Dark matter nature at electron-positron colliders: scalar or fermionic?

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Abstract. In this work, we investigated the possibility of identifying DM nature, at the future electron-positron colliders such as the International Linear Collider (ILC) and Compact Linear Collider (CLIC), at $\sqrt{s} = 500$ GeV and 1 TeV, using the final state $b\bar{b} + E_T$. For this purpose, we consider two models in which DM could be either a real scalar or a heavy right-handed neutrino (RHN). So we considered two parameter values sets for both models, and we defined and investigated the different recent experimental constraints. After that we define a set of kinematical cuts that suppress the background, and generate different distributions that are useful in identifying the DM nature. The use of polarized beams $P(e^-, e^+) = [+0.8, -0.3]$ at the ILC makes the signal detection easier and the DM identification more clear, where the statistical significance gets enhanced by twice (five times) for scalar (fermionic) DM.

1. Introduction
The standard model (SM) does not explain many questions such as baryogenesis, dark matter (DM), dark energy, despite the last discovery at the Large Hadron Collider (LHC) of a Higgs boson [1, 2]. In reality, the strong experimental evidence that the SM is inadequate as an ultimate theory was the neutrino oscillation observation [3, 4].

The seesaw mechanism is one of the popular mechanisms to explain the smallness of neutrino masses [5, 6]. Another way is based on getting naturally small neutrino masses radiatively, where the suppression natural instead of a suppression by a large scale of new physics (NP) [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Some of these models address also the DM problem, where DM candidate is a heavy right-handed neutrino (RHN) [9, 12, 13, 14, 18, 19,
20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. These models predict interesting signature at colliders experiments [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44].

The International Linear Collider (ILC) and the Compact Linear Collider (CLIC) were proposed to discover new physics in the tera-scale, where the first is designed to scan the \( \sqrt{s} \) from 250 to 500 GeV, which can be raised up to 1 TeV [45, 46, 47] as for the second which is under development with \( \sqrt{s} \) from 380 GeV to 3 TeV, which high-luminosity [48]. Also has the advantage of polarized \( e^-e^+ \) beams, which may lead to an increasing the signal and suppress the background, directly enhances the NP signal strength. In Ref. [49], it has been found that the b-tagging efficiency is about 80% when the mis-identification efficiencies for c-jet and u/d/s-jet are below 10% and 1% respectively. This encourage any study that contain b-jets. For instance, in Ref. [50], it has been shown that by considering the final state \( b\bar{b} + E_T \) at the ILC, the \( hWW \) coupling can be measured with polarized beams \( P(e^-, e^+) = [+0.8, -0.3] \) at a precision of 4.8% and 1.2% at 250 GeV and 500 GeV, respectively.

We can also address the DM problem by expanding the SM with singlet scalar, that plays a DM candidate role. To ensure the DM stability, this scalar field has to obey a global Z\(_2\) symmetry [51, 52, 53, 54, 55, 56, 57, 58]. In this work, we consider the signal \( e^-e^+ \rightarrow b\bar{b} + E_T \), where b-tagged jets come from Z/\( \gamma^* \)/Higgs depending on the model considered: SM, scalar DM or RHN DM, where the missing energy is pair of DM and extract relevant cuts that reduces the background and use the polarization to identify the DM nature based on the distributions shape compared to background.

This paper is organized as follows. In section-2, we describe the models and different current experimental constraints such as invisible Higgs decay, muon anomalous magnetic moment, LFV, DM relic density and the LEP-II data. We propose two values for the both model parameters that respected the different bounds. In section-3, we describe the investigated process, and analysis the results in section-4, where we consider the cases with and without polarized beams. Finally we give our conclusion in section-5.

2. DM Models and the current experimental Constraints

In this work, we will consider two types of models where the DM could be either a real scalar or a fermionic. So we consider for the first case a generic case of the Higgs portal [59], and in the second case, we extended the SM with three heavy RHN’s, \( N_i(i = 1, 2, 3) \), and an electrically charged singlet field under \( SU(2)_L \) scalar. Where a Z\(_2\) symmetry was imposed to ensure the DM candidate stability, under which \( \{S, N_i\} \rightarrow \{-S, -N_i\} \) and all other fields are even [39, 40, 41, 42, 43].

