Macro- and Micro-structure of Trust Networks

A challenging problem in the social sciences is the characterization of the formation of “social capital” [1, 2]. It is believed that societies with more social capital are more democratic and economically developed than societies with little social capital [2]. However, social capital is a concept hard to quantify and measure in a “real-world” context. Here, we take advantage of an existing “web of trust” between users of the ”Pretty-Good-Privacy” (PGP) encryption algorithm [3, 4]. The PGP algorithm is used in digital communication to “sign” documents so that the recipient knows for sure who the author is. Our analysis reveals the coexistence in the web of trust of a macro scale-free structure with micro strongly-connected cells in a complex network [6, 7, 8, 9]. We also show that when this network is intentionally attacked [10] the scale-free structure [5] rapidly disintegrates and that the resulting network is partitioned into a large number of small strongly-connected cells that are resilient to the intentional attack.

In digital communication, as in clandestine organizations, there is the distinct possibility that identities will be forged. Hence, there is the need to develop systems by which one can trust that someone is who he claims to be. A possible solution is a centralized service that certifies users; another is a self-organized system in which users certify one another. In the PGP web of trust, a user certifies another by “signing” his public encryption key [3, 4]. For such a decentralized solution to work, the system has to build social capital—i.e., to add more signatures—otherwise it will not work efficiently. Hence, we suggest that the PGP web of trust is a social network for which one can measure the formation of social capital and also the structure of the network itself (Fig. 1a).

We calculate the in- and out-degree cumulative distributions for the web of trust, and find that this distribution decays as a power law $P(k) \propto k^{-\alpha}$, with exponents $\alpha_{in} = 1.8 \pm 0.2$ and $\alpha_{out} = 1.7 \pm 0.2$ (Fig. 1b). We also find that the network is not a connected graph but instead comprises many strongly-connected clusters with a very wide range of sizes (from 2 to approximately 10,000 nodes). This feature of the network enables us to study its structure in more detail by considering the properties of each cluster separately: We measure the clustering coefficient $C$—which quantifies to which extend nodes adjacent to a given node are also adjacent to each other [11]—for each of the clusters in the network and find that it remains approximately constant (Fig. 1b). This result is rather surprising since $C$ decreases
rapidly with cluster size for a “pure” scale-free network [5] or a random graph [6].

The strongly-connected clusters are cross-linked by means of a few high degree nodes that act as “hubs” (Fig. 1c). The hubs appear to be organized in a hierarchical fashion giving rise to the scale-free structure while the strongly-connected clusters give rise to the large value of $C$. The coexistence of the scale-free connectivity and the strongly-connected clusters has implications for the vulnerability of trust networks. To demonstrate this assertion, we apply to the largest cluster the attack method proposed in Ref. [10]. As shown in Fig. 1d, we find that: (i) The network is extremely fragile against the systematic removal of the most connected nodes—due to the highly skewed connectivity of the hub super-structure; and (ii) the “lower-level” strongly-connected structure remains essentially un-affected. These two consequences of the structure of the web of trust have implications regarding the potential recovery of the network after an attack targeting the most connected nodes. Indeed, because the network still comprises large strongly-connected cells even after the largest cluster has been broken, it would be possible to quickly rebuild a large fraction of the original network by creating just a few links.

An important point that does not escape our attention is that there are plausible similarities between the structure of the web of trust we analyze here and the structure of clandestine/secret organizations [11]. In clandestine organizations trust is an essential ingredient because each member has has to be confident that her associates are not going to betray her. Moreover, as we found for the web of trust, clandestine organizations comprise small strongly-connected cells which are then connected to one another through a few highly-connected individuals [11].
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FIG. 1: Growth and structure of the PGP web of trust. The links in the PGP data set refer to the signatures of users public keys by other users. 

(a) Number of public keys in two PGP databases (“ftp://ftp.uit.no/pub/crypto/pgp/keys/pubring.pgp” and “http://dtype.org”) and average path length between those keys. We find that the number of signatures increases exponentially while the average path length increases linearly. These results can be interpreted as an increase in the social capital of the PGP web of trust because the number of users—i.e., of public keys—connected efficiently—i.e., only a few degrees of separation apart—is increasingly rapidly with time.

(b) Cumulative distribution of in- and out-degrees for the PGP web of trust. We analyze data recorded on July 2001 at “http://dtype.org”, when the web comprised 191,548 individuals and 286,290 directed links between them. The data follow a power law dependency with exponents -1.8 for the in-degree and -1.7 for the out-degree. (Inset) Clustering coefficient $C$ for the different strongly-connected clusters. For comparison, we also show the values of $C$ for (i) the small-world network model of, (ii) the scale-free network model of, and (iii) a random graph. $C$ is approximately constant for the web of trust, while for (ii) and (iii) it decays rapidly with cluster size.

(c) A strongly-connected cluster comprising 21 nodes. This cluster is strongly connected because every node is reachable from any other node. White lines indicate bi-directional links while yellow arrows indicate uni-directional links. The red nodes indicate the two strongly-connected clusters that would be left if one removes the green node (the hub).

(d) Intentional attack on the nodes with the highest in-degree of (i) the largest strongly-connected cluster of the web of trust and (ii) a random graph with the same in- and out-degree distributions. Initially, both graphs have 9562 nodes with an average degree of 5.80. As the fraction $f$ of nodes removed increases, the cluster is split into smaller components: (Top) Relative size $S$ of the largest strongly-connected cluster. (Bottom) Average size $\langle s \rangle$ of the other strongly-connected clusters. Note that for the web of trust the largest strongly-connected cluster breaks down faster but that the other strongly-connected clusters have average sizes that remain unchanged up to the total destruction of the largest strongly-connected cluster. For the random graph, the small clusters formed by removing nodes are almost all isolated nodes, which explains the slower decrease of $S$ and also the constant value of $\langle s \rangle = 1$. 

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