Elastic Routing in Wireless Ad Hoc Networks With Directional Antennas

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Abstract

Throughput scaling laws of an ad hoc network equipping directional antennas at each node are analyzed. More specifically, this paper considers a general framework in which the beam width of each node can scale at an arbitrary rate relative to the number of nodes. We introduce an elastic routing protocol, which enables to increase per-hop distance elastically according to the beam width, while maintaining an average signal-to-interference-and-noise ratio at each receiver as a constant. We then identify fundamental operating regimes characterized according to the beam width scaling and analyze throughput scaling laws for each of the regimes. The elastic routing is shown to achieve a much better throughput scaling law than that of the conventional nearest-neighbor multihop for all operating regimes. The gain comes from the fact that more source-destination pairs can be simultaneously activated as the beam width becomes narrower, which eventually leads to a linear throughput scaling law. In addition, our framework is applied to a hybrid network consisting of both wireless ad hoc nodes and infrastructure nodes. As a result, in the hybrid network, we analyze a further improved throughput scaling law and identify the operating regime where the use of directional antennas is beneficial.

Index Terms

Ad hoc network, beam width, directional antenna, elastic routing, hybrid network, multihop routing, throughput scaling law.
I. INTRODUCTION

As the number of devices explosively increases for machine-to-machine (M2M) communications in the era of the internet of things (IoT), characterizing the aggregate throughput of large-scale wireless networks becomes more crucial in developing transmission protocols efficiently delivering a number of packets. While numerical results via computer simulations depend heavily on specific operating parameters for a given system or protocol, a study on the capacity scaling of large-scale networks with respect to the number of nodes provides a fundamental limit on the network throughput. Hence, one can obtain remarkable insights into the practical design of a protocol by characterizing the capacity scaling law.

A. Related Work

In [1], throughput scaling was originally introduced and characterized in a large-scale wireless ad hoc network. It was shown that, for a network having \(n\) nodes randomly distributed in an unit area (i.e., a dense network), the aggregate throughput scales as \(\Omega(\sqrt{n/\log n})\) by conveying packets in the nearest-neighbor multihop routing fashion.\(^1\) There have been further studies based on multihop routing in the literature [2]–[8], while the total throughput scales far less than \(\Theta(n)\). In [9], the aggregate throughput in the dense network was improved to an almost linear scaling, i.e., \(\Theta(n^{1-\epsilon})\) for an arbitrarily small \(\epsilon > 0\), by using a hierarchical cooperation strategy. Besides the hierarchical cooperation scheme [9]–[12], there has also been a steady push to improve the throughput of interference-limited networks up to a linear scaling by using node mobility [5], [13], interference alignment [14], infrastructure support [15]–[21], and directional antennas [22], [23].

To achieve such a linear scaling, there will be a price to pay in terms of delay [5], [9], [13], cost of channel estimation [9], [14], and infrastructure investment [16], [19]. On the one hand, the use of directional antennas [22]–[24] in ad hoc networks has recently emerged as a promising technology leading to the enhanced spatial reuse, the improved transmission distance, and the reduced interference level with relatively low cost in comparison to alternative technologies. Especially, for wireless systems using millimeter wave (mmWave) technologies operating in the 10-300 GHz band, which have been considered as one solution to enable gigabit-per-second data rates, equipping directional antennas at each node may be more challenging. This is because mmWave links are inherently directional and thus steerable antenna arrays can be easily implemented, thus resulting in a much higher link gain [25]. Due to these reasons, the interest in studies of more amenable networks using directional antennas has been greatly growing. In the literature, the previous work based on the protocol model [1] has shown that, for an infinitely large antenna gain (or equivalently, for an infinitely small beam width), the use of directional antennas provides a substantial throughput enhancement up to a linear scaling [26]. Similarly, the throughput scaling law was studied based on an interference model for directional antennas [23]. There have also been other research directions showing the capacity scaling when directional antennas are used under different assumptions as well as different situations [27], [28]. It is, however, still unclear how much throughput scaling gain is attainable by directional antennas under a more realistic wireless channel. More precisely, for all beam width scaling conditions, the analysis so far would not be suitable for fully understanding the effects of directional antennas on the throughput scaling since it

\(^{1}\)We use the following notation: i) \(f(x) = O(g(x))\) means that there exist constants \(C\) and \(c\) such that \(f(x) \leq Cg(x)\) for all \(x > c\), ii) \(f(x) = o(g(x))\) means that \(\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0\), iii) \(f(x) = \Omega(g(x))\) if \(g(x) = O(f(x))\), and iv) \(f(x) = \Theta(g(x))\) if \(f(x) = O(g(x))\) and \(g(x) = O(f(x))\).
does not reflect the physically attainable antenna gain that may be exploited to enhance the throughput scaling in the presence of interference.

B. Contributions

In this paper, when there are \( n \) randomly located nodes in a network equipping directional antennas at each node, we deal with a general framework in which the beam width of each node, \( \theta \), scales at an arbitrary rate with respect to \( n \), which provides a comprehensive understanding on fundamental limits of directional antennas for wireless ad hoc networks. Similarly as in [22], [26], we take into account a simplified but feasible directional antenna model having mainlobe and sidelobe gains. Then, by completely utilizing the characteristics of directional antennas, we introduce a new routing, termed elastic routing, and analyze its throughput scaling laws. The proposed routing protocol enables to increase the average transmission distance at each hop elastically according to the beam width, while setting the average signal-to-interference-and-noise ratio (SINR) at each receiver to a constant independent of \( n \). We identify two fundamental operating regimes characterized according to the \( \theta \) scaling and analyze throughput scaling laws for each of the regimes. Our main results demonstrate that the proposed elastic routing achieves a much better throughput scaling law compared to the conventional nearest-neighbor multihop routing for all operating regimes. The gain comes from the fact that more source–destination (SD) pairs can be activated simultaneously as the beam width \( \theta \) becomes narrower, which eventually provides up to a linear throughput scaling. Interestingly, it is further shown that the average delay is reduced by the proposed elastic routing while achieving an improved throughput scaling, which is in sharp contrast with the omnidirectional mode (i.e., \( \theta = \Theta(1) \)) [5].

In addition, our result is extended to a hybrid network scenario. Since there will be a long latency and insufficient energy with only wireless connectivity, it would be good to deploy infrastructure nodes, or equivalently base stations (BSs) in the network, while possibly improving the throughput scaling. In a hybrid network equipping directional antennas at each node, we analyze the impact and benefits of our elastic routing in further improving the throughput scaling.

Our comprehensive analysis sheds more light on the routing policies and on operational issues for ad hoc networks in the directional mode.

C. Organization

The rest of this paper is organized as follows. The system model is described in Section II. In Section III the elastic routing protocol is presented. In Section IV the throughput scaling laws are derived. In Section V our result is extended to a hybrid network model. Finally, Section VI summarizes the paper with some concluding remarks.

