Effect of Upstream Dam Geometry on Peak Discharge During Overtopping Breach in Noncohesive Homogeneous Embankment Dams; Implications for Tailings Dams

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Abstract Tailings dams differ from conventional water-retaining dams in design, materials retained behind the dam and in construction of the dam, and the inclination of the upstream face of the dam. In this paper, we isolate the effect of upstream slope angle on behavior during overtopping breach. Six 1 m high homogeneous fine sand dams were constructed with upstream slope angles varying between 10.0° and 30.0° and brought to failure by V notch overtopping. Slope angle was observed to define the height of flow over the erosional breach crest hydraulic control structure that forms in the upstream face of the dam, with higher peak outflow corresponding to steeper upstream slope angles. A semiellipse was observed to be an excellent approximation of the geometry of the breach throughout the rising limb of the hydrograph to peak outflow. This observation permitted the use of the Ramanujan approximation for perimeter of an ellipse to define a mathematical relationship between breach width and arc length. A simplified method to predict the rising limb of the outflow hydrograph was then proposed based on the assumption of linear growth of breach width coupled with a semielliptical breach geometry. These findings show that hazard analysis for overtopping failure should consider the effect of upstream slope angle on peak outflow.

Plain Language Summary Dam breach is the failure of a dam structure resulting in the uncontrolled release of the material retained behind the dam in a sudden or catastrophic manner. The outflow hydrograph, defined as the time rate of material released from the dam, is used in computer models to predict how far, how fast, and how deep inundation will occur. Tailings are waste products of the mining industry generally produced by crushing and grinding rock to extract valuable minerals. Tailings dams differ from water-retaining dams in design, materials retained and used in construction of the dam, and the inclination of the upstream face of the dam. In this paper, we isolate the effect of upstream slope angle on behavior during overtopping breach. Six 1 m high homogeneous fine sand dams with upstream slope angles varying between 10.0° and 30.0° were breached by water overtopping. The upstream slope angle was observed to define the height of flow over the erosional weir that forms in the upstream face of the dam, with higher peak outflow corresponding to steeper upstream slope angles. These findings show that hazard analysis for overtopping failure should consider the effect of upstream slope angle on peak outflow.

1. Introduction

Tailings are waste products of the mining industry generally produced by crushing and grinding rock to extract valuable minerals. Particle size and distribution of tailings are dependent on the characteristics of the ore body and the milling processes used. While tailings dam design varies widely, in general terms, tailings dams are raised embankments constructed throughout the operational mine life to retain these wastes at the mine site (Vick, 1990). Construction materials may include natural fills from local borrow sources as well as tailings themselves. The coarse sand fraction of the tailings (separated from the tailings stream using cyclone processes) are often used as an economical construction material (Klohn, 1984). Dam breach is the failure of a dam structure resulting in the uncontrolled release of retained volume in a sudden or catastrophic manner (Martin et al., 2015). Over the past century, almost 3,000 lives have been lost, as a result of tailings dam breach (Santamarina et al., 2019). Recent notable examples include the 2014 Mount Polley failure in British Columbia, Canada (Morgenstern et al., 2015), the 2015 Fundão failure in Minas Gerais, Brazil (Morgenstern et al., 2016; Palu & Julien, 2019), the slumping failure in 2018 at Cadia mine in New South Wales, Australia (Jefferies et al., 2019), and the 2019 failure at...
Córrego do Feijao in Brumadinho, Brazil (de Lima et al., 2020; Robertson et al., 2019). This paper reports the results of research focused on increasing our understanding of dam breach processes as they apply to tailings dams, specifically to explore the effect of tailings dam geometry on the outflow hydrograph used in dam breach analyses informing management of risk.

Dam breach analyses are conducted to limit hazard exposure and potential for loss of life imposed by dams by informing dam consequence rating, flood-routing analyses, land use planning, flood mitigation strategies, and emergency response plans (e.g., Canadian Dam Association, 2013). A prediction of the outflow hydrograph, defined as the time rate of outflow representing the breach event (West et al., 2018), is the primary input boundary condition in many flood-routing numerical models (e.g., TELEMAC-MASCARET [Hervouet & Ata, 2017], HEC-RAS [USACE, 2016], Fluent [Yuan et al., 2016], and Flow-3d [Flow Science Inc 2018]). Quantitative approaches to the prediction of the outflow hydrograph are generally divided in the literature into two classes: parametric and physically based. Parametric approaches rely on regression analysis of empirical breach behavior in historical failures to describe breach parameters as a function of dam or reservoir properties (e.g., Costa, 1985; MacDonald & Langridge-Monopolis, 1984; Xu & Zhang, 2009). In contrast, physically based models use fundamental principles of hydraulics, sediment transport, and soil mechanics to predict breach behavior (Dam Safety Interest Group, 2017). A comprehensive review of existing breach models conducted by West et al. (2018) for water-retaining dams concluded that physically based models are generally more accurate and reliable than parametric models. However, it is unclear whether physically based models derived for the case of water-retaining dams are also applicable to the tailings dam case given differences in dam geometry and rheology of the materials retained.

