A lattice estimate of the $g_{D^*D\pi}$ coupling

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We present the results of the first direct determination of the $g_{D^*D\pi}$ coupling using lattice QCD. From our simulations in the quenched approximation, we obtain $g_{D^*D\pi} = 18.8_{-2.0}^{+1.1} \pm 2.3_{-1.0}^{+1.1}$ and $\hat{g} = 0.67_{-0.06}^{+0.04}$. It is in agreement with a recent experimental result from CLEO.

Introduction

The strong coupling constant $g_{H^*H\pi}$ of heavy-light mesons$^1$ and a soft pion is one of the essential parameters entering the lagrangian which englobes both the chiral and the heavy quark symmetry [1]. Its precise value is useful in evaluating the nearest pole contribution to the shape of the $B \to \pi$ and $D \to \pi$ semileptonic decay form factors, since the residues are directly proportional to $g_{B^*B\pi}$ and $g_{D^*D\pi}$, respectively. As $g_{B^*B\pi}$ is not measurable experimentally due to phase space, its theoretical prediction is useful to restrict the uncertainties on quantities like the $|V_{ub}|$ CKM matrix element.

The coupling $g_{H^*H\pi}$ is related to the coupling constant $\hat{g}$ appearing in the chiral theory for heavy mesons,

$$ g_{H^*H\pi} = \frac{2\sqrt{m_{H^*}m_H}}{f_\pi} \hat{g}. $$

(1)

The coupling $\hat{g}$ is a constant up to small $O(1/m_H)$ corrections in the heavy mass. A large number of predictions for $\hat{g}$, using several methods, can be found in the literature (see [3] for a list of results). A surprising feature in these determinations is the discrepancy between QCD sum rules which find typically low values, $\hat{g} \sim 0.3$, and quark models which naturally accommodate larger values$^2$. Recently, the CLEO collaboration measured the coupling $g_{D^*D\pi}$ and deduced, from eq. (2), a value for $\hat{g}$ (see ref [3]):

$$ g_{D^*D\pi} = 17.9_{-1.9}^{+1.9}, \quad \text{i.e.} \quad \hat{g} = 0.59_{-0.07}^{+0.07}. $$

(2)

An exploratory lattice computation of $\hat{g}$ has been performed in ref. [4]. That computation has been made in the static limit of HQET. To be able to directly confront the lattice to the experimental result, we decided to make the lattice QCD simulation by using fully relativistic quarks in the region of the charm quark mass. Notice that, contrary to the $b$-quark, the charmed quarks are directly accessible from the currently available lattices. In other words, to confront to the experiment, no extrapolation in the heavy quark mass is needed.

1. Theoretical basis

To determine $g_{H^*H\pi}$, we compute the matrix element of the axial vector current $A_\mu = \bar{q} \gamma_\mu \gamma_5 q$, where

$^1$We use in a generic way the letter $H$ for the $D$ or the $B$ mesons.

$^2$The Adler-Weisberger sum rule sets the bound $\hat{g} < 1$. 

$^*\text{Presented by Gregorio Herdoiza.}$

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between the vector ($H^*$) and pseudoscalar ($H$) heavy-light mesons. This matrix element is parametrized by the three form factors $A_{0,1,2}(q^2)$, where $q$ is the momentum transfer between $H^*$ and $H$.

The coupling $g_{H^*H}^*$ is related to the form factors through the following expression (see ref.[5] for details):

$$g_{H^*H}^* = \frac{1}{f_\pi} \times \left[ (m_{H^*} + m_H) A_1(0) + (m_{H^*} - m_H) A_2(0) \right].$$

The first term ($\propto A_1(0)$) is dominant since the heavy masses $m_{H^*}$ and $m_H$ are almost degenerate and, in the kinematical region where $q^2 \sim 0$, $A_1$ and $A_2$ are of the same order of magnitude. Our simulation has confirmed that the subdominant term ($\propto A_2(0)$) is indeed small ($\lesssim 5\%$). In studying $A_1(q^2)$, the point $q^2 \sim 0$ is reached already at $q = 0$. On the lattice, it is known that the signals are better when no external momenta are given to the interacting hadrons. All these conditions make the determination of $g_{D^*D^\pi}$ rather clean.

2. Lattice simulation

The main results are obtained from a simulation on a $24^3 \times 64$ lattice, at $\beta = 6.2$, using 100 configurations produced in the quenched approximation with the non-perturbative $O(a)$ improvement for Wilson fermions.

We have used six values of the light quark masses, corresponding to a pseudoscalar meson mass $\in [0.59, 0.83]$ GeV, and three values of the heavy quark masses, corresponding to $\overline{m}_H \in [1.83, 2.32]$ GeV, where, $\overline{m}_H = (3m_{H^*} + m_H)/4$, is the spin averaged mass of the heavy meson. Note that this interval includes the value $\overline{m}_D = 1.97$ GeV, so that no heavy mass extrapolation is required to reach the $\overline{D}$-meson.

