Calculation of the thermodynamic characteristics of an upward swirling gas flow under conditions of side wind

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Abstract. The paper presents the results of numerical modeling of lateral wind effects on three-dimensional unsteady air flows in an upward swirling stream of an artificially created tornado after entering a stationary mode. Under the corresponding initial and boundary conditions, the solutions of the complete system of Navier-Stokes equations in the computational domain in the form of a rectangular parallelepiped are numerically obtained using the explicit difference scheme. At fixed points in time, the gas-dynamic characteristics of air flows are calculated under the action of gravity and Coriolis, as well as taking into account the constant horizontal directional wind speed. Numerical calculations showed that the resulting density, temperature, and pressure gradients lead to significant and asymmetric changes in air flows at different points in time and at different points in the computational domain. The total displacement of the vortex in the direction of the wind, the deformation and increase in the diameter of the vortex, which begin in the bottom part and pass into its vertical part, are established. In the considered range of time changes, the stability of the air vortex under study against wind action and the stable operation of the computational scheme used are noted.

1. Introduction

The application of numerical methods to the study of complex unsteady three-dimensional gas flows as a compressible viscous heat-conducting continuous medium poses interest for two reasons.

Firstly, despite a significant number of publications, a description of extremely complex gas flows in free atmospheric swirling flows and gas flows in pipelines, adequate to nature, cannot yet be considered complete, and especially when it comes to non-stationary three-dimensional flows. Therefore, from a scientific point of view, mathematical and numerical modeling can be considered as the most important and, perhaps, the only tool for studying this type of flows.

Secondly, the numerical simulation of such flows has a pronounced applied character, since it not only gives recommendations on the destruction or reduction of the destructive effects of natural atmospheric vortices [1], but also allows calculating all the gas-dynamic and energy characteristics of the flows for more efficient operation of gas pipelines.

The main reasons for the emergence and stable functioning of the ascending swirling air flow [2] are the long existence of the ascending air flow and the rotation of the Earth, which, through the action of the Coriolis force, gives significant peripheral speed to air particles in the bottom part of the ascending swirling flow. The fact of the occurrence of a twist and its direction is strictly mathematically proved at the level of the corresponding theorems [3, 4] and confirmed experimentally [5, 6].
Numerous observations of natural upward swirling flows allowed hypothesizing that in the middle-high vertical part of such flows there is a boundary separating the external resting air from air moving in an upward swirling flow. In gas dynamics, such a boundary is called a contact surface [7, 8]. Therefore, to create a stable upward swirling flow in laboratory conditions, an airtight vertical cylindrical pipe with an exhaust fan directing air through the pipe from the bottom up is used as the contact surface. Numerical calculations of such a gas flow made it possible to give concrete suggestions and recommendations on the possible conduct of a large-scale experiment on swirling large masses of air - an artificial tornado.

The numerical results obtained earlier [9–12] provide a good basis for further research in the gas-dynamic theory of destructive atmospheric vortices. In particular, it seems very interesting to study the effect of external wind action on the ascending swirling air flow. The aim of this work is to numerically simulate the functioning of an artificial tornado under conditions of side wind impact on it, as well as the calculation of its thermodynamic characteristics.

2. Mathematical model

As the mathematical model describing complex three-dimensional unsteady flows of a compressible viscous heat-conducting gas (such as atmospheric air), the complete system of Navier-Stokes equations was used, which in dimensionless variables—taking into account the action of gravity and Coriolis in vector form—has the following form [4]:

\[
\begin{align*}
\rho_t + \mathbf{V} \cdot \nabla \rho + \rho \text{ div } \mathbf{V} &= 0, \\
\mathbf{V}_t + (\mathbf{V} \cdot \nabla) \mathbf{V} + \frac{7}{\gamma} \nabla p + \frac{1}{\gamma} \nabla T &= -2 \Omega \times \mathbf{V} + \frac{\mu_0}{\rho} \left[ \frac{1}{4} \mathbf{V} \cdot \nabla \left( \text{div } \mathbf{V} \right) + \frac{3}{4} \nabla \mathbf{V} \right], \\
\mathbf{T}_t + \mathbf{V} \cdot \nabla \mathbf{T} + (\gamma - 1) \text{ div } \mathbf{V} &= \frac{\kappa_0}{\rho} \Delta T + \frac{\mu_0 (\gamma - 1)}{2\rho} \left[ (u_x - v_y) \right]^2 + \\
&+ (u_x - w_z)^2 + (v_y - w_z)^2 + \frac{3}{2} \left[ (u_x + v_y)^2 + (u_x + w_z)^2 + (v_y + w_z)^2 \right].
\end{align*}
\]

