The Evolution of Quantum Measuring Devices

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Abstract. A quantum measuring device is introduced through a projective operator of any complete set of states that span the Hilbert space. Consequently, even a "bizarre" basis such as a basis of states composed of superpositions between location states, is legitimate despite its incomprehensible interpretation of a particle located in some places simultaneously.

The collapse scenario that lies in the essence of any quantum measuring device, suggests that measurement is actually an interpretation process that translate reality into the predefined concepts determined by the particular selection of the basis of states. The very fact that there are bases that contradict "common sense" suggests that our brain by serving as a measuring and interpreting "device", selects only unique measuring processes. We suggest a procedure of nonlinear recursive maps that dominant an evolution of states toward few selected bases of states.

1. Introduction
Let us start with the motivated example that in some aspects resemble the description of the Feynman quantum double slits experiment [1]. We consider a Hilbert space that can be spanned by the two location states entitled as \(x_1\) and \(x_2\). In representing (and potentially measure) a corresponding particle states, a conceivable basis might be the \(|x_1⟩\) and \(|x_2⟩\) states that simply describes the particle as being in one of the two locations. On the other hand, the states \(|±⟩ = \frac{1}{√2} (|x_1⟩ ± |x_2⟩)\) which also generate a physically legitimate basis, possess the "unconceivable" interpretation of a particle that stays in both locations simultaneously. An attempt to solve this unimaginable description is by performing a direct measurement of the apparently real particle location. Although each measurement will indeed revile a location, this measurement corresponds with collapsing the \(|±⟩\) original states into one of the \(|x_1, x_2⟩\) states [2]-[7] and therefore it cannot serve as a true reviling measurement of the real nature of the \(|±⟩\) states.

A more appropriate interpretation for the \(|±⟩\) states is the measurement of the \(|x_1⟩-|x_2⟩\) relative phase (0 or π). Unfortunately, it seems that in the vocabulary of ordinary perceptions, the concept relative phase between location states of a single particle is absent. It seems to exist only in abstract theories such as Quantum-Mechanics. Moreover, even the so called appropriate interpretation, namely the relative phase between the locations is more likely a description of a relative state between the real well imaginable concepts namely the particle location, rather then a description of the \(|±⟩\) states true nature. We note that not all superposition between location states correspond with a strange interpretation. In extreme situations such as in superposition between infinite-coordinate-Hilbert space, an equal phase superposition between locations states, namely the state \(|p⟩ = \sum_e e^{-ikx} |x⟩\) give rise to a completely new concept- that is, the momentum concept which is easily associated with the well conceivable velocity concept.
We can conclude this argument by observing that although nature allows us many alternatives in describing a physical word that is formally described with the Hilbert space, yet, only few options are conceivable to the human mind. In this paper we demonstrate only the nonlinear procedure that leads to the unification of specific measuring tolls over the other.

2. The Single Parameter Complementary Maps

Complementary maps are defined as coupled recursive maps that determine the process of the concepts generation.

Assume we have a map defined by the function $f_R(\alpha_n)$ with a single control parameter $R$ such that

$$\alpha_{n+1} = f_R(\alpha_n), \quad x \in [0, 1]$$

(1)

where all values of $\alpha_n$ are within the interval $[0, 1]$.

We introduce the term maps coherence with the definition of the complementary map as follows

$$\beta_{n+1} = 1 - f_R(1 - \beta_n),$$

(2)

where it is easy to see that $\forall n \beta \in [0, 1]$. For example, the $\alpha$-$\beta$ maps for the logistic map are:

$$\alpha_{n+1} = R\alpha_n (1 - \alpha_n) \quad \beta_{n+1} = 1 - R\beta_n (1 - \beta_n).$$

(3)

In general the two complementary maps $\alpha_n$ and $\beta_n$ can evolve independently provided that the initial variables $\alpha_0$ and $\beta_0$ are determined separately. Otherwise, when the initial conditions are initially coordinated such that $\alpha_0 + \beta_0 = 1$ the maps are considered to be initially unitary correlated.

3. Concepts Evolution in a 1/2-Spin Space

Various models and in particular the Spin-Glass-Model[8], associate the brain activity with interacting spins model. We adapt this approach.

