An Output-Feedback PID-type Global Regulator for Robot Manipulators with Bounded Inputs

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Abstract: An output-feedback PID-type controller for the global stabilization of manipulators with bounded inputs is proposed. It guarantees the global regulation goal preventing input saturation while avoiding the exact knowledge of the system model and parameter values as well as the need for velocity measurements. Furthermore, it adopts an SPD-SI structure by keeping both the P and D actions together within a generalized saturation function while including an additional similar saturating integral action separately. So far, a formal analytical formulation for such a saturating PID-type control structure was not available under the absence of velocity measurements. Experimental results on a 2-degree-of-freedom direct-drive manipulator corroborate the efficiency of the proposed controller. Copyright © 2015 IFAC

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1. INTRODUCTION

Classical Proportional-Integral-Derivative (PID) control has been a usual practice for the stabilization of robot manipulators in actual applications (Rocco, 1996). This is mainly due to its effectiveness on the achievement of the regulation objective experienced through its simple linear structure which avoids involving accurate data from the system, such as parameter values or model expressions. Nevertheless, through such a simple linear structure, there is no analytical certainty that the experienced stability properties have a global character. For this reason, alternative nonlinear versions of the PID controller, aiming at guaranteeing global regulation, have been developed for instance in (Kelly, 1998; Santibanez and Kelly, 1998). However, these algorithms implicitly assume that actuators can generate any torque value. Unfortunately, this is unrealistic in view of the saturation phenomenon commonly observed in real actuators. Furthermore, disregarding such natural constraints may lead to undesirable system behaviors and/or degraded closed-loop performances (Krikelis and Barkas, 1984). For this reason, bounded PID-type approaches have been further developed. For instance, semiglobal regulators with different saturating PID-type structures have been developed in (Alvarez-Ramirez et al., 2003) and (Alvarez-Ramirez et al., 2008). Through the singular perturbation methodology, these works showed the existence of an appropriate tuning mainly characterized by the requirement of small enough integral action gains and sufficiently high proportional and derivative ones. As far as the authors are aware, the first bounded PID-type controller for global regulation was presented in (Gorez, 1999); the algorithm permits to include or disregard velocities in the feedback. Nevertheless, the structure of the developed scheme is quite complex. Other works have focused on the solution of the global PID position stabilization problem for manipulators with bounded inputs through simpler structures, giving rise to the SP-SI-SD type algorithm developed in (Meza et al., 2005) via passivity theory and later on in (Su et al., 2010) through Lyapunov stability analysis, and to the SPD-SI type scheme presented in (Santibanez et al., 2008). In particular, the work in (Su et al., 2010) includes a velocity-free version of the proposed SP-SI-SD algorithm through the conventional (linear) dirty derivative (dynamic) operator.

The above cited bounded PID-type approaches give a solution to the formulated problem under input constraints and restricted data. In this direction, output-feedback schemes, like the velocity-free extensions of the algorithms...
presented in (Gorz, 1999) and (Su et al., 2010), are particularly important since they achieve regulation not only without the need for the exact knowledge of the system structure and parameter values but also through the exclusive feedback of the position variables. This proves to be particularly useful when velocity measurements are unavailable which seems a common practical situation. However, it is not yet clear how can a bounded output-feedback version/extension of the SPD-SI structure could be analytically supported. A solution to such an open problem has not only been motivated by the implicated analytical challenge but also by the nice performance expectations generated by analog SPD-type structures in gravity-compensation-type state-feedback contexts (Zavala-Río and Santibáñez, 2006). Such a solution is developed in this work by contributing an output-feedback global regulator for robot manipulators with bounded inputs that adopts an SPD-SI structure by keeping both the P and D actions together within a generalized saturation function while including an additional similar saturating integral action separately. Moreover, the proposed scheme permits the choice of the saturation functions and releases the control gains from saturation avoidance conditions. The global regulation objective is guaranteed —avoiding input saturation— considerably reducing the system data involved in the feedback by releasing this not only from exact knowledge of the system model and parameter values but also from velocity measurements. Experimental tests on a 2-degree-of-freedom (DOF) direct-drive manipulator corroborate the contributed result.

2. PRELIMINARIES

Let $X \in \mathbb{R}^{m \times n}$ and $y \in \mathbb{R}^n$. Throughout this paper, $X_{ij}$ represents the element of $X$ at its $i^{th}$ row and $j^{th}$ column, and $y_i$ denotes the $i^{th}$ element of $y$. $O_n$ stands for the origin of $\mathbb{R}^n$ and $I_n$ represents the $n \times n$ identity matrix. $\| \cdot \|$ denotes the standard Euclidean norm for vectors, i.e. $\| y \| = \sqrt{\sum_{i=1}^n y_i^2}$, and induced norm for matrices, i.e. $\| X \| = \sqrt{\lambda_{\text{max}} (X^T X)}$ where $\lambda_{\text{max}} (X^T X)$ represents the maximum eigenvalue of $X^T X$. For a continuous scalar function $\psi : \mathbb{R} \rightarrow \mathbb{R}$, $\psi'$ denotes its derivative, when differentiable, $D^+\psi$ its upper right-hand (Dini) derivative, i.e. $D^+\psi(\cdot) = \limsup_{x \to t^+} \frac{\psi(x) - \psi(t)}{x - t}$, with $D^+\psi = \psi'$ at points of differentiability (Khalil, 2002, Appendix C.2), and $\psi^{-1}$ its inverse, when invertible.

