Inferring the magnetic field vector in the quiet Sun

I. Photon noise and Selection Criteria

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ABSTRACT

In the past, spectropolarimetric data from Hinode/SP has been employed to infer the distribution of the magnetic field vector in the quiet Sun. While some authors have found predominantly horizontal magnetic fields, others favor an isotropic distribution. In this paper, we investigate whether it is actually possible to accurately retrieve the magnetic field vector in regions with very low polarization signals (e.g.: internetwork), employing the Fe I line pair at 6300 Å. We first perform inversions of the Stokes vector observed with Hinode/SP in the quiet Sun at disk center in order to confirm the distributions retrieved by other authors. We then carry out several Monte-Carlo simulations with synthetic data where we show that the observed distribution of the magnetic field vector can be explained in terms of purely vertical (v = 0°) and weak fields (B < 20 G), that are misinterpreted by the analysis technique (Stokes inversion code) as being horizontal (v ≈ 90°) and stronger (B ≈ 100 G), due to the effect of the photon noise. This casts doubts as to whether previous results, presenting the distributions for the magnetic field vector peaking at v = 90° and B = 100 G, are actually correct. We propose that an accurate determination of the magnetic field vector can be achieved by decreasing the photon noise to a point where most of the observed profiles posses Stokes Q or U profiles that are above the noise level. Unfortunately, for noise levels as low as 2.8 x 10^−4 only 30% of the observed region with Hinode/SP have strong enough Q or U signals, implying that the magnetic field vector remains unknown in the rest of the internetwork.

Key words. Magnetic fields – Line: profiles – Polarization – Sun: photosphere – Sun: surface magnetism

1. Introduction

The magnetic field in the quiet Sun internetwork has been subject to an intense debate over the years. Its study has been motivated by the fact that much of solar magnetic flux could be contained in this region (Sánchez Almeida 2004), which has important consequences in our understanding of the solar activity cycle (Spruit 2003; Solanki et al. 2006). In addition, quiet Sun magnetic fields are subject to the strong influence of the granular convection, giving rise to a rich variety of phenomena such as magnetic flux emergence (Centeno et al. 2007; Martínez González & Bellot Rubio 2009; Danilovic et al. 2010), convective collapse (Bellot Rubio et al. 2001; Nagata et al. 2008; Fischer et al. 2009), supersonic velocities (Socas-Navarro & Manso-Sainz 2005; Shimizu et al. 2007; Bellot Rubio 2009; Borrero et al. 2010) and so forth, that play an important role on the heating of the upper layers of the solar atmosphere (Schrijver et al. 1997, 1998; Schrijver & Title 2003).

These studies have triggered the development of instruments with ever-increasing spatial resolution, observations in different spectral windows (Khomonko et al. 2003; Domínguez Cerdeña et al. 2006; Socas-Navarro et al. 2008), and using spectral lines subject to the Zeeman and Hanle effect (Manso Sainz et al. 2004, Trujillo Bueno et al. 2004; Sánchez Almeida, J. 2005). A mayor step has been accomplished with the deployment of the Japanese spacecraf Hinode. The spectropolarimeter (SP) attached to the Solar Optical Telescope (SOT; Tsuneta et al. 2008) on-board Hinode allows to achieve continuous observations of the solar atmosphere with an extremely high-spatial resolution. This advancement has favored the use of data from this instrument (Ichimoto et al. 2008) in the investigations of the quiet Sun magnetism over the last few years. The analysis of this data has revealed that a large portion of the magnetic flux in the internetwork presents itself in the form of horizontal magnetic fields (Orozco Suárez et al. 2007a, Lites et al. 2008). Other works, also analyzing Hinode/SP data favor an isotropic (e.g.: turbulent) distribution of the magnetic field vector (Asensio Ramos 2009; Stenflo 2010). Motivated by this controversy we have revisited these results with the hope of offering possible solutions. In Section 2 we describe our observations and data-sets from the Hinode/SP instrument. Section 3 describes the analysis method and how we infer the magnetic field vector from spectropolarimetric observations. Section 4 presents our results and compares them with previous works, whereas Sections 5 and 6 are fully devoted to study the effects that photon noise and the different selection criteria applied to the data, have in the inferred distribution of the magnetic field. Finally, Section 7 summarizes our findings.

