X(3872) as a hadronic molecule and its decays to charmonium states and pions

Yubing Dong\textsuperscript{1,2,3}, Amand Faessler\textsuperscript{1}, Thomas Gutsche\textsuperscript{1}, Sergey Kovalenko\textsuperscript{4}, Valery E. Lyubovitskij\textsuperscript{1}\textsuperscript{*}

\textsuperscript{1} Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D–72076 Tübingen, Germany

\textsuperscript{2} Institute of High Energy Physics, Beijing 100049, P. R. China

\textsuperscript{3} Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, P. R. China

\textsuperscript{4} Centro de Estudios Subatómicos (CES), Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

(Dated: April 27, 2009)

The X(3872) with quantum numbers \( J^{PC} = 1^{++} \) is considered as a composite hadronic state comprised of the dominant molecular \( D^0 D^{*0} \) component and other hadronic pairs – \( D^\pm D^{*\mp} \), \( J/\psi \omega \) and \( J/\psi \rho \). Applying the compositeness condition we constrain the couplings of the X(3872) to its constituents. We calculate two- and three-body hadronic decays of the X(3872) to charmonium states \( \chi_{cJ} \) and pions using a phenomenological Lagrangian approach. Next using the estimated \( XJ/\psi \omega \) and \( XJ/\psi \rho \) couplings we calculate the widths of X(3872) \( \rightarrow J/\psi + h \) transitions, where \( h = \pi^+ \pi^-, \pi^+ \pi^- \pi^0, \pi^0 \gamma \) and \( \gamma \). The obtained results for the decay pattern of the X(3872) in a molecular interpretation could be useful for running and planned experiments.

PACS numbers: 12.38.Lg, 12.39.Fe, 13.25.Jx, 14.40.Gx, 36.10.Gv

Keywords: charm mesons, pion, hadronic molecule, strong and radiative decay

\textsuperscript{*} On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia
I. INTRODUCTION

The $X(3872)$ is one of the new meson resonances discovered during the last years [1], whose properties cannot be simply explained and understood in conventional quark models. Several structure interpretations for the $X(3872)$ have been proposed in the literature (for a status report see e.g. Refs. [2, 3, 4]). In the context of molecular approaches [5-31] the $X(3872)$ can be identified with a weakly bound hadronic molecule whose constituents are $D$ and $D^*$ mesons. The reason for this natural interpretation is that its mass $m_X$ is very close to the $D^0 D^{*0}$ threshold and hence is in analogy to the deuteron — a weakly bound state of proton and neutron. Note, that the idea to treat the hidden charm states as hadronic molecules traces back to Refs. [5, 6]. Originally it was proposed that the state $X(3872)$ is a superposition of $D^0 D^{*0}$ and $D^0 D^{*0}$ pairs. Later (see e.g. discussion in Refs. [14, 16, 18]) also other structures, such as a charmonium or even other meson pair configurations, were discussed in addition to the $D^0 D^{*0}$ charge conjugate (c.c.) component (here and throughout the paper we use the convention that $\bar{\psi}$ does not change sign under charge conjugation. See detailed discussion in Ref. [30]). The possibility of two nearly degenerated $X(3872)$ states with positive and negative charge parity has been discussed in Refs. [22, 32].

This paper focuses on the hadronic $X \to \chi_{cJ} + (\pi^0, 2\pi)$, $X \to J/\psi + (2\pi, 3\pi)$ and radiative $X \to J/\psi + (\pi^0\gamma, \gamma)$ decays. The $X(3872)$ with quantum numbers $J^{PC} = 1^{++}$ is considered as a composite hadronic state including a dominant molecular $D^0 D^{*0}$ component and other hadronic pairs — $D^\pm D^{*\mp}$, $J/\psi\omega$ and $J/\psi\rho$. This idea was originally proposed in [14]. Applying the compositeness condition we constrain the couplings of $X(3872)$ to its constituents. We calculate two- and three-body hadronic decays of the $X(3872)$ to charmonium states $\chi_{cJ}$ and pions using a phenomenological Lagrangian approach. Next, using the estimated $X J/\psi\omega$ and $X J/\psi\rho$ couplings we calculate the widths of $X \to J/\psi + h$ transitions, where $h = \pi^+\pi^-, \pi^+\pi^-\pi^0, \pi^0\gamma$ and $\gamma$. Present experimental numbers for the ratios of observed decay modes of $X(3872)$ by the Belle [33] and BABAR [34] Collaborations are

$$\Gamma(X \to J/\psi\pi^+\pi^-\pi^0)/\Gamma(X \to J/\psi\pi^+\pi^-) = 1.0 \pm 0.4 \text{(stat)} \pm 0.3 \text{(syst)}$$

and

$$\Gamma(X \to J/\psi\gamma)/\Gamma(X \to J/\psi\pi^+\pi^-) = 0.14 \pm 0.05 \text{ (Belle); } 0.33 \pm 0.12 \text{ (BABAR).}$$

The theoretical analysis of hadronic and radiative decays of $X(3872)$ has been carried out using a charmonium interpretation [13, 15], different molecular approaches [11, 13, 18, 29, 31] with possible inclusion of charmonium and other hadronic components in the $X$ wave function, QCD sum rules in [35], multipole expansion in QCD and chiral properties of soft pions [36]. In particular, pionic transitions from $X(3872)$ to the charmonium states $\chi_{cJ}$ have been considered using a pure charmonium and four-quark [36] structure for the $X(3872)$ and later on in the molecular interpretation [31]. A conclusion was that the decay rates significantly depend on the structure interpretation of the $X(3872)$. It was also proposed that the $X(3872)$ to $J/\psi$ transitions are dominated by short–distance effects and in the mechanism of these transitions the $J/\psi\omega$ and $J/\psi\rho$ components of $X$ probably play the essential role [17, 19].

