Three Dimensional Nonlinear Guidance Law for Exact Impact Time Control

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ABSTRACT This paper presents a nonlinear guidance law for controlling the impact time of a missile-target engagement in three dimensions. The guidance law is initially formulated for a two-dimensional encounter before being extended to a three-dimensional scenario. The guidance law is constructed such that nonlinear kinematics are neither linearized nor approximated. The guidance law offers a precise closed-form expression for impact time, which is the principal contribution of this work. Choosing an integrable heading error profile is the central concept of this research for determining an accurate impact time. A two-step strategy is utilized to accomplish an interception at a predetermined time. Because the control parameter is related to the desired impact time with an exact expression, a simple and accurate control of impact time is attained. The method provides equations for calculating the minimum and maximum impact times, enabling the determination of the range of impact times that can be achieved. The proposed guidance law can ensure that the seeker field of view restriction is not violated, if the initial heading error is within this limit. The developed method is utilized in salvo attacks, in which multiple missiles simultaneously strike an enemy target, increasing the missiles’ effectiveness against the target’s self-defense mechanism. To validate the effectiveness of the proposed strategy, MATLAB-based numerical simulations of 2D and 3D engagements, including salvo attacks, are conducted.

INDEX TERMS Impact time control, salvo attack, three dimensional engagement, nonlinear guidance law, field of view.

I. INTRODUCTION
The basic goal of missile guidance algorithms is to attain zero miss distance. Modern strategic targets, on the other hand, are equipped with self-defense devices like close-in-weapon systems (CIWS), which allow the target to identify and destroy invading missiles at close range. A solution to this problem is the simultaneous attack of multiple missiles, known as a salvo attack, which can be achieved if the time at which the individual missile hits the target (impact time) is controlled [1].

The proportional navigation (PN) guidance law is a popular guidance law due to its efficiency, simplicity, and robustness. One method of controlling the impact time using PN guidance law is to modify the navigation gain as a function of impact time error, which is defined as the difference between the desired and computed impact times [2], [3]. The impact time can alternatively be controlled using a fixed navigation gain with a lateral acceleration consisting of a base term and a bias term, where the bias term is proportional to the impact time error [1], [4]–[6]. For controlling the impact time, guidance laws derived using Lyapunov methods [7] and sliding mode control techniques [8]–[12] are also discussed in the literature. A Lyapunov candidate function, which is a function of the square of the impact time error, is considered. In sliding mode based guidance laws, the sliding surface is a function of impact time error. The aforementioned control strategies reveal the requirement of impact time error which necessitates the estimation of impact time.

Various methods for estimating the impact time are discussed in the literature. Computing range over closing velocity is a straightforward approach for estimating impact time, but it can yield correct answers only if the closing velocity is constant and the initial heading error is small. Another method for estimating the time-to-go is the recursive algorithm proposed by [13]. In [3], [14], a numerical based recursive technique is proposed for estimating the time-to-go for a PN guidance law engagement. For encounters with
large initial heading errors, the recursive approaches fail to produce accurate results. The authors of [15] were successful in presenting a strategy for determining an accurate time-to-go estimate for PN guidance law. The method involves numerical solution of governing differential equations involving heading error.

In contrast to recursive and numerical methods, closed-form expressions provide quick responses and a greater understanding of how various parameters affect performance. They are notably useful in designing various guidance schemes since they can yield first and higher order derivatives. The authors of [1], [2], [5], [16] proposed a number of closed-form formulations for calculating PN guideline law impact times. However, because these approaches use Taylor series approximation to make integrating non-elementary anti derivatives easier, the formulas they present are approximate. An exact closed-form equation in terms of an incomplete beta function for computing the time-to-go of a PN guidance law is presented in [17]. A closed-form formula for a control parameter as a function of desired impact time cannot be obtained using this method. The authors of [18] propose a circular trajectory-based guidance law that delivers an exact impact time. The approach’s performance is determined by the controller parameters chosen, hence a tuning process is required to attain the optimum results. If the impact time could be controlled by a control parameter related to an exact closed form expression involving the desired impact time, it would be more convenient and yield better results. Because the control parameter is related to the desired impact time, the guidance laws described in [19]–[21] do not require estimation of impact time. The control parameters are derived from an exact impact time solution developed in these works.

The approaches stated above were developed for planar engagements. However, because many missile-target encounters in reality are three-dimensional, this work’s main concern is on the impact time control in three-dimensional engagements. A three-dimensional impact time control problem is discussed in [22], where a guidance command for the leader is issued in the line-of-sight (LOS) direction, causing the impact time error to converge to zero in a predetermined amount of time, allowing the leader to reach the target at the predetermined impact time. Following that, a guidance command for each follower is created based on information from adjacent interceptors to ensure that each follower’s impact time is in sync with the leader’s. This method uses an approximate expression based on the ratio of range to closing velocity to estimate time-to-go. The authors of [23] propose a barrier Lyapunov function-based guidance strategy for controlling the impact time of an interceptor. The approximate closed-form expression for impact time presented in [24] is used in this work. Another approach for 3D impact time control is the method presented by [24] where a sliding mode control-based guidance strategy is employed. This method is also based on an approximate impact time expression.

Unlike the approximate expressions, this research focuses on constructing a guidance law to control the impact time of a three-dimensional missile-target engagement by employing a control parameter, which is related to desired impact time by an exact closed-form expression. The control parameter is related to the desired impact time and initial conditions by an exact closed form equation. The guidance law is first developed for a planar engagement and then it is extended for three dimensional scenario. The following are the main contributions of the proposed method.

