Weak localization correction to the F/S interface resistance

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I. INTRODUCTION

Novel phenomena in mesoscopic systems have been the subject of intense theoretical and experimental interest for many years.\textsuperscript{3} Observed effects such as weak localization and universal conductance fluctuations are due to the quantum interference of electrons at low temperatures. An important branch of mesoscopic physics is that of hybrid nanostructures where the influence of a superconductor (S) on a phase coherent normal (N) region is studied. At low temperatures, the role of Andreev reflection is important, whereby particles in the N region with excitation energies $\varepsilon$ lower than the superconducting gap energy $\Delta$ are reflected from the N/S interface as holes. Studies of subgap transport have shown that superconducting condensate penetration into the N region, known as the proximity effect, may substantially change the resistance of an N/S circuit.

Lately, improvements in the microprocessing of metals has led to the fabrication of nanostructures combining ferromagnetic (F) and S materials.\textsuperscript{4} The presence of a large exchange field, $\varepsilon_{ex} \gg \Delta$, suppresses electron-hole correlations in ferromagnets so that the role of the proximity effect is reduced. However a separate mechanism has been predicted\textsuperscript{5} to produce a resistance increase in a circuit consisting of a mono-domain F wire connected to an S electrode instead of an N electrode. This is a robust, classical effect which originates from the need to match a spin-polarized current in the ferromagnet to a spinless current in the superconductor.

The present work analyses the weak localization correction to the contact F/S resistance. In a semiclassical language, weak localization arises from an enhanced backscattering caused by the quantum interference of pairs of coherent quasi-particles.\textsuperscript{6} Constructive interference of a quasi-particle paired with its time reversed partner is destroyed by a magnetic field so we assume that the width of the F wire, $L_{\perp}$, is small, $L_{\perp} \lesssim 100nm$, in order to limit the influence of the intrinsic magnetic field. Weak localization is affected by the boundary conditions of the system. Particles that escape into an N electrode suffer dephasing which reduces the return probability. However, multiple Andreev reflections from an S electrode may enable a particle to return coherently. For a polydomain F wire, with no net average polarization, the return probability of an F/S system is greater than that of an F/N system.\textsuperscript{7}

In general, the spin channels have different conductivities, $\sigma_\alpha = e^2 \nu_\alpha D_\alpha$, where $\nu_\alpha$ and $D_\alpha$ are the density of states and the classical diffusion coefficient for electrons in the spin state $\alpha = (\uparrow, \downarrow)$. The degree of spin polarization, $\zeta$, is defined as $\zeta = (\sigma_\uparrow - \sigma_\downarrow) / (\sigma_\uparrow + \sigma_\downarrow)$ and we calculate the return probability in an F/S system with an arbitrary degree of polarization, $0 \leq \zeta \leq 1$. In addition we introduce a finite spin relaxation length, $L_\alpha$, shorter than the length, $L$, of the F wire, $L_\alpha \ll L$. As the spin polarization increases, the majority spin channel experiences a further increase in return probability (as compared to the F/N case), whereas the minority spin channel suffers a reduction. Majority carriers at the superconducting interface cannot find minority carrier states to Andreev reflect into and are more likely to be reflected normally whereas minority carriers at the interface have an enhanced probability to escape. For $\zeta \rightarrow 1$ the reservoir appears to be totally insulating to majority spins but normal as far as minority spins are concerned.

After calculating the correction to the classical diffusion coefficient that is due to the enhanced return probability, we find the corresponding change in the spin polarized particle distribution and we determine the weak localization correction to the classical resistance. Generally this is a sum of a bulk term that is independent of the state of the reservoir and a contact term, $\delta_r^{FS(FN)}$, that depends on whether the reservoir is in the superconducting or the normal state. We find that $\delta_r^{FS} > \delta_r^{FN}$, so that a change in the state of the reservoir from N to S results in an increase of the weak localization contribution.
to the resistance,

$$\delta r_c^{FS} - \delta r_c^{FN} \approx \gamma \left( \frac{8e^2}{\pi} \right) \frac{(\sigma_\uparrow + \sigma_\downarrow)^2}{\sigma_\uparrow \sigma_\downarrow} \left( \frac{R_0}{L_s} \right)^2 L_\perp^2,$$

(1)

where $R_0 = 1/(\sigma_\uparrow + \sigma_\downarrow) L_\perp^{d-2}$ is the resistance of a cube of length equal to the width of the wire $L_\perp$. The combination $R_0 L_s / L_\perp$ corresponds to the resistance of a piece of ferromagnet of length $L_s$. We evaluate the prefactor, $\gamma$, numerically and find that it is almost constant, $\gamma \approx 1/2$, for all experimentally relevant values of spin polarization.

