Flow Mechanism of Granular Materials
Discharging from Bin-Hopper System

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Abstract

A new theory to estimate the state variables of flowing particles in a bin-hopper system was presented. The flowing behavior of particles in the system was simulated by the particle element method. It was suggested that convergence of the flow path of particles in a hopper leads to a change in the bulk density of flowing particles, whereby particles in a hopper flow intermittently. On the basis of the microscopic behavior of flowing particles observed through the simulation, a flow model was derived to analyze the flow of particles in a conical hopper. The values and fluctuation of dynamic pressure acting on the wall during the flow of particles in a bin-hopper system were estimated and the generation mechanism of overpressure during flowing of particles was also presented on the basis of the model.

1. Introduction

Conventionally, the flowing behavior of fine particles discharged from a bin-hopper system has been paid particular attention, and aggressively studied from different viewpoints. Although it was the object of such studies to find out a reasonable design for a bin-hopper system, interests in dynamic specificities found in such flowing behavior of fine particles constituted a strong motivation as well. The behavior of particles flowing in a bin-hopper system, in comparison with the flowing behavior of a fluid, shows such specificities that: (1) a very high pressure (overpressure) on hopper wall is generated immediately after initiation of flow; (2) the flow rate at discharge is independent of the height of the particle layer; (3) wall pressure and flow rate periodically pulsate; and (4) the flow rate is proportional to the orifice bore to the power 2.5 to 3.0. Regarding (1) and (2), although concepts of switching stress\(^1\) and dynamic arch\(^1\) were suggested, respectively, the position of the overpressure generated, the magnitude of the pressure and the physical meaning of dynamic arch remain unclear. Bransby et al.\(^2\) observed flowing particles during discharge by X-ray, and found that a slip line was periodically generated and dissipated, and a wall pressure thereby periodically fluctuated. However, the mechanism leading to such periodic generation of the slip line is unknown. A theory of Brown and Richards\(^3\) concerning (4) is relatively conceptual, and a relation with the flowing mechanism of particles is quite uncertain.

Thus, although a gravitational flow out of a bin-hopper system is the simplest manner of flowing in flowing phenomena of fine particles, the flowing mechanism is still unknown despite a lot of experimental and theoretical studies. It may represent a limit of conventional approach in dynamic studies of fine particles. Specifically in the flowing behavior of fine particles, an observable quantity is limited to a very macroscopic one, and an engineering approach by means of a mathematical model is, therefore, very difficult.

Accordingly, in this study, we conducted a two-dimensional simulation of the flowing behavior of particles in a bin-hopper system by using the particle element method, and first microscopically observed the flowing state. Then, based on the result of the observation, we suggested a new theory to estimate the state variables of flowing particles in a bin-hopper system. In order to prove that the model is reasonable and useful, we presented a method for predicting a wall pressure in the bin-hopper system and frequency of fluctuation (pulsation) of the pressure.
Computer simulation by the particle element method of the flowing behavior of particles discharged

Fine particles are an assembly of solid particles. Therefore, a particle constituting the assembly that is in motion has an independent rotational and translational displacement, and a discrete characteristic derived from them is found in the behavior of particles\(^{5}\). The particle element method\(^{4, 7-9}\) is a simulation taking such discrete characteristic of particles in consideration, and sufficiently expresses the intermittent flowing behavior specific to particles\(^{5}\).

Figure 1 is an example of the two-dimensional simulation by the particle element method of the behavior of particles flowing in a bin-hopper system which is provided with a hopper in its lower part. Material constants and computational conditions employed in the simulation are shown in Table 1\(^{1}\).

Table 1: Computational condition for simulation of flowing particles

| Parameter          | Value          |
|--------------------|----------------|
| Young’s modulus    | 7.36 \times 10^7 Pa |
| Poisson’s ratio     | 0.3            |
| Particle density    | 2650 kg/m\(^3\) |
| Coefficient of friction | 0.7 (between two particles) |
|                      | 0.176 (between particle and wall) |
| Radius of particle  | 5 mm           |
| Time step           | 1.0 \times 10^{-4} s |

* It is not required to use the material coefficient of a specific particle, because it is the object of the study to observe the general flowing behavior of particles. Thus, for facilitating a computational simulation, values shown in the table were used.

