Chiral quarks, chiral limit, nonanalytic terms and baryon spectroscopy

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Abstract

It is shown that the principal pattern in baryon spectroscopy, which is associated with the flavor-spin hyperfine interactions, is due to the spontaneous breaking of chiral symmetry in QCD and persists in the chiral limit. All corrections, which are associated with a finite quark (Goldstone boson) mass are suppressed by the factor $(\mu/\Lambda_{\chi})^2$ and higher.

In a recent work [1] Thomas and Krein questioned the foundations of the description of the baryon spectrum with a chiral constituent quark model [2,3]. They claim that “...the leading nonanalytic (LNA) contributions are incorrect in such approaches. The failure to implement the correct chiral behaviour of QCD results in incorrect systematics for the corrections to the masses.” The argument made was that the splitting pattern implied by the short-range Goldstone boson exchange (GBE) operator

\[-\vec{\tau}_i \cdot \vec{\tau}_j \vec{\sigma}_i \cdot \vec{\sigma}_j,
\]

should be inconsistent with chiral symmetry because it is inconsistent with the leading nonanalytic contribution to baryon mass predicted by heavy baryon chiral perturbation theory (HBChPT) [4]. I here show that the HBChPT has no bearing on this issue.

It is shown by Jenkins and Manohar [4] that the nonanalytic contributions to octet and decuplet masses, such as $\mu^3, \mu^2 \ln \mu^2, ...$, where $\mu$ stands for a Goldstone boson mass, within the HBChPT in the chiral limit arise only from the loop diagrams a) and b) of Fig. 1, where only diagonal octet-octet or decuplet-decuplet vertices with respect to baryon field contribute, and there is no contribution from the diagrams c) and d) of Fig. 1, where the intermediate baryon belongs to a different $SU(3)$ multiplet. This argument was used by Thomas and Krein, but it was not emphasized that this statement is valid only in the chiral limit. In this limit the $\Delta$ and $N$ (decuplet-octet) are well split and all the infrared divergences of the diagrams c) and d) disappear. Since it is infrared contributions which are an

1Through the whole paper I use pion-exchange. The transition to the whole GBE exchange within the $SU(3)_F$ limit implies a substitution of the $SU(2)$ flavor matrices by the $SU(3)$ ones.
origin for nonanalytic terms, such terms should vanish in the chiral limit in the case of c) and d), while they persist in the case of diagrams a) and b). Because these nonanalytic terms for $N$ and $\Delta$ from a) and b) have exactly the same spin-isospin factors, they cannot split $N$ and $\Delta$ in the chiral limit. Beyond the chiral limit there appears a contribution from the diagrams c) and d) as well.

Consider now in details what happens with the $\Delta - N$ splitting in the chiral limit within the chiral constituent quark model. There are two distinct pion contributions to the baryon mass of Fig. 2. The diagram a) is a constituent quark self-energy while the diagram b) represents the pion-exchange interaction between the constituent quarks. Consider the first one. The result is well known and coincides with that one for the nucleon in baryon ChPT: it contains, in particular, the nonanalytic terms. All the nonanalytic terms of the constituent quark self-energy diagram are hidden in the constituent quark mass $m$ and thus appear in the $3m$ contribution to the baryon mass within the quark model. Evidently they do not split $N$ and $\Delta$, in agreement with HBChPT. These nonanalytic terms appear at the loop level of the effective pion-nucleon and pion-delta Lagrangians.

There is, however, a small difference in the magnitude of these nonanalytic terms. The quark and nucleon axial coupling constants are related as $g_A^N = 3/5 g_A^N$ within the exact $SU(6)$ nucleon wave function. There are three constituent quarks in the nucleon and thus the total contribution within the quark model is proportional $\sim 3(g_A^N)^2$, while at the nucleon level it is given by $\sim (g_A^N)^2 = 25/9(g_A^N)^2$. One of the sources of this small difference is that the exact $SU(6)$ is used, which is in fact broken by the interaction (1) and thus the nucleon wave function contains an admixture of the components from other multiplets.

