Thermal bistability in local microwave heating of a superconductor

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We study a strongly disordered superconducting layer heated by near-field microwave radiation from a nanometric metallic tip. The microwaves heat up the quasiparticles, which cool by phonon emission, and conduction away from the heated area. We predict a bistability with two stable states of the electron temperature under the tip. The heating can be tuned to induce a sub-micron sized normal region bounded by a domain wall between high and low temperature states. We propose this as a local probe to access different physics from existing methods, for example, to map out inhomogeneous superfluid flow in the layer. The bistability-induced domain wall will significantly improve its spatial resolution.

I. INTRODUCTION

Local probes constitute a powerful toolkit for experimental solid state physics. Non-invasive local probes of electronic properties range from the well-known Scanning Tunneling Microscopy (STM) [1, 2] to Microwave Impedance Microscopy (MIM) [3, 4]. Non-invasive local probes of thermal properties have now emerged, such as scanning thermal microscopy [5], providing a new window into dissipative processes in quantum transport [6]. Intrusive local probes include scanning gate microscopy (SGM), which probes the spatial structure of inhomogeneous 2D electron gases by measuring changes in global transport properties induced by a local electrostatic perturbation (a charged tip) [7]. Unfortunately, it cannot probe systems with high electron density (metals or superconductors) because they screen out its electrostatic potential very efficiently. Our idea is to circumvent this difficulty by applying a local thermal perturbation.

Here we propose a novel local probe: the main idea is to create a local hot spot by applying a microwave drive to a small metallic tip placed near the sample, and to subsequently measure the global transport properties of the sample. Such a probe would be complementary to the SGM which produces an electrostatic perturbation, and to the MIM which does not modify the sample’s properties. The local heating probe might prove especially suitable for probing thin films of strongly disordered superconductors, where local heating can create a small normal region. Indeed, while single-electron STM probes the local superconducting gap [8] and Andreev state microscopy probes the global superconducting phase coherence [9], a local suppression of superconductivity allows one to map out where the supercurrent is flowing in the sample. One can do this by observing if suppressing the superconductivity at a given point affects the global super-current or not. This is especially important in view of the potential application of strongly disordered superconductors as superinductance, a key element of superconducting circuit-based quantum technology [10, 11].

The probe’s spatial resolution is improved by a fundamental physics effect that we find: a local overheating bistability for subgap microwave frequencies. It occurs because only quasiparticles get heated by the microwaves, but not the condensate. Hence, while quasiparticles are rare, the superconductor remains cold, and they remain rare. However, once the number of quasiparticles exceeds a threshold, the gap shrinks and the microwave can break the Cooper pairs, generating more quasiparticles, allowing more heating. This heating is opposed by heat dissipation to phonons, and heat conduction away from the hot spot. This leads to two different (hot and cold) stable steady states in the hot spot, similar to a global bistability previously known for spatially uniform microwave fields [12–14]. However, we show a spatially local bistability can exist so long as the thermal conductivity is not too big. The bistability means there is a sudden domain-wall-like switch from the hotter to the colder steady state at a certain distance from the center of the irradiated region, giving the hot spot a sharp boundary. Thus the spatial resolution of the microwave tip is determined by the temperature relaxation length of the sample (the domain wall width), which can be significantly smaller (∼20 nm) than the hot spot radius, of the order of the tip size (∼100 nm).
II. MODEL

We consider a tip at a distance $z_0$ above a strongly disordered superconductor (such as InO$_x$ or NbN) with short coherence length $\xi$. We assume $\xi$ to be smaller than other length scales of the problem (the tip-plane distance $z_0$ and the thermal relaxation length $\Lambda$ defined later), and adopt a model where the superconducting gap $\Delta$ and the quasiparticle distribution function at each point $r$ correspond to a local equilibrium with a position-dependent electron temperature $T_e(r)$. This implies fast electron-electron collisions and leaves out microwave-induced non-equilibrium effects [15–19]. The phonon temperature $T_{ph}$ in the layer is assumed to be fixed by the cryostat, due to a good contact with the substrate. This simple model, with its standard ingredients, contains only three material parameters: the normal state conductivity $\sigma_N$, the superconductor critical temperature $T_c$, and the electron-phonon cooling strength. They all are assumed to be the same as in the bulk material, so the role of the layer thickness $d$ is only to relate bulk and surface quantities.

Taking $T_e(r)$ as constant across the layer’s thickness, gives a two-dimensional heat transport equation:

\[
\frac{c(T_e) n d}{\partial t} = \nabla \cdot [K(T_e) d \nabla T_e] - Q(T_e, T_{ph}) d + \mathcal{H}(r) \frac{I_0^2}{8\pi^2} \text{Re} \frac{1}{\sigma(\omega, T_e)} d.
\]  

(1)

We only consider the stationary state ($\partial T_e/\partial t = 0$), so the specific heat $c(T_e)$ drops out. The bulk electronic thermal conductivity $K(T_e)$ is the textbook expression [20, 21] (see Appendix A); it is $T_e \sigma_N/e^2$ multiplied by a function of only $T_e/T_c$, and it reduces to the Wiedemann-Franz law for $T_e > T_c$.

$Q(T_e, T_{ph})$ is the power per unit volume, transferred from electrons to phonons. We adopt the standard model of electrons coupled to acoustic phonons [16] in which the effective electron-phonon coupling is $\alpha^2(\Omega) F(\Omega) \propto \Omega^{-3}$ for phonon energy $\Omega$. In particular, $n = 5$ when the electron mean free path is much larger than the typical phonon wavelength, and $n = 6$ in the opposite limit [22–30]. In the normal state this model yields $Q(T_e, T_{ph}) = \Sigma(T_e^w - T_{ph}^w)$ with a material-dependent coefficient $\Sigma$. The expression for $Q(T_e, T_{ph})$ in the superconductor is rather bulky and given in Appendix A; for each $n$, $Q(T_e, T_{ph})$ is given by $\Sigma T_e^w$ multiplied by a universal function of $T_e/T_c$ and $T_{ph}/T_c$. Our results for $n = 5$ and $n = 6$ are qualitatively similar, as expected [31]; the parameter important for our problem is the differential electron-phonon heat conductance at $T_e = T_{ph} = T_c$, $\partial Q(T_e, T_{ph})/\partial T_e|_{T_e = T_{ph} = T_c} = n \Sigma T_e^{n-1}$, which also defines a crucial length scale in our analysis; the thermal relaxation length $\Lambda \equiv |K(T_c)/(n \Sigma T_e^{n-1})|^{1/2}$.

