Effect of an applied magnetic field on sloshing pressure in a magnetic fluid

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Abstract. Sloshing of a magnetic fluid can be controlled through altering the sloshing natural frequency by applying magnetic fields. The magnetic field is applied by two permanent magnets. The intensity of the applied magnetic field is changed by varying the position of the magnets and the magnet type. The dynamic pressure responses on the inner wall surface were used to gain an understanding of the sloshing phenomenon in this investigation. Several frequency response spectra are obtained in each experimental condition. These results indicated that the free surface disturbance caused by the sloshing was suppressed by the applied magnetic field.

1. Introduction

The sloshing phenomenon is the oscillation of the free surface of a liquid in a container because of external excitation [1]. Sloshing generates impact pressure and changes the centre of gravity of the container. When the container is excited to a frequency that is near the sloshing natural frequencies, resonance is reached and the free surface is disturbed.

A magnetic fluid is a colloidal liquid in which many ferromagnetic nanoparticles are suspended. It is a functional fluid that responds to an applied magnetic field [2]. The apparent gravity acting on the magnetic fluid varies due to the magnetic body force produced by the magnetic field. Thus it is possible to change the sloshing natural frequencies via the external magnetic field. Therefore, it is possible that sloshing can be controlled through the sloshing natural frequency by applying a magnetic field.

Sloshing of a magnetic fluid has been studied by only a few groups. Sudo et al. [3] investigated the vertical sloshing of magnetic fluid droplets using both rectangular and cylindrical containers. Sawada et al. [4] investigated lateral sloshing of a magnetic fluid in a rectangular container. Also, lateral sloshing of a magnetic fluid in a vertically-applied non-uniform magnetic field was studied by Sawada et al. [5], and the experimental results were compared with those obtained from nonlinear theory. There was an experimental investigation of pressure distribution related to water sloshing by Akyildiz and Unal [6].

In this study, two permanent magnets and an oscillating rectangular container filled with a magnetic fluid subjected to a horizontal magnetic field were used to investigate the influence of an applied magnetic field on sloshing. Small disc-type pressure transducers were placed on the...
inner wall surface of the container. This experimental approach was based on the measurement of the pressure at several points using pressure transducers, and the real-time computation of a discrete Fourier transform. The dynamic pressure response was thus determined, and the influence of the magnetic field on the sloshing was investigated.

2. Linear theory

A linear theoretical model is used for sloshing in a rectangular container. A two-dimensional fluid domain is considered, which is bound at the bottom and at both sides by the walls of a rectangular container and by the fluid free surface at the top. The container oscillates laterally with a single-frequency excitation $a \cos \omega t$, where $a$ and $\omega$ are the amplitude and frequency of the excitation, respectively. The sloshing analytical model is illustrated in figure 1. Here $L$ is the width of the tank, $h$ is the height of the fluid and $\eta$ is the displacement of the free surface.

![Sloshing analytical model.](image)

The fluid is assumed to be incompressible and inviscid, and the flow is assumed to be irrotational. The mass conservation for an incompressible fluid is written as

$$\nabla \cdot \mathbf{V} = 0 \quad (1)$$

where $\mathbf{V}$ is the velocity vector. Equation (1) is referred as the continuity equation. Under the assumption of an irrotational flow, the following velocity potential $\Phi(x, z, t)$ exists:

$$\mathbf{V} = -\nabla \Phi \quad (2)$$

Substituting equation (2) into equation (1) leads to the Laplace equation

$$\nabla^2 \Phi = 0 \quad (3)$$

Equation (3) is the governing differential equation for the flow, which is satisfied in the whole domain. To obtain a solution for the sloshing problem, the Laplace equation (3) has to be integrated into the fluid domain.

In order to integrate the governing equation, four boundary conditions have to be considered. The first is the wall boundary condition:

$$\frac{\partial \Phi}{\partial x} \bigg|_{x=-\frac{L}{2}} = \frac{\partial \Phi}{\partial x} \bigg|_{x=\frac{L}{2}} = 0 \quad (4)$$
The second is the bottom boundary condition:

$$\frac{\partial \Phi}{\partial z} |_{z=-h} = 0$$ \hspace{1cm} (5)

