The Entropy Transformed Rayleigh Distribution: Properties and Applications

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Abstract. In this paper, we introduce a new one-parameter continuous model, called the T-Rayleigh distribution which is obtained by applying transform of entropy, the new function was investigated and found to meet the conditions of the probability function. Some mathematical properties of this distribution, such as density function, cumulative function, moments, moment generating function, mode, median, skewness, kurtosis, Shannon and Renyi entropy are derived. The model parameter is estimated by the maximum likelihood method. Finally, we illustrated the importance of this model by the means of three different real data sets, we conclude that the new model performs better than the original Rayleigh distribution, via the three applications.

Keywords: T-Rayleigh model; entropy; transform of entropy.

1. Introduction

Khodabin & Ahmadabadi in (2010) introduced a study includes some properties such as the entropy function for the general Gamma(GG) distribution, exponential distribution and Weibull distribution, and use the method of Moments in estimating the parameters and compare method of maximum likelihood [1]. She researcher presented (Aseel) in (2012) a research in which she discussed to find a probability distribution for the times of failure through entropy transformation function using function reliability and cumulative distribution function for Burr Type-XII distribution [2]. Hassan, Ibrahim & Abood in (2013) suggested a probability distribution model, which representing certain generalizations of two- parameters Weibull distribution, And suggested this function using entropy such as transformation using the associative distribution function F(t) and the reliability function R(t)[3]. We present during this study a new model of the Rayleigh distribution called the T-Rayleigh distribution, by applying entropy transform and some mathematical properties are derived.

To compare the performance of the new distribution models, we consider the statistical criteria like; Kolmogorov-Smirnov statistic (K-S), Cramer-von Mises statistic (CR-M),
Anderson-Darling statistic (AD), Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC), and -Log likelihood (-LL). However, the better distribution corresponds to the smaller values of K-S, CR-M, AD, AIC, BIC and -LL criteria. Formulae of these criteria are given by

- Akaike information: \[ AIC = 2k - 2\log L \]

- Consistent Akaike information: \[ CAIC = AIC + \frac{2k(k+1)}{n-k-1} \]

- Bayesian information: \[ BIC = -2L + k \log (n) \]

- Cramer-Von Mises (W) statistic \[ CR-M = \frac{1}{12} n + \sum_{i=1}^{n} \frac{2i-1}{2n} - F(Y_i) \]

- Anderson-Darling (AD) statistic \[ AD = -n - \sum_{i=1}^{n} \frac{2i-1}{2n} \left[ \ln (F(Y_i)) + \ln (F(Y_{n+1-i})) \right] \]

and Kolmogorov Smirnov (K-S) statistic: \[ K - S = \max \left| F(Y_i) - \frac{i}{N}, \frac{i}{N} - F(Y_i) \right| \]

Where \( L \) is the Log-likelihood function, \( n \) is the size of random sample, and \( k \) is the number of parameters in the model.

2. Entropy Concept and Information:

The concept of entropy in the field of information theory was defined by Shannon (1948) as a measure of the uncertainty of random variables, which also considers the content of information [4]. After 1948 [6], various extensions of the Shannon entropy, such as Renyi entropy (1961) and Tsallis entropy (1988), including the Shannon entropy, were introduced as a special case.

Suppose that \( X \in x_i \) indicates a discrete random variable with (pmf) \( p(x_i) \), then information content \( I(x_i) \) can be measured as follows:

\[
I(x_i) = \log_b \left( \frac{1}{p(x_i)} \right)
\]

Equation (1) achieves the following two axiom[5]:

i) The higher probability of the event Implies the less information. When the probability of its occurrence is 1, its occurrence gives us no information, i.e.:
\[
P(x_i) = 1 \implies I(x_i) = 0
\]
\[
P(x_i) < 1 \implies I(x_i) > 0
\]

ii) If the x and y events are independent of each other, it is:
\[
I(x_i, y_i) = I(x_i) + I(y_i)
\]

It is natural that the information content of two independent events is equal to their respective information.

The unit of information is based on the base of the logarithm, such bit for base (2) and not for base(e).

