KCQ: A New Approach to Quantum Cryptography

I. General Principles and Key Generation

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A new approach to quantum cryptography to be called KCQ, keyed communication in quantum noise, is developed on the basis of quantum detection and communication theory for classical information transmission. By the use of a shared secret key that determines the quantum states generated for different data bit sequences, the users may employ the corresponding optimum quantum measurement to decode the data. This gives them a better error performance than an attacker who does not know the key when she makes her quantum measurement, and an overall generation of a fresh key may be obtained from the resulting advantage. This principle is illustrated in the operation of a concrete qubit system. A general information-theoretic description of the overall approach will be presented, and contrasted with the detection/coding description necessary for specific protocols. It is shown that the attacker’s error probability profile is needed for a complete assessment of her information on the generated key. The criterion of protocol efficiency and its sensitivity to system parameter fluctuation is proposed as another benchmark on the evaluation of key generation protocols. For systems described by infinite-dimensional state spaces referred to as qumodes, KCQ key generation schemes with coherent states of considerable energy will be presented together with corresponding security analysis. Various advantage enhancement and randomization techniques are introduced for improving the security and efficiency of such protocols. A specific m-ary coherent orthogonal signaling scheme, CPPM, is presented that can yield efficient secure key generation over long-distance telecommunication fibers using conventional optical technology. The issue of secrecy in direct encryption using KCQ is also discussed in general and in connection with the $\alpha\eta$ protocol, on which experimental progress has been made. It is indicated that information-theoretic security against known-plaintext attack is possible, which has never been suggested for any cryptosystem. In particular, it is shown that CPPM offers information-theoretic security against known-plaintext attacks while the data are unconditionally secure. Some qualitative comparison among the different key generation schemes are made from both a fundamental and a practical viewpoint. Further quantitative development, the detailed analysis of direct encryption, and the effects of various advantage enhancement techniques would be presented in future papers of this series. Some apparent gaps in the unconditional security proofs of previous protocols are indicated in Appendix A. The core of the paper is contained in sections III and VI.

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Contents

I. Introduction

II. KCQ qubit key generation

III. General principles of KCQ key generation

A. Principle of Key Generation
B. Advantage Creation with Shared Secret Key
C. Eve’s Information and Error Profile
D. Key Rate of Secure Protocols
E. Specific Detection or Coding Scheme
F. Key Verification
G. Advantage Enhancement
H. Overall KCQ Protocol
I. Unconditional Security and System Implementation

IV. Performance efficiency of QKG protocols

V. KCQ Coherent-State key generation with binary detection

A. $\alpha\eta$ and its Extensions
B. Binary Detection KCQ Key Generation

VI. KCQ coherent-state key generation with m-ary detection
I. INTRODUCTION

Quantum cryptography, the study of cryptographic protocols with security built on the basis of quantum effects, has been mainly developed along the line of the original BB84 protocol \[1\] and its variations \[2\]. The focus is on key generation (key expansion \[64\]), the establishment of a fresh key \[3\] between two users, which is often referred to in the literature as quantum key distribution \[65\]. Without the use of quantum effects, it was known that (classical) key generation is possible whenever the user and the attacker have different observations (ciphertexts) from which the user can derive a performance advantage \[4, 5, 6\], a process to be referred to as advantage creation \[66\]. In BB84 type quantum cryptographic schemes, advantage creation is obtained through intrusion-level detection \[68\] that quantitatively assures the attacker’s observation to be inferior to the user’s, thus allowing privacy distillation (amplification) \[9\] to essentially eliminate the attacker’s information on the final key generated. Classically, this approach cannot succeed because the attacker can always, in principle, clone a copy identical to the user’s observation, and no advantage can possibly be created. Quantum mechanically, there is a general tradeoff between the attacker’s disturbance and her information on the user’s observation. By estimating the intrusion level, the user can (probabilistically) assure a better observation for decoding the original data, from which a fresh key may be generated.

There are several problems, in theory and in practice, with the BB84 type quantum protocols. Among them are the necessity of using weak but accurate signal source, a near perfect transmission line, sensitive and fast quantum detectors, as well as the difficulties of having appropriate amplifiers or repeaters to compensate loss, developing specific practical protocols with quantifiable security against all realistic attacks, and achieving reasonable efficiency with such protocols. These problems are summarized in Section VIII and a few hitherto unanalyzed problems are summarized in Appendix A. As a consequence, the usefulness of BB84 type protocols is severely curtailed, especially for commercial applications. Most of these problems can be traced to the need of measuring the intrusion level for balance between the user and attacker’s information on the data, and the necessity of using weak signals. In this paper, a new type of quantum protocols, to be called KCQ (keyed communication or keyed CDMA in quantum noise), is presented. They do not need to involve intrusion-level detection and permit the use of coherent states with considerable energy, thus alleviating the above problems. They can be implemented using optical technology and readily integrated with existing optical communication formats. It is hoped that they would quickly bring quantum cryptography to practical application.

The basic idea of KCQ is to utilize a shared secret key between the users to determine the quantum signal set \[70\] to be chosen separately for each information sequence, the quantum noise being inherent in the quantum signal set from quantum detection and communication theory \[10, 11, 12, 13\]. Such shared secret keys have also not been used in classical key generation \[6\]. On the other hand, the use of a shared secret key is necessary in BB84 type protocols and classical public discussion protocols \[6\] for the purpose of message authentication or realizing the public channel. In contrast, a shared secret key is utilized in an essential way on KCQ protocols, but a fresh key can be generated that is much larger than the secret key used during key generation. For KCQ key generation, advantage creation is obtained from the different optimal or near-optimal quantum receiver performance between the user who knows the key and the attacker who does not when she makes her quantum measurement, even when a copy of the quantum signal is granted to the attacker for the purpose of bounding her information without intrusion-level detection. This difference in performance has no classical analog. The KCQ approach evolves from the anonymous key encryption method described in Ref. \[14\]. Exactly how and why this approach

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works is explained both generally and concretely in this paper.

In practice, infinite dimensional state spaces to be referred as qumodes provide the standard framework for describing coherent-state laser signals. As in classical communication systems, qumode systems allow the suppression of errors with signal energy without error control coding that may complicate security proofs and hinder the development of specific protocols. Also, KCQ can be used for direct encryption, which has different security performance criteria for key generation apart from the inefficient one-time pad approach. The basic use of KCQ in binary and $m$-ary detection of coherent-state qumode systems will be described. In particular, a specific coherent pulse-position modulation scheme, to be called CPPPM, is shown to have many dramatic characteristics including automatic privacy distillation, secure key generation and data transmission over long-distance telecom fibers, and information-theoretic security against known-plaintext attacks. This last characteristics is impossible in conventional cryptography, the possibility of which has also never been suggested in quantum cryptography. These qumode results are presented in sections V and VI, and direct encryption briefly described section VII. They are qualitatively compared to other quantum key generation (QKG) schemes in section VIII. Detailed quantitative development of CPPPM and other schemes for operation in realistic environment will be given in the future.

Key generation via KCQ on qubits is developed in Sections II. In Section III, a general analysis of the KCQ approach and QKG security against joint attacks are presented with the more complete criterion of error profile instead of mere mutual information. A protocol efficiency criterion on QKG protocols is introduced in Section IV, which should be insensitive to system parameter fluctuation for a protocol to be realistically useful. In Appendix A, we briefly discuss some serious gaps in the QKG unconditional security proofs against joint attacks given in the literature. In Appendix B, we respond to several criticisms on $\alpha_q$, a coherent-state KCQ scheme for direct encryption upon which significant experimental progress has been made. For readers who want to go directly to the core of our new results, please read sections III, V, and VI.

II. KCQ QUBIT KEY GENERATION

We consider a specific KCQ qubit scheme for key generation, to be called $q_k$, to introduce and explain the characteristics of the KCQ approach to quantum cryptography.

Let an arbitrary qubit state be represented by a real vector on the Bloch-Poincare sphere. As depicted in Fig. 1, an even number of $M$ points uniformly distributed on a fixed great circle on the sphere, corresponding to $M/2$ possible orthonormal bases, are used as possible quantum signal states for the bit value $b = 0, 1$. The opposite points on a diameter of the circle for a given basis are the two orthonormal states for the two possible bit values. The two neighbors of each of the $M$ points are taken to represent a different bit value. A shared secret key $K$ between two users, Adam (A) and Babe (B), is used to select a specific basis for each qubit. A secret polarity bit may also be introduced to be added to the data bit for randomizing the polarity of the basis. Instead of using the same $K$ for each $b$, a long running key $K'$ obtained from the output of a standard (classical) encryption mechanism with $K$ as the input may be used to yield different basis selection and polarity bits for different $b$'s in an $n$-sequence of input data $X_n$. This is depicted in Fig. 1 where, e.g., the ENC box may represent a synchronous stream cipher or even just a linear feedback shift register (LFSR) for the key extension $[7]$. Thus, generally a total of $1 + \log_2(M/2)$ bits from $K'$ would be used to determine the polarity bit and the selection of one of $M/2$ possible bases.

The key generation process goes as follows. Adam picks a random $n$-bit data sequence $X_n$, modulating $n$ corresponding qubits by using $K'$ to determine the polarity and basis for each qubit. Babe generates the same $K'$ to decide on the quantum measurement basis for each $b$ in $X_n$, and decode the bit value by the corresponding measurement. A classical error correcting code (CECC) may be used on the $n$-sequence to eliminate noise in the system that may originate anywhere, including source, transmission line, and detector. Privacy distillation may then be employed to bring the attacker Eve (E)'s bit error rate $P_{k}$ on the final key $K^g$ to any desired small level. Intrusion-level detection is avoided by granting E a full copy of the quantum signal for the purpose of bounding her information. Advantage creation is obtained from the different (optimal) quantum receiver performance, in an individual or joint attack, between B who knows $K$ and E who does not. Further data and signal keyless randomization may be obtained by A to guarantee security against joint attack and to improve the key-bit generation efficiency $k_{eff}^g$. Finally, the new key $K^g$ is verified to be correct by the use of another short key $K_v$ which may be done openly (publicly, i.e., the ciphertext is available to E). All these steps and features would be described and explained in detail in this and the following sections. There are significant theoretical and practical advantage of the $q_k$ scheme of Fig. 1 as compared to BB84. In particular, it can handle all system imperfection and noise of any origin at the same time, and produce reasonable key-bit generation efficiency $k_{eff}$.

The substantive use of secret keys in the key generation process other than message authentication has been introduced before $[12, 15, 17]$. In particular, it has been used in $[12, 17]$ in selecting one of the two possible bases in BB84 for improving $k_{eff}$, while keeping the other protocol steps including intrusion-level detection intact. In contrast, the security of our KCQ scheme is derived from a different quantum principle, the difference in optimal
quantum receiver performance with and without the key, rather than the information/disturbance tradeoff underlying BB84 and related schemes. Our use of a long running key \( K' \) and large \( M \) is not only essential in obtaining high \( k'_{\text{eff}} \), it is also essential for obtaining complexity-based security against known-plaintext attacks when KQ is used for direct encryption. It also plays a role in yielding reasonable key generation rates for coding-theoretic protocols with unconditional security. In contrast, the unconditional security proof described in Ref. [17] is not complete even with intrusion-level detection, due to the quantum state correlation or memory among different qubits induced by the shared secret key, in addition to the problems in such security proofs described in Appendix A. Indeed, it is a major problem in using a secret key that one needs to show there is a net result in cutting key generated after subtracting the original key used, or that the system somehow worth the \( |K| \) cost. Most significantly, the use of shared secret keys in our KCQ approach makes possible the development of large signal schemes with conventional optical technology as described in sections V and VI.

We first analyze the security of the above \( qk \) scheme under joint attack on the seed key \( K \) and a specific kind of individual attack on the data \( X \), given the quantum ciphertext. In contrast to data encryption, there is no known-plaintext attack \([4]\) in key generation because one \textit{presumes} \( A \) can generate completely random data bits unknown to both \( B \) and \( E \) a priori. The problem of how this can be done at high rate is a separate issue common to every kind of cryptography. The term 'individual attack' is ambiguous, but the attack considered in the quantum cryptography literature under this label usually refers to the situation where \( E \) prepares the probe/interaction to each qubit of the quantum signal sequence individually and \textit{identically}, measures each of her probe individually and \textit{identically}, and processes the resulting information \textit{independently} \([72]\) from one qubit to the other. Since, for bounding \( E \)'s information, we grant \( E \) a copy of the quantum signal sequence, the optimal performance of which clearly provides an \textit{upper bound} on \( E \)'s performance with an actual inferior copy obtained via a probe, there is no question of probe/interaction in the attack on such systems, only individual-qubit versus collective measurements. Possible disruption of the signal by \( E \) will be discussed in Sections III.F. The individual attack on the data analyzed quantitatively in the following is of the same nature as that in the BB84 literature \([2]\), namely a constant measurement on each qubit and independent processing. We will call such attacks \textit{constant individual attacks}.

Such an attack does \textit{not} include all possible attacks within a reasonable limitation on \( E \)'s technology, in both the BB84 and our schemes. If one may limit \( E \)'s measurement to individual qubit ones, perhaps because measurement across many qubits is difficult to make \([72]\), there is no reason to limit \( E \)'s classical processing after measurement to qubit-by-qubit separately, except for ease of analysis! A more detailed general classification of attacks and their analysis will be provided in the future.

Generally, let \( \rho_k^x \) be the quantum state corresponding to the data \( x \) \([72]\) (single bit or a bit sequence) and running key sequence \( k \) \([72]\) that is used to determine the basis and/or polarity of the \( qk \) scheme for that length of \( x \). For \( M/2 \) possible bases and a single bit \( x \), there is \( 1 + \log_2(M/2) \) bits in \( k \) for both basis and polarity determination. Each \( \rho_k^x \) can be represented as a real vector \( |r_k^x \rangle \) of norm 1 on the great circle, the angle between any two nearest neighbor vectors is \( 2\pi/M \) radian. The quantum ciphertext available to \( E \) for upperbounding her performance is \( \rho_k^x \) where both \( x \) and \( k \) are random.

