Abstract

Lexical entailment, such as hyponymy, is a fundamental issue in the semantics of natural language. This paper proposes distributional semantic models which efficiently learn word embeddings for entailment, using a recently-proposed framework for modelling entailment in a vector-space. These models postulate a latent vector for a pseudo-phrase containing two neighbouring word vectors. We investigate both modelling words as the evidence they contribute about this phrase vector, or as the posterior distribution of a one-word phrase vector, and find that the posterior vectors perform better. The resulting word embeddings outperform the best previous results on predicting hyponymy between words, in unsupervised and semi-supervised experiments.

1 Introduction

Modelling entailment, such as hyponymy, is a fundamental issue in the semantics of natural language, and there has been a lot of interest in modelling entailment using vector-space representations, particularly for lexical entailment relations such as hyponymy. Entailment is the relation of information inclusion, meaning that \( y \) entails \( x \) if and only if everything that is known given \( x \) is also known given \( y \). As such, representations which support entailment need to encode what is known, versus what is unknown.

Although much work has used vector-space embeddings of words in models of entailment, few models have developed vector-space embeddings which intrinsically model entailment. The exceptions have been Vilnis and McCallum (2015), who use variances to represent the amount of information about a continuous space, and Henderson and Popa (2016), who use probabilities to represent the amount of information about a discrete space. In this work we use the framework from Henderson and Popa (2016) to develop new distributional semantic models of entailment between words.

In the framework of Henderson and Popa (2016), each dimension of the vector-space represents something that might be known, and continuous vectors represent probabilities of these features being known or unknown. Henderson and Popa (2016) illustrate their framework by proposing a reinterpretation of existing Word2Vec (Mikolov et al., 2013a) word embeddings, which successfully predicts hyponymy with an unsupervised model. To motivate this reinterpretation of existing word embeddings, they propose a model of distributional semantics and argue that, under this reinterpretation, the Word2Vec training objective approximates the training objective of this distributional semantic model.

In this paper, we implement this distributional semantic model and train new word embeddings using the exact objective. This results in embeddings which directly encode what is known and unknown given a word, thus not requiring any reinterpretation to predict hyponymy. The distributional semantic model postulates a latent pseudo-phrase vector for the unified semantics of a word and its neighbouring context word. This latent vector must entail the features in both words’ vectors and must be consistent with a prior over semantic vectors, thereby modelling the redundancy and consistency between the semantics of two neighbouring words.

Our analysis of this entailment-based model of a word in context leads us to hypothe-
sise that the word embeddings suggested by Henderson and Popa (2016) are in fact not the best way to extract information about the semantics of a word from this model. They propose using a vector which represents the evidence about known features given the word. We propose to instead use a vector which represents the posterior distribution of known features for a phrase containing only the word. This posterior vector includes both the evidence from the word and its indirect consequences via the constraints imposed by the prior. Our efficient implementation of this model allows us to test this hypothesis by outputting either the evidence vectors or the posterior vectors as word embeddings. 

To evaluate these word embeddings, we predict hyponymy between words, in both an unsupervised and semi-supervised setting. Given the word embeddings for two words, we measure whether they are a hypernym-hyponym pair using an entailment operator from Henderson and Popa (2016) applied to the two embeddings. We find that using the evidence vectors performs as well as reinterpretation Word2Vec embeddings, confirming the claims of equivalence by Henderson and Popa (2016). But we also find that using the posterior vectors performs significantly better, confirming our hypothesis that posterior vectors are better, and achieving the best published results on this benchmark dataset. In addition to these unsupervised experiments, we evaluate in a semi-supervised setting and find a similar pattern of results, again achieving state-of-the-art performance.

In the rest of this paper, section 2 presents the formal framework we use for modelling entailment in a vector space, the distributional semantic models, and how these are used to predict hyponymy. Section 3 discusses additional related work, and then section 4 presents the empirical evaluation on hyponymy detection, in both unsupervised and semi-supervised experiments. Some additional analysis of the induced vectors is presented in section 4.4.

