Forecasting Electricity Consumption Using the Second-Order Fuzzy Time Series

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Abstract. There is a great development of Universiti Tun Hussein Onn Malaysia (UTHM) infrastructure since its formation in 1993. The development will be accompanied by the increasing demand for electricity. Hence, there is a need to forecast UTHM electricity consumption accurately so that UTHM can plan for future energy demand and utility saving decisions. Previous studies on UTHM electricity consumption prediction have been carried out using time series models, multiple linear regression and first-order fuzzy time series (FTS). The first-order FTS yield the best accuracy among these three methods. Previous forecasting problem showed higher order FTS can yield better accuracy. Therefore, in this study, the second-order FTS with trapezoidal membership function was implemented on the UTHM monthly electricity consumption from January 2009 to December 2018 to forecast January to December 2019 monthly electricity consumption. The procedure of the FTS and trapezoidal membership function was described together with January data. The second-order FTS forecast UTHM electricity consumption better than the first-order FTS.

1. Introduction

Time series is a sequence of equally spaced discrete temporal data. It may consist of some or all of the components such as trend, cyclical, seasonal and irregular. A trend is a long term pattern, while a cyclical is a repeated up and down movements in a time series. On the other hand, seasonal is a regular fluctuation during the same month or quarter whereas irregular is unexplained random component [1].

Forecasting is predicting future values based on the trends of past and current time series data. Forecasting for future electricity consumption is crucial for future power system planning and control. Forecasting can be divided into short-term forecasting (STF), medium-term forecasting (MTF) and long-term forecasting (LTF). STF up to one day or several weeks for scheduling the generation and transmission of electricity, MTF ranges from one day to several months to plan the fuel purchases, whereas LTF forecasts more than a year ahead up to twenty 20 years for power system planning [2-3]. The concept of fuzzy set theory was first proposed by Zadeh [4] in 1965. Based on Zadeh’s [5-8] works, Song and Chissom [9] is the first to apply concept of the fuzzy set on time series and develop a first-order time-invariant Fuzzy time series (FTS) model in 1993. Their definition of FTS is as follows:

Let \( Y(t) \) \((t = \ldots, 0, 1, 2, \ldots)\), be a time series, a subset of \( R \) and be the universe of discourse on which fuzzy sets \( f_i(t) \) \((i = 1, 2, \ldots)\) are defined. Let \( F(t) \) be a collection of \( f_i(t) \). Then, \( F(t) \) is called a fuzzy time series on \( Y(t) \) \((t = \ldots, 0, 1, 2, \ldots)\).

Song and Chissom later applied time-invariant FTS [10] and time-variant [11] FTS on the enrolment of Alabama University from the years 1971-1992. Their proposed procedure using FTS to forecast are as follow:

1. Define the universe of discourse
   \[ U = [D_{\text{min}} - D_1, D_{\text{max}} + D_2]. \]
2. Partition the universe of discourse into several even length intervals as \( u_1, u_2, \ldots, u_m. \)
3. Define some fuzzy sets on the universe.
4. Fuzzify the historical data using the memberships of each year's enrollment in each fuzzy set $A_i$.

5. Derive a fuzzy logical relationship (FLR).

$$R_i = A_i^T \times A_i$$ for all $n$ relations $A_i \rightarrow A_j, \quad R_i = \bigcup_{i=1}^{n} R_i$$

where $\times$ is the min operator, $T$ is the transpose operator and $\bigcup$ is the union operator.

6. Forecasted output using $A_i = A_{i-1} \circ R_i$.

where $A_{i-1}$ and $A_i$ are the fuzzified enrollments of year $i-1$ and $i$ represented by a fuzzy set.

The symbol

$\circ$ denotes the Max-Min composition operator.

7. Defuzzify forecasted results.

Chen [12] simplified Song and Chissom [10-11] procedure to implement first-order FTS with triangular Fuzzy membership function on the enrolment of Alabama University from the years 1971-1992. Chen [12] work has sparked the researchers' interests to improve the accuracy of FTS forecasting. Eleruja, Mu’azu and Dajab [13] improved the determination of the universe of discourse by replacing $D_1$ and $D_2$ [10-11] using the revised standard deviation while Cheng, Chang and Yeh [14] using standard deviation. The length of intervals was studied by Huarng [15]. Poulsen [16] and Tay [17] used trapezoidal membership functions of fuzzy time series instead of the triangular membership function. Later, researchers [16, 18-19] proved that higher order FTS improve the accuracy of the forecasting.

