Unification of the $k_T$ and threshold resummations

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Abstract

We derive a resummation formula for a $k_T$-dependent parton distribution function at threshold, where $k_T$ is a parton transverse momentum. The derivation requires infrared cutoffs for both longitudinal and transverse loop momenta as evaluating soft gluon emissions in the Collins-Soper resummation framework. This unified resummation exhibits suppression at large $b$, $b$ being the conjugate variable of $k_T$, which is similar to the $k_T$ resummation, and exhibits enhancement at small $b$, similar to the threshold resummation.
1. Introduction

Recently, we have demonstrated [1] that both the $k_T$ resummation for a parton distribution function $\phi(x, k_T, p^+)$ and the threshold resummation for $\phi(x, p^+)$ can be performed in the Collins-Soper (CS) framework [2]. The distribution function $\phi(x, k_T, p^+)$, associated with a hadron of momentum $p^\mu = p^+ \delta^{\mu+}$, describes the probability that a parton carries the longitudinal momentum $xp^+$ and the transverse momentum $k_T$. The distribution function $\phi(x, p^+)$, which coincides with the standard parton model, comes from $\phi(x, k_T, p^+)$ with $k_T$ integrated out. In the $k_T$ resummation the double logarithms $\ln^2(p^+ b)$, $b$ being the conjugate variable of $k_T$, are organized. The result is Sudakov suppression, quoted as [3]

$$\phi(x, b, p^+) = \exp \left[ -2 \int_{1/b}^{p^+} \frac{dp}{p} \int_{1/b}^{p} \frac{d\mu}{\mu} \gamma_K(\alpha_s(\mu)) \right] \phi^{(0)}, \quad (1)$$

where the anomalous dimension $\gamma_K$ will be defined later, and $\phi^{(0)}$ is the initial condition of the double-logarithm evolution. In the threshold resummation the double logarithms $\ln^2(1/N)$, $N$ being the moment of $\phi(x, p^+)$, are organized. The result is an enhancement [4]:

$$\phi(N, p^+) = \exp \left[ -2 \int_{0}^{1} dz \frac{z^{N-1} - 1}{1 - z} \int_{1-z}^{1} \frac{d\lambda}{\lambda} \gamma_K(\alpha_s(\lambda p^+)) \right] \phi^{(0)}, \quad (2)$$

with the same anomalous dimension $\gamma_K$.

We have neglected the dependence of $\phi$ on the renormalization (or factorization) scale $\mu$, which denotes the single-logarithm evolution, and can be easily derived using renormalization-group (RG) equations. Equation (1) has been employed to evaluate the $p_T$ spectrum of direct photon production, $p_T$ being the transverse momentum of the direct photon [5]. It was observed that the Sudakov exponential, after Fourier transformed to the $k_T$ space, provides the necessary $k_T$ smearing effect, which resolves the discrepancy between experimental data and the next-to-leading-order QCD ($\alpha\alpha_s^2$) predictions [6]. Equation (2) is suitable for the analyses of dijet, direct photon and heavy quark productions in kinematic end-point regions [4].

It has been found that in the CS framework the two different types of double logarithms $\ln^2(p^+ b)$ and $\ln^2(1/N)$ are summed by choosing appropriate infrared cutoffs in the evaluation of soft gluon corrections. If transverse
degrees of freedom of a parton are included, $1/b$ will serve as an infrared cutoff for soft gluon emissions. Combined with the scale $p^+$ from the hadron momentum, the large double logarithms $\ln^2(p^+b)$ are generated. In the endpoint region with $x \to 1$, we keep the longitudinal cutoff $(1-x)p^+$, or $p^+/N$ in the moment space, and integrate out the transverse degrees of freedom of a parton. Combined with $p^+$, the double logarithms $\ln^2(1/N)$ are produced. Therefore, in the case with $p^+b \gg N$, we neglect the longitudinal cutoff, and sum $\ln^2(p^+b)$. In the case with $N \gg p^+b$, we neglect the transverse cutoff, and sum $\ln^2(1/N)$. Based on these observations, it is straightforward to develop a unified resummation formalism by retaining the longitudinal and transverse cutoffs simultaneously. It will be shown in this letter that this unified resummation exhibits suppression at large $b$, similar to the $k_T$ resummation, and exhibits enhancement at small $b$, similar to the threshold resummation.

