BATSE GAMMA-RAY BURST LINE SEARCH. V. PROBABILITY OF DETECTING A LINE IN A BURST

D. L. Band, L. A. Ford,1 and J. L. Matteson
Center for Astrophysics and Space Sciences, 9500 Gilman Drive, Code 0424, University of California, San Diego, La Jolla, CA 92093

AND

M. S. Briggs, W. S. Paciesas, G. N. Pendleton, and R. D. Preece
Department of Physics, University of Alabama in Huntsville, Huntsville, AL 35899

Received 1996 October 18; accepted 1997 March 24

ABSTRACT

The physical importance of the apparent discrepancy between the detections by pre-BATSE missions of absorption lines in gamma-ray burst spectra and the absence of a BATSE line detection necessitates a statistical analysis of this discrepancy. This analysis requires a calculation of the probability that a line, if present, will be detected in a given burst. However, the connection between the detectability of a line in a spectrum and in a burst requires a model for the occurrence of a line within a burst. We have developed the necessary weighting for the line detection probability for each spectrum spanning the burst. The resulting calculations require a description of each spectrum in the BATSE database. With these tools, we identify the bursts in which lines are most likely to be detected. Also, by assuming a small frequency with which lines occur, we calculate the approximate number of BATSE bursts in which lines of various types could be detected. Lines similar to the Ginga detections can be detected in relatively few BATSE bursts; for example, in only ~20 bursts are lines similar to the GB 880205 pair of lines detectable. Ginga reported lines at ~20 and ~40 keV, whereas the low-energy cutoff of the BATSE spectra is typically above 20 keV; hence BATSE's sensitivity to lines is less than that of Ginga below 40 keV, and greater above. Therefore, the probability that the GB 880205 lines would be detected in a Ginga burst rather than a BATSE burst is ~0.2. Finally, we adopt a more appropriate test of the significance of a line feature.

Subject headings: gamma rays: bursts — methods: statistical

1. INTRODUCTION

The continued absence of a line detection in the gamma-ray burst spectra accumulated by the Burst and Transient Source Experiment (BATSE) on the Compton Gamma Ray Observatory (Palmer et al. 1994, hereafter Paper I) has led us to continue not only the search for lines in the BATSE data (Briggs et al. 1996) but also our study of the detectability of lines by the BATSE detectors and the statistical implications of the current results. In particular, we are evaluating the consistency between the BATSE observations and those of previous missions, particularly those of Ginga. These calculations assume that the BATSE detectors function properly and that our models of their performance are accurate, assumptions that we test continually (Paciesas et al. 1996; Preece et al. 1997). Here we fill a major gap in our statistical methodology and implement it for the BATSE data.

The description of our statistical methodology is clearest using conditional probabilities and their associated notation. Thus \( p(a | b) \) means the probability of proposition \( a \) given proposition \( b \). A bar over a proposition denotes the negation of that proposition. Since we need to differentiate between quantities that refer either to a burst as a whole or to a specific spectrum accumulated over a portion of the burst, we use the convention that roman indices specify spectra and greek indices identify bursts. For example, \( l_{\alpha} \) represents the proposition that a line exists in the \( \alpha \)th burst while \( l_{i} \) is the proposition that a line is present in the \( i \)th spectrum; technically, the burst within which the spectrum was accumulated should also be indicated (e.g., \( l_{i, \alpha} \)), but the burst will be understood from the context. As a reminder that these probabilities rely on our understanding of the detectors and gamma-ray bursts, we include as one of the given the proposition \( f \), which represents our model of the detector response, our parameterization of the burst continuum, etc. Our calculations can be seriously in error if our assumptions expressed by \( I \) are incorrect. For example, we use the "GRB" spectral function (Band et al. 1993) to model the continuum, but this spectral shape is not based on the source physics and therefore must be incorrect at some level of accuracy.

Our analysis of the possible line content of a burst sample is based on a hierarchy of probabilities. Ultimately we want the probability \( p(D | HI) \) of obtaining the observed data (proposition \( D \)), assuming hypothesis \( H \) (Band et al. 1994, hereafter Paper II). Thus \( D \) might represent the statement that no lines have been detected in the BATSE database, and \( H \) might be the hypothesis that lines exist and that we are modeling BATSE correctly. The information that might be represented by \( I \) and \( H \) overlaps; in general, \( H \) should include the information that differs when hypotheses are compared. Also known as the likelihood of \( H \), \( p(D | HI) \) can be used in measures of the consistency between BATSE and previous detectors. Our methodology does not result in \( p(D | HI) \) directly but rather in \( p(D | fHI) \), where \( f \) is the line frequency, the probability that a line is found in a given burst. When necessary, this dependence on \( f \) is removed by the Bayesian process of "marginalization." Since bursts are presumably independent events, \( p(D | fHI) \) is the product of the probabilities of obtaining the observed detections or

1 Present address: Department of Physics and Astronomy, University of Wisconsin, Eau Claire, P.O. Box 4004, Eau Claire, WI 54702.
nondetections in each burst. Thus, for the $N_B$ BATSE bursts in which no lines have been detected,

$$p(D|fHI) = \prod_{\sigma=1}^{N_B} 1 - p(L_{\sigma}|fHI), \quad (1)$$

where $L_{\sigma}$ is the proposition that a line has been detected in the $\sigma$th burst and, therefore, $p(L_{\sigma}|fHI)$ is the probability of detecting a line in this burst. If present, a line may persist over a time range shorter than the burst's duration and will be found in the $\sigma$th spectrum accumulated during the burst. Therefore $p(L_{\sigma}|fHI)$ will be a function of the probability $p(L_{\sigma}|fHI)$ of detecting the line in the $\sigma$th spectrum; $L_{\sigma}$ is the proposition that the line was detected in the $\sigma$th spectrum. The connection between $p(L_{\sigma}|fHI)$ and $p(L_{\sigma}|fHI)$ is presented in this paper. The line detection may be real or false, and therefore

$$p(L_{\sigma}|fHI) = p(L_{\sigma}|l_{\sigma}HI)p(l_{\sigma}|fHI) + p(L_{\sigma}|\bar{l}_{\sigma}HI)p(\bar{l}_{\sigma}|fHI) \quad (2)$$

(Band et al. 1995, hereafter Paper III), where $l_{\sigma}$ is the proposition that the line is present in the $\sigma$th spectrum. Thus $p(L_{\sigma}|l_{\sigma}HI)$ is the probability of detecting a line in a spectrum, and $p(L_{\sigma}|\bar{l}_{\sigma}HI)$ is the probability of a “false positive.” Currently, we assume that our detection criteria are stringent enough to make the false-positive rate negligible; an assumption we will investigate in the future. In this paper, we also discuss the database describing the BATSE spectra necessary to calculate $p(L_{\sigma}|l_{\sigma}fHI)$; we present some results of utilizing this database.