2.1. Scalar Dark Matter

In this model we extended the SM by adding a real singlet scalar defined under \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) as \( \phi \sim (1, 1, 0) \). This scalar field has to obey a global Z\(_2\) symmetry and should not develop a vev, consequently, it could be a weakly-interacting massive particle (WIMP) and can self-annihilate into SM particles final states via the Higgs mediator. According to free parameters model, the scalar field mass \( m_\phi \) and its coupling to Higgs \( c_s \), we can get the relic density and eschew the direct detection cross section bound. The Lagrangian reads

\[
\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi, H),
\]

where \( H \) is the SM Higgs doublet and \( V(\phi, H) \) is the scalar potential, which after the electroweak symmetry breaking reads

\[
V(\phi, h) \geq \frac{1}{2} m_\phi^2 \phi^2 + \frac{c_s v}{2} h \phi^2,
\]
Table 1. The parameters values for Model 1 and Model 2.

| Model | free parameters                  |
|-------|----------------------------------|
| $M_1$ | $\{m_\phi, c_s\} = \{10 \text{ GeV}, 1.25 \times 10^{-2}\}$ |
| $M_2$ | $\{m_\phi, c_s\} = \{60 \text{ GeV}, 2.35 \times 10^{-2}\}$ |

where $\nu = 246$ GeV is the SM doublet vev, and $h$ is the usual SM Higgs field. At an $e^-e^+$ collider, it is possible to produce the real singlet scalar $\phi$ via the Z-fusion, or by the associate production, where the $Z$ gauge boson subsequently decays, primarily hadronically [60]. The invisible Higgs decay width reads

$$
\Gamma_{\text{inv}}(h \to \phi\phi) = \frac{c_s^2 \nu^2}{32\pi m_h} \left( 1 - \frac{4m_\phi^2}{m_h^2} \right)^{\frac{1}{2}}.
$$

(3)

The experimental constraint on the invisible branching ratio reads

$$
B_{\text{inv}}(h \to \phi\phi) = \frac{\Gamma_{\text{inv}}}{\Gamma_{\text{inv}} + \Gamma_{\text{tot}}^{\text{SM}}} \leq 0.16,
$$

(4)

where $\Gamma_{\text{tot}}^{\text{SM}} = 4.20$ MeV is the SM Higgs total width [61]. We can transformed this experimental constraint into a new constraint on the model free parameters $\{m_\phi, c_s\}$ as

$$
c_s \leq 1.2882 \times 10^{-2} \left( 1 - \left( \frac{m_\phi}{62.5 \text{ GeV}} \right)^2 \right)^{-\frac{1}{4}}.
$$

(5)

In our study, we focus on the case where light masses range $m_\phi \leq m_h/2$, and we choose two values of the model free parameters, where they respect the experimental constraint (4). We denote them Model 1 ($M_1$) and Model 2 ($M_2$):

2.2. Fermionic Dark Matter

In this case, the SM was extended with an electrically charged singlet scalar field $S^+ \sim (1, 1, 2)$ and three RHN’s, $N_i \sim (1, 1, 0)$, if the global symmetry is imposed the lightest RHNs gets stable and play role a good DM candidate [12, 62]. The Lagrangian reads [63]

$$
\mathcal{L} = \mathcal{L}_{\text{SM}} + \{g_{\alpha R}N_i^C \ell_{\alpha R}S^+ + \frac{1}{2}m_{N_i}N_i^C N_i + h.c\} - V,
$$

(6)

where $\ell_{\alpha R}$ is the RH charged lepton, $m_{N_i}$ are the heavy RHN’s masses, $C$ is the charge conjugation operator and $g_{\alpha}$ are the new Yukawa couplings. Here $V$ is the scalar potential. The Greek letters denote $\alpha = \mu, c, \tau$ and the fermion generations are labeled by $i = 1, 2, 3$.

The interactions (6) induce a new contribution to the muon’s anomalous magnetic moment [64] and LFV processes such as $\ell_\mu \to \ell_\mu + \gamma$, and $\ell_\mu \to \ell_\tau + \ell_\tau + \ell_\mu$, all are generated at one-loop via the exchange of the charged scalar $S^+$, in Ref. [65, 66, 29, 30], we find the branching ratios. In Ref. [67, 68, 69, 70, 71, 72], the models has the positive contribution to the muon anomalous magnetic moments, but on the contrary in Ref. [29, 30], which leads to the widening of the gap between the experimental measurement and the SM prediction [60].
Table 2. The current experimental limits for different LFV processes.