Konica and Hanelli [20] adopted time, historical and forecasting value of the temperature and previous day load as the input of fuzzy interference system toolbox in Matlab for a short-term load forecasting electricity consumption for Albania. They predicted the next-day electricity consumption.

University Tun Hussein Onn Malaysia (UTHM) is a developing Malaysian Technical University in south peninsular Malaysia. UTHM is located in Batu Pahat, Johor Malaysia. It has great development since its formation in 1993. There is a new campus set up in Pagoh, Johor in 2017. The development is accompanied by the usage of electricity consumption.

Forecasting of UTHM electricity consumption has been studied using time series analysis [21], multiple linear regression [22] and first-order FTS [17]. The forecasting work [17, 21] utilized monthly data from January 2011 to December 2017, while [22] used monthly data from January 2011 to August 2018. The mean absolute percentage errors (MAPEs) are 5.74%, 11.14% and 10.62% for the study of [17, 21-22] respectively.

Accurate electricity consumption forecasting is important for Development and Maintenance Office, UTHM. Based on the future forecast, they can plan for the strategy to reduce the electricity consumption to save on the electricity bill due to financial constraints facing by UTHM. Besides, they can apply for appropriate next year budget on electricity from bursary of UTHM based on the accurate future forecast. Hence, there is a need to reduce the MAPE obtained by previous works of UTHM electricity forecasting. From the previous three studies, it is noticed that FTS yields the smallest error of 5.74% and previous studies [16, 18-19] showed higher order FTS can yield better accuracy. Therefore, in this study, the second-order FTS with trapezoidal membership function was applied on monthly UTHM electricity consumption from January 2009 to December 2018 and year 2019 electricity consumption was forecasted. The accuracy of the second-order FTS was later compared with the first-order FTS.
2. Fuzzy time series (FTS)

The input of this study is the same month of electricity consumption from the year 2009-2018, for example with the input of January electricity consumption from 2009-2018, the January 2019 electricity consumption will be forecasted. The process was repeated for February, March, … till December. The following steps show the general procedure to be taken in order to forecast monthly 2019 electricity consumption.

Sort the values of the same month electricity consumption from 2009-2018 in ascending order.

1. Compute distance, $D_i$ between any two consecutive electricity consumptions in the sorted dataset as

$$D_i = |y_{i+1} - y_i|$$

(1)

2. Find average distance, $AD$ between any two consecutive electricity consumptions in the sorted dataset as

$$AD = \frac{1}{n-1} \sum_{i=1}^{n-1} D_i$$

(2)

3. Compute the corresponding standard deviation, $\sigma_{AD}$ of the average distance as

$$\sigma_{AD} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n-1} (D_i - AD)^2}$$

(3)

4. $D$ should be in the following interval

$$AD - \sigma_{AD} \leq D \leq AD + \sigma_{AD}$$

(4)

Eliminate the outlier $D_i$ which is not in the range.

5. Recalculate the revised average distance $AD_r$ from the remaining $D_i$.

6. Define the universe of discourse

$$U = [y_{\text{MIN}} - AD_r, y_{\text{MAX}} + AD_r]$$

(5)

where $y_{\text{MIN}}$ is the minimum value of electricity consumption while $y_{\text{MAX}}$ is the maximum value of electricity consumption.

7. Fuzzify the universe of discourse using the trapezoidal fuzzification approach. In the trapezoidal fuzzy set, there are four quartets of trapezoidal fuzzy numbers $(a, b, c, d)$. The leftmost value of the trapezoidal set is $y_{\text{MIN}} - AD_r$, whereas the rightmost value of the trapezoidal set is $y_{\text{MAX}} + AD_r$ and the distance for each number of $a, b, c$ and $d$ are the revised average distance, $AD_r$.

8. When data belong to two fuzzy sets, the trapezoidal Fuzzy number (6) will be used to find the membership degree. The highest membership degree will be used to determine the membership of data.

$$\mu_a(y_i) = \begin{cases} 
0 & y_i < a \\
\frac{y_i - a}{b - a}, & a \leq y_i \leq b \\
1 & b \leq y_i \leq c \\
\frac{d - y_i}{d - c}, & c \leq y_i \leq d \\
0 & y_i > d 
\end{cases}$$

(6)

9. Next, a Fuzzy set relationship will be determined. If the time series variable $F(t-2), F(t-1)$ and $F(t)$ are fuzzified as $A_i, A_j$ and $A_k$ respectively, then the first order Fuzzy set relationship is $A_i$.
is related to $A_j$ and denoted as $A_i \rightarrow A_j$. On the other hand, the second order Fuzzy set relationship is $A_i$ and $A_j$ are related to $A_k$ and are denoted as $A_i, A_j \rightarrow A_k$.

