On the Fate of Processed Matter in Dwarf Galaxies

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ABSTRACT

Two dimensional calculations of the evolution of remnants generated by the strong mechanical energy deposited by stellar clusters in dwarf galaxies ($M \sim 10^9 - 10^{10} \, M_\odot$) are presented. The evolution is followed for times longer than both the blowout time and the presumed span of energy injection generated by a coeval massive stellar cluster. The remnants are shown to end up wrapping around the central region of the host galaxy, while growing to kpc-scale dimensions. Properties of the remnants such as luminosity, size, swept up mass, and expansion speed are given as a function of time for all calculated cases.

The final fate of the swept-up galactic gas and of the matter processed by the central starburst is shown to be highly-dependent on the properties of the low density galactic halo. Superbubbles powered by star clusters, with properties similar to those inferred from the observations, slow down in the presence of an extended halo to expansion speeds smaller than the host galaxy escape velocity. Values of the critical luminosity required for the superbubbles to reach the edge of the galaxies with a speed comparable to the escape speed are derived analytically and numerically. The critical luminosities are larger than those in the detected sources and thus, the superbubbles in amorphous dwarf galaxies must have already undergone blowout and are presently evolving into an extended low density halo. This will inhibit the loss of the swept-up and processed matter from the galaxy.
1. Introduction

Massive stellar clusters violently eject their processed matter through supernova (SN) explosions and strong stellar winds, leading to the formation of large-scale remnants commonly known as superbubbles. These, when generated in dwarf galaxies, are thought to acquire a dimension that rapidly exceeds, at least along the galaxy minor axis, the scale height of the mass distribution. When this occurs, the remnants are believed to blow out of their host galaxy i.e. as the shell of swept up interstellar matter (ISM) accelerates outwards and fragments via Rayleigh-Taylor (R-T) instabilities, it allows the processed matter to freely stream as a wind into the intergalactic medium (IGM). Furthermore, the most energetic events are thought to cause not only the loss of their newly processed metals but to be able to strip dwarf galaxies of their entire ISM (see De Young & Heckman 1994 and references therein).

Here however, we show that despite the fact that remnants easily manage to blowout or burst out of the disk matter distribution, under a wide variety of circumstances the loss of matter could be inhibited by the hydrodynamic response of the rapidly streaming wind gas to the conditions found in the galaxy halo.

Analytical and numerical hydrodynamic studies of the remnants caused by a strong stellar energy input in a plane-stratified atmosphere have analyzed the blowout phenomenon (Kompaneets 1960; Tenorio-Tagle & Bodenheimer 1988; Mac Low et al. 1989; Igumenshehev, Tutukov & Shustov 1990; Tenorio-Tagle, Różyczka & Bodenheimer 1990). For a recent account see Koo & McKee (1992) who derived the characteristic wind luminosity \( L_b \) that determines whether a bubble remnant is able to blowout of the disk configuration. \( L_b \) is the luminosity required for the expanding shell of swept up matter to have a sonic velocity when it reaches a disk scale height. \( L_b = 2.4 \times 10^{36} P_4 H_{100}^2 a_{s,10} \) erg s\(^{-1}\), where \( P_4 = P_0/(10^4 k) \) cm\(^{-3}\)K, \( H_{100} \) is the disk scale height in units of 100 pc and \( a_{s,10} \) is
the disk sound speed in units of 10 km s$^{-1}$. Koo & McKee (1992) estimated also that if the cluster luminosity ($L_*$) exceeds $3L_b$ the bubble would expand supersonically when reaching a scale height and it will begin to accelerate to blowout of the disk into the halo of the galaxy. Once the bubble blows out, there are two possible end solutions depending on the wind luminosity. For $L_* >> L_b$ blowout would cause the venting of the hot bubble interior out of the planar atmosphere. Lower wind luminosities ($3 \leq L_* / L_b \leq 10$) result in the formation of stable hydrodynamical jets as the processed matter is re-shocked and accelerated to supersonic speeds through a nozzle generated by the shocked disk matter. In this particular case there is no doubt the processed matter is due to escape the galactic system. The more energetic events ($L_* >> L_b$) destroy upon expansion the possibility of a nozzle formation and the hot gas in such cases acquires a maximum speed similar to its own velocity of sound, as it streams through the fragmented shell into the halo. In dwarf galaxies, these remnants have recently been investigated by several groups. In particular, amorphous dwarf galaxies have been selected as ideal laboratories to study the impact of starbursts on the ISM, and to investigate the possible ejection and loss of their ISM and processed matter into the IGM. These galaxies are ideal laboratories because they have an ISM mass in the range $10^7 - 10^9 \, \text{M}_\odot$ and a concentrated central starburst, as opposed to Magellanic Irregulars where the star formation seems to extend over the whole body of the galaxies (Gallagher & Hunter 1987; Marlowe, Heckman, Wise & Schommer 1995 and references therein). Theoretically these issues have been address by De Young & Gallagher (1990) who estimated that about 2/3 of the processed matter ejected by supernova will escape the galaxy. This conclusion was reached from an analysis of 2-D numerical calculations in a 1 kpc$^2$ grid, after comparing the processed matter expansion speed with the escape velocity of their selected galaxy. In a further study, De Young & Heckman (1994), who considered dwarf galaxies with masses in the range $10^7 - 10^9 \, \text{M}_\odot$ and physical sizes of 1 - 3 kpc, found that the degree of flatness of the assumed density distribution is an important parameter.
This is because after shock acceleration and through the blowout phenomena, the energy of the starburst tends to escape along the galaxy minor axis imparting little momentum to the gas in the outer portions of the disk matter distributions. Thick disks however, were found to be completely disrupted and accelerated to speeds larger than the escape velocity of galaxies. Note however, that the approach of De Young & Heckman does not include the contribution of dark matter to the gravitational potential of the dwarf galaxy, nor the existense of an extended halo component. Note also that the range of energies used spans to up to $10^{44}$ erg s$^{-1}$, although the maximum assumed energy was never allowed to surpass the number of SN expected if the whole galaxy ISM ($M_{ISM}$) was turned into stars following a Salpeter IMF i.e. the maximum used number of supernovae $N_{SN} = M_{ISM}/100$. It is important to note that the best studied dwarf galaxies have an estimated mechanical energy deposition in the range of $10^{40} - 10^{41}$ erg s$^{-1}$ (Marlowe et al. 1995). This implies a SN rate of $3 \times (10^{-4} - 10^{-3})$ per year and a total expected energy injection of $1.3 \times (10^{55} - 10^{56})$ erg if the energy input rate is assumed to last for $4 \times 10^7$ yr, time after which all massive stars from a coeval starburst would have exploded as SN.

Unfortunately, there is little information about the gaseous structure of dwarf galaxies. However, in all well studied cases the size of the region photoionized by the cluster of massive stars, exceeds the size of the superbubbles present in these galaxies (see Marlowe et al. 1995; their tables 4 and 5). Furthermore, the few that have been looked at in HI (see for example Hoffman et al. 1993; Hunter, Van Worden and Gallagher, 1994; Meurer 1994) display large envelopes, several times larger than the optical size of the galaxies. One can obviously argue that it may just be a projection effect, however, the galaxies under consideration are of the "amorphous" type and thus it is rather unlikely that all observed matter is confined to a narrow disk. It thus seems perhaps more likely that the superbubbles observed in these galaxies (Marlowe et al., 1995; Martin 1996) which present a continuous horseshoe shape attached to the central star forming region and which have
dimensions $\sim 1$ kpc and expansion velocities $\sim 50$ km s$^{-1}$ are presently evolving into the extended halos of their host galaxies.

Here we present two dimensional calculations of multi-supernova remnants evolving in dwarf galaxies (see sections 2 and 3). We find a good agreement with the above mentioned studies although our conclusion (see sections 4 and 5) is that the final fate of the swept-up and processed matter is highly-dependent on the properties of the previously ignored galactic gaseous halos.

