We carry out a systematic analysis of a pair of coupled qubits, each of which is subject to its own dissipative environment, and argue that a combination of the inter-qubit couplings which provides for the lowest possible decoherence rates corresponds to the incidence of a double spectral degeneracy in the two-qubit system. We support this general argument by the results of an evolutionary genetic algorithm which can also be used for optimizing time-dependent processes (gates) and their sequences that implement various quantum computing protocols.

The list of the previously discussed decoherence suppression/avoidance techniques includes such proposals as error correction [1], encoding into decoherence-free subspaces (DFS) [2], dynamical decoupling/recoupling schemes (e.g., "bang-bang" pulse sequences) [3], and the use of the quantum Zeno effect [4].

However, none of the above recipes are entirely universal and their actual implementation may be hindered by such factors as a significant encoding overhead that puts a strain on the (still scarcely available) quantum computing resources (error correction), a stringent requirement of a completely symmetrical qubits’ coupling to the dissipative environment (decoherence-free subspaces), a need for the frequency of the control pulses to be well in excess of the environment’s bandwidth (dynamical decoupling/recoupling) or an ability to perform a continuous high precision measurement (quantum Zeno effect). While being more feasible in some (most notably, liquid-state NMR and trapped-ion) as compared to other quantum computing designs, the implementation of the above approaches might be particularly challenging in solid-state architectures. It is for this reason that augmenting the above techniques with a many-body physics-conscious engineering of robust multi-qubit systems and a systematic approach to choosing the optimal (coherence-wise) values of their microscopic parameters appears highly desirable.

To this end, in the present work we explore yet another possibility of thwarting decoherence by virtue of permanent (albeit tunable) inter-qubit couplings. We find that, despite its being often thought of as a nuisance to be rid of, the properly tuned qubits’ “cross-talk” may indeed provide for an additional layer of protection against decoherence. Specifically, we focus on the problem of preserving an arbitrary initial state (”quantum memory”) of a basic two-qubit register during its idling period between consecutive gate operations. Moreover, the optimization method employed in our work can be further extended to the case of time-dependent two-qubit gates which can alone suffice to perform universal quantum operations.

The previous analyses of the problem in question [5,6] have been largely limited to the symmetries of the underlying Hamiltonians and the parameter values corresponding to the presently available experimental setups [7–9]. As a result, they have not systematically addressed the issue of a possible role of the inter-qubit couplings in reducing decoherence. In contrast, our discussion pertains to a more general two-qubit Hamiltonian

\[
\hat{H} = \sum_{a=x,y,z} \sum_{i=1,2} \hat{\sigma}_a^i (B_a^i + \hat{h}_a^i(t)) + J_{ab} \hat{\sigma}_a^1 \otimes \hat{\sigma}_b^2, \tag{1}
\]

where each of the two qubits described by an independent triplet of the Pauli matrices \(\hat{\sigma}_a^i\) \((i = 1, 2)\) is subject to a local magnetic field comprised of the constant \(\hat{B}_a^i = (\Delta_a^i, 0, \epsilon_a^i)\) and fluctuating \(\hat{h}_a^i(t) = (0, 0, h_a^i(t))\) components, the latter representing two uncorrelated \((< h_1(t) h_2(0) > = 0)\) dissipative reservoirs. It is well known that the case of independent reservoirs present a significantly greater challenge than the greatly simplifying assumption of the highly correlated ones where the standard DFS can be readily found [2].

As regards the symmetry of the qubits’ interaction, we restrict ourselves to the diagonal terms \(\sigma_a^1 \otimes \sigma_b^2\) and the associated parameters \(J_{ab} = J_{ab} \hat{\sigma}_a^1 \otimes \hat{\sigma}_b^2\). Thus, we exclude all the non-diagonal couplings between \(\sigma_a^1\) and \(\sigma_b^2\) with \(a \neq b\) which, albeit possible in principle, do not normally occur in any of the known qubit designs.

As we argue below, the preferred symmetries of the inter-qubit couplings can be found solely from the spectral analysis of the noiseless part of the Hamiltonian (1). This suggests that our results should be largely insensitive to the approximation used for treating the effect of the noisy reservoirs.

