation. During the collapse, the core is largely spun up accompanied by an increase in the degree of differential rotation; just after bounce, the rotational period reaches \( \sim 1 \) ms in a considerable part inside a radius of 50 km, while it is initially at least \( \sim 1 \) s. A magnetic field, which initially plays a small role, is greatly amplified by compression during collapse. In addition, a twisting of poloidal magnetic field lines due to differential rotation causes a large generation of the toroidal component of the magnetic field around the time of bounce. An outward matter motion driven by bounce first decelerates, losing energy due to a photodissociation of heavy nuclei, but accelerates again, helped by the magnetic pressure of the toroidal field and centrifugal force. As a result, a strong eruption of matter occurs preferentially along the pole. The above dynamical sequence can be followed in Figure 4 in terms of energetics (see thick lines): i.e., a decrease of the gravitational energy results in an increase of the rotational and the magnetic energy; the rotational energy is partially converted into toroidal magnetic energy; then the toroidal magnetic energy is consumed to boost the positive kinetic energy. The top panels of Figure 5...
show the distributions of the velocity and magnetic field at 164 ms (21 ms after bounce) for model \( \text{B}_{\text{s}}-\Omega-\eta^{-\infty} \). A fast mass eruption \((v_r \gtrsim 5 \times 10^9 \text{ cm s}^{-1})\) is seen notably around the pole, where the ratio of magnetic pressure to matter pressure is large. This also implies that magnetic force plays an essential role for a fast mass eruption. Note that the dynamical features described here are quite similar to those found in previous works that employ similar strengths of magnetic field and rotation (see e.g., Yamada & Sawai 2004; Takiwaki et al. 2009).

In a resistive model \( \text{B}_{\text{s}}-\Omega-\eta_{14} \), the evolution proceeds in a qualitatively similar way to the ideal model \( \text{B}_{\text{s}}-\Omega-\eta^{-\infty} \). However, outgoing velocities are relatively slow compared with the ideal model, which results in a smaller shock radius at the same physical time (compare the left panels of Figure 5). The right panels of Figure 5 imply that this is due to a less strong magnetic pressure in model \( \eta_{14} \). As easily expected, a magnetic field amplification by differential rotation is ineffective under the presence of resistivity. This means that the rotational energy cannot be spent efficiently as an energy source for the explosion. Indeed, it is observed in Figure 4 that the rotational energy in model \( \eta_{14} \) decreases slowly compared with that of model \( \eta^{-\infty} \), and that both the positive kinetic energy and magnetic energy increase slowly.

### 4.1.1. Explosion Energy

Below, we will see the effect of resistivity on the explosion energy together with the detailed mechanism of the explosion. Figure 6 shows the evolutions of the explosion energies, \( E_{\text{exp}} \), in model series \( \text{B}_{\text{s}}-\Omega \). It is found that a larger resistivity results in a smaller explosion energy.

As preparation for detailed analyses, we consider dividing the volume inside the shock surface into two parts, namely, the eruption region and the infall region. The definitions of these parts are as follows. First, the volume inside the shock surface is equally cut up with respect to \( \theta \) into 30 volume segments with a \( \Delta \theta = 3^\circ \) opening angle. The eruption region is defined by the sum of the segments whose integrated radial momentum is positive, whereas the infall region is defined by the sum of those having negative radial momentum. For example, in each left panel of Figure 5, the infall region appears in the vicinity of
It may be helpful to compare models B corresponds to the eruption region. Over a positive one, while the other part inside the shock surface with an angular velocity equation of motion written on the frame rotating around the pole by the individual acceleration terms in the equation of motion written on the frame rotating around the pole with an angular velocity $\Omega$,

$$
\frac{D'v_r}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{\partial \Phi}{\partial r} + \frac{1}{\rho c} (j_\theta B_\phi - j_\phi B_\theta) + \Omega^2 r \sin^2 \theta,
$$

(26)

where $D'/Dt$ denotes the Lagrangian derivative on the rotating frame. On the right-hand side of Equation (26), each term represents, from left to right, the acceleration due to pressure, gravity, magnetic field, and rotation. In comparing the accelerations, we take an angular average in the eruption region and a time average during $t = t_1 - t_2$ ms, which are defined by

$$
\langle a \rangle (r) = \int_{t_1}^{t_2} \left[ \frac{\int_{\text{erup}} a \rho \sin \theta d\theta}{\int_{\text{erup}} \rho \sin \theta d\theta} \right] dt / [t_2 - t_1],
$$

(27)

where each term on the right-hand side of Equation (26) is to be assigned with $a$. Figure 7 shows the radial distributions of accelerations, angularly averaged in the eruption region and time averaged during $t = 147–155$ ms, in models $\eta_{-\infty}$ and $\eta_{14}$. The averages are taken inside $\sim 140$ km, the smallest shock radius at 147 ms in models $\eta_{-\infty}$ and $\eta_{14}$. It is found that, in both models, an acceleration is positive almost everywhere for $r \gtrsim 50$ km (top left panel). As shown in Figure 8, a matter ejection also occurs roughly in this radial range, which implies that the amplitude of an acceleration is a good measure for the resulting magnitude of the explosion energy. There, a pressure acceleration alone is always smaller than a gravitational deceleration (bottom left panel). The bottom right panel shows that it is a magnetic and centrifugal acceleration that makes a matter ejection possible.

In the top right panel of Figure 7, the differences in accelerations between the two models are plotted, which shows that a total acceleration is generally larger in model $\eta_{-\infty}$ for $r \gtrsim 50$ km. It is likely that this causes a larger $dE_{\text{exp}}/dr$ in model $\eta_{-\infty}$ as observed in Figure 8. The top right panel also indicates that a larger total acceleration in model $\eta_{-\infty}$ is primary due to that in a magnetic and centrifugal acceleration. Although, a pressure
acceleration is smaller in model $\eta_{-\infty}$, this is more than compensated for by them. Therefore, we conclude that resistivity makes the explosion less energetic due to a small magnetic and centrifugal acceleration.

While a smaller magnetic acceleration in model $\eta_{14}$ is simply due to a weaker magnetic field as a result of magnetic diffusion, that of centrifugal acceleration is related to a less efficient angular momentum transfer owing to weaker magnetic stress. In Figure 8, the distributions of specific internal momenta at 155 ms are plotted in mass coordinates. It is observed that a specific angular momentum in model $\eta_{14}$ is larger than in model $\eta_{-\infty}$ inside a radius of $\sim 25$ km, whereas it is smaller outside of this region. This is the consequence of a less efficient outward angular momentum transfer in model $\eta_{14}$. Then, at a large radius, a centrifugal acceleration in model $\eta_{14}$ is smaller than that in model $\eta_{-\infty}$.

We also examined reasons for a larger pressure acceleration in model $\eta_{14}$ observed in the top right panel of Figure 7. In the present situation, pressure is an increasing function of density and specific internal energy. Although pressure also depends on electron fraction, this dependence is weaker than that on the above two quantities. Hence, it is expected that the density or specific internal energy volume averaged over the eruption region is larger in model D$\Omega$-$\eta_{14}$. By calculating these two average values, we found that both the average density and specific internal energy are larger for model $\eta_{14}$ (see Figure 10). A higher density in model $\eta_{14}$ implies that the matter expansion rate is smaller, which is consistent with what is observed in the left panels of Figure 5. A larger specific internal energy in model $\eta_{14}$ also may come from the smaller expansion rate, but may be caused by Joule heating too. To estimate the amount of thermal energy produced by Joule heating in model D$\Omega$-$\eta_{14}$, the total heating rate of the eruption region, $\int_{\text{erup}} 4\pi \eta j^2/(\rho c^2) dm$, is time integrated from 147 ms when the infall region begins to appear clearly. Then it is divided by a mass of the expansion region at each instant of time and compared with the difference in average specific internal energy between the two models. The result is shown in the right panel of Figure 10, which indicates that the contribution of Joule heating to the difference in the specific internal energy is quite small, around 150 ms. Thus, it is not likely that the larger pressure acceleration in model $\eta_{14}$, observed in the top right panel of Figure 7, is caused by Joule heating. A larger pressure acceleration seems to be due only to a smaller expansion rate in model $\eta_{14}$. The panel implies that only in a later phase, a larger specific internal energy in model $\eta_{14}$ may be somewhat contributed by Joule heating. Note, however, that the thermal energy estimation made here may be crude, since a part of the thermal energy produced in one region may migrate into another.

As we have seen, an inefficiency in both magnetic field amplification and angular momentum transfer leads to a weaker explosion in a resistive model. From the energetic point of view, this corresponds to an inefficiency in consuming the rotational energy as a fuel. Thus one might think that the explosion energy of the ideal model and a resistive model would be similar in terms of the rotational energy consumed. If this is the case, the final explosion energies in the three models, after all the available rotational energy has been drained, would be comparable to each other. According to Figure 11 this is not true, however. In this figure, the evolutions of the explosion energies in the three models are plotted against the remaining rotational energy. Since the maximum rotational energy, which is reached at the time of bounce, is almost the same among the three models, $E_{\text{rot}} \approx 7.9 \times 10^{51}$ erg, each model will consume roughly the same amount of rotational energy with a similar position in the abscissa. It is found that the explosion energy in a resistive model is smaller than that of the ideal model, even though the same amount of rotational energy is expended. This implies that a part of the rotational energy is wasted in locations where the criterion for the explosion is not fulfilled.

4.1.2. Magnetic Field Amplification

In this section we analyze a magnetic field amplification. In Figure 12, the angular distributions of the magnetic energies per unit mass, averaged over 50 km $< r < 0.9 \times r_{\text{sh}}$ at $t = 145$ ms (2 ms after bounce) and $t = 160$ ms (17 ms after bounce), are shown for models B$\Omega$-$\eta_{-\infty}$ and B$\Omega$-$\eta_{14}$. The left panel indicates that the total magnetic energy is relatively stronger around the pole at 145 ms in each model, reflecting the initial magnetic field configuration (see Figure 2). Turning to the right panel, it is found that, in each model, the contrast between the total magnetic energy around the pole and that around $\theta \sim 40^\circ$–$70^\circ$ becomes stronger at 160 ms than at 145 ms. In model $\eta_{14}$, the strong contrast in the total magnetic energy at 160 ms is mainly due to that in the toroidal magnetic energy, while in model $\eta_{-\infty}$, the contrast both in the toroidal and poloidal magnetic energies is responsible for that.
in the poloidal magnetic energy in model \( \eta \rightarrow \infty \) does not vary much. It seems that this causes the strong contrast in the toroidal magnetic energy is strengthened around \( \sim 155–160 \) ms, while that in volume \( V_\text{sh} \) of the remaining rotational energy. The evolution proceeds roughly in the counterclockwise direction.

