Testing the SOC hypothesis for the magnetosphere

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Abstract

As noted by Chang, the hypothesis of Self-Organised Criticality provides a theoretical framework in which the low dimensionality seen in magnetospheric indices can be combined with the scaling seen in their power spectra and with observed plasma bursty bulk flows. As such, it has considerable appeal, describing the aspects of the magnetospheric fuelling:storage:release cycle which are generic to slowly-driven, interaction-dominated, thresholded systems rather than unique to the magnetosphere. In consequence, several recent numerical “sandpile” algorithms have been used with a view to comparison with magnetospheric observables. However, demonstration of SOC in the magnetosphere will require further work in the definition of a set of observable properties which are the unique “fingerprint” of SOC. This is because, for example, a scale-free power spectrum admits several possible explanations other than SOC. A more subtle problem is important for both simulations and data analysis when dealing with multiscale and hence broadband phenomena such as SOC. This is that finite length systems such as the magnetosphere or magnetotail will by definition give information over a small range of orders of magnitude, and so scaling will tend to be narrowband. Here we develop a simple framework in which previous descriptions of magnetospheric dynamics can be described and contrasted. We then review existing observations which are indicative of SOC, and ask if they are sufficient

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to demonstrate it unambiguously, and if not, what new observations need to be made?

1. Introduction: Few-parameter models for magnetospheric dynamics

There is growing evidence that the coupled solar wind-magnetosphere-ionosphere (SW-M-I) system, viewed as a whole, is non-equilibrium, driven, dissipative and nonlinear (Vörös, 1991). That this should be so is reasonable, given that the magnetosphere is a complex system, with multiple self-interacting phenomena, occurring on a vast range of length and time scales. A consequence of this view is that part or all of the observed magnetospheric phenomenology may be a manifestation of physics resulting from the underlying complexity of the whole system. Because of their analytical intractability, such systems in space physics are typically studied using a “large” (i.e. many-parameter) numerical simulation code. More recently, however, some systems of this type in nature have been shown to lend themselves to few-parameter descriptions (Hastings and Sugihara, 1993), which arise because, paradoxically, the complexity of the system gives rise to simplicity in some of its observable characteristics. Examples of such descriptions are shown in the top row of Table 1, adapted from figure 7.1 of Hastings and Sugihara, (1993). Starting with the simplest, “linear plus noise” description, applied to the magnetosphere by Bargatze et al., (1985), we go to low dimensional nonlinear “chaotic” models such as Baker et al. (1990)’s modified “dripping tap”. We then see fractional Brownian motion (fBm), used by Takalo et al., (1994), as a null hypothesis against which chaos could be tested and finally we have Self-Organised Criticality (SOC), the magnetospheric application of which is due to Chang, (1992). Few-parameter models of intrinsically complex systems have already demonstrated their value in space physics by their ability to describe reproducible aspects of the magnetosphere’s behaviour and to motivate nonlinear predictive filters for geomagnetic activity (see the reviews of Klimas et al., 1996 and Sharma, 1995). The extent to which such models are applicable bears directly on the extent to which magnetospheric (and other laboratory or astrophysical macroscopic plasma systems) may have predictable phenomenology. In consequence the study of few parameter models for energetically open but spatially confined plasma systems is a
highly topical subject both with respect to the magnetospheric confinement system (e.g. Angelopoulos et al., 1999; Baker et al., 1999; Horton et al., 1999) and to magnetically confined laboratory plasmas (e.g. Dendy and Helander, 1997; Carreras et al., 1999; Pedrosa et al., 1999). One possible new approach of considerable current interest is the SOC paradigm introduced by Bak et al. (1987).

There is a natural hierarchy of few-parameter descriptions, ordered by the extent to which the many coupled degrees of freedom of the system manifest themselves. Broadly speaking, as we go from left to right along Table 1, we move down the hierarchy of description and the large number of degrees of freedom become increasingly explicit in the description. In consequence the importance of an underlying theory to define the model fully tends to also increase. To make effective use of the theory-model correspondence in such a table, however, theories must be falsifiable, as otherwise the parameters of the simple model may simply be “tuned” to bring it into closer and closer agreement with data. This may be a two-way process, as for example, casting a given model in falsifiable form by defining which phenomena must be tested for helps to clarify the underlying theory.

To make these abstract points more concrete, consider Table 1. The first row shows some notable examples of various simple models that have been applied both to the complex magnetospheric system, and to other such complex systems. The first column shows various properties that these models have which could be tested for in data, provided that suitable variables are measurable. If a given property can be shown not to be present in data then we can eliminate models which depend on it from consideration. In this paper we first describe the construction of Table 1 by describing the four levels of description which it encapsulates. As models based on the SOC hypothesis are of current interest for the SW-M-I coupling problem, we then specifically address the tests necessary to cast SOC models in falsifiable form.

