Scatterer Identification by Atomic Norm Minimization in Vehicular mm-Wave Propagation Channels

HERBERT GROLL\textsuperscript{1} (Member, IEEE), PETER GERSTOFT\textsuperscript{2} (Senior Member, IEEE), MARKUS HOFER\textsuperscript{3} (Member, IEEE), JIRI BLUMENSTEIN\textsuperscript{4} (Member, IEEE), THOMAS ZEMEN\textsuperscript{3} (Senior Member, IEEE), AND CHRISTOPH F. MECKLENBRÄUKER\textsuperscript{1} (Senior Member, IEEE)

\textsuperscript{1}Institute of Telecommunications, TU Wien, 1040 Vienna, Austria
\textsuperscript{2}Marine Physical Laboratory, University of California at San Diego, La Jolla, CA 92093, USA
\textsuperscript{3}AIT—Austrian Institute of Technology GmbH, 1210 Vienna, Austria
\textsuperscript{4}Department of Radio Electronics, Brno University of Technology, 61600 Brno, Czech Republic

Corresponding author: Herbert Groll (herbert.groll@tuwien.ac.at)

This work was supported in part by the Institute of Telecommunications through the Doctoral School “5G Internet of Things,” and in part by TU Wien Bibliothek through its Open Access Funding Program. The work of Markus Hofer and Thomas Zemen was supported by the Project DEDICATE under Principal Scientist Grant through the AIT—Austrian Institute of Technology. The work of Jiri Blumenstein was supported by the Project “Passive Crowd Monitoring System with Privacy by Design” under Grant VJ01030008.

\begin{abstract}
Sparse scatterer identification with atomic norm minimization (ANM) techniques in the delay-Doppler domain is investigated for a vehicle-to-infrastructure millimeter wave propagation channel. First, a two-dimensional ANM is formulated for jointly estimating the time-delays and Doppler frequencies associated with individual multipath components (MPCs) from short-time Fourier transformed measurements. The two-dimensional ANM is formulated as a semi-definite program and promotes sparsity in the delay-Doppler domain. The numerical complexity of the two-dimensional ANM limits the problem size which results in processing limitations on the time-frequency sample matrix size. Subsequently, a decoupled form of ANM is used together with a matrix pencil, allowing a larger sample matrix size. Simulations show that spatial clusters of a point-scatterers with small cluster spread are suitable to model specular reflection which result in significant MPCs and the successful extraction of their delay-Doppler parameters. The decoupled ANM is applied to vehicle-to-infrastructure channel sounder measurements in a sub-urban street in Vienna at 62 GHz. The obtained results show that the decoupled ANM successfully extracts the delay-Doppler parameters in high resolution for the channel’s significant MPCs.
\end{abstract}

\begin{IEEEkeywords}
Beyond 5G mobile communication, millimeter wave propagation, multipath channels, time-varying channels, vehicle-to-infrastructure connectivity.
\end{IEEEkeywords}

\section{I. INTRODUCTION}
Vehicular millimeter wave (mm-Wave) communication attracts increased interest recently. We note the efforts by IEEE 802.11bd and 3GPP NR V2X \cite{1, 2} to augment the available sub-6 GHz vehicular connectivity standards by high rate links. The available radio resources within the millimeter-wave spectrum enable high-throughput sensor data sharing among vehicles without consuming the limited sub-6 GHz resources for intelligent transport systems.

Many vehicular wireless communication channels have short stationarity times and limited stationarity bandwidths \cite{3, 4}. They are adequately modeled as randomly time-varying due to significant movements of receiver, transmitter, and interacting objects in the propagation environment, commonly known as scatterers at high-level characterization. The signal travels from transmitter (Tx) to receiver (Rx) via multiple propagation paths, where each path is a multipath component (MPC) \cite{5}. Any movements cause...
Doppler effects, which is a key characteristic of vehicular linear time-variant (LTV) channels [6]. Many movements, however, remain unknown and are fundamentally non-repeatable. This explains the need to characterize the vehicular LTV channel from a single measurement data set and raises challenges for channel identification and its validation from measurements [7].

Various low-complexity LTV channel models have been proposed to approximately describe the propagation effects on a transmitted waveform [8], [9]. Further, models with few parameters can be adequate for some propagation scenarios. For this reason, sparse estimation methods are attractive. For band-limited signals the LTV channel is well approximated by a basis expansion model with two-dimensional discrete prolate spheroidal basis and a low number of coefficients independent of the number of MPCs, which is advantageous in channel emulation [10] and estimation [11], [12]. However, these coefficients do not represent properties of individual MPCs.

Vehicular mm-Wave channels may potentially be modeled as (approximately) sparse in several domains: delay, Doppler, and directions of departure and arrival. Apart from the propagation scenario, the applicability of sparse models also depends on the separability of individual MPCs in those domains. Given sufficiently high bandwidth for resolving components in the delay domain, the significant MPCs may exhibit a sparse structure in delay. Conversely, given sufficiently large channel snapshot period for resolving components in the Doppler domain, the significant MPCs exhibit a sparse structure in Doppler [13].

The delay-Doppler domain allows a physically intuitive characterization of LTV channels with time-delays and Doppler shifts [6], where MPCs might be caused by physical objects present in the environment. In this domain, the channel is often compressible or approximately sparse [14], such that compressed sensing signal processing methods can leverage the sparse structure for estimation.

One sparsity-aware method with the aim to recover simple models from limited measurements is the framework of atomic norm minimization (ANM), a non-greedy method. Advantages of the ANM method are its gridless approach to the estimation of sparse models, its capability of automatic model selection, and its suitability for compressed sensing [15]. It can be understood as a generalization of the grid-based least absolute shrinkage and selection operator (LASSO) estimator to a gridless approach [15], [16]. ANM is also known as total-variation minimization [15] or the Beurling LASSO [17] in literature. Further, the formulation of ANM as an optimization problem allows to easily incorporate additional constraints to the original ANM [18]. Disadvantages of ANM are its unfavorable scaling of numerical complexity with problem size and the necessary signal separation conditions [17] to reach theoretical performance guarantees.

Popular sparse signal processing techniques for estimation are restricted to a predefined grid: e.g., orthogonal matching pursuit, basis pursuit, and LASSO, where an overcomplete basis or dictionary is essential for sparse recovery [19, Ch. 13]. Sparse methods also provide valuable tools in the noisy non-compressed sensing case for signal approximations [20], denoising [16], even for long-tailed data statistics [21]. Sparse channel estimation for time-varying channels is considered in [14] and [20]. Sparse approximations of vehicle-to-vehicle measurements with c-LASSO and oversampled discrete Fourier transform (DFT) basis is presented in [22]. The sparsity of wireless channels is often based solely on intuitive analysis and the lack of proper measures is discussed in [23] and [24] where a combination of indicators is employed to estimate channel sparsity.

State of the art wireless systems employ a probing signal to aid in channel estimation. The probing signal has finite bandwidth and snapshot period which leads to a fundamental uncertainty in the delay-Doppler domain [6]. Prior knowledge about the delay-Doppler characteristics of the propagation channel enables optimal design of such probing signal. Imposing a suitable sparsity constraint as prior information on the propagation channel enhances its recovery with super-resolution capability in the delay-Doppler domain [15], [25]. High-resolution delay-Doppler estimates enable improved cyclic prefix OFDM (CP-OFDM) channel estimates when only a few significant MPCs are present [26]. Furthermore, inferred propagation scenario geometry from delay-Doppler estimates aid in target tracking [27] and antenna beam alignment [28].

As the resolution of individual MPCs in delay becomes feasible due to increased bandwidth, delay estimation utilizes techniques similar to direction of arrival (DOA) estimation in array processing [29]. A gridless estimation of the DOA of sources from a sensor array via ANM is possible with an additional sparsity constraint [30]. An extension to non-uniform linear arrays via irregular Vandermonde decomposition is shown in [31]. The application of ANM for real array measurements is shown in [32]. DOA estimation with deep learning methods from single delay-Doppler snapshots in automotive radar is investigated in [33].

Delay-Doppler estimation from passive CP-OFDM radar signals is shown in [34] with basis pursuit. The authors of [18] and [35] employ the ANM for simulated data where model mismatch from varying delays is ignored. In contrast, we demonstrate delay-Doppler estimation with ANM on real data and simulation data. We think the employed simulation model based on spatial clusters of point-scatterers is a relevant scenario for vehicular mm-Wave communication.

A. SCIENTIFIC CONTRIBUTION
In this work we propose and develop ANM and decoupled ANM (D-ANM) for application to joint delay-Doppler estimation of MPCs for non-stationary vehicular mm-Wave channels. A summary of our contributions is:

- We formulate the delay-Doppler estimation as two-dimensional ANM and D-ANM problem based on channel sounding data.
We empirically analyze its performance by numerical channel simulations in the presence of a small number of spatial clusters of point-scatterers and in application to real-world mm-Wave measurement data, acquired in a vehicle-to-infrastructure environment.

We show, that ANM suffers from leakage effects similar to spectral analysis, possibly due to model mismatch.

We show that D-ANM is suitable for path identification in the delay-Doppler domain for cluster spreads smaller than the resolution of the system.

The known empirical value for the regularizer in D-ANM is a reasonable choice for good performance in scatterer identification.

Based on the vehicular mm-Wave measurement data, we show that the paths identified with D-ANM accurately model the time-variant channel parameters mean delay, delay spread, mean Doppler and Doppler spread.

**B. ORGANIZATION OF THE PAPER**

We define the sparse linear time-variant propagation channel and the delay-Doppler domain in Section II-A and introduce the channel sounding procedure in Section II-B. We formulate the atomic norm minimization algorithm for the delay-Doppler domain in Section II-C and the decoupled atomic norm minimization algorithm in Section II-D. The deduced parameter estimates from a sparse representation are defined in Section II-E. We define the simulation model for the joint delay-Doppler investigation in Section III-B and review a relevant vehicular measurement campaign in Section III-C. The corresponding results are shown in Section IV and we conclude the paper in Section V.

**C. NOTATION**

Bold lowercase denotes column vectors or sequences. Bold uppercase denote matrices.

denotes the Dirac delta function.

denotes the Kronecker product.

denotes the pseudo-inverse of a matrix.

denotes the complex conjugate transpose of .

denotes the complex conjugate transpose of .

denotes the Frobenius norm of .

denotes the trace of .

stacks all columns of to a single column vector.

constrains to be positive semidefinite (PSD).

is a Toeplitz matrix constructed from sequence .

is a block-Toeplitz matrix with Toeplitz blocks constructed from sequence .

**II. FUNDAMENTALS**

**A. LINEAR TIME-VARIANT CHANNEL MODEL**

For the input signal with corresponding Fourier transform the narrowband LTV channel input-output relation leads to the output signal [5, eq. (6.9)–(6.11)]

where is additive white Gaussian noise. The LTV channel is described by the time-varying impulse response and, equivalently, in the time-frequency domain by the time-varying transfer function . For a narrowband LTV system, it is adequate to approximate the Doppler effect as a frequency shift . The spreading function is a different representation which emphasizes time dispersion and frequency dispersion of the channel. A two-dimensional Fourier transform connects with [6, eq. (1.11)]

where and are both Fourier pairs.

A noise-free sparse LTV channel is modeled by a multipath delay-Doppler formulation of its impulse response with discrete MPCs [6, eq. (1.17)]

with complex-valued channel amplitude , delay , and Doppler frequency of the th MPC. This allows a simple spreading function such that (1) simplifies to the noise-free sparse time-varying transfer function

It is common to specialize the general model in (3) to a model with distinct groups of paths, e.g., 1 ≤ s < S are MPCs with at least a single bounce each and s = S is the line of sight (LOS) path (if present). This is a simple interpretation of plane waves arriving from different physical paths, where each path has a distinct complex-valued path amplitude , propagation delay , and Doppler shift . Our goal is to estimate the parameters , , , , , , , and , to completely describe the LTV channel within .
B. CHANNEL SOUNDING

Our approach to channel sounding is based on transmitting a known pilot signal and correlating it with the received signal at the receiver side. A multicarrier system’s transmit pilot signal \( s(t) \) is composed of known pilot symbols \( p[k, l] \) as

\[
s(t) = \sum_{k=0}^{N_v-1} \sum_{l=0}^{N_t-1} p[k, l] g_{k, l}(t),
\]

with transmit pulse \( g_{k, l}(t) = g(t-k\tau) e^{i2\pi f_{\Delta f} t}, \) where \( k \) is the discrete time index, \( l \) is the frequency index, \( f_{\Delta f} \) is the subcarrier spacing, and \( T = T_{CP} + \frac{1}{2\Delta f} \) is the symbol duration including cyclic prefix duration \( T_{CP} \). The receiver calculates the demodulated symbols \( r[k, l] \) from received signal \( r(t) \) with matched filtering with receive pulse \( g_{k, l}(t) = \gamma(t-k\tau) e^{i2\pi f_{\Delta f} t} \). For a multicarrier system with rectangular pulses, and a sufficiently dispersion-underspread channel [6, eq. (1.82)], estimates \( \hat{H}[k, l] \) for the LTV propagation channel transfer function \( H[k, l] \) are readily calculated from

\[
\hat{H}[k, l] = \frac{r[k, l]}{p[k, l]} = H[k, l] + z'[k, l],
\]

where the error term \( z'[k, l] \) includes noise, and self-interference. For channel sounding, a calibration procedure allows to compensate for the transfer functions of Tx and Rx analog frontends [13]. The symbol duration \( T \) limits the maximum unambiguous propagation delay, i.e., the maximum excess delay \( \tau_{\max} \) defined by the difference between maximum and minimum delays [5, Sec. 6.2.2]. On the other hand, shorter \( T \) reduces self-interference due to time-variance.

