Cosmic Ray Scattering in Compressible Turbulence

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\section*{ABSTRACT}
We study the scattering of low-energy Cosmic Rays (CRs) in a turbulent, compressible MHD fluid. We show that compressible MHD modes—fast or slow waves with wave lengths smaller than CR mean free paths induce cyclotron instability in CRs. The instability feeds the new small-scale Alfvénic wave component with wave vectors mostly along magnetic field, which is not a part of the MHD turbulence cascade. This new component gives feedback on the instability through decreasing the CR mean free path. We show that the ambient turbulence fully suppresses the instability at large scales, while wave steepening constrains the amplitude of the waves at small scales. We provide the energy spectrum of the plane-parallel Alfvénic component and calculate mean free paths of CRs as a function of their energy. We find that for the typical parameters of turbulence in the interstellar medium and in the intercluster medium the new Alfvénic component provides the scattering of the low energy CRs that exceeds the direct resonance scattering by MHD modes. This solves the problem of insufficient scattering of low-energy CRs in the turbulent interstellar or intracluster medium that was reported in the literature.

\textbf{Key words:} turbulence, cosmic rays, MHD, instabilities, scattering

\section*{1 INTRODUCTION}
Cosmic rays (CRs) and magnetic fields are essential components for many astrophysical ecosystems, including galaxies and clusters of galaxies (see Schlickeiser 2002). In many instances, e.g. Milky Way, the pressure of CRs and magnetic fields is larger than the gas pressure. As a rule, astrophysical magnetic fields are frozen in turbulent plasma and move together with it. As a result, CRs interacting with turbulent magnetic fields get scattered and accelerated (see Melrose 1968, Schlickeiser 2002).

The magnetohydrodynamic (MHD) approximation is widely used to describe the actual magnetized plasma turbulence over scales that are much larger than both the mean free path of the particles and their Larmor radius (see Kulsrud 2004). The theory of MHD turbulence has become testable recently due to numerical simulations (see Biskamp 2003) and this provided reliable foundations for describing turbulence-CRs interactions. The simulations (see Cho & Lazarian 2005 and ref. therein) confirmed the prediction of magnetized Alfvénic eddies being elongated along magnetic field (see Shebalin, Mattheau & Montgomery 1983, Higdon 1984) and provided results consistent with the quantitative relations for the degree of eddy elongation obtained in Goldreich & Sridhar (1995, henceforth GS95).

Scattering of CRs is an essential part of both CR propagation modes and models of CR acceleration. Efficient scattering is usually postulated (see Schlickeiser 2002), which ensures high degrees of coupling of CRs and magnetized plasma. In addition, efficient scattering provides appreciable second order Fermi acceleration and enables the return of CRs into the shock to ensure the first order Fermi acceleration.

This cornerstone of CR physics has been challenged recently when it became clear that Alfvénic eddies are stretched along magnetic field direction. As the interaction between CRs and such elongated eddies is weak (see discussion in Lerche & Schlickeiser 2001), this resulted in the prediction of long mean free paths for Milky Way CRs (Chandran 2000, Yan & Lazarian 2002, henceforth YL02). YL02 and Yan & Lazarian (2004) attempted to remedy the situation by appealing to CR scattering by isotropic sound-like fast modes. However, plasma-dependent damping of fast modes made the scattering very different in different parts of the interstellar medium. Is such a radical change of the CR scattering picture absolutely necessary?

We note, that the problem of CR scattering goes well beyond the Milky Way physics. Brunetti (2006) discussed the implications of suppressed CR scattering on the acceleration of CRs in the clusters of galaxies. Is there any process through which scattering by fast modes can provide high efficiency of CR scattering? Below we consider such a process that is related to CR feedback on MHD turbulence. We show that compression of CRs induces instability that results in the generation of modes that are parallel to the magnetic field. Such modes that are also frequently referred
to as slab modes have been long employed in the models of CR propagation (see, e.g., Jokipii 1966), their origin, however, was somewhat mysterious. This paper provides a physically motivated mechanism for the generation of slab modes and quantify the efficiency of their generation.

In what follows we discuss the properties of compressible MHD turbulence in § 2. We describe the kinetic instability that develops in CRs when the magnetic field is compressed on scales less than the CR mean free path in § 3. We consider the non-linear saturation of the instability in § 4 and its large-scale cut-off that follows from the interaction of the instability waves with the ambient turbulence in § 5. The feedback of CRs to compressible turbulence is considered in § 6. The implications of our work for various ISM phases and intra-cluster medium (ICM) are considered in § 7. The short summary of this work is presented in § 8.

