Generalized mass formula for non-strange and hyper nuclei with SU(6) symmetry breaking

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A simultaneous description of non-strange nuclei and hypernuclei is provided by a single mass formula inspired by the spin-flavour SU(6) symmetry breaking. This formula is used to estimate the hyperon binding energies of Lambda, double Lambda, Sigma, Cascade and Theta hypernuclei. The results are found to be in good agreement with the available experimental data on ‘bound’ nuclei and relativistic as well as quark mean field calculations. This mass formula is useful to estimate binding energies over a wide range of masses including the light mass nuclei. It is not applicable for repulsive potential.

Keywords : Hypernuclei, Hyperon binding energy, Exotic nuclei, Mass formula, Separation Energy.

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The hypernuclear physics is of great importance in many branches of physics. Of particular interest is the understanding of strange particles in baryonic matter, since many questions in heavy-ion physics, particle physics and astrophysics are related to the effect of strangeness (S) in nuclear matter. Moreover, the contribution of the hyperons strongly influences the mass of neutron stars as well. In the past decade considerable amount of spectroscopic informations were accumulated experimentally on the $\Lambda^0$ ($S=-1$)hypernuclei. The $\Lambda$ separation energies were determined for the ground states of about 40 $\Lambda$ hypernuclei including several double-$\Lambda$ hypernuclei [1], [2]. Doubly strange hypernuclei also arise in a form of $\Xi^-$ ($S=-2$) hypernuclei and were studied both theoretically [3] and experimentally [1], [4], [5] in a limited number of nuclei. Sigma hyperons exhibit interesting property in its interaction with nuclei. Several studies suggest that due to strongly repulsive $\Sigma$-nucleus potential Sigmas are unbound in nuclei, except for the very special case of nuclei with mass number $A=4$ [6]. Existence of a $T=1/2$, $S=0$ bound state in $^6\Sigma He$ was first predicted by Harada et al. [7] and has been found experimentally [8], [9], [10], [11]. The $1/A$ dependence of the Lane term present in the isospin-dependent $\Sigma$-nucleus potential [7] reduces the possibility of finding bound $\Sigma$-hypernuclei with large $A$.

In this scenario, an exotic hyperon $\Theta^+$ ($S=+1$, mass $\sim 1530$ MeV, width $< 15$ MeV) with exotic pentaquark structure was predicted in 1997 [12]. Announcement of its discovery at Spring-8, Japan [13] sparked an avalanche of activities in the field of hyperons and hypernuclei [14]. Now a question has arisen whether the $\Theta^+$ hyperon really exists [15] and if so, whether it will be bound in a nucleus [16], [17]. Calculations in a relativistic mean-field formalism (RMF) suggest that as there is an attractive $\Theta^+$-nucleus interaction, the $\Theta^+$ particle can be bound in nuclei and, the $\Theta^+$ hypernuclei would be bound more strongly than $\Lambda$ hypernuclei [18]. Recent calculations in quark mean-field (QMF) model also support existence of bound $\Theta^+$ hypernuclei and predict that in comparison to $\Lambda$ hypernuclei more bound states are there in $\Theta^+$ hypernuclei [19]. While a search for bound Theta hypernuclei is on, for a large number of hypernuclei, including double Lambdas, Cascade and Sigmas, more experimental data are needed. One also needs to have an apriori estimation of their possible binding energies in a wide mass range for planning of the experiment and to locate the peaks in the experimental missing mass spectra.

Although the $\Lambda$, $\Sigma$, $\Xi$ and $\Theta$ hyperons are all baryons, no single mass formula exists which can predict the binding energies of all of them on the same footing as the non-strange nuclei. In this work we present a generalized mass formula for both the strange and non-strange nuclei which does not disregard the normal nuclear matter properties. It is a straight forward equation and binding energy of non-strange as well as bound hypernuclei can be estimated by using this single equation. Its predictions compare well with the available experimental data. However it is not applicable for repulsive potential.