### II. CLASSICAL RESISTANCE

We begin by briefly describing the approach of Refs. [3,4] which enables one to calculate the classical contact resistance of an F/S interface and we refer the reader to those papers for further details. We consider a single domain, disordered ferromagnetic wire of length $L$ as depicted in Fig. 1. The coordinate $x$ is used to describe the position along the wire and throughout the paper we consider there to be a normal reservoir at the left hand side, $x = -L$, whereas the reservoir at the right hand side, $x = 0$, is either N or S. The resistance of the disordered F wire is determined by solving diffusion equations for the isotropic part of the electron distribution function $n_0(\varepsilon, x) = \int d\varepsilon_0 n_0(p, x)$. Using electron-hole symmetry, it is possible to consider only the symmetrized function $N_0(\varepsilon, x) = [n_0(\varepsilon, x) + n_0(-\varepsilon, x)]/2$, where $\varepsilon$ is the energy determined with respect to the chemical potential in the S (N) reservoir. The current density in any given spin channel is related to the spatial derivative of the distribution function as

$$j_\alpha = \sigma_\alpha \int_0^\infty d\varepsilon \partial_x N_\alpha(\varepsilon, x).$$

(2)

The distribution function obeys a differential equation describing diffusion in the ferromagnetic wire,

$$D_\alpha \partial_x^2 N_\alpha(\varepsilon, x) = w_{\uparrow \downarrow} \mu_\uparrow [N_\downarrow(\varepsilon, x) - N_\uparrow(\varepsilon, x)],$$

(3)

where $\mu = (\downarrow, \uparrow)$ for $\alpha = (\uparrow, \downarrow)$. This equation may be expressed conveniently as

$$\partial^2_x [\sigma_\uparrow N_\downarrow + \sigma_\downarrow N_\uparrow] = 0,$$

(4)

$$\left( \partial^2_x - L_s^{-2} \right) [N_\uparrow - N_\downarrow] = 0.$$  

(5)

Spin relaxation, which may arise from spin-orbit scattering or spin-flip scattering at defects, is described by the right-hand side of Eq. (3). One may define an effective spin relaxation rate $L_s$, as $L_s^{-2} = w_{\uparrow \downarrow} \mu_\uparrow / D_\uparrow + \mu_\downarrow / D_\downarrow$ which accounts for the relaxation of the difference between the spin channels. In a similar way, the spin relaxation length in an individual channel, $L_{\alpha}$, is defined as $L_{\alpha}^2 = w_{\uparrow \downarrow} \mu_\uparrow / D_\alpha$ such that $L_s^{-2} = L_\uparrow^{-2} + L_\downarrow^{-2}$. An equivalent expression is $L_\alpha^2 = L_s^2 (\sigma_\alpha + \sigma_\uparrow) / \sigma_\uparrow$.

The above equations should be complemented by four boundary conditions, two at each end of the ferromagnetic wire. The boundary conditions at the left hand side of the wire are given by the equilibrium distribution of electrons in the normal reservoir,

$$N_\alpha(\varepsilon, -L) = \frac{1}{2} \left[ n_T(\varepsilon - eV) + n_T(\varepsilon - eV) \right],$$

(6)

such that the distribution of up and down spins is equal. The boundary conditions at the right hand side of the ferromagnetic wire at the superconducting reservoir have been discussed in detail in Refs. [3,4]. For quasiparticles with energies below the superconducting gap they may be written as

$$\sigma_\uparrow \partial_x N_\uparrow|_0 = \sigma_\downarrow \partial_x N_\downarrow|_0,$$

(7)

$$N_\uparrow(\varepsilon, 0) + N_\downarrow(\varepsilon, 0) = [n_T(\varepsilon) + n_T(-\varepsilon)].$$

(8)