2. Dwarf Galaxies

2.1. The ambient gas density distribution

Massive galaxies ($M_{Gal} \geq 10^{10}$ M$_{\odot}$), such as the Milky Way, are capable of retaining a hot ($T_{Gal} \sim$ a few $10^6$K) and extended massive halo completely bound to their gravitational potential. This is frequently re-heated by a collection of superbubbles, caused by aging OB associations, that blowout of the plane HI disk density stratification. Less massive galaxies, on the other hand, are unable to keep such massive and hot halos. Instead, the smaller the galaxy total mass, the cooler and less extended their possible resultant halos become. Thus these are usually idealized as a low density turbulent medium (or clouds) that extends up to a maximum radius $R_G$ at which its characteristic velocity dispersion exceeds the galaxy escape speed. As stated by Suchkov et al. (1994, hereafter SBHL), ”in a real galaxy, much of the hydrostatic support of the interstellar matter is provided by the random motions of interstellar clouds”.

Following Li & Ikeuchi (1992), Tomisaka & Ikeuchi (1988, hereafter TI), Tomisaka & Bregman (1993, hereafter TB), and SBHL, we model our dwarf galaxies with three different
isothermal components.

\[ \rho_g = \rho_{NM} + \rho_{IM} + <\rho_{halo}> \quad (1) \]

where \( \rho_{NM}, \rho_{IM} \) and \( <\rho_{halo}> \) are the gas densities of the neutral and ionized components and the mean density of the extended turbulent halo material, respectively. The model corresponds to the bound-cool type halo in the notation of Li & Ikeuchi (1992). The total gas pressure is then given by

\[ P_{ext} = \frac{1}{\gamma} \rho_{NM} C_{NM}^2 + \frac{1}{\gamma} \rho_{IM} C_{IM}^2 + \frac{1}{3} <\rho_{halo}> C_{halo}^2, \quad (2) \]

where \( C_{NM}, C_{IM} \) and \( C_{halo} \) are the sound velocities of the neutral and ionized gas and the velocity dispersion of the extended halo component. \( \gamma \) is the ratio of specific heats.

Let \( R_B \) be the maximum extent of the stars and dark matter, and \( V_B \), the escape velocity at this surface. Following TI, and TB we assume that a fraction \((1 - e^2)\) of the radial component of gravity is balanced by pressure gradients. Then the initial gas density distribution follows from the relation:

\[ \frac{\rho_g}{\rho_0} = \alpha_{NM} \exp \left[ \frac{\gamma}{2} \left( \frac{V_B}{C_{NM}} \right)^2 \chi \right] + \alpha_{IM} \exp \left[ \frac{\gamma}{2} \left( \frac{V_B}{C_{IM}} \right)^2 \chi \right] + \alpha_{halo} \exp \left[ \frac{3}{2} \left( \frac{V_B}{C_{halo}} \right)^2 \chi \right], \quad (3) \]

where \( \rho_0 \) is the total gas density at the galactic center, \( \alpha_{NM} = \rho_{NM,0}/\rho_0 \), \( \alpha_{IM} = \rho_{IM,0}/\rho_0 \) and \( \alpha_{halo} = <\rho_{halo,0}>/\rho_0 \) are the density ratios of the various components to the total density at the galactic center. The function \( \chi \) equals

\[ \chi = F(\omega) - e^2 F(r) - (1 - e^2)F(0), \quad (4) \]

where \( r = \sqrt{x^2 + y^2} \) and \( \omega = \sqrt{x^2 + y^2 + z^2} \) are the cylindrical and spherical radii, and the function \( F \) is defined by equation (10). The rotation velocity of the ISM component comes from the relation:

\[ \frac{V^2}{r} = e^2 (g_x^2 + g_y^2)^{1/2}, \quad (5) \]

where \( g_x \) and \( g_y \) are the \( x \) and \( y \) components of the gravity field.
We include a transition from a rotating disk component to a non-rotating spherical halo and radial gradients in the disk density distribution by assuming

\[ e = e_0 \exp \left[ \left( \frac{z}{H_z} \right)^2 + \left( \frac{r}{H_r} \right)^2 \right], \tag{6} \]

where \( H_r \) and \( H_z \) are the characteristic scale heights along and perpendicular to the plane of the galaxy. Then (3) is not self-consistent if a hydrostatic gaseous disk is assumed, but it is asymptotically correct for a hydrostatic gaseous halo (TI, SBHL). Typical dwarf galaxy density distributions assuming a total mass \( M_B = 10^{10} M_\odot \) (models A and B) and \( 10^9 M_\odot \) (Models C) and a 10% gaseous component are shown in the Figure 1 (see also Table 1). Note that the last contour drawn represents, in each case, the gaseous edge of the galaxy \( R_G \), i.e. material further out with a velocity dispersion \( C_{\text{halo}} \) would exceed the escape velocity and thus it would not be bound to the system. Figure 2 gives a global impression of how the galactic ISM is distributed for different values of the assumed velocity dispersion of the extended component (also used to identify the models; see Table 1) and thus it shows how for larger values of \( C_{\text{halo}} \), the smaller, or more compact that the bound gaseous component results. The value of \( P_{\text{ext}}(R_G)/k \) varies in our models within a range 0.8 cm\(^{-3}\) K (Model B60) - 5.4\( \times \) 10\(^4\) cm\(^{-3}\) K (Model A120). This pressure should balance the extragalactic gas pressure and in fact covers the range often used in the literature (eg. Babul & Rees 1992).

Several calculations were also made for different ratios of \( M_{\text{ISM}}/M_B \), as well as for different dimensions of the dark matter distribution (see section 5).

2.2. The gravitational field

We have assumed that the main contribution to the gravitational field results from spherically symmetric stellar and dark matter components, disregarding the contribution of
a stellar disk component, and the self-gravity of the gaseous matter. A King model was
assumed for the background stars and a distribution inversely proportional to the square of
the radius has been adopted for a dark matter halo:
\[ \rho_{\text{tot}}(\omega) = \frac{\rho_1}{\left(1 + \left(\frac{\omega}{R_1}\right)^2\right)^{3/2}} + \frac{\rho_2}{1 + \left(\frac{\omega}{R_2}\right)^2}, \]  
(7)
where \( R_1 \) and \( R_2 \), \( \rho_1 \) and \( \rho_2 \) are characteristic scales and densities of the star and dark
matter components, respectively.

The total mass of these components may be expressed as
\[ M_B = 4\pi \int_0^{R_B} \rho_{\text{tot}}(\omega) \omega^2 \, d\omega = M_1 \left[ \ln (x_B + \sqrt{1 + x_B^2}) - \frac{x_B}{\sqrt{1 + x_B^2}} \right] + M_2 [y_B - \arctan (y_B)], \]  
(8)
where \( x_B = R_B/R_1 \), \( M_1 = 4\pi \rho_1 R_1^3 \), \( y_B = R_B/R_2 \), \( M_2 = 4\pi \rho_2 R_2^3 \). We fix a constant at
the gravity potential by assuming that it becomes Newtonian at the galactic boundary
\( R_B \). Then the gravity potential corresponding to the adopted total density distribution is
expressed as follows:
\[ \Phi(r, z) = -\frac{V_B^2}{2} F(\omega), \]  
(9)
where function \( F(\omega) \), is defined by the expression (see also SBHL)
\[ F(\omega) = 1 + \frac{R_B}{R_2} \frac{M_2}{M_{\text{tot}}} \left[ \frac{1}{2} \left( \ln (1 + y_B^2) - \ln (1 + y_\omega^2) \right) + \frac{\arctan (y_B)}{y_B} - \frac{\arctan (y_\omega)}{y_\omega} \right] + \]  
\[ \frac{R_B}{R_1} \frac{M_1}{M_{\text{tot}}} \left[ \frac{\ln (x_\omega + \sqrt{1 + x_\omega^2})}{x_\omega} - \frac{\ln (x_B + \sqrt{1 + x_B^2})}{x_B} \right], \quad \omega \leq R_B, \]  
(10)
\[ F(\omega) = \frac{R_B}{\omega}, \quad \omega > R_B, \]  
where \( x_\omega = \omega/R_1 \), \( y_\omega = \omega/R_2 \). The value of \( V_B \) is given by
\[ V_B = \sqrt{\frac{2GM_B}{R_B}}, \]  
(11)
whereas the escape velocity at the current position \((r, z)\) is

\[
V_{\text{esc}} = V_B \sqrt{F(\omega)}. \tag{12}
\]

