TEMPERATURE DEPENDENCE OF HADRON MASSES FROM THE ZIMANYI-MOSZKOWSKI MODEL

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ABSTRACT

An extended version of the Zimanyi-Moskowski (ZM) model has been used to calculate the temperature dependence of the hadronic masses. The calculation of meson masses in the Random Phase Approximation (RPA) gives an increase in the $\sigma$, $\omega$ and $\pi$ effective masses with temperature.

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In an earlier work \[1\] we have calculated the density dependence of hadron masses from a newly proposed model: Zimanyi-Moskowski (ZM) model \[2\]. The original ZM model was extended to include pions and the density dependence of the meson masses were calculated, from the extended model, in the Random Phase Approximation (RPA). This model differs from the popular Walecka model \[3\] in the form of coupling between the scalar meson and the nucleon, which is a derivative coupling in the new model. As a result of this derivative coupling the model reproduces some of the experimental results nicely. It yields the incompressibility $K = 224.49\text{MeV}$, which is much closer to the experimental value ($K = 210 \pm 30\text{MeV}$) compared to the Walecka model ($K = 524\text{MeV}$), and a nucleon effective mass $M^*_N = 797.64\text{MeV}$ at the nuclear matter saturation density ($\rho = 0.17\text{fm}^{-3}$). But, the price we have to pay is that the model looses its renormalisability due to the derivative coupling. This one can accept, at least as far as the discussion of the effective models goes, since the description of hadronic matter need only be valid upto the temperatures $T \leq 200\text{MeV}$, provided the deconfinement phase transition to QCD matter is a reality.

In \[1\] we restricted ourselves to the zero temperature finite density scenario. But the temperature dependence of hadron masses, at zero density, is also very interesting to study as such a situation is expected to form in the central rapidity region of high energy heavy ion collisions. In this work, we would like to calculate the temperature dependence of hadron masses from the ZM model. We shall briefly describe the different variants of the extended ZM model and then calculate the temperature dependence of hadron masses.

The Lagrangians for the different variants of the ZM models along with the Walecka model can be written in a unified way, once we rescale different fields \[1, 4\]:

\[\text{Equations and expressions...}\]
\[ \mathcal{L}_R = \bar{\psi} i \gamma_\mu \partial^\mu \psi - \bar{\psi} (M_n - m^* g_\sigma \sigma) \psi - m^* \left[ g_v \bar{\psi} \gamma_\mu \psi V^\mu - \frac{1}{4} G^\mu\nu G^\mu\nu + \frac{1}{2} m_v^2 V^\mu V^\mu \right] \\
+ \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m^2 \sigma^2) \]  

(1)

As in ref. [1] we can extend the model to include pion and the total Lagrangian becomes

\[ \mathcal{L} = \mathcal{L}_R + \mathcal{L}_\pi \]  

(2)

where

\[ \mathcal{L}_\pi = \frac{1}{2} (\partial_\mu \pi \partial^\mu \pi - m^2 \pi^2) + m^* \left[ \frac{f_{\pi NN}}{m_\pi} \right] \bar{\psi} \gamma_\mu \gamma_5 \tau. \psi \partial^\mu \pi \]  

(3)

where \( \psi, \sigma, \pi \) and \( V \) are respectively the nucleon, the scalar meson, the pion and the vector meson field; \( M_N, m_\sigma, m_\pi \) and \( m_v \) are the corresponding masses; \( g_\sigma \) and \( g_v \) are the couplings of nucleon to scalar and vector mesons respectively; \( G^{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \); \( f_{\pi NN} \) the pion decay constant and \( m^* \) is given as \( m^* = (1 + g_\sigma / M_N)^{-1} \).

The parameters \( \alpha \) and \( \beta \) pick up the following values for different models: Walecka: \( \alpha = 0, \beta = 0 \); ZM: \( \alpha = 0, \beta = 1 \); ZM2: \( \alpha = 1, \beta = 1 \); ZM3: \( \alpha = 2, \beta = 1 \).

In the Mean Field Approximation (MFA), where the meson fields are replaced by their ground state expectation values, the Lagrangian density is given as

\[ \mathcal{L}_R^0 = \bar{\psi} \left[ i \gamma_\cdot \partial - (M_N - m^* g_\sigma \sigma_0) - m^* g_\gamma_0 V^0 \right] \psi + m^* \frac{1}{2} m_v^2 V_0^2 - \frac{1}{2} m^2 \sigma_0^2 \]  

(4)

The energy density, calculated from \( \mathcal{L}_R^0 \), in the usual fashion, is given by

\[ \mathcal{E} = \frac{g_v^2}{2 m_v^2} m^* \rho_B^2 + \frac{m_\sigma^2}{2 g_\sigma^2} \left[ 1 - m^* \right]^2 + \frac{2 \gamma}{\pi^2} \int_0^{k_F} p^2 (p^2 + M_N^2)^{1/2} f(E_p^*) dp \]  

(5)

We fit the energy density to the nuclear ground state energy density at zero temperature and nuclear saturation density to obtain the different coupling constants. The values of different parameters may be obtained in ref. [1].