To that end, we resort to the standard Bloch-Redfield (BR), i.e. a weak-coupling and Markovian approximation, as does most of the previous work on the subject [5]. Although the BR approximation is known to become potentially unreliable in the (arguably, most important for quantum computing-related applications) short-time (as compared to...
the pertinent decoherence times) limit, the main advantage of the BR framework is a relative physical transparency of its results. Moreover, in light of the usual robustness of any symmetry-related conclusions, such as those to be described below, we anticipate that a more sophisticated analysis (e.g., akin that of Refs. [10]) would fully corroborate the BR results.

In the standard singlet/triplet basis formed by the states: \(|↑↑⟩=(1,0,0,0)\), \(|↓↓⟩=(0,0,0,1)\), and \(|↑↓⟩=(0,1,0,0)\), \(|↓↑⟩=(0,0,1,0)\), the noiseless part of the Hamiltonian (1) takes the form

\[
\hat{H}_0 = \begin{pmatrix}
J_z + \epsilon & \Delta & J_y - J_z & -\Delta^-

J_x - \epsilon & -\Delta^- & J_x + J_y & \Delta^-

J_y - J_z & J_x - J_y & -\Delta^- & \epsilon

-\Delta^- & \Delta^- & \epsilon & J_y - J_z - J_x
\end{pmatrix}
\]  

(2)

where \(\Delta = (\Delta_1 + \Delta_2)/\sqrt{2}\), \(\Delta^- = (\Delta_1 - \Delta_2)/\sqrt{2}\), \(\epsilon = \epsilon_1 + \epsilon_2\) and \(\epsilon^- = \epsilon_1 - \epsilon_2\).

The BR equations for the reduced two-qubit (size \(4 \times 4\)) density matrix \(\hat{\rho}(t)\) takes a particularly simple form in the basis of eigenstates of Eq.(2) \([5,11]\)

\[
\dot{\rho}_{nm}(t) = -i\omega_{nm} \rho_{nm}(t) - \sum_{k,l} R_{nmkl} \rho_{kl}(t)
\]  

(3)

where \(\omega_{nm} = (E_n - E_m)/\hbar\) are the transition frequencies, and the partial decoherence rates

\[R_{nmkl} = \delta_{lm} \sum_r \Lambda_{nrrk} + \delta_{nk} \sum_r \Lambda^*_{lrrm} - \Lambda^*_{lmnk} - \Lambda^*_{kmln}\]  

(4)

are given by combinations of the matrix elements of the relaxation tensor \([11]\)

\[
\Lambda_{lmnk} = \frac{1}{8} S(\omega_{nk})[\sigma^1_{z,lm} \sigma^1_{z,nk} + \sigma^2_{z,lm} \sigma^2_{z,nk}] + \frac{i}{4\pi} P \int_{0}^{\omega_{nc}} S(\omega) d\omega \frac{\omega_{nk}}{\omega^2 - \omega_{nk}^2} [\sigma^1_{z,lm} \sigma^1_{z,nk} + \sigma^2_{z,lm} \sigma^2_{z,nk}]
\]  

(5)

determined by the products between the matrix elements \(\sigma^i_{z,nk}\) computed in the eigenbasis of the noiseless Hamiltonian (2) and the spectral density of the reservoirs \(S(\omega) = \int_{0}^{\omega_{nc}} d\omega e^{i\omega t} \langle \{h_i(t), h_i(0)\} \rangle (i = 1, 2)\).

As in Refs. [5], we choose \(S(\omega)\) to be Ohmic, \(S(\omega) = \alpha \omega \coth(\omega/2T)\), thus justifying the applicability of the Fermi’s Golden rule-based expression (5) (hereafter we use the units \(\hbar = k_B = 1\)).

Obviously, the optimal choice of the Hamiltonian (2) would be the one that provides for the lowest decoherence rates composed of the matrix elements (5). The analysis of the expressions (5) reveals that their real parts tend to decrease monotonically with the increasing length \(J = |\vec{J}|\) of the vector \(\vec{J} = (J_x, J_y, J_z)\). However, in all the practically important cases (especially, in the before mentioned Josephson junction-based designs [7–9]) where the qubits emerge as effective (as opposed to genuine) two-level systems, an unlimited increase of \(J\) is not possible without leaving the designated "qubit subspace" of the full Hilbert space of the system. Besides, the coupling strength errors that are likely to occur in any realistic setup tend to increase with \(J\) as well. Therefore, in what follows we fix the length of the vector \(\vec{J}\) and search for an optimal configuration of its components, allowing for either sign of the latter.

