A POSSIBLE CORRELATION BETWEEN MASS RATIO AND PERIOD RATIO IN MULTIPLE PLANETARY SYSTEMS

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ABSTRACT

We report on a possible correlation between the mass ratio and the period ratio of pairs of adjacent planets in extrasolar planetary systems. Monte Carlo simulations show that the effect is significant to a level of 0.7%, as long as we exclude two pairs of planets whose periods are at the 1 : 2 resonance. Only the next few multiple systems can tell if the correlation is real.

Subject headings: planetary systems — solar system: general — stars: individual (GJ 876, HD 82943) — stars: statistics

1. THE CORRELATION BETWEEN THE MASS RATIO AND THE PERIOD RATIO

As of 2003 March, 101 extrasolar giant planets have been discovered, with minimum masses between 0.12 and 15 Jupiter masses and orbital periods between 2.986 and 5360 days.1 The periods and the masses of the planets are apparently correlated. The emerging population has shown indications of a correlation that corresponds to a paucity of massive planets with short orbital periods. Furthermore, this correlation does not appear in the population of planets that have been found in stellar binary systems (Zucker & Mazeh 2002).

The known extrasolar planets include a special subgroup of 22 planets found in 10 multiple systems (Fischer et al. 2003), two of which consist of three planets (u And and 55 Cnc). In this Letter, we focus on the masses and orbital periods of the planets found in the multiple systems and present a distinctive correlation that characterizes this subsample.

The multiple systems provide us with a unique feature—the mass and period ratios between planets in adjacent orbits, ratios that can reflect general characteristics of the multiple systems. Although only the minimum masses are known for each planet, we assume that the orbital inclinations of the planets in the same multiple system are similar and therefore that the ratio of the minimum masses is very close to the ratio of the actual masses. We therefore studied here the correlation between the (minimum) mass and the period ratios of all pairs of planets in adjacent orbits.

For each multiple system with two planets, we derived one mass ratio and one orbital-period ratio. For each of the two systems with three known planets, we derived two sets of ratios—one set of ratios between the intermediate planet and the innermost one, and one set of ratios between the outermost and the intermediate one. Altogether, we have 12 such pairs of extrasolar planets. In Figure 1, we plotted their mass ratios as a function of their period ratios.

The solar system includes two giant planets, Jupiter and Saturn, within the mass range of the known extrasolar planets. We added an open circle to the figure to represent the mass and period ratios of the Saturn/Jupiter pair. The mass and period ratios are very similar to those of 47 UMa, as already pointed out by Fischer et al. (2002).

Figure 1 shows an intriguing correlation between the two ratios. Except for two points that lie exactly at the 1 : 2 orbital-period resonance, all points seem to fit a straight line in a log-log plot. Considering the extrasolar planets alone, the correlation between the logarithms of the two ratios is 0.9415, and the best-fit line, which is also plotted in Figure 1, has a slope of 0.92 ± 0.10—suspiciously close to unity. When the fit includes the solar system point, the correlation rises to 0.9498.

2. SIGNIFICANCE

The number of points in Figure 1 is extremely small. We have altogether only 12 points (excluding Saturn/Jupiter), out of which, we claim, two points should be excluded because of their unique period ratio. On top of that, the data are subject to a strong selection effect that thwarts the detection of extrasolar planets with small masses and long periods. On the other hand, the correlation is intriguingly high, even for such a small number of points.

To estimate the significance of our findings, we performed two randomization tests. In the first one, we have used the masses and orbital periods of all known planets, a set that is supposedly subject to similar observational selection effects. We chose at random eight pairs and two triples of orbits from the 101 extrasolar orbits, and we calculated the correlation between their mass ratios and period ratios. We removed the two pairs that maximized the correlation for the remaining 10 pairs. Out of 1,000,000 random choices, only 7921 (0.8%) yielded correlations higher than that obtained for the true data of the extrasolar pairs of planets.

The Monte Carlo simulation described above shows the significance of the linear relation for the extrasolar planets alone. As we have seen in the previous section, the Saturn/Jupiter pair seems to agree with the same rule. In order to take account of this fact, we repeated the simulations, this time adding this pair to the randomly drawn 10 pairs and two triples, before rejecting two points. The significance implied by these modified simulations is 0.3%!

Note, however, that the selection effects of the single planets could have been quite different from the selection effects of the multiple systems. This might be the case mainly because once a planet has been discovered in a system, the observation strategy changes, leading to a different set of biases when additional planets are discovered in the same system. In general, a large spread of a few radial velocity measurements of the stars in the sample is the first hint of the first planet, which

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usually entails a further extensive series of observations. Large residuals relative to the derived orbit are a sign of a second planet, which can be the drive for additional observations that might detect and measure the orbital motion of the second (and third) planet.

