random-interaction study on linear systematics of $I^π = 11/2^-$ electromagnetic moments in Cd isotope chain

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In the random-interaction ensembles, electromagnetic moments of Cd $I^π = 11/2^-$ isomers predominately present linear systematics as changing the neutron number, which has been reported in realistic nuclear system. Quadrupole-like and δ-like pn interaction are responsible for such linear systematics of quadrupole and magnetic moments, respectively.

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I. INTERACTION

The low-lying spectra and magnetic properties of doubly even nuclei are highly regulated with simple patterns. For examples, they always have $I^π = 0^+$ ground states, and $I^π = 2^+$ second excited states with few exceptions; their quadrupole moments of $2^+_1$ and $2^+_2$ states generally present a strong correlation with similar magnitudes and different signs across the whole nuclide chart [1]. On the other hand, these two regularities robustly exist in an ensemble of nuclear models with random numbers as two-body interaction matrix elements [2]. These two and other robust regularities in the random-interaction ensemble demonstrates how simple regularity emerges out of complex nuclear system, even with interactions mostly deviating from reality [1][8].

Recently, it was reported that the $I^π = 11/2^-$ electromagnetic moments of neutron rich Cd isotopes are also simply regulated with an obvious linear systematics as changing neutron number [3]. Several theoretical investigations tried to understand this linear systematics based on the BCS [11], density functional theory [11], and schematic Shell Model [12]. However, it’s still the challenge to explain how such simplicity survives out of the complex nuclear structure [13]. We believe that the random-interaction ensemble may provide some clue. Furthermore, most previous random-interaction works focused on the robust properties of a single doubly even nucleus. It’s novel to apply random interaction to a nuclear-systematics study on an odd-mass isotope chain. Therefore, our work aims to probe and understand the robustness of such linear systematics in the random-interaction ensemble.

II. CALCULATION FRAMEWORK

Our random-interaction calculations covers $^{112-130}$Cd, whose single-particle orbits are limited to $^π0g9/2, v2s_{1/2}, v1d_{3/2}$ and $v0h_{11/2}$ with $Z = 40 \sim 50$ and $N = 64 \sim 82$ shell closures. No further truncation is introduced. Single-neutron energies are set to be degenerated, almost as reality. The two-body interaction is randomized within the two-body random ensemble (TBRE) [14][15]. In other words, any two-body interaction element denoted by $V_{jj',jj''}$ follows the Gaussian distribution with $(\mu = 0, \sigma^2 = 1 + \delta_{jj',jj''})$, where $j_1, j_2, j_3$ and $j_4$ represent the four single-particle orbits, and the superscript $J$ labels the rank.

We generate 3 000 000 sets of two-body interaction elements, and input them into the shell-model code [17]. With each set of two-body interaction elements, we first calculate corresponding low-lying spectra of even-mass Cd. If the calculation produces $I^π = 0^+$ ground states for all the even-mass Cd isotopes, we further calculate the $I^π = 11/2^-$ electromagnetic moments of odd-mass Cd with the same set of elements. To simplify our description, these quadrupole and magnetic moments are denoted by $Q$ and $\mu$, respectively. For $Q$ calculations, effective charges are set as $e_π = 1.5e$ and $e_ν = 0.5e$; For $\mu$ calculations, single-particle $g$ factors are set as $g_π = 5.586 \times 0.7\mu_N$, $g_{1l} = 1\mu_N$, $g_{νs} = -3.826 \times 0.7\mu_N$ and $g_{νl} = 0$, where the spin $g$ factors are conventionally quenched by 0.7.

To quantitatively describe the $Q$ and $\mu$ systematics, we introduce the Pearson correlation coefficient (denoted by $ρ$) [18] as a measure of linear correlation between electromagnetic moment and neutron number. This coefficient has a value between ±1, where 1, 0 and -1 correspond to totally positive linear correlation, no linear correlation, and totally negative linear correlation, respectively. For instance, according to Table I of Ref. [9], the experimental $Q$ values present $ρ = 0.997$ linear systematics, and $μ$ values present $ρ = 0.862$ smaller than $Q$. Thus, the $Q$ linearity is more evident than $μ$ one as observed.
this more straightforwardly, we compare the background of our analysis in Fig. 1. From the normal distribution, we perform a sampling in the TBRE before the $Q$ systematics and $0^+$ ground states for all the even-mass Cd isotopes. Sampling (I) corresponds to the sampling with $\rho > 0.9$ $Q$ systematics and $0^+$ g.s. sampling are relatively more attractive for each single even-mass Cd isotope in the TBRE. The error bar is determined by statistic error.