2.1 Linear models with optional noise term

Column 1 of Table 1 is the “linear + noise” model, typically a linear differential equation with optionally a stochastic noise term $\Delta w(t)$, (adopting the notation of Hastings and Sugihara (1993)) to which we may also add a
driving term \( F(t) \):

\[
\frac{dx(t)}{dt} = g(x, t) + \Delta w(t) + F(t) \tag{1}
\]

where \( g(x, t) \) can only be linear in the variables \( x \). Physically an input-output system is linear if the form of a system’s response closely resembles that of the forcing terms. This was the first level of approximation used in the input-output analysis of the SW-M-I system (Bargatze et al., 1985). The second and third rows of the table, labelled “Short-” and “long-term predictable” refer to the fact that in the absence of noise (\( \Delta w(t) = 0 \)) the short-and long-term behaviour of equation (1) is completely deterministic, while even if an additive stochastic noise term is present, closely-spaced initial conditions do not show exponential divergence. Such systems typically show relatively narrow-band spectral behaviour if the \( g \) term is dominant i.e. characteristic frequencies, and so we have “no” in the “global scaling” row for this model (row 4, column 2) to indicate that they would then not be scale free across the whole frequency range. They may, however, show regions of scale free behaviour in their frequency spectra, indicated in the table by the “sometimes” in the “scaling regions” row (column 2, row 5). The entry “no” for “low G-P dimension” (sixth row, column two), refers to the fact that such a system will usually appear high dimensional to the Grassberger-Proccacia (GP) algorithm (Grassberger and Procaccia, 1983), because of many degrees of freedom of the noise term.

### 2.2 Nonlinear deterministic models

Bargatze et al. (1985) confirmed the presence of nonlinearity in the \( AE \) family of indices, (\( AE, AU \), and \( AL \)) and hence the need for a next level of approximation. A prototype for differential equation models which exhibit nonlinear but deterministic dynamics (see the reviews of Sharma (1995) and Klimas et al. (1996)) is the “dripping faucet” of Shaw (1984), which was adapted to the magnetospheric problem by Baker et al. (1990). These models are of the form

\[
\frac{dx(t)}{dt} = g(x, t)) + F(t) \tag{2}
\]

where unlike equation (1) the term \( g(x, t) \) now contains nonlinear terms. In the hierarchy of Table 1, this is a nonlinear model, which can exhibit
low-dimensional, chaotic dynamics (column 3). Familiar examples of such systems in nonlinear physics include the (continuous) driven nonlinear pendulum and (discrete) logistic map (see e.g. Rowlands, 1990). In the magnetosphere this description was inspired by an analogy between a dripping tap and plasmoid ejection during substorms. The analogy was developed into a simplified magnetospheric model by estimating the large-scale electrical properties of the M-I system and combining these electrical components into a driven nonlinear oscillator circuit model (Klimas et al., 1992). It has been further developed into a plasma physics model by Horton and Doxas (1996).

In the case of a dissipative, driven, autonomous low dimensional system such as the Lorenz model, the dynamics, rather than exploring all of phase space ergodically, collapses onto a low dimensional region called an attractor. This attractor has fractional dimension (i.e. it is a strange attractor, in contrast for example with the 2D ellipse described in phase space by a simple linear 1D pendulum). A time series drawn from such a system will thus also have low fractional dimension when tested with the Grassberger-Procaccia algorithm, so we write “yes” against “Low G-P dimension” in column 3, row 6 of table 1. A strange attractor has the property that closely-spaced trajectories, with initial conditions identical to within measurement error, will diverge strongly if they traverse certain regions of the attractor i.e. repulsive fixed points (see figure 1 of Palmer, (1993) for a clear illustration of this). We thus write “no” against “long-term predictability” (column 3, row 3), because in this sense, measured by a positive Lyapunov exponent (e.g. Rowlands, 1990), it is now not present. The significance of this “new” low-dimensional, deterministic, chaos is that sensitive dependence on initial conditions arises from the \( g \) term and so exists without the presence of “old fashioned” stochasticity i.e. we need no \( \Delta w \) term. Such a model can generate wide-band, scale free “1/f” spectral behaviour when near a tangent bifurcation leading to intermittency (Lichtenberg and Lieberman, 1992), but this requires choice of certain values of the control parameters i.e. tuning. We thus write “sometimes” for against “global scaling” and “scaling regions”. “Tuning” in this sense has been considered a weakness in the applicability of low dimensional chaos to any complex natural system (Bak, 1997), not only the magnetosphere. A second practical question with such methods is that because the model definition usually starts from the observables, the map to which one applies nonlinear dynamics must be derived from data rather than given \textit{a priori} from theory. One might however see this as a strength,
and in practice this is addressed by an iterative process whereby the parameters suggested by observation and theory are being brought closer together (Klimas et al., 1996).