The nucleon effective mass at finite temperature, in the MFA, is given by

\[ M_N^* = M_N - g_\sigma m^* \sigma_0 = M_N - \frac{4 M_N G^2 m^*}{\pi^2 m_\sigma^2} \int \frac{p^2 dp}{E_p^*} f(E_p^*) \]  

(6)
where $E_p^* = \sqrt{p^2 + M_N^2}$ is the effective energy of the nucleon. A self consistent solution of the above equation gives the temperature dependence of the effective nucleon mass. Such a calculation has also been reported in ref. [4] very recently. Now, using the temperature dependence of effective baryon mass we shall calculate the meson masses, at finite temperature, in the RPA.

The calculation of meson masses is done using the following prescription: the Dyson’s equation relates the total Green’s function $D(p)$ to the free Green’s function $D_0(p)$ as

$$D^{-1}(p) = D_0^{-1}(p) + \Pi(p)$$

(7)

where $\Pi(p)$ is the polarisation function. The effect of interaction is embedded in the polarisation function. The pole of the full propagator then gives the effective mass. As in ref. [1], we restrict ourselves to only the pole mass and ignore the screening mass or the Landau mass. We have omitted the detailed calculation of the polarisation functions. The standard method for doing them can be obtained in ref. [1]. In figures [1-3] we have plotted the temperature dependence of the effective meson masses.

The motivation behind this work was to look for the temperature dependence of the hadron masses from a class of newly proposed models, the Zimanyi-Moskowski models. In the ultrarelativistic heavy ion collisions, the matter formed in the central rapidity region is expected to be at a very high temperature and either at zero or very low baryon density. So the temperature dependence of hadron masses is important as far as the background of the QGP signal is concerned. The ZM models differ from the usual Walecka model in the form of the coupling of the scalar mesons to the nucleons. The model has been extended to include the pseudoscalar pion which is the lightest of all mesons. Here we have incorporated, as in ref. [1], the pseudovector coupling of pions with nucleons. This model has been used to calculate the temperature dependence of the hadron masses. In the symmetric matter approximation, the equation of motion for the nucleon has been determined. This equation of motion has been used to calculate the nucleon propagator. We have used the nucleon propagator
to calculate the nuclear energy density at the nuclear saturation density to calculate the coupling constants. The effective nucleon mass has been calculated in the MFA which is found to decrease with temperature for all the cases but, for the Walecka model the change is much more compared to the ZM models. We do not show this result here as they have also been reported by Delfino et.al. very recently. We, however, go much beyond this and use the temperature dependence of the nucleon mass to calculate the temperature dependence of the meson masses, in the RPA, self consistently. The $\sigma, \omega$ and $\pi$ masses are found to increase with temperature, both in the ZM and Walecka models. Special attention may be paid to the pion mass. The main problem of treating pions in the Walecka model is that its mass increases with temperature abnormally as seen from fig. 3. One, however, expects that the pion mass should not change much, as the pion, being a Goldstone boson, is protected by chiral symmetry. As seen from fig.3 here, the pion mass in the ZM models does not change much with temperature and one can expect that the problem of abnormal change in pion mass, which is observed in the Walecka model, can be overcome in the ZM models. It should of course be mentioned in this context that neither the Walecka model nor the ZM models incorporate chiral symmetry and thus the preceding argument is somewhat extraneous. The $\sigma$-meson mass should decrease in a chiral model and becomes degenerate with the pion mass as a signature of the restoration of chiral symmetry at high temperature. But, as far as only the behaviour of the pion is concerned, a model of nuclear matter with ”correct” behaviour of the pion mass is quite appealing and in this respect the ZM models appear to perform better. If one carries the calculation beyond the temperature range of 200$MeV$ the same problem might appear in the ZM models also. But, at that high temperature the deconfinement phase transition is expected to occur and hadrons are no longer the elementary particles. Hence the results at such a high temperature range are not reliable at all.

So, to conclude, the temperature dependence of hadron masses is still a very controversial subject. In this specific model, the Zimányi-Moskowskï model, the
temperature dependence of the meson masses, especially the pion, comes out to be much more realistic compared to other models. The present model can be extended to include $\rho$ and $a_1$ mesons and the meson-meson couplings. Such calculations are in progress and will be communicated shortly.

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Figure 1: Temperature dependence of $\sigma$-meson mass; $W$ refers to the Walecka model and $ZM$, $ZM2$ and $ZM3$ refer to the different versions of the Ziman-Moskowski model.

Figure 2: Temperature dependence of $\omega$-meson mass; the nomenclature is same as fig.1.
Figure 3: Temperature dependence of $\pi$-meson mass; the nomenclature is same as fig.1.