A Learning-Based Method for Automatic Operator Selection in the Fanoos XAI System

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Abstract. We describe an extension of the Fanoos XAI system ([3, 5]) which enables the system to learn the appropriate action to take in order to satisfy a user’s request for description to be made more or less abstract. Specifically, descriptions of systems under analysis are stored in states, and in order to make a description more or less abstract, Fanoos selects an operator from a large library to apply to the state and generate a new description. Prior work on Fanoos predominately used hand-written methods for operator-selection; this current work allows Fanoos to leverage experience to learn the best operator to apply in a particular situation, balancing exploration and exploitation, leveraging expert insights when available, and utilizing similarity between the current state and past states. Additionally, in order to bootstrap the learning process (i.e., like in curriculum learning), we describe a simulated user which we implemented; this simulation allows Fanoos to gain general insights that enable reasonable courses of action, insights which later can be refined by experience with real users, as opposed to interacting with humans completely from scratch. Code implementing the methods described in the paper can be found at https://github.com/DBayani/Operator_Selection_Learning_Extends_For_Fanoos.

1 Introduction

Explainable artificial intelligence (XAI) has garnered increasing attention over the last decade, a surge often attributed to the increased power — and therefore desire to use — AI systems based on machine learning (ML), whose decision making processes are typically difficult for human users to understand. In many fields of practice, due to ethical, safety, and legal concerns, there has been hesitance to adopt the latest ML technology, in no small part due to the inability of area experts to check that the software is acting in an appropriate manner. Inspired by scientific and practical needs, Fanoos is an XAI system developed to produce explanations at multiple levels of abstract to suit a user’s situational needs. Fanoos allows users to interactively ask questions about an ML system’s behavior and receive explanations which, at the user’s request, can be made either more or less abstract. Further still, explanations provided by Fanoos come in multiple strength which users may freely choose between: Fanoos can provide descriptions that are guaranteed to reflect the system’s true behavior in all circumstances (including extremely rare pathological situations that may be useless to consider in practice), or, at the user’s request, Fanoos can attempt to explain the typical behavior of the system, capturing common-case occurrences without being bogged-down by pathological cases or circumstances that have zero probability of occurring.

In this work, we detail an extension of Fanoos which allows the system to learn what actions to take in order to better satisfy a request for greater or lesser abstraction. In particular, explanations shown to users are stored in states tracked by Fanoos, and modifying explanations is done by Fanoos selecting then apply an operator on an state in order to generate a new state. In this paper, we overview our approach to enabling Fanoos to learn which operator to apply to a given state in order to satisfy a user.
request, automatically balancing insights from previous user interactions, advise from expert-provided heuristics, and the need for sufficient exploration. Taking inspiration from curriculum learning, we also implement a simulated user which serves to bootstrap Fanoos’s learning process, allowing time with real humans to be spent fine-tuning what it has learned as opposed to starting from scratch.

Code for our approach can be found at https://github.com/DBay-ani/Operator_Selection_Learning_Extensions_For_Fanoos.

2 Brief Aside: Some Comments on the History of this Paper

The content of this paper is based on the ideas-document from early 2020 found under UUID d7fecc3b-93bb-424e-a838-3f000f3715cf at https://github.com/DBay-ani/FanoosFurtherMaterials/blob/master/manifest.xml. Publicly, brief written mention of these endeavors appeared in Appendix A.1.1 of [3] and A.1 of [4]; I discussed them in greater detail during personal interactions at ICAPS-XAIP 2020 and IJCAI-XAI 2020.

3 Overview

In this section, we provide an overview of how operators are selected in Fanoos, as well as the design criteria that governed them. The approach allows for leveraging expert rules for selecting operators as well as learning which operator to take. It supports classic logic-based rules, as well as approaches more closely related to soft classifiers.

The basic idea is as follows:

- There is an indexed set of operators, $S_O$, which may be applied to a state.

- There are an indexed set of selectors, $S_s$, which each produce a (normalized) distribution over $S_O$. Each selector takes in a variety of information, including the entire history of use in Fanoos, and the current state being applied to. For the sake of simplicity, we will write this as $s_i(*)$ where $s_i \in S_s$ — the point here is that * is used to represent the variety of arguments $s_i$ may have.

- A process weighs and combines the distributions produced by members of $S_s$ to produce a final distribution over $S_O$.

- The final distribution is used to inform the selection of operator. This selection process is conducted in a way that promotes a healthy balance between exploration and exploitation.

- After operator application, state formation, and receipt of the user’s next request, Fanoos is internally provided a numeric score as feedback, this score being based on the user’s request in the context of the queries preceding it.

- The numeric feedback is used to adjust the weights given to each member of $S_s$, and, in order to inform our exploration process, other bookkeeping is done to keep track of how often each operator has been used.

- This process repeats until the user exits.

We list now some of the high-level ideas that have shaped this design:

Generally, there are three categories of information that are available which we would like to leverage:

Number of times each operator was tried - we want to make sure we explore sufficiently

Success rate - we want to try and pick operators that work well, for our notion of what “working well” is.

Distance - In addition to having access to a state which we want to apply and operator on, and we have all the prior states, operators applied to them, and the results of operator application from the past. We would like to leverage knowledge of previous state’s structure to decide what to do, as opposed to simply choose operators based on what tends to work well when averaged across all states. As such, knowledge of how far different a state is from each prior state can
be used to help inform the decisions as to which operator is best applied in the current situation.

Ideally, the approach could allow for both of the following to be worked-in effectively:

(B1) Learning of which operator to apply
(B2) Expert knowledge and guidance on which operator to use

The approach taken addresses each of (A1) (A3) and (B1) (B2) above.

4 Operators

Fanoos produces descriptions in response to user’s questions based on values it stores in its most recent state. Inside states are stored CEGAR-refinement parameters, constraints on which predicates are allowable inside descriptions, various settings for parameters that influence the description generation process, and the content of the state’s user-facing description itself. In order to accommodate a user’s request that a description’s abstraction level be changed, Fanoos selects then applies an operator to the most recently used state, generating a new state from which to base a new description. In general, we view the process of responding to a user’s requests for changing abstraction level as being analogous to a binary tree search — where nodes are states and paths are determined by user’s requests. Operators act as the actual mechanism that moves current attention from a parent node to the child node most suitable for the corresponding user request, in particular triggering the generation of the state-description in the process.

We divide the 101 operators in our implementation into three categories: special operators, parameter-adjustment operators, and predicate constraining operators.

4.1 Special Operators

In our implementation, we consider two special operators: the start operator and the blank operator. The start operator is used exclusively to generate the initial description following a user’s question, using default settings for all internal parameters of the operators we have implemented, the start operator is the only one that may fill this role. The blank operator functions by simply re-running the description generation procedure over the state without modifying any aspect of the state except for the fresh generation of all reachability results and the associated description content. Given the non-determinism present in components of Fanoos, the blank operator helps measure the natural variance of the system’s behavior and establishes a baseline for how often improvements occur purely due to chance. Providing insights for testing aside, the blank operator is a reasonable course of action for situations where Fanoos has “essentially correct” settings in the state, but the description would benefit from slightly different choices among alternatives that prima facie seem equally good.

4.2 Parameter-Adjusting Operators

Parameter-adjusting operators, as the name entails, modify internal parameters of states, in turn influencing various aspects of the process that ultimately lead to the description Fanoos presents to users. The parameter-adjusting operators implemented modify or set various combinations of the following parameters:

- The sampling radius scaling parameter used during the process of determining the subset of predicates consistent with a box that are most specific (α in appendix E of [3]).
- Whether to reuse previous reachability results as a starting place for the reachability analysis needed by the current state, or to freshly compute all results.

3 In principle, one could attempt to tune the starting state values to maximize the proportion of cases where users are satisfied by the first description provided and do not request any further adjustments. We do not pursue such an extension in this work, however.

4 Unless otherwise specified by parameters of a state, Fanoos reuses reachability results that are stored in the previous state, both for increased efficiency and to better control sources of variability in description generation.
• Whether during the abstract state refinement process, abstract states (i.e., boxes in our current implementation) should have their axes split only along variables that appear in the question, or whether all variables should be candidates for splitting, regardless of whether they are involved in a user query. This determines whether the variable \( h \) from appendix F.6.2 of [3] should be allowed to range over all possible values or only a particular subset.

• Whether to attempt merging boxes after the reachability analysis and if so, how many iterations of merging should be tried. In our primary box-merging algorithm, one iteration examines each of the box-corners available that have at least two boxes incident on it to see if any of the incident boxes can be merged; multiple iterations repeat this process, each time using the updated list of available corners from the previous timestep.