The marginal entropy of \( X \) can be expressed by:

\[
H(X) = E[I(x_i)] = - \sum_{i=1}^{n} p(x_i) \log_2 p(x_i).
\]

2
Where \( p(x_i) \) is the probability of random variable \( x_i \), and \( n \) is the number of events. Equation (2) can be promoted to define the joint entropy of \( Y \) given \( X \) (or vice versa), expressed by \( H(X, Y) \) as follows:

\[
H(X, Y) = -\sum_{i=1}^{m} \sum_{j=1}^{n} p(x_i, y_j) \log_2 p(x_i, y_j).
\]

where, \( p(x_i, y_j) \) is the joint probability of random variables \( X,Y \) with \( m \) and \( n \) events. We can also define the mutual information of \( Y \) given \( X \) (or vice versa), as follows:

\[
I(X,Y) = -\sum_{i=1}^{m} \sum_{j=1}^{n} p(x_i, y_j) \log_2 \frac{p(x_i, y_j)}{p(x_i)p(y_j)}.
\]

3. The entropy transformed- Rayleigh distribution:

The cumulative distribution function \( (c. d. f) \) for the random variable \( X \) of Rayleigh distribution is given by[7]:

\[
F(x) = 1 - e^{-\frac{x^2}{2a^2}}, x \geq 0 \quad a > 0
\]

And also the Reliability function \( R(x) \) for the random variable \( X \) of Rayleigh distribution can be obtained as:

\[
R(x) = 1 - F(x) = e^{-\frac{x^2}{2a^2}}, \quad a > 0
\]

Soleha and Sewilam[8] considered the random variable \( X \) which represents the appropriate runtime for any component, and introduced the following expression:

\[
g(x) = F(x) + R(x) \ln R(x),
\]

Where, \( F(x) \) and \( R(x) \) are the cumulative and reliability function of a positive continuous variable \( X \), respectively. They called this shape of \( g(x) \) an "entropy-like transformation", possibly because of the term \( \{ R(x) \ln R(x) \} \), which is similar to the entropy expression associated with \( f(x) \) is the probability density function \( (p. d. f) \) of a continuous random variable \( X \)

\[
H(f) = -\int f(x) \ln f(x) \, dx,
\]

When equation(7) is employed, and derivation \( g(x) \) with respect to \( x \), the probability density function \( (P. d. f) \) in the following be obtained \( \{ g'(x) = R'(x) \ln R(x) \} \) because it fulfills the condition:

\[
\int_{0}^{\infty} g'(x)dx = \int_{0}^{\infty} R'(x) \ln R(x)dx = 1.
\]

Substituting of cumulative function in equation (6) and reliability function in equation (5) in equation (6) we obtain:

\[
g(x) = 1 - e^{-\frac{x^2}{2a^2}} + e^{-\frac{x^2}{2a^2}} \ln \left( e^{-\frac{x^2}{2a^2}} \right),
\]

And when derivation a function \( g(x) \) we obtain:

\[
g'(x) = 0 + \frac{x}{a^2} e^{-\frac{x^2}{2a^2}} + \frac{x}{a^2} e^{-\frac{x^2}{2a^2}} \left( \frac{x^2}{2a^2} \right) - \left( \frac{x}{a^2} \right) e^{-\frac{x^2}{2a^2}},
\]
The T-Rayleigh P. d. f is

\[ h(x) = g'(x) = \frac{x^3}{2a^4} e^{\frac{-x^2}{2a^2}}, \quad x >, a > 0. \quad (10) \]

We can prove that \( \int_0^\infty h(x) \, dx = 1. \)

With T-Rayleigh C. d. f

\[ G(x) = 1 - \frac{x^2}{2a^2} e^{\frac{-x^2}{2a^2}} - e^{\frac{-x^2}{2a^2}}. \quad (11) \]

Figure (1): Plots of the (P. d. f) of the T-Rayleigh model.

Figure (1) above illustrates some of the possible graphs of the (P. d. f) of a transformed Rayleigh distribution for different values of \( a \). The symmetry of T-Rayleigh distribution depends on the graph parameter \( a \). The distribution is positively skewed for \( a < 1 \), when \( a = 1 \), the distribution is symmetric. As can be seen from the graphs above, we observed that the skewness value(0.06848).This means that the T-Rayleigh model is positive-skewness, and also the kurtosis value (0.00427) the curve of this distribution is tapered and the degree of kurtosis increases by the decrease in the value of \( a \).
We also notice from Figure (2) above illustrates that the corresponding distribution function is an increasing function of $X$.

