On Generalized Interval Valued Fuzzy Soft Matrices

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Abstract: Interval valued fuzzy soft set was a combination of the interval valued fuzzy set and soft set, while in generalized interval valued fuzzy soft set a degree was attached with the parameterization of fuzzy sets in defining an interval valued fuzzy soft set. In this paper we introduced the concept of generalized interval valued fuzzy soft matrices. We discussed some of its types and some operations. We also discussed about the similarity of two generalized interval valued fuzzy soft matrices.

Keywords: Interval Valued Fuzzy Soft Set (IVFSS), Generalized Fuzzy Soft Set, Generalized IVFSS, Similarity

1. Introduction

A lot of problems in our real life in economics, social sciences, medical sciences, environmental sciences and engineering etc. involve various uncertainties. Many theories have been developed to deal with these uncertainties. Some of these theories are probability theory [1], fuzzy set theory (FST) [2], rough set theory (RST) [3], interval mathematics [4] and intuitionistic fuzzy set theory (IFST) [5] etc. Molodtsov [6] pointed out that all these theories have some inherent difficulties. He proposed soft set theory (SST) to overcome these difficulties. It was a generic mathematical tool for dealing problems having uncertainty. Later Maji and Biswas [7] defined soft subset and soft super set. They also defined absolute soft set and null soft set. They introduced some operations on soft set and De Morgan’s laws are also verified by them. Ali et al [8] pointed out some errors of the previous work and introduced some new operations. They further studied more and discussed some algebraic structures of soft sets. Maji et al. [9] proposed fuzzy soft set (FSS), an improvement of the SST by combining (FST) and (SST). Roy and Maji [10] gave an application of fuzzy soft set in decision making. Yang et al. [11] introduced the interval-valued fuzzy soft set (IVFSS) which was a combination of the IVFS and SST.

Majumdar and Samanta [12] introduced the concept of generalized fuzzy soft sets (GFSS). B. K. Saikia et al. [13] defined generalized fuzzy soft matrix (GFSM) and applied it to a decision making (DM) problem. Shawkat Alkhazaleh and Abdul Razak Salleh [14] introduced generalized interval valued fuzzy soft set (GIVFSS). In their generalization of FSS, they attached a degree with the parameterization of fuzzy sets in defining an IVFSS. They discussed various operations and properties of GIVFSS. Some of these are GIVFSS subset, GIVFSS equal set, generalized null interval valued fuzzy soft set (GNIVFS), generalized absolute interval valued fuzzy soft set (GAIVFS), compliment of GIVFSS, union of GIVFSS’s and intersection of GIVFSS’s. They defined AND and OR operations on GIVFSS and similarity measure of two GIVFSS’s. They also give some applications of GIVFSS in DM problem and medical diagnosis.

Mi Jung Son [15] introduced interval valued fuzzy soft set and defined some of its types. P. Rajarajeswari and P. Dhanalakshmi [16] developed interval-valued fuzzy soft matrix theory. Zulqarnain. M and M. Saeed [17] defined some new types of interval valued fuzzy soft matrix and gave an application of IVFSM in a decision making problem. Anjan Mukherjee and Sadhan Sarkar [18, 19] introduced Similarity measures for interval-valued intuitionistic fuzzy soft sets and gave applications in medical diagnosis problems. B. Chetia and P. K. Das [20] used interval-valued fuzzy soft sets and Sanchez’s approach for medical diagnosis. In recent years many researchers [21-25] have been worked on applications of interval valued fuzzy soft sets.

In this paper we extended the concept of GIVFSS and introduced generalized interval valued fuzzy soft matrix (GIVFSM). We defined different types of GIVFSM’s and
studied some properties. We also discussed some operators on the basis of weights and some of their properties.

2. Some Basic Definitions

Definition 2.1. [1] Let X be a universal set, P be set of parameters and \( I^P \) be the set of all fuzzy subsets of X. Let \( A \subseteq P \). A pair \((F, A)\) is a fuzzy soft set over X, where \( F : A \rightarrow I^P \).

Definition 2. [11] An IVFSS \( X \) on a universe \( U \) is a mapping such that \( X : U \rightarrow \text{Int}(0,1] \), where \( \text{Int}(0,1] \) represents the set of all closed subintervals of \([0,1)\), the set of all interval valued fuzzy sets on \( U \) is denoted by \( P(U) \). Suppose that \( X \in P(U) \), \( \forall x \in U \).

Let \( \mu_x(x) = [\mu^-_x(x), \mu^+_x(x)] \) is the degree of membership of \( x \) to \( X \), where \( \mu^-_x(x) \) and \( \mu^+_x(x) \) are the lower and upper degrees of membership of \( x \) to \( X \) respectively, such that

\[
0 \leq \mu^-_x(x) \leq \mu^+_x(x) \leq 1
\]

Definition 2.3. [26] Let \( X \) be the universal set and \( P \) be the set of parameters. Suppose that \( A \subseteq P \) and \((F, A)\) be a fuzzy soft set. Then the matrix form of the fuzzy soft set \((F, A)\) is given as

\[
A = [a_{ij}]_{m \times n}, \quad i = 1, 2, 3, \ldots, m \quad \text{and} \quad j = 1, 2, 3, \ldots, n
\]

where

\[
a_{ij} = \begin{cases} (u_j(p_i), p_j \in A) \\ 0, p_j \notin A \end{cases}
\]

Definition 2.4. [13] Let \( X \) be the universal set, \( P \) be the set of parameters and \( A \subseteq P \). Let \((F_A, p)\) be a GFSS over \((X, P)\).

