Testing Flavor Changing Neutrino Interactions in Long Baseline Experiments

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Abstract

We have investigated the possibility of discerning mass from flavor changing neutrino interactions induced $\nu_\mu \rightarrow \nu_\tau$ oscillations in the long baseline neutrino experiments K2K and MINOS. We have found that for virtually any value of the flavor conserving parameter $\epsilon'$ it will be possible to, independently, distinguish these two mechanisms at K2K, if the flavor changing parameter $\epsilon$ is in the range $\epsilon \gtrsim 0.77$, and at MINOS, if $\epsilon \gtrsim 0.2$. Moreover, if K2K measures a depletion of the expected $\nu_\mu$ flux then MINOS will either observe or discard completely flavor changing neutrino oscillations.

1 Introduction

Although over three decades of solar neutrino experiments [1] and one decade of atmospheric neutrino data [2,3] have confirmed, beyond any reasonable doubt, that

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neutrinos have indeed an oscillating nature, the question about which is the dynamical mechanism responsible for such oscillations still remains an open one.

We can find in the literature today a variety of different schemes [4–12], going from mass induced to gravitationally induced oscillations passing by decaying neutrinos, that can successfully account for the solar and/or atmospheric neutrino results. It is therefore important to investigate the possibility of distinguishing different solutions in experiments where neutrino beams are manufactured by accelerators in the Earth. These experiments have the advantage that one can control what is being produced (neutrino flux, flavor, energy distribution) as well as what is being detected. We therefore believe that they must provide the definite proof, not only of neutrino conversion, through the observation of neutrino flavor appearance or disappearance, but also of the dynamical nature of the oscillation process. Besides, it is always important to have an independent check of the operative neutrino oscillation mechanism.

It has been recently suggested [13] that flavor changing neutrino interactions (FCNI) could induce $\nu_\mu \rightarrow \nu_\tau$ oscillations that would explain rather well the sub-GeV and multi-GeV data reported by the Super-Kamiokande atmospheric neutrino experiment. Since then some controversy about the quality of this solution when one also includes the upward-going muon data in the FCNI analyses has come up [14–16]. In any case, regardless of the discussion on the goodness of the fit to the atmospheric neutrino data, the proposed FCNI solution selects a parameter space region that will be well in the reach of the forthcoming long baseline experiments. We will in what follows refer to this type of oscillation simply as flavor changing induced oscillation (FCIO).

It has been pointed out in Ref. [17] that a fine tuning of model parameters would be necessary in order to reconcile the FCIO solution to the atmospheric neutrino problem with current experimental limits on lepton number violation coming from lepton decays. However it will be up to nature to confirm or disclaim our theoretical preferences and prejudices. Having this in mind we have investigated the possibility of using the long baseline K2K [18] and MINOS [19] neutrino accelerator experiments in order to discriminate mass induced $\nu_\mu \rightarrow \nu_\tau$ oscillations (MIO) from FCIO. Many authors have suggested and discussed the physical capabilities of these experiments [20,21]. Both of them will have neutrino beams that will travel through a certain amount of Earth matter before reaching the corresponding neutrino detector, making them specially suited to probe the FCNI mechanism.

The outline of the paper is as follows. In Section 2, we briefly revise the two oscillation formalisms and recall, in each case, the distinct features of the $\nu_\mu \rightarrow \nu_\tau$ oscillation probability. In Section 3 we define the K2K and MINOS observables that we have used in our analysis and describe in detail how we have estimated their values as a function of the free oscillation parameters. In Section 4 we present and discuss our
results for K2K and MINOS. Our conclusions are presented in Section 5.

2 Review of the MIO and FCIO Formalisms

2.1 Mass Induced Oscillations

Maki, Nakagawa and Sakata [22] were the first to argue that if neutrinos have mass their states that are created and detected by weak interactions may not be the eigenstates of propagation and therefore neutrino flavor oscillations can occur. This is the simplest neutrino oscillation mechanism and perhaps the most likely one to take place in nature.

