Adaptive Fault-Tolerant Attitude-Tracking Control of Spacecraft With Quantized Control Torque

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ABSTRACT In this article, the problem of fault-tolerant attitude-tracking control of spacecraft with quantized control torque is addressed. Actuator faults/failures, an uncertain inertia matrix and unknown disturbances are considered in the attitude controller design of the spacecraft. A dynamical quantization strategy is developed to quantize the signals of the control torque, which can reduce the data transmission rate. An adaptive fault-tolerant controller based on sliding mode techniques is constructed to address the impacts of the actuator faults/failures, quantization errors, inertia matrix uncertainties and unknown disturbances. The developed control strategy with a quantizer can ensure that the entire closed-loop system is asymptotically convergent and achieves satisfactory attitude-tracking performance. Finally, simulation results are provided to show the effectiveness of the proposed method.

INDEX TERMS Attitude-tracking, fault-tolerant control (FTC), sliding mode control, signal quantization.

I. INTRODUCTION The attitude-tracking problem of a spacecraft system is a challenging issue involving highly coupled nonlinearity, an uncertainty inertia matrix and external system disturbances [1], [2]. In the past few decades, this problem has attracted the attention of many researchers, and various nonlinear approaches, e.g., sliding mode control (SMC), backstepping control and disturbance rejection adaptive control, have been developed to address it. Among these methods, SMC is known to provide a powerful tool against matched parameter uncertainties and external disturbances [3]–[7]. In [8], the problem of SMC for a class of singular T-S fuzzy systems was proposed, and good performance was achieved. However, previous studies have addressed the attitude tracking problem with normal functioning actuators only.

In fact, the actuators of spacecraft attitude control systems may undergo partial loss of effectiveness or complete failure, degrading the system performance potentially to the point of instability [9], [10]. Therefore, fault-tolerant control (FTC) problems of attitude control systems of spacecraft have received much research attention. Various control techniques such as robust control, SMC, adaptive model reference control and optimal control have been employed to design fault-tolerant controllers to ensure that spacecraft attitude control systems can maintain acceptable performance [11]–[18]. Based on the existing literature, SMC approaches have been verified as effective methods to handle the FTC problems for control systems and extensively applied to spacecraft attitude control systems [19]–[23]. In [24], considering the actuator faults/failures of the attitude control system of a flexible spacecraft, an adaptive sliding mode FTC scheme was developed, which can effectively compensate the effect of actuator faults/failures. In [25], the authors assumed that there are actuator faults/failures and external disturbance in the attitude control system of a rigid spacecraft. A sliding mode controller was proposed to address this fault-tolerant attitude-tracking problem. Theoretical analysis and simulation results showed that the designed controller can make the system converge to a small domain near the origin in finite time. In [26], the attitude-tracking problem of rigid spacecraft was addressed. With the control allocation technique and SMC technique, an adaptive sliding mode fault-tolerant controller was developed, which can eliminate the effects of actuator faults, mass and inertia uncertainties, and unknown external disturbances. In [27], a modified adaptive integral-type terminal sliding mode FTC scheme was proposed to solve the spacecraft attitude-tracking problem.

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In [28], an adaptive fixed-time sliding mode FTC strategy was designed to stabilize a spacecraft attitude-tracking system in the presence of actuator faults and unknown disturbances. With SMC techniques, other FTC schemes for spacecraft attitude control systems were proposed in [29]–[32].

Recently, with the wide application of functional plug-and-play components in micro spacecraft systems, a micro spacecraft can be built to be faster, lighter, and less expensive [33], [34]. Among these functional components, data transmission is implemented via wireless data communication techniques. Since the data communication rate of a low-cost wireless network is limited, a few unexpected phenomena, such as communication delay and data packet losses, may happen, leading to system performance degradation. Therefore, it is required to adapt an effective communication strategy to reduce the data transmission rate. Quantitative communication strategy, which has proven an effective communication strategy that can reduce the data transmission rate, has been widely applied in networked control systems. However, due to the introduced large quantization errors of the attitude control torque, the attitude control performance is degraded [35]. Hence, the quantization errors should be handled in the attitude controller design. In [36], the attitude stabilization control problem of spacecraft systems with quantized control torque was studied. However, the effects of the actuator faults/failures and unknown disturbances were not addressed in [37]. To our knowledge, few results have addressed the attitude-tracking control issue of spacecraft with actuator faults/failures, quantized control torque, uncertain inertia matrix and external disturbances, which motivates our current study.