2 Distributional Semantic Entailment

Distributional semantics uses the distribution of contexts in which a word occurs to induce the semantics of the word (Harris, 1954; Deerwester et al., 1990; Schütze, 1993). The Word2Vec model (Mikolov et al., 2013a) introduced a set of refinements and computational optimisations of this idea which allowed the learning of vector-space embeddings for words from very large corpora with very good semantic generalisation. Henderson and Popa (2016) motivate their reinterpretation the Word2Vec Skipgram (Mikolov et al., 2013a) distributional semantic model with an entailment-based model of the semantic relationship between a word and its context words. We start by explaining our interpretation of the distributional semantic model proposed by Henderson and Popa (2016), and then propose our alternative models.

Henderson and Popa (2016) postulate a latent vector $y$ which is the consistent unification of the features of the middle word $x_e'$ and the neighboring context word $x_r$, illustrated on the left in figure 1.1 We can think of the latent vector $y$ as representing the semantics of a pseudo-phrase consisting of the two words. The unification requirement is defined as requiring that $y$ entail both words, written $y \Rightarrow x_e'$ and $y \Rightarrow x_e$. The consistency requirement is defined as $y$ satisfying the constraints imposed by a prior $\theta(y)$. This approach models the relationship between the semantics of a word and its context as being redundant and consistent. If $x_e'$ and $x_e$ share features, then it will be easier for $y$ to satisfy both $y \Rightarrow x_e'$ and $y \Rightarrow x_e$. If the features of $x_e'$ and $x_e$ are consistent, then it will be easier for $y$ to satisfy the prior $\theta(y)$.

2.1 The Reinterpretation of Word2Vec

Henderson and Popa (2016) formalise the above model using their entailment-vectors framework. This framework models distributions over discrete vectors where a one in position $i$ means feature $i$ is known and a zero means it is unknown. Entailment $y \Rightarrow x$ requires that the ones in $x$ are a subset of the ones in $y$, so $1 \Rightarrow 1$, $0 \Rightarrow 0$ and $1 \Rightarrow 0$, but $0 \not\Rightarrow 1$. Distributions over these discrete vectors are represented as continuous vectors of log-odds $X$, so $P(x_i=1) = \sigma(X_i)$, where $\sigma$ is the logistic sigmoid. The probability of entailment $y \Rightarrow x$ between two such “entailment vectors” $Y, X$ can be mea-

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1 Note that “$x_e$” is being used here as the name of a whole vector, not to be confused with “$x_e$”, which refers to element $i$ in vector $x$. 

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expected according to $P$

For each feature

sured using the operator $\otimes$:

$\log P(y \Rightarrow x \mid Y, X) \approx Y \otimes X \equiv \sigma(-Y) \cdot \log \sigma(-X) \quad (1)$

For each feature $i$ in the vector, it calculates the expectation according to $P(y_i)$ that, either $y_i=1$ and thus the log-probability is zero, or $y_i=0$ and thus the log-probability is $\log P(x_i=0)$ (given that $\sigma(-X_i) = (1 - \sigma(X_i))$).

Henderson and Popa (2016) formalise

the model on the left in figure 1 by first inferring the

constraints have been satisfied (equation (2)).

$\max_y (E_{Y, X'_e, X_e} \log P(y \Rightarrow x'_e, y \Rightarrow x_e, y))$

$\approx Y \otimes X'_e + Y \otimes X_e + (- \sigma(-Y)) \cdot \theta(Y) \quad (2)$

where

$Y = - \log \sigma(-X'_e) + - \log \sigma(-X_e) + \theta(Y)$

(3)

where $E_{Y, X'_e, X_e}$ is the expectation over the distribution defined by the log-odds vectors $Y, X'_e, X_e$, and log and $\sigma$ are applied componentwise. The term $\theta(Y)$ is used to indicate the net effect of the prior on the vector $Y$. Note that, in the formula $(3)$ for inferring $Y$, the contribution $- \log \sigma(-X)$ of each word vector is also a component of the definition of $Y \otimes X$ from equation $(1)$. In this way, the score for measuring how well the entailment has been satisfied is using the same approximation as used in the inference to satisfy the entailment constraint. This function $- \log \sigma(-X)$ is a non-negative transform of $X$, as shown in figure 2. Intuitively, for an entailed vector $x$, we only care about the probability that $x_i=1$ (positive log-odds $X_i$), because that constrains the entailing vector $y$ to have $y_i=1$ (adding to the log-odds $Y$).

