Reconstructing Quintom from Ricci Dark Energy

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Holographic dark energy with Ricci scalar as IR cutoff called Ricci dark energy (RDE) probes the nature of dark energy with respect to the holographic principle of quantum gravity theory. Scalar field dark energy models like quintom are often regarded as an effective description of some underlying theory of dark energy. In this letter, we find how the generalized ghost condensate model (GGC) that can easily realize quintom behavior can be used to effectively describe the RDE and reconstruct the function $h(\phi)$ of GGC.
1 Introduction

The accelerating cosmic expansion first inferred from the observations of distant type Ia supernovae [1] has strongly confirmed by some other independent observations, such as the cosmic microwave background radiation (CMBR) [2] and Sloan Digital Sky Survey (SDSS) [3]. An exotic form of negative pressure matter called dark energy is used to explain this acceleration. The simplest candidate of dark energy is the cosmological constant $\Lambda$, whose energy density remains constant during the evolution of the universe. The cosmological model that consists of a mixture of the cosmological constant and cold dark matter is called LCDM model, which provides an excellent explanation for the acceleration of the universe phenomenon and other existing observational data. However, this model suffers the ‘fine-tuning’ problem and the ‘cosmic coincidence’ problem. To alleviate or even solve these two problems, many dynamic dark energy models were proposed such as quintessence mentioned, k-essence, tachyons, phantoms, ghost condensates and quintom etc.. Generically, we regard the scalar field as an effective description of an underlying theory of dark energy, which we still do not known, because we do not understand entirely the nature of dark energy before a complete theory of quantum gravity is established, since the dark energy problem may be in principle a problem belongs to quantum gravity [4].

Although we are lacking a quantum gravity theory today, we can still make some attempts to probe the nature of dark energy according to some principle of quantum gravity. It is well known that the holographic principle is an important result of the recent researches for exploring the quantum gravity(or string theory) [5]. So that the holographic dark energy model (HDE) constructed in light of the holographic principle possesses some significant features of an underlying theory of dark energy [6]. Recently, Gao et.al [7] have proposed a holographic dark energy model in which the future event horizon is replaced by the inverse of the Ricci scalar curvature, and they call this model the Ricci dark energy model(RDE). Of course, this model also respect the holographic principle. For this model with proper parameters, the equation of state crosses $-1$, so it is a ‘quintom’. It has been shown in ref.[8] that there is really a simple one-field model that can realize the quintom model, called the generalized ghost condensate model(GGC). In this letter, we find some equivalence between the GGC and RDE model, and we reconstruct the function $h(\phi)$ of GGC from RDE. In Section II, we will briefly
review RDE model and GGC model, and reconstruct the function $h(\phi)$ of GGC from RDE model in Section III. In the last section we will give some conclusions.

2 Briefly Review on RDE and GGC

Holographic principle [9] regards black holes as the maximally entropic objects of a given region, so a self-consistent effective field theory with UV cutoff $\Lambda$ in a box of size $L$ should satisfy the Bekenstein entropy bound [10]

$$(L\Lambda)^3 \leq S_{BH} = \pi L^2 M_{pl}^2,$$

where $M_{pl}$ is the Planck mass, $S_{BH}$ is the entropy of black hole, and $L$ acts as an IR cutoff. Furthermore, Cohen et.al. [11] found that the total energy should not exceed the mass of a black hole of the same size either, namely $L^3\Lambda^4 \leq L M_{pl}^2$. Under this assumption, Li [12] proposed the holographic dark energy as follows

$$\rho_\Lambda = 3 c^2 M_p^2 L^{-2}$$

where $c^2$ is a dimensionless constant. Since the holographic dark energy with Hubble horizon as its IR cutoff does not give an accelerating universe [13], Li suggested to use the future event horizon instead of Hubble horizon and particle horizon, then this model gives an accelerating universe and is consistent with current observation[12, 14]. For the recent works on holographic dark energy, see ref. [15, 16, 17].

