Optical Model description of fusion and elastic scattering of $^6\text{Li} + (^{58}\text{Ni}, \, ^{64}\text{Zn})$ at sub-barrier energies

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Abstract. Recently reported fusion cross sections for $^6\text{Li} + (^{58}\text{Ni}, \, ^{64}\text{Zn})$ have been reduced and compared with each other. Fairly good consistency was found but with some data spread at low energies, possibly due to contributions of direct reactions to the respective inclusive data. This possibility is investigated by doing Optical Model calculations that simultaneously fit the experimental elastic scattering and fusion data. In addition to a realistic bare potential, fusion and direct polarization potentials are introduced which in principle allow to separate the contributions of the corresponding reaction mechanisms. Good fits are obtained for elastic scattering data at all energies but the fusion data for both systems are underpredicted at the lowest energies. These results are consistent with a possibly dominant contribution of direct reactions to the corresponding inclusive data.

1. Introduction

There has been considerable interest in studying reactions with loosely bound nuclei, stable or radioactive [1, 2]. Among the stable weakly bound nuclei, $^6\text{Li}$ is perhaps the most studied one, with strong current interest [3–12]. Two particular systems having $^6\text{Li}$ as projectile and medium mass targets will be studied in the present work. Fusion data at energies around the barrier were published for the $^6\text{Li} + ^{64}\text{Zn}$ system in 2013 [13] and for the $^6\text{Li} + ^{58}\text{Ni}$ system in 2017 [7]. In the latter reference, the respective barrier parameters were extracted from the São Paulo potential, for both systems, and used to reduce the corresponding data. With this procedure, a direct comparison between the two systems can be done.

The comparison, restricted to the region where the data overlap with each other, showed a fairly good consistency but with some spread of the data at the lower energies (see Fig. 4 of Ref. [7], or the experimental points in Fig. 7 below). This spread may come from some direct channel contributions that could not be separated in the respective inclusive measurements. In the case of $^6\text{Li} + ^{58}\text{Ni}$, where inclusive proton yields were measured, the Direct Cluster Transfer (DCT) of deuterons or alpha particles to the target may produce protons undistinguishable from the evaporation protons of interest. For example, the transfer of a deuteron to states above the proton emission threshold could lead to protons undistinguishable from the evaporation protons.

Similarly, in the case of $^6\text{Li} + ^{64}\text{Zn}$ the authors concluded that the heavy residues that they measured could possibly include contributions from 1-proton or 1-neutron transfer as well as from DCT of deuterons or alphas. In fact, these authors actually mentioned that such contributions...
may become dominant in the sub-barrier region. One interesting question is whether there is any way of getting information about these direct-channel contributions using only the data that have been published so far.

To shed some light on this issue, it would help to make a simultaneous analysis of the respective elastic scattering and fusion data for each system. By a proper determination of fusion and direct polarization potentials one should be able to obtain fusion cross sections which are consistent with the elastic scattering data and with the dispersion relation. As a matter of fact, such an analysis was recently performed for the $^6\text{Li} + ^{58}\text{Ni}$ system [14]. As a result, it was shown that the fusion excitation function could be reproduced only partially, with discrepancies at the two lowest energies, where the experimental values were underpredicted. This result would be consistent with direct channel contributions in the respective inclusive data. The aim of the present work is to perform the complementary analysis for the $^6\text{Li} + ^{64}\text{Zn}$ system and compare both results.

The optical model extended to describe fusion is briefly described in Sec. 2 while the respective fits to the data are presented in Sec. 3. The experimental and predicted fusion cross sections for the two present systems are compared with each other in Sec. 4 and, finally, the conclusions are given in Sec. 5.

2. Extended Optical Model

It’s been long known that the parameter-free São Paulo Potential (SPP) [15] can be used as an appropriate bare potential to describe fusion reactions. Crema et al. [16, 17], for example, have shown that the SPP gives reliable results in the analysis of fusion reactions with stable weakly-bound projectiles. Among other results, this fact has been used to show that the experimental fusion excitation functions for neutron-halo projectiles are suppressed with respect to the corresponding SPP predictions for energies above the respective Coulomb barriers, while those for proton-halo systems are enhanced [2,18].

Within the framework of the optical model, fusion has been described as the absorption in a short-range imaginary potential interior to the barrier, $W_{\text{int}}$. Such a potential effectively simulates an incoming-wave boundary condition, thus giving a proper barrier penetration model (BPM) prediction. The optical model wave function should correctly describe the respective elastic scattering angular distributions if the fusion cross section calculated this way is to be realistic. So, an appropriate polarization potential should most probably be added to the bare potential. Assumedly, this polarization potential would result from respective couplings to fusion and direct channels. It has been shown for many systems that it is reasonable to decompose the total polarization potential into a direct and a fusion part [19–27]. With this in mind, the following general form for the total optical potential is proposed:

$$U_{\text{TOT}} = SPP + V_{\text{Coul}} - [iW_{\text{int}} + U_F + U_D],$$

where $V_{\text{Coul}}$ is the Coulomb potential, $W_{\text{int}}$ is a short range Woods-Saxon potential (interior to the barrier: $W_0 = 50$ MeV, $r_0 = 1.0$ fm, $a_0 = 0.2$ fm), and $U_F, U_D$ are complex polarization potentials that can be associated to couplings to the fusion and direct channels, respectively.

