Pulsar Double Lensing Sheds Light on the Origin of Extreme Scattering Events

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Abstract

In extreme scattering events, the brightness of a compact radio source drops significantly, as light is refracted out of the line of sight by foreground plasma lenses. Despite recent efforts, the nature of these lenses has remained a puzzle, because any roughly round lens would be so highly overpressurized relative to the interstellar medium that it could only exist for about a year. This, combined with a lack of constraints on distances and velocities, has led to a plethora of theoretical models. We present observations of a dramatic double-lensing event in pulsar PSR B0834+06 and use a novel phase-retrieval technique to show that the data can be reproduced remarkably well with a two-screen model: one screen with many small lenses and another with a single, strong one. We further show that the latter lens is so strong that it would inevitably cause extreme scattering events. Our observations show that the lens moves slowly and is highly elongated on the sky. If similarly elongated along the line of sight, as would arise naturally from a sheet of plasma viewed nearly edge-on, no large overpressure is required and hence the lens could be long-lived.

Unified Astronomy Thesaurus concepts: Interstellar medium (847); Radio pulsars (1353); Interstellar scattering (854); Radio astronomy (1338); Radio spectroscopy (1359); Warm ionized medium (1788)

1. Introduction

Extreme scattering event (ESE) propagation-produced variations in quasar flux density have been a puzzle since their discovery in 1987 (Fiedler et al. 1987). ESEs manifest as frequency-dependent changes in the observed flux of quasars, usually a sharp spike followed by a dip, for a period of several weeks to months.

It is now widely agreed that ESEs cannot be explained by intrinsic variations of the source (Fiedler et al. 1994). Instead, refraction effects from a dense plasma structure in the interstellar medium (ISM), hereafter referred to as a plasma lens, with a length scale of a few astronomical units can explain both the observed flux curve as well as the duration of such events (Clegg et al. 1998).

Yet one difficulty remains: the required electron density and temperature for a roughly round plasma lens implies an overpressure compared to the diffusive ISM by a factor of \(10^{3}\) (Walker & Wardle 1998; Gaensler & Slane 2006). Such a high pressure indicates that the plasma lens would evaporate on the timescale of 1 yr. This, combined with a lack of constraints on distances and velocities, has led to a plethora of theoretical models (Dong et al. 2018).

It was realized early on that pulsars might be powerful probes of these lenses; pulsars scan the sky quickly and, because of their compact sizes, scintillate due to multipath scattering in the ISM, yielding many new observables (Cordes & Wolszczan 1986; Goodman et al. 1987; Cognard et al. 1993; Rickett et al. 1997). In this paper, we present a multiepoch observation of a double-lensing event in pulsar PSR B0834+06. We use a novel phase-retrieval technique to show that the data can be reproduced remarkably well with a two-screen scattering model: one screen with many small lenses and another with a single, strong one (Liu et al. 2016). Then, we measure the magnification, size, and velocity of the latter lens, and show that it would inevitably cause ESEs if it passed by the line of sight to a quasar.

The paper is organized as follows: In Section 2 we summarize our observations and review the theory of pulsar scintillation. We describe the phase-retrieval technique we adopted in Section 3. The double-lensing model and its agreement with the data are presented in Section 4. In Sections 5 and 6, we extract parameters of the lens and demonstrate that it is capable of causing ESEs. Finally, we discuss our results and conclude in Section 7.

2. Observation and Pulsar Scintillation

From 2005 October to December, we took seven weekly observations of pulsar PSR B0834+06 at 318–319 MHz with the 305 m William E. Gordon Telescope at the Arecibo Observatory. At each session, we took 45 minutes of data using the 327 MHz receiver with a bandwidth of 0.78 MHz centered at 318.5 MHz. We created power spectra with 2048 frequency channels, summing the two circular polarizations. The spectra were then integrated according to the pulsar rotational phase with an integration time of 10 s (i.e., averaging over roughly eight pulses), and the resulting power as a function of frequency, time (10 s chunks), and pulsar rotational phase written out.