Figure 10. Left: evolution of the average density for \( r > 50 \) km in the eruption region in models \( B_\Omega \eta \rightarrow \infty \) and \( B_\Omega \eta_{14} \). Right: evolution of the average specific internal energy for \( r > 50 \) km in the eruption region in model \( \eta \rightarrow \infty \) (red line), model \( \eta_{14} \) (blue), and the difference between them (black). The magenta line shows the time-integrated Joule heating produced in the above region for model \( \eta_{14} \).

(A color version of this figure is available in the online journal.)

Figure 11. Evolutions of the explosion energies in model series \( B_\Omega \) in terms of the remaining rotational energy. The evolution proceeds roughly in the counterclockwise direction.

(A color version of this figure is available in the online journal.)

To understand how the contrast is strengthened, we follow the evolution of magnetic energy per unit mass in two representative volumes \( V_{25.5} \) and \( V_{58.5} \), where \( V_{\theta,\phi} \) is defined by 50 km \( \leq r \leq 0.9 \times r_{\text{sh}} \) and \( 1.5 \leq \theta \leq \theta_\star + 1.5 \). Figure 13 shows the evolution of the average magnetic energy per unit mass in the two volumes. It is observed both in models \( \eta \rightarrow \infty \) and \( \eta_{14} \), that the toroidal magnetic energy in both the volumes increases around 150 ms and maintains an almost constant value afterward. The increase rate is higher in volume \( V_{25.5} \). That is, the contrast in the toroidal magnetic energy is strengthened around 150 ms, and is maintained afterward. What is also found is that in model \( \eta \rightarrow \infty \), the poloidal magnetic energy in volume \( V_{25.5} \) increases during \( t \sim 155–160 \) ms, while that in volume \( V_{58.5} \) does not vary much. It seems that this causes the strong contrast in the poloidal magnetic energy in model \( \eta \rightarrow \infty \) at 160 ms, as shown in the right panel of Figure 12.

In order to study the amplification mechanisms of the magnetic field, we write down the evolution equations of the average magnetic energies per unit mass in volume \( V_{\theta,\phi} \), with mass \( M \), \( E_{[r, \theta, \phi]} = \int (B_{r, \theta, \phi}^2 / 8\pi) dV / M \):

\[
\frac{dE_{[r, \theta, \phi]}}{dt} = \dot{E}_{[r, \theta, \phi], \text{adv}} + \dot{E}_{[r, \theta, \phi], \text{cmp}} + \dot{E}_{[r, \theta, \phi], \text{shr}} + \dot{E}_{[r, \theta, \phi], \text{rst}} + \dot{E}_{[r, \theta, \phi], \text{VM}},
\]

where the terms on the right-hand side mean, from the left, the change of \( E_{[r, \theta, \phi]} \) due to advection, compression, velocity shear along magnetic field lines, resistivity, and the variation in the volume and mass. They are defined by

\[
\dot{E}_{[r, \theta, \phi], \text{adv}} = -\frac{1}{M} \int \left[ \frac{v_r}{r} \frac{\partial B_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial B_\theta}{\partial \theta} \right] \frac{B_{r, \theta, \phi}^2}{8\pi} dV,
\]

\[
\dot{E}_{[r, \theta, \phi], \text{cmp}} = -\frac{1}{M} \int \left[ 2v_r r + \frac{1}{r} \partial (\sin \theta v_\theta) \right] \frac{B_{r, \theta, \phi}^2}{4\pi} dV,
\]

\[
\dot{E}_{[r, \theta, \phi], \text{shr}} = -\frac{1}{M} \int \left[ \frac{1}{r} \partial (r v_r) + \frac{\cos \theta v_\theta}{r} \right] \frac{B_\theta^2}{4\pi} dV,
\]

\[
\dot{E}_{[r, \theta, \phi], \text{rst}} = \frac{1}{M} \int \left[ \frac{1}{r} \partial (r v_r) + \frac{1}{r} \partial (v_\theta/r \sin \theta) \right] \frac{B_{r, \theta, \phi}^2}{4\pi} dV,
\]

\[
\dot{E}_{[r, \theta, \phi], \text{VM}} = \frac{\int (B_{r, \theta, \phi}^2 / 8\pi) v_{\text{surf}} \cdot dS}{M} - \frac{\int (B_{r, \theta, \phi}^2 / 8\pi) dV dM}{M^2} \frac{dM}{dt},
\]
These definitions are different from the commonly used ones, where the resistive terms are written for constant resistivity. By evaluating these terms we can see which factors are essential for the magnetic field amplification in a given volume. The results are shown in Figures 14 and 15.

We first focus on the amplifications of the toroidal magnetic energy in model $\eta_{-\infty}$. In volume $V_{25.5}$, the toroidal magnetic energy is primarily amplified by an advection (top right panel of Figure 14). We found that a radial advection dominates over an angular advection, i.e., the amplification is due to an outward advection of large toroidal energy at small radii. As expected, velocity shear along poloidal magnetic field lines, viz., a twisting due to differential rotation also substantially contributes to the amplification. In volume $V_{58.5}$, the toroidal magnetic energy is also amplified due to an advection and shear (see the bottom right panel of Figure 14), but the amplitude of each term is much smaller than in volume $V_{25.5}$, which seems due to an a priori weaker magnetic field.

The amplification of the radial magnetic energy in volume $V_{25.5}$ is also dominated by radial advection. The shear term, which is far smaller than that of the advection term, also seems to play an important role after $\sim 160 \text{ ms}$, since it is comparable to a total $\hat{\mathcal{E}}$ (see the top left panel of Figure 14). As seen in the bottom left panel of Figure 14, the amplification mechanism of radial magnetic energy in volume $V_{58.5}$ is rather complex, contributed by several terms. As in the case of toroidal magnetic energy, the amplification of radial magnetic energy is also weaker in volume $V_{58.5}$ than in volume $V_{25.5}$.

Since the present model series involves a rotation, magneto-rotational instability (MRI) may occur and may play an important role in magnetic field amplification (Balbus & Hawley 1991; Akiyama et al. 2003; Masada et al. 2006). Signs for MRI growth are found in some previous MHD core-collapse simulations initially assuming a magnetar-class magnetic field (Yamada & Sawai 2004; Takiwaki et al. 2004; Obergaulinger et al. 2006; Shibata et al. 2006). Also, Obergaulinger et al. (2009) carried out simulations of MRI with a rather weak initial magnetic field, which is still stronger than that of ordinary pulsars by an order of magnitude, using a local simulation box, and found that the MRI exponentially amplifies the seed magnetic field. Note, however, that the effect of accretion is not considered in their local box simulations. According to Foglizzo et al. (2006), the neutrino-driven convection in the gain region can be stabilized or slowed down by accretion. This may also hold for MRI in the post-shock region.

We investigated whether, in models $\eta_{-\infty}$ and $\eta_{14}$, a region emerges where the criterion for the MRI (Balbus 1995) is satisfied and the growth timescale is short enough. In the top left panel of Figure 16, we plot the distribution of an MRI linear
Figure 14. Evolution of each $\dot{E}_r$ and $\dot{E}_\phi$ (see Equation (30)) in volumes $V_{25.5}$ and $V_{58.5}$ in model $B_s-\Omega-\eta_{-\infty}$. (A color version of this figure is available in the online journal.)

Figure 15. Same as Figure 14 but for $B_s-\Omega-\eta_{14}$. Note that the vertical scales of each panel in Figures 14 and 15 are different except for the bottom right panel. (A color version of this figure is available in the online journal.)
growth timescale, roughly estimated by $4\pi/|\sigma d\Omega/d\sigma|$, in the $\theta-r$ plane at 160 ms (17 ms after bounce) for model $\eta_{-\infty}$. It is shown that in a considerable part for $\theta \lesssim 40^\circ$, the growth timescale of MRI is a few ms to 10 ms, while in a larger $\theta$ the growth timescale is generally longer. We found that, in the above part, the growth timescale of $\sim 10$ ms is kept after bounce ($t = 143$ ms) until the end of the simulation. However, the top right panel of Figure 16 does not show MRI-like field-line bending as observed in some models in Yamada & Sawai (2004) (models MF3 and MF8). This may be because the present model leads to a stronger eruption of matter in the radial direction than those of Yamada & Sawai (2004) because of a rather mild initial rotation speed. In the present model, field-line bending produced by MRI may become invisible due to a dominant radial flow, and the absence of that will not necessarily mean non-operation of MRI. Note that the absence of field-line bending does not seem to be due to poor spatial resolution: field-line bending is observed even in the computations of model series Bm-$\Omega$ in which a magnetic field is generally weaker than in the present case and thus the resolution for capturing MRI is poorer. In the present case, the fastest growing wavelength is resolved everywhere with several tens of numerical cells (see the bottom panels of Figure 16). Although the number of numerical cells required to capture one wavelength depends on the numerical scheme, several tens of cells will be sufficient.

The growth of MRI will lead to an increase of the shear term in Equation (28), since MRI produces a shear of velocity along a magnetic field line. The left panel of Figure 14 shows that the shear term in volume $V_{25.5}$ becomes relatively large after $\sim 160$ ms ($\sim 20$ ms after bounce). Given that the MRI growth timescale there is $\sim 10$ ms, it may be reasonable to consider that the increase of the shear term is due to the operation of MRI. Even if this is the case, however, it is unlikely that MRI greatly amplifies a magnetic field, since the radial magnetic energy is

\footnote{It is known that a mildly rapid rotation, $T/|W| \sim 0.5\%$, is favorable for an energetic explosion (see Yamada & Sawai 2004).}
nearly constant after $\sim 160$ ms (see the left panel of Figure 13). The MRI seems at best to maintain the strength of the magnetic field in volume $V_{25.5}$. Since the fastest MRI growth timescale is comparable at each $\theta$ for $\theta \lesssim 40^\circ$, the situation will be more or less similar in this angular range.

