Strong quantum memory at resonant Fermi edges revealed by shot noise

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Studies of non-equilibrium current fluctuations enable assessing correlations involved in quantum transport through nanoscale conductors. They provide additional information to the mean current on charge statistics and the presence of coherence, dissipation, disorder, or entanglement. Shot noise, being a temporal integral of the current autocorrelation function, reveals dynamical information. In particular, it detects presence of non-Markovian dynamics, i.e., memory, within open systems, which has been subject of many current theoretical studies. We report on low-temperature shot noise measurements of electronic transport through InAs quantum dots in the Fermi-edge singularity regime and show that it exhibits strong memory effects caused by quantum correlations between the dot and fermionic reservoirs. Our work, apart from addressing noise in archetypical strongly correlated system of prime interest, discloses generic quantum dynamical mechanism occurring at interacting resonant Fermi edges.

Non-equilibrium electronic shot noise is a powerful diagnostic tool revealing properties of mesoscopic systems inaccessible by the mean current measurements1–3. For example, recent noise measurements in the Kondo impurities4,5 have brought new insights into strongly correlated transport. Shot noise is sensitive to the presence of non-Markovian dynamics6 intensively studied in broad context ranging from photosynthesis to quantum information7–12. However, most theoretical proposals as well as the newest quantum-optical experimental study12 rely on extensive engineering and control of the system and/or environment (bath) and a clear observation and identification of the quantum memory effects in “natural”, i.e., routinely fabricated solid-state systems has not been reported yet.

In resonant tunneling, which is ubiquitous in quantum electronic transport, the charge dynamics of the resonant level can be described by a simple Markovian master equation as long as the relaxation time related to the inverse of the transfer rates is long compared to the characteristic memory time of the fermionic bath (leads) given by the inverse temperature and/or the detuning of the level from the chemical potentials of the leads. Comparable time scales for system relaxation and bath memory break down the conventional description for low-temperature on-resonance transport and indicate strong non-Markovian features which, together with many-body-interactions typical for small nanostructures, influence the low-temperature width of the resonant steps in the current-voltage characteristics14, the decay of the level occupations15, or the noise6. For the noise, significant deviations from the conventional master equation description have already been observed16, although their origin has not been identified.

In this work we present new experimental results on the low-temperature noise measurements in the Fermi-edge singularity (FES) regime16–20 together with a theoretical analysis clearly revealing the presence of strong quantum memory around the edge. The noise-around-the-edge puzzle16 is briefly introduced in Fig. 1e, where the measured points are contrasted with the standard Markovian theory (black line) showing large deviations of ~15% in the Fano factor $F = S/2I$ (a convenient dimensionless measure of the shot noise $S$), far beyond the experimental uncertainty. Moreover, the measured dip breaks the Markovian lower bound of 1/2, which is a clear witness of strong memory. The blue line, nicely coinciding with the data, is our new theory accounting for the memory effects.

The FES, a paradigmatic exactly solvable many-body problem21,22, which originates from the Coulomb interaction of conduction electrons with those on a localised discrete level represented by core shell electrons or quantum dot (QD) levels, was first predicted in the X-ray spectra of metals23, but its signatures are observed also in resonant tunneling set-ups as a (truncated) power-law singularity of the mean current $I$ around, e.g., the emitter Fermi energy17–20. The interacting resonant level model describing the FES transport
setup has served recently as an important benchmark for novel quantum transport techniques including the noise calculation at its exactly solvable self-dual point (different from our experimental regime).

