Higher curvature self-interaction corrections to Hawking Radiation

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The purely thermal nature of Hawking radiation from evaporating black holes leads to the information loss paradox. A possible route to its resolution could be if (enough) correlations are shown to be present in the radiation emitted from evaporating black holes. A re-analysis of Hawking’s derivation including the effects of self-interactions in GR shows that the emitted radiation does deviate from pure thermality, however no correlations exist between successively emitted Hawking quanta. We extend the calculations to Einstein-Gauss-Bonnet gravity and investigate if higher curvature corrections to the action lead to some new correlations in the Hawking spectra. The effective trajectory of a massless shell is determined by solving the constraint equations and the semi-classical tunneling probability is calculated. As in the case of general relativity, the radiation is no longer thermal and there is no correlation between successive emissions. The absence of any extra correlations in the emitted radiations even in Gauss-Bonnet gravity suggests that the resolution of the paradox is beyond the scope of semi-classical gravity.

I. INTRODUCTION

Hawking radiation is an intriguing feature of quantum field theory in curved space-time. Hawking’s calculations predict a pure thermal spectrum from black holes suggesting that collapsing matter in an initial pure state evolves non-unitarily into a thermal mixed state. This leads to the information loss paradox in the semi-classical description of black holes. A full resolution of the paradox remains elusive although there are several plausible mechanisms that salvage unitarity. One possibility is to identify small deviations from thermality in the Hawking spectrum that arise from correlations between successive Hawking quanta which could encode information about the internal structure of the black hole. The usual derivation of Hawking radiation assumes a fixed space-time background and therefore neglects back reaction of the emitted radiation on the space-time metric. Note that a consistent treatment of back reaction is necessary for the conservation of energy to be valid. In principle, the back reaction could also induce other correlations in the Hawking spectrum causing deviations from the pure black body form and opening up the possibility that the final state is rendered pure again.

The back-reaction problem in its full generality is perhaps a difficult problem because of the non-linearity of the field equations. A possible way to extract the essential physics of the problem is to make use of the simplicity arising from considering spherically symmetric configurations, i.e., s-wave emission. This approach was developed by [1] who considered a spherically symmetric shell of radiation propagating in a spherically symmetric black hole space-time. Eliminating the gravitational constraints subject to spherical symmetry allowed them to find an effective action for the shell (radiation) degrees of freedom. Surprisingly, the trajectory of the shell, in the massless limit turned out to be interpretable as a null geodesic of a black hole geometry with a shifted ADM mass. This simple but remarkable result was used to quantise the shell action in a WKB approximation. An explicit calculation of the Bogoliubov coefficients showed that the emitted radiation deviated from a pure blackbody spectrum.

Subsequently, the calculation of [1] was recast in [2] as the computation of a semiclassical tunneling probability for a (spherical) shell of energy $E$ from behind the horizon (see also [3, 4]). Once the effective trajectory of the shell is determined, the tunneling probability calculated by evaluating the imaginary part of the semi-classical action is found to be,

$$\Gamma(E) \propto e^{\Delta S},$$

where $\Delta S = S(M) - S(M - E)$, the change in the Hawking-Bekenstein entropy because of the emission of the shell. This factor is the change of the phase space volume before and after the emission of the Hawking radiation and is exactly what is expected if the underlying microscopic theory is unitary. Obviously, the semi-classical approach can not determine proportionality factor which depends on the matrix element between the initial and the final state of the black hole and therefore requires a full quantum theory of gravity.
The emission probability in Eq. (1) contains corrections due to the self-gravity of the shell. But, because of the exponential nature of the function, it is immediately clear that,

\[ \Gamma(E_1) + \Gamma(E_2) = \Gamma(E_1 + E_2), \] (2)

where \( E_1 \) is the energy of a shell tunneling from the hole of initial mass \( M \) and \( E_2 \) is that of a subsequent shell tunneling out of the black hole whose mass is \( M - E_1 \) \[5\]. Hence, although the emission spectrum is corrected by the self-gravitational effect of the shell, successive shells tunneling out of the black hole are not correlated. As a result, the hope of resolving information paradox using back reaction calculations seems impossible as long as we are within the semi-classical regime. This result remains true for the case of a charged shell \[6\] and also for the case of AdS boundary conditions \[7\].

However, because general relativity is a perturbatively non-renormalizable theory - it only makes sense as an effective theory with the Lagrangian written as a series of irrelevant higher curvature terms. This will, of course, change the dynamics of the shell plus gravity system raising the question of whether such higher derivative terms could lead to the correlated emission of Hawking radiation. In fact, from the point of view of an effective theory, all terms consistent with diffeomorphism invariance could be present in the effective Lagrangian. But, only a subset of such terms may provide a consistent low energy description. For example, consider the most general second order higher curvature theory of gravity in \( D \) dimensions. The action of such a theory can be expressed as,

\[ A^G = \frac{1}{16\pi} \int d^D x \sqrt{-g} \left( R + \alpha R^2 + \beta R_{ab} R^{ab} + \gamma R_{abcd} R^{abcd} \right). \] (3)

Here the coefficients \( \alpha, \beta, \gamma \) are constants which measure the departure from general relativity. For generic values of these coefficients, these higher curvature terms introduce problematic features in the classical theory. The field equations contain time derivatives higher than second order creating difficulties for a well-defined initial value formalism. Moreover, the constraint structure of such a theory could be different from general relativity with the addition of possible second class constraints. This could lead to ghosts for quantum perturbations around the flat space. Also, there is no guarantee of the existence of a black hole solution of arbitrary values of these coefficients. However, for particular values of these constants, the theory is free of these unpleasantries. In five dimensions, one such choice is the Einstein-Gauss-Bonnet (EGB) gravity described by the action,

\[ A^G = \frac{1}{16\pi} \int d^5 x \sqrt{-g} \left[ R + \alpha \left( R^2 - 4 R_{ab} R^{ab} + R_{abcd} R^{abcd} \right) \right]. \] (4)