3.1. Statistical Properties

In this section, we introduce some important statistical and mathematical properties of T-Rayleigh such as moments and variance, moment generating function, mode, median, Shannon and Renyi entropy.

3.1.1. Moments

The moment about the point of origin is given by

$$\mu_r = E(x^r) = \int_0^\infty x^r h(x, a) \, dx,$$

$$= \frac{1}{2a^2} \int_0^\infty x^{r+3} e^{-\frac{x^2}{2a^2}} \, dx,$$

Let $\gamma = \frac{x^2}{2a^2} \Rightarrow x^2 = 2a^2\gamma$

$$x = a\left(2\gamma\right)^{1/2}$$

$$\left[ j \right] = \frac{dx}{dy} = \frac{a}{2^{1/2} \gamma^{1/2}},$$

$$= 2^{r+1-1} a^r \int_0^\infty \gamma^{r+1} e^{-\gamma} \, dy,$$

Then the moment generating function of the T-Rayleigh model is given by:

$$E(x^r) = 2^{r} a^r \Gamma \left( \frac{r}{2} + 2 \right). \quad (12)$$

The mean and second moment of the T-Rayleigh can be obtained by putting $r=1,2$ in equation (12):
\[ E(X) = \mu_1 = \frac{3\sqrt{2\pi} a}{4}. \]  
\[ E(X^2) = \mu_2 = 4a^2. \]

The variances is given by
\[ \text{var}(X) = E(X^2) - E(X)^2 = 0.46570826 \text{ a}^2. \]

3.1.2. Moment Generating Function
The moment generating function of X, say \( M_X(t) \), is
\[ M_X(t) = E(e^{tX}) = \int_0^\infty e^{tx} h(x, a) \, dx, \]
\[ = \int_0^\infty \left( 1 + tx + \frac{(tx)^2}{2!} + \cdots \right) h(x, a) \, dx, \]
\[ = \int_0^\infty \sum_{i=1}^\infty \frac{t^i}{i!} X^i h(x, a) \, dx, \]
\[ = \sum_{i=1}^\infty \frac{t^i}{i!} E(X^i), \]
Then the moment generating function of T-Rayleigh model is given by:
\[ M_X(t) = \sum_{i=1}^\infty \frac{t^i}{i!} \frac{1}{2^2a^i} \Gamma \left( \frac{i}{2} + 2 \right). \]

3.1.3. Mode
The mode can be obtained as
\[ \frac{\partial}{\partial x} \log h(x, a) = 0, \]
\[ \frac{\partial}{\partial x} \log \left[ \frac{1}{2a^3} x^3 e^{-\frac{x^2}{2a^2}} \right] = 0, \]
\[ x = a\sqrt{3}. \]

3.1.4. Median
The median can be obtained as
\[ \int_0^m h(x, a) \, dx = 0.5, \]
\[ \int_0^m \frac{x^3}{2a^4} e^{-\frac{x^2}{2a^2}} \, dx, \]
\[ \text{Let } \gamma = \frac{x^2}{2a^2} \implies x = a \left( \frac{1}{2} \gamma \right)^{\frac{1}{2}} \implies [j] = \frac{dx}{d\gamma} = \frac{a}{\gamma^{3/2} \gamma^{1/2}}. \]
\[ \int_0^m \gamma e^{-\gamma} d\gamma = 0.5, \]
And use integration by parts which takes the form; \( \int udv = uv - \int vdu \) we obtain the median:
\[ m \approx 1.15874. \]
3.1.5. Quantile function

The quantile function is obtained by the inverse distribution function of T-Rayleigh model. We equate the \((C, d, f)\) for T-Rayleigh model, \(G(x_q) = q\) for \(x_q\), as follows:

\[
q = 1 - x^2 \frac{x_q}{2a^2} e^{-x^2} - e^{-x^2}, \quad \text{suppose that } z = \frac{x^2}{2a^2},
\]

\[
\Rightarrow q - 1 = -e^{-z}(z + 1)
\]

Such that \(u = -z - 1\), as well as \(z = -u - 1\)

\[
\Rightarrow q - 1 = -e^{(-u-1)}((-u - 1) + 1),
\]

\[
\Rightarrow q - 1 = ue^{u+1}
\]

