Isospin violation in mixing and decays of \( \rho \)- and \( \omega \)-mesons

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Abstract

Influence of the isospin-violating \((\rho^0, \omega)\)-mixing is discussed for any pair of decays of \(\rho^0, \omega\) into the same final state. It is demonstrated, in analogy to the \(CP\)-violation in neutral kaon decays, that isospin violation can manifest itself in various forms: direct violation in amplitudes and/or violation due to mixing. In addition to the known decays \((\rho^0, \omega) \to \pi^+\pi^-\) and \((\rho^0, \omega) \to \pi^0\gamma\), the pair of decays to \(e^+e^-\) and the whole set of radiative decays with participation of \(\rho^0, \omega\) (in initial or final states) are shown to be also useful and perspective for studies. Existing data on these decays agree with the universal character of the mixing parameter and indirectly support enhancement of \(\rho^0 \to \pi^0\gamma\) in respect to \(\rho^\pm \to \pi^\pm\gamma\). Future precise measurements will allow to separate different forms of isospin violation and elucidate their mechanisms.

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1 Introduction

It is widely known that the isospin symmetry is violated. But nobody knows why and how it is violated. There are at least two possible sources of the violation:

- QED does not respect the isospin, since different members of any isomultiplet always have different electric charges. As a result, the photon can be considered as a two-component object with isospins $I = 0, 1$. Therefore, presence of photons, real or virtual, inevitably spoils the symmetry. The corresponding effect for processes without real photons is expected to be $\mathcal{O}(\alpha)$ in the amplitude.

- QCD can also violate isospin, due to different properties of $u$- and $d$-quarks. Most popular here are references to different quark masses, but other properties, not always directly related to masses, may also be efficient (as examples, I can mention magnetic moments, or difference of quark wave functions inside hadrons). Estimates of the expected effect in such approaches are rather ambiguous.

Experiments demonstrate isospin violation (e.g., hadron mass differences) mostly at the relative level of order $10^{-2}$ or less. This does not allow even to discriminate between the two above mechanisms. Thus, further studies, both theoretical and experimental, are necessary to elucidate the underlying physics.

A favourable site for such studies may be provided by mixing of $\rho^0$- and $\omega$-mesons, where some enhancement becomes possible due to $M_\omega \approx M_\rho$. A well- and long-known example is the decay $\omega \to \pi^+\pi^-$. The isospin symmetry totally forbids it (initial $I = 0$, final $I = 1$), but the mixing opens the cascade transition $\omega \to \rho^0 \to \pi^+\pi^-$. The resulting branching ratio achieves 2% [1], instead of $\mathcal{O}(\alpha^2)$.

A more recent example of possible manifestation of the mixing is given by decays $\rho \to \pi \gamma$. There are experimental evidences for enhancement of the neutral decay in respect to charged one (see [1]; exact value is still uncertain, as evident from comparison of the corresponding numbers in the neighbouring issues of Particle Data Tables [1, 2]). Meanwhile, the isospin conservation admits only the isoscalar photon component to participate in those decays, and so probabilities for $\rho^0 \to \pi^0\gamma$ and $\rho^\pm \to \pi^{\pm}\gamma$ were expected to be the same. Their inequality (either enhancement or suppression of the neutral decay) may emerge from contribution of the cascade $\rho^0 \to \omega \to \pi^0\gamma$ which is impossible for the charged decay (see [3] and references therein).

In a recent paper [4] I suggested to broaden the set of decays under consideration, since any pair of the decays $\omega, \rho^0 \to (the \ same \ final \ state)$ should be sensitive to the $(\rho, \omega)$-mixing. This talk gives a brief presentation of ideas and results of the paper [4].
2 Vector meson mixing

