Singletons, Doubletons and M-theory

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Abstract

We identify the two dimensional AdS subsupergroup $OSp(16/2, R)$ of the M-theory supergroup $OSp(1/32, R)$ which captures the dynamics of $n$ D0-branes in the large $n$ limit of Matrix theory. The $Sp(2, R)$ factor in the even subgroup $SO(16) \times Sp(2, R)$ of $OSp(16/2, R)$ corresponds to the AdS extension of the Poincare symmetry of the longitudinal directions. The infinite number of D0-branes with ever increasing and quantized values of longitudinal momenta are identified with the Fourier modes of the singleton supermultiplets of $OSp(16/2, R)$, which consist of 128 bosons and 128 fermions. The large $n$ limit of $N = 16 U(n)$ Yang-Mills quantum mechanics which describes Matrix theory is a conformally invariant $N = 16$ singleton quantum mechanics living on the boundary of $AdS_2$. We also review some of the earlier results on the spectra of Kaluza-Klein supergravity theories in relation to the recent conjecture of Maldacena relating the dynamics of $n$ $Dp$-branes to certain AdS supergravity theories. We point out the remarkable parallel between the conjecture of Maldacena and the construction of the spectra of $11 - d$ and type $IIB$ supergravity theories compactified over various spheres in terms of singleton or doubleton supermultiplets of corresponding AdS supergroups.

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1 Introduction

In the late twenties and early thirties the works of Dirac, Heisenberg, Jordan, Pauli and other physicists lay the foundations of relativistic quantum field theory. Feynman, Schwinger, Tomonaga and Dyson developed the successful theory of quantum electrodynamics in the late forties. In 1954 Yang and Mills formulated non-Abelian gauge theories and in the early seventies ’t Hooft proved the renormalizability of spontaneously broken non-Abelian gauge theories. The successful standard model of elementary particles is a local relativistic spontaneously broken non-Abelian gauge theory in which local fields transform covariantly under the Lorentz group.

About a decade after the foundations of relativistic quantum field theories were laid Wigner constructed and classified all unitary irreducible representations (UIR) of the Poincare group \[1\]. In Wigner’s point of view, particles are represented by states in a positive definite Hilbert space which carries UIRs of the Poincare group. The transition from Wigner’s representation of the Poincare group to the relativistic fields transforming in the finite dimensional non-unitary representations of the Lorentz group is achieved by Fourier expanding the covariant fields in terms of particle annihilation and creation operators that operate in a positive definite Hilbert space \[2\]. The particle states carry the unitary representations of the Poincare group.

Similarly, in a spacetime with constant curvature (de Sitter and anti-de Sitter (AdS) space-times) \[3\] the particle states form the basis of a unitary representation of de Sitter and AdS groups. It is well known that de Sitter groups \(SO(d,1)\) for \(d > 2\) in \(d\)-dimensional space do not admit unitary irreducible representation of the lowest (or highest) weight type. On the other hand AdS groups \(SO(d - 1,2)\) do admit UIR of the highest (or lowest weight) type (see \[4\] and references therein). The operator whose spectrum is bounded from below in a lowest weight representation is the energy operator. In general, local spacetime supersymmetry in a space of constant curvature (for \(d > 2\)) requires that the spacetime be of AdS type \[5\]. In a relativistic quantum field theory in an AdS background the fields transforming covariantly under the Lorentz group can be expanded in terms of particle creation and annihilation operators. The states created by the particle creation operators carry unitary representation of the AdS group \(SO(d,2)\).

Simple AdS supergroups exist in \(d\) dimensional space-times with \(d \leq 7\) \[6\]. In \(d = 3\) the AdS group is not simple and decomposes as \(SO(2,2) \approx SO(2,1)_+ \times SO(2,1)_-\). As a consequence the AdS group in three dimensions
admits \((p, q)\) type supersymmetric extensions which correspond to \(p\) and \(q\) supersymmetric extensions of \(SO(2, 1)_+\) and \(SO(2, 1)_-\), respectively. A complete list of AdS supergroups in \(d = 3\) was given in \([7]\).