- The proposed guidance law provides an exact closed-form solution for impact time, from which an exact closed-form expression for a control parameter is derived in terms of the desired impact time and initial conditions.
- The time-to-go error, defined as the difference between actual and computed time, is zero for the entire duration of the engagement; therefore, impact time estimation is not required. Here, the impact time is controlled by calculating the control parameter, a function of the desired impact time.
- Since heading errors are either constant or decrease monotonically to zero (never increase), the seeker’s look angle limits are respected without the need for additional control strategies.

II. PROBLEM FORMULATION

Consider a three-dimensional interceptor engagement with a stationary target, as depicted in Fig. 1. When the target mobility is small in comparison to interceptor agility, the assumption of a fixed target is justified. Let the origin of the inertial frame $X_I - Y_I - Z_I$ be the initial position of the interceptor. Let $X_m - Y_m - Z_m$ denotes the interceptor’s frame. The target position is denoted by the letter T. The line traced from interceptor to target is known as line-of-sight (LOS), and the distance along LOS from the missile position to target is known as range-to-target or simply range, which is denoted by $R$. $R_x$ is the projection of $R$ on the X-Y plane. The angle from the X-axis to $R_x$ is designated as $\psi_x$, while the angle from $R_y$ to $R$ is designated as $\gamma$. Let’s assume...
the interceptor velocity is $V$. The velocity lead angles in azimuth and elevation directions are designated as $\psi_m$ and $\gamma_m$, respectively. The lateral acceleration of the interceptor in the yaw and pitch directions are $a_y$ and $a_p$, respectively. The interceptor is assumed to be a point mass with a constant speed. The heading error, also known as the look angle, is the angle between the velocity vector and the range vector. It is denoted by $\sigma$ and can be calculated from

$$\cos \sigma = \frac{\vec{V} \cdot \vec{R}}{VR} = \cos \gamma_m \cos \psi_m$$

where

$$\vec{V} = Vi_m$$
$$\vec{R} = R \left( \cos \gamma_m \cos \psi_m \vec{r}_m + \cos \gamma_m \sin \psi_m \vec{j}_m + \sin \gamma_m \vec{k}_m \right)$$

(3)

where $\vec{r}_m, \vec{j}_m,$ and $\vec{k}_m$ are the unit vectors along $X_1, Y_1,$ and $Z_1$ directions and $\vec{V}, \vec{R}$ is the dot product of the vectors $\vec{V}$ and $\vec{R}$, respectively.

For such a scenario, the interceptor-target engagement kinematic [25] is given as follows:

$$\dot{i} = -V \cos \gamma \cos \psi$$  \hspace{1cm} (4)
$$\dot{\gamma} = -V \tan \gamma$$  \hspace{1cm} (5)
$$\dot{\psi} = -V \tan \gamma \tan \psi$$ \hspace{1cm} (6)
$$\dot{\gamma}_m = \frac{a_p}{V} - \dot{\gamma}_l \sin \gamma \tan \gamma_m \cos \gamma_{m}$$ \hspace{1cm} (7)
$$\dot{\gamma}_m = \frac{a_p}{V} - \dot{\gamma}_l \tan \gamma_m \sin \gamma_m$$ \hspace{1cm} (8)

The impact time, $t_f$ is the time at which the interceptor hits the target and time-to-go, $t_{go}$ is the time that is required for the interceptor to reach the target point from its current position. Thus, the actual time-to-go is

$$t_{god} = t_f - t$$ \hspace{1cm} (9)

where $t$ is the instantaneous time.

The goal of this study is to design a nonlinear guidance law governing the two lateral accelerations $a_p$ and $a_y$ that ensures a desired impact time $t_d$ for a three-dimensional missile engagement against a stationary target by tuning a control parameter that is related to the desired impact time and the initial conditions by an exact closed form solution. The developed guidance method is then able to handle multiple interceptors arriving at the target position simultaneously.

This research first develops a guidance law for impact time control of planar interaction, which is then extended to a three-dimensional scenario.

III. GUIDANCE LAW FORMULATION FOR PLANAR ENGAGEMENT

Fig. 2 depicts the planar engagement of an interceptor against a stationary target $T$. Initially, let the interceptor be at the origin of the X-Y frame. Let $M$ represent the interceptor’s current position. The heading angle, $\gamma_m$, is the angle made by the velocity vector with the X-axis, $\gamma_m$, is the angle formed by the line-of-sight (LOS) with X-axis direction. The heading error, or look angle, is the angle between the velocity vector and the LOS. It is expressed as

$$\sigma = \gamma_m - \theta$$ \hspace{1cm} (10)

From the engagement geometry, the kinematics can be obtained as

$$\dot{R} = -V \cos \gamma$$  \hspace{1cm} (11)
$$\dot{\theta} = -\frac{V \sin \gamma}{R}$$ \hspace{1cm} (12)
$$\dot{\gamma}_m = \frac{a}{V}$$ \hspace{1cm} (13)

(13)

where $a$ is the lateral acceleration. The instantaneous position $(x, y)$ is governed by

$$\dot{x} = V \cos \gamma_m$$ \hspace{1cm} (14)
$$\dot{y} = V \sin \gamma_m$$ \hspace{1cm} (15)

Differentiating (10) and substituting in (13), the lateral acceleration can be obtained as

$$a = V \left(\dot{\theta} + \dot{\sigma}\right)$$ \hspace{1cm} (16)