Earlier, Dover et al [3] prescribed two separate mass formulae for $\Lambda$ and $\Xi$ hypernuclei by introducing several volume and symmetry terms in Bethe-Weizsäcker mass formula (BW). The BW formulae given in Ref. [3] were developed

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for a much broader context (multi-Lambda systems or strange hadronic matter) with parameters inspired on the strengths of the YN and YY interactions. In this respect, the single-Lambda BW equation given in [3] is a limit of a more general formula. The BW equation proposed in Ref. [20] is inspired by the spin-flavour SU(6) symmetry and the pairing term is replaced by the expectation value of the space-exchange or, Majorana operator. The Majorana operator in the ground state configurations of single- and double-Lambda hypernuclei were expressed in terms of N, Z, and Lambda number. A strangeness dependent symmetry breaking term was also incorporated. This prescription gave reasonable description of the experimental data on single-and double-Lambda hypernuclei separation energies. As none of these formulations had explicit hyperon mass consideration, they can not be used for binding energy calculation of other hypernuclei. Both the formulae have some other limitations which will be discussed later.

The Wigners SU(4) symmetry arises as a result of the combined invariance in spin (I) and isospin (T). In order to incorporate the strangeness degree of isospin, SU_F(2) is replaced by SU_F(3) and the combined spin(I)-flavour(F) invariance gives rise to the SU(6) classification of Gursey and Radicati [21]. The SU_F(3) symmetry breaks by explicit consideration of a mass dependent term in a mass formula. The SU(6) symmetry breaking is related to different strengths of the nucleon-nucleus and hyperon-nuclear interactions and has important consequences. For example, although small, the $\Sigma - \Lambda$ mass difference figures prominently in the smallness of the $\Lambda$-nuclear spin-orbit interaction [22].

In this work the non-strange normal nuclei and strange hypernuclei are treated on the same footing with due consideration to SU(6) symmetry breaking. The generalization of the mass formula is pursued starting from the modified-Bethe-Weizs¨acker mass formula (BWM) preserving the normal nuclear matter properties. The BWM is basically the Bethe-Weizsacker mass formula extended for light nuclei [23], [24], [25], [26] which can explain the gross properties of binding energy versus nucleon number curves of all non- strange normal nuclei from Z=3 to Z=83. In BWM, the binding energy of a nucleus of mass number A and total charge Z is defined as

$$B(A, Z) = a_v A - a_s A^{2/3} - a_c Z(Z - 1)/A^{1/3} - a_{sym} (N - Z)^2/[(1 + e^{-A/k})A] + \delta_{new},$$

where for normal nuclei N and Z are the number of neutrons and protons respectively and

$$a_v = 15.777 \text{ MeV}, \ a_s = 18.34 \text{ MeV}, \ a_c = 0.71 \text{ MeV}, \ a_{sym} = 23.21 \text{ MeV and } k = 17,$$

and the pairing term,

$$\delta_{new} = (1 - e^{-A/c})\delta, \ \text{where } c = 30,$$

and

$$\delta = 12A^{-1/2} \ \text{for even neutron - even proton number}, \ = -12A^{-1/2} \ \text{for odd neutron - odd proton number}, \ = 0 \ \text{when total neutron plus proton number is odd.}$$

Hypernuclei are found to be more bound than normal nuclei. A systematic search of experimental data of hyperon separation energy ($S_Y$) for $\Lambda$, $\Lambda\Lambda$, $\Sigma^0$ and $\Xi^-$ hypernuclei leads to a generalised mass formula for hyper and non-strange nuclei which will be, henceforth, called the BWMH. The experimental $S_Y$ of the $\Lambda$-hypernuclei (for which experimental data are available over a wide mass range) is found to follow a relation $S_Y \propto A^{-2/3}$ [27]. The available data for Cascade hypernuclei also follow a similar $S_Y \propto A^{-2/3}$ trend but with different slope. Unlike Levai et al. [20], the SU(6) symmetry breaking term represented by the strangeness is taken here to be inversely proportional to $A^{2/3}$. Explicit inclusion of the pairing term partly accounts for the Majorana term while still preserving the nuclear saturation properties. An additional mass dependent term breaks the SU_F(3) symmetry.

In BWMH the hypernucleus is considered as a core of normal nucleus plus the hyperon(s) and the binding energy is defined as

$$B(A, Z) = a_v A - a_s A^{2/3} - a_c Z(Z - 1)/A^{1/3} - a_{sym} (N - Z_c)^2/[(1 + e^{-A/k})A] + \delta_{new}$$

$$+ n_Y [c_0 (m_Y - c_1 - c_2) |S| / A^{2/3}],$$

where $n_Y$ = number of hyperons in a nucleus, $m_Y$ = mass of the hyperon in MeV, $S$ = strangeness of the hyperon and mass number $A = N + Z_c + n_Y$ is equal to the total number of baryons. $N$ and $Z_c$ are the number of neutrons and protons respectively while the $Z$ in eqn.(5) is given by

