Strategy-Proof Prediction Markets

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ABSTRACT

Prediction markets aggregate agents’ beliefs regarding a future event, where each agent is paid based on the accuracy of its reported belief when compared to the realized outcome. Agents may strategically manipulate the market (e.g., delay reporting, make false reports) aiming for higher expected payments, and hence the accuracy of the market’s aggregated information will be in question. In this study, we present a general belief model that captures how agents influence each other beliefs, and show that there are three necessary and sufficient conditions for agents to behave truthfully in scoring rule based markets (SRMs). Given that these conditions are restrictive and difficult to satisfy in real-life, we present novel strategy-proof SRMs where agents are truthful while dismissing all these conditions. Although achieving such a strong form of truthfulness increases the worst-case loss in the new markets, we show that this is the minimum loss required to dismiss these conditions.

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Theory, Algorithms, Economics

Keywords
Prediction Markets, Scoring Rules, Mechanism Design

1. INTRODUCTION

Prediction markets have been used widely as a powerful tool to elicit the beliefs of agents about a future event; see [23,24,21,6]. In such markets, an agent reports a probability distribution (i.e., an estimate) over the set of mutually exclusive and exhaustive possible outcomes of a future event. When the outcome of this future event is realized, the agents are paid based on the accuracy of their reports when compared to the realized outcome. It has been shown that prediction markets produce better estimates for future events compared to polls and expert opinions [2,24].

Given that agents are paid based on their reports, an agent can maximize its expected payoff by strategically manipulating the market (e.g., delaying its report, and/or making false reports), and here, the accuracy of the market’s aggregated information will be in question. This fact highlights the fundamental relation between prediction markets and mechanism design [9,6].

In mechanism design [17], eliciting the private information of agents is required to determine an outcome that reflects their conflicting interests, where an agent’s private information defines its value of each possible outcome for the problem. However, a prediction market problem differs slightly from a mechanism design problem in the sense that eliciting the agents’ private beliefs is the end goal and no outcome will be determined. The prediction market problem is more close to an interdependent valuations mechanism design problem [13] (i.e., an agent’s value of an outcome depends on the private information of other agents in addition to its private information) than a classical mechanism design problem (i.e., an agent’s value of an outcome depends only on its private information), since a realistic model for the prediction market problem – as the one we consider here – should assume that an agent’s belief (therefore its report and expected payoff) is influenced by other agents. Without such influence, scoring rule based markets (SRMs) [14,15] are merely considering the report of the last participating agent, and may not converge to a final estimate that encapsulates the wisdom of the crowd.

In both mechanism design problems and prediction markets, truthfulness is achieved under a game-theoretic solution concept (i.e., a truth-telling equilibrium) such as dominant strategy (i.e., strategy-proof) which is the strongest form of truthfulness where an agent will be truthful even if other agents are not, or ex-post incentive compatibility which is a weaker form of truthfulness where an agent will be truthful if and only if all other agents are truthful. Unlike mechanism design problems, prediction markets normally operate at a loss that is considered the price for aggregating the agents’ beliefs. Several studies have addressed the strategic behavior of agents in prediction markets. They can be categorized as follows:

1. Adopting a game theoretic perspective that views the market as a game and investigates its truth-telling equilibrium under different models, such as conditionally dependent or independent beliefs of agents [2,11,3,20], betting games [19] and decision making markets where an outcome will be determined based on the aggregated information [4];

2. Investigating agents’ strategic behavior empirically by evaluating the effect of manipulators [16];

3. Adopting a mechanism design framework that produces a mechanism rather than a market where agents not only report their beliefs but also report the reasons behind their beliefs [9].

To the best of our knowledge, there have been no attempts to design prediction markets which maintain a stronger form of truthfulness, since a realistic model for the prediction market problem – as the one we consider here – should assume that an agent’s belief (therefore its report and expected payoff) is influenced by other agents. Without such influence, scoring rule based markets (SRMs) [14,15] are merely considering the report of the last participating agent, and may not converge to a final estimate that encapsulates the wisdom of the crowd.

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1Also known as market scoring rules (MSR). We use the “scoring rule based markets” terminology since it is more expressive.
fulness compared to existing markets. In this study, we define a general belief model that captures any possible influences between the agents either inside or outside the market. We show that there are three sufficient and necessary conditions for traditional (i.e., presently known) SRMs to be truthful. These conditions are the predefined participation order condition (i.e., agents report their beliefs to the market in a predefined order), the one participation and market influence condition (i.e., an agent can report only once and can influence other agents’ beliefs only by this single report), and the non-negative influence condition (i.e., an agent assumes that being influenced by other agents’ beliefs doesn’t decrease its expected value). Given that these conditions are very restrictive and difficult to satisfy in real-life, we present novel strategy-proof SRMs that achieve truthfulness while dismissing all the previous conditions. However, there is a trade-off between achieving such a strong form of truthfulness and the payments made to the agents, since these strategy-proof prediction markets make additional payments compared to traditional SRMs. We investigate dismissing each condition separately to evaluate its contribution to the market’s loss, and we show that these losses are the minimum possible losses to relax the previous conditions. Also, we show that our contribution can be extended to cost function based markets (CFMs) \[5\]. In the next section, we define the prediction market problem and our belief model. In section 3, we discuss scoring rules and SRMs. In section 4, we define solution concepts for truthfulness and the necessary and sufficient conditions for SRMs to be truthful. In section 5, we present the strategy-proof SRMs and extend our work to CFMs. Section 6 concludes the study.

2. PREDICTION MARKETS

Problem Statement. Consider a future event \(X\) that has a set \(\Omega = \{1, \ldots, N\}\) of mutually exclusive and exhaustive outcomes, and a set of agents who have privately known beliefs regarding that future event. Each agent’s belief is a probability distribution (i.e., an estimate) \(p = [p_1, \ldots, p_N]\) over the set \(\Omega\) of all possible outcomes for the event \(X\), where \(p_i\) is the agent’s probability that the outcome \(i \in \Omega\) will be realized. Let \(\Delta_N = \{p \in \mathbb{R}^N : 0 \leq p_i \leq 1, \sum_{i=1}^{N} p_i = 1\}\) be the probability simplex that contains all possible probability distributions \(p\). Assuming that it is required to obtain information about the event \(X\) by eliciting the agents’ beliefs regarding that event, a market maker may establish a prediction market where agents can report their probability estimates and get paid based on the accuracy of their reports when compared to the realized outcome \(i\) for the event \(X\).