Engagement time is the ratio of path length to velocity, so it can be controlled by adjusting the path length, which can be done by varying the velocity turn rate, implying that impact time can be controlled by varying the lateral acceleration. By selecting a desired heading error profile ($\dot{\sigma}$), the lateral acceleration can be adjusted to a desired value, according to (16). In this study, the guidance law is developed such that the heading profile follows a desired variation. The selection of an integrable heading error variation yields a closed-form solution for the impact time. Let the desired heading error profile be

$$\dot{\sigma} = -\frac{cV}{R_0} e^{-\sigma}$$ \hspace{1cm} (17)

(17)

where $c$ is a parameter that satisfies the boundary condition. The lateral acceleration is obtained by substituting (12) and (17) in (16) as

$$a = -V^2 \left(\frac{\sin \sigma}{R} + \frac{ce^{-\sigma}}{R_0}\right)$$ \hspace{1cm} (18)

(18)
A. RANGE VARIATION AND DETERMINATION OF THE PARAMETER \( C \)

In this section, an expression for range is derived that can be used to determine the parameter \( C \). Dividing (11) by (17) yields

\[
\frac{dR}{d\sigma} = \frac{R_0}{c} e^\sigma \cos \sigma \tag{19}
\]

Integrating (19), where \( R \) varies from \( R_0 \) to \( R \) and \( \sigma \) varies from \( \sigma_0 \) to \( \sigma \) yields the expression for range as

\[
R = R_0 + \frac{R_0}{2c} \left[ e^\sigma (\cos \sigma + \sin \sigma) - e^{\sigma_0} (\cos \sigma_0 + \sin \sigma_0) \right] \tag{20}
\]

The guidance objectives of impact time control are (i) the interceptor should hit the target and (ii) the hitting should be at a desired time \( t_d \). The first objective is achieved if

\[
R = 0 \text{ at } \sigma = 0 \tag{21}
\]

The parameter \( c \) is found so as to satisfy (21), which is evaluated as follows:

\[
c = \frac{e^{\sigma_0} (\cos \sigma_0 + \sin \sigma_0) - 1}{2} \tag{22}
\]

which is a function of initial conditions. Substituting (22) in (20) gives the expression for range as

\[
R = R_0 \frac{e^\sigma (\cos \sigma + \sin \sigma) - 1}{e^{\sigma_0} (\cos \sigma_0 + \sin \sigma_0) - 1} \tag{23}
\]

Proposition 1: For a missile-target engagement governed by the guidance command (18), the range and the heading error diminishes monotonically as the engagement progresses if \( \sigma_0 \in (\pi/2, 0) \) and interception occurs when the heading error is zero.

Proof: Derivative of \( R \) with respect to \( t \) can be obtained by applying chain rule of differentiation as

\[
\frac{dR}{dt} = \frac{dR}{d\sigma} \frac{d\sigma}{dt} = \frac{dR}{d\sigma} \tag{24}
\]

Differentiating (23) with respect to \( \sigma \) yields

\[
\frac{dR}{d\sigma} = \frac{2R_0 e^\sigma \cos \sigma}{e^{\sigma_0} (\cos \sigma_0 + \sin \sigma_0) - 1} > 0 \text{ if } \sigma \in (\pi/2, 0) \tag{25}
\]

Eq. (17) implies that

\[
\dot{\sigma} < 0 \Rightarrow \sigma \text{ decreases as } t \text{ increases}
\]

Hence, \( \sigma_0 \in (\pi/2, 0) \Rightarrow \sigma \in (\pi/2, 0) \tag{26} \]

It can be concluded from (25) and (26) that the range decreases as the heading error decreases. Also, substituting (17) and (25) in (24) gives

\[
\frac{dR}{dt} < 0 \tag{27}
\]

illustrating that the range diminishes monotonically as time increases.

Substituting \( \sigma = 0 \) in (23) gives \( R = 0 \) leading to the conclusion that the interception occurs at \( \sigma = 0 \).

Proposition 2: The magnitude of the lateral acceleration given by (18) increases monotonically and its maximum value is

\[
|a_{\text{max}}| = \frac{2cV^2}{R_0} \tag{28}
\]

Proof: Use (22) in (23) and substitute the obtained result in (18) leads to

\[
a = -\frac{cV^2}{R_0} \left( \frac{2 \sin \sigma}{e^\sigma (\cos \sigma + \sin \sigma) - 1 + e^{-\sigma}} \right) \tag{29}
\]

Hence, the magnitude of the lateral acceleration is

\[
|a| = \frac{cV^2}{R_0} \left( \frac{2 \sin \sigma}{e^\sigma (\cos \sigma + \sin \sigma) - 1 + e^{-\sigma}} \right) \tag{30}
\]

Differentiating (30) with respect to \( \sigma \) yields

\[
\frac{da}{d\sigma} = -\frac{2cV^2}{R_0} \left[ \left( e^\sigma (\cos \sigma + \sin \sigma) - 1 \right) e^{-\sigma} - e^\sigma \right] \tag{31}
\]

Since \( \cos \sigma - 1 < 0 \) if \( \sigma \in (0, \pi/2) \),

\[
\frac{da}{d\sigma} < 0
\]

It implies that when the heading error \( \sigma \) reduces, the magnitude of the lateral acceleration increases. It has already been established in Section (1) that the heading error diminishes monotonically as the engagement progresses and reaches zero at the interception. Thus, the lateral acceleration increases as the engagement progresses, reaching a maximum at \( \sigma = 0 \). Hence, inserting \( \sigma = 0 \) in (30) yields the maximum lateral acceleration.

\[
|a_{\text{max}}| = \frac{2cV^2}{R_0} \tag{32}
\]