Recently, Gao et.al [7] proposed the Ricci dark energy model(RDE), in which they take the Ricci scalar as the IR cutoff. The Ricci scalar of FRW universe is given by $R = -6(\dot{H} + 2H^2 + k/a^2)$, where dot denotes a derivative with respect to time $t$ and $k$ is the spatial curvature. The energy density of RDE is

$$\rho_X = \frac{3\alpha}{8\pi G} \left( \dot{H} + 2H^2 + \frac{k}{a^2} \right) \propto R$$

where the dimensionless coefficient $\alpha$ will be determined by observations. Solving the Friedmann equation they find

$$\frac{8\pi G}{3H_0^2}\rho_X = \frac{\alpha}{2 - \alpha} \Omega_{m0} e^{-3x} + f_0 e^{-(4 - \frac{2}{\alpha})x}$$

where $\Omega_{m0} \equiv 8\pi G \rho_{m0}/3H_0^2$, $x = \ln a$ and $f_0$ is an integration constant. Substituting the expression of $\rho_X$ into the conservation equation of energy,
namely \( p_X = -\rho_X - \frac{1}{3} \frac{dp_X}{dx} \) they get the pressure of RDE:

\[
p_X = -\frac{3H_0^2}{8\pi G} \left( \frac{2}{3\alpha} - \frac{1}{3} \right) f_0 e^{-\frac{4-2\alpha}{\alpha}x} \tag{4}
\]

Taking the observation values of parameters they find the \( \alpha \simeq 0.46 \) and \( f_0 \simeq 0.65 \) \footnote{7}. The the equation of state of RDE at high redshifts is closed to zero and approaches \(-1\) at present, and in the future RDE will becomes a phantom. The energy density of RDE during big bang nucleosynthesis (BBN) is much smaller than that of other components of the universe \( (\Omega_X|_{1\text{ MeV}} < 10^{-6} \text{ when } \alpha < 1) \), so it does not affect BBN procedure.

The SN analysis using the Gold data\footnote{11} indicates that the parametrization of \( H(z) \) which crosses the cosmological-constant boundary \( (w = -1) \) shows a good fit to data. Consider the Lagrangian density of a general scalar field \( p(\phi, X) \), where \( X = -(1/2)(\partial \phi)^2 \) is the kinetic energy term. The energy density can be derived by identifying the energy momentum tensor of the scalar field with that of a perfect fluid.

\[
\rho_{de} = 2X p_X - p , \tag{5}
\]

where \( p_X \equiv \partial p/\partial X \). Then, the dynamic equations for the scalar field in the flat FRW universe read

\[
H^2 = \frac{8\pi G}{3} (\rho_m + 2X p_X - p) \tag{6}
\]

\[
\dot{H} = -4\pi G (\rho_m + 2X p_X) , \tag{7}
\]

where \( X = \dot{\phi}^2/2 \) in the cosmological context. Making use of the energy conservation law of matter \( \dot{\rho}_m + 3H \rho_m = 0 \), we find \( \dot{\rho}_m = \Omega_{m0} \rho_c e^{-3x} \), where \( \rho_c \equiv 3H_0^2/(8\pi G) \) represents the present critical density of the universe. One can rewrite eq.\(6\) and \(7\) by using a dimensionless quantity \( r \equiv H^2/H_0^2 \) as follows

\[
p = -\rho_c \left( r + \frac{r'}{3} \right) \tag{8}
\]

\[
\phi^2 p_X = -\frac{3}{8\pi G r} \left( \frac{r'}{3} + \Omega_{m0} e^{-3x} \right) \tag{9}
\]

where prime denotes a derivative with respect to \( x \equiv \ln a \). The equation of state of dark energy is given by

\[
w = \frac{p}{\phi^2 p_X - p} = \frac{p}{r \rho_c - \rho_m} = \frac{r + r'/3}{\Omega_{m0} e^{-3x} - r} . \tag{10}
\]
It should be noticed that if we establish a correspondence between RDE model and the scalar field dark energy, we should choose a scalar field model, which can cross the cosmological-constant boundary \[18\].

