Fuzzy nonsingular terminal sliding mode control for rigid flexible manipulator

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Abstract. In order to solve the precise position control problem of a rigid and flexible manipulator with underactuated variables, the controller of the rigid and flexible manipulator was designed based on the fuzzy nonsingular terminal slip control strategy combined with state feedback. The simulation results show that, compared with the existing basic sliding mode position control methods, the proposed control method has less overshoot, less adjustment time, less input torque jitter, less maximum amplitude of end vibration and less error of end position, which can effectively improve the control quality of the rigid flexible manipulator in the working process.

Keywords: Non-singular terminal sliding mode control; rigid flexible manipulator; Feedback linearization.

1. Introduction

The progress of science and technology has promoted the development of arm technology, the mechanical arm has both rigid and soft elastic deformation mechanical arm are less, the advantages of light quality and flexible mechanical arm, can save the advantages of lower energy consumption by motion [1, 2], in the field of medicine and space science has been widely used, such as robot assistant and spacecraft, etc.

Because just dynamic model of flexible manipulator system has the characteristics of nonlinear, strong coupling, time-varying [3], and composed of a flexible manipulator each order modal dynamic subsystem not only affected by control inputs, also the influence of the input and output subsystem, the dynamic behaviour is very complex [4], so the traditional control methods can't achieve good tracking performance, need to study for just the nonlinear control of flexible manipulator method [5, 6]. Literature [5] proposed a position control method based on system energy for single-link flexible manipulator, and suppressed the residual vibration of the system by controlling the total energy of the system to converge to zero. Literature [7] proposed a fuzzy compensation sliding mode control method for two-link flexible manipulator. Literature [8] proposed a non-singular terminal sliding mode control (NN-NTSMC) method based on neural network approximation to solve the trajectory tracking problem of robotic airship, which effectively suppressed chattering and had faster convergence speed and better tracking accuracy.

Ying Sun et al. Literature [9] decompose the rigid-flexible manipulator dynamics model into input-output subsystem and zero-dynamics subsystem, and realize trajectory tracking based on fuzzy...
control. Due to the existence of underactuated variables in the rigid-flexible manipulator system, it is difficult to design the controller directly for the underactuated variables of the system to make it convergent in the process of studying the position control, which leads to the elastic vibration of the end-effector in the process of motion and reduces the control precision of the system.

Based on the above research foundation, this paper takes the two-link rigid-flexible coupling manipulator as the research object, and proposes a controller based on the non-singular terminal sliding mode control strategy combined with state feedback to design the rigid-flexible manipulator, and studies the vibration suppression problem of the end position of the system. Firstly, the dynamics model of the two-link rigid-flexible coupling manipulator is established by using the Euler-Lagrange method and the hypothetical mode method. Then, analysis of system dynamics model, dynamic model of the system input/output linearization processing, two input/output subsystem of manipulator, the rest for the dynamic subsystems, according to input and output subsystem use nonsingular terminal sliding mode control method, the dynamic subsystem using PD state feedback control method, finally through the comparative analysis with traditional sliding mode controller, it is concluded that this method is superior to the effect of vibration suppression.

2. Dynamic modeling of rigid flexible manipulator

2.1. Kinematic description

In this paper, a rigid arm of a flexible manipulator and the coupled two-link manipulator as the research object, the mechanical arm connected to the base of the rotating joint rigidity, flexible mechanical arm and robotic arm driven by motor shaft connection rigidity, ignore the vertical deformation of flexible manipulator, assuming that the flexible mechanical arm can be free in the plane of the bend, cross section plane after deformation and deformation after the vertical axis, as shown in figure 1.

In Figure 1, the OXY coordinate system is an inertial coordinate system fixed to the base. o1x1y1 and o2x2y2 are local coordinate systems fixed to the rigid manipulator and the flexible manipulator base, respectively, which will rotate with the rotation of the manipulator. θ1 is the rotation angle of o1x1y1 relative to OXY, and θ2 is the rotation angle of o2x2y2 relative to o1x1y1. The length of the rigid and flexible manipulators is l1 and l2, respectively. According to Euler-Bernoulli beam theory, the deformation displacement of any point of the flexible manipulator is represented by a space-time function w(x,t).

According to the basic principle of the hypothetical mode method, the flexible body is discretized [9], and the deformation amount w(x,t), can be expressed as:

$$w = \sum_{i=1}^{N} \Phi_i(x) \cdot q_i(t)$$

(1)
Where, \( N \) is the order number of mode shapes, \( \Phi_i(x) \) is the \( i \)th mode shapes, and \( q(t) \) is the corresponding mode coordinates.

