Compensation of B-L charge of matter with relic sneutrinos

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Abstract

We consider massless gauge boson connected to B-L charge with and without compensation to complete the investigation of the gauging of B and L charges. Relic sneutrinos predicted by SUSY and composite models may compensate B-L charge of matter. As a consequence of this possible compensation mechanism we have shown that the available experimental data admit the range of the B-L interaction constant, $10^{-29} < \alpha_{B-L} < 10^{-14}$, in addition to $\alpha_{B-L} < 10^{-49}$ obtained without compensation.

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Recently, there has been renewed interest in gauging baryon (B) and lepton (L) numbers [1–6]. Historically these numbers have been proposed to explain non-observation of certain processes such as proton and neutrinoless double beta decays. By analogy with QED it is natural to consider possible existence of gauge bosons connected to baryon and lepton charges. Massless baryonic and leptonic photons has been considered long time ago by T.D. Lee and C.N. Yang [7] and L.B. Okun [8], respectively. However, the experiments on checking the equality of inertial and gravitational masses by Eötvös and his colleagues [9] have put very strong upper limits on the value of baryonic and leptonic coupling constants: $\alpha_B < 10^{-47}$, $\alpha_L < 10^{-49}$ [10]. By comparing these values with electromagnetic coupling constant, $\alpha_{em} \sim 10^{-2}$, it has been concluded that the existence of such baryonic and leptonic photons are unrealistic. These statements can be weakened, if the mechanism for compensation of the baryonic and leptonic charges of matter exists. In [11] has been shown that leptonic charge of matter can be compensated by the relic scalar anti-neutrinos predicted by SUSY or composite (preonic) models [12–19].

Concerning the baryon charge alone there is no realistic candidate which can provide compensation. Massive gauge bosons connected to baryon and lepton charges have been considered in [20–25]. Massive gauge boson connected to B-L is considered in [26–29]. To complete the gauging of B and L charges in this paper we consider massless gauge boson connected to B-L charge with and without compensation.

For baryogenesis it is necessary the breaking of baryon, (B) and lepton, (L), symmetry in universe ($\triangle B = B - \bar{B} \neq 0, \triangle L \neq 0$). In massless case the conservation of B-L charge requires the Baryon and Lepton-asymmetry, to be equal $\triangle B = \triangle L \neq 0$.

In case of the massless gauge boson, the upper limits on the equality of inertial and gravitational masses obtained by Eötvös [9], improved by Braginsky and Panov [30] and updated by Eöt-Wash group from Washington University [31, 32] give a restriction on the strength of $\alpha_{B-L}$ coupling constant. By adopting the expression given in ref [33]

$$\frac{\alpha_{B-L}}{G} \left( \frac{A - Z}{M} \right) \left\{ \frac{(A - Z)_{Be}}{M_{Be}} - \frac{(A - Z)_{Ti}}{M_{Ti}} \right\} < \left\{ \begin{array}{l} (0.3 \pm 1.8) \times 10^{-13} Earth \\ (-3.1 \pm 4.7) \times 10^{-13} Sun \end{array} \right. \quad (1)$$

where $\alpha_{B-L}$ is interaction constant for B-L charge, A, Z, M are baryon number, atomic number and mass respectively, of the corresponding objects, namely earth, sun and test objects made by Beryllium and Titanium, $G = 6.10^{-39} m_p^{-2}$ is the Newton gravitational constant ($m_p$
we obtain upper limiting values for B-L coupling constant $\alpha_{B-L}$, namely, 

\[
\alpha_{B-L} < \left\{ \begin{array}{l}
2.1 \times 10^{-49} Earth \\
5.3 \times 10^{-49} Sun
\end{array} \right. \ .
\] 

(2)

In the calculation of baryon number the composition of earth and sun has been taken into account [34, 35].

These upper values for $\alpha_{B-L}$ indicate that B-L photons $\gamma_{B-L}$ are unrealistic. To overcome these limiting values for $\alpha_{B-L}$ we propose the compensation of the B-L charge of matter with relic scalar neutrinos which has been regarded as dark matter candidate [36–41]. The B-L charge of matter is solely that of neutrons since the B-L charge of proton and electron cancels each other’s effect.

By the analogy with electrodynamics we write down an equation which gives the B-L potential of an external B-L charge in relic scalar neutrino and antineutrino background [42]:

\[
\nabla^2 \phi (x) = -4\pi \alpha_{B-L} \left( n_n (x) + n_0 e^{-\phi(x)/kT_0} - n_0 e^{\phi(x)/kT_0} \right) dV.
\] 

(3)

Where $\phi (x)$ is the B-L potential, $n_n (x)$ is the neutron density of the object, $n_0 e^{(\pm)\phi(x)/kT_0}$ is the density distribution of relic scalar (anti)neutrinos in a B-L potential, $k$ is Boltzmann constant and $T_0$ is temperature of relic scalar neutrino and anti neutrino background. The net compensated B-L charge of the macroscopic object is given as:

\[
b = \int_V \left( n_n (x) + n_0 (x) e^{-\phi(x)/kT_0} - n_0 e^{\phi(x)/kT_0} \right) dV,
\] 

(4)

where the integration is over the volume of the object. Since it is very difficult to find the analytical solution of the differential equation (3) due to improper values of parameter ($n_n/n_0 \sim 10^{21}$), we restrict ourselves to a qualitative analysis at this stage.