We will consider that oscillations between $\nu_\mu \rightarrow \nu_\tau$ can occur in a two family scenario with two massive neutrinos. In this case neutrino mass eigenstates and flavor eigenstates can be related by a Cabibbo-like mixing matrix, that can be parameterized by a single angle. The neutrino evolution Hamiltonian in vacuum is rather trivial and the corresponding equations can be solved analytically to compute the oscillation probability between the two flavor eigenstates $\nu_\mu \rightarrow \nu_\tau$, $P_{\nu_\mu \rightarrow \nu_\tau}^\text{mass}$, to obtain the well known formula:

$$P_{\nu_\mu \rightarrow \nu_\tau}^\text{mass} \equiv P_{\nu_\mu \rightarrow \nu_\tau} \left( \sin^2 2\theta, \Delta m^2, E_\nu, L \right) = \sin^2(2\theta) \sin^2 \left( \frac{\pi L}{L_{\text{osc}}^m} \right),$$

where $\theta$ is the mixing angle, the two non-degenerate neutrinos have mass $m_1$ and $m_2$, $\Delta m^2 = |m_1^2 - m_2^2|$ is measured in $\text{eV}^2$, $L$ is the distance between the neutrino source and the detector in km, and $E_\nu$ the neutrino energy, in GeV. The neutrino oscillation length, which is also measured in km and grows linearly with the neutrino energy, is defined as

$$L_{\text{osc}}^m = \frac{\pi E_\nu}{1.27\Delta m^2}.$$  \hfill (2)

We have here two free parameters, $\sin^2 2\theta$ and $\Delta m^2$. Inspecting Eq. (1) we see that the maximum of the conversion probability, for a constant amplitude, happens when $L/L_{\text{osc}}^m = 1/2$. This condition is satisfied for fixed $L$ and an averaged neutrino energy $\langle E_\nu \rangle$ for

$$\Delta m^2_e = \frac{\pi}{2 \times 1.27} \left( \frac{\langle E_\nu \rangle}{L} \right),$$

(3)
2.2 Flavor Changing Induced Oscillations

The fact that FCNI can induce neutrino oscillations in matter was first investigated by Wolfenstein [23] who pointed out that interactions in a medium modify the dispersion relations of particles traveling through. Wolfenstein effect generates quantum phases in the time evolution of phenomenological neutrinos eigenstates which consequently can oscillate.

In a two-flavor mixing scheme the presence of flavor changing neutrino-matter interactions implies a non-trivial structure for the neutrino evolution Hamiltonian in matter, even if massless neutrinos and no mixing in the vacuum is assumed. The evolution equations describing the $\nu_\mu \to \nu_\tau$ transitions are given by [24]:

$$i \frac{d}{dr} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \sqrt{2} G_F \begin{pmatrix} 0 & \epsilon' n_f(r) \\ \epsilon n_f(r) & \epsilon' n_f(r) \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix},$$

(4)

where $\nu_\mu \equiv \nu_\mu(r)$ and $\nu_\tau \equiv \nu_\tau(r)$, are the probability amplitudes to find these neutrinos at a distance $r$ from their creation position, $\sqrt{2} G_F n_f(r) \epsilon f$ is the flavor-changing $\nu_\mu + f \to \nu_\tau + f$ forward scattering amplitude with the interacting fermion $f$ (electron, $d$ or $u$ quark) and $\sqrt{2} G_F n_f(r) \epsilon' f$ is the difference between the flavor diagonal $\nu_\mu - f$ and $\nu_\tau - f$ elastic forward scattering amplitudes, with $n_f(r)$ being the number density of the fermions which induce these processes. From now on we will consider FCNI only with a single fermion type and drop the label $f$ attached to $\epsilon$ and $\epsilon'$.

For constant matter density Eq. (4) can be analytically solved to give a conversion probability, $P^{\text{FCNI}}_{\nu_\mu \to \nu_\tau}$, that can be written as

$$P^{\text{FCNI}}_{\nu_\mu \to \nu_\tau} \equiv P_{\nu_\mu \to \nu_\tau}(\epsilon, \epsilon', L) = \frac{4\epsilon'^2}{4\epsilon^2 + \epsilon'^2} \sin^2 \left( \frac{\pi L}{L_{\text{osc}}} \right),$$

(5)

where $L$ is the neutrino flight length from the production source to the detector and $L_{\text{osc}}^f$ is the oscillation length, both measured in km. $L_{\text{osc}}^f$, which in contrast to Eq. (1), does not depend on the neutrino energy, can be written explicitly as follows:

$$L_{\text{osc}}^f = 2707.4 \times \frac{3}{C} \times \frac{(2 \text{ mol/cm}^3)}{n_e} \frac{1}{\sqrt{4\epsilon^2 + \epsilon'^2}},$$

(6)

where $C = 3$, for FCNI with $u$- or $d$-quarks, $C = 1$, for FCNI with electrons, and $n_e$ is the Earth’s electron density in mol/cm$^3$. For $n_e = 2$ mol/cm$^3$ and FCNI only with
\( u \) or \( d \) quarks \( L_{\text{osc}} = 2707.4 \) km. We have in this scheme also two free parameters, \( \epsilon \) and \( \epsilon' \). In this paper we will only consider FCNI involving \( d \)-quarks. If we were to consider electrons as the interacting fermions, instead of \( d \)-quarks, this would mean a simple rescaled of our results by a factor three and so there is no need to show this explicitly here.