The third is the kinematic free surface boundary condition. After linearization, the boundary condition assuming a small surface wave can be written as

$$V \cdot n = \frac{\partial \eta}{\partial t}$$ \hspace{1cm} (6)

where \( n \) is the unit vector normal to the surface. Finally, the fourth is the dynamic free surface boundary condition. An unsteady irrotational Bernoulli equation is considered to be:

$$-\frac{\partial \Phi}{\partial t} + \frac{1}{2} (u^2 + w^2) + gz + \frac{p}{\rho} = C(t)$$ \hspace{1cm} (7)

where \( C(t) \) is a time-dependent integration constant referred to as Bernoulli constant, \( \rho \) is the density and \( g \) is gravitational acceleration. It is necessary to consider the inertia force due to external excitation. Pressure \( p \) can be treated as constant because \( p \) is equal to the atmospheric pressure at the free surface. Furthermore, the \( x \) and \( z \) components of velocity \( u \) and \( w \) are assumed to be minute. Then, the dynamic free surface boundary condition can be written as

$$\frac{\partial \Phi}{\partial t} + gz - ax \omega^2 \cos \omega t = 0$$ \hspace{1cm} (8)

After solving the Laplace equation (3) using separation of variables, the following dispersion relation is obtained.

$$\omega_1^2 = gk \tanh kh$$ \hspace{1cm} (9)

where \( k \) is the wave number associated with the sloshing mode. \( \omega_1 \) is the first resonant angular frequency for the sloshing mode \( k \). The experimental results are described using dimensionless excitation oscillation frequency \( (f^* = f/f_1) \), where \( f_1 \) is the first resonant frequency obtained from dispersion relation.

![Figure 2. Experiment apparatus.](image-url)
Table 1. Characteristics of the magnetic fluid.

| Characteristic                        | Value       |
|---------------------------------------|-------------|
| Density at 25 °C                      | $1.16 \times 10^3$ kg/m$^3$ |
| Viscosity at 27 °C                    | 2.0 mPa·s   |
| Saturation magnetization at 25 °C     | 16.3 mT     |

3. Experiment

3.1. Experimental set-up

Figure 2 shows a schematic diagram of the experimental apparatus. The inner dimensions of the rectangular container are 200 mm $\times$ 30 mm $\times$ 250 mm, and it is made of a transparent acrylic resin with a thickness of 4 mm. The container is filled with a water-based magnetic fluid. The characteristics of the magnetic fluid used for the experiments are shown in table 1. A slider mechanism under the rectangular container is connected to a vibration exciter via two joints and a stainless cylindrical bar. The vibration exciter is controlled by measurement control software (LabVIEW) on a computer. When the vibration exciter generates a vibration, the container oscillates and the free surface of the magnetic fluid is disturbed. There are three small disc-type pressure transducers on the inner wall surfaces of the container, as shown in figure 3. Sensor 1 is placed on the bottom plate and sensor 2 and 3 are placed on the side wall. Pressure signals measured by the transducers are sent to the computer, and a fast Fourier transform (FFT) is used to obtain the amplitude of the pressure signal.

The magnetic fluid depth $h$ was 100 and 60 mm for all the experiments and the excitation frequency $f$ ranged from 0.5 to 3.5 Hz in increments of 0.01 Hz. The amplitude of the oscillation $a$ was 0.5, 1.0, and 1.5 mm for all experiments. The magnetic field was applied by two rectangular parallelepiped permanent magnets of the same type, composed of neodymium or ferrite. These magnets were maintained inside a box made of transparent acrylic resin with a thickness of 3 mm. Figure 4 shows the arrangement of the two permanent magnets. We used couples of neodymium and ferrite magnets. There were two spatial parameters for changing the applied magnetic field intensity. $y_m$ represents the distance between the inner wall surface of the container and the surface of the permanent magnet, and was varied by inserting 5 mm thick acrylic boards in
between the wall and the magnet. \( z_m \) represents the displacement from the free surface of the magnetic fluid at rest to the centre of the magnet. When the permanent magnet is placed below the free surface, the sign of \( z_m \) is negative. In the experiments, the excitation frequency, magnetic fluid depth, excitation amplitude, magnet type, parameters \( y_m \) and \( z_m \) were varied.

### 3.2. Distribution of the magnetic field

Four types of magnetic fields generated by the neodymium and ferrite magnets were used in the experiments. The neodymium magnets were in a heteropolar facing arrangement with different \( y_m \) values of 7.0 mm. The ferrite magnets were in a heteropolar facing or heteropolar parallel arrangement, and both had \( y_m \) values of 7.0 mm and 12.0 mm. The arrangements of the two magnets are shown in figure 5. The distribution of the magnetic field intensity between two permanent magnets along their centre line for the four types of magnetic fields is illustrated in figure 6. Here \( y \) is the displacement from the centre point of the two magnets, and \( B \) is the magnetic flux density. The magnetic field intensity increases near each permanent magnet, and decreases with increasing distance from the permanent magnet. The weakest point of \( B \) appears at the centre point between the two magnets. The magnetic flux density is almost zero in the magnetic field generated by two ferrite magnets placed in a heteropolar parallel arrangement.