Consider the $n$-DOF serial rigid manipulator dynamics with viscous friction (Kelly et al., 2005)

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + F_q + g(q) = \tau$$ (1)

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ are, respectively, the position (generalized coordinates), velocity, and acceleration vectors, $H(q) \in \mathbb{R}^{n \times n}$ is the inertia matrix, and $C(q, \dot{q})\dot{q}, F_q, g(q), \tau \in \mathbb{R}^n$ are respectively the vectors of Coriolis and centrifugal, viscous friction, gravity, and external input generalized forces, with $F_q \in \mathbb{R}^{n \times n}$ being a positive definite constant diagonal matrix whose entries $f_i > 0, i = 1, \ldots, n$, are the viscous friction coefficients, and $g(q) = \nabla U(q)$, with $U(q)$ being the gravitational potential energy, or equivalently

$$U(q) = U(q_0) + \int_{q_0}^q g^T(r)dr$$ (2a)

with

$$\int_{q_0}^q g^T(r)dr = \int_{q_{n0}}^{q_1} g_1(r_1, q_{02}, \ldots, q_{0n})dr_1 + \ldots + \int_{q_{n0}}^{q_n} g_n(q_1, \ldots, q_{n-1}, r_n)dr_n$$ (2b)

for any $1 \leq q, q_0 \leq \mathbb{R}^n$. Some well-known properties characterizing the terms of such a dynamical model are recalled here (Kelly et al., 2005, Chap. 4). Subsequently, we denote $\dot{H}$ the rate of change of $H$, i.e., $\dot{H} : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ : $(q, \dot{q}) \mapsto \frac{\partial H}{\partial q}(q, \dot{q})$. 

Property 1. $H(q)$ is a continuously differentiable matrix function being positive definite, symmetric and bounded on $\mathbb{R}^n$, i.e. such that $\mu_M I_n \leq H(q) \leq \mu_M I_n, \forall q \in \mathbb{R}^n$, for some constants $\mu_M \geq \mu_M > 0$.

Property 2. The Coriolis matrix $C(q, \dot{q})$ satisfies:

1. $\| C(q, \dot{q}) \| \leq k_C \| \dot{q} \|, \forall (q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$, for some constant $k_C \geq 0$;

2. for all $(q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^q, \dot{q}^T \left[ \frac{1}{2} \dot{H}(q, \dot{q}) - C(q, \dot{q}) \right] \dot{q} = 0$ and actually $\dot{H}(q, \dot{q}) = C(q, \dot{q}) + C^T(q, \dot{q})$.

Property 3. The viscous friction coefficient matrix satisfies $f_m \| \dot{q} \|^2 \leq \dot{q}^T F_q \leq f_M \| \dot{q} \|^2, \forall q \in \mathbb{R}^n$, where $0 < f_m \leq f_M \leq \max_j \{f_j\} \leq \max_i \{f_i\} = f_M$.

Property 4. The gravity force term $g(q)$ is a continuously differentiable bounded vector function with bounded Jacobian matrix $\frac{\partial g}{\partial q}$. Equivalently, every element of the gravity force vector, $g_i(q), i = 1, \ldots, n$, satisfies:

1. $|g_i(q)| \leq B_{gi}, \forall q \in \mathbb{R}^n$, for some constant $B_{gi} > 0$;

2. $\frac{\partial g_i}{\partial q_j}, j = 1, \ldots, n$, exist and are continuous and such that $\frac{\partial g_i}{\partial q_j}(q) \leq k_{gi}, \forall q \in \mathbb{R}^n$, for some positive constant $k_{gi}$, and consequently $|g_i(x) - g_i(y)| \leq \|g(x) - g(y)\| \leq k_{gi} \|x - y\|, \forall x, y \in \mathbb{R}^n$.

Let us suppose that the absolute value of each input $\tau_i$ is constrained to be smaller than a given saturation bound $T_i > 0$, i.e., $|\tau_i| \leq T_i, i = 1, \ldots, n$. More precisely, letting $u_i$ represent the control variable (controller output) relative to the $i^{th}$ degree of freedom, we have that

$$\tau_i = T_i \text{sat}(u_i/T_i) \quad (3)$$

where sat$(\cdot)$ is the standard saturation function, i.e. sat$(\cdot) = \text{sign}(\cdot) \min \{|\cdot|, 1\}$.

From Eqs. (1),(3), one sees that $T_i \geq B_{gi}, i = 1, \ldots, n$, is a necessary condition for the robot to be stabilizable at any desired equilibrium configuration $q_d \in \mathbb{R}^n$. This important fact is integrated to the analytical framework of the present work as follows.

Assumption 1. $T_i > \alpha B_{gi}, i = 1, \ldots, n$, for some $\alpha \geq 1$.

Functions fitting the following definition will be involved.

1 Since $g(q)$ is the gradient of the gravitational potential energy $U(q)$, a scalar function, then, for any $q, q_0 \in \mathbb{R}^n$, the inverse relation in (2a) is independent of the integration path (Khalil, 2002, p. 120). Eq. (2b) considers integration along the axes. This way, on every axis (i.e. at every integral in the right-hand side of (2b)), the corresponding coordinate varies (according to the specified integral limits) while the rest of the coordinates remain constant.

2 Property 4 is satisfied e.g. by robots having only revolute joints (Kelly et al., 2005, §4.3).
Definition 1. Given a positive constant $M$, a non-decreasing Lipschitz-continuous function $\sigma: \mathbb{R} \to \mathbb{R}$ is said to be a generalized saturation with bound $M$ if

(a) $\zeta \sigma(\zeta) > 0, \forall \zeta \neq 0$;
(b) $|\sigma(\zeta)| \leq M, \forall \zeta \in \mathbb{R}$.

If in addition
(c) $\sigma(\zeta) = \zeta$ when $|\zeta| \leq L$,
for some positive constant $L \leq M$, $\sigma$ is said to be a linear saturation for $(L, M)$.

