

LARGE RAINBOW MATCHINGS IN LARGE GRAPHS

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Abstract. A rainbow subgraph of an edge-colored graph is a subgraph whose edges have distinct colors. The color degree of a vertex \( v \) is the number of different colors on edges incident to \( v \). We show that if \( n \) is large enough (namely, \( n \geq 4.25k^2 \)), then each \( n \)-vertex graph \( G \) with minimum color degree at least \( k \) contains a rainbow matching of size at least \( k \).

1. Introduction

We consider edge-colored simple graphs. A subgraph \( H \) of such graph \( G \) is monochromatic if every edge of \( H \) is colored with the same color, and rainbow if no two edges of \( H \) have the same color. In the literature, a rainbow subgraph is also called totally multicolored, polychromatic, and heterochromatic.

In anti-Ramsey theory, for given \( n \) and a graph \( H \), the objective is to find the largest integer \( k \) such that there is a coloring of \( K_n \) using exactly \( k \) colors that contains no rainbow copy of \( H \). The anti-Ramsey numbers and their relation to the Turán numbers were first discussed by Erdős, Simonovits, and Sós [4]. Solutions to the anti-Ramsey problem are known for trees [9], matchings [6], and complete graphs [15], [1] (see [7] for a more complete survey). Rödl and Tuza proved there exist graphs \( G \) with arbitrarily large girth such that every proper edge coloring of \( G \) contains a rainbow cycle [14]. Erdős and Tuza asked for which graphs \( G \) there is a \( d \) such that there is a rainbow copy of \( G \) in any edge-coloring of \( K_n \) with exactly \( |E(G)| \) colors such that for every vertex \( v \in V(K_n) \) and every color \( \alpha \), \( v \) is the center of a monochromatic star with \( d \) edges and color \( \alpha \). They found positive results for trees, forests, \( C_4 \), and \( K_3 \) and found negative results for several infinite families of graphs [5].

For \( v \in V(G) \) and a coloring \( \phi \) on \( E(G) \), \( \hat{d}(v) \) is the number of distinct colors on the edges incident to \( v \). This is called the color degree of \( v \). The smallest color degree of all vertices in \( G \) is the minimum color degree of \( G \), or \( \hat{\delta}(G, \phi) \). The largest color degree is \( \hat{\Delta}(G, \phi) \).

Local anti-Ramsey theory seeks to find the maximum \( k \) such that there exists a coloring \( \phi \) of \( K_n \) that contains no rainbow copy of \( H \) and \( \hat{\delta}(K_n, \phi) \geq k \).

The topic of rainbow matchings has been well studied, along with a more general topic of rainbow subgraphs (see [10] for a survey). Let \( r(G, \phi) \) be the size of a largest rainbow matching in a graph \( G \) with edge coloring \( \phi \). In 2008, Wang and Li [17] showed that \( r(G, \phi) \geq \)

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Theorem 1. Let $G$ be an $n$-vertex graph and $\phi$ be an edge-coloring of $G$ with $n > 4.25\hat{\delta}^2(G, \phi)$. Then $(G, \phi)$ contains a rainbow matching with at least $\hat{\delta}(G, \phi)$ edges.

Our result gives a significantly weaker bound on the order of $G$ than the bounds in [3] but for a significantly wider class of edge-colorings.

Several ideas in the proof came from Diemunsch et al.’s paper [2]. The full proof is presented in the next section.

2. Proof of the Theorem

Let $(G, \phi)$ be a counter-example to our theorem with the fewest edges in $G$. For brevity, let $k := \hat{\delta}(G, \phi)$. Since $(G, \phi)$ is a counter-example, $n := |V(G)| > 4.25k^2$. The theorem is trivial for $k = 1$, and it is easy to see that if $\hat{\delta}(G) = 2$ and $(G, \phi)$ does not have a rainbow matching of size 2, then $|V(G)| \leq 4$. Therefore $k \geq 3$.

Claim 1. Each color class in $(G, \phi)$ forms a star forest.

Proof. Suppose that the edges of color $\alpha$ do not form a star forest. Then there exists an edge $uv$ of color $\alpha$ such that an edge $ux$ and an edge $vy$ also are colored with $\alpha$ (possibly, $x = y$). Then the graph $G' = G - uv$ has fewer edges than $G$, but $\hat{\delta}(G', \phi) = k$. By the minimality of $G$, $r(G', \phi) \geq k$. But then $r(G, \phi) \geq k$, a contradiction. \qed

We will denote the set of maximal monochromatic stars of size at least 2 by $S$. Let $E_0 \subseteq E(G)$ be the set of edges not incident to another edge of the same color, i.e. the maximal monochromatic stars of size 1.