Meanwhile, in the angular range of $\theta \gtrsim 40^\circ$, even the retention of magnetic field strength by the MRI will be modest, because a growth timescale reaches $\sim 10$ ms only in some limited locations (see the top left panel of Figure 16). Although, in volume $V_{58.5}$, the shear term helps to increase or keep the radial magnetic energy (the bottom left panel of Figure 14), this begins long before the MRI is expected to grow. At a later time (e.g., after $\sim 160$ ms), the shear term might contain some modest contribution from MRI.

The amplification mechanism in model $\eta_{14}$ is qualitatively similar to that in model $\eta_{-\infty}$, although the amplitude of each $\xi$ is usually smaller due to resistivity (see Figure 15). The top left panel of Figure 17 plotted for $t = 160$ ms shows that in a considerable part for $\theta \lesssim 45^\circ$, the MRI growth timescale is 10 ms to a few 10 ms, which is kept from the time of bounce (143 ms) until the end of the simulation. Figure 18 displays the distribution of the resistive timescale divided by the MRI growth timescale in the $\theta-r$ plane in model $B_3-\Omega_{-\eta_{14}}$ at $t = 160$ ms.

(A color version of this figure is available in the online journal.)
timescale, $R_{\text{MRI}} \equiv \tau_{\text{res}}/\tau_{\text{MRI}}$, at 160 ms. This indicates that the growth of the MRI is more or less hampered by resistivity. Especially in the vicinity of the pole, there is almost no chance for MRI growth. We found that the growth of MRI is possible through the simulation in the angular range of $20^\circ \lesssim \theta \lesssim 30^\circ$, although a growth rate will be somewhat decreased by resistivity in some locations (blue-colored regions for $20^\circ \lesssim \theta \lesssim 30^\circ$ in Figure 18). In this model, the fastest growing wavelength is resolved with several tens of numerical cells everywhere except for $r < 20$ km (see the bottom panels of Figure 17).

The top left panel of Figure 15 also shows that the shear term in volume $V_{25.5}$ becomes large after $\sim 160$ ms. According to the above discussion, this is possibly due to MRI, but not important for a magnetic field amplification. Again, MRI at best may contribute to maintaining the radial magnetic energy in model $\eta_{14}$.

In light of the above discussion, it can be said that the contrast in magnetic energy per unit mass over $\theta$ observed both in models $\eta_{-\infty}$ and $\eta_{14}$ is strengthened or kept by an outward advection of magnetic energy, twisting of poloidal magnetic field lines, and possibly by the MRI, all of which efficiently occur in a small-$\theta$ region.

### 4.1.3. Aspect Ratio

From Figure 5 it is found that the shape of the shock surface is prolate both in model $B_s-\Omega-\eta_{-\infty}$ and model $B_s-\Omega-\eta_{14}$, in which the latter shows a less prolate feature than the former. Defining the aspect ratio by the maximum-$z$ position of ejected matter, $z_{\text{ej,max}}$, divided by the maximum-$\sigma$ position, $\sigma_{\text{ej,max}}$, we found that it exceeds two at the end of each simulation (see Figure 19).

We first focus on the aspect ratio in model $\eta_{-\infty}$. Figure 19 indicates that the aspect ratio becomes larger than unity soon after bounce ($t = 143$ ms). Since a centrifugal force is relatively stronger around the equator, it hampers the collapse and weakens the bounce there. This is one reason for the prolate matter ejection. Figures 20 and 21 show the radial distributions of the accelerations, time averaged during $t = 155$–165 ms, in volumes $V_{25.5}$ and $V_{58.5}$, respectively. It is found that in volume $V_{25.5}$, matter is greatly accelerated in the radial range of 50 km $\lesssim r \lesssim 160$ km (see the solid line in the top left panel of Figure 20). Meanwhile, in volume $V_{58.5}$, the acceleration of matter is generally smaller (see the solid line in the top left panel of Figure 21). This implies that the acceleration of matter is larger for a smaller $\theta$, which causes a further increase of the aspect ratio. Comparing the bottom panels of the two figures, a larger magnetic and centrifugal acceleration in volume $V_{25.5}$ seems responsible for the larger acceleration. Keeping in mind that a stronger magnetic field leads to a larger amplitude of these two accelerations (see Section 4.1.1), it is likely that a polarly concentrated distribution of magnetic energy per unit mass (Section 4.1.2) is essential to generate a large aspect ratio.

Comparing the aspect ratio among the three models in the left panel of Figure 19, it is the smallest in model $\eta_{14}$, while in models $\eta_{-\infty}$ and $\eta_{13}$ the value is similar. The right panel of Figure 19 shows that the difference comes from the fact that $z_{\text{ej,max}}$ is largely affected by resistivity, while $\sigma_{\text{ej,max}}$ does not change so much. With this and the above speculation that a prolate matter ejection is caused by a polarly concentrated magnetic energy distribution, the smaller aspect ratio in model $\eta_{14}$ is readily understood. In Figure 20, it is shown that a total acceleration in volume $V_{25.5}$ is smaller in model $\eta_{14}$ due to smaller contribution from a magnetic and centrifugal acceleration. Meanwhile, a total acceleration in volume $V_{58.5}$ is not very different between the two models (see Figure 21), because a magnetic and centrifugal acceleration is not as important as in volume $V_{25.5}$ due to a small magnetic energy per unit mass. This leads to a large difference only in $z_{\text{ej,max}}$, and thus in the aspect ratio.

### 4.1.4. Diffusion and Dissipation Sites

In this section, we will see in which sites resistivity works efficiently. In Figure 22, the distribution of magnetic Reynolds number $R_m$, the ratio of the resistive timescale to dynamical timescale, in models $B_s-\Omega-\eta_{13}$ and $B_s-\Omega-\eta_{14}$ is displayed for $t = 164$ ms. We define the resistive and dynamical timescales, respectively, by $L^2/\eta$ and $|L||\mathbf{B}|/(\nu \times |\mathbf{B}|)$, where the scale length of the magnetic field is defined by $L \equiv |\mathbf{c}B|/[4\pi j]$. With these definitions, the magnetic Reynolds number is also the ratio of the size of the first term to the second term on the right-hand side of Equation (5). If we define a diffusion–dissipation site by the location where the magnetic Reynolds number is $\lesssim 100$, the right panel of Figure 22 shows that the diffusion–dissipation sites in model $\eta_{14}$ are around the pole, the equator, and the shock surface, and in the blue filaments inside the shock surface. In these sites, a magnetic Reynolds number is small since the scale length of the magnetic field is short: that of the toroidal field is short around the pole, while that of the poloidal field is short in the other sites. Comparing the right panels of Figure 5, an intense magnetic pressure dominance seen around the pole in model $\eta_{-\infty}$ is not found in model $\eta_{14}$. This implies that diffusion

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**Figure 19.** Evolutions of aspect ratios (left panel) and the maximum ejecta positions in $z$ and $\sigma$ (right panel) in model series $B_s-\Omega$. The dotted line in the left panel is for a different resolution run for model $B_s-\Omega-\eta_{14}$ (see Section 4.1.5).

(A color version of this figure is available in the online journal.)
and dissipation around the pole are essential for producing the dynamical differences between models $\eta_{-\infty}$ and $\eta_{14}$.

In the case of model $\eta_{13}$, resistivity also works efficiently around the pole, the equator, and the shock surface, but the volumes of these sites are much smaller than those in the former case due to a lower resistivity. Besides, a magnetic Reynolds number there is generally at least $\sim 30$, although in some very limited regions it reaches an order of unity. Hence, model $\eta_{13}$ is
rather close to the ideal model. The explosion energy and aspect ratio in model $\eta_{13}$ are not much different from those in model $\eta_{-\infty}$.

4.1.5. Convergences of Results

As a final remark for model series $Bs-\Omega$, we mention the convergences of results with respect to the size of the numerical cells. As a representative model, we carry out another run for model $Bs-\Omega-\eta_{14}$ with a different spatial resolution. Keeping the total number of cells the same, the size of the innermost cells is changed from 400 m to 200 m, where a cell size increases outward both in the $\sigma$-direction and the $z$-direction with a constant ratio of 1.0069. With this distribution, the resolution is higher than the original one for $z, \sigma < 200$ km, and lower otherwise. In Figures 4, 6, 12, 13, and 19, the results obtained with this different resolution are shown for model $Bs-\Omega-\eta_{14}$ by the dotted lines. Although slight deviations from the original results are observed in some figures, they do not seem to affect the discussions given above either qualitatively or quantitatively.

The error in the total energy conservation in the different resolution run is 14% as mentioned at the beginning of Section 4. The error in the total energy conservation in the normal resolution run of model $Bs-\Omega-\eta_{14}$, 26%, is roughly twice as large as that in the different resolution model. Nonetheless, as we have seen, only slight deviations are found between results of the two resolution runs. Hence, although the total energy error of 26% may be a little too large, this will not spoil the results of the simulation. We expect that this is also the case for the other models.

4.2. Moderate Magnetic Field and Rapid Rotation—Model Series $Bm-\Omega$

The dynamical evolutions in model series $Bm-\Omega$ are qualitatively similar to those found in model series $Bs-\Omega$, i.e., the twisting of magnetic field lines via a differential rotation increases magnetic and centrifugal acceleration, which play a key role in the ejection of matter. Owing to a weaker initial magnetic field, the explosion occurs less energetically than in the former model series. The distributions of velocity and magnetic field at $t = 181$ ms in model $Bm-\Omega-\eta_{-\infty}$ and $Bm-\Omega-\eta_{14}$ are depicted in Figure 23. Compared to model series $Bs-\Omega$ with the same resistivity (see Figure 5), this model series shows a slower outward velocity and a generally weaker magnetic pressure. As in the former model series, a comparison between the upper and lower panels of Figure 23 also indicates that the eruption of matter becomes weaker with the presence of resistivity, with which we expect a lower explosion energy for a resistive model.

4.2.1. Explosion Energy

Figure 24 plots the evolutions of the explosion energies in the present model series. The resulting explosion energy in each model appears to be about a factor of three smaller than in the corresponding model in model series $Bs-\Omega$. It is shown that resistivity also decreases the explosion energy as in the former case. This can be understood the same way as we have seen in Section 4.1.1 for model series $Bs-\Omega$.

Figure 25 shows the radial distribution of accelerations, angularly averaged in the eruption region and time averaged during 160–170 ms, according to Equation (27). The top left panel shows that an acceleration of matter mainly takes place around $r \sim 100$ km. There, although the pressure acceleration alone does not overcome the inward gravitational acceleration (see the bottom left panel), the total acceleration is positive with the help of magnetic and centrifugal acceleration (see the bottom right panel). From the top right panel, it is found that the total acceleration there is generally larger in model $\eta_{-\infty}$ and that this is primarily due to larger magnetic and centrifugal acceleration.