In analogy to the discussion made for MIO it is instructive to point out the condition that has to be satisfied in order to the conversion probability to be at a maximum for the FCIO mechanism. If we impose \( L/L_{\text{osc}} = 1/2 \) then

\[
\sqrt{4\epsilon_s^2 + \epsilon_s'^2} = \left( \frac{2707.4}{2 \times L} \right) \left( \frac{3}{C} \right) \left( \frac{(2 \text{ mol/cm}^3)}{n_e} \right),
\]

and if \( \epsilon' \) and \( \epsilon \) have comparable sizes

\[
\epsilon_s \sim \left( \frac{2707.4}{4 \times L} \right) \left( \frac{3}{C} \right) \left( \frac{(2 \text{ mol/cm}^3)}{n_e} \right).
\]

### 3 Description of the MIO and FCIO analysis for K2K and MINOS

#### 3.1 K2K Experiment

K2K [18] is a long baseline experiment were a \( \nu_\mu \) beam is produced by the Japanese KEK accelerator, driven through a certain amount of the Earth’s crust before reaching the K2K detector.

In our approach we only consider \( \nu_\mu \rightarrow \nu_\tau \) transitions, disregarding as completely negligible the channel \( \nu_\mu \rightarrow \nu_e \), and since the K2K experiment is unable, due to its range in energy, to observe \( \nu_\tau \) production, the only information which is valuable to us is a possible measurement of the reduction of the \( \nu_\mu \) flux. For this reason, we have computed the mean oscillation probability \( \langle P_{\nu_\mu \rightarrow \nu_\tau}(\sin^2(2\theta), \Delta m^2) \rangle \) that can be measured by K2K as a function of the MIO free parameters \( \sin^2 2\theta \) and \( \Delta m^2 \) defined as

\[
\langle P_{\nu_\mu \rightarrow \nu_\tau}(\sin^2(2\theta), \Delta m^2) \rangle = \frac{\int \int dx \, dE_\nu \, h(E_\nu) \, f(x, L_{\text{K2K}}) \, P_{\nu_\mu \rightarrow \nu_\tau}^{\text{mass}}}{\int h(E_\nu) \, dE_\nu},
\]

where \( L_{\text{K2K}} = 250 \) km, \( h(E_\nu) \) is the predicted neutrino energy spectrum for \( \nu_\mu \) in the far detector that can be found in the Ref. [18] and \( P_{\nu_\mu \rightarrow \nu_\tau}^{\text{mass}} \) is the probability...
showed in Eq. (1). In order to take into account the uncertainties in the distance $L$ we use in Eq. (9) the following Gaussian smearing function $f(x, L)$:

$$f(x, L) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left[ -\frac{(x - L)^2}{2\sigma^2} \right],$$

(10)

where we have assumed $\sigma = 0.05 L_{K2K}$.

In analogy, we have obtained the mean oscillation probability $\langle P_{\nu_\mu \rightarrow \nu_\tau} (\epsilon, \epsilon') \rangle$ for FCIO as a function of the free parameters $\epsilon$ and $\epsilon'$ defined by

$$\langle P_{\nu_\mu \rightarrow \nu_\tau} (\epsilon, \epsilon') \rangle = \int dx \, f(x, L_{K2K}) \, P_{\nu_\mu \rightarrow \nu_\tau}^{FCNI} ,$$

(11)

where $f(x, L_{K2K})$ is the smearing function given in Eq. (10) and $P_{\nu_\mu \rightarrow \nu_\tau}^{FCNI}$ is the probability given in Eq. (5). We have used in our calculations the Earth’s electron density profile for K2K given in Ref. [25], from this profile one can calculate that the mean electron density will be $n_e = 2.35 \text{ mol/cm}^3$. We use this mean electron density to compute the oscillation probability. Note that since the FCIO is energy independent, we do not need to take the average of Eq. (11) over the neutrino energy spectrum.

### 3.2 MINOS Experiment

The MINOS experiment [19] is a part of the Fermilab NuMI Project. The neutrinos which constitute the MINOS beam will be the result of the decay of pions and kaons that will be produced by the 120 GeV proton high intensity beam extracted from the Fermilab Main Injector. There will be two MINOS detectors, one located at Fermilab (the near detector) and another located in the Soudan mine in Minnesota, about 730 km away (the far detector). MINOS will be thus a $L_{\text{MIN}} = 730$ km long baseline experiment.

