Renormalization Group Analysis of Large Lepton Flavor Mixing and the Neutrino Mass

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The Superkamiokande experiment suggests large flavor mixing between $\nu_\mu$ and $\nu_\tau$. We show that both the second and third generation neutrino masses are larger than $O(0.1 \text{ eV})$ when the mixing angle receives significant corrections from the renormalization group equation (RGE). This implies that this mixing angle must be small at the right-handed neutrinos' decoupling scale when the results of the LSND experiments are correct and when the atmospheric neutrino anomaly is accounted for by $\nu_\mu$-$\nu_e$ oscillation.

§1. Introduction

Recent Superkamiokande data suggest large neutrino flavor mixing between $\nu_\mu$ and $\nu_\tau$. 1) According to this experimental result, there have been many theoretical attempts to explain why large flavor mixing is realized in the lepton sector. 2) One interesting approach concentrates on the effects of the renormalization group equation (RGE). The RGE enhances the neutrino flavor mixing in some situations. 3)−6) In this paper we analyze the RGE of the neutrino flavor mixing between $\nu_\mu$ and $\nu_e$ in the minimal supersymmetric standard model (MSSM) with right-handed neutrinos. Here we take the viewpoint that the smallness of neutrino masses is accounted for by the seesaw mechanism. 7) We consider the situation that $m_{\nu_\tau}$ is much smaller than $m_{\nu_\mu}$ and $m_{\nu_e}$, and we expect that the solar neutrino problem 8) is solved by the oscillation between $\nu_e$ and a sterile neutrino $\nu_s$. 9) This situation is the so-called “four light neutrinos scenario” 10), 11) with a mass spectrum satisfying $m_{\nu_s} \approx m_{\nu_e} \ll m_{\nu_\mu} \approx m_{\nu_\tau}$. The seesaw mechanism can also explain the results of the LSND, 12) which suggest small mixing between $\bar{\nu}_\mu$ and $\bar{\nu}_e$ with $m_{\nu_\mu}^2 - m_{\nu_e}^2 \sim 1 \text{ eV}^2$. 1) In this mass spectrum, we find that the mixing angle between $\nu_\mu$ and $\nu_e$ undergoes significant corrections due to the renormalization group equation (RGE), and the mixing angle at high energy must be small as long as the mixing at low energy is maximal. From the viewpoint of model building, we must find a fundamental theory which predicts a small mixing angle at high energy in models which have the same

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†) The confirmation of LSND results still awaits future experiments. Recent measurements in the KARMEN detector exclude part of the LSND allowed region. 13)
neutrino mass spectrum as that of the four light neutrinos scenario.

§2. The RGE effects of the neutrino flavor mixing

2.1. The RGEs of the Yukawa couplings

In this section we give the RGEs of the MSSM with right-handed neutrinos. The superpotential of the MSSM is given by

\[ W = y^u_{ij}Q_iH_u\bar{U}_j + y^d_{ij}Q_iH_d\bar{D}_j + y^e_{ij}L_iH_d\bar{E}_j + y^\nu_{ij}L_iH_u\bar{N}_j + \mu_H H_uH_d + \frac{1}{2}M_R^{ij}\bar{N}_i\bar{N}_j, \]  

(2.1)

where the indices indicate the generation number \((i, j = 2, 3)\). In this paper we neglect Yukawa couplings of the first generation, since we consider the case in which \(m_{\nu_e}\) is much smaller than \(m_{\nu_\mu}\) and \(m_{\nu_\tau}\). \(Q_i, L_i, \bar{U}_i, \bar{D}_i, \bar{E}_i, \bar{N}_i\) and \(H_u, H_d\) are quark doublet, lepton doublet, right-handed up-sector, right-handed down-sector, right-handed charged lepton, right-handed neutrino and Higgs fields, respectively. \(M_R^{ij}\) is the Majorana mass matrix of the right-handed neutrinos, which is symmetric under exchange of the generation indices \(i, j\). Here, \(\mu_H\) is the supersymmetric mass parameter of Higgs particles.

