Focused crossed Andreev reflection

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Abstract – We consider non-local transport mediated by Andreev reflection in a two-dimensional electron gas (2DEG) connected to one superconducting and two normal metal terminals. A robust scheme is presented for observing crossed Andreev reflection (CAR) between the normal metal terminals based on electron focusing by weak perpendicular magnetic fields. At slightly elevated temperatures the CAR signature can be easily distinguished from a background of quantum interference fluctuations. The CAR-induced entanglement between electrons can be switched on and off over large distances by the magnetic field.

Andreev reflection (AR) is a signature sub-gap scattering phenomena at normal-superconductor (NS) interfaces. Two electrons (at energies symmetrically around the chemical potential of the superconductor) enter the superconducting condensate as a Cooper pair, resulting in a retro-reflected hole on the normal side of the interface. The superconducting coherence length $\xi$ determines the spatial extent of the Cooper pairs and is therefore believed to govern the scale of the largest possible separation between the incoming electron and the retro-reflected hole. When two normal metal contacts $N_1$ and $N_2$ are connected to a superconductor, the Andreev reflected holes due to incoming electrons in $N_1$ may leave the structure through $N_2$ [1–4]. This non-local process, called crossed Andreev reflection (CAR), creates a spatially separated phase-coherent electron-hole pair, and is therefore a candidate for a solid-state entangler [5]. In this letter, we show that CAR can be enhanced using electron focusing. In most previous works [1–8], the superconductor is attached to the two normal contacts in such a way that electron transfer (ET) between the normal electrodes only happens through co-tunneling via virtual states in the superconductor. This process normally competes with CAR and typically dominates in linear response [3,9,10] such that CAR can dominate only beyond linear response or in the presence of interactions [8,11–13]. In contrast, we spatially separate the superconductor and normal contacts, thus suppressing electron co-tunneling between the normal contacts, while magnetic focusing allows to control the respective weights of ET and CAR processes.

Under direct coupling between the normal contacts and the superconductor, CAR can only be detected when the separation between the normal contacts is smaller than the superconducting coherence length $\xi$ [2,3,5,7,14,15]. Here we use the term CAR to describe all processes causing electron-hole entanglement between two normal metal contacts that are mediated by Andreev reflection, even when the contact separation is much larger than $\xi$. In our device, CAR competes with electron transfer (ET), in which electrons travel between the normal contacts either directly (direct ET) or via multiple Andreev reflections. The contribution from multiple Andreev reflections is negligible for devices studied in this work due to the small width of the contacts compared to other system dimensions.

Our system of choice is a high-mobility two-dimensional electron gas (2DEG) [16]. The large Fermi wavelength in a 2DEG facilitates our numerical calculations. However, our conclusions should also be valid for clean metallic systems at low temperature where long mean free paths can be achieved. For a setup similar to ours refs. [17,18] reported electron focusing-induced negative non-local signal produced by resonant enhancement of AR. The resonances depend on the magnetic field such that fluctuations in the non-local signal due to AR are difficult
to distinguish from quantum interference effects. Our approach is shown below to be more robust and better suited for entanglement creation.

Andreev reflection in the presence of a magnetic field has been thoroughly studied in the literature [17–22]. Electron focusing was used for the first direct observation of Andreev reflection at an NS interface [19,20]. Recently, an Andreev interferometer was used to demonstrate phase coherence of CAR and ET [14].

Our scheme is illustrated in fig. 1, which shows a 2DEG connected to a single superconducting contact $S$ between two normal contacts $N_1$ and $N_2$. Electrons are injected from the left contact $N_1$ by a small voltage bias. For weak magnetic fields, the motion of the electrons and holes can be understood in terms of semi-classical cyclotron orbits [16,20]. For certain magnetic fields (fig. 1(a)), the electrons from $N_1$ are focused on the superconducting center contact $S$, at which an Andreev reflected hole is emitted. Since AR changes the sign of both charge and effective mass, the holes will feel the same Lorentz force as the electrons and are therefore focused on contact $N_2$ to the right of $S$ at the same distance as $N_1$ [20–22]. At these magnetic fields, direct ET is suppressed in favor of CAR. A contribution to ET at these fields from multiple Andreev reflections is suppressed by the magnetic field together with back-scattering towards the superconductor. On the other hand, ET is enhanced when the incoming electrons are focused on $N_1$ such that the skipping orbits do not interact with the superconductor (fig. 1(b)).

