Little Higgs models provide us a new solution of the hierarchy problem of the SM. The problem is associated with the large difference in scales of SM and plank scale. One of the main features of these models is the existence of a vector-like top quark. The CP violating $K_L \to \pi^0 \nu \bar{\nu}$ process in the SM model is dominated by top quark penguin & box graphs. We examine this process in the Little Higgs model where the top quark sector has significant differences from the Standard Model.

PACS numbers:

I. INTRODUCTION

The Standard Model (SM) of particle physics is a extremely successful theory. Electroweak precision tests have probed SM at quantum level and have confirmed predictions of SM. The symmetry breaking sector (Higgs sector) of the SM has also been investigated by these precision measurements and it indicates the existence of weakly coupled Higgs sector and light Higgs boson whose mass is $m_H \leq 200$ GeV. This existence of weakly coupled Higgs sector creates what is known as hierarchy problem. This problem is related to the origin and radiative stability of two widely different mass scales namely the Electroweak (EW) scale and Plank scale. To solve this problem the Higgs sector has to be fine tuned from EW scale to plank scale. There have been many suggestions to avoid this problem. In one of the attractive solutions namely Supersymmetry (SUSY) the quadratic divergences in Higgs mass are canceled between fermionic and bosonic loops provided the SUSY breaking scale is near TeV. Extra dimensions theories use the geometry of higher dimensional space-time to address the problem.

An alternative approach to solve this hierarchy problem has been recently considered in generically called “Little Higgs model” \[1,2\]. The basic idea in these models is to realize Higgs boson doublets (and other scalars) as Goldstone modes in a globally SU(5) symmetric theory, spontaneously broken at a scale $f$ in the TeV range much higher than the vev of the SM Higgs ($v$) \[2,3\]. In the simplest version of the theory known as Littlest Higgs model \[2\], the effective theory at low energies involves many more particles in addition to the SM particles. Thus apart from SM spectrum there are charged heavy vector bosons ($W_H$), neutral heavy vector boson ($Z_H$), heavy photon ($A_H$), triplet of charged Higgs ($\Phi^{++}, \Phi^+, \Phi^0$) and a heavy top quark ($T$). The masses of these heavy particles are expected in the TeV region \[2,5\]. All these particles are expected to provide $O(v^2/f^2)$ corrections to all Flavor Changing Neutral Current (FCNC) amplitudes which are generated through loops. In addition because of mixing of the SM t-quark and its heavier counterpart $T$, we expect $O(v^2/f^2)$ violation of the CKM unitarity relation. A host of processes \[2,4,5,6,7\] have been evaluated in this model providing constraints on the vast parameter space of the LH, which will be useful in experimental search for the validity of the model.

The CP violating FCNC process $K_L \to \pi^0 \nu \bar{\nu}$ with an expected SM branching ratio of $\sim 10^{-10}$ is another process which is of special interest from theoretical viewpoint. The importance of this process in SM is because of two fold reasons firstly its proceeds through direct CP violation and secondly is totally dominated by short distance top-quark loops and charm quark plays no role in it \[3\]. Because of these reasons this process is believed to be the most ideal
one for extracting out the CP violating Wolfenstein parameter \( \eta \). Further, in view of the large mass of the top quark, QCD corrections are small and calculable in perturbation theory with the result that one expected the basic SM graphs to reproduce the amplitude quite accurately \cite{8}. The article by Buchalla & Buras \cite{9} reviews and updates the SM predictions for this process with a more complete set of references. A recent review by Isidori \cite{10} covers the same ground with a summary of new physics possibilities. This process has been extensively studied in literature for finding out signatures in new physics \cite{3,11,12,13,14}.