In Refs. [29, 37, 38] we developed the formalism for the study of recently observed exotic meson states (like $D_{s0}^*$(2317), $D_{s1}(2460)$, $X(3872)$, ... ) as hadronic molecules. In Ref. [29] we extended our formalism to the decay $X \to J/\psi\gamma$ assuming that the $X$ is the $S$–wave, positive charge parity ($S^{0} D^{*0} + D^{*0} D^{0}$)/$\sqrt{2}$ molecule. As for the case of the $D_{s0}$ and $D_{s1}$ states, a composite (molecular) structure of the $X(3872)$ meson is defined by the compositeness condition $Z = 0$ [39, 40, 41] (see also Refs. [29, 37, 38]). This condition implies that the renormalization constant of the hadron wave function is set equal to zero or that the hadron exists as a bound state of its constituents. The compositeness condition was originally applied to the study of the deuteron as a bound state of proton and neutron [32]. Then it was extensively used in low–energy hadron phenomenology as the master equation for the treatment of mesons and baryons as bound states of light and heavy constituent quarks (see e.g. Refs. [40, 41]). By constructing a phenomenological Lagrangian including the couplings of the bound state to its constituents and the constituents with other particles we calculated one–loop meson diagrams describing different decays of the molecular states (see details in [29, 37, 38]). In Ref. [29] we estimated the role of a possible charmonium component in the $X(3872)$. We showed that the charmonium contribution to the $X \to J/\psi\gamma$ decay width is suppressed in comparison with the molecular $D^0 D^{*0}$ component. As already stressed before, here we consider the $X(3872)$ as a superposition of the molecular $D^0 D^{*0}$ component and other hadronic pairs — $D^\pm D^{*\mp}$, $J/\psi\omega$ and $J/\psi\rho$. Because of the dominance of the $D^0 D^{*0}$ component in the transitions of $X$ into charmonium states $\chi_{cJ}$ and pions we estimate these decays using only that component. In the analysis of the decay widths with $J/\psi$ in the final state we will use the effective couplings $X J/\psi\omega$ and $X J/\psi\rho$ deduced from the compositeness condition.

In the present paper we proceed as follows. In Sec. II we first discuss the basic notions of our approach. We discuss the effective mesonic Lagrangian for the treatment of the $X(3872)$ meson as a superposition of the $D^0 D^{*0} + D^{*0} D^{0}$...
molecular component with the additional \(D^+D^{*-} + D^-D^{*+}\) and \(J/\psi\omega\) and \(J/\psi\rho\) hadronic pairs. Second, we consider the two–body hadronic decays \(X(3872) \rightarrow \chi_{cJ} + \pi^0(2\pi^0)\). Third, we discuss decays with \(J/\psi\) in the final state. In Sec. III we present our numerical results and perform a comparison with other theoretical approaches. Finally, in Sec. IV we present a short summary of our results.

II. APPRAOCH

A. Structure of the \(X(3872)\) meson

In this section we discuss the formalism for the study of the \(X(3872)\) meson. We adopt the convention that the spin and parity quantum numbers of the \(X(3872)\) are \(J^{PC} = 1^{++}\). Its mass we express in terms of the binding energy \(\epsilon_{D^0\bar{D}^{*0}} > 0\) with

\[
m_X = m_{D^0} + m_{D^{*0}} - \epsilon_{D^0\bar{D}^{*0}},
\]

where \(m_{D^0} = 1864.85\) MeV and \(m_{D^{*0}} = 2006.7\) MeV are the \(D^0\) and \(D^{*0}\) meson masses, respectively.

Following Ref. [14] we consider this state as superposition of the dominant molecular \(D^0\bar{D}^{*0}\) component and other hadronic configurations – \(D^{\pm}\bar{D}^{\mp}\), \(J/\psi\omega\), and \(J/\psi\rho\):

\[
|X(3872)\rangle = \frac{Z^{1/2}_{D^0\bar{D}^{*0}}}{\sqrt{2}}(|D^0\bar{D}^{*0}\rangle + |D^*\bar{D}\rangle) + \frac{Z^{1/2}_{D^{\pm}\bar{D}^{\mp}}}{\sqrt{2}}(|D^{\pm}\bar{D}^{\mp}\rangle + |D^{-}\bar{D}^{+}\rangle) + Z^{1/2}_{J/\psi\omega}|J/\psi\omega\rangle + Z^{1/2}_{J/\psi\rho}|J/\psi\rho\rangle,
\]

where \(Z_{H_1H_2}\) is the probability to find the \(X\) in the hadronic state \(H_1H_2\) with the normalization \(Z_{D^0\bar{D}^{*0}} + Z_{D^{\pm}\bar{D}^{\mp}} + Z_{J/\psi\omega} + Z_{J/\psi\rho} = 1\). For convenience, here and in the following we denote \(J/\psi\) by \(J_\psi\). The probabilities \(Z_{H_1H_2}\) have been estimated in [14] as function of the binding energy \(\epsilon\). Our approach is based on an effective interaction Lagrangian describing the couplings of the \(X(3872)\) to its meson constituents. We apply two forms of such Lagrangians – a local Lagrangian and a nonlocal form containing the correlation functions \(\Phi(y^2)\) characterizing the distribution of the constituents in the \(X(3872)\)). The simplest local Lagrangian reads

\[
\mathcal{L}_X^1(x) = g_{XD^0\bar{D}^{*0}}X_\mu(x)J^\mu_{D^0\bar{D}^{*0}}(x) + g_{XD^{\pm}\bar{D}^{\mp}}X_\mu(x)J^\mu_{D^{\pm}\bar{D}^{\mp}}(x)
+ \frac{g_{XJ/\psi\omega}}{m_X}\epsilon_{\mu\alpha\beta}\partial^\alpha X_\mu(x)J^\alpha_{J/\psi\omega}(x) + \frac{g_{XJ/\psi\rho}}{m_X}\epsilon_{\mu\alpha\beta}\partial^\alpha X_\mu(x)J^\alpha_{J/\psi\rho}(x),
\]

where \(g_{XH_1H_2}\) is the coupling of \(X(3872)\) to the constituents \(H_1\) and \(H_2\); \(X\) is the field describing \(X(3872)\); \(J^\mu_{H_1H_2}\) is the current composed of the hadronic fields \(H_1\) and \(H_2\):

\[
J^\mu_{DD*}(x) = \frac{1}{\sqrt{2}}(D(x)\bar{D}^{*\mu}(x) + \bar{D}(x)D^{*\mu}(x)) ,
\]

\[
J^\alpha_{J/\psi V} = J^\alpha_{J/\psi V},
\]

where \(V = \rho, \omega\).

