Distributed switched model-based predictive control for distributed large-scale systems with switched topology

Morteza Alinia Ahandani, Hamed Kharrati, Farzad Hashemzadeh and Mahdi Baradarannia

Department of Control Engineering, Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz, Iran; Department of Electrical Engineering, Langoard Branch, Islamic Azad University, Langoard, Iran; Control and Automation Engineering Department, Istanbul Technical University, Istanbul, Turkey

Abstract

Distributed switched large-scale systems are composed by dynamically coupled subsystems, in which interactions among subsystems vary over time according a switching signal. This paper presents a distributed robust switched model-based predictive control (DSwMPC) to control such systems. The proposed method guarantees stabilising the origin of the whole closed-loop system and ensures the constraints satisfaction in the presence of an unknown switching signal. In the distributed model-based predictive control (DMPC) used in this work, by considering the interactions among subsystems as an additive disturbance, the effect of the switch is reflected on the dynamic equation, local, and consistency constraint sets of the nominal subsystems. To compensate the effect of switching signal which creates a time-varying network topology, a robust tube-based switched model-based predictive control (RSwMPC) with switch–robust control invariant set as the target set robust to unknown mode switching is used as local controller. The scheme performance is assessed using three typical examples. The simulation results show that the input and state constraints are satisfied by the proposed DSwMPC at all times. They also validate that the closed-loop system converges to the origin. Also, a comparison of the DSwMPC with a centralised SwMPC (CSwMPC) and a decentralised SwMPC (DeSwMPC) are performed.

1. Introduction

Distributed large-scale systems such as electric power systems, manufacturing systems, process plants, communication networks and swarms of robots are those of high-dimensional systems composed of interconnections of several lower-dimensional subsystems. In the distributed large-scale systems, the subsystems can interact with neighbouring subsystems by both inputs, states, and outputs and they can have their local and/or global constraints and goals. A group of this kind of system is the systems with switched topology. Switched systems as a sub-class of hybrid dynamic systems can be used to model systems that are subject to known or unknown abrupt parameter variations such as synchronously switched linear systems, networks with periodically varying switchings, and sudden change of system structures (Ahandani et al., 2020). They appear naturally in the study of multi-rate sampled-data systems. Also, the switched systems can also be employed to describe the overall system of a single process controlled utilising multi-controller switching such as the hybrid control scheme for non-holonomic systems, which are not stabilisable by means of any individual continuous state feedback controller. Hence, switching among different system structures is an essential feature of many engineering and practical real-world systems (Sun & Ge, 2005), especially in large-scale systems due to the existence of a wide variety of actuators, sensors and communication networks. In the distributed switched large-scale systems studied in this research, interactions among subsystems vary over time according a switching signal. They also can be modelled as networked control systems with time-varying network topology.

A centralised control implementation for distributed large-scale systems is typically not realistic. In a decentralised control system, no information is exchanged between the controllers and every
controller makes its own decision. Still in the distributed control systems, controllers are allowed to share their information with local controllers. This paper focuses on distributed model-based predictive control (DMPC) strategy. In a DMPC control structure, the overall system under control is divided into many interacted subsystems, where each of them is controlled using a separate local controller based on model-based predictive control (MPC) strategy. This class of controllers combines the advantage of a decentralised control structure, e.g. high flexibility and good error tolerance, and benefits of a centralised control structure, e.g. good global performance (Li & Zheng, 2016). The online solving an optimisation problem at each sampling time in MPC increases the computational burden. So, the standard centralised MPC cannot be implemented for plants with fast dynamics and/or a large number of control variables. A natural solution to this problem is to decompose the plant into smaller subsystems and then to design local controllers (Hernandez et al., 2017). This can be considered as another advantage of the distributed implementation of MPC than its centralised version.

On the other side, the structure of the many practical systems could be flexible, and it could therefore be impossible to have a constant model structure. Moreover, the uncertainty in the system structure is a serious restriction in the implementation of classical centralised or hierarchical control schemes. This restriction is appeared for example, in plug-and-play systems (Rivero et al., 2013), such as smart grids or building automation systems, in which subsystems can be connected and/or disconnected at any time. In this situation, it is hard to construct the centralised model of the system required for centralised MPC in the first place and maintain its validity over time in real time in the second place. This issue is highlighted if the number of subsystems is large and the events of connecting and disconnecting appear in a high frequency. An example of this situation can be found in power networks, whose evolution leads to an increasing number of agents with many small producers involved and where properties as adaptability, efficiency, and reliability are necessary (Negenborn & Maestre, 2014). Motivated by those mentioned above, the current research employs a DMPC for control of distributed switched large-scale systems.

The different versions of DMPC proposed in the literature (Christofides et al., 2013; Maestre & Negenborn, 2014; Negenborn & Maestre, 2014; Scattolini, 2009) can be classified according to various criteria. The DMPC can be typically categorised based on the protocol of information exchange into non-iterative or iterative algorithms. In the DMPC with a non-iterative-based algorithm, each local MPC controller communicates only once per time-step with other local MPC controllers and solves the finite horizon optimal control problem once in a control period (Vaccarini et al., 2009; Zheng et al., 2011, 2012). In the DMPC with an iterative-based algorithm, each local MPC controller communicates with other neighbouring agents several times per time-step (Giselsson & Rantzer, 2013; Li et al., 2005; Zheng et al., 2009).

Also, they can be categorised based on the type of cost function, which is optimised per time-step into cooperative or non-cooperative algorithms. In the non-cooperative version of DMPC, each local MPC controller optimises the cost function of its corresponding subsystem (Dunbar, 2007; Farina & Scattolini, 2012; Maxim et al., 2018; Yuan et al., 2017). In its cooperative version, to obtain a better performance, each local controller optimises a global cost function (Giselsson & Rantzer, 2013; Zheng et al., 2018). So, in this version, each MPC agent needs a network to exchange information with all other subsystems. In another version of cooperative DMPC, to provide a trade-off between performance and communication cost, a novel coordination strategy was proposed in Zheng et al. (2009) and Zhang and Li (2007). Here each MPC agent minimises the cost function of its own subsystem and the cost function of the subsystems directly impacts them.

Another classification of DMPC can be done in terms of the topology of the communication network into fully connected or partially connected networks. In the DMPC with a fully connected structure (Chen, 2016; Li et al., 2005; Liu et al., 2010; Stewart et al., 2010), exchange of information can be done among of all local controllers. In contrast, in DMPC with a partially connected structure (Alessio & Bemporad, 2007; Farina & Scattolini, 2012; Rawlings & Stewart, 2008; Zhang & Li, 2007), it is required that send and receive of information are only done among neighbouring subsystems (Scattolini, 2009).

For a distributed implementation of MPC regulators, various approaches have been proposed in the literature. The most widely employed idea is to consider the interactions between subsystems as local
disturbances and then to employ the local robust MPC (RMPC) controllers to handle them. Farina and Scat-tolini (2012) and Betti et al. (2014a) employed a non-cooperative, non-iterative DMPC for control of a system composed of linear discrete-time dynamically interconnected subsystems. The satisfaction of state and input constraints, guaranteeing convergence of the closed-loop system, obtaining the local control inputs of each subsystem without needing the dynamical models of the other subsystems and to be the limited transmission of information are the main traits of their proposed DMPC. Also, some realisation issues for practical use of their proposed method, such as the automatic selection of some tuning parameters, the initialisation of the algorithm, or its response to unexpected disturbances, have been discussed and solved in Betti et al. (2014b). The current research employs their proposed DMPC for control of distributed switched large-scale systems. For the reason of the existence of switch in the communication network of this kind of system, it is required to use a switched RMPC controller instead of a simple RMPC for each subsystem.

Recently, some researches have been applied the MPC to switched systems. In Zhang et al. (2016) a switched MPC (SwMPC) of a class of discrete-time switched linear systems with idea of mode-dependent dwell-time (MDT) of variable lengths was investigated. Qi et al. (2021) investigated observer-based model MPC for switched systems with a mixed time/event-triggering mechanism. Piecewise Lyapunov function technique and average dwell time approach were employed to guarantee asymptotical stability. A stabilising MPC scheme for discrete-time switched linear systems with at least one stabilisable subsystem was designed in Augustine and Patil (2022). Tian et al. (2022) proposed a force-driven switched MPC path tracking control strategy of autonomous vehicles based on the system prediction model. An observer-based MPC for the discrete-time switched systems suffered by event-triggered mechanism and denial-of-service (DoS) attacks presented in Yan, Shi et al. (2023). To save network resources, they designed an event-triggered mechanism based on dwell time and triggered error. An upper bound of attack duty cycle was obtained so as the exponential convergence of the system could still be archived. Yan, Xue et al. (2023) studied a mixed time/event-triggered quantised model predictive security control (MPSC) with denial-of-service attacks for the switched systems. They transformed the MPC design into an optimisation problem minimising the constant upper bound of DoS. On this basis, the sufficient conditions were deduced for the exponential convergence of closed-loop systems. This research uses an SwMPC proposed in Danielson et al. (2019). By defining a new type of control invariant sets, called switch-robust control invariant (switch-RCI) sets, which are robust to unknown mode switching and under restrictions of minimum dwell-time and admissible mode transitions, it derived necessary and sufficient conditions for guaranteeing constraint satisfaction to control of constrained switched systems. The switch-RCI sets were used in the framework of an SwMPC to ensure constraint satisfaction in the presence of unknown mode switching with known minimum dwell-time.

Motivated by the above discussion, We employ a distributed SwMPC (DSwMPC) for the distributed switched constrained large-scale systems with input and state constraints.

### 1.1. Switch and large-scale systems: a literature review

The published works about the relation of the switch and large-scale systems can be categorised into three main groups. The first one includes those of researches in which the switch exists in the network topology of the system. One of the published studies on distributed control of distributed switched large-scale system is related to Zhang et al. (2017). They investigated sensor-network-based distributed control of networked control systems, in which the communication constraint and topology switching problems were addressed. Based on the Lyapunov direct method and the switched system approach, a sufficient condition was established to ensure the exponential stability of the closed-loop system. Also, Schiff er et al. (2017) studied large-scale systems with the dynamic communication topology. In this research, they extended a strict Lyapunov function to a common Lyapunov–Krasovskii functional to provide sufficient delay-dependent conditions for robust stability of a distributed averaging-based integral-controlled power system with dynamic communication topology as well as heterogeneous constant and fast-varying delays. The single published research in which the MPC was employed for the distributed large-scale systems with switched topology is related to Shi et al. (2020). They
proposed a controller with a hierarchical structure for a class of distributed networked control systems with quantisation and switching topology. An upper control layer receives the system-wide information and solves the MPC optimisation problem in a centralised fashion, while the local controllers are implemented in a distributed way. Based on the switched control system theory and the Lyapunov direct method, the desired controller is designed by solving a constrained LMI optimisation problem at each time step.