II. SYSTEM MODEL

We consider a two-dimensional wireless ad hoc network consisting of \( n \) nodes that are uniformly distributed at random on a square. The nodes are grouped into \( n/2 \) SD pairs at random. Each node operates in half-duplex mode and is equipped with a single directional antenna. The network area is assumed to be one and \( n \) in dense and extended networks, respectively.
We define a hybrid antenna model whose mainlobe is characterized as a sector and whose sidelobe forms a circle (backlobes are ignored in this model). As illustrated in Fig. 1, the antenna beam pattern has a gain value $G_m$ for the mainlobe of beam width $\theta \in [0, 2\pi)$, and also has a sidelobe of gain $G_s$ of beam width $2\pi - \theta$. The parameters $G_m$ and $G_s$ are then related according to

$$\frac{\theta}{2\pi} G_m + \frac{2\pi - \theta}{2\pi} G_s = 1,$$

where $0 \leq G_s \leq 1 \leq G_m$. In our work, we assume that $G_m = \Theta(1/\theta)$ and $G_s = \Theta(1)$, which do not violate the law of conservation of energy. For simplicity, we assume unit antenna efficiency, i.e., no antenna loss. Each node can steer its antenna for directional transmission or directional reception.

Similar to [26], for a given time instance, suppose that node $i \in \{1, \ldots, n\}$ transmits to node $k \in \{1, \ldots, n\} \setminus \{i\}$ and they beamform to each other according to the above assumption. Let $\mathcal{I}_1$, $\mathcal{I}_2$, and $\mathcal{I}_3$ denote three different sets of nodes transmitting at the same time, where both nodes $i_1 \in \mathcal{I}_1$ and $k$ beamform to each other, either node $i_2 \in \mathcal{I}_2$ or $k$ beamforms to the other node (but not both), and neither node $i_3 \in \mathcal{I}_3$ nor $k$ beamforms to the other node, respectively. Note that node $i$ is in $\mathcal{I}_1$ since nodes $i$ and $k$ beamform to each other.

Under the directional antenna model, the received signal of node $k$ at the given time instance is represented by

$$y_k = \sum_{i_1 \in \mathcal{I}_1} h_{k_{i_1}i_1}x_{i_1} + \sum_{i_2 \in \mathcal{I}_2} h_{k_{i_2}i_2}x_{i_2} + \sum_{i_3 \in \mathcal{I}_3} h_{k_{i_3}i_3}x_{i_3} + n_k,$$

where $x_{i_1}$, $x_{i_2}$, and $x_{i_3} \in \mathbb{C}$ are the signals transmitted by nodes $i_1$, $i_2$, and $i_3$, respectively, and $n_k$ denotes the circularly symmetric complex Gaussian noise with zero mean and variance

\[^3\text{Instead of directional antennas modeled as a cone in a three-dimensional view [26], we use a rather simple two-dimensional antenna model since simplifying the shape of the antenna pattern will not cause any fundamental change in terms of capacity scaling law.}\]

\[^4\text{Note that } G_m = G_s = 1 \text{ in the omnidirectional mode.}\]
The channel coefficients $h_{ki1}$, $h_{ki2}$, and $h_{ki3}$ are given by
\[
h_{ki1} = \frac{G_m e^{j\phi_{ki1}}}{r_{ki1}^{\alpha/2}},
\]
\[
h_{ki2} = \sqrt{G_m G_s} e^{j\phi_{ki2}} \frac{1}{r_{ki2}^{\alpha/2}},
\]
\[
h_{ki3} = \frac{G_s e^{j\phi_{ki3}}}{r_{ki3}^{\alpha/2}},
\]
respectively, where $\phi_{kj}$ represents the random phase uniformly distributed over $[0, 2\pi)$ and independent for different $j$, $k$, and time (transmission symbol), i.e., fast fading [9], [19], [29]. The parameters $r_{kj}$ and $\alpha > 2$ denote the distance between nodes $j$ and $k$, and the path-loss exponent, respectively. Each node should satisfy the average transmit power constraint $P > 0$ during transmission, which is a constant. Channel state information (CSI) is assumed to be available at all receivers, but not at transmitters.

In the following, we formally define the per-node and aggregate throughputs used throughout the paper.

**Definition 1 (Throughput):** A per-node throughput $R(n)$ is said to be achievable with high probability (w.h.p.) if all sources can transmit at the rate of $R(n)$ bits/sec/Hz to their destinations with probability approaching one as $n$ increases. Accordingly, the achievable aggregate throughput is at least given by $T(n) = (n/2)R(n)$. ♦

For the rest of this paper, we will analyze throughput scaling laws of wireless ad hoc networks equipping directional antennas at each node. Throughout the paper, $E[\cdot]$ and $Pr(\cdot)$ denote the statistical expectation and the probability, respectively. Unless otherwise stated, all logarithms are assumed to be to base two.

### III. Elastic Routing Protocol

In this section, we describe our elastic routing protocol, which can be designed with the help of the directional antennas. Under the protocol, we perform multihop (or even single-hop) transmission by elastically increasing per-hop distance as a function of the scaling parameter $\theta$, which ultimately enhances the throughput performance compared to the conventional multihop [1], [5]. We focus primarily on the dense network configuration, but the overall procedure of our protocol can be directly applied to the extended network configuration.

Let us first show how to perform beam steering at each node using directional antennas. At each hop, as illustrated in Fig. 2, the antennas of each selected transmitter–receiver pair are steered so that their beams cover each other, which enables to achieve the maximum antenna gain $G_m^2$ at each transmitter–receiver pair.

We now describe how each SD pair performs the elastic routing based on the beam steering technique described above. Let $T_S$ be the total number of scheduling time slots. We assume that, at each time slot $s \in \{1, \cdots, T_S\}$, randomly chosen $M(n)$ SD pairs are activated simultaneously, where $M(n)$ scales as $\Omega(\sqrt{n/\log n})$ and $O(n)$\footnote{It is not desirable that $M(n)$ scales as $o(\sqrt{n/\log n})$ since, in this case, the throughput scaling achieved by our elastic routing is less than that of the conventional multihop scheme.} We denote the set of scheduling time slots to which the $p$th SD pair belongs by $\Phi_p$, where $p \in \{1, \cdots, n/2\}$. For instance, if the $p$th SD pair is scheduled at time slots 1, 3, and 5, $\Phi_p$ is given by $\Phi_p = \{1, 3, 5\}$. We further denote $T_p \triangleq |\Phi_p|$, where $|\Phi_p|$ denotes the cardinality of $\Phi_p$.\footnote{It is not desirable that $M(n)$ scales as $o(\sqrt{n/\log n})$ since, in this case, the throughput scaling achieved by our elastic routing is less than that of the conventional multihop scheme.}
The following lemma shows that each of $n/2$ SD pairs can be served with almost the same fraction of time in the limit of large $n$.

**Lemma 1 (Strong typicality):** Suppose that $T_S = n^4$. Then, for sufficiently large $n$, $\frac{T_p}{T_S}$ is lower-bounded by

$$\frac{2M(n)}{n} - \frac{1}{n}$$

w.h.p. for all $p \in \{1, \cdots, n/2\}$. 

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**Fig. 2.** Beam steering at each transmitter–receiver pair.

**Fig. 3.** An example of an SD path passing through its associated routing regions in the dense network, where the 9-TDMA scheme is used.
Proof: Since $M(n)$ SD pairs among $n/2$ are scheduled at random for each time slot, the probability that a specific SD pair is served at each time slot is given by $2M(n)/n$. Hence, by applying the result of [30, Lemma 2.12], we show that, for any $\xi > 0$, the probability that 

$$\left| \frac{T_p}{T_S} - \frac{2M(n)}{n} \right| \leq \xi$$

for all $p \in \{1, \cdots, n/2\}$ is greater than $1 - \frac{n}{4\xi^2}$. Then, by setting $\xi = \frac{1}{n}$, we have $\frac{T_p}{T_S} \geq \frac{2M(n)}{n} - \frac{1}{n}$ with probability greater than $1 - \frac{1}{4n}$, which converges to one as $n$ tends to infinity. This completes the proof of the lemma.

