Constraints on planet X/Nemesis from Solar System’s inner dynamics

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Accepted 2009 July 28. Received 2009 July 8; in original form 2009 May 29

ABSTRACT
We use the corrections $\Delta \dot{\omega}$ to the standard Newtonian/Einsteinian perihelion precessions of the inner planets of the Solar system, recently estimated by E.V. Pitjeva by fitting a huge planetary data set with the dynamical models of the EPM ephemerides, to put constraints on the position of a putative, yet undiscovered large body $X$ of mass $M_X$, not modelled in the EPM software. The direct action of $X$ on the inner planets can be approximated by a elastic Hooke-type radial acceleration plus a term of comparable magnitude having a fixed direction in space pointing towards $X$. The perihelion precessions induced by them can be analytically worked out only for some particular positions of $X$ in the sky; in general, numerical calculations are used. We show that the indirect effects of $X$ on the inner planets through its action on the outer ones can be neglected, given the present-day level of accuracy in knowing $\Delta \dot{\omega}$. As a result, we find that Mars yields the tightest constraints, with the tidal parameter $K_X = \frac{GM_X}{r_X^3} \leq 3 \times 10^{-24} \text{s}^{-2}$. To constrain $r_X$ we consider the case of a rock-ice planet with the mass of Mars and the Earth, a giant planet with the mass of Jupiter, a brown dwarf with $M_X = 80m_{\text{Jupiter}}$, a red dwarf with $M = 0.5 \text{M}_\odot$ and a Sun-mass body. For each of them we plot $r_{X}^{\text{min}}$ as a function of the heliocentric latitude $\beta$ and longitude $\lambda$. We also determine the forbidden spatial region for $X$ by plotting its boundary surface in the three-dimensional space; it shows significant departures from spherical symmetry. A Mars-sized body can be found at not less than 70–85 au: such bounds are 147–175 au, 1006–1200 au, 4334–5170 au, 8113–9524 au and 10 222–12 000 au for a body with a mass equal to that of the Earth, Jupiter, a brown dwarf, red dwarf and the Sun, respectively.

Key words: gravitation – Celestial mechanics – ephemerides – Kuiper Belt – Oort Cloud – planets and satellites: general.

1 INTRODUCTION

Does the Solar system contain an undiscovered massive planet or a distant stellar-sized companion of the Sun?

The history of a hypothetical planet X dates back to the early suggestions by the astronomer P. Lowell (Lowell 1915), who thought that some glitches in the orbit of Uranus might be caused by what he dubbed Planet X. In 1930, the search that Lowell initiated led to the discovery of Pluto (Tombaugh 1961). The various constraints on the mass and position of a putative planet X, as candidate to accommodate the alleged orbital anomalies of Uranus (Brunini 1992), were summarized by Hogg, Quinlan & Tremaine (1991). Later, the claimed residuals in the orbit of Uranus were explained by Standish (1993) in terms of small systematic errors as an underestimate of the mass of Neptune by 0.5 per cent. However, the appeal of a yet undiscovered Solar system’s body of planetary size never faded. Indeed, according to Lykawka & Tadashi (2008), a ninth planet as large as the Earth may exist beyond Pluto, at about 100–170 au, to explain the architecture of the Edgeworth–Kuiper Belt. Previously, Brunini & Melita (2002) proposed the existence of a Mars-size body at 60 au to explain certain features in the distribution of the Trans-Neptunian Objects (TNOs) like the so-called Kuiper Cliff where low-eccentricity and low-inclination Kuiper Belt objects (KBOs) with semimajor axes greater than 50 au rapidly falls to zero, although several problems in explaining other features of the Kuiper Belt with such a hypothesis were pointed out later (Melita et al. 2004). According to Matose, Whitman & Whitmire (1999), a perturber body of mass $m \approx 1.5m_{\text{Jupiter}}$ at 25 kau would be able to explain the anomalous distribution of orbital elements of approximately 25 per cent of the 82 new class I Oort cloud comets. A similar hypothesis was put forth by Murray (1999); for a more sceptical view, see Horner & Evans (2002). Gomes, Matose & Lissauer (2006) suggested that distant detached objects among the TNOs (perihelion distance $q > 40$ au and semimajor axis $a > 50$ au) may have been generated by a hypothetical Neptune-mass companion having semimajor axis $b \leq 2000$ au or a Jupiter-mass companion with $b \leq 5000$ au on significantly inclined orbits. However, generally

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speaking, we stress that we still have bad statistics about the TNOs and the Edgeworth–Kuiper Belt objects (Sébastien & Morbidelli 2007; Schwamb, Brown & Rabinowitz 2009).