Figure (a) shows 1638 particles of equal size filled evenly in a bin-hopper system before initiation of flow. A straight line through the contact point between particles represents, by its length and direction, the magnitude and the direction of the force acting between the particles. A force acting against the wall of the system in a static state was generally consistent with the tendency observed in the distribution of particle pressures obtained by Jenssen’s formula. Figure (b) shows the state 0.015 sec after opening an orifice in the lower end of the hopper to allow the particles to flow out of the hopper. In Figure (b) and succeeding ones, in order to avoid complication, only straight lines representing the forces between the particles are shown. When the orifice is opened, particles within a triangular zone immediately above the orifice flow out, while the direction of the force between particles is changed from the vertical to the horizontal direction. A change from such active stress to passive stress is referred to as switching\(^{12}\), and it is known that the wall pressure in the location of switching is intermittent, as shown in Figure (c) and successive figures.

The boundary of the triangular zone is a collapsing position of the particle layer, that is, a so-called “slip line”. A flowing region surrounded by the slip line grows in the upper direction as time elapses, and the force against the wall of the hopper is much greater than that in a static state in the location where the slip line is generated. When the switching approaches the upper end of the hopper part (Figures (e) to (j)), a significantly high force acts in the vicinity of the connection between the upright wall and the hopper wall. The force is highest throughout the entire flowing process, and constitutes an overpressure that causes a problem immediately after discharge from the bin-hopper system. Referring to Figure (e) again, the particles below the slip line that is moving upward in the hopper are in a flowing state, and the interacting force between the particles and the wall is very low in such state. However, in Figure (f) showing a state immediately after the flow has increased, the force between particles in the vicinity immediately above the orifice is increased, and consolidation of particles has already begun in that area. Although the consolidated layer gradually grows upward (Figure (g)), when the layer reaches a certain height, the consolidated layer satisfies the collapsing conditions in a certain location thereof, and a collapsing flow is initiated (Figure (h)). Succeedingly, a similar consolidation starts in an area above that location (Figure (i)), and collapse is caused in that area (Figure (j)). Also in figures after Figure (j), it is observed that the particles in the hopper part are periodically subjected to the consolidation and collapse of the particle layer by the same mechanism during flowing (Figures (k) to (o)). Immediately before the collapse of the consolidated layer, the horizontal component of the force between the particles is increased, and the force against the wall becomes several times higher than that in the static state. On the contrary, after the collapse, it is observed that the force against the wall is significantly reduced in the flowing particles, and the wall pressure of the hopper periodically fluctuates due to generation and dissipation of the slip line.

In Figures 2 (a), (b) and (c), the distribution of the velocity of the particles in the states of Figures 1 (k), (l) and (o) is shown. The velocity of the particles during consolidation in the hopper part, as shown in Figure (k), is practically low, and it is confirmed in Figure (l) that the particles are in a flowing state, as described above, when the force...
Fig. 1  Two dimensional simulated results on the flowing behavior of particles in a bin

Fig. 2  Velocity distribution of flowing particles in a bin
between the particles is low. Figures 3 (a) and (b) show a fluctuation of the wall pressure during discharge of the particles through the connection between the wall of the hopper and the upright wall and in the vicinity of the orifice. The wall pressure periodically fluctuates in response to the periodic collapse of the consolidated layer (generation of the slip line) in the hopper part, and the fluctuation of the pressure is more significant, and the frequency of fluctuation is longer in the upper part of the hopper.

The simulation is of the flowing behavior of a small group of particles during a short period. However, the result of the simulation is sufficiently consistent with a lot of prior findings obtained through experiments of flowing particles that are discharged from a bin-hopper system. Specifically, the behavior of an entire particle layer was identical to the result obtained by Bransby et al. through a direct observation by X-ray. Thus, the microscopic behavior of flowing particles observed in the flowing and discharging simulation can be considered as being generally consistent with the basic flowing behavior of particles in a practical three-dimensional bin-hopper system. In other words, a practical flowing mechanism of particles in a bin-hopper system can be discussed by using the findings of this simulation.