• The degree of precision to use when comparing coordinate values if merging boxes. As explained in section 2.2 of [3], multiple boxes may be merged into a single box of slightly larger net volume, with the amount of permissible expansion determined by a precision threshold; this process allows us to merge boxes that roughly align (but might not exactly align), while preserving the soundness of our guarantees.

• The side-length used to determine when refinement should stop (that is, \( \epsilon \) in equation 2 of [3]).

• The value for Boolean variable "produceGreaterAbstraction" (the function "generateDescription") of [3].

The complete list of operators and how they modify the state parameters can be found at the public GitHub repo containing the code (https://github.com/DBay-ani/Operator_Selection_Learning_Extensions_For_Fanoos).

4.3 Predicate Constraining Operators

Predicate-constraining operators effect what predicates are allowed to be used in forming descriptions, either disallowing certain predicates or re-allowing predicates that were previously barred. In total, there are four such operators, accounting for all combinations of either allowing / disallowing a predicate and whether the aim is to increase or decrease the abstraction level. We will detail how predicates are selected for removal, then comment on how the process differs when making decisions to re-add them.

Let \( q_T \) be the state whose description, \( D_T \), the user currently wants altered. Let \( \text{quest}(q) \) be the specific question instance,\(^6\) for which, in the process of producing replies, a state \( q \) was generated. To determine which named predicate occurring in \( D_T \) to remove, the records of previous interactions are examined in order to select a candidate that best balances exploration with exploitation.\(^7\) Given a state \( q \) that occurred in the past, \( q_t \), let:

- \( \omega(q_t, p) \) be the number of times a named predicate, \( p \), occurs in the description of state \( q_t \). This may be greater than one if, for instance, \( p \) occurs in multiple conjuncts.\(^8\)
- \( rm(q_t) \) and \( rl(q_t) \) be predicates indicating that the user requested the description to become, respectively, more abstract and less abstract (\( rm: "\text{request more}" \))
- \( rb(q_t) \) indicate that the user requested to exit the inner QA-loop (i.e., "b" in Listing 1.3 of [3]) after seeing \( q_t \)'s description (\( rb: "\text{request break}" \))

Further, let \( r_T = rm \) and \( r_{T+1} = rl \) if the user requested that \( D_T \) (the current description) become more abstract, and \( r_T = rl \), \( r_{T+1} = rm \) if the user requested lower abstraction. The predicate to remove

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6 Here, if the same question is asked later, it is considered a different instance.
7 Exploration: trying the available options often enough to be informed of each potential outcome; Exploitation: choosing the option that, based on the information accumulated so far, seems most likely to result in the outcome the user requested — changing the abstraction level in the desired direction.
8 Naturally, one can consider a variant of this where, in what follows, one uses \( \omega(q_t, p) > 0 \) instead of \( \omega \) raw. Our implementation does not work in such a fashion, but one can of course implement such an operator instead of — or in addition to — what we currently have in our code.
is determined using the index returned by

\[
\text{UCB}(\langle \text{occ}(p) \mid p \text{ occurs in } D_T \rangle),
\]

where UCB is the deterministic Upper Confidence Bound algorithm [1] and

\[
\text{occ}(p) = \{q_t \in H_t | r_T(q_t) \land (\omega(q_t, p) > \omega(q_{t+1}, p))\}
\]

\[
\text{succ}(p) = \{q_t \in \text{occ}(p) | r_{T+1}(q_t) \lor r_b(q_{t+1})\}
\]

where "q_t ∈ H_t" is a slightly informal reference to accessing q_t from all previous interaction records (i.e., not just replies about \text{quest}(q_t) or records from this user session). q_{t+1} indicates the state that followed q_t \text{ while responding to the same question, } \text{quest}(q_t) \text{ (i.e., it is not simply any state that comes chronologically after q_t in database records); in the cases where q_{t+1} does not exist, we substitute infinity for } \omega(q_{t+1}, p), \text{ and false for both } r_{T+1}(q_t+1) \text{ and } r_b(q_{t+1}).

Operators that re-allow predicates follow a very similar process as the above, except the direction of inequality in the definition of \text{occ}(p) is reversed, and instead of considering the predicates that do occur in \text{D_T} in Eq. (1), only the predicates that were forbidden from occurring in \text{D_T} are considered.

While alternatives to the adopted method could be used — for example, approaches with greater stochasticity — we believe our choice of a UCB algorithm is most likely appropriate at this stage, considering its relative data efficiency and the likely nature of the environment. Future improvements or novel operators may introduce different or more sophisticated methods for predicate selection, such as attempts to further leverage joint-relationships present between predicates in a descriptions and/or contexts.

5 Inference

This section provides details on how operators are selected; that is, how inference is performed.

Let \(w_i \in \mathbb{R}\) be the weight that selector \(s_i \in S_s\) is given by the system (this value may be negative), and for any non-negative integer \(m\) let

\[
[m] = \{m' \in \mathbb{N} \setminus \{0\} | m' \leq m\}
\]

We form the following distribution, \(D_{\text{samp}}(\ast, \overrightarrow{w})\), which we will use momentarily to inform the selection of operator:

\[
D_{\text{samp}}'(\ast, \overrightarrow{w}) = \sum_{i \in [S_s]} w_is_i(\ast)
\]

\[
d' = \min_{i \in [S_s]} (\{D_{\text{samp}}'(\ast, \overrightarrow{w})\}) \leq 0 \{D_{\text{samp}}'(\ast, \overrightarrow{w})\})
\]

\[
D_{\text{samp}}(\ast, \overrightarrow{w}) = D_{\text{samp}}'(\ast, \overrightarrow{w}) - d' \rightarrow 1
\]

(2)

The subscript “samp” on \(D_{\text{samp}}\) is short for “sample”, a name reflective of roughly how we use it next. In the above, we only subtract the minimum weight when it is negative — so a glut of positive weights can actually flatten the distribution. The Upper Confidence Bound (UCB) algorithm ([1]), widely used from one-arm bandits, is then applied to \(D_{\text{samp}}(\ast, \overrightarrow{w})\) as though each member of \(D_{\text{samp}}(\ast, \overrightarrow{w})\) gave a success rate; this algorithm is responsible for producing the index of the operator to use. Notice that by using the (UCB) algorithm, we address [A1] while potentially respecting [A2] and [A3]. That is, the selectors themselves will address [A2] and [A3] when forming their vote distributions, while we address [A1] at the very end by using the UCB algorithm to balance the selectors’ suggestions with necessary exploration.

Observe that in our inference procedure, the only way we know anything about the state is through the votes provided by the selectors. Another fact worth highlighting is that, from the standpoint of theoretical concerns, we violate the assumptions of the UCB algorithm. The UCB algorithm provides guarantees under the assumption that the world state does not change — i.e., that the “success rate” for each operator, while not directly observed, is constant. We use this basic bandit algorithm in order to facilitate a simple implementation that explicitly considers the factors we highlighted (e.g., Item [A1]); we have few quibbles if one wishes to substitute-in a more sophisticated method at this inference phase (e.g., contextual bandit methods), so long as it satisfies our general requirements. On balance, here we have provided a straight-forward approach that is reasonable from a mechanical standpoint.

If an operator is ultimately selected that is not applicable — for example, an operator that tries to re-
move named-predicates when the state it is being applied to has a description that lacks any — then \( q_{t+1} \) copies the description and pertinent parameters from \( q_t \) (some internal content must differ between \( q_{t+1} \) and \( q_t \), such as the histories and bookkeeping data used in our specific implementation).

5.1 Supporting Further Featurization

Ignoring the details of the normalization done to form it, \( D_{\text{samp}}( \beta, \omega) \) can be seen as essentially a product between the weight vector, \( \omega \), and a matrix containing the distribution of votes for each selector. Thus, on its surface, it would seem our aggregation scheme is more-or-less linear, and thus would be unable to leverage joint-behavior of selectors. While accurate on a shallow assessment, we detail in this subsection how we have enabled more sophisticated inferences by properly crafting of the vote-gathering process and the scope of information visible to selectors. Essentially, we introduce “features” based on a subset of selectors’ votes, making a scheme that is linear in respect to the feature space but potentially non-linear in respect to the original space of selector votes.

In order to gather votes from selectors, each selector is queried to determine if it is ready to provide its vote-distribution over the operators. This occurs in a loop: for each iteration of the loop, at least one selector must cast its vote, and selectors may not re-deliver, modify, or rescind their votes once they are cast. Since selectors have privy to a broad base of information, they may see the vote-distributions issued thus far by their co-patriots. Using this fact, we can effectively support featurization by specifying a base-set of selectors that produce the “raw signals”, and a set of meta-selectors whose votes are purely functions of the votes provided by the base-set. See, for instance, the implementation of second-order selectors discussed in Section 5.

