Modeling investigation of coupling between damage and permeability for sandstone

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Abstract: Rock damage due to microcrack propagation leads to changes in seepage characteristics. Therefore, the coupled mechanism of rock damage and permeability plays a significant role in some underground engineering projects, such as mining, excavation, and hydraulic activity. In this study, a rock micro-unit is divided into damaged and undamaged parts. The damaged part only consists of cracks, and the undamaged part includes pores and matrices. A damage evolution function was proposed on the basis of statistical damage theory considering water pore pressure to demonstrate the mechanical behavior under triaxial compression conditions. The damage variable could be obtained with the Weibull distribution function. Then, a novel sandstone permeability model was developed by introducing the damage evolution function. The proposed coupled damage and permeability model covered the matrix and fracture permeabilities. A series of triaxial seepage experimental data of sandstone under different pore water pressures was cited to validate the coupled model. Results showed that the coupled damage and permeability model was remarkably consistent with experimental findings.

1. Introduction

The interaction mechanism between fluid and solid is necessary to ensure safety in underground engineering activities, such as hydropower project, mining project, and nuclear waste disposal [1]. For instance, during sandstone reservoir excavation, excavation-induced stress alters the stress state of the sandstone reservoir. The vertical stress of a nearby rock mass in front of the free face is enhanced; consequently, cracks rapidly propagate, and seepage behavior in a sandstone reservoir obviously changes [2,3].

Sandstone is a typical inhomogeneous rock material that contains two parts: the matrix system and the fracture system [4,5]. The connected pores in the matrix system and through cracks in the fracture system are the main seepage paths of fluid in a sandstone reservoir. Therefore, the coupled characteristics of the damage evolution and seepage behavior of sandstone should be elucidated. Numerous researchers attempted to reveal the damage-induced permeability evolution law of rock materials through experimental [6,7], numerical [8,9], and theoretical methods [3,10,11].
The great majority permeability model is established on the basis of elastic theory \cite{12-14}, which focuses on the variation in permeability in the elastic deformation stage. When external loading exceeds the yield stress threshold, these models may no longer be applicable to describe the variation in permeability. Therefore, further studies have focused on damage-induced permeability. Chen et al. \cite{15} introduced a growth function to modify the classical exponential permeability function of coal. This modeling method successfully reflects the complete variation in permeability during the whole stress–strain process. Ren \cite{10} proposed a novel damage-induced permeability model under coal compaction and fracturing conditions. This model couples damage variable and permeability through an exponential function.

In this study, a statistical damage model is proposed to couple a matrix and a fracture permeability function. The coupled model of damage and permeability is divided into undamaged and damaged permeabilities. Then, a series of triaxial compression experiments at different pore water pressures is conducted to validate the coupled model.

2. Damage constitutive function

One essential hypothesis is that the nominal stress-induced strain on a damaged material is equal to that of the effective stress-induced strain on an undamaged material \cite{11}. The damage variable and all mechanical properties are considered isotropy. The constitutive relation only considers Hooke law, and the plastic constitutive behavior is ignored. The effect of the pore water pressure on the external loading and the stress–strain constitutive function of sandstone is obtained as follows:

$$\sigma_{ij} - p\delta_{ij} = (1 - D)C_{ijkl}\varepsilon_{kl},$$

where $\sigma_{ij}$ is the stress tensor; $C_{ijkl}$ is the stiffness tensor of a material; $\varepsilon_{kl}$ is the strain tensor; $D$ is the damage variable; $p$ is the pore pressure; and $\delta_{ij}$ is the Kronecker delta.

This section focuses on the formation of the damage variable. An effective method is widely applied to define the damage variable based on the statistical damage mechanics. A basic assumption is that all microcracks and voids in rocks are randomly distributed, and these defects are randomly induced because of external loading \cite{16}. In other words, these defects in a certain strain range are also randomly generated \cite{17}. Furthermore, in this study, the effect of damage on the deformation of a solid material is isotropic. As such, only the axial stress–strain constitutive relation is established, and the axial strain is regarded as the random distribution variable. Therefore, the probability of defect generation can be expressed as the Weibull probability density function:

$$P(\varepsilon_1) = \frac{m_0}{F_0} \left( \frac{\varepsilon_1}{F_0} \right)^{m_0 - 1} \exp \left[ - \left( \frac{\varepsilon_1}{F_0} \right)^{m_0} \right] ,$$

where $\varepsilon_1$ is the axial strain, and $m_0$ and $F_0$ are the Weibull-distributed parameters. Then, $D$ can be replaced with the Weibull-distributed function and obtained as follows:

$$D = 1 - \exp \left[ - \left( \frac{\varepsilon_1}{F_0} \right)^{m_0} \right] .$$