The first of these, Eq. (7), describes Andreev reflection such that the spin-up current must equal the spin-down current and the second, Eq. (8), states that the sum of the distributions is equal to the equilibrium value. On solving the above differential equations with the appropriate boundary conditions it is possible to calculate the current density in the wire. In particular, the spatial derivative of the distribution is

$$\partial_x N_\alpha(\varepsilon, x) = \frac{(1 - 2N_\downarrow)}{2\Gamma L} \times \left[ 1 + \frac{(\sigma_\uparrow - \sigma_\downarrow)}{2\sigma_\alpha} \cosh[(x + L)/L_s] \right],$$

(9)

where

$$\Gamma = 1 + \frac{(\sigma_\uparrow - \sigma_\downarrow)^2}{4\sigma_\alpha \sigma_\downarrow} \frac{L_s}{L} \tanh(L/L_s),$$

(10)

and $N_L = N_\alpha(\varepsilon, -L)$. The effect of the superconducting reservoir, for strong spin relaxation $L \gg L_s$, is expressed in terms of a contact resistance $r_c^{FS}$ so that the total classical resistance, $R_{cl}$, may be written as a sum of resistances in series

$$R_{cl} = \frac{L}{L_\perp} R_0 + r_c^{FS},$$

(11)

$$r_c^{FS} \approx \frac{L_s}{L_\perp} \frac{\sigma_\uparrow - \sigma_\downarrow}{4\sigma_\alpha \sigma_\downarrow}.$$

(12)

For a normal reservoir on the right hand side, the boundary conditions are similar to those in Eq. (6), namely $N_L(\varepsilon, 0) = [n_T(\varepsilon) + n_T(-\varepsilon)]/2$. In this case, there is no contact resistance. A fall in temperature, precipitating a change in state of the reservoir from N to S, would therefore result in a resistance increase of the circuit. This is a robust classical effect which originates from the need to match a spin-polarized current in the ferromagnet to a spinless current in the superconductor. It is specific to mono-domain wires since in poly-domain wires the transport properties of spin-up and spin-down particles do not differ and, on the average, the current is not spin-polarized.
III. WEAK LOCALIZATION CORRECTION TO THE CONTACT RESISTANCE

The weak localization correction to the classical diffusion coefficient, δD\textsubscript{α}, is related to a two particle Green's function known as the Cooperon propagator. The Cooperon consists of a quasi-particle following a diffusive path that interferes with a quasi-particle traversing the same path in the opposite direction. When the particles follow a closed path, the interference results in an enhanced return probability. In a ferromagnet, singlet and S\textsubscript{z} = 0 triplet Cooperons are suppressed since the Fermi momentum for spin-up and spin-down particles is different, producing a difference in the phase accumulated by the particles along the path. However, there is no such suppression of the phase correlation in the triplet channel where same spin particles are paired. Thus we assume that the return probability is given by the triplet Cooperon propagator, C\textsubscript{αα}(x,x'), for equal spatial coordinates, which obeys a diffusion equation in the disordered ferromagnet,

\[
\frac{\delta D\textsubscript{α}(x)}{D\textsubscript{α}} \approx -\frac{8}{\pi L_\perp} C\textsubscript{αα}(x,x),
\]

which provides new equations for the correction to the distribution function, δN\textsubscript{α}, in terms of the correction to the classical diffusion coefficient, δD\textsubscript{α}, are

\[
\delta N\textsubscript{↑}(x) = 0, \quad \delta N\textsubscript{↓}(x) = 0.
\]

We stress that the existence of a Cooperon is not a result of the proximity effect as we do not consider triplet pairing induced in the ferromagnet by the presence of the superconductor. Nevertheless, weak localization is affected by the boundary conditions of the system. A schematic of the change in the boundary conditions of the Cooperon at the F/S interface as compared to the F/N interface is shown in Fig. 2. A process which pairs two spin-up quasi-particles is depicted in Fig. 2a. When the reservoir is in the N state, particles escaping into it suffer dephasing, thus destroying the Cooperon. At the left hand reservoir (x = -L), which is in the N state, we therefore have

\[
C\textsubscript{↑↑}(-L,x') = C\textsubscript{↓↓}(-L,x') = 0.
\]

When the right hand reservoir (x = 0) is in the S state we apply

\[
\sigma_\uparrow \partial_x C\textsubscript{↑↑}(x,x')|_{x=0} = -\sigma_\uparrow e^{i\chi} C\textsubscript{↓↓}(x,x')|_{x=0},
\]

where \( \chi \) is a phase accumulated on reflection from the superconductor. These boundary conditions, similar to those for the density Eqs. (18), account for Andreev reflection. Multiple Andreev reflections at an S electrode may enable a particle to return coherently to the original place in the F wire with the same spin polarization. Fig. 2b shows a process whereby a pair of spin-up quasi-particles are Andreev reflected as spin-down quasi-holes which are subsequently Andreev reflected as spin-up quasi-particles. The calculation of δD\textsubscript{α} by solving the Cooperon diffusion equation with the above boundary conditions is described in Appendix A.