A subset of \( X \times P \), \( R_A = \{(x, p), p \in P, x \in F_A (p)\} \) is a relation form of \((F_A, p)\), where

\[
\mu_{R_A} : X \times P \rightarrow [0,1] \quad \text{and} \quad \lambda_{R_A} : X \times P \rightarrow [0,1],
\]

such that

\[
\mu_{R_A} : (x, p) \in [0,1], \forall x \in X, p \in P
\]

and

\[
\lambda_{R_A} : (x, p) \in [0,1], \forall x \in X, p \in P.
\]

If \([\mu_{ij}, \lambda_{ij}]_{m \times n} = \mu_{R_A} ((x_i, p_j), \lambda (x_i, p_j))\), then we can define an \( m \times n \) generalized fuzzy soft matrix (GFSM) of GFSS over \((X, P)\) as

\[
[\mu_{ij}, \lambda_{ij}]_{m \times n} =
\]

\[
\begin{bmatrix}
(\mu_{11}, \lambda_{11}) & (\mu_{12}, \lambda_{12}) & \cdots & (\mu_{1n}, \lambda_{1n}) \\
(\mu_{21}, \lambda_{21}) & (\mu_{22}, \lambda_{22}) & \cdots & (\mu_{2n}, \lambda_{2n}) \\
\vdots & \vdots & \ddots & \vdots \\
(\mu_{m1}, \lambda_{m1}) & (\mu_{m2}, \lambda_{m2}) & \cdots & (\mu_{mn}, \lambda_{mn})
\end{bmatrix}
\]

Definition 2.5. [16] Let \( U = \{c_1, c_2, c_3, \ldots, c_m\} \) be the Universe set and \( E \) be the set of parameters given by \( E = \{e_1, e_2, e_3, \ldots, e_q\} \). Let \( A \) be a subset of \( E \) and \((F, A)\) be an interval valued fuzzy soft set over \( U \) and \( F \) is a mapping given by \( F : A \rightarrow I^U \), where \( I^U \) denotes the collection of all Interval valued fuzzy subsets of \( U \). Then the Interval valued fuzzy soft set can be expressed in matrix form as

\[
A = [a_{ij}]_{m \times n} \quad \text{or} \quad \bar{A} = [a_{ij}]_{m \times n}
\]

where

\[
a_{ij} = \begin{cases} \mu_{ij}(e_j) \quad \text{if} \ e_j \in A \\ 0 \quad \text{if} \ e_j \notin A \end{cases}
\]

The interval \([\mu_{ij}(e_j), \mu_{ij}(c_i)]\) represents the membership of \( c_i \) in the interval valued fuzzy set \( F(e_j) \).

If \( \mu_{ij}(c_i) = \mu_{ij}(c_i) \) then the Interval-valued fuzzy soft matrix (IVFSM) reduces to a FSM.

Definition 2.6. [11] Let \( U \) be an initial Universe set and \( E \) be the set of parameters, let \( A \subseteq E \). A pair \((F, A)\) is called Interval valued fuzzy soft set over \( U \) where \( F \) is a mapping given by \( F : A \rightarrow I^U \), where \( I^U \) denotes the collection of all Interval valued fuzzy subsets of \( U \).

Definition 2.7. [14] Let \( U \) be the Universal set and \( E \) be the set of parameters. Let \( A \subseteq E \) and \( \mu \) be a fuzzy subset of \( A \). Let \( F : A \rightarrow \mathcal{P}(U) \) and \( \mu : A \rightarrow [0,1] \) where \( \mathcal{P}(U) \) is the collection of all Interval valued fuzzy subsets of \( U \).

Define a function \( F_{\mu} : A \rightarrow \mathcal{P}(U) \times I \), such that

\[
F_{\mu}(e) = F(e) = (F(e), \mu(e))
\]

where \( F(e)(x) \) is an interval value and is called degree of membership of an element \( x \) to \( F(e) \) and \( \mu(e) \) is called the degree of possibility of this belongingness. Then \( F_{\mu} \) is a GIVFSS.

3. Generalized Interval Valued Fuzzy Soft Matrices

Definition 3.1 Let \( U \) be the Universal set and \( E \) be the set of parameters. Let \( A \subseteq E \) and \( \mu \) be a fuzzy subset of \( A \). Let \( F : A \rightarrow \mathcal{P}(U) \) and \( \mu : A \rightarrow [0,1] \) where \( \mathcal{P}(U) \) is the collection of all Interval valued fuzzy subsets of \( U \). A function \( F_{\mu} : A \rightarrow \mathcal{P}(U) \times I \) defined as

\[
F_{\mu}(e) = F(e) = (F(e), \mu(e)), \quad \text{where} \quad F_{\mu}(e) = (F(e)(x), \mu(e)), \forall e \in E
\]