The second group includes large-scale system of interconnected switched subsystems. In this kind of systems, each subsystem has its own switching signal which can be common or different from switching signal of other subsystems. Long and Zhao (2016) developed a switched-dynamic-surface-based decentralised adaptive neural output-feedback control approach for a class of switched large-scale uncertain nonlinear systems. Their method guarantees that all the signals in the resulting closed-loop system are semiglobal uniformly ultimately boundedness under a class of switching signals with average dwell time, and the tracking errors converge to a small neighbourhood of the origin. A switched decentralised adaptive control scheme was developed in Zhai et al. (2018) for switched interconnected nonlinear systems under arbitrary switching. Li et al. (2021) proposed a decentralised adaptive fuzzy sampled-data control for switched large-scale nonlinear systems with time-varying delays. In their strategy, the switching signal of each investigated large-scale system’s subsystem is allowed to be different by selecting a suitable common Lyapunov function.

The third group includes the researches in which the switch has been inserted in the network topology of controllers to provide a reconfiguration capability. In this group, the topology of the applied system does not include an online switching. Monasterios et al. (2019) studied the nesting properties of feasible regions and robust positive invariant sets for different partitions of the large-scale system. It investigated how subsystems may be grouped together into coalitions. A coalition of subsystems is a non-empty subset of all subsystems. A coalitional controller replaces local subsystem controllers and may achieve better performance, albeit at a higher cost of complexity and communication. Also a coalitional control framework based on a switching MPC architecture aimed at large-scale dynamically-coupled systems was employed by Fele et al. (2018). Their proposed game theoretical framework provides a dynamic establishment of cooperation in the control of a multi-agent system. The obtained results showed that the reconfiguration capabilities provided to the system through the proposed framework were suited for fault-tolerance needs or plug-and-play settings.

A switching mechanism for communication between local MPC controllers for large-scale systems was employed in Núñez et al. (2015). They proposed a time-varying scheme for non-centralised MPC of large-scale systems. In this strategy, several possible control structures for the communication between subsystems are considered and the hierarchical control system implements the one with the best performance according to a set of given objectives. Barreiro-Gomez et al. (2019) studied two fundamental components of the design of distributed optimisation-based controllers for large-scale systems, i.e. system partitioning and distributed optimisation algorithms. They combined the DMPC with the distributed partitioning algorithm with static and dynamical system partitioning. The obtained results of these two DMPC controllers demonstrated the effectiveness of both the density-dependent population game approach and the partitioning for a large-scale system.

Also, in some researches, the MPC is employed to handle plug and play operations in the structure of large-scale systems. Plug-and-play operation means that we exchange a subsystem in the overall system with another, possibly different, one or we add or remove a subsystem to the overall system. Lucia et al. (2015) proposed a DMPC to control coupled, possibly large-scale, systems for the case of plug-and-play operations. They explicitly consider the case in which a subsystem of the network is exchanged by a new one, but, the same physical interconnections. In their proposed method, when an exchange request is received, it is checked if the new subsystem can fulfill the contracts of the old subsystem, given the contracts of its future neighbours and its own dynamics and degrees of freedom. If this is possible, there exists no need to perform any redesign of the controllers. If the new subsystem cannot comply directly with the
existing contracts, the controllers of all the neighbours can be virtually redesigned/tuned at the time which the request takes place. If the virtual redesign is infeasible, the exchange operation is rejected; otherwise it is performed. Hou et al. (2021) designed a DMPC for networked systems where certain subsystems may be removed or inserted. In their proposed DMPC for this kind of reconfigurable systems, once the switch of topology is commanded, some additional optimisation problems required to be solved in the related subsystems’ controllers to ensure the feasibility of removal or plugging-in operation. Bai et al. (2020) investigated the DMPC of linear systems whose network topologies are changeable by the way of inserting new subsystems, disconnecting existing subsystems, or merely modifying the couplings between different subsystems. They firstly presented a distributed reconfiguration control strategy applying to any reconfigurable requirements. Then, based on the alternating direction method of multipliers (ADMM) algorithm, the way to redesign the reconfigured DMPC controller was provided and the iterative formulas employed in solving the reconfiguration optimisation problem via ADMM algorithm were derived.

Based on categories mentioned above, in the current research, a switch exists in both the network topologies of controllers and system. The contribution of the paper is to apply the robust SwMPC (RSwMPC) controllers in a distributed fashion on the distributed large-scale systems with switched topology, input and state constraints, in which the subsystems interact with each other by states and inputs. In comparison with Shi et al. (2020), this work considers a more general system that includes both input and state constraints and both input and state interactions. Also, the structure of the controller in Shi et al. (2020), is hierarchical and the MPC optimisation problem has to be solved in a centralised fashion. So its computational complexity increases when the modes of switching signal and the number of the subsystems increase. In our proposed DSwMPC, the MPC optimisation problems are locally solved for each controller, so the computational burden is considerably less than a centralised structure. Also, we already successfully applied a decentralised SwMPC (DeSwMPC) on distributed switched large-scale systems in Ahandani et al. (2020). In comparison with Ahandani et al. (2020), structure of the controller proposed in the current research is distributed and the interactions among subsystems in Ahandani et al. (2020) were only based on states, whereas the proposed approach in the current research can handle states and input interactions. Also our other main contribution is that we propose a modified switch-RCI set, called stabiliser switch-RCI to guarantee stability and also feasibility when a switch occurs which at the same time provides closed-loop constraint satisfaction.

The rest of the paper is organised as follows. In the next section, distributed switched large-scale systems are formulated. In Section 3, the employed DMPC strategy is presented. Section 4 contains the used MPC algorithm for switched constrained systems. The proposed switched DMPC is presented in Section 5. The simulation results are presented and analyzed in Section 6. Section 7 concludes the paper.

1.2. Notations and definitions

The Minkowski addition and subtraction of sets are denoted by the symbols $\oplus$ and $\ominus$, respectively, and they are defined as follows: $C = A \oplus B = \{c = a + b : \text{for all } a \in A, b \in B\}$ and $C = A \ominus B = \{c : c + b \in A, \text{for all } b \in B\}$. A set $\mathcal{O}$ is a control invariant set for the system $x(t + 1) = f(x, u)$ if there exists an admissible control law $u(x) \in \mathcal{U}$ such that for all $x \in \mathcal{O}$, $f(x, u) \in \mathcal{O}$ and for all $t \geq 0$. Also, $\mathcal{O}$ is positive invariant set for $x(t + 1) = f(x)$ if for all $x \in \mathcal{O}$, $f(x) \in \mathcal{O}$ and for all $t \geq 0$. A necessary and sufficient condition to be control invariance and as well as positive invariance is $\mathcal{O} \subseteq \text{Pre}(\mathcal{O})$ where Pre($\mathcal{S}$) is the predecessor operator that is the set of states that can be directed to the target set $\mathcal{S}$ under the dynamics of the system $x(t + 1) = f(x, u)$, i.e. $\text{Pre}(\mathcal{S}) = \{x|\exists u, f(x, u) \in \mathcal{S}\}$ without violating constraints. Also the set of $\text{Pre}^k(\mathcal{S}) \subseteq \mathcal{X}$ is the set of states $x \in \mathcal{X}$ that can be directed to the target set $\mathcal{S}$ under the dynamics of the system in $k$ discrete-time instances without violating the constraints. The set $\Omega \subseteq \mathcal{R}^{nx}$ is robust positively invariant (RPI) set for the system $x(t + 1) = f(x, w)$ where $w \in \mathcal{W} \subseteq \mathcal{R}^{nw}$ is a disturbance vector, if $f(x, w) \subseteq \Omega$ for all $x \in \Omega$ and all $w \in \mathcal{W}$ and for all $t \geq 0$, i.e. if only if $f(\Omega, \mathcal{W}) \subseteq \Omega$. The RPI set $\Omega$ is minimal if it is contained in every other RPI $\Omega$ verifies $\Omega \subseteq \Omega$.

2. Problem statement

Large-scale systems have traditionally been characterised by large numbers of variables, the structure of interconnected subsystems, and other features that
complicate the control models such as nonlinearities, time delays, and uncertainties. The decomposition of this kind of system into interconnections of a set of smaller-scale and more manageable subsystems allows for implementing effective decentralisation and coordination mechanisms (Filip & Leiviska, 2009). This decomposition or partitioning is often performed for either computational or practical reasons. In order to design a control structure based on robust control for large-scale systems, this research used the modelling proposed in Li and Zheng (2016). This paper considers the constrained discrete-time LTI distributed large-scale system, described by the following state space model:

\[
x(t + 1) = Ax(t) + Bu(t)
\]

\[
x(t) \in \mathcal{X}, \quad u(t) \in \mathcal{U}
\]

where \(x(t) \in \mathbb{R}^{n_x}\) and \(u(t) \in \mathbb{R}^{n_u}\) are the state and input of system, respectively. \(\mathcal{X}\) and \(\mathcal{U}\) are state and control constraint sets. It is assumed that the distributed large-scale system \(\mathcal{S}\) can be decomposed of \(M\) discrete-time linear subsystems \(\mathcal{S}_i, i \in \mathcal{P} = \{1, \ldots, M\}\), each of which is controlled by controller \(C_i\). Also, subsystems are dynamically interacted through states and inputs. Then, the subsystem \(\mathcal{S}_i\) has the following model:

\[
x_i(t + 1) = A_{ii}x_i(t) + B_{ii}u_i(t) + \sum_{j \in \mathcal{N}_i} A_{ij}x_j(t)
\]

\[+ \sum_{j \in \mathcal{N}_i} B_{ij}u_j(t)
\]

where \(x_i(t) \in \mathbb{R}^{n_{x_i}}\) and \(u_i(t) \in \mathbb{R}^{n_{ui}}\) are the state and input vectors of i-th subsystem, respectively. \(A_{ii} \in \mathbb{R}^{n_{x_i} \times n_{x_i}}, B_{ii} \in \mathbb{R}^{n_{ui} \times n_{ui}}, A_{ij} \in \mathbb{R}^{n_{x_i} \times n_{x_j}}\) and \(B_{ij} \in \mathbb{R}^{n_{ui} \times n_{ui}}\). \(\mathcal{N}_i\) is the set of neighbours of \(\mathcal{S}_i\) defined as:

\[
\mathcal{N}_i = \{j \in \mathcal{P} : A_{ij} \neq 0 \text{and/or} B_{ij} \neq 0, i \neq j\}
\]

It is said that subsystem \(\mathcal{S}_i\) is a dynamic neighbour of subsystem \(\mathcal{S}_i\) if only if at least one of the matrices \(A_{ij}\) or \(B_{ij}\) is non-zero, so \(\mathcal{N}_i\) will be non-empty. The subsystem \(\mathcal{S}_i\) includes the local (uncoupled) state and control constraints

\[
x_i(t) \in \mathcal{X}_i \subseteq \mathbb{R}^{n_{x_i}}, \quad u_i(t) \in \mathcal{U}_i \subseteq \mathbb{R}^{n_{ui}}
\]