Lemma 1. Let $\sigma: \mathbb{R} \to \mathbb{R}$ be a generalized saturation with bound $M$ and let $k$ be a positive constant. Then

1. $\lim_{\zeta \to \infty} D^+ \sigma(\zeta) = 0$;
2. $\exists \zeta_M' \in (0, \infty)$ such that $0 \leq D^+ \sigma(\zeta) \leq \zeta_M', \forall \zeta \in \mathbb{R}$;
3. $|\sigma(\zeta + \eta) - \sigma(\eta)| \leq \zeta_M' \zeta \eta$, $\forall \zeta, \eta \in \mathbb{R}$;
4. $\zeta_M' \zeta \eta^2 \leq \int_0^\zeta \sigma(k \eta) \, dk \leq \zeta_M'^2 \zeta \eta^2$, $\forall \zeta, \eta \in \mathbb{R}$;
5. $\int_0^\zeta \sigma(k \eta) \, dk \to 0, \forall \zeta > 0$;
6. $\int_0^\infty \zeta \eta \, d\eta \to \infty$ as $|\zeta| \to \infty$;
7. With $\sigma$ strictly increasing, for any constant $a$, $\sigma(a + \zeta) - \sigma(a)$ is a strictly increasing generalized saturation function with bound $M = M + |\sigma(a)|$.

Proof. Items 1, 2, 5–8 are proven in (López-Araujo et al., 2013). As for items 3 and 4, see Appendix A. □

3. THE PROPOSED CONTROL SCHEME

The proposed control law is defined as

$$u(q, \vartheta, \phi) = -s_p(K_p q + K_D \vartheta) + s_i(K_I \phi) \tag{4}$$

where $\dot{q} = q - q_d$, for any constant desired equilibrium position vector $q_d \in \mathbb{R}^n; \vartheta, \phi \in \mathbb{R}^n$ are the output vector variables of the integral-action dynamics, defined as

$$\dot{\phi} = -\varepsilon K_I^{-1} s_p(K_p q) \tag{5a}$$

and the velocity estimation subsystem, defined as

$$\dot{\vartheta} = -A[\vartheta + B \dot{q}] \tag{6a}$$

$$\dot{q} = \vartheta + B \dot{q} \tag{6b}$$

respectively; for any $x \in \mathbb{R}^n$, $s_p(x) = (\sigma_{P_1}(x_1), \ldots, \sigma_{P_n}(x_n))^T$ and $s_i(x) = (\sigma_{I_1}(x_1), \ldots, \sigma_{I_n}(x_n))^T$, with $\sigma_{P_i}(\cdot), \sigma_{I_i}(\cdot), i = 1, \ldots, n$, being linear saturation functions for $(L_{P_i}, M_{P_i})$ and $\sigma_{I_i}(\cdot), i = 1, \ldots, n$, being strictly increasing generalized saturation functions with bounds $M_{I_i}$, such that

$$L_{P_i} > 2B_{gi} \tag{7a}$$
$$M_{I_i} > B_{gi} \tag{7b}$$
$$M_{P_i} + M_{I_i} \leq T_i \tag{7c}$$

for $i = 1, \ldots, n; K_p = \text{diag}[k_{P_1}, \ldots, k_{P_n}], K_D = \text{diag}[k_{D_1}, \ldots, k_{D_n}], K_I = \text{diag}[k_{I_1}, \ldots, k_{I_n}], A = \text{diag}[a_1, \ldots, a_n]$ and $B = \text{diag}[b_1, \ldots, b_n]$, with $k_{P_i} > 0, \forall i = 1, \ldots, n$, and the rest of the control gains being positive constants such that

$$k_{P_i} \triangleq \min_{i} \{k_{P_i} \} > k_g \tag{8a}$$

(see Property 4.2) and

$$\beta_d \triangleq \min_{i} \left\{ \frac{a_i}{b_i} \right\} > \frac{\kappa}{2f_m} \tag{8b}$$

with $\kappa \triangleq \max_{i} \{\sigma_{P_i M} k_{P_i}, \sigma'_{P_i M} \}$ being the positive bound of $D^+ \sigma_{P_i}(\cdot)$, in accordance to item 2 of Lemma 1; and $\varepsilon$ (in Eq. (5a)) is a positive constant satisfying

$$\varepsilon < \varepsilon_M \triangleq \min\{\varepsilon_1, \varepsilon_2, \varepsilon_3\} \tag{9}$$

where

$$\varepsilon_1 \triangleq \frac{\beta_0 \beta D \mu_m}{\mu_M^2}, \quad \varepsilon_2 \triangleq \frac{\beta_0 \beta_D k_{Pm}}{k} \tag{10a}$$

(observe that by inequality (8b): $f_m - \frac{\kappa}{2f_m} > 0$), with $\beta_0 \triangleq 1 - \max\{k_{P_m}, \max\{2B_{gi}, T_{gi}\}\}$ (observe that by inequalities (8a) and (7a): $0 < \beta_0 < 1$, $\beta_M \triangleq k_{C} B_{P} + \mu_{M} \sigma'_{P M}$, $\beta_P \triangleq \min\{\frac{k_{P_m}}{\sigma'_{P M}}, 1\}$, $B_P \triangleq \sqrt{\sum_{i=1}^{n} \left(\frac{M_{P_i}}{k_{P_i}}\right)^2}$, $\sigma'_{P M} \triangleq \max\{\sigma_{P M}, 1\}$, and $\mu_{P}, \mu_{M}, k_{C}, f_{m}, M_{B}, B_{gi}$ and $k_g$ as defined through Properties 1–4.

