EXTENDED SIEGERT THEOREM IN THE RELATIVISTIC INVESTIGATION OF THE DEUTERON PHOTODISINTEGRATION REACTION

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The contribution of the two-body exchange current is investigated for the reaction of the deuteron photodisintegration in the framework of the Bethe-Salpeter formalism and with using extended Siegert theorem. This theorem allow to express the reaction amplitude in terms of extended electric and magnetic dipole moments of the system. The resultant analytical expression is faultless with respect to both translation and gauge invariance. It permits to perform calculations of the deuteron photodisintegration cross section and polarization observables taking into account two-body exchange current implicitly.

Keywords: deuteron photodisintegration, Siegert theorem, Bethe-Salpeter formalism.

I. INTRODUCTION

One of the most important method of the nucleon-nucleon interaction investigation is the reaction of the electrons and photons scattering on the deuteron. In the previous works [1], [2], [3] the reaction of deuteron photodisintegration has been studied on basis of the Bethe-Salpeter formalism within the framework of one-body approximation. Final state interaction has been considered besides. However it was insufficient for complete description of experimental data in the wide range of energy. With increasing of photon energy the significant contribution to differential cross section comes from meson exchange currents. An appropriate method of description of such effects is the extended Siegert theorem [4], [5]. Let’s consider it briefly.

II. THE EXTENDED SIEGERT’S THEOREM

Generalization of Siegert’s theorem is made without conventional decomposition of the EM current into two parts – the convection current associated with the motion of nucleus as a whole and the intrinsic current [6].

First on needs to construct the matrix element for the absorption of a real photon with three-momentum \( q \), while the nuclear system makes a transition from an internal state \( |P_i\rangle \) to a final internal state \( |P_f\rangle = |P_i + q_f\rangle \).

The required matrix element is given by

\[
S_{gf} = \int d^4x \langle P_f f | j_\mu (x) | P_i i \rangle \langle 0_\gamma | A^\mu (x) | 1_\gamma \rangle. \tag{1}
\]

Applying the invariance with respect to four translation we may write

\[
j_\mu (x) = e^{iP \cdot x} j_\mu (0)e^{-iP \cdot x}, \tag{2}
\]

where \( P \) is the operator of total four momentum of the system, i.e.

\[
P|P_i i\rangle = P_i|P_i i\rangle, \quad P|P_f f\rangle = P_f|P_f f\rangle. \tag{3}
\]

Introducing Eq. (2) into Eq. (1) we find

\[
S_{gf} = \langle P_f f | j_\mu (0) | P_i i \rangle \langle 0_\gamma | A^\mu (P_f - P_i) | 1_\gamma \rangle. \tag{4}
\]
with

\[ A^\mu(q) = \frac{1}{\sqrt{2\omega}} \sum_{s=0,1,2,3} \epsilon_\mu^s(q) (a_s(q) + a_s^+(q)) e^{i\omega t}, \]

(5)

where \( a_s(q) \) and \( a_s^+(q) \) are the destruction and creation operators, respectively, for a photon of three-momentum \( q \) and unit four-polarization vector \( \epsilon \). The transversality condition is \( \epsilon \cdot q = 0 \).

Now we will separate the matrix element into two parts using the identity

\[ \epsilon e^{-i\mathbf{q}\cdot\mathbf{x}} = \int_0^1 d\lambda \left\{ \nabla \cdot \left( \epsilon \mathbf{x} e^{-i\lambda \mathbf{q}\cdot\mathbf{x}} \right) + i\omega \mathbf{x} \times \epsilon e^{-i\lambda \mathbf{q}\cdot\mathbf{x}} \right\}, \]

(6)

where \( \epsilon \) and \( \epsilon' = \mathbf{q} \times \epsilon/\omega \) represent the unit electric and magnetic polarization vectors, respectively. This identity is equivalent to a gauge transformation of the EM potential in QED

\[ A^\mu_T(x) \rightarrow A^\mu(x) = A^\mu_T(x) - \partial_\mu \Lambda(x), \]

