Current-Target Correlations as a Probe of $\Delta G/G$
in Polarized Deep Inelastic Scattering

I. V. Akushevich $^a$ and S. V. Chekanov $^b$

$^a$ National Center of Particle and High Energy Physics, Bogdanovich str. 153, 220040 Minsk, Belarus.
Email: akush@hermes.desy.de

$^b$ Argonne National Laboratory, 9700 S.Cass Avenue, Argonne, IL 60439, USA
Email: chekanov@mail.desy.de

Abstract

The measurement of the polarized gluon distribution function $\Delta G/G$ using current-target correlations in the Breit frame of deep inelastic scattering is proposed. The approach is illustrated using a Monte Carlo simulation of polarized $ep$-collisions for HERA energies.

1 Introduction

A direct determination of the polarized gluon distribution is of importance for understanding the spin properties of the nucleon. Recent experimental measurements of spin-dependent structure functions $[1–3]$ have shown that the valence quarks account for only a small fraction of the nucleon spin. One of the possible explanations for this observation, known as “spin crisis”, is to assume a large contribution from the gluon spin. There exist many theoretical models which explain this phenomenon, but there is no experimental evidence to favor one of them.

A direct way to solve this puzzle is to measure the gluon density in polarized lepton-nucleon scattering. One of the suggested methods is based on the detection of the correlated high-$p_t$ hadron pairs in polarized deep inelastic scattering (DIS) $[4]$. This measurement has already been performed by the HERMES Collaboration $[5]$. Another possibility, which is planned to be studied at the COMPASS experiment, is to analyze events with open/closed charm $[6]$. At the HERA $ep$ collider, after possible upgrade to polarized beams, the polarized gluon density can also be measured from dijet events $[7]$.

In this paper a possibility to measure $\Delta G/G$ from the multiplicity correlations in neutral-current DIS is discussed. The proposed method is based on the Breit frame $[8]$. For

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1 On leave from the Institute of Physics, AS of Belarus, Skaryna av.70, Minsk 220072, Belarus.
Figure 1: A schematic representation of the Breit frame for the quark-parton model.

In this frame, in the quark-parton model (QPM), the incident quark carries $Q/2$ momentum in the positive $Z$-direction and the outgoing struck quark carries the same momentum in the negative $Z$-direction. The phase space of the event can be divided into two regions (see Fig. 1). All particles with negative $p^Z_{\text{Breit}}$ components of momenta form the current region. In the QPM, all these particles are produced from hadronization of the struck quark. Particles with positive $p^Z_{\text{Breit}}$ are assigned to the target region, which is associated with the proton remnants.

For the QPM in the Breit frame, the phase-space separation between the particles from the struck quark and the proton remnants is maximal. Therefore, it is expected that there are no correlations between the current-region particles and particles in the target region of the Breit frame. Such a separation does not exist when the leading-order QCD processes, known as Boson-Gluon Fusion (BGF) and QCD-Compton (QCDC) scattering, are involved. The kinematics of these processes lead to current-target anti-correlations predicted in [9] and experimentally measured by the ZEUS Collaboration [11].

The magnitude of these correlations at small $x$ is mainly determined by the BGF processes.

2 Current-Target Correlations. Unpolarized Case

Recently, it was noticed that the BGF events in non-polarized DIS can be studied without involving the jet algorithms [3]. For this one can measure a linear interdependence between the current- and target-region multiplicities in the Breit frame. The approach involves the measurement of the particle multiplicities in large phase-space regions, rather than clustering separate particles at small resolution scales. Therefore, high-order QCD and hadronization effects are expected to be smaller than for the dijet studies that use clustering algorithms with specific resolution scales. In addition, the method does not depend on the jet transverse energy $E_T$ used in jet reconstruction and is well suited for low $Q^2$ regions where the jet algorithms are less reliable.