The method used to segment blocks of selectors can be compared to a variety of techniques from ML and rule-based systems. In Appendix A, we describe some interpretations of this approach; each interpretation, while referring to the same implementation and raw facts, does provide different intuitions, different connections to prior work, and different insights for further exploration.

5.2 General Comments and Future Work for Our Vote Aggregation Scheme

We are not overly committed to the use of Eq. (2) for vote aggregation, and may modify it further in the future. Equation (2) is largely just a normalized weighted sum, and as such has some natural motivations and interpretations. This said, it is arguable that the form of Eq. (2) does not appropriately match how it is utilized in the UCB algorithm — for instance, it provides a sum that is normalized in respect to all operators, when the UCB algorithm is designed to deal with the individual success rate of each operator.

While there is obviously room for an inference rule more grounded in theory or for deeper connections (and justification) from existing literature, at this time we content ourselves with a procedure for this step that is generally sensible (while perhaps imperfect) and effectively incorporates the categories of information we wish to consider.

An aspect of the approach taken in Eq. (2) which we would like to retain is the fact that we essentially get “reverse selectors” for free — that is, if \( s_j \) is a selector whose pattern of voting is negatively correlated with good actions, the system uses votes from \( s_j \) to reduce the likelihood of doing what \( s_j \) suggests. An alternative we may consider is to use a Winnow-like update (e.g., [8]), as opposed to the rule adopted in Eq. (2); such a modification would keep all weights positive and more comfortably ignore “irrelevant” selectors, but would require the incorporation of additional selectors explicitly providing “reverse votes”.

10 Mechanically this is not a show-stopping issue, and most likely is not problematic from a theoretic standpoint in the limit. However, it does open-up more scenarios where the final operator is decided primarily based on exploration considerations as opposed to exploitation (i.e., when the addition of uncertainty bounds changes the ranking of operators).

11 These “reverse selectors” could be implemented as higher-order selectors that take the vote distribution of one selector, \( s_i(*) \), and output

\[
\langle f(j) \left( \sum_{j \in |S_o|} f(j) \right)^{-1} | j \in |S_o| \rangle
\]

where

\[
f(j) = U(\alpha \{p_i(*)\}_j + (1 - \alpha)U)^{-1}
\]
in order to emulate the ability to leverage selectors whose votes tend to negatively correlate with proper actions. Another option worth bearing in mind that has similarly shaped parts is the update method used in [17].

In future work, we may further examine the use of boosting-based approaches (e.g., soft-classifier variants of AdaBoost [11]) to produce a vote aggregation scheme that results in stronger overall outputs. In terms of engineering improvements, our current vanilla implementation of the selectors-voting framework is entirely serial in execution, but the process is trivially parallelizable, a fact that ideally would be leveraged.

6 Learning

We will break our discussion of learning from feedback into two parts: how the selector-weights are updated given a reward signal, and how the reward signal is derived from user feedback.

6.1 Updating Selector Weights Given the Reward Signal

Suppose we are given a reward signal \( y_t \in \{-1, 1, 0\} \) at iteration \( t \in \mathbb{N} \setminus \{0\} \) (in the next section, this signal is derived from the user feedback and the history, but for now we suppose a numerical score is already derived). Let \( O_t \) be the operator that was actually selected for iteration \( t \) (i.e., the one chosen at the end of inference). Tweaking notation slightly to incorporate a time-index, we update the weight of each selector as follows:

\[
w_{i,t+1} = w_{i,t} + \alpha y_t (s_i(s_t))(O_t) - U
\]

for some hyperparameter \( \alpha \in (0, 1) \).

\( ^{12} \) Up to the explicit dependency of higher-order selectors on lower-order selectors to vote first, of course. If we view these dependencies between selectors as a DAG, then each layer can run in parallel. For our implementation, and most reasonable extensions we can foresee, the width of layers (which is parallelizable work) is far larger than the number of layers (serial bottleneck). In combination with parallelization, probabilistically ignoring selectors based on their weight may also be an option if the run-time of this part of the procedure ever becomes a concern.

\( ^{13} \) We use “directly” in reference to the fact that, relative to the updates given to other selectors and given how votes are aggregated (Eq. (2)), a selector may loose or gain sway over which operator is ultimately selected even if its individual weight is not modified.

\( ^{14} \) Consider how a selector that consistently votes the uniform distribution would be treated were subtraction of \( U \) absent, such as when the system as a whole is more often successful than not. Bear in mind that while the addition of a uniform value would not alter the ranking of operators under the aggregated vote (Eq. (2)), the contribution would serve to "flatten" \( D_{samp} \).
corporates considerations for exploration. The approach proposed in this write-up was constructed prior to making this sort of connection, but it is no surprise that similar proposals might appear in similar problem settings — however, more work is to be done in order to certify that there is more than a superficial connection, one that offers more insight than citing the general relevance of the entire sub-field of model-free RL.

Further modifications to Eq. (3) may come by more fully approximating an error gradient based off of Eq. (2), which would require Eq. (3) to have a multiplier of roughly \( \sum_{j,k} w_{j,t}^2 \) for most \( i \in |S_i| \) (ignoring special effects of negative terms); in addition to the standard mathematics that would motivate this, it also plays into the intuitive story of punishing or rewarding selectors based on “how much they caused an outcome”, where a selector’s weight certainly plays a role in its blame. Space for modifications and alternatives abound, depending on how much of the rest of the system one is willing to modify — some such options we discussed in Section 5.2. Here, as we did there, we consider an effective approach that meets our general criteria, but which we do not claim is optimal or beyond criticism.

6.2 Producing the Reward Signal

We now discuss how, in this initial implementation, we form the \( y_t \) used in the previous subsection. Below, for ease of discussion, we will use "iteration t" and "time t" interchangeably.

At a high-level, the process in Fanoos follows the following overall flow:

(1) Produce the state for time \( t \).
(2) Display a response for time \( t \) by extracting relevant information from iteration-t’s state.
(3) Receive user-feedback for time \( t \), \( f_t \).
(4) Choose an operator at iteration \( t \) to be used to form the state for time \( t + 1 \).

The types of user feedback currently supported are listed in Table 1. Once we apply the operator to produce a new state and show it to the user, we will receive more user feedback, \( f_{t+1} \). While many different methods for forming \( y_t \) can be supported, our implementation uses the reward function described in Table 2. The idea behind this reward function is that we want to learn which operators do in fact produce more (respectively, less) abstract descriptions on user demand. We take the user "reversing" their request (e.g. \( f_t = l \) and \( f_{t+1} = m \)) to be a sign that abstraction definitely went in the correct direction and to a non-trivial degree. Similarly, if \( f_{t+1} = b \), we take it as a sign that we satisfied the user’s previous request. Naturally, the simplicity of this approach comes at the cost of potentially missing more nuanced patterns, such as what the appropriate feedback should be in hypothetical cases where we "got close to the right abstraction level but overshot/undershot". At the very least, however, our proposed reward function appears to be a reasonable way of guiding operator selection for cases that are less murky.

7 Bootstrapping the Learning Process: The Autouser

7.1 Purpose

Prior to spending the time, capital and patience needed for a human to train the operator selection system from scratch, we bootstrap the learning process by defining an oracle to act approximately as we expect a human user would; one can view this proposal as a form of curriculum learning \( \text{[15]} \), where the automated-user (“autouser” or “AU”) provides the first set of tasks (and hence lessons) to Fanoos so that Fanoos can more easily and rapidly fine-tune itself to satisfy real users’ requests. This proposal bares a few points worth clarifying:

First, the fact that we define an "autouser" to evaluate responses of Fanoos does not itself entail that we could hard-code an operation-selection method that

15 For those interested, it would be trivial to extend the user interface with an option, perhaps under the submenu invoked by entering “u”, to exit the description adjustment loop but interpret the exiting as failure. Ideally, the user would not have to be concerned about such things and any confusion caused to Fanoos by not having this additional option would be minimal.
| Types of User Feedback |
|------------------------|
| shorthand description |
| l | make the description less abstract |
| m | make the description more abstract |
| b | ends the user’s interrogation regarding the current question. |
| u | allows the user to specify operators to apply or, in the case of the history-travel or manual predicate review operators, interact with. |

Table 1: Description of the types of user responses Fanoos currently supports

| Reward Function |
|-----------------|
| $f_t$ | m | m | l | 1 | m/l | u | b |
| $f_{t+1}$ | m | l | b | l | m | b | u | (any) | (none) |
| $y_t$ | -1 | 1 | 1 | -1 | 1 | 1 | 0 | 0 | (none) |

Table 2: Description of the reward function Fanoos currently uses, showing the reward signal as a function of consecutive user requests. Note that when the user enters “b” at iteration $t$, the session of question-and-answering breaks, so there is not feedback or reward signal from iteration $t + 1$, since there is no iteration $t + 1$.

optimizes it directly. The fact the one knows how to evaluate a result does not mean they know how to produce a result — this distinction, for instance, is clearly visible in NP-complete problems. To some, this point may seem obvious (indeed, it is at the heart of much of reinforcement learning). However, we highlight it since, all to often, the first problem that comes to mind when brainstorming “how to deliver the right thing to a user” is trying to properly infer what a user wants; our point here is that, even if we had a perfect user model detailing every pertinent aspect of human cognition, determining what steps to take in order to satisfy that request would still, in general, need to be solved and can be far from trivial.