| LFV process       | Current constraint |
|-------------------|--------------------|
| $B(\mu \to e + \gamma)$ | $4.2 \times 10^{-13}$ [73] |
| $B(\tau \to \mu + \gamma)$ | $4.4 \times 10^{-8}$ [60] |
| $B(\tau \to e + \gamma)$ | $3.3 \times 10^{-8}$ [74] |
| $B(\tau \to e^- + e^+ + e^-)$ | $2.7 \times 10^{-8}$ [75] |
| $B(\mu \to e^- + e^+ + e^-)$ | $1.0 \times 10^{-12}$ [76] |
| $B(\tau \to \mu^- + \mu^+ + \mu^-)$ | $2.1 \times 10^{-8}$ [75] |

In Table 2, we present the current experimental limits for different LFV processes. The interactions (6) must be respected the current experimental bounds in Table 2 and the DM relic density $\Omega_{DM}h^2 = 0.1186 \pm 0.0020$ [77], when $N_1$ is considered as DM candidate, where the principal annihilation channel would be the $S^\pm$-mediated process $N_1N_1 \to \ell_\alpha\ell_\beta$.

In this work, we consider the process $e^-e^+ \to b\bar{b} + \bar{T}_T$ without polarized beams at $\sqrt{s} = 500$ GeV and 1 TeV, after that, we reanalyze the same process by using the different polarization combination at the $e^-e^+$ linear colliders like the ILC and CLIC. In the Background (SM), the same process have three sub-processes, where $\bar{T}_T^{(SM)} = \nu_\alpha\nu_\alpha$ and $\alpha = \mu,e,\tau$. In the fermionic case, when heavier RHN’s pair, $N_{2,3}$, could be created inside the collider but its can decay inside them and outside, where they decay into pairs of charged leptons $\ell_\alpha R \ell_\beta R$ ($\alpha,\beta = \mu,e,\tau$) and $N_{1,2}N_{1,2}$ via $S^\pm$-mediated processes.

The heavier RHN’s $N_{2,3}$ has a three-body decay, consequently may decay outside of the detector with a bigger distance when $m_{N_{2,3}} < m_S$. In inverse case $m_{N_1} < m_S < m_{N_{2,3}}$, $N_{2,3}$ has a two-body decay with a larger decay width and a smaller distance, that should be inside the detector. Then, we can defined the missing energy in the process mentioned above for the three cases as:

1. If $N_2$ and $N_3$ decays inside the detector, $\bar{T}_T = N_1N_1$,
2. If only $N_3$ decays inside the detector, $\bar{T}_T = N_1N_1, N_1N_2, N_2N_2$,
3. If the heavier RHN’s $N_{2,3}$ decay outside the detector, $\bar{T}_T = N_1N_1, N_1N_2, N_1N_3, N_2N_2, N_2N_3, N_3N_3$.

In order to check whether these three cases correspond to: $m_{N_{1,2,3}} < m_S$, $m_{N_{1,2}} < m_S < m_{N_3}$, and $m_{N_1} < m_S < m_{N_{2,3}}$, respectively, one should estimate the distance traveled by the heavier RHN’s $N_{2,3}$.

The distance $D_i$ traveled by the heavier RHN’s $N_{i=2,3}$ can be defined by

$$\frac{D_i}{1\,cm} = 1.98 \times 10^{-4} \left(\frac{\Gamma_i}{10^{-7}\,MeV}\right)^{-1} \left(\frac{E_i^2}{m_{N_i}^2} - 1\right)^{1/2},$$

where $\Gamma_i$, $m_{N_i}$, $E_i$ are the heavy RHN’s decays width estimated using LanHEP/CalcHEP [78, 79], masses and energies respectively.

In Fig. 1 we show the traveled distance $D_i$ as a function of $m_{N_{2,3}}$ for three the aforementioned cases for 500 benchmark points that agree the current bounds.

We can see clearly from Fig. 1 that $N_3$ decays mostly inside the detector except for few benchmark points when it has a three-body decay. The RHN $N_2$ decay inside the detector when has a two-body decay with a larger decay. In the inverse case, it could decay either inside or outside of the detector depending on the couplings.

