Possible $H$-like dibaryon states with heavy quarks

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Possible $H$-like dibaryon states $\Lambda_c\Lambda_c$ and $\Lambda_b\Lambda_b$ are investigated within the framework of quark delocalization color screening model. The results show that the interaction between two $\Lambda_c$’s is repulsive, so it cannot be bound state by itself. However, the strong attraction in $\Sigma_c\Sigma_c$ and $\Sigma_c^*\Sigma_c^*$ channels and the strong channel coupling, due to the central interaction of one-gluon-exchange and one-pion-exchange, among $\Lambda_c\Lambda_c$, $\Sigma_c\Sigma_c$, and $\Sigma_c^*\Sigma_c^*$ push the energy of system below the threshold of $\Lambda_c\Lambda_c$ by 22 MeV. The corresponding system $\Lambda_c\Lambda_b$ has the similar properties as that of $\Lambda_c\Lambda_c$ system, and a bound state is also possible in $\Lambda_b\Lambda_b$ system.

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I. INTRODUCTION

The $H$ dibaryon, a six quark $(uuddss)$ state corresponding asymptotically to a bound $\Lambda\Lambda$ system, was first proposed by Jaffe in 1977 [1]. This hypothesis initiated a worldwide activity of theoretical studies and experimental searches for dibaryon states [2]. In 1987, M. Oka et al. claimed that a sharp resonance appears in $^1S_0$ $\Lambda\Lambda$ scattering at $E_{c.m.} = 26.3$ MeV, which might correspond to the $H$ dibaryon state [3]. Moreover, M. Oka also proposed several $J^P = 2^+$ dibaryons in the quark cluster model without meson exchange [4]. Despite numerous claims, no dibaryon candidate has been confirmed experimentally so far. Recently, the interest in the $H$ dibaryon have been revived by lattice QCD calculations of different collaborations, NPLQCD [5] and HALQCD [6]. These two groups reported that the $H$ particle is indeed a bound state at pion masses larger than the physical ones. Then, Carames and Valcarce examined the $H$ dibaryon within a chiral constituent quark model and obtained a bound $H$ dibaryon with binding energy $B_H = 7$ MeV [7].

Understanding the hadron-hadron interactions and searching exotic quark states are important topics in temporary hadron physics. Recently observed many near-threshold charmonium-like states, such as $X(3872)$, $Y(3940)$, and $Z^+(4430)$, triggered lots of studies on the molecule-like bound states containing heavy quark hadrons. Such a study may help us to understand further the hadron-hadron interactions. In the heavy quark sector, the large masses of the heavy baryons reduce the kinetic of the system, which makes it easier to form bound states. One may wonder whether a $H$-like dibaryon state $\Lambda_c\Lambda_c$ exist or not.

In particular, the deuteron is a loosely bound state of a proton and a neutron, which may be regarded as a hadronic molecular state. The possibility of existing deuteron-like states, such as $N\Sigma_c$, $N\Sigma_c^*$, $N\Omega_c$, $\Xi\Xi_c$, and so on, were investigated by several realistic phenomenological nucleon-nucleon interaction models [8, 9]. The $N\Lambda$ system and relevant coupled channel effects were both studied on hadron level [10] and on quark level [11]. However, some different results were obtained by these two methods. On hadron level [10], it is found that molecular bound states of $N\Lambda_c$ are plausible in both the one-pion-exchange potential model and the one-boson-exchange potential model. On quark level [11], our group found the attraction between $N$ and $\Lambda_c$ is not strong enough to form any $N\Lambda_c$ bound state within our quark delocalization color screening model (QDCSM). Whereas the attraction between $N$ and $\Sigma_c$ is strong enough to form a bound state $N\Sigma_c$, it becomes a resonance state by coupling to the open $N\Lambda_c D$-wave channels. We also explored the corresponding states $N\Lambda_b$, $N\Sigma_b$ and the similar properties as that of states $N\Lambda_c$, $N\Sigma_c$ were obtained. Recently, the possible $\Lambda_c\Lambda_c$ molecular state was studied in the one-boson-exchange potential model [12] and in the one-pion-exchange potential model [13] on hadron level. Different results were obtained by these two models. The $\Lambda_c\Lambda_c$ does not exist in the former model, whereas the molecular bound state of $\Lambda_c\Lambda_c$ is possible in the later model. So the quark level study of the $\Lambda_c\Lambda_c$ system is interesting and necessary.