Remark 1. Note that $\dot{q}$ is not involved in any of the expressions in Eqs. (4)–(6). In fact, $\dot{q}$ is estimated on-line through the auxiliary subsystem in Eqs. (6), driven by $\dot{q}$ as input variable. Its output variable $\vartheta$ gives the estimated vector value of $q$. In fact, the auxiliary subsystem in Eqs. (6) gives rise to the so-called dirty derivative of $q$. This is the derivative of $q$ (or the velocity vector $\dot{q}$) with every of its components going through a first-order low pass filter. This is commonly done in practice to bound the high-frequency gains, giving rise to a causal (approximated) derivative operator.

Remark 2. Let us note that inequalities (7) (stating conditions on the saturation function parameters) require the satisfaction of Assumption 1 with $\alpha = 3$. A similar condition on the control input bounds has been required by other approaches where input constraints have been considered (Colbaugh et al., 1997). In saturating PID-type schemes from previous references, a similar or analog condition on the control input bounds remains implicit by requiring corresponding parameters to be high enough to satisfy conditions coming from the stability analysis and simultaneously low enough to fulfill the input-saturation-avoidance inequalities.

4. CLOSED-LOOP ANALYSIS

Consider system (1),(3) taking $u = u(q, \vartheta, \phi)$ as defined through Eqs. (4)–(6). Define the variable transformation

$$\begin{pmatrix} \dot{q} \\ \dot{\vartheta} \end{pmatrix} = \begin{pmatrix} q - q_d \\ \vartheta + B(q - q_d) \end{pmatrix} \quad \begin{pmatrix} \phi - \phi^* \end{pmatrix} \tag{10}$$

with $\phi^* = (\phi^*_1, \ldots, \phi^*_n)^T$ such that $s_f(K_f \phi^*) = g(q_d)$, or equivalently $\phi^*_i = \sigma_{I_i}^{-1}(g_i(q_d))/k_{I_i}$, $i = 1, \ldots, n$ (notice...
that their strictly increasing character renders the generalized saturation functions $\sigma_i$ invertible. Observe that, for every $i \in \{1, \ldots, n\}$ and all $(q, \vartheta, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$, by (7c) and the strictly increasing character of $\sigma_i$, we have that $|u_i(q, \vartheta, \phi)| \leq |\sigma_i(k_Pq_i + k_D\vartheta_i)| + |\sigma_i(k_I\dot{q}_i)| < M_{P_i} + M_{I_i} \leq T_i$. From this and (3), one sees that

$$T_i > |u_i(q, q, \vartheta, \phi, \phi^*)| = |u_i| = |\tau_i| \quad i = 1, \ldots, n \quad \forall (q, \vartheta, \phi) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \quad (11)$$

Hence, under the consideration of the variable change (10), the closed-loop dynamics adopts the (equivalent) form

$$H(q, \dot{q}) + C(q, \dot{q}) + g(q) + \bar{q}$$

$$= -s_p(K_Pq + K_D\vartheta) + \bar{q} + g(q_0)$$

$$= -A\dot{\bar{q}} + B\bar{q}$$

where $\bar{q}(\vartheta) = s_i(K_I\dot{\vartheta} + K_I\vartheta) - s_i(K_I\dot{\vartheta})$. Observe that, by item 8 of Lemma 1, the elements of $\vartheta_i(q_0)$, i.e., $\sigma_i(k_I\dot{\vartheta}_i) = \sigma_i(k_I\dot{\vartheta}_i) + k_I\dot{\vartheta}_i - \sigma_i(k_I\dot{\vartheta}_i)$, $i = 1, \ldots, n$, turn out to be strictly increasing generalized saturations.

**Proposition 1.** Consider the closed-loop system in Eqs. (12), under the satisfaction of Assumption 1 with $\alpha = 3$ and inequalities (7). Thus, for any positive definite diagonal matrices $K_I$, $K_P$, $K_D$, $A$ and $B$ such that inequalities (8) are satisfied, and any $\varepsilon$ fulfilling inequality (9), global asymptotic stability of the closed-loop trivial solution $(q, \dot{q}) = (0, 0)$ is guaranteed with $|\tau_i(t)| = |u_i(t)| < T_i$, $i = 1, \ldots, n$, $\forall t \geq 0$.

**Proof.** By (11), one sees that, along the system trajectories, $|\tau_i(t)| = |u_i(t)| < T_i$, $\forall t \geq 0$. This proves that, under the proposed scheme, the input saturation values, $T_i$, are never reached. Now, in order to carry out the stability analysis, a scalar function $V(q, \dot{q}, \vartheta, \phi)$ is defined as follows

$$V = \frac{1}{2}q^TH(q, \dot{q}) + \varepsilon s_p(K_Pq)K_P^{-1}H(q, \dot{q}) + V(q, q_0) - U(q)$$

$$- g^T(q_0)q + \int_0^q s_p(K_Pq)dr + \int_0^\phi s_q^T(r)dr + \frac{\varepsilon}{2}g^T\dot{q}B^{-1}\dot{q}$$

where $\int_0^\phi s_q^T(r)dr = \sum_{i=1}^n \int_0^\phi \sigma_i(I_i)dr_i$, $\int_0^\phi s_q^T(K_Pq)dr = \sum_{i=1}^n \sigma_p(k_P(I_i)dr_i$, and recall that $U$ represents the gravitational potential energy. Note, by recalling Eqs. (2), that the defined scalar function can be rewritten as

$$V = \frac{1}{2}q^TH(q, \dot{q}) + \varepsilon s_p(K_Pq)K_P^{-1}H(q, \dot{q}) + U_\gamma(q, \dot{q})$$

$$\quad + \gamma_0 \int_0^\phi s_p(K_Pq)dr + \int_0^\phi s_q^T(r)dr + \frac{\varepsilon}{2}g^T\dot{q}B^{-1}\dot{q}$$

where

$$U_\gamma(q, \dot{q}) = \int_0^\phi \left[ q(r - \dot{q}) - g(q_0) - (1 - \gamma_0)s_p(K_Pq) \right] dr$$