Below we list the AdS supergroups in \(d \leq 7\) and their even subgroups:

\[
\begin{align*}
d = 7 & \quad OSp(8^*/2N) \supset SO^*(8) \times USp(2N) \\
d = 6 & \quad F(4) \supset SO(5, 2) \times SU(2) \\
d = 5 & \quad SU(2, 2/N) \supset SU(2, 2) \times U(N) \\
d = 4 & \quad OSp(N/4, R) \supset SO(N) \times Sp(4, R) \\
d = 3 & \quad G_+ \times G_-, \\
d = 2 & \quad G_+, \\
\end{align*}
\]

where \(G_+\) and \(G_-\) can be any one of the following supergroups:

\[
\begin{align*}
i) & \quad OSp(N/2, R) \supset O(N) \times Sp(2, R) \\
ii) & \quad SU(N/1, 1) \supset U(N) \times SU(1, 1), N \neq 2 \\
& \quad SU(2/1, 1) \supset SU(2) \times SU(1, 1) \\
iii) & \quad OSp(4^*/2N) \supset O^*(4) \times USp(2N) = SU(2) \times USp(2N) \times SU(1, 1) \\
iv) & \quad G(3) \supset G_2 \times SU(1, 1), \text{ with } G_2 \text{ compact} \\
v) & \quad F(4) \supset Spin(7) \times SU(1, 1) \text{ with } Spin(7) \text{ compact} \\
vi) & \quad D^1(2, 1, \alpha) \supset SU(2) \times SU(2) \times SU(1, 1) \\
\end{align*}
\]

Some of the above AdS supergroups arise as the symmetry supergroups of ground states of gauged supergravity theories in the respective dimensions as well as the Kaluza-Klein compactifications on manifolds with non-trivial isometry groups. For example, the ground state of the \(S^7\) (or \(S^4\)) compactification of the eleven-dimensional supergravity is \(AdS_3 \times S^7\) (or \(AdS_7 \times S^4\)). Their respective symmetries are \(OSp(8/4, R)\) and \(OSp(8^*/4)\). Similarly II B supergravity in ten dimensions admits compactification down to five dimensions on \(S^5\). The symmetry of the corresponding ground state is \(SU(2, 2/4)\).

\section{Singleton and Doubleton Supermultiplets of AdS Supergroups in Supergravity and Superstring Theories}

In \([4]\) a general oscillator method was developed for constructing the unitary irreducible representations (UIR) of the lowest (or highest) weight type of non-compact groups. This method was generalized in \([12]\) to the construction of UIRs of non-compact supergroups. This simple oscillator method is quite powerful and yields readily the UIRs of lowest weight type of all non-compact groups and supergroups. Among these representations there
exist some fundamental ones, in terms of which all the other ones can be con-
structed by simple tensoring procedure. These representations can be looked
upon as the "quarks" of various non-compact groups and supergroups. In
the case of $d = 4$ AdS group $SO(3,2)$ these fundamental representations
were discovered by Dirac long time ago [13], which he referred to as the re-
markable representations of AdS group. These representations of Dirac were
later called singletons [14] (in the oscillator language these representations
require a single set of oscillators transforming in the fundamental represen-
tation of the maximal compact subgroup $SU(2) \times U(1)$ of the covering group
$Sp(4,R)$ of $SO(3,2)$ [9, 15, 16]. The corresponding fundamental representa-
tions of $d = 7$ and $d = 5$ AdS groups $SO^*(8)$ and $SU(2,2)$, respectively,
were named doubletons in [10, 11] since they require two sets of oscillators
transforming in the fundamental representations of corresponding maximal
compact subgroups.

The oscillator method for non-compact groups $G$ that admit lowest
weight representations works as follows: $G$ has a maximal compact sub-
group $G^0$ which is of the form $G^0 = H \times U(1)$ with respect to whose Lie
algebra $g^0$ we have a three grading of the Lie algebra $g$ of $G$,
\[ g = g^{-1} \oplus g^0 \oplus g^1 \] (2 - 1)
which simply means that the commutator of elements of grade $k$ and $l$
satisfies
\[ [g^k, g^l] \subseteq g^{k+l} \] (2 - 2)

For example, for $SU(1,1)$ this corresponds to the standard decomposi-
tion
\[ g = L_+ \oplus L_0 \oplus L_- \] (2 - 3)
where
\[
\begin{align*}
[L_0, L_\pm] &= \pm L_\pm \\
[L_+, L_-] &= 2L_0
\end{align*}
\] (2 - 4)

The three grading is determined by the generator $E$ of the $U(1)$ factor
of the maximal compact subgroup
\[
\begin{align*}
[E, g^+] &= g^+ \\
[E, g^-] &= -g^- \\
[E, g^0] &= 0
\end{align*}
\] (2 - 5)
In most physical applications $E$ turns out to be the energy operator. In such cases the unitary lowest weight representations correspond to positive energy representations.