Belief Model. There are several ways to model agents’ reasoning in prediction markets. We propose a belief model which focuses on how agents influence each other’s beliefs. Our belief model captures such influence by assuming that each agent receives two signals (i.e., probability estimates). The first signal is a private signal \(p^{prv}\), which represents the agent’s own reasoning about the future event without any external influences. The second signal is a public signal \(p^{pub}\), which represents the influence of other agents, either through their reports inside the market or through announcements and discussions outside the market. This public signal is the only way an agent is influenced by other agents. Each agent receives a predefined private and public signals, and thus, the agent cannot affect their contents. However, an agent can influence the public signals of other agents as we will show later.

Given the agent’s private and public signals, the agent needs to form a final true belief \(p\) about the future event. To do that, the agent must decide whether to use its public signal \(p^{pub}\) along with its private signal \(p^{prv}\) to produce a final belief \(p\) or to consider its private signal as its final true belief (i.e., \(p = p^{prv}\)). As we will show later, this decision depends on what the agent thinks about how its public signal \(p^{pub}\) will affect its expected payoff. Each agent has a predefined merging function \(M: \Delta_N \times \Delta_N \rightarrow \Delta_N\) that the agent will use if it decided to incorporate its public signal \(p^{pub}\) to produce its final belief \(p\), i.e., the merging function \(M(p^{prv}, p^{pub}) = p\) merges the agent’s private and public signals – in an arbitrary but predefined way – producing the final belief \(p\). For each agent, its \(p^{prv}\), \(p^{pub}\), and \(M(p^{prv}, p^{pub})\) are predefined and privately known to the agent, i.e., the agent’s private type is \((p^{prv}, p^{pub}, M)\). Each agent knows nothing about the types of other agents, and types may differ from one agent to another. We assume that each agent receives only one private signal and only one public signal, and we will later discuss relaxing this assumption.

Figure 1 summarizes an agent’s reasoning about reaching its final belief \(p\) as we discussed earlier, and then, the agent needs to decide whether it will participate in a truthful manner or manipulate the market as we will discuss below.

Truthfulness and Manipulation. Different types of prediction markets use different protocols for how agents express their beliefs in the market and how agents get paid. For instance, in SRMs, agents report their beliefs directly to the market and get paid using a particular scoring rule. While in CFMs, agents trade securities (e.g., a ticket that pays $1 if a particular outcome \(i\) is realized and $0 otherwise) in the market and their beliefs are inferred from their trading behavior. Describing a prediction market in terms of its protocol (i.e., how it works) only provides a partial picture of the market’s dynamics, since in order to elicit the agents’ beliefs quickly and accurately other conditions (e.g., when and how many times agents will participate in the market) are imposed to guarantee that agents will behave truthfully.

Definition 1. An agent behaves truthfully if once it receives its private signal \(p^{prv}\) and public signal \(p^{pub}\), it uses its merging function \(M(p^{prv}, p^{pub})\) to produce its final belief \(p\) and it reports \(p\) to the market without any delays.

However, an agent may manipulate the market either intentionally if this increases its expected payoff, or due to irrational, malicious, or any other behavior that contradicts the rationality assumption (i.e., an agent never behaves in a way that decreases its expected payoff). We define market manipulation as follows.

Definition 2. An agent can manipulate the market by: 1. delay reporting its true belief \(p = M(p^{prv}, p^{pub})\); 2. reporting a false belief \(p' \neq p\); and/or 3. misleading other agents either by participating more than once in the market and reporting false beliefs \(p' \neq p\), or by spreading false information outside the market.\(^2\)

\(^2\)We don’t detail how agents interact outside the market since how an agent is affected by other agents is encapsulated in the public signal the agent receives.
When the market is manipulable, the accuracy of the market’s aggregated information will be in question. Similar to mechanism design, truthfulness is achieved in prediction markets under a game-theoretic solution concept, and we define two solution concepts in the prediction markets context as follows.

**Definition 3.** A prediction market achieves truthfulness in

Dominant Strategy (also known as strategy-proofness): For any rational agent, being truthful (Definition 1) always maximizes the agent’s expected payoff even if other agents are manipulating the market (Definition 2).

Ex-Post Incentive Compatibility (also known as ex-post Nash): For any rational agent, being truthful (Definition 1) maximizes the agent’s expected payoff if other agents are rational and truthful (Definition 1).

We consider truthfulness in ex-post incentive compatibility to be an unrealistic solution concept for real-life applications in spite of its frequent usage in the mechanism design literature, because assuming that an agent—who has the ability to behave strategically—will always assume that other agents are truthful is unreasonable. Strategy-proofness is the strongest and most preferred form of truthfulness, since an agent will not manipulate the market irrespective of other agents’ behavior. Moreover, achieving strategy-proofness is motivated by the fact that it is not always possible to assume that all involved agents are fully rational.

3. SCORING RULE BASED MARKETS

Hanson [14][15] introduced SRMs as markets for aggregating the agents’ estimates, where scoring rules are used to pay agents for their reported beliefs.

**Scoring Rules.** Scoring rules have been used extensively in aggregating and evaluating the accuracy of reported probabilistic forecasts regarding future events [12]. A scoring rule is a function \( s : \Delta_N \times \Omega \rightarrow [−\infty, \infty] \). Given a probability distribution \( p \in \Delta_N \), a scoring rule \( s(p) = [s_1(p), \ldots, s_N(p)] \) assigns a score \( s_i(p) \) that takes a value in the extended real line \([−\infty, \infty]\) for each outcome \( i \in \Omega \). The score \( s_i(p) \) serves as a reward (penalty) that the agent will receive (pay) for predicting the distribution \( p \), and the outcome \( i \) was realized. From now on, we will use \( p \) to denote the probability estimate that reflects the true belief of an agent, and \( p' \) to denote the probability estimate that the agent will report, which may or may not be equivalent to \( p \). Given a scoring rule \( s \), an agent’s expected payoff from reporting \( p' \) while having a true belief \( p \) is \( E(s, p, p') = \sum_{i=1}^{N} p_i s_i(p') \).