Einstein-Gauss-Bonnet gravity is the unique theory in five dimensions which is free from perturbative ghosts \[8\] and leads to a well-defined initial value formalism. The Gauss-Bonnet correction term also appears as a low energy \( \alpha' \) correction in case of heterotic string theory \[8, 9\]. As in GR, the EGB theory admits spherically symmetric vacuum black hole solutions of the form \[10\],

\[ ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \] (5)

where the metric function is given by,

\[ f(r) = 1 + \frac{r^2}{4\alpha} \left[ 1 - \sqrt{1 + \frac{8\alpha M}{r^4}} \right]. \] (6)

Such black holes are analog of asymptotically flat, Schwarzschild black holes in general relativity and the zero of the metric function \( f(r) \) determines the location of the horizon. It is also possible to formulate the first law for these black holes for small perturbations and it can be expressed as \[11, 12\],

\[ \left( \frac{\kappa}{2\pi} \right) \delta S = \delta M, \] (7)

where \( \delta M \) represents the variation of the ADM mass and \( \kappa \) is the surface gravity of the hole, if we identify \( \kappa/2\pi \) as the Hawking temperature associated with the horizon, \( \delta S \) represents the change of the black hole entropy where the entropy \( S \) is,
\[ S = \frac{1}{4} \int_B \left( 1 + 2 \alpha (3) R \right) dA. \]  

(8)

The entropy of Einstein-Gauss-Bonnet (EGB) black holes are not proportional to the area but contain correction terms proportional to the intrinsic curvature \((3) R\) of the three-dimensional cross-section \(B\) of the horizon. This entropy also obeys the second law for linearized perturbations around a stationary black hole solution \([13]\).

The aim of this paper is to revisit the original calculation of \([1]\) by including a Gauss-Bonnet term in the gravitational action. We consider only the five-dimensional case, although the generalization to higher dimensions appears straightforward. We investigate if the tunneling probability still has the form given by Eq. (1) where \(S\) is the appropriate choice of black hole entropy in Einstein-Gauss-Bonnet gravity, namely the Jacobson Myers entropy in Eq. (8) \([13]\). The motivation of this study is to understand whether the modification of the gravitational dynamics due to the introduction of a Gauss Bonnet term can lead to new correlations in the black hole radiation spectrum. In fact, we will show that higher curvatures terms, at least in the form of Gauss-Bonnet gravity do not introduce any new correlations in the tunneling probability and the resolution of the paradox requires new physics.

To calculate the correction of the Hawking spectrum, we first consider the Hamiltonian formulation of EGB gravity. We follow \((-+,+,+,+,+)\) signature and our sign conventions are that of \([14]\).

II. REVIEW OF HAMILTONIAN FORMULATION OF EINSTEIN-GAUSS BONNET GRAVITY

We start with the standard spherically symmetric ADM form of the metric in five dimensions,

\[ ds^2 = -(N^t)^2 dt^2 + L^2 (dr + N^r dt)^2 + R^2 \left\{ d\theta^2 + \sin^2 \theta \left( d\phi^2 + \sin^2 \phi d\chi^2 \right) \right\}. \]  

(9)

The action of a massive shell propagating in this background can be written as,

\[ A^s = -m \int dt \sqrt{\hat{N}^2 - \hat{L}^2 (\dot{\bar{N}} + \bar{N})^2}, \]  

(10)

where the caret over various quantities indicates that those are evaluated on the world line of the shell. When the background geometry is fixed, the trajectory of the shell is simply a time-like geodesic of the background spacetime. To understand the effects of self-interaction, we need to solve the field equations in the presence of the shell. The restriction to spherical symmetry allows us to solve this problem because the dynamical degrees of freedom of the gravitational field decouple. The 'Coulomb' part of the gravitational field is completely fixed by the constraints in the presence of the shell. Thus, we may 'integrate out' the gravitational fields and obtain an effective Lagrangian for the shell alone. Note that, since EGB field equations are also second order in time, the dynamical degrees of freedom of EGB gravity are same as general relativity. Also, the structure and nature of the constraints are similar to GR. Moreover, Birkhoff’s theorem holds equally good for spherically symmetric space-times in EGB gravity \([15, 16]\). Hence, it may be expected that the analysis of Kraus and Wilczek \([1]\) can be generalized for EGB gravity in a straightforward manner. Our analysis shows that is indeed the case except with a minor modification related to the discontinuity conditions at the location of the shell.

To begin, we evaluate the EGB action for the ADM form of the metric. Spherical symmetry allows us to integrate over the angular directions and ignoring surface terms, the action takes the form,

\[ A^G = \int dt \int dr \mathcal{L}, \]  

(11)

where the Lagrangian is given by \([17]\),

\[ \mathcal{L} = - \left\{ \frac{\hat{L} - (N^r L')}{N^t} \left( \hat{R} - N^r R' \right) \right\} \left\{ R^2 + \lambda \left[ 1 - \left( \frac{R'}{L} \right)^2 + \frac{(\hat{R} - N^r R')^2}{3N^t} \right] \right\} 
- \frac{(\hat{R} - N^r R')^2}{N^t} \left[ LR - \lambda \left( \frac{R'}{L} \right)' \right] 
+ N^t LR \left[ 1 - \left( \frac{R'}{L} \right)^2 \right] - N^t \left( \frac{R'}{L} \right)' \left\{ R^2 + \lambda \left[ 1 - \left( \frac{R'}{L} \right)^2 \right] \right\} \}, \]  

