Pressure Response of a Horizontal Well in Tight Oil Reservoirs with Stimulated Reservoir Volume

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Stimulated reservoir volume is an effective stimulation measure and creates a complex fracture network, but the description and characterization of fracture network are very difficult. Well test analysis is a common method to describe the fracture network, and it is the key to build a proper interpretation model. However, most published works only consider the shape of the fractured area or the stress sensitivity effect, and few works take both factors into account. In this paper, based on reservoir properties and flow law after a stimulated reservoir volume, an interpretation model is established with an arbitrary shape of the fractured area and stress sensitivity effect of different flow areas. The model is solved to conduct the pressure response using Laplace transform, point source function, and boundary element theory. The influence of fractures’ parameters and stress sensitivity effect is analyzed on the pressure behavior. Results from this study show that the special flow regimes for a horizontal well with a stimulated reservoir volume are (1) bilinear flow dominated by hydraulic fractures, (2) linear flow dominated by formation around the hydraulic fractures, (3) crossflow from a matrix system to the fractured area, and (4) radial flow control by properties of the fractured area. Parameters of hydraulic fractures mainly affect the early stage of pressure behavior. On the contrary, the stress-sensitive effect mainly affects the middle and late stages; the stronger the stress sensitivity effect is, the more obvious the effect is. The findings of this study can help for better understanding of the fracture network in a tight oil reservoir with a stimulated reservoir volume.

1. Introduction

During the development of tight reservoirs, the stimulated reservoir volume has been widely used, which greatly increases the production of oil and gas wells [1–4]. Well testing is the effective means to evaluate the properties of the reservoir and the stimulation effect, which is widely used to characterize the fracture network in tight oil reservoirs with the stimulated reservoir volume [5–8]. The stimulated reservoir volume is mainly to form a complex network of fractures around the wellbore, which greatly improves the permeability of the fractured area and reduces the flow resistance of the fluid to increase well production. According to the properties of the reservoir, some scholars have designed a series of flow models to characterize the properties of the fractured area and calculate relevant parameters [9–11]. Based on microseismic data, Harikesavanallur et al. divided the reservoir into three regions and constructed a numerical model for a reservoir with a stimulated reservoir volume [12]. Considering the characteristics of dual porous media in the fractured area, Brohi et al. simplified the fractured horizontal well into three linear flow models and analyzed the influence of the storativity ratio, crossflow coefficient, and fractured area on well production [13]. Because of the heterogeneity of the fractured area, the trilinear flow model can be extended to a five-linear flow model, and the applicable conditions of the model were discussed [14]. Combined with the dual porosity feature and fluid seepage law in the fractured area, Deng et al. further extended the five-linear flow model and discussed the bottom pressure response characteristics of the reservoir [15]. In the practical volumetric fractured reservoirs, the fluid flow is not completely in a linear flow, and
different composite models are used to characterize the properties of the fractured area [16–19]. Extending the numerical model of the traditional multistage fractured horizontal well to the volumetric fractured horizontal wells, Zhao et al. simplified the fractured area to a circle and gave a semianalytic solution of the bottom pressure [16]. Fan et al. divided a reservoir with a stimulated reservoir volume into three areas, simplified the fractured area to a rectangular shape, and constructed a mathematical model of the bottom pressure [19]. Based on results of microseismic monitoring during the process of the stimulated reservoir volume, Chen et al. simplified the fractured area into an irregular shape, discussed the influence of the fractured area shape, and showed that the shape of the fractured area had obvious influence on the bottom pressure characteristic curve [20]. The existing production data show that the shape of the volumetric fractured area is irregular; it is necessary to study a method for characterizing this feature in tight reservoirs.

Due to the extremely low porosity and permeability of the tight reservoir, the scale of the pore is extremely small, and the small change of the pore size will cause a significant decline in mobility. As the effective overburden pressure first increases and then decreases, the permeability loss in tight reservoirs is as high as 60%, and the loss is irreversible [21]. The stress sensitivity of the matrix and artificial fractures can reduce the ultimate recovery by 18% [22]. Lacy et al. tests the curve of the fracture width with stress in a soft sandstone reservoir by computer-controlled simulation experiment technology and shows that the fracture width first decreases rapidly and then tends to be stable during the increase of effective stress, and the fracture width can be reduced by more than 60% [23]. These above studies verified that there are stress sensitivity effects in hydraulic fractures, the fractured area, and the matrix reservoir in the development of tight reservoirs with volumetric fractured horizontal wells. However, it is rarely seen that this feature is considered in the current reports of the bottom pressure response model [24, 25]. Zhu et al. construct a mathematical model of the bottom pressure with stress sensitivity effects, but this model is only suitable for vertical wells with a rectangular fractured area [25]. Jiang et al. gives a semianalytical model for predicting the transient pressure behavior of a hydraulically fractured horizontal well with stress-sensitive effects, but it is not suitable for volumetric fractured horizontal wells [26]. Therefore, it is of great significance to study the bottom pressure transient analysis model of a horizontal well in a tight oil reservoir with a stimulated reservoir volume and stress-sensitive effects.

In this paper, a mathematical model is firstly established, which is suitable for volumetric fractured horizontal wells with stress sensitivity effects in tight reservoirs. Then, using Laplace transform, point source function, and boundary element theory, the model is solved, and the irregular shape of the volumetric fractured area is characterized. Finally, the influence of relevant fractures and reservoir parameters on the response characteristic curve of the bottom pressure is discussed. The results have a certain guiding role for the fracturing design and fracturing effect evaluation.

