Research on control technology of tip-tilt mirror system in adaptive optics

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Abstract: In order to solve the control problem of the tip-tilt mirror under the unknown disturbance, a nonlinear disturbance observer with adaptive ability based on the sliding mode control is designed. Firstly, the sliding mode control method of the tip-tilt mirror system is established with Lyapunov functions. Secondly, an adaptive nonlinear disturbance observer is developed on a basis of observer model. Finally, the proposed sliding mode control method is combined with a nonlinear observer with adaptive capability to achieve the goal of improving the control accuracy of the system, while also reducing the chattering caused by the system. The experiment proves that this method is achievable. The experimental results show that the tracking error of the azimuth axis is reduced from 1.637μrad to 1.083μrad, and the accuracy is improved by about 51.2%. The tracking error of the pitch axis is reduced from 1.966μrad to 1.614μrad, and the accuracy is improved by about 21.8%. This method can greatly weaken the inherent chattering and external disturbance of the system, and improve the stability of the tip-tilt mirror system.

1.Introduction

In the wireless laser communication system, the adaptive optical tip-tilt mirror system is widely used, which can correct the beam deviation in real time and align the receiving end stably in real time, with low motion inertia, fast response speed, high angle resolution accuracy and strong anti-electromagnetic interference ability. The tracking accuracy and response speed of the precision tracking system affect the performance of the entire control system, while the performance of the tip-tilt mirror system determines the performance of the precision tracking system to a large extent. The overall tilt caused by atmospheric turbulence accounts for about 87% of the disturbance error. Therefore, the tip-tilt mirror system plays a crucial role in correcting the first-order tilt.

In order to obtain accurate information about space targets, Zhou Rui [4] and Wang Jiaying [5] et al. respectively identified the transfer function and disturbance model of tip-tilt mirror system, analyzed the dynamic characteristics of optical axis jitter more accurately, and designed the control algorithm of the system specifically. At the same time, a two-stage series control system was proposed. The
front-stage stabilization system uses an LMS adaptive filter to correct narrow-band disturbances, and the latter-stage stability system uses a proportional-integral controller to correct wide-band disturbances. The tip-tilt mirror system is also applied in liquid crystal adaptive optics, using an open-loop optical path design to improve the correction frequency and the energy utilization rate of polarized light, thus improving the universality of the system. However, due to the low sampling frequency and large time delay of the image sensor, the closed-loop performance is greatly affected. The prominent problems are as follows: the dynamic behavior is complex and it is difficult to obtain an accurate model. An additional controller suppression method is proposed that is not limited by the accurate model, and a new filter is formed by combining the low-pass filter and bandpass filter to improve the closed-loop control performance.

Aiming at the tracking accuracy problem of the piezoelectric tip-tilt mirror caused by the system chattering, the mathematical model of the tip-tilt mirror was analyzed, and the sliding mode control method was used to propose the sliding mode control method of the system model, which was independent of the deterministic system. In order to weaken chattering on the impact of the system precision, a nonlinear disturbance observer is designed to suppress jitter. In this paper, the sliding mode control of the nonlinear observer combined with adaptive optics, the results show that the system accuracy has been greatly improved, the corresponding steady-state error is decreased obviously, and the stability problem caused by the nonlinear system is solved.

2. Sliding mode control (SMC) design for adaptive nonlinear observer

2.1. Analysis of Tip-Tilt mirror system

In order to find out the law of the deflection of the tip-tilt mirror surface due to the action of driving voltage, the deflection of the mirror surface with the change of operating voltage under a single piezoelectric actuator is studied at first. Taking the piezoelectric actuator group \( a_1, a_2 \) as an example, the corresponding tip-tilt mirror structure is shown in Figure 1.

