SUPERALLOWED FERMI BETA DECAY – A STATUS REPORT

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ABSTRACT

Data on superallowed Fermi beta decay in nuclei are compiled and electromagnetic corrections discussed. Recommended values for the weak vector coupling constant and the leading element of the Cabibbo-Kobayashi-Maskawa mixing matrix are given. Comments on neutron decay and limits extracted on extensions to the Standard Model are made.

1. Introduction

The intensity of superallowed Fermi $\beta$-transitions between $0^+, T = 1$ nuclear states, as expressed through their $f_I$ values, is expected to be the same for all nuclei. This statement, based on the conserved vector current (CVC) hypothesis, follows from two considerations. First, in the allowed approximation of $\beta$-decay theory, the Fermi operator being the isospin ladder operator, $\tau_+$, leads to identically-valued matrix elements for all Fermi decays so long as isospin is an exact symmetry. Second, CVC ensures the weak vector coupling constant is not renormalized in a nuclear many-body medium; it remains a true constant. However, before these statements can be tested against experimental data, certain theoretical corrections have to be applied. For example, bremsstrahlung processes lead to radiative corrections, and the breakdown of analogue symmetry by the presence of charge-dependent forces between nucleons leads to Coulomb corrections in the nuclear matrix elements.

The nine best-known transitions have reached sufficient experimental precision that the CVC test can be passed at the level of a few parts in $10^4$ and the three-generation Standard Model at the level of its quantum corrections. World data on $Q$-values, lifetimes and branching ratios were thoroughly surveyed in 1989 and updated again this year. The principal new measurements are: some $Q$-value determinations by Barker et al., an $(n, \gamma)$ and $(p, \gamma)$ $Q$-value difference measurement, and some lifetime and branching-ratio measurements at Chalk River.

Once the CVC test is successfully passed, a value of the weak vector coupling constant for semi-leptonic decays is determined. A comparison with the same quantity for the purely leptonic $\mu$-decay yields the up-down quark-mixing matrix element, $V_{ud}$, of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The CKM matrix is unitary, so the sum of the elements in the first row of the matrix should satisfy

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1,$$

(1)
if the Standard Model for three generations is correct. The accuracy with which this relation can be tested is dominated by the accuracy with which $V_{ud}$ can be determined, which in turn depends on the accuracy with which the radiative and Coulomb corrections can be evaluated.

2. Radiative Corrections

In the present application we are only interested in the difference in radiative corrections between nuclear $\beta$-decay and $\mu$-decay, as only these differences impact on the deduced value of $V_{ud}$. To first order in the fine-structure constant, the uncorrected $\beta$-decay rate $\Gamma_0^\beta$ is modified to

$$\Gamma_\beta = \Gamma_0^\beta \left\{ 1 + \frac{\alpha}{2\pi} \left[ 3 \ln(m_W/m_P) + \bar{g}(E_m) + \ln(m_\omega/m_\lambda) + 2C - 4 \ln(m_\omega/m_Z) + A_g \right] \right\},$$

where $E_m$ is the maximum electron energy in the $\beta$-decay, and $m_W, m_P, m_Z$ are the masses of the $W$-boson, proton and $Z$-boson. We discuss $m_\lambda$ shortly. The first two terms in the square brackets in Eq. (2), $3 \ln(m_\omega/m_P) + \bar{g}(E_m)$, are the universal photonic contributions arising from the weak vector current in the $V-A$ theory. The function $g(E, E_m)$ is defined in Eq. (20b) of Sirlin and is here averaged over the electron spectrum. The next terms, $\ln(m_\omega/m_A) + 2C$, represent the asymptotic and nonasymptotic photonic corrections induced by the weak axial-vector current, which we discuss shortly. The fifth term arises from $Z$-exchange graphs, while the sixth term is a small perturbative QCD correction estimated by Marciano and Sirlin to be $A_g = -0.37$. It is convenient to gather the leading logarithms together and recast Eq. (2) as

$$\Gamma_\beta = \Gamma_0^\beta \left\{ 1 + \frac{\alpha}{2\pi} [4 \ln(m_Z/m_P) + \bar{g}(E_m) + \ln(m_\nu/m_\lambda) + 2C + A_g] \right\}. \tag{3}$$