The \(x, y, z\) components of gravity for \(\omega \leq R_B\) derived from (9) and (10) are

\[
g_x = -\frac{GM_1 x}{\omega^2} \frac{x_\omega}{\omega} \left[ \ln \left( x_\omega + \sqrt{1 + x_\omega^2} \right) - \frac{x_\omega}{\sqrt{1 + x_\omega^2}} + \frac{M_2}{M_1} (y_\omega - \arctan y_\omega) \right], \tag{13}
\]

\[
g_y = -\frac{GM_1 y}{\omega^2} \frac{x_\omega}{\omega} \left[ \ln \left( x_\omega + \sqrt{1 + x_\omega^2} \right) - \frac{x_\omega}{\sqrt{1 + x_\omega^2}} + \frac{M_2}{M_1} (y_\omega - \arctan y_\omega) \right], \tag{14}
\]

\[
g_z = -\frac{GM_1 z}{\omega^2} \frac{x_\omega}{\omega} \left[ \ln \left( x_\omega + \sqrt{1 + x_\omega^2} \right) - \frac{x_\omega}{\sqrt{1 + x_\omega^2}} + \frac{M_2}{M_1} (y_\omega - \arctan y_\omega) \right]. \tag{15}
\]

For larger distances \(\omega > R_B\) they are simply

\[
g_x = -\frac{GM_B x}{\omega^2} \frac{x_\omega}{\omega}, \tag{16}
\]

\[
g_y = -\frac{GM_B y}{\omega^2} \frac{x_\omega}{\omega}, \tag{17}
\]

\[
g_z = -\frac{GM_B z}{\omega^2} \frac{x_\omega}{\omega}. \tag{18}
\]

3. The Calculations

Here we present two-dimensional calculations of the evolution of the remnants generated by multi-supernova explosions from an aging massive stellar cluster evolving in a low-mass galaxy \((M_B \leq 10^{10} M_\odot)\). The remnant evolution is calculated by means of the thin layer approximation (Kompaneets 1960) a method developed in 2D and 3D by Bisnovatyi-Kogan & Blinnikov (1982); Mac Low & McCray (1988); Bisnovatyi-Kogan & Silich (1991); Palouš (1992); Silich (1992) and Silich et al. (1996), and now applied to a variety of cases (Bisnovatyi-Kogan & Silich 1995). The evolution is followed up to 60 -
170 Myr, long past the time at which the SN activity from the evolved - assumed coeval - massive cluster has come to an end.

3.1. The energy input

Based on the observations of dwarf galaxies (see Marlowe et al. 1995 and references therein) and the synthetic properties of starburst galaxies derived by Leitherer & Heckman (1995), the energy input rates for the calculations were set within the range $L_\ast = 10^{40} - 10^{41}$ erg s$^{-1}$, although in section 5 much more energetic clusters are considered. The assumed constant energy input rate $L_\ast$ depends on the total energy released by massive stars ($E_{\text{burst}}$) and the duration of the supernova phase ($t_{\text{burst}}$) and can be written as

$$L_{38} = 3.17 \times 10^3 \frac{E_{\text{burst}}/10^{56}\text{ergs}}{t_{\text{burst}}/10^7\text{year}}.$$  \hspace{1cm} (19)

Here the energy supply rate is $L_{38} = L_\ast / 10^{38}$ erg s$^{-1}$. We have also assumed that massive stars release an average $m_{SN} = 10M_\odot$ during their explosion, and that this is dispersed into the hot superbubble interior. Thus the rate of mass ejecta due to SN explosions, $\dot{M}_{SN}$, is assumed to be

$$\dot{M}_{SN} = N_{SN}m_{SN} = \frac{L_\ast m_{SN}}{E_{SN}},$$  \hspace{1cm} (20)

where $E_{SN}$ is the energy released by each SN explosion (typically $E_{SN} = 10^{51}\text{erg}$).

3.2. Initial conditions

All the calculations start with a small adiabatic spherical bubble of initial radius $R_e$ at the time

$$t_0 = \left[\frac{2\pi(9\gamma + 5)}{75(\gamma - 1)} \frac{\rho_g(x_0, y_0, z_0) R_e^5}{L_\ast}\right]^{1/3}.$$  \hspace{1cm} (21)
The initial expansion velocity, thermal energy, and mass within the bubble are

\[ u_0 = 0.6 \frac{R_e}{t_0}, \]
\[ E_{th}(t_0) = \frac{14}{9 \gamma + 5} L_s t_0, \]
\[ M_{in}(t_0) = \dot{M}_{SN} t_0. \]

Table 1 provides the main parameters of the simulated galaxies, as well as the energetics of their assumed bursts of star formation. Several parameters were unchanged in the calculations e.g. the metallicity of the galactic gas has been assumed to be \( \xi = Z/Z_\odot = 0.3 \); the duration of the supernova phase \( t_{\text{burst}} = 40 \) Myr; the constant \( e_0 = 0.9 \) in (eq 6). The parameter \( n_{NM,0} = 20 \, \text{cm}^{-3} \), (see (3)) the characteristic scale heights for the gas density distribution have been taken as \( H_z = 1 \, \text{kpc}, H_r = 0.5 R_B \), whereas the characteristic scale height for the King’s mass distribution was taken to be \( R_1 = 0.75 \, \text{kpc} \), and the parameter \( R_2 \) in equation (7) has been kept as half the truncation radius \( R_B \). The temperature of the ionized component was taken as \( 2 \times 10^4 \, \text{K} \) throughout the calculations. To satisfy the high pressure condition at the starburst region we also set the neutral gas temperature equals to \( 6 \times 10^3 \, \text{K} \). We relaxed the high pressure constraint for the low mass galaxies (models C), for which a temperature for the neutral gas of 100 K was assumed.

The method of solution and the procedure to calculate various remnant properties such as the bolometric and X-ray luminosities are given in Appendices A and B.

4. A typical evolutionary sequence

Figure 3 shows the evolutionary sequence of models A100 (see Table 1) for which the initial ISM density follows the distribution shown in Figure 1b with a central density value of 20.2 cm\(^{-3}\) and a halo velocity dispersion of 100 km s\(^{-1}\). The central starburst was
assumed to deposit $7.9 \times 10^{40}$ erg s$^{-1}$ during the first 40 Myr of evolution. As expected, the remnant grows at first elongated in the direction perpendicular to the disk of the galaxy and blows out into the halo. Soon however, as a result of blowout a secondary shell of newly swept up halo gas forms. This results from the stream of the high pressure processed matter that bursts out of the superbubble interior while causing a symmetrical and larger twin superbubble (see Figure 3a). This immediately begins to wrap around and ends up surrounding the inner densest part of the disk as it grows to large dimensions into the halo. Note that more elongated structures result if steeper density distributions are assumed (see Figure 3b). At later times, the leading shock manages to plough through the outer less dense sectors of the central disk and merges with its symmetrical counterpart to form a continuous structure and a more spherical single giant superbubble. During the process of shock merging, a considerable amount of the shell mass is left behind strangling a dense toroid carved in the initial density distribution. All of this matter stops then participating in the general outward motion and is instead strongly compressed towards the galaxy plane to remain completely engulfed by the almost spherical giant superbubble.