Using the Taylor series of the Lambert W function:

\[
W_0(x) = \sum_{j=0}^{\infty} \frac{(-j)^{j+1}}{j!} x^j.
\]

\[
\Rightarrow q - 1 = e(x - x^2)
\]

We can write the above expression as:

\[
ex^2 - ex + (q - 1) = 0,
\]

Solve the equation above for \((z)\), using General Law:

\[
z = \frac{e + \sqrt{e^2 - 4e(q - 1)}}{2e},
\]

Substituting the value of \((z)\) and simplifying we obtain:

\[
x_q = a \left[ 2 \left( \frac{e + \sqrt{e^2 - 4e(q - 1)}}{2e} \right) \right]^2.
\]

Thus, Equation (20) is the quantum function of the T-Rayleigh model. For \(q = 0.25, 0.5\) and 0.75 we obtain the first, second median and third quartile of the model respectively.

3.1.6. Skewness and Kurtosis

The skewness and kurtosis of T-Rayleigh model were found we obtain:

\[
S = \frac{Q(0.75) + Q(0.25) - 2Q(0.5)}{Q(0.75) - Q(0.25)} = 0.06848.
\]

and

\[
K = \frac{Q(0.375) - Q(0.125) + Q(0.875) - Q(0.625)}{Q(0.75) - Q(0.25)} = 0.00427.
\]

3.1.7. Shannon and Renyi entropy

The Shannon entropy can be obtained as

Taking the Natural-Logarithm of the T-Rayleigh model as in equation (10), we obtain:

\[
Ln h(x) = 3lnx - ln2 - 4lna - \frac{x^2}{2a^2}
\]

It is denoted by \(H(x)\) and is defined as follows
\[ H(x) = E[-\ln h(x)] = E \left[ -3 \ln x + \ln 2 + 4 \ln a + \frac{1}{2a^2}x^2 \right], \]
\[ = E \left[ \ln 2 + 4 \ln a + \frac{1}{2a^2}l_1 - 3l_2 \right], \]
where \( l_1 = E(x^2) \) and \( l_2 = E(\ln x) \).

Now \( l_1 = E(x^2) = \int_0^\infty \ln x \cdot x^3 e^{-\frac{x^2}{2a^2}} dx \)

Let \( y = \frac{x^2}{2a^2} \Rightarrow x = 2a^2y \)

\[ [j] = \left\{ \frac{dx}{dy} \right\} = \frac{a}{2^{1/2} y^{1/2}} \]

\[ l_1 = 2a^2 \int_0^\infty y^2 e^{-y} dy = 4a^2 \]

\[ l_2 = \ln a \int_0^2 ye^{-y} dy + \ln 2 \int_0^\infty ye^{-y} dy + \int_0^\infty \ln y e^{-y} dy, \quad = \ln a + \frac{\ln 2}{2} + \Gamma(1). \]

\[ H(x) = \ln a + \ln 2 - \frac{3 \ln 2}{2} - 3\Gamma(1) + 2. \quad (23) \]

Further, the Renyi entropy of the T-Rayleigh model is given by

\[ l_R(\rho) = \frac{1}{1 - \rho} \ln \left[ \int_0^\infty \left( \frac{x^3}{2a^4} \right)^\rho \left( e^{-\frac{x^2}{2a^2}} \right) dx \right]. \quad (24) \]

Suppose that \( f(x) = \int_0^\infty (h(x,a))^\rho dx \).

Let \( y = \frac{x^2}{2a^2} \Rightarrow x = a (2y)^{1/2} \Rightarrow [j] = \left\{ \frac{dx}{dy} \right\} = \frac{a}{2^{1/2} y^{1/2}} \)

\[ f(x) = \int_0^\infty \frac{3^{\rho-1}}{2^{\rho-1}} a^{3\rho+1} \frac{3^{\rho-1}}{2^{\rho-1}} e^{-\rho y} dy = \frac{\Gamma(\frac{3\rho-1}{2} + 1)}{\rho^{3\rho-1/2}}. \quad (25) \]