2 COMPRESSIBLE MHD TURBULENCE

In this section we provide a summary of the current knowledge of compressible MHD turbulence that we appeal to in our work. As we mentioned earlier, we describe an alternative mechanism for the scattering of low-energy CRs that provides rather small mean free paths for CRs. At scales larger than the mean free path CRs are well coupled to the flow of the thermal plasma. Therefore to identify new effects we concentrate on the MHD fluctuations on scales equal or smaller than the mean free path.1

The GS95 model quantifies turbulence anisotropy, introducing the so-called critical balance relation $k_\parallel \sim l^{2/3}$, where $k_\parallel$ and $k_\perp$ correspond to, respectively, the parallel and perpendicular wavenumber of eddies measured in respect to the local magnetic field. This scaling is true for both Alfvénic and pseudo-Alfvénic motions, which are the incompressible limit of the slow modes. GS95 and later Lithwick & Goldreich (2001) argued that the slow modes are passively advected by the Alfvén modes, while the energy exchange between the modes is small. Numerical studies (Cho & Vishniac 2000, Maron & Goldreich 2001, Cho, Lazarian & Vishniac 2002, Müller, Biskamp & Grappin 2003) provided reasonable support for the critical balance condition.2 We introduce the outer scale $L_A$ and use GS95 scaling in the form

$$k_\parallel \sim k_\perp^{2/3} L_A^{-1/3}, \quad \delta v \sim v_A (k_\parallel L_A)^{1/3},$$

We feel that GS95 provides a good starting point for studies of mildly compressible, and even supersonic MHD turbulence. Indeed, numerical calculations in Cho & Lazarian (2002, 2003, henceforth CL02, CL03) showed that scalings of the slow and Alfvén modes in compressible MHD are similar to their scalings in the incompressible case. The fast mode perturbations, on the other hand, are found to be mostly isotropic with a power-law index of about $-3/2$ (see CL02), which is the index of so-called acoustic wave turbulence. The coupling of the fast and Alfvén modes was shown to be weak, which allows separate studies of the corresponding cascades provided that the Alfvénic turbulence is strong, i.e. it evolved to develop the critical balance.

In the following treatment we will be primarily interested in two manifestations of compressible MHD turbulence, one of which is perturbations of the magnitude of the magnetic field and the other is wave damping through cascading by the ambient Alfvénic turbulence. Let us briefly discuss how much of our results depend on the adopted model of MHD turbulence.

As we are dealing with very small perturbations down in the inertial range, Alfvénic mode, having magnetic field perturbations perpendicular to the local magnetic field, has very little effect on the magnetic field magnitude, therefore, as far as the magnetic field compression is concerned, we are dealing with compressible modes. We introduce the power-law energy spectrum for the velocity perturbations of the compressible modes, $E(k) \sim k^{-\beta}$, where $\beta = 5/3$ for the Kolmogorov-type, and 3/2 for the acoustic type spectrum. Such a spectrum translates into velocities at the scale of $l$ which is $\delta v \sim v_A (l/L_A)^{\mu}$, where $\mu = (\beta - 1)/2$ and $L = l$.

A notable exception from this rule, MHD shocks, relevant for the critical balance condition

$$L \equiv (\delta v / v_A),$$

The Alfvén wave damping by Alfvénic turbulence we use in § 4 assumes GS95 scaling and anisotropy with some outer scale $L_A$. This scale might correspond to the isotropic injection of energy at scale $L$ and the injection velocity of $v_A$, i.e. the Alfvén Mach number $M_A \equiv (\delta v / v_A) = 1$. This model can be easily generalized for both $M_A > 1$ and $M_A < 1$ at the injection. Indeed, if $M_A > 1$, instead of the driving scale $L_D$ for $L_A$ one can use the scale at which the turbulent velocity gets equal to $v_A$. For $M_A > 1$ magnetic fields are not dynamically important at large scales and the turbulence follows the Kolmogorov cascade $v_t \sim l^{1/3}$ over the range of scales $[L_D, L_A]$. This provides $L_A \sim L_D M_A^{-3}$. If $M_A < 1$, the turbulence obeys GS95 scaling (also called “strong” MHD turbulence) not from the scale $L_D$, but from a smaller scale $l' \sim L_D M_A^{-3}$ (Lazarian & Vishniac 1999), while in the range $[L_D, l']$ the turbulence is “weak”. The velocity at scale $l'$ is expressed as $v_{l'} \sim v_A M_A^{-3}$, so that the “effective” value of $L_A$ will be $L_A = L_D M_A^{-3}$. All in all, given the strength and the nature of driving in a particular astrophysical environment one may estimate the two parameters, $L$ and $L_A$ that determine the velocity perturbations of the compressible and Alfvénic mode at small scales.

In § 3 we deal with the fluctuations of the magnetic field magnitude squared. The normalized amplitude of these fluctuations denoted as $A$ will depend on the plasma $\beta$ which is
the ratio of the gas pressure to the magnetic field pressure. In high-β plasmas $B^2$ will be perturbed mostly by slow waves (CL03) and the value of $A$ will be equal to $2(\delta v/v_A)\sin \theta$, where $\theta$ is the angle between the wave vector and the magnetic field. In the inertial range of strong turbulence the slow mode exhibits the same anisotropy as the Alfvénic mode. Therefore $\theta$ is close to 90°, so we can disregard the angular factor, i.e. $A = 2\delta v/v_A$. The perturbation made by the fast mode in high beta plasmas is smaller by a factor of $v_A/c_s$, where $c_s$ is sound velocity. In low-β plasmas this situation is reversed, with the slow mode only marginally perturbing $B$, but we may use the same expression $A = 2(\delta v/v_A)\sin \theta$ for the fast mode. We estimate the angular factor as of the order of unity, since fast modes are almost isotropic (CL02).

In other words, the expression $A = 2\delta v/v_A$ is interpreted as the compression factor for the most compressive mode which is the slow wave in high-β plasmas and the fast wave in low-β plasmas.

