CP Violation in $B \to \pi K$ Decays

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Abstract

I briefly review CP violation in the $B$ system, concentrating on $B \to \pi K$ decays. I discuss how to deal with electroweak-penguin contributions to these decays using flavour SU(3). With these, I show that the entire unitarity triangle can be extracted from measurements of $B \to \pi K$ decays. Finally, I examine the signals for new physics in these decays and the possibilities for measuring them.

\footnotesize
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1 Introduction

There are many known methods for extracting the CP-violating parameters $\rho$ and $\eta$ of the Cabibbo-Kobayashi-Maskawa (CKM) quarks mixing matrix, or, equivalently, $\alpha$, $\beta$ and $\gamma$ of the unitarity triangle (for a review of CP violation in the Standard Model (SM), see Ref. [1]). However, most of these make significant assumptions, leading to large associated theoretical errors. Also, in the last few years, the BaBar and Belle experiments have measured the branching ratios of rare $B$ decays with a precision sufficient to challenge many theoretical calculations. This is particularly true for $B \to \pi K$ decays. It is possible to extract CP-violating parameters from these decays alone using the old Nir and Quinn (NQ) analysis [2]. I will give a brief review of this method in Sec. 3. Unfortunately, we know that the NQ method is incorrect since their assumption of neglecting electroweak-penguin (EWP) contributions is clearly false.

The main purpose of this paper is to describe an extension of the NQ analysis. I will show in Sec. 3 how to use SU(3) flavor symmetry to take into account the effect of EWPs in $B \to \pi K$ decays, and thus, to resuscitate the NQ method. Finally, I will discuss briefly the possibility of detecting and measuring new physics (NP) in Sec. 5.

2 Preliminaries

I begin with some preliminaries concerning $B \to \pi K$ decays. We are interested in 8 decays: $B^0 \to \pi^0 K^0$, $B^0 \to \pi^- K^+$, $B^+ \to \pi^0 K^+$, $B^+ \to \pi^+ K^0$, and their CP-conjugate processes. Their measurement leads to a total of 9 experimental quantities:

- 4 averaged branching ratios
  \[ \Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f}) , \]  
  (1)

- 4 time-independent direct CP asymmetries
  \[ \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})} , \]  
  (2)

- 1 time-dependent indirect CP asymmetry
  \[ \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to \bar{f})} = -\text{Im}(\lambda) \sin(\Delta M_B t) , \]  
  (3)

where $\lambda \equiv e^{-2i\beta} \bar{A}/A$, and $A \equiv \Gamma(B \to f)$, $\bar{A} \equiv \Gamma(\bar{B} \to \bar{f})$ involve the CKM phase $\beta$. 


Recent experimental measurements are presented in Table 1 for completeness. Indirect CP violation has been measured recently by BaBar: $S_{\pi^0 K_S} = 0.48^{+0.38}_{-0.47} \pm 0.06$. It is clear that uncertainties are big, and we cannot hope for a miracle for resulting constraints on CKM parameters.

Table 1: Averaged branching ratios and direct CP asymmetries.

| Branching ratio ($10^{-6}$) | $A_{CP}$ |
|----------------------------|----------|
| $B^0 \to \pi^0 K^0$       | 11.7 ± 1.4 | 0.11 ± 0.23 |
| $B^0 \to \pi^- K^+$       | 18.2 ± 0.8  | −0.095 ± 0.028 |
| $B^+ \to \pi^0 K^+$       | 12.5$^{+1.1}_{-1.0}$ | −0.00 ± 0.05 |
| $B^+ \to \pi^+ K^0$       | 21.8 ± 1.4  | 0.03 ± 0.04 |

Another important aspect of $B \to \pi K$ decays is the isospin quadrilateral. Under isospin symmetry, mesons form isospin multiplets, e.g. $(B^+, B^0)$, $(\pi^+, \pi^0, \pi^-)$ and $(K^+, K^0)$. The effective hamiltonian for $B \to \pi K$ can then be written as a linear combination of the $(I = 0, I_3 = 0)$ and $(I = 1, I_3 = 0)$ isospin components. It is then easy to show, using the Wigner-Eckart theorem and a table of Clebsch-Gordan coefficients, that amplitudes of $B \to \pi K$ decays obey a simple quadrilateral rule in the complex plane:

\[ A^{i+0} + \sqrt{2} A^{i0+} = \sqrt{2} A^{i00} + A^{i-0} , \]
\[ \bar{A}^{i+0} + \sqrt{2} \bar{A}^{i0+} = \sqrt{2} \bar{A}^{i00} + \bar{A}^{i-0} , \]

where $A^{ij} = A(B \to \pi^i K^j)$ and $\bar{A}$’s are CP-conjugated amplitudes.