The nonlocal version of the Lagrangian is obtained from the local one by introducing the correlation function into the hadronic current \(J^\mu_{H_1H_2}\) as

\[
J^\mu_{DD*}(x) \rightarrow J^\mu_{DD*}(x) = \frac{1}{\sqrt{2}} \int d^4y \Phi_{DD*}(y^2)\left(D(x + y/2)\bar{D}^{*\mu}(x - y/2) + \bar{D}(x + y/2)D^{*\mu}(x - y/2)\right),
\]

\[
J^\alpha_{J/\psi V} \rightarrow J^\alpha_{J/\psi V} = J^\alpha_{J/\psi V}(x) \int d^4y \Phi_{J/\psi V}(y^2)V^{\alpha}(x + y).
\]

Here \(\Phi_{DD*}\) is the correlation function describing the distribution of \(DD*\) inside \(X\). The function \(\Phi_{J/\psi V}\) describes the distribution of the light vector meson \(V = \rho\) or \(\omega\) around the \(J_\psi\), which is located at the center of mass of the \(X(3872)\). Since \(m_V \ll m_{J_\psi}\), this description is like in heavy–light mesons where the heavy quark \(Q\) is surrounded by a light quark \(q\) in the heavy quark limit of \(m_q \ll m_Q\). A basic requirement for the choice of an explicit form of the correlation function \(\Phi\) is that its Fourier transform vanishes sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. We adopt an identical Gaussian form for both correlation functions \(\Phi_{DD*} = \Phi_{J/\psi V} = \Phi_X\) in order to reduce the number of free parameters. The Fourier transform of the universal vertex function \(\Phi_X\) is given by

\[
\tilde{\Phi}_X(p^2_E/\Lambda^2) = \exp(-p^2_E/\Lambda^2_X),
\]
where \( p_E \) is the Euclidean Jacobi momentum. Here, \( \Lambda_X \) is a size parameter. In Ref. 29, the parameter was varied in the region 2 – 3 GeV, a typical scales for \( D \) and \( D^* \) mesons - constituents of \( X(3872) \). In the present paper we fix the value to \( \Lambda_X = 2 \) GeV which is close to the masses of \( D \) and \( D^* \) mesons. One should remark, up to now we have no strong and direct justification for the value of the \( \Lambda_X \). The final conclusion about its magnitude can be drawn when we have more precise data on \( X(3872) \). Note, the local limit corresponds to the substitution of \( \Phi_X \) by the Dirac delta-function: \( \Phi_X(y^2) \rightarrow \delta^4(y) \).

The coupling constants \( g_{H_1 H_2} \) are determined by the compositeness condition \([37, 39, 40, 41]\). It implies that the renormalization constant of the hadron wave function is set equal to zero with

\[
Z_X = 1 - \Sigma_X(m_X^2) = 0 .
\]  

(10)

Here, \( \Sigma_X(m_X^2) = d\Sigma_X(p^2)/dp^2|_{p^2=m_X^2} \) is the derivative of the transverse part of the mass operator \( \Sigma_X^{\mu \nu} \), conventionally split into the transverse \( \Sigma_X \) and longitudinal \( \Sigma_X^L \) parts as:

\[
\Sigma_X^{\mu \nu}(p) = g_\perp^{\mu \nu}\Sigma_X(p^2) + \frac{p_\mu p_\nu}{p^2}\Sigma_X^L(p^2) ,
\]

(11)

where \( g_\perp^{\mu \nu} = g^{\mu \nu} - p^\mu p^\nu/p^2 \) and \( g_\perp^{\mu \nu} p_\mu = 0 \). The mass operator of the \( X(3872) \) receives contribution from four hadron–loop diagrams

\[
\Sigma_X(m_X^2) = \Sigma_{D^0 D^0}^{D^0 D^0}(m_X^2) + \Sigma_{D^\pm D^+}(m_X^2) + \Sigma_{J/\psi \omega}(m_X^2) + \Sigma_{J/\psi \rho}(m_X^2) .
\]

(12)

induced by the interaction of \( X \) with the corresponding hadronic pairs \( H_1H_2 \) given in Eqs. (3) and (6). A typical diagram contributing to \( \Sigma_X^{\mu \nu}(p) \) is shown in Fig.1. Using Eq. (4) and the compositeness condition \( \text{Eq.} \, 10 \) we get four independent equations to determine the coupling constants \( g_{XH_1 H_2} \):

\[
Z_{H_1 H_2} = \Sigma_{H_1 H_2}(m_X^2) .
\]

(13)

In order to evaluate the couplings \( g_{XH_1 H_2} \) we use the standard free propagators for the intermediate particles \( H_1 \) and \( H_2 \):

\[
iS_P(x - y) = \langle 0|TP(x)P^1(y)|0 \rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)}S_P(k), \quad S_P(k) = \frac{1}{m_P^2 - k^2 - i\epsilon}
\]

(14)

for pseudoscalar fields \( P \) and

\[
iS_V^{\mu \nu}(x - y) = \langle 0|TV^{\mu}(x)V^{\nu\dagger}(y)|0 \rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)}S_V^{\mu \nu}(k), \quad S_V^{\mu \nu}(k) = \frac{-g^{\mu \nu} + k^\mu k^\nu/m_V^2}{m_V^2 - k^2 - i\epsilon}
\]

(15)

for vector fields \( V \).

Following Eqs. (10) and (13) in the nonlocal case the coupling constants \( g_{XH_1 H_2} \) are given by

\[
\frac{Z_{D^0 D^0}}{g_{X D^0 D^0}^{D^0 D^0}} = \frac{1}{(4\pi \Lambda_X)^2} \int_0^1 dx \int_0^\infty \frac{dx}{\alpha} \frac{P(\alpha, x)}{(1 + \alpha)^3} \left( 1 + \frac{1}{4m_{D^0}^2(1 + \alpha)} \right) \exp(z_1) ,
\]

(16)

\[
\frac{Z_{J/\psi \omega}}{g_{X J/\psi \omega}^{J/\psi \omega}} = \frac{1}{(4\pi \Lambda_X)^2} \int_0^1 dx \int_0^\infty \frac{dx}{\alpha} \frac{Q(\alpha, x)}{(1 + \alpha)^3} \left( 1 + \frac{1}{4m_{\omega}^2(1 + \alpha^2)} \right) \exp(z_2) ,
\]

(17)

where

\[
P(\alpha, x) = \frac{\alpha}{2} \left( 1 + 2\alpha x(1 - x) \right), \quad Q(\alpha, x) = \alpha x(1 + \alpha(1 - x)), \quad \mu_i = \frac{m_i}{\Lambda_X} ,
\]

\[
z_1 = -2\mu_D^2 x - 2\mu_D^2 \alpha(1 - x) + \frac{P(\alpha, x)}{1 + \alpha} , \quad z_2 = -2\mu_D^2 x - 2\mu_D^2 \alpha(1 - x) + \frac{Q(\alpha, x)}{1 + \alpha} \mu_D^2 .
\]

(18)

The expression for \( g_{X D^\pm D^\mp} \) is obtained from (16) by the corresponding replacement of masses and probability parameter \( Z_{H_1 H_2} \).