Furthermore, the difficulty of extracting multiple signatures from the same data set probably introduces certain biases when looking for low-mass planets in multiple systems. It might be more difficult to identify a low-mass planet in a system that already contains a large planet if the orbital periods are similar rather than if they differ by a factor of 10 or more.

To try and overcome these difficulties, we ran another randomization test in which we considered only the planets found in multiple systems. Again, we chose at random eight pairs and two triples of orbits only from the 22 extrasolar orbits found at the multiple systems, and we calculated the correlation between their mass ratios and period ratios. The period and mass distributions of the inner and outer planets in our simulated population are very similar to those of the actual pairs. For each set of simulated pairs, we removed the two pairs that maximized the correlation for the remaining 10 pairs. Out of 1,000,000 random choices, only 6543 (0.65%) yielded correlations higher than that obtained for the true data of the extrasolar pairs of planets.

Note that our simulations rejected the points whose removal would have caused the correlation to be relatively high. In the true data, the rejected points shared the property of lying exactly on the 1 : 2 resonance. Any theory of planetary formation and/or migration would have to explain these findings, if verified.

Three other possibilities to interpret the possible correlation is to assume that the present periods reflect the original distances of the formation sites of the planets from their parent stars. Thus, the correlation that we found may be related to a correlation between the location of the formation site of a planet and its mass. Massive planets might migrate slower (e.g., Ward 1997; Trilling et al. 1998; Nelson et al. 2000) and therefore are left far away when the disk evaporates. A similar effect could have caused the paucity of the massive planets with short periods (Zucker & Mazeh 2002). If this is true, any point that represents a pair of planets in our parameter space slides to the right during migration, when the period of the smaller planet gets shorter.

Figure 1 shows that multiple systems with a large period ratio (and therefore a large orbital-radius ratio) also show a large ratio between their masses, with the more massive planet on the outside. Within the migration paradigm (e.g., Goldreich & Tremaine 1980; Lin, Bodenheimer, & Richardson 1996), the present orbital radii of the planets are substantially smaller than the distance of their formation sites from their parent stars. Therefore, the correlation we found, if verified, could be the result of some correlation between the migration range of a planet and its mass. Massive planets might migrate slower (e.g., Ward 1997; Trilling et al. 1998; Nelson et al. 2000) and therefore are left far away when the disk evaporates. A similar effect could have caused the paucity of the massive planets with short periods (Zucker & Mazeh 2002). If this is true, any point that represents a pair of planets in our parameter space slides to the right during migration, when the period of the smaller planet gets shorter.

The above general considerations do not explain why the correlation that we found holds for more than two decades and, in particular, why the exponent in the power law is so close to unity. Any theory of planetary formation and/or migration would have to explain these findings, if verified.

One other possibility to interpret the possible correlation is to assume that the present periods reflect the original distances of the formation sites of the planets from their parent stars. Thus, the correlation that we found may be related to a correlation between the location of the formation site of a planet and its mass. After all, the larger the disk radius at the formation site, the more mass is available for planetary accretion within a certain fraction of the planet’s original radius.

Interestingly enough, a similar idea has been proposed already by Laskar (2000), in a paper titled “On the Spacing of Planetary Systems.” In that paper, he suggested a power-law relation

$$\frac{m_1}{m_2} = \left(\frac{a_1}{a_2}\right)^{(2\nu+3)/6},$$

where $\nu$ is a number that depends on the mass ratio, period ratio, and migration rate. For the case of the present data set, $\nu = 0.99$, which is in good agreement with the power-law fit of the data. The value of $\nu$ for the simulation data is 0.92, also in good agreement with the power-law fit of the data.
which translates into a power-law relation between the mass ratio and the period ratio. However, the Laskar theory differs from the present data in two crucial details. Laskar considered cases in which $p$ is between 0 and $-3/2$, which translates into an exponent between 1/3 and 0 for the mass ratio–period ratio relation. The present data suggest an exponent that is close to unity. Furthermore, an extension of the Laskar relation goes, naturally, through the $(m/m_1, p/p_1) = (1, 1)$ point, whereas ours does not. Therefore, the Laskar theory by itself cannot explain the possible relation that we suggest here, and we probably need a combination of models for formation and migration.

The present Letter suffers from two drawbacks. First, although we have shown that the effect seems to be statistically significant, the number of points in Figure 1 is still quite small. A few additional planetary systems are critically needed in order to establish the reality of the effect. Second, the authors do not claim to fully understand the possible correlation. If the effect is established in the future, we will need theoretical studies to understand the mechanism behind it.

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