2. Formalism

Consider a quark distribution function for a hadron in the minimal subtraction scheme,

$$\phi(x, k_T, p^+) = \int \frac{dy^-}{2\pi} \int \frac{d^2b}{(2\pi)^2} e^{-ixp^+y^- + ik_T \cdot b} \langle p | \bar{q}(y^-, b) \frac{1}{2} \gamma^+ q(0) | p \rangle , \quad (3)$$

where $\gamma^+$ is a Dirac matrix, and $|p\rangle$ denotes the hadron with the momentum $p$. Averages over spin and color are understood. The above definition is given in the axial gauge $n \cdot A = 0$, where the gauge vector $n$ is assumed to be arbitrary with $n^2 \neq 0$. Though this definition is gauge dependent, physical observables, such as hadron structure functions and cross sections, are gauge invariant. It has been shown that the $n$ dependences cancel among the convolution factors, i.e., among parton distribution functions, final-state jets, and nonfactorizable soft gluon exchanges, in the factorization formula for a DIS structure function.

The key step in the CS technique is to obtain the derivative $p^+ d\phi/dp^+$. In the axial gauge $n$ appears in the gluon propagator, $(-i/l^2)N^{\mu\nu}(l)$, with

$$N^{\mu\nu}(l) = g^{\mu\nu} - \frac{n^\mu l^\nu + n^\nu l^\mu}{n \cdot l} + n^2 \frac{l^\mu l^\nu}{(n \cdot l)^2} . \quad (4)$$
Because of the scale invariance of $N^\mu\nu$ in $n$, $\phi$ depends on $p^+$ through the ratio $(p \cdot n)^2/n^2$, implying that the differential operator $d/dp^+$ can be replaced by $d/dn_\alpha$ using a chain rule,

$$ p^+ \frac{d}{dp^+} \phi = -\frac{n^2}{v \cdot n} \nu_\alpha \frac{d}{dn_\alpha} \phi , $$  

where $v = (1, 0, 0)$ is a dimensionless vector along $p$. The operator $d/dn_\alpha$ applies to $N^\mu\nu$, leading to

$$ -\frac{n^2}{v \cdot n} \nu_\alpha \frac{d}{dn_\alpha} N^\mu\nu = \hat{\nu}_\alpha (N^{\mu \alpha \nu} + N^{\alpha \nu \mu} ) , $$  

with the special vertex

$$ \hat{\nu}_\alpha = \frac{n^2 \nu_\alpha}{v \cdot nn \cdot l} . $$  

The momentum $l^\mu$ ($l^\nu$) is contracted with the vertex the differentiated gluon attaches, which is then replaced by the special vertex in Eq. (7). For each type of vertices, there exists a Ward identity, which relates the diagram with the contraction of $l^\mu$ ($l^\nu$) to the difference of two diagrams [7]. A pair cancellation occurs between the contractions with two adjacent vertices. Summing the diagrams containing different differentiated gluons, the special vertex moves to the outer end of a parton line. We arrive at the derivative,

$$ p^+ \frac{d}{dp^+} \phi(x, k_T, p^+) = 2 \bar{\phi}(x, k_T, p^+) , $$

described by Fig. 1(a), where the square in the new function $\bar{\phi}$ represents the special vertex $\hat{\nu}_\alpha$. The coefficient 2 comes from the equality of the two new functions with the special vertex on either side of the final-state cut.

To obtain a differential equation of $\phi$ from Eq. (8), we need to factorize the subdiagram containing the special vertex out of $\bar{\phi}$. The factorization holds in the leading regions of the loop momentum $l$ that flows through the special vertex. The collinear region of $l$ is not leading because of the factor $1/(n \cdot l)$ in $\hat{\nu}_\alpha$ with nonvanishing $n^2$. Therefore, the leading regions of $l$ are soft and hard, in which the subdiagram is factorized from $\hat{\phi}$ into a soft function $K$ and a hard function $G$, respectively. The remaining part is the original distribution function $\phi$. That is, $\bar{\phi}$ is expressed as the convolution of the functions $K$ and $G$ with $\phi$. 

