SUSTAINED MAGNETOSHEAR INSTABILITIES IN THE SOLAR TACHOCLINE

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ABSTRACT

Linear theory has demonstrated that toroidal magnetic fields in the solar tachocline are destabilized by the presence of latitudinal differential rotation. Previous nonlinear investigations of these global magnetoshear instabilities have only considered freely evolving scenarios, which will eventually dissipate after the instabilities saturate. Here we consider more realistic nonlinear scenarios in which the rotational shear is maintained indefinitely by mechanical forcing. When a broad toroidal field profile is specified as an initial condition, a so-called clamshell instability ensues, which is the dominant mode predicted by linear theory. After the initial nonlinear saturation, the residual mean fields are apparently too weak to sustain the instability indefinitely despite the mechanical forcing. However, when a mean poloidal field is imposed in addition to the rotational shear, a statistically steady state is achieved in which the clamshell instability is operating continually. This state is characterized by a quasi-periodic exchange of energy between the mean toroidal field and the instability mode with a longitudinal wavenumber $m = 1$. This quasi-periodic behavior has a timescale of several years and may have implications for tachocline dynamics and field emergence patterns throughout the solar activity cycle.

Subject headings: instabilities — MHD — stars: interiors — Sun: interior — Sun: magnetic fields — Sun: rotation

1. INTRODUCTION

The solar dynamo is thought to operate as an interface dynamo in which toroidal magnetic flux is generated by rotational shear in the tachocline and poloidal magnetic flux is generated in the convection zone by helical convection or by the decay of active regions (e.g., Charbonneau 2005). As the field strength grows, toroidal flux in the tachocline destabilizes and rises due to magnetic buoyancy, eventually emerging from the photosphere as active regions.

Toroidal flux in the solar tachocline is also susceptible to global instabilities induced by the latitudinal differential rotation. This was first demonstrated for two-dimensional spherical surfaces (Gilman & Fox 1997, 1999; Dikpati & Gilman 1999; Gilman & Dikpati 2000; Cally et al. 2003; Dikpati et al. 2004a) and was later extended to three-dimensional spherical shells under the shallow-water approximation (Gilman & Dikpati 2002; Dikpati et al. 2003) and the thin-shell approximation (Cally 2003; Miesch et al. 2007, hereafter MGD07; Gilman et al. 2007). For strong fields (such that the magnetic energy is comparable to the kinetic energy contained in the differential rotation), the most unstable modes have longitudinal wavenumber $m = 1$, which corresponds to a tipping of toroidal flux loops such that their central axis becomes misaligned with the rotation axis.

This class of global magnetoshear instabilities occurs both for concentrated bands of toroidal field and for broad field profiles. The preferred mode for broad field profiles, which are antisymmetric about the equator, is the clamshell instability whereby toroidal rings in the northern and southern hemispheres tip out of phase, reconnecting at the equator on one side of the sphere and opening up on the other side (Cally 2001; Cally et al. 2003; MGD07). In the absence of external forcing, the instability proceeds until loops of field become perpendicular to the equatorial plane and the latitudinal shear is nearly eliminated.

However, the solar tachocline is not an isolated system. Rotational shear is continually maintained by stresses from the overlying convection zone and the underlying radiative interior (Gough & McIntyre 1998; Talon et al. 2002; Miesch 2005). Meanwhile, according to the interface dynamo paradigm, magnetic flux is continually being replenished from above by penetrative convection and meridional circulation (Tobias et al. 2001; Dikpati et al. 2004b; Charbonneau 2005; Browning et al. 2006). If clamshell instabilities are indeed occurring in the tachocline, their environment is likely much more dynamic than the freely evolving scenarios considered thus far.

