Intrinsic-Correlation Quantum Key Generation

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Abstract

A new conceptual key generation scheme is presented by using intrinsic quantum correlations of single photons between Alice and Bob. The intrinsic bi-partite correlation functions allow key bit to be generated through high level communication language i.e. a key bit is directly encoded to shared correlation functions not to the state and detection of a photon at Bob does not mean key bit. These make the scheme robust against intercept-resend attack because Alice and Bob can always check the errors in their measurements and reveal the presence of Eve in their channel without leaking any key bit information. The scheme is strictly relied on the perfect beam splitter and mean photon number less than 1, where more than one photon in a coherent pulse will introduce more errors in Bob even without the presence of Eve. From the percentage of errors in Bob, we can estimate the amount of information will be leaked to Eve in the photon-number splitting attack. This scheme can preserve the randomness of phase-randomized light source for doubling the communication distance as in original Ekert’s protocol and providing the raw key generation rate a factor of 2 higher than weak coherent light protocols.
I. INTRODUCTION

The quantum mechanics without probability amplitude was proposed [1], leading to the possibility of quantum information processing more directly in terms of probability i.e discrete Wigner function [2]. In the same vein, it is exciting to explore quantum key generation [3, 4] with polarization correlation function or expectation value of two observers. In general, two observers $A$ and $B$ will have eigenvalues $\pm 1$ corresponding to photon detection at the transmitted or reflected port of their analyzers. The correlation function shared by them is $C(\theta_a, \theta_b) = \sum_{\pm 1}(AB)P(\theta_a, \theta_b)$, where $P(\theta_a, \theta_b)$ is the joint probability of detecting a photon in observer $A$ and $B$ with their analyzers at $\theta_a$ and $\theta_b$, respectively. The maximum/minimum values of $C(\theta_a, \theta_b) = \pm 1$ are corresponding to correlated/anti-correlated outcomes of $A$ and $B$ such that the $C = +1(A = +1, B = +1; A = -1, B = -1)$ and the $C = -1(A = +1, B = -1; A = -1, B = +1)$. There are totally four correlation functions $C_n = \pm \cos(\theta_a \pm \theta_b)$, where $n = 1, 2, 3, 4$, available from intrinsic polarization correlation of weak coherent states [5], coherent light mixed with random noise field [6] and coherent light fields [7, 8]. These four correlation functions have been demonstrated and used to perform bits correlations between two parties over a distance of 10 km, through a transmission fiber by using weak coherent states in Ref.5. One can predict the outcome of the other by guessing the $C_n$ through their projection angles and photon detection. The secure key bits between two observers $A$ and $B$ are then generated through sharing four bi-partite correlation functions and photon counting.