Figure 4 shows the total amount of matter swept by the remnant as a function of time. This steadily grows to values of several times $\sim 10^8 M_\odot$ with most of the material being collected as the remnant evolves into the halo. The sharp decrease shown in Figure 4a at about $25 \times 10^6$ yr is due to the process of outer shock merging above described. Figure 4 also shows the total amount of matter evaporated from the outer shell into the remnant interior, and the amount of material injected by the collection of SN explosions. In all cases, the evaporated mass amounts to only a few percent of the swept up mass but it constitutes the most significant mass input into the superbubble interior; at least an order of magnitude larger than that provided by the sequential SNe.

Meanwhile, the hot bubble interior, as well as the shell of swept up matter, cause the
X-ray emission from the nebulae. Figure 4e displays the total X-ray emission (see Appendix B), and the shell contributions to this luminosity. The bolometric bubble luminosity is shown in Figure 4d. In the conditions of the A100 model the remnant evolves mainly in the radiative mode and thus the shell contribution to the X-ray emission is negligible compared to the X-ray emission from the hot superbubble interior.

5. The fate of the processed matter

The central issue however, is whether or not one can estimate without uncertainties the possibility of mass ejection from the galaxy. For this, the obvious and clear cut procedure is to compare the fastest expansion speed of the remnant (say along the symmetry axis) with the galaxy escape velocity at different radii, or evolutionary times. The maximum speed \( u_{\text{exp}} \) of the sector of the remnant evolving into the steepest density gradient shows at first a continuous deceleration (see Figure 5), for \( R \)'s less than 1 kpc. Then during blowout into the galaxy halo, it suffers a sudden increase towards large speeds surpassing even the galaxy escape velocity values (dashed lines in Figure 5). The dynamic halo however, makes eventually an impact on the evolving superbubble forcing it to slow down again, once a significant amount of halo material has been incorporated into the remnant shell. These trends are shown in Figure 5 where the largest expansion remnant speeds (solid lines) for all A and B cases (see Table 1, and Figures 1 and 2) are compared with the galaxy escape velocity. Note that the maximum expansion speed after blowout into the halo, as well as the slowest deceleration occurs for the more spread out distributions of matter or less compact galaxies considered; which are those with the smallest \( C_{\text{halo}} \) values. All superbubbles attain large expansion speeds, even larger than \( V_{\text{esc}} \), however, they all end up with a velocity well below the galaxy escape speed.

An important point to notice here is that just as old supernova remnants are
disrupted once their expansion speed drops below the ISM random speed of motions, once a superbubble expansion becomes slower than the assumed velocity dispersion of the host galaxy halo, it should begin to be disrupted and eventually lose its identity. Giant, kpc-scale, filaments associated with the host galaxy, may be the only long lasting trace of the energetics from old stellar generations and their disrupted giant remnants. Thus important during and after disruption is to decide how much further can the hot superbubble interior expand. Note that its expansion would lead to further cooling and this to a further drop in pressure. The final fate, i.e. whether it will be lost or trapped by the galaxy, can be addressed by comparing its pressure ($P_{in}$) to the host galaxy outer boundary pressure $P_G = P_{ext}(R_G)$. With this aim in mind, the calculations were continued for times that largely exceeded the onset of shell disruption (i.e. the time when the expansion speed becomes smaller than the random speed of motions in the halo), and which provided us with a handling on the run of $P_{in}$. This, as shown in Figure 6, in most cases becomes smaller than $P_G$ with the implication that the processed matter will remain bound to the system. For the case A60, although $P_{in}$ exceeds $P_G$, it does not exceed the local pressure value at the shell stall radius (dotted line in Figure 6), and this leads to the contraction of the superbubble and the retention of the processed material.

Table 1 indicates several other calculated cases for which we assumed for example a different proportion of dark matter with respect to $M_{ISM}$ (D and E cases), or runs without thermal evaporation (A*100), or with a lower total mass (C cases). The results are nevertheless very similar to the above described cases.

Clearly, the expulsion of matter from a dwarf galaxy will only be possible if one could freely increase the energy input rate ($L$). The problem however, is to know which is the correct or sufficient amount for each of the galaxies. A good formalism could be device if one notices from Figures 1 and 2 that the extended low density halos are in fact rather
uniformly spread out and that they constitute a large fraction of the galaxies ISM. Thus, given the continuous energy input rate (at least for the first 40 Myr) one would expect that the relationships developed for the evolution of stellar wind bubbles in a uniform density medium (see Weaver et al. 1977) will give reasonable estimates. The functions $R(t)$ and $dR/dt$ lead to the time independent expression

$$ R = 16.6 \left( \frac{L_{38}}{n_0} \right)^{1/2} \left( \frac{u}{kms^{-1}} \right)^{-3/2} \text{kpc}. \quad (25) $$

¿From this we would like to know the amount of energy required for a superbubble to reach the outer radius of a galaxy with a speed comparable to the escape velocity of the system. We shall call this the critical luminosity ($L_{\text{crit}}$). Thus, if one sets $R = R_G$, $u = V_{\text{esc}}$ and the mean number density $n_0 \sim < n_{\text{halo}} > = fM_{\text{ISM}}/\frac{4\pi}{3}R_G^3 \mu_H$; where f is the fraction of the galaxy ISM swept during the evolution of the superbubble and $\mu_H$ is the mean mass per particle, it leads to:

$$ L_{\text{crit,38}} = 2.75 \times 10^{-2} \left( \frac{fM_{\text{ISM}}}{10^9 M_\odot} \right) \left( \frac{V_{\text{esc}}}{kms^{-1}} \right)^3 \left( \frac{R_G}{\text{kpc}} \right)^{-1}. \quad (26) $$

where $L_{\text{crit,38}} = L_{\text{crit}}/10^{38}$ erg.

A similar expression can also be found for late evolutionary times ($t > t_{\text{burst}}$) for which an instantaneous energy release may seem a better approximation. Thus, from Chevalier (1974) and Blinnikov et al. (1982) one obtains:

$$ R = 22 \left( \frac{\epsilon E_{50}}{n_0} \right)^{5/21} t_5^{2/7} \text{pc}, \quad (27) $$

where $E_{50}$ is the total amount of energy deposited by the star cluster in units of $10^{50}$ erg, and $t_5$ is the evolutionary time in $10^5$ years. For the fraction of the total input energy which is transformed into the remnant thermal energy $\epsilon = 5/11$ (Weaver et al. 1977). Relation (27) leads to the critical mechanical luminosity $L_{\text{crit}} = E_{\text{burst}}/t_{\text{burst}}$:

$$ L_{\text{crit,38}} = 8.89 \left( \frac{fM_{\text{ISM}}}{10^9 M_\odot} \right) \left( \frac{V_{\text{esc}}}{kms^{-1}} \right)^{6/5}. \quad (28) $$
Both relationships when the fraction of the swept up interstellar mass $f$ was set to 0.7 are displayed in Figure 7a,b for galaxies with an $M_{ISM} = 10^9 M_\odot$ and $10^8 M_\odot$, respectively.

Figure 7a (solid line) shows the $L_{crit}$ trend for $R_B = 5$ kpc and the dashed line for $R_B = 10$ kpc (cases A and B). The critical luminosity, although by definition it is sufficient to cause the ejection of processed matter out of the galaxy, it may only do so, depending on the host galaxy mass distribution, after a long evolutionary time. In some cases this can be larger than $t_{burst}$ and in fact much larger than the expected HII region lifetime ($\sim 10^7$ yr). Several of our numerical calculations in search of $L_{crit}$ are also shown in Figure 7. Diamonds in Figure 7a represent the numerical results for galaxies with $R_B = 5$ kpc, and triangles for $R_B = 10$ kpc models. In all cases the time required for the superbubble to reach the edge of the galaxy are indicated.

