Mathematical model of the vehicle taking into account the turn and reduction of tire friction

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Abstract. This work presents a mathematical model that takes into account a turn of a vehicle and the movement of the vehicle on a supporting surface with various friction properties in order to research the effect of a kinematic mismatch and friction decline to a coefficient reflecting the losses on wheel slip of a wheeled vehicle. According to the suggested approach, it is possible to select the type of an interaxle drive by comparing effectiveness of quantity indexes of the interaxle drive, while analyzing the full range of road conditions.

1. Introduction
When the vehicle is moving, there are two factors, which impose conflicting requirements towards an interaxle drive: kinematic disparity between the wheels and a reduced tyre friction to the supporting surface of the wheels of a drive axle [1]. A number of researches [2-6] describe mathematical models of all-wheel-drive vehicles. For a mathematical model in this research, it is necessary to take into consideration both factors listed above. The greatest value of the kinematic disparity is caused by turning the vehicle; therefore, to study this effect, we adapt the vehicle model described in [7], so that this model takes into account the kinematic disparity appearing when a vehicle is making a turn. Another factor affecting the choice of a type of an interaxle drive is the tyre friction decline with the support surface of one of the axle wheels. Therefore, it is also necessary to describe the movement of the vehicle on the supporting surface with various friction properties in the mathematical model.

2. Methods

2.1. Description of a mathematical model of the supporting surface with various adhesion properties
As it has been noted above, we accept the assumption that the same axle wheels have identical tyre friction conditions. The supporting surface consists of sections whose tyre friction coefficients are $\varphi_{\text{max}}$, $\varphi_{\text{min}}$. A real wheel has a contact area with length $2a$ (Figure 1); therefore, both of these values act when the border of sectors with different tyre friction coefficients is crossed. When designing the model, we assume that the tyre friction coefficient possesses intermediate values. For the sake of simplification, let us assume that the friction coefficient changes linearly when the border of sections with different coefficients is crossed.
Mathematically, the tyre coefficient of adhesion is expressed as follows:

$$
\varphi = \begin{cases} 
\varphi_{\text{min}}, & \text{if } x \leq L_1 - a; \\
\frac{\varphi_{\text{max}} - \varphi_{\text{min}}}{2a} (L_1 - a - x), & \text{if } L_1 - a < x < L_1 + a; \\
\varphi_{\text{max}}, & \text{if } x \geq L_1 + a;
\end{cases}
$$

(1)

where $\varphi_{\text{max}}$, $\varphi_{\text{min}}$ are correspondingly friction coefficients of section with standard and low value of friction; $L_1$, $L_2$ are length of sections with standard and low value of friction correspondingly; $a$ is a half of contact area length; $x$ is a coordinate of the longitudinal movement of the wheel center.

2.2. Description of the mathematical model of the vehicle taking into account the turn

Let us define the forces acting on the four-wheel drive vehicle while turning.

When taking into account the kinematics of turning an all-wheel drive vehicle, we neglect lateral sliding and wheel slipper. The wheels of each axle are replaced with one conventional middle wheel. The rotation angle of the front middle wheel $\theta$ is equal to the average rotation angle of the front wheels. The turning speed of steered wheels is $\dot{\theta}$. The calculation model for a turn of an all-wheel drive vehicle is shown in Figure 2.

According to the scheme in Figure 2, $V_1$ is the actual speed of the conventional front wheel, $V_1 = \frac{V}{\cos \theta}$; $V_2$ is the actual speed of the conditional rear wheel, $V_2 = V$; $V$ is the actual speed of the frame of the vehicle along the $x$ axis, $V=\dot{x}$. 
Let us introduce the formal inertia force $\Phi$ in the calculation model.