We then created dynamic spectra—power as a function of frequency and time—by subtracting the background, off-pulse spectrum from the integrated on-pulse signal. We further divided each time bin of the dynamic spectra by its mean over frequency to mitigate pulse-to-pulse variability of the pulsar. The spectra show the rich scintillation structures characteristic of scattering in the interstellar medium (Figure 1).
To highlight this structure we created conjugate spectra; the Fourier transforms of dynamic spectra, which are functions of Doppler frequency and differential delay; and the Fourier conjugates of time and frequency, respectively (Cordes et al. 2006). We show the modulus of four of our conjugate spectra in the top row of Figure 2. The power in each is concentrated in a broad parabola, called a scintillation arc, which consists of upside-down parabolas called inverted arclets.

At a delay of about 1 ms, there is an island of inverted arclets in the conjugate spectrum that migrates consistently down and to the right during the 7 weeks. This “1 ms feature” was first reported by Briskey et al. (2010) in a single-epoch VLBI observation made in the middle of our set of observations, and follow-up analysis by Liu et al. (2016) suggested it might arise from double lensing of the pulsar. As will become clear below, we find that the conjugate spectra can be modeled in detail using a double-lensing interpretation, and that the feature arises from a surprisingly strong lens.

The main scintillation arc can be understood from considering pairs of scattered images of the pulsar. In terms of their (complex) magnifications $\mu_{j,k}$ and angular offsets $\theta_{j,k}$ from the line of sight, the relative Doppler frequency $f_D$, geometric delay $\tau$, and brightness $I$ of a given pair $j,k$ are given by

$$\tau = f_\nu = \frac{d_{\text{eff}}}{2c} |(\theta_j|^2 - |\theta_k|^2),$$

$$f_D = f_i = \frac{1}{\lambda} (\theta_j - \theta_k) \cdot \nu_{\text{eff}},$$

$$I = |C(\tau, f_D)| = |\mu_j \mu_k^*|,$$

where $d_{\text{eff}}$ and $\nu_{\text{eff}}$ are the effective distance and velocity of the pulsar–screen–Earth system, $C$ is the conjugate spectrum, $c$ is the speed of light, and $\lambda$ is the observation wavelength (see Appendix A for details).

Generally, the magnifications are largest near the line of sight, and thus the brightest signals arise when one member of the pair has $|\theta| \approx 0$. Considering this, one infers $\tau \propto f_D^2$, thus reproducing the main parabola, as long as $d_{\text{eff}}$ and $\nu_{\text{eff}}$ are roughly constant, which would happen in a thin-screen scattering geometry with scattering localized along the line of sight (Stinebring et al. 2001; Cordes et al. 2006).

The inverted arclets arise from mutual interference between the scattered images, and their sharpness is a signature of highly anisotropic scattering, where the scattered images of the pulsar lie along a (nearly) straight line (Walker et al. 2004). Like in previous observations, we find that the arclets move along the scintillation arc at a constant speed for long periods of time (Hill et al. 2005; Marthi et al. 2020). This implies that the scattered images giving rise to the arclets must arise from a large group of parallel and elongated structures in the scattering screen, e.g., turbulence elongated along a given direction (Goldreich & Sridhar 1995), waves on a plasma sheet seen in projection as folds (Romani et al. 1987; Pen & Levin 2014), or magnetic needles of plasma stabilized by reconnection (Gwinn 2019).

### 3. Phase Retrieval

Scattering by the interstellar medium can be well described as a linear filter. Hence, the observed signal is the convolution between the impulse response function of the ISM and the intrinsic pulsar signal (Walker et al. 2008; Walker & Stinebring 2005; Walker et al. 2013; Pen et al. 2014). However, because pulsar emission is like amplitude-modulated noise, for slow pulsars whose pulse width is longer than the scattering time, the observed signal contains no useful phase information. Instead, via the dynamic spectrum one only has a measurement of the squared modulus of the impulse response function.

In general, retrieving the phases from just the amplitudes is an ill-posed problem. However, when the scattering is highly anisotropic, phase retrieval becomes possible. The method we use is described in detail in Baker et al. (2022), but, briefly, it relies on two realizations. First, for highly anisotropic scattering, the vector offsets $\theta_{j,k}$ in Equations (1) and (2) become effectively one-dimensional and for any given $\eta$ in $\tau = nf_D^2$ it becomes possible to remap the conjugate spectrum $C(f_D, \tau)$ to $C(\theta_j, \theta_k)$. For the correct quadratic constant of proportionality $\eta$, which is also the curvature of the scintillation arc, one then finds that the main arc and the arclets are aligned with the cardinal directions in $\theta-\theta$ space (Sprenger et al. 2021).