Flume and field experiments have played a central role in improving our understanding of dam breach processes and the development of physically based models for clear water dams. Notable studies include key contributions from the University of Auckland (Coleman et al., 2002), the IMPACT project (Morris et al., 2007; Samuels & Morris, 2010), the United States Department of Agriculture-Agricultural Research Service (USDA-ARS) research program (Hanson et al., 2005), and United States Geological Survey (USGS; Walder et al., 2015). A key observation for noncohesive dams from Coleman et al. (2002) and explored in further detail by Walder et al. (2015) is that the outflow due to overtopping is hydraulically controlled by the geometry of the breach crest that forms as an erosional feature in the upstream face of the dam. During a breach event, the breach crest is defined by connecting a series of high points along streamlines flowing over the breach channel in which flow is perpendicular to the breach crest at all points along its length (Coleman et al., 2002). Figure 1 illustrates the upstream slope of a noncohesive dam experiencing dam breach, including the erosional breach crest feature formed in the upstream face of the dam and the parameters defining the instantaneous breach geometry including breach width, defined along the crest of the dam, $B^*$, and the breach arc length projected on a horizontal plane, $B$. Due to the curved shape of the breach crest, $B^*$ will be shorter than $B$ (Coleman et al., 2002). The depth of water flowing over the breach, $d$, varies in height from zero at the intersection of the breach crest with the reservoir water elevation to its maximum point at the arc apex. This location acts as a weir that separates subcritical flow upstream from supercritical flow downstream. As the volumetric flow rate discharged over the breach crest is governed by the size and shape of the breach crest (Walder et al., 2015), a prediction of the evolution of the breach geometry on the upstream side of the dam is essential for the prediction of the outflow hydrograph.

Tailings dams differ from conventional water-retaining, or clear water dams, in design, materials retained behind the dam and used in construction of the dam, and of particular importance to breach analyses, the inclination of the upstream face of the dam. Tailings dams are typically constructed in stages using the coarse fraction of the tailings, consisting of noncohesive sand-sized particles. To reduce the risk of seepage induced failure (i.e., piping), shallow sloping tailings beaches are often constructed from the finer fraction of the tailings material deposited into the impoundment from the dam crest to reduce the hydraulic gradient and the quantity of seepage flow at the dam face (McLeod & Murray, 2003). The typical upstream geometry of a tailings dam therefore is inclined much shallower than would be constructed for a water-retaining dam. Despite these differences, relationships for clear water dams are often used in hazard analyses for tailings dams as no tailings-specific alternatives exist.

In this paper, we isolate the effect of upstream slope angle on behavior during overtopping breach as a first step toward building tailings-specific relationships (i.e., water will still be used as the retained fluid to isolate the effect of upstream slope angle). Past work on the influence of dam shape and erosion for the specific case of granular dykes (Müller et al., 2016; Schmitz et al., 2020) has made use of a dyke shape parameter, $\mu$, where
μ = (½ [upstream slope ratio + downstream slope ratio] + [crest breadth/dam height])/2.5 to capture the volume of the dam per unit width. These studies indicate that dykes with a larger volume per unit width lead to a more gradual increase in breach discharge and in breach width during the first stage of breach expansion. Here, we explore the specific mechanism by which the geometry of the upstream face of a dam influences the outflow breach hydrograph for the specific boundary condition of a finite reservoir rather than a dyke. Given that the shape of the breach crest in the upstream dam face has been observed to quantitatively define the outflow hydrograph (e.g., Walder et al., 2015), it is reasonable to hypothesize that the outflow hydrograph for the shallowly inclined upstream dam face associated with tailings dam geometry will differ from that of a clear water dam. It is currently unclear whether the use of models derived for relatively steep noncohesive clear water dams is a conservative approach (i.e., overestimating of peak discharge and/or breach outflow volume) for estimating breach for a noncohesive tailings dam geometry, and if so, to what degree.

The objective of this study is to use detailed observations of breach initiated by overtopping in large physical models of noncohesive dams of varying upstream slope angles ranging from steep (e.g., clear water dams, landslide dams) to shallow (e.g., tailings geometry) to define, for the first time, the effect of the upstream slope angle on the peak discharge for a finite reservoir. In the remainder of this paper, we describe the experimental program and instrumentation, including a novel strategy to highlight and capture the evolution of the geometry of the upstream breach crest acting as hydraulic control section. Based on these observations, we introduce a conceptual framework to quantify the instantaneous geometry of the breach using a simplifying assumption of a semieliptical shape of the breach crest. We then illustrate how this framework can be paired with a discharge equation to generate quantitative estimates of peak breach outflow for differing upstream slope angles including those relevant to the calculation of the breach outflow hydrograph for the shallowly sloping upstream slope angles associated with tailings dam geometry. Finally, we highlight the remaining research questions required to more fully define tailings dam breach, including the need to model breach of retained tailings rather than clear water.
2. Experimental Methods and Materials

2.1. Experimental Program

Experiments planned in the present study are part of the wider CanBreach research program (https://canbreach.ca) aimed at exploring tailings dam-specific breach processes. A significant benefit of physical modeling is the ability to isolate the effect of individual variables, with subsequent experimental campaigns adding the possibility of gradually layering on more tailings-specific complexity. In this, the first of a planned series of physical model experiments of the CanBreach study, we designed the experimental program to isolate the effect of upstream slope angle on peak discharge during notch overtopping of noncohesive dams by water flow. Thus, rather than attempt to model every characteristic of a tailings dam (e.g., construction method, tailings materials, deposition methods, and beach), experiments were conducted using an idealized homogeneous geometry, dam material (fine sand), and fluid retained (clear water). Six large physical model dams, 1 m in height, were constructed with upstream slope angles ranging from 10.0° to 30.0° from horizontal and downstream slope inclination of 3H:1V or 18.4° (Figure 2). The dyke shape parameter ($\mu$) as defined by Müller et al. (2016), ranged from 1.9 to 1.1. Details of the test matrix are presented in Table 1, including the context of experiments performed by Walder et al. (2015) and Coleman et al. (2002).

