The problem of the solution formalizing of determining the error of turning due to elastic deformations of the technological system

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Abstract. The paper raises the problems of formalizing the solution to the task of calculating the accuracy of turning due to the elastic displacements of the technological system elements. It is proposed to present the accuracy parameter of turning in the form of a technological limitation, which should be taken into account when optimizing processing modes. This allows getting a clear algorithm that can be used as part of CAD TP or digital production system. The paper also discusses ways to address the issue related to ensuring the accuracy of the calculation of turning errors.

1. Introduction
In the conditions of modern engineering production, information technologies, computer-aided design systems are widely used, and the concept of Industry 4.0 involves the exchange of data between individual production units and decision-making without human intervention. CNC machines equipped with special sensors allow receiving and processing data on various parameters of the cutting process in real time, quickly responding to their change with the appropriate correction input. However, to ensure reliable and trouble-free operation of the equipment in automatic mode, clear algorithms for working out various scenarios are necessary, which implies the presence of a formalized solution to the problems associated with controlling the cutting process.

2. The direct task of determining the error of turning due to the elastic displacements of the elements of the technological system
The result of solving the direct problem is the total displacement $y_{sup i}$ of the elements of the technological system during turning in the coordinate of the cutter $x_i$ under the action of the radial component of the cutting force $P_y$, which depends on the flexibility $w_{blank}$ of the workpiece, the rigidity of the headstock $j_f$, tailstock $j_b$, and the support $j_{sup}$ [1]:

$$y_{sum i} = P_y \left[ w_{blank i} + \left( \frac{L - x_i}{L} \right)^2 \cdot \frac{1}{j_f} + \left( \frac{x_i}{L} \right)^2 \cdot \frac{1}{j_b} + \frac{1}{j_{sup}} \right], \quad (1)$$

where $L$ – workpiece length between fixing points, mm.
Moreover, in literature [1, 2] there is no clear algorithm for further actions: how to consider the influence of these biases theoretically and in practice. In [2], it is stated that the actual size $d_{\text{real}}$ of machining should be considered as:

$$d_{\text{real}} = d_0 + 2 \cdot y_{\text{max}},$$

where $d_0$ – diameter of adjustment, mm.

I.e., it is understood that this error should be compensated by the dimensional setting of the machine. However, in addition to dimensional accuracy, there are shape errors (for example, deviation from cylindricity) that are not taken into account in this approach. Moreover, when machining parts of low rigidity (for example, long shafts), the actual deviation of the shape, due to the deflection of the workpiece, can reach values comparable to the tolerance of the shape and even the dimensional tolerance. So, with longitudinal turning of a workpiece made of 40X steel with a diameter of 40 mm and a length of 500 mm, a carbide cutter from T5K10 in centers with cutting modes: $t = 1$ mm, $S = 0.3$ mm / rev, $v = 110$ m / min (see Figure 1).

**Figure 1.** The results of calculating the elastic displacements of the elements of the technological system

The graph shows that the magnitude of the elastic displacements of the headstock and tailstock, the deflection of the workpiece are unstable and, at the same time, significantly affect the total amount of displacement. In this case, this effect is almost 35% of $y_{\text{max}}$. The absolute value of the displacement of these elements ($y_f, y_b, y_{\text{blank}}$) in radial terms is 47 μm, and the shape tolerance for the normal level of relative geometric accuracy at IT10 will be 30 μm. Thus, only due to the effect of these factors, the processing error will not allow to fulfill the specified shape tolerance, and other errors will also be superimposed on it, i.e., temperature deformations, random errors, errors due to wear of the cutting tool, etc.

### 2.1. Inaccuracies in design models of cutting force

Another problem is the determination of the radial component of the cutting force. According to modern concepts [3, 4], the cutting force depends on the following parameters:

- cutting modes ($v$ - cutting speed, m / min; $S$ - feed, mm / rev; $t$ - cutting depth, mm);
• physico-mechanical and thermophysical properties of the materials of the tool and the workpiece (strength, hardness, thermal conductivity);
• geometric parameters of the cutting tool (sharpening angles, radius at the tip of the cutter, etc.).