This article addresses the FTC problem for spacecraft attitude-tracking control systems with multiplicative and additive actuator faults, an uncertain inertia matrix, unknown external disturbances and limited communication bandwidth. A quantized adaptive fault-tolerant SMC law is developed. The contributions of this article are: (1) The FTC problem of attitude tracking of spacecraft control systems with actuators faults/failures, an uncertain inertia matrix, unknown external disturbances and limited communication bandwidth is considered; (2) A dynamic quantitative communication strategy is designed to reduce the burden of data transmission from the attitude control module to the actuator module; (3) A new sliding surface is designed, which can be reached faster than the traditional sliding surface; (4) A quantized adaptive fault-tolerant SMC scheme is proposed, which can stabilize the overall closed-loop attitude-tracking control system. Moreover, the effect of the actuator faults/failures, external disturbances and quantization errors is completely compensated by the designed controller.

This article is organized as follows. In section II, the problem of fault tolerant control for attitude tracking control systems of spacecraft is introduced. In section III, a sliding mode fault tolerant controller for the control systems with quantized control torque is designed. In section IV, an example is given to show the effectiveness of the proposed controller. In section V, the conclusions and potential future work are discussed.

II. PROBLEM FORMULATION

Considering an uncertain rigid spacecraft system, its uncertain attitude dynamic model with actuator faults/failures, quantized control torque, unknown disturbances and inertia matrix uncertainty is as follows:

$$\dot{\mathbf{q}} = \frac{1}{2} \mathbf{q}^T \mathbf{u}_{\mathbf{q}} \quad \dot{\mathbf{q}}_{\mathbf{q}} = \frac{1}{2} \mathbf{q}^T \mathbf{\omega}$$

where the attitude unit-quaternion $[\mathbf{q}^T \mathbf{q}_{\mathbf{q}}]^T \in \mathbb{H}^4$ that satisfies the constraint $\mathbf{q}^T \mathbf{q} + \mathbf{q}^T = 1$ is used to describe the attitude orientation of spacecraft in $F_0$ with respect to $F_1$, where $F_B$ and $F_I$ are the body fixed and inertial frames, respectively; $\mathbf{\omega} \in \mathbb{H}^3$ is expressed in $F_B$ and is the body angular velocity of the spacecraft with respect to $F_I$. The parameters of spacecraft inertia matrix $J \in \mathbb{H}^{3 \times 3}$ are uncertain and bounded; $d \in \mathbb{H}^3$ is the external disturbance, $I_3 \in \mathbb{H}^{3 \times 3}$ is the identity matrix, and $h \in \mathbb{H}^{3 \times 3}$ is used to present the cross-product matrix of a vector $\mathbf{h} = [h_1 \ h_2 \ h_3]^T$, which is of the form

$$
\begin{bmatrix}
0 & -h_3 & h_2 \\
-h_3 & h_1 & -h_2 \\
-h_2 & h_1 & 0
\end{bmatrix}
$$

The actuator configuration matrix $D \in \mathbb{H}^{3 \times N}$ has full row rank, i.e., $\text{rank}(D) = 3$, where $N$ is the number of actuators. $E = \text{diag}(\rho_1, \rho_2, \cdots, \rho_N) \in \mathbb{H}^{N \times N}$, where $\rho_j, j = 1, 2, \cdots, N$ shows the effectiveness of the actuator. If $\rho_j = 1$, the $i$th actuator is normal. If $\rho_j \in (0, 1)$, the $j$th actuator encounters partial loss of effectiveness. If $\rho_j = 0$, the $i$th actuator undergoes complete failure. In addition, $u \in \mathbb{H}^N$ is the control torque, and $\hat{u} \in \mathbb{H}^N$ is the additive fault. $Q(u) = [Q(u_1) \ Q(u_2) \ \cdots \ Q(u_N)]^T \in \mathbb{H}^N$ provides the quantized values of the control torque. It is well known that the quantitative communication strategy can reduce the communication burden of the network. There are two types of quantizers: the static logarithmic quantizer and the dynamical uniform quantizer [36]. In this article, the dynamical uniform quantizer is employed because the quantization error can be adjusted online. Suppose that the dynamic uniform quantizer has the following form:

$$Q(v) = \mu \text{round}(v), \quad \mu > 0$$

where $\mu$ is the online adjusting quantizer parameters that must be designed and round($\cdot$) is the rounding operation.

