Large Direct CP Violation in $B^0 \to \pi^+\pi^-$ and an Enhanced Branching Ratio for $B^0 \to \pi^0\pi^0$

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Abstract

Recent measurements of $B^0 \to \pi\pi$ decays reveal two features that are in conflict with conventional calculations: the channel $B^0(B^0) \to \pi^+\pi^-$ shows a large direct CP-violating asymmetry, and the channel $B^0(B^0) \to \pi^0\pi^0$ has an unexpectedly high branching ratio. We show that both features can be understood in terms of strong-interaction mixing of $\pi\pi$ and $D\bar{D}$ channels in the isospin-zero state, an effect that is important because of the large experimentally observed ratio $\Gamma(B^0/B^0 \to D^+D^-)/\Gamma(B^0/B^0 \to \pi^+\pi^-) \approx 50$. Our dynamical model correlates the branching ratios and the CP-violating parameters $C$ and $S$, for the decays $B^0(B^0) \to \pi^+\pi^-$, $B^0(B^0) \to \pi^0\pi^0$, $B^0(B^0) \to D^+D^-$ and $B^0(B^0) \to D^0\bar{D}^0$.

The Belle collaboration has presented new data [1] which support their original evidence [2] for large direct CP violation in the decays $B^0(B^0) \to \pi^+\pi^-$, the asymmetry parameter $C$ (= $-A$) being measured to be $C = -0.58 \pm 0.15 \pm 0.07$. In a related development, both the Babar [3] and Belle [4] collaborations have reported a sizable branching ratio for the decay $B^0(B^0) \to \pi^0\pi^0$, with an average value $\text{Br}(B^0/B^0 \to \pi^0\pi^0) = (1.9 \pm 0.6) \times 10^{-6}$. Both of these observations are unexpectedly large from the standpoint of conventional calculations [5, 6, 7] based on a short-distance, effective weak Hamiltonian and the assumption of factorization of products of currents in matrix elements for physical hadron states. In this paper, we carry out a calculation based upon the idea [8] of final-state interactions involving the mixing of $\pi\pi$ and $D\bar{D}$ channels. This dynamics provides a natural, correlated explanation of the new experimental facts, and leads to several further predictions.

To fix notation, we write the three $B \to \pi\pi$ amplitudes as

\[
\begin{align*}
A(B^0 \to \pi^+\pi^-) &= N(\lambda_u a_1 + \lambda_c a_p) \\
A(B^0 \to \pi^0\pi^0) &= N(\lambda_u a_2 - \lambda_c a_p)/\sqrt{2} \\
A(B^- \to \pi^-\pi^0) &= N\lambda_u (a_1 + a_2)/\sqrt{2}
\end{align*}
\] (1)

Here $a_1$, $a_2$, $a_p$ are, in general, complex numbers and $N$ is a positive normalization factor. The parameters $\lambda_u$ and $\lambda_c$ are CKM factors, defined as $\lambda_u = V_{ub}V_{ud}^*$, $\lambda_c = V_{cb}V_{cd}^*$, with magnitudes

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\[ |\lambda_u| \approx 3.6 \times 10^{-3}, \ |\lambda_c| \approx 8.8 \times 10^{-3} \] and phases given by \[ \lambda_u = |\lambda_u| e^{-i\gamma}, \ \lambda_c = -|\lambda_c|, \] with \( \gamma \approx 60^\circ \) [3]. The amplitudes in Eq. (i) are defined so that their absolute square gives the branching ratio, and they satisfy the isospin relation [10]

\[
\frac{1}{\sqrt{2}} A(B^0 \to \pi^+ \pi^0) + A(B^0 \to \pi^0 \pi^0) = A(B^0 \to \pi^- \pi^0)
\] (2)

From the results of the models discussed in [5, 6, 7], the parameters appearing in Eq. (i) have the following rough representation. The constants \( a_1, a_2, a_p \) are approximately real (to within a few degrees), with magnitudes \( a_1 \approx 1.0, a_2 \approx 0.2, a_p \approx -0.1 \). The normalization factor is \( N \approx 0.75 \); it is here fixed by the empirical branching ratio for \( B^- \to \pi^- \pi^0 \). The fact that the parameters \( a_1, a_2, a_p \) are nearly real implies immediately that there is very little direct \( CP \)-violating asymmetry between \( B^0 \to \pi^+ \pi^- \) and \( B^0 \to \pi^- \pi^+ \), as well as in the channels \( \pi^0 \pi^0 \) and \( \pi^+ \pi^- \). Furthermore, the absolute branching ratios following from the above parametrization are as follows (with experimental values given in parentheses):