There is a better than 30% agreement between the derived formulae and the numerical results. For superbubbles that require a time smaller or comparable to the starburst time $t_{burst}$ to reach the galaxy edge, our numerical estimates agree with relation (26), and for those that take a time longer than $t_{burst}$ equation (28) leads to an asymptotic value of the critical energy input rate $L_{crit}$.

An extended halo can therefore inhibit the loss of the swept-up and processed matter from a galaxy even if the remnants undergo blowout from the dense central gaseous component into the halo. Note also that the more compact ISM distributions demand more powerful stellar clusters (a larger $L_{crit}$) to allow their superbubbles to reach the outskirts of galaxies and eject their metals into the IGM. One can therefore conclude that the presence of an extended halo may dominate the evolution of the remnants, and that the final fate of the swept-up galactic gas and of the matter processed by the starburst, basically depend on the properties of the low density galactic halo rather than on the parameters of the gaseous central disk. Important is also the extent of the dark matter component: the more compact
this is, the larger the energy required to reach the critical luminosity.

Note also that all calculations with an energy input rate \( \leq 10^{41} \text{ erg s}^{-1} \) and a halo velocity dispersion \( C_{\text{halo}} \geq 60 \text{ km s}^{-1} \), as inferred from the observations of amorphous dwarf galaxies (Marlowe et al. 1995, Martin 1996, etc.), all lie below the curve of critical luminosity, and thus have led to superbubbles expanding with speeds smaller than the random speeds of motions of their host galaxy halos, implying their disruption long before they reach the edges of their galaxies, and thus have led to no loss of their processed matter. Such galaxies in order to lose their metals into the intergalactic medium, require of energetic starburst, more energetic than what they presently have. One could in this respect be tempted to think of a larger energy, larger than observed, being deposited in an extended dwarf galaxy and thus more easily surpass the critical luminosity value. However, the time required for such superbubbles to reach the galaxy edge and eject their metals into the IGM would exceed by large factors the HII region lifetime and thus most probably be undetectable events.

6. Conclusions

Based on the scarce but definite observations of amorphous dwarf galaxies, which show that the HI distribution outgrows by large factors the optical size of these galaxies, we have constructed realistic models which account for an extended cool halo component bound to the system. Models of amorphous galaxies with an \( M_{\text{ISM}} \sim 10^9 M_\odot \) and an energy input up to \( 10^{41} \text{ erg s}^{-1} \), as inferred from the observations, lead to blowout from the disk matter stratification, allowing the superbubbles to vent their hot matter into the galaxy halo. The outflowing gas immediately searches and follows the steepest density gradients affecting a large almost spherical volume. We have shown here that in the case of small systems, superbubbles wrap around and envelope the densest central region of the host galaxy. Such
remnants are strongly decelerated as they evolve into the halo, sufficiently to remain bound to their host galaxy.

Our calculations confirm the results of De Young and Gallagher (1990), and De Young and Heckman (1994) during the early superbubble evolution. However, our ultimate conclusions are different as the final fate of the remnants basically depends on the properties of the galactic halo and on the total energy of the starburst. The presence of the dark matter component also enhances the ultimate retention of the processed material. Under these circumstances total retention (i.e. expansion speeds $\leq$ escape velocity) of the remnants and thus of the matter processed by the exploding massive stars is unavoidable for galaxies with an ISM mass $\geq$ than a few $10^9 \, M_\odot$ and an energy input rate of up to $10^{41} \, \text{erg s}^{-1}$.

We have also derived analytical expressions for the critical luminosity required for a superbubble to be able to reach the outskirts of a galaxy and expulse the processed matter into IGM. The critical limit has been cross-checked with several numerical calculations for a large range of galaxy masses, dimensions and thus surface escape velocities.

¿From the calculations, we then conclude that the thoroughly studied dwarf galaxies with a gaseous mass $\sim 10^9 \, M_\odot$ and presenting giant remnants produced by the mechanical energy ($L \leq 10^{41} \, \text{erg s}^{-1}$) from massive stars (see Marlowe et al., 1995; Martin 1996; and Meurer 1994) are most likely to retain their ISM and their processed matter. The observed remnants present dimensions smaller than a few kpc and expansion speeds $\sim 50 \, \text{km s}^{-1}$ and thus based on our calculations, we predict that they have already undergone blowout from their disk matter distribution and that presently are evolving into the extended low density halo of their parent galaxy. Our results also predict that these superbubbles will be ultimately dispersed by the turbulent motions in the extended galactic halo.

The consequences of these conclusions are important in a variety of fields and we shall analyse these in a forthcoming communication. Clearly, the kind of dwarf galaxies that
we have analysed, are not responsible for the contamination of the IGM. Dwarf galaxies with an ISM mass of the order of $10^9 \, M_\odot$ will retain their metals unless they undergo an overwhelming burst of star formation much larger than those presently observed. We thus have to understand why do they present low metallicities. The secret must probably lies on the physics of mixing (see Tenorio-Tagle 1996).

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Appendix A: Main equations

Following TI, Li & Ikeuchi (1992), TB, SBHL our approach includes three co-existing ISM phases: a neutral, an ionized component, and a low density extended halo characterized by a turbulent velocity; also thought by other authors as a collection of low densities clouds with a velocity dispersion $C_{\text{halo}}$.

**Mass conservation.** The expanding shell caused by the star cluster energy deposition accumulates interstellar mass according to

\[
\frac{d\mu_L}{dt} = \frac{d\mu_g}{dt} - \frac{d\mu_{RT}}{dt} - \frac{d\mu_{\text{cusp}}}{dt},
\]  

(A1)

where the first term represents the mass growth due to swept-up interstellar gas, the second one the loss of mass resultant from the development of Rayleigh-Taylor (RT) instabilities after the shell blows out of the galaxy disk, and the third one describes possible shell self intersections following cusp formation (see below). We have assumed that the swept turbulent halo sticks to the shell. Then for any Lagrangian element with a surface area $d\Sigma$ and a unit vector normal to the shell $n_i$, the terms in equation (A1) take the form:

\[
\frac{d\mu_g}{dt} = \frac{d\mu_{ic}}{dt} + \frac{d < \mu_{\text{halo}} >}{dt}
\]  

(A2)

\[
\frac{d\mu_{ic}}{dt} = \begin{cases} 
(\rho_{NM} + \rho_{IM})(u_i - v) \cdot n_i d\Sigma, & u_n \geq a_s \\
0, & u_n < a_s
\end{cases}
\]  

(A3)

\[
\frac{d < \mu_{\text{halo}} >}{dt} = < \rho_{\text{halo}} > (u_i - v_{\text{halo}}) n_i d\Sigma,
\]  

(A4)
An effective sound velocity $a_s = (A_n C_{NM}^2 + A_i C_{IM}^2)^{1/2}$, where $A_n = \rho_{NM}/(\rho_{NM} + \rho_{IM})$, $A_i = \rho_{IM}/(\rho_{NM} + \rho_{IM})$, $a_{sh}$ is the normal component of the shell acceleration. $t_{cusp}$ is the time of cusp formation, $t_{RT}$ marks the time for the beginning of the acceleration phase, $\mu_L(t = t_{RT})$ is the mass of the Lagrangian mesh at the beginning of blow-out, and $\tau_{RT}$ is the characteristic growth time for RT instabilities

$$\tau_{RT} = \left( \frac{\lambda}{2\pi a_{sh}} \right)^{1/2}$$

where $\lambda$ is the same order of magnitude as the characteristic scale for the ambient gas density distribution, $\lambda \approx 2H_z$ (Koo & McKee, 1990). For simplicity we set here $\lambda$ equal to the characteristic scale for the gas density distribution at the galactic center $\lambda = H_{z0}$.