The first term in the square brackets gives a universal correction to nuclear $\beta$-decay of 2.1%, while the second term contributes a further 1% but varies slightly from nucleus to nucleus. The third and fourth terms have been estimated in 1986 to be

$$\frac{\alpha}{2\pi} [\ln(m_\nu/m_\lambda) + 2C] = (0.12 \pm 0.18)\%, \tag{4}$$

and it is the error here that contributes the principal error in the evaluation of the radiative correction. The last term in Eq. (3) contributes $-0.04\%$ to the $\beta$-decay rate and is negligible.

Since the first correction term in Eq. (3) is the largest $O(\alpha)$ correction, Marciano and Sirlin have approximated the effect of higher orders by summing all leading-logarithmic corrections of $O(\alpha^n \ln^n m_Z)$, $n = 1, 2 \cdots$ via a renormalization-group analysis. Such a resummation replaces Eq. (3) with
\[ \Gamma_\beta = \Gamma_\beta^0 \left\{ 1 + \frac{\alpha}{2\pi} \left[ \ln(m_p/m_\lambda) + 2C \right] + \frac{\alpha(m_p)}{2\pi} \left[ \gamma(E_m) + A_g \right] \right\} S(m_p, m_\lambda), \quad (5) \]

where \( S(m_p, m_\lambda) \) satisfies an appropriate renormalization-group equation and is evaluated to be 1.0225. Further, \( \alpha(\mu) \) is a running QED coupling constant having a value at the proton mass of \( \alpha^{-1}(m_p) = 133.93 \). Finally, there are higher-order radiative corrections of order \( Z^2 \alpha^2, Z^3 \alpha^3 \cdots \) where \( Z \) is the atomic number of the daughter nucleus in nuclear \( \beta \)-decay, which we will denote by \( \delta_2 \) and \( \delta_3 \), respectively. These terms yield a correction of order 0.5% to the \( \beta \)-decay rate.

A recent advance has been a closer look at the photonic corrections induced by the weak axial-vector current, the third and fourth terms in Eq. (2), and the numerical estimates given in Eq. (4). It is convenient to model these corrections by two graphs. In the first, the nucleon emits a \( \rho \)-meson, which in turn converts to an \( A_1 \)-meson and a photon. The \( A_1 \)-meson leads to a weak axial-vector transition producing an electron-neutrino pair, and the photon interacts with the electron. The graph has a closed-loop integration, comprising the electron, photon, \( A_1 \)-meson and \( W \)-boson lines, which is logarithmically divergent in the limit \( m_\lambda \to 0 \). The role of the \( A_1 \)-meson is to provide a low-energy cut-off. Hence the factor \( \ln(m_\omega/m_\lambda) \) appears in Eq. (2). Originally Marciano and Sirlin suggested a conservative range \( 400 \text{ MeV} < m_\lambda < 1600 \text{ MeV} \), which spans mass scales from \( 3m_\pi \) through the \( A_1 \) resonance region. More recently, Sirlin has used a range \( m_\lambda/2 < m_\lambda < 2m_\lambda \) with the central value given by the \( A_1 \) resonance mass, \( m_\lambda = 1260 \text{ MeV} \), a reasonable procedure since the \( A_1 \) resonance is now well established. We will adopt this latter suggestion for the leading logarithm in Eq. (4).