Papers [3, 4] report that the errors of traditional calculation models [5] of the cutting force can reach 100% or more with respect to the experimentally obtained values. The causes of these errors are described in detail in [3]. In particular, the following points can be noted:

- reference calculation models do not take into account the influence of the thermal conductivity of materials of contact pairs (cutting tools and workpieces) on the value of cutting force;
- the theoretical calculation does not take into account the real physico-mechanical and thermophysical properties of the materials of contact pairs, the spread of which is caused, for example, by the metallurgical tolerance on the chemical composition of steel and carbide;
- reference models, among other things, give inconsistent results, i.e. depending on the literature used, completely different calculation results can be obtained (and the difference can reach 100% or more). Possible reasons for such mutual differences between the models presented in various reference books are: features of statistical processing and grouping of data, sample sizes, quality of materials used (steels and hard alloys), the number of suppliers and supply lots, etc.

Therefore, one of the key areas in solving the problem of formalizing precision calculations is the development of reliable and adequate models for calculating the cutting force, which will allow design calculations, as well as adjust the cutting parameters during processing.

In particular, in [3, 4], models are proposed for calculating the radial component of the cutting force based on preliminary diagnostics of the properties of contact pairs of materials (workpiece and tool). Such diagnostics are supposed to be carried out at the stage of input control, after which the workpieces and tools should be sorted, marked and completed for entry to workplaces by means of an electronic warehouse system (which fully corresponds to the provisions of the industry 4.0 concept).

For example, according to [4], when machining structural steels with a carbide tool, it is recommended to determine the radial cutting force by the formula:

\[ P_y = 156 \cdot E_{emf}^{1.3} \cdot f_l \cdot S^{0.5} \cdot \nu^{-0.3}, \]  

where \( E_{emf} \) – the magnitude of the thermoEMF of the test passage, mV.

In design calculations, the upper limit reference value \( E_{emf} \), and, consequently, the cutting force should be used, since the most unfavorable outcome must be taken into account.

Of course, there are other methods for determining the cutting force, for example, direct measurement during processing. However, to create a high-quality system consisting of CAD and a digital production system, according to the authors of this article, it is advisable to use models that can be used both at the design stage and as part of adaptive cutting process control systems.

2.2. Problems of accounting for rigidity parameters of machine equipment

Another important problem when calculating the processing error is the lack of a clear and unambiguous method for tracking the state of the stiffness parameters of the elements of the technological system. It is known [1, 5–7] that there are parameters of dynamic \( j_{dyn} \) (determined during the process or from the results of processing [6]) and static \( j_s \) (determined during the loading by static force) of rigidity of machine elements. In the calculations, it is convenient to use the parameter of static rigidity, since this allows one to take into account the element-wise influence of all component errors on the result. Static stiffness is related to dynamic as follows:

\[ j_s = k_{dy} \cdot j_{dy}, \]  

Moreover, in the literature [1] it is indicated that \( j_s = 1.2 \ldots 2.0 j_{dy} \). The exact ratio largely depends on the technical condition of the machine tool equipment, wear of the rubbing parts, clearances in the
joints, etc. In addition, there are difficulties in decomposing the total value of $j_{\text{dyn}}$ into components (stiffness of the headstock, tailstock, support) [6, 7].