The lower limits to the scales we described are determined by the damping of MHD modes. While the Alfvénic mode in fully ionized media is supposed to be damped at the thermal Larmor radius, the compressive modes are damped by more efficient collisional and collisionless damping (Ginzburg 1961, Barnes & Scargle 1973, YL04). In this paper we introduce the value of the most compressive mode cut-off scale as $l_{\text{cut}}$.

3 INSTABILITY OF COMPRESSED CRS

It is obvious that at scales less than their mean free path CRs can be treated as a collisionless fluid. Particles in the collisionless fluid preserve the adiabatic invariant $p_i^2/B$, where $B$ is the magnetic field strength and $p_i$ is the momentum perpendicular to the magnetic field. MHD compressive modes change the magnitude of $B$ so that the distribution in momentum space becomes anisotropic (see Chew, Goldberger & Low, 1956).

Such a distribution is subject to a number of instabilities, some of which are hydrodynamic, i.e. involve the change of the entire distribution function, while others are kinetic, i.e. involve a chance in a fraction of particles, that is resonant with a particular wave-mode. Well-known examples of hydrodynamic instabilities are firehose and mirror instabilities (see e.g. Mikhailovskii, 1975). Hydrodynamic instabilities are typically fast with the largest wavenumber growing almost as fast as the gyrofrequency, but have a threshold, i.e. small deviations from isotropy do not induce instability.

While compressive motions can generally induce rather large changes in the magnitude of $B$ on scales at the injection scale of turbulence $L$, in §3 we estimate mean free path and show that it is much smaller than $L$. Therefore the compressions of the magnetic field we deal with are too small to induce hydrodynamic instabilities.

It has been well known that the momentum distribution functions with $p_s = p_0 > p_v$ are subject to kinetic instability called gyroresonance instability (Sagdeev & Shafranov, 1961, Mikhailovskii, 1975, Gary, 1993). This instability received less attention as its hydrodynamic counterparts. However, it is pretty fast for a power-law distribution of CRs, as we demonstrate below.

For a power-law distribution of CRs the growth rate of the cosmic-ray-Alfvén gyroresonance instability (henceforth GI) can be estimated as (see Appendix):

$$\gamma_{\text{CR}}(k_i) = \pm \omega_{\mu} \frac{n_{\text{CR}}(p > m\omega_{\mu}/k_i)}{n} AQ,$$

(2)

where $n_{\text{CR}}(p > m\omega_{\mu}/k_i)$ is the number density of CRs with momentum larger than the minimal resonant momentum for a wave vector value of $k_i$, $m$ is the proton mass, $n$ is the density of the thermal plasma, $\omega_{\mu}$ is the ion plasma frequency, $Q$ is a numerical factor, defined in the Appendix. The ± sign corresponds to the two MHD modes. We shall concentrate on the Alfvén mode, corresponding to the plus sign, as those are less subjected to linear damping (see §2, §6).

As we will demonstrate in the next chapter, when anisotropy is created by compressive turbulence, the anisotropy factor $A = (p_{\perp} - p_{\parallel})/p_{\parallel}$ will be small and will change its sign on the scale of the mean free path, depending on two competitive mechanisms – scattering which tends to isotropize momentum distribution, and magnetic field compression which tends to make it oblate or prolate.

We assumed that the unperturbed distribution of CRs is isotropic and follows a power law i.e. $F_0 \sim p^{-\alpha-2}$ where $\alpha$ is conveniently defined as the power-law index for a one-dimensional distribution (or particle density). For example, around the Earth $\alpha \sim 2.6$ up to the energies of $10^{14}$ eV. Note, that in order for the total energy to converge at high energies, $\alpha$ should be larger than 2.

The expression for the instability rate, assuming $A = 2\delta v/v_A$ (see §2), could be written as

$$\gamma_{\text{CR}}(r_p) = \frac{\delta \nu}{L_4} \left( \frac{r_p}{r_0} \right)^{\alpha+1},$$

(3)

where $r_p$ is a Larmor radius of a CR resonant with a particular wave vector $k_\parallel = m\Omega/p$, $r_0$ is the 1 GeV proton Larmor radius and

$$L_4 = 3.7 \cdot 10^{-2} \frac{1}{Q} \left( \frac{B}{5 \cdot 10^{-6} \text{G}} \right) \left( \frac{4 \cdot 10^{-10} \text{cm}^{-3}}{n_{\text{CR}}(r_p > r_0)} \right) \text{ pc.}$$

(4)

4 NON-LINEAR SUPPRESSION AND SATURATION

We introduce the CR mean free path $\lambda$ below which CRs could be treated as collisionless and the instability described in §3 is active. In the absence of other scattering processes the CRs are scattered by the slab-type motions generated by the instability above. Let us estimate $\lambda$ following Longair (1994). If the change of magnetic field direction is $\phi \sim \delta B/B$ the scattering that is a random walk requires $N \sim 1/\phi^2$ interaction and

$$\lambda \sim N r_p \sim r_p/\phi^2 \sim r_p B^2/(\delta B)^2,$$

(5)

where we designated $\delta B$ as the magnetic field perturbation pertaining to a particular wavenumber, i.e. $\delta B^2 \approx E(k)/k$.

