Nucleosynthesis Bounds on Small Dirac Neutrino Masses due to Chiral Symmetry Breaking

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Abstract

A Higgs doublet which has a positive mass squared term and Yukawa couplings to the quarks will acquire a vacuum expectation value typically of the order of $10^2\text{eV}$ or less, as a result of chiral symmetry breaking. We consider nucleosynthesis constraints on models which use this fact as a basis for understanding very small Dirac neutrino masses without requiring very small Yukawa couplings. The simplest such model requires the introduction of a second Higgs doublet plus right-handed neutrinos together with a global horizontal symmetry $U(1)_H$ which ensures that the neutrinos are naturally massless in the chiral symmetric limit. We show that present big-bang nucleosynthesis constraints impose a well-defined upper bound of 0.1eV on the neutrino masses. This bound may become several orders of magnitude more stringent in the future as our understanding of the observational constraints on nucleosynthesis improves. We discuss the phenomenological implications for neutrino dark matter, the solar neutrino problem and the atmospheric neutrino deficit.
1. Introduction

The question of whether neutrinos have mass has become particularly important in the light of evidence from various experiments of a deficit in the number of electron neutrinos from the Sun \cite{1, 2, 3, 4, 5}, which cannot be explained by modification of the standard solar model \cite{6} but which can readily be explained by neutrino oscillations between massive neutrino species \cite{7, 8}. However, these experiments and others designed to directly observe neutrino masses also require that neutrino masses are much smaller than those of the charged leptons; in particular, the electron neutrino mass must be less than 5eV, about $10^{-5}$ times the mass of the electron \cite{9}.

In order for the SU(3)$_c \times$SU(2)$_L \times$U(1)$_Y$ Standard Model to be able to account for neutrino masses, it must be extended by the addition of new particles. The simplest and most commonly considered possibility is the addition of right-handed weak isosinglet neutrinos. There are then essentially two ways in which the small neutrino masses can be understood. The simplest possibility is to couple the right-handed neutrinos to the Higgs doublet via extremely small Yukawa couplings, of magnitude $O(10^{-11})$ or less for the case of the electron neutrino. This will give rise to small Dirac masses for the neutrinos, with no lepton number (L) violation. The second possibility is to introduce large L violating Majorana masses for the right-handed neutrinos, together with Yukawa couplings of the right-handed neutrinos to the Higgs doublet of a strength unsuppressed relative to that of the charged lepton Yukawa couplings. This results in the so-called "see-saw" mechanism for small neutrino masses \cite{10}, in which mixing between heavy right-handed neutrinos and left-handed neutrinos results in essentially left-handed neutrinos with a small, lepton-number violating Majorana mass. These are the only possibilities for the case of the right-handed neutrino extension of the Standard Model with only one scalar doublet. However, if we consider models in which there is a second scalar doublet, then there is another possibility. The right-handed neutrinos might couple only to the second scalar doublet, which might in turn acquire only a very small vacuum expectation value (VEV). The question of the smallness of the neutrino masses is
then related to the reason for the smallness of the second Higgs doublet’s VEV.

One interesting possibility, originally suggested by Thomas and Xu [11], is that this small VEV could be induced by chiral symmetry breaking. They suggested a two Higgs doublet model with a global horizontal symmetry in order to ensure the masslessness of the neutrinos in the chiral symmetric limit. An upper bound was put on the resulting Dirac neutrino masses by using limits on the rate of helicity flipping processes in the supernova SN 1987A [11, 12, 13]. In the present paper, we will reconsider this two Higgs doublet model. In particular, we will consider the constraints on such models coming from big-bang nucleosynthesis [14, 15, 16]. We will show that the present nucleosynthesis upper bound on the neutrino masses is of the same order of magnitude as that coming from SN 1987A, but is much more clearly defined than the supernova upper bound, which is difficult to state precisely because of the complexity of the physics of supernovae. Such a clearly defined upper bound is important in order to be able to unambiguously assess the implications of the model for neutrino phenomenology, which is often sensitive to the mass squared of the neutrinos. In addition, the nucleosynthesis upper bound has the possibility of becoming much tighter in the future as our understanding of the observational constraints on big-bang nucleosynthesis improves. The supernova upper bound, on the other hand, is unlikely to be improved by much more than an order of magnitude.