Second, the notion of “bootstrapping” a learner on a reasonable but imperfect surrogate problem (here, trying to please the autouser) is far from silly. Unlike a human, an autouser is cheap, perfectly repeatable, and entirely transparent in regards to its evaluation method. While we doubt that an autouser we implement would capture all the trends in human desires with extreme accuracy, we expect it to more often than not agree with humans, particularly in cases where a human user would have strong sentiments regarding the comparative quality of results. This pattern of pre-training a learner on a closely related, easily accessible set of circumstances is core to curriculum learning and many sim-to-real works in robotics. Better models would provide us better confidence, but reasonable models can be useful even if imperfect, and we do not need to perfectly replicate a human mind in software in order to yield worthwhile, positive outcomes by taking this approach.¹⁷

7.2 Method

In what follows, we may refer to the autouser we implemented as AU. Let $q_t$ be that state shown to AU at time $t$, and $H_t$ be the full history of interactions Fanoos has had up to and including time $t$, whether that be with the AU or actual humans. $H$ contains the full records of all states, all requests, and any other pertinent side-information involved in prior interactions up to and include time $t$ — in short, a complete transcription of all past information potentially relevant. AU determines what feedback it should provide to Fanoos at time $t + 1$ (i.e.,

¹⁷This should be especially apparent to any who believe that abstraction level is objective and/or human-independent to any degree.
In our implementation, we use reservoir sampling to access the difference between the new state and the prior state’s values as opposed to the values themselves.

In our current implementation, the autouser considers the following criteria (i.e., the range of $\pi$):

- $j=1$ The total volume (after normalization) of abstract states covered by user-defined atomic predicates (a.k.a., “named” predicates)
- $j=2$ The total volume (after normalization) covered by box-range predicates
- $j=3$ The number of unique named predicates used in the description
- $j=4$ The number of conjunctions that appear in the description
- $j=5$ The number of box-range predicates used

The volume information used by Item [[1]] and Item [[2]] comes from the first component (“csToTV”) returned by algorithm 9 (“getVolumesCoveredInformation”) in [3]: as highlighted by our verbiage in the above bullets, the volumes used are normalized by the total volume covered by all the abstract states, thus bounding their ranges and aiding in their interpretation independent of the specific domain and question-type in use. With this mapping of $j$ to specific attributes, we use $\gamma_1 = \{2, 3, 4, 5\}$ and $\gamma_2 = \{1, 2, 3, 4\}$.

Among our experiments, we intend to conduct ablation studies, making sure to include analysis where the fields available to the selectors are disjoint from those allowed to the autouser. Various schemes for allocating information to the selectors (and operators) versus the autouser provide worthwhile insights — arrangements such as having both be able to see the same fields, allowing one to see a subset of what the other has, or insisting that their sources are (at least in name) disjoint. One must be careful in all scenarios, of course, to hedge interpretation of performance by consideration of how much “label leakage” there might have been (e.g., directly optimizing the

\[ \psi(q_{t-1}, q_t, H_t) = \text{step}_{0.5, 0.5}(1(f_t = "m")) \times \text{step}_{0.5, 0.5}(1(j \notin \gamma_1)) \times \begin{cases} \text{ECDF}(q_{t-1}.v_{\pi(j)} - q_t.v_{\pi(j)}, \delta.v_{\pi(j)}(H_t)) & \text{if } j \in \gamma_2 \\ \text{ECDF}(q_{t-1}.v_{\pi(j)} - q_t.v_{\pi(j)}, \delta.v_{\pi(j)}(H_t)) & \text{if } j \notin \gamma_2 \end{cases} \]

where

\[
\text{step}_{a,b}(x) = \begin{cases} 1 & x > b \\ 0 & x \in [a, b] \\ -1 & x < a \end{cases}
\]

In the above, $\pi$ is an injective map from $[k_{au}]$ to a subset of the fields present in states that AU bases its decisions on, and both $\gamma_1$ and $\gamma_2$ are subsets of $[k_{au}]$ used to gate the behavior of $\psi$. Notice that AU only accesses the difference between the new state and the prior state’s values as opposed to the values themselves.

That is, since the statistics in question reflect parts of the same state and system, there may be a “spiritual sense” where the information is “not disjoint”—our concern is ensuring that any connection between the fields is “meaningful” and a correlation inherit in the properties of the system, not a trivial connection equivalent to label-leakage.
difficulty meeting requests. One could say that, conversely is more lenient when Fanoos is having to modify descriptions in the fashion requested, and harshness as Fanoos demonstrates increased ability AU judges success in this scenario with increasing.

Taking general inspiration from curriculum learning, we will refer to this as the debatable case.

that are majority positive but not unanimously pos-
tive; we will refer to this as the debatable case. Taking general inspiration from curriculum learning, AU judges success in this scenario with increasing foolishness as Fanoos demonstrates increased ability to modify descriptions in the fashion requested, and conversely is more lenient when Fanoos is having difficulty meeting requests. One could say that, as Fanoos’s performance improves, the AU’s “expectations” increase, raising the bar for what is considered acceptably good. Adopting this adaptive criteria helps provide signals to Fanoos which highlight actions with effects that, while beneficial, would lead to insufficiently compelling changes if taken alone. At the same time, the approach applies long-term pressure in hopes that Fanoos can eventually uncover how to generate fully satisfying alterations to descriptions — alterations that may be extremely difficult to stumble upon if absolute perfection was demanded from the beginning.

In our implementation, AU handles the debatable case by randomly choosing whether to repeat its previous request or request something different, with odds increasingly favoring the former as the degree of success increases. This randomization is natural in the sense that, for humans, decisions for which evidence does not provide a clear, dominating answer often are influence by momentary mood and other arbitrary factors, inducing some randomness on the outcome. Mathematically, this randomization allows AU to indicate the relative effectiveness of each strategy attempted by Fanoos via the long-term proportion of success a strategy incurs — this despite the limited set of individual requests AU can make. Further, randomness in the user requests help the system as a whole explore more widely and reduces the likelihood of “getting stuck” during an encounter with pathological situations (such as cases where a particular request cannot in principle be satisfied or certain types of hypothetical, undesirable cycles in interactions occur).

As the success rate increases, the bounds on the acceptable $S_1$-to-$S_2$ ratio increase and become more narrow, placing higher demands on AU and providing less leniency. In Algorithm 1, in order to determine the rate at which the range of cutoffs narrow (approaching a deterministic cutoff), we introduce a parameter $\ell$. We tune $\ell$ so that at a global success rate of 60% (i.e., the success rate taken across all of $H_t$, not just across that session), Fanoos must pro-

\[ \ell = \frac{1}{2} \]  

\[ \ell = \frac{1}{2} \]  

Respectively, indicating that the AU is (a) unsatisfied and hence Fanoos failed or that (b) Fanoos succeeded.

21 See Appendix B for further comments on cycles and our perspective on them if they occur in our system.
duce a description that achieves higher than the minimum possible “improvement” — that is, higher than the lowest possible positive value of $\frac{k_{au}}{S_2}$, which is no less than $\frac{1}{k_{au}}$. From this, trivial calculation gives that

$$\ell \leq - \frac{\log(k_{au} - 1)}{\log(0.6)}$$

In particular, we use:

$$\ell = - \frac{\log(k_{au} - 1)}{\log(0.6)}$$

We highlight that $\ell$ influences only the lower bound of the judgment threshold; since $\alpha$ is chosen uniformly at random over [0, 1] in Algorithm 1 at a 60% success rate, the expected threshold is:

$$0.3 + 2^{-1}(k_{au} - 1)^{-1}$$

At this expected threshold, there would need to be roughly $1.3$ as many changes to the description in the proper direction as there are changes in the wrong direction — that is, the value of $\sum_{i \in [k_{au}]} \{P_i; \lambda > 0\}$ versus $\sum_{i \in [k_{au}]} \{P_i; \lambda < 0\}$ (where $P$ is from Line 1) must (in expectation) be roughly at least $\frac{1}{k_{au}}$ in order for AU to consider the description to have changed in the desired ways. In simplest terms, a success would need at least roughly twice as many positive changes as negative.