Concerning the existence of a putative stellar companion of the Sun, it was argued with different approaches. As an explanation of the peculiar properties of certain pulsars with anomalously small period derivatives, it was suggested by Harrison (1977) that the barycentre of the Solar system is accelerated, possibly because the Sun is a member of a binary system and has a hitherto undetected companion star. Latest studies by Zakamska & Tremaine (2005) constrain such a putative acceleration at a \( A_{\text{bary}} \lesssim 1.2 \times 10^{-9} \text{ m s}^{-2} \text{ level} \) \((4 \times 10^{-18} \text{ s}^{-1} \text{ in units of } A_{\text{bary}}/c\), where \( c \) is the speed of light in vacuum). Whitmire & Jackson (1984) and Davis, Hut & Muller (1984) suggested that the statistical periodicity of about 26 Myr in extinction rates on the Earth over the last 250 Myr reported by Raup & Sepkoski (1984) can be explained by a yet undetected companion star (called Nemesis) of the Sun in a highly elliptical orbit that periodically would disturb comets in the Oort cloud, causing a large increase in the number of comets visiting the inner Solar system with a consequential impact in increase events on the Earth. In a recent work, Muller (2002) used the hypothesis of Nemesis to explain the measurements of the ages of 155 lunar spherules from the Apollo 14 site. The exact nature of Nemesis is uncertain; it could be a red dwarf (Muller 2002) \((0.075 \leq m \leq 0.5 \text{M}_\odot)\) or a brown dwarf (Whitmire & Jackson 1984) \((m \approx 75–80 \text{m}_\odot)\). For some resonant mechanisms between Nemesis and the Sun triggering Oort cloud’s comet showers at every perihelion passage, see Vandervoort & Sather (1993). Recently, Foot & Silagadze (2001) and Silagadze (2001) put forth the hypothesis that Nemesis could be made up of the so-called mirror matter, whose existence is predicted if parity is an unbroken symmetry of nature.

In this paper, we constrain the distance of a very distant body for different values of its mass in a dynamical, model-independent way by looking at the gravitational effects induced by it on the motions of the inner planets orbiting in the 0.4–1.5 au range. The same approach was followed by Khriplovich & Pitjeva (2006) and Khriplovich (2007) to put constraints on density of diffuse dark matter in the Solar system. In Section 2, we calculate the acceleration imparted by a distant body X on an inner planet P and the resulting perihelion precession averaged over one orbital revolution of P. In Section 3, we discuss the position-dependent constraints on the minimum distance at which X can exist for several values of its mass, and depict the forbidden regions for it in the three-dimensional space. In Section 4, we compare our results to other constraints existing in the literature and summarize our findings.

## 2 THE PERIHELION PRECESSIONS INDUCED BY A DISTANT MASSIVE BODY

The gravitational acceleration imparted by a body of mass \( M_X \) on a planet P, with respect to some inertial frame, is

\[
A_{\text{PX}} = \frac{GM_X}{|r_X - r_P|^3} (r_X - r_P). \tag{1}
\]

If, as in our case, it is supposed that \( r_X \gg r_P \), by neglecting terms of order \( O(r_P^2/r_X^2) \), it is possible to use the approximated formula

\[
\frac{1}{|r_X - r_P|^3} \approx \frac{1}{r_X^3} \left[ 1 + \frac{3 (r_P \cdot \hat{n}_X)}{r_X} \right] + O \left( \frac{r_P}{r_X} \right)^2, \tag{2}
\]

where

\[
\hat{n}_X \equiv \frac{r_X}{r_X}. \tag{3}
\]

is the unit vector of X which will be assumed constant over one orbital revolution of P. By inserting equation (2) in equation (1) and neglecting the resulting term of order \( O(r_P^2/r_X^2) \) one has

\[
A_{\text{PX}} \approx -\frac{GM_X}{r_X^2} r_P + \frac{3GM_X}{r_X^2} (r_P \cdot \hat{n}_X) \hat{n}_X + \frac{GM_X}{r_X^2} \hat{n}_X. \tag{4}
\]

