Interaction of pressure waves with a bubble layer near free surface of a liquid

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Abstract. The radiation of pressure waves by a bubble layer near the free surface of a liquid excited by a high-intensity sound pulse has been studied. It was shown that the bubble layer exhibits the properties of a resonator with one partially transparent boundary. Bubbles on the boundary of the layer due to the interaction through the self-consistent field with the bubbles inside the layer perform collective pulsations. It was found that the emission spectrum of the layer is ruled. The lines of the emission spectrum of a layer have a regular composition and lie above the frequency of the intrinsic pulsations of bubbles.

1. Introduction

The importance of researches of sound propagation in a liquid with bubbles is determined by many applications. The relevance of research is determined not only by their influence on the work of sonars, but also by the fact that these fields contain information about the processes taking place in the Ocean [1, 2]. Similar questions arise in applied sonochemistry [3]. Typical objects of research are bubble clouds or bubble layers. In liquid with bubbles attenuation of a sound and distortion of a form of signals considerably increases. These properties should be able to be considered at a sound hydrolocation in the ocean. This leads to the need to increase the intensity of the probing signals. An increase in sound intensity leads to various nonlinear effects that provide additional information about the bubble layer [4].

The mathematical models of this problem consist of a set of two differential equations. The first of them, the linear non-dissipative wave equation, describes acoustic pressure changes in a suspension of gas bubbles in liquid [2, 3, 5]. This equation was previously obtained in [6, 7]. The second one is the Rayleigh–Plesset equation. The linearized right-hand side of the wave equation does not reflect a nonlinear change in the volumetric gas content in the bubble layer and, therefore, only partially describes nonlinear effects. The paper [8] presents a method for calculating the nonlinear acoustic cross-sections, scatter, attenuations and sound speeds from bubble clouds which may be inhomogeneous. The method was used for sound with an amplitude of 40 kPa.

The aim of this work is to analyze the radiation characteristics of a bubble layer near the free surface of a liquid excited by an intense pressure wave.

2. Physical formulation of the problem

A bubble layer is located near the free surface of the liquid. The layer thickness is chosen equal to the formation distance of the resonant soliton [9, 11]. In liquid, a plane wave of large amplitude falls on the bubble layer. The incident wave is partially reflected from the layer and partially enters inside. The
excited bubbles in the layer perform volume collective pulsations and emit sound waves into the liquid. The free surface is totally reflective boundary.

3. Nonlinear wave system of equations
The radiation characteristics of the bubble layer were studied using the wave model [9 – 11]. The wave system of equations in the one-dimensional case has the form

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = -\frac{1}{c^2} \frac{\partial}{\partial t} \left( p \frac{\partial}{\partial t} \ln (1-\alpha) \right),$$

(1)

$$R_k \frac{d^2 R_k}{dt^2} + \frac{3}{2} \left( \frac{dR_k}{dt} \right)^2 + \frac{4\mu}{\rho_0 R_k^2} \frac{dR_k}{dt} + \frac{2\sigma}{\rho_0 R_k} \frac{1}{R_k} \left( \frac{p}{R_0} + \frac{2\sigma}{R_k} \right) \left( \frac{R_{\infty}}{R_k} \right)^{\gamma} - \frac{p}{\rho_0} - \frac{p(x,t)}{p_0},$$

(2)

$$\alpha(x,t) = \sum_k \frac{v_k(t)}{\Omega},$$

(3)

$$v_k(t) = \frac{4}{3} \pi R_k^3(t),$$

(4)

where $k = 1, ..., N$, determines the total number of bubbles in the area under investigation $\Omega$, $\delta(x-x_k)$ characterizes the position of the $k$-th bubble from the region occupied by the bubbles, $p(x,t)$ is pressure in the wave, $P_0$ is initial pressure in the medium, $c_0$ is the speed of sound in pure liquid, $\alpha$ is volumetric gas content, $R_k$ is radius of the $k$-th bubble, $v_k$ is volume of the $k$-th bubble, $p$ is density of liquid, $\sigma$ is surface tension of liquid, $\mu$ is water viscosity, $\gamma$ is isentropic exponent, $t$ is time, $x$ is space coordinate. Water under normal conditions was chosen as liquid for calculations.