Substitution of the equation (25) in equation (24) we obtain:

\[ l_R(\rho) = \ln a - \frac{\ln 2}{2} + \frac{1}{1 - \rho} \ln \left[ \frac{\Gamma(\frac{3\rho-1}{2} + 1)}{\rho^{3\rho-1/2}} \right]. \quad (26) \]

3.2. Estimation of Parameter and Fisher’s information

If \((x_1, x_2, ..., x_n)\) for a random sample of size \(n\), taken from a community with a probability function \(h(x, a)\), where \(a\) is the parameter of the distribution to be estimated. Then the Likelihood function \(L(x; a)\) is

\[ L(x; a) = \prod_{i=1}^n \frac{x_i^3}{2a^4} e^{-\frac{x_i^2}{2a^2}}. \quad (28) \]

Taking the natural logarithm of the two parties in the equation above:

\[ \ln L(x; a) = 3 \sum_{i=1}^n \ln x_i - n \ln 2 - 4n \ln a - \frac{1}{2a^2} \sum_{i=1}^n x_i^2. \quad (29) \]
With the partial derivation of the equation (29) relative to the \(a\) parameter, this partial derivative is equal to zero, to obtain the maximum likelihood estimator of the parameter \(\hat{a}\).

\[
\frac{\partial \ln L(x; a)}{\partial a} = -\frac{4n}{a} + \frac{1}{a^3} \sum_{i=1}^{n} x^2_i = 0 ,
\]

\[
\hat{a}_{ML} = \sqrt{\frac{1}{4n} \sum_{i=1}^{n} x^2_i} . \quad (30)
\]

The Fisher’s information, denoted by \(I(a)\) is obtained by second derivative of the loglikelihood function as follows:

\[
\frac{\partial^2 \ln L(x; a)}{\partial a^2} = -\frac{4n}{a^2} - \frac{1}{a^3} \sum_{i=1}^{n} x^2_i = 0
\]

\[
I(a) = -E\left[\frac{\partial^2 \ln L(x; a)}{\partial a^2}\right] = -E\left[\frac{4n}{a^2} - \frac{1}{a^3} \sum_{i=1}^{n} x^2_i\right] = \frac{8n}{a^2}. \quad (31)
\]

Further, the information content of the T-Rayleigh model is given by:

\[
I(x) = -\log h(x) = -\left[3 \log x - 0.7 - 4 \log a - \frac{x^2}{2a^2} (0.43)\right]. \quad (32)
\]

From Table 1, we observed that the greater the value of the parameter, the less information about parameter \(a\). Further, the greater the value of the parameter, the greater information about variable \(a\) in Table 2.

**Table 1**: Fisher’s information for different values of the parameter.

| \(a\) | 0.5 | 1   | 2   | 4   | 6   |
|------|-----|-----|-----|-----|-----|
| \(I(a)\) | 32  | 8   | 2   | 0.5 | 0.2 |

**Table 2**: information content for different values of the variable.

\(x = 6\)

| \(a\) | 1   | 2   | 4   | 5   |
|------|-----|-----|-----|-----|
| \(I(x)\) | 6.1 | 1.50| 1.25| 1.47|


data1 is as follows [9]: (1.6, 2.0, 2.6, 3.0, 3.5, 3.9, 4.5, 4.6, 4.8, 5.0, 5.1, 5.3, 5.4, 5.6, 5.8, 6.0, 6.1, 6.3, 6.5, 6.7, 7.0, 7.1, 7.3, 7.3, 7.7, 7.7, 7.8, 7.9, 8.0, 8.1, 8.3, 8.4, 8.4, 8.5, 8.7, 8.8, 9.0).
Besides, the second dataset refers to the time to failure (in hours) of 400 μm electromigration specimens; this is referred to as electromigration Ozel, Alizadeh, Cakmakyapan, Hamedani, Ortega & Cancho (2017). The data are as follows:

\[\text{Data2} = (6.545, 9.289, 7.543, 6.956, 6.492, 5.459, 8.12, 4.706, 8.687, 2.997, 8.591, 6.129, 11.038, 5.381, 6.958, 4.288, 6.522, 4.137, 7.459, 7.495, 6.573, 6.538, 5.589, 6.087, 5.807, 6.725, 9.632, 6.369, 7.024, 8.336, 9.218, 7.945, 6.869, 6.352, 5.57, 6.725, 8.532, 9.663, 6.369, 7.024, 8.336, 9.218, 7.945, 6.869, 6.352, 5.57).