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The lowest-order contribution to $K$ is extracted from Fig. 1(b), whose contribution is written as

$$\bar{\phi}_s(x, k_T, p^+) = \bar{\phi}_{sv}(x, k_T, p^+) + \bar{\phi}_r(x, k_T, p^+) ,$$  \(9\)

with

$$\bar{\phi}_{sv} = \left[ ig^2 C_F \mu^\epsilon \int \frac{d^4-l}{(2\pi)^{4-\epsilon}} N_{\nu \beta}(l) \hat{v}^\beta v'^\nu \frac{1}{v \cdot l} - \delta K \right] \phi(x, k_T, p^+) ,$$  \(10\)

$$\bar{\phi}_{sr} = ig^2 C_F \mu^\epsilon \int \frac{d^4-l}{(2\pi)^{4-\epsilon}} N_{\nu \beta}(l) \hat{v}^\beta v'^\nu \frac{1}{v \cdot l} 2\pi i \delta(l^2)$$

$$\times \phi(x + l^+/p^+, |k_T + l_T|, p^+) ,$$  \(11\)

corresponding to the virtual and real gluon emissions, respectively. $C_F = 4/3$ is a color factor, and $\delta K$ an additive counterterm. The ultraviolet pole in Eq. (10) is isolated using the dimensional regularization.

To work out the $l_T$ integration explicitly, we employ the the Fourier transform from the $k_T$ space to the $b$ space. Inserting the identities

$$\int_x^1 \frac{d\xi}{\xi} \delta(\xi - x) = 1 , \quad \int_x^1 \frac{d\xi}{\xi} \delta(\xi - x - l^+/p^+) = 1 ,$$  \(12\)

into Eqs. (10) and (11), respectively, Eq. (9) becomes

$$\bar{\phi}_s(x, b, p^+) = \int_x^1 \frac{d\xi}{\xi} K \left( \left( 1 - \frac{x}{\xi} \right) \frac{p^+}{\mu}, b\mu, a_s(\mu) \right) \phi(\xi, b, p^+) ,$$  \(13\)

with

$$K = ig^2 C_F \mu^\epsilon \int \frac{d^4-l}{(2\pi)^{4-\epsilon}} N_{\nu \beta}(l) \hat{v}^\beta v'^\nu \frac{\delta(1 - x/\xi)}{l^2}$$

$$+ 2\pi i \delta(l^2) \delta \left( 1 - \frac{x}{\xi} - \frac{l^+}{p^+} \right) e^{i l_T \cdot b} - \delta K \delta \left( 1 - \frac{x}{\xi} \right) .$$  \(14\)

To obtain the above expression, we have adopted the approximation

$$\delta \left( 1 - \frac{x}{\xi} - \frac{l^+}{\xi p^+} \right) \approx \delta \left( 1 - \frac{x}{\xi} - \frac{l^+}{p^+} \right) ,$$  \(15\)
which is appropriate in the threshold region with \( x \to 1 \). The exponential \( \exp(i lT \cdot b) \) in the second term arises from the Fourier transform of the real gluon contribution.

To work out the \( \xi \) integration explicitly, we further employ the Mellin transform from the momentum fraction \( (x) \) space to the moment \( (N) \) space:

\[
\bar{\phi}_s(N, b, p^+) \equiv \int_0^1 dx x^{N-1} \bar{\phi}_s(x, b, p^+) ,
\]

\[
= K(p^+/(N\mu), b\mu, \alpha_s(\mu))\phi(N, b, p^+). \tag{16}
\]

with

\[
K(p^+/(N\mu), b\mu, \alpha_s(\mu)) = \int_0^1 dzz^{N-1}K((1-z)p^+/\mu, b\mu, \alpha_s(\mu)). \tag{17}
\]

The convolutions between \( K \) and \( \phi \) in \( l^+ \) and \( l_T \) are then completely simplified into a multiplication under the Mellin and Fourier transforms, respectively.

Equation (17) is evaluated in the Appendix, and the result is

\[
K(p^+/(N\mu), b\mu, \alpha_s(\mu)) = \frac{\alpha_s(\mu)}{\pi}C_F \left[ \ln \frac{1}{b\mu} - K_0 \left( \frac{2\nu p^+ b}{N} \right) \right], \tag{18}
\]

