Time Varying RF Phase Noise for Longitudinal Emittance Blow-up

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Abstract.
RF phase noise was shown to be effective for controlled longitudinal emittance blow-up in the Proton Synchrotron Booster (PSB) at CERN during beam tests in 2017, with further developments in 2018. At CERN, RF phase noise is used operationally in the Super Proton Synchrotron (SPS) and Large Hadron Collider (LHC). In this paper we show that it is suitable for operation with a variety of beam types in the PSB. In the PSB the synchrotron frequency changes by approximately a factor 4 during the 500 ms acceleration ramp, requiring large changes in the frequency band of the noise. During 2018, a new method of calculating the noise parameters has been demonstrated, which gives upper and lower bounds to the noise frequency band that are smoothly varying through the ramp. The new calculation method has been applied to operational beams accelerated in both single and double RF harmonics, the final results are presented here.

1. Introduction
Band limited phase noise is used operationally in the Large Hadron Collider (LHC) and Super Proton Synchrotron (SPS) at CERN for controlled longitudinal emittance blow-up [2, 1]. In contrast, the Proton Synchrotron (PS) and Proton Synchrotron Booster (PSB) use single frequency modulation of a high harmonic RF system, this method is not discussed here but details can be found in Ref. [3]. The use of band limited phase noise has been demonstrated in the PSB [4], here we give details of a new approach to calculating the noise program and the results of a 2018 reliability run.

Figure 1 shows the synchrotron frequency distribution at the start of the cycle in a harmonic \( h = 1 \) bucket with RF Voltage \( V_{h1} = 8 \) kV. To target a 1 eV s emittance (vertical red line), a noise band is applied with an upper bound at the small amplitude synchrotron frequency \( f_{s,0} \) and a lower bound at the synchrotron frequency at 1 eV s \( f_{s,1} \), shown by the two horizontal black lines. Any particle with a synchrotron frequency within that band will be excited to higher amplitudes, therefore allowing a direct targeting of the required longitudinal emittance [5].

During acceleration both \( f_{s,0} \) and \( f_{s,1} \) change significantly, as does the distance between them as shown in Fig. 2. In order to cleanly blow up the bunch the noise band must follow these changing limits as closely as possible. The noise is typically calculated by taking white noise and setting a bandwidth in the frequency domain, which is then converted to the time domain by an inverse Fourier Transform. This process is repeated at several stages along the ramp to follow the changing frequency spread [4]. The duration of each noise segment must be carefully
Figure 1: Synchrotron frequency distribution inside a single harmonic bucket ($8\,\text{kV}$, $h = 1$) at PSB injection (blue line) with the 1 eV s emittance (red line) and synchrotron frequencies at $f_{s,0}$ and 1 eV s (black lines).

Figure 2: Small amplitude and 1 eV s synchrotron frequency through the acceleration cycle with $V_{h1} = 8\,\text{kV}$.

chosen as a longer segment gives more resolution in frequency space, but shorter segments allow the changing frequency spread to be followed more closely. For the 2018 reliability run, a new approach to the calculation was used to better follow the changing conditions, which is the subject of the following section.
2. Noise Program Calculation

In time domain a sine wave with time varying frequency can be readily calculated as

\[ A(t) = \sin \left[ \int 2\pi f(t) dt + \varphi \right], \]  

(1)

where \( A(t) \) is the waveform, \( f(t) \) is the time dependent frequency and \( \varphi \) is an initial phase offset. Rather than transforming from frequency domain to time domain the noise program can be calculated by taking a sum over any number \( N \) versions of Eq. 1. For a given value of \( f(t) \) and \( \varphi \) this gives an \( A(t) \) of non-constant frequency that smoothly varies with time, as shown in Fig. 3, where \( f(t) \) varies linearly from 1 kHz at \( t = 0 \) ms to 10 kHz at \( t = 3 \) ms.

![Smoothly varying waveform with time dependent frequency.](image)

Figure 3: Smoothly varying waveform with time dependent frequency.

It follows that for any combination of waveforms of the type given by Eq. 1, the sum of these waveforms will also smoothly vary with time and contain the frequencies given by each value of \( f(t) \). Therefore, selecting values of \( f(t) \) to follow the synchroton frequency within a bunch will allow time dependent noise programs that are smoothly varying with time. In the example shown here, a longitudinal emittance of 0.9 eV s is targeted with blow-up applied from 350 ms to 550 ms after the cycle start (C350 to C550), the value of \( f(t) \) is given by interpolating between the upper and lower frequency bounds within that time range as shown by the black lines in Fig. 4.