$U_F$ ($U_D$) is taken as having a volume (derivative) Woods-Saxon shape, where in each case the radius and diffusenesses are the same for both, the respective real and imaginary parts. To be more specific, $U_{F,D}$ can be written as

$$U_{F,D} = V_{F,D} + iW_{F,D}.$$
\[
\sigma_{\text{fus}} = \frac{2}{\hbar v} \left| \chi^{(+)} | W_{\text{int}} + W_F | \chi^{(+)} \right| ,
\]

where \( \chi^{(+)} \) is the distorted wave function that satisfies the Schrödinger equation with the full optical potential of Eq. 1.

3. Simultaneous fit of elastic scattering and fusion data

Elastic scattering measurements were published in 2009 for \(^6\text{Li} + ^{58}\text{Ni} \) [29] at 5 near-barrier energies and also for \(^6\text{Li} + ^{64}\text{Zn} \) [30] at 8 energies. As mentioned in Sect. 1, a full analysis of the first system has been done already [14]. In the following, a simultaneous analysis of elastic [30] and fusion [13] data for the \(^6\text{Li} + ^{64}\text{Zn} \) system will be performed. As a first step, the geometric parameters of the polarization potentials will be fixed. For the fusion polarization potential, the values that have been published in the literature for \(^6\text{Li} \) induced reactions were adopted, that is, a reduced radius of 1.4 fm and a diffuseness of 0.43 fm. These are the same parameter values that were used for \(^6\text{Li} + ^{58}\text{Ni} \) [14].

As for the geometric parameters of the direct polarization potential, the following method was introduced: the elastic scattering data were plotted as a function of the distance of closest approach on a Rutherford trajectory, \( d \), according to the expression

\[
D = d \left( \frac{1}{3} A_p + \frac{1}{3} A_t \right) = \frac{1}{2} D_0 \left( 1 + \frac{1}{\sin(\theta_{\text{c.m.}}/2)} \right), \quad \text{with} \quad D_0 = \frac{Z_p Z_t e^2}{E_{\text{c.m.}}}.
\]

The corresponding plot is shown in Fig. 1 for the \(^6\text{Li} + ^{64}\text{Zn} \) system, where the data for all measured energies are included. It can be seen that, for low distances, they nicely overlap with each other. The strong absorption distance, \( d_S \), corresponding to the point where the ratio \( d\sigma/d\sigma_{\text{Ruth}} \) is 0.25, is therefore well defined, with a value \( d_S = 1.6 \) fm. If one assumes that the falling down of the data with decreasing \( d \) is due only to the surface potential \( U_D \), it is reasonable to take the radius of this potential as \( r_D = 1.6 \) fm, with an appropriate diffuseness to reproduce the respective slope. The dashed curve in Fig. 1 corresponds to a derivative Woods-Saxon shape with radius 1.6 fm. In order to fit the falling-down part of the data, a diffuseness \( a_D = 0.85 \) fm is needed. A similar analysis for the \(^6\text{Li} + ^{58}\text{Ni} \) system renders a radius of 1.61 fm and a diffuseness of 0.68 fm [14].

![Figure 1. Plot of \( d\sigma/d\sigma_{\text{Ruth}} \) vs \( d \) for the data of Ref. [30]. The solid curve corresponds to a three parameter function fit to describe the data while the dashed curve is explained in the text.](image_url)
work to perform all optical model calculations. After a \( \chi^2 \) analysis for each of the 8 energies measured, the strengths shown in Fig. 2 were found for the direct part of the potential. The black circles (upper panel) are the imaginary depths while the red ones are the corresponding real depths (lower panel). It is possible to check consistency of these potentials with the dispersion relation [32–34]:

\[
V(E) = V(E_S) + \frac{E - E_S}{\pi} P \int_0^\infty \frac{W(E')}{(E' - E_S)(E' - E)} dE'.
\]

(5)

In this expression, \( P \) stands for the principal value and \( V(E_S) \) is the value of \( V(E) \) at a reference energy \( E = E_S \).

The imaginary depths were approximated by straight line segments and the red dashed curve is the result of applying the dispersion relation to this approximation. It can be seen that the dispersion relation is well satisfied by these direct polarization potentials.

**Figure 2.** Real \( V_{D0} \) and imaginary \( W_{D0} \) parts of the direct polarization potential found for the \( ^6\text{Li} + ^{64}\text{Zn} \) system. The dashed curve is the result of applying the dispersion relation to the black curve.