As usually, $Q^2 = -q^2 = -(k - k')^2$ ($k$ and $k'$ denote the 4-momenta of the initial and final-state lepton, respectively) and the Bjorken scaling variable $x$ is defined as $Q^2/(2P \cdot q)$, where $P$ is the 4-momentum of the proton.
The correlation between the current and target region multiplicities can be measured with the covariance

$$\text{cov} = \langle n_c n_t \rangle - \langle n_c \rangle \langle n_t \rangle, \quad (1)$$

where $n_c$ ($n_t$) is the number of final-state particles in the current (target) region and $\langle \ldots \rangle$ is the averaging over all events. If $h$ particles from the hard QCD processes are emitted in the target region, one can rewrite (1) as

$$\text{cov} = \langle (\tilde{n}_c - h) (\tilde{n}_t + h) \rangle - \langle \tilde{n}_c - h \rangle \langle \tilde{n}_t + h \rangle. \quad (2)$$

Here $\tilde{n}_c$ is the total number of particles coming from zero and first-order QCD processes ($h \leq \tilde{n}_c$) and $\tilde{n}_t$ is the multiplicity of the proton remnants without counting the particles due to the hard QCD processes. From (1) one obtains

$$\text{cov} = \langle \tilde{n}_c h \rangle - \langle \tilde{n}_c \rangle \langle h \rangle - \langle h^2 \rangle. \quad (3)$$

In this expression, the contribution from the remnant multiplicity $\tilde{n}_t$ cancels since we consider the case when $\tilde{n}_t$ is independent of $n_c$ and $h$. This key assumption means that the only important effects leading to the correlation between $n_c$ and $n_t$ are the hard QCD radiation. The validity of this assumption has been tested in [9] for a Monte Carlo model based on the parton shower, describing higher than first order QCD effects, and the LUND string model for hadronization.

At small $x$, the BGF is the only dominant first-order QCD process. Let us define the BGF rate as

$$R_{\text{BGF}} = \frac{N_{\text{BGF}}}{N_{\text{ev}}}, \quad (4)$$

where $N_{\text{BGF}}$ is the number of the BGF events and $N_{\text{ev}}$ is the total number of events in the limit $N_{\text{ev}} \to \infty$. According to the assumption discussed above, $h = 0$ for the QPM. For the BGF, however, one or two quarks are emitted in the target hemisphere, so that their parton radiation gives $h > 0$. The averaging in (1-3) is performed over all possible DIS events, despite the fact that $\text{cov} \neq 0$ for the BGF events only. Therefore, one can apply the averaging over the relevant BGF events using the relation:

$$\langle f(h) \rangle = R_{\text{BGF}} \langle f(h) \rangle_{\text{BGF}}, \quad (5)$$

where $f(h)$ is any function of $h$ so that $f(h = 0) = 0$ and $\langle \ldots \rangle_{\text{BGF}}$ is the average over the BGF events. Using this relation, one has

$$\text{cov} = \langle (\tilde{n}_c h)_{\text{BGF}} - \langle \tilde{n}_c \rangle_{\text{BGF}} \langle h \rangle_{\text{BGF}} - \langle h^2 \rangle_{\text{BGF}} + R_{\text{BGF}} \langle h^2 \rangle_{\text{BGF}} \rangle R_{\text{BGF}}. \quad (6)$$

The last term can be neglected since it contains a small contribution. Therefore, in the linear approximation, the covariance can be expressed as

$$\text{cov} \simeq -A(Q^2, x) R_{\text{BGF}} \quad (7)$$

with

$$A(Q^2, x) = \langle \tilde{n}_c \rangle_{\text{BGF}} \langle h \rangle_{\text{BGF}} + \langle h^2 \rangle_{\text{BGF}} - \langle \tilde{n}_c h \rangle_{\text{BGF}}. \quad (8)$$

A more detailed form of this expression has been obtained in [9].

The function $A(Q^2, x)$ depends on:
1) A number of the final-state hadrons in the jets initiated by the quarks in the BGF processes.

2) Kinematics of the BGF jets in the Breit frame, which depend on $Q^2$ and $x$. Using LEPTO Monte Carlo [12] with the tuning described in [13] and GRV94 HO [14] parameterization of the parton distribution functions, we have found that for $5 < Q^2 < 50$ GeV$^2$, the fraction of BGF events with both quarks moving to the target region increases from 52% at $\langle x \rangle \simeq 0.2 \cdot 10^{-1}$ to 61% at $\langle x \rangle = 0.5 \cdot 10^{-3}$ (100% corresponds to all possible jet configurations of the BGF events in the Breit frame). Therefore, $A(Q^2, x) \simeq A(Q^2)$ in (7) is a good approximation. Note an $x$-dependence of the cov due to the BGF kinematics is rather small compared to the $x$-dependence of the BGF rate itself: $R_{\text{BGF}}$ increases from 7% at $\langle x \rangle \simeq 0.2 \cdot 10^{-1}$ to 21% at $\langle x \rangle = 0.5 \cdot 10^{-3}$.

