Soft dipole mode in $^{11}\text{Li}$ and three body continuum

Yu. A. Lurie and A. M. Shirokov∗
Institute for Nuclear Physics at Moscow State University,
Moscow 119899, Russia
and
Yu. F. Smirnov†
Instituto de Física, UNAM, Apartado postal 20-364,
Delegation Alvaro Obregon, 01000 México D.F., México

Abstract

Properties of the neutron rich $^{11}\text{Li}$ nucleus are calculated in the framework of the
cluster model $^{9}\text{Li}+n+n$. The formalism of the harmonic oscillator representation is
used for the description of bound and continuum spectrum states in the three-body-
democratic-decay approximation. It is shown that this approach allows one to take
into account adequately the long asymptotic tail of the $^{11}\text{Li}$ wave function (neutron
halo) and to reproduce correctly the binding energy, radius and $^{11}\text{Li}$ electromagnetic
dissociation cross-section on target nuclei. The shape and the energy position of
the $B(E1)$ peak corresponding to the soft dipole mode are also in agreement with
experiment.

Introduction

Recently secondary beams of radioactive heavy ions become available. As a result one get
a tool for experimental studies of light neutron-excess $\beta$-unstable nuclei. $^{11}\text{Li}$ is one of the
most interesting nuclei of the type, and its properties have being intensively studied both
experimentally [1]–[3] and theoretically (see, e.g., [4]–[24] and references therein).

The main peculiarity of the $^{11}\text{Li}$ nucleus is the so-called neutron halo formed by two
weakly-bound neutrons (two-neutron separation energy $\varepsilon_{2n} = 247\pm80$ keV; note, that there
are no bound states in two-body subsystems ($^{9}\text{Li}+n$) and ($n+n$) [25]). As the binding
energy of the neutron pair is small, the wave function decreases slowly at large distances.
It results in the anomalously large value of the $^{11}\text{Li}$ r.m.s. radius, $<r^2>_{^{11}\text{Li}}^{1/2} = 3.16 \pm 0.11$
fm [3] (to be compared to the one of $^{9}\text{Li}$, $<r^2>_{^{9}\text{Li}}^{1/2} = 2.32 \pm 0.02$ fm [25]). Evidently, the
anomalously large electromagnetic dissociation (ED) cross section of $^{11}\text{Li}$ beam on heavy

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†On leave of absence from Institute for Nuclear Physics at Moscow State University, Moscow 119899, Russia.
target nuclei is the manifestation of the neutron halo (the $^{11}$Li ED cross section on the Pb target is 20 times the one of the carbon beam [1]). To explain this effect it was suggested [7, 8] that oscillations of the halo neutrons with respect to the core $^9$Li give rise to the specific low energy branch of the giant dipole resonance (soft dipole mode) with excitation energy less than 1 MeV. Soft dipole mode is supposed to exhaust about 10% of the dipole energy-weighted sum rule (EWSR).

The shell-model [10, 11, 15, 19, 20], cluster [13, 17, 18, 22, 23, 24] and combined cluster-shell-model [12, 14, 24] approaches have been used in theoretical studies of the $^{11}$Li neutron halo properties. In some of these calculations (see, e.g., [10]–[14] a variational approach has been used which does not allow for continuum spectrum effects. There are also calculations with an allowance for two- [9, 15, 20] or three-body [17, 18, 22, 23, 24] continuum.

