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Projections and fractional dynamics of COVID-19 with optimal control strategies

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A B S T R A C T

When the entire world is eagerly waiting for a safe, effective and widely available COVID-19 vaccine, unprecedented spikes of new cases are evident in numerous countries. To gain a deeper understanding about the future dynamics of COVID-19, a compartmental mathematical model has been proposed in this paper incorporating all possible non-pharmaceutical intervention strategies. Model parameters have been calibrated using sophisticated trust-region-reflective algorithm and short-term projection results have been illustrated for Bangladesh and India. Control reproduction numbers ($R_c$) have been calculated in order to get insights about the current epidemic scenario in the above-mentioned countries. Forecasting results depict that the aforesaid countries are having downward trends in daily COVID-19 cases. Nevertheless, as the pandemic is not over in any country, it is highly recommended to use efficacious face coverings and maintain strict physical distancing in public gatherings. All necessary graphical simulations have been performed with the help of Caputo–Fabrizio fractional derivatives. In addition, optimal control strategies for fractional system have been designed and the existence of unique solution has also been showed using Picard–Lindelof technique. Finally, unconditional stability of the fractional numerical technique has been proved.

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1. Introduction

Mass-vaccination campaigns have already been launched in several countries amid coronavirus surges. However, many scientists have expressed their concerns regarding the efficacy of approved vaccines potential emergence of virus variants. As worldwide distribution of COVID-19 vaccines is indeed a tedious process, non-pharmaceutical intervention strategies are the realistic and effective solutions to control the new spikes of cases. Generally, an effective vaccine would take years, if not decades, to develop. Lack of transparency could be a vital issue in upcoming days and a false sense of security could evolve among general people if the vaccine does not work effectively.

Mathematical models can always provide considerable insights of the transmission dynamics and complexities of any infectious diseases, which eventually help government officials design overall epidemic planning. Importantly, mathematical analysis always plays a notable role in making vital public health decisions, resource allocation and implementation of social distancing measures and other non-pharmaceutical interventions. From the beginning of the COVID-19 outbreak, mathematicians and researchers are working relentlessly and have already done tremendous contributions in limiting the spread of the coronavirus in different parts of the world [1–5]. In an early contribution, Ferguson et al. [1] showed the impact of different non-pharmaceutical intervention strategies on COVID-19 mortality by developing an agent-based model. In another study, Ngonghala et al. showed that effective and comprehensive usage of face coverings can significantly limit the spread of the virus and reduce the COVID-induced mortality in different states of the USA in general in the absence of community lockdown measures and stringent social distancing practice. On the other hand, Nabi [6] projected the future dynamics of COVID-19 for various COVID-19 hotspots by proposing a compartmental mathematical model and concluded that early relaxation of lockdown measures and social distancing could bring a second wave in no time. As a matter of reality, inhabitants in several countries compelled to violate containment measures due to prolonged lockdown measures and severe economic recession [3]. Netherlands having one of the best health care systems in the world, is grappling with continuous spikes in daily cases due to aversion to masks.
In this study, in the absence of a safe, effective and world-wide approved vaccine, a new compartmental mathematical COVID-19 model has been designed incorporating all possible non-pharmaceutical intervention strategies such as wearing face coverings, social distancing, home or self-quarantine and self or institutional isolation. In addition, the impacts of different interventions scenarios have been analysed rigorously. The aim of this work is to project the future dynamics of COVID-19 outbreak in two countries namely Bangladesh and India, which are one of the worst-hit countries in the world. Estimation of parameters has been performed by using real-time data, followed by a projection of the evolution of the disease. For fractional simulations, we used the well known non-integer order derivative called Caputo–Fabrizio (CF) fractional derivative [7]. Since last few decades, there are so many epidemic models have been solved by non-integer order derivatives [8]. Recently some applications of non-integer order derivatives in mathematical epidemiology can be seen from [9–11]. There are so many research papers have been come to study the outbreaks of coronavirus, in which some are [12–16]. We performed the optimal control problem in CF derivative sense and provided the existence of unique solution by well-known technique named as Picard–Lindelof technique. We also proved the unconditionally stability of the given fractional numerical technique. We used the numerical data of the two given countries and perform the all necessary graphical simulations.

The entire chapter is organized as follows. Materials and methods are presented in Section 2. Section 3 is solely devoted to properties of solutions and asymptotic stability of the proposed model. In Section 4, estimation of model parameters and projection results have been discussed using daily COVID-19 data of Bangladesh and India. In Section 5, numerical and graphical simulations have been illustrated using Caputo–Fabrizio fractional derivatives. Later, optimal control problem has been designed in fractional sense in Section 6. The chapter ends with some insightful findings and strategies, which could significantly control the transmission dynamics of COVID-19.

2. Materials and methods

2.1. Model formulation

A compartmental mathematical has been proposed describing the transmission dynamics of the novel coronavirus incorporating all possible real-life interactions and social behavior. Considering different infection status, the entire human population (denoted by N(t) at time t) has been stratified into nine mutually-exclusive compartments of susceptible individuals (S(t)), early-exposed individuals (E1(t)), pre-symptomatic individuals (E2(t)), symptomatically-infectious (I(t)), asymptotically-infectious or infectious individuals with mild-symptoms (A(t)), quarantined infectious (Q(t)), hospitalised or isolated individuals (L(t)), recovered individuals (R(t)), disease-induced death cases (D(t)). Hence,