(12)
where, we have redefined the Gauss-Bonnet coupling constant as, \( \lambda = 4\alpha \). We shall denote \( b(L) = \left[ 1 - \left( \frac{R'}{L} \right)^2 \right] \) in what follows. The conjugate momenta corresponding to metric variables \( L \) and \( R \) respectively are,

\[
\pi_L = \frac{\partial \mathcal{L}}{\partial \dot{L}} = -\frac{\lambda}{3} y^3 - y \left\{ R^2 + \lambda b(L) \right\}, \tag{13}
\]

and,

\[
\pi_R = \frac{\partial \mathcal{L}}{\partial \dot{R}} = -\frac{\left( L - (N^r R')^t \right)}{N^t} \left\{ \lambda y^2 + R^2 + \lambda b(L) \right\} - 2y \left[ LR - \lambda \left( \frac{R'}{L} \right)^t \right] \left\{ R^2 + \lambda b(L) - \lambda y^2 \right\}. \tag{14}
\]

where we have defined \( y(t) = \left( \dot{R} - N^r R' \right) / N^t \). In the GR limit (i.e. when \( \lambda \to 0 \)), the conjugate momentum \( \pi_L = -yR^2 \). In EGB gravity, the relationship is given by Eq. (13) which is a cubic equation showing the possibility of many branches of solutions, of which some may not even have a smooth GR limit. We can rewrite the Lagrangian using the variable \( y \) as,

\[
\mathcal{L} = -y \left[ \dot{L} - (N^r L')^t \right] \left\{ R^2 + \lambda b(L) + \frac{\lambda y^2}{3} \right\} + N^t LR \left( b(L) - y^2 \right) - N^t \left( \frac{R'}{L} \right)^t \left\{ R^2 + \lambda b(L) - \lambda y^2 \right\}. \tag{15}
\]

Rearranging the above expression and discarding boundary terms, the action can be expressed in the standard Hamiltonian form,

\[
\mathcal{A}^G = \int dt \int dr \left\{ \pi_L \dot{L} + \pi_R \dot{R} - N^t H_t^G - N^r H_r^G \right\}, \tag{16}
\]

where

\[
H_t^G = y\pi_R - LR \left( b(L) - y^2 \right) + \left( \frac{R'}{L} \right)^t \left\{ R^2 + \lambda b(L) - \lambda y^2 \right\}, \quad \text{and} \quad H_r^G = R' \pi_R - L \pi'_L. \tag{17}
\]

Note that the quantity \( y \) is regarded as a function of the conjugate momentum \( \pi_L \) as in Eq. (13). Thus, the total action of the shell and gravity system is,

\[
\mathcal{A} = \int dt \int dr \left[ \pi_R \dot{R} + \pi_L \dot{L} - N^t (H_t^s + H_t^G) - N^r (H_r^s + H_r^G) \right] - \int dt M_{ADM}, \tag{18}
\]

with

\[
H_t^s = \left( \sqrt{\left( \frac{p}{L} \right)^2 + m^2} \right) \delta(r - \hat{r}) \quad ; \quad H_r^s = -p \delta(r - \hat{r}), \tag{19}
\]

and \( H_t^G \) and \( H_r^G \) are given by Eq. (17). The full constraint equations are then given by:

\[
H_t = H_t^s + H_t^G = 0; \quad H_r = H_r^s + H_r^G = 0. \tag{20}
\]

The last term in Eq. (13) represents the ADM mass of the total system which is a functional of the metric variables and needs to be included for a well-defined variational principle. To obtain an expression for \( M \), following [1], we consider a linear combination of constraints.
\[ \frac{R'}{L} \mathcal{H}_t - \frac{y}{L} \mathcal{H}_r = 0. \]  

(21)

Using Eq. (13), away from the shell, this can be written as \( \mathcal{M}' = 0 \), where

\[ \mathcal{M} = \frac{1}{2} (y^2 + b(L))^2 \left( R^2 + \frac{\lambda}{4} (y^2 + b(L))^2 \right). \]  

(22)

Therefore, \( \mathcal{M} \) is a constant away from the shell and the shell causes a discontinuity in the value of \( \mathcal{M} \). To understand this better, consider a static slice \( (\pi_L = 0) \). We can use Eq. (13) to solve for \( y \) and it turns out the only solution which has a smooth GR limit is \( y = 0 \). Then, using a Schwarzschild type gauge condition \( R' = 1 \) and \( R = r \), we can then solve for the metric variable \( L \) from Eq. (22) and obtain,

\[ \frac{1}{L^2} = 1 + \frac{y^2}{\lambda} \left[ 1 - \sqrt{1 + \frac{4\lambda M}{r^4}} \right]. \]  

(23)

This exactly matches with the static slice of the spherically symmetric vacuum solution of EGB gravity in Eq. (6) with \( \mathcal{M} \) as the corresponding mass parameter. Therefore outside the shell (i.e. \( r > \hat{r} \)), we have \( \mathcal{M} = M_+ \), the ADM mass of the total shell-gravity system. Inside the shell (i.e. \( r < \hat{r} \)), let us denote \( M = M_0 \). The relationship between \( M \) and \( M_+ \) is obtained by solving the constraints at the position of the shell.

### III. EFFECTIVE ACTION OF THE SHELL

We shall now follow a procedure similar to [1], and integrate out the gravitational degrees of freedom to find the equation of motion for the shell. The guiding principle is that, after elimination of the Lagrange multipliers \( N^t, N^r \), under a variation of the fields subject to the lapse and shift constraints, the variation of the effective action over all spacetime should take the Hamilton-Jacobi form,

\[ \delta \mathcal{A} = \int dt p_C \dot{\hat{r}} + \int dt dr (\pi_R \delta R + \pi_L \delta L) - \int dt M_+. \]  

(24)

In this phase space of configurations (away from the shell), \( \pi_L \) and \( \pi_R \) have already been determined in terms of the \( L \) and \( R \) as in Eq. (13) and Eq. (14) respectively. Therefore, we can find \( \mathcal{A} \) by substituting for \( \pi_{L,R} \) in the above equation and integrating over field space.

