A note on a new paradox in superluminal signalling

V.F. Perepelitsa
ITEP, Moscow

Abstract
The Tolman paradox is well known as a base for demonstrating the causality violation by faster-than-light signals within special relativity. It is constructed using a two-way exchange of faster-than-light signals between two inertial observers who are in a relative motion receding one from another. Recently a one-way superluminal signalling arrangement was suggested as a possible construction of a causal paradox. In this note we show that this suggestion is not correct, and no causality principle violation can occur in any one-way signalling by the use of faster-than-light particles and signals.
1 Introduction

In 1959 E.C.G. Sudarshan has developed a hypothesis of faster-than-light particles which was published in [1]. This publication was followed by a paper by G. Feinberg who coined a word *tachyon* to name these particles [2].

Not surprisingly, the hypothesis of faster-than-light particles has encountered strong objections related to the principle of causality. It has been shown in several papers [3, 4, 5, 6], in agreement with an earlier remark by Einstein [7] (see also [8, 9, 10]), that by using tachyons as information carriers one can build a causal loop, making possible the information transfer to the past of an observer, thus creating a causal paradox. The base for the construction of casual loops is the so called Tolman paradox [8, 9] in which two inertial observers, receding one from another, communicate via faster-than-light signals. This paradox, known for a while, prohibits faster-than-light particles and signals within Einstein’s special relativity (SR) [7].

Recently, in a paper by M. Fayngold, a simpler construction of a causal paradox was suggested based on a one-way superluminal signalling [15]. The new paradox is based on an assumption about a non-invariance of the information flow direction when using tachyons as information carriers. Unfortunately, this assumption is wrong from both common sense and the information theory applied to superluminal communications. A correct approach to the problem is considered in this note.

2 One-way superluminal signalling

Let us consider the exchange of tachyon signals between two inertial observers (counterparts) receding one from another with the constant velocity $u$. In principle, under the term “tachyon signal” one can assume a modulated tachyon beam carrying the arbitrarily rich information, but without loss of generality we adopt for the moment an approach that the signal can be transferred by a single tachyon.

Let us assume that the both observers are equipped with tachyon emitters, capable to emit fast tachyons having velocities $v > c^2/u$ to his counterpart and tachyon detectors (say, time-of-flight systems allowing to detect and identify tachyons, at least those which go along the line connecting the observers A and B). But first we consider a one way superluminal signalling, namely, a launching at $t_A^0$ of a fast tachyon $\alpha$ from the observer A to the observer B (Fig. 1). At the moment $t_A^1$ the tachyon $\alpha$ reaches the observer B and is detected by his apparatus. The sequence of these events is trivial from the point of view of the observer A.

However, it is not so trivial from the point of view of the observer B. For him, due to Lorentz transformation, the moment $t_B^1$ precedes the momentum $t_B^0$, and therefore he interprets the sequence of the events as a spontaneous emission by his detector of the

---

1The solution of this paradox in the framework of a model of a spontaneous breaking of SR is given in [11, 12]. While conserving causality, this model confines the effects of the violation of the Lorentz symmetry by tachyons, interacting with ordinary particles, within the free tachyon sector (i.e. within the sector of asymptotic tachyon states), thus avoiding strong experimental restrictions on such violations, compiled e.g. in [13, 14].

2At such velocities, under an appropriate Lorentz transformation, tachyons become antitachyons going backward in time.
tachyon signal (antitachyon $\pi$), which propagates to the observer $A$ and is absorbed by the apparatus of the latter at $t_0^B$.

For some reason, this effect is considered as a violation of causality in many books on the SR. The reason is the orthodox formulation of the principle of causality: cause always precedes the effect.

Fig. 1. Exchange of a tachyon $\alpha$ as seen a) by the observer $A$ to be an emission of this tachyon launched by him to reach the observer $B$, and b) as seen by the observer $B$ to be a spontaneous emission of an antitachyon $\pi$ moving to the observer $A$. The observer’s world lines are shown by solid lines, those for tachyons are shown by dashed ones, the directions of tachyon motion (in space and time) being indicated by arrows. The “dead time interval” preceding the emission of the tachyon $\alpha$ at $t_0^A = 0$, symbolized by a wave line, will be used later in the construction of the Tolman paradox.

