Toroidal plasma conditions where the p-11B fusion Lawson criterion could be eased

Yueng-Kay Peng (pengyuankai@enn.cn)
ENN Science and Technology Development Co., Ltd. https://orcid.org/0000-0003-2948-1058

Yuejiang Shi
ENN Science and Technology Development Co., Ltd.

Mingyuan Wang
ENN Science and Technology Development Co., Ltd.

Bing Liu
ENN Science and Technology Development Co., Ltd.

Xueqing Yan
Peking University

Article

Keywords:

DOI: https://doi.org/10.21203/rs.3.rs-93644/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License
Abstract

We examine the theoretical conditions in which the Lawson ignition criterion for p-\(^{11}\)B fusion in a magnetized toroidal plasma can be reduced substantially. It is determined that a velocity differential between the protons and the boron ions of the order of the plasma sound speed (Mach number of 1 or 2 at a plasma temperature of \(\sim 10^2\) keV) could raise the p-\(^{11}\)B fusion reaction rate to \(\sim 2 \times 10^{-22}\) m\(^3\)/s or \(\sim 6 \times 10^{-22}\) m\(^3\)/s, respectively, from the \(\sim 1 \times 10^{-22}\) m\(^3\)/s level in a static plasma. The Lawson triple product \((n_i \tau_e T_i)\) required for ignition can thereby be reduced to as low as \(\sim 10^{23}\) m\(^{-3}\) s keV, which is one order of magnitude above the ITER requirement for D-T burn. Since order-unity Mach numbers in velocity differentials between deuterons and impurity carbon ions have been maintained in tokamak plasmas under excellent confinement conditions, similar levels of velocity differentials between protons and minority boron-11 ions could in principle be maintained also. A theoretical possibility of achieving p-\(^{11}\)B fusion ignition in a toroidal plasma of \(\sim 10^2\) keV in ion temperature is hereby presented. Similar p-\(^{13}\)C plasmas, for example, will introduce a possibility of measuring the CNO fusion chain reaction rates in a laboratory.

Introduction

Neutron- and tritium-free fusion reactions, such as the CNO chain,\(^1\)\(^2\) have powered the stars. The aneutronic p-\(^{11}\)B fusion reaction has lured the imagination of fusion energy researchers of magnetic confinement systems since the 70’s.\(^3\)\(^-\)\(^5\) Research and development toward this clean fusion energy from a magnetically confined plasma, however, due to its very high required plasma temperatures (greater than \(\sim 10^2\) keV in the case of p-\(^{11}\)B), is an area of research imbued with great challenges. Overcoming them would create opportunities to change and possibly ease the engineering and technology requirements of a power producing fusion reactor. There is no guarantee that a practical solution for p-\(^{11}\)B fusion plasma exists or could be found. But if so, it would create a realistic chance to achieve the great goal of a universally available, inexhaustible, and environmentally benign power source. In this paper we introduce analysis showing that this challenge could in theory be met by creating a velocity differential between the protons and the boron ions that is of the order of the plasma sound speed.

In recent years however, the theoretical possibility of net energy gain from a magnetized p-\(^{11}\)B fusion plasma has remained elusive,\(^6\) and has been considered by many to be theoretically impossible. The basis for this view has been known to include the following:

1. Bremsstrahlung radiation from a plasma increases strongly with the electron temperature, density and the effective ion charge \(Z_{\text{eff}}\).\(^7\)\(^-\)\(^8\) As the plasma temperature is raised beyond 50 keV, relativistic effects would strongly enhance this radiation.\(^9\) The presence of a magnetic field can change the orbit of the electrons as they encounter Coulomb interactions with the ions. The magnetic field can substantially change the bremsstrahlung radiation when the Larmor radius \(r_B\) of an electron is smaller than the characteristic impact parameter \(r_s\) of close Coulomb collisions.\(^10\) According to analysis in [11], magnetic fields would reduce the bremsstrahlung radiation power. When \(r_B\) is larger
than \(r_s\), the effect of magnetic field on bremsstrahlung radiation becomes negligible. For the magnetic confined plasma discussed in this paper, \(r_B\) is much larger than \(r_s\). So the traditional calculation for the bremsstrahlung without magnetic field will be applied in this paper. Where the electron and ion temperatures are close to each other, the Bremsstrahlung radiation loss power density at a few hundreds of keV in plasma electron and ion temperatures would exceed the local power density produced by p-\(^{11}\)B fusion.\(^6\)

2. In the case of the tokamak, its energy confinement time\(^{12}\) scales unfavorably with the total plasma heating power \(P (\mu P^{0.69})\) required to reach such high temperatures. To maintain a plasma temperature at \(\sim 10^2\) keV, a tokamak would require increased physical parameters (radii, magnetic field, plasma current, etc.) to compensate for the degradation in energy confinement when the heating power (including fusion self-heating) is raised. However, sustained fusion plasma burn would become impossible under large increases in confinement when fusion ash would accumulate, dilute the fuel concentration, and discontinue fusion burn.\(^{13}\)