Similarly, the dispersion relation is also satisfied by the fusion part of the polarization potential, as indicated in Fig. 3

**Figure 3.** Real \( V_{F0} \) and imaginary \( W_{F0} \) parts of the fusion polarization potential found for the \( ^6\text{Li} + ^{64}\text{Zn} \) system. The dashed curve is the result of applying the dispersion relation to the black curve.

With the above direct and fusion polarization potentials, the predicted angular distributions are displayed by the curves in Fig. 4, which provide a reasonably good description of the experimental data.
The predicted fusion cross sections, calculated from equation 3, are indicated in Fig. 5 by the empty squares, while the circles represent the experimental data. Within the range of energy overlap, the experimental fusion cross sections are well reproduced only for the highest energies, but there is an under prediction for the lowest energies. It is important to mention that the $\chi^2$ analyses at these lower energies showed that no other reasonable potential depths can be found that reproduce the experimental cross sections. Similar to the results for the $^6$Li + $^{58}$Ni system [14], it is speculated that the discrepancies may be explained by a possible direct channel contribution to the respective inclusive measurements. This possibility was considered as quite probable by the authors of the respective experimental paper [13].

4. Comparison of both systems
The same type of analysis, performed for the $^6$Li + $^{58}$Ni system [14], rendered the predicted fusion cross sections shown with black diamonds in Fig. 6. The importance of introducing the fusion polarization potential is illustrated by comparing with respective predictions corresponding to $U_F = 0$ (red diamonds). Although including the fusion polarization potential increases considerably the fusion cross sections, there are still discrepancies with the reported data at the two lowest energies. It is possible that such discrepancies may be ascribed to DCT contributions present in the inclusive proton data of Ref. [7].
Figure 6. Experimental fusion cross sections for the $^6\text{Li} + ^{58}\text{Ni}$ system [7] (circles) and respective predictions from the optical model analysis [14]. Black diamonds correspond to the full polarization potential, while only the direct part was used for the red diamonds. The upper (lower) panel corresponds to a logarithmic (linear) Y-scale.

Finally, Fig. 7 shows a comparison of reduced cross sections for both systems, including experimental (filled symbols) and predicted (empty symbols) values. The reduction prescription is indicated in the axes labels and UFF stands for the so called Universal Fusion Function $\text{UFF} = \frac{2E}{(\hbar\omega_0 R_b^2)} \sigma^W$, where $\sigma^W$ represents Wong’s function [35]. The barrier parameters used to make the reduction, obtained from the $\text{SPP}$, were ($V_b = 12.36 \text{ MeV}, R_b = 9.02 \text{ fm}, \hbar\omega_0 = 3.63 \text{ MeV}$) for $^6\text{Li} + ^{58}\text{Ni}$ and ($V_b = 13.05 \text{ MeV}, R_b = 9.17 \text{ fm}, \hbar\omega_0 = 3.67 \text{ MeV}$) for $^6\text{Li} + ^{64}\text{Zn}$.

It is worth emphasizing that the respective predictions were obtained individually for each energy and for each system. In addition, the respective polarization potentials did always comply with the dispersion relation, consistent with previous findings for many systems [19–22,26,27]. It is nice to see that in this reduced plot the theoretical predictions for both systems do actually follow a common path (dashed line), which would be expected for the actual fusion cross sections corresponding to similar systems. This fact adds confidence to the present method of calculating the fusion cross sections. According to these results, assuming that the hypothesis of direct channel contributions in the inclusive data is true, such contributions would be fractionally more significant in the case of $^{64}\text{Zn}$ than in the case of the $^{58}\text{Ni}$ target.

Figure 7. Comparison of reduced fusion cross sections for the $^6\text{Li} + (^{58}\text{Ni}, ^{64}\text{Zn})$ systems. Both the experimental (Refs. [7] and [13], respectively) and the predicted values are shown.
5. Conclusions
Discrepancies observed in the reduced fusion cross sections for the systems $^6\text{Li} + (^{58}\text{Ni}, ^{64}\text{Zn})$ are investigated by performing extended optical model analyses that provide a consistent description of elastic scattering and fusion data. In the present manuscript the $^6\text{Li} + ^{64}\text{Zn}$ system is investigated, complementing the results of a previous work, where an analysis for $^6\text{Li} + ^{58}\text{Ni}$ was done [14].

The SPP was used as a realistic bare potential. Polarization potentials $U_F$ and $U_D$ associated to fusion and direct channel couplings, respectively, were introduced. A method was developed to fix the geometric parameters for $U_D$, while those for $U_F$ were taken from the literature. After fitting simultaneously the elastic scattering and the fusion data, the obtained polarization potentials complied with the dispersion relation. Good descriptions for the elastic scattering angular distributions were obtained. The predicted fusion cross sections agreed with the reported experimental values at the higher energies but discrepancies were observed at low energies.

When compared in a reduced plot, the predictions for both systems agree with each other even though they were derived independently for each system and for each energy. Possible ambiguities in the obtained optical potentials cannot be discarded but, if confirmed, the results would be consistent with a dominant contribution of direct channels in the respective inclusive data at the lowest energies. Exclusive measurements at such energies are desirable.

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