In the calculations, it was assumed that the gas in the bubbles obeys the adiabatic law and for a liquid the condition of linear acoustics $p(x,t) = \rho(x,t)c_0^2$ is satisfied. On the free surface the condition $p=0$ is satisfied.

Calculations were made for values of the parameters of the medium: $P_0 = 0.1$ MPa, $R_0 = 0.25 \times 10^{-3}$ m, $\gamma = 1.4$, $\rho = 10^3$ kg/m$^3$, $c_0 = 1500$ m/s. The width of the layer is $h = 3.89 \times 10^{-2}$ m. The wave amplitude is $P_0 = 0.5$ MPa, the duration of the incident wave is $\tau = 30 \times 10^{-6}$ s. The volumetric gas content is $\alpha = 10^{-3}$.

The system of equations (1) - (4) is reduced to a dimensionless form using the relations

$$\tilde{p} = p / P_b, \quad R_k = R_k / R_0, \quad t = t / \tau, \quad x = x / (\tau c_0), \quad \tau = R_0 \sqrt{\rho_0 / (3\gamma (P_0 + P_b))}.$$  

(5)

The numerical solution of the system of equations (1) - (4) is carried out by finite difference methods. To solve the wave equation (1), the implicit Crank-Nicholson scheme was used. The scheme gives convergent results with a second order accuracy. The solution of the resulting difference equations was carried out by the sweep method. The Rayleigh equation for each bubble was solved by the fourth-order Runge – Kutta method. Verification of the system of equations (1) - (4) was carried out on the basis of comparison with the experimental results [4] and is given in [9]. The equations are applicable for wave amplitudes of $\sim 2$ MPa.
4. Discussion of results

Fig. 1 presents the pressure pulse exciting the bubble layer. The pulse energy is $E_w = 2.89\, J$.

![Figure 1. Structure wave pulse exciting the bubble layer.](image1.png)

![Figure 2. The structure of the wave field in the layer with the same bubbles near the free surface.](image2.png)

During the interaction of a pulse with a bubble layer, the part of energy is carried away by the reflected wave. The remaining energy is trapped by the layer and then radiated into the liquid. Fig. 2 shows the spatial profile of the wave field for the layer with monodisperse bubbles. The spatial profile is represented at the moment when a resonant soliton is formed inside the bubble layer. The distance of soliton formation for the selected parameters of the medium is $0.04\, m$. From this moment of time the bubble layer begins to radiate the captured energy. Under the condition $p = 0$, the wave is completely reflected from the free surface.

The energy of the reflected wave is $E_w = 0.68\, J$. Under these conditions, 24% of the wave energy is reflected from the layer. 76% of the energy is captured by the layer. The energy in the layer is distributed between the bubbles and the wave. At the moment, the energy in the wave is $E_w = 1.2\, J$. The energy in the bubbles is $E_b = 1.0\, J$. The energy in the bubbles is the sum of the potential energy of the gas in the bubble and the kinetic energy of the motion of the added mass. The total energy in the layer at the moment is equal to $E = 2.2\, J$. The energy is continuously redistributed between bubbles and wave over time. The overall decrease in energy in the layer is due to the emission of sound into the liquid.

For low intensity waves, most of the energy of the incident wave is reflected from the layer than for intense waves. For example, for a wave with an amplitude of 500 Pa, the reflected wave contains 56% of the energy of the incident wave. The transparency of the boundary of the bubble layer increases by a factor of $2.43$ as the amplitude of the incident wave increases from 500 Pa to 0.5 MPa.

Fig. 3 presents the temporal dependence of the wave field in liquid. At large times, the layer emits sound in a linear range. The radiation of the bubble layer can be divided into three phases. The first phase has a short duration. At this stage, the bubble layer quickly loses energy due to intense nonlinear radiation. At the second stage, the bubble layer produces a weakly non-linear sound. This phase is longer than the first one. Then, the phase of the emission of linear sound comes. The radiation has a low-frequency modulation and lasts several orders of magnitude longer than the first two phases. This is due to the interaction of bubbles inside the layer between themselves and the wave field. In the first phase, the main part of energy is radiated by the bubble layer in the form of pulses.
Figure 3. Wave field in a fluid at a large distance from the layer as a function of time.