For the purpose of attacking the key, the quantum ciphertext reduces to \( \rho^k = \sum_x p_x \rho_x^k \) where \( p_x \) is the a priori probability of the data \( x \). By an optimal measurement on the qubits, the probability of correctly identifying the key is obtained from \( \rho^k \) via quantum detection theory. We may let \( x \) be any \( n \)-sequence, so that \( \rho_x^k = \rho_{x_1}^k \otimes \cdots \otimes \rho_{x_n}^k \). Let \( p_x = 1/2^n \) for each \( n \)-bit sequence. It is easily seen from the quantum modulation format that \( \sum_x p_x \rho_x^k = \otimes^n(I/2) \) where \( I \) is the qubit

![Diagram](image-url)
identity operator, irrespective of the nature of $k$. Thus, $\rho^k$ is independent of $k$ and the quantum ciphertext provides no information on $k$ at all. Specifically, any processing on $\rho^k$ yields an a posteriori probability on $k$ equal to the a priori probability $p_k$, which may be chosen to be uniform for maximum security.

Consider now the attack on $x$, through the decision on $x$ from measurement on the quantum ciphertext in state $\rho_x = \sum_k p_k \rho^k$. For individual attacks of the kind described above, one obtains $\rho_x$ corresponding to a single bit by tracing out the rest of the data sequence. When a polarity bit is used from $k$, it is easily seen that E’s bit error probability is $P_E^k = 1/2$ by averaging on the two polarities alone. When the polarity bit is not used, $p_1$ and $p_0$ are different. The optimum $P_E$ is given by

$$(1/2) - (1/4) ||p_1 - p_0||_1$$

in terms of the trace distance $||p_1 - p_0||_1$ between $p_1$ and $p_0$, which can be explicitly evaluated for a single qubit, with resulting

$$P_E = \frac{1}{2} \left( 1 - \frac{1}{M} \frac{1 - \cos(\pi/M)}{2 \sin^2(\pi/M)} \right)^{1/2}. \quad (1)$$

This $P_E$ goes as $\frac{1}{2} - \frac{1}{4M}$ for large $M$ and can thus be made arbitrarily small.

That it is unreasonable to consider only such limited individual attacks can be seen as follows. Suppose a key $k$ of $1 + \log_2 (M/2)$ bits is used repeatedly to determine the polarity and basis choice of each qubit state in a sequence. Even though the above individual attack error rate for E is the ideal 1/2, in actuality E has a probability $2^{-|k|}$ that $|k|$ \equiv number of bits in $k$, of completely decrypting $x$ by guessing $k$ and using it on every qubit $\rho^k$. In a similar way, the same problem arises for a running key obtained from a short seed key $K$. That is, the correlation between bits in $x$ due to $k$ can be exploited by joint classical processing on individual qubit measurements. The presence of the encryption box in Fig. 1 does not improve the situation fundamentally for information-theoretic security even if the seed key $K$ is long, because E can generate $K'$ from a guessed $K$ as the encryption mechanism is openly known. Before we discuss general joint attacks in the next section, a number of issues on the above development would first be cleared up.

First, aside from information-theoretic security issue, the encryption box in Fig. 1 always increases the security of $q_k$ through physical and computational complexity, and it also increases the efficiency of key-bit generation fundamentally. Since there is often a trade-off between security and efficiency, increasing the efficiency without compromising security is in a sense increasing security (for a given efficiency). Suppose the encryption box is a stream cipher that outputs a long running key $K'$ from a seed key $K$. If it is a maximum length LFSR of $|K|$ stages, then $K'$ up to length $2^{|K|}$ is ‘random’ in various sense, even though an exact knowledge of $K'$ of length $2^{|K|}$ is sufficient to determine $K$ uniquely from the Berlekamp-Massey algorithm. If $K$ is used repeatedly without $K'$ in Fig. 1, the $\rho^k$ would be correlated by the repeated $k$ for short $x$-sequence, with any reasonable $|K|$ and $M$. A joint (measurement) attack can then be launched much more easily because the physical complexity - in this case the correlated qubit measurement - needed is much smaller than the one that comes from a long $K'$ from the same $K$. While $K'$ is not open to observation in the present case, computational complexity obtains in any event when one attempts to correlate the different bits in $x$ through the unknown key $K'$. Computational complexity is an excellent security mechanism if can be shown to be exponential, as the Grover search can only reduce the exponent by a factor of 1/2. Long keys with $|K| \sim 10^8$ can readily be used in stream ciphers, and searching $2^{100}$ items is already far beyond the capability of any imagined quantum computer. In conventional cryptography, many stream ciphers are used by themselves as the complete security mechanism. It can be incorporated in schemes such as Fig. 1 or the qumodes schemes of Refs. 18, 29 to increase the overall security in direct encryption.

Second, if cloning is possible so that $2^{|K|}$ copies of the quantum ciphertext are available, there is no possibility of key generation in principle. This is because E can use the $2^{|K|}$ different keys on the different copies, narrowing down the data to exactly $2^{|K|}$ possibilities corresponding to the key uncertainty. She can then follow whatever processing the users employ on her own data, and the users have succeeded only in obtaining a derived key, not a fresh one, whose randomness comes entirely from the original key without forward secrecy. This also explains why no key generation is possible when the user and the attacker have the same observation. Note, however, the difference between cloning and having one full copy. Having one copy is equivalent to the classical situation where many identical copies can be made, because together they do not tell the input data better than just one copy. Quantum mechanically, the quantum uncertainty goes down with the number of copies available from the laws of quantum physics - indeed the state is in principle determined exactly, say by quantum tomography, with an infinite supply of identical copies. Thus, the classical analog of ‘cloning’ is the granting of one identical copy, not many ones as in full cloning.

Third, the possibility of attacking the data $x$ is not completely described by $\rho_x$ in general. This is because E knows there is a $\rho^k$ representation. Thus, she can attack the key via quantum measurement on part of the qubit sequence, and then attack the data with any knowledge she may thus learn. A similar but less serious situation occurs for attacks on the key via partial attacks on the data first. These possibilities, however, do not arise in the case of constant individual attack.

Before turning to joint attacks, it should be noted that security results against individual attacks are far from useless. In practice, a joint attack may require correlated qubit measurement, as the qubit states are correlated through the running key $K'$. Thus, the physical
complexity of such quantum measurement and the computational complexity introduced through the encryption box would provide very significant security against attacks that can be realistically launched in the foreseeable future. When joint processing of the individual measurement results is performed, the quantum noise introduced in such individual measurement already yields a noisier copy for the attacker that allows advantage distillation by the user. This would distinguish our cryptosystem as a truly quantum one that has no classical analog, and give real meaning to the security claim against individual attacks. Performance under different types of such ‘individual attacks’, including ones involving adaptive qubit-by-qubit measurements, will be presented in the future.

Consider the operation of qk of Fig. 1 for a perfect qubit channel, under joint (collective) attack by E where, as a performance bounding technique, she is supposed to have a full identical copy of the n-sequence quantum state as B. She has to make a quantum measurement, however, without knowing the key which she may possess later — see section III.B for a complete description. Here we would observe that she would not be able to make a perfect decryption with probability equal to 1 for any finite n. After whatever quantum measurement on the n-qubits she made, she still would not be able to make a perfect decryption if K is then given to her. This is because a perfect decryption occurs when and only when the measurement she made is exactly that prescribed by K. This shows that an advantage is created which may lead to an unconditionally secure protocol with or without the use of further channel coding, as shown in section III.H. When channel noise is present, a classical error correcting code (CECC) on the quantum states may be employed and separate privacy amplification may be required. The exact quantitative performance will be detailed elsewhere.

III. GENERAL PRINCIPLES OF KCQ KEY GENERATION

In this section, the basic principles underlying key generation via KCQ will be explained. The general principles of key generation will first be reviewed and analyzed, extending the usual framework to include shared secret keys and the more appropriate criterion of error profile in addition to and in place of mutual information. The quantum nature of KCQ will be pinpointed. The overall steps and structure of a KCQ key generation protocol will be exhibited, and the conditions for key generation established. Various basic conditions on key generation will be discussed, particularly in relation to the usual QKG approach. This section III and the later qumode key generation section VI that gives the specific qumode QKG protocol CPPM may be regarded as the heart and brain of this paper. For a detailed review of the background in classical as well as quantum detection and communication theory for classical information transmission that is crucial for a complete understanding of these sections, please see Ref. [36] and references cited therein. For a development of optical communication theory that is important in fully comprehending the details of how KCQ protocols work, please see also Ref. [5]

Consider an entire joint process of data transmission and encryption/decryption as described in Fig. 2. A sends an l-bit sequence U_i and encrypt/encode it into an n-qubit or n-qumode sequence in state \( \rho^E_x \) with the possible use of a shared secret key k with B, which may include a source code key \( K_s \), a channel code key \( K_c \), and a quantum state modulation code key \( K_m \). Classically, \( \rho^E_x \) would be replaced by just an n-bit channel input sequence \( X_n \) corresponding to the x in \( \rho^E_x \). The ‘channel’ represents all the interference from the system one has to suffer, with Ch_i giving output qubit states for \( i = E, B \). For E who does not know k, the state is \( \tilde{\rho}^*_z \) upon which she picks a measurement on the basis of that and her later knowledge from all sources including public discussion to produce an estimate \( \tilde{\rho}^*_z \) of \( \rho^E_x \), the final key generated by A and B. For B who knows k, the channel output state is \( \rho^B_k \) from which she uses her knowledge of k to obtain an estimate of \( \tilde{\rho}^*_B \) of \( U_i \). Classically, the states would be replaced by the observations \( Y^E_x \) and \( Y^B_n \), the disturbed output of \( X_n \). Quantum mechanically, they are the results of corresponding optimal or near-optimal measurements on the qubits or qumodes from which the estimates \( \tilde{U}^B_i \) are made. One may first consider, for simplicity, that \( Y^E_x \) is obtained without knowledge of \( K_m \). More generally, one may split \( \rho^E_x \) into parts from which attacks on x and on k are interwoven. Privacy distillation may already be incorporated in this process, or may be added to \( U_i \) and \( \tilde{U}^B_i \). The use of such an approach for direct encryption is briefly treated in section VII.

The essential steps in the operation of a KCQ key generation protocol involve

(i) The use of a shared secret key \( K_m \) between A and B that determines the quantum states generated for the data bit sequences in a detection/coding scheme between A and B that gives them a better error performance over E who does not know \( K_m \) when she makes her quantum measurement;

(ii) A way for A and B to extract a fresh key from the above performance advantage:

(iii) A key verification process using another shared secret key \( K_v \) between A and B.

The main novelty and power of this approach, in principle, consists of

(a) Performance advantage is derived from the different quantum receiver performance between B who knows the key \( K_m \) when she performs her quantum measurement and E who knows \( K_m \) only after she has made her quantum measurement.

(b) No intrusion-level detection or even intrusion de-
FIG. 2: General keyed communication in quantum noise.

The users derive from the generated string a generated key $K^g$ on which E’s error probability profile satisfies a given security level.

As a consequence, this approach makes possible the development of an efficient, secure key generation protocol over long-distance telecomm fibers using commercial optical technology. In the following, these points will be fully explained and explicated. The contrast between KCQ and the usual QKG approach, both in theory and in practice, will be highlighted.

A. Principle of Key Generation

A key generation protocol with information-theoretic security, whether it is based on classical or quantum randomness, would consist of the following three logical steps:

(i) Advantage Creation:
The users A and B create a communication situation between themselves with an observed random variable $Y_n^B$ for B that leads to a better error performance than that obtained by E from her observed random variable $Y_n^E$ and all her side information.

(ii) Error Correction:
The users agree on a generated string that is free of error with high probability if E is absent.

(iii) Privacy Distillation:
The first step (i) may be achieved classically in the presence of different noises for B and E’s communication channels with respect to A’s data, using perhaps the help of public discussions between A and B. In the quantum key generation approaches so far, (i) is achieved via intrusion-level detection, explicit or implicit, that guarantees that A and B have a better communication line than A and E in the sense of mutual information, which is also the privacy distillation criterion used in step (iii). The steps (ii) and (iii) could be combined by an error correcting code, quantum or classical, that simultaneously performs privacy distillation. This is indeed the way the usual QKG unconditional security (existence) proofs involving CSS codes [22, 23, 24, 25, 57] are carried out. In QKG experiments to date, these steps are distinct and a separate privacy distillation code is employed whenever step (iii) is implemented. In our KCQ protocol called CPPM in section VI, step (iii) is automatically achieved in an ideal fashion from the $m$-ary signaling scheme employed. More generally, we will show that privacy amplification is unnecessary in most cases when the proper criterion of E’s optimal error probability is used in place of her mutual information.
B. Advantage Creation with Shared Secret Key

In the literature [1 2 3], it was shown that if a situation is obtained in which the mutual information between A and B, \( I(X_A; Y_B) \) for the random variables \( X_A \) and \( Y_B \) in A and B’s possessions, is bigger than that between A and E, \( I(X_A; Y_E) \), or that of B and E from the symmetrically-inter-changeable roles of A and B, key generation is possible. That is, an information-theoretic existence proof is given under the condition

\[
I(X_A; Y_E) < I(X_A; Y_B) \tag{2}
\]

with the conclusion that an asymptotic key generation rate \( \Delta I = \max_{p(X_A)}[I(X_A; Y_B) - I(X_A; Y_E)] \) is possible between A and B with the (total) amount of mutual information E has on the key generated being arbitrarily small. In these results, there is no shared secret key between A and B.