In this Letter, we present the first nonlinear simulations of global magnetoshear instabilities in the solar tachocline in which the instabilities are continually maintained against nonlinear saturation and dissipation through external forcing. We use the same nonlinear, three-dimensional thin-shell model employed by MGD07 to study the freely evolving case. The linear stability of this thin-shell system has been investigated by Gilman et al. (2007). The reader is referred to these two papers for a much more comprehensive discussion of the unstable modes and their nonlinear saturation under freely evolving conditions. These papers also contain a more comprehensive discussion of related work and a more detailed description of the thin-shell model and the numerical algorithm.

2. THE THIN-SHELL MODEL

Our numerical model is based on the thin-shell approximation, which is described in detail by Miesch & Gilman (2004) and MGD07. The layer is assumed to be stably stratified, and the aspect ratio $\delta = D/R$ is assumed to be much less than unity, where $D$ and $R$ are the width and radial location of the computational domain. The system is made nondimensional through the use of horizontal and vertical length scales $R$ and $D$, a velocity scale based on the equatorial rotation rate, and the background density and entropy gradient. We neglect pres-
where \( \hat{z} \) is a unit vector in the vertical dimension. This field is maintained by adding a source term to the longitudinally averaged induction equation similar to equation (1), with the same replenishment timescale \( \tau \). This field is intended to represent poloidal flux generated by dynamo processes in the convection zone and pumped downward (§ 1).

Imposing \( \mathbf{B}_a \) together with \( u_o \) is an indirect way of imposing a mean toroidal field; the latitudinal shear in the upper portion of the layer stretches and amplifies the poloidal field through what is known as the \( \Omega \)-effect, resulting in a mean toroidal field \( \langle a \rangle \), which is antisymmetric about the equator. The peak amplitude of \( \langle a \rangle \) will in general depend on the poloidal field strength \( C_{\Omega} \), on the SGS diffusion, and to some extent on \( F_z \).

3. SUSTAINED MAGNETOSHEAR INSTABILITIES

Our simulations are initiated as described in MGD07, with an equilibrium state defined by the zonal velocity \( u_o \), and a broad toroidal field profile of the form \( a(\theta) = \alpha \cos \theta \sin \theta \). The initial pressure and temperature are chosen such that the initial state is in magnetohydrostatic and magnetogeostrophic balance. The parameter \( \alpha \) is set to unity for the simulations presented here, corresponding to a peak dimensional field strength of about 40 kG, toward the high end of the range expected to exist in the tachocline (MGD07). Global magnetoshear instabilities also occur for weaker fields, but they take longer to develop. The initial equilibrium state is perturbed by adding a random, small-scale velocity field.

Unlike the simulations presented in MGD07, we maintain the differential rotation through the forcing term defined in equation (1). An example of the subsequent evolution is shown in Figure 1. The spatial resolution used for this case is \( N_p = 128 \), \( N_z = 256 \), \( N_r = 210 \).

Figure 1 shows the components of the magnetic energy integrated over the volume of the shell. One time unit in the nondimensional system corresponds to about 4 days (MGD07). At early times, the mean toroidal field magnetic energy (TFME) dominates but the nonaxisymmetric magnetic energy (NAME) grows rapidly as the clamshell instability develops. Most of the NAME is in the \( m = 1 \) mode, which dominates the total magnetic energy after the instability saturates at \( t \sim 400 \).

The evolution shown in Figure 1 is very similar to the unforced cases discussed at length in MGD07. However, in the absence of mechanical forcing, the saturation of the clamshell instability induces a global redistribution of angular momentum, which reverses the sense of the differential rotation (MGD07). The forcing suppresses this, leaving the differential rotation profile unchanged after saturation.

After saturation, the magnetic energy in the mean toroidal field decreases as in the unforced case, despite the persistent rotational shear. The TFME oscillates with a period of about 15 time units, corresponding to about 2 months. At later times, the amplitude of the oscillation decreases and the period increases slightly, to about 18 time units. This oscillation appears to be induced by a standing Alfvén wave excited by the initial saturation of the instability. However, low-wavenumber Rossby waves are also excited and have a comparable period.