AU continues to request changes in description until one of three conditions are met: (1) the number of user requested adjustments reaches a pre-specified maximum (determined by a human-set parameter, e.g., 200 user adjustment requests), (2) the description has not changed for more than a human-set number of consecutive requests, $c_{au}$, or (3) the description contains only box-range predicates for $c_{au}$ many consecutive adjustments. As per our reward function described in Table 2 when AU issues the exit command, Fanoos will interpret this as a success signal, despite cases (2) and (3) more clearly being failures; as is the case with a real user, Fanoos is not informed as to what motivates AU’s exit. Since occasions of concern should be rare when using the autoruser — not only because of natural circumstances but also due to $c_{au}$ being sufficiently high — they should not be overly influential; empirical demonstration of such may be prudent. We are not overly concerned regard-

Input: $\psi$ as defined in Eq. (4); the history up to this time step, $H_t$; the current state, $q_t$; the previous state $q_{t-1}$; AU’s previous request to Fanoos, $r_{t-1}$; $\ell$, the value tuned in Eq. (5)

Output: A new request from AU, $r_t$, in response to the description Fanoos provided for the most recent state, $q_t$.

Algorithm 1: Pseudocode for the process implemented by the autoruser (AU) to judge responses from Fanoos.

1. $P \leftarrow \psi(q_{t-1}, q_t, H_t)$
2. $S_1 \leftarrow \sum_{i=1}^{k_{au}} \{P_i\};$
3. $S_2 \leftarrow \sum_{i=1}^{k_{au}} \{P_i\};$
4. if $S_2 == 0$ then
   5. /* No relevant aspect of the description substantially changed after AU’s most recent request. */
   6. return $r_{t-1}$
7. end
8. $\alpha \sim$ uniform([0, 1]);
9. $g \leftarrow \text{get\_global\_success\_rate}(H_t)$;
10. if $S_2 \geq \alpha g + (1 - \alpha)g^t$ then
11. return opposite($r_{t-1}$)
12. end
13. return $r_{t-1}$

8 Selectors Used In Current Implementation

Having described how selectors are used in Section 3, we now describe further the selectors we have implemented.

Recall that selectors are the components in our process that can incorporate expert knowledge (B1), knowledge of how useful operators are (A2), and how much the current state looks like previous states (A3). The output distributions that selectors provide are expressive, in that they can represent classical logic-based rules and more — the former by
providing verdicts that only have support on a subset of the operators.

In our implementation, we consider four categories of selectors: uninformed selectors, applicability selectors, history-informed selectors, and second-order selectors.

8.1 Uninformed and Applicability Selectors

Uninformed selectors are those that do not consider any aspect of the state, either directly or indirectly. In order to provide a baseline, help interpret our later results, and further encourage exploration, we provide a selector that simply places a uniform vote on each operator (i.e., places equal weight on all operators in all circumstances). Additionally, for the same reasons, for each operator we create a unique corresponding selector whose sole purpose is to vote exclusively for its assigned operator (i.e., the selector outputs a distribution with support only on its assigned operator).

Applicability operators produce votes that only indicate when a specific subset of operators are unexpected to be applicable in a given situation. As such, they assist in regard to consideration (B2). For instance, consider the set of operators, \( S_{O,1} \), that function by re-allowing a named predicate that had earlier been disallowed from appearing in descriptions; if all named predicates are allowable at time \( t \), then no member of \( S_{O,1} \) is applicable at time \( t \). In the circumstances where members of \( S_{O,1} \) are unapplicable, then a specific applicability selector, \( op_{a,1} \), would put all its support uniformly over \( S_O \setminus S_{O,1} \).

In the case where the members of \( S_{O,1} \) are applicable, \( op_{a,1} \) puts a uniform vote on all operators. In short, when an operator is unapplicable, it is clearly undesirable to use them and an applicability operator indicates such; when an operator is applicable, there might still be better alternatives in the specific circumstances, and as such an applicability operator then makes no claim as to which option is better. In our implementation, we provide applicability selectors that indicate when each of the operators in Section 4.3 would be sensible to use.

While applicability selectors are useful in their own right — potentially helping tip the balance in favor or disfavor of certain operators via their direct, additive influence in Eq. (2) — they are particularly useful when used in higher-order selectors (Section 8.3), which combine together the insights of multiple selectors in non-linear fashions.

8.2 History-Informed Selectors

History-informed selectors dig deeper into the values stored in a state than applicability selectors, casting votes based on a state’s reachability analysis results, description content, and historical similarities. Each of the selectors in this category learn — via their own methods — which operator is best to apply in a given circumstance, and thus these selectors are yet another component of our system that fill desire (B1).

For an initial and reasonable implementation, we consider history-informed selectors that proceed via the following steps:

1. Compute a distance between the current state and a relevant subset of those seen in the past,
2. Use the distance from step 1 to rank states (e.g., the state from the past closest to the current state, the past state second-closest, etc.),
3. Determine the mass to give each operator based on the operator’s success rate weighed by how close (in ranking) states it previously operated on are to the current state.

First we will overview what schemes we implemented for determining the distance-based ranking between states (step 1 and step 2), then we will overview how the resulting ranking is used to divvy out proportions of a vote across each operator (step 3). Prior to this, however, we take a moment to overview what information is available to base distances on.

8.2.1 State Fields Used in Distance Calculations

Let \( v_j \) be the value of field \( j \) in the state; that is, some observable aspect of the state we know about. The set of \( v_j \) considered by history-informed selectors include:

- The minimum, maximum, mean, median, standard deviation, total, first quartile and third
quartile of the scaled input boxes’ volumes. The scaled input boxes are those whose axis-lengths have been divided by the respective axis of the universal bounding box, allowing the measures to have increased independence from the specific domain used at a given time.

- We use the same summary statics as the previous bullet, except over the set:
  \[ \{ \text{sum\_side\_length}(b) | b \in \text{scaled\_input\_boxes} \} \]
- The logarithm of the total number of input boxes divided by the maximum possible number of boxes generally possible (e.g., not considering the question asked) in our refinement scheme given the refinement parameters used and given the domain information of the model being analyzed. Specifically:
  \[ \log_3(\text{total\_number\_boxes}) + \zeta \lfloor \log_3(\epsilon) \rfloor \]
  where \( \zeta \) is here taken to be the dimension of the input space. We provide a derivation of this in Appendix C.
- Information on the named predicates that occur in a description, namely the number of unique named predicates as well as the total number of times named predicates occur (recall that the same named predicate may appear more than once in a description if it occurs within multiple different conjuncts).
- The number of conjunctions, disjuncts and box-range predicates.
- Both the total volume and unique volume covered by each of the named predicates, box-range predicates, and conjuncts, as based on the results from algorithm 9 in Appendix B. As commented on in Section 7.2, the individual values that make up these sums are normalized.

The fields used above are applicable across all question types and domains. As such, using these fields allow our selectors to leverage all prior experiences — in a not entirely trivial sense, transferring what it learns between question types and domains. This said, it is not a general requirement of our approach that such broadly applicable fields be used — the information provided to selectors can be tailored for specific question types or domains — we simply choose the broadest net to cast in our current implementation, and save further specialization for some other venture.

While we currently do not inform selectors as to the question type or domain, future work could examine the proper methods of providing such information, balancing transfer of experiences between qualitatively different circumstances with insights provided by the specific setting.

8.2.2 Computing Distances Let \( Q_{f_t} \) be the set of states recorded in the history such that for each \( q \in Q_{f_t} \), the feedback from the user prior to forming \( q \) is the same as the user’s current request; e.g., if the user requested “m” now, then \( Q_{f_t} \) would contain states, \( q_t \), that were previously presented to users at a time \( t \) following the user’s “m” request at time \( t - 1 \).

We order the members of \( Q_{f_t} \) by descending distance from the current state, based on \( \langle v_j | j \in [k] \rangle \) (i.e., the distance between state A and state B is base on \( d(\langle A.v_j | j \in [k] \rangle, \langle B.v_j | j \in [k] \rangle) \), for some metric \( d \)). Let \( \text{rank}(q_t) \) be the rank assigned to \( q_t \) in \( Q_{f_t} \) by this ordering.

Currently, the filtered set of states that we use to inform decisions — here, \( Q_{f_t} \) — neither incorporates (i.e., filters by ) the question type nor by information about the specific domain. While this comes with the benefit of having a larger volume of experience to draw upon from a larger net of circumstances (arguably a simple form of “transfer learning” between situations), it comes at the cost of admitting “less precise” information that could “blur the view”. Future work may consider this further information (the question type and specific domain), likely in addition to — as opposed to a replacement of — \( Q_{f_t} \) (e.g., as another set of selectors to use, or as terms influencing the distance measures).