![Fig. 1 Schematic diagram of Tip-Tilt mirror structure](image)

In the figure, \( \theta \) is the deflection angle of the tip-tilt mirror following the stretching of the piezoelectric actuator. Plane \( P \) is the plane position after the mirror deflection, and the forces generated by a group of piezoelectric actuators \( a_1 \) and \( a_2 \) that constitute the differential structure are denoted as \( F_{a1} \) and \( F_{a2} \) respectively. According to the working mode of a single piezoelectric actuator, regarded as a second-order typical spring-mass-damping system. Then, the relationship between the rotation torque of the tip-tilt mirror (denoted as \( R \)) and the deflection angle \( \theta \) of the mirror with the voltage changes is:

\[
R = [J + (m_{a1} + m_{a2})d^2]\dot{\theta} + (c_{a1} + c_{a2})d^2\dot{\theta} + (k_{a1} + k_{a2})d^2\theta
\]  

(1)
In the formula, \( J \) represents the moment of inertia of the actuator, \( m_{a_1} \) and \( m_{a_2} \) represent the masses of the two piezoelectric actuators, \( c_{a_1} \) and \( c_{a_2} \) represent the damping coefficients of the two piezoelectric actuators, \( k_{a_1} \) and \( k_{a_2} \) represent the elastic coefficients of the two piezoelectric actuators, and \( d \) represents the distance between the piezoelectric actuators and the center position of the mirror. Since the models and specifications of the two actuators of a group of piezoelectric actuators are the same, it can be considered that two piezoelectric actuators will generate two forces of equal magnitude and opposite directions under the drive of voltage, then:

\[
F_{a_1} = -F_{a_2}
\]  
(2)

Therefore, the relationship between torque \( R \) and force \( F_{a_1} \) can be expressed as:

\[
R = 2F_{a_1}d
\]  
(3)

Let \( F(\theta) = 2F_{a_1} \), then:

\[
F(\theta) = M(\theta)\ddot{\theta} + C(\theta)\dot{\theta} + K(\theta)\theta
\]  
(4)

Where, \( M(\theta) = [J + (m_{a_1} + m_{a_2})d^2] / d \), \( C(\theta) = (c_{a_1} + c_{a_2})d \) and \( K(\theta) = (k_{a_1} + k_{a_2})d \) are equivalent to the mass, damping coefficient and elastic coefficient of piezoelectric actuator group \((a_i, a_j)\) respectively.

2.2. Design of sliding mode controller

Sliding mode control can give nonlinear system good control performance, and good robustness is its biggest advantage. When the controlled object is in sliding mode, it is not sensitive to the unknown disturbance outside, changes in the controlled object model parameters or the uncertainty of the controlled object model, and the control effect will not be easily affected.

According to formula (3), the torque \( R \) of the mirror surface is composed of the control torque \( \tau \) and disturbance torque \( \Delta \), which can be obtained as follows:

\[
M(\theta)\ddot{\theta} + C(\theta)\dot{\theta} + K(\theta)\theta = \tau + \Delta
\]  
(5)

Where, \( D \) is the distance generated by the mirror surface caused by disturbance acting on the driver, and \( \Delta \) is the force generated by disturbance. Let the tracking error be: \( e(t) = \theta - \theta_d \), \( \theta_d \) is the reference angle. A control law is developed for tip-tilt mirror system to make the tracking error \( e(t) \) converge to zero when the unknown disturbance exists, and the sliding variable is selected as:

\[
s = \dot{\theta} + \Lambda e
\]  
(6)

Where: \( \Lambda \) normal diagonal matrix. When the sliding mode \( s = 0 \), \( e = 0 \) is the attractor of the system error dynamic \( \dot{e} + \Lambda e = 0 \). Substitute it into equation (5) to obtain:

\[
M(\theta)(\ddot{\theta} - \dot{s}) + C(\theta)\dot{\theta} + K(\theta)\theta = \varphi(\theta)
\]  
(7)

Among them: \( \varphi(\theta) = M(\theta)\ddot{\theta}_d + C(\theta)\dot{\theta}_d + K(\theta)\theta_d = \dot{\theta}_d - \Lambda e \).

According to equation (5), rewrite (7) as:

\[
M(\theta)s + C(\theta)s = \tau - \varphi(\theta) + \Delta
\]  
(8)

Consider the Lyapunov function:

\[
V = \frac{1}{2}s^T M(\theta)s
\]  
(9)

Given the basic properties of the matrix, \( M(\theta) - 2C(\theta, \dot{\theta}) \) is an oblique symmetric matrix, and substituting \( \dot{V} \) into equation (8) can be written as:

\[
\dot{V} = s^T M(\theta)\ddot{s} + s^T C(\theta)s = s^T (\tau - \varphi(\theta) + \Delta)
\]  
(10)

Then, the basic sliding mode control (BSMC) rate can be deducted as:

\[
\tau = \varphi(\theta) - \rho_0 \text{sgn}(s)
\]  
(11)

