Modeling of embolism and malformation phenomena in the brain

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Abstract. In a previously published mathematical model of blood circulation in the cerebral cortex, it was assumed that the circulatory system of the brain consists of a distribution network of relatively large arteries and a collector network of veins similar in structure [1]. The connection between the networks is carried out through a network of numerous small capillaries, in which arterial blood as a result of metabolic processes with the cellular tissue of the brain turns into venous. A system of equations of viscous fluid motion in a heterogeneous medium consisting of three interpenetrating, but not intersecting, continuums is written out. Intersections are pathology for a healthy human organism. Multiple calculations show that pathologies localized even in small areas of the brain significantly affect the nature of the distribution of blood pressure in the arterial and venous channels. The finite element method is used in the calculations. The network of elements is refined in the subdomains of pathologies in the form of malformation or embolism with a decrease in the conductive properties of a heterogeneous environment. The numerical solution of the planar problem was carried out with parameters close to the real conditions of blood circulation in the human brain.

1. Introduction. Governing equations
The body of mammals, including humans, is a complex natural system of elements in which various kinds of physicochemical and electrical processes occur. It contains many cells, bacteria, viruses and other protozoa that are in close interconnection. One of the basic laws is the assimilation and dissimilation, division and death of cells, which in essence are also living organisms. A decisive role in these processes belongs to blood circulation. Nutrition and removal of the vital products of cells and bacteria is carried out by a directed flow of blood from the arterial bed to the venous bed through a network of small capillaries. Violation of capillary blood flow leads to the death of cells.

The blood flow pattern in the selected unit organ element can be represented in the form of the scheme represented by figure 1. Digits 1 and 2 denote the shares of arterial and venous blood flows, respectively. Arrows indicate capillary blood flow from the arterial network to the venous network, the volume of organ cells is represented by light rectangles.
Governing the blood flow process in continua, the system of equations consists of the laws of motion in the form of a linear dependence of the fluid velocities \( v_i \) in each continuum on the gradients of pressure \( p_i \) acting in them as well as the conservation law fluid mass subject to continuum compressibility \( \beta_i \):

\[
v_i = -\frac{k_i}{\mu} \text{grad} \ p_i ; \quad \frac{\partial(m_i \rho)}{\partial t} + \text{div}(\rho \nu_i) + q_i = 0 \quad (i = 1, 2, \ldots).
\]  

Symbols \( k_i, m_i, \rho, \mu, t \) denote continuum permeability and porosity, fluid density and viscosity, and time, respectively.

In hemodynamic problems, values \( q_i \) are the specific volumes of blood flowing from the arterial network through an extensive network of small capillaries into the venous network, as shown in the diagram (figure 1).

2. Formulation of the problem
Let consider the process of movement of a homogeneous incompressible fluid (blood), which occurs through three interpenetrating continua: 1 - network of arteries, 2 ─ network of veins, and 3 ─ network of capillaries. If in medium 1 or 2 there are no other external sources or drains, then the condition for preserving the volumes of flowing liquid from the distribution network 1 to the collector network 2 must be fulfilled.

Based on the laws (1), the system of equations has the form:

\[
\frac{k_i}{\mu} \Delta p_i = \beta_i \frac{\partial p_i}{\partial t} + q_i \quad (i = a, v).
\]  

Here \( \beta_i = dm_i / dp_i \) are compressibility coefficients of the pore space of embedded media, \( \Delta \) is Laplace operator, indices \( i = a, v \) denote values related to the arterial and venous network, respectively. In the model of blood circulation, the coefficients \( \beta_i \) characterize the elasticity of the network relative to large vessels. The compressibility of the blood can be neglected, and the elasticity of the arterial and venous network can naturally be assumed to be approximately equal \( \beta_a = \beta_v = \beta \), since they are contained in the same substantial space. Specific blood flow \( q \) through the capillary network is proportional to the pressure gradient \( (p_a - p_v)/l \) and inversely proportional to the viscosity of the blood. Here \( l \) means the length of the middle path that red blood cells pass in the process of converting arterial blood into venous blood. Thus, the value \( \alpha l \) is the usual conductivity of
the capillary network as a porous medium in which the fluid is filtered, described by an analog of Darcy’s law.

We write the system (2) in expanded form

\[
\frac{k_a}{\mu} \Delta p_a = \beta \frac{\partial p_a}{\partial t} + \alpha (p_a - p_v), \quad \frac{k_v}{\mu} \Delta p_v = \beta \frac{\partial p_v}{\partial t} - \alpha (p_a - p_v). \tag{3}
\]

The four parameters included in the system reflect the physiological state of the circulatory system of the body: vascular elasticity \( \beta \), as well as the conductivity \( k_a, k_v, \alpha \) of the arterial, venous networks and the network of small capillaries, respectively. The values of conductivities \( k_a, k_v \) can be estimated if the density distribution of the diameters of the vessels in living tissue by their size is known [1].