Now consider the interaction diagram b) of Fig. 2. This diagram is not a loop diagram. All effects related to this process are beyond the baryon ChPT, which deals with structureless baryons, and its effect is absorbed into a tree-level baryon mass within the effective baryon-meson Lagrangian. Effect of these meson exchanges can be systematically studied within the large $N_c$ approach. Note that the large $N_c$ nucleon wave function is a quark model wave function with the infinite number of quarks and with the FS Young diagram consisting of one row (with infinite number of boxes). What is important is that both large $N_c$ and simple quark model nucleon wave function with $N_c = 3$ are described by the one-row Young diagram (i.e. they belong to a completely symmetric $SU(6)_{FS}$ representation), and that the pion-exchange diagram b) satisfies all the necessary large $N_c$ counting rules. Both these circumstances is one of the origins of a success of the chiral constituent quark model in baryon spectroscopy.

\(^2\)Obviously the authors of ref. forget about these nonanalytic terms.

\(^3\)Only when this diagram is used to evaluate a matrix element perturbatively it also becomes a loop diagram, but of different kind. Its contribution is determined by the $SU(6)$ and radial structure of the baryon zero-order wave-function, in contrast to diagram a).
Unfortunately it is notoriously difficult to treat this interaction in a consistent relativistic manner, but since the constituent quark mass is rather large, \( \sim 300 - 400 \text{ MeV} \), one hopes that at least qualitative features can be understood using the \( 1/m \) expansion of the constituent quark spinors. To leading nonvanishing order \( (1/m^2) \) the structure of the \( Q_i - Q_j \) pion exchange interaction in momentum representation is given as

\[
V_\chi \sim \vec{\sigma}_i \cdot \vec{q} \vec{\sigma}_j \cdot \vec{q} \vec{\tau}_i \cdot \vec{\tau}_j D(q^2) F^2(q^2),
\]

where \( \vec{q} \) is pion 3-momentum, \( D(q^2) \) is dressed pion Green function, which generally includes both the nonlinear terms of the chiral Lagrangian and fermion loops, and \( F(q^2) \) is a pion-quark form factor, which takes into account the internal structure of both pion and constituent quark and thus provides natural ultraviolet cut-off. This form factor should be normalized to 1 at the time-like momentum \( q^2 = \mu^2 \). For the interaction of two different particles in static approximation only space-like momenta of the pion are important. Approaching \( \vec{q} \to 0 \) the pion Green function approaches at a free static Klein-Gordon Green function \( D_0 = -(\vec{q}^2 + \mu^2)^{-1} \) and form factor does not have any singularity. It then follows from (2) that

\[
V_\chi(\vec{q} = 0) = 0.
\]

This result is rather general and does not rely on any particular form of the chiral Lagrangian and pion-quark form factor. The only necessary ingredient is that pion is a pseudoscalar and hence the pion-quark vertex vanishes with \( \vec{q} \). The requirement (3) is equivalent in coordinate representation to

\[
\int d\vec{r} V_\chi(\vec{r}) = 0.
\]

The sum rule (4) is trivial for the tensor component of the pseudoscalar exchange interaction since the tensor force automatically vanishes on averaging over the directions of \( \vec{r} \). But for the spin-spin component of the pion-exchange interaction the sum rule (4) indicates that there must be a strong short-range term. Indeed, at large interquark separations the spin-spin component is represented by the Yukawa tail \( \sim \vec{\tau}_i \cdot \vec{\tau}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \mu^2 e^{-\mu r}/r \), it then follows from the sum rule (4) that at short interquark separations the spin-spin interaction must be opposite in sign compared to Yukawa tail and very strong, of the form (4). The concrete radial form of the interaction (1) should be determined by the explicit form of the chiral Lagrangian and pion-quark form factor, which are unknown. It is this short-range part of the GBE interaction between the constituent quarks which is of crucial importance for baryons: it has the sign appropriate to reproduce the level splittings and dominates over the Yukawa tail in baryons. Within the oversimplified consideration with a free Klein-Gordon Green function and without the pion-quark form factor, one obtains the well-known pion-exchange potential

\[
V = \frac{g^2}{4\pi} \frac{1}{34m_i m_j} \vec{\tau}_i \cdot \vec{\tau}_j \vec{\sigma}_i \cdot \vec{\sigma}_j \left\{ \frac{\mu^2 e^{-\mu r}}{r} - 4\pi \delta(\vec{r}) \right\},
\]

