Hologram of a single photon

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The spatial structure of single photons\(^1\)–\(^3\) is becoming an extensively explored resource to facilitate free-space quantum communication\(^4\)–\(^7\) and quantum computation\(^8\) as well as for benchmarking the limits of quantum entanglement generation\(^9\) with orbital angular momentum modes\(^10\) or reduction of the photon free-space propagation speed\(^10\). Although accurate tailoring of the spatial structure of photons is now routinely performed using methods employed for shaping classical optical beams\(^2\)–\(^11\), the reciprocal problem of retrieving the spatial phase-amplitude structure of an unknown single photon cannot be solved using complementary classical holography techniques\(^12,13\) that are known for excellent interferometric precision. Here, we introduce a method to record a hologram of a single photon that is probed by another reference photon, on the basis of a different concept of the quantum interference between two-photon probability amplitudes. As for classical holograms, the hologram of a single photon encodes the full information about the photon’s ‘shape’ (that is, its quantum wavefunction) whose local amplitude and phase are retrieved in the demonstrated experiment.

The complete characterization of a quantum wavefunction of an unknown photon presents a challenging task, the difficulty of which lies in the retrieval of its local phase variations. This is caused by the fundamental property of single photons, that is, their entirely indeterminate global phase indicated by the perfect rotational symmetry of their Wigner functions in the phase space\(^14\). This precludes the application of interferometric techniques such as optical holography, which uses the fixed phase relation between the investigated and reference light. Therefore the characterization of a photon’s spatial structure has never benefited from the precision and a simplicity provided by holography methods\(^12,13\) but as yet has only been tackled using tomographic techniques\(^15\) or weak measurements approach\(^2,16\).

In this Letter, we experimentally show that the hologram of a single photon (HSP) that encodes the full information about its spatial structure (given by the quantum wavefunction \(\psi(x) = \chi(x)\phi\); ref. 2) can be recorded if the first-order interference of the optical fields is replaced by the non-classical interference of spatially varying two-photon probability amplitudes. The idea of HSP, sketched in Fig. 1a, relies on overlapping the unknown photon \(|\psi_u(x)\rangle\) with an arbitrary local phase profile \(\varphi(x) = \arg(\psi_u(x))\) with a reference photon \(|\psi_r(x)\rangle\) that has the constant local phase profile on a beam splitter, both photons occupying similar spectral (temporal) modes. We then measure the transverse positions \(x\) and \(x'\) of the photons that coincidently left the two distinct output ports of the beam splitter. Any feature that distinguishes the photons, such as the local difference between their quantum wavefunctions \(\psi_u(x)\) and \(\psi_r(x)\) prevents the ideal two-photon coalescence known as the Hong–Ou–Mandel effect\(^15\), thus the observation of spatially localized coincidences \((x, x')\) serves as a sensitive probe of the spatial structure of the unknown photon. As we visualize in Fig. 1b, such a coincidence event can originate either from the transmission or reflection of both photons at the beam splitter. These two fundamentally indistinguishable events amount simultaneously to a two-photon probability amplitude \(\Psi(x, x')\) that describes one photon localized at position \(x\) and the other at \(x'\), which can be expressed in Feynman’s path integral formalism as

\[
\Psi(x, x') = \frac{1}{2} \langle \langle x | \psi_u(x') \rangle \langle x | \psi_r(x') \rangle - \langle x | \psi_r(x') \rangle \langle x | \psi_u(x') \rangle \rangle
\]  

(1)

Thanks to the recent advances in single-photon-sensitive intensified cameras\(^16\) we were able to measure the joint probability distribution \(|\Psi(x, x')|^2\) with a resolution high enough to reveal the spatial variations that originate from the non-destructive interference of the quantum paths of the unknown and the reference photons. Remarkably, this joint probability distribution provides information about the local phase profile of the unknown photon \(\psi(x):\)

\[
|\Psi(x, x')|^2 = \frac{1}{4} (|\psi_u(x)|^2 |\psi_r(x')|^2 + |\psi_r(x)|^2 |\psi_u(x')|^2) - \frac{1}{2} |\psi_u(x)| \langle \psi_r(x') \rangle \langle \psi_u(x) | \psi_r(x') \rangle \langle \psi_r(x) | \psi_u(x') \rangle \times \cos (\varphi(x) - \varphi(x'))
\]  

(2)

