Mass and radius estimation for the neutron star in X-ray burster 4U 1820-30

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ABSTRACT
We present a new method for determining masses and radii of neutron stars residing in thermo-nuclear X-ray burst sources. To illustrate this method we apply it to a burst from the source 4U 1820-30 recorded by the Rossi X-Ray Timing Explorer. Fits of the observed X-ray spectra to grids of Comptonised model atmospheres yield estimates for the mass and radius of the neutron star, $M = 1.3 \pm 0.6 M_\odot$ and $R = 11^{+3}_{-2}$ km, respectively.

Key words: Stars: individual (4U 1820-30) - stars: neutron - X-ray: bursts

1 INTRODUCTION
The low-mass X-ray binary (LMXB) 4U 1820-30 is located in the core of the globular cluster NGC 6624 (Giacconi et al. 1974). Using optical observations the distance is estimated to be $7.6 \pm 0.4$ kpc (Kuulkers et al. 2003). Thermo-nuclear X-ray bursts (so-called type I X-ray bursts, see, e.g., Lewin, van Paradijs & Taam 1993, Strohmayer & Bildsten 2006, for reviews) from this source were discovered by Grindlay et al. (1976). 4U 1820-30 has an extremely short binary orbital period of 685 sec (Stella, Priedhorsky & White 1987). Such a short orbital period and the X-ray burst activity imply that it is a low-mass binary system with a $\sim 0.06 - 0.07 M_\odot$ secondary (donor) star orbiting a neutron star. The donor is most probably a helium white dwarf (Rappaport et al. 1987).

Almost all of the X-ray bursts from 4U 1820-30 release so much energy, that the surface luminosity of the neutron star reaches the Eddington limit (see Vacca et al. 1986, Haberl et al. 1987, Kuulkers et al. 2003, Galloway et al. 2008). The atmosphere then expands due to radiation pressure and the photospheric radius increases by a factor of up to $\sim 20$. At the time of expansion and subsequent contraction the X-ray luminosity of 4U 1820-30 is expected to remain almost constant near the Eddington limit. Such a scenario implies that at first the effective temperature $T_{\text{eff}}$ of the compact star decreases when the radius increases. At the subsequent stage, the radius decreases and the $T_{\text{eff}}$ increases.

X-ray spectra taken during such strong burst can be used to estimate mass and radius for the compact star in 4U 1820-30, and hence, for determining the equation of state (EOS) for ultradense matter hidden in the stellar interior (cf. Haberl et al. 1987; Lewin et al. 2003).

Measurements of masses and radii of neutron stars are necessary for better understanding of the populations of all stellar objects and the latest stages of stellar evolution. Moreover, it is possible that some of them are made out of strange matter with the equation of state (EOS) different than that for neutrons. In such a case mass $M$ and radius $R$ of the compact star can be apparently lower than for a neutron star (see e.g. Haensel et al. 2007). Mass and radius determination for a compact star is understood as an intermediate step to constrain EOS in many recent papers (Özel 2006; Lattimer & Prakash 2007 for example).

At present, masses and radii of weakly magnetized compact stars in X-ray burst sources are rather poorly constrained, due to several reasons. First, usually there exists several sources of X-ray emission contributing to the total flux: thermonuclear explosions in the stellar envelope, accretion disk and accretion stream, and the transition layer between disk and the stellar surface. Second, observed satellite X-ray spectra are of poor quality and their interpretation with spectral fitting software is ambiguous. Third, available models of X-ray emission (and model atmospheres of hot compact stars) mostly are schematic and inadequate for fitting of non-Planckian spectra. The advantage of this work is that we use much more sophisticated model spectra than those used in previous papers (see the last paragraph of Section 2 and the beginning of Section 4).
2 OUTLINE OF THE METHOD

Our analysis uses published Rossi X-ray Timing Explorer (RXTE) Proportional Counter Array (PCA) X-ray spectral data of this source (Kuulkers et al. 2003), which were obtained during a strong burst just after the phase corresponding to the maximum effective temperature \( T_{\text{eff}} \). We assume, that the radius of the neutron star photosphere had decreased to its normal value. The cooling phase had just started, but the bolometric (X-ray) luminosity is still very close to the Eddington luminosity.

At first, we determined the surface gravity \( \log g \) and the surface gravitational redshift \( z \) for 4U 1820-30. Both parameters were constrained by the best fits of the observed spectra to grids of hundreds of theoretical X-ray spectra of hot neutron stars. Finally, the mass \( M \) and radius \( R \) of the compact object were determined from simple algebraic relations corresponding to a non-rotating star and the Schwarzschild metric (Majczyna & Madej 2005).

