Long-range thermoelectric effects in mesoscopic superconductor-normal metal structures.

A.F. Volkov$^{1,2}$ and V. V. Pavlovskii$^2$

$^{(1)}$Theoretische Physik III, Ruhr-Universität Bochum, D-44780 Bochum, Germany
$^{(2)}$Institute of Radioengineering and Electronics of the Russian Academy of Sciences, 103907 Moscow, Russia

We consider a mesoscopic four-terminal superconductor/normal metal (S/N) structure in the presence of a temperature gradient along the N wire. A thermoeomf arises in this system even in the absence of the thermoelectric quasiparticle current if the phase difference between the superconductors is not zero. We show that the thermoeomf is not small in the case of a negligible Josephson coupling between two superconductors. It is also shown that the thermoelectric voltage has two maxima: one at a low temperature and another at a temperature close to the critical temperature. The obtained temperature dependence of the thermoeomf describes qualitatively experimental data.

I. INTRODUCTION

Transport phenomena in mesoscopic superconductor-normal metal (S-N) structures have attracted a great interest in last years (see for example reviews [1,2] and references therein). Due to the proximity effect (PE) such properties of the normal metal N as the density-of-states etc are changed, and therefore transport properties of S-N structures are changed as well. For example, the resistance of a normal wire a part of which is a segment of a superconducting loop is a periodic function of a magnetic flux $\Phi$ threading this loop. The period of the oscillations is the magnetic flux quantum $\Phi_0 = \hbar c/2e$. The amplitude of the resistance oscillations $\delta R_m$ depends on temperature $T$ in a non-monotonic way reaching a maximum at $T_m = c_1 \epsilon_L$, where $c_1$ is a numerical factor and $\epsilon_L = D_N/L^2$ is the Thouless energy $[3,4]$. A lot of publications are devoted to the study of the electric conductance of S-N mesoscopic structures. Much less attention was paid to the study of thermoelectric phenomena in such structures. However the situation has changed recently. A number of papers has been published in which the results of both experimental $[5,6,8]$ and theoretical $[11–15,17]$ studies are presented. It has been established experimentally that the thermoeomf $V_{th}$ in a S-N structure with a superconducting loop also is an oscillating function of the magnetic flux in the loop and the amplitude of oscillations depends on temperature in a non-monotonic way $[5,6,8]$. In Refs. $[14,15,7]$ the thermoconductance in S-N mesoscopic structures was calculated and measured. These calculations generalized the results for the thermoconductivity in superconductors in the intermediate state obtained by Andreev a long time ago $[16]$. The thermoelectric effects in superconductors caused by the thermoelectric component $j_{th} = \eta \nabla T$ in the quasiparticle current $j_{qp}$ were considered in many papers starting from the Ginzburg paper $[9]$ (for more references see $[10]$). These effects in a short bridge between two superconductors were studied recently in Ref. $[10]$. The authors established that due to a charge imbalance the thermoeomf in the superconducting state may be comparable with that in the normal state. The thermoeomf $V_{th}$ induced in mesoscopic S-N structures with normal reservoirs kept at different temperatures was theoretically studied in Refs. $[11–13,17]$. Using a scattering matrix approach, Claughton and Lambert $[11]$ studied the dependence of the thermoelectric voltage on the phase difference $\varphi$ between the superconductors S taking into account the thermoelectric current $j_{th} = \eta \nabla T$. They showed that this dependence is periodic.