B. IMPACT TIME

As per the proposition 1, during the engagement, the heading error varies from \( \sigma_0 \) to 0 as range-to-target varies from \( R_0 \) to 0. Hence, the impact time can be found by integrating (17) where \( \sigma \) varies from \( \sigma_0 \) to 0 and \( t \) from 0 to \( t_f \)

\[
\int_{\sigma_0}^{0} e^\sigma d\sigma = -\int_{0}^{t_f} \frac{cV}{R_0} dt \tag{33}
\]

which yields

\[
t_f = \frac{R_0}{cV} (e^{\sigma_0} - 1) \tag{34}
\]

which is an exact closed-form solution for the impact time. The time-to-go can be found by integrating (17) where \( \sigma \) varies from \( \sigma \) to 0 and \( t \) from \( t \) to \( t_f \)

\[
\int_{\sigma}^{0} e^\sigma d\sigma = -\int_{t}^{t_f} \frac{cV}{R_0} dt \tag{35}
\]

which leads to

\[
t_{go} = t_f - t = \frac{R_0}{cV} (e^\sigma - 1) \tag{36}
\]
IV. IMPACT TIME CONTROL

It can be noted from (22) that the parameter \( c \) is independent of desired engagement time and is derived to satisfy the main objective of intercepting the missile with the target. Hence, a new parameter is necessary to control the impact time. It is obvious from (34) that for a given initial heading error, the engagement time is proportional to initial range \( R_0 \). Thus different engagement times are possible if \( R_0 \) is varied. However, for a given initial engagement scenario, \( R_0 \) is fixed. A possible solution to get a suitable \( R_0 \) to vary the engagement time is to adopt a two phase guidance strategy where the objective of initial phase is to find an appropriate control parameter which is a function of desired impact time, missile speed, and initial conditions. It may be noted that the initial range for phase 2 engagement has been modified to \( R_s \) whereas the initial heading error remains unchanged as \( \sigma_0 \). Hence, the lateral acceleration and maximum limit of lateral acceleration for phase-2 are obtained from (18) and (28) by substituting \( R_0 = R_s \) as

\[
a_2 = -V^2 \left( \frac{\sin \sigma}{R} + \frac{c e^{-\sigma}}{R_s} \right)
\]

and

\[
|a_{\text{max}2}| = \frac{2cV^2}{R_s}
\]

a: FIELD OF VIEW CONSTRAINT

Homing interceptors employ their own seekers, which are mounted on them, to acquire, track, and then to pursue a target. Such seekers often have limited field-of-view (FOV) capabilities. This constraint is achieved if the heading error never rises and the initial heading error is within the FOV limit. Thus, the FOV restriction is met if heading error either decrease monotonically or remain constant. For phase-1, \( \dot{\sigma} = 0 \), implying that the heading error is constant. The heading error profile varies as per (17) in phase 2, implying monotonically diminishing features. As a result, the proposed guidance law ensures FOV constraint if the initial heading error is within the FOV limit.

A. MINIMUM AND MAXIMUM IMPACT TIMES

Equation (43) implies that by adjusting the parameter \( R_s \), the total engagement time for a given initial state can be varied. To understand the dependence of engagement time on \( R_s \), consider the derivative of (43) with respect to \( R_s \),

\[
\frac{\partial t_d}{\partial R_s} = \frac{1}{V} \left( -\sec \sigma_0 + \frac{e^{\sigma_0} - 1}{c} \right)
\]

Substituting for \( c \) from (22) in (47) yields

\[
\frac{\partial t_d}{\partial R_s} = \frac{1}{V} f(\sigma_0)
\]

where

\[
f(\sigma_0) = -\sec \sigma_0 + 2 \frac{e^{\sigma_0} - 1}{e^{\sigma_0} (\cos \sigma_0 + \sin \sigma_0) - 1}
\]

The sign of \( f(\sigma_0) \) determines the sign of \( \frac{\partial t_d}{\partial R_s} \). The variation of \( f(\sigma_0) \) given by (49) versus initial heading error for all values in the range \((0, \pi/2)\) is shown in Fig. 3. This figure shows that the sign of \( f(\sigma_0) \), and hence the sign of \( \frac{\partial t_d}{\partial R_s} \), is always negative if \( \sigma_0 \in (0, \pi/2) \). Thus, it is possible to conclude that the engagement time decreases as \( R_s \) increases. As a result, the minimum and maximum feasible engagement times with the given initial condition are obtained by putting \( R_s = R_0 \) and \( R_s = 0 \), respectively, into (43), and the results are as follows:

\[
t_{\text{min}} = \frac{R_0}{cV} (e^{\sigma_0} - 1)
\]

\[
t_{\text{max}} = \frac{R_0}{V \cos \sigma_0}
\]
The maximum engagement time is feasible if the engagement consists solely of phase 1. In this scenario, the final heading error is not zero, hence the lateral acceleration is infinite, according to (16). As a result, the maximum engagement time as defined by (51) is the theoretical limit of the feasible impact time. Thus, in a realistic scenario, both phases of engagement scenarios are necessary to obtain maximum engagement time. Let $R_{\text{max}}$ be the range at which guidance command switches from phase 1 to phase 2, corresponding to the maximum lateral acceleration. Then $R_{\text{max}}$ can be computed from (46) as

$$ R_{\text{max}} = \frac{2cV^2}{|a_{\text{max}}|} $$

Substituting $R_s = R_{\text{max}}$ in (43), the maximum limit of possible engagement time considering the upper bound of lateral acceleration is obtained as

$$ t_{\text{fmax}} = \frac{R_0 - R_{\text{max}}}{V \cos \sigma_0} + \frac{R_{\text{max}}}{cV} \left(e^{\sigma_0} - 1\right) $$

Algorithm 1 Algorithm for Impact Time Control (Planar Engagement)