Natural or artificial causes of pathologies associated with the occurrence of foci of embolism or vascular malformation can have a significant effect on the redistribution of pressure in the arterial and venous network. Six different combinations can be made of the three parameters included in system (3), which determine the conductivities of interpenetrating continua. Each of the combinations corresponds to a certain type of pathology.

In relation to the pulse movement of blood in the cerebral cortex [2-4], the boundary conditions are set for the new system

\[
z = z_w: p_a = A \cos \omega t + B, \quad p_v = p_0; \\
z = z_p: \frac{\partial p_a}{\partial n} = \frac{\partial p_v}{\partial n} = 0. \tag{4}
\]

In problem (3), (4) without initial data \( A = (p_s - p_d)/2, \ B = (p_s + p_d)/2 \), \( p_s \) is systolic pressure, \( p_d \) is diastolic pressure, \( p_0 \) is venous blood absorption pressure, \( \omega \) is the heartbeat circular frequency, \( z_w = (x_w, y_w) \), \( z_p = (x_p, y_p) \) are points on the contours of the Willis circle and the periphery of the brain, respectively, \( \frac{\partial}{\partial n} \) is derivative normal to the periphery. The last boundary conditions means the blood does not leak through the external border (periphery) of the brain. The calculated area is presented in figure 2.

![Figure 2. Configuration of the calculated area (cerebral cortex).](image)

3. Numerical implementation

The system of equations (3) contains four independent parameters: vascular elasticity \( \beta \), a parameter \( \alpha \) that reflects the relative magnitude of capillary conductivity, and network conductivities \( k_a, k_v \). Various pathological cases reduce to choosing a certain combination of parameters composed of
conductivities $\alpha$ and $k_v, k_a$. In general, the problem (3), (4) is solved numerically as an initial-boundary-value one by finite element method involving an iterative process for each equation of the system.

Values $\beta = \beta_\mu / k = 0.01$, $\eta = \alpha / k = 0.01$, can be taken as the average parameters of a healthy body. Dashed lines indicate the possible location of pathologies in a circle with a radius of 1.5 cm and centered at a point (11, 6).

Figure 3 shows the pressure isolines for the case of malformation at $\eta_1 = 1$ in the circle, which corresponds to the situation when the conductivities of the arterial and venous networks equal to the conductivity of the capillary network.

A comparison of the calculations with the results presented in figure 3 for the normal state of the circulatory system shows a significant effect of even a small area of malformation on the redistribution of the total blood pressure in the brain.

![Figure 3](image1.png)

**Figure 3.** The isolines $p_a$ (a) and $p_v$ (b) in the case of pathology with $\eta_1 = 1$ in the circle.

The blood pressure distribution in the channels corresponds to the case of malformation, when the arterial and venous channels practically intersect at all points of the pathology area. This case of pressure equality in the channels is shown in figure 4.

![Figure 4](image2.png)

**Figure 4.** Isolines $p_a$ (a) and $p_v$ (b) for the case of malformation.
The results of the influence of embolism pathology in the local area for the venous vessel network are presented in the figure 5. Venous conduction \( k_v \) in the circle is taken a hundred times less than conduction of arteries \( k_a \). The rest of the brain \( k_b = k_a = k \). In this case, other parameters were taken \( \beta = 0.01 \) and \( \eta = 0.01 \). From figure 5 the behaviour of isolines shows that embolism in the venous network of vessels has little effect on the movement of arterial blood.

![Figure 5](image1.png)

**Figure 5.** Isolines \( p_a \) (a) and \( p_v \) (b) for the case of local venous embolism.

Numerical results for the case of embolism in the arterial network demonstrate that a blood pressure change in the local area where the arterial conductivity is taken one hundred times less than the conductivity of the veins has a strong effect for both blood networks. The pressure distribution for the arterial network (a) and venous network (b) in the presence of a weakly conducting brain area in the arteries is shown in figure 6.

![Figure 6](image2.png)

**Figure 6.** Isolines \( p_a \) (a) and \( p_v \) (b) for the case of local arterial embolism.
4. Conclusions
A series of calculations of human cerebral blood circulation in the presence of pathologies such as embolism or malformation in a relatively small circular region of the brain was carried out. The calculations are based on the mathematical model proposed in [1]. The results show an adequate correspondence of the calculations to medical conception about the nature of the physiological hemodynamic processes. The influence of pathologies on pressure fields in the arterial and venous networks is evaluated by comparing with a similar pressure distribution in a “healthy” body.

References
[1] Korsakova N, Pen’kovskij V, Shilova A and Shevchenko V 2016 Ser. Biomech. 30 (2) 24–31
[2] Penkovskii V I and Korsakova N K 2014 Proc. Int. Russian-Kazakhst. Conf. in Math. model. scince-tech. ecolog. problems oil-and-gas producing industry (Atyrau) pp 291–5
[3] Penkovskii V I and Korsakova N K 2018 Appl. Mech. Tech. Phys. 59 (3) 460–5
[4] Pen’kovskiy V I and Korsakova N K 2018 AIP Conf. Proc. 1939 020049