Such results can be generalized to include the use of a shared secret key \( K \) as follows. E is going to observe her channel output \( Y_E \) without the benefit of knowing \( K \). However, one has to make sure that the resulting \( K^9 \) generated between A and B is fresh, i.e., statistically independent of \( K \) and E’s observation \( Y_E \), i.e. \( I(K^9; Y_E K) \sim 0 \). Indeed, E can try every possible \( 2^{|K|} \) keys on her observation \( Y_E \) to determine the possible \( X_A \)’s. A conceptually convenient way to characterize this situation is to give E the key \( K \) after she made her observation \( Y_E \). Using the notation \( I(X_A; Y_E K) \) to denote her information in this situation where \( Y_E K \) denotes the joint random variables \( Y_E \) and \( K \), (3) generalizes to

\[
I(X_A; Y_E K) < I(X_A; Y_B) \tag{3}
\]

In [4], B is of course supposed to know \( K \) when she plans to observe \( Y_B \). Classically, the condition [4] may result if there is a limit on the data storage so that E cannot have the same observation as B who can just store the relevant data using the knowledge of \( K \), as in the broadcast scheme of Maurer. While the shared secret key \( K \) is what occurs in the above \( I(X_A; Y_E K) \) with which E may use to estimate \( K^9 \) in KCQ protocols, \( K \) may be interpreted as all the side information E obtains in other QKG protocols such as BB84, or as the missing information that allows the conditional entropy \( H(X|Y_E, K) = 0 \) in a classical random channel protocol. These useful interpretations would be used later.

The following important relation between any three random variables should be noted:

\[
I(X; Y K) = I(X; Y | K) + I(X; K) \tag{4}
\]

In the KCQ context, \( I(X; Y K) = 0 \) and \( I(X; Y_E|K) \) can be used in lieu of \( I(X; Y_E K) \). However, the distinction is important and has various ramifications in the BB84 key generation when \( K \) is interpreted as \( E \)’s side information.

Quantum mechanically, the knowledge of \( K_m \) that specifies the mapping from classical data to quantum states would allow B to choose the optimum or near-optimum quantum measurement to discriminate among the data. Without knowing \( K_m \), on the average E would need to pick a quantum measurement that would allow her to make reasonable estimates for different \( k_m \)’s, which leads to an inferior performance compared to B for a specific \( k_m \). This situation clearly obtains when E does not have long-term quantum memory to hold her copy of the quantum signal before she has to make a quantum measurement to extract the information without knowing the key. In practice, the key \( K_m \) can be erased or kept secret indefinitely from E, as E really would never have \( K_m \), and she has to make a quantum measurement without knowing \( K \) even if she has long-term quantum memory.

Classically, there is no need for E to know \( K \) in order for her to be able to correlate in a definite manner the different data connected by \( K \), such as the different session keys generated from a master key. For example, with the observations of \( x_1 \oplus k \) and \( x_2 \oplus k \) for two independent bits \( x_1 \) and \( x_2 \) and a secret bit \( k \), one knows exactly \( x_1 \oplus x_2 \) while knowing nothing about \( k \). In this quantum situation, however, such correlation cannot be obtained without quantum measurement on the quantum signals that E possesses. Thus, E has to suffer the uncertainty of picking her quantum measurement without knowing \( K \) even if she has long-term quantum memory. This is the principle underlying advantage creation in KCQ protocols. Note that this is a quantum effect with no classical analog, because classically E can always make a complete observation of her received signal in principle. There would be no incompatible measurements as in the quantum case. The following intuitive result related to the advantage of a shared secret key, as well as the average effect of side information or missing information, is useful in the analysis of classical and quantum key generation.

**Lemma 1**: For any three joint random variables \( X, Y, K \),

\[
I(X; Y K) \leq I(X; Y) + H(K). \tag{5}
\]

**Proof**: From \( I(X; Y K) = H(X, K) - H(X|Y, K) \), \( \text{(6)} \) is equivalent to \( H(Y) + H(K) + H(X|Y, K) \geq H(Y|X) + H(X) \). The left side of the last inequality is \( \geq H(X, Y, K) = H(X) + H(K|Y) + H(X|Y, K) \geq H(X, Y) = H(X) + H(Y|X) \), completing the proof.

According to \( \text{(6)} \), if \( K_m \) represents the missing classical information in a channel with random parameter that together with \( Y_E \) yields correct decryption, or \( H(X_A|Y_E, K_m) = 0 \), then \( H(K_m) \) is indeed the maximum possible amount of missed information, which may not be large. For example, \( K_m \) may represent the signal phase or amplitude variation caused by a classically random channel. In the quantum case, however, no missing information can make up totally the loss from \( Y_E \) to \( Y_E \). There is irretrievable loss from the inferior quan-
C. Eve’s Information and Error Profile

The security criterion for key generation, classical or quantum, has so far been limited to \( I(K^g; Y^E_n) \), the mutual information between Eve’s observation \( Y^E_n \) and the final key \( K^g \) generated, with the provision that E’s side information needs to be accounted for \([33]\). Except in the limit \( I(K^g; Y^E_n) = 0 \) which says that \( K^g \) and \( Y^E_n \) are statistically independent, the information-theoretic quantity \( I(K^g; Y^E_n) \) has no clear quantitative operational significance with regard to the usefulness of \( Y^E_n \) for Eve in a eavesdropping context. One such operational criterion is given by Eve’s trial complexity \( C_t \) as measured by the average number of trials she needs to successfully use trial keys on the basis of her information. For example, when she knows nothing about \( K^g \), which she would guess in successive trials to say, open a safe, she would need an average of \( C_t = 2^{[K^g]} - 1 + 1/2 \) trials to succeed. It would also describe E’s ability when she launches a known-plaintext attack on \( K^g \) used in a standard cipher. In general, \( C_t \) depends on her exact error profile \( p(K^g, K^E_n = Y^E_n) \), the probability that given her information, each of the \( 2^{[K^g]} \) guessed sequence \( K^E_n \) is correct, as given in the following equation (6). This error profile \( E \) can be obtained herself from the conditional probability \( p(X_A | Y^E_n) \), which is in turn specified by her channel transition probability, the a priori data probability, and the overall coding/communication format including possible deliberate randomization by A. Furthermore, many different \( p(K^g, K^E_n = Y^E_n) \) leads to the same \( H(K^g | Y^E_n) \) or \( I(K^g; Y^E_n) = |K^g| - H(K^g | Y^E_n) \), which alone is not sufficient to capture E’s ability to use her information. In the case \( I(K^g; Y^E_n) = 0 \), \( p(K^g, K^E_n = Y^E_n) = 2^{-|K^g|} \) for any \( K^g \). It is not known how \( I(K^g; Y^E_n) \) is related to \( C_t \) in general except in the asymptotic limit when E can encode or interpret \( I(K^g; Y^E_n) \) via asymptotic equipartition \([34]\), which she cannot since she does not control A’s data transmitter. One may lower bound E’s average bit-error probability \( P^E_n(K^g) \) by Fano’s inequality \([34]\), which is valid for any n-sequence \( Y^E_n \) that may possess correlations among its bit values. Let \( H_2 \) be the binary entropy function. Fano’s inequality gives, in the present situation,

\[
H_2[P^E_n(K^g)] \geq 1 - I(K^g; Y^E_n)/n. \tag{6}
\]

If \( I(K^g; Y^E_n) \) is exponentially small with exponent \( \Lambda \), it follows from \(\text{[9]}\) that \( 1/2 - P^E_n(K^g) \) is exponentially small with exponent \( \Lambda/2 \). However, the bit error rate is not really meaningful in this connection because the bits may be highly correlated in the way they affect \( C_t \).

The following analysis shows that \( I(K^g; Y^E_n) \) is not a sufficient measure of E’s capability unless it is really extremely small. One needs to supplement it at least by \( \bar{p}_E = \max p(\hat{K}^g_n = K^g | Y^E_n) \), which is an important criterion in our KCQ approach. The error profile gives the probabilities \( p_1 \geq p_2 \cdots \geq p_N, N = 2^{[K^g]} \), for each of the \( N \) guesses \( \hat{K}^g_n \). Thus, \( \bar{p}_E = p_1 \) and the trial complexity \( C_t \) is

\[
C_t = \sum_{n=1}^{N} np_n. \tag{7}
\]

Given that \( E \) has \( I_E \) bits of information on \( K^g \), with \( |K^g| = n \), the largest \( p_1 \) that can be obtained is given by the error profile that spreads \( 1 - p_1 \) among the \( 2^n - 1 \) other possibilities uniformly, i.e., it is determined by the equation

\[
H_2(p_1) + (1 - p_1) \log(2^n - 1) = n - I_E. \tag{8}
\]

Thus, \( p_1 \sim 2^{-l} \) for \( I_E \sim n \cdot 2^{-l} \) and large \( n \), and \( E \) needs only about 1 bit of information out of \( |K^g| = 100 \) for a possible error profile with \( p_1 = 10^{-2} \), a disastrous security breach. This most favorable situation for \( E \) with a given \( I_E \) may be contrasted with her most unfavorable situation, where her 1 bit knowledge corresponds to knowing one bit of the \( |K^g| \)-bit sequence exactly.

As we have just seen from \(33\), \( p_1 \) can be made about as large as the fraction of bits \( |K^g| \) that she knows through \( I_E \). To ensure \( p_1 \leq 2^{-l} \) through \( I_E \), one needs to impose a strong requirement that \( I(K^g; Y^E_n) < n \cdot 2^{-l} \). In addition to the necessity of assessing the actual minimum \( C_t \) attainable with a given \( I(K^g; Y^E_n) \), the above condition on \( p_1 \) alone would lead to \( l \sim 100 \) for a truly secure system, which is practically very difficult to obtain on the basis of reducing \( I(K^g; Y^E_n) \) for realistic \( n \). On the other hand, a more detailed assessment of the error profile or just \( p_1 \) would give a more accurate estimate of E’s true ability to use her information than the mere \( I(K^g; Y^E_n) \) as shown in the following.

Under the condition \( p_1 \leq 2^{-l} \), one obtains

**Lemma 2:**

When E’s optimum estimate of \( K^g \) has an error probability \( P^E_n \geq 1 - 2^{-l} \), her information on \( K^g \) satisfies \( I(K^g; Y^E_n) \leq n - l, n = |K^g| \).

**Proof:**

Since \( p_1 \leq 2^{-l} \), it follows that \( l \leq n \). The maximum \( I_E \) is obtained when \( H(p_i) \) is minimized, which occurs at \( p_1 = \cdots = p_m = 2^{-l} \) for \( m = 2^l \), \( p_{m+1} = \cdots = p_{2^m} = 0 \), from the Schur-concavity \([34]\) of \( H \) (which can actually be seen to follow from concavity directly).

Although this bound is weak, it may be used to establish the existence of secure KCQ protocols under rather general situations to be described in section III.H. Specifically, we take the approach that instead of \( I_E \), \( P_E \leq 2^{-l} \) has to be imposed, which leads to \( C_t \geq (2^l + 1)/2 \) similar to Lemma 2.

**Lemma 3:**

...
Let E’s optimal estimate of $K^g$ have a success probability $\bar{p}_E \leq 2^{-t}$, then her trial complexity $C_t$ is lower bounded by $C_t \geq (2^t + 1)/2$.

Thus, regardless of her mutual information $I_E$, Lemma 3 guarantees E’s trial complexity at a level that may already be satisfactory, and does that without privacy distillation. Note that a bound on $C_t$ for given $I_E$ can be obtained via (8) and Lemma 3. It follows from Lemmas 2 and 3 that it is much more useful to impose a bound on $p_1 \leq 2^{-80}$ for $n = 100$ that may correspond to $I_E \sim 20$, than a bound on $I_E \leq 1$ that may corresponds to $p_1 \sim 10^{-2}$. Furthermore, e.g., a system with $|K| \sim 20, n \sim 200, l \sim 100$ would be quite useful regardless of what $I_E$ actually is.

In view of our replacement of a constraint on $I_E$ by that of $\bar{p}_E$ together with the corresponding elimination of privacy distillation, the following comment is in order. Privacy distillation moves E’s uncertainty among its randomness to E as already observed in Ref. [8]. In as much as the qumode case) to launch this attack once. This corresponds to the above error profile with $p_1 = 2^{-|K|n}$ if no more than one trial. Note that the seed key $K_m$ has to be guessed in total as it is not being used in a bit by bit or segment by segment manner.

D. Key Rate of Secure Protocols

As is evident form Fig. 2, E has to deal with the $X_n$ chosen by A and has no independent way to encode her own channel $I(X_n;Y_n^E)$. Thus, by choosing a rate $R$ in between (2) and (8), one may be able to force the second term in $\Delta I$ to be zero and obtain $\Delta I = \max_{Y_n}(X_n) I(X_n;Y_n^E)$ as a consequence of the Shannon Coding Theorem and its Strong Converse for memoryless channels. The Strong Converse states that for memoryless channels. The Strong Converse states that the block error rate goes to 1 exponentially in the block length $n$ at rates above capacity, which may already imply $I(X_n;Y_n^E) \to 0$ as will be seen in the following proof of a code that generates arbitrarily close to $m(X_n;Y_n^E)$ bits.

It should be noted that condition (10) already includes the cost of the key $|K|$ for net key generation. If one uses the net-key generation condition

$$I(X_n;Y_n^E)/n < R < I(X_n;Y_n^B)/n$$

It is more stringent than (9) from Lemma 1. If the system is information-theoretic secure against known-plaintext attacks for direct encryption, briefly discussed in section VII and treated extensively in Part II, the keys may be re-used because the different fresh keys generated from two different uses of $K$ are independent of each other. In such a case, only condition (10) needs to be satisfied. If the $K_i$ are not re-used, a net key generation rate is obtained under (8) or (10) and (11).