Among our selectors, 54 use exclusively a single-feature distance. To be precise, for each \( v_j \), we create an operator selector that computes the distance between state A and state B as \( |A.v_j - B.v_j| \).

In addition to our simple single-feature approach, we consider the use of random projection \([13, 7]\), a relatively straight-forward method of dimensionality reduction that has nice theoretical properties, good time-complexity, and which has found widespread application. Further, in contrast to the selectors we detailed in the prior paragraph, random
projections produce their rankings based on multiple state-features.

First we produce five random vectors in \( \mathbb{R}^k \) with unit Euclidean norm, generated by sampling each component uniformly at random on \([0, 1]\) then normalizing the result. Let \( u_{p,i} \) be the \( i^{th} \) such vector produced (the \( p \) subscript is to remind us this is for “projection”). Let \( \phi_j(x) : \mathbb{R} \rightarrow \mathbb{R} \), \( i \in [k] \), be a strictly increasing function of \( x \) — we will specify what these are and why we use them in just a moment. For a selector using random projection with projection vector \( u_{p,i} \) and featurization functions \( \phi_j \), the distance between states A and B is computed as

\[
| \sum_{j=1}^{k} \{u_{p,i}\}_j \phi_j(A.v_j) - \sum_{j=1}^{k} \{u_{p,i}\}_j \phi_j(B.v_j) | \tag{6}
\]

or, equivalently, as

\[
| \mathcal{L}^2(u_{p,i} \cdot \{ \phi_j(A.v_j) - \phi_j(B.v_j) | j \in [k] \}) | \tag{7}
\]

where we use \( \mathcal{L}^2(\cdot, \cdot, \cdot) \) to be the \( L^2 \) inner-product.

Our featurization functions serve to put disparate fields on some common ground so that it is sensible to compare or combine them. We consider two methods to do this: the first is standardization, namely:

\[
\phi_j(A.v_j) = \frac{A.v_j - \text{mean}(\{q.v_j| q \in Q\})}{\text{std}(\{q.v_j| q \in Q\})} \tag{8}
\]

Standardization causes each \( A.v_j \), to have the same mean and standard deviation (zero and one, respectively), but preserves distances between values, in the sense that, for states A, B, and C such that \( A.v_j \neq B.v_j \), we have:

\[
A.v_j - C.v_j = \phi_j(A.v_j) - \phi_j(C.v_j) \]

\[
A.v_j - B.v_j \neq \phi_j(A.v_j) - \phi_j(B.v_j) \tag{9}
\]

Our second approach is \( \phi_j(A.v_j) = \text{ECDF}(A.v_j, \{q.v_j| q \in Q\}) \) — that is, in the distribution of \( v_j \) values, this featurization gives the proportion of the distribution that have value no greater than \( A.v_j \). Unlike standardization, the ECDF value has an a priori bounded image of \([0, 1]\), and random variables transformed by the ECDF have very well understood behavior (using the universality of the uniform distribution and Dvoretzky–Kiefer–Wolfowitz inequality to uniformly bound the error between the true CDF and the ECDF). The main distinction we draw between standardizing and using the ECDF, for our application, is whether or not the "raw" feature distance (as given by standardization) is more meaningful than the "distance across the distribution" (as given by taking the ECDF). In situations where the significance of a "raw" distance value varies substantially with where that distance is based in the distribution, the ECDF may provide a more sensible ranking of values. More broadly, ECDF can be used to measure distance in a way that is less affected by common artefacts of data representations that may be chosen; specifically, given a collection of data in one variable, applying a strictly increasing function to the data can radically change the distance between a pair of points, but the ECDF will be unaffected. Further, while the random projection method we adopt is neither a copula nor a difference between copulas, aspects of what make copulas attractive for modeling similarly motivate our use of ECDFs. Ultimately, whatever the case, we’d like to use whichever of the distribution’s properties best correspond with our “class label” — the operator(s) that is/are best to apply to the state in order to move the description’s abstraction level sufficiently in the proper direction (which, as an effect, should satisfy a request of type \( f_t \)). In the next subsection (Section 8.2.3), we go over how we leverage previously seen “labels”.

For efficiency, both in terms of placement in the memory hierarchy and time complexity at scale, we compute the ECDF using reservoir sampling. The mean and std are computed efficiently by tracking the sum, squared sum, and number of entries for each field.

8.2.3 Computing Vote Distributions Given State-Distances Let \( S' \in S_n \) be an arbitrary history-informed selector, and let \( \alpha \) be a fixed value in \((0, 1)\) specified ahead of time for use by \( S' \). Let

\[
\text{weight}'(O') = \sum_{q_t \in Q_{f_t}} y_t \mathbb{1}(O' = O_t) \alpha^{\text{rank}(q_t)} \tag{9}
\]

\[22\] Notice that in both of the approaches, the transform is based on \( Q \) as opposed to \( Q_{f_t} \); the latter would have been preferred, but for a variety of reasons, our implementation does not at this time use \( Q_{f_t} \) in those capacities.

\[25\] Also known as the probability integral transform.
where \( O' \in S_O \). The vote distribution returned by \( S' \) is produced by running \( \text{weight}' \) over each element of \( S_O \), then normalizing the resulting list of values.

Notice that the above description explicitly uses operator success rates and notions of distances between states, covering both \( A2 \) and \( A3 \). Further, to a very limited extent, the filtering used to form \( Q_{f_i} \) in Section 8.2.2 aids in pursuit of consideration \( B2 \). For each field considered, we create two selectors, one whose value for \( \alpha \) is 0.811 and one whose value for \( \alpha \) is 0.896. Assuming there are no ties and that the use-history is sufficiently long, an \( \alpha \) value of 0.811 places roughly 90% of the mass in the top 10 positions, while an \( \alpha \) value of 0.896 places roughly 90% of the mass in the top 20 positions.

While many reasonable alternatives for weighing operators would incorporate information about how often an operator has been used (i.e., Item \( A1 \)) in addition to its success rate (i.e., Item \( A2 \)), we limit ourselves here to considering only the latter in part because the final step in selecting an operator incorporates the former via the UCB algorithm.

An alternative that was implemented, though currently deactivated in our code, is the utilization of KNN-like approaches for weighing.\(^{26}\) For these selectors, we limit consideration to the top \( z \)-neighboring states (for some integer \( z \) that is a fixed parameter of the selector) and vote for each operator in proportion to its success rate over these states, providing no voting mass to other operators. KNN approaches can be implemented efficiently (especially when over a single variable), and offer a different profile of bases compared to the exponentially weighting (e.g., how they treat operators that are frequently used, but only for states distant from the current one). While interesting to consider, we have opted against using this KNN-like method within our current implementation due to unencouraging empirical observations thus far, suggesting that the approach does not sufficiently leverage available information while attempting to utilize the distance score. Our exponential weighing scheme seems capable of leveraging all pertinent information, and arguably utilizes distance in a more natural manner. However, we might reverse this decision at a later time, after further experimentation with appropriate resources.

### 8.3 Higher Order Selectors

Via the infrastructure described in Section 5.1, we support modeling joint-behavior between selectors by constructing a new selector that acts as a function of other selectors’ output. Among the benefits this provides, it enables combining the opinions of multiple selectors that each examine independent information streams. In total, 1552 out of the 1731 selectors we have implemented fall into this category.

In our implementation, we produce higher-order selectors whose votes are the normalized product of two history-informed and/or applicability selectors; in the case where the support of the product is empty, the uniform distribution is returned. For each pair of selectors that vote based on the single-feature distance of distinct fields, we form a unique new selector. Additionally, we form a unique new selector for each pairing of a history-informed selector\(^ {27} \) and an applicability selector.

### 9 Experiments

Our experiments are on-going. Currently, we using two simple domains — the testing domains, as described at \( \text{[2]} \) — as fast and generic arenas that we understand well, and thus allow us to better interpret and extend results. In contrast, if we used complex domains and learned systems from real-world scenarios for our initial evaluations, not only would it require greater resources, it would introduce numerous confounding factors and make it more difficult to attribute outcomes to causes. Further, we would like to be able to learn lessons from one domain, make improvements, then be able to evaluate on a new domain, to help ensure that any attempted modifications were not “overfit” by the experimenters to the characteristics of one domain.

\(^{26}\) KNN: K-Nearest Neighbors (“K” here being unrelated to variables that appear elsewhere in this document).