According on the negative search coming from LEP-II about single photon with missing energy signal [80], we will constrain our parameters space with keeping same cuts used by

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| $B(\tau \to e^- + e^+ + e^-)$ | $2.7 \times 10^{-8}$ [75] |
| $B(\mu \to e^- + e^+ + e^-)$ | $1.0 \times 10^{-12}$ [76] |
| $B(\tau \to \mu^- + \mu^+ + \mu^-)$ | $2.1 \times 10^{-8}$ [75] |
In this paper, we want to probe possible phenomenological implications for interactions (1) and (6) through the final state $b\bar{b} + E_T$ at lepton collider and to search for new Physics. Since the $m_h = 125.09$ GeV Higgs has the dominant decay mode $B(h \to b\bar{b}) = 57.7\%$ \cite{60}, while the $Z$ branching ratio $B(Z \to b\bar{b}) = 15.12\%$ \cite{60} is also significant. Then the choice of the

3. **The final state $b\bar{b} + E_T$ at $e^-e^+$ Colliders**

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**Table 3.** The parameter values for Model 3 and Model 4.

| Models | Parameters |
|--------|------------|
| $M_3$  | $m_{N_1}$ (GeV) | 25.788, 28.885, 36.274, |
|        | $m_S$ (GeV)    | 196.75, |
|        | $g_{\alpha/2}$ | $\begin{pmatrix} 75.063 - i0.14367 & 0.0026819 - i0.015758 & -136.030 - 70.675 \\ -3.6203 - i35.9460 & -0.0035368 + i0.041316 & 120.47 - i286.100 \\ -3.0602 - i0.49553 & 0.057628 - i0.2462700 & -235.27 + i33.529 \end{pmatrix}$, |
| $M_4$  | $m_{N_1}$ (GeV) | 62.184, 76.275, 95.736, |
|        | $m_S$ (GeV)    | 126.78, |
|        | $g_{\alpha/2}$ | $\begin{pmatrix} -60.008 + i2.4015 & -0.55187 - i1.1133 & -32.641 + i41.313 \\ 5.0213 + i22.533 & 3.5209 - i2.2480 & -112.35 - i32.473 \\ 4.2829 + i3.7764 & -2.2562 + i2.3886 & -171.25 - i94.890 \end{pmatrix}$, |

**Figure 1.** The distance $D_i$ traveled by the heavier RHN’s versus their masses for the three cases: $m_{N_{1,2,3}} < m_S$ (left), $m_{N_{1,2}} < m_S < m_{N_3}$ (middle) and $m_{N_1} < m_S < m_{N_{2,3}}$ (right). Here, we consider the typical energy for $N_{2,3}$ to be 200 GeV.

LEP-II. In Fig. 2, we display the significance of the signal $e^-e^+ \to \gamma + E_T$ (in the palette) for different values of the coupling $|g_{1e}|$ and the charged scalar mass for 3000 benchmark points that are in agreement with the bounds from the muon anomalous magnetic moment and the LFV for two cases with $m_{N_1} = \{25, 30, 35$ GeV$\}$ and $m_{N_1} = \{50, 60, 70$ GeV$\}$.

From Fig. 2, for the case when $N_1$ heavier than 50 GeV , we can see that the LFV bounds are respected, the bound from LEP-II is also satisfied, in the contrary case the LEP-II could exclude some benchmark points, especially using the analysis with $\sqrt{s} = 207.2$ GeV. For our analysis, we consider the numerical values shown in Table. 3, which we call Model 3 ($M_3$) and Model 4 ($M_4$).

According Table. 3, for both $M_3$ and $M_4$ the heavy RHN’s $N_{2,3}$ decay via a three-body process $N_{2,3} \to N_1 + \ell_\alpha + \ell_\beta$ where the traveled distance $D_i$ is very small. So the both decay inside the detector, this means that the missing energy defined as, $E_T = N_1 N_1$. 
The coupling $|g_{1e}|$ as a function of $m_S$ for 3000 benchmark points with the two cases $m_N_i = \{25, 30, 35 \text{ GeV}\}$ (Left) and $m_N_i = \{50, 60, 70 \text{ GeV}\}$ (Right). The palette represents the signal significance $S$ of the process $e^- e^+ \rightarrow \gamma + E_T$ for the CM energy values $\sqrt{s} = 188.6$ (up) and $\sqrt{s} = 207.2$ (down) with the integrated luminosity $L = 176 \text{ pb}^{-1}$ and $L = 130.2 \text{ pb}^{-1}$, respectively. The solid (dashed) line corresponds the new constraint at LEP II which makes the signal significance smaller than $S = 3$ ($S < 2$). For most of the benchmark points used here, the missing energy is identified as $E_T = N_1 N_1$, which justifies the choice of $|g_{1e}|$ in the y-axes.