The quark delocalization color screening model (QDCSM) was developed in the 1990s with the aim of explaining the similarities between nuclear and molecular forces [14]. The model gives a good description of $NN$ and $YN$ interactions and the properties of deuteron [15]. It is also employed to calculate the baryon-baryon scattering phase shifts in the framework of the resonating group method (RGM), and the dibaryon candidates are also studied with this model [16, 17]. Recent study also show that the intermediate-range attraction mechanism in the QDCSM, quark delocalization and color screening, is an alternative mechanism for the $\sigma$-meson exchange in the most common quark model, the chiral quark model [16, 17]. In the frame of QDCSM, the $H$ dibaryon were also obtained [18]. Therefore, it is very interesting to investigate whether a $H$-like dibaryon state $\Lambda_c\Lambda_c$ exist or not in QDCSM.

In present work, QDCSM is employed to study the properties of $\Lambda_c\Lambda_c$ systems. the channel-coupling effect of
Here, a phenomenological color screening confinement constant. All other symbols have their usual meanings.

The summary is shown in the last section. After the introduction, we present a brief introduction of the quark models used in section II. Section III devotes to the numerical results and discussions. The detail of QDCSM used in the present work can be found in the references [14–17]. Here, we just present the salient features of the model. The model Hamiltonian is:

\[
H = \sum_{i=1}^{6} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_c + \sum_{i<j}^{N} \left[ V^G(r_{ij}) + V^x(r_{ij}) + V^C(r_{ij}) \right],
\]

\[
V^G(r_{ij}) = \frac{1}{4} \alpha_{i} \lambda_{i} \cdot \lambda_{j} \left[ \frac{1}{r_{ij}} - \frac{\pi}{2} \left( \frac{1}{m_{i}^{2}} + \frac{1}{m_{j}^{2}} + \frac{4\sigma_{i} \cdot \sigma_{j}}{3m_{i}m_{j}} \right) \delta(r_{ij}) - \frac{3}{4m_{i}m_{j}r_{ij}^{3}} S_{ij} \right],
\]

\[
V^x(r_{ij}) = \frac{1}{3} \alpha_{ch} \frac{\Lambda^{2}}{\Lambda^{2} - m_{\chi}^{2}} \left[ \left( Y(m_{\chi}r_{ij}) - \frac{\Lambda^{3}}{m_{\chi}^{3}} Y(\Lambda r_{ij}) \right) \sigma_{i} \cdot \sigma_{j} \right.
\]
\[
+ \left[ H(m_{\chi}r_{ij}) - \frac{\Lambda^{3}}{m_{\chi}^{3}} H(\Lambda r_{ij}) \right] S_{ij} \right] F_{i} \cdot F_{j}, \quad \chi = \pi, K, \eta
\]

\[
V^C(r_{ij}) = -a_{\sigma} \lambda_{i} \cdot \lambda_{j} [f(r_{ij}) + V_0],
\]

\[
f(r_{ij}) = \begin{cases} r_{ij}^{2} & \text{if } i, j \text{ occur in the same baryon orbit} \\ \frac{1 - e^{-r_{ij}^{2}/\mu_{ij}^{2}}}{e^{-r_{ij}^{2}/\mu_{ij}^{2}}} & \text{if } i, j \text{ occur in different baryon orbits} \end{cases}
\]

\[
S_{ij} = \frac{(\sigma_{i} \cdot r_{ij})(\sigma_{j} \cdot r_{ij})}{r_{ij}^{3}} - \frac{1}{3} \sigma_{i} \cdot \sigma_{j}.
\]