2. Model Construction and Solution

After stimulating the reservoir volume in tight oil reservoirs, multiple main fractures and a series of secondary fractures are formed in the fractured area. According to the physical property and flow laws of reservoirs, after stimulating the reservoir volume, the reservoir can be divided into three areas: unfractured area, fractured area, and hydraulic fractures (Figure 1). Due to the difference of flow channels in each area, its flow laws and stress sensitivity effects are significantly different. Here, the mechanism of stress sensitivity effects is discussed and the flow models are constructed separately in these areas.
2.1. Construction of Flow Model in Unfractured Area. In tight reservoirs, the pore structure is complex and the pore throat radius is small. As the formation pressure decreases, their stress sensitivity is stronger than conventional reservoirs (Figure 2). Many scholars have studied stress sensitivity effects in tight reservoirs through physical simulation experiments and numerical simulation techniques and have built corresponding permeability models [27–29]. Based on the relationship model between porosity and permeability, Mckee et al. proposed an exponential model between permeability and effective stress using rock mechanics theory [30]. It can be expressed as

\[ k_m = k_0 e^{-\alpha \phi_m P_{eff}}, \tag{1} \]

where \( k_m \) is the current permeability of unfractured area; \( k_0 \) is the initial permeability of unfractured area; \( \alpha \) is the porosity index, which indicates the exponential relationship between permeability and porosity during the decrease of the pore pressure; \( C_p \) is the compression coefficient; and \( P_{eff} \) is the effective pressure, which can be seen as the difference between the initial pressure and the current pressure.

Using the exponential model between permeability and effective stress to simplify the porous reservoir into a capillary model, Wang and Cheng introduced the fractal theory to obtain the expression of effective permeability during rock compression [31]. It can be written as

\[ k_m = k_0 e^{-\gamma P_{eff}}, \tag{3} \]

where \( \gamma \) is the permeability modulus. The expressions of initial permeability and permeability modulus are as follows:

\[ k_0 = \frac{4(2 - D_{f_m}) \phi_m^{-1} \lambda_0^{1+D_{f_m}}}{128(3 + 4D_{f_m} - D_{f_m})(1 - \phi_m)^{3+D_{f_m}} P_{eff}^{-1}} \left[ 1 - \left( \frac{\lambda_0 \phi_m^D}{\lambda_0^{max}} \right)^{3+D_{f_m}} \right] e^{-((3+D_{f_m})/2)C_p P_{eff}}, \tag{4} \]

\[ \gamma = \frac{3}{2} D_{f_m} C_p. \tag{5} \]

Based on the formation characteristics of the unfractured area, the outer boundary is defined as the infinite boundary, and the inner boundary is defined as the junction with the fractured area. The flow model of the unfractured area can be obtained by supplementing the initial

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Figure 2: Diagram of the stress sensitivity in the capillary bundle model [31].
conditions and boundary conditions. The flow model is as follows (Appendix B):

\[
\frac{\partial^2 \xi_{1D}}{\partial \tau_D^2} + \frac{1}{r_D} \frac{\partial \xi_{1D}}{\partial \tau_D} = \xi_{1D},
\]

\[
\xi_{1D}(t_D = 0) = 0,
\]

\[
\xi_{1D}(r_D \rightarrow \infty) = 0,
\]

\[
\partial \xi_{1D} \bigg|_{r_D = \infty} = \xi_{2D} \bigg|_{r_D = \infty}.
\]

\[
\frac{\partial \xi_{1D}}{\partial \tau_D} \bigg|_{r_D = \infty} = M \frac{\partial \xi_{2D}}{\partial \tau_D} \bigg|_{r_D = \infty}.
\]

(6)

2.2. Construction of Flow Model in Fractured Area. Because of low permeability in tight reservoirs, it is difficult to obtain industrial oil production, so the stimulated reservoir volume is generally used in well completion. During the process of the stimulated reservoir volume, a series of secondary fractures are formed, which greatly increases the contact area between the fracture wall and the reservoir, minimizes the seepage distance from pore to hydraulic fractures, and greatly improves well production [32]. There is a complex fracture network in the fractured area, and the dual porosity medium model is widely used to study the flow mechanism in the fractured area (Figure 3) [33, 34]. Wang and Cheng used the slab model and fractal theory to build a mathematical model between the permeability and the effective stress of fracture system [31]:

\[
k_f = \phi_f \frac{(2 - D_{ff}) \eta_{0}\eta_{max}}{12 \left(3 + D_{Tf} - D_{ff}\right)^{2+D_{ff}}} \left\{ \frac{1 - \left(a_{max}/a_0\right)^{3+D_{ff}-D_{ff}}}{\left(1 - a_{min}/a_0\right)^{2-D_{ff}}}, \right.\]

where \( k_f \) is the current permeability of the fracture system in the fractured area, \( \phi_f \) is the porosity of the fractured area, \( D_{ff} \) is the fractal dimension of the fracture system, \( D_{Tf} \) is the tortuosity fractal dimension, \( a_{min} \) is the maximum spacing of the fracture system, \( a_{max} \) is the minimum spacing of the fracture system, and \( C_p \) is the fracture compression coefficient.

According to the concept of permeability modulus, combined with the permeability model of the fracture system in the plate model, the effective permeability model can be simplified as follows:

\[
k_f = k_{f0} e^{-\gamma_f P_{ai}},
\]

(8)

where \( \gamma_f \) is the permeability modulus of the fracture system and \( k_{f0} \) is the initial permeability of the fracture system. The expressions of the initial permeability and the permeability modulus are as follows:

\[
k_{f0} = \frac{\phi_f (2 - D_{ff}) \eta_{0}\eta_{max}}{12 \left(3 + D_{Tf} - D_{ff}\right)^{2+D_{ff}}} \left\{ \frac{1 - \left(a_{max}/a_0\right)^{3+D_{ff}-D_{ff}}}{\left(1 - a_{min}/a_0\right)^{2-D_{ff}}}, \right.\]

(9)

\[
\gamma_f = (2 + D_{Tf}) C_p.
\]

(10)

Based on the property and fluid flow character of the fractured area, a flow model can be obtained combining the initial conditions and boundary conditions, which can be written as (Appendix B)

\[
\frac{\partial^2 \xi_{FD}}{\partial \tau_D^2} + \frac{1}{r_D} \frac{\partial \xi_{FD}}{\partial \tau_D} = M_{12} \frac{\partial \xi_{FD}}{\partial \tau_D} \left\{ \frac{1 - \left(a_{max}/a_0\right)^{3+D_{ff}-D_{ff}}}{\left(1 - a_{min}/a_0\right)^{2-D_{ff}}}, \right.\]

(11)

\[
\frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty} = \xi_{FD} \bigg|_{r_D = \infty},
\]

\[
\frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty} = \frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty},
\]

\[
\frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty} = \frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty},
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\frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty} = \frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty},
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\frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty} = \frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty},
\]

\[
\frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty} = \frac{\partial \xi_{FD}}{\partial \tau_D} \bigg|_{r_D = \infty}.
\]

(11)
2.3. Construction of Flow Model in Hydraulic Fractures. Regardless of the density change in the flow process, the flow equation in hydraulic fractures can be written as

$$\frac{\partial}{\partial t_F} \left( \frac{k_F \omega_F}{\mu} \frac{\partial p_F}{\partial l_F} \right) + q_F(l_F) = 0,$$

where $l_F$ is the fracture length, $k_F \omega_F$ is the fracture conductivity, $p_F$ is the pressure in hydraulic fractures, and $q_F(l_F)$ is the production of the hydraulic fracture microelement $l_F$.

