SPONTANEOUS VIOLATION OF LORENTZ AND CPT SYMMETRY

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1. INTRODUCTION

The standard model as well as many of its modern day extensions preserves Lorentz and CPT symmetry. In fact, symmetry under the Lorentz group is a basic assumption in virtually any fundamental theory used to describe elementary particle physics. Under very mild assumptions, the postulates of a point particle theory that preserves Lorentz invariance imply that CPT is also preserved [1].

In this proceedings, I will discuss the construction of quantum field theories that break Lorentz and CPT symmetry. There are both experimental and theoretical motivations to develop such theories.

Many sensitive experimental tests of Lorentz and CPT symmetry have been performed. For example, high precision tests involving atomic systems [2, 3], clock comparisons [4, 5], and neutral meson oscillations [6, 7] provide stringent tests of Lorentz and CPT symmetry. Recently, a pendulum with a net macroscopic spin angular momentum has been constructed and used to investigate spin-dependent Lorentz and CPT violation [8]. In the past, such experiments have placed bounds on phenomenological parameters that lack any clear connection with the microscopic physics of the standard model. One motivation of constructing a theory in the context of the standard model that allows for Lorentz and CPT violation is the desire to have a single theory within the context of conventional quantum field theory that could relate various experiments and be used to motivate future investigations.

This begs the question as to how such effects might arise naturally within the current framework of quantum field theory. The main idea is that miniscule low-energy remnant effects that violate fundamental symmetries may arise in theories underlying the standard model. One example is string theory in which nontrivial structure of the vacuum solutions may induce observable Lorentz and CPT violations [9, 10, 11].

Rather than attempting a construction based directly on a specific underlying model, such as string theory, we proceed using the generic mechanism of spontaneous symmetry violation to implement the breaking. Terms involving standard model fields that violate Lorentz and CPT symmetry are assumed to arise from a

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general spontaneous symmetry breaking mechanism in which vacuum expectation values for tensor fields are generated in the underlying theory \[12\]. The approach then is to construct all possible terms that can arise through spontaneous symmetry breaking that are consistent with the gauge invariance of the standard model and power-counting renormalizability. These conditions are imposed on the model to limit the deviation from the conventional standard model by preserving gauge symmetries and renormalizability. It is somewhat analogous to imposing R-parity in supersymmetry to eliminate pesky lepton number violating interactions.

The resulting terms lead to modified field equations that can be analyzed within the context of conventional quantum field theory. In this proceedings, I will develop the modified Feynman rules for a model theory and will explore some possible consequences for quantum electrodynamics. Other topics in Lorentz and CPT violation including a detailed analysis of causality and stability issues \[13\] and an investigation of effects on neutrino oscillations \[14\] are also being discussed at this meeting.

2. SPONTANEOUS BREAKING OF LORENTZ SYMMETRY

In this section, a general spontaneous symmetry breaking mechanism is applied to the fermion sector to generate an example of the types of interactions that arise. The conventional mechanism of this type occurs in theories that contain scalar field potentials with nontrivial minima, such as the conventional Higgs mechanism of the standard model in which Yukawa couplings generate the fermion masses after spontaneous symmetry breaking of the scalar Higgs field. In conventional theories of this sort, internal symmetries of the original Lagrangian such as gauge invariance may be violated, but Lorentz symmetry is always maintained.

The key element in preserving Lorentz invariance when symmetry is broken spontaneously is the fact that a Lorentz scalar obtains an expectation value. Spontaneous Lorentz breaking may occur in a fundamental theory containing a potential for a tensor field that has nontrivial minima. For example, consider a lagrangian describing a fermion $\psi$ and a tensor $T$ of the form

$$L = L_0 - L'$$

where

$$L' \supset \frac{\lambda}{M^k} T \cdot \bar{\psi} \Gamma (i\partial)^k \psi + \text{h.c.} + V(T)$$

In this expression, $\lambda$ is a dimensionless coupling constant, $M$ is some heavy mass scale of the underlying theory, $\Gamma$ denotes a general gamma matrix element in the Dirac algebra, and $V(T)$ is a potential for the tensor field (indices are suppressed for notational simplicity). The potential $V(T)$ is assumed to arise in a more fundamental theory underlying the standard model. Note that terms contributing to $V(T)$ are precluded from conventional renormalizable four-dimensional field theories, making this type of violation impossible. However, these terms are naturally generated in the low-energy limit of more general theories such as string theory \[9, 10\].