The probability $p(L_{\sigma}|l_{\sigma}HI)$ of detecting a line that is present in the $\sigma$th spectrum is the foundation of our methodology. If they exist, lines are undoubtedly characterized by a currently unknown distribution of energy centroids, line widths, intensities (e.g., equivalent widths), and perhaps other parameters. The detectability of each line type can be considered separately; we generally have been using the lines reported by Ginga in the S1 segment of GB 870303 (Graziani et al. 1992) and in GB 880205 (Murakami et al. 1988) as archetypes, although a generic set of lines can be used. We find (Paper III) that the detectability of a given line is a function of the strength of the continuum (i.e., the signal-to-noise ratio, or S/N) and the angle between the detector normal and the burst (the burst angle). This study of line detectability shows that BATSE would have detected the lines reported by Ginga, assuming of course that BATSE functions as modeled.

Next we need $p(L_{\sigma}|l_{\sigma}HI)$, the probability of detecting a line somewhere in the $\sigma$th burst given that the line is indeed present, which requires a relationship between the presence of a line in the $\sigma$th burst and its presence in the $\sigma$th spectrum within this burst. Thus we may calculate that there is a very high probability of detecting a line in a given spectrum if it is present [$p(L_{\sigma}|l_{\sigma}HI) \approx 1$] but may conclude that, even if a line is present somewhere in the burst, it is most likely not in the spectrum in question [$p(L_{\sigma}|\bar{l}_{\sigma}HI) \ll 1$]. For example, based on empirical evidence or theoretical prejudice, we may believe that lines do not persist for the length of time over which the spectrum was accumulated. The few line detections from all the burst missions are insufficient to map out the line distribution (e.g., energies, intensities, widths, persistence times), and therefore we must model the probability $p(l_{\sigma}|l_{\sigma}H)$ that a line occurs in the $\sigma$th spectrum, regardless of whether it is detectable.

With $p(L_{\sigma}|l_{\sigma}HI)$ for the bursts observed by different missions we can evaluate the consistency between these missions. Both BATSE and Ginga provided sufficient data to carry out such an analysis. We have developed a number of measures of the consistency between these two missions, using both standard “frequentist” (Paper I) and Bayesian statistics (Paper II). In addition, values of $p(L_{\sigma}|l_{\sigma}HI)$ for the bursts in the BATSE database can be used to identify the most promising bursts for further analysis. We first apply new line-search techniques (Schaefer et al. 1994; Briggs et al. 1996) to bursts in which lines are most likely to be detected.

The statistical analysis outlined above requires a characterization of all the spectra from BATSE and Ginga and measures of line detectability for both instruments. Paper III provides the line detection probabilities for the BATSE Spectroscopy Detectors (SDs), the relevant BATSE detectors. Below we describe the database of BATSE spectra created to characterize the BATSE bursts. Fenimore et al. (1993) performed a preliminary evaluation of the Ginga data for the GB 880205 line set; a more extensive extraction of the necessary Ginga information is planned.

Here we consider a “real” line to be a true feature in the spectrum that arrives at the detector; most likely a real feature was emitted by the burst, although the feature may possibly have been imposed on the spectrum by astrophysical processes between the burst source and the detector. A “detection” is a feature that satisfies the detection criteria and is therefore considered to be a real line. Note that we treat the detection of a line as a binary conclusion: a feature is either considered to be a detected line or it is ignored in subsequent analysis. Unlike frequentist statistics, in which a hypothesis is either true or false, Bayesian statistics (which is used in Paper II) allows our confidence in a hypothesis to be quantified via a probability that the hypothesis is true. In principle, we could develop a formalism that permits a fractional detection through the probability that a feature is real. We could then develop a methodology of using the entire spectral database to tease out the line distribution from all the spectral bumps and wiggles, most of which are undoubtedly fluctuations but a small fraction of which might result from an underlying distribution of real lines. Specifically, deviations from the distribution of line significances expected from mere fluctuations could be used to estimate the distribution of true spectral lines; this assumes that the fluctuation distribution can be estimated accurately. However, most scientists are more comfortable working with definite detections and nondetections, and that is the route we have taken.

Our detection criteria are (1) that a spectral feature is significant in the spectrum from one detector and (2) that all spectra from the detectors that observed the burst are consistent. Until recently, significance was defined using the $F$-test. However, as discussed in Appendix A, the $F$-test is appropriate when the uncertainties on the measured quantities—here the counts in each channel—are unknown (Eadie et al. 1971), which is not the case here. We have therefore adopted a maximum likelihood ratio test, which uses $\Delta \chi^2$ as the relevant statistic (D. Lamb 1995, private communication). In practice, these two tests usually yield comparable results.

This paper begins with the method for connecting the probabilities of detecting a line in a given spectrum and anywhere in a burst (§ 2). The resulting methodology requires the line detection probability for every burst spec-
trum accumulated by BATSE; a database of parameters
describing these spectra is required to calculate these prob-
abilities (§ 3). These data are used to rank the BATSE bursts
by the maximum signal-to-noise ratio of any spectrum
accumulated during a burst. This database is also utilized to
find the number of bursts in which different line types could
be detected under the assumption of a small line-occurrence
frequency (§ 4); this number is crucial to the study of consist-
cy between the BATSE and Ginga observations (§ 5). In
Appendix A, we discuss the maximum likelihood ratio test
that we have adopted to evaluate the significance of an
observed line feature. Appendix B lists the large number of
symbols used in this work.