Then the generalized interval valued fuzzy soft set \( F_{\mu} \) can be expressed in matrix form as

\[
(F_{\mu}(e_i), \mu) = [a_{ij}]_{m \times n}
\]

where

\[
a_{ij} = \begin{cases} (F(e_j)(x_i), \mu(e_j)) \quad \text{if} \ e_j \in A \text{ and } i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \\ (0,0) \quad \text{if} \ e_j \notin A \text{ and } i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \end{cases}
\]

i.e. \([a_{ij}]_{m \times n} = \)
Example 3.3. Consider a set of three motorbikes $U = \{b_1, b_2, b_3\}$ and a set of parameters, 

$E = \{e_1, e_2, e_3\}$, where $e_1, e_2$ and $e_3$ stands for cheap, expansive and comfortable respectively. The GFSS’s $F_\mu$ and $G_\lambda$ are defined as

$$F_\mu(e_1) = \begin{pmatrix} 0.2,0.4 \end{pmatrix}, \begin{pmatrix} 0.3,0.5 \end{pmatrix}, \begin{pmatrix} 0.1,0.2 \end{pmatrix}$$

$$F_\mu(e_2) = \begin{pmatrix} 0.2,0.4 \end{pmatrix}, \begin{pmatrix} 0.3,0.5 \end{pmatrix}, \begin{pmatrix} 0.1,0.2 \end{pmatrix}$$

$$F_\mu(e_3) = \begin{pmatrix} 0.2,0.4 \end{pmatrix}, \begin{pmatrix} 0.3,0.5 \end{pmatrix}, \begin{pmatrix} 0.1,0.2 \end{pmatrix}$$

and

$$G_\lambda(e_1) = \begin{pmatrix} 0.3,0.5 \end{pmatrix}, \begin{pmatrix} 0.2,0.4 \end{pmatrix}, \begin{pmatrix} 0.1,0.2 \end{pmatrix}$$

$$G_\lambda(e_2) = \begin{pmatrix} 0.2,0.4 \end{pmatrix}, \begin{pmatrix} 0.3,0.5 \end{pmatrix}, \begin{pmatrix} 0.1,0.2 \end{pmatrix}$$

$$G_\lambda(e_3) = \begin{pmatrix} 0.2,0.4 \end{pmatrix}, \begin{pmatrix} 0.3,0.5 \end{pmatrix}, \begin{pmatrix} 0.1,0.2 \end{pmatrix}$$

Then the matrix representation of $F_\mu$ and $G_\lambda$ are given as

$$F_\mu = \begin{bmatrix} [0.1,0.4] & [0.4,0.7] & [0.2,0.4] \\ [0.3,0.5] & [0.5,0.8] & [0.3,0.6] \\ [0.4,0.5] & [0.2,0.4] & [0.3,0.6] \end{bmatrix}$$

$$G_\lambda = \begin{bmatrix} [0.1,0.4] & [0.4,0.7] & [0.2,0.4] \\ [0.3,0.5] & [0.5,0.8] & [0.3,0.6] \\ [0.4,0.5] & [0.2,0.4] & [0.3,0.6] \end{bmatrix}$$

It is clear that $F_\mu$ is GIVFS sub matrix of $G_\lambda$.

Definition 3.4. Let $F_\mu$ and $G_\lambda$ be two GIVFSM’s. Then $F_\mu$ is called GIVFS equal matrix of $G_\lambda$ if

$$F_\mu(e_i) = G_\lambda(e_i), \mu(e_i) = \lambda(e_i), \forall i = 1,2,...,m$$

and $j, k = 1,2,...,n$.

Definition 3.5. Let $F_\mu$ and $G_\lambda$ be two GIVFSM’s. Then $F_\mu$ is called GIVFS sub matrix of $G_\lambda$ if

$$F_\mu(e_i) \leq G_\lambda(e_i), \mu(e_i) \leq \lambda(e_i), \forall e_i, F_\mu(e_i) < G_\lambda(e_i),$$

and $\mu(e_i) < \lambda(e_i)$, for at least one $e_i$.

Definition 3.6. Let $F_\mu$ and $G_\lambda$ be two GIVFSM’s. Then $F_\mu$ is called GIVFS sub matrix of $G_\lambda$ if

$$F_\mu(e_i) < G_\lambda(e_i), \mu(e_i) < \lambda(e_i), \forall i = 1,2,...,m$$

and $j, k = 1,2,...,n$.

Definition 3.7. A GIVFS $F_\mu$ is called GIVFS rectangular matrix if

$$F_\mu = [F_\mu(e_i), \mu(e_i)]_{m \times n}, m \neq n, \forall i, j$$

Definition 3.8. A GIVFS $F_\mu$ is called GIVFS square matrix if

$$F_\mu = [F_\mu(e_i), \mu(e_i)]_{m \times m}, m = n, \forall i, j$$

Definition 3.9. A GIVFS $F_\mu$ is called GIVFS diagonal matrix if

$$F_\mu = [F_\mu(e_i), \mu(e_i)]_{m \times m}, m = n \text{ and } F_\mu = 0, \forall i \neq j$$

Definition 10. A GIVFS $F_\mu$ is called GIVFS scalar matrix if

$$F_\mu = [F_\mu(e_i), \mu(e_i)]_{m \times n} \text{ and } (F_\mu(e_i), \mu(e_i)) = (k, \mu), \forall i, j$$