$$N(t) = S(t) + E_1(t) + E_2(t) + I(t) + A(t) + Q(t) + L(t) + R(t) + D(t)$$

The schematic diagram of the proposed model is illustrated in Fig. 1, where susceptible individuals can become infected by an effective contact with individuals in the pre-symptomatic (E2(t)) or symptomatically-infectious (I(t)), asymptotically-infectious (A(t)), quarantined-infectious (Q(t)) and isolated-infectious (L(t)) compartments. Effective contact rates are $\lambda_{E_1}$, $\lambda_{E_2}$, $\lambda_{Q}$, $\lambda_{I}$, and $\lambda_{L}$ respectively and the expressions are defined in (2). Importantly, the compartment $E_1(t)$ consists of early-infected individuals who are still not infectious, whereas the individuals in pre-symptomatic cohort $E_2(t)$ have the capability of transmitting coronavirus before the end of the disease incubation period. A proportion of individuals in newly-exposed compartment ($E_1(t)$) progress to pre-symptomatic class ($E_2(t)$) at a rate $\kappa_1$. After the completion of disease mean incubation period, at a rate $\rho \kappa_2$, a fraction of individuals who have clear clinical symptoms of COVID-19 progress to $I(t)$ compartment. Individuals in $E_2(t)$ class who do not have any clear symptoms progress to $A(t)$ class at a rate $(1 - \rho) \kappa_2$. Pre-symptomatic individuals are assumed to be self-quarantined at a rate $q$. With the help of diagnostic or surveillance testing approaches, symptomatically-infectious individuals and asymptotically-infectious individuals are brought under institutional or home isolation at rates $\tau_A$ and $\tau_I$ respectively. Moreover, the parameter $\gamma(I(Q)\gamma(I))$ represents the recovery rate for individuals in the I(A)(Q)(L) class. Finally, the disease-induced mortality rate for individuals in the I(Q)(L) compartment is defined by the parameter $\delta_I(\delta_Q)(\delta_L)$. Considering all the above-mentioned interactions, the transmission dynamics of COVID-19 can be described by the following system of nonlinear ordinary differential equations.
\[
\begin{align*}
\frac{dS}{dt} &= - (\lambda_I + \lambda_A + \lambda_Q + \lambda_L + \lambda_{E_2}) S, \\
\frac{dE_1}{dt} &= (\lambda_I + \lambda_A + \lambda_Q + \lambda_L + \lambda_{E_2}) S - \kappa_I E_1, \\
\frac{dE_2}{dt} &= \kappa_I E_1 - (\kappa_2 + q) E_2, \\
\frac{dI}{dt} &= \rho \kappa_2 E_2 - (\tau_1 + \gamma_1 + \delta_1) I, \\
\frac{dA}{dt} &= (1 - \rho) \kappa_2 E_2 - (\tau_1 + \gamma_1 + \delta_2) A, \\
\frac{dQ}{dt} &= q E_2 - (\gamma_Q + \delta_Q) Q, \\
\frac{dL}{dt} &= \tau_1 I + \tau_4 A - (\delta_4 + \gamma_4) L, \\
\frac{dR}{dt} &= \gamma I + \gamma_A A + \gamma_Q Q + \gamma_L L, \\
\frac{dD}{dt} &= \delta I + \delta_4 L + \delta_Q Q.
\end{align*}
\]

where the forces of infection are defined below

\[
\begin{align*}
\lambda_{E_2} &= \beta_{E_2} (1 - m_{E_2}) \frac{E_2}{N}, \\
\lambda_I &= \beta_I (1 - m_I) \frac{I}{N}, \\
\lambda_A &= \beta_A (1 - m_A) \frac{A}{N}, \\
\lambda_Q &= \beta_Q (1 - m_Q) \frac{Q}{N}, \\
\lambda_L &= \beta_L (1 - m_L) \frac{L}{N}.
\end{align*}
\]

The parameters are described in Table 1.

We set \(x = (S, E_1, E_2, A, Q, L, R, D)\) the vector of state variable. Let \(f : \mathbb{R}^9 \rightarrow \mathbb{R}^9\) the the right hand side of system (1), which is a continuously differentiable function on \(\mathbb{R}^9\). According to [17, Theorem III.10.VI], for any initial condition in \(\Omega\), a unique solution of (1) exists, at least locally, and remains in \(\Omega\) for its maximal interval of existence [17, Theorem III.10.XVI]. Hence, model (1) is biologically well-defined.

2.2. Data sources

From the beginning of the COVID-19 outbreak, Center of Disease Control and Prevention (CDC) is providing authoritative and genuine data of daily confirmed COVID-19 cases continuously and it is indeed a trustworthy data repository source. Daily reported data of Bangladesh and India have been compiled using the source. Johns Hopkins University Center for Systems Science and Engineering (JHU CSSE) is carefully maintaining the data repository supported by ESRI Living Atlas Team and the Johns Hopkins University Applied Physics Lab (JHU APL). The repository is really convenient to use and publicly available [18].

3. Mathematical analysis

3.1. Positivity and boundedness of solutions

Here, we prove that all state variables of model (1) are non-negative for all time, i.e., solutions of the model (1) with positive initial data remain positive for all time \(t > 0\). The following result can be obtained.

**Lemma 1.** Solutions of COVID-19 model (1) with positive initial conditions are positive for all \(t \geq 0\).

**Proof.** Assume that \(\phi(t) = (S(t), E_1(t), E_2(t), I(t), A(t), Q(t), L(t), R(t), D(t))\) is a solution of (1) with positive initial conditions. Let us consider \(E_2(t)\) for \(t \geq 0\). It follows from the third equation of system (1) that

\[
E_2(t) = E_2(0) \exp \left[ \int_0^t (\kappa_1 E_1(\tau) - (\kappa_2 + q) E_2(\tau)) d\tau \right].
\]

Since \(E_2(0) > 0\), it follows that \(E_2(t) \geq 0\) for \(t \geq 0\). We proceed with the same for \(S(t), E_1(t), I(t), A(t), Q(t), L(t), R(t)\) and \(D(t)\). \(\Box\)

**Lemma 2.** Solutions of COVID-19 model (1) with positive initial conditions are bounded by the total population \(N_0\).

**Proof.** Let \(N(t) = S(t) + E_1(t) + E_2(t) + I(t) + A(t) + Q(t) + L(t) + R(t) + D(t)\). Thus, in absence of disease, we have

\[
\frac{dN(t)}{dt} = 0 \Leftrightarrow N(t) = cte := N_0
\]

where \(N_0\) is equal to the total population. It follows that for all \(t \geq 0\), we have \(S(t) \leq N_0, E_1(t) \leq N_0, E_2(t) \leq N_0, I(t) \leq N_0, A(t) \leq N_0, Q(t) \leq N_0, L(t) \leq N_0, R(t) \leq N_0\) and \(D(t) \leq N_0\) with \(N(t) = S(t) + E_1(t) + E_2(t) + I(t) + A(t) + Q(t) + L(t) + R(t) + D(t) \leq N_0\). \(\Box\)