2. PROBABILITY THAT A LINE IS PRESENT IN A GIVEN
BURST

Here we consider the probability of detecting a line in a
single burst, and therefore we suppress the (greek) indices
specifying the burst. Also, we consider the probability of
detecting a line of a given type, specified by parameters such as
energy centroid, intrinsic width, and intensity (but not the
time over which the line is present); the resulting calcu-
mation must be performed for each line type. BATSE accu-
culates a series of consecutive spectra from four different
SDs (the Spectroscopy High Energy Resolution Burst—
SHERB—data type); we refer to these basic spectra that
provide the finest time resolution available as SHERB
spectra. We assume that there are N SHERB spectra for a
given detector across the burst, from which it is possible to
construct N(N + 1)/2 different averaged spectra composed
of consecutive SHERB spectra; quantities describing these
averaged spectra are specified by roman indices (e.g., l indicates
the proposition that a line exists in the ith averaged
spectrum).

The probability of detecting a line in the burst as a whole is the probability of detecting a line in at least one of the
spectra that can be searched,

\[ p(L \mid fHI) = 1 - \prod_{i=1}^{N(N+1)/2} 1 - p(L_i \mid fHI) \]  

(3)

(the probability of at least one detection is 1 minus the probability of no detections). As will be discussed below, the
line frequency \( f \) is the probability that a line is present some-
where in the burst, \( f = p(l \mid HI) \). A line detection in a given
spectrum results either from the detection of a real line or a
spurious detection (i.e., a "false positive"):

\[ p(L_i \mid fHI) = p(L_i \mid l, HI)p(l_i \mid fHI) + p(L_i \mid \bar{l}, HI)p(l_i \mid fHI) \]

\[ = p(L_i \mid l, HI)p(l_i \mid fHI) + p(L_i \mid \bar{l}, HI)[1 - p(l_i \mid fHI)], \]  

(4)

where we have used the fact that \( p(l_i \mid fHI) \) and \( p(l_i \mid fHI) \) are exhaustive. Paper III calculated \( p(L_i \mid l, HI) \) for the BATSE
SDs; the probability of a spurious detection, \( p(L_i \mid \bar{l}, HI) \), will be studied further but is clearly dependent on the detection
threshold.

Our focus here is \( p(l_i \mid fHI) \), which is a statement of how
lines occur in burst spectra. Is the probability that a line
occurs in a burst the same for all bursts, or does it depend
on duration, spectral hardness, or other burst properties?
Do lines persist for a long time or for short intervals?
Unfortunately, since there have been very few detections, we
know very little about \( p(l_i \mid fHI) \). Therefore we have to con-
struct reasonable models of \( p(l_i \mid fHI) \), which we will use for
further calculations.

Let \( dp(l_i \mid t_b, t_c, fHI) = g(t_b, t_c)dt_bdt_c \) be the probability
density for a line beginning at \( t_b \) and ending at \( t_c \). If we assume
that the probability depends only on the time a line persists, and does not favor the beginning or end of the
burst, then \( g(t_b, t_c) \) will depend only on the persistence time
\( t_c - t_b \), i.e., \( g(t_b, t_c) = g(t_c - t_b) \). Since the data consist of discrete spectra, we cannot isolate the spectrum over the
precise interval during which a line is present (if such exists,
since the line intensity may vary). Instead, the line will be
attributed to a particular sum of consecutive SHERB
spectra with an accumulation period overlapping the time
the line was actually present; conversely, a spectrum
summed from a number of SHERB spectra may show lines
with a variety of beginning and end times. The probability
that a line begins between \( t_{b,1} \) and \( t_{b,2} \) and ends between \( t_{c,1} \) and \( t_{c,2} \) would be attributed to the ith spectrum
accumulated between \( t_i \) and \( t_k \) \( (t_{b,1} \leq t_j \leq t_{b,2}, t_{c,1} \leq t_k \leq t_{c,2}) \), is

\[ p(l_i \mid fHI) = 1 - \exp \left[ - \int_{t_{b,1}}^{t_{b,2}} \int_{t_{c,1}}^{t_{c,2}} dt_b dt_c g(t_b, t_c) \right], \]  

(5)

the probability of a line existing in the burst is

\[ p(l_i \mid fHI) = 1 - \exp \left[ - \int_0^T dt_b \int_0^T dt_c g(t_b, t_c) \right], \]  

(6)

where \( T \) is the burst duration. These expressions are derived from
\( \int_1 - p(l_i \mid fHI) = \exp \left[ - \sum_j \int_0^T \int_0^T dt_j \right] \) \( \int_0^T \int_0^T dt_k g(t_j, t_k)dt_jdt_k \) where
\( \int_0^T \int_0^T dt_j \) is valid because \( p(l_i \mid fHI) = g(t_b, t_c)dt_bdt_c \) [i.e., \( p(l_i \mid fHI) \) is small]. Finally,
\( \sum_j p(l_i \mid fHI) \) is \( \int g(t_b, t_c)dt_bdt_c \). In practice, if the ith spectrum
begins at \( t_i \) and ends at \( t_k \), we use \( t_{b,1} = (t_{i-1} + t_i)/2, \)
\( t_{b,2} = (t_i + t_{i+1})/2, \quad t_{c,1} = (t_{k-1} + t_k)/2, \quad t_{c,2} = (t_k \quad t_{k+1})/2 \), with a somewhat more complicated expression for a single SHERB spectrum.

As examples, we consider three different functional forms for
\( g(t_c - t_b) \). In each case there are two major variants. The first (eqs. [7], [9], and [11] below) assumes that \( g \) is the same function of the persistence time \( t_c - t_b \), with the same normalization for all bursts, and thus the line frequency
varies from burst to burst (i.e., lines are more likely to occur in
long bursts). The second variant (eqs. [8], [10], and [12] below) assumes that the line frequency is the same for all
bursts, and therefore the normalization of \( g \) varies from
burst to burst. In practice we use the second case.

**Model 1**: \( g(t_c - t_b) = c \). If \( c \) is the same for all bursts, then the first variant of this model is

\[ p(l_i \mid cHI) = 1 - \exp \left[ -c(t_{b,2} - t_{b,1})(t_{c,2} - t_{c,1}) \right], \]

\[ p(l_i \mid cHI) = 1 - \exp \left( -cT^2/2 \right), \]  

(7)

where \( 0 \leq c \leq \infty \). Note that \( f \) increases with the duration \( T \). On the other hand, if \( p(l_i \mid c) = f \) for each burst, then the second variant is

\[ p(l_i \mid fHI) = 1 - \left( 1 - f \right)^2(t_{b,2} - t_{b,1})(t_{c,2} - t_{c,1})/T^2, \]  

(8)

where \( 0 \leq f \leq 1 \). Note that there is no dependence on the persistence time \( t_c - t_b \).

