The development of statistical analysis methods for the study of correlations and statistical memory effects in the recorded data of physical experiments

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Abstract. In this paper, we discuss the prospects for the use of statistical analysis methods in the study of correlations and statistical memory effects in the experimental data of physical experiments. Developed methods under consideration: Memory Functions Formalism and Flicker-Noise Spectroscopy allow obtaining a large set of quantitative parameters and qualitative characteristics directly from temporal signals generated by complex physical systems. Additionally, for the study of collective phenomena and effects, one- and two-parameter cross-correlation functions are proposed, which allow studying cross-correlations between simultaneously recorded signals in spatially separated areas of the object under study. The introduced analytical relations and numerical algorithms can be applied to solve the problems of metrology of surface structures at the nano- and microlevels, to study collective phenomena in the plasma of astrophysical objects, the structure of molecules, and their complexes.

1. Introduction. Physics of complex systems

In the second half of the 20th century, interdisciplinary areas that can be described as “sciences of complexity” acquired relevance. Beginning with the works of Henri Poincaré, the theory of dynamical systems has been actively used in the study of nonlinear physical systems that are sensitive to even small changes in the initial parameters. Later, the theory of dynamic chaos, the theory of catastrophes, the theory of strange attractors appeared in this area, developing a mathematical apparatus for describing a wide range of natural phenomena [1, 2]. Starting with the work of Ludwig von Bertalanffy, general systems theory outlined methods and approaches to the study of processes, regardless of their nature, based on the discovery of relationships between parts of complex systems, as well as their reactions to external influences [3]. “Black box” cybernetics and neurocybernetics contributed to the development of artificial intelligence and machine learning systems [4, 5]. Due to the mutual penetration of these areas and the wide dissemination of the results obtained in the scientific community, it became possible to study the unique properties of complex both physical and non-physical systems.

The scientific literature highlights such properties of complex systems as complexity, openness, nonlinearity, adaptability, emergence, self-organization, and critical transitions [6, 7]. A complex system is understood as an object consisting of a significant number of interacting elements. The interaction between the parts of the whole leads to the enumerated unique properties that make systems “difficult” to study by classical methods. When studying such an interaction, special attention is paid
to collective effects, statistical memory, and correlations. Collective phenomena have a significant impact on the evolution of complex systems: from the physics of accelerators (beam intensity limitation) and condensed matter physics (collective effects) to collective behavior in wildlife [8].

2. The application of Memory Functions Formalism and Flicker-Noise Spectroscopy in the analysis of autocorrelations and statistical memory effects

In this paper, we discuss the prospects for using the author's methods: Memory Functions Formalism (MFF) and Flicker-Noise Spectroscopy (FNS) for the analysis of correlations and statistical memory effects in the experimental data of physical experiments. In addition, cross-correlators are discussed, which allow analyzing cross-correlations in simultaneously fixed sets of experimental data.

Figure 1 shows the theoretical methods and numerical algorithms developed by the team of authors, as well as the results of applying these methods in the analysis of the temporal dynamics of complex physical and non-physical systems.

![Diagram showing theoretical approaches and numerical algorithms](image)

**Theoretical approaches and numerical algorithms**

- Memory Functions Formalism
- Flicker-Noise Spectroscopy

**Results**

- Analysis of fractal dimensions
- Possibilities of probability theory and mathematical statistics

- Quantitative description of temporal dynamics and evolution of complex systems – «parameterization» of time series
- Detection and analysis of synchronization effects and collective phenomena of mutual dynamics of complex systems’ components
- Identification of precursors or predictors of sudden evolutionary changes or structural changes in complex systems

Figure 1. Developed methods for the analysis of experimental data sets of physical experiments and the results obtained using them.

2.1. Memory Functions Formalism

Memory Functions Formalism is a finite-difference generalization of the Zwanzig-Mori projection formalism [9, 10] for analyzing the discrete dynamics of complex systems. The method is based on the representation of the temporal dynamics of the process under study in the form of a multidimensional state vector that obeys the equation of motion written in a discrete form. The use of the Zwanzig-Mori projection technique and the Gram-Schmidt orthogonalization procedure allows shortening the description. Within the framework of the method, for the studied time series, a chain of finite-
difference linked kinetic equations of the Zwanzig-Mori type is constructed for the time correlation function and statistical memory functions for interrelated variables [11–13]. Due to the presented concept, it becomes possible to describe the aftereffects at different relaxation levels. The method provides a large set of characteristics and dependences: time dependences of orthogonal dynamic variables, phase portraits of combinations of dynamic variables, relaxation and kinetic parameters, statistical memory functions and their power spectra, frequency dependences of statistical memory measures calculated directly from sequences of dynamic variables of complex systems.

