Crust breaking and the limiting rotational frequency of neutron stars

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The limiting rotational frequency of neutron stars may be determined by the strength of their crusts. As a star spins up from accretion, centrifugal forces will cause the crust to fail. If the crust breaks unevenly, a rotating mass quadrupole moment will radiate gravitational waves (GW). This radiation can prevent further spin up and may be a promising source for continuous GW searches. We calculate that for a breaking strain (strength) of neutron star crust that is consistent with molecular dynamics simulations, the crust may fail at rotational frequencies in agreement with observations.

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Some neutron stars (NS’s) are known to rotate rapidly.[1] Initially slowly rotating NS’s in accreting systems can increase their rotational rate as material falling from a binary companion transfers angular momentum. However, most of the observed rapidly rotating NS’s spin only at about half of the Keplerian break-up frequency.

They are thought to have reached a spin equilibrium, where the angular momentum gained from accretion is balanced out by the angular momentum radiated via either gravitational waves (GW’s) or magnetic dipole radiation (See Ref.[2] and references therein). In this Letter, we argue that the limit on the spin frequency of NS’s could be set by the strength of the NS crust.

In the last few decades of the 20th century many works have been devoted to crust breaking which was explored as a possible source of starquakes and pulsar glitches.[3–5]. As the NS in the accreted binary system spins up—or spins down in the case of isolated pulsars—its equilibrium shape changes and stresses develop in the crust.[6, 7]. A sufficiently large stress can break the crust and change the moment of inertia of the star causing glitches in its rotation rate. It was recently found, however, that the crust is likely to be very strong, with a breaking strain (fractional deformation) of $\sigma \sim 0.1$.[8, 9]. The crust is strong because high pressure suppresses the formation of voids and fractures and because long range coulomb interactions insure that each ion is “bound” to many other ions. The breaking strain is much larger than previously thought $\sigma = 10^{-4} - 10^{-2}$[10]. The failure of a weak crust could explain some (but not all) of the sudden jumps in the rotational frequencies of pulsars. However, a strong crust allows the star to sufficiently deform before it may undergo failure. To the best of our knowledge, no previous work has examined the change in rotational frequency that is necessary for a strong crust to fail.

A strong crust may allow a star to be spun up considerably before the crust fails. When a region of the crust fails, it may move under centrifugal force to larger radii and produce a significant ellipticity $\epsilon = (I_{xx} - I_{yy})/I_{zz}$. This is the fractional difference in the principle moments of inertia. A strong crust can support an ellipticity as large as $\epsilon \sim 10^{-5}$.[8, 11].

When the crust starts to break, we do not know how much of the crust will fail. If even a small fraction of the crust fails, it could produce an $\epsilon \sim 10^{-8}$. This $\epsilon$ can radiate GW and lead to torque balance where the angular momentum gained from accretion is radiated as GW.[2]. This will prevent the star from spinning up further.

In this Letter we suggest that the strength of the crust determines the limiting rotational frequency of neutron stars. A star can spin up to a frequency where the crust first starts to fail. When the crust does break, it is likely to break unevenly, for example only in a small region on one side of the star. This produces a nonzero $\epsilon$ so that the star radiates GW and is prevented from spinning up further. Below we calculate that for $\sigma$ of order 0.1, the crust may first break at a limiting rotational frequency of 300-700 Hz. This is consistent with observations. Note that our crust breaking mechanism is distinct from Bildsten’s idea of deformations from nonuniform electron captures[12, 13] and does not require a temperature gradient.

There have been many searches for continuous GW from rotating stars[14], including from known pulsars, see for example[15], and from accreting neutron stars[16, 17]. These searches will be extended in the near future with Advanced LIGO and VIRGO data[15, 18] or by the use of third generation GW detectors[19].

We follow the same formalism of quaking neutron stars as outlined in Ref.[27]. The star is modeled as a two-component homogeneous spheroid: a crust of uniform density that rests on top of a core of an incompressible fluid. As the star spins up (or down) a solid crust attempts to re-adjust its shape, which in turn builds stresses in the crust. The local distortion of the solid
is given by the strain tensor \[20^\text{\textsuperscript{\textcircled{a}}}:\]

\[u_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right),\]

(1)

where \(\mathbf{u}(\mathbf{r})\) is the displacement field in a material. The strain tensor can be diagonalized in a local coordinate system, where positive eigenvalues represent compression whereas negative eigenvalues represent dilation along the respective axes. One can then define the strain angle as the difference between the largest and the smallest eigenvalues \(\lambda_{\text{ii}},\) \(\text{\textit{i.e.}}\)

\[\varepsilon \equiv \lambda_{\text{max}} - \lambda_{\text{min}}.\]