channel $b\bar{b} + E_T$ is interesting since the b-tagging efficiency is shown to be about 80% when the misidentification efficiencies for c-jet and u/d/s-jet are below 10% and 1%, respectively, at both the ILC and CLIC [49]. This is encouraging to consider the $b\bar{b}$ final state for our studied models $M_i$ due to possible clear signal. Where the signal (Fig. 3-d and Fig. 3-e) and the background contributions $e^- e^+ \rightarrow Z(Z, h, \gamma^*) \rightarrow b\bar{b} + E_T$ (Fig. 3-a and -b) in addition to the W-fusion diagrams (Fig. 3-c).

Future experiments such as the ILC [45, 46] and CILC [81] has the advantage of polarized beams of $e^- e^+$ that help as to identify the DM nature fermionic, vector or scalar?. Here, we will consider both cases with and without polarized beams at $\sqrt{s} = 500 \text{ GeV}$ and 1 TeV. The general signal significance definition is given by [82]

$$S = \sqrt{2} \times [(N_S + N_{BG}) \times \log(1 + N_S/N_{BG}) - N_S],$$

where $N_S$ ($N_B$) are the signal (background) events number. Here $N_S$ is given by

$$N_{S,BG} = \epsilon_b^2 \times \mathcal{L}_{\text{int}} \times \sigma_{S,BG},$$
with $\epsilon_b = 0.8$ is the $b$-tagging efficiency factor, $\mathcal{L}_{\text{int}}$ is the integrated luminosity, and $\sigma_{S,BG}$ is the signal or background cross section value.

4. Results & Analysis & Discussion

With help the LanHEP packages [78] we implement the models and generate their Feynman rules, also we use CalcHep [79] to estimate the cross section and produce all differential cross section at $\sqrt{s} = 500 \text{ GeV}$ and 1 TeV. For extract the cuts on the kinematic variables that raise the significance, we produce different distributions and look for ranges where the background is supressed while keeping the signal value, we produce different distributions and look for ranges where the background is reduced while keeping the signal value. So we generate different distributions using the following pre-cuts:

- The transverse momentum of the bottom quark and the bottom antiquark must satisfy: $p_T > 15 \text{ GeV}$,
- The missing energy $E_T > 30 \text{ GeV}$,
- The invariant mass of the bottom quark and the bottom antiquark must be in the range: $71 \text{ GeV} < M_{b\bar{b}} < 145 \text{ GeV}$,
- The jet separation radius must satisfy: $\Delta R_{b\bar{b}} = \sqrt{\Delta \phi^2 + \Delta \eta^2} > 0.4$.

Firstly, we consider unpolarized beams of $e^-$ and $e^+$ to generate the differential cross section for the background and the signal at $\sqrt{s} = 500 \text{ GeV}$ and 1 TeV. Then we look for kinematical variables regions where the background is reduced and the signal is maintained as possible. Then, the full set of cuts is given in Table-4.

4.1. Analysis without polarized Beams

By imposing the full set of cuts in Table-4 at $\sqrt{s} = 500 \text{ GeV}$ and 1 TeV, using unpolarized beams, we get the results shown in Table-5.

Through the results presented in Table-5, the signal cross section within the full set of cuts gets reduced a bit with respect to the case within the pre-cuts for all models at $\sqrt{s} = 500 \text{ GeV}$ and 1 TeV. Whereas, the background cross section gets reduced by about 83.5% (79%) at
Table 4. The full set of cuts for the process $e^-e^+ \rightarrow \bar{b}b + \slashed{E}_T$ at both CM energies at $\sqrt{s} = 500$ GeV and 1 TeV. Here $p_T^b$ is the transverse momentum of the bottom quark ($b$), $\slashed{E}_T$ is the missing energy, $M^{b\bar{b}}$ is the invariant mass of the bottom quark ($b$) and the bottom antiquark ($\bar{b}$), $\triangle R_{b,\bar{b}}$ is the jet cone angle, $E_T^{b,\bar{b}}$ is the transverse energy for the bottom quark ($b$) and bottom antiquark ($\bar{b}$), $M_T^{E_T}$ is the transverse mass of bottom-missing energy. All masses and energies are given in GeV.