2. Observations

The data used in this work correspond to spectropolarimetric observations (full Stokes vector: I, Q, U, V) around the Fe I line pair at 6301.5 Å and 6302.5 Å, recorded with the spectropolarimeter on-board the Japanese satellite Hinode (Lites et al. 2001; Kosugi et al. 2007; Tsuneta et al. 2008; Suematsu et al. 2008; Ichimoto et al. 2008). These two lines have Landé factors of g_{eff} = 1.67 and g = 2.5. The image stabilization system in Hinode (Shimizu et al. 2008) allows the slit spectropolarimeter...
to achieve a spatial resolution of 0.32". Here we analyze several data sets corresponding to quiet Sun observations. The maps employed are the same as the ones already analyzed by Orozco Suárez et al. (2007a, 2007b) and Lites et al. (2008). Hereafter these three papers will be referred to as: OS07a, OS07b and LIT08. Further analysis of some subsections of the same dataset have also been carried out by Asensio Ramos (2009), Martinez González et al. (2010) and Stenflo (2010). In the following subsections we briefly describe each selected map. A summary of the main their properties can be also found in Table 1.

2.1. Normal Hinode/SP map

This map contains 2047×1024 pixels with a pixel size of 0.15" and 0.16" in the horizontal and vertical directions, respectively. The field of view is centered at (-53.1",7.9") from disk center ($\mu = 0.998$). The exposure time at each slit position is 4.8 s, yielding a noise level of $\sigma_s \approx 10^{-3}$ (in units of the average quiet Sun intensity). Here, the index $s$ refers to any of the four components of the Stokes vector. Hereafter we will refer to this map as Map A. The data was recorded in March 10, 2007, from 11:37 UT to 14:36 UT. A map of the continuum intensity and magnetic field strength (retrieved by the inversion; Sect. 4) in a subfield of this map is displayed in Figure 1. The subfield has been chosen to coincide with Figure 2 in OS07a.

2.2. Deep Hinode/SP map

This map contains 727×1024 pixels with a pixel size of 0.15" and 0.16" in the horizontal and vertical directions, respectively. In this case, the slit did not scan across the solar surface, but instead it was kept at the same location (-22.4",7.8") on the sun

![Map A](image)
from 00:20 UT to 02:19 UT on February 27, 2007. The exposure time at each slit position is 9.6 s, yielding a noise level of $\sigma_s \approx 7.5 \times 10^{-4}$ (in units of the average quiet Sun intensity). Hereafter we will refer to this map as Map B. Figure 2 shows a map of the continuum intensity and magnetic field strength.

### 3. Inversion of Stokes profiles and magnetic field vector retrieval

The magnetic field vector was retrieved from the Stokes vector using the VFISV (Very Fast Inversion of the Stokes Vector; Borrero et al. 2010) inversion code. We employ a parametrized atmospheric model that contains a magnetic and a non-magnetic atmosphere next to each other. The magnetic atmosphere is characterized by the following parameters: magnetic field strength $B$, inclination of the magnetic field with respect to the observer $\varphi$, azimuthal angle of the magnetic field in the plane perpendicular to the observer $\phi$, line-of-sight velocity $V_{\text{LOS}}$, damping parameter $a$, ratio of the absorption coefficient between the line-core and the line-continuum $\eta_0$, Doppler-width $\Delta V_{\text{Dep}}$, source function at the observer’s location $S_{\text{S}}$, and the gradient of the source function with optical depth $S_{\text{I}}$. The non-magnetic atmosphere has one single parameter: fractional area of the pixel filled with non-magnetic plasma (aka non-magnetic mm filling factor) $\alpha_{\text{nm}}$.

Using initial values for these parameters, VFISV obtains the theoretical Stokes vector $I_{\text{th}} = (I, Q, U, V)$ as:

$$I_{\text{tot}} = (1 - \alpha_{\text{nm}})I_{\text{mag}} + \alpha_{\text{nm}}I_{\text{nm}},$$

(1)

where $I_{\text{mag}}$ is the polarized contribution of the magnetized atmosphere, which is obtained by solving the radiative transfer equation under the assumptions of the Milne-Eddington (M-E) approximation. $I_{\text{nm}}$ corresponds to the non-magnetic contribution, which is thus unpolarized ($Q_{\text{nm}} = U_{\text{nm}} = V_{\text{nm}} = 0$: see Section 3.1). The total theoretical Stokes vector $I_{\text{th}}$ is then compared with the observed one using a merit function $\chi^2$, and by means of a non-linear least-squares-fitting algorithm (Levenberg-Marquardt method; Press et al. 1986) we iterate our initial 10 free parameters until a minimum in $\chi^2$ is reached. As an example, Figures 1 and 2 show the resulting magnetic field strength (from the inversion) for Maps A and B.

### 3.1. Treatment of the non-magnetic atmosphere

In order to compare with OS07b, we have adopted for our inversion a treatment of the non-magnetic atmosphere that similar to theirs: a different $I_{\text{nm}}$ profile is used in the inversion of each pixel in the map $I_{\text{nm}}$ is obtained for each pixel by averaging only the intensity profiles $I$ (polarization not included) from the neighboring pixels in a 1° square around the inverted one. This method attempts to mimic a narrow spectrograph’s point spread function. OS07b interprets the need for a non-magnetic component as the reduction in the amplitude of the polarization signals (reduction $\propto (1 - \alpha_{\text{nm}})$) cause by diffraction. Danilovic et al. (2008) ascribes it as being caused by a small defocus in the SP instrument.