The last term of Eq. (1) represents Joule heating of the electrons in the layer by the near-field microwaves at frequency $\omega$. We parametrize its strength by $I_0$, the amplitude of the total ac displacement current, flowing through the effective capacitor formed by the tip and the layer, due to the applied microwave voltage. The function $\mathcal{H}(r)$ is determined by the spatial distribution of the induced surface currents in the layer; its exact form depends on the tip shape. Still, for any axially symmetric tip whose radius $R_{tip}$ does not strongly exceed the tip-sample distance $z_0$, $\mathcal{H}(r)$ has the same qualitative form: $\mathcal{H}(r = 0) = 0$, it reaches a maximum value $\sim 1/z_0^2$ at $r \sim z_0$, and decays at $r \gg z_0$. In the calculations we use the expression corresponding to a spherical tip of radius $R_{tip} \ll z_0$ (see Appendix B)

\[
\mathcal{H}(r) = \frac{r^2}{(z_0^2 + r^2)} \left(1 - \frac{r^2}{z_0^2}ight)^2.
\]  

(2)

The 2D conductivity $\sigma(\omega, T_e)$ of the layer also appears in Eq. (1), with the bulk conductivity $\sigma(\omega, T_c)$ given by the standard Mattis-Bardeen expression [34] (see Appendix A). Typical material parameters for two commonly used disordered superconductors, NbN and InO$_x$, are given in Table I.

| Material | $T_c$, K | $1/\sigma_N$, $\Omega$ m | $n$ | $\Sigma$, W K$^{-n}$ m$^{-4}$ | $\Lambda$, nm |
|----------|----------|--------------------------|-----|------------------|-------------|
| InO$_x$  | 3.5      | $5 \times 10^{-6}$       | 6   | $2 \times 10^4$  | 17          |
| NbN      | 10.0     | $4 \times 10^{-6}$       | 5   | $5 \times 10^4$  | 16          |

TABLE I. Material parameters used for the numerical calculations, typical for InO$_x$ and NbN [32, 33].

III. BISTABILITY IN ABSENCE OF HEAT CONDUCTION

It is instructive to start with the local version of Eq. (1), setting $K = 0$. Then, at each point $r$, the electron temperature $T_e$ is found from the algebraic equation

\[
j^2 \text{Re} \left(\frac{1}{\sigma(\omega, T_e)} \right) = Q(T_e, T_{ph}),
\]  

(3)

with $j^2 = I_0^2 \mathcal{H}(r)/(8\pi^2 d^2)$. Equation (3) contains only bulk quantities, and is analogous to the heat balance equation for a superconductor in a spatially uniform microwave field. For that problem, the heat balance equation may have two stable solutions for $T_e$ [12–14].

The peculiar $T_e$ dependence of the heating term on the left-hand side of Eq. (3), sketched in Fig. 2, is a rather common origin for bistabilities related to electron overheating. In our case, it is due to the physics of quasiparticle heating, summarized in the Introduction. As a result, Eq. (3) has three solutions for $T_e$, as sketched in Fig. 2; only the high- and the low-temperature solutions are stable, the middle one is unstable. Fig. 2 shows that $j^2$ controls the vertical scale of the heating curve, so the bistability disappears if $j^2$ is too large or too small.

When $h\omega$ and the solutions for $T_e$ are all of the order of $T_c$, the typical scale of $j$ which governs the bistability,
is \( j_\ast = \sqrt{\sigma N \Sigma T_c^2} \). It is the current density needed to maintain the electrons at \( T_e = T_c \) when the phonons are at \( T_{ph} = 0 \). It is important that \( j_\ast \ll j_c \), the critical current density; this condition is necessary to justify the calculation of the dissipated power using the linear response conductivity. The condition \( j_\ast \ll j_c \) is naturally satisfied when electron-phonon coupling is weak. Indeed, we obtain \( j_\ast/j_c \sim \sqrt{\hbar}/(T_c \tau_{ph}) \), using (i) the expression \( j_c \approx 1.5 \sigma_N \Sigma N_0 (2e \hbar / D \Delta_0)^{-3/2} \) [35] with \( 2\Delta_0 \approx 3.53 T_c \) being the zero-temperature gap and \( D \) the electron diffusion coefficient, (ii) the Einstein relation \( \sigma_N = 2N_0 e^2 D / N_0 \) being the density of states per spin at the Fermi level, and (iii) \( \Sigma T_c^2 \sim N_0 T_c^2 / \tau_{ph} \) where \( \tau_{ph} \) is the time the electron with energy \( \sim T_c \) spends before emitting a phonon. The ratio \( j_\ast/j_c \) must necessarily be small in any material well-described as a gas of electronic quasiparticles.

Fig. 3 shows the curves of numerical solutions of Eq. (3). The curves are universal when plotted in the appropriate units (\( T_e \) and \( j_\ast \)), i.e., valid for any material with the electron-phonon cooling exponent \( n = 5 \) or \( n = 6 \). We see that they are very weakly sensitive to whether \( n = 5 \) or \( n = 6 \). The bistable region exists only for frequencies below \( 2\Delta(T_{ph}) \), the gap at \( T_{ph} \). At low frequencies, (i) it is bounded from below by the current \( j_\ast \sqrt{1-(T_{ph}/T_c)^n} \), (ii) it extends to high currents \( \propto 1/\omega^2 \) since \( \Re[1/\sigma(\omega, T_c)] \propto \omega^2 \) is small. Of course, at high currents the validity of the theory is limited by the condition \( j \ll j_c \), which is not included in the model. The high-temperature solution lies below \( T_c \) only in a small part of the bistable region with \( j < j_\ast \sqrt{1-(T_{ph}/T_c)^n} \); at higher \( j \), the high-temperature solution is normal.

