Evaluate Asymmetric Peristaltic Pumping Drug Carrying Image in Biological System: Measure Multiphase Flows in Biomedical Applications

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Abstract: The proposed model of drug delivery has been developed as a medication methodology for the direct treatment of diseased body tissues. The mathematical model is built upon the particulate peristaltic transport of an electrical conducting Jeffrey fluid within an asymmetric duct. The flow takes place under the action of slip effects due to the occurrence of magnetohydrodynamics, which is generally known as electrical resistance and the energy released by charged particles as they make collisions with other particles. Transportation of drug particles along with Jeffrey fluid due to peristaltic pumping in a rectangular duct is demonstrated. Magnetic force is utilized for the control of the process of pumping to the flow path at the right position. Taking into consideration the flow conditions and assumptions, the derivation of the system of partial differential equations of the flow is described. The eigenfunction expansion method is used to establish the solutions, and then the data are graphically displayed to imagine the effects of different parameters. It can be professed that the velocity component for Jeffrey fluid flow is decreased because of magnetic force, volume fraction size, and wall compliance. Heat and mass transfer with nanoparticles of different shapes of particles to extend this work.

Keywords: asymmetric traveling wave; eigen expansion method; bio-bi-phase flow; magnetohydrodynamics; peristaltic pumping; slip effects

1. Introduction

Recent literature has brought peristaltic movements to theoretical attention. Attitudes concerning the classification of peristaltic reflux also vary. A few biomedical mechanisms, such as dialysis and lung machines, follow this assumption. Stimulated by this, several tools, such as the peristaltic micropump (PDMS), heart–lung machine, etc., have been established. These are utilized in products such as sanitary transportation, blood siphons, pharma transportation systems, etc. Such shapes are manifested in various essential physical characteristics in the most important and useful sections of humans by having a rhythmic beating movement. They act in the same way as the sensory mechanism in human kidneys. They extract information from the adjacent liquid and transmit it to prepare the cells for urine transport. Specific examples such as ketchup, polymer rheology, blood flow, and stoneware are non-Newtonian fluids. At present, non-Newtonian fluids are more applicable relative to other fluids in the physiological field. Power-law fluid models exhibit shear thickening as one of the major classes of fluids. Comparing Jeffrey fluid and the power-law fluid model type, Jeffrey fluid is more useful. A pioneering study by Latham [1] reported...
experimental work on peristaltic transport. Heat transfer analysis with a nanofluid over a wavy channel was analyzed by Akbar et al. [2]. Peristaltic pumping of Jeffrey fluid in a rectangle was investigated by Nadeem et al. [3]. Ellahi et al. [4] studied heat and mass transfer in a wavy duct in a rectangular region of viscous non-Newtonian fluid. Several analytical, numerical, and experimental studies were discussed in [5–8]. De Vries et al. [9] observed that the intravascular flow for myometrial shrinkages is peristaltic flow, and myometrial contractions may appear in both asymmetric and symmetric paths. In their assessment, Eytan and Elad [10] mathematically established a model of peristaltic transport fluid flow in a two-dimensional channel. For numerous biological fluids, magnetohydrodynamic is also beneficial for controlling the flow because of the Lorentz force. Later on, Mekheimer [11] discovered the magnetohydrodynamic effects of couple stress on the peristaltic movement of non-Newtonian fluid. Numerous theoretical experiments [12–22] have been undertaken by several investigators on peristaltic movements under more than one simplified hypothesis of small amplitude ratio, low Reynolds number, small wave number, long wavelength, etc. Srivastava and Saxena [23] examined Casson fluid due to two-layer stenotic vessels that are appropriate for the cardiovascular system. After that, Srivastava [24] investigated a particulate movement fluid model of blood flow in stenotic vessels. Once more, Srivastava and Saxena [25] described a multiphase non-Newtonian fluid flow model for blood flow stimulated by peristaltic transport. Using the qualitative perspective finalized by fuzzy Delphi, Ali and Kaur [26] investigated warehousing practices and performance of Technology 4.0 of the Saudi Arabian Organization. Zeeshan et al. [27] investigated the effect of heat and mass transfer of multiphase flow on an asymmetric peristaltic transport in a curved configuration. The objective of another study was to investigate the social sustainability practices adopted by warehouse operation organizations by using mixed methods based on expert opinion and survey data [28]. Mahmoud et al. [29] successfully employed the effects of the Hartmann number on the peristaltic pumping of Jeffrey fluid flow using a porosity-applied Adomian decomposition method. This can help firms migrate to a circular economy (CE) and bring economic and environmental benefits. Khan et al. [30] used a modified Delphi method to find ten factors that affect the adoption of remanufacturing to propose a framework that would support the circular economy. Mekheimer et al. [31] considered the effects of MHD with peristaltic transport through porosity due to a surface auditory wavy channel. Several related experiments on the subject can be obtained from [32–39].

