Acoustic Optimization of a Power Take-off Gear Box Using Numerical Transfer Path Analysis

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Abstract. In this contribution, a method is proposed which allows for a target-oriented structural optimization of drive train systems using transfer path analysis on a multi body system simulation with respect to the airborne sound. It combines the existing approaches of multi body simulation (MBS) and experimental transfer path analysis (TPA) with sound radiation calculations thereby creating a new tool chain to not only identify critical operation points but also critical transfer paths of an MBS model. This approach combines the advantages of elastic MBS (i.e. information about modes and forces acting between different components in critical operating points) with the advantages of TPA (i.e. identifying critical paths from the excitation to the receiver) thereby enabling the designer to numerically deduct and evaluate target-oriented structural optimizations already in early stages of the development process. It therefore reduces development times and costs by reducing iteration loops and the manufacturing of physical prototypes. The method is demonstrated on a power take-off (PTO) gear box, of which the sound pressure level was reduced by 6 dB within three virtual optimization loops that each only take a few hours by applying the proposed method.

1. Introduction

The continuous shortening of product lifecycles has created a demand for faster time to market and thus for faster and more efficient development processes. Therefore, methods of virtual product development (VPD) have become increasingly established, as they reduce development time and cost by incorporating numerical methods into the development process, thereby reducing the number of iteration loops with expensive, physical prototypes. Current trends in VPD include the integration of simulation models of different domains and detail levels into seamless multi-domain system models which is referred to as model-based systems engineering (MBSE) and allows for the evaluation and optimization of features of complex systems. Within this framework, elastic multi body simulation (eMBS) is used to analyze the structural dynamic properties of drive train systems. eMBS models have enabled the design engineer to efficiently evaluate and quantify the effect of design changes on the dynamic behavior at system level, which can be quantified in terms of structure borne and airborne sound that both play an important role for the customer acceptance of a product. However, the eMBS method alone does not allow for the target-oriented deducting of optimization measures to the structure as this requires the identification of critical transfer paths of numerical models which still remains a challenge. TPA is a well-known method for experimentally identifying these critical paths on physical prototypes. As this method however is a purely experimental approach, it leaves the advantages of the simulation unused. The aim of this work is therefore to propose a method that enables the design engineer to identify...
optimizations to an existing virtual prototype with regard to the airborne sound by combining eMBS and TPA and extending it by incorporating the sound radiation calculation into the TPA-formulation.

Based on the state of the art described in chapter 2 an approach to incorporate the experimental approach of transfer path analysis (TPA) into the simulation domain is presented in chapter 3. Special focus will be directed towards the analysis of airborne sound as the most important NVH quantity. By doing so, critical transfer paths can be identified and optimization measures deduced. The method is demonstrated on a power take-off (PTO) gear box in chapter 4. It yields a 6 dB optimization of the total sound pressure level within three virtual optimization loops. Conclusions and outlook will be presented in chapter 5.

2. State of the art
Two fundamentals are essential to understanding the presented work: The widely used experimental TPA-Method as well as the calculation of airborne sound radiation as part of system level simulation models based on the eMBS method. Both are presented in greater detail in what follows:

2.1. Transfer path analysis
In the experimental world, TPA is a widely established method for identifying critical transfer paths for different operation points [1–4]. Its basic concept is shown in figure 1. A structural dynamic system is divided into an active system which contains the excitations \( \mathbf{f}_1 \) (e.g. from gear boxes, combustion or electrical engines or road-tire noise), denoted “A” in the following, and a passive system, denoted “B”, that contains the structure linking the active system to a receiver point (e.g. the driver’s ear). These two systems are connected at discrete interface points, where forces are transferred from the active to the passive subsystem.