The essence of the paper is to propose a new scheme for key generation based on single photons bi-partite correlation functions. The proposed experiment setting is similar to simplified EPR protocol (BBM92) [9], where the EPR pairs are replaced with two weak coherent states prepared from a light source in the middle of Alice and Bob. The light source is two modes ($x, y$) weak coherent states combined through a beam splitter producing two spatially separated quantum channels. We have the product states at the input of the beam splitter such that $|\alpha\rangle_x(\pm i|\beta\rangle_y) = e^{-\frac{n_x}{2}} e^{-\frac{n_y}{2}} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{n^n m^m}{\sqrt{n!} \sqrt{m!}} |n, m\rangle \propto |0\rangle_x |0\rangle_y \pm i\sqrt{n_x} |1\rangle_x |0\rangle_y \pm i\sqrt{n_y} |0\rangle_x |1\rangle_y + \ldots$, where $\pm i$ is provided by random phase modulator $\pm \frac{\pi}{2}$ along the y-direction and $n_x, n_y \leq 1$. The density matrix of the output state from the beam splitter is dependant on the integration over the phase space of $P$ representation for the input product state. Since the $|\alpha\rangle_x$ and $|\beta\rangle_y$ are intrinsically correlated from the same laser,
the integration over the phase spaces of $|\alpha_x\rangle$ and $|\beta_y\rangle$ can produce the output state that is intrinsically entangled [10]. One can see that, the beam splitter transforms the input state $|\alpha_x(\pm i|\beta_y\rangle)\rangle$ into inseparable output state $\left(\frac{|\alpha_x+i\epsilon\beta_y\rangle}{\sqrt{2}}\right)_{1,2} (\frac{|\alpha_x+i\epsilon\beta_y\rangle}{\sqrt{2}})$, where $\phi_m = \pm 90^\circ$ is random modulated phase. The inseparable output state can provide four types of bi-partite correlation functions ($C_{1,2,3,4}$) by manipulating the linear phase shifters in Box 1 of Alice. The security of this scheme is guaranteed by the fact that the nonorthogonality of the product coherent states, which cannot be identified by a single measurement. The protocol is strictly depended on equal mean photon number in the mode $(x, y)$ i.e. $n_x = n_y = n_c < 1$. We will discuss how this condition can restrict more than one photon in a coherent pulse in this protocol and hence to prevent the photon-number splitting attack. The single photons sent to Alice and Bob are called ancilla photon and signal photon, respectively. The scheme requires coincidence detection of ancilla photon at Alice and signal photon at Bob. The ancilla photon passes through/reflects from an analyzer at Alice is registered as bit ’1’/’0’. The signal photon passes through/reflects from an analyzer at Bob is denoted as ’yes’/’no’ of his guess on Alice’s key bit. ’Yes’ did not mean that Bob will have bit ’1’. It can be bit ’0’. Alice and Bob generate key bits through secure communication in the sense that the bit information is encoded on the $\{C_n\}$ not the state. We will discuss how the protocol is robust against intercept-resend attack. The security analysis of this scheme may well be different from current weak light protocols [11–16].

In this paper, we first outline the sequence of steps for implementing key generation between Alice and Bob. Then, we discuss the physics and experimental detail of each step. Finally, we briefly discuss how the protocol can prevent the photon-number splitting and intercept-resend attacks.

II. THE IC-QKD PROTOCOL

The proposed experiment setup is shown in Fig.1. The setting is similar to the BBM92 protocol, where the EPR pairs are replaced with two mode and phase-modulated weak coherent states, which are intrinsic-correlated from a laser source in the middle of Alice and Bob. The two weak coherent states are distributed through two pulses within a bit period, which is phase modulated in an asymmetric Mach-Zehnder interferometer (MZI).

Four bi-partite correlation functions ($C_n, n = 1, 2, 3, 4$) are assigned into two groups
FIG. 1. The proposed experiment setup for implementing key generation using a signal photon at Bob and an ancilla photon at Alice. The laser source is located in the middle of Alice and Bob, and secured by a trusted third party. The two weak coherent states are distributed through two pulses within a bit period. The dotted boxes (Box 1 and Box 2) are the wave plates used for preparing bi-partite correlation between Alice and Bob. HWP: Half-Wave Plate, QWP: Quarter-Wave Plate, SPD: Single Photon Detector, PBS: Polarization Beam Splitter, BS: Beam Splitter.

\((\Psi, \Phi)\), where \(C_{1,2} \to \Psi\) and \(C_{3,4} \to \Phi\). Alice will have the key bit '0' and '1' for the correlation functions \(C_{1,3}\) and \(C_{2,4}\), respectively. Alice can randomly choose the correlation function \(\{C_n\}\) and then her key bits. Bob guesses the correlation function based on his polarization angle of his analyzer and records the outcome of his measurement i.e. 'yes' or 'no' of his guess. After the signal transmission, Alice then tells Bob through classical channel about the sequence of groups \(\{\Psi, \Phi\}\) that she has randomly chosen. Bob can then know Alice’s key bit by the outcome of his measurement regardless he made a right or wrong guess, so doubling the raw key generation rate.

To illustrate the scheme more systematically, we will discuss an example of the key generation as shown in Fig.2.

**Step − 1:** Alice randomly chooses the \(\Psi\{C_1, C_2\}\) or \(\Phi\{C_3, C_4\}\) by choosing the waveplates combination in Box 1 as shown in Fig.1. (Alice randomly chooses the \(C_n\) is analog to choosing a random basis in BB84.)
### FIG. 2. The scheme for key generation between Alice and Bob.

