Supermembrane on the PP-wave Background

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Abstract

We study the closed and open supermembranes on the maximally supersymmetric pp-wave background. In the framework of the membrane theory, the superalgebra is calculated by using the Dirac bracket and we obtain its central extension by surface terms. The result supports the existence of the extended objects in the membrane theory in the pp-wave limit. When the central terms are discarded, the associated algebra completely agrees with that of Berenstein-Maldacena-Nastase matrix model. We also discuss the open supermembranes on the pp-wave and elaborate the possible boundary conditions.

Keywords: supermembranes, matrix theory, M-theory, pp-waves
1 Introduction

For the past years, many works toward the investigation of the M-theory has been done, and in particular the matrix model approach seems greatly successful [1]. These are attempts to describe the scattering in the eleven dimensional supergravity theory. For many years, the eleven-dimensional supergravity backgrounds have been studied, and the Minkowski-space, $AdS_4 \times S^7$, $AdS_7 \times S^4$ and Kowalski-Glikman (KG) pp-wave solution [2] are only known cases as the maximally supersymmetric backgrounds. They are possible candidates for the M-theory backgrounds. In particular, by taking a certain limit called Penrose limit [3,4], the KG solution can be obtained from the $AdS_4 \times S^7$ or $AdS_7 \times S^4$ backgrounds [5]. Also, the maximally supersymmetric IIB supergravity background has been lately found [6] and it has been shown that the Green-Schwarz (GS) type IIB superstring theory on the pp-waves is exactly solvable [7–9]. The pp-wave background used in the works [7–9] can be also obtained by taking Penrose limit in the $AdS_5 \times S^5$ [6].

The fact that the maximally supersymmetric pp-wave backgrounds are obtained by taking the Penrose limit in the $AdS$ background leads to the work [10], where the IIB string theory on the pp-wave is used for investigating the $AdS$/CFT correspondence [11, 12] in the string theoretic analysis. That is the exactly solvable model with nontrivial string background and it provides an interesting area to study properties of strings with background fluxes. Moreover, Banks-Fischler-Shenker-Susskind (BFSS) matrix model on the maximally supersymmetric pp-wave (which we refer to Berenstein-Maldacena-Nastase (BMN) matrix model) has been proposed from the considerations for the superparticles. The action of the BMN matrix model has been also derived directly from the membrane theory on the maximally supersymmetric pp-wave [13].

In this paper we consider the closed and open supermembranes on the eleven-dimensional maximally supersymmetric pp-wave background. We calculate the supercharges and associated algebra. In contrast with the algebra in the BMN model, surface terms are included in our membrane case. It is the central extension of the superalgebra in the BMN model and we discuss the extended objects contained in the membrane theory on the pp-wave.

Next we discuss the boundary conditions for the open supermembrane on the pp-wave by calculating surface terms under the variations of the supersymmetry transformations. In the case of flat background, the open supermembrane can end on the $p$-dimensional hypersurface only for the values $p = 1, 5$ and $9$. However, we show that some additional surface terms arise
in the pp-wave case and only the value \( p = 1 \) is allowed for the open supermembrane on the pp-wave.

This paper is organized as follows. In section 2, as a short review we provide an explanation of the action of the supermembrane and supersymmetries on the maximally supersymmetric pp-wave background. In section 3 we will calculate the supercharges and associated algebra by the use of the Dirac bracket procedure. In order to discuss the extended objects, we carefully analyze the surface terms. In section 4, the boundary conditions for the open supermembranes on the pp-wave will be considered. Section 5 is devoted to considerations and discussions. In appendix, our notation is summarized.

## 2 Supermembrane on Maximally Supersymmetric PP-wave

We consider the (closed and open) supermembranes \([14–16]\) (for the review, see \([17–23]\)) on the eleven-dimensional maximally supersymmetric pp-waves (Kowalski-Glikman (KG) solution) \([2]\). Its metric is given as

\[
ds^2 = -2dx^+dx^- + G_+(dx^+)^2 + \sum_{\mu=1}^{9}(dx^\mu)^2, \tag{2.1}
\]

\[
G_+ \equiv -\left(\left(\mu \over 3\right)^2 (x_1^2 + x_2^2 + x_3^2) + \left(\mu \over 6\right)^2 (x_4^2 + \cdots + x_9^2)\right),
\]

where the constant 4-form flux for +, 1, 2, 3 directions,

\[
F_{+123} = \mu, \quad (\mu \neq 0) \tag{2.2}
\]

is equipped.

