TWO-BODY RELAXATION DRIVEN EVOLUTION OF THE YOUNG STELLAR DISK IN THE GALACTIC CENTER

LADISLAV ŠUBR and JAROSLAV HAAS
Charles University in Prague, Faculty of Mathematics and Physics, Astronomical Institute, V Holešovicích 2, Praha CZ-18000, Czech Republic
Received 2014 January 20; accepted 2014 March 19; published 2014 April 24

ABSTRACT
The center of our Galaxy hosts almost two hundred very young stars, a subset of which is orbiting the central supermassive black hole (SMBH) in a relatively thin disk-like structure. First analyses indicated a power-law surface density profile of the disk, \( \Sigma \propto R^{\beta} \) with \( \beta = -2 \). Recently, however, doubts about this profile arose. In particular, it now seems to be better described by a sort of broken power law. By means of both analytical arguments and numerical N-body modeling, we show that such a broken power-law profile is a natural consequence of the two-body relaxation of the disk. Due to the small relative velocities of the nearby stars in co-planar Keplerian orbits around the SMBH, two-body relaxation is effective enough to affect the evolution of the disk on timescales comparable to its estimated age. In the inner, densest part of the disk, the profile becomes rather flat (\( \beta \approx -1 \)) while the outer parts keep imprints of the initial state. Our numerical models show that the observed projected surface density profile of the young stellar disk can result from two-body relaxation driven evolution of a disk with initial single power-law profile with \(-2 \lesssim \beta \lesssim -1.5 \). In addition, we suggest that two-body relaxation may have caused a significant radial migration of the S-stars toward the central SMBH, thus playing an important role in their formation scenario.

Key words: Galaxy: nucleus – methods: numerical – stars: kinematics and dynamics

1. INTRODUCTION

It is already textbook knowledge that galactic nuclei in general contain an extended cluster of old stars surrounding a central supermassive black hole (SMBH). The properties of such a star cluster in the Galactic center (GC), as well as the mass of the SMBH (\( \approx 4 \times 10^{6} M_\odot \); e.g., Ghez et al. 2008) and its distance from the Sun (\( \approx 8 \) kpc; e.g., Gillessen et al. 2009), have been discussed many times in the literature (e.g., Buchholz et al. 2009; Do et al. 2009, 2013). Improvement of observational instruments and techniques over the past few decades, however, led to the surprising discovery of a significant number of young stars in the immediate vicinity of the SMBH, the so-called S-stars, have been identified as quite typical, \( \approx 20 \) Myr old stars of spectral type B with randomly oriented orbits (e.g., Ghez et al. 2005). Stars that are observed farther out (projected distance \( \gtrsim 1 '' \)) have been identified as massive, \( \approx 6 \) Myr old OB- or WR-stars (e.g., Paumard et al. 2006; Bartko et al. 2009, 2010; Do et al. 2013) which appear to be orbiting the central SMBH on near-Keplerian orbits. Those in the immediate vicinity of the SMBH, the so-called S-stars, have been identified as quite typical, \( \approx 20 \) Myr old stars of spectral type B with randomly oriented orbits (e.g., Ghez et al. 2005). Stars that are observed farther out (projected distance \( \gtrsim 1 '' \)) have been identified as massive, \( \approx 6 \) Myr old OB- or WR-stars (e.g., Paumard et al. 2006). In contrast to the S-stars, it has been suggested (Levin & Beloborodov 2003) that a non-negligible subset of these massive stars form a coherently rotating stellar structure around the SMBH—a stellar disk, which is commonly called the clockwise system (CWS). This exciting possibility inspired many authors to study various aspects of the dynamical evolution of such stellar disks.