**Results**

We first describe cross-correlation measurements of current shot noise in self-assembled InAs QDs in the FES regime. The experimental set-up is depicted in Fig. 1a and explained in more detail in the Methods section. At zero bias voltage the ground state energy level of the InAs dots lies far above the emitter Fermi energy (see Fig. 1b, left). Therefore, a large threshold voltage bias $V_{th} = 170 \text{ mV}$ applied to the collector lead is required to shift $\varepsilon_D$ to resonance with the emitter Fermi energy by electrostatic gating with the leverage factor $\eta = \frac{\varepsilon_D - \varepsilon_F}{\varepsilon_D}$ (excluded double occupancy due to strong onsite Coulomb interaction implies usage of $2\gamma_E$ instead of just $\gamma_E$ as for a noninteracting resonant level). While the collector rate $\gamma_C \approx 8 \times 10^9 s^{-1}$ is basically constant, $\gamma_E$ reflects the energy dependence of the emitter DOS and exhibits an expected asymmetry of the tunneling barriers with $\gamma_E/\gamma_C$ ranging between 0.06 and 0.22. Plausibly assuming constant $\gamma_C$ throughout the resonance we can analogously to Ref. 16 extrapolate the $\gamma_E$ to the resonance (shaded) region in Fig. 1d (solid lines) from the expression for the current. The resulting Markovian prediction based on these extrapolated rates (black curve in Fig. 1e) clearly exhibits substantial deviations from the measurement inexplicable by experimental errors. Obviously, the resonant transport regime calls for a radically new theoretical understanding.

**Discussion**

Using the procedure briefly described in the Methods section for $B = 0$, we arrive at a non-Markovian generalised master equation (GME) for the occupations of the resonant level, where $p(t)$ is the probability that the level is occupied by an electron, while $p_0(t) = 1 - p(t)$ denotes the probability of the dot being empty,

$$\frac{dp(t)}{dt} = \gamma C p(t) + \int_0^t dt' \left[ \gamma_E^2(t') p_0(t-t') - 2 \gamma_E(t') p_0(t-t') \right] .$$

Far enough from the edge, i.e., outside of the shaded region of Fig. 1c, we can use the standard Markovian master equation and evaluate the emitter and collector tunneling rates, Fig. 1d, from formulas $I = 2 e \gamma_E/\gamma_C$ and $F = 1 - 4 \gamma_E/\gamma_C (2 \gamma_E + \gamma_C)^2$ (excluded double occupancy due to strong onsite Coulomb interaction implies usage of $2\gamma_E$ instead of just $\gamma_E$ as for a noninteracting resonant level). While the collector rate $\gamma_C \approx 8 \times 10^9 s^{-1}$ is basically constant, $\gamma_E$ reflects the energy dependence of the emitter DOS and exhibits an expected asymmetry of the tunneling barriers with $\gamma_E/\gamma_C$ ranging between 0.06 and 0.22. Plausibly assuming constant $\gamma_C$ throughout the resonance we can analogously to Ref. 16 extrapolate the $\gamma_E$ to the resonance (shaded) region in Fig. 1d (solid lines) from the expression for the current. The resulting Markovian prediction based on these extrapolated rates (black curve in Fig. 1e) clearly exhibits substantial deviations from the measurement inexplicable by experimental errors. Obviously, the resonant transport regime calls for a radically new theoretical understanding.
In the zero-temperature limit the formula simplifies to electron transfer rates across the QD/emitter-lead interface caused by sequential occupying and emptying of the QD. Being of at the emitter junction is included the GME memory kernel corresponding to equation (1) is the dimensionless energy/voltage distance from the resonant edge, \( z \) is the FES critical exponent, and \( B(x, y) \) denotes the beta-function. In the zero-temperature limit the formula simplifies to the explicit form of the rates namely two independent critical exponents were fixed by fitting simultaneously curves at various temperatures, shown in Fig. 2. In the finite magnetic field case free parameters were fitted by fitting simultaneously curves at various temperatures, namely two independent critical exponents \( x_1 = 0.40, \ x_2 = 0.43 \) and magnetic field. We start by fitting the experimental data for the mean current (insensitive to memory) around the edge(s) with a straightforward extension of the above theory to the case of two spin-split levels due to the magnetic-field with resulting \( 3 \times 3 \) (double occupancy excluded) memory kernel analogous to equation (2) as shown in Fig. 2. In the finite magnetic field case free parameters were fitted by fitting simultaneously curves at various temperatures, namely two independent critical exponents \( x_1 = 0.40, \ x_2 = 0.43 \) and thresholds \( V_{th,1} \) together with an overall prefactor to the emitter rates and the leverage factor \( \eta \), while in the \( B = 0 \) case only 4 parameters due to a single resonance peak were fixed with \( x = 0.28 \) in qualitative agreement with in-depth investigations.