Define the quantization error as

$$e_u = Q(u(t)) - u(t)$$

Consequently, the quantization errors satisfy the following constraint:

$$\|e_u\| = \|Q(u(t)) - u(t)\| \leq \Delta \mu, \quad \Delta = \frac{\sqrt{N}}{2}$$

where \( N \) is the number of actuators and \( \mu \leq k_3 \|Q(v)\| \). Here, \( k_3 > 0 \) is a parameter to be selected later.

In this article, we aim to develop an effective FTC scheme that can address the attitude-tracking issue of the spacecraft with the quantized control torque. In addition, the system is assumed to have actuator faults/failures, unknown disturbances and an uncertain inertia matrix. The framework of the overall closed-loop attitude control system is displayed in Fig. 1. Suppose that the desired trajectories of the spacecraft attitude are given by

\[
\hat{q}_d = \frac{1}{2}(q_{d4}I_3 + q_d \times \omega_d)
\]

\[
\dot{q}_{d4} = -\frac{1}{2}q_d^T \omega_d
\]

(5)

where the unit-quaternion \( \{ q_{d1}^T q_d \}^T \) that satisfies the constraint \( q_{d1}^T q_d + q_{d4}^2 = 1 \) is the attitude quaternion to be tracked; \( \omega_d \) is the desired angular velocity.

The attitude-tracking error \( \{ q_e^T q_{e4} \}^T \) is utilized to describe the relative orientation between the attitude quaternion \( \{ q_{d1}^T q_d \}^T \) and reference attitude quaternion \( \{ q_{d1}^T q_d \}^T \) and can be calculated by

\[
q_e = q_{d4}q - q_d \times q_{d4}q_d
\]

\[
q_{e4} = q_{d4}^T q_d + q_{d4}q_d
\]

(6)

The rotation matrix is defined as

\[
R(q_e, q_{e4}) = (q_{e1}^T q_{e4})I_3 + 2q_{e1}q_{e4}^T - 2q_{e4}q_e \times
\]

(7)

\[
\|R(q_e, q_{e4})\| = 1 \text{ and } \dot{R}(q_e, q_{e4}) = -\omega_e \times R(q_e, q_{e4}) \text{, where } \omega_e = \omega - R(q_e, q_{e4}) \omega_d \text{ is the relative velocity. In the following discussion, we use } R \text{ to refer to } R(q_e, q_{e4}) \text{ for the convenience of presentation.}
\]

According to the attitude dynamics (1)–(4), the dynamics of the attitude-tracking error are

\[
\dot{q}_e = \frac{1}{2}(q_{e4}I_3 + q_e \times \omega_e)
\]

\[
\dot{q}_{e4} = -\frac{1}{2}q_e^T \omega_e
\]

\[
J \dot{\omega}_e = -(\omega_e \times R \omega_d) + J(\omega_e + R \omega_d)
\]

\[
+J(\omega_e \times R \omega_d - R \omega_d) + d + D(EQ(u) + \bar{u})
\]

(8)

As discussed in [1], this tracking issue of a spacecraft attitude control system is equivalent to designing a controller so that the attitude error dynamic response (5) is asymptotically stable, or \( \lim_{t \to \infty} q_e = 0 \) and \( \lim_{t \to \infty} \omega_e = 0 \). Hence, the aim of this article is twofold: to design a quantization rule for \( u(t) \) to reduce the data transmission frequency via the communication channel and to design an FTC scheme \( u(t) \) to maintain a stable closed-loop attitude-tracking control of the spacecraft with the proposed quantized control torque.

### III. SLIDING MODE FAULT-TOLERANT CONTROL WITH QUANTIZED CONTROL TORQUE

Since SMC is strongly robust against system uncertainties and nonlinearities, it has received much attention and been broadly used to address the attitude-tracking issues of spacecraft. In this section, we aim to develop a new adaptive SMC scheme to address the attitude-tracking problem with actuator faults/failures, quantized control torque and external disturbances. First, to facilitate the control law design, suppose that the following assumptions hold.

**Assumption 1:** The uncertain symmetric positive definite inertia matrix \( J \) satisfies \( \|J\| \leq \lambda \), where \( \lambda > 0 \) is an unknown constant parameter.

**Assumption 2:** There are two unknown constant parameters \( \alpha_1 > 0 \) and \( \alpha_2 > 0 \) such that \( \|\omega_d\| \leq \alpha_1 \) and \( \|\dot{\omega}_d\| \leq \alpha_2 \) hold.

**Assumption 3:** Disturbances \( d(t) \) and additive faults \( \omega(t) \) are unknown and bounded, satisfying \( \|d(t)\| + \|\dot{\omega}(t)\| \leq \epsilon_1 + \epsilon_2 + \|q_e(t)\| + k_2 \|\dot{\omega}_d(t)\| \), where \( \epsilon_1, \epsilon_2 \) and \( k_2 \) are unknown positive constant parameters.