| $E_{CM}$ (GeV) | Selection cuts |
|----------------|----------------|
| 500            | 15 < $p_T^b$, 30 < $\slashed{E}_T$, 71 < $M^{b\bar{b}}$ < 145, 0.4 < $\triangle R_{b,\bar{b}}$, 90 < $E_T^{b,\bar{b}}$ < 230, 210 < $M_T^{E_T}$, |
| 1000           | 15 < $p_T^b$, 30 < $\slashed{E}_T$, 71 < $M^{b\bar{b}}$ < 145, 0.4 < $\triangle R_{b,\bar{b}}$, 125 < $E_T^{b,\bar{b}}$, 240 < $M_T^{E_T}$. |

Table 5. The cross section values of the background and the signal for each models within the pre-cuts are $\sigma^{BG}$, $\sigma^S$ and after applying the full cuts set given in Table 4 $\sigma^{BG}$, $\sigma^S$ at $\sqrt{s} = 500$ GeV and 1 TeV, and the corresponding signal significance for the luminosity values $L = 100$ fb$^{-1}$, 500 fb$^{-1}$.

| $E_{CM}$ (GeV) | $\sigma^{BG}$ (fb) | Models | $\sigma^{S}$ (fb) | $\sigma^{BG}$ (fb) | $\sigma^{S}$ (fb) | $S_{100}$ | $S_{500}$ |
|----------------|---------------------|--------|-----------------|-------------------|-----------------|-----------|-----------|
| 500            | 108.19              | $M_1$  | 1.475           | 0.520             | 0.9808          | 2.1936    |           |
|                |                     | $M_2$  | 1.479           | 17.804            | 0.638           | 1.2024    | 2.6888    |
|                |                     | $M_3$  | 1.425           | 0.956             | 1.7960          | 4.0168    |           |
|                |                     | $M_4$  | 1.338           | 1.070             | 2.0088          | 4.4912    |           |
| 1000           | 233.27              | $M_1$  | 0.352           | 0.282             | 0.3216          | 0.7192    |           |
|                |                     | $M_2$  | 0.353           | 0.292             | 0.3328          | 0.7448    |           |
|                |                     | $M_3$  | 1.265           | 49.072            | 0.942           | 1.0720    | 2.3976    |
|                |                     | $M_4$  | 0.954           | 0.760             | 0.8656          | 1.9352    |           |

$\sqrt{s} = 500$ GeV (at 1 TeV). For luminosity $L = 500$ fb$^{-1}$ only we remark a deviation from the SM at $\sqrt{s} = 500$ GeV for $M_{3,4}$. In case of large luminosity values that allow the signal to be seen, we show relevant normalized distributions in Fig. 4 and Fig. 5, for at $\sqrt{s} = 500$ GeV and 1 TeV, respectively. The relevant distributions here are the polar angle between bottom-antibottom jets $\cos(\theta^{b,\bar{b}})$, the jet energy $E^b$, the jet transverse energy $E_T^b$, the jet transverse momentum $p_T^b$, the transverse mass of the bottom-antibottom jets $M_T^{b,\bar{b}}$, the invariant mass of the missing energy with a jet $M_T^{E_T}$, the jet pseudo rapidity $\eta^b$, the two-jets pseudo rapidity $\eta^{b,\bar{b}}$, and the polar angle between both the jets in the boost direction $\cos(\Theta^{b,\bar{b}})$. At $E_{CM} = 500$ GeV (Fig. 4), for scalar DM ($M_{1,2}$), the normalized distributions have different shapes with respect to both the background and the fermionic DM case. For fermionic DM case, the normalized distributions have the same shape with respect to the background with a remarkable shift. However, at $E_{CM} = 1$ TeV (Fig. 5), the two cases of scalar and fermionic DM could be easily distinguished due to the different normalized distributions shapes.
In search of new physics, the use polarized beams at future $e^- e^+$ colliders such as ILC and CLIC, could reduce the background and/or enhance the signal [45, 46, 47]. At the ILC, the polarization degree of the $e^- (e^+)$ beams could reach 80% (30%) [46]. The $e^+$ polarization could be improved up to 60% at CLIC [81].

Here, we reanalyze the same process at the same CM energy values within the polarization $e^- (e^+)$ at $\sqrt{s} = 500 \text{ GeV}$ and 1 TeV. One can also see the cross section value for $\sqrt{s} = 500 \text{ GeV}$, 1 TeV, respectively. When considering the polarization beams, the background cross section gets decreased sharply due to the vertices suppression of the $e^- e^+$ with gauge bosons unlike the vertices of charged scalar-Majorana fermion-charged lepton (for $M_{3,4}$) which enhances the cross section.

