Measuring Systemic Risk: 
Robust Ranking Techniques Approach

Amirhossein Sadoghi 
Frankfurt School of Finance & Management, Frankfurt am Main Email: a.sadoghi@fs.de

Abstract

The recent economic crisis has raised a wide awareness that the financial system should be considered as a complex network with financial institutions and financial dependencies respectively as nodes and links between these nodes. Systemic risk is defined as the risk of default of a large portion of financial exposures among institution in the network. Indeed, the structure of this network is an important element to measure systemic risk and there is no widely accepted methodology to determine the systemically important nodes in a large financial network. In this research, we introduce a metric for systemic risk measurement with taking into account both common idiosyncratic shocks as well as contagion through counterparty exposures. Our focus is on application of eigenvalue problems, as a robust approach to the ranking techniques, to measure systemic risk. Recently, the efficient algorithm has been developed for robust eigenvector problem to reduce to a nonsmooth convex optimization problem. We applied this technique and studied the performance and convergence behavior of the algorithm with different structure of the financial network.

Keywords: Financial Network, Systemic Risk, Nonsmooth Convex Problem

1. Introduction

The recent financial crisis has raised a wide awareness of contagion through counterparty exposures and requires of quantitative tools for assessing and monitoring contagion and systemic risk in a financial system. Systemic risk can be defined as the risk of default of a large fraction of the financial system concerning to interlinked fiscal exposures across the system. The main problem in financial systems concerns the determination of the so-called systemically important institution; however, there is no generally accepted method to determine the major institution in a financial system.

The network perspective provides a set of techniques for analyzing the structure of whole entities and a range of theories to explain mechanisms behind a complex system. In a financial network, ”vertices” represent organizations, and directed links represent financial operations between these institutions. This approach gives a possibility to model financial dependencies between financial firms and it shows their economic activities of various lending mechanisms to invest in other fiscal firms. As mentioned before, modern financial system is a concentrated and interconnected system. In consequence, it becomes less decomposable and vulnerable to systemically fall down. In this interconnected system, Contagion Dynamic can be defined as follows:

When a single bank starts to hoard liquidity: it creates challenges for banks that were beforehand borrowing from this bank to meet their own liquidity conditions. Default bank which is suffered a liquidity loss makes potentially other banks which are connected by interbank lending have liquidity shortage. Therefore, liquidity hoarding can spread across the system due to the structure and connectivity of the interbank network.

The aim of this paper is to introduce an Impact Index as a metric of measuring the systemic risk for application to default contagion on (directed) interbank networks and how robust ranking technique can contribute to the quantitative estimation of systemic risk. We also explore analytically the propagation mechanism of funding contagion which can lead to systemic liquidity risks. Later on, we discuss about computational performance and assess in which practical situations this metric is relevant. Results show that how the complexity of financial linkages can increase the systemic risks which can threat financial network resilience.

The remainder of the paper is organized as follows: section two provides a background to the problem we address and gives a brief overview of systemic risk in a financial system. In section three, we review some models of financial network and their general properties. Section four introduces an impact index as a metric for measuring systemic risk and constructs a problem and discusses about an approximation solution. In section five, we develop an algorithm to estimate this impact index. We present results of applying this algorithm to solve corresponding problem and investigate the efficiency and accuracy of the solutions obtained for different network structures and compare with optimum method in section seven. Finally, this research is summarized in section eight.

2. Systemic risk

Systemic risk defines as the risk of default of a large fraction of the financial system as a whole due to the spread of financial
exposures through system. It refers to the contagion risk to the
whole of financial system as a result of the default of one or
some of organizations; it has become a known phenomenon
after the failure of Lehman Brothers in 2008.
When the total of the decreasing in the market value of the
external assets of a bank exceeds its net worth, and then
the total liabilities of a bank exceed the total assets. In this
situation, the bank will be incapable to repay its debtors and to
fulfill its investor’s requirements. The bank will be in default
position; here we use distressed term for a defaulted bank.
One of the main sources of systemic risk is the contagion
economic distress in the financial system. By reason of
interlinked financial exposures among institutions, this distress
can spread throughout the whole of financial system in a
domino fashion. This cascade causes a disturbance in the entire
financial network and it maybe spillover to the larger financial
sector (?).