Claim 2. For every edge $v_1v_2 \in E(G)$, there is an $i \in \{1, 2\}$, such that $\bar{d}(v_i) = k$ and $v_1v_2$ is the only edge of its color at $v_i$. 2
Proof. Otherwise, we can delete the edge and consider the smaller graph.  

Claim 3. All leaves $v \in V(G)$ of stars in $S$ have $\hat{d}(v) = k$. 

Proof. This follows immediately from Claim 2.

For the sake of exposition, we will now direct all edges of our graph $G$. With an abuse of notation, we will still call the resulting directed graph $G$. In every star in $S$, we will direct the edges away from the center. All edges in $E_0$ will be directed in a way such that the sequence of color outdegrees in $G$, $\hat{d}_0^+ \geq \hat{d}_1^+ \geq \ldots \geq \hat{d}_n^+$ is lexicographically maximized. Note that by Claim 1, the set of edges towards $v$ forms a rainbow star, and so $d^-(v) \leq \hat{d}(v)$.

Let $C$ be the set of vertices with non-zero outdegree and $L := V \setminus C$. Let $S^* \subseteq S$ be the set of maximal monochromatic stars with at least two vertices in $L$, and let $E_0^* \subseteq E_0 \cup S$ be the set of maximal monochromatic stars with exactly one vertex in $L$. For a color $\alpha$, let $E_H[\alpha]$ be the set of edges colored $\alpha$ in a graph $H$. If there is no confusion, we will denote it by $E[\alpha]$.

Claim 4. For every $v \in V(G)$ with $\hat{d}(v) \geq k + 1$, $d^+(v) = 0$. In particular, $d^+(v) \leq k$ for every $v \in V(G)$. Moreover, for all $w \in L$, $d(w) = k$.

Proof. Suppose that $\hat{d}(v) \geq k + 1$, and let $w_i v$ be the edges directed towards $v$. By Claim 2 and (1), $\hat{d}(w_i) = k$ and $w_i v \in E_0$ for all $i$. Then $d^+(w_i) \leq \hat{d}(w_i) = k$. Reversing all edges $w_i v$ would increase the color outdegree of $v$ with a final value larger than $k$ while decreasing the color outdegree of each $w_i$, which was at most $k$. Hence the sequence of color outdegrees would lexicographically increase, a contradiction to the choice of the orientation of $G$.

By the definition of $L$, if $w \in L$, then $d^+(w) = 0$. So in this case by the previous paragraph, $k \leq \hat{d}(w) \leq d^-(w) \leq k$, which proves the second statement.

Claim 5. No color class in $(G, \phi)$ has more than $2k - 2$ components.

Proof. Otherwise, remove the edges of a color class $\alpha$ with at least $2k - 1$ components, and use induction to find a rainbow matching with $k - 1$ edges in the remaining graph. This matching can be incident to at most $2k - 2$ of the components of $\alpha$, so there is at least one component of $\alpha$ not incident to the matching, and we can pick any edge in this component to extend the matching to a rainbow matching on $k$ edges.

We consider three cases. If $n > 4.25k^2$, then at least one of the three cases will apply. The first two cases will use greedy algorithms.

Case 1. $|S^*| + \frac{1}{2}|E_0^*| \geq 2.5k^2$.

For every $S \in S^*$, assign a weight of $w_1(e) = 1/|S \cap L|$ to each of the edges of $S$ incident to $L$. Assign a weight of $w_1(e) = 1/2$ to every edge $e \in E_0^*$. Edges in $G|C|$ receive zero weight. Let $G_0 \subseteq G$ be the subgraph of edges with positive weight. For every set of edges $E' \subseteq E(G)$, let $w_1(E')$ be the sum of the weights of the edges in $E'$. For every vertex, let $w_1(v) = \sum_{a \in N^+(v)} w_1(va) + \sum_{b \in N^-(v)} w_1(bv)$. Note that $G_0$ is bipartite with partite sets $C$ and $L$ and that $w_1(e) \leq 1/2$ for every edge $e \in E(G)$. Furthermore,

$$\frac{1}{2} \sum_{v \in V(G)} w_1(v) = \sum_{e \in E(G)} w_1(e) = |S^*| + \frac{1}{2}|E_0^*| \geq 2.5k^2.$$
Claim 6. For every \( v \in V(G) \), \( w_1(v) \leq 2(k - 1) \).