When a reservoir exploits, the formation pressure decreases, and the effective stress acting on proppants increases, which results in proppant compression deformation and falling into the formation on the fracture surface (Figure 4). The decrease of the fracture width will lead to the decrease of flow capacity of hydraulic fractures [35–39]. According to results of physical simulation, the relationship between conductivity and effective stress can be expressed as [40]

$$F_{CD} = F_{CDO} e^{-Y_{FD} \rho \rho_D},$$

where $F_{CDO}$ is the initial dimensionless conductivity of hydraulic fractures and $Y_{FD}$ is the dimensionless permeability modulus of hydraulic fractures.

The flow equation of hydraulic fractures is dimensionless (Appendix A), and the perturbation transformation and Laplace transformation are used at the same time. The equation can be written as

$$\frac{\partial^2 \xi_{FD}}{\partial r_{FD}^2} - 2 \pi \frac{2}{F_{CDO}} q_{FD}(l_{FD}) = 0.$$

Combining the initial conditions of hydraulic fractures and other related conditions, the flow model can be written as

$$\left\{ \begin{array}{l}
\frac{\partial^2 \xi_{FD}}{\partial r_{FD}^2} - 2 \pi \frac{2}{F_{CDO}} q_{FD}(l_{FD}) = 0,
\xi_{FD}(t_D = 0) = 0,
\bar{q}_{FDj} = \frac{\partial \bar{q}_{FD}}{\partial r_D} \bigg|_{r_D},
\bar{q}_{FDj} = \frac{\partial \bar{q}_{FD}}{\partial l_D} \bigg|_{l_D = 0} = \frac{2 \pi}{s} \frac{\rho}{F_{CDO}}.
\end{array} \right.$$

2.4. Model Solution. Analytical or semianalytical methods are generally used to solve flow models of most regular composite reservoirs, but it is difficult to solve the flow model of irregular composite reservoirs. In this paper, the boundary element method is used to solve the flow model of a volumetric fractured horizontal well considering stress sensitivity. If the basic solutions of the flow model in the unfractured area and fractured area are $G_1(P, Q, s)$ and $G_2(P, Q, s)$, respectively, they satisfy the following equations:

$$\frac{\partial^2 G_1(P, Q, s)}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial G_1(P, Q, s)}{\partial r_D} - s G_1(P, Q, s) + \delta(P, Q) = 0,$$

$$\frac{\partial^2 G_2(P, Q, s)}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial G_2(P, Q, s)}{\partial r_D} - \frac{M_{12} \omega_m (1 - \omega_m) s + \lambda}{(1 - \omega_m) s + \lambda} G_2(P, Q, s) + \delta(P, Q) = 0.$$

The basic solutions of the flow equation in the unfractured area and fractured area can be written as

$$G_1(P, Q, s) = \frac{1}{2 \pi} K(r_D \sqrt{s}),$$

$$G_2(P, Q, s) = \frac{1}{2 \pi} K \left( \frac{\omega_m (1 - \omega_m) s + \lambda}{(1 - \omega_m) s + \lambda} M_{12} r_D \right).$$

Multiply the basic solution equations by $\tilde{\xi}_{1D}$ and $\tilde{\xi}_{2D}$, respectively; there are

$$\tilde{\xi}_{1D} \frac{\partial^2 G_1(P, Q, s)}{\partial r_D^2} + \tilde{\xi}_{1D} \frac{\partial G_1(P, Q, s)}{\partial r_D} - s \tilde{\xi}_{1D} G_1(P, Q, s) + \tilde{\xi}_{1D} \delta(P, Q) = 0,$$

$$\tilde{\xi}_{2D} \frac{\partial^2 G_2(P, Q, s)}{\partial r_D^2} + \tilde{\xi}_{2D} \frac{\partial G_2(P, Q, s)}{\partial r_D} - \frac{M_{12} \omega_m (1 - \omega_m) s + \lambda}{(1 - \omega_m) s + \lambda} \tilde{\xi}_{2D} G_2(P, Q, s) + \tilde{\xi}_{2D} \delta(P, Q) = 0.$$

In the same way, multiplying the flow equations in the unfractured area and fractured area by the basic solutions $G_1(P, Q, s)$ and $G_2(P, Q, s)$, respectively, the equations can be written as

$$G_1(P, Q, s) \frac{\partial^2 \tilde{\xi}_{1D}}{\partial r_D^2} + G_1(P, Q, s) \frac{\partial \tilde{\xi}_{1D}}{\partial r_D} - s \tilde{\xi}_{1D} G_1(P, Q, s) = 0,$$

$$G_2(P, Q, s) \frac{\partial^2 \tilde{\xi}_{2D}}{\partial r_D^2} + G_2(P, Q, s) \frac{\partial \tilde{\xi}_{2D}}{\partial r_D} - \frac{M_{12} \omega_m (1 - \omega_m) s + \lambda}{(1 - \omega_m) s + \lambda} \tilde{\xi}_{2D} G_2(P, Q, s) = 0.$$
\[ G_2(P, Q, s) \frac{\partial^2 \tilde{\varepsilon}_{ID}(P, s)}{\partial r_D^2} + G_1(P, Q, s) \frac{\partial \tilde{\varepsilon}_{ID}}{\partial r_D} \]
\[ - \frac{M_1 \omega_m (1 - \omega_m) s + \lambda}{(1 - \omega_m) s + \lambda} \tilde{\xi}_{ID} G_2(P, Q, s) = \frac{q_{D,1}}{M_1 s} \delta(Q, P). \]

