Abstract—Algorithms for Blind Source Separation (BSS) of acoustic signals require efficient and fast converging optimization strategies to adapt to non-stationary signal statistics and time-varying acoustic scenarios. In this paper, we derive fast converging update rules from a negentropy perspective, which are based on the Majorize-Minimize (MM) principle and eigenvalue decomposition. The presented update rules are shown to outperform competing state-of-the-art methods in terms of convergence speed at a comparable runtime due to the restriction to unitary demixing matrices. This is demonstrated by experiments with recorded real-world data.

Index Terms—Independent Vector Analysis, fast convergence, MM Algorithm, FastIVA

I. INTRODUCTION

Blind Source Separation (BSS) aims at separating sources from an observed mixture by using only very weak assumptions about the underlying scenario. Hence, such methods are applicable in a variety of situations [1]–[4]. One important aspect in the design of BSS algorithms is the development of fast converging and at the same time computationally simple optimization strategies. For Independent Component Analysis (ICA) the FastICA update rules based on a fixed-point iteration scheme represent the gold standard in this research field [5]. This update scheme is derived by maximizing the so-called negentropy, i.e., by maximizing the non-gaussianity of each separated source. Several variants of these updates including the extension to complex-valued data have been proposed [6].

In this contribution, we consider mixtures of acoustic sources [3], [4]. The most important difference of BSS methods for acoustic mixtures relative to instantaneous problems [1] is the mixture model: Observed acoustic signals undergo propagation delay and multipath propagation. Hence, a convolutive mixture model is needed. A well-established concept is to transform the problem into the Short-Time Fourier Transform (STFT) domain and solve instantaneous BSS problems in each frequency bin [7]. However, this causes the well-known inner permutation problem which has to be resolved by additional heuristic measures to obtain decent results [8]. As an alternative which aims at avoiding the inner permutation problem, Independent Vector Analysis (IVA) has been proposed [9]. A fast fixed-point algorithm called FastIVA has been developed following the ideas of FastICA [5] for the optimization of IVA [10]. Fast and stable update rules have been developed based on the Majorize-Minimize (MM) principle and the iterative projection technique [11] and methods for accelerating its convergence have been investigated [12]. Even faster update rules for the specific case of two sources and two microphones based on a Generalized Eigenvalue Decomposition (GEVD) have been presented in [13].

For source extraction [14], [15], i.e., the separation of a desired source from a set of multiple interfering sources, update rules based on an Eigenvalue Decomposition (EVD) of a weighted microphone covariance matrix have been proposed [16] and spatial prior knowledge about the source of interest has been introduced in these update rules in [17]. Recently, priors on the source signal spectra for IVA were proposed based on deep neural networks [18], [19].

In this contribution, we propose a new update scheme for IVA described in terms of the negentropy of the demixed signals and based on the MM principle. The optimization of the upper bound of the MM algorithm is posed as an eigenvalue problem, which allows for fast convergence of the algorithm. In comparison to [13], our EVD-based update scheme allows for the separation of an arbitrary number of sources instead of only two. In [16], a structurally similar update scheme has been derived for the extraction of a single source from a different perspective. Here, we derive update rules which are also capable of separating an arbitrary number of sources. Update rules for extracting a single source are included in the proposed method as a special case. We note that FastIVA [10] uses the same cost function but uses a fixed-point algorithm for its optimization. The superiority of our proposed method over FastIVA and AuxIVA in terms of convergence speed and separation performance after convergence is demonstrated by experiments using real-world data created from measured Room Impulse Responses (RIRs).

II. COST FUNCTION

We consider a determined scenario, in which $K$ source signals are captured by $K$ microphones with microphone signals described in the STFT domain as

$$\tilde{x}_{f,n} := [\tilde{x}_{1,f,n}, \ldots, \tilde{x}_{K,f,n}]^T \in \mathbb{C}^K, \quad (1)$$

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where \( f \in \{1, \ldots, F\} \) indexes the frequency and \( n \in \{1, \ldots, N\} \) the time frame. The aim of the developed algorithm is to estimate the demixed signals
\[
y_{f,n} := [y_{1,f,n}, \ldots, y_{K,f,n}]^T \in \mathbb{C}^K
\]
from the microphone signals \( \tilde{x}_{f,n} \). For notational convenience, we introduce the broadband vector of the demixed signal of channel \( k \) and time frame \( n \)
\[
\bar{y}_{k,n} := [y_{k,1,n}, \ldots, y_{k,F,n}]^T \in \mathbb{C}^F,
\]
which is modeled to follow a multivariate supergaussian Probability Density Function (PDF) \( p(y_{k,n}) \), where all frequency bins are modeled to be uncorrelated but statistically dependent. Examples for such PDFs, which are typically used for IVA include the multivariate Laplacian PDF [20].