In what follows, we study the model (1) in the following set

\[
D = \left\{ (S, E_1, E_2, A, Q, L, R, D) \in \mathbb{R}_+^9 : S + E_1 + E_2 + I + A + Q + L + R + D \leq N_0 \right\}
\]

which is positively-invariant and attracting region for the model (1).
3.2. Asymptotic stability of disease-free equilibria

The disease-free equilibrium point denoted by \( x_0 \) can be defined as follows:

\[
x_0 = (S_0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T = (N_0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^T
\]

Using notations in Rießche and Watmough [19], matrices \( F \) and \( V \) for the new infection terms and the remaining transfer terms are, respectively, given by

\[
F = \begin{pmatrix}
0 & \beta_e(1-m\xi)\frac{S}{N} & \beta_i(1-m\xi)\frac{S}{N} & \beta_A(1-m\xi)\frac{S}{N} & \beta_Q(1-m\xi)\frac{S}{N} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[
V = \begin{pmatrix}
\kappa & 0 & 0 & 0 & 0 & 0 \\
-k_1 & \kappa + q & 0 & 0 & 0 & 0 \\
0 & -k \kappa_2 & \tau_i + \gamma_i + \delta_i & 0 & 0 & 0 \\
0 & -(1-\rho)k_2 & 0 & \tau_A + \gamma_A & 0 & 0 \\
0 & -q & 0 & -\tau_i & -\tau_A & \gamma_0 + \delta_0 \\
0 & 0 & 0 & \gamma_0 & \gamma_0 + \delta_0 & 0 \\
\end{pmatrix}
\]

Then, the control reproduction ratio is defined, following [19,20], as the spectral radius of the next generation matrix, \( FV^{-1} \):

\[
R_c = \rho(FV^{-1}) = R_{E_5} + R_i + R_A + R_Q + R_L
\]

where,

\[
R_{E_5} = \frac{\beta_e(1-m\xi)}{k_2 + q}
\]

\[
R_1 = \frac{\rho k \beta_2(1-m\xi)}{(k_2 + q)(\gamma_A + \tau_A + \delta_A)}
\]

\[
R_A = \frac{\kappa_2(1-\rho)\beta_A(1-m\xi)}{(k_2 + q)(\gamma_A + \tau_A)}
\]

\[
R_Q = \frac{q \beta_Q(1-m\xi)}{(k_2 + q)(\gamma_0 + \delta_0)}
\]

\[
R_L = \frac{\kappa_2 \beta_2(1-m\xi)(\tau_i + \delta_i)(1-\rho) + \tau_i(\rho \gamma_A + \tau_A)}{(k_2 + q)(\gamma_0 + \delta_0)}
\]

where \( \rho (\cdot) \) represents the spectral radius operator.

The formula for control reproduction number has been formulated. Indeed, the insightful epidemic threshold, \( R_c \) calculates the average number of new secondary COVID-19 cases generated by a COVID-19 positive individual in a population a portion susceptible people are using effective face coverings. Different non-pharmaceutical measures are acting as control measures which lead to bring down \( R_c \) under unity [19]. Hence, we claim the following result followed by a direct consequence of the next generation operator method [19, Theorem 2], where \( \rho (\cdot) \) represents the spectral radius operator. The insightful epidemic threshold, \( R_0 \) calculates the average number of new secondary COVID-19 infections generated by an COVID-19 positive patient in a completely susceptible population. The control of COVID-19 pandemic passes by the application of some control measures which contribute to decrease until \( R_0 \) less than one [19]. Hence we claim the following result.

**Theorem 1.** The COVID-19 transmission dynamics is influenced by the basic reproduction number \( R_0 \) as follows:

1. If \( R_0 < 1 \), then a sufficiently small flow of infected individuals will not generate an outbreak of the COVID-19, i.e. the disease-equilibrium \( E_0 \) is locally asymptotically stable on \( \omega \).

2. If \( R_0 > 1 \), then a sufficiently small flow of infected individuals will generate an outbreak of the COVID-19, and the disease-equilibrium \( E_0 \) is unstable.

**Lemma 3.** If \( R_c < 1 \), the disease-free equilibrium \( x_0 \) is locally asymptotically stable and unstable if \( R_c > 1 \). [5]

**Remark 2.** If \( R_c < 1 \), then a sufficiently small flow of infected individuals will not generate an outbreak of COVID-19, whereas for \( R_c > 1 \), epidemic curve reaches a peak by growing exponentially and then decreases to zero as \( t \to \infty \).

The better control of the COVID-19 can be established by the fact that the DFE \( E_0 \) is globally asymptotically stable (GAS). In this context, we claim the following result.

**Theorem 3.** if \( R_c < 1 \), then the manifold, \( W \), of disease-free equilibrium points of the model (1) is GAS in \( \mathbb{D} \).

In the absence of use of face coverings, i.e. \( m = 0 \). \( R_c \) converges to the basic reproduction number, \( R_0 \). Now, we will study the global stability of the disease-free equilibrium whenever the basic reproduction number is less than one \( (R_c < 1) \). For this, we use the following Lyapunov function

\[
\mathcal{L} = a_1 E_1 + a_2 E_2 + a_3 I + a_4 A + a_5 Q + a_6 L.
\]

By deriving this function along the trajectories of the system (1), we obtain

\[
\dot{\mathcal{L}} = a_1 [\beta_i(1-m\xi)I + \beta_A(1-m\xi)Q + \beta_Q(1-m\xi)Q + \beta_{E_5}(1-m\xi)E_5 + \eta(1-m\xi)E_5]
\]

\[
+ a_2 [\kappa_2 E_1 - (k_2 + q)E_2 + a_1 \kappa_2 E_2 - (\tau_i + \gamma_i + \delta_i)I]
\]

\[
+ a_4 [(1-\rho)k_2 E_2 - (\tau_A + \gamma_A)A] + a_5 [qE_2 - (\gamma_0 + \delta_0)Q]
\]

\[
+ a_6 [\tau_A + \tau_A A - (\delta_A + \gamma_A)\mathcal{L}]
\]

\[
= (a_1 \kappa_1 + a_2 k_1)E_1
\]

\[
+ [a_1 \beta_i(1-m\xi) - a_2 (k_2 + q) + a_3 \rho k_2 + a_4 (1-\rho)k_2 + \eta qE_2]E_5
\]

\[
+ [a_1 \beta_A(1-m\xi) - a_3 (\tau_A + \gamma_A) + a_4 \tau_A A]E_2
\]

\[
+ [a_1 \beta_Q(1-m\xi) - a_4 (\gamma_0 + \delta_0)]Q + [a_1 \beta_{E_5}(1-m\xi) - a_5 (\delta_A + \gamma_A)]\mathcal{L}
\]

