Unphysical Gauge Fixing in Higgs Mechanism

Takehisa Fujita\textsuperscript{1}, Atsushi Kusaka\textsuperscript{1}, Kazuhiro Tsuda\textsuperscript{1}, and Sachiko Oshima\textsuperscript{1}
Department of Physics, Faculty of Science and Technology, Nihon University, Tokyo, Japan

Abstract

The unitary gauge in the Higgs mechanism is to impose the condition of $\phi = \phi^\dagger$ on the Higgs fields. However, this is not the gauge fixing but simply a procedure for producing the massive vector boson fields by hand. The Lagrangian density of the weak interactions should be reconsidered by starting from the massive vector boson fields which couple to the fermion currents as the initial ingredients.

1 Introduction

The whole idea of the symmetry breaking has been critically examined in the recent textbook \cite{1}, and the physics of the spontaneous symmetry breaking is, by now, understood in terms of the standard knowledge of quantum field theory. In particular, if one wishes to understand the vacuum state in a field theory model of fermions, then one has to understand the structure of the negative energy states of the corresponding field theory model. The importance of the negative energy state in the Dirac field can be easily understood if one looks into the Dirac equation. In the Dirac equation, the energy eigenvalue itself is the physical observable since it is written as

$$(-i \nabla \cdot \alpha + m\beta) \psi(r, t) = E\psi(r, t) \quad (1.1)$$

which means that the energy eigenvalue $E$ must be physical. Therefore, if one obtains the energy eigenvalue which is negative, then this negative energy itself must be physical. For this, we have to always carry out the field quantization with anti-commutation relation, and this is equivalent to the realization of the Pauli principle. Under this Pauli principle, one can construct the vacuum state of the corresponding field theory model by filling out all the negative energy states completely. If there is any hole in the vacuum state, then this corresponds to the anti-particle state which is an observed fact.

However, it is normally very difficult to construct the vacuum state of the interacting system, and in fact, the exact solution of the model field theory is practically impossible.

\footnotetext[1]{e-mail: fffujita@phys.cst.nihon-u.ac.jp}
\footnotetext[2]{e-mail: kusaka@phys.cst.nihon-u.ac.jp}
\footnotetext[3]{e-mail: nobita@phys.cst.nihon-u.ac.jp}
\footnotetext[4]{e-mail: oshima@phys.cst.nihon-u.ac.jp}
in four dimensions. Nevertheless, the physics of the spontaneous symmetry breaking is now clearly understood and some of the model field theory prefer the vacuum state which breaks the chiral symmetry though there exists no massless (Goldstone) boson. In this respect, one can say that the symmetry breaking physics can be understood only after one solves the whole system of the corresponding field theory model. One cannot understand its physics by rewriting the Lagrangian density into a new shape since the Lagrangian density itself cannot be any physical observables. From the Lagrangian density, one can learn a symmetry property of the model field theory.

In this respect, the physics of the Higgs mechanism [2] is very vague, and as we will show below, the whole procedure of the Higgs mechanism was carried out with simple-minded mistakes. This is mainly connected to the misunderstanding of the gauge fixing where one degree of freedom of the gauge fields must be reduced in order to solve the equations of motion of the gauge fields. Therefore, one cannot insert the condition of the gauge fixing into the Lagrangian density. This is clear since the Lagrangian density only plays a role for producing the equation of motions. Indeed, the Lagrangian density itself is not directly a physical observable, and the Hamiltonian constructed from the Lagrangian density is most important after the fields are quantized. For the field quantization, one has to make use of the gauge fixing condition which can determine the gauge field \( A_\mu \) together with the equation of motions. This means that only the final Lagrangian density is relevant to the description of physical observables, and thus the success of the Glashow-Weinberg-Salem model [3, 4, 5] is entirely due to the final version of the weak Hamiltonian which is not at all the gauge field theory but is a model field theory of the massive vector fields which couple to the fermion currents.

## 2 Gauge Fixing

The Lagrangian density of the Higgs mechanism is given as

\[
\mathcal{L} = \frac{1}{2} (D_\mu \phi) \dagger (D^\mu \phi) - \frac{1}{4} u_0 \left( |\phi|^2 - \lambda^2 \right)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{2.1}
\]

where

\[
D_\mu = \partial_\mu + igA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.2}
\]

Here, we only consider the U(1) case since it is sufficient for the present discussions. The above Lagrangian density is indeed gauge invariant, and in this respect, the scalar field may interact with gauge fields in eq.(2.1). However, it should be noted that there is no experimental indication that the fundamental scalar field can interact with any gauge fields in terms of the Lagrangian density of eq.(2.1).