**Figure 1 | Quantum interference of two spatially structured photons.**

\(a:\) In analogy to classical holography we repeatedly overlap an unknown photon \(|\psi_u(x)\rangle\) with a reference (known) photon \(|\psi_r(x)\rangle\) with a constant local phase profile on a 50/50 beam splitter. Coincidence events localized in \(x\) and \(x'\) provide the joint probability distribution \(|\Psi(x, x')|^2\), which is sensitive to any differences between the quantum wavefunctions of the photons \(\psi_u(x)\) and \(\psi_r(x)\), including the local variations of their phases. \(b:\) The spatially localized coincidence events \((x, x')\) originate from the non-destructive interference of the probability amplitudes of two classically exclusive, but quantum mechanically coexisting scenarios. Left: The unknown photon in \(x\) and the reference photon in \(x'\) have passed through the beam splitter. Right: Both photons localized conversely in \(x'\) and \(x\) have been reflected from the beam splitter.
The HSP given by $|\psi(x, x')|^2$ is entirely insensitive to any constant offset of the local phase profile of the unknown photon in contrast to optical holograms, which, while being recorded, are extremely sensitive to a phase shift between reference and unknown fields. The visibility of the HSP fringes $V$ is defined by a spectral (temporal) mode overlap that can be high and stable for photons that are generated by different sources, such as two independent spontaneous parametric down-conversion (SPDC) sources\textsuperscript{15}, quantum dots\textsuperscript{19} or even dissimilar sources\textsuperscript{21}. In Fig. 2 we visualize the HSP structures for the examples of local phase profiles. We can resort to one of the numerous methods of phase retrieval\textsuperscript{22} to infer $\phi(x)$ from equation (2), as the detection probability distributions $|\psi_u(x)|^2$, $|\psi_r(x)|^2$ are quantities that are directly measurable.

We selected the situation depicted in Fig. 2a for experimental demonstration of an HSP where the unknown photon has the quadratic local phase $\phi(x) = kx^2/2R$ resulting in a cross-shaped HSP. Here $R$ is the radius of curvature and $k = 2\pi/800$ nm is the wave number. Both the unknown and the reference photons were generated via a type II SPDC process realized in a periodically poled potassium titanyl phosphate (PPKTP) nonlinear crystal pumped with 400 nm light from a continuous-wave diode laser. The high distinguishability of their spectral (temporal) modes was confirmed in an independent HOM dip measurement, yielding a visibility of 91%.

As depicted in Fig. 3, the photons were spatially filtered by a single-mode fibre, separated by a polarization beam splitter and directed individually to two arms of a delay line and phase-imprinting system. The lengths of the arms were adjusted to overlap the photons temporally and to set the constant-phase waists of the mode exiting a fibre collimator on the mirror surfaces. We inserted a cylindrical lens (focal length, $f = 75$ mm) in proximity to one of the mirrors, thus imprinting the quadratic local phase profile in a horizontal direction on the unknown photon during its propagation back and forth. As the reference and unknown photons that propagated through the different arms of the delay line were orthogonally polarized, no interference occurred at the delay line output.

The key part of the experimental set-up was an intensified scientific complementary metal-oxide–semiconductor (I-sCMOS) camera system with parameters suitable to detect spatially resolved photon pairs with a negligible dark count background (see Methods for camera operation details and a discussion of the signal-to-noise ratio). We imaged the delay-line mirror surfaces on the camera using a standard unit-magnification optical system that preserves both the amplitude and the phase of the spatial wavefunctions of the impinging photons. A cylindrical lens (CL2) placed in front of the camera reduced the mode size in the vertical direction perpendicular to the plane of the set-up, decreasing the frame reading time.

The beam splitter transformation was implemented in the collinear configuration as a half-wave plate followed by a calcite polarization displacer with its two output ports mapped onto two distinct regions of the camera. In the experiment we retained frames containing two detected photons for analysis, registering their positions in a horizontal dimension parameterized by $x$, $x'$ coordinates in respective regions of the I-sCMOS sensor. The high spatial resolution allowed us to record the subtle variations of the detected photon positions and thus directly measure the empirical coincidence probability distribution $|\psi(x, x')|^2$.

The measured HSP, which consists of approximately $2.2 \times 10^3$ detected photon pairs is presented in Fig. 4a; it closely resembles the theoretical cross-like shape shown in Fig. 2a. Following equation (2) we
decoded the phase \( \varphi(x) \) using a numerical method (see ref. 22 and Methods for details) that finds the local phase profile that yields the closest coincidence probability distribution to the measured data as displayed in Fig. 4b. The procedure was fed with virtually identical wavefunction amplitudes of the unknown photon \( |\psi_u(x)\rangle \) presented in Fig. 4c, and the reference photon \( |\psi_r(x)\rangle \) measured independently using the coincidence imaging scheme (see ref. 18 and Methods for details).

