Superposition models and the multiplicity fluctuations in heavy ion collisions

K. FIAŁKOWSKИ, R. WIT

M. Smoluchowski Institute of Physics
Jagellonian University
30-059 Kraków, ul.Reymonta 4, Poland

Abstract

A class of simple superposition models based on the Glauber picture of multiple collisions is compared with the data on the centrality dependence of the multiplicity distributions in a central rapidity bin. We show how the results depend on the specific assumptions concerning the distributions in the number of participants and their relations to the distributions of the number of produced hadrons in various phase space bins. None of the versions of the model describes satisfactorily the centrality dependence of the scaled dispersion.

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\(^1\)e-mail address: fialkowski@th.if.uj.edu.pl
\(^2\)e-mail address: romuald.wit@uj.edu.pl
1 Introduction

In a recent note [1] we have analyzed the PHENIX data [2] on the centrality dependence of the multiplicity distributions in a central pseudorapidity bin for heavy ion collisions. We have shown that a superposition model which presents the final state as a simple superposition of the final states from nucleon-nucleon collisions (described by the PYTHIA 8 generator [3]) fails to describe the data. Even if the average number of nucleon-nucleon collisions is treated as a free parameter to be fitted to the average multiplicity for each centrality bin, the dispersion values for these bins disagree with the data.

We have suggested two possible reasons for this failure. First, the multiplicity distributions for the pp collisions measured by PHENIX are not reproduced correctly by PYTHIA with the default parameter values; the fluctuations in data are significantly bigger than in the model. Second, it is well known that the average multiplicities in heavy ion collisions are not exactly proportional to the number of "wounded" nucleons [4] in colliding nuclei [5]. Thus it does not seem reasonable to assume that the final state may be considered as a simple superposition of final states from nucleon-nucleon collisions.

One may try to remove the deficiencies listed above taking into account two following remarks. First, it is well known that the multiplicity distributions in any pseudorapidity bin for the pp collisions may be successfully parametrized by negative binomial distribution [6]. Such a distribution with the parameters fitted to the data should be thus used instead of that from PYTHIA with the default parameter values. Second, the wounded nucleon model is known to work much better when modified to the "wounded quark model" [7]. In a simple version of this model presented recently [8] each nucleon consists of a quark and a diquark. The nucleon-nucleon collision may be well approximated by the interaction of just one component from each nucleon (both contributing similarly to the multiplicity of the final state). However, for the nucleon interacting more than once in the heavy ion collision, both components are likely to interact and one gets a double contribution to the multiplicity of the final state.

Another possible improvement of the model is the inclusion of fluctuations in the number of participants for a given impact parameter. The reliable description of such fluctuations requires the detailed knowledge of the nuclear structure, but one may estimate the influence of this effect by considering simple distributions "bracketing" the physically plausible distributions from below and above.

However, such modifications are difficult to be built into the full-fledged Monte Carlo generator (e.g., PYTHIA). Therefore we construct a class of simple generators which, for consecutive values of the impact parameter, produce the samples of "events" consisting only of two numbers. These are the numbers of charged hadrons in the phase space regions corresponding to the two detectors used in the PHENIX experiment. Such "events" are then processed exactly in the same way as the PHENIX data.

In the next section we describe the construction of such generators, introducing gradually the improvements discussed above and comparing the consecutive versions with the data. In the last section the results are summarized and some conclusions are drawn.

2 Simple models and the PHENIX data

The PHENIX data for the multiplicity distributions in a small central pseudorapidity bin are collected for various beams, energies and the centrality classes defined by multiplicity ranges in the auxiliary "BBC counters" covering the pseudorapidities \( \eta \) from the range...
3 < |η| < 3.9. As before [1], we concentrate here on the AuAu data for 200 GeV, as the dependence on the energy and atomic number is not very difficult to reproduce.