In this model the RGEs of Yukawa couplings are given by

\[ \frac{d}{dt}y^u = \frac{1}{(4\pi)^2} \times \left[ \text{tr} \left( 3y^u y^u + y^\nu y^\nu \right) + 3y^u y^u + y^d y^d - 4\pi \left( \frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{15}\alpha_1 \right) \right] y^u, \]

\[ \frac{d}{dt}y^d = \frac{1}{(4\pi)^2} \times \left[ \text{tr} \left( 3y^d y^d + y^e y^e \right) + 3y^d y^d + y^u y^u - 4\pi \left( \frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{7}{15}\alpha_1 \right) \right] y^d, \]

\[ \frac{d}{dt}y^e = \frac{1}{(4\pi)^2} \left[ \text{tr} \left( 3y^e y^e \right) + 3y^e y^e - 4\pi \left( 3\alpha_2 + \frac{9}{5}\alpha_1 \right) \right] y^e, \]

\[ \frac{d}{dt}y^\nu = \frac{1}{(4\pi)^2} \left[ \text{tr} \left( 3y^\nu y^\nu \right) + 3y^\nu y^\nu - 4\pi \left( 3\alpha_2 + \frac{3}{5}\alpha_1 \right) \right] y^\nu, \]

(2.2)

where \(t = \ln \mu\) and \(\mu\) is the renormalization point. These equations are meaningful in the energy region in which \(\mu > M_R\), where \(M_R\) denotes the energy scale of the Majorana mass.

Below the scale of \(M_R\), we should take the decoupling effects of heavy neutrinos into account. The effective theory is described without heavy neutrinos. The superpotential of Eq. (2.1) is modified as

\[ W = y^u_{ij}Q_iH_u\bar{U}_j + y^d_{ij}Q_iH_d\bar{D}_j + y^e_{ij}L_iH_d\bar{E}_j - \frac{1}{2}\kappa_{ij}\nu_i\nu_jH_uH_u. \]

(2.3)
Here the $\nu_i$ are the light modes of neutrinos which remain after integrating out the heavy ones. The coupling constant $\kappa_{ij}$ is defined as

$$\kappa_{ij} = (y^\nu M^{-1}_R y^\nu)^{ij}.$$  

(2.4)

It relates to the mass matrix of the light neutrinos as

$$m^\nu_{ij} = \left(\frac{v^u}{2}\right)^2 \kappa_{ij} = \left(\frac{v^2 \sin^2 \beta}{2}\right) \kappa_{ij},$$  

(2.5)

where

$$\tan \beta \equiv \frac{v_u}{v_d}, \quad v^2 = v^2_u + v^2_d,$$  

(2.6)

with $\langle H_u \rangle = v_u$ and $\langle H_d \rangle = v_d$. The value of $v$ is given by

$$v = M_Z \frac{\sin 2\theta_W}{2} \sqrt{\frac{\alpha}{\pi}} = 245.4 \text{ (GeV)},$$  

(2.7)

with $M_Z = 91.187 \text{ GeV}$, $\alpha = 127.9$, and $\sin^2 \theta_W = 0.23$. 14)

For $\mu < M_R$, the RGEs of the Yukawa couplings Eq. (2.2) become

$$\frac{d}{dt} y^u = \frac{1}{(4\pi)^2} \left[ \text{tr} \left( 3 y^u y^u + y^u y^u + y^d y^d - 4\pi \left( \frac{16}{3} \alpha_3 + 3 \alpha_2 + \frac{13}{15} \alpha_1 \right) \right) \right] y^u,$$

and

$$\frac{d}{dt} y^d = \frac{1}{(4\pi)^2} \left[ \text{tr} \left( 3 y^d y^d + y^e y^e + y^u y^u - 4\pi \left( \frac{16}{3} \alpha_3 + 3 \alpha_2 + \frac{7}{15} \alpha_1 \right) \right) \right] y^d,$$