A regular scoring rule is a scoring rule where an agent’s expected payoff \( E(s, p, p') \) takes a value in \([−\infty, \infty]\) for all \( p, p' \in \Delta_N \), and the expected payoff \( E(s, p, p) \) for reporting the true estimate \( p \) takes a value in \([−\infty, \infty]\). This implies that \( s_i(p) \) is finite whenever \( p_i > 0 \). A proper scoring rule is a scoring rule where a risk neutral agent has no incentive to report any distribution \( p' \) other than its true belief estimate \( p \), i.e., \( E(s, p, p') \geq E(s, p, p) \), \( \forall p, p' \in \Delta_N \). The scoring rule is said to be strictly proper if the previous inequality holds with equality only when \( p = p' \). Let \( b > 0 \) and \( a_1, \ldots, a_N \) be parameters. An example of a regular strictly proper scoring rule is the logarithmic scoring rule [13].

\[
s_i(p) = a_i + b \ln(p_i).
\]

Given a proper scoring rule \( s \), we can define an uncertainty function and a discrepancy function [10]. The uncertainty function \( S(s, x) = \sum_{i=1}^{N} x_i s_i(x) \) for \( x \in \Delta_N \) measures the uncertainty (i.e., lack of precision) associated with the distribution \( x \). When an agent faces a proper scoring rule \( s \), the uncertainty function \( S(s, p) \) is equal to the agent’s maximum expected payoff \( E(s, p, p) \) that results from reporting its true belief \( p \). The discrepancy function

\[
D(s, x, y) = \sum_{i=1}^{N} x_i s_i(x) - \sum_{i=1}^{N} x_i s_i(y)
\]

for any \( x, y \in \Delta_N \) measures the distance between the two distributions \( x \) and \( y \) using the scoring rule \( s \) as the metric for measurement (i.e., the distance differs accordingly with the \( s \) in use). This distance reflects the difference between the uncertainty associated with \( x \) and the uncertainty associated with \( y \) compared to \( x \).

**Scoring Rule Based Markets.** A scoring rule based market (SRM) can be viewed as a sequentially shared proper scoring rule \( s \) that works as described in Procedure 1. The market always keeps a current probability estimate \( p_i \), which is defined by the market maker when the market begins by an initial estimate \( p_0 \) (Step 1).

Until the market closes (Steps 2-5), any agent can change that current estimate \( p_i \) to its reported estimate \( p'_i \) (Step 3). The market maker saves the initial estimate and all the reported estimates in a vector \( \theta \) (Steps 1 and 4). The market’s closing (i.e. the last) current estimate represents the market’s elicited information from all the agents. Once outcome \( i \) is realized, each agent who made a report \( p_i \) receives \( s_i(p_i) \) and pays \( s_i(p_0) \), where \( p_0 \) is the current market estimate that immediately precedes the agent’s report (Steps 6-9).

\[ \]

**Procedure 1** SRM Protocol.

1: \( m \leftarrow 1, t \leftarrow 0, p_c \leftarrow p_0 \) and \( \theta(t) \leftarrow p_c \).
2: repeat
3: An agent \( j \) reports an estimate \( p'_i \) and \( p_c \leftarrow p'_i \).
4: \( t \leftarrow t + 1 \) and \( \theta(t) \leftarrow p'_i \).
5: until Market closes and outcome \( i \) is realized.
6: while \( m \leq t \) do
7: \( p'_i \leftarrow \theta(m), p_c \leftarrow \theta(m - 1) \) and \( m \leftarrow m + 1 \).
8: The payment of agent \( j \) who reported \( p'_i: s_i(p'_i) - s_i(p_0) \).
9: end while

In a SRM that uses a proper scoring rule \( s \), if an agent’s true belief estimate is \( p \) and it changed the market current probability estimate \( p_c \) to \( p' \), then its expected payoff is

\[
E(s, p, p', p_c) = \sum_{i=1}^{N} p_i (s_i(p') - s_i(p_0)).
\]

Prediction markets normally run at a loss, at least theoretically. The market maker’s loss is the price it pays to elicit the agents’ beliefs about the future event. In SRMs when outcome \( i \) is realized, each agent pays \( s_i(p_0) \) for the immediate previous report \( p_c \), which is the same amount \( s_i(p') \) received by the agent who reported \( p' = p_c \). Thus, all the intermediate payments to and from the agents offset each other, and the market maker is left with receiving \( s_i(p_0) \) from the first participating agent and paying \( s_i(p') \) for the last participating agent who reported \( p' \). But because any outcome \( i \in \Omega \) can be realized, and the last report \( p' \) can be any estimate in the simplex \( \Delta_N \), the market maker’s worst case loss (WCL) is

\[
\max_{i \in \Omega} \sup_{p \in \Delta_N} (s_i(p) - s_i(p_0)),
\]

where \( p_0 \) is the initial estimate. In Eq. 3 \( s_i(p) - s_i(p_0) \) reflects the highest possible price that the market maker will pay for its additional gained knowledge compared to its initial estimate \( p_0 \), given any possible outcome \( i \).

...
Example 1. A software house wants to predict the probable release date of a certain software, either in March (outcome A) or in June (outcome B), and will establish a logarithmic SRM (LSRM) that uses a logarithmic scoring rule (Eq. 1) with \( a_i = 0 \), \( \forall i \in \Omega \) and \( b = 1 \). The market starts by an initial estimate \( p_0 = [p_{APB}] = [0.5 \ 0.5] \) where \( p_A \) and \( p_B \) are the probabilities that outcomes A and B will be realized, respectively. To illustrate how this LSRM works, Table 1 shows the participating agents (Column 1), and for each agent the table shows its true belief \( p \) (Column 2), its participation order (PO) (Column 3), the market’s current estimate \( p_c \) when it participated (Column 4), its reported belief \( p' \) (Column 5), and its expected payoff (EP) (Column 6). According to Table 1 we have that agent 1 and agent 2, and both are truthful (i.e., reported \( p' = p \)).

