The Study of the Polarization Errors of All Fiber Optical Current Transformers

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Abstract. In-line Sagnac fiber optical current transformer is a new kind of all fiber current sensors. By using the Jones Matrices formalism, the polarization errors of the FOT is studied. By using the Jones Matrices formalism, the expression of the main optical parts of the FOT are built, and the the expression of interference is got. The precision which is affected by the parameters of the main optical parts is analyzed.

1. Introduction
All fiber-optical current transducer (FOCT) is the device of measuring the current of the high voltage net based on Faraday effect. Because it transmits signals by means of optical fiber, it has many advantages such as having better insulaion, higher reliability and better Anti-EMI than the traditional transformers. So many countries’ researchers pay high attentions to it. However, the fiber sensing coil has the character of the inherent linear birefringence, which has the unfavourable effect of application. As the technology of the optical fiber develops rapidly, FOCT obtains prodigious development due to the solution of the problem of the optical fiber’s birefringence.

The reflection-type FOCT has the excellent nonreciprocal features[1,2], Anti-interference from outside environment. So it has the strong practical value. It’s the virtual interferometer of the polarization, and requires that the light should keep the state of the polarization during the transmission. And that the unsatisfactory optical components cause the crosstalk of the polarized light and has the bad effect of the measuring accuracy. This paper will analyse the polarized properties of the reflection-type FOCT, and find out the main error sources.

2. The Fundamental Principle of All Fiber Optical Current Transformer (FOCT)
The structure of FOCT is like this as shown in Figure 1[3,4,5],
its fundamental principle is explained as follows: The light source emits the light which then goes through the fiber coupler and Polarizer, and becomes the linearly polarized light, then gets into the polarization-maintenance fiber at an angle of 45°, and then it transfers averagely in the direction of X-axis and Y-axis, the two orthogonal modes of light go through λ/4 optical Wave-plate and change into the right and left circularly polarized light respectively, enter into the sensing optic fiber. The magnetic field generated by the transmission current produces the Faraday effect (also called magneto-optic effect) in the sensing fiber, Two circular polarized light propagates at the different speeds, and interchange the two polarization modes (the right circularly polarized light becomes the left circularly polarized light, the left circularly polarized light becomes the right circularly polarized light) because of the mirror reflection at the end face of the sensing fiber, then goes through the sensing fiber and interacts with the magnetic field again, the phase becomes twice as before. The beams of light restore to the linearly polarized light when they go through the λ/4 optical Wave-plate again, and their interference occur at the fiber polarizer. Finally, two beams of the linearly polarized light carrying the phase information couple into the detector via the optical coupler. Because the two interference beams both go through the X-axis and Y-axis of the polarization-maintenance fiber and the left and right polarized state in the sensing fiber, they only have slight difference of the time and carry the nonreciprocal phase error due to the Faraday effect.

Now let’s establish idealized the Jones matrix of the main optical devices respectively as follows[6,7]: the Jones matrix expression of the polarizer is

$$\hat{L}_p = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

(1)

The Jones matrix expression of the 45° fusion point between the fiber pigtails of the polarizer and the phase modulator is:

$$\hat{L}_{45°} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

(2)

The Jones matrix expression of the phase modulator is:

$$\hat{L}_{PMn} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i(\psi(t-c)+2\pi\Delta t)} \end{pmatrix}$$

(3)
The Jones matrix expression of the $\lambda/4$ Wave-plate is:

$$\hat{L}_{\lambda/4} = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

(4)

The Jones matrix expression of the Faraday effect of the optical sensor head is:

$$\hat{L}_{Flin} = \begin{pmatrix} \cos F & -\sin F \\ \sin F & \cos F \end{pmatrix}$$

(5)

As above the expression: $F = NVI$, $N$ is the number of the sensing fiber turns, $V$ is Verdet constant, $I$ is the conductor current.

The Jones matrix expression of the mirror reflection of the sensing fiber end face is:

$$\hat{L}_{mirror} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(6)

The Jones matrix expression of reflecting through these optical devices is:

$$\hat{L}_{Fout} = \begin{pmatrix} \cos F & -\sin F \\ \sin F & \cos F \end{pmatrix}$$

$$\hat{L}_{PMout} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ e^{i(\psi(t)+2\pi\Delta\tau)} \end{pmatrix}$$

(7)

$$\hat{L}_{45^\circ\text{out}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

(8)

Suppose the input light is natural light, its expression is:

$$\vec{E}_{in} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

(10)

The expression of the output light is:

$$\vec{E} = \hat{L}_p \cdot \hat{L}_{45^\circ\text{out}} \cdot \hat{L}_{PMout} \cdot \hat{L}_{\lambda/4} \cdot \hat{L}_{Fout} \cdot \hat{L}_{mirror} \cdot \hat{L}_{Flin} \cdot \hat{L}_{PMin} \cdot \hat{L}_{April} \cdot \hat{L}_p \cdot \vec{E}_{in}$$