To implement the superposition idea we use the code based on the Glauber formalism [9] for the heavy ion collision which calculates for each impact parameter the number of interacting nucleons in two colliding nuclei [10]. This code was already used for comparison of the wounded nucleon and the wounded quark models [11]. In the following we assume that the average multiplicities in the BBC counters \(< n_{BBC} >\) and in the central counter \(< n_c >\) are proportional to the global number of interacting nucleons in both nuclei \(N_p\):

\[< n_{BBC} > = \alpha N_p, \quad < n_c > = \beta N_p.\]

### 2.1 Wounded nucleons

Let us assume a simple geometric distribution of the impact parameter, in which the number of events is proportional to \(b\). To relate the generator to the experimental data we need the values of the two constants \(\alpha\) and \(\beta\). To reproduce the experimental distribution of \(n_{BBC}\) presented by PHENIX [12] we take \(\alpha = 5.2\). A fit to the dependence of \(< n_c >\) on \(N_p\) shown in ref. [2] gives the value of \(\beta = 0.18\).

In the PHENIX data each centrality class (defined by the limits on \(n_{BBC}\)) was labeled by the value of \(N_p\) corresponding to this range of \(n_{BBC}\) in the HIJING event generator. In our model this correspondence is only slightly different; the average values of \(N_p\) in all classes (defined by the same bounds on \(n_{BBC}\)) here and in the next subsections are shifted in the worst case by a few percent. For consistency, in what follows, we use our values of \(N_p\) on the \(x\) axes of the plots.

The simplest assumption on the multiplicity distributions in the BBC and central counters (for fixed \(b\) and \(N_p\)) is to describe them by the Poisson distribution. Then one may calculate the scaled dispersion of the distribution in \(n_c\) (for the central counter) for each centrality class as defined by a cut in \(n_{BBC}\). More precisely, we calculate for each value of \(b\) the number of the events \(N_{ev}\) to be generated (proportional to \(b\)) and the values of \(N_p\), \(< n_{BBC} >\) and \(< n_c >\). Then we generate \(N_{ev}\) values of \(n_{BBC}\) according to the Poisson distribution with average \(< n_{BBC} >\) and register the numbers \(N_{ev}^i\) of the values falling in the ranges which define consecutive centrality classes\(^3\). Finally, these values of \(N_{ev}^i\) are used to decide how many values of \(n_c\) should be generated according to the Poisson distribution for each class. The final distribution of \(n_c\) for each class is the sum of the distributions generated for all values of \(b\).

This procedure by definition reproduces correctly the experimental dependence of the value of \(< n_c >\) on the corresponding average value of \(N_p\) defined for the consecutive centrality classes. This is illustrated in Fig.1, where for transparency the big experimental error bars are omitted. The real test of the model is thus the centrality dependence of the scaled variance

\[\omega = (\langle n_c^2 > - \langle n_c >^2)/\langle n_c >.\]

The results are shown and compared with the PHENIX data in Fig.2 together with the results from two other versions of the model (to be discussed later). Here and in the following the errors are comparable with the size of the data points.

\(^3\)An independent consistency check of our procedure is the fact that the numbers of events in all classes defined by the limiting values of \(n_{BBC}\) are almost the same (with a few percent accuracy); remember that these values were selected by PHENIX to divide the global sample of events into the equally populated bins. We repeat this check in the following for all versions of the model.
Figure 1: The average multiplicity in the central detector for the PHENIX pp and AuAu data (+ marks) and the superposition model (x marks). The results of other versions of the model, shown as squares and stars, will be commented upon later.

We see that the fluctuations are strongly underestimated and increase monotonically with $N_p$ in disagreement with the data. In fact, the values are quite similar (although slightly lower) to the results obtained by superimposing the $pp$ events generated from PYTHIA. This is so because the Poisson distribution is only slightly narrower than the distribution predicted by PYTHIA for the $pp$ collisions.

