Fixture Locating Precision Pre-Analysis for One-plane-dual-pins fixture System

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Abstract. Type two assembly is mainly fixed by additional fixture but not its own features, and the one-plane-dual-pins fixture system is commonly used for type two assembly assurance; hence, the locating pre-analysis for one-plane-dual-pins fixture system is introduced. The locating variation in one-plane-dual-pins fixture system leads to the gesture displacement of the workpiece in space. Variation source that affects the fixture locating precision mainly includes the fixture locating error and the fabrication error of the fitting holes. While the fixture error and fabrication error of the fitting holes are confirmed, the calculation for the fixture locating error of assembled workpiece is introduced. The fixture locating error simulation is further researched, and one example for fixture locating error propagation is illustrated.

1. Introduction
There are generally two types of assembly during the assembly process: 1) the assembly between two parts or components by fitting operation, where the latter part or component is located according to the first part basis without specific fixture; 2) the assembly between two parts or components by riveting or welding, and each parts or components is located by specific fixture individually. The typical specific fixture layout for pre-locating the assembled workpiece is one-plane-dual-pins layout system. One-plane is usually the large surface for fixing the part, and one of the dual-pins is cylindrical pin, the other pin is the prism pin.

Pre-locating analysis of one-plane-dual-pins layout system is significant for ensuring the assembly accuracy of type two assembly. Wu et al. [1] introduced models for different types of fixture layout system based on the mechanism mapping, and it provides the foundation model for the analysis of locating accuracy. Luo et al. [2] studied the locating problem of any space surface, and analyzed the positioning problem by distance function. Liu et al [3] established a positioning mapping model between the element fixture and the workpiece to minimize the locating error. Tsai et al. [4] proposed a matrix analysis method for the positioning design of prismatic parts.

For actual practice application, the locating accuracy problem of one-plane-dual-pins layout system is discussed in this paper. Size error of the locating pins and holes are considered, and pre-locating analysis of one-plane-dual-pins layout system is introduced.

2. Error source of one-plane-dual-pins fixture system
For type two assembly, the one-plane-dual-pins fixture system is shown in Fig. 1. The locating basis is Oxyz, the cylindrical pin is P1, and the prism pin is P2. The workpiece can translate along the positioning groove that is $\alpha$ angle with x-axis.

The factors that affecting the locating error of one-plane-dual-pins fixture system include:
(1) The locating error of $P_1$ and $P_2$: $\Delta X_{P_1}, \Delta Z_{P_1}, \Delta X_{P_2}$ and $\Delta Z_{P_2}$.

(2) The locating error of the fitting holes of workpiece. The hole fitting with $P_1$ is $Y_1$, and the hole fitting with $P_2$ is $Y_2$; the locating error of the fitting holes of workpiece are: $\Delta X_{Y_1}, \Delta Z_{Y_1}, \Delta X_{Y_2}$ and $\Delta Z_{Y_2}$.

(3) While the assembled workpieces are more than three, the repeated locating error of the part is included.

![Figure 1. fixture locating of the part in x-z plane](image)

3 Fixture error and locating hole error

3.1 Assembly displacement caused by fixture error

Suppose that the fixture error in process $k$ caused by $P_1$ and $P_2$ can be recorded as:

$$\Delta P(k) = [\Delta X_{P_1}(k), \Delta Z_{P_1}(k), \Delta X_{P_2}(k), \Delta Z_{P_2}(k)]^T$$  \hspace{1cm} (1)

According to reference [5], the locating error of workpiece $A$ caused by the fixture error of $P_1$ and $P_2$ in process $k$ is:

$$X_A^{(k)} = M_{A P_1} Q_{P_1 P_2}(k) \Delta P(k)$$  \hspace{1cm} (2)

Where $Q_{P_1 P_2}(k)$ is

$$Q_{P_1 P_2}(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\alpha & -\cos\alpha & 0 & 0 \\ \frac{L}{L'} & \frac{L}{L'} & \frac{L}{L'} & \frac{L}{L'} \end{bmatrix}$$  \hspace{1cm} (3)

$$M_{A P_1} = \begin{bmatrix} 1 & 0 & -L_x(A, P_1) \\ 0 & 1 & -L_y(A, P_1) \\ 0 & 0 & 1 \end{bmatrix}$$  \hspace{1cm} (4)

Where $L'$ is the distance between $P_1$ and $P_2$; $L_x(A, P_1) = Z_n - Z_A$, $L_y(A, P_1) = X_n - X_A$, which are the distances along $x$ and $z$ directions between the centroid point of workpiece and $P_1$.

3.2 Assembly displacement caused by error of the fitting hole and groove

The locating accuracy also affected by the fabricating error of the fitting hole $Y_1$ and the fitting groove $Y_2$. As shown in Fig. 2, suppose that $Y_1, Y_2$ are the ideal positions of fitting hole and groove, and $Y_1', Y_2'$ are actual positions of fitting hole and groove, the rotation offset of the workpiece is $\epsilon$.