1. Input missile speed, $V$, Maximum limit of lateral acceleration $a_{\text{max}}$, and initial conditions, $R_0$, $\theta_0$ and $\sigma_0$
2. Find minimum impact time $t_{f\text{min}}$ using (50)
3. Calculate $R_{\text{max}}$ corresponding to upper limit $a_{\text{max}}$ using (52)
4. Compute maximum impact time $t_{\text{fmax}}$ using (53)
5. Get admissible impact time set $S = [t_{f\text{min}}, t_{\text{fmax}}]$
6. Choose a desired impact time from the set $S$
7. Compute the parameter, $c$ using (22)
8. Compute control parameter $R_s$ using (44)
9. **repeat**
   10. **if** $R > R_s$, **then**
11. Generate guidance command using (38)
12. **else**
13. Generate guidance command using (45)
14. **end if**
15. **until** $R=0$

The algorithm for the impact time control strategy for the planar engagement is shown in Algorithm 1.

V. THREE DIMENSIONAL IMPACT TIME CONTROL GUIDANCE LAW

The main goal of this study is to present a three-dimensional nonlinear guidance law for impact time control that provides an exact closed-form expressions for both the impact time and the control parameter. The strategy employed in this work is to develop yaw and pitch lateral accelerations such that the combined effect of these two accelerations produces the same heading error profile as described by (17) for planar engagement. The equation (1) is differentiated to determine the relationships between the heading error rate ($\dot{\sigma}$), yaw rate ($\dot{\psi}_m$), and pitch rate ($\dot{\psi}_m$), which is found as

$$ \sin \sigma \dot{\sigma} = \sin \gamma_m \cos \psi_m \dot{\gamma}_m + \cos \gamma_m \sin \psi_m \dot{\psi}_m $$

Substituting for $\dot{\gamma}_m$ and $\dot{\psi}_m$ from (7) and (8), respectively in (54) yields

$$ \sin \sigma \dot{\sigma} = \sin \gamma_m \cos \psi_m \left[\frac{a_p}{V} - p_1\right] + \cos \gamma_m \sin \psi_m \left[\frac{a_y}{V \cos \gamma_m} - y_1\right] $$

where

$$ p_1 = \dot{\psi}_l \sin \gamma_l \sin \psi_m + \dot{\gamma}_l \cos \psi_m $$

$$ y_1 = \dot{\psi}_l \left(\cos \gamma_l - \tan \gamma_m \cos \psi_m \sin \gamma_l\right) + \dot{\gamma}_l \tan \gamma_m \sin \psi_m $$

A. YAW LATERAL ACCELERATION

The yaw lateral acceleration is designed such that a desired variations for yaw angle described by

$$ \dot{\psi}_m = K_\psi \sin \sigma $$

where

$$ K_\psi = \frac{\psi_0}{\sin \sigma_0} $$

is achieved. The yaw turn rate is then obtained by differentiating (59) as

$$ \dot{\psi}_m = K_\psi \cos \sigma \dot{\sigma} $$

Substituting (61) and (58) into (8) yields the yaw lateral acceleration as

$$ a_y = (K_\psi \cos \sigma \dot{\sigma} + y_1)V \cos \gamma_m $$

It should be noted that the yaw lateral acceleration is a function of the heading error profile provided by (17) for the planar engagement.

B. PITCH LATERAL ACCELERATION

As previously stated, the pitch lateral acceleration is derived so as to satisfy the condition that the combined action of the yaw lateral acceleration given by (62) and the pitch lateral acceleration results in the desired variation of heading error given by (17). This can be accomplished by substituting (62) in (55) and calculating the pitch acceleration as follows:

$$ a_p = V \left(\frac{1}{\sin \gamma_m \cos \psi_m} \left(\sin \sigma - \cos \gamma_m \sin \psi_m K_\psi \cos \sigma \dot{\sigma} + p_1\right)\right) $$
It is worth noting that the formulae for the impact time and the control parameter are the same as for the planar engagement since the heading error and range variations are the same in both scenarios. Algorithm 2 depicts the methodology for impact time control in a three-dimensional engagement.

1) YAW AND PITCH LATERAL ACCELERATIONS FOR PHASE-1
During Phase-1, \( \dot{\sigma} = 0 \) and hence (62) and (63) will become

\[
a_y = V f_{1n} \cos \gamma_{in} \\
fa = V f_{1n}
\]

Algorithm 2 Algorithm for Impact Time Control (Three Dimensional Engagement)

1. Input missile speed, \( V \), Maximum limit of lateral acceleration \( a_{\text{max}} \), and initial conditions, \( \theta_0 \), \( \gamma_{i0} \), \( \psi_{i0} \), \( \gamma_{in0} \), and \( \psi_{in0} \)
2. Compute \( \sigma_0 \) from (1)
3. Find minimum impact time \( t_{\text{fmin}} \) using (50)
4. Calculate \( R_{\text{max}} \) corresponding to upper limit \( a_{\text{max}} \) using (52)
5. Compute maximum impact time \( t_{\text{fmax}} \) using (53)
6. Get admissible impact time set \( S = \{ t_{\text{fmin}}, t_{\text{fmax}} \} \)
7. Choose a desired impact time from the set \( S \)
8. Compute the parameter, \( R \), using (22)
9. Compute control parameter \( R \), using (44)
10. repeat
11. if \( R > R \), then
12. Generate guidance command using (64) and (65)
13. else
14. Generate guidance commands using (62) and (63)
15. end if
16. until \( R=0 \)