(7)
We choose \( a_i = 1, 2, \ldots, 6 \), such that coefficients of \( E_1, I, A, Q, \) and \( L \) become zero. That is
\[
\begin{align*}
-a_1 \kappa_1 + a_2 \kappa_1 &= 0 \\
a_1 \beta_1 (1 - m \xi) - a_3 (\tau_1 + \gamma + \delta_1) + a_6 \tau_1 &= 0 \\
a_1 \beta_2 (1 - m \xi) - a_4 (\tau_1 + \gamma) + a_6 \tau_2 &= 0 \\
a_1 \beta_3 (1 - m \xi) - a_5 (\gamma + \delta_2) &= 0 \\
a_1 \beta_4 (1 - m \xi) - a_6 (\delta_1 + \gamma + \delta) &= 0
\end{align*}
\]
which the non-zero solution is given by
\[
\begin{align*}
a_6 &= a_1 \beta_6 (1 - m \xi) + \delta_1 + \gamma_1 \\
a_5 &= a_1 \beta_5 (1 - m \xi) + a_6 \tau_1 \\
a_4 &= a_1 \beta_4 (1 - m \xi) + a_6 \tau_1 \\
a_3 &= a_1 \beta_3 (1 - m \xi) + a_6 \tau_1 \\
a_2 &= a_1 \\
\end{align*}
\]
Plugging (9) into (7) gives
\[
\dot{E} \leq a_1 (k_2 + q) \left[ \frac{\beta_2 \chi_2 (1 - m \xi)}{k_2 + q} + \frac{\rho \kappa_2 \beta_1 (1 - m \xi)}{(k_2 + q) (\gamma + t + \delta_1)} \right] + \frac{k_2 (1 - \rho) \beta_6 (1 - m \xi)}{(k_2 + q) (\gamma + t + \delta_1)} + \frac{q \beta_2 (1 - m \xi)}{(k_2 + q) (\gamma_2 + \delta_2)} \left[ \frac{k_2 (1 - m \xi) \tau_1 (\gamma + \delta_1)}{(k_2 + q) (\gamma + \delta_1)} - 1 \right] E_2. \\
\]
Setting \( a_1 = \frac{1}{k_2 + q} \), we finally obtain
\[
\dot{E} \leq (Rc_1 + Rc_2 + R_A + R_B + R_L - 1) E_2 = (Rc_1 - 1) E_2. \\
\]
From (11), it follows that \( \dot{E} < 0 \) if \( Rc_1 < 1 \), and \( \dot{E} = 0 \) if and only if \( E_1 = E_2 = I = A = Q = L = 0 \). Therefore, \( E \) is a Lyapunov function for system (1). Moreover, the maximal invariant set contained in \( \{G(t), E_1(t), E_2(t), I(t), A(t), Q(t), L(t), R(t), D(t) \} \in \Omega : \dot{E} = 0 \) is the continuum of the disease-free equilibrium \( \epsilon_0 \). Thus, from Lyapunov theory, we deduce that the disease-free equilibrium \( \epsilon_0 \) is GAS if \( Rc_1 < 1 \). Hence, it follows, by the LaSalle’s Invariance Principal, that the continuum of disease-free equilibria of the model (1) is a stable global attractor in \( \Omega \) whenever \( Rc_1 < 1 \). The previous analysis can be summarised as follows:

**Theorem 4.** If \( Rc_1 < 1 \), then the disease-free equilibrium \( \epsilon_0 \) is globally asymptotically stable on \( \Omega \).

4. Model calibration and forecasting

The model (1) calibration has been performed using a newly developed optimization algorithm based on trust-region-reflective (TRR) algorithm, which can be regarded as an evolution of Levenberg-Marquardt algorithm [6]. This robust optimization procedure can be used effectively for solving nonlinear least-squares problems. This algorithm has been implemented using the `lsqcurvefit` function, which is available in the Optimization Toolbox in MATLAB. Necessary model parameters have been estimated using this optimization technique. Daily infected cases data have been collected from a trusted data repository, which is available online. A 7-day moving average of the daily reported cases has been used for our model calibration due to moderate volatile nature of real data. It has been observed that the number of daily testing in Bangladesh and India have been really inconsistent. With an aim to capture the real outbreak scenario, the 7-day moving average has been used in this regard.

4.1. Bangladesh

Due to prolonged lockdown measures and severe economic recession, inhabitants of Bangladesh have started violating safety measures like wearing face coverings and maintaining physical distancing. Figs. 2 and 3 illustrate the model fitting performance with observed data from early March 2020 to early January 2021 for Bangladesh. The estimated error is found to be hovering around 10% for daily new cases. The actual outbreak scenario in Bangladesh is still a puzzle to be solved for the health officials due to scant COVID-19 testing program. The control reproduction number \( (R_c) \) is estimated to be \( \sim 0.73 \) (95%CI : 0.62 – 0.84) as of January 05, 2021 and prior established findings for this metric go well with the estimation [6,21]. Figs. 4 and 5 enlighten that the tally of cumulative infected cases is projected to reach 436K around March 31, 2021 and the estimated total death cases could reach 11,400 by the end of March, 2021. Table 2 illustrates the key features used to calibrate this scenario, which have been justified in prior clinical studies and relevant literature.

4.2. India

When India is celebrating a busy festival season, the tally of fresh COVID-19 cases continued to rise. Relaxation in protective and social-distancing measures could result in a significant upsurge in daily cases in upcoming days. Fig. 6 illustrates the fact that India is witnessing a downward trend after having peak. According to our projection results, India could reel under the second wave of infection unless non-pharmaceutical interventions strategies are followed comprehensively. As of January 09, 2021 the country’s caseload now stands at 10,448,134 and it’s death toll has mounted to 151,000. Figs. 6 and 7 illustrate the fitting performance of our proposed model for India from late January 2020 to early January 2021. Historical data from January 30, 2020 to January 05, 2021 have been considered to calibrate the model parameters. As we can see from the figures, model-fitting is exceptionally well for the historical observed data. Based on our projection results from Fig. 8, the number of daily cases could be brought under 1000 cases if mass-level efficacious face coverings is strictly maintained. The control reproduction number \( Rc_1 \) is estimated to be \( \sim 1.1 \) as of January 09, 2021 complementing the prior studied observations [6,21]. Fig. 9 depicts that the tally of cumulative infected cases is projected to reach 11.259K by the end of March 2021 if current trend continues. It has also been enlightened in a recent study [25,26] that, comprehensive usage of efficacious face coverings could be the most influential strategy to control the COVID-19 airborne transmission. In addition, country’s death toll could mount to 172K over the period. Table 3 illustrates the key features used to calibrate this scenario.