Rotational distortion of a compact star depends both on the rate of rotation and details of its internal structure including the EOS. None of them are known a priori. Also there are no available models of flattened compact stars. Therefore, assumption of a nonrotating star is the only practical approximation for our research.

We also assume that the burst emission from the source is isotropic. The assumption means that the eruption exhibits spherical symmetry, and anisotropy caused by the disk was neglected. Moreover, properties of the accretion disk (e.g. its thickness), possible existence of the accretion disk corona and the unknown inclination angle of the disk do not alter the observed X-ray spectrum by assumption.

Grids of helium-dominated model atmospheres and X-ray spectra of hot neutron star were computed with the ATM21 code, which computes model atmospheres with the account of Compton scattering on free electrons. The code takes into account angle-averaged Compton scattering of X-ray photons with initial energies approaching the electron rest mass. Our models atmospheres were computed with rich set of bound-free and free-free energy-dependent opacities. A detailed description of the equations and numerical methods were given in a long series of earlier papers (Madej 1991a,b; Joss and Madej 2001; Madej, Joss and Różanska 2004; Majczyna et al. 2002, 2005).

3 OBSERVATIONAL DATA

Our analysis makes use of the series of 113 X-ray spectra taken during a strong burst from 4U 1820-30 recorded by the PCA onboard the RXTE satellite on UT 1997 May 2, which started at 17:32:45. The burst spectra are at a 0.25 sec time resolution, and are numbered from 5 to 120. They are corrected for deadtime. All PCUs were on during the observation. We subtracted the pre-burst persistent emission from the burst emission. The whole sequence of spectra was described and analyzed in detail in the paper by Kuulkers et al. (2003). For this paper we took into account the latest response matrix, using MARFRMP version 3.2.6.

For our analysis we have chosen only 2 spectra of that series, which correspond to the phase of the burst just after the maximum blackbody temperature \( T_{\text{bb}} \), see Kuulkers et al. (2003). These spectra are labeled No. 21 and 22. The single burst of 4U 1820-30 was chosen as trial and validation of the method.

4 MODELS

Model atmospheres and theoretical spectra of our grid were parametrized by

- effective temperature \( T_{\text{eff}} \) in the range \( 1.0 \times 10^7 < T_{\text{eff}} < 3.0 \times 10^7 \) K in steps of \( 0.2 \times 10^6 \), and
- decimal logarithm of gravitational acceleration in the atmosphere of a neutron star, \( \log g \). The lowest gravity model of \( log g_{\text{min}} \) just touches the critical gravity, when the atmosphere loses hydrostatic equilibrium (models at \( \log g \) equal to \( \log g_{\text{min}} - 0.1 \) or less were dynamically unstable). The highest gravity equals to \( \log g_{\text{max}} = 15.0 \) for all effective temperatures \( T_{\text{eff}} \). E.g. for \( T_{\text{eff}} = 2.6 \times 10^7 \) K we precomputed a set of theoretical spectra corresponding to \( \log g = 14.3, 14.4, 14.5, 14.6, 14.7, 14.8, 14.9 \) and 15.0. Our grid consists of 288 models.

We assumed for all analyzed spectra, that the photospheric radius of the compact object has already decreased to its normal value. The cooling phase has just started, but the bolometric luminosity is still close to the Eddington luminosity. In this regime model atmospheres and their X-ray spectra are more sensitive for changes of both the effective temperature \( T_{\text{eff}} \) and gravity \( \log g \). The farther off the Eddington luminosity, the less accurate is our method of determining both parameters of the compact star. In practice, it turned out that we can use it only for the spectra No. 21 and 22. To make sure that our minimum of \( \chi^2 \) is a global, we have used an extensive grid of models.

For a given \( T_{\text{eff}} \) calculated X-ray spectra exhibit peak fluxes always at an energy higher than the blackbody of the same temperature. It is a well-known effect that the \( T_{\text{bb}} \) from black-body fits to the burst spectra (called color temperature \( T_{\text{col}} \)) are always greater than \( T_{\text{eff}} \). In other words, color factor \( f = T_{\text{col}}/T_{\text{bb}} > 1 \) (Madej 1974; Lewin et al. 1993; Shapiro & Titarchuk 2004). We draw attention of the reader to the fact that this difference is implicitly taken into account in our calculated burst spectra.

The important issue is the choice of the chemical composition of the model atmospheres. We know that the material accreted by the neutron star is dominated by helium with small amounts of hydrogen (Cumming 2003). Moreover, at the phase of a burst just after the maximum luminosity some ashes from nuclear burning are still exposed at the neutron star surface. That matter was ejected by the radiation driven wind during the phase of radius expansion (Weinberg, Bildsten & Schatz 2006). However, we accepted a rather simple chemical composition for our grids of model atmospheres, i.e., we assumed helium with small amounts of iron. In this way iron was used as the "average metal" and replaced a mixture of heavy elements which must be created during a thermonuclear burst.