\( K_0 \) being the modified Bessel function. The gauge factor \( \nu = \sqrt{(v \cdot n)^2/|n|^2} \) confirms our argument that \( \phi \) depends on \( p^+ \) via the ratio \( (p \cdot n)^2/n^2 \). It is easy to examine the large \( p^+ b \) and \( N \) limits of the above expression. For \( p^+ b \gg N \), we have \( K_0 \to 0 \) and

\[
K \to \frac{\alpha_s}{\pi}C_F \ln \frac{1}{b\mu}, \tag{19}
\]

which is exactly the soft function with the characteristic scale \( 1/b \) appearing in the \( k_T \) resummation \[1\], \[3\]. For \( N \gg p^+ b \), we have \( K_0 \approx -\ln(\nu p^+ b/N) \) and

\[
K \to \frac{\alpha_s}{\pi}C_F \ln \frac{\nu p^+}{N\mu}, \tag{20}
\]

which is the soft function with the characteristic scale \( p^+/N \) for the threshold resummation \[1\]. Hence, Eq. (18) is indeed appropriate for the unification of the \( k_T \) and threshold resummations.
The lowest-order contribution to $G$ from Fig. 1(c) is given by

$$\bar{\phi}_h(N, b, p^+) = G(p^+/\mu, \alpha_s(\mu))\phi(N, b, p^+) ,$$

(21)
in the $b$ and $N$ spaces, with

$$G = -ig^2C_F\mu^\varepsilon\int \frac{d^d-l}{(2\pi)^{d-2}}N_{\nu\beta}(l)\hat{\nu}^\beta(l)\left[\frac{\hat{p}^\mu I + \hat{v}^\mu}{(p-l)^2}\right] - \delta G ,$$

(22)
in the limit $x \to 1$, where $\delta G$ is an additive counterterm. The second term, whose sign is opposite to that of $\bar{\phi}_sv$, is a soft subtraction. This term avoids double counting, and ensures a hard momentum flow in $G$. A straightforward calculation gives

$$G(p^+/\mu, \alpha_s(\mu)) = -\frac{\alpha_s(\mu)}{\pi}C_F\ln \frac{p^+\nu}{\mu} ,$$

(23)
which is the same as the hard functions with the characteristic scale $p^+$ appearing in the $k_T$ and threshold resummations [1, 3].

Using $\bar{\phi} = \bar{\phi}_s + \bar{\phi}_h$, Eq. (8) becomes

$$p^+ \frac{d}{dp^+}\phi(N, b, p^+) = 2\left[K(p^+/\nu\mu, b\mu, \alpha_s(\mu)) + G(p^+/\mu, \alpha_s(\mu))\right] \times \phi(N, b, p^+) .$$

(24)

The functions $K$ and $G$ possess ultraviolet divergences individually as indicated by their counterterms. These divergences, both from the virtual gluon contribution $\bar{\phi}_sv$, cancel each other, such that the sum $K + G$ is RG invariant. The single logarithms contained in $K$ and $G$ are organized by the RG equations

$$\mu \frac{d}{d\mu} K = -\gamma_K = -\mu \frac{d}{d\mu} G .$$

(25)
The anomalous dimension of $K$, $\lambda_K = \mu d\delta K/d\mu$, is given, up to two loops, by [8]

$$\gamma_K = \frac{\alpha_s}{\pi}C_F + \left(\frac{\alpha_s}{\pi}\right)^2 C_F \left[C_A \left(\frac{67}{36} - \frac{\pi^2}{12}\right) - \frac{5}{18} n_f\right] ,$$

(26)
with $n_f$ the number of quark flavors, and $C_A = 3$ a color factor.

As solving Eq. (25), we allow the variable $\mu$ evolves from the characteristic scale of $K$ to the scale of $G$. We discuss the cases for $p^+ b \gg N$ and for
\( N \gg p^+b \) first, which will help the derivation of the unified resummation. The solution of \( K+G \) is written as

\[
K(p^+/(N\mu), b\mu, \alpha_s(\mu)) + G(p^+/\mu, \alpha_s(\mu))
\]

\[
= -\int_{1/b}^{p^+} \frac{d\mu}{\mu} \gamma_K(\alpha_s(\mu)) , \quad \text{for } p^+b \gg N ,
\]

\[
-\int_{p^+/N}^{p^+} \frac{d\mu}{\mu} \gamma_K(\alpha_s(\mu)) , \quad \text{for } N \gg p^+b ,
\]

(27)

where the initial conditions of \( K \) and \( G \) of the RG evolution have been neglected, since they are irrelevant to the double-logarithm summation. Equation (27) indicates that the distribution function \( \phi(N, b, p^+) \) involves \( \ln(p^+b) \) and \( \ln(1/N) \) in the \( p^+b \to \infty \) and \( N \to \infty \) limits, respectively, as stated before. This can be easily understood by ignoring the variation of \( \gamma_K \), and performing the \( \mu \) integration directly.

