On the gravitational stability of D1-D5-P black holes

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We examine the stability of the nonextremal D1-D5-P black hole solutions. In particular, we look for the appearance of a superradiant instability for the spinning black holes but we find no evidence of such an instability. We compare this situation with that for the smooth soliton geometries, which were recently observed to suffer from an ergoregion instability, and consider the implications for the fuzzball proposal.

I. INTRODUCTION

Despite being well-understood classically, black holes still pose a number of unanswered questions at the quantum level, such as the information paradox. A radical new approach to describe stringy black holes, now known as the ‘fuzzball’ proposal, was first suggested some years ago by Mathur and collaborators [1]. They advocated that the microstates underlying a black hole are individually described by horizon-free geometries and that the black hole geometry only emerges in a coarse-grained description which ‘averages’ over the $e^{S_{BH}}$ microstate geometries. In this approach, the effective horizon of a black hole appears as a surface at a radius where the individual microstate geometries start to ‘differ appreciably’ from one another. Therefore quantum gravity effects are not confined close to the black hole singularity but rather the entire interior of the black hole is ‘filled’ by fluctuating geometries – hence the nomenclature: the ‘fuzzball’ description of black holes.

Finding evidence to support this conjecture is not easy though, especially because it entails finding families of horizon-free geometries sufficiently extensive to describe all of the $e^{S_{BH}}$ microstates for a given black hole. Most of the studies to date focus on supersymmetric configurations, namely the BPS D1-D5 system [2], the BPS D1-D5-P system [3], and the BPS D1-D5-P-KK system [4]. However, if the fuzzball proposal is to be useful, it must also be extended to non-supersymmetric systems. This then poses the extremely difficult problem of finding non-supersymmetric (smooth) horizon-free geometries that can be associated with the microstates of a non-BPS black hole. So far, the only known solutions in this class are those of Jejjala, Madden, Ross and Titchener [6], hereafter referred to as JMaRT solitons. The JMaRT solutions comprise a five-parameter family of D1-D5-P non-supersymmetric smooth geometries which are asymptotically flat. These solutions may be parameterized by the D1-brane and D5-brane charges, the (asymptotic) radius of the internal circle with Kaluza-Klein momentum, and by two integers $m$ and $n$ which fix the remaining physical parameters. These integers also determine a spectral flow in the CFT which allows the underlying microstate to be identified. For $m = n + 1$, the JMaRT solitons reduce to supersymmetric solutions found previously in [2, 3]. The geometry of these solutions was also recently examined in more detail in [7].

In a previous paper [8], we have shown that the non-supersymmetric JMaRT solitons are classically unstable against an ergoregion instability [9]. This kind of instability is generic to geometries with an ergoregion but no horizon, as was first noticed in [9]. Thus this instability should be a robust feature of any smooth horizon-free geometry corresponding to a non-BPS microstate with angular momentum. At first sight then, this seems to pose a challenge for the fuzzball proposal: if horizon-free non-supersymmetric solitons are expected to describe the microstates of non-BPS black
holes, then the nonextremal D1-D5-P black hole which results from averaging over an ensemble of such solutions should be unstable against an analogous instability. The purpose of the present work is to address this issue. While the presence of an event horizon eliminates the possibility of an ergoregion instability, there is, however, an obvious candidate instability in the black hole case: the superradiant instability [8, 10, 11, 12].

In general, spinning nonextremal black holes will exhibit superradiant scattering, whereby an incident wave packet can be reflected with a stronger amplitude. Superradiance by itself does not provide a classical instability, but an instability can arise if the waves are reflected back and forth. Stated in other words, a superradiant instability is present if there are bound states subjected to superradiance. Some examples of black hole systems unstable against such mechanism are: i) the black hole bomb where an artificial mirror surrounds a Kerr black hole [13]; ii) a massive scalar field in a Kerr background [14]; iii) small Kerr-AdS black holes [15]; iv) rotating black strings [10, 11, 12]. This instability seems to be the natural extension, to black holes, of the ergoregion instability found in [8]. Therefore, according to the fuzzball proposal, one might expect that the black hole family of D1-D5-P non-supersymmetric geometries should be superradiantly unstable — however, we find they are not! Nevertheless, as we will discuss, this does not present a sharp contradiction with the fuzzball proposal.

The remainder of this paper is organized as follows: In Section III we briefly review some of the relevant properties of the D1-D5-P family of supergravity solutions. In Sections III and V we study minimally coupled scalar waves in the general D1-D5-P background. We write the wave equation in a Schrödinger form and perform an extensive search for unstable free scalar modes in the D1-D5-P black hole geometry. We find no unstable modes. The same conclusion applies for a massive scalar field. In Section V we discuss our results and consider some of their implications for the fuzzball proposal. In particular, we discuss possible ways the fuzzball proposal can consistently incorporate our results. The paper contains two appendices: In Appendix A we show that the wave equation for the general D1-D5-P system written in Section III reduces to the form presented in [6, 8] when we restrict our analysis to the JMaRT solitons. In Appendix B we give the explicit expression for the Schrödinger potentials introduced in section IV.

II. PROPERTIES OF THE D1-D5-P FAMILY OF SOLUTIONS

The black hole and solitonic configurations considered here are solutions of the type Iib supergravity equations. The solutions all carry the three charges of the D1-D5-P system and so are expected to give a strong-coupling description of different (ensembles of) microstates of this system. The supergravity solution is comprised of a (ten-dimensional) metric and also a nontrivial dilaton and RR two-form potential. The system is compactified to five dimensions on $M^4 \times S^1$ with the D5-branes wrapping the full internal space and the D1-branes and KK-momentum on the distinguished $S^1$. The other component of the compactification $M^4$ is a Ricci-flat four-manifold, which we take to be either a four-torus $T^4$ or $K3$. The notation is best understood by considering the construction of these solutions. One begins with the general solutions of [3, 16, 18] which contain eight parameters: a mass parameter, $M$; spin parameters in two orthogonal planes, $a_1, a_2$ (which we assume are non-negative without loss of generality); three boost parameters, $\delta_1, \delta_5, \delta_p$, which fix the D1-brane, D5-brane and KK-momentum charges, respectively; the radius of the $S^1$, $R$; the volume of the four-manifold, $V_4$ (which plays no role in the following). The geometry is described by the six-dimensional line element which is given below (see also Equation (2.12) of [16]) and which is parameterized by a time coordinate $t$; a radial coordinate $r$; three angular coordinates $\theta, \phi, \psi$; and the coordinate on the $S^1$, $y$.

The (ten-dimensional) dilaton and two-form RR gauge potential which support the D1-D5-P configuration are [3, 6].

\[
\begin{align*}
\epsilon^{2\Phi_{10}} & = \frac{\tilde{H}_1}{\tilde{H}_5}, \\
C^{(2)} & = \frac{M \cos^2 \theta}{\tilde{H}_1} \left[ (a_2 c_1 s_5 c_p - a_1 s_1 c_5 s_p)dt + (a_1 s_1 c_5 c_p - a_2 c_1 s_5 s_p)dy \right] + d\psi \\
& + \frac{M \sin^2 \theta}{\tilde{H}_1} \left[ (a_1 s_1 s_5 c_p - a_2 s_1 c_5 s_p)dt + (a_2 s_1 c_5 c_p - a_1 c_1 s_5 s_p)dy \right] + d\phi \\
& - \frac{Ms_1 c_1}{\tilde{H}_1}dt \wedge dy - \frac{Ms_5 c_5}{\tilde{H}_1} (r^2 + a_2^2 + Ms_1^2) \cos^2 \theta d\psi \wedge d\phi. 
\end{align*}
\]

However, these will only play an ancillary role in the present discussion. Central to our analysis will be the six-dimensional geometry consisting of the noncompact space, as well as the $y$-circle. The contravariant components of

\[1\] In the following, only the asymptotic D1-brane charge $Q_1$ appearing in the supergravity fields will be relevant. Of course, with $M^4 = K3$, $Q_1 \propto N_1 - N_5$ where $N_1$ and $N_5$ are the numbers of constituent D1- and D5-branes, respectively, comprising the system [10]. Ref. [20] describes how this technical point produces an interesting physical effect for the D1-D5-P black holes considered here.
the string-frame metric are\textsuperscript{2}
\begin{align*}
g^{tt} &= \frac{\tilde{c}^2}{r^2} + 2s_p c_p g^{\tilde{t}\tilde{y}} + s_p^2 g^{\tilde{y}\tilde{y}}, \\
g^{ty} &= s_p c_p \left( g^{tt} + g^{\tilde{y}\tilde{y}} + (c_p^2 + s_p^2) g^{\tilde{t}\tilde{y}} \right), \\
g^{yy} &= s_p^2 c_p^2 + 2s_p c_p g^{\tilde{y}\tilde{y}} + c_p^2 g^{\tilde{y}\tilde{y}}, \\
g^{\phi\phi} &= s_p g^{\tilde{t}\tilde{y}} + s_p g^{\tilde{y}\tilde{y}}, \\
g^{r\phi} &= c_p g^{\tilde{t}\tilde{y}} + s_p g^{\tilde{y}\tilde{y}}, \\
g^{rr} &= \frac{1}{\sqrt{H_1 H_5}} \left( f(r) + M + Ms_1^2 + Ms_2^2 + \frac{M^2 c_1^2 c_2^2 r^2}{g(r)} \right), \\
g^{i\tilde{y}} &= -\frac{1}{\sqrt{H_1 H_5}} \left( f(r) + Ms_1^2 c_1 c_2 a_1 a_2 \right) / g(r), \\
g^{\tilde{y}\tilde{y}} &= \frac{1}{\sqrt{H_1 H_5}} \left( f(r) + Ms_1^2 + Ms_2^2 + \frac{M^2 c_1^2 c_2^2 r^2}{g(r)} \right), \\
g^{\phi\phi} &= \frac{1}{\sqrt{H_1 H_5}} \left( r^2 + a_1^2 + a_2^2 - M a_1^2 \right), \\
g^{r\phi} &= \frac{1}{\sqrt{H_1 H_5}} \left( r^2 + a_1^2 + a_2^2 - M a_1^2 \right).
\end{align*}

where\textsuperscript{3}
\begin{align*}
g^{tt} &= -\frac{1}{\sqrt{H_1 H_5}} \left( f(r) + M + Ms_1^2 + Ms_2^2 + \frac{M^2 c_1^2 c_2^2 r^2}{g(r)} \right), \\
g^{i\tilde{y}} &= -\frac{1}{\sqrt{H_1 H_5}} \left( f(r) + Ms_1^2 c_1 c_2 a_1 a_2 \right) / g(r), \\
g^{\tilde{y}\tilde{y}} &= \frac{1}{\sqrt{H_1 H_5}} \left( f(r) + Ms_1^2 + Ms_2^2 + \frac{M^2 c_1^2 c_2^2 r^2}{g(r)} \right), \\
g^{\phi\phi} &= -\frac{1}{\sqrt{H_1 H_5}} \left( Ms_1 s_2 a_1 (r^2 + a_1^2 - M) \right) / g(r), \\
g^{r\phi} &= \frac{1}{\sqrt{H_1 H_5}} \left( Ms_1 s_2 a_1 (r^2 + a_1^2 - M) \right).
\end{align*}