We were able to constrain the iron abundance by estimating the quality of the trial fits, which was given by the value of \( \chi^2 \) for 1 degree of freedom. We tested fits to the grid of almost pure helium atmosphere spectra with very small amount of hydrogen and to few grids of model atmo-
spheres adding iron of various abundance. (Hydrogen must be included in the ATM21 input data set due to numerical reasons.) For the final fitting we used the grid of model atmospheres and spectra computed for the best chemical composition, as shown in Table 1.

| $Z$ | $N_Z$ |
|-----|-------|
| 1   | 1.00e+00 |
| 2   | 1.00e+06  |
| 26  | 1.11e+03  |

Table 1. Number abundances of elements, $N_Z = N_{el}/N_H$

5 FITTING PROCEDURE

All fits of the observed X-ray spectra to the numerical models depend on the following parameters:

- hydrogen column density $N_H$. We assumed that $N_H$ is identical for all spectra and equals to $N_H = 0.16 \times 10^{22} \pm 0.003 \text{ cm}^{-2}$ (see Kuulkers et al. 2003),
- redshift $z$, from 0.1 to 0.6, and
- normalization parameter.

For each of the combination ($T_{\text{eff}}$, log $g$) of the available grid of numerical models of X-ray spectra we determined the redshift $z$, the normalization parameter and $\chi^2$. $\chi^2$ was determined from the fits using the XSPEC software, version 11.3.2. Within XSPEC, we used the spectral fitting model 'wabs atable' in the energy range $2.9 - 20.0 \text{ keV}$.

1. We ignored models with limiting redshift $z = 0.1$ and $z = 0.6$. Remaining values of log $g$ and the corresponding fits were rejected.
2. We have chosen only models with $\chi^2$ in the range $[\chi^2_{\text{min}}, \chi^2_{\text{min}} + \Delta \chi^2]$, where $\chi^2_{\text{min}}$ is the minimum value $\chi^2$ and $\Delta \chi^2$ represents the increase of $\chi^2$ small enough to be in the $1\sigma$ confidence range. We applied here the estimate for $\Delta \chi^2$ taken from Press et al. (1996), page 692, which give in our case $\Delta \chi^2 = 2.3/23 \text{ d.o.f} \approx 0.1$. For the spectrum No. 21 we obtained $\chi^2_{\text{min}} = 1.257$ (see Fig. 1) and for the spectrum No. 22 we obtained $\chi^2_{\text{min}} = 0.816$ (see Fig. 2).
3. For each model which has passed the above procedure we obtained a single pair of values for mass $M$ and radius $R$ from the algebraic equations corresponding to nonrotating star and the Schwarzschild metric, cf. Majczyna & Madej (2005). The gravitational redshift is given by:

$$1 + z = \left( 1 - \frac{2GM}{Re^2} \right)^{-1/2}$$

where $G$ is the gravitational constant, $M$ is the neutron star mass, $R$ is the radius measured on the neutron star surface, $c$ denotes the speed of light. Gravitational acceleration on the neutron star surface equals to:

$$g = \frac{GM}{R^2} \left( 1 - \frac{2GM}{Re^2} \right)^{-1/2}$$

We solve Eqs. 1-2 for mass $M$ and radius $R$, and obtain the following explicit expressions:

$$M = \frac{z^2c^4}{4gG} \frac{(2 + z)^2}{(1 + z)^3}$$

Both mass and radius of a neutron star are functions only of the surface gravity $g$ and the gravitational redshift $z$. The effective temperature $T_{\text{eff}}$ does not directly influence neither $M$ nor $R$.

We assumed here that for each fitted X-ray spectrum the apparent radius of the photosphere $R_{\text{ph}}$ at the touchdown phase of that burst is equal to the true radius $R$ of the neutron star in 1820-30. This is a widely accepted assumption, that both radii are equal at the touchdown phase of strong X-ray bursts with radius expansion. Note, that it was critically rediscussed recently by Steiner et al. (2010).

Extreme masses and radii collected in the above set determine the errors of mass and radius of the $1\sigma$ confidence range.

4. The best values of mass $M$ and radius $R$ are arithmetic averages of individual determinations. We decided to use the mean value rather than the determination based on the

Figure 1. Sample fit of our redshifted model spectrum of $T_{\text{eff}} = 2.5 \times 10^7 \text{ K}$, log $g = 14.2$ and the redshift $z = 0.24$ to the RXTE spectrum No. 21 of this X-ray burst.