The paper is organized as follows. In section 2 we discuss the minimal two Higgs doublet model with a global horizontal symmetry which can naturally generate a small Dirac neutrino mass via chiral symmetry breaking, as originally suggested by Thomas and Xu [11]. We also suggest a four Higgs doublet extension of the model which has the advantage of being compatible with supersymmetry whilst not introducing large flavour changing neutral current effects. In section 3 we consider the big bang nucleosynthesis constraints on the resulting neutrino masses. In section 4 we will consider the implications of these constraints for various aspects of neutrino phenomenology. In section 5 we give our conclusions.
2. Models for Dirac Neutrino Masses Due To Chiral Symmetry Breaking

We first consider the "minimal" model for Dirac neutrino masses from chiral symmetry breaking, which was first discussed in reference [11]. Consider the following two Higgs doublet extension of the Standard Model. We will refer to both scalar doublets as Higgs doublets, since they will both acquire vacuum expectation values; however, only one of them will have a negative mass squared term. Let $H_+ = \begin{pmatrix} \phi_+^0 \\ \phi_+^+ \end{pmatrix}$ be the second Higgs doublet, assumed to have a positive mass squared, and let $H = \begin{pmatrix} \phi^-_0 \\ \phi^-_+ \end{pmatrix}$ be the conventional Standard Model Higgs doublet responsible for the quark and lepton masses. In order that the neutrino masses can be induced by chiral symmetry breaking we must impose on the model that $H$ does not couple to the right-handed neutrinos and in addition that there are no scalar couplings of $H$ to $H_+$ of the form $H_+^\dagger H$, which would otherwise lead to a large VEV for $H_+$. We will discuss shortly the necessary form of symmetry required to achieve this. The most general form of Yukawa couplings is then given by

$$h_u \, \bar{u}_R H Q + h_d \, \bar{d}_R \tilde{H} Q + h_e \, \bar{e}_R \tilde{H} L + h_c.$$  \(2.1\)

where $\tilde{H}_i = \epsilon_{ij} H^*_j$, with $i$ and $j$ being SU(2)$_L$ indices and where we have suppressed the generation indices. We will consider all couplings to be real in the following. Once chiral symmetry breaking occurs, the light quark condensates $<\bar{u}u>$, $<\bar{d}d>$ and $<\bar{s}s>$ will become non-zero, with a value given by $[18]$

$$<\bar{q}q> = \frac{f_\pi^2 m_q^2}{\sqrt{2}(m_u + m_d)} \quad (2.2),$$

where $q = u, d$ or $s$, $f_\pi$ is the pion decay constant, $m_\pi$ is the pion mass and $m_{u,d}$ are the current masses of the up and down quarks. As a result, the leading terms in the scalar potential for $H_+$ will be of the form $[19]$

$$V(H_+) = m_+^2 |\phi_+^0|^2 - (\tilde{\lambda}_q <\bar{q}q_L> \phi_+^0 + \text{h.c.}) + \ldots \quad (2.3).$$
(Throughout this paper we will denote the Yukawa couplings in the mass eigenstate basis by a tilde. By a choice of basis the neutrino Yukawa coupling matrix $\lambda_\nu$ can be made diagonal throughout). Thus the additional Higgs will gain a VEV

$$< \phi^0_+ > = \frac{\tilde{\lambda}_q < \overline{q} q >}{m^+_+} \quad (2.4).$$

With $f_\pi \approx 90\text{MeV}, m_u + m_d \approx 15\text{MeV}$ and $m_\pi = 135\text{MeV}$ [9] this gives

$$< \phi^0_+ > \approx 700\tilde{\lambda}_q \left( \frac{100\text{GeV}}{m^+_+} \right)^2 \text{eV} \quad (2.5),$$

and so the neutrinos will gain a mass given by

$$m_\nu \approx 700\lambda_\nu\tilde{\lambda}_q \left( \frac{100\text{GeV}}{m^+_+} \right)^2 \text{eV} \quad (2.6).$$

We see that with Yukawa couplings of magnitude $\lesssim 0.1$ we would very naturally obtain neutrino masses of the order of 1eV or less.