Where \( S_{ij} \) is quark tensor operator, \( Y(x) \), \( H(x) \) and \( G(x) \) are standard Yukawa functions [19], \( T_c \) is the kinetic energy of the center of mass, \( \alpha_{ch} \) is the chiral coupling constant, determined as usual from the \( \pi \)-nucleon coupling constant. All other symbols have their usual meanings. Here, a phenomenological color screening confinement potential is used, and \( \mu_{ij} \) is the color screening parameter. For the light-flavor quark system, it is determined by fitting the deuteron properties, \( N \) \( N \) scattering phase shifts, \( \Lambda \) \( \Lambda \) and \( \Sigma \) \( \Sigma \) scattering phase shifts, respectively, with \( \mu_{uu} = 1.2, \mu_{us} = 0.3, \mu_{ss} = 0.08 \), satisfying the relation \( \mu_{uu}^{2} = \mu_{uu} \ast \mu_{ss} \) [17]. When extending to the heavy quark case, there is no experimental data available, so we take it as a common parameter. In the present work, we take \( \mu_{cc} = 0.001 \) and \( \mu_{uc} = 0.0346 \), also satisfy the relation \( \mu_{cc}^{2} = \mu_{uu} \ast \mu_{cc} \). All the other parameters are taken from [11].

The quark delocalization in QDCSM is realized by specifying the single particle orbital wave function of QDCSM as a linear combination of left and right Gaussians, the single particle orbital wave functions used in the ordinary quark cluster model,

\[
\psi_{\alpha}(s_{i}, \epsilon) = (\phi_{\alpha}(s_{i}) + \epsilon \phi_{\alpha}(-s_{i})) / N(\epsilon),
\]

\[
\psi_{\beta}(-s_{i}, \epsilon) = (\phi_{\beta}(-s_{i}) + \epsilon \phi_{\beta}(s_{i})) / N(\epsilon),
\]

\[
N(\epsilon) = \sqrt{1 + \epsilon^2 + 2\epsilon e^{-\epsilon^2/4b^2}}.
\]

\[
\phi_{\alpha}(s_{i}) = \left( \frac{1}{\pi b^2} \right)^{3/4} e^{-\frac{1}{2b^2}(r_{a} - s_{i}/2)^2}
\]

\[
\phi_{\beta}(s_{i}) = \left( \frac{1}{\pi b^2} \right)^{3/4} e^{-\frac{1}{2b^2}(r_{a} + s_{i}/2)^2}.
\]

Here \( s_{i}, i = 1, 2, \ldots, n \) are the generating coordinates, which are introduced to expand the relative motion wavefunction [13]. The mixing parameter \( \epsilon(s_{i}) \) is not an adjusted one but determined variationally by the dynamics of the multi-quark system itself. This assumption allows the multi-quark system to choose its favorable configuration in the interacting process. It has been used to explain the cross-over transition between hadron phase and quark-gluon plasma phase [20].

**III. THE RESULTS AND DISCUSSIONS**

Here, we perform a dynamical investigation of the \( \Lambda_{c} \) \( \Lambda_{c} \) system with \( IJ^{P} = 00^{+} \) in the QDCSM. The channel coupling effects are also considered. The labels of all coupled channels are listed in Table I.
Because an attractive potential is necessary for forming bound state or resonance, we first calculate the effective potentials of all the channels listed in Table I. The effective potential between two colorless clusters is defined as, \( V(s) = E(s) - E(\infty) \), where \( E(s) \) is the diagonal matrix element of the Hamiltonian of the system in the generating coordinate. The effective potentials of the \( S \)-wave and \( D \)-wave channels are shown in Fig. 1(a) and (b) respectively. From Fig. 1(a), we can see that the potentials are attractive for the \( ^1S_0 \) channels \( \Sigma_c \Sigma_c \), \( N\Xi_{cc} \) and \( \Sigma_c \Sigma_c^* \). While for the channel \( \Lambda_c \Lambda_c \), the potential is repulsive and so no bound state can be formed in this single channel. However, the attractions of \( \Sigma_c \Sigma_c \) channel and \( \Sigma_c^* \Sigma_c^* \) channel are very large, the channel coupling effects of \( \Sigma_c \Sigma_c \) and \( \Sigma_c^* \Sigma_c^* \) to \( \Lambda_c \Lambda_c \) will push the energy of \( \Lambda_c \Lambda_c \) downward, it is possible to form a bound state. For the \( ^5D_0 \) channels shown in Fig. 1(b), the potentials are all repulsive.

In order to see whether or not there is any bound state, a dynamic calculation is needed. Here the RGM equation is employed. Expanding the relative motion wavefunction between two clusters in the RGM equation by gaussians, the integro-differential equation of RGM can be reduced to algebraic equation, the generalized eigen-equation. The energy of the system can be obtained by solving the eigen-equation. In the calculation, the baryon-baryon separation (|s_n|) is taken to be less than 6 fm (to keep the matrix dimension manageable small).