VI. SALVO ATTACK
Salvo attack is the simultaneous interception on same target by multiple missiles. Fig. 4 shows \( n \) interceptors \( I_1, I_2, I_3, \ldots, I_{n-1}, I_n \) located respectively at \((x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_{n-1}, y_{n-1}), (x_n, y_n)\). Let \( V_i, i = 1, 2, 3, \ldots, n \) and \( \sigma_{in}, i = 1, 2, 3, \ldots, n \) be the velocities and initial heading errors of the interceptors \( I_i, i = 1, 2, 3, \ldots, n \), respectively. All the interceptors are launched at \( t = 0 \) and the objective is that all hit the target at the same instant \( t_f \). To achieve the salvo attack, each interceptors is commanded with the same impact time \( t_f \) and all are guided as per the Algorithm 1. Another approach is to launch individual interceptors at \( t = t_{in0}, i = 1, 2, \ldots, n \) and use Algorithm 1 with desired impact time \( t_{il} = t_f - t_{in0} \) so that all reaches target point at same instant \( t = t_f \). These two approaches can also be used for three dimensional engagement but with Algorithm 2.

While designing the impact time guidance law, it was assumed that \( \sigma \in [0, \pi/2] \). However \( \sigma \in [-\pi/2, \pi/2] \) is the mirror image of \( \sigma \) in \( 0 \). Hence the proposed guidance law is applicable to the larger range \( \sigma \in (-\pi/2, \pi/2) \). Considering the sign, the heading error can be written as

\[
\sigma = \text{sgn}(\sigma)|\sigma|
\]

where

\[
\text{sgn}(\sigma) = \begin{cases} 
1, & \sigma > 0 \\
-1, & \sigma < 0 
\end{cases}
\]

Hence, the heading error given by (17) will be modified as

\[
\dot{\sigma} = -\text{sgn}(\sigma)\frac{cV}{R}e^{-|\sigma|}, \sigma \in (-\pi/2, \pi/2)
\]

Also, the parameters \( c \) and \( R \), and the minimum and maximum impact times will be modified as

\[
c = \frac{e^{\sigma_0}(\cos|\sigma_0| + \sin|\sigma_0| - 1)}{2}
\]

\[
R_{s} = \frac{c(R_{0} - V_{f_{il}} \cos \sigma_{0})}{c - (e^{\sigma_0} - 1) \cos|\sigma_0|}
\]

\[
t_{dl} = \frac{R_{0} - R_{s}}{V \cos|\sigma_0|} + \frac{R_{s}}{cV} (e^{\sigma_0} - 1)
\]

\[
t_{\text{fmin}} = \frac{R_{0}}{cV} (e^{\sigma_0} - 1)
\]

\[
t_{\text{fmax}} = \frac{R_{0} - R_{\text{fmax}}}{V \cos|\sigma_0|} + \frac{R_{\text{fmax}}}{cV} (e^{\sigma_0} - 1)
\]

VII. NUMERICAL SIMULATIONS
In this section, the efficacy of the proposed guidance strategy is evaluated using numerical simulations for various engagement scenarios. The simulations are done using MATLAB 2021a. For all simulation scenarios, it is assumed that the interceptor speed is 300 m/s and the bound on the interceptor’s lateral acceleration is 15g, where g is the acceleration due to gravity.

A. PERFORMANCE OF THE PLANAR ENGAGEMENT
Consider the following planar engagement scenario: \( R_{0} = 7000 \text{ m}, \theta_{0} = 0^\circ \), and \( \gamma_{0} = 60^\circ \). The initial heading error is calculated as \( \sigma_{0} = 60^\circ \) from (10). The minimum impact time found from (50) is \( t_{\text{fmin}} = 29.84 \text{ s} \). Similarly, the theoretical upper limit of impact time is \( t_{\text{fmax}} = 46.66 \text{ s} \), which is calculated using (51). However, taking into account the constraint imposed by the maximum lateral acceleration limit, \( t_{\text{fmax}} \) is calculated from (53) as 44.53 s. As a result, the set of possible impact times is \( S = [29.84, 44.53] \). And the desired impact time \( t_{dl} = 32.8 \text{ s} \) is chosen from this set \( S \).
A. Saleem, E. R. Lisy: Three Dimensional Nonlinear Guidance Law for Exact Impact Time Control

FIGURE 5. Impact time control for planar engagement.

The parameters $c$ and $R_s$ are calculated from (22) and (44), respectively, and the values $c = 1.4463$ and $R_s = 5768.5$ are obtained. The range variation during the engagement is depicted in Fig. 5. At $t = 32.8$ s, the range becomes zero, implying that the interceptor reaches the target at the desired impact time. The simulations reveal that with the proposed guidance law an exact control of impact time is possible. The range-to-target decreases monotonically satisfying (11). At $t = 8.2$ s, the condition $R = R_s = 5768.5$ is met. Thus, the phase-1 engagement corresponds to $0 \leq t \leq 8.2$, and the engagement follows the phase-2 guidance command over the remaining period. The heading error is constant during phase-1, and the heading error profile follows (17) during phase-2, as shown in Fig. 5b. The lateral acceleration profile for both phases is illustrated in Fig. 6a satisfying (38) and (45) during the two stages, respectively. The maximum lateral acceleration computed by (28) is $45.13$ which perfectly matches the value obtained by simulation, as shown in Fig. 6a. The interceptor trajectory is illustrated in Fig. 6b, implying that the interceptor impacts the target at $(7000, 0)$ which is the target location.

FIGURE 6. Impact time control for planar engagement.