Where \( \rho_0 \) is a constant, usually \( \Delta \) is bounded, and \( \|\Delta\| \leq \rho \). According to previous experience \( \rho_0 > \rho \), a boundary \( \rho_0 \) is obtained, and \( \dot{V} \) can be simplified as:
\[ V = s^T (D\Delta - \rho_b \text{sgn}(s)) \leq - (\rho_b - \rho) \| s \| \leq 0 \] (12)

Obviously, if \( s \neq 0 \), then \( V \) is negative. Using Barbalat Lemma, we can conclude that at \( t \to +\infty \), the trajectory of system equation (5) can be driven to the sliding plane \( s(t) = 0 \). Conservative upper bound is usually used in control design. In addition, it can be seen from equation (11) that unnecessary chattering can be reduced by adjusting parameter \( \rho_b \). The \text{sgn}(x) function is replaced by continuous approximation \( s/(s + \varepsilon) \) to reduce chattering in experimental tests, where \( \varepsilon \) is a small normal number. The smaller \( \varepsilon \) is, the better approximate performance.

2.3. Nonlinear observer design

Observer-based sliding mode control (OBSMC) is developed in the observer-based control design framework. The objectives of the system are two folds: (1) Design a new nonlinear observer to estimate the unknown perturbations of the tip-tilt mirror; (2) Design an OBSMC, so that the system output can accurately track the reference angle when capturing unknown disturbances. Therefore, the adaptive nonlinear disturbance observer (ANDO) of dynamic equation (8) is designed as follow:

\[ \dot{\hat{\Theta}} = -I(t)D\hat{\Theta} - l(t)(DL(t)M(\theta)\dot{\Theta} + f(t)) - (p(t)M(\theta) + l(t)M(\theta))\dot{\Theta} \] (13)

Where \( \hat{\Delta} \) is the estimate of unknown disturbance \( \Delta \), \( z \) is the state vector of the nonlinear observer, \( l(t) \) is the gain of the nonlinear observer, \( p(t) \) is the design parameter matrix, \( f(t) = \tau - C(\theta)\dot{\theta} - K(\theta)\dot{\theta} \)

Now, the estimation error is defined as:

\[ \hat{\Delta} = \Delta - \hat{\Delta} \] (14)

From the equation (14) can be:

\[ \hat{\Delta} = \Delta - \hat{\Delta} \] (15)

Substitute equation (5) and equation (13) into equation (15) above to obtain:

\[ \hat{\Delta} = l(t)Dz + l(t)(DL(t)\dot{\Theta} - l(t)DA + \hat{\Delta}) \] (16)

So let \( z = \hat{\Delta} - l(t)M(\theta)\dot{\Theta} \), we derive

\[ \hat{\Delta} = l(t)D\hat{\Theta} - l(t)DA + \hat{\Delta} \] (17)

The results show that the estimation error is affected by equation (18):

\[ \hat{\Delta} = -l(t)D\hat{\Theta} + \hat{\Delta} \] (18)

To continuous bounded disturbance hypothesis that equation (5) in the derivative of unknown external disturbances are bounded, as \( \hat{\Delta} \to 0 \)

Assuming that the above assumptions are met and the observer gain is chosen as: \( \hat{\Delta} = -l(t)D\hat{\Theta} \), then the estimated error system equation (18) is asymptotically stable.

2.4. SMC design of nonlinear observer

If the disturbance estimation error in equation (18) satisfies

\[ \gamma^* = \sup_{t>0} \left\| \Delta - \hat{\Delta} \right\| \] (19)

By using the appropriate estimation of the disturbance, the control law that the system state can drive on the sliding plane \( s(t) = 0 \) is:

\[ \tau = \theta - D(\hat{\Delta} + \gamma \text{sgn}(s^T D)) \] (20)

Here \( \gamma > \gamma^* \).

