Selection of method and means of measuring resonant frequency of serial oscillatory circuit

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Abstract. When developing a measurement technique for the resonant frequency of a series oscillatory circuit, an important problem is to select the method and means of measurement. In this paper, calculations are carried out with using two methods based on different principles. The first was made by means of the universal bridge (bridge method) with the subsequent calculation of indications, the second - by means of the measuring installation realizing the resonant principle of measurement of frequency. Relative errors were calculated ($\delta_1 = 0.56\%$ and $\delta_2 = 0.08\%$ respectively) and the confidence intervals of the frequency value were determined for each of the methods. Despite the high labor costs, a method based on the resonance principle was chosen to develop the measurement technique, as it has the necessary accuracy.

1. Introduction

One of the most important characteristics of periodic processes is the frequency, which is determined by the number of complete cycles (periods) of oscillations per unit time interval. Frequency measurement is necessary in many fields of science and technology, especially in radio electronics, which covers a wide area of electrical oscillations from infra-low to ultra-high frequencies inclusive [1].

When the electrical capacitor and the inductor are connected in series with a sinusoidal signal of a certain frequency, a zero reactive resistance will be observed. This effect is called the resonance of the oscillating circuit, and the scheme of capacitor and inductor - a serial oscillatory circuit, and the frequency at which this effect is manifested - the resonance frequency. The phenomenon of voltage resonance is used in electrical filters of various kinds, for example, if it is necessary to eliminate a current component of a certain frequency from the transmitted signal [2]. For example, in [3] the authors developed a method of induction resonance control of the mass fraction of iron in magnetite ore. Another good example of the use of power series and parallel oscillation circuits is a power resonant filter for obtaining a sinusoidal voltage.

Accurate calculation and measurement of resonant frequency is an important task, because if the frequency is different from the resonant one, then the scheme loses its marvelous qualities and manifests itself as a coil and a capacitor. Therefore, when developing a technique for performing measurements of the resonant frequency of a serial oscillatory circuit, it is necessary to choose the method with the least error as the result.

2. Materials and methods

When developing the procedure for performing measurements of the resonant frequency of a serial
oscillatory circuit, certain parameters must be taken into account, to which the measurement method must correspond. In our case, the value of the inductance L should not exceed 1 mH, and the capacitance of the circuit should not exceed 200 pF. The relative error of the result of measuring the resonant frequency should be no more than ± 0.12%.

The measurements were carried out by 2 methods. In the first case, the readings were taken with the help of a commercially available universal bridge (analog or digital) with subsequent calculation of the readings. The E7-9 meter was used, in which the measurement of inductance and capacitance is carried out by the resonance method with an indication of the resonance at zero beats. The error in measuring the device is ± 1% and ± 0.5% when measuring inductance and capacitance, respectively. In the second - with the help of a measuring device that implements the resonant principle of frequency measurement (including a voltage source of sinusoidal shape with adjustable frequency of the object under study – the resonant circuit; resonance indicator; frequency). In this case, the influence of the input capacitance of the resonance indicator and the cable capacitance were taken into account. When using the digital frequency meter Ch3-54 as a means of measuring the frequency, the reference time is chosen equal to 10 s. To study the shape of electrical signals by visual observation and measurement of their amplitude and time parameters, the universal oscilloscope C1-65A was used, the error in measuring the signal amplitude and time intervals did not exceed 5% (GOST 22737 and GOST 23158). These methods were used to measure the resonant frequency of the serial connection of the electric capacitor and the inductor, with a sinusoidal signal of a certain frequency.

3. Calculation and comparison of the accuracy of methods for measuring the resonance frequency of the serial oscillatory circuit

Let us consider the first version of the solution of this measurement problem. In this case, for example, when using the universal bridge E7-9, the limits in which the measured resonant frequency of the serial oscillatory circuit is located can be represented as:

$$f_{\text{ind}} - \Delta f \leq f \leq f_{\text{ind}} + \Delta f,$$

where the indications $f_{\text{ind}}$ are calculated by the formula:

$$f_{\text{ind}} = \frac{1}{2\pi \sqrt{L_{\text{ind}} \cdot C_{\text{ind}}}},$$

where $L_{\text{ind}}$ and $C_{\text{ind}}$ - respectively, the readings of the universal bridge E7-9 in the mode of measuring inductance and capacitance.

The maximum permissible absolute deviation of the reading from the measured frequency:

$$\pm \Delta f = 0.5 f_{\text{ind}} \cdot \sqrt{\delta L^2 + \delta C^2},$$

where $\delta L, \delta C$ is the maximum relative deviation of the bridge readings from the measured inductance and capacitance.

The relative error of the frequency in considering the first solution of the measurement problem:

$$\delta_{1f} = \frac{\pm \Delta f}{f_{\text{ind}}} \cdot 100\%.$$

The calculation example of $f_{\text{ind}}$ is given below:

$$f_{\text{ind}} = \frac{1}{2\pi \sqrt{L_{\text{ind}} \cdot C_{\text{ind}}}} = \frac{1}{2\pi \sqrt{1 \cdot 10^{-3} \cdot 200 \cdot 10^{-12}}} = 356.061 \text{ kHz}$$
where \( L_{\text{ind}} \) and \( C_{\text{ind}} \) are the initial data of the measurement task.

