Explanation of the mass of the muon

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The difference of the rest masses \( m(\pi^\pm) - m(\mu^\pm) \) is nearly equal to \( 1/4 \) of the rest mass of the \( \pi^\pm \) mesons and is equal to the sum of the rest masses of the \( 0.7 \cdot 10^9 \) muon neutrinos (respectively anti-muon neutrinos) which are in the cubic lattice of the \( \pi^\pm \) mesons according to the standing wave model. In the decay of a \( \pi^+ \) or \( \pi^- \) meson all muon neutrinos, respectively anti-muon neutrinos, of the cubic lattice of the \( \pi^\pm \) mesons are emitted. The sum of the oscillation energies of all neutrinos in the \( \pi^\pm \) mesons is the same as the sum of the oscillation energies of the remaining neutrinos in the \( \mu^\pm \) mesons. Consequently the mass of the \( \mu^\pm \) mesons is equal to \( m(\pi^\pm) - 0.7 \cdot 10^9 m(\nu_\mu) \approx 0.75 m(\pi^\pm) \), within 1% in agreement with the measured ratio \( m(\mu^\pm) / m(\pi^\pm) = 0.757028 \).

1 Introduction

In a previous paper [1] we have found that the ratios of the masses of the so-called stable mesons and baryons of the \( \gamma \)-branch of the spectrum of the stable elementary particles are integer multiples of the mass of the \( \pi^0 \) meson within, on the average, 0.73%. We have explained this empirical fact with the standing wave model [2]. The ratios of the masses of the neutrino branch of the stable particle spectrum are integer multiples of the mass of the \( \pi^\pm \) mesons times a common factor \( 0.86 \pm 0.02 \) [1]. We have explained the ratios of the masses of the particles of the \( \nu \)-branch with the standing wave model in [3]. Surprisingly we can also explain the ratio of the mass of the \( \mu^\pm \) mesons to the mass of the \( \pi^\pm \) mesons with the standing wave model.

2 The mass of the muons

The mass of the \( \mu^\pm \) mesons is \( m(\mu^\pm) = 105.658389 \pm 3.4 \cdot 10^{-6} \) MeV, according to the Particle Physics Summary [4]. The muons are stable, their lifetime is \( \tau_\mu = 2.19703 \cdot 10^{-6} \pm 4 \cdot 10^{-11} \) sec, about a hundred times as long as the
lifetime of the \( \pi^\pm \) mesons, that means longer than the lifetime of any other elementary particle, but for the electrons, protons and neutrons. The muons are part of the lepton family which is distinguished from the mesons and baryons not so much by their mass as the name lepton implies, actually the mass of the \( \tau \) meson is about twice the mass of the proton, but rather by the absence of strong interaction with the mesons and baryons. The masses of the leptons are not explained by the standard model of the particles.

Comparing the mass of the \( \mu^\pm \) mesons to the mass of the \( \pi^\pm \) mesons
\[
m(\pi^\pm) = 139.56995 \text{ MeV}
\]
we find that
\[
m(\mu^\pm)/m(\pi^\pm) = 0.757028 \approx 3/4
\]
or that
\[
m(\pi^\pm) - m(\mu^\pm) = 33.91156 \text{ MeV} = 0.24297 \cdot m(\pi^\pm)
\]
The mass of the electron is approximately 1/206 of the mass of the muon, the contribution of \( m(e) \) to \( m(\mu) \) will therefore be neglected in the following.

We assume, as we have done in [2] and [3] and as appears to be natural, that the particles, including the muons, consist of the particles into which they decay. The \( \mu^+ \) meson decays via
\[
\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e.
\]
The muons are apparently composed primarily of the neutrinos which are already present in the cubic neutrino lattice of the \( \pi^\pm \) mesons according to the standing wave model [3]. The \( \pi^+ \) meson decays via
\[
\pi^+ \to \mu^+ + \nu_\mu
\]
or the conjugate particles in the decay of the \( \pi^- \) meson, with 99.988% of the \( \pi^\pm \) decays in this form. The energy \( m(\pi^\pm) - m(\mu^\pm) \approx 1/4 \cdot m(\pi^\pm) \) is lost when a \( \mu^+ \) meson and a muon neutrino \( \nu_\mu \), respectively a \( \mu^- \) and an anti-muon neutrino \( \bar{\nu}_\mu \), are emitted by the \( \pi^\pm \) mesons. The rest of the energy in the rest mass of the \( \pi^\pm \) mesons passes to the rest mass of the \( \mu^\pm \) mesons. In the standing wave model of the particles of the neutrino branch [3] the \( \pi^\pm \) mesons are composed of a cubic lattice consisting of \( N/2 = 0.71 \cdot 10^9 \) muon neutrinos \( \nu_\mu \) and the same number of anti-muon neutrinos \( \bar{\nu}_\mu \), \( m(\nu_\mu) = m(\bar{\nu}_\mu) \), as well as of \( N/2 \) electron neutrinos \( \nu_e \) and the same number of anti-electron neutrinos \( \bar{\nu}_e \), \( m(\nu_e) = m(\bar{\nu}_e) \), plus the oscillation energy of these neutrinos. We found in [3] that the mass of a single muon neutrino should be 50 milli-eV/c\(^2\), and the mass of a single electron neutrino should be 5 meV/c\(^2\). Using these values of \( N \) and of the neutrino masses we find two intriguing “coincidences”.

