Comment on ”A simple explanation of the non-appearance of physical gluons and quarks”

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Abstract

In a recent paper by Johan Hansson it is claimed that the non-appearance of quarks and gluons as physical particles is an automatic result of the nonabelian nature of the color interaction in quantum chromodynamics. It is shown that the arguments given by Hansson are insufficient to support his claim by giving simple counter arguments.

PACS: 11.10.-z Field theory
1 Introduction

The investigation of quark confinement has a long history in theoretical physics. In [1] it is claimed that the non-appearance of gluons and quarks in the physical spectrum is a trivial consequence of the nonabelian structure of the corresponding Yang-Mills theory. It is the aim of this comment to show that this assumption is an oversimplification which can be rejected by very simple counter arguments.

2 Counter arguments

It is a well known fact that confinement can also be observed in the case of abelian gauge theories. The simplest example is given by the Schwinger model [2, 3, 4], i.e. quantum electrodynamics in 1+1 dimensions with massless electrons and photons. This model can be solved exactly and it turns out that the fermions and the photon disappear from the physical spectrum. What remains in the physical sector is a scalar particle which has a mass that is proportional to the square of the coupling constant. The Schwinger model and other similar theories [5, 6] have served as toy models in order to explain the confinement mechanism and the infrared behaviour of quantum field theories.

In [1] it is argued that confinement is due to nonperturbative properties of QCD by considering the equations of motion of the gluon field. It is important to note that one should not mix up the two different notions of classical and nonperturbative. Classical solutions of the equations of motion of fields are clearly nonperturbative, but this insight is only of restricted value as far as the quantized theory is concerned. Confinement is an intrinsically quantum field theoretical problem.

In [1] it is argued that the representation of the gluonic field with color index $b$

$$A^b_\mu(x) = \int \frac{d^3k}{2\omega} \left( a^b_\mu(k)e^{-ikx} + a^b_\mu(\bar{k})e^{ikx} \right), \quad b = 1, \ldots, 8$$

(1)
does no longer hold in the nonabelian case, due to the nonlinear structure of the equations of motion

$$(\delta_{ab}\partial^\mu + gf_{abc}A^c_\nu)(\partial_\mu A^b_\nu - \partial_\nu A^b_\mu + gf_{bde}A^d_\mu A^e_\nu) = 0.$$ 

(2)

But scattering theory constructs an $S$-matrix which relates asymptotic initial and final states, where eq. (2) does not cause any problems: The trivial plane wave solution of a gluon field with 'fixed' color $a$

$$A^a_\mu(x) = \delta_{ab}\epsilon^b_\mu e^{-ikx}$$

(3)
solves the equation of motion, since $f_{abc}$ is totally antisymmetric such that the gluon field with color index $b$ cannot couple to itself. Therefore we have no reason to assume that plane wave solutions cannot be quantized in a canonical way from a naive point of view. Furthermore, additional ”colored” but equivalent solutions can be constructed from (3) by gauge transformations.

I mention the fact that every solution of the Maxwell equations for the photon field is also a solution of the corresponding equations in the case of purely gluonic QCD, if the color of the gluon field is held constant throughout space. The confinement problem, which is still one of the most fundamental problems in theoretical physics, must be tackled by nonperturbative methods. One possibility is lattice gauge theory, which allows the computation of correlation functions in quantum field theory [7] by numerical methods.
3 Conclusions

It is clear that confinement is most probably realized in nature, and it is also well known that perturbative QCD is a valuable tool only for the high energy regime of the theory. Therefore, the simple Fock space structure which is used as a basis of perturbation theory must lead to mathematical inconsistencies.

Unfortunately, no proof of these statements is given in [1]. The classical equations of motion do not rule out plane wave solutions in the case of purely gluonic QCD, and arguments which go beyond the classical level are missing in [1].

The assertion that the criterion for confinement is the nonabelian structure of QCD (which leads to nonlinear equations of motion) is already ruled out by the observation that there are exactly solvable theories which are abelian and which show the phenomenon of confinement. Additionally, the equations of motion of coupled fields (e.g. in quantum electrodynamics or the standard model) are nonlinear, but we do observe asymptotic states in the experiment which can be described by perturbation theory in a satisfactory way.

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