2.3 Stochastic descriptive models

Osborne and Provenzale (1989) showed that time series taken from certain random “coloured noise” processes, when tested with the G-P algorithm, would exhibit low dimensionality, and thus behave in this respect as a low dimensional chaotic system would. This led to the application by Takalo et al., (1994 and references therein) of a third type of model, fractional Brownian motion denoted by fBm in Table 1 (e.g. Malamud and Turcotte, 1999), as a hypothesis against which to test the low dimensional nonlinear models described by the previous column. The suggestion of fBm recognised the possibility that the apparent low dimensionality and fractality of the magnetospheric indices was the consequence of their being the output of an otherwise intrinsically many-degree of freedom stochastic system, identified by a particular “coloured noise” power spectrum (hence “sometimes” against “Low G-P dimension” row 6, column 4). Effectively the model is:

\[
\frac{dx(t)}{dt} = \Delta w(t)
\]  

(3)

A simple example is Brownian motion where the time evolution is discrete, and each step (\( \delta x = \delta t \Delta w \)) is drawn from a white Gaussian distribution (e.g. Malamud and Turcotte, 1999). We then find that neither short term nor long term prediction is possible because each step is entirely stochastic, giving us “no” against “short-” and “long-term predictable” (rows 2 and 3 of column 3). We note however that closely-spaced initial conditions diverge algebraically rather than exponentially, i.e. the impossibility of long term forecasting here arises from the external stochasticity in \( \Delta w(t) \) rather than intrinsic chaos from \( g \). Global scaling (row 4, column 4) must arise, irrespective of any free parameters, because there is no time scale in such a model. More complex time evolutions, where successive steps are taken from a fractional Gaussian noise, are called fractal Brownian motions. A subset of such motions has been shown to have low G-P dimension (Osborne and Provenzale, 1989). We note that the presence of global scaling or scaling regions in the power spectra drawn from time series, or low G-P dimension, cannot distinguish between

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nonlinear low dimensional models (column 3) and fBm (column 4), because they are shared properties. The differences will only be unambiguous when one notes the different physical origins of the low G-P dimension between chaos and coloured noise, see Takalo et al., (1994), or when one uses another discriminator such as short term predictability.

2.4 Sandpile models of self organised criticality

The most recently introduced class of models (column 5) in Table 1 are those motivated by the hypothesis of Self-Organised Criticality (Chang, 1992; 1999, see also Vörös, (1991); Chen and Holland, (1993); Robinson, (1993)). SOC was first identified in (Bak et al., 1987), and can be modelled by, numerical cellular automaton “sandpile models” (Katz, 1986; Bak et al., 1987) These discrete-variable models (Consolini, 1997; Uritsky and Pudovkin, 1998) and the closely-related continuous-variable discrete-space time models (Chapman et al, 1998;2000, Takalo et al., 1999a;1999b; Watkins et al., 1999b) are currently being studied for their possible magnetospheric application. Consideration of SOC in our context was motivated in particular by the fact that SOC can account for known magnetospheric phenomenology such as low dimensionality (Chang, 1992) and scale free power spectra, while providing a framework for understanding observed properties of the magnetosphere such as bursty transport in the tail (c.f. “Bursty bulk flows” (Angelopoulos, 1996)). It may be that the long term value of SOC to plasma physics will be as a starting point for more realistic “avalanche” models of turbulent transport (see Dendy and Helander, 1997, for more on this question, as applied to laboratory plasmas). However, at this stage of its consideration with respect to understanding in magnetospheric physics it remains useful to consider Bak et al.’s original, sandpile model-based, definition of SOC in the framework of our table of observables, as it is used explicitly or implicitly by much of the current work on SOC in this and other fields.

Bak et al. (1987, henceforth BTW87) originally proposed SOC to explain the apparent ubiquity of both spatial fractals and “1/f” spectra in nature. They observed it in a class of numerical cellular automata, called “sandpile models” for which analogous continuous thresholded diffusion equations have since been shown to exist (Lu, 1995). The equations are modified from stochastic diffusion equations (Pelletier and Turcotte, 1999) which have a
form such as
\[ \frac{\partial g(x, t)}{\partial t} = \nabla^2 g(x, t) + \Delta w(x, t) \] (4)

by the introduction of a thresholding process (Jensen, 1998), e. g.
\[ \frac{\partial g(x, t)}{\partial t} = \nabla^2 g(x, t) \Theta(g(x, t) - g_c) + \Delta w(x, t) \] (5)

where the step function \( \Theta \) initiates diffusive transport when the variable \( g \) reaches a critical gradient \( g_c \). The main debate centres on how to motivate and satisfactorily introduce this \textit{ad hoc} thresholding term (see Lu, 1995; Jensen, 1998) but several properties, such as the low frequency power spectrum may be dependent only on equation (4) and the nature of the boundary terms (Jensen, 1998), and so may be common to both SOC and stochastic diffusion.

The behaviour classified by BTW87 as SOC is the evolution of the medium described by the cellular automaton or differential equation models from arbitrary initial conditions to a non-equilibrium but steady state, “self-organisation”. The medium then evolves by dissipating energy on all scales via thresholded reconfiguration/energy release events called “avalanches”.

The assertion by Bak et al. (1987;1988) that the observed scale free, and hence power law, distribution of the size of these energy release events (the “avalanche distribution”) measured the arbitrary response of a self-organised fractal structure in the medium to perturbations introduced by random fuelling was the reason for their use of the term “critical”. Their analogy was with the scale free critical state associated with phase transitions in critical phenomena (Huang, 1987). The common observable features cited by Bak et al. were that both systems were globally scale free (hence we may write “yes” in column 5, row 4 of table 1), and also that a finite size scaling analysis gave a good data collapse, as it would in a \textit{bona fide} critical system (Cardy,1996).