Before going to a polarized case, let us again remind all approximations made in (7).

1. We neglect the QCD Compton scattering, considering low $Q^2$ regions. Note that even if a small fraction of the QCDC is present, some QCDC events cannot contribute to the correlations since singularities in the QCDC cross section favor the event topology with two jets in the current region, which does not produce the correlations (see, for example, Ref. [15]).

2. Effects from high-order QCD and hadronization are already assumed to contribute to a value of $A(Q^2, x)$, an exact form of which is beyond the scope of the present study. However, the assumption made to derive (7) was that the high-order QCD processes and hadronization do not change significantly the LO dijet kinematics in the Breit frame. This assumption leads to the factorization of $A(Q^2, x)$ from the BGF rate in the linear approximation.

Using a numerical simulation [9], it was shown that the effects quoted above do not strongly contribute to the relation (7) with an $x$-independent $A(Q^2)$ in the range $5 < Q^2 < 50$ GeV$^2$ and $0.5 \cdot 10^{-3} < x < 0.2 \cdot 10^{-1}$. According to the LUND model implemented in JETSET, the hadronization does not produce large correlations during the formation and independent breaking of the strings stretched between the current-region partons and the remnant.

3 Polarized Case

The ratio of spin dependent gluon density $\Delta G(x)$ to spin averaged gluon density $G(x)$ at a fixed $Q^2$ is proportional to the asymmetry of the BGF cross sections,

$$\langle a_{LL} \rangle \frac{\Delta G(x)}{G(x)} = \frac{\sigma_{\uparrow \downarrow}^{\uparrow \downarrow} - \sigma_{\uparrow \downarrow}^{\uparrow \downarrow}}{\sigma_{\uparrow \downarrow}^{\uparrow \downarrow} + \sigma_{\uparrow \downarrow}^{\uparrow \downarrow}} = \frac{\sigma_{\text{tot}}^{\uparrow \downarrow} R_{\text{BGF}}^{\uparrow \downarrow} - \sigma_{\text{tot}}^{\uparrow \downarrow} R_{\text{BGF}}^{\uparrow \downarrow}}{\sigma_{\text{tot}}^{\uparrow \downarrow} R_{\text{BGF}}^{\uparrow \downarrow} + \sigma_{\text{tot}}^{\uparrow \downarrow} R_{\text{BGF}}^{\uparrow \downarrow}},$$

where $\langle a_{LL} \rangle$ is the value of the BGF asymmetry at partonic level [16], $\sigma_{\text{tot}}^{\uparrow \downarrow}$ ($R_{\text{BGF}}^{\uparrow \downarrow}$) and $\sigma_{BGF}^{\uparrow \downarrow}$ ($R_{BGF}^{\uparrow \downarrow}$) are the BGF cross sections (BGF rates) in case of the ant-parallel ($\uparrow \downarrow$) and parallel ($\uparrow \uparrow$) polarizations of the incoming lepton and nucleon. $\sigma_{\text{tot}}^{\uparrow \downarrow}$ and $\sigma_{\text{tot}}^{\uparrow \downarrow}$ are the total cross sections which can be obtained by counting the number of events in a given kinematic bin for the different spin configurations, normalized to an integrated luminosity.
The expression (9) can further be rewritten as

\[ \langle a_{LL} \rangle \frac{\Delta G(x)}{G(x)} \simeq \Delta \rho + A_\parallel, \quad \Delta \rho = \frac{R_{\uparrow\downarrow} - R_{\uparrow\uparrow}}{R_{\uparrow\downarrow} + R_{\uparrow\uparrow}}, \quad (10) \]

\[ A_\parallel = \frac{\sigma_{\text{tot}}^{\uparrow\downarrow} - \sigma_{\text{tot}}^{\uparrow\uparrow}}{\sigma_{\text{tot}}^{\uparrow\downarrow} + \sigma_{\text{tot}}^{\uparrow\uparrow}}, \quad (11) \]

where \( A_\parallel \) is an inclusive polarized asymmetry, which is expected to be small compared to the asymmetry \( \Delta \rho \) of the BGF rates. The relationship between (9) and (10) can easily be seen after rewriting \( \Delta \rho + A_\parallel \) as