We show a concept generation in a single spin system. Assume the 1/2-spin-space. We observe the initial condition basis of states:

$$|0\rangle = |\uparrow\rangle = A_0 |\uparrow\rangle + B_0 |\downarrow\rangle, \quad |\bar{0}\rangle = |\downarrow\rangle = -B_0 |\uparrow\rangle + A_0 |\downarrow\rangle$$

(4)

and under the maps $n$-iterations the initial coefficients $A_0$ $B_0$ evolve into the $n$-coefficients $A_n$ $B_n$ to generate the states

$$|n\rangle = A_n |\uparrow\rangle + B_n |\downarrow\rangle, \quad |\bar{n}\rangle = -B_n |\uparrow\rangle + A_n |\downarrow\rangle$$

(5)

where the coefficients that follow the recursive maps function $f$ are:

$$\alpha_n = |A_n|^2 \quad \beta_n = |B_n|^2$$

(6)

with the recursive maps of eqs. 1 and 2. We demonstrate our formalism with the logistic formula:

$$\alpha_{n+1} = R\alpha_n (1 - \alpha_n), \quad \beta_{n+1} = 1 - R\beta_n (1 - \beta_n).$$

(7)

The logistic maps posses fixed points such that $\forall n \alpha_n = 0$ and $\beta_n = 1$ meaning that if we start with the initial condition $|0\rangle = |\uparrow\rangle$ and $|\bar{0}\rangle = |\downarrow\rangle$ then the states will be constantly fixed. Thus, a very limited observer who experience reality under logistic maps dominated brain, notifies the uniqueness of the up – down states among the other rotated states and he associate them with the concept vertical states.

Recursive maps such as the logistic map have also the tendency of reaching constant values,
provided that they are not in the chaotic regimes. In our example of the logistic maps, if $R < 3$ we obtain that for $n \to \infty$, $\alpha_n \to \frac{R-1}{R}$ and $\beta_n \to \frac{R}{R}$. Consequently we obtain states that except for the $\alpha_n = 0, 1$-values, always converge into the defined basis of states:

$$|\infty\rangle_R = \sqrt{\frac{R-1}{R}} |\uparrow\rangle + \sqrt{\frac{1}{R}} |\downarrow\rangle, \quad |\infty\rangle_R = -\sqrt{\frac{1}{R}} |\uparrow\rangle + \sqrt{\frac{R-1}{R}} |\downarrow\rangle$$  \hspace{1cm} (8)

To simplify our discussion furthermore, we assume that $R = 2$ to obtain:

$$|\infty\rangle_{1/2} = \frac{1}{\sqrt{2}} |\uparrow\rangle + \frac{1}{\sqrt{2}} |\downarrow\rangle \equiv |\rightarrow\rangle, \quad |\infty\rangle_{1/2} = -\frac{1}{\sqrt{2}} |\uparrow\rangle + \frac{1}{\sqrt{2}} |\downarrow\rangle \equiv |\leftarrow\rangle$$  \hspace{1cm} (9)

Going back to the limited observer who view and interpret the word in according to the elementary $R = 2$-logistic-map. Now in addition to the vertical basis, his brain is also capable of generating the horizontal concepts. These two concepts are the only way that our “limited” observer experiences the word. Other options collapse by his brain interpretation tools into one of these two options.

In general we can select complementary maps that converge into the constant values $A_\infty$ and $B_\infty$ (see eqs. 5). This yields always to the same universal states representation that is eventually associated by the observer with a concept.

### 3.1. Double Bifurcation Recursive Maps and Concepts Evolution

Let us slightly extend our analysis to the double period bifurcation maps behavior \[9\], that is, maps that converge into the two sets:

$$\tilde{\alpha}_- = \lim_{n \to \infty} \alpha_n, \quad \tilde{\alpha}_+ = \lim_{n \to \infty} \alpha_{n+1} \quad \tilde{\beta}_- = \lim_{n \to \infty} \beta_n = 1 - \tilde{\alpha}_- \quad \tilde{\beta}_+ = \lim_{n \to \infty} \beta_{n+1} = 1 - \tilde{\alpha}_+$$  \hspace{1cm} (10)

Although now each map converges into two values, it is possible to redefine a double $g$-maps which retrieves a single fixed point

$$g_R = f (f_R (\alpha_n)) \equiv f_R \circ f_R \quad \tilde{g}_R \equiv (1 - f_R) \circ (1 - f_R),$$  \hspace{1cm} (11)

yielding

$$\alpha_{n+2} = g_R (\alpha_n) \quad \beta_{n+2} = \tilde{g}_R (\beta_n)$$  \hspace{1cm} (12)

where the composed $g$-maps corresponds with a time interval which is twice the original $f$-time. Under the $g$-maps we obtain two types of sets; The firsts converges into the $|\alpha_-, \beta_-\rangle$ and the others reach the output $|\alpha_+, \beta_+\rangle$.