The single channel calculation shows that the energy of \( \Lambda_c \Lambda_c \) is above its threshold, the sum of masses of two \( \Lambda_c \)'s. It is reasonable, because the interaction between the two \( \Lambda_c \)'s is repulsive as mentioned above. For \( N\Xi_{cc} \) channel, the attraction is too weak to tie the two particles together, so it is also unbound. At the same time, due to the stronger attraction, the energies of \( \Sigma_c \Sigma_c \) and \( \Sigma_c^* \Sigma_c^* \) are below their corresponding thresholds. The binding energy of \( \Sigma_c \Sigma_c \) and \( \Sigma_c^* \Sigma_c^* \) states are listed in Table II in which 'ub' means unbound. For the \( ^5D_0 \) channels, they are all unbound since the potentials are all repulsive, so we leave them out from Table II. We also do a channel-coupling calculation and a bound state, which energy is below the threshold of \( \Lambda_c \Lambda_c \), is obtained. The binding energy is also shown in Table II under the head 'c.c.' There are several features which are discussed below.

First, the individual \( S \)-wave \( \Lambda_c \Lambda_c \) channel is unbound in our quark level calculation, which is consistent with the conclusion on the hadron level [12, 13]. For other individual channels, there are some different results. In Ref. [13], the calculation shows all the individual channels are unbound and in Ref. [12], \( \Sigma_c \Sigma_c \) is bound. While in our quark level calculation, the individual \( \Sigma_c \Sigma_c \) and \( \Sigma_c^* \Sigma_c^* \) are deeply bound.

Secondly, by taking into account the channel-coupling effect, a bound state is obtained for the \( \Lambda_c \Lambda_c \) system, which is also consistent with the conclusion on the hadron level [13]. However, the channel-coupling effect is different between our quark level calculation and their hadron level calculation. In Ref. [13], the coupling of \( \Lambda_c \Lambda_c \) to the \( D \)-wave channels \( \Sigma_c \Sigma_c^* \) and \( \Sigma_c^* \Sigma_c^* \) are crucial in binding two \( \Lambda_c \)'s. This indicates the importance of the tensor force. This conclusion is the same as their calculation of \( N\Lambda_c \) system [10]. While in our quark level calculation, the coupling between \( \Lambda_c \Lambda_c \), \( N\Xi_{cc} \), \( \Sigma_c \Sigma_c \) and \( \Sigma_c^* \Sigma_c^* \) channels is through the central force. The transition potentials of these four channels are shown in Fig. 2(a). It is the strong coupling among these channels that makes the \( \Lambda_c \Lambda_c \) \( ^1S_0 \) be unbound. The transition potentials of three \( D \)-wave channels are shown in Fig. 2(b). To see the effects of tensor force, the transition potentials for the \( S \) and \( D \) wave channels are shown in Fig. 3(a) and (b). From which one can see that the effects of tensor force are much small compared with that of the central force. Thus the \( S \) and \( D \) wave channels are unbound in our quark model calculation. This conclusion is consistent with our calculation of \( N\Lambda_c \) system [11], in which the effect of the \( N\Sigma_c^* (^5D_0) \) channel coupling to \( N\Lambda_c \) \( ^1S_0 \) is very small.

Thirdly, the properties of the \( \Lambda_c \Lambda_c \) system in our quark model is similar to that of the \( \Lambda\Lambda \) system. Our group has calculated the \( H \)-dibaryon before [18], in which the single channel \( \Lambda\Lambda \) is unbound, when coupled to the channels \( N\Xi \) and \( \Sigma\Sigma \), it becomes a bound state. Here, we extend our model to study the heavy flavor dibaryons, we find it is possible to form a bound state in the \( \Lambda\Lambda \) system, which it is a \( H \)-like dibaryon state.