According to Ref. [26], MINOS will be able to measure independently the rates and the energy spectra for neutral current (nc) and charged current (cc) reactions. About 3000 $\nu_\mu$ cc-events/kt/year are expected in the MINOS far detector for the highest energy configuration. We will compute here the ratio $R_{\text{nc/cc}}$ that should be expected at MINOS for MIO and FCIO as a function of the respective free parameters. This ratio has the advantage that it does not require the understanding of the relative fluxes at the near and far detectors, it is also quite sensitive to oscillations since when they occur not only cc-events are depleted but nc-events are enhanced.
In the MIO or FCIO hypothesis this ratio can be written as [27]:

\[
R_{\text{nc/cc}}^{\text{osc}} = \frac{\int dE_\nu N_{\text{nc}}^{\text{osc}}(E_\nu)}{\int dE_\nu N_{\text{cc}}^{\text{osc}}(E_\nu)},
\]

(12)

where

\[
N_{\text{nc}}^{\text{osc}}(E_\nu) = N_{\text{cc}}^{\text{no-osc}}(E_\nu)(1 - P_{\nu_\mu \to \nu_\tau}) + N_{\text{cc}}^{\text{no-osc}}(E_\nu)\eta(E_\nu)BP_{\nu_\mu \to \nu_\tau},
\]

(13)

describes the two possible ways that a muon can be produced; the first term representing the contribution of surviving \(\nu_\mu\) and the second the contribution of \(\tau \to \nu_\mu \nu_\tau \nu_\mu\) decays, from taus generated by \(\nu_\tau\) interactions in the detector after \(\nu_\mu \to \nu_\tau\) conversion and

\[
N_{\text{cc}}^{\text{osc}}(E_\nu) = N_{\text{cc}}^{\text{no-osc}}(E_\nu) + N_{\text{cc}}^{\text{no-osc}}(E_\nu)\eta(E_\nu)(1 - B)P_{\nu_\mu \to \nu_\tau},
\]

(14)

is the nc contribution with

\[
\eta(E_\nu) = \frac{\sigma_{\nu_\tau}^{\text{cc}}(E_\nu)}{\sigma_{\nu_\mu}^{\text{cc}}(E_\nu)},
\]

(15)

where \(B = 0.18\), the branching ratio for \(\tau\) leptonic decay, \(N_{\text{cc}}^{\text{no-osc}}\) is the energy spectrum for \(\nu_\mu\) cc-events in the MINOS far detector in the case of no oscillation [26]. The high-energy wide-band beam was assumed and the spectrum has already been smeared by the expected detector resolution [26]. From this spectrum we have inferred \(N_{\text{cc}}^{\text{no-osc}}\), the expected energy spectrum for \(\nu_\mu\) nc-events in the MINOS far detector in the case of no oscillation, using the approximation:

\[
N_{\text{cc}}^{\text{no-osc}}(E_\nu) = N_{\text{cc}}^{\text{no-osc}}(E_\nu)\frac{\sigma_{\nu_\tau}^{\text{cc}}(E_\nu)}{\sigma_{\nu_\mu}^{\text{cc}}(E_\nu)}.
\]

(16)

The cross sections \(\sigma_{\nu_\mu}^{\text{cc}}, \sigma_{\nu_\tau}^{\text{cc}}\) and \(\sigma_{\nu_\mu}^{\text{nc}}\) were taken from Refs. [28–31] and the conversion probability, \(P_{\nu_\mu \to \nu_\tau}\), used for the MIO case is defined by

\[
P_{\nu_\mu \to \nu_\tau} = \int dx f(x, L_{\text{MIN}}) P_{\nu_\mu \to \nu_\tau}^{\text{mass}},
\]

(17)

and for FCIO case by

\[
P_{\nu_\mu \to \nu_\tau} = \langle P_{\nu_\mu \to \nu_\tau}(\epsilon, \epsilon') \rangle = \int dx f(x, L_{\text{MIN}}) P_{\nu_\mu \to \nu_\tau}^{\text{FCIO}},
\]

(18)

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where \( f(x, L_{\text{MIN}}) \) is the smearing function given in Eq. (10) with \( \sigma = 0.05 L_{\text{MIN}} \) and \( P_{\nu_{\mu} \rightarrow \nu_{\tau}}^{\text{mass}}, P_{\nu_{\mu} \rightarrow \nu_{\tau}}^{\text{FCNI}} \) are the probabilities given in Eqs. (1) and (5) respectively.