The maximum permissible absolute deviation of the readings from the measured frequency was calculated taking into account the maximum relative deviations of the bridge readings from the measured inductance \( \delta_L = 0.01 \) and the capacitance \( \delta_C = 0.005 \). An example of calculating the absolute deviation and the relative frequency error is presented below.

\[
\pm \Delta f = 0.5 f_{\text{ind}} \sqrt{\delta_L^2 + \delta_C^2} = 0.5 \cdot 356.061 \sqrt{0.005^2 + 0.01^2} = 1.990 \text{ kHz}
\]

\[
\delta_{f} = \frac{\pm \Delta f}{f_{\text{ind}}} \cdot 100\% = 0.56\%.
\]

From here, we express the obtained range of values of the resonant frequency of the serial oscillatory circuit, when measuring by the bridge method:

\[
356.061 - 1.990 \leq f \leq 356.061 + 1.990
\]

\[
354.071 \leq f \leq 358.051 \text{ kHz}.
\]

Let us consider the second version of the solution of this measurement problem. Figure 1 shows an equivalent installation scheme that implements the resonant principle of measuring the frequency of a serial oscillatory circuit.

**Figure 1.** Equivalent circuit of the measuring unit realizing the resonance principle of frequency measurement

With this measurement of the resonant frequency of the circuit, a voltmeter or an electron beam oscilloscope is used as a resonance indicator. However, their input capacitance \( C_{\text{in}} \), as well as the capacitance of the coaxial cable \( C_{\text{cab}} \), connecting the input of the resonance indicator to the circuit, can have a significant effect on the measurement result, since they form an additional capacitance \( C_{\text{add}} \), connected in parallel to the capacitance of the circuit and changing its resonance frequency [4]. The experimentally obtained value of the resonant frequency is determined by the relation:

\[
f_{\text{ex}} = \frac{1}{2\pi \sqrt{L(C + C_{\text{add}})}}
\]
where L and C are respectively the inductance and capacitance of the circuit. The experimentally obtained value of the resonance frequency will differ from the natural resonance frequency of the circuit, determined in accordance with the expression:

\[ f_c = \frac{1}{2\pi\sqrt{LC}}. \]

With constant \( C_{in} = 32 \text{ pF} \) and \( C_{cab} = 20 \text{ pF} \), the relative correction to the frequency reading will be:

\[ \beta = \frac{f_c - f_{ex}}{f_c} = 1 - \frac{C}{\sqrt{C + C_{add}}}. \]

The additional capacitance is calculated as the sum of the capacitance of the coaxial cable and the input capacitance. The absolute correction to the frequency meter reading caused by the presence of \( C_{add} \) will be:

\[ \theta = \beta \cdot f_c = \frac{\beta}{1 - \beta} \cdot f_{ex}. \]

Then the eigenfrequency of the circuit can be determined from the ratio:

\[ f_c = \frac{f_{ex}}{1 - \beta}. \]

The influence of the output impedance of the source of the signal (generator) of the active component of the complex input resistance of the resonance indicator due to the "blurring" of the resonance curve introduces an element of subjectivism in the search for an extremum [5]. Randomness in determining the voltage extremum can be taken into account in multiple frequency measurements with subsequent statistical processing of the data [6, 7]. Further calculations are made based on the results obtained experimentally using this method.

\[ f_i = f_{exi} + \theta_i. \]

Let us calculate the average value of the result of the frequency:

\[ \hat{f} = \frac{\sum f_i}{n} = 355.714 \text{ kHz}. \]

Next let us calculate the estimate of the root-mean-square deviation of the measurement result \( S_f = 0.175 \) and check the array of data obtained for the presence of gross errors (misses) according to the "3-sigma rule" \( 3S_f = 0.5268 \text{ kHz} \).

We calculate the estimate of the standard deviation of the arithmetic mean:

\[ S_{\hat{f}} = \frac{S_f}{\sqrt{n}} = 0.136 \text{ kHz}. \]

With confidence probability \( P = 0.95 \), the Student's coefficient is determined as \( t_p = 2.14 \) for \( n = 15 \). Half of the confidence interval is calculated as:

\[ \epsilon = t_p \cdot S_{\hat{f}} = 0.291 \text{ kHz}; \]
\( \delta_{2f} = \frac{\varepsilon}{f} \cdot 100\% = 0.08\% \).

Then the limits of the measured resonant frequency of the circuit are determined:

\[
\hat{f} - \varepsilon \leq f \leq \hat{f} + \varepsilon;
\]

\[
355.714 - 0.291 \leq f \leq 355.714 + 0.291.
\]

4. Conclusion

The limits in which the resonant frequency of the series oscillatory circuit is found, measured by the bridge method, have the following form: \(354.071 \leq f \leq 358.051\) kHz. Relative error - \(\delta_{1f} = 0.56\%\).

The second method of solving the problem requires more time and effort. Despite this, the confidence interval of the value of the resonant frequency found using the method based on the resonance principle has a much smaller scatter. The relative error is also lower than in the first version of the calculation. The limits in which the value of the resonant frequency of the series oscillatory circuit is found are: \(355.423 \leq f \leq 356.005\) kHz. Relative error - \(\delta_{2f} = 0.08\%\).

Since the first (bridge method) method of measuring frequency is not accurate enough and its relative error is higher in comparison with the second method of measuring the resonant frequency of a serial oscillatory circuit, then we will use it to develop the measurement procedure. Even in spite of the higher labor costs of the second method, only it provides the necessary accuracy, because the bridge method by relative error does not fit into the threshold value necessary for the development of the methodology for performing measurements.

References

[1] Bessonov L A 1978 *Theoretical Foundations of Electrical Engineering: Electric Chains* 528
[2] Gonorovskii I S 1986 *Radio circuits and signals* 512
[3] Bazhenov I N, Basov O O 2018 *Journal of Mining institute* 230 123-130
[4] Dubov G M, Dubinkin D M 2011 *Methods and means of measurement, testing and control* 224
[5] Divin A G, Ponomarev S V 2011 *Methods and means of measurement, testing and control* 104
[6] Medyakova E I 2009 *Methods and means of measurement, testing and control* 101
[7] Oleshchuk V A, Vereshchagina A S 2015 *Methods and means of measurement, testing and control* 92