Superbubble Activity in Star-Forming Galaxies

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Abstract.

Mechanical feedback from massive stars, primarily from supernovae, can dominate ISM structuring and phase balance, thereby profoundly affecting galactic evolutionary processes. Our understanding of mechanical feedback is based on the adiabatic, wind-driven bubble model, applied on size scales ranging over three decades. Tests of the model, and our consequent understanding of feedback, are reviewed. While the model is broadly successful, critical unknowns still prevent a comprehensive understanding of the consequences of feedback.

Keywords: galaxies: evolution, intergalactic medium, ISM: bubbles, ISM: general, supernova remnants

1. Introduction

Mechanical feedback from massive stars is a dominant driver of evolutionary processes in galaxies, and takes place on scales ranging from individual wind-driven bubbles to galactic superwinds. Our understanding of the feedback process is based on the standard evolutionary model for stellar wind- and supernova-driven bubbles (Pikel’ner 1968; Weaver et al. 1977): hot (log T/K ~ 6 – 7), low-density (n ~ 0.01 cm⁻³) gas is generated within a double-shock structure, and the pressure of this hot gas, chemically enriched by the stellar products, drives the growth of the thin, radiatively-cooled shell. In the adiabatic model, energy loss from the hot gas is negligible, yielding simple analytic expressions for the shell radius R and expansion velocity v as a function of time t:

\[ R \propto \left( \frac{L}{n} \right)^{1/5} t^{3/5}, \]

\[ v \propto \left( \frac{L}{n} \right)^{1/5} t^{-2/5}. \]  

(1)

For a stellar wind-driven bubble, the mechanical luminosity \( L = \frac{1}{2} \dot{M}v_{\infty}^2 \), where \( \dot{M} \) and \( v_{\infty} \) are the wind mass-loss rate and terminal velocity, respectively. For OB associations, supernovae (SNe) quickly dominate over winds, in which case \( L = N_s E_{51}/t_e \) (e.g., Mac Low & McCray 1988), where \( N_s \) and \( E_{51} \) are the total number of SNe and SN energy, respectively, and \( t_e \) is the time over which the SNe occur.

The fate of the interior hot gas is crucial to the phase balance and enrichment of the interstellar and intergalactic media. Is stellar feed-
back indeed the source of the diffuse, hot gas in the ISM? Do galactic superwinds from starbursts eject metal-enriched gas from galaxies? How does mechanical feedback affect the structure of the ISM and porosity, for example, to ionizing radiation? Do superbubbles trigger renewed star formation?

Determining the relevance of the standard, adiabatic model for shell evolution is clearly critical in answering these fundamental questions. Several tests can be applied: 1) Comparing the observed vs predicted dynamics of individual bubbles and superbubbles; 2) Comparing the observed vs predicted statistical properties of superbubble populations, for example size, velocity, and energy distributions; 3) Identifying spatial correlations of superbubbles with the progenitor OB associations and their relics; and 4) Testing the observed vs predicted dynamics and properties of galactic superwinds. The last is presently more difficult and the subject of entire reviews in its own right (e.g., Heckman 2002). Therefore I will discuss here only the first three tests.

2. Individual shell systems

2.1. Single star bubbles

Few studies exist of single star bubbles from isolated OB stars. Oey & Massey (1994) examined two nebular examples in M33, and spectroscopically classified the parent O stars. The inferred stellar masses and ages implied wind parameters that were consistent with the observed sizes and shell ages predicted by the adiabatic model. However, the parameters were loosely constrained. H\textsc{i} shells with radii of several tens of pc have been identified as wind-blown bubbles around a number of Galactic O and Of stars (Cappa & Benaglia 1998; Benaglia & Cappa 1999). These are largely consistent with the standard model, and probe a specific subset of fairly evolved stars with old shells that have essentially stopped expanding.