Suppose that the fabricating error of the fitting hole and groove in process $k$ caused by $Y_1$ and $Y_2$ can be recorded as:

$$\Delta Y = [\Delta X_{Y_1}(k), \Delta Z_{Y_1}(k), \Delta X_{Y_2}(k), \Delta Z_{Y_2}(k), \beta(k)]^T$$  \hspace{1cm} (5)

Where $\Delta X_{Y_1}(k), \Delta Z_{Y_1}(k)$ are translation displacement along the $x$-axis and $z$-axis direction caused by $Y_1$, and $\Delta X_{Y_2}(k), \Delta Z_{Y_2}(k)$ are translation displacement along the $x$-axis and $z$-axis direction caused by $Y_2$, and $\beta(k)$ is the rotation displacement of the fitting groove.
Figure 2. locating accuracy affected by the fabricating error of the fitting hole and groove

The locating error of workpiece A caused by the fabricating error of the fitting groove Y₂ in process k is shown in Fig. 3. The ideal position of the prism pin is P₂ and the actual position of the prism pin is P₂’. The rotation offset of the workpiece A is ε, and the lengths of Y₁’ P₂ and Y₁’ P₂’ are r.

Figure 3. locating error of workpiece A caused by the fabricating error of fitting groove Y₂

As shown in Fig. 3, the displacement of Y₂’ referring to Y₁’ in the x-axis direction is ΔX₁(k) = ΔX_Y(k) - ΔX_Y₁(k), in the z-axis direction is ΔZ₁(k) = ΔZ_Y(k) - ΔZ_Y₁(k). In addition, the following six equations exist:

\[ l = \sqrt{\Delta X_Y^2 + \Delta Z_Y^2} \]  \hspace{1cm} (6)

\[ \phi = \arctan \frac{\Delta X_Y}{\Delta Z_Y} \]  \hspace{1cm} (7)

\[ \beta + \gamma + \phi = \frac{\pi}{2} \]  \hspace{1cm} (8)

\[ \frac{m}{\sin \gamma} = \frac{l}{\sin \theta} \]  \hspace{1cm} (9)

\[ \theta = \beta + \frac{\pi - \epsilon}{2} \]  \hspace{1cm} (10)
\[ m \approx 2r \sin \frac{\varepsilon}{2} \]

Solving the equations, the rotation offset of the workpiece is \( \varepsilon \):

\[
\varepsilon = \beta + \arcsin \left( \frac{\pi}{2} - \beta - \arctan \frac{\Delta X_y}{\Delta Z_y} \right) \sqrt{(\Delta X_y)^2 + (\Delta Z_y)^2} - \sin \beta
\]

The locating error of workpiece A caused by the fitting hole \( Y_1 \) in process \( k \) is:

\[
X^{k\varepsilon}_{Y_1}(k) = [\Delta X_{Y_1}, \Delta Z_{Y_1}, \varepsilon]^T
\]

(1) If \( \Delta Z_y = 0.1 \text{cm}, r = 50 \text{cm}, \) the \( \Delta X_y - \varepsilon \) function curve is shown in Fig.4. As shown in Fig. 4, the \( \Delta X_y - \varepsilon \) function curve is distributed in the first and third quadrant. The larger value of \( \Delta X_y \), can result in the smaller value of \( \varepsilon \); while \( \Delta X_y < -0.0001 \), the larger value of \( \beta \), can result in the larger value of \( \varepsilon \); while \( \Delta X_y > -0.0001 \), the larger the value of \( \beta \), can result in the smaller value of \( \varepsilon \).

![Figure 4. \( \Delta X_y - \varepsilon \) function curve](image)

(2) If \( \Delta X_y = 0.1 \text{cm}, r = 50 \text{cm}, \) the \( \Delta Z_y - \varepsilon \) function curve is shown in Fig.5. As shown in Fig. 5, the \( \Delta Z_y - \varepsilon \) function curve is distributed in the first and second quadrant. In the second quadrant, the larger value of \( \Delta X_y \), can result in the smaller value of \( \varepsilon \); while \( \Delta X_y < -3.161 \), the larger value of \( \beta \), can result in the smaller value of \( \varepsilon \); while \( \Delta X_y > -3.161 \), the larger the value of \( \beta \), can result in the larger value of \( \varepsilon \). In the first quadrant, the larger value of \( \Delta X_y \), can result in the larger value of \( \varepsilon \); the larger value of \( \beta \), can result in the larger value of \( \varepsilon \).
4. Case Study

An assembly with three workpieces is illustrated as an example to verify the calculation of locating accuracy. Fig. 6 is the individual station for this assembly; the dotted line shows the nominal position of parts, and the solid line shows the actual position of parts.