### 3 Reconstructing GGC from RDE

Consider the generalized ghost condensate model proposed in ref.[8] with Lagrangian density

\[ p = -X + h(\phi)X^2, \tag{11} \]

where \( h(\phi) \) is a function in terms of scalar field \( \phi \), and \( h(\phi) = ce^{\lambda \phi} \) corresponds to the dilatonic ghost condensate model. From eq.\((8)\) and \((9)\), we get the following dynamic equations for GGC

\[
\begin{align*}
\rho' &= \frac{3}{8\pi G r} (4r + r' - \Omega_{m0}e^{-3x}) \\
\phi' &= \frac{3}{(4\pi G)^2 \phi^4 r^2} (-3r - r' + 4\pi G r \phi^2) \rho_{c0}^{-1}.
\end{align*}
\tag{12, 13}
\]

In the following, we construct the correspondence between GGC and RDE, namely \( \rho_{de} = \rho_X \), which provides that

\[ r = \frac{H^2}{H_0^2} = \Omega_{m0}e^{-3x} + \frac{8\pi G}{3H_0^2} \rho_{de}. \tag{14} \]

And by using eq.\((3)\), we get

\[ r = \frac{2}{2 - \alpha} \Omega_{m0}e^{-3x} + f_0 e^{-(4 - \frac{2}{\alpha})x} \tag{15} \]

and

\[ r' = -\frac{6}{2 - \alpha} \Omega_{m0}e^{-3x} - \left(4 - \frac{2}{\alpha}\right) f_0 e^{-(4 - \frac{2}{\alpha})x}. \tag{16} \]

Thus, we can numerically solve \((12)\) and \((13)\) to obtain the function \( h(\phi) \), which plotted in Fig.1, and the scalar field \( \phi \) is also plotted in Fig.2.
Reconstruction of $h(\phi)$

**Figure 1.** Reconstruction of the generalized ghost model according to the Ricci dark energy model. The function $h(\phi)$ is plotted in unit of $\rho_0^{-1}$ with $\phi$ in unit of $(4\pi G)^{-1/2} = \sqrt{2}M_p$.

Reconstruction of $\phi(z)$

**Figure 2.** Evolution of the scalar field $\phi(z)$ in unit of $\sqrt{2}M_p$ reconstructed according to the Ricci dark energy.

From Fig.1 and Fig.2, one can see that $h(\phi)$ vanishes when $\phi$ is larger, which corresponding to the earlier universe. From eq.(10), we know that when $p_X = 0$, i.e. $hX = 1/2$, the equation of state will cross the cosmological-constant boundary, and the universe enter the phantom region ($p_X < 0$) without discontinuous behavior of $h$ and $X$, see Fig.3 and Fig.4. Furthermore,
our results are consistent with that of ref. [8], in which the authors have reconstructed the generalized ghost condensate model from the SN Gold data [19]. And the shapes of the curvatures seems much better than the reconstruction from HDE [18] to compare with that of ref [8].

Figure 3. The sign of $p_X = -1 + 2hX$ changed at $z = 0$, and the universe enter the phantom region.

The reconstructed evolutions of equation of state $w$ in terms of $\phi$ are plotted in Fig.4.

Figure 4. Equation of state in term of $\phi$ for reconstructed GCC with $\alpha = 0.46$ and $f_0 = 0.65$. 
From Fig. 4, one can see clearly that GGC mimic the evolving behavior of equation of state in RDE well. The equation of state in GGC crosses \(-1\) at \(z = 0\) and enter phantom region in the future, so it does realize a quintom-like dark energy.

4 Conclusions

In conclusion, we adopt a correspondence between the Ricci dark energy model and a scalar field dark energy (GGC) suggested in ref.[18]. The current observation data imply RDE as quintom-like dark energy, i.e. the equation of state of dark energy crosses the cosmological-constant boundary \(w = -1\) during the evolution of the universe. If the scalar field theory is regarded as an effective description of the dark energy, we should be capable of using the scalar field model to mimic the evolving behavior of RDE and reconstructing the scalar model. The generalized ghost condensate models (GGC) is a good choice for such a reconstruction since it can realize the quintom behavior easily. Thus, we reconstructed the function \(h(\phi)\) of the GGC using parameters \(\alpha \approx 0.46\) and \(f_0 \approx 0.65\) in RDE given in ref.[7]. We find that our reconstruction results in Fig.1 and Fig.2 are consistent with that of ref.[8], in which the authors have reconstructed GGC from the SN Gold data[19]. And the shapes of the curvatures in Fig.1 and Fig.2 seems much better than the reconstruction from HDE [18] to compare with that of ref.[8]. We also hope that the future high precision observation data may be able to determine the parameters of RDE and reveal some significant features of the underlying theory of dark energy.

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