Dams were constructed in the 36 m long horizontal portion of the Queen's University Landslide Flume (e.g., Bullard et al., 2019; Coombs et al., 2020; Miller et al., 2017). The crest of the dam was positioned upstream of the midpoint of the horizontal portion of the flume to create a larger volume downstream of the dam than upstream (Figure 2). In other words, the experiment was designed to permit an upstream reservoir to overtop the dam at an elevation of approximately 1 m, and to enable the observation of breach flows up until the point of peak outflow while maintaining a sufficiently low downstream water level to minimize the effect of tailwater submergence. At stages of the breach later than the peak outflow (i.e., not the focus of this study), the upstream and submerged downstream water levels reach equilibrium at an approximate reservoir elevation of 0.5 m. A drained downstream toe boundary condition was not possible due to the large volume of fluid retained (16 m$^3$) released in a short duration breach (~120 s).

![Figure 2. Physical model geometry and instrumentation layout to investigate the effect of upstream slope angle, $\alpha$, on peak discharge. The total horizontal flume length is 36 m, with the dam positioned upstream of center to minimize the effects of tailwater submergence on peak discharge. Dam breach, initiated from notch overtopping, will result in the depletion of the upstream reservoir into the initially dry downstream flume volume until water levels equilibrate, resulting in tailwater submergence (units: m).](image-url)
Mindful of the tailwater submergence boundary condition inherent in the design of the present study, the first experiment was designed to provide a baseline of comparison of our results to a case from the drained-toe dam breach of Walder et al. (2015). The geometry of the dam physical models adopted in the present study was selected to replicate this prior study as closely as possible, with the same dam height (1.0 m), a nominally identical dam width (2.09 m) and crest breadth (0.3 m), and for the base case, an identical upstream slope angle (30.0°). Notable differences between these studies include the downstream slope angle (18.4° vs. 30.0° by USGS), a different reservoir stage–volume relationship (discussed later in the paper in Section 3.7), and tailwater submergence in the present study. Since the work of Walder et al. (2015) indicates that the outflow hydrograph is hydraulically controlled by the upstream geometry of the breach crest and the reservoir storage function, it is hypothesized that the tailwater submergence in the present study will not impact peak outflow if peak outflow occurs while the downstream reservoir elevation is well below the elevation of the upstream breach crest (i.e., free-flow conditions exist from breach onset to peak outflow; the tailwater elevation is below the elevation of the breach crest [Rantz, 1982]). The validity of this hypothesis will be revisited later in the manuscript during the discussion of the experimental observations.

### 2.2. Materials and Construction Methods

Dams were constructed from uniform fine sand (#730 Silica Sand by Weldon Company which is comparable to F110 Ottawa sand) of mean grain size, $d_{50}$ of 0.12 mm and friction angle of 30.5° (Beddoe & Take, 2016). This angle therefore defines the upper bound of slope angles possible in the study (i.e., slope stability of dry downstream dam face or submerged upstream dam face). This sand is sufficiently close in grain size to the sand of Walder et al. (2015) (mean grain size, $d_{50}$ of 0.21 mm) to argue that a direct comparison of experimental results can be made between the two test programs. The particle size distribution of this sand is also comparable to the cyclone copper tailings sand materials presented by Whitehead & Witte (2019) where the average $d_{50}$ was about 0.2.

The construction process adopted in this study is captured through a series of photographs presented in Figure 3. The base of the concrete flume prior to dam construction is illustrated in Figure 3a. The most prominent feature in the photograph is a 10.2 cm diameter toe drain constructed from two sections of corrugated high density polyethylene drainage pipe that was installed under the dam crest, flush to the flume base, and secured to the floor with metal strapping. A perforated 2.0 m long pipe section was located parallel to the crest and wrapped in geotextile fabric to allow for water ingress while providing a barrier to the sand material. The perforated pipe section was attached to a solid (nonperforated) 3.5 m long pipe section with an L-joint, extending from the dam.