Inserting Eq. (27) into (24), we obtain the solutions

\[
\phi(N, b, p^+) = \exp \left[ -2 \int_{1/b}^{p^+} \frac{dp}{p} \int_{1/b}^{p} \frac{d\mu}{\mu} \gamma_K(\alpha_s(\mu)) \right] \phi^{(0)} ,
\]

(28)

\[
\phi(N, b, p^+) = \exp \left[ -2 \int_{p^+/N}^{p^+} \frac{dp}{p} \int_{p^+}^{p} \frac{d\mu}{\mu} \gamma_K(\alpha_s(\mu)) \right] \phi^{(0)} ,
\]

(29)

for \( p^+b \gg N \) and \( N \gg p^+b \), respectively. Note that we have replaced the derivative \( p^+d/dp^+ \) by \( (p^+/N)d/d(p^+/N) \) as deriving Eq. (29) \(^1\), since we intend to resum \( \ln(1/N) \). Obviously, Eq. (28) is identical to Eq. (4) with \( x \to 1 \), and Eq. (29) is equivalent to Eq. (2) up to corrections suppressed by a power \( 1/N \) \(^1\).

Equation (18) implies the characteristic scale of \( K \) for the unified resummation,

\[
\frac{1}{b} \exp \left[ -K_0 \left( \frac{p^+b}{N} \right) \right] ,
\]

(30)

where the gauge factor \( 2\nu \) has been suppressed. Hinted by Eqs. (14) and (20), we rewrite Eqs. (28) and (29) in terms of the above scale as

\[
\phi(N, b, p^+) = \exp \left[ -2 \int_{\exp[-K_0(p^+b/N)/b]}^{p^+} \frac{dp}{p} \int_{1/b}^{p} \frac{d\mu}{\mu} \gamma_K(\alpha_s(\mu)) \right] \phi^{(0)} ,
\]

(31)

\[
\phi(N, b, p^+) = \exp \left[ -2 \int_{\exp[-K_0(p^+b/N)/b]}^{p^+} \frac{dp}{p} \int_{p^+}^{p} \frac{d\mu}{\mu} \gamma_K(\alpha_s(\mu)) \right] \phi^{(0)} ,
\]

(32)
At last, to unify the above expressions, we replace the lower bound $s$ of $\mu$ by
\[ \frac{1}{b} \exp[-K_0(p^+b)], \quad (33) \]
which is motivated by Eq. (30). The unified resummation is then given by
\[
\phi(N, b, p^+) = \exp \left[ -2 \int_0^{\mu^+} \frac{d\mu}{\mu} \int_{\exp[-K_0(p^+b)/b]}^{\mu} d\mu' \gamma_K(\alpha_s(\mu)) \right] \phi^{(0)},
\]
which is appropriate for arbitrary $p^+b$ and $N$. It is easy to justify that
Eq. (34) approaches the $k_T$ resummation in Eq. (28) as $b \to \infty$, and ap-
proaches the threshold resummation in Eq. (29) as $b \to 0$.

3. Discussion

Comparing Eq. (34) with (28) and (29), we have the following remark. In the $k_T$ resummation it is the logarithms of the large scale $p^+$ from $G$ that are organized, and the result is a suppression. In the threshold resummation it is the logarithms of the small scale $p^+/N$ from $K$, as shown in Eq. (24), that are organized. The result is an enhancement. The opposite effects of these two resummations are attributed to the opposite directions of the double-logarithm evolution: from $1/b$ to the large $p^+$ in the former case, and from $p^+$ to the small $p^+/N$ in the latter case. The unified resummation for a $k_T$-dependent parton distribution function at threshold exhibits both behaviors: it is a suppression in the large $b$ region ($p^+b \gg N$), and turns into an enhancement in the small $b$ region ($N \gg p^+b$). That is, Eq. (34) displays the opposite effects of the $k_T$ and threshold resummations at different $b$.