(a) The difference of the rest masses of the \( \pi^\pm \) and \( \mu^\pm \) mesons is nearly equal to the sum of the rest masses of all muon neutrinos respectively anti-muon neutrinos in the \( \pi^\pm \) mesons.
\[
m(\pi^\pm) - m(\mu^\pm) = 33.912 \text{ MeV} \quad \text{versus} \quad N/2 \cdot m(\nu_\mu) = 35.675 \text{ MeV}.
\]

(b) The energy in the oscillations of all neutrinos in the \( \pi^\pm \) mesons is
nearly the same as the energy in the oscillations of all $\bar{\nu}_\mu$, respectively $\nu_\mu$, and $\nu_e$ and $\bar{\nu}_e$ neutrinos in the $\mu^\pm$ mesons.

$$E_{osc}(\pi^\pm) = m(\pi^\pm) - N[m(\nu_\mu) + m(\nu_e)] = 61.09 \text{ MeV} \quad \text{versus} \quad E_{osc}(\mu^\pm) = m(\mu^\pm) - N/2 \cdot m(\bar{\nu}_\mu) - Nm(\nu_e) = 62.85 \text{ MeV}.$$  

Both statements are, of course, valid only within the accuracy with which the number $2N$ of all neutrinos in the $\pi^\pm$ lattice is known, as well as within the accuracy with which the masses $m(\nu_\mu)$ and $m(\nu_e)$ have been determined in [3]. It cannot be expected that this accuracy is better than a few percent, considering in particular the uncertainty of the lattice constant.

In the two-body decay of the $\pi^\pm$ mesons each decay product has the same and specific momentum $p$ which is $p = 30 \text{ MeV}/c$ in the rest frame, according to the Particle Physics Summary [4]. The momentum of the decay particles translates into a kinetic energy which for the $\mu$ mesons is $E_k(\mu^\pm) = 4.1 \text{ MeV}$, and for the single emitted muon neutrino with a practically vanishing rest mass $E_k(\nu_\mu) = 30 \text{ MeV}$. Hence the kinetic energies of the two decay products of the $\pi^\pm$ meson is $34.1 \text{ MeV}$, which is, as must be, practically equal to $m(\pi^\pm) - m(\mu^\pm) = 33.9 \text{ MeV}$. In terms of our model of the neutrino lattice in the $\pi^\pm$ mesons and the statement (a) above this means that the rest masses of all muon neutrinos in the $\pi^\pm$ mesons are converted into kinetic energy. All but one of the $0.7 \cdot 10^9$ muon neutrinos disappear in the decay of the cubic neutrino lattice of the $\pi^\pm$ mesons. We learn from the high energy collision $e^+ + e^- \to \mu^+ + \mu^-$ that $\mu$ mesons, and the neutrinos which are part of the $\mu$ meson masses, can be created directly out of the kinetic energy of the electrons and positrons in the $e^+ + e^-$ collision. What we observe in the $\pi^\pm$ decay is the reverse process, the conversion of the energy in neutrino rest masses into kinetic energy.

We note that the difference of the rest masses $m(\pi^\pm) - m(\mu^\pm)$ provides an independent check of the value of the rest mass of the muon neutrino. Using $N/2 = 0.7135 \cdot 10^9$ and $m(\pi^\pm) - m(\mu^\pm) = 33.912 \text{ MeV}$ it follows that the rest mass of the muon neutrino should be $m(\nu_\mu) = 47.53 \text{ meV}/c^2$, whereas we found $m(\nu_\mu) = 50 \text{ meV}/c^2$ in [3].