Second, if aligned, $C(\theta_j, \theta_k)$ can be factorized using eigenvector decomposition, and the largest eigenvector will be an estimate of the impulse response function (Baker et al. 2022).

In our determinations of the wavefields from each of our dynamic spectra, we follow the procedure of Baker et al. (2022) in detail. In particular, we reduce the frequency resolution of the dynamic spectra in order to avoid including information from the 1 ms feature (for which the assumption of a single one-dimensional screen does not hold). We then map the resulting impulse response function estimates back to estimates of the dynamic wavefield, interpolating to the original resolution, and replace the estimated amplitudes with those given by the observed dynamic spectrum. Finally, Fourier transforms of the estimated dynamic wavefields yield the wavefields in the frequency domain, presented in the second row of Figure 2.

We see that in Figure 2 all arclets are reduced to single points in the wavefields, with most lying along the parabola from the main screen, and a few in the 1 ms feature. The power in the 1 ms feature concentrates along a blizzard shape that resembles a portion of a parabola, offset from the origin. We will show that such structure is exactly as predicted by the double-lensing geometry.
4. Double-lensing Model

With the wavefield, it becomes possible to test the double-lensing model directly. We construct a simple but detailed model, illustrated in Figure 3, in which we represent individual lenses as linear structures that can bend light only perpendicular to their extent. We assume two lensing planes, with velocities,
Table 1
Lensing Parameters of Pulsar-Screen-Earth System.

| Parameter | Value |
|-----------|-------|
| \(d_{\text{psr}}\) | 620 ± 60 pc |
| \(v_\alpha\) | 2.2 ± 0.2 mas yr\(^{-1}\) |
| \(v_\delta\) | 51.6 ± 0.1 mas yr\(^{-1}\) |
| \(d_1\) | 389 ± 5 pc |
| \(d_2\) | 415 ± 11 pc |
| \(v_{\theta 1}\) | −23 ± 1 km s\(^{-1}\) |
| \(v_{\theta 2}\) | −3 ± 3 km s\(^{-1}\) |
| \(\alpha_1\) | 155° ± 1° |
| \(\alpha_2\) | 136° ± 1° |

**Note.** The pulsar distance \(d_{\text{psr}}\) and angular velocity towards East and North \((\mu_\alpha, \mu_\delta)\) are measured from VLBI Observations. The distances to the main scattering screen \(d_1\) and that of the 1 ms lens \(d_2\) are calculated assuming the pulsar distance and velocity, and their uncertainties reflect only the uncertainty in the relative distance and velocity. For the velocities, we can only constrain the component parallel to the images (i.e., along the direction normal to the linear lenses). The position angles \(\alpha_1\) and \(\alpha_2\) are between the lines of images and north (through east). The central value of \(\alpha_2\) was adjusted slightly (within the uncertainty) from that given in Liu et al. (2016) to best reproduce the wavefields presented in Figure 2.

distances, and orientations of the linear lenses taken from Liu et al. (2016; see Table 1). Next, for each epoch we use the local maxima along the scintillation arc in the wavefields to determine the location of the linear lenses on the main lensing screen. Given those locations, the doubly refracted rays can then be solved and the corresponding relative Doppler frequency and delay calculated (see Appendix B).

In Figure 2, we show with faint blue dots the points on the main arc that we use to define linear lenses on the main scattering screen, and with red dots the corresponding double-lensed rays inferred from our model (blue and red rays in the model in Figure 3, respectively). As can be seen in Figure 2, the model perfectly reproduces the observed 1 ms feature, including its evolution over a period of 50 days, indicating the 1 ms feature indeed arises from double refraction by highly anisotropic lenses.

As noted above, the images making up the 1 ms feature form part of a parabola offset from the origin. That the parabola is incomplete implies that not all lenses in the main screen participate in the double refraction, confirming the conclusion of Liu et al. (2016) that the lens that causes the 1 ms feature terminates. The required geometry is illustrated in the right panel of Figure 3: the two orange lenses at the bottom do not contribute double-lensed rays (red, dashed lines) because of the termination of the 1 ms (red) lens. As time progresses, not only the overall delay decreases while the pulsar moves toward the 1 ms lens, but also more and more of the bottom of the parabola appears. This entire evolution of the 1 ms feature is captured by our simple double-lensing model, as shown in the bottom row of Figure 2.