**Remark 1:** Assumptions 1 and 2 are typical assumptions, and similar assumptions are found in [10] and [19]. Assumption 3 is reasonable because external disturbances such as the gravitation, aerodynamic drag, and magnetic forces are bounded, and the uncertain additive faults are bounded in the engineering system.

Then, to achieve the defined performance \( \lim_{t \to \infty} q_e = 0 \) and \( \lim_{t \to \infty} \omega_e = 0 \), the following sliding surface is defined:

\[
s(t) = \omega_e(t) + (k - \alpha_0 e^{-\beta_0 t})q_e(t) + \alpha_1 e^{-\beta_1 t}
\]

(9)

where \( s(t) = [s_1(t) s_2(t) s_3(t)]^T \in \mathbb{R}^3 \) and \( k > 0 \) is arbitrary. Parameters \( \alpha_0 > 0, \beta_0 > 0, \alpha_1 \) and \( \beta_1 \) should be selected to satisfy \( s(0) = 0 \). The designed sliding surface \( s(t) \) can be reached faster than the sliding surface proposed in [1].

**Remark 2:** It was proved in [1] and [35] that if we can develop a feedback control law \( u(t) \) to ensure that \( s(t) = 0 \) is achieved, then the defined objective performance relations \( \lim_{t \to \infty} q_e = 0 \) and \( \lim_{t \to \infty} \omega_e = 0 \) are guaranteed. Hence, in the following discussion, to address the attitude-tracking problem, we should attempt to design a feedback control scheme \( u(t) \) that can drive the state trajectories of the overall closed-loop system (8) to the sliding surface \( s(t) = 0 \).

**Lemma 1:** There are unknown constant parameters \( c > 0 \) and \( k_2 > 0 \) such that inequality constraint (10) holds:

\[
\left\| (kq_{e} - R \omega_d) \times + \frac{1}{2} k \|q_{e}I_3 + q_{e} \times R \omega_d\| \right\| J \right\| \|\dot{\omega}_d\|
\]

\[
+ \| (kq_{e} - R \omega_d) \times JR \omega_d - J \dot{\omega}_d \|
\]

\[
\leq (k_2 - k_0) \|\dot{\omega}_d\| + (c - \epsilon_0)
\]

(10)

where \( c \) and \( k_2 \) are unknown positive constants.
Proof: Since \( \begin{bmatrix} q_1^T & q_4^T \end{bmatrix} \) satisfies \( q_1^T q_4 + q_4^T q_4 = 1 \), \( q_4 \) and \( q_4 \) are bounded. Then, \( R \) is bounded. Moreover, the desired angular velocity \( \omega_d \), its time derivative \( \dot{\omega}_d \) and the uncertain inertia matrix \( J \) are bounded. Therefore, we can always find unknown sufficiently large positive scalars \( \kappa_1 \) and \( \kappa_2 \) such that

\[
\| \|(k_0 - a_0 e^{-\beta_0 t})q_4 + a_1 e^{-\beta_1 t} - \omega_d(t) \| \| + \frac{1}{2} k \| q_4 e^T \| + \| \omega_4 \| + \| (k_0 - a_0 e^{-\beta_0 t})q_4 + a_1 e^{-\beta_1 t} \| - \omega_d(t) J \| \|| R \| \omega_d \| - J R \| \dot{\omega}_d \| \leq \kappa_1 \| \omega_4 \| + \kappa_2
\]

(11)

Clearly, there are unknown constant parameters \( k_2 \geq \kappa_1 + k_0 \) and \( c \geq \kappa_2 + c_0 \) such that the following constraint holds:

\[
\| \|(k_0 - a_0 e^{-\beta_0 t})q_4 + a_1 e^{-\beta_1 t} - \omega_d(t) \| \| + \frac{1}{2} k \| q_4 e^T \| + \| \omega_4 \| + \| (k_0 - a_0 e^{-\beta_0 t})q_4 + a_1 e^{-\beta_1 t} \| - \omega_d(t) J \| \|| R \| \omega_d \| - J R \| \dot{\omega}_d \| \leq (k_2 - k_0) \| \omega_4 \| + (c - c_0)
\]

(12)

This completes the proof.

In the following discussion, we provide a theorem to obtain an adaptive SMC law \( u(t) \) to ensure the asymptotic stability of the entire closed-loop control system (8).