The LH models has substantial modification to top-quark loops, both in terms of Unitarity relation violation and extra loops arising out of replacing \( t \) by \( T \)-quark \cite{4,6}. As its known that FCNC processes vanishes if we have a unitarity of CKM and complete horizontal symmetry of the masses of the quarks i.e., masses of all the quarks are same. In SM although CKM is unitary but we the large t-quark mass breaks the horizontal symmetry of quark masses which results in low rates of FCNC processes. Many of these low energy FCNC processes crucially depends on the top quark mass. Another theoretically very important process which depends on top quark mass in \( K_L \rightarrow \pi^0 \nu \bar{\nu} \) because of the reasons mentioned above. Experimentally, at 90\% CL, we have an indirect limit for the Branching ratio for the process \( Br(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 1.7 \times 10^{-9} \) \cite{10} which in principle can be achieved at SM level in the future dedicated experiments \cite{10}. As the LH models predicts a new top quark with mass in TeV range so it is worthwhile to test the effects which LH type models have on this decay.

This paper is organized as follows : in Section II we will present the effective Hamiltonian for the process in SM. In sections III we will present the results of the corrections due to \( T \)-quark. In Section IV results of corrections due to extra scalars is given and finally in Section V we will present the results of the correction due to heavy photon \( A_H \).

In the last section VI we will conclude with numerical analysis of our results and discussion.

## II. EFFECTIVE HAMILTONIAN

The basic quark level graphs in SM responsible for \( K_L \rightarrow \pi^0 \nu \bar{\nu} \) are shown in Figures 1, 2, 3. The effective Hamiltonian for the process can be written as :

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i} \frac{\alpha}{2\pi Sin^2 \theta_W}(V^*_{td}V_{td})(\bar{s}d)\nu_A(\bar{\nu}\nu)\nu_AX(x_t) + h.c. \tag{2.1}
\]

where \( x_t = \left( \frac{m_t^2}{m_W^2} \right) \) and the function \( X \) in SM has been worked out to \( O(\alpha_s) \) in \cite{8}. We will evaluate the additional contributions to \( X(m_t) \) given by particles of LH model. Since the result will be the corrections to the SM value, we do not calculate the \( O(\alpha_s) \) corrections to this. For our calculations we have used unitary gauge, unlike the original SM calculation \cite{13} but check that the total SM contribution matches with the results given by Buchalla & Buras \cite{16}. Further we retain terms up to order \( O(v^2/f^2) \) and consequently drop all terms in any diagram which is of order higher than \( O(v^2/f^2) \). We also drop terms independent of the internal quark mass since the CKM unitarity is valid up to this order when the T-quark is also considered in LH. For the results of the calculation of the individual diagrams we drop the divergent parts as we have checked that these divergences cancel when all the diagrams are added in LH model.

We have used Feynman rules given in Han et.al., \cite{2}. Before presenting our results first we will define our convention. In our convention the indices \( i, k, l, a \) have values \( 1, 2 \). In our convention

\[
\begin{align*}
\text{Masses} & \\
& m_1 = m_t \\
m_2 = m_T \\
m_3 = m_{W_L} \\
m_4 = m_{W_H} \end{align*}
\]

The vertices’s involved in our calculations are summarized in Appendix B.

In LH model the SM function \( X(x_t) \) given in eqn. (2.1) gets additional contributions from new graphs having LH
particles. The new value of the function in LH model becomes *:

\[
X^{LH} = X^{SM}(x_t) + X_{ZL, ZH}(TW_L, TW_H, tW_L) + X_{A_H}(tW_L) + X_{ZL, H}(\Phi_t, \Phi_T)
+ X_{Box}(W_k W_l t_i \text{ except } k = l = i = 1)
\]

where \(X^{SM}\) represents the SM contribution, the other terms represents the extra contribution in LH and are defined in sections III, IV, V.

### III. CORRECTIONS DUE TO EXTRA SM TOP AND T QUARKS, W AND Z BOSONS

The Feynman diagrams corresponding to the contribution of \(t, T\) quarks, \(W_{L, H}\) and \(Z_{L, H}\) are given in Figures 1(a)(b), Fig. 2(a)(d) and Fig 2. We are not including the effects of scalars (\(\Phi\)) and heavy photon (\(A_H\)), these results we will present in next section. In eqn (2.2) this contribution is represented by term \(X_{ZL, ZH}(TW_L, TW_H, tW_L)\).