5. Solution of the model in Caputo–Fabrizio fractional derivative sense

5.1. Preliminaries

Here we recall the definitions of Caputo and Caputo–Fabrizio fractional derivatives.

**Definition 1.** Podlubny [27] The Caputo definition of non-integer order derivative of order \( \varphi > 0 \) of a function \( G : (0, \infty) \to \mathbb{R} \) is defined by
\[
D_{\varphi}^{\varphi} G(t) = \frac{1}{\Gamma(n - \varphi)} \int_{0}^{t} (t - \tau)^{n - \varphi - 1} G^{n}(\tau) d\tau \\
\]
where \( n = [\varphi] + 1 \) and \( [\varphi] \) is the integer part of \( \varphi \).
Fig. 2. Fitting performance of the model for daily infected cases in Bangladesh from March 08, 2020 to January 05, 2021.

Fig. 3. Fitting performance of the model for cumulative infected cases in Bangladesh from March 08 to January 05, 2021.

Fig. 4. Projection results for daily new confirmed cases for Bangladesh from early March 2020 to late March 2021.

**Definition 2** (Jajarmi and Baleanu [28], Losada and Nieto [29]). For $G \in H^1(c,d)$ and $0 < \varrho < 1$, the Caputo–Fabrizio (CF) fractional derivative (FD) of order $\varrho$ is defined by

$$\frac{\text{CF}}{c} D^\varrho_c G(t) = \frac{1}{1 - \varrho} \int_c^t \frac{dG(\eta)}{d\eta} \exp[-\theta(t - \eta)]d\eta$$

where $\theta = \frac{\varrho}{1 - \varrho}$

The CF non-integer order integral is defined as

$$\frac{\text{CF}}{c} I^\varrho_c G(t) = (1 - \varrho) G(t) + \varrho \int_c^t G(\eta)d\eta.$$

5.2. Existence and uniqueness analysis

Now, we prove the existence of unique solution for the given COVID-19 model in the sense of Caputo–Fabrizio fractional deriva-
Fig. 5. Projection results for cumulative cases for Bangladesh early March 2020 to late March 2021.

Table 2
Calibrated parameters of the proposed model (1) using trust-region-reflective algorithm and daily COVID-19 cases data of Bangladesh.

| Parameter | Range (Unit) | Baseline value | TRR output | Reference |
|-----------|--------------|----------------|-------------|-----------|
| $\beta_1$ | 0.1–1.5 day$^{-1}$ | 0.55 | 0.15 | [6,22] |
| $\beta_2$ | 0.1–0.9 day$^{-1}$ | 0.3 | 0.1 | [6,22] |
| $\beta_3$ | 0.1–0.9 day$^{-1}$ | 0.5 | 0.1 | [6,22] |
| $\beta_4$ | 0.05–0.3 day$^{-1}$ | 0.3 | 0.12 | [6,22] |
| $\beta_5$ | 0.1–0.9 day$^{-1}$ | 0.3 | 0.1 | [6,22] |
| $\beta_6$ | 0.05–0.3 day$^{-1}$ | 0.3 | 0.11 | [5] |
| $m$ | 0.01–0.3 (dimensionless) | 0.1 | 0.3 | [5] |
| $\zeta$ | 0.5 (dimensionless) | 0.5 | 0.5 | [5] |
| $k_1$ | $\frac{1}{7}$ day$^{-1}$ | $\frac{1}{7}$ | $\frac{1}{7}$ | [2,5] |
| $k_2$ | 1 day$^{-1}$ | 1 | 1 | [2] |
| $q$ | 0.1–0.6 day$^{-1}$ | 0.3 | 0.47 | Estimated |
| $\rho$ | 0.6–0.7 (dimensionless) | 0.65 | 0.65 | [5] |
| $r_1$ | $\frac{1}{7}$ day$^{-1}$ | 1/10 | 1/8 | [5,6] |
| $r_2$ | $\frac{1}{10}$ day$^{-1}$ | 1/10 | 1/8 | [5,6] |
| $\gamma_1$ | $\frac{1}{10}$ day$^{-1}$ | 1/7 | 1/12 | [23,24] |
| $\gamma_2$ | $\frac{1}{10}$ day$^{-1}$ | 1/10 | 1/12 | [23,24] |
| $\gamma_3$ | 1/21 day$^{-1}$ | 0.071 | 0.071 | [23,24] |
| $\gamma_4$ | 1/21 day$^{-1}$ | 0.071 | 0.071 | [23,24] |
| $\delta_1$ | 0.0001–0.01 day$^{-1}$ | 0.001 | 0.0004 | [1,6] |
| $\delta_2$ | 0.0001–0.01 day$^{-1}$ | 0.001 | 0.0009 | [1,6] |
| $\delta_3$ | 0.0001–0.01 day$^{-1}$ | 0.001 | 0.0007 | [1,6] |

Fig. 6. Fitting performance of the model for daily infected cases in India from January 30 to January 05, 2021.
Fig. 7. Fitting performance of the model for cumulative infected cases in India from January 30 to January 05, 2021.

Fig. 8. Projection results for daily new confirmed cases for India from late January 2020 to late March 2021.