As depicted in Fig. 3, we divide the whole area into $1/A(n)$ square routing cells with per-cell area $A(n)$, where $A(n)$ is assumed to scale as $\Omega(\log n/n)$ and $O(1)$. We draw the straight line connecting a source to its destination, termed an SD line. Then, packets for each SD pair travel horizontally or vertically along its SD line by hopping along adjacent routing cells of area $A(n)$ until they reach the corresponding destination. While travelling along its SD line, a certain node in each routing cell is arbitrarily selected as a relay forwarding the packets. As in the earlier work [1], [5], in the dense network, a transmit power of $P(\log n/n)^{\alpha/2}$ is used at each node.

We further divide each routing cell with area $A(n)$ into smaller square regions of area $2\log n/n$, which guarantees that each smaller region has at least one node w.h.p. (see [1] for the details). As in [1], [5], the 9-time division multiple access (TDMA) scheme is used between smaller regions to avoid huge interference (see also Fig. 3). Then, only a single node in each activated smaller region transmits its packets.

Figure 3 illustrates the packet transmission of a particular SD pair, where the source and destination nodes are indicated by blue circles, the relay nodes are indicated by green circles, and the interfering nodes that simultaneously transmit packets of other SD pairs are indicated by red circles, respectively.

Let $d_{\text{hop}}$ and $\bar{h}$ denote the average transmission distance at each hop and the average number of hops per SD pair, respectively. Then, from the above routing, it follows that

$$A(n) = \Theta \left( d_{\text{hop}}^2 \right) = \Theta \left( \frac{1}{\bar{h}^2} \right)$$

in the dense network.

Now, let us turn to how to decide per-cell area $A(n)$ according to given beam width $\theta$, which plays an important role in determining the throughput of the proposed elastic routing. If $A(n)$ is given by a function of $n$, then the average per-hop distance $d_{\text{hop}}$ can also be determined using (2). We note that $d_{\text{hop}}$ is elastically increased as much as possible while the average SINR at each receiver is set to $\Theta(1)$. Such $d_{\text{hop}}$ is shown according to the value of $\theta$ as follows:

$$d_{\text{hop}} = \Theta \left( \min \left\{ \sqrt{\frac{\log n}{n}} \theta^{-\frac{2}{\alpha}}, 1 \right\} \right)$$

$$= \begin{cases} 
\Theta \left( \sqrt{\frac{\log n}{n}} \theta^{-\frac{2}{\alpha}} \right) & \text{if } \theta^{-1} = o \left( \left( \frac{n}{\log n} \right)^{\alpha/4} \right) \\
\Theta \left( \frac{n}{\log n} \right)^{\alpha/4} & \text{if } \theta^{-1} = \Omega \left( \left( \frac{n}{\log n} \right)^{\alpha/4} \right),
\end{cases}$$

which will be verified later. Note that $d_{\text{hop}} = \Theta(\sqrt{\log n/n})$ in the omnidirectional mode (i.e., $\theta = \Theta(1)$).
As expressed in (3), according to the beam width $\theta$, the whole operating regimes are divided into two fundamental regimes. The two operating regimes and their corresponding routing schemes in each regime are summarized as follows.

- **Regime I**: $\theta^{-1} = o\left(\left(\frac{n}{\log n}\right)^{\alpha/4}\right)$.

  In the regime, the elastic routing outperforms the nearest-neighbor multihop routing. As $\theta^{-1}$ increases, a long-range transmission is performed at each hop owing to an enhanced antenna gain at each transmitter–receiver pair.

- **Regime II**: $\theta^{-1} = \Omega\left(\left(\frac{n}{\log n}\right)^{\alpha/4}\right)$.

  The single-hop transmission is performed in the regime, where per-hop distance can reach up to $O(1)$.

In the extended network, a full transmit power $P$ is used at each node. Then, it follows that

$$d_{\text{hop}} = \Theta\left(\min\left\{\sqrt{\log n}\theta^{-2/\alpha}, \sqrt{n}\right\}\right)$$

$$= \begin{cases} 
\Theta\left(\sqrt{\log n}\theta^{-2/\alpha}\right) & \text{if } \theta^{-1} = o\left(\left(\frac{n}{\log n}\right)^{\alpha/4}\right) \\
\Theta\left(\sqrt{n}\right) & \text{if } \theta^{-1} = \Omega\left(\left(\frac{n}{\log n}\right)^{\alpha/4}\right) 
\end{cases}$$

which turns out to be scaled up by a factor of $\sqrt{n}$, compared to the dense network case in (3).

Since, unlike the nearest-neighbor multihop routing [1], packets can travel much farther at each hop for the proposed elastic routing in the directional mode, the average number of hops, $\bar{h}$, can be reduced significantly, thereby resulting in an improved throughput up to a linear scaling, which will also be specified in the following section.

### IV. MAIN RESULTS

This section presents our main result, which shows throughput scaling laws achievable by elastic routing for both dense and extended networks using directional antennas.

#### A. Throughput Scaling in Dense Networks

In this subsection, the throughput scaling for dense networks under the elastic routing protocol is analyzed. Let $tx_{p,l}$ and $rx_{p,l}$ denote the transmitter and the corresponding receiver of the $l$th hop of the $p$th SD pair, respectively, where $l \in \mathcal{H}_p$ and $p \in \{1, \cdots, n/2\}$. Recall that $\mathcal{H}_p = \{1, 2, \cdots, \beta_p h\}$ denotes the set of hops for the $p$th SD pair, where $\beta_p > 0$ is a parameter that scales as $O(1)$. Here, $tx_{p,l}$ and $rx_{p,l}$ are fixed during the entire scheduling time since they are solely determined by the network geometry. Let $\text{SINR}_{p,l}(s)$ denote the
instantaneous received SINR of $rx_{p,l}$ at time slot $s \in \{1, \cdots, T_S\}$ for the $l$th hop of the $p$th SD pair. Then, we have

$$\text{SINR}_{p,l}(s) = \frac{P_{p,l}(s)}{N_0 + I_{p,l}(s)},$$

(5)

where $P_{p,l}(s)$ denotes the received signal power of $rx_{p,l}$ from the desired transmitter $tx_{p,l}$ at time slot $s$ and $I_{p,l}(s)$ denotes the total interference power of $rx_{p,l}$ from all interfering nodes at time slot $s$.