Let us begin with bare states $\omega^{(0)}$ and $\rho^{(0)}$. They have bare (complex) masses
\[ M_\omega^{(0)} = m_\omega^{(0)} - \frac{i}{2} \Gamma_\omega^{(0)}, \quad M_\rho^{(0)} = m_\rho^{(0)} - \frac{i}{2} \Gamma_\rho^{(0)} \]
and bare propagators
\[ [D_\omega^{(0)}(k^2)]_{\mu\nu} = \frac{g_{\mu\nu} - k_\mu k_\nu}{k^2 - M_\omega^{(0)2}}, \quad [D_\rho^{(0)}(k^2)]_{\mu\nu} = \frac{g_{\mu\nu} - k_\mu k_\nu}{k^2 - M_\rho^{(0)2}}. \tag{1} \]
Mixing arises if there exist transitions $\omega^{(0)} \rightarrow \rho^{(0)}$ and $\rho^{(0)} \rightarrow \omega^{(0)}$. Corresponding transition vertices may be described by transition amplitudes $G_{\omega\rho}$ and $G_{\rho\omega}$ respectively\(^2\). Summation over all mutual transitions provides four different propagators for bare states:
\[ D_{\rho\rho}(k^2), \quad D_{\rho\omega}(k^2), \quad D_{\omega\rho}(k^2), \quad D_{\omega\omega}(k^2), \]
which describe all reciprocal transformations of $\rho^{(0)}$ and $\omega^{(0)}$. Together they may be considered as a $2 \times 2$ matrix propagator. Its diagonalization picks out physical propagators $D_\omega(k^2)$ and $D_\rho(k^2)$ with physical masses
\[ M_\omega^2 = M^2 + K\delta M^2, \quad M_\rho^2 = M^2 - K\delta M^2, \tag{2} \]
where
\[ \delta M^2 = \frac{M_\omega^{(0)2} - M_\rho^{(0)2}}{2}, \quad M^2 = \frac{M_\omega^{(0)2} + M_\rho^{(0)2}}{2}, \]
\[ K = \sqrt{1 + \tilde{G}_{\rho\omega}\tilde{G}_{\omega\rho}}, \quad \tilde{G}_{\rho\omega} = \frac{G_{\rho\omega}}{\delta M^2}, \quad \tilde{G}_{\omega\rho} = \frac{G_{\omega\rho}}{\delta M^2}. \]

Now we can consider a process $i \rightarrow f$ where $\rho^0$ and/or $\omega$ appear as the intermediate states. Its amplitude in terms of bare states is
\[ A_{if} = A_{i\rho}^{(0)} D_{\rho\rho} A_{\rho f}^{(0)} + A_{i\omega}^{(0)} D_{\rho\omega} A_{\omega f}^{(0)} + A_{i\omega}^{(0)} D_{\omega\rho} A_{\omega f}^{(0)} + A_{i\rho}^{(0)} D_{\omega\omega} A_{\rho f}^{(0)}, \tag{3} \]
where $A_{i\rho}^{(0)}$, $A_{i\omega}^{(0)}$ are production amplitudes for bare $\rho^{(0)}$-, $\omega^{(0)}$-states, while $A_{\rho f}^{(0)}$, $A_{\omega f}^{(0)}$ are their decay amplitudes. The whole amplitude may be rewritten in terms of physical states in the simple form
\[ A_{if} = A_{i\rho} D_{\rho} A_{\rho f} + A_{i\omega} D_{\omega} A_{\omega f}, \tag{4} \]
where the physical propagators $D_{\rho}(k^2)$, $D_{\omega}(k^2)$ are used together with the physical amplitudes
\[ A_{i\rho} = \sqrt{\frac{K + 1}{2K}} \left( A_{i\rho}^{(0)} - A_{i\omega}^{(0)} \tilde{G}_{\omega\rho} \frac{K + 1}{K} \right), \quad A_{i\omega} = \sqrt{\frac{K + 1}{2K}} \left( A_{i\omega}^{(0)} + A_{i\rho}^{(0)} \tilde{G}_{\rho\omega} \frac{K + 1}{K} \right) \tag{5} \]
\(^2\)See \(^3\) for more detailed description of the vertices.
for the $\rho^0$, $\omega$-meson production and

$$A_{\rho f} = \sqrt{\frac{K + 1}{2K}} \left( A_{\rho f}^{(0)} - \frac{\tilde{G}_{\rho\omega}}{K + 1} A_{\omega f}^{(0)} \right), \quad A_{\omega f} = \sqrt{\frac{K + 1}{2K}} \left( A_{\omega f}^{(0)} + \frac{\tilde{G}_{\omega\rho}}{K + 1} A_{\rho f}^{(0)} \right)$$

(6)

for the meson decays.