$$Z = Z_c + n_Y q$$

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where $q$ is the charge number (with proper sign) of hyperon(s) constituting the hypernucleus. For non-strange (S=0) normal nuclei, $Z = Z_c = Z$ as $n_Y = 0$. The choice of $\delta_{\text{new}}$ value depends on the number of neutrons and protons in both normal and hypernuclei. For example, in case of $^9\Lambda Li$ the neutron number $N = 5$(odd), proton number $Z_c = 3$(odd), and $n_Y = 1$. Therefore, $\delta = -12A^{-1/2}$ as the $(N, Z_c)$ combination is odd-odd, although the total baryon number $A = A = N + Z_c + n_Y = 9$(odd). Whereas, for non-strange normal $^9Li$ nucleus $\delta = 0$ for $A = 9$(odd).

In eqn.(5), the constants $c_0, c_1$ and $c_2$ have been fixed from an empirical fit to the experimental values of $S_Y$ for thirty-five $\Lambda$, three $\Lambda - \Lambda$ and six $\Xi^-$ hypernuclei (Table 1).

| Hypernuclei | $S_Y$ MeV | $\Delta S_Y$ MeV | $n_Y$ | $NS$ | Ref. |
|-------------|-----------|------------------|-------|------|-----|
| $^4\Lambda H$ | 2.04 | 0.04 | 1 | -1 | [1] |
| $^4\Lambda He$ | 2.39 | 0.03 | 1 | -1 | [1] |
| $^5\Lambda He$ | 3.12 | 0.02 | 1 | -1 | [1] |
| $^6\Lambda He$ | 4.18 | 0.10 | 1 | -1 | [1] |
| $^7\Lambda He$ | 5.23 | 0.00 | 1 | -1 | [28] |
| $^8\Lambda He$ | 7.16 | 0.70 | 1 | -1 | [1] |
| $^6\Lambda Li$ | 4.50 | 0.00 | 1 | -1 | [1] |
| $^7\Lambda Li$ | 5.58 | 0.03 | 1 | -1 | [1] |
| $^8\Lambda Li$ | 6.80 | 0.03 | 1 | -1 | [1] |
| $^9\Lambda Li$ | 8.50 | 0.12 | 1 | -1 | [1] |
| $^7\Lambda Be$ | 5.16 | 0.08 | 1 | -1 | [1] |
| $^8\Lambda Be$ | 6.84 | 0.05 | 1 | -1 | [1] |
| $^9\Lambda Be$ | 6.71 | 0.04 | 1 | -1 | [1] |
| $^{10}\Lambda Be$ | 9.11 | 0.22 | 1 | -1 | [1] |
| $^9\Lambda B$ | 8.29 | 0.18 | 1 | -1 | [1] |
| $^{10}\Lambda B$ | 8.89 | 0.12 | 1 | -1 | [1] |
| $^{11}\Lambda B$ | 10.24 | 0.05 | 1 | -1 | [1] |
| $^{12}\Lambda B$ | 11.37 | 0.06 | 1 | -1 | [1] |
| $^{12}\Lambda C$ | 10.76 | 0.19 | 1 | -1 | [1] |
| $^{13}\Lambda C$ | 11.69 | 0.12 | 1 | -1 | [1] |
| $^{14}\Lambda C$ | 12.17 | 0.33 | 1 | -1 | [1] |
| $^{14}\Lambda N$ | 12.17 | 0.00 | 1 | -1 | [1] |
| $^{15}\Lambda N$ | 13.59 | 0.15 | 1 | -1 | [1] |
The convention used in Table 1 (as well as in the text) is $A^Y_qZ$ where $Z$ is the net charge, $Y$ is the hyperon type with charge $q$ and, $A$ is the total number of baryons [1,7]. There exists another convention in the literature in which one uses $A^Y_qZ_c$ for denoting the hypernuclei, where $Z_c$ is the number of protons. Therefore, according to this other convention, the hypernucleus $^4\Sigma^+\text{He}$ of the present manuscript will be $^4\Sigma^+H$. Also, $^{10}\Theta^+\text{Li}$ would be $^{10}\Theta^+\text{He}$, and $^{11}\Theta^+\text{Be}$ would be $^{11}\Theta^+\text{Li}$.
Here we calculate the root mean square deviation i.e., r.m.s. \( \langle \sigma \rangle \) for the hyperon separation energies where, 

\[
\sigma^2 = \left( \frac{1}{N}\sum [(S_Y)_{Th.} - (S_Y)_{Exp.}]^2 \right).
\]