**Model 2**: \( g(t_c - t_b) = cb^2 \exp \left[ -b(t_c - t_b) \right] \). This model would result from a sequence of independent line
occurrences, that is, the probability of the line’s occurring in
any given time interval does not affect the probability of its presence in the next time interval. If \( c \) is constant for all bursts, then
\[
p(l|cH) = 1 - \exp\left\{ c(e^{bt_2} - e^{-bt_2})(e^{-bt_2} - e^{bt_2})\right\},
\]
\[
p(l|cH) = f = 1 - \exp\left\{ -c(e^{-bT} - (1 - bT))\right\},
\]
where \( 0 \leq c \leq \infty \). If we assume \( p(l|cH) = f \), then
\[
p(l|fH) = 1 - (1 - f)[\exp(bt_{b,2}) - \exp(bt_{b,1})]
\times \left[ \exp(-bt_{e,2}) - \exp(-bt_{e,1}) \right]\left( (1 - bT) - e^{-bt_2} \right),
\]
where \( 0 \leq f \leq 1 \). In this case there is a strong dependence on the persistence time \( t_e - t_b \) since \((e^{bt_2} - e^{-bt_2})(e^{-bt_2} - e^{bt_2}) = \exp(-b(t_e - t_b))(1 - e^{-b(t_e - t_b)})\).

Model 3: \( g(t_e - t_b) = (t_e - t_b)^{-b} \).

3. DATABASE

In Paper III we found that the probability \( p(L_i|fH) \) of detecting a line in a spectrum was a function of the spectrum's S/N and the burst angle. Therefore we need these quantities for each spectrum from all the detectors for which there are data for a given burst. Because \textit{Ginga} reported lines at \( \sim 20 \) and \( \sim 40 \) keV, we use S/Ns calculated between 25 and 35 keV. Thus the S/N measures the strength of the continuum in the energy range of interest, which should mitigate the effect of differently shaped continua.

We would like to search spectra with arbitrary beginning and end times, but the telemetry only provides spectra with discrete beginning and end times. Our search is meant to find the combination of consecutive SHERB spectra in which a candidate feature has the greatest significance. Thus, if \( N \) SHERB spectra span a burst, we need to consider \((N + 1)/2\) spectra. However, the database does not need to store parameters for all \((N + 1)/2\) possible spectra, since they can be calculated from a smaller set of data. Here we assume that the burst angle and background count rate \( R_b \) are constant for the entire burst for the detector providing the SHERB spectra, a reasonable assumption since the burst durations are usually less than 100 s (the timescale over which the background rate might change significantly enough to affect our results; the burst angle will change on much longer timescales). The S/N for each possible spectrum can be calculated from the counts and accumulation time for each SHERB spectrum. Thus
\[
\left( \frac{S}{N} \right)_i = \frac{C_i - R_b \Delta t_i}{\sqrt{C_i \Delta E}},
\]
where \( C_i \) is the number of detected counts summed over all the SHERB spectra of which the \( i \)th spectrum consists, \( R_b \) is the background count rate, \( \Delta t \) is the time over which the spectrum was accumulated (i.e., the sum of the accumulation times of the constituent SHERB spectra), and \( \Delta E \) is the size of the energy range (\( \Delta E \sim 10 \text{ keV} \)). Both \( R_b \) and \( C_i \) are accumulated over \( \Delta E \). The factor of \( \Delta E^{-1/2} \) converts the S/N from a ratio using the counts over an energy range (\( \Delta E \) will vary in size from detector to detector and burst to burst) to a ratio using the counts per keV. Note that a live-time correction is not made. Thus the database need contain parameters only for each SHERB spectrum, as discussed in detail below.

For a burst to be included in our database, it had to have a peak count rate in the Large-Area Detectors (LADs) of over \( \sim 7500 \text{ s}^{-1} \) in the 50-300 keV energy band; of the 1550 bursts on which BATSE triggered between 1991 April and 1996 May, 297 met this criterion. After identifying the channels between 25 and 35 keV, we extracted the number of counts in these channels for each SHERB spectrum for all the detectors that provided data. The background count rate is the time average from a series of SHERB spectra after the burst, if available, and SHER spectra (Spectroscopy High Energy Resolution; background spectra accumulated when BATSE is not in burst mode) before and after the burst, if necessary. Calculating higher accuracy backgrounds is unnecessary for our purposes since here we only need a measure of the strength of the burst, not an accurate background-subtracted spectrum for spectral fitting. In some cases the calculated background was clearly too high—indicated by a large number of negative background-subtracted count rates—or low—found by inspecting weak bursts with large S/Ns. Incorrect background rates were recalculated, often using stretches of background in the middle of, or just after, a burst. A burst for which the S/N is sensitive to the background level is usually too weak to harbor detectable spectral lines. Spectra from all detectors were included in the database, but we ignored data from detectors set at low gain or with burst angles greater than \( \sim 85° \) low-gain detectors have a low-energy cutoff \( E_{\text{low}} \) above the energies at which lines have been observed, and the spacecraft shields the detectors for very large burst angles. Line detectability depends on the energy range covered; a line at 20 keV will not be detected if the spectrum begins at 20 keV. Therefore we also characterized each spectrum by its low-energy edge \( E_{\text{low}} \), which we define as the upper end of the SD low-energy distortion (SLED—an electronic artifact just above a spectrum’s true low-energy cutoff; see Band et al. 1992). The database therefore consists of the following data for each detector for each burst: the time interval over which the SHERB spectra were accumulated; the number of counts in the 25–35 keV range for each SHERB spectrum; and additional information for each burst-detector pair such as the burst angle, the energy \( E_{\text{low}} \) of the upper end of the SLED, and the exact energy width \( \Delta E \) of the 25–35 keV range.

The product of the methodology and database described above is the probability, for each burst, of detecting a line if present, \( p(L|fH) \). The primary purpose of this probability
is to assess the consistency between BATSE and other missions, and to estimate the frequency with which lines occur. However, this probability can also be used to identify the bursts in which lines are most detectable. Our search should therefore focus on those bursts. As an example, we characterized each burst by the maximum S/N for any spectrum during the burst. Figure 1 presents the cumulative distribution. Since the gain, and thus the energy range included in the spectrum, varies from burst to burst and detector to detector, we show distributions by maximum \( E_{\text{low}} \). Thus a line at 20 keV would be detectable in those bursts with a detector for which \( E_{\text{low}} < 15 \) keV. From Paper III we find that the GB 880205 line set (19.4 and 38.8 keV) would have been detected half the time by BATSE for S/N \( \approx 7 \). The line at 38.8 keV appears to determine the detectability of this line set in the BATSE spectra, and therefore we require \( E_{\text{low}} \lesssim 25 \) keV. We see that the line at \( \approx 40 \) keV would have been detectable in the highest S/N spectrum in about 65 bursts.