In this contribution we present the study of the neutron halo properties of $^{11}$Li in a three-body cluster model $^9$Li+n+n. We make use of a three-body wave function expansion in six-dimensional harmonic oscillator eigenfunctions. Our attention is focused on the effects of three-body continuum. The continuum spectra effects are accounted for in the framework of the oscillator representation of the scattering theory [26]. This method has been generalized [27] on the case of the true three-body scattering [28, 29] and has been successfully used recently in the study of the $^{12}$C monopole excitations in the cluster model $\alpha + \alpha + \alpha$ [30]. As it has been noted, there are no bound two-body subsystems like ($^9$Li+n) or (n+n) in the system. Therefore, only the so-called ”democratic” three-body decay [23, 31] is inherent to this system. It can be adequately described in the framework of the ”true” three-body scattering theory (democratic decay approximation) [29]. In the study of the ground and excited states properties we go beyond the pure diagonalization of the Hamiltonian matrix on the basis of oscillator functions. We search for the $S$-matrix pole corresponding to the ground state, and construct an infinite series expansion in oscillator functions for the ground and excited state wave functions. It enables us not only to calculate the $^{11}$Li ground state energy with high accuracy, but to describe also the wide spatial distribution of two valence neutrons in $^{11}$Li (neutron halo).

1 The model

The $^{11}$Li ground state and the continuum spectrum wave functions are calculated in the framework of the three-body cluster model $^9$Li+n+n. The model assumptions are the following ones:

1) The cluster $^9$Li is supposed to be structureless and the excitations of its internal degrees of freedom are not considered.

2) We don’t account for non-central components of the interaction between two valence neutrons and between valence neutron and the cluster $^9$Li. Therefore, the wave function can be characterized by the three-body orbital angular momentum $J$ and its projection $M$.

3) The states with the total spin of the valence neutron pair $S=0$ are only considered, and the ground state three-body orbital angular momentum is supposed to be equal to zero: $L=0$.

4) $n$-$^9$Li interaction is described by the shallow potential of Johansen et al. [13] effec-
tively simulating the Pauli principle \cite{32}. \( NN \)-interaction is described by the Gaussian potential \cite{13}.

5) Only democratic decay channels are allowed for.

The wave function of the system \( ^9Li+n+n \), \( \psi_{JM}(x,y) \), is expanded in three-body hyperspherical functions, \( \Phi^{Jy}_{Kx} \),

\[
\psi_{JM}(x,y) = \sum_{Kxly} \psi^{(J)}_{Kxly}(\rho) \Phi^{Jy}_{Kx} \Phi^{JM}_{K}(\hat{\rho}),
\]

where \( K \) is hypermomentum, \( l_x \) and \( l_y \) are the angular momenta corresponding to the Jacobi coordinates

\[
x = \sqrt{\frac{m\omega}{2\hbar}}(r_1 - r_2), \quad y = \sqrt{\frac{18m\omega}{11\hbar}}\left(\frac{r_1 + r_2}{2} - r_3\right),
\]

respectively, \( m \) is the neutron mass, \( r_i \) are coordinates of the valence neutrons \((i = 1, 2)\) and the cluster \( ^9Li \) \((i = 3)\), \( \rho = (x^2 + y^2)^{1/2} \) is a three-body hyperradius, \( \hat{\rho} \) stands for the set of angular variables in the six-dimensional space.

In the c. m. frame the Hamiltonian is of the form:

\[
H = T + V_{12} + V_{13} + V_{23},
\]

where \( T \) is the three-body relative motion kinetic energy operator, and \( V_{ij} \) are the two-body potentials. For the radial wave functions \( \psi^{(J)}_{Kxly}(\rho) \) we have the usual set of the \( K \)-harmonic method coupled equations (see, e.g., \cite{29}). The equations are solved by expanding the radial wave function,

\[
\psi^{(J)}_{Kxly}(\rho) = \sum_{n=0}^{\infty} D^{(J)}_{nKxly}(E) \varphi_{nK}(\rho),
\]

in the six-dimensional harmonic oscillator eigenfunctions,

\[
\varphi_{nK}(\rho) = (-1)^n \sqrt{\frac{2n!}{\Gamma(n+K+3)}} \rho^K L_n^{K+2}(\rho^2) \exp(-\rho^2/2),
\]

where \( L_n^{\alpha}(x) \) is the associated Laguerre polynomial.