Studies of Wolf-Rayet ring nebulae suggest shells that are too small, equivalent to an overestimate in $L/n$ by an order of magnitude (e.g., Treffers & Chu 1982; García-Segura & Mac Low 1995; Drissen et al. 1995). However, the progenitor star produces several wind phases, including both fast and slow winds, with extreme changes in $L$. Their cumulative effect on shell morphology is complex and poorly understood. It is therefore unsurprising to find significant discrepancies between the predictions and observations of shell parameters. Hence, more studies of the simpler, single OB star bubbles are needed.
2.2. Superbubbles

Superbubbles around OB associations are more prominent than single-star bubbles, and thus have been studied more extensively. Soft X-ray emission has been detected within many objects, which is qualitatively consistent with the adiabatic evolution model. Two classes of X-ray emission have been identified: objects with X-ray luminosity $L_x$ in excess of the model’s prediction (Chu & Mac Low 1990; Wang & Helfand 1991), and objects that remain undetected in X-rays (Chu et al. 1995). The X-ray–bright objects are thought to be overluminous because of SNR impacts on the shell walls. Upper limits on the X-ray–dim objects remain consistent with $L_x$ predicted by the adiabatic model. It will thus be of great interest to determine $L_x$ for these objects with XMM-Newton or Chandra. Also, an interface region between the hot gas and cooler shells should generate intermediate temperatures and ions. Chu et al. (1994) searched a number sightlines through LMC superbubbles and confirmed the existence of C\textsc{iv} and Si\textsc{iv} absorption in all cases.

A stringent test of the adiabatic model is to compare the predicted and observed shell kinematics in cases where the input mechanical power and other parameters are well-constrained. This was carried out for eight, young, wind-dominated LMC superbubbles by Oey & Massey (1995), Oey (1996), and Oey & Smedley (1998). The predicted growth rate for the shells was higher than implied by their observed $R$ and $v$, equivalent to an overestimate in $L/n$ by an order of magnitude. However, even after adjusting $L/n$ in the models, over half the objects still showed observed expansion velocities that were typically a factor of two higher than predicted for the given $R$. Similar discrepancies were reported for Galactic objects by Saken et al. (1992) and Brown et al. (1995).

SNR impacts on the shell wall are the favored explanation for the high-velocity shells, since these also exhibit the anomalously high X-ray emission and elevated [S II]/H$\alpha$ ratios. However, a sudden drop in the ambient density can induce a “mini-blowout” with shell kinematics that can easily reproduce the anomalous velocities (Oey & Smedley 1998; Mac Low et al. 1998; Silich & Franco 1999). Indeed, were it not for the X-ray and nebular diagnostics, it would be impossible to distinguish the shell acceleration mechanism from the kinematics alone.

Thus we see that the ambient properties are critical in determining the shell evolution. An underestimate in $n$ could, for example, contribute to the growth rate discrepancy described above, that is seen in all the objects. To clarify the ambient gas distribution, Oey et al. (2001) mapped the H\textsc{i} distribution within a $\sim 40'$ radius of three nebular LMC superbubbles at $30''$ resolution. The results show neutral
environments that vary to an extreme, despite morphologically similar optical nebulae. It is therefore essentially impossible to infer properties of the ambient material without direct, multi-wavelength observations.

Another vital parameter for shell evolution is the ambient pressure, \( P_0 \), which determines whether and when the superbubble growth becomes pressure-confined. While \( P_0 \) is usually unimportant in young, high-pressure superbubbles like the nebular objects mentioned above, it is of vital importance in the mid- to late-stage evolution. It may also be relevant in high-pressure, ionized environments like dense star-forming regions (e.g., García-Segura & Franco 1996). Ultimately, \( P_0 \) determines the final size of the shells, and conditions relative to blowout. The uniformity and distribution of \( P_0 \) in the multiphase ISM is therefore especially relevant to a global understanding of superbubbles in galaxies (see below).

Finally, if the hot gas within superbubbles does not blow out and merge into the hot, ionized medium (HIM), it is likely that the objects will cool and depart from energy conservation. Indeed, whether and how the hot interior cools has long been a major question for superbubble evolution and the fate of the hot gas. Thermal conduction at the interface between the cool shell wall and hot gas should cause a high rate of mass-loading into the interior. The evaporated shell material dominates the mass in the hot region, which could be further supplemented by evaporation and ablation from small clouds overrun by the expanding shocks (e.g., Cowie & McKee 1977; McKee et al. 1984; Arthur & Henney 1996). If the interior density is sufficiently increased, radiative cooling will dominate, and the shells will no longer grow adiabatically. In addition, Silich et al. (2001) point out the importance of enhanced metallicity in the superbubble interiors, caused by the stellar and SN yields. Preliminary investigation for individual objects by Silich & Oey (2001) shows enhancement in \( L_x \) by almost an order of magnitude for low-metallicity (\( Z = 0.05 Z_\odot \)) objects. This increase in the cooling rate could facilitate a transition from adiabatic to momentum-conserving evolution.