The picture of mixed $\rho^{(0)}$, $\omega^{(0)}$-states is similar to the well-known picture of mixing for $K^0, \bar{K}^0$, as described by Lee, Oehme, Yang [5]. It corresponds to diagonalization of the mass squared matrix of the ($\rho, \omega$)-system

$$\mathcal{M}^2 = \begin{pmatrix} M_{\rho}^{(0)2} & G_{\omega\rho} \\ G_{\rho\omega} & M_{\omega}^{(0)2} \end{pmatrix}$$

(7)

(and its matrix propagator $\mathcal{D} = (k^2 - \mathcal{M}^2)^{-1}$) in the form

$$\mathcal{M}^2 = \sqrt{\frac{K + 1}{2K}} \begin{pmatrix} 1 & \frac{G_{\rho\omega}}{K + 1} \\ -\frac{G_{\rho\omega}}{K + 1} & 1 \end{pmatrix} \cdot \begin{pmatrix} M_{\rho}^2 & 0 \\ 0 & M_{\omega}^2 \end{pmatrix} \cdot \sqrt{\frac{K + 1}{2K}} \begin{pmatrix} 1 & -\frac{G_{\omega\rho}}{K + 1} \\ \frac{G_{\omega\rho}}{K + 1} & 1 \end{pmatrix}.$$  

(8)

The bare states $|\rho^{(0)}\rangle$ and $|\omega^{(0)}\rangle$ appear to be analogs of flavour states $|K^0\rangle$ and $|\bar{K}^0\rangle$, while the physical states

$$|\rho\rangle = N_{\rho} \left( |\rho^{(0)}\rangle - \frac{\tilde{G}_{\rho\omega}}{K + 1} |\omega^{(0)}\rangle \right), \quad |\omega\rangle = N_{\omega} \left( \frac{\tilde{G}_{\omega\rho}}{K + 1} |\rho^{(0)}\rangle + |\omega^{(0)}\rangle \right)$$

(9)

play the role of $|K_S\rangle$ and $|K_L\rangle$ (compare with expressions (6); $N_{\rho}$ and $N_{\omega}$ are normalizing factors). The essential difference, however, is the nonvanishing $\delta M^2$, which would imply CPT-violation in the case of $(K^0\bar{K}^0)$. As for the neutral kaons, there is a possibility of rephasing for $\rho^{(0)}$ and $\omega^{(0)}$. $T$-invariance makes possible to fix their phases so that

$$\tilde{G}_{\rho\omega} = \tilde{G}_{\omega\rho} \equiv \tilde{G}.$$

Analogy between the two systems would be more evident if one could observe oscillating time distributions of $\rho$- and $\omega$-decays. This is, however, quite unrealistic, and we can study only time-integrated double-pole distributions in $k^2$. More detailed discussion of similarity and difference between $(\rho^0, \omega)$ and $(K^0, \bar{K}^0)$ may be found in [4]. I would like, nevertheless, to mention here one unfamiliar point: while the bare states are orthogonal, the physical $(\rho^0, \omega)$-states are orthogonal only if $\tilde{G}$ is real.

## 3 Mixing and isospin violation in decays

Symmetry violations in decays of neutral kaons are known to reveal themselves in two forms: mixing violation, manifested in mixing parameters of eigenstates; and direct violation, seen
as a property of one or another particular amplitude for kaon decays. Isospin violation for
the \((\rho, \omega)\)-system may also have two forms. It can be direct violation, seen in production
or decay amplitudes for bare states; or it can be mixing violation due to dimensionless
parameters \(\tilde{G}_{\rho\omega}\) and \(\tilde{G}_{\omega\rho}\). Existing experience allows to expect relative effects in amplitudes
\(\sim 0.01\) for the direct violation, while \(|\tilde{G}|\) might be up to 0.1. This apparent enhancement
of \(\tilde{G}\) arises due to the denominator \(\delta M^2\), small at the hadron mass scale. Nevertheless, the
difference is not very strong, and future accurate description may require to account for the
both kinds of isospin violation.

Let us compare a pair of decays \((\omega, \rho^0) \to f\) with the same final state. The ratio of their
amplitudes is
\[
a_{\omega/\rho^0 f} \equiv \frac{A_{\omega f}}{A_{\rho^0 f}} = a_{\omega/\rho^0 f}^{(0)} \left(1 + \frac{\tilde{G}}{(K+1) a_{\omega/\rho^0 f}^{(0)}}\right) \left(1 - \frac{\tilde{G} a_{\omega/\rho^0 f}^{(0)}}{K+1}\right)^{-1},
\]
where we assume \(T\)-invariance and define
\[
a_{\omega/\rho^0 f}^{(0)} \equiv \frac{A_{\omega f}^{(0)}}{A_{\rho^0 f}^{(0)}}.
\]
Now, neglecting the difference of phase spaces in the decays, we can easily describe the
measurable quantity
\[
r_{\omega/\rho^0 f} \equiv \frac{\Gamma(\omega \to f)}{\Gamma(\rho^0 \to f)} = \left|a_{\omega/\rho^0 f}\right|^2.
\]
Each pair of decays has its own parameter \(a_{\omega/\rho^0 f}^{(0)}\), while \(\tilde{G}\) is universal.