The acceleration of equation (4) consists of three terms: a non-constant radial Hooke-type term, a non-constant term directed along the fixed direction of \( \hat{n}_X \) and a constant (over the typical time-scale of P) term having the direction of \( \hat{n}_X \) as well. An equation identical to equation (4) can be written also for the Sun by replacing everywhere \( r_P \) with \( r_s \); thus, since we are interested in the motion of the planet P relative to the Sun, the constant term cancels out, and by posing \( r = r_P - r_s \) one can write down the heliocentric perturbing acceleration felt by the planet P due to X as

\[
A_X = A_{\text{Hooke}} + A_{\text{NX}} = -\frac{GM_X}{r_X^2} r + \frac{3GM_X}{r_X^2} (r \cdot \hat{n}_X) \hat{n}_X. \tag{5}
\]

In fact, the planetary observation-based quantities we will use in the following have been determined in the frame of reference of the presently known Solar system’s barycentre (SSB). By the way, equation (5), where \( r \) is intended as the position vector of P with respect to the known SSB, is valid also in this case. Indeed, by repeating the same reasonings as before, we are led to a three-term equation like equation (4) in which the vectors entering it refer to the SSB frame; now, if X existed, the SSB frame would be uniformly accelerated by

\[
A_{\text{SSB}} = \frac{GM_X}{r_X^2} \hat{n}_X, \tag{6}
\]

so that P, referred to such a non-inertial SSB frame, would also be acted upon by an inertial acceleration

\[
A_{\text{in}} = -\frac{GM_X}{r_X^2} \hat{n}_X, \tag{7}
\]

in addition to the gravitational one of equation (4), which just cancels out the third constant term in equation (4) leaving us with equation (5). Note that, in general, it is not legitimate to neglect \( A_{\text{NX}} \) with respect to \( A_{\text{Hooke}} \) since

\[
\frac{A_{\text{NX}}}{A_{\text{Hooke}}} = 3 \cos \psi, \tag{8}
\]

where \( \psi \) is the angle between \( r \) and \( \hat{n}_X \), which, in general, may vary within \( 0 \leq \psi \leq 2\pi \) during an orbital revolution of P. Moreover, if the acceleration of a distant body could only be expressed by a Hooke-type term, this would lead to the absurd conclusion that no bodies exist at all outside the Solar system because the existence of an anomalous Hooke-like acceleration has been ruled out by taking the ratios of the perihelia of different pairs of planets (Iorio 2008).

The acceleration of equation (5) is in agreement with the potential

\[
U_X = \frac{GM_X}{2r_X} \left[ r^2 - 3 (r \cdot \hat{n}_X)^2 \right] \tag{9}
\]

proposed by Hogg et al. (1991). The action of equation (5) on the orbit of a known planet of the Solar system can be treated perturbatively with the Gauss equations (Bertotti et al. 2003) of the variations of the Keplerian orbital elements

\[
d\tilde{a}/dt = \frac{2}{n\sqrt{1-e^2}} \left[ eA_t \sin f + A_t \left( P_t \right) \right], \tag{10}
\]

\[
d\tilde{e}/dt = \frac{\sqrt{1-e^2}}{na} \left\{ A_t \sin f + A_t \left[ \cos f + \frac{1}{e} \left( 1 - \frac{r}{a} \right) \right] \right\}, \tag{11}
\]
do not include the force imparted by a distant companion of the Sun, such corrections are, in principle, suitable to constrain the unmodelled action of such a putative body accounting for its direct action on the inner planets themselves and, in principle, the indirect one on them through the outer planets; we will discuss such an issue later.

Concerning the calculation of the perihelion precession induced by equation (5), as already noted, \( r_X \) can certainly be considered as constant over the orbital periods \( P_0 \lesssim 1.5 \) yr of the inner planets. Such an approximation simplifies our calculations. In the case of the Hooke-type term of equation (5)

\[
A_{\text{Hooke}} = -K_X r, \quad K_X = \frac{GM_X}{r_X^3},
\]

the task of working out its secular orbital effects has already been performed several times in literature (see e.g. Iorio 2008 and references therein). The result for the longitude of perihelion is

\[
\langle \Delta \varpi \rangle_{\text{Hooke}} = \frac{3}{2} K_X \sqrt{1 - e^2} \frac{\sqrt{1 - e^2}}{n} \left[ -\frac{1}{2} \frac{GM}{a^3} \right].
\]