The equations of motion for the scalar field \( \phi \) become

\[
\partial_\mu (\partial^\mu + igA^\mu) \phi = -u_0 \phi \left( |\phi|^2 - \lambda^2 \right) - igA_\mu (\partial^\mu + igA^\mu) \phi \tag{2.3}
\]

\[
\partial_\mu (\partial^\mu - igA^\mu) \phi^\dagger = -u_0 \phi^\dagger \left( |\phi|^2 - \lambda^2 \right) + igA_\mu (\partial^\mu - igA^\mu) \phi. \tag{2.4}
\]
On the other hand, the equation of motion for the gauge field $A_\mu$ can be written as

$$\partial_\mu F^{\mu\nu} = g J^\nu$$

where

$$J_\mu = \frac{1}{2} i \left\{ \phi^\dagger (\partial_\mu + ig A_\mu) \phi - \phi (\partial_\mu - ig A_\mu) \phi^\dagger \right\}.$$  \hspace{1cm} (2.6)

One can also check that the current $J_\mu$ is conserved, that is

$$\partial_\mu J^\mu = 0.$$ \hspace{1cm} (2.7)

This Lagrangian density of eq.(2.1) has been employed for the discussion of the Higgs mechanism.

### 2.1 Gauge Freedom and Number of Independent Equations

Now, we should count the number of the degrees of freedom and the number of equations. For the scalar field, we have two independent functions $\phi$ and $\phi^\dagger$. Concerning the gauge fields $A_\mu$, we have four since there are $A_0$, $A_1$, $A_2$, $A_3$ fields. Thus, the number of the independent fields is six. On the other hand, the number of equation is five since the equation for the scalar fields is two and the number of the gauge fields is three. This three can be easily understood, even though it looks that the independent number of equations in eq.(2.5) is four, but due to the current conservation the number of the independent equations becomes three. This means that the number of the independent functions is six while the number of equations is five, and they are not equal. This is the gauge freedom, and therefore in order to solve the equations of motion, one has to put an additional condition for the gauge field $A_\mu$ like the Coulomb gauge which means $\nabla \cdot A = 0$. In this respect, the gauge fixing is simply to reduce the redundant functional variable of the gauge field $A_\mu$ to solve the equations of motion, and nothing more than that.

### 2.2 Unitary Gauge Fixing

In the Higgs mechanism, the central role is played by the gauge fixing of the unitary gauge. The unitary gauge means that one takes

$$\phi = \phi^\dagger.$$  \hspace{1cm} (2.8)

This is the constraint on the scalar field $\phi$ even though there is no gauge freedom in this respect. For the scalar field, the phase can be changed, but this does not mean that one can erase one degree of freedom. One should transform the scalar field in the gauge transformation as

$$\phi' = e^{-ig\chi} \phi$$

but one must keep the number of degree of freedom after the gauge transformation. Whatever one fixes the gauge $\chi$, one cannot change the shape of the scalar field $\phi$ since it
is a functional variable and must be determined from the equations of motion. The gauge freedom is, of course, found in the vector potential $A_\mu$ as we discussed above. In this sense, one sees that the unitary gauge fixing is a simple mistake. The basic reason why people overlooked this simple-minded mistake must be due to their obscure presentation of the Higgs mechanism. Also, it should be related to the fact that, at the time of presenting the Higgs mechanism, the spontaneous symmetry breaking physics was not understood properly since the vacuum of the corresponding field theory was far beyond the proper understanding. Indeed, the Goldstone boson after the spontaneous symmetry breaking was taken to be almost a mysterious object since there was no experiment which suggests any existence of the Goldstone boson. Instead, a wrong theory prevailed among physicists. Therefore, they could assume a very unphysical procedure of the Higgs mechanism and people pretended that they could understand it all.

### 2.3 Final Lagrangian Density

After an improper gauge fixing, one arrives at the final Lagrangian density

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{4} u_0 \left( |\lambda + \eta(x)|^2 - \lambda^2 \right)^2 + \frac{1}{2} g^2 (\lambda + \eta(x))^2 A_\mu A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (2.9)$$

where we rewrite the Higgs field as

$$\phi = \phi^\dagger = \lambda + \eta(x). \quad (2.10)$$

Since the real scalar field $\eta$ is supposed to be small and besides a real scalar field is unphysical [6, 7], it may be set to zero, that is, $\eta = 0$. In this case, we arrive at the following Lagrangian density

$$\mathcal{L} = \frac{1}{2} g^2 \lambda^2 A_\mu A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (2.11)$$

This should be the final Lagrangian density of the Higgs theory, and it is nothing but the massive vector boson field which has nothing to do with the gauge theory.