In the next step we replace the Poisson distribution by the negative binomial distribution (NBD):

$$P_{NBD}^{<n>,k}(n) = \frac{\Gamma(n + k)}{\Gamma(k)\Gamma(n + 1)} \left(\frac{<n>}{<n>+k}\right)^n \left(\frac{k}{<n>+k}\right)^k$$

with parameters fitted to the PHENIX pp data. More precisely, if we assume that a single participant nucleon yields (for some part of the phase space) a charged hadron distribution described by the NBD with the parameters $<n>$ and $k$, we expect for the $pp$ collisions the NBD with parameters $2 <n>, 2k$, and for a heavy ion collision with $N_p$ participants the NBD with parameters $N_p <n>$ and $N_p k$. The rest of the procedure remains unchanged.

There is an uncertainty connected with this prescription for the NBD parameters. The PHENIX pp data for the central bin are collected on condition that there is at least one charged hadron in each of the two BBC counters. Thus we cannot assume that the exact values of the measured parameters of this distribution should be used to predict the distribution for given value of $N_p$, where obviously not all the participants must produce the hadrons falling into these counters. Therefore in the following we compare the results for different values of the parameters for the "elementary" NBD in $n_c$ (and $n_{BBC}$).

We have performed the calculations with the average multiplicities of the $n_c$ and $n_{BBC}$ distributions calculated for each $N_p$ as before and with three different choices of the $k$ parameter. In the first calculation, for all values of $N_p$ we have taken approximately the same ratio of $k_c$ to $<n_c>$ as in the PHENIX pp data (about 6) and assumed that for the distribution of $n_{BBC}$ this ratio is the same. In the second one, we assumed much broader distribution of $n_{BBC}$, with $k/ <n>\sim 1$. Finally, we have chosen the distribution of $n_c$. 

with the ratio of $k_c$ to $<n_c>$ smaller than in the PHENIX $pp$ data by a factor of 0.5. The last two choices may be regarded as maximizing the fluctuations due to the spread of multiplicities which define the centrality classes. In fact, the third choice gives such a broad distribution of $n_c$ for a given $N_p$ that it may be regarded as breaking the basic assumption of our class of models: building the heavy ion collisions from the elementary $pp$ collisions. Indeed, $k_c/ <n_c> \simeq 3$ corresponds to the value of $\omega - 1$ twice as big as in the PHENIX $pp$ data.

The first two choices give very similar results, significantly higher than the Poisson distributions, but the fluctuations in heavy ion collisions are still underestimated (see Fig.2). The scaled dispersion increases monotonically with the number of participants, contrary to the data. For the third choice we match the data for a low number of participants, but not for the central events, where the model results keep increasing while the data fall down.

2.2 Wounded quarks

In this subsection a more refined model based on the "wounded quark" idea [7, 8] is developed. We shall modify the code [10] which was used to calculate for each value of the impact parameter $b$ the global number of participants in two colliding nuclei

$$N_p(\vec{b}) = \int d^2s T_A(\vec{s}) \{ 1 - [1 - \frac{\sigma_{NN}T_B(\vec{s} - \vec{b})}{B}]^B \} + \int d^2s T_B(\vec{s}) \{ 1 - [1 - \frac{\sigma_{NN}T_A(\vec{s} - \vec{b})}{A}]^A \}.$$

Similar arguments allow to calculate the global number of nucleons which interacted exactly once

$$N_{p}^1(\vec{b}) = \int d^2s T_A(\vec{s})\sigma_{NN}T_B(\vec{s} - \vec{b})\left[ 1 - \frac{\sigma_{NN}T_B(\vec{s} - \vec{b})}{B} \right]^{B-1} + \int d^2s T_B(\vec{s})\sigma_{NN}T_A(\vec{s} - \vec{b})\left[ 1 - \frac{\sigma_{NN}T_A(\vec{s} - \vec{b})}{A} \right]^{A-1}.$$
and the number of those, which interacted at least twice

\[ N_p^2(\vec{b}) = \int d^2 s T_A(\vec{s}) \left\{ 1 - \left[ 1 - \frac{\sigma_{NN} T_B(\vec{s} - \vec{b})}{B} \right]^B \right\} + \sigma_{NN} T_B(\vec{s} - \vec{b}) \left[ 1 - \frac{\sigma_{NN} T_B(\vec{s} - \vec{b})}{B} \right]^{B-1} \]