The Lagrangian of supermembrane on the maximally supersymmetric pp-wave is given as a sum of \(L_0\) and Wess-Zumino term \(L_{WZ}\) \([\square]\)

\[
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{WZ}, \quad \mathcal{L}_0 = -\sqrt{-g(X, \theta)}, \tag{2.3}
\]

where the induced metric \(g_{ij}\) is given by

\[
g_{ij} = \Pi_i^\alpha \Pi_j^\beta \eta_{\alpha\beta}, \quad g = \det g_{ij} \tag{2.4}
\]

*Our notation and convention are summarized in Appendix.*
and the supervielbein $\Pi^A$ and covariant derivative $D_\theta$ for $\theta$ are defined by using vielbein $e_\mu^\hat{\nu}$ and spin connection $\omega^{\hat{\nu}\hat{\alpha}}$

$$
\Pi^\hat{\nu} = dX^\hat{\mu} e_\mu^\hat{\nu} - i\bar{\theta}\Gamma^{\hat{\nu}} D\theta ,
$$

$$\Pi^\bar{\alpha} = (D\theta)^\bar{\alpha} \equiv d\theta^\bar{\alpha} + e^\hat{\nu}(T_\hat{\nu} \bar{\delta}^{\hat{\nu}\bar{\alpha}})F_{\bar{a}i\bar{a}u} - \frac{1}{4}\omega^{\hat{\nu}\bar{\alpha}}(\Gamma_f \bar{\delta}_f)\bar{\theta} ,
$$

$$
T_\hat{\nu} \bar{\delta}^{\hat{\nu}\bar{\alpha}} = \frac{1}{288}(\Gamma_f \bar{\delta}^{\hat{\nu}\bar{\alpha}} - 8\delta_f^{\hat{\nu}\bar{\alpha}}). 
$$

The maximally supersymmetric pp-wave background is achieved by taking Penrose limit \cite{5} in the $AdS_4 \times S^7$ or $AdS_7 \times S^4$ where the supervielbeins are given by \cite{23,24}

$$\Pi^\bar{\alpha} = \left(\frac{\sinh M}{M}D\theta\right)^\bar{\alpha} ,
$$

$$\Pi^\hat{\nu} = dx^\hat{\mu} e_\mu^\hat{\nu} - i\bar{\theta}\Gamma^{\hat{\nu}}\left(\frac{2}{M}\frac{\sinh M}{2}\right)^2 D\theta ,
$$

$$
iM^2 = 2(T_\hat{\nu} \bar{\delta}^{\hat{\nu}\bar{\alpha}})F_{\bar{a}i\bar{a}u}(\bar{\theta}\Gamma^{\hat{\nu}}) - \frac{1}{288}(\Gamma_f \bar{\delta}_f)\bar{\theta}(\Gamma^{\hat{\nu}\bar{\delta}^{\hat{\nu}\bar{\alpha}}}F_{\bar{a}i\bar{a}u} + 24\Gamma_{\bar{a}u}F^{\hat{\nu}\bar{\delta}^{\hat{\nu}\bar{\alpha}}}].
$$

When we take the light-cone gauge in the Penrose limit, $M^2 = 0$ is satisfied. In addition the $D\theta$ becomes a simple formula

$$D\theta = d\theta + e^+ T^+_{+123}\theta F_{+123} ,
$$

and supervielbeins are dramatically simplified, though the action in the $AdS$ background has non-trivial interaction terms. In this gauge we write down the Wess-Zumino term $L_{WZ}$

$$L_{WZ} = \frac{1}{6} \epsilon^{ijk}C_{\mu\nu\rho}\partial_i X^\mu \partial_j X^\nu \partial_k X^\rho
$$

$$+ \frac{i}{2} \epsilon^{ijk}\bar{\theta}\Gamma^\mu D_i \bar{\theta} \left(\Pi^\gamma_{\mu k} + i\Pi^\rho_{\nu \rho} \bar{\theta}\Gamma^\rho D_k \bar{\theta} - \frac{1}{3} \bar{\theta}\Gamma^{\mu \rho} D_{k \theta}\right) .
$$

Here the supervielbeins on the maximally supersymmetric pp-wave are given by Eqs. (2.5) and (2.11). $C_{\mu\nu\rho}$ is the 3-form potential and its field strength is described by Eq. (2.12). The above supermembrane action is difficult to analyze directly, and so we shall rewrite Lagrangian (2.3) following the work \cite{10} in the light-cone gauge in terms of $SO(9)$ spinor $\psi$

$$w^{-1}L = \frac{1}{2}D_\tau X^\tau D_\tau X^\tau - \frac{1}{4}(\{X^r, X^s\})^2 - \frac{1}{2}(\mu_3)^2 \sum_{i=1}^{3} X_i^2 - \frac{1}{2}(\mu_6)^2 \sum_{i'=4}^{9} X_i^2
$$

$$- \frac{\mu_6}{6} \sum_{i,j,k=1}^{3} \epsilon_{ijk} X^K \{X^i, X^j\} + i\psi^r \gamma^r \{ X^r, \psi \} + i\psi^r D_\tau \psi + i\frac{\mu_8}{4} \psi^r \gamma_{123} \psi .
$$
We used a convention $P_0^+ = 1$. Here "$\tau$" is the time coordinate on the worldvolume and $\{ , \}$ is Lie bracket given by using an arbitrary function $w(\sigma)$ of worldvolume spatial coordinates $\sigma^a$ ($a = 1, 2$)

$$\{ A, B \} \equiv \frac{1}{w} \epsilon^{ab} \partial_a A \partial_b B , \quad (a, b = 1, 2).$$

with $\partial_a = \frac{\partial}{\partial \sigma^a}$. Also this theory has large residual gauge symmetry called the area-preserving diffeomorphism (APD) and the covariant derivative for this gauge symmetry is defined by a gauge connection $\omega$

$$D_\tau X^r \equiv \partial_\tau X^r - \{ \omega, X^r \} . \quad (2.14)$$

In this model, if we replace the variables in the Lagrangian (2.13) following the rule with

$$X(\xi^i) \rightarrow X(\tau)$$
$$\psi(\xi^i) \rightarrow \psi(\tau)$$
$$\int d^2 \sigma w(\sigma) \rightarrow \text{Tr}$$
$$\{ , \} \rightarrow -i[ , ] ,$$

we can obtain the BMN matrix model, starting from the Lagrangian for supermembrane on the maximally supersymmetric pp-wave.