If the function $V(T)$ has nontrivial minima, a nonzero expectation value of $T$ will be generated in the vacuum. The lagrangian will then contain a term of the form

$$L' \supset \frac{\lambda}{M^k} \langle T \rangle \cdot \bar{\psi} \Gamma (i\partial)^k \psi + \text{h.c.}$$
that is bilinear in the fermion fields and can violate Lorentz invariance and various
discrete symmetries C, P, T, CP, and CPT.

3. RELATIVISTIC QUANTUM MECHANICS

To illustrate the techniques for treating terms of the form generated in Eq. (3),
we examine a subset of all the possible terms. An example applicable to the standard
model fermions is furnished by the choice \( k = 0 \) (no derivatives) and \( \Gamma \sim \gamma^\mu \) or
\( \Gamma \sim \gamma^5 \gamma^\mu \), the most general nonderivative terms that violate CPT symmetry.
With these restrictions, the model lagrangian for a single fermion \( \psi \) becomes

\[
L = \frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi - a_\mu \bar{\psi} \gamma^\mu \psi - b_\mu \bar{\psi} \gamma^5 \gamma^\mu \psi - m \bar{\psi} \psi ,
\]

(4)

where the parameters \( a_\mu \) and \( b_\mu \) are real constant coefficients that denote the tensor
expectation values and coupling constants that are present in (3).

Several features of this theory can be immediately deduced from the structure
of the lagrangian. First, the lagrangian is hermitian and therefore preserves proba-
bility. This means that conventional quantum mechanics can be used to evolve the
particle states in time. The model lagrangian is invariant under translations and
U(1) gauge transformations which leads to conservation of energy, momentum, and
charge. The resulting Dirac equation

\[
(i \gamma^\mu \partial_\mu - a_\mu \gamma^\mu - b_\mu \gamma^5 \gamma^\mu - m) \psi = 0 .
\]

(5)

obtained by minimizing the variation of the action with respect to the fermion field
is linear in \( \psi \). Equation (3) can be solved exactly using the plane-wave solutions

\[
\psi(x) = e^{\pm ip_\mu x^\mu} w(p) ,
\]

(6)

where \( p^0(p) \equiv E(p) \) is the energy (defined as the magnitude of the eigenvalue of
the hamiltonian acting on the state) determined by setting the determinant of the
matrix acting on \( w(p) \) equal to zero.

The general form of the resulting dispersion relation is complicated, so we
content ourselves here by investigating the special case \( \vec{b} = 0 \). The exact solutions
for the energies are found to be

\[
E_+(\vec{p}) = \left[ m^2 + (|\vec{p} - \vec{a}| \pm b_0)^2 \right]^{1/2} + a_0 ,
\]

(7)

\[
E_-(\vec{p}) = \left[ m^2 + (|\vec{p} + \vec{a}| \mp b_0)^2 \right]^{1/2} - a_0 .
\]

(8)

Some interesting consequences of the breaking is apparent. Note that the conventional
energy degeneracy of the fermion and antifermion states is broken by \( a_\mu \) while
\( b_0 \) splits the degeneracy of the helicities. These energy splittings are indicative of
the effect of the general Lorentz violating terms in the standard model extension.
The corresponding spinor solutions form an orthogonal basis of states as a result of
the hermiticity of the hamiltonian.

An interesting feature of the above dispersion relations is the modified rela-
tionship that exists between the velocity of a wave packet and its corresponding
momentum. For instance, a wave packet formed from a superposition of positive hel-
city fermions with a four-momentum \( p^\mu = (E, \vec{p}) \) has a corresponding expectation
value for the velocity operator \( \vec{v} = i[H, \vec{x}] = \gamma^0 \vec{\gamma} \) of

\[
\langle \vec{v} \rangle = \left( \frac{|\vec{p} - \vec{a}| - b_0}{(E - a^0)} \right) \frac{\vec{p} - \vec{a}}{|\vec{p} - \vec{a}|} .
\]

(9)
Note that the velocity of the packet and the conserved momentum are not in the same direction. Examination of the velocity using a general nonzero $b_\mu$ reveals that $|v_j| < 1$, and that the limiting velocity as $\vec{p} \to \infty$ is 1. This implies that effects due to the CPT violating terms are mild enough to preserve causality.\footnote{Note that while causality is preserved, there can be problems with stability at energies nearing the heavy mass scale of the underlying theory \[15\]. Also see Kostelecký, these proceedings.} This will be verified independently from the perspective of quantum field theory that will now be discussed.