The weak localization correction to the diffusion coefficient, δD\textsubscript{α}, leads to a correction to the current density, \( \delta j_{\uparrow} = e^2 \nu_\alpha \delta D\textsubscript{α} \int_0^\infty dx \partial_x N_\alpha(\varepsilon,x) \). However, δD\textsubscript{α} also produces a correction to the particle distribution function. We take this into account perturbatively by assuming that the correction, δD\textsubscript{α}, to the classical diffusion coefficient, D\textsubscript{α}, is small and that there is a corresponding small correction, δN\textsubscript{α}, to the classical particle distribution, N\textsubscript{α}. Expressions for the total diffusion coefficient, D\textsubscript{α} = D\textsubscript{α0} + δD\textsubscript{α}, and the total particle distribution, N\textsubscript{α} = N\textsubscript{α0} + δN\textsubscript{α}, are substituted into the previous differential equations describing diffusion in the ferromagnet, Eqs. (6), and the boundary conditions, Eqs. (18), to provide new equations for δN\textsubscript{α}. Hence the differential equations for the correction to the distribution function, δN\textsubscript{α}, in terms of the correction to the classical diffusion coefficient, δD\textsubscript{α}, are

\[
\partial_x^2 \left[ \sigma_\uparrow \delta N\textsubscript{↑} + \sigma_\downarrow \delta N\textsubscript{↓} \right] = -\partial_x \left[ \delta J\textsubscript{↑} + \delta J\textsubscript{↓} \right],
\]

\[
\left( \partial_x^2 - L^2 \right) \left[ \delta N\textsubscript{↑} - \delta N\textsubscript{↓} \right] = -\partial_x \left[ \frac{\delta J\textsubscript{↑}}{\sigma_\uparrow} - \frac{\delta J\textsubscript{↓}}{\sigma_\downarrow} \right],
\]

where δJ\textsubscript{α}(x) = e^2 ν_α δD\textsubscript{α}(x) ∂_x N\textsubscript{α0} (x). The boundary conditions at the ferromagnetic reservoir on the left hand side are

\[
\delta N\textsubscript{↑}(\varepsilon,-L) = \delta N\textsubscript{↓}(\varepsilon,-L) = 0.
\]

whereas the boundary conditions at the superconducting reservoir on the right hand side are

\[
\sigma_\uparrow \partial_\varepsilon \delta N\textsubscript{↑}|_0 - \sigma_\downarrow \partial_\varepsilon \delta N\textsubscript{↓}|_0 = \delta J\textsubscript{↑} - \delta J\textsubscript{↓},
\]

\[
\delta N\textsubscript{↑}(\varepsilon,0) + \delta N\textsubscript{↓}(\varepsilon,0) = 0.
\]

General solutions to the differential equations, Eqs. (18,19), are

\[
\sigma_\uparrow \delta N\textsubscript{↑} + \sigma_\downarrow \delta N\textsubscript{↓} = U x + V - \int^y_x \left[ \delta J\textsubscript{↑}(y) + \delta J\textsubscript{↓}(y) \right] dy,
\]

and

\[
\delta N\textsubscript{↑} - \delta N\textsubscript{↓} = W e^{-x/L_s} + Y e^{x/L_s} - \int^y_x \left[ \frac{\delta J\textsubscript{↑}(y)}{\sigma_\uparrow} - \frac{\delta J\textsubscript{↓}(y)}{\sigma_\downarrow} \right] \cosh \left( \frac{x-y}{L_s} \right) dy.
\]

where U,V,W, and Y are unknown factors to be determined by the four boundary conditions. It is thus possible to evaluate δN\textsubscript{α} in terms of the correction to the
diffusion coefficient, $\delta D_\alpha$, and the classical part of the
distribution, $N^0_\alpha$.