According to the boundary conditions of the cantilever beam, the modal shape function of the flexible arm can be expressed as:

\[
\Phi_i(x) = \cosh \gamma_i x - \cos \gamma_i x - k_i (\sinh \gamma_i x - \sin \gamma_i x)
\]  

(2)

In the formula, \( \gamma_i \) is the eigenvalue, and \( i \) is the mode order.

According to the results in Literature [10], the first two modes can be used to describe the lateral deformation of the flexible body. Therefore, the deformation quantity \( W \) can be described as:

\[
W = \Phi_1(x) \cdot q_1(t) + \Phi_2(x) \cdot q_2(t)
\]

(3)

2.2. Establishment of kinetic model

When the flexible manipulator is not deformed, the coordinate of point P in the moving reference coordinate system \( o_2x_2y_2 \) is \( (x,0) \), \( 0 \leq x \leq l_2 \), and the coordinate of any point of the flexible manipulator is:

\[
Rp' = \begin{bmatrix}
 x_1 \\
y_1 \\
x_2 \\
y_2
\end{bmatrix}
= \begin{bmatrix}
l_1 c_1 + xc_{12} - wx_{12} \\
l_1 s_1 + xs_{12} + wc_{12}
\end{bmatrix}
\]

(4)

In the formula, \( \cos \theta_1 \) and \( \cos \theta_2 \), \( c_{12}, s_{12} \) are \( \cos(\theta_1 + \theta_2) \) and \( \sin(\theta_1 + \theta_2) \) respectively. The velocity of this point in the inertial coordinate system is:

\[
\dot{Rp}' = \begin{bmatrix}
 -l_1 s_1 \dot{\theta}_1 - xs_{12}(\dot{\theta}_1 + \dot{\theta}_2) - wc_{12}(\dot{\theta}_1 + \dot{\theta}_2) - \dot{w}x_{12} \\
l_1 c_1 \dot{\theta}_1 + xc_{12}(\dot{\theta}_1 + \dot{\theta}_2) - wc_{12}(\dot{\theta}_1 + \dot{\theta}_2) - \dot{w}c_{12}
\end{bmatrix}
\]

(5)

The kinetic energy of the rigid-flexible manipulator is:

\[
T = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} \rho \dot{R}p'^T \cdot \dot{R}p' \cdot dx + \frac{1}{2} J_2 (\dot{\theta}_1 + \dot{\theta}_2)^2 + \frac{1}{2} M_l \dot{\theta}_1^2 + \frac{1}{2} M \dot{R}p'^T \cdot \dot{R}p',
\]

(6)

Where: \( J_1 \) is rigid mechanical arm and the rotary inertia of the rotating parts center of relative \( o_1 \), \( J_2 \) is a flexible robotic arm and moment of inertia of rotating parts relatively center \( o_2 \), \( \rho \) is the linear density of flexible manipulator, \( M_l \) is the flexible mechanical arm and the total mass of the rotating parts, \( MP \) is terminal load quality, \( Rp' \) is the end of the flexible manipulator coord.

Since the influence of gravitational potential energy is not considered, the total potential energy of the system is equal to the total elastic potential energy, which can be expressed as:

\[
U = \frac{1}{2} E(x, t)^2 dx
\]

(7)

Substituting (6) and (7) into the Lagrange equation, the dynamics equation of the rigid-flexible coupling manipulator is obtained as follows:

\[
MZ + VZ + KZ + F = T
\]

(8)

Where, \( M \) is the \( 4 \times 4 \) generalized mass matrix, \( V \) is the generalized damping matrix, \( K \) is the stiffness matrix, \( F \) is the nonlinear term of centrifugal force and Coriolis force, \( T \) is the generalized driving moment, and \( Z = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_1, \dot{\theta}_2]^T, V = \text{diag}(\delta_1, \delta_1, \delta_1, \delta_1)^T, K = [k_1, k_1, k_1, k_1]^T, T = [T_1, T_2, 0]^T \).