We have taken the light-cone gauge and so original symmetries are not seen manifestly but the Lagrangian (2.13) still has residual supersymmetries,

$$\delta_\epsilon X^r = 2\psi^r \gamma^r \epsilon(\tau) , \quad \delta_\epsilon \omega = 2\psi^r \epsilon(\tau) ,$$
$$\delta_\epsilon \psi = -i D_\tau X^r \gamma^r \epsilon(\tau) + \frac{i}{2} \{ X^r , X^s \} \gamma_{rs} \epsilon(\tau)$$
$$+ \frac{\mu}{3} i \sum_{I=1}^3 X^I \gamma_I \gamma_{123} \epsilon(\tau) - \frac{\mu}{6} \sum_{I'=4}^9 X^{I'} \gamma_{I'} \gamma_{123} \epsilon(\tau) ,$$
$$\epsilon(\tau) = \exp \left( \frac{\mu}{12} \gamma_{123} \tau \right) \epsilon_0 \quad (\epsilon_0 : \text{constant spinor}).$$

These transformation rules are 16 linearly-realized supersymmetries on the maximally supersymmetric pp-wave. In taking the limit, $\mu \rightarrow 0$, we recover the supersymmetry transformations on the flat space. In the context of the eleven dimensional supersymmetry, this corresponds to the dynamical supersymmetry. The Lagrangian (2.13) has other 16 nonlinearly realized
supersymmetries,

\[ \delta \eta X^r = 0, \quad \delta \eta \omega = 0, \]
\[ \delta \eta \psi = \eta(\tau), \quad \eta(\tau) = \exp \left( -\frac{\mu}{4} \gamma_{123} \tau \right) \eta_0, \quad (\eta_0 : \text{constant spinor}). \]

It corresponds to the kinematical supersymmetry in the eleven dimensional theory.

3 Supercharge Algebra from the Supermembrane

To begin, we derive supercharges for the supersymmetries (2.15) and (2.16), and then study associated superalgebra by the use of the Dirac bracket. We discuss the extended objects on the pp-wave from the viewpoint of the central charges of the superalgebra.

Supercharges \( Q^+ \) and \( Q^- \) of the linearly and non-linearly realized supersymmetries, respectively, are obtained as Noether charges

\[
Q^+ = \int d^2 \sigma \left[ -2e^{-\frac{\mu}{4} \gamma_{123} \tau} \left( DX^r \gamma_r \psi + \frac{1}{2} \{ X^r, X^s \gamma_{rs} \psi \right) \\
\frac{1}{3} \sum_{i=1}^{3} X'^i \gamma_i \gamma_{123} \psi + \frac{3}{6} \sum_{i'=4}^{9} X'^{i'} \gamma_{i'} \gamma_{123} \psi \right],
\]
\[ (3.1) \]

\[
Q^- = \int d^2 \sigma \left[ -2ie^{\frac{\mu}{4} \gamma_{123} \tau} \psi \right] \\
= -2ie^{\frac{\mu}{4} \gamma_{123} \tau} \psi_0,
\]
\[ (3.2) \]

where \( \psi_0 \) is the zero-mode of \( \psi \) and we have used the normalization with \( \int d^2 \sigma \ w(\sigma) = 1 \).

Next we shall calculate the superalgebra satisfied by (3.1) and (3.2). The supermembrane theory contains the fermionic field \( \psi^\alpha \) and this leads to the second class constraint \( \Xi^\alpha \approx 0 \) for the theory

\[
\Xi^\alpha = S^\alpha - iw\psi^T = 0, \quad \left( S^\alpha \equiv \frac{\partial L}{\partial (\partial_\omega \psi^\alpha)} = iw\psi^T \right).
\]
\[ (3.3) \]

The fermionic field is the SO(9) spinor with 16 components. We have to deal properly with this second class constraint by the use of the Dirac bracket. The calculation of the Dirac bracket needs only a constraint matrix \( C_{\alpha\beta} \)

\[
C_{\alpha\beta} \equiv \{ \Xi^\alpha(\sigma), \Xi^\beta(\sigma') \}_{PB} = -2iw\delta_{\alpha\beta}\delta^{(2)}(\sigma - \sigma'),
\]
\[ (3.4) \]
and its inverse matrix \((C^{-1})_{\alpha\beta}\) is given by
\[
(C^{-1})_{\alpha\beta} = \frac{i}{2w} \delta_{\alpha\beta} \delta^{(2)}(\sigma - \sigma').
\] (3.5)

