Simulation of wave mitigation by coastal vegetation using smoothed particle hydrodynamics method

Iryanto¹, P H Gunawan²

¹ Computational Science, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Jalan Ganesha 10, Bandung 40132, Indonesia.
² Department of Computational Science, School of Computing, Telkom University, Jalan Telekommunikasi Terusan Buah Batu, Bandung 40257, Indonesia.
E-mail: ¹iryanto.math@yahoo.com; ²harry.gunawan.putu@gmail.com

Abstract. Vegetation in coastal area lead to wave mitigation has been studied by some researchers recently. The effect of vegetation forest in coastal area is minimizing the negative impact of wave propagation. In order to describe the effect of vegetation resistance into the water flow, the modified model of framework smoothed hydrodynamics particle has been constructed. In the Lagrangian framework, the Darcy, Manning, and laminar viscosity resistances are added. The effect of each resistances is given in some results of numerical simulations. Simulation of wave mitigation on sloping beach is also given.

1. Introduction
Coastal forest vegetation (such as Mangrove forest) becomes really important in the structure of coastal environmental. This forest has many advantages, for instance to keep the natural habitat and also as a wave barrier. The role of coastal forest in the mitigation of tsunami impacts is a debatable topic and numerical simulation can provide an important contribution to this debate. For instance, several articles have been devoted to the numerical modeling of tsunami mitigation by mangroves in the past few years [1, 2, 3, 4].

One way of modeling the coastal forest resistances is to add a friction term to the governing equation of fluid motion. Another way of modeling the coastal forest resistances is to consider the vegetation trees structures and to take into account the drag and inertial forces they induce (using the Morison equation for instance [4]), or the forest resistances can be assumed as porous media [1]. However, the use of porous media represents the coastal vegetation in the governing equation of fluid motion is not straightforward. Hence, the modification of governing equation is needed for modeling fluid flow in porous media.

Here, we would like to elaborate the model of wave mitigation using smoothed particle hydrodynamics (SPH) introduced in [5, 6, 7, 8]. The mass, momentum and particle position equations of SPH framework for fluid flow are governed in the Lagrangian form as:
\[
\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{v},
\]
(1)
\[
\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho} \nabla P + \mathbf{g} + \mathbf{F}_{\text{force}},
\]
(2)
\[
\frac{D\mathbf{x}}{Dt} = \mathbf{v},
\]
(3)

where \(\rho\) is fluid density, \(t\) time, \(\mathbf{v}\) flow velocity, \(\mathbf{x}\) position of particles, \(P\) pressure, \(\mathbf{g}\) gravitational force \((0, -9.81)^T\) and \(\mathbf{F}_{\text{force}}\) additional force (such as surface tension, friction, etc). In order to use the original equations of SPH framework (1-3) for modeling the vegetation resistances directly, the term \(\mathbf{F}_{\text{force}}\) in Eq. (2) can be defined as the Darcy, Manning, and laminar viscosity resistances. Therefore, the use of vegetation resistance as a porous media in this paper is not considered since the modification of SPH framework (1-3) is needed.

The goals of this paper are; (a) to investigate further the modeling of wave mitigation by coastal vegetation, by comparing the different vegetation resistances and (b) to design SPH method for model with resistance terms.

This paper is organized as follows, in Section 2, the description of SPH and with the vegetation force will be given. The numerical results with some friction terms are elaborated in Section 3. Finally, in Section 4 the conclusions are drawn.

2. The model of coastal vegetation resistance in SPH
In this section, we would like to elaborate the governing equations of vegetation friction in Lagrangian fluid motion and its SPH approach.

2.1. Governing equation with vegetation friction
In modeling the resistance term in fluid flow, some equations of friction term are already proposed [9, 4, 10, 11, 12]. For instance, in [4] the friction term in water flow is modeled by Darcy or Manning friction. In Darcy friction, the friction term is given as

\[
\mathbf{F}_{\text{force}} = -\mu \frac{\mathbf{v} \mathbf{v}}{8g},
\]
(4)

where \(\mu\) is the Darcy coefficient. Following Gunawan, et al., [9], the Manning friction is given as

\[
\mathbf{F}_{\text{force}} = -\frac{C_f \mathbf{v} \mathbf{v}}{H^{1/3}},
\]
(5)

where \(C_f\) is the Manning coefficient, and \(H\) water depth. Another friction term can be modeled as the laminar viscous stress by Lo and Shao [13]:

\[
\mathbf{F}_{\text{force}} = \nu_0 \nabla^2 \mathbf{v}
\]
(6)

where \(\nu_0\) is the kinetic viscosity.