The physics of electron focusing can be best understood in a semi-classical picture. The length scale associated with the motion of electrons with momentum $\hbar k_F$ in a magnetic field $B$ is the cyclotron diameter $d_c = 2\hbar k_F/eB$, where we assume ballistic kinetics or $d_c \ll l_{nf}$ [16]. Electron focusing between the normal contacts in fig. 1 occurs when the distance $2L$ between $N_1$ and $N_2$ obeys $2L = nd_c$, where $n$ is a positive integer. ET is enhanced for odd and CAR for even $n$. The field [16]

$$B_{\text{focus}} = \frac{2\hbar k_F}{eL}$$

(1)
determines the scale for which focusing features can be expected.

For strong magnetic fields the system enters the quantum Hall (QH) regime, in which the charge carriers are better described as chiral edge states than as semi-classical skipping orbits [23]. The characteristic length scale associated with the QH regime is the magnetic length $\lambda_B$, which is the radius of the disc that encloses one flux quantum, $\pi l_B/2 = \Phi_0 = h/2e$. In the semi-classical regime the magnetic flux density $n_B = 1/(\pi l_B^2)$ should be substantially lower than the electron density $n = \frac{k_F^2}{(2\pi)}$, giving $B = \frac{n}{\frac{n_B}{2}} \approx 7T$, for typical values for the electron density in a 2DEG, $n \approx 3.5 \times 10^{15} \text{ m}^{-2}$ (corresponding to $\lambda_B \approx 40 \text{ nm}$) [16]. We expect CAR to be enhanced also in the QH regime, since the edge states will be forced to interact with the superconductor on the way from $N_1$ to $N_2$. This regime should be experimentally accessible since superconductors with upper critical fields above $10^4 \text{T}$ are readily available [24].

We will now confirm the semi-classical predictions by a numerical quantum simulation of the non-local transport properties of the device shown in fig. 1. The competition between CAR and ET is studied through the non-local conductance [3,9,10],

$$G_{21} = \frac{-\partial I_2}{\partial V_1} = G_{\text{ET}}^2 - G_{\text{CAR}}^2,$$

(2)

where $I_2$ is the current response in contact $N_2$ due to the application of a voltage $V_1$ in the normal metal contact $N_1$, while $N_2$ and $S$ are grounded. The overall minus sign is due to the definition of the currents to be positive when electrons leave the reservoirs. The difference in sign of $G_{\text{ET}}^2$ and $G_{\text{CAR}}^2$ in eq. (2) is due to the fact that the outgoing current in $N_2$ produced by ET consists of negatively charged electrons, while CAR contributes with positively charged holes.

In our calculation we employ the standard 2DEG Hamiltonian

$$\mathcal{H}(r) = \frac{\mathbf{p}^2}{2m} + V(r) - \mu,$$

(3)
where \( p = -i\hbar \nabla + e A(\mathbf{r}) \) is the momentum and \( m \) the effective mass. The Hamiltonian (3) is extended it to Nambu space [25]

\[
H = \int d\mathbf{r} \Psi^\dagger(\mathbf{r}) \begin{pmatrix} \mathcal{H}(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -\mathcal{H}^*(\mathbf{r}) \end{pmatrix} \Psi(\mathbf{r}),
\]

where at the contact \( S \) the superconducting pair potential \( \Delta(\mathbf{r}) \) is assumed to vary abruptly on the scale of the Fermi wavelength \( \lambda_F \), and is therefore modelled as step function which is non-zero only inside the center contact \( S \). All energies are measured from the chemical potential \( \mu \) of the superconductor. The Nambu spinor \( \Psi \) is defined in terms of the field operators \( \psi \) as \( \Psi = (\psi, \psi^\dagger)^T \). A perpendicular magnetic field \( \mathbf{B} = \nabla \times \mathbf{A} = Be_z \) is included everywhere except in the superconductor [26]. We consider only elastic scattering.

At zero temperature, quantum interference due to scattering at the sharp boundaries close to the contacts can mask the electron focusing effect [16]. We therefore calculate the non-local differential conductance at finite temperature, using the standard formula,

\[
G_{21} = \int d\varepsilon \mathcal{G}_{21}(\varepsilon) \left( -\frac{\partial n_F(\varepsilon)}{\partial \varepsilon} \right),
\]

where \( n_F \) is the Fermi-Dirac distribution function.

We use the knitting algorithm presented in ref. [27] to calculate the self-energies and retarded and advanced Green functions. Standard expressions relate the conductance and current density to these quantities. The device used in the simulations is sketched in the inset of fig. 1(a), where the two auxiliary contacts \( N_3 \) and \( N_4 \) are drains for the electrons that do not contribute to the resonances. All edges cause specular electron scattering only.