(2.8)

and

$$\frac{d}{dt} \kappa = \frac{1}{8\pi^2} \left[ \left\{ \text{tr} \left( 3 y^u y^u \right) - 4\pi \left( 3 \alpha_2 + \frac{3}{5} \alpha_1 \right) \right\} \kappa + \frac{1}{2} \left\{ \left( y^e y^e \right) \kappa + \kappa \left( y^e y^e \right)^T \right\} \right].$$  

(2.9)

From Eq. (2.8), we can see that the RGEs of the quark and charged lepton do not include the neutrino Yukawa couplings, in contrast to the case for $\mu > M_R$, as in Eq. (2.2). Hence, below $M_R$, the running of the Yukawa couplings of the quark and charged lepton can be determined independently of that of the neutrinos.

2.2. The RGEs of neutrinos in the effective theory

From this point, we concentrate on the RGE effects below the scale of $M_R$, which are given by Eqs. (2.8) and (2.9). Since the RGEs of the Yukawa couplings for the quark and charged lepton of Eq. (2.8) can be solved without information concerning the neutrino sector, as mentioned above, the renormalization point dependences of $y_\mu$ and $y_\tau$ are completely determined by the boundary conditions of the RGEs,
which we take as the masses of quark and charged lepton, the Cabibbo-Kobayashi-Maskawa matrix,\textsuperscript{15} and $\tan \beta$ at the weak scale. Then we have only to concentrate on Eq. (2.9). Here we neglect CP phases in the flavor mixing matrices of the quark and the lepton sector for simplicity.

For convenience, we consider three independent parameters, $\kappa_r \equiv \kappa_{22}/\kappa_{33}$, $\sin^2 2\theta_{23}$ and $\delta \kappa^2$, instead of the $\kappa_{ij}$ [ $\kappa_{22}$, $\kappa_{33}$ and $\kappa_{23}$ ( $=\kappa_{32}$)]. Here, $\sin \theta_{23}$ and $\delta \kappa^2$ are determined from the $\kappa_{ij}$ by the equations

$$
\kappa = \begin{pmatrix} \cos \theta_{23} & \sin \theta_{23} \\ -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \kappa_2 & 0 \\ 0 & \kappa_3 \end{pmatrix} \begin{pmatrix} \cos \theta_{23} & -\sin \theta_{23} \\ \sin \theta_{23} & \cos \theta_{23} \end{pmatrix},
$$

(2.10)

$$
\delta \kappa^2 \equiv \kappa_3^2 - \kappa_2^2,
$$

(2.11)

where

$$
\kappa_3 = \frac{\sqrt{\delta \kappa^2}}{2} \left( \sqrt{\alpha} + \frac{1}{\sqrt{\alpha}} \right), \quad \kappa_2 = \frac{\sqrt{\delta \kappa^2}}{2} \left( \sqrt{\alpha} - \frac{1}{\sqrt{\alpha}} \right),
$$

(2.12)

with

$$
\alpha \equiv \left| \frac{1 + \kappa_r}{1 - \kappa_r} \cos 2\theta_{23} \right|.
$$

(2.13)

By using this relation, the RGE of Eq. (2.9) can be rewritten into the following three equations:**

$$
\frac{d}{dt} \kappa_r = -\frac{1}{8\pi^2} (y_r^2 - y_\mu^2) \kappa_r,
$$

(2.14)

$$
\frac{d}{dt} \sin^2 2\theta_{23} = -\frac{1}{8\pi^2} \sin^2 2\theta_{23}(1 - \sin^2 2\theta_{23})(y_r^2 - y_\mu^2) \frac{1 + \kappa_r}{1 - \kappa_r},
$$

(2.15)

$$
\frac{d}{dt} \delta \kappa^2 = \frac{1}{8\pi^2} \left[ 2 \left\{ \text{tr} \left( 3y_u^u y_u^u \right) - 4\pi \left( 3\alpha_2 + \frac{3}{5}\alpha_1 \right) \right\} + y_r^2 + y_\mu^2 + (y_r^2 - y_\mu^2) \left( \frac{1 + \kappa_r^2}{1 - \kappa_r^2} - \frac{1}{2} \frac{1 + \kappa_r}{1 - \kappa_r} \sin^2 2\theta_{23} \right) \right] \delta \kappa^2.
$$

(2.16)

Both $\sin^2 2\theta_{23}$ and $\delta \kappa^2$ are directly related to the observed quantities in neutrino oscillation experiments.\*** The mass squared difference can be written as $\delta m^2 = v^4 \sin^4 \beta \delta \kappa^2/4$ by using Eq. (2.5).