The basic equation of dynamics for the movement of a vehicle in the horizontal plane is as follows:

$$J_{xy}\ddot{\omega} = (T_1 - P_{f1})L\sin \theta + R_1 L\cos \theta - \Phi b\cos \Omega - M_c - P_{kr}c\sin \Psi,$$

where $J_{xy}$ is moment of inertia of the frame of the vehicle on a horizontal plane relative to the center of the rear axle; $\Psi$ is the angle between the perpendicular, dropped from the center of the turn to the longitudinal axis of the vehicle, and the direction to the point of hook load application.

$$\Psi = \tan^{-1}\left(\frac{c}{L}\tan \theta\right). \quad (2)$$

Let us introduce a formal inertia moment:

$$M_u = J_{xy}\ddot{\omega}. \quad (3)$$

Consequently, we can apply kinetostatics methods. In this case, the sum of torques around the center of the rear axle will be equal to 0.

$$(T_1 - P_{f1})L\sin \theta + R_1 L\cos \theta - \Phi b\cos \Omega - M_c - P_{kr}c\sin \Psi - M_u = 0. \quad (4)$$

Hence the lateral soil reaction to the front wheels is as follows:

$$R_1 = \frac{(T_1 - P_{f1})L\sin \theta + \Phi b\cos \Omega + M_c + P_{kr}c\sin \Psi + M_u}{L\cos \theta}, \quad (5)$$

Where $\Phi$ is inertia force, in our terms [8]

$$\Phi = \frac{mv_c^2}{R} + \frac{mv_c b\theta}{L} + \frac{m\ddot{v}_c}{L}, \quad (6)$$

Where $V_c$ is a vehicle center of mass speed, $V_c = \frac{v}{\cos \Omega}$;

$\Omega$ is the angle between the perpendicular dropped from the center of rotation to the longitudinal axis of the vehicle and the direction to the center of mass,

$$\Omega = \tan^{-1}\left(\frac{b}{L}\tan \theta\right);$$
the derivative of this quantity will be equal to
\[
\dot{\Omega} = -\frac{1}{\frac{1}{L} + b \sin \theta} \frac{b}{L} \dot{\theta}.
\]

The first derivative of the center of mass velocity
\[
\dot{V}_c = \left( \frac{V}{\cos \Omega} \right)';
\]
or
\[
\dot{V}_c = \frac{V \cos \Omega + V \sin \Omega \dot{\Omega}}{(\cos \Omega)^2}.
\]

The distance between the center of rotation and the center of mass of the vehicle is as follows:
\[
R_c = \frac{b}{\sin \Omega}.
\]

Turn resistance moment \([9]\) is as follows:
\[
M_c = \sum M_{cl} \cdot 10^{-0.1V},
\]
where
\[
M_{cl} = \frac{2}{3} \mu R_{zl} \frac{R_{z1}}{\pi P_w}.
\]

\(R_{zl}\) is a vertical reaction on wheels; \(P_w\) is the air pressure in the tyre \([10]\);

\[
\mu = \mu^{\max} / [0.5a + (1 - a) R/B + 0.5];
\]
given and are soil parameters.

Hence
\[
M_c = \frac{2}{3} \mu 10^{-0.1V} \left( R_{z1} \frac{R_{z1}}{\pi P_{w1}} + R_{z2} \frac{R_{z2}}{\pi P_{w2}} \right) \tag{7}
\]

Inertia moment is
\[
M_i = J_{xy} \ddot{\omega},
\]
where \(J_{xy}\) is the moment of inertia of the car frame in a horizontal plane relative to the center of the rear axle; \(J_{xy} = J_{0,xy} + mb^2\);

\(J_{0,xy}\) is the moment of inertia of the car frame on a horizontal plane relative to the center of its mass.

