The Non-Isolated Resolving Number of Some Corona Graphs

W Abidin, A N M Salman and S W Saputro

Combinatorial Mathematics Research Group, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Indonesia.

Corresponding author: wahyuniabidin@uin-alauddin.ac.id

Abstract. An ordered set \( W = \{w_1, w_2,..., w_k\} \subseteq V(G) \) and a vertex \( v \) in a connected graph \( G \), the representation of \( v \) with respect to \( W \) is the ordered \( k \)-tuple \( r(v|W) = (d(v,w_1), d(v,w_2),..., d(v,w_k)) \), where \( d(x,y) \) represents the distance between the vertices \( x \) and \( y \) in \( G \). The set \( W \) is called a resolving set for \( G \) if every vertex of \( G \) has distinct representations. A resolving set with the minimum number of vertices is called a basis for \( G \) and its cardinality is called the metric dimension of \( G \), denoted by \( \text{dim}(G) \). A resolving set \( W \) is called a non-isolated resolving set if the induced subgraph \( (W) \) has no isolated vertices. The minimum cardinality of a non-isolated resolving set of \( G \) is called the non-isolated resolving number of \( G \), denoted by \( nr(G) \). The corona product between a graph \( G \) and a graph \( H \), denoted by \( G \circ H \), is a graph obtained from one copy of \( G \) and \( |V(G)| \) copies \( H_1, H_2,..., H_n \) of \( H \) such that all vertices in \( H_1 \) are adjacent to the \( i \)-th vertex of \( G \). We study the non-isolated resolving sets of some corona graphs. We determine \( nr(G \circ H) \) where \( G \) is any connected graph and \( H \) is a complete graph, a cycle, or a path.

1. Introduction

All graphs in this paper are finite, simple, and connected. Let \( G = (V,E) \) be a graph. The distance \( d(u,v) \) between two vertices \( u \) and \( v \) in a connected graph \( G \) is the length of a shortest \( u - v \) path in \( G \). For \( W = \{w_1, w_2,..., w_k\} \subseteq V(G) \) and a vertex \( v \) in a connected graph \( G \), the representation of \( v \) with respect to \( G \) is the ordered \( k \)-tuple \( r(v|W) = (d(v,w_1), d(v,w_2),..., d(v,w_k)) \). If every distinct vertices \( x, y \in V(G) \) satisfy \( r(x|W) \neq r(y|W) \), then \( W \) is called a resolving set. A resolving set with the minimum cardinality is called a basis of \( G \). Its cardinality is called the metric dimension of \( G \), denoted by \( \text{dim}(G) \).

The metric dimension problem was first studied by Harary and Melter [8] and independently by Slater [15]. Slater considered the minimum resolving set of a graph as the location of the placement of a minimum number of sonar/loran detecting devices in a network. Thus, the position of every vertex in the network can be uniquely describe by its distances to the devices in the set.

The metric dimension problem is a difficult problem. Garey and Johnson [7] have shown that determining the metric dimension of any graph is an NP-Problem. However, some results for certain classes of graph has been obtained, which can be seen in [2,5,9,10,11,12,13,16].

Now in this paper, let us consider the other version of the resolving set problem, which is called a non-isolated resolving number. In this version, if an induced subgraph of \( G \) by a resolving set \( W \) does not contain an isolated vertex, then \( W \) is called a non-isolated resolving set. The non-isolated resolving
set with minimum cardinality is called an \( nr \)-set. The non-isolated resolving number of \( G \), denoted by \( nr(G) \), is the cardinality of \( nr \)-set of \( G \).

This non-isolated resolving problem was introduced by Chitra and Arumugam [6]. They have proven that \( nr(G) \leq 2 \dim(G) \). They have characterized all connected graphs of order \( n \geq 3 \) with \( nr(G) = n - 1 \). They have also determined an exact value of the non-isolated resolving number of some graphs included paths, complete graphs, friendship graphs, complete bipartite graphs, and the Cartesian product of graphs.

In another paper, Yunika et al. [17] determined the non-isolated resolving number of some exponential graphs. Meanwhile, Avadayappan et al. [1] determined the non-isolated resolving number of double broom graphs, the join of a complete graph and a path.