We can consider $\delta B$ as a function of either $k$ or the resonant Larmor radius $r_p$ (see Longair 1994). As the instability grows, $\delta B$ will grow, which reduces $\lambda$. On the other hand, it is the mean free path $\lambda$ which determines the scale at which compressions of the magnetic field are important. This can be understood as follows: the CR distribution “remembers” the perturbed value of the magnetic field and its anisotropy only during the time the typical particle travels its mean free
path. Once particles scatter significantly, the anisotropy of the distribution is effectively “reset”. As a result only low amplitude motions on scales less than λ excite the instability, or in other words, the degree of anisotropy LA is determined by the local perturbation of the magnetic field on the scale λ. We call this process non-linear suppression.

The instability grows as d(δB²)/dt = γCR(δB²) where the injection of energy is happening at the scale of the mean free path, i.e.

\[ γ_{CR} \approx \frac{v_A}{L_i} \left( \frac{r_0}{L} \right)^\mu \left( \frac{δB}{B} \right)^{-2\mu} \left( \frac{r_p}{r_0} \right)^{-1-α} , \]  

where eqs. (3) and (5) were used. We see that according to the above equations δB perturbations will grow as λ₁/2, thus reducing λ virtually to r_p. In other words, the non-linear suppression is not able to constrain the development of instability and we have to consider other non-linear processes, such as wave steepening.

Steepening does not occur for a monochromatic circularly polarized Alfvén wave as the amplitude of the magnetic field stays the same. However, for a collection of waves with different wavelengths the amplitude of the magnetic field fluctuates in space and time and therefore the steepening effect is present. The steepening rate can be estimated as

\[ γ_{steep} \approx -\frac{d(δB/δt)}{k_∥ v_A} , \]  

where the “−” sign reflects the fact, that steepening damps the instability.

By comparing (3) and (7) we get the equilibrium or saturated amplitude of the instability-induced perturbations

\[ \frac{δB}{B} \approx \frac{r_0^{1/2}}{L_i^{1/(2µ+2)} L_p^{µ/(2µ+2)} r_0^{-α/2}} (r_p/r_0)^{(α-1)/µ} , \]  

which for α = 2.6 and µ = 1/3 produces a rather shallow spectrum of perturbations, E(κ) ≈ (δB)²/κ k⁻⁰.⁸

Combining Eqs. (5) and (3) one gets that the energy of the slab modes at κ⊥ ∼ r_p is supplied from the compressions at scale

\[ λ \approx L_i^{1/(µ+1)} L_p^{µ/(µ+1)} r_0^{(α-1)/µ} . \]  

So far we assumed that the turbulent compressible motions are not damped. This is a good approximation until λ is larger than the compressive mode cutoff scale l_cut. If, on the other hand, l_cut > λ the compression for the instability is supplied from the eddies at the damping scale, namely, δε/νA ∼ (l_cut/L)¹/³(λ/l_cut). The modification of our formulae is self-evident. Instead of eq. (5) one gets

\[ \frac{δB}{B} \approx \left( \frac{r_0^{1/2}}{L_i^{1/4} L_p^{(4α-1)/4} l_{cut}^{-1/4}} \right)^{1/4} (r_p/r_0)^{(3-α)/4} , \]  

which, for the same value of α = 2.6, corresponds to a steeper spectrum of E(κ) ~ κ⁻¹.².

5 DAMPING BY ALFVÉNIC TURBULENCE

The instability we considered in § 3 has the largest growth rates for the wave vector parallel to the field. This is due to the fact, that the phase of a resonance can be kept constant for a long time only if κ⊥ is small. The ambient turbulence non-linearly damps the instability through a process that is analogous to the suppression of the streaming instability (YL 02, Farmer & Goldreich 2004). In what follows we find the lower limit on κ⊥ using the approach similar to that in Farmer & Goldreich (2004). We also provide the results of numerical calculations that validate this approach.

For Alfvénic turbulence we adopt the GS95 scaling, (I), which reflects the tendency of eddies to get elongated along the magnetic field. For the sake of simplicity we take the scale L_A, introduced in §2, equal to the scale L. This is not necessarily true for any astrophysical environment, however our formulae are trivially generalized for the case of L_A ≠ L.

Consider a wavepacket of Alfvén waves that moves nearly parallel to the magnetic field with the dispersion of angles δκ⊥/κ∥ ∼ θ_k. The individual waves follow the local direction of the magnetic field lines. As a result, the dispersion in angles of the wave packet cannot be less than the dispersion of angles due to the ambient Alfvénic turbulence, θ_k > θ_hk. The latter for the GS95 model (see eq. (I)) is θ_hk ∼ δB/k_⊥ B_0 ~ (κ⊥ L)⁻¹/³. The modes with minimal θ_k are the fastest growing ones. As we establish below (see Eq. (III)), they are the least damped. Therefore for our simplified treatment we shall limit our attention to the wavepackets with resonant κ⊥⁻¹ ∼ r_p and θ_hk ∼ θ_k. One can determine the characteristic perpendicular wavenumber κ⊥ ∼ δk⊥ ∼ r_p⁻¹(r_p/L)¹/₄ of the “most parallel modes” that are created by streaming CRs.
The strong Alfvénic turbulence decorrelates the wave-packet with \( k_\perp \) on the time scale of \( v_\perp k_\perp \). Thus using the above expression for \( k_\perp \) and Eq. (11) we get
\[
\tau_{\text{turb}} \sim -k_\perp v_\perp \sim -v_A k_\perp^{2/3} L^{-1/3} \sim -v_A r_p^{-1/2} L^{-1/2},
\]
which, up to the “~” that we used to denote the damping nature of the process, coincides with the damping rate obtained in Farmer & Goldreich (2004) and with the results of our numerical simulations shown in Fig. 1.