The angular velocity of rotation of the frame of the vehicle is
\[
\omega = \frac{V}{R'},
\]
where \(R\) is the radius of rotation, the distance from the center of rotation to the longitudinal axis of the car,
\[
R = \frac{L}{\tan \theta},
\]
the angular acceleration when turning the car is as follows:
\[
\ddot{\omega} = \frac{VR - VR'}{R^2}, \tag{8}
\]
where \(R' = \left( \frac{L}{\tan \theta} \right)'\).
Having performed a series of transformations in series, we will obtain the following:

\[ \dot{R} = -\frac{L}{(\sin \theta)^2} \dot{\theta}. \]

Substituting in the formula (8), we will obtain the following:

\[ \dot{\omega} = \frac{\psi \tan \theta - \frac{\gamma \phi}{L}}{L}. \]

Hence the inertial moment is as follows:

\[ M_\text{u} = (J_0_{xy} + m b^2) \dot{\omega}. \quad (9) \]

In this case, the system of differential equations describing the movement of a vehicle equipped with a differential interaxle drive, taking into account the turn, will be as follows:

\[
\left\{ \begin{align*}
J_1 \ddot{\varphi}_1 &= M_e - \frac{M_{12}}{i_{12}}; \\
(J_2 + J_3 + J_4) \ddot{\varphi}_2 &= M_{35} - \frac{M_{46}}{i_{46}}; \\
J_5 \ddot{\varphi}_5 &= M_{35} - T_1 \dot{r}_{k_1}; \\
J_6 \ddot{\varphi}_6 &= M_{46} - T_2 \dot{r}_{k_2}; \\
\left( J_3 + \frac{J_4}{u_d^2} \right) \dot{\varphi}_\text{rel} &= \frac{M_{46}}{u_d i_{46}} - \frac{M_{35}}{i_{35}}; \\
m \ddot{x} &= (T_1 - P_{f_1}) \cos \theta - R_1 \sin \theta + T_2 - P_{f_2} - P_{kp} \cos \Psi; \\
J_{xy} \ddot{a} &= R_{z_1} a - R_{z_2} b + \left( (T_1 - P_{f_1}) \cos \theta - R_1 \sin \theta + T_2 - P_{f_2} \right) h - P_{kp} (h - h_1) \cos \Psi; \\
m \ddot{z} &= R_{z_1} + R_{z_2} - G; \\
\end{align*} \right. \quad (10)
\]

For all-wheel-drive vehicle the following generalized coordinates are taken:

\( x, z \) - respectively horizontal and vertical movement of vehicle frame; \( \varphi_1, \varphi_2, \varphi_5, \varphi_6, \varphi_\text{rel} \) - angular coordinates of the flywheel of the engine, carrier of central differential, front and rear wheels and rotation angle of sun gear of central differential against carrier; \( \alpha \) - The angle of rotation of the vehicle frame around a horizontal axis passing through the center of mass.

The system of differential equations is supplemented by the equations of non-holonomic constraints [11]

\[
\begin{align*}
\dot{\chi}_1 &= r_{k_1} \varphi_5 - \dot{x} / \cos \theta - (\lambda_1 + k_{\delta_1}) \varphi_5 T_1; \\
\dot{\chi}_2 &= r_{k_2} \varphi_6 - \dot{x} - (\lambda_2 + k_{\delta_2}) \varphi_6 T_2; \\
\end{align*}
\]

(11)

If the value of the angle of rotation \( \theta \) is equal to zero, the system of equations will take the form corresponding to a mathematical model of a vehicle moving in a rectilinear way. It means that the mathematical model is universal and can be used to describe the movement of the vehicle in the presence of a kinematic mismatch of the front and rear wheels, as well as in its absence.

A number of researchers suggest using various energy efficiency criteria for evaluation in order to select the type of the interaxle drive [11-14]. In [15], an efficiency coefficient is proposed; it reflects the loss of slipping of the wheels as a single criterion for evaluating the effectiveness of the type of the interaxle drive.