As major effect of the complexity of the contemporary
financial network and lack of an adequate methodology for
measuring systemic risk, anticipating the impact of defaults in
financial system is challenging work. For long time, size of
financial institution’s balance sheet has been used to rank the
institutions in the system. However, the recent economic crises
have indicated that small institutions relatively can have large
impact on the system and large institutions as terms of their
balance sheet size are stated as "Too Big to Fail”. This financial
crisis also indicates that the interconnectedness of the financial
can be a major reason of failing entire economic system.
The interconnections cause system to be more complex and
concentrated and consequently less decomposable and system
becomes noticeably more vulnerable to systemically fall down.
Financial system as an integrated system needs to be modular
to be stable over time. Interconnections of financial system
play a main role in transmission of liquid risk.
All these issues have shown that in order to accurate assess the
systemic importance of financial institutions; we should taking
the complex and interconnected structure of the financial
system into account and focused on the effect of connectivity
of financial network as risk absorbers or amplifiers.
Most methods measure systemic risk without considering to
interaction of counter-parties; (?); (?)) measured systemic risk
with measuring the expected shortfall per bank conditional on
a distressed financial institution as systemic capital shortfall.
((?)) introduced CoVaR to measure systemic risk of the
financial system conditional on system being in distress, based
on the value at risk (VaR) in the median state of the institution.
Interconnectedness of financial institutions has been high-
lighted in recent researches; (?;?. ?) used network model
to study the probability of contagion of financial distress
interbank networks. ? use simulation studies based on network
models to calculating the expected size of contagion events
models.
? analyzed the potentials for contagion and systemic risk in an
interlinked financial network and introduced a methodology for
analyzing the default of an institution, metrics for detecting the
systemic importance of institutions in a network of interlinked
financial institutions. ? study complex networks analysis of
the controversial US Federal Reserve Bank (FED) emergency
program dataset and introduce DebtRank metric as a recursive
way the to determine the impact of the distress of one or some
financial institutions through their counterparties through the
whole network, based on feedback centrality. ? introduce
SinkRank metric to predict the significance of disturbance
caused by the collapse of a bank in a payment system and
identify most affected banks in the system. ? purpose a simple
model of cross-holdings to examined cascades in financial
networks; they conclude that diversification and integration of
financial institutions have non-monotonic effects on financial
contagions.
However, ? indicate the lack of widely accepted method to
verify the systemically important financial institution in a
counterparties network.

3. Financial systems as complex networks

As mentioned in previous section, the recent economic crisis
has raised a wide awareness of the financial system should be
studied as a complex network whose nodes are financial insti-
tutions and links are their financial dependencies. In this ap-
proach, systemic risk defined as the default of a large part of
the financial system due to contagion through counterparty ex-
posures in the network.
Network approach plays a central role in modeling the trans-
mission of information and determining how it spreads. Net-
work Modeling provides a better view to analysis the dynami-
cal progress of the financial network structure and explains how
information and financial flow passing through system. In this
perspective, systemic risk can be measured and quantified from
the analysis of the dynamical structure of the network.
Network approach is mathematically formalized and some the-
ories and methods are built to study of financial interactions
patterns.
In this section, we provide basic concepts and definitions that
are basis for language network financial analysis. Different
structure of networks came from different presentation of re-
lationships in different application; ones can find more infor-
mation about economic network in ?.
General speaking, a financial network consists the set of nodes
\( N = \{1, \ldots, n\} \in N \times N \) referred to as financial institutions
or firms and forms as direct or indirect relations. The canonical
form a network is an undirected graph, which nodes are con-
ected without direction. This type of network model can rep-
resent economic relations or partnership, friendships in social
sciences.
The second type modeled as direct network which a node can be
connected to other node with second node not being con-
ected to first node. In this directed network topology, node \( i \)
and the link \( A_{ij} \) represent institutions and the financial interac-
tions between them respectively, i.e. at time of the default of
institution \( j \), the institution \( i \) faces a failure of \( A_{ij} \).
Total value of the asset invested by institution \( i \) in banking ac-
In this network, the out-degree of one node represents its number of creditors, the in-degree of a node is given by:

\[ \text{In-Degree} = \sum_i A_{ij} \]