The total energy of the compensating scalar neutrino in a spherical macroscopic object is negative, so that the potential energy of this neutrinos is greater than their kinetic energy $U > KE$. If we assume that scalar neutrinos obey the Boltzmann distribution at temperature $T_0 = \mathcal{O}(\sim 300^\circ K)$ for Earth and $T_0 = \mathcal{O}(\sim 10^{4\circ}K)$ for Sun, where the temperatures are thermalization temperatures for the sneutrinos within the earth and the sun, respectively, then their mean kinetic energy is $< KE > = (3/2)kT_0$ and their potential energy in terms
of the compensated B-L charge is given as \( U = -\alpha_{B-L}(N_n - N_\nu + N_\bar{\nu})/R \), where \( R \) is the object radius, \( N_n, N_\nu \) and \( N_\bar{\nu} \) are the B-L charges due to neutrons, scalar neutrinos and scalar antineutrinos within the object. The contribution of \( N_\bar{\nu} \) is negligible, so that we can write the following equation:

\[
\alpha_{B-L} \frac{N_n - N_\nu}{R} > \frac{3}{2}kT_0.
\] (5)

The neutrino emission from the sun will constantly decrease the Sun’s lepton number. This loss is compensated by the relic sneutrinos present in the environment around the Sun.

The massless hypothetic photon \( \gamma_{B-L} \) should produce a sort of effective repulsive “Coulomb” force around the earth. Therefore, the interaction between the earth and a test object with mass \( m_i \) and compensated B-L charge \( b_i \) can be expressed as follows:

\[
F = -G \frac{Mm_i}{r^2} + \alpha_{B-L} \frac{Bb_i}{r^2},
\] (6)

where \( B \) and \( M \) are the compensated B-L charge and mass of the earth. If we define the compensation rate as \( \beta = (N_n - N_\nu)/N_n \), using eq. (5), \( \beta \) becomes

\[
\beta > 6,6 \frac{(R/cm)}{\alpha_{B-L}N_n}(T_0/^oK)
\] (7)

If we write \( B \) and \( b_i \), in terms of \( \beta \) and \( \beta_i \), then the force equation (6) can be written as

\[
F = -G \frac{Mm_i}{r^2} + \beta \beta_i \alpha_{B-L} \frac{N_nN_{n_i}}{r^2}.
\] (8)

In Table 1 the total neutron number, compensation rates and effective interaction constants (\( \alpha^{eff} = \beta_i \beta \alpha_{B-L} \)) of the earth and the sun \([34,35]\) are given. In the experiments, the compensation rates of the test object and earth must be taken to equal each other. By using the data given in Table 1, lower limit for the B-L interaction constant can be obtained as

\[
\alpha_{B-L} > \begin{cases} 
0.27 \times 10^{-29} \text{Earth} \\
0.38 \times 10^{-31} \text{Sun}
\end{cases}
\] (9)

The contribution to the \( \bar{\nu}_e e \) interaction cross section from the \( \gamma_{B-L} \) proposed by the model should be less than W and Z boson contribution, so that the upper limit on the B-L interaction constant may be determined by \( \bar{\nu}_e e \) interaction. Whereof it follows that
Table I: Some parameters and compensation rates for the earth and the sun

|                  | Earth       | Sun        |
|------------------|-------------|------------|
| $R (cm)$         | $6.4 \times 10^8$ | $7 \times 10^{10}$ |
| $N_n$            | $1.66 \times 10^{51}$ | $1.72 \times 10^{56}$ |
| $\beta \alpha_{B-L}$ | $7.63 \times 10^{-40}$ | $2.68 \times 10^{-41}$ |
| $\alpha_{eff}$   | $< 2.1 \times 10^{-49}$ | $< 5.3 \times 10^{-49}$ |

$\alpha_{B-L} < G_F s / \sqrt{2 \pi}$ with $E_\nu$ 10 MeV and $s \approx 2 E_\nu m_e \ 10^{-5} GeV^2$ we have approximately $\alpha_{B-L} < 10^{-12}$ (more accurate estimations lead to $\alpha_{B-L} < 10^{-13} - 10^{-14}$).

To form the B-L model it has been added an $U(1)'_{B-L}$ group to the Standard Model (SM) gauge group as $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)'_{B-L}$. Hence the covariant derivative has been as follows:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ig_2 T \cdot A_\mu - ig_1 \frac{Y}{2} B_\mu - ig'_1 b B'_\mu.$$ (10)

Where $g_1, g_2$ and $g'_1 = \sqrt{4 \pi \alpha_{B-L}}$ are gauge coupling constants; $T$ is an isospin operator of a corresponding multiplet of fermion or Higgs fields; $Y$ is the weak hypercharge, $b$ is the B-L charge of the corresponding multiplet, $A_\mu, B_\mu$ and $B'_\mu$ are gauge fields. The B-L charge is 1/3 for quarks, −1 for leptons [26].

The interaction Lagrangian can be written as

$$L = L_{SM} + g'_1 J^\mu_{fer} B'_\mu,$$ (11)

where $L_{SM}$ is the Lagrangian of the Standard Model, with right-handed neutrino, $\nu_R$, added

$$J^\mu_{fer} = \sum_f b_f \bar{f} \gamma^\mu f$$ (12)

and $b_f$ is the B-L charge of the corresponding fermion. The model proposed is anomaly free.

In conclusion, we have shown that the gauging of B-L charge without compensation admits coupling constant to be $\alpha_{B-L} < 10^{-49}$. If the compensation mechanism takes place, the interaction constant of the B-L charge may have another interval, namely,
$10^{-29} < \alpha_{B-L} < 10^{-14}$.

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