In system (1):

- \( t \) is time;
- \( x, y, z \) are Cartesian coordinates;
- \( \rho \) is gas density;
- \( \mathbf{V} = (u, v, w) \) is gas velocity vector with projections on corresponding Cartesian axes;
- \( T \) is gas temperature;
- \( \mathbf{g} = (0, 0, -g) \) is gravitational vector;
- \( \gamma = 1.4 \) is polytropic indicator for air;
- \( -2 \Omega \times \mathbf{V} = (av - bw, -au, bu) \) is Coriolis force acceleration vector;
- \( a = 2\Omega \sin\psi, \ b = 2\Omega \cos\psi, \ \Omega = |\Omega| \); \( \Omega \) is Earth angular velocity vector;
- \( \psi \) is the latitude of point \( \mathcal{O} \), the origin of the Cartesian coordinate system rotating with the Earth;
- \( \mu_0 = 0.001, \ \kappa_0 \approx 1.46\mu_0 \) are constant values of dimensionless viscosity and thermal conductivity coefficients.

The computational domain (Figure 1) is a rectangular parallelepiped with side lengths \( x^0 = 1, y^0 = 1 \) and \( z^0 = 0.04 \) along axes \( Ox, Oy \) and \( Oz \), respectively, which is filled with a three-dimensional grid of nodes of intersection of three families of planes \( x = x_i, \ y = y_j, \ z = z_k \), where \( x_i = i \cdot \Delta x, \ y_j = j \cdot \Delta y, \ z_k = k \cdot \Delta z, \ 0 \leq i \leq L, \ 0 \leq j \leq M, \ 0 \leq k \leq N \). Difference steps in three spatial variables \( \Delta x = x^0 / L, \ \Delta y = y^0 / M, \ \Delta z = z^0 / N \), \( L = 200, \ M = 200, \ N = 20 \).
The initial conditions in all internal nodes of the computational domain are the values of the five functions sought that characterize the air flow in an artificially created tornado at the time $t = t_s$ it enters the stationary mode of operation [12]:

$$ u = u(x, y, z, t_s), \quad v = v(x, y, z, t_s), \quad w = w(x, y, z, t_s), \quad T = T(x, y, z, t_s), \quad \rho = \rho(x, y, z, t_s). $$

The boundary conditions on the faces of the computational domain are set as follows.

The density on the four side faces of the calculated parallelepiped $x = 0, \, x = x^0, \, y = 0, \, y = y^0$, is taken equal to the values from the stationary distribution of the density of atmospheric air [4]

$$ \rho_{10} = \rho_0(x, y, z), \quad \rho_{y0} = \rho_0(x, y, z), $$

$$ \rho_0(0) = (1 - k z)^{v-1}, \quad k = \frac{\rho_{x0}}{\rho_{y0}}, \quad l = 0.0065 \ \text{K/m}, \quad x_{00} = 50 \ \text{m}, \quad T_00 = 288^\circ \text{K}, \quad \nu = \frac{\gamma g}{k}. $$

The density on the lower $z = 0$ and upper $z = z^0$ faces obeys the continuity of the flow. This means that the density values at the boundaries of the region are carried out by linear extrapolation along the normal to the given boundary surface from the inner part of the computational domain [4].

For the temperature on all lateral faces, values are set from the stationary distribution of atmospheric air temperature

$$ T_{10} = T_0(x, y, z), \quad T_{y0} = T_0(x, y, z), \quad T_0(z) = 1 - k z. $$

The temperature on the lower $z = 0$ and upper $z = z^0$ faces corresponds to the condition of symmetry. In this case, the temperature values are calculated from the condition that their derivative is normal to the given face.

On the upper and lower faces, the conditions of non-leakage are set. In this case, the third component of the velocity is equal to zero $w_{x=0, \, z=0} = 0$, and the first and second components of the velocity vector are determined from the symmetry condition, that is, they are calculated from the condition that their derivative to normal of the given face is zero. In addition, vertical speed $w = 0.03$ is set through a square hole with the size of $0.1 \times 0.1$ which is in the center of the upper face and simulates vertical air blowing at a speed of 10 m/s through a pipe with a diameter of 5 m.