Comparative analysis of the multiphase peristaltic pumping of both Newtonian \( \lambda' \neq 0 \) and non-Newtonian \( \lambda' = 0 \) fluids under the contemplation of the MHD in an asymmetric duct is presented. In the current article, noncompressible Jeffrey fluid flow is peristaltically inspired and considered as a drug particle with nonconducting steady fluid. The eigenfunction expansion strategy is utilized to handle the problem of the liquid stage and the particle stage. The blouse is discussed in detail, and outcomes are examined with the help of diagrams.

2. Formulation of the Problem

In the present analysis, the authors studied the viscous intertemporal effects for particulate Jeffrey fluid in a rectangular duct enclosure through an asymmetric sinusoidal progressive wave produced by the peristaltic motion of the wall. Moreover, it was assumed that the steady, incompressible flow of Jeffrey fluid in the particulate peristaltic flow with an inclined magnetic field is applied perpendicular to the X-axis. The flow of the given problem is established in the Cartesian coordinate system, with the X-axis reflected parallel to the irrotational velocity \( c \). Figure 1 shows the flow pattern and variables.
Figure 1 can be mathematically expressed as [40]:

$$\Pi_1(x,t) = a_1 \sin\left(2\pi\lambda^{-1}(X - c\tilde{t})\right) - d_1,$$  \hspace{1cm} (1)

$$\Pi_2(x,t) = b_1 \left(\varphi + \sin\left(2\pi\lambda^{-1}(X - c\tilde{t})\right)\right) - d_2,$$  \hspace{1cm} (2)

where the velocity $\overline{U}$, $\overline{V}$ and $\overline{W}$ are components with dimensions $X$, $Y$ and $Z$, correspondingly. The governing equations of continuity and momentum for the particulate and fluid phase in a rotating frame are:

$$\nabla = \overline{\nabla}(X, Y, Z, \overline{t}), \overline{V}(X, Z, \overline{t}), \overline{W}(X, Z, \overline{t}).$$  \hspace{1cm} (3)

Jeffrey fluid $\overline{\Pi}$ as a base fluid stress tensor is [41]. In addition, $\dot{\gamma}$, $\ddot{\gamma}$ are substantial derivatives.

$$\overline{\Pi} = \frac{\mu_f}{1 + M} (\dot{\gamma} + \lambda^2 \ddot{\gamma}),$$  \hspace{1cm} (4)

The governing equations are in the laboratory frame $(X, Y, Z)$, which illustrates the flow of an incompressible Jeffrey fluid, given as follows:

$$\dot{u}_{f,p} + c = \overline{U}_{f,p}\dot{x} + c\tilde{t} = X_{f,p}, \dot{\varphi} = \overline{V}_{f,p}, \dot{\gamma} = \overline{V},$$

$$\nabla = \overline{\nabla}(X, Y, Z, \overline{t}), \overline{W}(X, Z, \overline{t}).$$  \hspace{1cm} (5)

Outlining the dimensionless parameters:

$$x = \delta\lambda^{-1}, y = \delta d^{-1}, z = \delta a^{-1}, u_{f,p} = \tilde{u}_{f,p} c^{-1}, w_{f,p} = \tilde{w}_{f,p}(c\delta)^{-1}, t = c\tilde{t}\lambda^{-1}, h_{1,2} = \Pi_{1,2} a^{-1},$$

$$\varphi = \delta a^{-1}, p = a^2\rho(uc\lambda)^{-1}, \delta = a\lambda^{-1}, \beta = ad^{-1}, \text{Re} = \rho\alpha\delta\lambda^{-1}, M = \sqrt{\delta\mu^{-1}},$$

$$M_i = S' a^2\mu a^{-1}(1 - C)^{-1}, \Pi_{\bar{g}z} = a(\mu c)^{-1} \Omega_{xw}, \Pi_{\bar{g}z} = a(\mu c)^{-1} \Omega_{xw}, \Pi_{\bar{g}y} = a(\mu c)^{-1} \Omega_{xw},$$

$$\Pi_{\bar{g}z} = d(\mu c)^{-1} \Omega_{gx}, \Pi_{\bar{g}z} = \lambda(\mu c)^{-1} \Omega_{xw}, \Pi_{\bar{g}y} = \lambda(\mu c)^{-1} \Omega_{xw}.$$  \hspace{1cm} (6)
Fluid phase:
\[
\frac{\partial u_f}{\partial x} = -\frac{\partial w_f}{\partial z},
\]
(7)