2. Dynamic model of flowing behavior of particles

One of the important functions of a bin-hopper system is to discharge particles contained therein into a specific container. Therefore, the path of particles flowing in the bin-hopper system is necessarily reduced toward the outlet end. The result of the simulation described above shows that a bulk density of particles flowing in a bin-hopper system is changed with a geometric change of the path, and the constant flow of the particles is disturbed. In other words, it was found that the intermittent flow associated with consolidation and collapse of a particle layer was caused by a change in density, and the flow rate and the force against the wall thereby periodically fluctuated. By carefully examining the behavior of particles obtained through the simulation, such dynamic model as described below is established for the flowing behavior of particles discharged from a circular bin-hopper system that is most commonly used.

The stress of particles created before discharge can be approximated by a so-called active stress. As soon as discharge of the particles is initiated, a slip line is generated in the lower end, and switching to a passive state occurs. In a location where the slip line is generated, the wall of the system is subjected to a passive pressure higher than the pressure present in a static state. The slip line generated in a hopper part generally moves upwards, and the flow area is thereby increased. As described above, however, because the slip line is generated by the reduction of the flow path of the particles, the slip line in the upper end of the hopper, at which the reduction of the flow path begins, becomes the final one. At the beginning of the flow, particles in the cylindrical part above the final slip line are at a bulk density generally equivalent to that at initial loading, since they are still substantially immobile, and the connection between the cylinder and the hopper is, therefore, subjected to a very high passive pressure. This constitutes an overpressure at the beginning of the flow, and the higher bulk density at initial loading causes a higher pressure. It is clear that such overpressure is generated in a connection between the cylinder and the hopper at the beginning of the flow.

In the simulation, in order to clearly demonstrate the change in state during the flow of particles discharged from the system and generation of an over-
pressure, the particles were evenly loaded before discharge. Therefore, a transition period was required to form a structure of particles allowing a constant flow from the evenly loaded state. Figures 1 (a) to (k) correspond to the transition period, and a sufficient flowing condition is achieved in Figure (l). Accordingly, examination of the mechanism of constant flow in the bin-hopper system should be reasonably started from the state of Figure (l). Now, the bulk density of particles in a sufficient flowing condition in a bin-hopper system as shown in Figure 4 is defined by $\gamma_d$ and the mean vertical mass velocity of the particles in such condition by $U(\gamma_d)$. In such condition, as the flow path is reduced in the downward direction of the hopper, a consolidated particle layer of bulk density $\gamma_d$ is formed by particles in the proximity of the orifice, and the flow velocity is reduced [Figure 4 (b)]. Because the particles forming the consolidated layer are built up with an inertial force corresponding to the velocity $U(\gamma_d)$ from above, a horizontal force apt to separate the particles from each other dominantly acts on the particles that are falling, and the ratio of the radial component $\sigma_r$ to the vertical component $\sigma_z$ of the force is at $\sigma_r/\sigma_z > 1$. As soon as the consolidated particle layer reaches a height $h_2$ (Figure (c)), collapse conditions are met at a certain position within the consolidated layer, and slip collapse occurs. The slip line generated in the consolidated layer at that time is indicated by $a_1$-slip line entering from the right side of the maximum main stress, and the height required to create the critical stress $\sigma_{c1}$ on the slip line by $h_2$. After the collapse, particles below the $a_1$-slip line return to the flowing state of bulk density $\gamma_d$ again. On the other hand, because a consolidated layer of a lower flowing velocity is already formed at the height $h_2$ above the slip line before the collapse, an $a_2$-slip line is formed similarly in the layer, causing collapse of the layer. While the formation of consolidated layer and the slip collapse are sequentially transferred upward in such manner, an $a_3$-slip line is finally formed in the upper end of the wall of the hopper, at which the reduction of the flow path begins. The particles flowing in the hopper part come out of the hopper, as generation and dissipation of the slip line are periodically repeated according to the mechanism of density variation due to the reduction of the flow path. Here, an interval of generation of an $a_n$-slip line corresponds to the time required for a consolidated layer to be built up from $h_{n+1}$ to $h_n$. Therefore, the shape and interval of generation of a slip line depend on the shape of the bin-hopper system and on the characteristics of the particles. The fluctuation of the wall pressure caused by the slip line generated in the upper end of the hopper is highest and takes the longest interval, and causes the most significant effect on the flow in the hopper.