Agent 1 participated first, and changed the current market estimate \( p_c = p_0 = [0.5 \ 0.5] \) to \( p' = [0.4 \ 0.6] \) and its expected payoff will be 0.0201. Then, agent 2 participated and changed the current probability \( p_c = [0.4 \ 0.6] \) to \( p' = [0.7 \ 0.3] \), and its expected payoff will be 0.1838.

Agent 2 is a programmer who can’t affect the software’s release date, and we will illustrate how he came up with his final belief \( p \). Once he heard about this market, he formed his own belief \( p^{\text{pub}} = [0.8 \ 0.2] \). However, after a discussion with colleagues over lunch, he became aware of some technical problems facing the development team, and he knew that the project manager responsible for this software was on a sick leave. Furthermore, the programmer observed a previous report [0.4 0.6] made to the market (i.e., agent 1). To capture all these external effects – other than the agent’s initial belief \( p^{\text{pub}} \), we assume that agent 2 receives a public signal \( p^{\text{pub}} = [0.45 \ 0.55] \). We stress that agent 2 can’t affect the content of its public signal, e.g., the agent can’t choose which colleagues to talk to or which information it receives. However, an agent can affect the public signals of other agents, e.g., the report of agent 1 affected the public signal of agent 2. We assume here that agent 2 thinks that using \( p^{\text{pub}} \) to produce its final belief will increase its expected value, and it will use a predefined merging function to come up with its final belief \( p \). Agent 2 used a simple merging function \( M(p^{\text{pub}}, p^{\text{pub}}) \) that works for an event with two outcomes as follows: the final belief \( p \) is \( p^{\text{pub}} \) after increasing the probability of the outcome that had a higher probability in \( p^{\text{pub}} \) by 0.1, and decreasing the the probability of the other outcome by 0.1. Given that outcome B (i.e., software released in June) has a higher probability in \( p^{\text{pub}} \), then \( p = [(0.8-0.1) (0.2+0.1)] = [0.7 \ 0.3] \) (i.e., the true belief \( p \) of agent 2 in Table 1). It is clear here that the reported belief of agent 2 is the final current estimate in the market which reflects the elicited beliefs of all agents. If agents don’t influence the beliefs of each other (e.g., agent 2 wasn’t influenced by the report of agent 1), then the market would only have elicited the belief of the last participating agent.

Table 1: Example 1

| Agent | \( p \) | PO | \( p_c \) | \( p' \) | EP |
|-------|-------|----|-----|-----|----|
| 1     | [0.4 0.6] | 1st | [0.5 0.5] | [0.4 0.6] | 0.0201 |
| 2     | [0.7 0.3] | 2nd | [0.4 0.6] | [0.7 0.3] | 0.1838 |

4. SRMS Truthfulness

Unfortunately, existing prediction markets don’t even achieve truthfulness in ex-post incentive compatibility (Definition 3), and they operate under restrictive and unrealistic conditions to achieve truthfulness. In SRMs literature and prediction markets in general, it is stated that an agent will be truthful (Definition 1) under myopic participation, i.e., “It is optimal for traders to report their true beliefs provided that they ignore the impact of their reports on the profit they might garner from the future trades ...” Other studies (e.g., [7]) discuss truthfulness conditions more explicitly. Given our belief model, we will require three separate conditions in order to analyze and develop strategy-proof prediction markets.

Definition 4. The SRM truthfulness conditions are:

1. Prefixed Participation Order (PPO): Agents participate in the market in a predefined order.
2. One Participation and Market Influence (OP-MI): Each agent participates only once in the market, and can influence the public signals of other agents only through its single report (i.e., any external communication (e.g., discussions) is not allowed).
3. Non-Negative Influence (NNI): Considering the public signal \( p^{\text{pub}} \) doesn’t decrease the agent’s expected payoff.

The first two conditions are easy to understand, but the NNI condition needs more elaboration. Consider the probability distribution \( p^{\text{nat}} \) defined by “nature” over the set \( \Omega \) of all possible outcomes for the future event, where \( p^{\text{nat}} \) dictates the “real” probability that each outcome \( i \in \Omega \) will be realized. Considering any arbitrary market current estimate \( p_i \) in any SRM, the absolute (or global) maximum expected payoff any agent can ever get in principle is attained by changing \( p_i \) to \( p^{\text{nat}} \). No agent knows the estimate \( p^{\text{nat}} \) for certain, but each agent thinks that its true belief \( p = M(p^{\text{pub}}, p^{\text{pub}}) \) is equal to \( p^{\text{nat}} \) or at least hopes that \( p \) is very close to \( p^{\text{nat}} \), since the closer \( p \) is to \( p^{\text{nat}} \) the higher the agent’s expected payoff will be. Given a proper scoring rule \( s \), this can be expressed by the discrepancy value \( D(s, p, p^{\text{nat}}) \), where the smaller the discrepancy value the closer is \( p \) to \( p^{\text{nat}} \). The NNI condition means that each agent believes that when using its public signal \( p^{\text{pub}} \) to produce its true belief \( p \), the true belief \( p \) is closer to \( p^{\text{nat}} \) than \( p^{\text{pub}} \), i.e., \( D(s, p, p^{\text{nat}}) \leq D(s, p^{\text{pub}}, p^{\text{nat}}) \). This means that using the public signal \( p^{\text{pub}} \) to produce the final belief \( p \) has a non-negative influence on (i.e., doesn’t decrease) the agent’s expected payoff, as \( p^{\text{pub}} \) enhances the agent’s private belief \( p^{\text{pub}} \) about the future event. We stress that the NNI condition does not necessarily hold if all agents are truthful (Definition 1), e.g., an agent may report its true belief, but this belief will negatively influence other agents. That’s why we state the NNI condition – whether other agents are truthful or not – rather than stating an ex-post incentive compatibility condition (Definition 3).

To show that the conditions in Definition 4 are necessary and sufficient for agents in SRMs to be truthful (Definition 1), we will show that by dismissing each condition separately, an agent can maximize its expected payoff by manipulating the market (Definition 2). Then, we will show that if these three conditions hold simultaneously, then an agent has no incentive to manipulate the market.