The essence of the oscillator method is to realize the generators of $G$ as bilinears of bosonic annihilation and creation operators transforming typically in the fundamental (and its conjugate) representation of $H$. In the Fock space $\mathcal{H}$ of all the oscillators one chooses a set of states $|\Omega\rangle$ which transform irreducibly under $H \times U(1)$ and are annihilated by all the generators in $g^{-1}$. Then by acting on $|\Omega\rangle$ with generators in $g^+1$ one obtains an infinite set of states

$$|\Omega\rangle, \quad g^1|\Omega\rangle, \quad g^1g^1|\Omega\rangle, \ldots$$

which form the UIR of the lowest weight type (positive energy) of $G$. The infinite set of states thus obtained corresponds to the decomposition of the UIR of $G$ with respect to its maximal compact subgroup.

As we have already mentioned, whenever we can realize the generators of $G$ in terms of a single set of oscillators transforming in an irreducible representation of $H$ the corresponding UIRs of the lowest weight type will be called singleton representations and there exist two such representations for a given group $G$. For AdS groups the singleton representations correspond to scalar and spinor fields. In certain cases we need a minimum of two sets of oscillators transforming irreducibly under $H$ to realize the generators of $G$. In such cases the corresponding UIRs are called doubleton representations and there exist infinitely many doubleton representations of $G$ corresponding to fields of different "spins". For example, the non-compact group $Sp(2N, R)$ admits singleton representations. On the other hand, non-compact groups $SO^*(2N)$ and $SU(N, M)$ admit doubleton representations.

The oscillator method can similarly be used to construct unitary lowest type UIR’s of non-compact supergroups [12, 17]. They also admit singleton or doubleton supermultiplets corresponding to some fundamental lowest weight unitary irreducible representations. For example, the non-compact supergroup $OSp(2N/2M, R)$ with the even subgroup $SO(2N) \times Sp(2M, R)$ admits singleton supermultiplets, while $OSp(2N^*/2M)$ and $SU(N, M/P)$ with even subgroups $SO^*(2N) \times USp(2M)$ and $SU(N, M) \times SU(P) \times U(1)$ admit doubleton supermultiplets.

As is well known the AdS group $SO(d - 1, 2)$ admits an In"on"u-Wigner type contraction to the Poincare group in $d$-dimensional Minkowskian spacetime limit. What is quite surprising is the fact that the singleton representations of Dirac in $d = 4$ are singular when the Poincare limit is taken! This
is due to the remarkable property noticed by Dirac \[13\], that the wave functions corresponding to the singleton representations do not depend on the radius of the AdS space-time. Singletons can be thus properly understood as fields living on the boundary of the corresponding AdS space-time. Yet the boundary singleton field theory controls the physics of the "bulk" AdS space-time! The AdS group $SO(3,2)$ should be interpreted as the conformal group of the singleton theory living on the boundary of the AdS space-time.

That the "remarkable" representations of Dirac \[13\] do not have a Poincare limit has been known for a long time \[14\]. One way to see that singleton irreducible representations can not have a Poincare limit is to look at the singleton supermultiplets. For example, the singleton supermultiplet of $OSp(8/4,R)$ has 8 scalar and 8 spin $\frac{1}{2}$ fields \[9, 16, 15\]. On the other hand, it is well known that the shortest $N = 8$ Poincare supermultiplets have spin range two. Similarly one can show that singleton or doubleton supermultiplets of extended AdS supergroups in various dimensions can not have a Poincare limit.

The unitary supermultiplets of AdS supergroups in $d = 3$ were constructed in \[7\] and in \[18\] it was shown that the light-cone Green-Schwarz II A, II B and heterotic superstring theories can be interpreted as the $(8c,8s), (8c,8c)$ and $(8c,0)$ superconformally invariant singleton field theories of the $AdS_3$ supergroups $OSp(8/2,R)_c \times OSp(8/2,R)_s$, $OSp(8/2,R)_c \times OSp(8/2,R)_c$, and $OSp(8/2,R)_c \times Sp(2,R)$, respectively.