4. CONSTRUCTION OF THE FREE FIELD THEORY

The approach to quantization taken here is similar to the conventional one in which the quantization conditions on the fields are deduced from the requirement of positivity of the energy. The wave function $\psi$ is expanded in terms of its four solutions as

$$
\psi(x) = \int \frac{d^3p}{(2\pi)^3} \sum_{\alpha=1}^{2} \left[ \frac{m}{E_\alpha^{(\alpha)}} b^{(\alpha)}(\vec{p}) e^{-ip^{(\alpha)\cdot x} u^{(\alpha)}(\vec{p})} + \frac{m}{E_\alpha^{(\alpha)}} d^{(\alpha)}(\vec{p}) e^{ip^{(\alpha)\cdot x} v^{(\alpha)}(\vec{p})} \right],
$$

and is promoted to an operator acting on a Hilbert space of basis states.

Translational invariance is used to define the conserved energy and momentum as

$$
P_\mu = \int d^3x \Theta_0^\mu = \int d^3x \frac{i}{2} [\bar{\psi}^{\upsilon_0} \partial_\mu \psi].
$$

The time component $P_0$ is interpreted as the energy (after normal ordering of the operators) and is positive definite (for $|a^0| < m$) provided the following anticommutation relations are imposed:

$$
\{b^{(\alpha)}(\vec{p}), b^{(\alpha')}_{\beta}(\vec{p}'')\} = (2\pi)^3 \frac{E_\alpha^{(\alpha)}}{m} \delta_{\alpha\alpha'} \delta^3(\vec{p} - \vec{p}'') ,
$$

$$
\{d^{(\alpha)}(\vec{p}), d^{(\alpha')}_{\beta}(\vec{p}'')\} = (2\pi)^3 \frac{E_\alpha^{(\alpha)}}{m} \delta_{\alpha\alpha'} \delta^3(\vec{p} - \vec{p}'') .
$$

The resulting equal-time anticommutators of the fields are

$$
\{\psi_\alpha(t, \vec{x}), \psi^\dagger_{\beta}(t, \vec{x}')\} = \delta_{\alpha\beta} \delta^3(\vec{x} - \vec{x}') ,
$$

$$
\{\psi_\alpha(t, \vec{x}), \psi_{\beta}(t, \vec{x}')\} = 0 ,
$$

$$
\{\psi^\dagger_{\alpha}(t, \vec{x}), \psi^\dagger_{\beta}(t, \vec{x}')\} = 0 .
$$

These relations show that conventional Fermi statistics remain unaltered by the CPT violation.

The conserved charge $Q$ and conserved momentum $P_\mu$ are now computed explicitly as

$$
Q = \int \frac{d^3p}{(2\pi)^3} \sum_{\alpha=1}^{2} \left[ \frac{m}{E_\alpha^{(\alpha)}} b^{(\alpha)}(\vec{p}) b^{(\alpha)}(\vec{p}) - \frac{m}{E_\alpha^{(\alpha)}} d^{(\alpha)}(\vec{p}) d^{(\alpha)}(\vec{p}) \right] ,
$$

\[14\]
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\[ P_{\mu} = \int \frac{d^3p}{(2\pi)^3} \sum_{\alpha=1}^2 \left[ \frac{m}{E_\mu} p_{\mu}^{(\alpha)} b_{(\alpha)} (\vec{p}) b_{(\alpha)} (\vec{p}) + \frac{m}{E_{(\alpha)} \gamma_5} d_{(\alpha)} (\vec{p}) d_{(\alpha)} (\vec{p}) \right] . \] (15)

From these expressions, it is observed that the charge of the fermion is unperturbed and the energy and momentum satisfy the same energy-momentum relations that were found using relativistic quantum mechanics.

To preserve causality, it is necessary that the anticommutation relations of the fermion fields at unequal times are zero for spacelike separations. Explicit integration for the special case of \( \vec{b} = 0 \) reveals that

\[ \{ \psi_\alpha (x), \overline{\psi}_\beta (x') \} = 0 , \] (16)

for spacelike separations \( (x - x')^2 < 0 \). This result indicates that physical observables separated by spacelike intervals will in fact commute. This agrees with our previous results regarding the velocity obtained using the relativistic quantum mechanics approach. An analysis of causality and stability issues for other Lorentz- and CPT-violating terms in the fermion sector of the standard model extension has recently been performed [15]. Some similar issues pertaining to causality in the photon sector have subsequently been addressed [16].