However, the above discussion does not address the question of the conditions under which the neutrinos remain massless in the chiral symmetric limit. The clearest approach is to impose a symmetry which can prevent the coupling of the conventional Higgs to the right-handed neutrinos or to $H_+$, whilst allowing $H_+$ to couple to $\overline{u}_R Q$ in order that chiral symmetry breaking can induce a small VEV for $H_+$. Since the conventional Higgs must also couple to $\overline{u}_R Q$, the only possible way to have such a symmetry is to consider a symmetry that distinguishes between different quark generations. The simplest possibility is to consider a global horizontal symmetry $U(1)_H$ under which $H_+$, the first generation right-handed up-type quark, $u_{R_1}$, and the right-handed neutrinos, $\nu_{R_i}$, transform according to $(H_+, u_{R_1}, \nu_{R_i}) \rightarrow e^{i\eta}(H_+, u_{R_1}, \nu_{R_i})$, with all other fields invariant. In this case the allowed Yukawa couplings from (2.1) are given by

$$h_{u_{R_1}j} \overline{u}_{R_1} H Q_j + h_{d_{R_1}i} \overline{d}_{R_1} \tilde{H} Q_j + h_{e_{R_1}i} \overline{e}_{R_1} \tilde{H} l_j$$

$$+ \lambda_{\nu_{R_i} j} \overline{\nu}_{R_i} H l_j + \lambda_{u_{R_1}i} \overline{u}_{R_1} H Q_i + \text{h.c.} \quad (2.7),$$

where $i, j = 1, 2, 3$ and $\alpha = 2, 3$. In this case the Dirac neutrino masses originate from the up quark condensate. This model was originally suggested by Thomas and
Xu (equation (12) of reference [11]). The most important feature of the Yukawa couplings in (2.7) is that, in addition to the neutrinos, the lightest up-type quark will also be massless in the limit of unbroken chiral symmetry. This feature is an unavoidable consequence of ensuring the masslessness of the neutrinos in the chiral symmetric limit via a symmetry. However, as far as is known from the present understanding of non-perturbative effects in QCD, such a possibility is not inconsistent with hadron phenomenology [20, 21]. It is possible that an up quark current mass could be generated by QCD instanton effects. (A massless down quark, on the other hand, would be inconsistent with the observed pseudoscalar meson masses [20]).

The form of the Yukawa couplings in (2.7) has an important phenomenological advantage. In general one would expect that adding a second Higgs doublet could result in large flavour changing neutral current (FCNC) effects due to tree-level exchange of the additional neutral Higgs scalar in H+ [22, 23]. In particular, one would expect strong constraints to be imposed on \( \lambda_q \) by limits from \( \Delta m_K \) and \( \Delta m_D \). However, it is easy to see that with the form of the Yukawa coupling matrix \( \lambda_u \) in (2.7) such tree-level processes do not occur. This is because only the first generation right-handed up quark couples to \( \lambda_u \). Therefore we only have a coupling to \( \overline{u}_R \) but not to \( \overline{c}_R \), as would be necessary in order to have a tree-level contribution to \( \Delta m_D \). So potentially dangerous tree-level FCNC effects are naturally suppressed in this model.

Before discussing the cosmological constraints, we briefly consider a supersymmetric version of the model. The minimal two Higgs doublet model given by equation (2.7) would not be compatible with supersymmetry. A supersymmetric version would require two more Higgs doublets; one in order to give masses to both the up-type and down-type quarks and one in order to maintain anomaly freedom once the Higgsinos corresponding to H+ are introduced [24]. Thus a supersymmetric version of the model will have four Higgs doublets: \( H_u \) and \( H_d \), responsible for the up and down-type quark masses, and \( H_+ \) and \( H_- \), where \( H_- \) has the negative of the hypercharge of \( H_+ \). Although it would be possible for the additional Higgs doublet \( H_- \)
not to couple to the quarks and leptons, in the most general case we would expect both $H_d$ and $H_-$ to couple to $\overline{e}_R L$ and $\overline{d}_R Q$. In this case, for reasonable values of the mass of the additional Higgs doublet (say $m_+ \lesssim 1$ TeV), there would be a danger of a large tree-level contribution to $\Delta m_K$ \[ \text{[22, 23].} \]

In order to avoid this danger we will extend the $U(1)_H$ symmetry such that $H_-, d_{R_i}$ and $e_{R_i}$ transform according to $(H_-, d_{R_1}, e_{R_i}) \rightarrow e^{i\eta'} (H_-, d_{R_1}, e_{R_i})$. In this case the Yukawa couplings of the four Higgs doublet model would be given by

$$
\lambda_{\nu_{ij}} \overline{\nu}_R \lambda H_+ L_j + \lambda_{u_{ij}} \overline{u}_R q_1 H_+ q_i + h_{u_{ij}} \overline{q}_R \Delta_u H_u Q_j \\
+ \lambda_{e_{ij}} \overline{e}_R \lambda H_- L_j + \lambda_{d_{ij}} \overline{d}_R q_1 H_- q_i + h_{d_{ij}} \overline{q}_R \Delta_d H_d Q_j \quad (2.8).
$$

We see that this model requires that $H_-$ develops a large VEV in order to give a mass to the charged leptons and to the down quark. In order for $U(1)_H$ to prevent $H_+$ from coupling to $H_-$, it is necessary that the charge of $H_-$ under $U(1)_H$ should not equal the negative of the charge of $H_+$ ($\eta' \neq -\eta$).