Obviously, one can see that $P_{p,l}(s) = 0$ if $s \notin \Phi_p$ (6) from the proposed elastic routing. For $s \in \Phi_p$, the $p$th SD pair is activated and nodes $tx_{p,l}$ and $rx_{p,l}$ direct their beams to each other to maximize the antenna gain during the $l$th hop transmission. Hence, we have

$$P_{p,l}(s) = |h_{rx_{p,l}tx_{p,l}}|^2 P \left( \frac{\log n}{n} \right)^{\alpha/2}$$

$$= \Omega \left( G_m^2 \left( \frac{\log n}{d_{hop} n} \right)^{\alpha/2} \right) \text{ if } s \in \Phi_p.$$

(7)

Now, we turn to computing the total interference power $I_{p,l}(s)$. For analytical convenience, we first divide $I_{p,l}(s)$ into two parts, $I_{p,l}^{[1]}(s)$ and $I_{p,l}^{[2]}(s)$, which indicate the intra-pair interference power and the inter-pair interference power at time slot $s \in \{1, \cdots, T_S\}$, respectively. Due to our multihop-based elastic routing characteristics, the beam directions at the transmitters and receivers belonging to the same SD pair may be highly correlated to each other. Figure 4 illustrates how the intra-pair interference is generated along with correlated beam directions when multiple transmitters in each SD pair are simultaneously activated. For this reason, we treat the intra-pair interference separately from the inter-pair interference. Then, it follows that$^5$

$$I_{p,l}^{[1]}(s) \leq 2 P \left( \frac{\log n}{n} \right)^{\alpha/2} \sum_{t=1}^{\beta_p h} \frac{G_m^2}{(l \cdot d_{hop} n)^{\alpha}}$$

$$= O \left( G_m^2 \left( \frac{\log n}{d_{hop} n} \right)^{\alpha/2} \right),$$

(8)

where the inequality holds since at most two nodes can generate the intra-pair interference between $(l - 1)d_{hop}$ and $ld_{hop}$ apart from node $rx_{p,l}$. Each receiver also suffers from the inter-pair interference, which is generated by other activated SD pairs. Unlike the intra-pair interference case, the routing path of all SD pairs is determined independently of each other since $M(n)$ SD pairs are chosen uniformly at random at each time slot $s$. As illustrated in Fig. 5 when the 9-TDMA scheme is used, the nodes generating the inter-pair interference can be computed by using the layering technique$^9$, $^{19}$ with the concept of tier indexed by $t$, where $in_{p,l,t,i}(s)$ denotes the $i$th inter-pair interferer placed on the $i$th tier that causes the inter-pair interference to $rx_{p,l}$ at time slot $s$. Note that $in_{p,l,t,i}(s)$ may vary over time slots

$^5$Due to the 9-TDMA scheme, even if the distance between $rx_{p,l}$ and each intra-pair interferer needs to be more carefully considered, it does not fundamentally change the scaling law result for $I_{p,l}^{[1]}(s)$. 


Fig. 5. The inter-pair interferers that affect $\text{rx}_{p,l}$ when the 9-TDMA is used in the dense network.

since $M(n)$ SD pairs are uniformly chosen at random for each time slot $s \in \{1, \cdots, T_S\}$. Then, the total amount of inter-pair interference of $\text{rx}_{p,l}$ at time $s$ is upper-bounded by

$$I_p[l](s) \leq \sum_{t=1}^{\infty} \sum_{i=1}^{8t} I_p[l,t,i](s) \leq P \left( \frac{\log n}{n} \right)^{\alpha/2} \sum_{t=1}^{\infty} \sum_{i=1}^{8t} \left( t \sqrt{\frac{\log n}{n}} \right)^{-\alpha} X_{p,l,t,i}(s) = P \sum_{t=1}^{\infty} \sum_{i=1}^{8t} t^{-\alpha} X_{p,l,t,i}(s)$$

where $I_p[l,t,i](s)$ denotes the inter-pair interference power of $\text{rx}_{p,l}$ caused by the inter-pair interferer $\text{in}_{p,l,t,i}(s)$ at time slot $s$ and $X_{p,l,t,i}(s) \in \{G_m^2, G_m G_s, G_s^2\}$ denotes the antenna gain between $\text{rx}_{p,l}$ and $\text{in}_{p,l,t,i}(s)$ at time slot $s$.

Before presenting our main result, we start from the following lemma, which establishes an upper bound on the expected inter-pair interference power at each receiver.

**Lemma 2:** Consider the $p$th SD pair whose packets travel over $\beta_p \bar{h}$ hops from the source to its destination. Then, the expectation of the inter-pair interference power $I_p[l](s)$ is upper-bounded by

$$\mathbb{E} \left[ I_p[l](s) \right] = O(1).$$

**Proof:** First of all, similarly as in [22], [26], the direction of the receive beam at node $\text{rx}_{p,l}$ is independent of the direction of the transmit beam at $\text{in}_{p,l,t,i}$ for $t \in \{1, 2, \cdots\}$ and $i \in \{1, \cdots, 8t\}$ since inter-pair interferers are only concerned. Thus, from [2], the expectation
of the inter-pair interference power is upper-bounded by

$$\mathbb{E} \left[ I_{p,l}^{(i)}(s) \right] \leq \mathbb{E} \left[ P \sum_{t=1}^{\infty} \sum_{i=1}^{8t} t^{-\alpha} X_{p,l,t,i}(s) \right]$$

$$= P \mathbb{E} \left[ X_{p,l,t,i}(s) \right] \sum_{t=1}^{\infty} \sum_{i=1}^{8t} t^{-\alpha}$$

$$= 8P \mathbb{E} \left[ X_{p,l,t,i}(s) \right] \sum_{t=1}^{\infty} t^{-\alpha}, \quad (11)$$

where the first equality holds since $$\mathbb{E} \left[ X_{p,l,t,i}(s) \right]$$ is the same for all $$t \in \{1, 2, \cdots \}$$ and $$i \in \{1, \cdots, 8t\}$$. By using the fact that

$$\text{Pr} \left( X_{p,l,t,i}(s) = G_m^2 \right) = \frac{\theta^2}{4\pi^2}, \quad (12a)$$

$$\text{Pr} \left( X_{p,l,t,i}(s) = G_m G_s \right) = \frac{(2\pi - \theta)\theta}{2\pi^2}, \quad (12b)$$

$$\text{Pr} \left( X_{p,l,t,i}(s) = G_s^2 \right) = \frac{(2\pi - \theta)^2}{4\pi^2}, \quad (12c)$$

we have

$$\mathbb{E} \left[ X_{I2}^{k(p,t,i)}(s) \right] \leq \frac{\theta^2 G_m^2}{4\pi^2} + \frac{(2\pi - \theta)\theta}{2\pi^2} G_m G_s + \frac{(2\pi - \theta)^2 G_s^2}{4\pi^2}$$

$$= \Theta(1), \quad (13)$$

which comes from the fact that $$G_m = \Theta(1/\theta)$$ and $$G_s = \Theta(1)$$. Finally, from (11) and (13) and the fact that $$\sum_{t=1}^{\infty} t^{-\alpha}$$ is upper-bounded by a constant for $$\alpha > 2$$, we have (10), which completes the proof of the lemma.

In the following theorem, we state the aggregate throughput achievable by elasting routing in the dense network.

**Theorem 1:** In the dense ad hoc network of unit area, the aggregate throughput achieved by elastic routing is given by

$$T(n) = \Omega \left( \min \left\{ \sqrt{n\theta^{-2/\alpha}}, n \right\} n^{-\epsilon} \right)$$

$$= \begin{cases} \Omega \left( n^{1/2-\epsilon} \theta^{-2/\alpha} \right) & \text{if } \theta^{-1}=o \left( \left( \frac{n}{\log n} \right)^{\alpha/4} \right) \\ \Omega(n^{1-\epsilon}) & \text{if } \theta^{-1}=\Omega \left( \left( \frac{n}{\log n} \right)^{\alpha/4} \right) \end{cases} \quad (14)$$

w.h.p., where $$\epsilon > 0$$ is an arbitrarily small constant.