In shallow-water systems that have a deformable upper boundary, global magnetoshear instabilities can possess significant kinetic helicity, suggesting they may serve to generate poloidal field from toroidal field and thus drive a self-sustained dynamo contained entirely within the tachocline (Dikpati & Gilman 2001a; Gilman & Dikpati 2002; Dikpati et al. 2003).

The simulation shown in Figure 1 does not extend more than a diffusive timescale \((\sim 10^5 \text{ time units})\), so it is uncertain
whether or not it may be classified as a dynamo. The energy in the mean fields may be leveling off beyond \( t \approx 800 \), but it is unclear whether this state will persist indefinitely. A more diffusive analog of this case (\( R_c = 10^3 \)) was certainly not a dynamo since the total magnetic energy decayed steadily after the initial saturation.

In any case, it appears either that the clamshell instability is no longer operating beyond \( t \approx 500 \) or that it is operating on a much longer timescale. The total magnetic energy remains dominated by the \( m = 1 \) component for at least 5 yr in dimensional time units and decreases steadily for the duration of the simulation. The mean field appears to be a remnant of the initial saturation of the instability at \( t \approx 400 \) as opposed to a dynamo-generated field induced by ongoing instabilities.

The situation changes dramatically if a mean poloidal field is imposed as described in § 2. Figure 2 illustrates the evolution of the magnetic energy components in a simulation with both mechanical and magnetic forcing (\( N_s = 64, N_p = 128, N_v = 210 \)). The time span shown covers a period beyond the initial saturation of the instability, after a statistically steady state has been reached.

The time evolution shown in Figure 2 reflects a quasi-periodic exchange of energy between the mean toroidal field and the nonaxisymmetric field components, the latter dominated by the \( m = 1 \) mode. The first two oscillations shown each span about 340 time units, which corresponds to 3.7 yr. However, the TFME drops lower in the subsequent cycle, reaching a minimum at \( t \approx 1500 \), leading to a longer cycle of about 400 time units (4.4 yr). The following cycle is then much shorter, lasting only about 230 time units (2.5 yr). The shape of each cycle is asymmetric, with a relatively slow rise in the TFME followed by a sharper drop as the clamshell instability sets in.

The nonaxisymmetric magnetic field exhibits quasi-periodic cycles similar to the mean toroidal field but phase-shifted such that the maxima in the NAME occur as the TFME is decreasing. Again, this reflects the repeated development of the clamshell instability, which transfers energy from the mean toroidal field to the \( m = 1 \) components. After the instability saturates, the mechanical and magnetic forcing reestablish the mean fields and the next cycle proceeds.

In the simulations reported here, as in the freely evolving cases reported in MGD07, the magnetic field remains predominantly horizontal and may be represented by a scalar magnetic potential \( J \) defined such that \( B \approx \hat{\varphi} \times \nabla J \). Thus, to a good approximation, contours of \( J \) trace the horizontal field lines as illustrated in Figure 3.

The changing patterns shown in Figure 3 illustrate the competing effects of the forcing and the instabilities. At \( t = 1800 \) (Fig. 3a), the mean toroidal field dominates the magnetic energy, although nonaxisymmetric structure is evident. By \( t = 1950 \), the clamshell instability has transferred much of this energy to the \( m = 1 \) mode and horizontal field lines are oriented more north-south (Fig. 3b). The imposed shear then operates on these fields as well as the imposed poloidal field to rebuild the mean toroidal field, which again dominates by \( t = 2150 \) (Fig. 3c).

The vertical structure of the flow is similar to analogous freely evolving cases with vertical shear discussed in MGD07. The velocity and magnetic fluctuations remain predominantly horizontal, and the instability proceeds most vigorously near the top of the layer where the latitudinal shear is strongest.