In all the calculations involving FCIO we have assumed for MINOS a constant density of \( n_e = 2.80 \text{ mol/cm}^3 \), which is the typical value for rock [32,33].

4 Presentation and Discussion of Results

We have computed the region in the \( \epsilon \times \epsilon' \) plane that will be attainable by K2K and by MINOS. In order to do this, we have compared the probabilities given in Eqs. (11) and (18) with the sensitivity limit, \( P_{\nu_{\mu} \rightarrow \nu_{\tau}}^{\text{lim}} \), for each one of these experiments and selected the pairs of values \((\epsilon, \epsilon')\) that satisfied the condition:

\[
\langle P_{\nu_{\mu} \rightarrow \nu_{\tau}}(\epsilon, \epsilon') \rangle \geq P_{\nu_{\mu} \rightarrow \nu_{\tau}}^{\text{lim}},
\]

where \( \langle P_{\nu_{\mu} \rightarrow \nu_{\tau}}(\epsilon, \epsilon') \rangle \) and \( P_{\nu_{\mu} \rightarrow \nu_{\tau}}^{\text{lim}} \) are respectively given by Eq. (11) and equal to 0.25 in the case of K2K and given by Eq. (18) and equal to 0.010 in the case of MINOS.

We display in Figs. (1)(a) and (b) the observable values for the parameters \( \epsilon \) and \( \epsilon' \) obtained by applying the condition given by Eq. (19) to K2K and MINOS respectively. We see that for K2K \( \epsilon \gtrsim 0.77 \) for any value of \( \epsilon' \), while for MINOS one can reach much smaller values since \( \epsilon \gtrsim 5 \times 10^{-2} \). In fact we can see from Figs. (1)(b) that virtually all values of \( \epsilon \) that are allowed by the FCIO analysis of the Super-Kamiokande data [13] can in principle be tested by the MINOS long baseline experiment.

4.1 K2K

In Figs.2 we show the averaged probabilities \( \langle P_{\nu_{\mu} \rightarrow \nu_{\tau}}(\sin^2(2\theta), \Delta m^2) \rangle \) for MIO and \( \langle P_{\nu_{\mu} \rightarrow \nu_{\tau}}(\epsilon, \epsilon') \rangle \) for FCIO that can be measured by the K2K experiment. In Fig.2(a) the averaged probabilities for mass induced oscillation with \( \sin^2(2\theta)=0.8 \) and 1.0 are shown as a function of \( \Delta m^2 \). These two curves give the maximum and minimum probabilities as a function of \( \Delta m^2 \) that can be reached if the oscillation parameters are chosen inside the 90\% C. L. allowed region for Super-Kamiokande solution to the atmospheric neutrino problem [34], i.e. \( 0.8 \leq \sin^2(2\theta) \leq 1.0 \) and \( 2.0 \times 10^{-3} \text{ eV}^2 \leq \Delta m^2 \leq 9.0 \times 10^{-3} \text{ eV}^2 \). We note that in the range of parameters considered here the averaged oscillation probability of K2K can vary from a minimum value around 0.25, which is the sensitivity limit, to a maximal value around 0.69, which corresponds to
Fig. 1. Region in the $\epsilon \times \epsilon'$ plane that will be observable in (a) K2K and (b) MINOS in the case of FCNI induced $\nu_\mu \rightarrow \nu_\tau$ oscillations.

$$\Delta m^2_\ast \sim \langle E_\nu \rangle / L = 6 \times 10^{-3} \text{ eV}^2,$$
just at the sensitivity of K2K, as can be expected from Eq. (3) for an averaged neutrino energy of about 2 GeV.

In the Fig. 2(b) we present two curves for the averaged probability in the case of FCIO, as a function of $\epsilon$ in the observable region of the K2K experiment. These curves give the maximum and minimum averaged probabilities as a function of $\epsilon$ that can be reached if $0.1 \leq \epsilon' \leq 1$. We observe that for this range of $\epsilon'$ these curves are almost indistinguishable one from the other. This reflects the fact that
the probability given in Eq. (5) can be written, when the argument of the sine is small, which is just the case for K2K ($L_{K2K} = 250$ km), simply as

$$P_{\nu_\mu \to \nu_\tau}^{\text{FCNI}} \sim \frac{4e^2}{4e^2 + e'^2} \left[ \pi \left( \frac{L}{2707.4} \right) \left( \frac{C}{3} \right) \left( \frac{n_e}{2 \text{ mol/cm}^3} \right) \sqrt{4e^2 + e'^2} \right]^2,$$

(20)

which is independent of $e'$ and grows proportionally to $e^2$. This is the reason why
we have chosen to present in Fig. 2(b) the averaged probability as a function of $\epsilon$ instead of $\epsilon'$, which would be a more natural choice to compare with Fig. 2(a). If FCIO are to be observed in K2K the averaged conversion probability will lay in the range 0.25-0.4.