**Proof:** First of all, we set $$T_S = n^\epsilon$$, which satisfies the condition in Lemma 1. Since the per-node throughput $$R(n)$$ in Definition 1 is defined as the average rate per each SD pair over the entire time slots,
\[ R(n) \geq \frac{1}{T_S} \min_{p,l} \left\{ \sum_{s \in \{1, \ldots, T_S\}} \log (1 + \text{SINR}_{p,l}(s)) \right\} \]

\[ = \frac{1}{T_S} \min_{p,l} \left\{ \sum_{s \in \Phi_p} \log (1 + \text{SINR}_{p,l}(s)|s \in \Phi_p) \right\} \quad (15) \]

is achievable, where the minimum is taken over all pairs \( p \in \{1, \ldots, n/2\} \) and hops \( l \in \mathcal{H}_p \). Here, the equality holds since \( \text{SINR}_{p,l}(s) = 0 \) if \( s \notin \Phi_p \) (see (5) and (6)).

Then, from (5), (7)–(9), \( \text{SINR}_{p,l}(s) \) can be lower-bounded by

\[ \text{SINR}_{\text{min}}(s) \triangleq \frac{c_0 G_m^2 \left( \frac{\log n}{d_{\text{hop}}^m} \right)^{\alpha/2}}{N_0 + c_1 G_m^2 \left( \frac{\log n}{d_{\text{hop}}^m} \right)^{\alpha/2} + I_{\text{max}}^{|2|}(s)}, \quad (16) \]

which does not rely on parameters \( p \) and \( l \), if \( s \in \Phi_p \) for all \( p \in \{1, \ldots, n/2\} \) and \( l \in \mathcal{H}_p \), where

\[ I_{\text{max}}^{|2|}(s) \triangleq P \sum_{t=1}^{\infty} \sum_{i=1}^{8t} t^{-\alpha} X_{p,l,t,i}(s). \]

Hence, one can obtain

\[ R(n) \geq \frac{1}{T_S} \sum_{s \in \Phi_p} \log (1 + \text{SINR}_{\text{min}}(s)|s \in \Phi_p) \]

w.h.p. \( \frac{T_p}{T_S} \mathbb{E} \left[ \log (1 + \text{SINR}_{\text{min}}(s)|s \in \Phi_p) \right] \geq \frac{T_p}{T_S} \log \left( 1 + \frac{c_0 G_m^2 \left( \frac{\log n}{d_{\text{hop}}^m} \right)^{\alpha/2}}{N_0 + c_1 G_m^2 \left( \frac{\log n}{d_{\text{hop}}^m} \right)^{\alpha/2} + \mathbb{E} \left[ I_{\text{max}}^{|2|}(s) \right]} \right) \]

w.h.p. \( \frac{T_p}{T_S} \log \left( 1 + \frac{c_0 G_m^2 \left( \frac{\log n}{d_{\text{hop}}^m} \right)^{\alpha/2}}{N_0 + c_1 G_m^2 \left( \frac{\log n}{d_{\text{hop}}^m} \right)^{\alpha/2} + c_2} \right) \]

w.h.p. \( \left( \frac{2M(n)}{n} - \frac{1}{n} \right) \log \left( 1 + \frac{c_0 G_m^2 \left( \frac{\log n}{d_{\text{hop}}^m} \right)^{\alpha/2}}{N_0 + c_1 G_m^2 \left( \frac{\log n}{d_{\text{hop}}^m} \right)^{\alpha/2} + c_2} \right) \quad (17) \]

is achievable w.h.p., where \( d_{\text{hop}} \) is given by (3) and \( c_0, c_1, \) and \( c_2 \) are some positive constants. Here, the first inequality holds since \( \text{SINR}_{\text{min}}(s) \) is the same for all \( p \in \{1, \ldots, n/2\} \) and \( l \in \mathcal{H}_p \), the second inequality holds since \( I_{\text{max}}^{|2|}(s) \) is independent and identically distributed (i.i.d.) for \( s \) and \( T_p \) in an increasing function of \( n \) (refer to Lemma 1), the third inequality holds by Jensen’s inequality since \( \log(1 + a/x) \) is convex in \( x \) for all \( a > 0 \), the fourth inequality holds from Lemma 2 and the fifth inequality holds from Lemma 1.
Fig. 6. The aggregate throughput scaling $T(n)$, achieved by elastic routing, with respect to the inverse of the beam width, $1/\theta$.

Since

$$G_m^2 \left( \frac{\log n}{d_{\text{hop}} n} \right)^{\alpha/2} = \begin{cases} \Theta(1) & \text{if } \theta^{-1} = o \left( \frac{n}{\log n} \right)^{\alpha/4} \\ \Omega(1) & \text{if } \theta^{-1} = \Omega \left( \frac{n}{\log n} \right)^{\alpha/4} \end{cases}$$

(18)

from (3), by substituting (18) into (17), it is shown that $R(n) = \Omega \left( \frac{M(n)}{n} \right)$ is achievable w.h.p. and, as a consequence, $T(n) = \Omega \left( M(n) \right)$ is achievable w.h.p. Finally, from the fact that

$$M(n) = \Theta \left( \frac{n}{h \log n} \right) = \Theta \left( \frac{d_{\text{hop}} n}{\log n} \right)$$

and (5), the aggregate throughput achievable by elastic routing is given by (14) w.h.p. This completes the proof of the theorem.

The aggregate throughput scaling $T(n)$ is illustrated in Fig. 6 according to the scaling parameter $1/\theta$ (the terms $\epsilon$ and $\log n$ are omitted for notational convenience). From Theorem 1, each operating regime is now closely scrutinized. When $\theta^{-1} = o \left( \frac{n}{\log n} \right)^{\alpha/4}$ (Regime I), the throughput scaling gets increased as the beam width of the directional antenna becomes narrower. This is the regime where the proposed elastic routing provides a significant throughput improvement over the nearest-neighbor multihop transmission with increasing antenna gain. If $\theta^{-1} = \Omega \left( \frac{n}{\log n} \right)^{\alpha/4}$ (Regime II), then a linear throughput scaling is achieved, which corresponds to the fundamental limit of the network under consideration and is consistent with the previous work (including the beam width scaling condition) based on the protocol model [26]. This is because all SD pairs can be simultaneously activated with no degradation on the received SINR, which scales as $\Omega(1)$, while maintaining per-node throughput as a constant. Our result is thus general in the sense that the achievable scheme and its throughput are shown for all operating regimes with respect to $\theta$ (i.e., for an arbitrary scaling of $\theta$).

From the achievability result, the following two interesting discussions are also shown.

**Remark 1 (Delay analysis):** An improved aggregate throughput leads to a delay reduction owing to elastic routing, whereas, by using the conventional multihop scheme in the omnidirectional mode, the delay increases proportionally with throughput $T(n)$ [5].

**Remark 2 (Ideal antenna model):** For comparison, let us consider an ideal antenna model.
It can be straightforwardly shown that Lemma 2 also holds when there is no sidelobe gain \( G_s = 0 \), i.e., \( G_s = 0 \). This reveals that, under this ideal model, the aggregate throughput \( T(n) \) is given by (14) as long as \( G_s = O(1) \). Therefore, the existence of sidelobe beams, whose gain scales as \( O(1) \), does not cause any throughput loss in scaling law.

To be specific, with regard to Remark 1 when we define the network delay \( D(n) \) as the average number of hops per SD pair, denoted by \( \bar{h} \), a trade-off between the network delay scaling and the aggregate throughput scaling can be illustrated in Fig. 7. In the omnidirectional mode, the network throughput increases up to \( \Theta(\sqrt{n}) \) while the network delay gets also increased [5]. On the other hand, with the use of directional antennas, the proposed elastic routing enables to increase the per-hop distance during the packet transmission, resulting in the reduced network delay and the increased number of simultaneously active SD pairs.