In the following, signal vectors without frame index \( n \) denote Random Vectors (RVs) and signal vectors with frame index their realizations. As the PDF of a mixture of multiple independent non-Gaussian source signals tends toward a Gaussian, maximizing the negentropy [11] of the RV of the demixed signals \( y := [y_1^T, \ldots, y_K^T]^T \) is an intuitive and widely used BSS cost function. The negentropy, i.e., Kullback-Leibler divergence between the PDFs of the RVs \( y \) and \( z \), where the latter is normally distributed with same mean vector and covariance matrix as \( y \) is defined by
\[
N(y) := 
\sum_{k=1}^{K} N(y_k) = 
\sum_{k=1}^{K} \mathcal{H}(z_k) - \mathcal{H}(y_k),
\]
where \( z_k \) is defined analogously to \( y_k \). In the following, we will consider the maximization of the sum of the channel-wise negentropies \( N(y_k) \) as a surrogate for the maximization of \( N(y) \). The requirement \( \mathbb{E} \{ y_{f,n} y_{f,n}^H \} = I_K \) can be accomplished by whitening the observed signals
\[
x_{f,n} := Q_f \tilde{x}_{f,n} \quad \text{with} \quad Q_f := \left( \mathbb{E} \{ \tilde{x}_{f,n} \tilde{x}_{f,n}^H \} \right)^{-\frac{1}{2}}
\]
and estimating the demixed signals
\[
y_{f,n} = W_f x_{f,n},
\]
with a unitary demixing matrix (cf. [11])
\[
W_f := [w_1, \ldots, w_K]^H \in \mathbb{C}^{K \times K}.
\]
Here, \( w_{k,f} \) denotes the demixing filter which extracts the \( k \)th source signal sample \( y_{k,f,n} \) at frequency \( f \) and time frame \( n \). By using the definition of the differential entropy \( \mathcal{H}(\cdot) \) and the source model \( G \),
\[
\mathcal{H}(y_k) := \mathbb{E} \{ G(y_k) \} \quad \text{with} \quad G(y_k) := - \log p(y_k),
\]
we obtain the following optimization problem by assuming i.i.d. signal frames (cf. [11], [10])
\[
\begin{aligned}
\text{minimize} & \quad \sum_{k=1}^{K} \mathbb{E} \{ G(y_{k,n}) \} \\
\text{subject to} & \quad W_f W_f^H = I_K \quad \forall f.
\end{aligned}
\]
Here, (10) reflects the maximization of channel-wise negentropies (5) and (11) realizes the unitarity constraint on the demixing matrices \( W_f \). In (10), we introduced the approximation of the expectation operator by arithmetic averaging over all available time frames \( \mathbb{E} \{ \cdot \} := \frac{1}{N} \sum_{n=1}^{N} \{ \cdot \} \). The optimization problem of (10) and (11) is closely related to the IVA cost function [9]
\[
J_{\text{IVA}}(W) := \sum_{k=1}^{K} \mathbb{E} \{ G(y_{k,n}) \} - 2 \sum_{f=1}^{F} \log |\det W_f|,
\]
where \( W \) denotes the set of demixing vectors \( w_{k,f} \) of all frequency bins \( f \) and channels \( k \). The first term of the IVA cost function (12) corresponds to (10). The second term of (12) is a regularizer on the demixing matrices \( W_f \) ensuring linearly independent demixing filter vectors \( w_{k,f} \). For unitary \( W_f \) this term is constant and, hence, is irrelevant for optimization. In the optimization problem of (10) and (11), the role of the regularizer is taken by the (stronger) constraint of unitarity of \( W_f \). However, the assumption of unitary demixing matrices is a significant restriction w.r.t. the IVA cost function as will become obvious in the experimental evaluations.