Proof. Suppose \((G, \phi)\) has a vertex \( v \) with \( w_1(v) > 2(k - 1) \). Let \( G' = G - v \). Then \( \delta(G', \phi) \geq k - 1 \) and \( |V(G')| = n - 1 > 4.25(k - 1)^2 \). By the minimality of \((G, \phi)\), the colored graph \((G', \phi)\) has a rainbow matching \( M \) of size \( k - 1 \). At most \( k - 1 \) of the stars \( v \) is incident to have colors appearing in \( M \); each of them contributes a weight of at most 1 to \( w_1(v) \). As \( w_1(v) > 2(k - 1) \), there are at least \( 2k - 1 \) edges incident to \( v \) with colors not appearing in \( M \). At least one of these edges is not incident to \( M \). Thus \((G, \phi)\) has a rainbow matching of size \( k \), a contradiction. \qed

We propose an algorithm that will find a rainbow matching of size at least \( k \). For \( i = 1, 2, \ldots \), at Step \( i \):

0) If \( G_{i-1} \) has no edges or \( i - 1 = k \), then stop.

1) If a vertex \( v \) of maximum weight has \( w_1(v) > 2(k - i) \) in \( G_{i-1} \), then set \( G_i = G_{i-1} - v \) and go to Step \( i + 1 \).

2) If the largest color class \( E[\alpha] \) of \( G_{i-1} \) has at least \( 2(k - i) + 1 \) components, then set \( G_i = G_{i-1} - E[\alpha] \) and go to Step \( i + 1 \).

3) If \( w_1(v) \leq 2(k - i) \) for all \( v \in V(G_{i-1}) \) and every color class has at most \( 2(k - i) \) components, then set \( G_i = G_{i-1} - x - y - E[\phi(xy)] \) for some edge \( xy \in E(G_{i-1}) \).

We will refer to these as options (1), (2), and (3) for Step \( i \). We call the difference in the total weight of the remaining edges between \( G_{i-1} \) and \( G_i \) the weight of Step \( i \) or \( W_1(i) \). When both options (1) and (2) are possible, we will pick option (1). Option (3) is only used when neither of options (1) and (2) are possible.

Let \( G_r \) be the last graph created by the algorithm, i.e., \( r = k \) or \( G_r \) has no edges. We will first show by reversed induction on \( i \) that

\[
(\text{II}) \quad G_i \text{ has a rainbow matching of size at least } r - i. 
\]

This trivially holds for \( i = r \). Suppose (II) holds for some \( i \), and \( M_i \) is a rainbow matching of size \( r - i \) in \( G_i \). If we used Option (1) in Step \( i \), then there is some edge \( e \in E(G_{i-1}) \) incident with \( v \) that is not incident with \( M_i \) and whose color does not appear on the edges of \( M_i \), similarly to the proof of Claim 6. If we used Option (2) in Step \( i \), then there is some component of \( E_{G_{i-1}}[\alpha] \) that is not incident with \( M_i \), and we let \( e \) be an edge of that component. If we used Option (3) in Step \( i \), then let \( e = xy \). In each scenario, \( M_i + e \) is a rainbow matching of size \( r - i + 1 \) in \( G_{i-1} \). This proves the induction step and thus (II).

So, if \( r = k \), then we are done.

Assume \( r < k \). Then the algorithm stopped because \( E(G_{r+1}) = \emptyset \). This means that

\[
(\text{III}) \quad \sum_{i=1}^{r} W_1(i) = \sum_{e \in E(G)} w_1(e) \geq 2.5k^2. 
\]

We will show that this is not the case. Suppose that at Step \( i \), we perform Option (3). By the bipartite nature of \( G_0 \), we may assume that \( y \in L \). By Claim 4 \( w_1(y) - w_1(xy) \leq \frac{k-1}{2} \). Because Options (1) and (2) were not performed at Step \( i \), \( w_1(x) + w_1(E_{G_{i-1}}[\phi(xy)]) \leq 4(k - i) \). Therefore the weight of Step \( i \) is at most \( \frac{k-1}{2} + 4(k - i) < 4.5k - 4i \).