B. Throughput Scaling in Extended Networks

We now analyze the throughput scaling for extended networks using the elastic routing protocol. From the fact that Lemma 2 also holds for extended networks, the following theorem establishes our second main result.

**Theorem 2:** In the extended ad hoc network of unit node density, the aggregate throughput achieved by elastic routing is identical to the dense network case as in Theorem 1.

**Proof:** In the extended network, by following the same analysis as in (15)–(17), we have

\[
R(n) \xrightarrow{\text{w.h.p.}} \left( \frac{2M(n)}{n} - \frac{1}{n} \right) \log \left( 1 + \frac{c_3G^2_m}{d_{\text{hop}}^\alpha} \right),
\]

where \( d_{\text{hop}} \) is given by (14) and \( c_3, c_4, \) and \( c_5 \) are some positive constants. Since

\[
\frac{G^2_m}{d_{\text{hop}}^\alpha} \begin{cases} \Theta \left( \frac{1}{(\log n)^{\alpha/4}} \right) & \text{if } \theta^{-1} = o \left( \left( \frac{n}{\log n} \right)^{\alpha/4} \right) \\ \Omega \left( \frac{1}{(\log n)^{\alpha/4}} \right) & \text{if } \theta^{-1} = \Omega \left( \left( \frac{n}{\log n} \right)^{\alpha/4} \right) \end{cases}
\]
It follows that $R(n) = \Omega \left( \frac{M(n)}{n} \right)$ and $T(n) = \Omega (M(n))$ are achievable w.h.p., which completes the proof of the theorem.

As in the achievability result based on the nearest-neighbor multihop in the omnidirectional mode [1], note that, when elastic routing is used in the network, the aggregate throughput scaling for the extended network is the same as the dense network configuration for all operating regimes with respect to $\theta$. That is, the network configuration type does not essentially change our scaling result as long as packet forwarding protocols such as the nearest-neighbor multihop routing and elastic routing are concerned.

V. EXTENSION TO HYBRID NETWORKS: THE USE OF INFRASTRUCTURE

In this section, we consider hybrid networks by deploying infrastructure aiding wireless nodes. Such hybrid networks, consisting of both ad hoc nodes and infrastructure nodes, have been introduced, and their throughput scaling laws were analyzed in [15]–[21]. In a hybrid network equipping directional antennas at each node, we analyze the impact and benefits of the proposed elastic routing in further improving the throughput scaling.

A. System Model

The whole network area is divided into $b(n)$ square cells, each of which is covered by one BS equipped with a single directional antenna at its center (see Fig. 8), which similarly follows the system model in [15]–[18]. For analytic convenience, let us state that parameters $n$ and $b(n)$ are related according to $b(n) = n^\gamma$ for $\gamma \in [0, 1)$. Moreover, as in [15]–[19], it is assumed that BSs are connected to each other by wired infrastructure with infinite bandwidth (i.e., infinite capacity) and that they are neither sources nor destinations. We assume that each BS has an average transmit power constraint $P$ (constant) and CSI is available at the receive side including the receive BSs, but not at the transmit side including the transmit BSs. We do not assume the use of any sophisticated multiuser detection schemes at each receiver, thereby resulting in an easier implementation.

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The throughput degradation up to a polylogarithmic term, coming from a power limitation, is omitted since it is negligible.
B. Infrastructure-Supported Elastic Routing Protocol

We state our result based on the dense network model, but the overall procedure can be straightforwardly applied to the extended network configuration. Now, we describe a new elastic routing scheme utilizing infrastructure, i.e., BSs. For the infrastructure-supported elastic routing scheme, each square cell is tessellated into several square routing cells having the area of \( A_{\text{infra}}(n) \) (which will be specified later). As illustrated in Fig. 8, the infrastructure-supported elastic routing consists of three phases: access routing, BS-to-BS transmission, and exit routing. In the access routing, a source node transmits its packet to the nearest BS. The packet is then delivered to another BS that is nearest to the destination of the source via wired link. The packet is received at the destination from the BS in the exit routing. To avoid a large amount of interference, different time slots are used between not only routing schemes with and without infrastructure support but also access and exit routings. The infrastructure-supported elastic routing protocol is described more specifically as follows:

- Divide the network into equal square cells of area \( 1/b(n) \), each having one BS at the center of each cell, and again divide each cell into smaller square routing cells of area \( A_{\text{infra}}(n) \), specified in \((20)\).
- In the access routing, one source in each cell transmits its packets to the corresponding BS via the elastic routing, using one of the nodes in each adjacent routing cell. We draw the straight line connecting a source to its nearest BS. For convenience, we term the line a “source–BS line”. Then, packets for an SD pair travel horizontally or vertically along its source–BS line by hopping along adjacent routing cells of area \( A_{\text{infra}}(n) \) until they reach the nearest BS. Similarly as in the pure ad hoc transmission case, while travelling along a source–BS line, a certain node in each routing cell is arbitrarily selected as a relay forwarding the packets. As in the omnidirectional mode, at each hop, a transmit power of \( P(\log n/n)^{\alpha/2} \) is used, and the antennas of each selected transmitter–receiver pair are steered so that their beams cover each other.
- The BS that completes decoding its packets transmits them to the BS closest to the corresponding destination by wired BS-to-BS links.
- In the exit routing, similarly as in the access routing, the infrastructure-supported elastic routing from a BS to the corresponding destination is performed, where each BS uses power \( P(\log n/n)^{\alpha/2} \) that satisfies the power constraint. We draw the straight line connecting a destination to its nearest BS. Then, packets for an SD pair travel horizontally or vertically along the drawn BS–destination line by hopping along adjacent routing cells of area \( A_{\text{infra}}(n) \) until they reach their destination. At each hop, the antennas of each selected transmitter–receiver pair are also steered so that their beams cover each other.

When \( b(n) \) BSs are deployed over the hybrid network, the maximum distance that packets travel through air interface is limited by \( O\left( \sqrt{\frac{1}{b(n)}} \right) \) if an infrastructure-supported routing protocol is used. Thus, similarly as in the ad hoc network case, the area of the routing cell, \( A_{\text{infra}}(n) \), is given by

\[
A_{\text{infra}}(n) = \Theta \left( d_{\text{infra-hop}}^2 \right) = \Theta \left( \frac{1}{b(n)\bar{h}_{\text{infra}}^2} \right)
\]  

\((20)\)

in the dense hybrid network, where \( d_{\text{infra-hop}} \) and \( \bar{h}_{\text{infra}} \) denote the average per-hop distance and the average number of hops, respectively, in the infrastructure-supported elastic routing. For given \( A_{\text{infra}}(n) \), the average per-hop distance \( d_{\text{infra-hop}} \) can be determined using \((20)\) and
is shown according to the value of \( \theta \) as follows:

\[
d_{\text{infra-hop}} = \begin{cases} 
\Theta \left( \sqrt{\log n / \theta^{-2}} \right) & \text{if } \theta^{-1} = o \left( \frac{n}{b(n) \log n} \right)^{\alpha/4} \\
\Theta \left( \sqrt{1 / b(n)} \right) & \text{if } \theta^{-1} = \Omega \left( \frac{n}{b(n) \log n} \right)^{\alpha/4},
\end{cases}
\]

(21)

where \( b(n) = n^\gamma \) for \( \gamma \in [0, 1) \), which will be verified in the next subsection.

Note that, for the given transmit power (i.e., the transmit power \( P(\log n / n^{\alpha/2}) \)), per-hop distance in the infrastructure-supported elastic routing is longer than that in the conventional infrastructure-supported multihop of the omnidirectional mode [15]–[18] for all \( \theta \).