Unlike Ref. $[11]$ in Refs. $[12,13]$ the thermoelectric current $j_{th}$ was completely neglected because the thermoelectric coefficient $\eta$ contains a small parameter: $\eta \propto T/\epsilon_F$, where $\epsilon_F$ is the Fermi energy. It was shown that even in the absence of the thermoelectric current $j_{th}$ a temperature gradient in the N wire leads to the build-up of a thermoelectric voltage $V_{th}$ if the N wire is in contact with two superconductors and the phase difference $\varphi$ between the superconductors is not zero. The voltage $V_{th}$ in the S-N structure may be of the order $V_{th} \approx c_2 \delta T/\epsilon$, where $\delta T = T_r - T_l$, $T_r,l$ is the temperature of the right (left) normal reservoir (see Fig.1), $c_2$ is a numerical factor (in the case under consideration $c_2 \approx 0.1$). Therefore the thermoeomf $V_{th}$ below $T_c$ is much larger than the ordinary thermoeomf $V_{ord}$ ($V_{ord} \sim (T/\epsilon_F) \delta T/\epsilon$) above the critical temperature $T_c$. This effect can be called a giant thermoelectric effect in S-N mesoscopic structures. It was shown in Refs. $[12,13]$ that the voltage $V_{th}$ oscillates with increasing the phase difference: $V_{th} \propto \sin \varphi$, that is, the function $V_{th}(\varphi)$ is shifted by $\pi/2$ with respect to the phase dependence of the resistance variation $\delta R_N$ of the normal wire: $\delta R_N(\varphi) \propto \cos \varphi$. The amplitude of these oscillations $V_{th}(\pi/2)$ is a non-monotonic function of temperature with a maximum at a temperature $T_m$ of the order of the Thouless energy $\epsilon_L$. These results are obtained in Refs. $[12,13]$ on the basis of microscopic equations for the quasiclassical Green’s functions. It was assumed that the S-N interface
transmittance is low (due to a mismatch of the Fermi surfaces or to the presence of a potential barrier) and therefore the PE is weak. In this case the problem can be solved analytically.

Another limit of perfect S-N interfaces was considered in Ref. [17] by using the same microscopic equations. Solutions for these (kinetic and Usadel) equations were found in Ref. [17] numerically and the obtained results are similar to those found in Refs. [12,13]. In the theoretical publications [12,13,17] the physics of this giant thermoelectric effect is explained in terms of the temperature-dependent Josephson current which, in the presence of a temperature gradient, has different values at different S-N interfaces. In order to compensate this difference, a quasiparticle current driven by the voltage $V_{th}$ arises in the system. An approximate formula for the thermoelectric was presented in Ref. [17], where in the main approximation the voltage $V_{th}$ is expressed in terms of the Josephson critical current $I_c(T)$. A correction to this expression for the voltage $V_{th}$ is small. Although in some limiting cases the representation of $V_{th}$ through $I_c(T)$ is possible (the authors assumed that the order parameter $\Delta$ is much larger than the Thouless energy $\epsilon_L$), in a general case this can not be done. The point is that the Josephson current is a thermodynamical quantity (it can be presented as a derivative of a free energy with respect to the phase difference), whereas the voltage $V_{th}$ is not. To make this point quite clear, one can consider a limiting case of temperatures $T$ larger than $\epsilon_L$ (to be more exact, the ratio $2\pi T/\epsilon_L$ should be much larger than 1). In this case the Josephson current is exponentially small and the Josephson coupling is negligible. However, generally speaking, the voltage $V_{th}$ is not exponentially small. Its value is determined by the ratio of other parameters: $\Delta$ and $\epsilon_L$. The aim of this paper is to consider the case when the Josephson current is small, but the thermoelectric effect is not small in comparison with its maximal value. We will show that if the ratio $2\pi T/\epsilon_L$ is large, the Josephson coupling between superconductors is almost negligible, but the thermoeomf $V_{th}$ is not small and can be measured. In this case one can say about a long-range thermoelectric effects.

II. MODEL AND BASIC ASSUMPTIONS

As inRefs. [12,13,17], we consider a system shown schematically in the inset of Fig. 3. A normal wire $N$, or a thin film with a width narrower than the coherence length $\xi_F = \sqrt{D_N/2\pi T}$, connects two normal reservoirs $N_{l,r}$. The left reservoir $N_l$ is kept at a temperature $T$ and the right reservoir $N_r$ has a temperature $T + \delta T$. We assume that the S-N interface resistance $R_b$ is larger than the resistance $R_L$ of the N wire so that the ratio $R_L/R_b$ is a small parameter. In this case the problem allows an analytic solution because there is a small parameter, the amplitude of the condensate function $|F^{R(A)}|$, where $F^{R(A)}$ is the retarded (advanced) quasiclassical Green’s function induced in the N wire due to the PE. Therefore the PE is assumed to be weak and the characteristics of the N wire deviate only slightly from those in the normal state.