where the tensor force component which is irrelevant to discussion here, has been dropped.
The pion-exchange interaction makes a sense only at momenta below the chiral symmetry breaking scale $\Lambda_\chi \sim 1\text{GeV}$, where both pions and constituent quarks exist as effective quasiparticle degrees of freedom. The ultraviolet cut off is provided by the pion-quark form factor and thus the $\delta$-function term in (5) is substituted by the finite function with the range $\Lambda_\chi^{-1}$. Note that the short-range interaction of the same form comes also from the $\rho$-exchange [5], which can also be considered as a representation of a correlated two-pion exchange [6], since the latter has a $\rho$-meson pole in t-channel. There are phenomenological reasons to believe that these contributions are also important [5,6].

What happens with the pion-exchange potential in the chiral limit, $\mu = 0$? In this case the sum rule (3) - (4) is no longer valid since the $\vec{q}^2$ behaviour of the numerator is exactly cancelled by the pion Green function, $-\vec{q}^{-2}$. As a result the $\mu$-dependent long-range part of the interaction vanishes, while the $\Lambda_\chi$-dependent short-range part survives. Note that while the volume integral (3) - (4) is discontinuous and in the chiral limit the right-hand side of equations (3) - (4) is not a zero, the approaching chiral limit in the interaction potential (5) is continuous. That this is so can be easily seen from (5) applying the limit $\mu = 0$.

Thus the contribution of the interaction (5) via its short-range part appears at the leading order, $m_0$, within the chiral perturbation theory, where $m_c$ stands for current quark mass. This simple observation has by far-going consequences: while the physics of baryons does not change much in the chiral limit (e.g. the $\Delta - N$ mass splitting persists), the long-range spin-spin nuclear force vanishes (the tensor interaction in this limit is $\sim r^{-3}$). Note, that approaching the chiral limit does not cause any infrared problems (there are no infrared divergences) and this limit can be safely reached by a substitution $\mu = 0$ in the pion Green function. It also implies that there are no nonanalytic in $\mu$ contributions to baryon masses and, in particular, to $\Delta - N$ mass splitting from the long-range Yukawa tail in the chiral limit. The crucial difference between the loop-diagram a) and the interaction diagram b) as far as the infrared behaviour is concerned is obvious.

The leading contribution from the long-range part of the interaction (5) appears at the order $\mu^2 \sim m_c$ and thus is suppressed by a small factor $\left(\frac{\mu}{\Lambda_\chi}\right)^2$ compared to the contribution of the short-range part. The contribution at the order $\mu^3 \sim m_c^{3/2}$ is suppressed by the third power of this small factor.

This is perfectly consistent with the large $N_c$ analysis [7] up to the order $N_c^{-2}$ and also with analysis which incorporates in addition the ChPT [8]. These authors find the following relations between the octet-decuplet masses at the tree level (taking into account the $SU(3)$ breaking):

\[ M_\Delta - M_N = M_{\Sigma^*} - M_\Sigma + \frac{3}{2}(M_\Sigma - M_\Lambda), \]
\[ M_{\Xi^*} - M_{\Xi} = M_{\Sigma^*} - M_\Sigma, \]
\[ M_{\Omega} - M_\Delta = 3(M_{\Xi^*} - M_{\Sigma^*}), \]
which are very well satisfied empirically. Note that exactly the same relations have been found within the chiral constituent quark model \[2\] (see eq. (7.5)). It is also found in ref. \[8\] that the loop corrections to the relations above appear at the order $\mu^2$, which is consistent with our analysis.

The main merit of the hyperfine interaction (\[1\]) is not that it is able to explain the octet-decuplet splitting, which can be also explained in other picture, but that it solves at the same time the long-standing problem of the relative position of the lowest positive-negative parity excited states \[2,3,5\]. What is interesting, even an analysis of the negative parity states alone within a careful phenomenological approach \[9\] or within the large $N_c$ study \[10\] give an additional credibility to the interaction (\[1\]).