Definition 11. A GIVFS $F_\mu$ is called GIVFS row matrix if

$$F_\mu = [F_\mu(e_i), \mu(e_i)]_{m \times n} \text{ and } m = 1, \forall i, j$$

Definition 12. A GIVFS $F_\mu$ is called GIVFS column matrix if

$$F_\mu = [F_\mu(e_i), \mu(e_i)]_{m \times n} \text{ and } n = 1, \forall i, j$$

Definition 13. Let $F_\mu$ be a GIVFSM, then scalar multiple of $F_\mu$ by a scalar $k$ is defined as

$$kF_\mu = [kF_\mu(e_i), k\mu(e_i)]_{m \times n}, \forall i, j \text{ and } 0 \leq k \leq 1$$

Definition 14. A GIVFS $A_\mu$ is called generalized absolute IVFSM, if

$$A_\mu = [A_\mu(e_i), \mu(e_i)]_{m \times n}, \text{ where } A_\mu(e_i) = 1 \text{ and } \mu(e_i) = 1, \forall i, j$$

Definition 15. A GIVFS $\phi_\mu$ is called generalized null IVFSM, if

$$\phi_\mu = [\phi_\mu(e_i), \mu(e_i)]_{m \times n}, \text{ where } \phi_\mu(e_i) = 0 \text{ and } \mu(e_i) = 0, \forall i, j$$

4. Some Operations on GIVFSM’s

Definition 1. Let $F_\mu = [F_\mu(e_i), \mu(e_i)]_{m \times n}$ be a GIVFSM, where

$$F_\mu(e_i) = [F_\mu(e_i), F_\mu(e_i)]_{m \times n}, \text{ then compliment of } F_\mu \text{ is denoted by } F_\mu^c = G_\lambda \text{ and is given by }$$

$$F_\mu^c = G_\lambda = \begin{bmatrix} 1 - F_\mu(e_i), 1 - F_\mu(e_i), 1 - \mu(e_i) \end{bmatrix}, \forall i, j.$$
Example 4.2. Consider a GIVFSM $F_{\mu}$ as in example 3.3,

\[
F_{\mu} = \begin{bmatrix}
(0.1,0.4),0.3 & (0.4,0.7),0.3 & (0.2,0.4),0.3 \\
(0.2,0.3),0.4 & (0.3,0.4),0.4 & (0.2,0.5),0.4 \\
(0.3,0.5),0.1 & (0.1,0.2),0.1 & (0.0,2),0.1 \\
\end{bmatrix}
\]

Then compliment of $F_{\mu}$ is given by

\[
F_{\mu}^c = \begin{bmatrix}
(0.6,0.9),0.7 & (0.3,0.6),0.7 & (0.6,0.8),0.7 \\
(0.7,0.8),0.6 & (0.7,1),0.6 & (0.5,0.8),0.6 \\
(0.5,0.7),0.9 & (0.8,0.9),0.9 & (0.8,1),0.1 \\
\end{bmatrix}
\]

Proposition 4.3. Let $F_{\mu}$ be a GIVFSM, then $(F_{\mu}^c)^c = F_{\mu}$.

Proof: From Definition, we have

\[
F_{\mu} \cup F_{\mu} = F_{\mu}
\]

Proposition 4.4. The union of two GIVFSM’s $F_{\mu} = [F(e_j(x_i), \mu(e_j))]_{m \times n}$ and $G_{\lambda} = [G(e_x(x_i), \lambda(e_x))]_{m \times n}$, denoted by $F_{\mu} \cup G_{\lambda}$ is a GIVFSM $[H(e_x(x_i), \gamma(e_x))]_{m \times n}$ such that

\[
H(e_x(x_i), \gamma(e_x)) = \sup(F(e_x(x_i)), G(e_x(x_i))) \cup \inf(F(e_x(x_i)), G(e_x(x_i))]
\]

Example 4.5. Consider two GIVFSM’s $F_{\mu}$ and $G_{\lambda}$ as in example 3.3,

\[
F_{\mu} = \begin{bmatrix}
(0.1,0.4),0.3 & (0.4,0.7),0.3 & (0.2,0.4),0.3 \\
(0.2,0.3),0.4 & (0.3,0.4),0.4 & (0.2,0.5),0.4 \\
(0.3,0.5),0.1 & (0.1,0.2),0.1 & (0.0,2),0.1 \\
\end{bmatrix}
\]

and

\[
G_{\lambda} = \begin{bmatrix}
(0.3,0.5),0.4 & (0.5,0.8),0.4 & (0.3,0.6),0.4 \\
(0.4,0.5),0.5 & (0.2,0.4),0.5 & (0.4,0.6),0.5 \\
(0.5,0.7),0.3 & (0.3,0.5),0.3 & (0.1,0.3),0.3 \\
\end{bmatrix}
\]

Then the union of $F_{\mu}$ and $G_{\lambda}$ is given by

\[
F_{\mu} \cup G_{\lambda} = \begin{bmatrix}
(0.4,0.5),0.4 & (0.5,0.8),0.4 & (0.3,0.6),0.4 \\
(0.4,0.5),0.5 & (0.2,0.4),0.5 & (0.4,0.6),0.5 \\
(0.5,0.7),0.3 & (0.3,0.5),0.3 & (0.1,0.3),0.3 \\
\end{bmatrix}
\]