In addition, we can describe the decays in term of Feynman diagrams. To lowest order, there are 6 diagrams involved in $B \to \pi K$ decays: a gluonic penguin amplitude ($P$), a color-favored tree amplitude ($T$), a color-suppressed tree amplitude ($C$), an annihilation amplitude ($A$), a color-favored electroweak-penguin amplitude ($P_{EW}$) and a color-suppressed electroweak-penguin amplitude ($P_{EW}^C$). We can easily derive the following relations:

\[ A^{i00} = P - Te^{i\gamma} - Ce^{i\gamma} - P_{EW} - \frac{2}{3} P_{EW}^C - Ae^{i\gamma} , \]
\[ \sqrt{2} A^{00+} = -P - Te^{i\gamma} - Ce^{i\gamma} - P_{EW} - \frac{2}{3} P_{EW}^C - Ae^{i\gamma} , \]
\[ \sqrt{2} A^{000} = P - Ce^{i\gamma} - P_{EW} - \frac{1}{3} P_{EW}^C , \]
\[ A^{-+} = -P - Te^{i\gamma} - \frac{2}{3} P_{EW}^C , \]

where the weak phase $\gamma$ is written explicitly, and the strong phases are included in the amplitude definitions for notation convenience. For CP-conjugated amplitudes,
we have exactly the same relations as in Eq. (3), but with $e^{-i\gamma}$ instead of $e^{i\gamma}$. Note that isospin quadrilateral equations (Eq. (4)) are respected by Eq. (5).

Since our main goal is to extract CKM weak phases from $B \rightarrow \pi K$ decays alone, let us ask the following question: do we have enough information to extract CKM weak phases? A simple counting exercise tells us that the answer is no. We have 13 independent theoretical parameters: 6 magnitudes ($|P|, |T|, |C|, |P_{EW}|, |P_{EW}^{C}|$ and $|A|$), 5 relative strong phases and 2 CKM weak phases. But we have only 9 experimental measurements, which is not sufficient to solve the full system of equations. A natural approach for such a deadlock is to neglect some amplitudes until we can solve. This leads us to the NQ analysis.

3 Nir and Quinn Analysis

I present here a very summarized version of the NQ method [2]. The basic assumption of NQ is to neglect all EWP’s, reducing the number of theoretical parameters to 9. The system is now solvable in principle.

We have seen that the decay amplitudes form two isospin quadrilaterals in the complex plane. Let us assign them the names quadrilateral and quadrilateral bar. The first step is to rotate quadrilateral bar by an angle of $2\gamma$. I will use the term quadrilateral tilde for the result:

$$\tilde{A}^{ij} = e^{2i\gamma} \bar{A}^{ij}. \quad (6)$$

This rotation done, there are two important observations:

1. The quadrilateral and the quadrilateral tilde have a common diagonal:

$$D_1 = A^{-+} + \sqrt{2} A^{00} = (T + C)e^{i\gamma},$$
$$\tilde{D}_1 = \tilde{A}^{-+} + \sqrt{2} \tilde{A}^{00} = e^{2i\gamma}(T + C)e^{-i\gamma} = D_1. \quad (7)$$

2. The other two diagonals bisect one another:

$$A^{00} + A^{0+} = \tilde{A}^{00} + \tilde{A}^{0+}. \quad (8)$$

With these two observations, there is enough information to fix the quadrilaterals, up to a discrete ambiguity. The CKM weak phase $\alpha$ is then extracted using the time-dependent CP asymmetry and the angle between $Arg(\tilde{A}^{00}/A^{00})$ taken from quadrilaterals. Our goal seems to be reached, but unfortunately, this analysis is incorrect since EWP’s are not negligible. It was shown [7] using factorization that amplitudes obey the hierarchy

$$O(1) : |P|,$$
$$O(\epsilon) : |T|, |P_{EW}|,$$
$$O(\epsilon^2) : |C|, |P_{EW}^{C}|,$$
$$O(\epsilon^3) : |A|, \quad (9)$$
where $\epsilon \approx 0.2$. Even if this hierarchy is very rough, it is clear that neglecting EWP's implies large theoretical errors. This problem is known as EWP pollution. For this reason, the extraction of CKM weak phases from $B \to \pi K$ decays alone was abandoned for a decade. To solve this problem, we need more theoretical assumptions.

