Two-Dimensional Sub-diffraction-limited Imaging by an Optimized Multilayer Superlens

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An optimized multilayer superlens is designed, using a rigorous and efficient approach based on the method of moments (MoM) in conjunction with a simulated annealing (SA) algorithm. For the MoM solution, fast evaluation of closed-form Green’s functions (GFs) in the spatial domain is performed by applying the complex-image (CI) technique, which obviates the time-consuming numerical evaluation of Sommerfeld integrals. The imaging capability of the superlens is examined with the correlation coefficient; results show that using circular polarization for the incident wave can improve this coefficient. To validate the proposed method, finite-element-based simulations are exploited, which reveal the method’s accuracy and computational efficiency. Simulation results indicate that the designed structure is capable of producing two-dimensional sub-diffraction-limited images in the visible range, which may make it more versatile for practical applications. Finally, as a considerable finding, it is demonstrated for the proposed design that using circularly polarized illumination provides improved super-resolving performance, compared to linearly polarized illumination.

Keywords: Complex-image technique, Method of Moments, multilayer superlens, Simulated Annealing algorithm

OCIS codes: (000.4430) Numerical approximation and analysis; (050.1755) Computational electromagnetic methods; (050.2065) Effective medium theory; (050.6624) Subwavelength structures

I. INTRODUCTION

With conventional lenses, focusing of light below the diffraction limit is fundamentally impossible. The diffraction limit restricts imaging resolution to about half of the exposing wavelength, due to the exponential decay of the evanescent waves that carry the high-spatial-frequency information [1]. On the other hand, subwavelength imaging has great potential in various applications, including but not limited to nano-lithography [2], biomedical imaging, and data storage [1]. In 2000, Pendry proposed that a metal slab with negative permittivity could be used to achieve near-field imaging with subwavelength resolution. This structure, termed a near-field superlens, can improve resolution beyond the diffraction limit due to resonant excitation of surface plasmons and evanescent-wave enhancement [3]. According to this finding, several practical implementations of the near-field superlens have been reported [4-6]. The imaging of the superlens depends on the matching condition, which dictates permittivities of equal amplitudes but opposite signs for the metal layer and its surrounding dielectric. This condition can be satisfied in the ultraviolet (UV) frequency range for common metal-dielectric couples. Despite numerous implementations of superlenses in the UV range, regular application of superlenses in the visible and near-infrared ranges has been more exciting. To this end, the use of metal-dielectric composites and multilayered structures has been proposed. Although using both of these structures can tune the real part of the permittivity, and consequently the operational wavelength, fabrication results indicate that significant loss

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limits the actual application of superlenses made of composite structures [7].

Multilayered structures can act at the plasmon resonance of the individual layers, where the permittivity matching condition must be satisfied [8]. In this regime, incorporation of additional layers improves transmission characteristics. However, these structures show large impedance mismatch, and compared to a single-layer superlens they do not always offer better resolving power [9]. Using these structures, subwavelength resolution can also be achieved at frequencies different from the plasmon resonances of the individual layers. In this case the imaging mechanism is different from the permittivity-matching condition, and the operational wavelength can be tuned by altering the relative thicknesses of the metal and dielectric layers [7].

Most studies of multilayered superlenses have been focused on transfer-function analysis [10-13]. While this analysis reveals a lens’s transmission coefficients over a range of spatial frequencies, it does not provide a measure of image-reconstruction capability, which is required for practical imaging [9]. Due to near-field operation, superlens performance is sensitive to the coupling between object and lens, but these interactions have not attracted considerable attention in the literature [14]. Alternatively, imaging performance of a multilayered structure depends on various parameters, such as total superlens thickness, number of layers, and individual layer thicknesses [10, 11, 15, 16]. Therefore, an efficient method for modeling a multilayered superlens that includes the object-lens interactions and can consider the effects of various parameters sorely needed. In this paper, an integral-equation method in conjunction with an optimization algorithm is used to achieve a multilayered structure optimized for sub-diffraction-limited imaging. The proposed procedure deals with two-dimensional (2D) objects, while previous studies have been focused on one-dimensional periodic structures [6, 10, 11, 14].