For \( \tau_{\max} \) and maximum Doppler shift \( v_{\max} \), the channel is sufficiently underspread if \( \tau_{\max} v_{\max} \ll 1 \), which is generally the case for vehicular wireless channels [6, Sec. 1.5.1].

From the discrete transfer function estimates \( \hat{H}[k, l] \) in (5) we define the time-frequncy limited time-varying discrete transfer function

\[
\hat{H}[t_b; k, l] = \hat{H}[k + \frac{l}{T_R} - \frac{N_v}{2}, l], \quad k = 0, \ldots, N_v - 1,
\]

with \( N_v \) channel snapshots in time at channel snapshot period \( T_R \), \( N_t \) samples in frequency, limited to observation time \( T_{obs} = N_v T_R \) and centered at time \( t_b \) and frequency \( f_c \). For each time-frequency region \( t_b \), we construct the corresponding sample matrix \( \mathbf{Y}[t_b] \in \mathbb{C}^{N_v \times N_t} \)

\[
\mathbf{Y} = \begin{bmatrix}
\hat{H}[t_b; 0, 0] & \ldots & \hat{H}[t_b; 0, N_t - 1] \\
\vdots & \ddots & \vdots \\
\hat{H}[t_b; N_v - 1, 0] & \ldots & \hat{H}[t_b; N_v - 1, N_t - 1]
\end{bmatrix},
\]

where we treat each time-frequency region \( t_b \) independent and write only \( \mathbf{Y} \) for ease of notation.

With regard to the discrete MPC model in (2), the delay parameter \( \tau(t) \) associated with MPCs varies only slowly with time. It follows from \( \tau_{\max} \ll 1 \) that phase drifts due to Doppler shifts in (3) allow a piecewise approximation \( e^{2\pi i v t} \approx e^{2\pi i v t}, \) for \( t \in [(k-1)T, k T] \).

For \( \tau_{\max} \ll 1 \), self-interference is negligible during the duration of one symbol [34], [36, Assumption A3]. Therefore, multiple measurements taken sequentially in time need to be acquired to resolve multiple Doppler frequencies sufficiently well [37].

Compared to (1), the sampled doubly selective channel \( H[k, l] \) is the two-dimensional DFT of a smoothed spreading function \( S_{t,v}[\nu', \nu''] \)

\[
H[k, l] = \sum_{\nu=0}^{N_v-1} \sum_{\nu'=0}^{N_v-1} S_{t,v}[\nu', \nu''] e^{2\pi i (\Delta v \nu' \tau - \Delta f' \nu'')},
\]

\[
S_{t,v}[\nu', \nu''] = \sum_{\nu=0}^{N_v-1} \sum_{\nu'=0}^{N_v-1} H[k, l] e^{2\pi i (\Delta v \nu' \tau - \Delta f' \nu'')},
\]

with delay index \( \nu' \), delay resolution \( \Delta \tau = \frac{1}{R} \), signal bandwidth \( B \), Doppler index \( \nu'' \), number of time snapshots \( N_v \), number of frequency samples \( N_t \), and Doppler resolution \( \Delta v = \frac{1}{N_v T_R} \). The DFT relation in (8) implies, that the channel is two-dimensional periodic, which is most likely not the case. Similar to \( S_{t,v}(\tau, v) \), the smoothed spreading function \( S_{t,v}[\nu', \nu''] \) describes the dispersion of the signal in the delay-Doppler domain and was found to have an approximate sparse support in vehicular mm-Wave scenarios due to employed directive antennas [38] and sufficient bandwidth for resolving arrivals in time [20].

Finite symbol lengths, limited bandwidth, employed pulse-shaping, and point-scatterer model mismatch lead to practical limits of a sparse representation of the spreading function [14]. Already for point-scatterers, grid-mismatch of the sampled wireless channel manifests itself as DFT leakage since their discrete delay and Doppler naturally are not exactly located on DFT bins.

A sparse representation of the approximate sparse spreading function is nonetheless desirable for simpler description of the propagation channel. A structured sparse channel model is advantageous in developing simpler channel tracking algorithms [20].

While the channel needs to fulfill the underspread property, the choice of sampling parameters for pilot-based channel sounding method in (6) define the limits for identifying delay and Doppler. For a lower rate of pilots in time, a channel snapshot period \( T_R \geq T \) is sufficient as long as \( v_{\max} \leq \frac{1}{T_R} \) is fulfilled [5, eq. (8.5)].

A suitable time-varying transfer function formulation of the discrete MPC model in (2) for \( v_{\max} T \ll 1 \) is

\[
\hat{H}(k T_R, l f) = \sum_{s=1}^{S} \sum_{v=0}^{N_v-1} \sum_{\nu=0}^{N_v-1} C_s e^{2\pi i (v k T_R - \nu l f)},
\]

For a bandwidth \( B \) and observation time \( T_{obs} \leq T_{stat} \), which ensures short time validity of the channel model (3) for the measured channel, we define the matrix \( \mathbf{X} \in \mathbb{C}^{N_v \times N_t} \)

\[
\mathbf{X} = \begin{bmatrix}
\hat{H}[0, 0] & \ldots & \hat{H}[0, N_t - 1] \\
\vdots & \ddots & \vdots \\
\hat{H}[N_v - 1, 0] & \ldots & \hat{H}[N_v - 1, N_t - 1]
\end{bmatrix},
\]
where $\tilde{H}[k, l] = H(kT_R, l\Delta f)$. With respect to the sampling parameters, we define the normalized delay $\bar{\tau} = \tau \Delta f \in [0, 1)$ and normalized Doppler $\bar{v} = v T_R \in [-0.5, 0.5)$, and define complex exponential basis vectors

\[
\begin{align*}
\mathbf{a}_\nu(\bar{v}) &= \begin{bmatrix} e^{j2\pi \bar{v}} & \ldots & e^{j2\pi (N_v-1)\bar{v}} \end{bmatrix}^T, \quad \bar{v} \in [-\frac{1}{2}, \frac{1}{2}), \\
\mathbf{a}_{\bar{\tau}}(\bar{\bar{v}}) &= \begin{bmatrix} e^{-j2\pi \bar{\bar{v}}} & \ldots & e^{-j2\pi (N_{\bar{\tau}}-1)\bar{\bar{v}}} \end{bmatrix}^T, \quad \bar{\bar{v}} \in [0, 1),
\end{align*}
\]

for time and frequency samples, respectively. The discrete MPC model (10) is then equivalent to the sum of $S$ two-dimensional complex exponentials

\[
\mathbf{X} = \sum_{s=1}^{S} c_s \mathbf{a}_\nu(\bar{v}_s) \mathbf{a}_{\bar{\tau}}(\bar{\bar{v}}_s)^T. \tag{11}
\]

### C. ATOMIC NORM

The linear signal model for an observed two-dimensional sample matrix $\mathbf{Y} \in \mathbb{C}^{N_v \times N_{\bar{\tau}}}$ is

\[
\mathbf{Y} = \mathbf{X} + \mathbf{Z},
\]

with unobserved two-dimensional complex exponential signal $\mathbf{X} \in \mathbb{C}^{N_v \times N_{\bar{\tau}}}$ as in (11) and additive noise $\mathbf{Z} \in \mathbb{C}^{N_v \times N_{\bar{\tau}}}$. The framework of atomic norm minimization helps in finding the sparsest representation of $\mathbf{X}$ with atoms from the atomic set $\mathcal{A}$, i.e., an overcomplete parametric dictionary defined as [39]

\[
\mathcal{A} = \{e^{j\varphi} \mathbf{a}_\nu(\bar{v}) \mathbf{a}_{\bar{\tau}}(\bar{\bar{v}})^T : \bar{v} \in [-\frac{1}{2}, \frac{1}{2}), \bar{\bar{v}} \in [0, 1), \varphi \in [0, 2\pi]\}.
\]

The continuous parameters $\bar{\tau}$ and $\bar{v}$ are restricted to the unambiguous intervals related to normalized delay and Doppler, see Sec. II-B. The atomic norm $\|\mathbf{X}\|_{\mathcal{A}}$, induced by the atomic set $\mathcal{A}$, is the function

\[
\|\mathbf{X}\|_{\mathcal{A}} = \inf\{w \geq 0 | \mathbf{X} \in w \cdot \text{conv}(\mathcal{A})\} \tag{13}
\]

which gauges the model order to express $\mathbf{X}$ as a convex combination with atoms of $\mathcal{A}$, i.e., the atomic decomposition (11), where $\text{conv}(\mathcal{A})$ is the convex hull of $\mathcal{A}$. Lagrangian duality theory is an important aspect in understanding the atomic norm. The dual of the atomic norm involves finding the maximum absolute value of a bounded complex trigonometric polynomial defined with respect to $\mathcal{A}$, for which there is an approximate semi-definite program (SDP) optimization solution for two-dimensions [15, 39].

The atomic norm in (13) is approximated by solving a SDP [39]

\[
2 \cdot \|\mathbf{X}\|_{\mathcal{A}} \approx \min_{\mathbf{X}, \mathbf{u}, \mathbf{w}} \frac{1}{N_v N_{\bar{\tau}}} \text{Tr}(\mathbf{T}_2(\mathbf{u})) + w, \quad \text{s.t.} \quad \begin{bmatrix} \mathbf{T}_2(\mathbf{u}) & \mathbf{\hat{X}} \end{bmatrix} \begin{bmatrix} \mathbf{\hat{X}}^H & \mathbf{w} \end{bmatrix} \succeq 0, \quad \mathbf{\hat{X}} = \mathbf{X}, \tag{14}
\]

where $\mathbf{T}_2(\mathbf{u}) \in \mathbb{C}^{N_v N_{\bar{\tau}} \times N_v N_{\bar{\tau}}}$ is a block-Toeplitz matrix with Toeplitz blocks, further described together with sequence $\mathbf{u}$ in Section II-C2, $w \in \mathbb{R}^+$ is a free optimization variable, ($\frac{1}{N_v N_{\bar{\tau}}} \text{Tr}(\mathbf{T}_2(\mathbf{u})) + w$) is the trace-norm$^1$ of the PSD matrix $\mathbf{X}$.

$^1$Also known as nuclear norm.

The atomic norm $\|\cdot\|_{\mathcal{A}}$ in (13) is already a convex relaxed version similar to the convex relaxation from $\ell_0$-norm to $\ell_1$-norm in sparse recovery algorithms [15], e.g. LASSO.

The underlying atoms of (11), i.e. $\mathbf{a}_\nu(\bar{v}) \mathbf{a}_{\bar{\tau}}(\bar{\bar{v}})^T$ from estimates $\bar{\tau}$ and $\bar{v}$, are retrieved only after evaluating a dual polynomial of the dual problem, finding the roots of the dual polynomial, or from Vandermonde decomposition [40] of $\mathbf{T}_2(\mathbf{u})$. For the last two cases, there is no need for model selection [15]. For Vandermonde decomposition, the number of atoms $S$ corresponds to the rank of $\mathbf{T}_2(\mathbf{u})$ [40].

After scaling of the normalized estimates according to the system parameters (see Sec. II-B), we derive the non-normalized estimates $\bar{\tau} = \bar{\tau}/\Delta f$ and $\bar{v} = \bar{v}/T_R$, which we don’t explicitly state from here on.

1) ATOMIC SOFT THRESHOLDING

In the presence of noise $\mathbf{Z}$, atomic norm minimization is accompanied with a denoising term [16] where a trade-off between measurement reconstruction of the observed sample matrix $\mathbf{Y}$ and sparsity is set with a regularization parameter $\mu$ which trades sparsity for data reconstruction. The ANM problem in (14) is modified through additional denoising, or soft thresholding, and can approximately be retrieved with the SDP [18, 39]

\[
\begin{array}{ll}
\min_{\mathbf{X}, \mathbf{u}, \mathbf{w}} & \mu' \text{Tr}(\mathbf{T}_2(\mathbf{u})) + \mu' w + \|\mathbf{\hat{X}} - \mathbf{Y}\|_F^2 \\
\text{s.t.} & \begin{bmatrix} \mathbf{T}_2(\mathbf{u}) & \mathbf{\hat{X}} \end{bmatrix} \begin{bmatrix} \mathbf{\hat{X}}^H & \mathbf{w} \end{bmatrix} \succeq 0,
\end{array} \tag{15}
\]

where $\|\mathbf{\hat{X}} - \mathbf{Y}\|_F$ measures quality of reconstruction and the remaining terms promote sparsity of the optimal solution. Although no model order selection is needed, the choice of regularizer $\mu'$ depends on both the noise model and noise level as shown for the one-dimensional case in [16] and empirically set to

\[
\mu'_e = \sigma Z \sqrt{N_v N_{\bar{\tau}}} \ln(N_v N_{\bar{\tau}}), \tag{16}
\]

for the two-dimensional case [18] for complex Gaussian matrix $\mathbf{Z}$ with independent and identically distributed (i.i.d.) entries $Z_{ij} \sim CN(0, \sigma^2 Z)$. In the following, we refer to ANM with soft thresholding in (15) solely as ANM, as our focus is on noisy observations.