By Claims 5 and 6 Option (3) is performed at Step 1. If \( W_1(i) < 4.5k - 4i \) for all \( i \), then \( \sum_{i=1}^{r} W_1(i) < \sum_{i=1}^{r} 4.5k - 4i = 4.5kr - 2r(r+1) \leq 2.5k(k - 1) \), a contradiction to (III). Let \( i \) be the first time that \( W_1(i) \geq 4.5k - 4i \), and \( j < i \) be the last time Option (3) is performed.
prior to $i$. By the choice of $i$, $W_1(a) < 4.5k - 4a$ when $a \leq j$. Because Option (1) and (2) were not chosen at Step $i$, $W_1(i') \leq 2(k - j)$ for each Step $i'$ such that $i' > j$ and Option (1) or (2) is used. Note that by choice of $i$ and $j$, this bound applies for all steps between $j + 1$ and $i$. Furthermore, by the choice of $i$, $2(k - j) > 4.5k - 4i' - 1$ for $i' > i$. It follows that $W_1(b) \leq 2(k - j)$ for each $b > j$, and so

$$\sum_{a=1}^{r} W_1(a) \leq \sum_{a=1}^{j} (4.5k - 4a) + 2(k - j)(r - j) \leq 4.5kj - 2j(j + 1) + 2(k - j)(k - 1 - j)$$

$$= k(0.5j + 2k - 2) < 2.5k^2,$$

a contradiction to (2).

**Case 2.** $|C| \geq 1.75k^2$.

We will use a different weighting: For every vertex $v \in C$ and outgoing edge $vw$, if $vw \in E_0$, we let $w_2(vw) = 1/d^+(v)$, where $d^+(v)$ is the color outdegree of $v$, and if $vw$ is in a star $S \in \mathcal{S}$, then we let $w_2(vw) = 1/(d^+(v)||S||)$. For a vertex $v \in V(G)$, let $w^+(v)$ and $w^-(v)$ denote the accumulated weights of the outgoing and incoming edges, respectively, and $w_2(v) = w^+(v) + w^-(v)$. By definition, $w^+(v) = 1$ for each $v \in C$. Then

$$\sum_{e \in E(G)} w_2(e) = \sum_{v \in V(G)} w^-(v) = \sum_{v \in V(G)} w^+(v) = |C| \geq 1.75k^2.$$

**Claim 7.** Let $uv$ be a directed edge in $G$ and $e$ an edge incident to $u$ that is not $uv$. Then $w_2(e) \leq 1/2$.

**Proof.** The result is easy if $e$ is in a monochromatic star with size at least 2, so assume $e \in E_0$. If $e$ is directed away from $u$, then $d^+(u) \geq 2$ and the claim follows. Suppose now that $e$ is directed towards $u$, say $e = uw$, and $w_2(e) = 1$. Then $d^+(w) = 1$, and reversing $e$ we obtain the orientation of $G$ where the outdegree of $w$ decreases from 1 to 0, and the outdegree of $u$ increases from $d^+(u) \geq 1$ to $d^+(u) + 1$. The new orientation has a lexicographically larger outdegree sequence, which is a contradiction. \hfill \Box

**Claim 8.** For every color $\alpha$, we have $w_2(E[\alpha]) \leq 1.5(k - 1)$.

**Proof.** Otherwise, remove the edges of a color class $E[\alpha]$ with $w_2(E[\alpha]) > 1.5(k - 1)$, and use induction to find a rainbow matching with $k - 1$ edges in the remaining graph. For every directed edge $vw \in M$, $v$ can be incident to a component of $E[\alpha]$ of weight at most 1/2, and $w$ can be incident to a component of $E[\alpha]$ of weight at most 1, so there is at least one component of $E[\alpha]$ not incident to the vertices of $M$, and we can pick any edge in this component to extend $M$ to a rainbow matching of $k$ edges. \hfill \Box

We will use the following greedy algorithm: Start from $G$, and at Step $i$, choose a color $\alpha$ with the minimum value $w_2(E[\alpha]) > 0$, and pick any edge $e_i \in E[\alpha]$ of that color, and put it in the matching $M$, and then delete all edges of $G$ that are either incident to $e_i$ or have the same color as $e_i$. Without loss of generality, we may assume that edge $e_i$ has color $i$. If we can repeat the process $k$ times, we have found our desired rainbow matching, so assume that we run out of edges after $r < k$ steps, and call the matching we receive $M$. Let $h \leq k - 1$ be the first step after which only edges with colors present in $M$ remain in $G_h$. Let $\beta$ be a
color not used in \( M \) such that the last edges in \( E[\beta] \) were deleted at Step \( h \). Such \( \beta \) exists, since \( G \) has at least \( k \) colors on its edges.