In solving problems related to a multilayered structure, even though methods based on differential equations have a straightforward implementation, discretization of the object and the surrounding space is still required, while in methods based on integral equations only the object is discretized, and no absorbing boundary condition is required [17, 18]. In this work, an efficient formulation of the method of moments (MoM) is applied to solve the volume integral equation that governs the multilayered superlens structure. This study uses the mixed-potential integral equation (MPIE) with the lower-order singularities in the kernels of the resulting integrals, which requires the spatial-domain Green’s functions (GFs) for vector and scalar potentials in multilayered media [19]. The spatial-domain GFs in multilayered media can be expressed in the form of Sommerfeld integrals [19]. Here, by applying the complex-image (CI) technique, which obviates the time-consuming numerical evaluation of the Sommerfeld integral, GFs in the spatial domain are obtained in closed-form [18]. Subsequently, a simulated annealing (SA) algorithm [20] is applied as a robust global-optimization algorithm, to achieve a superlens structure with optimized parameters at a wavelength of 633 nm. In the optimization procedure, the correlation coefficient is used as a quantitative measure of the imaging performance of the lens [9, 21].

The rest of the paper is organized as follows. In section II, the numerical method for superlens design is described in detail. In section III, the design procedure that gives rise to the optimized superlens introduced, in which simulations based on the finite-element method (FEM) are employed to verify the imaging results of the designed structure. Afterward, the subwavelength resolution capability of the proposed superlens structure in 2D imaging is demonstrated. It is shown that by applying circular polarization, resolving power in both x and y directions is improved. Concluding remarks are presented in section V.

II. NUMERICAL METHOD

To characterize the imaging performance of a multilayered superlens, the volume integral equation is used. For a non-magnetic multilayered medium, as shown in Fig. 1, the electric field in each area satisfies the following equation:

$$\vec{E} = \vec{E}_{\text{inc}} + \int_{\Omega'} G_E(\vec{r}, \vec{r}') \vec{J}_{\text{eff}}(\vec{r}') d\vec{r}'$$

where $\vec{J}_{\text{eff}}$ is the equivalent volume electric current density inside the etched area, determined by

$$\vec{J}_{\text{eff}} = j\omega \varepsilon_0 (\varepsilon_{\text{re}} - \varepsilon_{\text{rb}}) \vec{E}.$$  

Here $\vec{E}_{\text{inc}}$ is the incident electric field, $\vec{E}$ is the unknown total field, $\varepsilon_{\text{re}}$ and $\varepsilon_{\text{rb}}$ are the permittivities of the etched area and the background layer, respectively, and $G$ is the dyadic GF associated with multilayered media.

To derive the MPIE formulation, the electric-field expression in terms of vector and scalar potentials is used as

$$\vec{E} = -j\omega \vec{A} - \nabla \varphi$$

FIG. 1. Schematic of the multilayered superlens.
With the aid of the Lorenz gauge [22], Eq. (1) can be written as

\[
\tilde{E} = \tilde{E}_{\text{inc}} - j \omega \int \overline{G}_A(\vec{r}, \vec{r}') \overline{J}_{\text{eff}}(\vec{r}') d\vec{r}' + \nabla \int \frac{\nabla \cdot \overline{G}_A(\vec{r}, \vec{r}')}{j \omega \varepsilon} \overline{J}_{\text{eff}}(\vec{r}') d\vec{r}' (4)
\]

where the Sommerfeld choice of potentials, also known as Michalski III [22, 23], is used for the spatial-domain vector potential dyadic GF as

\[
\overline{G}_A = \begin{bmatrix} G^A_{xx} & 0 & 0 \\ 0 & G^A_{yy} & 0 \\ G^A_{zx} & G^A_{zy} & G^A_{zz} \end{bmatrix}
\]

(5)

Subsequently, with the scalar potential integral representation in terms of the induced electric charge

\[
\varphi = \int_{\overline{v}} G_{\varphi}(\vec{r}, \vec{r}') \rho(\vec{r}') d\vec{r}' = \int_{\overline{v}} \overline{\nabla} \overline{G}_A(\vec{r}, \vec{r}') \overline{J}_{\text{eff}}(\vec{r}') d\vec{r}' (6)
\]

and the continuity equation [24], MPIE representation of the integral equation can be obtained as

\[
\tilde{E} = \tilde{E}_{\text{inc}} - j \omega \int \overline{G}_A(\vec{r}, \vec{r}') \overline{J}_{\text{eff}}(\vec{r}') d\vec{r}' - \nabla \int \frac{\overline{\nabla} \cdot \overline{G}_A(\vec{r}, \vec{r}')}{j \omega \varepsilon} \overline{J}_{\text{eff}}(\vec{r}') d\vec{r}' (7)
\]

where \( \overline{G}_A \) and \( \overline{G}_{\varphi} \) are GFs for the vector and scalar potentials respectively.