We are using the notation $c_i \equiv \cosh \delta_i$ and $s_i \equiv \sinh \delta_i$. Throughout these expressions, we also use the functions:
\begin{align*}
g(r) &= (r^2 + a_1^2)(r^2 + a_2^2) - Mr^2 \equiv (r^2 - r_+^2)(r^2 - r_-^2), \\
f(r) &= r^2 + a_1^2 \sin^2 \theta + a_2^2 \cos^2 \theta, \\
\tilde{H}_i(r) &= f(r) + Ms_i^2, \text{ with } i = 1, 5.
\end{align*}

Without loss of generality, we will assume $a_1 \geq a_2 \geq 0$ in the following.

Depending on the values of the parameters, this solution can describe a black hole or a naked curvature singularity. However, it was also realized that in a third parameter regime\textsuperscript{[6]}, this geometry corresponds to a smooth soliton – denoted the JMaRT soliton in\textsuperscript{[3]} – or a conical singularity. The best way to identify each one of these branches of solutions is to look at the $g^{rr}$ component of the general metric, which is proportional to the function $g(r)$, given in eq.\textsuperscript{[3]} above. The roots of $g(r)$, $r_+$ and $r_-$, are given by\textsuperscript{[6]}
\begin{equation}
r^2_\pm = \frac{1}{2} (M - a_1^2 - a_2^2) \pm \frac{1}{2} \sqrt{(M - a_1^2 - a_2^2)^2 - 4a_1^2 a_2^2},
\end{equation}
and they are real whenever $|M - a_1^2 - a_2^2| > 2a_1 a_2$. We can naturally divide the general family of solutions into three branches, namely\textsuperscript{[6]}:
\begin{equation}
\begin{cases}
M \leq (a_1 - a_2)^2 & \Rightarrow r_+^2 < 0, \\
(a_1 - a_2)^2 < M < (a_1 + a_2)^2 & \Rightarrow r_+^2 \notin \mathbb{R}, \\
M \geq (a_1 + a_2)^2 & \Rightarrow r_+^2 > 0.
\end{cases}
\end{equation}

In the first case the system can describe either a smooth soliton or a conical singularity. (In Appendix\textsuperscript{[A]} we present the constraints which must be satisfied to produce a smooth geometry.) In the second case, the function $g(r)$ does not have real roots and the system describes a naked singularity, with curvature singularities where $\tilde{H}_i(r)$ vanishes. Finally, in the third case, the system describes a black hole with outer horizon at $r^2 = r_+^2$ and inner horizon at $r^2 = r_-^2$. The upper bound of the first branch, $M^2 = (a_1 - a_2)^2$ (for which $r_+^2 = r_-^2 = -a_1 a_2$), includes as special cases the full set of supersymmetric smooth solitons\textsuperscript{[2, 3]}. The lower bound of the third branch, $M = (a_1 + a_2)^2$, corresponds to an extremal black hole with $r_+^2 = r_-^2 = a_1 a_2$. The supersymmetric limit of the above three-charge system corresponds

\textsuperscript{2}The boosted coordinates ($\tilde{t}, \tilde{y}$) are related to the unboosted coordinates $(t, y)$ by $\tilde{t} = tc_p - ysp, \tilde{y} = ycp - ts_p$ – see Appendix A of\textsuperscript{[4]}. 
\textsuperscript{3}Eqs. (A.2), (A.3) and (A.4) of\textsuperscript{[4]} have a typo and must be multiplied by minus one.
to taking the limit $M \to 0$ and $\delta_1 \to \infty$, while keeping the other parameters fixed, including the conserved charges $Q_i = M s_i c_i$. One gets, in the non-singular case, the black hole solutions of \[\text{[21]}\] or the supersymmetric solitons of \[\text{[2, 3]}\] – see \[\text{[5]}\].

In this paper, the key feature of interest in this geometry is the ergosphere. In particular, we will be considering instabilities that arise due to the existence of an ergoregion. To verify the presence of the ergoregion, one may take the norm of the Killing vector $V = \partial_t$ yielding

$$g_{AB}V^AV^B = -\frac{f - M r^2}{\sqrt{H_1 H_5}}.$$  (9)

This result shows that $V = \partial_t$ becomes space-like for $f(r) < M c_p^2$ and thus one would conclude that an ergosphere appears at $f(r) = M c_p^2$. However, in the supersymmetric limit, this norm (9) becomes $|V|^2 = -(f - Q_5)/\sqrt{H_1 H_5}$. So using this measure, one arrives at the counter-intuitive conclusion that the ergoregion persists even for the supersymmetric backgrounds. The resolution of this puzzle is evident in the discussion of section 6.2 of \[\text{[6]}\]. The key point is that the present geometry ‘rotates’ along the internal $y$-direction, as well as along the angles $\phi$ and $\psi$. Hence for the purposes of defining the ergosphere, we have a continuous family of asymptotically time-like Killing vectors: $V = \partial_t + v^p \partial_p$ with $|v^y| < 1$.\(^4\) Now one can push the position of the ergosphere to smaller radii by adjusting the free parameter $v^y$. In particular, the position seems to be minimized by the choice $v^y = \tanh \delta_p$, for which $\vec{V} = \partial_t$ (i.e., we have undone the boost of footnote\[\text{[2]}\] and

$$g_{AB}\vec{V}^A\vec{V}^B = -\frac{f - M}{\sqrt{H_1 H_5}}.$$  (10)

Further we find that this choice of $\vec{V}$ matches the Killing vector arising from the square of the covariantly constant Killing spinor appearing in the BPS limit. Of course, the ergoregion now disappears in this supersymmetric limit where $M \to 0$ and $f \to r^2$. A perspective relevant for the following discussion is that in the supersymmetric backgrounds, $\vec{V}$ provides a globally time-like Killing vector that ensures there is a rotating or ‘boosted’ frame where all energies can be defined to be positive.

### III. SEPARATION OF THE WAVE EQUATION IN THE GENERAL D1-D5-P SYSTEM

The JMaRT solitons were found to be classically unstable in ref. \[\text{[5]}\]. What we wish to do now is apply a similar stability analysis to three-charged black hole geometry to ascertain whether or not these solutions are also unstable. A stability analysis starts by disturbing slightly the system and then letting it evolve freely. Stable systems return to their original ‘position’ whereas in unstable ones, the perturbations will grow without bound in time. Ideally one would like to consider perturbations by any of the type IIb supergravity fields, e.g., metric, dilaton or RR two-form perturbations. In Sections \[\text{[III]}\] and \[\text{[IV]}\], we will begin by considering only free scalar perturbations that obey the Klein-Gordon equation. These perturbations are expected to capture the essential physics while at the same time simplifying enormously the calculations. However, as we discuss in Section \[\text{[V]}\] our results for the free scalar apply directly to certain supergravity fields.

So we first consider the Klein-Gordon equation for a massless scalar field propagating in a general three-charge geometry. The results of this section are valid for the full range of the parameters defining the configuration space. Let us note that our strategy will be to solve the wave equation in the six-dimensional background formed by the non-compact directions and the distinguished $S^1$ circle. That is, we limit our analysis so that the directions along internal manifold $M^4$ play no role. This amounts to having a wave with no excitations along these internal directions. We must mention that a related analysis of this equation in the effective five-dimensional geometry coming from also reducing on the $S^1$ circle first appeared in \[\text{[12]}\].\(^5\)

We must preceed our analysis of the wave equation with the observation that for the D1-D5-P backgrounds, the effective six-dimensional dilaton vanishes: The ten-dimensional dilaton is given in eq. \[\text{[11]}\] while the string-frame metric

\(^{4}\) We note that $|v^y| = 1$ would yield an asymptotically null Killing vector. Of course, the norm of the angular Killing vectors $\partial_\phi$ and $\partial_\psi$ diverges asymptotically and so we cannot consider further linear combinations by including either of these vectors.

\(^{5}\) The present analysis is made in six, rather than five, dimensions because the Kaluza-Klein momentum along the $S^1$ plays an important role, as discussed at the end of that section \[\text{[15]}\]. Further the coordinates in \[\text{[14]}\] are adapted to calculating the absorption cross-section of the black hole rather than producing the Schrödinger form of section \[\text{[16]}\].
on the internal space \( M^4 \) takes the form

\[
G_{ab} = \left( \frac{H_1}{H_5} \right)^{\frac{1}{2}} \tilde{G}_{ab} = e^{\Phi_{10}} \tilde{G}_{ab},
\]

where \( \tilde{G}_{ab} \) is a fiducial metric with unit four-volume. Hence upon reducing the supergravity action to six dimensions, we find the effective six-dimensional dilaton becomes:

\[
e^{-2\Phi_6} \equiv e^{-2\Phi_{10}} \sqrt{\det \tilde{G}_{ab}} = 1.
\]

It follows then that in the effective six-dimensional theory, there is no distinction between the string-frame and Einstein-frame metrics. In particular, the six-dimensional metric has precisely the same components as those of the ten-dimensional string-frame metric, given in eq. (3).