If K2K observes a depletion of the $\nu_\mu$ flux their experimental result can be translated into a certain allowed region of averaged probability in Figs. 2. It is possible that this region will be far enough from 0.4, the maximum allowed probability by FCIO, so that K2K may be able to rule out FCIO in the entire observable parameter region given in Fig. 1(a). On the other hand if this allowed region turns out to be statistically compatible with values in the range 0.25-0.4 this observable on its own will not be conclusive since both type of oscillations will be able to explain the data. Nevertheless K2K can measure the energy spectrum of $\nu_\mu$ cc-events, thus we further investigated to what extent one could use this information to disentangle the two mechanisms of oscillation considered here.

In Figs. 3 we show the ratio, $N_{\text{osc}}^{\nu_\mu cc}/N_{\text{no-osc}}^{\nu_\mu cc}$, of the energy spectrum of $\nu_\mu$ cc-events in the hypothesis of oscillation over the expected energy spectrum without oscillation. In Fig. 3(a) MIO is considered with maximal oscillation amplitude ($\sin^2(2\theta) = 1.0$) and three different values of $\Delta m^2$ in the range where the measured averaged oscillation probability cannot be conclusive. In Fig. 3(b) we show a plot for FCIO, for $\epsilon' = 1.0$ and three different values of $\epsilon$. While MIO cause a pronounced spectral distortion at low energy, FCIO, which are energy independent, do not. According to the K2K detector simulation [35] they will be able to determine the neutrino energy in the region $E_\nu = 0.5 \sim 3.0$ GeV with better precision than 10% so that we can expect that they will have enough accurate data points in the low energy region ($E_\nu \lesssim 1.5$ GeV) to carefully examine possible spectral distortions and perhaps reach a definite conclusion about which is the mechanism responsible for $\nu_\mu \rightarrow \nu_\tau$ conversion at K2K.

4.2 MINOS

We show in Figs. 4(a)-(c) the behavior of the ratio $N_{\nu_\mu cc}^{\text{osc}}/N_{\nu_\mu cc}^{\text{no-osc}}$, which is essentially the $\nu_\mu$ survival probability for MINOS, for the MIO and FCIO mechanisms. In Fig. 4(a) we see that this ratio for MIO is not very affected by $\sin^2(2\theta)$, in the range allowed by the Super-Kamiokande experiment, and is always above $\sim 0.7$ in the range of $\Delta m^2$ we have considered. On the other hand it depends rather strongly on the values of $\epsilon$ and of $\epsilon'$ as can be seen in Figs. 4(b) and (c). In particular for some values of $\epsilon$ there is a very strong suppression, when the conversion probability is close to its maximum, nevertheless the ratio is never zero due to the contribution of $\tau$ decays as shown in Eq. (13).
Fig. 3. Ratio of the energy spectrum of $\nu_\mu$ cc-events for (a) MIO with $\sin^2(2\theta) = 1.0$ and (b) FCIO with $\epsilon' = 1.0$ over the expected energy spectrum in the absence of neutrino oscillations for K2K.

For MINOS we have calculated the ratio, $R_{\text{osc/nc}}$, which is given in Eq. (12). This is shown in Fig. 5(a) for the case of MIO. Again we have chosen to display this ratio as a function of $\Delta m^2$ for the minimum and maximum amplitudes allowed in the 90% C. L. region for Super-Kamiokande solution to the atmospheric neutrino problem [34]. We see that this mechanism gives rise to $R_{\text{osc/nc}}$ in the range 0.3-0.52. The ratio $R_{\text{osc/nc}}$ in the hypothesis of FCIO is shown in Fig. 5(b) as a function of $\epsilon$ for two different values of $\epsilon'$. We also show in this figure, as a reference, a flat solid
Fig. 4. Ratio of the number of $\nu_\mu$ cc-events for (a) MIO as a function of $\Delta m^2$, (b) FCIO as a function of $\epsilon'$ and (c) FCIO as a function of $\epsilon$ over the number of $\nu_\mu$ cc-events in the case of no oscillation. The solid line with the arrow in (b) and (c) marks the sensitivity of MINOS.

line corresponding to the maximum value of $R_{\text{osc}}^{\text{ne}/\text{cc}}$ for MIO (0.52). Note that the lowest point of the curves in Fig. 5(a) is exactly the MINOS sensitivity limit for the quantity $R_{\text{osc}}^{\text{ne}/\text{cc}}$. From Eq. (8) one can infer that for $L = L_{\text{K2K}}$, $n_e \sim 2.35 \text{ mol/cm}^3$ and $C \sim 3$ we have the maximum of the conversion probability at $\epsilon_* \sim 0.6$. The peaks we observe in Fig. 5(b) are due to the fact that we pass by this maximum
of the conversion probability. Note that even at this point, the contribution due to muons generated by tau decays prevents this ratio from going to infinity.