\[
\begin{align*}
\text{Br}(B^0 \to \pi^+ \pi^-) &= 5.3 \times 10^{-6} \quad \text{[exp. (5.3 \pm 0.8) \times 10^{-6}]} \\
\text{Br}(B^0/B^0 \to \pi^+ \pi^-) &= 9.2 \times 10^{-6} \quad \text{[exp. (4.6 \pm 0.4) \times 10^{-6}]} \\
\text{Br}(B^0/B^0 \to \pi^0 \pi^0) &= 0.2 \times 10^{-6} \quad \text{[exp. (1.9 \pm 0.6) \times 10^{-6}]} 
\end{align*}
\] (3)

The most striking feature is the strong enhancement of the \( \pi^0 \pi^0 \) rate compared to this model expectation.

It was pointed out in Ref. [8] that the \( CP \)-violating asymmetries and branching ratios in the \( B \to \pi \pi \) system would be strongly affected by final-state interactions involving the mixing of the \( \pi \pi \) and \( D\bar{D} \) channels in the isospin \( I = 0 \) state, as a consequence of the large ratio of partial decay widths \( \Gamma(B^0 \to D^+ D^-)/\Gamma(B^0 \to \pi^+ \pi^-) \approx 3/14 |V_{cb}|^2/|V_{ub}|^2 \approx 26 \) expected in the Bauer-Stech-Wirbel model [5]. A large ratio has now been confirmed by the Belle measurement [11] of the branching ratio \( \text{Br}(B^0/B^0 \to D^+ D^-) = 2.5 \times 10^{-4} \), which is about 50 times larger than \( \text{Br}(B^0/B^0 \to \pi^+ \pi^-) \). This fact gives new urgency to an investigation of \( \pi \pi \leftrightarrow D\bar{D} \) mixing as a way of resolving the puzzling observations in \( B \to \pi \pi \) decays.

The \( \pi \pi \) system exists in the states \( I = 0 \) or \( I = 2 \), while the \( D\bar{D} \) system has \( I = 0 \) or \( I = 1 \). Mixing can occur between the isospin-zero states

\[
\begin{align*}
|\pi \pi\rangle_0 &= \sqrt{\frac{2}{3}}|\pi^+ \pi^-\rangle - \sqrt{\frac{1}{3}}|\pi^0 \pi^0\rangle \\
|D\bar{D}\rangle_0 &= \sqrt{\frac{1}{2}} \left[ |D^+ D^-\rangle + |D^0 \bar{D}^0\rangle \right]
\end{align*}
\] (4)

By contrast, the \( I = 2 \) \( \pi \pi \) state and the \( I = 1 \) \( D\bar{D} \) state, given by

\[
\begin{align*}
|\pi \pi\rangle_2 &= \sqrt{\frac{1}{3}}|\pi^+ \pi^-\rangle + \sqrt{\frac{2}{3}}|\pi^0 \pi^0\rangle \\
|D\bar{D}\rangle_1 &= \sqrt{\frac{1}{2}} \left[ |D^+ D^-\rangle - |D^0 \bar{D}^0\rangle \right]
\end{align*}
\] (5)

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are unaffected by mixing. The physical decay amplitudes of $\overline{B^0}$ to the above four states are

\[
A^{(0)}_{\pi\pi} = \sqrt{\frac{2}{3}} A_{\pi^+\pi^-} - \sqrt{\frac{1}{3}} A_{\pi^0\pi^0}
\]
\[
A^{(2)}_{\pi\pi} = \sqrt{\frac{1}{3}} A_{\pi^+\pi^-} + \sqrt{\frac{2}{3}} A_{\pi^0\pi^0}
\]
\[
A^{(0)}_{D\overline{D}} = \sqrt{\frac{1}{2}} \left[ A_{D^+\overline{D}^-} + A_{D^0\overline{D}^0} \right]
\]
\[
A^{(1)}_{D\overline{D}} = \sqrt{\frac{1}{2}} \left[ A_{D^+\overline{D}^-} - A_{D^0\overline{D}^0} \right]
\]