In this paper, we consider the corona product between two connected graphs \( G \) and \( H \). The corona graph \( G \odot H \) is a graph obtained from one copy of \( G \) and \( V(G) \) copies \( H_{1}, H_{2},...,H_{|V(G)|} \) of \( H \) where every vertex of \( H_{i} \) is adjacent to the \( i \)-th vertex of \( G \) We recall that a graph \( G + H \) is a graph with \( V(G + H) = V(G) \cup V(H) \) and \( E(G + H) = E(G) \cup E(H) \cup \{xy| x \in V(G), y \in V(H)\} \). Note that \( G \odot H \) contains an induced subgraph which is isomorphic to \( K_{1} + H \). Some of our results provide a connection between \( nr(G \odot H) \) and \( \dim(K_{1} + H) \). In order to prove that, we use the following useful lemma, which has been proved in [14].

**Lemma 1 [14]** Let \( Q \) be a connected graph. Then there exists a basis \( S \) of \( G + K_{1} \) such that \( S \subseteq V(G) \).

2. Main Results
In this section, we study the non-isolated resolving set of \( G \odot H \) where \( G \) is a connected graphs and \( H \) is either a complete graph, a path, or a cycle as stated in Theorem 1,2,3, respectively.

**Theorem 1** Let \( m \) and \( n \) be two positive integers. Let \( G \) be a connected graph \( G \) of order \( n \geq 1 \). Then

\[
\text{nr}(G \odot K_{m}) = \begin{cases} 
  nm, & \text{if } m = 2, \\
  n(m - 1), & \text{if } m \geq 3. 
\end{cases}
\]

*Proof.*

Let \( V(G) = \{a_{1}, a_{2}, ..., a_{n}\} \) and \( H = (G \odot K_{m}) \), where \( V(H) = V(G) \cup \{v_{l}^{l} v_{j}^{l}|1 \leq l \leq n, 1 \leq i < j \leq m\} \) and \( E(H) = E(G) \cup \{v_{l}^{l} v_{j}^{l}|1 \leq l \leq n, 1 \leq i < j \leq m\} \cup \{a_{i} v_{l}^{l}|1 \leq l \leq n, 1 \leq i < j \leq m\} \). We distinguish two cases.

**Case 1.1 :** \( m = 2 \)

For \( l \in \{1,2,...,n\} \), we define the set \( W_{l} = \{v_{l}^{1}, v_{2}^{l}\} \). Let \( W = \bigcup_{l=1}^{n} W_{l} \). Note that \( |W| = 2n \). Since \( v_{l}^{1} v_{2}^{l} \in E(H) \), it is clear that \( W \) does not contain an isolated vertex.

Now, we will show that \( W \) is a resolving set of \( H \). Let \( x \) and \( y \) be two distinct vertices in \( V(H) - W \). Then \( x \) and \( y \) are in \( G \). Let \( x = a_{p} \) \( x = a_{p} \) and \( y = a_{q} \) for some \( p, q \in \{1,2,...,n\} \) with \( p \neq q \). Since \( d(x, v_{2}^{p}) = 1 \neq d(y, v_{2}^{p}), r(x|W_{p}) \neq r(y|W_{p}) \). It implies that \( r(x|W) \neq r(y|W) \).

By contradiction, suppose that \( \text{nr}(G \odot K_{m}) \leq 2n - 1 \).

Let \( W \) be an \( nr \)-set of \( G \odot K_{m} \) with \( |W| \leq 2n - 1 \). For \( l \in \{1,2,...,n\} \), let \( W_{l} = \{v_{l}^{i} \in W| i \in \{1,2,...,m\}\} \). Since \( |W| \leq 2n - 1 \), there exists \( l \in \{1,2,...,n\} \) such that \( |W_{l}| \leq 1 \). If \( |W_{l}| = 0 \), then for \( w \in W \) \( d(v_{l}^{1}, w) = d(v_{2}^{l}, w) \), which implies \( r(v_{l}^{1}|W) = r(v_{2}^{l}|W) \), a contradiction.

If \( |W_{l}| = 1 \), then \( W \) contains an isolated vertex, a contradiction.

**Case 1.2 :** \( m \geq 3 \)

For \( l \in \{1,2,...,n\} \), we define the set \( W_{l} = \{v_{l}^{1}, v_{2}^{l}, ..., v_{m-1}^{l}\} \). Let \( W = \bigcup_{l=1}^{n} W_{l} \). Note that \( |W| = n(m - 1) \). Since \( v_{l}^{i} v_{j}^{l} \in E(H) \) for \( 1 \leq i < j \leq m - 1 \), it is clear that \( W \) does not contain an isolated vertex.

