Determining Direction-of-Arrival Accuracy for Installed Antennas by Postprocessing of Far-Field Data

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Abstract  Direction-of-arrival (DoA) estimation accuracy can be degraded due to installation effects, such as platform reflections, diffraction from metal edges, and reflections and refraction in the radome. To analyze these effects, this paper starts with a definition of the term installation error related to DoA estimation. Thereafter, we present a postprocessing method, which can be used to determine the DoA estimation accuracy for installed antennas. By computing synthetic signals from the installed far-field data, it is possible to analyze the installation errors described above, in addition to analyzing array model errors. The method formulation is general, thus allowing generic array configurations, installation configurations, and direction-finding algorithms to be studied. The use of the presented method is demonstrated by a case study of a wideband four-quadrant array. In this case study, we investigate the installation errors due to a single-shell radome. Thereafter, the effects of platform reflections are also analyzed, for an antenna placement in the tail of a fighter aircraft. Simulation results are presented for both the monopulse and the MUltiple SIgnal Classification direction-finding algorithms.

1. Introduction

Direction-of-arrival (DoA) estimation is an important task for a number of active and passive sensors (see, e.g., Filik & Tuncer, 2009; Tuncer & Özgen, 2009; Tuncer et al., 2007). Radar systems implemented with direction-finding (DF) algorithms are classified as active DF systems, since they contain both a transmitter and a receiver. In radar meteor science, the DoA estimates are used to calculate meteoroid trajectories and orbits (Kastinen, 2018; Kero et al., 2012). Passive DF systems, on the other hand, only receive signals. Passive DF systems are used in a number of electronic warfare (EW) applications, such as surveillance of radar signals in electronic support measures and electronic intelligence (De Martino, 2012). Similarly, the surveillance and DoA estimation of communications signals are classified as communications electronic support measures and communications intelligence. The DoA estimation accuracy requirements vary between these applications. As an example, an electronic intelligence system typically has strict requirements on DoA estimation accuracy, while a radar warning receiver instead has strict requirements on fast processing and detection speed, with less strict requirements on DoA estimation accuracy. These requirements directly influence the choice of antenna configuration and DF algorithm and how the antennas are installed on the platform.

There are a number of factors that can negatively impact the DoA estimation accuracy. The effects of array model errors, such as sensor position errors and the mutual coupling between antennas, have been studied previously (Elbir, 2017; Friedlander & Weiss, 1991; Ye & Liu, 2008; Zhang et al., 2005). Several DF algorithms compensate for mutual coupling (see, e.g., Zhang et al., 2005). In Swindlehurst and Kailath (1992), a theoretical expression is presented for the MUltiple SIgnal Classification (MUSIC) DoA estimation error due to array model errors.

In addition to array model errors, installation errors are introduced when the DF system is installed on a platform, such as an aircraft, satellite, ship, or an antenna mast. The installation errors are caused by the following three effects. First, reflections in metal structures on the platform cause multipath effects. These effects can be observed as a rapidly oscillating “ripple” in the antenna far-field patterns. Secondary installation effects include diffraction from metal edges and creeping waves on curved metal surfaces (see, e.g., Kipp et al., 2015; Kipp & Capoglu, 2014). Third, most installations also include a radome, that is, a shell consisting of dielectric materials (and possibly a frequency-selective screen), which cover the antennas to protect them from the environment (Nair & Jha, 2014). The radome design is typically a trade-off between structural, thermodynamic, aerodynamic, and electromagnetic considerations (Kozakoff, 2010). As a consequence,
the radome may have suboptimal performance from an electromagnetic point of view, resulting in a pointing error. The radome-induced pointing error for monopulse systems has previously been studied using a high-frequency analysis in Burks et al. (1982). The error in DoA due to refraction in a radome is commonly referred to as the boresight error (Kozakoff, 2010). Radomes also degrade system performance by increasing the cross-polarization and the side-lobe level.

There are multiple interesting references on array calibration (see, e.g., Aumann et al., 1989; Gupta et al., 2003; Thoma et al., 2004); however, note that this paper does not present a method for array calibration. Rather, the background motivating this research is the need to compare different antenna placements for an EW system on an aircraft. The presented method is useful for estimating the installed system performance for a given antenna placement and installation, in order to determine during the predesign phase if it is possible to satisfy the system requirement specification with the suggested antenna placement and radome design.

All of the three installation effects described above can be captured in the installed far-field data. The embedded pattern element (EEP) is the far-field amplitude of an antenna element, where the mutual coupling to the neighboring elements is taken into account (Pozar, 1994). EEPs data, which also include the installation effects, are here referred to as installed EEPs. The computation of installed EEPs has historically been considered to be a very challenging task. These challenges are mainly due to the electrical size of the platforms, which makes the numerical problems extremely large (Malmström, 2017; Macnamara, 2010). This has resulted in the development of high-frequency approximate methods, such as the shooting and bouncing rays (SBR) method. The installation problem is thereby considered as a scattering problem similarly to the analysis of radar cross section, although the source is an antenna installed on the platform, rather than a plane wave as typically used for radar cross section analysis. Further development of the SBR method for installed antenna performance has included extensions for creeping waves, in addition to diffraction models such as the uniform theory of diffraction (Kipp et al., 2015; Kipp & Capoglu, 2014). Due to the increase in computer power during the last decade, it is now possible to determine installed EEPs from full-wave methods, such as the method of moments or the finite difference time domain method, at least up to a few GHz. Using modern GPU computing, it is possible to perform full-wave analysis of small- and middle-sized radomes (Frid & Jonsson, 2018a). This trend also enables large array antennas to be modeled in a full-wave manner, whereby the effects of mutual coupling and “edge effects” can be analyzed in detail. The trend of increasing computer power is expected to continue, which opens the possibility to determine installed EEPs accurately for increasingly large problems. As a result, new postprocessing methods for analyzing the installed EEPs data have recently emerged. In particular, it was shown in Frid et al. (2015) that the installed far-field data can be used to estimate the isolation between antennas, whereby the risk for unintentional interference between multiple systems on the same platform can be assessed. Case studies have been used to investigate the accuracy of the computed installed far fields (Malmström et al., 2018).