Fig. 9. Projection results of cumulative cases for India from late January 2020 to late March 2021.
tive by the application of fixed-point theory. In this concern, the proposed system can be rewritten in the equivalent form as follows:

\[ \begin{align*}
&\text{CF } D^\ast D^\ast S(t) = g_1(t, S(t)), \\
&\text{CF } D^\ast D^\ast E_1(t) = g_2(t, E_1(t)), \\
&\text{CF } D^\ast D^\ast E_2(t) = g_3(t, E_2(t)), \\
&\text{CF } D^\ast I(t) = g_4(t, I(t)), \\
&\text{CF } D^\ast A(t) = g_5(t, A(t)), \\
&\text{CF } D^\ast Q(t) = g_6(t, Q(t)), \\
&\text{CF } D^\ast L(t) = g_7(t, L(t)), \\
&\text{CF } D^\ast R(t) = g_8(t, R(t)), \\
&\text{CF } D^\ast D(t) = g_9(t, D(t)). \\
\end{align*} \]

(13)

By applying the CF non-integer order integral operator, the above system (13), reduces to the following integral equation of Volterra type of order \(0 < \varrho < 1\).

\[ S(t) - S(0) = (1 - \varrho)g_1(t, S) + \varrho \int_0^t g_1(\chi, S)d\chi. \]

\[ E_1(t) - E_1(0) = (1 - \varrho)g_2(t, E_1) + \varrho \int_0^t g_2(\chi, E_1)d\chi. \]

\[ E_2(t) - E_2(0) = (1 - \varrho)g_3(t, E_2) + \varrho \int_0^t g_3(\chi, E_2)d\chi. \]

\[ I(t) - I(0) = (1 - \varrho)g_4(t, I) + \varrho \int_0^t g_4(\chi, I)d\chi. \]

\[ A(t) - A(0) = (1 - \varrho)g_5(t, A) + \varrho \int_0^t g_5(\chi, A)d\chi. \]

\[ Q(t) - Q(0) = (1 - \varrho)g_6(t, Q) + \varrho \int_0^t g_6(\chi, Q)d\chi. \]

\[ L(t) - L(0) = (1 - \varrho)g_7(t, L) + \varrho \int_0^t g_7(\chi, L)d\chi. \]

\[ R(t) - R(0) = (1 - \varrho)g_8(t, R) + \varrho \int_0^t g_8(\chi, R)d\chi. \]

\[ D(t) - D(0) = (1 - \varrho)g_9(t, D) + \varrho \int_0^t g_9(\chi, D)d\chi. \]

(14)

Now, we get the subsequent iterative algorithm

\[ S_{n+1}(t) = (1 - \varrho)g_1(t, S_n) + \varrho \int_0^t g_1(\chi, S_n)d\chi, \]

\[ E_{1n+1}(t) = (1 - \varrho)g_2(t, E_{1n}) + \varrho \int_0^t g_2(\chi, E_{1n})d\chi. \]

\[ E_{2n+1}(t) = (1 - \varrho)g_3(t, E_{2n}) + \varrho \int_0^t g_3(\chi, E_{2n})d\chi. \]

\[ I_{n+1}(t) = (1 - \varrho)g_4(t, I_n) + \varrho \int_0^t g_4(\chi, I_n)d\chi. \]

\[ A_{n+1}(t) = (1 - \varrho)g_5(t, A_n) + \varrho \int_0^t g_5(\chi, A_n)d\chi. \]

\[ Q_{n+1}(t) = (1 - \varrho)g_6(t, Q_n) + \varrho \int_0^t g_6(\chi, Q_n)d\chi. \]

\[ L_{n+1}(t) = (1 - \varrho)g_7(t, L_n) + \varrho \int_0^t g_7(\chi, L_n)d\chi. \]

\[ R_{n+1}(t) = (1 - \varrho)g_8(t, R_n) + \varrho \int_0^t g_8(\chi, R_n)d\chi. \]

\[ D_{n+1}(t) = (1 - \varrho)g_9(t, D_n) + \varrho \int_0^t g_9(\chi, D_n)d\chi. \]

(15)

Here we assume that we can get the exact solution by taking the limit as \(n\) tends to infinity.

5.2.1. Existence analysis by using Picard–Lindelof approach

Let us consider

\[ F_1 = \sup_{c_1} \| g_1(t, S) \|, \quad F_2 = \sup_{c_2} \| g_2(t, E_1) \|, \quad F_3 = \sup_{c_3} \| g_3(t, E_2) \|. \]

\[ R_1 = \sup_{c_4} \| g_4(t, I) \|, \quad R_2 = \sup_{c_5} \| g_5(t, A) \|, \quad R_3 = \sup_{c_6} \| g_6(t, Q) \|. \]

\[ F_4 = \sup_{c_7} \| g_7(t, L) \|, \quad F_5 = \sup_{c_8} \| g_8(t, R) \|, \quad F_6 = \sup_{c_9} \| g_9(t, D) \|. \]

(16)

where

\[ C_{c_1} = | t - c, t + c | \times [ S - z_1, S + z_1 ] = D_1 \times B_1, \]

\[ C_{c_2} = | t - c, t + c | \times [ E_1 - z_2, E_1 + z_2 ] = D_1 \times B_2, \]

\[ C_{c_3} = | t - c, t + c | \times [ E_2 - z_3, E_2 + z_3 ] = D_1 \times B_3, \]

\[ C_{c_4} = | t - c, t + c | \times [ I - z_4, I + z_4 ] = D_1 \times B_4, \]

\[ C_{c_5} = | t - c, t + c | \times [ A - z_5, A + z_5 ] = D_1 \times B_5, \]

\[ C_{c_6} = | t - c, t + c | \times [ Q - z_6, Q + z_6 ] = D_1 \times B_6, \]

\[ C_{c_7} = | t - c, t + c | \times [ L - z_7, L + z_7 ] = D_1 \times B_7, \]

\[ C_{c_8} = | t - c, t + c | \times [ R - z_8, R + z_8 ] = D_1 \times B_8, \]

\[ C_{c_9} = | t - c, t + c | \times [ D - z_9, D + z_9 ] = D_1 \times B_9. \]

(17)
considering the Picard operator as
\[ \phi(t) = C(D_1(t), B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9), \]
given as follows:
\[ \phi_0(t) = \zeta_0(t) + \Delta(t, \zeta(t)) \]
where \( t \in [0, T] \). Clearly, we have the numerical approximation of \( I(t) \) as
\[ I(t_{k+1}) = I(t_k) + \left( 1 - \theta - \frac{\theta \Delta t}{2} \right) G_4(t_k, I(t_k)) \]
Then by linear interpolation about \( G_4(t, I(t)) \) and applying trapezoid rule for integration on the integral term, we can then write
\[ \int_{t_k}^{t_{k+1}} G_4(t, I(t)) dt \approx \frac{3 \Delta t}{2} G_4(t_k, I(t_k)) - \frac{\Delta t}{2} G_4(t_{k-1}, I(t_{k-1})) \]
ture of infectious $I(t)$ is same as for India, as when the population of infected individuals increases then asymptomatic infectious $A(t)$ also increases with same nature. In sub-Figs. 11d and 12 b, we see that the fractional order does not play any big role because the nature of the classes is nearly same at all different fractional order values $\rho$. Initial values of given classes for Bangladesh are $S(0) = 164689383, E_1(0) = 10, E_2(0) = 4, I(0) = 2, A(0) = 1, Q(0) = 0, L(0) = 0, R(0) = 0$ and $D(0) = 0$. We have used the total population of the country for $S(0)$.