Hence we consider the Klein-Gordon equation in the effective six-dimensional theory\(^6\) coming from the reduction of the general three-charge geometry:

\[
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^A} \left( \sqrt{-g} g^{AB} \frac{\partial}{\partial x^B} \Psi \right) = 0,
\]

where \( g^{AB} \) is the six-dimensional metric explicitly written above, in section II. Further \( g \) is the determinant of this six-dimensional metric, which is given by

\[
\sqrt{-g} = r \sin \theta \cos \theta \sqrt{H_1 H_5}.
\]

This is the same equation studied in the stability analysis of [8]. Introducing the separation ansatz

\[
\Psi = \exp \left[ -i \omega \frac{t}{R} - i \lambda \frac{y}{R} + im_p \psi + im_p \phi \right] \chi(\theta) h(r),
\]

and the separation constant \( \Lambda \), the wave equation separates. The angular equation is

\[
\frac{1}{\sin 2\theta} \frac{d}{d\theta} \left( \sin 2\theta \frac{d\chi}{d\theta} \right) + \left[ \frac{\omega^2 - \lambda^2}{\sin^2 \theta} - \frac{m_p^2}{\sin^2 \theta} + \frac{\omega^2}{R^2} (a_1^2 \sin^2 \theta + a_2^2 \cos^2 \theta) \right] \chi = 0,
\]

and the radial equation is

\[
\frac{1}{r} \frac{d}{dr} \left[ g(r) \frac{d}{dr} h \right] - \Lambda h + \left[ \frac{\omega^2}{R^2} (r^2 + M s_1^2 + M s_2^2) + (\omega c_p + \lambda s_p)^2 \right] h \\
+ \frac{1}{g(r)} \left\{ -\omega^2 \frac{M^2}{R^2} \left[ -c_1^2 s_2^2 p^2 - 2 s_1 s_5 s_p c_1 c_5 c_p a_1 a_2 + s_1^2 s_2^2 s_p^2 (r^2 + a_1^2 + a_2^2 - M) \right] \\
+ \frac{2 \omega M^2}{R^2} \left[ -s_1 s_5 s_p \left[ -c_1^2 s_2^2 p^2 + s_1^2 s_2^2 (r^2 + a_1^2 + a_2^2 - M) \right] + (c_1^2 + s_1^2) s_1 s_5 s_p c_1 c_5 c_p a_1 a_2 \right] \\
+ \frac{2 \omega m_p}{R} \left[ c_1 c_5 c_p a_2 (r^2 + a_1^2) - s_1 s_5 s_p a_2 (r^2 + a_1^2 - M) \right] \\
+ \frac{2 \omega m_p}{R} \left[ c_1 c_5 c_p a_1 (r^2 + a_2^2) - s_1 s_5 s_p a_1 (r^2 + a_2^2 - M) \right] \\
- \frac{\lambda^2 M^2}{R^2} \left[ -c_1^2 s_2^2 p^2 - 2 s_1 s_5 s_p c_1 c_5 c_p a_1 a_2 + s_1^2 s_2^2 s_p (r^2 + a_1^2 + a_2^2 - M) \right] \\
+ \frac{2 \lambda m_p}{R} \left[ c_1 c_5 s_p a_2 (r^2 + a_1^2) - s_1 s_5 c_p a_2 (r^2 + a_1^2 - M) \right] \\
+ \frac{2 \lambda m_p}{R} \left[ c_1 c_5 s_p a_1 (r^2 + a_2^2) - s_1 s_5 c_p a_1 (r^2 + a_2^2 - M) \right] \\
- \frac{m_p^2}{\omega} \left[ (r^2 + a_2^2) (a_1^2 - a_2^2) - Ma_1^2 \right] - \frac{m_p^2}{\omega} \left[ (r^2 + a_1^2) (a_2^2 - a_1^2) - Ma_2^2 \right] + 2 m_p m_p M a_1 a_2 \right\} h = 0,
\]

\(^6\) This wave equation is equivalent to that which would be produced in reducing the equation of motion of a massless scalar field which is minimally coupled to the Einstein-frame metric (and no other fields) in ten dimensions.
This radial wave equation is valid for the general three-charge geometries, including the black hole solutions and the JMaRT solitons. However, as shown in Appendix A when the JMaRT constraints (A1)-(A2) are imposed, the wave equation can be rewritten in a considerably simplified way, namely as in eq. (10).

The angular equation (10), plus the appropriate regularity requirements, defines a Sturm-Liouville problem, and the solutions are known as higher dimensional spin-weighted spheroidal harmonics [22]. We can label the corresponding eigenvalues $\Lambda$ with an index $l$, $\Lambda(\omega) = \Lambda_{l m_m \omega}(\omega)$ and therefore the wavefunctions form a complete set over the integer $l$. In the general case, the problem at hand consists of two coupled second-order differential equations: given some boundary conditions, one has to compute simultaneously both values of $\omega$ and $\Lambda$ that satisfy these boundary conditions. However, for vanishing $a_2^2$ we get the (five-dimensional) flat space result, $\Lambda = l(l + 2)$, and the associated angular functions are simply given by Jacobi polynomials [22]. For non-zero, but small $a_2^2$, we have

$$\Lambda = l(l + 2) + O\left(\frac{a_2^2 \omega^2 - \lambda^2}{R^2}\right).$$

(18)

The integer $l$ is constrained to be $l \geq |m_\psi| + |m_\phi|$. We will always assume $a_1^2 \omega^2 \lambda^2 \ll \max(m_\psi^2, m_\phi^2)$ (with $i = 1, 2$) and thus $\Lambda \approx l(l + 2)$. Making this assumption implies we may neglect the terms proportional to $a_i$ in the angular equation. But given the way $\Lambda$ and $\omega$ appear in the radial equation, the corrections to $\Lambda$ may not be negligible when we determine $\omega$. To ensure that setting $\Lambda = l(l + 2)$ is consistent in both the angular and radial equations, we must additionally require [see first line of (17)]:

$$a_1^2 \ll \max(|r_+^2 + M(s_1^2 + s_2^2)|, M^2 p^2).$$

IV. WAVE EQUATION IN THE SCHRÖDINGER FORM

In [8], we have shown that the non-supersymmetric JMaRT solitons are unstable against the ergoregion instability. The ingredients for this instability are the existence of an ergoregion in a geometry without horizon. If there is a counterpart of the ergoregion instability on the black hole side of the configuration space, it seems likely to be the so-called superradiant instability. The ingredients for this instability are the existence of an ergoregion around a horizon and the presence of ‘bound’ states within the superradiant regime [10, 11, 12, 13, 14, 15].

The simplest way to find if the non-supersymmetric black hole solutions of the D1-D5-P system are superradiantly unstable is to rewrite the radial wave equation in the form of an effective Schrödinger equation, and to study the corresponding Schrödinger potentials. We now apply this approach in the following.

In [8] we found that the introduction of dimensionless coordinate

$$x = \frac{r^2 - r_+^2}{r^2 - r_-^2},$$

(19)

along with a new wavefunction $H$

$$h(x) = \frac{1}{\sqrt{x(1 + x)}} H(x).$$

(20)

transformed the radial wave equation (17) to

$$\frac{\partial^2 H}{\partial x^2} + \frac{m_\psi^2}{4x^2(1 + x)^2}(\Sigma_\psi - U_+)(\Sigma_\psi - U_-) H = 0,$$

(21)

where $P$, $U_-$, and $U_+$ are presented in [8]. However, we find that this form is no longer appropriate to study the black hole sector, since in this case both $U_-$ and $U_+$ take on complex values, sufficiently close to the black hole horizon $r_+$. Instead, we introduce the ‘tortoise’ coordinate

$$\frac{dr_*}{dr} = \frac{r^4}{g(r)},$$

(22)

\footnote{Note that this coordinate is not appropriate to analyze the $M \leq (a_1 - a_2)^2$ branch of solutions since $r_+^2 < 0$ in this case, and thus $r_*$ becomes complex near the origin $r^2 = r_+^2$.}
where \( g(r) \) was defined in eq. (5) and the new wavefunction \( \Phi \) is given by
\[
h = r^{-3/2} \Phi.
\]

Then (17) can be written as a Schrödinger equation,
\[
\frac{d^2 \Phi}{dr^2} - V \Phi = 0,
\]
with
\[
V = -\frac{g(r)}{r^{10}} \left( r^4 W(r) - \frac{3}{2} r g'(r) - \frac{21}{4} g(r) \right).
\]

Here the prime denotes a derivative with respect to \( r \), and \( W(r) \) is defined by writing (17) in the form \( \frac{1}{r} \partial_r \left[ \frac{g(r)}{r} \partial_r h \right] + W(r) h = 0 \). As discussed earlier, we assume \( \Lambda \approx l(l + 2) \) independent of \( \omega \) and therefore \( V \) is a quadratic function of \( \omega \)
\[
V = -\gamma (\omega - V_+) (\omega - V_-).
\]

The explicit forms of \( \gamma \) and of the Schrödinger potentials \( V_\pm \) is given in Appendix B. It can be checked that the asymptotic behavior of the potentials is
\[
\lim_{r \to r_+} V_\pm = \omega_{\text{sup}},
\]
\[
\lim_{r \to \infty} V_\pm = \pm |\lambda|.
\]

where we defined the superradiant factor,
\[
\omega_{\text{sup}} = m_\phi \Omega_\phi R + m_\psi \Omega_\psi R - \lambda \Omega_y.
\]