![Figure 1. General concept of TPA](image)

The receiver response, which can be a displacement, velocity, acceleration or sound pressure level, denoted \( u_3 \), can be calculated depending on the interface forces \( g_2 \) acting between the two subsystems when assembled and the frequency response functions (FRF) of the passive System \( Y^B_{32} \) as shown in equation (1).

\[
u_3 = Y^B_{32} \cdot g_2 (1)\]

The multiplication of an interface force in one degree of freedom (DOF) with the corresponding FRF to the receiver, results in the transfer path contribution. As these interface forces are typically difficult to measure in an experimental setup, different methods have been developed for reconstructing the interface forces [5, 6]. The advantage of the simulation however is, that the interface forces can directly be obtained from time-domain analysis (e.g. a rpm runup) so that the experimental methods (e.g. matrix-inverse methods) don’t have to be applied on a simulation model.

While some efforts have been made to extend TPA-methodology into the simulation domain, their application remains limited to structure borne sound [7, 8] or requires the computational intensive export of all surface node’s velocities in the time-domain for every interface force and every time step [9]. A
computationally efficient extension to airborne sound in the frequency domain therefore is necessary to deduce optimization measures for a system’s NVH behavior.

2.2. Calculating airborne sound pressure levels

The toolchain for calculating airborne sound levels is depicted in figure 2. [10]

Based on modeling the components in a finite element (FE) environment, a modal reduction [11] is carried out which allows for the flexible modeling of these components in an eMBS environment. After defining the interaction between these components (in form of force elements like bearings and gear mesh behavior as well as boundary conditions like external torque and rotational speed) a time-domain analysis of the entire system is carried out. The system model has to be solved in the time domain and cannot be represented in the frequency domain as nonlinearities (e.g. nonlinear bearing stiffnesses and clearances, gear mesh excitations and load depending gear mesh stiffnesses) have to be considered. The time-dependent deformation of the bodies (in form of elongation of modal states) is subsequently exported. Together with the recovery matrix of the flexible body, which links the modal states to nodal DOFs for a selected node set on the radiating surface, the time-dependent surface velocities can be calculated [10]. To calculate the sound pressure level at a receiver point, an acoustic transfer vector (ATV) matrix needs to be generated, which links the surface node velocities to the sound pressure levels at the receiver point [12]. This is done in an FE-environment in the frequency domain by modeling the surrounding air as a fluid and exciting this fluid individually at every surface node in all translational directions. By using a fast Fourier transform (FFT) to transform the surface into the frequency domain and multiplying these with the ATV matrix, the sound pressure level at the receiver point is obtained.

3. Problem formulation and solution approach

The existing approach does not allow for the tracking of transfer paths from the excitation at the interface points to the response at the receiver point, as the nodal surface velocities are exported as time-domain values and only represent the response of the entire system, see figure 2. Therefore, information about the transfer paths is not included in the radiation calculation. For a TPA to be carried out, both the time-domain values of the interface forces and the transfer paths from the interfaces to the receiver points (formulated in a frequency domain) are necessary. As the interface points are within the eMBS model and as the transfer paths from the interface points to the surface velocities are not available in the frequency domain in current radiation calculations, a TPA cannot be executed based on the toolchain of figure 2. Therefore, the design engineer can extract no knowledge about transfer paths and optimization methods out of the airborne sound signal. Hence, a new modeling approach is suggested, see figure 3.
Figure 3. Proposed method and toolchain for performing TPA on airborne sound

The general concept of the proposed method is to set up a model structure compliant to the TPA formulation. This requires the interface forces of the assembled model to be exported as time-domain values and the passive system (containing both the structural dynamic model and the radiation calculation) to be modeled in the frequency domain. By combining the passive system models’ frequency responses (structural dynamic and radiation), a TPA can be subsequently carried out. In detail, this leads to the following adaptions to the original toolchain for airborne sound (see figure 2):

First of all, instead of using the surface velocities as a result of the system model (marked in blue) in time-domain, the time-domain values of the interface forces are used as this is the necessary input for the calculation of the TPA. By doing so, the discontinuity between the different simulation models is shifted to the interface points to match the model structure required for a TPA.