\[
\text{Re} \left( u_f \frac{\partial u_f}{\partial x} + v_f \frac{\partial u_f}{\partial x} + w_f \frac{\partial u_f}{\partial z} \right) + \frac{\partial p}{\partial x} = \delta \frac{\partial}{\partial x} \Omega_{x}\delta + \beta^2 \frac{\partial}{\partial y} \Omega_{xy} + \delta \frac{\partial}{\partial z} \Omega_{xz} + M_1 \left( u_p - u_f \right) C - M^2 \left( u_f + 1 \right) (1 - C)^{-1},
\]
(8)

\[
\frac{\partial p}{\partial y} = \delta \frac{\partial}{\partial x} \Omega_{x}\delta + \delta \frac{\partial}{\partial y} \Omega_{yy} + \delta \frac{\partial}{\partial z} \Omega_{yz} + M_1 C \left( v_p - v_f \right),
\]
(9)

\[
\text{Re} \delta^2 \left( u_f \frac{\partial w_f}{\partial x} + v_f \frac{\partial w_f}{\partial x} + w_f \frac{\partial w_f}{\partial z} \right) + \frac{\partial p}{\partial z} = \delta^2 \frac{\partial}{\partial x} \Omega_{x}\delta + \delta \beta^2 \frac{\partial}{\partial y} \Omega_{xy} + \delta \frac{\partial}{\partial z} \Omega_{zz} + \delta M_1 C \left( w_p - w_f \right).
\]
(10)

Particulate phase:
\[
\frac{\partial u_p}{\partial x} = -\frac{\partial w_p}{\partial z},
\]
(11)

\[
\text{Re} \left( u_p \frac{\partial u_p}{\partial x} + w_p \frac{\partial u_p}{\partial z} \right) + \frac{\partial p}{\partial x} = M_1 \left( u_f - u_p \right) (1 - C),
\]
(12)

\[
\frac{\partial p}{\partial y} = \delta M_1 \left( 1 - C \right) \left( v_f - v_p \right),
\]
(13)

\[
\text{Re} \delta \left( u_p \frac{\partial w_p}{\partial x} + w_p \frac{\partial w_p}{\partial z} \right) + \frac{\partial p}{\partial z} = M_1 \left( w_f - w_p \right) \delta (1 - C).
\]
(14)

Using Equation (6), the nondimensional parameters from Equation (7) to Equation (14) described. The developing equations with long wavelength and creeping flow for the fluid phase can be described as:

\[
\frac{dp}{dx} - CM_1 \left( u_f - u_p \right) + M^2 = \frac{\partial^2 u}{\partial y^2} \beta^2 \left( 1 + \lambda' \right)^{-1} + \frac{\partial^2 u}{\partial z^2} \left( 1 + \lambda' \right)^{-1} - M^2 u_f,
\]
(15)

Particulate phase:

\[
M_1^{-1} (1 - C)^{-1} \frac{dp}{dx} = -\left( u_p - u_f \right).
\]
(16)

where \( \lambda' \) represents the drag coefficients, and \( u_0 \) is used for the empirical relationship of fluid viscosity, which can be defined as:

\[
S' = 4.5 a^{-2} \mu_0 \lambda_1 \left( C \right), \quad \lambda_1 \left( C \right) = \frac{4 + 3 \sqrt{8 C - 3 C^2}}{4 + 9 C^2 - 12 C}, \quad \mu_S = (1 - \mu_0) \lambda \left( C \right), \quad \lambda = \frac{7}{100} \left( \frac{2.49 C + 107}{1 + 100 C} \right).
\]
(17)

Boundary conditions are defined in the nondimensional form.

\[
\begin{align*}
\text{at } z = h_1, & \quad u = -1, \quad y = y_{\text{slip}}, \quad \frac{\partial u}{\partial z} = -1, \\
\text{at } z = h_2, & \quad u = -1 + \frac{y_{\text{slip}}}{1 + \lambda'}, \quad \frac{\partial u}{\partial z} = 0.
\end{align*}
\]
(18)

The formulation of the complaint wall was expressed by [42].

\[
\begin{align*}
\frac{\partial^2 u}{\partial x^2} E_1 + \frac{\partial^2 u}{\partial y^2} E_2 + \frac{\partial^2 u}{\partial z^2} E_3 - \frac{\partial^2 u}{\partial x \partial y} E_4 - \frac{\partial^2 u}{\partial x \partial z} E_5 - CM_1 \left( u_f - u_p \right) + M^2 = \\
\frac{\partial^2 u}{\partial x^2} \beta^2 \left( 1 + \lambda' \right)^{-1} + \frac{\partial^2 u}{\partial z^2} \left( 1 + \lambda' \right)^{-1} - M^2 u_f,
\end{align*}
\]
(19)
3. Solution of the Problem