For the simulation shown in Figures 2 and 3, the amplitude of the imposed poloidal field, \( C_p \), is such that the integrated poloidal field magnetic energy (PFME) is about twice the equipartition value \( \int \frac{1}{2} |u_i^p|/2dV \), which is probably unrealistically large for the solar tachocline. Weaker imposed fields could potentially produce mean toroidal fields of comparable strength, but this can only be achieved in a simulation if the SGS diffusion is sufficiently low. Indeed, an analogous simulation with a weaker imposed field (PFME/DRKE \( = 0.02 \), where DRKE is the differential rotation kinetic energy) and the same diffusion coefficients (\( R_c = 10^3, R_t = 10^4 \)) produced weaker mean toroidal fields (TFME/DRKE \( \sim 3 \)). This system also exhibits sustained clamshell instabilities with some quasi-periodic behavior on timescales of several years, but longer term trends are also evident, comparable to or longer than the duration of the simulation (~10 yr). Such longer term evolution is to be expected since the growth rate of the clamshell instability decreases with decreasing field strength.

Achieving substantially lower SGS diffusion would require higher resolution, which is a challenge because of the long integration times necessary to capture multiple cycles. The relatively short forcing timescale used for the simulations shown in Figures 1–3 (\( \tau = 0.1 \), corresponding to about 10 hr) is also in some sense required by the setup of our numerical experiments. In a simulation similar to that shown in Figures 2 and 3 but with \( \tau = 2 \) (8 days), the mechanical forcing is insufficient to overcome the magnetic tension associated with the imposed poloidal field. As a result, the DRKE is only 20% of the target value associated with \( u_o \) and the clamshell instability is suppressed. The field does exhibit \( m = 1 \) structure, but TFME dominates the magnetic energy and the evolution is quasi-steady, with a slow retrograde propagation and no episodic opening up of the clamshell pattern.

Forcing timescales in the solar tachocline are longer, but the diffusion is lower, so strong toroidal fields could be produced with lower poloidal field strengths and the differential rotation could be maintained with weaker mechanical forcing. Since helioseismic inversions indicate that the rotational shear in the tachocline is indeed continually maintained and linear analysis suggests that even weak toroidal fields are unstable in the presence of such shear, it is likely that the tachocline is indeed continually undergoing global magnetoshear instabilities.

In this Letter, we have demonstrated that the clamshell instability can operate continually when rotational shear and magnetic fields are continually replenished. The temporal evolution is quasi-periodic as mean fields alternately build up and destabilize. This has the character of a critical phenomenon, but
it is not self-organized criticality in the technical sense because there is a characteristic time and spatial scale associated with the growth rate and wavenumber of the instability (Jensen 1998). Although we have focused on the clamshell instability, it is likely that other instability modes may similarly be operating continually in the tachocline, most notably the $m = 1$ tipping instability, which exists for banded toroidal field profiles (Dikpati & Gilman 1999; Cally et al. 2003; MGD07).

If global magnetoshear instabilities are indeed occurring in the solar tachocline, they would have wide-ranging implications for tachocline dynamics and for the coupling between the convective envelope and the radiative interior. Angular momentum transport induced by these instabilities may influence the differential rotation profile in the convection zone and the longer term rotational evolution of the Sun (Charbonneau & MacGregor 1993; Gilman 2000). Chemical transport across the tachocline also has implications for solar evolution, helioseismic structural inversions, and photospheric abundance measurements (Christensen-Dalsgaard 2002; Pinsonneault 1997).

Although the simulations reported here suggest that clamshell instabilities may not be capable of sustaining a dynamo localized entirely within the tachocline, they may still contribute to poloidal field generation. Using a flux-transport dynamo model, Dikpati & Gilman (2001b) showed that including a weak $\alpha$-effect due to tachocline instabilities in addition to a stronger Babcock-Leighton source term operating near the surface helps to select an antisymmetric toroidal field parity, as suggested by solar observations. If active regions do indeed arise from the buoyant rise of toroidal flux in the tachocline (§ 1), then nonaxisymmetric structure induced by tachocline instabilities should be reflected in flux emergence patterns. Some evidence for $m = 1$ structure in the distribution of sunspots has recently been reported by Norton & Gilman (2005).

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