3. Prediction of pulsation frequency of flow rate and wall pressure

3.1 Shape and critical stress of slip line occurring in the hopper

Based on the model derived above, the pulsation frequency of the flow rate and wall pressure and the magnitude of the wall pressure are predicted. For this purpose, it is required first to obtain the shape of the slip line generated in the hopper part and the critical stress $\sigma_c$ acting on the slip line. Thus, assuming that a fine particle layer forms a continuous mass satisfying Coulomb's breakage criteria, the shape of the slip line is estimated using the method of Takahashi et al. In the hopper shown in Figure 5, the $r$- and $z$-axes coordinates are established, and the radius of the hopper at a given location is defined by $R$. Now, assuming that the mean main stress $\sigma_m$ acting on the slip line is constant for simplification, a coordinate of the-slip line generated in a consolidated
layer is given by:

\[
\frac{r}{R} = \frac{R_x}{R} \cdot \frac{F(\alpha_w, \alpha, \eta)}{F(\alpha_w, \alpha, \eta)} - \frac{F(\alpha_w, \alpha, \eta)}{F(\alpha_w, \alpha, \eta)} + 1
\]

\[
\frac{z}{R} = \frac{z_x}{R} - \left( \frac{R_x}{R} - 1 \right) \cdot \frac{G(\alpha_w, \alpha, \eta)}{F(\alpha_w, \alpha, \eta)}
\]

\[
\alpha_w = (\phi_{ws} + \chi_p)/2 + \theta, \quad \alpha_c = 0
\]

\[
\chi_p = \sin^{-1}\left( \frac{\sin\phi_{ws}}{\sin\phi_s} \right), \quad \eta = \frac{\pi}{4} - \frac{\phi_s}{2}
\]

where

\[
F(x, y, \eta) = (x-y) \cos 2\eta - \sin 2\eta \ln |\cos(y-\eta)|/\cos(x-\eta)\]

\[
G(x, y-\eta) = (x-y) \sin 2\eta + \cos 2\eta \ln |\cos(y-\eta)|/\cos(x-\eta)\]

\(\alpha\) is the angle between the plane of the maximum main stress and the z-axis, \(\alpha_w\) and \(\alpha_c\) are boundary values at the wall and central axis, respectively, and \(\phi_s\) and \(\phi_{ws}\) are static friction angles of the particle layer and the wall.

In the central axis of the hopper, the critical stress \(\sigma_c\) acting on the slip line is a vertical stress, that is, the minimum main stress, and is given by:

\[
\sigma_c = \frac{r_x R (1 - \sin \phi_s)}{2 \sin \phi_s F(\alpha_w, \alpha, \eta)}
\]

Thus, using the friction angle \(\phi_s\) of a consolidated layer, the friction angle \(\phi_{ws}\) of the wall, and an open angle \(\theta\) of the hopper, the shape and critical stress \(\sigma_c\) of the slip line generated at a given location of the hopper part can be predicted.

3.2 Height \(h_n\) of consolidated particle layer creating critical stress \(\sigma_c\)

The mean vertical stress \(\bar{\sigma}_z\) acting in a given plane within the bin-hopper system is obtained by Walters’s formula for predicting a particle pressure\(^{12}\), and given for the cylindrical part and hopper part, respectively, by the following formulae.

Cylindrical part

\[
\frac{\bar{\sigma}_z}{\gamma gd} = \frac{1}{4BD} (1 - \phi^{4BD(z/d)})
\]

Hopper part

\[
\frac{\bar{\sigma}_z}{\gamma gd} = \frac{1 - 2 (z/d) \tan \theta}{2 \tan \theta (K-1)} \left( 1 - (1 - 2(z_0/d) \tan \theta)^{K-1} \right)
\]

\[
+ \frac{\sigma_{z,0}}{\gamma gd} \left( 1 - 2 (z_0/d) \tan \theta \right)^K
\]

where

\[
BD = \frac{\tan \phi_w \cos^2 \phi}{1 + \sin^2 \phi} \pm 2y \sin \phi, \quad y = \frac{2}{3} \left( 1 - (1 - c)^{3/2} \right)
\]

\[
c = \left( \frac{\tan \mu}{\tan \phi} \right)^2
\]

\[
D = \cos \mu \left( 1 + \sin^2 \phi \cos \phi \right) - 2 \left( \sin^2 \phi - \sin^2 \mu \right)^{1/2}
\]