10. The fuzzy linear relation group (FLRG) will be determined by grouping the same fuzzy set which is related to more than one set.
11. The midpoint of each of the FLRG will be computed.
12. The forecasted value will be the average value of the midpoint of the FLRG values.

3. Error analysis
The performance of the above time series methods can be measured by mean absolute percentage error (MAPE) as below:

$$\text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100\%,$$

(7)
where $y_i, \hat{y}_i$ are real and forecasted data respectively, $n$ is the number of real data.

4. Results and Discussion
The forecasting process of January 2019 electricity consumption will be started on the January data from the year 2009 to 2018 which is shown in table 1 as the following steps:

| Year | Jan  | sort          | $D$     | $|D-AD|^2$ |
|------|------|---------------|---------|-----------|
| 2009 | 496.379 | 496.379 |         |           |
| 2010 | 1800.342 | 1757.133 | 1260.754 | 997281.4 |
| 2011 | 1757.133 | 1800.342 | 43.209  | 47919.5  |
| 2012 | 2646.807 | 2117.226 | 316.884 | 2999.729 |
| 2013 | 2855.407 | 2256.096 | 138.87  | 15189.14 |
| 2014 | 2379.815 | 2376.343 | 3.472   | 66895.8  |
| 2015 | 2774.32  | 2379.815 |        |           |
| 2016 | 2376.343 | 2646.807 | 266.992 | 2379272  |
| 2017 | 2256.096 | 2774.32  | 127.513 | 18117.49 |
| 2018 | 2117.226 | 2855.407 | 81.087  | 32770.86 |

| AD  | 262.1142 | $AD-\sigma_{AD}$ | -103.236 |
|-----|----------|--------------------|----------|
| $\sigma_{AD}$ | 365.3498 | $AD+\sigma_{AD}$ | 627.464 |

1) The sorted values of the January electricity consumption from the year 2009-2018 in the current dataset in ascending order are shown in column 3 of table 1.
2) Compute distance, $D$ using Eq. (1) and the results are shown in the fourth column of table 1.
3) The average distance, $AD$ between any two consecutive electricity consumption in the sorted dataset was found as 262.114.
4) The corresponding standard deviation, $\sigma_{AD}$ of the average distance was obtained as 365.359.
5) $D$ should be in the interval $-103.236 \leq D \leq 627.464$.
   Eliminate the outlier $D = 1260.754$ which is not in the range.
6) The revised average distance $AD_r$ from the remaining $D$ was calculated as $AD_r = 137.284$.
7) Define the universe of discourse $U = [359.095, 2992.691]$.
   Hence, the lower boundary of $U$ is 359.095 MWh, while the upper boundary of $U$ is 2992.691 MWh.
8) The trapezoidal fuzzy sets were obtained as given in Table 2:

| Fuzzy set | $a$      | $b$      | $c$      | $d$      |
|-----------|----------|----------|----------|----------|
| $A_1$     | 359.0948 | 496.379  | 633.6633 | 770.9475 |
| $A_2$     | 633.6633 | 770.9475 | 908.2318 | 1045.516 |
| $A_3$     | 908.2318 | 1045.516 | 1182.8   | 1320.085 |
| $A_4$     | 1182.8   | 1320.085 | 1457.369 | 1594.563 |
| $A_5$     | 1457.369 | 1594.563 | 1731.937 | 1869.222 |
| $A_6$     | 1731.937 | 1869.222 | 2006.506 | 2143.79  |
| $A_7$     | 2006.506 | 2143.79  | 2281.074 | 2418.359 |
| $A_8$     | 2281.074 | 2418.359 | 2555.643 | 2692.927 |
| $A_9$     | 2555.643 | 2692.927 | 2830.211 | 2967.496 |
| $A_{10}$  | 2830.211 | 2967.496 | 3104.78  | 3242.064 |