### Table 1

**Slope Angle Sensitivity Test Series Details**

| Research group                                | Test ID | Dam height (m) | Crest breadth (m) | Dam width (m) | Upstream slope (°) | Downstream slope (°) | Pilot notch location | $d_{50}$ (mm) | $Q_{peak}$ (m³/s) |
|-----------------------------------------------|---------|----------------|------------------|---------------|-------------------|---------------------|---------------------|---------------|-----------------|
| Queen's University (present study)            | S4      | 1.00           | 0.30             | 2.09          | 10.0              | 18.4                | Flume center         | 0.12          | 0.07            |
|                                               | S1      | 1.00           | 0.30             | 2.09          | 18.4              | 18.4                | Flume center         | 0.12          | 0.14            |
|                                               | S2      | 1.00           | 0.30             | 2.09          | 18.4              | 18.4                | Flume center         | 0.12          | 0.16            |
|                                               | S5      | 1.00           | 0.30             | 2.09          | 25.0              | 18.4                | Flume center         | 0.12          | 0.17            |
|                                               | S6      | 1.00           | 0.30             | 2.09          | 30.0              | 18.4                | Flume center         | 0.12          | 0.24            |
| USGS (Walder et al., 2015)                    | 1       | 1.00           | 0.36             | 2.00          | 30.0              | 30.0                | Flume center         | 0.21          | 0.13            |
|                                               | 2       | 1.00           | 0.36             | 2.00          | 30.0              | 30.0                | Flume center         | 0.21          | 0.13            |
|                                               | 3       | 1.00           | 0.36             | 2.00          | 30.0              | 30.0                | Flume center         | 0.21          | 0.14            |
|                                               | 4       | 1.00           | 0.36             | 2.00          | 30.0              | 30.0                | Flume center         | 0.21          | 0.17            |
|                                               | 5       | 1.02           | 0.33             | 2.00          | 30.0              | 30.0                | Flume center         | 0.21          | 0.15            |
|                                               | 10      | 1.00           | 0.36             | 2.00          | 30.0              | 30.0                | Flume center         | 0.21          | 0.13            |
| University of Auckland (Coleman et al., 2002) | N/A     | 0.30           | 0.07             | 2.21          | 20.3              | 20.3                | Flume sidewall       | 0.50          | 0.20            |
Figure 3. Images of the construction process. (a) Photograph of L-shaped toe drain drilled into the flume base prior to sand placement looking downstream; (b) materials handling with Bobcat skid steer; (c) sand placement in progress; (d) Slurry pumping sand between geotextile-wrapped plywood forms looking upstream; (e) downstream view of finished dam, post surface smoothing; and (f) side profile view of cameras mounted to the rotating support beam, in its upright position.
crest to beyond the downstream toe of the dam. The outline of the downstream face of the dam was defined using aluminum angle brackets anchored to the concrete sidewalls of the flume. Sand was deposited into the flume with a Bobcat skid steer loader (Figure 3b) for later placement. The construction methods of the dams combined a variety of techniques, including hydraulic placement, shoveling, and hand tamping (Figure 3c). Hydraulic placement was a method that allowed for relatively consistent construction across tests, while not requiring the sand to be dried to an optimal moisture content following each test or utilization of traditional compaction techniques (e.g., construction in lifts, vibratory compaction). In order to contain the sand in the desired dam geometry, formwork was constructed from plywood sheets. The plywood sheets were saw cut to create drainage openings before being wrapped in geotextile fabric to serve in a separation role. Aluminum angle brackets were drilled into the flume side walls, inclined at 18.4° from horizontal on each of the downstream sidewalls and inclined at angles ranging from 10.0° to 30.0° from horizontal on each of the upstream sidewalls. The plywood formwork was then sequentially bolted to the brackets as sand was deposited (Figure 3d). Sand was liquefied into a 33% solids content slurry for hydraulic placement by jetting the sand with high pressure water. The slurry was pumped into place with a Tsurumi HSD 2.55S submersible single-phase portable slurry pump, and a 186 W submersible puddle pump was utilized within the formwork area to quickly evacuate ponded water. Following construction, dams were left to drain for several days prior to removing the formwork. After the formwork was removed, any additional sand required to touch up the dam geometry was shoveled into place, hand tamped, and smoothed with a trowel (Figure 3e). Density was measured prior to reservoir impoundment with a nuclear density gauge at three points on the dam crest and one point on the downstream slope 1 m from dam crest. The average measured dry density of the dam material achieved through this method, 1,517 kg/m$^3$, corresponds to loose placement and is similar to the dry density reported by Walder et al. (2015) of 1,514 kg/m$^3$.

2.3. Instrumentation

The primary function of the instrumentation network of cameras and sensors was to capture the instantaneous geometry of the erosional breach hydraulic control section and outflow through the breach. The evolution of the breach crest was measured from aerial (plan view) images captured by four Canon EOS Rebel T5 Digital Single-Lens Reflex (DSLR) cameras (18 megapixel APS-C 22.3 mm by 14.9 mm sensor with 18–55 mm focal length lenses) mounted 2.5 m above the dam crest elevation (Figure 3f), capturing a combined field of view of approximately 8.0 m in length. The cameras were controlled with a wired trigger cable and images were acquired every 5 s during breach with a synchronized remote intervalometer.

A novel strategy using dyed sand was adopted in the study to permit the extent of the erosional weir formed in the upstream dam face during breach to be identified readily in overhead images. Given that the homogeneous dam was constructed entirely from the same light colored fine sand, a thin surficial layer of sand dyed black with India ink was placed on the surface of the upstream dam slope to permit high-contrast visual identification of the erosional hydraulic control structure. The current extent of the breach at the time of every photo can therefore be defined as the boundary between the dark sand (uneroded upstream dam surface) and light sand (eroded). Additional steps were then taken to translate these measurements in pixels to standard physical units accounting for refraction through a changing depth of water. The falling height of water in the reservoir during the test results in a variable scale factor between linear distances and pixels due to refraction. A coordinate system was marked on aluminum angle brackets drilled into the flume side walls, allowing for calculation of a scale factor for each image analyzed that accounted for the differences in refraction as the reservoir water level decreased. As the primary objective of this network of cameras was to capture the instantaneous geometry of the erosional weir on the upstream face, a decision was made to not use seeding particles to calculate flow velocity using particle image velocimetry (PIV) as they would occlude the submerged erosion feature of interest.

Additional qualitative video documenting the process of dam breach was captured throughout each test with a Canon EOS Rebel T5 camera positioned downstream of the dam, oblique to the downstream face. Plan view video was recorded with a GoPro Hero 3 camera. Additional plan view video was recorded with a Canon EOS Rebel T5 camera, mounted to a tall extendable tripod, located against the exterior wall of the flume, near the dam crest.

Reservoir elevation was monitored with five Akamina AWP-24 wave capacitance height gauges, distributed longitudinally upstream of the dam along the reservoir centerline, set to a sampling frequency of 100 Hz. Discharge over the dam during breach was calculated from the change in reservoir elevation over the known reservoir
geometry. One wave height capacitance gauge was also positioned 1.0 m downstream of the dam toe to monitor tailwater submergence.

2.4. Test Procedure

A V notch approximately 6 cm wide by 6 cm deep with side slopes of 45° was incised into the center of the dam crest with a piece of aluminum angle bracket prior to filling the flume reservoir. This notch size was sufficiently small to encourage a slow initiation phase of breach, whereas a substantially larger initial notch would have resulted in a shorter initial phase. As the reservoir was filled, an in-test verification of the calibration of the capacitance wave height gauges was conducted. When the reservoir reached the elevation at which water entered the V notch to a depth of approximately 30 mm, the water supply was turned off, and data acquisition with the capacitance wave height gauges and plan view cameras was started simultaneously.