Proposition 4.6. Let $F_{\mu}$ be a GIVFSM, then

\[
F_{\mu} \cup F_{\mu} = F_{\mu}
\]

Proof: From Definition, we have

\[
[F(e_j(x_i), \mu(e_j))]_{m \times n} \cup [F(e_j(x_i), \mu(e_j))]_{m \times n}
\]

Such that

\[
[F(e_j(x_i), \mu(e_j))]_{m \times n}
\]

\[
= [F(e_j(x_i), \mu(e_j))]_{m \times n}
\]

Proposition 4.7. Let $F_{\mu} = [F(e_j(x_i), \mu(e_j))]_{m \times n}$ and $G_{\lambda} = [G(e_x(x_i), \lambda(e_x))]_{m \times n}$ be two GIVFSM’s, then

a) $F_{\mu} \cup G_{\lambda} = G_{\lambda} \cup F_{\mu}$

b) $F_{\mu} \cap A_{\mu} = A_{\mu}$

c) $F_{\mu} \cup \phi_{\mu} = F_{\mu}$

Proof:

(a) From Definition, we have

\[
[F(e_j(x_i), \mu(e_j))]_{m \times n} \cup [G(e_x(x_i), \lambda(e_x))]_{m \times n}
\]

Such that

\[
[H(e_x(x_i), \gamma(e_x))]_{m \times n}
\]

But $H_{\gamma} = F_{\mu} \cup G_{\lambda} = G_{\lambda} \cup F_{\mu}$ (since union of GIVFSM’s is commutative)

and $\gamma(e_x) = s(\mu(e_j), \lambda(e_x)) = s(\lambda(e_x), \mu(e_j))$, (since $s = norm$ is commutative)

Then,

\[
F_{\mu} \cup G_{\lambda} = G_{\lambda} \cup F_{\mu}
\]

The proof of (b) and (c) are straight forward from definition.

Definition 4.8. The intersection of two GIVFSM’s $F_{\mu} = [F(e_j(x_i), \mu(e_j))]_{m \times n}$ and $G_{\lambda} = [G(e_x(x_i), \lambda(e_x))]_{m \times n}$, denoted by $F_{\mu} \cap G_{\lambda}$ is a GIVFSM $[H(e_x(x_i), \gamma(e_x))]_{m \times n}$ such that

\[
[H(e_x(x_i), \gamma(e_x))]_{m \times n} = \inf(F(e_x(x_i)), G(e_x(x_i))) \cup \sup(F(e_x(x_i)), G(e_x(x_i))]
\]

Example 4.9. Consider two GIVFSM’s $F_{\mu}$ and $G_{\lambda}$ as in example 3.3,

\[
F_{\mu} = \begin{bmatrix}
(0.1,0.4),0.3 & (0.4,0.7),0.3 & (0.2,0.4),0.3 \\
(0.2,0.3),0.4 & (0.3,0.4),0.4 & (0.2,0.5),0.4 \\
(0.3,0.5),0.1 & (0.1,0.2),0.1 & (0.0,2),0.1 \\
\end{bmatrix}
\]

and

\[
G_{\lambda} = \begin{bmatrix}
(0.3,0.5),0.4 & (0.5,0.8),0.4 & (0.3,0.6),0.4 \\
(0.4,0.5),0.5 & (0.2,0.4),0.5 & (0.4,0.6),0.5 \\
(0.5,0.7),0.3 & (0.3,0.5),0.3 & (0.1,0.3),0.3 \\
\end{bmatrix}
\]
and
\[
G_A = \begin{bmatrix}
(0.3, 0.5, 0.4) & (0.5, 0.8, 0.4) & (0.3, 0.6, 0.4) \\
(0.4, 0.5, 0.5) & (0.2, 0.4, 0.5) & (0.4, 0.6, 0.5) \\
(0.5, 0.7, 0.3) & (0.3, 0.5, 0.3) & (0.1, 0.3, 0.3)
\end{bmatrix}
\]

Then the intersection of \( F_\mu \) and \( G_A \) is given by
\[
F_\mu \cap G_A = \begin{bmatrix}
(0.1, 0.4, 0.3) & (0.4, 0.7, 0.3) & (0.2, 0.4, 0.3) \\
(0.2, 0.3, 0.4) & (0.3, 0.3, 0.4) & (0.2, 0.5, 0.4) \\
(0.3, 0.5, 0.1) & (0.1, 0.2, 0.1) & (0.2, 0.2, 0.1)
\end{bmatrix}
\]

Proposition 4.10. Let \( F_\mu \) be a GIVFSM, then
\[
F_\mu \cap F_\mu = F_\mu
\]
Proof: From Definition, we have
\[
[ F(e_\mu)(x_i), \mu(e_\mu) ]_{m \times n} \cap [ F(e_\mu)(x_i), \mu(e_\mu) ]_{m \times n} = [ H(e_\mu)(x_i), \gamma(e_\mu) ]_{m \times n}
\]

such that
\[
[ H(e_\mu)(x_i), \gamma(e_\mu) ]_{m \times n}
= \inf[ F(e_{\mu_l}), F(e_{\mu_j}) ]_{m \times n}
\]