A comparison between eqs. (6) and (11) indicates that for the spectral index of CRs \( \alpha > 3/2 \) the ambient Alfvénic turbulence provides an upper limit on the scale of perturbations that arise from compressible-induced instabilities even without accounting for nonlinear suppression.

If we use nonlinear suppression, the critical scale can be obtained by using eqs. (6), (8) and (11). For \( \alpha > 5/3 \) our instability is damped for all scales, larger than \( r_p,\text{crit} \approx r_0 \left( L^{1-\mu} + 1 L_{\lambda q}^{-2} \right)^{1/(2\alpha - \mu - 3)} \).

Therefore the spectrum of plane Alfvén waves given by eq. (5) will protrude from \( r_p,\text{crit} \) down to \( r_p,\text{min} \) which corresponds to minimum energies of CRs.

6 FEEDBACK ON COMPRESSIBLE TURBULENCE

Linear instability theory does not describe the energy transfer between particles and CRs. Moreover, in calculating linear response we have not included the CR input into the real part of the dielectric tensor. Speaking of our description of CR scattering by waves, we can consider each resonant scattering event in the rest frame of the wave where it is a pure pitch-angle scattering, i.e. there is no energy transferred between the particle and the wave. Basically, the model of linear instability and its nonlinear suppression and saturation developed in \( \S 4 \) is unable to fully describe the transfer of energy from compressible modes to CRs and from CRs to waves.

Let us first figure out the processes of transfer of energy from CRs to waves. As we noted before, there is no transfer of energy in the frame moving with the wave. Therefore, if we have waves moving in only one direction, particles will pitch-scatter and establish the drift equilibrium with such waves. The drift velocity will be equal to the Alfvén velocity. This is the case of the well-known streaming instability if we can neglect the wave damping. In our case, however, we have waves moving in both directions, so the particle can lose or gain energy in the lab frame by multiple scattering. If each scattering event changes pitch angle by \( \phi \sim \delta B / B \) (c.f. eq (5)) the energy is changed by \( 2p_L V_A \delta B / B \). This occurs at a Larmor frequency, i.e. \( c / r_p \). Now, starting with two above equations we can talk about two processes, one of which is the well-known diffusion in energy space, described by the diffusion coefficient \( D_{pp} \) estimated as
\[
D_{pp} \approx \frac{p^2 v_p^2}{r_p c} \left( \frac{\delta B}{B} \right)^2.
\]

This process is mostly describe the particle-wave equilibrium state where energy is slowly redistributed between particles. Suppose, however, that we efficiently drain energy from the waves so that the situation become non-equilibrium. In this case we talk about the second process which is a directed transfer of energy from particles to waves. The rate of this process depends on the degree of non-equilibrium, but generally cannot exceed the inverse of the minimum time at which particles can lose the energy available from anisotropy, i.e.
\[
\gamma_{\text{ex}} = 1 / \tau_{\text{ex},\text{min}} \approx \frac{v_A \delta B}{r_p \lambda A} \frac{1}{B}.
\]

This rate is actually \( B / (\delta BA) \) times higher than a steepening rate. More careful estimate show that the volumetric rate of the energy exchange between particles and waves divided by the volumetric steepening energy rate is equal to \( B / (2B \delta BAQ) \) which is much larger then unity. Therefore we can conclude that if the energy is drained from waves at a steepening rate, it will be efficiently resupplied by CRs.

The anisotropy of CR distribution is, in turn, supplied by the compressive perturbations. CR pressure is usually neglected in the dispersion relation for the compressive perturbations. However we know that it is there as CRs and the perturbation are connected by the magnetic field. When a fraction of the CR pressure is lost due to the process described above, some of the perturbation energy is lost too. This provides a new mechanism for the damping of compressive perturbations in a medium with CRs.

As we noted previously, each particular scale of compressive motions \( \lambda \) transfers energy to a smaller scale of motions at \( r_p \). Also, we know that the flow of energy in Kolmogorov-type turbulence is a constant. The steepening rate for the reasonable range of parameters is, however, growing rather fast, as we show in the next section. This could lead to a model where compressive motions are fully damped at a particular scale \( \lambda_0 \) where Kolmogorov energy transfer rate is equal to the steepening rate at the scale \( r_p (\lambda_0) \). More careful consideration, however, shows a different picture. At the end of \( \S 4 \) we showed that scales larger than \( \lambda \) could also provide some compression for the CRs. If this compression is higher than the one produced at \( \lambda \), it will also provide higher energy rate. Therefore, the compres-
This spectrum of compressive motions will protrude from handy expression and equations from lower scales will be determined by formula (10). The following frequency slab Alfvénic motions, corresponding to these and mean free path falls below compressive motions cutoff or feedback the mean free path scale to the CR Larmor radius scale. If the mean free path are not damped fully, but rather to the state where they suppress scale, the spectrum of slab waves becomes steeper. 