### 2.4 Proper Gauge Fixing

For the equations of motion eqs.(2.3-5), one can make a proper gauge fixing such as the Coulomb gauge ($\nabla \cdot A = 0$) or the temporal gauge ($A_0 = 0$). In this case, one can solve the equations of motion properly, and one can calculate any physical observables which are, in this case, gauge invariant quantities like $F_{\mu\nu}$ in the classical field theory. Further, we can quantize the fields $\phi$ and $A^\mu$ after the gauge fixing, and we can carry out the perturbation theory since we have now the quantized Hamiltonian. But this must be completely different from the Higgs mechanism.
2.5 Higgs Potential

In the Lagrangian density of the Higgs mechanism, one assumes a self-interacting field potential

\[ U(\phi) = \frac{1}{4} u_0 \left( |\phi|^2 - \lambda^2 \right)^2. \]  

(2.12)

This was originally introduced in the discussion of the spontaneous symmetry breaking physics as a toy model [8]. But in the mean time, this scalar field potential is considered to be a fundamental potential. However, one may ask a question as to where this potential \( U(\phi) \) is produced from since the scalar field is obviously not a free field. Unless one can understand the basic origin of this potential \( U(\phi) \), it is extremely difficult to understand the Higgs mechanism itself from the fundamental physics point of view.

3 What shall we do ?

When experiments suggested that there might be heavy vector bosons exchanged between leptons and baryons in the weak processes, people wanted to start from the massive vector bosons. However, it was somehow believed among educated physicists that only gauge field theories must be renormalizable. We do not know where this belief came from. In fact, there is no strong reason that only the gauge field theory is renormalizable. It is clear that QED is a good gauge theory which is renormalizable, and there is no conceptual problem in QED which can describe all the experiments related to the electromagnetic processes. However, this does not mean that other non-gauge field theory models are not renormalizable. In fact, the basic condition of the renormalizability must be concerned with the coupling constant \( g \) which must be dimensionless [1].

3.1 Renormalizability of Non-abelian Gauge Field

However, one should be careful for the renormalizability of the non-abelian gauge field theory. As one can easily convince oneself, the non-abelian gauge theory has an intrinsic problem of the perturbation theory [9]. This is connected to the fact that the color charge in the non-abelian gauge field depends on the gauge, and therefore it cannot be physical observables. This means that the free gauge field which has a color charge is gauge dependent, and thus one cannot develop the perturbation theory in a normal way. In QCD, this is exhibited as the experimental fact that free quarks and free gluons are not observed in nature. No free field is a kinematical constraint and thus it is beyond any dynamics. Therefore, one cannot discuss the renormalizability of the non-abelian gauge field theory models due to the lack of the perturbation scheme in this model field theory [1] [9].
3.2 Massive Vector Field Theory

Even though the Higgs mechanism itself is meaningless, the final Lagrangian density may well be physically interesting. This is clear since, from this Lagrangian density, one can construct the Hamiltonian which can describe the experimental observables. In this respect, we may write the simplest Lagrangian density for two flavor leptons which couple to the SU(2) vector fields $W^a_\mu$

$$\mathcal{L} = \bar{\Psi} (i\partial_\mu \gamma^\mu + m) \Psi - g J^a_\mu W^{a,\mu} + \frac{1}{2} M^2 W^a_\mu W^{a,\mu} - \frac{1}{4} G^{a}_{\mu\nu} G^{a,\mu\nu}$$

(3.1)

where $M$ denotes the mass of the vector boson. The fermion wave function $\Psi$ has two components. The fermion current $J^a_\mu$ and the field strength $G^{a}_{\mu\nu}$ are defined as

$$J^a_\mu = \bar{\Psi} \gamma_\mu \tau^a \Psi, \quad G^{a}_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu.$$  

(3.4)

This Lagrangian density is almost the same as the standard model Lagrangian density, apart from the Higgs fields.

3.3 Renormalizability

The most important of all is to examine the renormalizability of the massive vector boson fields which couple to the fermion currents [10]. In this sense, this is just similar to checking the final Lagrangian density of the standard model itself whether it can be renormalizable or not. This is beyond the scope of the present paper, but it should be done in future.

References

[1] T. Fujita, "Symmetry and Its Breaking in Quantum Field Theory", (Nova Science Publishers, 2007)

[2] P.W. Higgs, Phys. Lett. 12, 132 (1964)

[3] S.L. Glashow, Nucl. Phys. 22, 579 (1961)

[4] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967)

[5] A. Salam, In Elementary particle physics (Nobel Symposium No. 8), Ed. N. Svartholm; Almqvist and Wilsell, Stockholm (1968)
[6] T. Fujita, S. Kanemaki, A. Kusaka and S. Oshima, “Mystery of Real Scalar Klein-Gordon Field”, physics/0610268

[7] S. Kanemaki, A. Kusaka, S. Oshima and T. Fujita, ”Problems of scalar bosons”, in New Fundamentals in Fields and Particles, (Research Signpost, 2008)

[8] J. Goldstone, Nuovo Cimento 19, 154 (1961)

[9] T. Fujita, ”Physical observables in gauge field theory”, in New Fundamentals in Fields and Particles, (Research Signpost, 2008)

[10] K. Nishijima, “Fields and Particles”, (W.A. Benjamin, INC, 1969)