We will integrate along a contour in \( L \)-space and \( R \)-space (over which the constraints are satisfied) starting from the given geometry specified by \( L, R, \pi_L, \pi_R \) up to some fiducial geometry which has \( \pi_R, L = 0 \). Here we implicitly assume that \( L, R \) configuration sub-space is homotopically trivial (otherwise the choice of the contour of integration will matter). This is done in two stages- integrate \( \int \pi_L dL \) keeping \( R \) fixed with \( \pi_L \) determined by Equation (13) until \( \pi_L = 0 \). Subsequently, we integrate along the \( \pi_L = 0 \) contour (varying both \( L \) and \( R \)) to the fiducial geometry.

Before proceeding to perform the integrals, we rewrite Eq. (13) as,

\[ \pi_L = - \left[ \frac{\lambda y^3}{3} + \lambda y \left( a(R) - y^2 \right) \right], \]  

(25)

where \( a(R) = \sqrt{\frac{4\lambda M}{R^4} + \frac{R^2}{\lambda}} \).

Here we have used the definition Eq. (22), of ADM mass which can also be written as

\[ \left( b(L) + y^2 + \frac{R^2}{\lambda} \right)^2 = \left( \frac{4M}{\lambda} + \frac{R^4}{\lambda^2} \right). \]  

(26)

From this, assuming that \( R \) is being held fixed we get \( dL = -(yL^3/R^2) dy \) where we consider \( L = L(y) \). This allows us to change the variable of integration from \( L \) to the field \( y \). At the lower limit of integration, the value of \( y \) is
determined by requiring that \( \pi_L = 0 \) - which is a cubic equation for \( y \). But we have already observed that \( y = 0 \) gives the GR limit. Therefore, we integrate along the real \( y \)-contour until \( y = 0 \).

Thus we get

\[
\mathcal{A} = -\int_{r_{\min}}^{\infty} dr \int_0^y \frac{R' \pi_L(y) y \, dy}{(k^2 + y^2)^{\frac{3}{2}}},
\]

where we have defined \( k^2 = 1 - a(R) + (R^2/\lambda) \).

The integral can be done exactly and we get

\[
\mathcal{A} = -\int dr \left( Ly \left( \lambda + R^2 + \frac{\lambda y^2}{3} \right) - R' \left( \lambda + R^2 \right) \log \left( \frac{R' + y}{k} \right) \right),
\]

where the variable \( y \) is an implicit function \( y(L, R, R') \) as determined from the constraint Eq. \( (28) \). The above derivation works away from the shell, and hence the coordinate \( r \) is to be integrated over all space excluding the location of the shell at \( r = \hat{r} \).

We can complete the evaluation of the action by integrating along the \( \pi_L = 0 \) contour by varying \( R \) until we attain the fiducial metric. But since \( \pi_L = 0 \), the second constraint \( \pi_R = (L/R') \pi_L' \) implies that \( \pi_R = 0 \) - hence, this integral doesn’t contribute anything to the action as in [1]. It is obvious that the \( \lambda \to 0 \) limit reduces to the GR action (the powers of \( R \) are different from [1] since we are in five dimensions).

By construction, the action in Eq. \( (28) \) satisfies \( \pi_L = (\delta \mathcal{A}/\delta L) \). But since derivatives of \( R \) appear in the action, an integration by parts is required in determining \( (\delta \mathcal{A}/\delta R) = \pi_R \). Remarkably, one can show that the second constraint \( \pi_R = (L/R') \pi_L' \) holds automatically, modulo terms at the location of the shell which spoil this identity. These surface terms at \( r = \hat{r} \) are similar to the ones in [1],

\[
\left[ \left( \frac{\partial \mathcal{A}}{\partial R'} \right)(\hat{r} + \epsilon) - \left( \frac{\partial \mathcal{A}}{\partial R'} \right)(\hat{r} - \epsilon) \right] dR.
\]

We can evaluate these terms, \( (\partial \mathcal{A}/\partial R') \) and subtract them from the action which will ensure that \( \pi_R = (\delta \mathcal{A}/\delta R) \) everywhere.

Similarly, under an arbitrary variation \( \delta R \), the ADM mass term in the action changes as well because the \( y \)-field implicitly depends on \( R \). Including these surface terms, we arrive at

\[
\frac{\partial \mathcal{A}}{\partial M_+} dM_+ = \int_{\hat{r} + \epsilon}^{\infty} dr \frac{Ly}{(k^2 + y^2)^{\frac{3}{2}}} dM_+.
\]

Combining all the terms given above, the action becomes,

\[
\mathcal{A} = \int_{r_{\min}}^{\hat{r} + \epsilon} dr \left[ -L y \left( \frac{\lambda y^2}{3} + \lambda + R^2 \right) + R' \left( R^2 + \lambda \right) \log \left( \frac{y + R'}{k} \right) \right] + \int_{\hat{r} + \epsilon}^{\infty} dr \left[ -L y \left( \frac{\lambda y^2}{3} + \lambda + R^2 \right) \right]
\]

\[
+ R' \left( R^2 + \lambda \right) \log \left( \frac{y + R'}{k} \right) - \int dt \frac{dR}{dt} \left\{ (R^2 + \lambda) \log \left( \frac{y_{t+ \epsilon} + R'}{k} \right) - \lambda y_{t+ \epsilon} \left( \frac{R'}{L} \right) \right\}
\]

\[
+ \int dt \frac{dR}{dt} \left\{ (R^2 + \lambda) \log \left( \frac{y_{t+ \epsilon} + R'}{k} \right) - \lambda y_{t+ \epsilon} \left( \frac{R'}{L} \right) \right\} + \int dt \left[ \int_{r+ \epsilon}^{\infty} \frac{Ly}{y^2 - \left( \frac{R'}{L} \right)} M_+ - \int dt M_+. \right]
\]

This \textit{reduced} action still has considerable gauge redundancy coming from coordinate transformations. We may use this to our advantage and simplify terms by choosing a particular form for the fields \( R, L \) - however, we have to ensure that the jump constraint coming from \( H_t = 0 \) is respected. Put in another way, not all coordinate transformations are allowed by the constraints - especially in the region around the shell.