The second graph represents the sequential emission from a nucleon first of a \( W \)-boson to produce an electron-neutrino pair and second a photon to be reabsorbed by the electron. This, together with the graph with the photon and \( W \)-boson emitted in reversed order, are the Born graphs and their contribution is written as \((\alpha/\pi)C\) in Eq. (2). If the nucleus is taken as a collection of non-interacting nucleons and the graphs evaluated from a single nucleon, then a universal contribution of \( C = 3g_\lambda (\mu_p + \mu_n) I \) is obtained, where \( I \) is a loop integral, providing the axial-vector coupling constant and the nucleon isoscalar magnetic moment are taken at their free-nucleon values, \( g_\lambda = 1.26 \) and \( (\mu_p + \mu_n) = 0.88 \), respectively. The integral, \( I \), depends on the choice of form factors introduced at the hadron vertices required to render the loop integration finite. Towner has studied this dependence and recommends a value \( C = 0.881 \pm 0.030 \).

An important advance was the observation of Jaus and Rasche that the emission of the \( W \)-boson and photon do not necessarily have to be from the same nucleon. Thus we write

\[ C = C_{\text{Born}} + C_{\text{NS}}, \quad (6) \]

where the departure of the value of \( C \) from the universal single-nucleon value of \( C_{\text{Born}} = 0.881 \pm 0.030 \) is attributed to both the nuclear-structure dependent two-nucleon graphs.
and a quenching in the single-nucleon graphs due to the renormalization of the coupling constants $g_A$ and $(\mu_p + \mu_n)$ in the nuclear medium. Two-nucleon graph calculations are to be found in refs.\cite{14,16} and quenching results in ref.\cite{17}. With the exception of $^{10}\text{C}$ and $^{14}\text{O}$, the correction is generally small, $(\alpha/\pi)C_{\text{NS}}$ being less than 0.09%. For the ensuing analysis, we adopt the values from Towner\cite{17}.

It has been convenient in the past to separate the radiative correction into those terms that are nuclear-structure dependent, $\delta_R$, and those that are not, $\Delta^\nu_R$. We will continue with this separation, although it is cumbersome with the term $\overline{g}(E_m)$ contained in the renormalization-group extrapolation. Thus we define

$$\Gamma_\beta = \Gamma_{\beta 0} (1 + \delta_R)(1 + \Delta^\nu_R), \quad (7)$$

with

$$1 + \delta_R = 1 + \frac{\alpha}{2\pi} \left[ \overline{g}(E_m) + \delta_2 + \delta_3 + 2C_{\text{NS}} \right], \quad (8)$$

$$1 + \Delta^\nu_R = \left\{ 1 + \frac{\alpha}{2\pi} \left[ \ln(m_p/m_A) + 2C_{\text{Born}} \right] + \frac{\alpha(m_p)}{2\pi} \left[ \overline{g}(E_m) + A_g \right] \right\} \times S(m_p, m_z) \left\{ 1 - \frac{\alpha}{2\pi} \overline{g}(E_m) \right\}. \quad (9)$$

We persevere with this separation in Eq. (7) because only the nucleus-dependent radiative corrections are required for the testing of the CVC hypothesis through the constancy of the $ft$ values. The values of $\delta_R$ adopted for this test are recorded in Table 1. For the nucleus-independent correction, we obtain

$$\Delta^\nu_R = (2.40 \pm 0.08)\%, \quad (10)$$

where the error reflects the range adopted for $m_A$ for the low-energy cut-off.

### 3. Coulomb Corrections

Both Coulomb and charge-dependent nuclear forces destroy isospin symmetry and reduce the value of the Fermi matrix element between analogue states from its simple value of $\langle M_V \rangle^2 = 2$ to a corrected value of $2(1 - \delta_C)$. Two physical phenomena can contribute to $\delta_C$. First, the degree of configuration mixing in the shell-model wavefunctions varies from member to member within an isospin multiplet, leading to a correction $\delta_{IM}$. Second, protons are typically less bound than neutrons; thus the tail of the radial wavefunction for protons extends further than that for neutrons, so the radial overlap of the parent and daughter nucleus is reduced from the normally assumed value of unity – a correction $\delta_{RO}$. Generally, $\delta_{RO}$ is larger than $\delta_{IM}$.