Also, \(n_x = \sum_{i \in \mathcal{P}} n_{x_i}, \quad n_u = \sum_{i \in \mathcal{P}} n_{ui}, \quad \mathcal{X} = \prod_{i=1}^{M} \mathcal{X}_i\) and \(\mathcal{U} = \prod_{i=1}^{M} \mathcal{U}_i\). Also the interacted subsystems can be subject to global (coupled) constraints described in collective form by

\[
\sum_{i=1}^{M} (\psi_{i}^X x_i(t) + \psi_{i}^B u_i(t)) \leq 1_p
\]

where the matrices \(\psi_{i}^X \in \mathbb{R}^{p \times n_{x_i}}\) and \(\psi_{i}^B \in \mathbb{R}^{p \times n_{u_i}}\) define the global constraint for subsystem \(\mathcal{S}_i\) and \(1_p\) is a \(p\)-vector of all ones. By definition of the interaction vector \(w_i\), named as perturbation or disturbance vector, Equation (2) is rewritten as follows:

\[
x_i(t + 1) = A_{ii}x_i(t) + B_{ii}u_i(t) + w_i(t)
\]

where

\[
w_i(t) = \sum_{j \in \mathcal{N}_i} A_{ij}x_j(t) + \sum_{j \in \mathcal{N}_i} B_{ij}u_j(t)
\]

In the distributed switched large-scale systems, a switching signal changes the network topology over time. So this kind of system is described by the following state space model:

\[
x(t + 1) = A^{\sigma(t)}x(t) + B^{\sigma(t)}u(t)
\]

\[
x(t) \in \mathcal{X}^{\sigma(t)}, \quad u(t) \in \mathcal{U}^{\sigma(t)}
\]

where the switching sequence \(\sigma : \mathcal{N} \rightarrow \mathcal{I}\) is a known or unknown exogenous input that switches network topology between a finite number of modes \(\mathcal{I} \subseteq \mathcal{N}\). From the viewpoint of the dynamic of the \(i\)-th subsystem, due to switching signal varies the interactions among subsystems, it affects only the interaction vector \(w_i\). In other words, a switch can only change the set of neighbours of \(\mathcal{S}_i\), i.e. \(\mathcal{N}_i\). So the dynamic equation of subsystem \(\mathcal{S}_i\) in such a system can be expressed as

\[
x_i(t + 1) = A_{ii}x_i(t) + B_{ii}u_i(t) + w_i^{\sigma(t)}(t)
\]

where

\[
w_i^{\sigma(t)}(t) = \sum_{j \in \mathcal{N}_i^{\sigma(t)}} A_{ij}x_j(t) + \sum_{j \in \mathcal{N}_i^{\sigma(t)}} B_{ij}u_j(t)
\]

Equations (10) and (11) clearly show that the local constraints of subsystems have not been affected by switch in network topology but the global or interaction constraints can be varied by a switching signal (Ahandani et al., 2020). So, the system employed in this research
to be controlled is a distributed switched large-scale system described by the state space model of (9). This system includes \( M \) coupled subsystems with dynamics shown in (10) and (11). In this networked modelling with time-varying topology, the role of switching signal \( \sigma(t) \) is that it provides a possibility to vary the set of neighbours of subsystems. This paper, based on this modelling, aims to design a distributed control structure to be applied on such systems taking into account that the switched interaction term among the subsystems can be considered as disturbances to be rejected.

3. Distributed robust tube-based MPC

This section describes the distributed robust tube-based MPC proposed in Farina and Scattolini (2012), Betti et al. (2014a) and Betti et al. (2014b) for control of large-scale constrained discrete-time linear systems consisting dynamically coupled subsystems. This DMPC in combination with SwMPC will be adopted to switch to the network topology of the system (see Sect. 5).

The DMPC presented in Farina and Scattolini (2012), Betti et al. (2014a) and Betti et al. (2014b) guarantees stability and convergence properties of the system (1) and constraints satisfaction under mild assumptions. The dynamic equations of subsystems, in (2), can be subjected to local and global state and control constraints. In their proposed DMPC, at each time instant, the subsystem \( S_i \) sends information about its future state \( \tilde{x}_i \) and input \( \tilde{u}_i \) reference trajectories, calculated based on the solution of optimal control problem in the previous time instant, to its interconnecting subsystems. The DMPC solves the optimal control problem of current time instant as its actual trajectories \( x_i \) and \( u_i \) lie within certain regions in the neighbourhood of the reference ones. The DMPC limits the error between reference trajectories, i.e. \( \tilde{x}_i \) and input \( \tilde{u}_i \), and predictive states calculated in the current sampling time within certain regions by adding some new constraints to the optimal control problem of each subsystem. So the DMPC considers the differences \( x_i - \tilde{x}_i \) and input \( u_i - \tilde{u}_i \) as unknown bounded disturbances and it employs a tube-based RMP controller motivated by the discussion in Mayne et al. (2005) for each subsystem to reject them.

Consider the dynamic of a subsystem \( S_i \) as (2) subjected to constraints of (3) where \( \mathcal{N}_i \) and \( \mathcal{U}_i \) are convex neighbourhoods of the origin. Also, for the system (2), it can be considered the global static constraints as follows

\[
H_i(x(t), u(t)) \leq 0
\]

where \( s = 1, \ldots, n_c \). The subsystem \( S_i \) is subjected to constraint \( H_i \) if \( x_i \) and/or \( u_i \) are arguments of \( H_i \), while \( D_i = \{ s \in \{1, \ldots, n_c \} : H_i \text{is constraint on} \} \) denotes the set of constraints on \( S_i \). The subsystem \( S_i \) is a constraint neighbour of subsystem \( S_i \) if there exists \( s \in D_i \) such that \( x_j \) and/or \( u_j \) are arguments of \( H_s \), while \( \mathcal{H}_i \) denotes the set of the constraint neighbours of \( S_i \) (Farina & Scattolini, 2012), Betti et al. (2014a) and Betti et al. (2014b) In general, \( S_j \) is called a neighbour of \( S_i \) if \( j \in \mathcal{P}_i \) where \( \mathcal{P}_i = \mathcal{N}_i \cup \mathcal{H}_i \). Concerning system (1) and its partition, the following primary assumption on decentralised stabilisability is introduced.

**Assumption 3.1:**

(i) The matrices \( F_{ii} = A_{ii} + B_{ii}K_i \)

\( i \in \mathcal{P}_i \) are Schur.

(ii) The matrix \( F = A + BK \) is Schur.

where \( K = \text{diag}\{K_1, \ldots, K_M\} \).

Let \( \tilde{x}_i(t + k) \) and \( \tilde{u}_i(t + k) \) are future state and input reference trajectories transmitted by \( S_i \) to its neighbour subsystems. In DMPC some added constraints to finite-horizon optimal control problem ensure that real trajectories of each subsystem lie in the specified time-invariant neighbourhoods of their reference trajectories, i.e. \( x_i(t) \in \tilde{x}_i(t) + \mathcal{E}_i \) and \( u_i(t) \in \tilde{u}_i(t) + \mathcal{E}_i^u \), where \( 0 \in \mathcal{E}_i \) and \( 0 \in \mathcal{E}_i^u \). So the dynamic equation of the subsystem \( S_i \) in (2) can be expressed as

\[
x_i(t + 1) = A_{ii}x_i(t) + B_{ii}u_i(t) + \sum_{j \in \mathcal{N}_i} A_{ij}\tilde{x}_j(t) + \sum_{j \in \mathcal{N}_i} B_{ij}\tilde{u}_j(t) + w_i^f(t)
\]

(13)

where

\[
w_i^f(t) = \sum_{j \in \mathcal{N}_i} A_{ij}(x_j(t) - \tilde{x}_j(t)) + \sum_{j \in \mathcal{N}_i} B_{ij}(u_j(t) - \tilde{u}_j(t)) \in W_i
\]

(14)

and \( W_i = \bigoplus_{j \in \mathcal{N}_i} (A_{ij}\mathcal{E}_j + B_{ij}\mathcal{E}_j^u) \). In DMPC, a local robust MPC is employed for each subsystem with the dynamic of (13), where the term \( \sum_{j \in \mathcal{N}_i} A_{ij}\tilde{x}_j(t + k) + \sum_{j \in \mathcal{N}_i} B_{ij}\tilde{u}_j(t + k) \) is a prior known input over the
prediction horizon \( k = 0, \ldots, N_i - 1 \) and \( w_i^E(t) \) is a bounded disturbance to be rejected. In order to produce predictions, a nominal model for subsystem \( S_i \) is obtained by removing the disturbance term of \( w_i^E(t) \) in (13):

\[
x_i(t + 1) = A_{ii}x_i(t) + B_{ii}u_i(t) + \sum_{j \in N_i} A_{ij}\hat{x}_j(t) + \sum_{j \in N_i} B_{ij}\hat{u}_j(t)
\]

(15)

So the local controller \( C_i \) when the current state of subsystem \( S_i \) is \( x_i(t) \), is defined in the form of an implicit MPC law as follows:

\[
u_i(t) = \hat{u}_i(t) + K_i(x_i(t) - \hat{x}_i(t))
\]

(16)

where \( K_i \in \mathbb{R}^{n_{ui} \times n_{xi}} \), \( i \in \mathcal{P}_i \) must be chosen to satisfy Assumption 3.1. A local MPC controller is employed to obtain two unknown variables of \( \hat{u}_i(t) \) and \( \hat{x}_i(t) \).

Given (13), (15) and (16), and since \( F_{ii} = A_{ii} + B_{ii}K_i \) is Schur and \( w_i^E \in \mathcal{W}_i \) is bounded, they define the non-empty RPI sets, \( \mathcal{Z}_i \) (see Farina & Scattolini, 2012). Based on \( \mathcal{Z}_i \), two new sets, neighbourhoods of origin, \( \Delta E_i \) and \( \Delta U_i \) for \( i = 1, \ldots, M \) are defined in a way that \( \Delta E_i \oplus \mathcal{Z}_i \subseteq E_i \) and \( \Delta U_i \oplus K_i\mathcal{Z}_i \subseteq U_i \) respectively (Mayne et al., 2005).

By assuming that each subsystem \( S_i \) knows future reference trajectories of its neighbours over the entire prediction horizon, i.e. \( \bar{x}_i(t + k) \) and \( \bar{u}_i(t + k) \), \( k = 0, \ldots, N_i - 1 \) and \( j \in N_i \cup H_i \cup \{ i \} \), the unknown variables of (16) are computed by online solving the MPC local performance index of subsystem \( S_i \) described in (17a) at each sampling time \( t \):

\[
\min_{\hat{x}_i(0) \in N_i - 1} \sum_{k=0}^{N_i-1} (\|\hat{x}_i(k|t)\|^2_Q + \|\hat{u}_i(k|t)\|^2_R) + \|\hat{x}_i(N_i|t)\|^2_R
\]

(17a)

subject to (13),

\[
x_i(t) - \hat{x}_i(t) \in \mathcal{Z}_i
\]

(17b)

\[
\hat{x}_i(t + k|t) - \bar{x}_i(t + k|t) \in \Delta E_i \quad k = 0, \ldots, N_i - 1
\]

(17c)

\[
\hat{u}_i(t + k|t) - \bar{u}_i(t + k|t) \in \Delta U_i \quad k = 0, \ldots, N_i - 1
\]

(17d)

\[
\hat{x}_i(t + k|t) \in \hat{X}_i, \quad \hat{u}_i(t + k|t) \in \hat{U}_i
\]

(17e)

\[
\hat{x}_i(t + N_i|t) \in \hat{X}_i
\]

(17f)

where \( N_i \in \mathcal{N} \) is the prediction horizon, \( l_i(\hat{x}_i(k|t)), \hat{u}_i(k|t)) = (\|\hat{x}_i(k|t)\|^2_Q + \|\hat{u}_i(k|t)\|^2_R) \) is stage cost, \( V_{fi}(\hat{x}_i(t + N_i|t)) = \|\hat{x}_i(N_i|t)\|^2_{R_{fi}} \) is terminal cost and \( \hat{X}_i \) is terminal region.