More generally, since the above strategy yields a block error rate $P^E > 1 - e^{-me}$ even when the error-free data is used as $K^g$ without additional privacy distillation. Using lemma 2, we have also $I(K^g;Y_{nm}^E|K) \leq m(n - \varepsilon)$. To summarize, using also lemma 3 we have

**Theorem 1:**

Under condition (10), unconditionally secure protocols may be obtained via error correction coding but without further privacy distillation that satisfy

$$\bar{p}_1 \leq e^{-me}, \quad I(X_n;Y_{nm}^E|K) \leq m(n - \varepsilon), \quad C_t \geq (2^m + 1)/2$$

where $\varepsilon > 0$ is determined from the channel specification.
Note that a net key $K^g$ can be taken as $X_n$ or its privacy distilled version with $I(K^g; Y_n^E, K) \to 0$ guaranteed from \[ \text{[12]} \] only when $|K| < \varepsilon$. On the other hand, the security level \[ \text{[12]} \] itself may be satisfactory already. We use the phrase ‘unconditional security’ above in the usual sense, except that the security level is measured by $\bar{p}_E$ and $C_l$ rather than $I_E$. This represents a more efficient approach because, as we have seen in III.C, a bound on $\bar{p}_E$ has to be imposed in any case.

E. Specific Detection or Coding Scheme

As in all the unconditional security proofs of QKG protocols so far presented in the literature, the above development can only yield an existence proof of a secure protocol, because no specific code has been given. For actual application, one would need to provide a specific coding or signaling scheme, in either classical or quantum key generation, and show quantitatively that Eve has little knowledge on $K^g$. While $I(K^g; Y_n^E)$, to be supplemented by E’s side information, is a measure on E’s information, it is not the most useful quantity to deal with as we have already seen above because E cannot encode. Indeed, as discussed in III.C, it is her error profile on $K^g$, not just $I(K^g; Y_n^E)$, that really matters. The appropriate measure to this end is her optimum block error probability $\bar{P}_E^c(X_n|Y_n^E, K)$ given her observation and side information. For advantage creation, one wants to obtain the situation where

\[
\bar{P}_E^c(X_n|Y_n^E, K) \to 1, \quad \bar{P}_E^B(X_n|Y_n^B) \to 0. \tag{13}
\]

Condition \[ \text{[13]} \] is equivalent to the above coding-theoretic existence result when one argues, as in \[ \text{[12]} \], by coding on the $n$-bit symbols. In actual application, it may well occur that the users work with only a single $n$-bit symbol at a time, especially when it is to correspond to n-bit coding. In such case, \[ \text{[4]} \] or \[ \text{[9]} \] can be ignored and \[ \text{[13]} \] is the appropriate condition for advantage creation. Similar to Theorem 1, we obtain via Lemma 2 and Lemma 3.

**Theorem 2:**

For a detection scheme that has $\bar{P}_E^c(X_n|Y_n^E, K) > 1 - 2^{-l}$ while $\bar{P}_E^B(X_n|Y_n^E)$ is error-free in the use, an unconditionally secure protocol for the following fixed security level is obtained without further privacy distillation and with a key cost $|K|$, 

\[
\bar{p}_E \leq 2^{-l}, \quad I(X_n; Y_n^E|K) \leq n - l, \quad C_l \geq (2^l + 1)/2. \tag{14}
\]

Note that Theorem 2 is valid for a single use of the block detection scheme without further coding. As in Theorem 1, the scheme is certainly useful whenever $|K| < l$ if the resulting security level is satisfactory. To decrease $I(K^g; Y_n^E K)$, further privacy distillation may be needed, while no guarantee is offered in the theorem that a net key with $I_E \to 0$ can be obtained that is larger than $|K|$. However, in an appropriately designed scheme, it may already happen that $I(X_n; Y_n^E K) \to 0$ with a secure error profile without further privacy distillation. In such a case, the following bit-error rate condition holds with independent bit errors,

\[
P_b^E(X|Y^E, K) \to \frac{1}{2}, \quad P_b^B(X|Y^B) \to 0. \tag{15}
\]

See section VI for an example of a specific protocol that displays such behavior under rather general attacks. Condition \[ \text{[15]} \] implies \[ \text{[13]} \], and can be taken as the advantage creation condition at the bit level that requires no further privacy distillation for generating $K^g$.

Observe that in the security proof of a specific detection/coding scheme, one must be careful to ascertain E’s optimum block error by including her adaptive attacks, in particular ones via attacks on the key. Also, for a full security proof one needs to solve the novel quantum detection problem in which one chooses the optimum quantum measurement in anticipation of making a future decision on the basis of further information not available at the time of quantum measurement. However, such problems also occur in the usual QKG protocols even in the context of individual attacks, whenever a specific scheme is employed rather than a mere coding-theoretic existence claim. They have yet to be dealt with in the literature.

F. Key Verification

A final key verification process is needed in KCQ protocols as compared to BB84. In this case, A or B uses the generated key $k^g$ to encrypt a fixed shared secret bit sequence $K$, which is perhaps the extended output of a fixed known transformation on some separate shorter shared secret key $K_n$, $|K_n| \leq |K^g|$, and sends it to the other party through reliable communication. The encryption mechanism for getting $K$, via $K^g$ may be the same as Fig.2 but used for direct encryption, thus no privacy distillation and key verification would be involved there. It is also possible to reverse the above roles of $K_n$ and $K^g$ in the verification, making it similar to a message authentication protocol on $K^g$ with a secret key $K_n$. When $K_n$ is used as one-time pad to check an openly chosen random unkeyed hashed version of $K^g$ of size $|K_n|$, the average probability that two different $K^g$ are mistakenly agreed upon is $2^{-|K_n|}$. This results from a random coding argument similar to the proof of the Shannon Channel Coding Theorem or the privacy distillation code performance theorem in Ref. \[ \text{[3]} \]. The standard results on hashing collision \[ \text{[3]} \] can also be used instead. If the users believe there is only a small number of errors in $k^g$, they may try to correct them via open discussion as in some BB84 protocols, via parity check or other methods.

Given that a common key $k^g$ is generated between A and B, it can be seen that E’s information cannot be more than what she can get from one full copy of the quantum
ciphertext, which may be granted to her for the purpose of bounding her information. This is because whatever probe she introduced that may mess up the state B receives, she cannot obtain more information than that of a full copy although she may introduce enough errors to make $K^g$ agreement between A and B impossible. Such a mess up, however, is not something A and B could avoid in the presence of even a ‘passive’ attacker that takes energy out of the signal by tapping. Thus, there is no loss of generality in not being able to establish a key in the presence of E, so long as the protocol is not sensitive as to be discussed in section IV. Also, E cannot correlate her own states and B’s states to obtain information, as she does not know $K$ and what states to correlate with. The probability of such successful attack is small exponentially in the number of bits $n$. Thus, perfect forward secrecy of $K^g$ with respect to $K$ is obtained, while both intrusion detection and intrusion-level detection are avoided.

G. Advantage Enhancement

Given that advantage can be created, it is possible to enhance it, i.e., decreasing the attacker’s performance for a fixed user performance, by various techniques. These include first of all data bit randomization (DBR), the use of a randomly chosen open or secret source code that re-arranges the $U_i$ in Fig. 2. Secondly, one may employ deliberate error randomization (DER), the addition of error bits to B introduced deliberately by A with a corresponding error correcting mechanism such as a CECC on the quantum states. Thirdly, one may introduce chaining among the data and keys, i.e., the use of local feedback to make future transmissions dependent on past ones. Especially when used in conjunction, such techniques could lead to a flattening of E’s error profile for any fixed $n$, and hence her trial complexity $C_t$ whether her entropy $H(X_n|Y^n,E,K)$ is affected or not.

In the classical situation where a fixed amount of missing information represented by $\tilde{K}_m$ can be used to uniquely specify the channel suffered by E as described in III. B, all such techniques cannot produce $H(X_n|Y^n,E,K)$ more than $H(\tilde{K}_m)$ from lemma 1. Nevertheless, such technique may still be useful in a given application because $|\tilde{K}_m|$ may be large or it may be difficult to ascertain exactly. Furthermore, this is not the amount needed to be provided as shared secrets between A and B. Quantum mechanically, there is no $\tilde{K}_m$ that can restore the data perfectly for E in a KCQ protocol. Thus, these techniques could increase the key generation rate beyond the original $\Delta I$. In both the classical and quantum cases, such possibility arises because the joint probability distribution of the random variables $(X_A,Y_B,Y_E)$ are not specified a priori, but rather subject to the creation of communication lines between (A, B) and (A, E) in a given communication situation.

Another technique for enhancing the advantage may be obtained as follows. Let $1 - \lambda$ be the fraction of system splitted off by E and $\lambda$ the one remaining for B. This can be easily quantified in qumode system via the signal energy, so that $\eta\lambda$ is the total fraction received by B under the line transmittance $\eta$. Let $p^B_\eta$ be the probability that A and B verify that their generated keys agree with a fraction $\eta\lambda$ of the signal received by B, and $p^{E}_{1-\lambda}$ the probability that E correctly obtains the key with her fraction $1 - \lambda$. The strategy of granting E a copy of the quantum signal to bound her information is equivalent to the condition that E’s probability of successful cheating $p^E_\eta$, $p^E = p^B_\eta \cdot p^{E}_{1-\lambda}$, is small when A and B proceeds as if E were not interrupting in a KCQ protocol described in III. F. By making $p^B_\eta$ small for $\lambda \leq \lambda_0$, a set threshold, $p^E$ is modified to

$$p^E = p^B_\eta \cdot p^{E}_{1-\lambda_0}. \quad (16)$$

This represents advantage enhancement since (16) is smaller than $p^B_\eta \cdot p^{E}_{1-\lambda}$. Two remarks on this technique are in order. First, under the use of (16) which is a kind of pre-set automatic intrusion detection, the cryptosystem becomes more sensitive and thus loses some of the robustness characteristics of KCQ protocols. Second, this technique may be considered as one of advantage creation, because the user’s performance may be correspondingly lowered with the decrease of the attacker’s performance.

H. Overall KCQ Protocol

Schematically, a KCQ protocol corresponding to the communication situation of Fig. 2 may be summarized as follows.

Generic KCQ Protocol:

(i) A picks a random bit sequence $U_i$, encodes and modulates the corresponding $n$ qubits or $n$ qumodes as in Fig. 2, with a total secret key $K = (K_x,K_c,K_m,K_v)$ shared with B.

(ii) From $K_m$, advantage creation is achieved via the different error performance obtainable by B and E who does and does not know $K_m$ at the time of their quantum measurements.

(iii) Advantage enhancement and privacy distillation may be achieved with appropriate system design, deliberate randomization and chaining techniques, so that a substantial net key can be generated on which E has as little information or as large an optimum error probability as desired.

(iv) A and B verify that they agree on a common $K^g$ by using it with the secret key string $K_v$.

It is important to note the crucial role of the key verification process in the overall protocol. If E messed up the state B receives, it produces no effect on the security of the protocol if the key is verified because $E$ cannot in any case have more information on the correctly agreed
key than what she can get from one full copy of the quantum signal, exponentially probabilistically. If the system is designed to be not sensitive to small disturbance, as any practical system must be, it is perfectly fine that the presence of E would disrupt the key generation process so that the key is not verified. On the other hand, there are problems in QKG protocols with intrusion-level detection that the key is not verified. On the other hand, there are problems in QKG protocols with intrusion-level detection that the key is not verified.

We have explained the above steps that may enhance the advantage and improve the efficiency and security of the protocol. A full treatment of specific schemes will be given in the following and in Part II. By combing the analysis of sections II and III, we have

**Theorem 3:**
In the absence of channel noise, the protocol $qk$ of Fig. 1 allows unconditionally secure key generation for any fixed $n$-sequence with a security level given in the form $\rho^k$ without further privacy distillation.

**Proof:** We have seen in section II that E’s error probability is bounded from zero for any $n$, while B’s is exactly zero, and the key $K$ is completely hidden with the quantum ciphertext alone. Thus, the above generic KCQ protocol would generate a fresh key. By coding as in Theorem 1, the security level $\rho^k$ is obtained where $\varepsilon$ depends on the encryption mechanism and the state $\rho_x$.

Intuitively, one may expect that $H(K|X_n, Y_n^E)$ would remain substantial in $qk$ even for large $n$, and also that $H(X_n|Y_n^E, K)$ is large. However, in the absence of either a security proof against known-plaintext attack or a proof that $I(X_n; Y_n^E K)/n$ can be made sufficiently small, Theorem 3 is not sufficient to guarantee a nonzero net key generation rate that E knows essentially nothing about. On the other hand, the bounds on $\rho_x$ and $C_v$ should be sufficient. A similar result would hold in the presence of channel noise, but a rigorous proof requires new techniques to be presented in Part II.

More generally, secure protocols can be created by using quantum entanglement as follows. For each $n$-bit data sequence $x$, let $2^{[K]}$ mutually orthogonal states in $\otimes_{i=1}^n H_i$ be the possible $\rho_x^k$ corresponding to differnet values of $K$, $|K| < n$. Each individual state space $H_i$ may be of any dimension. There exist many such modulation formats where the resulting $\rho_x = \sum_k \rho_x^k$ are not mutually orthogonal for different $x$. We have

**Theorem 4:**
In the absence of channel noise, a KCQ protocol employing the above state modulation allows unconditionally secure key generation for any fixed $n$-sequence with security level given by $\rho^k$ without further privacy distillation.