More recently, however, ideas were introduced that would mitigate certain aspects of these difficulties, such as:

1. It was observed in tokamaks under intense Neutral Beam Injection (NBI) heating,\(^{14-16}\) that during the so-called energetic or hot-ion mode, the central ion temperatures \(T_i(0)\) were as high as 35keV while maintaining a ratio to the central electron temperature \(T_i(0) / T_e(0)\) higher than a factor of 3. This created a plasma condition where the Bremsstrahlung radiation loss power density could be reduced while maintaining the fusion power density.\(^6,17\)

2. By increasing the proton fraction \(x_p\) of the plasma ions, reducing the effective charge \((Z_{eff})\) of the plasma, the Bremsstrahlung radiation loss would be reduced further\(^6,17\) without reducing the fusion power substantially.

**Method And Results**

Following these leads, the \(n_{i}t_E\) vs. \(T_i\) values required for ignition \((Q_{pB} = 1)\), as \(T_i/T_e\) and \(x_p\) are varied from 2 to 8 and 0.5 to 0.9, respectively, and are calculated using the common approaches.\(^6,18\) Recent re-evaluated \(^{11}\)B(p,3a) fusion reaction cross sections,\(^19\) the accuracy of which was more recently further refined,\(^{15}\) are included. The kinetic effects that may cause an increase of the number of protons at higher energies (with respect to a pure Maxwellian distribution), leading to a net effect of approximately 30% increase of the fusion yield for the same global plasma parameters\(^9\) are not included in the calculation.

Here, \(Q_{pB} = 1\) is defined where the fusion a self-heating power exactly replaces the externally applied heating power that is required in the first place to bring the plasma to ignition. This definition is appropriate since p-\(^{11}\)B produces 100% of its fusion energy in the form of near-3-MeV a particles, which can in principle be well confined in a tokamak or another magnetic configuration of sufficient size and
field. Note that Lawson's net energy gain model requires that, where $Q$ is the plasma fusion energy gain and $\eta$ is the fusion energy conversion efficiency back to a fusion plasma in a power reactor. It is seen that $Q_{pB} = 1$ corresponds to the requirement of high $\eta > 0.5$. Here one could rely on some form of direct conversion of the plasma kinetic energy to electricity.

Figure 1 shows that under these conditions, $p^{11}\text{B}$ fusion ignition could in theory be achieved if the Lawson triple product ($n_i t_E T_i$) can reach $\sim 10^{24}$ m$^{-3}$ s keV. This is however over two orders of magnitude higher than the ITER D-T fusion goal of $\sim 6 \times 10^{21}$ m$^{-3}$ s keV.

More recently, toroidal rotation velocities of ions of different charges (Ne-X, C-VI) were measured and compared with the calculated velocities of the main deuteron ions in a “Quiescent Double-Barrier” plasma with an excellent energy confinement time on the DIII-D tokamak. Similar results of plasmas with a strong “Internal Transport Barrier” were found on the JET tokamak. The impurity velocities were found to be proportional to $\tilde{N} P_i Z_i$, where $P_i$ is the pressure and $Z_i$ the charge state of the species $i$. This is consistent with the theoretical neoclassical transport model of a tokamak plasma. A substantial difference, $V_d$, in the toroidal velocities of the impurity ions and the deuterons was observed, the corresponding differences in Mach numbers ($M = V_d/C_s$) being near order-unity in magnitude. This points to a possibility of maintaining a substantial velocity differential between the protons and the boron ions in a future high-temperature $p^{11}\text{B}$ plasma.

We assume that these ions on the average rotate with a velocity differential $V_d$. We further assume that these ions can be approximately described by a Maxwellian velocity distribution with a temperature $T$, written as:

More recently, toroidal rotation velocities of ions of different charges (Ne-X, C-VI) were measured and compared with the calculated velocities of the main deuteron ions in a “Quiescent Double-Barrier” plasma with an excellent energy confinement time on the DIII-D tokamak. Similar results of plasmas with a strong “Internal Transport Barrier” were found on the JET tokamak. The impurity velocities were found to be proportional to $\tilde{N} P_i Z_i$, where $P_i$ is the pressure and $Z_i$ the charge state of the species $i$. This is consistent with the theoretical neoclassical transport model of a tokamak plasma. A substantial difference, $V_d$, in the toroidal velocities of the impurity ions and the deuterons was observed, the corresponding differences in Mach numbers ($M = V_d/C_s$) being near order-unity in magnitude. This points to a possibility of maintaining a substantial velocity differential between the protons and the boron ions in a future high-temperature $p^{11}\text{B}$ plasma.