Finally the full momentum equation (2) with for example the Manning friction can be written as

\[
\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho} \nabla P + \mathbf{g} - \frac{C_f \mathbf{v} \mathbf{v}}{H^{1/3}}.
\]
(7)
2.2. SPH approach

The main feature of the SPH method is described in detail in Monaghan ([5, 6]) and Liu ([7, 8]). The general fundamental of SPH is to approximate any function \( \varphi(x) \), where \( \varphi : \mathbb{R} \to \mathbb{R} \) by an integral interpolation:

\[
\varphi(x) = \int \varphi(x') W(x - x', h) \, dx
\]

where \( h \) is the length of weighting function or kernel function \( W(x - x', h) \). In discrete form, this interpolation of point \( i \) becomes:

\[
\varphi(x_i) = \sum_j \omega_j \frac{\varphi_j}{\rho_j} W(x_i - x_j, h)
\]

where this summation is acting over all the particles \( j \) within a compact support region, fixed by \( h \) (see Figure 1). The volume of particle is defined by \( V_j = \omega_j/\rho_j \), with \( \omega_j \) and \( \rho_j \) are the mass and density of particle \( j \) respectively.

In [7], several conditions for weighting or kernel function \( W \) are given in order to increase the performance of SPH method such as the positivity, compact support, smoothness, normalization, etc. The kernel function are defined by the non-dimensional distance between particles \( i \) and \( j \) by \( q = r/h \), where \( r \) is the distance between particles. Some different kernel definitions can be found in the literature, for instance, see [6, 7, 14]. In this research, we use a common Cubic spline kernel function:

\[
W(r, h) = \alpha_D \begin{cases} 
1 - \frac{3}{2} q^2 + \frac{3}{4} q^3 & 0 \leq q \leq 1 \\
\frac{1}{4} (2 - q)^3 & 1 \leq q \leq 2 \\
0 & q \geq 2
\end{cases}
\]

where \( \alpha_D \) is the normalization parameter defined by \( 10/(7\pi h^2) \) or \( 1/(\pi h^3) \) for two- or three-dimensional space respectively. Next, the function of particle approximation for spatial derivative of particle \( i \) is given as

\[
\nabla \cdot \varphi(x_i) = -\sum_j \omega_j \frac{\varphi_j}{\rho_j} \nabla W(x_i - x_j, h),
\]

where

\[
\nabla W(x_i - x_j, h) = \nabla_i W_{ij} = \frac{x_i - x_j}{r_{ij}} \frac{\partial W_{ij}}{r_{ij}}, \quad r_{ij} = ||x_i - x_j||.
\]
2.3. Discrete space of continuity equation

From the SPH approach above, the discrete form of continuity equation (1) are given as

\[
\frac{d\rho_i}{dt} = \sum_j \omega_j (v_i - v_j) \nabla_i W_{ij},
\]

(13)

where \(\rho_i\) is the density corresponding to particle \(i\) and \(v_k\) is the velocity corresponding to the particle \(k\), evaluated at \(i\) or \(j\). This form follows the discrete form of mass equation as in the SPHysics open source code [14], in order to prevent artificial density near free surfaces and boundaries.

2.4. Discrete space of momentum equation

In the discrete form of momentum equation (2), we follow the discrete form with artificial viscosity proposed by Monaghan [5] and in detail can be found in [7]. The discrete form can be written as,

\[
\frac{dv_i}{dt} = - \sum_j \omega_j \left( \frac{p_j}{\rho_j} \frac{p_i}{\rho_i} + \Pi_{ij} \right) \nabla_i W_{ij} + g + F_{\text{force}}^i,
\]

(14)

where \(p_i\) and \(p_j\) are the pressure of particle \(i\) and \(j\) respectively. The term \(\Pi_{ij}\) denotes the artificial viscosity defined by

\[
\Pi_{ij} = \begin{cases} 
-\alpha_{\Pi} \bar{c}_{ij} \phi_{ij} \frac{v_{ij} \cdot x_{ij}}{\bar{\rho}_{ij}} & v_{ij} \cdot x_{ij} < 0 \\
0 & \text{otherwise} 
\end{cases}
\]

(15)

where

\[
\phi_{ij} = \frac{h_{ij} v_{ij} \cdot x_{ij}}{r_{ij}^2 + (0.1 h_{ij})^2}, \quad v_{ij} = v_i - v_j, \quad x_{ij} = x_i - x_j,
\]