Momentum conservation. We consider four effects leading to changes in the shell momentum: the pressure difference between the inside gas and the surrounding medium $\Delta P$, the galactic gravitational force $g$, the accumulation of the ambient gas momentum, and the momentum loss during the RT break-out. We assume that RT clumps separate from the shell with the same speed as the current shell expansion velocities. Also that intercloud mass accumulation stops wherever the expansion velocity decreases below the sound speed $a_s$, and thus the interaction with the ambient intercloud gas modifies only the normal component of the momentum but the shell does not accumulate mass any further.

With these simplifications, the equation of motion for any Lagrangian element with mass $\mu_L$ is (see also Ostriker & McKee 1988, Silich et al. 1996):

$$\frac{d}{dt}(\mu_L \mathbf{u}) = \Delta P n d\Sigma + \mathbf{v} \frac{d\mu_c}{dt} + \mathbf{v}_{halo} \frac{d\mu_{halo}}{dt} - \mathbf{u} \frac{d\mu_{RT}}{dt} +$$
Then the equation for the shell expansion velocity can be written as
\[
\frac{\text{d}u}{\text{d}t} = \mu^{-1}_L \Delta P n d\Sigma + (v - u) \mu^{-1}_L \frac{\text{d}\mu_{ic}}{\text{d}t} + (\mathbf{v}_{CM} - u) \mu^{-1}_L \frac{\text{d}\mu_{halo}}{\text{d}t} + \\
+ g - \rho_{ic}(x, y, z) \mu^{-1}_L [(u - v) \cdot \mathbf{n}]^2 n d\Sigma,
\]
(A9)
\[
\frac{\text{d}r_i}{\text{d}t} = u
\]
(A10)

In the current calculations we have assumed that the intercloud gas and halo clouds move with the same regular velocities \((v = v_{halo})\).

**The energy balance.** The total energy balance for a remnant caused by the strong energy input from the central star cluster reads as:
\[
E_{SN} + E_{k,g} + E_{th,g} = E_{th} + E_{k,sh} + E_{th,sh} + E_g + E_{r,in} + E_{r,sh} + E_{k,RT},
\]
(A11)
where the left-hand terms are the energy deposited by massive stars \((E_{SN})\), and the initial kinetic and thermal energies of the accumulated ISM \((E_{k,g} \text{ and } E_{th,g})\). The right-hand terms are the bubble interior thermal energy \((E_{th})\), the kinetic and the thermal energies of the shell \((E_{k,sh} \text{ and } E_{th,sh})\), the shell gravitational energy \((E_g)\), the energy losses from the hot bubble interior \((E_{r,in})\) and from the shell \((E_{r,sh})\), and the kinetic energy of RT clumps \((E_{k,RT})\). The derivative of the bubble thermal energy could be expressed as follows
\[
\frac{\text{d}E_{th}}{\text{d}t} = L_{\text{burst}} - L_{in} - P_{in} \frac{\text{d}\Omega}{\text{d}t},
\]
(A12)
where
\[
\frac{\text{d}\Omega}{\text{d}t} = \sum_{i=1}^{N} u_n d\Sigma,
\]
(A13)
\(L_{in}\) is the energy loss from the hot bubble interior, and \(u_n\) is the normal component of the expansion velocity. As the main contribution for the interior bubble cooling comes from the
outer dense layers, we approximate \( L_{in} \) as

\[
L_{in} = L_R \frac{\Sigma^R}{\Sigma} = \varepsilon_2 L_R,
\]

where \( \Sigma \) is the total surface of the remnant, and the index \( R \) denotes the radiative part of the shell, \( \varepsilon_2 = \Sigma^R/\Sigma \). \( L_R \) is the energy loss from the bubble interior if all the remnant becomes radiative (for the calculations of \( L_R \) see Silich et al. 1996).

One can then calculate the bolometric shell luminosity from the total energy balance:

\[
L_{sh} = P_{in} \frac{d\Omega}{dt} + \frac{dE_{k,g}}{dt} + \frac{dE_{th,g}}{dt} - \frac{dE_{th,sh}}{dt} - \left( \frac{dE_{k,sh}}{dt} + \frac{dE_{k,RT}}{dt} \right) - \frac{dE_g}{dt},
\]

where

\[
\frac{dE_{k,sh}}{dt} = \frac{1}{2} \sum_{i=1}^{N} \left( \dot{\mu}_L u^2 + 2\mu_L \frac{du}{dt} \right)
\]

\[
\frac{dE_{k,g}}{dt} = \begin{cases} \frac{1}{2} \sum_{i=1}^{N} (\dot{\mu}_{ic} + \dot{\mu}_{halo}) v^2, & u_n \geq a_s \\ \frac{1}{2} \sum_{i=1}^{N} (\dot{\mu}_{ic}(V_n^2 - u_n^2) + \dot{\mu}_{halo} u^2), & u_n < a_s \end{cases}
\]

\[
\frac{dE_{th,g}}{dt} = \sum_{i=1}^{N} \frac{\dot{\mu}_g}{\gamma(\gamma - 1)} \left[ \frac{\rho_{NM}}{\rho_g} C_{NM}^2 + \frac{\rho_{IM}}{\rho_g} C_{IM}^2 + \frac{\gamma(\gamma - 1)}{2} \frac{\rho_{halo}}{\rho_g} C_{halo}^2 \right]
\]

\[
\frac{dE_g}{dt} = -\sum_{i=1}^{N} \mu_L u g
\]

\[
\frac{dE_{k,RT}}{dt} = \frac{1}{2} \sum_{i=1}^{N_{RT}} \dot{\mu}_{RT} u^2,
\]

(A15) gives a good estimate of the radiative shell luminosity if one sets

\[
\frac{dE_{th,sh}}{dt} = 0.
\]

However, in the thin layer approximation there is no satisfactory procedure for the calculation of an adiabatic shell thermal energy. Therefore for quasi-adiabatic shell sectors
we use:

\[ L_{sh}^{A} = \xi \sum_{i=1}^{N_A} n_{sh,a}^2 \Lambda_t(T_{sh,a}) \Delta R d\Sigma^A, \]  

(A22)

where \( \xi \) is the galactic gas metallicity, \( \Lambda_t(T) \) is cooling function, \( n_{sh,a} \) and \( T_{sh,a} \) are the mean atomic particle number density and temperature within the adiabatic part of the shell, \( \Delta R = 0.14 R_s \) is the quasi-adiabatic shell thickness, \( d\Sigma^A \) is the surface area of the adiabatic Lagrangian mesh, and \( N_A \) is the number of the "adiabatic" Lagrangian meshpoints.

\[ n_{sh,a} = \frac{\mu_L}{\mu_{in} \Delta R d\Sigma^a} \]  

(A23)

\[ T_{sh,a} = \frac{1}{k n_{sh,a}} \]  

(A24)

The total shell luminosity then follows from the expression

\[ L_{sh} = L_{sh}^a + L_{sh}^R, \]  

(A25)

whereas the bolometric bubble luminosity is

\[ L_{bol} = L_{in} + L_{sh}, \]  

(A26)
Appendix B: The inner bubble structure

Our approximate description of the bubble structure is based on the similarity solution of Weaver et al. (1977). We assume that the density and temperature distribution can be approximated by:

\[ n = n_c(1 - x_i)^{-\lambda}, \quad (B1) \]
\[ T = T_c(1 - x_i)^{\lambda}, \quad (B2) \]

where \( x_i = r/R_i \) is the dimensionless distance from the cavity center to the particular Lagrangian element \( i \), and \( \lambda = 2/5 \). To determine \( n_c \) (and \( T_c \)) we assume that the total mass interior to superbubble at all times is:

\[ M_{\text{tot}} = M_{\text{SN}} + M_{\text{ev}}, \quad (B3) \]

where

\[ M_{\text{SN}} = \int_0^{t_{\text{burst}}} \dot{M}_{\text{SN}} dt, \quad (B4) \]
\[ M_{\text{ev}} = \int_0^t \dot{M}_{\text{ev}} dt. \quad (B5) \]

The value of \( \dot{M}_{\text{SN}} \) is defined by (20), and the shell thermal evaporation is (Mac Low & McCray, 1988):

\[ \dot{M}_{\text{ev}} = \begin{cases} \frac{4}{35} \mu m u C T_c^{5/2} \sum_{i=1}^{N_R} \frac{\delta E_i}{R_i}, & \text{radiative phase}, \\ 0, & \text{adiabatic phase}, \end{cases} \quad (B6) \]

where \( k \) is the Boltzmann’s constant, \( C = 6 \times 10^{-7} \text{ erg s}^{-1} \text{ cm}^{-1} \text{ K}^{-2/7} \) is the constant thermal conductivity coefficient for a fully ionized hydrogen plasma (Cowie & McKee 1977).