Here, \( \Omega_\phi, \Omega_\psi, \) and \( \Omega_y \) are, respectively, the angular velocities along \( \phi, \psi \) and the velocity along the \( S^1 \) given by
\[
\Omega_{\phi, \psi} = -\frac{a_2 r_+^2}{(r_+^2 + a_2^2)} \left( r_+^2 c_1 c_5 c_p + a_1 a_2 s_1 s_5 c_p \right), \quad \Omega_y = \frac{r_+^2 c_1 c_5 c_p + a_1 a_2 s_1 s_5 c_p}{r_+^2 c_1 c_5 c_p + a_1 a_2 s_1 s_5 c_p}.
\]

Typical forms of the potentials \( V_\pm \) are displayed in Fig. 1. In these plots, the ‘allowed’ regions where the solutions have an oscillatory behavior are those where \( \omega \) is above \( V_+ \) or below \( V_- \), i.e., above or below both curves \( V_\pm \). In those intervals where \( \omega \) is in between the curves of \( V_+ \) and \( V_- \) (forbidden regions), the solutions have a real exponential behavior. From these plots we will infer the stability of the system. We impose only ingoing-wave boundary conditions near the horizon and outgoing at infinity. This means that the asymptotic behavior of the solutions of (23)-(24), is
\[
h(r) \sim \begin{cases} 
(\omega^2 - r_+^2)^{-1/2} e^{-i \varpi \ln(r^2 - r_+^2)}, & \text{as } r \to r_+, \\
(\omega^2 - r_+^2)^{-1/2} e^{-i \varpi \ln(r^2 - r_+^2)} \sqrt{\omega^2 - \lambda^2} \, r, & \text{as } r \to \infty,
\end{cases}
\]

where we have defined
\[
\varpi = \frac{M}{2 R} \left( \frac{r_+^2 c_1 c_5 c_p + a_1 a_2 s_1 s_5 c_p}{r_+^2 c_1 c_5 c_p + a_1 a_2 s_1 s_5 c_p} \right) (\omega - \omega_{\text{sup}}).
\]

When the frequency of the wave is such that \( \varpi \) is negative,
\[
|\omega| < |\omega_{\text{sup}}|,
\]
one is in the superradiant regime, and the amplitude of any incident wave is amplified upon scattering from the black hole.

The simplest way to appreciate this result is through the following argument. From (14) and (31) one has that at the horizon the wave solution behaves as \( \Psi(t, r)|_{r \to r_+} \sim e^{-i \omega t / R} e^{-i \varpi \ln(r^2 - R^2)} \). The phase velocity of the wave is
FIG. 1: Typical Schrödinger potentials for the black hole branch, $M \geq (a_1 + a_2)^2$ or $r_+^2 > 0$, and for modes with no KK momentum, $\lambda = 0$. Figure a) refers to the case $m_\psi > 0$, while figure b) corresponds to case $m_\psi < 0$. Superradiant modes (dashed line) with $|\omega| < |\omega_{\text{sup}}|$ do exist but there are no trapped bound states and thus the geometry is not afflicted by the superradiant instability.

then $v_{\text{ph}} \propto -\frac{\omega}{\alpha}$. Now, the value of this phase velocity can be positive or negative depending on the value of $\omega$ (when we fix the other parameters), so one might question if the first line of (31) really describes always an ingoing wave. What is relevant for the discussion is not the phase velocity, but the group velocity of the waves. The normalized group velocity, $v_{\text{gr}}$, at the horizon is $v_{\text{gr}} = 2(r_+^2 - r_-^2) r_+ \alpha^{-1} d(-\omega) d\omega = -1$. This is a negative value that signals that the near-horizon wave solution in (31) always represents an ingoing wave independently of the value of $\omega$, and thus we have the correct physical boundary condition. However, note that in the superradiant regime (33), the phase velocity is positive and so waves appear as outgoing to an inertial observer at spatial infinity. Therefore energy is in fact being extracted from the black hole.

FIG. 2: Examples of potentials that would correspond to black hole geometries afflicted by the superradiant instability. Unstable modes are superradiant modes with $|\omega| < |\omega_{\text{sup}}|$ that are also bound states (dashed line). Case a) corresponds to $\lambda = 0$ while case b) corresponds to $\lambda \neq 0$.

The relevance of superradiance in the present context is that it may give rise to instabilities. The only extra ingredient one needs for that to happen is to somehow ‘trap’ the waves near the horizon. An artificial way of doing

\[9 \text{ Note that for } M \geq (a_1 + a_2)^2 \text{ one has } r_+^2 \geq a_1 a_2. \text{ Moreover, } c_i^2 = 1 + s_i^2 > s_i^2.\]
this would be to enclose the black hole inside a mirror \[13\]. Any initial perturbation will get successively amplified near the black hole event horizon and reflected back at the mirror, thus creating an instability. This instability is caused by the mirror, which is an artificial wall, but one can also devise natural mirrors. For example, consider a massive scalar field \[14\]. Imagine a wavepacket of the massive field in a distant circular orbit. The gravitational force binds the field and keeps it from escaping or radiating away to infinity. But at the event horizon some of the field goes down the black hole, and if the frequency of the field is in the superradiant region then the field is amplified. Hence the field is amplified at the event horizon while being confined away from infinity. Yet another way to understand this, is to think in terms of wave propagation in an effective potential. If the effective potential has a well, then waves get ‘trapped’ in the well and amplified by superradiance, thereby triggering an instability. In the case of massive fields on a four-dimensional Kerr background, the effective potential indeed has such a well. Consequently, the massive field grows exponentially and is unstable. It is the presence of a bound state that simulates the mirror, and so without a bound state we should never get an instability.

It was found in \[10,11,12\] that introducing KK momentum for a massless field in black string (or brane) geometries can be equivalent to having a mass and so can also trigger superradiant instabilities. Hence we pay particular attention to KK momentum in our analysis below.

The strategy here is to look for bound states in the effective potential to ascertain whether or not the geometry is superradiantly unstable. We start by considering the case in which the waves have no KK momentum, \(\lambda = 0\). In this case, at infinity the potentials go to zero, as indicated by (28). In Fig. 1 we sketch two typical examples of potentials that can occur when \(\lambda = 0\). The specific parameters that yield these plots are black holes with \((a_1 = 32, a_2 = 16, M = 1.01(a_1 + a_2)^2, c_1 = 5, c_5 = 1.517, c_p = 5, R = 1)\), and modes with \((l = 10, m_\phi = 0, \lambda = 0)\) and \(m_\phi = 10\) [Fig. 1a] and \(m_\psi = -10\) [Fig. 1b]. The geometry has superradiant modes that satisfy (33), and examples of these are represented by a dashed line in these plots. In Fig. 1a, an incident wave with frequency \(\omega_{\text{sup}} < \omega < 0\) is reflected in the potential \(V_-\) and returns back to infinity with an amplified amplitude. Similarly, in Fig. 1b, superradiant modes are those with \(0 < \omega < \omega_{\text{sup}}\) that are reflected in the potential \(V_+\). In both cases, the pattern speed of the superradiant modes, \(\Sigma_\psi = \omega/(R m_\psi)\), is negative. This means that to be amplified, the waves must satisfy (33) and, in addition, rotate in the same sense as the black hole rotation (the angular velocity of the black hole \(\Omega_\psi\) is negative – see (30)). The second necessary condition for the superradiant instability – the existence of bound states – is however absent in these plots. We have done an extensive search (by varying the parameters \(a_1, M, c_1, R \) and \(l, m_\psi, m_\phi\)) for superradiant bound states and have found none with \(\lambda = 0\). For the sake of clarity, a typical example of Schrödinger potentials that would allow superradiant bound states is sketched in Fig. 2a.

We now turn our attention to modes with KK momentum, \(\lambda\), i.e., with \(\lambda \neq 0\). The KK momentum provides a potential barrier of weight \(\lambda\) at infinity [see (28)] and thus are the most promising when looking for instabilities. However, not even in this case does there seem to be an instability, as shown in Fig. 3. This figure represents the several shapes of potentials \(V_\lambda\) that we can get when \(\lambda \neq 0\). None of these plots have bound states and so it seems that the D1-D5-P black holes are not afflicted by the superradiant instability. In Fig. 3b, we plot an example of a case for which the system with \(\lambda \neq 0\) would have superradiant instabilities. As we just stated, we found no such case. In Table I we give specific examples of parameters that have the Schrödinger potentials presented in Fig. 3.

As discussed in \[11\] the absence of bound states in the potential seems to be closely related to the absence of stable circular orbits in the geometries, which is a generic feature of higher dimensional spaces \[10,11,12\]. It is not so surprising then to find that higher dimensional rotating black holes are stable against the superradiant mechanism.

We end this section by considering perturbations of a massive free scalar. Massive scalar fields, with mass \(\mu\), obey the equation

\[
\frac{1}{\sqrt{-g}} \partial_A \left( \sqrt{-g} g^{AB} \partial_B \Psi \right) = \mu^2 \Psi,
\]

with the determinant given by Eq. (14). It is easy to see that, under the same separation ansatz (15), the wave equation will be separable if and only if \(\sqrt{-g}\) itself separates. This happens in particular if \(\tilde{H}_1 = \tilde{H}_5\), i.e., if

| \(\lambda\) | 0 | \(\pm 0.0005\) | 0.005 | \(-0.05\) | 0.05 | \(-0.5\) | 0.5 | \(-1\) | 1 | \(-5\) | 5 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Figure** | Fig. 1a | Fig. 2a | Fig. 3a | Fig. 3b | Fig. 3c | Fig. 3d | Fig. 3e | Fig. 3g | Fig. 3d | Fig. 3c | Fig. 3b |

TABLE I: Some examples of modes that have the Schrödinger potentials plotted in Fig. 3. The black hole geometry is described by the parameters \((a_1 = 32, a_2 = 16, M = 1.01(a_1 + a_2)^2, c_1 = 5, c_5 = 1.517, c_p = 5, R = 1)\). The modes have \((l = m_\phi = 10, m_\psi = 0)\) and \(\lambda\) indicated in the first row of the table. Similar Schrödinger potentials are obtained when \(m_\psi\) is switched on.
$Q_1 = Q_5.^{10}$ In this case, under the assumption of small mass $\mu$, the angular equation is still given by (16) with the angular eigenvalue $\Lambda$ defined in (18), and the radial wave equation (17) receives now a source term on its right-hand side given by $\mu^2(r^2 + M s^2) h$. We followed the same procedure to turn this equation into a Schrödinger-like ODE. Again, we did not find evidence of bound states. Therefore the inclusion of a scalar mass does not seem sufficient to make these geometries unstable through the superradiant mechanism.