In order to model the transfer from the interface points to the receiver point (i.e. for the modeling of the passive system), two separate models are necessary, both of which are formulated in the frequency domain. The radiation is still calculated using the ATV-matrices (marked in green). However, to link the interface forces to the surface velocities, an additional model is required (marked in magenta). This model is the structural dynamics model of the passive system. It is excited at the interface points and yields the frequency response functions from the interface forces to the surface velocities. To minimize the data output, the modal states are exported and using the recovery matrix of the passive subsystem the full transfer functions are calculated. The required frequency response matrix of the entire passive system (consisting of the structural passive system and the radiation calculation) is eventually calculated by multiplying the FRF from the interface forces to the surface nodes’ velocities $Y_{32, \text{struct}}^B$ with the ATV matrix $Y_{32, \text{rad}}^B$ according to equation (2).

$$Y_{32}^B = Y_{32, \text{struct}}^B \cdot Y_{32, \text{rad}}^B$$  \hspace{1cm} (2)
4. Application to a power take-off gearbox

The proposed method is applied to a PTO gearbox, depicted in figure 4. It consists of an input shaft, an auxiliary PTO shaft, an intermediate shaft (also allowing a PTO), an output shaft and the surrounding housing. Between these shafts, gear stages transmit the power. A receiver point is defined at the rear side of the housing, at a distance of 0.5 m from the surface.

In order to perform a TPA on this system, the boundaries between active and passive subsystem have to be defined (see also figure 1). The active subsystem contains the shafts and the gear stages, see figure 5. The passive system contains the housing and the radiation model. The 8 bearings (two per shaft) form the interface points.

![Figure 4](image_url1)

**Figure 4.** Demonstrator for airborne sound TPA calculation: power take-off gearbox

![Figure 5](image_url2)

**Figure 5.** Subsystem definition of the power take-off gearbox
According to the method described in chapter 3, three different models are generated: First, an eMBS model of the entire mechanical system is created (highlighted in figure 3 in blue), which yields the time-domain values of the interface forces. The model is loaded with a constant torque and an rpm runup is conducted in the time domain. Bearing stiffnesses are taken into account by the Hertz’ian contact theory, bearing clearances are also considered. The gear mesh excitations are modeled based on the load-dependent Weber/Banaschek model [13]. Secondly, a model of the housing is created in an eMBS environment (highlighted in figure 3 in magenta). The housing is modeled linearly. The material properties of the cast iron are identified by fitting the model to a conducted experimental modal analysis. This model’s purpose is to yield the frequency domain transfer paths from the bearings to the radiating surface. Lastly, an FE model to calculate ATV matrices is created. The air is modeled as a linear fluid and the boundary nodes are copied to the housing model, where they are tied to the radiating surface. For reference, the modeling method according to figure 2 is also applied – the surface velocities are exported from the systems model and used to calculate a reference sound pressure level at the receiver point. Measurement data were available for the validation of the model.

The measured sound pressure level is compared to the calculated one (according to the modeling tool chain described in figure 2), see figure 6.