The local state and input constraints for the nominal system (15) are \( \hat{X}_i \) and \( \hat{U}_i \), respectively. These constraints, used in (17e), can be obtained after computing the RPI set \( \mathcal{Z}_i \) as

\[
\hat{X}_i = X_I \oplus \mathcal{Z}_i
\]

(18)

\[
\hat{U}_i = U_I \oplus K_i\mathcal{Z}_i
\]

(19)

The positive-definite stage and terminal matrices in (17), i.e. \( Q_i \), \( R_i \) and \( P_i \), are chosen based on algorithms discussed in Betti et al. (2014a) to ensure the stability of MPC. Also, other design parameters employed in DMPC i.e. the sets \( \mathcal{Z}_i \), \( \Delta E_i \) and \( \Delta U_i \), \( \hat{X}_i \) and the gain matrices \( K_i \) are calculated as proposed in Betti et al. (2014a).

4. Switched model-based predictive control

Danielson et al. (2019) studied the control of switched constrained linear systems by defining a new kind of control invariant sets for switched constrained systems robust to unknown mode switching, named switch-RCI sets. These switch-RCI sets are used to derive necessary and sufficient conditions for the existence of a control-law to ensure constraint satisfaction in the presence of unknown mode switching with known minimum dwell-time. The switch-RCI sets are also employed to design a recursively feasible MPC that enforces closed-loop constraint satisfaction for switched constrained systems. This section briefly describes the SwMPC proposed by Danielson et al. (2019).

Consider a switched constrained system described as follows:

\[
x(t + 1) = f_{\sigma(t)}(x(t), u(t))
\]

\[
x(t) \in \mathcal{X}_{\sigma(t)}, \quad u(t) \in \mathcal{U}_{\sigma(t)}
\]

(20)

where \( x(t) \in \mathbb{R}^{n_x} \) is the state and \( u(t) \in \mathbb{R}^{n_u} \) is the input. The switching sequence \( \sigma : \mathcal{N} \rightarrow \mathcal{I} \) is an unknown exogenous input that switches the dynamics \( f_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_x} \) and the constraint sets \( \mathcal{X} \in \)
\( \mathcal{R}^{n_s} \) and \( \mathcal{U} \in \mathcal{R}^{n_u} \) between a finite number of modes \( \mathcal{I} \subseteq \mathcal{N} \). The number of inputs \( n_u \) may depend on the mode \( i \in \mathcal{I} \). Danielson et al. (2019) implemented two common constraints on the switching sequence, i.e. dwell-time and mode transition restrictions. The set of switching signals \( \sigma \) that satisfies these restrictions is described as follows:

\[
\sum (d, G) = \{ \sigma : \mathcal{N} \rightarrow \mathcal{I} : \text{dwell}_{i}(\sigma) \geq d_i, (\sigma(\tau_j), \sigma(\tau_{j+1})) \in E, \forall s \in \mathcal{N} \} \tag{21}
\]

where

\[
dwell_i(\sigma) = \min(\tau_{s+1} - \tau_s : \sigma(\tau_s) = i, s \in \mathcal{N}) \tag{22}
\]

and \( \text{dwell}_{i}(\sigma) \) is dwell-time of a mode \( i \in \mathcal{I}, \tau_s \in \mathcal{N} \) are the discrete-times at which the mode changes \( \sigma(\tau_s) \neq \sigma(\tau_s - 1) \) and \( G = (\mathcal{I}, E) \) is a directed graph in which the graph nodes \( \mathcal{I} \) denote the modes of the switched system (20) and each directed edge \((i, j) \in E \) indicates that a switch from mode \( \sigma(\tau_s) = i \) to mode \( \sigma(\tau_{s+1}) = j \) is allowed. For brevity, we will use the short-hand \( \sum = \sum (d, G) \) later.

The system (20) has a property of the time-dependent dwell-time denoted by \( \delta(t) \), where

\[
\delta(t + 1) = \begin{cases} 
\max(\delta(t) - 1, 0) & \text{if } \sigma(t + 1) = \sigma(t) \\
\frac{d_{\sigma(\tau_{t+1})}}{\delta(\tau_{t+1})}, & \text{otherwise}
\end{cases} \tag{23}
\]

The objective of Danielson et al. (2019) is to determine whether there exists a control-law \( u(t) \in \mathcal{U}_{\sigma(t)} \) and \( x(t) \in X_{\sigma(t)} \) for all allowable switching signals \( \sigma(t) \in \Sigma \) and subsequent times \( t \geq 0 \) provided that initial states, modes, and remaining dwell-times are \( x_0, \sigma_0 \) and \( \delta_0 \), respectively.

They defined a new class of control invariant sets for switched constrained systems (20) named switch-RCI sets. Switch-RCI sets \( \{C_i\}_{i \in \mathcal{I}} \) are constraint admissible control invariant sets that are robust to unknown mode switching provided that switching sequences are contained in (21). They are defined in Definition 4.1.

**Definition 4.1 (Switch-RCI sets):** A collection of sets \( C_i \subseteq X_i \) for \( i \in \mathcal{I} \) is named switch-RCI if they are control invariant, i.e. \( C_i \subseteq \text{Pre}_{i}^{d_j}(C_j) \), for each mode \( i \in \mathcal{I} \) and mutually reachable \( C_i \subseteq \text{Pre}_{j}^{d_j}(C_j) \). The dwell-time \( d_j \) for each allowable mode transition \( (i, j) \in E \).

Then they by defining maximal switch-RCI sets in Definition 4.2, combined the sufficient and necessary conditions for the existence of a control-law to ensure constraint satisfaction.

**Definition 4.2 (Maximal Switch-RCI Sets):** A collection of sets \( \{C_i\}_{i \in \mathcal{I}} \) is called maximal switch-RCI if they are switch-RCI and, for any group of switch-RCI sets \( \{C_i\}_{i \in \mathcal{I}} \), we have \( C_i \subseteq C_i^\infty \) for each mode \( i \in \mathcal{I} \).

**Theorem 4.1:** A control-law can be defined as \( k(x, \sigma, \delta) \) to guarantee constraint satisfaction for all times \( t \in \mathcal{N} \) and for all allowable switching signals \( \sigma \in \Sigma \) if and only if the initial state \( x(t_0) \), mode \( \sigma(t_0) \), and remaining dwell-time \( \delta(t_0) \) are contained in the following initial conditions set for the maximal switch-RCI sets \( \{C_i\}_{i \in \mathcal{I}} \) :

\[
\mathcal{I}C = \{(x(t_0), \sigma(t_0), \delta(t_0)) : (x(t_0) \in \text{Pre}_{\sigma(t_0)}(C_{\sigma(t_0)}) \}
\tag{24}
\]

Pseudocode of Figure 1 computes the maximal switch-RCI sets \( \{C_i\}_{i \in \mathcal{I}} \) (Danielson et al., 2019). Danielson et al. (2019) employed the maximal switch-RCI sets \( \{C_i\}_{i \in \mathcal{I}} \) to obtain terminal constraint set in the SwMPC to generate control inputs that satisfy state and input constraints. They formulated the SwMPC where (i) the prediction horizon is longer than the dwell-time of each mode \( i \in \mathcal{I} \) (i.e. \( N \geq d_i \)), named long-horizon SwMPC, (ii) the prediction horizon is shorter than the dwell-time of each mode \( i \in \mathcal{I} \) (i.e. \( N \leq d_i \)), named short-horizon SwMPC. In this section, we concentrate on long-horizon SwMPC. The following proposed relations can be extended for short-horizon SwMPC with some modifications based on Danielson et al. (2019).

1. for each mode \( i \in \mathcal{I} \) do
2. \( \Omega_i^0 = X_i \)
3. end for
4. repeat
5. for each mode \( i \in \mathcal{I} \) do
6. update sets
7. \( \Omega_i^{k+1} = \Omega_i^k \cap \text{Pre}_{\sigma_i}(\Omega_j^k) \cap \text{Pre}_{j}^{d_j}(\Omega_j^k) \)
8. end for
9. until \( \Omega_i^{k+1} = \Omega_i^k \) for all \( i \in \mathcal{I} \)
10. \( C_i^\infty = \Omega_i^k \) for all \( i \in \mathcal{I} \)

**Figure 1.** Pseudocode for computation of maximal switch-RCI sets (Danielson et al., 2019).
For the long-horizon SwMPC, the control input \( u(t) \) is computed by solving the following open-loop constrained finite horizon optimal control problem:

\[
\min_{u(0:N-1)} \sum_{k=0}^{N-1} P^\sigma(t)(x(k|t), u(k|t)) + V_f^\sigma(t)(x(N|t))
\]

\[ (25a) \]

\[ x(t + k + 1) = f(x(t), u(t + k|t)) \quad k = 0, \ldots, N - 1 \]

\[ (25b) \]

\[ x(t + k|t) \in \mathcal{X}^\sigma(t), \quad u(t + k|t) \in \mathcal{U}^\sigma(t) \quad k = 0, \ldots, N - 1 \]

\[ (25c) \]

\[ x(t + k|t) \in \mathcal{T}^\sigma(t) \quad \text{for} \quad k \geq \delta(0|t) \]

\[ (25d) \]

in which \( x(0|t) = x(t) \) is the initial condition of the switched system defined in (20), \( x(t + k|t) \) is the predicted state trajectory of the system driven by \( u(t + k|t) \) over the prediction horizon \( N \geq \delta_k \), \( \sigma(0|t) = \sigma(t) \) is the current mode of the system, \( \delta(0|t) = \delta(t) \) is the remaining dwell-time, and \( \mathcal{T}^\sigma(t) \) is the switch-RCI set. By solving (25), the obtained control input guarantees the constraints satisfaction and the terminal cost, \( V_f^\sigma(t)(\cdot) \), and the stage cost, \( P^\sigma(t)(\cdot, \cdot) \), are assigned to satisfy secondary control objectives such as stability or reference tracking for the individual modes. In solving optimisation problem (25), it is assumed that the mode \( \sigma(t) \in \mathcal{I} \) is constant \( \sigma(k|t) = \sigma(0|t) \) over the entire horizon \( k = 0, \ldots, N - 1 \). The control input is the first element \( u^*(0|t) \) of the optimal open-loop input sequence \( u^*(0|t), \ldots, u^*(N - 1|t) \) i.e.

\[ u(t) = u^*(0|t) \]

\[ (26) \]

In contrast to traditional MPC, due to being depend on the current state \( x(t) \), mode \( \sigma(t) \), and the remaining dwell-time \( \delta(t) \), the controller (26) is mode-varying and time-varying (Danielson et al., 2019). Also for more details about stability and the recursively feasible of (25) see Danielson et al. (2019).