In Figure 26, the distributions of $dE_{\exp}/dr$ at 170 ms are shown for models $\eta_{-\infty}$ and $\eta_{14}$. It is found that an eruption of matter occurs for $r \gtrsim 100$ km, and $dE_{\exp}/dr$ is larger everywhere in model $\eta_{-\infty}$. It is expected that a larger total acceleration around $r \sim 100$ km is responsible for this. Thus, it is likely that in this model series, resistivity also makes the explosion less energetic by decreasing magnetic and centrifugal acceleration as in model series $Bs-\Omega$.

4.2.2. Magnetic Field Amplification

A magnetic field amplification in the present model series goes on in a way similar to that the stronger magnetic field case $Bs-\Omega$. Figure 27 shows the distributions of magnetic energies per unit mass, averaged over 50 km < $r$ < 0.9 x $r_{sh}$ at $t = 143$ ms (2 ms after bounce) and $t = 180$ ms (39 ms after bounce), for models $Bm-\Omega-\eta_{-\infty}$ and $Bm-\Omega-\eta_{14}$. Again, it is observed that the contrast in a magnetic energy per unit mass over
Here, we also evaluated each term in the evolution equations of magnetic energies per unit mass (see Equations (28) and (30)). The results are shown in Figures 29 and 30. It is found in both models \( \eta_{-\infty} \) and \( \eta_{14} \) that the amplification mechanisms in volume \( V_{13.5} \) are similar to those found in volume \( V_{25.5} \) of model series \( B_s-\Omega \), i.e., the toroidal and radial magnetic energies are mainly amplified or kept due to an advection and shear (see Section 4.1.2). As in the former model series, we found here that the amplification is weaker in a volume with a larger \( \theta_s \), viz., weaker in volume \( V_{55.5} \).

The top left panel of Figure 31 shows the distributions of the MRI growth timescale in the \( \theta-r \) plane for model \( Bm-\Omega-\eta_{-\infty} \) at \( t = 170 \text{ ms} \) (29 ms after bounce). It is found that the MRI growth timescale is short, \( \sim \) a few ms to 10 ms, in a considerable part for \( r \lesssim 20^\circ \), while for a larger \( \theta \), the growth timescale is generally much longer. We found that the short growth timescale mentioned above is maintained from 148 ms (7 ms after bounce) until the end of the simulation.

In this model, the growth of MRI is also assured by magnetic field-line bending, which starts appearing shortly after 150 ms. The top right panel of Figure 31, which depicts the structure of a poloidal magnetic field for \( t = 170 \text{ ms} \), shows apparent...
field-line bending especially around the pole, where the growth timescale is generally short. In Section 4.1.2, we have seen that in both models $Bm$-$\Omega$-$\eta_{-\infty}$ and $Bm$-$\Omega$-$\eta_{14}$, the shear term for volume $V_{13.5}$ starts increasing around the time when the growth of the MRI is expected, and thus we speculated that the increase is caused by the MRI. In the present model, $Bm$-$\Omega$-$\eta_{-\infty}$, the shear term for volume $V_{13.5}$ does not behave like that. As seen in the top left panel of Figure 29, it has a large negative value around $\sim155$–$160$ ms and a large positive value around $\sim170$–$180$ ms. With this rather complicated behavior, it is difficult to guess when MRI makes a substantial contribution to the shear term. Nonetheless, we could at least state that MRI does not play a crucial role in amplifying a magnetic field. This is because the radial magnetic energy has already grown strong enough by the period of $\sim170$–$180$ ms, while only during this period of time, the shear term works to amplify the magnetic field (see Figures 28 and 29).

In model $\eta_{14}$, a region with a short growth timescale is not as limited to a small $\theta$ as in model $\eta_{-\infty}$, albeit the growth timescale is generally longer, typically a few 10 ms. As seen in the top left panel of Figure 32, although a small-$\theta$ region is a more favorable site for the MRI, a large $\theta$ region is still subject to fast MRI. However, as noted in Section 4.1.2, we should take into account that the growth of MRI may be suppressed by resistivity. In Figure 33, the distribution of $R_{MRI}$, the resistive timescale divided by the MRI growth timescale, is depicted at $t = 170$ ms for model $\eta_{14}$. It is shown that $R_{MRI}$ is smaller than unity in the vicinity of the pole and equator, which means no MRI growth occurs there. We found that, for $20^\circ \lesssim \theta \lesssim 40^\circ$, $R_{MRI}$ is always larger than unity in a considerable part of the fast MRI regions. Hence, MRI also still seems to work in model $\eta_{14}$. In the top right panel of Figure 32, MRI-like field-line bending is also found around the pole, albeit less prominent than in the ideal model.

In model $\eta_{14}$, the shear term for volume $V_{13.5}$ becomes large at a late time, $\sim190$ ms (the top left panel of Figure 30). Since, as in the former models, the radial magnetic energy does not largely increase after this time (the left panel of Figure 28), the shear term, and thus MRI, does not seem to be very important for amplification of a magnetic field, although they may play some role in maintaining the strength of a magnetic field.

Compared with model series $Bm$-$\Omega$, the computations of this model series have poorer spatial resolution for capturing MRI (see the bottom right panels of Figures 31 and 32). Specifically, in model $Bm$-$\Omega$-$\eta_{-\infty}$, the fastest growing wavelength is resolved with less than 10 numerical cells in some locations even for $r > 50$ km, where a magnetic field plays a dynamically important
role. However, since these locations are limited to small areas, we do not expect that the dynamics would drastically change if the growth of MRI were fully resolved there.

Comparing the right panels of Figures 12 and 27, the distribution of magnetic energy in model $Bm\Omega-\eta_{-\infty}$ is more concentrated toward small $\theta$ than in model $Bs\Omega-\eta_{-\infty}$. Although the two figures are depicted at different times, the above-mentioned feature is still observed for the distributions compared at the same physical time after bounce (e.g., 37 ms after bounce for both models). This may be explained as follows. Because a magnetic field in model $Bm\Omega-\eta_{-\infty}$ is weaker, a strong eruption of matter supported by magnetic force is restricted to a small region in the vicinity of the pole where a magnetic field is relatively strong. Then, due to amplification of a magnetic field by radial advection, the contrast between magnetic energy in the vicinity of the pole and the other region becomes increasingly stronger.

4.2.3. Aspect Ratio

The aspect ratio of a shock surface also becomes smaller for larger resistivity in model series $Bm\Omega$ (see the left panel of Figure 34). As seen in the right panel of Figure 34, the difference in the aspect ratio is attained due to that in $z_{ej,max}$ as in the case of model series $Bs\Omega$. The same discussion as before will explain this feature, i.e., a stronger magnetic field and thus a larger magnetic and centrifugal acceleration in a small $\theta$ compared with in a large $\theta$ make a shock surface prolate, which is, however, less standout with the presence of resistivity.

A trait of the present model series is a larger impact of resistivity, i.e., the aspect ratio is larger in model $Bm\eta_{-\infty}$ than in model $Bs\eta_{-\infty}$, while it is smaller in model $Bm\eta_{14}$ than in model $Bs\eta_{14}$.

The larger aspect ratio in model $Bm\Omega-\eta_{-\infty}$ than in $Bs\Omega\eta_{-\infty}$ would be related to the distribution of the magnetic energy, which is more notably concentrated toward small $\theta$ in the former model, as discussed in Section 4.2.2. The smaller aspect ratio in model $Bm\Omega\eta_{14}$ compared to $Bs\Omega\eta_{14}$ may be understood as follows. Although the distributions of magnetic energy are similar in their shape between the two models (compare the right panels of Figures 12 and 27), the magnitude is generally weaker in model $Bm\Omega\eta_{14}$, reflecting the initial field strength, and the role of a magnetic field in erupting matter is less important than in $Bs\Omega\eta_{14}$. As a result, the shape of a shock surface is not affected very much by the magnetic field, and the aspect ratio becomes smaller.

4.3. Very Strong Magnetic Field and No Rotation—Model Series $Bss-\Omega$

The dynamical evolutions found in model series $Bss\Omega$ are qualitatively different from those in the former two model series. Without rotation, neither the generation of a toroidal magnetic field nor an angular momentum transfer occurs. However, a magnetic field still seems to play some role. The initially strong magnetic field affects the dynamics even during the collapse. Typical evolution proceeds as follows. Although the

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure27}
\caption{Angular distributions of magnetic energies per unit mass averaged over 50 km $< r < 0.9 \times r_{ej}$ at $t = 143$ ms (2 ms after bounce; left) and $t = 180$ ms (39 ms after bounce; right). The solid and dashed lines are for models $Bm\Omega-\eta_{-\infty}$ and $Bm\Omega-\eta_{14}$, respectively. (A color version of this figure is available in the online journal.)}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure28}
\caption{Evolutions of the average magnetic energies per unit mass of volumes $V_{13.5}$ (left) and $V_{55.5}$ (right). The solid lines are for model $Bm\Omega-\eta_{-\infty}$ while the dashed lines are for $Bm\Omega-\eta_{14}$. The graphs are plotted from $t = 143$ ms (2 ms after bounce). (A color version of this figure is available in the online journal.)}
\end{figure}
total magnetic energy per unit mass is a priori smaller around the equator, that of the $B_\theta$ is conversely larger there. A strong $B_\theta$ attenuates a matter infall around the equator, and leads to a weak bounce in the lateral direction. Due to the weak bounce, falling matter soon prevails outgoing matter and then the infall region forms around the equator. Meanwhile, in the other part, a bounce occurs which is strong enough to form the eruption region, and with the help of magnetic force, especially magnetic...
pressure of $B_\theta$, the shock surface propagates outward to reach 3000 km at the end of the simulation. Figure 35 shows the distributions of velocity and magnetic field in models Bss-$\tilde{\Omega}$-$\eta_{-\infty}$ and Bss-$\tilde{\Omega}$-$\eta_{14}$ at $t = 195$ ms (40 ms after bounce). In each of the left panels, the formation of the eruption region and infall region are well observed. Each right panel shows that magnetic pressure is mildly important in the eruption region. As shown in Figure 36 the magnetic energies in the two models are $\sim 10^{51}$ erg. This implies that a magnetic field has a potential to somewhat boost the explosion. Although no significant difference between models $\eta_{-\infty}$ and $\eta_{14}$ can be found in Figure 35, as we will see, resistivity still plays a certain role in the dynamics of this model series.