A natural choice of coordinates that fixes this gauge freedom is \( R = r \) and \( L = 1 \). However, if we assume it for \( r > (\hat{r} + \epsilon) \), because of \( H_t = 0 \), we cannot assume \( R = r \) in a region \( r_c < r < \hat{r} - \epsilon \) inside of the shell. In this region the field \( R' \) is not an independent variable - i.e., it is determined by the jump discontinuity \( H_t = 0 \) in terms of the variables in the region \( r > \hat{r} + \epsilon \).
Keeping this in mind, we shall calculate the time derivative of the above action to obtain the Lagrangian for this system. The derivative w.r.t $L$ gives \( \frac{\partial A}{\partial L} \frac{dL}{dt} = \pi_L \dot{L} \). The contribution from the derivative w.r.t $R$ involves three terms

\[
\frac{\partial A}{\partial R} \frac{dR}{dt} + \frac{\partial A}{\partial R'} \frac{dR'}{dt} = \frac{\partial A}{\partial R} \frac{dR}{dt} - \frac{\partial}{\partial R'} \left( \frac{\partial A}{\partial R} \right) \dot{R} + \frac{\partial}{\partial R'} \left( \frac{\partial A}{\partial R'} \dot{R} \right).
\]

The first two terms combine to give $\pi_R \dot{R}$, but the second term involves varying $R'$ over all spacetime - as we have argued, due to the constraint $\mathcal{H}_t = 0$, in the range $r_\epsilon < r < \dot{r} - \epsilon$, we are not allowed to vary $R'$. Thus we subtract the integral of this term over this range. By an explicit computation, we find that only the term involving the second derivative of $R$ contributes in this range \([18]\) which is

\[
\int_{r_\epsilon}^{\dot{r} - \epsilon} \frac{R'}{yL} \left[ \lambda \left[ 1 - \left( \frac{R'}{L} \right)^2 \right] + R^2 - \lambda y^2 \right] \dot{R}.
\]

In this region, we may as well assume that $y, L$ are essentially constant, and $\dot{R} = R' \dot{r}$. We can now perform the integral to get \([18]\).

\[
\int_{r_\epsilon}^{\dot{r} - \epsilon} \frac{R'}{yL} \lambda \left[ 1 - \left( \frac{R'}{L} \right)^2 \right] + R^2 \dot{R}.
\]

The remaining part of the integration is quite straightforward, and we find that the terms rearrange nicely to give the Lagrangian,

\[
\mathcal{L} = \dot{\hat{L}} \{ \pi_L(y_\epsilon - \pi_L(y_\epsilon)) \} - \dot{R} \left( R^2 + \lambda \right) \log \left( \frac{y_{r_\epsilon} + \frac{R'}{L}}{y_\epsilon + \frac{R'}{L}} \right) + \int_{r_\epsilon}^{\dot{r} - \epsilon} \frac{\pi_R \dot{R} + \pi_L \dot{L}}{} + \int_{r_\epsilon}^{\dot{r} + \epsilon} \frac{\pi_R \dot{R} + \pi_L \dot{L}}{} - M_+.
\]

To proceed further and obtain a Lagrangian with a single particle interpretation, we need to use the jump conditions and determine the relationship between $y_\epsilon$ and $y(\dot{r} - \epsilon)$ and similarly for $R'$.

### IV. THE JUMP DISCONTINUITIES

As mentioned earlier, we shall use a coordinate system where the functions $R, L, N^t$ and $N^r$ are continuous across the shell while $R', L', \pi_L$, and $\pi_R$ are allowed to have finite discontinuities. The discontinuity in $\pi_L$ can be calculated by integrating the constraint equation $\mathcal{H}_t^e + \mathcal{H}_t^G = 0$ across the shell, which gives

\[
\pi_L(\dot{r} + \epsilon) - \pi_L(\dot{r} - \epsilon) = -\frac{p}{L}. \tag{36}
\]

This equation is exactly same as that of general relativity \([1]\). In case of GR, the second junction condition which determines the jump discontinuity of $R'$ can also be found by integrating the $\mathcal{H}_t = 0$ equation in a straightforward manner. But in the case of EGB gravity, the situation is different. To understand the difficulty, we write the explicit form of the constraint equation:

\[
0 = \mathcal{H}_t^e + \mathcal{H}_t^G = \left( \sqrt{\left( \frac{p}{L} \right)^2 + m^2} \right) \delta(r - \dot{r}) + y \left\{ \pi_R + y \left[ LR - \lambda \left( \frac{R'}{L} \right)' \right] \right\} - LR \left[ 1 - \left( \frac{R'}{L} \right)^2 \right] + \left( \frac{R'}{L} \right)' \left( R^2 + \lambda \left[ 1 - \left( \frac{R'}{L} \right)^2 \right] \right).
\]

All the terms can be easily integrated across the shell except the term $\lambda y^2$ $(R'/L)'$ since both $y$ and $R'$ have jump discontinuities at the location of the shell. Unless we know the exact dependence of $y$ in terms of other quantities, the second jump condition cannot be determined.