5. Properties of the 1 ms Lens

5.1. Velocity and Aspect Ratio

From the VLBI observations of Brisken et al. (2010), Liu et al. (2016) inferred a low velocity of both lenses. We can confirm this from our wavefield spectra, as those spectra are essentially holographic images of the pulsar in delay-Doppler-shift space. This means that for a single screen, given knowledge of the velocities and distances of the pulsar, screen, and Earth, the wavefield can be mapped to the lensed image of the pulsar on the sky, with only a reflection ambiguity around the direction of motion. One generally cannot map a second screen using the same procedure, but in our case the two scattering planes are known to be much closer to each other than they are to Earth or the pulsar (see Table 1), and hence the holographic mapping still yields a reasonable approximation also for the 1 ms feature.

The construction of the holographic images makes use of the fact that for the wavefield one has (see Appendix A)

\[
\tau = \frac{d_{\text{eff}}}{2c} |\theta|^2, \quad (4)
\]

\[
f_D = \frac{\theta \cdot v_{\text{eff}}}{\lambda}. \quad (5)
\]

Thus, geometrically, the Doppler frequency \(f_D\) constrains the scattered image to a line on the sky perpendicular to the effective velocity, whereas the differential delay \(\tau\) constrains it to a circle centered on the line-of-sight image. Since the line intersects with the circle twice, at two points symmetric around the direction of the effective velocity, the mapping has a two-fold ambiguity. In our data set, this ambiguity is resolved by the previous VLBI observation of Brisken et al. (2010).\(^9\)

To produce the scattered images of the pulsar, we first rebinned our wavefield power spectra by a factor of 4 in delay to increase the signal-to-noise ratio. Then, for each delay, we subtracted the noise floor and calculated average fluxes and Doppler frequencies inside masked regions around the main parabola (on both positive and negative sides) and around the 1 ms feature, and then converted delay and Doppler frequency to angles along and perpendicular to the direction of \(v_{\text{eff}}\) using the above equations.

We present the resulting approximate pulsar images of the first and last epochs in Figure 4. The scattered images mostly lie along a straight line, but the 1 ms feature is mapped onto a line segment in a different direction. While both structures are linear, the mechanisms for producing them differ. The main linear group of images arises from singly scattered rays by a parallel set of linear lenses. Each linear lens creates an image at a location closest to the line of sight, so the images move with the pulsar, and the lenses are extended perpendicular to this linear group of images (see Figure 3). On the other hand, the 1 ms feature is created by double lensing, in which rays are partially bent by the 1 ms lens and then further bent toward the observer by lenses on the main screen. As seen from Earth, these images can move along the 1 ms lens as the pulsar moves, but not perpendicular to it. Without the main scattering screen, the 1 ms lens would only be capable of producing a single scattered image (an image that we do not see because the 1 ms lens terminates, although it should have appeared shortly after our campaign, when the pulsar crossed the termination point).

Since the multiple images we see from the double lensing trace out part of the 1 ms lens, they reveal directly that it has a high aspect ratio. The lack of movement between the first and

\(^9\) A sign error in the analysis of Brisken et al. (2010) caused their VLBI images to be flipped along both axes. As a result, the 1 ms feature was mapped to the south of the pulsar, which is inconsistent with its delay decreasing with time. This sign error is important here, but does not influence the discussion in Brisken et al. (2010).
6. Interpretation and Discussion

We argue that the parameters we measured for the 1 ms lens imply that it would cause ESEs if it passed in front of a quasar. First, for any lens to cause a significant drop in flux, it must deflect a radio source by more than half of its angular width. The 1 ms lens, given the largest impact parameter $δβ$ seen in our observations, is able to deflect radiation by at least 83 mas at the observation frequency 318 MHz, where the deflection angle is given by $βδβ/s$, $s$ being the fractional distance of the scattering screen defined in Appendix A. Given that the bending angle scales as the square of the wavelength, it can thus bend light by at least half its width up to 3.4-GHz, covering the range in frequencies where extreme scattering events are observed. Second, given its low velocity, the crossing time would be roughly $w/v_0 ≈ 40$ days (where $v_0 = 30 \text{ km s}^{-1}$ is the orbital velocity of the Earth), in agreement with observed ESEs.