The values of static and dynamic stiffness can be determined experimentally for specific equipment in a specific period of time. They must not be less than the permissible values specified in the instruction manual (passport). The dynamic coefficient $k_{\text{dyn}}$ is also determined experimentally:

$$ k_{\text{dyn}} = \frac{j_{\text{st test}}}{j_{\text{dyn test}}}. $$

Thus, after the initial (after the installation of a new machine), next (periodic) or extraordinary (after breakdown, malfunction, and other reasons) technical diagnostics of the machine equipment, information on the real values of $j_{\text{dyn test}}$ and $j_{\text{st test}}$ should be entered in the passport or in the machine operation log. If these values turned out to be less than the passport ones, then equipment repair or replacement is necessary. Based on the foregoing, the following conclusion can be made - a clear methodology for tracking the current state of machine equipment is needed, which minimizes the likelihood of real values of rigidity of machine elements beyond the certified values. At present, in real production in Russia, this issue is quite acute, because there is no regulatory and technical documentation that strictly regulates the frequency of technical diagnostics of the equipment of the machine park and its methodology, and the requirements of operating manuals are far from being always met.

3. The algorithm for solving the inverse problem

From the point of view of formalization, it is convenient to talk about solving the problem in its inverse formulation: to determine the permissible cutting conditions and the size of the setting at which the specified processing accuracy will be ensured. In this case, the cutter feed $S$ and the cutting depth $t$ can act as such parameters. As a parameter with which you can control the accuracy of processing (including adjusting it directly during the cutting process), it is advisable to take the feed. The cutting depth can be used at the stage of design calculations, if with the chosen fixing scheme, accuracy cannot be ensured only by limiting the feed rate, and changing the fixing scheme for any reason is undesirable.

An enlarged algorithm for solving the inverse problem is presented in Figure 2. It should be noted that this approach is convenient in that it allows you to embed the solution algorithm as part of the optimization module for cutting conditions in the form of an additional limitation ‘technological limit because of accuracy’ $S_{\text{ly}}$. In this form, this algorithm can be used both in CAD TP and in digital production systems.
Figure 2. Algorithm for solving the inverse problem of ensuring the accuracy of turning.

The algorithm provides the ability to automatically obtain source data from the corresponding "electronic passports", which are a kind of database and are components of a data bank of a digital production system of an enterprise (workshop, site). For example, an electronic passport of a technological process includes data on cutting conditions, a workpiece fixing scheme, part geometrical parameters, required accuracy, etc.

It should be noted that in addition to technological limitations, the algorithm involves calculating the dimensional static setting $\Delta d_0$, which depends on the elastic displacement of the machine support under the action of $P_y$. This parameter is proposed to be used when setting the machine to size. Thus, this approach gives recommendations for specific actions to account for and compensate for errors, while direct calculation gives only information about what the amount of displacement under the influence of the cutting force will be.

3.1. Processing accuracy as a technological limitation when optimizing cutting conditions

In view of the foregoing, for example, for the case of fixing in the centers during turning of structural steel with a carbide cutter, you can write an expression to determine the allowable force $P_{yi}$ in the current coordinate of the cutter:

$$P_{yi} \leq \frac{[y]}{y_{blank} + y_f + y_b}.$$  \hspace{1cm} (6)

Expressing the feed from formula (3) and writing it on the left side of the inequality, we can obtain a valid feed value in the current coordinate of the cutter according to the conditions of processing accuracy:

$$S_i \leq S_{[y_i]} = 0.5 \cdot \left[ \frac{[y]}{ (y_{blank} + y_f + y_b) \cdot 156 \cdot E_{emf} \cdot t^1 \cdot v^{0.3}} \right].$$  \hspace{1cm} (7)

By analogy, expressions can also be obtained for other pinning schemes. In the case of the design calculation, it is advisable to talk about the minimum allowable feed:

$$S \leq S_{[y]} = \min \{S_{[y_i]} \}.$$  \hspace{1cm} (8)

It should be noted that the minimum feed values will be obtained at the points of maximum displacements $y_{max}$ of the elements of the technological system. So when processing a relatively non-
rigid workpiece (the shaft length is greater than the accepted diameter by 10 or more times) for the case of fixing in the centers, the maximum displacement will be observed in the coordinate \( x = L / 2 \).

When machining a very stiff workpiece - in the coordinates \( x = 0 \) or \( x = L \), depending on the ratio of the stiffness values of the front and tailstock, respectively. Other options may be observed, depending on the ratio of the stiffness of the elements of the technological system (machine, tool and workpiece).