The noise program is therefore calculated with

\[ \varphi(t) = \alpha \times \sum_{i=0}^{N} \sin \left[ \int 2\pi f_i(t) dt + \varphi_i \right], \]  

(2)

where \( \alpha \) is an amplitude term. \( f_i(t) \) and \( \varphi_i \) are selected as an interpolation between \( f_{s,0} \) and \( f_{s,1} \), and from the range \([0, 2\pi]\) respectively via random sampling. Figure 5 shows \( \varphi(t) \) as used to provide blow-up to 0.9 eV s in the upper plot, with a sliding window FFT shown on the lower plot.
Figure 4: Upper and lower frequency bounds required to reach 0.9 eV s with $V_{h1} = 8 \text{kV}$ in the PSB, with a selection of sampled time varying frequencies (black lines).

Figure 5: Noise program (top) and sliding window FFT (bottom) of the noise program used to give blow-up to 0.9 eV s with $V_{h1} = 8 \text{kV}$, the white lines show $f_{s,0}$ and $f_{s,1}$.
3. Reliability Run

As phase noise had not previously been used operationally in the PSB, the decision was taken to perform a reliability run during 2018 with three different operational beams, which are representative of the main operational parameter space of the PSB. These beams were the BCMS, a low intensity ($\approx 85 \times 10^{10}$ ppb) cycle for the LHC accelerated using a single harmonic ($V_{h1} = 8\, \text{kV}$); the LHC25, an intermediate intensity ($\approx 160 \times 10^{10}$ ppb) cycle with $V_{h1} = 8\, \text{kV}$ and $V_{h2} = 6\, \text{kV}$; and the MTE beam, a high intensity ($\approx 500 \times 10^{10}$ ppb) cycle with $V_{h1} = 8\, \text{kV}$ and $V_{h2} = 8\, \text{kV}$, which is split prior to extraction. Unlike previous studies of phase noise in the PSB [4], the new method does not require applying voltage at the second harmonic.

![RF Buckets at capture for BCMS, LHC25 and MTE type beams](image)

Figure 6: RF Buckets at capture for BCMS, LHC25 and MTE type beams

As well as the large intensity span given by these variants, the different voltage ratios give very different effects in the single particle regime. Figure 6 shows the separatrices formed for the three cycle types at the start of the ramp, the dashed lines show the inner separatrices that occur as a result of having $V_{h2} > V_{h1}/2$.

An additional complication in the PSB is that the machine is composed of four superposed rings, which are nominally identical but with small unavoidable differences. These differences mean that from ring-to-ring there needs to be slight variations in the noise program to achieve optimal results. This did not pose any significant problems.

Figure (7) shows the tomographic reconstructions [6] of the longitudinal phase space at extraction for BCMS (7a), LHC25 (7b), and MTE left (7c) and right (7d) after longitudinal splitting. These four examples show that for the full parameter space of interest the bunches can be blown-up to produce a very smooth longitudinal distribution with the required parameters.

Figure (8) shows the bunch length, RMS $dp/p$, and horizontal and vertical emittances at extraction from each ring for the BCMS beam over the course of the run. The reliability run started on the 15th of May (vertical red line) and continued until the end of year shutdown. As can be seen the inter-ring variations are small and do not change noticeably with the start of the reliability run, the variation with time is also minimal suggesting there is no significant change in the performance of the blow-up over the course of the year as should be expected. Equivalent stability and inter-ring variation was also seen for the LHC25 and MTE beams.
Figure 7: Example tomographic reconstructions of longitudinal phase space at extraction for the three beam types used in the reliability run, MTE left and right are the two bunches produced by longitudinal splitting.
Figure 8: Measured bunch length, RMS $\frac{dp}{\rho}$, $\varepsilon_H$ and $\varepsilon_V$ for BCMS from the four rings of the PS Booster during 2018, the phase noise reliability run started at the red line.
4. Conclusion
Following from the successful demonstration of band limited phase noise in the Proton Synchrotron Booster (PSB) at CERN in 2017, a reliability run was performed during 2018. For the reliability run, RF phase noise was calculated by summing time varying waveforms to allow easier tracking of the changing synchrotron frequency distribution. Three different beam types were included in the reliability run covering the main parameter spaces of the PSB; BCMS (≈ 85 × 10¹⁰ ppb, V₁ = 8 kV, V₂ = 0 kV); LHC25 (≈ 160 × 10¹⁰ ppb, V₁ = 8 kV, V₂ = 6 kV); and MTE (≈ 500 × 10¹⁰ ppb, V₁ = 8 kV, V₂ = 8 kV). In all cases the beams were produced within required specifications over the course of the run and inter-ring variations were comparable to those seen with the standard operational blow-up using single tone modulation of a high harmonic. The method of calculating phase noise shown here should be applicable to any accelerator, but is particularly suitable to those where f₁ is large.

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