**Theorem 1:** Considering the attitude error dynamic system (8) with a quantized control torque, the actuator faults/failures, uncertain inertia matrix and unknown disturbances satisfy Assumptions (1)–(3). The following adaptive SMC law \( u(t) \) is implemented.

\[
u(t) = -D^T(\sigma(s(t)) + (\beta + \gamma^{-1})((\beta + \gamma^{-1}) + \| \omega_4 \| \| s(t) \|) + \| J_R \| \omega_4 \| \| s(t) \|
\]

(13)

and

\[
\begin{cases}
0, & \text{if } \hat{c} \geq c_{\max} \\
p_0 \| s(t) \|, & \text{otherwise}
\end{cases}
\]

(14)

\[
\begin{cases}
0, & \text{if } \hat{k}_1 \geq k_{1\text{max}} \\
p_1 \| q_4 \| \| s(t) \|, & \text{otherwise}
\end{cases}
\]

(15)

\[
\begin{cases}
0, & \text{if } \hat{k}_2 \geq k_{2\text{max}} \\
p_2 \| \omega_4 \| \| s(t) \|, & \text{otherwise}
\end{cases}
\]

(16)

In addition, the quantization sensitivity parameter \( \mu \leq k_3 \| Q(s(t)) \| \), where \( k_3 \) should satisfy

\[
k_3 \leq \min \left\{ \frac{2\varepsilon}{\sqrt{n} \| DE \| \| D \| + \varepsilon}, \frac{2\varepsilon \beta}{\sqrt{n} \varepsilon + \| DE \| \| D \| (\beta + \gamma^{-1})} \right\}
\]

(18)

Here, \( c_{\max}, \hat{k}_{1\text{max}}, \hat{k}_{2\text{max}} \) and \( \| E \|_{\text{max}} \) are the bounds of \( c, \hat{k}_1, \hat{k}_2 \), and \( \| E \| \) respectively; \( p_0, p_1, p_2, p_3, \sigma, \beta, \varepsilon_1 \) and \( \varepsilon_2 \) are positive constant parameters to be determined; \( \hat{\gamma}, \hat{k}_1, \hat{k}_2 \) and \( \hat{\gamma} \) are the estimation of unknown parameters \( c, k_1, k_2, \) and \( \theta \), respectively; and we assume that \( 0 < \xi \leq \theta \leq \lambda_{\min}(DE^T) \).

According to (5), we can obtain the time derivative of \( V(t) \), which is given by

\[
\dot{V}(t) = \frac{1}{2} s^T(t)(s(t) - (k_0 - a_0 e^{-\beta_0 t})q_4 - a_1 e^{-\beta_1 t} + R \omega_d(t)) + D(EQ(u(t)) + J(q_4) + q_4 \times \omega_4)
\]

(19)

\[
\dot{V}(t) = \frac{1}{2} (k_0 - a_0 e^{-\beta_0 t})q_4 - a_1 e^{-\beta_1 t} + R \omega_d(t)) \times J(\omega_4 + R \omega_d) \times (E Q(u(t)) + J q_4 I)
\]

(20)

Note that

\[
s^T(t) \| s(t) \| = 0
\]

(21)

Substituting (21) into (20) yields

\[
\dot{V}(t) = \frac{1}{2} \| s(t) \| (k_0 - a_0 e^{-\beta_0 t})q_4 - a_1 e^{-\beta_1 t} + R \omega_d(t)) \times J(\omega_4 + R \omega_d) \times (E Q(u(t)) + J q_4 I)
\]

(22)
According to Assumptions 1-3 and Lemma 1, we have
\[
\dot{V}(t) \leq (c+k_1 \|q_c(t)\|+k_2 \|\omega(t)\|) \|s(t)\| + \frac{1}{2} \|\alpha_0 e^{-\beta_0 t}\| 
\times \|J(q_{a4}+q_e)\| \|\omega_e(t)\| + \|\alpha_0 e^{-\beta_0 t}\| \|q_e\| 
\times \|s(t)+\alpha_1 \beta_1 e^{-\beta_1 t}\| \|s(t)+DEQ(u)\| + \frac{1}{p_0} \tilde{c} \tilde{e}_1 - \frac{1}{p_1} \tilde{k}_1 \tilde{k}_1 - \frac{1}{p_2} \tilde{k}_2 \tilde{k}_2 + \frac{1}{p_3} \tilde{\gamma} \tilde{\gamma}^{-2} \dot{\gamma} 
\]
(23)