Figure 2 shows the calculated non-local conductance from eq. (5) as a function of perpendicular magnetic field at a temperature \( T = 1 \text{K} \). The value chosen for \( \Delta \) corresponds to Pb, which has a critical temperature \( T_c \approx 7 \text{K} \) [28]. Also, since \( T < T_c/2 \), we disregard the temperature dependence of the pair potential, \( \Delta(T) \approx \Delta(0) \) [29].

The injector \( N_1 \), superconducting \( S \), and collector \( N_3 \) contacts are point contacts with width \( W \approx \lambda_F / 2 \), so that only a single mode contributes to the current [23]. The distance \( L = 500 \text{nm} \) between the contacts corresponds to a focusing field of \( B_{\text{focus}} = (0.39 \pm 0.02) \text{T} \), where the uncertainty is due to the finite width \( W \) of the contacts relative to \( L \). The value found in the simulation agrees with the expectations within this uncertainty.

In fig. 2 the total non-local conductance \( G_{21} \) is shown together with the conductance contribution due to CAR. The negative peaks in \( G_{21} \) at integer values of the focusing field are consistent with the semi-classical interpretation presented earlier, and demonstrate that ET is completely dominated by CAR for such fields. The expected enhancement of ET at half-integer \( B/B_{\text{focus}} \) is somewhat masked by quantum interference, but positive peaks in \( G_{21} \) when \( B/B_{\text{focus}} \) equals 1/2 and 3/2 are clearly visible. The field associated with focusing can easily be adjusted to be well separated from the scale of quantum interference by changing the distance \( L \) between the contacts. As the magnetic field increases beyond \( 2.5 B_{\text{focus}} \), the system gradually enters the QH regime.

The enhancement of CAR at \( B/B_{\text{focus}} = 1/2 \) can be visualized by calculating the charge current density due to electrons injected from contact \( N_1 \). This is shown in fig. 3, where we have set \( B \approx 2B_{\text{focus}} \). A skipping orbit between \( N_1 \) and \( S \) is clearly visible. Also visible is the diffraction of the incoming current through \( N_1 \), which leads to a broadening of the skipping orbit trajectories. In fig. 3(a)
the center contact $S$ is normal ($\Delta = 0$) and a large portion of the injected current leaves through $S$. In contrast, when $S$ is in the superconducting state, as shown in fig. 3(b), the current density increases substantially between $S$ and $N_2$ due to CAR.

Electron focusing over length scales for which AR-mediated electron-hole correlations can be observed is limited by the mean free path $l_{\text{inf}}$ rather than by the superconducting coherence length $\xi$ [17]. For typical superconductors, $\xi \sim 10-100 \text{ nm}$, [12], whereas $l_{\text{inf}}$ can reach several microns in 2DEGs [23]. Very high mobilities have also been reported for graphene [30,31], which is another candidate for focused CAR. The tuning between CAR and ET is possible only below the critical magnetic field of the superconductor, and should also not introduce spin selectivity of the contacts. Since electron focusing clearly discriminates between CAR and ET, our device can maximize entanglement generation in artificial solid-state devices.

Contacts between superconducting metals and 2DEGs have been fabricated for several types of heterostructures [32,33]. Although experimentally challenging due to the presence of Schottky barriers, fairly high transparencies have been reported (for instance transmission probability $\sim 0.55$ with a critical field of 2T in the In-GaAs heterostructures presented in ref. [33]). In fig. 4, we plot the height of the CAR peak at $B/B_{\text{focus}} = 2$ (at $T = 0$) as a function of the transmission probability of the NS contact. The CAR peak diminishes with decreasing quality of the interface but not dramatically so. We conclude that the effect should be observable with the available technology.

In conclusion, we have shown that electron focusing can be used to enhance CAR relative to quantum interference effects over the length scale of the mean free path $l_{\text{inf}}$ [17], which can be several orders of magnitude larger than $\xi$ [12,16]. CAR is enhanced at the cost of ET for magnetic fields that are integer multiples of the focusing field in eq. (1), producing a clear, negative non-local conductance signal. At half-integer multiples of the focusing field, CAR plays a negligible role since the electron orbits avoid the superconducting contact. Instead ET is enhanced as in normal electron focusing [16]. The necessary magnetic field is relatively weak, and should be an easily accessible experimental “knob” for controlling the CAR enhancement.

CAR has been proposed as a means to create a solid-state entangler, using the natural entanglement of Cooper pairs. However, in most systems quasiparticle backscattering into the injector contacts is a serious limitation [34]. This difficulty does not exist in our scheme.