As shown above, $y_\mu(\mu)$ and $y_r(\mu)$ are determined without knowing the neutrino Yukawa coupling. Thus we can obtain the values of $\sin^2 2\theta_{23}$ and $\kappa_r$ at the weak scale by using only Eqs. (2.14) and (2.15). We analyze the RGEs of Eqs. (2.14) and (2.15) by inputting various values of $\sin \theta_{23}$ and $\kappa_r$ for the initial conditions at $\mu = M_R$.\textsuperscript{13}

\textsuperscript{13} We use $m_t = 174.5$ GeV, $m_c = 0.657$ GeV, $m_b = 3.02$ GeV, $m_s = 9.935 \times 10^{-2}$ GeV, $m_q = 1.746$ GeV, and $m_\mu = 1.0273 \times 10^{-3}$ GeV at $\mu = M_Z$.\textsuperscript{14}

\textsuperscript{**} Equation (2.15) was first derived by Babu, Leung and Pantaleone in Ref. 3).

\textsuperscript{***} Since we use a diagonal base of the charged lepton, $\sin^2 2\theta_{23}$ in this paper is an observable quantity.

\textsuperscript{13} As shown below, we do not need to calculate Eq. (2.16) in our analysis.
2.3. Numerical results for the RGEs

Now let us give the numerical results for the RGEs. Figure 1 plots the energy dependences of the values of \([y_\tau^2 - y_\mu^2]\), which are the coefficients of the RGEs of \(\kappa_r\) in Eq. (2.14) and \(\sin^2 \theta_{23}\) in Eq. (2.15). The four lines correspond to the various values of \(\tan \beta\), which we take to be 5, 20, 35, and 50. The values of \([y_\tau^2 - y_\mu^2]\) do not experience significant RGE corrections when \(\tan \beta\) is small.

Figure 2 plots the energy dependence of \(\kappa_r\) for \(\tan \beta = 50\) with various initial conditions at \(M_R = 10^{14}\) GeV. As seen here, \(\kappa_r\) is a monotonically decreasing...
function of the energy. If we take a small value of tan β, the slope tends to be flat, because the value of $[y_T^2 - y_B^2]$ in Eq. (2.14) decreases as shown in Fig. 1.

Figure 3 plots the energy dependence of the mixing angle with the same values of $\kappa_r(M_R)$ as Fig. 2. There are three curves corresponding to the values of $\kappa_r(M_R)$, and all of them have the same boundary condition, $\sin^2 2\theta_{23}(M_R) = 0.1$. From these figures we can see that the mixing angle at the weak scale changes with $\kappa_r(M_R)$. By comparing Fig. 3 with Fig. 2, we can easily understand that this difference is due to the factor $[(1 + \kappa_r)/(1 - \kappa_r)]$ on the right-hand side of the RGE of Eq. (2.15). In the case $\kappa_r(M_R) = 0.8$, where $\kappa_r$ does not exceed 1 at any energy scale, the mixing angle does not experience significant RGE corrections. This is because the right-hand side of Eq. (2.15) does not become so large as to enhance the mixing angle. On the other hand, in the case $\kappa_r(M_R) = 0.9$, $\kappa_r$ exceeds 1 near the weak scale, which results in significant enhancement of the mixing angle by the factor $[(1 + \kappa_r)/(1 - \kappa_r)]$. Finally, in the case $\kappa_r(M_R) = 0.99$, $\kappa_r$ exceeds 1 above the weak scale. Then, the mixing angle becomes maximal at high energy. However, beyond that point, it decreases rapidly, because the sign of $[(1 + \kappa_r)/(1 - \kappa_r)]$ changes.