By this procedure all parameter values are fully determined and the predicted Fano factor curves in the lower panel of Fig. 3 are free of any ambiguity. Considering this, the correspondence between the measurements (points with error-bars) and our non-Markovian theory (lines) is quite remarkable in all cases encompassing two magnetic field values and various temperatures. We also compare the Markovian, i.e., with the derivative terms in equation (3) omitted (dashed lines), and non-Markovian (solid lines) predictions in the insets and the detail of Fig. 3 with clear demonstration of the already mentioned non-Markovian features in the low-temperature Fano factor, namely, the significantly more pronounced dip on the high-voltage side of the FES and the potentially super-Poissonian peak on thresholds.
the low-voltage side with fast destruction of the non-Markovian corrections with temperature or distance from the resonant edge. All these features are clearly seen in the experimental data as well. The super-Poissonian Fano factor due to quantum coherence at the lower edge is not reliably confirmed experimentally because of associated large errors resulting from a ratio of very small values of both the current and noise (tunneling regime). Nevertheless, the experimentally observed peak just below the upper edge (see the detail in Fig. 3), although sub-Poissonian, is caused by the very same mechanism and is thus an indirect confirmation of the purely quantum memory effect. Altogether, the importance of the non-Markovian corrections due to quantum memory is established both qualitatively and quantitatively.

Methods

Experimental details. The studied InAs QDs are embedded in a GaAs-AlAs-GaAs resonant tunneling device patterned into pillars with a cross section of 9 × 9 μm sufficiently small to resolve single dot tunneling. The effective AlAs barrier widths of 4 and 3 nm are slightly asymmetric. The measurements, whose schematic of the electronic setup is shown in Fig. 1a, were performed in a dilution refrigerator at temperatures down to 70 mK and magnetic fields up to 13 T. The DC part of the source drain current I is measured with a transimpedance amplifier which also biases the sample. Two 4.7 kΩ resistors convert the fluctuating current to voltages which are measured in a cross-correlation configuration. Together with parasitic capacitances these resistors form RC-circuits which define the bandwidth of our experiment. To increase this bandwidth we use home-built coaxial cables thereby lowering the total parasitic capacitance to 20 pF.

The voltage fluctuations are amplified by a two-stage low temperature amplifier based on the ATF34143 HEMT with a gain of 22 dB, followed by a room temperature amplifier with a gain of 60 dB. The amplified signal is filtered and digitised, and from the Fourier spectra the cross-correlation noise power $S_{\text{SH}}$ is calculated and averaged over 8 minutes. To retrieve the shot noise power the real part of $S_{\text{SH}}$ at the frequency range from 500 kHz to 3 MHz is evaluated. A technical background-noise, largely dominated by thermal noise sources like the first transistor stage of the cryogenic amplifiers and the conversion resistors, is measured at zero current and subtracted. At finite sample impedances the partially correlated thermal background is estimated and also subtracted. The correlation gain parameters are determined by noise thermometry. The error bars in Fig. 1e and Fig. 3 consist of the statistical error, the error of the estimated background and the calibration error.