Where the leftmost is $y_{\text{MIN}} - AD_r = 496.379 - 137.284 = 359.095$. Then the following values will be added with $AD_r$. Next, the values of $a$ and $b$ for the following fuzzy set are equal to the preceding values of $c$ and $d$ from preceding Fuzzy set. The rightmost value of the fuzzy set will not necessarily equals to $y_{\text{MAX}} + AD_r = 2855.407 + 137.284 = 2992.691$ but will be greater than it. Here $y_{\text{MIN}}$ and $y_{\text{MAX}}$ are the lowest and highest values in the real data.
9) The fuzzified membership set of January electricity consumption is given in Table 3. It is clearly shown that January 2009, 2015, 2017 belong to fuzzy set $A_1$, $A_5$ and $A_7$. January 2010-2014, 2016 and 2018 belong to both fuzzy sets, hence the formula (6) was used to determine their higher membership degree as given in columns 3 and 4 in table 3. The fuzzied membership set was determined based on the highest membership degree. For example, electricity consumption in January 2010 is 1800.342MWh belongs to fuzzy sets $A_5$ at interval $c$ to $d$ and $A_6$ at the interval $a$ to $b$. The membership degrees of $A_5$ and $A_6$ are shown in row 3, columns 3 and 4 in table 3. The highest membership degree is $A_5$ and thus the January 2010 data is fuzzified as Fuzzy set $A_5$.

| Year | Jan | FL | FU | Fuzzified  |
|------|-----|----|----|------------|
| 2009 | 496.379 | 0  | 0  | $A_1$      |
| 2010 | 1800.342 | 0.501729 | 0.498271 | $A_5$ |
| 2011 | 1757.133 | 0.81647  | 0.18353 | $A_5$ |

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Table 2. Trapezoidal Fuzzy numbers ($a$, $b$, $c$, $d$)

Table 3. Fuzzified membership set
10. The fuzzy logical relationships (FLR) was obtained as:

First-order FTS:

\[ A_1 \rightarrow A_5, \ A_5 \rightarrow A_9, \ A_9 \rightarrow A_9, \ A_9 \rightarrow A_8, \ A_8 \rightarrow A_8. \]

Second-order FTS:

\[ A_1, A_5 \rightarrow A_9, \ A_5, A_9 \rightarrow A_9, \ A_9, A_8 \rightarrow A_8, \ A_8, A_9 \rightarrow A_9. \]

11. Next, a fuzzy logical relation group (FLRG) were determined by grouping the same group of fuzzy set relationships as shown in table 4 for first-order FTS and table 5 for second order FTS.

Table 4. The first-order FTS FLRG

| Year | Jan       | Fuzzified | First-order | FLRG |
|------|-----------|-----------|-------------|------|
| 2009 | 496.379   | \( A_1 \) |             |      |
| 2010 | 1800.342  | \( A_1 \) | \( A_1 \rightarrow A_5 \) | \( A_5 \rightarrow A_5 \) |
| 2011 | 1757.133  | \( A_1 \) | \( A_1 \rightarrow A_5 \) | \( A_5 \rightarrow A_5, A_5 \rightarrow A_9 \) |
| 2012 | 2646.807  | \( A_1 \) | \( A_1 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2013 | 2855.407  | \( A_1 \) | \( A_1 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2014 | 2379.815  | \( A_1 \) | \( A_1 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2015 | 2774.32   | \( A_1 \) | \( A_1 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2016 | 2376.343  | \( A_1 \) | \( A_1 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2017 | 2256.096  | \( A_1 \) | \( A_1 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2018 | 2117.226  | \( A_1 \) | \( A_1 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2019 |           |           | \( A_1 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |

Table 5. The second-order FTS FLRG

| Year | Jan       | Fuzzified | Second-order | FLRG |
|------|-----------|-----------|--------------|------|
| 2009 |           | \( A_1 \) | \( A_1 \rightarrow A_5 \) \( A_5 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2010 |           | \( A_1 \) | \( A_1 \rightarrow A_5 \) \( A_5 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2011 |           | \( A_1 \) | \( A_1 \rightarrow A_5 \) \( A_5 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2012 |           | \( A_1 \) | \( A_1 \rightarrow A_9 \) \( A_9 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2013 |           | \( A_1 \) | \( A_1 \rightarrow A_9 \) \( A_9 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2014 |           | \( A_1 \) | \( A_1 \rightarrow A_9 \) \( A_9 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2015 |           | \( A_1 \) | \( A_1 \rightarrow A_9 \) \( A_9 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2016 |           | \( A_1 \) | \( A_1 \rightarrow A_9 \) \( A_9 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2017 |           | \( A_1 \) | \( A_1 \rightarrow A_9 \) \( A_9 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2018 |           | \( A_1 \) | \( A_1 \rightarrow A_9 \) \( A_9 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
| 2019 |           | \( A_1 \) | \( A_1 \rightarrow A_9 \) \( A_9 \rightarrow A_9 \) | \( A_9 \rightarrow A_9, A_9 \rightarrow A_9, A_9 \rightarrow A_9 \) |
12. The midpoint of each of the FLRG is given in table 6 and table 7. The midpoint is calculated based on the average value of the right-hand side of FLRG fuzzy set.