The combination of a scale free response to perturbations with the release of this energy by random unloading events, was expected to give rise to a power law frequency spectrum. This was expected to be “\(1/f\)” and hence to explain the ubiquity in nature of noise with correlations on all time-scales. Unlike a “\(1/f^2\)” spectrum above a characteristic frequency which can be explained in many systems simply by random switching of levels, the appearance of “\(1/f\)” \((f^{-\beta})\) spectra where the spectral index is between about 0.8 and 1.4 is a long standing problem in many branches of physics (Jensen, 1998).
In summary, in this picture, the SOC hypothesis would be that: “extended driven systems will tend to self-organise to fractal structures which dissipate energy on all scales in space and time, and hence give rise to scale-free “avalanche” energy burst distributions and “1/f” noise”. More recent sandpile algorithms which allow fuelling to continue while unloading occurs have a broken power law spectrum and so we have added “yes” to column 5, row 5 as well (see also section 3.1.2).

SOC behaviour, as diagnosed by scale-free energy release and/or “1/f” power spectra, has since been claimed for many systems in nature (see chapter 3 of Jensen (1998) for a compact review, and Rodriguez-Itubé and Rinaldo (1997) for a longer exposition in the particular context of fractal river networks). At this point, we simply note that a definition of SOC in terms of what an SOC system does can only be used to identify an SOC system if no other system does exactly the same things. Identification of global scaling, shared by SOC, fractal Brownian motion and low dimensional chaotic systems when intermittent is, for example, thus not an unambiguous test. It is for this reason that we have used Table 1, as a guide to how the “footprint” of SOC may be more unambiguously defined. We have left the other rows as question marks because BTW87’s sandpile model definition of SOC was not unambiguous in these respects, and these issues are still under study. The wider definition of SOC used by Chang, (1992), predicts low dimensional behaviour i.e. “yes” in column 5, row 6, at least close to criticality, while many workers have taken the predictions for column 5, rows 1 and 2 to be “no” e.g. the remarks of Consolini, (1997): “In fact, if the magnetospheric dynamics could be the result of a low-dimensional dimensional chaotic dynamics, we could have some hope to forecast the evolution. On the contrary, the existence of a critical state removes this possibility, because the fluctuations of the system at a critical point are completely unpredictable”. This is equivalent behaviour to fBm. See also Bak (1994) on this point where sandpile models are asserted not to show sensitive dependence on initial conditions i.e. they are unpredictable on both long and short timescales but not chaotic. The similarity of SOC to fBm with regard to the phenomena in Table 1 might suggest that SOC adds nothing to fBm. However, there are differences. One is the fact that an SOC system releases energy by means of avalanches, effectively a new observable property, which we have thus indicated by adding a row in Table 1 to those used by Hastings and Sugihara (1993). We are indebted to a referee for the suggestion that avalanche models may also have
different phase spectra to the (usually random) phase behaviour of noise. A second advantage is that SOC can be explained in terms of an underlying theory and encapsulated in terms of sandpile models, which begin to allow explanation in terms of the underlying plasma physics of the system. A third advantage is that the release of energy by avalanches is suggestive both of bursty transport in plasma confinement systems (e.g. Carreras et al., 1999; Pedrosa et al., 1999), and, possibly, the substorm problem (Chang, 1992; Consolini, 1997).

The study of SOC in solar terrestrial-physics has proceeded initially through comparison of signatures in data, particularly the AE/AU/AL indices, with analogous signatures in “sandpile model” realisations of SOC. However, as we will now show, these signatures are not all unique to SOC, and the combinations in which they appear may be model dependent. Furthermore, we recall that the original proposal of the relevance of SOC to the SW-M-I system (Chang, 1992) was not predicated exclusively on a definition of SOC derived from sandpile models. To minimise possible confusion in this rapidly developing area, two questions are addressed. These are i) which experimental signatures will be needed to distinguish unambiguously between SOC and, for example, deterministic chaos and ii) what are the predictions of SOC models which will be robust against fluctuations in the input, or limited station or satellite coverage etc?

3 Towards unambiguous tests of SOC

Having decided what the predictions of SOC are which may be confidently entered in Table 1, we now go on to see what the observations currently available enable us to say. The question immediately arises as to whether, Picture A), the SOC system is seen as being the complete magnetosphere (“global SOC”), in which case \( x \) in equation (5) are the system state variables, for which the AE indices (Davis and Sugiura, 1966) have been taken as proxies; or, Picture B), SOC is more local in scope (“local SOC”), and plays a role in generating, stabilising and destabilising the magnetotail current sheet, in which case \( x \) might be a locally-measured magnetic field or the field seen in an MHD-derived sandpile simulation. Picture A) is closer to that given in Uritsky and Pudovkin (1998) and Consolini (1997; 1999), while Picture B) seems to us to be the motivation for Takalo et al., (1999a; b; c). Because any
approximation will have a natural maximum scale of applicability, the idea of “local SOC” is not the contradiction it may at first appear to be.