In the local case we neglect the longitudinal part \( k^\mu k'^\nu/m_V^2 \) of the vector meson propagator for the calculation of the coupling constants \( g_{XH_1 H_2} \) in order to have finite results. When writing the mass \( m_{H} \) of the hadronic molecule
the form $m_X = m_{H_1} + m_{H_2} - \epsilon_{H_1H_2}$, where $\epsilon_{H_1H_2}$ represents the binding energy specific to a hadronic pair $(H_1H_2)$, we can perform an expansion of $g_{XH_1H_2}^2$ in powers of $\epsilon_{H_1H_2}$. The leading-order $O(\sqrt{\epsilon_{H_1H_2}})$ result of

$$g_{XH_1H_2}^2 = \frac{Z_{H_1H_2} C_{H_1H_2}}{(m_{H_1} + m_{H_2})^{5/2}} \sqrt{32\epsilon_{H_1H_2}}$$

(19)

is in agreement with the ones derived in Refs. [38, 39, 42] also based on the compositeness condition $Z_X = 0$. Here we have the factor $C_{H_1H_2} = 1$ for $H_1H_2 = D^0D^{*0}$, $D^+D^{*-}$ and $C_{H_1H_2} = 1/2$ for $J_{\psi\omega}$, $J_{\psi\rho}$.

The numerical determination of the couplings $g_{XH_1H_2}$ for a specific hadron pair $H_1$ and $H_2$ shows that values obtained in the local and nonlocal case are very similar to each other. For example, for a binding energy of $\epsilon_{D^0D^{*0}} = 0.3$ MeV which corresponds to $m_X = 3.87151$ GeV, $\epsilon_{D^0D^{*0}} = 8.38$ MeV, $\epsilon_{J_{\psi\omega}} = 8.056$ MeV, and $\epsilon_{J_{\psi\rho}} = 0.896$ MeV we get in terms of the probability factors $Z_{H_1H_2}$

$$g_{XD^0D^{*0}} = 7.13 \text{ GeV} \sqrt{Z_{D^0D^{*0}}} \text{ (nonlocal)}, \quad 4.33 \text{ GeV} \sqrt{Z_{D^0D^{*0}}} \text{ (local)},$$

$$g_{XD^0D^{*0}} = 11.93 \text{ GeV} \sqrt{Z_{D^0D^{*0}}} \text{ (nonlocal)}, \quad 9.98 \text{ GeV} \sqrt{Z_{D^0D^{*0}}} \text{ (local)},$$

$$g_{XJ_{\psi\omega}} = 6.59 \text{ GeV} \sqrt{Z_{J_{\psi\omega}}} \text{ (nonlocal)}, \quad 7.79 \text{ GeV} \sqrt{Z_{J_{\psi\omega}}} \text{ (local)},$$

$$g_{XJ_{\psi\rho}} = 4.93 \text{ GeV} \sqrt{Z_{J_{\psi\rho}}} \text{ (nonlocal)}, \quad 4.50 \text{ GeV} \sqrt{Z_{J_{\psi\rho}}} \text{ (local)}.\quad(20)$$

We point out that for the three couplings $g_{XD^0D^{*0}}$, $g_{XJ_{\psi\omega}}$, and $g_{XJ_{\psi\rho}}$ there is no big difference between the nonlocal and local case. The reason is that the local couplings scale as $\epsilon_{H_1H_2}^{1/4}$. Therefore, a sizable deviation of the local coupling from the nonlocal one will only be relevant for values of $\epsilon_{H_1H_2} < 1$ MeV. For the nonlocal couplings the dependence on $\epsilon_{H_1H_2}$ is less pronounced. To illustrate this effect, in Table 1 we indicate the dependence of $g_{XD^0D^{*0}}/\sqrt{Z_{D^0D^{*0}}}$ as a function of $\epsilon_{D^0D^{*0}}$ both for the local and nonlocal case. The nonlocal coupling changes slowly when $\epsilon_{D^0D^{*0}}$ is varied from 0.3 to 3 MeV. This is not the case for the local coupling: its value changes significantly when $\epsilon_{D^0D^{*0}}$ is increased from 0.3 to 1 MeV, but it remains more stable and gets closer to the result of the nonlocal case for $\epsilon_{D^0D^{*0}} \geq 1$ MeV. (This corresponds to the case of the other couplings $g_{XH_1H_2}$ calculated at $\epsilon_{H_1H_2} \geq 1$ MeV).

### B. $X \rightarrow \chi_{cJ} + \pi^0$ transitions

In this subsection we consider the formalism for the two-body $X(3872) \rightarrow \chi_{cJ} + \pi^0$ transitions. Here the values of $J = 0, 1, 2$ correspond to the $J^P = 0^+, 1^+, 2^+$ quantum numbers of the charmonium states. The decays are described by the $(D^0D^{*0})$ loop diagram shown in Fig.2. A further inclusion of the charged $(D^\pm D^{*\mp})$ loops approximately gives the following correction to the decay widths

$$\Gamma_0 \rightarrow \Gamma \simeq \Gamma_0 \left(1 + \frac{Z_{XD^0D^{*0}}}{Z_{XD^0D^{*0}}} \right)^2.\quad(21)$$

The diagrams of Fig.2 are generated by a phenomenological Lagrangian which contains two main parts: i) the first part is the Lagrangian derived in our approach describing the coupling of $X(3872)$ to its constituents; ii) the second part is the set of interaction Lagrangians describing the possible couplings of $D(D^*)$ mesons to pions and charmonia states. This second part can be taken from heavy hadron chiral perturbation theory (HHChPT) [43, 44, 45] (for convenience we use a relativistic normalization of the meson states and write the Lagrangians in manifestly Lorentz covariant form):

$$\mathcal{L}_{D^*D^*} = \frac{g_{D^*D^*}}{\sqrt{2}} \left(D^*_{\mu\nu} D^*_{\mu\nu} D^*_{\nu\nu} + H.c. \right),\quad(22)$$

$$\mathcal{L}_{D^*D^*} = \frac{g_{D^*D^*}}{2\sqrt{2}} \epsilon_{\mu\nu\alpha\beta} \left(D^*_{\mu\nu} D^*_{\nu\alpha} D^*_{\alpha\beta} + H.c. \right),\quad(23)$$