We show the complex quantum wavefunction of an unknown photon, that is, its measured amplitude and the phase extracted from its HSP along with the uncertainty ranges in Fig. 4c. We found that the radius of curvature of the reconstructed local phase profile of the photon (\( R = 34.0 \pm 1.5 \) mm) was in good agreement with the value expected from a double pass through the phase-imprinting lens, which has been confirmed in an independent measurement by interfering classical beams in this set-up. The uncertainty of the reconstructed phase is below \( 2\pi/25 \) in the central region and it diverges only on the edges of the wavefunction due to the scarcity of registered counts outside the central region. The high resolution of the phase retrieval translates into the high fidelity of the quantum state reconstruction (see Methods for details).

The HSP method transfers hologram recording techniques into the field of quantum optics, presenting a compelling and promising way for retrieving quantum wavefunctions. The technique can be readily adapted to the more general configurations where a reference photon has an unknown structure by spatially shearing the photons in the second measurement run (see Methods for details). The HSP technique can be also extended to the two-dimensional case, requiring the efficient detection of a four-dimensional coincidence probability distribution (see Methods for details). The parallel development of low-jitter, time-resolving detectors would allow the implementation of HSPs in the mathematically equivalent spectral (temporal) domain, for which the local phase sensitivity of the non-classical interference has been observed and several wavefunction reconstruction techniques have been presented. Finally we emphasize that because our scheme relies solely on multiparticle bosonic interference, it can be generalized for all bosons. The measurement of the hologram of a single atom and the retrieval of its wavefunction could be achieved using the scheme recently reported in the first experimental realization of two-boson interference, which relies on a detection technique similar to the one employed in this Letter.

Methods

Methods and any associated references are available in the online version of the paper.

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Author contributions
W.W. proposed the idea of wavefunction phase retrieval. R.C. designed and programmed the experiment, developed HSP methods, analysed the data and prepared figures. M.J. built the set-up and performed the measurements. R.C. and M.J. wrote the manuscript assisted by W.W. and K.B, who supervised the work and contributed to data analysis.

Additional information
Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to M.J.

Competing financial interests
The authors declare no competing financial interests.
Methods
Photon source. The photon pairs (consisting of the unknown and the reference photons) were generated via a type II degenerate SPDC process realized in 5-mm-long periodically poled KTP crystals (with a poling period of 9.2 μm) pumped with 8 mW of 400 nm light from a single-mode continuous-wave diode laser. The temperature of the crystal was stabilized to 24.1 °C to ensure maximal and stable overlap between the spectral modes of the generated photons. The photons were spectrally filtered by a narrowband 3 nm full-width at half-maximum interference filter, spatially filtered by a single-mode fibre and temporarily overlapped after a polarization beam splitter by means of an optical delay line where a double-pass through quarter-wave plate (Λ/4) rotated the photons’ polarization by 90°.

We characterized the indistinguishability of the photons used in the experiment with the standard avalanche photodiode coincidence system by measuring the Hong–Ou–Mandel dip, yielding a visibility of 91%.

Single-photon localization with an I-sCMOS camera. To localize photons with a high spatial resolution we used an I-sCMOS camera system, assembled by our group29. The image intensifying process begins with a gallium arsenide photocathode that converts the impinging photons into electrons with a quantum efficiency of about 20%. Each electron then enters the microchannel plate, where it triggers the charge avalanche that hits the phosphor screen, resulting in a bright green flash with a decay time below 200 ns. A typical phosphor screen has a diameter of 66 μm and is highly random brightness determined by the stochastic avalanche process. The flashes are imaged on the I-sCMOS camera sensor via a bright relay lens and real-time localized by a software algorithm that retrieves the central positions of the flashes from a raw image with subpixel accuracy. We acquired data from 1,000 × 20 pixel regions of interest selected on the I-sCMOS camera sensor with a frame rate of 7 kHz. We set the time gate of the image intensifier to 30 ns, thereby ensuring that virtually no accidental coincidences or more than two photons per frame were detected. See the Supplementary Information for further details of the camera construction and operation.