According to Eqs. (1) and (3) and Hooke law, the stress–strain relationship can be expressed as follows:

$$\sigma_1 - p = E\varepsilon_1 \exp \left[ - \left( \frac{\varepsilon_1}{F_0} \right)^{m_0} \right] + 2\nu\sigma_3,$$

where $E$ is the elastic modulus, MPa; $\mu$ is the Poisson ratio; and $\sigma_1$ is the axial stress.
Then, \( m_0 \) and \( F_0 \) are obtained by fitting the experimental data in accordance with previously described methods \[18\].

3. Coupled model of damage and permeability

In this study, the seepage channel can be divided into two parts. One part includes the connected pores in the matrix system of sandstone, and the other part comprises the fracture system of sandstone. According to the essential assumption of the damage mechanics, the matrix system is defined as the undamaged region of sandstone, and the fracture system is defined as the damaged region of sandstone. Sandstone permeability can be divided into the undamaged and damaged permeabilities. Therefore, the relationship between the damage variable and the sandstone permeability is coupled on the basis of the above assumption. The coupled model of damage and permeability of sandstone can be expressed as follows:

\[
k = k^u + k^d = (1 - D)k_m + Dk_f,
\]

where \( k^u \) is the undamaged permeability of sandstone, \( 10^{-3} \mu m^2 \); \( k^d \) is the undamaged permeability of sandstone, \( 10^{-3} \mu m^2 \); \( k_m \) is the matrix permeability of sandstone, \( 10^{-3} \mu m^2 \); and \( k_f \) is the fracture permeability of sandstone, \( 10^{-3} \mu m^2 \).

The matrix system of sandstone contains the pores and the skeleton of solid particles \[19\]. The connected pores are the main seepage channel of the matrix system of sandstone. Therefore, matrix permeability is obtained in accordance with the cubic law \[20\]:

\[
k_m = k_{m0} \left( \frac{\phi_m}{\phi_{m0}} \right)^3,
\]

where \( k_{m0} \) is the initial matrix permeability of sandstone, \( 10^{-3} \mu m^2 \); \( \phi_m \) is the matrix porosity of sandstone, \%; and \( \phi_{m0} \) is the initial matrix porosity of sandstone, \%.

Changes in porosity are mainly controlled by the external stress and the pore water pressure and defined as follows \[21\]:

\[
\frac{\phi_m}{\phi_{m0}} = 1 - \frac{R_m}{\phi_{m0}} \varepsilon_1,
\]

where \( R_m \) is the elastic modulus reduction ratio, and \( \varepsilon_1 \) is the effective strain.

Then, the matrix permeability of sandstone can be given as

\[
k_m = k_{m0} \left( 1 - \frac{R_m}{\phi_{m0}} \varepsilon_1 \right)^3,
\]

According to the above assumption and the results of Ren et al. (2020), the fracture permeability of the fracture system can be obtained by the following function:

\[
k_f = k_{f0} \left( 1 + \frac{2(1 - R_m)}{\phi_{f0}} \varepsilon_1 \right)^3,
\]

According to Eqs. (3), (5), (8), and (9), the coupled model of the damage variable and the permeability of sandstone can be expressed as

\[
k = (1 - D)k_{m0} \left( 1 - \frac{R_m}{\phi_{m0}} \varepsilon_1 \right)^3 + Dk_{f0} \left( 1 + \frac{2(1 - R_m)}{\phi_{f0}} \varepsilon_1 \right)^3.
\]
4. Experimental methodology

A series of triaxial compression experiments under different pore water pressures was carried out using the TAW-1000 servo-controlled rock mechanics testing apparatuses to reveal the coupled characteristic between the damage and permeability of sandstone. The stress, strain, and permeability data of sandstone specimens were measured during the whole stress-strain process. In these experiments, sandstone was used as the testing target. The details of the testing apparatuses and specimen preparation are presented in [6].