2) TOEPLITZ STRUCTURE

The Hermitian block-Toeplitz matrix $\mathbf{T}_2(\mathbf{u})$, where each block is a Toeplitz matrix, arises from a single atom $\mathbf{a}_\nu(\bar{v}) \otimes \mathbf{a}_{\bar{\tau}}(\bar{\bar{v}})$ out of $\mathcal{A}$ in the form $\mathbf{a}_\nu(\bar{v}) \mathbf{a}_{\bar{\tau}}(\bar{\bar{v}})^H \otimes (\mathbf{a}_\nu(\bar{v}) \mathbf{a}_{\bar{\tau}}(\bar{\bar{v}})^H)^H$ [39]. The extension to the vectorized two-dimensional complex exponential matrix $\mathbf{vec}(\mathbf{X})$ leads to [40], a deterministic version of eq. (9)

\[
\mathbf{vec}(\mathbf{X})\mathbf{vec}(\mathbf{X})^H = \frac{1}{N_v N_{\bar{\tau}}} \text{Tr}(\mathbf{T}_2(\mathbf{u})) + w.
\]
where a block structure can be readily seen. Although the complex sequence block-Toeplitz matrix \( T \) itself a Toeplitz matrix \( T \), where \( \{a_t(\tau)\} \) is an \( N \times 1 \) vector parameterized \( N \) shots in time \( \tau \) off-diagonal and zeros elsewhere. Note that, \( T \) is unknown in (17), we can find a expression of (18) is

\[
T(\mathbf{u}) = \begin{bmatrix}
T_1(\mathbf{u}_0) & T_1(\mathbf{u}_1) & \ldots & T_1(\mathbf{u}_{N_v-1}) \\
T_1(\mathbf{u}_{-1}) & T_1(\mathbf{u}_0) & \ldots & T_1(\mathbf{u}_{N_v-2}) \\
\vdots & \vdots & \ddots & \vdots \\
T_1(\mathbf{u}_{-(N_v-1)}) & T_1(\mathbf{u}_{-(N_v-2)}) & \ldots & T_1(\mathbf{u}_{0})
\end{bmatrix}
\]  

(18)

where \( \mathbf{u}_k = \{u_{n_k}, n_{-k}\} \) is a subsequence of \( \mathbf{u} \) and each block itself a Toeplitz matrix \( T_1(\mathbf{u}_k) \), \( k = 0, \ldots, N_v - 1 \) fully parameterizes \( T_2(\mathbf{u}) \). The sum-of-Kronecker-products form of \( T_1(\mathbf{u}_k) \) is

\[
T_1(\mathbf{u}_k) = (u_{n_k}, n_{-k}) I_{N_v} + \sum_{l=1}^{N_v} (u_{l,k}\Theta^{(l)}_{N_v} + (u_{-l,k}\Theta^{(-l)}_{N_v})
\]  

(19)

where \( \Theta^{(k)}_{N_v} \in \mathbb{R}^{N_v \times N_v} \) is a matrix with all ones on the \( k \)-th off-diagonal and zeros elsewhere. Note that, \( \Theta^{(-k)}_{N_v} \) is the transpose of \( \Theta^{(k)}_{N_v} \). Due to the Hermitian property, \( T_1(\mathbf{u}_k) = T_1(\mathbf{u}_{-k})^H \). The sum-of-Kronecker-products form of (18) is

\[
T_2(\mathbf{u}) = \sum_{s=1}^{S} |c_s|^2 \text{vec}(a_s(\tilde{v}_s) a_s(\tilde{r}_s)^T) \\
\times \text{vec}(a_s(\tilde{v}_s) a_s(\tilde{r}_s)^T)^H
\]  

\[
= \sum_{s=1}^{S} |c_s|^2 [a_s(\tilde{r}_s) \otimes a_s(\tilde{v}_s)]^H
\]  

\[
= \sum_{s=1}^{S} |c_s|^2 [a_s(\tilde{r}_s) a_s(\tilde{r}_s)^T]^H
\]  

\[
\otimes [a_s(\tilde{v}_s) a_s(\tilde{v}_s)]^H.
\]  

(17)

The parameterization of \( T_2(\mathbf{u}) \) through the sequence \( \mathbf{u} \) limits the degrees of freedom of the PSD matrix in the optimization (15).

**D. DECOUPLED ATOMIC SOFT THRESHOLDING**

The SDP (15) quickly grows in complexity with both snapshots in time \( N_v \) or frequency samples \( N_r \), such that the Toeplitz matrix size in (18) increases beyond feasible size (See also Sec. III-D and Sec. IV-C).

The atomic norm minimization problem is efficiently approximately solved in a decoupled way [41], where instead of the atomic norm of the vectorized model matrix \( \text{vec}(\mathbf{X}) \) in (13), the atomic norm in matrix formulation is used. Taking \( N_v \) sequential snapshots equidistant in time of a signal with one-dimensional complex exponentials

\[
x[n_v] = \sum_{s=1}^{S} c_s[n_v] a_s(\tilde{r}_s),
\]  

(21)

the multiple measurement snapshot \( x[0] \ldots x[N_v - 1] \approx \mathbf{X} \) exhibits a two-dimensional structure as in (11) for \( c_s[0] \ldots c_s[N_v - 1] \approx c_s a_s(\tilde{v}_s) \) and stationary \( \tau_s \). This is applicable to the sample matrix \( \mathbf{Y} \) and model matrix \( \mathbf{X} \) and thus interpreted as a two-dimensional snapshot.

For the number of paths \( S \leq \min(N_v, N_r) \), the solution for the D-ANM problem is found with the SDP [41, eq. (28)]

\[
\begin{align*}
\text{argmin}_{\mathbf{X}, \mathbf{a}_s, \mathbf{a}_t} & \frac{\mu}{\sqrt{N_v N_r}} [\text{Tr}(T_1(\mathbf{u}_s)) + \text{Tr}(T_1(\mathbf{u}_t))] \\
& + ||\hat{\mathbf{X}} - \mathbf{X}||_F^2 \\
\text{s.t.} & \left[ T_1(\mathbf{u}_s) \hspace{1cm} \hat{\mathbf{X}} \hspace{1cm} T_1(\mathbf{u}_t) \right] \succeq 0.
\end{align*}
\]  

(22)

where \( T_1(\mathbf{u}_s) \in \mathbb{C}^{N_v \times N_v} \) and \( T_1(\mathbf{u}_t) \in \mathbb{C}^{N_v \times N_v} \) denote single level Toeplitz matrices with recoverable delay \( \tilde{r} \) and Doppler frequency \( \tilde{v} \) by separate Vandermonde decomposition of each Toeplitz matrix. The choice of regularizer \( \mu \) in (22) trades sparsity for data reconstruction, similar to ANM in (15). Furthermore, for D-ANM we employ the same empirical value \( \mu = \mu \) as for ANM.

Note the reduced size of the PSD constraint in (22) is \((N_v + N_r) \times (N_v + N_r) \) compared to the ANM formulation in (15) where it is \((N_v N_r + 1) \times (N_v N_r + 1) \). The model order \( S \) depends on rank(\( T_1(\mathbf{u}_s) \)), rank(\( T_1(\mathbf{u}_t) \), and on overlapping delay or Doppler of paths [41]. We use a matrix pencil method [40, eq. (33)] for Vandermonde decomposition and parameter recovery where the poles of the characteristic polynomial of \( T_1 \) are found as a solution of a generalized eigenvalue problem [42, eq. (13)]. An additional pairing step is necessary where we try all possible combinations of \( (\tilde{r}_i, \tilde{v}_j) \), \( i = 1, \ldots, S_r \), \( s = 1, \ldots, S_v \) and keep only the pairs with strongest contribution \( |a_s(\tilde{v}_j)^H \mathbf{X} a_s(\tilde{r}_i)^\ast| \), where each \( \tilde{r}_i \) and \( \tilde{v}_j \) appear only once. This approach is similar in the main peaks in the two-dimensional conventional beamformer (CBF). Estimates of \( S_r \) and \( S_v \) can be based on the significant eigenvalues of \( T_1(\mathbf{u}_s) \), and \( T_1(\mathbf{u}_t) \) respectively.

The applicability of D-ANM on the estimated time-varying transfer function matrix (see Sec. II-B) is justified due to the interpretation of the multiple snapshots of one-dimensional channel transfer functions as a single snapshot in a time-frequency region, where Doppler modulation introduces phase changes only visible across multiple snapshots, i.e. the second dimension.

**E. SPARSE REPRESENTATION**

The goal is to estimate delay and Doppler of the significant MPCs. As there is no ground truth other than the measured data, we construct a sparse representation in (9) based on
two-dimensional complex exponentials \( \mathbf{X} (11) \) and assess its similarity with the measured propagation channel based on the sample matrix \( \mathbf{Y} (7) \). Based on \( \mathbf{Y} \) in the SDP (15), we retrieve a block-Toeplitz matrix with Toeplitz blocks \( \mathbf{T}_2(\mathbf{u}) \) and \( \mathbf{X} \), a denoised version of the sample matrix. We assume \( \mathbf{T}_2(\mathbf{u}) \) to have rank \( \text{rank}(\mathbf{T}_2(\mathbf{u})) = R < \min(N_v, N_\nu) \) such that a Vandermonde decomposition exists [40]. We recover \( S = R \) delay-Doppler pairs \( \{\dot{\tau}_s, \dot{\nu}_s \mid s = 1, \ldots, S\} \) from \( \mathbf{T}_2(\mathbf{u}) \) with \( s = 1, \ldots, S \) with the matrix pencil and auto-pairing (MAP) method and construct a basis based on \( \mathbf{A}_f = [\mathbf{a}_1(\dot{\tau}_1) \ldots \mathbf{a}_r(\dot{\tau}_S)] \) and \( \mathbf{A}_\nu = [\mathbf{a}_1(\dot{\nu}_1) \ldots \mathbf{a}_r(\dot{\nu}_S)] \). For D-ANM, we construct the basis from the paired parameters as described previously.

To retrieve the still missing complex-valued path amplitudes \( \mathbf{c} = [c_1 \ldots c_S]^T \), it is common to employ a least-squares approach. An estimate \( \hat{\mathbf{c}} = [\hat{c}_1 \ldots \hat{c}_S]^T \) is retrieved from the sample matrix \( \mathbf{Y} \) as

\[
\hat{\mathbf{c}} = \mathbf{B}^T \text{vec}(\mathbf{Y}),
\]

where \( \mathbf{B} = [\mathbf{b}(\dot{\tau}_1, \dot{\nu}_1) \ldots \mathbf{b}(\dot{\tau}_S, \dot{\nu}_S)] \) with column-wise Kronecker products \( \mathbf{b}(\dot{\tau}_s, \dot{\nu}_s) = \mathbf{a}_1(\dot{\tau}_s) \otimes \mathbf{a}_r(\dot{\nu}_s) \).

An estimate for the discrete MPC model in (9), limited in time-frequency as in (11), is then

\[
\hat{\mathbf{H}} = \mathbf{A}_f \cdot \text{diag}(\hat{\mathbf{c}}) \mathbf{A}_\nu^T.
\]

The sparse representation of the matrix is now described by the parameter set \( \{\hat{\tau}_s, \hat{\nu}_s \mid s = 1, \ldots, S\} \) only. We define the normalized approximation error (NAE) as

\[
\text{NAE}(\hat{\mathbf{H}}; \mathbf{Y}) = \sqrt{E \left\{ \frac{\| \mathbf{Y} - \hat{\mathbf{H}} \|_F^2}{\| \mathbf{Y} \|_F^2} \right\}},
\]

as an error metric for measuring the quality of reconstruction.

### III. JOINT DELAY-DOPPLER SPARSITY

A low-complexity approximation of mm-Wave vehicular wireless channels is of interest for numerically efficient simulation and real-time emulation [43]. We observe from the local scattering function (LSF) of measurement data, that the channel is approximately sparse (“compressible”) in the delay-Doppler domain. A sparse approximation in delay-Doppler domain was shown with the c-LASSO in [44]. Vehicular wireless channels feature more discrete MPCs in the Doppler domain for communication at mm-Wave bands compared to the sub-6 GHz bands [13]. Since the support set of the LSF is only approximately sparse, the question arises how well (9) approximates the observed data for various choices of \( S \). How well this works with a sparse representation is answered in Section II-E. The parameter set should be compressed and still adequately describe the observed data.

For vehicular channels, geometry-based stochastic models divide the channel into different parts including LOS, discrete, and diffuse MPCs [45], [46]. For mm-Wave propagation, specular reflectivity is often lower in favor of diffuse reflectivity [47]. We suspect a significant influence of the scatterer model on the approximation performance. Therefore, we evaluate the performance on a geometric channel model where MPCs are modeled by spatial clusters of point-scatterers, which cause fading and broadening of MPCs in the delay and Doppler domain.

### A. RESOLUTION LIMIT

The capability of identifying MPCs depends on employed antennas, received signal strength, noise, and the resolution limit in the delay-Doppler domain, among others. Frequency and time spacing parameters \( \Delta f \) and \( T_R \) of the channel sounder define the delay-Doppler grid in this dual domain. Parameter estimation performance from spectral analysis of measurements in terms of resolution depends on the spectral properties of the employed frequency and time windows. The observed channel spectrum is then the convolution of the received signal and a window. For small window length, interpolation of the grid can lead to finer resolution when searching for the maximum peak of the spectrum.

This approach does not necessarily lead to better estimates for more than one closely located sources as their spectral peaks combine and become unresolvable. In array processing, the Rayleigh resolution limit describes the ability to resolve two plane waves of same magnitude impinging on a sensor array [29, p. 48]. Subspace methods like MUSIC and ESPRIT overcome the Rayleigh resolution limit [15], but they need multiple snapshots of the same time-frequency region which are not available for the vehicular setting.

Increasing the number of samples in time and frequency to increase resolution is desirable. For a fixed bandwidth and given maximum Doppler shift, sufficient subcarrier spacing \( \Delta f \) is guaranteed by ensuring \( \Delta f \gg v_{\text{max}} \). The multicarrier system’s employed rectangular transmit and receive filters (see Sec. II-B) implicitly set a frequency window. Therefore, power from one delay tap leaks to neighboring taps only with quadratic decay [14, eq. (22)]. The Rayleigh resolution limit in the delay domain is \( \Delta \tau = \frac{f}{N_v} \).