### 5. Distributed switched model-based predictive control

In order to adopt the DMPC of Section 3 for the large-scale systems with switched topology, this section presents the DSwMPC. In this method, the local RSwMPC controllers are used to handle the existence of switch in the network topology, instead of employing the RMPC regulators. The DSwMPC preserves all characteristics of DMPC such as asymptotic stability of the closed-loop system and constraints satisfaction. Because of employing the switch-RCI sets, the DSwMPC is robust to unknown mode switching. In other words, the DSwMPC is robust to unknown switching times and unknown next topologies, but as mentioned for SwMPC, it is assumed that a minimum dwell-time and allowable mode transitions are known in prior. Restriction of mode transitions for the distributed switched large-scale systems means that it is not necessary to know neighbours of a subsystem in prior but a set of allowable neighbourhoods has to be known in prior. These assumptions, minimum dwell-time and mode transition restrictions, are two common constraints on the switched sequence of switched systems (Chen & Lazar, 2022; Danielson et al., 2019; Pepe, 2021).

#### 5.1. DSwMPC: implementation

Let the dynamic equation of a distributed switched large-scale system, \( \mathcal{S} \), as defined in (9). Suppose \( \sigma(t) \) is an unknown switching signal that changes the network topology. Based on modelling proposed in (10), (11) and (13) for dynamic equation of \( i \)-th subsystem, the state space model of subsystem \( \mathcal{S}_i \) can be expressed as:

\[ x_i(t + 1) = A_{ij}x_i(t) + B_{ij}u_i(t) + \sum_{j \in \mathcal{N}_{i}^\sigma(t)} A_{ij} \tilde{x}_j(t) \]

\[ + \sum_{j \in \mathcal{N}_{i}^\sigma(t)} B_{ij} \tilde{u}_j(t) + w_{i}^\sigma(t)(t) \]

\[ x(t) \in \mathcal{X}_\sigma(t), \quad u(t) \in \mathcal{U}_\sigma(t) \]

\[ (27) \]

where

\[ w_{i}^\sigma(t)(t) = \sum_{j \in \mathcal{N}_i} A_{ij}(x_j(t) - \tilde{x}_j(t)) \]

\[ + \sum_{j \in \mathcal{N}_i} B_{ij}(u_j(t) - \tilde{u}_j(t)) \in \mathcal{W}_i^\sigma(t) \]

\[ (28) \]

where \( w_{i}^\sigma(t)(t) \in \mathcal{W}_i^\sigma(t) = \bigoplus_{j \in \mathcal{N}_{i}^\sigma(t)}(A_{ij}e_j + B_{ij}e_u) \). Observe that according to (27) and (28), the effect of a switch in the network topology appears in the terms of future state and input reference trajectories and also the term of additive disturbance in dynamic equation of \( \mathcal{S}_i \). Also, we know that when a switch occurs, due to the new possible neighbourhood set, \( \mathcal{Z}_{i,t} \in \mathcal{P} \), must be updated based on the new network topology. On the
other side, based on (17b), (17c), (17d), (18) and (19), \( S_i \) is utilised for computation of feasible regions of \( \hat{x}_i(0), \hat{\xi}_i(t), \hat{u}_i(t) \) and also tightened constraints \( \hat{X}_i \) and \( \hat{U}_i \). So a switch in the network topology affects both the dynamic equation and constraints of the nominal subsystem in the structure of the proposed DS\( \text{wMPC} \) and therefore its effect appears in the optimisation problem of (17). From the model defined in (27) for the dynamic equation of subsystem \( S_i \), its corresponding nominal subsystem is considered as follows:

\[
\begin{align*}
\hat{x}_i(t+1) &= A_{ii}\hat{x}(t) + B_{ii}\hat{u}_i(t) + \sum_{j \in \mathcal{N}^\sigma_i(t)} A_{ij}\hat{x}(t) \\
&+ \sum_{j \in \mathcal{N}^\sigma_i(t)} B_{ij}\hat{u}_j(t) \\
\hat{\xi}(t) &\in \hat{X}_i(t), \quad \hat{u}(t) \in \hat{U}_i(t)
\end{align*}
\]

where

\[
\begin{align*}
\hat{X}_i(t) &= X_i \ominus \hat{Z}_i(t) \\
\hat{U}_i(t) &= U_i \ominus K^\sigma_i(t) \hat{Z}_i(t)
\end{align*}
\]

\( \hat{Z}_i(t) \) is the RPI set of \( S_i \) under switching signal of \( \sigma(t) \) and \( K^\sigma_i(t) \) satisfies Assumption 3.1 for the current topology.

So, the controller \( C_i \) of current topology by assumption that the current state of \( S_i \) is \( x_i \), is defined by

\[
u_i(t) = \hat{u}_i(t) + K^\sigma_i(t) \hat{x}_i(t) - \hat{x}_i(t)\]

where two unknown variables of \( \hat{x}_i(t) \) and \( \hat{u}_i(t) \) are calculated using the MPC controller for the model defined in (29). The variables \( \hat{x}_i(t) \) and \( \hat{u}_i(t) \) are equal to optimisation parameters of \( \hat{x}_i(0) \) and \( \hat{u}_i(0) \), respectively. These parameters of local MPC controller, for the long-horizon version, are obtained by optimising the following MPC performance index:

\[
\begin{align*}
\min_{\hat{x}_i(0), \hat{u}_i(0), N_i-1} & \sum_{k=0}^{N_i-1} \left( \|\hat{x}_i(k|t)\|_{Q_i}^2 + \|\hat{u}_i(k|t)\|_{P_i}^2 \right) \\
&+ \|\hat{x}_i(N_i|t)\|^2_{P_0}
\end{align*}
\]

\[
\hat{x}_i(t+1) = A_{ii}\hat{x}_i(t) + B_{ii}\hat{u}_i(t) + \sum_{j \in \mathcal{N}^\sigma_i(t)} A_{ij}\hat{x}_j(t) \\
+ \sum_{j \in \mathcal{N}^\sigma_i(t)} B_{ij}\hat{u}_j(t) \quad k = 0, \ldots, N_i - 1
\]

\[
x_i(t) - \hat{x}_i(t) \in Z_i^\sigma(t)
\]

Note that the effect of the switch is reflected in all constraints of the abovementioned optimal control problem. Constraint (33g) is a combination of constraint (17f) for the optimal control problem of DMPC and constraint (25d) for the optimal control problem of SwMPC. It ensures that before a switch occurs, the states of each subsystem in the current topology will reach a feasible region for the next topology. To guarantee stability and also feasibility when a switch occurs, we propose a modified switch-RCI set, called stabiliser switch-RCI, for the current topology in (33g). The stabiliser switch-RCI set, \( T_i^{\sigma(0)}(t) \) in (33g), must be computed in a way that it satisfies both the required conditions of a terminal region and also a switch-RCI set. This is performed by revising stage 2 of algorithm proposed in Figure 1, i.e. by considering \( \Omega^0_i = \hat{X}_f^i \), where \( \hat{X}_f^i \) is a terminal region of the nominal subsystem (29) computed as presented in Farina and Scattolini (2012), Betti et al. (2014a) and Betti et al. (2014b). Structure of the DS\( \text{wMPC} \) algorithm for each subsystem is summarised in an algorithm presented in Figure 2.

It should be noticed that when a switch occurs, all matrices include stage and terminal matrices in (33), i.e. \( Q_i^0, R_i^0 \) and \( P_i^0 \) and all sets in (33) include \( Z_i, \Delta E_i^\sigma \) and \( \Delta E_i^\mu \), and the gain matrices \( K_i \) of the DS\( \text{wMPC} \) have to be updated based on the new topology. Note that for the case where the next network topology is unknown in prior, \( Z_i(t) \) is updated based on the maximum applicable disturbance. This value can be obtained with the participation of all neighbours in the allowable neighbourhood set of each subsystem.

This proposed method does not imposed any restrictions on the structure of applied system, including its dimensions, the number of subsystems, the degree of interaction, and so on. But the main challenge of proposed DS\( \text{wMPC} \) is that how the matrices of \( Q_i^0, R_i^0 \) and \( P_i^0 \) and sets of \( Z_i, \Delta E_i^\sigma \) and \( \Delta E_i^\mu \), and the gain matrices \( K_i \) can be computed using effective
ways in a reasonable time and how the computational burden of this computation can be decreased as much as possible, especially when the neighboring subsystems are not known in advance and the number of possible neighboring subsystems for a subsystem is large. On the other side, as proposed in Betti et al. (2014a), some employed algorithms to compute aforementioned matrices and sets, can only provide sufficient conditions, meaning that other criteria and algorithms can be devised and adopted, which can be even more efficient, especially when applied to specific case studies.

5.2. Stability and feasibility

In the DMPC, existence of a non-empty RPI set of $Z_i$ guarantees stability, feasibility and constraints satisfaction of the system (1). As proposed in Farina and Scattolini (2012), Betti et al. (2014a) and Betti et al. (2014b), their DMPC guarantees asymptotic closed-loop stability of the origin of the distributed large-scale system and constraints satisfaction by employing the RMPC controllers as local regulators. They showed that if there exists a feasible control input for each subsystem to ensure stability and feasibility conditions, so the stability and feasibility of overall system can be guaranteed. In the DMPC, interactions among subsystems of system (1) have been modelled as an added disturbance term. Based on this kind of modelling, in the distributed large-scale systems with switched topology, Equation (9), the term include switch emerges only in the disturbance term of each subsystem. So when a switch is occurred, only the disturbance term is changed. This disturbance term is reflected in the dynamic equation of subsystems by employing the RPI set of $\hat{Z}_i(t)$.

The current research, to address existence of time-varying network topology, replaces the RSwMPC instead of the RMPC as local regulators. The RSwMPC controllers use an implicit MPC law (16) as control input, and employ a SwMPC for a dynamic equation of nominal subsystems. As proposed in Danielson et al. (2019), the SwMPC by defining the switch-RCI sets, determines necessary and sufficient conditions for the existence of a control-law that guarantees constraints satisfaction and recursive feasibility in the presence of unknown mode switching. To ensure stability, we propose the stabiliser switch-RCI sets as they can satisfy both the required conditions of a terminal region and also a switch-RCI set. Since existence of a non-empty stabiliser switch-RCI set shows that there is a valid control input to drive states into a terminal

---

![Figure 2](image-url)  
**Figure 2.** Structure of the DSwMPC algorithm for each subsystem.