Considering (2), the MPIE of (7) for different components of \( \tilde{E} \) can be written as

\[
E_x = E_{x,\text{inc}} + \omega \varepsilon_x (\varepsilon_x - \varepsilon_{\text{in}}) \int G_x(\vec{r}, \vec{r}') E_x(\vec{r}') d\vec{r}' - \varepsilon_x (\varepsilon_x - \varepsilon_{\text{in}}) \frac{\partial}{\partial x} \int G_x(\vec{r}, \vec{r}') \frac{\partial}{\partial x} E_x(\vec{r}') d\vec{r}'
\]

(9)

\[
= E_{x,\text{inc}} + \omega \varepsilon_x (\varepsilon_x - \varepsilon_{\text{in}}) \int G_{xy}(\vec{r}, \vec{r}') \frac{\partial}{\partial y} E_y(\vec{r}') d\vec{r}' + \int G_{x\alpha}(\vec{r}, \vec{r}') \frac{\partial}{\partial z} E_z(\vec{r}') d\vec{r}'
\]

(9)

where \( \alpha_s, \alpha_y, \) and \( \alpha_z \) are unknown coefficients for the \( x, y, \) and \( z \) components of the electric field respectively. Also, \( M, N, \) and \( L \) are the numbers of segments used respectively in the \( x, y, \) and \( z \) directions to discretize the etched area, and \( P(.) \) and \( T(.) \) are pulse and triangular basis functions used to approximate the electric field in the etched area, as shown in Fig. 2. According to the pulse function representation for the derivative of the triangular basis function, this implementation helps to achieve a convenient and efficient solution.

Eq. (9) can be recast with the simple operator representations as follows:

\[
L_x \left\{ \frac{\partial}{\partial x} \tilde{E} \right\} = E_{x,\text{inc}}
\]

(10)

\[
L_y \left\{ \frac{\partial}{\partial y} \tilde{E} \right\} = E_{x,\text{inc}}
\]

\[
L_z \left\{ \frac{\partial}{\partial z} \tilde{E} \right\} = E_{x,\text{inc}}
\]

FIG. 2. Pulse and triangular basis functions with triangular derivative as pulse functions.
where the $L$ operators can be obtained from (8). Subsequently, Galerkin’s method with the following testing functions is applied:

$$W_x = \sum_{l,m=1}^{L,M-1} T_m(x) P_x(y) P_l(z)$$

$$W_y = \sum_{l,m=1}^{L,M-1} P_m(x) T_y(y) P_l(z)$$

$$W_z = \sum_{l,m=1}^{L,M-1} P_m(x) P_z(y) T_z(z)$$

where $W_x$, $W_y$, and $W_z$ are the testing functions for the $x$, $y$, and $z$ components of the electric field respectively.

Implementing Galerkin’s method results in this set of equations:

$$\langle W_x, L_x \{ E \} \rangle = \langle W_x, E_{inc,x} \rangle$$ \hspace{1cm} (12a)

$$\langle W_y, L_y \{ E \} \rangle = \langle W_y, E_{inc,y} \rangle$$ \hspace{1cm} (12b)

$$\langle W_z, L_z \{ E \} \rangle = \langle W_z, E_{inc,z} \rangle$$ \hspace{1cm} (12c)

where $\langle \ldots \rangle$ is the inner-product symbol. Eq. (12a) is the set of $N_c$ equations with $N_c = N_x + N_y + N_z$ unknown coefficients. The numbers of equations in Eq. (12b) and Eq. (12c) are $N_c$ and $N_c$ respectively, both with $N_c$ unknown coefficients, where $N_c = (M-1) \cdot N_x \cdot L$, $N_y = M \cdot (N-1) \cdot L$, and $N_z = M \cdot N_y \cdot (L-1)$ are the numbers of basis functions used in the $x$, $y$, and $z$ directions respectively.

Eq. (12) can now be written in a matrix form as

$$Z \mathbf{I} = \mathbf{V}$$ \hspace{1cm} (13)

where $Z$ is a $N_c \times N_c$ impedance matrix, and $\mathbf{V}$ is the known vector related to the incident electric field and $\mathbf{I}$ is the vector containing the unknown coefficients, both with $N_c$ elements.

The unknown coefficients of the electric field, and hence the equivalent volume current density distribution, are obtained by solving this matrix equation. Finally, the electric field can be obtained in each region of the structure from (8).