4. SIMPLIFIED LIKELIHOOD CALCULATION

The Bayesian consistency measures and related quantities require \( p(D \mid f HI) \), the probability of obtaining the observed results \( D \) assuming a hypothesis \( H \) about bursts and the detectors (Paper II); \( p(D \mid f HI) \) is also known as the likelihood of \( f \) and \( H \). Thus \( D \) might represent the absence of a BATSE line detection or the Ginga line detections in specific bursts, while \( H \) might stand for the hypothesis that lines exist, the BATSE detectors are modeled correctly, and the BATSE and Ginga results are consistent. In our formulation we explicitly separate out the line frequency \( f \). A burst with a detection contributes to \( p(D \mid f HI) \) a factor of \( p(L_n \mid f HI) \), while a burst with no line detection contributes \( 1 - p(L_n \mid f HI) \). Note that, as before, roman and greek indices specify spectra and bursts, respectively. Thus if there are line detections in \( n_b \) bursts in a database of \( N_b \) bursts, then

\[
p(D \mid f HI) = \prod_{\sigma=1}^{N_b} \left[ 1 - p(L_n \mid f HI) \right] \prod_{\rho=1}^{n_b} p(L_n \mid f HI) ,
\]

where the detections have been placed at the beginning of the database. The line frequency \( f \) is not a quantity of interest to the consistency issue, and therefore it is “marginalized,”

\[
p(D \mid HI) = \int df p(f \mid HI) p(D \mid f HI) .
\]

The “prior” for \( f \), \( p(f \mid HI) \), is our assessment of the likely value of \( f \) before the data \( D \) were obtained. In general we assume that \( f \) could be any value between 0 and 1, and therefore \( p(f \mid HI) = 1 \).

We saw in § 2 that \( p(L_n \mid f HI) = 1 - (1 - f)^n \) (e.g., eq. [8], [10], or [12]). Consequently, the probabilities of detecting a line in a given burst \( p(L_n \mid f HI) \) and of obtaining the observed database \( p(D \mid f HI) \) are complicated functions of \( f \). Thus the integral over \( f \) in equation (15) will be a time-consuming numerical calculation since information from all the bursts must be included in evaluating the integrand at each value of \( f \). However, we can make some simplifying assumptions. First, we assume that the false-positive probability \( p(L_n \mid f HI) \) is very small and can be neglected. Second, the absence of a detection in the BATSE data set indicates that the line frequency \( f \) is probably small, and therefore

\[
p(L_n \mid f HI) = 1 - (1 - f)^n \approx \gamma_n f .
\]

Consequently,

\[
p(L_n \mid f HI) = \left( \frac{N_b(N_b + 1)/2}{1 - M(L_n \mid f HI)} \right) f ,
\]

\[
p(L_n \mid f HI) = \left[ \frac{N_b N_b (N_b + 1)/2}{\sum_{i=1}^{N_b} p(L_n \mid f HI) \gamma_i} \right] f ,
\]

\[
p(D \mid f HI) = 1 - \left( \sum_{\sigma=1}^{N_b} \sum_{i=1}^{n_b} p(L_n \mid f HI) \gamma_i \right) f ,
\]

\[
M(L_n \mid f HI) = \left[ \sum_{\sigma=1}^{N_b} \sum_{i=1}^{n_b} p(L_n \mid f HI) \gamma_i \right] .
\]

\( N_n \) is the number of SHERB spectra spanning the \( n \)th burst. We approximated \( p(D \mid f HI) \) in equation (19) for the case of no detections, which is currently relevant for BATSE (i.e., \( N_n = 0 \) in eq. [14]). The quantity \( M(L_n \mid f HI) \) is the sum of each burst’s detection probability; thus \( M(L_n \mid f HI) \) will be nearly equal to the number of bursts if all bursts are uniformly strong, whereas weak bursts will not contribute to this statistic. Since \( M(L_n \mid f HI) \) is the first-order expansion in \( f \), it is valid for small values of \( f \), i.e., under the assumption that a line is unlikely to be present in a given burst. The small-\( f \) approximation in equation (19) is valid only for \( f \ll 1/M(L_n \mid f HI) \); note that \( (1 - f)^n \approx 1 - mf \) is not accurate for \( f \geq 1/m \) even if \( 1/m \) is small. However, we shall use this approximation to \( f \sim 1/M(L_n \mid f HI) \). We can now marginalize \( f \) to obtain

\[
p(D \mid HI) = \int df p(f \mid HI) p(D \mid f HI) = \frac{1}{2M(L_n \mid f HI)} ,
\]

where we set \( p(D \mid f HI) = 0 \) for \( f \geq 1/M(L_n \mid f HI) \). Using the expression in Paper II for the probability distribution for \( f \) given the new data \( D \) (here the absence of a BATSE line detection), we find

\[
p(f \mid D HI) = p(f \mid HI) p(D \mid f HI) p(D \mid HI)
\]

\[
= \begin{cases} 
2M(L_n \mid f HI)[1 - M(L_n \mid f HI) f] , & \text{if } f \leq 1/M(L_n \mid f HI) , \\
0 , & \text{if } f \geq 1/M(L_n \mid f HI) .
\end{cases}
\]
of S/Ns. The calculations assumed low-energy angles and that the transition from 0 to 1 occurs over a range of energy. In we found that this probability is insensitive to the low-energy cutoff as long as sufficient continuum is included below the line candidate (in the example there, 15–20 keV). We have been using the two Ginga detections to characterize the unknown line distribution. For the line at 21.1 keV in the S1 segment of GB 870303, the transition between a detection probability of 0 and 1 occurs at an S/N of ~2, while for the harmonic lines in GB 880205 at 19.4 and 38.8 keV the transition occurs at an S/N of ~7; in both cases the detectability is also angle dependent. However, for greater generality we present, in Figure 3, \( M(L_0 | l_p HI) \) for a range of S/Ns and low-energy cutoffs. As can be seen, \( M(L_0 | l_p HI) \) is 50 for detecting lines similar to the one in GB 870303, assuming a low-energy cutoff of 15 keV will suffice. If the detectability of the GB 880205 lines is dominated by the 38.6 keV line (see Fig. 1 of Paper III), and thus a low-energy cutoff less than 25 keV is necessary, then \( M(L_0 | l_p HI) \approx 20 \). It is clear from these curves that despite the large number of bursts observed by BATSE in the past 5 years, lines would be detectable in relatively few bursts.