To appropriate eliminate unmodeled errors, improve firm - soft movement and the control precision of the mechanical arm, the friction between the need to consider the joint effect of coulomb viscous friction model, this paper adopts a continuous function to approximate the Stribeck friction model, and conducive to the realization of the control of each joint motor, so the cullen viscous friction model for friction torque of each joint mechanical arm just [11] :

\[
\tau_\beta = f_{\text{cf}} \text{sign}(\dot{\theta}_j) + f_{\text{vf}} \dot{\theta}_j
\]

(9)

Where, \( i=1,2 \), \( f_{\text{cf}} \) is the Coulomb friction moment coefficient, \( f_{\text{vf}} \) is the viscous friction moment coefficient, and \( T_\tau = [T_{\tau_1}, T_{\tau_2}, 0] \). Therefore, after considering the joint friction, the dynamics model of the rigid-flexible manipulator is modified as follows:
\[ M\ddot{Z} + V\dot{Z} + KZ + F = T - T_f \]  

(10)

3. The input/output linearization of the system model

3.1. System output redefinition

In the process of rigid and flexible manipulator motion control, not only should the position of the flexible arm end be considered to be able to accurately track the desired motion trajectory, but also the elastic deformation of the flexible arm should be suppressed during the motion process. When the position of the flexible arm end is taken as the output of the system, it is a non-minimum phase system. The output redefinition method is an effective method to overcome the non-minimum phase system. In this paper, the output redefinition method is used to reconstruct the observed values of the flexible arm end. Since the dynamics equation (10) of the rigid-flexible manipulator system deduced above is a second-order nonlinear strongly coupled differential equation concerning generalized coordinates, it is necessary to transform the equation (10) into:

\[
\begin{bmatrix}
M_{\theta\theta} & M_{\theta q} \\
M_{q\theta} & M_{qq}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\dot{q}
\end{bmatrix} +
\begin{bmatrix}
E_{\theta} \\
E_{q}
\end{bmatrix}
\dot{\theta} +
\begin{bmatrix}
F_{\theta}(\theta, q) \\
F_{q}(\theta, q)
\end{bmatrix} =
\begin{bmatrix}
t_r - \tau_{j_1} \\
0
\end{bmatrix}
\]  

(11)

Multiply the left and right sides of Equation (11) by M-1, and let 

\[
-1 
\begin{bmatrix}
H_{\theta\theta} & H_{\theta q} \\
H_{q\theta} & H_{qq}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\dot{q}
\end{bmatrix} = \begin{bmatrix}
\theta \\dot{\theta}
\end{bmatrix}
\]

The output equation of Angle \( \theta \) and modal coordinate \( q \) be obtained as follows:

\[
\begin{aligned}
\dot{\theta} &= -H_{\theta\theta}(F_{\theta}(\theta, q) + E_{\theta}\dot{\theta}) - H_{\theta q}(F_{q}(\theta, q) + E_{q}\dot{q}) + H_{\theta q}(\tau_r - \tau_{\beta}) \\
\dot{q} &= -H_{q\theta}(F_{\theta}(\theta, q) + E_{\theta}\dot{\theta}) - H_{qq}(F_{q}(\theta, q) + E_{q}\dot{q}) + H_{qq}(\tau_r - \tau_{\beta})
\end{aligned}
\]  

(12)

According to the derivation results in reference [10], the end position of the rigid and flexible manipulator can be expressed by the output redefinition method vercom:

\[
y = \theta + A\cdot q
\]  

(13)

Where: \( y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \), \( \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \), \( q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \), \( A = \begin{bmatrix} 0 & 0 \\ I_4 & \Phi_1(l) & \Phi_2(l) \end{bmatrix} \) and \( \alpha \) is the coefficient value redefined on the output.

3.2. System input/output linearization

In traditional manipulator in the design of the controller, the control algorithm based on the nonlinear compensation system controller design, this method requires accurate dynamics model, in addition, the nonlinear compensation is complex and the cost is high, in order to avoid this difficulty, need to the system input/output feedback linearization model [12]. The basic idea of input/output linearization is to find a nonlinear controller called inner-loop control, and introduce a nonlinear feedback term or dynamic compensation term. Under this control, the system will be transformed into a linear input/output relationship, and then the outer loop control can be designed to achieve the desired output tracking.

According to the output equation of Equation (12) above, it can be known that the rigid-flexible manipulator is a dynamic system with two inputs and four outputs. The state variables of the system are defined as follows: \( x = \begin{bmatrix} \theta & \dot{\theta} & q & \dot{q} \end{bmatrix} \). Therefore, the dynamic equation (12) of the system can be translated into:

\[
\dot{x} = f(x) + g(x)\cdot u
\]  

(14)

Where: \( f(x) = \begin{bmatrix} \dot{\theta} \\ \dot{q} \end{bmatrix} \)

\[
g(x) = \begin{bmatrix} 0 & 0 & H_{\theta\theta} & H_{\theta q} \\ -H_{\theta\theta}(F_{\theta}(\theta, q) + E_{\theta}\dot{\theta}) - H_{\theta q}(F_{q}(\theta, q) + E_{q}\dot{q}) \\ -H_{q\theta}(F_{\theta}(\theta, q) + E_{\theta}\dot{\theta}) - H_{qq}(F_{q}(\theta, q) + E_{q}\dot{q}) \end{bmatrix}
\]