The permissible offset \([y]\) should be assigned depending on the specified tolerance of the size \( T_d \) and the tolerance of the shape \( T_f \) (cylindricity or radial run-out). If the deviation of the shape is not indicated, then for the normal level of relative geometric accuracy, the following formula can accept [5]:

\[
[y] \leq T_f \approx 0.3 \cdot T_d.
\]  \hspace{1cm} (9)

Naturally, it must be understood that in addition to the elastic displacements of the elements of the technological system under the action of the radial component of the cutting force, there are other processing errors: random, errors due to temperature deformations, errors due to wear of the cutting tool, etc. The influence of some of them can be reduced: for example, the error due to temperature deformations can be reduced through the use of coolant; the influence of errors due to tool wear can be compensated for by using special routines and sensors of CNC equipment, etc. At the same time, errors in calculating the error due to deformations caused by the radial component of the cutting force must be taken into account, since their influence is difficult to compensate for, and the absolute values of elastic displacements are comparable with the shape tolerance, and sometimes with the dimensional tolerance.

3.2. Static machine setting

The static setting should be calculated taking into account the feed restriction (8), as well as according to the results of optimization of cutting conditions, which is provided by the algorithm (Figure 2). Static tuning is the difference between the nominal size indicated in the drawing and the actual size for which the machine should be tuned. The value of the static setting is proposed to be determined as follows:

\[
\Delta d_0 = 2 \cdot \frac{P_d}{J_{sup}}.
\]  \hspace{1cm} (10)

3.3. Analysis of the results of solving the inverse problem

The initial data are as follows: workpiece fixing scheme – ‘in the centers’; straight turning tool with geometry: \( \varphi = \varphi_1 = 45^0 \), \( \gamma = 0^0 \), \( \lambda = 0^0 \), \( r = 1 \text{ mm} \); cutter material: T5K10; workpiece material: steel 40X; processing modes: \( V = 110 \text{ m} / \text{min} \), \( S_0 = 0.3 \text{ mm} / \text{rev} \), \( t = 1 \text{ mm} \); workpiece geometry: \( \phi 40 \times 500 \text{ mm} \); the rigidity of the elements of the technological system is adopted according to the passport of a screw-cutting machine 16K20. Size tolerance is according to IT10.

As a result of the calculation, the following data were obtained: estimated allowable feed \( S_{\sup{y}} = 0.06 \text{ mm} / \text{rev} \); size adjustment \( \Delta d_0 = 64 \mu \text{m} \); setting diameter \( d_0 = 39.936 \text{ mm} \).

The calculated maximum absolute value of the displacements not taken into account by the static setting is 21 \mu \text{m}, which is less than the shape tolerance \( T_f = 30 \mu \text{m} \).

The experimental verification was carried out on a 16K20F3 lathe, the static rigidity of which was tested for compliance with the passport values. During the test, the radial runout was measured in two mutually perpendicular sections of the shaft with a pitch of 25 mm in the longitudinal direction. The range of maximum values of radial runout with a tenfold repetition of the experiment was 18 ... 33 \mu \text{m}, which generally corresponds to a given shape tolerance of 30 \mu \text{m}.

The spread is due to the effect of unaccounted for errors and uneven machining allowance, as well as the use of a static stiffness indicator, rather than a dynamic one. It would be incorrect to evaluate the relative and absolute errors in this case, since we are not talking about precisely calculated values, but about limit and permissible values.
4. Conclusion
The article discusses the problems of formalizing the solution to the problem of calculating the errors of turning caused by elastic deformations of the technological system. Options for solving these issues are proposed, in particular, the use of alternative modern approaches for calculating the radial component of the cutting force. It is shown that one of the key is the problem of determining the static and dynamic stiffness of machine tool equipment, as well as the issue of accounting and compensation for the movements of various components of the technological system.

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