The thermoeomf $V_{th}$ arising in the system at $\delta T \neq 0$ is an integral over energies $\epsilon$ from a distribution function $f_-(\epsilon)$ [12,13,17]. This distribution function called a transverse part of the distribution function in Ref. [18] and denoted by $f_1$ in Ref. [19] is a difference between electron- and hole-like excitations: $f_- = n_1 - p_1$ (see, for example, [20] and [21]). This function determines the voltage and is related to a so-called charge-imbalance [22]. Another distribution function $f_+ = 1 - (n_1 + p_1)$ determines, for example, the supercurrent and the order parameter (in the superconductor). These distribution functions obey kinetic equations [18-21]. The kinetic equations were applied to the study of transport properties of mesoscopic S/N structures [23]. The function $f_-$ obeys a kinetic equation, which being written in notations of Ref. [12,13] has the form

$$M_- \partial_x f_-(x) + J_S f_+(x) - J_{an} \partial_x f_+(x) = J$$

where all the coefficients are expressed in terms of the retarded (advanced) Green’s functions: $\widehat{G}^{R(A)} = G^{R(A)} \hat{\sigma}_z + \widehat{F}^{R(A)}$; $M_- = (1 - G^{R} G^{A} - (\widehat{F}^{R} \widehat{F}^{A})) / 2$; $J_{an} = (\widehat{F}^{R} \widehat{F}^{A})_z / 2$, is an anomalous current, and the notation $(\widehat{F}^{R} \widehat{F}^{A})_z$ means $(\widehat{F}^{R} \widehat{F}^{A})_z = Tr(\hat{\sigma}_z \widehat{F}^{R} \widehat{F}^{A}) / 2$. The function $J_S$ determines the Josephson current and can be expressed through the values of $\widehat{F}^{R(A)}$ at the S/N interface

$$J_S = - (i/(4R_b \sigma)) Tr(\hat{\sigma}_z (\widehat{F}^{R} \widehat{F}^{R} - \widehat{F}^{A} \widehat{F}^{A}))$$

The Green’s functions $\widehat{F}^{R(A)}$ in the superconductors S are not disturbed by the PE due to a low S/N interface transmittance, and therefore in the right superconductor, they are equal to

$$\widehat{F}^{R(A)} = [i \hat{\sigma}_y \cos(\varphi/2) + i \hat{\sigma}_x \sin(\varphi/2)] \Delta / \xi^{R(A)}$$
where $\xi^{R(A)}_e = \sqrt{(\epsilon + i\gamma)^2 - \Delta^2}$; a phenomenological parameter $\gamma$ describes a damping in the superconductors; the matrices $\hat{F}^{R(A)}_S$ in the left superconductor have the same form if one makes a replacement: $\phi \mapsto -\phi$. The coefficient $M_-$ is also expressed through the retarded (advanced) Green's functions $F^{R(A)}$ in the N wire, but the corrections due to the weak PE are small and approximately we have $M_- \approx 1$. The partial total current $J$ is constant over each separate piece of the N wire. Since the “currents” $J_S$ and $J_{an}$ are proportional to small parameters $|F^{R(A)}|^2$, the distribution function $f_+$ (longitudinal in terms of Ref. [18]) may be taken in zero order approximation, that is, equal to its value in the absence of the PE. It has the form

$$f_+ \cong \frac{\delta f_{eq}}{2}(1 + x/L_N)$$

(4)

where $\delta f_{eq} = [\tanh(\epsilon/2(T + \delta T)) - \tanh(\epsilon/2T)]$. Eq.(1) should be solved with the boundary conditions at the points $x = \pm L_N: f_-(\pm L_N) = F_-(V_r,t)$, where $F_-(V) = [\tanh((\epsilon + eV)/2T) - \tanh((\epsilon - eV)/2T)]/2$. In addition, one has to use the boundary conditions at the S/N interfaces (see [12,13]). The voltages $V_\pm = (V_r \pm V_t)/2$ are found from the condition of the absence of the total current through the N reservoirs (the condition of a disconnected circuit). They are equal to [13,30]

$$eV_+ \cong -\delta T(L_1/L_N) \int d\epsilon (\epsilon) g_{z+}(\epsilon, L_1) f'_{eq} \int d\epsilon ([\nu_S + g_{1+}(\epsilon, L_1)] f'_{eq});$$