The treatment of the second term \( A_{\text{Hooke}} \) of equation (5) is much more complex because it is not simply radial. Indeed, its direction is given by \( \hat{\mathbf{r}}_X \) which is fixed in the inertial space \( \{x,y,z\} \) during the orbital motion of \( P \). Its components are the three direction cosines, so that the Cartesian components of

\[
A_{\text{Hooke}} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}
\]

are

\[
\begin{align*}
A_x &= 3K_X (sx_x + y_n + zn_x) n_x, \\
A_y &= 3K_X (sx_y + y_n + zn_y) n_y, \\
A_z &= 3K_X (sx_z + y_n + zn_z) n_z.
\end{align*}
\]

In terms of the standard heliocentric angular coordinates \( \lambda, \beta \), we can write the components of \( \hat{\mathbf{n}}_X \) as

\[
\begin{align*}
n_x &= \sqrt{1 - \sin^2 \beta_X \cos \lambda_X}, \\
n_y &= \sqrt{1 - \sin^2 \beta_X \sin \lambda_X}, \\
n_z &= \sin \beta_X,
\end{align*}
\]

with \( n_x^2 + n_y^2 + n_z^2 = 1 \). In order to have the radial, transverse and normal components of \( A_{\text{Hooke}} \) to be inserted into the right-hand-sides of the Gauss equations

\[
\begin{align*}
A_i &= A_{\text{Hooke}} \cdot \hat{\mathbf{r}}_x, \\
A_i &= A_{\text{Hooke}} \cdot \hat{\mathbf{i}}, \\
A_n &= A_{\text{Hooke}} \cdot \hat{\mathbf{n}}_x.
\end{align*}
\]

1 We used \( \dot{a} = \dot{f} \) because over one orbital revolution the pericentre \( \omega \) can be assumed constant.

2 Only their standard \( N \)-body mutual interactions have been, indeed, modelled in the EPM ephemerides.

3 Recall that \( \lambda \) is the usual longitude \( \phi \) of the standard spherical coordinates, while \( \beta = 90^\circ - \phi \), where \( \phi \) is the usual co-latitude of the standard spherical coordinates.

Table 1. Estimated corrections \( \Delta \varpi \), in milliarcsec cty\(^{-1} \) (1 arcsec cty\(^{-1} \) = 1.5 \times 10^{-15} \) s\(^{-1} \), to the standard Newton/Einstein perihelion precessions of the inner planets according to table 3 of Pitjeva (2005) (Mercury, Earth, Mars). The result for Venus has been obtained by recently processing radio-metric data from Magellan spacecraft (E.V. Pitjeva, private communication). The errors are not the formal, statistical ones. The SSB frame, assumed as inertial, i.e. without modelling the action of a putative body \( X \), has been used.

| Planet   | Correction |
|----------|------------|
| Mercury  | 3.6 ± 5.0  |
| Venus    | −0.4 ± 0.5 |
| Earth    | −0.2 ± 0.4 |
| Mars     | 0.1 ± 0.5  |
we need the Cartesian components of the comoving unit vectors \( \hat{r}, \hat{i}, \hat{n} \): they are (Montenbruck & Gill 2000)

\[
\hat{r} = \left( \cos \Omega \cos u - \cos I \sin \Omega \sin u \right) \sin i \sin u
\]

\[
\hat{i} = \left( \sin \Omega \cos u + \cos I \cos \Omega \sin u \right) \sin i \cos u
\]

\[
\hat{n} = \left( -\sin I \cos \Omega \right) \cos u
\]

(31)

(32)

(33)

In our specific case, \( \Omega \) is the longitude of the ascending node which yields the position of the line of the nodes, i.e. the intersection of the orbital plane with the mean ecliptic at the epoch (J2000), with respect to the reference \( x \)-axis pointing toward the Aries point \( \Upsilon \).

Since, according to equations (22)–(24), the planet’s coordinates \( x, y, z \) enter the components of \( A_{X} \), we also need the expressions for the unperturbed coordinates of the planet (Montenbruck & Gill 2000)

\[ x = r (\cos \Omega \cos u - \cos I \sin \Omega \sin u), \]

\[ y = r (\sin \Omega \cos u + \cos I \cos \Omega \sin u), \]

\[ z = r \sin I \sin u. \]

Thus, \( A_{1}, A_{2}, A_{3} \) are non-linear functions of the three unknown parameters \( K_{X}, \lambda_{X}, \beta_{X} \) of \( X \); by computing the averaged perihelion precessions induced by \( A_{\text{Hooke}} \) and \( A_{\text{aX}} \) and comparing them with the estimated corrections \( \Delta \phi \) to the usual perihelion precessions, it is possible to have an upper bound on \( K_{X} \), and, thus, a lower bound on \( r_{X} \) for given values of \( M_{X} \), as a function of the position of \( X \) in the sky, i.e. \( r_{X}^{\min} = r_{X}^{\text{min}}(\lambda_{X}, \beta_{X}) \).