**Step − 2**: Alice fixes her polarizer at angle \( \theta_1 = +45^\circ \) and records the valid detections of the ancilla photon in '+' SPD3 and '-' SPD4 as bit '1' or '0'.

**Step − 3**: Bob guesses on the \( \mathcal{C}_n \) by projecting his polarizer at angles \( \theta_2 = \pm 45^\circ \). The \( \theta_2 = +45^\circ \) means Bob’s guess on \( \{\mathcal{C}_1, \mathcal{C}_4\} \). The \( \theta_2 = -45^\circ \) means Bob’s guess on \( \{\mathcal{C}_2, \mathcal{C}_3\} \).

(Alice only chooses one polarization angle \( \theta_1 = +45^\circ \) and Bob chooses two polarization angles \( \theta_2 = \pm 45^\circ \) because of the shared \( \mathcal{C}_n \). We do not need to perform the violation of Bell inequality in Ekert’s protocol. And also, not like BBM92, Alice and Bob do not need to measure the basis \((0^\circ, 90^\circ)\) of the incoming photon.)

**Step − 4**: The outcomes of Bob’s guess through the valid detections of signal photon. 'Yes' means that the '+' SPD1 'click' or his guess is right. 'No' means that the '-' SPD2 'click' or his guess is wrong. However, Bob did not know the Alice’s key bit yet.

(Alternative: Alice and Bob can detect the presence of Eve by broadcasting part of the results obtained by Bob in step 3 and 4. They did not leak any bit information to Eve because Alice will not announces the group information in step 5 for the exposed data. Note that in BB84, Alice and Bob announce their choice of bases, and Bob never broadcast his result which is the key bit.)

**Step − 5**: After the signal transmission, Alice announces to Bob through classical channel about the group of her choice \( (\Phi, \Psi) \), not revealing her choice of correlation function.

**Step − 6**: Bob knows Alice’s key bit after he received the group information and verified with the record of his valid detection. To illustrate this, let’s say for the first column,

|   | \( \Phi(\mathcal{C}_3, \mathcal{C}_4) \) | \( \Psi(\mathcal{C}_1, \mathcal{C}_2) \) | \( \Phi(\mathcal{C}_3, \mathcal{C}_4) \) | \( \Psi(\mathcal{C}_1, \mathcal{C}_2) \) |
|---|---|---|---|---|
| 1 |   |   |   |   |
| 2 | 0 | 0 | 1 | 1 |
| 3 | +45° (\( \mathcal{C}_1, \mathcal{C}_4 \)) | +45° (\( \mathcal{C}_1, \mathcal{C}_4 \)) | -45° (\( \mathcal{C}_2, \mathcal{C}_3 \)) | -45° (\( \mathcal{C}_2, \mathcal{C}_3 \)) |
| 4 | No or '-' | Yes or '+' | No or '-' | Yes or '+' |
| 5 | \( \Phi \) | \( \Psi \) | \( \Phi \) | \( \Psi \) |
| 6 | 0 | 0 | 1 | 1 |
Alice announces to Bob that the group is \( \Phi \). Bob’s guess on the group \( \Phi \) is \( C_4 \), which is corresponding to bit ‘1’. However, he knows that the guess is wrong through his valid detection in step 4. From here, he knows Alice’s key bit is bit ‘0’.

(The random choices of \( C_n \) in Alice and projection angle \( \theta_2 = \pm 45^\circ \) in Bob did not need to discard half of the raw keys compared to BB84 where Alice and Bob have the wrong bases.)

We believe that this scheme can provide the raw key generation rate a factor of 2 higher than the BB84 [17] and B92 [18], and double the communication distance as in the simplified EPR (BBM92) [9] and the original Ekert’s protocol [19]. We would outline the detail explanation of each step.