Attention has been mostly paid to the evolution of eccentricities and inclinations of the stellar orbits in the disk (e.g., Alexander et al. 2007; Cuadra et al. 2008) due to the still ongoing debate about the origin of those young stars that are observed off the suggested disk plane. Evolution of another important quantity, the semi-major axes of the orbits in the disk, which determine its radial structure has been, however, usually neglected. This question might have been overlooked because of the two following reasons, among others. First, raw estimates of the classical two-body relaxation time, which is supposed to be adequate for changes of orbital energy, gave orders of magnitude larger values than the age of the young stars when the parameters of the old star cluster were considered. In contrast, the characteristic timescale of resonant relaxation (either vector or scalar) that is relevant for changes of angular momentum, and consequently, eccentricity and inclination, is estimated to be of the order of millions of years. Second, analyses of the observational data usually led to the conclusion that the surface (number) density profile of the CWS is compatible with a single power law \( \Sigma \propto R^{\beta} \), with \( \beta = -2 \) (Paumard et al. 2006; Lu et al. 2009), which is widely accepted to be a natural outcome of the likely formation scenario of such stellar disks (Lin & Pringle 1987; Levin 2007) and, therefore, there seems nothing interesting to be done.

Nevertheless, pieces of evidence that a broken power-law surface density profile may better fit the observational data have been recently reported. Buchholz et al. (2009) find values of \( \beta = -1.08 \) and \(-3.46 \) with a break point at 0.4 pc (10") for which they do not provide any explanation. Furthermore, although they do not give explicit values of the broken power-law indices, Do et al. (2013) find a sort of plateau in their surface density profile which is followed by a sharp drop of density beyond 0.16 pc (4"). Is there any dynamical reason why a broken power law is a better description of an evolved stellar disk around an SMBH? In this paper, we address this question.

2. THEORETICAL CONSIDERATIONS

A classical equation for two-body relaxation time within a self-gravitating system of stars of mass \( M_\ast \), number density \( n \), and velocity dispersion \( \sigma \) is (Binney & Tremaine 1987)

\[
\tau_\text{R} \approx 0.34 \frac{\sigma^3}{G^2 M_\ast^2 n \ln \Lambda}, \tag{1}
\]

where \( G \) stands for the gravitational constant and \( \ln \Lambda \) is the Coulomb logarithm. Equation (1) gives values of the order of
$10^6$ yr for the old star cluster in the GC when replacing $\sigma$ with the Keplerian orbital velocity around the SMBH, $\sigma \approx v_K \equiv \sqrt{GM_*/R}$:

$$t_R(R) \approx \frac{0.2}{\ln \Lambda} \left( \frac{N(R)}{10^6} \right)^{-1} \left( \frac{M_*/4 \times 10^6 M_\odot}{R} \right)^{-1/2} \times \left( \frac{M_*/M_*}{10^6} \right)^2 \left( \frac{R}{0.04 \text{ pc}} \right)^{3/2} \text{ Gyr.} \quad (2)$$

Here, $N(R) \approx n R^3$ denotes the number of stars below radius $R$. The situation is, however, different if we consider a thin stellar disk whose density (both mass and number) exceeds the density of the old nuclear star cluster. At the same time, and more importantly, the relative velocities of stars within the coherently rotating disk are much smaller than their orbital velocity. Both these facts lead to a considerable (orders of magnitude) shortening of the two-body relaxation timescale within a thin stellar disk with respect to what could be inferred from the parameters of the old star cluster.

The theory of relaxation of thin disks of bodies driven by their mutual gravitational interaction has already been well established in the context of protoplanetary disks. We point the reader to the paper by Stewart & Ida (2000), which summarizes the different theoretical approaches to the problem. The authors derive Equation (4.10) for the temporal evolution of the mean square eccentricity, which can be rewritten in the form

$$\frac{d\langle e^2 \rangle}{dt} = \langle e^2 \rangle \frac{T_{\text{char}}}{T_{\text{char}}^*} \text{ with } T_{\text{char}} = \frac{0.06 \langle e^2 \rangle^2}{\alpha \Omega \Sigma R^2 \left( \frac{M_*}{M_*} \right)^2}, \quad (3)$$

where $M_*$ is the mass of the individual bodies in the disk (stars in our case), $M_*$ is the mass of the central body which dominates the gravitational potential (the SMBH), $\Sigma$ is the surface density of the (stellar) disk, and $\Omega \equiv \sqrt{GM_*/R^3}$ is the Keplerian angular velocity. $\alpha$ is a numerical factor of order unity which can be tuned by comparing with $N$-body integration. Stewart & Ida (2000) report $\alpha = 2$ as a suitable choice and we will adopt this value hereafter.