Taking into account both $\delta D_\alpha$ and $\delta N_\alpha$, the weak
localization correction to the current density summed over
both spin channels is

$$
\delta j = \int_0^\infty d\varepsilon \left[ \delta J_\uparrow + \delta J_\downarrow + \sigma_\uparrow \partial_x \delta N_\uparrow + \sigma_\downarrow \partial_x \delta N_\downarrow \right].
$$

(25)

This is determined by the factor $U$ in the general solution
Eq. (24). Using the boundary conditions, Eqs. (20,21,22),
to evaluate $U$, the weak localization correction to the cur-
rent density may be expressed as

$$
\delta j = \int_0^\infty \frac{dz}{\Gamma} \int_{-L}^0 \frac{dx}{L} \left\{ \delta J_\uparrow + \delta J_\downarrow \right\} + \\
- \frac{(\sigma_\uparrow - \sigma_\downarrow)}{2} \left[ \frac{\delta J_\uparrow}{\sigma_\uparrow} - \frac{\delta J_\downarrow}{\sigma_\downarrow} \right] \left[ \cosh \left( (x + L)/L_s \right) \right].
$$

(26)

The term in brackets contains spatial dependences
arising from the cosh term and from $\delta J_\alpha(x) = e^2\nu_\alpha \delta D_\alpha(x) \partial_x N^0_\alpha(x)$, where $\partial_x N^0_\alpha(x)$ depends on
\cosh[(x + L)/L_s], Eq. (2), and $\delta D_\alpha(x)$ is given in Ap-
pendix A. Taking these terms into account, it is straight-
forward to perform the integration with respect to the
coordinate $x$. The final step is to evaluate $\delta D_\alpha(x)$ nu-
merically as detailed in Appendix A. For a long wire,
we express the weak localization correction as a sum of
resistances,

$$
\delta R = \delta R^B + \delta r^{FS}
$$

(27)

where $\delta R^B$ is the bulk contribution, $\delta R^B = (8e^2/\pi)R^2_\alpha L(L_\uparrow + L_\downarrow)/L^2_\alpha$ and $\delta r^{FS}$ is a contact term,

$$
\delta r^{FS} \approx -(1 - \gamma) \left( \frac{8e^2}{\pi} \right) \frac{(\sigma_\uparrow + \sigma_\downarrow)^2}{\sigma_\uparrow \sigma_\downarrow} \left( \frac{R^\circ}{L_s} \right)^2,
$$

(28)

The parameter $\gamma$ is plotted as a function of the degree
of polarization, $\zeta$, in Fig. 3. The solid line is $\gamma$ for
$L/L_s = 20$ whereas the dashed line is the contribution of
the first two terms in Eq. (25) only, neglecting the in-
fluence of density variations. It is worth noting that the
analytic result for an F/N interface is $\gamma = 0$ and for a poly-
domain wire connected to an S reservoir it is $\gamma = 1/3$.

In these cases, $\delta N = 0$. For moderate values of $\zeta$, we
find that there is no large change in $\gamma$, $\gamma \approx 1/2$, and
the influence of density variations is small. However, for
very large spin polarization, $\zeta \gtrsim 0.8$, $\gamma$ increases dramat-
ically and the role of density variations is vital. Drawing
on the interpretation of the results for the return prob-
ability mentioned in Appendix A, we naively estimate the
resistance of a system with $\zeta = 1$ by considering the
majority (up) spin channel to behave as if connected to an
insulating barrier and the minority channel to behave as
if connected to a normal barrier. Such a procedure
gives $\gamma \sim 1 - \sigma_\downarrow/(\sigma_\uparrow + \sigma_\downarrow) \sim 1$. This rough estimation
of the value of $\gamma$ at $\zeta = 1$ appears to be in agreement
with Fig. 3. However, some notes of caution about the
results for large polarization need to made. Firstly, the
graphs are shown as a function of $\zeta$ for fixed $L/L_s = 20$,
where the spin relaxation lengths, $L_\alpha$, in the individual
spin channels depend on $\zeta$. In particular, $L_\uparrow \gg L_\downarrow$ for
large $\zeta$. Secondly, the error in the numerical evaluation
of the diffusion coefficient increases rapidly as $\zeta \to 1$.
Finally, values of $\zeta \gtrsim 0.8$ are unlikely to be realised in
real materials and, even if they were, the intrinsic mag-
netic field in such materials would destroy any quantum
interference.