For luminosity $L = 100 \text{ fb}^{-1}$, one remarks a discovery for $M_{3,4}$ at $\sqrt{s} = 500 \text{ GeV}$ and 1 TeV, however, for $L = 500 \text{ fb}^{-1}$ one can see also a discovery for all models at at $\sqrt{s} = 500 \text{ GeV}$
Figure 5. The relevant normalized distributions of the process $e^- e^+ \rightarrow b \bar{b} + \not{E_T}$ at $E_{CM} = 1$ TeV.

Table 6. The cross section values for the background $\sigma^{BG}$ and the signal $\sigma^S$ estimated for the considered energies within the full set of cuts given in Table-4, without and with polarized beams at both CM energies $E_{CM} = 500$ GeV and 1 TeV. The significance $S_{100}$ and $S_{500}$ correspond to the two integrated luminosity values $L = 100$ fb$^{-1}$ and 500 fb$^{-1}$, respectively.

| $E_{CM}$ (GeV) | $\sigma^{BG}$ (fb) | $P (e^-, e^+)$ | $\sigma^S$ (fb) | $S_{100}$ | $S_{500}$ | $P (e^-, e^+)$ | $\sigma^{BG}$ (fb) | $\sigma^S$ (fb) | $S_{100}$ | $S_{500}$ |
|---------------|-----------------|--------------|----------------|----------|-----------|--------------|-----------------|---------------|----------|-----------|
| 500           | 17.804          | $M_1$        | 0.520          | 0.9808   | 2.1936    | 0.558        | 1.9488         | 4.3584        |
|               |                 | $M_2$        | 0.638          | 1.2024   | 2.6888    | 0.685        | 2.3832         | 5.3304        |
|               |                 | $M_3$        | 0.956          | 1.7960   | 4.0168    | 2.166        | 7.2328         | 16.1736       |
|               |                 | $M_4$        | 1.070          | 2.0088   | 4.9121    | 2.570        | 8.4944         | 18.9944       |
|               |                 | $M_1$        | 0.282          | 0.3216   | 0.7192    | 0.303        | 0.7640         | 1.7096        |
|               |                 | $M_2$        | 0.292          | 0.3328   | 0.7448    | 0.313        | 0.7896         | 1.7656        |
| 1000          | 49.072          | $M_3$        | 0.942          | 1.0720   | 2.3976    | 9.950        | 5.472          | 12.8312       | 28.6912     |
|               |                 | $M_4$        | 0.760          | 0.8656   | 1.9352    | 4.219        | 10.0520        | 22.4784       |

except Model $M_1$. At 1 TeV within the same luminosity, we could not even see a deviation from the SM for $M_{1,2}$ unlike $M_{3,4}$ where one can see clearly a discovery. In Fig. 6 and Fig. 7,
we show the relevant normalized distributions at $\sqrt{s} = 500$ GeV and 1 TeV, respectively, with the polarized beams.

![Normalized Distributions](image)

**Figure 6.** The relevant normalized distributions of the process $e^-e^+ \rightarrow b\bar{b} + \not{E}_T$ at $E_{CM} = 500$ GeV with polarized beams $P(e^-, e^+) = [+0.8, -0.3]$.

From the Fig. 6, for scalar DM, the normalized distributions have different shapes with respect to the background. However, also for fermionic DM, the distributions shape is different. From the Fig. 7, one notices that the normalized distributions: $E^b$, $E_T^b$, $p_T^b$, $M^b + \not{E}_T$ and $\cos(\Theta^{b,\bar{b}})$, have different shapes between the background, the scalar DM and the fermionic DM case. One remarks also, that for the background and the fermionic DM case, the normalized distributions have the same shape especially for the polar angle between bottom-antibottom jets $\cos(\Theta^{b,\bar{b}})$, the jet pseudorapidity $\eta^b$, the two-jet pseudo rapidity $\eta^{b,\bar{b}}$.