Another feature of our inversions is that we do not consider the free parameter $\alpha_{\text{nm}}$ (Eq. 1), but rather we use a different variable $\beta_{\text{nm}}$, where:

$$\beta_{\text{nm}} = \tanh^{-1}(2\alpha_{\text{nm}} - 1)$$

(2)

Since $\alpha_{\text{nm}}$ varies from 0 (pixel fully filled with the magnetic atmosphere) to 1 (pixel fully filled with non-magnetized plasma), $\beta_{\text{nm}}$ varies (through Eq. 2) between $(-\infty, +\infty)$. The reason for employing $\beta_{\text{nm}}$ instead of $\alpha_{\text{nm}}$ is to prevent the inversion from reaching a local minimum in the $\chi^2$-hypersurface where $\alpha_{\text{nm}}$ is either 0 or 1, resulting in an unrealistic number of pixels harboring these two values. The use of $\beta_{\text{nm}}$ is supported by the fact that $\chi^2$ improves on average by about 10 % when using it instead of $\alpha_{\text{nm}}$ (see 5th and 6th columns in Table 1).

Due to our treatment of the non-magnetic atmosphere some caution should be exercised when speaking about field strengths in this work. In our modeling of the Stokes parameters (Eq. 1) we assume that the light we observe at each pixel has a polarized contribution from the pixel itself, plus a non-polarized contribution from the surrounding pixels. From this point of view we cannot talk about intrinsic field strengths but rather about mean field strength inside the resolution element.

An alternative interpretation of Eq. 1 assumes that inside the resolution element there are two kinds of atmospheres: a magnetized one and a non-magnetized one, where the magnetized one possesses only one magnetic field. From this point of view it is possible to refer to $B$ as the intrinsic field strength. However, Hinode/SP’s resolution is not sufficient to fully resolve all magnetic structures in the quiet Sun. Therefore our treatment oversimplifies the problem, as we are assuming that the magnetic field is constant inside the magnetized atmosphere. In this respect, $B$ should be interpreted as the mean field strength in the magnetized atmosphere. This implies that both interpretations of Eq. 1, despite being different from a physical point of view, are indistinguishable from each other. Because of this, in the following we will refer to the magnetic field as $B$, in order to indicate its averaged nature.

It is important to mention that the results presented in this paper, in particular those in Section 5, do not depend on the scheme employed to characterize the stray-light. As a matter of fact, the same results are obtained if we consider a global stray-light profile, or even if it is completely neglected.

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1 This inversion code is freely available for download at the Community Spectro-Polarimetric Analysis Center (CSAC; Lites et al. 2007), which is an initiative at the High Altitude Observatory and National Center for Atmospheric Research (NCAR): [http://www.csac.hao.ucar.edu/](http://www.csac.hao.ucar.edu/)

2 Note that we have dropped the vector character of the stray-light profile (Equation 1) because of its non-polarized character. This means that $Q_{\text{nm}} = U_{\text{nm}} = V_{\text{nm}} = 0$ and therefore only the first component of the Stokes vector $I_{\text{nm}}$ (total intensity) remains.

3 The word global refers to a stray-light profile $I_{\text{nm}}$ that is common for all inverted pixels, and that is obtained as an average of all pixels, in the FOV, where the polarization signals are below the noise level.
3.2. One-line vs two-lines inversion

One limitation of the VFISV inversion code is that it only allows to analyze the data from one spectral line at a time. Hinode/SP records the full Stokes vector for the Fe I line pair at 6301.5 Å and 6302.5 Å across 112 spectral positions. From these two lines the obvious choice, due to its larger sensitivity to magnetic fields (larger Landé factor; see Sect. 2), is Fe I 6302.5. Therefore VFISV only makes use of 50 spectral points around the second spectral line. This results in an obvious loss of information, that will yield slightly larger errors in the determination of our model parameters (Sect. 3; see also Orozco Suárez et al. 2010).

In order to make sure that this shortcoming is not critical for our study we have repeated the inversions of map A, using the HELIX++ inversion code (Lagg et al. 2004, 2009), which allows to consider both spectral lines simultaneously. The resulting histograms for velocity $V_{\text{LOS}}$, magnetic field strength $B$, inclination $\gamma$, et cetera, are very similar to those obtained from the 1-line inversions using VFISV (see Figure 3). This makes us confident that our results are not affected by the fact that only one spectral line is being employed, or at least not to the point of invalidating our results. We will return to this point in Section 5.