The length of the time window is lower bounded due to \( T_R \geq T \) and upper bounded by the approximation error through stationary delays \( \tau_s(t) \approx \tau_s \). Similarly to the frequency window, a rectangular time window leads to a quadratic power decay for the spectral window in the Doppler domain [48, eq. (21c)]. The Rayleigh resolution limit in the Doppler domain is \( \Delta \nu \approx \frac{1}{N_v T_R} \) for the rectangular time window. Decreasing the number of snapshots in time \( N_v \) at constant \( T_R \), the decreasing Doppler resolution suffers additionally from increased spectral leakage due to the rectangular window and thus employing a time window with a trade-off between resolution and leakage is desirable.

For successful separation of components and estimation of their parameters \( \nu_i \) and \( \tau_i \) with ANM a sufficient condition is [41]

\[
\Delta_{\text{min}, \nu} \geq \frac{N_v \Delta \nu}{[N_v - 1/4]}, \quad \text{or} \quad \Delta_{\text{min}, \tau} \geq \frac{N_r \Delta \tau}{[N_r - 1/4]},
\]

(26)
where $\Delta_{\text{min}, v}$ and $\Delta_{\text{min}, \tau}$ are the minimum wraparound distances of all $v_j$ and $\tau_i$, respectively. Although the separation condition is quite restrictive compared to the limits of a spectral approach, for ANM parameter values are not restricted to lie on the grid. Furthermore, the separation is not dependent on the dynamic range of the source magnitudes.

**B. NUMERICAL SIMULATION MODEL**

We simulate a non-stationary radio channel based on the geometry-based stochastic channel model illustrated in Fig. 1. The simulated propagation environment consists of a static Rx at the origin, a Tx moving with constant magnitude velocity vector $v_{\text{TX}}$, and several static point-scatterers with coordinates drawn from two independent spatial distributions. The location parameters of the spatial distributions are random with $r_1, r_2 \sim U(r_{\text{min}}, r_{\text{max}})$ and $\phi_1, \phi_2 \sim U(0, 2\pi)$. We define a cluster as a group of point-scatterers with parameters from the same distribution and the cluster center as the distribution’s location parameter. Each cluster consists of $N_{\text{MPC}}$ point-scatterers, i.i.d. Gaussian in two-dimensional space with cluster spread $a$, i.e. point-scatterer positions are at spatial coordinate $x \sim N(r_x \cos \phi_x, a_x^2)$, $y \sim N(r_y \sin \phi_y, a_y^2)$. We consider multipath propagation where the LOS component is associated with a direct path from Tx to Rx and several MPCs are each associated with a path from Tx to Rx trough a single bounce at a single point-scatterer. The MPC amplitudes are complex Gaussian distributed. For large $N_{\text{MPC}}$ this leads to a Rayleigh distributed envelope of the received signal, which is a suitable model for spatially uniform terrain for mm-Wave [49]. A similar time-varying scattering model for diffuse scattering is shown in [50]. The MPCs due to point-scatterers located within a small area are later identified as a single MPC due to insufficient resolution.

At $t = 0$, the Tx starts at distance $d_{\text{LOS}}$ from Rx and moves in a randomly chosen direction $\phi_{\text{TX}}$ drawn from the uniform distribution $U(0, 2\pi)$. The movement of Tx leads to a change in path delays for the LOS component and all scattered MPCs. At constant velocity, the rate of change is proportional to the effective velocity of the corresponding paths [51], i.e. $v_{\text{TX}}$ projected onto the unit vector from Tx to Rx or to any of the interacting scatterers.

The frequency domain channel model is defined in terms of the band-limited, complex valued base-band formulation

$$H_{\text{SC}}[k, l] = H_{\text{LOS}}[k, l] + \sum_{i=1}^{S-1} H_{\text{Cl}(i)}[k, l], \quad (27)$$

where the $k$-th transfer functions corresponds to time $t = kT_{\text{R}}$, $l$ is the frequency index, $H_{\text{LOS}}[k, l]$ is the LOS component, and $H_{\text{Cl}(i)}[k, l]$ is the $s$-th cluster of several MPCs. We define the components as

$$H_{\text{LOS}}[k, l] = g_{\text{LOS}}[k]e^{-j2\pi f_{\text{LOS}} l}, \quad (28)$$

$$H_{\text{Cl}(i)}[k, l] = \sum_{m=1}^{N_{\text{MPC}(i)}} g_{\text{MPC}(s,m)}[k]e^{-j2\pi f_{\tau_{\text{MPC}(s,m)}} l}, \quad (29)$$

where $g_{\text{LOS}}[k]$ and $g_{\text{MPC}(s,m)}[k]$ are complex-valued amplitudes, $\tau_{\text{LOS}}[k]$ is LOS propagation delay, $\tau_{\text{MPC}(s,m)}[k] = \tau_{\text{Cl}(i)}[k] + \Delta_{\text{MPC}(s,m)}[k]$ is the propagation delay of a multipath centered around cluster delay $\tau_{\text{Cl}(i)}[k]$ of the geometric model with deviations $\Delta_{\text{MPC}(s,m)}[k]$, and $f_j = f_c + l\Delta f - l\frac{B}{2}$ is the subcarrier frequency. We approximate the time-varying delays for $0 \leq kT_{\text{R}} < T_{\text{obs}}$ as

$$\tau[k] = \tau - kT_{\text{R}} \frac{v}{f_c} \quad (30)$$

such that for $\frac{B}{f_c} \ll 1$

$$e^{-j2\pi f_j \tau[k]} = e^{-j2\pi (f_c + l\Delta f - \frac{B}{2}) (\tau - kT_{\text{R}} \frac{v}{f_c})} \approx e^{2\pi (kT_{\text{R}} + l\Delta f) \tau - j\frac{B}{2} \tau} \quad (31)$$

and insert it into (28) and (29) to get the approximations

$$H_{\text{LOS}}[k, l] \approx g_{\text{LOS}}[k]e^{2\pi (kT_{\text{R}} + l\Delta f) \tau_{\text{LOS}}} \approx g_{\text{LOS}}[k]e^{2\pi (kT_{\text{R}} + l\Delta f) \tau_{\text{LOS}}}, \quad (32)$$

$$H_{\text{Cl}(i)}[k, l] = g_{\text{Cl}(i)}[k, l] e^{2\pi (kT_{\text{R}} + l\Delta f) \tau_{\text{Cl}(i)}} \approx g_{\text{Cl}(i)}[k, l] e^{2\pi (kT_{\text{R}} + l\Delta f) \tau_{\text{Cl}(i)}}, \quad (33)$$

where $v_{\text{LOS}}$ and $\nu_{\text{Cl}(i)}$ are Doppler shifts for the LOS component and the cluster, respectively. The complex-valued amplitudes are combined in the fluctuating cluster amplitude

$$g_{\text{Cl}(i)}[k, l] = \sum_{m=1}^{N_{\text{MPC}(i)}} g_{\text{MPC}(s,m)}[k] e^{-j2\pi 1\Delta f \Delta_{\text{MPC}(s,m)}}[k]. \quad (34)$$

The fluctuations in $g_{\text{LOS}}[k]$ and $g_{\text{Cl}(i)}[k, l]$ are important factors for a deviation of (27) from the model in (3), where the number of discrete MPCs is $S$. It includes one LOS component and $S-1$ clusters.

We model the path amplitude for the LOS component as

$$g_{\text{LOS}}[k] = \frac{1}{\sqrt{L_{\text{LOS}}[k]}} e^{j\phi_0}, \quad (35)$$
where \( L_P[k] = (4\pi d_{LOS}[k] f_c / c_0)^2 \) is the free-space path loss of the LOS component, \( \varphi_0 \) is an initial phase offset, and \( c_0 \) is the speed of light. For the MPCs

\[
g_{MPC}(k,m)[l] = g_{LOS}[l] a_{MPC}(k,m) \frac{g_s}{\sqrt{N_{MPC}(k)}} \quad (36)
\]

with \( a_{MPC}(k,m) \sim CN(0,1) \) and an additional coefficient \( g_s \) due to additional path loss and reflectivity. The phase offsets \( \varphi_{LOS} \) and \( \varphi_{TX}(k) \) are absorbed into \( \varphi_0 \) and \( a_{MPC}(k,m) \). \( N_s \) snapshots in time, and \( N_v \) frequency samples of \( H_{SC}[k,l] \) are combined into the time-frequency matrix \( X_{SC} \in \mathbb{C}^{N_v \times N_s} \). For each realization of \( X_{SC} \), only the position of Tx changes. We aim in estimating the parameters for the LOS component and clusters of a realization \( X_{SC} \approx X \), where the discrete MPC model in (11) serves as an approximation. The intention for this geometric model is to be close to the following vehicle-to-infrastructure scenario but not specific to the actual measurement site.

C. MEASUREMENT CAMPAIGN

In this work, 155 MHz bandwidth at 62.35 GHz are used from the vehicular multiband measurement campaign in [13]. A car drives along a street towards a cross-roads in an urban street environment and transmits simultaneously at 3.2 GHz, 34.3 GHz, and 62.35 GHz with a multicarrier system (see Sec. II-B) employing omni-directional antennas. A receiver is placed at the sidewalk and has directive antennas aimed towards the traffic light as shown in Fig. 2 and Fig. 3. The velocity vector \( \mathbf{v}_{TX} \) of the car, projected onto the unit vector of the direction from the Tx position to the Rx position, defines the effective velocity \( \mathbf{v}_{TX} \), which is the source of the Doppler shift for the LOS component. The car transmits pilot symbols with a bandwidth of \( B = 155 \) MHz, \( N_v = 155 \) subcarriers with spacing \( \Delta f = 1 \) MHz.

We obtain time-frequency snapshots of the propagation channel \( \hat{H}[t_b; k, l] \) (6) at a snapshot period of \( T_R = 125 \) μs, limited to \( T_{obs} = N_v T_R \), and centered at time \( t_b \) and frequency \( f_c \). For each time-frequency snapshot \( t_b \), there is a corresponding sample matrix \( Y \).

We assume wide-sense stationary uncorrelated scatterers (WSSUS) approximately for the channel, locally within \( T_{obs} \). An estimate of the non-stationary spectral process of the channel, known as the LSF \( \hat{C}[t_b; l', k'] \), is calculated with the local multitaper spectral estimates [52], [53]

\[
\hat{C}[t_b; l', k'] = \frac{1}{I} \sum_{i=0}^{I-1} |\hat{G}[t_b; l', k']|^2,
\]

\[
\hat{G}[t_b; l', k'] = \frac{1}{\sqrt{N_v N_s}} \sum_{i=0}^{N_v-1} \sum_{k=0}^{N_s-1} \hat{H}[t_b; k, l] \cdot D_{t}[k - k', l - l'] e^{2\pi i T_R k' \Delta f l'} \quad (37),
\]

where \( D_{t}[k, l] \) with \( i = 1, \ldots, I \) are suitable time-frequency windows, i.e. the data tapers. Employing a single data taper reduces estimation bias due to leakage. However, it also reduces the effective sample size where the multitaper method counteracts the increase in estimation variance [52].

Generally, the LSF describes the energy shift in the time-frequency domain locally at specific time and frequency. Delay and Doppler power profiles are derived from the LSF for a suitable local description of the energy shift in only time or frequency, respectively. We estimate the delay power profiles as the expectation of the LSF estimate (37) with respect to Doppler [13]

\[
\hat{P}_d[t_b; l'] = E_{l'} \{ \hat{C}[t_b; l', k'] \} = \frac{1}{N_v} \sum_{k'} \hat{C}[t_b; l', k'] \quad (38),
\]

and the Doppler power profiles as the expectation with respect to delay

\[
\hat{P}_\nu[t_b; l'] = E_{l'} \{ \hat{C}[t_b; l', k'] \} = \frac{1}{N_r} \sum_{l'} \hat{C}[t_b; l', k'] \quad (39).
\]

We have dropped the dependency on frequency for the LSF and the power profiles, where we assume stationarity in the
frequency domain within the occupied bandwidth locally at $f_c$. The first and the second central moments of the time-varying delay power profile (38) are the time-varying mean delay and the time-varying RMS delay spread [4]. Correspondingly, the moments of the time-varying Doppler power profile (39) are the time-varying mean Doppler and the time-varying RMS Doppler spread. Those moments are well-known wireless channel parameters.

Based on the measurement parameters, delay resolution is $\Delta \tau = 6.45 \text{ ns}$, corresponding to 1.93 m separation in propagation distance.

We observe the wireless channel of the $f_c = 62.35 \text{ GHz}$ band during the passing maneuver for an observation time of $T_{\text{obs}} = N_v T_R = 64 \text{ ms}$, i.e. $N_v = 512$ snapshots of channel transfer functions. The LSFs $\hat{C}[t_1, l', k']$ and $\hat{C}[t_2, l', k']$ for two selected time regions $t_1$ and $t_2$ are shown in Fig. 4(a) and Fig. 4(b), respectively. The LOS component is clearly visible and other most significant MPCs could be from a nearby fence on the western side of the road [13] and from light posts.

At the beginning of the recording, the first arrived path has minimum delay as the transmitter car passes the receiver and enters the main lobe of the receive antenna pattern. The car heads towards the cross-roads and a maximum negative Doppler shift is observed in $\hat{C}[t_2, l', k']$ at $t_1$ with MPCs reaching $\nu > 2 \text{ kHz}$. The difference in delay between the first arrival and the next MPCs is still relatively high at $t_1$. The first arrival and follow-up MPCs start to merge in delay domain for $\hat{C}[t_2, l', k']$ at $t_2$ and the given bandwidth. We assume the presence of a LOS component and several MPCs in the delay-Doppler domain, by inspection of the LSFs at $t_1$ and $t_2$. Single bounce paths allow a trivial geometric interpretation based on delay-Doppler estimates and positions of Rx and Tx. A possible location of the single bounce for the path, with the earliest arrival time after the line of sight (LOS) component, is close to a lamp post, as shown in Fig. 2.