1. Step 1: Initialization.
2. Let $t=0$, $p=0$, Determine minimum dwell-time $d_i$
3. Determine $N_{i}^{\gamma(t)}$, $K_i$, $Q_i^0$, $R_i^0$, $P_i^0$, $Z_i$, $\Delta E_i$ and $\Delta E_i^u$ based on initial topology on each DMPC controller in parallel.
4. Choose control horizon $N_i$ long enough such that $N_i \geq d_i$
5. Step 2:
6. **while** $p \leq d_i$ **do**
7. Measure current state $x_i(t)$
8. Exchange reference trajectories $\tilde{x}_i$ and $\tilde{u}_i$ according to current topology.
9. Solve the MPC performance index (33) and determine $\hat{x}_i(0)$ and $\hat{u}_i(0 : N_i - 1)$
10. Apply $u_i(t)$ according to (32)
11. Let $t = t + 1$, $p = p + 1$, repeat Step 2
12. **end while**
13. Step 3:
14. **while** $p > d_i$ **do**
15. **if** switch occurred **then**
16. Update $N_{i}^{\gamma(t)}$, $K_i$, $Q_i^0$, $R_i^0$, $P_i^0$, $Z_i$, $\Delta E_i$ and $\Delta E_i^u$ with the updated topology
17. Let $p = 0$
18. **end if**
19. Go to Step 2
20. **end while**
constraint set at end of the considered prediction horizon, then in fact its existence guarantees the stability of DSwMPC. So, existence of local non-empty stabiliser switch-RCI sets are what a DSwMPC needs to ensure stability and feasibility.

In an overall view, the DSwMPC does not impose any new constraints than the original DMPC. The DSwMPC modifies the DMPC to be adopted to switch in network topology by replacing the MPC with a SwMPC controller. The SwMPC is similar to MPC only with a change in determining a terminal region. The stabiliser switch-RCI set in SwMPC by modification of $\Omega_{i}^0 = \hat{X}_i$, on one side, it has all necessary

| Time interval | $N_1$ | $N_2$ | $N_3$ | $N_4$ |
|---------------|-------|-------|-------|-------|
| 0–4           | {2}   | {1}   | {4}   | {3}   |
| 5–9           | {2,3} | {3,4} | {3,4} | {1,3} |
| 10–14         | {4}   | {4}   | {2}   | {1}   |

Table 1. Neighbourhood set of each subsystem in three considered topologies.

Figure 3. Three considered network topologies. (a) Initial topology, (b) Second topology, (c) Third topology.

Figure 4. The response curves of the states trajectories in Example 6.1. (a) subsystem 1 (b) subsystem 2 (c) subsystem 3 (d) subsystem 4.
conditions of a terminal region, and the other side it can ensure a feasible switching among different modes of system. So, all discussions and results about convergence, stability, feasibility and constraints satisfaction in the DMPC and the SwMPC can be generalised for the DSwMPC. In other words, by solving (33), the asymptotic stability of the closed-loop distributed switched large-scale system, recursive feasibility and constraints satisfaction are guaranteed.

6. Results and discussions

In this section, the proposed DSwMPC is tested by being applied on a discrete-time distributed switched large-scale system, $S$, consisting of four interacting subsystems. The models of these four subsystems are given by

$$S_1 : x_1(t+1) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x_1(t) + \begin{pmatrix} 0.5 \\ 1 \end{pmatrix} u_1(t)$$

$$A_{12} = 0.08 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A_{13} = 0.05 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A_{14} = 0.06 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$(-1.2, -1.2) \leq (x_{11}(t), x_{12}(t)) \leq (1, 1) - 0.5 \leq u_1(t) \leq 0.5$$

(34a)

$$S_2 : x_2(t+1) = \begin{pmatrix} 2 & -0.96 \\ 1 & 0 \end{pmatrix} x_2(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u_2(t)$$

$$+ \sum_{j \in \mathcal{N}_2^{\sigma(t)}} A_{2j} x_j(t)$$

$$A_{21} = 0.05 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A_{23} = 0.04 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Figure 5. The response curves of the input trajectories in Example 6.1. (a) subsystem 1 (b) subsystem 2 (c) subsystem 3 (d) subsystem 4.
\begin{align}
A_{24} &= 0.04 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
A_{24} &= 0.04 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 0.6 \leq u_2(t) \leq 0.6 \\
S_3 : x_3(t + 1) &= \begin{pmatrix} 1.2 & 0.51 \\ 0.1 & 1 \end{pmatrix} x_3(t) + \begin{pmatrix} 0.5 \\ 1 \end{pmatrix} u_3(t) \\
&+ \sum_{j \in N_3(t)} A_{3j} x_j(t) \\
A_{31} &= 0.04 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad A_{32} = 0.04 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
A_{34} &= 0.1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
\begin{pmatrix} -1.3 \\ -1.3 \end{pmatrix} \leq \begin{pmatrix} x_{31}(t) \\ x_{32}(t) \end{pmatrix} \leq \begin{pmatrix} 1.3 \\ 1.3 \end{pmatrix} - 1.1 \leq u_3(t) \leq 1.1 \\
(34b)
\end{align}

\begin{align}
S_4 : x_4(t + 1) &= \begin{pmatrix} 1.1 & 2 \\ 0 & 0.95 \end{pmatrix} x_4(t) + \begin{pmatrix} 0 \\ 0.7787 \end{pmatrix} u_4(t) \\
&+ \sum_{j \in N_4(t)} A_{4j} x_j(t) \\
A_{41} &= 0.03 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad A_{42} = 0.03 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
A_{43} &= 0.05 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
\begin{pmatrix} -1.4 \\ -1.4 \end{pmatrix} \leq \begin{pmatrix} x_{41}(t) \\ x_{42}(t) \end{pmatrix} \leq \begin{pmatrix} 1.4 \\ 1.4 \end{pmatrix} - 1 \leq u_4(t) \leq 1 \\
(34d)
\end{align}

In the aforementioned system, after switching times, it is required that $N_i^{\sigma(t)}$ is updated based on the new network topology. Simulation time is considered equals to 15 time instances and it is assumed that the switching signal changes $N_i^{\sigma(t)}$ in $t = 4$ and $t = 9$. The initial

![Image](image_url)

**Figure 6.** The response curves of the states trajectories in Example 6.2. (a) subsystem 1 (b) subsystem 2 (c) subsystem 3 (d) subsystem 4.
conditions are chosen to be

\[
\begin{align*}
    x_1(0) &= \begin{pmatrix} -0.55 \\ 0.9 \end{pmatrix}, \\
    x_2(0) &= \begin{pmatrix} -0.8 \\ -0.9 \end{pmatrix}, \\
    x_3(0) &= \begin{pmatrix} -0.8 \\ -0.6 \end{pmatrix}, \\
    x_4(0) &= \begin{pmatrix} -0.7 \\ 0.8 \end{pmatrix}
\end{align*}
\]

The constrained finite-time optimal control problem of (33) is solved for each subsystem using the fmincon function in the MATLAB environment. The chosen control horizon \( N_i \) and minimum dwell-time \( d_i \) are 5 and 3 samples, respectively (Ahandani et al., 2020). It can be noticed that the dimension of overall system does not impose any restrictions on application of the DSwMPC. But it is natural that a problem with a large dimension leads to an increase in computational time. Our simulation study is divided into two parts. In the first part, in order to test of the DSwMPC under different work conditions and to provide the comprehensive simulations, three examples are designed based on system \( S \) under various scenarios and the performance of DSwMPC is tested on them. In the second part, the effectiveness of DSwMPC is verified by comparing its results with those obtained by a centralised SwMPC (CSwMPC) and a DeSwMPC (Ahandani et al., 2020).

6.1. Illustrative examples

Example 6.1: Consider a distributed switched system as illustrated in (34a)–(34d). In this scenario, let the switching times are unknown in prior but the next neighbourhood sets, \( N_i^{\sigma(t)}, \) are known in prior. The neighbourhood set of each subsystem in three considered topologies are shown in Table 1 and Figure 2. It can be noticed that it is not necessary to have the same number of neighbours for all the subsystems and size of \( N_i^{\sigma(t)} \) in each subsystem can be different. The size of \( N_i^{\sigma(t)} \) can be restricted by (30) and (31) where a big

![Figure 7](image_url)

Figure 7. The response curves of the input trajectories in Example 6.2. (a) subsystem 1 (b) subsystem 2 (c) subsystem 3 (d) subsystem 4.
neighbourhood set imposes a big $\mathcal{Z}_i^{\sigma(t)}$ and so it is possible that the tightened constraints of $\hat{\mathcal{X}}_i^{\sigma(t)}$ and $\hat{\mathcal{U}}_i^{\sigma(t)}$ are empty sets. (Figure 3)

The state responses and the inputs of the four subsystems are presented in Figures 4 and 5. It can be observed that constraints on states and inputs are always fulfilled by the proposed DSwMPC. The simulation results also confirm that the closed-loop system successfully converges to the origin.

**Example 6.2:** In Example 6.2, let both switching times and the next neighbouring sets to be unknown in prior. All simulation parameters and characteristics are the same with Example 6.1. Only the possible neighbours of different subsystems are restricted to be: $\mathcal{N}_1^{\sigma(t)} = \{2, 3\}$, $\mathcal{N}_2^{\sigma(t)} = \{1, 3, 4\}$, $\mathcal{N}_3^{\sigma(t)} = \{1, 2\}$ and $\mathcal{N}_4^{\sigma(t)} = \{1, 3\}$ where for example $\mathcal{N}_1^{\sigma(t)} = \{2, 3\}$ means that the neighbours of subsystem 1 under the switching signal can be subsystem 2 or 3 or both of them. Since the neighbourhood sets have been supposed to be unknown in prior, the maximum RPI set $\mathcal{Z}_i$, $i = 1, \ldots, 4$, of each subsystem must be computed corresponding the maximum applicable disturbance to it in all possible topologies. For example for $S_1$, the maximum RPI set $\mathcal{Z}_1$ is computed with this assumption that its neighbourhood set is composed of both subsystems 2 and 3. It can be computed off-line and is used to provide a feasible switch among topologies.

The evolutions of states and control input of the four subsystems for Example 6.2 are depicted in Figures 6 and 7, respectively. The achieved results show that under this control strategy the closed-loop system converges to the origin. In other words, they confirm that the DSwMPC successfully stabilises closed-loop system. Clearly, it can be observed that the constraints on the input and states are satisfied by the DSwMPC.

**Figure 8.** The response curves of the states trajectories in Example 6.3. (a) subsystem 1 (b) subsystem 2 (c) subsystem 3 (d) subsystem 4.
at all times. However, in comparison with Example 1, the convergence speed of curves is slower than the corresponding curves in Figures 4 and 5. These observations as mentioned above, clearly verify the robustness characteristics of the DSwMPC against unknown switching times and especially against unknown next topology.

**Example 6.3:** In this example such Example 6.2, it is assumed that both of switching time and the neighbourhood sets, $\mathcal{N}_i^\sigma(t)$, after switch are unknown in prior. The neighbourhood set of each subsystem in every topology and switching times are the same as Example 6.1. In comparison with Example 6.2, the sets of possible neighbourhoods are unknown in this example and the controller have no information about the next possible topologies. In other words, it is assumed that $\mathcal{N}_1^\sigma(t) = \{2, 3, 4\}$, $\mathcal{N}_2^\sigma(t) = \{1, 3, 4\}$, $\mathcal{N}_3^\sigma(t) = \{1, 2, 4\}$ and $\mathcal{N}_4^\sigma(t) = \{1, 2, 3\}$ where for example $\mathcal{N}_1^\sigma(t) = \{2, 3, 4\}$ means that the neighbours of subsystem 1 under switching signal $\sigma(t)$ can be subsystem 2 or 3 or 4 or a combination of two or three neighbours. So such Example 6.2, the maximum RPI set $\mathcal{Z}_i$, $i = 1, \ldots, 4$, of each subsystem must be computed corresponding the maximum applicable disturbance to it in all possible topologies. It means

**Figure 9.** The response curves of the input trajectories in Example 6.3. (a) subsystem 1 (b) subsystem 2 (c) subsystem 3 (d) subsystem 4.