4 Taking into account EWP’s

I begin this section with a quick review of the effective hamiltonian. Using the renormalization group and operator-product expansion, the effective hamiltonian can be written as a linear combination of simple operators [8]:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \left( (V_{ub}^* V_{uq} (c_1 O_1 + c_2 O_2) - V_{tb}^* V_{tq} \sum_{i=3}^{10} c_i O_i) \right) , \quad (10)$$

where

$$O_1 = (\bar{b}_\beta u_\alpha)_{V-A} (\bar{u}_\alpha q_\beta)_{V-A} , \quad O_2 = (\bar{b}_u u_\alpha (\bar{u} q)_{V-A} ,$$
$$O_3 = (\bar{b} q)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A} , \quad O_4 = (\bar{b}_\beta q_\alpha)_{V-A} \sum_{q'} (\bar{q}' q_\beta)_{V-A} ,$$
$$O_5 = (\bar{b} q)_{V-A} \sum_{q'} (\bar{q}' q')_{V+A} , \quad O_6 = (\bar{b}_\beta q_\alpha)_{V-A} \sum_{q'} (\bar{q}' q_\beta)_{V+A} ,$$
$$O_7 = \frac{3}{2} (\bar{b} q)_{V-A} \sum_{q'} (\bar{q}' q')_{V+A} , \quad O_8 = \frac{3}{2} (\bar{b}_\beta q_\alpha)_{V-A} \sum_{q'} (\bar{q}' q_\beta)_{V+A} ,$$
$$O_9 = \frac{3}{2} (\bar{b} q)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A} , \quad O_{10} = \frac{3}{2} (\bar{b}_\beta q_\alpha)_{V-A} \sum_{q'} (\bar{q}' q_\beta)_{V-A} . \quad (11)$$

Above, the $c$'s are the well known Wilson coefficients, $O_1$ and $O_2$ are tree operators, $O_3$ to $O_6$ are gluonic penguin operators and $O_7$ to $O_{10}$ are EWP operators. There are two useful observations to make here. First, the Wilson coefficients $c_7$ and $c_8$ are small compared with $c_9$ and $c_{10}$.

$$c_7 = 3.49 \times 10^{-4} , \quad c_8 = 3.72 \times 10^{-4} ,$$
$$c_9 = -9.92 \times 10^{-3} , \quad c_{10} = 2.54 \times 10^{-3} . \quad (12)$$

The second observation is that, under the assumption of neglecting $O_7$ and $O_8$, EWP operators are purely $(V-A) \times (V-A)$ and have exactly the same structure as the tree operators after a Fiertz transformation. We can therefore guess that there exist relations relating trees and EWP’s. In fact, using SU(3) flavor symmetry and neglecting annihilation amplitude, we can derive the following explicit relations.
\[ P_{EW} = \frac{3c_9 + c_{10}}{4c_1 + c_2} R(T + C) + \frac{3c_9 - c_{10}}{4c_1 - c_2} R(T - C), \]
\[ P_{EW}^C = \frac{3c_9 + c_{10}}{4c_1 + c_2} R(T + C) - \frac{3c_9 - c_{10}}{4c_1 - c_2} R(T - C), \]

where \( R = |V_{tb}^* V_{ts}/V_{ub}^* V_{us}| = (\lambda^2 \sqrt{\rho^2 + \eta^2})^{-1} \). We have therefore written EWP’s in terms of trees, CKM factors and known quantities.

Let us repeat the counting exercise using these new relations: we have 3 magnitudes (\(|P|, |T|, |C|\)), 2 relative strong phases and 2 CKM weak phases for a total of 7 independent theoretical parameters. We have a total of 8 measurements (neglecting annihilation amplitudes implies that \( |A^+| = |\bar{A}^+| \)). The consequence is that, in principle, we can solve for the full unitarity triangle with \( B \to \pi K \) decays alone. Note also that the time-dependent CP asymmetry is not necessary. Naturally there are many discrete ambiguities, but the measurement of the indirect CP asymmetry or other outside inputs (e.g. the measurement of \( \sqrt{\rho^2 + \eta^2} \)) reduces these significantly.