We have done the graphical simulations for India which is the second highest populous country in the world and also the second worst-hit country by COVID-19. To study the outbreak of COVID-19 in India, in the family of Figs. 13–15, we exemplified the all necessary graphs of given classes to observe the dynamics of COVID-19. To perform numerical simulations, we took the numerical values from the Table 3. In the family of Fig. 13, we analysed the plots of $S(t), E_1(t), E_2(t), I(t), A(t)$ and $Q(t)$. We observed that the nature of peaks is mostly same as for other above analysed data, for different fractional order values peaks are well defined and when we decrease the fractional order then the peaks sifted towards the later time period. In the collection of Figs. 14 and 15, first we show the nature of $L(t), R(t), D(t)$ and then analysed the plots of $I(t)$ versus $S(t), A(t), Q(t), L(t)$ and $R(t)$. When we compare the given classes with $I(t)$, we again observed that when the population of infected individuals increases then asymptomatic infectious $A(t)$ also increases with same nature. In sub-Figs. 14d and 15 b, we observed that at the different fractional order values the nature of the classes is nearly same. Initial values of given classes for India are $S(0) = 414001316, E_1(0) = 10, E_2(0) = 4, I(0) = 2, A(0) = 1, Q(0) = 0, L(0) = 0, R(0) = 0$ and $D(0) = 0$. We have used the 30% of the total population of India for $S(0)$.
Fig. 11. Plots of $L(t)$, $R(t)$, $D(t)$ and relationship of $I(t)$ versus $S(t)$, $A(t)$, $Q(t)$, $L(t)$, $R(t)$ for Bangladesh data.

Fig. 12. $I(t)$ versus $L(t)$, $R(t)$ for Bangladesh data.
From the all above graphical observations we found that the Caputo–Fabrizio fractional derivative playing well to study the outbreaks of coronavirus in the aforesaid two countries.

6. Optimal control problem formulation

In this concern, our main aim is to decrease the number of infected individuals with COVID-19 at the same time decrease the cost \( J(v) \) associated with their strategies. For this purpose, we use a control function \( v = (v_1, v_2, v_3) \), where \( v_1(t) \) is for introducing the public education or aware the public with health-care measures, \( v_2(t) \) is the control function for enhancement of the strength of treatment for the infected individuals, \( v_3(t) \) is the control function for the necessary suggestions of health care measures for those who are in asymptomatic infectious class and yet not admitted in the hospital.

\[
\begin{align*}
CT D_t^\nu S(t) &= -(1 - v_1)(\lambda_1 + \lambda_A + \lambda_Q + \lambda_L + \lambda_{E_2})S, \\
CT D_t^\nu E_1(t) &= (1 - v_1)(\lambda_1 + \lambda_A + \lambda_Q + \lambda_L + \lambda_{E_2})S - \kappa_1 E_1, \\
CT D_t^\nu E_2(t) &= \kappa_1 E_1 - (\kappa_2 + q)E_2, \\
CT D_t^\nu I(t) &= \rho \kappa_2 E_2 - (\tau_I + \gamma_I + \delta_I + v_2)I, \\
CT D_t^\nu A(t) &= (1 - \rho)\kappa_2 E_2 - (\tau_A + \gamma_A + v_2)A, \\
CT D_t^\nu Q(t) &= qE_2 - (\gamma_Q + \delta_Q)Q, \\
CT D_t^\nu L(t) &= \tau_L + \tau_A A - (\delta_L + \gamma_L)L, \\
CT D_t^\nu R(t) &= \gamma_I A + \gamma_Q Q + \gamma_L L.
\end{align*}
\]

(26)

To define the optimal control problem (OCP), we are excluding the death equation \( D(t) \), because there is no significance of deaths in optimal controls. Now consider the state system given in (26) in \( \mathbb{R}^6 \), with the set of admissible control func-
Fig. 14. Plots of $L(t)$, $R(t)$, $D(t)$ and relationship of $I(t)$ versus $S(t)$, $A(t)$, $Q(t)$, $L(t)$, $R(t)$ for India data.

Fig. 15. $I(t)$ versus $L(t)$, $R(t)$ for India data.
tion. $\Omega = \{(v_1(.), v_2(.), v_3(.))|v_i$ is Lebesgue measurable on $[0, 1]| 0 \leq (v_1(.), v_2(.), v_3(.)) \leq 1$ So the objective functional is defined by

$$J(v_1(.), v_2(.), v_3(.)) = \int_0^T \left[ l(t) + \frac{1}{2} \sum_{i=1}^3 (k_i v_i^2(t) + k_{2i} v_i^2(t) + k_{3i} v_i^2(t)) \right] \, dt$$

where the constants $k_1$, $k_2$ and $k_3$ are a measure of associative cost with the controls $v_1$, $v_2$ and $v_3$. Then we find the optimal controls $v_1$, $v_2$ and $v_3$ to minimize the cost function.

$$J(v_1, v_2, v_3) = \int_0^T \mu(S, E_1, E_2, I, A, Q, L, R, v_1, v_2, v_3, t) \, dt$$

subject to constraint, $\frac{\partial}{\partial v_1} S(t) = \phi_1$, $\frac{\partial}{\partial v_2} E_2(t) = \phi_2$, $\frac{\partial}{\partial v_3} R(t) = \phi_3$ where $\phi_1 = \phi_2 = \phi_3 = 0$ and the given initial coordination are agreed $S(0) = S_0$, $E_1(0) = E_1$, $R(0) = R_0$.