A. Step 1, 2: Prepare and Measure

The light source is located in the middle of Alice and Bob. A pulsed, 45\(^\circ\)-polarized laser light with a polarizing beam splitter (PBS) is used to provide a weak coherent state \( |\alpha\rangle_x \) with horizontal polarization and another weak coherent state \( |\beta\rangle_y \) with vertical polarization in an asymmetric Mach-Zehnder interferometer (MZI). These coherent states are located at two time slots and combined through a perfect 50/50 beam splitter (BS1), producing two spatially separated beams, i.e., beam 1 and beam 2. The input state of the beam splitter is a product state of two mode weak coherent states \( |\alpha\rangle_x |\beta\rangle_y \), which is the prepared state for this protocol. The annihilation operators \( \hat{a} \) and \( \hat{b} \) are the input field operators for the beam splitter (BS1) and for the coherent states \( |\alpha\rangle_x \) and \( |\beta\rangle_y \). The operator \( \hat{b} \) is phase-modulated with the phase \( \pm \phi_a \) as \( \hat{b} e^{\pm i\phi_a} \), where \( \phi_a = \phi_m + \phi_A \), and \( \phi_m = \pm 90^\circ \) is randomly chosen. The \( \phi_a \) is produced by a phase modulator at the short arm of the MZI. The annihilation operators \( \hat{c} = \frac{1}{\sqrt{2}}(\hat{a}\hat{x} + i\hat{b}\hat{y}) \) and \( \hat{d} = \frac{1}{\sqrt{2}}(i\hat{a}\hat{x} + \hat{b}\hat{y}) \) are the output field operators for the BS1, which are sent through beam 2 to Bob and beam 1 to Alice, respectively. To distribute the operators \( \hat{d} \) and \( \hat{c} \) to Alice and Bob, we first place a polarizer at 45\(^\circ\) at each output of the BS1 so that two pulses are co-polarized within a bit period. The reason is two orthogonal polarized pulses may be vulnerable to Eve attacks. To create single photon quantum channel between the source and Bob, the beam 2 is attenuated to single photon level with the mean photon number per pulse \( (n_x, n_y) \leq 1 \). Similarly, the beam 1 is further attenuated to single photon level with mean photon number per pulse \( (n_x, n_y) \leq 1 \). The co-polarized two pulses are sent
through transmission fibers to Alice’s and Bob’s MZI, where the phase demodulators at the short arms induced a phase of $\theta_B$. Four pulses (2 in the x-mode and 2 in the y-mode) are created within one bit period. The x-y mode ($e^{i(\pm 90^\circ + \phi_A) }, e^{i\phi_B}$) in the middle time slot are used for key generation. The phase ($\phi_A - \phi_B = 0$) can be accomplished by making use of the phase coding $\alpha \eta$ system [20,23], which is proven to be more efficient secure encryption system over long distances. Note that we apply phase coding $\alpha \eta$ system on a single photon level instead of mesoscopic coherent state.

The random phase $\phi_m = \pm 90^\circ$ is unknown to Bob and Alice. With the assumption of low mean photon numbers for $n_x$ and $n_y$, the two weak coherent states can be treated as $|\tilde{\alpha}_x, \tilde{\beta}_{y}\rangle = e^{\frac{i}{2}(n_x+n_y)[|0\rangle_x|0\rangle_y + \sqrt{n_x} |1\rangle_x|0\rangle_y + \sqrt{n_y} |0\rangle_x|1\rangle_y,...]}$, where we have considered the vacuum and one photon number state in mode x or y. The phase $\phi_m = \pm 90^\circ$ is not included here because we have treated the $\phi_m$ in the annihilation operator $\hat{b}e^{i(\pm \phi_m + \phi_A)}$. Four types of intrinsic bi-partite correlation functions $C_{1,2} = \{-\cos(\theta_1 - \theta_2), +\cos(\theta_1 + \theta_2)\}$ and $C_{3,4} = \{-\cos(\theta_1 + \theta_2), +\cos(\theta_1 - \theta_2)\}$ can be established through the combination of wave plates as shown in the dotted boxes (Box 1 and Box 2) in Fig.1. The half-wave plates (HWP3 and HWP5) before the polarizing beam splitters (PBS1 and PBS2) at Bob and Alice are used for projecting polarization angles $\theta_1$ and $\theta_2$, respectively, so that the maximum correlation $C_{1,2,3,4} = \pm 1$ is obtained.