It has been objected that if A) were true, all system-level outputs should show global scaling and that some are observed not to have this property (Borovsky and Nemzek, 1994). However Chapman et al. (1998), used the 1-dimensional avalanche model of Dendy and Helander (1998), to illustrate a system in which the internal energy release showed scaling while energy flowing out of the system (“systemwide”) did not, a feature seen in some other sandpile models (e.g. Pinho and Andrade, 1998). Until pictures A and B can either be distinguished or reconciled, care must be taken not to justify one using measurements consistent with the other and vice versa. We thus first (section 3.1) examine those system level outputs in which evidence of SOC has been claimed, and then (section 3.2) briefly consider evidence for SOC on more internal scales.

3.1 Remote Observations of system outputs

So far the main global dataset for testing for SOC has been the AE indices. This is because, since Bargatze et al. (1985), a candidate dynamical variable for all the models discussed above has been the energy dissipated by the magnetosphere into the ionosphere, for which most workers have taken the Auroral Electrojet Index \( AE \) to be a proxy.

3.1.1 Global scaling: Power law power spectra

The work of Tsurutani et al. (1990) described the power spectrum of \( AE \) as “broken power law”, in that the high frequency behaviour is approximately \( 1/f^2 \) while the lower frequency behaviour is approximately \( 1/f \), with a break at \( (1/5) \) hours\(^{-1} \). This “1/f” behaviour has been cited as evidence of SOC in the magnetosphere (Consolini, 1997; 1999; Uritsky and Pudovkin, 1998). Two main scenarios have been advanced, in one the “1/f” spectrum is seen as arising from interactions between correlated avalanches, which would then be interpreted as substorms (Consolini, 1997); while in the second the “1/f” behaviour (Uritsky and Pudovkin, 1998) is related in part to the input, which is allowed to modulate the threshold values in the sandpile algorithm.

The first apparent complication in this interpretation is that the \( AE \) spectrum is “broken” i.e. a “1/f^2” high frequency part has been reported, whereas
criticality in BTW87’s original picture was expected to give long-period correlations and hence a “1/f” spectrum (Jensen, 1998). The resolution, due to Consolini (1997), is discussed in the section 3.1.2. The second, more fundamental, problem is that a “1/f” spectrum in the output of a system could only be an unambiguous indicator of SOC if this spectrum is not being passed through from the input. The fact that the input spectrum of the solar wind follows $AE$ closely over the low frequency “1/f” range that concerns us here (Tsurutani et al., 1990; Freeman et al., 1998) suggests that the power spectrum should not be used for this purpose. In this context, the high degree of predictability of $AE$ from the solar wind input is suggestive (e.g. Baker et al., 1997), as is the fact that the power spectrum of the signal from a neural network prediction of $AE$ has “1/f” form (Takalo et al., 1996). We return to this question in section 3.1.3 when we discuss the avalanche statistics. The possible ability of some avalanche algorithms to emulate a nonlinear filter, and show sensitivity to the distribution of the input fuelling rate (Takalo et al., 1999c); or conversely to eliminate traces of fluctuating input (Watkins et al., 1999b), increases the relevance of this question.

### 3.1.2 Scaling Regions: Spectral breaks

If the system is known to be SOC a priori or from other tests, the presence of a high frequency “1/f^2” component is understandable. This is because recent work (notably Hwa and Kardar, 1992) has shown that a “running” sandpile (and hence SOC differential equation models such as the example used by (Takalo et al., 1999a)) can show this type of “broken” power spectrum. The reason is that allowing the fuelling to occur on a similar time scale to the unloading events permits a bursty “1/f^2” power spectrum of individual avalanches to co-exist with the “1/f” power spectrum which is ascribed to interactions between events (see Jensen, 1998). If the bursts are identified with substorms then the break at 5 hours will be related to the maximum duration of a substorm. Furthermore, the original Bak et al. (i.e. “non-running”) sandpile model was quickly shown (Jensen et al., 1989) to produce a “1/f^2” spectrum in its energy release events, illustrating that the although the pile is in critical state, shown in particular by the finite size scaling of the avalanche distribution (Bak et al., 1988), the critical state is not revealed by the energy release power spectrum.

The complications in this very appealing simple interpretation arise for
two main reasons. One is that we do not know a priori that the system is SOC, so mapping an output variable of the sandpile model to the observed AE spectrum is not a unique process. An alternative way to get a broken spectrum of the form shown by Tsurutani et al. (1990) for AE is in a boundary driven 2D sandpile of the BTW87 type (see figure 4.6 of Jensen, 1998). In this case the variable whose spectrum is obtained is not the transport of sand over the edge of the pile but the sum of the dynamical variable $g$ across the pile i.e.

$$<g(t)> = \sum_{i,j} g_{ij} \equiv \int d\mathbf{x} \ g(\mathbf{x},t)$$

As with Tsurutani et al, 1990, the spectrum shown by Jensen (1998) is $\sim 1/f$ below a critical frequency and $\sim 1/f^2$ above, with the break set by a time scale $T_{\text{max}}(L)$ corresponding to the longest avalanche possible in the system. This would imply that $T_{\text{max}}$ is related to the system size $L$, and furnish a possible test if the system’s value of $L$ could be varied significantly. In other words, the robust property is the break itself rather than the variable whose broken spectrum is being calculated.