We denote by \( C_{pa_i} \), the capital of institution \( i \) which is used against financial risk. Similar settings for InterBank (IB) networks constitute financial systems has been used in other researches, \(?\) In this network setting, matrix \( W \) is defined as adjacency matrix with elements \( w_{ij} = a \) if node \( i \) and node \( j \) are connected, 0 otherwise. Here, we use weighted network associated weights that represent the strengths of nodes relative to another one. This adjacency matrix with entries that are not only zero or one represents an interbank position points from the debtor to the creditor:

\[ w_{ij} = \begin{cases} a & \text{interbank relation (debtor } i \text{ to creditor } j) \\ 0 & \text{no interbank relation} \end{cases} \]

For a financial network, one might wonder how the structure of network affects the amount of risk is captured by the each institution. This approach can help to create another network using this information and compare this new network with the original one regarding to different scenarios. With given network, one can add up the incoming connections as number of creditors and outgoing connections as number of debtors separately, obtaining two numbers for the degree of each node (institution) in the system.

**Definition 1. In-Degree**

In the network setting, in-degree of one node represents the number of its creditors, the in-degree of a node \( i \) is given by:

\[ \text{degree}_{\text{in}}^i = \# j W w_{ji} > 0 \]

**Definition 2. Out-Degree**

In this network, the out-degree of one node represents its number of debtors and it is given by:

\[ \text{degree}_{\text{out}}^i = \# j W w_{ij} > 0 \]

The degree distribution of network, \( P_{\text{degree}}(\text{degree}_{\text{in}}, \text{degree}_{\text{out}}) \) is the empirical distribution of the fraction of nodes in a network with in-degree \( \text{degree}_{\text{in}} \) and out-degree \( \text{degree}_{\text{out}} \).

The marginal \( P_{\text{degree}} \) involving just the in-degree and the out-degree which is easier to study. The correlation between in- and out-degree can make a large difference in financial network to spread contagion of economic distress entire financial system. The degree of network capture how nodes are connected, however, it cannot show position of nodes and distance between them. In order to know which node needs to have attention, we need to look at connectivity and centrality of network.

The other important characteristics of network are how close a node to other nodes and how easy reaching other nodes.

**Definition 3. Closeness Centrality**

Closeness Centrality (CC) measures how far a node from other nodes with measuring relative distances between nodes. It is defined as:

\[ CC_i = \frac{n - 1}{\sum_j A_{ij}} \]

**Definition 4. Betweenness Centrality**

The total number of paths between node \( i \) and node \( j \) is called \( P_{ij} \) and \( P_{ik} \) is number of paths pass through node \( k \). Betweenness Centrality (BC) of node \( ik \) is defined by:

\[ BC_k = \sum_{i,j, k} \frac{P_{ik}}{P_{ij}} \]

Betweenness Centrality measure the intermediary characteristics of a node.

All of these measurements capture different aspect of network structure and none of them dominate other measurements.

### 3.1. Network structures of real world banking systems

Due to lack of access to real-world banking network, there exist few empirical researches in this field. \(?\) define a methodology to verify potentials for contagion and systemic risk and they apply it on unique data set of mutual exposures and of Brazilian financial institutions to analyze the role of network structure in estimation of systemic risk.

The topology of interbank payments network between commercial banks over the Fedwire Funds Service has been studied by \(?\) \(?\) assessed the Swiss interbank network and measure systemic risk via mutual interbank exposure and spillover effects of a bank on other banks in the system.

These empirical studies of structural, topological, aspect of real-world networks show that in-degree and out-degree have heavy tailed distributions and these distributions follow power laws with the heterogeneous interconnections structure and correlation between them are shown to be the significant features.
to verify the effect of contagion. They found these properties of the network changed considerably in the immediate after financial crisis.