Being proportional to the noise spectral density evaluated at frequencies corresponding to the transitions between different pairs of energy levels, the matrix elements (5), just like \(S(\omega_{mn})\) itself, attain their minimum values (which are proportional to the reservoirs’ temperature \(T\)) at \(\omega_{mn} = 0\). Thus, one might expect that the relaxation processes will be quenched and the decoherence rates will become suppressed, should some of the transition frequencies happen to vanish.

Such a behavior would occur if any two of the energy levels of the Hamiltonian (2) became degenerate or, better yet, all the four levels of (2) became doubly (or even four-fold) degenerate. In this case, the lower pair of the degenerate levels could be viewed as an effective ("logical") qubit which is protected from decoherence by an energy gap separating it from the upper pair of levels, resulting in an exponential suppression of the relaxation rates at low temperatures. Then, by encoding quantum information into this logical qubit’s subspace, one can greatly suppress its decay, any residual decoherence being solely due to pure dephasing controlled by \(S(0) \approx aT\).

Next, we apply this general argument to the practically important case of identical qubits (\(\Delta_1 = \Delta_2 = \Delta/2\)) which are both tuned to the "co-resonance" point (\(\epsilon_1 = \epsilon_2 = 0\)).

We note, in passing, that a special significance of this parameter regime as a potentially most coherence-friendly one is also evidenced by the experiments on the Josephson charge-phase qubits [9].

At the co-resonance point, the spectrum of Eq.(2) and the corresponding eigenvectors are given by the following simple expressions: \(E_1 = J_x + K, E_2 = -(J_x + J_y + J_z), E_3 = -J_x + J_y + J_z, E_4 = J_x - K\) and
\[ |\chi_1 > = (1, (J_y - J_z + K)/\Delta, 1, 0)/\sqrt{2K(K + J_y - J_z)}, \quad |\chi_2 > = (0, 0, 0, 1), |\chi_3 > = (-1, 0, 1, 0)/\sqrt{2}, \quad |\chi_4 > = (1, (J_y - J_z - K)/\Delta, 1, 0)/\sqrt{2K(K - J_y + J_z)}, \]

The only attainable incidence of double spectral degeneracy (\(E_1 = E_3, E_2 = E_4\) or \(E_1 = E_2, E_3 = E_4\)) can occur for \(J_{y,z} > 0\) or \(J_{y,z} < 0\), respectively. Resolving the above conditions, we find that the conjectured optimal configuration of the inter-qubit couplings must obey the equations

\[ J_{opt}^x = 0, \quad 2J_{opt}^y J_{opt}^z = \Delta^2 \] (6)

By taking into account the normalization of the vector \(\mathbf{j}\) and Eq.(6) we finally obtain \(J_{opt}^x = \pm \frac{1}{2}(\sqrt{J_z^2 + \Delta^2} - \sqrt{J_z^2 - \Delta^2}), \quad J_{opt}^y = \pm \frac{1}{2}(\sqrt{J_z^2 + \Delta^2} + \sqrt{J_z^2 - \Delta^2}).\) Notably, this result shows that the conjectured optimal regime can only be achieved for sufficiently strong couplings (\(J > \Delta\)).

Having elucidated the physical content of the expected \(J_{opt}\), we now confirm our predictions by applying a direct optimization procedure based on a genetic algorithm [12]. The latter starts out with random sets of parameters \(J\) which, in the language of Ref. [12], constitute an initial "population". In the course of the optimization procedure, the worst configurations are discarded and certain recombination procedures ("mutations") are performed on the rest of the population. The new solutions ("offsprings") are selected according to the values of the chosen fitness function. The iterations continue until the set of parameters converges to a stable solution representing the sought-after optimal configuration \(J_{opt}\).

As a fitness function we choose such a customary quantifier of the register’s performance as purity \(P(t) = \frac{1}{16} \sum_{j=1}^{16} Tr(|\rho(t)|^2(t))\) averaged over the solutions \(\rho(t)\) of Eq.(3) with the initial conditions \(\rho(0) = |\psi^\dagger_0| < |\psi_0^\dagger|\) given by the 16 product states \(|\psi_a\rangle = |\psi_{a,1}\rangle \otimes |\psi_{b,2}\rangle\). \(\psi_1 := |↓\rangle, |\psi_2 := |↑\rangle, |\psi_3 := 1/\sqrt{2}|↓\rangle + |↑\rangle, |\psi_4 := 1/\sqrt{2}|↓\rangle - |↑\rangle\).