For the last dataset, consists of 100 carbon fiber stress-breaking observations in (Gba) provided by Ijaz and Asim (2019).

\[\text{Data3} = (3.70, 2.74, 2.73, 2.50, 3.60, 3.11, 3.27, 2.87, 1.47, 3.11, 4.42, 2.41, 3.19, 3.22, 1.69, 3.28, 3.09, 1.87, 3.15, 4.90, 3.75, 2.95, 2.97, 3.39, 2.96, 2.53, 2.67, 2.93, 3.22, 3.39, 2.81, 4.20, 3.33, 2.55, 3.31, 3.31, 2.85, 2.56, 3.56, 3.15, 2.35, 2.55, 2.59, 2.38, 2.81, 2.77, 2.17, 2.83, 1.92, 1.41, 3.68, 2.97, 1.36, 0.98, 2.76, 4.91, 3.68, 1.84, 1.59, 3.19, 1.57, 0.81, 5.56, 1.73, 1.59, 2.00, 1.22, 1.12, 1.71, 2.17, 1.17, 5.08, 2.48, 1.18, 3.51, 2.17, 1.69, 1.25, 4.38, 1.84, 0.39, 3.68, 2.48, 0.85, 1.61, 2.79, 4.70, 2.03, 1.80, 1.57, 1.08, 2.03, 1.61, 2.12, 1.89, 2.88, 2.82, 2.05, 3.65).

The analysis of the data sets is performed through R software. Table 3 provides the parameter estimate, Standard errors with their -LL of the model parameter (a) whereas errors the values of K-S, CR-M, AD, AIC and BIC are given in Table 4.

| Table 3: Estimated parameters of the Rayleigh and T-Rayleigh model for data sets. |
|---------------------------------------------------------------|
| **Model** | **Parameter Estimate** | **Standard Error** | **- LL** |
|-----------------|------------------|------------------|---------|
| Data Set I      | Rayleigh         | 4.62721          | 0.3658124 | 91.88274 |
|                 | T-Rayleigh       | 3.271936         | 0.1829067 | 85.3659 |
| Data Set II     | Rayleigh         | 5.06371          | 0.3296194 | 137.4123 |
|                 | T-Rayleigh       | 3.580455         | 0.1647949 | 120.9336 |
| Data Set III    | Rayleigh         | 1.986129         | 0.09930628 | 149.5009 |
|                 | T-Rayleigh       | 1.404387         | 0.0496515 | 141.9504 |

| Table 4: The K-S, CR-M, AD, AIC and BIC values for data sets. |
|---------------------------------------------------------------|
| **Model** | **K-S** | **CR-M** | **AD** | **AIC** | **BIC** |
|-----------------|---------|----------|--------|--------|--------|
| Data Set I      | Rayleigh | 0.22679  | 0.67230 | 3.49190 | 185.7655 | 187.4544 |
|                 | T-Rayleigh | **0.125946** | 0.203818 | 1.188843 | 172.7318 | 174.4207 |
| Data Set II     | Rayleigh | 0.3127813 | 1.7084403 | 8.6575971 | 276.8247 | 278.9022 |
|                 | T-Rayleigh | **0.1930492** | 0.5820038 | 3.3055708 | 243.8672 | 245.9448 |
| Data Set III    | Rayleigh | 0.1383341 | 0.6340038 | 3.5460256 | 301.0018 | 303.6070 |
|                 | T-Rayleigh | **0.06322810** | 0.08093368 | 0.55904285 | 285.9009 | 288.5060 |
5. Conclusions and Recommendation

1) In this paper, we have presented a new model of the Rayleigh distribution, called the T-Rayleigh distribution. This probability function $h(x; a)$ is obtained by applying entropy transformation. Some of its mathematical properties are derived as well as maximum likelihood estimation. The figures in the tables 1 and 2 respectively show that the T-Rayleigh model has lesser values of K-S and CR-M for the three data sets considered in comparison with the Rayleigh model actually studied. As such, it can be inferred that our model is much more flexible and it could be chosen as the best model.

2) We recommended to use the new model when data that have depression and also that a new probability T-Rayleigh model be applied in such scientific aspects like medicine aspects, engineering and other.

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