These studies also show that in the counterparty networks central part all other banks have been connected to all other banks. These networks can be modeled with direct weighted scale free network with power law distributions for in-degree and out-degree of nodes.

Based on these findings, we use a random network model to simulate banking networks, and use this model for simulation studies of contagion and systemic risk in financial networks.

4. Methodology

4.1. Problem Setup

This model studies the network effects on liquidity hoarding. Particularly, it shows how liquidity shortages can spread through the financial system via interbank linkages. Model examines the structure of financial network, such as, joint distribution of lending and borrowing links and its average degree in determining how shocks spread.

In this section, we introduce an index to quantify investment relations among financial institutions, regarding to the impact of the distress of institutions to their counter-parties across the entire system in a recursive way. In this approach, we estimate the systemically important institutions with analyzing the structure of dependencies among them in network approach.

We use a directed network in which the vertices represent financial institutions and the edges represent financial relations. The total value of the asset invested by institution $i$ in backing activities is: $A_i = \sum_j A_{ij}$ and $W$ is weighted adjacency matrix with entries as an interbank position points from the debtor to the creditor is represented with $C_{pa_i}$ the capital of institution $i$ which used against financial risk.

**Definition 5. Vulnerability Weight**

In the counterparties network, we introduce the Vulnerability Weight of a node which is the fraction of capital decreased by the default of one counterparty:

$$w_i = \frac{A_{ij}}{C_{pa_i}}$$  \hspace{1cm} (7)

4.1.1. Market Liquidity

In banking, liquidity is the ability to full fill requirements when banks come due without happening improper losses. Market liquidity defined as the potential of market to liquidate an asset quickly with keeping its value.

Financial institutions have several ways for generating liquidity, such as, having access to from other banks (interbank loans) or borrowing from a central bank, selling loans and raising capital. It requires constantly banker monitor cash flows to guarantee that sufficient liquidity is maintained. There are theoretical and empirical evidences that illiquid can lead to being insolvent and banks can’t pay their debts.

Banks also are required to monitor major individual liquidity risks personally as well as diversifying counterparty risk. Stress testing against extreme scenarios helps to identify abnormal market liquidity circumstances and to avoid sudden liquidity shifts. Losing liquidity can be happed to a bank if it lost some fraction of its deposits due to hoarding counterparty. In order to avoid suffering liquidity problems, banks immediately ask for available InterBank liquidity. With this mechanism, illiquidity spread through the whole system.

Theoretical and empirical researches suggest that banks need to invest on higher return assets with lower market liquidity to compensate for trading assets cost. A bank to be satiable in the financial system needs to attract significant liquid funds with low costs.

4.1.2. Solvency

Solvency is defined as the ability of a bank to accomplish long-term expansion and it becomes insolvent when its current assets exceed the current liabilities e.g. due to the default one of counter-parties. As result these banks are under-capitalized and threatened with insolvency and insufficiency to discharge all debts. Insolvency of a large player in banking system can pose systemic risks to whole system. Therefore, it is important to quantitatively model the contagion effect and insolvency of banks to define the basic underlying requirements for controlling the solvency risk of each individual bank.

**Definition 6. Solvency Index**

Solvency Index defined as the default of individual institutions over a short term cash flows horizon since debts are collected from its debtors:

$$Solv_i = \sum_j A_{ij} - \sum_j A_{ij}$$  \hspace{1cm} (8)
4.1.3. Relative Impact

The economic value of the impact of one institute on others is calculated by multiplying the impact by relative economic value of its counterparts. We define Relative Impact as fraction of Solvency Index one institute on accumulative solvency indexes of its neighbors.