D. LIMITED OBSERVATION TIME

Long observation times can lead to problems for practical implementations in the vehicular scenario, e.g. when the stationarity time of the channel is exceeded or when memory depth of the receiver system is limited. Therefore, a sample reduction is often necessary. Reducing the number of channel snapshots to $N_v = 16$, i.e. $T_{\text{obs}} = 2 \text{ ms}$, reduces the resolution in the Doppler domain by a factor 512/16=32. When selecting only every second subcarrier such that $N_v = 77$ and $\Delta f = 2 \text{ MHz}$, the occupied bandwidth remains similar, thus also the delay resolution is similar. However, reducing the subcarriers by two reduces $\tau_{\text{max}}$ by two, which is an issue if excess delays of MPCs exceed $\tau_{\text{max}}$.

When applying the ANM method on the measured channel, the PSD matrix in (13), which is of size $(N_v N_r + 1)^2$, decreases from $6.3 \cdot 10^9 (N_r = 155, N_v = 512)$ to $1.52 \cdot 10^6 (N_r = 77, N_v = 16)$ entries, acceptable for (13) with standard SDP solvers.

Fig. 5(a) and Fig. 5(b) show LSFs with lower Doppler resolution $\Delta \nu = 500 \text{ Hz}$ and half range of alias free delay. The distinction between a LOS component and several MPCs by identification in the LSFs becomes more difficult due to limited resolution in delay-Doppler.

For D-ANM, larger problem sizes become feasible with standard solvers. In case of $N_r = 155, N_v = 128$, the PSD matrix has more than $393 \cdot 10^6$ entries.

E. MODEL MISMATCH

The model (9) has its limitations for joint delay-Doppler estimation with vectorized ANM (15). A potential concern is the independence assumption between delay and Doppler shift in the approximations. In the continuous model (2), the rate of change in delay for a single path with time-varying propagation delay $\tau_c(t)$ is proportional to an effective velocity $v_s(t) = -\frac{\partial \tau_c(t)}{\partial t}$. The sampled effective velocity is

$$v_s[k] = -c_0 \frac{\partial \tau_c(kT)}{\partial t}.$$  (40)

For a multicarrier system at center frequency $f_c$ and subcarriers at $f = f_c + \Delta f \left( l - \frac{L-1}{2} \right)$, the Doppler shift from a single path is [51]

$$v_s[k, l] = \frac{\nu_c[k]}{c_0} f_l,$$  (41)

where $\nu_c[k, l] \approx v_s[k]$ for small $B/f_c$ due to the narrow relative bandwidth designated to mm-Wave bands w.r.t. the

![FIGURE 4. Local scattering function (LSF) with $\Delta \nu = 15.625 \text{ Hz}$ Doppler resolution due to 64 ms observation time ($N_v = 512, T_R = 125 \mu s$) at $f_c = 62.35 \text{ GHz}$ for selected times (a) $13.11 \text{ s}$ and (b) $13.75 \text{ s}$.](image-url)
FIGURE 5. Local scattering function (LSF) with lower Doppler resolution due to 2 ms observation time \((N_r = 16, \tau_{\text{R}} = 125 \mu s)\) at \(f_c = 62.35 \text{ GHz}\) for selected times (a) 13.11 s and (b) 13.75 s.

center frequencies. From (40) and (41),

\[
v_{\nu}[k] = -f_c \frac{\partial \tau_{\nu}(kT)}{\partial t}.
\]

Thus, with Doppler shift present, delay estimates from multiple snapshots acquired sequentially in time suffer from a systematic drift due to the channel’s non-stationarity, which introduces errors due to the first order approximation of \(\tau_r(t)\). Furthermore, the piecewise approximation in (9) assumes constant Doppler shift and introduces errors when the effective velocity is not constant, e.g. due to a varying velocity vector of the transmitter car or variations in the enclosed angle \(\phi_{\nu}[k]\) between direction of heading and receiver or scatterer. The tolerable error due to constant acceleration, i.e. linearly increasing velocity, is investigated for an emulated channel in [10], where there is a trade-off between maximum Doppler shift and stationarity time.

Although the approximations leading to independence between delay and Doppler in (9) are justified, its implications to the ANM approach are still unknown.

Another issue is a spread in delay and Doppler due to extended scatterers. The geometric model in Section III-B models the spread in delay through the spatial distribution of the point-scatterers, which increases the variation in propagation delays between MPCs attributed to the same spatial distribution. The spread in Doppler follows from its connection to the change in delay as defined in (40).

The simulation model in [35] is based on (9) and thus neglects these model mismatch issues.

IV. RESULTS

A. SIMULATION DATA

We choose the simulation parameters for a vehicular communication system designed for maximum resolvable velocity \(v_{\text{max}} = 20 \text{ m/s}\), maximum excess delay \(\tau_{\text{max}} = 0.3 \mu s\), center frequency \(f_c = 60 \text{ GHz}\), and bandwidth \(B = 160 \text{ MHz}\), which results in a maximum Doppler frequency of \(v_{\text{max}} f_c = 20 \times \frac{60}{160} = 4 \text{ kHz}\) for single bounce paths. The simulation environment assumes the Tx at distance \(d_{\text{LOS}} = 5 \text{ m}\). The location parameters \(r_1\) and \(r_2\) of the point-scatterers’ spatial distributions are within a radius of \(r_{\min} = 5 \text{ m}\) and \(r_{\max} = 10 \text{ m}\). They consist of \(N_{\text{MPC}} = 1000\) point-scatterers each with a cluster spread \(\alpha_c \in [0, 8 \text{ m}]\). According to [54, Sec. 9.1 p. 239], 6 MPCs are sufficient for a Rayleigh distributed envelope. However, when the point-scatterers’ spatial density decreases and become resolvable, 6 MPCs might not suffice. For MPCs to become resolvable by ANM or D-ANM, they have to fulfill the minimum separation condition (26), i.e. \(\Delta_{\text{min},r} \geq \frac{N_r \Delta_{\nu}}{(N_r - 1) / 4} = \frac{50.625 \text{ ns}}{1(4)} = 26.0 \text{ ns}\) for \(N_r = 50\) or \(\Delta_{\text{min},\nu} \geq \frac{N_{\nu}}{(N_r - 1) / 4} = \frac{0.125 \text{ kHz}}{125 \text{ Hz}} = 2.67 \text{ kHz}\) for \(N_{\nu} = 16\). Therefore, realizations of the simple cluster model are discarded if the minimum separation condition (26) is violated regarding the cluster centers.

To prevent aliasing in delay, \(N_r \geq \max \Delta_{\nu} / 0.3 \times 160 = 48\). Each cluster is attenuated by \(-g_s = 6 \text{ dB}\) relative to the LOS component.

We generate multiple LTV realizations \(X_{\text{SC}}\) of the model in Sec. III-B and add complex Gaussian noise \(Z \in \mathbb{C}^{N_r \times N_t}\), with i.i.d. entries \(Z_{ij} \sim \mathcal{CN}(0, \sigma_Z^2)\). The velocity vector and scatterer positions are constant during one realization \(X_{\text{SC}}\). We define the array signal-to-noise ratio (SNR) as \(\text{SNR} = 10 \log_{10} (E_{X_{\text{SC}}} / E_{Z^2})\). We set \(\text{SNR} = 30 \text{ dB}\) and normalize the sample matrix \(Y = X_{\text{SC}} + Z\) such that \(\|Y\|_F^2 = N_r N_t\). The solution of the SDP in (15) is a denoised matrix \(\hat{X}\) and a generated block-Toeplitz matrix \(T_2(\mu)\), where each block is a Toeplitz matrix itself. The regularizer \(\mu\) defines the trade-off between sparsity of the model structure and reconstruction error of the simulation data with added noise and model mismatch. A Vandermonde decomposition of \(T_2(\mu)\) with the MAPP method [40] recovers delay-Doppler pairs \((\tau_{\nu}, v_{\nu})\). The number of pairs is known for the simulation and limited to \(S = 3\).

Next we employ D-ANM to estimate delay-Doppler pairs. The sample matrix \(Y\) for D-ANM is normalized as in the ANM case. The solution of the SDP in (22) is a denoised matrix \(\hat{X}\) and two Toeplitz matrices \(T_1(\mu_r)\) and \(T_1(\mu_\nu)\). For recovering the parameters from \(T_1(\mu_r)\) and \(T_1(\mu_\nu)\), the number of estimated parameters \(S\) (significant MPCs) corresponds to the number of significant eigenvalues of the Toeplitz matrices. We set \(S_{\nu} = \min(3, N_{\nu}(\Delta_{\nu}))\) and \(S_{\tau} = \min(3, N_{\tau}(\Delta_{\tau}))\), where \(N_{\tau}(\Delta_{\tau})\) and \(N_{\nu}(\Delta_{\nu})\) are the number of significant eigenvalues of \(T_1(\mu_r)\) and \(T_1(\mu_\nu)\) with magnitude not
denoising is less pronounced in Fig. 6(c). Although the LSF in the background. Compared to Fig. 6(b), the effect of Fig. 6(c) are in high agreement with the LSF of the denoised Fig. 6(c). The delay-Doppler estimates with D-ANM in delay and Doppler parameters is described in Section II-D.

FIGURE 6. Simulation with cluster spread 2 mm, $N_t = 50$, $N_r = 16$. (a) The true cluster parameters (+) $(\tau_{\text{Cl}(s)}, \nu_{\text{Cl}(s)})$. Recovered delay-Doppler pairs (o) $(\hat{\tau}_s, \hat{\nu}_s)$ with (b) atomic norm minimization (ANM) (15) with $\mu' = \mu_e$ and (c) decoupled ANM (D-ANM) (22) with $\mu = \mu_e$. Background colors are the LSFs, normalized to maximum, with the samples from (a) $X_{\text{SC}}$, (b) ANM denoised $\hat{X}$, and (c) D-ANM denoised $\hat{X}$, respectively.

smaller than 20 dB of the largest eigenvalue each. Pairing of delay and Doppler parameters is described in Section II-D.

Fig. 6(a) shows the true cluster parameters (+) $(\tau_{\text{Cl}(s)}, \nu_{\text{Cl}(s)})$ for the simulation with cluster spread 2 mm, $N_t = 50$, and $N_r = 16$. Delay-Doppler pairs (o) $(\hat{\tau}_s, \hat{\nu}_s)$ are shown in Fig. 6(b) for recovery with with atomic norm minimization (ANM) (15) with $\mu' = \mu_e$ and in Fig. 6(c) for recovery with decoupled ANM (D-ANM) (22) with $\mu = \mu_e$. Although the strong LOS component is modeled as a single discrete path, two delay-Doppler estimates are quite near to the strong LOS component (Fig. 6(b) (20 ns, 0.5 kHz)). We observe this effect in all our simulations with ANM (15), which could be due to spectral leakage (see Sec. III-A). A further significant MPC corresponding to the remaining cluster is not identified. The delay-Doppler estimates with D-ANM in Fig. 6(c) are in high agreement with the true cluster parameters.

For random LTV channels, we estimate the non-stationary spectral process with the LSF in (37) based on the time-varying transfer function $\hat{H}[t_k; k, l]$ (6) limited to a region in time and frequency. Similarly, we now calculate an equivalent LSF with the samples of the time-frequency regions $X_{\text{SC}}$ and $\hat{X}$, respectively, for their description in the delay-Doppler domain. The background color in Fig. 6(a) shows the LSF of a single realization $X_{\text{SC}}$ from the simulation model (27), the LSF of the denoised $\hat{X}$ with ANM (15) in Fig. 6(b), and LSF of the denoised $\hat{X}$ with D-ANM (22) in Fig. 6(c). The delay-Doppler estimates with D-ANM in Fig. 6(c) are in high agreement with the LSF of the denoised $\hat{X}$ in the background. Compared to Fig. 6(b), the effect of denoising is less pronounced in Fig. 6(c). Although the LSF of the denoised $\hat{X}$ in Fig. 6(b) looks remarkably similar to the ground truth in Fig. 6(a), the estimated parameters for ANM do only partially agree with the true cluster parameters. Therefore, we consider the vectorized ANM to be of little use for the employed model. Furthermore, the high computational complexity of the SDP (15) makes it susceptible to numerical and convergence issues [55].

Therefore, we restrict the remaining discussion of simulations to D-ANM, the decoupled SDP in (22). Fig. 7(a)–(d) show the D-ANM recovered delay-Doppler parameter pairs (o) for $N_t = 50$, $N_r = 16$ and the LSFs of D-ANM denoised $\hat{X}$ (22) of the same single time-frequency realization $X_{\text{SC}}$ as in Fig. 6. The intensity of the denoising effect depends on the regularizer $\mu$ as shown in Fig. 7(a) and Fig. 7(b) (see also Fig. 6(c)). The empirical value $\mu = \mu_e$ (16) as in Fig. 6(c) is a reasonable choice according to our observations with the simulation model. An increase in cluster spread from $\alpha_s = 2$ mm in Fig. 6 to 100 $\times$ $\alpha_s = 0.2$ m in Fig. 7(c) still achieves good estimation of the cluster centers. At larger cluster spreads 1000 $\times$ $\alpha_s = 2$ m in Fig. 7(d) the algorithm fails to identify one of the clusters because the cluster spread is too large.