3. Observations of Dam Breach

3.1. Breach Evolution

The breach process for noncohesive earthen embankments, where overtopping is initiated with a V notch incision in the crest, has been described in the literature through the definition of three sequential stages of breach behavior (Coleman et al., 2002; Visser et al., 1995; Walder et al., 2015). These stages are explored using photographs capturing breach evolution, observed from a downstream oblique view, which are presented in Figure 4.

Stage 1, typically lasting approximately 40–85 s for the 1 m dams in the present study, consists of the initial flow through the V notch. With erosion, the V notch slowly begins to increase in size; however, since the downstream face of the dam is not fully saturated, some of the water and sediment exiting the notch infiltrates and is deposited on the downstream dam face. Eventually, as the downstream face of the dam in the vicinity of this infiltration becomes saturated, the notch outflow incises a narrow channel into the downstream face. The stage ends when the incised channel reaches the downstream toe (Figure 4a).

In Stage 2, the incised channel begins to form steps beginning at the toe of the dam, retrogressing toward the crest. The channel is widened by slumping blocks of undercut material, usually alternating from one side of the channel to the other. After approximately a duration of 60 s, the stage is defined to reach its conclusion when the stepping channel intersects with the breach crest (Figure 4b).

In Stage 3, the final stage, lasting 145–275 s duration for the dams in the present study, discharge increases rapidly, as a pathway between the stored volume in the reservoir and downstream dam toe has been opened. The breach channel widens quickly as increasing outflow continues to undercut the channel banks, and the channel is eroded both laterally and vertically. Peak discharge is reached, and as the outflow is decreasing, the channel continues to erode until the end of the breach. The end of the stage coincides with the end of the breach process (Figures 4c and 4d), defined as either the full loss of reservoir containment, or in the case of the present study, tailwater submergence as the water elevation equilibrates on both sides of the dam.

The definition of the time scale associated with dam breach is very important in practice as it has direct implications for emergency response time. In this paper, we adopt the definition of the time datum of Walder et al. (2015). Of the 11 breach tests reported by these authors, 6 replicate breach tests were used to explore the reproducibility of the time scale of the outflow hydrograph. They showed that by selecting the time datum, $T = 0$, as the onset of Stage 3, the time of rapid increase in outflow, test to test variations in outflow hydrographs were small and breach behavior could be directly compared. This approach was found to be useful in the present study as differences in the duration of morphological Stages 1 and 2, arising from intertest variability of the degree of saturation of the dam downstream face (i.e., evaporation and gravity drainage after slightly different lengths of time between construction and testing) were observed.

3.2. Outflow Hydrographs

Discharge with time throughout the breach process was calculated from reservoir elevation data and the reservoir storage function for each of the six dams. The resulting outflow hydrographs, using the onset of Stage 3 as the time datum ($T = 0$), are presented in Figure 5a. A strong relationship between upstream slope angle and the
Figure 4. Downstream isometric and plan view of dam breach morphology for a typical test. In the plan view image, the V notch is located at the bottom of the image and the upstream surface of the dam covered with a thin layer of black dyed stand. Stage 1 (a): an initial channel is incised into the downstream dam face. Stage 2 (b): the breach channel forms step like features, retrogressing backward toward the dam crest. Stage 3 onset (c): the breach channel intersects the upstream dam crest, marked by an exponential increase in outflow and the time datum $T = 0$ is set. Stage 3 (d): rate of discharge increases to a maximum value.
Figure 5. (a) Breach outflow hydrographs for each test in the series (discharge vs. time); (b) reservoir elevation versus discharge during breach for all tests in the series; and (c) peak discharge versus dam upstream slope angle. United States Geological Survey (USGS) data shown are the high and low range from six replicate tests reported by Walder et al. (2015).
outflow hydrograph is apparent in the data. Peak discharge was observed to increase as the upstream slope angle increases, ranging from 0.07 m$^3$/s for a 10.0° upstream slope to 0.24 m$^3$/s for a 30.0° upstream slope (Figure 5a). Repeatability between experiments was investigated with two replicate tests performed at each of the 18.4° and 30.0° upstream slope angles configurations. The peak discharge observed in the second replicate test was within 14% and 4% of the first, respectively. The time to reach peak discharge was observed to decrease as slope angle increases, ranging from 120 s for a 10.0° upstream slope to 75 s for 30.0° upstream slope.

The influence of the flooded tailwater on the outflow hydrograph is explored in Figure 5b, in which the reservoir elevation during Stage 3 of the breach is plotted against discharge. In each experiment, Stage 1 of the breach was initiated when the reservoir level reached the bottom of the notch (∼0.95 m) resulting in slightly different reservoir elevations at Stage 3 depending on the volume of flow required for Stage 1 and 2 erosion. Peak outflow occurred after a drop in reservoir elevation of approximately 0.2 m, with discharge then slowly reducing until an abrupt halt when water levels achieve equilibration on both sides of the dam at approximately 0.45 m elevation. These data provide evidence that peak outflow occurred prior to the influence of tailwater submergence on the outflow hydrograph.

Peak discharge observed in each experiment is plotted against upstream slope angle in Figure 5c. A strong linear relationship is observed between the four different slope angles tested in the Queen's flume, with a coefficient of determination, $r^2$, of 0.97. This finding indicates that upstream slope angle should be considered when predicting peak outflow during breach of tailings dams. For the sake of comparison, the maximum and minimum peak discharge from the series of six replicate tests performed with an upstream slope angle of 30° by Walder et al. (2015) are presented on the same figure. Although the upstream dam geometry between the USGS and Queen's experiments is identical, the storage functions of their reservoirs are not. This illustrates that upstream slope angle alone cannot fully predict peak discharge. The effect of storage function will be explored in more detail later in Section 3.7 of this paper.