**Figure 6.** Comparison between measured (left) and simulated (right) sound pressure levels

First of all, the comparison in the waterfall diagram shows a good match in the frequency content. Both first gear stage orders (76th and 98.9th) can clearly be identified in both the simulation results and the measurement results. The measurement results additionally show pulse-width-modulation-orders, induced by the testbench, which are not considered in the simulation. Looking at the amplitudes of the waterfall diagrams it can be deduced that several resonance peaks are clearly identified. However, the simulation proved to be about 10% too stiff. The first resonance occurs at 1180 rpm in the measurement and at 1300 rpm in the simulation and is accompanied by sidebands modulating the exciting order of the second gear stage. Both peaks show similar amplitude values of around 100 dB. Also, the second dominant resonance of the second gear stage at 1600 rpm can be identified in the simulation, again at slightly higher rotational speeds of about 1650 rpm. The resonance peaks of the first gear stage order (76th) at 1300 rpm are also represented in the simulation. However, several additional peaks in the simulation can be found (e.g. at 2000 rpm in the second gear stage or 1800 rpm in the first gear stage) which seem to be significantly more damped in the real system. The well matching rpm band between 1000 rpm and 1500 rpm is used for further validation of the proposed method. The sum levels between simulation and measurement are compared in figure 7. It can clearly be seen that while the absolute values are differencing slightly, the three peaks at ca. 1100 rpm, 1300 rpm and 1500 rpm are all found in the simulation as well. The model therefore can be considered as qualitatively valid and will be used for the further analysis.
To validate the proposed method, a comparison between the sound pressure levels calculated according to figure 2 and the sound pressure levels created from superimposing all transfer paths is shown in figure 8. The reconstructed signal matches the original signal both in frequency content and in absolute amplitudes very well, so that it can be deduced, that no transfer paths have been missed or miscalculated.
Now, the transfer path contributions can be calculated by multiplying the ATV matrix with the FRF matrix of the structural dynamic passive system and with the force of each interface DOF. The path contributions are depicted in figure 9. Each row shows the aggregated path contributions of one bearing to the sound pressure level. It can clearly be seen that the two tapered bearings supporting the output shaft are the dominant factors for the sound pressure level in the rpm range of up to 1200 rpm, while at higher rpm values the intermediate shaft cylinder bearing becomes dominant. It is also evident, that the two bearings supporting the auxiliary PTO shaft are not contributing to the sound pressure level. This is to be expected, as no power is transmitted through this shaft in the chosen verification setup.

![Path contributions](image)

**Figure 9.** Path contributions of the eight bearings to the sound pressure level of the power take-off gearbox

4.1. **Target-oriented optimization – Strength of the proposed method**

The proposed method allows for the identification of critical transfer paths and therefore enables the design engineer to deduct optimization measures. If necessary, the identification of optimization measures can be assisted by the deformation shapes of the radiating surface in the critical operation points, which can directly be obtained from the eMBS model. In what follows, optimization measures will be derived for the power take-off gearbox and will be evaluated based on the effect on the airborne sound pressure level.

Based on the critical transfer paths identified in figure 9, the optimization measure has been derived to increase the mass of the housing at the bearing support of the output shaft by increasing the thickness of the support, see figure 10, left. This adds a mass of 2.5% to the housing’s original weight. The sum levels of the sound pressure level at the receiver point are shown in figure 10 on the right. While a significant reduction of 5 dB lower peak values can be achieved in the rpm range of up to 1200 rpm, only little optimization is to be seen at higher rpm values, as the output shaft bearings are not the critical paths at this point. Therefore, in a second optimization, additional mass was added to the cylinder bearing of the intermediate shaft as well. It can clearly be seen how especially at rotational speeds higher than 1200 rpm a significant decrease in peak values of 6 dB can be achieved.
This shows how with using TPA on numerical models a target-oriented optimization of individual peaks can be carried out, as transfer paths for every operational point can be identified and compared. By closing the gap of TPA towards airborne sound in the numerical domain, a direct evaluation with the most dominant NVH quantity at relevant receiver points as the driver’s ear can be carried out for the first time.

5. Summary and Outlook
In this work, a method has been proposed to use transfer path analysis (TPA) to identify critical transfer paths to airborne sound in an elastic multi body system simulation (eMBS). It incorporates a newly developed toolchain which overcomes the constraint of exporting time-domain velocities of the eMBS model, which doesn’t allow conclusion to the critical transfer path. Instead, an additional model of the passive structure in the frequency domain allows for the interface forces to be exported from the system model and the frequency response function of the entire passive system (consisting of the passive structure and the radiation properties) to be calculated. Thus, the transfer path contributions to the airborne sound can be obtained. This allows for the target-oriented optimization of the NVH behavior on a virtual prototype, thereby reducing development time and cost. The method is demonstrated on a power take-off (PTO) gear box and yields a 6 dB decrease in the sound pressure level.

Further work an experimental TPA needs to be done with regard to rotational degrees of freedom. Existing methods for deducing interface forces need to be extended to include the information about torques and moments acting on the interface.

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