$T_{\text{char}}$, introduced in Equation (3), approximately corresponds to the time required for $\langle e^2 \rangle$ to grow by a factor of two and, due to its dependence on $\langle e^2 \rangle$, it may be arbitrarily short. A better sense of the characteristic timescale is obtained by integrating Equation (3) which gives the time for the mean square eccentricity to grow from initial (nearly zero value) to $\langle e^2 \rangle$:

$$t\left( \langle e^2 \rangle \right) = \int_0^{\langle e^2 \rangle} \frac{dt}{d\langle e^2 \rangle} d\langle e^2 \rangle = 0.015 \left( \frac{\langle e^2 \rangle^2}{\Omega \Sigma R^2 \left( \frac{M_*}{M_*} \right)^2} \right). \quad (4)$$

For a stellar disk of total mass $M_d$ and radial surface density profile $\Sigma \propto R^{-2}$ ranging from $R_{in}$ to $R_{out}$, Equation (4) gives

$$t\left( \langle e^2 \rangle \right) \approx 1.2 \ln \left( \frac{R_{\text{out}}}{R_{\text{in}}} \right) \left( \frac{R_{\text{in}}}{0.04 \text{ pc}} \right)^{3/2} \left( \frac{M_*/M_*}{10^6} \right) \left( \frac{M_*/M_*}{4 \times 10^6 M_\odot} \right)^{-1/2} \left( \frac{\langle e^2 \rangle}{0.01} \right)^2 \text{ Myr.} \quad (5)$$

We see that at least for the inner parts of the young stellar disk in the GC, the characteristic time for orbital elements to change is comparable to its age.

One of the methods discussed by Stewart & Ida (2000), which led to the derivation of Equation (3), is based on the evaluation of the diffusion coefficients in the Fokker–Planck equation, which is quadratic in velocity. The Fokker–Planck equation was then transformed to equations for changes in the square of the eccentricity and inclination. As the orbital energy is also a quadratic form of velocity, it is reasonable to expect that the characteristic time for evolution of the radius of the orbit is similar to that expressed in Equations (4) and (5). Consequently, we expect that the radial density profile of the stellar disk in the GC may have changed during its lifetime. We note that our assumption about the evolution of the semi-major axes goes beyond the analytical work of Stewart & Ida (2000) and it requires justification by a suitable numerical model which we present below.

The rate of radial migration has to be a function of the radial density profile, which determines the strength of the twobody relaxation. A stationary solution requires a bulk accretion rate independent of radius $R$, i.e., $\dot{\Sigma} (dR/dt) = \text{ const.}$ One possible candidate for the stationary disk density profile is the Bahcall & Wolf (1976) solution with the distribution function of the semi-major axes $N_\alpha \propto a^{1/4}$, which implies $\Sigma(R) \propto R^{-3/4}$. Despite this solution being derived under the assumption of spherical symmetry, underlying energetic balance arguments are not affected by this assumption. Bearing this caveat in mind we may further speculate that due to the rather strong radial dependence of the characteristic time, the stationary solution would settle in the inner parts of the disk, while the outer parts would still keep imprints of the initial state. Hence, for an initial $\Sigma(R) \propto R^{-2}$ that clearly differs from the expected stationary state, we would obtain a broken power-law surface density profile of the stellar disk.

### 3. NUMERICAL MODEL

In order to investigate evolution of the disk density profile, we set up a model integrated numerically using the direct $N$-body integration code NBODY6 (Aarseth 2003). We added a fixed Keplerian potential representing the central SMBH of mass $4 \times 10^6 M_\odot$ to the original publicly available version of the code. The young stellar disk is represented by 1200 point-like particles with mass function $N_m \propto M^{-1.5}$ in the range $(1 M_\odot, 150 M_\odot)$; the total mass of the disk is $M_d = 14700 M_\odot$. The adopted values of the parameters of the mass function are compatible with the recent work of Lu et al. (2013), who investigated the properties of the population of young stars in the GC. The stars were initially set on circular orbits with normalized angular momentum vectors distributed uniformly with a maximum deviation of 1° from the mean (i.e., the initial disk opening angle is 2°). We investigated models with initial power-law surface density profile $\Sigma(R) \propto R^\beta$ with several different values of index $\beta \in (-3, -1)$. The radial extent of the disk is set to an interval $(0.04 \text{ pc}, 0.4 \text{ pc})$.