Now, we show that \( W \) is a resolving set of \( H \). Let \( x \) and \( y \) be two distinct vertices in \( V(H) - W \). We distinguish three subcases.
Suppose now that following three facts.

Let $x = v^p_m$ and $y = v^q_m$ for $p, q \in \{1, 2, ..., n\}$ with $p \neq q$. Since $d(x, v^p_m) = 1 \neq 3 \leq d(y, v^q_1)$, we have $r(x|W_p) \neq r(y|W_p)$.

Subcases 1.2.2:
Let $x = a_p$ and $y = a_q$ for $p, q \in \{1, 2, ..., n\}$ with $p \neq q$. Since $d(x, v^p_m) = 1 \neq 2 \leq d(y, v^q_1)$, we have $r(x|W_p) \neq r(y|W_p)$.

Subcases 1.2.3:
Let $x = v^p_m$ and $y = a_q$ for $p, q \in \{1, 2, ..., n\}$. Since $d(x, v^p_m) = 1 \neq 2 \leq d(y, v^q_1)$, we obtain $r(x|W_p) \neq r(y|W_q)$.

All subcases above imply that $r(x|W) \neq r(y|W)$.

Now, we will prove that $nr(G \otimes K_m) \geq n(m-1)$. Let $W$ be an $nr$-set of $G \otimes K_m$. For every $l \in \{1, 2, ..., n\}$, let $W_l = \{v^l_1 \in W \mid l \in \{1, 2, ..., m\}\}$. We have claim that $|W_l| \geq m - \frac{1}{2}$.

Otherwise, we have two vertices $v^l_s$ and $v^l_t$ for some $s, t \in \{1, 2, ..., m\}$ such that $v^l_s \notin W_t$ and $v^l_t \notin W_s$. Since $d(v^l_s, x) = d(v^l_t, x) \forall x \in W$, we obtain $r(v^l_s|W) = r(v^l_t|W)$, a contradiction. Hence, $|W| \geq n(m - 1)$.

Next, we consider the corona graph $H = G \otimes Q_n$, where $G$ is any connected graph and $Q_n$ is a path or a cycle with order $n$. Let $S$ be a set of two or more vertices of $Q_n$. Let $v^l_1, v^l_2 \in S$ be two distinct vertices of $Q_n$. Let $P(v^l_1, v^l_2)$ be a path in $Q_n$ from $v^l_1$ to $v^l_2$. We define a gap between $v^l_1$ and $v^l_2$ as $V(P(v^l_1, v^l_2)) = \{v^l_1, v^l_2\}$, where every vertex in a gap is not element of $S$.

The vertices $v^l_1$ and $v^l_2$ we called as end points of gap between $v^l_1$ and $v^l_2$. Two different gaps are called neighboring gaps if they have common end point.

In case $Q_n$ is a cycle, if $|S| = r$, then $S$ has $r$ gaps. In case $Q_n$ is a path, if $|S| = r - 1$, then $S$ has $r$ gaps. Note that, for both cases, some of gaps maybe empty. This definition was first introduced by Buczowski et al. [3] to prove the metric dimension of the wheel graph. In addition M. Bača et al. [2] using this gap technique to prove metric dimensions of complete bipartite graph minus its Hamiltonian cycle.

Now, let $V(H_1) = V(\{a_1\} + Q_n^l)$, where $a_1 \in V(G), Q_n^l \subseteq Q_n$ and $W_l$ be basis of $H_l$. We observe the following three facts.

(i) Every gap of $W_l$ contains at most three vertices. Otherwise, there is a containing four vertices $v^l_j, v^l_{j+1}, v^l_{j+2}, v^l_{j+3}$ of $Q_n^l$, where $1 \leq j \leq n, 1 \leq l \leq m$. However, $r(v^l_{j+1}|W_l) = r(v^l_{j+2}|W_l) = (2, 2, ..., 2)$, a contradiction.

(ii) At most one gap of $W_l$ contains three vertices. Otherwise, there exist distinct two gaps $\{v^l_j, v^l_{j+1}, v^l_{j+2}\}$ and $\{v^l_k, v^l_{k+1}, v^l_{k+2}\}$. However, $r(v^l_{j+1}|W_l) = r(v^l_{k+2}|W_l) = (2, 2, ..., 2)$, a contradiction.