The value impact of one institute on its neighbors can be defined as:

\[ I_i = \sum_j w_{ij} r_i \]  

where:

\[ r_i = \frac{Solv_i}{\sum_j Solv_j} \quad j \in s | A_{ij} > 0 \]  

In addition, its counter-parties also have impact, so we need to add the second term for the indirect impact via neighbors.

\[ \bar{I} = \frac{\sum j \in s I_j}{\sum j | A_{ij} > 0} \]  

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In addition, its counter-parties also have impact, so we need to add the second term for the indirect impact via the neighbors defined corresponding Contagion Index as follows:

\[ I_i = \alpha \sum_j w_{ij} I_j + \beta \sum_j w_{ij} r_i \]  

The first term is measuring the proportional of the expected loss generated by the failure of financial institution. The second term represents the proportional of the risk a financial institution has from equation (12):

\[ M \in R_{nn} \]  

4.2. Problem Formulation

We can rewrite equation (11) in a matrix form:

\[ I = \alpha W \cdot I + \beta Wr \]  

Let’s define matrix \( M \in R_{nn} \) as a non-negative (positive) matrix from equation (12):

\[ M = \alpha W + \beta Wr \]  

We are looking for \( \bar{I} \) as a robust extension solution of the dominant Eigenvector of matrix \( M \). Indeed, finding central nodes in contagion and risk dispersion can be used to measure systemic risk.

**Proposition 1. Robust Eigenvector solution**

The vector \( \bar{I} \) is defined as

\[ \bar{I} = M \]  

is a robust solution of the Eigenvector problem on \( M \) if

\[ I \in \text{Argmin}_{|I|} (\max_{M \subset P} ||I - M||) \]  

where \( \sum \{u^2 \in R_{nn} | \sum_i u_i = 1\} \)

\[ \bar{I} \in \text{Argmin}_{1 \leq i \leq n} (\max_{M \subset P} ||I - M||) + \epsilon \]  

\[ \bar{I} \] coincides with \( I \) if \( \epsilon \) is small enough and all eigenvalues lie inside the \( 1 - \lambda_i > 0 \); \( i = 2, \ldots, n \) one can find main proof in [A. Juditsky and B. Polyak,2012, Proposition 2]. In the case norm 2 vector \( I \) is unique due to the strict convexity on \( R_{nn} \)

The basic Eigenvector problem is formulated as a problem of finding the score vector \( x \) to satisfy following equation:

\[ x = Px \]  

where \( P \) is an adjacency matrix of a graph with \( n \) connected nodes with outgoing and incoming links.

They are several issues related using standard Eigenvector centrality and PageRank methods (?) to apply defined problem:

1. Matrix \( M \) is neither column-stochastic nor row-stochastic i.e. it is not a square matrix of nonnegative with each row or each column summing to one.
2. Vector \( I \) is not unique: typically in the real-world financial network, there are some disconnected subgraphs as well as cycles and solution for cyclic matrix it may converge slowly.
3. Financial networks are large weighted networks, which are often perceived as being harder to analyze than their unweighted counterparts.
4. Some Eigenvector centrality and standard PageRank cannot detect cascade in the financial system.

4.3. Solving Optimization Problem

Matrix \( M \) is the irregular matrix i.e. there is no robust dominant vector \( \bar{I} \) in explicit form. Instead we need to solve structured convex optimization problem (14). For medium-size problems in a medium size financial network, one can apply interior-point methods as a continuous nonlinear optimization.

Large scale financial network can be formulated as large-scale problems, which can be solved with some methods like mirror-descent family (?). Huge financial counterparties network, if adjacency matrix is sparse enough, the method which is explained in ? can be applied.

\( \bar{I} \) find numerical approximations to similar eigenvector problem, it works based on the power method. This method can be used in the case adjacency matrix has linearly independent eigenvectors and the eigenvalues can be ordered in magnitude a \( |A_1| > |A_2| > \cdots > |A_n| \). The power method can be described as follows:

**Step 1:** \( I_0 \) should be chosen appropriately.

**Step 2:** \( \sigma_{k+1} \) dominate term in \( MI_k \).
Step 3: $I_{k+1} = (1/\sigma_{k+1})MI_k$.
Whenever this ordering has be done, algorithm converges to largest eigenvalue $I$ is called the dominant eigenvalue of matrix $M$. One of main advantage of this method is it can use any norm and it is not necessary to normalized matrix.