In order to clearly distinguish general trends from random fluctuations, we typically set up 20 numerical realizations of each model, showing results averaged over all realizations below. The reliability of our numerical approach has been tested by comparing it with qualitatively similar models from Stewart & Ida (2000); see the Appendix.

### 4. RESULTS

We followed the dynamical evolution of the stellar disk to $t > 10^7$ Myr. In terms of the mean (square) eccentricity
and inclination, the evolution is similar for all models (which differ by the value of the index $\beta$). In Figure 1, we show the results for $\beta = -2$ which corresponds to the commonly accepted theoretically plausible surface density profile of a stellar disk formed by self-gravitational fragmentation of the parent gaseous disk. The mean values of both the squared eccentricity and inclination, $\langle e^2 \rangle$ and $\langle i^2 \rangle$, grow gradually, i.e., the velocity dispersion within the disk evolves toward isotropy and the disk becomes geometrically thicker. At later stages, the evolution of eccentricity follows the theoretically predicted relation $\langle e^2 \rangle^{1/2} \propto t^{1/4}$ (Equation (4); see also Stewart & Ida 2000). According to our numerical model, inclinations evolve somewhat faster than eccentricities and the expected relation $\langle i^2 \rangle^{1/2} \approx 0.5 \langle e^2 \rangle^{1/2}$ holds only approximately. This apparent discrepancy is due to the theoretical prediction not considering the temporal evolution of the disk surface density. Stewart & Ida (2000) have shown (and we have also verified with appropriate models; see the Appendix) that after renormalization to constant surface density, the evolution of the mean square eccentricity and inclination matches the theoretical lines much better. The renormalization to constant density is, however, not possible for models with a radially dependent surface density that differs by the order of magnitude spread of the semi-major axes in the inner parts of the disk holds for a wide range of initial values of index $\beta$. This suggests a possibility that there exists an equilibrium state toward which the two-body relaxation of a thin stellar disk around a dominating central mass leads. The equilibrium state inferred from our numerical model, $N_u \approx \text{const.}$, appears to be close to the Bahcall & Wolf (1976) solution mentioned in Section 2. Due to the radial dependence of the relaxation time, the flat distribution of the semi-major axes settles first in the inner parts of the disk and gradually propagates outward. In the outer parts of the disk, where the two-body relaxation timescale is still longer than the evolution time, the distribution is affected much less.

Figure 3 demonstrates that the flattening of the distribution of the semi-major axes in the inner parts of the disk holds for a wide range of initial values of index $\beta$. This suggests a possibility that the numerical model already shows evident disk evolution up to $R \approx 0.1$ pc at $t \approx 6$ Myr is likely due to the broad mass spectrum which is known to accelerate the two-body relaxation driven evolution of self-gravitating systems.

A generic consequence of the shear that acts in differentially rotating disks is their radial spreading (both inward and outward). We attribute two features of the distribution shown in Figure 3 which equals $\approx 12 M_\odot$ in our case. The fact that the numerical model already shows evident disk evolution up to $R \approx 0.1$ pc at $t \approx 6$ Myr is likely due to the broad mass spectrum which is known to accelerate the two-body relaxation driven evolution of self-gravitating systems.
Figure 4. Projected radial surface density profile, $\Sigma_p(R_p)$, of the evolved stellar disk with initial value of index $\beta = -1.5$ within the interval $a \in (0.03\, \text{pc}, \ 0.6\, \text{pc})$ (solid line) and $\beta = -2$ for $a \in (0.04\, \text{pc}, \ 0.4\, \text{pc})$ (dashed line). The thin dotted line shows the initial state of the model with $\beta = -1.5$. The data points represent the projected density as determined by Do et al. (2013) with scaling $1^\ast = 0.04\, \text{pc}$. Note that the rightmost data point was not displayed in Figure 13 of the original paper as the authors restricted themselves to the region below $12^\circ$. The modeled density profiles are normalized arbitrarily to match the observational data.