\(^{27}\) In multiple dimensions, a KD-tree (e.g., \( \text{[6, 18]} \)) allows for efficient search for nearest neighbors, albeit approximate. In a single dimension, a sorted list trivially allows efficient and exact answers.

\(^{28}\) I.e., both those based on single-feature distance and those using random projection.
Currently, trials are produced on an on-going basis by having questions randomly generated for the target domains (similar to the generation process described in section 4.2 of [3]). We bound the length of time each question-response session may run, as well as the amount of memory allowed to Fanoos; if these bounds are exceeded, our experiment harness forces the sessions to exit.

After an extended period of running our process, we plan to examine:

(R1) Which selectors receive the largest (or smallest) magnitude weight, and whether the weight is positive or negative. Discussing the distribution of weights also may be of interest.

(R2) Which operators were used most often. Also worth examination here are the weights given to the uninformed selectors that vote exclusively for one particular operator (see Section 8.1), since the occurrence of related activities are reflected in the weights of those selectors (those activities being the number of times its targeted operator was not used, the number of times the operator was used and succeeded, and the number of times the operated failed when used).

(R3) How quickly Fanoos — using the learning method described in this paper with the reward function (i.e., AU) prescribed — learned which operators are appropriate to use in different circumstances.

(R4) The running success rate across time (including bounds to account for discretization effects on the value)

(R5) Distribution statistics (e.g., mean and variance) for the change in pertinent measures that occur after application of the blank operator. This provides a baseline to judge the performance of other operators and evaluate other measures.

(R6) Analysis based on human-provided word-labels, similar to section 4.3.3 (“Human Word Labeling”) of [3]. The labels are seen neither by Fanoos nor by AU, allowing us to use the information as a separate evaluation for both with minimal explicit contamination.

(R7) Qualitative analysis of descriptions (similar to the experiments in [3]) generated under different regimes of training and at different points in time.

(R8) If space, resources, and interest are amenable to it, case-studies of interactions taken from different durations into the learning process (e.g., at the beginning, after UCB bounds become “small”, mid-way before performance plateaus, and after performance stabilizes).

Examination of the final vote distributions (i.e., Eq. (2)) at each step. For example, we may plot the per-timestep entropy and the time-averaged entropy (the latter with appropriate bounds). We expect the entropy of the final vote distribution to generally decrease from initial values as learning progresses. We also expect the time-averaged entropy will plateau at a lower-bound eventually, presumably around the time of when roughly maximum performance is reach; we would take this to indicate that Fanoos has developed stronger ideas/opinions as to a what operators are worthwhile to apply in various circumstances.

A plot over time of the “strength” of AU’s opinion on a subject, as shown by the value of:

\[
\frac{S_1 - g^t}{g - g^t}
\]

where the variables in question come from Algorithm 1. Note that this value is negative when AU will reissue the same request, greater than or equal to one when the AU issues the opposite request, and is in [0,1) when the response provided by AU is influence by randomization. Both raw values and the time average with variance bounds may be shown. Comparison to the plot of success rate over time (Item [R4]) may aid interpretation — namely that as \(g\) increases, the strength of AU’s opinion should on average be lower.

(R9) Analysis of the correlation between final vote distributions in pairs of consecutive timesteps, \(t\) and \(t + 1\). Consideration of this topic may benefit from (a) further stratifying analysis buckets by whether the timesteps represent a successful adjustment (i.e, by whether \(f_t \neq f_{t+1}\)) or (b) a scatter plot of the correlations versus the “strength or the AU’s opinion” (as defined in Item [R10]) at timestep \(t + 1\).

Recall from Algorithm 1 that randomization plays a role in determining the final feedback that AU provides,
(R12) Resources and interest permitting, ablation studies as state-fields used by the autouser and/or selectors are removed and/or added. We already provided some comments regarding the utility of these experiments in Section 7.2.

(R13) Resources and interest permitting, examination of behavior when selectors and/or operators are added or removed, whether it be at the beginning of the learning process or after some time into it. In the case of selectors, it would make sense to categorize them by their weight (strong negative, strong positive, near zero, or other) and show the behavior when each category is removed. Motivation for these trials come not only from the desire to build a scientific understanding, but also from engineering considerations, such the desire to enable and understand the ramifications of adding novel innovations in a hot-swap fashion.

(R14) Resources and interest permitting, examination of the overall effects of interpreting user-invoked exiting of the description adjustment loop in the fashion described in Section 6.2. Our comments at the end of Section 7.2 are relevant here. Experiments under this heading could include performance analysis after the reward for $f_t = \text{“b”}$ in Table 2 is changed to 0 and/or 1, in addition to other analysis of Fanoos’s sensitivity to the value of $c_{au}$ used by the autouser.

(R15) Sanity checks on the frequency that cycles (if any) occur. This would also include analysis of the distribution of cycles’ periods and content. Comments regarding cycles in Section 7.2 and Appendix B pertain here.

After demonstrating on the test models and gaining insights from those trials, naturally we also aim to showcase on real models, performing similar, rigorous analysis to the extent possible and practical.

Ideally — resources providing — user studies will also be conducted to demonstrate:

- Improvements garnered by the learning process (and therefore also the AU), as shown by human blind evaluations on before-learning and after-learning comparisons
- Measurements of human satisfaction with the adjustments provided by the system after it has been trained to the point of plateaued reward under AU.
- Examination of whether AU causes the average success rate of Fanoos, after fine-tuning with a human, to be as high or higher than if Fanoos was trained from scratch by a human
- Examination of whether the time required to fine-tune Fanoos for humans to perform satisfactorily or maximally after training with AU is substantially less than the time required for a human to train Fanoos from scratch to achieve similar levels of stable, average performance.

10 Further Extensions

Comments regarding possible extensions of this work that may be of interest for readers to consider can be found in Appendix D.

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A Several Interpretations of the Operator Selection Process

The method used to segment blocks of selectors can be compared to a variety of techniques from ML and
rule-based systems. We have already exposed how the connection to featurization in generic ML is relevant; by having features that are non-linear functions of the base-set of selectors, we can easily form non-linear decision boundaries even if the final aggregation method is linear in respect to the features — this is common practice in generic ML, for instance, while using SVMs.

Beyond seeing selectors as barely informed, observable aspects of phenomena — as basically just data without deeper processing — we may also view them as experts, in the sense often used to motivate randomized weighted majority voting ([14]) or other simple forms of zero-regret learning. While easily interchanged from a purely mathematical outlook, this latter perspective of the inputs provides a different sensibility with which to view the selectors. Similarly, considering the approach from the standpoint of literature for general ensemble-methods[30] provides still a slightly different lens to view the process and, worth emphasizing, yet another tool-bag of tricks that may be applicable. Motivated by the desire to perform better than the single best selectors, we suspect the ensemble viewpoint would lead down more roads of benefit than placing concerns over minimizing regret in the front of mind[31].

Outside of ML-based approaches, literature on scalable rule-based system architectures provide another perspective for comparison, particularly those based on blackboard architectures [12, 10, 19].

An immediate question that may spring to the reader’s mind is what utility, if anything, pointing to these connections provides. Let us first note that, regardless of the perspective taken, all ways of viewing our work would be in regard to the same concrete facts — namely, our work. As tautological as this sounds, the distinction being drawn is similar if not the same as the difference between a particular mathematical structure (in the sense the work “structure” is used in classical logic), and various logics that may consider that structure, but under differing rules and syntax. That is, while all the mentioned literature-lenses consider the same concrete object, they provide differing evaluation criteria, either explicitly or implicitly via the history, trends, social-structure, and connotations of the area. Of particular interest to a problem solver (e.g., the researcher or engineer trying to extend this work), these different perspectives provide different paths and (implicitly if not explicitly) heuristics from which to work off of. For those interested in a deeper discussion of the almost “meta-research” notions presented here, the beginning chapters of [16] provide a reasonable starting place.

B Further Comment on How We Currently View Any Cycles that Occur in the Fanoos-Autouser Interactions

Arguably, not all types of cycle are bad. For instance, if Fanoos produces a repeating series of descriptions in response to similarly repeating requests by the autouser, so long as AU is being satisfied and the cycle has a period of considerable length, this may well help strengthen Fanoos’s knowledge of how to satisfy requests on cases that may be particularly frequent. While we admit that the possibility of the autouser (as we currently implement it) being involved in cycles is a point of departure from ideal human behavior (namely, searching through descriptions like a binary tree, and using the special, user-invokeable history-travel operator[32] in case they wish to return to an earlier state / description), the benefits just mentioned may still be present and might outweigh any detrimental effects. This all said, if this is ultimately deemed a problem, it would be trivial to modify the autouser to issue punishments to Fanoos if the latter ever repeated a description; this in-and-of-itself, however, would not prevent future cycles from occurring and may cause Fanoos to “become

30 While we have been aware of general connections to ensemble methods such as bagging, boosting, cascades etc., we very recently came upon the lead to stacked methods ([21] [9]) that use combiners based on linear models. We are still in the process of adequately exploring this particular connection, but early indications suggest it is a fruitful tie.