The duty cycle of ESEs is about 0.007 (Fiedler et al. 1987), which means that if lenses like the 1 ms lens are responsible, they cannot be rare: their typical separation, assuming a high aspect ratio, would be $ω/0.007 ≈ 200$ mas. For PSR B0834+06, given its proper motion of 50 mas yr$^{-1}$, one would expect it to cross a similar lens roughly every 4 yr. Of course, many such crossings would be missed, but we note that an event that is (in hindsight) similar to ours that occurred in the 1980s (Rickett et al. 1997).

One may wonder whether some of the more distant lenses on the main screen would also be capable of causing ESEs, since they deflect pulsar radiation by similar angles and some are at least as bright as the 1 ms feature in the holographically reconstructed images shown in Figure 4. However, for the 1 ms lens, the observed brightness of images is much lower than the magnification because rays are refracted twice, while for the lenses on the main screen the brightness is a direct measure of their magnifications. For those with a large bending angle, we find $μ < 0.1\%$, and hence infer angular widths below 0.03 mas. Such widths are small compared with the angular widths of quasars (Gurvits et al. 1999; Koay et al. 2018), and hence these lenses cannot cause the significant dimming seen in ESEs.

If the 1 ms lens is typical of structures causing ESEs, it excludes a number of models. In particular, the low velocity eliminates any models that demand the velocity to be orders of magnitude higher, like those that appeal to structures in the Galactic halo (Fiedler et al. 1987). Furthermore, the high aspect ratio undermines isotropic models like large clouds of self-gravitating gas (Walker & Wardle 1998).

Indeed, the high aspect ratio may help solve the largest conundrum in ESEs, which is that simple estimates, based on spherical symmetry, give a very high electron density, of $∼10^4 \text{ cm}^{-3}$, which implies an overpressure by 3 orders of magnitude compared to the general ISM (Fiedler et al. 1987; Clegg et al. 1998; Walker & Wardle 1998). For the 1 ms lens, the observed largest bending angle $α ≈ 83$ mas implies a gradient of the electron column density (Clegg et al. 1998)

$$\frac{dN_e}{dx} = \frac{2πα}{Xr_e} ≈ 900 \text{ cm}^{-2} \text{ cm}^{-1}. \quad (7)$$

where $r_e$ is the classical electron radius. Thus, under spherical symmetry, one would infer an electron density similar to the problematic ones mentioned above (Bannister et al. 2016). Given that the lens is elongated on the sky, however, it may well be elongated along the line of sight too, i.e., be sheet-like. If so,
the inferred electron density within the lens decreases by the elongation factor, and the overpressure problem can be avoided if the lens is elongated by about a factor of 10\(^3\).

The above supports the idea that lensing could arise from plasma sheets viewed under grazing incidence. Both overdense (Romani et al. 1987) and underdense (Pen & King 2012) sheets have been suggested, and those could be distinguished based on how their behavior scales with frequency (Simard & Pen 2018). Unfortunately, for this purpose, our bandwidth of 1 MHz is too limited, but we believe a reanalysis of the VLBI data on this event will likely yield an answer (D. Baker et al. 2023, in preparation).

A different test of whether ESEs are caused by structures like our 1 ms lens can be made using quasi-flux monitoring surveys: if a lens is highly anisotropic, Earth’s orbital motion would cause the lens to be traversed multiple times. For lenses stationary relative to the local standard of rest, we perform a simulation and find that the likelihood for repeats approaches 1.

7. Conclusions

In conclusion, we report a strong, slowly moving and highly anisotropic lens from pulsar scintillation observations. The observation used only 6 hr of telescope time yet revealed detailed and novel plasma structures in the interstellar medium. Lenses with properties similar to the 1 ms lens cannot be rare in the galaxy. Extended pulsar and quasar monitoring with the next-generation survey telescopes like CHIME will further ascertain the statistics of these scatterers, while detailed studies with large telescopes like FAST promise to determine their physical properties. This will have benefits beyond understanding the lensing properties, since refraction by discrete and dense structures in the ISM could pose significant challenges for the detection of gravitational waves with a pulsar timing array (Cognard et al. 1993; Coles et al. 2015). Deeper understanding will help develop mitigation strategies, which can increase the sensitivity of pulsar timing arrays (Burke-Spolaor et al. 2019).

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Code Availability

The code used for phase retrieval and generating the wavefields is integrated into the scintools package developed by Reardon et al. (2020), available at https://github.com/danielreardon/scintools. The ray-tracing code used for modeling the double-lensing geometry is part of the screens package developed by van Kerkwijk & van Lieshout (2022) at https://github.com/mhvk/screens.