In \[9\] the unitary supermultiplets of $N = 8$ AdS supergroup $OSp(N/4,R)$ in $d = 4$ were constructed and the spectrum of the $S^7$ compactification of eleven dimensional supergravity was fitted into its unitary supermultiplets. The singleton supermultiplet sits at the bottom of an infinite tower of Kaluza-Klein modes and appears as gauge degrees of freedom that decouple from the spectrum \[9\]. However, in a related work it was shown that the infinite tower of Kaluza-Klein states of any given spin fall into irreducible representations of a spectrum generating non-compact group $SO(8,1)$ \[14\]. This infinite tower of Kaluza-Klein states contains all the physical modes as well as gauge modes including the singleton modes. Thus singleton modes are essential for fitting the Kaluza-Klein states into unitary representations of a spectrum generating group. Furthermore, even though the singleton supermultiplet decouples from the spectrum as gauge modes, by tensoring the singleton supermultiplets repeatedly and restricting oneself to CPT self-conjugate supermultiplets in the tensor products one generates the entire tower of Kaluza-Klein supermultiplets of 11 dimensional supergravity over the seven sphere!
The compactification of 11-d supergravity over the four sphere $S^4$ down to seven dimensions was studied in \cite{10, 21}. The ground state of this compactification is $AdS_7 \times S^4$ and has the symmetry $OSp(8^*/4)$ with the even subgroup $SO(6,2) \times USp(4)$. The infinite Kaluza Klein modes fall into unitary supermultiplets of $OSp(8^*/4)$ \cite{10}. Again the doubleton supermultiplet appears as gauge modes and decouples from the spectrum. It consists of five scalars, four fermions and a self-dual two form field. The infinite set of Kaluza-Klein modes of a given spin together with some gauge modes form the basis of a unitary representation of a spectrum generating group $SO(5,1)$ \cite{13, 20}. The doubleton supermultiplet sits at the bottom of the infinite tower of $SO(5,1)$ modes corresponding to scalar, spinor and the self-dual antisymmetric tensor fields.

The unitary supermultiplets of $2N$ extended $AdS_5$ supergroups $SU(2,2/N)$ were studied in \cite{11, 12}. The spectrum of the $S^5$ compactification of ten dimensional IIB supergravity was calculated in \cite{11, 22}. The entire spectrum falls into an infinite tower of unitary supermultiplets of $N = 8$ $AdS_5$ superalgebra $SU(2,2/4)$ \cite{11}. The CPT self-conjugate doubleton supermultiplet of $N = 8$ $AdS$ superalgebra again decouples from the physical spectrum and corresponds to some gauge modes. In \cite{11} it was pointed out that $N = 8$ doubleton field theory is the conformally invariant $N = 4$ super Yang-Mills theory in $d = 4$. Again, even though the doubleton supermultiplet decouples from the Kaluza-Klein spectrum of ten dimensional IIB supergravity on $S^5$, by tensoring the doubleton supermultiplet with itself repeatedly and restricting oneself to the CPT self-conjugate supermultiplets one generates the entire spectrum of Kaluza-Klein states of ten dimensional IIB theory on $S^5$. As in the case of $S^7$ and $S^4$ compactifications of 11-d supergravity one can show that the Kaluza Klein modes of a given spin together with some gauge modes form unitary representations of a spectrum generating $SO(6,1)$ in the $S^5$ compactification of ten dimensional IIB supergravity \cite{13, 20}.

Even though the Poincare limit of the singleton representations of the $d = 4$ $AdS$ group $SO(3,2)$ is singular the tensor product of two singleton irreps of $SO(3,2)$ decomposes into an infinite set of massless irreps which do have a smooth Poincare limit \cite{14, 15, 1}. Similarly, the tensor product of two singleton supermultiplets of $N$ extended $AdS$ supergroup $OSp(N/4,R)$ decomposes into an infinite set of massless supermultiplets which do have a Poincare limit \cite{1, 13}. The simple $AdS$ groups in higher dimensions than four that do admit supersymmetric extensions have doubleton representations only. The doubleton supermultiplets of extended $AdS$ supergroups in $d = 5$ ($SU(2,2/N)$) and $d = 7$ ($OSp(8^*/2N)$) share the remarkable fea-
tures of the singleton supermultiplets of $d = 4$ AdS supergroups in that the tensor product of any two doubleton supermultiplets decompose into an infinite set of massless supermultiplets \[10, 11\]. Based on these facts and some other arguments one of the authors made the following proposal for defining massless representations \[16\] in AdS spacetimes:

A representation (or a supermultiplet) of an AdS group (or supergroup) is massless if it occurs in the decomposition of the tensor product of two singleton or two doubleton representations (or supermultiplets).