III. INTERACTION PROPERTY WITH $I^\pi = 0^+$ GROUND STATES

Our $Q$ and $\mu$ calculations are based on the interactions, which can provide $I^\pi = 0^+$ ground states (denoted by $0^+$ g.s.) for all the even-mass Cd isotopes. In other words, we perform a sampling in the TBRE before the $Q$ and $\mu$ calculations. Only $\sim 1\%$ interactions can survive such sampling, although $0^+$-g.s. predominance is still preserved for each single even-mass Cd isotope in the TBRE. We present $V_{J_1J_2J_3J_4}^{J}$ average values (denoted by $\langle V_{J_1J_2J_3J_4}^{J} \rangle$) after this $0^+$-g.s. sampling as the background of our analysis in Fig. 1.

According to Fig. 1 all the $\langle V_{J_1J_2J_3J_4}^{J} \rangle$ values with $J = 0$ after the $0^+$-g.s. sampling are relatively more attractive than others, corresponding to the short-range property of nuclear force in realistic nuclear system. To visualize this more straightforwardly, we compare the $\langle V_{g_2g_2g_2g_2}^{J} \rangle$ and $\langle V_{h_1h_1h_1h_1}^{J} \rangle$ values with two-body interactions elements of a typical short-range interaction, i.e. the $\delta$ force, in Fig. 2. The similarity between them is obvious. Therefore, we conclude that even through the interaction origin of the $0^+$-g.s. predominance for a single nucleus in the TBRE is unclear, the short-range property of nuclear force is still the key to keep all the doubly even nuclei have $0^+$ g.s. in both TBRE and realistic nuclear system.

IV. Q LINEARITY

After the $0^+$-g.s. sampling, we calculate the $\rho$ distribution of the $Q$ systematics after the $0^+$-g.s. sampling and in the whole TBRE compared with the background distribution (denoted by $P_{\text{bkg}}$, see text for definition). The error bar is determined by statistic error.
where the predominance of the $Q$ linearity is more obvious with $P(\rho > 0.9)/P_{bkg} \simeq 300$.

One may argue that, the linear $Q$ systematics out of the $0^+\text{-g.s.}$ sampling is trivial, because the $0^+\text{-g.s.}$ sampling favors the $\delta$-like interaction, and thus should enhance the seniority scheme, which has been proposed to be the origin of the linearity of $Q$ systematics in Ref. [3].

To examine this argument, we also calculate the $\rho$ distribution of the $Q$ systematics in the whole TBRE without the $0^+\text{-g.s.}$ sampling as shown in Fig. 3(b) $P(\rho > 0.9)$ with the $0^+\text{-g.s.}$ sampling is 3 times of that without this sampling according to Fig. 3(a), which agrees with the claim in Ref. [3], that the seniority scheme indeed enhances the linearity of the $Q$ systematics. However, in Fig. 3(b), the predominance of $P(\rho > 0.9)/P_{bkg}$ is still obvious, even without the $0^+\text{-g.s.}$ sampling. This means that the $Q$ linearity is robust in the whole TBRE, which can not be totally attributed to the seniority scheme here.

To search the origin of this $Q$ linearity in the TBRE, we perform two additional samplings:

(I) the sampling with $\rho > 0.9$ and $0^+$ ground states for all the even-mass Cd isotopes;

(II) the sampling with $\rho > 0.9$ regardless of even-mass Cd ground states.

The $\langle V_{j_i j_2 j_3 j_4} \rangle$ values of these two samples are presented in Fig. 4 compared with those after the $0^+\text{-g.s.}$ sampling. Sampling (I) and the $0^+\text{-g.s.}$ sampling share the same short-range property of like-nucleons interaction, i.e., relatively attractive interaction elements with rank $J = 0$, which can be trivial, because sampling (I) is actually based on the $0^+\text{-g.s.}$ sampling. Furthermore, after sampling (I), the proton-neutron ($pn$) interaction elements between $\pi_0 g_9/2$ and $\nu_0 h_{11/2}$ orbits obviously follow the parabolic rule [14], as increasing rank $J$, corresponding to the quadrupole interaction [21]. Sampling (II) also favors a quadrupole-like $pn$ interaction, even though the rank $J = 0$ interaction elements after this sampling present not short-range property. Therefore, we conclude that the quadrupole $pn$ interaction is responsible to induce the $Q$ linearity, and the seniority scheme is a boost to this linearity in random-interaction ensemble.