For large values of the total number of oscillator quanta, \( N = 2n + K \), the potential energy matrix elements are small compared to the matrix elements of kinetic energy which increase linearly with \( N \). In the framework of the oscillator representation of scattering theory the matrix of the potential energy, \( V = V_{12} + V_{13} + V_{23} \), on the basis of functions \( \varphi_{nK}(\rho) \) is truncated, i.e. all matrix elements \( V_{N't't'}^{N'T'T'} \) for \( N > \tilde{N} \) and/or \( N' > \tilde{N} \) are neglected \( (\tilde{N} \) denotes the truncation boundary). The kinetic energy matrix, \( T_{N'T'T'}^{N'T'T'} \), is an infinite tridiagonal matrix for any value of the multi-index \( \Gamma = \{K, l_x, l_y\} \) which labels different decay channels in the hyperspherical representation. The eigenvectors of this matrix which are important for the description of the wave function asymptotical behavior at \( \rho \rightarrow \infty \), in the oscillator representation are of the form \cite{27}:

\[
D^{(J)}_{nKxly}(E) = \frac{1}{2} \left[ \delta_{\Gamma\Gamma'} C_{nK}^{(+)}(E) - S_{\Gamma\Gamma'} C_{nK}^{(-)}(E) \right] \quad \text{for } E \geq 0,
\]

\[
D^{(J)}_{nKxly}(E) = \alpha_{\Gamma} C_{nK}^{(+)}(E) \quad \text{for } E < 0,
\]
where $S_{\Gamma'}$ is the matrix element of the $S$-matrix, and $\alpha_{\Gamma}$ is the normalization constant of the bound state wave function for the channel $\Gamma$.

There are analytical expressions for the functions $C_{nK}(E)$ \[27\],

$$C_{nK}(E) = C_{nK}(E) \pm i S_{nK}(E),$$ \hspace{1cm} (8)

where

$$S_{nK}(E) = \frac{1}{\sqrt{\overline{\hbar \omega}}} \sqrt{\frac{2n!}{\Gamma(n+K+3)}} q^{K+2} L^{K+2}_n(q^2) \exp \left(-\frac{q^2}{2}\right) ,$$ \hspace{1cm} (9)

$$C_{nK}(E) = -\frac{1}{\pi S_{0K}(E)} P.V. \int_0^{\infty} dE' \frac{S_{nK}(E') S_{0K}(E')}{E - E'},$$ \hspace{1cm} (10)

and the total energy of the system, $E = \frac{1}{2} q^2 \bar{\hbar} \omega$. The functions $S_{nK}(E)$ and $C_{nK}(E)$ give rise to functions with the following asymptotics in the coordinate space:

$$\sum_{n=0}^{\infty} S_{nK}(E) \varphi_{nK}(\rho) = \frac{1}{\sqrt{\overline{\hbar \omega}} \rho^2} J_{K+2}(q \rho),$$ \hspace{1cm} (12)

$$\sum_{n=0}^{\infty} C_{nK}(E) \varphi_{nK}(\rho) \simeq \frac{1}{\sqrt{\overline{\hbar \omega}} \rho^2} N_{K+2}(q \rho),$$ \hspace{1cm} (13)

The asymptotic expressions \[11\] for $D^{(J)}_{nK_{\Gamma\Gamma'}}(E)$ correspond to the account of democratic decay channels only \[29\], i.e. the wave function in the asymptotic region is a superposition of an outgoing six-dimensional spherical wave in the channel $\Gamma'$ and ingoing six-dimensional spherical waves in all channels $\Gamma$. The applicability of the democratic decay approximation for various nuclear reactions with few particles in a final state has been discussed in \[29, 31\]. In the framework of the oscillator representation of scattering theory this approximation has been used \[30\] in the study of monopole excitations of $^{12}\text{C}$ in the cluster model $\alpha + \alpha + \alpha$. The analysis of experimental data for $2^+$ states in $A = 6$ nuclei within the $\alpha + N + N$ cluster model assumptions has been performed using this approximation in ref. \[33\]. This analysis has demonstrated that the approximation is an adequate tool for description of the decay states of Barromean nuclei.