In order to discuss these models further, we must consider the observational constraints on the product $\lambda_{\nu} \lambda_q$ appearing in the expression for the neutrino masses (2.6). These are imposed by primordial nucleosynthesis \[ [14, 15, 16] \] and by limits on neutrino helicity-changing processes from the supernova SN 1987A \[ [11, 12, 13] \].

Since the neutrino masses and the observational constraints in the case of the supersymmetrizable four Higgs doublet model will be essentially the same as those in the case of the "minimal" two Higgs doublet model, we will focus our attention on the two Higgs doublet model in the following.
3 . Nucleosynthesis Constraints on the Neutrino Masses

The success of the primordial nucleosynthesis calculation of the abundances of light elements (D, $^3$He, $^4$He and $^7$Li) in the Standard Model [14, 15] imposes strong constraints on the addition of new light particles of mass less than O(1)MeV, such as light right-handed neutrinos. There has recently been some controversy over exactly what the nucleosynthesis upper bound on the number of additional light degrees of freedom is [16]. We may discuss this in terms of the effective number of massless left-handed neutrinos at nucleosynthesis, $N_\nu$. The controversy is related to what observational constraints on light element abundances should be imposed. Previously an indirect bound on the primordial D+$^3$He abundance was used, inferred by using chemical evolution models combined with measurements of the abundance in the solar neighbourhood. This gave an upper bound $N_\nu \lesssim 3.3$ [14]. However, recent evidence from planetary nebulae implies a need for $^3$He production in low mass stars, suggesting that the primordial D+$^3$He density inferred from chemical evolution models is incorrect [16]. Using instead the more reliable estimate of the primordial $^7$Li density, Olive et al give a 95%c.l. upper limit $N_\nu < 3.9$ with a central value for $N_\nu$ equal to 3.02 [16]. Kernan and Sarkar conclude that the upper bound can be as large as 4.53, taking observational uncertainties into account [17]. The constraints on the neutrino masses following from these upper bounds on $N_\nu$ will depend on how many neutrinos are effectively massless at nucleosynthesis. Present experimental constraints give $m_{\nu_e} < 5.1$eV, $m_{\nu_\mu} < 160$keV and $m_{\nu_\tau} < 24$MeV [18]. Thus it is possible that $m_{\nu_\tau}$ could be heavier than 1MeV and so not affect nucleosynthesis. However, bounds from the supernova SN 1987A combined with constraints from nucleosynthesis imply that $m_{\nu_\tau}$ must be less than 0.4MeV if the dominant $\nu_\tau$ decay is to electromagnetic final states [19], as would be the case in the class of model we are discussing here. Thus from the point of view of nucleosynthesis we will consider all three neutrino species be effectively massless.

The condition $\Delta N_\nu < 1.53$ (where we define $\Delta N_\nu$ by $N_\nu = 3 + \Delta N_\nu$) requires that the right-handed neutrinos freeze out of chemical equilibrium prior to the quark-
hadron phase transition. This is because each right-handed neutrino species contributes the equivalent of $n_{\text{eff}}$ left-handed neutrino species to the energy density at nucleosynthesis, where $n_{\text{eff}}$ is related to the freeze-out temperature $T_{\text{fr}}$ of the right-handed neutrinos by

$$n_{\text{eff}} = \left( \frac{g(T_{\text{nucl}})}{g(T_{\text{fr}})} \right)^{4/3},$$