Measurement details. We measured the amplitude of the wavefunctions |ψu(x)| and |ψr(x)| by setting the half-wave plate (Λ/2; HWP) to θ = 0° and θ = 45° to direct photons into different output ports of the calcite displacer. The nearly identical squared amplitudes of both photons were recovered following our coincidence imaging scheme18. Then we proceed to the HSP measurement by setting the HWP to θ = 22.5° and interchangeably measuring the wavefunction amplitudes and HSP by rotating the HWP after each set of 5 × 106 frames were taken out of a total of 1.8 × 109 frames collected.

Signal-to-noise ratio. Besides the two-photon events described by equation (2), we may also detect accidental coincidences caused by thermal emission of dark counts that reduce the signal-to-noise ratio (SNR). Although the contribution of these accidental coincidences can be neglected in the presented experiment, as we show below, the incorporation of noise in data analysis can be important when other types of measurement technique, such as scanning multiphoton counting for quantum imaging applications or ghost-imaging techniques as illustrated in Supplementary Fig. 5. See the Methods for the following optimization problem:

$$\min_{\psi} \left( | \langle \psi^T | x, x' \rangle |^2 - | \langle \psi^T | x, x' \rangle |^2 \right)$$

(5)

where |ψ(x, x')⟩ is the measured empirical distribution, |ψ(x, x')⟩ is a functional defined by equation (2) and constructed from the measured amplitudes |ψu(x)| and |ψr(x)| depending on the vectorized phase profile ψ(x) that is being searched for and |1⟩ is the Frobenius norm of the matrix. As the general global search is a computationally hard problem, we divided our optimization into two simpler subsequent steps. We assumed that ψ(x) is a general fourth-order polynomial and we ran a global scan to find its coefficient and the visibility parameter V. Then we performed the local optimization with unconstrained values of the discretized ψ(x), starting from the result obtained using the global search.

Uncertainties of the results. To account for the uncertainty of the empirical HSP in the phase-retrieval procedure, we applied the Monte Carlo approach. We repeated the phase retrieval procedure 5,000 times, each time randomizing |ψ(x, x')⟩ by drawing the initial counts values at each pixel from the corresponding Poissonian distributions. In each realization we obtained |ψ(x, x')⟩ and the corresponding vector of the discretized phase. The Monte Carlo approach resulted in the mean reconstructed HSP presented in Fig. 4b and the phase profiles whose mean and standard deviation, after unifying their convexes and constant phase offset, are presented in Fig. 4c.

Fidelity of quantum state reconstruction. The fidelity of the quantum state reconstruction $F = \frac{\langle \psi_{rec} | \psi \rangle}{\langle \psi_{rec} | \psi_{rec} \rangle}$ in the HSP method depends in general on a state of an unknown photon. The fidelity of the reconstruction procedure for our experimentally realized scenario can be estimated numerically in a Monte Carlo simulation, where we draw from Poissonian statistics data sets that are then fed into the phase-retrieval algorithm. The fidelity of the quantum state reconstruction varies with the number of coincidences recorded in the HSP and the spectral visibility V, as presented in Supplementary Fig. 8. For our experiment the expected fidelity of the reconstruction is above $F > 98.5\%$ and would approach unity with increasing data samples.

Possible practical implementations and generalizations of the HSP method. Owing to its simplicity the HSP method could be performed in currently available experimental settings for coincidence imaging using scanning single-pixel detectors30,31, camera devices32,33 or single-photon detector arrays34,35. Moreover, certain straightforward generalizations offer realistic paths for further experiments beyond a proof-of-principle demonstration.

First of all, if both photons are unknown, we could retrieve the difference between the reference local phase structures $\phi_1(x) - \phi_2(x)$ used the same procedure as presented before. Using spatial shearing, that is, by applying a small transverse displacement δx of the reference photon $\psi_1(x) → \psi_1(x + δx)$, in the second measurement run $\psi_2(x) → \psi_2(x + δx)$ could be retrieved, providing information about the local phase structure of the reference photon and consequently about the local phase of the unknown photon.

Second, the HSP method can be analogously employed for bidimensional transverse photon structures, yielding four-dimensional coincidence distributions defined by equation (2) under the substitution $(x, x') → (r, r')$, where r and r’ are two-dimensional vectors in space. Practical measurements of the bidimensional local phase structure of photons would require either faster intensified cameras such as emerging intensified time-stamping CMOS sensors36, or spatially modified ghost-imaging techniques as illustrated in Supplementary Fig. 5. See the Supplementary Information for a detailed explanation of the generalization schemes.

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