Two approximations are then used. We assume that a portion of the total mass \( (M_1) \) is enclosed within the radius of the contact discontinuity \( (R_{CD} = \varepsilon_1 R_s) \)

\[ M_1 = \mu_m n_c \int_0^{2\pi} d\phi \int_0^\pi \sin \theta d\theta \int_0^{R_{CD}} (1 - r/R_s)^{-\lambda} r^2 dr = 3\mu_m n_c I_{m1}(\lambda)\Omega, \quad (B7) \]
\[ I_{m1}(\lambda) = \int_{0}^{\varepsilon_1} (1 - x)^{-\lambda} x^2 dx = \frac{1}{1 - \lambda} \left[ 1 - (1 - \varepsilon_1)^{1-\lambda} \right] \]
\[ + \frac{1}{3 - \lambda} \left[ 1 - (1 - \varepsilon_1)^{3-\lambda} \right] - \frac{2}{2 - \lambda} \left[ 1 - (1 - \varepsilon_1)^{2-\lambda} \right]. \]

\( M_1 = M_{tot} \) during the adiabatic phase but as sectors of the shock become radiative, \( M_{tot} \) fills a larger volume. In such case, the fraction of \( M_{tot} \) that now reaches up to \( R_s \), assuming a collapsed thin shell with radius equal to that of the shock, is simply

\[ M_2 = \varepsilon_2 (M - M_1), \]

(\( B9 \))

\[ M = 3\mu_{in} n_c I_m(\lambda) \Omega, \]

(\( B10 \))

\[ I_m(\lambda) = \int_{0}^{1} (1 - x)^{-\lambda} x^2 dx = \frac{1}{1 - \lambda} + \frac{1}{3 - \lambda} - \frac{2}{2 - \lambda}. \]

(\( B11 \))

The mass distribution of the (adiabatic-radiative) remnant interior can be defined as

\[ M_1 + M_2 = \varepsilon_2 M + (1 - \varepsilon_2) M_1 = M_{tot}. \]

(\( B12 \))

Equations (B3 - B12) define the central superbubble density and temperature:

\[ n_c = \frac{1}{3\mu_{in}[\varepsilon_2 I_m(\lambda) + (1 - \varepsilon_2) I_{m1}(\lambda)]} \frac{M_{tot}}{\Omega}, \]

(\( B13 \))

\[ T_c = \frac{3\mu_{in}[\varepsilon_2 I_m(\lambda) + (1 - \varepsilon_2) I_{m1}(\lambda)] P_{in}}{k} \frac{\Omega}{M_{tot}}. \]

(\( B14 \))

6.0.1. X-ray luminosity

The total bubble X-ray luminosity includes the X-ray emission from the hot bubble interior and the X-ray emission of the swept-up interstellar gas. We conventionally divide the X-ray emission from the bubble interior into an emission \( L_{x1} \), from the matter within the radius \( R_{CD} \), and that arising from the gas between the ”adiabatic CD” surface and the X-ray cut-off surface, \( L_{x2} \). The first part is defined by the equation

\[ L_{x1} = \int_{0}^{2\pi} d\phi \int_{0}^{\pi} \sin \theta d\theta \int_{0}^{\varepsilon_1 R_s} \xi n_a^2(r) \Lambda_x r^2 dr = 3\xi \lambda^{-1} n_{ac}^2 \Omega I_{x1} \]

(\( B15 \))
where

\[ I_{x1} = \frac{1}{T_c} \int_{T_{CD}}^{T_c} \Lambda_x(T) \left( \frac{T}{T_c} \right)^{\frac{1-2\lambda}{\lambda}} \left[ 1 - \left( \frac{T}{T_c} \right)^{\frac{1}{\lambda}} \right]^2 dT \]  

(B16)

The second part we approximate by the value

\[ L_{x2} = (L_{xr} - L_{x1}) \varepsilon_2, \]  

(B17)

where \( L_{xr} \) is the X-ray emission from the interior of the fully radiative bubble (see Silich et al. 1996):

\[ L_{xr} = \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta \int_0^{R_s} \xi n_r^2(r) \Lambda_x r^2 dr = 3 \xi \lambda^{-1} n_{ac}^2 \Omega I_{xr}, \]  

(B18)

\[ I_{xr} = \frac{1}{T_c} \int_{T_{x, cut}}^{T_c} \Lambda_x(T) \left( \frac{T}{T_c} \right)^{\frac{1-2\lambda}{\lambda}} \left[ 1 - \left( \frac{T}{T_c} \right)^{\frac{1}{\lambda}} \right]^2 dT. \]  

(B19)

Here \( T_{CD} \) is the temperature at the ”adiabatic CD” surface, and \( T_{x, cut} \approx 5 \times 10^5 K \) is the X-ray cut-off temperature (see Chu & Mac Low, 1990; also SBHL). The X-ray luminosity of the hot bubble interior is then

\[ L_{x, in} = \varepsilon_2 L_{xr} + (1 - \varepsilon_2) L_{x1}. \]  

(B20)

The X-ray luminosity from the shell includes both the X-ray emission from the quasi-adiabatic sectors of the shell \( L_{x, sh}^a \) and the X-ray emission from the outer layers of the radiative shell \( L_{x, sh}^R \). We approximate \( L_{x, sh}^a \) by the expression for the homogeneous shell luminosity

\[ L_{x, sh}^A = \xi \sum n_{sh,a}^r \Lambda_x(T_{sh,a}) \Delta R d\Sigma^A, \]  

(B21)

and use the expression

\[ L_{x, sh}^R = \begin{cases} 0, & T_s < T_{x, cut}, \\ \sum_{i=1}^{N_R} \left( \frac{T_{max}}{T_s} \right)^2 \frac{\Lambda_x(T_s)}{\Lambda_{tot}(T_{max})} L_{b, sh}^R, & T_s \geq T_{x, cut}, \end{cases} \]  

(B22)

to estimate the X-ray luminosity from the outer shock front. Here the mean shell density and temperature are defined by formulae (A23) - (A24), \( d\Sigma^a \) is the surface area of the...
adiabatic Lagrangian mesh, \( T_{\text{max}} \approx 10^5 K \) is the temperature at the peak of the cooling function \( \Lambda_{\text{tot}} \). The shell X-ray emission then equals

\[
L_{x,\text{out}} = L_{x,\text{sh}}^A + L_{x,\text{sh}}^R
\]  

(B23)

and the total bubble X-ray luminosity reads as

\[
L_x = L_{x,\text{in}} + L_{x,\text{out}}.
\]  

(B24)