(23)

There are \( M \) source sinks in the solution area. In order to apply Green’s function, the flow equations in the unfractured area \( \Omega_1 \) and the fractured area \( \Omega_2 \) are subtracted to obtain under any boundary conditions. The flow equations are [20]

\[ \int_{\Omega_2} \left[ \tilde{\varepsilon}_{ID}(P, s) \nabla^2 G_1(P, Q, s) - \nabla^2 \tilde{\varepsilon}_{ID}(P, s) G_1(P, Q, s) + \delta(P, Q) \tilde{\varepsilon}_{ID}(Q, s) \right] d\Omega = - \frac{1}{M} \sum_{i=1}^{M} \tilde{q}_{ID,i} G_1(W_{ii}, Q, s) = 0, \]

(24)

\[ \int_{\Omega_1} \left[ \tilde{\varepsilon}_{ID}(P, s) \nabla^2 G_2(P, Q, s) - \nabla^2 \tilde{\varepsilon}_{ID}(P, s) G_2(P, Q, s) + \delta(P, Q) \tilde{\varepsilon}_{ID}(Q, s) \right] d\Omega = - \frac{1}{M} \sum_{i=1}^{M} \tilde{q}_{ID,i} G_2(W_{ii}, Q, s) = 0. \]

(25)

Since there is no source-sink term in the unfractured area \( \Omega_2 \), the flow equations are transformed into a line integral by Green’s formula:

\[ \frac{1}{2} \tilde{\varepsilon}_{ID}(P, Q, s) = \left[ C_1 \int_{\Gamma_1} \left( G_1(P, Q, s) \frac{\partial \tilde{\varepsilon}_{ID}(P, s)}{\partial n} - \tilde{\xi}_{ID}(P, s) \frac{\partial G_1(P, Q, s)}{\partial n} \right) d\Gamma \right]. \]

(26)

Because it is impossible to solve the problem by direct integration, it needs to be discretized to obtain the numerical solution. The composite boundary and the outer boundary are discretized into \( N \) segments, and each hydraulic fracture is discretized into \( N_F \) segments (Figure 5). The formula (28) can be written as

\[ \frac{1}{2} \tilde{\varepsilon}_{ID}(P, Q, s) = \sum_{i=1}^{N} \int_{\Gamma_i} \left[ G_1(P, Q, s) \frac{\partial \tilde{\varepsilon}_{ID}(P, s)}{\partial n} - \tilde{\xi}_{ID}(P, s) \frac{\partial G_1(P, Q, s)}{\partial n} \right] d\Gamma \]

(27)

\[ + \frac{1}{2} \sum_{i=1}^{N_F} \tilde{q}_{ID,i} G_2(W_{ii}, Q, s). \]

(28)

Because the above formula cannot be solved by direct integration, it needs to be discretized to obtain the numerical solution. The boundary and hydraulic fractures are discretized, the boundary is discretized into \( N \) segments, and each hydraulic fracture is discretized into \( N_F \) segments (Figure 5). The formula (28) can be written as

\[ \frac{1}{2} \tilde{\varepsilon}_{ID}(P, Q, s) = \sum_{i=1}^{N} \int_{\Gamma_i} \left[ G_1(P, Q, s) \frac{\partial \tilde{\varepsilon}_{ID}(P, s)}{\partial n} - \tilde{\xi}_{ID}(P, s) \frac{\partial G_1(P, Q, s)}{\partial n} \right] d\Gamma \]

(29)

\[ + \frac{1}{2} \sum_{i=1}^{N_F} \tilde{q}_{ID,i} G_2(W_{ii}, Q, s). \]

According to the boundary integral method, the relationship between the wellbore pressure and the fracture can be deduced as follows:

\[ \tilde{b}_{mD} - \tilde{b}_{MFD} = \frac{2\pi}{s_F CD} \left( x_D - \int_{0}^{r_D} \int_{0}^{r_D} q_F(x_D') dx_D' dx_D' \right). \]

(30)

The integral equation of the vertical fracture with finite conductivity is discretized [35, 36]:

\[ \int_{0}^{r_D} \int_{0}^{r_D} q_F(x_D') dx_D' dx_D' = \frac{\Delta x_D^2}{8} q_{FD}(x_{mD}) + \sum_{i=1}^{n-1} q_{ID}(x_{mD-i}) \]

(31)

\[ \cdot \left( \frac{\Delta x_D^2}{2} + \Delta x_D(x_{mD} - \Delta x_D \cdot i) \right). \]

According to the law of mass conservation, the total flow of fracture discrete elements is equal to well production. Combined with the discrete model of the unfractured area, fractured area, and hydraulic fractures, the solution matrix of the flow equation for volumetric fractured horizontal wells in tight oil reservoirs can be obtained:

\[
\begin{pmatrix}
A_1 & A_{12} & B_{12} & 0 & 0 \\
0 & A_{21} & B_{21} & C_2 & E \\
0 & 0 & 0 & D_F & 0
\end{pmatrix}
\begin{pmatrix}
\tilde{\varepsilon}_{D,1} \\
\tilde{\varepsilon}_{D,2} \\
\tilde{\varepsilon}_{FWD}
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
\frac{1}{s}
\end{pmatrix}.
\]

(32)

By solving equation (32), the bottom pressure can be obtained. According to the Duhamel principle, the bottom pressure of the volumetric fractured horizontal well in
tight reservoirs considering well storage and skin effect is obtained:

\[
\bar{\xi}_{\text{FWDO}} = \frac{s\xi_{\text{FWD}} + S}{s + C_D s^2 \left( \xi_{\text{FWD}} + S \right)},
\]

(33)

where \(\xi_{\text{FWDO}}\) is the bottom pressure considering the wellbore storage effect and skin effect, \(S\) is the skin effect, and \(C_D\) is the dimensionless well storage coefficient.

According to the calculated bottom pressure of the volumetric fractured horizontal well in tight reservoirs, the dimensionless bottom pressure in real space can be obtained by Pedrosa transformation and Stehfest numerical inversion [41, 42].