Theorem 1. A SRM without the PPO condition is manipulable even if the OP-MI and NNI conditions hold.

Proof. We prove this theorem using a counter-example [7] that shows that a SRM is manipulable even when agents participate only once, report their true beliefs and we neglect any influences.

\(^3\)The expected payoff is approximated to four-decimals precision and is calculated using Eq. 4 given \( p \), \( p' \) and \( p_{\text{prv}} \): \( (0.4) \ln(0.4) - \ln(0.5) + (0.6) \ln(0.6) - \ln(0.5) = 0.0201 \).

\(^4\)It is sufficient to show that an agent has incentive to manipulate the market given a particular private type of another agent, without assuming that the agent knows the private type of that other agent.
between the agents’ beliefs (i.e., OP-MI and NNI conditions hold). Consider Example 1, and assume that agents there participated in a predefined order.

**Table 2: Example 2.**

| Agent | p   | PO | prv | p'  | EP  |
|-------|-----|----|-----|-----|-----|
| 2     | [0.7 0.3] | 1st | [0.5 0.5] | [0.7 0.3] | 0.0825 |
| 1     | [0.4 0.6] | 2nd | [0.7 0.3] | [0.4 0.6] | 0.1920 |

In Example 2 (Table 2), we assume that agent 1 has the chance to report after agent 2. First, agent 2 changes prv = p0 = [0.5 0.5] to p' = [0.7 0.3], then agent 1 changes prv = [0.7 0.3] to p' = [0.4 0.6]. Agent 1 expected payoff will be 0.1920, which is higher than its expected payoff (i.e., 0.0201 in Table 1) when it participated before agent 2. □

**Theorem 2.** A SRM without the OP-MI condition is manipulable even if the PPO and NNI conditions hold.

**Proof.** We prove this theorem by using a counter-example (recall Footnote 4) that shows that a SRM is manipulable when agents participate in a predefined order (i.e., PPO condition holds) and when agents believe that considering their public signals doesn’t decrease their expected payoff (i.e., NNI condition holds). Using example 2, agent 1 has – after considering its private and public signals – a true belief of [0.4 0.6] and will report after agent 2 according to a predefined order. Agent 2 has a private signal prv = [0.7 0.3], and considers the market current estimate as its public signal ppub.

Agent 2 uses the merging function M' of Example 1. As shown in Table 2, the true belief p of agent 2 will be its private signal pprv = [0.7 0.3], because the market current estimate prv = p0 = [0.5 0.5] is the public signal and it doesn’t favor any outcome.

**Theorem 3.** A SRM without the NNI condition is manipulable even if the PPO and OP-MI conditions hold.

**Proof.** Relaxing the NNI condition implies that an agent will believe that its public signal ppub doesn’t enhance its private belief pprv, which means that considering ppub while formulating its true belief p = M(pprv, ppub) may decrease its expected payoff, i.e., D(s, ppprv, ppub) < D(s, ppprv, pprv). In this case, the agent is better off by neglecting its public signal ppub, and will report its private belief pprv. Reporting p = pprv and not p = M(pprv, ppub) violates the agent’s truthfulness (Definition 1), and is considered a strategic misreporting. This can happen under the PPO and the OP-MI conditions. □

**Theorem 4.** In a SRM, the PPO, OP-MI and NNI conditions are necessary and sufficient for agents to be truthful.

**Proof.** In theorems 1, 2 and 3, we showed that relaxing each of the PPO, OP-MI and NNI conditions separately makes a SRM manipulable, and thus, they are necessary conditions for achieving truthfulness. We will now show that they are collectively sufficient conditions by showing the effect of each condition. NNI Condition Effect: an agent will assume that its public signal ppub doesn’t decrease its expected payoff (i.e., D(s, p, ppub) ≤ D(s, ppprv, pprv)) and will use it to produce its final true belief p = M(pprv, ppub). This will hold for any arbitrary merging function M, since the NNI condition encapsulated the effect of ppub on an agent’s expected payoff regardless of M. PPO Condition Effect: an agent is participating in a predefined order and will be paid based on pc that will be fixed according to this predefined order. OP-MI Condition Effect: Given any estimate pc, the agent can’t influence pc because the agent couldn’t have reported it, and the agent couldn’t have affected the public signal of the agent who reported it. Given the effects of the PPO and OP-MI conditions, the agent has no control over the estimate pc based on which it will be paid and s1(pc), ∀i ∈ Ω in Eq. 4 are considered constants from the agent’s perspective. Given the effect of the NNI condition and that a SRM uses a proper scoring rule, reporting p' = p maximizes the agent’s expected payoff by maximizing ∑i=1N c1s1(p'). □

**5. STRATEGY-PROOF SRMS**

We will present strategy-proof SRMs that achieve truthfulness in dominant strategy while dismissing all the previously mentioned conditions. We will start dismissing one condition at a time to illustrate the condition’s effect on the market maker’s WCL.

**Relax PPO.** Consider a SRM with any arbitrary proper scoring rule s, it is easy to see in Eq. 3 that the agent’s expected payoff E(s, p, pc) (i.e., Eq. 3) when changing the market current estimate pc to its true belief (i.e., report p' = p) is simply the discrepancy function D(s, p, pc) of the proper scoring rule s (i.e., Eq. 3).

\[
E(s, p, p, pc) = \sum_{i=1}^{N} c_{i} (s_{i}(p) - s_{i}(pc))
\]

(5)

This implies that in a SRM, an agent’s expected payoff is the discrepancy distance between its report and the previous report made to the market (i.e., the market’s current estimate pc). Thus, an agent can maximize its expected payoff by choosing to go after the estimate pc that has the greatest discrepancy value compared to its own report. We present an arbitrary participation (AP) SRM (Procedure 2) that works without the PPO condition, while an agent will not benefit from delaying its report.