2.3. Supergiant shells

The very largest \( \text{H I} \) shells, having sizes of order 1 kpc, emphasize some of the problems with the mechanical feedback model, and also highlight possible alternative shell-creating mechanisms.

The existence of infalling high-velocity clouds (HVCs) suggests that the impact of these objects could be an important contributor to supergiant shell populations. This suggestion is consistent with galactic fountain models for disk galaxies (e.g., Shapiro & Field 1976), which are
ultimately also powered by mechanical feedback in the disk. A number of hydrodynamical simulations of infalling HVCs confirm that these impacts result in shell-like structures (e.g., Tenorio-Tagle et al. 1986; Rand & Stone 1996; Santillán et al. 1999).

In addition, tidal effects, which dominate energetics and structure formation at the largest length scales, could also create HI features that resemble shells. Note that many SN-driven shells will not exhibit expansion velocities if they have become pressure-confined by the ambient medium, thus a lack of observed expansion velocities cannot distinguish between the feedback model and other models. It has been suggested that some of the largest holes in, e.g., M33 are simply morphologically-suggestive inter-arm regions (Deul & den Hartog 1990). The same may be true of the giant hole identified by de Blok & Walter (2000) in NGC 6822. Simple self-gravity effects have also produced shell- and hole-like structures in numerical simulations (Wada et al. 2000), although morphologically these structures appear more filamentary than the observations.

While such alternative mechanisms for creating shell-like structures undoubtedly contribute to the supergiant shell population, the conventional mechanical feedback model nevertheless also appears to apply in many situations. Meaburn’s (1980) LMC-4 is a well-known example that is unambiguously linked to Shapley’s Constellation III, a large, extended complex of young stars. Kim et al. (1999) are able to identify an evolutionary sequence for supergiant shells in the LMC, based on the relative loci of Hα and HI emission. In addition, Lee & Irwin (1997) considered formation mechanisms for supergiant shells in the edge-on SBc galaxy NGC 3044. They found no evidence of HVCs, and since the galaxy is isolated, tidal interactions are also unable to explain the supergiant shells. They therefore conclude that the active star formation seen in NGC 3044 most likely explains its supergiant shell structures.

Thus, probably both mechanical feedback and other mechanisms form supergiant shell structures. Presumably different processes dominate under different circumstances, and these remain to be understood.

3. Statistical properties of superbubble populations

The statistical properties of the superbubble populations offer another test of the standard evolutionary model for the shells. Oey & Clarke (1997) derived expressions for the differential size distribution $N(R) dR$ of superbubbles in a uniform ISM, using the analytic expressions for adiabatic evolution (equation 1). We considered a power-law mechani-
cal luminosity function for the parent OB associations,

$$\phi(L) dL = A L^{-\beta} dL,$$  \hspace{1cm} (2)

with $\beta \simeq 2$, which is robustly associated with the \textsc{H}\textsc{ii} region luminosity function (e.g., Kennicutt et al. 1989; Oey & Clarke 1998a). The superbubble growth is taken to be pressure-confined when the interior pressure $P_i = P_0$. Star formation is assumed to be coeval within each OB association, with SNe therefore exploding over a period $t_e = 40$ Myr, the lifetime of the lowest-mass SN progenitors. For constant star-formation rate $\psi$ and power-law $\phi(L)$, we found that:

$$N(R) \propto R^{1-2\beta},$$  \hspace{1cm} (3)

effectively yielding $N(R) \propto R^{-3}$ for $\beta = 2$. Oey & Clarke (1997) also derive $N(R)$ for other combinations of $\psi$ and $\phi(L)$.

This result agrees well with the \textsc{H}\textsc{i} shell catalog for the Small Magellanic Cloud (SMC) compiled by Staveley-Smith et al. (1997). This is by far the most complete sample of \textsc{H}\textsc{i} shells obtained for any galaxy, as evidenced by the fact that the relative number counts of \textsc{H}\textsc{ii} regions and \textsc{H}\textsc{i} shells are in excellent agreement with their relative life expectancies. For shells having $R \geq 100$ pc, the fitted power-law slope $\alpha = 1 - 2\beta$ is $2.7 \pm 0.6$, in excellent agreement with the general prediction of $\alpha = 3$.