One way is to solve Eq.\([13]\) and express $y$ in terms of $\pi_L$ which can always be done at least in a series expansion. We can then substitute the result into the integral and determine the jump condition (order by order). Such a procedure although possible in principle is non-trivial to implement because of the presence of several branches of the solution.
To solve this problem, we instead use a physically motivated condition. First of all, we note that in GR, there is no such problematic term and the jump condition is (Eqn. 3.12 of [1], generalized to five dimensions),

$$R'(\dot{r} + \epsilon) - R'(\dot{r} - \epsilon) = -\sqrt{p^2 + m^2 L^2} \over R^2.$$  

(38)

Also, in case of GR, we have $y = -\left(\pi_L/R^2\right)$. Let us consider the case of a massless shell and define $\eta = sgn(p) = \pm$. Then the jump condition maybe rewritten as,

$$\left(\frac{R'}{L} + \eta y\right)_{\dot{r}+\epsilon} - \left(\frac{R'}{L} + \eta y\right)_{\dot{r}-\epsilon} .$$  

(39)

To understand the implication of the continuity of the quantity $\eta y + (R'/L)$ across the shell, we consider the quantity $dR/dt$ which represents the velocity of the shell, in our choice of gauge $R = r$. We demand that the shell is modeled such a way that this velocity of the shell is constant across the shell i.e, we are considering a structureless shell without any internal stresses. We will show that this implies the continuity of the quantity $\eta y + (R'/L)$ [19].

So, we restrict ourselves to the solutions which satisfy the condition,

$$\frac{dR}{dt}(\dot{r} + \epsilon) = \frac{dR}{dt}(\dot{r} - \epsilon).$$  

(40)

This immediately implies,

$$\dot{R}(\dot{r} + \epsilon) + R'(\dot{r} + \epsilon)\dot{r} = \dot{R}(\dot{r} - \epsilon) + R'(\dot{r} - \epsilon)\dot{r}.$$  

(41)

We write this as $\dot{r}\Delta R' + \Delta \dot{R} = 0$ where $\Delta$ represents the jump across the shell. We can also replace the jump in $\dot{R}$ in terms of the discontinuity of $y$ using the definition $y = (R - N^t R')/N^t$. Since all the metric functions like $R$, $L$, $N^t$ and $N^r$ are continuous across the shell, we obtain,

$$\dot{r} = -\frac{\Delta y}{\Delta R'} \dot{N}^t - \dot{N}^r.$$  

(42)

This represents the trajectory of the shell in terms of various jump conditions. But the trajectory of the shell can also be obtained directly by varying the action of the shell in the geometry described by Eq. [18, 19, 19]. In the massless limit, the variation gives,

$$\dot{r} = \frac{\dot{N}^t}{\eta L} - \dot{N}^r.$$  

(43)

Comparing Eq. (42) and Eq. (43) we get the condition,

$$\Delta \left(\frac{R'}{L} + \eta y\right) = 0 \implies \left(\frac{R'}{L} + \eta y\right)(\dot{r} + \epsilon) = \left(\frac{R'}{L} + \eta y\right)(\dot{r} - \epsilon).$$  

(44)

Therefore, the continuity of the quantity $\eta y + (R'/L)$ is equivalent to the statement that the thin shell has velocity $\dot{r}$. In GR, the jump condition is automatically compatible with this condition. In case of EGB gravity, we will impose this as an extra condition and this will allow us to derive the second jump condition (without explicitly determining $y$ in terms of $\pi_L$).

Using this condition, we can rewrite the offending terms in the constraint equation as,

$$y^2 \left(\frac{R'}{L}\right)' = \left[\left(\frac{R'}{L}\right)^2 - 2\left(\frac{R'}{L}\right)\left(\frac{R'}{L} + \eta y\right) + \left(\frac{R'}{L} + \eta y\right)^2\right] \left(\frac{R'}{L}\right).$$  

(45)

As is evident, if we use the continuity of $\eta y + (R'/L)$, we can easily integrate this across the shell. In fact, in Appendix [VIII] we show how to explicitly integrate this can obtain a consistent solution of both the constraint equations.

So in conclusion, we will finally consider only a massless shell obeying continuity of $\eta y + (R'/L)$ across the shell. The effective trajectory of the shell will be determined subjected to this condition. In [18, 19], such continuity conditions were shown to lead to consistent equations of motion of the shell, and in particular, were necessary in order to determine the equation of motion for the momentum of the shell.
V. EQUATION OF MOTION AND TUNNELING PROBABILITY

Inserting the continuity condition Eq. (44) in the massless limit into the Lagrangian in Eq. (35), we get

\[ \mathcal{L} = iL \left\{ \pi_L(y_<) - \pi_L(y_>) \right\} - \eta \dot{R} (R^2 + \lambda) \log \left[ \frac{\pi'_L + \eta y_>} {\pi'_L + \eta y_<} \right] + \int_{r_{\min}}^{r_0} dr \left[ \pi_R \dot{R} + \pi_L \dot{L} \right] + \int_{r_0}^{\infty} dr \left[ \pi_R \dot{R} + \pi_L \dot{L} \right] - M_+. \] (46)

If we set \( R = r \) and \( L = 1 \) as a gauge choice in the Lagrangian, the time derivative terms drop out, and we obtain the canonical momentum of the particle as,

\[ p_e = \pi_L(y_<) - \pi_L(y_>) - \eta (r^2 + \lambda) \log \left[ \frac{1 + \eta y_>} {1 + \eta y_<} \right]. \] (47)

As in the case of Einstein gravity (1), we find that \( M_+ \) is the Hamiltonian of the shell (since the Lagrangian has the structure of \( L = p_e \dot{r} - H \)). It is also possible to obtain an explicit expression for \( M_+ \).