These physical decay amplitudes are related to the “bare” amplitudes calculated in the absence of final-state interactions, i.e. with no mixing, which we denote by $\tilde{A}$:

\[
\begin{pmatrix}
    A^{(0)}_{\pi\pi} \\
    A^{(0)}_{D\overline{D}}
\end{pmatrix} = S^1 \begin{pmatrix}
    \tilde{A}^{(0)}_{\pi\pi} \\
    \tilde{A}^{(0)}_{D\overline{D}}
\end{pmatrix}
\]

\[
A^{(2)}_{\pi\pi} = \tilde{A}^{(2)}_{\pi\pi}
\]
\[
A^{(1)}_{D\overline{D}} = \tilde{A}^{(1)}_{D\overline{D}}
\]

Here $S$ denotes the strong-interaction $S$ matrix connecting the isospin-zero states $|\pi\pi\rangle_0$ and $|D\overline{D}\rangle_0$ which can be written generally as

\[
S = \begin{pmatrix}
    \cos 2\theta e^{i2\delta_1} & i \sin 2\theta e^{i(\delta_1+\delta_2)} \\
    i \sin 2\theta e^{i(\delta_1+\delta_2)} & \cos 2\theta e^{i2\delta_2}
\end{pmatrix}
\]

where $\theta$ is a mixing angle, and $\delta_1$ and $\delta_2$ are the strong-interaction phase shifts for the elastic scattering of $\pi\pi$ and $D\overline{D}$ systems in the $I = 0$ state, at $\sqrt{s} = M_B$. For any choice of these three parameters, the matrix $S^\dagger$ can be calculated numerically, and the set of four equations (7) solved to obtain the physical amplitudes $A_{\pi^+\pi^-}$, $A_{\pi^0\pi^0}$, $A_{D^+D^-}$ and $A_{D^0\overline{D}^0}$ in terms of the bare amplitudes. The bare amplitudes are identified with those calculated in the factorization model [5, 6, 7], which we list below

\[
\begin{aligned}
\tilde{A}_{\pi^+\pi^-} &= N(\lambda_u a_1 + \lambda_c a_p) \\
\tilde{A}_{\pi^0\pi^0} &= N(\lambda_u a_2 - \lambda_c a_p)/\sqrt{2} \\
\tilde{A}_{D^+D^-} &= N'\lambda_c a_1 \\
\tilde{A}_{D^0\overline{D}^0} &= 0
\end{aligned}
\]

where the first two equations are as in Eq. (11), and the factor $N'$ is determined from the empirical branching ratio $\text{Br}(B^0/\overline{B^0} \to D^+D^-) = N'^2|\lambda_c|^2a_1^2 = 2.5 \times 10^{-4}$ to be $N' = 1.79$.

In order to show, in a transparent way, how the mixing mechanism gives rise to large direct $CP$ violation in $B^0 \to \pi^+\pi^-$, as well as an enhanced branching ratio for $B^0 \to \pi^0\pi^0$, we

1 The two-channel $S$-matrix has been discussed, in particular in [12, 13]. The $S^\dagger$ prescription is given in [6, 12]. An alternative prescription, using $\frac{1}{2}[1+S]$ in place of $S^\dagger$, has been discussed by Kamal [14], and was used in Ref. [8].
consider, for illustration, the case where the elastic phases $\delta_1$ and $\delta_2$ in the $S$ matrix (Eq. (8)) are neglected, so that $S^{12}$ may be written as

$$S^{12} = \begin{pmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{pmatrix}$$ (10)

The amplitudes $A_{\pi^+\pi^-}$ and $A_{\pi^0\pi^0}$ for $B^0$ decay are then given by

$$A_{\pi^+\pi^-} = \frac{1}{3} \left( 1 + 2 \cos \theta \tilde{A}_{\pi^+\pi^-} + \sqrt{2} \frac{1 - \cos \theta}{3} \tilde{A}_{\pi^0\pi^0} \right) + i \sin \frac{\theta}{\sqrt{3}} \left( \tilde{A}_{D^+D^-} + \tilde{A}_{D^0\bar{D}^0} \right)$$ (11)

$$A_{\pi^0\pi^0} = \sqrt{2} \frac{1 - \cos \theta}{3} \tilde{A}_{\pi^+\pi^-} + \frac{2 + \cos \theta}{3} \tilde{A}_{\pi^0\pi^0} - i \sin \frac{\theta}{\sqrt{6}} \left( \tilde{A}_{D^+D^-} + \tilde{A}_{D^0\bar{D}^0} \right)$$