(7)

where the superscript \( T \) on \( A \) indicates it is in the transverse gauge, i.e. \( A^T = (0, \epsilon^T e^{-i\mathbf{q}\cdot\mathbf{x}}) \), and \( \Lambda(x) \) is such an function of \( x \) that the current has the Siegert limit. Suppressing the time dependence it means that

\[ \lim_{\omega \rightarrow 0} \Lambda(x) = \epsilon \cdot x. \]

(8)

Thus the identity (6) is a gauge transformation with the following choice for \( \Lambda(x) \) (Foldy gauge [7])

\[ \Lambda(x) = \epsilon \cdot x \int_0^1 d\lambda \epsilon e^{-i\lambda \mathbf{q}\cdot\mathbf{x}} \cong \epsilon \cdot \mathbf{x} (1 - \frac{1}{2} \mathbf{q}\cdot\mathbf{x} - \frac{1}{6} \mathbf{q}\cdot\mathbf{q}^2 + O(\mathbf{q}^3)). \]

(9)

In principle, it should make no difference for the final result at low energies. However, in approximate many-body calculations with \( j_\mu \) not necessarily conserved, the choice of gauge does become important. The Foldy gauge has the good theoretical property that \( A(x) \) projects out from \( j(x) \) only the magnetic part, i.e. all magnetic effects are contained in \( j_m(x) \). It means that the knowledge of the nonrelativistic one-body nucleon charge density \( \rho(1) \) is optimized in the Foldy gauge.

Substituting Eq. (6) into Eq. (4), one has

\[ \langle \mathbf{P}_f \mid j_\mu(0) \mid \mathbf{P}_i \rangle = i(E_f - E_i) D_{ij}(q) - i\mathbf{q} \times M_{ij}(q), \]

(10)

where \( q = P_f - P_i \), \( E_i \) and \( E_f \) is the total energy of the nuclear system in the initial and final states. The quantities \( D_{ij}(q) \) and \( M_{ij}(q) \) has the form

\[ D_{ij}(q) = \int_0^1 d\lambda \int d\mathbf{x} \rho_{ij}(\mathbf{x}; \mathbf{P}_i) e^{i\lambda \mathbf{q}\cdot\mathbf{x}}, \]

(11)

\[ M_{ij}(q) = \int_0^1 d\lambda \int d\mathbf{x} \lambda \left[ \mathbf{x} \times j_{ij}(\mathbf{x}; \mathbf{P}_i) \right] e^{i\lambda \mathbf{q}\cdot\mathbf{x}}. \]

(12)

with

\[ \langle \mathbf{P}_f \mid j(0) \mid \mathbf{P}_i \rangle = \int d\mathbf{x} e^{i(\mathbf{P}_f - \mathbf{P}_i) \cdot \mathbf{x}} j_{ij}(\mathbf{x}; \mathbf{P}_i), \]

(13)

\[ \langle \mathbf{P}_f \mid \rho(0) \mid \mathbf{P}_i \rangle = \int d\mathbf{x} e^{i(\mathbf{P}_f - \mathbf{P}_i) \cdot \mathbf{x}} \rho_{ij}(\mathbf{x}; \mathbf{P}_i). \]

(14)

Reversing Eqs. (14) and (13), we verify that

\[ j_{ij}(\mathbf{x}; \mathbf{P}_i) = \frac{1}{(2\pi)^3} \int d\mathbf{p} e^{-i\mathbf{p}\cdot\mathbf{x}} (\mathbf{p} + \mathbf{P}_i f|j(0)|\mathbf{P}_i \rangle), \]

(15)

\[ \rho_{ij}(\mathbf{x}; \mathbf{P}_i) = \frac{1}{(2\pi)^3} \int d\mathbf{p} e^{-i\mathbf{p}\cdot\mathbf{x}} (\mathbf{p} + \mathbf{P}_i f|\rho(0)|\mathbf{P}_i \rangle). \]

(16)
From Eqs. (11) and (14) we immediately find

$$\mathbf{q} \cdot \mathbf{D}_{ij}(\mathbf{q}) = \langle \mathbf{P}_f f | \rho(0) | \mathbf{P}_i i \rangle - \langle \mathbf{P}_i f | \rho(0) | \mathbf{P}_f i \rangle. \quad (17)$$