The computation of the spatial-domain GFs for the multilayered structure, namely $G_{Lxx}$, $G_{Lxy}$, $G_{Lzx}$, $G_{Lzy}$, $G_{Qx}$, $G_{Qy}$, and $G_{Qz}$ in (8), is needed in the matrix-filling step of the MoM. It should be mentioned that for a symmetrical, infinite multilayered structure, $G_{Lxx} = G_{Lxy}$, $G_{Lzx} = G_{Lzy}$ and $G_{Qx} = G_{Qy}$ are satisfied.

The spatial-domain GFs are expressed in terms of the Sommerfeld integrals [19] as

$$G(\rho,z) = \frac{1}{4\pi} \int -G(k_{\rho},z) H_{1}^{(2)}(k_{\rho}\rho) k_{\rho} dk_{\rho}$$ \hspace{1cm} (14)

where $\hat{G}$ is the spectral-domain GF, $k_{\rho}$ is the radial spectral variable, and $H_{1}^{(2)}$ is the zeroth-order Hankel function of the second kind.

The spectral-domain GFs for the multilayered structure are obtained using the equivalent transmission-line model. This approach results in simple, closed-form expressions for structures with any number of layers [25, 26].

In this work, the CI technique is used to find the spatial-domain GFs for the multilayered media. In this technique, the spectral-domain functions are approximated by a series of complex exponentials. The CI technique gives rise to closed-form GFs, without the need for any numerical integration; hence the computational efficiency of the MoM solution can be improved. It should be noted that this fast evaluation of GFs is also crucial for the optimization process.

To achieve high accuracy in the near- and far-field regions, it is necessary to extract the quasistatic and surface-wave terms respectively, before applying the CI approximation. However, in this work there is an insignificant surface-wave contribution for the multilayered structure, and simulation results show that it is not necessary to extract these terms, which become dominant only in the far-field region.

In the CI method, the spectral-domain functions are approximated by a finite series of exponentials as:

$$\tilde{F}(k) = \sum_{n=1}^{N_f} a_n \exp(k b_n)$$ \hspace{1cm} (15)

where $k_{\rho b} = \sqrt{k_0^2 - k_{\rho}^2}$, $k_{\rho}$ is the propagation coefficient of the background (Cr) layer in the etched area of the imaging structure, and $N_f$ is the number of complex images used in the approximation. It is worth noting that in the CI method, (15) is satisfied over the following specific path in the $k_{\rho b}$ plane, as shown in Fig. 3:

$$k_{\rho b} = k_{\rho} \left[ -jt + \frac{k_{\rho}}{k_{\epsilon}} \left( 1 - \frac{t}{T} \right) \right], \quad 0 \leq t \leq T$$ \hspace{1cm} (16)

where $k_{\epsilon}$ is the propagation coefficient of the dielectric (ZnS) layer and $T$ is the truncation point with respect to $k_{\epsilon}$.

This path, shown in Fig. 3, is denoted by C. The complex coefficients $a_n$ and $b_n$ are calculated using the generalized

\[ \text{FIG. 3. Approximation path in the } k_{\rho b} \text{ plane.} \]
The complex images are located in the background layer. Table 1 lists the appropriate truncation parameter $T$ and the number of complex images $N$ for different GF components for the proposed multilayered structure.

The accuracy of the CI approximation can be verified by numerical evaluation of the Sommerfeld integrals of (14) [29]. It is noticeable that the speed and efficiency of the CI method facilitate the optimization algorithm that follows. Due to the computational efficiency of the proposed numerical method, it is possible to achieve an optimized superlens structure with the desired imaging performance.

### III. OPTIMIZED SUPERLENS STRUCTURE

The metal-dielectric multilayer is used as a superlens in the visible range. A schematic of such a multilayered superlens is shown in Fig. 1. A multilayered structure with sufficiently thin layers can be considered an anisotropic medium. According to effective-medium theory (EMT), the parallel and perpendicular permittivities are described as [7]

$$
\varepsilon_x = \varepsilon_y = \frac{\varepsilon_m + \eta \varepsilon_d}{1 + \eta}, \quad \varepsilon_z = \left(\frac{\varepsilon_m^{-1} + \eta \varepsilon_d^{-1}}{1 + \eta}\right)^{-1}
$$