The trajectory of the massless shell is obtained from the effective Lagrangian as,

\[ \frac{1}{\dot{r}} = \frac{\partial p_e}{\partial M_+} = \frac{1}{y + \eta} + \frac{\eta \lambda (y - \eta)}{y (\lambda y^2 + r^2)}. \] (48)

This represents the effective trajectory of a massless shell moving in a spherically symmetric black hole spacetime which is a solution of Einstein-Gauss-Bonnet gravity. For definiteness, let us now choose the case \( \eta = +1 \), corresponding to an outgoing shell. Also, for our choice of gauge, \( y = -N^r/N^t \) and solving the ADM mass equation Eq. (26) gives \( y = \pm \sqrt{1 - f(r)} \) where

\[ f(r) = 1 + \frac{r^2}{\lambda} \left[ 1 - \sqrt{1 + \frac{4 \lambda M_+}{r^4}} \right]. \] (49)

\( M_+ \) is a function of the momentum \( p \) of the shell which may be determined by using the jump conditions as detailed in the appendix. But we shall not require the explicit solution in what follows.

We fix all the signs using the case of general relativity. In general relativity, (when \( \lambda = 0 \)) the effective trajectory of the shell is a null geodesic in a Gullstrand-Painlevé form of the metric given by [1],

\[ ds^2 = -f(r) dt^2 + 2 \sqrt{1 - f(r)} dt dr + dr^2 + r^2 d\Omega^2, \] (50)

thus fixing the sign \( y = -\sqrt{1 - f} \). The back reaction effect due to the shell of energy \( E \) shifts the mass of the original solution from \( M \) to \( M_+ = M - E \). In the presence of the Gauss-Bonnet term, the trajectory Eq. (48) of the shell is modified and the shell no longer moves along a null geodesic of the original metric.

Once we obtain the trajectory of the shell, we can proceed to calculate the quantum mechanical emission probability. In [1], this is determined by quantizing the effective Lagrangian of the shell and finding the Bogoliubov coefficients using a WKB approximation. In principle, we can repeat the same calculation with the trajectory in Eq. (48). But, instead, we shall follow an equivalent but more physically transparent procedure using the results of [2, 20]. The key idea is to calculate the tunneling probability of such a shell moving in the geometry corrected by the back reaction by evaluating the imaginary part of the action due to the presence of the horizon.

We begin with the action and find the imaginary part of the action due to the presence of the horizon at \( f(r, M_+) = 0 \). The semi-classical action for an s-wave outgoing positive energy shell which crosses the horizon outwards from \( r_h - \epsilon \) to \( r_h + \epsilon \) is,

\[ A = \int p \, dr = \int_0^p dp' \, dr = \int_{r_h - \epsilon}^{r_h + \epsilon} dr \int_0^p \frac{dM}{r}. \] (51)

Where we have used the Hamiltonian equation \( (dM/dp) = \dot{r} \) and \( dM \) is the change in ADM Hamiltonian due to the tunneling of the shell, so in our case \( dM = M - M_+ \), the shift in the ADM mass of the spacetime due to the back
The crucial step is then to consider the trajectory of the shell near the horizon. The presence of the horizon leads to a pole in the semi-classical action and when extended to the complex plane, the pole gives rise to a residue contributing to the imaginary part of the action. Therefore, we only need to consider the trajectory in a near horizon approximation. In terms of the function \( f(r) \), the trajectory of an outgoing shell is,

\[
\frac{1}{r} = \frac{1}{1 - \sqrt{1 - f}} + \frac{\lambda(1 + \sqrt{1 - f})}{\sqrt{1 - f}(\lambda(1 - f) + r^2)}.
\]

The second term of the trajectory in Eq. (52) is finite. The contribution to the imaginary part is entirely due to the first term and near the horizon at \( r = r_h \), Eq. (51) can be written as,

\[
A = \int \int_{r_h - \epsilon}^{r_h + \epsilon} \frac{dM}{r} \frac{dr}{\kappa(r - r_h)},
\]

where we have defined the surface gravity as \( \kappa = f'(r_h)/2 \). To evaluate the imaginary part, we evaluate the residue at this pole,

\[
A = (-i\pi) \int \frac{dM}{\kappa} = (-i) \int \frac{\pi dM}{\kappa}.
\]

Then, the total semi-classical tunneling probability is,

\[
\Gamma \propto e^{-2 \operatorname{Im} A} \propto e^{\frac{\pi \Delta S}{\kappa}}.
\]

Using the first law for black holes in EGB gravity expressed in Eq. (7), we can express this tunneling probability as,

\[
\Gamma \propto e^{\int \delta S} = e^{\Delta S},
\]

where \( \Delta S \) is the change in the entropy as the ADM mass changes from \( M \) to \( M_a = M - E \), due to the tunneling of a shell of energy \( E \). Note that, here the entropy is no longer proportional to the area but given by the Jacobson-Myers expression in Eq. (8).

In conclusion, the effective trajectory of the shell in EGB gravity is no longer a null geodesic in the corresponding Gullstrand-Painlevé form of the metric. However, the pole structure due to the presence of the horizon remains same as in the case of general relativity and a calculation based on semi-classical tunneling gives the emission probability of a shell of energy \( E \) as,

\[
\Gamma(E) \propto e^{\Delta S},
\]

where \( \Delta S = S(M) - S(M - E) \), the change in entropy because of the change in the ADM mass due to the back reaction of the shell. Note that, the entropy \( S \) is exactly same as the expression which obeys the first law in Eq. (7). Both the background field equation, as well as the entropy pick up corrections due to the Gauss-Bonnet term, but the tunneling probability continues to be the same form as in the case of general relativity.

As discussed in the introduction, as long as the tunneling probability is given by the exponential of the change of the entropy, there will be no correlation between the emission of two successive shells since \( \Gamma(E_1 + E_2) = \Gamma(E_1) + \Gamma(E_2) \). Hence, the correction due to the Gauss-Bonnet term does not create any new correlation in the semi-classical Hawking spectrum and therefore any hope of solving the information loss paradox using modified dynamics of gravity may not be fruitful. As long as we are in the regime of the validity of semiclassical gravity, it seems that information loss is inevitable.