4.3.1. Explosion Energy and Diffusion of Magnetic Field

In Figure 37, the evolution of explosion energies in model series Bss-$\tilde{\Omega}$ is plotted. As shown there, the explosion energies are found to be $\sim 10^{50}$ erg, one order of magnitude smaller than that of a canonical supernova. The figure also indicates that the models involving resistivity produce a larger explosion energy compared with the ideal model, which is opposite to what is found in the model series involving rotation Bss-$\Omega$ and Bss-$\tilde{\Omega}$.

In Figure 38, we plot the radial distributions of $dE_{\text{exp}}/dr$ at $t = 201$ ms in models Bss-$\tilde{\Omega}$-$\eta_{-\infty}$ and Bss-$\tilde{\Omega}$-$\eta_{14}$, which shows that the explosion energy in model $\eta_{14}$ is greater than that in model $\eta_{-\infty}$ at most radial ranges. We examined whether a radial acceleration in model $\eta_{14}$ is larger than in model $\eta_{-\infty}$, but found no substantial difference between them. We also compared the two models by the radial distributions of radial velocity angularly averaged in the eruption region at 201 ms (see Figure 39). It is found that a radial velocity is mostly larger in model $\eta_{14}$. This seems a key factor in determining why the explosion energy is larger in model $\eta_{14}$.

Then the question is: Why is the radial velocity larger in model $\eta_{14}$, despite comparable strengths of radial accelerations? In Figure 40, the velocity distributions in the $\sigma-z$ plane at $t = 201$ ms are depicted for the two models with a slight...
zooming toward the center. For model $\eta_{-\infty}$, it is observed that matter flows from the infall region into the eruption region around $r \lesssim 200$ km, and damps an ejection of matter. In model $\eta_{14}$, on the other hand, the flow of matter into the eruption region is inhibited by a “positive velocity island” observed around $150$ km $\lesssim r \lesssim 350$ km in the infall region, and a coherent outflow of matter in the eruption region is maintained even for $r \lesssim 200$ km. In this way, the appearance of the positive velocity island seems to be related to the difference in the velocity distribution observed between the two models.

We found that the positive velocity island begins to emerge around 170 ms (15 ms after bounce) in model $\eta_{14}$. Figure 41 shows the radial distributions of radial velocities, angularly averaged in the infall region at 175 ms, for the two models. It is observed that in the radial range of $60$ km $\lesssim r \lesssim 180$ km, an infall velocity is slower in model $\eta_{14}$, which seems due to the positive velocity island. In Figure 42, the radial distributions of radial accelerations, angularly averaged in the infall region and time averaged during $t = 170$–175 ms, are plotted. As expected, a radial acceleration in the infall region is larger in model $\eta_{14}$ around a radial range similar to the above, $90$ km $\lesssim r \lesssim 180$ km.
We found that larger pressure and magnetic acceleration are responsible for this acceleration superiority (see the right panel of Figure 42). We speculate that a larger pressure acceleration in model $\eta_{14}$ will not be caused by Joule heating, since the heating timescale in the above range is too long, $\sim 1-10$ s, to produce extra pressure. It is more likely that the accumulation of falling matter stemming from magnetic acceleration causes pressure in the positive velocity island to become larger. It seems that a
magnetic acceleration is the primary factor for the formation of the positive velocity island.

In the absence of a toroidal magnetic field, a radial magnetic acceleration is written as

$$a_R = \frac{B_\theta}{\rho} \left( \frac{\partial B_\theta}{\partial r} + \frac{1}{r} \frac{\partial B_r}{\partial \theta} - \frac{B_\theta}{r} \right). \quad (30)$$

The radial distributions of the magnetic field in the infall regions at 175 ms plotted in the left panel of Figure 43 indicate that $B_\theta$ is far greater than $B_r$ in the above radial range of 90–180 km, and thus the second term on the right-hand side of Equation (30), a part of magnetic tension, is dominant. The left panel of Figure 43 also shows that, in the above radial range, $B_\theta$ is larger in model $\eta_{14}$ than in model $\eta_{-\infty}$, while $B_r$ is smaller in model $\eta_{14}$. We found that the superiority in $B_\theta$ overshadows the inferiority in $B_r$, i.e., $B_\theta(r)B_\theta(r)$ is generally larger in model $\eta_{14}$ in the above radial range. To put it simply, a larger $B_\theta$ in model $\eta_{14}$ is crucial to the formation of a positive velocity island.

The radial distribution of $B_\theta$ in model $\eta_{14}$ shown in the left panel of Figure 43 compared with that of model $\eta_{-\infty}$ invokes a magnetic field diffusion. Indeed, it is found that the magnetic Reynolds number is smaller than unity almost everywhere for $r \lesssim 85$ km in the infall region (see the right panel of Figure 44), viz., a diffusion effectively advects a magnetic field outward against matter infall. The right panel of Figure 43 shows that the gradient of $B_\theta$ in model $\eta_{-\infty}$ is steep inside the central region of the 30 km radius, while that in model $\eta_{14}$ is shallow even below a radius of 30 km. This implies that in the latter model, resistivity effectively diffuses $B_\theta$ out of the central region. In model $\eta_{-\infty}$, the gradient of $B_\theta$ suddenly becomes shallow around $r = 30$ km. Then with resistivity, the incoming diffusion flux there, $\eta \nabla B_\theta$, would be larger than the outgoing one, which would result in an increase of $B_\theta$ outside the radius of 30 km. In this way, a magnetic field diffused out from deep inside the core suppresses the infalling of matter and is responsible for a larger explosion energy in model $\eta_{14}$.

A similar mechanism also works in model $\eta_{13}$. As shown in the left panel of Figure 44, diffusion of the magnetic field in the infall region also occurs well in this model. However, due to a smaller resistivity, diffusion sites are limited to a smaller volume compared with model $\eta_{14}$. Nonetheless, the explosion energy in model $\eta_{13}$ is comparable to that of model $\eta_{14}$ at the end of the simulations (see Figure 37).

4.3.2. Aspect Ratio

The aspect ratios in model series Bss-Ω are also larger than unity, i.e., the shape of ejecta is prolate, but are not as large as in the rotating cases (see Figure 45). Since the initial magnetic field assumed in the present model series is very strong, it affects the dynamics even before bounce. Due to the dipole-like configuration, a magnetic field hampers a matter infall especially well in the lateral direction. This causes a weak bounce around the equator, and thus a prolate ejection of matter.
The difference from the former model series is that the aspect ratios do not grow drastically after bounce (see Figure 45). We expect that this is caused by a roughly flat angular distribution of magnetic energy per unit mass in the eruption region as indicated in Figure 46 for models Bss-$\Omega$-$\eta_{-\infty}$ and Bss-$\Omega$-$\eta_{14}$, keeping in mind that a polarly concentrated magnetic field distribution is essential for a large aspect ratio in the rotating cases.

We found that the change from the rather polarly concentrated distribution of the magnetic field at the beginning to a roughly flat angular distribution occurs during the collapsing phase, i.e., before bounce. Figure 47 shows the angular distributions of $\dot{\varepsilon}_r$, radially averaged over 50 $< r < 1000$ km at 122 ms (33 ms before bounce) in model Bss-$\Omega$-$\eta_{-\infty}$. It is shown that a total $\dot{\varepsilon}_r$ is larger around the equator, and compression of $B_r$ in the $\theta$-direction plays an important role. Since $B_\theta$ is a priori strong around the equator due to the assumed magnetic field configuration, the infall of matter there is slightly deflected into the non-radial direction by a Maxwell stress of $B_r B_\theta / 4 \pi$, viz., the $\theta$-component of velocity is generated. In this way, $B_\theta$ is compressed into the $\theta$-direction, which leads to an increase of $B_r$ preferentially around the equator and a rather flat angular distribution of the magnetic field.

A notable point here is that a rather flat distribution of the magnetic field, which causes a low value of the aspect ratio, is established before bounce when resistivity does not work well because of the large-scale height of the magnetic field. This is why the aspect ratio for different resistivity results in a similar value.

4.4. Numerical Resistivity

In the preceding sections, we have seen how resistivity impacts the dynamics of a strongly magnetized supernova. However, we should remember that the numerical results are not only affected by physical resistivity but also may be influenced by numerical resistivity. Here, we estimate the possible impact of numerical resistivity on the dynamics.

First, we directly compared the strengths of physical resistivity and numerical resistivity. We consider dividing the numerical flux into two terms, the advection term and the numerical diffusion term, where the former may account for an advection of a physical flux, while the latter may account for a numerical diffusion. For example, in the equation of numerical fluxes $F^\nu$ in Equations (10), the first and second terms on the right-hand side are the advection term and the numerical diffusion term, respectively. Accordingly, the ideal part of a numerical flux vector for the induction equations $\vec{E}$, which are defined by a combination of 6–8 components of numerical flux vectors $\dot{\mathbf{F}}^\nu$ and $\dot{\mathbf{F}}^\nu$ (see Equations (15)), is also divided into the advection term and the numerical diffusion term. Then, the total numerical flux vector for the induction equations can be written as $\dot{\mathbf{E}} = \dot{\mathbf{F}}_{\text{adv}} + \dot{\mathbf{F}}_{\text{nd}} + \mathbf{G}$, where subscripts “adv” and “nd” stand for advection and numerical diffusion, respectively. Recall that a term $\mathbf{G}$ arises due to physical resistivity.

Now, to compare the strengths of physical resistivity and numerical resistivity we evaluate $\zeta \equiv |\mathbf{G}|/|\dot{\mathbf{F}}_{\text{adv}}|$, in which physical resistivity is dominant for $\zeta \gg 1$ while numerical resistivity is dominant for $\zeta \ll 1$. The distribution of $\zeta$ for model Bss-$\Omega$-$\eta_{13}$ at 164 ms is shown in Figure 48. This implies that physical resistivity dominates over numerical resistivity in a considerable volume inside the shock surface, but in some locations numerical resistivity is not negligible.