$$\mathcal{L}_{\chi_{cJ}D^*D^*} = \chi_{c0} \left(g_{\chi_{c0}DD} D^*_i D^*_i + g_{\chi_{c0}D^*D^*} D^*_i D^*_i \right) + ig_{\chi_{c1}D^*D^*} \chi_{c1} \left(D^*_{\mu\nu} D^*_i + H.c. \right) + g_{\chi_{c2}D^*D^*} \chi_{c2} D^*_i D^*_i.\quad(24)$$
Here $\pi = \pi^\pm$ is a $2 \times 2$ matrix containing the pion fields; $D$ and $D^*$ are the doublets of charm pseudoscalar and vector $D$ mesons; $\chi_{cJ}$ are the fields describing the charmonium states; $i, j$ are the isospin indices. The hadronic coupling constants are expressed in terms of the universal HCHCPT couplings $g, g_1$ and the hadronic masses as \[ g_{D^*D^\pi} = \frac{g_{D^*D\pi}}{\sqrt{m_D m_{D^*}}} = \frac{g}{F_\pi} \sqrt{2}, \]

\[ g_{X_{c0}DD} = \frac{3m_D}{m_{D^*}} g_{X_{c0}D^*D^*} = -2g_1 m_D \sqrt{3m_{c0}}, \]

\[ g_{X_{c1}D^*D} = g_1 \sqrt{2m_{c1} m_D m_{D^*}}, \]

\[ g_{X_{c2}D^*D} = 2g_1 m_D \sqrt{m_{c2}}, \]

where $F_\pi = 92.4$ MeV is the leptonic decay constant. The coupling $g = 0.59$ (central value) is fixed from the data on the $D^{*0} \to D^{0}\pi$ branching ratio \[1\]. The coupling $g_1$ is related to the constant $f_{X_{c0}}$ parametrizing the matrix element $\langle 0|\bar{c}c|X_{c0}(p)\rangle = f_{X_{c0}} m_{X_{c0}}$ \[2\] as

\[ g_1 = \sqrt{\frac{m_{X_{c0}}}{3}} \frac{1}{f_{X_{c0}}}. \]

Using the estimate for $f_{X_{c0}} = 510$ MeV from QCD sum rules \[3\] we obtain for the coupling $g_1 = -2.09$ GeV$^{-1/2}$.

Evaluation of the diagrams in Fig.2 allows to write down an effective Lagrangian corresponding to the $X(3872) \to \chi_{cJ}\pi^0$ transitions with

\[ \mathcal{L}_{X_{c0}\pi} = g_{X_{c0}\pi} X^\mu \partial_\mu \chi_{c0}\pi^0, \]

\[ \mathcal{L}_{X_{c1}\pi} = \frac{g_{X_{c1}\pi}}{m_X} \partial^\alpha X^\beta \chi_{c1}^\mu \partial^\nu \chi_{c1}^\nu \epsilon_{\mu\nu\alpha\beta}, \]

\[ \mathcal{L}_{X_{c2}\pi} = \frac{g_{X_{c2}\pi}}{m_X} X_{c2}^\mu \chi_{c2}^\mu \partial^\alpha \pi^0. \]

In terms of the effective couplings $g_{X_{cJ}\pi}$ the decay widths of the $X(3872) \to \chi_{cJ}\pi^0$ transitions are determined according to the expression:

\[ \Gamma(X(3872) \to \chi_{cJ}\pi^0) = \frac{P_\pi^2}{24\pi m_X^2} c_J g_{X_{cJ}\pi}^2, \]

where $c_J = 1$ for $J = 0$; 2 for $J = 1$ and $5/3(1 + 2P_\pi^2/5m_{c2}^2)$ for $J = 2$. Here $P_\pi = \lambda^{1/2}(m_X^2, m_{X_{cJ}}^2, m_\pi^2)/(2m_X)$ is the pion momentum in the $X(3872)$ rest frame and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$ is the Källen function.

### C. $X \to \chi_{cJ} + 2\pi$ transitions

For the three-body decays $X(3872) \to \chi_{cJ} + 2\pi^0$ we evaluate the diagrams of Fig.3. In our notation $p, p_1, p_2$ and $p_3$ are the momenta of $X, \chi_{cJ}$ and the two pions, respectively. We introduce the invariant variables $s_i (i = 1, 2, 3)$:

\[ p = p_1 + p_2 + p_3, \]

\[ s_1 = (p_1 + p_2)^2 = (p - p_3)^2, \]

\[ s_2 = (p_2 + p_3)^2 = (p - p_1)^2, \]

\[ s_3 = (p_1 + p_3)^2 = (p - p_2)^2, \]

\[ s_1 + s_2 + s_3 = m_X^2 + m_{X_{cJ}}^2 + 2m_{cJ}^2. \]

The decay widths are calculated according to the formula:

\[ \Gamma(X(3872) \to \chi_{cJ} + 2\pi^0) = \frac{1}{1536\pi^3 m_X^2} \int_{4m_{cJ}^2}^{(m_X - m_{cJ})^2} \frac{ds_2}{ds_1} \sum_{\text{pol}} |M_{\text{inv}}|^2, \]

where

\[ s_1^\pm = m_{cJ}^2 + \frac{1}{2} \left( m_X^2 + m_{X_{cJ}}^2 - s_2 \pm \lambda^{1/2}(s_2, m_X^2, m_{X_{cJ}}^2) \sqrt{1 - \frac{4m_{cJ}^2}{s_2}} \right), \]

\[ M_{\text{inv}} \] is corresponding invariant matrix element.
D. Hadronic and radiative $X \to J/\psi + h$ decays

To get estimates for the decay widths of $X(3872) \to J/\psi + h$ with $h = \pi^+\pi^-\pi^0, \pi^+\pi^0, \pi^0\gamma, \gamma$ we use the results of Ref. [18], which are based on the assumption that these decays proceed through the processes $X \to J/\psi\omega$ and $J/\psi\rho$. In particular, it was shown that the $X(3872) \to J/\psi + h$ decay widths can be expressed in terms of $G_{XJ_\psi V}$ couplings as

$$
\Gamma(X \to J/\psi\pi^+\pi^-) = |G_{XJ_\psi\rho}|^2 \cdot 223 \text{ keV},
\Gamma(X \to J/\psi\pi^+\pi^0) = |G_{XJ_\psi\omega}|^2 \cdot 19.4 \text{ keV},
\Gamma(X \to J/\psi\pi^0\gamma) \approx |G_{XJ_\psi\omega}|^2 \cdot 3.24 \text{ keV},
\Gamma(X \to J/\psi\gamma) = |G_{XJ_\psi\rho}| + 0.30|G_{XJ_\psi\omega}|^2 \cdot 5.51 \text{ keV}.
$$

(32)

The couplings $G_{XJ_\psi V}$ introduced in [18] are related to our set of couplings $g_{XJ_\psi V}$ as:

$$
G_{XJ_\psi V} = \frac{g_{XJ_\psi V}}{m_V}.
$$

(33)

In our approach, based on the representation [21] for the $X$, we deduced the effective couplings $g_{XJ_\psi\omega}$ and $g_{XJ_\psi\rho}$ in terms of the unknown probabilities $Z_{XJ_\psi\omega}$ and $Z_{XJ_\psi\rho}$. These results we use in Eqs. (32)-(33). Note that Eqs. (32)-(33), corresponding to the $X \to J/\psi + h$ decays, only take into account short-distance effects [18]. To be consistent one should also include long-distance effects due to the contribution of the molecular $D^0D^{*0}$ component. Such a detailed analysis goes beyond the scope of the present work. Here we estimate both short and long-distance effects only for the $X \to \gamma J/\psi$ decays using our previous results on the molecular contribution obtained in Ref. [29].