(11)

The output interference light intensity is:

$$I_{out} = \vec{E}^* \cdot \vec{E}$$

(12)

By computing the expression, its simplification is:

$$I_{out} = 0.5 \cdot E^2_x \cdot (1 + \cos(\psi(t-\tau) - \psi(t) + 4F))$$

(13)

In the condition of the square wave modulation, if

$$\psi(t) - \psi(t-\tau) = \pm \frac{\pi}{2} + \phi_f$$

(14)
Then

\[ I_{\text{out}} = 0.5 \cdot E_{i}^{2} \cdot (1 \pm \sin(4F - \phi_{f})) \]  

(15)

\( \phi_{f} \) is feedback phase shift, under the condition of closed loop, it’s obtained as follows:

\[ \sin(4F - \phi_{f}) = 0, \quad \text{then} \quad \phi_{f} = 4F \]

(16)

The (16) expression indicates that the output phase of the fiber optical current transformer is only related to Faraday effect, and thus eliminate the adverse effect of the power fluctuation of light source.

3. Major Polarization Error Sources

The major polarization error sources of the reflective Sagnac interferometer-type fiber optic current transformer include the polarizer error, phase modulator error, \( \lambda/4 \) wave-plate error and the linear birefringence error of the fibersensor head.

3.1. Polarizer Error

For the undesirable polarizer, its Jones matrix expression is:

\[ \hat{L}_{p} = \begin{pmatrix} 1 & 0 \\ 0 & \varepsilon \end{pmatrix} \]

(17)

Here \( \varepsilon^{2} \) is the extinction ratio of the polarizer. It is defined as the ratio of the minimum transmitted light intensity to the maximum transmitted light intensity through the polarizer, which is an important parameter to measure the quality of polarizer. The expression is:

\[ \varepsilon^{2} = \frac{I_{\text{min}}}{I_{\text{max}}} \]

By substituting the equation (17) into the equation (11) given, the result of the output light intensity is:

\[ I_{\text{out}} = 0.5E_{i}^{2}[(1 + \varepsilon^{2})^{2} + (1 - \varepsilon^{2})^{2}] \cdot \cos(\psi(t - \tau) - \psi(t) + 4F) \]

(18)

It can be seen from formula (18) that the extinction ratio of the polarizer has a certain effect on measuring accuracy of the FOCT. Especially, the unstable extinction ratio can reduce the measurement sensitivity of the FOCT.

3.2. Phase Modulator Error

The function of phase modulator is to produce the bias modulation phase and improve the signal detection sensitivity. The phases of two beams transmitted through the ideal phase modulator would be changed separately, there by achieve the function of the phase modulation. However, two beams of light along two optical axes would intercouple. This kind of parasitic effect can cause the variation of the interference intensity detected. Because the frequency of light intensity varies with the frequency of modulated square wave, the detection circuit cannot distinguish which one is the true output signal. This can cause the measurement error.

3.3. \( \lambda/4 \) Wave-Plate Error

In the reflective-style fiber optical current transform, nonideal \( \lambda/4 \)wave-plateis welded with the polarization maintaining optical fiber at an angle of \( \theta \), its corresponding Jones matrix is:
\[
L_{\lambda/4} = \cos \frac{\phi}{2} \left( \frac{1 + itg \phi}{2} \cos 2\theta \quad itg \frac{\phi}{2} \sin 2\theta \right) \\
\left( itg \frac{\phi}{2} \sin 2\theta \quad 1 - itg \frac{\phi}{2} \cos 2\theta \right)
\]

(19)

Here \( \phi \) is phase delay, the ideal value is 90°, \( \theta \) is axis angle, the ideal value is 45°.

Supposing the equation \( h = 1 - \sin \phi \sin 2\theta \), if the \( \lambda/4 \) wave-plate is ideal and the axis angle is 45°, then there is \( h = 0 \). By substituting it into the equation (11), the corresponding output value of interferometer is:

\[
I_{out} = 0.5 \cdot E_c^2 \cdot (1 + h \cos(\psi(t - \tau) - \psi(t))) \\
+ \cos(\psi(t - \tau) - \psi(t) + 4F))
\]

(20)

By substituting it into the equation (14) and simplify the equation, the result is:

\[
I_{out} = K[1 \pm h \sin \Phi_f \pm \\
(1 - h) \sin(4F - \Phi_f)]
\]

(21)

When the FOCT system satisfies the closed loop condition, the AC component of the interference signal is zero, as it is:

\[
\pm h \sin \Phi_f \pm (1 - h) \sin(4F - \Phi_f) = 0
\]

(22)

Let’s consider only one case of the equation (8), and think like this: \( \sin \Phi_f \approx \Phi_f \), \( \sin(4F - \Phi_f) \approx 4F - \Phi_f \), therefore

\[
\Phi_f = (1 - h)4F
\]

(23)

It’s can be seen from the equation (23) that the axis angle and phase delay of the \( \lambda/4 \) wave-plate have a direct impact on the scale factor of FOCT.