### 3.3. Breach Geometry

Breach geometry throughout dam failure was measured on the plan view images captured at 5 s intervals during Stage 3 of the breach. Figure 6a shows the breach evolution captured during a typical test, in this case with an upstream slope angle of 30.0°. The breach crest, manually identified in the images, is shown as a red line corresponding to the boundary between the uneroded upstream face (black sand) and current leading edge of the
eroding beach crest (white sand). For the sake of comparison, a similar series of images for the flattest upstream slope angle tested (10.0°) is presented in Figure 6b. Qualitatively, the dam breach process of all dams from steep to shallow upstream slope angles resembled a growing semielliptical weir.

These raw measurements were then converted from pixels into physical units and used to quantify the instantaneous breach geometry including breach width, defined along the crest of the dam, $B^*$, and the breach arc length between time $T = 0$ (start of Stage 3) and the point where the breach shape was observed to be limited by the flume side walls (Figures 7a and 7b, respectively). For the period of the breach process where data were collected, the breach width and arc length growth rate can be broadly described as constant with respect to time since initiation of Stage 3.

Another measure of breach geometry is the ratio of $B/B^*$, presented in Figure 7c. The results observed at the Queen’s University laboratory were within the range reported by Al-Riffai (2014) and Walder et al. (2015) of 1.05–1.45, with the exception of the dam with an upstream slope of 10.0°, which exceeded this threshold. This observation is consistent with the rounder shape and comparatively longer breach crest length for this test, illustrated in the image sequence of Figure 6.

### 3.4. Ellipticity Index

Review of the plan view images shows that the breach shape resembles a semiellipse. This is illustrated in Figure 8a, in which the shape of the growing breach is fitted with an equation of a semiellipse. The ellipticity index has been used in the literature as a quantitative measure of how well an observed shape fits to the geometric definition of an ellipse (e.g., Taylor et al., 2018). Here, we adopt this index to quantify the degree of semiellipticity of the breach crest:

$$e_B = 1 - 2 \frac{A_B - A_{B\cdot SE}}{A_B}$$

(1)

where $e_B$ is the breach semiellipticity index, $A_B$ the original breach area of closed polygon defining breach geometry, and $A_{B\cdot SE}$ is the area of geometrical intersection between original breach shape and its idealized semielliptical shape.

In this definition of ellipticity index, a perfect fit would occur when the area of the breach exactly overlaps with the area of a semiellipse, $A_B = A_{B\cdot SE}$, so $e_B = 1$. As shown in Figure 8b, the degree of semiellipticity of the dam breaches in our study is generally greater than 0.9 at all times during breach for all test configurations. This provides tangible evidence that an equation of a semiellipse successfully captures the evolving geometry of a growing breach crest in the upstream slope as it erodes from the start of Stage 3 to peak outflow.

### 3.5. Breach Ellipse Parameters

The observation that the breach geometry is a semiellipse permits the use of mathematical approximations for the perimeter of an ellipse to permit the arc length and width of breach geometry to be linked mathematically. A review of available approximations described by Michon (2020) led to the use of the approximation formulated by Srinivasan Ramanujan in 1914 for the
Figure 8. (a) Typical evolution of upstream breach crest geometry illustrating semielliptical shape maintained over each 5 s increment to peak outflow; (b) ellipticity index of the breach crest shape, measured during each test, where $e_c = 1$ means the breach crest is perfectly semielliptical.
perimeter of an ellipse. This equation, modified by a factor of 2 to calculate the arc length of a semiellipse (breach crest arc length) is

\[ B \approx \pi \left[ 3(a + b) - \sqrt{(3a + b)(a + 3b)} \right] / 2 \]  \hspace{1cm} (2)

The ellipse shape is typically described by its major axis \( a \) and minor axis \( b \) (Figure 9). Note that the major axis \( a \), aligns with the breach width, \( B^* \), resulting in \( a = B^*/2 \). The ellipse minor axis \( b \), measures the maximum perpendicular distance from the intersection of the breach crest with the reservoir water elevation, to the intersection with the breach crest. Thus for a breach of known width and aspect ratio, Equation 2 permits the calculation of the arc length of the hydraulic control section. Furthermore, from geometry of the upstream slope and use of a simplifying assumption of a flat reservoir surface, the depth flowing over the breach crest can then be approximated at any point along the crest by

\[ d = x \times \tan \alpha \]  \hspace{1cm} (3)

where \( d \) is the depth over breach (m), \( x \) the perpendicular distance from point of water surface intersection with upstream dam face to breach crest (m), and \( \alpha \) is the dam upstream slope (°).

The evolution of breach geometry for dams of each of the upstream slope angles is presented in terms of the major axis \( a \), minor axis \( b \), aspect ratio \( a/b \), and average water depth over the breach crest in Figure 10. As a reminder, the breach crest horizontal distance, \( B^* \), equals \( 2a \), where \( a \) is the major ellipse axis (Figure 9). Ellipse parameters \( a \) and \( b \), respectively, produce largely linear trends, which is to be expected as they are a function of \( B^* \), which was shown earlier to increase linearly with time. The aspect ratio, \( a/b \), describes the proportional relationship between the major and minor axes of the ellipse. Data of the aspect ratio observed in dams of each upstream slope angle are presented in Figure 10c from the onset of Stage 3 until the breach width growth was
limited by the 2.09 m width of the flume. The ratio of \(a/b\) ranged from about 0.9 to 2.2 across all tests, with a mean value of 1.43 ± 0.28. In this work, the simplifying assumption of a constant aspect ratio of the breach event will be used, implying that as a breach crest grows larger, it does so proportionately along both axes. Finally, the average depth flowing over the breach crest throughout the breach event, calculated with Equation 3, is shown in Figure 10d. Following Equation 3, the upstream slope angle controls the available depth of water over the breach crest hydraulic control structure, with smaller average flow depths for shallower upstream slopes. Considering this structure to act as a weir, the upstream slope angle therefore directly impacts the discharge. This will be explored quantitatively next using the concept of a rating curve.