$$= \sum_{i=1}^n \int_0^\phi \left[ q_i(r_i - \dot{q}_i) - g(q_0) - (1 - \gamma_0)s_p(k_Pq_i) \right] dr_i$$

with $\tilde{g}_1(r_1) = g_1(r_1 + q_1r, q_2, \ldots, q_dn)$

$$\tilde{g}_2(r_2) = g_2(q_1, r_2 + q_2, q_3, \ldots, q_dn)$$

$$\vdots$$

$$\tilde{g}_n(r_n) = g_n(q_1, q_2, \ldots, q_{n-1}, r_n + q_dn)$$

and $\gamma_0$ is a constant satisfying

$$\beta_0 \frac{\varepsilon^2}{T_i^2} < \gamma_0 < \beta_0$$

(observe, from inequality (9) and the definition of $\beta_0$, that $0 < \beta_0 \varepsilon^2/T_i^2 < \beta_0 < 1$). Under this consideration, $U_\gamma(q, \dot{q})$ turns out to be lower-bounded by

$$W_{10}(\bar{q}) = \sum_{i=1}^n u_{10}(\tilde{q}_i)$$

where

$$u_{10}(\tilde{q}_i) \equiv \begin{cases} \frac{k_{1i}}{2} \tilde{q}_i^2, & \text{if } |\tilde{q}_i| \leq \tilde{q}_i^* \\ \frac{k_{1i}q_i^*}{2} & \text{if } |\tilde{q}_i| > \tilde{q}_i^* \end{cases}$$

with $0 < k_{1i} \leq (1 - \gamma_0)k_{P_i} - k_\gamma$ and $\tilde{q}_i^* = \left[ L_{P_i} - 2B_{P_i}/(1 - \gamma_0)\right]/k_{P_i}$ (note that by inequality (13) and the definition of $\beta_0$: $0 < (1 - \gamma_0)k_{P_i} - k_\gamma$ and $\tilde{q}_i^* > 0$). This is proven in (Mendoza et al., 2015, Appendix 2). From this, Property 1 and item 5 of Lemma 1, we have

$$V \geq \frac{\mu_m}{2} ||\dot{q}||^2 - \varepsilon \mu_M ||K_P^{-1}s_p(K_Pq)||^2||q|| + W_{10}(\bar{q})$$

$$\quad + \gamma_0 \sum_{i=1}^n \sigma_p^2(k_Pq_i) + \int_0^\phi s_q^T(r)dr + \frac{\varepsilon}{2}g^T\dot{q}B^{-1}\dot{q}$$

$$\geq W_{11}(\tilde{q}, \dot{q}) + W_{10}(\bar{q}) + \int_0^\phi s_q^T(r)dr + \frac{\varepsilon}{2}g^T\dot{q}B^{-1}\dot{q}$$

where $W_{11}(\tilde{q}, \dot{q})$

$$= \frac{\mu_m}{2} ||\dot{q}||^2 - \varepsilon \mu_M ||K_P^{-1}s_p(K_Pq)||^2||q||$$

$$\quad + \gamma_0 \frac{\varepsilon}{2} ||K_P^{-1}s_p(K_Pq)||^2$$

$$\quad + \frac{1}{2} \left( ||K_P^{-1}s_p(K_Pq)|| \right)^T \left( ||K_P^{-1}s_p(K_Pq)|| \right)$$

with $Q^{11} = \left( \gamma_0 \frac{\varepsilon}{2} - \varepsilon \mu_M \right) Q^{11} - \varepsilon \mu_M Q^{11}$. By inequality (13), $W_{11}(\tilde{q}, \dot{q})$ is positive definite (since with $\varepsilon < \varepsilon_M \leq \varepsilon_1$, in accordance to inequality (9), any $\gamma_0$ satisfying (13) renders $Q^{11}$ positive definite) and note that $W_{11}(0, \tilde{q}) \rightarrow \infty$ as $||\tilde{q}|| \rightarrow \infty$ while, from Eqs. (14) and items 6-7 of Lemma 1, it is clear that $W_{10}$ and the integral term in the right-hand side of (15) are radially unbounded positive definite functions of $q$ and $\phi$ respectively. Thus, $V(\tilde{q}, \dot{q})$ is concluded to be positive definite and radially unbounded. Its upper right-hand derivative along the system trajectories, $\dot{V} = D^+V$ (Michel et al., 2008, §6.1A), is given by

$$\dot{V} = -\tilde{q}^TF\tilde{q} - \tilde{q}^Ts_\gamma(\tilde{q}, \varphi) - \varepsilon s_p^2(K_Pq)K_P^{-1}s_p\tilde{q}$$

$$\quad - \varepsilon s_p^2(K_Pq)K_P^{-1}s_p\tilde{q} + \varphi + \varepsilon \tilde{q}^T C(q, \dot{q})K_P^{-1}s_p(K_Pq) - \kappa \tilde{q}^T B^{-1}A\dot{q} + \kappa \dot{q}T\tilde{q}$$

where $H(q, \dot{q})$ and $\tilde{q}$ have been replaced by their equivalent expressions from the closed-loop dynamics in Eqs. (12), Property 2.2 has been used and

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Note that, in the error variable space, $q = \tilde{q} + q_d$. Consequently $H(q) = H(\bar{q} + \bar{q}_d)$, $C(q, \bar{q}) = C(\bar{q} + \bar{q}_d, \bar{q})$, and $g(q) = g(\bar{q} + \bar{q}_d)$. However, for the sake of simplicity, $H(q)$, $C(q, \bar{q})$, and $g(q)$ are used throughout the paper. Moreover, the arguments of $V$ and its derivative along the system trajectories, $\dot{V}$, will be dropped throughout the developments.
\[ s'_p(Kp\bar{q}) \triangleq \text{diag}[D^+\sigma_p(k_p\bar{q}_1), \ldots, D^+\sigma_p(k_p\bar{q}_n)] \]
\[ s_d(\bar{q}, \vartheta) \triangleq s_p(Kp\bar{q} + K_D\vartheta) - s_p(Kp\bar{q}) \]