Proposition 4.11. Let \( F_\mu = [ F(e_\mu)(x_i), \mu(e_\mu) ]_{m \times n} \) and \( G_A = [ G(e_\mu)(x_i), \lambda(e_\mu) ]_{m \times n} \) be two GIVFSM's, then
(a) \( F_\mu \cap G_A = G_A \cap F_\mu \)
Proof:
(a) Consider \( F_\mu \cup G_A = ( [ F(e_{\mu_l})(x_i), F(e_{\mu_l})(x_i) ], 1 - \mu_\mu(e_\mu) ) \cup ( [ G(e_{\mu_l})(x_i), G(e_{\mu_l})(x_i) ], 1 - \lambda(e_\mu) )
= \inf[ F(e_{\mu_l}), G(e_{\mu_l}) ]_{m \times n}
(1 - \mu_\mu(e_\mu)) \cup (1 - \lambda(e_\mu))
= [ F(e_{\mu_l})(x_i), F(e_{\mu_l})(x_i) ] \cup [ G(e_{\mu_l})(x_i), G(e_{\mu_l})(x_i) ]
(1 - \mu_\mu(e_\mu)) \cup (1 - \lambda(e_\mu))
= [ F(e_{\mu_l})(x_i), \mu(e_\mu) ] \cup [ G(e_{\mu_l})(x_i), \lambda(e_\mu) ]

(b) The proof is similar to proof of (a).
Proposition 4.14. Let \( F_\mu = [ F(e_\mu)(x_i), \mu(e_\mu) ]_{m \times n} \) be a GIVFSM and \( H_{\gamma} = [ H(e_\mu)(x_i), \gamma(e_\mu) ]_{m \times n} \)
be three GIVFSM's, then
(a) \( F_\mu \cap ( G_A \cap H_{\gamma} ) = ( F_\mu \cap G_A ) \cap H_{\gamma} \)
(b) \( F_\mu \cap ( G_A \cap H_{\gamma} ) = ( F_\mu \cap G_A ) \cap ( F_\mu \cap H_{\gamma} ) \)
Proof:
(a) \( F_\mu \cup ( G_A \cap H_{\gamma} ) = \sup[ F(e_{\mu_l}), G(e_{\mu_l}) ] \cap H(e_{\mu_l}) ]_{m \times n}
= \inf[ \sup(F(e_{\mu_l}), G(e_{\mu_l}) ), \sup(F(e_{\mu_l}), H(e_{\mu_l}) ) ]
= [ \inf(\sup(F(e_{\mu_l}), G(e_{\mu_l}) ), \sup(F(e_{\mu_l}), H(e_{\mu_l}) ) ]
\[ = (F(e_{j,}F(e_{j})) \cup (G(e_{k,}G(e_{k})) \cap \left( (F(e_{j,}F(e_{j})) \cup (H(e_{t,}H(e_{t})) \right) \]

and
\[
(\mu(e_{j}) \cup (\lambda(e_{k}) \cap \gamma(e_{j})) = \max\{\mu(e_{j}), \lambda(e_{k}) \cap \gamma(e_{j})\} = \max\{\mu(e_{j}), \min\lambda(e_{k}) \cap \gamma(e_{j})\} = \min\{\max\{\mu(e_{j}), \lambda(e_{k}) \cap \gamma(e_{j})\}, \max\{\mu(e_{j}), \lambda(e_{k}) \cap \gamma(e_{j})\}\} = \min\{\mu(e_{j}) \cup \lambda(e_{k}), \mu(e_{j}) \cup \gamma(e_{j})\} = (\mu(e_{j}) \cup \lambda(e_{k})) \cap (\mu(e_{j}) \cup \gamma(e_{j}))\}
\]

(b) The proof is similar to proof of (a).

Definition 4.15. Let \( F_{\mu} = [F(e_{j})(x_{i}), \mu(e_{j})]_{m \times n} \) and \( G_{3} = [G(e_{k})(x_{i}), \lambda(e_{k})]_{m \times n} \) be two GIVFSM's, then AND product of \( F_{\mu} \) and \( G_{3} \) is denoted by \( F_{\mu} \land G_{3} \) and is defined as
\[ F_{\mu} \land G_{3} = H_{\gamma} \]

Such that \( H_{\gamma} = [H(e_{j})(x_{i}), \gamma(e_{j})]_{m \times n} \)

where
\[ H(e_{j})(x_{i}) = F(e_{j})(x_{i}) \cap G(e_{j})(x_{i}), \ F(e_{j}), G(e_{k}) \in F(e_{j}) \times G(e_{k}) \forall x_{i}, e_{j}, e_{k}. \]

and \( \gamma(e_{j}) = s(\mu(e_{j}), \lambda(e_{k})), where \mu(e_{j}), \lambda(e_{k}), \mu(e_{j}) \times \lambda(e_{k}) \forall j, k and s is an s - norm. \)