Indeed, such an ordering of cause and effect allows avoiding problems with causality in the world of ordinary particles. But as we shall see later, this formulation should be changed when considering faster-than-light particles and signals.

3 Construction of a one-way causal paradox and its failure

3.1 The construction

The main idea of a construction of a one-way causal paradox suggested in [15] is the introduction of a third observer, $C$, positioned between the observers $A$ and $B$, who can control the passage of the superluminal signal through its position (in both directions), for example, by intercepting the signal by a plug which does not transmit tachyons. Then the sequence of events is considered in both observer frames, $A$ and $B$, and an appearance
of a logical inconsistency in this sequence is declared in the cases when the plug blocks
the passage of the tachyon signal at the position of C.

Indeed, if one follows strictly the orthodox principle of causality, as the author of [15]
does, the introduction of the plug, viewed from the frame A, removes the part of the
tachyon world line from C to B. On the other hand, this action, viewed from the frame
of the observer B (who, according to [15] with the reference to the orthodox principle
of causality, hosts the cause in this case) should remove the part from C to A. Such an
ambiguity is considered in [15] as a logical paradox which can be used to ban faster-than-
light particles and signals.

3.2 The failure

What is wrong in this construction? The answer is: it violates the principle of the
invariance of the information flow direction.

It is easy to prove that this principle holds in any process of the information transfer,
whatever could be the time order of the sending and receiving of the information (as we
have seen, this order can be reversed in the case of the tachyon exchange). For the proof
we may turn our consideration from a single-tachyon signal to the signal containing much
information (e.g. transmitted by a modulated tachyon beam). Then two straightforward
arguments can be used in order to prove the invariance of the information flow direction.

First, we note that any information message presents a sequence of symbols (e.g.
letters and numbers) separated by time-like intervals. Therefore the time ordering of the
symbols in this sequence is invariant, i.e. it does not change even when the message is
sent with the faster-than-light (spacelike) carriers, which can go backward in time.

Second, each information message can bear an identifier of its sender, so the source of
the message can be determined without doubt in any reference frame, whatever could be
the time order of the message sending and receiving.

The application of the principle of the invariance of the information flow direction
results in doubtless identification of a cause and its effect in any causally related sequence
of events. Applying it to the consideration of a concrete one-way signalling described
above, we arrive to four different situations, all of them being logically self-consistent:

a) If the observer A issues a superluminal signal and it is blocked by the observer C,
i.e. at the position of C, all the observers record the pair of the events (A, C).

b) If the observer B issues a superluminal signal and it is blocked at the position of C,
all the observers record the pair (B, C).

c) If the observer A issues a superluminal signal and it is not blocked by the observer
C, i.e. it goes to B and is absorbed there, all the observers record the pair of the
events (A, B).

d) If the observer B issues a superluminal signal and it is not blocked at the position
of C, all the observers record the pair (B, A).

The time ordering inside the pairs, i.e. the time ordering of a cause, invariantly associ-
ated with the signal source (and presented by the first letter in parentheses in the items
above), and its effect can be different for different observers, but unless a two-way sig-
nalling is constructed (in the spirit of the Tolman paradox, described in Sect. 5) no logical
contradiction in these situations exists.
4 The principle of causality

In the view of inapplicability of the orthodox principle of causality when considering faster-than-light particles and signals, it has to be reformulated. However, the modified principle of causality should reduce to the orthodox one when being applied to ordinary particles.

Therefore, to get a consensus, the causality principle should be reformulated as follows: any cause has an unalterable own origin. In other words, the causality principle appears to be a requirement of the impossibility of the creation of causal loops (i.e. causal chains containing closed world lines) which admit the change of the conditions of their own creation. Implemented in the world of ordinary particles the modified causality principle reproduces the orthodox one, while in the world with faster-than-light particles it allows a consideration of a superluminal signalling by the exchange of tachyons, conserving at the same time the principle of the invariance of the information flow direction.

The modified principle of causality automatically excludes one-way causal paradoxes. Referring to [11, 12], we note that accepting the modified causality principle one can rehabilitate the tachyon hypothesis, removing from it the causality problem and the problem of the tachyon vacuum instability, though at the price of abandoning the SR postulate about the equivalence of inertial frames when dealing with the faster-than-light particles and signals (by introducing a tachyon preferred reference frame), which has to be replaced by the requirement of a covariant formulation of a tachyon theory (from the point of view of the present author, the price is not tremendously high).