By the use of the matrix \((C^{-1})_{\alpha\beta}\), we can introduce the Dirac bracket \(\{ , \}_DB\) in terms of the Poisson bracket \(\{ , \}_PB\)
\[
\{F, G\}_DB \equiv \{F, G\}_PB - \{F, \Xi_\alpha\}_PB (C^{-1})^{\alpha\beta} \{\Xi_\beta, G\}_PB,
\]
\[
= \{F, G\}_PB - \frac{i}{2w} \{F, \Xi_\alpha\}_PB (C^{-1})^{\alpha\beta} \{\Xi_\beta, G\}_PB.
\] (3.6)

Thus, we can define the commutation relations on the bosonic and fermionic fields with their canonical momenta
\[
\{X^\tau(\sigma), P_s(\sigma')\}_DB = \delta^\tau_s \delta^{(2)}(\sigma - \sigma'),
\] (3.7)
\[
\{\psi_\alpha(\sigma), S^\tau_\beta(\sigma')\}_DB = \frac{1}{2} \delta_{\alpha\beta} \delta^{(2)}(\sigma - \sigma'),
\] (3.8)
\[
P_\tau = wD_\tau X_\tau.
\]

The commutation relations (3.7),(3.8) are rewritten in terms of \(\psi^{\tau}\) and \(D_\tau X_s\)
\[
\{X^\tau, D_\tau X_s\}_DB = \frac{1}{w} \delta^\tau_s \delta^{(2)}(\sigma - \sigma'),
\] (3.9)
\[
\{\psi_\alpha(\sigma), \psi^{\tau}(\sigma')\}_DB = -\frac{i}{2w} \delta_{\alpha\beta} \delta^{(2)}(\sigma - \sigma').
\] (3.10)

The superalgebra is calculated by the use of the Dirac bracket (3.7),(3.8),(3.9),(3.10) and we obtain the results
\[
i \left\{ \frac{1}{\sqrt{2}} Q_\alpha, \frac{1}{\sqrt{2}} (Q^-)^\beta \right\}_DB = -\delta_{\alpha\beta},
\] (3.11)
\[
i \left\{ \frac{1}{\sqrt{2}} Q^+_\alpha, \frac{1}{\sqrt{2}} (Q^-)^\beta \right\}_DB = i \sum_{I=1}^3 \left[ \left( P^I_0 + \frac{\mu}{3} X_0^I \gamma_{123} \right) \gamma_I e^{-\frac{3}{2} \gamma_{123} \gamma} \right]_{\alpha\beta}
\]
\[
+ i \sum_{I'=4}^9 \left[ \left( P^I' - \frac{\mu}{6} X_0 X_{123} \gamma_{123} \right) \gamma_I e^{-\frac{3}{2} \gamma_{123} \gamma} \right]_{\alpha\beta} - i \sum_{I,J=1}^3 \int d^2 \sigma \partial_a S^{a}_{II'} \left( \gamma^I e^{-\frac{3}{2} \gamma_{123} \gamma} \right)_{\alpha\beta}
\]
\[
- i \sum_{I',J'=4}^9 \int d^2 \sigma \partial_a S^{a}_{II'} \left( \gamma^I e^{-\frac{3}{2} \gamma_{123} \gamma} \right)_{\alpha\beta}.
\] (3.12)
\[ i \left\{ \frac{1}{\sqrt{2}} Q^+_\alpha \frac{1}{\sqrt{2}} (Q^+)^T_\beta \right\}_{DB} = 2H \delta_{\alpha \beta} \] (3.13)

\[ + \frac{\mu}{3} \sum_{I,J=1}^{3} M_{0}^{IJ} (\gamma_{IJ} \gamma_{123})_{\alpha \beta} - \frac{\mu}{6} \sum_{I',J'=4}^{9} M_{0}^{I'J'} (\gamma_{I'J'} \gamma_{123})_{\alpha \beta} \]

\[ - 2 \sum_{I=1}^{3} \int d^2 \sigma \varphi X_{I} (\gamma_{I})_{\alpha \beta} - 2 \sum_{I'=4}^{9} \int d^2 \sigma \varphi X_{I'} (\gamma_{I'} \varepsilon^{123} \gamma)_{\alpha \beta} \]

\[ + 2 \sum_{I=1}^{3} \int d^2 \sigma \partial_{\alpha} S_{I} (\gamma_{I})_{\alpha \beta} + 2 \sum_{I'=4}^{9} \int d^2 \sigma \partial_{\alpha} S_{I'} (\gamma_{I'} \varepsilon^{123} \gamma)_{\alpha \beta} \]

\[ + 2 \sum_{I=1}^{3} \sum_{I'=1}^{9} \int d^2 \sigma \partial_{\alpha} S_{I} (\gamma_{I} \gamma_{I'})_{\alpha \beta} + 2 \sum_{I'=4}^{9} \int d^2 \sigma \partial_{\alpha} S_{I'} (\gamma_{I'} \varepsilon^{123} \gamma)_{\alpha \beta} \]

\[ + 2 \mu \sum_{I,J=1}^{3} \sum_{I'=1}^{9} \int d^2 \sigma \partial_{\alpha} U_{I} (\gamma_{I} \gamma_{I'})_{\alpha \beta} \]

\[ + 2 \mu \sum_{I,J=1}^{3} \sum_{I'=1}^{9} \int d^2 \sigma \partial_{\alpha} U_{I'} (\gamma_{I'} \gamma_{I})_{\alpha \beta} \]