Next, we examined how much numerical resistivity affects the dynamics. This is done by evaluating $R_{\text{num}} \equiv |\dot{\mathbf{F}}_{\text{nd}}|/|\dot{\mathbf{F}}_{\text{adv}}|$, which may corresponds to a magnetic Reynolds number for numerical resistivity. Even if numerical resistivity dominates
over physical resistivity, it does not make a substantial impact on the dynamics provided $R_{m,\text{num}} \gg 1$. Figure 49 shows the distribution of $R_{m,\text{num}}$ for model $B_{\Omega}-\eta_{13}$ at 164 ms. It is found that $R_{m,\text{num}}$ is much larger than unity almost everywhere except around the shock surface, where a steep variation of a magnetic field exists. Meanwhile, we have seen in Section 4.1.4 that a magnetic Reynolds number for physical resistivity in model $B_{\Omega}-\eta_{13}$ is $\sim 30$ around the equator (see the left panel of Figure 22). These facts imply that numerical resistivity is mostly too small to influence the dynamics, while physical resistivity affects the
dynamics, albeit slightly. Thus, it seems that physical resistivity “effectively” dominates over numerical resistivity in this model.

From the above discussion, it is likely that a comparison between \( R_m \) and \( R_{m,\text{num}} \) is more meaningful than that between physical resistivity and numerical resistivity themselves in order to assess the influence of numerical resistivity. Thus, we also compare \( R_m \) and \( R_{m,\text{num}} \) for models Bm-\( \Omega \)-\( \eta_\text{13} \) and Bss-\( \Omega \)-\( \eta_\text{13} \). In model Bm-\( \Omega \)-\( \eta_\text{13} \), \( R_{m,\text{num}} \) is much larger than unity almost everywhere except around the shock surface, while a relatively small \( R_m \sim 1-10 \) is found around the equator (see Figure 50). Hence, physical resistivity also effectively dominates over numerical resistivity in this model. In model Bss-\( \Omega \)-\( \eta_\text{13} \), a mildly small \( R_{m,\text{num}} \sim 10 \) appears not only around the shock surface but also at other locations (see the left panel of Figure 51). This means that numerical resistivity somewhat affects the dynamics in this model. However, the right panel of Figure 51 shows that physical resistivity is more influential than numerical resistivity, especially at small radii. Recalling that the essential role of resistivity in this model is to diffuse a magnetic field from a small radius to a larger radius, physical resistivity seems to play a primary role in yielding a different result for Bss-\( \Omega \)-\( \eta_\text{13} \) from that of the ideal model. For confirmation,
we carried out the above analysis in the models with $\eta = 10^{14} \text{ cm}^2 \text{s}^{-1}$, and found that the "effective" dominance of physical resistivity over numerical resistivity is more pronounced than the counterpart models with $\eta = 10^{13} \text{ cm}^2 \text{s}^{-1}$.

5. DISCUSSION AND CONCLUSION

We have conducted MHD simulations of CCSNe to understand the role of turbulent resistivity in the dynamics. As a result, we found that resistivity possibly has a great impact on the explosion energy, the magnetic field amplification, and the aspect ratio. Exactly how and how much resistivity affects these things depends on the initial strengths of magnetic field and rotation together with the strength of the resistivity.

In the rotating cases (model series B$\Omega$ and B$m\Omega$), resistivity results in a small explosion energy. This is mainly ascribed to a small magnetic and centrifugal acceleration due to an inefficiency in the magnetic field amplification and angular momentum transfer, respectively. Meanwhile, in the case of a very strong magnetic field and no rotation (model series B$s\Omega$), resistivity enhances the explosion energy. In the ideal model, an inflow of negative radial momenta from the infall region into the eruption region inhibits a powerful mass eruption. With resistivity, a magnetic field in the infall region diffuses outward from deep inside the core to counteract the negative momentum invasion.

The ideal models involving rotation show a polarly concentrated distribution of magnetic energy per unit mass. Since an initial magnetic energy per unit mass is somewhat larger around the pole than the equator, an amplification preferentially occurs there, which makes the contrast stronger. We found that the main mechanism of amplification around the pole is an outward advection of magnetic energy from a small radius and the twisting of poloidal magnetic field lines by differential rotation. In a resistive model, amplification occurs in a qualitatively similar way to the corresponding ideal model, but proceeds less efficiently. As a result, the angular distribution of the magnetic field becomes less concentrated toward the pole. Although we
found signs of MRI growth in these models, it does not seem to play a substantial role in amplifying the magnetic field. The MRI will at best help to maintain the strength of a magnetic field. We should note here that in our two-dimensional axisymmetric simulations, the MRI may not be followed correctly, since non-axisymmetric modes are also important in the MRI. A comparison between a two-dimensional axisymmetric and three-dimensional simulations in the behavior of the MRI will be made and presented elsewhere in the near future.

Every model involving a rotation shows a large aspect ratio, \( >2.5 \), at the end of the simulation, where a model with larger resistivity results in a smaller aspect ratio. The impact of resistivity on the aspect ratio is more apparent in the moderate magnetic field case, model series \( Bm-\Omega \). The aspect ratios obtained in the rotation models are expected to grow even larger later on, since a radial velocity is still faster around the pole than around the equator at the end of the simulations. Further long-term computations are necessary to predict the aspect ratio after the shock surface breaks out of the stellar surface. In a model without rotation, on the other hand, the aspect ratio maintains a low value, \( \sim 1.5 \), through each simulation. No significant difference is found among models with different resistivity. It is found that the aspect ratios attained in the present simulations are related to the angular distribution of magnetic energy per unit mass. That is, the more a magnetic energy is concentrated toward a small \( \theta \), the larger the aspect ratio of ejected matter.

In the literature, we have found eight CCSNe in which both the upper and lower limits of the aspect ratio are measured: SN1987a, SN1993J, SN1997X, SN1998S, SN2002ap, SN2005bf, SN2007gr, and SN2010jl. The observed aspect ratios are at most \( \approx 3 \), e.g., \( \approx 2–3 \) for SN1987A (Papalilioosios et al. 1989), \( \approx 3 \) for 1997X, and 1.2–2.5 for SN1998S (Wang et al. 2001). It seems that aspect ratios obtained in our rotational models are too large compared with these observations, keeping in mind that ours will grow larger later on. However, we note that the above sample does not necessarily contain supernovae that leave a magnetar. Applying the SGR/AXP birth rate of \( \sim 0.1–0.2 \) per century (Leahy & Ouyed 2007) and the Galactic CCSN rate of \( \sim 0.8–3.0 \) per century (Diehl et al. 2006), the rate of magnetar production among all CCSNe is \( \sim 3\%–25\% \).

The constant resistivity that we assume in our computations may not be natural. Resistivity arising from turbulent convective motions may appear only around convectively unstable regions and may take a different value at a different position. Computations implementing such non-constant resistivity should be performed in future works. The effects of viscosity, which is expected to coincide with resistivity in turbulence, also have to be investigated. Although the ideal way to study the effects of turbulent resistivity and viscosity is through a direct numerical simulation of turbulence itself, this is currently quite infeasible. This kind of simulation may demand a computational facility dozens of orders of magnitude more powerful than what is currently possible.

Although we carried out two-dimensional MHD simulations with the assumptions of axisymmetry and a purely poloidal magnetic field at the beginning, it is well known that a purely poloidal and a purely toroidal magnetic field are both unstable against non-axisymmetric perturbations. According to Braithwaite (2009), the ratio of the poloidal magnetic energy to the total magnetic energy, \( E_p/E \), should be less than 0.8 for stability. In each of our rotational models, we find that \( E_p/E < 0.8 \) is always satisfied after bounce. Although the stability criterion is not fulfilled before bounce, the instability will not grow substantially, since it takes at most one Alfvén timescale until bounce occurs. Thus, we expect that the results obtained in our rotating models will not change drastically even if we carry out three-dimensional MHD simulations. In fact, three-dimensional core-collapse simulations with a strong magnetic field and a rapid rotation performed by Kuroda & Umeda (2010) show results qualitatively similar to those of our two-dimensional simulations. However, a purely poloidal magnetic field at the beginning may itself be unnatural. Also, in our non-rotating models, the above stability criterion is not satisfied at all. The toroidal component of a magnetic field should be added to the initial condition to deal with more realistic situations. It may be necessary in future work to investigate how such an additional component affects the results presented here.

In the present work, we studied the role of resistivity in the limited parameter range of the magnetic field and rotation. As we found that resistivity affects the dynamics differently for a different strength of magnetic field and rotation, it may be interesting to carry out a systematic study with a wide range of parameters in the future.

H.S. gratefully thanks Kenta Kiuchi, Hiroki Nagakura, and Nobutoshi Yasutake for their helpful advice in developing the numerical MHD code. H.S. is also grateful to Ken’ichiro Nakazato and Kosuke Sumiyoshi for providing Shen’s EOS table and for instructing him in handling that. This work is supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science, and Technology, Japan (21840050, 24244036, 23540323, 23340069, 20105004).

APPENDIX

Here we present numerical results of various HD and MHD test problems calculated with *Yamaazakura*. Among them, the numerical solution of a one-dimensional Riemann problem, Sedov’s self-similar point source explosion, Yahil’s self-similar collapse, the linear wave propagation problem, and the magnetic field diffusion problem can be compared with the analytic solutions. The others are multi-dimensional problems, and no reference solution exits. One can find numerical results of such problems in the literature, and may check whether our results match up with those. In each test calculation, the ideal gas EOS, \( p = (\gamma - 1)e \), is used. The distribution of numerical cells is uniform unless otherwise stated.

A.1. Riemann Problem in One Dimension

A one-dimensional Riemann problem, which is also referred as a shock tube problem or Sod problem, is carried out to check that one-dimensional HD equations are properly solved. The initial set-up is as follows:

\[
\begin{pmatrix}
p
\end{pmatrix}_{\text{left}} = \begin{pmatrix}
1.0 \\
0.0 \\
1.0 
\end{pmatrix} \quad \text{for} \quad x < 0.5,
\]

\[
\begin{pmatrix}
p
\end{pmatrix}_{\text{right}} = \begin{pmatrix}
0.0 \\
0.1 \\
0.0 
\end{pmatrix} \quad \text{for} \quad x > 0.5 
\]

with the computational domain \( x \in [0, 1] \). The adiabatic index is set as \( \gamma = 1.4 \). Numerical results with 800 cells are shown in
Figure 52. Numerical results (red crosses) of the one-dimensional Riemann problem for the reference solutions (black lines) at \( t = 0.164 \).