The annihilation operator $\hat{d}$ at beam 1 is transformed through the combination of wave-plates in Box 1 and a polarization analyzer consists of a half waveplate (HWP5) and a polarizing beam splitter (PBS2). Alice can randomly choose the $C_n$, $n=1,2,3,4$ through the settings of QWP2 and HWP1 as shown in the Box 1. For the correlation functions $C_{1,2,3,4}$, the photon number operator $\hat{d}^\dagger \hat{d}$ at Alice is given by,

$$\hat{d}^\dagger \hat{d} = \frac{1}{2} [\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b} \pm \hat{a}^\dagger \hat{b} e^{\pm i \phi_m} + \text{c.c}]. \quad (1)$$

The expectation values for the correlation $\{ C_{1,2} \}$ are given by,

$$C_1 \rightarrow \langle \tilde{\alpha} \tilde{\beta} | \hat{d}^\dagger \hat{d} | \tilde{\alpha} \tilde{\beta} \rangle \propto [n_x + n_y - 2\sqrt{n_xn_y}\cos(2\theta_1 + \phi_m)]$$

$$C_2 \rightarrow \langle \tilde{\alpha} \tilde{\beta} | \hat{d}^\dagger \hat{d} | \tilde{\alpha} \tilde{\beta} \rangle \propto [n_x + n_y + 2\sqrt{n_xn_y}\cos(2\theta_1 - \phi_m)], \quad (2)$$

respectively. While the expectation values for the correlation $\{ C_{3,4} \}$ are given by,

$$C_3 \rightarrow \langle \tilde{\alpha} \tilde{\beta} | \hat{d}^\dagger \hat{d} | \tilde{\alpha} \tilde{\beta} \rangle \propto [n_x + n_y - 2\sqrt{n_xn_y}\cos(2\theta_1 - \phi_m)]$$

$$C_4 \rightarrow \langle \tilde{\alpha} \tilde{\beta} | \hat{d}^\dagger \hat{d} | \tilde{\alpha} \tilde{\beta} \rangle \propto [n_x + n_y + 2\sqrt{n_xn_y}\cos(2\theta_1 + \phi_m)], \quad (3)$$
respectively. We have neglected the $\frac{1}{2}e^{-(n_x+n_y)}$ and higher photon number states for the simplicity of the following discussions. Note that the condition $n_x = n_y$ (perfect beam splitter) has made the higher photon number states having the same form of interference terms. Alice will observe the interference term of $-\cos(2\theta_1 + \phi_m)$ for the $C_1$, $+\cos(2\theta_1 - \phi_m)$ for the $C_2$, $-\cos(2\theta_1 - \phi_m)$ for the $C_3$, and $+\cos(2\theta_1 + \phi_m)$ for the $C_4$. For example, if Alice chooses $C_1$, she has the interference term of $-\cos(2\theta_1 + \phi_m)$. For the mean photon number per pulse $n_x = n_y = n_c \leq 1.0$. The expectation value $\langle \hat{d}^\dagger \hat{d} \rangle$ will have the maximum value of $2n_ce^{-2n_c}$ and minimum value of 0. The maximum value of $\langle \hat{d}^\dagger \hat{d} \rangle$ is corresponding to the interference term with value +1 and the minimum value (0) of $\langle \hat{d}^\dagger \hat{d} \rangle$ is corresponding to the interference term with value -1. In other words, the interference term with value +1 indicates that the ancilla photon passed through a PBS2. While the interference term with value -1 indicates that the ancilla photon reflects from a PBS2. The interference term with the value +1 (-1) is corresponding to the detection of the ancilla photon at Alice’s detector '+'SPD3 ('-'SPD4) or bit '1' (bit '0'). Note that we are not going to multiply Alice’s and Bob’s interference term and measure the mean value of the multiplied interference intensity as previously demonstrated in ref [5], i.e., $C_{1,2,3,4} = \pm \cos(\theta_1 \pm \theta_2)$, where the phase term $\phi_m$ is varied from $0^\circ \rightarrow 2\pi$.