The second problem is that the broken power law spectrum for AE cannot be uniquely interpreted as the output of an SOC system because other types of physical system can produce power spectra which show global scaling or scaling over a region or regions. The example of global scaling, i.e. scaling over a very wide bandwidth, discussed in section 2.3 was simple Brownian motion which has an $f^{-2}$ spectrum at all values of $f$. A less well known example of scaling over a restricted region is the “random telegraph”, a random sequence of square pulses (i.e. states +1 or -1) switched at Poisson distributed intervals which gives $f^{-2}$ for frequencies higher than the inverse correlation time but has a flat spectrum (because uncorrelated) for lower frequencies (Bendat, 1958; Jensen, 1998). It is very important to note that the “$1/f^2$” part of the spectrum here is due entirely to the exponential autocorrelation of the pulses, and is not the same as the intrinsically scale free, and wideband, behaviour of a coloured noise source such as Brownian motion. If the lifetimes of the correlated pulses extend over two orders of magnitude in time then so will the “$1/f^2$” spectrum, and so a test such as the second order structure function (see Takalo et al., 1994 and references therein), or a variance histogram (i.e. Fourier power spectrum) will be unable to distinguish this “trivial” apparent scaling from the “interesting” scaling resulting
from coloured noise. A similar problem whereby level changes with a $1/f^2$ spectrum might mask an intrinsic Kolmogorov spectrum was treated for solar wind turbulence by Roberts and Goldstein (1987).

The fact that the high frequency scaling region in the spectrum of $AE$ might arise from a cause other than SOC is important in our application because $AE$ is known a priori to be a compound index which mixes driven and unloading effects (Kamide and Baumjohann, 1991). This mixed origin is reflected by its power spectrum (Freeman et al., 1998), structure function (Takalo et al., 1994), and avalanche distribution (section 3.1.3 and Freeman et al., 2000). In consequence a suitable “null hypothesis” for the power spectrum against which the SOC models so far proposed should be evaluated is that $AE$ consists of a solar wind driven “1/f” component - arising from the $DP2$ convection electrojet (Kamide and Baumjohann, 1991) - and a random unloading $DP1$ substorm electrojet component which looks like “1/$f^2$” over two orders of magnitude in frequency and appears predominantly in $AL$. By analogy with section 3 we may call this Picture C (“no global SOC”).

A possible avenue for testing Consolini’s (1997) “interacting burst” interpretation of the “1/f” spectrum would then be to see if the correlation properties of the “1/f” part of the power spectrum of $AE$ differ in any way from those of Akasofu’s $\epsilon$ parameter, which estimates the component of solar wind Poynting flux entering the magnetosphere. If they do, this adds support to the possibility that the bursts may be correlated with each other as a result of a process which occurs in the magnetosphere itself; rather than the “1/f” behaviour being explained by the long-period correlation already present in the solar wind’s power spectrum.

3.1.3 Global scaling: Avalanche distributions

Because of the above concerns, we see a better candidate for an unambiguous indicator of SOC as being the statistical distribution of energy released by individual “events”. Since the work of Bak et al. an SOC system has been expected to show a “power law” probability distribution for this quantity. Consolini, (1999) has plotted the distribution $D(s)$ of a burst measure $s = \int_{\Omega} (AE(t) - L_{AE}) dt$, formed from $AE$ where $L_{AE}$ was a running quiet time background level of $70 \pm 30$ nT, and each integration was taken over a period $\Omega$ where the integrand was positive (a burst). The $AE$ data used covered the period from 1975 and 1978-1987. The distribution obtained could be
described by an exponentially cutoff power law (Consolini, personal communication, 1998) extending the result previously obtained by Consolini, (1997) for data from 1978.

The presence of such a power law suggested a magnetospheric analogue of the Gutenberg-Richter law in seismology, and has played a significant role in generating the interest in SOC in magnetospheric physics. Both its existence and its apparent robustness with activity level require confirmation and explanation whether by an SOC theory or another one. Although both a simple power law, or the above exponentially cut off power law are possible fits, Consolini (1999) has recently demonstrated that a better description for the burst size distribution of $AE$ is an exponentially modified power law with a small lognormal “bump” component. If the system is SOC, this requires an explanation of the “bump”, which may be found in the different behaviour of internal and systemwide dissipation in some sandpile models (Chapman et al., 1998) or in subcritical dynamics in the SOC system (Consolini, 1999).

### 3.1.4 Global scaling: Lifetime distributions

The SOC hypothesis had earlier led Takalo (1993) and Consolini (1999) to examine the distribution of lifetimes of the bursts. This is potentially a stronger indicator of SOC than the burst size, because exponentially modified power-law burst size distributions can also be generated by randomly quenched, exponentially growing instabilities in an otherwise non-critical medium (Aschwanden et al., 1998). $AE$ was found (Consolini, 1999) to show a exponentially modified power law distribution of lifetimes, but with evidence of a “bump” at around 100 minutes. The “null hypothesis” mentioned in 3.1.2 (Picture C) led Freeman et al., (2000) to calculate the analogous burst lifetime distributions for $AU$, $AL$ and the solar wind quantities $\epsilon$ and $vB_s$. They found exponentially modified power laws with very similar slopes for all quantities, but the “bump” was only found in the AL and AE, magnetospheric component. This suggests that the “bump” is of intrinsically magnetospheric origin (due to the DP1 current (Kamide and Baumjohann, 1991)) while the scale-free burst lifetime distribution may actually be of solar wind origin (Freeman et al., 2000), if the DP2 current system (Kamide and Baumjohann, 1991) is transparent to the driver.
3.1.5 Other tests: Predictability, low dimensionality