(2)

The NS crust breaks when the local strain angle in the stress plane reaches a critical value set by the breaking strain of the material, \(\varepsilon = \sigma.\)

The accumulation of strain in spin-down (-up) neutron stars is described in detail in Ref. \[7^\text{\textsuperscript{\textcircled{b}}}\]. Here we will outline the main equations needed to perform our calculations. We assume that initially the material is under great internal pressure but is unstrained. For self-gravitating objects, the equation for hydrostatic equilibrium of a rotating star depends on the material’s stress tensor, the local gravitational potential, and the centrifugal force. As the star spins slower (faster) the centrifugal force changes. The condition for hydrostatic equilibrium for this new configuration is then derived by considering a Lagrangian perturbation of the initial state. It is shown that the displacement field in the crust then can be solved using these expressions \[7^\text{\textsuperscript{\textcircled{b}}}:

\[u_r(r, \theta) = \left( ar - \frac{A}{7} r^3 - \frac{B}{2 r^2} + \frac{b}{r^4} \right) P_2(\theta),\]

(3)

\[u_\theta(r, \theta) = \left( \frac{ar}{2} - \frac{5}{42} A r^3 - \frac{b}{3 r^4} \right) \frac{dP_2(\theta)}{d\theta},\]

(4)

where \(r\) is the radial distance, \(P_2(\theta) = (3 \cos^2 \theta - 1)/2\) is the second Legendre polynomial, \(a, b, A,\) and \(B\) are constants that are determined using the following boundary conditions at the outer and inner boundaries of the crust:

\[a - \frac{8}{21} A R^2 - \frac{B}{2 R^3} + \frac{8 b}{3 R^5} = 0,\]

(5)

\[a - \frac{8}{21} A R^2 - \frac{B}{2 R^3} + \frac{8 b}{3 R^5} = 0,\]

(6)

\[-2 f'(R) = -\frac{2 v_K^2 f(R)}{5 c_t^3} = \frac{R^2 \Omega_i^2 - \Omega_i^2}{3 c_t^3} = \frac{A R^2 + \frac{B}{R^3}}{R^2},\]

(7)

Here \(R\) and \(R'\) are the total and core radii of the neutron star, respectively, \(v_K = \sqrt{GM/R}\) is the Keplerian velocity, \(c_t = \sqrt{\mu/\rho}\) is the transverse sound speed, \(\Omega_i\) and \(\Omega_t\) are the initial and final angular velocities, and \(f(r)\) is the radial part of the displacement field \(u_r(r, \theta).\) The transverse sound speed depends on the elastic shear modulus \(\mu\) \[21^\text{\textsuperscript{\textcircled{b}}}\] and the density \(\rho.\) Our calculations show that \(c_t \simeq 10^6\) m/s in most of the crust.

Solving the system of equations consisted of boundary conditions \[3^\text{\textsuperscript{\textcircled{c}}} - \[5\] we obtain the four coefficients \((a, A, b, B)\) that go into the equations for the displacement field, \(\text{i.e.\; Eqns. \[3^\text{\textsuperscript{\textcircled{c}}} - \[4\].\)}}\ The result is then applied to the equation for the strain tensor \[1.\) One can then find a coordinate system in which the strain tensor is diagonal, which gives the strain angle \(\varepsilon\) as described in Eq. \[2.\)

Instead of fixing the NS radius and crust thickness \[7^\text{\textsuperscript{\textcircled{b}}}\] we will consider some realistic cases by choosing several possible configurations stemming from current uncertainties in the equation of state (EOS) of neutron-star matter. The size of both the core and the crust is therefore determined by the EOS. In particular, we select the soft and the stiff EOS models by Hebeler et al. \[22.\] Here we consider the four models IU-FSU, IU-FSU\textsuperscript{max}, HLPS\textsuperscript{soft} and HLPS\textsuperscript{stiff}, respectively \[22.\] These two models should bracket current uncertainties in existing models of the nuclear EOS. The HLPS\textsuperscript{soft} predicts a compact star with \(R_{1.4} = 9.94\) km, whereas HLPS\textsuperscript{stiff} predicts a large star with \(R_{1.4} = 13.59\) km, where the index \(1.4\) refers to a canonical \(1.4M_\odot\) neutron star.