where $g(T) = g_b + \frac{7}{8}g_f$ is the number of effectively massless degrees of freedom in thermal equilibrium at temperature $T$, with $g_b = 2$ for the photon and $g_f = 4$ for Dirac fermions [15]. The reduction of $n_{\text{eff}}$ from 1 is due to the adiabatic expansion of the Universe when the number of effectively massless degrees of freedom changes, for example during a confining phase transition or when a particle species becomes non-relativistic and annihilates away; this will dilute a particle species which is out of chemical equilibrium relative to those in equilibrium, which maintain their equilibrium densities. At temperatures above the quark-hadron phase transition, one has free quarks and gluons in thermal equilibrium, and $g(T) = 61.75$ at $T \approx T_{\text{qh}}$. Below the temperature of the quark-hadron phase transition, one has quarks and gluons confined in hadrons, with $g(T_{\text{qh}}) = 17.25$ and $g(T_{\text{nucl}}) = 10.75$. From this we see that for $T_{\text{fr}}$ slightly below $T_{\text{qh}}$ we have $n_{\text{eff}} = 0.53$, whilst for $T_{\text{fr}}$ slightly above $T_{\text{qh}}$ this becomes $n_{\text{eff}} = 0.097$. Thus for the case where we have 3 effectively massless right-handed neutrinos we find that for $T_{\text{fr}}$ slightly above $T_{\text{qh}}$ we have $\Delta N_{\nu} = 0.29$, whilst for $T_{\text{fr}}$ slightly below $T_{\text{qh}}$ we have $\Delta N_{\nu} = 1.60$. Thus we see that $T_{\text{fr}} < T_{\text{qh}}$ is ruled out. In Table 1 we list the values of $\Delta N_{\nu}$ as a function of the known Standard Model particle thresholds. (The inclusion of the Higgs boson thresholds due to the physical Higgs of the Standard Model and the additional doublet $H_+$ in the two Higgs doublet model will only reduce the smallest possible value of $\Delta N_{\nu}$ from 0.14 to 0.13).

Imposing the condition $T_{\text{fr}} > T_{\text{qh}}$ allows us to put an upper bound on the couplings entering in the cross-sections for processes changing the number of right-handed neutrinos via $\phi_+^0$ exchange: (i) $\bar{\nu}_R\nu_R \leftrightarrow \bar{\nu}_L\nu_L$ (Figure 1) and (ii) $\bar{\nu}_R\nu_L \leftrightarrow \bar{q}_R q_L$ and related inelastic scattering processes (Figure 2). In addition there are...
analogous processes formed by replacing $\phi_0^+$ by $\phi_+^+$. Adding these simply multiplies the rate of $\nu_1$ annihilation due to $\phi_0^+$ by a factor of 2. From the diagram of Figure 1 we obtain for the $\phi_0^+$ contribution to the $\nu_1$ annihilation cross-section

$$\sigma(\nu_R \nu_R \rightarrow \nu_L \nu_L) = \frac{E^2}{24\pi m_+^4} \sum_j (\lambda_{\nu_1} \lambda_{\nu_1})^2$$

where $E$ is the energy of each annihilating neutrino in the centre of mass frame. In this we have assumed that $E$ is small compared with $m_+^2$. If $E$ were to approach or exceed $m_+^2$ then the effect of the s-channel pole terms or of the direct production of $H_+$ Higgs scalars by neutrino annihilations would simply be to tighten the upper bounds we derive below. Including the $\phi_+^+$ exchange contribution, the annihilation rate in the early Universe is then given by

$$\Gamma_{\text{ann}} = 2 < n \sigma v > \approx \frac{9T^5}{8\pi^2 m_+^4} \sum_j (\lambda_{\nu_1} \lambda_{\nu_1})^2$$

where $n$ is the number density of scattering particles in the thermal background ($n = \frac{12g'}{\pi^2} T^3$, with $g' = g_B + \frac{3}{4} g_F$ being the number of light degrees of freedom in thermal equilibrium), $v$ is the relative velocity of the scattering particles ($v = 1$) and we have used for the average energy of the annihilating fermions $E \approx 3T$ [15]. Requiring that this is less than the expansion rate $H$ of the Universe at the temperature of the quark-hadron phase transition $T_{\text{qh}}$ ($H = \frac{kT_{\text{qh}}^2}{M_{\text{Pl}}}$, where $k_T = \left(\frac{4\pi^2 g(T_{\text{qh}})}{45}\right)^{1/2} \approx 13$) then gives the upper bound

$$\left(\sum_j (\lambda_{\nu_1} \lambda_{\nu_1})^2\right)^{1/4} \lesssim 0.024 \left(\frac{m_+}{100\text{GeV}}\right) \left(\frac{0.2\text{GeV}}{T_{\text{qh}}}\right)^{3/4} \left(\frac{g(T_{\text{fr}})}{g(T_{\text{qh}})}\right)^{1/8} \left(\frac{T_{\text{qh}}}{T_{\text{fr}}}\right)^{3/4}$$