IV. BISTABILITY IN THE FULL PROBLEM

Returning to the steady state of the full Eq. (1), we consider \( j = j(r) = I_0 \sqrt{\rho N(8\pi r^2 d^2)} \) which vanishes at \( r \to 0 \), \( \infty \), and reaches a maximum \( j_{max} \approx 0.120 I_0/(\sqrt{2z_0 d}) \) at \( r_{max} \approx 1.27 z_0 \). Then, for given \( \hbar \omega / T_c \) and \( T_{ph}/T_c \), the solutions are determined by two dimensionless parameters, \( j_{max}/j_\ast \) and \( \Lambda/z_0 \). Dividing Eq. (1) by \( nd \Sigma T_{ph}^2 \) and measuring \( r \) in the units of \( z_0 \), we cast the last two terms of Eq. (1) [those entering Eq. (3)] in the dimensionless parameters \( (T_e/T_c) \) and \( j/j_\ast \) at each \( r \), while the gradient term is proportional to \( (N_0 \Sigma)^2 \). From the values in Table I we see that \( \Lambda \sim 20 \text{nm} \) can be significantly smaller than \( z_0 \sim 100 \text{nm} \).

The total power, needed to maintain \( j_{max} = j_\ast \), is given by the spatial integral of the last term in Eq. (1), \( 5.5 \Sigma T_e^2 z_0^2 d \ln[(4\pi \sigma_N \omega)/(d/z_0)] \) (see Appendix B). This gives a few nanowatts for \( \text{InO}_x \) and a few microwatts for \( \text{NbN} \) of thickness \( d = 10 \text{nm} \) and \( z_0 = 100 \text{nm} \). For \( \Lambda/z_0 \to 0 \), the stationary profile \( T_e(r) \) is deter-
mined by the simple mapping of \( j(r)/j_\ast \) to the \( j/j_\ast \) axes on Fig. 3. Let \( \hbar \omega/T_c \) and \( T_{ph}/T_c \) be such that the uniform system is bistable in the interval \( j_1 < j < j_2 \) with some \( j_1, j_2 \); that is, Eq. (3) has three solutions \( T_1(j) < T_u(j) < T_\ast(j) \) (see Fig. 3a) for \( j_1 < j < j_2 \), only \( T_1(j) \) for \( j < j_1 \), and only \( T_\ast(j) \) for \( j > j_2 \). Thus, for \( j_\ast < j_1 \), only the solution \( T_1(j(r)) \) is possible for any \( r \). For \( j_1 < j_\ast < j_2 \), there are three local solutions \( T_1(j(r)) < T_u(j(r)) < T_\ast(j(r)) \) in an interval of \( r \) around \( r_{max} \) where \( j(r) > j_1 \), so the dependence \( T_e(r) \) consists of a low-temperature branch and a disconnected closed contour (dotted curve in Fig. 4a).

For \( j_\ast > j_2 \), only the \( T_\ast(j(r)) \) solution is possible around \( r_{max} \), where \( j(r) > j_3 \), but there are two regions with three solutions on both sides (dotted curve in Fig. 4b). Thus the system must switch between the stable low- and high-temperature local solutions creating a “domain wall” (whose width is vanishing in the limit \( \Lambda/z_0 \to 0 \)) at some position \( r \) on each side of \( r_{max} \). This position can be found by noting that any global stable stationary solution of Eq. (1) represents a minimum of a certain functional \( \mathcal{F}[T_e(r)] \), explicitly given in Appendix C. Minimizing the functional with respect to the domain wall position \( R \), we identify it as the point where \( j(R) \) satisfies the condition

\[
\int_{T_1(j(R))}^{T_\ast(j(R))} \left[ Q(T,T_{ph}) - \frac{j^2(R)}{\sigma_0(\omega,T)} \right] \mathcal{K}(T) \, dT = \int_{T_\ast(j(R))}^{T_u(j(R))} \left[ \frac{\text{Re} \, j^2(R)}{\sigma_0(\omega,T)} - Q(T,T_{ph}) \right] \mathcal{K}(T) \, dT, \tag{4}
\]

which resembles the Maxwell construction (equal-area rule) for the Van der Waals isotherms. This construction is valid for \( \Lambda/z_0 \to 0 \), but it remains qualitatively correct for realistic values of \( \Lambda/z_0 \), as we will see below.

Then for \( j_\ast > j_2 \), the system has only one global solution for \( T_e(r) \). However, it is the underlying bistability that causes the domain walls in that solution, whose width \( \Lambda \) can be significantly smaller than the size of the hot spot (solid black curve in Fig. 4b). This rapid spatial variation of \( T_e(r) \) will be useful for creating a local thermal perturbation of the system with submicron resolution. Intriguingly, \( T_e \) exceeds \( T_c \) in a ring, leaving a small superconducting core precisely below the tip.

In contrast, for \( j_1 < j_\ast < j_2 \) the disconnected low-temperature branch \( T_1(r) \) represents a stable global solution of Eq. (1) in the limit \( \Lambda/z_0 \to 0 \). In addition, there is another stable solution with two domain walls. Then, the system exhibits a global bistability [multiple solutions of Eq. (1) for the whole profile \( T_c(r) \)], inherited from the local bistability (multiple solutions of Eq. (3) for \( T_c \)) at a given point \( r \). The global bistability is accompanied by a hysteretic behaviour of \( T_e(r) \) as one changes the microwave strength, \( j_\ast/j_\ast \).