There are two comprehensive sets of calculations of these corrections: by Towner, Hardy and Harvey\cite{18,19,21} (THH), and by Ormand and Brown\cite{20,21}. Their relative merits are discussed in ref.\cite{1} and will not be repeated here. Ormand and Brown have just
|   | \( ft \) | \( \delta_R(\%) \) | \( \delta_C(\%)^a \) | \( \mathcal{F}t \) |
|---|---|---|---|---|
|\(^{10}\)C | 3040.1(51) | 1.30(4) | 0.16(3) | 3074.4(54) |
|\(^{14}\)O | 3038.1(18) | 1.26(5) | 0.22(3) | 3069.7(26) |
|\(^{26m}\)Al | 3035.8(17) | 1.45(2) | 0.31(3) | 3070.0(21) |
|\(^{34}\)Cl | 3048.4(19) | 1.33(3) | 0.61(3) | 3070.1(24) |
|\(^{38m}\)K | 3047.9(26) | 1.33(4) | 0.62(3) | 3069.4(31) |
|\(^{42}\)Sc | 3045.1(14) | 1.47(5) | 0.41(3) | 3077.3(23) |
|\(^{46}\)V | 3044.6(18) | 1.40(6) | 0.41(3) | 3074.4(27) |
|\(^{50}\)Mn | 3043.7(16) | 1.40(7) | 0.41(3) | 3073.8(27) |
|\(^{54}\)Co | 3045.8(11) | 1.39(7) | 0.52(3) | 3072.2(27) |

^aTabulated uncertainties represent “statistical” scatter only.

released a revised calculation of \( \delta_C \), and we will adopt these values in what follows. A close examination indicates that the two sets exhibit very similar nucleus-to-nucleus variations: both predict the largest correction for \(^{38m}\)K and the smallest for \(^{10}\)C. However, there is a systematic difference, with the THH values being larger than the OB values by 0.08% on the average. Accordingly, we adopt for \( \delta_C \) of each transition the unweighted average of the two independent calculations for that transition and then analyze the scatter of all nine pairs of \( \delta_C \) calculations about their respective averages to obtain a standard deviation of \( \pm 0.03\% \) for the purposes of the CVC test. After the CVC test has been satisfied and data on the nine nuclear transitions are averaged together, a further “systematic” error of \( \pm 0.04\% \) is incorporated to represent the systematic difference between the two calculations. The \( \delta_C \) corrections are listed in Table 1.

Three recent developments can have a bearing on the \( \delta_C \) calculations: (a) Wilkinson suggests that the experimental transition rates, when corrected for radiative and Coulomb effects, still display a weak dependence on the atomic number, \( Z \), of the daughter nucleus, which he attributes to some inadequacy in the Coulomb-correction calculation. His largely empirical approach to this is to remove the nucleus-to-nucleus fluctuations evident in both the THH and OB calculations from the experimental \( ft \) values and then fit a smooth curve through the resultant values, which can be extrapolated back to \( Z = 0 \) to obtain a recommended value. It should be remembered, however, that this is a heuristic procedure, not based on an \textit{ab initio} calculation. Initially, it was thought that the inadequacy in the \( \delta_C \) calculation might be an inadequate allowance for the effects of the spectator nucleons (the core nucleons in
shell-model approaches) in the radial overlaps, but recent analyses show such effects are negligible.

(b) Barker has computed a full set of $\delta_C$ values in an $R$-matrix formulation in which the configuration space is divided into internal and external regions. This produces an obvious dependence on the choice of channel radius, $a$; smaller $\delta_C$ values arise with larger values of $a$. Since physically-motivated values of $a$ increase with nuclear mass number as $A^{1/3}$, small $\delta_C$ values are obtained in heavier nuclei. In addition, the computations produce overall small values with $\delta_C < 0.2\%$ in all cases and $< 0.05\%$ in the $fp$-shell nuclei $^{46}$V, $^{50}$Mn and $^{54}$Co. The results lead to corrected $ft$ values that are not consistent with the CVC hypothesis.