(5)

$$2eV_- \cong r_S \delta T \int d\epsilon (\epsilon) g_{z-}(\epsilon, L_1) f'_{eq}$$

(6)

where $g_{z\pm} = (1/4)[(\hat{F}^R - \hat{F}^A)(\hat{F}^R_S \pm \hat{F}^R_b)], g_{1\pm} = (1/4)[(\hat{F}^R + \hat{F}^A)(\hat{F}^R_S + \hat{F}^R_b)]$, $f'_{eq} = \cosh^{-2}(\epsilon/2T)$, and $\nu_S = \text{Re}G^R_S(\epsilon)$ is the density-of-states of a BCS superconductor, $r_S = R_S/R_b$ is a small parameter, $R_b$ is the S/N boundary resistance, $R_S = L_1/\sigma$ is the resistance of a piece of the N wire in the normal state. The denominator in Eq.(5) is proportional to the conductance of the S/N interface [23–25]. The first term in the integrand of the denominator describes the conductance due to quasiparticles above the gap $\Delta$, whereas the second term describes the subgap conductance ($|\epsilon| < \Delta$). Therefore the integrand is not zero at all energies.

Thus the voltages $V_\pm$ are expressed in terms of the Green’s functions $F^{R(A)}$ which can be found from the linearized Usadel equation. Discuss now some general properties of the expressions (5) and (6). The condensate functions $F^{R(A)}$ induced in the N wire have the same matrix structure as the functions $\hat{F}^{R(A)}_S$ (see Eq.(3))

$$\hat{F}^R = i\sigma_y \cos(\varphi/2) F^R_y + i\sigma_x \sin(\varphi/2) F^R_x$$

(7)

Taking into account that $F^R = -(F^A)^*$, the expressions for $g_{z\pm}$ and for $g_{1+}$ can be represented in the form

$$g_{z+} = \frac{1}{2} \text{Re}(F_y - F_x) \text{Im} F_S \sin \varphi; \; g_{z-} = \frac{1}{2} \text{Im}(F_y - F_x) \text{Re} F_S \sin \varphi$$

(8)

$$g_{1+} = \frac{1}{2} \text{Im}[(F_y + F_x) \cos \varphi] \text{Im} F_S$$

(9)

Here we dropped the index $R$: $F^R_y = F_y$. Note also that $\text{Im} F_S = -\Delta/|\xi(\epsilon)|$ at $|\epsilon| \leq \Delta$ and $\text{Im} F_S = 0$ at $|\epsilon| \geq \Delta$, whereas $\text{Re} F_S = \Delta/\xi(\epsilon)$ at $|\epsilon| \geq \Delta$ and $\text{Re} F_S = 0$ at $|\epsilon| \leq \Delta$. Therefore at low temperatures ($T << \Delta$) the voltage $V_-$ is much smaller than the voltage $V_+$. It is useful to compare Eqs.(8) and the expression for $J_S$ that can be represented as

$$J_S = (r_S/2L_1) \text{Im}[(F_y - F_x) F_S] \sin \varphi$$

(10)
FIG. 1. Temperature dependence of the normalized thermoemf (the solid lines, the left scale) \( v_+ = e(V_r + V_I)/2\delta T \) and the normalized critical current (the dashed lines, the right scale) \( I_c(T)/I_c(0) \) for different values of the ratio \( \Delta/\epsilon_S \) and \( r_S = 0.5 \); where \( \epsilon_S = D/L_2 \) and \( \Delta \) is the energy gap at zero temperature. Results are shown for \( L_1/L_2 = \sqrt{3}; L_1/L_{SN} \equiv L_1/(L_N - L_1) = \sqrt{0.3} \); \( \gamma = 0.1\epsilon_S \).