V. DISCUSSION

In this paper, we studied a general family of supergravity solutions for the D1-D5-P system which contains two special branches: one with smooth horizon-free geometries (JMaRT solitons) and a black hole branch. In general, when rotating, these solutions have an ergoregion. This is absent only in the supersymmetric case (in both branches). However, we note that if these backgrounds are regarded as solutions of type I supergravity, some of the ergo-free backgrounds are non-supersymmetric (on both branches), e.g., certain solutions are extreme but non-supersymmetric black holes $^{23, 25}$. With an ergoregion, the background exhibits superradiant phenomena, however, superradiant scattering per se is harmless. It will only extract a small amount of rotational energy from the geometry and transfer it to a wave that transports it to infinity. There are, however, two situations where the addition of extra ingredients leads to a catastrophic instability. One is when we have a spacetime geometry with an ergoregion but no horizon. This situation generically leads to the so-called ergoregion instability $^{9}$ in which there are modes which are outgoing

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$^{10}$ This condition is also satisfied to a good approximation, if $a_1/r_+$ and $a_2/r_+$ are very small.
at infinity, regular at the origin, and growing unboundedly with time. The negative energy that is stored in the ergoregion core, by energy conservation, will then also grow negative without bound. In a previous article, we found that the D1-D5-P smooth horizon-free geometries are afflicted by this instability [8].

The other scenario which produces a catastrophic instability is more well-known. A rotating black hole with an ergosphere and with a ‘reflecting wall’ can lead to multiple superradiant scattering/reflection that extracts rotational energy from the black hole without bound. This reflecting wall can be provided by an artificial mirror [13], by the mass of the propagating field [14], by an asymptotic anti-de Sitter geometry [15] or by the KK momentum of a black string geometry [10, 11, 12]. The key feature, necessary for the activation of this instability, is that the effective potential that describes the field propagation in the given background must have a well where bound states can be trapped. The D1-D5-P black hole could potentially be afflicted by this superradiant instability. Indeed, it has both an ergoregion and KK momentum along the distinguished $S^1$ circle, which might create the mentioned reflecting boundary. However, in our extensive search over the parameters of the solution, we have found that the other ingredient – the potential well – is absent. Hence the D1-D5-P black holes do not appear to suffer from any superradiant instability.

In sections III and IV, we verified the absence of a superradiant instability for a minimally coupled scalar field (both massless and massive) propagating in the D1-D5-P black hole background. Of course, it would be most interesting to verify that the black hole remains stable when we perturb it by the fields of the type IIB supergravity theory. Hence, we now address the question of to what extent our analysis applies to the supergravity fields. Of course, the same discussion applies to our instability analysis of the JMaRT solitons [8] which was also explicitly carried out with a massless minimally coupled field.

As discussed at the beginning of section III, we are considering the propagation of scalars in the six-dimensional space comprised of the non-compact directions and the distinguished $S^1$ circle. Hence let us consider the scalars appearing in the reduced six-dimensional supergravity. After compactifying the type IIB supergravity down to six dimensions, the scalars parameterize the moduli space

$$\mathcal{M}_0 = SO(5, n; R)/SO(5) \times SO(n)$$

(35)

to additional global identifications, where $n = 5$ or 21 for $M^4 = T^4$ or $K3$, respectively [26]. Hence the six-dimensional theory contains a total of $5n$ independent scalars. However, when the D1-D5 string is introduced in the six dimensions, several of these scalars interact with the RR two-form [2] sourced by the string. These ‘fixed’ scalars acquire an effective mass in the $AdS_3 \times S^3$ core of the black hole. Setting aside these fixed scalars, the residual moduli space is

$$\mathcal{M} = SO(4, n; R)/SO(4) \times SO(n)$$

(36)

again up to additional global identifications [26]. Hence we are left with $4n$ ‘minimal’ scalars in the six-dimensional D1-D5-P black string background.

In the six-dimensional effective action, the kinetic term for the scalars can be written as [27]:

$$I_{\text{scalar}} = \int d^6x \sqrt{-g} g^{\mu\nu} L_{ij} \partial_\mu M^{jk} \partial_\nu M^{kl},$$

(37)

where the scalar fields are represented by the $(5 + n) \times (5 + n)$ matrix $M^{ij}$ taking values in the coset [35]. These then satisfy the following identities:

$$M^T = M \quad \text{and} \quad M L M^T = L \quad \text{with} \quad L_{ij} = \begin{pmatrix} 0 & 1_n & 0 \\ 1_n & 0 & 0 \\ 0 & 0 & 1_{n-5} \end{pmatrix}$$

(38)

where $1_d$ are $d \times d$ unit matrices and the superscript $T$ indicates matrix transposition. As discussed in section III, the six-dimensional dilaton vanishes and hence in eq. (37), $g_{\mu\nu}$ corresponds to both the string-frame or Einstein-frame metric. The full scalar action also includes couplings to the gauge fields and form fields in six dimensions [27], as well as to internal fluxes [28, 29]. As discussed above, in the present background, these interactions are irrelevant for the $4n$ minimal scalars and so eq. (37) compromises the entire effective action for these fields. Now this action (37) has a deceptively simple form and so one might conclude that all of the minimal scalars satisfy the massless Klein-Gordon equation (12). However, in fact, the action is implicitly non-linear since the scalars take values on the coset [35] and so in general, this conclusion is mistaken. For example, with the torus compactification, i.e., $M^4 = T^4$, the equation of motion for scalars originating from the internal components of the ten-dimensional metric is

$$\nabla^2 G^{ab} - G^{ac} G^{bd} \nabla^2 G_{cd} = 0.$$  

(39)
when the other fields are set to zero. Carefully examining an explicit representation of \( M \) \cite{27}, one finds that relatively few of the scalars actually satisfy eq. \( \) \cite{13} in general. Recall that we must focus on the minimal scalars of \( \) \cite{35} and then there are just three scalars comprising the antisymmetric tensor parameterizing the \( 4 \times 4 \) block just above the diagonal in the top-right corner of \( M \) \cite{27}, i.e., in the same position as \( 1, \) appears in \( L \). In the torus compactification, these three scalars correspond to the internal components of the RR two-form, \( C_{ab}^{(2)} \) and beginning with the \( d = 10 \) supergravity action, one can easily demonstrate that these field obey the ordinary massless Klein-Gordon equation in six dimensions. The general discussion here confirms that in fact these three fields are the only scalars satisfying this simple equation of motion, in general. Of course, there is one case which deserves special attention: six dimensions. The general discussion here confirms that in fact these three fields are the only scalars satisfying this supergravity action, one can easily demonstrate that these field obey the ordinary massless Klein-Gordon equation in six dimensions. The general discussion here confirms that in fact these three fields are the only scalars satisfying this equation. For example, for the \( T^4 \) compactification, the internal moduli may be written as \( G_{ab} = \delta_{ab} + h_{ab} \) with traceless perturbations \( h_{ab} \) and the eq. \( \) \cite{35} reduces to \( \nabla^2 h_{ab} = 0 \).

The lesson we derive from the above discussion is that there are precisely three fields in the six-dimensional supergravity whose fluctuations are described by the Klein-Gordon equation \( \) \cite{13} analyzed in Sections \[ \] \cite{14} and \[ \] \cite{15} as well as in \[ \] \cite{8}. However, in the special case \( Q_1 = Q_5 \), all \( 4n \) minimal scalars satisfy this equation. Therefore, our conclusions that we derived for the minimally coupled (massless) scalar field apply straightforwardly for these fields above. Further then, and as the main conclusion, perturbations of certain supergravity fields can drive the JM\textit{a}RT geometries \[ \] unstable due to the ergoregion instability \[ \] but on the other hand, these same fields do not seem to produce a comparable (superradiant) instability for the D1-D5-P black holes.

While our analysis does not apply in general to the remaining \( 5n - 3 \) scalars, we are tempted to discuss these in qualitative terms. Quite generally, we expect that the superradiant instability will not appear for these scalars, independent of most of the details of their wave equation. First, we observe that for very short wavelengths, we expect wave packets to propagate along the characteristic curves of the wave equation. For example, with the Klein-Gordon equation \( \) \cite{13}, high-frequency wave packets travel along null geodesics. These characteristics are determined by the principle part of the wave equation, i.e., the second order terms. Now for a general scalar, we expect that these characteristics will in fact match the null geodesics of the Klein-Gordon field. The relevant term in the action will be precisely the kinetic term of a given scalar \( \) \cite{27}. Hence we can note here that the interactions with the background RR two-form will not affect the characteristics. Next the background scalars will only modify the kinetic term of any given scalar excitation by multiplying the latter with a nontrivial overall factor. Hence the principle part of the resulting wave equation is only modified by an overall factor which leaves the characteristics unchanged. Hence we expect that short-wavelength wave packets of all of \( 5n \) scalars propagate along null geodesics of the six-dimensional geometry.

Next we consider the analysis of [13, 11] which considered the superradiant instability in black strings and black branes in arbitrary dimensions – these were solutions of the vacuum Einstein equations. It was found that while an instability appears with four (noncompact) dimensions, no such instability appears in five and higher dimensions. Only in four dimensions did the background provide an effective potential which trapped bound states. However, it was pointed that in the high-frequency limit such a bound state would be following a stable circular orbit in the black hole background and so in higher dimensions, the absence of bound states can be related to the absence of stable circular orbits, as mentioned in section \[ \] \cite{15}. In the present case, we again are studying black strings in six dimensions or effectively five-dimensional black holes. Hence we should not expect to find stable orbits in these backgrounds. While a complete proof would require a new detailed analysis, the absence of a trapping potential for Klein-Gordon scalars certainly suggests the absence of any such orbits. The absence of such orbits can then be used to argue the absence of bound states and hence the absence of a superradiant instability for a general supergravity scalar.