We rewrite this to form:

\[
X_{ZL, ZH}(TW_L, TW_H, tW_L) = X_Z(W(t) + X_t(t) + X^{SE}(t))
\]

where \(X^W\) and \(X^t\) are the contributions of the penguin diagrams where the neutral boson (\(Z_{L, H}\)) is emitted from charged vector boson (\(W_{L, H}\)) and top quark respectively whereas \(X^{SE}\) indicates the contribution of the self energy diagram.

![Feynman Diagrams](image.png)

**FIG. 1: Penguin diagrams**

The contribution of the diagram in Fig 1(b) where \(Z\) is emitted from charge vector boson is:

\[
X^W(t) = \sum_{i,k,l,a} \left\{ \frac{8\pi^2M_Z^2\cos^2\theta_W}{M_Z^2} g_{tsW}(i, k) g_{tdW}(i, l) g_{WWZ}(k, l, a) g_{\nu\nu Z}(a) \right\} \left[ 3m_i^2 F(m_i, M_k, M_l) - \frac{3}{4} \frac{m_i^2(M_i^2 + M_l^2)}{M_k^2 M_l^2} \left( F(M_k, M_l) + m_i^2 F(m_i, M_k, M_l) \right) + \frac{1}{128\pi^2} \frac{m_i^2(M_i^2 + M_l^2)}{M_k^2 M_l^2} \right]
\]

* in writing this we have only retained terms of order \(O(v^2/f^2)\)
where the functions $F$'s are defined in the appendix A.

Contribution of the diagram Fig. 1(a) where $Z$ is emitted from top quark line is:

$$X^t(t) = - \sum_{i,j,k,a} \left\{ \frac{8\pi^2 M_Z^2 \cos^2 \theta_w}{M_Z^2} g_{\ell W}^l(j,k) g_{\nu W} (i,k) g_{\nu \nu Z} (a) \right\}$$

$$\left[ g_{t Z}^l(i,j,k) \left\{ F(m_i, m_j, M_k) \left( M_k^2 + \frac{m_i^2 m_j^2}{M_k^2} - (m_i^2 + m_j^2) \right) + F(m_i, m_j) \left( 1 - \frac{(m_i^2 + m_j^2)}{M_k^2} \right) \right\} + g_{t Z}^R(i, j, k)m_i m_j \left\{ - \frac{3}{2} F(m_i, m_j, M_k) + \frac{1}{2 M_k^2} F(m_i, m_j) - \frac{1}{4 M_k^2 16 \pi^2} \right\} \right]$$

$$X^{t}(t) = - \sum_{i,j,k,a} \left\{ \frac{8\pi^2 M_Z^2 \cos^2 \theta_w}{M_Z^2} g_{\ell W}^l(j,k) g_{\nu W} (i,k) g_{\nu \nu Z} (a) \right\}$$

$$\left[ g_{t Z}^l(i,j,k) \left\{ F(m_i, m_j, M_k) \left( M_k^2 + \frac{m_i^2 m_j^2}{M_k^2} - (m_i^2 + m_j^2) \right) + F(m_i, m_j) \left( 1 - \frac{(m_i^2 + m_j^2)}{M_k^2} \right) \right\} + g_{t Z}^R(i, j, k)m_i m_j \left\{ - \frac{3}{2} F(m_i, m_j, M_k) + \frac{1}{2 M_k^2} F(m_i, m_j) - \frac{1}{4 M_k^2 16 \pi^2} \right\} \right]$$

The contribution of the self energy diagram given in Figure 3(a) is:

$$X^{SE}(t) = - \sum_{i,k,a} \left\{ \frac{8\pi^2 M_Z^2 \cos^2 \theta_w}{M_Z^2} g_{\ell W}^l(i,k) g_{\nu \nu Z} (a) g_{\nu \nu Z}^L (a) \right\}$$

$$\left[ - \frac{1}{16 \pi^2} \frac{m_k^2}{12 M_k^2} + \int_0^1 dx (-2(1-x)) F_1(C_{i,k}) + \int_0^1 dx \left\{ 2x + \frac{(1-x)}{2} \right\} \frac{F(C_{i,k})}{M_k^2} \right]$$

FIG. 2: Box diagram.