The size of the crust is crucial in our investigation. Although the core-crust transition pressures in both models are the same, \(P_i = 0.4054\) MeV fm\(^{-3}\), the corresponding crustal sizes differ because of the pressure gradient at the crust-core interface. Compact stars predicted by a soft EOS have larger pressure gradients at the crust-core interface due to their strong local gravity and therefore produce a thinner crust. Whereas the crust of large stars becomes thick as a result of the weaker gravity at the crust-core boundary. Alternatively, one can get a thick crust by using a moderate EOS that is not relatively stiff, but predicts a crust-core transition pressure that is large. For this reason, we also select two relativistic mean-field models, IU-FSU and IU-FSU\textsuperscript{max} \[23, 24\] which predict the same maximum stellar masses, but substantially different crust-core transition pressures, hence crust thicknesses. In particular, the IU-FSU predicts \(P_i = 0.2890\) MeV fm\(^{-3}\) and \(R_{1.4} = 12.49\) km, whereas the IU-FSU\textsuperscript{max} predicts \(P_i = 0.5184\) MeV fm\(^{-3}\) and \(R_{1.4} = 12.80\) km.

We note that the crust thickness is not only important for our problem, but also is important in modeling of the cooling of quiescent low-mass X-ray binaries, as the cooling timescale is proportional to the square of the crust thickness \[25.\] In Fig. 1 we plot the crustal mass \(M_{\text{crust}}\) (a) and the fraction of crustal thickness \(\Delta R_{\text{crust}}/R_{\text{tot}}\) (b) as a function of the total mass of neutron stars predicted by the four models. As evident from the figure, the crustal mass can be as low as about \(0.01M_\odot\) for compact stars and as large as \(0.04M_\odot\) for stars that have a thick crust.

Now we consider crust breaking for isolated stars that are born with some initial frequency and are spinning down due to \(\text{e.g.}\) magnetic dipole radiation. As the star loses angular momentum, the fluid in the core responds by reducing the equatorial bulge and it becomes
FIG. 1: (Color online). The neutron star crust mass $M_{\text{crust}}$ (a) and crust thickness fraction $\Delta R_{\text{crust}}/R_{\text{tot}}$ (b) as a function of the neutron star mass $M$.

FIG. 2: (Color online). (Left) Crust failure on accreting neutron star in arbitrary region $\phi$. (Right) Crust failure on isolated slowing down star. Motion in radial direction under centrifugal force is exaggerated.

more spherical. Strain develops in the solid crust that may eventually fail after losing support from the core, see Fig. 2 (Right). However, if the crust is strong enough, then there is a certain maximum initial frequency below which the crust will never break, even when the star has (nearly) stopped rotating. In Fig. 3 we plot this maximum initial frequency as a function of stellar mass for various EOS models, where we assume $\sigma = 0.1$ \cite{8, 9}. In Table I we provide predicted values of the maximum initial frequency for several NS masses. The solution suggests that the limiting spin frequency is proportional to the square root of the breaking strain, $f_{\text{max}} \propto \sqrt{\sigma}$, therefore results presented in Table I as well as Fig. 3 can be easily scaled to other values of $\sigma$. In particular, the crust of a canonical $1.4M_\odot$ NS, assuming the HLPSStiff EOS and a breaking strain of $\sigma = 0.1$, will never break if it is born with $f \lesssim 460$ Hz or correspondingly with the spin period of $P \gtrsim 2.17$ ms. This suggests that the crust of most isolated neutron stars may never have failed from slowing down.

Alternatively, if a NS is born with a large spin frequency above this value, the crust could break after the star has spun down significantly. A very large initial spin frequency could occur, for example, in a merger of two low mass NS that produces a stable very rapidly rotating NS as a remnant. If the crust then failed asymmetrically, the star will have a nonzero $\epsilon$ and will radiate gravitational waves. Thus young NSs, produced in exotic events with large initial spins, may be promising sources for continuous GW searches.

We now consider NSs in accreting systems that increase their rotational frequency via the transfer of angular momentum from the companion. For example, we consider the case of recycled pulsars. We consider a very simple model of how strain develops in the crust as the crust is being replaced. We assume the crust starts from zero strain at some initial frequency $f_{\text{in}}$ such that the critical strain is developed by the time the crust is fully replaced and the star has been spun up to $f_{\text{fin}}$.
work should explore the stress in partially replaced crusts in more detail.