The formula for determining the efficiency, taking into account the loss on slipping of an all-wheel drive vehicle, given in [2] is as follows:

\[
\eta_\delta = 1 - \frac{T_1 v_{1-\delta_1} T_2 v_{1-\delta_2}}{T_1 v_{1+\delta_1} T_2 v_{1+\delta_2}}, \quad (12)
\]
The formula for determining the efficiency, taking into account the loss of slipping, taking into account the rotation will be as follows:

$$\eta_\delta = 1 - \frac{T_1 \frac{1}{\cos \theta} \frac{\delta_1}{1 - \delta_1} + T_2 \frac{\delta_2}{1 - \delta_2}}{T_1 \frac{1}{\cos \theta} \frac{1}{1 - \delta_1} + T_2 \frac{1}{1 - \delta_2}}.$$  \hspace{1cm} (13)

3. Results

Formula (13) was further used to determine the instantaneous value of slipping efficiency at each point of the considered time period. This will allow calculating the average value for the slipping efficiency over a certain period, which is necessary to assess the traction and economic qualities of a vehicle with various types of the interaxle drive. The findings of these researches are given in [16].

To investigate the efficiency in the entire range of road conditions, it is necessary to set a range of road conditions. As coordinates, we choose: the coefficient of adhesion of the slipping axle $\phi'$ and the kinematic disparity, $m_H$, where

$$m_H = \left( \frac{V' - V''}{V''} \right) \cdot 100\%,$$  \hspace{1cm} (14)

where $V'$ – the speed of the advancing axle, $V''$ – the speed of the lagging axle, $V' > V''$.

In Figure 3 you can see the dependence of the efficiency mean value reflecting losses on wheel slip, $\eta_\delta^{\text{mean}}$, on coefficient of disparity $m_H$ and coefficient of friction of the slipping axle $\phi'$ for the vehicle equipped with differential with the limited transfer relation.

![Figure 3](image.png)

Figure 3. Dependence of the efficiency mean value, reflecting losses on wheel slip, for the vehicle equipped with differential with the limited transfer ratio.

4. Conclusion

The mathematical model developed in accordance with the dynamic model allows modeling the movement of a vehicle order to research the effect of a kinematic mismatch and friction decline to the efficiency of vehicle.
References
[1] Andreev A, Kabanau V, Vantsevich V 2010 Driveline systems of ground vehicles. Theory and design (Boca Raton: CRC Press)
[2] Cheli F, Pedrinelli M, Zorzutti A 2006 ASME 8th Biennial Conference on Engineering, Systems Design and Analysis 2 235
[3] Vantsevich V, Shyrokau B 2008 ASME 2008 Dynamic Systems and Control Conference 891
[4] Deur J, Hancock M, Assadian F 2008 ASME 2008 International Mechanical Engineering Congress and Ex-position 17 427
[5] Keller A, Gorelov V, Anchukov V 2015 Procedia Engineering 129 280
[6] Shafaei S, Loghavi M, Kamgar S 2019 Information Processing in Agriculture 6(2) 183
[7] Efimov A, Kireev S, Korchagina M 2018 Problems of machine building and machine reliability 1 94
[8] Troyanovskaya I 2010 Materials of the international scientific-technical conference of Association of automobile engineers (Moscow) p 490
[9] Troyanovskaya I, Pozin B 2015 Procedia Engineering 129 156
[10] Babulal Yasheen, Joachim Stallmann M, Schalk Els P 2015 Journal of Terramechanics 61 77
[11] Vantsevich V 2008 Journal of Terramechanics 45(3) 89
[12] Vantsevich V 2007 Journal of Terramechanics 44(3) 239
[13] Keller A, Murog I, Shafikov D, Usikov V, Ushnurtsev S 2013 Avtomobilnaya promyshlennost 4 14
[14] Keller A, Murog I, Aliukov S 2015 SAE 2015 World Congress & Exhibition
[15] Efimov A, Kireev S, Korchagina M 2018 XIV International Scientific-Technical Conference “Dynamic of Technical Systems” (DTS-2018)
[16] Efimov A, Kireev S, Korchagina M, Lebedev A, Kaderov Kh 2019 Technologies in Environmental Science and Education 135