We compare the estimation accuracy between estimated and simulated model in terms of estimated delay-Doppler pairs $(\hat{\tau}_s, \hat{\nu}_s)$ with the cluster parameters $(\tau_{\text{Cl}(s)}, \nu_{\text{Cl}(s)})$ in a squared Euclidean distance

$$d^2(s, k) = \left(\frac{\tau_{\text{Cl}(s)} - \hat{\tau}_s}{\tau_{\text{max}}}\right)^2 + \left(\frac{\nu_{\text{Cl}(s)} - \hat{\nu}_s}{2\nu_{\text{max}}}\right)^2,$$

(43)
If the number of detected components based on (44) is insufficient in approximating the MPCs caused by a cluster of point-scatterers as a single MPC with increased delay-Doppler spread. In Fig. 9 we see two types of mismatch: On the one hand, for low levels of cluster spread we see an error floor in (c) due to violation of the WSSUS assumption because the Tx is moving. On the other hand, for high cluster spreads starting at 7 m, the Fourier basis does not provide a good expansion up to delay resolution, i.e. $\Delta \tau_{c0} = 1.87 \text{m}$, where the effect is most dramatic for problem size (c). However, for problem size (c), cluster spreads near $d_s = 0.4 \text{m}$ result in a very low RMSE.

Next, we evaluate the applicability of the sparse representation $\Tilde{H}$ (24) on the noiseless sampled time-varying frequency transfer function with different previously defined problem sizes (a)–(c). We denote $\Tilde{H}$ with parameters from the geometric model as $\Tilde{H}_{SC}$ and $\Tilde{H}$ with estimated parameters through D-ANM (22) as $\Tilde{H}_{DANM}$. The NAE($\cdot; X_{SC}$) in (25), of the matrix $X_{SC} \in \mathbb{C}^{N_{\tau} \times N_t}$ depending on the cluster spread is shown in Fig. 9 for $\Tilde{H}_{SC}$ and $\Tilde{H}_{DANM}$.

$\text{NAE}(\Tilde{H}_{SC}; X_{SC})$ increases with cluster spread as the limited number of parameters becomes insufficient in approximating the MPCs caused by a cluster of point-scatterers as a single MPC with increased delay-Doppler spread. In Fig. 9 we see two types of mismatch: On the one hand, for low levels of cluster spread we see an error floor in (c) due to violation of the WSSUS assumption because the Tx is moving. On the other hand, for high cluster spreads starting at $10^{-1} \text{m}$, the Fourier basis does not provide a good expansion of the channel. Furthermore, for very large spreads of point-scatterers, the clusters are less suitable in modeling specular reflection and more likely modeling diffuse scattering [50].

In the case of the sparse model with D-ANM estimated parameters, $\text{NAE}(\Tilde{H}_{DANM}; X_{SC})$ start for low cluster spreads at medium to high $\text{NAE}(>0.3)$ and approach $\text{NAE}(\Tilde{H}_{SC}; X_{SC})$ with increased cluster spread, see Fig. 9. Although absolute delay drifts get larger with (c) longer observation times, the sparse representation from estimated parameters results

with $\tau C(S) = \tau_{LOS}$ and $\nu C(S) = \nu_{LOS}$.

Due to an association problem, we choose the best pairing i.e.

$$d_s^2 = \min_k d^2(s,k). \quad (44)$$

If the number of detected components $K < S$, $d^2(s,k) = 2$ for $k = K + 1, \ldots, S$. This serves as a penalty for undetected clusters. We use the root mean squared error (RMSE) based on (44)

$$\text{RMSE} = \sqrt{E \left[ \frac{1}{S} \sum_s d_s^2 \right]}, \quad (45)$$

where the expectation is with respect to all realizations of the simple cluster model (see Sec. III-B) and different noise realizations.

The RMSE over 10 model realizations each with 4 noise realizations for different problem sizes is shown in Fig. 8 for problem sizes (a) $N_c = 50$, $N_s = 16$, (b) $N_c = 100$, $N_s = 8$, and (c) $N_c = 155$, $N_s = 128$. The RMSE fluctuates strongest for problem size (a) and increases for all problem sizes (a)–(c) with $\mu$, where $\mu$ close to the empirical $\mu_e$ (16) gives the best results on average. The value of $\mu$ and cluster spread shows only minor changes for RMSE of problem size (b).

The cluster spreads do not influence the RMSE significantly until spreads reach the order of the propagation distance due to delay resolution, i.e. $\Delta \tau_{c0} = 1.87 \text{m}$, where the effect is most dramatic for problem size (c). However, for problem size (c), cluster spreads near $d_s = 0.4 \text{m}$ result in a very low RMSE.

FIGURE 8. Root mean squared error (RMSE) (45) over 10 realizations of the simple cluster model each with 4 noise realizations with different cluster spreads and regularization of D-ANM in (22). Problem size (a) $N_c = 50$, $N_s = 16$, (b) $N_c = 100$, $N_s = 8$, and (c) $N_c = 155$, $N_s = 128$. At 60 GHz, a cluster spread 1 m corresponds to 200 $\lambda$. For propagation delay $\Delta \tau = 1/8 = 1/160 \text{MHz}$, propagation distance is $\Delta \tau_{c0} = 1.87 \text{m}$.

FIGURE 9. Normalized approximation error $\text{NAE}(\cdot; X_{SC})$ (25) vs cluster spread over 10 realizations of the simple cluster model each with 4 noise realizations for $\mu = \mu_e$. (a) $N_c = 50$, $N_s = 16$, (b) $N_c = 100$, $N_s = 8$, (c) $N_c = 155$, $N_s = 128$. Sparse representation with cluster parameters $H_{SC}$ and estimated parameters $H_{DANM}$ from decoupled ANM (D-ANM).
in a better approximation of matrix $X_{SC}$ with low cluster spreads than for (a)–(b) shorter observation times. Furthermore, for (c) the difference between $\text{NAE}(\hat{H}_{DANM}; X_{SC})$ and $\text{NAE}(\hat{H}_{SC}; X_{SC})$ is small (<0.1).

### B. MEASUREMENT DATA

#### 1) MEASUREMENT EVALUATION WITH ANM

The weak influence of small cluster spreads in the simulation model to identify clusters as discrete MPCs motivates further investigation of D-ANM on measurement data. Nonetheless, we first analyze two selected time regions $t_1$ and $t_2$, and the corresponding sample matrices $Y$ (7) of the measurement at $f_c = 62.35 \text{ GHz}$, further with vectorized ANM (15) applied to $Y$. The solution of the SDP is a ANM denoised matrix $\hat{X}$ and a block-Toeplitz matrix with Toeplitz blocks $T_2(u)$. Fig. 10 shows the eigenvalues of recovered $T_2(u)$ for $N_e = 16, N_t = 77$, and different regularizer values $\mu'$. With increasing $\mu'$, the SDP generates $T_2(u)$ with fewer eigenvalues, where their magnitudes are close to the largest eigenvalue magnitude. A breakpoint in eigenvalue magnitude is clearly visible in $t_1$ at eigenvalue index 16, which could indicate the maximum rank of the Toeplitz block-Toeplitz structure. From a noise power estimate $\hat{\sigma}^2$ from measurement data, empirical values $\mu' = \mu_e (16)$ are $\mu_e[t_1] = 4.31$ and $\mu_e[t_2] = 6.37$ for the given problem size. Higher $\mu'$ often lead to a failure in solving the SDP with the employed solver.

The number of parameter estimates corresponds to the significant eigenvalues of $T_2(u)$. We estimate the delay-Doppler pairs with the two-dimensional MAP algorithm $T_2(u)$ and limit the number of pairs to $S = \min(N_v, N_e) = 16$ for MAPP, based on the rank argument of $T_2(u)$ in Section II-E. We sort the delay-Doppler pairs in decreasing order, based on their corresponding power estimates with MAPP [40].

![Figure 10. Eigenvalues of recovered Toeplitz matrix $T_2(u)$ for different regularizer values $\mu'$. Top: $t_1 = 13.11$ s, bottom: $t_2 = 13.75$ s recording time. Normalized with first eigenvalue.](image)

Fig. 11 shows the $S = 16$ largest peaks of the LSF $\hat{C}[t_0; l', k']$ (37) for time-frequency snapshots (+) and the recovered delay-Doppler pairs $(\hat{t}_v, \hat{v}_b)$ from $T_2(u)$ derived with ANM and $\mu' = 0.5 \mu_e, \mu' = \mu_e$, and $\mu' = 4 \mu_e$. At recording time $t_1$, the largest LSF peak (60 ns, $-1.8 \text{ kHz}$) corresponds to the strong LOS component where neighboring peaks with similar delay or Doppler are likely due to spectral leakage despite data tapering. Further LSF peaks with delay $> 100$ ns and Doppler $> 0 \text{ kHz}$ correspond to other scatterers. The delay-Doppler ANM estimates in Doppler domain for all shown $\mu'$ suffer from a leakage effect similar to the leakage effect from spectral analysis for strong LOS. ANM identifies only one additional scatterer to the LOS. At recording time $t_2$, less LSF peaks are in the neighborhood corresponding to the LOS with moderate power. ANM identifies now more scatterers for $\mu' = 0.5 \mu_e$ and $\mu' = \mu_e$ additional to the LOS, where delay-Doppler estimates still suffer mildly from the leakage effect. For high $\mu' = 4 \mu_e$, the ANM fails to identify most of the scatterers.

We now calculate an equivalent LSF with the samples of the time-frequency region described by the ANM denoised matrix $\hat{X}$, for a description in the delay-Doppler domain. The background color in Fig. 11 shows the LSFs $\hat{C}[t_0; l', k']$ based on the sample matrices $Y$ and denoised $\hat{X}$, respectively. Increasing $\mu'$ shows an intensification of the denoising effect in the LSF, where the strong differences between the LSF from $Y$ and the LSF from denoised $\hat{X}$ at $t_2$ and at $\mu' = 4 \mu_e$ coincides with failing identification of scatterers in the later.

Next, we evaluate the applicability of the sparse representation $\hat{H}$ (24) on the sample matrix $Y$ defined in (7). We denote $\hat{H}$ with estimated parameters through ANM as $\hat{H}_{ANM}$. The model order is unknown for the measurement data and estimation is limited to the previously set maximum value of 16. We show the approximation quality with $\text{NAE}(\cdot; Y)$ (25) for reconstructing $Y$ with the sparse representation $\hat{H}_{ANM}$ defined in (24) with recovered parameters $[\hat{\gamma}_e, \hat{\gamma}_b, \hat{v}_b | s = 1, \ldots, S]$. The trade-off between sparsity and measurement reconstruction is shown in Fig. 12 where the trace-norm is $\text{Tr}(T_2(u))/\text{Tr}(N_e N_t)$. The $\text{NAE}(\hat{X}; Y)$ through the ANM denoised $\hat{X}$ serves as a comparison. $\text{NAE}(\hat{X}; Y)$ decreases with decreasing $\mu'$ as the trace-norm has only insignificant weight for the objective function in (15). $\text{NAE}(\hat{H}_{ANM}; Y)$ through the estimate $\hat{H}_{ANM}$ of the discrete MPCs model also decreases with decreasing $\mu'$, where $S = 16$ paths lead to $\text{NAE} < 0.16$ in both cases for $\mu' = \mu_e$. For a further decrease of $\mu'$, $\text{NAE}(\hat{H}_{ANM}; Y)$ saturates and does not decrease anymore. Reducing the number of paths to $S = 14$ paths slightly increases the error for $t_2$, but dramatically increases it for $t_2$ and thus leads to a bad approximation for $t_1$. Note, $\text{NAEs} > 0.6$ are not shown.

#### 2) MEASUREMENT EVALUATION WITH D-ANM

The ANM method (15) is impractical for large problem sizes. Therefore, we now analyze the previously selected time regions $t_1$ and $t_2$ at $f_c = 62.35 \text{ GHz}$ with D-ANM (22) and are able to select a higher number of time-frequency samples.
FIGURE 11. Identified scatterer with atomic norm minimization (ANM) for time-frequency limited measurement data. The largest peaks (+) of the local scattering function (LSF) \( \hat{C}_{t;b,l';k'} \) (37). Recovered delay-Doppler pairs (o) with D-ANM (15) applied to the sample matrix \( Y \) (see Sec. II-B) for different \( \mu' \) followed by MAPP. The number of pairs is limited to \( S = 16 \). \( Y \) has \( N_\nu = 16 \) snapshots in time and \( N_\tau = 77 \) samples in frequency centered around \( t_b \) and \( f_c = 62.35 \) GHz, respectively. \( T_{ob} = N_\nu T_R = 2 \) ms, \( \Delta f = 2 \) MHz. Background colors are the LSFs \( \hat{C}_{t;b,l';k'} \), normalized to maximum, with the samples from \( Y \), and ANM denoised matrix \( \hat{X} \), respectively. Top: \( t_1 = 13.11 \) s, bottom: \( t_2 = 13.75 \) s recording time.

FIGURE 12. L-curve shows trade-off between sparsity with trace-norm \( \text{Tr}(T_2(u)) / (N_\nu N_\tau) \) and normalized approximation error (NAE) (25) of measurement reconstruction with atomic norm minimization (ANM) denoised \( X \) or with a sparse representation \( \tilde{H}_{ANM} \), each for different regularizer values \( \mu' \). The sparse channel consists of 14 or 16 paths maximum. Top: \( t_1 = 13.11 \) s, bottom: \( t_2 = 13.75 \) s recording time.

Here, a longer duration \( N_\nu = 128 \), i.e. \( T_{ob} = 16 \) ms, and more subcarriers for similar bandwidth, \( N_\tau = 155 \), are taken from the measurement. First, we estimate parameters in delay domain and Doppler domain independently with a matrix pencil method from the recovered Toeplitz matrices \( T_1(\nu) \) and \( T_1(\nu) \), respectively.