Equations (15) and (16) using the eigenfunction expansion method can be used to solve the coupled partial differential equations [43] as follows:

\[
\begin{align*}
    u_f &= \left( \frac{dp}{d\tau} S_1 \text{Cosh}[S_8] - \left( (-1 + C) S_2 + \frac{dp}{d\tau} \right) S_1 \text{Cosh} \left[ \frac{1}{2} S_2 (h_1 + h_2 - 2\pi S_1) \right] + S_2 \frac{dp}{d\tau} S_{\text{slip}} \text{Sinh}[S_8] \right) \\
    &\quad / \left( (-1 + C) S_2 \left( S_1 \text{Cosh}[S_8] + S_2 S_{\text{slip}} \text{Sinh}[S_8] \right) \right)
\end{align*}
\]

\[
\begin{align*}
    2(2\pi + 4n\pi) \text{Cos}[S_4] \text{Cosh}[yS_7] \text{Sech}[S_7] \left( \sin \left[ \frac{2}{S_6} S_8 \right] - \sin \left[ \frac{1}{2} S_6 \right] \right) \\
    \left( 1 + \left( \frac{dp}{d\tau} S_1 \text{Cosh} \left[ \frac{1}{2} S_8 \right] \right) - \left( (-1 + C) S_2 + \frac{dp}{d\tau} \right) S_1 \text{Cosh} \left[ \frac{1}{2} S_2 (h_1 + h_2 - 2\pi S_1) \right] + S_2 \frac{dp}{d\tau} S_{\text{slip}} \text{Sinh}[S_8] \right) / \\
    \left( (-1 + C) S_2 \left( S_1 \text{Cosh}[S_8] + S_2 S_{\text{slip}} \text{Sinh}[\frac{1}{2} S_8] \right) \right)
\end{align*}
\]

\[
\begin{align*}
    u_p &= \frac{1}{M_1(1 - C)} \left( \text{Cosh}[S_8] \frac{dp}{d\tau} S_1 - \left( \frac{dp}{d\tau} + S_2 (C - 1) \right) S_1 \text{Cosh} \left[ \frac{1}{2} (h_1 + h_2 - 2\pi S_1) S_1 S_2 \right] + \text{Sinh}[S_8] S_2 \frac{dp}{d\tau} \right) / \\
    &\quad \left( S_2 (C - 1) \left( S_{\text{slip}} \text{Sinh} \left[ \frac{1}{2} S_8 S_2 + \text{Cosh}[S_8] S_1 \right] \right) \right)
\end{align*}
\]

\[
\begin{align*}
    &\quad / \left( (2n - 1) \pi (h_1 S_2 - h_2 S_1 - 2h_1 S_2 + 2h_2 S_1 + \text{Sin}[h_1 (1 - 2n) \pi]) - \text{Sin}[h_2 (1 - 2n) \pi) \right)
\end{align*}
\]

where

- \(S_1 = \sqrt{1 + \lambda'}, S_2 = \sqrt{1 + M^2}, S_3 = h_1 - h_2, S_4 = \frac{1}{2} (-1 + 2n) \pi z,
- S_5 = \frac{1}{2} h_1 (1 - 1 + 2n) \pi S_6 = \frac{1}{2} h_2 (-1 + 2n) \pi,
- S_7 = \sqrt{\frac{\lambda' (-1 + 2n)^2 \pi^2 + M^2 (1 + \lambda')}{M^2}}, S_8 = \frac{1}{2} (h_1 - h_2) \sqrt{1 + M^2} \sqrt{1 + \lambda'}

4. Results and Discussion

In this section, a detailed explanation is provided for the flow problem of graph developments and the resulting numerical assessment model utilizing a multiphase fluid flow formulation for volume fraction. A comparison study is graphically shown in Figure 2 of the velocity profile between Newtonian (\(\lambda' \neq 0\)) and non-Newtonian fluid (\(\lambda' = 0\)).