While array model errors are already well described in the literature, little attention has been given to the DoA estimation error caused by the antenna installation. Most of the previous work has focused specifically on the analysis of radomes for monopulse systems (see Arpin & Ollevier, 2007; Burks et al., 1982; Siwiak et al., 1979), while other installation configurations and DF algorithms have received less attention. In most practical cases, the installed DoA accuracy is verified by measurements during the integration and verification process. However, measurement results can only be obtained at a late stage in the integration process for an installed DF system. At this point, it is usually difficult and costly to change the installation configuration or antenna placement. There is therefore a demand for general simulation methods that are capable of estimating all installation errors prior to installation.

The separation of the array model error and the installation error is of interest for systems-engineering (SE) and requirements management. In SE (INCOSE, 2015), the DF system is a subsystem to the platform system (e.g., the aircraft system). Note that the array model error is a property of the DF system, while the installation error is a property of the platform system, and these errors are therefore typically handled in two different requirement specifications. This SE reasoning for separating the two errors is also typically reflected in the organization and the time plan for the project, that is, the organization that produce DF systems is often not the same organization which is responsible for producing the platform and installing the systems onboard. Therefore, the effects of the array model error can be measured in the antenna measurement range once the DF system is manufactured, before it is installed on the platform. The effects of the installation error
can be determined at a later time, for example, using flight tests or sea acceptance tests. As a final example, note that the same DF system may be sold to and installed on multiple platforms, for example, when using the same EW system on two different aircraft. In this example, the array model error is unchanged, while the installation error is changed. In conclusion, there is a practical need to separate the array model error from the installation error, since they are properties of different systems, are possibly managed by different organizations, and are typically tested at different times.

This paper starts with a definition of the term installation error \( \epsilon_{\text{inst}} \), as applied to DoA estimation. This definition is applicable to generic array configurations and multiple DF algorithms. Thereafter, we present a postprocessing method for determining the DoA estimation accuracy from installed EEPs data. One of the advantages in using installed EEPs data is that both installation errors and array model errors are taken into account. The presented method is relatively straightforward: First, synthetic signals are computed from the installed EEPs data, and these signals are then used as input to the DF algorithm. The accuracy is thereafter determined by comparing the estimated and the known directions. The main originality of this paper centers on the installation error and our method to determine the installed DoA accuracy in the presign phase, at which point neither the DF system or the platform is ready for verification measurements. The focus on the installation error, as a difference to the more well-studied array model error, is mainly due to that the installation error can be larger than the array model error, as demonstrated here with a case study of airborne EW antennas. The method presented here thus provides essential information for antenna placement studies that can be used either to accept the antenna placement and installation configuration or to search for a better antenna placement on the platform. Some results for the monopulse DF algorithm have been presented at the 2018 Atlantic Radio Science Conference (Frid & Jonsson, 2018b), which resulted in the invitation to present this extended version. In this paper, we extend this method to be applicable to multiple DF algorithms. To demonstrate this applicability to multiple DF algorithms, we present simulation results for the MUSIC algorithm, in addition to the monopulse algorithm. This extended version also includes new simulation results, as described below.

The presented postprocessing method is applicable to generic arrays, as long as the installed EEPs and the locations of the antenna elements are known. The use of this postprocessing method is exemplified by considering a simulation model of a wideband four-quadrant array. First, the installation error due to a single-shell radome is considered. Thereafter, an additional example presents the installation error for this array with radome, when installed in the tail of the fighter aircraft Viggen. This example is of particular interest since EW antennas are also placed in the tail of other fighter aircraft such as Gripen E (e.g., Augustsson, 2017; Scott, 2019). This antenna placement is also interesting since it results in significant reflections in the aircraft, notably affecting the DoA estimation accuracy. To our knowledge, no other paper has presented a similar method applied to installed antenna performance nor has any previous paper considered a realistic antenna placement for a DF system on an aircraft similar to the example presented here.

2. Method

Consider the generic array antenna illustrated in Figure 1a. The antenna numbered \( n \) is represented by a coordinate \( \vec{r}_n \) and an EEP \( \hat{f}_n \). The theoretical presentation in this section will be based on this generic formulation, and the method can therefore be applied to any array antenna with known coordinates and EEPs. The antennas may therefore be of different type, size, or orientation, since this information is captured in the EEP data. It is possible for one antenna in Figure 1a to be a sub-array corresponding to one receiver channel, due to power combining at the sub-array level in the antenna feed network. In general, we therefore consider a total of \( M \geq N \) antenna elements connected to \( N \) receiver channels. A common DF array is the uniform circular array, which is typically implemented with \( N = M \). Another example, which will be discussed in detail below, is the four-quadrant array with \( N = 4 \) and \( M \gg N \) (see Figures 1b and 1c). Sub-array configurations will be further described in section 2.3.

An emitter, which is transmitting the signal \( s \), is located at the coordinate \( \vec{r} \). We use a system of coordinates with the origin close to the DF system, and \( \vec{r} = \vec{r} / r \) is therefore interpreted as the DoA. The DoA can be expressed in terms of direction cosines, that is, \( \vec{r} = u \hat{x} + v \hat{y} + w \hat{z} \). The direction cosines \( u, v, \) and \( w \) are related to the spherical angles \( \theta \) and \( \phi \) according to \( u(\theta, \phi) = \sin \theta \cos \phi, v(\theta, \phi) = \sin \theta \sin \phi \) and \( w(\theta, \phi) = \cos \phi \). Once the DoA has been estimated, the spherical angles can be calculated from the estimated direction cosines \( u_{\text{est}} \).
Figure 1. (a) Illustration of a generic array antenna with $N$ antenna elements (connected to $N$ receiver channels) located at coordinates $\vec{r}_1$, $\vec{r}_2$, …, $\vec{r}_N$, with far-field amplitudes $f_1$, $f_2$, …, $f_N$. The antenna elements do not need to be identical and may have different orientations, which is illustrated by two different antenna element symbols. In the case where one receiver channel is used for one sub-array, the total number of antenna elements $M$ will be greater than the number of receiver channels $N$ (see section 2.3). (b) Illustration of the four-quadrant array ($N = 4$) with aperture dimensions $w \times h$. (c) BoR array antenna with $M = 48$ elements divided into corresponding sub-arrays ($N = 4$), with the $z$ axis in the system of coordinates is aligned with the boresight direction.

and $v_{est}$ according to

$$\phi_{est} = \arctan(v_{est}/u_{est}), \quad (1)$$

followed by

$$\theta_{est} = \arcsin(u_{est}/\cos\phi_{est}). \quad (2)$$

This section is organized as follows. Section 2.1 presents the frequency-domain signal model used to compute synthetic signals from installed far-field data. The corresponding time-domain signal model is presented in section 2.2. Section 2.3 shows how to compute the EEP of a sub-array from the EEPs of the antenna elements in the sub-array. Section 2.4 presents the definition of installation error $\epsilon_{m}^{(n)}$ applied to DoA estimation. The relation between the installation error and the array model error $\epsilon_{m}^{(n)}$ is also described. The four-quadrant configuration is considered below in section 2.5. The monopulse and MUSIC algorithms are briefly reviewed in sections 2.6 and 2.7, respectively. Finally, section 2.8 summarizes the method to determine the DoA accuracy from the installed far-field data. For sake of clarity, a list of variable definitions is presented in Table 1.