**B. Step 3, 4: Guess and Verify**

The annihilation operator $\hat{c}$ at beam 2 is transformed through a quarter waveplate (QWP4) at 45° and a polarization analyzer (HWP3 and PBS1). The photon number operator at Bob is given by,

$$\hat{c}^\dagger \hat{c} = \frac{1}{2} [\hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b} + \hat{a}^\dagger \hat{b} e^{i2\theta_2 + i\phi_m} + \hat{b}^\dagger \hat{a} e^{-i2\theta_2 - i\phi_m}], \quad (4)$$

and then the expectation value is given by

$$\langle \hat{a} \hat{b} | \hat{c}^\dagger \hat{c} | \hat{a} \hat{b} \rangle \propto [n_x + n_y + 2\sqrt{n_xn_y}\cos(2\theta_2 + \phi_m)]. \quad (5)$$

Bob has the same interference term $\cos(2\theta_2 + \phi_m)$ for all the $C_n$ because the QWP4 is projected at the fixed angle +45° in Box 2. At Bob, if the single photon detector (SPD) at '+' or '-' port detects the signal photon, he then assigned the valid detection as 'Yes' or 'No' for his guesses on $C_n, n = 1, 2, 3, 4$. 
In this scheme, the $\theta_1 = +45^\circ$ at Alice is fixed. Let’s discuss the ideal situation where Alice chooses the $\{C_n\}$ with the right $\phi_m = \pm 90^\circ$ and Bob guesses the right $\{C_n\}$ with his projection angle $\theta_2$. For example, if Alice chooses $\{C_1; \theta_1 = +45^\circ\}$ with the $\phi_m = -90^\circ$, the interference term in Alice has the value -1 indicating her '-' SPD4 will click. She records the detection as bit '0'. While interference term in Bob with his projection angle $\theta_2 = +45^\circ$ has the value +1 indicating his '+' SPD1 will 'click'('yes'). This means that his guess on $C_1$ is right. If Alice chooses $\{C_2, \theta_1 = +45^\circ\}$ with the $\phi_m = +90^\circ$, her interference term indicates that her '+' SPD3 will 'click' or bit '1' is registered. The interference term in Bob with $\theta_2 = -45^\circ$ has the value +1 indicating his '+' SPD1 will 'click'('yes'). This means that his guess on $C_1$ is right. If Alice chooses $\{C_2, \theta_1 = +45^\circ\}$ with the $\phi_m = +90^\circ$, her interference term indicates that her '+' SPD3 will 'click' or bit '1' is registered. The interference term in Bob with $\theta_2 = -45^\circ$ has the value +1 indicating his '+' SPD1 will 'click'('yes'). This means that his guess on $C_1$ is right. Similarly, if Alice chooses $\{C_3, \theta_1 = +45^\circ\}$ with the $\phi_m = +90^\circ$ and $\{C_4, \theta_1 = +45^\circ\}$ with the $\phi_m = -90^\circ$, she will register bit '0' and '1', respectively. Bob will have the right guess on $C_{3,4}$ with his projection angles $\theta_2 = +45^\circ, -45^\circ$, respectively.

We formulated the above discussion by assigning the $C_n$ into two groups i.e. $C_{1,2} \rightarrow \Psi$ and $C_{3,4} \rightarrow \Phi$. Then, we can denote the groups $\{\Psi, \Phi\}$ as a function of the $C_n$, projection angle $\theta$, and random phase $\phi_m = \pm 90^\circ$ for Alice and Bob to generate the shared key bits. If Alice and Bob share the group $\Psi\{C_1, \theta_1 = +45^\circ, \theta_2 = +45^\circ, \phi_m = -90^\circ\}$ which is corresponding to bit '0', then the '-' SPD4 in Alice will detect the ancilla photon and Bob’s '+' SPD1 will detect the signal photon meaning 'yes'. Similarly, for $\Psi\{C_2, \theta_1 = +45^\circ, \theta_2 = -45^\circ, \phi_m = +90^\circ\} \rightarrow$ bit'1', $\Phi\{C_3, \theta_1 = +45^\circ, \theta_2 = +45^\circ, \phi_m = -90^\circ\} \rightarrow$ bit'0', $\Phi\{C_4, \theta_1 = +45^\circ, \theta_2 = -45^\circ, \phi_m = -90^\circ\} \rightarrow$ bit'1'.