(iii) If a gap of $W_l$ contains at least two vertices, then any neighboring gaps contain at most one vertex. Otherwise, there exist five consecutive vertices $v^l_j, v^l_{j+1}, v^l_{j+2}, v^l_{j+3}, v^l_{j+4}$ of $Q_n^l$, such that $v^l_{j+2}$ is the only vertex of $W_l$. However, $r(v^l_{j+1}|W_l) = r(v^l_{j+3}|W_l)$, a contradiction.

Suppose now that $W_l$ is any set of vertices (a basis or not) of $Q_n^l$ that satisfies (i)-(iii), and let $v$ be any vertex of $V(H_l) - W_l$. There are four possibilities.

(1) $v$ belongs to a gap of size 1 of $W_l$. Let $v^l_1$ and $v^l_2$ be the neighboring vertices of $W_l$ that determine this gap. Then $v$ is adjacent to $v^l_1$ and $v^l_2$ and has distance 2 from all other vertices of $W_l$. Since $n \geq 7$, no other vertices of $H_l$ has this property and so $r(v|W_l) \neq r(x|W_l)$ for $v \neq x$.

(2) $v$ belongs to a gap of size 2 of $W_l$. Then we may assume that $v^l_1, v^l_{j+1} = v, v^l_{j+2}, v^l_{j+3}$ are vertices of $Q_n^l$, where $v^l_{j+1}, v^l_{j+3} \in W_l$ and $v^l_{j+2} \notin W_l$. Then $v$ is adjacent to $v^l_2$ and has distance 2 from
all other vertices of $W_i$. By property (iii), only $v$ has this property and so $r(v|W_i) \neq r(x|W_i)$ for $v \neq x$.

(3) $v$ belongs to a gap of size 3 of $W_i$. Then there exist vertices $v^i_j, v^i_{j+1}, v^i_{j+2}, v^i_{j+3}, v^i_{j+4}$ of $Q_n^i$, only $v^i_{j+1}$ and $v^i_{j+4}$ which of belong to $W_i$. Assume first that $v = v^i_{j+1}$. Then $v$ is adjacent to $v^i_j$ and has distance 2 from all other vertices of $W_i$. By property (iii), $v$ is the only vertex of $H_i$ with this property and so $r(v|W_i) \neq r(x|W_i)$ for $v \neq x$. Next, we assume that $v = v^i_{j+2}$. Then $r(v|W_i) = (2, 2, 2, \ldots, 2)$. By properties (i) and (ii), no other vertex of $H_i$ has this representation.

(4) $u = a_i$. Then $r(u|W_i) = (1, 1, \ldots, 1)$ and $u$ is the only vertex of $H_i$ with this representation. Consequently, any set $W$ having properties (i)-(iii) is a resolving set of $H_i$.

The following lemma will be used to prove the upperbound of the Theorem 2 and Theorem 3.

**Lemma 2** For $n \geq 7$, let $Q_n$ be a path or a cycle. Then very basis $S$ of $K_1 + Q_n$ contains an isolated vertex.

**Proof.**

Suppose there is a basis $S$ of $K_1 + Q_n$ that does not contain an isolated vertex.

**Case 2.1:** $|S|$ is even.

Let $|S| = 2q$

**Subcases 2.1.1:** $Q_n$ is a cycle.

For some integer $q \geq 1$. By (iii) at most $q$ gaps contain more than one vertex and, by (i) and (ii), all contain at most two vertices except possibly one containing three vertices. So, the number of vertices belonging to the gaps of $S$ is at most $2q + 1$. Since $S$ does not contain an isolated vertex, we have $q$ empty gaps. Hence $n - 2q \leq 2q + 1$, which implies that $|S| = 2q \geq \frac{n+1}{2}$. In [3], Buczkowski et al. has been proven that $\dim(C_n + K_1) = \left[\frac{2n+2}{5}\right]$. Since $\left[\frac{n-1}{2}\right] = \left[\frac{5n-5}{10}\right] > \left[\frac{4n+4}{10}\right] = \left[\frac{2n+2}{5}\right]$, we have a contradiction.