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**Figure 4.** Application of the two permanent magnets.

**Figure 5.** Arrangement of the two magnets.

**Figure 6.** Distribution of the magnetic field on the centre line \((z = 0)\).

**Figure 7.** Distribution of the magnetic field (Ferrite, Heteropolar parallel).
Figure 7 shows the 3-D distribution of magnetic field for ferrite magnets in a heteropolar parallel arrangement. The distribution of the magnetic field in the z direction is asymmetrical.

4. Results and discussion
The frequency responses of the dimensionless pressure amplitude on the inner wall surface of the container are shown in figure 8. Here the following dimensionless pressure is introduced.

\[ P^* = \frac{P}{\rho g L} \]  

(10)

* is the pressure amplitude. \( P \) is taken from the root means square (RMS) amplitude of the signal wave form. The FFT harmonic is considered to be the same as the excitation vibration. The results without the magnetic field are represented by a solid line in figure 8. The relationships between positions of the permanent magnets \( z_m \) and either the resonance frequency \( f_r \) or dimensionless pressure amplitude at resonance \( P^*_r \) were obtained from the frequency response. Figure 9 shows the resonant frequency shift as a function of \( z_m \) for \( h = 100 \text{ mm} \) and \( a = 1.0 \text{ mm} \) at Sensor 1. Figure 10 shows the dimensionless pressure amplitude at resonance with \( z_m \) at Sensor 1.

**Figure 8.** Frequency responses of the pressure amplitude at Sensor 1 (Neodymium).

**Figure 9.** Resonant frequency shift with \( z_m \) at Sensor 1.

**Figure 10.** Dimensionless pressure amplitude at resonance with \( z_m \) at Sensor 1.

**Figure 11.** Resonant frequency shift with \( z_m \) at Sensor 1.
at resonance for $h = 100$ mm and $a = 1.0$ mm at Sensor 1, and figure 11 shows the resonant frequency shift as a function of $y_m$ for $y_m = 7.0$ mm at Sensor 1. The broken line in these figures indicates the resonant frequency without a magnetic field. In the previous study, the difference in $f_r$ and $P^*_r$ between $y_m = 7$ mm and 12 mm under a magnetic field generated by neodymium magnets was investigated [7]. In this study we focus on the difference in magnet type, excitation amplitude, and the depth of the magnetic fluid.

When the magnets are above the free surface, the resonant frequency is lower than that without a magnetic field for each condition, as shown in figures 8 and 9. On the other hand, the resonant frequency becomes greater than that without a magnetic field when the magnets are placed below the free surface of the magnetic fluid. These different results are caused by a change in the apparent gravitational force. However, the resonant frequency shift when the ferrite magnets are in a parallel arrangement shows an opposite tendency to the other results because of the distribution of the magnetic field. This was caused by difference in the distribution of magnetic flux between the two magnets and difference in direction of the magnetic flux above and below the magnets. Further detailed analysis of the distribution of the magnetic field is necessary in order to explain this behaviour.

The dimensionless pressure amplitude at resonance $P^*_r$ shows lower values than those without a magnetic field, as shown in figure 10. This is because the movement of a magnetic fluid is restricted by a magnetic field. Therefore, it can be said that sloshing in a magnetic fluid can be controlled by a magnetic field from the point of view of the dynamic pressure variation.

The influence of the excitation amplitude $a$ and the magnetic fluid depth $h$ on the resonant frequency are shown in figure 11. There are no large differences caused by excitation amplitude. There are different values of $f_r$ near $z_m = 0$ mm; however, these are caused by instability of feedback control of excitation amplitude. On the other hand, $f_r$ is changed with the magnetic fluid depth. When the fluid depth is large, the resonant frequency is also large with or without magnetic fluids. Therefore, the magnetic fluid depth has no relevance to controlling sloshing by a magnetic field.

5. Conclusion
The dynamic pressure change of a magnetic fluid sloshing in a rectangular container under lateral excitation was investigated experimentally. The resonant frequency was changed by varying the positions of the magnets, the type of magnetic field and the arrangement of magnets. Notably, the range of the resonant frequency and the dimensionless pressure amplitude at resonance was changed significantly by varying $z_m$. Based on these results, it can be concluded that a magnetic field applied near the free surface of a magnetic fluid can be used to control the sloshing phenomenon of the fluid. It is important to note that sloshing does not depend on the excitation amplitude or on the magnetic fluid depth.

Acknowledgements
We would like to thank Mr. Jean Klingler of Souriau Japan K. K. for his experimental help. This work was partially supported by a Grant-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science.

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