Due to small number of data, simultaneous 3 parameter search does not yield any meaningful result. After many trial searches we fixed the value of \( c_0 = 0.0335 \) and made two parameter search using 35 \( \Lambda \) hypernuclei data which yielded \( c_1 = 27.59 \) (45) and \( c_2 = 48.6 \) (21) with r.m.s. = 1.416 MeV. For better results from actual plotting of the graphs, \( c_1 = 26.7 \) was chosen and, keeping it fixed, a two parameter search with 35 \( \Lambda \) hypernuclei data lead to \( c_0 = 0.0327 \) (4) and \( c_2 = 48.6 \) (21) with r.m.s. deviation 1.416 MeV. Finally we fixed \( c_1 = 26.7 \) and \( c_2 = 48.7 \) and performed a one parameter search with 35 \( \Lambda \) hypernuclei data. This leads to \( c_0 = 0.0327 \) (2) with r.m.s. = 1.375 MeV. We fixed \( c_0 = 0.0335 \) which yielded best results for the two parameter fit. Finally, from the overall best fit to all the data (r.m.s. = 1.4 MeV), we choose \( c_0 = 0.0335; c_1 = 26.7 \) and \( c_2 = 48.7 \) in eqn. (5).

\[
B(A, Z) = a_e A - a_s A^{2/3} - a_c Z (Z - 1) / A^{1/3} - a_{sym} (N - Z_c)^2 / (1 + e^{-A/k}) A + \delta_{new} \\
+ n_Y [0.0335M_Y - 26.7 - 48.7 | S |_/A^{2/3}],
\]

(7)

The hyperon separation energy \( S_Y \) defined as

\[
S_Y = B(A, Z)_{hyper} - B(A - n_Y, Z_c)_{core},
\]

(8)

is the difference between the binding energy of a hypernucleus and the binding energy of its non-strange core nucleus.

It is interesting to note that this single equation yields the values of \( S_Y \) in reasonable agreement with the available experimental data of all known bound hypernuclei. Fig. [1] shows plots of \( S_Y \) versus \( A \) for \( \Lambda \) and \( \Lambda \Lambda \) hypernuclei which are in good agreement with the experimental data [1], [2]. The recent discovery of the \( ^{10} \Lambda \) nucleus points to a value of \( S_Y \approx 10 - 12 \) MeV [35] where the BWMH predicts \( S_Y(^{10} \Lambda Li) = 11.4 \) MeV. The BWMH prediction of \( S_Y \) for the \( ^{3} \Lambda \) He also agrees well [Fig. 1(b)] with the recent experimental value 7.3 \pm 0.2 MeV of Takahashi et al. [2].

Harada et al. investigated the structure of \( ^{3} \Lambda \) He by the coupled-channel calculation between the \( ^{3} \)He + \( \Sigma^+ \) and the \( ^{3} \)He + \( ^{3} \)He channels and predicted the \( \Sigma^+ \) binding energy in \( ^{3} \)He to be 3.7 to 4.6 MeV [7]. Available experimental binding energy values are \( 4.4 \pm 0.3 \) ± 1 MeV [8], \( 2.8 \pm 0.7 \) MeV [9], \( 4 \pm 1 \) MeV [10]. The BWMH predicts binding energy of \( ^{3} \)He (\( m_Y = 1192.55 \) MeV) and \( ^{3} \)He (\( m_Y = 1189.37 \) MeV) in \( ^{3} \)He (= \( ^{3} \)He + \( ^{3} \)He) as 2.69 MeV and \( ^{3} \)He (= \( \Sigma^+ + ^{3} \)He) as 1.6 MeV respectively. Search for heavier \( \Sigma \) hypernuclei has been carried out by several authors [6], [8], [9], [10], [11], [36] without success. P. K. Saha et al. studied inclusive \((\pi^-, K^+)\) spectra on C, Si, Ni, In and Bi targets and concluded that a \( \Sigma \)-nucleus potential is strongly repulsive in such relatively heavy nuclei [6]. It has been suggested that the binding of the Sigma in the light hypernuclear species like \( ^{3} \Lambda \) He is mainly due to a strong isovector term of the Sigma-nucleus potential which emphasizes the differences of the Sigma-N interaction in the isospin \( T = \frac{1}{2} \) and the \( T = \frac{3}{2} \) channels. This term is not present in the BWMH formula. Nevertheless, without changing any parameter, this single general equation (BWMH) reproduces the binding energy of the 'bound' \( ^{3} \Lambda \) He hypernuclei. As mentioned before, the theoretical formulation suggests that the binding energy of the light Sigma hypernuclear species is provided by the isospin dependent Lane term (associated to the isospin asymmetry of the core) not included in the BWMH formula. Thus the binding energy of BWMH obtained for light hypernuclei comes from the isospin-averaged (i.e. isospin independent) term which provides attraction, while recent analysis on heavier Sigma hypernuclei (dominated by this isospin independent term of the Sigma-nucleus potential) suggest repulsion. For \( ^{3} \)He, \( ^{3} \)He, \( ^{3} \)He, and \( ^{3} \)He hypernuclei the separation energies predicted by BWMH are -3.53 MeV, -4.62 MeV and -3.37 MeV respectively indicating that these light Sigma hypernuclei would be unbound, even if the potential is attractive. In fact, so far no bound state of these hypernuclei could be found in the experiment. Indeed, further data on Sigma hypernuclei are necessary to determine more conclusively whether the Sigma feels attraction or repulsion.