The energy spectrum and the wave functions are calculated in the following way \[27\]. First of all, one should find eigenvalues, $E_{\lambda}^{(J)}$, and eigenvectors, $\{\gamma^{(\lambda)}_{n\Gamma'}(J)\}$, of the truncated Hamiltonian matrix, $\{H_{n\Gamma'}^{\lambda}(J)\}$, $2n + K \leq \tilde{N}$, $2n' + K' \leq \tilde{N}$; i.e. one should solve an eigenproblem for the set of equations

$$\sum_{n', \Gamma'}^{2n' + K' \leq \tilde{N}} [H_{n\Gamma}^{\lambda'}(J) - \delta_{n'n'} \delta_{\Gamma\Gamma'} E_{\lambda}^{(J)}] \gamma^{(\lambda)}_{n'\Gamma'}(J) = 0 , \hspace{1cm} 2n + K \leq \tilde{N} .$$ \hspace{1cm} (14)

In the usual variational approach, the lowest eigenvalue, $E_0^{(J=0)}$, and the corresponding eigenvector, $\{\gamma^{(0)}_{n\Gamma}(J=0)\}$ are treated as the ground state energy and the corresponding wave function in the oscillator representation. Such approximation is unable to describe the slowly dying asymptotic tail of the halo neutron space distribution. Thus, in order to
describe the neutron halo properties of the $^{11}\text{Li}$ nucleus, one is pushed to account for the asymptotic region $N \geq \tilde{N}$. For bound states this account is equivalent to the location of $S$-matrix poles. The $S$-matrix can be calculated by the equation [27]

$$S = (A^{(+)})^{-1}A^{(-)}.$$  \hspace{1cm} (15)

The matrices $A^{(+)}$ and $A^{(-)}$ have the following matrix elements:

$$A^{(\pm)}_{\Gamma' \Gamma} = \delta_{\Gamma' \Gamma} C^{(\pm)}_{\bar{n}, K}(E) + P^{\bar{n}', \Gamma'}_{\bar{n}, \Gamma}(E) T^{\Gamma'}_{\bar{n}', \bar{n}'+1} C^{(\pm)}_{\bar{n}'+1, K'}(E),$$ \hspace{1cm} (16)

the kinetic energy matrix elements,

$$T^{\Gamma}_{\bar{n}, n+1} = -\frac{1}{2} \hbar \omega \sqrt{(n+1)(n+K+3)}, \quad \bar{n} = \frac{1}{2}(\tilde{N} - K), \quad \bar{n}' = \frac{1}{2}(\tilde{N} - K'),$$

and

$$P^{\bar{n}', \Gamma'}_{\bar{n}, \Gamma}(E) = \sum_{\lambda} \frac{\gamma^{(\lambda)}_{\bar{n}' \Gamma'} \gamma^{(\lambda)}_{\bar{n} \Gamma}}{E_{\lambda} - E},$$ \hspace{1cm} (17)

To calculate the bound state energy, i.e. to locate the corresponding $S$-matrix pole, one should solve the nonlinear equation [27]

$$\det A^{(+)} = 0,$$ \hspace{1cm} (18)

which can be easily obtained from (13). Asymptotic normalization constants of the bound states, $\alpha_{\Gamma}$, can be found by numerical solution of the set of linear homogeneous equations

$$\sum_{\Gamma'} A^{(+)}_{\Gamma' \Gamma} \alpha_{\Gamma'} = 0.$$ \hspace{1cm} (19)

The set of $\alpha_{\Gamma}$ can be multiplied by any common multiplier. This uncertainty is eliminated by numerical normalization of the wave function. Now the coefficients $D^{(J)}_{n K l_x l_y}(E)$ of the expansion (4) in the asymptotic region $N \geq \tilde{N}$ can be easily obtained using (7).

For the continuum spectrum states we calculate $S$-matrix for any positive energy $E$ using (15)–(17), and than calculate the set of $D^{(J)}_{n K l_x l_y}(E)$ in the asymptotic region $N \geq \tilde{N}$ using (7).