It will be necessary to examine the sandpile models and other realisations of SOC in more detail before we can say with certainty what they predict for the remaining rows of column 5, table 1. Bak (1994)'s arguments about long term prediction are based on the assertion that $\delta$, the separation of two initially infinitesimally close trajectories in a BTW87-type sandpile model grows with time quadratically, $\delta = at^2$, rather than exponentially, $\delta = e^{\lambda t}$, as would be the case in a chaotic system such as that of column 3 (where $\lambda$ is the Lyapunov exponent). This assertion needs to be tested in other sandpile models and in data from candidate systems. It is a potential discriminator between chaos and SOC.

The demonstration that an SOC system can show low dimensional behaviour was given by Chang (1992;1999) on the basis of a more general formulation of SOC than the sandpile-inspired one of Bak et al. It thus remains to be seen in general what sandpile models predict for dimensionality.

3.1.6 New tests: Intermittency and laminar time

The open questions described in the previous section and the ambiguities necessarily present in the data reviewed in sections 3.1.1 to 3.1.4 mean that at present it is not possible to completely eliminate any of the models discussed in Table 1, except the artificially simple linear model which was included for completeness. New tests are thus required. An example of such a test is the degree of intermittency present in the time series. Consolini et al. (1996) showed that the $f^{-2}$ spectral regime of $AE$ might be described as an $f^{-1.8}$ regime corresponding to the inertial range of a turbulent system, with an exponent modified from the Kolmogorov value by the presence of intermittency. These authors showed a good fit to the p-model of turbulence, also shown in the solar wind by Horbury and Balogh (1997). We are thus not presently able to distinguish between intermittency intrinsic to $AE$ and that due to the solar wind driver. Further work on this topic is likely to prove valuable (see also Vörös et al., 1998).

More recently, it has been claimed (Boffetta et al., 1999) that the probability density $D(\tau)$ of time intervals $\tau$ between bursts (the "laminar time") can be used to distinguish an SOC system of the BTW87 type, which has exponential $D(\tau)$, from a shell model of turbulence, which has power law
$D(\tau)$. It might seem that the power law $D(\tau)$ for AE shown by Consolini (1999) would rule out SOC in the global magnetosphere. However, as emphasised by Einaudi and Velli, (1999), the predictions for $D(\tau)$ are not in general known either for more realistic SOC models or for all turbulence models. The relevance of this work to the issue of “sympathetic flaring” (Boffetta et al., 1999 and references therein) in solar physics is likely to give rise to further exploration of this topic, and hence magnetospheric application.

3.2 Local observations of current sheet dynamics

It is fair to say that, for the above reasons, the evidence of SOC in the largest scale outputs of the magnetospheric system, measured by $AE$ and $AL$, is not yet persuasive. The main problem is that the behaviour of the $AE$ indices is similar to that of the solar wind in a number of respects. If an intrinsic and a solar wind component are always present, then testing for SOC in these compound indices will always be problematic, as will the interpretation of results based on them (c.f. Consolini, 1999; Freeman et al., 2000).

It may be more instructive to study regions of the magnetosphere where the effect of the solar wind input is less directly visible, and recent attention has been focused on SOC as a model of the magnetotail current sheet. So far this has been achieved by truncating the ideal MHD equations i.e. replacing the convection term in

$$\frac{\partial B(x,t)}{\partial t} = \nabla^2 B(x,t) + \nabla \times (v \times B)$$

by a source term, resulting in an equation analogous to (4)

$$\frac{\partial B(x,t)}{\partial t} = \nabla^2 B(x,t) + \Delta w(x,t)$$

and then introducing one of several possible thresholding terms c.f. equation (5) (Vassiliadis et al., 1998; Takalo et al., 1999a;b;c) to map the problem onto a cellular automaton or a differential equation like that of (Lu, 1995). In view of the limited applicability of such non-self consistent models it is encouraging that reduced MHD simulations of turbulence are also demonstrating SOC phenomenology (Einaudi and Velli, 1999).

Consideration of SOC as a model for the magnetotail may also be motivated by the suggestion by of Zelenyi et al., (1998) that the tail exists as a
critical percolation cluster. Critical percolation, whereby an avalanche can extend exactly to the maximum scale length of a system rather than just below or just above it, was the original proposed explanation for the relevance of criticality in SOC (Bak et al., 1988). The idea has recently been further developed to explain how self-organisation occurs in sandpiles (Zapperi et al., 1997) in a picture whereby the edge fluctuations drive the system back to a critical percolation state.