For decays \((\omega, \rho^0) \to \pi^+\pi^-\) we can assume absence of direct isospin violation, \(i.e.,\)
\(a_{\omega/\rho^0 (2\pi)}^{(0)} = 0\), and obtain the simple relation
\[
r_{\omega/\rho^0 (2\pi)} \equiv \frac{\Gamma(\omega \to 2\pi)}{\Gamma(\rho^0)} = \frac{1}{4} \left|\tilde{G}\right|^2,
\]
where deviation of \(K\) from unity has been neglected. On the basis of Tables \[1\] it gives
\[
\left|\tilde{G}\right| = (6.2 \pm 0.5) \cdot 10^{-2}.
\]
Large variation of this quantity when extracting it from the consequent issues of the Particle
Data Tables \[1\, 2\] shows that the realistic error should be taken at least twice higher.

Photonic decays \((\omega, \rho^0) \to \pi^0\gamma, \eta\gamma, e^+e^-\) and \(\eta' \to (\omega, \rho^0)\gamma\) contain the real or virtual
photon and have, therefore, nonvanishing direct isospin violation. It can be assumed, however,
to have very simple form based on the structure of the photon coupling to light quarks:
\[
e_u \bar{u} u + e_d \bar{d} d = \frac{e_u + e_d}{\sqrt{2}} \frac{\bar{u} u + \bar{d} d}{\sqrt{2}} + \frac{e_u - e_d}{\sqrt{2}} \frac{\bar{u} u - \bar{d} d}{\sqrt{2}}.
\]
Evidently, the effective isovector charge is 3 times more than the isoscalar one. We can append this fact by assumption that $uu$ and $dd$ components of the mesons produce the same matrix elements. Then for the ratios

$$
r_{\eta'/\rho}/\omega \equiv \frac{\Gamma(\eta' \rightarrow \rho^0 \gamma)}{\Gamma(\eta' \rightarrow \omega \gamma)}, \quad r_{\rho^0}/\omega \eta \equiv \frac{\Gamma(\rho^0 \rightarrow \eta \gamma)}{\Gamma(\omega \rightarrow \eta \gamma)}, \quad r_{\rho^0}/\omega (ee) \equiv \frac{\Gamma(\rho^0 \rightarrow e^+ e^-)}{\Gamma(\omega \rightarrow e^+ e^-)}
$$

we obtain the same expression

$$
r = 9 \left| \frac{1 - \frac{1}{6} \tilde{G}}{1 + \frac{3}{2} \tilde{G}} \right|^2.
$$

A given value of $r$ corresponds to a circle in the complex plane of $\tilde{G}$, which should intersect another circle, related to eq.(12), and determine $\tilde{G}$ up to the sign of $\text{Im} \tilde{G}$.

Data of Tables [1] provide the values

$$
r_{\eta'/\rho}/\omega = 9.74 \pm 1.05, \quad r_{\rho^0}/\omega \eta = 10.3 \pm 2.6, \quad r_{\rho^0}/\omega (ee) = 11.42 \pm 0.42,
$$

which do not contradict each other. Experimental errors transform all the corresponding circles into circular bands shown in fig.1. Though the errors are large, the picture looks consistent with the value of $\tilde{G}$ being universal for various decays and having $\text{Re} \tilde{G} < 0$.

In the same approach we can write

$$
r_{\rho^0}/\rho^{\pm} \equiv \frac{\Gamma(\rho^0 \rightarrow \pi^0 \gamma)}{\Gamma(\rho^{\pm} \rightarrow \pi^{\pm} \gamma)} = \left| 1 - \frac{3}{2} \tilde{G} \right|^2,
$$

which shows that interference of direct and cascade contributions may either suppress or enhance the neutral radiative decay. $\text{Re} \tilde{G} < 0$ leads to enhancement of the neutral vs. charged decay, in agreement with experimental evidences [1]. This demonstrates both the role of mixing in pairs of $(\rho^0, \omega)$-decays and consistency of the discussed approach to description of the isospin violation.