\footnote{Misleading other agents is not always beneficial, and an agent must balance its loss when bluffing (e.g., agent 1 negative expected payoff from its first false report) with its future gains.}

\footnote{We stress that an agent’s public signal is predefined and the agent can’t affect the signal’s content by altering its participation time.}
Procedure 2 AP SRM Protocol.
1: \( m \leftarrow 1, t \leftarrow 0, p_c \leftarrow p_0 \) and \( \theta(t) \leftarrow p_c \).
2: repeat
3: An agent \( j \) reports an estimate \( p' \) and \( p_c \leftarrow p' \).
4: \( t \leftarrow t + 1 \) and \( \theta(t) \leftarrow p' \).
5: until Market closes and outcome \( i \) is realized.
6: while \( m \leq t \) do
7: \( p' \leftarrow \theta(m) \), compute \( p_c' = \arg\max_{p_c \in \Theta} \left[ \sum_{i=1}^{N} p_i' s_i(p_c') - \sum_{i=1}^{N} p_i's_i(p_c) \right] \), and \( m \leftarrow m + 1 \).
8: The payment of agent \( j \) who reported \( p' \): \( s_i(p') - s_i(p'_c) \).
9: end while

Theorem 5. Given an AP SRM, an agent will be truthful under the OP-MI and NNI conditions.

Proof. The AP SRM (Procedure 2) is similar to a SRM (Procedure 1) except for Step 7. The NNI condition has the same effect as in theorem 4. The OP-MI condition has the same effect as in theorem 4, and thus, the agent can’t affect the vector \( \theta \) that holds all the reported estimates to the market from its beginning till its end. In an AP SRM, an agent is paid based on the estimate \( p'_c \) that maximizes its expected payoff given its reported estimate \( p' \), where \( p'_c \) is chosen from vector \( \theta \) and has the greatest discrepancy value compared to \( p' \). In other words, the AP SRM market will pay each agent assuming that it reported directly after the \( p_c \), that corresponds to the greatest discrepancy value compared to the agent’s report \( p' \). Thus, an agent has no incentive to delay its participation time. By substituting \( p'_c \) in Eq. 4, an agent’s expected payoff is \( E(s, p^0, p') = \sum_{i=1}^{N} p_i s_i(p_c') - s_i(p'_c) \), we can consider \( s_i(p_c') \), \( \forall i \in \Omega \) as the constants that maximize \( E(s, p^0, p', p'_c) \). Given that an AP SRM is using a proper scoring rule, reporting \( p^0 = p \) maximizes the agent’s expected payoff by maximizing \( \sum_{i=1}^{N} s_i(p_i) \).

In principle, a SRM (Procedure 1) pays and receives payments from every agent, but because the agents’ payments offset each other (i.e., each agent pays what the previous agent receives), the SRM pays only the last participating agent. This is not the case anymore in an AP SRM, and what an agent pays is not what the previous agent receives because \( p'_c \) is not necessarily the previous report made to the market. This implies that we must consider a payment for each agent, and this increases the market’s WCL by the factor of the number \( n \) of participating agents as follows.

\[
\max_{i \in \Omega} \sup_{p_c, p \in \Delta_N} (s_i(p) - s_i(p_c)) = (6)
\]

However, this is the minimum WCL required to dismiss the PPO condition as we now show.

Lemma 6. The AP SRM is truthful without the PPO condition and has the minimum WCL.

Proof. In a SRM (Procedure 1) without the PPO condition, an agent is free to choose its participation time. The maximum expected payoff an agent can get by altering its participation time is by reporting after \( p'_c \) that has the greatest discrepancy value compared to its belief \( p \). And thus, the minimum expected payoff that prevents the agent from altering its participation time is to pay the agent based on \( p'_c \), which is the AP SRM payment.

Relax OP-MI. We present a non-myopic (NM) SRM (Procedure 3) which works without the OP-MI condition, while an agent will not benefit from participating more than once and/or misleading other agents. The NM SRM is similar to a SRM (Procedure 1) except for Steps 6-9 concerning how agents are paid, and for Step 3 where agents report their private beliefs \( p^{prv} \) along with their final beliefs \( p = M(p^{prv}, p^{pub}) \). However, the market current estimate \( p_c \) changes according to the final beliefs. Let \( \theta^{prv} \) denote the vector that holds all reported private estimates.

Procedure 3 NM SRM Protocol.
1: \( p_c \leftarrow p_0 \).
2: repeat
3: An agent \( j \) reports an estimate \( p' \), a private estimate \( p^{prv} \) and \( p_c \leftarrow p' \).
4: Add \( p^{prv} \) to \( \theta^{prv} \).
5: until Market closes and outcome \( i \) is realized.
6: for all Participating Agents do
7: Let \( p' \) be the last report by agent \( j \) made to the market, and \( p^{prv} \) is the immediate previous private belief from \( \theta^{prv} \) reported to the market before \( p' \) by any agent \( k \neq j \).
8: The payment of agent \( j \): \( s_i(p') - s_i(p^{prv}) \).
9: end for

Theorem 7. Given a NM SRM, an agent will be truthful under the PPO and NNI conditions.

Proof. The NNI condition has the same effect as in theorem 4. Without the OP-MI condition, an agent can get the maximum expected payoff by: 1. report to the market again and again every time it realizes that reporting yields a positive expected payoff; and 2. by influencing the public signals of other agents for future gains (e.g., example 3 in theorem 2), either inside the market by making misleading reports or outside the market. For the first point, the NM SRM eliminates the incentive to do this by paying an agent based on its last report to the market (Step 7), and will neglect all the agent’s previous reports. For the second point, the NM SRM eliminates the incentive to do that by paying the agent (Steps 7-8) based on the private beliefs of other agent which is not affected by their public signals. Given the PPO condition and that agent \( j \) is paid based on \( p^{prv} \) that was reported by another agent \( k \neq j \) immediately before the last report made by agent \( j \), an agent can’t choose the \( p^{prv} \) which will be used in its payment. By substituting \( p^{prv} \) in Eq. 4, an agent’s expected payoff is \( E(s, p^0, p', p^{prv}) = \sum_{i=1}^{N} p_i s_i(p'_c) - s_i(p^{prv}) \), and given the previous, \( s_i(p^{prv}) \), \( \forall i \in \Omega \) are considered constants from the agent’s perspective. Given that a NM SRM is using a proper scoring rule, reporting \( p^0 = p \) maximizes the agent’s expected payoff by maximizing \( \sum_{i=1}^{N} s_i(p_i) \).