Finite \( \Lambda/z_0 \) causes a broadening of the domain walls; the solid curves in Fig. 4 show this for realistic parameters. Fig. 5 shows that increasing \( \Lambda/z_0 \) causes the high-temperature and unstable solutions to approach each other, and annihilate at a certain \( \Lambda/z_0 \) (e.g., \( \Lambda/z_0 \approx 0.6 \) for the parameters in Fig. 5). The low-temperature solution survives at larger \( \Lambda/z_0 \), because it is favoured by the faster heat evacuation from the below the tip.
V. LOCAL HEATING WITH A HOT TIP

Finally, we briefly discuss another possible setup, when the sample is heated not by the external microwave drive, but by thermal radiation from the tip, held at high temperature $T_{\text{tip}}$. The corresponding microwave field can be modelled by that of a thermally fluctuating magnetic dipole, as discussed in detail in Appendix D. The heating power is then given by an integral over all frequencies. Crucially, both the typical frequency and the strength of the microwave field are controlled by the same parameter $T_{\text{tip}}$, while in the previous setting the microwave strength $J_0$ and frequency $\omega$ could be controlled independently. For typical material parameters, to produce a noticeable change in temperature can be tuned to locally destroy the superconductivity (unless $\Lambda \ll z_0$, the tip-sample distance (or the tip size, if larger). This is the case for strongly disordered superconductors such as NbN or InO$_x$, if $\Lambda \ll z_0$, the tip-sample distance (or the tip size, if larger). This is the case for strongly disordered superconductors such as NbN or InO$_x$. Then the heating is due to absorption of photons with $\omega \gg T_c/\hbar$, so it is not sensitive to the superconductivity. This results in a single solution, a smooth profile $T_e(r)$ exceeding $T_{\text{ph}}$ in a region whose size is a few $z_0$ at least. Moreover, for realistic parameters, its effect is too weak to locally destroy the superconductivity (unless $T_{\text{ph}} \rightarrow T_c$), so it would be a much worse local probe than a tip with microwave driving.

VI. CONCLUSIONS

We propose a local thermal probe based on a submicron-sized hot spot created in a thin superconducting layer by microwave radiation produced by a small metallic tip. Our simple model shows how the electron temperature is locally driven away from the substrate (phonon) temperature, assuming the electrons remain in local thermal equilibrium among themselves. We have shown that the hot spot can have two possible stable states, similarly to a bulk bistability, discussed earlier for spatially uniform microwave fields.

We have identified the superconductor’s thermal relaxation length $\Lambda$, and shown that global bistability requires $\Lambda \lesssim z_0$, the tip-sample distance (or the tip size, if larger). This is the case for strongly disordered superconductors such as NbN or InO$_x$. Then the hot spot has a sharp boundary corresponding to a domain wall between two local stable solutions. The hot spot temperature can be tuned to locally destroy the superconductivity. We thus propose it as a scanning probe with sub-micron resolution, ideal for mapping out where the supercurrent flows in such disordered superconductors.

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Appendix A: Electric and thermal conductivity, electron-phonon cooling

Here, we briefly summarize how the various parameters in Eq. (1) are modeled.

For the temperature dependence of the gap $\Delta(T_e)$ we use an empirical expression [36],

$$\frac{\Delta(T_e)}{\Delta_0} = \sqrt{\cos\left(\frac{\pi T_e^2}{T_c^2}\right)},$$

which agrees within 3% with the BCS expression. $\Delta_0$ is related to $T_c$ by the weak coupling relation ($\gamma = 0.577\ldots$ is the Euler-Mascheroni constant):

$$\frac{\Delta_0}{T_c} = \frac{\pi}{\gamma} \approx 1.76.$$

The ac conductivity of a superconductor is [34]

$$\sigma(\omega) = \frac{1}{\hbar \omega} \int_{\Delta}^{\infty} d\epsilon \frac{\tanh(\frac{\epsilon + \hbar \omega}{2T_e})}{2T_e}$$

where

$$\Upsilon(\epsilon) = \frac{\epsilon(\epsilon + \hbar \omega) + \Delta^2}{\sqrt{|\epsilon^2 - \Delta^2|} \sqrt{(\epsilon + \hbar \omega)^2 - \Delta^2}},$$

Here $\sigma_N$ is the bulk electrical conductivity of the material in the normal state. The electronic contribution to the thermal conductivity of the superconductor also depends $\sigma_N$ via [20, 21]:

$$\kappa(T_e) = \frac{\sigma_N}{\epsilon^2} \int_{\Delta(T_e)}^{\infty} \frac{e^2 d\epsilon}{2T_e^2 \cosh^2[\epsilon/(2T_e)]},$$

The power per unit volume transferred from electrons to phonons can be written using the collision integral in Ref. [16] as

$$Q(T_e, T_{\text{ph}}) = \frac{\Sigma}{4(n-1)! c(n)} \int_{-\infty}^{\infty} d\epsilon \int_{-\infty}^{\infty} d\Omega \Omega^{n-2} \times$$

$$\times \left( \coth \frac{\Omega}{2T_e} - \coth \frac{\epsilon}{2T_{\text{ph}}} \right) \times$$

$$\times \left( \tanh \frac{\epsilon + \Omega}{2T_e} - \tanh \frac{\epsilon}{2T_e} \right) \times$$

$$\times \frac{\Theta(|\epsilon| - \Delta)}{\sqrt{\epsilon^2 - \Delta^2}} \frac{\Theta(|\epsilon + \Omega| - \Delta)}{\sqrt{(\epsilon + \Omega)^2 - \Delta^2}} \times$$

$$\times \left[ \epsilon(\epsilon + \Omega) - \Delta^2 \right] \text{sign}[\epsilon(\epsilon + \Omega)].$$
Appendix B: Heating by a microwave drive

We model the superconducting layer as an infinitely thin 2D sheet with a 2D conductivity \( \sigma_{2D}(\omega) = \sigma(\omega) d \), given by the standard Mattis-Bardeen expression (A3) multiplied by the layer thickness \( d \). We represent the tip as a small spherical particle of radius \( R_{\text{tip}} \), placed at a distance \( z_0 \) from the layer, assuming both \( d, R_{\text{tip}} \ll z_0 \). While the first inequality is quite realistic (we have in mind \( z_0 \sim 100 \text{ nm}, d \sim 10 - 20 \text{ nm} \)), the second will be used to simplify the calculations and obtain relatively simple final expressions; in the end we will set \( R_{\text{tip}} \sim z_0 \), so these expressions will be valid only as qualitative estimates (which would be the case anyway, since in reality the tip is not spherical). This geometry determines the tip-layer capacitance \( C_{\text{tip}} \sim R_{\text{tip}} \).