The nonlinear phase of the radiation lasts $\sim 10^{-2}$ seconds under the given conditions. This range is approximately 100 times longer than the duration of the incident wave. The amplitude of the radiation wave decreases to a value of $0.01 P_0$. A feature of this phase is the emission of sound pulses by a bubble layer. In this case, soliton-like structures are formed inside the layer, which, leaving the layer, form the pulses observed in the time dependence of the sound radiation $P(t)$. Fig. 4 shows the time dependence of the initial phase of radiation of a pressure wave by a bubble layer.

Fig. 5 presents the spatial profile of the pressure field, which shows the first pulse emitted by the layer and the profile of the radii of bubbles in the bubble layer. A comparison of the graphs of the distribution of the pressure field and the distribution of the radii of bubbles shows that soliton-like wave structures [9] have been formed in the layer at the moment, which after exiting the bubble layer form pulses in a pure liquid.

Figure 4. The initial phase of the bubble layer radiation as a function of time. Figure 5. The spatial profile of the initial phase of radiation of a bubble layer.

During the period of the nonlinear phase, the layer emits $\sim 90$% of the stored energy. Fig. 6 is a graph of bubble pulsations located on the boundary of the bubble layer as a function of time. As follows from the graph, bubble pulsations are highly non-linear.
The second phase of the radiation layer can be characterized as a transition from a strongly nonlinear phase of radiation to a linear phase. In this intermediate phase, the emission spectrum has not yet stabilized. Next, we consider the phase of linear radiation, which occurs at times of $\sim 1 - 2 \times 10^{-2}$ sec.

Fig. 7 presents the fragments of emitted sound wave and pulsation of a bubble in the linear range. The frequency of free pulsations of the bubble is described by formula $f_0 = 1 / (2\pi R_0) \sqrt{\gamma P_0 / \rho_0}$. For the considered bubbles with $R_0 = 0.25 \times 10^{-3} m$, the frequency of free pulsations is $f_0 = 7536$ Hz or in a dimensionless form $f_0 = 0.0375$. This frequency is the only parameter with which the spectral emission lines of the layer can be compared.

Fig. 8 shows the emission spectra of the bubble layer. Figure 8a shows the spectrum of the nonlinear phase of the radiation. The spectrum starts from zero and has pronounced highs. In addition, the spectrum has the characteristic features of the spectra of pulsed signals. This suggests that the layer in this phase periodically emits pulsed signals. This is consistent with the graph in Figure 5. The maximum line of the spectrum is at a frequency of 0.061.

In the linear phase, the amplitude of the emitted sound wave is $2 \times 10^{-5}$ of the magnitude of the excitation wave.

Fig. 8b shows the spectrum of the linear phase of the radiation. The spectrum consists of individual lines with an inhomogeneous distribution of lines within the spectrum. In the low-frequency part of the spectrum, the lines are separated. In the high-frequency part of the spectrum, the lines are located close to each other and are not completely separated. The maximum line is located at a frequency of 0.05. The value of this line is 7 times less than the maximum line in the spectrum of the nonlinear phase of the radiation.
Figure 8. The emission spectrum of a layer with monodisperse bubbles. Volumetric gas content $\alpha=10^{-3}$.

a) Spectrum of the nonlinear phase of radiation. b) Linear phase spectrum.

The lower limit of the emission spectrum frequencies for a layer is 0.055, and the upper limit is limited by the frequency of 0.065.

In all the cases studied, the lower limit of the emission spectra has a significantly higher frequency than the frequency of free pulsations of the bubble. Thus, it follows that the radiation of a bubble layer is determined solely by the properties of the layer as a resonator and associated collective pulsations of bubbles inside this resonator.

The above results show that the interaction of large-amplitude pulses with a bubble layer gives additional information about the layer. This can be used for underwater sonar.

Conclusion
Studies have shown that the bubble layer is a complex resonator with a dynamic internal structure. The boundary of the layer with a liquid is a bubble emitter, the pulsations of which depend on the coordinated collective dynamics of bubbles inside the layer. The collective dynamics in the layer is determined by the self-consistent dynamics of the wave field and reaction field of the bubbles. The spectral characteristics of radiation are not associated with resonant characteristics of bubbles. All lines of the spectrum lie above the frequency of the intrinsic pulsations of the bubbles.

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