From (13)–(16), we have
\[
\dot{V}(t) \leq (c+k_1 \|q_c(t)\|+k_2 \|\omega(t)\|) \|s(t)\|+\|DE\| \|s(t)\| \Delta \mu 
- \tilde{\gamma} \tilde{\gamma}^{-2} \dot{\gamma} \leq (\tilde{c}+k_1 \|q_c(t)\|+k_2 \|\omega(t)\|) \|s(t)\|+(\tilde{c}+k_1 \|q_c(t)\|+k_2 
\times \|\omega(t)\|) \|s(t)\|+\|DE\| \|s(t)\| \Delta \mu 
\times (\sigma \|s(t)+k_1 \|q_c(t)\|+k_2 \|\omega(t)\|) \|s(t)\| \frac{s(t)}{\|s(t)\|} - \frac{1}{p_0} \tilde{c} \tilde{e}_1 
\]
(24)

From (4), we know that
\[
\|Q(t)\| \leq \frac{1}{1 - (\sqrt{n}/2)k_3} \|u(t)\| 
\]
(25)

Then, we can obtain
\[
\Delta \mu \leq \frac{(\sqrt{n}/2)k_3}{1 - (\sqrt{n}/2)k_3} \|u(t)\| 
\]
\[
\leq \frac{(\sqrt{n}/2)k_3}{1 - (\sqrt{n}/2)k_3} \|DE\| + \|s(t)\| + (\beta + \dot{\gamma}) 
\times (\tilde{c}+k_1 \|q_c(t)\|+k_2 \|\omega(t)\|) \|s(t)\| 
\times (\tilde{c}+k_1 \|q_c(t)\|+k_2 \|\omega(t)\|) \|s(t)\| 
\times \|J(q_{a4}+q_e)\| \|\omega_e(t)\| + \|\alpha_0 e^{-\beta_0 t}\| \|q_e(t)\| 
\times \|s(t)+\alpha_1 \beta_1 e^{-\beta_1 t}\| \|s(t)+DEQ(u)\| + \frac{1}{p_0} \tilde{c} \tilde{e}_1 - \frac{1}{p_1} \tilde{k}_1 \tilde{k}_1 - \frac{1}{p_2} \tilde{k}_2 \tilde{k}_2 + \frac{1}{p_3} \tilde{\gamma} \tilde{\gamma}^{-2} \dot{\gamma} 
\]
(26)

Recall that \(\tilde{\gamma} = -\dot{\gamma}^{-1}\). Substituting (16), (17) and (26) into (24) yields
\[
\dot{V}(t) \leq (\tilde{c}+k_1 \|q_c(t)\|+k_2 \|\omega(t)\|) \|s(t)\| 
\times \|\alpha_1 \beta_1 e^{-\beta_1 t}\| \|q_e(t)\| + (\tilde{k}_1 \|q_e(t)\|+\tilde{k}_2 \|\omega(t)\|) \|s(t)\| \|
\]
(27)

Note that
\[
k_3 < \min\{\frac{2\xi}{\sqrt{n}(\|DE\| \|D\|+\xi)}, \frac{2\xi\beta}{\sqrt{n}(\|DE\| \|D\| \beta + \lambda_{\min}(DE^T))}\} 
\]
(28)

Thus, substituting (28) into (27) yields
\[
\dot{V}(t) \leq -\epsilon_1 \|s(t)\|^2 - \epsilon_2 \|s(t)\| 
\]
(29)

where \(\epsilon_1 = \sigma - \frac{1}{1 - (\sqrt{n}/2)k_3} \|DE\| \|D\| \sigma > 0\) and \(\epsilon_2 = \tilde{\sigma} \dot{\gamma} - \frac{1}{1 - (\sqrt{n}/2)k_3} \|DE\| \|D\| \beta \dot{\gamma} > 0\).

Then, the sliding surface \(s(t) = 0\) can be reached in finite time, which indicates that the defined tracking goals \(\lim_{t \to \infty} q_c = 0\) and \(\lim_{t \to \infty} \omega_e = 0\) can be achieved. This completes the proof.

Remark 3: Generally, to weaken the chattering phenomenon of the control signals, control scheme (13) should be replaced by
\[
u(t) = -\tilde{D}^T(\sigma \|s(t)+k_1 \|q_c(t)+k_2 \|\omega(t)\|) \|s(t)\| \|\alpha_0 e^{-\beta_0 t}\| 
\times (\tilde{k}_1 \|q_c(t)\|+\tilde{k}_2 \|\omega(t)\|) \|s(t)\| + \psi 
\]
where \(\psi > 0\) is a small constant. Moreover, the initial estimates of \(\tilde{\gamma}, \tilde{k}_1, \tilde{k}_2\) and \(\dot{\gamma}\) are selected as \(\tilde{\gamma}(0) > 0, \tilde{k}_1(0) > 0, \tilde{k}_2(0) > 0, \) and \(\dot{\gamma}(0) > 0\), respectively.