Here \( M^{IJ} \) and \( M^{I'J'} \) are defined by

\[ M^{IJ} = X^{I} P^{J} - P^{I} X^{J} - \frac{1}{2} S^{T} \gamma^{IJ} \psi, \] (3.14)

\[ M^{I'J'} = X^{I'} P^{J'} - P^{I'} X^{J'} - \frac{1}{2} S^{T} \gamma^{I'J'} \psi, \] (3.15)

and the \( SO(3) \times SO(6) \) Lorentz generators \( M_{0}^{IJ} \) and \( M_{0}^{I'J'} \) are given as

\[ M_{0}^{IJ} \equiv \int d^2 \sigma M^{IJ}, \] (3.16)

\[ M_{0}^{I'J'} \equiv \int d^2 \sigma M^{I'J'}. \] (3.17)

They satisfy the \( SO(3) \times SO(6) \) Lorentz algebra,

\[ \{ M_{0}^{IJ}, M_{0}^{KL} \}_{DB} = \delta^{IK} M_{0}^{JL} - \delta^{IL} M_{0}^{JK} - \delta^{JK} M_{0}^{IL} + \delta^{JL} M_{0}^{IK}, \] (3.18)

\[ \{ M_{0}^{I'J'}, M_{0}^{K'L'} \}_{DB} = \delta^{I'K'} M_{0}^{J'L'} - \delta^{I'L'} M_{0}^{J'K'} - \delta^{J'K'} M_{0}^{I'L'} + \delta^{J'L'} M_{0}^{I'K'}. \] (3.19)

The zero-modes of \( P_{r}^{r} (= w D_{r} X^{r}) \) and \( X^{r} \) are written by

\[ P_{0}^{r} \equiv \int d^2 \sigma w D_{r} X^{r}, \quad X_{0}^{r} = \int d^2 \sigma w X^{r}. \] (3.20)
Also, the Hamiltonian $H$ is expressed as

$$H = \int d^2 \sigma \; w \left[ \frac{1}{2} \left( \frac{P^r}{w} \right)^2 + \frac{1}{4} \{X^r, X^s\}^2 + \frac{1}{2} \left( \frac{\mu}{3} \right)^2 \sum_{i=1}^3 (X^i)^2 + \frac{1}{2} \left( \frac{\mu}{6} \right)^2 \sum_{i'=4}^9 (X^{i'})^2 \right] + \frac{\mu}{6} \sum_{i,j,k=1}^3 \epsilon_{ijk} X^k \{X^i, X^j\} - w^{-1} \frac{\mu}{4} S^r \gamma_{123} \psi - w^{-1} S^r \gamma_r \{X^r, \psi\}. \quad (3.21)$$

Other quantities in the above algebra are defined by

$$S_{rs}^a \equiv -\frac{1}{2} e^{ab} X^{|r|} \partial_b X^s, \quad (3.22)$$

$$\varphi \equiv w \{ w^{-1} P^r, X^r \} + i w \{ \psi^T, \psi \}, \quad (3.23)$$

$$S_r^a \equiv e^{ab} \left( w^{-1} X_r P_s \partial_b X^s + X_r i \psi^T \partial_b \psi + \frac{3}{8} i X^s \partial_b (\psi^T \gamma_{rs} \psi) \right),$$

$$S_{rstu}^a \equiv \frac{i}{48} e^{ab} X^{|r|} \partial_b \left( \psi^T \gamma_{stu} \psi \right), \quad (3.24)$$

$$U_{JKI'}^{aJ'I'} \equiv -\frac{1}{6} \sum_{i=1}^3 \epsilon_{ijk} e^{ab} X^{i'} \partial_b (X^i X^{i'}), \quad (3.25)$$

$$U_I^{a} \equiv \frac{1}{2} \epsilon^{ab} X^{i'} \partial_b \left[ \frac{1}{3} \sum_{i=1}^3 (X^i)^2 - \frac{1}{6} \sum_{i'=4}^9 (X^{i'})^2 \right].$$

The above superalgebra (other than the central charges) completely agrees with that of the BMN matrix model \[10\]. Also, in the $\mu \to 0$ limit, the above algebra realize the superalgebra of the supermembrane in the flat space given in \[10\].

Also, the above superalgebra includes some central charges. These charges indicate the existence of extended objects in supermembrane theories on the maximally supersymmetric pp-wave. First, the charges $S_{rs}^a$ and $S_r^a$ correspond to the transverse M2-brane (D2-brane in type IIA string theory) and longitudinal M2-brane (fundamental string in type IIA string theory), respectively. Next, $S_{rstu}^a$ corresponds to the longitudinal M5-brane charge (D4-brane in type IIA string theory). As is well-known, these charges have appeared in the supermembrane theory on the flat eleven-dimensional Minkowski space. In addition, in our supermembrane theory the superalgebra includes the additional central charges, $U_{JKI'}^{aJ'I'}$ and $U_I^{a}$. We do not properly confirm the physical interpretation of these extra extended objects only living on the pp-wave. These might be related to the fuzzy membrane and giant graviton discussed in \[14\], or another new extended object due to a certain kind of the Myers effects on the pp-wave \[26\].

\[1\]We can absorb the factor $1/\sqrt{2}$ in front of the supercharge in the definition of the fermions $\psi$. 