The coefficients $D^{(J)}_{n K l_x l_y}(E)$ in the inner region $N \leq \tilde{N}$ are calculated by the expression:

$$D^{(J)}_{n K l_x l_y}(E) = -\sum_{\Gamma'} P^{\bar{n}', \Gamma'}_{\bar{n} \Gamma}(E) T^{\Gamma'}_{\bar{n}', \bar{n}'+1} D^{(J)}_{\bar{n}'+1, K' l_x' l_y'}(E),$$ \hspace{1cm} (20)

where $\bar{n}' = \frac{1}{2}(\tilde{N} - K')$ and for calculation of $D^{(J)}_{n+1, K l_x l_y}(E)$ eqs. (4) or (3) are used for continuum or bound state, respectively.

## 2 The results of the calculations

The interactions of the valence neutrons with each other and with the cluster $^9\text{Li}$ are described by the potentials $V_{12}(r_{12})$ and $V_{13}(r_{13}) = V_{23}(r_{23})$, respectively. We use the
following parametrization of the potentials \[13\]:

\[
V_{ij}(r) = V_{ij}^{(1)} \exp \left[ - \frac{r}{b_{ij}^{(1)}} \right]^2 + V_{ij}^{(2)} \exp \left[ - \frac{r}{b_{ij}^{(1)}} \right]^2,
\]

\[
V_{12}^{(1)} = -31 \text{ MeV}, \quad V_{12}^{(2)} = 0, \quad b_{12}^{(1)} = 1.8 \text{ fm};
\]

\[
V_{13}^{(1)} = -7 \text{ MeV}, \quad V_{13}^{(2)} = -1 \text{ MeV}, \quad b_{13}^{(1)} = 2.4 \text{ fm}, \quad b_{13}^{(2)} = 3.0 \text{ fm}.
\]

In the external asymptotic region \(N \geq \bar{N}\) we consequently allow for channels \(\Gamma\) characterized by \(K = K_{\text{min}}, K_{\text{min}} + 2, \ldots\) \((K_{\text{min}}\) is the minimal possible value of \(K\) for a given \(J\)) until the convergence for all physical properties under consideration is achieved. The convergence is found to be very good, and the allowance for the decay channels with \(K > K_{\text{min}} + 2\) do not yield any visual variation of the results. So, we consider in the external asymptotic region \(N > \bar{N}\) the channels with \(K \leq K_{\text{min}} + 2\) only. Note, that components with all possible values of \(K \leq \bar{N}\) are accounted for in the calculation of the wave function in the inner region \(N \leq \bar{N}\).

The parameter \(\hbar \omega\) is set to be equal to 7.1 MeV in our calculations. This value corresponds approximately to the minimum of ground state energy \(E_0\).

2.1 The ground state

The results for the \(^{11}\text{Li}\) ground state for different values of the truncation parameter \(\bar{N}\) are presented in the table 1. The variational ground state energies, \(E_0^{(d)}\), obtained by the pure diagonalization of the truncated Hamiltonian matrix are listed in the second column, while the \(J\)-matrix results, \(E_0\), which are the solutions of the eq. \([18]\) are listed in the third column. It is seen, that by locating the \(S\)-matrix pole using eq. \([18]\) that is equivalent to the allowance for the long asymptotic tail of the wave function, we improve essentially the convergence for the binding energy.

The results presented in the table 1 have been obtained using Lanczos smoothing of the three-body potential energy matrix \([34]\). It has been shown in ref. \([34]\) that the smoothing improves the convergency of calculations of two-body scattering phases. The results of the full \(J\)-matrix calculations of the ground state energy of \(^{11}\text{Li}\) obtained with the use of the smoothing, \(E_0\), and without it, \(E_0^{(w.s.)}\), for various values of the truncation boundary \(\bar{N}\) are presented in table 2. It is seen from the table 2 that the smoothing causes underestimation of the binding energy of a three-body system. Nevertheless, \(E_0^{(w.s.)}\) shows a staggering as \(\bar{N}\) increases. The smoothing results in a usual-type monotone decrease of \(E_0\) as \(\bar{N}\) increases and in a better convergence of continuum spectrum calculations. At the same time, the ground state wave functions obtained with the smoothing and without it differ very slightly. So, below we shall discuss only the results of calculations with the smoothed potential energy matrix. The role of the smoothing in three-body calculations we shall discuss in more detail elsewhere \([22]\).