In the previous discussion, the \( \Lambda\Lambda \) system is investigated and a \( H \)-like dibaryon state is found. Because of the heavy flavor symmetry, we also extend the study to the bottom case of \( \Lambda_b \Lambda_b \) system. The numerical results

| Channels | \( \Sigma_c \Sigma_c \) | \( N\Xi_{cc} \) | \( \Lambda_c \Lambda_c \) | \( \Sigma_c^* \Sigma_c^* \) | c.c. |
|----------|----------------|----------------|----------------|----------------|-----|
| B.E.(MeV) | -157 | ub | ub | -91 | -22 |

### Table I: The \( \Lambda_c \Lambda_c \) and \( \Lambda_c \Lambda_b \) states and the channels coupled to them.

| Channels | \( J^P = 0^+ \) | \( J^P = 0^+ \) |
|----------|----------------|----------------|
|          | \( \Sigma_c \Sigma_c (^1S_0) \) | \( \Sigma_c \Sigma_c (^1S_0) \) |
|          | \( N\Xi_{cc} (^1S_0) \) | \( \Lambda_c \Lambda_c (^1S_0) \) |
|          | \( \Sigma_c^* \Sigma_c^* (^1S_0) \) | \( \Sigma_c^* \Sigma_c^* (^1S_0) \) |
|          | \( N\Xi_{cc} (^3D_0) \) | \( \Sigma_c \Sigma_c (^3D_0) \) |
|          | \( \Sigma_c^* \Sigma_c^* (^3D_0) \) | \( \Sigma_c \Sigma_c (^3D_0) \) |

### Table II: The binding energy of every \( ^1S_0 \) channels of \( \Lambda_c \Lambda_c \) system and with channel coupling (c.c.)
FIG. 1: The potentials of different channels for the $J^P = 0^+$ case of the $\Lambda_c\Lambda_c$ system.

FIG. 2: The transition potentials of (a): $S$-wave channels and (b): $D$-wave channels for the $J^P = 0^+$ case of the $\Lambda_c\Lambda_c$ system.

for the $NA_b$ system are listed in Figs. 4 and Table III. The results are similar to the $\Lambda_c\Lambda_c$ system. From Table III we also find there is also a $H$-like dibaryon state in the $\Lambda_b\Lambda_b$ system in our quark model.

IV. SUMMARY

In this work, we perform a dynamical calculation of the $\Lambda_c\Lambda_c$ system with $IJ^P = 00^+$ in the framework of QDCSM. Our results show that the interaction between two $\Lambda_c$'s is repulsive, so it cannot be a bound state by itself. The attractions of $\Sigma_c\Sigma_c$ and $\Sigma_c^*\Sigma_c^*$ channels are strong.
enough to bind two $\Sigma_c$'s and two $\Sigma^*$'s together. It is possible to form a $H$-like dibaryon state in the $\Lambda_c\Lambda_c$ system with the binding energy 22 MeV in our quark model by including the channel-coupling effect. This result is consistent with the result of the calculation on the hadron level [13]. However, the effect of the channel coupling is different between these two approaches. The role of the central force is much more important than the tensor force in our quark level calculation, while in the calculation on the hadron level [13], the tensor force is shown to be important and the $D$-wave channels are crucial in binding two $\Lambda_c$'s. Further investigation should be done to understand the difference between the approaches on the hadron level and the quark level. It will help us to understand the quark-duality and exotic quark states.

Extension of the study to the bottom case has also

FIG. 3: The transition potentials of $S - D$ wave channels for the $J^P = 0^+$ case of the $\Lambda_c\Lambda_c$ system.

FIG. 4: The potentials of different channels for the $J^P = 0^+$ case of the $\Lambda_b\Lambda_b$ system.
TABLE III: The binding energy of every $^1S_0$ channels of $\Lambda b\Lambda b$ system and with channel coupling (c.c.).

| Channels | $\Sigma_b\Sigma_b$ | $N\Xi_{bb}$ | $\Lambda_b\Lambda_b$ | $\Sigma^*_b\Sigma^*_b$ | c.c. |
|----------|-------------------|-----------|---------------------|---------------------|------|
| B.E.(MeV) | $-162$ | $ub$ | $ub$ | $-79$ | $-19$ |

been done. The results of $\Lambda_b\Lambda_b$ system is similar to the $\Lambda_c\Lambda_c$ system, and there exits a $H$-like dibaryon state in the $\Lambda_b\Lambda_b$ system with a binding energy of 19 MeV in our quark model. On the experimental side, finding the $H$-like dibaryon states $\Lambda_c\Lambda_c$ and $\Lambda_b\Lambda_b$ will be a challenging subject.

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