### 2.1. Frequency-Domain Signal Model

We will first consider the signal model in the frequency-domain, with the time convention $e^{j\omega t}$, and thereafter consider the signal model in the time-domain in the subsequent section. The signal model below can also be used for an active DF system (radar), by interpreting $s$ as the signal reflected off a radar target. By using superposition, it is straightforward to extend the signal model below to multiple simultaneous emitters.

We assume far-field transmission from the emitter to the DF system. The complex-valued frequency-domain signal measured in receiver channel $n$ is therefore given by Friis’ transmission equation on the general form
where $\vec{f}$ is the far-field amplitude of the transmitter antenna and $\vec{r}_n$ is the distance vector from the transmitter antenna to receiver antenna $n$ located at $\vec{r}_n$. Note that $\tilde{f}_t$ and $\tilde{f}_n(\hat{r})$ are frequency dependent, but the

$$x_n(\omega) = \left(2\pi j f_n(-\vec{R}_n) \cdot \tilde{f}_t(\vec{R}_n) e^{-j \omega c / \vec{R}_n} \right) s(\omega), \quad (3)$$

Note. In all cases, the index $n$ spans from 1 to $N$, and the index $m$ spans from 1 to $M$. 

### Table 1

| Variables | Definitions |
|-----------|-------------|
| $N$ | Total number of receiver channels in the DF system |
| $M$ | Total number of antenna elements, including elements in all sub-arrays |
| $q_{nm}$ | Sub-array coefficient |
| $\vec{r}_n$ | Coordinates of antenna (or sub-array) $n$ in the DF system |
| $\vec{r}_m$ | Coordinates of antenna element $m$ inside a sub-array |
| $\hat{r}$ | Location of emitter (or radar target) |
| $\hat{r}$ | DoA, that is, the unit direction vector pointing at the emitter, $\hat{r} = \hat{r}/r$ |
| $u, v, \psi$ | Direction cosines, defined by $\hat{r} = u\hat{x} + v\hat{y} + \psi\hat{z}$, where $u(\theta, \phi) = \sin \theta \cos \phi, v(\theta, \phi) = \sin \theta \sin \phi$ and $\psi(\theta) = \cos(\theta)$ |
| $\theta$ | Polar angle, measured from the $z$ axis |
| $\phi$ | Azimuth angle, measured from $x$ axis toward the $y$ axis |
| $\hat{r}_0$, $\theta_0$, $\phi_0$ | Scan direction and corresponding scan angles |
| $\hat{r}_{est}$, $\hat{u}_{est}$, $\hat{v}_{est}$ | Estimated DoA with corresponding direction cosines |
| $\theta_{est}$, $\Phi_{est}$ | Estimated angles of arrival |
| $x, y, z$ | Cartesian coordinates |
| $\omega$ | Angular frequency $\omega = 2\pi f$, where $f$ is the frequency |
| $t$ | Time |
| $k$ | Wavenumber: $k = \omega / c$, where $c$ is the speed of light in vacuum |
| $s(t)$ or $S(\omega)$ | Signal transmitted by emitter in the time or frequency domain |
| $x_n(t)$ or $X_n(\omega)$ | Signal received at channel $n$ in the DF system |
| $\tilde{s}(t)$ or $\tilde{S}(\omega)$ | Synthetic signals determined from installed far-field data |
| $P(\hat{r}, \omega)$ | Factor describing properties of emitter or target |
| $L(\hat{r})$ | Common far-field amplitude used in the approximation (11) |
| $a_n(\hat{r}, \omega)$ | Component of steering vector corresponding to receiver channel $n$ |
| $\tilde{a}_n(\hat{r}, \omega)$ | Approximate representation of $a_n(\hat{r}, \omega)$ by data set or model |
| $b_m$ | Excitation coefficient for antenna element $m$ |
| $\tilde{f}_n(\hat{r})$ | Far-field amplitude (EEP) of receiving antenna (or sub-array) $n$ |
| $\tilde{f}_t(\hat{r})$ | Far-field amplitude of transmitting antenna |
| $\tau_n$ | True time delay in receiver channel $n$ |
| $\epsilon_n$ | Error due to approximations in the signal model used for the DF algorithm, including both the array model error $\epsilon_n^{(m)}$ and the installation error $\epsilon_n^{(i)}$ |
| $\hat{h}$ | Unit polarization vector for the plane wave incident on the DF system |
| $\sigma, d_e, d_a$ | Sum, elevation difference and azimuth difference signals |
| $\kappa_e, \kappa_a$ | Monopulse slope coefficients in elevation and azimuth, respectively |
| $h, w$ | Height and width of rectangular array aperture |
| $K$ | Number of time samples in sampled time-domain signal |
| $X$ | Covariance matrix |
| $G$ | MUSIC pseudo-spectrum |
| $E$ | Matrix with column vectors determined by the basis for noise space |
explicit frequency dependence (e.g., $\tilde{f}_n(-\hat{R}_n, \omega)$) is omitted in the notation here. By invoking the assumption that the emitter is in the far-field of the DF system, that is, $r_n / \hat{r} \ll 1$, the usual far-field approximation (Mailloux, 2005) can be applied to simplify (3). With all $N$ antennas located close the origin, the amplitude factor can be approximated as $1/(kR_n) \approx 1/(kr)$. In the exponential factor, the approximation $R_n \approx r - t_n (\pm \hat{r})$, where $+$ is used in the transmitting situation and $-$ is used in the receiving situation, is applied for the phase (Kerns, 1981; Mailloux, 2005). In conclusion, (3) can be simplified to