### 6.0.2. Phase transitions

During the evolution in a disk-halo system, superbubbles undergo several transitions (from quasi-adiabatic to radiative and vice versa). The time for phase transition can be estimated by comparing the characteristic cooling time

\[
\tau_{\text{cool}} = \frac{3\mu_a^2 kT}{2\mu_{in} \xi n_a \Lambda_{\text{tot}}}
\]  

(B25)

with the dynamic time \( t \). Here \( n_a \) is the atomic number density and the ratio of \( \mu_a/\mu_{in} \) takes into account gas ionization. We assume the mean shell density and temperature (see Appendix A) to estimate the time for the adiabatic - radiative transition, and use Gaetz and Salpeter (1983) cooling function up to \( 4 \times 10^7 K \). The \( T^{1/2} \) growth of the cooling rate for higher temperatures is assumed. The sudden loss of the thermal energy during the adiabatic to radiative transition (see Chevalier 1974; and Falle 1981), is taken into account and is assumed to be in a direct proportion to the surface of the rapidly cooling part of the shell:

\[
\Delta E_{\text{th}} = \frac{\varepsilon_2}{2} E_{\text{th}}.
\]  

(B26)

Thus, if the adiabatic-radiative transition occurs for a spherical shell, the remnant looses half it’s current thermal energy.
6.0.3. Cusps formation and grid rearrangement

As the shell reaches several characteristic scales $H_z$, the acceleration phase begins, and the shell experiences Rayleigh-Taylor instabilities. Once the accelerated sector of the shell is disrupted, a secondary shell forms in the low density halo. Soon the bottom parts of the "cap" begin to overlap and intersect the vertically elongated main body of the remnant and a ring-like cusp forms. Some of the Lagrangian particles then move into the bubble interior what makes the later calculations impossible. To avoid this difficulty we have introduced a simple geometric criterion. The cylindrical radius of any Lagrangian element at any Lagrangian $\theta$-slice (with the exception of top, bottom, and those, which have maximum radii) should be either greater than the cylindrical radius of the previous Lagrangian mesh element and smaller than the radius of the next one, or it should be smaller and greater than the radii of the previous and following Lagrangian mesh particles. If this is not the case, we assume cusp formation and the mass of the corresponding particle goes into the remnant interior and is not considered any further.

We also exclude from future consideration those Lagrangian zones which evolve and interpenetrate the galaxy midplane. To maintain a sufficient space resolution we regrid the shell, if the total number of Lagrangian zones, which are parallel to the galaxy plane falls below the initial value $N_z$. 
Figure captions

1) The ISM of dwarf galaxies. a-d show the density distribution for models A120, A100, A80, and A60. For comparison purpose e shows model B60 and f shows the distribution obtained for model C50 (see Table 1). In all frames the outer contour was drawn at the edge of the galaxy.

2) The ISM mass distribution. The distribution of the $10^9 M_\odot$ and $10^8 M_\odot$ ISM for models A and C (see Table 1), respectively.

3) The time evolution. Evolutionary sequence showing the shape and development of the superbubble for the cases A100 and B60 (see Table 1). The various contours are drawn at $t = 14 \times 10^6$ yr, $26 \times 10^6$ yr and $50 \times 10^6$ yr.

4) The time evolution. Panel a displays the amount of ISM swept by the superbubble calculated for the case A100 (see Table 1), as a function of time, while b and c display the matter deposited into the superbubble interior by thermal evaporation from the shell of swept up matter and that deposited by the sequential SNe. d and e present the calculated bolometric and X-ray luminosity of the superbubble. The latter one shows the contribution from the swept up shell being much smaller than that produced by the remnant interior which accounts for most of the total X-ray luminosity.

5) The fate of processed matter. Panels a and b compare the maximum expansion speed, measured along the symmetry axis (solid lines), with the escape velocity as a function of distance to the centre of the galaxy (dashed lines), for all A and B cases. The dotted lines mark the moment when the fastest expansion speed drops below the assumed halo velocity dispersion.

6) The fate of processed matter. The ratio of the pressure within the superbubble (for models A) to that at the edge of its host galaxy, as a function of the superbubble size. The
sudden change in slope is, in all cases, due to the sudden cut in energy injection at $t_{\text{burst}} = 40 \times 10^6$ yr. The dotted line indicates the run of the superbubble interior pressure for the model A60 normalized to the ISM pressure at the shell stall radius.

7) The critical luminosity. Panels a and b show the critical luminosity values (derived from eq 26; solid lines) required for a superbubble evolving into a $M_{\text{ISM}} = 10^9 \, M_\odot$ (a) or $M_{\text{ISM}} = 10^8 \, M_\odot$ (b) to reach the edge of its host galaxy with a velocity similar to the escape velocity measured at the edge of the system. Larger luminosities would produce faster remnants and thus their processed matter would be lost from the galaxy. Panel (a) also shows the luminosity values obtained from the same eq 26 (dashed line) for all B cases; i.e. the cases for which the dark matter component is assumed to be distributed within a radius of 5 kpc, instead of 10 kpc. The dotted line (in a and b) indicates the luminosity values derived from eq 28, regarded as a better approximation when the evolutionary time exceeds the $t_{\text{burst}} = 40 \times 10^6$ yr. The diamond and triangular symbols indicate the critical luminosity values derived from our numerical simulations, after a number of iterations. For each of these, the indicated ages mark the evolutionary time required for the superbubble to reach the edge of the system. Those exceeding $t_{\text{burst}}$ should then be compared with the solution represented with the dotted line.
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Table 1
Model parameters

| N      | $M_{tot}$ | $M_{gas}$ | $R_B$ | $R_G$ | $C_{halo}$ | $n_0$ | $E_{burst}$ | $L$  | Shell evap. |
|--------|-----------|-----------|-------|-------|------------|-------|-------------|------|-------------|
|        | $10^9 M_{\odot}$ | $10^9 M_{\odot}$ | kpc   | kpc   | km/s$^{-1}$ | cm$^{-3}$ | $10^{55}$ ergs | $10^{40}$ erg s$^{-1}$ |
| A120   | 10.0      | 1.0       | 10    | 3.9   | 120        | 20.3  | 10          | 7.9  | yes         |
| A100   | 10.0      | 1.0       | 10    | 8.5   | 100        | 20.2  | 10          | 7.9  | yes         |
| A80    | 10.0      | 1.0       | 10    | 13.6  | 80         | 20.2  | 10          | 7.9  | yes         |
| A60    | 10.0      | 1.0       | 10    | 24.0  | 60         | 20.2  | 5           | 3.95 | yes         |
| A*100  | 10.0      | 1.0       | 10    | 8.5   | 100        | 20.2  | 10          | 7.9  | no          |
| B160   | 10.0      | 1.0       | 5     | 2.8   | 160        | 20.6  | 10          | 7.9  | yes         |
| B120   | 10.0      | 1.0       | 5     | 6.0   | 120        | 20.3  | 10          | 7.9  | yes         |
| B100   | 10.0      | 1.0       | 5     | 8.7   | 100        | 20.3  | 10          | 7.9  | yes         |
| B60    | 10.0      | 1.0       | 5     | 24.0  | 60         | 21.6  | 5           | 3.95 | yes         |
| C50    | 1.0       | 0.1       | 5     | 3.0   | 50         | 20.05 | 1.07        | 0.85 | yes         |
| C40    | 1.0       | 0.1       | 5     | 5.4   | 40         | 20.20 | 0.57        | 0.45 | yes         |
| C30    | 1.0       | 0.1       | 5     | 9.7   | 30         | 20.20 | 0.42        | 0.33 | yes         |
| D80    | 5.0       | 1.0       | 10    | 5.7   | 80         | 20.26 | 10          | 7.9  | yes         |
| E80    | 5.0       | 1.0       | 5     | 6.8   | 80         | 20.36 | 10          | 7.9  | yes         |
Model A60

![Graph showing R(kpc) vs Z(kpc) with contour lines for different values of R and Z.](image-url)