Theoretical details. Hamiltonian of simplified spin-less model of a resonant level tunnel-coupled to two leads (emitter $E$ and collector $C$) and Coulomb-coupled just to the emitter reads $H = H_{\text{QD}} + H_{\text{E}} + H_{\text{T}} + V_{\text{dd}} d$ with $H_{\text{QD}} = \varepsilon z d d$, $H_{\text{E}} = \sum_{\beta=E,C} \varepsilon_{\beta} c_{\beta}^\dagger c_{\beta}$, $H_{\text{T}} = \sum_{\beta=E,C} \{ \gamma_{\beta} d^\dagger c_{\beta} + \gamma_{\beta}^* c_{\beta}^\dagger d \}$ and $V_{\text{dd}} = \sum_{\beta=E,C} V_{\alpha_{\beta}} c_{\beta}^\dagger c_{\beta}$, where $\beta = E, C, d$ and $\alpha_{E}$ and $\alpha_{C}$ are QD and lead annihilation operators, $\varepsilon_{\beta}$ and $\varepsilon_{\beta}$ are the energies of the QD level and of the electrons in the leads, respectively, while $\gamma_{\beta}$ describe the tunneling between the QD and the leads. The last term describes the scattering of emitter lead electrons on an electron in the QD and is responsible for the FES phenomenon. Since the Fermi level of the collector lead is far below the resonant level and $\gamma_{\beta}$ does not depend on energy close to the edge (Fig. 1d) we can use the method by Gurvitz and Prager to exactly integrate out the collector lead. This leads to the equation of motion for the density operator $\rho (t,n)$ of the dot and the emitter resolved with respect to the number $n$ of passed electrons through the emitter/QD interface. After introducing the counting field $\gamma$ as a conjugate variable to $n$, one can
write the equation of motion for $\sigma(t; \chi) = \sum_\alpha \sigma(t; n)e^{i\alpha}$, partly expressed in the block form as
$$d\sigma(t, \chi) = -i[H(z, \chi)\sigma(t; \chi) - \sigma(t; \chi)H(z, \chi)] dt \pm i(\gamma_c \sigma(t, \chi) - \gamma_c/2 \sigma(t, \chi)) \pm i\gamma_c \sigma(t, \chi),$$

where $H(z, \chi) = H_{QD} + H_e + V_x d^2 + \sum_\alpha \{ J_\alpha e^{i\theta_\alpha/2} c_\alpha + J_\alpha^* e^{-i\theta_\alpha/2} a_\alpha^\dagger d \}$ is the appropriately modified Hamiltonian of the emitter and QD including the counting field.

The emitter lead can then be handled perturbatively in the tunnel coupling $t_i$ following closely the derivation for dissipative double quantum dot from Ref. 31 by first separating equation (4) into four equations for the elements of $\sigma$. Tracing out the electron states of the emitter lead in the two equations for the evolution of the diagonal elements $\sigma_{ij}(t; \chi)$ we find the evolution equations for the generalised QD occupations $p_i(t; \chi) = \text{Tr}_q(\sigma_{ii}(t; \chi))$

$$dp_i(t; \chi) = \pm i\sum_{\alpha} \text{Im}(t_i Tr_{q}(\sigma_{ii}(t; \chi)))$$

The equation governing the evolution of $\sigma_{ij}(t; \chi)$ which enters Eqs. (5) contains the diagonal elements $\sigma_{ij}(t; \chi)$ of the total density matrix. In order to close the equations for $p(t; \chi)$ we perform physically motivated state-resolved perturbative decoupling of the density matrix into $\sigma_{ij}(t; \chi) = \rho_{ij}(t; \chi)/\Omega_{ij}$ with $\rho_{ij} = \exp[-(H_{QD} + H_e + V_x d^2 + \sum_\alpha \{ J_\alpha e^{i\theta_\alpha/2} c_\alpha + J_\alpha^* e^{-i\theta_\alpha/2} a_\alpha^\dagger d \})/\hbar T]$ being the grand-canonical density matrix of the emitter lead at temperature $T$ and chemical potential $\mu_T$ when the QD is empty ($\gamma = 0$) or occupied ($\gamma = 1$). Thus, after multiplying the forward rate by 2 due to the interplay of spin and Coulomb blockade28, we find equation (2) (reducing to equation (1) for $\gamma = 0$) with FES Green’s functions reading

$G_0(t) = \sum_{\alpha \beta} J_{\alpha} J_{\beta} tr_{q} e^{i[H_e + V_x d^2]} c_{\alpha} e^{-i[H_e + V_x d^2]} c_{\beta} d_t$

$G_1(t) = \sum_{\alpha \beta} J_{\alpha} J_{\beta} tr_{q} e^{i[H_e + V_x d^2]} c_{\alpha} e^{-i[H_e + V_x d^2]} c_{\beta} d_t$

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Author contributions

The experiment was designed by F.H. and R.J.H. and carried out mostly by N.U. and N.M. Theory was designed and developed by K.R. and T.N. N.U. and K.R. jointly analyzed and interpreted the data. Manuscript was written mainly by T.N. with contributions from N.U. and K.R. and with steady input and feedback from F.H. and R.J.H.

Additional information

Competing financial interests: The authors declare no competing financial interests.

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