B. IMPACT TIME CONTROL OF THREE DIMENSIONAL ENGAGEMENT

Consider a three-dimensional engagement with $R_0 = 10000$ m, $\gamma_{00} = 30^\circ$, $\psi_{01} = 0^\circ$, $\gamma_{01} = 40^\circ$, and $\psi_{01} = 20^\circ$. The initial heading error is computed using (1) as $\sigma_0 = 43.95^\circ$. The minimum and maximum impact times are calculated as $t_{f_{\text{min}}} = 37.6$ s, and $t_{f_{\text{max}}} = 46.3$ s, respectively, using (50) and (51). The impact time set $S = [37.6, 46.3]$ is used to select a desired impact time $t_d = 40$ s. The parameters $c$ and $R_s$ are calculated and obtained as $c = 1.0227$ and $R_s = 7247.9$ using (22) and (44). The variation of range-to-target during the encounter is depicted in Fig. 7. This figure clearly shows that the range becomes zero at $t = 40$ s, implying that the interceptor reaches the target at the desired time $t = 40$ s. The range at which the second phase of guidance law begins is the range specified by the control parameter $R_S$ which is $7247.9$ and occurs at $t = 12.7436$ s. Hence the period from $t = 12.7436$ to $t = 40$ s corresponds to the second phase of the guidance strategy. The heading error is constant from $t = 0$ to $t = 12.7436$ s, as seen from Fig. 7b which corresponds to phase-1, for which $\dot{\sigma} = 0$. The heading error varies as (17) from $t = 12.7436$ to $t = 40$ s and becomes zero at the interception time. This plot and Fig. 5b show that the seeker look angle (heading error) is
not increasing and so the FOV constraint is satisfied. The pitch and yaw lateral accelerations are shown in Fig. 8 and Fig. 9, respectively. In both cases, the duration \(0 \leq 12.7436\) s corresponds to phase-1. The lateral accelerations vary according to (64) and (65) for phase-1 and (62) and (63) for phase-2, respectively. The target point is \((8660, 0, 5000)\) since \(\gamma_0 = 30^\circ\) and \(\psi_0 = 0^\circ\). The trajectory is depicted in Fig. 10, and it is clear from this figure that the interceptor strikes the target point \((8660, 0, 5000)\). Fig 11 shows the estimated and actual time from which it is obvious that both the estimated and actual times-to-go exactly matches throughout the engagement.

**TABLE 1.** Initial conditions of the interceptors participating in salvo attack - 2D engagement.

| Interceptor | \(x_0\) (m) | \(y_0\) (m) | \(\gamma_0\) (deg) | \(R_0\) (m) | \(\theta_0\) (deg) |
|------------|-------------|-------------|-------------------|-------------|-----------------|
| \(I_1\)    | -6000       | 4000        | 20                | 7311.1      | -33.7           |
| \(I_2\)    | 4250        | 5800        | -80               | 7190.4      | -126.13         |
| \(I_3\)    | 4200        | -6300       | 170               | 7517        | 123             |
| \(I_4\)    | -5000       | -4500       | 90                | 6726.8      | 42              |

**FIGURE 7.** Impact time control of a three dimensional engagement.

**FIGURE 8.** Pitch Lateral acceleration profile (3D engagement).

**FIGURE 9.** Yaw Lateral acceleration profile (3D engagement).

**FIGURE 10.** Trajectory of three dimensional engagement.

**FIGURE 11.** Estimated and actual time.

**C. SALVO ATTACK: 2D ENGAGEMENT**

Salvo attack is the simultaneous attack on a target by a number of interceptors. It is a many to one attack. This can be achieved if all interceptors hits the target simultaneously.

1) INTERCEPTORS LAUNCHED FROM DIFFERENT LOCATIONS

If all interceptors are launched at \(t = 0\), the desired impact time for all interceptors is the same. Consider four interceptors in Table 1 with initial conditions \((x_0, y_0, \gamma_0)\). Let \((0, 0)\) is the coordinates of the target. The initial range-to-target \((R_0)\) and LOS angle \((\theta_0)\) are calculated using

\[
R_0 = \sqrt{(x_T - x_0)^2 + (y_T - y_0)^2}
\]
and
\[
\theta_0 = \tan^{-1} \frac{y_T - y_0}{x_T - x_0}
\]
which are also shown in Table 1.

Table 2 shows the minimum and maximum possible impact times for each interceptor. The set \( S = [29.05 \text{ } 33.51] \) is the intersection set of all conceivable impact time sets. Now chose a desired impact time from this set. A desired impact time of \( t_d = 30 \text{ s} \) is selected, and all interceptors are launched at \( t = 0 \) and commanded with the same desired time of \( t_d = 30 \text{ s} \). The range variation of all interceptors is depicted in Fig. 12. The range-to-target of all interceptors becomes zero at \( t = 30 \text{ s} \), demonstrating that simultaneous attack at the desired time is achieved. The trajectory of each interceptors is illustrated in Fig. 12b, which demonstrates that all of them hit the target.