Recent experimental data for \( \Xi^- + ^{12}C \) and \( \Xi^- + ^{11}B \) are not conclusive and predict values in the range of 3 to 10 MeV depending on the \( \Xi^- \) well depth [4], while, Khaustov et al. suggests potential depth to be less than 20 MeV [5]. In absence of any new conclusive data, we have used the available data tabulated in Ref. [1]. BWMH estimations for the \( \Xi^- \) separation energies compare well with the experimental values (Fig. 2a). As no experimental data exists so far on the bound Theta hypernuclei, the \( \Theta^+ \) separation energies are compared with the recent theoretical predictions and the \( S_Y \) of \( \Theta^+ \) are found to be close to the quark mean field (QMF) calculations [19].

Analysis of each term of eqn. (8) reveals that the \( \delta \) term has very little contribution (positive) in \( S_Y \). The \( a_{sym} \) term difference also contributes positive but, quite small except at very high \( |(N - Z_c)| \). The \( a_e \) term difference (asd) is always negative and being more so at small \( A \) (Fig. 3). For \( q = 0 \), the Coulomb term difference (acd) contributes positive but very small for all \( A \). For \( q = 1 \), the Coulomb term difference (cst) is positive and increases rapidly with \( A \) while for \( q = -1 \), the acd is negative and decreases rapidly with \( A \). In the \( \Theta^+ \) plot (Fig. 2b) a maximum at low \( A \) and decrease in \( S_Y \) values at large \( A \) arise mainly due to increasing contribution from the "strangeness term" (i.e., \( stern1 = -48.7 | S |_/A^{2/3} \)), surface term (asad) and decreasing (negative) contribution from the Coulomb term (Fig. 3c). For \( q = 0 \) and \( 1 \) such
maxima in Lambda plot [Fig. 1(a),1(b)] and Ξ− plot [Fig. 2(a)] are absent as asd, acd and sternl terms make increasing contributions with $A$ (fig.3a, 3b).

Interestingly, the large masses of hyperons make some strongly bound hypernuclei whose non-strange core nuclei might be unbound. For example, BWMH shows that the prediction for the hyperon separation energy $S_V$ for the $^{10}_8 Li$ hypernucleus is 26.4 $MeV$ and that for the $^{11}_8 Be$ hypernucleus is 25.1 $MeV$, the cores of which (i.e., $^9 He$ and $^{10}_8 Li$ respectively) are known to be unbound. BWMH predicts the $\Theta^+ + ^{12} C$ binding energy to be 23.24 $MeV$ where as, the same for the $\Lambda + ^{12} C$ binding energy is 12.65 $MeV$. This shows that compared to the $\Lambda$ hyperons, the $\Theta^+$ hyperons will be more strongly bound in a nucleus. This finding is in consonance with the prediction of Zhong et al. [18] and QMF calculation [19].