Figure 4 displays the contour plot of the mixing angle $\sin^2 2\theta_{23}$ at the weak scale. This plot is obtained by solving Eq. (2.15) with the various values of $\kappa_r(M_R)$ (horizontal axis) and $\sin^2 2\theta_{23}(M_R)$ (vertical axis). Here $\kappa_c$ in Fig. 4 is the value at $M_R$ that results in $\kappa_r(weak) = 1$. In the parameter region $\kappa_r(M_R) < 0.6$, there are no significant RGE corrections, and there the mixing angle does not change drastically. The energy dependence of the mixing angle in this case is similar to that represented by the solid lines in Fig. 3. Next, in the case $\kappa_r(M_R) \simeq \kappa_c$, which implies $\kappa_r(weak) \simeq 1$, the mixing angle at the weak scale is strongly enhanced near the weak scale, as shown by the dashed lines in Fig. 3. Then the mixing angle at the weak scale realizes a maximum, independently of the mixing angle at $M_R$. Finally, in the case $\kappa_r(M_R) > \kappa_c$, the mixing angle at the weak scale becomes small even if
large mixing is realized at $M_R$, where the energy dependence of the mixing angle is similar to that represented by the dotted line in Fig. 3. From these arguments we can easily see that there is large enhancement of the mixing angle from the RGE around $\kappa_c \simeq 1$.

Figure 5 displays the contour plot of the heaviest neutrino mass $m_3$ at the weak scale. The horizontal and vertical axes here are the same as those of Fig. 4. We determine the masses of the neutrino by substituting the parameters in Eq. (2.12) with the result of Fig. 4 and the experimental value $\delta m^2_{23} \simeq 1.3 \times 10^{-3}$ eV$^2$. Since we use the experimental value of $\delta m^2_{23}$ instead of evaluating the RGE of Eq. (2.16), we can determine the masses without any additional input parameters. As $m_3$ becomes large, the region of $\kappa_r(M_R)$ is limited around $\kappa_c$. By comparing with Fig. 4, it is found that the region where the mixing angle at the weak scale is always larger than 0.9 despite the small mixing at the $M_R$ scale corresponds to the region where the heaviest mass $m_3$ is larger than $O(0.1 \text{ eV})$.

Figures 6 and 7 correspond to Figs. 4 and 5 with the value of $\tan \beta = 35$. Compared with the case $\tan \beta = 50$, the values of $\kappa_c$ are just shifted to the right in the case $\tan \beta = 35$. Around $\kappa_r(M_R) \simeq \kappa_c$, the value of $m_3$ is slightly larger than that in the case $\tan \beta = 50$ at the same value of $\sin^2 2\theta_{23}$. In general, $\kappa_c$ approaches 1 as $\tan \beta$ becomes smaller, and the value of $m_3$ around $\kappa_r(M_R) \simeq \kappa_c$ becomes larger. When $m_3$ is $O(1 \text{ eV})$, the maximal enhancement of the mixing angle is derived by the RGE even in the case of small $\tan \beta$. The value of $m_3$ around $\kappa_c$ becomes larger as the value of $\tan \beta$ becomes smaller. In the case $\tan \beta = 5$, neutrino masses satisfying $m_3 \simeq m_2 \simeq 2 \text{ (eV) }$ result in maximal mixing in the weak scale from the small initial mixing of $\sin^2 2\theta_{23}(M_R) \simeq 0.04$. 

Fig. 4. Contour plot of the mixing angle $\sin^2 2\theta_{23}$ at the weak scale with $\tan \beta = 50$ and $M_R = 10^{14}$ GeV. The horizontal axis corresponds to the value of $\kappa_r$ at the $\mu = M_R$ scale, and the vertical axis corresponds to the mixing angle at $\mu = M_R$. The values of $\sin^2 2\theta_{23}$ at the weak scale are determined by inputting the initial values of $\sin^2 2\theta_{23}(M_R)$ and $\kappa_r(M_R)$. Around $\kappa_c$, the RGE makes the weak scale mixing large for any initial conditions.
Fig. 5. Contour plot of the heaviest neutrino mass $m_3$ at the weak scale, with $\tan \beta = 50$ and $M_R = 10^{14}$ GeV. The horizontal and vertical axes are the same as in Fig. 4. The values of $m_3$ at the weak scale are determined by inputting the initial values of $\sin^2 2\theta_{23}(M_R)$ and $\kappa_r(M_R)$, and the experimental value of $\delta m_{23}^2 = 1.3 \times 10^{-3}$ eV$^2$. As the mass becomes large, the region of $\kappa_r(M_R)$ is limited around $\kappa_c$.