The \(^{11}\text{Li}\) r.m.s. radius, \(<r^2>_{11}^{1/2}\), can be calculated by the following equation:

\[
<r^2>_{11}^{1/2} = \frac{9}{11} <r^2>_{9} + \frac{\hbar}{11 m \omega} <p^2>, \tag{21}
\]

\[\downarrow\]

Note, that \(J\)-matrix calculation is not a variational one, so, the ground state energy may behave non-monotonically as \(N\) increases.
Table 1: $^{11}$Li ground state properties (see text for details).

| Truncation boundary $\tilde{N}$ | Ground state energy, MeV | Neutron halo mean square radius $<r^2>_{\frac{1}{2}}$, fm |
|---------------------------------|--------------------------|-----------------------------------|
|                                 | $E_0^{(d)}$  | $E_0$                         | $<r^2>_{\frac{1}{2}}^{11}$  | $<r^2>_{\frac{1}{2}}^{11}$  |
| 12                              | -0.012       | -0.150            | 2.83                        | 3.31                        |
| 16                              | -0.116       | -0.199            | 2.91                        | 3.29                        |
| 20                              | -0.171       | -0.225            | 2.98                        | 3.31                        |
| 24                              | -0.202       | -0.240            | 3.04                        | 3.32                        |
| Experiment                      | -0.247±0.080 |                  | 3.16±0.11                   |

Table 2: The effect of the Lanczos smoothing on the ground state energy of $^{11}$Li (see text for details).

| $\tilde{N}$ | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
|-------------|----|----|----|----|----|----|----|----|
| $E_0$       |   |    |    |    |    |    |    |    |
| $E_0^{(w.s.)}$ | | | | | | | | |

where $<r^2>_{\frac{1}{2}}^{9}$ is the $^9$Li r.m.s. radius and the mean square value of the hyperradius, $<\rho^2>$, can be easily calculated using the ground state wave function. The values of $<r^2>_{\frac{1}{2}}^{11}$ and $<r^2>_{\frac{1}{2}}^{11}$ obtained by the pure diagonalization of the truncated Hamiltonian matrix and with the allowance for the asymptotic tail of the wave function, respectively, are presented in the 4-th and the 5-th columns of the table 1.

Figure 1 presents the transverse momentum distribution of the cluster $^9$Li in $^{11}$Li. This momentum distribution is currently supposed (see, e.g., [16]) to be proportional to the $^9$Li transverse momentum distribution $dN/dp_{\perp}$ in the fragmentation of high-energy $^{11}$Li beams on target nuclei. The experimental data for $^{11}$Li fragmentation [4] are also presented on the figure. The results of $J$-matrix calculations with $\tilde{N} = 16$ and $\tilde{N} = 24$ are so close to each other that their plots on fig.1 coalesce and cannot be distinguished.