Since the decay of the $\pi^\pm$ mesons seems to mean the removal of all $\nu_\mu$, respectively, all $\bar{\nu}_\mu$ neutrinos from the neutrino lattice of the $\pi^\pm$ mesons, the muons should contain the remaining neutrinos of the original cubic lattice, that means $N/2$ anti-muon neutrinos $\bar{\nu}_\mu$, respectively, $N/2$ muon neutrinos $\nu_\mu$, plus $N/2$ electron neutrinos $\nu_e$ as well as $N/2$ anti-electron neutrinos $\bar{\nu}_e$. It can be shown immediately that the concept that the muons consist of three
types of neutrinos and their oscillation energies leads to the correct mass of
the muons. The energy $E$ in the rest mass of the $\mu$ mesons must be equal to
the oscillation or average kinetic energy of all neutrinos in the particle plus
the energy in the rest masses of the neutrinos. From $E = E_k + \Sigma m(\nu)c^2$
follows the elementary formula Eq.(18) in [3], which applied to the case of
the muons is

$$\frac{E(\mu^\pm)}{E(\pi^\pm)} = \frac{E_k(\mu^\pm)}{E_k(\pi^\pm)} \cdot \frac{1 + \Sigma m'(c^2/E_k(\mu^\pm))}{1 + \Sigma mc^2/E_k(\pi^\pm)}. \quad (1)$$

With the empirical $E_k(\mu^\pm) = m(\mu^\pm)c^2 - \Sigma m'c^2 = 62.9$ MeV and the
empirical $E_k(\pi^\pm) = m(\pi^\pm)c^2 - \Sigma mc^2 = 61.1$ MeV, where $\Sigma m'c^2 = N/2 \cdot 50$
meV $+ N \cdot 5$ meV (the sum of the energies of the rest masses of the anti-muon
neutrinos, electron neutrinos and anti-electron neutrinos in the $\mu^+$ meson),
and $\Sigma mc^2 = N \cdot 50$ meV $+ N \cdot 5$ meV (the sum of the energies of the rest
masses of the four neutrino types in the $\pi^\pm$ mesons), it follows that

$$E(\mu^\pm)/E(\pi^\pm) = 0.75763, \quad (2)$$

whereas the measured ratio is $E(\mu^\pm)/E(\pi^\pm) = 0.757028$. That means that
the concept of the muons consisting of three neutrino types plus their oscilla-
tion or kinetic energy leads to the correct ratio of the mass of the $\mu^\pm$ mesons
to the mass of the $\pi^\pm$ mesons.

3 The oscillation energies of the neutrinos

The neutrinos in the body of a muon must oscillate because the collision
$e^+e^- \rightarrow \mu^+\mu^-$ tells that a continuum of frequencies must be present in the
muons, if Fourier analysis holds. The continuum of frequencies from Fourier
analysis can be absorbed by the continuous spectrum of the oscillations in
a neutrino lattice. The energy of the oscillations of the neutrinos in the
muon lattice is the sum of the energies of a plane oscillation in an isotropic,
diatomic lattice consisting of $N/2$ $\bar{\nu}_\mu$ neutrinos and $N/2$ $\nu_e$ neutrinos, part of
the remains of the diatomic neutrino lattice of the $\pi^\pm$ mesons, plus the energy
of the diatomic oscillations of $N/2$ $\bar{\nu}_\mu$ and $N/2$ $\bar{\nu}_e$ neutrinos which neutrinos
were likewise in the lattice of the $\pi^\pm$ mesons. The latter oscillations are
likely to be perpendicular to the first mentioned diatomic oscillations in the
muon, because the $\bar{\nu}_e, \nu_e$ neutrino pairs are oriented perpendicular to the $\bar{\nu}_\mu$,
neutrino pairs in the original cubic lattice of the $\pi^\pm$ mesons, see Fig. 1 of [3].