\[ \delta v \sim \lambda \text{ law, or spectrum of turbulence } E^{-3}. \]

This spectrum of compressive motions will protrude from the scale \( \lambda_{d} \) down to scale \( \ell_{\text{cut}} \). The amplitudes of the high-frequency slab Alfvénic motions, corresponding to these and lower scales will be determined by formula (10). The following handy expression and equations from §4 could be used to derive \( \lambda_{d} \):

\[ \frac{\lambda_{d}}{L} \approx \left( \frac{\delta B}{B} \right)^2. \] 

Fig. 2 shows energy densities of the compressive motions and Alfvén slab-type waves in two cases: first, when the CR feedback is unimportant, but compressive motions are damped at \( \ell_{\text{cut}} \); second, when the CR feedback become important at some scale \( \lambda_{d} \). Spectral slopes correspond to the case with \( \mu = 1/3 \). 

7 ASTROPHYSICAL CONSEQUENCES AND DISCUSSION

7.1 CRs in ISM and galaxy clusters

Let’s consider CRs in galaxy clusters. The magnetic field magnitude and the density of CRs are somewhat uncertain there (see Esslin et al. 2005), so we adopt values similar to our galaxy, namely \( B = 5 \mu G, n_{\text{CR}}(E > 1 \text{ GeV}) = 4 \cdot 10^{-10} \text{ cm}^{-3} \) and \( \alpha = 2.6 \). This corresponds roughly to equipartition between CR and magnetic field energies. In clusters these energy densities are around 5 per cent of the thermal energy density. We will have then \( L_{1} \approx 6 \cdot 10^{-7} \text{ pc} \). The reference Larmor radius of 1 GeV proton is \( r_{0} \approx 2 \cdot 10^{-7} \text{ pc} \). We take the scale \( L = 1 \text{ kpc} \), which, being Alfvénic at this scale, corresponds roughly to driving with the virial velocity at the scale of 30 kpc. For these numerical values and \( \mu = 1/3 \) we will have, from Eq. [5], \( \delta B/B = 0.04(r_{p}/r_{0})^{-0.1}, \) almost independent on scale, \( r_{p,\text{crit}} \approx 10^{3}r_{0} \approx 2 \cdot 10^{-4} \text{ pc, } \lambda \approx 10^{-4}(r_{p}/r_{0})^{1.2} \text{ pc,} \) and the mean free path corresponding to the turbulent damping \( (r_{p} = r_{p,\text{crit}}) \) is 0.4 pc which is much smaller than the outer scale. We estimate collisionless cutoff as \( \ell_{\text{cut}} \approx 10^{-12} \text{ pc} \). The feedback mean free path will be around 0.3 pc, so the spectrum of Alfvénic slab motions will be mostly steeper, \( k^{-1.2} \) and the mean free paths will be modified according to (10) and (5). The efficient CR scattering entails efficient second-order Fermi acceleration, see eq. (13), the process that may be important for clusters of galaxies (Cassano, Brunetti, 2005).

In our galaxy one can assume same values for \( B, \alpha \) and \( n \) and value of \( L \) around 50 pc. We assume an acoustic turbulence spectrum for fast waves, taking \( \mu = 1/4 \). We generally get a smaller range of Alfvénic slab motions, from scales of \( r_{0} \) to about 600\( r_{0} \) with \( \delta B/B = 0.093(r_{p}/r_{0})^{-0.14} \). The resulting mean free paths \( \lambda \) vary from 2.3 \( \cdot \) \( 10^{-5} \text{ pc} \) to 8 \( \cdot \) \( 10^{-2} \text{ pc} \). In the Galactic Corona, fast waves will be damped by collisionless damping (see, e.g., Ginzburg 1961) with a cutoff of around 1.6 \( \cdot \) \( 10^{-3} \text{ pc} \), which is within the range of \( \lambda \) that we deal with. In the warm ionized medium (WIM) the collisional damping cutoff will be around \( 10^{-4} \text{ pc} \). The feedback mean free path will be around \( 7 \cdot 10^{-2} \text{ pc} \). Again the spectrum of slab motions becomes steeper and the mean free paths of CRs are modified accordingly.

In § 2 we assumed that the compression factor \( A \) is larger than \( v_{A}/c \). This assumption is satisfied in galaxy clusters, as, from the previously adopted values and \( n \approx 10^{-3} \text{ cm}^{-3}, v_{A}/c \approx 10^{-3} \), while compression factors for scales between 2 pc and 2 \( \cdot \) \( 10^{-4} \text{ pc} \) are between 0.12 and 5 \( \cdot \) \( 10^{-3} \). For the Milky Way ISM this condition is satisfied much better, as for \( n \approx 1, v_{A}/c \approx 4 \cdot 10^{-5} \), and compression factors are generally larger, due to the fact that the minimum \( \lambda /L \) is smaller.

The slab Alfvén modes had been a part of the CR paradigm from the very start of the research in the field (see Jokipii 1966). Together with anisotropic components they are part of some of the modern models of CR propagation (see Zank & Matthaeus 1992, Bieber et al. 1994, Shalchi et al. 2006). In our model the slab plane-parallel modes emerge naturally as the result of the interaction of compressible turbulence with CR. Although this mechanism is different from the earlier considered processes, it may justify some of the earlier calculations invoking slab modes. Unlike earlier theories we predict the dependence on the amount of the slab mode energy on the relative pressure of the CRs.