Fig. 5. Ratio of nc/cc events at the MINOS far detector for (a) MIO as a function of $\Delta m^2$ and (b) FCIO as a function of $\epsilon$.

We see clearly from the comparison of Figs. 5(a) and (b) that for $0.35 \lesssim \epsilon \lesssim 0.9$ it is possible to use $R^{\text{nc/cc}}$ to distinguish MIO from FCIO, for virtually any value of $\epsilon' \leq 1.0$. Nevertheless if the measured nc/cc ratio, with its corresponding error, is consistent with the MIO range, 0.3-0.52, MINOS will be able to rule out a large region in the $\epsilon \times \epsilon'$ plane, but this measurement on its own will not be sufficient
to completely distinguish between the two oscillation mechanisms since for $\epsilon \lesssim 0.3$ FCIO also predicts a nc/cc ratio within this interval. In this case one can try to use the spectral information for cc-events and nc-events that will be available at MINOS.

First one can look for distortions in the energy spectrum for cc-events. It can be seen in Fig. 6(a) that this is very sizable for MIO at lower energies as long as
\[ \Delta m^2 \gtrsim 0.004 \text{eV}^2, \] but becomes increasingly difficult to measure as \( \Delta m^2 \) decreases. On the other hand for FCIO, although the mechanism is energy independent, we perceive a extremely mild spectral distortion in Fig. 6(b). This happens due to the presence of the second term in Eq. (13) which carries the cross section energy dependence through the \( \eta(E_{\nu}) \) parameter described in Eq. (15). We have plotted in Fig. 6(b) curves corresponding to four different values of \( \epsilon \) fixing \( \epsilon' = 1.0 \). The ratio \( N_{\text{osc}}^{\text{cc}} / N_{\text{no-osc}}^{\text{cc}} \), for \( \epsilon \lesssim 0.3 \), is completely independence of \( \epsilon' \) and is not shown here. The four \( \epsilon \) values selected belong to the region in Fig. 5 where the \( R_{\text{osc/cc}}^{\text{nc}} \) test cannot discriminate between MIO and FCIO. For \( \epsilon \lesssim 0.2 \) these flat curves may be experimentally indistinguishable from the MIO case with \( \Delta m^2 \approx 0.002 \text{ eV}^2 \). To try to separate these two solutions a precise measurement of the spectrum around 5 GeV seems to be crucial.

We also can look for distortions in the energy spectrum for nc-events. As can be seen in Figs. 7(a)-(b) this presents a quite different behavior for MIO and FCIO, but again the case \( \Delta m^2 \approx 0.002 \text{ eV}^2 \) can be hardly distinguished from the case \( \epsilon \lesssim 0.1 \).

Now if we compare Fig. 6(a)-(b) with Figs. 7(a)-(b) we notice a possible smoking gun to discriminate the two mechanisms: their different energy dependence for nc- and cc-events. MIO in general will cause a pronounced energy distortion for cc-events and a milder energy distortion for nc-events, at low energy and decreasing with energy. The FCIO mechanism, in contrast, presents practically no distortion for cc-events and a smooth distortion for nc-events, which increases with energy. Again the latter behavior reflects basically the energy dependence in \( \eta(E_{\nu}) \) for FCIO is an energy independent effect. This cross check will be useful for \( \Delta m^2 \gtrsim 0.003 \text{ eV}^2 \) and \( \epsilon \gtrsim 0.2 \).

5 Conclusions

We have studied the possibility of distinguishing mass from flavor changing induced \( \nu_{\mu} \to \nu_{\tau} \) conversion in the long baseline experiments K2K and MINOS. We have performed a series of estimations of a number of observables that will be measured by those two experiments, for these two oscillation hypothesis.