If a microwave voltage is applied between the sample and the tip, this situation can be analyzed in terms of an effective circuit which includes, in series with the microwave voltage source at frequency \( \omega \), and the tip-sample capacitance \( C_{\text{tip}} \), an effective impedance \( Z_{\text{tip}}(\omega) \) representing the sample, as well as an external impedance \( Z_{\text{ext}} \) corresponding to the external circuit used to connect the voltage source.

Typically, the impedance of the capacitor, \( Z_{\text{tip}} = -1/(i\omega C_{\text{tip}}) \), is much larger than the sample impedance \( Z_0 \). Indeed, associating the latter with the resistance per square \( 1/\sigma_{2D} \) of the 2D layer with thickness \( d \ll z_0 \), estimating the capacitance to be of the order of the tip radius, and taking the latter to be \( 50 \text{ nm} \), for \( \hbar \omega = 1 \text{ K} \) we obtain \( Z_{\text{tip}} = 1.4 \text{ M}\Omega \). This is much larger than \( 1/\sigma_{2D} \) even for \( \sigma_{2D} = e^2/h = 1/(4.11 \text{ k}\Omega) \), about the smallest possible sheet conductivity allowed for a superconductor, below which a superconductor-insulator transition occurs [37, 38]. This means that the oscillating charge distribution in the layer is determined by the oscillating charge \( q(t) \) on the tip, so the currents in the layer follow to maintain this oscillating charge distribution in the layer. The small in-plane electric field is the one required to drive these currents.

The external impedance \( Z_{\text{ext}} \) depends on the specific experimental setup. If \( Z_{\text{ext}} \gg Z_{\text{tip}} \), the tip is effectively current-biased. An applied current \( I(t) = I_0 \cos \omega t + dq/dt \) corresponds to the charge on the tip \( q(t) = (I_0/\omega) \sin \omega t \). If \( Z_{\text{ext}} \ll Z_{\text{tip}} \), the tip is biased by a voltage \( V(t) = V_0 \cos \omega t \), so the charge on the tip is \( q(t) = C_{\text{tip}} V_0 \cos \omega t \).

In electrostatics, a point charge \( q \) placed at a distance \( z_0 \) from a conducting plane, induces a 2D screening charge density on the plane [39]

\[
\rho(r) = -\frac{q}{2\pi(r^2 + z_0^2)^{3/2}}. \tag{B1}
\]

where \( r = (x, y) \) is the in-plane position, and \( r = |r| \). When the tip charge \( q(t) \) is oscillating, we assume that the density \( \rho(r) \) simply follows Eq. (B1) instantaneously. This instantaneous approximation breaks down at sufficiently large distances where the charges are no longer able to follow. The charge density relaxation time at a distance \( r \) can be estimated as the \( RC \) relaxation time of an effective circuit with the capacitance \( \sim r \) and the resistance \( \sim 1/\sigma_{2D} \) (since in 2D the resistance per square does not scale with the size). Requiring this relaxation time to be smaller than \( 1/\omega \), we arrive at the length scale \( a_{\epsilon}(\omega) = 2\pi \sigma_{2D}/\omega \) [see Eq. (D5) for the rigorous definition]. Another obvious cutoff scale is \( c/\omega \), which becomes more relevant if \( 2\pi \sigma_{2D}/c > 1 \). For the frequencies we are interested in (of the order of the superconductor critical temperature, a few Kelvins), both cutoff scales are much larger than \( z_0 \), the typical length scale of Eq. (B1). Thus they play almost no role in the physics discussed in this work, they only enter via a logarithmic cutoff in Eq. (B5) below.

The 2D current density \( \mathbf{J}(r, t) = j(r, t)d \) can be found from the continuity equation \( \nabla \cdot \mathbf{J} = -\partial \rho/\partial t \) with \( \rho(r, t) \) given by Eq. (B1). Since the whole picture is axially symmetric, \( \mathbf{J} \) has only the radial component \( J_r \) which satisfies a first-order ordinary differential equation with \( I = dq/dt \):

\[
\frac{\partial J_r}{\partial r} + \frac{J_r}{r} - \frac{I}{2\pi(r^2 + z_0^2)^{3/2}} = 0. \tag{B2}
\]

The solution of this equation which is finite at \( r \to 0 \), reads

\[
J_r(r, t) = \frac{I(t)}{2\pi r} \left(1 - \frac{z_0}{\sqrt{z_0^2 + r^2}}\right). \tag{B3}
\]

For the current \( I(t) = I_0 \cos \omega t \), the in-plane electric field is related to this current via \( \sigma_{2D}(\omega) \), so the Joule dissipation per unit area is given by

\[
P(r) = \frac{1}{(2\pi r)^2} \left(1 - \frac{z_0}{\sqrt{z_0^2 + r^2}}\right)^2 \frac{I_0^2}{2} \frac{1}{\sigma_{2D}(\omega)} \text{Re} \frac{1}{\sigma_{2D}(\omega)}. \tag{B4}
\]

The total power injected into the sample is the integral of \( P(r) \) over the sample area,

\[
\int_0^\infty P(r) 2\pi r dr = \frac{I_0^2}{4\pi} \text{Re} \frac{1}{\sigma_{2D}(\omega)} \ln \frac{\min\{|e/\omega|, |a_{\epsilon}(\omega)|\}}{z_0}, \tag{B5}
\]

where the logarithmic divergence at large distances is cut off at the scale discussed in the previous paragraph. For a voltage-biased tip, one should replace \( I_0 \to \omega C_{\text{tip}} V_0 \) in Eqs. (B4,B5).
Appendix C: Variational form of the heat transport equation