The resulting expression can be rewritten as
\[ \dot{V} = \frac{\dot{q}^T[k\vartheta - s_d(\bar{q}, \vartheta)]}{\varepsilon \sigma_p(Kp\bar{q})K_p^{-1}F\dot{q} - \varepsilon \dot{\gamma}_1\sigma_p(Kp\bar{q})K_p^{-1}K_p^{-1}K_p^{-1}S_p(Kp\bar{q}) - \varepsilon \dot{W}_{\gamma_1}(\bar{q}) - \varepsilon \dot{s}_p(Kp\bar{q})\dot{K}_p^{-1}S_p(Kp\bar{q}) H(\dot{q})\dot{q} + \varepsilon \dot{q}^T \dot{C}(\dot{q}, \dot{q})\dot{K}_p^{-1}S_p(Kp\bar{q}) - \kappa \dot{\theta} B^{-1} A \vartheta \]

where
\[ W_{\gamma_1}(\bar{q}) = \sigma_p(Kp\bar{q})K_p^{-1}\left[1 - (1 - \gamma) s_p(Kp\bar{q}) + g(q) - g(q_d)\right] \]
\[ = \sum_{i=1}^{n} \left[1 - \left(1 - \gamma\right) \frac{\sigma_p(k_p\bar{q}_i)}{k_p} \right] \left[ g_i(q) - g_i(q_d) \right] \]

and \( \gamma_1 \) is a constant satisfying
\[ \beta_0 \left[ \max \left\{ \varepsilon / \varepsilon_2, \left( \varepsilon - \varepsilon_3 \overline{c}_3 / \varepsilon_4 - \varepsilon \right) \right\} \right] < \gamma_1 < \beta_0 \]

(16)

(from inequality (9) and the definition of \( \beta_0 \), one verifies, after simple developments, that \( \beta_0 \left[ \max \left\{ \varepsilon / \varepsilon_2, \left( \varepsilon - \varepsilon_3 \overline{c}_3 / \varepsilon_4 - \varepsilon \right) \right\} \right] < \beta_0 < 1 \). Under this consideration, \( W_{\gamma_1}(\bar{q}) \) turns out to be lower-bounded by
\[ W_{20}(\bar{q}) = \sum_{i=1}^{n} w_{20}^{20}(\bar{q}_i) \]

where
\[ w_{20}^{20}(\bar{q}_i) = \begin{cases} c_i \dot{q}_i^2 & \text{if } |\bar{q}_i| \leq L_{p_i}/k_p \\ \overline{w}_i(\bar{q}_i) & \text{if } |\bar{q}_i| > L_{p_i}/k_p \end{cases} \]

with \( \overline{w}_i(\bar{q}_i) = \frac{d}{d\bar{q}_i} \left[ (\sigma_p(k_p\bar{q}_i) - L_{p_i}) + c_i \left( \frac{L_{p_i}}{k_p} \right)^2 \right] \), \( d_i = (1 - \gamma_1)L_{p_i} - 2B_{p_i}, c_i = \min \left\{ h_i, \frac{dk_p}{k_p} \right\} \) and \( h = (1 - \gamma_1)K_{p_{\text{im}}}-k_{\text{r}} \) (notice, from inequality (16) and the definition of \( \beta_0 \), that \( d_i > 0 \) and \( h > 0 \), hence \( c_i > 0 \)); this is proven in (Mendoza et al., 2015, Appendix 3). From this, Properties 1, 2.1 and 3. items 2 of Lemma 1 and (b) of Definition 1, and the positive definite character of \( K_p \), we have that
\[ \dot{V} \leq \frac{\|q\|^2}{\kappa \dot{\theta} - s_d(\bar{q}, \vartheta)} + \varepsilon \beta_0 \beta_1 \left[ K_p^{-1}S_p(Kp\bar{q}) \right] \frac{\|q\|^2}{\kappa \dot{\theta} - s_d(\bar{q}, \vartheta)} + \varepsilon \beta_0 \beta_1 \left[ K_p^{-1}S_p(Kp\bar{q}) \right] \frac{\|q\|^2}{\kappa \dot{\theta} - s_d(\bar{q}, \vartheta)} - \varepsilon \dot{W}_{\gamma_1}(\bar{q}) \]

Let us note that by item 4 of Lemma 1, we have that \( \kappa \dot{\theta} - s_d(\bar{q}, \vartheta) \|^2 \leq \left[ \kappa \dot{\theta} - s_d(\bar{q}, \vartheta) \right]^T \left[ \kappa \dot{\theta} - s_d(\bar{q}, \vartheta) \right] = \kappa^2 \dot{\theta}^T \dot{\theta} - 2\kappa \dot{\gamma}_1 \dot{\gamma}_1 + \dot{s}_d^2(\bar{q}, \vartheta) \leq \kappa^2 \|\dot{\theta}\|^2 - \|s_d(\bar{q}, \vartheta)\|^2 \leq \kappa^2 \|\dot{\theta}\|^2, i.e., \( \|\dot{\theta} - s_d(\bar{q}, \vartheta)\| \leq \kappa \|\dot{\theta}\|, \forall (\bar{q}, \vartheta) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \). From this and item 4 of Lemma 1, we get
\[ \dot{V} \leq \frac{\|q\|^2}{\kappa \dot{\theta} - s_d(\bar{q}, \vartheta)} + \varepsilon \dot{W}_{M}(\bar{q}) \frac{\|q\|^2}{\kappa \dot{\theta} - s_d(\bar{q}, \vartheta)} + \varepsilon \dot{W}_{\gamma_1}(\bar{q}) \frac{\|q\|^2}{\kappa \dot{\theta} - s_d(\bar{q}, \vartheta)} - \varepsilon \dot{W}_{\gamma_1}(\bar{q}) \leq -\varepsilon W_{21}(\bar{q}, \vartheta) - \varepsilon W_{22}(\bar{q}, \vartheta) - \varepsilon W_{20}(\bar{q}) \]