(16)

\[
\bar{c}_{ij} = 0.5(c_i + c_j), \quad \bar{\rho}_{ij} = 0.5(\rho_i + \rho_j), \quad h_{ij} = 0.5(h_i + h_j).
\]

(17)

Moreover, the coefficient \(\alpha_{\Pi}\) is a constant that is commonly set at 1.0 ([7, 5]). The terms \(c_i\) and \(c_j\) represent the speed of sound of particles \(i\) and \(j\) respectively. Note that in Monaghan [15], in order to get the SPH formalism as weakly compressible, the fluid pressure is defined as an equation of state. The relation between pressure and density in equation of state is given as

\[
p(\rho) = c_0^2 \rho_0 \gamma \left[ \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \right],
\]

(18)

where \(\gamma = 7\), \(\rho_0 = 1000\, \text{kg m}^{-3}\) is the reference density and \(c_0 = c(\rho_0) = \sqrt{(\partial p/\partial \rho)|_{\rho_0}}\) the speed of sound at the reference density.

The main concern in this paper is to discretize the resistance term denoted by \(F_{\text{force}}\) due to the presence of vegetation forest on fluid flow. In the case of Darcy friction,

\[
F_{\text{force}}^i = -\frac{\mu}{8g} v_i |v_i|.
\]

(19)

Manning friction,

\[
F_{\text{force}}^i = -C_f \frac{v_i |v_i|}{H_i^{1/3}}.
\]

(20)

Laminar viscosity,

\[
F_{\text{force}}^i = (\nu_0 \nabla^2 v) = \sum_j \omega_j \left( \frac{4\nu_0 x_{ij} \cdot \nabla_i W_{ij}}{(\rho_i + \rho_j)(r_{ij}^2 + (0.1 h_{ij})^2)} \right) v_{ij}.
\]

(21)
2.5. **Time stepping method**

Several numerical schemes have been introduced to solve the time integration in SPH approach [14]. Monaghan in [16] is introduced the predictor-corrector scheme, Verlet in [17] is introduced splitting scheme, the Symplectic scheme which is known as time reversible scheme with the accuracy up to $O(\Delta t^2)$ is given by Leimkuhler, et al., [18] and the Beeman scheme (uses a Beeman predictor step and Adams-Bashforth-Moulton corrector step) is elaborated in [19]. However, here we are interested in using the Leapfrog scheme which is the velocity and position leap-over each other as described in [7]. In [7], the velocity is updated at half-time step and the position is updated at full-time step. Nonetheless, in our research, we choose the opposite way namely the position is updated at half-time step whereas the velocity is updated at full-time step. Let’s we denote $\Delta t$ is the time step and the current time $t^n = n \Delta t$, thus for one iteration we compute the following

\[ x^* = x^n + v^n \frac{\Delta t}{2}, \]  
\[ v^{n+1} = v^n + D v^* \Delta t, \]  
\[ \rho^{n+1} = \rho^n + D \rho^* \Delta t, \]  
\[ x^{n+1} = x^* + v^{n+1} \frac{\Delta t}{2}. \]

where $v^*$, $x^*$ and $\rho^*$ denote the intermediate particle velocity, position and density.

3. **Numerical results**

In this section, we would like to elaborate some numerical simulations of wave mitigation by vegetation forest. First, the validation of our SPH code by the comparison of numerical results with the analytic solutions and experiments will be given. Second, we will show the effect of three friction terms in the SPH method such as Darcy, Manning, and laminar viscosity. Third, the energy profile for each simulations will be presented to establish the energy reduction around the coastal vegetation. Finally, the numerical simulation of wave mitigation on sloping beach will be elaborated.

3.1. **SPH validation**

3.1.1. **Comparison with analytic solution**  Here, we consider two cases of dry bed dam break simulation in order to verify our SPH code run well. The first case is the dry dam break with flat bottom and the second case is the dam break with inclined bottom. Follows [20], we consider the computational domain is $\Omega = [-10, 10]$ and the initial condition of water height is defined as $1 - 10 \leq x \leq 0$ for all cases. For the second case, the inclined bottom is given as a linear function with slope 0.01 with starting point $x = -10$.

The comparisons of simulation results with analytic solutions of the dam break problem are shown Figure 2. The analytic solutions of both cases can be found in [20, 21] for more detail. Note that, the analytic solutions for both cases are based on depth averaged model. Nevertheless, Figure 2 shows that our results have in a good agreement with the analytic solutions of dam break with flat and inclined bottom.

