Resonant light scattering from dielectric ring of rectangular cross section

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Abstract. In this paper we present the results of numerical calculations of electromagnetic properties for cylindrical ring resonators (RRs) with rectangular cross-section and a dielectric permittivity corresponding to silicon $\varepsilon = 12$. The calculation of the scattering spectrum (Radar Cross Section) and the field distribution of the modes were performed at in-plane polarized light excitation. The presence of four side walls in the RRs determines a richer spectrum of eigenmodes in comparison with cylindrical whispering gallery modes of disk resonators and creates more possibilities and diversity for their practical applications.

1. Introduction.
Well-known Whispering Gallery Modes (WGMs) can be considered as electromagnetic modes confined within the structure by total reflections from the curved side surface of the resonator with the proper phase condition after circling along the whole resonator. WGM resonators have been investigated in detail and are widely used in sensors and detectors because of their huge Q-factor and a narrow resonance line width [1–3]. In contrast, studies of RRs are far from complete, although they have significant advantages over WGM [4–9]. The presence of four sides in RRs with a square or rectangular cross section leads to strong light localization and the existence of volume resonance modes, which cannot be obtained in cylindrical resonator.

In the scattering spectrum of ring dielectric structure, we obtained a series of resonances, which we will hereinafter call the Ring Gallery Modes (RGMs) by analogy with the WGMs in disk dielectric microresonators. In this work, we found that, unlike WGMs, which form a continuous sequence of quasi-equidistant modes, RGMs are divided into several series, each consisting of one intense wide band with complex structure and several narrow high-Q lines, the intensity of which decreases rapidly according to the logarithmic law.

2. Scattering spectrum of a dielectric RR.
We consider here a dielectric RR with homogeneous dielectric permittivity $\varepsilon = 12$ (that corresponds to the dielectric constant of silicon) and zero attenuation. The ring is embedded in the air ($\varepsilon = 1$). The length of the center line $L = \pi (r_{out} + r_{in})$, the ratio of the inner radius $r_{in}$ to the outer radius $r_{out}$ is 0.6, width $g = r_{out} - r_{in}$ and thickness $h$ satisfy the relation $g/h = 5$. In our calculations, the electric field of the incident wave is linearly polarized in the plane of...
the ring (TE polarization). All the computations were performed based on finite-difference frequency-domain (FDFD) method using the commercial software COMSOL.

Figure 1. Total Radar cross-section spectrum of dielectric ring under normal incidence of an electromagnetic wave on the lateral face. The ring permittivity $\varepsilon = 12$. $L = 61.9\, cm$, $g/h = 5$. The ring is embedded in air, $\varepsilon_{air} = 1$.

A complete electromagnetic description of the ring resonance structure was obtained, containing information about the spectrum (eigenvalues) and the distribution of mode fields (eigenfunctions). This information made it possible to fully interpret the scattering spectrum using the standard notation of the eigenfunctions, which are characterized by the azimuthal ($m$), radial ($r$), and axial ($z$) mode indices, forming ordered triple $(m, r, z)$.

Figure 1, which shows the calculated RCS spectrum for the dielectric constant $\varepsilon = 12$, allows us to analyze the main features of light scattering by a dielectric RR. The spectrum begins with a broad low-frequency band of low intensity, which corresponds to a dipole resonance. The dipole band is followed by a complex and, at first glance, chaotic set of wide and narrow resonance lines. However, a careful analysis of the spectrum clarifies the periodic sequence of line sets and their interpretation, which is confirmed by the shape of the calculated field patterns for each resonance, discussed below.

To interpret the results and discuss the spectra, we introduce the concept of RGM by analogy with the term WGM. Figure 1 shows the spectrum containing two full galleries and the beginning of the third gallery. The gallery begins with a wide band of complex shape and continues with a limited number of constantly narrowing symmetrical lines until it ends when the next wide band appears, belonging to the next gallery. Let us compare the mechanisms of the appearance of resonances in a ring resonator (RGM gallery) and a disk resonator (WGM gallery). WGM arise in a disk due to total reflections from the rim with the proper phase condition after circling along the entire resonator. In WGM - resonator the azimuthal index $m$ has very large values;
in real devices, \( m \) is usually in the range from about 102 to 105. In the case of RGM, which are considered in this work, the situation is completely different both with respect to the mechanism of the formation of resonances and in terms of the value of the azimuthal index. The boundary conditions are determined by the presence of all four walls and the electromagnetic field fills the entire volume of the ring resonator. The key to interpreting the spectrum in figure 1 is the presence of two side walls in the ring instead of one side wall in the disk. For simplicity, the calculations considered the case of a thin rectangular ring with the ratio of the width of the ring (the difference between the outer and inner radii) to its thickness equal to 5. Therefore, in the low-frequency region of the spectrum shown in figure 1, the axial mode index will not change (half the wavelength, \( z = 0.5 \)), and the electromagnetic mode will be determined by a pair of azimuthal and radial indices \((m, r)\).

However, our RR is thin, but not so narrow that with increasing frequency, i.e. with a decrease in the wavelength, the radial index \( r \) remained constant in the discussed spectral range. With decreasing wavelength, the boundary conditions in the radial direction, determined by the two side walls, change, leading to an increase in the radial index \( r \). This is how the galleries of the RR are formed, each of which begins with a change in the radial index by 0.5, which corresponds to an increase in the transverse distribution of the electromagnetic field by half the wavelength. In this case, the boundary conditions on the side walls of the resonator remain unchanged and correspond to zero field intensity.

Figure 2. The calculated field patterns in the \((x, y)\) cross sections (resonator top view) for ring permittivity \( \varepsilon = 12 \). The azimuthal, radial, and axial mode indices \((m, r, z)\) are indicated for each resonance. The vertical scale corresponds to the field amplitude \(|E|\). The inset shows the scattering geometry: the incident wave is polarized in the plane of the ring.

Figure 2 confirms the above interpretation of the scattering spectrum in Figure 1. Detailed maps of the calculated resonant electromagnetic field distribution inside the ring depending on the wavelength are presented. The first gallery corresponds to the radial index \( r = 0.5 \). In the middle of the ring in the cross section, there is a single maximum \(|E|\) (yellow-red color) and at
the side walls of the ring the field disappears (blue color) which corresponds to a half-wave. In the second gallery, in the cross section there are two field maxima $|E|$ of and three minima (in the center and at the edges), which corresponds to one full wavelength, $r = 1$. And so on, with a successive increase in the radial index by $\Delta r = 0.5$.

3. Conclusion

Devices based on silicon micro RRs are already indispensable for a wide range of applications and fundamental research. However, due to the limited design freedom of the RR (dielectric permittivity and roundtrip length), the functionality cannot always be fully explored and optimized. In our work, we have demonstrated that such resonators have a much more complex spectrum of eigenmodes in comparison with WGM. Structures with RGM contain both wide and narrow lines of various shapes, which will allow realizing new functionality in silicon ring resonators. Our results provide a new understanding of the phenomena of light localization and the relationship between the geometry and physical properties of microscopic photonic structures.

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