We discuss the physical impacts of numerous parameters such as concentration of volume \(C\), Jeffrey parameter \(\lambda'\), velocity slip parameter \(S_{\text{slip}}\), magnetohydrodynamics \(M\), and complaint walls \(E_1, E_2, E_3, E_4, E_5\), as shown in Figures 3–12 for both fluid and particulate velocities.
Figures 3 and 4 are plotted to identify the effect of $S_{Slip}$ and $C$ as a volume fraction on fluid phase velocity $u_f$. It is noticed that when we increase the values of the slip parameter $S_{Slip}$ and volume fraction $C$ in the velocity profile, the flow of both liquid and particulate phases will continue to be particularly tightly combined for extremely dilute concentrations, where the rate of collisions among particulates is minimal. At greater concentrations, collisions among particles may lead to variations between the two phases of the wave propagating in the rectangular duct, with an increasing downturn with the passage of time. In Figure 5, the graph of the velocity profile for many values of magnetohydrodynamics $M$ shows a decrease by increasing the value of the parameter. The effect of magnetohydrodynamics rises, then there is an extensive decrement percieved in the velocity distribution due to the greater transverse magnetohydrodynamics creating more Lorentz force. Figure 6 shows that the velocity of the profile rises with the growing effects of the Jeffrey parameter $\lambda'$, which is an indication that the motion of the fluid is enhanced. Further, it is disclosed that the traveling of the consistent segment is in the position along with that of non-Newtonian Jeffrey fluid flow. Figure 7 demonstrates that by expanding the elastic parameters $E_1, E_2, E_3, E_4, E_5$, the velocity increases. It is additionally intriguing to take note of the fact that the velocity profile is illustrative of the fixed estimations of the parameters, and its size is the most extremely close to the focal point of the channel. From this, it is seen that the progression of residue particles corresponds with that of the fluid flow. Figures 8–12 define the steering of all the above parameters for the particulate velocity. It is visible that each one of the above-mentioned functions is extremely parallel to $U_p$, as visible in the fluid velocity section. The most effective distinction that can be simply shown from the figures of the particulates is that the velocity is distinctly substantially equal in this section with fluid phase speed for equal values of the different parameters.

![Figure 2](image_url)

**Figure 2.** Comparison of velocity profile between Newtonian ($\lambda' \neq 0$) and non-Newtonian fluid ($\lambda' = 0$).
Figure 3. $u_f$ curves of the different values of $S_{\text{Slip}}$.

With $\lambda' = 1.0, C = 0.1, M = 1.0$.

Figure 4. $u_f$ curves of the different values of $C$.

With $S_{\text{Slip}} = 0.4, M = 1.0, \lambda' = 1.0$. 
Figure 5. $u_f$ curves of the different values of $M$.

$S_{Slip} = 0.4, \lambda' = 1.0$ and $C = 0.1$. (20)

Figure 6. $u_f$ curves of the different values of $\lambda'$. 

With $M = 1.0, S_{Slip} = 0.4$, and $C = 0.1$. 

$\lambda' = 0.5, 1.0, 1.5, 2.0$
Figure 7. $u_f$ curves of the different values of $E_1, E_2, E_3, E_4, E_5$.

With $\lambda' = 1.0, M = 1.0, S_{Slip} = 0.4, \text{ and } C = 0.1$.

Figure 8. $u_p$ curves of the different values of $S_{Slip}$.

With $\lambda' = 1.0, C = 0.1, \text{ and } M = 1.0$. 
Figure 9. $u_p$ curves of the different values of $C$.

With $S_{Slip} = 0.4$, $M = 1.0$, and $\lambda' = 1.0$.

Figure 10. $u_p$ curves of the different values of $M$.

$S_{Slip} = 0.4$, $\lambda' = 1.0$ and $C = 0.1$. 
\[ \lambda' = 0.5, 1.0, 1.5, 2.0 \]

Figure 11. \( u_p \) curves of the different values of \( \lambda' \).

With \( M = 1.0, S_{\text{slip}} = 0.4 \), and \( C = 0.1 \).

\[ E_1 = 1.0, E_1 = 1.0, E_1 = 1.0, \]
\[ E_2 = 2.0, E_2 = 1.0, E_2 = 1.0, \]
\[ E_3 = 3.0, E_3 = 1.0, E_3 = 1.0, \]
\[ E_4 = 4.0, E_4 = 1.0, E_4 = 1.0, \]
\[ E_5 = 1.0, E_5 = 1.0, E_5 = 1.0, \]

Figure 12. \( u_p \) curves of the different values of \( E_1, E_2, E_3, E_4, E_5 \).

With \( \lambda' = 1.0, M = 1.0, S_{\text{slip}} = 0.4 \), and \( C = 0.1 \).

5. Streamlines

Figures 13–18 display the impact of different parameters such as slip, Jeffrey, and magnetic effect on the trapping for fluid and volume fraction velocities. The stream function with the level curve is supposed to move with the character of the wave boundaries of the walls. The volume of the bolus shrinks for the greater quantities of the slip parameters represented in Figure 13. In Figure 14, it is detected that the bolus magnitude is substantially accelerated by \( \lambda' \). The distribution bolus is enhanced with the spread in the Jeffrey parameter. Opposite results are examined for the magnetic effect, as shown
in Figure 15. However, Figures 16 and 18 demonstrate the bolus size is diminished for increasing standards of particulate fluid.