Only for the rare strong bursts are lines detectable in spectra accumulated by Ginga and the BATSE SDs. Since BATSE detects bursts with a much larger detector than the SDs, whereas Ginga detected bursts with the same detector that accumulated spectra, the BATSE burst database includes a larger fraction of weak bursts. In addition, Ginga reported lines at ~20 and ~40 keV. The Ginga burst detector was sensitive down to 2 keV (Murakami et al. 1989), whereas BATSE’s \( E_{\text{low}} \) is typically ~20 keV. On the other hand, each BATSE SD has an area twice that of the Ginga detector. Therefore BATSE is usually less sensitive than Ginga to lines below ~40 keV, and more sensitive above 40 keV.

The necessary Ginga data have not yet been extracted to complete the study of the consistency between the BATSE and Ginga observations as presented in Paper II. However, the small values of \( M(L_0 | l_p HI) \) for the BATSE bursts and

cutoff of 10 keV, which is rarely achieved because of the SLED electronic artifact, which raises the effective cutoff (Band et al. 1992) and the gain settings of the SDs. However, in Band et al. (1996, hereafter Paper IV) we showed that the detectability of a line is insensitive to the low-energy cutoff as long as sufficient continuum is included below the line candidate (in the example there, 15–20 keV). We have been using the two Ginga detections to characterize the unknown line distribution. For the line at 21.1 keV in the S1 segment of GB 870303, the transition between a detection probability of 0 and 1 occurs at an S/N of ~2, while for the harmonic lines in GB 880205 at 19.4 and 38.8 keV the transition occurs at an S/N of ~7; in both cases the detectability is also angle dependent. However, for greater generality we present, in Figure 3, \( M(L_0 | l_p HI) \) for a range of S/Ns and low-energy cutoffs. As can be seen, \( M(L_0 | l_p HI) \) is 50 for detecting lines similar to the one in GB 870303, assuming a low-energy cutoff of 15 keV will suffice. If the detectability of the GB 880205 lines is dominated by the 38.6 keV line (see Fig. 1 of Paper III), and thus a low-energy cutoff less than 25 keV is necessary, then \( M(L_0 | l_p HI) \approx 20 \). It is clear from these curves that despite the large number of bursts observed by BATSE in the past 5 years, lines would be detectable in relatively few bursts.

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where we have used the prior \( p(f | HI) = 1 \). In both equations (21) and (22), we extend the approximation in equation (16) to the regime \( f \approx 1/\langle L_0 | l_p HI \rangle \), where the approximation will have broken down. In Figure 2, we compare \( p(f | DHI) \) in equation (22) to \( p(f | DHI) = (m + 1)(1 - f)^m \), which results from the absence of a line detection in \( m \) bursts in which lines could have been detected with 100% probability [i.e., \( p(L_0 | l_p HI) = 1 \)]. As can be seen, the small-\( f \) approximation is accurate to a factor of ~2 in normalization and extent. Given the uncertainties and other approximations in this analysis, this accuracy is sufficient. In part, the small-\( f \) approximation demonstrates the utility of \( M(L_0 | l_p HI) \) as a diagnostic statistic.

5. DISCUSSION

The quantity \( M(L_0 | l_p HI) \) characterizes the detectability of spectral lines in a burst database and, thus, our ability to learn about lines from the database. Primarily, \( M(L_0 | l_p HI) \) is the approximate number of bursts in which lines could be detected. We have seen that its inverse is twice \( \langle L_0 | l_p HI \rangle \), the likelihood of the hypothesis \( H \), and that it is the width of the distribution for the line frequency \( f \). Using the burst database described in § 3, we calculated \( M(L_0 | l_p HI) \) for the BATSE spectra. Since the burst distribution—the frequency of lines of different types and where within the burst they occur—is unknown, we made a number of modeling assumptions. These assumptions, along with the supposition that lines exist and that the modeling of the BATSE detectors is correct, constitute the hypothesis \( H \). First, we assume that the line frequency \( f \) is the same for all bursts and that a line can occur in any spectrum with equal probability (the second variant of model 1 in § 2). Thus we use equation (8) to define \( \gamma_i \), i.e., \( \gamma_i = 2(t_e - t_o)(t_e - t_o)/T^2 \). Second, we approximate \( p(L_i | l_p HI) \), the probability of detecting a line in a spectrum, as \( \gamma_i \) for S/Ns above a threshold value if the low-energy cutoff is less than a certain energy. In Paper III, we found that this probability \( p(L_i | l_p HI) \) was a function of both the S/N and the burst angle and that the transition from 0 to 1 occurs over a range of S/Ns. The calculations in Paper III assumed a low-energy cutoff of 10 keV, which is rarely achieved because of the SLED electronic artifact, which raises the effective cutoff (Band et al. 1992) and the gain settings of the SDs. However, in Band et al. (1996, hereafter Paper IV) we showed that the detectability of a line is insensitive to the low-energy cutoff as long as sufficient continuum is included below the line candidate (in the example there, 15–20 keV). We have been using the two Ginga detections to characterize the unknown line distribution. For the line at 21.1 keV in the S1 segment of GB 870303, the transition between a detection probability of 0 and 1 occurs at an S/N of ~2, while for the harmonic lines in GB 880205 at 19.4 and 38.8 keV the transition occurs at an S/N of ~7; in both cases the detectability is also angle dependent. However, for greater generality we present, in Figure 3, \( M(L_0 | l_p HI) \) for a range of S/Ns and low-energy cutoffs. As can be seen, \( M(L_0 | l_p HI) \) is 50 for detecting lines similar to the one in GB 870303, assuming a low-energy cutoff of 15 keV will suffice. If the detectability of the GB 880205 lines is dominated by the 38.6 keV line (see Fig. 1 of Paper III), and thus a low-energy cutoff less than 25 keV is necessary, then \( M(L_0 | l_p HI) \approx 20 \). It is clear from these curves that despite the large number of bursts observed by BATSE in the past 5 years, lines would be detectable in relatively few bursts.