### 3. Analysis and Discussion

In order to understand the influence of the fracturing scale and formation properties on pressure response characteristics, the bottom pressure under different parameters is calculated, the pressure and pressure derivative curves are drawn, and influences of relevant parameters on the \(f\) pressure response characteristic curve are discussed. The relevant data in the calculation process is shown in Table 1.

#### 3.1. Flow Stage of Volumetric Fractured Horizontal Well in Tight Reservoirs

The model is used to calculate the bottom pressure in the development of volumetric fractured horizontal wells in tight reservoirs, and the pressure and pressure derivative curves are drawn using results of the model (Figure 6). According to the characteristics of the curve, the flow can be divided into eight stages in tight oil reservoirs with volumetric fractured horizontal wells. The first flow stage is the follow-up section, which is mainly affected by storage effect and skin effect. Since this stage lasts for a period of time, the response characteristics of the hydraulic fracture are masked in this example. The second stage is the bilinear flow stage, which reflects the linear flow characteristics of hydraulic fractures and formation around fractures, and its reflection on the pressure and pressure derivative curve is a parallel line with a slope of \(1/4\). The third stage is the linear flow stage, which reflects the characteristics of linear flow in the fractured area. The pressure and pressure derivative curves are parallel lines with a slope of \(1/2\). The fourth stage is the radial flow stage in the fracture system of the fractured area, which is characterized by a horizontal line of \(0.5\) in the pressure derivative curve. The fifth stage is the crossflow period from the matrix system to the fracture system in the fractured area, which mainly shows the concave feature in the pressure derivative curve. The sixth stage is the radial flow stage of the whole fractured area, which occurs after

| Parameters                  | Values | Parameters                  | Values |
|-----------------------------|--------|-----------------------------|--------|
| Dimensionless fracture length | 60     | Number of fractures         | 6      |
| Dimensionless fracture spacing | 100    | Crossflow coefficient      | 0.00001|
| Dimensionless composite radius | 1500   | Elastic storage ratio      | 0.2    |
| Dimensionless permeability modulus | 0     | Skin factor                | 0.03   |
| Dimensionless storage coefficient | 20     | Dimensionless conductivity | 400    |
the matrix system reaches equilibrium flow with the fracture system. The seventh stage is the transition flow from the fractured area to the unfractured area. The eighth stage is the radial flow period in the unfractured area.

3.2. Influence of Hydraulic Fracture Number. The number of hydraulic fractures mainly affects the bilinear flow stage and the linear flow stage. The more hydraulic fractures present, the longer the bilinear flow and linear flow lasts (Figure 7). The larger the number of hydraulic fractures is, the larger the contact area between hydraulic fractures and formation and the smaller the pressure to flow from formation to wellbore under the same condition. As the number of hydraulic fractures increases, the fractured area increases and the duration of the bilinear flow and linear flow increases in the early stage of the test, but which has no effect on the later flow characteristics of the fractured area and unfractured area.

3.3. Influence of Hydraulic Fracture Length. The length of hydraulic fractures mainly affects the bilinear flow stage and the linear flow stage (Figure 8). The longer the length of hydraulic fractures is, the longer the duration of the bilinear flow and linear flow is. The length of hydraulic fractures is long, the contact area is large between hydraulic fractures and formation, and the pressure to flow from formation to wellbore is relatively small. Under the same conditions, the

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**Figure 6:** Type curves of a volumetric fracturing horizontal well in tight reservoirs.

**Figure 7:** Characteristic curve of bottom pressure under different numbers of fractures.
production pressure difference is small and the pressure drop is relatively slow in the early production. Since the length of hydraulic fractures is long, the area of bilinear flow and linear flow increases, and the duration of these stages increases.

3.4. Influence of Hydraulic Fracture Spacing. The spacing of hydraulic fractures mainly affects the linear flow stage. The larger the spacing of hydraulic fractures is, the longer the linear flow lasts (Figure 9). As the spacing of hydraulic fractures increases, the interaction between fractures occurs later, and reaching the linear flow for the entire fractured area is also later. At the same time, the area affected by fractures increases, the duration of linear flow increases, and radial flow appears later.

3.5. Influence of Crossflow Coefficient. The crossflow coefficient mainly affects the crossflow stage from the matrix system to the fracture system in the fractured area. The larger the crossflow coefficient is, the earlier the crossflow stage appears (Figure 10). When the crossflow coefficient is large, the fluid of the matrix system is more likely to flow to the fracture system. It leads to crossflow from the matrix system to the fracture system under low differential pressure; the
crossflow stage appears earlier. Due to the difference of crossflow coefficient, the time of the crossflow stage appears different. In the well test, it may appear simultaneously with the fracture linear flow and the transition flow between the fractured area and the unfractured area, which results in the unobvious crossflow stage in the characteristic curve.

3.6. Influence of Storativity Ratio. The storativity ratio mainly affects the bilinear flow stage, the linear flow stage, and the crossflow stage of the matrix system to the fracture system (Figure 11). The larger the storativity ratio is, the longer the duration of bilinear flow and linear flow is and the smaller the depression amplitude is in the crossflow stage. The larger the storativity ratio is, the more the fluid is contained in the fracture system, and the larger the elastic energy is, so the longer the duration of the bilinear flow and linear flow continues. As the percentage of fluid increases in the fracture system and decreases in the matrix system, the energy supplement to the fracture system is reduced, and the depression amplitude of the pressure derivative curve is reduced.

3.7. Influence of Fractured Area Radius. The radius mainly affects the end time of the radial flow in the fractured area (Figure 12). The larger the radius is, the longer the duration of the radial flow stage continues. If the radius is larger, it takes the pressure wave more time to reach the boundary,
so the radial flow of the fractured area continues for a long time.

3.8. Influence of Permeability Modulus. The permeability modulus mainly affects the radial flow stage in the fractured area, the transition flow stage from the fractured area to the unfractured area, and the linear flow stage in the unfractured area (Figure 13). The larger the permeability modulus is, the greater the elastic modulus, which indicates that stress sensitivity is more likely to occur in tight reservoirs. Under the same pressure drop, the permeability declines more in strong stress-sensitive reservoirs, the flow resistance increases more, and the upwarping appears more obviously in the characteristic curve.