Since each group $\{\Psi, \Phi\}$ has bit '0' and '1' associated with its correlation function $C_n$ and depends on random phase $\phi_m = \pm 90^\circ$, then how Alice knows her choice of $C_n$ and valid detection in her detectors can generate key bit in agreement with Bob’s guess on $C_n$ through his projection angle and valid detection in his detectors. The beauty of this scheme is Alice and Bob can generate the shared key bits through the shared correlation functions without requiring any information about the random phase $\phi_m = \pm 90^\circ$ in the source located in the middle between them. To understand this more clearly, let’s consider the case where the phase $\phi_m = -90^\circ$ in the source. Alice chooses the $\Psi\{C_1, \theta_1 = +45^\circ, \phi_m = -90^\circ\} \rightarrow$ bit'0', Alice has the interference term $-\cos(2\theta_1 + \phi_m) \rightarrow -\cos(2(45^\circ) - 90^\circ) \rightarrow -1$ or '-' SPD4 will click registering bit '0'. Bob has no idea about what Alice’s choice on the $\{C_n\}$ and the phase $\phi_m = -90^\circ$. Bob has the interference term $\cos(2\theta_2 + \phi_m)$. Bob can only project the HWP3 to $\theta_2 = \pm 45^\circ$. Bob chooses the $\theta_2 = +45^\circ(\theta_2 = -45^\circ)$ for his guess on $\Psi\{C_1\} \rightarrow$ bit'0'.
or $\Phi\{C_1\} \rightarrow \text{bit}'1'$ ($\Psi\{C_2\} \rightarrow \text{bit}'1'$ or $\Phi\{C_3\} \rightarrow \text{bit}'0'$). Bob knows his guess is right or wrong through the valid detection in his detector '+' SPD1 or '-' SPD2, respectively. Bob did not know the key bit until Alice announces which group $\{\Psi, \Phi\}$ in Step 5. Let’s say, Bob chooses $\theta_2 = +45^\circ$, the interference term $\cos(2(+45^\circ) - 90^\circ) \rightarrow +1$ meaning his '+' SPD1 will 'click'('yes') or his guess on $\Psi\{C_1\} \rightarrow 0'$ or $\Phi\{C_4\} \rightarrow '1'$ is right. After the signal transmission, Alice announces the group $\Psi$ of her choice (Step 5). Bob knows the $C_1$ is right. Then, Alice and Bob share the key bit '0' (Step 6). If Bob chooses $\theta_2 = -45^\circ$, the interference term $\cos(2(-45^\circ) - 90^\circ) \rightarrow -1$ meaning his '-' SPD2 will 'click' or his guess on $\Psi\{C_2\} \rightarrow '1'$ or $\Phi\{C_3\} \rightarrow 0'$ is wrong. After the signal transmission, Alice announces the group $\Psi$ of her choice (Step 5). Bob knew that his guess on $\Psi\{C_2\}$ or bit '1' is wrong. Bob knows the bit is '0', then Alice and Bob will share the key bit '0' (Step 6). Bob knows the key bits of Alice regardless his guess is right or wrong. The above scenario is Alice’s choice of $C_1$ with right phase $\phi_m = -90^\circ$. The most intriguing part of this scheme is Alice knows her key bit regardless her choice of $C_n$ in agreement with the phase $\phi_m = \pm 90^\circ$. To discuss this, let’s consider the case where the phase $\phi_m = +90^\circ$, Alice still chooses the $\Psi\{C_1, \theta_1 = +45^\circ, \phi_m = -90^\circ\}$ or bit '0', the interference term in Alice $-\cos(2(45^\circ) + 90^\circ) \rightarrow +1$ meaning that the '+' SPD3 will 'click' or bit '1'. Alice knew then her choice of $C_1$ is wrong. She also knew that the phase $\phi_m = +90^\circ$ was meant for the $\Psi\{C_2, \theta_1 = +45^\circ, \phi_m = +90^\circ\} \rightarrow \text{bit}'1'$, so she records the bit '1' as registered by her valid detection. Amazingly, Bob will have the bit '1'. As discussed above, if Bob chooses $\theta_2 = +45^\circ$, the interference term $+\cos(2(+45^\circ) + 90^\circ) \rightarrow -1$ meaning that his '-' SPD2 will 'click' or 'No' on his guess. After Alice announces the group $\Psi$, Bob knew that his guess on $C_1$ or bit '0' is wrong, so he knows that Alice has bit '1'. The similar description is applied for the other $\Psi\{C_2\}$, $\Phi\{C_3\}$, and $\Phi\{C_4\}$.