It is seen that in calculation of the ground state, the allowance for the wave function asymptotics is very important for a weekly-bound system like $^{11}$Li. The terms of expansion (4) with the number of total oscillator quanta $N \simeq 100$ that cannot be obtained in the usual oscillator-basis variational calculations, play an essential role in the formation of the transverse momentum distribution, r.m.s. radius, etc. The convergence of $<r^2>_{\frac{1}{2}}^{11}$, transverse momentum distribution and other properties of the wave function (e.g., of the weights of its components) in the full $J$-matrix calculation is very good. Nevertheless, it is seen that the r.m.s. radius converges to a value that is somewhat larger than the experimental one, and the calculated transverse momentum distribution is narrower than the experimental one. These shortcomings can be overcome by the adjustment of $n$-$^9$Li potential. We have not aimed to fit the potential to the $^{11}$Li properties, we have just take its parameters from ref. [13]. The differences between the calculated and experimental values of r.m.s. radius arise from the fact that in ref. [13] the potential has been fitted to the $^{11}$Li r.m.s. radius and binding energy in variational calculations that are unable to reproduce these quantities with high accuracy.
Figure 1: The $^9$Li transverse momentum distribution in the ground state of $^{11}$Li. 1 — variational calculations, 2 — $J$-matrix calculations. Experimental data for the $^9$Li transverse momentum distribution in the fragmentation of $^{11}$Li 800 MeV/nucleon beam are taken from ref. [2].

2.2 The soft dipole mode

The dipole transition operator in our model is of the form

$$\mathcal{M}(E_1 \mu) = -\frac{N_v Z}{A} e \gamma Y_{1\mu}(\hat{y}),$$

where $e$ is the proton charge, $A = 11$, $Z = 3$ and the number of valence neutrons, $N_v = 2$. The operator (22) corresponds to the excitation of the three-body cluster modes only. The excitation energy of the first excited state of $^9$Li is relatively high ($\sim 4$ MeV). So, low-energy $E1$-transitions correspond to the excitation of the cluster degrees of freedom only and should be described by the operator (22).

The cluster reduced probability of the $E1$-transition, $\mathcal{B}(E1; E_f - E_0)$, associated with the operator (22), is displayed on the figure 2. In $J$-matrix approach, zero-width pure variational peaks of $\mathcal{B}(E1; E_f - E_0)$ gain finite widths and shift to lower energies. It is seen that in calculation of $\mathcal{B}(E1; E_f - E_0)$ it is important to allow for the asymptotics not only for continuum states, but for the ground state as well. Due to the wide spatial distribution of the halo neutrons density, large distances contribute essentially to the $E1$ strength in the low-energy region. To account for this contribution and to get a convergence, one should allow for ground and continuum state wave function components with very large values of...
Figure 2: Cluster $B(E1; \text{g.s.} \rightarrow \text{continuum})$ for $^{11}\text{Li}$. Zero-width peaks with diamonds on the top (1) given in arbitrary units have been obtained in the pure variational approach for both ground and exited states, i.e. the wave functions for the initial and the excited states have been obtained by the diagonalization of the truncated Hamiltonian matrix. The curve with dots (2) has been calculated with the variational ground state wave function, the continuum spectrum wave function has been obtained by the $J$-matrix approach. The solid curve (3) is the result of $J$-matrix calculations for both the ground and continuum spectrum states.

The total number of oscillator quanta $N \simeq 2000$ in calculations of $B(E1; E_f - E_0)$ . The large-distance $E1$-transitions enhance the peak of the cluster $B(E1; E_f - E_0)$ and shift it to a lower energy. Obviously, this peak should be associated with the soft dipole mode. Thus, the neutron halo manifests itself in the appearance of the soft dipole mode that arises from the shift and enhancement of the $B(E1; E_f - E_0)$ peak. This effect appears to be very important in the calculation of the $^{11}\text{Li}$ electromagnetic dissociation cross section (see below).

Large distances contribute essentially to $E2$- and $E0$-transitions, too. The shift and the enhancement of $B(E0; E_f - E_0)$ and of $B(E2; E_f - E_0)$ are even more pronounced. Nevertheless, $E2$- and $E0$-transitions do not play an important role in the electromagnetic dissociation, and we shall not discuss them here.

Figure 3 shows the comparison of the results of our calculations of cluster $B(E1; E_f - E_0)$ with the parametrization of experimental data of ref. [6]. The agreement is reasonable. The form of the $B(E1; E_f - E_0)$ peak is well reproduced, the discrepancy in the position of the $B(E1; E_f - E_0)$ maximum is supposed to be eliminated by the adjustment of the potentials. The results of the $B(E1; E_f - E_0)$ calculations of refs. [19, 23] are also depicted on fig. 3. The three-body cluster calculations with the allowance for democratic decay channels of ref.