**Proof:**
From Lemma 1, $I(X_n; Y_n|K)$ as obtained by optimizing over all E’s possible quantum measurements is bound by $H(K) + I(X_n; Y_n)$, obtained by the same measurement. From Holevo’s inequality $\rho^k$, $I(X_n; Y_n) \leq S(\sum_x p_x \rho_x) - \sum_x p_x S(\rho_x)$. From the above modulation format, $S(\sum_x p_x \rho_x) < n$ while $S(\rho) = |K| = H(K)$. Thus, condition $\rho^k$ is satisfied and unconditionally secure protocols exists, via coding $n$-bit blocks described in III.D, with a security level given by $\rho^k$.

For both Theorems 3 and 4, the system could be useful for any $|K| < n$ while the $|K|$ cost is negligible for large $m$, but the net key generation rate is not guaranteed to be nonzero if $I(K^g; Y_n^E)$ has to be made extremely small. Note that the possibility of security proof for QKG relies on the claim that all the possible useful actions of E have been exhausted. That this is true in any particular protocol has to be ascertained carefully. We assert that this is the case for KCQ protocols described in this section and section VI, as long as the model is taken to be a valid description of the real situation $\rho^k$.

Note that although security against attacks on the key is automatic in $qk$, it is not in a general KCQ system. Even with key security on quantum ciphertext-only attacks, one needs to consider situations where some knowledge on the data $x$ is obtained from a partial attack on $\rho_x^k$, and then an attack on the key $k$ is launched with such knowledge, and then on $x$ again, etc. The system is not fully secure unless the key is secure against such ‘statistical attack’. This problem also occurs in BB84 type QKG when the generated key $K^g$ is used later for direct encryption. Even for the one-time pad mode, a known-plaintext attack can be launched to learn part of $K^g$, and knowledge on the rest may be obtained by E from this and her probe information. These problems will be analyzed in detail in part II.

I. Unconditional Security and System Implementation

Unconditional security (US) in QKG is usually taken to mean security against all possible attacks allowed by the laws of physics (and logic), at a level that can be made arbitrarily close to perfect. In particular, it would imply security against an attacker that has unlimited computational power and hence can perform any exhaustive search. Unconditional security must therefore be information-theoretic, not complexity-based, security. The term has, lately, often been used in a weakened sense, such as ‘unconditional security against individual attacks’. As long as the security claim is precisely spelled out, the terminology issue is secondary. Since the security is only as good as the mathematical model being valid at the time of system use, it is useful to consider various different qualitative degrees of unconditional security, especially in commercial type applications. As a matter of fact, the experimental development of QKG is still struggling within the realm of individual attacks, the
justification being that it is physically complex, and currently practically impossible, to launch more general attacks. In this paper, various weaker claims of security are also considered, especially for specific quantifiable protocols.

In the presence of system imperfection including those arising from the source, transmission line, and detector, we separate out E’s disturbance and call the rest ‘channel noise’ and ‘channel loss’, as is common in communication theory. In a situation intended for cryptographic applications, one first determines what this actual channel including all these imperfections is, as characterized by the channel parameters in a canonical representation with some confidence interval estimates on these parameters. To guarantee security, advantage creation is to be obtained under the following Advantage Creation Principle for unconditional security.

**{(US) Advantage Creation Principle:**

All the noise and loss suffered by the users are assumed absent to the attacker, except those arising from a fundamentally inescapable limit or introduced deliberately by the user at the transmitter.

Generally, advantage creation is obtained from the difference between $I(X_n, Y_n^B)$, which includes all the channel disturbance not counting E’s for QCQ schemes and which is obtained from $\rho^{AB}$ after disturbance by E for BB84 type schemes, and $I(X_n; Y_n^E K)$ which includes all the irreremovable system disturbance to E. There should be a sufficient margin between $I(X_n; Y_n^B)$ and $R$ of (22) in a coding based KCQ scheme so that the protocol is not sensitive to small fluctuation in channel parameters and small disturbance by E. Thus, it is evident that even if one can in principle get $R$ close to $I(X_n; Y_n^B)$, $\Delta I$ determines the levels of channel parameter fluctuation and E’s disturbance that can be tolerated for an efficient protocol (high $P_{eff}$ in Section IV). What counts as an irreremovable disturbance to E is a matter of technology state and reality constraints, as well as fundamental obstacles. For unconditional security, such fundamental obstacles may include the laws of physics, deliberate actions by A at the transmitter, shared secret between A and B, as well as facts of nature as we know them. Only these should be included in the above US Advantage Creation Principle. For many applications, practical advantage creation such as those obtained from security against individual attacks, may be quite sufficient. Thus, in a weakened form, one may modify the US Advantage Creation Principle by imposing on E whatever constraint that may seem reasonable in a given application.

**IV. PERFORMANCE EFFICIENCY OF QKG PROTOCOLS**

The performance of a QKG or key generation scheme for useful real-life application is gauged not only by its security level, but also its efficiency in at least two senses to be elaborated in the following. The security level is most usefully measured in terms of E’s error profile on the final key as averaged over the random parameters of the system and minimized over E’s possible attacks. Since there is generally a trade-off between security and efficiency, raising the efficiency for a given security level is equivalent to raising the security level for a given efficiency. In addition, for a protocol to be useful the efficiencies cannot be too low.

The first type of efficiency that should be considered is protocol efficiency, denoted by $P_{eff}$, which has not been treated in the QKG literature. It can be defined as the probability that the protocol is not aborted for a given channel and a fixed security level in the absence of an attacker E. It is essential to consider the robustness of $P_{eff}$ with respect to channel parameter fluctuation, e.g., how sensitive $P_{eff}$ is to small changes in channel parameter $\lambda$, which may denote, e.g., the independent qubit noise rate of any kind. In practice, $\lambda$ is known only approximately for a variety of reasons, and imperfection in the system can never be entirely eliminated. If $P_{eff}$ is sensitive to such small changes, the protocol may be practically useless as it may be aborted almost all the time. Sensitivity issues are crucial in engineering design, and there are examples of ‘supersensitive’ ideal system whose performance drops dramatically in the presence of small imperfection. Classical examples include detection in nonwhite Gaussian noise and image resolution beyond the diffraction limit. Superposition of ‘macroscopic’ quantum states is supersensitive to loss. This crucial sensitivity issue is one of fundamental principle, not mere state of technology. It has thus far received little attention in the field of quantum information.

As will be shown in sections V–VII, our qumode KCQ key generation protocols are robust to channel parameter fluctuations. On the other hand, the Lo-Chau protocol is supersensitive at high security level. This is because any amount of noise in the system would be mistaken as E’s action in the parity-check hashing, and the protocol would be aborted according to its prescription. The situation is particularly severe in view of our discussion in III.C concerning E’s ability to use her information. The reverse reconciliation protocol in Ref. 38, which supposedly can operate in any loss, is supersensitive in high loss. Let $\eta$ be the transmittance so that $\eta \ll 1$ corresponds to the high loss situation. In the presence of a small additive noise of $\eta/2$ photons in the system, the protocol becomes completely useless because the noise induced by the attacker cannot be distinguished from excess noise. Apparently, the modified Lo-Chau or Shor-Preskill type protocols can be operated without such supersensitivity. Note that high security
level often decreases $P_{\text{eff}}$ and it is important to quantify the tradeoff.

Even when the scheme is not supersensitive, the sensitivity level has to be quantified in a QKG scheme involving intrusion-level detection for a complete protocol with quantifiable security, for the following reason that has not been discussed in the literature. The security proofs of such a scheme always have a conclusion of the following form: If $E$ has information $I_{E_i} > \delta$ on $K^g$ from any attack $a_i$, then the probability that $a_i$ would pass the test is $P_{E_i} < 1 - \epsilon_\delta$ for small $\delta$ and $\epsilon_\delta$. This may be put in form of a conditional probability statement

$$P(\text{pass}|I_{E_i} > \delta) < 1 - \epsilon_\delta. \quad (17)$$

However, the reverse conditional probability statement is required for a real security proof:

$$P(I_{E_i} > \delta|\text{pass}) < 1 - \epsilon_\delta. \quad (18)$$

It is easy to see that $P(I_{E_i} > \delta|\text{pass})$ can be large under \[(17),\] say if $E$ keeps attacking with $a_i$ that yields significant $I_{E_i}$. For example, in standard BB84, $E$ can learn each bit with probability $\sim 0.85$ in an opaque attack. This shows that the users must employ a 'stopping strategy' in a complete protocol, by adopting a stopping rule which stops the whole process after a number of test results that lead to aborting the protocol, and another rule to re-start the process. To evaluate \[(18),\] one would need $P(\text{pass})$ in addition to \[(17)\] which would involve in turn

$$P(\text{pass}|I_{E_i} < \delta). \quad (19)$$

This probability \[(19)\] depends on the quantitative sensitivity level just discussed, corresponding to the case of no attack or $I_{E_i} = 0$. In addition, $E$'s optimal attack on the sequence of key generation trials depends on the user's stopping and re-starting strategy, and it appears one cannot bound the overall $P(I_{E_i} > \delta|\text{pass})$ before such a strategy is spelled out. It should be clear that much remains to be done to obtain quantitative results on the overall protocol security and efficiency. In this connection, it may also be observed that in all security proofs involving the use of an error correcting code, it was not shown that an efficient (non-exponential) decoding algorithm exists. Thus, on many levels it has not been demonstrated that an efficient protocol exists with quantifiable security levels even for standard BB84. On the other hand, these problems do not arise in specific KCQ schemes.

After the protocol goes forward, there is clearly the question of key-bit generation efficiency (or rate) $k_{\text{eff}}^g$ which may be defined via the number of generated key bits subtracted by the number of key bits used in the protocol, i.e., $K_{\text{eff}}^g = (|K^g| - |K|)/n$ when an $n$-bit data sequence was used to obtain $K^g$ with a total key $K = (K_s, K_e, K_m, K_v)$ that is not re-used. This should be distinguished from the final effective key generation rate $k^g$, which includes all the operations in the protocol to give the actual speed of key generation, a subject not discussed in this paper. When the modulation key $K_i$ with $|K_i| = |K_m|$ is not re-used in the coded system of section III.4 in an $m$-block of $n$-bit symbols,

$$k_{\text{eff}}^g = R - |K_m|/n - I_E/n - |K_v|/mn, \quad (20)$$

where $I_E$ is $E$'s information rate that needs to be eliminated. If the protocol is secure under known-plaintext attacks, the $|K_m|$ term can be omitted. The $I_E$ term may be omitted if the coding scheme automatically forces $I(X_n; Y_n E K)$ to be negligible.

V. KCQ COHERENT-STATE KEY GENERATION WITH BINARY DETECTION

In this section we describe the use of KCQ on qumodes, quantum modes with infinite-dimensional Hilbert state spaces, for key generation via coherent states of intermediate or large energy. In most of the current experimental developments [2, 38] of QKG, coherent states are employed in BB84 type protocols that are limited in energy to $\sim 0.1$ photon, if only because of the photon-number splitting attack that $E$ can launch near the transmitter [39, 40]. With KCQ, we will in this and the next section show that much larger energy can be employed, line amplifiers and pre-amplifiers can be used, and conventional optical technology on the sources, modulators, and detectors can be utilized. Furthermore, direct encryption coherent-state KCQ in what is called the $\alpha\eta$ scheme has already been experimentally observed [17, 18], which will integrate smoothly with the corresponding key generation schemes that are currently under experimental development.

A. $\alpha\eta$ and its Extensions

The usual description of a single coherent state already involves an infinite dimensional space, referred to as a qumode. Similar to the qubit case in Fig. 1, we may consider $M$ possible coherent states $|\alpha_l\rangle$ in a single-mode realization,

$$|\alpha_l\rangle = \alpha_l (\cos \theta_l + i \sin \theta_l), \quad \theta_l = \frac{2\pi l}{M}, \quad l \in \{1, ..., M\}, \quad (21)$$

where $\alpha_0^2$ is the energy (photon number) in the state, and $\frac{2\pi l}{M}$ is the angle between two neighboring states. In a two-mode realization, the states are products of two coherent states

$$|\alpha_0 \cos \theta_1 \rangle_1 |\alpha_0 \sin \theta_1 \rangle_2, \quad \theta_1 = \frac{2\pi l_1}{M}, \quad l_1 \in \{1, ..., M\}, \quad (22)$$

The qumodes may be those associated with polarization, time, frequency, or any type of classical mode. Any two basis states form a phase reversal keying (antipodal) signal set, which are nearly orthogonal for $\alpha_0 \geq 3$. The optimal quantum phase measurement [41] yields a
root-mean-square phase error $\Delta \theta \sim 1/\alpha_0$. Thus, when $M \gg \alpha_0$, the probability of error $P^E_b \sim 1/2$ when the basis is not known which has been confirmed numerically [13], while $P^B_\text{het} \sim \exp(-\alpha_0^2) \to 0$ when the basis is known.

This scheme can be used for key generation as follows. An attacker not knowing the key $K$ has to make a measurement to cover all possible angles for different possible $K'$ in her effort to pin down the data $X$. From such measurement result, she can then try to determine all the possible $x$ corresponding to the different possible $K'$. For each running key $k'$ from her trial key $k$ that selects a particular basis for a particular bit, $E$ has a classical binary decision problem for two, not $m$, possible signal points. The more practical heterodyne measurement, which may be forced on the attacker with a signal set of varying amplitudes in addition to the varying phase of $\alpha_0$, is 6 dB worse in energy than the optimal measurement $\bar{P}$.

The optimal phase measurement, which has no known physical realization, is worse than the optimal quantum measurement of antipodal signals by $\sim 3$ dB in signal energy $A$ detailed binary-decision numerical evaluation on this performance is under way, but the 3 dB estimate follows from the amplitude/phase and conjugate quadratures (heterodyne/demodulation) analogy, and is supported by known results [42]. In any event, the precise number is important in an actual design of real system and in bringing out the intrinsic limitation of the system, but is not as important for illustrating the possibility and basic principle involved. In this case, the principle is that there is a substantial difference in performance due to a quantum effect that has no classical analog, viz, different incompatible quantum measurements versus a single complete measurement in the classical case.