Figure 13. Streamlines for fluid flow for distinct values of $S_{\text{Slip}} = 0.1, 0.2, 0.3, 0.4$.

Figure 14. Cont.
Figure 14. Streamlines for fluid flow for distinct values of $\lambda' = 0.5, 1.0, 1.5, 2.0$.

Figure 15. Streamlines for fluid flow for distinct values of $M = 0.2, 0.4, 0.6, 0.8$. 
Figure 16. Streamlines for particulate flow for distinct values of $S_{slip} = 0.1, 0.2, 0.3, 0.4$.

Figure 17. Cont.
Conclusions

In the present study, a methodical study analyzing the impacts of magnetohydrodynamics on the peristaltic movement of a non-Newtonian Jeffry fluid model with an embedded solid particle due to an asymmetric rectangular duct intermittent sinusoidal wave on the barrier is modeled. The movement of the multiphase fluid combination through the compliant wall along with the uniform rectangular duct is supposed. The
mathematical problem that governs the multiphase is expressed in terms of a variety of physical parameters, such as lubrication supposition and arterial aspects assumed for the enclosed space. Partial differential equations were solved numerically, and results were obtained in the final closed form. The following accumulative outcomes were examined. The velocities of \( u_f \) and \( u_p \) are diminished because of wall compliance and magnetic force. It was observed that the Jeffrey parameter \( \lambda' \) and the slip parameter \( S_{\text{Slip}} \) measures improve the fluid velocity parameter \( u_f \), which is acceptable. It was noticed that as \( u_f \) and \( S_{\text{Slip}} \) increase, the volume and quantity of the trapped boluses reduce, but a reverse inspection is accumulated in the instance of \( u_p \) and the magnetic field number. It was also found that a similar type of evidence is achieved for the bolus occurrence of the particulate phase for the different parameters analyzed.

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**Nomenclature**

- \( \mathbf{U, V, W} \): Velocity components
- \( \mathbf{X, Y, Z} \): Cartesian coordinate
- \( C \): Volume fraction
- \( u_f \): Fluid phase velocity
- \( a_1 \): Wave amplitude
- \( S' \): Drag coefficient
- \( c \): Wave velocity
- \( R_e \): Reynold number
- \( d_1, d_2 \): Height of duct
- \( d_3 \): Width of duct
- \( S \): Drag force
- \( B_0 \): Magnetic field
- \( P \): Pressure in fixed frame
- \( S \): Stress tensor
- \( M \): Hartmann number
- \( E_1 \): Tension wall
- \( E_2 \): Mass wall
- \( E_3 \): Damping nature
- \( E_4 \): Rigidity
- \( E_5 \): Elasticity

**Greek symbols**

- \( \mu_s \): Viscosity of the fluid
- \( \sigma \): Electric conductivity of the fluid
- \( \lambda \): Wavelength
- \( \lambda' \): Jeffrey parameter
- \( \rho \): Fluid density
- \( \gamma, \dot{\gamma} \): Substantial derivatives
- \( \Phi \): Amplitude ratio

**Subscripts**

- \( f \): Fluid phase
- \( p \): Particulate phase
References

1. Latham, T.W. Fluid Motions in a Peristaltic Pump. Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 1966.

2. Sher Akbar, N.; Wahid Butt, A.; Noor, N.F.M. Heat Transfer Analysis on Transport of Copper Nanofluids Due to Metachronal Waves of Cilia. *Curr. Nanosci.* 2014, 10, 807–815. [CrossRef]

3. Nadeem, S.; Riaz, A.; Akbar, N.S. Mathematical model for the peristaltic flow of Jeffrey fluid with nanoparticles phenomenon through a rectangular duct. *Appl. Nanosci.* 2014, 4, 613–624. [CrossRef]

4. Ellahi, R.; Bhatti, M.M.; Vafai, K. Effects of heat and mass transfer on peristaltic flow in a non-uniform rectangular duct. *Int. J. Heat Mass Transf.* 2014, 71, 706–719. [CrossRef]

5. Pozrikidis, C. A study of peristaltic flow. *J. Fluid Mech.* 1987, 180, 515–527. [CrossRef]

6. Hina, S.; Mostafa, M.; Hayat, T.; Alsaedi, A. Peristaltic flow of couple-stress fluid with heat and mass transfer: An application in biomedicine. *J. Mech. Med. Biol.* 2015, 15, 1550042. [CrossRef]