\[ \frac{\sigma_{\text{tot}}^{\uparrow\downarrow} R_{\uparrow\downarrow}^{\text{BGF}} - \sigma_{\text{tot}}^{\uparrow\uparrow} R_{\uparrow\uparrow}^{\text{BGF}}}{(\sigma_{\text{tot}}^{\uparrow\downarrow} R_{\uparrow\downarrow}^{\text{BGF}} + \sigma_{\text{tot}}^{\uparrow\uparrow} R_{\uparrow\uparrow}^{\text{BGF}})(1 - a)}, \quad \]

\[ a = \frac{(R_{\uparrow\downarrow}^{\text{BGF}} - R_{\uparrow\uparrow}^{\text{BGF}})(\sigma_{\text{tot}}^{\uparrow\downarrow} - \sigma_{\text{tot}}^{\uparrow\uparrow})}{2(\sigma_{\text{tot}}^{\uparrow\downarrow} R_{\uparrow\downarrow}^{\text{BGF}} + \sigma_{\text{tot}}^{\uparrow\uparrow} R_{\uparrow\uparrow}^{\text{BGF}})}, \]

which is approximately equal to (9) by neglecting a small contribution from \( a \).

According to (10),

\[ \text{cov}^{\uparrow\downarrow} \simeq -A(Q^2) R_{\uparrow\downarrow}^{\text{BGF}}, \quad \text{cov}^{\uparrow\uparrow} \simeq -A(Q^2) R_{\uparrow\uparrow}^{\text{BGF}}, \quad (12) \]

where \( A(Q^2) \) is considered to be independent of the polarization, since this function is mainly determined by the number of the final-state hadrons in jets, i.e. by multiple-gluon radiation and hadronization. From (12) it is easy to see that the theoretical \( \Delta \rho \) can be determined via the covariances

\[ \Delta \rho \simeq \Delta \text{cov} = \frac{\text{cov}^{\uparrow\downarrow} - \text{cov}^{\uparrow\uparrow}}{\text{cov}^{\uparrow\downarrow} + \text{cov}^{\uparrow\uparrow}}. \quad (13) \]

Thus the asymmetry \( \Delta G(x)/G(x) \) can experimentally be obtained from \( \Delta \text{cov} \) and directly measurable \( A_\parallel \). Below we shall numerically estimate the BGF asymmetry \( \Delta \rho \) (14). We calculate this quantity directly from the BGF rates and then reconstruct it by measuring the \( \Delta \text{cov} \) from the final-state hadrons of polarized DIS.

### 4 Numerical Studies

In order to study the asymmetry using the current-target correlations we use the PEPSI Monte Carlo generator [17] for the polarized leptoproduction. This model is based on the LEPTO 6.5 for DIS together with JETSET 7.4 describing the fragmentation. Two DIS samples with the opposite spin configurations were generated in the region \( 5 < Q^2 < 50 \) GeV\(^2\). The electron and proton beam energies were taken to be 27.5 and 920 GeV, respectively. For simplicity, both beam polarizations were assumed to be 100%. The total number of the generated DIS events is 1M for each polarization sample for the given \( Q^2 \) range. The covariances for each polarization were determined from charged final-state hadrons according to (11). Hadrons with lifetime \( c\tau > 1 \) cm are declared to be stable.

Fig. 2a shows the behavior of \( \Delta \text{cov} \) calculated from final-state charged hadrons compared to the theoretical prediction for \( \Delta \rho \), generated also with the PEPSI using information on the generated type of the LO process. The GS-A [18] polarized parton distribution
Figure 2: The asymmetry calculated using correlations between current- and target-region charged multiplicities generated with PEPSI (full symbols) compared to the theoretical expectations for $\Delta \rho$ of the same model (solid lines). The figures show the $x$-dependence of the asymmetry in the range $5 < Q^2 < 50$ GeV$^2$: (a) The asymmetry for the GS-A model implemented in PEPSI; (b) The same but when $\Delta G$ is set to zero.

was used. From this figure one can conclude that the experimentally measured asymmetry $\Delta \text{cov}$ does reproduce the theoretically expected behavior of the $\Delta \rho$. This is especially evident for a small $x$, where the contribution from the QCDC can be neglected. It is also seen that typical statistics expected at HERA are sufficient to reliably determine the sign and the magnitude of the asymmetry.