$$x_n(\omega) = P(\hat{r}, \omega) \hat{n} \cdot \tilde{f}_n(\hat{r}) e^{-j\omega/c \hat{r} \cdot \hat{s}(\omega)}.$$  

where the simplified notation

$$P(\hat{r}, \omega) \hat{n} = 2\pi \int \tilde{f}_n(-\hat{n}) e^{-j\omega/c \hat{r} \cdot \hat{s}(\omega)}$$

is used. Here, $\hat{n}$ is interpreted as the unit polarization vector of the plane wave incident on the DF system, and $P(\hat{r}, \omega)$ is a complex-valued coefficient which describes the transmitter antenna properties, which is independent of the receiver index $n$. There are some algorithms that enable simultaneous estimation of DoA $\hat{r}$ and polarization $\hat{n}$ (see, e.g., Li & Compton, 1991). By comparing the signal model (4) to the signal model used in, for example, Krim and Viberg (1996) and Schmidt (1986), the models are identical if the following notation is introduced for the steering vector components $a_n$:

$$a_n(\hat{r}) = \hat{n} \cdot \tilde{f}_n(\hat{r}) e^{-j\omega/c \hat{r} \cdot \hat{s}(\omega)},$$

Equation (6) is a useful identity, which will be studied further in section 2.4.

It is convenient to explicitly state the following result observed from (4); the far-field amplitude of an antenna with phase-reference point in the origin is given by

$$\tilde{f}_{\text{ref}}(\hat{r}) = \tilde{f}_n(\hat{r}) e^{j2\pi \hat{r} \cdot \hat{s}_n},$$

where $\tilde{f}_n(\hat{r})$ is evaluated with phase-reference point in $\hat{r}_n$ and the same convention for $\pm$ as above. The transmitting situation, with $+$ in (7), is most prevalent in the literature, since the far-field amplitude is defined in the transmitting situation (IEEE Standard for Definitions of Terms for Antennas, 2014). Note that the array factor can be directly derived from (7) (see Balanis, 1997 and section 2.4). Due to the exponential factor, the phase will be slowly varying (ideally constant) as a function of the angle for $\tilde{f}_n(\hat{r})$ but rapidly varying for $\tilde{f}_{\text{ref}}(\hat{r})$.

Beam steering is typically used when the DF array has high gain. Beam steering enables the capability to track the emitter with the main beam, which increases the antenna gain in the direction of the emitter. Furthermore, a guard function, or side-lobe blanker, can be used to resolve ambiguities (see, e.g., Toland, 2001). In an active electronically scanned array, each antenna element is connected to an individual transmit/receive module containing both an amplifier and a true time delay or a phase shifter. It is useful to deembed the effects of the transmit/receive modules by introducing the notation $\tilde{f}_n = e^{j\varphi_n} \tilde{f}_n$, where $\tau_n = \varphi_n / \omega$ is the time delay for the receiver channel $n$.

### 2.2. Time-Domain Signal Model

The frequency-domain signal model (4) can be transformed to the time-domain. When $s$ is a narrowband signal, we approximate $P(\hat{r}, \omega) \hat{n} \cdot \tilde{f}_n(\hat{r})$ as constant within the bandwidth centered around $\omega$. This is consistent with the narrowband assumption described in, for example, Krim and Viberg (1996). By noting that $\mathcal{F}^{-1}[e^{-j\omega/c \hat{r} \cdot \hat{s}(\omega)}] = s(t - (1/c)\hat{r}_n \cdot \hat{r} + \tau_n)$, where $\mathcal{F}^{-1}$ is the inverse Fourier transform, we get

$$x_n(t) = P(\hat{r}, \omega) \hat{n} \cdot \tilde{f}_n(\hat{r}) s(t + \Delta t_n + \tau_n).$$

The time difference $\Delta t_n = -\hat{r} \cdot \hat{r}_n / c$ can be understood physically as the time difference for antenna (or sub-array) $n$ relative to an antenna located in the origin (see, e.g., Friedlander & Weiss, 1991). This relation between time delay and antenna placement is of critical significance for most DF algorithms. As expected, when $\tau_n = +\hat{r}_0 \cdot \hat{r}_n / c$ is used to steer the beam to the direction $\hat{r}_0$, then $\Delta t_n$ and $\tau_n$ will cancel if $\hat{r}_0 = \hat{r}$ (Mailloux, 2005).
2.3. Embedded Element Patterns, Mutual Coupling, and Sub-arrays

The \( n \)-th EEP \( f_n \) is determined from simulation or measurement by stimulating port \( n \) and terminating the remaining ports in matched loads (Pozar, 1994). Since mutual coupling effects are included in the EEPs, it follows that the signal model above includes the mutual coupling effects if the EEPs are used to represent the antenna far-field amplitudes. This property of the EEP is also useful for array optimization (Frid & Jonsson, 2018a).

If the DF system is implemented with \( N \) sub-arrays in an array antenna consisting of \( M \) antenna elements, then \( f_n \) can be determined by the following approach. Let antenna element \( m \) be located at \( \vec{r}_m \) and have the EEP \( \bar{g}_m \) evaluated with phase-reference point in the origin. Note that \( f_n \) is the EEP of sub-array \( n \) in an array of \( N \) sub-arrays, whereas \( \bar{g}_m \) is the EEP of antenna element \( m \) in an array of \( M \) antenna elements. The sub-array pattern can thus be computed by (Frid & Jonsson, 2018a)

\[
\bar{g}_m = \sum_{m=1}^{M} b_m q_{nm} \bar{g}_m(\vec{r}).
\]

The sub-array pattern can thus be computed by (Frid & Jonsson, 2018a) \( \sum_{m=1}^{M} b_m q_{nm} \bar{g}_m(\vec{r}) \), where \( b_m \) is the excitation coefficient describing the amplitude and phase tapering for antenna element \( m \). The sub-array coefficient \( q_{nm} \) equals 1 if \( m \) is located in sub-array \( n \) and 0 otherwise. Note that the origin is used as phase reference point for this sum expression, and the phase reference point \( \vec{r}_n \) is used for \( f_n \) in the signal model above. With the standard (IEEE Standard for Definitions of Terms for Antennas, 2014), \( \bar{g}_m \) are defined in the transmitting situation. Thus, (7) can be inverted to obtain

\[
f_n(\vec{r}) = e^{-j k f_n r} \sum_{m=1}^{M} b_m q_{nm} \bar{g}_m(\vec{r}).
\]

Since (9) is evaluated with phase reference point in \( \vec{r}_n, f_n \) may be directly inserted in the frequency-domain signal model (4) and the narrowband time-domain signal model (8).