The behavior of the unified resummation can be explained as follows. For an intermediate $x$, virtual and real soft gluon corrections cancel exactly in the small $b$ region, since they have almost equal phase space. Hence, there are only single collinear logarithms, namely, no double logarithms. In this case the Sudakov exponential approaches unity as $b < 1/p^+$ \[9\], indicating the soft cancellation stated above. However, at threshold ($x \to 1$), real gluon emissions still do not have sufficient phase space even as $b \to 0$, and soft virtual corrections are not cancelled exactly. In this case the double logarithms $\ln^2(1/N)$ persist and become dominant. Sudakov suppression then transits into an enhancement, instead of unity, smoothly as $b$ decreases.
The result obtained in this work will be useful for the study of, say, the production of large $p_T$ direct photons or jets. Phenomenological applications of Eq. (34) will be published elsewhere.

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Appendix

In this Appendix we present the detail of deriving Eq. (18). A simple investigation of Eq. (9) shows that the first term $g_{\nu\beta}$ in $N_{\nu\beta}$ gives a vanishing contribution because of $v^2 = 0$, and the contribution from the second term $-n_\nu l_\beta / n \cdot l$ cancels the contribution from the fourth term $n^2 l_\nu l_\beta / (n \cdot l)^2$. Hence, we concentrate only on the third term $-n_\beta l_\nu / n \cdot l$, which leads Eq. (17) to

$$K = -i g^2 C_F \mu^\epsilon \int_0^1 dzz^{-1} \int \frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}} \frac{n^2 (1-z)}{l^2} \left[ \delta(1-z) + 2\pi i \delta(l^2) \delta \left(1 - z - \frac{l^+}{p^+}\right) e^{i l_T \cdot b} \right] - \delta K \delta(1-z).$$

(35)

Performing the integrations over $l^-$ and $l^+$, we have

$$K = -g^2 C_F \mu^\epsilon \int_0^1 dzz^{-1} \int \frac{d^{2-\epsilon}l}{(2\pi)^{3-\epsilon}} \left[ \delta(1-z) + 2\pi i \delta(l^2) \delta \left(1 - z - \frac{l^+}{p^+}\right) e^{i l_T \cdot b} \right] - \delta K \delta(1-z).$$

(36)

The first term in the integral has been evaluated in [3], and the result with its ultraviolet pole subtracted by $\delta K$ is quoted as

$$\frac{\alpha_s}{\pi} C_F \ln \frac{a}{\mu},$$

(37)

where constants of order unity have been neglected. The infrared regulator $a$, introduced by the replacement of the denominator $l_T^2$ by $l_T^2 + a^2$, will approach zero at last. The second term, after performing the integration over $l_T$, gives

$$2\frac{\alpha_s}{\pi} C_F \nu p^+ b \int_0^1 dzz^{-1} K_1 \left( 2\sqrt{(1-z)^2 \nu^2 p^+ + a^2 b} \right),$$

(38)
where $K_1$ is the modified Bessel function, and the infrared regulator $a$ for the divergence at $z \to 1$ is also introduced by the same replacement.

We adopt the identity

$$
\int_0^1 dz z^{N-1} K_1 \left( 2 \sqrt{(1-z)^2 + a^2 b} \right) = \int_0^1 dz (z^{N-1} - 1) K_1 (2(1-z)\nu p^+ b) + \int_0^1 dz K_1 \left( 2 \sqrt{(1-z)^2 + a^2 b} \right),
$$

(39)

where $a$ in the first term has been dropped, since the $z$-integral is infrared finite. To perform the integration, we employ the relation

$$
\int_0^1 dz (z^{N-1} - 1) K_1 (2(1-z)\nu p^+ b) = -\int_0^{1/N} dz K_1 (2(1-z)\nu p^+ b),
$$

(40)

which is is valid up to corrections suppressed by $1/N$. It is then trivial to show that Eq. (38) leads to

$$
\frac{\alpha_s}{\pi} C_F \left[ \ln \frac{1}{ab} - K_0 \left( \frac{2\nu p^+ b}{N} \right) \right].
$$

(41)

At last, combining Eqs. (37) and (41), we obtain Eq. (18).
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Figure Captions

**Fig. 1.** (a) The derivative $p^+ d\phi / dp^+$ in the axial gauge. (b) The $O(\alpha_s)$ function $K$. (c) The $O(\alpha_s)$ function $G$. 
$$p^+ \frac{d}{dp^+} \phi = 2 \phi$$

(a)

(b) +

(c) -

FIG. 1