Analysis of progressive cavity pumps specific wear processes using Finnie models

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Abstract. Progressive cavitation pumps are designed to work in aggressive environments thus their wear is inevitable. The specific tribological coupling of these pumps is composed of a helical rotor with a single outward helix and a stator with a double inside helix. These elements are in relative motion and direct contact to each other and also in direct contact with the pumped oil. Therefore the main forms of wear of rotor-stator coupling elements are the abrasive wear and the erosion wear. In this paper is presented the analysis of erosion with Finnie models. The results are useful for estimation of progressive cavity pumps specific abrasive-erosive wear.

1. Introduction
The specific tribological coupling of progressive cavitation pump is composed of a helical rotor with a single outward helix and a stator with a double inside helix.

The rotor-stator coupling wear resistance and working life are ensured by high-performance solutions concerning the design and manufacture of progressive cavity pumps.

The pumps are designed to work in aggressive environments so their wear is inevitable.

Specific tribological processes of progressive cavity pumps can be studied considering the followings:

- rotor and stator materials;
- the characteristics of pumped fluid;
- contamination of oil pumped under pressure.

The rotor is made of steel (alloy steel or stainless steel with high hardness) and the stator is made of an elastomer with well-defined properties.

The specific couplings components are in relative motion and direct contact with each other and in direct contact with the pumped oil. The wear of the specific tribological coupling of the progressive cavitation pumps (rotor-stator) must be approach in this context.

The manner in which is archived the sealing between the helix of the rotor and the stator in order to ensure the high pressure of the fluid in the cavities formed, thus avoiding the reverse flow of discharge chamber to the aspiration chamber is of great tribological interest.
The fluids circulated by progressive cavitation pumps are characterized by the followings:

- viscosity and specific weight differentiated in a range of relatively high values;
- the content of solid particles (in some cases can reach up to 60% sand);
- water content which can reach up to 90%);
- the temperature of the pumped liquid that can be relatively high: 60 ÷ 120 °C for fluids without sand and 40 ÷ 90 °C for fluids containing sand;
- the existence of corrosive agents in the pumped oil (CO₂, H₂S or O₂).

Abrasive wear and erosion wear are the main forms of wear of the coupling components (rotor and stator). Essentially, abrasion and erosion are caused by the impact of the rigid particle on the active surfaces of the pump. The impact is mainly differentiated by the nature of the material subjected to stress: elastic impact and plastic impact.

According to Johnson [1] the elastic impact occurs at a speed nearly 0.1 m/s, and the full plastic penetration occurs at a speed of 5 m/s in progressive cavitation pumps case. For plastic materials the plastic deformation of the target surface occurs at a very low speed (for hard steel with σₖ = 1000 N/mm², the impact speed is 0.14 m/s [2]).

Pumped oil contains abrasive solid particles in suspension. The particles entrained by the rotor spiral are pressed on the surface of the stator which is made of an elastomer (synthetic rubber) that deforms and fixes the particles during the contact. In this way, it is achieved an efficient sealing of pumping chambers and thus the effects of abrasion are reduced. After the removal of the rotor from the stator the particle of sand that was interposed falls under its own weight into the pumping chamber (cavity). In this case the particles circulated by the fluid with certain speeds are directed to the surface of the rotor and the stator and become active in terms of abrasion and erosion.

The arguments previously presented underline the requirement of studying wear processes specific to progressive cavity pumps.

2. Analytical models of abrasive erosion

Theoretical and experimental research of erosive wear processes have been and are currently a constant concern of many experts in the field of tribology [3-8]. The research results of erosion were materialized by the development of mathematical models specific to these tribological process. In his study, Meng [9] presented and analyzed 28 such models. Essentially the mathematical models of erosion suggest calculation equations for the intensity of erosive wear.

Development of abrasive erosion models is obviously marked by technical level of theoretical and experimental investigation resources which are currently in an accelerated dynamic of improvement.

It is an unanimous assessment of experts in the field that 1990 marks an important milestone in the research development history of erosive wear [10-11].

The pioneering stage in development of erosion mathematical models is marked by classic models foundation, characterized as the "spine" of the research of erosive processes.

Thus, the classical models of erosion, developed before 1990 are those proposed by Finnie in 1960 [12] and in 1972 [13], Bitter in 1963 [14], Hutchings (in 1974, 1979 and 1981) [15], Johnson and Cook (in 1983 and 1985) [16], and Sundararajan and Shewmon in 1983 [17]. These models have been validated by research performed by specialists in the fields and undeniably reflects the true image of the essence of erosion mechanism.