8
4 Open Supermembrane on PP-wave

In the case of the open membrane, which has the boundary on the worldvolume toward the spatial directions $\sigma^1$ and $\sigma^2$, the surface terms do not vanish automatically. Thus we must properly treat the total derivative terms under the variation of the above supersymmetry transformations, and consider the boundary conditions in order for the surface terms to vanish. Let us recall that the membrane $p$-branes are allowed for $p = 1, 5, 9$ in the flat background due to the boundary conditions \([27]\). M5-brane corresponds to $p = 5$. The case of $p = 9$ is related to “the end of the world” in Hořava-Witten’s works \([29]\).

In our pp-wave case, we obtain the total derivative terms for the linear supersymmetry \((2.15)\) explicitly

\[
\begin{align*}
&\left\{ X^r, D_\tau X^s \psi^r \gamma_s \gamma_r \epsilon(\tau) + \frac{1}{2} \{ X^s, X^t \} \psi^r \gamma_{st} \gamma_r \epsilon(\tau) \right\} \\
&- \frac{\mu}{3} \sum_{i,j=1}^3 w \{ X^i, X^j \psi^r \gamma_{ij} \gamma_r \epsilon(\tau) \} - \frac{\mu}{6} \sum_{i',j'=4}^9 w \{ X^{i'}, X^{j'} \psi^r \gamma_{i'j'} \gamma_r \epsilon(\tau) \} \\
&+ \frac{\mu}{3} \sum_{i=1}^3 \sum_{i'=4}^9 w \{ X^i, X^{i'} \psi^r \gamma_{i'} \gamma_r \epsilon(\tau) \} - \frac{\mu}{6} \sum_{i=1}^3 \sum_{i'=4}^9 w \{ X^i, X^{i'} \psi^r \gamma_i \gamma_{i'} \gamma_r \epsilon(\tau) \},
\end{align*}
\]

and for the nonlinear supersymmetry \((2.16)\), we can calculate the corresponding term

\[
w \{ X^r, i \eta \gamma_r \psi \}.
\]

This surface term for the nonlinear supersymmetry has the same form as the flat space case. However, some additional terms proportional to $\mu$ appear for the linear supersymmetry in addition to the surface terms in the flat background. The variations of the action under the linear and nonlinear supersymmetry transformations can be written as

\[
\begin{align*}
\delta S &= \delta_\epsilon S + \delta^{(\mu)} S + \delta_\eta S, \\
\delta_\epsilon S &= - \int d\tau \int d\xi \left[ \partial_\epsilon X^r \cdot \left( D_\tau X^s \psi^r \gamma_s \gamma_r \epsilon(\tau) + \frac{1}{2} \{ X^s, X^t \} \psi^r \gamma_{st} \gamma_r \epsilon(\tau) \right) \right], \\
\delta^{(\mu)} S &= - \int d\tau \int d\xi \left[ - \frac{\mu}{3} \sum_{i,j=1}^3 \partial_\epsilon X^r \cdot X^i \psi^r \gamma_{i} \gamma_r \gamma_{i23} \epsilon(\tau) - \frac{\mu}{6} \sum_{i',j'=4}^9 \partial_\epsilon X^r \cdot X^{i'} \psi^r \gamma_{i'} \gamma_r \gamma_{i'23} \epsilon(\tau) \\
&- \frac{\mu}{3} \sum_{i=1}^3 \sum_{i'=4}^9 \partial_\epsilon X^r \cdot X^i \psi^r \gamma_{i} \gamma_{i'} \gamma_r \gamma_{i23} \epsilon(\tau) - \frac{\mu}{6} \sum_{i=1}^3 \sum_{i'=4}^9 \partial_\epsilon X^r \cdot X^{i'} \psi^r \gamma_{i'} \gamma_{i} \gamma_{i23} \epsilon(\tau) \right], \\
\delta_\eta S &= - i \int d\tau \int d\xi \partial_\epsilon X^r \cdot \eta(\tau) \gamma_r \psi,
\end{align*}
\]
where $\partial \Sigma$ is the boundary of the open supermembrane worldvolume and $\xi$ is the coordinate for the tangent direction of the boundary. Note that the tangential derivative $\partial_t$ and normal derivative $\partial_n$ on the boundary are defined by

$$\partial_t X^r \equiv \epsilon^{ab} n_a \partial_b X^r,$$

$$\partial_n X^r \equiv n^a \partial_a X^r.$$  \hspace{1cm} (4.6) (4.7)

Here $n^a$ is the unit vector toward the normal direction on the boundary. We would like to consider the $p$-dimensional hypersurface (membrane $p$-brane) on which supermembranes can end, and investigate the condition that such a surface can exist. First, by following the discussion of the $p$-brane in string theory, the boundary conditions for our membrane are classified

Neumann : $\partial_n X^{\overline{m}} = 0$, \ (overline{m} = 0, 10 and some $p - 1$ coordinates) \hspace{1cm} (4.8)

Dirichlet : $\partial_t X^{\overline{m}} = 0$, \ (overline{m} = other $10 - p$ coordinates). \hspace{1cm} (4.9)

By applying these boundary conditions to (4.3) and (4.5), the constraints

$$\eta_0 \gamma^{\overline{m}} \psi = \epsilon_0 \gamma^{\overline{m}} \gamma^{\overline{n}} \psi = \epsilon_0 \gamma^{\overline{m}} \gamma^{\overline{n}} \gamma^{\overline{n}} \psi = 0,$$ \hspace{1cm} (4.10)

can be obtained. These are the same conditions as the flat case and (4.10) leads us to the well-known results $p = 1, 5$ and $9$. However, in the pp-wave case we also need to take account of the constraints coming from the additional surface terms (4.4).