4. Results

In Figures 3a, 4a and 5a we present histograms for the number of pixels whose unsigned polarization signals $|V|$ or $\sqrt{Q^2 + U^2}$ have peak values $N$ times larger than or equal to the noise level in each map. This is will be referred to as SNR: signal-to-noise ratio. The vertical lines in these figures indicate the different SNR-thresholds that we have considered in our analysis: 3, 4.5 and 6. For instance, the vertical dashed-dotted line in Fig. 3a corresponds to SNR $= 3$. This means that all pixels in map A, where at least one of the polarization profiles (Stokes $Q$, $U$ or $V$) possesses a SNR $\geq 3$, are included in the histograms of the remaining panels and represented also by the dashed-dotted line. Likewise for SNR $\geq 4.5$ (solid lines) and SNR $\geq 6$ (dashed lines). The rest of the panels in Figs. 3-5 show the normalized histograms for the magnetic field strength $B$ (b-panels), magnetic field inclination with respect to the observer $\gamma$ (c-panels), and the magnetic filling factor $\alpha_{\text{mag}} = 1 - \alpha_{\text{am}}$ (d-panels), which is interpreted as the fractional area of the pixel filled with magnetized plasma. The normalization is done to ensure that the total area under each histogram equals to 1. In all these histograms we have avoided network regions by leaving out of the analysis those pixels where the mean field strength in the resolution element is larger than 750 Gauss.

Note that in all a-panels in Figs. 3-5 the histograms for the circular and linear polarization peak at SNR $\approx 3$. This is due to photon noise. The probability that photon noise produces a signal, at a given wavelength position, that it is above $3\sigma_n$ is only $0.3\%$. However, we must take into account that we are taking the peak value for all 112 (in circular polarization or Stokes $V$) or 224 (in linear polarization or Stokes $Q$ and $U$; see Sect. 3.2), and therefore it is almost guaranteed that at every observed spatial pixel, photon noise will yield a polarization signal that is larger or equal than $3\sigma_n$.

Since all considered maps are close to disk center, we can interpret the retrieved inclination with respect to the observer, $\gamma$, as the inclination with respect to the normal direction to the solar surface. A value of $\gamma = 0^\circ$, $180^\circ$ indicates a vertical (pointing upwards or downwards, respectively) magnetic field, whereas a value of $\gamma = 90^\circ$ indicates a magnetic field that is contained in the solar surface.

The resulting histograms are rather similar in all three (A, B and C) maps. For all SNR-thresholds considered, the distributions of the average magnetic field strength $B$, magnetic field inclination $\gamma$, and magnetic filling factor $\alpha_{\text{mag}}$, peak at around...
for the magnetic filling factor
the inclination of the magnetic field vector
trinsic noise level is
atively. These thresholds are also indicated with vertical lines in
dashed lines display the histograms obtained from those pix-
ograms for the magnetic field strength\(\bar{B}\). Panel-c: histogram for
the magnetic field strength \(\bar{B}\). Panel-d: histogram for
the magnetic field strength and magnetic filling-factor that shift
ward at higher values. In addition, the central peak at \(\gamma = 90^\circ\) in
the histogram for the inclination becomes less pronounced and
almost disappears. Still, the magnetic field still remains rather
horizontal: \(\gamma \in [60, 120]^\circ\). One can interpret this trend in terms
of the Zeeman effect in the weak-field regime, through which the
plitude of the polarization signals is proportional to the
strength of the magnetic field. From this point of view, imposing
larger SNR-thresholds in the polarization immediately
sects stronger fields. The histograms for \(\gamma\) (inclination) and \(\alpha_{\text{mag}}\)
(magnetic filling-factor) indicate that these stronger magnetic
fields tend to be more vertical and to occupy a larger fraction of the
resolution element.

The second observed trend in the retrieved histograms is
related to the map-to-map differences or, equivalently, to the
oise level \(\sigma_{\text{rms}}\) in the four components of the Stokes vector.
Comparing figures 3, 4 and 5 reveals that the histogram of the
magnetic field strength shifts towards lower values from Map A
\((\sigma_r = 10^{-3})\) to Map C \((\sigma_r = 2.8 \times 10^{-4})\). Again, this behavior can
be explained as a consequence of being able to detect smaller po-
larization signals, hence weaker magnetic fields, when the noise
is reduced. In addition, the disappearance of the peak at \(\gamma = 90^\circ\)
for large SNR-thresholds is much less pronounced in map C than
in maps A or B. An explanation for this will be offered in Section
6.