The phase tapering \( b_m = e^{-j k h_m} \) is applied to steer the main beam of each sub-array to the scan direction \( h_m = u_0 \hat{x} + v_0 \hat{y} + w_0 \hat{z} \), where \( \vec{r}_m \) is the location of antenna element \( m \) (Mailloux, 2005). This corresponds to the time delay \( r_m \) described in section 2.2 in the average sense, that is, \( r_m \) is the effective time delay applied to sub-array \( n \), by applying individual time delays at all antenna elements in that sub-array. Amplitude tapering can also be included in \( b_m \) (Frid & Jonsson, 2018a).

2.4. Array Model Errors and Installation Errors

A DF algorithm needs some representation of the DF antennas in order to estimate the DoA, and this representation is commonly expressed in terms of steering vectors. Note that any data set or mathematical model used to represent the steering vectors in the DF algorithm will include some errors, due to simulation or measurement accuracy or due to modeling assumptions (Swindlehurst & Kailath, 1992). We therefore use the notation

\[
a_n(\vec{r}) = \bar{a}_n(\vec{r}) + e_n(\vec{r}),
\]

where \( \bar{a}_n \) is the representation of \( a_n(\vec{r}) \), based on a mathematical model or a data set, and \( e_n \) is the resulting error term.

It is common to assume that the DF antennas (or sub-arrays) are identical, that is,

\[
\bar{a}_n(\vec{r}, \omega) = L(\vec{r}) e^{-j/(\omega/c) \vec{k} \cdot \vec{r}}.
\]

We introduce the notation \( L(\vec{r}) \) to express this approximation of identical far fields. From (6), it follows that \( h \cdot f_n(\vec{r}) = L(\vec{r}) \). Note that \( L \) is independent of the antenna index, while \( f_n \) has index \( n \). This approximation gives rise to an error term \( e_n \), as described above. This is the same approximation that is made in the derivation of the array factor (Balanis, 1997; Mailloux, 2005). It is common to implement DF algorithms for radar and EW using the simple approximation \( L(\vec{r}) = 1 \) (see, e.g., Kastinen, 2018; Mailloux, 2005), that is,

\[
\bar{a}_n(\vec{r}, \omega) = e^{-j/(\omega/c) \vec{k} \cdot \vec{r}}.
\]

Equation (12) can be interpreted as representing each sub-array by an isotropic point antenna located in the center of the sub-array.

The sources of error in the approximation (11) can be understood by comparison to (6). Theoretically, the far-field patterns are identical in the case of an infinite periodic array or in the idealized case of zero mutual
coupling between the antennas (Mailloux, 2005). For all other cases, the far-field patterns are not identical since the electromagnetic environment, including mutual coupling, is different for all antenna elements (Ellgardt, 2009). This is particularly clear when comparing an element located at the edge of the array to an element located in the center of the array, as shown in Frid and Jonsson (2018a). Further errors arise due to antenna placement accuracy, as well as phase and gain errors between the receiver channels. With a sub-array configuration, the sub-array tapering can also contribute to the sub-array patterns being dissimilar. All of the above are referred to as array model errors (see, e.g., Swindlehurst & Kailath, 1992). Additional errors arise due to the antenna installation, that is, due to reflections in the platform and diffraction from metal edges, in addition to reflection and refraction in the radome. As an example, the effects of the radome on the EEP will differ from antenna element to antenna element, thus introducing an error with respect to the approximation (11). This can be summarized by partitioning the error term into

\[ e_n = e_n^{(m)} + e_n^{(i)}, \]  

(13)

where \( e_n^{(m)} \) is the array model error and \( e_n^{(i)} \) is the installation error. Note that the installation error defined by (13) is an error in terms of steering vector components. The resulting pointing error (or DoA accuracy) in terms of angles or direction cosines will be described in section 2.8. While the array model error \( e_n^{(m)} \) and the installation error \( e_n^{(i)} \) both appear directly in (10), there is a practical need to separate the two errors. This practical need to separate the array model error and the installation error is described from the SE perspective in the introduction of this paper.

### 2.5. Four-Quadrant Arrays

The four-quadrant array is a sub-array configuration where an array is divided in four sub-arrays, as illustrated in Figures 1b and 1c. The far-field amplitude of sub-array \( n \) can be determined by the sum (9). The geometrical center of the sub-array is used as phase-reference point, that is, \( \hat{r}_1 = -w\hat{x}/4 + h\hat{y}/4, \hat{r}_2 = w\hat{x}/4 + h\hat{y}/4, \hat{r}_3 = -w\hat{x}/4 - h\hat{y}/4 \) and \( \hat{r}_4 = w\hat{x}/4 - h\hat{y}/4 \). The signals received by the sub-arrays after the power combining network are labeled \( x_1, x_2, x_3, \) and \( x_4 \).

It is well known that DF algorithms will only give an ambiguity-free estimation under certain conditions (Kastinen, 2018; Godara & Cantoni, 1981; Tan et al., 1996). As will be demonstrated in section 2.6, the monopulse method applied to a four-quadrant array is only ambiguity-free in the main lobe of the sum pattern. A DF system usually does not need to be ambiguity-free, as long as additional information can be provided to resolve the ambiguities, for example, by using a guard antenna. To handle ambiguities, the array can be steered such that the emitter is within the main lobe, as described in section 2.1.

### 2.6. The Monopulse DF Algorithm Applied to Four-Quadrant Arrays

In this section, we briefly review the derivation of the monopulse DF algorithm applied to four-quadrant arrays. Compared to standard references such as Nickel (2006), we will here focus on the approximations which are made in the derivation and how these approximations relate to the accuracy of the DF system. The derivation of the monopulse method is often presented based on an array factor expression. However, since the array factor is defined in the transmitting mode (IEEE Standard for Definitions of Terms for Antennas, 2014), the resulting expressions will be related to the correct expressions by a complex conjugate. This common mistake also appears in our conference paper (Frid & Jonsson, 2018b), and it is corrected below.