Software: astropy (Astropy Collaboration et al. 2013, 2018, 2022), numpy (Harris et al. 2020), matplotlib (Hunter 2007), screens (Van Kerkwijk & van Lieshout 2022).

Appendix A

Theory of Pulsar Scintillation

Observations suggests that a significant amount of pulsar scattering is dominated by thin screens (for a detailed account, see Stinebring et al. 2001). We consider a pulsar behind a single scattering screen, with distances \(d_{\text{psr}}\) and \(d_{\text{sc}}\) from the observer, respectively.

The geometric delays for rays that pass through the screen are then the same as for the case that the source is at infinity and the screen is at what is known as the “effective distance,”

\[
    d_{\text{eff}} = \frac{d_{\text{psr}}d_{\text{sc}}}{d_{\text{psr}} - d_{\text{sc}}} = d_{\text{psr}} \left(1 - \frac{s}{s}\right), \quad (A1)
\]

where \(s = 1 - d_{\text{psr}}/d_{\text{sc}}\) is the fractional distance from the pulsar to the screen. For two paths at angles \(\theta_j\) and \(\theta_k\) between the observer and a screen at \(d_{\text{eff}}\), one then recovers the differential geometric delay:

\[
    \tau = f_\nu (\theta_j^2 - \theta_k^2). \quad (A2)
\]

Because of the different delays along different paths, the pulsar’s radiation interferes with itself and casts a diffraction pattern onto the observer plane. The velocity of this diffraction pattern with respect to the observer is called the effective velocity, which determines the differential Doppler shifts:

\[
    f_D = f_j = \frac{1}{\lambda} (\theta_j - \theta_k) \cdot v_{\text{eff}}, \quad (A3)
\]

where the effective velocity is given by

\[
    v_{\text{eff}} = -\frac{1 - s}{s} v_{\text{psr}} + \frac{1}{s} v_{\text{sc}} - v_\oplus, \quad (A4)
\]

and \(v_{\text{psr}}, v_{\text{sc}},\) and \(v_\oplus\) are the components perpendicular to the line of sight of the pulsar, screen, and Earth velocities, respectively.

Under the stationary phase approximation, we can treat the pulsar as scattered into \(N\) images with (complex) magnifications \(\mu_j\) and geometric phase delay \(\exp\{i[f_D j + \tau_j]\nu\}\), where \(\tau_j = (d_{\text{eff}}/c)\theta_j^2\) and \(f_D = (\theta_j \cdot v_{\text{eff}})/\lambda\) are the geometric delay and Doppler frequency with respect to the line-of-sight image. We normalize the flux so that \(\sum_j |\mu_j|^2 = 1\).

The dynamic spectrum, which encodes the interference of the \(N\) images, is given by

\[
    D(t, \nu) = \left| \sum_j \mu_j \exp\{2\pi i f_D j + \tau_j]\nu\} \right|^2 \quad (A5)
\]
$$=\sum_{j,k}\mu_j\mu_k \exp[2\pi i (f_{D_j} - f_{D_k}) \tau + (\tau_j - \tau_k)\nu]. \quad (A6)$$

Note that the dynamic spectrum defined above is entirely real as the phase is antisymmetric under the exchange of $j$ and $k$. The Fourier transform of the dynamic spectrum is the conjugate spectrum; its square modulus, called the secondary spectrum, is given by

$$|C(f_D, \tau)|^2 = 2\sum_{j,k}\mu_j^2\mu_k^2\delta(f_{D_j} - f_{D_k})\delta(\tau, \tau_j - \tau_k). \quad (A7)$$

**Appendix B**

**Double Refraction by Two Linear Lenses**

Consider two screens with linear features between the telescope and the pulsar, and a cylindrical coordinate system in which $z$ is along the line of sight, with direction $\hat{z}$ pointing toward the pulsar. For a scattering screen $i$ at distance $d_i$, a line, representing a linear lens on screen $i$, can be written as

$$d_i \hat{z} + r_i + \sigma_i \hat{u}_i,$$  

where $r_i$ is a cylindrical radius from the line of sight to the line (i.e., $\hat{r} \cdot \hat{z} = 0$), $\hat{u}_i = \hat{z} \times \hat{r}$ a unit vector perpendicular to it in the plane of the screen, and $\sigma$ is the position along the line.