III. UPDATE RULES

In the literature, predominantly fixed-point algorithms (FastICA, FastIVA) have been used for the optimization of negentropy-based BSS cost functions [11], [5], [6], [10]. Motivated by the success of MM-based approaches [22] for BSS based on the minimum mutual information principle [11], [20], [23], we exploit the MM principle for the optimization of the negentropy-based IVA cost function (10), (11) in this contribution.

In the following, \( l \in \{1, \ldots, L\} \) denotes the iteration index and \( W(l) \) is the set of the \( l \)th iterates of all demixing vectors. The main idea of the MM principle [22] is to define an upper bound \( U \), which fulfills the properties of majorization and tangency, i.e., equality iff \( W = W(l) \),
\[
J(W) \leq U \left( W_W(l) \right) \quad \text{and} \quad J \left( W(l) \right) = U \left( W(l) | W(l) \right),
\]
w.r.t. the cost function \( J \). The upper bound \( U \) should be designed such that its optimization is easier than the iterative optimization of the cost function itself, or, ideally, solvable in closed form. As minimization of the upper bound
\[
W(l+1) = \arg \min_W U \left( W_W(l) \right)
\]
forces monotonically decreasing values of \( U \), the following ‘downhill property’ of MM algorithms is obtained
\[
J \left( W(l+1) \right) \leq U \left( W(l+1) | W(l) \right) \leq U \left( W(l) | W(l) \right) = J \left( W(l) \right).
\]
For the construction of the upper bound, we use the inequality \( \|X_k\|_2 \) for supergaurous source models \( \hat{G}(r_{k,n}) = G(y_{k,n}) \) dependent on the norm of the \( k \)th demixed signal \( r_{k,n} := \|Y^l_{k,n}\|_2 \)

\[
\hat{E}\{ \hat{G}(r_{k,n}) \} \leq \frac{1}{2} \sum_{j=1}^{F} w_{k,f}^{H} V_{k,f} w_{k,f} + \text{const.} \quad (15)
\]

Here, we introduced the weighted covariance matrix of microphone observations

\[
V_{k,f} := \hat{E}\left\{ \frac{G'(r_{k,n})}{r_{k,n}} x_{f,n} x_{f,n}^{H} \right\}. \quad (16)
\]

Combining (10) and (15) and neglecting constant terms yields a surrogate for the optimization problem (10), (11) defined by

\[
\begin{align}
\text{minimize} & \quad \frac{1}{2} \sum_{k=1}^{K} \sum_{f=1}^{F} w_{k,f}^{H} V_{k,f} w_{k,f} \leq U \left( W | W^{(l)} \right) \quad (17) \\
\text{subject to} & \quad w_{f}^{H} w_{f} = I_{K} \forall f,
\end{align}
\]

with equality of (17) to (10) iff \( W = W^{(l)} \). Equality up to a constant is denoted by \( \leq \). By inspection of (17), we see that the optimization w.r.t. the demixing matrices \( W_f \) is now expressed by the optimization of demixing filter vectors \( w_{k,f} \) separately for different frequency bins and channels. However, the channel-wise demixing filters \( w_{k,f} \) are coupled within one frequency bin due to the constraint (18). To simplify the problem, we divide the optimization of (17) and (18) into two steps: a) Relaxing the constraint (18), which allows for solving (17) for each demixing filter \( w_{k,f} \) without being influenced by the other demixing filters. b) Imposing (18) by projecting the results from a) onto the set of unitary matrices, the so-called complex Stiefel manifold.

For Step a), we replace the unitarity constraint (18) by a unit norm constraint for the demixing filters and obtain an optimization problem which is now only dependent on a single output channel \( k \)

\[
\begin{align}
\text{minimize} & \quad \frac{1}{2} w_{k,f}^{H} V_{k,f} w_{k,f} \quad (19) \\
\text{subject to} & \quad \|w_{k,f}\|_2^2 = 1. \quad (20)
\end{align}
\]