Proposition 2. Contagion Index
Contagion Index for each node of a (directed) interbank networks for each iteration of solution procedure can be determined as:

$$I_{k+1} = \frac{k}{k+1}WI_k + \frac{1}{k+1}WR_1$$

Proof. We apply the power method with averaging to minimize:

$$\|\bar{I} - M\bar{I}\|_2 = \frac{\|MKI_1 - I_1\|_2}{k} \leq \frac{\text{Constant}}{k}$$  

(18)

With $M = \alpha W + \beta Wr$ we have:

$$I_k = \frac{I_1 + M I_1 + \cdots + M^{k-1}I_1}{k}$$

(19)

and with choosing $I_1 = WR_1$ we will get this recurrent form:

$$I_{k+1} = \frac{k}{k+1}WI_k + \frac{1}{k+1}WR_1$$

(20)

And following previous equation $M = \alpha W + \beta Wr$ we can define matrix $M_k$ as :

$$M_k = \frac{k}{k+1}W + \frac{1}{k+1}WR$$  

(21)

That means $\alpha = \frac{k}{k+1}$ and $\beta = \frac{1}{k+1}$ are weights coefficients of matrix.

5. Algorithm

After we formulate the finding systemically important financial institutions in a investment network as convex optimization problem, we explain the approximation solution. We specify an algorithm to model mathematical process in solving above problem.

In the initial step of this algorithm, we create different type of financial network like complete network, scaled free random network and we calculate adjacency matrix corresponding to each network.

We now want to estimate the impact of one node on its indirect successors; we define this impact in terms of a recursive equation:

$$M_k = \frac{k}{k+1}W + \frac{1}{k+1}WR$$ (k is the iteration number of solution procedure).

The first step of this procedure is specifying the initial value of impact index. As explain earlier we define $I_1$ as $WR_1$, as already explained, matrix $W$ is the weighted adjacency matrix of the investment network.

Following algorithm explains this procedure:

Algorithm 1

Step 1: begin: $I_1$ as initial value for impact index

$$I_1 = WR_1$$

Step 2: k-th iteration: $I_k = MI_{k+1}$

where

$$M = \frac{k}{k+1}W + \frac{1}{k+1}WR$$

Step 3: Stop condition: $|I_k - \bar{I}| < \text{Tolerance}$

6. Financial Networks Example

Here, we demonstrate how to estimate the systemically importance of the institutions by applying algorithm 1 to different dependencies link structures in network approach. Since financial institutions failures are atypical and also corresponding data mostly are confidential and is not available publicly, we use small sample data from \text{?}.

More specifically, we analyze the interbank payment as the backbone for financial transactions with using FNA simulator platform (www.fna.fi). It creates network with nodes with size scale of capital value and sets payment transaction for each link with widths scale as value of payments, see Figure 4.

Figure 5 Indicates the distribution of total value received and sent and capital value of each banks. It shows that these distributions are relatively positive skew with 1.62 skewness is the degree of asymmetry of mean i.e. the tail on the right side is longer than the left side and the average located on the right of the median value. Figure 4.1 demonstrates the result obtained when algorithm 1 was applied to given dataset; as you can see there is positive correlation between number of in-degree and out degree with result of systemic risk index in the network.
In the past, size of financial institution’s balance sheet like total capital or total financial transaction has been used to rank the institutions in the system. Recent economic crises have indicated that small institutions relatively can have large impact on the system therefore the interconnectedness of the financial system can be a major reason of failing the whole of system.

Results of applying algorithm 1 on example network dataset indicate two systemically important banks; these banks are systemically important not just because of their balance sheet size but also they have interactions to most of nodes in the system.

7. Numerical Experiments

7.1. Simulation model of financial systems

The first step of generating of banking network model, we need to create network consists of N nodes (to represent banks) and directed edges (representing the interbank positions) may exist in both directions. The structure of desired network should be a realization of the empirical studies which are explained in previous sections.