Eq. (1) can be written in terms of the variational derivative of a “free energy” functional $\mathcal{F}[T_e(r)]$:

$$
\mathcal{F} = \int d^2r \left[ \frac{\mathcal{K}(T_e)}{2} \nabla T_e^2 + \mathcal{Q}(T_e) - j^2(r) \mathcal{R}(T_e) \right],
$$

where

$$
\mathcal{K}(T_e) = \frac{\partial T_e}{\partial t} - \frac{\delta \mathcal{F}[T_e(r)]}{\delta T_e(r)},
$$

$$
\mathcal{Q}(T_e) = \int_0^{T_e} \mathcal{K}(T') Q(T', T_{ph}) dT',
$$

$$
\mathcal{R}(T_e) = \int_0^{T_e} \text{Re} \frac{\mathcal{K}(T')}{\sigma(\omega, T^*)} dT'.
$$

A stable solution of Eq. (1) corresponds to a local minimum of the functional (C1b).

We now use this to explain the domain wall’s position, taking the example of the situation sketched in Fig. 6. There we assume $h\omega/T_c$ and $T_{ph}/T_c$ are such that the system is bistable for $\mathcal{K} = 0$ in the interval $j_1 < j < j_2$ with some $j_1, j_2$. In other words, for $j_1 < j < j_2$, there are three $T_e$ which are solutions of Eq. (3), these being $T_h(j) < T_u(j) < T_l(j)$. Let $R_1, R_2$ be such that $j(R_{1,2}) = j_{1,2}$, and for definiteness we are assuming $R_1 < R_2$. If we take the limit $\Lambda/z_0 \to 0$ and neglect the gradient term, the position $R$ where the switching between the two solutions occurs can be found by minimizing the “free energy”

$$
\int_{R_1}^R 2\pi r dr \left[ j^2(r) \mathcal{R}(T_h(j^2(r))) - \mathcal{Q}(T_h(j^2(r))) \right] + \int_R^{R_2} 2\pi r dr \left[ j^2(r) \mathcal{R}(T_l(j^2(r))) - \mathcal{Q}(T_l(j^2(r))) \right]
$$

with respect to $R$. This determines $R$ as the position where $j(R)$ satisfies the condition

$$
\int_0^{T_h(j(R))} \left[ \text{Re} \frac{j^2(R) \mathcal{K}(T)}{\sigma(\omega, T)} - \mathcal{K}(T) Q(T, T_{ph}) \right] dT = \int_0^{T_l(j(R))} \left[ \text{Re} \frac{j^2(R) \mathcal{K}(T)}{\sigma(\omega, T)} - \mathcal{K}(T) Q(T, T_{ph}) \right] dT,
$$

which is equivalent to Eq. (4).

Including the weak gradient term, we can estimate the width $w$ of the transition region by minimizing

$$
\min_w \left\{ 2\pi R w \left[ \frac{\mathcal{K}(T_h) T_h - \mathcal{K}(T_l) T_l}{w} \right]^2 + 2\pi R w \int_{T_h}^{T_l} \left[ \text{Re} \frac{j^2(T) \mathcal{K}(T)}{\sigma(\omega, T)} - \mathcal{K}(T) Q(T, T_{ph}) \right] dT \right\}.!
$$

If all temperatures are a fraction of $T_c$, all quantities are of the same order as in the normal state. Then $j^2 \sim \sigma Q$, which gives $w \sim \Lambda$.

Appendix D: Heating by thermal radiation from a hot tip

The body of the manuscript described in detail the use of the tip to apply near-field microwaves to the superconductor, however we only briefly mentioned heating the tip. Naively, one would expect the physics to be similar in both cases, because both processes are intended to locally heat the superconductor. However, we find that heating the tip is a very ineffective way of heating the superconductor, compared to microwave driving. For realistic experimental parameters, we find that a hot tip does not drive the superconductor into the normal state, nor create bistability. Here we explain in detail how we model the hot tip, and how we arrive at these conclusions.

1. Hot tip as a fluctuating dipole

As in Sec. B, we model the superconducting layer as an infinitely thin 2D sheet with a 2D conductivity $\sigma_{2D}(\omega) = \sigma(\omega) d$, and the tip as a small spherical metallic particle of radius $R_{tip}$ placed at a distance $z_0$ from the layer, assuming both $d, R_{tip} \ll z_0$. The tip material
is characterized by its bulk conductivity $\sigma_{tip}$, which we assume to be frequency-independent.

A similar problem was studied in Ref. [40] in the framework of fluctuational electrodynamics [41–43], which we will also adopt here. Namely, we represent the tip as a thermally fluctuating dipole (either electric or magnetic), which produces a fluctuating field which is heating up the electrons in the sheet. Below we analyze the electric and magnetic contributions separately, and find that the magnetic one dominates, in agreement with Ref. [40]. To simplify the calculations, we assume the dipole to oscillate only along the $z$ direction (perpendicular to the sample plane). Contribution of the in-plane fluctuations is of the same order, so our results will remain valid as qualitative estimates.

In this appendix, since we are handling an essentially 3D problem, we adopt the notation $\mathbf{r} = (x, y, z) \equiv (r_\parallel, z)$. In the rest of the paper, $\mathbf{r}$ refers to the in-plane position, that is, the subscript "\( \parallel \)" is omitted for brevity. For the in-plane wave vector we use the notation $\mathbf{k} \equiv (k_x, k_y)$. All equations are written in the CGS units.