3.6. Rating Curve

In hydraulics, a rating curve communicates the integrated effect of stream geometry and hydraulic parameters, converting flow height to volume (e.g., Chubak & McGinn, 2002).

The discharge equation to describe flow over the breach crest employed by Walder et al. (2015) was

\[
Q = C_d \left( \frac{2}{3} \right)^{5/2} g^{1/2} B < E >^{3/2}
\]  

(4)
where $Q$ is the discharge ($\text{m}^3/\text{s}$), $C_d$ the discharge coefficient (dimensionless), $g$ the acceleration due to gravity ($\text{m}/\text{s}^2$), $B$ the breach crest arc length ($\text{m}$), and $<E>$ is the mean specific energy ($\text{m}$).

At an average depth along the breach crest, $d_{avg}$, $<E>$ can be written as

$$<E> = \frac{v^2}{2g} + d_{avg}$$  \hspace{1cm} (5)

where $v$ is the mean flow velocity ($\text{m}/\text{s}$) and $d_{avg}$ is the average depth flowing over breach crest ($\text{m}$).

The Froude number, $Fr$, is a hydraulic relationship of the ratio of gravitational forces to inertial forces (e.g., Yalin & da Silva, 2001) and provides a method of classifying flow conditions, defined as

$$Fr = \frac{v}{\sqrt{g \times d_{avg}}}$$  \hspace{1cm} (6)

The breach crest is a location of critical flow, separating upstream subcritical flow and downstream supercritical flow. However, as discussed by Walder et al. (2015), due to the strong curvature of the streamlines at the breach crest, relationships for steady, uniform, rectilinear flow cannot be applied to the elliptically shaped weir; specifically, the critical Froude number does not equal 1 at this location (Castro-Orgaz & Chanson, 2009). Walder et al. (2015) calculated the critical Froude number at the breach crest to be $0.74 \pm 0.14$ from their laboratory data. In other words, the Froude number will equal 1 slightly downstream of the breach crest location.

The flow velocity was not measured during this experiment as full-field PIV conducted on images of the reservoir surface would have required dense seeding of particles that would have occluded the breach geometry that was the primary focus of this study. Therefore, a Froude number at the breach crest of $Fr = 0.74$ was adopted based on the work by Walder et al. (2015), due to comparable dam dimensions and materials. $Fr = 0.74$ can be substituted into Equation 5 to solve for the average flow velocity, and Equation 6 can be rewritten as

$$<E> = 1.274d_{avg}$$  \hspace{1cm} (7)

A rating curve has been developed for this test series, comparing measured outflow to the outflow calculated utilizing Equation 4, without a discharge coefficient value (Figure 11). This plot also includes data collected by Coleman et al. (2002) and Walder et al. (2015) for context. This comparison illustrates that a similar rating curve was obtained in the present work and these past studies. Slightly more variation is observed in the Queen’s experiments, with two of the replicate tests falling slightly below the relationship observed for the other testing scenarios. These two tests correspond to the longest duration of impoundment prior to testing—indicating that future work should investigate the mechanism for this variation (i.e., possible changes to the upstream geometry due to wetting collapse of the unsaturated sand comprising the dam, or other possible mechanisms). For all other observations, the slope of the rating curve, equal to the discharge coefficient, $C_d$, is remarkably consistent. $C_d$ can be thought of as a correction factor, which accounts for observed differences between discharge measured in the laboratory and the theoretical discharge calculated with the discharge equation. The $C_d$ value for this test series was found to be 0.98 (exclusive of the two outlying replicate tests), close to the value of $C_d = 0.96$ reported by Walder et al. (2015). The rating curve analysis suggests that the outflow equation and associated assumption of Froude number is a reasonable representation of the outflow over an elliptically shaped breach crest.

### 3.7. Peak Outflow Calculation

The elliptical weir framework explored in this manuscript suggests that discharge up to peak outflow could be estimated if one adopts a series of simplifying assumptions of (a) linear growth of breach width with time (a parameterization that substitutes for a description of the processes by which sand grains are mobilized and entrained by the water flow); (b) arc length and average depth calculated considering a semielliptical breach geometry of constant aspect ratio; (c) rating curve quantifying flow over the weir; and, finally, (d) a method for quantitative inclusion of the reservoir storage function and an end-condition identifying the time of peak flow.

This approach is explored in a time-stepping model in which breach width is linearly increased with time for the breach events conducted in the Queen’s and USGS flumes (Figure 12). For both scenarios, the breach at the
onset of Stage 3 and growth rate with time were based on a linear regression of $B^*$ versus time, as shown by the dashed line on Figure 7a. Since the breach crest was found to be elliptical, the ellipse aspect ratio can be used to calculate the breach crest arc length, $B$, based on the horizontal breach crest width, $B^*$, and the aspect ratio, $a/b$. The mean aspect ratio, $a/b = 1.43$ has been used in this analysis. Next, the depth flowing over the breach crest can be calculated based on the upstream slope angle. Finally, incremental discharge can be calculated with the discharge equation and $C_d = 0.98$ from the rating curve in Figure 11. At each timestep, the outflow through the breach was then used to estimate the increment of depletion of the reservoir volume. The shape of the reservoir storage functions for the two flumes is presented in Figure 13, illustrating the total volume available for release during breach was about 13 m$^3$ at the USGS facility, compared to roughly 16 m$^3$ at the Queen's facility. As will be detailed below, the available reservoir volume is critical in determination of peak discharge, as expected from empirical relationships derived from field events (e.g., Evans, 1986). This updated reservoir elevation is then used in the next time step.