As we see, in both clusters of galaxies and ionized gas in Milky Way the instability within CR fluid limits the CRs mean free path. Like in scenario discussed in YL04, where the compressible fast modes were identified as the major CRs scattering agent, compressive modes are essential for scattering. However in this treatment, unlike YL04, we show that compressions at scales much larger than the resonance scale are important. This difference is crucial for scattering of low-energy CRs, as the fast mode have collisional or collisionless cut-offs which, depending on the media, may be larger than the low-energy CR gyroradius. In this case YL04 appealed to Transient Time Damping (TTD) processes, which are less efficient for scattering than gyroresonance\(^5\). Our present work shows that the slab Alfvén mode

\(^5\) In fact the gyroresonance instability can be the major source of isotropization during the TTD acceleration.
discussed in the present paper can be responsible for efficient scattering. Another important difference fromYL04 is that our new mechanism require relatively large total pressure of CRs (see §1). In the case when the pressure of CRs is negligible the fast modes could stay the major scattering agent (see Petrosh et al., 2006).

Our model predicts rather small mean free paths, but this does not contradict the estimates on the average lifetime of the CR in the Galaxy. These lifetimes are estimated to be around Galaxy thickness divided by the Alfvén velocity, which is a powerful support for the models with streaming instability. Our model will predict similar lifetimes, because it includes turbulence which advects CRs on outer scale comparable with Galaxy thickness with velocity of around Alfvén velocity. In fact, the turbulence itself could be generated on these scales by Parker instability.

### 7.2 Partially Ionized Gas

Previous discussion is also applicable to partially ionized gas, if the degree of ionization is larger than ~ 90%. Indeed, for such high ionization degrees the Alfvénic turbulence cascades to scales less than the ion-neutral decoupling scale (see Lithwick & Goldreich 2001).

If, on the other hand, the degree of ionization is lower, we assume that Alfvénic turbulence is fully damped by ion-neutral collisions at the scale $l_{\text{damp}}$, and it would not be able to provide turbulent damping for $k_{\perp}l_{\text{damp}} < 1$. As we saw in § 5 the damping for slab waves with $k_{\parallel}$ is provided by turbulent eddies with $k_{\perp} \sim k_{\parallel}^{3/4}$, therefore, our slab-type component arising from CR instability will prudere to scales as large as $l_{\text{damp}}^{1/3}/L^{1/3}$. This scale could be substantially larger than the $r_{\text{p, crit}}$ derived in § 5.

According to Lazarian, Vishniac & Cho (2004) the regime of viscosity damped turbulence emerges for Alfvénic turbulence at scales less than $l_{\text{damp}}$. This regime is characterized by a shallow $k^{-1}$ spectrum of magnetic perturbations and it persists down to the ion-neutral decoupling scale where it reverts to intermitten Alfvénic turbulence that involves only ions. The detailed treatment of the interactions of CRs with turbulence in partially ionized medium is beyond the scope of this paper, however.

### 7.3 Thermal plasma mean free paths in galaxy clusters

In the paper above we considered the CR component of the ISM or ICM, which are the high energy particles that interact with the rest of the medium via the magnetic fields. These particles have a power-law distribution that arises from the acceleration terms that are proportional to the CR momenta. The astrophysical plasma, on the other hand, is assumed to have a Maxwellian distribution and provides us with both conductivity and mass density which are required for a MHD treatment. In a fully ionized plasma particle-to-particle collisions are Coulomb scattering and the rate of the collisions becomes smaller with temperature. With high temperature and small density these mean free paths can be huge. For example, in galaxy clusters it could be as large as 4kpc. This lead to apparent contradiction, as particles with such a huge mean free path will be subjected to acceleration and will not be Maxwellian.

Schekochihin and Cowley (2005) proposed that thermal particles will be scattered by instabilities. They considered hydrodynamic as well as kinetic instabilities and considered the evolution of the cluster from initial state with no magnetic field. Their argument is that the Reynolds number, being initially very low, will increase with increasing magnetic field and the dynamo will self-accelerate. They predict folded magnetic fields due to high-Prandtl number dynamo and their mean free paths are between viscous scale and the reversal scale.

In this subsection we estimate mean free paths of thermal particles in a way similar to the rest of our paper, keeping in mind that there are quite a few other plasma effects and some MHD dynamo effects that might be important, so that these estimates are still rather speculative. This may be excused by the fact that thermal mean free path, viscosity and thermal conductivity are very important for cluster dynamics.

Anisotropic distributions of thermal particles will excite waves with inverse wavevectors of the order of thermal Larmor radius, $r_{T} \approx 10^{-9}$ pc as the instability is exponentially slow for smaller wavevectors (see Mikhailovskii 1975, eq. 10.7). All particles will have approximately the same mean free path, and the value of $\delta B/B$ that provides scattering will now refer to the total perturbed magnetic field, in contrast with its definition in § 4. Apparently the energy-transfer arguments of §6 will be most important, as the steepening is very fast on thermal Larmor scales. By equating steepening and turbulent energy transfer rates we have $r_{T}/L \approx (\delta B/B)^{4}$, which gives $\delta B/B \approx 10^{-3}$, $\lambda \approx 10^{-3}$ pc.