(c) In a recent preprint, Sagawa, van Giai and Suzuki compute the isospin mixing in the ground states of several odd-odd $N=Z$ nuclei in the Hartree-Fock approximation taking into account forces that break charge symmetry and charge independence. It must be stressed that isospin mixing in ground states is not the same as analogue-symmetry breaking in Fermi beta decay: the latter depends on the difference in isospin mixing in adjacent nuclei. Sagawa et al. find considerable isospin mixing arising from the core nucleons, and obtain values as large as $1\%$ for $^{54}$Co, which they cite as a warning that considerable core effects may be important in $\delta_C$ calculations.

4. The CVC test and the effective vector coupling constant

The test of the conserved vector current (CVC) hypothesis is that the $ft$ values for superallowed beta decay, when suitably corrected for electromagnetic effects as just discussed, should all be constant. We write

\[ ft(1 + \delta_R)(1 - \delta_C) \equiv Ft = K/(2G'_V^2), \]
\[ K = 2\pi^3 \ln 2\hbar^2/(m^0_e c^4), \]
\[ G'_V^2 = G^2_V (1 + \Delta^y_V R), \]

where $G'_V$ is the effective vector coupling constant. It includes the nucleus-independent radiative correction, $\Delta^y_V$, which is not relevant for the test of CVC. The status of the experimental data and the theoretical corrections $\delta_R$ and $\delta_C$ are summarised in Table 1. The weighted average of the $Ft$ values (with statistical uncertainty only) is $3072.3 \pm 1.0$ with a corresponding $\chi^2$ per degree of freedom of 1.20. A two-parameter fit to the function, $Ft = Ft(0) [1 + a_1 Z]$, gives only a marginal decrease in the $\chi^2/\nu$ to 1.00 with the parameter $a_1 = (0.7 \pm 0.5) \times 10^{-4}$. Statistically there is insufficient evidence for a residual $Z$-dependence. The CVC hypothesis is therefore verified at the level of $4 \times 10^{-4}$. The nine $Ft$ values are displayed in Fig. 1.

Having substantiated the consistency of the experimental $Ft$ values, we now turn to the extraction of the effective nuclear vector coupling constant, $G'_V$, which is directly related by Eq. (11) to the average $Ft$ value. It is important now, however, to include in the $Ft$-value uncertainty the effect of “systematic” as well as “statistical” uncertainties in the charge-correction calculation, as described in Sect.3. These “systematic”
or model-dependent effects were estimated to be ±0.04%. Thus the average $\mathcal{F}t$ value (with both "statistical" and "systematic" errors included) becomes

$$\mathcal{F}t = 3072.3 \pm 2.0 \text{ s},$$

(14)

and so

$$G'_\nu/(hc)^3 = (1.14959 \pm 0.00038) \times 10^{-5} \text{ GeV}^{-2}. \quad (15)$$

The error bars in Eqs. (14) and (15) are dominated by the theoretical uncertainties in $\delta_C$.

5. The value of $V_{ud}$ and the unitarity test

The superallowed nuclear beta transitions are examples of semi-leptonic weak decays, which, when compared with the pure leptonic muon decay, yield an experimental value for $V_{ud}$ via the relationship

$$G'^2_\nu/G^2_\rho = V^2_{ud}(1 + \Delta^\nu_R). \quad (16)$$

Here, $\Delta^\nu_R$ is the nucleus-independent radiative correction given in Eq. (10). Thus with

$$G_\rho/(hc)^3 = (1.166387 \pm 0.000021) \times 10^{-5} \text{ GeV}^{-2}, \quad (17)$$

derived from muon decay, we obtain
\[ V_{ud} = 0.9740 \pm 0.0005. \]  

The quoted uncertainty is dominated by uncertainties in the theoretical corrections, \( \Delta^V_R \) and \( \delta_C \); experimental errors on the input data for the \( \mathcal{F}t \)-value contribute less than \( \pm 0.0002 \) to the uncertainty in Eq. (18).