Clearly for $\theta = 0$, the physical amplitudes reduce to the bare amplitudes. Inserting the bare amplitudes from Eq. (9), we can rewrite $A_{\pi^+\pi^-}$ and $A_{\pi^0\pi^0}$ as linear combinations of $\lambda_u$ and $\lambda_c$:

$$A_{\pi^+\pi^-} = N \left[ \lambda_u \left\{ \frac{1 + 2 \cos \theta}{3} a_1 + \frac{1 - \cos \theta}{3} a_2 \right\} + \lambda_c (a_p \cos \theta + a_m) \right]$$ (12)

$$A_{\pi^0\pi^0} = \frac{N}{\sqrt{2}} \left[ \lambda_u \left\{ \frac{2(1 - \cos \theta)}{3} a_1 + \frac{2 + \cos \theta}{3} a_2 \right\} - \lambda_c (a_p \cos \theta + a_m) \right]$$

where

$$a_m = i \frac{1}{\sqrt{3}} \sin \theta \frac{N'}{N} a_1$$ (13)

Note that the isospin relation in Eq. (2) continues to be fulfilled. The important new feature of the amplitudes in Eq. (12) is the appearance of the imaginary term $a_m$ in the coefficient of $\lambda_c$, in striking contrast to the real term $a_p$. The imaginary nature of this dynamical term is an inescapable consequence of $S$-matrix unitarity, which enforces the factor $i$ in the off-diagonal matrix element in Eq. (10). The term $a_m$, given in Eq. (13), has a magnitude $|a_m| \approx 1.39 \sin \theta$, and dominates the term $a_p \cos \theta$ even for a modest mixing angle $\sim 0.1$. We will now show that the mixing term $a_m$ has profound consequences for direct $CP$ violation in the decays $B^0 \to \pi^+\pi^-$, and for the branching ratio of the channel $B^0 \to \pi^0\pi^0$.

1 \hspace{1cm} **C and S Parameters for $B^0 \to \pi^+\pi^-$ and $B^0 \to \pi^0\pi^0$**

The $C$ and $S$ parameters derived from the time-dependent asymmetry between $\overline{B}^0$ and $B^0$ decays into $\pi^+\pi^-$ are defined as

$$C_{+-} = \frac{1 - |\lambda_{+-}|^2}{1 + |\lambda_{+-}|^2}$$ (14)

$$S_{+-} = \frac{2 \text{Im}\lambda_{+-}}{1 + |\lambda_{+-}|^2}$$
Using the amplitude $A_{\pi \pi^-}$ of the $\pi \pi^-$ functions of $\theta$ close to its bare value, and can be lowered slightly with the introduction of phases $q$. The parametrization in Eq. (1), based on the models [5, 6, 7], gives $B_{\text{2 Branching Ratio for } B^0 \to \pi^+\pi^-}$, where $B_{\text{2}}$ for $B^0 \to \pi^+\pi^-$, the role of $\Pi C$ Table 1, where we also list $B_{\text{2}}$ for $B^0 \to \pi^+\pi^-$, the numerical results for $\text{Br}(\pi \to \pi^-)$ and $\text{Br}(\pi^+\pi^-)$ derived from the matrix $S_{\text{2}} \text{CP}$ violation. CP indicate in Figs. 1 and 2 two examples, obtained with the values data [1]. In our approach, the role of $\Pi C$ of these models, and is responsible for the prediction $B_{\text{2}}$ and $B_{\text{2}}$ in our approach, the role of $\Pi C$ Table 1, where we also list $B_{\text{2}}$ for $B^0 \to \pi^+\pi^-$, the numerical results for $\text{Br}(\pi \to \pi^-)$ and $\text{Br}(\pi^+\pi^-)$ derived from the matrix $S_{\text{2}} \text{CP}$ violation. CP indicate in Figs. 1 and 2 two examples, obtained with the values $\delta_1 = \pm 10^\circ$, $\delta_1 + \delta_2 = -30^\circ$. Table 1 gives numerical values for a few choices of parameters. In all cases, there is a large direct $CP$ violation.