Relatively simple analytical expressions can be found only for particular positions of the body \( X \); for example, by assuming \( n_{1} = n_{2} = 0 \), i.e. by considering \( X \) located somewhere along the \( z \)-axis, it is possible to obtain

\[
\langle \dot{\phi} \rangle_{\text{ax}} = \frac{9K_{X} \sqrt{1 - e^{2}} \sin^{2} I}{4n} \left( 1 + \frac{5}{3} \cos 2\omega \right). \]

(37)

In Table 2, we quote the lower bound on \( r_{X} = z_{X} \) for several values of \( M_{X} \) obtained from the maximum of the estimated correction to the standard rate of the perihelion of Mars applied to equations (20) and (37). In general, the problem must be tackled numerically; also in this case, it turns out that the data from Mars yield the tightest constraints. However, equations (20) and (37) are already enough to show that the effect of a body \( X \) could not be mimicked by a correction to the Sun’s quadrupole mass moment \( J_{2} \) accounting for its imperfect modelling in the EPM ephemerides. Indeed, it is easy to show that the secular precession of the longitude of pericentre of a test body moving along an orbit with small eccentricity around an oblate body of equatorial radius \( R \) is

\[
\langle \dot{\phi} \rangle_{J_{2}} = \frac{3}{2} a \left( \frac{J_{2}}{a} \right)^{2} \left( \frac{5}{2} \cos^{2} I \right). \]

(38)

Now, let us consider the case in which \( X \) is directed along the \( z \)-axis and the planet \( P \) has \( I = 0 \); only the retrograde Hooke-type precession of equation (20) would be non-vanishing, while equation (38) would induce a prograde rate.

### 3 THE THREE-DIMENSIONAL SPATIAL CONSTRAINTS ON THE LOCATION OF \( X \)

In Fig. 1 we plot the maximum value of \( K_{X} \), obtained from the perihelion of Mars which turns out to yield the most effective constraints, as a function of \( \beta \) and \( \lambda \). It turns out that \( K_{X}^{\text{max}} \) is of the order of \( 10^{-24} \text{ s}^{-2} \).

The minimum distance \( r_{X} \) can be obtained for different values of \( M_{X} \) as a function of \( \beta \) and \( \lambda \) as well according to

\[
r_{X}^{\min} = \left( \frac{GM_{X}}{K_{X}^{\text{max}}} \right)^{1/3}. \]

(39)

We will consider the case of two rock-ice bodies with the masses of Mars and the Earth, a giant planet with the mass of Jupiter, a brown dwarf with \( M_{X} = 80m_{\text{Jupiter}} \), or a red dwarf with \( M_{X} = 0.5M_{\odot} \) and a Sun-mass body with \( M_{X} = M_{\odot} \), not necessarily an active main-sequence star.

It is also possible to visualize the forbidden region for \( X \) in the three-dimensional space by plotting its delimiting surface whose parametric equations are

\[
x = r_{X}^{\text{min}}(\beta, \lambda) \cos \beta \cos \lambda, \]

(40)

\[
y = r_{X}^{\text{min}}(\beta, \lambda) \cos \beta \sin \lambda, \]

(41)

\[
z = r_{X}^{\text{min}}(\beta, \lambda) \sin \beta. \]

(42)