FIG. 3: Self energy
where $x$ is the Feynman parameter and $C_{ik}$ is the function of Feynman parameter $x$ and masses $m_i$ and $M_k$ as given in eqn (A6). The contribution of Box diagram in Figure 2 can be written as:

$$X_{Box} = \sum_{i,k,l} \left\{ -4\pi^2 M_Z^2 \cos^2 \theta_w g_{\gamma \gamma} (i,k) g_{\text{t} \text{d} \text{w}} (i,l) g_{\nu \nu W} (i,l) \right\} \left[ 4F(m_i, M_k, M_l) + \frac{m_i^2}{4 M_k^2 M_l^2} F(M_k, M_l) \right] \left[ \begin{array}{c} m_i^2 \left( \frac{1}{M_k^2} + \frac{1}{M_l^2} \right) F(m_i, M_k, M_l) + \frac{1}{16\pi^2} \frac{m_i^2}{8 M_k^2 M_l^2} \end{array} \right]$$

(3.5)

In addition to above calculated diagrams there should be another self energy diagram represented by Figure 3(d). But this is proportional to $p_d$ is $p_d$ is the external momenta of $d$ quark and hence this contribution vanishes when $m_d \to 0$.

IV. SCALAR BOSON CONTRIBUTIONS

Little Higgs models also have doubly charged Higgs scalars but they can’t couple to SM fermions so they won’t give any contributions to $X^{LH}$ but the singly charged Higgs ($\Phi^\pm$) can give contributions in a manner similar to the contributions given by charged vector bosons ($W_L^\pm, H$). The relevant Feynman diagrams for the contribution of the charged scalars are given in Figures 1(c)(d) and 2(b). There won’t be any extra box diagram contribution in LH due to charged Higgs if neutrinos are taken to be massless.

We can write down the charged scalar contribution given in eqn (2.2) as:

$$X_{Z_L,H} (\Phi t, \Phi T) = X^\Phi (\Phi) + X^\nu (\Phi) + X^{SE} (\Phi)$$

(4.1)

where $X^\Phi (\Phi)$ and $X^\nu (\Phi)$ are the contributions of charged Higgs penguins where $Z$ is emitted from charged Higgs line and top quark line respectively. $X^{SE}$ is the self energy contribution.

$$X^\Phi (\Phi) = \sum_{i,a} \left\{ \frac{8\pi^2 M_Z^2 c_w^2}{g^2} \left( a - 2s_w^2 \right) \frac{g_{\nu \nu Z}(a)}{2M_Z^2} \right\} \left[ F_1 (m_h) + m_i^2 F(m_i, m_h, m_h) + \frac{1}{32\pi^2} \right]$$

(4.2)

where $m_h$ is the mass of the charged Higgs Boson and $\mu$ is the $M_W$ mass scale.

$$X^\nu (\Phi) = \sum_{i,j,a} \left\{ \frac{8\pi^2 c_w^2 M_Z^2}{g^2} \frac{m_i^2 v^2}{v_F^2 M_Z^2} \frac{1}{g_{\nu \nu Z}(a)} \right\} \left[ g_{\nu \nu Z}(i,j,a) m_i m_j F(m_i, m_j, m_h) - \frac{g_{\nu \nu Z}(i,j,a)}{2} \right]$$

(4.3)

$$X^{SE} (\Phi) = \sum_{i,a} \left\{ \frac{8\pi^2 c_w^2 M_Z^2}{g^2} \frac{m_i^2 v^2}{v_F^2 M_Z^2} \frac{1}{g_{\nu \nu Z}(a)} \right\} \left[ \int_0^1 dx \frac{1}{16\pi^2} (1-x) \log \left( \frac{m_i^2 x + \mu^2}{m_h^2 (1-x)} \right) \right]$$

(4.4)

V. HEAVY PHOTON CONTRIBUTION

For heavy photon ($A_H$), we don’t have to consider the Higgs diagrams because the Higgs couplings is already of order $O(v/f)$, as the mass of $A_H$ will come in the denominator so these diagrams would be higher order diagrams in $f/v$.