C. Throughput Scaling

In the following theorem, we establish our third main result, which presents the aggregate throughput \( T(n) \) with respect to \( \theta^{-1} \) when the proposed elastic routing protocol is used in the dense hybrid network equipping directional antennas.

**Theorem 3:** In the dense network using infrastructure, the aggregate throughput achieved by elastic routing with and without BS support is given by

\[
T(n) = \begin{cases} 
\Theta \left( \frac{1}{b(n)} \right) & \text{if } \theta^{-1} = o \left( \frac{n}{b(n) \log n} \right)^{\alpha/4} \\
\Theta \left( \frac{1}{b(n)} \right) & \text{if } \theta^{-1} = \Omega \left( \frac{n}{b(n) \log n} \right)^{\alpha/4},
\end{cases}
\]

(22)

w.h.p., where \( \epsilon > 0 \) is an arbitrarily small constant.

**Proof:** We provide a brief sketch of the proof since the proof essentially follows almost the same line as in Theorem 1. Under the hybrid network, the aggregate throughput \( T(n) \) is lower-bounded by

\[
T(n) \geq \max \{ T_{\text{infra}}(n), T_{\text{adhoc}}(n) \},
\]

where \( T_{\text{infra}}(n) \) and \( T_{\text{adhoc}}(n) \) denote the aggregate throughputs achieved by the infrastructure-supported and pure ad hoc elastic routing protocols, respectively.

From the fact that \( T_{\text{adhoc}}(n) \) is given by (14) in Theorem 1, let us now focus on computing \( T_{\text{infra}}(n) \). We start from dealing with the access routing. The transmission rate of each hop is expressed as a function of the SINR value at each receiver that the packet of a source goes through via the infrastructure-supported elastic routing until it reaches the corresponding BS. Let \( I_{p,l}^{[1]}(s) \) and \( I_{p,l}^{[2]}(s) \) denote the intra-pair and inter-pair interference powers of \( r_{x_{p,l}} \) (either an ad hoc node or a BS), respectively, for the \( l \)th hop of the source–BS line corresponding to the \( p \)th SD pair at time slot \( s \in \{1, \cdots, T_s\} \).
Then, by following the same analysis as in (8) and (9), we have

\[
I^{[3]}_{p,l}(s) = O \left( G^2_m \left( \frac{\log n}{d^{2}_{\text{infra-hop}} n} \right)^{\alpha/2} \right),
\]

\[
I^{[4]}_{p,l}(s) \leq P \sum_{t=1}^{\infty} \sum_{i=1}^{8t} t^{-\alpha} Y_{p,l,t,i}(s),
\]

where \( Y_{p,l,t,i}(s) \in \{ G^2_m, G_m G_s, G^2_s \} \) denotes the antenna gain between \( \text{rx}_{p,l} \) and \( \text{in}_{p,l,t,i}(s) \) at time slot \( s \). Under the infrastructure-supported routing protocol, there are \( b(n) \) source-BS lines that are active simultaneously at each time slot. Thus, similarly as in the proof of Theorem 1, the achievable per-node transmission rate during the access routing, \( R_{\text{access}}(n) \) is lower-bounded by

\[
R_{\text{access}}(n) \geq \left( \frac{2b(n)}{n} - \frac{1}{n} \right) \cdot \log \left( 1 + \frac{c_6 G^2_m \left( \frac{\log n}{d^{2}_{\text{infra-hop}} n} \right)^{\alpha/2}}{N_0 + c_7 G^2_m \left( \frac{\log n}{d^{2}_{\text{infra-hop}} n} \right)^{\alpha/2} + c_8} \right),
\]

w.h.p., where \( d_{\text{infra-hop}} \) is given by (21) and \( c_6, c_7, \) and \( c_8 \) are some positive constants. By substituting (21) into (23), it follows that

\[
R_{\text{access}}(n) = \Omega \left( \frac{b(n)}{n} \right)
\]

is achievable w.h.p. In a similar fashion, it can also be shown that \( R_{\text{exit}}(n) = \Omega \left( \frac{b(n)}{n} \right) \) is achievable w.h.p. during the exit routing. Hence, it is shown that \( R_{\text{infra}}(n) = \Omega \left( \frac{b(n)}{n} \right) \) is achievable w.h.p. and, as a consequence, \( T_{\text{infra}}(n) = \Omega \left( b(n) \right) \) is achievable w.h.p. In conclusion, the aggregate throughput \( T(n) \) is given by the result in (22), which completes the proof of the theorem.

From Theorem 3 it is shown that, as the number of BSs \( b(n) \) scales slower than \( n^{1/2} \), the throughput scaling and operating regimes remain the same as those of no infrastructure model (i.e., the ad hoc network model). However, when \( b(n) \) scales faster than \( n^{1/2} \), the operating regimes and the best achievable schemes in each regime are considerably changed and are summarized as follows:

- **Regime III**: \( \theta^{-1} = o \left( \frac{b(n)}{n} \right)^{\alpha/4} \).
  The infrastructure-supported elastic routing is used, and its throughput scaling is given by \( b(n) \). In the regime, the use of directional antennas is not useful in terms of throughput scaling laws.

- **Regime IV**: \( \theta^{-1} = \Omega \left( \frac{b(n)}{n} \right)^{\alpha/4} \) and \( \theta^{-1} = o \left( \frac{n}{\log n} \right)^{\alpha/4} \).
  In Regime IV, the ad hoc elastic routing outperforms the infrastructure-supported elastic routing. Thus, the use of infrastructure is not helpful in this regime. The aggregate throughput scaling \( \Omega \left( n^{1/2 - \theta^{-2/\alpha}} \right) \) is achieved, which is the same as that in Regime I assuming no BSs.

- **Regime V**: \( \theta^{-1} = \Omega \left( \frac{n}{\log n} \right)^{\alpha/4} \)
Fig. 9. The aggregate throughput scaling $T(n)$, achieved by elastic routing, with respect to the inverse of the beam width, $1/\theta$, where $b(n) = \Omega(n^{1/2})$ is assumed.

The single-hop transmission is performed in this regime where a linear aggregate throughput scaling is achieved, which shows the same scaling as Regime II.

In the hybrid network assuming that $b(n) = \Omega(n^{1/2})$, the aggregate throughput scaling $T(n)$ is illustrated in Fig. 9. Note that, when $b(n) = o(n^{1/2})$, the throughput scaling is the same as Theorem 1 which is shown in Fig. 6. When the beam width $\theta$ of each directional antenna is not sufficiently narrow (Regime III), the throughput scaling is given by the number of BSs, $b(n)$, where the throughput scaling is improved with the help of the BSs but the use of directional antennas is not helpful in further enhancing the throughput performance. As $\theta$ becomes narrower (Regime IV), the throughput scaling is improved with increasing $\theta^{-1}$ and thus the directional antenna gain can be attainable, where the pure ad hoc elastic routing is dominant in the regime. When $\theta^{-1} = \Omega\left(\left(\frac{n}{\log n}\right)^{-\alpha/4}\right)$ (Regime V), a linear throughput scaling is achieved, which eventually approaches the fundamental limit of the network.

In addition, we remark that, since each source/BS can reach the corresponding BS/destination via one hop using a transmit power of $Pb(n)^{-\alpha/2}\theta^2$, an infrastructure-supported single-hop directional transmission in each cell can also be performed while achieving the same throughput scaling law as the infrastructure-supported elastic routing case.