III. DISCUSSION

Weak coherent state is usually vulnerable to photon-number splitting attack i.e Eve can tap one signal photon from beam 2 and perform the same measurement as Bob did. In this protocol, we make use of the phase coding $\alpha\eta$ system [20][23] to secure beam 1 and beam 2 from Eve attacks. One can also use differential phase shifted method to distribute operators $\hat{c}$ and $\hat{d}$ at the output of the BS1. The protocol can prevent more than one
photon in a coherent pulse by applying the condition $n_x = n_y = n_c < 1$. For example, 2 photons in beam 2 can be attributed to higher photon number in (x, y) mode such that $|1\rangle_x|1\rangle_y, |0\rangle_x|2\rangle_y, |2\rangle_x|0\rangle_y$, which will have the expectation value of $\langle \hat{c}'\hat{c}'\rangle$ as given by

$$\propto (n_x^2 + n_y^2 + 2n_xn_y + 2n_x\sqrt{n_xn_y}\cos(2\theta_1 + \phi_m + 2n_x\sqrt{n_xn_y}\cos(2\theta_1 + \phi_m)).$$

It is more sensitive to the condition $n_x = n_y$ or if it will introduce more errors on Bob if the condition $n_x = n_y$ is not fulfilled, even though its contribution is in the order of mean photon number $n_c$ less than the one photon number case ($|1\rangle_x|0\rangle_y$ and $|0\rangle_x|1\rangle_y$). In other words, the existence of multiple photons will introduce the percentage of errors in Bob even without the presence of Eve. The percentage will also be the amount information leaking to Eve in the photon-number splitting attack.

In intercept-resend attacks, Eve knows that the bit information is not encoded in the state. He can perform the same measurement as Bob did. Even though we assume that Eve break the security of the $\alpha\eta$, Eve cannot resend a perfect copy of the original signal photon because the input product of two mode weak coherent states is a nonorthogonal state. For example, the input product state $|\alpha\rangle_x|\beta\rangle_y$ of the beam splitter (BS1) will have the output product state $(|\alpha\rangle_x + e^{i\phi_m}|\beta\rangle_y)\sqrt{2}(|\alpha\rangle_x + e^{i\phi_m}|\beta\rangle_y)\sqrt{2}$, where the state $(|\alpha\rangle_x + e^{i\phi_m}|\beta\rangle_y)\sqrt{2}$ is sent to Bob. The non-orthogonality of coherent states and the $\phi_m = \pm 90^\circ$ cannot be measured in a single measurement. Bob will reveal the present of Eve by sending part of his observations (Step 3 and 4) to Alice. Let’s say in the second column in Fig.2, Alice knows that Bob is supposed to observe 'Yes' with his guess through his projected angle $\theta_2 = +45^\circ$. Because of the imperfect copy from Eve, Bob might observe 'No' with the $\theta_2 = +45^\circ$, so Alice will aware of the error and stop the transmission. There is no leakage of information to Eve because Alice has not implemented Step 5 yet.

IV. CONCLUSION

In conclusion, we have presented unique feature of a new scheme based on intrinsic quantum correlation of weak coherent states for generating key bits between Alice and Bob through high level communication language. The scheme preserve the randomness of the light source for doubling the communication distance, and also increase the success rate of raw key generation compared to Ekert’s protocol, and double the sift key rate compared to
weak coherent light protocol.

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