Optimization by using the lagrangian multiplier \( \lambda_{k,f} \) yields the following eigenvalue problem

\[
V_{k,f} w_{k,f} = \lambda_{k,f} w_{k,f}, \quad (21)
\]

which shows that the eigenvalues of \( V_{k,f} \) are the critical points of the optimization problem (19), (20). By multiplication of (21) with \( w_{k,f}^{H} \) from the left, we obtain

\[
w_{k,f}^{H} V_{k,f} w_{k,f} = \lambda_{k,f} w_{k,f}^{H} w_{k,f} = \lambda_{k,f}. \quad (22)
\]

Hence, the optimal \( w_{k,f} \) is the eigenvector of \( V_{k,f} \) corresponding to the smallest eigenvalue \( \lambda_{k,f} \) (as \( V_{k,f} \) is Hermitian, its eigenvalues are real-valued and can be ordered). If the smallest eigenvalue \( \lambda_{k,f} \) has algebraic multiplicity one, the choice of \( w_{k,f} \) is unique up to an arbitrary phase term, i.e., all elements of the set

\[
\{ w_{k,f} \in \mathbb{C}^K | w_{k,f} = e^{j\phi} \tilde{w}_{k,f} \}, \quad (23)
\]

where \( \phi \) denotes an arbitrary phase and \( \tilde{w}_{k,f} \) is a solution of (19) and (20), represent equivalent solutions. Under the natural assumption of distinct temporal variance patterns of the source signals, the eigenvalues of \( V_{k,f} \) can be assumed to be distinct and, hence, the solution for \( w_{k,f} \) is unique up to an arbitrary phase term.

For Step b), i.e., to impose the unitarity constraint (18) on the demixing matrices \( W_f \) obtained from collecting the demixing filter vectors from Step a), the closest unitary matrix in terms of the Frobenius distance is calculated

\[
W_f = \arg\min_{T_f \in \mathcal{O}_{K \times K}} \| W_f - T_f \|_{F}^2, \quad (24)
\]

where \( \mathcal{O}_{K \times K} \) denotes the set of \( K \times K \) unitary matrices. This results in [24]

\[
W_f = \left( W_f \tilde{W}_f^H \right)^{\frac{1}{2}} \tilde{W}_f. \quad (25)
\]

The MM algorithm alternates now between two steps: construction of the upper bound by parameterization of the proposed surrogate optimization problem (17), (18) with the weighted covariance matrix \( V_{k,f} \) (see (16)) and minimization of it by calculating the demixing filters \( w_{k,f} \) by eigenvalue decomposition and orthogonalization of the demixing matrices (25). This is summarized in Alg. 1.

**Algorithm 1 FasterIVA**

**INPUT:** Microphone signals \( x_{f,n} \forall f,n \)

**Whitening:** Estimate \( Q_f \forall f \) and \( x_{f,n} = Q_f \tilde{x}_{f,n} \forall f,n \)

**Initialize:** \( y^{(0)}_{f,n} = x_{f,n} \forall f,n \)

for \( l = 1 \) to \( L \) do

\( r_{k,n} = \| \hat{y}^{(l)}_{k,n} \|_2 \forall k,n \)

for \( f = 1 \) to \( F \) do

Estimate \( V_{k,f} \) by (16)

Compute eigenvector \( w^{(l)}_{k,f} \) corresponding to smallest eigenvalue \( \lambda^{(l)}_{k,f} \) of \( V_{k,f} \)

end for

\( W^{(l)}_f \leftarrow \left( W^{(l)}_f (W^{(l)}_f)H \right)^{\frac{1}{2}} W^{(l)}_f \) (see (23))

\( y^{(l)}_{f,n} = W^{(l)}_f \tilde{x}_{f,n} \)