However, within the SR the tachyon hypothesis remains incompatible even with the modified principle of causality, as can be demonstrated via the Tolman paradox construction.

5 The Tolman paradox

In order to make the demonstration obvious, let us introduce a mandatory condition for the emitting of the tachyon signal α by the observer A: the signal should be sent at \( t_A^0 \) if and only if there were no other tachyon signal coming from the observer B and registered by the apparatus of the observer A during a certain preceding period \( T \). The duration of \( T \), equal to the distance between the observers A and B at the moment \( t_A^0 \) divided by \( c \), would be sufficient for the argumentation.

As we have described above, the observer B receives this signal as coming from his future, though from the space-like separated region, as shown in Fig. 1b. However, if the observer B could not produce any influence to the signal sending (namely, to the

\[ 3 \]In particular, the modified principle of causality excludes the possibility of the realization of time-machine solutions of Einstein equations, first considered by K. Gödel in [16]. On the other hand, it agrees with the Novikov self-consistency principle [17].

\[ 4 \]An example of an originally parallel tachyon beam diverging due to mutual interactions and/or due to interactions with other particles inside an isolated system (presenting a particular thermodynamic process involving tachyons), considered first in the tachyon preferred reference frame and then in a moving frame in which the time sequence of individual tachyon interactions can be reversed, shows that the concept of entropy, associated intrinsically with the processes of the information transfer, should share the principle of the invariance of the information flow direction, presented in this note. The author is grateful to Prof. O. V. Kancheli for raising the question about the increase/decrease of entropy of an isolated system in processes involving faster-than-light particles.
emission of tachyon $\alpha$), i.e. the observer $A$ would be inaccessible to the observer $B$, after detecting by the latter the tachyonic signal $\alpha$, during a whole interval $T$ preceding the $t_0^A$, as it occurs in the model suggested in [11, 12] in which all the acausal tachyon states are confined within the tachyon vacuum, then no problem with causality would appear.

Unfortunately, such a confinement is not possible within the SR. The observer $B$ in our example has an access to the observer $A$ during the above interval since he can possess a tachyon emitter equivalent, according to Lorentz symmetry, with that of the observer $A$. So, at the time $t_2^B$ (see Fig. 2b) he sends a faster-than-light signal (tachyon $\beta$) towards the observer $A$. If the velocity of the tachyon $\beta$ in the $B$ frame is higher than that of the antitachyon $\bar{\alpha}$, the trajectory of the tachyon $\beta$ intersects the trajectory of the antitachyon $\bar{\alpha}$ somewhere in the space between the $B$ and $A$, and then tachyon $\beta$ will reach the observer $A$ and will be detected by him at the time $t_3$ which, in the both frames, precedes the time $t_0$. One can see that the possibility of a causal loop is realized.

Let us consider this loop in the frame of the observer $A$ (Fig. 2a). He will detect a tachyonic signal (interpreted by him as an emission of the antitachyon $\bar{\beta}$ by his detector) at time $t_3^A$, i.e. inside the time interval $T$ preceding the launching time $t_0^A$. But the mandatory condition for launching the tachyon $\alpha$ (specified at the beginning of this Section) was the absence of any tachyonic signal from $B$ during that interval. We have a capital logical paradox, which was the main reason of a rejection of the possibility of faster-than-light signals during a century, beginning with Einstein’s formulation of this rejection [7]. We note that often this rejection produces a mental barrier, preventing constructive discussion of any problem related to the faster-than-light particles and signals, though alternative approaches to the subject also exist.

---

**Fig. 2.** A causal loop (the Tolman paradox) as seen a) in the frame of the observer $A$ and b) in the frame of the observer $B$. We emphasize once more that the arrows on the tachyon world lines indicate the tachyon motion direction, which can be opposite (as in the cases of antitachyons $\bar{\alpha}$ and $\bar{\beta}$) to the information flow direction, the latter being always directed from a cause to its effect.
6 Conclusion

Turning to the concluding section of [15] we can reformulate several statements related to the case when the observer C blocks the tachyon signal, not allowing a passage of tachyons through his plug, to make these statements correct (the corrections to the original items of [15] are given in italic):

(a) *If the observer A issues a superluminal signal* both observers record the pair of the events (A, C) (albeit in the opposite ordering)

(b) *If the observer B issues a superluminal signal* both record the pair (B, C), also in the opposite ordering

The outcomes (c) and (d) considered in the concluding section of [15] should be discarded as mutually controversial.