The sum \( \sigma \), elevation difference \( d_e \), and azimuth difference \( d_a \) signals are defined as \( \sigma = x_1 + x_2 + x_3 + x_4 \), \( d_e = x_1 + x_2 - x_3 - x_4 \), and \( d_a = x_1 - x_2 + x_3 - x_4 \). Note the sub-array numbering in Figures 1b and 1c. This definition of sum and difference signals is standard for monopulse estimation (see, e.g., Nickel, 2006; Sherman & Barton, 2011), and it results in a simple yet effective method for directly extracting the DoA from the measured signals, as described below. By application of (4), the signals at sub-array level are determined by

\[ x_n(\omega) = P(\hat{r}, \omega)(d_n(\hat{r}, \omega) + e_n)s(\omega). \]  

(14)

Invoking the approximation (11), including time delays for beam steering following section 2.1, and omitting the error term results in

\[ x_n(\omega) = P(\hat{r}, \omega)L(\hat{r})s(\omega)e^{-j\omega \varepsilon n(\hat{r} - \hat{r}_0)}. \]  

(15)
By direct application of (15), we find
\[
d_\sigma(\hat{r}) = P(\hat{r}, \omega) L(\hat{r}) s(\omega) \left[ e^{-j\hat{k}_r u} + e^{-j\hat{k}_r v} - e^{-j\hat{k}_r u} - e^{-j\hat{k}_r v} \right].
\] (16)

This expression can be simplified by application of Euler's formula:
\[
d_\sigma(\hat{r}) = -4j P(\hat{r}, \omega) L(\hat{r}) s(\omega) \cos(kw(u - u_0)/4) \sin(kh(v - v_0)/4),
\] (17)

where \( \hat{r}_n \) are determined according to section 2.5. The azimuth difference \( d_u \) and sum \( \sigma \) signals are found by analogy:
\[
d_u(\hat{r}) = 4j P(\hat{r}, \omega) L(\hat{r}) s(\omega) \sin(kw(u - u_0)/4) \cos(kh(v - v_0)/4),
\] (18)
\[
\sigma(\hat{r}) = 4P(\hat{r}, \omega) L(\hat{r}) s(\omega) \cos(kw(u - u_0)/4) \cos(kh(v - v_0)/4).
\] (19)

Since the array factor is defined in the transmitting situation, the corresponding array factors are found as the complex conjugate of (17)–(19), when omitting the factor \( P(\hat{r}, \omega) L(\hat{r}) s(\omega) \). The difference array factors, and thereby also the difference patterns, are odd, with a null in the direction of the main lobe of the sum pattern, that is, when \( (u, v) = (u_0, v_0) \). The difference patterns have two major lobes, located on opposite sides of this null, with opposite phase. The approximately linear slope through the null is used for the DoA estimation.

By dividing (17) and (18) by (19), we have the following relations between the monopulse ratios and the DoA \( (u, v) \):
\[
\frac{d_u}{\sigma} = -j \tan(\kappa_u(v - v_0)),
\] (20)
\[
\frac{d_\sigma}{\sigma} = j \tan(\kappa_\sigma(u - u_0)).
\] (21)

The advantage of this normalization is that the monopulse ratios are independent of the sub-array patterns \( f_\sigma(\hat{r}) \). However, one should keep in mind that this only holds for the idealized case with \( \epsilon = 0 \). The monopulse slope coefficients are determined by \( \kappa_u = kw/4 \) and \( \kappa_\sigma = kh/4 \). When tapering is applied in such a way that the far-field amplitudes of the sub-arrays are not identical, (20) and (21) can still be applied approximately by replacing the monopulse slope coefficients \( \kappa_u \) and \( \kappa_\sigma \) by their corresponding tapered values. By tapering the array excitation, the beamwidth is increased, and the monopulse slope coefficients are therefore decreased (see, e.g., Frid & Jonsson, 2018a).

It is interesting to note that the DoA information resides in the imaginary part of the monopulse ratios. Furthermore, the DoA information in the monopulse ratio is related to the phase shifts between the sub-arrays (see, e.g., (16)), and all sub-array patterns have the same scan direction. These properties are often collected into the category phase monopulse, as a contrast to amplitude monopulse, where the DoA information is in the real part of the monopulse ratios (Sherman & Barton, 2011).

In conclusion, the unknowns \( u \) and \( v \) can be directly determined from the imaginary parts of the measured monopulse ratios by inverting (20) and (21). Therefore, \( \theta, \phi, \) and \( \hat{r} \) can be calculated according to (1) and (2). In principle, this enables the DoA to be estimated from a single pulse, hence the name monopulse. Since (20) and (21) are periodic, the estimate of \( u \) and \( v \) is ambiguous outside the ambiguity-free region, which is given by
\[
\pi/2 > \min \left\{ \frac{\kappa_u |v - v_0|}{\kappa_\sigma |u - u_0|} \right\}.
\] (22)

Therefore, a guard antenna is often used to verify that the measured signal was received within the ambiguity-free region and not in the side-lobes (see, e.g., Toland, 2001).

### 2.7. The MUSIC DF Algorithm Applied to Four-Quadrant Arrays

The MUSIC algorithm is a well-known subspace method, described in a number of references (see, e.g., Krim & Viberg, 1996; Schmidt, 1986). The input data used by the MUSIC algorithm are the measured signals.
3. Results

3.1. Installed Far-Field Data

To demonstrate the use of the method in Figure 2, we consider a wideband array antenna of dimensions \( w = 50 \text{ mm} \) and \( h = 43 \text{ mm} \), designed for 5–24 GHz (Figure 3a). The body-of-revolution (BoR) array (Holter, 2007) is a relevant example antenna, since it is popular for airborne EW applications. First, the installation accuracy. The method is illustrated in Figure 2. First, the installed far-field patterns \( \hat{f}_n \) are obtained either from simulation or measurement. If needed, the phase-reference point may be moved using (7). In the case of a sub-array configuration, the sub-array far-field amplitude is calculated from the installed EEPs by application of (9). Thereafter, synthetic signals \( \hat{x} \) are determined using the signal model in the frequency-domain (4) or the time-domain (8) for some specified DoA \( \hat{\phi} \). These signals are used as input for the DF algorithm, which produces an estimated DoA \( \hat{\phi}_{est} \). The DoA accuracy is determined by comparing \( \hat{\phi}_{est} \) and \( \hat{\phi} \). With the application of (1) and (2), the results can be presented in terms of spherical angles.