Figure 3 to this process: first, it is the steepening of the initial density profile, visible particularly for the model with $\beta = -3$ beyond $R \approx 0.1\, \text{pc}$, which is due to stars that migrated outward from the inner, very dense region. Second, as the rate of radial migration is proportional to the disk density, we see a more prominent inward migration of stars across the initial disk inner radius for steeper density profiles (lower $\beta$). We will discuss this effect in Section 6 in the context of the S-stars. Distribution of the semi-major axes of the young stars in the GC cannot be determined from the observational data without some a priori assumptions about their configuration. Hence, instead of attempting to compare the output of our model with the (biased) deprojected structure of the system of young stars in the GC, we plot the projected surface density of the system in our model in Figure 4. For this purpose, we choose the angle $127^\circ$ between the line of sight and the symmetry axis of the disk which corresponds to the inclination of the young stellar disk in the GC with respect to the plane of the sky given by Paumard et al. (2006); other authors report only slightly different values, e.g., $123^\circ$ is the best fit of Levin & Beloborodov (2003) who identified the young stellar disk for the first time. Together with the lines determined from our numerical models, we show the projected surface density of all young stars in the GC (points in Figure 4) as given in Do et al. (2013). We do not consider it reasonable to fit the models with the surface density determined from observations, which suffers from errors that are too large. Still, according to a by-eye comparison, the model with initial density profile $\Sigma(R) \propto R^{-2}$ in $(0.04\, \text{pc}, \ 0.4\, \text{pc})$ differs somewhat from the observational data. Hence, we also show results for another model with $\beta = -1.5$ and $R \in (0.03\, \text{pc}, \ 0.6\, \text{pc})$ which matches the observational data reasonably well. Comparison of the two models further indicates that the profile below $\approx 0.1\, \text{pc}$ is not much affected by the manipulation of the model parameters, though these may be eventually used to fit the observational data in the outer parts of the disk. In this context, the most recently reported smaller outer radius of the disk at $\approx 0.13\, \text{pc}$ (Yelda et al. 2014) can also be straightforwardly accommodated.

5. DISCUSSION

In previous sections, we learned that two-body relaxation can lead to rapid changes of the density profile of a stellar disk around an SMBH. The question now is whether this process cannot be marginalized by some other, stronger effect. In Section 2 we have indirectly argued that the two-body relaxation within the whole central star cluster is likely to act on an orders-of-magnitude longer timescale than the two-body relaxation within the disk. Another form of relaxation, in particular the vector resonant relaxation, however, may be a stronger opponent. The characteristic timescale of the vector resonant relaxation is $(Rauch & Tremaine 1996; Hopman & Alexander 2006)$

$$t_{RR\times}(a) \approx 0.6 \left( \frac{M_\bullet}{M_*} \right) \frac{P(a)}{N^{1/2}(a)}$$

(6)

where $P(a) = 2\pi a^2 \sqrt{G M_\bullet} / a^3$ is the orbital period of a star on orbit with semi-major axis $a$ and $N(a)$ is the number of stars with semi-major axis $<a$. For a star cluster with radial density profile $\rho(R) \propto R^{-\gamma}$ this roughly implies

$$t_{RR\times}(R) \approx 2 \left( \frac{M_*}{M_{N,0.04}} \right)^{1/2} \left( \frac{M_*}{M_\bullet} \right)^{1/2} \left( \frac{M_\bullet}{10^6} \right) \frac{P(a)}{N^{1/2}(a)}$$

(7)

with $M_{N,0.04}$ denoting the mass of the nuclear star cluster below 0.04 pc. The process of vector resonant relaxation generally leads to changes in the orientations of the orbits, i.e., it tends to disrupt (thicken) the disk. Similarly to the two-body relaxation within the disk, it is efficient mainly at small radii. A full $N$-body model including the old star cluster is needed to determine, whether the resonant relaxation among the young and old stars could dominate over the two-body relaxation within the disk and, consequently, suppress the evolution of its radial density profile.