It is easy to identify each set as follows: Suppose that we start with initial conditions that are already fixed at the final states $\tilde{\alpha}_-$ or $\tilde{\alpha}_+$. For $n = 0, 1, 2, 3...$ the maps split into two types:

#### Even series iteration:

$$\alpha_{2n+2} = g_R (\alpha_{2n}) \quad \beta_{2n+2} = 1 - g_R (\beta_{2n})$$  \hspace{1cm} (13)

The series $\alpha_0, \alpha_2, \alpha_4...$

#### Odd series iteration:

$$\alpha_{2n+3} = g_R (\alpha_{2n}) \quad \beta_{2n+3} = 1 - g_R (\beta_{2n})$$  \hspace{1cm} (14)

The series $\alpha_1, \alpha_3, \alpha_5...$

where the even and odd series converge into the values $|\tilde{\alpha}_-, \tilde{\beta}_-\rangle$ and $|\tilde{\alpha}_+, \tilde{\beta}_+\rangle$, respectively.

Since the two maps operate separately, we associate the maps with independent states $|o\rangle$ and $|e\rangle$ types, respectively.

For the logistic map $\alpha_{n+1} = R\alpha_n (1 - \alpha_n)$ we obtain:

$$|\alpha_-, \beta_-\rangle = \left(\frac{2R - 4}{R}, \frac{4 - R}{R}\right) \quad |\alpha_+, \beta_+\rangle = \left(\frac{2}{R}, \frac{R - 2}{R}\right)$$  \hspace{1cm} (15)
In general the initial states converge into multi kinds of representations, for example:

**Even basis:** \(|\infty\rangle_e = \sqrt{\frac{1}{4}}|\uparrow\rangle + \sqrt{\frac{1}{2}}|\downarrow\rangle|\infty\rangle_e = -\sqrt{\frac{1}{4}}|\uparrow\rangle + \sqrt{\frac{1}{2}}|\downarrow\rangle

**Odd basis:** \(|\infty\rangle_o = \sqrt{\frac{1}{4}}|\uparrow\rangle + \sqrt{\frac{1}{2}}|\downarrow\rangle|\infty\rangle_o = -\sqrt{\frac{1}{4}}|\uparrow\rangle + \sqrt{\frac{1}{2}}|\downarrow\rangle

where for the logistic map we have four representations:

1. **Standard basis:** Refers to \(\alpha_0 = 0\) and \(\beta_0 = 1\) \(\forall n, \tilde{n} |n\rangle = |\uparrow\rangle |\tilde{n}\rangle = |\downarrow\rangle

2. **Single value basis**

   For \(1 \leq R \leq 3\) \(|\infty\rangle_s = \sqrt{\frac{R-1}{R}}|\uparrow\rangle + \sqrt{\frac{1}{R}}|\downarrow\rangle|\infty\rangle_s = -\sqrt{\frac{1}{R}}|\uparrow\rangle + \sqrt{\frac{R-1}{R}}|\downarrow\rangle

3. **Even basis, \(n = 0, 2, 4, \ldots\)** \(|\infty\rangle_e = \sqrt{\frac{2}{R}}|\uparrow\rangle + \sqrt{\frac{R-2}{R}}|\downarrow\rangle|\infty\rangle_e = -\sqrt{\frac{2}{R}}|\uparrow\rangle + \sqrt{\frac{R-2}{R}}|\downarrow\rangle

4. **Odd basis, \(n = 1, 3, 5, \ldots\)** \(|\infty\rangle_o = \sqrt{\frac{2R-4}{R}}|\uparrow\rangle + \sqrt{\frac{R-4}{R}}|\downarrow\rangle|\infty\rangle_o = -\sqrt{\frac{2R-4}{R}}|\uparrow\rangle + \sqrt{\frac{R-4}{R}}|\downarrow\rangle

Numerical calculations show[10] that even in the bifurcation regime \((3 \leq R \leq 1 + \sqrt{6})\), if we start with the initial condition \(\alpha_0 = \alpha_{\infty} = \frac{R-1}{R}\), although we are in the double-bifurcation regime, the maps will remain in the same single value \(\alpha_{n\rightarrow\infty} = \frac{R-1}{R}\) as in eq. 7. It follows that an observer who experience maps in the bifurcation regime is familiar with all the four bases as expressed in eq. 17 including the Single Value Basis.

4. **summary**

An important insight of quantum mechanics concerns the way we experience reality. Clearly, by selecting a unique measuring device, an observer enforces reality to exhibits a face that is consistent with his basic understanding that is scalded in his measuring tool preference. In other words, the observer edits reality according to his interpretation tools.

In this paper we explored the procedure that leads to that specific observer selection and we showed that nonlinear maps can converge into values that lead to the unique measuring tools definitions. In that sense our procedure provides a formalism of how concepts concerning reality can be formed in the observer mind.

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