More precisely, for discrimination of two equally likely coherent states $\{|\alpha_0\rangle, |\alpha_0\rangle\}$, the optimum quantum receiver yields an error rate $\bar{P}_b$ that may be compared to the heterodyne result $P^\text{het}_b$ and the phase measurement result $P^\text{ph}_b$, with $S = \alpha_0^2$,

$$\bar{P}_b = \frac{1}{4}e^{-4S}, \quad P^\text{het}_b \sim \frac{1}{2}e^{-S}, \quad P^\text{ph}_b \sim \frac{1}{2}e^{-2S} \quad (23)$$

Here, $S$ measures the average number of photons received in the detector and [28] applies in the so-called quantum-limited detection regime — unity detector quantum efficiency, infinite detector bandwidth, all device noise suppressed. Under [28] and dropping the factors in front of the exponentials for a numerical estimate of the bit-error rate (BER), which is required to be $\leq 10^{-9}$ per use in a typical communication application, we have, for a mesoscopic level $S \sim 10$, $\bar{P}_b \sim 10^{-12}$, $P^\text{het}_b \sim 10^{-3}$, $P^\text{ph}_b \sim 10^{-6}$. If the data arrives at a rate of 1 Gbps, the user $B$ is likely to have $10^6$ error-free bits in 1 sec, while $E$ would have $\sim 10^3$ errors among her $10^9$ bits with the optimum phase measurement. By the usual privacy distillation approach [8], the users can generate $\sim 10^3$ secure key bits by eliminating $E$’s information. Thus, in principle, $\alpha_0$ in its original form is capable of secure key generation against individual attacks that employs the optimal phase measurement on each qumode. Similar to all cases of specific QKG schemes to date, that there is no full security proof against even constant individual attacks in contrast to the claim of unconditional security in existence proofs, the above analysis does not prove there is no other individual or collective measurement, particularly adaptive ones utilizing the seed key information, that would yield a substantially better BER for $E$ than the optimal phase measurement. Intuitively, we feel that is quite unlikely, but new techniques in classical and quantum detection theory are being developed to give precise quantitative treatment on such problems. Note that whatever the final result may turn out to be, it only affects the quantitative advantage level but not the possibility of advantage creation. In this connection, we may mention that $\alpha_0$ in its original form was proposed for key generation in Ref. 45, with no consideration of information-theoretic key security against meaningful attacks. In the way $\alpha_0$ was run, actually no fresh key can be generated because $P^\text{het}_b$ in [28] is very small.

More serious limitations on the use of $\alpha_0$ for key generation, I believe, arise from the US Advantage Creation Principle when the above scheme is to be utilized in practice. In the first place, device thermal noise is significant at high data rate and small signals, thus optical pre-amplifiers need to be used. For the usual erbium amplifier this would already take out the advantage over $E$. On this issue, it may be pointed out that the optimum binary quantum receiver has not been implemented so far in $\alpha_0$, but the near-optimum Kennedy receiver [14], with $P'_b = (1/2)e^{-4S}$ is currently under development. (The factor 1/2 difference between $\bar{P}_b$ and $P'_b$ can be recovered in a Dohmner receiver described in Ref. 45, which is the first systematic investigation of optical receiver performance improvement via feedback.) On the other hand, the photon number amplifier (PNA) [39, 46] could lead to an ideal Kennedy receiver in principle although PNA is far from practical at present. Secondly, in the presence of a line loss $\eta$ from A to B, one would need to compare $P^\text{het}_b \sim e^{-4nS}$ to $P^E_b \sim e^{-2S}$ according to the Advantage Creation Principle when $E$ attacks near A. Even if one uses the advantage creation technique of accounting for $E$’s energy splitting described in II. G, and the postdetection selection technique of Ref. 8, in conjunction it appears difficult to create an advantage over $E$ that would allow key generation over truly long-distance telecommunication fibers. Thus, a more powerful approach via $m$-ary detection is developed in the following. Before we turn to this advantage creation technique, it is useful to introduce a number of other techniques that may improve the security and efficiency of KCQ schemes, and to demonstrate a general limitation on the binary detection approach to coherent-state KCQ for key generation.

It is important to note that $\alpha_0$ represents a new type of cipher even when it is operated in a completely classical setting, and even in the absence of any channel noise. This is because deliberate randomization may be introduced by $A$ in many ways. Consider the situation where
the circle of Fig. 1 represents a classical two-dimensional signal space, say corresponding to the two quadratures of a single frequency. In the absence of any noise, the circle would represent the different possible phase-shifted signals of a given energy. Thus, the cryptosystem can be run in the same way as \( \alpha \eta \) with or without classical noise, in hardware and even in software. Indeed, in the direct encryption experiments on \( \alpha \eta \) reported thus far \( \text{[18, 29]} \), the performance obtained via the coherent-state quantum noise can also be achieved by high-speed deliberate partial randomization of the signals by A corresponding to the coherent-state noise effect. When \( \alpha \eta \) is used for key generation, one may employ the technique of deliberate signal randomization (DSR) for which each signal state at the output of the encryption box of Fig. 1 is further randomized so that it is uniformly distributed on the semi-circle centered at the state chosen by the running key \( K' \). Similar to the qubit case, one may readily show the intuitively obvious fact that the key \( K \) is completely hidden form E who does not know \( K \) and \( X \), even if she possesses one copy of \( \rho_x^b \) corresponding to an arbitrarily long data sequence \( x \). The general interwined case described in section III will be treated in Part II, as it involves security against known-plaintext attack on direct encryption. This is true classically also as just discussed. In the presence of quantum or classical noise, one needs to use a proper randomization if all the error control is built in the antipodal signal set only, as above. When a CECC is used on top of the antipodal signals, deliberation error randomization can be introduced to improve security/efficiency as described in section III.G.

In addition to providing complete protection against ciphertext-only attack on the key, DSR also improves the efficiency of key generation. If the state is rotated by an angle \( \theta \) away from the one set by \( K' \) and is unknown to the detector which knows \( K' \), the optimum phase measurement BER as a function of \( \theta \) is yet to be evaluated, but the corresponding Kennedy and heterodyne receiver performance are

\[
\bar{P}_b(\theta) \sim e^{-\frac{x^2 \cos^2 \theta}{1-\cos^2 \theta}}, \quad P^{\text{het}}_b(\theta) \sim \frac{1}{2} e^{-S \cos^2 \theta}. \tag{24}
\]

The \( \bar{P}_b(\theta) \) in \( \text{(24)} \) is the Chernov bound \( \text{[17]} \) for the Kennedy receiver when \( \cos \theta \ll 1 \), while \( P^{\text{het}}_b(\theta) \) is the usual upper bound on Gaussian errors \( \text{[4]} \). It is expected that in a more exact evaluation of \( \bar{P}_b(\theta) \), the energy advantage is closer to the original 6 dB than the 3 dB one of \( \text{(24)} \), and similarly for \( P^{\text{het}}_b(\theta) \). Thus, even for large signal energy, A can control the BER to B and E causes more errors to E through B’s advantage. In practice, some CECC should be used for reliable system operation, and channel code key \( K_c \), chaining, and other techniques could be used to enhance the advantage already created for efficiency improvement.

With DSR on \( \alpha \eta \), one may obtain secure key generation against constant individual attacks as follows. Let the angle \( \theta \) be randomized so that it appears uniform over the whole circle with respect to E’s optimum (phase) constant qumode measurement. In this way, the key \( K \) is completely hidden even in a known-plaintext attack, in a way exactly similar to the classical noiseless case where a semicircle is sufficient to protect against ciphertext-only attacks. If the signal strength \( S \) is not large enough, this would also introduce error to B. However, a CECC can be designed to correct only up to the BER B then suffers, which is smaller than that of E due to B’s error performance advantage. By using Theorem 1, one may generate fresh keys, with no cost for each \( n \)-bit symbol due to security against known-plaintext attacks. We summarize

**Theorem 5:**

With proper use of DSR just described, a coded \( \alpha \eta \) scheme leads to unconditionally secure net key generation against constant individual attacks with security level given by \( \text{[12]} \).

Again, with the development of proper bounding techniques, we believe the restriction to constant individual attacks in Theorem 5 can be simply removed. The detailed quantitative dependence of the security level as a function of \( S \) and other system parameters will be given for both individual and joint attacks in the future.

### B. Binary Detection KCQ Key Generation

For binary coherent-state signals, the optimal quantum receiver performance cannot be better than that of heterodyne by 6 dB in energy or error exponent. This is a known fact among all the usual binary coherent state systems, but there is no general proof in the literature. A proof can be supplied, which is not difficult, but is omitted here for brevity. Also, it may be proved that antipodal signals lead to optimal BER under energy constraint on coherent states. Furthermore, it is not possible to increase the error advantage by utilizing bandwidth, or more generally any multimode system, for the following reason.

Consider an optical quantum field of arbitrary bandwidth \( E(x,t) \) where \( x \) is the transverse spatial dimension. On her copy of the field, whether it is the one she split off by tapping or the hypothetical one we grant her for bounding her information, she can always in principle make a heterodyne measurement to obtain the classical readout \( \varepsilon(x,t) \), which is described by \( \text{[48]} \),

\[
\varepsilon(x,t) = \varepsilon_s(x,t) + n(x,t), \tag{25}
\]

where \( \varepsilon_s(x,t) \) is the amplitude of the coherent-state signal and \( n(x,t) \) is an additive Gaussian noise in \( t \) with spectral density \( hf \) at frequency \( f \). All quantum fluctuation in every space-time mode has already been included. In a binary detection system involving two classical signals in additive white Gaussian noise (AWGN), one can always extract one signal dimension (one quadrature out of one mode) that contains all the information for optimal
discrimination \[47\]. The whiteness approximation that all \(f \sim f_0\), a single frequency, is very good for all practical optical signals. Thus, even though \[24\] contains many quantum noise photons—one from every mode—in the optimal receiver only one such mode is to be extracted by appropriate spatial-temporal filtering. As a consequence, we are back to the single-mode situation where there is just one noise photon from heterodyne. Indeed, heterodyne is a ‘universal’ measurement whose result captures all aspects of the field mode: amplitude, phase, quadratures, etc., that allows E to try all possible K to identify all possible data. One may understand this result from the important fact that a multimode coherent-state excited field from vacuum is equivalent to a single-mode coherent-state excited field.

The result just quoted is not true for nonclassical light, i.e., optical fields in quantum states that is not classical, not a coherent state or a classically random superposition of coherent states. Clearly, there can be huge improvement between the optimal and heterodyne detection of a nonclassical state. For number states, ideal photon counting yields \(P_b = 0\) for on-off signals. For squeezed states, homodyne detection along the maximum squeezing direction sees the minimum noise as compared to one that may see a large noise without knowledge of that direction. Thus, by using K to determine such directions, the users would obtain huge error advantage over E even in a binary detection system. One can similarly use number states and other orthogonal states in conjunction with coherent states to create other binary systems that give arbitrarily small BER for \(b\) but large ones for E.

We would not go into the details for such development because intermediate or large-energy nonclassical states do not have much practical significance as data source in long-distance communication \[13\]. This is because the inevitable system imperfection, especially linear loss, would quickly transform such nonclassical states into classical ones. As a consequence, the initial energy or error advantage disappears quickly over a lossy communication line. For realistic application of mesoscopic or macroscopic energy signals, we may want to limit ourselves to coherent states.

VI. KCQ COHERENT-STATE KEY GENERATION WITH \(m\)-ARY DETECTION

The above limitation on the binary detection advantage of an optimal quantum receiver versus heterodyne can be overcome in \(m\)-ary detection. The use of \(m\)-ary systems, in fact, is one form of coding. As will be seen in the following, it indeed corresponds to driving the system at a rate between B’s and E’s mutual information with respect to A as in \[11\]. Amazingly, for the particular CPPM system we now turn, such a rate choice by A automatically makes \(I_E\) go to zero with a flat error profile, with also full information-theoretic security against known plaintext attack on the key. This is proved against the universal heterodyne attack, and is likely to be true against all possible attacks. Thus, not only the data enjoy unconditional security at the near perfect level, the key has security that has never even been suggested possible before in either standard or quantum cryptography.

A. CPPM—Coherent Pulse Position Modulation

An \(m\)-ary coherent-state pulse position modulation system has the following signal set for \(m\) possible messages,

\[
|\phi_i\rangle = |0\rangle_1 \cdots |\alpha_0\rangle_i \cdots |0\rangle_m, \quad i \in \{1, \ldots, m\}.
\]

In \[25\], each \(|\phi_i\rangle\) is in \(m\) qmodes all of which are in the vacuum state except the \(i\)th mode, which is in a coherent state \(|\alpha_0\rangle_i\). The corresponding classical signals are orthogonal pulse position modulated if each mode is from a different time segment, but generally the modes can be of any type. For brevity, we retain the term ‘pulse position’ even through ‘general mode position’ is more appropriate.