There are, nevertheless, two obvious limitations in the use of the potential picture (\[3\]): (i) it relies on the leading term in the $1/m$ expansion of the constituent quark spinors, and (ii) it uses a static approximation for a pion Green function, and thus all retardation effects are neglected. How important these retardation effects are is an interesting issue and deserves a special study. However, in order to treat the retardation effects in a nonperturbative calculation one would solve a Bethe-Salpeter-like equation in the 3-body system... Within the static approximation the nonperturbative treatment is straightforward \[3\]. It is important to realize, however, that the successes of the GBE interaction in baryon spectroscopy are based not on details of the dynamical space-time treatment, but on the flavor-spin structure and sign of the short-range interaction (\[1\]) \[2\], which is rather general and persists with any dynamical treatment.

In conclusion, I will summarize. The idea of the chiral constituent quark model \[2,3,5\] is that the main features of the baryon spectrum are supplied by the spontaneous breaking of chiral symmetry, i.e. by the constituent mass of quarks and the interaction (\[1\]) between confined constituent quarks. As a consequence the $N$ and $\Delta$ are split already in the chiral limit, as it must be. The expressions (in the notation of ref. \[2\])

\[
M_N = M_0 - 15P_{00}^\pi,
\]

\[
M_\Delta = M_0 - 3P_{00}^\pi,
\]

(6)

where $P_{00}^\pi$ is positive, arise from the interaction (\[1\]). The long-range Yukawa tail, which has the opposite sign represents only a small perturbation. It is in fact possible to obtain a near perfect fit of the baryon spectrum in a dynamical 3-body calculation neglecting the long-range Yukawa tail contribution, with a quality even better than that of \[3\].

The implication is that baryon ChPT has no bearing on the interactions (\[1\]) nor on the expressions (\[3\]), which should be considered as leading order contributions ($\sim m_c^0$, where $m_c$ is current quark mass) within the chiral perturbation theory. This does not mean, however, that the systematic corrections from the finite meson (current quark) mass should be ignored.
A rough idea about importance of the finite meson mass corrections for the $N$ and $\Delta$ can be obtained from the comparison of the contributions of the first and second terms in (5) in nonperturbative calculations [3]. The former one turns out to be much smaller than the latter. This is because of a small matter radius of the $N$ and $\Delta$ [11]. For highly excited states, however, the role of the Yukawa tail increases because of a bigger baryon size and thus the importance of the ChPT corrections should be expected to increase. To consider these corrections systematically one definitely needs to consider the loop contributions to the interactions between constituent quarks as well as the couplings to decay channels, which is rather involved task. This task is one for constituent quark chiral perturbation theory which is awaiting practical implementation.

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REFERENCES

[1] A. W. Thomas and G. Krein, Preprint “Chiral corrections in hadron spectroscopy”, nucl-th/9902013 (version 2), Phys. Lett. B, in print.
[2] L. Ya. Glozman and D.O. Riska, Physics Reports 268, 263 (1996).
[3] L. Ya. Glozman, W. Plessas, K. Varga, R. F. Wagenbrunn, Phys. Rev. D58, 094030 (1998).
[4] E. Jenkins and A. Manohar, in: “Effective Field Theories of the Standard Model”, ed. Ulf-G. Meišner, World Scientific, 1992, p. 113; Phys. Lett. B255 (1991), 558.
[5] L. Ya. Glozman, Surveys in High Energy Physics (1998) - in print, hep-ph/9805345.
[6] D.O. Riska and G. E. Brown, hep-ph/9902319.
[7] R. Dashen, E. Jenkins and A.V. Manohar, Phys. Rev. D49 (1994) 4713, (E) D51 (1995) 2489.
[8] Y. Oh and W. Weise, hep-ph/9901354.
[9] H. Collins and H. Georgi, Phys. Rev. D59 (1999) 094010.
[10] C.E. Carlson, C.D. Carone, J.L. Goity, and R.F. Lebed, Phys. Rev. D59 (1999) 114008.
[11] L. Ya. Glozman and K. Varga, hep-ph/9901439.

Figure captions

Fig.1 Pion loop contributions to the baryon mass within the baryon chiral perturbation theory.

Fig.2 Pion loop a) and pion exchange b) contributions to the baryon mass within the chiral constituent quark model.