**Subcases 2.1.2:** $Q_n$ is a path.

For some integer $q \geq 1$. By (iii) at most $q$ gaps contain more than one vertex and, by (i) and (ii), all contain at most two vertices except possibly one containing three vertices. So, the number of vertices belonging to the gaps of $S$ is at most $2q - 1$. Since $S$ does not contain an isolated vertex, we have $q$ empty gaps. Hence $n - 2q \leq 2q - 1$, which implies that $|S| = 2q \geq \frac{n+1}{2}$. In [4], Cáceres et al. has been proven that $\dim(P_n + K_1) = \left[\frac{2n+2}{5}\right]$. Since $\left[\frac{n+1}{2}\right] = \left[\frac{5n+5}{10}\right] > \left[\frac{4n+4}{10}\right] = \left[\frac{2n+2}{5}\right]$, we have a contradiction.

**Case 2.2:** $|S|$ is odd.

Let $|S| = 2q + 1$.

**Subcases 2.2.1:** $Q_n$ is a cycle.

For some integer $q \geq 1$. By (iii) at most $q$ gaps contain more than one vertex and, by (i) and (ii), all contain at most two vertices except possibly one containing three vertices. So, the number of vertices belonging to the gaps of $S$ is at most $2q + 1$. Since $S$ does not contain an isolated vertex, we have $q + 1$ empty gaps. Hence $n - 2q - 1 \leq 2q + 1$, which implies that $|S| = 2q \geq \frac{n-2}{2}$. In [3], Buczkowski et al. has been proven that $\dim(C_n + K_1) = \left[\frac{2n+2}{5}\right]$. Since $\left[\frac{n-2}{2}\right] = \left[\frac{5n-10}{10}\right] > \left[\frac{4n+4}{10}\right] = \left[\frac{2n+2}{5}\right]$, we have a contradiction.

**Subcases 2.2.2:** $Q_n$ is a path.

For some integer $q \geq 1$. By (iii) at most $q$ gaps contain more than one vertex and, by (i) and (ii), all contain at most two vertices except possibly one containing three vertices. So, the number of vertices belonging to the gaps of $S$ is at most $2q - 1$. Since $S$ does not contain an isolated vertex, we have $q + 1$ empty gaps. Hence $n - 2q - 1 \leq 2q - 1$, which implies that $|S| = 2q + 1 \geq \frac{n+2}{2}$. In [4], Cáceres et al.
al. has been proven that $\dim(P_n + K_1) = \left\lceil \frac{2n+2}{5} \right\rceil$. Since $\frac{n+2}{2} = \left\lceil \frac{5n+10}{10} \right\rceil > \left\lceil \frac{4n+4}{10} \right\rceil = \left\lceil \frac{2n+2}{5} \right\rceil$, we have a contradiction.

**Theorem 2** For $n \geq 7$, let $G$ be a connected graph of order $m \geq 1$ then, 
\[ nr(G \hat{\circ} C_n) = \left\lceil \dim(K_1 + C_n) + 1 \right\rceil m \]

**Proof.** Let $V(G) = \{a_1, a_2, ..., a_m\}$, $V(C_n) = \{v_1, v_2, ..., v_p\}$. For $n \geq 7$, let $H = (G \hat{\circ} C_n)$, $V(H) = V(G) \cup \{v_1 \mid 1 \leq i \leq n, 1 \leq l \leq m\}$ and $E(H) = E(G) \cup \{v_i v_{i+1}^1, v_i v_{i+1}^1 \mid 1 \leq i < j \leq n - 1\} \cup \{a_i v_j^1, 1 \leq l \leq m\}$. Let $V(K_1 + C_n) = \{v_0\} \cup V(C_n)$ and $E(K_1 + C_n) = E(C_n) \cup \{v_0 v_i \mid 1 \leq i \leq n\}$. Let $B$ be a basis of $K_1 + C_n$. In [14] it is proven that there exists a basis $B$ of $K_1 + Q$ for a connected graph $Q$, such that all vertices of $B$ are from $Q$.

For $l \in \{1, 2, ..., m\}$, we define the set $W_l = \{v_i^l \mid v_i \in B\} \cup \{a_i\}$. Note that, $|W_l| = \dim(K_1 + C_n) + 1$. Let $W = \bigcup_{l=1}^{m} W_l$. Since $v_i^l a_i \in E(H)$, then $W$ does not contain an isolated vertex.