In the extended hybrid network, a full transmit power $P$ is used at each node (including each BS) since the network is also power-limited. Then, one can easily see that the throughput scaling and fundamental operating regimes are the same as those in the dense network.

VI. CONCLUSION

This paper has analyzed throughput scaling laws of ad hoc networks equipping directional antennas, each of which has a scalable beam width $\theta$ with respect to the number of nodes, $n$. To fully utilize the characteristics of directional antennas, the elastic routing protocol was proposed, where per-hop distance is increased elastically according to $\theta$ while the average received SINR is maintained as a constant. Under the proposed routing protocol, fundamental operating regimes with respect to $\theta$ and the corresponding throughput scaling laws were identified. It was proved that the elastic routing protocol exhibits a much higher throughput scaling result compared to the conventional multihop scheme as $\theta^{-1}$ increases. Moreover, our result was generalized to the hybrid network scenario with infrastructure. The impacts and
benefits of the elastic routing protocol were comprehensively analyzed in further improving throughput scaling laws in the hybrid network.

REFERENCES

[1] P. Gupta and P. R. Kumar, “The capacity of wireless networks,” IEEE Trans. Inf. Theory, vol. 46, no. 3, pp. 388–404, Mar. 2000.
[2] ——, “Towards an information theory of large networks: an achievable rate region,” IEEE Trans. Inf. Theory, vol. 49, pp. 1877–1894, Aug. 2003.
[3] O. Dousse, M. Franceschetti, and P. Thiran, “On the throughput scaling of wireless relay networks,” IEEE Trans. Inf. Theory, vol. 52, pp. 2756–2761, June 2006.
[4] F. Xue, L.-L. Xie, and P. R. Kumar, “The transport capacity of wireless networks over fading channels,” IEEE Trans. Inf. Theory, vol. 51, pp. 834–847, Mar. 2005.
[5] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, “Optimal throughput-delay scaling in wireless networks-Part I: The fluid model,” IEEE Trans. Inf. Theory, vol. 52, no. 6, pp. 2568–2592, June 2006.
[6] A. El Gamal and J. Mammen, “Optimal hopping in ad hoc wireless networks,” in Proc. IEEE INFOCOM, Barcelona, Spain, Apr. 2006, pp. 1–10.
[7] M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, “Closing the gap in the capacity of wireless networks via percolation theory,” IEEE Trans. Inf. Theory, vol. 53, no. 3, pp. 1009–1018, Mar. 2007.
[8] W.-Y. Shin, S.-Y. Chung, and Y. H. Lee, “Parallel opportunistic routing in wireless networks,” IEEE Trans. Inf. Theory, vol. 59, no. 10, pp. 6290–6300, Oct. 2013.
[9] A. Özgür, O. Lévêque, and D. N. C. Tse, “Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks,” IEEE Trans. Inf. Theory, vol. 53, no. 10, pp. 3549–3572, Oct. 2007.
[10] U. Niesen, P. Gupta, and D. Shah, “On capacity scaling in arbitrary wireless networks,” IEEE Trans. Inf. Theory, vol. 55, pp. 3959–3982, Sept. 2009.
[11] ——, “The balanced unicast and multicast capacity regions of large wireless networks,” IEEE Trans. Inf. Theory, vol. 56, no. 5, pp. 2249–2271, May 2010.
[12] S.-W. Jeon and M. Gastpar, “Capacity scaling of cognitive networks: Beyond interference-limited communication,” IEEE Trans. Inf. Theory, vol. 60, pp. 7824–7840, Dec. 2014.
[13] M. Grossglauser and D. N. C. Tse, “Mobility increases the capacity of ad hoc wireless networks,” IEEE/ACM Trans. Networking, vol. 10, no. 4, pp. 477–486, Aug. 2002.
[14] V. R. Cadambe and S. A. Jafar, “Interference alignment and degrees of freedom of the K-user interference channel,” IEEE Trans. Inf. Theory, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.
[15] O. Dousse, P. Thiran, and M. Hasler, “Throughput capacity of random ad hoc networks with infrastructure support,” in Proc. ACM MobiCom, San Diego, CA, Sept 2003, pp. 55–65.
[16] A. Zemlianov and G. de Veciana, “Capacity of ad hoc wireless networks with infrastructure support,” IEEE J. Select. Areas Commun., vol. 23, no. 3, pp. 657–667, May 2005.
[17] B. Liu, Z. Liu, and D. Towsley, “On the capacity of hybrid wireless networks,” in Proc. IEEE INFOCOM, San Francisco, CA, Mar./Apr. 2003, pp. 1543–1552.
[18] B. Liu, P. Thiran, and D. Towsley, “Capacity of a wireless ad hoc network with infrastructure,” in Proc. ACM MobiHoc, Montréal, Canada, Sept. 2007.
[19] W.-Y. Shin, S.-W. Jeon, N. Devroye, M. H. Vu, S.-Y. Chung, Y. H. Lee, and V. Tarokh, “Improved capacity scaling in wireless networks with infrastructure,” IEEE Trans. Inf. Theory, vol. 57, no. 8, pp. 5088–5102, Aug. 2011.
[20] P. Li and Y. Fang, “The capacity of heterogeneous wireless networks,” in Proc. IEEE INFOCOM, San Diego, CA, Mar. 2010.
[21] S.-W. Jeon and S.-Y. Chung, “Capacity scaling of single-source multiantenna wireless networks without CSIT,” IEEE Trans. Inf. Theory, vol. 58, pp. 6870–6878, Nov. 2012.
[22] S. Yi, Y. Pei, and S. Kalyanaraman, “On the capacity improvement of ad hoc wireless networks using directional antennas,” in Proc. ACM MobiHoc, Annapolis, Maryland, June 2003, pp. 108–116.
[23] J. Zhang and S. C. Liew, “Capacity improvement of wireless ad hoc networks with directional antennas,” SIGMOBILE Mobile Comput. Commun. Rev., vol. 10, no. 4, pp. 911–915, Oct. 2006.
[24] R. Ramanathan, J. Redi, C. Santivanez, D. Wiggins, and S. Polit, “Ad hoc networking with directional antennas: A complete system solution,” in Proc. ACM MobiHoc, Annapolis, Maryland, June 2003, pp. 76–87.
[25] A. Thornbug, T. Bai, and R. W. Heath, “Performance analysis of mmwave ad hoc networks,” IEEE Trans. Signal Processing, submitted for publication, available at http://arxiv.org/abs/1412.0765.
[26] P. Li, C. Zhang, and Y. Fang, “The capacity of wireless ad hoc networks using directional antennas,” IEEE Trans. Mobile Comput., vol. 10, no. 10, pp. 1374–1387, Oct. 2011.
[27] C. Peraki and S. Servetto, “On the maximum stable throughput problem in random networks with directional antennas,” in Proc. ACM MobiHoc, Annapolis, Maryland, June 2003, pp. 2097–2106, July 2010.
[28] A. Özgür, R. Johari, D. N. C. Tse, and O. Lévêque, “Information-theoretic operating regimes of large wireless networks,” IEEE Trans. Inf. Theory, vol. 56, no. 1, pp. 427–437, Jan. 2010.
[30] U. Csiszár and J. Körner, *Information Theory: coding Theorems for Discrete Memoryless Systems*. New York: Academic, 1981.

[31] J. Yoon, W.-Y. Shin, and S.-W. Jeon, “Elastic routing in wireless networks with directional antennas,” in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Honolulu, HI, June/July 2014, pp. 1001–1005.