\[
\cos \mu \left[ \cos \phi \pm y \sin \phi \right]
\]

\[
K = 2 \left( \frac{ED}{\tan \theta} + D - 1 \right)
\]

\[
\mu = \tan^{-1} \frac{\sin \phi \sin \left( 2\phi + \theta \right)}{1 + \sin \phi \cos \left( 2\phi + \theta \right)}
\]

\[
E = \frac{\cos \phi \sin \left( 2\phi + \theta \right)}{1 + \sin \phi \cos \left( 2\phi + \theta \right)}, \quad \phi = \frac{\phi_w + \chi_p}{2}
\]

\(BD\) is equal to the coefficient of particle pressure of Janssen multiplied by the coefficient of friction of the wall \((\tan \phi_w)\), and \(\sigma_{z,0}\) is a downward created stress. \(E\) is the ratio of the vertical to the shearing stresses, and \(D\) the coefficient of distribution for correlating the vertical stress of the wall and the mean vertical stress. They are obtained by a geometric study of Mohr’s circle shown in Figures 6 (a) and (b), and the positive symbol is for a flowing range, while the negative values are for a consolidated particle layer, considering the ratio of particle pressure. From Mohr’s circle for a consolidated layer, the mean vertical stress \(\bar{\sigma}_{z'}\) in a sectional plane creating the critical stress \(\sigma_c\) in the central axis of the bin-hopper system is given by:

\[
\frac{(\sigma_{z})_{T}}{\gamma gd} = \frac{1 + \sin^2 \phi_s - 2 \sin \phi_s \cdot Y_s}{1 + \sin^2 \phi_s - 2 \sin \phi_s}
\]

Thus, using the friction angle \(\phi_s\) of a consolidated layer, the friction angle \(\phi_{ws}\) of the wall and an open angle \(\theta\) of the hopper, the shape and critical stress \(\sigma_c\) of the slip line generated at a given location of the hopper part can be predicted.
The maximum wall pressure $\sigma_w$ at formation of a consolidated layer can be predicted from $(\sigma_c)_T$ in by the following formula.

$$
\sigma_w = \frac{ED(\sigma_c)_T \sin 2\varepsilon}{\sin (2\theta + 2\varepsilon) \tan \phi_{ws}}
$$

Then, by using formulae, the height of the consolidated layer creating the critical stress $\sigma_c$ at a given location within the bin-hopper system can be calculated in the following manner. Now, it is supposed that a consolidated particle layer is formed from the upper end of the hopper to a position $z_3$ in a bin-hopper system shown in Figure 7, and an $a_w$-slip line is generated at a position $z_4$. First, the mean particle pressure $\sigma_{w1}$ in the bottom part of the cylinder is calculated by substituting the characteristics of the flowing particle in Eq. (4). Then, by using the positive symbol in Eq. (5), and taking $\sigma_{z1}$ as the load stress, the mean stress $\sigma_{z3}$ acting in a plane at a depth $z_3$ is obtained. Finally, by taking $\sigma_{z3}$ as the load stress, and using the negative symbol and the characteristic values of the consolidated particle layer, $\sigma_{z4}$ acting in the consolidated layer at a depth $z_4$ is obtained. Although the vertical stress at a given depth and height of a consolidated layer can be obtained in such manner, a static fine particle layer under a critical stress has been analyzed above. However, in the flow model, the flow velocity of particles changes as the consolidated layer is formed, and it is required to take the dynamic pressure $\sigma_D$ associated with the change into consideration. Then, by assuming that the particles at a mean flow velocity $\bar{U}$ are decelerated and momentarily stopped in the consolidated layer, $\sigma_D$ is given from the change of momentum by:

$$
\sigma_D = \frac{\gamma_4 \bar{U}}{3} \cdot \frac{(z_0 - z_4 + h)^3 - (z_0 - z_4)^3}{(z_0 - z_4)^2}
$$

Then, by repeating the numerical calculation, $h_n$ is obtained when the particle pressure $(\sigma_{z4})_E$ given by the sum of Eqs. (4), (5) and (8) and the particle pressure $(\sigma_{z4})_T$ equal to the critical stress $\sigma_c$ meet the condition of the following formula, and are substantially equal.