For the lateral faces of the computational domain, for all velocity vector components that are normal to the faces, continuity conditions are set, while two tangential velocity components are calculated by the condition of symmetry. Additionally, on the left (western) face $x = 0$, with
The normal component of the velocity is set equal to the wind speed \( u = 0.03 \). Thus, a horizontally directed wind action is simulated on an ascending swirling stream of an artificially created tornado.

The calculation of the three-dimensional unsteady flow is carried out according to an explicit difference scheme by switching from the next \( n - \text{th} \) time layer to the next time layer \( n + 1 \) with a constant given step \( \Delta t \). The values of all the desired functions are calculated at all the internal points of the rectangular parallelepiped. After that, the values of the desired functions are determined at all internal points of each of the six faces and twelve edges.

The calculations were carried out with the following input parameters: scaled dimensional values of density, speed, distance, and time are respectively \( \rho_{00} = 1.2928 \text{ kg/m}^3 \), \( u_{00} = 333 \text{ m/s} \), \( x_{00} = 50 \text{ m} \), \( t_{00} = x_{00}/u_{00} = 0.15 \text{ s} \). Difference steps in three spatial variables \( \Delta x = \Delta y = 0.005 \text{, } \Delta z = 0.002 \text{, while time step } \Delta t = 0.001 \).

3. Calculation results

Figures 2-3 show graphs of the density function \( \rho(x, y) \) for a height \( z = 0.2 \text{ m} \) and two fixed points in time \( t_1 = 1 \text{ s} \), \( t_2 = 10 \text{ s} \). Along axes \( Ox \) and \( Oy \) are the numbers of nodes of the computational grid, along axis \( Oz \) is the gas density in dimensionless quantities.

![Figure 2. Density for \( t_1 = 1 \text{ s} \)](image1)

![Figure 3. Density for \( t_2 = 10 \text{ s} \)](image2)

The behavior of the gas density at the indicated height, as follows from the calculations, can be characterized as follows. By the estimated time \( t_1 = 1 \text{ s} \), the density in the western part of the computational domain increases to \( 1.004 \text{ (1.29797 kg/m}^3 \text{ in dimensional values)} \), while in the eastern part the region of reduced density is characteristic of the stationary state of the ascending swirl flow (Figure 2). Then, with a decrease in its values, the region of increased density propagates along the peripheral regions, covering the center of the vortex. Gradually, it begins to deform with a simultaneous decrease in its size, giving way to an increasing region of lower values (Figure 3).

Figures 4-5 show the temperature distribution as graphs of the functions of two variables at the same height and at the same determined points in time.
Figure 4. Temperature for $t_1 = 1$ s

Figure 5. Temperature for $t_2 = 10$ s

At the beginning of the wind action (Figure 4), the temperature on the left (western) face of the computational domain is $1.005 \ (289.44^\circ K \text{ in dimensional values})$. At subsequent time instants, the elevated temperatures begin to cover the funnel-shaped central region of the lower temperature, where the bulk of the vortex is located. Subsequently, elevated temperatures are concentrated near the western $x = 0$ and southern $y = 0$ faces of the computational domain (Figure 5). It should be noted that the excess of temperature over the scale value for all calculated moments of time is about $1.5^\circ K$. Nevertheless, the visualization program allows recording such a temperature difference and its dynamics.

Figures 6-7 show the calculated pressure distributions as graphs of the functions of two variables corresponding to the same height and the same fixed points in time. The behavior of pressure is similar to the behavior of density and temperature.

Figure 6. Pressure for $t_1 = 1$ s

Figure 7. Pressure for $t_2 = 10$ s
Calculations showed that the resulting density, temperature, and pressure gradients lead to significant and asymmetric changes in air flows at different points in time and at different points in the computational domain. There is a general displacement of the vortex and its deformation in the direction of the wind. In the considered range of time changes, the stability of the air vortex under study against wind action and the stable operation of the computational scheme used are noted.

4. Conclusions
A numerical simulation of the lateral wind effect on three-dimensional unsteady air flows in an upward swirling stream of an artificially created tornado in a stationary mode of operation was performed. Using an explicit difference scheme, numerically obtained solutions of the complete system of Navier-Stokes equations in the computational domain in the form of a rectangular parallelepiped. The thermodynamic characteristics of currents are calculated at fixed times under the action of gravity and Coriolis, as well as when constant horizontal wind speed is taken into account.

Numerical calculations showed that the resulting density, temperature, and pressure gradients lead to corresponding changes in air flows at different times and at different points in the computational domain. The displacement of the vortex in the direction of the wind, as well as its uneven deformation, is established.

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