At this point we are ready to write the $S$-matrix element in terms of the matrix elements operators $\mathbf{D}(\mathbf{q})$ and $\mathbf{M}(\mathbf{q})$. Substituting Eq. (10) into Eq. (4) and using the relation (17) we obtain

$$S_{ij}^\gamma = \langle \mathbf{P}_i f | \rho(0) | \mathbf{P}_i i \rangle \langle 0, \gamma | \phi(q) | 1, \gamma \rangle + i \mathbf{q} \cdot \mathbf{M}_{ij}(\mathbf{q}) \cdot \langle 0, \gamma | \mathbf{A}(q) | 1, \gamma \rangle \cdot \mathbf{D}_{ij}(\mathbf{q}) \quad (18)$$

Finally introducing the strength of the electric and magnetic fields (in momentum space)

$$\mathbf{E}(\mathbf{q}) = \omega \mathbf{A}(q) - \mathbf{q} \phi(q), \quad (19)$$

$$\mathbf{H}(\mathbf{q}) = \mathbf{q} \times \mathbf{A}(q), \quad (20)$$

we cast the $S$-matrix element in the manifestly gauge independent form

$$S_{ij}^\gamma = -\langle 0, \gamma | \mathbf{E}(\mathbf{0}) | 1, \gamma \rangle \cdot \mathbf{D}_{ij}(\mathbf{q}) - \langle 0, \gamma | \mathbf{H}(\mathbf{0}) | 1, \gamma \rangle \cdot \mathbf{M}_{ij}(\mathbf{q}) \quad (21)$$

The scattering amplitude for the photon absorption is written in gauge independent form

$$T_{ij}^\gamma = -\langle 0, \gamma | \mathbf{E}(\mathbf{0}) | 1, \gamma \rangle \cdot \mathbf{D}_{ij}(\mathbf{q}) - \langle 0, \gamma | \mathbf{H}(\mathbf{0}) | 1, \gamma \rangle \cdot \mathbf{M}_{ij}(\mathbf{q}) \quad (22)$$

with

$$\langle 0, \gamma | \mathbf{E}(\mathbf{0}) | 1, \gamma \rangle = \frac{1}{\sqrt{2\omega}} \left( \omega \varepsilon(q) - q \varepsilon_0(q) \right), \quad (23)$$

$$\langle 0, \gamma | \mathbf{H}(\mathbf{0}) | 1, \gamma \rangle = \frac{1}{\sqrt{2\omega}} \mathbf{q} \times \varepsilon(q). \quad (24)$$

The first term in Eq. (21) is equal to zero, since it is proportional to the matrix element of the total charge between orthogonal states. This equation is consistent with requirements of gauge invariance as well as invariance with respect to translations.

### III. APPLICATION TO RELATIVISTIC FORMALISM

Now we shall write down $T$-matrix in Siegert’s form within Bethe-Salpeter formalism (21)

$$T_{ij}^\gamma = -\frac{1}{\sqrt{2\omega}} \left( \omega \varepsilon(q) - q \varepsilon_0(q) \right) \cdot \mathbf{D}_{ij}(\mathbf{q}) - \frac{1}{\sqrt{2\omega}} |q \times \varepsilon(q)| \cdot \mathbf{M}_{ij}(\mathbf{q}). \quad (25)$$

The operators $\mathbf{D}_{ij}(\mathbf{q})$ and $\mathbf{M}_{ij}(\mathbf{q})$ could be obtained from Eqs. (11) and (12) with the use of Eqs. (15) and (10)

$$\mathbf{D}_{ij}(\mathbf{q}) = -i \int_0^1 \frac{d\lambda}{\lambda} \nabla_{\mathbf{q}} \langle \lambda \mathbf{q} + \mathbf{P}_i f | \rho(0) | \mathbf{P}_j i \rangle, \quad (26)$$

$$\mathbf{M}_{ij}(\mathbf{q}) = -i \int_0^1 \frac{d\lambda}{\lambda} \nabla_{\mathbf{q}} \times \langle \lambda \mathbf{q} + \mathbf{P}_i f | \mathbf{j}(0) | \mathbf{P}_j i \rangle. \quad (27)$$