The photon counting as well as heterodyne error performance of \[26\] are well known \[21\]. The block error rate from direct detection is exponential optimum for large \(m\).

\[
P^e_{\text{dir}} = (1 - \frac{1}{m}) e^{-S}, \quad \tilde{P}_e \rightarrow e^{-S}.
\]

The optimum block error rate \(\tilde{P}_e\) for \[26\] is known exactly \[11\], and given in \[27\] asymptotically. In contrast, for large \(m\) the heterodyne block error rate \(P^\text{het}_e\) approaches 1 exponentially in \(n = \log_2 m\), which is a general consequence of the Strong Converse to the Channel Coding Theorem as discussed in section III.D. For the present Gaussian channel case for heterodyne receivers, explicit lower bound on the block error rate \(P^\text{het}_e\), conditioned on any transmitted \(i\), can be obtained in the form (p382 of \[31\])

\[
P^\text{het}_e > (1 - [\Phi(y)]^m) \Phi(y - \sqrt{2S}),
\]

where \(\Phi\) is the normalized Gaussian distribution. By choosing \(y > \sqrt{2S}\), \[28\] yields explicitly \(P^\text{het}_e \rightarrow 1\) exponentially in \(n\) for any given \(S\). It is a main characteristic of classical orthogonal or simplex signals in AWGN that whenever an error is made, it is equally likely to be decoded by the optimal receiver to any of the \(m - 1\) other messages. Thus, under the condition \(P^\text{het}_e \rightarrow 1\), the error profile is uniform, viz., \(p_i = 1/m\) or the BER \(P_b = 1/2\) with independent errors.

The KCQ qmode key generation scheme CPPM works as follows. Consider \(m = 2^n\) possible \(n\)-bit sequences, and possible coherent-states

\[
|\psi_i\rangle = \otimes_{j=1}^m |\alpha_{ij}\rangle_j, \quad i, j \in \{1, \ldots, m\}
\]

in correspondence with \(|\{\phi_i\}\rangle\) of \[29\]. For simplicity, one may set \(\sum_j |\alpha_{ij}|^2 = |\alpha_0|^2 = S\) for every \(i\). Let \(f_k\)
be a one-to-one map between $|26\rangle$ and $|29\rangle$ indexed by a key $K$. As an example of physical realization, the connection between $|26\rangle$ and $|29\rangle$ could be through a set of $N$ beam-splitters with transmission coefficients $\sqrt{\eta}$ for complex numbers $\eta$, $l \in \{1,\ldots,N\}$, determined by $k$. Such a physical realization combines the $\alpha_{ij}$ of $|21\rangle$ coherently through the $\eta$’s, and is represented by a unitary transformation between the two $m$-tensor product state spaces $\otimes_{i=1}^{m} H_i$ and $\otimes_{i=1}^{m} H'_i$ for the input and the output. The states $|\psi_i\rangle$ of $|29\rangle$ are used to modulate the data $i$ by $A$, and $B$ demodulates by first applying $f_k$ to transform it to $|\phi_i\rangle$ of $|26\rangle$ and then use direct detection on each of the $m$ modes $H_i$.

Without knowing $f_k$ or $\eta_l$ so that there are both amplitude and phase uncertainties for each $l$, it is expected that an attacker can do very little better than heterodyne on all the $H'_i$ modes, which is equivalent to heterodyne on all the $H_i$ modes, and then apply the different $f_k$’s on the classical measurement result $|26\rangle$. As presented above, by making $m$ large one can then make not only $\bar{p}_E = 2^{-l}$ for any $l$ but E’s error profile is in fact nearly uniform, with $p_i = (1 - 2^{-2l})/(m - 1)$ for $i \geq 2$, thus no need for further privacy distillation. As a consequence, the system is not only completely secure against ciphertext-only attack on the key but also fully secure against known-plaintext attacks. This is because given an input-output pair $(X_n, Y_n^E)$, the heterodyne output $Y_n^E$ has no relation to $X_n$ for any $k$ from E’s uniform error profile. The exact quantitative behavior may be bounded via $|28\rangle$. We summarize:

**Theorem 6**

Against E’s universal heterodyne attack, the $m$-ary CPPM KCQ protocol is unconditionally secure with asymptotic key generation rate $n = \log_2 m$ per use and $E, I_E$ going to zero exponentially in $n$.

The only easy way to remove the restriction to heterodyning for E is to note that the optimum quantum receiver for discrimination among the states $|26\rangle$ is unique $|40\rangle$. Thus, there is a gap between it and the receiver performance that does not know $K$ at the time of quantum measurement, which translates into a mutual information statement $|40\rangle$ that can be used to show the existence of codes upon further describing, as described in III.E, that yields security in the sense of Theorem 1. Although this does not seem to give useful practical protocol for actual implementation and does not guarantee key bit generation, it is of interest in principle to record the following.

**Theorem 7:**

Against any attack by Eve, the CPPM scheme may be further coded to provide unconditional security with levels given by $|12\rangle$.

**B. Further Outlook**

The direct detection or optimal detection performance $|29\rangle$ is affected by the presence of device noise so that there is no more vacuum state in $|26\rangle$. However, ordinary pre-amplifier could be used that suppresses all the device noise with a resulting performance degradation that amounts to a less factor $1/4 \leq \eta < 1$ in m-ary PPM. Furthermore, in principle a photon-number amplifier mentioned in VI.A can be used as a noiseless pre-amplifier. Quantitative evaluations of the resulting performance are, however, yet to be carried out. One major advantage of coherent-state KCQ scheme is that they can be used through a limited number of amplifiers and switching nodes in a properly designed system with appropriate amplifiers. In general, quantum amplifiers degrade the user’s error performance due to the fundamental quantum noise they introduce $|41\rangle$. In a properly designed chain with appropriately chosen amplifier gains and lengths of lossy line segments, one can obtain a linear $|50\rangle$ instead of an exponential degradation in the signal-to-noise ratio (SNR) as a function of total line length. There is no need to decrypt and re-encrypt at the input of an amplifier as in a repeater, as long as the degradation introduced by the amplifier still leaves B with performance advantage over E. Recall the overall general Advantage Creation Principle for key generation that B must have performance advantage on the decoded information-bit sequence after all system imperfections including loss and noise are taken into account, as compared to E’s decoded information-bit sequence for no loss and no imperfection other than unavoidable ones. In the case of CPPM, B’s performance would be scaled by the total transmittance $\eta$, so that $S$ is replaced by $\eta S$ in $|27\rangle$. In principle, it is still a better performance compared to $|28\rangle$ with $\eta = 1$ for large enough $m$. Thus, CPPM can be secure for arbitrarily long-distance fiber communication.

It may be mentioned that the possible use of amplifier in a quantum cryptosystem has been introduced previously for weak coherent states and heterodyne/homodyne detection $|51\rangle$ that traces back to Ref. $|52\rangle$ that describes both coherent-state and squeezed-state cryptosystems. In particular, the usual-state scheme in Ref. $|51\rangle$ employs conjugate-variable measurement detection of intrusion level similar to the schemes of Ref. $|53\rangle$, while also allowing a limited use of amplifiers as described. However, all such schemes are inefficient because weak or small energy signals have to be used to avoid good performance in determining the actual signal state via optimal quantum detection by E, and via attacks similar to the USD attack on coherent-state realization of BB84 type systems $|39,40\rangle$.

The CPPM scheme is also ideal for direct data encryption because it automatically produces a near uniform error profile on E corresponding to near-perfect bit-by-bit security. Indeed, from the constant inner product $\langle \phi_i | \phi_j \rangle$ for every $i \neq j$ which is the quantum analog of the classical orthogonal or simplex signal behavior that is
responsible for their near uniform error profile in AWGN, it would be possible to prove that such a property persists under E’s optimal attack. It would then appear that all problems are solved in principle as arbitrarily large error exponent advantage can be obtained between and by making large.

Unfortunately, as in a classical orthogonal signaling scheme, large in CPPM means exponential growth of bandwidth, not to mention growth in physical complexity. Indeed, itself is an infinite-bandwidth result for large . One the other hand, it is known that if the signal-to-quantum noise per unit bandwidth is small, coherent-state direct detection systems do have larger capacity than heterodyne ones. Thus, it may be expected that properly designed error correcting codes, usually employed for bandlimited systems for such reasons, could be developed to retain much of the CPPM advantage for a large given bandwidth.

VII. KCQ AND DIRECT ENCRYPTION

For direct encryption, one needs to consider ciphertext-only attack on the key, on the data, and known-plaintext attack on the key. In conventional cryptography one has the Shannon bound, , on the conditional entropy of the data given the ciphertext via the key entropy. In the quantum case or in the presence of irreducible classical noise to E, the corresponding bound

is no longer valid where is the classical ciphertext available to E. In the quantum case, is obtained via a quantum measurement. If is valid as is the case in conventional cryptography, it does not mean that E knows all the bits in X except for of them. That would be disastrous as it often happens that while . The operational meaning of has never been analyzed in conventional cryptography, to my knowledge. It is usually not considered a problem because it is presumed that E would get many information bits in X wrong knowing only Y and not . However, a more detailed analysis is needed for a security proof with respect to whatever chosen criteria, as we have done for key generation in section III.C. But that has never been provided in conventional cryptography other than the trivial one-time pad case. When is violated, Lemma 1 implies , a condition that allows key generation via as in Theorem 1. Direct encryption, such violation has the important implication that very high level of data-bit security may be obtained without using the inefficient one-time pad. Indeed, we have seen how this may occur in CPPM treated in section VI. Note that Theorems 1 and 2 can also be used to describe the data quality in direct encryption.

It is easy to protect from ciphertext-only joint attacks in with the use of, e.g., DSR, discussed in section V.A. The technique can be extended to cover known-plaintext attacks in two different ways, to be presented in Part II. Without DSR, the bound obtains with the right-hand side. The usual security problem is known-plaintext attack, in which E tries to determine from data-output sequence pairs with statistical correlation information on the data (of varying degree). Security against known-plaintext attacks is always at best computational complexity-based against exponential search in conventional cryptography. For noisy system, we suggest that it is possible to have information-theoretic security, i.e.,

which has never been suggested before and is clearly impossible in conventional cryptography where . Full security would correspond to , which again can be closely approximated in CPPM systems, at least for the universal heterodyne attack.

Even when is as is the case in conventional cryptography for sufficiently long X, the system may be secure in the sense of high search complexity. In particular, in the original form without DSR provides an additional search problem to E, as compared to just the encryption box, that is exponential in , at least for brute-force search. For increasing the search complexity, one may make sure the input data can never be perfectly known in several ways. One is to use polarity or padding bits as described in Section II. Another is to generate the polarity bits through a running key obtained from another encryption mechanism with the same K or another different key as the seed key. Note that the proper use of DSR would introduce inevitable coherent-state quantum noise for E, which may even lead to information-theoretic security already if the energy in the coherent state is not too large. No proof of any cryptosystem has ever been given in conventional cryptography, to my knowledge, that establishes rigorous exponential lower bound on the search complexity. And we have not (yet) succeeded in proving that necessitates an exponential search either - it is just an added search burden as compared to just the encryption box and appears exponential. In general, such multi-variable correlated classical statistical problem has the full mathematical complexity of many-body problems and quantum field theory in physics. Useful lower bound is also notoriously difficult to obtain in computation problems. Perhaps these explain why no rigorous security proof is available on such complexity-based security.

On the other hand, exponential search complexity should be good enough for any application. We have mentioned in section II that Grover’s search only reduces the exponent by a factor of two, which is easily compensated by increasing the key size by a factor of two in many standard schemes as well as in our KCQ schemes, or qubit or coherent-state.

As noted previously in this paper, data security against attacks on the key with statistical knowledge on data
that are not completely random is required for a complete proof of key generation security with KCQ. An extensive general theoretical development of direct encryption security analysis will be provided in Part II, where conditional probabilities will be used in addition to entropies for more precise quantitative estimate of specific coding/detection scheme performance. The behaviors of $\alpha$, CPPM, and their refinement and extension will be developed for concrete cryptosystem design.

VIII. COMPARISON AMONG QKG SCHEMES

We present below a brief qualitative comparison between QKG schemes of the BB84 type, of the qubit KCQ type, and the coherent-state KCQ type. Detailed quantitative comparisons will be given after rigorous evaluation of the quantitative characteristics of these schemes for finite $n$.

Theoretically, BB84 type protocols suffer from the following classes of problems as a matter of fundamental principle.

(i) It is hard to bound Eve’s error rate on the key generated due to the difficulties of intrusion-level estimation under joint attacks with side information on the error correcting and privacy distillation codes.

(ii) It is difficult to produce a complete protocol that can be practically implemented with quantifiable security and efficiency, due to the decoding problem and the stopping-rule problem.

(iii) It is hard to include the various system imperfections in an unconditional security proof, and to build protocols robust with respect to fluctuations in the magnitude of these imperfections.

(iv) The necessary use of weak signals and the difficulty of repeating the signal without decryption imply low throughput even with just moderate loss.

(v) The intrinsic small quantum effect of a single photon necessitates an accurate sensitivity and analysis of respect to the system imperfection and environmental perturbations, that would result in a low interference tolerance threshold in commercial applications.

Corresponding to these problems are related practical one including

(i’) It is difficult or impossible to rigorously ascertain the quantitative security level of the generated key.

(ii’) The throughput or key-generation rate would be low, especially in the presence of substantial loss.

(iii’) The cryptosystem is sensitive to interference, and needs to be controlled and checked with a high precision difficult to achieve practically.

(iv’) High-precision components corresponding to a fundamentally new technology are required, including the source, transmission line or repeater, and detector.

With the exception of (i) and (ii), which are further discussed in Appendix A, all these problems are evident and well known in BB84 although there are disagreements on how readily they can be overcome. Nevertheless, in the foreseeable future it seems clear that BB84 type schemes cannot be made to operate in a commercial type environment with any reasonable level of security and efficiency for even moderately long distance. The weak coherent-state schemes of Ref. [59] also suffer from all these problems except (iv’), and that of Ref. [61] is only slightly better.

With the use of qubit KCQ type schemes, the theoretical problems in (i) and (ii) can be largely overcome, but not the ones in (iii) and (iv) except perhaps with very low key-bit generation efficiency. Except for (i’), the practical difficulties (ii’)-(iv’) also remain. Again, it may be possible to alleviate these problems with strong error correction that implies a low $k_{eff}$.