where we have used $T_{\text{qh}} = 200\text{MeV}$ as a typical value [20] and we have shown explicitly the dependence on $T_{\text{fr}}$. (Note that, for the case $i = j$, this gives an upper bound on $\lambda_{\nu_1}$ itself). We can also obtain an upper bound on the product $\lambda_{\nu_1} \tilde{\lambda}_u$ which enters in the expression for the neutrino mass. Adding the rates for the processes shown in Figure 2, (i) $\nu_R \nu_R \rightarrow \nu_L \nu_R$, (ii) $\nu_R \nu_R \rightarrow \nu_L \nu_L$ and (iii) $\nu_R \tilde{\nu}_L \rightarrow \nu_L \tilde{\nu}_R$, and including the factor of 2 for the analogous $\phi_+^+$ exchange processes, we obtain

$$\Gamma \approx \frac{135T^5}{8\pi^3 m_+^4} \lambda_{\nu_1}^j \lambda_{\nu_1} \lambda_{\nu_1} \lambda_{\nu_1} \lambda_{\nu_1} \tilde{\lambda}_u \tilde{\lambda}_u$$

(3.5).
where the trace is over quark flavours. Requiring that this be less than the expansion rate of the Universe at the quark-hadron phase transition then gives the upper bound

\[(\lambda_{\nu i}^\dagger \lambda_{\nu i})^{1/2}(\text{Tr}[\lambda_u^\dagger \lambda_u])^{1/2} \lesssim 1.6 \times 10^{-4} \left(\frac{m_\nu}{100\text{GeV}}\right)^2 \left(\frac{0.2\text{GeV}}{T_{\text{qh}}}ight)^{3/2} \left(\frac{g(T_{\text{fr}})}{g(T_{\text{qh}})}\right)^{1/4} \left(\frac{T_{\text{qh}}}{T_{\text{fr}}^3}\right)^{3/2}\]  \tag{3.6}

Note that, when combined with the upper bound on \(\lambda_{\nu i}\) from (3.4), this gives an upper bound on \(\text{Tr}[\lambda_u^\dagger \lambda_u]\) itself. Using (3.6), and noting that \(\lambda_u \leq \text{Tr}[\lambda_u^\dagger \lambda_u]^{1/2}\), we see that the upper limit on the neutrino masses is then given by

\[m_\nu \lesssim 0.11\text{eV} \left(\frac{0.2\text{GeV}}{T_{\text{qh}}^3}\right)^{3/2} \left(\frac{g(T_{\text{fr}})}{g(T_{\text{qh}})}\right)^{1/4} \left(\frac{T_{\text{qh}}}{T_{\text{fr}}}\right)^{3/2}\]  \tag{3.7}

Thus we see that the present nucleosynthesis constraint \(\Delta N_\nu < 0.9\) implies that the neutrino masses must be less than about 0.1eV in this class of model. The nucleosynthesis upper bound on the neutrino masses could become much more stringent in the future if the upper bound on \(N_\nu\) were to approach the Standard Model value \(N_\nu = 3\). From Table 1 we see that once the limit on \(\Delta N_\nu\) is less than 0.29, the freeze-out temperature must be larger than \(\frac{m_\nu}{3}\) (corresponding to \(E_\nu \approx 3T > m_\nu\)), leading to an upper bound on the neutrino masses of 2.6x10^{-2}eV, whilst if the limit on \(\Delta N_\nu\) were to become less than 0.20, the freeze-out temperature would have to be larger than \(\frac{m_\nu}{3}\), giving an upper bound on the neutrino masses of about 6.3x10^{-5}eV. If \(\Delta N_\nu\) was constrained to be below 0.14 (or 0.13, including Higgs thresholds in the two Higgs doublet model), then it would not be possible for three massless right-handed neutrinos to be consistent with nucleosynthesis when only the thresholds of the Standard Model or its two Higgs doublet extension are considered. The four Higgs doublet supersymmetric extension could, however, allow for a significantly smaller \(\Delta N_\nu\) if the freeze-out temperature was larger than the masses of the supersymmetric partners of the Standard Model particles.