Fig. 13 shows the eigenvalues of the recovered Toeplitz matrices in the Doppler domain (\( T_1(\nu) \)) and in delay domain (\( T_1(\nu) \)) for different regularizer values \( \mu' \). Threshold for pairing is at 20 dB. Normalized with first eigenvalue.
(\(T_1(u_i)\)) for different regularization parameter \(\mu\). As in the ANM case, with increasing \(\mu\), the SDP generates \(T_1(u_i)\) and \(T_1(u_t)\) with fewer eigenvalues each, where their magnitudes are close to the largest eigenvalue magnitude. The empirical values for \(\mu\) according to (16) are \(\mu_e[f_1] = 20.76\) and \(\mu_e[f_2] = 29\) based on estimated noise power of the measurement. The number of estimated parameters \(S\) corresponds to the number of significant eigenvalues of the Toeplitz matrices. For further evaluation, we limit the number of parameters \(S = S_x = S_y = \min(32, N_{\lambda(t)}, N_{\nu(t)})\), where \(N_{\lambda(t)}\) and \(N_{\nu(t)}\) are the number of significant eigenvalues of their respective Toeplitz matrices \(T_1(u_i)\) and \(T_1(u_t)\) with magnitudes not smaller than \(20\) dB from the largest eigenvalue each.

We pair the separate parameter estimates (see Sec. II-D) and get \(S\) delay-Doppler pairs \((\hat{t}, \hat{\nu})\) as shown in Fig. 14. A choice of lower \(\mu\) leads to more significant eigenvalues and thus more detected delay-Doppler pairs. However, those pairs do not agree with the peaks of the LSF in Fig. 14. We now analyze a larger part of the car’s trajectory as shown in Fig. 2. After the Tx passed the Rx, the separation in delay \(\Delta t\) and in Doppler \(\Delta \nu\) between the LOS component and several MPCs decreased significantly. The distances between the LOS component and the MPC with earliest arrival change from delay \(\Delta t[f_1] = 43.7\) ns and Doppler \(\Delta \nu[f_1] = 3.6\) kHz (Fig. 4(a)) to delay \(\Delta t[f_2] = 22.2\) ns and Doppler \(\Delta \nu[f_2] = 1.43\) kHz (Fig. 4(b)).

Further, the estimated signal-to-noise ratio \(\text{SNR} = 10 \log_{10}(\|Y\|^2 / (N_{y} N_{e} \sigma_f^2) - 1)\) decreased, see Fig. 16. We employ the NAE to compare \(\hat{H}_{\text{DANM}}\) from estimated parameters with the sample matrix \(Y\) for different recording times \(t_b\). NAE depends further on the number of paths utilized for the sparse representation \(\hat{H}_{\text{DANM}}\). Fig. 16 shows the NAE(\(\hat{H}_{\text{DANM}}; Y\)) over time at fixed \(\frac{\mu}{\mu_e}\) ratios, where the number of paths of the sparse representation is limited to 11 pairs.
32. Although lower \( \mu \) lead to more detected paths compared to cases with higher \( \mu \), NAE does not necessarily decrease accordingly. At \( \mu = \mu_e \) and \( \mu = 4 \cdot \mu_e \), the detected paths show an NAE(\( \hat{\mathbf{H}}_{\text{DANM}}; \mathbf{Y} \)) < 0.15 at \( t_1 \), NAE(\( \hat{\mathbf{H}}_{\text{DANM}}; \mathbf{Y} \)) < 0.2 at \( t_2 \), and the approximation is sustainable for recording time larger than 14 s.

As an alternative method we denote \( \hat{\mathbf{H}} \) with estimated parameters based on the largest peaks of the multitaper spectral estimator (37) as \( \hat{\mathbf{H}}_{\text{peaks}} \). We employ separable windows \( D_{i k l} = u_i[k] v_l[l] \) where \( u_i[k] \) and \( v_l[l] \) are time and frequency windows, respectively [53], [56]. The choice of the windows is a trade-off between bias, variance, and resolution. We need high spectral resolution of the largest peak positions and select the first Slepian sequence [57] with \( N_{W_l} = 1.5 \) as time window \( u_i[k] \) and the first Slepian sequence with \( N_{W_f} = 1.5 \) as frequency window \( v_l[l] \), where \( N_{W_l} \) and \( N_{W_f} \) are the normalized time \( \times \) half bandwidth products of the employed spectral windows in time and frequency, respectively. This leads to a single time-frequency window. Note, that we employ a lower number of windows in each domain than the empirical maximum \( 2 N_{W_l} - 1, 2 N_{W_f} - 1 \) respectively, to combat leakage [58, Ch. 8.2]. The spectral resolution reduces to \( R_f = 2 N_{W_f} \Delta f = 19.3 \) ns in delay and \( R_d = 2 N_{W_l} \Delta f = 187.5 \) Hz in Doppler due to the employed windows. Estimating LSF with additional DFT oversampling by zero padding (ZP) allows a finer grid of the peak positions of each spectral main lobe in delay and Doppler. A trivial two-dimensional peak search selects the \( S \) largest peaks of the LSF and thus determines the paths and parameters for the sparse representation \( \hat{\mathbf{H}}_{\text{peaks}} \). The NAE(\( \hat{\mathbf{H}}_{\text{peaks}}; \mathbf{Y} \)) is slightly higher than the sparse representations based on D-ANM for recording time smaller than 14 s, where SNR > 20 dB. Overall NAE(\( \hat{\mathbf{H}}_{\text{peaks}}; \mathbf{Y} \)) is close to the D-ANM case for larger recording time. For further statistical evaluation of the LSF, derived by the measurement or by the sparse representations, we allow lower resolution for estimator of the LSF to achieve lower variance. We select two Slepian sequences with \( N_{W_f} = 2 \) as time windows \( u_i[k] \) and the first two Slepian sequences with \( N_{W_f} = 2.5 \) as frequency windows \( v_l[l] \). This leads to a total number of \( I = 4 \) orthogonal windows, which is again lower than the empirical maximum to combat leakage. The spectral resolution reduces further to \( R_f = 2 N_{W_f} \Delta f = 32.2 \) ns in delay and \( R_d = 2 N_{W_l} \Delta f = 250 \) Hz in Doppler due to the employed windows. Prior to the calculation of the moments, thresholding is applied to the delay (38) and Doppler power profiles (39) to exclude values 30 dB lower than the peak power or values lower than 5 dB above the estimated noise power of the corresponding delay or Doppler power profiles.

Fig. 17(a)–(d) show the first and second central moments in delay and Doppler domain of the LSF of the measurement data along the car’s trajectory and their sparse representations \( \hat{\mathbf{H}}_{\text{DANM}} \) and \( \hat{\mathbf{H}}_{\text{peaks}} \).

The filled areas for \( \hat{\mathbf{H}}_{\text{DANM}} \) and \( \hat{\mathbf{H}}_{\text{peaks}} \) show the moments when the number of paths is varied between a single path

\[ \mu = 0.1, \mu_e, \mu = 0.5, \mu_e, \mu = 1, \mu_e, \mu = 5, \mu_e, \mu = 10, \mu_e, \mu = 25, \mu_e \]
to a maximum of 32 paths, if detected. The mean delays in Fig. 17(a) and mean Doppler shifts in Fig. 17(c) follow closely the first moments of the LSF of the measurement data. With only a single path for $\tilde{H}_{\text{DANM}}$ and $\tilde{H}_{\text{peaks}}$, mean delay and mean Doppler most likely coincide with delay and Doppler shift of the dominant LOS component. For $\tilde{H}_{\text{peaks}}$ with a single path, mean delay and mean Doppler are step functions due to the employed delay-Doppler grid for calculating the LSF (37). The RMS delay spreads in Fig. 17(b) and RMS Doppler shift spreads in Fig. 17(d) depend heavily on the number of paths where for the maximum number of paths, there is only a slight deviation to the LSF of the measurement data. Despite lower spreads in Fig. 17(b) and Fig. 17(d) for the D-ANM case compared to the measurement data, their trends follow the measurement data closely and nearly as good as $\tilde{H}_{\text{peaks}}$. With only a single path for $\tilde{H}_{\text{DANM}}$ and $\tilde{H}_{\text{peaks}}$, the spreads reduce to a residual value close to the delay resolution $\Delta\tau$ and Doppler resolution $\Delta\nu$, respectively.

### C. SDP SOLVER

We solve the SDPs (15) and (22) with CVXPY [59] utilizing an alternating direction method of multipliers-based cone splitting solver [60]. It employs a first-order method which scales better to larger problems and can solve them with modest accuracy more quickly than interior point methods [60]. However, the complexity for the SDPs grows quickly with problem size such that also first-order methods become insolvable even for the decoupled case (22). Solvers for SDPs with first-order and interior point methods work iteratively. Available complexity analysis for the later case estimate $O(P^3)$ steps for each iteration [41], where $P$ is the size of the PSD matrix constraint of (15) and (22) and complexity of first-order methods is usually smaller. At most $O(\sqrt{P} \ln(1/\epsilon))$ iterations are necessary for recovery precision $\epsilon$, thus $O(P^{3.5} \ln(1/\epsilon))$ of overall time complexity [41]. $P = N_\tau N_\nu + 1$ for ANM in (15) and the following Vandermonde decomposition with MAPP of the Toeplitz block-Toeplitz matrix [40] has complexity $O(P^2 R)$ for rank $R$ [41]. $P = N_\tau + N_\nu$ for D-ANM in (22) and the two separate matrix pencil method have complexity $O(N_\tau^2)$ and $O(N_\nu^2)$.

The sample matrix $Y$ as input to the solver is scaled such that $\|Y\|^2 = N_\tau N_\nu$. For given $N_\tau$ and $N_\nu$, the empirical value for the regularizer $\mu_e$ depends only on $\sigma_Z$, (See (16)), thus it is scaled correspondingly.

The time for solving the SDP depends not only on the problem size but also on the data and the regularizer $\mu_e$ for (15) and $\mu$ for (22). Increasing values usually lead to slower convergence of the solver and longer processing time. For large values, e.g., larger than right-most column in Fig. 11, the SDP solver will likely not converge in our experiments. The mean elapsed processing time for the different algorithms and problem sizes of the measurement data is shown in Table 1 for

### TABLE 1. Elapsed time for solving the semi-definite programs (SDPs) of atomic norm minimization (ANM) for $\mu = \mu_e$ and decoupled ANM (D-ANM) for $\mu = \mu_e$ with utilized solver [60] and different problem size ($N_\tau \times N_\nu$).

| Algorithm | Problem Size | Time   |
|----------|--------------|--------|
| ANM      | $16 \times 77$ | > 2 hours |
| ANM      | $32 \times 77$ | > 3 days   |
| D-ANM    | $16 \times 77$ | 1.5 s   |
| D-ANM    | $32 \times 77$ | 1.6 s   |
| D-ANM    | $128 \times 155$ | 22 s   |
\( \mu^c = \mu_e \) and \( \mu = \mu_e \) when run on a single core of an AMD EPYC 7302 processor.

The processing time for ANM is already nearly an hour for a small problem size of \( N_v = 16 \) and \( N_r = 77 \). The processing time decreases dramatically with D-ANM, i.e. from hours to seconds for \( N_v = 16 \) and \( N_r = 77 \), such that it is possible to solve larger problems.

### V. CONCLUSION

We show an identification method of sparse scatterers with atomic norm minimization (ANM) techniques in the time-delay Doppler-frequency domain for vehicle-to-infrastructure millimeter wave communication. The decoupled ANM together with a matrix pencil method allows for estimation with larger sample size and thus good accuracy.

We studied the approximation error versus cluster spread and verified through simulation, that small cluster spreads lead to acceptable estimation accuracy for delay and Doppler parameters. Analysis of the vehicular mm-Wave measurement data shows a good agreement between identified paths and verified through simulation, that small cluster spreads for estimation with larger sample size and thus good accuracy.

The processing time for ANM is already nearly an hour and future challenges,''

IEEE Veh. Technol. Mag., vol. 15, no. 1, pp. 13–22, May 2020, doi: 10.1109/MVT.2020.2993327.

L. Zheng and X. Wang, “Super-resolution delay-Doppler estimation for OFDM passive radar,” IEEE Trans. Signal Process., vol. 65, no. 9, pp. 2197–2210, May 2017, doi: 10.1109/TSP.2017.2659650.

K. P. Murphy, Machine Learning—A Probabilistic Perspective, London, U.K.: MIT Press, 2012.

W. Li and J. C. Preisig, “Estimation of rapidly time-varying sparse channels,” IEEE J. Ocean. Eng., vol. 32, no. 4, pp. 927–939, Oct. 2007, doi: 10.1109/JOE.2007.906409.

C. F. Mecklenbrauker, P. Gerstoft, and E. Ollila, “DOA M-estimation using sparse Bayesian learning,” in Proc. IEEE Int. Conf. Acoust., Speech Signal Process. (ICASSP), Singapore, May 2022, pp. 4933–4937.

T. Blazek and C. F. Mecklenbrauker, “Sparse-time-variant impulse response estimation for vehicular channels using the c-LASSO,” in Proc. IEEE 28th Ann. Int. Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC), Oct. 2017, pp. 1–5.

R. He, B. Ai, G. Wang, M. Yang, C. Huang, and Z. Zhong, “Wireless channel sparsity: Measurement, analysis, and exploitation in estimation,” IEEE Wireless Commun., vol. 28, no. 4, pp. 113–119, Aug. 2021, doi: 10.1109/MWC.001.2000378.

H. Zhang, R. He, B. Ai, S. Cui, and H. Zhang, “Measuring sparsity of wireless channels,” IEEE Trans. Cognit. Commun. Netw., vol. 7, no. 1, pp. 133–144, Mar. 2021, doi: 10.1109/TCN.2020.3013270.

