Mechanism of Liquid Bridges Stretched out of a Liquid Bath

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Abstract. In the present paper, we investigate liquid bridges stretched out of a liquid bath. The liquid bridge is formed between a perfectly wetting rod and a liquid bath, and can be stretched vertically at a constant velocity. We record the results with a high-speed camera and analyze the stretching process before the rod stops at a maximum height. The stretching process is mainly controlled by two dimensionless parameters, namely the Froude number and the Ohnesorge number, respectively representing the influence of inertia forces and viscous forces. The profile development, the pinch-off phenomenon and the volume of extracted liquid are studied in detail, particularly on the influences of stretching velocities and liquid viscosities. Finally, we reveal the underlying mechanism using the dimensionless momentum equation.

1. Introduction
The stretching of liquid bridges are common phenomena in everyday experience, such as drinking strategies of cats and dogs [1, 2] and ligament fragmentation. Liquid bridges have also been applied in various industries: in printing processes, the fragmentation of stretching liquid bridges could lead to the undesirable little drops [3]; in food processing, stretching of liquid bridges could influence the production as well as the consumer perception [4]. More recently, a new method named as the continuous liquid interface production of the 3D printing technology [5] has been proposed, which can increase the printing speed by dozens of times. With faster lifting velocity of the printing object out of a resin bath, the stretching of liquid bridges could become an important subject in the fast 3D printing processing.

A few studies can be found on the liquid bridges stretched out of a liquid bath [1, 2, 6, 7]. The previous investigations are mainly focus on the low viscosity bridges, influences of liquid viscosities have never been studied. In this paper, we stretch the liquid bridge using a cylindrical rod, and focus on the process before the rod stops at the highest position, in particular the influences of stretching velocities and liquid viscosities. We will first introduce the experimental setup, then the results and the discussions, and finally the conclusions.

2. Experimental setup
The liquid bridges are stretched out of a liquid bath by a steel cylindrical rod of radius $R=5$ mm. As shown in Figure 1 (a), the rod is attached on a vertical translation plate, which can be lifted up at a constant velocity $0.1 \text{ m/s} \leq U \leq 0.8 \text{ m/s}$ by a motorized stage. The silicone oil liquid is contained in a fixed container with a radius and a depth much larger than the rod radius. The properties of silicone oils we use here are density $\rho = 970 \text{ kg/m}^3$, surface tension $\gamma = 0.021 \text{ N/m}$ and viscosities $0.01 \text{ Pa s} \leq \mu \leq 1 \text{ Pa s}$. The results of liquid bridge stretching are recorded by a high-speed camera (Phantom V2512,
USA) fitted with a Nikkro 60 mm microlens at frames rates 1000-5000 frames per second and 68 μm per pixel. A high-intensity LED lamp is used to produce the flickerless backlighting. The rod is initially just attached to the flat surface of the liquid bath and perfectly wetted, then lifted up at a constant velocity. Considering the similar condition of liquid stretched by the tongues of cats [1] and dogs [2], we stop the rod at a height \( H_{\text{max}} = 30 \text{ mm} \) and define the liquid bridge height as \( H \) (see Figure 1 (b)). We focus on the process before the rod reaches the height \( H_{\text{max}} \).

**Figure 1.** (a) Experimental setup, (1) the high-speed camera, (2) the LED lamp, (3) the vertical translation plate, (4) the cylindrical rod, (5) the liquid container, (6) the silicone oil. (b) The sketch of the liquid bridge, \( H \) is the height, \( H_{\text{max}} \) is the maximum height. The dotted line represents the position of flat surface.

### 3. Results

Before we present the experimental results, it is important to introduce the dimensionless parameters. First we regard the rod diameter as the length scale of the stretching system, non-dimensionalised by the capillarity length, yielding \( \varepsilon = R/l_c = 3.36 \), where the capillarity length is \( l_c = (\gamma/\rho g)^{1/2} = 1.49 \text{ mm} \). The dimensionless height of the liquid bridge is defined as \( H' = H/R \), we have \( 0 \leq H' \leq 6 \) during the whole process. Define \( V \) as the volume of the liquid bridge, the dimensionless volume is introduced as \( V' = V/R^3 \). The liquid properties, in particular the viscosity, is represented by the Ohnesorge number \( Oh = \mu/(\rho \gamma R)^{1/2} \). And finally, the Froude number \( Fr = U/(gR)^{1/2} \), which represents inertia forces against gravitational forces.

**Figure 2.** The silicone oil of \( Oh = 0.03 \) with \( Fr = 0.45, 1.81, 3.62 \) for (a), (b) and (c), \( Oh = 3.13 \) with \( Fr = 0.45, 1.81, 3.62 \) for (d), (e) and (f). The numbers in the figures are \( H' \).

The liquid bridges stretched out of a low viscosity bath are shown in Figure 2 (a-c), the figures presented are before the pinch-off time, after which the liquid bridge is not stretched anymore. A liquid bridge is formed between the bath and the moving rod, and is stretched longer and thinner as the rod goes up. The pinch-off happens before the rod reaches the highest position, for low stretching velocity with \( Fr = 0.45 \), it occurs at the bottom of the liquid bridge, while for high velocity with \( Fr = 3.62 \) it happens at the top region close to the rod. The liquid bridges stretched out of a high viscosity
liquid bath \((Oh = 3.13)\) are shown in Figure 2 (d-f). Different from the low viscosity ones, the high viscosity liquid bridges do not break before the rod reach the highest position. The bottom part of the high viscosity liquid bridge remains stationary while the top part extends longer, which indicates the liquid keeps being extracted from the bath during the stretching. According to the Rayleigh-Plateau instability [8], the breakup timescales are \(t_i = (\rho R^3/\gamma)^{1/2}\) and \(t_v = \mu R/\gamma\) for low and high viscosity bridges. The dimensionless pinch-off height scales are therefore \(H_i^* = Fr\epsilon\) and \(H_v^* = Fr\epsilon\cdot Oh\). For the low viscosity bridge of \(Oh=0.03\), we have \(1.52 \leq H_i^* \leq 12.20\) generally approximate \(H_{\text{max}}^* = 6\), meaning it would pinch off during the stretching. For the high viscosity bridge of \(Oh = 3.13\), we have \(4.76 \leq H_v^* \leq 38.10\) generally larger than 6, meaning it will not pinch-off during the stretching.