31 Notice that in the preceding discussion, we were commenting on lenses to view the inputs of the system, not necessarily commenting on additional desiderata we want to adopt.

32 See “u” in Table I.
confused” as to the reason it is receiving negative feedback. Risk for “confusion” may be particularly high if none of Fanoos’s selectors are sufficiently sensitive to the structure of the interaction history — essentially, how could Fanoos know it should not repeat descriptions if it lacks “long-enough-term memory” to know it repeated itself? It may be possible to detect a cycle via latent factors in the current timestep’s state but that seems doubtful and at best not sufficiently reliable. Naturally, a way to help address these concerns is to either modify existing selectors or add new ones that incorporate this additional knowledge, voting in a way that aim to avoid cycles while also pursuing the other pertinent objectives.

C Explanation and Derivation of How the Total Number of Boxes are Used in Section 8.2.1

In our refinement scheme, per the description in [3], we bisect or trisect the axes of the input boxes until no pertinent axis is longer than $\epsilon$. Under such operation, that makes the maximum number of boxes:

$$3^{\lceil \log_3(\epsilon) \rceil}$$

Under a typical normalization, then, we would have:

$$\frac{\text{total\_number\_boxes}}{3^{\lceil \log_3(\epsilon) \rceil}}$$

However, we use a logarithm of the above, which, after trivial simplification, gives the formula shown in Section 8.2.1:

$$\log_3\left(\frac{\text{total\_number\_boxes}}{3^{\lceil \log_3(\epsilon) \rceil}}\right)$$

We were motivated to use a logarithm first and foremost by what would make sense for the distance measure: given how our refinement scheme works and the multi-dimensional nature of our abstract states, the proportional change in the number of boxes is typically more significant than the raw change in number.

D Other Future Works and Future Additions

Fanoos and the extensions of it detailed in this paper are amenable to the addition of numerous components, features, and improvements. In this work, we have described a sensible and effective method of allowing Fanoos to learn to select appropriate operators from a large collection of options in order to respond to a user’s request for greater or lesser abstraction of a particular state’s description. While this work can be even further extended in interesting ways, we (almost by necessity) leave a number of such promising items as future work. In the body of this paper, we have already highlighted some avenues for further improvements and capabilities; here, we provide a small sample of additional add-ons that did not fit into the main body or would be too far distracting if included there. Listed in not particular order:

FE1 Statistics about the abstract states that reflect their spatial distribution in a way that is not tied to one specific domain. Such information would help further inform selectors. Potential statistics include summary values for (a) the distribution of radii between the box-centers and the center of the universal bounding box or (b) the distribution of distances between pairs of box-centers (possibly found through random sampling, for the sake of efficiency).

FE2 For use by selectors when dealing with abstract states over the output space: the addition of statistics derived from the output-box volumes normalized by the approximate image of the learned system. The approximate image for the

33 Bear in mind that a state need not repeat in order for a description to repeat. In fact, taken fully, states cannot repeat since each contains the history of interaction — however, it is possible that all other variables in a state at timestep $t$ match those of some prior timestep.

34 Recall from Section 4.2 that the operators may change the set of axes that are candidates for refinement in some circumstances.

35 The text “FE” that appears next to the numbering stands for “Further Extensions.”
learned system would be found by pushing the input-space’s universal bounding box through the learned system via our abstract domain analysis.

Possible modification to the reward function to include an exponential decay based on the amount of time it took to produce a result — for example, including a term such as the value \( \exp\left(-\frac{\text{time\_taken} - 60\text{minutes}}{10}\right) \) normalized between zero and one. This would help promote generation of faster descriptions — balanced, of course, with other performance criteria.

Addition of selectors that leverage knowledge of the operator’s internal structure. The motivation for this is also reflected in Item [A3], but in this work, we have primarily focused on leveraging similarity between states. Our framework immediately facilitates the addition of such information about operators. The operators do have internal structures that selectors in Fanoos could in principle access; in the case of parameter adjusting operators (Section 4.2), computing some sort of similarity between operators should be straightforward. For instance, while we’re not necessarily advocating for use of a Euclidean distance here, it is the case that such a metric could be directly used for parameter adjusting operators.

One can easily brainstorm ways with which selectors can aggregate over similarity between operators after an initial aggregation over states in order to produce a final vote distribution including both knowledge sources. Even more simply, as a starting place, one can leverage the infrastructure established for the applicability-based selectors (Section 5.1): for instance, a selector can be made that adds more weight to operators that re-form boxes with larger refinement parameters when the request is “m”, and does the opposite when the request is “l”. In addition to home-brewed methods, literature pertaining to action selection in continuous action spaces may yield some insight (such spaces often have natural notions of distances between their members), though caution must be taken since our setting lacks most arithmetic properties that are useful in continuous settings (e.g., generally speaking, it would not be possible to “average” over our operators — with current arrangements, even if such arithmetic was defined, our set of operators would likely not be closed under it).

Methods for learning to apply multiple operators in a row prior to querying a user for further feedback (perhaps lumping the multiple basic operators into one “meta-operator”).

The addition of predicate constraining operators (Section 4.3) that select predicates based on information regarding the volume that the predicate covers.

Simple but not yet included in the standard library for facilitating generation of predicates: the inclusion of a negation sign on predicates, operating using the similar framework as our conjunction to act on predicates. Negated predicates can easily be included as though they are primitive predicates (i.e., from a mechanical standpoint, they could be handled the same way when forming descriptions). A tweak that might be desired if this is pursued (other than trying to leverage what we know to boost efficiency compared to a simple implementation) is to add a heuristic that prefers non-negated predicates over negated predicates when making final decisions. Such a heuristic would likely be unneeded in most circumstances, however, by nature of how we filter for predicates that are sufficiently specific to a box (that is, in most circumstances where there are candidate literals that are not negated, we expected a negated predicate to not be among those that are most specific to a box); see algorithm 1 in [3].

Operators and facilitating infrastructure in our refinement process allowing users to select predicates in a description to be replaced with different description content, the latter generated via changing the refinement of the chosen predicate’s underlying abstract states. This proposal is in contrast to the predicate constraining operators in Section 4.3 which work on the syntactic level, disallowing or reallowing use of a symbol. The process to incorporate this proposal’s feedback would be to find those boxes consistent with the sub-condition (predicate, conjunct, a subset of disjuncts, etc.), sufficiently refine those boxes
Learning-Based Operator Selection in Fanoos

more/less, then ideally leave the rest of the description generation process alone. An example of the process envisioned: suppose the user asks a question \( \textit{quest}_A \) and Fanoos says that such a thing occurs when conditions \( B, C, \) or \( D \) happen. The user then may ask for a more concrete description of the occurrences that \( B \) refers to. Fanoos then refines the boxes consistent with \( \textit{quest}_A \land B \) further and does not modify abstract states that are outside that collection (such as those consistent with \( \textit{quest}_A \land \neg B \)).

Additional experiments that demonstrate Fanoos elaborating the behavioral differences between over-, under-, and correctly trained policies that operate in the same domain.

Per discussion in Appendix B, modifying Fanoos and/or the autouser to prevent and/or be punished for repeating a description during a single description adjustment loop — if, that is, such a modification is ultimately determined to be desirable.

For the autouser, those curious might like to look into dynamically changing the cutoffs used in Eq. (4) for \( j \in \gamma_2 \), motivated by similar reasoning as the dynamic threshold used in Algorithm 1.

An additional selector that functions similarly to the random projection (see Section 8.2.2) but whose vector undergoes a perceptron-like updated based on feedback. That is, if \( s_i \) is the selector and the operator chosen at the end of the process is \( O_t \), then the reward signal used in a perceptron-update of \( s_i \)'s projection vector is

\[
y_i(\mathbb{1}( (s_i(*_t))(O_t) > U ) - \mathbb{1}( (s_i(*_t))(O_t) < U ) )
\]

(see Section 6.1 for definition of terms).

The reward signal is intentionally zero when \((s_i(*_t))(O_t) = U\). It is perhaps possible that such an arrangement could cause the projection vector to get stuck at an undesirable value in pathological cases (e.g., the projection vector becomes zero), but given how we divy votes after establishing the distances, it is very unlikely that a “stuck-position” would be stable over the long-term if it ever were to occur. That is, in pathological cases the projection vector may spend more time in the neighborhood of “bad” values than we’d like, but eventually the vector would move away from it with great likelihood.