Remark 4: Parameters \(\tilde{\gamma}, \tilde{k}_1, \) and \(\tilde{k}_2\) are the estimates of parameters \(c, k_1, \) and \(k_2, \) respectively. These estimates are used to compensate the effects of the unknown disturbances, parameter uncertainties and additive fault. Parameter \(\dot{\gamma}\) is the estimate of parameter \(\dot{\gamma}\), is used to eliminate the effects of the actuator failures, and \(k_3\) is the quantization sensitivity parameter and should be selected to make the quantization error \(e_\alpha(t)\) satisfy condition \(\|e_\alpha(t)\| = \|Q(t)-u(t)\| \leq k_3 \|Q(t)\|\).
Remark 6: In controller (13), the matrix $D$ and parameters $\sigma$ and $\beta$ are constant, and parameters $\hat{c}$, $\hat{k}_1$, $\hat{k}_2$ and $\hat{\gamma}$ can be obtained by recursive calculation. Moreover, there are only approximately 30 multiplications and additions in the calculations, and the control law can be easily obtained.

Remark 7: In controller (13), parameters $\alpha_0$, $\beta_0$, $k$, $\alpha_1$ and $\beta_1$ should be selected to satisfy $\varsigma(0) = 0$. Parameter $k_3$ should be selected to satisfy condition (18). Other parameters can be arbitrarily selected according to the system performance.

IV. SIMULATION RESULTS

In this section, to illustrate the good performance of the developed FTC scheme for the attitude-tracking problem of the spacecraft with multiplicative and additive actuator faults/failures, a control torque, an uncertain inertia matrix and unknown disturbances, a numerical example is provided. Suppose that the uncertain inertia matrix of spacecraft $J$ is of the form

$$J = \begin{bmatrix}
    140 & 5.2 & 3.9 \\
    5.2 & 150 & 4.4 \\
    3.9 & 4.4 & 135
\end{bmatrix} \text{kgm}^2$$

and

$$\Delta J = \text{diag}[9 \sin(t/10), 7 \sin(t/15), 12 \sin(t/10)] \text{kgm}^2$$

and the unknown disturbances $d(t)$ are assumed as follows:

$$d(t) = 10^{-4}[2 \sin(5t) 1.5 \cos(5t) + 0.5 3 \sin(5t)]^T \text{Nm}.$$

To strengthen the reliability of the system, four reaction wheels (RWs) are considered as actuators, and the configuration matrix is set as:

$$D = \frac{1}{\sqrt{3}} \begin{bmatrix}
    -1 & -1 & 1 & 1 \\
    1 & -1 & -1 & 1 \\
    1 & 1 & 1 & 1
\end{bmatrix}$$

Suppose that the four actuators undergo a partial loss from an effective fault or complete fault; we define detailed fault scenarios as follows:

$$\rho_1 = \begin{cases}
    1, & t \leq 3.5 \\
    0.3, & \text{otherwise}
\end{cases}, \quad \rho_2 = \begin{cases}
    1, & t \leq 5.5 \\
    0.4+0.1 \sin(t), & \text{otherwise}
\end{cases},$$

$$\rho_3 = 0, \quad \rho_4 = \begin{cases}
    1, & t \leq 7.5 \\
    0.5+0.1 \sin(t), & \text{otherwise}
\end{cases},$$

$$\vec{u}_1 = \vec{u}_3 = 0, \quad \vec{u}_2 = \vec{u}_4 = 0.$$

In this simulation example, the initial states of $[q^T q_4]^T$ and $\omega$ are set as:

$$[q^T(0) q_4(0)]^T = [0.3 -0.2 -0.3 0.8832]^T$$

and

$$\omega(0) = [0 0 0]^T \text{deg/s}$$

Assume that the target-tracking angular velocity is

$$\omega_d = 0.01[\sin(t/30) \cos(t/40) \sin(t/30)]^T \text{rad/s}$$

and the initial value of the reference unit quaternion is

$$[q_d^T(0) q_{dd}(0)]^T = [0 0 0 1]^T.$$

Design parameters $p_0$, $p_1$, $p_2$ and $p_3$ are selected as $p_0 = p_1 = p_2 = p_3 = 2$. The controller parameters are provided as follows: $\sigma = 5$, $k = 0.1$, $\beta = 1$, $\hat{c}(0) = \hat{k}_1(0) = \hat{k}_2(0) = 1$ and $\hat{\gamma}(0) = 0.1$.