In passing we note that the existence of negative-energy geodesics trapped in the ergoregion of the JM\textit{a}RT solutions can be argued on general grounds, as discussed in \[ \] \cite{8}. Hence a similar reasoning to that above suggests that any of the supergravity scalars can initiate the ergoregion instability in these backgrounds. That is, these bound geodesics would correspond to trapped states in the context of a field theory analysis. The key question then becomes whether the corresponding scalar field modes of the field fit inside the ergoregion or whether they leak out to infinity, i.e., whether they correspond to a true negative-energy bound state or to a mode producing the ergoregion instability. The detailed analysis of [8] showed that both kinds of modes existed for a Klein-Gordon scalar but in particular the spinning modes were generically associated with the ergoregion instability. While we have not extended this detailed analysis to the complete collection of six-dimensional supergravity scalars, we expect that the similar results would be found. That is, the ergoregion instability will generically be initiated by such modes that are ‘trapped’ by their angular momentum.

Of course, the primary motivation for the present study were the possible implications for Mathur’s fuzzball proposal [11]. We have already presented an extensive discussion on this topic in \[ \] \cite{8} and will only comment on a few of the salient points here. According to Mathur’s proposal, the individual microstates of a black hole are described by horizon-free solitons and the black hole geometry only appears after ‘averaging’ over these microstate solutions. Much of the
evidence for these ideas comes from studying certain supersymmetric solutions in five \[2,3,4\] and four \[5\] dimensions. However, if the fuzzball proposal is to have any substance, it must also extend to non-supersymmetric black holes. The JMaRT solitons provide the first family of smooth horizon-free geometries which are non-supersymmetric and so correspond to non-BPS microstates \[6\]. However, as mentioned in the introduction, these solutions also present an apparent contradiction with the fuzzball proposal. That is, the JMaRT solitons suffer from a classical instability, namely an ergoregion instability, and further it can be argued that this instability should be robust feature of any smooth horizon-free geometry carrying angular momentum \[g\]. However, the nonextremal rotating D1-D5-P black holes exhibit no comparable instability. In particular, we have shown here that these black holes do not exhibit a superradiant instability. Hence there is a possible contradiction for the fuzzball proposal since one would expect that if the ergoregion instability is common to all of the rotating non-BPS microstate geometries, then this instability should be reflected in the black hole geometry which is supposed to arise from averaging over the microstate solutions.

However, this reasoning is not definitive and this puzzle still has physically sensible resolution, at least in principle. In particular, there are two observations which we must make about the JMaRT solitons. First, the mass and spin of the JMaRT and black hole solutions are in very different regimes, as described in \(a\): \(M \leq (a_1 - a_2)^2\) for the JMaRT branch and \(M \geq (a_1 + a_2)^2\) for the black hole branch. Hence the JMaRT solutions should be expected to represent at best a very small contribution to the microstate ensemble underlying a nonextremal D1-D5-P black hole. Second, the JMaRT solutions are very symmetric spacetimes. In particular, they have all of the same Killing symmetries as the D1-D5-P black holes, since these are simply two branches of a common family of supergravity solutions. In contrast, generic microstate geometries are expected to contain complex throats which do not respect these Killing symmetries \[1\]. Hence the physical characteristics of the JMaRT solutions are likely not representative of those for a typical microstate in the black hole ensemble. While the typical microstate geometries should still suffer from an ergoregion instability, one might expect that the instability timescale becomes very long \[8\], especially in the ‘classical limit’ where the string coupling is taken to zero \[30\]. In particular, the complex throat at the core of the typical microstate geometries should emulate the absorptive behaviour of the black horizon in this limit, making it difficult to distinguish physics in these backgrounds from that in a black hole background, except on very large time scales. It is reasonable then that the timescale of the ergoregion instability should be a scale that grows parametrically as \(g_s \rightarrow 0\).

Given that the ergoregion instability should be a generic feature of nonsupersymmetric microstate geometries, it would be interesting to study the dual non-BPS microstates at weak coupling for evidence of such an instability. As a particularly simple set of microstates have been identified to correspond to the JMaRT solitons \[6\], these may provide a good framework to initiate such a line of investigation. For more general microstates, \(e.g.,\) those expected to describe a near-extremal spinning D1-D5-P black hole \[31\], one must be careful to distinguish the expected Hawking radiation from radiation related to the ergoregion instability. In this regime, the latter is likely to be related to the ‘nonthermal’ radiation that is expected to produce the spin-down of the black hole \[32\]. It may be, however, that when considering non-BPS configurations that the ergoregion instability provides a signature by which microstate geometries are more easily distinguished from their black hole counter-parts. Hence this seems a promising direction of research.

Of course, another challenging problem which remains is the construction of a more or less complete family of microstate geometries beyond the BPS sector. While the existence of the JMaRT solitons indicates that at least certain non-BPS states can be described by horizon-free geometries, it is not at all clear that this property should be shared by all such states. However, this is certainly another intriguing research direction.

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APPENDIX A: SIMPLIFICATION OF THE WAVE EQUATION FOR THE JMaRT SOLITON

In this appendix we clarify the connection between the present perturbation analysis on the black hole branch of the D1-D5-P system and the stability analysis of the JMaRT solitons done in [1].

In either case, we require that the function $g(r)$, given in eq. (4), has real roots. The JMaRT solitons then appear in the low-mass regime of [3], $M^2 \leq (a_1 - a_2)^2$, where $r_+^2 < 0$. While the metric may appear singular at $r^2 = r_+^2$, one can impose a series of constraints that ensure that the solutions are free of singularities, horizons and closed time-like curves. This task leads to the construction of the JMaRT solitons [2]. These solitonic solutions have an appropriate circle that shrinks to zero at the origin and the constraints ensure that this happens smoothly. First, $M$ and $R$ are re-expressed in terms of the remaining parameters – see Eqs. (3.15) and (3.20) of [3],

$$R = \frac{Ms_1c_5c_5\sqrt{\frac{s_1s_5s_5sp}{s_1s_5s_5sp}}}{\sqrt{a_1a_2(c_1c_2^2 - s_1^2s_2^2)}}$$

$$M = a_1^2 + a_2^2 - a_1a_2\frac{c_1c_2^2 + s_1^2s_2^2}{s_1c_5s_5spcp}$$  \hspace{1cm} (A1)

Then ensuring the geometry remains smooth requires imposing to ‘quantization’ conditions

$$\frac{s_pcp}{a_1c_5cp - a_2s_1s_5sp} R = n$$

$$\frac{s_pcp}{a_2c_5cp - a_1s_1s_5sp} R = m$$  \hspace{1cm} (A2)

where $m, n$ are both integers [6]. These two constraints can be put in a more elegant form by introducing the dimensionless quantities,

$$j = \left(\frac{a_2}{a_1}\right)^{1/2} \leq 1, \quad s = \left(\frac{s_1s_5s_p}{(c_1c_5cp)}\right)^{1/2},$$  \hspace{1cm} (A3)

with which the constraints (A1) can be re-expressed as

$$\frac{j + j^{-1}}{s + s^{-1}} = m - n, \quad \frac{j - j^{-1}}{s - s^{-1}} = m + n.$$  \hspace{1cm} (A4)

Again, without loss of generality, we have assumed $a_1 \geq a_2 \geq 0$, which further implies $m > n \geq 0$. We also note here that the special case $m = n + 1$ corresponds to supersymmetric solutions. In this case one also has: $M = 0, s = 1, j = 1, a_1 = a_2$.

Imposing the constraints (A2) leaves a five-parameter family of smooth solitonic solutions. We can think of the independent parameters as the D1-brane and D5-brane charges, $Q_1, Q_5$; the (asymptotic) radius of the $y$-circle, $R$; and the two integers, $m$ and $n$, which fix the remaining physical parameters as [6]

$$Q_p = nm\frac{Q_1Q_5}{R^2}, \quad J_\phi = -m\frac{Q_1Q_5}{R}, \quad J_\psi = n\frac{Q_1Q_5}{R}.$$  \hspace{1cm} (A5)

Of course, depending on the specific application, it may be more appropriate and/or simpler to describe the solutions using a different set of quantities. To conclude our discussion of the JMaRT case, we note that the roots (7) of $g^{-\tau}$ can be rewritten as

$$r_+^2 = -a_1a_2\frac{s_1s_5sp}{c_1c_5cp}, \quad r_-^2 = -a_1a_2\frac{s_1c_5cp}{s_1s_5s_5sp},$$  \hspace{1cm} (A6)