The considered photonic decays can be easily described in the framework of the additive quark model. Its simplest form provides exactly the same expression as in eq.(14). To check that they have more general meaning we can consider many-particle decays $(\rho^0, \omega) \rightarrow \pi^0 \pi^0 \gamma$ which are not easy for application of the additive quark model. However, we can use the fact of "isotopic separation" in these decays: only isovector (isoscalar) component of the photon would participate in the decay of $\rho^0$ $(\omega)$ in the absence of some additional isospin violation, because of mixing or any direct effects. Therefore, the quantity

$$
r_{\rho^0}/\omega(\pi\pi) \equiv \frac{\Gamma(\rho^0 \rightarrow \pi^0 \pi^0 \gamma)}{\Gamma(\omega \rightarrow \pi^0 \pi^0 \gamma)}
$$

should satisfy the same eq.(14). Experimentally [1], $r_{\rho^0}/\omega(\pi\pi) \approx 11$. Uncertainty is still large, but we see just the expected tendency $(r_{\rho^0}/\omega(\pi\pi)$ seems to be higher than the unmixed numerical value of 9).
Up to now we have neglected any really direct violation of the isospin symmetry. In photonic decays this meant that the arising matrix elements were assumed to be the same for \(u\)- and \(d\)-components of the mesons, and violation emerged only due to difference of \(e_u\) and \(e_d\). However, the slight difference of \(r_{\eta'\rho_0/\omega}\) and \(r_{\rho_0/\omega(ee)}\) may be viewed as an evidence for existence of some additional direct violation, giving different matrix elements for \(u\)- and \(d\)-components. Another possible evidence for such violation comes from the ratio

\[
  r_{\omega/\rho^{\pm\pi}} \equiv \frac{\Gamma(\omega \rightarrow \pi^0\gamma)}{\Gamma(\rho^{\pm\pi} \rightarrow \pi^0\gamma)},
\]

which experimentally equals \((10.9 \pm 1.3)\). This exceeds expectation based on the expression

\[
  r_{\omega/\rho^{\pm\pi}} = 9 \left| 1 + \frac{1}{6} \tilde{G} \right|^2,
\]

with \(\tilde{G}\) satisfying eq.(13) and having \(\text{Re} \tilde{G} < 0\). Sources of additional (direct) violations are still to be discussed.

### 4 Conclusion

The above examples demonstrate that the \((\rho, \omega)\)-mixing reveals itself not only in decays \(\omega \rightarrow \pi^+\pi^-\) and \(\rho^0 \rightarrow \pi^0\gamma\). It also affects all pairs of decays of \(\rho^0, \omega\) to the same final state and decays of heavier particles with production of \(\rho^0, \omega\).

Though the current precision is still insufficient for firm conclusions, existing data on radiative decays of \((\rho^0, \omega)\) and decays to \(e^+e^-\) are shown to agree with the regular, correlated manner expected for the influence of mixing. The whole set of decays gives additional indirect support for enhancement of \(\rho^0 \rightarrow \pi^0\gamma\) in comparison with \(\rho^{\pm} \rightarrow \pi^{\pm}\gamma\).

Future, more precise measurements of those and other decays will help to separate isospin violation due to \((\rho, \omega)\)-mixing from direct violation in various processes and to study them in detail. This will allow to pick out the underlying physics and construct adequate models for isospin violation.

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Figure 1: Properties of various \((\rho^0, \omega)\) decay pairs as seen on the complex plane of \(\tilde{G}\) when using values (15). The long-dashed uncovered band is for \((\rho^0, \omega) \rightarrow \eta \gamma\); the short-dashed band with left-inclined hatching is for \((\rho^0, \omega) \rightarrow e^+ e^-\); the dotted band with right-inclined hatching is for \(\eta' \rightarrow (\rho^0, \omega) \gamma\). The solid band with double hatching is for \((\omega, \rho) \rightarrow \pi \pi\), eq.(13) with the doubled error. The area to the left/right of the solid line corresponds to \(r_{\rho^0/\rho^\pm\pi} > 1\) more/less than unity, \(i.e.,\) to enhancement/suppression of \(\rho^0 \rightarrow \pi^0 \gamma\) in respect to \(\rho^\pm \rightarrow \pi^\pm \gamma\).