Quantization parameter $k_3$ is selected as $k_3 = 0.15 < \min \left\{ \frac{2 \times 0.1}{2 \times (0.5+0.1)}, \frac{2 \times 0.1}{2 \times (0.1+0.55)} \right\}$. Thus, constraint (18) is satisfied. Then, we have $\mu = 0.15 \frac{\|Q(u(t))\|}{\|\varsigma(t)\|}$. To protect the control signals from chattering, function $\frac{\varsigma(t)}{\|\varsigma(t)\|}$ is replaced by $\frac{\varsigma(t)}{[\|\varsigma(t)\|+\psi]}$, where $\psi = 0.001$.

The simulation results of the proposed adaptive FTC approach under actuator faults and a quantized control torque are plotted in Figs. 2–8. For a clear explanation, Euler angles converted from the unit quaternion are used to express the attitude-tracking errors. In Figs. 2 and 3, we observe that both tracking errors of the angular velocity and attitude are bounded in a small domain near the origin. Thus, the angular velocity and attitude effectively track the trajectories of the desired angular velocity and attitude. In addition, the control torques and quantized control torque of four actuators are shown in Figs. 4 and 5, respectively, and the trajectories of the sliding surface are shown in Fig. 6. Figs. 2–6 verify that the communication data rate is reduced and that the system performance can be ensured. Thus, the developed
adaptive FTC law for the spacecraft system with actuator faults/failures, quantized control torque, uncertain inertia matrix and unknown disturbances is effective. Adaptive parameters $\hat{c}$, $\hat{k}_1$, $\hat{k}_2$ and $\hat{\gamma}$ are plotted in Fig. 7, and the quantizer parameter is shown in Fig. 8. The adaptive parameters are obviously bounded, which indicates the effectiveness of the proposed adaptation law. Comparisons between our method and the proposed method in [1] are shown in Figs. 9-11. Our method exhibits good performance.

Moreover, the sizes of data transmission from the controller to the actuators with and without the designed dynamical uniform quantizer are compared. First, the storage sizes of both quantized control torque and traditional control torque sequence are selected as 4 bytes. Second, to achieve an equivalent attitude-tracking precision with the traditional controller, the sampling time of the controller is set as $T = 0.25 \, s$. Communication must occur only when the quantization levels of the control torque change. The simulation results show that during the entire simulation of attitude tracking, the quantization levels of control torque change 2125 times. Thus, the size of the communication data set for the attitude-tracking mission with the proposed dynamical uniform quantizer is 8500 bytes. In the traditional attitude control, the control torque must be sent to the actuators for each control cycle. Since the time duration for the simulation is 500 $s$ and the sampling time of the controller is set as $T = 0.25 \, s$, the size of the communication data during the entire simulation time without the dynamical uniform quantizer is 32000 bytes. Thus, the size of the communication data set is greatly reduced by 74.5% with the proposed control strategy.
scheme design. A dynamical quantization strategy is developed to quantize the signals of control torque. To eliminate the influence of quantization errors and compensate the influence of actuator faults/failures, an adaptive SMC scheme is proposed, which can stabilize the overall closed-loop attitude tracking control system. Simulation results are provided to illustrate the efficiency of our developed method. The size of transmission data transmission from the attitude control module to the actuator module is greatly reduced, and satisfactory attitude tracking performance is achieved. Future work will focus on investigating the problem of attitude-tracking control for rigid spacecraft systems with actuator faults and limited communication bandwidth via an output feedback approach.

V. CONCLUSION

In this article, the FTC issue for attitude tracking of a spacecraft attitude control system is investigated. Multiplicative and additive actuator faults/failures, a quantized control torque, an uncertain inertia matrix and unknown external disturbances have been accommodated in the process of FTC design. A dynamical quantization strategy is developed to quantize the signals of control torque. To eliminate the influence of quantization errors and compensate the influence of actuator faults/failures, an adaptive SMC scheme is proposed, which can stabilize the overall closed-loop attitude tracking control system. Simulation results are provided to illustrate the efficiency of our developed method. The size of transmission data transmission from the attitude control module to the actuator module is greatly reduced, and satisfactory attitude tracking performance is achieved. Future work will focus on investigating the problem of attitude-tracking control for rigid spacecraft systems with actuator faults and limited communication bandwidth via an output feedback approach.

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