A true full $N$-body model of the inner parsec of the GC is beyond our current computational capabilities. Nevertheless, in order to obtain some test of the influence of the self-gravitating nuclear star cluster on the dynamical evolution of the disk, we introduce a simplified model. The stellar disk is represented by 200 equal-mass stars of mass $100 M_\odot$, forming an initially thin disk with opening angle $4^\circ$ which spans from 0.04 pc to 0.4 pc with surface density $\Sigma(R) \propto R^{-2}$ (i.e., with the distribution of the semi-major axes falling as $a^{-1}$). We followed the evolution of the standalone disk in either the dominant potential of the central SMBH of mass 4 x $10^6 M_\odot$ or with the central Keplerian potential superposed with a smooth potential of mass with radial density profile $\rho(R) \propto R^{-7/4}$ (i.e., the Bahcall & Wolf 1976 solution), and also the stellar disk embedded in a spherical cluster of 8000 stars of mass 50 $M_\odot$ with a Bahcall & Wolf (1976) distribution of the semi-major axes and thermal ($N_\omega \propto e$) distribution of eccentricities. The evolved distributions of the semi-major axes are shown in Figure 5. The individual models give results that are close enough to each other to make the conclusion that the presence of the spherical old star cluster only marginally affects the evolution of the stellar disk.

Aside from the random torques from individual stars of the old star cluster, a systematic influence of massive sources of gravity on the young stellar disk could also weaken the impact of two-body relaxation. Such sources have already been discussed...
in the literature. Nayakshin et al. (2006) and Löckmann & Baumgardt (2009) considered the evolution of two stellar disks due to mutual gravitational torques, Šubr et al. (2009) and Haas et al. (2011a, 2011b) took into account the torque exerted on the stellar disk by the external massive gaseous torus and Mapelli et al. (2013) dealt with the torques due to the remnants of the gaseous disk which gave birth to the young stars. A general statement about the effect of systematic torques from massive, roughly axisymmetric sources of gravity on the young stellar disk is that it leads to a change in the orientation of the orbits of the stars in the disk. The strength of the effect rapidly decreases with the separation of the particular star and the perturbing source. In accordance with this, Šubr et al. (2009), Haas et al. (2011a, 2011b), and Mapelli et al. (2013) concluded that the perturbations they considered mainly affect the outer parts of the disk, i.e., the two-body relaxation is likely to be dominant in the inner parts of the stellar disk. In the case of two, mutually interacting disks of similar radii, their mass determines whether this interaction will lead to a fast puff-up (i.e., lowering their densities) which could make the two-body relaxation marginal.

6. CONCLUSIONS

We showed that two-body relaxation may play an important role in the evolution of the radial density profile of a thin stellar disk around an SMBH given that the disk is initially near-circular. This assumption keeps the relative velocities of the stars in the disk low enough to enable them to effectively interact. In particular, considering the parameters of the young nuclear cluster in the GC, such a disk evolves on a timescale comparable to its age.

The question of the evolution of the semi-major axes of the stars forming a self-gravitating disk around a SMBH was also addressed by Gualandris et al. (2012) who did not find any significant evolution of the radial density profile in their numerical models. This is likely due to the fact that their distribution of initial eccentricities of the orbits in the disk peaked at $\langle e^2 \rangle \approx 0.3$. From the point of view of dynamical evolution, the results of Gualandris et al. (2012) are in accordance with our findings as our calculations confirm that after $\sqrt{\langle e^2 \rangle} \approx 0.3$ is reached, the evolution of the distribution of the semi-major axes is negligible. In addition, Löckmann & Baumgardt (2009) also noticed accelerated energy relaxation in their models of stellar disks with small initial eccentricities ($\langle e \rangle \approx 0.03$), attributing it (as we do) to low relative velocities and high density.

We emphasize that it is important to be cautious when interpreting the observational data with respect to the theoretical expectations about the initial state of the system under consideration. In particular, in this paper we showed that starting from a rather simple initial configuration, the dynamical evolution of a young stellar system in the GC over a few millions of years may lead to a state compatible with recent observational data. Within the framework of our model, the characteristic timescale of the disk evolution grows with radial distance from the center (for reasonable initial profiles). We thus suggest that only the outer parts of the disk ($\gtrsim 0.1$ pc) may help determine its initial state. In the inner parts, where two-body relaxation is more efficient, the distribution of the semi-major axes tends to be flat, implying that the surface density decreases roughly as $R^{-1}$ which is remarkably close to the value reported by Buchholz et al. (2009).