7. Maiti, S.; Misra, J.C. Peristaltic transport of a couple stress fluid. Some applications to hemodynamics. *J. Mech. Med. Biol.* 2012, 12, 1250048. [CrossRef]

8. Shapiro, A.H.; Jaffrin, M.Y.; Weinberg, S.L. Peristaltic pumping with long wavelengths at low Reynolds number. *J. Fluid Mech.* 1969, 37, 799–825. [CrossRef]

9. Hayat, T.; Wang, Y.; Hutter, K.; Asghar, S.; Siddiqui, A.M. Peristaltic transport of an Oldroyd-B fluid in a planar channel. *Math. Probl. Eng.* 2004, 2004, 347–376. [CrossRef]

10. Hayat, T.; Mahomed, F.M.; Asghar, S. Peristaltic flow of magnetohydrodynamic Johnson-Segalman fluid. *Nonlinear Dyn.* 2005, 40, 375–385. [CrossRef]

11. Mekheimer, K. Effect of the induced magnetic field on peristaltic flow of a couple stress fluid. *Phys. Lett. A* 2008, 372, 4271–4278. [CrossRef]

12. Mekheimer, K.; Husseny, S.Z.-A.; El Lateef, A.I.A. Effect of Lateral Walls on Peristaltic Flow through an Asymmetric Rectangular Duct. *Appl. Biosci. Biochem.* 2011, 8, 295–308. [CrossRef]

13. Seadawy, A.R.; Ali, A.; Albarakati, W.A. Analytical wave solutions of the (2+1)-dimensional first in-tegro-differential Kadomtsev-Petviashivili hierarchy equation by using modified mathematical methods. *Results Phys.* 2019, 15, 102775. [CrossRef]

14. Alharbi, K.A.M.; Ijaz, N.; Riaz, A.; Ellahi, R. Peristaltic flow of couple-stress fluid with heat and mass transfer: An application in biomedicine. *J. Mech. Med. Biol.* 2015, 15, 1550042. [CrossRef]

15. Bhatti, M.M.; Zeeshan, A.; Ijaz, N.; Ellahi, R. Heat transfer and inclined magnetic field analysis on peristaltically induced motion of small particles. *J. Braz. Soc. Mech. Sci. Eng.* 2017, 39, 3259–3267. [CrossRef]

16. Almaleki, D. Stability of the Data-Model Fit over Increasing Levels of Factorial Invariance for Different Features of Design in Factor Analysis. *Eng. Technol. Appl. Sci. Res.* 2021, 11, 6849–6856. [CrossRef]

17. Zeeshan, A.; Ijaz, N.; Riaz, A.; Mann, A.B.; Hobiny, A. Flow of nonspherical nanoparticles in electromagnetohydrodynamics of nanofluids through a porous medium between eccentric cylinders. *J. Porous Media* 2020, 23, 1201–1212. [CrossRef]

18. Ahmed, I.; Seadawy, A.R.; Lu, D. Kinky breathers, W-shaped and multi-peak solitons interaction in (2 + 1)-dimensional nonlinear Schrödinger equation with Kerr law of nonlinearity. *Eur. Phys. J. Plus* 2019, 134, 120. [CrossRef]

19. Zeeshan, A.; Bhatti, M.M.; Ijaz, N.; Beg, O.A.; Kadir, A. Biologically inspired transport of solid spherical nanoparticles in an electrified conducting viscoelastic fluid with heat transfer. *Therm. Sci.* 2020, 24, 1251–1260. [CrossRef]

20. Reddy, M.G.; Makinde, O.D. Magnetohydrodynamic Peristaltic Transport of Jeffrey Nanofluid in an Asymmetric Channel. *J. Mol. Liq.* 2016, 223, 1242–1248. [CrossRef]

21. Almaleki, D.A. Accuracy of Data-Model Fit Using Growing Levels of Invariance Models. *Int. J. Comput. Sci. Netw. Secur.* 2021, 21, 157–164. [CrossRef]

22. Ijaz, N.; Zeeshan, A.; Rehman, S.U. Effect of electro-osmosis and mixed convection on nano-bio-fluid with non-spherical particles in a curved channel. *Mech. Ind.* 2018, 19, 108. [CrossRef]

23. Srivastava, V.; Saxena, M. Two-layered model of Casson fluid flow through stenotic blood vessels: Applications to the cardiovascular system. *J. Biomech.* 1994, 27, 921–928. [CrossRef]

24. Srivastava, V. Particle-fluid suspension model of blood flow through stenotic blood vessels with applications. *Int. J. Bio-Med. Comput.* 1995, 38, 141–154. [CrossRef]