Finally, we made several assumptions to get this result. Let us examine them one by one and analyse roughly the associated errors. First, we have neglected annihilation contribution. According to the hierarchy of Eq. (9), this is an error at the order of 1%. Second, I did not mention it previously, but there are in fact three gluonic penguins and three EWP’s (depending on the internal u, c or t quark in the loop). We have supposed that all penguins are dominated by the internal t quark. From CKM factors, we can estimate this error to be about 2%. Third, we have neglected the operators \( O_7 \) and \( O_8 \) in the effective hamiltonian making about a 4% error. Finally, the SU(3)-breaking is estimated to be about 5% for \( B \to \pi K \) decays \( \text{[10]} \) (though it is typically larger than this). Adding these estimates, we find that the total error is roughly 10%. With such a theoretical error, it is clear that the extracted values of CKM phases are not clean. This method alone cannot provide precise values of CKM parameters. In practice, this method is applicable in parallel with other methods in a more global fit (e.g. see Ref. \( \text{[11]} \)).

## 5 New Physics

Some work has been done analyzing \( B \to \pi K \) decays in the framework of SM. Everything doesn’t seem to be fine. As an example, there is a \( 2.4\sigma \) deviation of the Lipkin sum rule and some ratios of decay rates are not as expected \( \text{[13]} \). Also, there are some stronger signs of discrepancy in other decays, such as the \( 3.5\sigma \) discrepancy between the measurement of the CP asymmetry in \( B_d^0(t) \to J/\psi K_s \) and that in \( B_d^0(t) \to \phi K_s \) from Belle. All of these have in common \( \bar{b} \to \bar{s} \) transitions. Thus, even if the discrepancies are not huge, the door is open for some scenarios of new
physics (NP). These are only hints for NP, but it might be interesting to go further and to measure theoretical parameters in some NP scenarios. It is possible in the \( B \) system (e.g. see Ref. [12]).

In the previous section, we have not exploited the full potential of the \( SU(3) \) flavor symmetry. In fact, under this symmetry, \( B \to \pi\pi, B \to \pi K \) and \( B \to KK \) decays (and others involving \( \eta \)'s) are related to one another [14]. This adds many experimental measurements, so that there are many new constraints on the system. It is true that flavor symmetry is not a perfect symmetry, but even if theoretical errors associated with this assumption are large, it gives us the possibility to add many new theoretical parameters. Thus, the \( B \) system gives us enough information to test some scenarios of NP.

It has been shown recently [15] that, to a good approximation, NP amplitudes have negligible strong-phase differences. This is a consequence of the hypothesis that strong phases are due mainly to rescattering. Since NP rescattering is estimated to be about 5% of the NP contributions, we can neglect the NP strong phases. Using this simple assumption, we can find a model-independent parametrization of NP amplitudes.

As an example [16], to make a minimal usage of the previous hypothesis, we can combine \( B \to \pi K \) and \( B \to \pi\pi \) decays, and take \( \beta \) from the time-dependent CP asymmetry from \( B \to J/\psi K_S \). (It is assumed that NP affects only \( \bar{b} \to \bar{s} \), so that \( B \to \pi\pi \) is not affected.) There are a total of 16 experimental measurements. With this, there is enough information to solve for the NP parameters. In the case of totally model-independent NP, there are 16 independent theoretical parameters, so that the discrete ambiguities make this process a bit discouraging. On the other hand, for more constrained NP scenarios (e.g. isospin-conserving NP or \( Z \)-mediated flavor-changing neutral currents), the situation seems to be more realistic in practice. However, in addition to the theoretical errors discussed in Sec. 4, there is \( SU(3) \) breaking in the relation of \( B \to \pi K \) and \( B \to \pi\pi \) decays, plus the assumption of neglecting the NP strong phases. Thus, without a better understanding of \( SU(3) \)-breaking, the resulting theoretical errors are big (\( \gtrsim 25\% \)). Still, even in this case, the method could help us qualitatively to prefer some scenarios of NP and maybe eliminate some if we are lucky enough. In the next few years, with a better experimental precision of branching ratios and CP violation, this procedure might be possible.

6 Conclusion

There are three points to take from this talk. First, some reasonable assumptions allow us to remove the EWP pollution in \( B \to \pi K \) decays. Second, once removed, these decays alone suffice to extract the full unitarity triangle with or without time-dependent asymmetry. Finally, in principle, there is enough information in the \( B \) system to detect NP and also to measure its parameters in a model-independent
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