Only for the rare strong bursts are lines detectable in spectra accumulated by Ginga and the BATSE SDs. Since BATSE detects bursts with a much larger detector than the SDs, whereas Ginga detected bursts with the same detector that accumulated spectra, the BATSE burst database includes a larger fraction of weak bursts. In addition, Ginga reported lines at ~20 and ~40 keV. The Ginga burst detector was sensitive down to 2 keV (Murakami et al. 1989), whereas BATSE’s \( E_{\text{low}} \) is typically ~20 keV. On the other hand, each BATSE SD has an area twice that of the Ginga detector. Therefore BATSE is usually less sensitive than Ginga to lines below ~40 keV, and more sensitive above 40 keV.

The necessary Ginga data have not yet been extracted to complete the study of the consistency between the BATSE and Ginga observations as presented in Paper II. However, the small values of \( M(L_0 | l_p HI) \) for the BATSE bursts and

![Fig. 2.—Distribution for the line frequency \( f \), \( p(f | HI) \). In the first case (solid curve), the sum of the probabilities for each burst that a line would be detected if present is \( M(L_0 | l_p HI) = 40 \). This case is calculated in the approximation that \( f \) is small, which breaks down for \( f \sim 1/\langle L_0 | l_p HI \rangle \). In the second case (dashed curve), there are \( m = 40 \) bursts in which lines are always detectable.](image1)

![Fig. 3.—\( M(L_0 | l_p HI) \) as a function of the threshold signal-to-noise ratio ("SNR") for different maximum low-energy cutoffs. A line is detectable in a spectrum if the spectrum’s signal-to-noise ratio in the 25–35 keV band exceeds the threshold and the low-energy cutoff \( E_{\text{low}} \) is less than the maximum value labeling each curve. \( M(L_0 | l_p HI) \) is a measure of the number of bursts in which lines could have been detected, the inverse of the likelihood, and the width of the distribution for the line frequency.](image2)
the preliminary value of $M(L_p) | I_p HI \approx 5.4$ for a Ginga detection of lines similar to the one in GB 880205 (Fenimore et al. 1993) indicate that the apparent discrepancy between BATSE and Ginga is not severe. For example, the probability that the one detection of a line similar to GB 880205 occurred in the Ginga bursts and not in the BATSE data is (Papers I and II) $P \approx M_{\text{Ginga}} (M_{\text{Ginga}} + M_{\text{BATSE}}) \approx 5.4/(5.4 + 20) \approx 0.2$, which is hardly an improbable event.

6. SUMMARY

We now have a methodology that provides the probability for detecting a line in the bursts observed by BATSE. This probability can be used to evaluate the consistency between the line detections and nondetections by BATSE and other burst missions, to estimate the frequency with which lines occur, and to identify bursts in which lines are likely to be discovered by new search techniques. The new element in the methodology is the weighting of the probabilities for detecting a line in each of the spectra spanning the burst, which can be formed from the SHERB spectra provided by the telemetry. This weighting is model dependent; we have explored three models in which the line occurrence depends only on the time a line persists.

Implementing this methodology requires parameter values characterizing each spectrum in the bursts under consideration. To this end, we built a database of the necessary information. We have been using this database to identify the bursts in which lines may be detected.

We calculated the number of bursts in which lines of various types, as parameterized by the minimum S/N and maximum low-energy cutoff necessary for a detection, could have been detected if the lines were indeed present. This calculation assumes a small line frequency. These quantities are necessary for the probability that no lines would be detected in the BATSE data and for the distribution for the line frequency, and therefore these numbers are essential for measures of the consistency between the BATSE and Ginga line observations. Ginga-like lines can be detected in relatively few of the large number of bursts BATSE has observed; for example, lines similar to the GB 880205 pair are detectable in only $\sim 20$ BATSE bursts. Although comparable Ginga data are not yet available, the discrepancy between Ginga and BATSE does not appear to be severe. For example, a simple calculation shows that the probability that the GB 880205 line set would be detected in a Ginga burst is 20%, which is hardly improbable.

Finally, to evaluate the significance of line candidates, we have adopted the maximum likelihood ratio test, which is more appropriate than the $F$-test. The $F$-test should be used when the uncertainties on the data points are unknown. These two tests yield similar significances when the reduced $\chi^2$ is of order unity. Indeed, we find little change in the significances given by the two tests for the line candidates identified by the visual search of BATSE spectra.

We thank C. Graziani, D. Lamb, and T. Loredo for insightful discussions about statistics. We are grateful for the detailed comments of the referee, A. Connors. The work of the University of California, San Diego group is supported by NASA contract NAS 8-36081.

APPENDIX A

SIGNIFICANCE STATISTIC

To evaluate the significance of a given spectral feature, we have been using the $F$-test, which compares fits with nested models (i.e., one model is a subset of the other). Assume that $\chi^2_1$ results from fitting a spectrum of $N_1$ channels by a continuum model with $r_1$ parameters (thus $v_1 = N_1 - r_1$ degrees of freedom) and that $\chi^2_2$ results from fitting the spectrum by a continuum-plus-line(s) model with $r_2$ parameters ($v_2 = N_2 - r_2$ degrees of freedom). In the continuum-plus-line(s) model, the $r_1$ continuum parameters are the same as for the continuum model; thus an additional $\Delta v = r_2 - r_1$ parameters have been added by modeling the line(s). If the continuum model is correct and there are actually no lines, then the quantity

$$F_0 = \frac{\chi^2_1 - \chi^2_2}{\Delta v} \sqrt{\frac{\chi^2_2}{v_2}}$$

is distributed as $F(\Delta v, v_1)$. Consequently, $P(F \geq F_0)$ is the probability of finding $F$ greater than or equal to $F_0$ when a continuum-plus-line(s) model is fitted to a count spectrum resulting from a photon spectrum correctly described by the continuum model. This is thus the probability that the improvement in $\chi^2$ by adding the additional $\Delta v$ line parameters is a fluctuation.