Discussions of the direct $CP$-violating parameter $B_{\text{2}}$ are often based on an amplitude for $B^0 \to \pi^+\pi^-$ written in the form

$$A_{\pi^+\pi^-} \sim \left[ e^{-i\gamma} + \frac{P_{\pi\pi}}{T_{\pi\pi}} \right]$$

The parametrization in Eq. (1), based on the models [5] [6] [7], gives $|P_{\pi\pi}/T_{\pi\pi}| = 0.24$, and $\text{arg}(P_{\pi\pi}/T_{\pi\pi}) = 0$. The small phase of the “penguin-to-tree” ratio $P_{\pi\pi}/T_{\pi\pi}$ is a generic feature of these models, and is responsible for the prediction $B_{\text{2}} \approx 0$, which is now contradicted by data [1]. In our approach, the role of $P_{\pi\pi}/T_{\pi\pi}$ is played by the ratio

$$|P/T| = \frac{|\lambda_c| (a^0 \cos \theta + a_m)}{|\lambda_u| (1 + \frac{1+2 \cos \theta}{3} a_1 + \frac{1+2 \cos \theta}{3} a_2)}$$

For a typical value $\theta = 0.2$, this ratio has the modulus $|P/T| \approx 0.77$, and a phase $\text{arg}(P/T) \approx -70^\circ$. The difference is a consequence of the term $a_m$ in Eq. (1), which reflects the physical final-state interaction of the $\pi\pi$ system, as implemented in our model through $\pi\pi \leftrightarrow D\bar{D}$ mixing.

2 Branching Ratio for $B^0 \to \pi^0\pi^0$ and $B^0 \to \pi^+\pi^-$

The branching ratios (averaged over $B^0$ and $\bar{B}^0$) may be calculated in our model by taking the absolute square of the $B^0$ decay amplitudes in Eq. (12), and the corresponding amplitudes for $B^0$ decay. The results are shown in Fig. 2. It is remarkable that the empirical branching ratio for $B^0 \to \pi^0\pi^0$ is accurately reproduced, using the same value $\theta \approx 0.2$ which accounts for the asymmetry parameter $B_{\text{2}}$. We also note that the branching ratio $B^0 \to \pi^+\pi^-$ remains close to its bare value, and can be lowered slightly with the introduction of phases $\delta_1$ and $\delta_2$. Numerical results for $\text{Br}(B^0 \to \pi^0\pi^0)$ and $\text{Br}(B^0 \to \pi^+\pi^-)$ are listed in Table 1.
3 Branching Ratio for $B^0 \to D^0 \overline{D^0}$

Since our model treats the $\pi \pi$ and $D\overline{D}$ states with $I = 0$ as a coupled system, it also produces predictions for branching ratios and asymmetry parameters in $B^0 \to D^+ D^-$ and $B^0 \to D^0 \overline{D^0}$. The amplitudes after mixing are

$$A_{D^+ D^-} = \frac{1}{2} \left[ i \sin \theta \sqrt{\frac{2}{3}} \left( \sqrt{2} \tilde{A}_{\pi+\pi^-} - \tilde{A}_{\pi^0\pi^0} \right) 
+ (\cos \theta + 1) \tilde{A}_{D^+ D^-} + (\cos \theta - 1) \tilde{A}_{D^0 \overline{D^0}} \right]$$

$$A_{D^0 \overline{D^0}} = \frac{1}{2} \left[ i \sin \theta \sqrt{\frac{2}{3}} \left( \sqrt{2} \tilde{A}_{\pi+\pi^-} - \tilde{A}_{\pi^0\pi^0} \right) 
+ (\cos \theta - 1) \tilde{A}_{D^+ D^-} + (\cos \theta + 1) \tilde{A}_{D^0 \overline{D^0}} \right]$$

(20)

Of particular interest is the branching ratio for $B^0 / B^0 \to D^0 \overline{D^0}$, since it vanishes at the level of the bare amplitude ($\tilde{A}_{D^0 \overline{D^0}} = 0$), and is induced by mixing with the $\pi \pi$ system. For $\theta = 0.2$, ignoring the phases $\delta_1$, $\delta_2$, our model predicts

$$\text{Br}(B^0 / B^0 \to D^0 \overline{D^0}) = 1.45 \times 10^{-7}$$

(21)

(At this low level, one must assume that other sources of final-state interaction or a non-zero bare amplitude could raise this branching ratio further.) Direct $CP$ violation follows from $A_{D^0 \overline{D^0}}$ in Eq. (20): $C_{D^0 \overline{D^0}} = -0.50$ for $\theta = 0.2$. Direct $CP$ violation in $D^+ D^-$ (and in $\pi^0 \pi^0$) is small, because these decays are dominated by a single amplitude. There is little mixing in $A_{D^+ D^-}$ in Eq. (20) (and none in the $I = 2$ amplitude for $\pi^- \pi^0$).