By comparing the results produced at $\sqrt{s} = 500$ GeV using polarized beams (Fig. 6) with those without polarization (Fig. 4), one remarks a clear difference. For instance, if the DM is a scalar, the maximum of the normalized distributions of $\eta^b$ get shifted into $-1 < \eta^b < 0.2$, with respect to the case without polarization. At 1 TeV, for the fermionic DM case, the maximum of the normalized distributions of $\cos(\Theta^{b,\bar{b}})$ get shifted also into $|\cos(\Theta^{b,\bar{b}})| < 0.4$, respectively. In Table-7, we summarize the events number for the background and the signal using polarized and unpolarized beams at $\sqrt{s} = 500$ GeV and 1 TeV.

The results presented in Table-7 gives evidences that with the polarization $P(e^-, e^+) = [+0.8, -0.3]$, suppresses the background $N_{BG}$ events number by 72% and by 80% for at $\sqrt{s} = 500$ GeV and 1 TeV, respectively. Simultaneously, the signal $N_S$ number of events for $M_3$ ($M_4$) gets improved by 127% (140%) and by 481% (455%) for $\sqrt{s} = 500$ GeV and
Figure 7. The relevant normalized distributions of the process $e^-e^+ \rightarrow b\bar{b}+\mathbb{E}_T$ at $E_{CM} = 1$ TeV with polarized beams $P(e^-, e^+) = [+0.8, -0.3]$.

Table 7. The background and signal events number $N_{BG}$, $N_S$ estimated for the considered energies within the full set of cuts given in Table 4, without and with polarized beams at both CM energies $E_{CM} = 500$ GeV and 1 TeV. The significance $S_{100}$ and $S_{500}$ correspond to the two integrated luminosity values $L = 100$ fb$^{-1}$ and 500 fb$^{-1}$, respectively.

| $E_{CM}$ (GeV) | $N_{BG}$ | $P(e^-, e^+)$ | $[0, 0]$ | $N_S$ | $S_{100}$ | $P(e^-, e^+)$ | $[+0.8, -0.3]$ | $N_S$ | $S_{500}$ |
|----------------|---------|---------------|--------|-------|---------|---------------|----------------|-------|---------|
| 500            | 1139.456| $M_1$         | 33.2864| 0.9808| 2.1936  | 35.7120       | 1.9488         | 4.3584|
|                |         | $M_2$         | 40.8320| 1.3284| 2.6888  | 43.8400       | 2.3832         | 5.3304|
|                |         | $M_3$         | 61.1840| 1.7960| 4.0168  | 138.6368      | 7.2328         | 16.1736|
|                |         | $M_4$         | 68.4800| 2.0088| 4.4912  | 164.4846      | 8.4944         | 18.9944|
| 1000           | 3140.608| $M_1$         | 18.0608| 0.3216| 0.7192  | 19.4048       | 0.7648         | 1.7104|
|                |         | $M_2$         | 18.6944| 0.3328| 0.7448  | 20.0320       | 0.7896         | 1.7656|
|                |         | $M_3$         | 60.2880| 1.0720| 2.3976  | 636.8064      | 350.2080       | 12.8312|
|                |         | $M_4$         | 48.6528| 0.8656| 1.9360  | 270.0224      | 10.0528        | 22.4784|

1 TeV, respectively. This (significant) excess of events numbers could be an indication of the DM nature, i.e., if the DM is a fermionic nature, the excess could be about 5 times.
5. Conclusions
In this paper, we have probe the possibility of detecting the signal significance of DM and identifying its nature scalar or fermionic? produced at future $e^−e^+\rightarrow b\bar{b} + \not{E}_T$ at $\sqrt{s} = 500$ GeV and 1 TeV. Where, we considered two types of models with two parameters values sets, and we determine and investigate different experimental constraints for each model, like the Higgs invisible decay, the muon anomalous magnetic moment, LFV, DM relic density and LEP-II data.

When we used the appropriate cuts (in Table-4), the background gets significantly suppressed and the signal significance gets elevate especially for fermionic DM case. In the unpolarized beams case at $\sqrt{s} = 500$ GeV , we can identified the nature of DM baesd the normalized distributions: $E_T^b, M^{b,\not{E}_T}, \eta^{b,\bar{b}}$ and $\cos(\Theta^{b,\bar{b}})$. However, a remarkable shift can be observed in most of the distributions for fermionc DM case. At 1 TeV, also based on different distributions we can be distinguished the DM nature whether it is scalar or fermionic.

Using polarized beams, the shape difference with respect to the background for most of the distributions is more clear, the background cross section gets reduced by about 80% and/or the signal gets raised. This leads to a significant enhancement on the statistical significance by double (five) if the DM is a scalar (fermionic).

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