\[ + \int d^2 s T_B(\vec{s}) \left\{ 1 - \left[ 1 - \frac{\sigma_{NN} T_A(\vec{s} - \vec{b})}{A} \right]^A \right\} - \sigma_{NN} T_A(\vec{s} - \vec{b}) \left[ 1 - \frac{\sigma_{NN} T_A(\vec{s} - \vec{b})}{A} \right]^{A-1} \]

with the obvious condition \( N_p = N_p^1 + N_p^2 \). Assuming that the multiple interaction results in doubling the average multiplicity and the value of the \( k \) parameter one gets for each value of \( \vec{b} \) the multiplicity distribution given by

\[ P(n) = \gamma P_{NBD}^{N_p^1 < n_0 >, N_p^1 k} (n) + (1 - \gamma) P_{NBD}^{N_p^2 < n_0 >, 2N_p^2 k} (n), \]

where \( \gamma = \frac{N_p^1}{(N_p^1 + N_p^2)} \). Note that the average multiplicity in this distribution is

\[ < n > = \left[ \gamma N_p^1 + 2(1 - \gamma) N_p^2 \right] < n_0 >. \]

To reproduce the dependence of \( < n_{BBC} > \) and \( < n_c > \) on \( N_p > \) we needed the values of \( < n_0 > \) of 2.7 and 0.095, correspondingly. The ratio \( k/# n_0 > \) was assumed about 6, as before. Now we generate the distributions in the BBC and central counters according to this distribution and proceed as before. The same procedure is also performed for the Poisson distribution. The dependence of the average multiplicity on \( N_p \), shown in Fig.1 as squares, is exactly the same as in the previous version of the model (shown as the \( x \)-signs). The results for the scaled dispersion are shown in Fig.3. The values are slightly lower, but the main feature of Fig.2 remains unchanged: the dispersion increases monotonically with the number of participants, in disagreement with the data.

![Figure 3](image.png)

**Figure 3:** The scaled dispersion for the PHENIX pp and AuAu data (+ marks) and the "wounded quark" model with the Poisson distributions (\( x \) marks) and NBD distributions (stars).

Thus, contrary to our speculations formulated in [1], the replacement of the "wounded nucleon" by the "wounded quark" model did not improve the situation. The reason is that the fluctuations of \( n_c \) for a given centrality class result mainly from the fluctuations in the number of sources contributing to the values of \( n_{BBC} \) in the range defining this class. It does not really matter if these sources are nucleons or quarks (diquarks); for
the linear relation between their number and average value of $n_{BBC}$ the results are quite similar. The slight reduction of the dispersion values is probably a consequence of the fact that the number of sources is now bigger.

The values of the scaled dispersion may be increased by decreasing the $k_c/ < n_c >$ value, as in the previous subsection. However, the centrality dependence remains monotonic in disagreement with the data.

2.3 Fluctuating number of participants

There is still one assumption in the simple model we use which may be modified without abandoning the basic superposition idea. Using the code [10] one can calculate a well defined value of $N_p$ (or $N_p^1$ and $N_p^2$) for each value of $b$. Obviously, in the real collision of nuclei this number may fluctuate and the calculated value should be treated rather as an average. Moreover, there is a simple reason why such fluctuations should not increase monotonically with centrality: for most central collisions almost all nucleons are "wounded" and the average is close to the maximal possible value. This may damp the fluctuations and eventually may lead to non-monotonic dependence of the scaled dispersion on the number of participants.