IV. CONCLUSION

The weak localization correction to the classical diffusion
coefficient, $\delta D$, is dependent on polarization, with
majority spins more likely to be reflected from the F/S
interface than minority spins. Taking into account the
change in the spin polarized particle distribution in the
F wire arising from $\delta D$, we found that the weak local-
ization correction to the contact resistance is related to
the square of the resistance of a piece of F wire with
length equal to the spin relaxation length (see Eq. (1))
with a numerical prefactor that is almost constant for all
experimentally relevant values of spin polarization.

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APPENDIX A: CORRECTION TO THE
DIFFUSION COEFFICIENT

This appendix describes the evaluation of the weak
localization correction to the classical diffusion coeffi-
cient, $\delta D_\alpha$, Eq. (13). In order to calculate the Cooperon,
Eq. (14), we consider spinor eigenvectors with compo-
ents $\psi_{\alpha \alpha}(x)$ that obey the following diffusion equa-
tions:

$$
\left( -\nu_\uparrow D_\uparrow \partial_x^2 + \nu_\uparrow \tau^\uparrow_1 \right) \psi_{\uparrow \alpha}(x) = \lambda_\alpha \psi_{\uparrow \alpha}(x),
$$

(A1)

$$
\left( -\nu_\downarrow D_\downarrow \partial_x^2 + \nu_\downarrow \tau^\downarrow_1 \right) \psi_{\downarrow \alpha}(x) = \lambda_\alpha \psi_{\downarrow \alpha}(x),
$$

(A2)

such that the Cooperon $C_{\alpha \alpha}$ is given by

$$
C_{\alpha \alpha}(x, x) = \sum_{n \neq 0} \frac{\left| \psi_{\alpha \alpha}(x) \right|^2}{\lambda_\alpha}.
$$

(A3)

The momenta in the spin channels, $Q_{\alpha \alpha}$, are related,
$\nu_\uparrow D_\uparrow Q^\uparrow_{\alpha \alpha} + \nu_\uparrow \tau^\uparrow_1 = \nu_\downarrow D_\downarrow Q^\downarrow_{\alpha \alpha} + \nu_\downarrow \tau^\downarrow_1 \equiv \lambda_\alpha$. At the
left hand reservoir \((x = -L)\) the Cooperon is zero since a particle that escapes into it suffers dephasing, Eq. \((15)\),

\[
\psi_{n\uparrow}(-L) = \psi_{n\downarrow}(-L) = 0.
\]  
\((A4)\)

whereas when the reservoir on the right hand \((x = 0)\) is a superconductor, the boundary conditions Eqs. \((A1,A2)\) give

\[
\psi_{n\uparrow}(0) = e^{i\xi} \psi_{n\downarrow}(0),
\]  
\((A5)\)

\[
\sigma_{\uparrow} \partial_x \psi_{n\uparrow}(x)|_{x=0} = -\sigma_{\downarrow} e^{i\xi} \partial_x \psi_{n\downarrow}(x)|_{x=0}.
\]  
\((A6)\)

The spinor eigenvectors also obey a normalisation condition, \(\int_{-L}^{0} (|\psi_{n\uparrow}(x)|^2 + |\psi_{n\downarrow}(x)|^2) dx = 1\). We find the general solutions to the diffusion equations and use the boundary conditions at the left hand lead \((x = -L)\), Eq. \((A4)\), to give

\[
\psi_{n\alpha}(x) = N_{n\alpha} \sin [Q_{n\alpha}(x + L)]
\]  
\((A7)\)

On substituting into the boundary conditions on the right hand side \((x = 0)\), we find

\[
N_{n\uparrow} \sin [Q_{n\uparrow} L] = e^{i\xi} N_{n\downarrow} \sin [Q_{n\downarrow} L],
\]  
\((A8)\)

\[
\sigma_{\uparrow} Q_{n\uparrow} N_{n\uparrow} \cos [Q_{n\downarrow} L] = -e^{i\xi} \sigma_{\downarrow} Q_{n\downarrow} N_{n\downarrow} \cos [Q_{n\downarrow} L].
\]  
\((A9)\)

Eliminating the normalisation constants leaves an equation for determining the eigenvalues,

\[
\sigma_{\downarrow} Q_{n\downarrow} \cos [Q_{n\downarrow} L] \sin [Q_{n\uparrow} L] + \sigma_{\uparrow} Q_{n\uparrow} \cos [Q_{n\uparrow} L] \sin [Q_{n\downarrow} L] = 0.
\]  
\((A10)\)

We solve this equation numerically to find the eigenvalues and determine their contribution to the Cooperon, Eq. \((A3)\).