**Table 2.** State square errors of DSwMPC for Examples 6.1–6.3.

| Example  | Sub1 $x_1$ | Sub1 $x_2$ | Sub2 $x_1$ | Sub2 $x_2$ | Sub3 $x_1$ | Sub3 $x_2$ | Sub4 $x_1$ | Sub4 $x_2$ | Sum  |
|----------|------------|------------|------------|------------|------------|------------|------------|------------|------|
| Example 6.1 | 0.5694     | 0.9693     | 0.8921     | 1.2341     | 1.3781     | 1.1156     | 1.2964     | 0.8580     | 8.3129 |
| Example 6.2 | 0.5574     | 0.9470     | 0.9124     | 1.2493     | 1.2198     | 1.2752     | 1.4370     | 0.8346     | 8.4326 |
| Example 6.3 | 0.8510     | 1.0143     | 1.0298     | 1.3331     | 1.3771     | 1.1054     | 1.2866     | 0.8607     | 8.8579 |
Figure 10. The comparison results of DSwMPC, DeSwMPC and CSwMPC in Example 6.1 in terms of the states trajectories. (a) subsystem 1 (b) subsystem 1 (c) subsystem 2 (d) subsystem 2 (e) subsystem 3 (f) subsystem 3 (g) subsystem 4 (h) subsystem 4.
that the maximum applicable disturbance to each subsystem can be computed by assumption that it is impacted by all of the other subsystems, e.g. for subsystem 1 we have $W_{1}^{\sigma}(t) = \bigoplus_{j \in N_{1}^{\sigma}(t)} (A_{1j} \xi_{j}^{\sigma} + B_{1j} u_{j})$.

The simulation results for evolutions of states and control input on Example 6.3 are depicted in Figures 8 and 9, respectively. It can be observed that despite considering the maximum applicable disturbance on each subsystem, the constraints on the input and states are addresses by the DSwMPC at all times. Moreover, as again shown by these figures, the closed-loop system converges to the origin. However, the convergence speed of curves is less than the corresponding curves in Figures 4–7. These figures in comparison with Example 6.2, clearly show the robustness characteristics of the DSwMPC against strong interactions among subsystems. It should be noted that when a stabiliser switch–RCI set, $T_{i}^{\sigma(0)}$, cannot be calculated for a substem, or in other words, this set is empty, it means that this set of neighbours is not allowed for the corresponding subsystem and it is not possible to stabilise the system and to ensure constraint satisfaction in the presence of unknown mode switching with this set of neighbours.

Also Table 2 summarises the numerical comparisons according to the state square errors of close-loop system under the control of the DSWMPC for Examples 6.1–6.3. As the obtained results of this table show, the value of the sum of square errors increases from Example 6.1 to Example 6.3.

6.2. Comparison study

To demonstrate the performance of the proposed DSwMPC more clearly, a comparison of the DSwMPC and results of DeSwMPC (Ahandani et al., 2020) and the CSwMPC is also provided. The CSwMPC has

![Figure 11](image-url)

Figure 11. The comparison results of DSwMPC, DeSwMPC and CSwMPC in Example 6.2 in terms of the input trajectories. (a) subsystem 1 (b) subsystem 2 (c) subsystem 3 (d) subsystem 4.
Figure 12. The comparison results of DSwMPC and DeSwMPC in Example 6.2 in terms of the states trajectories. (a) subsystem 1 (b) subsystem 1 (c) subsystem 2 (d) subsystem 2 (e) subsystem 3 (f) subsystem 3 (g) subsystem 3 (h) subsystem 4.
been designed as proposed in Section 4. All parameter values of CSwMPC are considered equal to parameters of the local SwMPC. This comparison study is carried out using Example 6.1 to Example 6.3. To apply the CSwMPC of Section 4 on a system, it is necessary that the operating modes of system to be known in advance (see algorithm of Figure 1). Since in Examples 6.2 and 6.3 the subsequent topology is not known in advance, so subsequent operating modes of the overall system are not known and this version of CSwMPC cannot be applied on such a system. For the DeSwMPC, all parameters were tuned based on those proposed in Ahandani et al. (2020).

The trajectories of the control inputs and the states associated with the four subsystems are depicted in Figures 9–12. Also, the sum of square errors of close-loop system for the DSwMPC, DeSwMPC and CSwMPC are shown in Table 3. It can be observed that the shapes of the state response curves under the CSwMPC are very similar to those obtained by the DSwMPC. Only the states of closed loop system under CSwMPC are a bit faster with respect to the DSwMPC, especially for states of subsystems 2 and 3. The achieved results of Table 3 highlight the effectiveness of DSwMPC. However, the CSwMPC has the best quality in Example 6.1 but in addition to overall problems of centralised control strategies, it can not be applied on systems with scenarios such as Examples 6.2 and 6.3. Also about the DecSwMPC, it is observed that however it achieves a faster convergence in Example 6.1, but its performance is dramatically decreased in Examples 6.2 and 6.3 when number of agents in the sets of possible neighbourhoods is increased so that this method can not find any feasible control actions on Example 6.3. From these results we can see that, compared to the CSwMPC and the DeSwMPC proposed in Ahandani et al. (2020), the DSwMPC proposed in this article has a considerable flexibility to be applied on distributed switched systems in which the system topology changes under various scenarios. Table 4 compares the DSwMPC, DeSwMPC and CSwMPC in terms of computational time. It is clear that due to the difference in the structure of these methods and especially the different algorithms used in them to calculate the required sets, their execution times are different from each other. In Example 6.1, the CSwMPC obtains the best computational time and the DSwMPC has a better performance than the DeSwMPC. Also, it achieves a better computational time than the DeSwMPC in Example 6.2. It should be noted that in this comparison, the delay times that occur in practical applications for the exchange of information between controllers or between controllers and system are not included and only the time required to execute the algorithms used in each method are compared.

### 7. Conclusions

A DSwMPC was presented in this study for handling dynamic interconnections among subsystems in a distributed switched large-scale system subjected to state/input constraints. In a distributed switched large-scale system, coupling among subsystems varies over time according to a switching signal. Such systems are described as the networked control systems with time-varying network topology in which a switching signal determines the neighbourhood set of each subsystem over time. The proposed DSwMPC ensured the stability of the origin of the whole closed-loop system and as well constraints satisfaction. In the employed DMPC, by modelling the coupling terms of subsystems as the additive disturbance, their effect appeared in the dynamic equation, local states and input control constraints of the nominal subsystem. The local controllers exploited tube-based MPC to guarantee robustness than interconnections among subsystems. To consider the effect of switching signal which creates a time-varying network topology, the DSwMPC employed a robust tube-based SwMPC with the switch-RCI set as the target set robust to unknown mode switching as local regulators. Since the switch of network topology changes interactions among subsystems, it affects the terms of future state and input reference trajectories and additive disturbance in the

| Method | Example 6.1 | Example 6.2 | Example 6.3 |
|--------|-------------|-------------|-------------|
| CSwMPC | 8.1124      | –           | –           |
| DeSwMPC| 8.2829      | 8.5480      | –           |
| DSwMPC | 8.3129      | 8.4326      | 8.8579      |

| Method | Example 6.1 | Example 6.2 | Example 6.3 |
|--------|-------------|-------------|-------------|
| CSwMPC | 5.01        | –           | –           |
| DeSwMPC| 31.82       | 36.62       | –           |
| DSwMPC | 16.16       | 21.37       | 30.12       |
dynamic of each subsystem, so the switching signal affects the dynamic equation and constraints of nominal subsystems. Three typical examples were employed to illustrate the merits and effectiveness of DSwMPC. In all of them, it was supposed that the switching times are unknown in prior, but the next neighbourhood sets were assumed to be known in prior in the first example and it was assumed to be unknown in the second and third ones. The simulation results demonstrated that the proposed DSwMPC satisfied the input and state constraints at all times. They also validated that the closed-loop system converges to the origin. It can be seen from the comparison results that, compared to the CSwMPC and the DeSwMPC, the DSwMPC proposed in this article has a flexible structure to be applied on distributed switched systems in which the system topology changes under various scenarios (Figure 13).

Figure 13. The comparison results of DSwMPC and DeSwMPC in Example 6.1 in terms of the input trajectories. (a) subsystem 1 (b) subsystem 2 (c) subsystem 3 (d) subsystem 4.

Disclosure statement
No potential conflict of interest was reported by the author(s).

Data availability statement
The authors confirm that the data supporting the findings of this study are available within the article.

References
Ahandani, M. A., Kharrati, H., Hashemzadeh, F., & Baradarania, M. (2020). Decentralized switched model-based predictive control for distributed large-scale systems with topology switching. Nonlinear Analysis: Hybrid Systems, 38, 100912. https://doi.org/10.1016/j.nahs.2020.100912
Alessio, A., & Bemporad, A. (2007). Decentralized model predictive control of constrained linear systems. In 2007 european control conference (ecc) (pp. 2813–2818). IEEE.
Augustine, M. T., & Patil, D. U. (2022). A stabilizing model predictive control scheme with arbitrary prediction horizon
for switched linear systems. *IEEE Control Systems Letters*, 6, 2461–2466. https://doi.org/10.1109/LCSYS.2022.3164768

Bai, T., Li, S., & Zou, Y. (2020). Distributed MPC for reconfigurable architecture systems via alternating direction method of multipliers. *IEEE/CAA Journal of Automatica Sinica*, 8(7), 1336–1344. https://doi.org/10.1109/JAS.2020.1003195

Barreiro-Gomez, J., Ocampo-Martinez, C., & Quijano, N. (2014). Coalitional control for self-organizing agents. *IEEE Control Systems Magazine*, 34(4), 87–97. https://doi.org/10.1109/MCS.2014.2320397

Betts, G., Farina, M., & Scattolini, R. (2014b). Realization of distributed MPC: A noncooperative approach based on robustness concepts. In *Distributed model predictive control made easy* (pp. 421–435). Springer.