3.1.2. **Comparison with experiment**  Another validation for our SPH code is to compare our numerical result with the benchmark experiment. Here we use the benchmark of dam break experiment provided by Koshizuka et. al., [22] and also it can be found in the paper of Doring et al., [23]. In this simulation, the initial water depth is given as $10 \leq x \geq 0.146$ within the computational domain $\Omega = [0, 0.584]$. The comparison of numerical result and experiment can be seen in Figure 3.
Figure 2. The comparasions between numerical results and analytic solutions of dry bed dam break simulation with flat bottom (left) and inclined bottom (right).

Figure 3. Dry bed dam break experimental data (top) and SPH approach (bottom) at time $t = 0.2, 0.6, 0.8$.

Figure 3 shows that our results from our SPH code are in a good agreement with the experiments. As we have shown in Figures 2 and 3, we can conclude that our SPH code is run well to simulate the water flow.

3.2. Wave mitigation

In this simulation, the computational domain is given as $\Omega = [0, 5]$. The boundary conditions on left and right domain are set to be wall boundary. At initial time $t = 0$, the initial water particles are located on the right side of a stroke position of wavemakers $x = 0.5$ (see Figure 4). Additionally, the coastal vegetation area is defined at location $3.5 \leq x \leq 5$. Here, we set the distance between particles is $256 \times 10^{-4}$ and a constant time step is $\Delta t = 4.25 \times 10^{-4}$. Moreover, the initial water depth is set to be 0.5.

Here, we use three of various friction terms that are Darcy, Manning and laminar viscosity. The coefficients for each friction terms are arbitrarily chosen. As the one of the goals of this paper, the choice of the coefficients for each friction terms is purely dedicated to show reduction of water velocity which is induced wave amplitude and energy due to the coastal forest existence. Therefore, here we do not consider to study the effect of the changes of coefficient values for various friction terms. In addition, the coefficients of friction terms can be changed in accordance with the observations from experiment.

When $t > 0$, the wavemakers moves horizontally to the right and to the left as a plane
wavemakers in shallow water proposed by Galvin [24] and can be found in [25] Chapter 6 for more detail. The effect of the movement of wavemakers causes the water particles pushed to the right and will be reflected or retained by wall boundary. And thus, the waves will be formed periodically. The numerical results of this simulation can be seen in Figure 5.

In Figure 5(b), (c) and (d), we can see clearly that the water height on coastal vegetation area is lower than in the pure water height without friction Figure 5 (a). Here, the friction coefficient in coastal vegetation area for Darcy, Manning and laminar viscosity are given as \( \mu = g \), \( C_f = 0.25 \), and \( \nu_0 = 0.1 \) respectively. Figure 5 does not give any information about the number of friction effect for each friction terms clearly. Therefore, the computation of total energy from all particles will be presented in order to see the level of influences of friction terms. Afterwards, we can observe the reduction of total energy of water on coastal vegetation area in Subsection 3.3.

### 3.3. Energy Profile

Here, authors try to show the total energy change due to the presence of resistance term i.e. coastal vegetation. Following [26], the formula to calculate the total energy of discretized system (1-3) is given as

\[
E_T = \sum_i \omega_i \left( \frac{1}{2} v_i + u_i \right), \quad \text{where} \quad \frac{\partial u}{\partial \rho} = \frac{p}{\rho^2}. \tag{26}
\]
This total energy consists of sum of kinetic and internal energy (see [26] for more detail). In Figure 6, the profiles of total energy (26) for each simulations in Subsection 3.2 are given. The total energy is calculated only on coastal vegetation area of simulation at $3.5 \leq x \leq 5$.

We can observe that, in Figure 6, the total energy of water particles on coastal vegetation area is reduced by the presence of friction term. Thus we can see clearly that the total energy without friction term is higher that the others where the friction term is added. And these results are very satisfactory in accordance with our expectation that is the energy reduction by friction term.

3.4. Wave mitigation on sloping beach

Since the real coastal bathymetry is not flat, thus we are interested to elaborate the numerical simulation using non-flat bathymetry. In this subsection, another interesting simulation is devoted to see how the propagation of wave mitigation by coastal vegetation on sloping beach. Here, we will only consider the simulation with Manning friction term as a coastal vegetation resistance. The configurations of this simulation are following, the inclination of bathymetry is given start from $x = 3$ with slope $1 : 2$ and under our computational domain $\Omega = [0, 5]$. The coastal vegetation area is defined at location $3 \leq x \leq 4$. Following our simulations in Subsection 3.2, similar initial wave will be generated by a wavemaker. The results of this simulation can be seen in Figure 7.