$^\dagger$ mass of Heavy Photon is of order $f/v$
Littlest Higgs model (v²/f²). As the WWĀH coupling is \( O(v²/f²) \) so the penguin diagrams where A_H is emitted from W± would also be higher order diagram and hence need not be considered. The coupling of the new vector type top quark (T) to W± is proportional to v/f and hence won’t contribute to A_H diagrams upto \( O(v²/f²) \). There won’t be any contribution from W_H also because in case of W_H diagrams the mass of heavy W will come in denominator and hence makes the diagram to be of higher order in (v/f).

Contribution of heavy photon diagrams can be written as:

\[
X_{A_H}(tW_L) = X^{SE}_{A_H} + X^{peng}_{A_H}
\]

\[X^{peng}_{A_H} = \left\{ \begin{array}{l}
\left( \frac{g^2}{f^2} \right) M_{A_H}^2 \left( \frac{1}{f^2} + \frac{c^2}{2} \right) \int_0^1 dx \left[ -2(1-x)X_1 \right]
\end{array} \right. \]

\[X^{SE}_{A_H} = \left\{ \begin{array}{l}
\left( \frac{g^2}{f^2} \right) M_{A_H}^2 \left( \frac{1}{f^2} + \frac{c^2}{2} \right) \int_0^1 dx \left[ -2(1-x)X_1 \right]
\end{array} \right. \]

where in above expressions \( m = m_t \) (mass of SM top quark) and \( M = M_{W_L} \) mass of SM W-boson and \( C^2 = (x m^2 + (1-x) M^2) \) and \( M_{A_H} \) is the mass of heavy photon.

VI. NUMERICAL ANALYSIS AND DISCUSSION

The Littlest Higgs model has a large spectrum of heavy particles other than SM particles. In case of LH model there is a global SU(5) symmetry which is broken at TeV range \( \Lambda_s = 4\pi f \) and in the process the scalar, doublet and triplet acquire vev’s \( v \) and \( v' \). In our work we have used a model which has considered model known as Littlest Higgs model where we have only a single light Higgs doublet but there are many variations of this model possible which can extend this and have possibility of two light Higgs doublets [17]. Some of the universal features of all Little Higgs models which are useful in phenomenology are:

- Existence of heavy gauge bosons like \( W_H^\pm \) and \( Z_H \) which are required to cancel W and Z loops.
- A new heavy fermion which is required to cancel the SM top quark divergence.
- Set of heavy scalars. This sector is heavily model dependent and some variations of LH may have many singlets, doublets and triplets.

The Littlest Higgs model have following input parameters above the SM one which can be parametrized as [2, 3]:

1. \( \tan \theta = s/c = g_1/g_2 \) ratio of new SU(2) coupling constants.
2. \( \tan \theta' = s/c = g_1'/g_2' \) ratio of new U(1) couplings.
3. \( f \) : scale at which SU(5) global symmetry is broken.
4. \( v' \) : vev of triplet Higgs. Triplet Higgs vev has an upper bound given by \( v' \leq \frac{\pi^2}{4f} \) where \( v \) is the SM Higgs vev.
5. \( m_H \) : mass of SM Higgs boson.
6. \( M_T \) : mass of the new vector type top quark.
$M_T$ and $m_t$ (mass of SM top) together fixes two Yukawa couplings $\lambda_1$ and $\lambda_2$.

For our numerical analysis we will be going to use $s, s', v/f, v'/f, m_H$ as input parameters. Regarding $M_T$ we will be going to use another combination, which is the mixing parameter of SM top quark and heavy vector like T quark, defined as $x_L = \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}$ as the input parameter.

Results of our numerical analysis are summarized in Figures 4 and 5. In these figures we have plotted $X^{LH}$ as the function of various LH model parameters.