Using the numerical calculation methods for describing the mechanism of erosion it opens a new stage of research in this field which is located between 1990-1995 and continued up to present date [10]. Using finite element method bi- and tridimensional models of erosion caused by abrasive particles specified forms (spherical in most cases) and different sizes which interacts with the surfaces of ductile and brittle materials were developed. Thus, based on the finite element method, Wang and Yang [10] model describe the effect of impact angles, impact speed and penetration depth of the particle on the intensity of erosive wear process.
3. Results of the erosion wear analysis using Finnie models. Discussions

Finnie models are applied for two erosion processes: the one in which the proportion of abrasive particles with microcutting effect ($p_a$) is 10% and the second one in which the percentage is 50%.

The proposed equations for the first Finnie model [12] (developed in 1960) are as following:

$$E_F = \frac{p_a \cdot \rho_m \cdot v^2 \cdot (1 - c_r^2)}{0.9272 \cdot H_s \cdot \psi \cdot K_F} \left( \sin 2\alpha - \frac{6}{K_F} \sin^2 \alpha \right), \text{ for } \tan \alpha \leq \frac{K_F}{6}$$  \hspace{1cm} (1)

$$E_F = \frac{p_a \cdot \rho_m \cdot v^2 \cdot (1 - c_r^2)}{0.9272 \cdot H_s \cdot \psi \cdot K_F} \left( \frac{K_F}{6} \cos^2 \alpha \right), \text{ for } \tan \alpha \geq \frac{K_F}{6}$$  \hspace{1cm} (2)

where: $E_F$ – dimensionless wear rate; $p_a$ – the percentage of abrasive particles with microcutting effects; $\rho_m$ – target material density; $v$ – particle impact speed (m·s$^{-1}$); $\alpha$ – the angle of incidence; $c_r$ – restitution coefficient; $H_s$ – static hardness; $\Psi$ – the ratio of the contact length ($L$) and cutting depth ($\delta$) of impact area; $K_F$ – the ratio of the horizontal component ($F_0$) and the vertical component ($F_v$) of the characteristic impact force.

The equations are inferred by considering the following assumptions:
- abrasive particles are sharp;
- cutting and recoil effects of the particles are significant;
- the material is plastically deformed by the impact of the abrasive particle.

The calculation formulas modified by Finnie (in 1972) [13] have the following structure:

$$E_F = \left( \frac{p_a \cdot \rho_m \cdot v^2 \cdot (1 - c_r^2)}{2K} \right) \frac{0.9272 \cdot H_s}{\sin 2\alpha - \frac{8}{K_F} \sin^2 \alpha}, \text{ for } \tan \alpha \leq \frac{K_F}{6}$$  \hspace{1cm} (3)

$$E_F = \left( \frac{p_a \cdot \rho_m \cdot v^2 \cdot (1 - c_r^2)}{16} \right) \frac{0.9272 \cdot H_s}{\cos^2 \alpha}, \text{ for } \tan \alpha \geq \frac{K_F}{6}$$  \hspace{1cm} (4)

The involved parameters in the calculation formulas, for both Finnie models, are listed in table 1, together with their associated values used in the analysis performed.

| parameter | calculation value |
|-----------|-------------------|
| $p_a$ – the percentage of abrasive particles with microcutting effects for a concentration of 3.3x10$^3$ mg/l sand | 50% - for the first model: $p_a$=1.65 kg/m$^3$; 10% - for the second model: $p_a$ = 0.33 kg/m$^3$; |
| $\rho_m$ – density of the target area [kg/m$^3$] | 1.19 |
| $\alpha$ – the angle of incidence [$^\circ$] | 30 $^\circ$ |
| $v$ – particle impact speed [m/s] | 4.71 |
| $H_s$ – surface hardness [N/m$^2$] | stator I: 80; stator II: 75; stator III: 70 |
| $c_r$ – restitution coefficient | 0.5 |
| $\rho_a$ – density of the particle material [kg/m$^3$] | 2590-2670 |
| $E_e$ – equivalent elastic modulus [N/m$^3$] | stator I: 4.2; stator II: 3.6; stator III: 1.7 |
| $\Psi$ – the ratio of the contact length ($L$) and the cutting depth ($\delta$) of the impact area | 0.7÷15 |
| $\mu$ – friction coefficient between the particle and the material | 0.1÷1 |
Analysis of wear rate variation was achieved for the following cases:

1. $E_F = f(\alpha)$ - the variation depending on the incidence angle, for the three analyzed stators (the surface hardness $H_s$ is: for stator I: 80 N/m$^2$; stator II: 75 N/m$^2$; stator III: 70 N/m$^2$), for three values of the friction coefficient ($\mu = 0.1; \ 0.5; \ 1$). The results are shown in figure 1.