The energy of the oscillations of the lattice is given by

$$E_{osc} = \frac{Nh\nu_0}{(2\pi)^2} \int \int_{-\pi}^{\pi} f(\phi_1, \phi_2) d\phi_1 d\phi_2,$$

as in Eq.(17) of [3] or in the original paper of Born and v.Karman [5], Eq.(50) therein. The oscillation energy of the diatomic lattice containing the $\bar{\nu}_\mu$ and $\nu_e$ neutrinos of the $\mu^\pm$ mesons is 1/2 of the oscillation energy of the diatomic lattice oscillations in the $\pi^\pm$ mesons because the number of the pairs $\bar{\nu}_\mu$ and $\nu_e$ in the $\mu^\pm$ mesons is 1/2 of the number of the corresponding pairs $\bar{\nu}_\mu$ and $\nu_e$ and $\nu_\mu$ and $\bar{\nu}_e$ in the $\pi^\pm$ mesons. The functions $f(\phi_1, \phi_2)$ in Eq.(3) describing the frequency spectrum of the oscillations are the same for the diatomic neutrino pairs in the $\pi^\pm$ mesons and the diatomic neutrino pairs in the $\mu^\pm$ mesons. The functions are given by Eq.(6) of [3]. Since both functions are the same the ratio of the energy in the diatomic $\bar{\nu}_\mu, \nu_e$ lattice oscillation of the $\mu^\pm$ mesons to the energy of the diatomic lattice oscillations in the $\pi^\pm$ mesons is $= 1/2$ according to Eq.(3). But since the same applies for the diatomic oscillations of the $\bar{\nu}_\mu, \bar{\nu}_e$ pairs in the $\mu^\pm$ mesons, the sum of the energies of both oscillations is equal to the oscillation energy of the $\pi^\pm$ mesons, as (b) says.

Since the frequencies of the diatomic oscillations are a quadratic function of $\nu$ there exists for each positive frequency also a negative frequency of the same absolute value. That means, according to (3), that the sum of the energies of the oscillations with negative frequencies is negative, but has the same absolute value as the sum of the energies of the oscillations with positive frequencies. That means that each muon has an antiparticle. In the antiparticle of a particular muon the oscillation energy of the neutrinos is replaced by the oscillation energy of the frequencies with the opposite sign, and the masses of the neutrinos are replaced by the masses of their antineutrinos, which have the same absolute value of the masses they replaced. The energy in the mass of a muon is consequently equal to the energy in the mass of its antiparticle, as it is with the masses of the particles of the $\gamma$-branch and of the $\nu$-branch, and with the masses of the $\mu^+$ and $\mu^-$ mesons.

Finally we ask why do the muons not interact strongly with the mesons and baryons? In [6] we have shown that a strong force emanates from the sides of a cubic lattice. This force is caused by the unsaturated weak forces of about $10^6$ neutrinos at a side of the surface of the neutrino lattice of the
mesons and baryons. The existence of such a force follows from the study of Born and Stern [7] which dealt with the forces between two parts of a cubic lattice cleaved in vacuum. If the muons have a lattice consisting of $\bar{\nu}_{\mu}$, respectively $\nu_{\mu}$, and $\nu_e$ and $\bar{\nu}_e$ neutrinos the lattice surface is not the same as the surface of the cubic $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_e$, $\bar{\nu}_e$ lattice of the mesons and baryons, as described by the standing wave model [2,3]. Therefore it does not seem likely that the muons interact in the same way with the mesons and baryons as the mesons and baryons interact with each other. To put this in another way, a triclinic lattice does not bond with a cubic lattice.

4 Conclusions

It has been shown that the mass of the muons can be explained as the sum of the rest masses of $0.71 \cdot 10^9$ muon neutrinos, respectively anti-muon neutrinos, and of the same number of electron neutrinos and anti-electron neutrinos, plus their oscillation energies. The three neutrino types in a muon are the remains of the cubic neutrino lattice of the $\pi^\pm$ mesons from which the muons are formed in the $\pi^\pm$ decay. The mass of the muons differs from the mass of the $\pi^\pm$ mesons by the sum of the rest masses of all muon neutrinos, respectively of all anti-muon neutrinos which are in the $\pi^\pm$ mesons. All muon neutrinos of one type are emitted when the $\pi^\pm$ mesons decay. Since the sum of the rest masses of all muon neutrinos of one type in the $\pi^\pm$ mesons is approximately $1/4$ of the rest mass of $\pi^\pm$ mesons, and since the oscillation energies in $\pi^\pm$ and $\mu^\pm$ are the same, the mass remaining in the $\mu^\pm$ mesons after the $\pi^\pm$ decay is $\approx 3/4 \cdot m(\pi^\pm)$, within 1% in agreement with the measured ratio $m(\mu^\pm)/m(\pi^\pm) = 0.757028$. We have also found that the muons do not interact with the mesons and baryons in the same strong way as the mesons and baryons interact with each other.

The explanation of the mass of the muons and of their weak interaction is a straightforward consequence of our standing wave model of the stable mesons and baryons.

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