Next, the issue of extending this free field theory to interacting theory is addressed. Much of the conventional formalism developed for perturbative calculations in conventional interacting field theory carries over essentially unchanged to the present case. The asymptotic in and out states are defined as in the usual case using the free field solutions. The LSZ reduction procedure is then used to express the transition-matrix elements in terms of Green’s functions for the theory. Dyson’s formalism is then used to express the time-ordered products of the interacting fields in terms of the asymptotic fields. Wick’s theorem remains unaffected by the modifications.

A central result is that the usual Feynman rules apply provided that the Feynman propagator is modified to

\[ S_F (p) = \frac{i}{p_\mu \gamma^\mu - m^2 + i\epsilon} , \] (17)

and the exact spinor solutions of the modified free fermion theory are used on external legs. The main reason that conventional techniques apply seems to be due to the fact that the Lorentz violating modifications are linear in the fermion fields.

5. QED EXTENSION AND THE PHOTON

In this section, some implications of Lorentz breaking for photon propagation are investigated. The conventional QED lagrangian is

\[ \mathcal{L}_{\text{electron}}^{\text{QED}} = \frac{i}{2} \overline{\psi} \gamma_\mu \gamma_5 D_\mu \psi - m_e \overline{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \] (18)

where \( \psi \) is the electron field, \( m_e \) is its mass, and \( F_{\mu\nu} \) is the photon field strength tensor. When all possible Lorentz-violating contributions from spontaneous symmetry breaking consistent with gauge invariance and power-counting renormalizability
are introduced into the standard model, the resulting modifications to the photon sector are \[12\]
\[
\mathcal{L}_{\text{photon}}^{\text{CPT-even}} = -\frac{1}{4}(k_F)_{\kappa \lambda \mu \nu} F^{\kappa \lambda} F_{\mu \nu} , \tag{19}
\]
and
\[
\mathcal{L}_{\text{photon}}^{\text{CPT-odd}} = \pm \frac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa \lambda \mu \nu} A^{\lambda} F_{\mu \nu} . \tag{20}
\]
The parameters \( k_F \) and \( k_{AF} \) are fixed background fields related to vacuum expectation values of tensors coupled to the photon in the underlying theory. These two couplings are even and odd under CPT respectively. The CPT-odd terms have been treated in detail elsewhere \[12, 17\]. Here the special case of \((k_{AF})^\kappa = 0\) (no CPT-odd piece), and \((k_F)_{\lambda j k} = -\frac{1}{2} \beta_j \beta_k\) is examined for the sake of a specific example.

The resulting modifications to the Maxwell equations are linear just as the modified Dirac equation is in the fermion case. Plane waves can therefore be used to solve the modified equations of motion. A solution exists provided \( p_\mu \) satisfies
\[
(p_o)^2 = 0 , \tag{21}
\]
\[
(p_e)^2 = -\left(\bar{\beta} \times \bar{p}_e\right)^2 \frac{1}{1 + \bar{\beta}^2} , \tag{22}
\]
where \( p_o \) denotes an ordinary mode and \( p_e \) denotes an extraordinary mode of photon propagation. The ordinary mode propagates as a conventional photon, while the extraordinary mode has a modified dispersion relation.

For the special direction of propagation for which \( \bar{\beta} \cdot \bar{p} = 0 \), the ordinary mode is polarized with \( \vec{A}_o \) along the direction of \( \bar{p} \times \bar{\beta} \) while the extraordinary mode \( \vec{A}_e \) is polarized along \( \bar{\beta} \). Both polarizations are perpendicular to the momentum vector of the wave \( \bar{p} \). The group velocities of wave packets \( \vec{v}_g \equiv \nabla_{\vec{p}} p^0 \) are calculated as
\[
\vec{v}_{g,o} = \hat{\bar{p}} , \quad \vec{v}_{g,e} = \frac{1}{\sqrt{1 + \bar{\beta}^2}} \hat{\bar{p}} . \tag{23}
\]
The extraordinary mode is seen to travel with a modified velocity that is slightly less than the velocity of the ordinary mode. As a result, an initially plane polarized wave will in general become elliptically polarized after traveling a distance
\[
r \simeq \frac{\pi}{2 \left(\sqrt{1 + \bar{\beta}^2} - 1\right)} p^0 \simeq \frac{\pi}{\bar{\beta}^2 p^0} , \tag{24}
\]
where the approximation holds for \( \bar{\beta}^2 \ll k_F \ll 1 \). The magnetic field behaves analogously. Terms of this form can also have implications for photon birefringence. In particular they can contribute to polarization rotation from distant quasars \[18\].
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