During the experimental process, the confining pressure remains constant ($\sigma_2 = \sigma_3 = 10$ MPa). The displacement-controlled method was applied to yield axial loading, and the axial stress was loaded at a rate of 0.02 mm/min. Measurement was performed via the steady flow method, and the water injection pressures (inlet water pressure $p_1$) were set to 1, 4, and 7 MPa, respectively. The outlet pressure $p_1$ was equivalent to atmospheric pressure. Therefore, the pore water pressure $p$ is equal to half of the sum of the inlet water pressure and outlet pressure, i.e., $p = (p_1 + p_1) / 2$. Sandstone permeability follows the Darcy law. The detailed experimental procedure is described in another study [7].

5. Results and model verification

5.1. Experimental results

![Figure 1. Relationship between strain and deviatoric stress at different pore water pressures.](image)

The stress–strain curves of sandstone specimens under different pore water pressures are illustrated in Fig. 1. The evolution law of these whole stress–strain curves is coincident, and the elastic modulus and Poisson ratio of sandstone specimens almost remain constant (Table 1 in Section 5.2). However, the peak strength increases as the pore water pressure decreases. In Fig. 2, the axial strain–permeability curves at different pore water pressures show a similar change in trend; in other words, permeability initially decreases as the axial strain increases. Then, permeability increases as the axial strain further increases. An increased pore water pressure has a stimulating effect on sandstone permeability. Therefore, the initial permeability of sandstone specimens increases as the pore water pressure increases.
5.2. Model verification

The stress–strain constitutive relation can be calculated with Eq. (4). In Fig. 3, the model curves are consistent with the experimental curves of sandstone specimens at different pore water pressures. The comparison of model and experimental curves has an obvious deviation before the peak stress. This deviation may be induced by two causes. One of them is that axial stress is terminated during permeability measurement, which can induce a small strain. Therefore, the elastic deformation stage of the experimental curves is not a straight line. The other is that microdefects are compacted in the initial loading stage, leading to an enhanced elastic modulus. The stress–strain relations in the initial compaction stage show a nonlinear concave curve. However, the proposed damage constitutive model can demonstrate the deformation evolution and the mechanical constitutive behavior.

The proposed coupled model of the damage and permeability of sandstone is described in Eq. (11). Sandstone permeability is divided into matrix and fracture permeabilities based on the essential assumption of damage mechanics. The model parameters $m_0$, $F_0$, $R_m$, $k_m0$, $k_f0$, $\phi_m0$, and $\phi_f0$ can be obtained by fitting the experimental data in this study. All the model parameters are presented in Table 1. In Fig. 4, the proposed coupled model of damage and permeability of sandstone matches the testing data successfully well during the whole stress–strain process.

![Graph showing relationship between axial strain and deviatoric stress at different pore pressures.](image)

**Figure 2.** Relationship between the axial strain and deviatoric stress at different pore pressures.

**Table 1.** Experimental and model parameters.

| $p$/MPa | $m_0$ | $F_0$ | $R_m$ | $k_m0$ ($10^{-3}$μm²) | $k_f0$ ($10^{-3}$μm²) | $\phi_m0$ | $\phi_f0$ |
|---------|-------|-------|-------|------------------------|------------------------|-----------|-----------|
| 0.5     | 5.19  | 0.016 | 0.8   | 10.91                  | 1.93                   | 0.019     | 0.063     |
| 2.0     | 4.00  | 0.015 | 0.94  | 26.39                  | 4.62                   | 0.026     | 0.071     |
| 3.5     | 3.1   | 0.015 | 0.85  | 11.66                  | 6.415                  | 0.120     | 0.074     |
Figure 3. Comparison of the experimental data and the damage model at different pore pressures.

Figure 4. Comparison of the experimental data and the permeability model at different pore pressures.

6. Conclusions
In this study, a novel coupled model of damage and permeability of sandstone was proposed, and the proposed model could reveal the variation in permeability during sandstone damage evolution.

(1) Triaxial compression experimental results showed that a higher pore water pressure resulted in enhanced peak strength of sandstone. Moreover, the effect of the pore water pressure on the elastic modulus and Poisson ratio was not obvious.

(2) The damage constitutive model of sandstone was established on the basis of classical statistical damage theory, and the axial strain was assumed as the Weibull-distributed variable. This model could remarkably match the experimental data.

(3) The proposed coupled model of damage and permeability of sandstone could be divided into the undamaged and damaged permeabilities controlled by the permeability of the matrix and fracture system, respectively. This coupled model was consistent with the experimental curves of sandstone permeability at different pore water pressures.
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