By Claim 7 one step can reduce the weight \( w_2(E[\beta]) \) by at most 1.5. It follows that \( w_2(E[\beta]) \) at Step \( i \leq h \) is at most \( 1.5(h - i + 1) \). As we always pick the color with the smallest weight, the color \( i \leq h \) also had weight at most \( 1.5(h - i + 1) \) when we deleted it in Step \( i \). Every color \( i > h \) which appears in \( M \) has weight at most \( 1.5(k - 1) \) by Claim 8. Thus, the total weight of colors in \( M \) at the moment of their deletion is at most \( 1.5 \sum_{i=1}^{h} i + 1.5(k - 1)(k - 1 - h) \).

Claim 9. For each vertex \( v \), \( w_2(v) \leq \frac{(k+1)}{2} \).

Proof. Suppose there are two edges, \( e_1 \) and \( e_2 \), incident with \( v \) such that \( w_2(e_1) = w_2(e_2) = 1 \). By Claim 7 both edges are directed towards \( v \) and are in \( E_0 \). Consider the orientation of \( G \) where the directions of \( e_1 \) and \( e_2 \) have been reversed. Then the outdegree of \( v \) has been increased by 2, while the outdegree of two other vertices changed from 1 to 0. This creates a lexicographically larger outdegree sequence, a contradiction.

By Claim 4 if \( \hat{d}(v) \geq k + 1 \), then \( w_2(v) = 1 \). If \( \hat{d}(v) = k \), then by the above \( w_2(v) \leq \frac{1 + (k - 1)}{2} \).

If an edge \( e \) has a color \( \beta \) not in \( M \) or has color \( i \leq h \) but was deleted at Step \( j \) with \( j < i \), then \( e \) is incident to the edges \( \{e_1, \ldots, e_h\} \). By Claim 9 the total weight of such edges is at most \( 2h(k+1)/2 \).

However, this is a contradiction because it implies

\[
|C| \leq h(k + 1) + \frac{3}{2} \sum_{i=1}^{h} i + \frac{3}{2}(k - 1)(k - 1 - h) = \frac{3k^2}{2} - 3k + \frac{3}{2} + \frac{3h^2}{4} - \frac{hk}{2} + \frac{13h}{4} < 1.75k^2.
\]

Case 3. \(|L| > |S^*| + 0.5|E_0^*|\).

We will introduce yet another weighting, now of vertices in \( L \). For every star \( S \in S^* \), add a weight of \( \frac{1}{|L \cap V(S)|} \) to every vertex in \( L \cap V(S) \). For every edge \( e \in E_0^* \), add a weight of \( 1/2 \) to the vertex in \( L \cap e \). For every \( v \in L \), let \( w_3(v) \) be the resulting weight of \( v \).

Since \( \sum_{v \in L} w_3(v) = |S^*| + 0.5|E_0^*| < |L| \), there is a vertex \( v \in L \) with \( w_3(v) < 1 \). Let \( S_1, S_2, \ldots, S_k \) be the \( k \) maximal monochromatic stars incident to \( v \) ordered so that \( |L \cap V(S_i)| \leq |L \cap V(S_j)| \) for \( 1 \leq i < j \leq k \) (where \( S_1 \in E_0 \) is allowed). Since \( v \notin C \), all these stars have different centers and different colors. Now we greedily construct a rainbow matching \( M \) of size \( k \), using one edge from each \( S_i \) as follows. Start from including into \( M \) the edge in \( S_1 \) containing \( v \). Assume that for \( \ell \geq 2 \), we have picked a matching \( M \) containing one edge from each \( S_i \) for \( 1 \leq i < \ell - 1 \). Since \( w_3(v) < 1 \), we know that \( |L \cap V(S_\ell)| > \ell \) for \( \ell \geq 2 \). As every edge in \( M \) contains at most one vertex in \( L \), we can extend the matching with an edge from the center of \( S_\ell \) to an unused vertex in \( L \cap V(S_\ell) \).

To finish the proof, let us check that at least one of the above cases holds. If Cases 1 and 2 do not hold, then \(|C| < 1.75k^2\) and \(|S^*| + 0.5|E_0^*| < 2.5k^2\). Then, since \( n > 4.25k^2 \), \(|L| > 4.25k^2 - 1.75k^2 = 2.5k^2 \), and we have Case 3. \(\Box\)

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