This should be taken as a working definition and agrees with some other definitions of masslessness in $d \leq 7$. Tensoring more than two singleton or doubleton representations leads to massive representations of AdS groups and supergroups \[3, 11, 11, 16\].

3 Singletons, Doubletons and p-branes

Gibbons and Townsend \[23\] found solutions of $d = 10$ and $d = 11$ supergravity equations of motion that interpolate between different supersymmetric vacua connected via a wormhole throat. In particular, in $d = 11$ the relevant p-branes are membrane and five-brane and in $d = 10$ II B self-dual three-brane configuration. These p-branes interpolate between d-dimensional Minkowski space-time and $AdS_{p+2} \times S^{d-p-2}$ space. The p-brane solutions are non-singular and break half of the supersymmetries.

Now, $AdS_{p+2} \times S^{d-p-2}$ are, as pointed out above, known to be maximally supersymmetric solutions of the corresponding $d = 10$ and $d = 11$ supergravity theories. Furthermore, effective world-volume actions for these p-brane solutions in $d = 10$ and $d = 11$ are related to singleton supermultiplets of the corresponding AdS supergroups. In particular, the fields of the various singleton supermultiplets, already listed above, show up as corresponding factors in the harmonic expansion of the d-dimensional fields on the $d-p-2$ spheres. As we have pointed out, these modes can be gauged away everywhere except on the boundary of the appropriate $AdS_{p+2}$ space-time. These boundaries represent nothing more than p-brane cores, or opening of the wormhole throat that connects d-dimensional Minkowski space-time with the corresponding AdS space-time \[23\]. The relevant singleton supermultiplets, as we have seen, are in 3-d the $N = 8$ scalar supermultiplet (for $AdS_4$) \[3, 24, 26\], in 6-d the $N = 2$ antisymmetric tensor supermultiplet (for $AdS_7$) \[10\], in 4-d the $N = 4$ Yang-Mills supermultiplet (for $AdS_5$)
The first two arise in the analysis of the quadratic fluctuations around eleven dimensional membrane solutions. The corresponding singleton superconformal theory controls the physical excitations of the eleven dimensional p-brane world-volume theories.

Based on these observations and closely related results of Maldacena, considered dynamics of \( n \) parallel \( Dp \)-branes in the limit when the field theory on the \( Dp \)-brane decouples from the bulk. Maldacena argued that in the limit of the near horizon geometry supergravity solutions pointed out by Gibbons and Townsend can be trusted if the large \( n \) limit is taken. The curvatures of \( AdS_{p+2} \) space and \( S^{d-p-2} \) scale as positive powers of \( 1/n \), so in the large \( n \) limit these supergravity solutions are valid. The Hilbert space of the induced (singleton or doubleton) superconformal field theories includes the Hilbert space of supergravity solutions of \( AdS_{p+2} \times S^{d-p-2} \). Based on this observation Maldacena then conjectured that compactifications of M-theory (or superstring theory) on \( AdS_{p+2} \times S^{d-p-2} \) are dual to various superconformal field theories. (These observations were extended in a number of recent works.)

Our analysis above show that these superconformal theories are simply the singleton or doubleton field theories of \( AdS \) supergroups in various dimensions. Furthermore we would like to point out that the conjecture of Maldacena has a beautiful counterpart in the construction of massless and massive irreducible \( N = 8 \) \( AdS_5 \) supermultiplets in terms of doubleton supermultiplets. If we make the assumption that the bound states of the doubleton field theory i.e. the \( N = 4 \) Yang-Mills in \( d = 4 \) (boundary of \( AdS_5 \)) are CPT self-conjugate supermultiplets then the two doubleton bound states give the massless \( N = 8 \) \( AdS_5 \) supergravity multiplet. These bound states have twice the \( AdS_5 \) energy of doubletons and correspond to massless particles in the Poincare limit! Bound states of three and more doubletons yield ever increasing tower of massive supermultiplets which correspond to the spectrum of ten dimensional IIB supergravity over \( S^5 \). What is remarkable is that these bound states of doubletons are bound states at threshold i.e. with no binding energy (in the \( AdS \) sense!!). This is exactly the picture proposed by Witten for the bound states of \( Dp \)-branes. Of course, as is well known, in ordinary QCD we do not expect to have bound states at threshold. The above picture suggests that the \( N = 4 \) \( U(n) \) Yang-Mills theory must have bound states at threshold as \( n \) goes to infinity.