Figure 1. Wear rate variation depending on the angle of incidence for: (a) and (b) stator I; (c) and (d) stator II; (e) and (f) stator III.
2. \( E_F = f(\mu) \) - the variation depending on the ratio of the contact length and the cutting depth of the impact area, for different values of the incidence angle (\( \alpha \) = 15°; 30°; 60°), for the three analyzed stators (the surface hardness \( H_s \) is: for stator I: 80 N/m\(^2\); stator II: 75 N/m\(^2\); stator III: 70 N/m\(^2\)), for the friction coefficient of \( \mu = 0.5 \). The results are shown in figure 2.

First Finnie model

Second Finnie model

Figure 2. Wear rate variation depending on the ratio of the contact length and the cutting depth of the impact area for: (a) and (b) stator I; (c) and (d) stator II; (e) and (f) stator III.
3. $E_p = f(v)$ - the variation depending on the impact speed, for the three analyzed stators (the surface hardness $H_s$ is: for stator I: 80 N/m$^2$; stator II: 75 N/m$^2$; stator III: 70 N/m$^2$), for the friction coefficient of $\mu = 0.5$. The results are shown in figure 3.

![Figure 3](image_url)

**Figure 3.** Wear rate variation depending on the impact speed for: (a) First Finnie model; (b) Second Finnie model.

In table 2 are shown the calculated values of the wear rate with Finnie models, for the angle of incidence $\alpha = 20^\circ$ and the speed of $v = 4.71$ [m/s], and the values experimentally determined for the three analyzed stators.

| Stator | The calculation model of the rate of wear | Rate of wear | Calculated value | Experimentally determined value [18] |
|--------|------------------------------------------|--------------|------------------|-------------------------------------|
| I      | Finnie I                                 |              | $6.454 \times 10^{-3}$ | $5.356 \times 10^{-3}$ |
|        | Finnie II                               |              | $4.841 \times 10^{-3}$ |                       |
| II     | Finnie I                                 |              | $6.885 \times 10^{-3}$ | $5.783 \times 10^{-3}$ |
|        | Finnie II                               |              | $5.163 \times 10^{-3}$ |                       |
| III    | Finnie I                                 |              | $7.376 \times 10^{-3}$ | $6.175 \times 10^{-3}$ |
|        | Finnie II                               |              | $5.532 \times 10^{-3}$ |                       |

The analysis of the obtained results for the two Finnie models leads to the following findings:
- the shape of wear rate variation curves with the angle of incidence is the same for different values of the friction coefficient and the same value of the impact speed;
- is distinguished an incidence angle at which the wear rate has a maximum value; this is considered the critical angle of incidence in relation to the erosive effects it can produces;
- wear rate variation depending on the angle of incidence is different in relation to the critical angle of incidence: from zero to the critical angle the wear rate increases rapidly, then decreases slowly becoming zero at incidence angle of 90$^\circ$;
- the maximum value rate of the wear rate and the angle of incidence at which is recorded this value it is changing depending on the friction coefficient: the increase of the friction coefficient leads to decreasing of the wear rate and to increasing of the critical angle of incidence;
- for angles of incidence higher than the critical value, erosive wear rate is not influenced by the friction coefficient;
- the wear rate decreases exponentially with the $\psi$ coefficient (the ratio between the contact length and the depth of the cut) for angles of incidence smaller than 45$^\circ$ - for the first model, and is maintained constant for any value of the $\psi$ coefficient – for the second model;
erosive wear rate increases exponentially with speed impact at the same angle of incidence (for the second analyzed model); the increase is not influenced by the friction coefficient;
for the erosion developed at the same speed and angle of incidence, the wear rate is not influenced by the friction coefficient between the erodent particle and the target area.

4. Conclusion
- in all analyzed cases, the estimated erosive wear rate with second Finnie model is less than the one assessed with the first Finnie model, which is explained by the fact that the percentage of abrasive particles with microcutting effects is 10% for the second model, versus 50% for the first model (the basic assumptions for the two models);
- the presented graphical results highlights the influence on the eroded surface hardness on the erosion wear: the increas hardness leads to a lower rate of erosion wear;
- the results achieved with the second Finnie model are the closest to the experimentally determined values of wear rate. It is therefore recommended to use the second model for the evaluation of erosive process of progression cavitation pumps.

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