4. What signatures of SOC are robust enough to be detectable in “real world” data?

SOC is of particular interest to magnetospheric physics because it is robust, in the sense that the characteristic observed behaviour does not necessarily change greatly over wide ranges of parameter space. This robustness is thus distinct from the scale invariant phenomena that arise near critical points such as phase transitions and hence in restricted regions of parameter space (Huang, 1987). SOC systems are in this way also distinct from chaotic systems such as the dripping tap which frequently show radically different types of behaviour as control parameters change, a point emphasised by Bak (1997, pages 29-31). They are not however, as easily distinguishable from fBm.

However we still need to ask what aspect or aspects of the magnetosphere’s behaviour would be both sufficient to uniquely identify SOC and yet also robust enough to be seen under the wide range of activity levels exhibited by the magnetosphere and the solar wind i.e., to pick the specific robust discriminator. The slowly driven condition is of particular interest in the magnetospheric context because most SOC simulations have been conducted in the limit where the rate of inflow is “slow” relative to dissipation. Watkins et al., (1999b); and Chapman et al., (1999) have recently studied the question of how robust the magnetospherically relevant aspects of SOC are to changes in the inflow rate. They found that the power law avalanche distribution was preserved for the largest values of internal energy release, and gave arguments as to why this should be so. This result may give confidence that such a distribution, if shown on other grounds to be unique to SOC, will be able to be used as a diagnostic. Similar studies for the power spectrum are being carried out in an MHD-derived model by Takalo et al.,
5. Conclusions

We have attempted to identify the distinguishing observable features of different few-parameter models applied to the magnetosphere. The “linear+noise” model was abandoned because of observed nonlinearity, low dimensionality and lack of long-term predictability in the auroral index time series. Low dimensional models have been questioned because the low dimensionality is not unique to them and because their scaling properties are not robust against changes in the input parameters. An alternative, fractional Brownian motion, which gives low dimensionality and robust scaling is unsatisfying because it does not lead to an underlying plasma physical description. The newest alternative, SOC, chosen for its robust scaling properties, can be seen as providing both a physical explanation for fBm and also accounting for the bursty nature of transport in the magnetosphere. SOC has yet to have its low dimensionality and predictability properties fully defined, but so far they seem to be similar to those of fBm. Thus attention must be focused on other means of distinguishing these last two, such as the observed intermittency and avalanching properties. Even so, questions about the application of fBm and SOC as models of the magnetosphere’s large scale output (picture A) rather than of its solar wind-driven aspects are raised by the similarity of input and output power spectra and burst distributions. Resolution of these issues is hampered by the narrow bandwidth of even the best available data series, which for example make it difficult to distinguish between wide-band coloured noise and random state changes as the origin of the $f^{-2}$ spectrum of AE (Watkins et al., 1999a).

This seems to leave four possibilities, not all of which are mutually exclusive:

i) The global “SOC”-like properties we have referred to come from outside the magnetosphere, i.e. the magnetosphere can be quite well described by a “weakly nonlinear plus coloured noise” model; weakly nonlinear to give the necessary degree of predictability of the output from the input while giving long-term unpredictibility, but with a coloured noise input from the fBm or SOC nature of the turbulent solar wind causing the scaling properties. This scenario appears to be compatible with picture C, the data in Freeman et al.
(2000), and alternative iii) below.

ii) Some SOC systems (Watkins et al, 1999b; Consolini, private communication, 1998) will destroy the information contained in their input. The scaling observed in their outputs is then independent of any present in the input, so any common scaling exponents between input and output are either coincidental or evidence of universality in certain confined plasma systems.

iii) Measuring global properties is the wrong thing to do, i.e. SOC is not an aspect of the global magnetosphere but relevant more locally to the magnetotail (compatible with picture B, and alternative i) above). This possibility is likely to be illuminated by further studies of SOC as a magnetotail model.

or iv) that another type of model is required (e.g. Chapman, 1999).

It is also important to emphasise that the extent to which SOC is observable, and distinguishable from other nonlinear physics paradigms (such as those presently used to study turbulence) is an important generic question in contemporary physics (including but going beyond, plasma physics). The diversity and quality of the existing ground-based and space-based magnetospheric databases provide a key testbed with which these intrinsically interdisciplinary questions can now be addressed; while ongoing investigations in astrophysical and laboratory confinement systems, both in plasma physics and elsewhere will continue to be applicable to the question of magnetospheric SOC.

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Table 1: Four examples of possible approaches to understanding magnetospheric time series, adapted from Hastings and Sugihara (1993)

| 1. Model:       | 2. Linear (plus noise) | 3. Low-dimensional nonlinear | 4. fBm nonlinear | 5. SOC sandpile |
|-----------------|------------------------|-------------------------------|------------------|------------------|
| 1. Property     |                         |                               |                  |                  |
| 2. Short term predictable | Yes                  | Yes                           | No                | ?                |
| 3. Long term predictable | Yes                  | No                            | No                | ?                |
| 4. Global Scaling | No                    | Sometimes                     | Yes               | Yes              |
| 5. Scaling Regions | Sometimes             | Sometimes                     | Yes               | Yes              |
| 6. Low G-P Dimension | No                   | Yes                           | Sometimes         | ?                |
| 7. Avalanches   | No                     | ?                             | ?                 | Yes              |
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