The wave equation in the background of the JMaRT solitons is still given by (17), but we can simplify it by using the JMaRT constraints (A1)-(A6). The results of this Appendix will make use of these constraints and so they will be valid only for the horizon-free JMaRT solutions; they no longer apply to the general case and in particular to the black hole case. Define

$$\rho = \frac{c_1c_2^2 - s_1s_2^2}{s_1s_5s_5sp},$$

$$\varrho = \frac{c_1c_2^2 - s_1s_2^2}{s_1s_5s_5sp} s_pcp,$$  \hspace{1cm} (A7)
and note that $r_+^2 - r_0^2 = \frac{\omega_\lambda m}{\rho^2}$. If and only if the JMaRT constraints are imposed, we can verify the following identities:

\[- \frac{\omega^2}{g(r)} \frac{M^2}{R^2} \left[ c_1^2 c_5^2 c_p^2 r^2 - 2 s_1 s_5 s_p c_1 c_5 c_p a_1 a_2 + s_1^2 s_5^2 s_p^2 (r^2 + a_1^2 + a_2^2 - M) \right] = \frac{\omega^2}{g(r)} (r_+^2 - r_-^2) \rho^2 (r^2 - r_+^2),\]

\[2 \lambda \omega \frac{M^2}{g(r)} \left[ - s_p c_p \left[ - c_1^2 c_5^2 r^2 + s_1^2 s_5^2 (r^2 + a_1^2 + a_2^2 - M) \right] + (c_p^2 + s_p^2) s_1 s_5 s_p a_1 a_2 \right] = 2 \lambda \omega \frac{M^2}{g(r)} (r_+^2 - r_-^2) \rho \theta (r^2 - r_+^2),\]

\[- \frac{2 \omega m_\theta}{g(r)} \frac{M}{R} \left[ - c_1 c_5 c_p a_2 (r^2 + a_1^2) + s_1 s_5 s_p a_1 (r^2 + a_1^2 - M) \right] = - \frac{2 \omega m_\theta}{g(r)} (r_+^2 - r_-^2) \rho n (r^2 - r_-^2),\]

\[- \frac{2 \omega m_\psi}{g(r)} \frac{M}{R} \left[ c_1 c_5 c_p a_1 (r^2 + a_2^2) - s_1 s_5 s_p a_2 (r^2 + a_2^2 - M) \right] = \frac{2 \omega m_\psi}{g(r)} (r_+^2 - r_-^2) \rho m (r^2 - r_-^2),\]

\[- \frac{\lambda^2}{g(r)} \frac{M^2}{R^2} \left[ - c_1^2 c_5^2 s_p^2 r^2 - 2 s_1 s_5 s_p c_1 c_5 c_p a_1 a_2 + s_1^2 s_5^2 s_p^2 (r^2 + a_1^2 + a_2^2 - M) \right] = - \frac{\lambda^2}{g(r)} (r_+^2 - r_-^2) \left[ (r^2 - r_0^2)^2 - \vartheta^2 (r^2 - r_-^2) \right],\]

\[2 \lambda m_\theta \frac{M}{g(r)} \frac{M}{R} \left[ c_1 c_5 s_p a_2 (r^2 + a_1^2) - s_1 s_5 s_p a_1 (r^2 + a_1^2 - M) \right] = 2 \lambda m_\theta \frac{M}{g(r)} (r_+^2 - r_-^2) \left[ - m (r^2 - r_-^2) - n \theta (r^2 - r_+^2) \right],\]

\[2 \lambda m_\psi \frac{M}{g(r)} \frac{M}{R} \left[ c_1 c_5 s_p a_1 (r^2 + a_2^2) - s_1 s_5 s_p a_2 (r^2 + a_2^2 - M) \right] = 2 \lambda m_\psi \frac{M}{g(r)} (r_+^2 - r_-^2) \left[ n (r^2 - r_-^2) + m \theta (r^2 - r_+^2) \right],\]

\[- \frac{m_\phi}{g(r)} \frac{M}{R} \left[ (r^2 + a_1^2) (a_1^2 - a_2^2) - M a_2 \right] = - \frac{m_\phi}{g(r)} (r_+^2 - r_-^2) \left[ m^2 (r^2 - r_-^2) - n^2 (r^2 - r_+^2) \right],\]

\[- \frac{2 \omega m_\theta}{g(r)} \frac{M}{R} \left[ a_2 (r^2 - r_-^2) \right] = \frac{2 \omega m_\theta}{g(r)} (r_+^2 - r_-^2) \left[ 2 n m (r^2 - r_-^2) \right],\]

\[- \frac{m_\psi}{g(r)} \frac{M}{R} \left[ (r^2 + a_1^2) (a_2^2 - a_2^2) - M a_2 \right] = - \frac{m_\psi}{g(r)} (r_+^2 - r_-^2) \left[ n^2 (r^2 - r_-^2) - m^2 (r^2 - r_+^2) \right].\]

So, in these equalities, the left-hand side was taken from (17) and the right-hand side is valid only if the JMaRT constraints (11)-(12) are imposed. Hence in the JMaRT case, we leave the first line of (17) unchanged but we can simplify all the terms proportionally to $g(r)$. Inserting (A8) into eq. (17) yields the simplified version of the wave equation for the JMaRT solitons,

\[
\frac{1}{r} \frac{d}{dr} \left[ \frac{g(r)}{r} \frac{d}{dr} h \right] - \Lambda h + \left[ \frac{(\omega^2 - \lambda^2)}{R^2} (r^2 + M s_1^2 + M s_5^2) + (\omega c_p + \lambda s_p) \frac{M}{R^2} \right] h
- (r_+^2 - r_-^2) \frac{\lambda - n m_\phi + n m_\psi \vartheta}{r^2 - r_+^2} \frac{M}{R^2} + (r_+^2 - r_-^2) \frac{\omega \vartheta + \lambda \theta - n m_\phi + m m_\psi \vartheta}{r^2 - r_-^2}) = 0,
\]

which is exactly Equation (6.4) of [8] and Equation (14) of [8].

APPENDIX B: THE SCHröDINGER POTENTIALS

In this appendix we give the explicit form of the function of the function $\gamma$ and of the Schrödinger potentials $V_\pm$ that are defined in (20):

\[
\gamma = \frac{M^2}{r^6 R^2} \left[ c_1^2 c_5^2 c_p^2 r^2 + \frac{g(r)}{M^2} (r^2 + M (r_0^2 + s_1^2 + s_5^2)) + 2 a_1 a_2 c_1 c_5 c_p s_1 s_5 s_p + (M - a_1^2 - a_2^2 - r^2) s_1^2 s_5^2 s_p^2 \right],
\]

\[
V_\pm = \frac{K_1}{2 \gamma} \pm \sqrt{\left( \frac{K_1}{2 \gamma} \right)^2 - \frac{K_0}{\gamma}}.
\]
with

\[ K_0 = \frac{g(r)}{4r^{10}} \left[ -3g(r) - 12 \left( -2r^2 r_+^2 + r^2 (r_+^2 + r_+^2) \right) + 4r^4 \left( l(l - 2) - \frac{\lambda^2}{R^2} (-M s_p^2 + r^2 + M s_1^2 + M s_2^2) \right) \right] \\
+ \frac{M^2 \Delta}{R^6 R_{p}^4} \left[ c_1 c_2 s_p^2 r_+^2 + 2a_1 a_2 c_1 c_2 c_5 c_6 s_1 s_2 s_3 s_5 - c_1 c_2 s_1 s_5 s_p^3 \right] \left( a_1^2 a_2^2 - M + r^2 \right) \\
+ \frac{2M \Delta}{R^6 R_{p}^4} \left[ c_1 s_2 s_5 s_p \left( a_1^3 + a_1 (r^2 - M) + a_2 s_p \left( r^2 - M + a_2^2 \right) \right) + a_1 a_2 s_1 s_5 s_p \left( a_1 + a_2 s_p \left( r^2 + a_1^2 \right) - a_2 m_p \right) \right] \\
- \frac{1}{r^2} \left[ r^2 (a_1^2 - a_2^2) (m_p^2 - m_\phi^2) + (a_1^2 m_p^2 - a_2 m_\phi^2) (a_1^2 - a_2^2 - M) \right], \\
K_1 = \frac{2M}{r^{10} R} \left[ c_1 c_2 c_5 c_6 \left[ a_1 a_2 (a_1 m_\phi + a_2 m_\psi) + r^2 (a_2 m_\phi + a_1 m_\psi) \right] \\
- \left[ a_1^3 m_\phi + a_1 m_\phi (r^2 - M) + a_2 m_\psi (a_2^2 - M + r^2) \right] s_1 s_5 s_p \right] \\
- \frac{2M \Delta}{R^6 R_{p}^4} \left[ a_1 a_2 c_1 c_2 c_5 s_1 s_2 s_3 M + c_1 s_p^2 \left[ c_1^2 c_2^2 M r^2 + g(r) - M s_1^2 s_p^3 (a_1^2 + a_2^2 - M + r^2) \right] + a_1 a_2 c_1 c_2 M s_1 s_5 s_p^2 \right]. \] (B2)