Evidently, \( V_{ud} \) is by far the largest of the three matrix elements needed for the unitarity test, Eq. (1), and its uncertainty is the most crucial. The value of \( V_{us} \) is taken to be \( 0.2205 \pm 0.0018 \), the average of two results, one from the analysis of \( K_{e3} \) decays, the other from hyperon decays. Finally, \( V_{ub} \) can be derived, with substantial uncertainty, from the semi-leptonic decay of \( B \) mesons, but is sufficiently small \( \pm 0.0032 \pm 0.0009 \), as to be negligible in the unitarity test. The result,

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9972 \pm 0.0013, \]

indicates a violation of the unitarity condition for three generations by more than twice the estimated error.

6. Neutron decay

Because of the complexities involved in finite nuclei, we briefly turn our attention to the study of neutron decay, where nuclear-structure corrections can be expected to be minimal and \( \delta_C \) is essentially zero. There have been impressive advances in the measurement of the neutron mean lifetime in the past six years; the average of eleven measurements is \( \tau_n = 887.0 \pm 2.0 \) s. The problem for the extraction of the weak vector coupling constant and hence \( V_{ud} \) in neutron decay is that the transition is a mix of vector and axial-vector components and a separate \( \beta \)-asymmetry experiment is required to separate the two. There are three recent accurate \( \beta \)-asymmetry experiments, not completely in accord; their weighted average leads to \( \lambda \equiv G'_V/G'_A = -1.2599 \pm 0.0025 \).

From these two data, the individual coupling constants can be deduced:

\[
G'_A/(\hbar c)^3 = -(1.4566 \pm 0.0018) \times 10^{-5} \text{ GeV}^{-2},
\]

\[
G'_V/(\hbar c)^3 = (1.1561 \pm 0.0023) \times 10^{-5} \text{ GeV}^{-2} \quad \text{[neutron].}
\]

Note the neutron result for \( G'_V \) is considerably less precise but, even so, it is in serious disagreement with nuclear decays, Eq. (13).

With the use of the nucleus-independent radiative correction, \( \Delta^V_R \) of Eq. (10), the neutron result for the vector coupling constant can be used to test the unitarity of the CKM matrix, yielding

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0082 \pm 0.0040, \]

which differs from unitarity, but in the opposite sense to the result from nuclear \( \beta \) decay. In the absence of any viable explanation within the Standard Model for the
discrepancy between the neutron and nuclear results, nor any reason to prefer one over
the other, we can do no better than take the weighted average of both results for \( G'_v \),
suitably inflating the resultant uncertainty. This leads to

\[
G'_v/(hc)^3 = (1.1498 \pm 0.0010) \times 10^{-5} \text{ GeV}^{-2} \quad [\text{average}],
\]

\[
|V_{ud}| = 0.9741 \pm 0.0010 \quad [\text{average}],
\]

and

\[
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9975 \pm 0.0020 \quad [\text{average}],
\]

which does not disagree significantly from unitarity.

7. Beyond the Standard Model

The failure of the experimental test to reproduce the unitarity result can be used
to set limits on extensions to the Standard Model. Two examples follow:
(a) The existence of additional neutral gauge bosons, beyond the usual \( \gamma \) and \( Z \) of the
Standard \( SU(2)_L \times U(1) \) Model would signal the presence of additional \( U(1) \) groups
and would lead to further contributions to the radiative correction \( \Delta_{V_R}^\chi \). This additional
contribution could then be responsible for the failure of the experimental test of uni-
tarity. Marciano and Sirlin\(^26\) give a quantitative estimate of this for the grand unified
models of the type \( SO(10) \rightarrow SU(3)_c \times SU(2)_L \times U(1) \times U(1)_{\chi} \). Writing

\[
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta,
\]

then for the grand unified model, \( SO(10) \), Marciano and Sirlin\(^26\) obtain

\[
\Delta = -\frac{3\alpha}{2\pi \cos^2 \theta_W} \frac{\ln x}{x-1},
\]