In Fig. 2, we plot the minimum distance at which a body of mass \( M_{X} = m_{\text{Mars}} \) can be found as a function of its latitude and longitude \( \beta \) and \( \lambda \). The largest value is about 85 au and occurs in the ecliptic plane \( (\beta = 0) \) at about \( \lambda = 0^\circ, 60^\circ, 150^\circ, 250^\circ, 345^\circ \). Note that for \( \beta = \pm 90^\circ \), i.e. for \( X \) along the \( z \)-axis, the minimum distance is as in Table 2, i.e. 70 au. For just a few positions in the sky, such a Mars-sized body could be at no less than 20 au. In Fig. 3, we depict the surface of minimum distance delimiting the spatial region in which such a body can exist according to the data from Mars. It has not a simple spherical shape, as it would have had if \( X \) exerted an isotropic force on Mars, and it has a precise spatial orientation with respect to the \( \{x, y, z\} \) frame. The analytical results of Table 2 obtained with equation (37) in the case \( \beta = \pm 90^\circ \) are confirmed by Figs 2 and 3. By slicing the surface with a vertical symmetry plane, it turns out that the shape of the central bull is rather oblate, approximately by \( \approx 0.77 \), contrary to the lateral lobes which are more spherical. Moreover, the largest forbidden regions in the ecliptic plane are two approximately orthogonal strips with a length of about 180 au (see Fig. 4 depicting the situation in the ecliptic plane), while in the vertical direction there is a strip approximately 120 au long.

Figures analogous to Figs 2–4 hold for bodies with the mass of the Earth, Jupiter, a brown dwarf \( (m = 80m_{\text{Jupiter}}) \), or a red dwarf with
Figure 1. Maximum value, in s$^{-2}$, of $K_X = GM_X/r_X^3$ as a function of the heliocentric longitude $\lambda$ and latitude $\beta$ of the putative body $X$; its upper bound is of the order of $3 \times 10^{-24}$ s$^{-2}$. The perihelion of Mars has been used (Table 1).

Figure 2. Minimum distance $r_{min}$ at which a rock-ice planet of mass $M_X = m_{\text{Mars}}$ can exist as a function of its heliocentric longitude $\lambda$ and latitude $\beta$. The perihelion of Mars has been used (Table 1).

$M = 0.5 \, M_\odot$ and a Sun-sized body. The results of Table 2 concerning the position of $X$ along the $z$-axis are confirmed. The same qualitative features of the case $M_X = m_{\text{Mars}}$ occur. Concerning an Earth-sized body, it could mainly be found at no less than 147–175 au, with a minimum distance of 35 au for just a few positions in the sky. The minimum distance of a Jupiter-like mass is 1006–1200 au, with about 200 au in some points. For a brown dwarf ($M_X = 80m_{\text{Jupiter}}$), the limiting distance is mainly 4334–5170 au, with a minimum of 861 au at some positions, while for a red dwarf ($M_X = 0.5 \, M_\odot$), it is 8113–9524 au, with a lowest value of 2000 au. Finally, a Sun-mass body cannot be located at less than 10 222–12 000 au for most of the sky positions, with 2520 au at just a few points. In the case of an Earth-sized body, the length of the largest ecliptical forbidden strip turns out to be about 400 au, while the vertical one amounts to about 300 au.

Concerning the strategy adopted so far, let us note that we compared the estimated $\Delta \sigma$ for the inner planets to their computed perihelion precessions due to $X$ through equation (5); in fact, one
should, in principle, also take into account the indirect effects of X on the inner planets of the Solar system through its direct action on the outer planets. Indeed, if one of them is perturbed by X, its action on the inner planets will differ from the standard N-body one, fully modelled in the EPM ephemerides. To roughly evaluate such an effect, let us reason as follows. The position of both X and an outer planet acted upon by X can approximately be considered as constant in space with respect to a rocky planet over a typical orbital period $P_b \lesssim 1$ yr. The maximum value of the disturbing acceleration imparted by X on a giant planet, G, like Jupiter will be
of the order of
\[ A_{GX} \leq 3 \kappa_X r_{G} \approx 7 \times 10^{-12} \text{m s}^{-2}; \]  
(43)
we used \( \kappa_X = 3 \times 10^{-24} \text{s}^2. \) \( A_{GX} \) will displace the position of \( G \) with respect to its usual position by an amount
\[ \Delta r_G \approx \frac{A_{GX}}{2} \beta X_3 p_s^2 \approx 3 \times 10^{-9} \text{m} \]  
(44)
over \( p_s \leq 1 \text{yr} \) during which \( A_{GX} \) can be considered as uniform. As a consequence, the maximum extra-acceleration \( \Delta A_{P} \) induced on an inner planet \( P \) by \( G \) during \( p_s \) can be evaluated as
\[ \Delta A_{P} \approx 12 \left( \frac{G m_G}{r_G^2} \right) r < 2 \times 10^{-15} \text{m s}^{-2}; \]  
(45)
we have just used equation (5) adapted to the present case. The perihelion precession induced by \( \Delta A_{P} \) can roughly be evaluated by dividing it by the orbital velocity of \( P \), i.e. \( an \), so that
\[ (\omega)_G \approx \frac{\Delta A_{P}}{an} < 10^{-5} \text{arcsec, ct}^{-1}. \]  
(46)
It is about one order of magnitude smaller than the present-day accuracy in knowing the precessions of the perihelia of the inner planets, as shown by Table 1. Thus, we conclude that the indirect effects of \( X \) on the inner planets through its action on the giant ones can be neglected, as we did.