With the use of the qumode KCQ schemes of intermediate to large energy, all the fundamental difficulties (i)-(iv) can be substantially reduced. Furthermore, each of the practical difficulties (i’)-(iv’) either disappears or is substantially alleviated. The exception is loss in long-distance fiber communication. In principle, a wideband coherent CPPM system presented in section VI could solve all problems. For practical application, new approaches are needed to deal with bandwidth limitation and coherence requirements. It is still a major problem to create enough advantage for unconditional information-theoretic security.

IX. CONCLUDING REMARKS

A new principle of quantum cryptography has been presented on the basis of optimal versus nonoptimal quantum detection when a seed key is known or not known. This possibility of yielding better performance for the users over an attacker is a quantum effect with no classical analog. In classical physics, a complete observation of the physical signal state can be made with or without the key. It would be misleading to phrase the basis of this possibility as no-cloning, which is trivially covered [52] by quantum detection theory that provides detailed quantitative limits on quantum state discrimination from the laws of quantum physics. A detailed development of the appropriate novel quantum detection theory will be given in the future for a complete quantitative assessment of cryptosystem efficiency and security. This will be done especially in terms of Eve’s optimal probability of guessing the generated key correctly, which is a more appropriate criterion than her mutual information.

A powerful new KCQ protocol CPPM that utilizes $m$-ary instead of binary detection has been presented that could, in principle, lead to secure key generation and data encryption over long-distance telecomm fibers. However, the problem of obtaining such a protocol under bandwidth and practical constraints remains both a theoreti-
Indeed, even for constant individual attacks, this is factorially under all possible attacks that do not lead to existence and the choice of a code that would perform satisfactorily under all possible attacks has only recently been rigorously dealt with in Ref. [57].

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APPENDIX A: PROBLEMS OF QKG UNCONDITIONAL SECURITY PROOFS

In the QKG literature, there are different types of security proofs that purport to show the existence of unconditionally secure protocols against any attack allowed by the laws of physics, while new proofs are continually emerging that we have not scrutinized. They are the proofs of Shor-Preskill [21], Lo-Chau [57], Mayers [55], Biham et al. [56] and recently a more complete approach by Hamada [57]. I believe that all of them have serious gaps that are difficult to close. The situation is rather confusing because the strategies of these proofs are different, and problems in one type may not arise in another. In the following, three major problems in these proofs are briefly described, at least one of them applies to any of the above proofs. In addition, three other problems in a complete protocol are briefly indicated, none of which has been addressed in the literature, to my knowledge.

The three major problems in these proofs are

(i) It is hard to rigorously and accurately estimate Eve’s disturbance on the qubits under a joint attack.

(ii) It is difficult to show there exists a universal error correcting code that would produce the desired security level for all attacks that pass the intrusion-level test in the protocol with significant probability.

(iii) The side information that Eve has from the public knowledge has not been properly taken into account in the estimate of her information on the key generated.

From an information-theoretic viewpoint, a joint attack from Eve creates in general a quantum channel with entanglement on the user’s qubits. It seems impossible to obtain a good estimate of Eve’s disturbance from merely one copy of the general channel. This difficulty, which may be called the inference problem, also affects the existence and the choice of a code that would perform satisfactorily under all possible attacks that do not lead to the protocol being aborted, as expressed in (ii) above. Indeed, even for constant individual attacks this coding problem has only recently been rigorously dealt with in Ref. [57].

When it applies, the inference problem is serious and does not seem to have a solution even in the asymptotic limit $n \to \infty$. It seems to be handled in Ref. [21] by the argument of quantum to classical reduction adopted from Ref. [55]. However, the local measurement actually performed by the users is not equivalent to the nonlocal degenerate Bell measurement needed for the reduction to go through, with respect to the determination of the state after the measurement that is needed in the next step of the proof. They are only equivalent, in both the cases of Ref. [21] and Ref. [57], with respect to the probabilities that govern the use of the measurement results. Perhaps the equivalence claim arose from interpreting the description of a measurement by the same $X \otimes X$ differently in two different contexts [24]. When $X \otimes X$ has a degenerate spectrum, the measurement as specified by a POM is not uniquely represented by the symbol $X \otimes X$. Furthermore, the inference of the test qubit results to the information qubits left cannot be justified by the quantum de Finetti Theorem [25, 59] because quantum entanglement leads to violation of the exchangeability premise of the theorem, and quantum entanglement is precisely what a joint attack can yield that an individual attack cannot. However, the inference problem does not arise in proofs where B makes measurements on all received qubits before proceeding. The problem of such proofs is how one may bound Eve’s information under her optimal attack.

Note that a security proof needs to answer this question for the user: given that a key $K^g$ is obtained by following the protocol on $n$ qubits, what one can rigorously say about the error profile or information that E has on $K^g$ as optimized over all her possible attacks. This question is especially serious for the realistic case when the statistical fluctuation due to a finite $n$ needs to be under control. It appears that new techniques need to be developed to handle such problem in this type of protocols with intrusion-level detection. Even asymptotically, the coding problem remains on what scheme one should employ that guarantees a bound on Eve’s information when she optimizes her probe/interaction in anticipation that she would receive side information later before she makes her measurement. This problem is coupled with the following side information problem, although the latter constitutes a problem al by itself.

In terms of our notation, the side information problem can be simply stated as follows. Let $S$ be E’s side information before she made her final measurement and estimate of the generated key $K^g$ from an observation on her ancilla. Then Eve’s mutual information on $K^g$ is given by $I(K^g; Y_E S)$. From (4), this is equal to $I(K^g; Y_E | S) + I(K^g; S)$. Most treatments just bound $I(K^g; Y_E)$. In [57], the (smaller) $I(K^g; Y_E | S)$ is bounded but $I(K^g; S)$ is ignored. However, $I(K^g; S)$ may grow with the number of qubits $n$ in $x$ and has to be subtracted from $K^g$ to show that a net positive key generation rate is in fact obtained. In this regard, we may recall that the fundamental superiority of quantum over standard cryptography is based almost exclusively on the availability of...
APPENDIX B: ON CRITICISMS OF $\alpha\eta$

There have appeared three papers \cite{17, 18, 60} in quantum this year that purport to show that the $\alpha\eta$ scheme reported in \cite{17, 18} is insecure in various ways. These criticisms are briefly summarized and responded to in this Appendix.

A general criticism seems to be made in Ref. \cite{60} that our claim in \cite{17, 18} on the possible use of amplifiers in coherent-state cryptosystems cannot be valid. It is not clear exactly what this objection is. In any event, we qualify such use in our papers by the statement that security must be guaranteed for $E$ attacking near the transmitter, since quantum amplifiers generally degrade the communication performance. There are three specific criticisms from Ref. \cite{60} that one can ascertain:

(i) In the presence of loss so large that $E$ can get $2^{|K|}$ copies by splitting the coherent-state signal at the transmitter, there can only be complexity-based security.

(ii) With just a 3 dB loss, use of the Grover Search implies there can only be complexity-based security.

(iii) The Grover Search is ‘powerful’ against complexity-based security.

In response, observe that only complexity-based security is ever claimed in \cite{17, 18} against joint attack on the key $K$. The other information-theoretic security claimed is on individual ciphertext-only attack on the data. As discussed in Section VII, it is quite sufficient to have complexity-based security if it can be proved exponential, which is only reduced by a factor of 2 with the Grover Search. Long keys of thousands of bits can be used in $\alpha\eta$ at high speed both in software and hardware implementation, making the exponential search completely ineffective.

Furthermore, the Grover Search cannot be launched against $\alpha\eta$ with a 3 dB loss. If it can, there is no need for the $2^{|K|}$ copies extensively discussed in Ref. \cite{60}. Indeed, there is no discussion there on how the Grover or any search can be launched with a 3 dB loss. There is a general misconstrue in some papers on quantum cryptography that a 3 dB loss on a coherent state cryptosystem renders it insecure because $E$ can obtain a copy of the quantum ciphertext identical to $B$. This is not true even without the use of a secret key. It is not true for B92 \cite{8} or YK \cite{7} or the usual-state scheme described in \cite{51}, although it renders a coherent-state BB84 scheme and some `continuous-variable’ schemes essentially insecure. When a secret key $K$ is used, it is not true at all. Indeed, the possibility of key generation while granting $E$ a full copy to bound her performance depends on this being not true. As explained in this paper, knowledge of the secret key allows $B$ to make a better measurement than $E$, who cannot attain the same performance as $B$ even if she knows the key later.

It is true that when $2^{|K|}$ copies are available to $E$, there is no information-theoretic security left in a known-plaintext attack on direct encryption, and key generation is impossible. But it is clear that no one would contemplate the operation of cryptosystem over such a huge loss $2^{-|K|}$ without intermittent amplifiers or other compensating devices. Numerically, $2^{-|K|}$ corresponds to the propagation loss over one thousand kilometers of low-loss fibers without amplifiers for just $|K| \sim 80$ bits in $\alpha\eta$ with $M \sim 10^3$. It is totally out of the realm of possibility to sustain such loss even in ordinary optical communication without cryptography. As the use of amplifiers is suggested in \cite{17, 18}, it is hard to see why such a criticism is relevant. As a matter of fact, $\alpha\eta$ in its original form is insecure at a much smaller loss than $2^{-|K|}$ for any reasonable $|K|$. In Ref. \cite{61}, it is claimed that a device can be found that would lead to a bit error rate $P_E^c$ much lower than the quantum detection theory result $\sim 1/2$ reported in \cite{17} for individual ciphertext-only attack on the data. As pointed out by several others including G. Barbosa and O. Hirota, such a device cannot exist because violating quantum detection theory means violating the laws of quantum physics. In Ref. \cite{62}, it is claimed that $\alpha\eta$ is merely a classical cipher. The exact nature of $\alpha\eta$ for key generation has been analyzed in section V, and for direct encryption in section VII. While $\alpha\eta$ can indeed
be run in the classical limit and even in just software, the blanket claim that $\alpha\eta$ is classical for intermediate and large signal energy, and hence presumably does not permit key generation, is incorrect because their equation (10) does not hold exactly. In particular, the discussion around 28 in our section V.A shows how $\alpha\eta$ in just its original form may allow key generation. For a further concise discussion, see Ref. 62.

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We distinguish the qualitative intrusion detection that tells the absence or presence of an attacker, from the quantitative intrusion-level detection that is needed in a BB84 protocol to generate a fresh final key that the attacker knows essentially nothing about.

The classical noise and B92 type protocols suffer from many of these problems also, due to the necessary use of weak signals. On the other hand, the post-detection selection technique in these protocols can be used on top of KCQ protocols to increase the security level at the expense of efficiency.

The term CDMA - code division multiple access - is used here as in cellular communication to denote an arbitrary signal set for communication.

We use the term ‘key extension’ to denote the process of getting a larger session key $K'$ from a seed key $K$, avoiding the term ‘key expansion’ for possible confusions.

We distinguish ‘independent’ from ‘uncorrelated’ in the standard mathematical and statistical sense, avoiding the common use of ‘uncorrelated’ to mean ‘statistically independent’ by many physicists.

Indeed, if there is already a good experimental demonstration of a complete Bell measurement over a single pair of single-photon qubits, there is none on 3-qubit systems.

Generally, we use capitals to denote a random variable in the standard mathematical and statistical sense, avoiding the term ‘key expansion’ for possible confusions.

We often use $k$ instead of $k'$ for values of $K'$ to simplify notation, which should not cause confusion.

Note that this possibility only obtains under our performance bounding assumption that E has a whole copy, which would not occur in practice without E disrupting the protocol so much that it would be aborted during key verification - see section III.F and III.G. Nevertheless, it demonstrates that it is sometimes possible for E to obtain a lot more information by collective rather than independent processing.

Similarly in BB84, joint classical processing on individual qubit measurements may be used to exploit the overall correlation between bits to optimize E’s information through the error correction information announced publicly. As to be discussed further later, this has not been properly accounted for in the security analysis in the literature for both individual and joint attacks.

This is the Kerckhoff’s Principle in cryptography which states that only the shared secret key can be assumed unknown to an attacker.

In some KCQ implementations, in particular the ones on qumodes reported in $\text{[17, 18]}$, $K'$ is open to partial observation. Then direct complexity-based security obtains from the need to inverting such imprecise $K'$ to $K$.

Note that there is no need to know $k$ to be able to correlate data bits through its repeated use. For example, in two uses of one time pad $x_1 \oplus k, x_2 \oplus k$ on two random data bits $x_1$ and $x_2$, we know $x_1 \oplus x_2$ without knowing $k$. Indeed, no information on $k$ is obtained from the observation of $x_1 \oplus k$ and $x_2 \oplus k$.

We use the term ‘conventional cryptography’ to denote the situation where E and B have the same observation, $Y_E = Y_B = Y$. It is distinguished from classical noise cryptography and from quantum cryptography.

In general, one may ignore insecurity claims against the classical analog.
security of a protocol that are made for reasons not intrinsic to the protocol, e.g., that a shared secret key is not really secret. Such a claim is common to all protocols, which always require some common secrecy between two users, say for agent authentication, that distinguishes them from other parties. Similarly, the record of a secret key in KCQ schemes can be assumed safeguarded or ‘destroyed’, as the situation is different from that of a public key distribution center which needs to use the public key repeatedly.

[83] Note that this description is neither the Schrodinger nor the Heisenberg picture, but is more convenient in problems of quantum system analysis.

[84] While only I am responsible for the assertions in this Appendix, they are made after extensive discussion in our group that include also G.M. D’Ariano, W.-Y. Hwang, R. Nair, and M. Raginsky who made important contributions and clarifications that make the writing of this Appendix possible. I also benefited from exchanges with M. Hamada, H.-K. Lo, and N. Lütkenhaus. I hope this Appendix would stimulate serious exchanges, and that it would be replaced by separate papers in the future.