### 2. Electric dipole

According to the fluctuation-dissipation theorem, the electric dipole moment,

$$ p(t) = \int \frac{d\omega}{2\pi} d_\parallel^2 e^{-i\omega t}, \quad (D1) $$

subject to thermal fluctuations at temperature $T_{tip}$, has the fluctuation spectrum

$$ \langle p_\omega p_{\omega'} \rangle = \frac{\hbar}{2} \text{Im} \alpha_e(\omega) \coth \frac{\hbar \omega}{2 T_{tip}} 2\pi \delta(\omega + \omega'), \quad (D2) $$

where

$$ \alpha_e(\omega) = R_{tip}^3 \frac{4\pi i \sigma_{tip}/\omega}{4\pi i \sigma_{tip}/\omega + 3} \quad (D3) $$

is the electric polarizability of a sphere of radius $R_{tip}$ with the dielectric function $1 + 4\pi i \sigma_{tip}/\omega$. For a tip made of a good metal, $4\pi \sigma_{tip} \sim 10^{18}$ $\text{s}^{-1}$ (corresponding to $1/\sigma_{tip} = 1.13 \times 10^{-7}$ $\text{Ω} \cdot \text{m}$), we have $\omega \ll 4\pi \sigma_{tip}$, so Im $\alpha_e \approx 3 R_{tip}^3 \omega/(4\pi \sigma_{tip})$.

To find the induced fluctuating electric field, we use the quasi-static approximation, since the dimensions of the structure are much smaller than the thermal photon wavelength. Namely, we write the Poisson equation for each Fourier component of the electrostatic potential $\varphi_{k_\omega}(z) e^{i k_\parallel \cdot \mathbf{r} - i \omega t}$:

$$ \left( \frac{\partial^2}{\partial z^2} - k^2 \right) \varphi_{k_\omega}(z) = -4\pi p_\omega \delta'(z - z_0) $$

$$ + \frac{4\pi i k^2}{\omega} \sigma_{2D}(\omega) \varphi_{k_\omega}(0) \delta(z). \quad (D4) $$

The charge density, appearing on the right-hand side, consists of two parts. The first one, $-p_\omega \delta'(z - z_0)$ with $\delta'(z)$ standing for the derivative of the Dirac delta function, is that of the point dipole $p_\omega$, placed at the point $\mathbf{r} = (0, 0, z_0)$ and oriented along $z$. The second part is the charge density induced in the superconducting layer at $z = 0$ by the oscillating in-plane electric field $-i k_\parallel \varphi_{k_\omega}(0)$. Indeed, this field induces the current $-i k_\parallel \varphi_{k_\omega}(0) \sigma_{2D}(\omega)$, which is related to the charge density by the continuity equation.

Solution of Eq. (D4) is sought as linear combinations of $e^{\pm ik z}$ in the three intervals $-\infty < z < 0$, $0 < z < z_0$, $z_0 < z < \infty$, decaying at $\pm \infty$. The coefficients should be matched to give the correct jump of $\varphi_{k_\omega}(z)$ at $z = z_0$ and the jump in $d\varphi_{k_\omega}(z)/dz$ at $z = 0$, needed to reproduce the right-hand side of Eq. (D4). The result is

$$ \varphi_{k_\omega}(z = 0) = \frac{2\pi p_\omega e^{-k z_0}}{1 + k \alpha_e(\omega)} \equiv a_e(\omega) \equiv \frac{2\pi i \sigma_{2D}(\omega)}{\omega}, \quad (D5) $$

Even for a rather small conductivity $\sigma_{2D} = e^2/h = 1/(4.11 \text{ kΩ})$ and a rather high frequency corresponding to the temperature $\hbar \omega = 100$ K, we obtain $|a_e(\omega)| \approx 1.05 \text{ μm}$, while the typical $k \sim 1/z_0 \sim (10^3 \text{ nm})^{-1}$. Thus, we can neglect unity in the denominator, and obtain the in-plane electric field $E_{k_\omega}(\mathbf{r}_\parallel, z = 0)$ by the inverse Fourier transform with respect to $k$:

$$ E_{k_\omega}(\mathbf{r}_\parallel, z = 0) = \frac{p_\omega}{\alpha_e(\omega)} \frac{r_\parallel}{(r_\parallel^2 + z_0^2)^{3/2}}. \quad (D6) $$

Finally, the heating power per unit area at a point $(\mathbf{r}_\parallel, 0)$ is found as $\langle \mathbf{J}(\mathbf{r}_\parallel, t) \cdot E_{k_\omega}(\mathbf{r}_\parallel, 0, t) \rangle$, where the current Fourier component $\mathbf{J}_\omega(\mathbf{r}_\parallel) = \sigma_{2D}(\omega) E_{k_\omega}(\mathbf{r}_\parallel, 0)$. The averaging is performed using Eq. (D2), and one should subtract the inverse heat flow from current fluctuations in the superconducting layer with electron temperature $T_e$, which is given by the same expression but with the replacement $\coth[\hbar \omega/(2T_{tip})] \rightarrow \coth[\hbar \omega/(2T_e)]$:

$$ P_{e}(\mathbf{r}_\parallel) = \frac{3}{8\pi^2} \frac{R_{tip}^3}{(r_\parallel^2 + z_0^2)^{3/2}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\hbar \omega^3}{4\pi \sigma_{tip}} \text{Re} \frac{1}{\sigma_{2D}(\omega)} \times \left( \coth \frac{\hbar \omega}{2T_{tip}} - \coth \frac{\hbar \omega}{2T_e} \right). \quad (D7) $$

### 3. Magnetic dipole

The fluctuations of the magnetic moment $\mu$ are fully analogous to Eq. (D2):

$$ \langle \mu_{k_\omega} \mu_{k_\omega'} \rangle = \frac{\hbar}{2} \text{Im} \alpha_m(\omega) \coth \frac{\hbar \omega}{2 T_{tip}} 2\pi \delta(\omega + \omega'), \quad (D8) $$

where

$$ \alpha_m(\omega) = \frac{R_{tip}^3}{30} \left( \frac{\omega R_{tip}}{c} \right)^2 \frac{4\pi i \sigma_{tip}}{\omega}, \quad (D9) $$
is the magnetic polarizability of a sphere of radius \(R_{\text{tip}}\) with the dielectric function \(1 + 4\pi \sigma_{\text{tip}}/\omega\) [40].