Substituting expression for the matrix element of the EM current between two relativistic states

$$\langle \mathbf{P}_f f | j^\mu(0) | \mathbf{P}_i i \rangle = i \int \frac{dsdk}{(2\pi)^8} \bar{\chi}_{P_f}(s) \Lambda(\mathbf{s}, \mathbf{k}; \mathbf{P}_f, \mathbf{P}_i) \chi_{P_i}(k), \quad (28)$$

into Eqs. (26) and (27) we obtain

$$\mathbf{D}_{ij}(\mathbf{q}) = \int_0^1 \frac{d\lambda}{\lambda} \nabla_{\mathbf{q}} \int \frac{dsdk}{(2\pi)^8} \bar{\chi}_{P_f}(s) \Lambda(\mathbf{s}, \mathbf{k}; \mathbf{P}_f, \mathbf{P}_i) \chi_{P_i}(k), \quad (29)$$

$$\mathbf{M}_{ij}(\mathbf{q}) = \int_0^1 d\lambda \nabla_{\mathbf{q}} \times \int \frac{dsdk}{(2\pi)^8} \bar{\chi}_{P_f}(s) \Lambda(\mathbf{s}, \mathbf{k}; \mathbf{P}_f, \mathbf{P}_i) \chi_{P_i}(k). \quad (30)$$
Using the transverse gauge in calculations, i.e. $\varepsilon_0^T = 0$ and $\varepsilon^T \cdot q = 0$, one finds that

$$T^\gamma_{if} = -\sqrt{\frac{\omega}{2}} \varepsilon^T_{\rho} \cdot D_{if}(q) - \frac{1}{\sqrt{2}} \varepsilon^T_{\rho} \cdot M_{if}(q), \quad \rho = \pm 1. \quad (31)$$

One concludes that this matrix elements gives response of the nuclear system in two transverse perpendicular directions (defined by the three-vectors $\varepsilon$ and $\varepsilon'$) with respect to the photon three-momentum $q$.

### IV. RESULTS AND DISCUSSIONS

The results of our calculations are depicted in the Fig. 1 and Fig. 2. The calculations has performed in nonrelativistic (Shrodinger equation) and relativistic (Bethe-Salpeter equation) models. In both cases we use Graz II potential of nucleon-nucleon interaction and carry out the computations with and without including two-body current effectively via extended Siegert theorem. We haven’t taken into account final state interaction in this work. Notations are explained in the figure’s captions.

It is seen in the first plot of Fig. 1 that two-body effects give a large contribution to the differential cross section even at the photon energy equal to 20 MeV. One-body approximation is 30% less than experimental data. Calculations with using extended Siegert theorem allow to agree with experimental data.

Relativistic effects give larger contribution to two-body current than to one body current. We can see that practically for all the plots at the figures, particularly for $T_{22}$ at 200 MeV. For $T_{20}$ we can see large contribution from relativistic effects even at photon energy equal to 20 Mev.

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[1] K.Yu. Kazakov and S.Eh. Shirmovsky, Phys. Rev. C. 2001. V. 63. P. 014002.
[2] K. Yu. Kazakov and D. V. Shulga, Phys. Rev. C. 2002. V. 65. P. 064002.
[3] S. G. Bondarenko, V. V. Burov, K. Yu. Kazakov, D. V. Shulga. Phys. Part. Nucl. Lett. 2004. V. 1. N. 4(121). P. 17.
[4] A.F. Siegert, Phys. Rev. 52 (1937) 787.
[5] J.L. Friar and S. Fallieros, Phys. Rev. C29 (1984) 1645.
[6] A.V. Shebeko, J. Nucl. Phys. 49 (1989) 1.
[7] L.L. Foldy, Phys. Rev. 92 (1953) 178.
[8] H. Arenhövel and M. Sanzone, Few Body Syst. Suppl. 3 (1991) 1.
FIG. 1: Differential cross section and tensor target asymmetry $T_{20}$. Notation of the curves: nonrelativistic one-body current (long-dashed); nonrelativistic two-body current (dash-dotted); relativistic one-body current (dotted); relativistic two-body current (full). Experimental data is taken from [8].
FIG. 2: Tensor target asymmetry $T_{22}$ and total cross section. Notation of the curves: nonrelativistic one-body current (long-dashed); nonrelativistic two-body current (dash-dotted); relativistic one-body current (dotted); relativistic two-body current (full).