We further showed that the large density at the inner edge of the disk leads to its significant spreading toward the center. For $\beta = -2$ we obtained the minimum value of the semi-major axis averaged over 20 realizations of the numerical model ($\langle a_{\text{min}} \rangle \approx 0.019$ pc at $t = 6$ Myr, i.e., less than one-half of the initial inner radius of the disk. For $\beta = -3$ we obtained an even lower value, $\langle a_{\text{min}} \rangle \approx 0.01$ pc. Hence, we speculate that two-body relaxation may have played an important role in dynamical evolution of the S-stars, which are orbiting with semi-major axes 0.005 pc $\lesssim a \lesssim 0.04$ pc around an SMBH. Although it seems that even a disk with an unrealistically steep initial density profile does not provide sufficiently tight orbits, the two-body relaxation may have worked for a longer period of time provided the S-stars originate from some previous star formation episode and, at the same time, if not placing the S-stars in their orbits, two-body relaxation may have helped to do so to other processes, e.g., the tidal disruption of binaries on highly eccentric orbits around the SMBH (see e.g., Löckmann et al. 2009; Perets & Gualandris 2010).

We appreciate kind help of the author of the NBODY6 code, Sverre Aarseth, in tuning the code with the additional Keplerian potential for a better performance and Tuan Do for providing us with the data plotted in Figure 4. This work was supported in part by the National Science Foundation under grant No. PHY-1066293 and the hospitality of the Aspen Center for Physics. We appreciate a financial support from the Czech Ministry of Education through the grant LD12065 (the European Action COST MP0905) and the Research Program MSMT0021620860.

APPENDIX

NUMERICAL TESTS

In order to test the robustness of the analytical results of Stewart & Ida (2000) and their applicability to our configuration as well as to test the reliability of the numerical code used, we performed several numerical tests. The first is a slightly modified configuration 1 introduced in Stewart & Ida (2000, Section V.A). In particular, we integrated the evolution of 1024 equal-mass particles of mass $M = 5 \times 10^{-19} M_\odot$ which initially
formed a thin disk with constant surface density ranging from 0.972 R₀ to 1.028 R₀. We set up initial inclinations such that $i_{H} \equiv \sqrt{\langle e^2 \rangle (3M_*/2M)^{1/3}} = 1$. For the sake of simplicity, we placed the stars on exactly circular orbits, which are, however, in this case nearly indistinguishable from orbits with eccentricities characterized by $e_{H} \equiv \sqrt{\langle (e^2) \rangle (3M_*/2M)^{1/3}} = 2$ (i.e., $\langle e^2 \rangle \approx 0.001$), the initial value of Stewart & Ida (2000).

The left panel of Figure 6 shows the temporal evolution of the surface density, $\sigma$, for the scaling $\sigma \propto M$ formed a thin disk with constant surface density ranging from $0.972 R_0$ to $1.028 R_0$. The system is scaled to solar system parameters in order to compare with Figure 2 of Stewart & Ida (2000). Right: evolution of $e_H$ and $i_H$ for three different systems with time scaling according to Equation (A1). The solid line stands for the model with 1024 particles of mass $M = 5 \times 10^{-3} M_\odot$, the dotted one for the model with 256 particles of mass $M = 5.12 \times 10^{-4} M_\odot$, and the dashed one for the case of 40 particles of mass $M = 3 \times 10^{-6}$. Thin dotted lines show the $t^{1/4}$ dependence for comparison.

We note that the numerical results can be directly rescaled to arbitrary $M_*$ and $R_0$ which imply a time scaling $t \rightarrow t_s$, with $t_s \equiv (R_0/1 AU)^{3/2} / (M_*/M_\odot)^{1/2}$. Besides this trivial scaling, Stewart & Ida (2000) state that the temporal evolution of $i_H$ and $e_H$ can be scaled for varying surface density of the disk and masses of individual bodies (planetesimals in their case, stars in the context of our paper) in the disk. In particular, they show that two disks with surface densities $\sigma_1$ and $\sigma_2$ and masses of the individual bodies in the disk $M_1$ and $M_2$ will give $i_{H1}(t) = i_{H2}(t')$ and, analogically, $e_{H1}(t) = e_{H2}(t')$ with

$$t' = \frac{\sigma_1}{\sigma_2} \left( \frac{M_2}{M_1} \right)^{1/3}. \quad (A1)$$