The critical current is an integral over all energies from the function \( J_S(\epsilon) \) which can be transformed into a sum over the Matsubara frequencies \( \omega = \pi T(2n + 1) \)

\[
I_c = \frac{\sigma r_S}{4L_1} \int d\epsilon \text{Im}[(F_y - F_x)F_S] \tanh(\epsilon/2T) = \frac{\pi T\sigma r_S}{4L_1} \sum_{\omega}[(F_y - F_x)F_S]_{\epsilon = i\omega} \tag{11}
\]

We will see that at large ratio \( T/\epsilon_L \), the difference \( (F_y - F_x) \propto \exp(-4\sqrt{2\omega/\epsilon_L}) \) is exponentially small and therefore the critical current \( I_c \) is very small (here we take \( \epsilon_L = D/L^2 \) with \( L = L_1 = L_2 \)). On the other hand, Eq.(8) for \( g_{z+} \), for example, can not be represented as a sum over the Matsubara frequencies. The point is that the critical current
$I_c$ can be written as an integral over energies from the product of the retarded (or advanced) Green’s functions only: $I_c \sim \Im \int d\epsilon |(F_y - F_x)F_S^\pm| \tanh(\epsilon/2T)$. Therefore one can enclose the contour of integration in the upper half-plane of $\epsilon$, where the retarded functions are analytic functions, and calculate the sum over the poles (the Matsubara frequencies) of the function $\tanh(\epsilon/2T)$. The functions $g_{\pm}$ contain the products of the type $(F_y - F_x)^R F_S^A$ (an anomalous function in terminology of Ref. [26]), which are not analytic functions both in the upper and lower half-plane of $\epsilon$. Thus the integral cannot be reduced to the sum over the Matsubara frequencies. The importance of the anomalous terms was emphasized by Gor’kov and Eliashberg [26] who showed that due to these terms the generalization of the Ginzburg-Landau equation to a nonstationary case, generally speaking, is not possible. In the case of mesoscopic S/N structures these terms lead to long-range effects [27].

Thus in a general case the statement about the smallness of $V_\pm$ is not valid. Only if the condition $T, \epsilon L \ll \Delta$ is fulfilled, one can regard $F_S$ as a constant (the integrand in Eq.(8) converges over energies of the order of $T$) and represent Eq.(6) as a sum over the Matsubara frequencies. In this case the voltages $V_\pm$ are also exponentially small. However if the Thouless energy $\epsilon L$ is not small in comparison with $\Delta$ (for example, in [8] $\epsilon S \approx 2.8 K$ and $\Delta \approx 2.28 \ K$), one can not neglect the dependence of $F_S$ on the energy $\epsilon$ and represent the integral in the form of the sum over the Matsubara frequencies. In this case the critical current $I_c$ may be exponentially small ($I_c \propto \exp(-\sqrt{2\pi T/\epsilon L})$), whereas the voltages $V_\pm$ are not small.

In order to calculate the voltages $V_\pm$ explicitly, we need to find the retarded (advanced) Green’s function $F^{R(A)}$. In the considered limit of the weak PE, these functions are easily found from a solution for the linearized Usadel equation

$$\partial^2 \bar{F}^{R(A)}/\partial x^2 - (\kappa^{R(A)})^2 \bar{F}^{R(A)} = 0$$

where $(\kappa^{R(A)})^2 = \mp 2i/\epsilon D_N$. We write out here the solutions for the functions $F_{x,y}$ (we again dropped the indices $R(A)$)

$$F_x = (rSF_y/\theta_y)[\tanh \theta_S \tanh \theta_{SN} + \tanh \theta_y(\tanh \theta_S + \tanh \theta_{SN})]/D_x$$

$$F_y = (rSF_y/\theta_y)[\tanh \theta_{SN} + \tanh \theta_y(1 + \tanh \theta_S \tanh \theta_{SN})]/D_y$$

where $D_x = \tanh \theta_y \tanh \theta_S \tanh \theta_{SN} + \tanh \theta_S + \tanh \theta_{SN}$; $D_y = 1 + \tanh \theta_{SN}(\tanh \theta_y + \tanh \theta_S)$; $r_S = R_1/R_b, R_1 = L_1/\sigma, \theta_y = \kappa L_2, \theta_S = \kappa L_1, \theta_{SN} = \kappa L_{SN}, L_{SN} = L_N - L_1$.

