Frequency dependent conductivity in the integer quantum Hall effect

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Abstract. Frequency dependent electronic transport is investigated for a two-dimensional disordered system in the presence of a strong perpendicular static magnetic field. The ac-conductivity is calculated numerically from Kubo’s linear response theory using a recursive Green’s function technique. In the tail of the lowest Landau band, we find a linear frequency dependence for the imaginary part of $\sigma_{xx}(\omega)$ which agrees well with earlier analytical calculations. On the other hand, the frequency dependence of the real part can not be expressed by a simple power law. The broadening of the $\sigma_{xx}$-peak with frequency in the lowest Landau band is found to exhibit a scaling relation from which the critical exponent can be extracted.

Keywords: frequency dependent conductivity, ac-QHE, frequency scaling

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1 Introduction

The understanding of the integer quantum Hall effect is intimately related to localization and quantum critical phenomena. In a single-particle description all electronic states are localized, except those at the critical points near the center of the Landau bands. As a consequence, at temperature $T = 0$ the static longitudinal conductivity $\sigma_{xx}$ vanishes. For finite system size or finite temperature small conductivity peaks appear at the transition points where the quantized Hall conductivity changes by $e^2/h$. A further way for obtaining a non-zero $\sigma_{xx}$ is to apply a time dependent electric field and to measure the frequency dependent conductivity. The broadening of the resultant $\sigma_{xx}(\omega)$-peaks with frequency $\omega$ has been detected to exhibit a scaling relation $\sim \omega^\kappa$ from which the product $z_\omega \mu = \kappa^{-1}$ of the dynamical scaling exponent $z_\omega$ and the critical exponent $\mu$ governing the divergence of the localization length can be obtained. However, the frequency-scaling has recently been questioned experimentally and theoretically.

2 Model and Method

We consider non-interacting electrons on a square lattice in the presence of disorder and a strong perpendicular magnetic field $B$ in a one-band tight-binding approximation. The disorder potentials $w$ are drawn at random from an interval $W/2 \leq w \leq W/2$.
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Fig. 1 The imaginary part of the longitudinal conductivity \( \sigma_{xx}(\omega, E) \) in units \( e^2/h \) versus frequency \( \omega \). For small filling factors \( \nu \) a linear frequency dependence is observed at low \( \omega \).

with constant probability distribution \( P(w) = 1/W \). The Hamilton matrix is

\[
(H\psi)(x, y) = w(x, y) \psi(x, y) + V [\psi(x + a, y) + \psi(x - a, y) + \\
\exp(-i2\pi\alpha_B x/a) \psi(x, y + a) + \exp(i2\pi\alpha_B x/a) \psi(x, y - a)],
\]

where the lattice constant \( a = 1 \) is taken as the unit of length and the transfer term \( V = 1 \) as the unit of energy. The magnetic field \( B \) is chosen to be commensurate with the lattice and \( \alpha_B = a^2 eB/h \) denotes the number of flux quanta per plaquette.

The Hamiltonian \( H^{(N+1)} \) of a lattice system of length \( N + 1 \) can be split into the Hamiltonian \( H^{(N)}_{m,n} \) of the lattice consisting of \( N \) slices, the part \( H_{N+1,N+1} \) of the newly added slice, and the coupling \( V_{N,N+1} + V_{N+1,N}^\dagger \). On this account it is possible to numerically calculate the frequency dependent conductivity using a recursive Green’s function technique developed previously [5, 6, 7].

\[
\sigma_{xx}(\omega, E_F) = \lim_{\varepsilon \to 0^+} \lim_{M \to \infty} \frac{e^2}{\hbar \Omega 1} \frac{1}{\hbar \omega} \int_{E_F - \hbar \omega}^{E_F} \text{Tr}\{(hz)^2 xG^+ xG^- + \} dE,
\]

where \( z = \omega + 2i\varepsilon/\hbar \). \( \Omega \) is the system area \( M \times N \), \( G^+ = G(E + \hbar \omega + i\varepsilon) \), and \( G^- = G(E - i\varepsilon) \). The small imaginary part of the energy is necessary for properly taking the thermodynamic limit. It has to be set to zero, \( \varepsilon \to 0^+ \), after increasing the system width \( M \) to infinity, or at least after out-ranging an effective system width introduced by the frequency, \( L_\omega = (1/\rho \hbar \omega)^{1/2} \) (\( \rho \) = density of states), which is relevant near the critical point. The Green’s functions of a system of length \( N + 1 \) can also be calculated recursively via Dyson’s equation with \( V_{N,N+1} \) as the interaction.