end for

**Backprojection**

**OUTPUT:** Demixed signals \( y^{(L)}_{f,n} \forall f,n \)

**IV. EXPERIMENTS**

For the experimental evaluation of the new negentropy-based IVA algorithm relative to other IVA algorithms, we convolved speech signals of 10 sec length with RIRs measured in three different rooms: two meeting rooms \( (T_{60} \in \{0.2 s, 0.4 s\}) \) and a seminar room \( (T_{60} = 0.9 s) \). For the measurements, we used a linear microphone array with 4.2 cm
spacing. To obtain representative results, we considered different measurement setups: source positions at 1 m and 2 m distance from the microphone array and at 40°/140° and 40°/90°/140° w.r.t. the microphone array axis. To simulate microphone noise, we added white Gaussian noise to the observed signals to obtain a Signal-to-Noise Ratio (SNR) of 30 dB. To address the effect of source variability, we chose the clean source signals randomly from a set of four male and four female speakers and repeated the experiments 20 times in this way. The simulated microphone signals are transformed into the STFT domain by a Hamming window of length 2048 and 50% overlap at a sampling frequency of 16 kHz. The performance of the investigated algorithms is measured in terms of Signal-to-Distortion Ratio (SDR), Signal-to-Interference Ratio (SIR) and Signal-to-Artefact Ratio (SAR) [25]. These performance measures are not directly connected to the cost function, but are closely related to the separation performance as experienced by a human listener. We used for all algorithms a Laplacian source model yielding \( G(r_{k,n}) = r_{k,n} \) [9], [11]. The scaling ambiguity of the frequency bin-wise estimates is resolved by the backprojection technique [11].

To benchmark the results, we compared the performance with two state-of-the-art algorithms: AuxIVA [11], which can be considered as the best performing algorithm in the field, and FastIVA [10], which is based on the same cost function as the proposed method [10], but uses a fixed-point algorithm for optimization. Results for the comparison of the investigated algorithms with IVA optimized by a natural gradient update scheme [9] are not shown here as its convergence turned out to be exceedingly slow and the final values are not better than for the competing methods. Note that a variation of experimental parameters such as STFT length, noise type, SNR etc. affected the discussed algorithms similarly. The restriction to unitary demixing matrices is well known to yield a fast initial convergence at the cost of inferior steady-state performance relative to methods that require only invertible demixing matrices [26]. Hence, a natural idea is to use the proposed method ‘FasterIVA’ until reaching the steady state and then relax the unitarity constraint by switching to the AuxIVA update rules (found to be superior to FastIVA in preliminary experiments) which do not constrain the demixing matrices to be unitary. The switching of this hybrid approach from FasterIVA to AuxIVA is triggered once FasterIVA reached a steady state characterized by only small changes of \( W_f \):

\[
\frac{1}{FK^2} \sum_{f=1}^{F} \left\| W_f^{(l-1)} - W_f^{(l)} \right\|_F^2 < \gamma.
\]

The threshold \( \gamma \) is chosen here to \( \gamma = 0.05 \). The experimental results including all three different rooms, both source-array distances and 20 repeated draws of source signals resulting

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**Fig. 1.** Performance of the investigated algorithms in terms of SDR and SIR improvement over the number of iterations. To create the plots, results for three different rooms (\( T_{60} = 0.2 \text{ sec}, 0.4 \text{ sec}, 0.9 \text{ sec} \)) and two different source-array distances (1 m, 2 m) have been used. Each of these experiments have been repeated 20 times, where the source signals have been chosen from a set of four male and four female speakers. The first row shows results for a determined scenario with two speakers, the second row a scenario with three speakers.
in 120 different experimental conditions for each number of sources are shown in Fig. 1 over the number of iterations. The slowest convergence among the discussed methods is obtained by FastIVA. Often, this algorithm did not even reach the steady state within the given number of iterations. However, even after convergence its final values were still not better than the competing methods in the vast majority of cases. The MM-based AuxIVA algorithm outperformed FastIVA in terms of convergence speed but also w.r.t. its final values. The proposed method FasterIVA showed much faster initial convergence than both FastIVA and AuxIVA and usually reached its steady state already after about five iterations. On the other hand, its final values were slightly worse than AuxIVA for the two-source scenarios, while it was the same for the three-source case. The ‘Hybrid’ approach, which switches after convergence of FasterIVA to the AuxIVA update rules, obtained in all scenarios the fastest convergence and the best final values at a comparable runtime. The values for SAR improvement have been omitted here due to a lack of space, but they showed comparable results for the discussed methods with a slight advantage for the hybrid approach. The runtime per iteration of the investigated methods, which is comparable in most cases, is given in Tab. 1. Note that, e.g., in the 3-source experiment, FasterIVA needs only 4 iterations to reach the ∆SIR value of FastIVA after 30 iterations, so that the complexity gain for FasterIVA for comparable performance amounts to a factor of approximately 5.

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