In Fig.1 we plot the purity decay rate \(|dP(t)/dt|\) as a function of \(J_y\) and \(J_z\) for \(\epsilon_j = J_z = 0\) and \(\alpha = 10^{-3}\). For different values of \(J\), the minima of this two-parameter function collapse onto a hyperbola in the \(J_y - J_z\) plane which appears to be described by Eq.(6) within the accuracy of our numerical solution of the BR equations (3).

By varying the be observed that, consistent with our preliminary insight into the mechanism of decoherence suppression, the effect of the interaction-induced double spectral degeneracy appears to be most pronounced at \(T \ll \Delta\), while at higher \(T\) the landscape shown in Fig.1 flattens out, thus making the optimization less effective.

We also observed that the use of an alternate performance measure, fidelity \(F = \frac{1}{16} \sum_{j=1}^{16} Tr\{\rho(t)|\rho_0(t)\}\) where \(\rho_0(t)\) represents the unitary evolution of an initial state \(\rho(0)\), yields the same results.

In order to further illustrate our findings, in Fig.2 we present the behavior of a typical matrix element \(R_{14123}\) as a function of a single parameter \(w\) introduced by the relations: \(J_z = 0, J_y = 0.25\Delta w, J_z = 1.98\Delta w\). It can be readily seen from Fig.2 that at the double degeneracy point \(w = 1\) which corresponds to \(J = \sqrt{J_y^2 + J_z^2} = 2\Delta\) the matrix element \(R_{14123}\) drops to its lowest value.

By contrast, in the non-degenerate case exemplified by the Ising configuration \(J_y = J_z = 0, J_z = 2\Delta w\) the above dramatic drops now absent, regardless of the value of \(J = J_z\). In Fig.3 we contrast the purity \(P(t)\) computed for the optimal configuration with the results obtained in the Ising-, XY-, and Heisenberg-symmetrical, as well as non-interacting cases. These results suggest a systematic way of constructing a class of highly robust input states that undergo a slow decay governed by pure dephasing (no relaxation), as compared to the case of general encoding.

For \(J_{y,z} > 0\) a natural candidate for an orthogonal pair of such states is presented by the (anti)symmetrical combination \(|\chi_{stable}^+ > = (|\chi_2 > \pm |\chi_4 > )/\sqrt{2}\) of the lower pair of the degenerate \((E_2 = E_4)\) energy levels. The corresponding initial density matrix is characterized by the only non-zero entries \(\rho_{22}(0) = \pm \rho_{44}(0) = \pm \rho_{42}(0) = 1/2\).

In Fig.4 we plot the decay rate of the purity \(dP(t)/dt\) as a function of the parameter \(w\) for the initial state \(|\chi_{stable}^+ >\). Note that at the double degeneracy point \((w = 1)\) the purity decay rate is reduced by several orders of magnitude.

The high robustness of a degenerate ground state facilitated by the presence of the energy gap in our two-qubit system is somewhat reminiscent of the notion of "supercoherent" subspaces introduced in the abstract setting in Ref. [13]. However, despite a certain resemblance between the two, our implementation of a robust logical qubit is different in a number of aspects.

For one, the construction proposed in Ref. [13] requires at least four physical qubits which interact by virtue of the Hamiltonian whose spectrum can be classified by the eigenvalues of the total angular momentum of the system. Then the degenerate singlet ground state behaves as an "error detecting code" that can only lose its coherence by absorbing energy from the reservoirs, the rate of which process appears to be exponentially suppressed for \(T \rightarrow 0\).
In contrast, our logical qubit consists of only two physical qubits interacting via a generic (not necessarily spin-rotationally invariant) Hamiltonian, and its decoherence rate is limited by pure dephasing whose contribution remains linear in $T$ even at the lowest temperatures. However, albeit not providing an equally strong protection for the logical qubit itself, in our case the double degeneracy of the two-qubit spectrum does improve the gate performance for general encoding, as manifested by the gate characteristics averaged over different initial states.

