Comment

Comment on ‘Improvements for drift-diffusion plasma fluid models with explicit time integration’

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Abstract

Recently, J Teunissen reported a fully explicit method, namely the current-limit approach, which claimed to overcome the dielectric relaxation time restriction for the drift-diffusion plasma fluid model. In this comment, we point out that the current-limit approach is not mathematically consistent, and discuss about the possible reason why the inconsistency was not visibly noticed.

Keywords: fluid model, plasma, current-limit approach, consistency

Recently, J Teunissen reported a fully explicit method which claimed to overcome the dielectric relaxation time restriction for the drift-diffusion plasma fluid model [1]. The dielectric relaxation time restriction results from the coupling between the Poisson equation and the charge carrier transport equations. This time restriction can be removed by semi-implicit schemes [2–5] at the price of solving a variable-coefficient elliptic equation which is generally more expensive than solving a constant-coefficient Poisson’s equation. Therefore, it would be valuable if one can use a fully explicit scheme to overcome the dielectric relaxation time restriction.

The main concern of this comment is that the fully explicit method reported by J Teunissen, namely the current-limit approach, is not mathematically consistent.

Quoted from [6], ‘Consistency of a discretization refers to a quantitative measure of the extent to which the exact solution satisfies the discrete problem. Stability of a discretization refers to a quantitative measure of the well-posedness of the discrete problem. A fundamental result in numerical analysis is that the error of a discretization may be bounded in terms of its consistency and stability’. A numerical scheme is consistent if its discrete operator converges towards the continuous operator of the PDE when \( \Delta x \to 0 \) and \( \Delta t \to 0 \), namely, the truncation error should vanish.

Without loss of generality, we omit the diffusion term in the fluid model. The transport equations in the fluid model may be written as

\[
\frac{\partial n}{\partial t} + \frac{\partial f}{\partial x} = s(n),
\]

and the semi-discretized form of equation (1) is,

\[
\frac{dn_i}{dt} + \frac{f_{i+1/2} - f_{i-1/2}}{\Delta x} = s(n_i).
\]

Then, equation (2) is consistent if it always converges to equation (1) as \( \Delta x \to 0 \), namely

\[
\frac{dn_i}{dt} + \frac{f_{i+1/2} - f_{i-1/2}}{\Delta x} = s(n_i) + O(\Delta x^p), \quad \text{with } p > 0.
\]

According to the current-limit approach, if the flux \( f_{i+1/2} > f_{\text{max}} \) where \( f_{\text{max}} = \frac{\varepsilon_0 E}{c \Delta t} \) (see equations (17) and (18) of [1]), the flux \( f_{i+1/2} \) is limited to be \( \hat{f}_{i+1/2} = f_{\text{max}} \), and
We solve equation (0.01 0) which is obviously different from the initial problem observed (figures 3 and 5 in [equation (4)]).

Table 1. Numerical convergence rate for equation (7) solved with equation (8).

| Δt  | Error (|exp(1) − ŷ(1)|) | Numerical convergence rate |
|-----|-----------------------|---------------------------|
| 0.04| 0.04988               |                           |
| 0.02| 0.02405               | 1.05                      |
| 0.01| 0.01079               | 1.16                      |
| 0.005| 0.004065             | 1.41                      |

Table 1 shows a numerically observed first order convergence. However, equation (8) is not consistent with equation (7), but is consistent with another different problem similar to equation (7):

\[
\frac{dy}{dt} = y + 0.001y, \quad y(0) = 1. \tag{9}
\]

Now we discuss the possible reason for why there were no visible differences between the current-limit approach and the explicit scheme shown in [1]. In the fluid model, \( f = nv \) and \( s(n) = \alpha n|v| = \alpha f \), with \( \alpha \) not small, and typically \( \alpha \gg 1 \). When both \( f_{i+1/2} \) and \( f_{i-1/2} \) are limited, \( f_{i-1/2} - f_{i+1/2} \) may cancel to a large degree; when \( f_{i-1/2} < f_{\text{max}} < f_{i+1/2} \) (i.e., only \( f_{i+1/2} \) is limited), because \( f_{i+1/2} \) and \( f_{i-1/2} \) are the fluxes of a same cell, they are generally close to each other, therefore, \( f_{\text{max}} \) is close to \( f_{i+1/2} \). In either case, the additional term may be much smaller than \( s(n_i) \). Therefore, the inconsistency may not be visibly noticed. This coincides with the observation in the example of equations (7) and (8).

Finally, we wish to emphasize that in this comment we focus on the mathematical characteristics of the current-limit approach itself, not on a possible way to get a visually similar result. We feel that a mathematically correct scheme is preferred for reliable simulations.

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References

[1] Teunissen J 2020 Improvements for drift-diffusion plasma fluid models with explicit time integration Plasma Sources Sci. Technol. 29 015010
[2] Ventzek P L G, Sommerer T J, Hoekstra R J and Kushner M J 1993 Two-dimensional hybrid model of inductively coupled plasma sources for etching Appl. Phys. Lett. 63 605–7
[3] Lin B, Zhuang C, Cai Z, Zeng R and Bao W 2020 An efficient and accurate MPI-based parallel simulator for streamer discharges in three dimensions J. Comput. Phys. 401 109026
[4] Villa A, Barbieri L, Gondola M and Malgesini R 2013 An asymptotic preserving scheme for the streamer simulation J. Comput. Phys. 242 86–102
[5] Villa A, Barbieri L, Gondola M, Leon-Garzon A R and Malgesini R 2017 An efficient algorithm for corona simulation with complex chemical models J. Comput. Phys. 337 233–51
[6] Arnold D N 2015 Stability, consistency, and convergence of numerical discretizations Encyclopedia of Applied and Computational Mathematics ed B Engquist (Berlin: Springer)