In NM SRM, the agents’ payments don’t offset each other, because each agent is paid by the market according to its report \( p' \) (i.e., \( s_i(p_i') \)) and pays the market according to \( p^{prv} \) (i.e., \( s_i(p^{prv}) \)), and \( p^{prv} \) is not the \( p \) based on which the previous agent is paid. This implies that we must consider a payment for each agent, and this increases the market’s WCL by a factor of the number \( n \) of agents (i.e., the same loss as in Eq. 6).

Lemma 8. The NM SRM is truthful without the OP-MI condition and has the minimum WCL.

Proof. Recalling the two points in the proof of theorem 7 about how an agent can increase its expected payoff, the NM SRM avoids the first point without an extra loss by paying an agent only for its last report. For the second point, the NM SRM eliminated any incentives for an agent from misleading other agents by paying it according to the private beliefs of other agents. In SRM (Procedure 1) without OP-MI, the maximum gain an agent can get is by misleading reports or outside the market. For the first point, the NM SRM eliminates the incentive to do this by paying misleading the previous agent to report a \( p_c \) that has the greatest discrepancy value for the agent’s report \( p' \). The minimum amount to eliminate such an incentive for an agent to mislead another agent is the WCL of the NM SRM.
Relax PPO and OP-MI. We present an arbitrary participation non-myopic (AP NM) SRM (Procedure [4]) which works without the PPO and the OP-MI conditions by combining the ideas behind the AP SRM and NM SRM.

**Theorem 9.** Given an AP NM SRM, an agent will be truthful under the NNI condition.

Proof. The proof here is similar to the proof of theorem 7. The NNI condition has the same effect as in theorem 4, and paying an agent according to the private beliefs of other agents has the same effect as in theorem 7. However when we dismiss the PPO condition, the agent can maximize its expected payoff by altering its participation time to report \( p' \) after the private signal \( p'_{prv} \) that has the greatest discrepancy value from \( p' \). But similar to the AP SRM, the AP NM SRM pays an agent based on \( p'_{prv} \) that is chosen from all the reported private estimates \( \theta_{prv}^{ij} \) of other agents and maximizes the agent’s expected value given its report \( p' \). Here, the agent has no incentive to alter its participation time. Given that agent \( j \) can’t influence the vector \( \theta_{prv}^{ij} \), and by substituting \( p'_{prv} \), an agent’s expected payoff is 
\[
E(s, p, p', p_{prv}) = \sum_{i=1}^{N} p_i (s_i(p') - s_i(p'_{prv})) ,
\]

and we can consider \( s_i(p'_{prv}), \forall i \in \Omega \) as the constants that maximize \( E(s, p, p', p_{prv}) \). Given that an AP NM SRM is using a proper scoring rule, reporting \( p' = p \) maximizes the agent’s expected payoff by maximizing \( \sum_{i=1}^{N} p_i s_i(p') \).

The WCL in an AP NM SRM is the same as the loss in an AP SRM, but it pays an agent based on the estimate that maximizes the agent’s expected payoff that is chosen from \( \theta_{prv}^{ij} \) rather than \( \theta_j \) in the AP SRM.

**Procedure 4** AP NM SRM Protocol.

1: \( p_0 \leftarrow p_0 \).
2: repeat
3: An agent \( j \) reports an estimate \( p' \), a private estimate \( p_{prv} \) and \( p_0 \leftarrow p' \).
4: Add \( p_{prv} \) to \( \theta_{prv} \).
5: until Market closes and outcome \( i \) is realized.
6: for all Participating Agents do
7: Let \( p' \) be the last report of agent \( j \) to the market, compute the vector \( \theta_{prv}^{ij} \) from \( p_{prv} \) to hold the reported private beliefs of all agents except agent \( j \), and compute
\[
p_{prv}' = \arg\max_{p_{prv}'} \left\{ \sum_{i=1}^{N} p_i s_i(p') - \sum_{i=1}^{N} p_i s_i(p_{prv}^{'ij}) \right\}.
\]
8: The payment of agent \( j \): \( s_i(p') - s_i(p'_{prv}) \).
9: end for

**Lemma 10.** The AP NM SRM is truthful without the PPO and OP-MI conditions and has the minimum WCL.

Proof. Directly follows from Lemmas 6 and 8. \( \square \)

Relax PPO, OP-MI and NNI. Finally, we present a strategy-proof SRM (Procedure[5]) that works without any conditions, and agents will behave truthfully.

**Theorem 11.** Given a strategy-proof SRM, an agent will be truthful.

Proof. Similar to the proof of theorem 7, any agent \( j \) is paid only for its last report, and based on the private signals reported by other agents, and thus, it has no incentive to participate more than once and/or manipulate other agents’ public signals either inside or outside the market. When we dismiss the NNI condition, an agent will not assume that using its public signal enhances its private signal and may decide to neglect its public signal. To avoid this, the strategy-proof SRM (Steps 7-8) pays agent \( j \) the maximum amount when outcome \( i \) is realized according to either its reported final belief \( p' \) or its reported private signal \( p_{prv}^{ij} \). Thus, the agent doesn’t care about the implications of using its public signal. Agent \( j \) is paid based on the private signals \( p_1 \) and \( p_2 \) in \( \theta_{prv}^{ij} \) that correspond to the maximum discrepancy value compared to the agent’s reported beliefs \( p' \) and \( p_{prv}^{ij} \), and thus, the agent has no incentive to alter its participation time. Given that agent \( j \) can’t influence the vector \( \theta_{prv}^{ij} \), and substituting \( p_1 \) and \( p_2 \) in Eq.4 results in two expected payoff equations \( E(s, p, p', p_{prv}) \) and \( E(s, p_{prv}^{ij}, p_{prv}^{ij}, p_2) \), agent \( j \) considers \( s_i(p_1^{'ij}) \) and \( s_i(p_2^{'ij}), \forall i \in \Omega \) in both equations as the constants that maximize these equations. Given that the market is using a proper scoring rule, reporting \( p' = p \) and \( p_{prv}^{ij} = p_{prv}^{ij} \) (where \( p_{prv}^{ij} \) is the true private signal of agent \( j \)) maximizes the agent’s expected payoff in the two equations by maximizing \( \sum_{i=1}^{N} p_i s_i(p') \) and by maximizing \( \sum_{i=1}^{N} p_i s_i^{'ij}(p_{prv}^{ij}) \). \( \square \)