Here, let us define the following operators

$$P_\pm \equiv \frac{1}{2} (1 \pm \gamma^{m_1} \gamma^{m_2} \ldots \gamma^{m_{10-p}}).$$  \hspace{1cm} (4.11)

These are the projection operators if and only if $\frac{1}{2}p(p+1)$ is odd. Thus, the value of $p$ is limited to $p = 1, 2, 5, 6$ and $9$. The requirement that boundary term should vanish leads to the constraints Eq. (4.10), and so it provides a further restriction for the value of $p$. If we assume that $1/2$ BPS boundary hypersurface, then the condition

$$P_- \psi = 0,$$ \hspace{1cm} (4.12)

is in our hand. Then we can write $\psi$ as $\psi = P_+ \psi$. To begin, from the second equation in (4.11), $P_+ \epsilon_0 = 0$ is followed. Next, we can read off from the third equation in (4.11) that $9 - p$ should be even. As a result, $p = 1, 5$ and $9$ are allowed in the flat case for the boundary hypersurface. However, the story does not end because the additional boundary terms exist in
the case of the pp-wave. We can easily check whether the additional surface terms (4.4) vanish or not in each \( p = 1, 5, 9 \) case. In the \( p = 1 \) case we can immediately see that the additional terms (4.4) vanish. Here, it can be seen from the constraints (4.10) that only the even number of gamma matrices with Neumann indices \( \overline{m} \)'s and arbitrary number of gamma matrices with Dirichlet indices \( m \)'s are allowed to appear between \( \epsilon^T_0 \) and \( \psi \). Equivalently, odd number of gamma matrices with Neumann indices \( \overline{m} \)'s cannot appear between \( \epsilon^T_0 \) and \( \psi \). However, it is found from the expression (4.4) that such a condition cannot be satisfied in the cases \( p = 5 \) and \( 9 \) because there are inevitably several terms including odd number of Neumann components. In conclusion, only \( p = 1 \) is allowed for membrane \( p \)-brane on the pp-wave, and \( p = 5, 9 \) membrane \( p \)-brane cannot exist.

This result would be also plausible from the viewpoint of the chirality matrix [30]. It is because that the flux is turned on the 1, 2, 3 directions on the pp-wave, and so the \( SO(4) \) and \( SO(8) \) chirality, which is important for \( p = 5 \) and \( p = 9 \) cases, cannot be respected. The reason that \( p = 1 \) case is allowed is unknown, since what \( p = 1 \) means physically has not been well understood.

In the above discussion, we have assumed that the 1/2 BPS boundary hypersurface, that is, the flat boundary hypersurface. However, it might be clear that such flat boundaries cannot exist, because the pp-wave background is curved. Possibly, the curved hypersurface as discussed in [31] might become the boundary of the supermembrane. But, we do not know how to treat such curved boundaries, and do not discuss the case here.

## 5 Conclusions and Discussions

In this paper, we have studied the supercharges and its associated algebra. In particular, by treating the surface terms carefully, its central extension has been derived. The superalgebra apart from the central charges completely agrees with that of the BMN matrix model. The central charges obtained in our derivation realize the flat space results in the \( \mu \to 0 \), and also include some additional ones. We do not confirm the physical interpretation of the additional central charges. These seem to indicate the extra extended objects coming from a kind of the Myers effect on the pp-wave background.

Moreover, we have discussed the boundary conditions of the open supermembrane on the maximally supersymmetric pp-wave background. It is well-known that the membrane \( p \)-branes
in the flat space are allowed to exist only for \( p = 1, 5 \) and 9. In our case on the pp-wave, more strict constraints for such hypersurfaces arise, and so only the value \( p = 1 \) is allowed. In our discussion, we have not included the 2-form which can couple to the boundary hypersurface. It might be possible by turning on the 2-form on the boundary that 5- and 9-dimensional hypersurfaces exist as the boundaries of the open supermembranes on the pp-wave.

In this paper, we have used the \( SO(9) \) formulation for the simplicity, but it is also interesting to work in the \( SO(10,1) \) covariant formulation, where the nature of longitudinal components are clear and more definite considerations would be possible. This is an interesting future work.

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Appendix

In this appendix, we summarize several notations used in the paper.

**Notation**

We consider supermembrane in eleven dimensional curved spacetime and use a notation of supercoordinates \((D = 11)\):

\[
X^M = (X^\hat{\mu}, \theta^\alpha), \quad \hat{\mu} = (+, -, \mu), \quad \mu = 1, \ldots, D - 2.
\]

The background metric is expressed as \(G_{MN}\).