where (arguments are dropped for simplicity)
\[ W_{21} = \frac{\gamma_1 k_{pm}}{2} \left[ \|K_p^{-1}S_p(Kp\bar{q})\|^2 + \kappa \beta_d \|\dot{\theta}\|^2 \right] \]

\[ W_{22} = \frac{\gamma_1 k_{pm}}{2} \left[ \|K_p^{-1}S_p(Kp\bar{q})\|^2 + \kappa \beta_d \|\dot{\theta}\|^2 \right] \]

5. EXPERIMENTAL RESULTS

In order to corroborate the efficiency of the proposed output-feedback SPD-SI control scheme, real-time tests were implemented using a 2-DOF direct-drive robot manipulator. The experimental setup is a 2-revolute-joint robot arm located at the Instituto Tecnológico de la Laguna, Mexico, previously used in (López-Araujo et al., 2013). The robot actuators are direct-drive brushless servomotors operated in torque mode, i.e., they act as torque sources and receive an analog voltage as a torque refer-
ence signal. Joint positions are obtained using incremental encoders on the motors. In order to get the encoder data and generate reference voltages, the robot includes a motion control board based on a DSP 32-bit floating point microprocessor. The control algorithm is executed at a 2.5 millisecond sampling period on a PC-host computer.

For the experimental manipulator, Properties 1–4 are satisfied with $\mu_c = 0.088$ kg/m², $\mu_M = 2.533$ kg/m², $k_C = 0.1545$ kg/m², $f_M = 0.175$ kg/m²/s, $f_M = 2.288$ kg/m²/s, $B_{g1} = 40.29$ Nm, $B_{g2} = 1.825$ Nm and $k_g = 40.373$ Nm/rad. The maximum allowed torques (input saturation bounds) are $T_1 = 150$ Nm and $T_2 = 15$ Nm for the first and second links respectively. From these data, one easily corroborates that Assumption 1 is fulfilled with $\alpha = 3$.

The saturation functions used for the implementation were $\sigma_{P_i}(\dot{q}) = M_{P_i}\text{sat}(\dot{q}/M_{P_i})$ and

$$
\sigma_{I_i}(\dot{q}) = \begin{cases} 
\dot{q} & \forall |\dot{q}| \leq L_{I_i} \\
\rho(\dot{q} L_{I_i}, M_{I_i}) & \forall |\dot{q}| > L_{I_i}
\end{cases}
$$

where $\rho(\dot{q}; L, M) = \text{sign}(\dot{q}) L + (M - L) \tanh \left( \frac{\text{sign}(\dot{q}) L}{M - L} \right)$, $i = 1, 2$, for $0 < L < M$. Note that $\sigma_{P_i M} = \sigma_{I_1 M} = 1$, $\forall i \in \{1, 2\}$. The saturation function parameters were selected in order to satisfy inequalities (7) as (all of them expressed in Nm): $M_{P1} = 81$, $M_{P2} = 7$, $M_{I1} = 48$, $M_{I2} = 5$ and $L_{I1} = 0.9 M_{I1}$, $i = 1, 2$.

For comparison purposes, additional experimental tests were implemented using the output-feedback version of the bounded PID-type controller presented in (Su et al., 2010) (choice made taking into account the analog nature of the compared algorithms; globally stabilizing via output feedback developed in a bounded-input context, and the recent appearance of (Su et al., 2010)), i.e.

$$
u = - K_P \text{Tanh}(\eta q) - K_D \text{Tanh}(\theta) - K_I \text{Tanh}(\phi)
$$

(18a)

$$
\dot{\varphi} = - A \varphi + B q
$$

(18b)

$$
\phi = \varphi + B q
$$

(18c)

where $\eta$ being a (sufficiently large) positive constant and

$$
\text{Tanh}(\eta q) = \left( \text{Tanh}(x_1), \ldots, \text{Tanh}(x_n) \right)^T
$$

for any $x \in \mathbb{R}^n$. For the sake of simplicity, this algorithm is subsequently referred to as the S10 controller.

The experiments were run taking the desired joint positions as $q_d = (q_{d1}, q_{d2})^T = \left( \pi/4, \pi/4 \right)^T$ [rad]. The initial conditions were $q(0) = \dot{q}(0) = 0$, and, for the SPD-SI type algorithm proposed in this work, $\phi_i(0)$ was taken so as to have $\phi(0) = 0$, while $\phi_i(0) = 0_2$ was taken for the S10 controller in view of the way how it