In the next step, we weight each link (financial relation) of the network with loan magnitudes as well as balance sheet of bank (node). The total assets a bank are the sum of its loans to other banks as its interbank assets and the sum of its external assets. The liability of a bank includes the sum of loans taken from other banks as interbank liabilities and its customer deposits. This show that the distribution of claims within the system and loan can be uniform distribution when bank wants maximum its diversity lending strategy with giving of equal loans size to all their debtors.

This simulation is performed in such as way as to ensure the generated network represents the real-world banking system. Dependent on characteristics of the interbank liability structure, we model different banking system. As mentioned in financial network section, empirical studies found that network have fat-tailed degree with Pareto distribution for the exposures and the in-degree and out-degree with power law distributions to be stable structure across time.

7.2. Financial Systems as Complete Network

From theoretical findings of ?, if there exists a directed financial link from each bank to all others, banks equally spread their claims as well as diversify equally their contagion risk. This structure gives us homogeneous analysis in terms of completeness and interconnectedness of the financial network and asset-liability structure. This type of network is called complete network, see Figure 9.

7.3. Financial Systems as Random Network

Empirical studies show that full connectivity of real-world financial system is unrealistic therefore we need to omit some directed edges within the financial network. ? used scale-free network to model a financial system. However, less complete system increase the possibility of contagion risk due to highly interconnection links and lack of potential for risk diversification (?).

7.3.1. Basic BA Algorithm

? purpose the Barabasi-Albert (BA) algorithm for generating random scale-free networks: this algorithm starts with a set
of small fully connected nodes and then add a new node at a
time with exactly k edges. Preferential attachment probability
used to attach a new node to an existing node in proportion to
the number of node’s edges. This process is continued until all
nodes are connected. Under this process, it is possible for mul-
tiple edges to exist a pair of nodes and loop in the network.In
financial network, financial institution are serially connected in
such a way that the preceding one is connected to the foremost
institution. Figure 10 shows a BA network in circle and cartes-
ian format.

7.4. Numerical Simulation

This paper examines the role of complexity and concentra-
tion to identify systemically important institution. We generate
financial network with BA scale-free directed networks with
empirical properties of Brazilian interbank networks [2]. In
the initial step in each experiment is to construct the network
of unsecured interbank from an underlying distribution of the
network structure.

Here is the Simulation steps:

Simulation Steps

Step 1: Generate network.
Step 2: Use Pareto distribution for connectivities and exposure
sizes: Heterogeneity.
Step 3: Contagion Index is computed for each node with
Algorithm 1 and interior-point method.

7.5. Numerical Results

The numerical results presented here were obtained by using
Matlab’s built on a PC running under processor (Intel Core 2.30
GHz) to analyze simulated network data of the BA algorithm
and complete network; we have firstly generated several net-
works following the ? (BA) algorithm and complete network.
Then we simulate financial transaction data as well as capital
value for each bank following specific distributions which are
described in previous section.

Optimal solution obtained from built-in function lsqlin solves
constrained linear least square problems with interior point
method. We define linear constraints from financial relations
in network data set and try to find an optimal solution of the
medium-scale optimization mentioned problem allows equal
upper and lower bounds.

In order to compare algorithm 1 and optimal algorithm (interior
point method), relative errors were defined as follows:

\[
\text{Rel}_{err} = \frac{(Err_{Alg1} - Err_{interior})}{Err_{interior}}
\]  

where \(Err_{Alg1}\) is the minimum objective value of algorithm 1
and \(Err_{interior}\) is objective value of optimal solution.

In tables 1 and 2, CPU time and relative error of algorithms
1 toward the above mentioned optimal solution are indicated
for both simulated networks of BA network and complete
network. Size is number of nodes in the network, “Interco” is
total number of interconnections and “Max in”, “Max out” are
maximum number of in-degree and out-degree, ”Ite” is number
of iterations of solution procedure to reach tolerance value.

These tables show the numerical result obtained from algorithm
1, with different network topologies data set while the number
of nodes and edges increases exponentially.