3 Results and discussion

The imaginary and real parts of the frequency dependent longitudinal conductivity \( \sigma_{xx}(\omega, E) \) are shown in Figs. 1 and 2 respectively. The Fermi-energy \( E \) is situated in
Fig. 2 The real part of the longitudinal conductivity \( \sigma_{xx}(\omega, E) \) in units \( e^2/h \) as a function of frequency \( \omega \). The location of the Fermi-energies \( E \) in the tail of the lowest Landau band correspond to filling factors \( \nu = 1.2 \cdot 10^{-2} (\circlearrowright), \nu = 1.0 \cdot 10^{-2} (+), \nu = 8.5 \cdot 10^{-3} (\bigcirc), \nu = 7.1 \cdot 10^{-3} (\times), \nu = 6.1 \cdot 10^{-3} (\triangle) \). The broken line represents a frequency dependence \( \sim \omega^2 \).

The lowest tail of the disorder broadened Landau band corresponding to small filling factors \( \nu \). The system parameters are taken to be as follows: width \( M = 32 \), length \( N = 1.3 \cdot 10^5 \), the imaginary part of the energy \( \varepsilon = 4 \cdot 10^{-4} \), disorder strength \( W = 0.1 \), and magnetic field \( \alpha_B = 1/8 \). The corresponding cyclotron energy is \( h\omega_c \approx 0.78 \) in our units. For \( \omega \) smaller than a crossover frequency \( \tilde{\omega}(E) \), a linear frequency dependence can be observed. Such a behavior has been suggested previously for the low frequency limit from analytical calculations based on a one-instanton approximation [8].

The real part of the frequency dependent longitudinal conductivity is in accord with the analytical predictions, this is not the case for the real part. According to the calculations of Refs. [8, 9], for \( E \) in the lowest localization regime, the real part of \( \sigma_{xx}(\omega, E) \) should obey a quadratic frequency dependence for small frequencies,

\[
\mathcal{R} \sigma_{xx}(\omega, E) = c\omega^2 \ln(1/\omega^2),
\]

where \( c \) is a constant. Our results shown in Fig. 2 are not compatible with this suggestion. For \( \omega \lesssim \tilde{\omega}(E) \), we observe an almost frequency independent conductivity, which for larger \( \omega \) is followed by a pronounced increase. The steepness of this increase strongly depends on the position of the Fermi-energy and thus can not be fitted by a single power law. Both the exponent and \( \tilde{\omega}(E) \), the starting point of the frequency dependence, increase with decreasing filling factor. We have checked that the frequency dependence shown in Figs. 1 and 2 is neither influenced by the choice of \( M \) nor \( L \). We believe that the absence of a single power-law behavior in \( \mathcal{R} \sigma_{xx}(\omega, E) \) is due to the

\[
\Im \sigma_{xx}(\omega, E) = -2e^2l_B\omega\rho(E),
\]
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\[ \sigma_{xx}(\omega, E) = \frac{e^2}{h} \]

Fig. 3 Frequency broadening of the conductivity peak \( \sigma_{xx}(\omega, E) \) in units of \( e^2/h \) versus Fermi-energy \( E \) for \( M = 32, L = 10^5, W = 1, \alpha = 1/8, \) and \( \varepsilon = 0.0004. \)

circumstance that the applied frequency is not small in comparison with the Landau band width in our calculations. A larger disorder \( W \) would improve the situation.

Turning now to the behavior of \( \Re \sigma_{xx}(\omega, E) \) near the Landau band center where frequency scaling has been observed in experiments [1]. In Fig. 3 the broadening of the conductivity peak with frequency is shown as a function of energy. With increasing \( \omega \) the halfwidth \( \Delta E \) increases while the peak height is reduced. We find a power law relation \( \Delta E \sim \omega^\kappa \) with \( \kappa = 0.2 \pm 0.01 \) which is close to \( (z_\omega \mu)^{-1} = 0.21 \) expected from \( z_\omega = 2 \) (non-interacting electrons) and \( \mu = 2.35 \) [1]. Thus, in contrast to Refs. [3, 4] our results describe a frequency scaling similar to what has been observed experimentally in Ref. [1]. While the origin of \( z_\omega \approx 1.19 \) reported for non-interacting electrons in Ref. [3] is completely unclear, the absence of scaling at low frequencies in Ref. [4] is attributed to the finite bandwidth of delocalized states in their model. Whether this can serve as an explanation for the experiment [2] remains to be seen.

In conclusion, we have presented results of numerical calculations for the frequency dependent longitudinal conductivity within the lowest Landau band. The broadening of the \( \sigma_{xx} \)-peak exhibits a power-law behavior in agreement with the notion of frequency scaling. For small \( \omega \), the imaginary part shows a \( \sigma_{xx}(\omega) \sim \omega \) relation in the lowest Landau band tail in accord with suggestions from analytical considerations.

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