Thus, the one-way superluminal signalling does not result in a causal paradox and cannot be used to ban faster-than light signals within the SR. Contrary, the use of a two-way construction of the Tolman paradox is crucial in the demonstration of the incompatibility of the SR with the existence of tachyons due to appearance of logical inconsistencies related to the principle of causality. This means, according to [11], that this theory must not be used when considering superluminal signalling.

Acknowledgements

The author is grateful to Profs. K. G. Boreskov, F. S. Dzheparov, and O. V. Kancheli for useful discussions, and to Dr. B. R. French for the critical reading of the manuscript.

References

[1] Bilaniuk O. M. P., Deshpande V. K., Sudarshan E. C. G., *“Meta”-relativity*, Amer. J. Phys. 30, 718-723 (1962)

[2] Feinberg G., *Possibility of faster-than-light particles*, Phys.Rev. 159, 1089-1105 (1967)

[3] Newton R. G., *Causality effects of particles that travel faster than light*, Phys. Rev., 162, 1274 (1967)

[4] Rolnick W. B., *Implications of causality for faster-than-light matter*, Phys. Rev. 183, 1105-1108 (1969)

[5] Benford G. A., Book D. L., Newcomb W. A., *The tachyonic antitelephone*, Phys. Rev. D 2, 263-265 (1970)

[6] Parmentola J. A., Lee D. D. H., *Peculiar properties of tachyon signals*, Phys. Rev. D 4, 1912-1915 (1971)
Einstein A., Über die vom Relativitätsprinzip geforderte Trägheit der Energie, Ann. d. Phys. (Vierte Folge) 23, 371-384 (1907). English translation: On the inertia of energy required by the relativity principle, in The Collected Papers of Albert Einstein, vol.2, 238-250, Princeton Univ. Press, Princeton, New Jersey.

Tolman R. C., The Theory of Relativity of Motion, p. 54, Univ. of California Press, Berkeley (1917)

Møller C., The Theory of Relativity, p. 52, Oxford Univ. Press (1952)

Bohm D., The Special Relativity, p. 155, Addison-Wesley, New York (1965)

Perepelitsa V. F., Looking for a theory of faster-than-light particles, arXiv:gen-ph/1407.3245

Perepelitsa V. F., Lorentz-invariant and Lorentz-non-invariant aspects of a scalar tachyon field Lagrangian and the scalar tachyon Feynman propagator, arXiv:gen-ph/1605.03425

D. Mattingly, Modern tests of Lorentz invariance, Living Rev. Rel., 5, 5-84 (2005) [arXiv:gr-qc/0502.097]

V.A. Kostelecký, N. Russel, Data tables for Lorentz and CPT violation, Rev. Mod. Phys., 83, 11-31 (2011) [arXiv:hep-ph/0801.0287]

Fayngold M., A new paradox in superluminal signaling, arXiv:gen-ph/1506.06721

Gödel K., An example of a new type of cosmological solutions of Einstein’s field equations of gravitation, Rev. Mod. Phys. 21, 447-450 (1949)

Novikov I. D. Evolution of the Universe, p. 169, Cambridge Univ. Press, (1983)

Friedman J., Morris M. S., Novikov I. D., Echeverria F., Klinkhammer G., Thorne K. S., Yurtsever, U., Cauchy problem in spacetimes with closed time-like curves, Phys. Rev. D 42, 1915-1930 (1990)

The impossibility of faster-than-light signals was formulated by Einstein in this paper as follows: “This result (sending a signal back in time - V. F. P.) signifies that we would have to consider as possible a transfer mechanism whose use would produce an effect which precedes the cause (accompanied by an act of will, for example). Even though, in my opinion, this result does not contain a contradiction from a purely logical point of view, it conflicts so absolutely with the character of all our experience, that impossibility of the assumption $W > V$ is sufficiently proved by this result” (here, in Einstein’s notations, $W$ is the velocity of signal transfer and $V$ is the velocity of light).