$X_p \approx - \log \sigma(-X_e) + \theta(Y) \quad (4)$

Both vectors $X_e$ and $X_p$ are parameters of the model, which need to be learned. Thus, there is no need to explicitly model the prior, thereby avoiding the need to choose a particular form for the prior $\theta$, which in general may be very complex.

This gives us the following score for how well the constraints of this model can be satisfied.

$\max_y (E_{Y, X'_e, X_e} \log P(y \Rightarrow x'_e, y \Rightarrow x_e, y))$

$\approx Y \otimes X'_e + (- \sigma(-Y)) \cdot X_p \quad (5)$

where

$Y = - \log \sigma(-X'_e) + X_p \quad (6)$

In (Henderson and Popa, 2016), score $(5)$ is only used to provide a reinterpretation of Word2Vec word embeddings. They show that a transformation of the vectors output by Word2Vec...
(“W2V u.d.⟨” below) can be seen as an approximation to the evidence vector \( X_e \). In Section 4, we empirically test this hypothesis by directly training \( X_e \) (“W2H evidence” below) and comparing the results to those with reinterpreted Word2Vec vectors.

### 2.2 New Distributional Semantic Models

In this paper, we implement distributional semantic models based on score (5) and use them to train new word embeddings. We call these models the Word2Hyp models, because they are based on Word2Vec but are designed to predict hyponymy.

To motivate our models, we provide a better understanding of the model behind score (5). In particular, we note that although we want \( X_p \) to approximate the effects \( \theta(Y) \) of the prior as in equation 4, in fact \( X_p \) is only dependent on one of the two words, and thus can only incorporate the portion of \( \theta(Y) \) which arises from that one word. Thus, a better understanding of \( X_p \) is provided by equation (7).

\[
X_p \approx -\log \sigma(-X_e) + \theta(X_p)
\]

In this framework, equation (7) is exactly the same formula as would be used to infer the vector for a single-word phrase (analogously to equation (3)).

This interpretation of the approximate model in equation 5 is given on the right side of figure 1. As shown, \( X_p \) is interpreted as the posterior vector for a single-word phrase, which incorporates the evidence and the prior for that word. In contrast, \( X'_e \) is just the evidence about \( Y \) provided by the other word. This model, as argued above, approximates the model on the left side in Figure 1. But the grey part of the figure does not need to be explicitly modelled.

This interpretation suggests that the posterior vector \( X_p \) should be a better reflection of the semantics of the word than the evidence vector \( X_e \), since it includes both the direct evidence for some features and their indirect consequences for other features. We test this hypothesis empirically in Section 4.

To implement our distributional semantic models, we define new versions of the Word2Vec code (Mikolov et al., 2013a,b). The Word2Vec code trains two vectors for each word, where negative sampling is applied to one of these vectors, and the other is the output vector. This applies to both the Skipgram and CBOW versions of training. Both versions also use a dot product between vectors to try to predict whether the example is a positive or negative sample. We simply replace this dot product with score (5) directly in the Word2Vec code, leaving the rest of the algorithm unchanged. We make this change in one of two ways, one where the output vector corresponds to the evidence vector \( X_e \), and one where the output vector corresponds to the posterior vector \( X_p \). We will refer to the model where \( X_p \) is output as the “posterior” model, and the model where \( X_e \) is output as the “evidence” model. Both these methods can be applied to both the Skipgram and CBOW models, giving us four different models to evaluate.

### 2.3 Modelling Hyponymy

The proposed distributional semantic models output a word embedding vector for every word in the vocabulary, which are directly interpretable as entailment vectors in the entailment framework. Thus, to predict lexical entailment between two words, we can simply apply the \( \otimes \) operator to their vectors, to get an approximation of the log-probability of entailment.

We evaluate these entailment predictions on hyponymy detection. Hyponym-hypernym pairs should have associate embeddings \( Y, X \) which have a higher entailment scores \( Y \otimes X \) than other pairs. We rank the word pairs by the entailment scores for their embeddings, and evaluate this ranked list against the gold hyponymy annotations. We evaluate on hyponymy detection because it reflects a direct form of lexical entailment; the semantic features of a hypernym (e.g. “animal”) should be included in the semantic features of the hyponym (e.g. “cat”). Other forms of lexical entailment would benefit from some kind of reasoning or world knowledge, which we leave to future work on compositional models.