Figures 11 and 12 illustrate the systemic risk (Impact Index)
versus in-degree and out-degree distributions. One can find
high correlation between impact index and interconnection of
nodes in the system.

We study the impact of network size, Out-degree and
In-degree on measuring systemic risk, we consider scale-free
network (BA Algorithm) and regress the Impact index on these
Table 1: Numerical results of BA Network

| Size  | 50   | 100  | 200  |
|-------|------|------|------|
| Interco | 2420 | 4821 | 10475|
| Max in   | 129  | 222  | 403  |
| Max out  | 135  | 214  | 394  |
| Tol     | $10^{-5}$ | $10^{-3}$ | $10^{-3}$ |
| time    | $10^{-3}$ | 0.015 | 0.0039 |
| Ite     | 18    | 58   | 182  |
| RelErr  | 0.034 | 0.043 | 0.050 |

Table 2: Numerical results of Complete Network

| Size  | 50   | 100  | 300  |
|-------|------|------|------|
| Interco | 9718 | 39474| 359039|
| Max in   | 234  | 443  | 1311 |
| Max out  | 223  | 453  | 1290 |
| Tol     | $10^{-3}$ | $10^{-3}$ | $10^{-3}$ |
| time    | 0.001 | 0.0013 | 0.0038 |
| Ite     | 16    | 52   | 164  |
| Rel-Err | 0.030 | 0.029 | 0.050 |

Figure 12: Systemic risk (Impact index) versus Total number of Out-connection quantities. Table 3 shows the result of these estimations: it is clear "Total In connection", "Total Out connection" both highly significant and they can explain most variation of impact index ($R^2=0.599$ and 0.69) but size of bank is not main effect of contagion effect. We can conclude the interconnectedness of the financial network can be a main cause of failing entire system.

8. Summary and Discussion

Having started with the definition of the systemic risk in this research, we have studied the Eigenvalue problem for problem verifying systemically important financial institutions in inter-bank network. An algorithm for reducing above problem to convex optimization is developed. Numerical results show that this algorithm is able to exploit efficiently information about the systemically important nodes in a large-scale financial network dataset. In order to study this algorithm and its effectiveness and to compare it with optimum method, network simulations with different network structure and balance sheet allocations have been performed. Computational results indicate that our algorithm requires less CPU time than optimum algorithm for producing much more accurate approximation to the optimal solution. Results show that CPU time does not increase exponentially when the number of nodes and interconnections of network grow, which is indicated algorithm has robust convergence behavior rate independent of structure of network. Robust Eigenvalue solution obtained without removing the cycles from the financial network which has strongly underestimating the impact index. The results suggest that process of investigation of systemically important institutions with high systemic impact should include the serious issue of hidden interconnection with major counterparties in a complex financial network. This experiment discovered that concentrated networks were highly susceptible to shock to key players. Higher connectivity and concentration in the network leads financial system more vulnerable to a systemic liquidity risk. This results consist with other researchers which confirm that fat-tailed networks be likely to be more robust to unsystematic shocks. Therefore, banks need higher liquidity requirements mostly from liquid asset rather than interbank assets. One of main conclusion of this study is keeping larger stock of high-quality liquid assets can improve stability of the system and makes the system less vulnerable to systemic liquidity risk. However, the unsecured interbank liabilities cannot cover with liquid assets. Complexity of financial system can cause increased risk exposures as well as bank’s capital. One way to reduce contagion is by netting-off gross exposures between participants within the financial network to shrink the number of interbank connections. At the present time, we don’t have sufficient knowledge about network structure of interbank system. Having to access to cor-
responding data allows us to map the entire financial system and assess its properties. Central banks can use this fast algorithm in conjunction with the interbank network structure to identify which organizations are systemically important and they might be involved in a cascade toward whole financial system. The main conclusion is that the specific aims of this research have been achieved: Algorithms for verifying the most systemically important institution in financial network has been developed. The performance of this algorithm testifies its efficiency and flexibility to deal with different financial network topologies.

9. References

References