In Figure 7(b), we can observe clearly that the elevation of water height on sloping beach is mitigated by the presence of coastal vegetation around the inclination of bathymetry. The elevation of water height in Figure 7(a) where there is no friction term, is reached around $x = 4.5$. 
Meanwhile, in Figure 7(b) the propagation of water is only reached around $x = 4.3$. Therefore from this simulation, we can see that the coastal vegetation is important for preventing the propagation of water elevation.

Conclusions
In this paper, the simulations of wave propagation through coastal vegetation using SPH method have been elaborated. Several friction terms in fluid model in Lagrangian form also have been described in order to bring to the SPH framework. The comparison of various resistances by numerical simulations has been shown. In addition, the validation of our SPH code is given to show our simulation produces the reasonable results. The results with friction terms are shown satisfactory in accordance with the reduction of the total energy (kinetic and internal energy) in coastal vegetation area. Moreover, as the results from the simulation of wave mitigation on sloping beach, we are also shown that the resistance term causes the mitigation of the propagation of water elevation.

Acknowledgments
Iryanto and P. H. Gunawan acknowledge the support of the Ensemble Estimation of Flood Risk in a Changing Climate project funded by The British Council through their Global Innovation Initiative. Partial support from Riset Desentralisasi ITB 2016 is also acknowledged.

References
[1] Pudjaprasetya S 2013 Nonlinear Processes in Geophysics 20 1023–1030
[2] Yanagisawa H, Koshimura S, Goto K, Miyagi T, Imamura F, Ruangrassamee A and Tanavud C 2009 Estuarine, Coastal and Shelf Science 81 27–37
[3] Koh H L, Teh S Y, Liu P L F, Ismail A I M and Lee H L 2009 Journal of Asian Earth Sciences 36 74–83
[4] Teh S Y, Koh H L, Liu P L F, Ismail A I M and Lee H L 2009 Journal of Asian Earth Sciences 36 38–46
[5] Monaghan J J 1992 Annual review of astronomy and astrophysics 30 543–574
[6] Monaghan J J 2005 Reports on progress in physics 68 1703
[7] Liu G R and Liu M B 2003 Smoothed particle hydrodynamics: a meshfree particle method (World Scientific)
[8] Liu G R and Liu M B 2003 Smoothed particle hydrodynamics: a meshfree particle method (World Scientific)
[9] Gunawan P H and Lhébrard X 2015 Computers & Fluids 121 44 – 50 ISSN 0045-7930
[10] Simpson G and Castellotti S 2006 Computers & Geosciences 32 1600–1614
[11] Kadlec R H 1990 Journal of Hydraulic Engineering 116 691–706
[12] Julien P and Simons D 1985 Transactions of the ASAE 28 755–762
[13] Lo E Y and Shao S 2002 Applied Ocean Research 24 275–286
[14] Gomez-Gesteira M, Rogers B D, Crespo A J, Dalrymple R, Narayanaswamy M and Dominguez J M 2012 Computers & Geosciences 48 289–299
[15] Monaghan J J 1994 Journal of computational physics 110 399–406
[16] Monaghan J J 1989 Journal of Computational physics 82 1–15
[17] Verlet L 1967 Physical review 159 98
[18] Leimkuhler B J, Reich S and Skeel R D 1996 Mathematical Approaches to biomolecular structure and dynamics (Springer) pp 161–185
[19] Beeman D 1976 Journal of Computational Physics 20 130–139
[20] Pudjaprasetya S and Magdalena I 2014 East Asian Journal on Applied Mathematics 4 152–165
[21] Chanson H 2008 Journal of Applied Fluid Mechanics 1 1–10
[22] Koshizuka S and Oka Y 2000 Proceedings of the European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS), Barcelona, Spain pp 11–14
[23] Doring M, Andrillon Y, Alessandrini B and Ferrant P 2003 18th International Workshop on Water Waves and Floating Bodies
[24] Galvin Jr C J 1964 Wave-height prediction for wave generators in shallow water Tech. rep. (No. CERC-TM-4).
[25] Dean R and Dalrymple R 1991 Water Wave Mechanics for Engineers and Scientists Advanced series on ocean engineering (World Scientific) ISBN 9789810204211
[26] Michael Owen J 2014 International Journal for Numerical Methods in Fluids 75 749–774