In Figure 4 we have plotted $X^{LH}$ as a function of $v/f$ for various values of $s$. The four different panels in the plot corresponds to four different $x_L$ values. The branching ratio of $K_L \to \pi \nu \bar{\nu}$ is proportional to square of $X^{LH}$. As we can see from the Fig 4 that the magnitude of $X^{LH}$ in some region of LH parameter space can get a enhancement of more than 100% with respect to SM value which effectively means a enhancement in branching ratio by a factor of four. We can also see from this figure that LH model predicts substantial deviation from SM results for higher $x_L$ values. The deviation from SM increases for low $s$ values. In Figure 5 we have plotted $X^{LH}$ as a function of $x_L$ for various values of $s$. Different panels corresponds to different values of $s'$. For all the graphs in Figure 5 we have chosen the value $v/f = 0.1$. As we can see from the figures that in certain region of parameter space we can get substantial enhancements in the branching ratios. This figure also emphasize that for higher $x_L$ values LH model can predict a enhancement in the branching ratio of $K_L \to \pi \nu \bar{\nu}$ more than 100%.

In this work we have confined ourselves to the Littlest Higgs model and have tried to investigate the effects extra top like quark on $K_L \to \pi \nu \bar{\nu}$. The reason for choosing this process was two fold firstly it is totally dominated by the top quark exchange in SM hence relatively free from uncertainties secondly the branching ratio of this process ($K_L \to \pi \nu \bar{\nu}$) scales with $m_t$ in SM. So in the sort of models which predict the existence of a new quark whose behavior is similar to SM top quark should effect this process the most. Heavier is the mass of extra top like quark
FIG. 5: Plot of $X^{LH}$ with $x_L$ for various values of $s$. Different panels corresponds to different value of $s'$. In above plots we have used $\frac{v_f}{f} = 0.1$

more increase it will predict to the branching ratio of this process. Generically all the variations of Little Higgs model predicts existence of a heavy top quark whose mass is in TeV range and hence our qualitative results will remain the same in those models also. Although this process is not yet observed but future dedicated experiments would be able to observe this process \cite{10} and hence would be able to put constraints of Little Higgs type models.

Its is useful to discuss the constraints on parameters of LH models imposed by various precision and decay measurements \cite{2, 5, 18, 19, 21}. In the Littlest Higgs model we don’t have SU(2) custodial symmetry \cite{2, 21}. This symmetry protects relation of W, Z masses and $\rho = 1$. The custodial SU(2) symmetry violating corrections mainly arises from the heavy U(1) gauge bosons \cite{21}. Using this philosophy many variations of little Higgs models which have approximate custodial SU(2) symmetry \cite{20} have been constructed which can relax the electroweak precision constraints on LH type models. These variations pushes the lower mass bounds on heavy W, Z bosons and new top quark. As concluded by Chang & Wacker \cite{20} that approximate custodial symmetry can actually bring down the breaking scale (scale at which global symmetry is spontaneously broken) upto 700 GeV. They also gave concluded that precision electroweak measurements can push the lower bound on heavy W and Z mass to around 2.5 TeV and mass of the top around 2 TeV. But as pointed out above that we have chosen $K_L \rightarrow \pi \nu \bar{\nu}$ precisely for the reason that it is dominated by top in SM and hence can have substantial modifications in these variations of LH which pushes the mass of new top up. Low energy precision data on $(g-2)_\mu$ of muon and atomic “weak charge” of cesium doesn’t impose any new constraint on model parameters \cite{18}.

In Littlest Higgs model which we have considered Hewett et.al. \cite{18} noted that considering precision electroweak measurement there exists a very small region of parameter space where we can lower the bound of $f$ to be around TeV. But considering Tevatron bounds also this bound on $f$ can be pushed to $f \approx 3.5$ TeV.

We finally comment on the relationship of our estimate with a recent paper by Buras et.al \cite{11} who have studied
this process in relation to the observed anomalies in the decays of B into $\pi\pi$ and $\pi K$ channels. The net conclusion of the paper was that in order to explain the $\pi\pi$, $\pi K$ anomaly one is led to an effective value of $X$ slightly higher than the SM value but more importantly with a phase of about $86^\circ$. The analysis is more or less model independent but the likely area of the phase is in the squark mass matrices. The LH model, which has been proposed as an alternative to SUSY, however results in possible change in $|X|$ but no additional phase.