For the monopulse method, \( \hat{\phi}_{est} \) is obtained by inverting (20) and (21). By comparison of \( \hat{\phi}_{est} \) and \( \hat{\phi} \), the effects of the approximation (11) can be determined. For the MUSIC algorithm, the synthetic signals \( \hat{x} \) are used in (23), and the steering vector components \( \hat{a}_n \) are used in (24). We have used the isotropic approximation (12) in the implementation of the MUSIC algorithm here. The resulting DoA accuracy will therefore include both array model errors (i.e., mutual coupling) and installation errors (i.e., reflection and refraction in the radome in addition to reflections and diffraction from the platform). To determine the effects of mutual coupling, we have used the following approach. First, the DoA accuracy is determined for a DF system in free space by application of the process in Figure 2. Thereafter, the DoA accuracy for an installed DF system is determined by repeating the process with a new data set representing installed antennas. By comparison of these two accuracy results, the relative importance of array model errors and installation errors can be estimated.

In the implementation, we have calculated the synthetic signals \( \hat{x}(t) \) from time-harmonic signals \( s(t) = e^{j\omega t} \). While the analysis can be extended to more generic signals, we have chosen to restrict the analysis to time-harmonic signals in order to analyze the DoA accuracy as a function of frequency. This is not a restriction, since wideband signals can be considered by superposition of multiple narrowband components (Krim & Viberg, 1996). White noise is not added to the synthetic signals here, since we are interested in determining the accuracy due to the installation configuration in the absence of noise. The DoA accuracy due to a specified SNR, without the installation effects taken into account, are analyzed in, for example, Barton (1988) and Sherman and Barton (2011).
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Figure 3. Illustration of the investigated cases: (a) BoR array in free space and (b) BoR array installed behind radome. Half of the radome is hidden to show the array beneath. (c) BoR array with radome installed in the tail of the aircraft Viggen. The equivalent currents on the Huygens box are shown at 20 GHz.

error due to a single-shell radome is investigated, and thereafter, the installation error due to platform reflections is investigated. To determine the array model errors, we first consider the BoR array in free space. To determine the combination of array model errors and installation errors, we thereafter consider the BoR array installed behind the radome (see section 2.8). The two cases are referred to as (a) and (b) and are illustrated in Figure 3. By using the same methodology, we thereafter investigate Case (c), that is, including the aircraft model.

The BoR array consists of $8 \times 6$ elements of vertical polarization, that is, the antenna elements have $\hat{y}$ polarization with the system of coordinates in Figure 1c. The single-shell radome has an extended hemispherical shape (similar to an extended hemispherical lens; Frid, 2016) of diameter 90 mm and extension 34 mm, made from quartz composite ($\varepsilon_r = 3.2, \tan \delta = 0.006$) of thickness 7.5 mm. The 48 installed EEPs $\vec{g}_{m}$ for the BoR array were computed using the finite integration technique (FIT) implemented in CST Microwave Studio. A postprocessing script implemented in Matlab was used to compute the sub-array patterns using (9). Thus, any combination of frequency, tapering, incident polarization, and scan direction can be studied efficiently using the method in Figure 2.

As a final example, we consider the installation of this array and radome in the tail of the Swedish aircraft Viggen, as illustrated in Figure 3c. The installed far-field data, including scattering in the metal aircraft...
Figure 4. Estimated angle of arrival $\theta_{\text{est}}$, estimated for the BoR array in Figure 3a, plotted versus true angle of arrival $\theta$ for (a) monopulse and (b) MUSIC. Within the ambiguity-free region, the curves collapse on the ideal dashed line.

and diffraction from metal edges, was computed using the SBR method implemented in CST Microwave Studio. As source for the SBR simulation, we used the equivalent currents on a box, referred to as a Huygens box, enclosing the array and radome. The representation using equivalent currents on a Huygens box is a standard procedure described in, for example, Malmström et al. (2018). Thus, the near-field data from the FIT simulation described above is used to form the 48 Huygens boxes. Using this method of domain decomposition, the effects from the radome are determined using the FIT simulation, while the reflections in the aircraft (including edge diffraction) are determined from the SBR method. The SBR simulation was carried out using a dedicated simulation computer with two Nvidia Tesla K80 GPUs.

3.2. Simulation Results

The first example illustrates the array model errors for Case (a). We estimate the polar angle $\theta_{\text{est}}$ for a set of angles $\theta$ in the elevation plane $\phi = 90^\circ$, for various frequencies and scan angles $\theta_0$. The results for Case (a) are shown in Figure 4. As expected, the relation between $\theta_{\text{est}}$ and $\theta$ collapses on a straight line with unity slope. The boundaries of the ambiguity-free region are visible in the figure due to the abrupt discontinuity in the curve. The ambiguity-free region is smaller in the upper part of the frequency band, as shown in (22) for the monopulse method. These ambiguities are also present in the MUSIC estimation, which can be observed as multiple peaks in the spectrum (see Figure 5). Since the radome is not considered for Case (a), any deviation from the straight line within the ambiguity-free region is due to array model errors, including mutual coupling. A comparison between Figure 4a and Figure 4b shows similar results for both monopulse and MUSIC algorithms, with only some minor differences close to the boundary of the ambiguity-free region, that is, close to the first null in the sum pattern.

Figure 6 shows the monopulse estimation error in polar angle, that is, $|\theta - \theta_{\text{est}}|$ for both Cases (a) and (b). The array model errors in Case (a) are due to mutual coupling, and the monopulse algorithm does not compensate for mutual coupling. The error obtained when the radome is installed (Case (b)) is a combination of array model errors and installation errors. From (13), it is expected that the error for Case (b) is greater than the error for Case (a). Indeed, Figure 6 shows a maximum error of $2^\circ$ for Case (b) and an error below $1^\circ$ for
Figure 5. MUSIC pseudo-spectrum determined from synthetic signals at 20 GHz for Case (b) in Figure 3. The ambiguity-free region is indicated with a red dashed rectangle of width $\Delta u = 0.6$ and height $\Delta v = 0.8$, centered around the scan direction $u_0 = 0.5$ and $v_0 = 0$. The true DoA is shown as a green circle, which agrees well with the corresponding peak in the pseudo-spectrum.