### 8 SUMMARY

All in all, in the paper above we have demonstrated that

1. Turbulent compressions of magnetic field result in the kinetic instability of CRs that drives Alfvénic perturbations of much higher frequency with wave vectors almost parallel to the magnetic field. These Alfvénic perturbations efficiently scatter and isotropize CRs.

2. The above effect is present over the limited energy range of the CRs. The high energy cut-off is determined by the ambient Alfvénic turbulence. The nonlinear reverse via limiting the CR mean free path and the steepening of the generated waves control the intensity of the new slab-type Alfvénic component. This intensity depends on both the amplitude of the compressible perturbations and CR pressure.

3. The presence of linear damping of compressible motions or the strong feedback damping effect modifies the instability and results in a slightly steeper spectrum of generated Alfvénic perturbations.

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APPENDIX A: CYCLOTRON INSTABILITIES OF COSMIC RAYS IN PLASMA

We follow the standard procedure of deriving the dispersion relations of electromagnetic waves in plasma. The field of the wave creates a perturbation $f_1$ in the particle distribution function $f_0$. We define the current density of the perturbation as $j_i = \sigma_{ik} E_k$ where $\sigma_{ik}$ is the conductivity tensor and

$$\epsilon_{ij} = \delta_{ij} + \frac{4\pi i}{\omega} \sigma_{ik}$$

is a dielectric tensor. The perturbation eigenmodes are determined by the so-called dispersion equation

$$\left| \epsilon_{\alpha\beta} - \left( \frac{c k_i}{\omega} \right)^2 \left( \delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2} \right) \right| = 0$$

Symmetries of the dielectric tensor are determined by symmetries of the initial particle distribution function. In our treatment we consider a two-component medium in which most of the contribution into the dielectric tensor comes from the thermal isotropic plasma, while the small contribution from CRs is responsible for the instability. Let us consider transverse, circularly polarized waves with wavevector parallel to the magnetic field. The dispersion relation will reduce to

$$\omega = c k_\perp v_A$$

for collisionless particles. The CR component $e^{(1)}_\perp$ is circularly polarized waves which are either stable or unstable (Mikhailovskii, 1975, Kulsrud 2004).

We shall limit ourselves to the plus sign as explained in §3. Depending on its sign the instability constitutes the growth or damping. It could be shown that in the limit of $v_A/c \ll 1$ this growth rate is equal to

$$\gamma_{CR} = \pi^2 e^2 v_A \int \frac{\sigma^2}{c^2} \left( \frac{\partial F}{\partial p_\parallel} - v_\parallel \frac{\partial F}{\partial p_\perp} \right) \delta(k_\parallel v_\parallel \pm \omega_C) d^3p$$

where $\omega_C = eB/mc\gamma$ is a particle gyration frequency and $F(p_\parallel, p_\perp)$ is a distribution function of CRs. The $\pm$ sign correspond to the two MHD modes. We shall limit ourselves to the plus sign as explained in § 3.

We introduce a small anisotropy factor $A$ and the unperturbed distribution function $F_0$ as

$$A = \frac{p_\perp - p_\parallel}{p_\parallel}, \quad F_0(p) \sim \left( p^2_\parallel + p^2_\perp \right)^{-\alpha/2-1}$$

where $\alpha$ is introduced in § 2. We assume $\alpha$ to be between 2 and 3 as for the CR distribution in our galaxy. The oblate distribution then will be described as

$$F(p) \sim \left( p^2_\parallel + p^2_\perp (1-A)^2 \right)^{-\alpha/2-1}$$

and we can, in the linear order to $A$, calculate that

$$p_\perp \frac{\partial F}{\partial p_\perp} - p_\parallel \frac{\partial F}{\partial p_\parallel} = (-\alpha - 2)AF.$$

Now the expression for the instability rate will be

$$\gamma_{CR} = \pi^2 e^2 n_{CR} v_A \left( -\alpha - 2 \right) A \frac{\omega_C}{\omega_C} \int \frac{\omega_C}{n_{CR}} \frac{v_\parallel}{c} F_0 \delta \left( k_\parallel p_\parallel + \omega_C \right) d^3p,$$

where we replaced $F$ with $F_0$ and introduced the cyclotron frequency $\omega_C$, and the total density of CRs $n_{CR}$. The integral in this expression is dimensionless. The total density of CRs is mostly determined by the low-energy cutoff of the distribution $F_0$ and is rather irrelevant for the instability where only resonant particles contribute. It is more useful to introduce the number of fast particles $n_{CR}(p > m\omega_B/k_\parallel)$ which is determined by the integration of $F_0$ over a region with momentum larger than the resonant momentum. After taking the integrals we denote the gamma function as $\Gamma$ and arrive at

$$\gamma_{CR}(k_\parallel) = \omega p_\parallel n_{CR}(p > m\omega_B/k_\parallel) AQ,$$

where $Q = \frac{\pi^{3/2}}{32} (\alpha + 2)(\alpha - 1) - \frac{\Gamma(\alpha/2)}{\Gamma(\alpha/2 + 3/2)}$.

6 Speaking more quantitatively, the degree of anisotropy should be larger than $v_A/c$ for instability to take place (Kulsrud 2004). In the astrophysical section we show that in the typical setting of the ISM or ICM our approximation is accurate enough (see § 6).
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