25. Srivastava, V.P.; Saxena, M. A two-fluid model of non-Newtonian blood flow induced by peristaltic waves. *Rheol. Acta* 1995, 34, 406–414. [CrossRef]

26. Ali, S.S.; Kaur, R. Exploring the Impact of Technology 4.0 Driven Practice on Warehousing Performance: A Hybrid Approach. *Mathematics* 2022, 10, 1252. [CrossRef]

27. Zeeshan, A.; Ijaz, N.; Majeed, A. Analysis of magnetohydrodynamics peristaltic transport of hydrogen bubble in water. *Int. J. Hydron. Energy* 2018, 43, 979–985. [CrossRef]

28. Ali, S.S.; Kaur, R. Effectiveness of corporate social responsibility (CSR) in implementation of social sustainability in warehousing of developing countries: A hybrid approach. *J. Clean. Prod.* 2021, 324, 129154. [CrossRef]

29. Mahmoud, S.R.; Afifi, N.A.S.; Al-Isede, H.M. Effect of porous medium and magnetic field on peristaltic transport of a Jeffrey fluid. *Int. J. Math. Anal.* 2011, 5, 1025–1034. [CrossRef]
30. Khan, S.; Ali, S.S.; Singh, R. Determinants of Remanufacturing Adoption for Circular Economy: A Causal Relationship Evaluation Framework. Appl. Syst. Innov. 2022, 5, 62. [CrossRef]
31. Mekheimer, K.; Salem, A.; Zaher, A. Peristatcally induced MHD slip flow in a porous medium due to a surface acoustic wavy wall. J. Egypt. Math. Soc. 2014, 22, 143–151. [CrossRef]
32. Ijaz, N.; Zeeshan, A.; Riaz, A.; Alhodaly, M.S. Transport of drugs using complex peristaltic waves in a biological system. Waves Random Complex Media 2022, 1–16. [CrossRef]
33. Seadawy, A.R.; Lu, D.; Iqbal, M. Application of mathematical methods on the system of dynamical equations for the ion sound and Langmuir waves. Pramana 2019, 93, 10. [CrossRef]
34. Zeeshan, A.; Ijaz, N.; Bhatti, M.M. Flow analysis of particulate suspension on an asymmetric peristaltic motion in a curved con-figuration with heat and mass transfer. Mech. Ind. 2018, 19, 401. [CrossRef]
35. Almaleki, D.A.; Alomrany, A.G. The Effect of Methods of Estimating the Ability on The Accuracy and Items Parameters Accordingly to 3PL Model. Int. J. Comput. Sci. Netw. Secur. 2021, 21, 93–102.
36. Ijaz, N.; Zeeshan, A.; Bhatti, M. Peristaltic propulsion of particulate non-Newtonian Ree-Eyring fluid in a duct through constant magnetic field. Alex. Eng. J. 2018, 57, 1055–1060. [CrossRef]
37. Nadeem, S.; Akram, S. Pristaltic flow of a Jeffrey fluid in a rectangular duct. Nonlinear Anal. Real World Appl. 2010, 11, 4238–4247. [CrossRef]
38. Almaleki, D. Examinee characteristics and their impact on the psychometric properties of a multiple-choice test ac-cording to the item response theory (IRT). Eng. Technol. Appl. Sci. Res. 2021, 11, 6889–6901. [CrossRef]
39. Zeeshan, A.; Ijaz, N.; Bhatti, M.M.; Mann, A. Mathematical study of peristaltic propulsion of solid–liquid multiphase flow with a biorheological fluid as the base fluid in a duct. Chin. J. Phys. 2017, 55, 1596–1604. [CrossRef]
40. Akram, S.; Athar, M.; Saeed, K.; Umair, M.Y. Nanomaterials effects on induced magnetic field and dou-ble-diffusivity convection on peristaltic transport of Prandtl nanofluids in inclined asymmetric channel. Nanomater. Nanotechnol. 2022, 12, 18479804211048630. [CrossRef]
41. Kothandapani, M.; Srinivas, S. Peristaltic transport of a Jeffrey fluid under the effectof magnetic field in an asymmetric channel. Int. J. Non-Linear Mech. 2008, 43, 915. [CrossRef]
42. Hayat, T.; Javed, M.; Ali, N. MHD peristaltic transport of a Jeffery fluid in a channel with complaint walls and porous space. Transp. Porous Media 2008, 74, 259–274. [CrossRef]
43. Ellahi, R.; Hussain, F. Simultaneous effects of MHD and partial slip on peristaltic flow of Jeffery fluid in a rectangular duct. J. Magn. Magn. Mater. 2015, 393, 284–292. [CrossRef]