Case (a). Similar results were obtained for the MUSIC algorithm, as further described below. These results show that the installation error (due to a radome in this example) can be as large as the array model error.

To identify if there are directions where a significant error occurs, it is convenient to use contour plots. Figure 7 presents contour plots of the elevation estimation error for the monopulse method applied to Cases (a)-(c). We start by considering the radome effects (Case (b)) and consider the full installation (Case (c)) below. Corresponding results for the MUSIC algorithm are presented in Figure 8. These plots correspond to 20 GHz and the scan direction $\theta_0 = 30^\circ$, $\phi_0 = 0^\circ$. As expected, the error for Case (b) is greater than the error for Case (a) for both monopulse and MUSIC algorithms. In Figures 7a and 7b, we note that the error is smallest at the center of the main lobe, and the error is increasing with increasing distance from the center. The maximum radome-induced error is below 0.05 in terms of direction cosines. We used $K = 64$ time samples for the results presented in Figure 8, and increasing $K$ beyond this value does not significantly affect the results. For Cases (a) and (b) considering array model errors and radome errors, the results for the monopulse algorithm in Figure 7 are similar to those for the MUSIC algorithm in Figure 8. There are only small differences between Figures 7a and 7b and Figures 8a and 7b. Some deviation between these figures is expected due to the differences between the DF algorithms and their implementations. Specifically, the monopulse algorithm is based on the approximation (11), while the MUSIC algorithm implemented here uses the isotropic approximation (12), which is more strict.

Figure 6. Monopulse estimation error in polar angle $|\theta - \theta_{\text{est}}|$, computed in the elevation plane ($\phi = 90^\circ$) at 20 GHz. The scan angle is set to $\theta_0 = 30^\circ$ in the elevation plane. Results are presented for both Case (a), that is, without radome, and Case (b), that is, with the radome installed.
Finally, we consider Case (c), that is, the array with radome installed on the aircraft (Figure 3c). Comparing Figure 7b and Figure 7c, we note that the reflections in the aircraft cause the error to increase. We also note that there is some rapid variations (“ripple”) in Figure 7c, due to reflections in the aircraft. The increase in error due to reflections in the aircraft is more significant for the MUSIC implementation (Figure 8c) compared to the monopulse implementation. This is expected since the monopulse implementation assumes that the sub-array far-field patterns are identical, while the MUSIC implementation here has a stricter
Figure 8. Elevation estimation error $|v - v_{est}|$ at 20 GHz for (a) BoR array without radome, (b) BoR array with radome, and (c) BoR array with radome installed on aircraft, determined for the MUSIC DF algorithm. The scan direction $(u_0, v_0)$ is marked by +, and the axis limits are set by (22).
Figure 9. Estimation error for the polar angle, that is, $|\theta - \theta_{\text{est}}|$, expressed in $^\circ$, for the BoR array with radome installed on the aircraft (Case (c)), for (a) monopulse and (b) MUSIC. The same scan direction as for Figures 7 and 8 is considered, but the results are expressed in spherical angles instead of direction cosines. The scan direction is marked by $\pm$.

assumption of isotropic far-field patterns. The maximum error (in terms of direction cosines) occurring in the work region $0.4 \leq u \leq 0.6$ and $-0.2 \leq v \leq 0.2$ is 0.17 for the MUSIC implementation for Case (c). Locally large errors are often considered acceptable as long as the average root-mean-square error is below some specified value. The root-mean-square error for the MUSIC implementation in the work region for Case (c) is 0.045. The importance of the installation error and antenna placement is evident by a comparison of Figure 8a and Figure 8c.

Most of the results so far are presented in terms of direction cosines, rather than spherical angles. While the use of direction cosines is standard in some fields of antenna engineering, it is often considered that the presentation in spherical angles is more intuitive. By application of (1) and (2), Figure 9 presents the estimation error for the polar angle, that is, $|\theta - \theta_{\text{est}}|$, as a function of the spherical angles $\theta$ and $\phi$. Note that while Figures 7 and 8 only present the estimation error in the elevation direction cosine $v$, Figure 9 considers also the horizontal direction cosine $u$ according to (1) and (2).

4. Conclusions

First, this paper presents a definition of the term installation error applied to DoA estimation. Based on (10) and (13), the installation error can be understood as an error in the representation of the antenna steering vectors, which are used by the DF algorithm. Specifically, the installed EEPs include some effects due to reflections in the platform, creeping waves, and diffraction from edges, as well as reflections and refraction in the radome. Second, this paper presents a method to determine the DoA estimation accuracy for an installed DF system, by using the installed EEPs as input (Figure 2). After the installed EEPs have been determined by simulation, this method can be applied as a postprocessing step.
It is preferred to apply this postprocessing method at an early stage, typically in the predesign phase, in order to find an installation configuration and antenna placement which will satisfy DoA accuracy requirements. Measurement results for the installed performance can only be obtained after the DF system is installed. At this point, it is very difficult and costly to change the antenna placement, and such measurements are therefore mainly useful for verifying system performance once the system is fully installed.

In the presented simulation results, we have demonstrated the applicability of this postprocessing method for both the MUSIC and the monopulse DF algorithms. Note that this is to demonstrate the generality of the method and not for sake of comparing the two DF algorithms. Although it is beyond the scope of this paper, it is perhaps possible using the analysis method presented in Figure 2, to do a comparison of different DF algorithms. As an example, a DF algorithm that includes calibration to compensate for mutual coupling could be compared to a DF algorithm that does not include such calibration, by exchanging the DF algorithm used in Step 3 of Figure 2 and comparing the results.

We have presented simulation results for a realistic installation of EW antennas on a fighter aircraft. These results demonstrate that installation errors (e.g., Figure 8c) due to reflections in the platform can be significantly larger than array model errors (e.g., Figure 8a). The emphasis of this paper is not on this specific array or radome design but rather on the application of the method in Figure 2. Indeed, the accuracy of this particular system can probably be improved by using either a thinner radome or a sandwich radome design or by changing the antenna placement. Such considerations can be investigated further by direct application of the method that we have presented here. In fact, the general method formulation allows any other array and installation configuration to be analyzed.

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