We then consider the volume of the liquid bridge. As the bottom part of the liquid bridge might fall back to the bath, we use the similar strategy shown in Gart et al. [2], namely calculating the volume of the liquid bridge in the range of \(0.5 \leq H^* \leq 6\). Results are shown in Figure 3 and Figure 4, non-dimensionalised by \(R^3\). In Figure 3 we show volumes for bridges of a certain liquid stretching at different velocities. In the initial stage, the bridge approximately a cylinder liquid column with radius \(R\), the dimensionless volume \(V^*\) follows the corresponding dashed line, as shown in Figure 3. When the stretching velocity increases, a larger maximum volume \(V_{\text{max}}^*\) occurs at a higher height. However, the enhancements of \(V_{\text{max}}^*\) is quite small for low viscosity bridges (see Figure 3 (a)) while those for high viscosity bridges are observable (see Figure 3 (b)). In Figure 4 we show the influence of the viscosity on the volumes, at a certain stretching velocity, \(Fr = 0.45\) in (a) and \(Fr = 3.62\) in (b). The initial stages follow the dashed line corresponding to a cylinder column. For both slow and fast stretching, the viscosity always improves the liquid extraction from the bath dramatically.

![Figure 3](image1.png)

*Figure 3.* Volume of liquid bridges, (a) different stretching velocity (with different \(Fr\)) for a certain viscosity \((Oh = 0.03)\), (b) different stretching velocity (with different \(Fr\)) for a certain viscosity \((Oh = 3.13)\). The dashed lines represent the volume of a cylinder with a radius \(R\).

![Figure 4](image2.png)

*Figure 4.* Volume of liquid bridges, (a) different viscosities (with different \(Oh\)) for a certain stretching velocity \((Fr = 0.45)\), (b) different viscosities (with different \(Oh\)) for a certain stretching velocity \((Fr = 3.62)\). The dashed lines represent the volume of a cylinder with a radius \(R\).*
4. Discussion

To find the underlying mechanism of liquid bridges stretched out a liquid bath, we introduce the momentum equation

\[ \rho \frac{D\mathbf{u}}{Dt} + \nabla p - \mu \Delta \mathbf{u} + \rho \mathbf{g} = 0, \quad (1) \]

where \( \mathbf{u} \) is the velocity, \( p \) is the pressure. The four terms from left to right represent inertia forces, capillary forces, viscous forces and gravitational forces, respectively. Using the following transformation,

\[ \mathbf{u} \rightarrow \mathbf{u}^* \cdot \mathbf{U}, \quad p \rightarrow p^* \cdot \frac{\gamma}{R}, \quad t \rightarrow t^* \cdot \frac{R}{U}, \]

\[ \nabla \rightarrow \nabla^* \cdot \frac{1}{R}, \quad \Delta \rightarrow \Delta^* \cdot \frac{1}{R^2}, \quad \mathbf{g} \rightarrow \mathbf{g}^* \cdot g, \quad (2) \]

we obtain the dimensionless momentum equation,

\[ \left( Fr^2 \right) \frac{D\mathbf{u}^*}{Dt} + \left( \frac{1}{\varepsilon^2} \right) \nabla^* p^* - \left( \frac{Oh}{\varepsilon} Fr \right) \Delta^* \mathbf{u}^* + \mathbf{g}^* = 0. \quad (3) \]

The four dimensionless number corresponding to the four forces are \( Fr^2 \), \( 1/\varepsilon^2 \), \( Fr \cdot Oh/\varepsilon \) and 1, respectively. In the present paper, \( 1/\varepsilon^2 = 0.09 << 1 \), hence capillary forces can generally be ignored in the stretching process. For low viscosity bridges with small \( Oh \), viscous forces are negligible (e.g. 0.02Fr for \( Oh = 0.03 \)), the system is dominated by inertia forces and gravitational forces. When we increase the viscosity, viscous forces increase and become comparable to inertia forces (e.g. 0.93Fr for \( Oh = 3.13 \)), viscous forces then help inertia forces to extract liquid form the bath.

5. Conclusion

In this paper, we present the experiments of liquid bridges stretched from a liquid bath and analyze the stretching process before the rod reaches the maximum height. Generally, a larger stretching velocity and a larger viscosity would induce a thicker bridge. For the low viscosity bridges, pinch-off phenomenon could happen before the rod reaches the maximum height, while for high viscosity ones it never happens in the period we concern. The enhancement of stretching velocities on the extracted volume is small for low viscosity liquid bridges, while it is observable for high viscosity ones. Viscosities can increase the extracted volume dramatically, for both slow and fast stretching. At last, using the dimensionless form of the momentum equation, we found that both inertia forces and viscous forces work against gravity in the extraction of liquid, while viscous forces are negligible for low viscosity liquid bridges. The results of this paper are of the references for various industrial applications, such as understanding the food processing, avoiding the undesirable little drops in liquid transport processing, and determining the entrained liquid volume in the 3D printing processing.

Acknowledgments

We acknowledge support from the National Natural Science Foundation of China (Grant No. 51875507).

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