where \( x = (m_{Z_{\chi}}/m_{W})^2, \sin^2 \theta_W = 0.23 \) and \( \alpha \) is the fine-structure constant. Thus, on the
one hand, with \( \Delta \) positive from Eq. (24), the correction has the wrong sign to provide
a limit on the \( Z_{\chi} \) boson mass, while, on the other hand, the neutron data in Eq. (22)
gives limits of \( 30 < m_{Z_{\chi}} < 85 \text{ GeV} \), which are unacceptable, since experiments\(^13,26\)
searching for departures from the Standard Model in neutrino-electron scattering or
searching directly in \( p\overline{p} \) colliders have concluded that \( m_{Z_{\chi}} > 320 \text{ GeV} \).

(b) Another example is an extension of the Standard Model into a left-right symmetric
form, \( SU(2)_L \times SU(2)_R \times U(1) \), where the \( W \)-boson associated with the group \( SU(2)_R \),
\( (W_R) \), is deemed to be much heavier than the traditional \( W \)-boson associated with
\( SU(2)_L \), \( (W_L) \), such that right-hand couplings in weak interactions are suppressed.
The mass eigenstates for the two bosons will not necessarily coincide with the weak-
interaction eigenstates, but will be a linear combination of them,
\[ W_1 = W_L \cos \zeta - W_R \sin \zeta, \]
\[ W_2 = W_L \sin \zeta + W_R \cos \zeta, \]  
(27)

where \( \zeta \) is known as the left-right mixing angle. The presence of right-hand currents will lead to a correction to the value of \( V_{ud} \) determined from \( \beta \)-decay under the assumption of left-hand currents only. If we assume that the failure of the unitarity test in Eq. (19) is entirely due to the presence of right-hand currents, then the value for the mixing angle, \( \zeta \), is obtained from the expression

\[ \Delta = 2\zeta. \]  
(28)

The result from superallowed beta decay for the mixing angle is \( \zeta = 0.0014 \pm 0.0006 \), which is small but non-zero. This is a much tighter limit than can be obtained from other nuclear physics experiments.

8. Conclusions

In summary, we note that:

(a) The average \( \langle Ft \rangle \) value of Eq. (14) for nine precision data with a \( \chi^2 \) per degree of freedom of 1.20 provides strong confirmation of the CVC hypothesis to four parts in \( 10^4 \).

(b) There is a serious discrepancy between \( G'_{\nu} \) determined from superallowed beta decay and from neutron decay, which has yet to be resolved.

(c) The significance of the failure of the unitarity test, Eq. (19), is not yet settled. It may, of course, indicate the need for some extension to the three-generation Standard Model. However, it may also reflect some undiagnosed inadequacy in the evaluation of \( V_{us} \) (as already suggested for other reasons in ref. [25]) or possibly in the \( \delta_C \) corrections used to determine \( V_{ud} \). There is little scope left in the data in Fig.1 for introducing significant additional Z-dependence in \( \delta_C \), as has been suggested recently [23].

References

[1] J.C. Hardy, I.S. Towner, V.T. Koslowsky, E. Hagberg and H. Schmeing, Nucl. Phys. A509, 429 (1990)
[2] I.S. Towner and J.C. Hardy, in The Nucleus as a Laboratory for Studying Symmetries and Fundamental Interactions, eds. E.M. Henley and W.C. Haxton (World-Scientific, Singapore, 1995) to be published
[3] S.A. Brindhaban and P.H. Barker, Phys. Rev. C49, 2401 (1994); S. Lin, S.A. Brindhaban and P.H. Barker, Phys. Rev. C49, 3098 (1994); P.A. Amundsen and P.H. Barker, Phys. Rev. C50, 2466 (1994)
[4] S.W. Kikstra, Z. Guo, C. van der Leun, P.M. Endt, S. Raman, T.A. Walkiewicz, J.W. Starner, E.T. Jueney and I.S. Towner, Nucl. Phys. A529, 39 (1991); T.A. Walkiewicz, S. Raman, E.T. Jueney, J.W. Starner and J.E. Lynn, Phys. Rev. C45, 1597 (1992)
[5] V.T. Koslowsky et al., to be published; preliminary results appear in E. Hagberg, V.T. Koslowsky, I.S. Towner, J.G. Hykawy, G. Savard, T. Shinozuka, P.P. Unger and H. Schmeing, Nuclei far from Stability/Atomic Masses and Fundamental Constants (Institute of Physics Conference Series # 132, ed. R. Neugart and A. Wöhr, 1994) p.783