It is pertinent to note that unlike the previous two mass formulae [3], [20] the BWMH mass formula does not jeopardize the normal nuclear matter properties. Fig.4 shows that the BWMH predicts the line of stability quite well while the mass formula of Dover et al. [3] shows mismatch for medium and heavy nuclei. The mass formula of Levai et al. [20] although reproduces the line of stability, it does not reproduce the binding energy versus mass number ($A$) plot (Fig. 5a) of non-strange light nuclei such as, $^6 Li$ [37]. Mass formula of Dover et al. also shows marked deviations for isotopes with larger neutron numbers. The sharp oscillations in the experimental neutron separation energy ($S_n$) versus $A$ plots (Fig. 5b) are reproduced by BWMH but, not by the other two as the pairing term is altogether absent in the mass formula of Dover et al. [3] and, in case of the mass formula of Levai et al. [20] it is too small. The incompressibility of infinite nuclear matter [38] obtained using the energy co-efficient $a_o$ of the volume term is about 300 $MeV$ [39] for BWM and BWMH which is within the limits of experimental values while the mass formula of Levai et al. [20] predicts values in the range of 400-480 $MeV$ which are too high to be realistic. The presence of Majorana term in the mass formula of Levai et al. (eqns. (6) and (8) of [20]) also poses a serious problem that the binding energy per nucleon diverges as mass number $A$ goes to infinity. This violates the nuclear saturation properties. As BWMH is not plagued with such divergence problems and nuclear saturation properties are well preserved for large $A$, this mass formula will be useful for extension to astrophysics related problems, like equation of state of a neutron star.

In summary, a single mass formula (BWMH) valid for both non-strange normal nuclei and strange hypernuclei is prescribed by introducing hyperon mass and strangeness dependent SU(6) symmetry breaking terms in the modified-Bethe-Weizsäcker mass formula (BWM). The BWMH preserves the normal nuclear matter properties. Due to delicate balance amongst the surface, strangeness and Coulomb contributions, a maximum in $q = +1$ hyperon separation energies is found near lower $A$ values along with a decreasing trend at larger $A$. This feature is absent in hypernuclei with $q = 0$ and $q = -1$ hyperon(s). Hyperon separation energies calculated by BWMH are in good agreement with the available experimental data on 'bound' nuclei, including $^4 He$. Heavier Sigma-hypernuclei were predicted to be unbound for repulsive Sigma-nucleus potential [6]. As BWMH does not account for the repulsive potential, it predicts high binding energies for the heavy Sigma hypernuclei which contradicts the present wisdom. On the other hand, several authors have predicted that Theta-hypernuclei would be bound more strongly than Lambda hypernuclei. Quark mean field estimates for the Theta-separation energies are now available for medium to heavy-mass Theta-hypernuclei [19] and those predictions are in close agreement with the BWMH predictions. It is noteworthy that BWMH can predict separation energies of both light and heavy hyper-nuclei. In view of the proposed search of the elusive $\Theta^+$ hypernuclei and for many other hypernuclei which are predicted to be 'bound' (but their binding energies are unknown) the present mass formula is expected to provide a useful guideline.

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FIG. 1. Hyperon separation energy $S_Y$ versus mass number $A$ for (a) single $\Lambda$ ($m_\Lambda = 1115.63$ MeV) predicted by BWMH (solid), Dover et al. [3] (top most, dashed-line) and Levai et al. [20] (dotted) and experimental values [1, 2] (rombus with error bars) and, (b) double $\Lambda$ predicted by BWMH (solid), and Levai et al. [20] (dotted) and experimental values [1, 2] (rombus with error bars). In the inset the three data points of double $\Lambda$ are shown with predictions of BWMH. In all the figures, lines are added only as guides to the eyes.
FIG. 2. Hyperon separation energy $S_Y$ versus mass number $A$ for (a) single $\Xi^-$ ($m_Y=1321.32$ MeV) and experimental values [1] and, (b) single $\Theta^+$ ($m_Y=1540$ MeV) separation and quark mean field calculations of Shen and Toki [19].
FIG. 3. Contribution from Surface (asd), Coulomb (acd) and Strangeness (sterm1) term differences to the hyperon separation energies (eqn.8) for (a) $\Lambda^0$, (b) $\Xi^-$ and (c) $\Theta^+$. 
FIG. 4. The BWMH, Dover et al. [3] and Levai et al. [20] predictions for line of stability and experimentally observed stable nuclei [37]. Predictions of BWMH and Levai et al are almost identical and those of Dover et al. deviate from the experimental values in heavier nuclei.
FIG. 5. Predictions of BWMH, Dover et al. [3] and Levai et al. [20] and the experimental data [37] for (a) binding energy versus mass number $A$ for $Z=3$ and (b) the one neutron separation energy versus $A$ for $Z=38$. 