We inserted to the current $A_H$ the momentum $\vec{q} = \{(0,0,0);(1,0,0)\}$ (in $2\pi/L$ units), and we have checked that the case $\vec{q} = 0$ corresponds indeed to small values of $q^2$ (we have $q^2 \lesssim 0.01$ GeV$^2$).

The matrix elements that we compute on the lattice are illustrated in fig. 1. We extract the form factors $A_1$ and $A_2$ by using the standard analysis of the two-point and three-point Green functions on the lattice.

In order to determine $g_{D^*D^\pi}$, we first have to extrapolate our simulated light quark masses down to the $u/d$-quark (corresponding to the physical pion) and then interpolate the heavy meson masses to the physical $D$ meson.

The chiral extrapolation has been performed according to several forms (linear, quadratic, “chiral log”; see fig. 2). In our region of light quark masses, we are not able to isolate non-linearities. Therefore, we shall use non-linear fits to estimate the systematic uncertainties.

The heavy mass interpolation function is guided by the heavy quark symmetry, $\hat{g} = a + b/\overline{m}_H$. The value of the parameter $a$ corresponds to the value of $\hat{g}$ in the static limit, which we note $\hat{g}_\infty$.

As a comment, we remark that the computation of the form factors $A_{1,2}$, and therefore $g_{D^*D^\pi}$
Table 1
For each heavy-light meson mass $m_H$, we show the values of $g_{H^* H\pi}$ and of the corresponding $\hat{g}$.

| $m_H$ (GeV) | 1.83(9) | 2.08(10) | 2.32(11) |
|-------------|---------|----------|----------|
| $g_{H^* H\pi}$ | 17.7 ± 2.2 | 20.1 ± 2.7 | 22.6 ± 3.3 |
| $\hat{g}$ | 0.669(72) | 0.668(79) | 0.673(88) |

Figure 3. Interpolation of $\hat{g}$ to the $\bar{D}$-meson (filled circle).

(eq. 3), do not depend on the improvement of the bare axial current $A_\mu$ (see ref. 1).

3. Results

We present in table 1 the values of $g_{H^* H\pi}$ and $\hat{g}$ for each heavy-light meson mass $m_H$. All these quantities are already linearly extrapolated to the $u/d$-quark mass.

The heavy quark interpolation of $\hat{g}$ to the $\bar{D}$-meson mass is shown in fig. 3. Our results indicate that the slope in $1/m_H$ for the coupling $\hat{g}$ is small. Assuming that the linear dependence in $1/m_H$ holds all the way to the static limit, $1/m_H \to 0$, we obtain that $\hat{g}_\infty$ is not more than 10% larger than $\hat{g}$ at the level of the charm quark mass.

Our final results, at $\beta = 6.2$, are:

$$g_{D^* D\pi} = 18.8 \pm 2.3^{+1.1}_{-2.0}, \quad \text{i.e.}$$

$$\hat{g} = 0.67 \pm 0.08^{+0.04}_{-0.06}. \quad (4)$$

4. Systematic uncertainties

In order to study the $\mathcal{O}(a)$ effects, we have performed in addition to the simulation at $\beta = 6.2$, another one at $\beta = 6.0$. The difference between the two results is smaller than one standard deviation. With two values of the lattice spacing we cannot attempt a continuum limit extrapolation, but, as it is observed for other similar quantities, we can hope that the results at $\beta = 6.2$ are close to those in the continuum limit. Further computations at larger $\beta$ would certainly improve the precision in the determination of $\hat{g}$.

Finite volume effects were studied by performing simulations at $\beta = 6.0$ with two different volumes, $16^3 \times 64$ and $24^3 \times 64$. Within our statistics, we do not see any evidence for the presence of finite lattice volume effects. From the comparison between these simulations, we add a conservative 6% contribution to the overall systematic uncertainty.

The chiral extrapolation is the major source of uncertainty in our results. We estimate this effect by comparing the linear fit, to the quadratic and “chiral log” ones (c.f. Section 2).

Conclusion

We have performed the first lattice measurement of the $g_{D^* D\pi}$ coupling. Our results (4) are in good agreement with the experimental value (2). The discrepancies with other theoretical predictions, like the smaller values of $\hat{g}$ given by the QCD sum rules, still need an explanation. Further improvements of our results would include, studying the continuum limit and attempting an unquenched calculation of $\hat{g}$.

Work supported by the European Community’s Human potential programme under HPRN-CT-2000-00145 Hadrons/LatticeQCD.

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