V. $\mu$ LINEARITY

In Fig. 4 we present $\rho$ distributions of the $\mu$ systematics with the $0^+\text{-g.s.}$ sampling and in the whole TBRE, normalized with $P_{bkg}$. Here $P_{bkg}$ for $\mu$ systematics should be the same as that for $Q$. In the TBRE, $P(\rho)/P_{bkg}$ is always close to 1, which demonstrates that the TBRE does not characterize the $\mu$ systematics. However, after the $0^+\text{-g.s.}$ sampling, relatively larger possibility for $|\rho| > 0.9$ emerges ($\sim 1\%$ in the $0^+\text{-g.s.}$ sample), corresponding to the the predominance of the $\mu$ linearity.

It seems that the $\mu$ linearity requires even-mass Cd $0^+$ ground states, i.e. the seniority scheme as we have explained. However, pure seniority scheme can only provide a constant $\mu$ as agued in Ref. [3]. Thus, we need to further probe other origin of the $\mu$ linearity beside the seniority scheme. We perform a sampling for $\rho > 0.9$ $\mu$ systematics based on the $0^+\text{-g.s.}$ sampling, and present $\langle V_{j_i j_2 j_3 j_4} \rangle$ values after such sampling in Fig. 5.

In Fig. 6 the $\rho > 0.9$ sampling and the $0^+\text{-g.s.}$ sampling share the short-range property of the like-nucleon interaction, which means that the seniority scheme is still
important for the $\mu$ linearity. Furthermore, the $\rho > 0.9$ sampling presents additional structure of $\langle V_{g_2 h_{11/2} g_2 h_{11/2}} \delta \rangle$.

We replot the detail of $\langle V_{g_2 h_{11/2} g_2 h_{11/2}} \rangle$ of the $\rho > 0.9$ sample in Fig. 6. The even-$J$ behavior of $\langle V_{g_2 h_{11/2} g_2 h_{11/2}} \rangle$ is different from odd-$J$ one: the even-$J$ $\langle V_{g_2 h_{11/2} g_2 h_{11/2}} \rangle$ values present an obvious parabolic evolution, while those with odd $J$ seem less regulated. This odd-even difference also characterizes the $pn$ interaction governed by the $\delta$ force. More specifically, the evolution of even-$J$ interaction between $\pi 0 g_9/2$ and $\nu 0 h_{11/2}$ orbit are only attributed to the $T = 0$ $\delta$ force as

$$V_{g_2 h_{11/2} g_2 h_{11/2}}^{J=\text{even}} \propto \left( \frac{9}{2} \begin{array}[]{c} 11/2 \\ 1/2 \\ -1/2 \\ 0 \end{array} \right)^2 \times V^{T=0} \left\{ \frac{1 + 121}{J(J+1)} \right\},$$

while the odd-$J$ $\delta$ interaction elements have both $T = 0$ and $T = 1$ contributions as

$$V_{g_2 h_{11/2} g_2 h_{11/2}}^{J=\text{odd}} \propto \left( \frac{9}{2} \begin{array}[]{c} 11/2 \\ 1/2 \\ -1/2 \\ 0 \end{array} \right)^2 \times \left\{ V^{T=1} + V^{T=0} \right\} \frac{1}{J(J+1)}.$$  

VI. SUMMARY

To summarize, the random-interaction ensemble predominantly reproduces the linear $Q$ and $\mu$ systematics in the Cd isotopes chain. The $pn$ interaction is the key to linearize the the $Q$ and $\mu$ systematics, although the seniority scheme is a significant boost. For the $Q$ linearity, the $pn$ interaction presents quadrupole-like feature. For the $\mu$ linearity, the $\delta$-like $pn$ interaction is required with repulsive $T = 1$ and attractive $T = 0$ components.

Our work also emphasizes that the short-range interaction between like nucleons is responsible to reproduce the $I^\pi = 0^+$ ground states for all the even-mass nuclei in both TBRE and realistic nuclear system, which may provides a new viewpoint to understand the predominance of $I = 0$ ground states in the TBRE.

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