In this magnetostatic problem, it is convenient to find the electric field from the vector potential \(A_{k\omega}(z)\). For the magnetic dipole \(\mu e_x\) directed along the \(z\) axis with the unit vector \(e_z\), one can seek the vector potential in the form \(A_{k\omega}(z) = ik \times e_z \psi_{k\omega}(z)\). Substituting it into the 3D Ampère’s law \(\nabla \times \nabla \times A = 4\pi j/c\), we obtain the following equation for the scalar function \(\psi_{k\omega}(z)\):

\[
\left( \frac{\partial^2}{\partial z^2} - k^2 \right) \psi_{k\omega}(z) = -4\pi \mu_x \delta(z - z_0) - \frac{4\pi i\omega}{c^2} \sigma_{\text{2D}}(\omega) \psi_{k\omega}(0) \delta(z).
\]

(D10)

Proceeding analogously to the electrostatic case, we find the solution

\[
A_{k\omega}(z = 0) = ik \times e_z \frac{2\pi \mu_x e^{-kz_0}}{k + 1/a_m(\omega)},
\]

(D11)

\[
a_m(\omega) \equiv -\frac{c^2}{2\pi i\omega \sigma_{\text{2D}}(\omega)}.
\]

Again, taking \(\sigma_{\text{2D}} = e^2/h\) and \(\hbar\omega = 100\) K, we obtain \([a_m(\omega)] = 0.5\) mm, so \(1/a_m\) in the denominator can be safely neglected even for much higher conductivities. The electric field,

\[
E_{\parallel\omega}(r_\parallel, z = 0) = -\frac{i\omega}{c} \frac{\mu_x e_x \times r_\parallel}{(r^2_\parallel + z^2_0)^{3/2}},
\]

(D12)

determines the heating power per unit area:

\[
P_m(r_\parallel) = \frac{1}{60} \frac{R_{\text{tip}}^5 d}{(r^2_\parallel + z^2_0)^{5/2}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\hbar \omega^3}{c^4} 4\pi \sigma_{\text{tip}} \Re \sigma(\omega) \times \left( \coth \frac{\hbar \omega}{2T_{\text{tip}}} - \coth \frac{\hbar \omega}{2T_e} \right).
\]

(D13)

4. Comparison: electric versus magnetic dipole

Expressions (D7) and (D13) are very similar. Neglecting \(\Im \sigma(\omega)\), one can estimate their ratio as

\[
\frac{P_m(r_\parallel)}{P_e(r_\parallel)} \sim \left( \frac{4\pi \sigma_{\text{tip}} \sigma_{\text{2D}} R_{\text{tip}}}{e^2} \right)^2.
\]

(D14)

For \(R_{\text{tip}} = 100\) nm and a rather small \(\sigma_{\text{2D}} = e^2/h\), but large \(4\pi \sigma_{\text{tip}} = 10^{18} \text{s}^{-1}\) (corresponding to \(1/\sigma_{\text{tip}} = 1.13 \times 10^{-7} \Omega \cdot \text{m}\)) this ratio is about \(6\). Thus, for materials and tip sizes we are interested in, heating via the magnetic dipole fluctuations dominates.

5. Electron temperature profile

As we will see shortly, the most interesting case is \(T_{\text{tip}} \gg T_e\). Then one can neglect the frequency dependence of \(\sigma(\omega)\) and take it to coincide with the normal state value. Then, the frequency integral in Eq. (D13) is calculated explicitly:

\[
P_m(r_\parallel) = \frac{\pi^3}{450} \frac{T_{\text{tip}}^4 - T_e^4}{(\hbar c)^4} \frac{r^2_\parallel R_{\text{tip}}^5}{(r^2_\parallel + z^2_0)^3} 4\pi \hbar \sigma_{\text{tip}} \sigma_{\text{2D}}.
\]

(D15)

We remind the reader that this equation is written in CGS units; to pass to SI units, it should be divided by \((4\pi \varepsilon_0)\) with \(\varepsilon_0\) being the vacuum permittivity.

Let us estimate the heating power at \(r_\parallel = z_0/\sqrt{2}\), corresponding to the maximum of \(P_m(r_\parallel)\). For \(z_0 = 100\) nm, \(R_{\text{tip}} = 50\) nm, \(1/\sigma_{\text{tip}} = 10^{-7} \Omega \cdot \text{m}\), \(\sigma_{\text{2D}} = e^2/h \approx 1/(4.1\text{kHz})\), and room temperature \(T_{\text{tip}} = 300\) K, we obtain \(P_m \approx 2\) W/m². Balancing it with the phonon cooling power \(\Sigma(T_e - T_{\text{ph}}) d\) for the parameters of InOx (Table 1 of the main text) and for \(T_{\text{ph}} = T_e = 3\) K, we obtain very little overheating, \(T_e - T_{\text{ph}} \sim 10^{-4}\) K, and for NbN it is even smaller. A noticeable electronic overheating can be obtained for lower \(T_{\text{ph}} \lesssim 1\) K, when the cooling power is suppressed by the presence of the superconducting gap (Fig. 7). Still, it is not sufficient to suppress the superconductivity and create a normal region in the center. Also, even for low \(T_{\text{ph}}\), the tip temperature \(T_{\text{tip}}\) should be quite high, so the heating occurs via absorption of photons with \(\hbar \omega \sim T_{\text{tip}} \gg T_e\). First, this validates Eq. (D15). Second, in this regime the heating power does not depend on \(T_e\), which eliminates any bistability.

The typical size of the overheated region is at least \(z_0\). Moreover, at low temperatures the phonon cooling power \(Q(T_e, T_{\text{ph}})\) vanishes faster than the electronic thermal conductivity \(\kappa(T_e)\). As a result, at temperatures significantly below \(T_e\), the size of the overheated region significantly exceeds \(z_0\), which makes this setting unsuitable for creating a localized thermal perturbation.

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