The exponential smallness of the critical current $I_c$ at high temperatures $T > D/L^2$ can be easily verified if one considers a simple case of equal distances $L_1 = L_2 = L_{SN} \equiv L$, that is, $\theta_S = \theta_y = \theta_{SN} \equiv \theta$. In this case we get for the difference $(F_y - F_x)$

$$F_y - F_x = (rSF_y/\theta)[\tanh \theta(1 - \tanh^2 \theta)^2/(2 + \tanh^2 \theta)(1 + 2 \tanh^2 \theta)]$$

It is seen from Eq.(15) that at $\theta >> 1$ we have $(F_y - F_x) \propto (1 - \tanh^2 \theta)^2 \propto (1/\theta) \exp(-4L\sqrt{2\pi T/D})$, that is, this difference is small if $2\omega \approx 2\pi T >> D/(2L)^2$. Therefore the critical Josephson current also is exponentially small (see Eq.(11)). On the other hand, neither $V_\pm$ nor $V_-$ can be represented as a sum over the Matsubara frequencies. Thus, one can not claim that, for example, $V_+ \pm$ should be exponentially small if the Josephson coupling is very weak.

III. RESULTS AND DISCUSSION

We can rewrite the expressions for $V_\pm$ in the form

$$eV_+ /\Delta T \equiv -(L_1/L_N) < (\cos \beta)F'_n F''_S >, \sin \varphi / < 2\nu_S + [F'_n + F''_n \cos \varphi]F''_S >,$$

$$2eV_- /\Delta T \equiv r_S < F''_n F'_S >, \sin \varphi.$$
In Fig.1 and Fig.2 we plot the dependence of the normalized voltage $v_+ \equiv eV_+/\delta T$ and the normalized critical current $i_c \equiv I_c(T)/I_c(0)$ as a function of the normalized temperature $T/\epsilon_S$. These curves are presented for $\varphi = \pi/2$, where $v_+$ reaches a maximum, and different values of the ratio $\Delta/\epsilon_S$. We choose the value of $r_S$ equal to: $r_S = 0.5$ (note that the curves for smaller $r_S$, $r_S \leq 0.3$, are similar to those plotted in Fig.1 and 2). We set $\gamma = 0.1\epsilon_S$ for the data in Fig.1 and $\gamma = 0.03\epsilon_S$ for the data in Fig.2. Note that the damping parameter $\gamma$ may essentially exceed the corresponding value in bulk superconductors because the PE leads to an additional damping of the order of $\alpha r_S \epsilon_S$, where $\alpha$ is a parameter which depends on geometry of the S and N films. We see that in both figures the critical current decays to zero smoothly and fast (faster than exponentially) when the temperature $T$ increases, but the magnitude of $v_+$ decreases not so fast. Close to the critical temperature $T_c$ the critical current is very small in comparison with $I_c(0)$, whereas the normalized thermoemf is not so small.

![FIG. 3. Temperature dependence of the normalized thermoelectric voltage between the normal reservoirs $v_+ = e(V_r - V_l)/2\delta T$ for different values of the ratio $\Delta/\epsilon_S$ and $r_S = 0.5$ (the values of other parameters are the same as in Fig.2). Inset: the structure under consideration; the normal wire connects two normal reservoirs kept at temperatures $T$ (left reservoir) and $T + \delta T$ (right reservoir) and two superconductors with the phases $\varphi/2$ and $-\varphi/2$. The distance between the normal reservoirs is: $L_N = L_1 + L_{SN}$. The electric potential of the superconductors is set to zero.

Although the value of $v_- \equiv eV_-/\delta T$ is much smaller than $v_+$, it depends on temperature quite differently being very small at low $T$ and having a maximum at a temperature near $T_c$. We represent the temperature dependence of $v_-$ in Fig.3. As we mentioned before, the voltage $eV_-$ is very small at low temperatures $T$ because the function $g_{-1}(\epsilon, L_1)$ is zero at $|\epsilon| < \Delta$ and grows for $T > \Delta$. At $T = T_c$ this voltage as well as $V_+$ turns to zero. Therefore, for some parameters the voltages $V_{r,l} = (V_+ \pm V_-)$ may have two maxima (or extrema): one at low $T$, where $V_+$ has a maximum, and another one at a temperature close to $T_c$, where $V_-$ has a maximum. It is worth mentioning that a possible asymmetry of the system (for example, different distances between the crossing point and the left and right N reservoirs) may increase the voltage $V_-$ [17].