In the Lorentz frame, we also use the coordinates \((D = 11)\):

\[
X^A = (X^\hat{r}, \theta^\bar{\alpha}), \quad \hat{r} = (+, -, r), \quad r = 1, \ldots, D - 2.
\]

The metric is flat and is described by \(\eta_{AB}\). In these notations, we introduced a set of light-cone coordinates \(X^\pm \equiv \frac{1}{\sqrt{2}} (X^0 \pm X^{D-1})\).

The membrane has three-dimensional worldvolume and its coordinates are parameterized by \(\xi^i = (\tau, \sigma^a), \quad a = 1, 2\), and its metric is given by \(g_{ij}\).

Next we shall summarize the \(SO(10,1)\) gamma matrices \((D = 11)\):

\[
\{\Gamma^\hat{\mu}, \Gamma^\hat{\nu}\} = 2G^{\hat{\mu}\hat{\nu}}, \quad \{\Gamma^\hat{r}, \Gamma^\hat{s}\} = 2\eta^{\hat{r}\hat{s}};
\]

\[
\Gamma^\hat{\mu} \equiv e^\hat{\mu}_r \Gamma^\hat{r}, \quad \Gamma^\hat{r} \equiv e^\hat{r}_\mu \Gamma^\mu,
\]

\[
\Gamma^\mu = \gamma^\mu \otimes \sigma_3 = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & -\gamma^\mu \end{pmatrix}, \quad \text{(real symmetric)}
\]

\[
\Gamma^0 = 1 \otimes i\sigma_2 = \begin{pmatrix} 0 & -I_{16} \\ I_{16} & 0 \end{pmatrix}, \quad \text{(real skew - symmetric)}
\]

\[
\Gamma^{D-1} = 1 \otimes \sigma_1 = \begin{pmatrix} 0 & I_{16} \\ I_{16} & 0 \end{pmatrix}, \quad \text{(real symmetric)}
\]

\[
\Gamma^\pm \equiv \frac{1}{\sqrt{2}} (\Gamma^0 \pm \Gamma^{D-1}), \quad \{\Gamma^+, \Gamma^-\} = -2I_{32},
\]

\[
\Gamma^+ = \sqrt{2} \begin{pmatrix} 0 & 0 \\ I_{16} & 0 \end{pmatrix}, \quad \Gamma^- = \sqrt{2} \begin{pmatrix} 0 & -I_{16} \\ 0 & 0 \end{pmatrix}.
\]
We take the light-cone gauge and decompose the 32 component $SO(10,1)$ spinor $\theta$ in terms of $SO(9)$ spinor $\psi$ with 16 components

$$X^+ = \tau, \quad \Gamma^+ \theta = 0, \quad (\bar{\theta} \Gamma^+ = 0),$$

$$\implies \theta = \frac{1}{2^{1/4}w} \begin{pmatrix} 0 \\ \psi \end{pmatrix},$$

$$\bar{\theta} = \theta^T (-\Gamma^0) = -\frac{1}{2^{1/4}w} (\psi, 0).$$

In the light-cone gauge, there are several useful identities

$$\bar{\theta} \Gamma^r \partial_i \theta = 0, \quad (\text{for } \hat{r} \neq -),$$

$$\bar{\theta} \Gamma_{rs} \partial_i \theta = 0,$$

$$\bar{\theta} \Gamma^{+r} \partial_i \theta = 0,$$

$$\bar{\theta} \Gamma^{+r} \partial_i \theta = 0.$$

In the pp-wave background, the vielbein is calculated as

$$e^{\hat{r}}_\mu : \quad e^+ = e^- = 1, \quad e^+ = 0, \quad e^- = -\frac{1}{4} G^{++}, \quad e^r_\mu = \delta^r_\mu,$$

$$e^{\hat{r}}_r : \quad e^+ = e^- = 1, \quad e^+ = 0, \quad e^- = \frac{1}{4} G^{++}, \quad e^\mu_r = \delta^\mu_r,$$

$$e^{\hat{r}}^{\hat{r}} : \quad e^{++} = e^{+r} = e^{-r} = e^{\mu+} = e^{\mu-} = 0,$$

$$e^{+-} = e^{-+} = -1, \quad e^{--} = -\frac{1}{4} G^{++}, \quad e^{\mu r} = \delta^{\mu r},$$

$$e^{\hat{r}}^{\hat{r}} : \quad e^{\mu+} = e^{\mu-} = e^{-+} = e^{++} = e^{--} = 0,$$

$$e^{+} = e^{--} = -1, \quad e^{++} = +\frac{1}{4} G^{++}, \quad e^{\mu r} = \delta^{\mu r},$$

and the spin connection is evaluated as

$$\omega^{\hat{r}}_{\hat{r}} \equiv \omega^{\hat{r}}_{\hat{r}} dx^{\hat{\mu}} \implies \omega^+ = \frac{1}{4} \partial^+ G_{++} dx^+, \quad \text{otherwise} = 0,$$

$$\omega^{\hat{r}}_{\hat{r}} \equiv \omega^{\hat{r}}_{\hat{r}} \omega^{\hat{r}}_{\hat{r}} \implies \omega^- = \frac{1}{4} \partial^- G_{++} dx^+, \quad \text{otherwise} = 0.$$
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