Now let us take the following modified cost function

$$\tilde{J} = \int_0^T \left[ H(S_1, E_1, ..., \theta, v_1, t) - \frac{1}{2} \sum_{j=1}^3 (\theta_j \phi_j(S_1, E_1, ..., \theta, v_1, t)) \right] \, dt$$

where $i = 1, 2, 3$ and $j = 1, 2, 3, ... 8$ Hence the Hamiltonian is defined as follows:

$$H(S_1, E_1, ..., R, v_1, t) = \mu(S_1, E_1, ..., R, v_1, t) + \frac{1}{2} \sum_{j=1}^3 (\theta_j \phi_j(S_1, E_1, ..., R, v_1, t))$$

where $i = 1, 2, 3$ and $j = 1, 2, 3, ... 8$ from Eqs. (29) and (30), the necessary and sufficient conditions for the functional optimal control problem (FOCP) are given as:

$$\begin{align*}
\frac{\partial}{\partial \theta_1} = \frac{\partial H}{\partial S_1} & , \quad \frac{\partial}{\partial \theta_2} = \frac{\partial H}{\partial E_1} & , \quad \frac{\partial}{\partial \theta_3} = \frac{\partial H}{\partial R} \\
\frac{\partial}{\partial \theta_4} = \frac{\partial H}{\partial \theta_1} & , \quad \frac{\partial}{\partial \theta_5} = \frac{\partial H}{\partial \theta_2} & , \quad \frac{\partial}{\partial \theta_6} = \frac{\partial H}{\partial \theta_3} \\
\frac{\partial}{\partial \theta_7} = \frac{\partial H}{\partial \theta_4} & , \quad \frac{\partial}{\partial \theta_8} = \frac{\partial H}{\partial \theta_5} & \\
\frac{\partial}{\partial \theta_9} = \frac{\partial H}{\partial \theta_6} & \quad \frac{\partial}{\partial \theta_10} = \frac{\partial H}{\partial \theta_7} & , \quad \frac{\partial}{\partial \theta_11} = \frac{\partial H}{\partial \theta_8} \\
\frac{\partial}{\partial \theta_12} = \frac{\partial H}{\partial \theta_9} & , \quad \frac{\partial}{\partial \theta_13} = \frac{\partial H}{\partial \theta_10} & , \quad \frac{\partial}{\partial \theta_14} = \frac{\partial H}{\partial \theta_11} \\
\frac{\partial}{\partial \theta_15} = \frac{\partial H}{\partial \theta_12} & , \quad \frac{\partial}{\partial \theta_16} = \frac{\partial H}{\partial \theta_13} & , \quad \frac{\partial}{\partial \theta_17} = \frac{\partial H}{\partial \theta_14} \\
\text{hence the} \begin{array}{l}
\begin{array}{c}
0 = \frac{\partial}{\partial \theta_1} \\
0 = \frac{\partial}{\partial \theta_2} \\
0 = \frac{\partial}{\partial \theta_3} \\
\frac{\partial}{\partial \theta_4} = \frac{\partial}{\partial \theta_5} = \frac{\partial}{\partial \theta_6} = 0 \\
\frac{\partial}{\partial \theta_7} = \frac{\partial}{\partial \theta_8} = \frac{\partial}{\partial \theta_9} = \frac{\partial}{\partial \theta_10} = \frac{\partial}{\partial \theta_11} = \frac{\partial}{\partial \theta_12} = \frac{\partial}{\partial \theta_13} = \frac{\partial}{\partial \theta_14} = \frac{\partial}{\partial \theta_15} = \frac{\partial}{\partial \theta_16} = \frac{\partial}{\partial \theta_17} = 0
\end{array}
\end{array}
\end{align*}$$

Moreover, $\theta_j(T) = 0$, $j = 1, 2, ..., 8$. are the lagranges multipliers Eqs. (31) and (32) express the necessary condition in terms of a Hamiltonian for the OCP defined above.

6.1. Optimality conditions for fractional order

Let us write the Hamiltonian function as follows:

$$H(S_1, E_1, ..., R, v_1, \theta) = l + \frac{1}{2} \sum_{i=1}^3 (k_i v_i^2 + k_{2i} v_i^2 + k_{3i} v_i^2) + \theta_1[-(1 - v_1) (l_1 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7)]$$

where $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 = 0$ and the characteristic of the fractional optimal control given by (35) is obtained by solving the Eqn $\frac{d \theta_j}{dt} = 0$, $j = 1, 2, ..., 8$, on the interior of the control set and using the property of control space $v$. □

7. Conclusions

Different mathematical paradigms can provide considerable insights and scientific evidences pertinent to any ongoing epidemic dynamics. Based on those valuable information, health officials and public health experts can set up potential control strategies to battle against any epidemic. From the emergence of the novel coronavirus in China, researchers and scientists are working relentlessly to develop various mathematical modeling approaches to gain a deeper understanding on the progression dynamics of COVID-19 in the world. In addition, in the absence of any safe, effective and widely available COVID-19 vaccine, different preventive measures are the most effective tool in combating against the virus. On the basis of robust forecasting results of reliable epidemiological models, government officials can deploy different public health intervention strategies to control the rapid transmission of the virus. In this paper, a compartmental mathematical model has been designed to describe the transmission dynamics of the COVID-19 incorporating all possible real-life interactions and effective non-pharmaceutical interventions. Disease-free equilibrium (DFE) of the proposed model is found to be globally asymptotically stable (GAS), whenever control reproduction number ($R_c$) than less than unity. In addition, advanced forecasting techniques have also been applied for Bangladesh and India to portray the future dynamics of the pandemic in near term. It has been enlightened in our study that mass-level using of highly effective face coverings could be a crucial factor in controlling the spread of coronavirus. Moreover, strict social-distancing measures and comprehensive contact-tracing are also effective strategies in battling against this pandemic. The public health implication of these insightful findings is government officials can undertake crucial clinical and...
public health decisions by analyzing all mathematical results and scientific evidences. Caputo–Fabrizio non-integer order derivative has been applied to solve the proposed mathematical model in fractional sense. We proved the existence of unique solution for the proposed fractional initial value problem. We proved the unconditional stability of the given technique. An important concern of fractional optimal control problem is given to suggest the health care measures for reducing the transmissibility of COVID-19 infection in the population.

Declaration of Competing Interest

This work does not have any conflicts of interest.

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