In Fig.4 and 5 we plot the temperature dependence $v_r = v_+ + v_-$ for the damping parameters $\gamma = 0.1$ and $\gamma = 0.03$. One can see that this dependence has two maxima. The resistance of a S/N system has a similar temperature dependence with two maxima [28]. Moreover, for $\Delta/\epsilon_S \gg 1.5$ the thermoemf drops to zero at a certain temperature, remains very small at larger $T$ and increases again at temperatures close to $T_c$. At $T = T_c$ the thermoemf drops to zero. A similar dependence of the thermoemf $V_r$ as a function of the magnitude of the dc heater current was observed in experiment [8].

Note that the voltage $V_l = V_+ - V_-$ of the left cool N reservoir (see Fig.6) has the second maximum near $T_c$ the sign of which is opposite to the sign of the first maximum. Such a behaviour was observed in experiment [29].

Obviously the average temperature in the N wire (or film) is proportional to the magnitude of the heater current. Thus one can say about a qualitative agreement between theory and experiment. It is difficult to carry out more precise comparison because the PE in the experiment seems to be strong. Perhaps, the strong PE is the reason for a high value of the second maximum of $V_r$ near $T_c$ (in the experiment, it is comparable with the value of the first
maximum at low $T$) because it is proportional to the parameter $r_S$.

![Figure 4](image1.png)

**FIG. 4.** Temperature dependence of the thermoelectric voltage $v_r = (v_+ + v_-) = eV_r/\delta T$ for different ratio $\Delta/\epsilon_S$ and $r_S = 0.5$ (the values of other parameters are the same as in Fig.1).

![Figure 5](image2.png)

**FIG. 5.** The same dependencies as in Fig.4 for $\gamma = 0.03\epsilon_S$.

We assumed that the parameter $r_S$ is small; however in the case of a strong PE this parameter should be taken of the order of 1 because the condensate functions $F^{R(A)}$ in the N film saturate with increasing $r_S$. Finally we give some estimations for parameters close to the experimental ones [8]: for $L_1 = 0.2\mu$ and $D = 150cm^2/s$, one has $\epsilon_S \approx 2.8K$. This value of $\epsilon_S$ is a little larger than $\Delta \approx 2.2K$ ($\Delta \approx 0.8\epsilon_S$). The estimations of the thermoemf give the values $eV_{th}/\delta T \approx 6 - 10\mu V/K$. These values are comparable with the voltage $V_{th}$ observed in Ref. [5], but much larger than $V_{th}$ measured in Refs. [6,8] (note that in Refs. [5,6] the voltage $V_-$, but not $V_+$ was measured; in the case $r_S \sim 1$ the magnitudes of both voltages are comparable). Perhaps the reason for this discrepancy is the energy relaxation processes in the N wire which were neglected in our calculations. Indeed, in order to neglect the inelastic scattering
the condition $\tau^{-1}_{in} \ll r_S(D/L^2)$ should be fulfilled, where $\tau^{-1}_{in}$ is the inelastic scattering rate. Otherwise in the system arises a strong depairing which suppresses the PE and destroys the phase coherent effects discussed above. The more exact comparison with experimental data is not possible at this stage; for example, a thermoemf symmetric with respect to the phase difference $\varphi$ was observed in experiments. The origin of this part of thermoemf still is unclear.

![Temperature dependence of the thermoelectric voltage $v_t = (v_+ - v_-) = eV_r/\delta T$ for different ratio $\Delta/\epsilon_S$, for $r_S = 0.5$. (the values of other parameters are the same as in Fig.2).](image)

**FIG. 6.** Temperature dependence of the thermoelectric voltage $v_t = (v_+ - v_-) = eV_r/\delta T$ for different ratio $\Delta/\epsilon_S$, for $r_S = 0.5$ (the values of other parameters are the same as in Fig.2).