**Procedure 5** Strategy-proof SRM Protocol.

1: \( p_0 \leftarrow p_0 \).
2: repeat
3: An agent \( j \) reports an estimate \( p' \), a private estimate \( p_{prv}^{ij} \) and \( p_0 \leftarrow p' \).
4: Add \( p_{prv}^{ij} \) to \( \theta_{prv}^{ij} \).
5: until Market closes and outcome \( i \) is realized.
6: for all Participating Agents do
7: Let \( p' \) be the last report of agent \( j \) to the market, \( p_{prv}^{ij} \) be the private belief of agent \( j \) reported with \( p' \), compute the vector \( \theta_{prv}^{ij} \) from \( p_{prv}^{ij} \) to hold the reported private beliefs of all agents except agent \( j \), compute
\[
p_{prv}' = \arg\max_{p_{prv}'} \left\{ \sum_{i=1}^{N} p_i s_i(p') - \sum_{i=1}^{N} p_i s_i(p_{prv}^{ij}) \right\},
\]
and compute
\[
p_{prv}'' = \arg\max_{p_{prv}'' \in \theta_{prv}^{ij}} \left\{ \sum_{i=1}^{N} p_i s_i(p_{prv}^{'ij}) - \sum_{i=1}^{N} p_i s_i(p_{prv}^{ij}) \right\}.
\]
8: The payment of agent \( j \): \( \max((s_i(p') - s_i(p_1)), (s_i(p_{prv}^{'ij}) - s_i(p_{prv}^{ij}))) \).
9: end for

Again, the WCL in the strategy-proof SRM is same as that of the AP SRM and the AP NM SRM.

**Remark 1.** We previously assumed that each agent receives only one private signal and one public signal. Our work extends to scenarios where an agent receives its private and public signals and reports to the market, and then receives new private and public signals and reports again to the market, as long as there is no strategic interaction between the two reports (e.g., the agent is not waiting to receive the new signals).

**Remark 2.** Our work extends to design strategy-proof convex CFMs. In CFMs [5], the market maker trades a security for each outcome \( i \in \Omega \) that pays \( s(i) \) if and only if outcome \( i \) was realized, and \( 0 \) otherwise. Let \( q_i \) denote the number of shares of security \( i \) held currently by the agents, and \( q = [q_1, \ldots, q_N] \) denote the vector of shares of all securities currently held by the agents where \( q \in \mathbb{R}^N \). The securities are priced based on a cost function
\[
C(q) : \mathbb{R}^N \rightarrow \mathbb{R},
\]
which describes the amount of money currently wagered in the market as a function of the quantity of shares \( q \) held by agents. The instantaneous price of buying an infinitesimal amount of security \( i \) is given by \( p_i (q) = \partial C(q) / \partial q_i \). Let \( p(q) = [p_1(q), \ldots, p_N(q)] \) denote the vector of prices for all the securities. An agent trades a bundle \( r = [r_1, \ldots, r_N] \in \mathbb{R}^N \), where \( r_i \) is the amount of shares purchased (or sold if negative) from security \( i \). When the agent purchases \( r_i \), the agent pays the market \( C(q + r) - C(q) \). Given an agent’s true belief \( p_i \), the agent’s expected payoff from purchasing \( r \) is
\[
E(C, p, q, r) = \sum_{i=1}^{N} p_i r_i.
\]
An agent maximizes its expected payoff by buying a bundle \( r \) such that the prices in the market after this purchase is equivalent to the agent’s true belief, i.e., \( p(q + r) = p \).

In [8] Theorem 3, a one-to-one mapping was shown between a set of convex CFMs and a class of strictly proper SRMs. This mapping guarantees that an agent who changes the market current estimate from \( p_c \) to \( p \) in a SRM has exactly the same expected payoff for every outcome \( i \in \Omega \) as the agent who changes the quantity vector \( q \) to \( q + r \) such that \( p(q) = p_c \) and \( p(q + r) = p \) in a convex CFM. This equivalence guarantees that all the strategic behaviors we indicated for SRMs and the measures taken to prevent them still hold for convex CFMs. We just need to illustrate how the extra payments in the strategy-proof SRMs are made in convex CFMs. In strategy-proof convex CFMs, agents need to report their private signals along with their purchases. Once an agent purchases a bundle, the market maker can realize the agent’s true belief by inspecting the market’s prices after the purchase. After the market closes, the market maker can determine at which point of time agent \( j \) would have preferred to make its purchase in order to maximize its expected payoff based on the private beliefs reported by other agents. Then, the market maker can issue (i.e., give for free) more securities to agent \( j \) in order to equate its expected payoff from its purchase with the expected payoff as expressed in any of the previous procedures. In practice, this will increase the market maker’s loss. The WCL of strategy-proof convex CFMs needs further investigation since the WCL in traditional SRMs is bounded irrespective of the number of participating agents and the equivalence in [8] doesn’t state that this bound holds for any convex CFMs. The WCL is bounded for a convex CFM if the conjugate of its \( C(q) \) is bounded over the convex hull of the probability simplex [1 Theorem 3].

6. CONCLUSIONS AND FUTURE WORK

We showed that designing prediction markets that have the strongest form of truthfulness (i.e., strategy-proofness) is possible, but comes with an unbounded WCL for SRMs. However, some commonly used markets (e.g., CFMs) may have unbounded loss. Moreover, assuming that markets will operate under very restrictive – and almost impossible to hold – conditions such as PPO, OP-MI or NNI is unrealistic, e.g., how a market maker will guarantee the order of the reports made to the market. As well, we have shown that this is the minimum possible WCL required to dismiss these conditions. This trade off between the money loss and achieving a strong form of truthfulness is very common in mechanism design and in prediction markets with more complicated settings (e.g., [22]). Extending current ideas to other types of prediction markets appears fruitful avenue of pursuit.

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