This picture also holds true for the doubleton field theory of \( N = 4 \) \( AdS_7 \).
supermultiplet in \(d = 6\). The massless and massive bound states (again assuming CPT invariance as above) yield the complete spectrum of eleven-dimensional supergravity compactified down to \(d = 7\) over \(S^4\). Similarly, the singleton field theory of \(N = 8\) AdS\(_4\) bound states in \(d = 3\) yields the spectrum of eleven dimensional supergravity over \(S^7\). In all these cases, the AdS bound states are bound states at threshold in the sense of AdS energy!

4 Singletons, Doubletons and Matrix Theory

M-theory \([44]\) has as its low energy theory the eleven dimensional supergravity. It includes in its spectrum the states of eleven dimensional supergravity and BPS states that correspond to the extension of eleven dimensional SUSY algebra by central charges. The eleven dimensional M-theory SUSY algebra with two-form and five-form central charges can be obtained as a contraction of the simple supergroup \(OSp(1/32, R)\) \([45], [46]\), with the even subgroup \(Sp(32, R)\). It can be written in the form

\[
\{Q_\alpha, Q_\beta\} = (\Gamma^m)_{\alpha\beta} P_m + \frac{1}{2} (\Gamma^{m_1 m_2})_{\alpha\beta} Z_{m_1 m_2} + \frac{1}{5!} (\Gamma^{m_1 ... m_5})_{\alpha\beta} Z_{m_1 ... m_5}
\]

(4 - 1)

where \(m_i = 0, ..., 10\) and \(\alpha, \beta = 1, ..., 32\); \(\Gamma^m\) are Dirac matrices, \(\Gamma^{m_1 ... m_k}(k = 2, 5)\) are their antisymmetrized products and \(C\) is the charge conjugation matrix.

Matrix theory as proposed by Banks, Fischler, Shenker and Susskind \([47]\) corresponds to a formulation of M-theory in the infinite momentum frame (IMF). In the IMF the Poincare symmetry is broken down to Galilean symmetry and the SUSY algebra takes the form \([48]\)

\[
\begin{align*}
\{Q^+_{\mu}, Q^+_\nu\} &= P^+ \delta_{\mu\nu} \\
\{Q^-_{\mu}, Q^-_\nu\} &= P^- \delta_{\mu\nu} + \Gamma^a_{\mu\nu} Z_a + \frac{1}{4!} \Gamma^{a_1 ... a_4}_{\mu\nu} Z_{a_1 ... a_4} \\
\{Q^+_{\mu}, Q^-_\nu\} &= \Gamma^a_{\mu\nu} P_a + \frac{1}{2} \Gamma^{a_1 a_2}_{\mu\nu} Z_{a_1 a_2} + \frac{1}{5!} \Gamma^{a_1 ... a_5}_{\mu\nu} Z_{a_1 ... a_5}
\end{align*}
\]

(4 - 2)

where the original thirty two supercharges split into a pair of sixteen supercharges \(Q^+_{\mu}\) and \(Q^-_{\mu}\) \((\mu = 1, ..., 16)\) and \(a_i\) are the transverse space indices.

In the IMF limit, as pointed out by Banks, Fischler, Shenker and Susskind, all mass scales disappear and the spectrum consist of D0-branes \([49]\) which correspond to plane eleven dimensional gravitational waves. They are described by the translation part of the algebra above.
The dynamics of \( n \) D0-branes on the other hand is described by \( N = 16 \) SUSY \( U(n) \) quantum mechanics corresponding to 128 bosons and 128 fermions (called supergravitons in \[17\], \[50\], \[43\], \[51\]) with the Lagrangian

\[
L = \frac{1}{2R} (D_0 X^a)^2 + \theta^a D_0 \theta_\alpha - \frac{R}{4} [X^a, X^b]^2 + i R \theta^a \Gamma_{\alpha\beta}[X^a, \theta^\beta]
\]

where \( R \) is the extent of the longitudinal direction, \( D_0 \equiv \partial_0 - [A_0, \cdot] \) and \( A_0, X^a (a = 1, \ldots, 9), \theta^\alpha (\alpha = 1, \ldots, 16) \) are hermitean \( n \times n \) matrices. The longitudinal momentum is \( P^+ = n/R \).