As an example we present results for the calculation of the spatially averaged return probability, \(\overline{\delta D}_{\alpha} = \int_{-L}^{0} \delta D_{\alpha}(x)(dx/L)\), for arbitrary spin polarization in the ferromagnet. For long wires, \(L \gg L_S\), we express it as

\[
\overline{\delta D}_{\alpha} \approx \delta D_{\alpha}^B \left(1 - \eta_{\alpha} \frac{L_{\alpha}}{L}\right),
\]  
\((A11)\)

where \(\delta D_{\alpha}^B\) is the bulk contribution, \(\delta D_{\alpha}^B = -8 L_{\alpha}/\pi \nu_{\alpha} L_S^2\). The numerical coefficient \(\eta_{\alpha}\) depends on the state of the reservoir and the degree of polarization in the ferromagnet. We compare with analytic results obtained in certain limits. When the reservoir on the right hand side of the wire is in the normal state, the boundary conditions are the same as those in the left hand reservoir and \(\eta_{\alpha} = 1\). For a poly-domain ferromagnetic wire connected to an S reservoir, where the classical contact resistance Eq. \((12)\) gives no effect, \(\eta_{\alpha} = 1/2\). The second term in the above equation represents Cooperon decay due to the probability of particle escape into the right hand reservoir. Multiple Andreev reflection at the boundary, illustrated in a sketch in Fig. 2b, causes a factor of two reduction in the case of a polydomain ferromagnetic wire connected to a superconducting reservoir. In the case of a ferromagnetic wire connected to an insulating reservoir, \(\eta_{\alpha} = 0\), since the probability of particle escape into the right hand reservoir is totally supressed.

In Fig. 4 we plot \(\eta_{\alpha}\) for a ferromagnetic wire with arbitrary polarization connected to a superconducting reservoir. It is shown for each spin channel separately and we assume that the up-spins are the majority spins, \(\sigma_{\uparrow} \geq \sigma_{\downarrow}\). For zero spin polarization, \(\eta_{\uparrow} = \eta_{\downarrow} = 1/2\), as described above. On increasing the spin polarization, \(\eta_{\uparrow} < 1/2\) whereas \(\eta_{\downarrow} > 1/2\). Majority carriers at the superconducting interface cannot find minority carrier states to Andreev reflect into and are normally reflected which increases the probability to return whereas majority carriers at the interface have an enhanced probability to escape. In the limit of total spin polarization, \(\eta_{\uparrow} \rightarrow 0\) and \(\eta_{\downarrow} \rightarrow 1\) which means that the reservoir appears to be totally insulating to majority spins but normal as far as minority spins are concerned.

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FIG. 1. Schematic of the ferromagnetic wire connected to a normal (N) reservoir on the left hand side, $x = -L$, and a superconducting (S) reservoir on the right hand side, $x = 0$.

FIG. 2. (a) Cooperon decay at the F/N boundary due to electron escape into the N reservoir. (b) Multiple Andreev reflection at the S reservoir changes the Cooperon boundary conditions at the interface.
FIG. 3. The prefactor, $\gamma$, of the increase, $\delta r_s^S - \delta r_s^N$, (Eq. (1)) in the weak localization contribution to the resistance as the reservoir changes from the normal state to superconducting, plotted as a function of the degree of polarization, $\zeta$, for $L/L_s = 20$. The solid line is the total weak localization contribution whereas the dashed line does not include the contribution of density variations. The error increases as $\zeta \to 1$ (see comments at the end of section III).
FIG. 4. The prefactor, $\eta_\alpha$, of the contact term in the spatially averaged return probability, $\overline{\delta D_\alpha}$, (Eq. (A11)) for the F/S system, plotted as a function of the polarization, $\zeta$, for $L/L_s = 20$. The solid line is for the majority spin channel whereas the dashed line is for minority spins.