4 DISCUSSION AND CONCLUSIONS

We will, now, compare our dynamical constraints with others obtained with different methods.

First, we compare the dynamical constraints of Table 2 and, more generally, of our full analysis with those obtainable as \( r_X = \sqrt{G M_X/r_{bar}^3} \) from the upper bound on the SSB’s acceleration \( A_{bar} \leq 1.2 \times 10^{-9} \text{m s}^{-2} \) recently derived by Zakamska & Tremaine (2005) with an analysis of the timing data of several millisecond pulsars, pulsars in binary systems and pulsating white dwarf. Indeed, an acceleration exerted on the known SSB would affect the observed value of the rate of the period change of astronomical clocks such as pulsars and pulsating white dwarfs. Such a method is able to provide a uniform sensitivity over the entire sky. The pulsar-based, isotropic \( 1/r_X^3 \) constraints are summarized in Table 3. By ‘isotropic’ we mean that the value of \( A_{bar} \) used can be ruled out for 100 per cent of the sky at 95 per cent confidence level, with practically all the methods used by Zakamska & Tremaine (2005); by using PSR B1913 + 6 the quoted acceleration is ruled out at 95 per cent confidence for 94 per cent of the sky. They are not competitive with those of Table 2 and the rest of our analysis for all the bodies considered. However, according to Eichler (2009), future timing of millisecond pulsars looking for higher order pulsar period derivatives should be able to extend such limits to several thousand au within a decade. A precise limit on the unmodelled relative acceleration between the Solar system and PSR J0437 – 4715 has recently been obtained by Deller (2008) by comparing a VLBI-based measurement of the trigonometric parallax of PSR J0437 – 4715 with the kinematic distance obtained from pulsar timing, which is calculated from the pulsar’s proper motion and apparent rate of change of orbital period. As a result, Jupiter-mass planets within 226 au of the Sun in 50 per cent of the sky (95 per cent confidence) are excluded. Proposals to search for primordial black holes (PBHs) with the Square Kilometre Array by using modification of pulsar timing residuals when PBHs pass within about 1000 au and impart impulse accelerations to the Earth have been put forth by Seto & Cooray (2007).

Other acceleration-type \( 1/r_X^3 \) constraints were obtained by looking for a direct action of \( X \) on the Pioneer 10/11 spacecraft from an inspection of their tracking data (Anderson 1988): the upper bound in the SSB acceleration obtained in this way is \( 4.2 \times 10^{-30} \text{m s}^{-2} \). This translates in a re-scaling of the values of Table 3 by a factor of 1.7, still not competitive with ours.

Hogg et al. (1991) looked for tidal-type \( 1/r_X^3 \) constraints, like ours, with a dynamical approach based on numerical simulations of the data of the four outer planets over the time-span 1910–1990. They found a relation among \( M_X \), in units of terrestrial masses, \( r_X \), in units of 0.1 au, and \( \sigma_0 \), which is the standard deviation of the assumed Gaussian random errors in \( \lambda \) and \( \beta \), in units of 0.1 arcsec; it is, for the ecliptic plane,
\[ \left( \frac{r_X}{100 \text{au}} \right) > \left[ \left( \frac{M_X}{m_\oplus} \right) \left( \frac{0.1 \text{arcsec}}{\sigma_0} \right) \frac{1}{(6 - 10)} \right]^{1/3}. \]  
(47)
By assuming, rather optimistically, \( \sigma_0 \approx 0.01 \text{arcsec} \), equation (47) yields for \( M_X = m_\oplus r_X \geq 120 \text{au} \), which is generally smaller than our limits for the ecliptic plane; a more conservative value of \( \sigma_0 \approx 0.1 \text{arcsec} \) yields \( r_X \geq 55 \text{au} \). It may be interesting to recall that Brunini (1992) predicted the existence of an alleged perturber of Uranus with \( M_X = 0.6 m_\oplus \) at a heliocentric distance of 44 au.