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REFERENCES

[1] BYERS J. M. and FLATTÉ M. E., Phys. Rev. Lett., 74 (1995) 306.
[2] DEUTSCHER G. and FEINBERG D., Appl. Phys. Lett., 76 (2000) 487.
[3] FALCI G., FEINBERG D. and HEKKING F. W. J., Europhys. Lett., 54 (2001) 255.
[4] FEINBERG D., Eur. Phys. J. B, 36 (2003) 419.
[5] RECHER P., SUKHORUKOV E. V. and LOSS D., Phys. Rev. B, 63 (2001) 165314.
[6] DEN HARTOG S. G., KAPTEYN C. M. A., VAN WEES B. J., KLAPWUIK T. M. and BORGHS G., Phys. Rev. Lett., 77 (1996) 4954.
[7] BECKMANN D., WEBER H. B. and V. LÖHNEYSEN H., Phys. Rev. Lett., 93 (2004) 197003.
[8] RUSSO S., KROUG M., KLAPWUIK T. M. and MORPURGO A. F., Phys. Rev. Lett., 95 (2005) 027002.
[9] MORTEN J. P., BRATAAS A. and BELZIG W., Phys. Rev. B, 74 (2006) 214510.
[10] MORTEN J. P., HUERTAS-HERNANDO D., BELZIG W. and BRATAAS A., Phys. Rev. B, 78 (2008) 224515.
[11] CADDEN-ZIMANSKY P. and CHANDRASEKHAR V., Phys. Rev. Lett., 97 (2006) 237003.
[12] LEVY YEYATI A., BERGERET F. S., MARTÍN-RODERO A. and KLAPWUIK T. M., Nat. Phys., 3 (2007) 455.
[13] GOLUBEV D. S., KALENKOVA M. S. and ZAKIN A. D., Phys. Rev. Lett., 103 (2009) 067006.
[14] CADDEN-ZIMANSKY P., WEI J. and CHANDRASEKHAR V., Nat. Phys., 5 (2009) 393.
[15] WEI J. and CHANDRASEKHAR V., Nat. Phys., 6 (2010) 494.
[16] VAN HOUTEN H., BEENACKER C. W. J., WILLIAMSON J. G., BROEKAART M. E. I., VAN LOOSDRECHT P. H. M., VAN WEES B. J., MOOI J. E., FOXON C. T. AND HARRIS J. J., Phys. Rev. B, 39(1989) 8556.
[17] POPLÁK P. K., LAMBERT C. J., KOLTAI J. AND CSERTI J., Phys. Rev. B, 74 (2006) 132508.
[18] RAKYTA P., KORMAYOS A., KAUFMANN Z. AND CSERTI J., Phys. Rev. B, 76 (2007) 064516.
[19] BOZHKO S. I., TSOI V. S. and YAKOVLEV S. E., Pis'ma Zh. Eksp. Teor. Fiz., 36 (1982) 123 (JETP Lett., 36 (1982) 152).
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[20] Benistant P. A. M., van Kempen H. and Wyder P., Phys. Rev. Lett., 51 (1983) 817.
[21] Tsol V. S., Bass J. and Wyder P., Rev. Mod. Phys., 71 (1999) 1641.
[22] Giazotto F., Governale M., Zülicke U. and Beltram F., Phys. Rev. B, 72 (2005) 054518.
[23] Beenakker C. W. J., van Houten H. and van Wees B. J., Superlattices Microstruct., 5 (1989) 127.
[24] Niu H. J. and Hampshire D. P., Phys. Rev. Lett., 91 (2003) 027002.
[25] Nambu Y., Phys. Rev., 117 (1960) 648.
[26] Hoppe H., Zülicke U. and Schönh G., Phys. Rev. Lett., 84 (2000) 1804.
[27] Kazymyrenko K. and Waintal X., Phys. Rev. B, 77 (2008) 115119.
[28] Lide D. P. (Editor), CRC Handbook of Chemistry and Physics, 90th edition (CRC Press) 2010, http://www.hbcnetbase.com.
[29] Bardeen J., Cooper L. N. and Schrieffer J. R., Phys. Rev., 108 (1957) 1175.
[30] Bolotin K., Sikes K., Jiang Z., Klima M., Fudenberg G., Hone J., Kim P. and Stormer H., Solid State Commun., 146 (2008) 351.
[31] Chen F., Xia J. and Tao N., Nano Lett., 9 (2009) 1621.
[32] Takayanagi H. and Kawakami T., Phys. Rev. Lett., 54 (1985) 2449.
[33] Boulay S., Dufouleur J., Roche P., Gennser U., Cavanna A. and Mailly D., J. Appl. Phys., 105 (2009) 123919.
[34] Morten J. P., Huertas-Hernando D., Belzig W. and Brataas A., EPL, 81 (2008) 40002.