\footnote{In place of Eqs. (18c), Su et al. (2010) define $\phi(t) = \eta^2 q(t) + \eta \int_0^t \text{Tanh}(\eta q(s)) ds$, which imposes the auxiliary initial condition $\phi(0) = \eta^2 \dot{q}(0)$ (or, equivalently, $\phi(0) = 0_n$ in the context of Eqs. (18c)). Instead, Eqs. (18c)—or their (equivalent) time representation $\phi(t) = \phi(0) + \eta \int_0^t \text{Tanh}(\eta q(s)) ds$—keeps the required auxiliary dynamics while permitting any initial condition for $\phi$ (or, equivalently, for $\phi_i$ in the context of Eqs. (18c)).}

is presented in (Su et al., 2010) (recall Footnote 5). The control parameters for the scheme proposed in this work were selected —taking into account the tuning conditions from inequalities (8) and (9)— so as to get fast responses. As for the S10 algorithm, the control parameters were tuned so as to get the best possible closed-loop responses while adhering to the saturation-avoidance inequalities and stability conditions (some of which had to be verified numerically) presented in (Su et al., 2010). The resulting tuning values were: $K_P = \text{diag}[6000, 500]$ Nm/rad, $K_D = \text{diag}[2, 2]$ Nm/rad, $K_I = \text{diag}[900, 1500]$ Nm/rad, $A = \text{diag}[60, 60]$ s⁻¹, $B = \text{diag}[5, 5]$ s⁻¹ and $\varepsilon = 0.024$ s⁻¹ for the proposed SPD-SI scheme, and $K_P = \text{diag}[108, 11.5]$ Nm, $K_D = \text{diag}[0.5, 0.1]$ Nm, $K_I = \text{diag}[40.5, 1.9]$ Nm, $A = \text{diag}[60, 40]$ s⁻¹, $B = \text{diag}[70, 20]$ s⁻¹ and $\eta = 170$ s/rad for the S10 controller.

Figs. 1 and 2 show the experimental results. Note that the proposed SPD-SI controller achieved the regulation objective—avoiding input saturation—with relatively low overshoot. The S10 controller is also observed to achieve the regulation objective preventing input saturation but with a higher overshoot that could not be lowered down under the tuning procedure presented in (Su et al., 2010). Note further that the control objective has been achieved with negligible effect (on the system trajectories) of the imminent measurement noise. Restricted effect of noise on the closed loop responses may be seen as a natural consequence of the output-feedback nature of the proposed
Table 1. Performance index evaluations

| perf. index | SPD-SI | S10 |
|-------------|-------|-----|
| $t_s$       |       |     |
| $t_0 = t_s$, $\Delta = 3.32$ s | 1.15 s | 1.68 s |
| ISE        | $t_0 = 0$, $\Delta = 5$ s | 0.001 | 0.004 |

approach since only position variables are considered in the control algorithm, avoiding additional noise corruption from speed measurements.

For further comparison, two performance indices were evaluated for every tested controller: the stabilization time, taken as $t_s = \inf \{t_s \geq 0 : \|q(t)\| \leq 0.05 \|q(0)\| \forall t \geq t_s\}$, and the integral of the square of the position error (ISE), i.e. $\int_{0}^{t_s+\Delta} \left[ \sum_{i=1}^{2} \q_i^2(t) \right] dt$. Table 1 shows the resulting values of such performance index evaluations, whence one concludes that the SPD-SI algorithm has achieved faster stabilization (shorter $t_s$), lower steady-state error (ISE with $t_0 = t_s$ and $\Delta = 3.32$ s) and lower ISE-valued mean position error (deviation) during the whole test (ISE with $t_0 = 0$ and $\Delta = 5$ s).

6. CONCLUSIONS

Global stabilization of robot manipulators with bounded inputs through PID-type controllers had been achieved with a considerable degree of complexity. Efforts on the simplification of such type of algorithms conducted to simple SP-SI-SD and SPD-SI approaches. While an output-feedback extension of the former could be developed, it was not clear how to release the latter from velocity measurements, which are not always available in practice. Such an analytical challenge has been overcome in this work, by contributing an output-feedback SPD-SI control scheme constructed by means of generalized saturation functions. The efficiency of the proposed scheme was corroborated through experimental results on a 2-DOF manipulator. Future work will focus on a generalization of the output-feedback PID-type control structure offering multiple options on the saturating structure, thus widening the design alternatives to improve the closed-loop performance.

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Appendix A

3. Let $\psi$, $\varsigma$, $\eta$ $\in \mathbb{R}$. Since $\sigma$ is nondecreasing, we have that $\sigma(\psi) \geq \sigma(\varsigma) \iff \psi \geq \varsigma$ and $\sigma(\psi) \leq \sigma(\eta) \iff \psi \leq \eta$. Let $\psi = \varsigma + \eta$. Then $\sigma(\varsigma + \eta) - \sigma(\eta) \geq 0 \iff \varsigma \geq 0$, $\forall \eta \in \mathbb{R}$, and $\sigma(\varsigma + \eta) - \sigma(\varsigma) \leq 0 \iff \varsigma \leq 0$, $\forall \eta \in \mathbb{R}$, whence it follows that $|\sigma(\varsigma + \eta) - \sigma(\varsigma)| \geq 0$, $\forall \varsigma, \eta \in \mathbb{R}$.

4. From Lipschitz-continuity of $\sigma$ and item 2 of the statement, we have $|\sigma(\varsigma + k\eta) - \varsigma - (\varsigma + k\eta)| \leq |\varsigma| |k\varsigma|$. By multiplying both sides of this inequality by $|\sigma(\varsigma + k\eta) - \varsigma - (\varsigma + k\eta)|$ and taking into account item 3 of the statement, we get $|\sigma(k\varsigma + k\eta) - \sigma(\varsigma + k\eta)| \leq |\varsigma| |k\varsigma| |k\varsigma|$, $\forall \varsigma, \eta \in \mathbb{R}$, while by the same arguments we get $|\sigma(k\varsigma + k\eta) - \varsigma - (\varsigma + k\eta)| = |\varsigma| |k\varsigma| |k\varsigma|$, $\forall \varsigma, \eta \in \mathbb{R}$, whence one concludes that $|\sigma(k\varsigma + k\eta) - \sigma(\varsigma)| \leq |\varsigma| |k\varsigma| |k\varsigma|^2$, $\forall k \in \mathbb{R}$.