Figure 6 shows histograms for the longitudinal component of
the magnetic field \(B_\parallel = B \cos \gamma\), and the transverse component of
the magnetic field \(B_\perp = B \sin \gamma\) for the three analyzed maps: A
(top panels), B (middle panels) and C (bottom panels). \(B_\parallel\) and \(B_\perp\)
can be identified with the projection of the magnetic field vector
on the vertical direction to the solar surface and the projection of
the magnetic field over the solar surface, respectively. Because
the retrieved magnetic fields are rather inclined \((\gamma \rightarrow 90^\circ)\), the
horizontal component of the magnetic field \(B_\perp\) is larger than the
vertical component \(B_\parallel\).

We have calculated the center-of-gravity (CoG) of the his-
tograms for \(B_\parallel\) and \(B_\perp\) in Fig. 6 and plotted them as a function of
the signal-to-noise ratio in Figure 7. If we take as a reference
the results obtained from considering only those pixels whose po-
larization levels are 4.5 times above the noise level: SNR = 4.5, we
then see that the CoG of \(B_\perp\) (dashed lines) is located at around
122.0 Gauss for the Map A \((\sigma_r = 10^{-3})\; \text{crosses}\), and it de-

50-100 Gauss, 80-100 degrees and 20-30\%, respectively. These
results involve the existence of rather weak and horizontal fields
in the internetwork, occupying about 20\% of the resolution ele-
ment at every pixel. This is in excellent agreement with the re-
results presented in OS07a and OS07b. In particular, our retrieved
histograms for Map A (Figs. 3) using a SNR-threshold of 4.5 are
very similar to those in Figs. 3 and 4 in OS07a and Fig. 7 in
OS07b.

A closer look to Fig. 3-5 reveals a couple of trends. The first
one involves histograms for the same map (noise-level \(\sigma_r\) fixed)
obtained with different signal-to-noise thresholds. Here we ob-
serve that increasing the SNR-threshold yields histograms for
the magnetic field strength and magnetic filling-factor that shift
wards at higher values. In addition, the central peak at \(\gamma = 90^\circ\) in
the histogram for the inclination becomes less pronounced and
almost disappears. Still, the magnetic field still remains rather
horizontal: \(\gamma \in [60, 120]^\circ\). One can interpret this trend in terms
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then see that the CoG of \(B_\perp\) (dashed lines) is located at around
122.0 Gauss for the Map A \((\sigma_r = 10^{-3})\; \text{crosses}\), and it de-
creases to 87.4 Gauss when the noise level is 2.8 × 10⁻⁴ (Map C; diamonds). Differences are even larger if we consider the peak-value (instead of the CoG) in the histograms: 90.6 Gauss in map A, and 57.1 Gauss in map C. Meanwhile, the CoG values in the histogram of $B_\parallel$ (solid lines) only decreases from 21.6 (Map A; crosses) to 11.9 Gauss (Map C; diamonds). Similar trends are observed for other signal-to-noise thresholds. This indicates that both $B_\parallel$ and $B_\perp$ are responsible for the decrease in the CoG and peak value of $B$ when the noise level is reduced (Figs. 3b-5b), with $B_\parallel$ being the main contributor. 

Figs. 6 and 7 indicate that the CoG and peak values in the histograms for $B_\parallel$ and $B_\perp$ become smaller as the noise level decreases. In the light of this result, it is possible that, the current noise levels in the polarization signals (in all three maps considered) is responsible for the large values of $B_\perp$, and therefore for the peak at $y = 90^\circ$ in the histograms for the inclinations? And if so: is it conceivable that, given a low enough level of noise in the polarization signals, the detection thresholds for $B_\parallel$ and $B_\perp$ would be similar, thereby opening the possibility for the detection of non-horizontal magnetic fields? In the following we will address these two questions.

5. Effect of the photon noise

In order to study the effect that the photon noise has on the retrieval of the inclination of the magnetic field vector in the quiet Sun, we have performed the following Monte-Carlo-like test: we first produce a large number of synthetic profiles (1024×2047) where we assume that the magnetic field is always vertical. Half of the synthetic profiles posses $\gamma_{\text{syn}} = 0^\circ$, while the other half have $\gamma_{\text{syn}} = 180^\circ$. The rest of the Milne-Eddington parameters needed for the synthesis: $\Delta_{\text{O} 6563}$, $V_{\text{LOS}}$, $\delta_{\text{b}}$, $\delta_{\text{a}}$, etcetera (see Sect. 3) are taken from the results of the inversion of map A. This is done in order to employ kinematic and thermodynamic parameters that are representative of the quiet Sun. The only exception is the magnetic field strength $B$: for the synthesis we assume $\bar{B}$ to be equal to the vertical component of the field as obtained from the inversion of map A: $B_{\parallel}^{\text{syn}} = \bar{B}_{\parallel}^{\text{syn}} \cos \gamma_{\text{syn}}$. Note that since $\gamma_{\text{syn}} = 0, 180^\circ$ this ensures that the vertical component of the magnetic field in this test is equal to the vertical component of the magnetic field from map A: $B_{\parallel}^{\text{syn}} = B_{\parallel}^{\text{syn}}$.