[1] For a review see S.D. Mathur, “The fuzzball proposal for black holes: An elementary review,” Fortsch. Phys. 53, 793 (2005), hep-th/0502056;
S.D. Mathur, “The quantum structure of black holes,” hep-th/0510180.
[2] V. Balasubramanian, J. de Boer, E. Keski-Vakkuri, and S.F. Ross, “Supersymmetric conical defects: Towards a string theoretic description of black hole formation,” Phys. Rev. D 64 (2001) 064011 [arXiv:hep-th/0011217];
J.M. Maldacena and L. Maoz, “De-singularization by rotation,” JHEP 12 (2002) 055 [arXiv:hep-th/012025];
O. Lunin and S.D. Mathur, “Metric of the multiply wind rotating string,” Nucl. Phys. B 610 (2001) 49 [arXiv:hep-th/0105136];
“AdS/CFT duality and the black hole information paradox,” Nucl. Phys. B 623 (2002) 342 [arXiv:hep-th/0109154];
“Statistical interpretation of Bekenstein entropy for systems with a stretched horizon,” Phys. Rev. Lett. 88 (2002) 211303 [arXiv:hep-th/0202072];
“The slowly rotating near extremal D1-D5 system as a ‘hot tube’,” Nucl. Phys. B 615 (2001) 285 [arXiv:hep-th/0107113];
O. Lunin, J.M. Maldacena, and L. Maoz, “Gravity solutions for the D1-D5 system with angular momentum,” hep-th/0212210;
H. Lin, O. Lunin and J.M. Maldacena, “Bubbling AdS space and 1/2 BPS geometries,” JHEP 0410 (2004) 025 [arXiv:hep-th/0409174];
M. Taylor, “General 2 charge geometries,” hep-th/0507223;
K. Skenderis and M. Taylor, “Fuzzball solutions and D1-D5 microstates,” Phys. Rev. Lett. 98 (2007) 071601 [arXiv:hep-th/0609154];
I. Kanitscheider, K. Skenderis and M. Taylor, “Holographic anatomy of fuzzballs,” JHEP 0704 (2007) 023 [arXiv:hep-th/0611171];
“Fuzzballs with internal excitations,” arXiv:0704.0690 [hep-th].
[3] O. Lunin, “Adding momentum to D1-D5 system,” JHEP 0404 (2004) 054 [arXiv:hep-th/0404006];
S. Giusto, S.D. Mathur, and A. Saxena, “Dual geometries for a set of 3-charge microstates,” Nucl. Phys. B 701 (2004) 357 [arXiv:hep-th/0405017].
[4] S.D. Mathur, A. Saxena and Y.K. Srivastava, “Constructing ‘hair’ for the three charge hole,” Nucl. Phys. B 680 (2004) 415 [arXiv:hep-th/0311092];
S. Giusto, S.D. Mathur, and A. Saxena, “3-charge geometries and their CFT duals,” Nucl. Phys. B 710 (2005) 425 [arXiv:hep-th/0406103];
S. Giusto and S.D. Mathur, “Geometry of D1-D5-P bound states,” Nucl. Phys. B 729 (2005) 203 [arXiv:hep-th/0409067];
J. Ford, S. Giusto and A. Saxena, “A class of BPS time-dependent 3-charge microstates from spectral flow,” arXiv:hep-th/0612227;
I. Bena and N.P. Warner, “One ring to rule them all ... and in the darkness bind them?,” hep-th/0408106; “Bubbling supertubes and foaming black holes,” hep-th/0505166;
I. Bena, C.W. Wang and N.P. Warner, “The foaming three-charge black hole,” Phys. Rev. D 75 (2007) 124026 [arXiv:hep-th/0604110]; “Mergers and typical black hole microstates,” JHEP 0611 (2006) 042 [arXiv:hep-th/0608217]; “Plumbing the Abyss: Black Ring Microstates,” arXiv:0706.3786 [hep-th];
P. Berglund, E.G. Gimon and T.S. Levi, “Supergravity microstates for BPS black holes and black rings,” hep-th/0503053;
[5] I. Bena and P. Kraus, “Microstates of the D1-D5-KK system,” Phys. Rev. D 72 (2005) 025007 [arXiv:hep-th/0503053];
“Microscopic description of black rings in AdS/CFT,” JHEP 0412 (2004) 070 [arXiv:hep-th/0408186]. I. Bena, P. Kraus and N.P. Warner, “Black rings in Taub-NUT,” Phys. Rev. D 72 (2005) 084019 [arXiv:hep-th/0504142];
H. Elvang, R. Emparan, D. Mateos and H.S. Reall, “Supersymmetric 4D rotating black holes from 5D black rings,” JHEP 0508 (2005) 042 [arXiv:hep-th/0504125];
A. Saxena, G. Potvin, S. Giusto and A.W. Peet, “Smooth geometries with four charges in four dimensions,” JHEP 0604 (2006) 010 [arXiv:hep-th/0509214];
V. Balasubramanian, E.G. Gimon and T.S. Levi, “Four Dimensional Black Hole Microstates: From D-branes to Spacetime Foam,” arXiv:hep-th/0606118.

[6] V. Jejjala, O. Madden, S.F. Ross and G. Titchener, “Non-supersymmetric smooth geometries and D1-D5-P bound states,” Phys. Rev. D 71, 124030 (2005), [arXiv:0504181].

[7] E.G. Gimon, T.S. Levi and S.F. Ross, “Geometry of non-supersymmetric three-charge bound states,” arXiv:0705.1238 [hep-th].

[8] V. Cardoso, O.J.C. Dias, J.L. Hovdebo and R.C. Myers, “Instability of non-supersymmetric smooth geometries,” Phys. Rev. D 73, 064031 (2006); [arXiv:hep-th/0512277].

[9] J.L. Friedman, “Ergosphere instability,” Commun. Math. Phys. 63, 243 (1978).

[10] V. Cardoso and J.P.S. Lemos, “New instability for rotating black branes and strings,” Phys. Lett. B 621, 219 (2005), [arXiv:0412078].

[11] V. Cardoso and S. Yoshida, “Superradiant instabilities of rotating black branes and strings,” JHEP 0507, 009 (2005), [arXiv:0502206].

[12] O.J.C. Dias, “Superradiant instability of large radius doubly spinning black rings,” Phys. Rev. D 73, 124035 (2006), [arXiv:hep-th/0602064].

[13] W.H. Press and S.A. Teukolsky, “Floating Orbits, super-radiant scattering and the black-hole bomb,” Nature 238, 211 (1972);
V. Cardoso, O.J.C. Dias, J.P.S. Lemos and S. Yoshida, “The black hole bomb and superradiant instabilities,” Phys. Rev. D 70, 044039 (2004) [Erratum-ibid. D 70, 049903 (2004)]; [arXiv:hep-th/0404096].

[14] T. Damour, N. Deruelle and R. Ruffini, “On Quantum Resonances In Stationary Geometries,” Lett. Nuovo Cim. 15, 257 (1976);
S. Detweiler, “Klein-Gordon Equation And Rotating Black Holes,” Phys. Rev. D 22, 2323 (1980);
H. Furushashi and Y. Nambu, “Instability of massive scalar fields in Kerr-Newman spacetime,” Prog. Theor. Phys. 112, 983 (2004), [arXiv:hep-th/0402037].

[15] V. Cardoso and O.J.C. Dias, “Small Kerr-anti-de Sitter black holes are unstable,” Phys. Rev. D 70, 084011 (2004), [arXiv:hep-th/0405006];
H. K. Kunduri, J. Lucietti and H. S. Reall “Gravitational perturbations of higher dimensional rotating black holes: Tensor Perturbations,” Phys. Rev. D 74 (2006) 084021, [arXiv:hep-th/0606076].

[16] J.C. Breckenridge, D.A. Lowe, R.C. Myers, A.W. Peet, A. Strominger and C. Vafa, “Macroscopic and Microscopic Entropy of Near-Extremal Spinning Black Holes,” Phys. Lett. B 381, 423 (1996), [arXiv:hep-th/9603078].

[17] M. Cvetic and F. Larsen, “General rotating black holes in string theory: Greybody factors and event horizons,” Phys. Rev. D 56, 4994 (1997), [arXiv:hep-th/9705192].

[18] M. Cvetic and D. Youm, “General Rotating Five Dimensional Black Holes of Toroidally Compactified Heterotic String,” Nucl. Phys. B 476, 118 (1996), [arXiv:hep-th/9603100].

[19] M. Bershadsky, C. Vafa and V. Sadov, “D-Strings on D-Manifolds,” Nucl. Phys. B 463, 398 (1996) [arXiv:hep-th/9510225];
M.B. Green, J.A. Harvey and G.W. Moore, “I-brane inflow and anomalous couplings on D-branes,” Class. Quant. Grav. 14, 47 (1997) [arXiv:hep-th/9605033];
Y.K. Cheung and Z. Yin, “Anomalies, branes, and currents,” Nucl. Phys. B 517, 69 (1998) [arXiv:hep-th/9710206].

[20] C.V. Johnson and R.C. Myers, “The enhancon, black holes, and the second law,” Phys. Rev. D 64, 106002 (2001) [arXiv:hep-th/0105159].

[21] A.A. Tseytlin, “Extreme dyonic black holes in string theory,” Mod. Phys. Lett. A 11, 689 (1996), [arXiv:hep-th/9601177];
J.C. Breckenridge, R.C. Myers, A.W. Peet and C. Vafa, “D-branes and spinning black holes,” Phys. Lett. B 391, 93 (1997), [arXiv:hep-th/9602065].

[22] E. Berti, V. Cardoso and M. Casals, “Eigenvalues and eigenfunctions of spin-weighted spheroidal harmonics in four and higher dimensions,” Phys. Rev. D 73, 024013 (2006), gr-qc/0511111.

[23] O.J.C. Dias and P.J. Silva, “Attractors and the quantum statistical relation for extreme (BPS or not) black holes,” arXiv:0704.1405 [hep-th].

[24] J.M. Bardeen, W.H. Press and S.A. Teukolsky, “Rotating Black Holes: Locally Nonrotating Frames, Energy Extraction, And Scalar Synchrotron Radiation,” Astrophys. J. 178, 347 (1972).

[25] O.J.C. Dias, R. Emparan and A. Maccarrone, in preparation (2007).

[26] N. Seiberg and E. Witten, “The D1/D5 system and singular CFT,” JHEP 9904, 017 (1999) [arXiv:hep-th/9903224].

[27] see, for example:
J. Maharana and J. H. Schwarz, “Noncompact symmetries in string theory,” Nucl. Phys. B 390, 3 (1993) [arXiv:hep-th/9207016];
A. Sen, “String String Duality Conjecture In Six-Dimensions And Charged Solitonic Strings,” Nucl. Phys. B 450, 103 (1995) [arXiv:hep-th/9404027].

[28] N. Kaloper and R.C. Myers, “The O(dd) story of massive supergravity,” JHEP 9905, 010 (1999) [arXiv:hep-th/9901045];
J. Shelton, W. Taylor and B. Wecht, “Nongeometric flux compactifications,” JHEP 0510, 085 (2005) [arXiv:hep-th/0508133].

[29] M. Grana, “Flux compactifications in string theory: A comprehensive review,” Phys. Rept. 423, 91 (2006) [arXiv:hep-th/0509003].

[30] J.M. Maldacena and A. Strominger, “Black hole greybody factors and D-brane spectroscopy,” Phys. Rev. D 55 (1997) 861
[arXiv:hep-th/9609026].

[31] J.C. Breckenridge, D.A. Lowe, R.C. Myers, A.W. Peet, A. Strominger and C. Vafa, “Macroscopic and Microscopic Entropy of Near-Extremal Spinning Black Holes,” Phys. Lett. B 381 (1996) 423 [arXiv:hep-th/9603078].

[32] D.N. Page, “Particle Emission Rates From A Black Hole. 2. Massless Particles From A Rotating Hole,” Phys. Rev. D 14 (1976) 3260.