Let us now identify an \( \text{OSp}(16/2, R) \) subsupergroup of \( \text{OSp}(1/32, R) \) that will be important to the understanding of the dynamics of D0 particles in the IMF. The Lie superalgebra of \( \text{OSp}(1/32, R) \) can be given a 5-graded structure with respect to its maximal compact subalgebra \( \text{U}(16) \) \[17\]

\[
\text{OSp}(1/32, R) = L_{ij} \oplus L_i \oplus L^i_j \oplus L^i \oplus L^{ij}
\]

\[
= g^{-2} \oplus g^{-1} \oplus g^0 \oplus g^1 \oplus g^2
\]

where \( L^i_j \) are \( \text{U}(16) \) generators and \( L_i (L^i) \) transform in the fundamental \( 16(16) \) representation of \( \text{U}(16) \). The generators \( L_{ij} = L_{ji} \) transform in the symmetric tensor representation of \( \text{U}(16) \). We also have \( L^i_{ij} = L^{ij} \) and \( L^{i\dagger}_i = L^i \).

The trace components of the grade \( \mp 2 \) and grade zero subspaces \((L_{ii}, L^i, L^i_i)\) form an \( \text{Sp}(2, R) \) subalgebra. The maximal subalgebra of \( \text{U}(16) \) that commutes with this \( \text{Sp}(2, R) \) is \( \text{SO}(16) \). The \( \text{SO}(16) \times \text{Sp}(2, R) \) subalgebra has an extension to the superalgebra \( \text{OSp}(16/2, R) \) which is a subalgebra of \( \text{OSp}(1/32, R) \). We shall identify this \( \text{Sp}(2, R) \) with the \( \text{AdS} \) extension of the Poincaré group of the light-cone coordinates. With this identification \( \text{OSp}(16/2, R) \) becomes the \( \text{AdS} \) symmetry of the M-theory algebra when it is compactified down to two dimensions.

The supergroup \( \text{OSp}(16/2, R) \) admits singleton supermultiplets \[7\]. The corresponding field theory exists only on the boundary of the two dimensional \( \text{AdS}_2 \) space and is simply the conformally invariant singleton field theory in one dimension (more precisely on the boundary of \( \text{AdS}_2 \)) with 16 supercharges. The singleton supermultiplets of \( \text{OSp}(16/2, R) \) consists of 128 bosons and 128′ fermions, transforming in the left-handed (128) and right-handed (128′) spinor representation of \( \text{SO}(16) \). In the other singleton supermultiplet the bosons transform in 128′ and fermions in 128 of \( \text{SO}(16) \).

Singleton representation of \( \text{Sp}(2, R) \) when expanded in a particle basis consists of an infinite tower of states with an ever increasing quantized \( \text{U}(1) \).
eigenvalues \[4\]. We propose to identify this infinite tower of states with an infinite tower of \(D0\)-branes \[49\] with quantized values of the longitudinal momentum as in the Matrix theory framework of \(M\)-theory in the infinite momentum frame \[17\] i.e. the singleton supermultiplets of \(OSp(16/2, R)\) can be identified with an infinite tower of \(128 + 128 D0\)-branes (supergravitons).

Note that to get a precise correspondence with singleton field theory we need an infinite number of supergravitons. This is also consistent with the claim that for large \(n\) the dynamics of \(D0\)-branes is related to the light-cone supermembrane \[28\]. More specifically, Matrix theory of \(N = 16\) Yang-Mills quantum mechanics in the limit of the infinite number \((n = \infty)\) of \(D0\)-branes is given by the \(2 + 1\) dimensional light-cone Hamiltonian for the classical supermembrane configuration. The Lorentz symmetry of the corresponding world-volume theory of the supermembrane \[29\] can then be identified with our \(Sp(2, R) \approx SO(2, 1)\) symmetry. Thus \(OSp(16/2, R)\) conformal singleton field theories represent the large \(n\) limit of Matrix theory.

This picture is in perfect agreement with Maldacena’s proposal \[37\] for the case of the large \(n\) number of \(D0\)-branes. Thus the singleton quantum mechanics living on the boundary of \(AdS_2\) controls the large \(n\) limit of Matrix theory which is the \(N = 16 U(n)\) Yang-Mills quantum mechanics. Furthermore this is a rather natural extrapolation of the correspondence between \(AdS_3\) singleton field theories and string theories (IIA, IIB and heterotic) in the light-cone frame \[18\].