\footnote{The classical turning point for the basis function with the total number of oscillator quanta $N = 1000$ is at a distance of $\approx 108$ fm from the origin.}
Figure 3: Comparison of our results for $\mathcal{B}(E1; \text{g.s.} \rightarrow \text{continuum})$ in $^{11}\text{Li}$ with results of other authors. 1 — this work ($J$-matrix method), 2 — ref. [23], 3 — ref. [19], 4 — experimental data parametrization of ref. [6].

[23] nicely reproduce the energy of the soft dipole mode. Nevertheless, the form of the peak in our calculations is reproduced better. Note, that the authors of ref. [23] used another set of the potential parameters. The calculations of ref. [19] with the allowance for two-body decay channels failed to reproduce both the position and the form of the $\mathcal{B}(E1; E_f - E_0)$ peak.

The soft dipole mode exhausts about 90% of the cluster EWSR associated with the operator (22), $S_{\text{clust}}(E1)$. Nevertheless, using the results of ref. [35] it is easy to obtain that

$$\frac{S_{\text{clust}}(E1)}{S_{\text{tot}}(E1)} = \frac{1}{12} \approx 8.3\%,$$

(23)

where $S_{\text{tot}}(E1)$ is the total EWSR accounting for excitations of all nucleons. So, the contribution from the soft dipole mode to the total EWSR is relatively small. In the vicinity of the sharp $\mathcal{B}(E1; E_f - E_0)$ maximum at the excitation energy $E \approx 1-2$ MeV only $\sim 8\%$ fraction of the total EWSR is exhausted. Nevertheless, the account for the soft dipole mode results in an essential increase of the electromagnetic dissociation cross section of 0.8 GeV/nucleon $^{11}\text{Li}$ beams on Pb and Cu targets. The wide space distribution of the halo neutrons density is also well-manifested in the electromagnetic dissociation of $^{11}\text{Li}$ beam. The contribution of the large-distance $E\lambda$-transitions to the cross section is about 50%. The calculations have been performed by the equivalent photon method [36]. The only parameter of the method is the minimal value of the impact parameter $b_{\text{min}}$. We use for $b_{\text{min}}$ the values of 9.0 fm for Pb and 6.8 fm for Cu target nuclei, respectively.
quantities are the sums of the $^{11}\text{Li}$ and target nucleus charge radii. With these values of $b_{min}$ we obtain for the electromagnetic dissociation cross sections the values of 0.966 barn for the Pb target and 0.132 barn for the Cu target; the corresponding experimental values are $0.890 \pm 0.110$ barn and $0.21 \pm 0.04$ barn, respectively [4]. $E0$- and $E2$-transitions give only 1.2% contribution in the cross sections.

$E1$-transitions in $^{11}\text{Li}$ have been studied in the framework of RPA+two-body-continuum model in refs. [15, 20]. Though our model assumptions differ significantly from the ones of refs. [13, 20], the results are in good agreement. For example, the excitation energy values corresponding to the peaks of the function $B(E1; E_f - E_0)$ displayed on the fig. 2, are very close to the values that one can find in refs. [15, 20].

Conclusions

It is shown, that cluster model $^9\text{Li}+n+n$ yields a good description of the ground state properties and $E1$-transitions in the $^{11}\text{Li}$ nucleus. The oscillator functions expansion technique may be used in the studies of weakly-bound systems with long-tailed wave functions, e.g., in the study of neutron halo properties. For both bound and continuum states the correct account of the wave function asymptotics in the framework of the oscillator representation of scattering theory is very important in such studies. Low-energy $E1$-transitions in $^{11}\text{Li}$ are of the cluster nature. The widths and the position of resonant states calculated in the democratic decay approximation are in a reasonable agreement with experiment.

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