### Table 6. The midpoint of the first-order FLRG

| Year | Jan | FLRG          | Midpoint           |
|------|-----|---------------|--------------------|
| 2009 | 496.379 | $A_1$               |                   |
| 2010 | 1800.342 | $A_3$           | 1663.29513         |
| 2011 | 1757.133 | $A_5$         | 1663.29513         |
| 2012 | 2646.807 | $A_9$         | 1663.29513         |
| 2013 | 2855.407 | $A_9$         | 1663.29513         |
| 2014 | 2379.815 | $A_9$         | 1663.29513         |
| 2015 | 2774.32 | $A_9$         | 2761.56913         |
| 2016 | 2376.343 | $A_9$         | 2761.56913         |
| 2017 | 2256.096 | $A_9$         | 2761.56913         |
| 2018 | 2117.226 | $A_9$         | 2761.56913         |
| 2019 | 77 | $A_7$         | 2212.43213         |

### Table 7. The midpoint of the second-order FLRG

| Year | Jan | FLRG          | Midpoint           |
|------|-----|---------------|--------------------|
| 2009 | 496.379 | $A_1$               |                   |
| 2010 | 1800.342 | $A_3$           | 2487.001           |
| 2011 | 1757.133 | $A_5$         | 2487.001           |
| 2012 | 2646.807 | $A_9$         | 2487.001           |
| 2013 | 2855.407 | $A_9$         | 2487.001           |
| 2014 | 2379.815 | $A_9$         | 2487.001           |
| 2015 | 2774.32 | $A_9$         | 2212.43213         |
| 2016 | 2376.343 | $A_9$         | 2212.43213         |
| 2017 | 2256.096 | $A_9$         | 2212.43213         |
| 2018 | 2117.226 | $A_9$         | 2212.43213         |
| 2019 | 77 | $A_7$         | 2212.43213         |
13. Finally, the forecasted January electricity consumption is the average of the midpoint in tables 6 and 7. Notice that the first-order uses the previous year to forecast next year, whereas the second-order uses the previous two years to forecast the third year.

| Year | Jan    | First-order FTS forecast | Second-order FTS forecast |
|------|--------|---------------------------|---------------------------|
| 2009 | 496.379|                           |                           |
| 2010 | 1800.342| 1663.29513               |                           |
| 2011 | 1757.133| 2212.43213               | 1663.2951                 |
| 2012 | 2646.807| 2212.43213               | 2761.5691                 |
| 2013 | 2855.407| 2624.28488               | 2761.5691                 |
| 2014 | 2379.815| 2624.28488               | 2487.0006                 |
| 2015 | 2774.32 | 2487.00063               | 2487.0006                 |
| 2016 | 2376.343| 2212.43213               | 2212.4321                 |
| 2017 | 2256.096| 2212.43213               | 2761.5691                 |
| 2018 | 2117.226| 2212.43213               | 2487.0006                 |
| 2019 | 2212.432| 2212.43213               | 2212.4321                 |

UTHM electricity consumption patterns versus month for years 2009-2018 is shown in Figure 1. It is noticed that electricity consumption has increased since the year 2009. The electricity consumption fluctuates for each month. The highest electricity consumption is 3228.53 MWh occurred in March 2015, while the minimum consumption is 496.379 MWh in January 2009. The electricity consumption for certain months are low than usual may because the month is mostly semester break of UTHM. There are fewer students in the campus and therefore, the electricity consumption will be less if compared to the months that are not semester break.
Figure 2 shows the time series of UTHM electricity consumption continuously from January 2009-December 2018. The electricity consumption is range from 500 MWh to 3500 MWh. The time series seems is not stationary and is increasing. The electricity consumption in the year 2019 will be forecasted by using the first and second-order FTS.