Imagine now a ray going from the observer to some point along a linear lens on the first screen, at distance $d_1$. Since it will be easiest to work in terms of angles relative to the observer, we use $\rho = r/d$ and $z = \sigma/d$ to write this trajectory, from $d = 0$ to $d = d_1$, as

$$d(\hat{z} + \rho_1 \hat{r}_1 + \sigma_1 \hat{u}_1). \quad (B2)$$

When the ray hits the lens, light can be bent only perpendicular to the lens, by an angle which we will label $\alpha_1$ (with positive $\alpha_1$ implying bending closer to the line of sight; Simard & Pen 2018). Hence, beyond the screen, for $d > d_1$, its trajectory will be

$$d(\hat{z} + \rho_1 \hat{r}_1 + \sigma_1 \hat{u}_1) - (d - d_1)\alpha_1 \hat{r}_1. \quad (B3)$$

If the ray then hits a lens on the second screen at a distance $d_2$, it will again be bent, by $\alpha_2$, and then follow, for $d > d_2 > d_1$,

$$d(\hat{z} + \rho_1 \hat{r}_1 + \sigma_1 \hat{u}_1) - (d - d_1)\alpha_1 \hat{r}_1 - (d - d_2)\alpha_2 \hat{r}_2. \quad (B4)$$

In order to specify the full trajectory, we need to make sure that the ray actually intersects the lens on the second screen, and ends at the pulsar, i.e.,

$$d_2(\hat{z} + \rho_1 \hat{r}_1 + \sigma_1 \hat{u}_1) - (d_2 - d_1)\alpha_1 \hat{r}_1 = d_2(\hat{z} + \rho_2 \hat{r}_2 + \sigma_2 \hat{u}_2), \quad (B5)$$

$$d_p(\hat{z} + \rho_1 \hat{r}_1 + \sigma_1 \hat{u}_1) - (d_p - d_1)\alpha_1 \hat{r}_1 = -(d_p - d_2)\alpha_2 \hat{r}_2 = d_\rho \hat{z}. \quad (B6)$$

These constraints can be simplified to

$$\sigma_1 \hat{u}_1 - (1 - d_1/d_2)\alpha_1 \hat{r}_1 - \sigma_2 \hat{u}_2 = \rho_2 \hat{r}_2 - \rho_1 \hat{r}_1, \quad (B7)$$

$$\sigma_1 \hat{u}_1 - (1 - d_1/d_p)\alpha_1 \hat{r}_1 - (1 - d_2/d_p)\alpha_2 \hat{r}_2 = -\rho_1 \hat{r}_1. \quad (B8)$$

Hence, one is left with a pair of two-dimensional vector equations with four scalar unknowns, viz., the bending angles $\alpha_{1,2}$ and angular offsets $\sigma_{1,2}$ along the two linear lenses. This set of equations can be extended easily to arbitrary number of screens and a possible non-zero origin. Since the equations are linear in the unknowns, the set can be solved using matrix inversion. We use the `screens` package (Van Kerkwijk & van Lieshout 2022) for this purpose, which also uses the inverted matrix to calculate time derivatives $\dot{\zeta}$ and $\dot{\alpha}$ given velocities of the observer, screens, and pulsar (and thus $\rho$ in the equations), as well as the implied delay $\tau$ and its time derivative $\ddot{\tau}$ for each ray.

We note that the same scattering geometry is obtained by considering the geometric limit of the two-screen scattering model in Sprenger et al. (2022). It is different, however, from the geometry considered by Simard et al. (2019), as those authors assumed that the scattering points were fixed on their respective screens, while we assume that light can only be bent perpendicular to the linear structures, which implies that the scattering images move along those structures as the relative positions of the observer, screens, and pulsar change.