For an illustration, Fig. 2b shows the same as Fig. 2a, but after switching the gluon polarization term off, i.e. $\Delta G = 0$. The quark polarization distributions were unchanged. In this case, both $\Delta \rho$ and $\Delta \text{cov}$ tend towards zero. A difference between $\Delta \rho$ and zero value is expected to come from a positive value of $A_{\parallel}$ [11]. It may also be noted a small discrepancy between $\Delta \text{cov}$ and $\Delta \rho$. This might mean that for this, in fact, unrealistic case, the values of $R_{\text{BGF}}^{\uparrow \downarrow}$ and $R_{\text{BGF}}^{\uparrow \uparrow}$ are so close to each other that an approximate nature of the expressions [12] makes a noticeable effect on the relationship between $\Delta \text{cov}$ and $\Delta \rho$.

**Dependence on cut-offs.** The LEPTO Monte Carlo used by PEPSI contains cut-offs to prevent divergences in the LO matrix elements. Therefore, the magnitude of the BGF rate in PEPSI is ambiguously defined. This may produce a systematic bias in the relation [13].

Let us remind that, in the BGF cross section, the singularities in the two-parton emission are given by $1/z(1-z)$ [13], where $z = (p' \cdot p)/(p' \cdot q)$ ($p$ is the momentum of the incoming parton and $p'$ is that of the final-state parton, $q^2 = -Q^2$). The LEPTO cut-offs are based on the so-called "mixed scheme" in which the parameter $z_{\text{min}}$, restricting the
values of the variable $z$, plays an important role in the determination of the probability for the BGF event to occur. The default value of this cut-off is set to 0.04. If one decreases this cut-off, the relative contribution of the BGF rates increases, with respect to the QCDC. In this case, the relation (7) is expected to be a good estimate. However, for a large $z_{\text{min}}$, the BGF rate is small and $\Delta \text{cov}$ might poorly reflect the behavior of $\Delta \rho$. Therefore, for our systematic checks, the value of $z_{\text{min}}$ was increased. We have verified that the relationship between $\Delta \text{cov}$ and $\Delta \rho$ still holds up to $z_{\text{min}} = 0.2$ for the given statistic uncertainties. Note that while for the default value $z_{\text{min}} = 0.04$ the BGF rate is about 0.6 in the smallest $x$-bin, the rate of this process drops down to 0.3 when $z_{\text{min}} = 0.2$ is considered.

We have also found that $\Delta \rho$ changes only a little by varying the cut-off. This can be explained by the normalisation $(R_{\text{BGF}}^{\uparrow \downarrow} + R_{\text{BGF}}^{\uparrow \uparrow})$ used in (10). Note that, from the experimental point of view, a similar normalisation $(\text{cov}^{\uparrow \downarrow} + \text{cov}^{\uparrow \uparrow})$ in (13) would help to measure the $\Delta \text{cov}$ reliably even if the detector track acceptance for the target-region is small (see [11] for details).

Dependence on the structure function. An important test for our method is to investigate the asymmetry measured with $\Delta \text{cov}$ for different types of the parameterization for the parton densities. Various types of polarized parton distributions included into the PEPSI package were analyzed. The method was found to be sensitive to input structure functions and $\Delta \text{cov}$ reproduces the trends of $\Delta \rho$.

5 Conclusions

In this paper we have proposed a new method to measure the asymmetry in polarized lepton-nucleon scattering. This method is based on the measurement of the current-target correlations in the Breit frame. The advantage of the method is that all inclusive DIS events with well reconstructed Breit frame can be used to determine the asymmetry, without specific constraints to select useful events for this measurement.

In this respect, it is important to emphasize that our approach is well suited for rather low $Q^2$ and $E_T$, i.e. for the regions where dijet reconstruction suffers from misclusterings and large hadronization corrections. Thus the suggested method compliments and extends the study of $\Delta G/G$ to kinematic regions of low $Q^2$ ($E_T$) where the method suggested in [7] is less reliable. We also expect different systematics for these two methods: While the method discussed in [7] is a subject of the systematic uncertainties as for the standard dijet studies, systematic effects for our method most probably would come from a low detector acceptance in the target region of the Breit frame.

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