where $\varepsilon_d$ and $\varepsilon_m$ are the permittivities of dielectric and metal respectively, and $\eta$ is the ratio of the thickness of the dielectric layer to that of the metal layer. A parallel permittivity equal to unity ($\varepsilon_x = \varepsilon_y = 1$) and an infinite perpendicular permittivity ($\varepsilon_z \rightarrow \infty$) yield the situation known as the canalization regime, where a multilayered structure can act as a superlens [7]. In the canalization regime, the operating wavelength can be tuned by altering the thickness ratio of metal to dielectric layers, whereas for the permittivity-matching condition ($\varepsilon_d \equiv |\varepsilon_m|$) superlens operation is limited to a single wavelength for a given metal-dielectric couple. In this regime the surface-plasmon mode splitting on the metal layers leads to subwavelength imaging that is different from the surface-plasmon resonance excitation at the permittivity-matching condition [30]. Both conditions for the canalization regime ($\varepsilon_x = \varepsilon_y = 1$ and $\varepsilon_z \rightarrow \infty$) cannot be satisfied simultaneously, due to practical restrictions on the available materials, and losses [7].

This work is concerned with superlens design in the canalization regime, and the impedance-matching condition ($\varepsilon_x = \varepsilon_y = 1$) is the starting point for design at the commonly used laser wavelength of 633 nm. The metal used is silver (Ag), since it has the lowest loss in the visible range, compared to other metals [7]. Initially $\text{SiO}_2$ with permittivity of 2.12 is chosen for the dielectric layers, while the permittivity of Ag at 633 nm is -17.9-0.477i [13]. In this case, the impedance-matching condition leads to $\eta = 16.9$ for the thickness ratio of the adjacent layer, using the permittivity of $\text{SiO}_2$ and the real part of Ag’s permittivity. For this thickness ratio (16.9) and the realizable thickness of the metal layers, the required thickness of the dielectric layers does not yield acceptable imaging performance. Our simulations show that thinner dielectric layers are required, due to the compact design needed for efficient transmission of components of higher spatial frequency.

To address this issue, the dielectric material is changed to ZnS, which has numerous applications in the visible range [31, 32]. ZnS has a high permittivity, 5.52, which gives rise to $\eta = 4.2$ for the impedance-matching condition. By applying only the calculated thickness ratio, the appropriate design cannot be expected, because of the approximate nature of the EMT, the unfulfilled second condition for the canalization regime ($\varepsilon_x \rightarrow \infty$), and the neglect of loss in the metal. In addition, the literature [7, 28] has indicated that other aspects of the multilayered structure, including the total metal thickness and the number of layers, can affect the imaging performance of the superlens. Thus optimizing the different parameters of the superlens is essential to achieve the best structure for subwavelength imaging. Hence, the SA algorithm is employed to optimize the superlens structure for imaging of the etched area of $\lambda_0/10$ by $\lambda_0/10$ in the 20-nm light-absorbing layer of Cr. The permittivity of Cr is -6.2-30i at the operating wavelength, and the etched area is filled with ZnS. Here the correlation coefficient is used as a quantitative criterion to examine the imaging performance.

To investigate the imaging performance, the field intensity distribution at 13 nm below the etched area in the absence of the superlens is assumed as an object, and the distribution below the final superlens layer in the air is considered as an image. The goal of the optimization process is to improve the correlation between these two distributions. Different parameters, including the total metal thickness, the number of layers, the thickness ratio of the adjacent layers, the thickness of the superlens first layer, and the image plane’s distance from the final layer, are tuned to achieve the optimal superlens structure. Although in general simulations show that good correlation can be achieved with thin metal layers, practical considerations limit the minimum thickness of metal layers to be considered in optimization.

The final superlens consists of 17 alternating metal and dielectric layers. This structure is employed for imaging the etched area in the chromium layer patterned on the

| GF       | N  | T  |
|----------|----|----|
| $G_{xx}$ | 18 | 10 |
| $G_{yy}$ | 50 | 22 |
| $G_{zz}$ | 20 | 6  |
| $G_{xe}$ | 40 | 22 |
| $G_{ye}$ | 30 | 22 |
silica substrate, as shown in Fig. 1. The thicknesses of the ZnS and Ag layers are 5.52 nm and 1.81 nm respectively. The first ZnS layer has a thickness of 15.7 nm, and the image plane’s distance from the final layer is 2.52 nm. This optimized structure has a compact design with thin layers, which is needed for imaging in the canaiization regime. To our knowledge, the commonly used deposition methods cannot be exploited to realize the proposed structure, so techniques with acceptable performance in thin-layer deposition, including atomic layer deposition (ALD) and molecular beam epitaxy (MBE), must be considered [33-35].