We tested this scaling by means of two additional models, one with 256 particles of mass $M = 5.12 \times 10^{-4} M_\odot$ and the other one with 40 particles of mass $M = 3 \times 10^{-6} M_\odot$. In both cases, the disk of constant surface density spread initially from 0.944 $R_0$ to 1.056 $R_0$. When rescaled to $M_* = 4 \times 10^5 M_\odot$ and $R_0 = 0.04$ pc, both models have surface density comparable to the inner parts of the disk described in Section 3. In addition, the masses of the bodies within the latter model are approximately equal to the mean mass of the stars in the models presented in this paper. The right panel of Figure 6 shows the temporal evolution of $i_H$ and $e_H$ for all three models introduced in this Appendix with time scaled according to Equation (A1). Note that, in evaluating $t'$, we considered the temporal evolution of $\sigma$, which we determined numerically. For both $i_H$ and $e_H$, the lines for the three different models lie roughly on top of each other, indicating that (1) the theory of Stewart & Ida (2000) holds for a wide range of the disk parameters and (2) numerical results obtained using the NBODY6 code are in agreement with this theory.

Last but not the least, the reliability of our numerical integrations is supported by energy conservation check—in all models presented in this work, the total relative energy error throughout the whole integration was $\lesssim 10^{-5}$.

REFERENCES

Aarseth, S. 2003, Gravitational N-Body Simulations (Cambridge: Cambridge Univ. Press)
Alexander, R. D., Begelman, M. C., & Armitage, P. J. 2007, ApJ, 654, 907
Bahcall, J. N., & Wolf, R. A. 1976, ApJ, 209, 214
Bartko, H., Martins, F., Fritz, T. K., et al. 2009, ApJ, 697, 1741
Bartko, H., Martins, F., Trippe, S., et al. 2010, ApJ, 708, 834
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton, NJ: Princeton Univ. Press)
Buchholz, R. M., Schödel, R., & Eckart, A. 2009, A&A, 499, 483
Cuadra, J., Armitage, P. J., & Alexander, R. D. 2008, MNRAS, 388, L64
Do, T., Lu, J. R., Ghez, A. M., et al. 2013, ApJ, 764, 154
Ghez, A. M., Salim, S., Horstine, S. D., et al. 2005, ApJ, 620, 744
Ghez, A. M., Salim, S., Weinberg, N. N., et al. 2008, ApJ, 689, 1044
Gillessen, S., Eisenhauer, F., Trippe, S., et al. 2009, ApJ, 692, 1075
Gualandris, A., Mapelli, M., & Perets, H. B. 2012, MNRAS, 427, 1793
Haas, J., Subr, L., & Kroupa, P. 2011a, MNRAS, 412, 1905
Haas, J., Subr, L., & Vokrouhlický, D. 2011b, MNRAS, 416, 1023
Hopman, C., & Alexander, T. 2006, ApJ, 645, 1152
Levin, Y. 2007, MNRAS, 374, 515
Levin, Y., & Beloborodov, A. M. 2003, ApJL, 590, L33
Lin, D. N. C., & Pringle, J. E. 1987, MNRAS, 225, 607
Lockmann, U., & Baumgardt, H. 2009, MNRAS, 394, 1841
Lockmann, U., Baumgardt, H., & Kroupa, P. 2009, MNRAS, 398, 429
Lu, J. R., Do, T., Ghez, A. M., et al. 2013, ApJ, 764, 155
Lu, J. R., Ghez, A. M., & Horstine, S. D. 2009, ApJ, 690, 1463
Mapelli, M., Gualandris, A., & Hayfield, T. 2013, MNRAS, 436, 3809
Nayakshin, S., Dehnen, W., Cuadra, J., & Genzel, R. 2006, MNRAS, 366, 1410
Paumard, T., Genzel, R., Martins, F., et al. 2006, ApJ, 643, 1011
Perets, H. B., & Gualandris, A. 2010, ApJ, 719, 220
Rauch, K. P., & Tremaine, S. 1996, NewA, I, 149
Stewart, G. R., & Ida, S. 2000, Icar, 143, 28
Subr, L., Schovancová, J., & Kroupa, P. 2009, A&A, 496, 695
Yelda, S., Ghez, A. M., Lu, J. R., et al. 2014, ApJ, 783, 131