4. Summary and Conclusion

(1) According to formation properties and the flow law, after stimulating the reservoir volume in tight reservoirs with a horizontal well, a mathematical model of fluid flow is constructed. The model is solved using Laplace transform, point source function, and boundary element theory, and the bottom pressure characteristic curve is obtained

(2) The model specially considers the irregular of the fractured area and stress-sensitive effects. This model can be used to evaluate the effect of the stimulated reservoir volume and stress sensitivity in tight reservoirs

(3) According to the characteristic curve of the volumetric fractured horizontal wells, there are eight flow stages in tight reservoirs: follow-up stage, bilinear flow stage, linear flow stage in the fracture system of the fractured area, crossflow stage, radial flow stage in the fractured area, transitional flow stage, and radial flow stage in the unfractured area

(4) The parameters of hydraulic fractures mainly affect the early flow stages and have no effect on the later flow stages. The stress sensitivity mainly affects the middle and late periods of the characteristic curve; the stronger the stress sensitivity is, the more obvious the effect is

(5) The permeability modulus of the model is assumed to be a constant; the change of permeability modulus should be further studied in a later flow model

Appendix

A. Dimensionless Definition

In the dimensionless process, the dimensionless parameters are defined as follows:

Dimensionless pressure: \( p_D = \frac{2\pi k_m h}{q \mu} (p_i - p) \) \hspace{1cm} (A.1)

Dimensionless production: \( q_D = \frac{q \mu}{2\pi k_m h (p_i - p_{wf})} \) \hspace{1cm} (A.2)

Dimensionless time: \( t_D = \frac{k_m}{\phi_m C_m \mu} t \) \hspace{1cm} (A.3)

Storativity ratio: \( \omega = \frac{\phi_m C_m}{\phi_f C_f + \phi_m C_m} \) \hspace{1cm} (A.4)

Crossflow coefficient: \( \lambda = \frac{C_m}{2\pi (\phi_m + \phi_f) h t_{ref}} \) \hspace{1cm} (A.5)

Mobility ratio: \( M_{12} = \frac{k_f / \mu}{k_m / \mu} \) \hspace{1cm} (A.6)

Dimensionless storage coefficient: \( C_D = \frac{C}{2\pi (\phi_m + \phi_f) h t_{ref}} \) \hspace{1cm} (A.7)

Dimensionless conductivity: \( F_{CF} = \frac{k_f \omega_f}{k_f \omega_f} \) \hspace{1cm} (A.8)

Dimensionless permeability modulus: \( \gamma_D = \frac{q \mu B}{2\pi k_f h} \) \hspace{1cm} (A.9)

B. Derivation of Flow Model in Unfractured Area

When the fluid flows in an unfractured area, it is assumed that it is a single seepage flow of a slightly compressible fluid, and the flow equation can be expressed as follows:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{m}{\mu} \frac{\partial p_1}{\partial r} \right) = \phi_m C_m \frac{\partial p_1}{\partial t},
\]  

(\text{B.1})

where \( r \) is the radius, \( \mu \) is the fluid viscosity, \( C_m \) is the comprehensive compressibility, \( t \) is the time, \( p \) is the pressure, and subscript 1 represents the unfractured area.

Considering the stress sensitivity, and integrating the effective permeability model into the flow equation, the flow equation in unfractured area can be obtained:

\[
\frac{\partial^2 p_1}{\partial r^2} + \frac{1}{r} \frac{\partial p_1}{\partial r} + \gamma \left( \frac{\partial p_1}{\partial t} \right)^2 = \phi_m C_m \frac{\partial p_1}{\partial t}.
\]  

(\text{B.2})

In order to simplify the calculation, the flow model of unfractured area is dimensionless (Appendix A), and the dimensionless flow equation can be obtained as

\[
\frac{\partial^2 p_{1D}}{\partial r_{TD}^2} + \frac{1}{r_D} \frac{\partial p_{1D}}{\partial r_{TD}} + \gamma_{1D} \left( \frac{\partial p_{1D}}{\partial t_{TD}} \right)^2 = \phi_m C_m \frac{\partial p_{1D}}{\partial t_{TD}}.
\]  

(\text{B.3})
Since stress sensitivity is taken into account, the dimensionless flow equation is a strongly nonlinear equation, which cannot be solved directly and must be linearized to obtain its analytical solution. In order to transform the nonlinear equations into linear equations, Pedrosa transformation is used for the flow equations [41]. It can be written as

\[ p_{1D} = -\frac{1}{y_{1D}} \ln (1 - y_{1D} \xi_{1D}), \]  

where \( \xi_{1D} \) is the dimensionless variable.

Based on the Pedrosa transformation, the flow equation can be written as follows:

\[ \frac{\partial^2 \xi_{1D}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \xi_{1D}}{\partial r_D} = \frac{1}{1 - y_{1D} \xi_{1D}} \frac{\partial \xi_{1D}}{\partial r_D}. \]  

Because the value of \( y_{1D} \) is very small, the accuracy of the 0-order perturbation transformation can meet the calculation [41]. Laplace transform is applied to the simplified equation; it is expressed as

\[ \frac{\partial^2 \tilde{\xi}_{1D}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \tilde{\xi}_{1D}}{\partial r_D} = \tilde{\xi}_{1D}. \]  

Since the outer boundary is infinite, its expression is in Laplace space:

\[ \tilde{\xi}_{1D}(r_e \rightarrow \infty) = 0. \]  

The inner boundary is the interface between the fractured area and the unfractured area, and the equation of the pressure and flow rate can be written as

\[ \tilde{\xi}_{1D}[r_i] = \tilde{\xi}_{2D}[r_i], \]

\[ \left. \frac{\partial \tilde{\xi}_{1D}}{\partial r_D} \right|_{r_i} = M_{12} \left. \frac{\partial \tilde{\xi}_{2D}}{\partial r_D} \right|_{r_i}. \]