Let us, now, focus on direct observational limits. A planetary body would reflect the solar light and, therefore, could be detected in the optical or near-infrared surveys. Tombaugh (1961) conducted an all-sky optical survey, in the plane of the ecliptic, that yielded the following constrain:
\[ \left( \frac{r_X}{100 \text{au}} \right) > \left[ \left( \frac{M_X}{m_\oplus} \right) \frac{1}{3.4} \right]^{1/6}, \]  
(48)
by assuming a visual albedo \( p_a = 0.02 \) (Hogg et al. 1991). For an Earth-sized body the limit is \( r_X \geq 81 \text{au} \). Another optical survey yielding similar results was performed by Kowal (1989). In addition to the reflected sunlight, the X’s own thermal emission would be detectable in the mid-to-far-infrared. The IRAS All-Sky Survey (Neugebauer et al. 1984) would have been able to discover a gas giant in the range \( 70 \leq r_X \leq 400 \text{au} \), but without success, in agreement with our bounds for Jupiter-sized bodies. A recent ecliptic survey was done by Larsen (2007) with the Spacewatch telescope. This survey was sensitive to Mars-sized objects out to 300 au and Jupiter-sized planets out to 1200 au; for low inclinations to the ecliptic, it ruled out more than one to two Pluto-sized objects out to 100 au and one to two Mars-sized objects to 200 au. Concerning the case of a Sun-mass body, the presence of a main-sequence star above the hydrogen-burning limit was excluded within 1 pc by the all-sky synoptic Tycho-2 survey (Hogg et al. 2000).

Table 3. Approximate minimum heliocentric distance \( r_X \), in au, at which a still unseen object having a mass \( M_X \) equal to that of the astronomical bodies listed here can be located according to the limit \( A_{bar} = G M_X/r_X^3 \leq 1.2 \times 10^{-9} \text{m s}^{-2} \) on the SSB’s acceleration recently obtained by Zakamska & Tremaine (2005) from pulsar timing data. We used the maximum value \( A_{bar} = 4 \times 10^{-18} \text{s}^{-1} \), where \( c \) is the speed of light in vacuum, of the SSB acceleration allowed by pulsar timing data for about 100 per cent of the sky at 95 per cent level of confidence (table 2 of Zakamska & Tremaine 2005).

| Mars | Earth | Jupiter | Sun |
|------|-------|---------|-----|
| 1.4  | 3.9   | 68.6    | 2224|
Observational constraints on the properties of X might be obtained by sampling the astrometric position of background stars over the entire sky with the future astrometric GAIA mission (Scott et al. 2005). Indeed, the apparent motion of X along the parallactic ellipse would deflect the angular position of distant stars due to the astrometric microlensing ('induced parallax'). A Jupiter-sized planet at 2000 au in the ecliptic plane could be detected by GAIA. Also the phenomenon of mesolensing (Di Stefano 2008) could be used to gain information on possible planets at distances >1000 au. Babich, Blake & Steinhardt (2007) proposed to use the spectral distortion induced on the cosmic microwave background (CMB) by putative distant masses to put constraints on the physical properties and distances of them. With the all-sky synoptic survey Pan-STARRS (Jewitt 2003) massive planets such as Neptune would be undetectable through reflected sunlight beyond about 800 au, while a body with $M_X = 0.1 M_{\odot}$ would be undetectable for $r_X > 2000$ au.

In conclusion, the dynamical constraints on a still undiscovered planet X in the outer regions of the Solar system that we dynamically obtained from the orbital motions of the inner planets of the Solar system with the extra-precessions of the perihelia estimated by Pitjeva with the EPM ephemerides are tighter than those obtained from pulsar timing data analysis and the outer planets dynamics; moreover, they could be used in conjunction with the future planned all-sky surveys in the choice of the areas of the sky to be investigated. If, and when, other teams of astronomers will independently estimate their own correction to the standard perihelion precessions with different ephemerides, it will be possible to repeat such tests. Moreover, a complementary approach to that presented here which could also be implemented consists of modifying the dynamical force models of the ephemerides by also including the action of X on all the planets of the Solar system, and repeating the global fitting procedure of the entire planetary data set estimating, among other parameters, also those which directly account for X itself and looking at a new set of planetary residuals.

ACKNOWLEDGMENTS

I gratefully thank C. Heinke and D. Ragozzine for their important critical remarks.

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