Betts, G., Farina, M., & Scattolini, R. (2014a). Distributed MPC: A time-varying partitioning for predictive control design: Density-games approach. *Journal of Process Control*, 25, 1–14. https://doi.org/10.1016/j.jprocont.2018.12.011

Betti, G., Farina, M., & Scattolini, R. (2014). Time-varying partitioning for predictive control design: Density-games approach. *Journal of Process Control*, 25, 1–14. https://doi.org/10.1016/j.jprocont.2018.12.011

Betti, G., Farina, M., & Scattolini, R. (2014b). Realization issues, tuning, and testing of a distributed predictive control algorithm. *Journal of Process Control*, 24(4), 424–434. https://doi.org/10.1016/j.jprocont.2014.02.016

Chen, W. (2016). Improved distributed model predictive control with control planning set. *Journal of Control Science and Engineering*, 2016, 8167931. https://doi.org/10.1155/2016/8167931

Chen, Y., & Lazar, M. (2022). An efficient MPC algorithm for switched systems with minimum dwell time constraints. *Automatica*, 143, 110453. https://doi.org/10.1016/j.automatica.2022.110453

Christofides, P. D., Scattolini, R., & Liu, J. (2013). Distributed model predictive control: A tutorial review and future research directions. *Computers and Chemical Engineering*, 51, 21–41. https://doi.org/10.1016/j.compchemeng.2012.05.011

Danielson, C., Bridgeman, L. J., & Di Cairano, S. (2019). Necessary and sufficient conditions for constraint satisfaction in switched systems using switch–robust control invariant sets. *International Journal of Robust and Nonlinear Control*, 29(9), 2589–2602. https://doi.org/10.1002/rnc.v29.9

Dunbar, W. B. (2007). Distributed receding horizon control of dynamically coupled nonlinear systems. *IEEE Transactions on Automatic Control*, 52(7), 1249–1263. https://doi.org/10.1109/TAC.2007.900828

Farina, M., & Scattolini, R. (2012). Distributed predictive control: A non-cooperative algorithm with neighbor-to-neighbor communication for linear systems. *Automatica*, 48(6), 1088–1096. https://doi.org/10.1016/j.automatica.2012.03.020

Fele, F., Debada, E., Maestre, J. M., & Camacho, E. F. (2018). Coalitional control for self-organizing agents. *IEEE Transactions on Automatic Control*, 63(9), 2883–2897. https://doi.org/10.1109/TAC.9

Filip, F. G., & Leivisiká, K. (2009). Large-scale complex systems. In *Springer handbook of automation* (pp. 619–638). Springer.

Giselsson, P., & Rantzer, A. (2013). On feasibility, stability and performance in distributed model predictive control. *IEEE Transactions on Automatic Control*, 59(4), 1031–1036. https://doi.org/10.1109/TAC.2013.2285779

Hernandez, B., Baldivieso, P., & Trodden, P. (2017). Distributed MPC: Guaranteeing global stability from locally designed tubes. *IFAC-PapersOnLine*, 50(1), 11829–11834. https://doi.org/10.1016/j.ifacol.2017.08.1995

Hou, B., Li, S., & Zheng, Y. (2021). Distributed model predictive control for reconfigurable systems with network connection. *IEEE Transactions on Automation Science and Engineering*, 1–12. https://doi.org/10.1109/TASE.2021.3058298

Li, S., C. K. Ahn, Chadli, M., & Xiang, Z. (2021). Sampled-data adaptive fuzzy control of switched large-scale nonlinear delay systems. *IEEE Transactions on Fuzzy Systems*, 30(4), 1014–1024. https://doi.org/10.1109/TFUZZ.2021.3052094

Li, S., Zhang, Y., & Zhu, Q. (2005). Nash-optimization enhanced distributed model predictive control applied to the shell benchmark problem. *Information Sciences*, 170(2-4), 329–349. https://doi.org/10.1016/j.ins.2004.03.008

Li, S., & Zheng, Y. (2016). *Distributed model predictive control for plant-wide systems*. John Wiley and Sons.

Liu, J., Chen, X., Muñoz de la Peña, D., & Christofides, P. D. (2010). Sequential and iterative architectures for distributed model predictive control of nonlinear process systems. *AIChE Journal*, 56(8), 2137–2149. https://doi.org/10.1002/aic.v56:8

Long, L., & Zhao, J. (2016). Decentralized adaptive neural output-feedback DSC for switched large-scale nonlinear systems. *IEEE Transactions on Cybernetics*, 47(4), 908–919. https://doi.org/10.1109/TCYB.2016.2533393

Lucia, S., Kögel, M., & Findeisen, R. (2015). Contract-based predictive control of distributed systems with plug and play capabilities. *IFAC-PapersOnLine*, 48(23), 205–211. https://doi.org/10.1016/j.ifacol.2015.11.284

Maestre, J. M., & Negenborn, R. R. (2014). Distributed mpc: A noncooperative approach based on robustness concepts. In *Distributed model predictive control made easy* (Vol. 69, pp. 421–435). Springer.

Maxim, A., Copot, D., De Keyser, R., & Ionescu, C. M. (2018). An industrially relevant formulation of a distributed model predictive control algorithm based on minimal process information. *Journal of Process Control*, 68, 240–253. https://doi.org/10.1016/j.jprocont.2018.06.004

Mayne, D. Q., Seron, M. M., & Raković, S. (2005). Robust model predictive control of constrained linear systems with bounded disturbances. *Automatica*, 41(2), 219–224. https://doi.org/10.1016/j.automatica.2004.08.019

Monasterios, P. B., Trodden, P. A., & Cannon, M. (2019). On feasible sets for coalitional MPC. In 2019 ieee 58th conference on decision and control (cdc) (pp. 4668–4673). IEEE.

Negenborn, R. R., & Maestre, J. M. (2014). Distributed model predictive control: An overview and roadmap of future research opportunities. *IEEE Control Systems Magazine*, 34(4), 87–97. https://doi.org/10.1109/MCS.2014.2320397

Núñez, A., Ocampo-Martinez, C., Maestre, J. M., & De Schutter, B. (2015). Time-varying scheme for noncentralized model predictive control of large-scale systems. *Mathematical Problems in Engineering*, 2015, 560702. https://doi.org/10.1155/2015/560702

Pepe, P. (2021). On Lyapunov methods for nonlinear discrete-time switching systems with dwell-time ranges. *IEEE
Transactions on Automatic Control, 67(3), 1574–1581. https://doi.org/10.1109/TAC.2021.3069661
Qi, Y., Yu, W., Huang, J., & Yu, Y. (2021). Model predictive control for switched systems with a novel mixed time/event-triggerring mechanism. Nonlinear Analysis: Hybrid Systems, 42, 101081. https://doi.org/10.1016/j.nahs.2021.101081
Rawlings, J. B., & Stewart, B. T. (2008). Coordinating multiple optimization-based controllers: New opportunities and challenges. Journal of Process Control, 18(9), 839–845. https://doi.org/10.1016/j.jprocont.2008.06.005
Riverso, S., Farina, M., & Ferrari-Trecate, G. (2013). Plug-and-play decentralized model predictive control for linear systems. IEEE Transactions on Automatic Control, 58(10), 2608–2614. https://doi.org/10.1109/TAC.2013.2254641
Scattolini, R. (2009). Architectures for distributed and hierarchical model predictive control—a review. Journal of Process Control, 19(5), 723–731. https://doi.org/10.1016/j.jprocont.2009.02.003
Schiffer, J., Dörfler, F., & Fridman, E. (2017). Robustness of distributed averaging control in power systems: Time delays and dynamic communication topology. Automatica, 80, 261–271. https://doi.org/10.1016/j.autmatica.2017.02.040
Shi, T., Shi, P., & Zhang, H. (2020). Model predictive control of distributed networked control systems with quantization and switching topology. International Journal of Robust and Nonlinear Control, 30(12), 4584–4599. https://doi.org/10.1002/rnc.v30.12
Stewart, B. T., Venkat, A. N., Rawlings, J. B., Wright, S. J., & Pannocchia, G. (2010). Cooperative distributed model predictive control. Systems and Control Letters, 59(8), 460–469. https://doi.org/10.1016/j.sysconle.2010.06.005
Sun, Z., & Ge, S. S. (2005). Analysis and synthesis of switched linear control systems. Automatica, 41(2), 181–195. https://doi.org/10.1016/j.automatica.2004.09.015
Tian, Y., Yao, Q., Wang, C., Wang, S., Liu, J., & Wang, Q. (2022). Switched model predictive controller for path tracking of autonomous vehicle considering rollover stability. Vehicle System Dynamics, 60(12), 4166–4185. https://doi.org/10.1080/00423114.2021.1999990
Vaccarini, M., Longhi, S., & Katebi, M. R. (2009). Unconstrained networked decentralized model predictive control. Journal of Process Control, 19(2), 328–339. https://doi.org/10.1016/j.jprocont.2008.03.005
Yan, J., Shi, L., Xia, Y., & Zhang, Y. (2023). Event-triggered model predictive control of switched systems with denial-of-service attacks. Asian Journal of Control. https://doi.org/10.1016/j.jfranklin.2023.08.039
Yan, J., Xue, H., Xia, Y., & Zhang, Y. (2023). Quantized MPSC for switched systems based on permissible type-switching mechanism. Journal of the Franklin Institute, 360(15), 10942–10971. https://doi.org/10.1016/j.jfranklin.2023.08.039
Yuan, F., Zou, Y., & Niu, Y. (2017). Event-triggered noncooperative distributed predictive control for dynamically coupled large-scale systems. Cogent Engineering, 4(1), 1422227. https://doi.org/10.1080/23319196.2017.1422227
Zhai, D., Liu, X., & Liu, Y. J. (2018). Adaptive decentralized controller design for a class of switched interconnected nonlinear systems. IEEE Transactions on Cybernetics, 50(4), 1644–1654. https://doi.org/10.1109/TCCYB.6221036
Zhang, D., Nguang, S. K., & Yu, L. (2017). Distributed control of large-scale networked control systems with communication constraints and topology switching. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 47(7), 1746–1757. https://doi.org/10.1109/TSMC.2017.2681702
Zhang, J., Li, S., Ahn, C. K., & Xiang, Z. (2021). Adaptive fuzzy decentralized dynamic surface control for switched large-scale nonlinear systems with full-state constraints. IEEE Transactions on Cybernetics, 52(10), 10761–10772. https://doi.org/10.1109/TCCYB.2021.3069461
Zhang, L., Zhuang, S., & Braatz, R. D. (2016). Switched model predictive control of switched linear systems: Feasibility, stability and robustness. Automatica, 67, 8–21. https://doi.org/10.1016/j.automatica.2016.01.010
Zhang, Y., & Li, S. (2007). Networked model predictive control based on neighbourhood optimization for serially connected large-scale processes. Journal of Process Control, 17(1), 37–50. https://doi.org/10.1016/j.jprocont.2006.08.009
Zheng, H., Wu, J., Wu, W., & Negenborn, R. R. (2019). Cooperative distributed predictive control for collision-free vehicle platoons. IET Intelligent Transport Systems, 13(5), 816–824. https://doi.org/10.1049/itrs.2013.5
Zheng, Y., Li, S., & Li, N. (2011). Distributed model predictive control over network information exchange for large-scale systems. Control Engineering Practice, 19(7), 757–769. https://doi.org/10.1016/j.conengprac.2011.04.003
Zheng, Y., Li, S., & Qiu, H. (2012). Networked coordination-based distributed model predictive control for large-scale system. IEEE Transactions on Control Systems Technology, 21(3), 991–998. https://doi.org/10.1109/TCST.2012.2196280
Zheng, Y., Li, S., & Wang, X. (2009). Distributed model predictive control for plant-wide hot-rolled strip laminar cooling process. Journal of Process Control, 19(9), 1427–1437. https://doi.org/10.1016/j.jprocont.2009.04.012