One can also identify the initial states whose decay can only get worse upon increasing the qubit coupling. These fragile states belong to the subspace spanned by the complementary pair of eigenstates $|\chi_1\rangle,|\chi_3\rangle$.

Lastly, a word of caution is in order. Conceivably, any incidence of degeneracy may invalidate the use of the Golden rule-based Eq.(5) whose (sufficient) conditions of applicability requires that $max|R_{mnkl}| \ll min|\omega_{mn}|$. However, we found that our numerical solution remains stable even in a close vicinity of the double degeneracy point.

The optimal configuration $J^{\text{opt}}$ shows no abrupt changes upon varying the strength of the coupling $\alpha$ from its lowest values that do comply with the above sufficient conditions all the way down to the parameter range where $min|\omega_{mn}| \ll max|R_{mnkl}| \ll max|\omega_{mn}|$.

To summarize, in the present work we demonstrated that the properly chosen permanent inter-qubit couplings can provide a new means of protecting quantum registers against decoherence. While being relatively unexplored in the mainstream quantum information proposals, a similar idea has been in the focus of a number of scenarios exploring the possibility of a natural emergence of logical qubits and the conditions facilitating fault-tolerant computations in strongly correlated spin systems.

In the existing proposals [14] the prototype logical qubits are envisioned as topological ground and/or quasi-particle bulk/edge states of rather exotic chiral spin liquids. Thus, despite their enjoying an exceptionally high degree of coherence, the (topo)logical qubits require an enormous overhead in encoding, since creating only a handful of such qubits takes a macroscopically large number of interacting physical ones. Besides, the nearly perfect isolation of (topo)logical qubits from the environment can also make initialization of and read-out from such qubits rather challenging. Nonetheless, our results show that a somewhat more modest idea of augmenting the other decoherence-suppression techniques with appropriate tailoring of the inter-qubit couplings might still result in a substantial improvement of the quantum register's performance.

The numerical optimization method employed in this work can be further extended to the case of time-dependent two-qubit gates as well as beyond the Bloch-Redfield approximation and/or the assumption of the Ohmic dissipative environments.

As regards the gate optimization, we find that in many cases the best performance of a two-qubit register can be achieved with a rectangular pulse $J_a(t) = J_a \Theta(T - t) \Theta(t)$ where the choice of the coupling parameters $J_a$ facilitates the degeneracy of the lowest pair of energy levels. These results will be presented elsewhere [15].

The rapid pace of the technological progress in solid-state quantum computing gives one a hope that the specific prescriptions towards building robust qubits and their assemblies discussed in this work can be implemented in future devices. In this regard, very promising appear to be the phase [7], charge [8] and charge-phase [9] superconducting qubit architectures where, in principle, one can achieve any desired symmetry of the interaction terms in Eq.(1) by merely tuning the capacitive and inductive couplings between different Josephson junctions which implement the physical qubits.

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FIG. 1. Purity decay rate $dP/dt$ as a function of $J_y$ and $J_z$ for $J_x = 0$ and $T = 10^{-5} \Delta$. 
FIG. 2. Partial dephasing rates exemplified by a typical matrix element $R_{4123}$ as a function of the parameter $w$ computed for $T = 10^{-5} \Delta$, $\vec{J} = (0, 0.25\Delta w, 1.98\Delta w)$ (solid line) and $\vec{J} = (0, 0, 2\Delta w)$ (dashed line).

FIG. 3. Purity as a function of time for zero coupling ($\vec{J} = 0$, thin solid line); Ising ($\vec{J} = (J, 0, 0)$, dotted line and $\vec{J} = (0, J, 0)$, dashed line), XY ($\vec{J} = (J/\sqrt{2}, J/\sqrt{2}, 0)$, dash-double dotted line), Heisenberg ($\vec{J} = (J/\sqrt{3}, J/\sqrt{3}, J/\sqrt{3})$, dash-dotted line), and the optimal configuration($\vec{J} = \vec{J}_{\text{opt}}$, solid line). Here $J = 2\Delta$ and $T = 10^{-7} \Delta$.

FIG. 4. Purity decay rate $dP/dt$ as a function of the parameter $w$ for $\vec{J} = (0, 0.25\Delta w, 1.98\Delta w)$for $T = 10^{-5} \Delta$ (solid line) and $T = \Delta$ (dashed line).