(A color version of this figure is available in the online journal.)

Figure 52, in which good agreement with the reference solution is found.

A.2. Riemann Problem in Two Dimensions

In order to test a multi-dimensional HD computation, a Riemann problem in two dimensions is carried out. A square numerical domain \((x, y) \in [0, 1] \times [0, 1]\) is divided into four parts, and constant hydrodynamical values are set in each part. The initial condition is

\[
\begin{bmatrix}
\rho \\
px \\
pv \\
py
\end{bmatrix} = \begin{cases}
(1.0, 0.0, 0.0) & \text{for } x < 0.5 \text{ and } y < 0.5, \\
(1.0, 1.0, 0.0) & \text{for } x < 0.5 \text{ and } y > 0.5, \\
(1.0, 1.0, 0.0) & \text{for } x > 0.5 \text{ and } y < 0.5, \\
(0.4, 0.5313, 0.0) & \text{for } x > 0.5 \text{ and } y > 0.5,
\end{cases}
\]

which is symmetric across a diagonal line \( y = x \). An adiabatic index of \( \gamma = 1.4 \) is used here. These set-ups are the same as Case 12 of Liska & Wendroff (2003). The calculation is done with 400 \( \times \) 400 numerical cells. A numerical result is shown in Figure 53, which is in good agreement with that of Liska & Wendroff (2003).

A.3. Point Source Spherical Explosion

Another HD test calculation that we have performed is a spherical point source explosion; the solution is known as Sedov’s solution. A one-dimensional spherically symmetric computation is performed with polar coordinates. We use non-uniform numerical cells with the spatial resolution of the \( i \)th cell being

\[
\Delta r_i = \Delta r_{\text{min}}^{2/3}, \text{ where } \Delta r_{\text{min}} = 10^{-4}, \alpha = 1.0056, \text{ and the maximum of } i \text{ is 720.}
\]

This cell distribution is similar to that of the MHD core-collapse simulations presented in the present work, where the same cell distributions are set along the \( \sigma - \) and \( z - \) directions separately. Hence, we may be able to speculate here how much a propagating shock wave diffuses in our MHD core-collapse simulations. We assume a static sphere of radius \( r = 1.0 \) with a uniform density of \( \rho_0 = 1.0 \). A point source explosion is initiated by injecting the internal energy \( E_0 = 3 \times 10^7 \text{ cgs} \) at the central sphere of radius \( r = 0.01 \). Numerical results are shown in Figure 54. It is found that the sharpness of the density peak in the numerical solution is maintained during propagation, despite the fact that the spatial resolution is lower for a larger radius. This implies that, in our computations, a propagating shock wave is not damped crucially due to numerical diffusion.

A.4. Self-Similar Collapse

In order to check whether gravity is correctly dealt with by Yamazakura, we tested a self-similar collapse; the solution is known as Yahil’s solution (Yahil 1983). Both a one-dimensional spherically symmetric calculation with polar coordinates and a two-dimensional axially symmetric calculation with cylindrical coordinates are performed. In describing Yahil’s solution, a radius, density, and velocity are written in dimensionless form:

\[
R = \kappa^{-1/2} G(\gamma - 1)/2 (t_0 - t)^{\gamma - 2},
\]

\[
D(R) = \rho G(t_0 - t)^2,
\]

\[
V(R) = u \kappa^{-1/2} G(\gamma - 1)/2 (t_0 - t)^{\gamma - 1},
\]

where \( G, \gamma, \kappa, \text{ and } t_0 \) are, respectively, the gravitational constant, adiabatic index, polytrope coefficient, and the time when the central density diverges. We set \( \gamma = 1.3, \kappa = 1.2435 \times 10^{17}/2^{4/3} \text{ cgs} \), and \( t_0 = 0.1 \text{ s} \). The collapse of a sphere with a \( 4000 \text{ km} \) radius follows. In the cylindrical coordinate calculations, two different spatial resolutions are tested, where the resolutions of the innermost cells are \( \Delta r_{\text{min}}, \Delta z_{\text{min}} = 2 \times 10^4 \text{ and } 4 \times 10^4 \text{ cm} \). The number of numerical cells is \( N_r \times N_z = 720 \times 720 \) in both calculations. In the polar coordinate calculation, only one spatial resolution, \( \Delta r_{\text{min}} = 4 \times 10^4 \text{ cm, with } N_r = 720, \) is taken.

The distributions of density and velocity when the central density reaches \( 2.3 \times 10^{14} \text{ g cm}^{-3} \) are presented in Figure 55, while errors at the inner-most numerical cells are shown in

Figure 53. Numerical result of the two-dimensional Riemann problem at \( r = 0.25 \) (density color map with contours and velocity field vectors). The computation is done with 400 \( \times \) 400 cells.

(A color version of this figure is available in the online journal.)
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**Figure 54.** Numerical results of a point source spherical explosion for density (top left), velocity (top right), and pressure (bottom left) distribution. The bottom right panel is shown to see, with linear scale, a sharpness of the numerical solution for pressure. In each figure, the red, green, and blue crosses represent solutions at $t = 0.05, 0.2$, and $0.4$, respectively. (A color version of this figure is available in the online journal.)

**Figure 55.** Results for the calculations of Yahil’s self-similar collapse with cylindrical coordinates (left) and polar coordinates (right). The solid lines and dotted lines are, respectively, for the analytic solution of $D$ and $V$, while the colored crosses represent the numerical ones. For the cylindrical coordinate calculations, the numerical solutions along the pole ($\sigma = 0$) are shown. (A color version of this figure is available in the online journal.)

| Coordinate | $\{\Delta \rho_{\text{min}}, \Delta v_{\text{min}}, \Delta z_{\text{min}}\}$ (cm) | $D$-error$^b$ (%) | $V$-error$^c$ (%) |
|------------|-------------------------------------------------|-------------------|-------------------|
| Cylindrical | $2 \times 10^4$ | 7.28 | 22.0 |
| Cylindrical | $4 \times 10^4$ | 13.1 | 22.9 |
| Polar | $4 \times 10^4$ | 4.18 | 1.22 |

**Table 2.** Errors in the Numerical Solution at the Innermost Cell

Notes.

$^a$ Spatial resolution of the innermost cell.

$^b$ Error of $D$ at the innermost cell.

$^c$ Error of $V$ at the innermost cell.

Table 2. Note that, in our MHD core-collapse simulations, a bounce occurs when the central density reaches $\sim 2 \times 10^{14}$ g cm$^{-3}$. Although the spherical collapse of the $15 M_\odot$ progenitor used in the present work is not very well described by Yahil’s solution$^{11}$, the results obtained here may indicate how much error our collapse simulations involve. From the left panel of Figure 55 one can see that a deviation from the analytic solution is large near the center in each calculation.$^{12}$ In the cylindrical coordinate calculation with $\{\Delta \rho_{\text{min}}, \Delta v_{\text{min}}\} = 4 \times 10^4$ cm, in which the grid construction is the same as our standard MHD collapse simulations, the errors in both $D$ and $V$ are $\sim 10\%$ (see the second row of Table 2). Although these errors seem too large to pursue a proper numerical simulation, as discussed in Section 4.1.5, varying the spatial resolution in an MHD collapse

---

$^{11}$ We found that when the central density is in the range of $\sim 10^{12}$–$10^{13}$ g cm$^{-3}$ in a spherical collapse of the progenitor, the distribution of density and velocity is roughly described by Yahil’s solution. Outside this range, the deviation from Yahil’s solution is large.

$^{12}$ The deviations from the analytic solution around the outer boundary are a numerical artifact due to a boundary condition.
simulation does not change the results very much. This implies that the errors around the center are not fatal to the extent of the discussions developed in this paper.

As seen in Figure 55 and in Table 2, the cylindrical coordinate calculations have larger errors than the polar coordinate calculation does, which may be due to the structure of numerical cells. However, we found that numerically evaluated electric currents behave better in cylindrical coordinates than in polar coordinates, and thus we adopt the former in the present MHD simulations.

A.5. MHD Riemann Problem

As a basic test for the ideal-MHD part of our code, we carried out a one-dimensional MHD Riemann problem that is known as the Brio–Wu Problem (Brio & Wu 1988). The initial condition is set as

\[
\begin{pmatrix}
\rho \\
\frac{\rho}{p} \\
\frac{B_x}{\sqrt{4\pi}} \\
\frac{B_y}{\sqrt{4\pi}}
\end{pmatrix} = \begin{pmatrix}
1.0 \\
1.0 \\
0.75 \\
1.0
\end{pmatrix} \quad \text{for} \quad x < 0.5,
\]

\[
\begin{pmatrix}
\rho \\
\frac{\rho}{p} \\
\frac{B_x}{\sqrt{4\pi}} \\
\frac{B_y}{\sqrt{4\pi}}
\end{pmatrix} = \begin{pmatrix}
0.125 \\
0.1 \\
0.75 \\
-1.0
\end{pmatrix} \quad \text{for} \quad x > 0.5,
\]

where the computational domain is \(x \in [0, 1]\). The initial velocities are set as zero. The adiabatic index here is 2.0. The results of the calculation at \(t = 0.1\) performed with 800 numerical cells are given in Figure 56 together with those obtained by a famous numerical code ZEUS-2D (Stone & Norman 1992). Although a slight difference can be found, the two results match well.

A.6. Linear Wave Propagation

In Section 2, we mentioned that the KT scheme adopted in our numerical code is third order in time and second order in space. Here, we will see whether \textit{Yamazakura} solves the MHD equations fairly with these degrees of accuracy, by carrying out a linear wave propagation problem in one-dimensional Cartesian coordinates. A right-going slow magnetosonic wave is adopted as a linear wave.

According to Gardiner & Stone (2005), we set the initial state of the conserved variables as

\[ u_{in} = u_0 + \epsilon \mathbf{R} \cos(2\pi x), \]  

where \(u_0\) is the background state, \(\epsilon = 10^{-6}\) is the wave amplitude, and \(\mathbf{R}\) is the right eigenvector for the right-going slow magnetosonic wave in conserved variables given by

\[
\mathbf{R} = \frac{1}{6\sqrt{5}} \begin{pmatrix}
12 \\
6 \\
4 \\
9 \\
0 \\
-4\sqrt{2} \\
-2
\end{pmatrix}.
\]
The background state is set as \( \rho_0 = 1, p_0 = 1, v_{0x} = v_{0y} = 0, B_{0x} = 1, B_{0y} = \sqrt{2}, \) and \( B_{0z} = 1/2, \) in which the slow magnetosonic speed is \( c_s = 1/2. \) The adiabatic index is set as \( \gamma = 5/3. \) The computational domain is \( x \in [0, 1], \) and the periodic boundary condition is adopted at each boundary. Each calculation is run until \( t = 4 \) when the wave has propagated the distance of two wavelengths.