In addition to the initial conditions, the flow governing equation of the unfractured area can be written as

\[ \begin{cases} \frac{\partial^2 \tilde{\xi}_{1D}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \tilde{\xi}_{1D}}{\partial r_D} = \tilde{\xi}_{1D}, \\
\tilde{\xi}_{1D}(r_D = 0) = 0, \\
\tilde{\xi}_{1D}(r_D \rightarrow \infty) = 0, \\
\tilde{\xi}_{1D}|_r = \tilde{\xi}_{2D}|_r, \\
\left. \frac{\partial \tilde{\xi}_{1D}}{\partial r_D} \right|_r = M_{12} \left. \frac{\partial \tilde{\xi}_{2D}}{\partial r_D} \right|_r. \end{cases} \]

C. Derivation of Flow Model in Fractured Area

Regarding the fractured area as a dual-porosity medium formation, the flow from the matrix system to the fracture system is considered. The flow equations in the matrix system and the fracture system can be written as

\[ -\alpha_{jm} \frac{k_m}{\mu} (p_m - p_f) = \phi_m C_m \frac{\partial p_m}{\partial t}, \]

\[ 1 \frac{\partial}{\partial t} \left( \frac{k_f^p}{\mu} p_f \right) + \alpha_{jm} \frac{k_m}{\mu} (p_m - p_f) = \phi_f C_f \frac{\partial p_f}{\partial t}. \]

In order to simplify the calculation, the flow model is dimensionless (Appendix A), and the dimensionless flow equations of the matrix system and the fracture system are obtained, which can be written as

\[ -\lambda (p_{md} - p_{fd}) = \frac{\partial p_{md}}{\partial t}, \]

\[ \frac{\partial^2 \bar{P}_{ID}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \bar{P}_{ID}}{\partial r_D} + \bar{Y}_{ID} \left( \frac{\partial \bar{P}_{ID}}{\partial r_D} \right)^2 - \lambda (\bar{P}_{ID} - p_{md}) \]

\[ = e^{\nu_{ID} t} \frac{1}{M_{12}} \frac{1 - \omega_m}{\omega_m} \frac{\partial \bar{P}_{ID}}{\partial t}. \]

Using disturbance transformation and Laplace transformation for the above equations, the flow equation can be obtained [41]:

\[ \frac{\partial^2 \bar{\xi}_{ID}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \bar{\xi}_{ID}}{\partial r_D} = M_{12} \omega_m (1 - \omega_m) s + \lambda \bar{\xi}_{ID}. \]

For the fractured area, the interface between the fractured area and the unfractured area is its outer boundary, and the equations of the pressure and flow rates can be written as

\[ \bar{\xi}_{ID}[r_i] = \bar{\xi}_{2D}[r_i], \]

\[ \left. \frac{\partial \bar{\xi}_{ID}}{\partial r_D} \right|_{r_i} = M_{12} \left. \frac{\partial \bar{\xi}_{2D}}{\partial r_D} \right|_{r_i}. \]

The inner boundary is the contact area between the fractured area and hydraulic fractures. The flow equation can be supplemented as

\[ \left. \frac{\partial \bar{\xi}_{ID}}{\partial r_D} \right|_{r_f} = \frac{\partial \bar{\xi}_{ID}}{\partial r_D} |_{r_f}. \]
According to the initial state of the reservoir, the fluid flow model can be written as

\[
\begin{align*}
\frac{\partial^2 \xi_{\text{fd}}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial \xi_{\text{fd}}}{\partial r_D} &= \frac{M_{12} \omega_m (1 - \omega_m) s + \lambda}{(1 - \omega_m) s + \lambda} \xi_{\text{fd}}, \\
\xi_{\text{fd}}(r_D = 0) &= 0, \\
\xi_{\text{fd}}(r_D) = \xi_{\text{fd}}(r_D) & \text{if } r_D = r, \\
\frac{\partial \xi_{\text{fd}}}{\partial r_D} \bigg|_{r_i} &= M_{12} \frac{\partial \xi_{\text{fd}}}{\partial r_D} \bigg|_{r_2}, \\
\frac{\partial^2 \xi_{\text{fd}}}{\partial r_D^2} \bigg|_{r_j} &= \frac{q_{ij}}{M_{12} s}.
\end{align*}
\]

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

We declare that we have no competing interests.