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APPENDIX A: LOOP FUNCTIONS :

\begin{align}
F(x) &= \frac{x^2}{16\pi^2} \left[ 1 + \log \left( \frac{\mu^2}{x^2} \right) \right] \tag{A1} \\
F_1(x) &= \frac{1}{16\pi^2} \log \left( \frac{\mu^2}{x^2} \right) \tag{A2} \\
F(x, y) &= \frac{F(x) - F(y)}{x^2 - y^2} \tag{A3} \\
F(x, y, z) &= \frac{1}{(x^2 - y^2)(x^2 - z^2)} \left[ F(x, z) - F(y, z) \right] \tag{A4} \\
F_1(x, y) &= -\frac{[F(x) - F(y)]}{(x^2 - y^2)^2} + \frac{F_1(x)}{(x^2 - y^2)} \tag{A5} \\
C_{ik}^2 &= x m_i^2 + M_k^2 (1 - x) \tag{A6}
\end{align}

APPENDIX B: COUPLING CONSTANTS :

Various vertices which we have used in our calculations are defined in Table 1, where $y_e = -\frac{2}{5}$ and $P_{L,R} = (\frac{1+\gamma_5}{2})$. The full expression of the coupling constants can be read off from Han et.al. 2.

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| Particles                          | Vertex                                                                 |
|----------------------------------|------------------------------------------------------------------------|
| $s_t \bar{W}_k$                  | $\frac{i\gamma^\mu}{\sqrt{2}}g_{sW}(i,k)\gamma_\alpha P_L$          |
| $d_t \bar{W}_k$                  | $\frac{i\gamma^\mu}{\sqrt{2}}g_{sW}(i,k)\gamma_\alpha P_L$          |
| $\nu\bar{Z}_a$                   | $\frac{i\gamma}{\sqrt{2}}g_{\nu Z}(a)\gamma_\alpha P_L$            |
| $d_d \bar{Z}_a$                  | $ig \left( g_{dZ}^L(a)\gamma_\alpha P_L + g_{dZ}^R(a)\gamma_\alpha P_R \right)$ |
| $t_t \bar{Z}_a$                  | $ig\gamma_\alpha \left[ g_{tZ}^L(i,j,a)P_L + g_{tZ}^R(i,j,a)P_R \right]$ |
| $s_t \Phi^\pm$                   | $\frac{-i\gamma^\mu}{\sqrt{2}}\gamma P_L g_{sW}(i)$                  |
| $d_t \Phi^\pm$                   | $\frac{-i\gamma^\mu}{\sqrt{2}}\gamma P_L g_{dW}(i)$                  |
| $d_d A_H$                        | $\frac{i\gamma^\mu}{\sqrt{2}}\gamma_\alpha \left[ g_{dA}^L P_L + g_{dA}^R P_R \right]$ |
| $t_t A_H$                        | $\frac{i\gamma^\mu}{\sqrt{2}}\gamma_\alpha \left[ g_{tA}^L P_L + g_{tA}^R P_R \right]$ |
| $\nu \bar{A}_H$                  | $\frac{i\gamma^\mu}{\sqrt{2}}\gamma_\alpha \left( y_e - \frac{2}{3} + \frac{1}{2}c^2 \right) P_L$ |
| $W_k(k)W_l(k)Z_a$                | $-igg_{WWZ}(k,l,a)\left[ k_3 g_{\chi \phi} - 2k_5 g_{\lambda \phi} + k_6 g_{\lambda \chi} \right]$ |
| $\Phi_k(k)\Phi_l(k)Z_a$          | $\frac{i\gamma^\mu}{\sqrt{2}}(a - 2s^3_i)k_5 g_{\chi \phi}(a). \quad a = 0 \text{ for LH.}$ |

TABLE I:

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