2) INTERCEPTORS LAUNCHED FROM SAME LOCATION
Consider a 2D engagement of four interceptors with initial conditions \( R_0 = 7000 \text{ m}, \theta_0 = 0^\circ \), and \( \gamma_0 = 60^\circ \). The interceptors are launched at different instants \( t_0 = t_d + (n - 1)t_g \), \( n = 1, 2, 3, 4 \) and let \( t_g = 2 \text{ s} \). The objective is that all should reach the target point at a desired time \( t_d \). The desired time is so chosen that \( t_{f_0} = t_d - (n - 1)t_g \in [t_{f_{\text{min}}} \text{ } t_{f_{\text{max}}}] \). Table 3 shows the launch time of different interceptors and their engagement time. For example, the interceptor \( I_3 \) is launched at \( t = 4 \text{ s} \) and its engagement time should be \( t_{f_3} = t_d - (n - 1)t_g = 34 \text{ s} \) so that it hits the target at \( t = 4 + 34 = 38 \text{ s} \). Similarly, it can be seen from this table that all reach the target point at \( t = 38 \text{ s} \) if they are launched at the instant and commanded for the engagement time shown in second and third column of the Table 3, respectively. Fig. 13a represents the range variation. The figure illustrates that the interceptors are launched at \( t = 0, 2, 4, \) and 6, respectively and the range-to-target of all interceptors becomes zero at \( t = 38 \text{ s} \) showing that simultaneous impact is achieved at the desired time. Fig. 13b shows the trajectories of all interceptors. The figure clearly shows that the path length of the interceptor that requires less engagement time is less than the path length of the interceptor that requires greater engagement time.

D. SALVO ATTACK: 3D ENGAGEMENT
Consider a 3D engagement with initial conditions shown in Table 4. Let the target be at origin. Table 5 shows the magnitude and orientation of the initial range and also the
The initial heading error is computed using (1). The magnitude and orientation of the initial range are computed using

\[ R_0 = \sqrt{(x_t - x_m)^2 + (y_t - y_m)^2 + (z_t - z_m)^2} \]

\[ \gamma_0 = \tan^{-1} \frac{z_t - z_m}{\sqrt{(x_t - x_m)^2 + (y_t - y_m)^2}} \]

\[ \psi_0 = \tan^{-1} \frac{y_t - y_m}{x_t - x_m} \]

where \((x_t, y_t, z_t)\) is the target position and \((x_m, y_m, z_m)\) is the initial interceptor position. Table 6 depicts the minimum and maximum impact time of the individual interceptors which are calculated using (72) and (73). The intersection set of impact time of all the three interceptors is

\[ S = [37.39, 52.94]. \]

A desired time \(t_d = 37.5\) is chosen from this set. Fig. 14 shows the range variation. Starting from their respective initial range, the range-to-target of all interceptors decreases and becomes zero at \(t = 37.5\) leading to the conclusion that simultaneous impact at desired time is achieved. Fig. 14b represents the trajectories of individual interceptors participating in the 3D salvo attack.

### E. COMPARISON WITH EXISTING METHODS

In this section, the performance of the proposed method is compared with two recently published three-dimensional guidance strategies for impact time control: (i) Kumar and Mukherjee [23] and (ii) Sinha et al. [24]. The time-to-go computed by the two methods and the proposed method is compared with the actual time-to-go, as shown in Fig. 15a. This figure demonstrates that the two existing methods produce a time-to-go error during the initial engagement, whereas the time-to-go error of the proposed method is zero throughout the engagement, as the actual and computed times-to-go of the proposed method are identical from the initial engagement to the final interception time. For both recent approaches, additional lateral acceleration is required to eliminate the time-to-go error. Nonetheless, this additional control effort is not necessary for the proposed method. This
fact is clear from Figs. (16a) and (16b). This is evident from Figs. (16a) and (16b). These two figures also demonstrate that the maximum lateral acceleration demand for both strategies is greater than that of the proposed method. Table 7 depicts the maximum lateral acceleration requirements for all three methods. This table demonstrates that the maximum lateral acceleration requirement for the proposed method is minimal. The heading error variation of the recent works and proposed method is shown in Fig. 15b. The heading error is increased for both existing methods in order to eliminate the time-to-go error. Therefore, these methods require an additional control strategy to ensure that the look-angle field-of-view constraint is satisfied. For the proposed method, however, the heading error never increases, as it is governed by $\sigma_{\dot{\theta}} = 0$ in the first phase and 17 in the second phase. According to the proven proposition 1, the heading error decreases monotonically as the engagement progresses. Consequently, if the initial heading error is within the prescribed limits, the proposed method satisfies the field of view constraint. As a summary of the comparison, the following points can be made: The time-to-go error is zero throughout the engagement for the proposed method as opposed to the non-zero initial time-to-go errors produced by existing methods, (ii) the maximum lateral acceleration demand is lowest for the proposed method, (iii) the heading error is never increased as opposed to an initial increase for other methods, and (iv) the proposed method inherently satisfies the field of view requirement.

VIII. CONCLUSION

This paper proposes a nonlinear guidance law for impact time control of three-dimensional interceptor-target engagement. First, the guidance law for a planar engagement is developed, followed by its extension to a three-dimensional scenario. Key to extending the guidance law to the three-dimensional case is the development of yaw and pitch lateral accelerations such that their combined effect produces the same heading error profile as proposed for the planar case. The heading error profile is selected so that the time derivative of heading error variation is integrable and can ensure a zero miss distance when the heading error is zero. A two-stage strategy is utilized to achieve an intercept at the desired time. For both the impact time and the control parameter, exact closed-form equations are derived. Because the control parameter is a function of the desired impact time and initial conditions, it can be precisely adjusted to provide impact time control. The expressions for the theoretical and practical lower and upper limits are derived, and a feasible impact time can be selected from this set. Another benefit of the proposed guidance law is that it does not violate the seeker’s restricted field of view. The proposed method is used to launch a salvo attack, in which multiple interceptors simultaneously strike a target, thereby increasing the survivability.
of interceptors against anti-ship missile self-defense mechanisms. Simulation studies conducted for both 2D and 3D engagements prove the utility and validity of the proposed guideline law. A future objective of this work would be to develop a three-dimensional nonlinear guidance law with a continuous lateral acceleration profile in which the impact time can be controlled by a tuning parameter that is directly proportional to the desired impact time and initial conditions via an exact closed-form expression.

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