[6] E. Hagberg, V.T. Koslowsky, J.C. Hardy, I.S. Towner, J.G. Hykawy, G. Savard and T. Shinozuka, Phys. Rev. Lett. 73, 396 (1994)

[7] G. Savard, A. Galindo-Uribarri, E. Hagberg, J.C. Hardy, V.T. Koslowsky, D.C. Radford and I.S. Towner, Phys. Rev. Lett. 74, 1521 (1995)

[8] A. Sirlin, Rev. Mod. Phys. 50, 573 (1978)

[9] A. Sirlin, Phys. Rev. 164, 1767 (1967)

[10] W.J. Marciano and A. Sirlin, Phys. Rev. Lett. 56, 22 (1986)

[11] A. Sirlin, Phys. Rev. D35, 3423 (1987); W. Jaus and G. Rasche, Phys. Rev. D35, 3420 (1987); A. Sirlin and R. Zucchini, Phys. Rev. Lett. 57, 1994 (1986)

[12] A. Sirlin, in Precision Tests of the Standard Electroweak Model ed. P. Langacker (World-Scientific, Singapore, 1994) to be published

[13] Particle Data Group, Phys. Rev. D50, 1173 (1994)

[14] I.S. Towner, Nucl. Phys. A540, 478 (1992)

[15] W. Jaus and G. Rasche, Phys. Rev. D41, 166 (1990)

[16] F.C. Barker, B.A. Brown, W. Jaus and G. Rasche, Nucl. Phys. A540, 501 (1992)

[17] I.S. Towner, Phys. Lett. B333, 13 (1994)

[18] I.S. Towner, J.C. Hardy and M. Harvey, Nucl. Phys. A284, 269 (1977)

[19] I.S. Towner, in Symmetry Violations in Subatomic Physics, eds. B. Castel and P.J. O’Donnel (World-Scientific, Singapore, 1989) p.211

[20] W.E. Ormand and B.A. Brown, Phys. Rev. Lett. 62, 866 (1989); Nucl. Phys. A440, 274 (1985)

[21] W.E. Ormand and B.A. Brown, Isospin-mixing corrections for fp-shell Fermi transitions, preprint, to be published

[22] D.H. Wilkinson, Nucl. Phys. A511, 301 (1990); Nucl. Inst. and Method 335, 172,182,201 (1993); Zeit. Phys. A348, 129 (1994)

[23] D.H. Wilkinson, Nucl. Phys. A587, 421 (1995)

[24] F.C. Barker, Nucl. Phys. A579, 62 (1994); F.C. Barker, Nucl. Phys. A537, 143 (1992)

[25] H. Sagawa, N. van Giai and T. Suzuki, Isospin mixing and sum rule of superallowed Fermi β decay, preprint, to be published

[26] W.J. Marciano and A. Sirlin, Phys. Rev. D35, 1672 (1987); P.Langacker and M Luo, Phys. Rev. D45, 278 (1992)

[27] A.S. Carnoy, J. Deutsch, R. Prieels, N. Severijns and P.A. Quin, J. Phys. G18, 823 (1992); A.S. Carnoy, J. Deutsch, T.A. Girard and R. Prieels, Phys. Rev. Lett. 65, 3249 (1990) and Phys. Rev. C43, 2825 (1991)

[28] A. Garcia, R. Huerta and P. Kielenowski, Phys. Rev. D45, 879 (1992)