We next compare the upper bound on the neutrino masses coming from nucleosynthesis with that coming from neutrino helicity-flip processes in the supernova SN 1987A [11, 12, 13]. Thomas and Xu give an upper bound on the neutrino masses of 0.05eV [11], based on an upper bound on the helicity flipping cross-section from the requirement that the energy of the supernova is not rapidly carried away by
right-handed neutrino emission, which would unacceptably shorten the observed neutrino pulse. However, it is difficult to give anything better than an order of magnitude estimate for the neutrino mass upper bound from the supernova [12]. The physics of the interaction of neutrinos with nucleons in the dense core of the supernova is a non-trivial problem in nuclear physics, which introduces an order of magnitude uncertainty in the upper bound on $(\lambda_{\nu} \tilde{\lambda}_q)^2$ [12]. The bound derived by Thomas and Xu is based on evaluating $< n |\bar{\nu} u | n >$ for a single isolated nucleon [11], whereas in the dense core of the supernova collective effects might increase this scattering rate by perhaps an order of magnitude [12], in which case the upper bound on the neutrino masses from the supernova becomes 0.16eV. Another order of magnitude uncertainty in the right-handed neutrino emission rate is introduced by the model of the supernova itself [12]. Thus we see that it is difficult to be very clear about exactly what the upper limit from the supernova is, beyond giving an order of magnitude estimate which is typically somewhere between 0.1eV and 0.01eV. In contrast with the supernova bound, the nucleosynthesis upper bound on the neutrino masses is much simpler to derive and is quite precise. Since the phenomenology of neutrinos is often sensitive to the neutrino mass squared, such precision in the neutrino mass bounds is important in order to be able to unambiguously discuss their phenomenological implications. The nucleosynthesis upper bound also has the possibility of becoming several orders of magnitude tighter in the future as our understanding of big-bang nucleosynthesis develops, whereas the supernova upper bound is unlikely to be improved by much more than an order of magnitude [12].
4. Aspects of the phenomenology of massive neutrinos with masses induced by chiral symmetry breaking.

We next compare the allowed range of neutrino masses from chiral symmetry breaking with the range of neutrino masses required in order to explain certain important observations in cosmology and astrophysics, namely a) the possibility of a hot component of dark matter due to a massive neutrino [27] b) the solar neutrino problem [7, 8] and c) the atmospheric neutrino deficit [28].

a) Hot dark matter: We can immediately see that there is no possibility of a significant contribution to cosmological dark matter from neutrinos in this model. This would require a neutrino mass in the 1eV to 10eV range [15, 27], whereas the present nucleosynthesis constraint gives an upper bound of about 0.11eV. This would allow a fractional contribution to the closure dark matter density of no more than \( \Omega_\nu \approx 0.053 \left( \frac{m_\nu}{5\text{eV}} \right) h^{-2} \lesssim 0.005 \), where \( h \) is the Hubble parameter in units of 100 km s\(^{-1}\) Mpc\(^{-1}\) (0.5 \( \lesssim h \lesssim 1 \)) [15]. Thus this mechanism for neutrino masses is not compatible with recent ideas which use a hot neutrino component of dark matter (with \( \Omega_\nu \approx 0.2 \)) to account for the discrepancies between observed large-scale structure and the large-scale structure predicted by \( \Omega = 1 \) cold dark matter models with a scale-invariant primordial fluctuation spectrum [27].

b) MSW solution of the solar neutrino problem: The Mikheyev-Smirnov-Wolfenstein (MSW) matter oscillation solution to the solar neutrino problem [7, 8] has three main regions of mass squared splitting between the neutrino masses; a solution corresponding to \( \Delta m_\nu^2 \approx 10^{-4}\text{eV}^2 \) for neutrino mixing angles \( \theta \) corresponding to \( \sin^2 2\theta \) increasing from \( 10^{-4} \) to \( 10^{-1} \) (adiabatic solution), a second solution corresponding to \( \Delta m_\nu^2 \) decreasing from \( 10^{-4}\text{eV}^2 \) to \( 10^{-7}\text{eV}^2 \) as \( \sin^2 2\theta \) increases from \( 10^{-4} \) to \( 10^{-1} \) (non-adiabatic solution) and a third region corresponding to \( \sin^2 2\theta \) of order 1 for mass squared splittings from about \( 10^{-7}\text{eV}^2 \) up to about \( 10^{-4}\text{eV}^2 \) (large-mixing solution). Thus the MSW solution is generally consistent with the present nucleosynthesis upper bound on the neutrino masses. However, from Table 1 we see that if the nucleosynthesis constraint on \( \Delta N_\nu \) should become established at \( \Delta N_\nu < 0.24 \),
then the upper limit on the neutrino masses would become $m_\nu < 4.1 \times 10^{-3}\text{eV}$, which would rule out the adiabatic solution, whilst if the upper bound was tightened to $\Delta N_\nu < 0.20$, then the upper bound would become $m_\nu < 6.3 \times 10^{-5}\text{eV}$ and the MSW solution to the solar neutrino problem would be ruled out completely.