We have calculated the region in the \( \epsilon \times \epsilon' \) plane that will be observable at K2K and MINOS long baseline experiments. Although K2K will not be able to completely test the region in the \( \epsilon \times \epsilon' \) plane which is allowed by the atmospheric neutrino FCIO solution given in Ref. [13], it will be able to cover well the region for high values of \( \epsilon \). K2K is not very sensitive to \( \epsilon' \) but is quite sensitive to \( \epsilon \gtrsim 0.8 \). MINOS however will be sensitive to \( \epsilon' \) as well as to \( \epsilon \) and will cover a wider range of these parameters.
Fig. 7. Ratio of the energy spectrum of $\nu_\mu$ nc-events for (a) MIO with $\sin^2(2\theta) = 1.0$ and (b) FCIO with $\epsilon' = 1.0$ over the expected energy spectrum in the absence of neutrino oscillation for MINOS.

From Figs. 2 we see that if K2K experiment measures a averaged survival probability well above 40% then it can completely exclude the FCIO as a possible solution to the atmospheric neutrino problem. But if the measured averaged survival probability is around or smaller than 40% K2K will have to do very good job in measuring the energy spectrum of $\nu_\mu$ cc-events to be able to discriminate between FCIO and MIO.

In the case of MINOS, the measurement of the ratio $R^{\text{osc}}_{\text{nc/\mu}}$ as well as of the spectrum
ratios \( \frac{N_{\text{osc}}}{N_{\text{no-osc}}^{\text{cc}}} \) and \( \frac{N_{\text{osc}}}{N_{\text{no-osc}}^{\text{nc}}} \) are very powerful tests. They will most certainly permit MINOS to explore all the region up to \( \epsilon \gtrsim 0.2 \).

K2K is now running and they will certainly have results before MINOS begins to operate. There are three possibilities that one can visualize. If K2K observes a signal compatible with no-oscillation this implies that we are in the worst possible situation for this experiment since the average probability observed by K2K would be around or lower than its expected experimental sensitivity. In principle, this would mean that we are in a bad shape, but a careful analysis of our results indicates that this is not so. The reason is that this negative result would put very strong limits on MIO and FCIO mechanisms that would have great consequences on our expectation for MINOS. This situation would mean either that \( \Delta m^2 < 2 \times 10^{-3} \text{ eV}^2 \) for MIO or that \( \epsilon < 0.77 \) for FCIO, so while for the MIO mechanism this would imply that MINOS should also see a signal compatible with no-oscillation for the FCIO mechanism MINOS could still see something. Looking at Fig. 5(b), we observe that all the range \( 0.32 < \epsilon < 0.77 \) can be tested by MINOS independently of the value taken by \( \epsilon' \). For \( \epsilon < 0.32 \) one would still expected some small distortion effect on the MINOS nc-event spectrum. By comparing the maximal values attained by \( \frac{N_{\text{osc}}}{N_{\text{no-osc}}^{\text{nc}}} \) we see that it may be still possible to cover a extended range of \( \epsilon \), depending on \( \epsilon' \). The more spectacular effect is expected to occur near the point \( \epsilon \sim 0.6 \), where an almost complete conversion of muon neutrinos to tau neutrinos occurs.

A second possibility is that K2K measures a positive signal of oscillation. It is clear that if K2K measures a positive signal of \( \nu_\mu \) flux suppression, which means \( \Delta m^2 > 2 \times 10^{-3} \text{ eV}^2 \) for MIO and \( \epsilon > 0.77 \) for FCIO, MINOS will be able to confirm the FCIO solution or discard it completely. We see in Fig. 5(b) that in this case one should expect a striking signal for the ratio \( R_{\text{osc}}^{\text{nc/cc}} \) at MINOS if \( 0.77 \leq \epsilon \leq 0.9 \), independent of \( \epsilon' \). Also if \( \epsilon > 0.9 \) the energy spectrum ratios \( \frac{N_{\text{osc}}^{\text{cc}}}{N_{\text{no-osc}}^{\text{cc}}} \) and \( \frac{N_{\text{osc}}^{\text{nc}}}{N_{\text{no-osc}}^{\text{nc}}} \) will present for MINOS a very distinct effect for FCIO.

Finally the third possibility. K2K observes a stronger signal than the maximum expected for any of the two mechanisms, i.e. if the averaged conversion probability is found to be much higher than 0.69. In this case neither MIO nor FCIO can explain such a copious \( \nu_\mu \) disappearance. This could be a signal of an additional disappearance channel for \( \nu_\mu \) and could imply in a not so negligible contribution of \( \nu_\mu \rightarrow \nu_e \) transitions.

Its clear that the calculations we have performed should be repeated by the experimentalists, taking into account efficiencies and other detector dependent corrections, when this will be known, to compute more realistic bounds. They should not be very far from the values given here. We conclude that K2K and MINOS will be able, in most envisaged situations, to pin down which of the two mechanisms FCIO or MIO really takes place in nature.
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