In order to check the degree of accuracy in space, we first tested two calculation series with uniform cell construction, each of which contains four calculations with different grid numbers. In the first series, series A, we fix \( \Delta x \) by controlling the Courant–Friedrichs–Lewy (CFL) number while varying \( \Delta t. \) The numbers of cells \( N \) and CFL numbers \( \nu \) for the four calculations are \((N, \nu) = (8, 49), (32, 0.49/16), (128, 0.49/4), \) and \((512, 0.49)\). In the second series, series B, we fixed the CFL number at \( \nu = 0.49 \) and tested the same set of cell numbers as before. Note that in series B, both \( \Delta x \) and \( \Delta t \) are halved by doubling the cell number. The distributions of \( v_x \) at \( t = 4 \) for series B are plotted in Figure 57, which shows that the numerical solution matches the analytic one well for \( N \geq 128. \) The error from the analytic solution is evaluated by the L1 error norm, where

\[
\delta u_i^m = \frac{1}{N} \sum_i |u_i^m - \{u_i^m\}|, \tag{A8}
\]

The left panel of Figure 58 plots the variation of the L1 error norm with respect to \( N \) for series A and B. The red plots (series A) show that the scheme is approximately second order in space. We found that the results from series B (blue plots) are almost identical to those in series A, despite the fact that \( \Delta t \) is larger in series B for a fixed \( N, \) a fixed \( \Delta x, \) except for \( N = 512. \) This indicates that an error due to \( \Delta t \) is dominated by an error due to \( \Delta x \) for \( \nu < 0.5 \) (the CFL condition for the KT scheme), and creates difficulty in properly checking the degree of accuracy in time. Nonetheless, we can still state that the scheme is at least approximately second order in time based on the fact that the L1 error norms in series B, varying both \( \Delta x \) and \( \Delta t \) at the same rate, result in \( \sim N^{-2} \) (see the blue plots in the left panel of Figure 58).

Next, we evaluate the degrees of accuracy of our code for cases of non-uniform cell construction. A structure of numerical cells we set here is such that the \((2n-1)\)th cell has a size \( L/N \times (1-\alpha) \) while the \(2n\)th cell has a size \( L/N \times (1+\alpha), \) where \( L \) is the size of the whole numerical domain and \( \alpha < 1 \) is the parameter representing the degree of non-uniformity. We tested three cases \( \alpha = 10^{-2}, 10^{-1}, \) and \( 5 \times 10^{-1}. \) In each case, four calculations fixing \( \nu = 0.49 \) with \( N = 8, 32, 128, \) and 512 are performed. The results are plotted in the right panel of Figure 58. It is shown that a second order in space and at least a second order in time are marginally kept even for a 200% size increase and 33% size decrease from the neighboring cell \( (\alpha = 5 \times 10^{-1}). \) Note that a numerical cell size in our MHD-collapse simulations increases outward by 0.56%.

### A.7. Rotor Problem

In order to check that the multi-dimensional ideal MHD equations are solved well with our code, we perform two test calculations. One is the so-called rotor problem to be presented here, and the other is a point source explosion with a magnetic field which will be shown in the next section.

The rotor problem was proposed by Balsara & Spicer (1999) and simulates the propagation of torsional Alfvén waves. The system initially consists of a rapidly rotating cylinder of dense fluid surrounded by lighter gas at rest, which is threaded...
of Ziegler (2004) performed with Cartesian coordinates.

Figure 60. This also shows good agreement with the calculation by a uniform magnetic field parallel to the $x$-axis. With a computational domain of $(x,y) \in [-0.5,0.5] \times [-0.5,0.5]$, the initial condition is

$$\begin{align*}
\rho &= 1 + 9f(r), \\
p &= 1, \\
v_x &= -2f(r)y/r, \quad v_y = -2f(r)x/r, \quad v_z = 0, \\
B_x &= 5, \quad B_y = B_z = 0, \\
\end{align*}$$

(A9)

where

$$f(r) = \begin{cases} 
1 & \text{if } r < 0.1, \\
\frac{200}{3}(0.115 - r) & \text{if } 0.1 < r < 0.115, \\
0 & \text{if } r > 0.115,
\end{cases}$$

$$r = (x^2 + y^2)^{1/2}.$$ 

The adiabatic index is $\gamma = 1.4$. The calculation is performed with 400 × 400 numerical cells. Results of the calculation are displayed in Figure 59, and show good agreement with those found in Ziegler (2004).

A.8. Point Source Three-dimensional Explosion with Magnetic Field

The calculation presented here differs from the spherical explosion test described above in that fluid is magnetized. The initial condition is given by

$$\begin{align*}
\rho &= 1, \\
p &= \begin{cases} 
10^4 & \text{if } x^2 + y^2 + z^2 < r^2, \\
1 & \text{otherwise}
\end{cases}, \\
v_x = v_y = v_z &= 0, \\
B_z/\sqrt{4\pi} &= 100, \quad B_x = B_y = 0. \\
\end{align*}$$

(A10)

The adiabatic index is chosen as $\gamma = 5/3$. We perform a calculation in cylindrical coordinates with a numerical domain of $(\sigma,z) \in [0,0.5] \times [0,0.5]$, taking the symmetric axis in the magnetic field direction. The number of numerical cells is $N_\sigma \times N_z = 96 \times 96$. The result of the calculation is given in Figure 60. This also shows good agreement with the calculation of Ziegler (2004) performed with Cartesian coordinates.

A.9. Magnetic Field Diffusion

In order to determine if Yamazakura properly handles the resistive terms in the induction equation, we test a magnetic field diffusion problem in one dimension. Here, we do not solve the full set of MHD equations but the magnetic diffusion equation with constant resistivity,

$$\frac{\partial B}{\partial t} = \eta \frac{\partial^2 B}{\partial x^2},$$

(A11)

by freezing fluid in the induction equation. Setting the initial condition with an anti-parallel magnetic field, $B = -B_0$ for $x < 0$ and $B = B_0$ for $x > 0$, the exact solution at time $t$ is written as

$$B(x,t) = \begin{cases} 
-\frac{2B_0}{\sqrt{\pi}} \int_0^{x/\sqrt{\eta t}} e^{-u^2} du & \text{for } x < 0, \\
\frac{2B_0}{\sqrt{\pi}} \int_0^{x/\sqrt{\eta t}} e^{-u^2} du & \text{for } x > 0.
\end{cases}$$

(A12)

We set $B_0 = 100$ and $\eta = 10$. The calculation is performed with 800 numerical cells for $x \in [-5,5]$. The results are displayed in Figure 61, in which one can find good agreement between the numerical and exact solutions.

A.10. Spontaneous Fast Reconnection

We finally check that Yamazakura properly deals with the multi-dimensional resistive-MHD equations using a simulation
of a spontaneous fast reconnection first conducted by Ugai (1999). The initial current system is constructed as follows:

\[
B_x(y)/\sqrt{4\pi} = \begin{cases} 
\sin(\pi y/2) & \text{for } y < 1, \\
1 & \text{for } 1 < y < 3.6, \\
\cos((y-3.6)\pi/1.2) & \text{for } 3.6 < y < 4.2, \\
0 & \text{for } y > 4.2, \\
-B_x(-y)/\sqrt{4\pi} & \text{for } y < 0,
\end{cases}
\]

\[p = (1 + \beta_0 - B^2_x/4\pi)/2,\]

\[\rho = 2p/(1 + \beta_0),\]  

(A13)

where \(\beta_0 = 0.15\). A velocity and the \(x, y\)-component of the magnetic field are initially zero. The adiabatic index is taken as \(\gamma = 5/3\). A reconnection of magnetic field lines is initiated by an anomalous resistivity:

\[
\eta(r, t) = \begin{cases} 
k_R[V_d(r, t) - V_c] & \text{for } V_d > V_c, \\
0 & \text{for } V_d < V_c,
\end{cases}
\]

(A14)

(A15)

where

\[
V_d(r, t) = \left| \frac{J(r, t)}{\rho(r, t)} \right|, \\
V_c = V_{c,0} \left[ \frac{2p}{\rho(1 + \beta_0)} \right]^\alpha.
\]

(A16)

The parameters are taken as \(k_R = 0.03\), \(V_{c,0} = 4\), and \(\alpha = 0.5\). For details of the resistivity model, see Ugai (1999). The anomalous resistivity model (A14) is assumed for \(t > 4\). During \(0 < t < 4\), localized resistivity is imposed to disturb the initial one-dimensional configuration:

\[
\eta(r) = \eta_d \exp[-(x/1.1)^2 - (y/1.1)^2],
\]

(A17)

where \(\eta_d = 0.02\). The computation is performed with \(800 \times 800\) numerical cells covering the domain of \((x, y) \in [-20, 20] \times [-6, 6]\).

Some important results of the computation are shown in Figures 62 and 63. The global magnetic-field distributions are displayed in Figure 62 for \(t = 18\) and 30. We found that they are similar to those obtained by Ugai (1999), and also those of Feng et al. (2006). The left panel of Figure 63 represents profiles of \(v_x\) and \(B_z\) along the \(x\)-axis at \(t = 24\) and \(t = 30\), while the right panel shows the evolution of the resistivity \(\eta\) and electric field \(E\) at the origin, \(v_y\) at \((x, y) = (0, 0.9)\), and a magnetic flux \(\Phi\) evaluated by

\[
\Phi = \int_{y \geq 0} B_x(x = 0, y) dy.
\]

For these results, we also find a rough agreement with the above two works, although there are some insignificant differences.
Figure 63. Results for the calculation of spontaneous fast reconnection. Left: profiles of $v_x$ and $B_y$ along the $x$-axis at $t = 24$ and $t = 30$. Right: temporal variations of the magnetic flux $\Phi$, the resistivity $\eta$, and electric field $E$ at the origin, and $v_y$ at $(x, y) = (0, 0.9)$.

(A color version of this figure is available in the online journal.)

With all of these results presented in this Appendix, we expect that the present MHD collapse simulations performed by Yamazakura offer proper results.

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