According to the system output of Equation (13), redefine the result, and carry out quadratic differentiation on Equation (13) to get the system output result:
\[ \dot{y}(t) = \alpha(\theta, q) + \beta(\theta, q) \cdot u \]  

(15)

Because the firm - soft mechanical arm system dimension is 4, but the input/output subsystem dimension is only 2, so the observation of dynamic subsystems within the dimension is 2, through internal subsystem design, can be as much as possible to reduce the influence on terminal position, flexible modes so should not only consider the system input and output subsystem, and need to consider the dynamic subsystem, tectonic system dynamic subsystems within the state equation is as follows:

\[
\begin{cases}
    \dot{A}_1 = A_2 \\
    \dot{A}_2 = \phi(\theta, q) + H_{\theta q} u
\end{cases}
\]  

(16)

Where: \( [A_1^T \quad A_2^T]^T = [q^T \quad \dot{q}^T]^T \), \( \phi(\theta, q, \dot{\theta}, \dot{q}) = -H_{\theta q}(F_1(\theta, q) + E_1\dot{\theta}) - H_{\theta q}(F_2(\theta, q) + E_2\dot{q}) \)

4. Design of fuzzy nonsingular terminal sliding mode controller

4.1. Design of input/output subsystem Fuzzy nonsingular terminal sliding mode controller

For the input and output subsystem of the rigid flexible manipulator, a non-singular terminal sliding mode controller is designed to ensure that the terminal position can quickly and asymptotically reach the given desired trajectory [13]. The internal dynamic subsystem is not only affected by the control input, but also the input and output subsystems. Here, pole assignment method of PD state feedback controller is adopted to stabilize the internal dynamic subsystem. The total control input consists of the input and output subsystem control quantity \( u_{ex} \) and the internal dynamic subsystem control quantity \( u_{in} \), and the control structure is shown in Figure 2.

\[
\begin{align*}
    \dot{y}(t) &= \alpha(\theta, q) + \beta(\theta, q) \cdot u \\
    \dot{A}_1 &= A_2 \\
    \dot{A}_2 &= \phi(\theta, q) + H_{\theta q} u
\end{align*}
\]  

(17)

The following is the analysis of the design principle of sliding mode surface in inverse dynamic sliding mode controller of input and output subsystem. In this paper, exponential reaching law is adopted \( \dot{S} = -\varepsilon \text{sgn} S - K_s S \), according to Equations (15) and (16), we can get:

\[
\begin{align*}
    \dot{S} &= \dot{y} - \dot{y}_d + \beta \dot{\theta} \dot{q} \\
    &= \beta \dot{\theta} \dot{q} (\alpha + \beta \dot{\theta} \dot{q} - Z(\alpha, \theta, q, \dot{\theta}, \dot{q})) + \dot{e} \\
    &= -\varepsilon \cdot \text{sgn} S - K_s S
\end{align*}
\]  

(18)
Where, $-K_S$ represents the exponential approaching term, which makes the approaching speed gradually decrease from a larger value to zero, which not only shortens the approaching time, but also makes the speed of the moving point reaching the switching surface very small. Pure index approach, moving point near the switching surface is a gradual process, it cannot guarantee a limited time to arrive, when not limited time arrived there is no switching surface sliding mode, so to increase a uniform approach $-\varepsilon \text{sgn} S$, when close to zero, approaching velocity is $\varepsilon$ not zero, which can guarantee system can reach the switching surface in finite time.

According to Equation (18), the control law of inverse dynamic sliding mode control of input and output subsystem can be designed as:

$$u_{ex} = Z^{-1}(\alpha, \theta, q)(\dot{y}_r^- - [\beta \frac{P}{v} \dot{e}^{(e-1)}] (-\varepsilon \text{sgn} S - K_S S - \dot{e}^-) - \Gamma(\alpha, \theta, q, \dot{\theta}, \dot{q}))$$

(19)

Where, $\varepsilon$ is the sliding mode gain matrix of $(2 \times 2)$, $K_S$ is the exponential approaching term parameter matrix of $(2 \times 2)$, and $\text{sgn}(S) = [\text{sgn}(S_1) \text{sgn}(S_2)]^T$ is the sign function vector.

According to reference [10] and the Lyapunov function [14], the state is stable.