Fig. 6. Contour plot of the mixing angle $\sin^2 2\theta_{23}$ at the weak scale with $\tan \beta = 35$ and $M_R = 10^{14}$ GeV. In this case, $\kappa_c$ is larger than that of Fig. 4.

We stress here that the large enhancement factor at the weak scale induced by $[(1 + \kappa_r)/(1 - \kappa_r)]$ in Eq. (2.15) is not a fine-tuning when $m_2 \simeq m_3$. In fact, Eqs. (2.12) and (2.13) suggest $\kappa_2 \simeq \kappa_3$ when $\alpha \gg 1$, which implies that the factor $[(1 + \kappa_r)/(1 - \kappa_r)]$ must be very large at the weak scale. It is worth noting that a large enhancement of the mixing angle can be derived by the RGE even in the case of small $\tan \beta$, and in this case the heaviest mass $m_3$ is of order 1 eV.
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Fig. 7. Contour plot of the heaviest neutrino mass at the weak scale with \( \tan \beta = 35 \) and \( \mathcal{M}_R = 10^{14} \) GeV. The region where \( m_3 \) is larger than \( O(0.1 \text{ eV}) \) corresponds to the region in which the maximal mixing angle is realized at low energy.

§3. Summary and discussion

In this paper we analyzed the RGE effects of the neutrino flavor mixing between \( \nu_\mu \) and \( \nu_\tau \) in the MSSM with right-handed neutrinos. We found that \( m_2 \) and \( m_3 \) are larger than \( O(0.1 \text{ eV}) \) when the mixing angle between \( \nu_\mu \) and \( \nu_\tau \) undergoes significant corrections due to the RGE. For the purpose of solving not only the atmospheric and the solar neutrino problems but also accounting for the LSND results with a neutrino mass spectrum characterized by \( m_{\nu_e} \approx m_{\nu_\mu} \ll m_{\nu_\tau} \approx m_{\nu_\mu} \) \( (\sim 0.1 \sim 1 \text{ eV}) \), this scenario is a candidate to explain the large mixing origin.

From the viewpoint of model building, we must find a fundamental theory which predicts a small mixing angle at the high energy scale in models which have the same neutrino mass spectrum as that in the four light neutrinos scenario.

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References

1) Superkamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81 (1998), 1562.
   T. Kajita, talk at the International Conference Neutrino '98, Takayama, Japan, June 1998.
2) See, for example, M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Rev. D57 (1998), 4429.
N. Haba, N. Okamura and M. Sugiura

K. S. Babu and S. M. Barr, Phys. Lett. B381 (1996), 202.
S. M. Barr, Phys. Rev. D55 (1997), 1659.
B. Brahmachari and R. N. Mohapatra, Phys. Rev. D58 (1998), 15003.
C. H. Albright and S. M. Barr, Phys. Rev. D58 (1998), 13002.
J. Sato and T. Yanagida, Phys. Lett. B430 (1998), 127.
S. L. Glashow, Phys. Lett. B256 (1991), 255.
E. J. Chun, C. W. Kim and U. W. Lee, Phys. Rev. D58 (1998), 93003.

Y. Chikira, N. Haba and Y. Mimura, hep-ph/9808254.
E. Ma, hep-ph/9807386.
M. Tanimoto, hep-ph/9807283.
Y. Nomura and T. Yanagida, hep-ph/9807325.
N. Haba, hep-ph/9807552.
S. Davidson and S. F. King, hep-ph/9808296.
R. N. Mohapatra and S. Nussinov, hep-ph/9808301.
Y. Grossman, Y. Nir and Y. Shadmi, hep-ph/9808355.
R. N. Mohapatra and S. Nussinov, hep-ph/9809415.