In what follows we consider different distributions which may be assumed for $N_p$ around this average. The reliable choice would require a thorough knowledge of the nuclear structure, and in particular of the two nucleon correlations in the position space. However, one may use simple lower and upper limits to the fluctuations. As the first one, we use a simple binomial distribution of the number of participants with the average $<N_p>$ calculated from the code [10]; its maximal value is obviously given by $2A$. Let us remind here that the dispersion squared of this distribution is given by

$$D^2 = <N_p> (1 - <N_p> / 2A).$$

![Scaled dispersion](figure4.png)

Figure 4: The scaled dispersion for the PHENIX $pp$ and $AuAu$ data (+ marks) and the superposition model with the binomial smearing in the number of participants and the Poisson distributions ($x$ marks) and the NBD distributions (stars)

Now the procedure is more involved. For each value of the impact parameter $b$ we calculate $<N_p>$ as before, then generate randomly the values of $N_p$ from the binomial
distribution with this average, calculate the average values of $n_{BBC}$ and $n_c$ (proportional to $N_p$) and generate the values of these variables according to the Poisson or NBD distributions. The results are shown in Fig.4.

Contrary to the naïve expectations, the fluctuations in the number of participants did not increase the values of the scaled dispersion. In fact, the values for the NBD distributions are significantly lower than in Fig.2 for the same values of the $k$ parameter. This may be interpreted as the result of the weaker correlation between the range of $n_{BBC}$ and of $n_c$, since these two values are generated independently and the range of $N_p$ is now broader for each "class of centrality".

The second choice, maximizing the spread of $N_p$ for a given $b$, is just a flat distribution. More precisely, we assume that for the $\langle N_p \rangle$ value below $A/4$ the probability of any value of $N_p$ between 0 and $2 < N_p >$ is the same, for the $\langle N_p \rangle$ value between $A/4$ and $3A/4$ the allowed range is between $\langle N_p \rangle - A/4$ and $\langle N_p \rangle + A/4$ and for $\langle N_p \rangle$ above $3A/4$ the allowed range of $N_p$ is between $2A - 2 \langle N_p \rangle$ and $2A$. Now the dispersion squared is given in these three ranges of $\langle N_p \rangle$ by

$2 < N_p > (2 < N_p > + 1)/6, \quad A(A/2 + 1)/24, \quad (2A - < N_p >)(4A - 2 < N_p > + 1)/6.$

These values are obviously much higher than in the case of the binomial distribution considered before. Repeating the procedure described above we get the scaled dispersion as shown in Fig.5. The results for the average multiplicity are shown as stars in Fig.1.

For the Poissonian distribution and for the NBD with realistic values of $k/<n>$ the results are almost the same as in Fig.4. If we choose the value of $k/<n>$ much lower than in the $pp$ data, as already used in Fig.2, we get non-monotonical dependence of $\omega$ on $\langle N_p \rangle$. However, the shape disagrees with the data and the values are still much too low. Thus we conclude that by increasing the fluctuations in the number of participants we are not able to reproduce the data.

Figure 5: The scaled dispersion for the PHENIX $pp$ and $AuAu$ data (+ marks) and the superposition model with the flat smearing in the number of participants and the Poisson distributions (x marks) and the NBD distributions (stars and squares)
3 Conclusions and outlook

We considered the multiplicity distributions in the central rapidity bin in the heavy ion collisions for various "centrality classes" defined by the multiplicity in another rapidity bin. A class of simple generators is constructed basing on the assumption that the final state is a superposition of states obtained separately from each participant nucleon. We considered the modifications resulting from counting the "wounded quarks" instead of the "wounded nucleons" and from the fluctuations in the number of participant nucleons for the given value of the impact parameter. None of the versions of the superposition model considered is compatible with the data. In particular, the observed slow increase of the scaled dispersion at moderate centralities and the decrease for the most central events is not reproduced.

The reason why the modifications introduced did not yield the expected improvement seems to be the particular definition of "centrality classes" used in the PHENIX experiment. Both the "wounded quark" idea and the fluctuations in the number of participants may give saturation (or even decrease) of the fluctuations for highest centrality assuming that the centrality is defined by a fixed range of the number of participants. However, when it is defined by the multiplicity of hadrons in some detector (as in the PHENIX data) these expectations are not justified, as we have seen in the model calculations.

Our conclusions are that it seems to be difficult (if not impossible) to describe the dependence of the scaled dispersion on centrality measured by PHENIX in the framework of superposition models. This suggests the presence of some collective effects in these data.

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