We then proceed to add noise at different levels and we employ the VFISV code (see Sect. 3) to invert the Stokes profiles of Fe I 6302.5 Å allowing all atmospheric parameters, including the inclination $\gamma$, to vary freely. Finally, we compare the original distributions for $\gamma_{\text{syn}}$, $B_{\parallel}^{\text{syn}}$ and $B_{\perp}^{\text{syn}}$, with the retrieved ones. In our tests we have employed four different levels of noise: $\sigma_{\gamma} = 10^{-3}$ (as in map A), $7.5 \times 10^{-4}$ (as in map B), $2.8 \times 10^{-4}$ (as in map C) and finally, as an example of extremely high signal-to-noise observations, $\sigma_{\gamma} = 10^{-5}$. All numbers are expressed in units of the continuum intensity in the quiet Sun. As in Section 4, our analysis considers only those profiles where the signal-to-noise ratio in at least one of the polarization profiles (Stokes $Q$, $U$ or $V$) is equal or larger than 3, 4.5 and 6.

The results for the vertical component of the magnetic field $B_\parallel$ are presented in Figure 8. Each of the four panels in this figure displays the results for the one of the four levels of noise under consideration. The original distribution of vertical magnetic fields $B_{\parallel}^{\text{syn}}$ is indicated by the black lines. The retrieved distributions, corresponding to the analysis of pixels where the

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4 We employ the random number generator algorithm for normally distributed variables by Levy (1992).
peak at around $\gamma \approx 90^\circ$. This happens in spite of having started with vertical fields, whose distribution is represented in Fig. 10 by the two vertical black arrows at $\gamma = 0, 180^\circ$. The fact that we have obtained horizontal fields out of purely vertical ones is due to the sole effect of the photon noise. This is also true for noise levels as low as $2.8 \times 10^{-5}$, which corresponds to the highest signal-to-noise data analyzed in OS07a, OS07b and LIT08. Even at noise levels as low as $10^{-5}$ (lower-right panel in Fig. 10) the correct distribution of inclinations is not properly recovered, although at least in this case, some amount of vertical fields remain after the inversion.

Another interesting feature of our Monte-Carlo simulation is that it also reproduces the drop in the central peak at $\gamma \approx 90^\circ$ when using a SNR-threshold of 4.5 and 6 or, in order words, when only considering pixels where the polarization signal are 6 times or more above the noise level. This was already seen from the inversions of real data: Figs. 3c-5c. However, this simulation fails to reproduce the fact that the observed drop at $\gamma \approx 90^\circ$ is less pronounced for smaller noise levels.

The tests presented in this section demonstrate that it is possible to deduce the existence of horizontal fields from purely vertical ones due to the sole effect of photon noise. Photon noise is also responsible for an overestimation in $B_\perp$ and therefore also in $B$. These effects occur for noise levels as low as $\sigma_v = 10^{-5}$. Therefore, results from this simulations cast doubt as to whether it is possible to correctly retrieve the distribution of the magnetic field vector in the quiet Sun internetwork. This will be addressed in the next section.

Finally it is important to mention a couple of consistency checks that have been carried out. The first one was performed in order to study if our results are biased as a consequence of the inability of the VFISV inversion code to invert both FeI 6301.5 Å and FeI 6302.5 Å spectral lines (see Sect. 3.2). We have repeated our Monte-Carlo experiments using the SIR inversion code (Ruzic Cobo & del Toro Iniesta 1992) and inverting both spectral lines simultaneously. Due to SIR’s limited speed we have run the experiment with only 5000 profiles, but this is sufficient to confirm the results presented throughout this section. The second consistency check is to repeat these experiments in the absence of photon noise. With this test we have confirmed that vertical magnetic fields are actually retrieved (initial $\delta$-Dirac distributions), and to test the convergence algorithm of the inversion code.

6. Selection effects

In the inversion of observed Hinode/SP data (Sect. 4) we have applied a selection criteria that considers in the analysis all those pixels where any of the polarization signals ($Q$, $U$ or $V$) are above a certain signal-to-noise ratio. This is the same criteria applied in the analysis of simulated data in Sect. 5 and also in OS07a and OS07b.

A very important consequence to be drawn from Sect. 5 is that the distributions for the magnetic field vector obtained from the inversion of the simulated data, are not trustworthy as long as Stokes $Q$ and $U$ are dominated by noise. This was clearly the case in the inversion of the simulated data in that section since there it had been assumed that the magnetic field was vertical.