According to equation (10), substitute equation (8) and equation (20) into equation (14) to obtain:

\[ \dot{V} = s^T (D\Delta - D(\hat{\Delta} + \gamma \text{sgn}(s^T D))) = -\gamma s^T D \text{sgn}(s^T D) + s^T D(\Delta - \hat{\Delta}) \leq -\gamma s^T D + \gamma^* \| s \| \] (21)
By the inference can be concluded that the estimation error $\|\Delta - \hat{\Delta}\|$ in unknown disturbance $\Delta$ under asymptotically converge to zero. So, there exists a positive control parameter $\gamma$ small enough for $\gamma > \gamma^*$. According to formula (21), when $s \neq 0$, $\dot{V} < 0$, therefore, the system satisfies Lyapunov stability theory. At this time, the system state can reach the sliding surface $s = 0$ under the condition of $t \to +\infty$. In the case of sliding mode $s = 0$, there exists a normal number diagonal matrix so that $\dot{e} + \Lambda e = 0$, the system equation of state satisfies $\lim_{t \to \infty} e(t) = 0$. Therefore, under the proposed control law equation (20), the system state will converge to the reference trajectory.

3. Results & Discussion

3.1 Simulation results and analysis

In order to verify the feasibility of the designed nonlinear observer with adaptive ability, a control model of the tip-tilt mirror system was built by MATLAB Simulink, and the control effects of nonlinear observer and non-nonlinear observer were compared. Step signal was input to the system, and the addition time was 0.1s. The step response curve was compared with the error curve after entering steady-state. As shown in Figure 2 and Figure 3.

According to the comparison, it can be seen that the dynamic response of the nonlinear observer does not appear overharmonic oscillation, the adjustment time to achieve stability is 0.02s, and the steady-state error can be considered as almost zero. When the nonlinear observer is not used, the overshoot is 10.8% and there is oscillation phenomenon. The adjusting time is 0.07s, and the steady-state error can also be considered as zero. Since chattering exists in the system, the chattering is obviously reduced by adopting nonlinear observer.

The tracking performance of tip-tilt mirror is further verified and the tracking simulation experiment of sinusoidal position signal is carried out. Input sinusoidal signal with amplitude of $3 \text{ mrad}$ and period of 325ms to the tip-tilt mirror system. Compare the two control schemes, and the results are shown in Figure 4(a), Figure 4(b), Figure 5(a), Figure 5(b) and Figure 6.
According to the sinusoidal tracking curve, the output effect of the two control schemes is good, which can track the input signal well. However, according to the local amplification, the tracking effect of the nonlinear observer is better than that of the non-nonlinear observer, and the output curve is closer to the sinusoidal signal curve of the input. According to the tracking error curve results, the tracking error of the nonlinear observer is two times smaller than that of the non-nonlinear observer,
and the chattering suppression effect is better, and the steady-state performance of the system is greatly increased.

3.2. Experimental results and analysis
In order to verify the actual tracking performance after adopting the observer scheme, an experimental device was set up to test the tilt correction of simulated atmospheric turbulence with tip-tilt mirror in an indoor environment. The experimental device is divided into two parts, the transmitting part and the receiving part. The emitter, as a simulation target source, reaches the tip-tilt mirror system after simulating atmospheric turbulence, which consists of a laser and a parallel light tube. The receiving device is the control system of optical terminal machine, which consists of tip-tilt mirror system, deformation mirror system, Hartmann sensor, CCD camera, precision tracking servo system and upper computer system.

In the experiment, the state of the system is stable, and the corrected state data is read by the upper computer system. Figure 7 is the tracking error curve without nonlinear observer in practical application. The azimuth axis error is 1.637μrad and the pitching axis error is 1.966μrad. Figure 8 is the tracking error curve with nonlinear observer in practical application, the azimuth axis error is 1.083μrad and the pitching axis error is 1.614μrad. By comparison, the tracking accuracy of azimuth axis is improved about 51.2%, and the pitching axis is improved about 21.8%.

4. Conclusions
In this paper, the basic principle and mathematical model of the tip-tilt mirror system are analyzed, and an OBSMC is developed. In order to improve the performance of the system, a new nonlinear observer with adaptive ability is designed, which can be used to approximate the uncertainties caused by unknown disturbances in the tip-tilt mirror system. For the tip-tilt mirror with unknown disturbance, the sliding mode control and nonlinear observer are combined. MATLAB simulation results show that the steady-state error of the system is two times smaller than that without the nonlinear observer. In practice, the azimuth axis tracking accuracy of the tip-tilt mirror system is improved by about 51.2% and the pitching axis by about 21.8%, ensuring the wavefront detection accuracy of the adaptive optical system.

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