**IV. CONCLUSION**

In conclusion, we show that the thermoemf arising in the four-terminal S/N structure (see inset in Fig.2) is measurable even in the case of a very weak Josephson coupling. One can say about a long-range thermoemf. The thermoemf $V_{l,r}$ (or thermopower $V_{l,r}/\delta T$) depends on $T$ in a non-monotonic way and may have two extrema: one at low temperatures and another at temperatures close to $T_c$. At some values of parameters and intermediate temperatures the thermoelectric voltage is negligible. This behavior qualitatively agrees with the experimental observations [8]. We would like to thank SFB 491 for a financial support. We are grateful to V. Chandrasekhar, V.Petrashov and I.Sosnin for valuable discussions.

[1] C. W. J. Beenakker, Rev. Mod. Phys. 69, 731 (1997).
[2] C. Lambert and R. Raimondi, J. Phys. Cond. Matt. 10 (1998).
[3] Yu. Nazarov and T. H. Stoof, Phys. Rev.Lett. 76, 823 (1996).
[4] P. Charlat, H. Courtois, Ph. Gandit, D. Mailly, A. F. Volkov, and B. Pannetier, Phys. Rev. Lett. 77, 4950 (1996).
[5] J. Eom, C.-J. Chien, and V. Chandrasekhar, Phys. Rev. Lett. 81, 437 (1998).
[6] D. A. Dikin, S. Jung, and V. Chandrasekhar, Europhys. Lett. 57, 564 (2002); Phys. Rev. B 65, 012511 (2002).
[7] Z. Jiang, V. Chandrasekhar, cond-mat/0501478; 0501477 (2005).
[8] A. Parsons, I. A. Sosnin, and V. T. Petrashov, Phys. Rev. B 67, 140502(R) (2003).
[9] V.L. Ginzburg, Zh. Eks. Teor. Fiz. 14, 177 (1944).
[10] V.L. Gurevich., V.I. Kozub, and A.L. Shelankov, cond-mat/0504116 (2005).
[11] N. R. Claughton and C. J. Lambert, Phys. Rev. B 53, 6605 (1996).
[12] R. Seviour and A. F. Volkov, Phys. Rev. B 62, 6116 (2000).
[13] V. R. Kogan, V. V. Pavlovskii, and A. F. Volkov, Europhys. Lett. 59, 875 (2002).
[14] E. McCann and V. I. Falko, Phys. Rev. B 68, 172404 (2003).
[15] E. V. Bezuglyi and V. Vinokur, Phys. Rev. Lett. 91, 137002 (2003).
[16] A.F. Andreev, Sov. Phys. JETP, 19, 1228 (1964).
[17] P. Virtanen and T. T. Heikkilä, Phys. Rev. Lett. 92, 177004 (2004); J. Low Temp. Phys. 136, 401 (2004).
[18] A. Schmid and G. Schön, J. Low Temp. Phys. 20, 207 (1975).
[19] A.I. Larkin, Yu.N. Ovchinnikov, JETP 46, 155 (1977).
[20] S. N. Artemenko and A. F. Volkov, Sov. Phys. Uspekhi 22, 295 (1979).
[21] N. B. Kopnin, Theory of Nonequilibrium Superconductivity, Clarendon Press, Oxford (2001).
[22] M. Tinhkam and J. Clarke, Phys. Rev. Lett. 28, 1366 (1972).
[23] A.F. Volkov, A.V. Zaitsev, and T.M. Klapwijk, Physica C 210, 21 (1993).
[24] F. W. J. Hekking and Yu. V. Nazarov, Phys. Rev. Lett. 71, 1625-1628 (1993).
[25] A.F. Volkov, Physica B 203, 267 (1994).
[26] L.P. Gor'kov and G.M. Eliashberg, Sov. Phys. JETP 27, 328 (1968).
[27] A.F. Volkov and H. Takayanagi, Phys. Rev. B 56, 11184 (1997).
[28] S. Shapira, E. H. Linfield, C. J. Lambert, R. Seviour, A. F. Volkov, and A. V. Zaitsev
  Phys. Rev. Lett. 84, 159 (2000)
[29] Private communication.
[30] Note that the density-of-states in the superconductors $\nu_S$ is dropped in the expression for $V_+$ in Ref. [13] because at low $T$ it is zero. For $T > \Delta$ it should be taken into account.