Example 4.16. Consider two GIVFSM's \( F_{\mu} \) and \( G_{3} \) as in example 3.3,
\[ F_{\mu} = \begin{bmatrix}
(0.1,0.4),0.3 & (0.4,0.7),0.3 & (0.2,0.4),0.3 \\
(0.2,0.3),0.4 & (0.3,0.4),0.4 & (0.2,0.5),0.4 \\
(0.3,0.5),0.1 & (0.1,0.2),0.1 & (0.0,0.2),0.1
\end{bmatrix} \]

then \( F_{\mu} \land G_{3} \) is given by
\[ F_{\mu} \land G_{3} = \begin{bmatrix}
(0.1,0.4),0.3 & (0.4,0.7),0.3 & (0.2,0.4),0.3 \\
(0.1,0.4),0.3 & (0.2,0.4),0.3 & (0.2,0.4),0.3 \\
(0.1,0.4),0.3 & (0.3,0.5),0.3 & (0.1,0.3),0.3 \\
(0.2,0.3),0.4 & (0.3,0.4),0.4 & (0.2,0.5),0.4 \\
(0.2,0.3),0.4 & (0.3,0.4),0.4 & (0.2,0.5),0.4 \\
(0.3,0.5),0.1 & (0.1,0.2),0.1 & (0.0,0.2),0.1 \\
(0.3,0.5),0.1 & (0.1,0.2),0.1 & (0.0,0.2),0.1 \\
(0.3,0.5),0.1 & (0.1,0.2),0.1 & (0.0,0.2),0.1
\end{bmatrix} \]

Definition 4.17. Let \( F_{\mu} = [F(e_{j})(x_{i}), \mu(e_{j})]_{m \times n} \) and \( G_{3} = [G(e_{k})(x_{i}), \lambda(e_{k})]_{m \times n} \) be two GIVFSM's, then OR product of \( F_{\mu} \) and \( G_{3} \) denoted by \( F_{\mu} \lor G_{3} \) and is defined as
\[ F_{\mu} \lor G_{3} = H_{\gamma} \]

Such that \( H_{\gamma} = [H(e_{j})(x_{i}), \gamma(e_{j})]_{m \times n} \)

\[ \gamma(e_{j}) = s(\mu(e_{j}), \lambda(e_{k})), where \mu(e_{j}), \lambda(e_{k}), \mu(e_{j}), \lambda(e_{k}) \forall j, k and s is an s - norm. \]
Definition 4.20. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{i,n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{i,n}$ be two GIVFSM's, then weighted geometric mean of $F_\mu$ and $G_\lambda$, denoted by $F_\mu \times^wG_\lambda$, is defined by

$$H^w(e_j)(x_i) = [H^w(e_{jL})(x_i), H^w(e_{jU})(x_i)]$$

$$H^w(e_{jL})(x_i) = \frac{w_2 F(e_{jL})(x_i) + w_2 G(e_{jL})(x_i)}{w_1 + w_2}, \quad H^w(e_{jU})(x_i) = \frac{w_1 F(e_{jU})(x_i) + w_2 G(e_{jU})(x_i)}{w_1 + w_2}$$

Where

$$H^w_y = [H^w(e_j)(x_i), y^w(e_j)]$$

such that

Definition 4.21. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{i,n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{i,n}$ be two GIVFSM's, then geometric mean of $F_\mu$ and $G_\lambda$, denoted by $F_\mu \times G_\lambda$, is defined by

$$F_\mu \times G_\lambda = \left(\left[F(e_{jL})(x_i) \times G(e_{jL})(x_i)\right]^{w_1}, \left[F(e_{jU})(x_i) \times G(e_{jU})(x_i)\right]^{w_2}\right)$$

Definition 4.22. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{i,n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{i,n}$ be two GIVFSM's, then weighted geometric mean of $F_\mu$ and $G_\lambda$, denoted by $F_\mu \times^wG_\lambda$, is defined by

$$F_\mu \times^wG_\lambda = \left(\left[F(e_{jL})(x_i) \times^w G(e_{jL})(x_i)\right]^{w_1 + w_2}, \left[F(e_{jU})(x_i) \times^w G(e_{jU})(x_i)\right]^{w_1 + w_2}\right)$$

Definition 4.23. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{i,n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{i,n}$ be two GIVFSM's, then harmonic mean of $F_\mu$ and $G_\lambda$, denoted by $F_\mu \& G_\lambda$, is defined by

$$F_\mu \& G_\lambda = \left(\left[\frac{F(e_{jL})(x_i) + G(e_{jL})(x_i)}{2}, \frac{F(e_{jU})(x_i) + G(e_{jU})(x_i)}{2}\right], \frac{\mu(e_j) + \lambda(e_k)}{2}\right)$$

Definition 4.24. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{i,n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{i,n}$ be two GIVFSM's, then weighted harmonic mean of $F_\mu$ and $G_\lambda$, denoted by $F_\mu \times^{wH}G_\lambda$, is defined by

$$F_\mu \times^{wH}G_\lambda = \left(\left[\frac{F(e_{jL})(x_i) + G(e_{jL})(x_i)}{w_1 + w_2}, \frac{F(e_{jU})(x_i) + G(e_{jU})(x_i)}{w_1 + w_2}\right], \frac{\mu(e_j) + \lambda(e_k)}{w_1 + w_2}\right)$$

Proposition 4.25. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{i,n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{i,n}$ be two GIVFSM's, then the following holds.