The $F$-test we have been using is based on a maximum likelihood ratio test in which the uncertainties are unknown. The original version defines $\chi^2$ without uncertainties, $S^2 = \sum_{i=1}^{N_c} (y_i - m_i)^2/N_c$, where $y_i$ is the observed value and $m_i$ is the model value. Then

$$F_1 = \frac{S^2_1 - S^2_2}{S^2_2} \frac{v_1}{\Delta v}$$

is distributed as $F(\Delta v, v_1)$ (Eadie et al. 1971, p. 238). Since $N_c$ is large, there is little difference between $v_1$ and $v_2$. Both $F_0$ and $F_1$ use ratios of $\chi^2$ and $S^2$, respectively. Thus the $F$ statistic eliminates the effect of a systematic multiplicative error in the uncertainties used in $\chi^2$ or of an unknown constant uncertainty on the data points in $S^2$. This demonstrates why the $F$-test is appropriate for the case in which the uncertainties are not known.

However, we find that the uncertainties in our data result predominantly if not exclusively from counting statistics, and consequently the uncertainties are known. Thus we can use the fundamental maximum likelihood ratio test (MLRT), from which the $F$-test is derived. This test states that $\Delta \chi^2 = \chi^2_1 - \chi^2_2$ is distributed as $\chi^2(\Delta v)$ if the continuum model is sufficient and no lines are present (Eadie et al. 1971, pp. 230–237). As with the $F$-test, a small value of $P(\Delta \chi^2 \geq \chi^2_0)$ indicates a small
probability that the continuum model alone describes the data. In Figure 4, we compare the MLRT with the F-test (using the $F_0$ statistic of eq. [A1]) for the same values of $\Delta \chi^2$ and different values of the reduced $\chi^2$. As Figure 4 shows, the two tests yield the same values for $\chi^2_0$ slightly less than 1, which is not surprising, since we expect $\chi^2_0 \sim 1$ if our spectral model is correct. A value of $\chi^2_0$ that differs significantly from unity may result from an incorrect value for the uncertainties used in $\chi^2$, which the F-test attempts to correct. However, other factors may cause $\chi^2_0$ to differ from unity, such as an incorrect continuum model and inaccuracies in the detector response model and the energy calibration. This is a major reason to favor the MLRT. To determine the continuum from which a candidate line deviates, we include all the spectral data, including continuum far from the line. However, we do not know the true continuum shape, which might raise $\chi^2_0$. In addition, the F-test depends on the total number of data points (the $F$ statistic has a distribution that is a function of the number of degrees of freedom). On the other hand, the MLRT is a function of the number of added parameters, $\Delta v$.

In most cases the MLRT and the F-test will lead to the same conclusion as to whether a feature is significant. Indeed, evaluating the line candidates from the visual search of BATSE SD spectra with the MLRT as opposed to the F-test (which was used in Paper IV) does not lead to a qualitative difference in significance. Figure 5 compares the probabilities given by these two tests. As concluded in Paper IV, none of the line candidates are significant.

**APPENDIX B**

**NOTATION**

The following is a list of the symbols used in this paper:

- $b, c$: Constants used in modeling $g(t_w, t_c)$
- $C_i$: Total counts over energy range $\Delta E$ in the $i$th spectrum
- $\chi^2_i$: The $\chi^2$ statistic for the $i$th spectral fit
The observations, specifically, whether lines were detected in a burst database.

\( \Delta \chi^2 \) The difference in \( \chi^2 \) between continuum and continuum-plus-line(s) fits to a spectrum with a candidate line feature.

\( \Delta E \) Width of the energy range over which the S/N is measured.

\( \Delta t \) Accumulation time of the \( i \)th spectrum.

\( \Delta v \) Number of parameters added by modeling a line.

\( E_{\text{low}} \) Low-energy edge of the usable energy range.

\( f \) The frequency with which a line type is present in any burst; the use of \( f \) assumes that each burst, regardless of its characteristics, has the same probability of hosting the line type.

\( g(t_p, t_e) \) The factor in \( p(D) / f_HI \) for the Ginga and BATSE burst databases, respectively.

\( \gamma_i \) The factor in \( p(D) / f_HI \) for the Ginga and BATSE burst databases, respectively.

\( H \) Hypothesis about the presence of lines and the operation of the BATSE and/or Ginga detectors.

\( I \) The proposition representing our understanding of the burst detector and other information known or assumed about the burst.

\( l_a \) The proposition that a line is present in the \( \sigma \)th burst.

\( l_i \) The proposition that a line is present in the \( i \)th spectrum (an index specifying the burst is suppressed).

\( L_a \) The proposition that a line is detected in the \( \sigma \)th burst.

\( L_i \) The proposition that a line is detected in the \( i \)th spectrum (an index specifying the burst is suppressed).

\( M(L_o | l_o HI) \) The number of SHERB spectra spanning the \( i \)th burst.

\( N \) Number of bursts in a burst database.

\( N_c \) Number of channels in a spectrum.

\( n_d \) Number of line detections in a burst database.

\( N_{HI} \) Number of bursts in the \( HI \) burst.

\( p(D) / f_HI \) The likelihood of \( H \) and \( f \), the probability of observing \( D \) given hypothesis \( H \) and line frequency \( f \).

\( p(D) / HI \) The likelihood of \( H \), the probability of observing \( D \) given hypothesis \( H \).

\( p(f) / HI \) “Prior” for the line frequency, the probability distribution for \( f \) given \( H \) and \( I \), but without knowledge of \( D \).

\( p(f) / DHI \) Probability distribution for \( f \) based on the observations \( D \) given \( H \) and \( I \).

\( p(L_o | l_o HI) \) Probability of detecting a line somewhere in the \( \sigma \)th burst when none is present.

\( p(L_o | l_o HI) \) Probability of detecting a line in the \( i \)th spectrum, assuming a line frequency \( f \).

\( p(L_o | l_o HI) \) Probability of detecting a line in the \( i \)th spectrum given that it is indeed present.

\( p(L_o | l_o HI) \) Probability of a false positive, i.e., of detecting a line in the \( i \)th burst when none is present.

\( p(l_o | f) / HI \) Probability that the line is in the \( i \)th spectrum, assuming a line frequency \( f \).

\( p(l_o | f) / HI \) Probability that if a line is present in the \( \sigma \)th burst, it is in the \( i \)th spectrum of that burst (an index specifying the burst is suppressed).

\( R_B \) Background count rate in a detector over energy range \( \Delta E \).

\( r_i \) Number of parameters in the \( i \)th spectral fit.

\( t_b \) Time line first becomes apparent.

\( t_e \) Time line is last apparent.

\( T \) Burst duration.

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