### 3 Related Work

In this paper we propose a distributional semantic model which is based on entailment. Most of the work on modelling entailment with vector space embeddings has simply used distributional semantic vectors within a model of entailment, and is therefore not directly relevant here. See (Shwartz et al., 2017) for a comprehensive review of such measures. Shwartz et al. (2017) evaluate these measure as unsupervised models of hyponymy detection and run experiments on a number of hy-
ponymy datasets. We report their best comparable result in Table 1.

Vilnis and McCallum (2015) propose an unsupervised model of entailment in a vector space, and evaluate it on hyponymy detection. Instead of representing words as a point in a vector space, they represent words as a Gaussian distribution over points in a vector space. The variance of this distribution in a given dimension indicates the extent to which the dimension’s feature is unknown, so they use KL-divergence to detect hyponymy relations. Although this model has a nice theoretical motivation, the word representations are more complex and training appears to be more computationally expensive than the method proposed here.

The semi-supervised model of Kruszewski et al. (2015) learns a discrete Boolean vector space for predicting hyponymy. But they do not propose any unsupervised method for learning these vectors.

Weeds et al. (2014) report hyponymy detection results for a number of unsupervised and semi-supervised models. They propose a semi-supervised evaluation methodology where the words in the training and test sets are disjoint, so that the supervised component must learn about the unsupervised vector space and not about the individual words. Following Henderson and Popa (2016), we replicate their experimental setup in our evaluations, for both unsupervised and semi-supervised models, and compare to the best results among the models evaluated by Weeds et al. (2014), Shwartz et al. (2017) and Henderson and Popa (2016).

4 Evaluation of Word Embeddings

We evaluate on hyponymy detection in both a fully unsupervised setup and a semi-supervised setup. In the semi-supervised setup, we use labelled hyponymy data to train a linear mapping from the unsupervised vector space to a new vector space with the objective of correctly predicting hyponymy relations in the new vector space. This prediction is done with the same (or equivalent) entailment operator as for the unsupervised experiments (called “map ⋀” in Table 2).

We replicate the experimental setup of Weeds et al. (2014), using their selection of hyponym-hyponym pairs from the BLESS dataset (Baroni and Lenci, 2011), which consists of noun-noun pairs, including 50% positive hyponymy pairs plus 50% negative pairs consisting of some other hyponymy pairs reversed, some pairs in other semantic relations, and some random pairs. As in (Weeds et al., 2014), our semi-supervised experiments use ten-fold cross validation, where each fold has items removed from the training set if they contain a word that also occurs in the testing set.

The word embedding vectors which we train have 200 dimensions and were trained using our Word2Hyp modification of the Word2Vec code (with default settings), trained on a corpus of half a billion words of Wikipedia. We also replicate the approach of Henderson and Popa (2016) by training Word2Vec embeddings on this data.

To quantify performance on hyponymy detection, for each model we rank the list of pairs according to the score given by the model, and report two measures of performance for this ranked lists. The “50% Acc” measure treats the first half of the list as labelled positive and the second half as labelled negative. This is motivated by the fact that we know a priori that the proportion of positive examples has been artificially set to (approximately) 50%. Average precision is a measure of the accuracy for ranked lists, used in Information Retrieval and advocated as a measure of hyponymy detection by Vilnis and McCallum (2015). For each positive example, precision is measured at the threshold just below that example, and these precision scores are averaged over positive examples. For cross validation, we average over the union of positive examples in all the test sets. Both these measures are reported (when available) in Tables 1 and 2.

4.1 Unsupervised Hyponymy Detection

The first set of experiments evaluate the different embeddings in their unsupervised models of hyponymy detection. Results are shown in Table 1. Our principal point of comparison is the best results from (Henderson and Popa, 2016) (called “W2V GoogleNews” in Table 1). They use the pre-existing publicly available GoogleNews word embeddings, which were trained with the Word2Vec software on 100 billion words of the GoogleNews dataset, and have 300 dimensions. To provide a more direct comparison, we replicate the model of Henderson and Popa (2016) but using the same embedding training setup as for our Word2Hyp model (“W2V Skip”). Both cases use their proposed reinterpretation of these vectors for predicting entailment (“u.d.⊤”). We also report
the best results from Weeds et al. (2014) and the best results from (Shwartz et al., 2017). For our proposed Word2Hyp distributional semantic models (“W2H”), we report results for the four combinations of using the CBOW or Skipgram (“Skip”) model to train the evidence or posterior vectors.