The actual UTHM electricity consumption is shown in blue colour from January 2009 to December 2018 and forecasted electricity consumption in red colour using the first-order FTS from January 2010 to December 2019 were depicted in Figure 3. The first-order FTS uses the one-year data in 2009 to forecast the electricity consumption for 2010 and so on. Hence, the forecasted results start from January 2010.
Figure 3. Actual Electricity Consumption and the first-order FTS

The actual UTHM electricity consumption is shown in blue colour from January 2009 to December 2018 and forecasted electricity consumption in red colour using the second-order FTS from January 2011 to December 2019 were depicted in Figure 3. The second-order FTS uses two years of data in 2009 and 2010 to forecast the electricity consumption for 2011 and so on. Hence, the forecasted results start from January 2011. It is noticed that the second-order FTS predict the pattern closer to the actual pattern if compared to the first-order FTS.

Figure 4. Actual Electricity Consumption and the second-order FTS

Table 9 gives the MAPE values for first-order and second-order FTS forecasting from January 2009-December 2018. The MAPE of the second-order FTS is 3.627 % which is much lower than the first-order FTS which is 7.240 %. Hence, in this case, the second-order FTS has a higher accuracy of prediction as compared to the first-order FTS.
Table 9. MAPE (%) from January 2009-December 2018

|               | The first-order FTS | The second-order FTS |
|---------------|---------------------|---------------------|
| The first-order FTS | 7.240               |                     |
| The second-order FTS | 3.627               |                     |

Table 10 gives the first four-month 2019 UTHM real electricity consumption recently collected and 2019 predicted values by the first and second-order FTS.

Table 10. Forecasted electricity consumption of UTHM for the year 2019 by using the first-order and second-order FTS

| Month  | UTHM electricity consumption, MWh | Actual | First-order FTS | Second-order FTS |
|--------|----------------------------------|--------|-----------------|-----------------|
| January| 2237.882                         | 2212.432 | 2212.432        |
| February| 1923.017                       | 1891.714 | 2016.595        |
| March  | 2503.558                         | 2592.221 | 2592.221        |
| April  | 2451.083                         | 2798.906 | 2646.204        |
| May    | 2430.637                         | 2627.157 |                 |
| June   | 2801.596                         | 1922.117 |                 |
| July   | 2692.683                         | 2222.102 |                 |
| August | 1923.774                         | 1996.489 |                 |
| September| 2169.027                      | 2169.027 |                 |
| October| 2783.073                         | 2691.283 |                 |
| November| 2172.361                      | 2334.711 |                 |
| December| 2515.648                         | 2344.703 |                 |

The MAPE values for the first and second-order FTS between the first four months of 2019 data is shown in Table 11. It is shown that for future forecast, the second-order FTS still performs better as it gives a smaller error of MAPE of 4.376 if compared to 5.124 for the first-order FTS.

Table 11. MAPE (%) for 2019 data

|               | The first-order FTS | The second-order FTS |
|---------------|---------------------|---------------------|
| The first-order FTS | 5.124               |                     |
| The second-order FTS | 4.376               |                     |

The overall MAPE from January 2009 to Apr 2019 for both the first and second-order FTS is given in Table 12. Again it is proved that the second-order FTS still performs better as it gives a smaller error of MAPE of 3.657 if compared to 7.164 for the first-order FTS.

Table 12. MAPE (%) from January 2009 – April 2019

|               | The first-order FTS |
|---------------|---------------------|
| The first-order FTS | 7.164               |
5 Conclusion
The first-order and second-order FTS using trapezoidal membership function and revised average distance to replace arbitrary numbers of $D_1$ and $D_2$ [10-11] was applied on monthly UTHM electricity consumption from January 2009-December 2018 to forecast monthly 2019 UTHM electricity consumption. The same month data was used to forecast the next year electricity consumption. The January data was utilised and a step-by-step procedure for the first-order and second-order FTS was demonstrated. The procedure was repeated twelve times for twelve months. Based on the error analysis made, the MAPE from January 2009 to December 2018 of the first-order FTS and second-order FTS is 7.240 % and 3.627 %, respectively, while the MAPE changes to 7.164% and 3.657 if included the recently collected four-month data of 2019. The MAPE for January 2019 to Apr 2019 is 5.124% and 4.376% for the first-order FTS and second-order FTS respectively. These MAPE results indicate that the second-order FTS has the highest accuracy and the best performance if compared to the first-order FTS. Hence, it is proven that better prediction can be achieved by applying higher-order FTS.

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