In [14, 15] correlations and statistical memory effects were studied in the sequence of Wolf numbers as one of the quantitative characteristics of solar activity, as well as the X-ray intensity of the GRS 1915+105 microquasar. The results obtained reveal individual physical mechanisms of X-ray generation, the energy released during accretion in the GRS 1915+105 binary system, and the formation of sunspots in the solar photosphere.

Subsequently, the MFF was adapted to the study of the statistical memory effects and cross-correlations in the dynamics of time series of simultaneously recorded experimental parameters characterizing the evolution of complex systems [16].

2.2. Flicker-Noise Spectroscopy
Flicker-Noise Spectroscopy is a method based on the representation of time signals as a combination of jump irregularities (low-frequency components) and burst irregularities (high-frequency chaotic components) [17–19]. These irregularities manifest themselves in correlation relationships at different levels of the spatio-temporal hierarchy. For this, the corresponding procedures and algorithms for parameterization are formulated in the form of general phenomenological expressions for the difference moments and power spectra. The method allows obtaining information about both low-frequency resonant and high-frequency chaotic components of the considered signals using power spectra and transient “structural” functions, as well as a set of parameters characterizing the duration of correlation times, the loss of correlations in sequences of irregularities of the studied signals.

The use of the FNS to introduce “passport parameters” of chaotic surface structures, i.e. roughness profiles studied by atomic force microscopy, makes it possible to approach the solution of problems of surface metrology at nanoscales [20]. In [21] the “certification” of sufficiently homogeneous surfaces of lithium fluoride single crystals grown from a melt was carried out. In [22] a study was made of inhomogeneous surfaces, which can be formed, for example, during the destruction of the material, in the processes of crystallization on the surface, chemical modification.

The FNS approach allows obtaining direct information about the dynamics of correlation relationships between simultaneously recorded signals – dynamic variables of the same essence, measured at spatially separated points of a complex system. In [23, 24] authors study the cross-correlations of the time signals of the radio emission of quasars recorded in different frequency ranges, as well as various indicators of solar activity. An analysis of the X-ray emission of microquasars, which is produced mainly by the inner hot layers of accreting disks, allows obtaining information about the dynamics of the processes occurring in these regions. The study of the dynamics of solar activity indicators allows us to establish the effects of frequency-phase synchronization and periodic patterns at different time intervals.

3. Conclusion
The methods of statistical analysis being developed: the Memory Functions Formalism and Flicker-Noise Spectroscopy, as well as the numerical algorithms and procedures obtained within their framework for calculating parameters, characteristics, and graphical dependencies, are based on the author's theoretical concepts and applied results in the field of nonequilibrium statistical physics, as well as physics of complex systems [25, 26]. The physics of complex systems is aimed at finding physical patterns in describing the behavior and properties of complex systems.
The Memory Functions Formalism allows introducing a set of statistical markers – information measures of memory, kinetic and relaxation parameters, linear measures of synchronization, which are determined directly from experimental signals, and to describe the behavior of these characteristics at different time intervals (scales). The results obtained contribute to the formation of detailed ideas about the role of statistical memory effects, correlations, and fluctuations of the initial time sequences in the description of the dynamic states of complex physical systems.

To extract sufficient quantitative information to make it possible to judge the nature of frequency-phase synchronization, the Flicker-Noise Spectroscopy method is used with two-parameter dependences of cross-correlators calculated based on digitized signals from experimental data sets recorded simultaneously at different frequencies. In addition, parameters introduced in the framework of the FNS allow detecting chaotic and resonant components of the initial discrete time series.

The proposed methods can be effectively applied to the study of autocorrelations, cross-correlations, and statistical memory effects in time sequences and spatial maps of experimentally recorded parameters of physical experiments (see for example [27, 28]), including the study of collective effects in plasma [29], the study of surface structures on nano- and microscales [30], the structure of molecules and their complexes according to the characteristic absorption bands.

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