It is therefore critical to study whether the aforementioned criteria, when applied to Hinode/SP data, select pixels where the linear polarization profiles ($Q$ and $U$) are above the noise or, on the contrary, where $V$ is the only Stokes parameter above the noise. For the simulated data in Sect. 5, the latter was certainly the case since the signals in $Q$ and $U$ were due only to photon noise. What about the observations from Hinode/SP employed in Sect. 4?

In Figure 11 we show the percentage of pixels (with respect to the total number of pixels in each Hinode/SP map) as a function of the SNR-threshold applied. The three maps with different noise levels considered in Sect. 4 are indicated by different symbols: crosses (map A: $\sigma_u = 10^{-3}$), squares (map B: $\sigma_u = 7.5 \times 10^{-4}$), and diamonds (map C: $\sigma_u = 2.8 \times 10^{-4}$). Here we have applied two different criteria. The first one is the same as in Sect. 4 and 5: a pixel is taken into account if any of three polarization profiles ($Q$, $U$ or $V$) has a given signal-to-noise ratio (solid lines). The second one considers only those pixels were either $Q$ or $U$ posses a certain signal-to-noise ratio (dashed lines).

Taking as a reference a SNR-threshold of 4.5 (same as in OS07a) we find that 30.2% of the pixels contained in map A posses at least one of polarization signal that is above this threshold. However, if we restrict our analysis to $Q$ or $U$ only, we realize that only 3.4% of the pixels fulfill this criteria. This implies that the vast majority of the pixels analyzed have linear polarization profiles that are dominated by noise and thus, as demonstrated in Sect. 5, this automatically results into horizontal fields (see Fig. 10). Consequently, the peak at $B = 100$ G and $\gamma = 90^\circ$ in Fig. 3 in this work and also in Fig. 3 in OS07a is not real but due to noise.

In map C, with a much lower noise level than map A, up to 80.0% of the pixels posses signals where at least one of the polarization profiles is above 4.5 times the noise level. However, the number drops to 31.6% if we consider only $Q$ or $U$. As a consequence, only $\approx 1/3$ of the peaks at $B = 50$ G and $\gamma = 90^\circ$ in Fig. 5 is real, with the remaining $\approx 2/3$ being due to noise. This explains why in map A the peak at $\gamma \approx 90^\circ$ disappears for large SNR-thresholds: it is mostly noise (Fig. 3c), while remaining in map C (Fig. 5c): part of the peak there is actually real.

What is then the inclination and strength of the magnetic field vector in the pixels where $Q$ and $U$ are not above the noise level? This represents 97% of all pixels included in the analysis of Fig. 3 (map A) and about 70% of all pixels included in the analysis of Fig. 5 (map C). As demonstrated in Sect. 5 it is possible that they are vertical $\gamma \approx 0^\circ$ with a field strength of $B \approx 10$ G. However, as long as $B_\perp$ remains below our detection threshold this is only speculation.

In order to improve the reliability of the histograms for the magnetic field vector it is possible to repeat the analysis in Sect. 4 but employing only those pixels where the linear polar-
ization signals (Stokes $Q$ or $U$) are 4.5 times above the noise level. In this case we still obtain the central peak at $\gamma = 90^\circ$ and $B = 60 - 120$ G in all three maps considered. This was to be expected because, $Q$ and $U$ signals sufficiently above the noise implies the existence of highly inclined magnetic fields ($\gamma \rightarrow 90^\circ$). This is so because in the weak-field regime, the linear polarization is a second order effect (Landi Degl'Innocenti, 1992) in the magnetic field as compared to the circular polarization.

However, it is important to clarify that histograms obtained applying the selection criteria to the linear polarization would only contain a minority of internetwork pixels, leaving out many of them (up to 90 % of internetwork pixels in map A or up to 70 % of pixels in map C) whose inclination is unknown. Our tests show that in order to correctly infer the inclination of those pixels where only Stokes $V$ is above the signal, a noise level better than $\times 10^{-5}$ (see Fig. 10) is needed.

According to Fig. 11 one might think that, even though only 3.4 % of the pixels in map A and 31.6 % in map C, posses a signal-to-noise larger or equal to 4.5, these numbers grow up to 69.6 % and 85.9 % respectively if a SNR-threshold of 3 is considered. However, as explained in Sect. 4, SNR $\geq$ 3 is not sufficient to reliably infer the magnetic field vector because photon noise produces signals that are above 3 times the noise level in most of the pixels.