The best unsupervised model of Weeds et al. (2014) and the two Word2Hyp models with evidence vectors perform similarly. The reinterpretation of Word2Vec vectors (“W2V GoogleNews u.d.”) performs better, but when the same method is applied to the smaller Wikipedia corpus (“W2V Skip u.d.”), this difference all but disappears. This confirms the hypothesis of Henderson and Popa (2016) that the reinterpretation of Word2Vec vectors and the evidence vectors from Word2Hyp are approximately equivalent.

However, even with this smaller corpus, using the proposed posterior vectors from the Word2Hyp model are significantly more accurate than the reinterpretation of Word2Vec vectors. This confirms the hypothesis that the posterior vectors from the Word2Hyp model are a better model of the semantics of a word than the evidence vectors suggested by Henderson and Popa (2016).

Using the CBOW model or the Skipgram model makes only a small difference. The average precision score shows the same pattern as the accuracy.

To allow a direct comparison to the model of Vilnis and McCallum (2015), we also evaluated the unsupervised models on the hyponymy data from (Baroni et al., 2012), which is not as carefully designed to evaluate hyponymy as the (Weeds et al., 2014) data. Both the evidence and posterior vectors of the Word2Hyp CBOW model achieved average precision (81%, 80%) which is not significantly different from the best model of Vilnis and McCallum (2015) (80%).

### 4.2 Semi-supervised Hyponymy Detection

The semi-supervised experiments train a linear mapping from each unsupervised vector space to a new vector space, where the entailment operator $\map$ is used to predict hyponymy (“map $\map$”).

The semi-supervised results (shown in Table 2) no longer show an advantage of GoogleNews vectors over Wikipedia vectors for the reinterpretation of Word2Vec vectors. And the advantage of posterior vectors over the evidence vectors is less pronounced. However, the two posterior vectors still perform much better than all the previously proposed models, achieving 86% accuracy and nearly 93% average precision. These semi-supervised results confirm the results from the unsupervised experiments, that Word2Vec embeddings and Word2Hyp evidence embeddings perform similarly, but that using the posterior vectors of the Word2Hyp model perform better.

### 4.3 Training Times

Because the similarity measure in equation 5 is more complex than a simple dot product, training a new distributional semantic model is slower than with the original Word2Vec code. In our experiments, training took about 8 times longer for the CBOW model and about 15 times longer for the Skipgram model. This meant that Word2Hyp CBOW trained about 8 times faster than Word2Hyp Skipgram. As in the Word2Vec code, we used a quadrature approximation (i.e. a
Table 3: Ranking of the abstractness \((0 \leq X \leq 1)\) of frequent words from the hyponymy dataset, using Word2Hyp-Skipgram-posterior embeddings.

| most abstract | least abstract |
|---------------|---------------|
| something     | fork          |
| necessity     | hockey        |
| anything      | housing       |
| sense         | republican    |
| end           | housing       |
| back          | elm           |
| inside        | primate       |
| saw           | cricket       |
| good          | fur           |

look-up table) to speed up the computation of the sigmoid function, and we added the same technique for computing the log-sigmoid function.

4.4 Discussion

The relative success of our distributional semantic models at unsupervised hyponymy detection indicates that they are capturing some aspects of lexical entailment. But the gap between the unsupervised and semi-supervised results indicates that other features are also being captured. This is not surprising, since many other factors influence the co-occurrence statistics of words.

To get a better understanding of these word embeddings, we ranked them by degree of abstractness. Table 3 shows the most abstract and least abstract frequent words that occur in the hyponymy data. To measure abstractness, we used our best unsupervised embeddings and measured how well they are entailed by the zero log-odds vector, which represents a uniform half probability of knowing each feature. For a vector to be entailed by the zero vector, it must be that its features are mostly probably unknown. The less you know given a word, the more abstract it is.

An initial ranking found that six of the top ten abstract words had frequency less than 300 in the Wikipedia data, but none of the ten least abstract terms were infrequent. This indicates a problem with the current method, since infrequent words are generally very specific (as was the case for these low-frequency words, *submissiveness, implementer, overdraft, ruminant, warplane, and londoner*). Although this is an interesting characteristic of the method, the terms themselves seem to be noise, so we rank only terms with frequency greater than 300.