III. RESULTS

We present our numerical results in terms of the probabilities $Z_{H_1H_2}$ and then substitute the typical values for $Z_{H_1H_2}$ based on the estimate of Ref. [14] for a binding energy of $\epsilon = 0.3$ MeV:

$$
Z_{D^0D^{*0}} = 0.92, \quad Z_{D^0D^{*+}} = 0.033, \quad Z_{J_\psi\omega} = 0.041, \quad Z_{J_\psi\rho} = 0.006.
$$

(34)

In Table 2 we present our results for the $X \to \chi_{cJ} + \pi^0(2\pi^0)$ decay widths and the ratios

$$
R_{cJ} = \frac{\Gamma(X \to \chi_{cJ} + 2\pi^0)}{\Gamma(X \to \chi_{cJ} + \pi^0)}.
$$

(35)

We also give predictions for the effective couplings $g_{XJ_\psi\omega}$. In the second column we indicate the contribution of the $D^0D^{*0}$ loop only. Results are given in terms of the $Z_{H_1H_2}$ factors and values in brackets are based on the explicit numbers of Eq. (34). The third column contains the results including both $D^0D^{*0} + D^-D^{**}$ loops, again based on the probability factors of Eq. (34). In the fourth column we give the predictions based on the approximate formula (21). We also introduce the notation $\beta = (Z_{D^0D^{*+}}/Z_{D^0D^{*0}})^{1/2}$ for the ratio of the probability factors. Again, values in brackets are deduced with the explicit values for $Z_{H_1H_2}$.

The $D^0D^{*0}$ molecular component gives (as naively expected) the dominant contribution to the $X \to \chi_{cJ} + \pi^0, 2\pi^0$ rates. Also, the results based on the approximate expression (21) including the charged $D^0D^{*+}$ component turn out to be quite close to the exact calculation. Comparing our predicted ratios of Table 2 to the results of Ref. [31]

$$
R_{c0} = 9.1 \times 10^{-6}, \quad R_{c1} = 6.1 \times 10^{-1}, \quad R_{c2} = 7.8 \times 10^{-6}
$$

larger differences occur. This is especially due to the nonrelativistic treatment of the $D^0$ and $D^{*0}$ mesons in Ref. [31]. The large value of $R_{c3}$ in Ref. [31] is sensitive to the treatment of the pole position of the nonrelativistic energy denominator and to the width of the $D^0$ meson.

In Table 3 we present our results for the $X \to J/\psi + h$ decays as based on the set of relations of Eq. (32). The predictions are given both for the local and nonlocal cases. Again, final results are given in terms of the relevant $Z_{H_1H_2}$ factors, using in addition the notation $\sigma = (Z_{J_\psi\rho}/Z_{J_\psi\omega})^{1/2}$, while numbers in brackets are based on Eq. (34). For the probability factors of Eq. (34) we also list our results for the ratios

$$
R_1 = \frac{\Gamma(X \to J/\psi\pi^+\pi^-\pi^0)}{\Gamma(X \to J/\psi\pi^+\pi^-)}, \quad R_2 = \frac{\Gamma(X \to J/\psi\gamma)}{\Gamma(X \to J/\psi\pi^+\pi^-)}.
$$

(37)
related to the present experimental situation given in Eqs. (11) and (2). One can see, that nonlocal and local cases are numerically similar to each other. To our mind only the decay width $\Gamma(X \rightarrow J/\psi\pi^+\pi^-\pi^0)$ and hence the ratio $R_1$ might be overestimated in the local case. Also note that the results for $R_1$ and $R_2$ in the more realistic, nonlocal case are consistent with present experimental findings displayed in Eqs. (11) and (2). Let us remark that the results obtained in the local case are close to the nonlocal case. As one can from the numbers, the local approximation including truncation of the vector meson propagator is reasonable approximation to the nonlocal case at $\Lambda_X = 2$ GeV. When $\Lambda_X$ is increasing the difference of two cases becomes more sizable.

Next we also want to comment on the result for the decay width $\Gamma(X \rightarrow J/\psi\gamma)$. In Ref. [29] we originally gave an estimate for this decay width including the molecular $D^0D^{\ast 0}$ and the $c\bar{c}$ charmonium components. We showed that the contribution of the charmonium component is strongly suppressed. For a cutoff value of $\Lambda = 2$ GeV our result for $\Gamma(X \rightarrow J/\psi\gamma)$ was 118.9 keV. In Ref. [29] we described the couplings of $J/\psi$ to $D^0D^0$ and $D^{\ast 0}D^{\ast 0}$ applying a phenomenological Lagrangian used in the analysis of $J/\psi$. We also did not include possible, additional form factors at the meson interaction vertices for reasons of simplicity and in order to have less free parameters. Inclusion of such form factors could lead to a further reduction of the predicted value for the $X \rightarrow J/\psi\gamma$ decay width. The importance of these form factors was recognized before in connection with different aspects of charm physics, in particular, with the suppression of the $J/\psi$ dissociation cross sections [43]. This implies that our result of Ref. [29] corresponds to an upper limit for the decay width $\Gamma(X \rightarrow \gamma J/\psi)$. Let us note that this value can be further reduced by the following four effects: i) by the probability factor $Z_{D^0D^{\ast 0}}$; ii) when using smaller values for the couplings of $J/\psi$ to the $D^0D^0$ and $D^{\ast 0}D^{\ast 0}$ pairs (in Ref. [29] we used $g_{J/\psi, DD} = g_{J/\psi, D^*D^*} = 6.5$); iii) by the inclusion of form factors in the $J/\psi D^0D^{\ast 0}$ and $J/\psi D^{\ast 0}D^{\ast 0}$ vertices; iv) when taking into account the short-distance mechanism of the $X \rightarrow J/\psi + V \rightarrow \gamma$ transition, considered presently, leading to destructive interference with the molecular contribution. Without introducing form factors at the $J/\psi D^0D^0$ and $J/\psi D^{\ast 0}D^{\ast 0}$ vertices and taking into account three additional suppression effects [i], [ii] and [iv] we now have for $\Gamma(X \rightarrow J/\psi\gamma)$ in terms of the coupling $g_{J/\psi} = g_{J/\psi, DD} = g_{J/\psi, D^*D^*}$:

$$\Gamma(X \rightarrow J/\psi\gamma) = (1.605 g_{J/\psi} - 2.354)^2 \text{ keV}. \quad (38)$$