We note that different models for ISM structure yield different predictions for $N(R)$. For example, Stanimirović et al. (1999) suggest a possible fractal structure for the neutral ISM. From the same \textsc{H}\textsc{i} dataset of the SMC, they find a fractal dimension implying a size distribution for \textsc{H}\textsc{i} holes of $\alpha = 3.5$. It is difficult to empirically differentiate this from our model, having $\alpha = 3$; but it is worth noting that the predictions are intrinsically different.

However, the superbubble size distribution presently is not a sensitive test in determining whether or not the objects evolve adiabatically. If all the internal energy is radiated away, the objects are predicted to follow the momentum-conserving law given by Steigman et al. (1975):

$$R \propto (L/n v_{\infty})^{1/4} t^{1/2}.$$  \hspace{1cm} (4)

The stall radius $R_f$ in this case is only 1.3 times larger than for the adiabatic model, and the size distribution follows the same law $N(R) \propto R^{1-2\beta}$ (Oey & Clarke 1997). The observations of hot gas are therefore vital confirmation that the adiabatic model applies to a significant fraction of superbubbles.

We can also derive the distribution of expansion velocities $N(v) dv$, which describes only the growing objects (Oey & Clarke 1998b):

$$N_{\text{grow}}(v) \propto v^{-7/2}, \hspace{1cm} \beta > 1.5$$  \hspace{1cm} (5)
This again compares well with the SMC H\textsuperscript{i} shell catalog: the fitted power-law slope is 2.9 ± 1.4. Thus, despite the crude assumptions in deriving the shell size and velocity distributions, the data suggest that the neutral ISM in the SMC is fully consistent with superbubble activity dominating the structure. Although most other available H\textsuperscript{i} shell catalogs are highly incomplete, preliminary results for a few other galaxies also show agreement with our model for the size distribution (Kim et al. 1999; Mashchenko et al. 1999; Oey & Clarke 1997).

4. Correspondence with Star-Forming Regions

One of the most obvious global tests of mechanical feedback is to identify the parent stellar populations, or their remains, with the superbubbles. M31 (Brinks & Bajaja 1986) and M33 (Deul & den Hartog 1990) both show correlations of OB associations with H\textsuperscript{i} holes. However, H\textalpha\ II shows contradictory results, based on the H\textsuperscript{i} hole catalog compiled by Puche et al. (1992). Tongue & Westpfahl (1995) found that the SN rate implied by radio continuum emission is consistent with the hole energetics in that galaxy. However, Rhode et al. (1999) carried out a direct, BVR search for remnant stellar populations within the H\textsuperscript{i} holes, and found little evidence for the expected stars. But Stewart et al. (2000) used far-UV data from the Ultraviolet Imaging Telescope and H\alpha imaging to conclude that a significant correlation between the H\textsuperscript{i} holes and recent star formation does indeed support a feedback origin for the holes.

It is perhaps unsurprising that studies of H\textalpha\ II yield these confusing results in view of that galaxy’s distance of 3 Mpc. The LMC, which is 60 times closer, presents much better spatial resolution and should therefore yield less ambiguous results. Kim et al. (1999) examined the correspondence between their H\textsuperscript{i} shell catalog, catalogued H\textsuperscript{ii} regions (Davies et al. 1976), and H\alpha imaging. Not only do they find a correspondence, but they are also able to identify an evolutionary sequence with respect to the relative sizes, expansion velocities, and H\alpha emission. Further investigation of the Magellanic Clouds should confirm and reveal more quantitative details of the mechanical feedback process (Oey, Gerken, & Walterbos, in preparation).

5. ISM porosity and galactic superwinds

The consequences of feedback for the global ISM can be evaluated quantitatively in terms of the interstellar porosity parameter $Q$, which
is the ratio: \((\text{total area or volume occupied by superbubbles}) / (\text{total area or volume of the galaxy})\). Thus it is essentially the filling factor of hot gas, assuming hot gas is contained within all of the superbubbles. Values of \(Q\) near unity indicate the HIM dominates the multiphase ISM, and values \(\gg 1\) imply an outflow, with the galaxy generating more hot gas than it can contain.