4.2. Design of fuzzy controller

From the point of view of sliding surface, fuzzy method is used to adjust the system dynamically in real time, so that the system can reach the sliding surface as soon as possible and the jitter is small. The specific design is as follows:

Firstly, the variables are fuzzified. Let $S_i$ be the input of the fuzzy controller and the output variable, the universe of discourse be set as $[-100, 100]$ and $[-80, 80]$, and the fuzzy variable be $\{\text{PL (positive)}, \text{ZR (zero)}, \text{NL (negative)}\}$. For $S$, the partition region corresponds to NL $[-100, -20]$, ZR $[-20, 20]$, PL $[20, 100]$; for $\beta$, the partition region corresponds to NL $[-80, -40]$, ZR $[-40, 40]$, PL $[40, 80]$.

Then, the fuzzy reasoning method adopts "If then", and the rules are designed as follows: 1) if $S$ is NL, then $\beta$ is PL; 2) if $S$ is ZR, then $\beta$ is ZR; 3) if $S$ is PL, then $\beta$ is NL.

Finally, the center of gravity method is used for fuzzy decision, as shown in the formula (20).

$$\sum_{\mu_{a}(u)} = \sum_{\mu_{a}(u)}$$

(20)

4.3. Stabilization of internal dynamic subsystem

In order to stabilize the internal dynamic subsystem (12), the flexible deformation generalized coordinate variables are bounded. Adopt state feedback control [15]:

$$u_{in} = K_q q + K_{q} \dot{q}$$

(21)

According to the controller designed above, the total input control quantity is:

$$u = u_{cs} + u_{in}$$

$$= Z^{-1}(\alpha, \theta, q)(\dot{y}_d^- - [\beta \frac{P}{v} \dot{e}^{(e-1)}] (-\varepsilon \text{sgn} S - K_S S - \dot{e}^-) - \Gamma(\alpha, \theta, q, \dot{\theta}, \dot{q})) + K_q q + K_{q} \dot{q}$$

(22)

The internal dynamic subsystem can be guaranteed to be asymptotically stable [15].

5. Analysis of simulation results

In order to analyze the control effect of fuzzy nonsingular terminal sliding mode (FNTSMC) of the input/output subsystem and the internal dynamic subsystem of the rigid-flexible manipulator designed in this paper, the dynamics simulation of the system control model is carried out, and the comparison with the traditional sliding mode controller(SMC) is made. The system parameters of the rigid flexible manipulator are shown in Table 1, given the step signal: $y_d = [0.5 \ 0.5]^T$, the sliding mode predictive control time : $T=1s$, the sampling time is 10-3s. The scale factor of fuzzy input variable $S$ is $[303,384]$.
and the scale factor of fuzzy output variable $\beta$ is $[13,15]$ The simulation results are shown in Figure 3-5.

| Table 1. System parameters of rigid-flexible manipulator |
|----------------------------------------------------------|
| Physical parameters | Rigid manipulator | Flexible manipulator |
|----------------------|------------------|----------------------|
| Length/m m           | $L_1=0.33$       | $L_1=0.30$           |
| Moment of inertia/kg·m$^2$ | $J_1=0.0812$    | $J_2=0.138$         |
| Linear density/kg·m$^3$ | $\rho_2 = 0.4865$ | $E_1=26.055$       |
| Elastic modulus/N·m$^2$ |                |                      |
| Mass of the end/kg   | $M_1=0.221$      | $M_0=0.5$           |

![Angular displacement change](image1)

(a) joint 1  (b) joint 2

**Fig. 3** Angular displacement change

![Input torque](image2)

(a) joint 1  (b) joint 2

**Fig. 4** Input torque

![End vibration](image3)

**Fig. 5** End vibration
By observing the results in Fig. 3, Fig. 4 and Fig. 5, the horizontal analysis and comparison of the first-order and second-order modal models showed that the response time of FNTSMC at the end position was slightly slower than that of SMC, but the overshoot and adjustment time were smaller. The jitter of the input torque is much smaller than that of the SMC, and the maximum amplitude of vibration at the end is smaller. Longitudinal comparison of the two methods shows that the first-order modal model has lower accuracy than the second-order modal model, and the terminal response has overshoot, and the input torque jitter and terminal vibration are large, but the structure is simple and easy to implement. This conclusion is of great significance for further research on the compensation method of the first-order modal model.

An effective control scheme based on fuzzy nonsingular terminal sliding mode control for rigid flexible manipulator system is proposed. First, set up a system dynamics model of joint friction factors, in order to overcome the non-minimum phase system, the output redefinition method is adopted to get at the end of the flexible manipulator are observed, and the system dynamics model of the input/output feedback linearization, this paper proposes a nonsingular terminal sliding mode control method for faster convergence speed and tracking error smaller, small vibration at the end, and can effectively avoid joint input torque chattering phenomenon, proved the superiority of this method.

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