The imaging performance of the optimized superlens structure is demonstrated in Fig. 4, where the structure is illuminated by an incident plane wave at 633 nm with the electric field in the $y$ direction. The electric-field intensity distribution at the image plane is shown in Fig. 4(c); it is similar to the object’s field distribution shown in Fig. 4(a), with a correlation coefficient of 0.978. In the absence of the superlens, the field intensity at the same distance is dispersed (Fig. 4(e)), which demonstrates the imaging capability of the proposed structure.

To validate the proposed numerical method, simulations are also performed using the commercial FEM-based software package COMSOL MULTIPHYSICS, showing good agreement between the results of two methods. It is worth noting that FEM simulation is performed using ~1.48 million degrees of freedom (DOFs), whereas the proposed method has only $N_t = 3424$ unknown coefficients.

According to the simulation results, lower dispersion is observed near the faces of the etched area that are perpendicular to the incident electric field, compared to the other faces. This phenomenon is associated with the excitation of plasmons on the metal surface, where the transmitted field is enhanced through the etched area. Next, the structure is illuminated with a plane wave with circular polarization, to excite plasmons on all metal surfaces. The field intensity distribution in this case is depicted in Fig. 5, which shows the superlens’s performance. This figure also shows good compatibility between solutions from FEM and the proposed MoM. A correlation coefficient of 0.988 for circularly polarized illumination indicates improved imaging capability, compared to the value of 0.978 for linearly polarized illumination.
Furthermore, simulations in the next section demonstrate that using circular polarization is a suitable means to enhance resolving power in both $x$ and $y$ directions.

Figure 6(a) and (b) show the field intensity distributions in the $yz$ plane across the center of the etched area, in the absence and presence of the superlens respectively. From these FEM simulation results, the light transfer to the far side of the superlens operating in the canalization regime is apparent.

**IV. SUB-DIFFRACTION-LIMITED IMAGING**

Linear and circular polarizations are employed to investigate the sub-diffraction-limited imaging performance of the optimized superlens.

**4.1. Imaging of a Single Etched Area**

Figures 7 and 8 represent the imaging results for an etched area of $\lambda_0/20$ by $\lambda_0/20$ with linear and circular polarizations respectively. The FEM simulations verify the results of the proposed CI-based MoM, as shown in these figures.
4.2. Pair Imaging with Lateral Distance

Furthermore, the proposed superlens is used to image a pair of etched areas of $\lambda/20$ by $\lambda/20$ separated by $\lambda/10$, which is much smaller than the diffraction limit. The imaging performance of the superlens for linear and circular polarizations is demonstrated in Fig. 9. As can be seen, using this superlens, two objects can be clearly resolved in the image plane, while in the absence of the lens the field intensities from the objects are mixed at this plane. Moreover, from these results and the field intensity comparison in Fig. 10, it is obvious that the superlens shows improved resolving power for circular polarization compared to linear.

4.3. Pair Imaging with Longitudinal Distance

As another example, consider two etched areas separated by a gap of $\lambda/10$ in the $y$ direction. Superlens imaging with resolution beyond the diffraction limit is indicated in Figs. 11 (a), (c), and (e).

4.4. Imaging of Four Etched Areas with Lateral and Longitudinal Distance

Finally, four etched areas with separation distance of $\lambda/10$ in both $x$ and $y$ directions are considered as an object.
The sub-diffraction-limited imaging performance of the superlens is clear from Fig. 11(d), whereas the etched areas cannot distinguish in the absence of the superlens, as shown in Fig. 11(f).

V. CONCLUSION

In this paper, an optimized multilayer superlens has been introduced for 2D sub-diffraction-limited imaging. The MoM solution of the MPIE formulation has been used for superlens analysis. The closed-form spatial-domain GFs have been calculated via the CI technique, which improves the efficiency of this method. The SA algorithm has been employed to optimize different parameters of the superlens structure, resulting in a superlens design with an appreciable correlation coefficient. Furthermore, simulations performed with FEM-based software have verified the accuracy of the superlens imaging results. Finally, the simulations associated with sub-diffraction-limited imaging have shown that, using the proposed superlens, objects as small as $\lambda/10$ can be resolved clearly. According to our findings, it is noticeable that an incident plane wave with circular polarization can improve the imaging capability and resolving power of the superlens.

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