It is straightforward to use the analytic expression for \(N(R)\) (equation 3) to derive \(Q\) in terms of a galaxy’s star-formation rate, \(\Psi\) (Oey et al. 2001; see also Clarke & Oey 2002):

\[
Q \simeq 16 \frac{\Psi(M_\odot \, \text{yr}^{-1})}{h R_g^2 / (\text{kpc}^3)} \propto \frac{1}{P_0} ,
\]

for \(\beta = 2\), a Salpeter (1955) IMF for stellar masses \(0.1 \leq m \leq 100 \, M_\odot\), and \(P_0/k = 9500\). \(R_g\) and \(h\) are the radius of the gaseous star-forming disk and gas scale height, respectively. We caution that \(Q\) depends on ambient interstellar parameters, for example, \(P_0^{-1}\) as indicated.

Oey et al. (2001) estimated \(Q\) for all the galaxies in the Local Group. The Milky Way yields \(Q \sim 1\) for some methods and \(Q < 1\) for others, consistent with the ambiguous results found in the past (e.g., McKee & Ostriker 1977; Slavin & Cox 1993). The LMC yields \(Q \sim 1\), implying that hot gas dominates the ISM volume. The remainder of the Local Group galaxies all show \(Q \ll 1\), with the sole exception of IC 10, a starburst galaxy for which \(Q \sim 20\), thereby unambiguously predicting an outflow. Oey et al. (2001) crudely estimate the mass-loss rate in this outflow \(\dot{M}_{\text{out}}\), assuming that the material is largely evaporated from shell walls by thermal conduction. We find that \(\dot{M}_{\text{out}} \sim \Psi\); indeed, absorption-line studies of local starburst galaxies by Heckman et al. (2000) also show that empirically, the outflow and star-formation rates have the same order of magnitude for that sample.

Since \(Q \sim 1\) represents a rough threshold for the escape of super-winds from the galactic disk, this also implies the simultaneous escape of newly-synthesized metals, which are contained in the hot gas. Likewise, the shredding of the ISM into filaments facilitates the escape of ionizing radiation, thus \(Q \sim 1\) also represents an escape threshold for ionizing photons (Clarke & Oey 2002). We finally note the extensive body of numerical work on superbubbles and blowout conditions. Mac Low (1999) and Strickland & Stevens (2000) provide overviews of this field. The details of the numerical predictions are presently difficult to confirm empirically, but observations with XMM-Newton and Chandra are beginning to constrain the dominant processes.
6. Summary

Observations of mechanical feedback ranging from individual stellar wind bubbles to galactic superwinds are all largely consistent with the conventional adiabatic model for shell evolution. Presently none of the discrepancies are of a magnitude that suggest any need for major revision of the conventional understanding. The existence and properties of multiphase gas and filamentary structure are broadly consistent with the adiabatic model. It is also a remarkable strength that the model succeeds across size scales ranging over at least three orders of magnitude.

However, characterizing the dominant parameters and their effects on the shell evolution is still highly problematic. For example, critical ambient ISM conditions like density, pressure, and ionization distributions remain elusive. The mechanisms and conditions for cooling of the interior energy need to be identified, and energy budgets reliably determined. Perhaps the most fundamental question is the fate of the hot gas generated within the superbubbles: Does it escape to constitute the HIM? Does it escape from starburst galaxies, and from their gravitational potentials? These issues have crucial consequences for galactic evolutionary processes, and our understanding depends on further clarifying the mechanical feedback process.