With a monotonically increasing breach width, discharge will increase indefinitely with time until fully eroded if left unconstrained by the reservoir volume. For a finite reservoir, an endpoint therefore needs to be included in the calculation to identify the point at which the reservoir is unable to meet the discharge demand—which is taken here to define the time of peak discharge. At the start of each time step of the calculation, a check was made to see if the reservoir elevation is sufficiently high to permit the full height of flow over the new, larger width of the weir. The first instance where this condition cannot be made is deemed to be the point of peak discharge. Given that further time steps of the breach will no longer hold to the initial assumption of a monotonically increasing breach width, this point marks the logical end point of the simplified calculation.

Comparisons of the rising limb of the outflow hydrograph calculated with the elliptical framework to the observations of the Queen's and USGS flumes are presented in Figures 14a and 14b, respectively. These comparisons are made for dams of 30° upstream slope angle and indicate that the simplified method provides a reasonable visual match to the outflow hydrographs observed in both facilities, and in particular, illustrate how the simplified method can account for different reservoir storage functions. However, for the scenario of breach of tailings dams, the most significant aspect of this simplified model is the inherent inclusion of upstream slope angle in

![Figure 11. Rating curve calculated utilizing the same weir rating curve equation presented by Walder et al. (2015). The gray points show data reported by Walder et al. (2015) and Coleman et al. (2002).]
the calculation of discharge. The time step model was therefore run for each of the four slope angles investigated in the present study to calculate the peak discharge from the model. As shown in Figure 14c, this model with simplifying assumptions of a constant breach growth rate and elliptical aspect ratio for all tests was sufficient to capture the effect of upstream slope angle on the peak outflow observed during breach for the range of 10°–30° upstream slope angles investigated in this study.

Postpeak discharge behavior has not been calculated in this study due to the limitation of the flume width. Generally, breach behavior transitioned from 3D to 2D soon after peak discharge was reached, and data collection stopped. As such, the 3D breach crest width growth rate behavior postpeak discharge remains an area of future study.

4. Conclusions

The objective of this study was to use detailed observations of breach initiated by overtopping in large physical model dams of varying upstream slope angles ranging from steep (e.g., representative of clear water dams) to shallow (e.g., tailings dams) to define, for the first time, the effect of the upstream slope angle on the peak

Figure 12. Calculation of peak outflow using the semiellipse calculation framework assuming a monotonically increasing breach width and constant aspect ratio. For given breach scenario (e.g., a breach of width $X$ evolves in $Y$ minutes), this framework directly includes the effect of upstream slope angle, $\alpha$, to enable comparisons to the physical model experiments in the differing reservoir volumes tested in the Queen’s and USGS flumes.
breach discharge for a finite reservoir boundary condition. Six large dams, 1 m in height, were constructed with upstream slope angles ranging from 10.0° to 30.0° from horizontal and brought to failure by V-notch overtopping using a reservoir of 16 m$^3$. A strong relationship between upstream slope angle and the resulting outflow hydrograph during breach was observed. Peak discharge was observed to increase as the upstream slope angle increases, ranging from 0.07 m$^3$/s for a 10.0° upstream slope to 0.24 m$^3$/s for a 30.0°. A novel strategy using dyed sand was adopted in the study to permit the extent of the erosional weir formed in the upstream dam face during breach to be identified readily in overhead images. Analysis of this image data indicates that a semiellipse is an excellent approximation of the geometry of the breach on the rising limb of the hydrograph to peak outflow. This observation permitted the use of the Ramanujan approximation for the perimeter of an ellipse to define a mathematical relationship to link breach width and arc length. A simplified method to predict the rising limb of the outflow hydrograph was then proposed based on the assumption of (a) linear growth of breach width with time; (b) arc length and average depth calculated considering a semielliptical breach geometry of constant aspect ratio; (c) rating curve quantifying flow over the weir; and, finally, (d) a method for quantitative inclusion of the reservoir storage function and an end-condition identifying the time of peak flow. For the scenario of tailings dam breach, the most significant aspect of this simplified model is the inherent inclusion of upstream slope angle in the calculation of discharge, sufficient to capture the effect of upstream slope angle on the peak outflow observed during breach for the range of 10°–30° upstream slope angles investigated in this study.

Tailings dams differ from typical water-retaining dams in design (e.g., upstream, central, downstream construction), materials retained behind the dam and used in construction of the dam, and the inclination of the upstream face of the dam. In this manuscript, we have isolated solely the variable of upstream inclination. This work indicates that the upstream slope angle plays a significant role in dam breach through the definition of the height of flow over the erosional breach crest hydraulic control structure that forms in the upstream face of the dam. Estimates of peak outflow for breach of tailings dams (and other breach events such as landslide dams) should therefore consider the effect of upstream slope angle in the generation of the outflow hydrograph used in dam breach analyses conducted to limit hazard exposure and potential for loss of life imposed by dams by informing dam consequence rating, flood-routing analyses, land use planning, flood mitigation strategies,
and emergency response plans. For empirically based models for prediction of peak outflow, this means that accuracy of predictions could be improved by considering only cases of similar upstream slope angle. For other physically based models for prediction of the outflow hydrograph, the data set provided in this manuscript provides experimental observations to assess the particular model's ability to capture the effect of upstream slope angle for tailings dam geometry. Finally, while the present study addressed a primary variable defining differences in breach behavior for tailings dams, additional research is needed to further explore other aspects of tailings dams that influence dam breach that were not studied in the present study. These factors include construction method (i.e., dam cross section and materials), more realistic tailings materials, potential for liquefaction and entrainment of retained tailings, differences arising from other failure mechanisms such as shear failure of the downstream face, and the additional geometric control of a low angle (i.e., nearly horizontal) tailings beach retained behind the dam.

**Data Availability Statement**

The data used in this research are archived and freely accessible for download from the CanBreach Data Archive hosted by Scholars Portal Dataverse (Walsh et al., 2021).
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