A diagram associated with the subconstituent algebra of a distance-regular graph

Supalak Sumalroj *

Department of Mathematics, Naresuan University, Phitsanulok, Thailand

Received 20 December 2017, accepted 6 June 2019, published online 18 September 2019

Abstract

In this paper we consider a distance-regular graph $\Gamma$. Fix a vertex $x$ of $\Gamma$ and consider the corresponding subconstituent algebra $T = T(x)$. The algebra $T$ is the $\mathbb{C}$-algebra generated by the Bose-Mesner algebra $M$ of $\Gamma$ and the dual Bose-Mesner algebra $M^*$ of $\Gamma$ with respect to $x$. We consider the subspaces $M, M^*, MM^*, M^*M, MM^*M, M^*MM^*, \ldots$ along with their intersections and sums. In our notation, $MM^*$ means Span$\{RS | R \in M, S \in M^*\}$, and so on. We introduce a diagram that describes how these subspaces are related. We describe in detail that part of the diagram up to $MM^* + M^*M$. For each subspace $U$ shown in this part of the diagram, we display an orthogonal basis for $U$ along with the dimension of $U$. For an edge $U \subseteq W$ from this part of the diagram, we display an orthogonal basis for the orthogonal complement of $U$ in $W$ along with the dimension of this orthogonal complement.

Keywords: Subconstituent algebra, Terwilliger algebra, distance-regular graph.

Math. Subj. Class.: 05E30

*The author would like to thank Professor Paul Terwilliger for many valuable ideas and insightful suggestions on my work. This paper was written while the author was an Honorary Fellow at the University of Wisconsin-Madison supported by the Development and Promotion of Science and Technology Talents (DPST) Project, Thailand.

E-mail address: supalaks@nu.ac.th (Supalak Sumalroj)
Diagram prirejen Terwilligerjevi algebri razdaljno-regularnega grafa

Supalak Sumalroj *

Department of Mathematics, Naresuan University, Phitsanulok, Thailand

Prejeto 20. decembra 2016, sprejeto 6. junija 2019, objavljeno na spletu 18. septembra 2019

Povzetek

V tem članku obravnavamo razdaljno-regularen graf $\Gamma$. Fiksiramo vozlišče $x$ grafa $\Gamma$ in obravnavamo prirejeno Terwilligerjevo algebro $T = T(x)$. Algebra $T$ je $\mathbb{C}$-algebra, generirana z Bose-Mesnerjevo algebro $M$ grafa $\Gamma$ in dualno Bose-Mesnerjevo algebro $M^*$ grafa $\Gamma$ glede na $x$. Obravnavamo podprostore $M, M^*, MM^*, M^* M, MM^* M, M^* MM^*, \ldots$ ter njihove preseke in vsote. Pri tem $MM^*$ pomeni $\text{Span}\{RS \mid R \in M, S \in M^*\}$ in tako dalje. Uvedemo diagram, ki opisuje, kako so ti podprostori povezani med seboj. Podrobno opišemo del tega diagrama vse do $MM^* + M^* M$. Za vsak podprostor $U$, prikazan v tem delu diagrama, podamo njegovo ortogonalno bazo ter njegovo dimenzijo. Za povezavo $U \subseteq W$ iz tega dela diagrama podamo ortogonalno bazo ortogonalnega komplementa podprostora $U$ v prostoru $W$ ter dimenzijo tega ortogonalnega komplementa.

Ključne besede: Podkonstitutivna algebra, Terwilligerjeva algebra, razdaljno-regularen graf.

Math. Subj. Class.: 05E30

* Avtorica bi se rada zahvalila profesorju Paulu Terwilligerju za mnoge dragocene ideje in pronicljive predloge v zvezi z mojim delom. Ta članek je bila napisana, ko je bila avtorica častna študijantka na University of Wisconsin-Madison, podprta s strani Development and Promotion of Science and Technology Talents (DPST) Project, Thailand.

E-poštni naslov: supalaks@nu.ac.th (Supalak Sumalroj)

To delo je objavljeno pod licenco https://creativecommons.org/licenses/by/4.0/