Figure 2. Sample fit of the spectrum No. 22 of 4U 1820-30 by our redshifted model with $T_{\text{eff}} = 2.2 \times 10^7 \text{ K}$, log $g = 14.0$ and $z = 0.12$. 

$$M = \frac{z^2c^4}{4gG} \frac{(2 + z)^2}{(1 + z)^3}$$
model with $\chi^2_{\text{min}}$, since our grid of models is extensive but was not fine enough.

For the spectrum No. 21 we obtained mass $M = 1.19^{+1.28}_{-0.75} M_\odot$ and radius $R = 9.40^{+8.91}_{-3.49}$ km. For the spectrum No. 22, $M = 1.36^{+0.75}_{-0.71} M_\odot$ and $R = 11.38^{+2.83}_{-2.23}$ km, respectively. Both determinations yield similar values, but the spectrum No. 22 yields much lower errors.

We had to average measurements with asymmetric errors. This difficult problem was analysed by Roger Barlow (arXiv:physics/0406120v1 and arXiv:physics/0401042v1). We used his Java applet which is available at http://www.slac.stanford.edu/~barlow/statistics.html and obtained averaged results: $M = 1.3 \pm 0.6 M_\odot$, $R = 11^{+3}_{-2}$ km (see Fig. 3).

6 CONCLUSIONS

In this paper we presented a new method to determine the mass and radius of the compact object in type I X-ray burst sources. We analyzed X-ray spectra of the burster 4U 1820-30 in phases when the luminosity of the source is still close to the Eddington luminosity and the compact star photosphere returned close to its normal size after radius expansion. The observed X-ray spectra were fitted to grids of theoretical spectra of model atmospheres in radiative and hydrostatic equilibrium computed with the precise treatment of Compton scattering on free electrons.

Blackbody fits allow us to determine radius only for the burster of known distance. Using our method we may obtain not only radius but also mass of the neutron star simultaneously. Furthermore, our $M$ and $R$ determinations are not directly dependent on the distance to the source. It is affected only by the value of the hydrogen column density $N_H$. This is a great advantage over blackbody fits.

Using our method, we constrained the mass $M = 1.3 \pm 0.6 M_\odot$ and radius $R \approx 11^{+3}_{-2}$ km for the compact star in 4U 1820-30. Mass and radius of the compact star in 4U 1820-30 obtained in this paper are close to the canonical mass $M = 1.4 M_\odot$ and radius $R \approx 10$ km of neutron star (see the monograph by Haensel, Potekhin & Yakovlev (2007)).

Our mass and radius determination are similar to the values published in Shaposhnikov & Titarchuk (2004), $R = 11.2^{+0.4}_{-0.5}$ km and $M = 1.29^{+0.12}_{-0.07} M_\odot$, respectively. Their estimates were based on a simple analytical model of X-ray spectrum formation on the neutron star. Authors also assumed that the distance to the burster equals to $d = 5.8$ kpc. However, our results differ from the conclusion obtained from analysis of quasi-periodic oscillations (QPO) in this source. It is thought that the highest observed QPO frequency from 4U 1820-30 is the marginally stable orbit frequency, therefore, it implied the estimated mass of $M \sim 2.2 M_\odot$ (Smale, Zhang & White 1997).

Güver et al. (2010) has just presented mass and radius estimation for neutron star in 4U 1820-30. They obtained the most probable values $M = 1.58 \pm 0.06 M_\odot$ and $R = 9.1 \pm 0.4$ km from five strong bursts of this source observed by RXTE. Results of our paper are consistent with that of Güver et al. (2010), though both papers presented different approaches for fitting of the observed spectra.

In their research Güver et al. (2010) fitted observed X-ray spectra of the source to the blackbody spectrum, corrected for interstellar absorption. They subsequently determined other parameters, like normalisation, bolometric flux and blackbody temperature. Mass and radius could be eventually obtained from set algebraic equations involving distance to 4U 1820-30 and the color correction factor $f_c$, the latter was known from model atmosphere calculations.

Our approach lies in the fact that shapes of both observed and numerical spectra differ from shape of the blackbody. Then we performed trial fits of a single observed X-ray count spectrum to hundreds of precomputed numerical spectra (each of them corrected for interstellar absorption $N_H$), to find the best fit $T_{\text{eff}}$ and $\log g$, redshifted by $z$. No informations on the distance, bolometric flux or color factors were applied here.

We note here that one cannot exclude, however, the possibility that the momentary radius of the photosphere $R_{\text{ph}}$ at touchdown phase is still larger than the 'reference' radius $R$ of the neutron star in 4U 1820-30 (Steiner et al. 2010, sec. 2.2). In such a case our transformation from $(z, \log g)$ to $M$ and $R$ is not strictly valid, cf. Eqs. 3-4. Then, errors of our $M$ and $R$ estimation must increase substantially, as compared e.g. with those drawn in Fig. 3.

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