c) Atmospheric neutrino deficit: The atmospheric neutrino deficit problem [28] refers to the observation that the electron and muon neutrinos arising from high energy cosmic rays incident on the upper atmosphere should be observed with the ratio $\frac{(\nu_\mu + \bar{\nu}_\mu)}{(\nu_e + \bar{\nu}_e)} \approx 2$. However, experimentally this ratio is observed to be approximately equal to 1, which may be interpreted as evidence of vacuum oscillations of muon neutrinos to $\nu_e$ or $\nu_\tau$ with a large vacuum mixing angle ($\sin^2 2\theta \gtrsim 0.4$) and $\Delta m^2_\nu \approx 10^{-3}\text{eV}^2$ to $10^{-1}\text{eV}^2$ [28]. From this we see that the range of mass squared splitting required to solve the atmospheric neutrino problem is consistent with the present nucleosynthesis constraint ($\Delta m^2_\nu \lesssim 10^{-2}\text{eV}^2$). However, should the nucleosynthesis constraint be improved to $\Delta N_\nu < 0.24$, then the upper bound on the neutrino masses would become $m_\nu < 4.1 \times 10^{-3}\text{eV}$, which would rule out a vacuum oscillation solution to the atmospheric neutrino deficit problem.
5. Conclusions.

We have considered the possibility that small Dirac neutrino masses in the Standard Model could arise as a result of a small expectation value induced in a second Higgs field by chiral symmetry breaking. Present big bang nucleosynthesis constraints imply that the neutrino masses are less than about 0.11eV. This upper bound may become much more stringent in the future, as our understanding of the primordial light element abundances improves and the constraint on the number of additional light degrees of freedom compatible with big bang nucleosynthesis becomes tighter. The constraints on the neutrino masses from the rate of helicity flip in the supernova SN 1987A are not significantly stronger than the present nucleosynthesis constraint and, unlike the nucleosynthesis constraint, are difficult to state precisely and not likely to become much tighter in the future.

In general big bang nucleosynthesis rules out neutrinos with masses due to chiral symmetry breaking as a significant contributor to cosmological dark matter, but does not at present significantly constrain the possibility of an MSW solution of the solar neutrino problem or a vacuum oscillation solution of the atmospheric neutrino deficit. However, should the nucleosynthesis constraint on the number of additional neutrinos become established in the future at $\Delta N_{\nu} < 0.24$, then the vacuum oscillation solution of the atmospheric neutrino deficit and the adiabatic MSW solution of the solar neutrino problem would be ruled out, whilst if the bound were to become established at $\Delta N_{\nu} < 0.20$ the MSW solution of the solar neutrino problem would be ruled out completely. Thus a substantial improvement in the present bound on $\Delta N_{\nu}$ could impose severe constraints on neutrino phenomenology in this class of neutrino mass model.

The author would like to thank Emilio Ribeiro and Pedro Bicudo for useful discussions regarding chiral symmetry breaking and Subir Sarkar for some informative comments. This research was funded by the Grupo Teorico da Altas Energias (GTAE), Portugal and by the PPARC, UK.
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Figure Captions.

Figure 1: Annihilation of $\nu_R \bar{\nu}_R$ pairs.

Figure 2: Inelastic scattering processes involving thermal background quarks which change the number of right-handed neutrinos.
Table 1. $\Delta N_\nu$ due to three right-handed neutrinos and the upper bound on $m_\nu$ as a function of $T_{fr}$.

| $\Delta N_\nu$ | $m_\nu (eV) <$ | $T_{fr} <$ |
|----------------|----------------|-----------|
| 3.00           | Ruled Out      | $m_\mu/3$ |
| 2.06           | Ruled Out      | $m_\pi/3$ |
| 1.60           | Ruled Out      | $T_{qh}$  |
| 0.29           | 0.11           | $m_c/3$   |
| 0.24           | $2.6 \times 10^{-2}$ | $m_b/3$ |
| 0.20           | $4.1 \times 10^{-3}$ | $m_W/3$ |
| 0.17           | $6.3 \times 10^{-5}$ | $m_Z/3$ |
| 0.16           | $5.2 \times 10^{-5}$ | $m_t/3$ |
| 0.14           | $2.0 \times 10^{-5}$ | ?         |