The fact that the singleton quantum mechanics is conformally invariant is consistent with the requirement that in the large \(n\) limit of Matrix theory longitudinal boost invariance (which is closely related to scale invariance in the longitudinal momentum) has to be recovered, thus restoring the full 11-d Lorentz invariance.

5 Discussion and Conclusions

Interpreting the simple supergroup \(OSp(1/32, R)\) as a ”generalized \(AdS\)” supergroup in \(d = 11\) \[52\] whose contraction gives the \(M\)-theory superalgebra \[15\, 14\], suggests immediately that it can also be understood as the generalized conformal group in ten dimensions \[46\]. Now, the noncompact supergroups \(OSp(2N + 1/2M, R)\) with the even subgroup \(SO(2N + 1) \times Sp(2M, R)\) do not in general admit three grading with respect to a compact subsuper group. However \(OSp(2N + 1/2M, R)\) does admit a five-grading with respect to its subsupergroup \(U(N | M)\) \[17\]. The oscillator method of \[12\] for con-
structing UIR’s of non-compact supergroups was generalized to supergroups that admit such five-grading in [17]. Using this method one can construct the unitary supermultiplets of $OSp(1/32, R)$ [53]. Positive energy UIR’s of $OSp(1/32, R)$ are uniquely characterized by the $U(16)$ quantum numbers of the corresponding lowest weight vectors. One finds that $OSp(1/32, R)$ admits a singleton supermultiplet consisting of a scalar field $\phi(1)$ and a spinor field $\psi(16)$.

Some of the lowest "massless" supermultiplets of $OSp(1/32, R)$ are [53]

\[
\phi(1) \oplus \psi_\mu(16) \oplus A_{\mu\nu}(120)
\]

\[
\psi_\mu(16) \oplus S_{\mu\nu}(136)
\]

\[
S_{\mu\nu}(136) \oplus \psi_{\mu\rho}(816)
\]

\[
\psi_{\mu\nu\rho}(816) \oplus S_{\mu\nu\rho\lambda}(3876)
\]

where $\mu, \nu, ... = 1, 2, ..., 16$ are $SU(16)$ indices.

To get a supermultiplet of $OSp(1/32, R)$ which includes the fields of 11-d supergravity one needs to tensor four copies of the singleton supermultiplet [53]. However, following the definition of "masslessness" stated in section two [16] these would correspond to massive supermultiplets! One possible resolution of this problem is that these "massive" supermultiplets become massless supermultiplets of 11-d Ponicare superalgebra when we contract $OSp(1/32, R)$. Furthermore, the definition of masslessness proposed in [16] may have to be modified in the presence of central charges. That the contraction of $OSp(1/32, R)$ to 11-d supergravity algebra may change the definition of masslessness is also consistent with the fact that there are no known 11-d supersymmetric field theories that correspond to "massless" supermultiplets above. Another possible resolution is to extend $OSp(1/32, R)$ to a larger symmetry such as $OSp(1/32, R) \times OSp(1/32, R)$ as suggested in some recent work [54] [55]. These issues will be addressed elsewhere [53].

In conclusion, in this article we have studied the singleton and doubleton supermultiplets of AdS supergroups in various dimensions in relation to the dynamics of Dp-branes and Matrix theory. We have identified the two dimensional AdS supergroup $OSp(16/2, R)$, with the even subgroup

\[3\] we indicate the $SU(16)$ representation of the fields in parenthesis while omitting their $U(1)$ charges.
SO(16) \times Sp(2, R), which determines the dynamics of \( n \) D0-branes in the large \( n \) limit of Matrix theory. The \( Sp(2, R) \) symmetry corresponds to the AdS extension of the Poincare symmetry of the longitudinal direction. The infinite number of D0-branes are the Fourier modes of the singleton supermultiplets of \( OSp(16/2, R) \) which consist of 128 bosons and 128 fermions. We have also shown that the recent conjecture of Maldacena relating the dynamics of \( n \) Dp-branes to AdS supergravity theories has a beautiful counterpart in the known constructions of the spectra of 10-d IIB and 11-d supergravity theories over various spheres in terms of singleton and doubleton supermultiplets. In all these cases the field theories of singleton and doubleton supermultiplets of AdS supergroups in various dimensions are conformally invariant theories living on the boundary of the AdS spacetime. In particular, the large \( n \) limit of \( N = 16 \) \( U(n) \) Yang-Mills quantum mechanics which describes Matrix theory is a conformally invariant \( N = 16 \) singleton quantum mechanics.

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