Acknowledgements

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References

Arthur, S. J. & Henney, W. J. 1996, ApJ, 457, 752
Benaglia, P. & Cappa, C. E. 1999, A&A, 346, 979
Brown, A. G. A., Hartmann, D., & Burton, W. B. 1995, A&A, 300, 903
Cappa, C. E. & Benaglia, P. 1998, AJ, 116, 1906
Chu, Y.-H., Chang, H.-W., Su, Y.-L., & Mac Low, M.-M., 1995, ApJ, 450, 157
Chu, Y.-H. & Mac Low, M.-M., 1990, ApJ, 365, 510
Chu, Y.-H., Wakker, B., Mac Low, M.-M., & García-Segura, G. 1994, AJ, 108, 1696
Clarke, C. J. & Oey, M. S. 2002, MNRAS, in press
Cowie, L. L. & McKee, C. F. 1977, ApJ, 211, 135
Drissen, L., Moffat, A. F. J., Walborn, N. R., & Shara, M. R. 1995, AJ, 110, 2235
García-Segura, G. & Franco, J. 1996, ApJ, 469, 171
García-Segura, G. & Mac Low, M.-M. 1995, ApJ, 455, 145
Heckman, T. M. 2002, in Extragalactic Gas at Low Redshift, eds., J. S. Mulchaey & J. Stocke, San Francisco: ASP, 292.
Heckman, T. M., Lehnert, M. D., Strickland, D. K., & Armus, L. 2000, ApJS, 129, 493
Kennicutt, R. C., Edgar, B. K., & Hodge, P. W. 1989, ApJ, 337, 761
Kim, S., Dopita, M. A., Staveley-Smith, L., & Bessell, M. S. 1999, AJ, 118, 2797
Mac Low, M.-M., in New Perspectives on the Interstellar Medium, eds. A. R. Taylor, T. L. Landecker, & G. Joncas, San Francisco: Astron. Soc. Pacific, 303
Mac Low, M.-M., Chang, T. H., Chu, Y.-H., Points, S. D., Smith, R. C., & Wakker, B. P. 1998, ApJ, 493, 260
Mac Low, M.-M. & McCray, R. 1988, ApJ, 324, 776 Smith, R. C. 1996, ApJ, 464, 829
Mashchenko, S. Y., Thilker, D. A., & Braun, R. 1999, A&A, 343, 352
McKee, C. F. & Ostriker, J. P. 1977, ApJ, 218, 148
McKee, C. F., Van Buren, D., & Lazareff, B. 1984, ApJ, 278, L115
Oey, M. S. 1996, ApJ, 467, 666
Oey, M. S. & Clarke, C. J. 1997, MNRAS, 289, 570
Oey, M. S. & Clarke, C. J. 1998a, AJ, 115, 1543
Oey, M. S. & Clarke, C. J. 1998b, in Interstellar Turbulence, eds. J. Franco & A. Carramiñana, Cambridge: Cambridge Univ. Press, 112
Oey, M. S., Clarke, C. J., & Massey, P. 2001, in Dwarf Galaxies and Their Environment, eds. K. S. de Boer, R.-J. Dettmar, & U. Klein, Shaker Verlag, 181.
Oey, M. S., Groves, B., Staveley-Smith, L., & Smith, R. C. 2001, in preparation
Oey, M. S. & Massey, P., 1994, ApJ, 425, 635
Oey, M. S. & Massey, P., 1995, ApJ, 452, 210
Oey, M. S. & Smedley, S. A. 1998, AJ, 116, 1263
Pikel’ner, S. B. 1968, Astrophys. Lett., 2, 97
Saken, J. M., Shull, J. M., Garmany, C. D., Nichols-Bohlin, J., & Fesen, R. A. 1992, ApJ, 397, 537
Salpeter, E. E. 1955, ApJ, 121, 161
Silich, S. A. & Franco, J. 1999, ApJ, 522, 863
Silich, S. A. & Oey, M.S. 2001, in Extragalactic Star Clusters, eds. E. K. Grebel, D. Geisler, & D. Minniti, San Francisco: ASP, in press
Silich, S. A., Tenorio-Tagle, G., Terlevich, R., Terlevich, E., & Netzer, H. 2001, MNRAS, 324, 191
Slavin, J. D. & Cox, D. P. 1993, ApJ, 417, 187
Stanimirović, S., Staveley-Smith, L., Dickey, J. M., Sault, R. J., & Snowden, S. L. 1999, MNRAS, 302, 417
Staveley-Smith, L., Sault, R. J., Hatzidimitriou, D., Kesteven, M. J., & McConnell, D. 1997, MNRAS 289, 225
Steigman, G., Strittmatter, P. A., & Williams, R. E. 1975, ApJ, 198, 575
Strickland, D. K. & Stevens, I. R. 2000, MNRAS, 314, 511
Treffers, R. R. & Chu, Y.-H. 1982, ApJ, 254, 569
Wang, Q. & Helfand, D. J. 1991, ApJ, 373, 497
Weaver, R., McCray, R., Castor, J., Shapiro, P., & Moore, R. 1977, ApJ, 218, 377