VI. DISCUSSION

The inclusion of higher curvature terms in the action changes the dynamics of gravity. Therefore, it seemed plausible that the self-interaction calculations in [1] could be modified significantly in the presence of higher curvature terms and lead to new correlations in the Hawking spectrum. This possibility motivated us to generalize the results of [1].
beyond general relativity. We consider an Einstein-Gauss-Bonnet theory in five dimensions and calculate the effective trajectory of a massless shell. The procedure is almost parallel to that in the case of general relativity, except that the jump conditions were handled differently. We imposed a continuity condition on the velocity of the shell which was automatic in GR (see the discussion in [18, 19]) and this enabled us to integrate the jump condition across the shell.

The characteristic feature of the Gauss-Bonnet terms was that the constraint structure is same as general relativity. There are no additional degrees of freedom and therefore the calculations turned out to be straightforward. The effective equation of a massless shell can be obtained and is given by Eq. (48). The trajectory is used to calculate the semi-classical tunneling probability and the result is the same form as of general relativity \( \Gamma(E) \propto e^{\Delta S} \) where the entropy \( S \) is now the Jacobson Myers entropy in Eq. (8) obeying the first law. Also, as in case of GR, \( \Gamma(E_1 + E_2) = \Gamma(E_1) + \Gamma(E_2) \) implying that there is no correlation between successive emission of two shells.

Although, it has been argued that small corrections in Hawking spectrum can not lead to the purification of the final state [21, 22], it is interesting to note that a spherically symmetric self-interaction calculation does not produce any useful correlation at all, even when the dynamics of gravity is modified by the Gauss-Bonnet term. Clearly, it will be worthwhile to extend our result beyond Lovelock theories and analyze the situation in the presence of extra degrees of freedom and/or extra constraints.

It has been suggested [23], that the original derivation of black hole radiation may not be sensitive to the details of the dynamical structure of field equations but rather depends only on the kinematical properties of the event horizon. Our result is in conformity with this point of view and seems to suggest that the proper resolution of information loss is beyond the scope of semi-classical gravity.

VII. ACKNOWLEDGEMENTS

The authors would wish to thank the organizers of the GIAN workshop at IIT Gandhinagar, where this work was initiated. SS is supported by the Department of Science and Technology, Government of India under the SERB Fast Track Scheme for Young Scientists (YSS/2015/001346). KPY likes to thank IIT, Gandhinagar for hospitality, where part of this work was carried out.

VIII. APPENDIX: A CONSISTENCY RELATION FOR THE CONSTRAINTS.

In section IV, we have shown how the assumption of that the velocity of the shell is well defined allows us to obtain the jump conditions for \( \pi_L \) and \( R' \) across the shell [14]. In this appendix, we show that the discontinuities of the field variables across the shell are all mutually consistent and show that there exist real solutions such that the constraints are satisfied.

As mentioned earlier, we shall use a coordinate system where the functions \( R, L, N^t \) and \( N^r \) are continuous across the shell while \( R', L', \pi_L, \) and \( \pi_R \) are allowed to have finite discontinuities. For simplicity, we consider the shell is located between \( r = \hat{r} \) and \( r = \hat{r} + \epsilon \) and we take a simple choice of the metric function across the shell as [18],

\[
R(r, t) = r - \epsilon \alpha g \left( \frac{\hat{r} - r}{\epsilon} \right); \quad L = 1, \tag{58}
\]

where \( g(x) \) is a continuous and differentiable function defined for all \( x \in (0, -1) \) and satisfies the following,

\[
g(0) = g(-1) = 0; \quad g'(0) = 1; \quad g'(-1) = 0,
\]

and time appears implicitly through \( \hat{r}, \alpha \). Differentiating \( R(r, t) \) w.r.t \( r \) gives,

\[
R'_\hat{r} = 1 + \alpha; \quad R'_{\hat{r}+\epsilon} = 1.
\]

The variable \( y \) is now determined so that the velocity continuity condition [14] is satisfied,

\[
y_{\hat{r}+\epsilon} = h; \quad y_{\hat{r}} = h - \eta \alpha; \quad \eta = \text{sign}(p).
\]

The value of \( h(t) \) can be readily obtained from the ADM condition Eq. (26) as,

\[
h = \pm \sqrt{1 - f(r, M_+)}; \quad f(r, M_+) = 1 + \frac{\kappa^2}{\lambda} \left[ 1 - \sqrt{1 + \frac{4 \lambda M_+}{r^4}} \right]. \tag{59}
\]
Now the discontinuity of $\pi_L$ can be written as,
\begin{equation}
\frac{2 \eta \lambda \alpha^3}{3} - \eta \lambda h^2 \alpha - 2 \lambda h \alpha - \eta \alpha X^2 + 2 \eta \lambda \alpha^2 = -p. \tag{60}
\end{equation}

The second constraint relation is obtained by integrating $\mathcal{H}_t^s + \mathcal{H}_t^G = 0$ across the shell \[37\]. We assume that the momentum $\pi_R$ and $y$ are both non-singular at the location of the shell. Using the velocity continuity condition as in Eq. (45), we may integrate across the shell and then substitute to get
\begin{equation}
\frac{2 \lambda \alpha^3}{3} + \lambda \alpha^2 - \eta \lambda \alpha^2 h + \lambda h^2 \alpha - \alpha X^2 = -\hat{V}. \tag{61}
\end{equation}

These two equations show that the values of the fields $R', y$ inside the shell are fixed by their values outside the shell.

Solving the above two equations in the massless limit leads to the relation
\begin{equation}
2 h^2 + \eta h(2 - \alpha) - \alpha = 0. \tag{62}
\end{equation}

This equation can be solved for $\alpha(t)$ in terms of $h(t)$ which, in turn, is determined as a solution of either cubic equations in terms of the $\hat{r}$ variables of the shell. Then $M_+$ is determined in by $h = \pm \sqrt{1 - f(r, M_+)}$.