(a) $F_\mu \times G_\lambda = G_\lambda \times F_\mu$
(b) $F_\mu \times G_\lambda = G_\lambda \times F_\mu$
(c) $F_\mu \& G_\lambda = G_\lambda \& F_\mu$

5. Similarity Between Two GIVESM's

Definition 5.1. Let $F_\mu = [F(e_j)(x_i), \mu(e_j)]_{i,n}$ and $G_\lambda = [G(e_k)(x_i), \lambda(e_k)]_{i,n}$ be two GIVFSM's, then similarity between $F_\mu$ and $G_\lambda$, denoted by $S(F_\mu, G_\lambda)$, is defined by

$$S(F_\mu, G_\lambda) = H_y, \text{where } H_y = [H(e_{jL})(x_i), y, H(e_{jU})(x_i), y].$$

$$H(e_{jL})(x_i) = \min \left(\phi(F(e_{jL}), G(e_{jL})), \phi(F(e_{jU}), G(e_{jU}))\right)$$

$$H(e_{jU})(x_i) = \max \left(\phi(F(e_{jL}), G(e_{jL})), \phi(F(e_{jU}), G(e_{jU}))\right)$$

Such that

$$\phi(F(e_{jL}), G(e_{jL})) = \begin{cases} \frac{\sum_i \max \{\min (F(e_{jL}), G(e_{jL}))\}}{\frac{\sum_i \max (F(e_{jL}))}{\sum_i \max (G(e_{jL}))}, & \text{if } F(e_{jL}) = 0, \forall e_j \\ \frac{\sum_i \max \{\min (F(e_{jL}), G(e_{jL}))\}}{\sum_i \max (F(e_{jL}))}, & \text{otherwise} \end{cases}$$
\[
\gamma = 1 - \frac{\sum_i |\mu(e_i) - \lambda(e_i)|}{\sum_i |\mu(e_i) - \lambda(e_i)|}, \quad j, k = 1, 2, \ldots, n.
\]

Definition 5.2. Two GIVFSM’s \(F_\mu\) and \(G_\lambda\) are called significantly similar if \(S(F_\mu, G_\lambda) \geq 1/2\).

Theorem 5.3. Let \(F_\mu = [\{F(e_i)(x_i), \mu(e_i)\}]_{i=1}^{m}\) and \(G_\lambda = [\{G(e_k)(x_i), \lambda(e_k)\}]_{i=1}^{n}\) be two GIVFSM’s, then
(a) \(S(F_\mu, G_\lambda) \neq S(G_\lambda, F_\mu)\)
(b) \(H(e_{\ell L})(x_i) \geq 0\) and \(H(e_{e L})(x_i) \leq 1\)
(c) If \(F_\mu = G_\lambda\), then \(S(F_\mu, G_\lambda) = 1\)

Proof:
(a) The proof is straightforward and follows from definition.
(b) From definition, we have
\[
\varphi(F(e_{j L}), G(e_{k L})) = \begin{cases} 
0, & \text{if } F(e_{j L}) = 0, \forall e_j \\
\frac{\sum_{i=1}^{m_n} \max \{ \min \{F(e_{j L}), G(e_{k L})\} \}}{\sum_{i=1}^{m_n} \max F(e_{j L})}, & \text{otherwise}
\end{cases}
\]

If \(F(e_{j L}) = 0, \forall e_j\), then \(H(e_{e L})(x_i) = 0\) and if \(F(e_{j L}) \neq 0\), for some \(e_j\), then it is clear that \(H(e_{\ell L})(x_i) \geq 0\). Since
\[
H(e_{\ell L})(x_i) = \max(\varphi(F(e_{j L}), G(e_{k L})), \varphi(F(e_{j L}), G(e_{k L})))
\]

Assume that \(\varphi(F(e_{j L}), G(e_{k L})) = 1\) and \(\varphi(F(e_{j L}), G(e_{k L})) = 1\), then \(H(e_{\ell L})(x_i) = 1\)
If \(\varphi(F(e_{j L}), G(e_{k L})) < 1\) and \(\varphi(F(e_{j L}), G(e_{k L})) < 1\), then \(H(e_{\ell L})(x_i) \leq 1\).
(a) The proof is straightforward and follows from definition.

Theorem 5.4. Let \(F_\mu = [\{F(e_i)(x_i), \mu(e_i)\}]_{i=1}^{m}\), \(G_\lambda = [\{G(e_k)(x_i), \lambda(e_k)\}]_{i=1}^{n}\) and \(H_\gamma = [\{H(e_i)(x_i), \gamma(e_i)\}]_{i=1}^{n}\) be three GIVFSM’s, then the following hold:
\[
F_\mu \subseteq G_\lambda \subseteq H_\gamma \Rightarrow S(F_\mu, H_\gamma) \leq S(G_\lambda, H_\gamma)
\]

Proof: The proof is straightforward and follows from definition.

6. Conclusion

We have introduced the concept of GIVFSM’s in this paper. Some of its types are defined. Some basic operations like union, intersection, compliment, AND operation and OR operation have been defined and exemplified. Arithmetic mean, geometric mean, harmonic mean and their weighted means are also defined and some properties of these operators are discussed. Furthermore, similarity between two GIVFSM’s is discussed. To future concern, GIVFSM’s can be used to solve decision making problems in situations where uncertainty involved.

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