**Appendix C**

**Flux Estimation and Magnification of the 1 ms Lens**

To find the flux of a scattered image $i$, which equals $\mu_i^2$, we integrate over a region corresponding to the interference of that image with all other images, i.e., the corresponding inverted arclet in the secondary spectrum, as defined in Equation (A7). Evaluating this integral, one finds that it yields twice the flux of image $i$:

$$\int \delta(j, i) 2\sum_{j,k} \mu_j^2 \mu_k^2 \delta(f_{D_j} - f_{D_k}) \delta(\tau, \tau_j - \tau_k) df_D d\tau = 2\mu_i^2 \sum_k \mu_k^2 = 2\mu_i^2. \quad (C1)$$

We measured the flux of the 1 ms feature by integrating around it (red region in Figure C1) in each of the seven secondary spectra (with appropriate noise floors subtracted). We

![Figure C1](image-url)
also measured the flux of the corresponding part of the main arc that participated in the double lensing, as inferred from our double-lensing model (blue region in Figure C1). In Figure C2, we show the resulting fractional fluxes, as well as their ratio, which we use as an estimate of the magnification of the 1 ms lens.

Generally, both fluxes increase with time, as expected since the images approach the line of sight. The one exception is the second epoch, in which the 1 ms feature is much fainter. We do not know the reason for this, but note that on that day the other side of the scintillation arc also appeared much fainter at high delay than it was in the first and third epochs. Neglecting the second epoch, we find an averaged magnification of $0.06 \pm 0.02$ for the 1 ms lens, which is what we used in the Section 5.2 to infer the width of the lens.

Note that one might have expected the magnification of the 1 ms lens to increase with time as its impact parameter got smaller, too. In our estimate, however, we have ignored that the magnification of the linear lenses on the main screen is not exactly the same for the singly and doubly scattered rays, since these have different bending angles. We leave a detailed analysis of the magnification as a function of bending angle (and perhaps location along the lens) for future work.

Appendix D

Repetition Likelihoods for Extreme Scattering Events

If, as our observations suggest, quasar ESEs are caused by slowly moving linear plasma lenses, one might expect to see repetitions due to Earth’s orbital motion. The likelihood depends on how the Earth’s motion projects on the screen, and thus on the screen orientation and location on the sky. To estimate the probabilities, we simulated screens over the entire sky, sampling on a Gaussian Legendre grid with 25 points along the decl. and 50 points along the R.A. axis. Then, using the astropy package (Astropy Collaboration et al. 2018, 2013), we calculated Earth’s trajectory relative to the local standard of rest (i.e., orbital motion plus the solar system systemic motion), as projected on the sky for each angular direction, for a 1 yr period.

We then generated large numbers of randomly oriented lines for each grid point to represent linear lenses, assumed stationary relatively to the local standard of rest. For each grid point, we counted the number of simulated lenses that intersected more than once with the projected trajectory of Earth, and then took the ratio with the number that intersected at least once as the likelihood for an ESE to repeat within 1 yr. We show the result in Figure D1. For the line of sight to the first ESE, QSO 0954+658, the (interpolated) probability is a modest 0.44, consistent with no repetition having been seen in the 3 yr the source was
monitored (Fiedler et al. 1987), but there are also regions on the sky for which the probability of repeats approaches unity.

Appendix E
Testing the Screen Geometry

The main scintillation arc persists for a long period of time, which offers the opportunity to test whether the velocity and distance to the underlying scattering screen remain consistent with what is inferred from VLBI work (McKee et al. 2021; F. Syed et al. 2023, in preparation),

$$\eta = \frac{d_{\text{eff}} \alpha^2}{2c (v_0 \cos \alpha)^2}, \quad (E1)$$

where $\alpha$ is the angle between the effective velocity (Equation (A4)) and the line defined by the scattered images.

Measured values of $\eta$ for a year-long observing campaign are presented in Figure D2, with the seven observations analyzed here highlighted. Overdrawn is the evolution predicted based on previous VLBI work (see Table 1) in the main text and the known motion of Earth. The prediction is qualitatively correct, but somewhat offset from the measurements. To correct for this offset, we performed a fit in which we only allowed the velocity of the main scattering screen to vary (which is the parameter with the largest fractional uncertainty). We found a better fit for $v_{1,1} = -24 \text{ km s}^{-1}$, which is within the reported uncertainty. Thus, overall our results confirm the values inferred from the previous analysis of VLBI data (Brisken et al. 2010; Liu et al. 2016).

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Figure D2. Measured quadratic coefficient of the scintillation arc over a year-long observation campaign of PSR B0834+06. The seven observations analyzed here are highlighted in orange. The variation predicted using the parameters from Liu et al. (2016) based on previous VLBI work (see Table 1) is slightly offset, but an adjustment of just the screen velocity, within the quoted uncertainty, makes it consistent with our measurements.

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