When varying $g_{J/\psi}$ from 5 to 6.5 we get

$$\Gamma(X \rightarrow J/\psi\gamma) = 32.2 - 65.3 \text{ keV}, \quad (39)$$

where a further possible reduction of this value can be obtained by including form factors at the $J/\psi D^0D^0$ and $J/\psi D^{\ast 0}D^{\ast 0}$ vertices. Note, that three different results for the $\Gamma(X \rightarrow J/\psi\gamma)$ are obtained using different approximation for the $X(3872)$ wave function: i) 64.4 - 118.9 keV was obtained for a mixture of molecular $DD^*$ and charmonium $c\bar{c}$ components; ii) 5.5 keV was obtained for pure $J/\psi V$ components; iii) 32.2 - 65.3 keV was obtained taking a destructive interference of molecular $DD^*$ and charmonium $c\bar{c}$ components with $J/\psi V$ components.

Our final comment concerns the $X \rightarrow \psi(2s) + \gamma$ decay width recently measured by the BABAR Collaboration [34]:

$$R_3 = \frac{\Gamma(X \rightarrow \psi(2s)\gamma)}{\Gamma(X \rightarrow J/\psi\gamma)} = 3.5 \pm 1.4 \quad (40)$$

In our opinion this value can be interpreted as a signal for mixing of the $D^0D^{\ast 0}$ and $J/\psi V$ components in the $X \rightarrow J/\psi\gamma$ mode. In the $X \rightarrow \psi(2s)\gamma$ transition only the molecular $D^0D^{\ast 0}$ component will contribute under the condition that a $\psi(2s)V$ component in the $X(3872)$ is completely absent or suppressed relative to the $J/\psi V$ configurations. In the future we plan to calculate all the decay modes $X \rightarrow J/\psi h$ including $X \rightarrow \psi(2s)\gamma$ using the HfHChPT Lagrangian [47].

### IV. SUMMARY

We have considered the $X(3872)$ resonance with $J^{PC} = 1^{++}$ as a composite hadronic state made up of a dominant molecular $D^0D^{\ast 0}$ component and other hadronic pairs $D\pm D^\mp$, $J/\psi\omega$ and $J/\psi\rho$. Applying the compositeness condition we constrained the couplings of $X(3872)$ to its constituents. We calculated two- and three-body hadronic decays of the $X(3872)$ to charmonium states $\chi_{cJ}$ and pions using a phenomenological Lagrangian approach. Then using the estimated $XJ/\psi\omega$ and $XJ/\psi\rho$ couplings we calculated the widths of $X(3872) \rightarrow J/\psi + h$ transitions, where $h = \pi^+\pi^-, \pi^+\pi^-\pi^0, \pi^0\gamma$ and $\gamma$. The full, structure-dependent decay pattern of the $X(3872)$ developed here can serve to possibly identify its hadronic composition in running and planned experiments.
Acknowledgments

This work was supported by the DFG under Contract No. FA67/31-1, No. FA67/31-2, and No. GRK683. This work is supported by the National Sciences Foundations No. 10775148 and by CAS Grant No. KJCX3-SYW-N2 (YBD). This is also part of the European Community-Research Infrastructure Integrating Activity “Study of Strongly Interacting Matter” (HadronPhysics2, Grant Agreement No. 227431) and of the President grant of Russia “Scientific Schools” No. 871.2008.2. Y.B.D. would like to thank the Theory Group of Universidad Tecnica Federico Santa Maria for its hospitality. V.E.L. would like to thank the University of Cuenca, Ecuador for its hospitality. This research is also part of the European Community-Research Infrastructure Integrating Activity “Study of Strongly Interacting Matter” (HadronPhysics2, Grant Agreement No. 227431) and of the President grant of Russia “Scientific Schools” No. 871.2008.2. Y.B.D. would like to thank the Theory Group of Universidad Tecnica Federico Santa Maria for its hospitality. This work was partially supported by the PBCT Project No. ACT-028 Center of Subatomic Physics.
Phys. Rev. D 60, 094002 (1999) [arXiv:hep-ph/9904421]; A. Faessler, T. Gutsche, M. A. Ivanov, V. E. Lyubovitskij and P. Wang, Phys. Rev. D 68, 014011 (2003) [arXiv:hep-ph/0304031]; A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, Phys. Rev. D 73, 094013 (2006) [arXiv:hep-ph/0602193]; A. Faessler, T. Gutsche, B. R. Holstein, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, Phys. Rev. D 74, 074010 (2006) [arXiv:hep-ph/0608015].

[42] V. Baru, J. Haidenbauer, C. Hanhart, Yu. Kalashnikova and A. E. Kudryavtsev, Phys. Lett. B 586, 53 (2004) [arXiv:hep-ph/0308129]; C. Hanhart, Yu. S. Kalashnikova, A. E. Kudryavtsev and A. V. Nefediev, Phys. Rev. D 75, 074015 (2007) [arXiv:hep-ph/0701214]; F. K. Guo, C. Hanhart and U. G. Meissner, Phys. Lett. B 665, 26 (2008) [arXiv:0803.1392 [hep-ph]].

[43] M. B. Wise, Phys. Rev. D 45, R2188 (1992); G. Burdman and J. F. Donoghue, Phys. Lett. B 280, 287 (1992); A. F. Falk and M. E. Luke, Phys. Lett. B 292, 119 (1992) [arXiv:hep-ph/9206241]; T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin and H. L. Yu, Phys. Rev. D 46, 1148 (1992) [Erratum-ibid. D 55, 5851 (1997)].

[44] E. E. Jenkins, M. E. Luke, A. V. Manohar and M. J. Savage, Nucl. Phys. B 390, 463 (1993) [arXiv:hep-ph/9204238].

[45] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Rev. D 69, 054023 (2004) [arXiv:hep-ph/0310084].

[46] P. Colangelo, F. De Fazio and T. N. Pham, Phys. Lett. B 542, 71 (2002) [arXiv:hep-ph/0207061].