7. Conclusions

In this paper we have studied the distribution of the magnetic field vector (strength $B$ and inclination with respect to the vertical direction $\gamma$) in internetwork regions with data from the spectropolarimeter (SP) on-board the Japanese satellite Hinode. We have selected three different maps recorded at disk center with different integration times, and therefore different noise levels. In all three cases, the retrieved inclination of the magnetic field peaks at $\gamma = 90^\circ$, in agreement with previous results (Orozco Suárez et al. 2007a, 2007b).

In addition, we have found that the distribution of vertical/longitudinal fields ($B_\parallel = \bar{B} \cos \gamma$) and horizontal/transverse fields ($B_\perp = \bar{B} \sin \gamma$) shift towards smaller values when we analyze data with lower levels of photon noise. In spite of this decrease, the relation $B_\parallel > \parallel B_\perp$ holds, which explains why all analyzed maps yield histograms for the inclination with a clear peak at $\gamma = 90^\circ$.

To understand this result we have carried out a Monte-Carlo-like simulation in which we produce a large number of synthetic profiles assuming vertical fields ($\gamma = 0, 180^\circ$) and, after adding photon noise at different levels, we invert the synthetic Stokes profiles to investigate whether the original distributions ($\delta$-Dirac functions) can be recovered. These tests show that, peaks at $\gamma = 90^\circ$ (horizontal fields) are obtained for noise levels as low as $\sigma_r = 2.8 \times 10^{-4}$ even though in the synthesis we had used only vertical ones. This means that the histograms for the inclination obtained with real data are contaminated to a large extent by the effect of the photon noise, which makes vertical and horizontal fields indistinguishable.

To what extent the histograms are contaminated depends on the selection criteria employed to select pixels that enter in the analysis. If we select pixels where the signal-to-noise ratio in any of the polarization profiles (Stokes $Q$, $U$ or $V$) is above 4.5, the contamination can be as large as 70-95% (depending on the noise level). This renders the inferred distributions for the magnetic field inclination and transverse component of the magnetic field, meaningless. This is the approach followed in Orozco Suárez et al. (2007a, 2007b). Other authors such as Lites et al. (2007) and Kobel (2009) impose a larger threshold for Stokes
A better approach is to analyze only those pixels where the linear polarization profiles (Stokes Q or U) have a signal-to-noise ratio larger than or equal to 4.5. This strategy has the advantage that the inclination of the magnetic field can be properly retrieved by the inversion of the radiative transfer equation applied to the observed Stokes profiles. A similar conclusion is reached by Asensio Ramos (2009). The main shortcoming of this selection procedure is that it considers only small regions within the internetwork: 5-30 % (depending on the noise level), leaving out large portions where the inclinations is not determined. We conclude therefore that the real distribution of inclinations in the quiet Sun internetwork remains unknown. A proper inference can only be carried out with noise level that are beyond the current capabilities of modern spectropolarimeters. Workarounds can be found by introducing temporal averages, but the required integration time exceeds the lifetime of granulation. It is therefore not clear what is being represented by histograms obtained with averages spanning several minutes.

The importance of the photon noise has been already acknowledged by Asensio Ramos (2009) and Stenflo (2010). In the latter work, results are interpreted in terms of an isotropic distribution of magnetic fields. However, these works do not analyze low-noise data, thus missing the contribution from pixels with strong Q and U signals: up to 30 % of the total amount of pixels. The question is therefore whether an isotropic distribution of fields can explain these regions of truly horizontal fields.

Asensio Ramos (2009; see also Martínez González et al. 2008) also conclude that the distribution of magnetic fields is quasi-isotropic, however, whereas the latter refers to pixels where the polarization signal is above the noise level, the former infers a quasi-isotropic distribution only for those pixels whose polarization signal is below the noise. Our work differs from these ones in that in our case we can explain many features of the resulting histograms for the inclination employing only vertical fields. In the future we intent to adress the possibility of isotropism in more detail.

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Fig. 10. Same as Fig. 8 but for the inclination of the magnetic field vector γ. The vertical black arrows indicates the original distribution employed to produce the synthetic Stokes profiles: it’s a double δ-Dirac centered at 0° and 180° because the synthetic profiles were obtained assuming that the magnetic field is longitudinal/vertical. Note however, that the effect of the noise induces the inversion code to retrieve highly inclined magnetic fields (color lines).

Fig. 11. Number of pixels (with respect to the total amount of pixels in each map) as a function of their signal-to-noise ratio. In solid lines it is showed the number of pixels when considering any of the polarization profiles (Stokes Q, U or V), whereas in dashed lines it is plotted the number of pixels when considering only the linear polarization (Stokes Q or U). Symbols indicate each of the three maps employed: crosses (map A; σs = 7.5 × 10−3), squares (map B; σs = 2.8 × 10−2), and diamonds (map C; σs = 2.8 × 10−2).