3) K. S. Babu, C. N. Leung and J. Pantaleone, Phys. Lett. B319 (1993), 191.

4) P. H. Chankowski and Z. Pluciennik, Phys. Lett. B316 (1993), 312.

5) M. Tanimoto, Phys. Lett. B360 (1995), 41.

6) N. Haba and T. Matsuoka, Prog. Theor. Phys. 99 (1998), 831.

7) T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, ed. O. Sawada and A. Sugamoto (KEK, report 79-18, 1979), p. 95.
M. Gell-Mann, P. Ramond and S. Slansky, in Supergravity, ed. P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315.
R. Mohapatra and S. Senjanović, Phys. Rev. Lett. 44 (1980), 912.

8) GALLEX Collaboration, Phys. Lett. B388 (1996), 384.

SAGE Collaboration, Phys. Lett. B328 (1994), 234.

Homestake Collaboration, Nucl. Phys. B38 (Proc. Suppl.) (1995), 47.

Kamiokande Collaboration, Nucl. Phys. B38 (Proc. Suppl.) (1995), 55.

Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81 (1998), 1158.

9) See, for example, N. Hata and P. Langacker, Phys. Rev. D50 (1994), 632; D56 (1997), 6107.

J. N. Bahcall, P. I. Krastev and A. Y. Smirnov, Phys. Rev. D58 (1998), 96016.

10) See, for example, D. O. Caldwell and R. N. Mohapatra, Phys. Rev. D48 (1993), 3259.

J. T. Peltoniemi and J. W. F. Valle, Nucl. Phys. B406 (1993), 409.

R. Foot and R. R. Volkas, Phys. Rev D52 (1995), 6595.

E. Ma and P. Roy, Phys. Rev. D52 (1995), 4780.

J. J. Gomez-Cadenas and M. C. Gonzales-Garcia, Z. Phys. C71 (1996), 443.

S. M. Bilenky, C. Giunti and W. Grimus, Proc. of Neutrino 96, Helsinki, June 1996, ed. K. Enqvist et al. (World Scientific, 1997).

S. M. Bilenky, C. Giunti and W. Grimus, Eur. Phys. J. C1 (1998), 247.

N. Okada and O. Yasuda, Int. J. Mod. Phys. A12 (1997), 3669.

S. C. Gibbons, R. N. Mohapatra, S. Nandi and A. Raychaudhuri, Phys. Lett. B430 (1998), 296.

S. M. Bilenky, C. Giunti and W. Grimus, hep-ph/9805387.
V. Barger, T. J. Weiler and K. Whisnant, Phys. Lett. B427 (1998), 97.
V. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, Phys. Rev. D58 (1998), 3016.
E. M. Lipmanov, hep-ph/9806442.
V. Barger, hep-ph/9808353.
S. M. Bilenky, C. Giunti and W. Grimus, hep-ph/9809368.
V. Barger, hep-ph/9809466.
R. N. Mohapatra, hep-ph/9809415.

11) B. Brahmachari and R. N. Mohapatra, hep-ph/9805429.

12) LSND Collaboration, C. Athanassopoulos et al., Phys. Rev. Lett. 75 (1995), 2650; 77 (1996), 3082; 81 (1998), 1774.
13) The LSND results will be tested by the KARMEN experiment, talk by B. Armbruster at 33rd Rencontres de Moriond: *Electroweak Interactions and Unified Theories*, Les Arcs, France, March 1998, and talk by B. Zeitnitz at the International conference *Neutrino '98*, Takayama, Japan, June 1998, and also by the BooNE experiment, E. Church et al., nucl-ex/9706011.

14) C. Caso et al., Eur. Phys. J. C3 (1998), 1.

15) N. Cabibbo, Phys. Rev. Lett. 10 (1964), 531.
   M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973), 652.