Suppose that the two positioning pins of part one are \( P_1, P_2 \); respectively, and the positioning groove are \( Y_1, Y_2 \). The two positioning pins of part two are \( P_3, P_4 \), and the positioning groove are \( Y_3, Y_4 \). Assuming that (1) the fixture error of part one is \( \Delta P_{11} = [1, 1, 1, 2] \), the angle \( \alpha_1 \) referring to the \( x \)-axis is 60°, and the distance between the two pins is \( L_{F1} = 500 \text{mm} \). (2) the locating error of part one is \( \Delta Y_1 = [1, 2, 1, 2, \frac{\pi}{180}] \), the distance between the fitting hole and fitting groove is \( R_1 = 500 \text{mm} \). (3) the fixture error of part two is \( \Delta P_{12} = [1, 1, 2, 1] \), the angle \( \alpha_2 \) referring to the \( x \)-axis is 60°, and the distance between the two pins is \( L_{F2} = 500 \text{mm} \). (4) the locating error of part two is \( \Delta Y_2 = [2, 1, 2, 1, \frac{\pi}{180}] \), the distance between the fitting hole and fitting groove is \( R_2 = 500 \text{mm} \).

![Figure 6. Schematic diagram of assembly within individual station](image)

Within the assembly, suppose that:

\[
M_{A1P1} = M_{A2P3} = M_{A1Y1} = M_{A2Y2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

The locating error of part one caused by the fixture error is \( X_{A1}^{(1)}(l) = [1.00002, 0.99998, 0.00002]^T \), and the locating error of part two caused by the fixture error is \( X_{A2}^{(1)}(l) = [0.99997, 1.00003, -0.00003]^T \). The locating error of part one caused by the fabricating error of fitting hole and groove is \( X_{A1}^{(2)}(l) = [-1.00197, -1.99803, -0.00197]^T \), and the locating error of part two caused by the fabricating error of fitting hole and groove is \( X_{A2}^{(2)}(l) = [-2.00197, -0.99803, -0.00197]^T \). According to equation reference [5], the variable flow of station one is:
Similarly, for individual station two, suppose that the subassembly after assembly station one is recorded as \{1,2\}; the two positioning pins of subassembly \{1,2\} are P₁, P₂; respectively, and the positioning groove are Y₁, Y₂. The two positioning pins of part three are P₃, P₄, and the positioning groove are Y₅, Y₆. Assuming that:

(1) the repeated locating error of part one is \(X_{A₁}^{12}(2) = [0.00195, 0.99805, 0.00199]^T\), the repeated locating error of part two is \(X_{A₂}^{12}(2) = [0.2185, 0.00305, 0.00199]^T\);

(2) the fixture error of subassembly \{1,2\} is \(ΔP_{21} = [1,1,1,2]\), the angle \(α₁\) referring to the x-axis is 60°, and the distance between the two pins is \(L^F₁ = 500\)mm; the locating error of part one is \(ΔY₁ = [1,2,1,2, \frac{π}{180}]\), the distance between the fitting hole and fitting groove is \(R₁ = 500\)mm.

(3) the fixture error of part three is \(ΔP_{32} = [0.8,0.3,0.6,0.4]\), the angle \(α₂\) referring to the x-axis is 60°, and the distance between the two pins is \(L^F₁ = 500\)mm. (4) the locating error of part three is \(ΔY₃ = [0.9,1.1,1,2, \frac{π}{180}]\), the distance between the fitting hole and fitting groove is \(R₃ = 500\)mm.

Within the assembly, suppose that:

\[
M_{A3PS} = M_{A3Y3} = \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & -1 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

The locating error of part one caused by the fixture error is \(X_{A₁}^{12}(2) = [1.00002, 0.49998, 0.00002]^T\); the locating error of part two caused by the fixture error is \(X_{A₂}^{12}(2) = [1.00028, 0.48823, 0.00002]^T\), and the locating error of part two caused by the fixture error is \(X_{A₁}^{12}(2) = [0.80001, 0.29999, 0.00001]^T\). The locating error of part one caused by the fabricating error of fitting hole and groove is \(X_{A₂}^{12}(2) = [-1.00197, -1.99803, -0.00197]^T\), the locating error of part two caused by the fabricating error of fitting hole and groove is \(X_{A₂}^{12}(2) = [-1.02166, -1.10154, -0.00197]^T\), and the locating error of part three caused by the fabricating error of fitting hole and groove is \(X_{A₁}^{12}(2) = [-0.93291, -1.06709, -0.03291]^T\) According to equation reference [5], the variable flow of station two is:

\[
X(2) = \begin{bmatrix}
-0.00195 & -0.99805 & -0.00195 \\
-0.80488 & -0.50376 & 0.00125 \\
-0.1329 & -0.7671 & -0.0039 \\
\end{bmatrix}
\]

5. Summary
The locating variation in one-plane-dual-pins fixture system is mainly caused by the fixture error of the positioning pin, and is caused by the fabricating error of fitting hole and groove of each part. For type two assembly, locating accuracy pre-analysis of one-plane-dual-pins fixture system is important for pre-analysis of the final functional assembly requirement. In this paper, the locating and direction error of two-pin positioning part are analyzed; the accuracy of the parts with fabricating error of fitting hole and groove are further calculated. Considering the assembly with three parts, an example is introduced for calculation verification.

6. References
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