The average spin frequency derivative of an accreting pulsar can be estimated using some simple accretion model [26]:

\[
\dot{f} = 2.3 \times 10^{-14} \sqrt{\xi} \mathcal{M}_{-10}^{5/7} M_{1.4}^{3/7} B_8^{2/7} R_{10}^{4/7} \text{Hz s}^{-1},
\]

where \(\xi \approx 0.3 - 1.0\) is a correction factor due to the non-spherical geometry of accretion, \(\mathcal{M}_{-10}\) is the mass accretion rate in units of \(10^{-10} M_\odot\) year\(^{-1}\), \(R_{10}\) is the radius of the neutron star in units of 10 km, and \(B_8\) is the surface magnetic field of the neutron star in units of 10\(^8\) Gs. For the sake of simplicity, we take \(\xi = 1, M_{-10} = 1,\) and \(B_8 = 1\). As neutron stars accrete materials, their initial spin frequency changes by \(\delta f \approx \dot{f} \Delta t\). The crust is fully replaced in a timescale of \(\Delta t \approx M_{\text{crust}} / M \text{ years}\). During this time period the spin frequency of the neutron star changes by \(\delta f\), and stresses develop in the newly formed crust. The largest strain angle develops at the crust-core boundary near the equator [7] and is therefore the first place where the crust fails. In Table II we present predictions for the initial \(f_{\text{in}}\) and final \(f_{\text{fin}}\) rotational frequencies of neutron stars for several stellar configurations and values of the breaking strain. The crust of compact stars—such as predicted by the HLPSSSoft—do not develop large strain angles and therefore require smaller values of the breaking strain in order to fail. In other words, with \(\sigma \approx 0.1\) the crust of a very compact star may never break. The effect of compactness is also clearly visible when results are compared between 1.4 and 1.8 \(M_\odot\) NSs whose radii are more or less constant. On the other hand, the crust of large stars (HLPSStiff), as well as those with a thick crust (IU-FSUmax) are more likely to fail even if the material breaking strain is large. In particular, for a canonical NS in the IU-FSU model, we find that an increase in rotational frequency from \(f_{\text{in}} = 349\) Hz to \(f_{\text{fin}} = 515\) Hz is needed before its crust is replaced and a strain angle of as large as \(\varepsilon = \sigma = 0.05\) is developed. Note that a larger value of the breaking strain shifts these values to higher frequencies (See Table II).

Instead of fixing the breaking strain, in Fig. 4 we now plot strain angle at the equatorial crust-core bound-

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Model} & \sigma & f_{\text{in}}^+ (\text{Hz}) & f_{\text{fin}}^+ (\text{Hz}) & f_{\text{in}}^- (\text{Hz}) & f_{\text{fin}}^- (\text{Hz}) \\
\hline
\text{HLPSStiff} & 0.05 & 0 & 326 & 35 & 368 \\
& 0.10 & 136 & 479 & 236 & 569 \\
& 0.05 & 349 & 515 & 909 & 1022 \\
\text{IU-FSU} & 0.10 & 781 & 947 & 1875 & 1988 \\
& 0.05 & 35 & 358 & 374 & 586 \\
& 0.10 & 232 & 555 & 854 & 1066 \\
\text{IU-FSUmax} & 0.05 & 0 & 326 & 35 & 368 \\
& 0.10 & 136 & 479 & 236 & 569 \\
& 0.05 & 349 & 515 & 909 & 1022 \\
\hline
\end{array}
\]

TABLE II: Initial and final rotational frequencies of accreting neutron stars whose crust breaks when the crust is fully replaced. The results are presented for breaking strains of \(\sigma = 0.05\) and 0.1 and for two different stellar masses of \(M = 1.4 M_\odot, 1.8 M_\odot\). The crust of compact stars as predicted by the HLPSSoft EOS never breaks for these values of \(\sigma\) and therefore results for this EOS are not listed.

FIG. 4: (Color online). The breaking strain \(\sigma\) as a function of the neutron star mass that allows the crust to fail when the final rotational frequency \(f_{\text{fin}} = 716.36\) Hz is equal to the maximum observed frequency.

In summary, we have explored the impact of the change in rotational frequency of NSs on the strong crust. We found that the crust of most isolated NSs may never fail. On the other hand, the crust of NSs in accreting systems is found to fail at rotational frequencies in agreement with observations. When the crust fails, a nonzero \(\epsilon\) is likely to be produced and the star will start to radiate GW, see Fig. 2 (Left). Torque from the GW radiation is a stiff function of the rotational frequency. If the initial GW torque is larger than the accretion torque, the star will continue to spin up slightly. Alternatively, if the initial GW torque is smaller than the accretion torque, the star will continue to spin down slightly. In both cases the star is expected to reach equilibrium where the GW torque balances the accretion torque. This will leave the accreting star a potential promising source for present and future searches for continuous GW.
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