[47] Z. W. Lin and C. M. Ko, Phys. Rev. C 62, 034903 (2000) [arXiv:nucl-th/9912046].

[48] S. G. Matinyan and B. Muller, Phys. Rev. C 58, 2994 (1998) [arXiv:nucl-th/9806027].

FIG. 1: $H_1H_2$ hadron–loop diagrams contributing to the mass operator of the $X(3872)$ meson.

FIG. 2: Diagrams contributing to the hadronic transitions $X(3872) \rightarrow \chi_{cJ}^+ + \pi^0$. 
FIG. 3: Diagrams contributing to the hadronic transitions $X(3872) \rightarrow \chi_{cJ} + 2\pi^0$. 
Table 1. Dependence of the couplings $G_{X^{D^0 D^*0}} = g_{X^{D^0 D^*0}} / Z_{D^0 D^*0}$ on the binding energy $\epsilon_{D^0 D^*0}$.

| $\epsilon_{D^0 D^*0}$ (MeV) | 0.3 | 0.5 | 0.7 | 1 | 1.3 | 1.5 | 1.7 | 2 | 2.3 | 2.5 | 2.7 | 3 |
|-----------------------------|-----|-----|-----|---|-----|-----|-----|---|-----|-----|-----|---|
| Local case: $G_{X^{D^0 D^*0}}$ (GeV) | 4.33 | 4.92 | 5.35 | 5.85 | 6.25 | 6.48 | 6.69 | 6.96 | 7.21 | 7.36 | 7.50 | 7.70 |
| Nonlocal case: $G_{X^{D^0 D^*0}}$ (GeV) | 7.13 | 7.25 | 7.37 | 7.54 | 7.72 | 7.83 | 7.94 | 8.11 | 8.28 | 8.39 | 8.49 | 8.65 |

Table 2. Properties of $X \rightarrow \chi_{cJ} + \pi^0(2\pi^0)$ decays. The numbers in brackets and for column $D^0 D^*0 + D^- D^{*-}$ [exact] result from explicit values for $Z_{D^0 D^*0}$ and $\beta = (Z_{D^0 D^*0}/Z_{D^0 D^*0})^{1/2}$ of Eq. [34].

| Quantity | $D^0 D^*0$ loop | $D^0 D^*0 + D^- D^{*-}$ [exact] | $D^0 D^*0 + D^- D^{*-}$ [Eq. (21)] |
|----------|------------------|---------------------------------|---------------------------------|
| $g_{X^{0,0}}$ | 0.826 $\sqrt{Z_{D^0 D^*0}}$ (0.792) | 1.007 | 0.826 $\sqrt{Z_{D^0 D^*0}}(1 + \beta)(0.942)$ |
| $g_{X^{0,1}}$ | 0.444 $\sqrt{Z_{D^0 D^*0}}$ (0.426) | 0.539 | 0.444 $\sqrt{Z_{D^0 D^*0}}(1 + \beta)(0.507)$ |
| $g_{X^{0,2}}$ | 0.655 $\sqrt{Z_{D^0 D^*0}}$ (0.628) | 0.797 | 0.655 $\sqrt{Z_{D^0 D^*0}}(1 + \beta)(0.747)$ |
| $\Gamma(X \rightarrow \chi_{c0} + \pi^0)$, keV | 41.1 $Z_{D^0 D^*0}$ (37.8) | 61.0 | 41.1 $Z_{D^0 D^*0}$ (1 + $\beta^2$) (53.5) |
| $\Gamma(X \rightarrow \chi_{c0} + 2\pi^0)$, eV | 63.3 $Z_{D^0 D^*0}$ (58.2) | 94.0 | 63.3 $Z_{D^0 D^*0}$ (1 + $\beta^2$) (82.4) |
| $R_{c0} \times 10^3$ | 1.54 | 1.54 | 1.54 |
| $\Gamma(X \rightarrow \chi_{c1} + \pi^0)$, keV | 11.1 $Z_{D^0 D^*0}$ (10.2) | 16.4 | 11.1 $Z_{D^0 D^*0}$ (1 + $\beta^2$) (14.5) |
| $\Gamma(X \rightarrow \chi_{c1} + 2\pi^0)$, eV | 743 $Z_{D^0 D^*0}$ (683.6) | 1095.2 | 743 $Z_{D^0 D^*0}$ (1 + $\beta^2$) (969.6) |
| $R_{c1} \times 10^2$ | 6.69 | 6.68 | 6.69 |
| $\Gamma(X \rightarrow \chi_{c2} + \pi^0)$, keV | 15 $Z_{D^0 D^*0}$ (13.8) | 22.1 | 15 $Z_{D^0 D^*0}$ (1 + $\beta^2$) (19.5) |
| $\Gamma(X \rightarrow \chi_{c2} + 2\pi^0)$, eV | 20.6 $Z_{D^0 D^*0}$ (19.0) | 30.4 | 20.6 $Z_{D^0 D^*0}$ (1 + $\beta^2$) (26.9) |
| $R_{c2} \times 10^3$ | 1.38 | 1.38 | 1.38 |

Table 3. Properties of $X \rightarrow J/\psi + h$ decays. The numbers in brackets and for the ratios $R_1$, $R_2$ from explicit values for $Z_{J/\psi}$, $Z_{J/\psi}$ and $\sigma = (Z_{J/\psi}/Z_{J/\psi})^{1/2}$ of Eq. [34].

| Quantity | Local case | Nonlocal case |
|----------|------------|--------------|
| $\Gamma(X \rightarrow J/\psi^* + \pi^-)$, keV | 7.5 $\times 10^3 Z_{J/\psi}$ (45.0) | 9.0 $\times 10^3 Z_{J/\psi}$ (54.0) |
| $\Gamma(X \rightarrow J/\psi^* + \pi^0)$, keV | 1.92 $\times 10^5 Z_{J/\psi}$ (78.9) | 1.38 $\times 10^3 Z_{J/\psi}$ (56.6) |
| $\Gamma(X \rightarrow J/\psi^0 + \gamma)$, keV | 0.32 $\times 10^5 Z_{J/\psi}$ (13.2) | 0.23 $\times 10^5 Z_{J/\psi}$ (9.4) |
| $\Gamma(X \rightarrow J/\psi + \gamma)$, keV | 49.18 $Z_{J/\psi}$ (1 + 1.94$\sigma$) (6.1) | 35.19 $Z_{J/\psi}$ (1 + 2.51$\sigma$) (5.5) |
| $R_1$ | 1.75 | 1.05 |
| $R_2$ | 0.14 | 0.10 |