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References

[1] M. J. Mayerhofer, E. P. Lolon, N. R. Warpinski, C. L. Cipolla, D. Walser, and C. M. Rightmire, "What is stimulated reservoir volume?,” SPE Production & Operations, vol. 25, no. 1, pp. 89–98, 2010.
[2] T. Suppachoknirun and A. N. Tutuncu, "Hydraulic fracturing and production optimization in eagle ford shale using coupled geomechanics and fluid flow model,” Rock Mechanics and Rock Engineering, vol. 50, no. 12, pp. 1–18, 2017.
[3] M. Nassir, A. Settari, and R. G. Wan, "Prediction of stimulated reservoir volume and optimization of fracturing in tight gas and shale with a fully elasto-plastic coupled geomechanical model,” SPE Journal, vol. 19, no. 5, pp. 771–785, 2014.
[4] T. G. Fan and G. Q. Zhang, "Laboratory investigation of hydraulic fracture networks in formations with continuous orthogonal fractures,” Energy, vol. 74, pp. 164–173, 2014.
[5] Z. M. Chen, H. S. Chen, X. W. Liao, L. B. Zeng, and B. Zhou, "Evaluation of fracture networks along fractured horizontal wells in tight oil reservoirs: a case study of Jimusar oilfield in the Junggar Basin,” Oil & Gas Geology, vol. 41, no. 6, pp. 1288–1298, 2020.
[6] M. Brown, E. Ozkan, R. Raghavan, and H. Kazemi, "Practical solutions for pressure-transient responses of fractured horizontal wells in unconventional shale reservoirs,” SPE Reservoir Evaluation & Engineering, vol. 14, no. 6, pp. 663–676, 2009.
[7] M. Meng, Z. Chen, X. Liao, J. Wang, and L. Shi, "A well-testing method for parameter evaluation of multiple fractured horizontal wells with non-uniform fractures in shale oil reservoirs,” Advances in Geo-energy research, vol. 4, no. 2, pp. 187–198, 2020.
[8] S. T. Lee and J. R. Brockenbrough, "A new approximate analytic solution for finite-conductivity vertical fractures,” SPE Formation Evaluation, vol. 1, no. 1, pp. 75–88, 1986.
[9] J. Wang, A. L. Jia, and Y. S. Wei, "A semi-analytical solution for multiple-trilinear-flow model with asymmetry configuration in multifractured horizontal well,” Journal of Natural Gas Science & Engineering, vol. 30, pp. 515–530, 2016.
[10] B. A. Mohammad, D. Morteza, and Z. Sohrab, "Semi-analytical solution for productivity evaluation of a multi-fractured horizontal well in a bounded dual-porosity reservoir,” Journal of Hydrology, vol. 581, p. 124288, 2020.
[11] M. Wei, Y. Duan, M. Dong, Q. Fang, and M. Dejam, "Transient production decline behavior analysis for a multi-fractured horizontal well with discrete fracture networks in shale gas reservoirs,” Journal of Porous Media, vol. 22, no. 3, pp. 343–361, 2019.
[12] A. K. Harikesavanallur, F. Deimbacher, M. V. Crick, Z. Xu, and P. G. Forster, "Volumetric fracture modeling approach (VFMA): incorporating microseismic data in the simulation of shale gas reservoirs,” in Paper presented at the SPE Annual Technical Conference and Exhibition, Florence, Italy, September 2010.
[13] I. Brohi, M. Pooladi-Darvish, and R. Aguilera, "Modeling fractured horizontal wells as dual porosity composite reservoirs – application to tight gas, shale gas and tight oil cases,” in Paper presented at the SPE Western North American Regional Meeting, Anchorage, Alaska, USA, May 2011.
[14] E. Stalogrova and L. Mattar, "Analytical model for unconventional multifractured composite systems,” SPE Reservoir Evaluation & Engineering, vol. 16, no. 3, pp. 246–256, 2013.
[15] Q. Deng, R. S. Nie, Y. L. Jia, X. Y. Huang, J. M. Li, and H. K. Li, "A new analytical model for non-uniformly distributed multifractured system in shale gas reservoirs,” Journal of Natural Gas Science and Engineering, vol. 27, pp. 719–737, 2015.
[16] Y. L. Zhao, L. H. Zhang, J. X. Luo, and B. N. Zhang, "Performance of fractured horizontal well with stimulated reservoir volume in unconventional gas reservoir,” Journal of Hydrology, vol. 512, pp. 447–456, 2014.
[17] J. J. Guo, H. T. Wang, and L. H. Zhang, "Transient pressure and production dynamics of multi-stage fractured horizontal wells in shale gas reservoirs with stimulated reservoir volume,” Journal of Natural Gas Science & Engineering, vol. 35, pp. 425–443, 2016.
[18] F. Dongyan, Y. Jun, S. Hai, Z. Hui, and W. Wei, "A composite model of hydraulic fractured horizontal well with stimulated reservoir volume in tight oil & gas reservoir,” Journal of Natural Gas Science and Engineering, vol. 24, pp. 115–123, 2015.
[19] R. H. Zhang, L. H. Zhang, R. H. Wang, Y. L. Zhao, and D. L. Zhang, "Research on transient flow theory of a multiple fractured horizontal well in a composite shale gas reservoir based on the finite-element method,” Journal of Natural Gas Science and Engineering, vol. 33, pp. 587–598, 2016.
[20] P. Chen, S. Jiang, Y. Chen, and K. Zhang, "Pressure response and production performance of volumetric fracturing horizontal well in shale gas reservoir based on boundary element method,” Engineering Analysis with Boundary Elements, vol. 87, pp. 66–77, 2018.
A. Reinicke, E. Rybacki, S. Stanchits, E. Huenges, and F. Haeri, M. Izadi, and M. Zeidouni, “A semianalytical model for predicting transient pressure behavior of a hydraulically fractured horizontal well in a naturally fractured reservoir, SPE Journal, vol. 24, no. 3, pp. 1322–1341, 2019.

A. F. Gangi, “Variation of whole and fractured porous rock permeability with confining pressure,” International Journal of Rock Mechanics and Mining Science & Geomechanics Abstracts, vol. 15, no. 5, pp. 249–257, 1978.

X. Tian, L. Cheng, R. Cao et al., “A new approach to calculate permeability stress sensitivity in tight sandstone oil reservoirs considering micro-pore-throat structure,” Journal of Petroleum Science & Engineering, vol. 2015, no. 133, pp. 576–588, 2015.

Y. Zhang, L. Wang, H. Li, Y. Zhang, and G. Fu, “Experimental study of the permeability of fractured sandstone under complex stress paths,” Energy Science & Engineering, vol. 8, no. 9, pp. 3217–3227, 2020.

C. R. Mckee, A. C. Bumb, and R. A. Koenig, “Stress-dependent permeability and porosity of coal,” SPE Formation Evaluation, vol. 3, no. 1, pp. 81–91, 1988.

F. Y. Wang and H. Cheng, “Effect of tortuosity on the stress-dependent permeability of tight sandstones: analytical modeling and experimentation,” Marine and Petroleum Geology, vol. 120, p. 104524, 2020.

T. Fischer, S. Hainzl, L. Eisner, S. A. Shapiro, and J. Le Calvez, “Microseismic signatures of hydraulic fracture growth in sediment formations: observations and modeling,” Journal of Geophysical Research, vol. 113, article B02307, 2008.

G. Fuentes-Cruz and P. P. Valko, “Revisiting the dual-porosity/dual-permeability modeling of unconventional reservoirs: the induced-interporosity flow field,” SPE Journal, vol. 20, no. 1, pp. 124–141, 2015.

F. Haeri, M. Izadi, and M. Zeidouni, “Unconventional multi-fractured analytical solution using dual porosity model,” Journal of Natural Gas Science and Engineering, vol. 45, pp. 230–242, 2017.

A. Reinicke, E. Rybacki, S. Stanchits, E. Huenges, and G. Dresen, “Hydraulic fracturing stimulation techniques and formation damage mechanisms—implications from laboratory testing of tight sandstone–proppant systems,” Geochemistry, vol. 70, no. 3, pp. 107–117, 2010.