$$
| (\sigma_{z4})_T - (\sigma_{z4})_E | \leq 10^{-5}
$$

3.3 Prediction of pulsation frequency $f$

The pulsation frequency $f$ is the inverse of the building-up time of the consolidated particle layer sufficient for the creation of the critical stress, and
Fig. 8 Calculated slip line and waveforms of dynamic pressure acting on hopper wall is given by the following formula, where \( W \) is the volume of the consolidated particle layer determined by \( h_n \).

\[
f = \frac{\pi \bar{U} \gamma_d \left( (z_0 - z_4) \tan \theta \right)^2}{W (\gamma_s - \gamma_d)}
\]  

(10)

In this way, the pulsation frequency and wall pressure at a given location in the hopper part can be predicted by using the geometric characteristics (\( \theta \), \( d \), \( R_0 \), \( L \)) of the bin-hopper system and characteristics (\( \gamma_s \), \( \gamma_d \), \( \phi_s \), \( \phi_d \), \( \phi_{ws} \), \( \phi_{wd} \)) of the particles.

Figure 8 shows a computational example of quartz sand (\( \gamma_s = 2650 \, \text{kg/m}^3 \)) flowing out of a small bin-hopper system at \( \theta = 20^\circ \), \( R_0 = 15 \, \text{mm} \), \( d = 0.5 \, \text{m} \), \( L = 0.6 \, \text{m} \). The pulsation frequency at the connection between the cylindrical and the hopper parts is 1.11 Hz, and is consistent with the component of low frequency observed with such small bin-hopper system\(^6\). The pulsation frequency immediately above the orifice is not as high as that in the lower part of the hopper at 3.42 Hz, showing a consistency with the tendency observed in the simulation. Since the shape of the bin-hopper system, specifically, an opening angle \( \theta \) and orifice bore \( R_0 \) determine the mean flow velocity \( U \) of particles flowing in the system, and the characteristics of the flow and those of the consolidated particle layer are determined by the velocity, a proper prediction of the characteristics of particles is essential. A detailed comparison of the pulsation frequency, flow rate and fluctuation of wall pressure according to the flow model suggested herein with actual measurements shall be reported separately.

**Conclusion**

We suggested a dynamic model for a flow mechanism of particles flowing in a bin-hopper system, and presented a method for predicting the particle pressure during discharge and the pulsation frequency of particles according to the model. First, a two-dimensional simulation was conducted for the flowing behavior of particles by using the particle element method. The result of the simulation was sufficiently consistent with prior findings experimentally known, and it was demonstrated that the flowing behavior of individual particles in a flowing state and the quantity of state can be microscopically observed using the simulation. Then, a model of flowing mechanism was obtained through a careful observation of the flowing behavior of particles using the simulation, and it was demonstrated that the flowing behavior of particles in a bin-hopper system can be dynamically analyzed by predicting the particle pressure fluctuating in a flowing state of the particles and the pulsation frequency as an example for confirming the usefulness of the model.

**Nomenclature**

- \( d \) = diameter of hopper [m]
- \( f \) = frequency of fluctuation of wall pressure [Hz]
- \( g \) = gravitational acceleration [m/s²]
- \( h \) = height of dense packed bed [m]
- \( L \) = height of cylindrical part in bin [m]
- \( R \) = radius of hopper [m]
- \( t \) = time [s]
- \( U \) = mean vertical velocity [m/s]
- \( W \) = volume of dense packed bed [m³]
- \( z \) = distance from upper surface of hopper [m]
- \( \alpha \) = angle between major principal plane and vertical axis in bin [deg]
- \( \gamma \) = bulk density [kg/m³]
- \( \eta \) = angle between failure plane and minor principal plane [deg]
- \( \theta \) = half angle of hopper [deg]
- \( \sigma \) = normal stress [Pa]
- \( \tau \) = shear stress [Pa]
- \( \phi \) = effective angle of friction [deg]

\(<\text{Subscript}>\)

- \( c \) = critical state
- \( D \) = dynamic condition
- \( d \) = dense packing state
- \( E \) = estimated value
- \( m \) = mean value
- \( o \) = orifice
= the theoretical value
r = radial direction
s = static condition (loose packing state)
w = wall
z = vertical direction

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