R. Heckel, V. I. Morgenstern, and M. Soltanolkotabi, “Super-resolution radar,” Inf. Inference J.IMA, vol. 5, no. 2, pp. 22–75, Mar. 2016, doi: 10.1093/imaiai/awu001.

R. Alieiev, T. Hehn, A. Kwoczek, and T. Kürner, “Predictive communications to line spectral estimation,” IEEE Trans. Signal Process., vol. 65, no. 9, pp. 2377–2392, Nov. 2020, doi: 10.1109/TSP.2020.2993327.

E. Zöchmann, V. Va, M. Rupp, and R. W. Heath, Jr., “Geometric tracking radar,” in Proc. IEEE PIMRC (PIMRC), London, U.K.: Academic, 2011.

C. F. Mecklenbrauker, P. Gerstoft, and E. Ollila, “DOA M-estimation using sparse Bayesian learning,” in Proc. IEEE Trans. Signal Process., vol. 66, no. 11, pp. 7237–7252, Nov. 2020, doi: 10.1109/TSP.2020.2993327.

L. Xenaki and P. Gerstoft, “Grid-free compressive beamforming,” J. Acoust. Soc. Amer., vol. 137, no. 4, pp. 1923–1935, 2002, doi: 10.1121/1.4762690.

H. L. Van Trees, Optimum Array Processing. Hoboken, NJ, USA: Wiley, 2002.

A. Xenaki and P. Gerstoft, “Grid-free compressive beamforming,” J. Acoust. Soc. Amer., vol. 137, no. 4, pp. 1923–1935, 2020, doi: 10.1121/1.4762690.
M. Wagner, Y. Park, and P. Gerstoft, “Gridless DOA estimation and root-MUSIC for non-uniform linear arrays,” *IEEE Trans. Signal Process.*, vol. 69, pp. 2144–2157, 2021, doi: 10.1109/TSP.2021.3068353.

M. Wagner, H. Groll, A. Dormian, V. Sathyaranayanan, C. Mecklenbrauker, and P. Gerstoft, “Phase coherent EM array measurement on a refractive environment,” *IEEE Trans. Antennas Propag.*, vol. 69, no. 10, pp. 6783–6796, Oct. 2021, doi: 10.1109/TAP.2021.3069516.

J. Fuchs, M. Gardill, M. Lubke, A. Dubey, and F. Lurz, “A machine learning perspective on automotive radar direction of arrival estimation,” *IEEE Access*, vol. 10, pp. 6775–6797, 2022, doi: 10.1109/ACCESS.2022.3141587.

C. R. Berger, B. Demiszis, J. Heckenbach, P. Willett, and S. Zhou, “Signal processing for passive radar using OFDM waveforms,” *IEEE J. Sel. Topics Signal Process.*, vol. 4, no. 1, pp. 226–238, Feb. 2010, doi: 10.1109/JSTSP.2009.2038977.

Y. Li, X. Wang, and Z. Ding, “Multi-target position and velocity estimation using OFDM communication signals,” *IEEE Trans. Veh. Technol.*, vol. 68, no. 2, pp. 1160–1174, Feb. 2020, doi: 10.1109/TVT.2019.2956928.

W. U. Bajwa, K. Gedeayibu, and Y. C. Eldar, “Identification of parametric underspread linear systems and super-resolution radar,” *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2548–2561, Jun. 2011, doi: 10.1109/TSP.2011.2114657.

S. Salous, Propagation Measurement and Channel Modelling, Hoboken, NJ, USA: Wiley, 2013, doi: 10.1002/9781118502280.

H. Groll, A. Prokes, A. E. Molisch, S. Cabano, E. Zochmann, S. Pratschner, M. Lerch, D. Schutzenhofer, M. Hofer, J. Blumenstein, S. Sangodiyon, and T. Zemen, “Sparsity in the delay-Doppler domain for measured 60 GHz vehicle-to-infrastructure communication channels,” in *Proc. IEEE Int. Conf. Commun. Workshops (ICC Workshops)*, May 2019, pp. 1–6, doi: 10.1109/ICC.2019.8756930.

Y. Chi and Y. Chen, “Compressive two-dimensional harmonic retrieval via atomic norm minimization,” *IEEE Trans. Signal Process.*, vol. 63, no. 4, pp. 1030–1042, Feb. 2015, doi: 10.1109/TSP.2014.2386283.

Z. Yang, L. Xie, and P. Stoica, “Vandemonde decomposition of multilevel Toeplitz matrices with application to multidimensional super-resolution,” *IEEE Trans. Inf. Theory*, vol. 62, no. 6, pp. 3685–3701, Jun. 2016, doi: 10.1109/TIT.2016.2555041.

T. Sarkar and O. Pereira, “Using the matrix pencil method to estimate the parameters of a sum of complex exponentials,” *IEEE Antennas Propag. Mag.*, vol. 37, no. 1, pp. 48–55, Feb. 1995, doi: 10.1109/74.370583.

E. Zochmann, T. Mathiesen, T. Blazek, H. Groll, and G. Ghiassi, “A bandwidth scalable millimetre wave over-the-air test system with low complexity,” in *Proc. 14th Eur. Conf. Antennas Propag. (EuCAP)*, Mar. 2020, pp. 1–5, doi: 10.23919/EUCAP4036.2020.9360131.

T. Blazek, H. Groll, S. Pratschner, and E. Zochmann, “Vehicular channel characterization in orthogonal time-frequency space,” in *Proc. IEEE Int. Conf. Commun. Workshops (ICC Workshops)*, May 2019, pp. 1–5, doi: 10.1109/ICC.2019.8756177.

J. Karedal, F. Tufvesson, N. Czink, A. Paier, C. Dumard, T. Zemen, C. A. M. Meker, and A. E. Molisch, “A geometry-based stochastic MIMO model for vehicle-to-vehicle communications,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3646–3657, Jul. 2009, doi: 10.1109/TWC.2009.080753.

C. Gustafson, K. Mahler, D. Bolin, and F. Tufvesson, “The COST IRACON geometry-based stochastic channel model for vehicle-to-vehicle communication in interactions,” *IEEE Trans. Veh. Technol.*, vol. 69, no. 3, pp. 2365–2375, Mar. 2020, doi: 10.1109/TVT.2020.2964277.

S. Salous, V. D. Estopist, F. Fuschini, R. S. Thomae, R. Mueller, D. Duplech, K. Haneda, J. M. M. Garcia-Pardo, J. P. Garcia, D. P. Gailiot, S. H. Hur, and M. Nekovee, “Millimetre-wave propagation: Characterization and modeling toward fifth-generation systems,” *IEEE Antennas Propag. Mag.*, vol. 58, no. 5, pp. 115–127, Dec. 2016, doi: 10.1109/MAP.2016.2609815.

F. J. Harris, “On the use of windows for harmonic analysis with the discrete Fourier transform,” *Proc. IEEE*, vol. 66, no. 1, pp. 51–83, Jan. 1978, doi: 10.1109/PROC.1978.10837.

F. T. Ulaby, T. F. Haddock, and R. T. Austin, “Fluctuation statistics of millimeter-wave scattering from distributed targets,” *IEEE Trans. Geosci. Remote Sens.*, vol. 26, no. 3, pp. 268–281, May 1988, doi: 10.1109/36.3030.

N. Czink, F. Kaltenberger, Y. Zhou, L. Bernadó, T. Zemen, and X. Yin, “Low-complexity geometry-based modeling of diffuse scattering,” in *Proc. 4th Eur. Conf. Antennas Propag. (EuCAP)*, 2010, pp. 1–4.

M. Pätzold and C. A. Gutierrez, “Modelling of non-WSSUS channels with time-variant Doppler and delay characteristics,” in *Proc. IEEE 7th Int. Conf. Commun. Electron. (ICCE)*, Jul. 2018, pp. 1–6.

D. J. Thomson, “Spectrum estimation and harmonic analysis,” *Proc. IEEE*, vol. 70, no. 9, pp. 1055–1096, Sep. 1982, doi: 10.1109/PROC.1982.12433.

G. Matz, “Doubly underspread non-WSSUS channels: Analysis and estimation of channel statistics,” in *Proc. 4th IEEE Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jun. 2003, pp. 190–194, doi: 10.1109/SPAWC.2003.1318948.

H. L. Van Trees, Detection, Estimation, and Modulation Theory: Part III. Radar-Sonar Signal Processing and Gaussian Signal in Noise. Hoboken, NJ, USA: Wiley, 2001.

B. O’Donoghue, E. Chu, N. Parikh, and S. Boyd, “Comon optimization via operator splitting and homogenious self-dual embedding,” *J. Optim. Theory Appl.*, vol. 169, no. 3, pp. 1042–1086, Jun. 2016, doi: 10.1007/s10957-016-0892-3.

A. Paier, T. Zemen, L. Bernado, G. Matz, J. Karedal, N. Czink, C. Dumard, F. Tufvesson, A. E. Molisch, and C. F. Mecklenbrauker, “Non-WSSUS vehicular channel characterization in highway and urban scenarios at 5.2 GHz using the local scattering function,” in *Proc. Int. ITG Workshop Smart Antennas*, Feb. 2008, pp. 9–15, doi: 10.1109/WSA.2008.4755530.

D. Sleipan, “Prolate spheroidal wave functions, Fourier analysis, and uncertainty—V: The discrete case,” *Bell Syst. Tech. J.*, vol. 57, no. 5, pp. 1371–1430, May 1978, doi: 10.1002/1538-7630.1978.tb02104.x.

D. B. Percival and A. T. Walden, *Spectral Analysis for Univariate Time Series* (Cambridge Series in Statistical and Probabilistic Mathematics). Cambridge, U.K.: Cambridge Univ. Press, 2020.

S. Diasgnd and S. Boyd, “CVXPY: A Python-embedded modeling language for convex optimization,” *J. Mach. Learn. Res.*, vol. 17, no. 1, pp. 2909–2913, Jan. 2016, [Online]. Available: http://stanford.edu/~boyd/papers/pdf/cvxpy_paper.pdf.

B. O'Donoghue, E. Chu, N. Parikh, and S. Boyd. (Sep. 2021). SCS: Splitting Conic Solver, Version 2.1.4. [Online]. Available: https://github.com/cvxgrp/scs.
MARKUS HOFER (Member, IEEE) received the Dipl.-Ing. degree (Hons.) in telecommunications from the Vienna University of Technology, Vienna, Austria, in 2013, and the Ph.D. degree, in 2019. From 2013 to 2015, he was a Researcher with the Signal and Information Processing Department, FTW Telecommunications Research Center Vienna. He has been with the AIT—Austrian Institute of Technology, Vienna, since 2015. He is currently working as a Scientist with the Research Group for Ultrareliable Wireless Machine-to-Machine Communications. His research interests include ultra-reliable low latency wireless communications, reflective intelligent surfaces, mmWave communications, cell-free massive MIMO, time-variant channel measurements, modeling and real-time emulation, time-variant channel estimation, 5G massive MIMO systems, software-defined radio rapid prototyping, cooperative communication systems, and interference management.

THOMAS ZEMEN (Senior Member, IEEE) received the Dipl.-Ing. degree (Hons.) in electrical engineering, the Ph.D. degree (Hons.), and the Venia Docendi (Habilitation) degree in mobile communications from the Vienna University of Technology, in 1998, 2004, and 2013, respectively. He is currently a Principal Scientist at the AIT—Austrian Institute of Technology, Vienna, Austria, leading the Reliable Wireless Communications Group. He joined AIT, in 2014, and took on the role of thematic coordinator for physical layer security, in 2018. From 2003 to 2014, he was with FTW Forschungszentrum Telekommunikation Wien heading the Signal and Information Processing Department, since 2008. From 1998 to 2003, he worked as a Hardware Engineer and the Project Manager of the Radio Communication Devices Department, Siemens Austria. He has authored four book chapters, 39 journal articles, more than 119 conference communications, and two patents. His research interest includes the interplay of the physical wireless radio communication channel with other parts of a communication system in time-critical applications.

JIRI BLUMENSTEIN (Member, IEEE) received the Ph.D. degree from the Brno University of Technology, in 2013. In 2011, he was a Researcher with the Institute of Telecommunications, TU Wien. He has cooperated with several companies, including Racom, Volkswagen, and ON Semiconductor in the area of applied research of wireless systems and in the area of the fundamental research funded by the Czech Science Foundation. He is currently a Researcher with the Department of Radio Electronics, Brno University of Technology. His research interests include signal processing, physical layer of communication systems, channel characterization and modeling, and wireless system design.

CHRISTOPH F. MECKLENBRÄUKER (Senior Member, IEEE) received the Dipl.-Ing. degree (Hons.) in electrical engineering from Technische Universität Wien, Vienna, Austria, in 1992, and the Dr.-Ing. degree (Hons.) from Ruhr-Universität Bochum, Bochum, Germany, in 1998. From 1997 to 2000, he worked at Siemens AG Austria and engaged in the standardization of UMTS. From 2000 to 2006, he held a senior researcher position with the Telecommunications Research Center Vienna (FTW), Vienna. In 2006, he joined TU Wien as a Full Professor. From 2009 to 2016, he has led the Christian Doppler Laboratory for Wireless Technologies for Sustainable Mobility. He has authored approximately 250 papers in international journals and conferences, for which he has also served as a reviewer and was granted several patents in the field of mobile cellular networks. His current research interests include 5G and 6G radio interfaces (vehicular connectivity and sensor networks) and antennas and propagation. He is a member of the Antennas and Propagation Society, the Intelligent Transportation Society, the Vehicular Technology Society, the Signal Processing Society, VDE, and EURASIP. His Ph.D. dissertation received the Gert-Massenberg Prize, in 1998. He is the Councilor of the IEEE Student Branch Wien.