Now, we will show that $W$ is a resolving set of $H$. Let $x$ and $y$ be two different vertices in $V(H) - W$.

i. Let $x = v_i^l$ and $y = v_p^l$, with $a, b \in \{1, 2, ..., n\}$, $a \neq b$. Since $x$ and $y$ are the vertices in a copy of $K_1 + C_n$ and $B$ is a basis of $K_1 + C_n$, then $x$ and $y$ resolve by $W_l$. Therefore, $r(x|W) \neq r(y|W)$.

ii. Let $x = v_i^l$ and $y = v_p^l$, with $a, b \in \{1, 2, ..., n\}$, $l, p \in \{1, 2, ..., m\}$, $l \neq p$. Since $d(x, a_i) = 1 \neq 2 \leq d(y, a_i)$ then $r(x|W) \neq r(y|W)$.

By contradiction, suppose that $nr(G \hat{\circ} C_n) \leq \dim(K_1 + C_n) + 1| m - 1$.

Let $W$ be an $nr$-set of $H$ with $|W| \leq \dim(K_1 + C_n) + 1| m - 1$. For $l \in \{1, 2, ..., m\}$, let $W_l = \{v_i^l, a_i \in W \mid i \in \{1, 2, ..., m\}\}$. Then there exists $l \in \{1, 2, ..., m\}$ such that $|W_l| \leq \dim(K_1 + C_n)$. Since an induced subgraph of $H$ by $\{a_i, v_i^l \mid 1 \leq i \leq n\}$ is isomorphic to $K_1 + C_n$, say $H_l$, then $|W_l|$ must be $\dim(K_1 + C_n)$. So, it is clear that every two different vertices in $H_l$ has different representation with respect to $W_l$. However, by Lemma 2, every basis $B$ of $K_1 + C_n$ contains an isolated vertex. Therefore, we have a contradiction.

**Theorem 3** For $n \geq 7$, let $G$ be a connected graph of order $m \geq 1$, then 
\[ nr(G \hat{\circ} P_n) = \left\lceil \dim(K_1 + P_n) + 1 \right\rceil m. \]

**Proof.** Let $V(G) = \{a_1, a_2, ..., a_m\}$, $V(P_n) = \{v_1, v_2, ..., v_n\}$. For $n \geq 7$, let $H = (G \hat{\circ} P_n)$, $V(H) = V(G) \cup \{v_i^l \mid 1 \leq i \leq n, 1 \leq l \leq m\}$ and $E(H) = E(G) \cup \{v_i v_{i+1}^1, v_i v_{i+1}^1 \mid 1 \leq i < j \leq n - 1\} \cup \{a_i v_j^1, 1 \leq l \leq m\}$. Let $V(K_1 + P_n) = \{v_0\} \cup V(P_n)$ and $E(K_1 + P_n) = E(P_n) \cup \{v_0 v_i \mid 1 \leq i \leq n\}$. Let $B$ be a basis of $K_1 + P_n$. In [14] it is proven that there exists a basis $B$ of $K_2 + Q$ for a connected graph $Q$, such that all vertices of $B$ are from $Q$.

For $l \in \{1, 2, ..., m\}$, we define the set $W_l = \{v_i^l \mid v_i \in B\} \cup \{a_i\}$. Note that, $|W_l| = \dim(K_1 + P_n) + 1$. Let $W = \bigcup_{l=1}^{m} W_l$. Since $v_i^l a_i \in E(H)$, then $W$ does not contain an isolated vertex.

Now, we will show that $W$ is a resolving set of $H$. Let $x$ and $y$ be two different vertices in $V(H) - W$.

i. Let $x = v_i^l$ and $y = v_p^l$, with $a, b \in \{1, 2, ..., n\}$, $a \neq b$. Since $x$ and $y$ are the vertices in a copy of $K_1 + P_n$ and $B$ is a basis of $K_1 + P_n$, then $x$ and $y$ resolve by $W_l$. Therefore, $r(x|W) \neq r(y|W)$.

ii. Let $x = v_i^l$ and $y = v_p^l$, with $a, b \in \{1, 2, ..., n\}$, $l, p \in \{1, 2, ..., m\}$, $l \neq p$. Since $d(x, a_i) = 1 \neq 2 \leq d(y, a_i)$ then $r(x|W) \neq r(y|W)$.