To conclude, we have demonstrated a mechanism of final-state interactions among physical hadrons in $B^0 \to \pi \pi$ decays which predicts a large direct $CP$-violating parameter $C_{+-}$. The same mechanism enhances the theoretical prediction for the branching ratio of $B^0 / B^0 \to \pi^0 \pi^0$ to the experimentally observed level. Predictions are made for the $C$ and $S$ parameters of $B^0 / B^0 \to \pi^0 \pi^0$ decays, and for the branching ratio of $B^0 / B^0 \to D^0 \overline{D^0}$. The model makes essential use of the large empirical ratio $\Gamma(B^0 / B^0 \to D^+ D^-) / \Gamma(B^0 / B^0 \to \pi^+ \pi^-) \approx 50$. Its success in the present context leads to the expectation that sizable direct $CP$ violation could be observed in other charmless $B$ decays, in which an amplitude of order $\lambda_u$ receives a dynamical contribution proportional to $\lambda_c$, through mixing with a channel possessing a large branching ratio. The resulting amplitude contains two pieces which are comparable in magnitude and have different weak-interaction and strong-interaction phases. We have treated earlier [15] the charged-particle decays $B^\pm \to \eta \pi^\pm$ (and $B^\pm \to \eta'/\pi^\pm$), which are influenced by mixing with the channel $B^\pm \to \eta_c \pi^\pm$, and have predicted significant direct $CP$ violation. Evidence for a sizable violation in $B^\pm \to \eta \pi^\pm$ has indeed been reported in one experiment [16], the first ever seen in a charged-particle decay.

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| Observable         | No mixing | With mixing | Data                                      |
|-------------------|-----------|-------------|-------------------------------------------|
|                   |           | $\theta = 0.2$ | $\theta = 0.17$ | $\theta = 0.2$ |
|                   |           | $\delta_1 = 0^\circ$ | $\delta_1 = -10^\circ$ | $\delta_1 = 10^\circ$ |
| $\delta_2 = 0^\circ$ |           | $\delta_2 = -20^\circ$ | $\delta_2 = -40^\circ$ |                          |
| $C_{+-}$           | $\pm 0.00$ | -0.65       | -0.66         | -0.81         | -0.58 $\pm$ 0.15 $\pm$ 0.07 (Belle [1]) |
| $S_{+-}$           | -0.60     | -0.63       | -0.55         | -0.40         | -1.00 $\pm$ 0.21 $\pm$ 0.07 (Belle [1]) |
| $\text{Br}(B^0/\bar{B}^0 \rightarrow \pi^0 \pi^0)$ | 0.2       | 1.8         | 1.7           | 1.6           | 1.7 $\pm$ 0.6 $\pm$ 0.2 (Belle [1]) |
| $\text{Br}(B^0/\bar{B}^0 \rightarrow \pi^+ \pi^-)$ | 9.3       | 12.2        | 10.5          | 9.9           | 4.4 $\pm$ 0.6 $\pm$ 0.3 (Belle [18]) |
| $C_{00}$           | $\pm 0.00$ | +0.48       | +0.51         | +0.56         |                                      |
| $S_{00}$           | +0.73     | -0.65       | -0.78         | -0.49         |                                      |

Table 1: Observables for different mixing angles $\theta$ and strong-interaction phases $\delta_1$ and $\delta_2$. All branching ratios are given in units of $10^{-6}$. 
Figure 1: $C$ and $S$ parameters for the decay $B^0(B^0) \rightarrow \pi^+\pi^-$. Full line is for $\delta_1 = \delta_2 = 0^\circ$, dotted line for $\delta_1 = -10^\circ, \delta_2 = -20^\circ$, dashed line for $\delta_1 = 10^\circ, \delta_2 = -40^\circ$. 
Figure 2: Average branching ratios for $B^0/B^0 \rightarrow \pi^0\pi^0$ and $B^0/B^0 \rightarrow \pi^+\pi^-$. Full line is for $\delta_1 = \delta_2 = 0^\circ$, dotted line for $\delta_1 = -10^\circ$, $\delta_2 = -20^\circ$, dashed line for $\delta_1 = 10^\circ$, $\delta_2 = -40^\circ$. 