The most abstract terms in table 3 include some clearly semantically abstract terms, in particular *something* and *anything* are ranked highest. Others may be affected by lexical ambiguity, since the model does not disambiguate words by part-of-speech (such as *end, good, sense, back, and saw*). The least abstract terms are mostly very semantically specific, but it is indicative that this list includes *primate*, which is an abstract term in Zoology but presumably occurs in very specific contexts in Wikipedia.

5 Conclusions

In this paper, we propose distributional semantic models for efficiently training word embeddings which are specifically designed to capture semantic entailment. This work builds on the work of Henderson and Popa (2016), who propose a framework for modelling entailment in a vector-space, and a distributional semantic model for reinterpreting Word2Vec word embeddings. Our contribution differs from theirs in that we train new word embeddings, and we choose different vectors in the model to output as word embeddings. Empirical results on unsupervised and semi-supervised hyponymy detection confirm that the model’s evidence vectors, which Henderson and Popa (2016) suggest to use, do indeed perform equivalently to their reinterpretation of Word2Vec vectors. But these experiments also show that the model’s posterior vectors, which we propose to use, perform significantly better, outperforming all previous results on this task.

The success of this distributional semantic model demonstrates that the entailment vector framework can be effective at extracting information about lexical entailment from the redundancy and consistency of words with their contexts in large text corpora. This result suggests future work on modelling other indirect evidence about semantics using the entailment vector framework.

References

Marco Baroni, Raffaella Bernardi, Ngoc-Quynh Do, and Chung-chieh Shan. 2012. Entailment above the word level in distributional semantics. In Proceedings of the 13th Conference of the European Chapter of the Association for Computational Linguistics (EACL). Association for Computational Linguistics, Avignon, France, pages 23–32. http://www.aclweb.org/anthology/E12-1004.

Marco Baroni and Alessandro Lenci. 2011. How we blessed distributional semantic evaluation. In Proceedings of the GEMS 2011 Workshop on GEometrical Models of Natural
Zellig S. Harris. 1954. Distributional structure. *Journal of Word* 10(2-3):146–162. https://doi.org/10.1080/00437956.1954.11659520.

James Henderson and Diana Popa. 2016. A vector space for distributional semantics for entailment. In *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*. Association for Computational Linguistics, Berlin, Germany, pages 2052–2062. http://www.aclweb.org/anthology/P16-1193.

Germn Kruszewski, Denis Paperno, and Marco Baroni. 2015. Deriving boolean structures from distributional vectors. *Transactions of the Association for Computational Linguistics* 3:375–388. https://tacl2013.cs.columbia.edu/ojs/index.php/tacl/article/view/616.

Tomas Mikolov, Kai Chen, Greg Corrado, and Jeffrey Dean. 2013a. Efficient estimation of word representations in vector space. *CoRR* abs/1301.3781. http://arxiv.org/abs/1301.3781.

Tomas Mikolov, Ilya Sutskever, Kai Chen, Greg S Corrado, and Jeff Dean. 2013b. Distributed representations of words and phrases and their compositionality. In C.J.C. Burges, L. Bottou, M. Welling, Z. Ghahramani, and K.Q. Weinberger, editors, *Advances in Neural Information Processing Systems 26*, Curran Associates, Inc., pages 3111–3119. http://papers.nips.cc/paper/5021-distributed-representations-of-words-and-phrases-and-their-compositionality.pdf.

Hinrich Schütze. 1993. Word space. In *Advances in Neural Information Processing Systems 5*. Morgan Kaufmann, pages 895–902.

Vered Shwartz, Enrico Santus, and Dominik Schlechweg. 2017. Hypernyms under siege: Linguistically-motivated artillery for hypernymy detection. In *Proceedings of the 15th Conference of the European Chapter of the Association for Computational Linguistics: Volume 1, Long Papers*. Association for Computational Linguistics, Valencia, Spain, pages 65–75. http://www.aclweb.org/anthology/E17-1007.

Luke Vilnis and Andrew McCallum. 2015. Word representations via Gaussian embedding. In *Proceedings of the International Conference on Learning Representations 2015 (ICLR)*. http://arxiv.org/abs/1412.6623.