By contradiction, suppose that $nr(G \hat{\circ} P_n) \leq \dim(K_1 + P_n) + 1| m - 1$.

Let $W$ be an $nr$-set of $H$ with $|W| \leq \dim(K_1 + P_n) + 1| m - 1$. For $l \in \{1, 2, ..., m\}$, let $W_l = \{v_i^l, a_i \in W \mid i \in \{1, 2, ..., m\}\}$. Then there exists $l \in \{1, 2, ..., m\}$ such that $|W_l| \leq \dim(K_1 + P_n)$. Since
an induced subgraph of $H$ by $\{a_i, v_i^1 | 1 \leq i \leq n\}$ is isomorphic to $K_1 + P_n$, say $H_t$, then $|W_t|$ must be $\dim(K_1 + P_n)$. So, it is clear that every two different vertices in $H_t$ has different representation with respect to $W_t$. However, by Lemma 2, every basis $S$ of $K_1 + P_n$ contains an isolated vertex. Therefore, we have a contradiction.

3. Conclusion

In this paper, we have studied non-isolated resolving set of the corona product $G \odot H$ where $G$ is any connected graphs and $H$ is complete graph, a cycle or a paths. We obtain an exact value of non-isolated resolving number of them.

References

[1] Selvam Avadayappan, M Bhuvaneshwari and P Jeya Bala Chitra 2017 More results on research International Journal of Appl. and Adv. Sci. Research Special Issue 49

[2] M Bača, ET Baskoro, A N M Salman, S W Saputro and R Simanjuntak 2011 The metric dimension of regular bipartite graphs Bull. Math. Soc. Sci. Math. Roumanie 54(102) 15

[3] P S Buczkowski, G Chartrand, C Posisson and P Zhang 2003 On k-dimensional Graphs and their bases Period. Math. Hungar 46(1) 9

[4] J Caceres, C Hernando, M Mora, I M Pelayo, C Seara and D R Wood 2000 On the metric dimension of some families of graphs Electronic Notes in Discrete Math. 105 99

[5] G Chartrand, L Eroh, M A Johnson and O R Oellermann 2000 Resolvability in graphs and the metric dimension of a graph Discrete Appl. Math. 105 99

[6] P. J. B. Chitra and S Arumugam 2015 “Resolving sets without isolated vertices”, Procedia Comput. Sci, 74, pp.38-42.

[7] M R Garey and D S Johnson 1979 Computers and Intractability: A Guide to The Theory of NP-Completeness (California:W.H. Freeman).

[8] F Harary and R A Melter 197 On the metric dimension of a graph Ars Combin 2 191

[9] H Iswadi, E T Baskoro, A N M Salman and R Simanjuntak 2010 The metric dimension of amalgamation cycles Far. East J. Math. Sci. 41(1) 19

[10] H Iswadi, E T Baskoro, A N M Salman and R Simanjuntak 2010 The resolving graph of amalgamation cycles Utilitas Mathematica 83 121

[11] H Iswadi, E T Baskoro and R Simanjuntak 2011 On the metric dimension of corona product of graphs Far. East J. Math. Sci. 52(2) 155

[12] H Iswadi, E T Baskoro, R Simanjuntak and A N M. Salman 2008 The metric dimension of graphs with pendant edges Combin. Math. Combin. Comput. 65 139

[13] S W Saputro, E T Baskoro, A N M Salman and D Suprijanto 2009 The metric dimension of a complete n-partite and its Cartesian product with a path Combin. Math. Combin. Comput. 7, 283-293.

[14] S W Saputro, R Simanjuntak, S Uttunggadewa, H Assiyatun, E T Baskoro, A N M Salman and M Bača 2013 The metric dimension of the lexicographic product of graphs Discrete Math 213 1045

[15] P J Slater 1975 Leaves of trees Congr. Numer 14 549

[16] I Tomescu and I Javaid 2000 On the metric dimension of the Jahangir graph Bull. Math. Soc. Sci. Math. Roumanie 4 371

[17] S M Yunika, Slamin, Dafi and Kusbudiono 2016 On the metric dimension with non-isolated resolving number of some exponentional graph Proceeding The 1st IBSC: Towards The Extended Use Of Basic Science For Enhancing Health, Environment Energy And Biotechnology (University of Jember, Indonesia) pp. 328-330.