3.4. Birefringence Error of the Sensing Head

The fiber sensor head has linear and circular birefringence’s, the transmission matrix could be established as this[8,9]:

\[
\hat{L}_{Fin} = \begin{pmatrix} A & -B \\ B & A^* \end{pmatrix}
\]

(24)

\[ A = \cos \frac{\alpha}{2} + i \sin \frac{\alpha}{2} \cos \chi \]

(25)

\[ B = \sin \frac{\alpha}{2} \sin \chi \]

(26)

\[ \alpha = 2 \sqrt{(F + T)^2 + \left(\frac{\delta}{2}\right)^2} \]

(27)

\[ \tan \chi = 2(F + T) / \delta \]

(28)
There into, parameter $T$ is the circular birefringence, parameter $\delta$ is the linear birefringence.

\[
\hat{L}_{\text{out}} = \begin{pmatrix} C & -D \\ D & C^* \end{pmatrix}
\]

(29)

\[C = \cos \frac{\beta}{2} + i \sin \frac{\beta}{2} \cos \xi \]

(30)

\[D = \sin \frac{\beta}{2} \sin \xi \]

(31)

\[\beta = 2 \sqrt{(F - T)^2 + \left(\frac{\delta}{2}\right)^2} \]

(32)

\[\tan \xi = 2(F - T) / \delta \]

(33)

By substituting it into the equation (11), and regarding $F$ as $F \ll \frac{\delta}{2} \ll T$, the output equation can be derivated as follows:

\[I_{\text{out}} = 0.5 \cdot E_x^2 \cdot (1 + \cos(\psi(t) - \psi(t - \tau))) \cdot (\cos \alpha \cos \beta - \sin \alpha \sin \beta \sin \chi \sin \xi) + \sin(\psi(t) - \psi(t - \tau)) \cdot (\sin \alpha \cos \beta \sin \chi + \cos \alpha \sin \beta \sin \xi) \]

(34)

Substitute it into equation (14), then

\[I_{\text{out}} = 0.5 \cdot E_x^2 \cdot (1 - \sin(\phi_f)) \cdot (\cos \alpha \cos \beta - \sin \alpha \sin \beta \sin \chi \sin \xi) + \cos(\phi_f) \cdot (\sin \alpha \cos \beta \sin \chi + \cos \alpha \sin \beta \sin \xi) \]

(35)

It can be obtained from closed-loop condition:

\[\phi_f = \arctan\left(\frac{\sin \alpha \cos \beta \sin \chi + \cos \alpha \sin \beta \sin \xi}{\cos \alpha \cos \beta - \sin \alpha \sin \beta \sin \chi \sin \xi}\right) \]

(36)

Thus, the error of the scale factor is:

\[K = \frac{[\arctan\left(\frac{\sin \alpha \cos \beta \sin \chi + \cos \alpha \sin \beta \sin \xi}{\cos \alpha \cos \beta - \sin \alpha \sin \beta \sin \chi \sin \xi}\right) - 4F]}{4F} \]

(37)

Calculate the relationship between the error of the scale factor and the birefringence (including circular birefringence and linear birefringence) as shown in the figure 2.
Figure 2. relationship between the error of the scale factor and the birefringence

Supposed that the linear birefringence of the sensor head is 16°, draw a relationship curve between the scale factor and the circular birefringence as shown in the figure 3.

Figure 3. relationship between the scale factor and the circular birefringence

I can be seen from figure (3), while the parameter $T$ of the fiber sensor head is big enough, on the basis of the approximation $\sin \chi \approx 0$, $\sin \xi \approx 0$, the equation (36) can be simplified as $\phi_f = 4F$ (38).

Formula (38) is consistent with the interference expression in ideal state, that is, a mass of the circular birefringence in the fibre sensor head can effectively suppress the influence of linear birefringence on the measurement error of FOCT. This can be achieved by twisting single-mode fibers.

4. Conclusion

The reflective-style FOCT essentially is a kind of polarization interferometer, there are strict requirements for the polarization of light in the optical path. The optical devices including polarizer, phase modulator, $\lambda/4$ wave-plate and the birefringence of the fibre sensor head impact on the output of FOCT. We can take steps to eliminate error source through the analysis of error mechanism, and further improve the measurement accuracy.
5. Reference

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