We assume that these ions on the average rotate with a velocity differential $V_d$. We further assume that these ions can be approximately described by a Maxwellian velocity distribution with a temperature $T$, written as:
In the mass-centered coordinates the fusion reaction rates can be written as:

\[ f_p(v_p) = \left( \frac{m_p}{2\pi T} \right)^{\nu/2} \exp\left( -\frac{m_p v_p^2}{2T} \right) \]

\[ f_r(v_r) = \left( \frac{m_r}{2\pi T} \right)^{\nu/2} \exp\left( -\frac{m_r (v_r^2 + V_j^2 - 2V_j v_r \cos(\theta))}{2T} \right) \]

In the mass-centered coordinates the fusion reaction rates can be written as:

\[ \langle \sigma v \rangle = 2\pi \left( \frac{\beta}{\pi} \right)^{3/2} \int \sigma(v) v^2 \exp\left( -\beta(v^2 + V_j^2 - 2V_j v \cos(\theta)) \right) \sin(\theta) d\theta dv, \]

where \( \nu = |v_p - v_e|, \beta = \frac{N_{PB}}{2\pi} \). Following integration in \( \theta \) one obtains:

\[ \langle \sigma v \rangle = \frac{1}{V_j} \left( \frac{\beta}{\pi} \right)^{1/2} \int \sigma(v) v^2 \left[ \exp\left( -\beta(x - V_j)^2 \right) - \exp\left( -\beta(x + V_j)^2 \right) \right] dv. \]

Figure 2 shows the variations of this p-\(^{11}\)B reaction rate, as a function of \( T_i \) and as \( V_d/C_s = 0, 1, 2, 3 \), where \( C_s = [(T_i + T_e)/M_{PB}]^{0.5} \). It is seen that a velocity differential between the protons and the boron ions of the order of the plasma sound speed (Mach number of 1 or 2 at a plasma temperature of \( \sim 10^2 \) keV) could raise the p-\(^{11}\)B fusion reaction rate to \( \sim 2 \times 10^{-22} \) m\(^3\)/s or \( \sim 6 \times 10^{-22} \) m\(^3\)/s, respectively, from the \( \sim 1 \times 10^{-22} \) m\(^3\)/s level in a static plasma. The results show that the maximum reaction rates below 300 keV would nearly double to \( \sim 7 \times 10^{-22} \) m\(^3\)/s as \( V_d \) is increased to \( 2C_s \); a large increase to a value of \( 7.5 \times 10^{-22} \) m\(^3\)/s would be obtained when \( T_i \) is reduced toward 50 keV, if \( V_d \) is increased to \( 3C_s \).

A pronounced effect can also be seen in the corresponding Lawson criterion,\(^{18}\) as shown in Figure 3.

The calculated \( n_i E \) vs. \( T_i \) values for \( Q_{PB} = 0.2, 1, 2 \) with \( T_i \) up to 300 keV are provided in Figure 3, where \( x_p = 0.9 \) and \( T_i/T_e = 4 \) are assumed. Here \( Q_{PB} \) is defined as the fusion power replacing part of or all the externally applied heating power, which is used to reach a specific \( Q_{PB} \) value in the first place. It is seen that the minimum triple product \( (n_i E T_i) \) required to obtain these \( Q_{PB} \) values would be lowered to \( \sim 1.4x \), \( \sim 7x \), \( \sim 24x \) \( 10^{22} \) m\(^3\) s keV, respectively. A theoretical possibility of a driven or a sustained p-\(^{11}\)B plasma fusion burn via high \( V_d \) is hereby indicated, reducing the \( Q_{PB} = 1 \) requirement to about one order of magnitude above the ITER Phase I target of \( \sim 6 \times 10^{21} \) m\(^3\) s keV.\(^{22}\)

**Discussion**

Another possible method of producing such large \( V_d \) values are introduced through injection of a compact laser plasma accelerator (CLAPA)\(^{27}\) proton stream into a toroidal plasma. When an ultra-intense laser beam is incident on a plastic or metal foil, an intense proton beam of up to 150 pC with an
energy spectrum of an exponential profile from 100 keV to 10MeV and a divergence angle of tens degrees can be generated.\textsuperscript{28} A velocity differential between the protons and the heavier ions (such as boron or carbon) in a toroidal plasma can be created during a CLAPA pulse. Although the time-duration of the proton bunch is as short as 100fs at the point of injection, it will lengthen to nanosecond or more due to the energy spread, depending on the flight path length.

Once integrated with a magnetic confinement configuration, CLAPA can reliably deliver intense proton beams of tens of pC, with such as 1\% energy spread of different energies up to 10MeV, and good spatial uniformity. When injected into a $^\text{11}\text{B}$ plasma of sufficient temperature, density and beam path-lengths, the theoretical effects of large $V_d$ on $^\text{11}\text{B}$ plasma fusion reaction rates can in principle be measured.

It is noted that ion acceleration driven by ultra-intense laser pulses has been an active field of research for years because it delivers ion bunches of multiple-MeV energy with ultrashort duration, originating from $\mu$m-scale spots. Near the source, such bunches can be $\sim10^{10}$ times denser than classically accelerated ion bunches.\textsuperscript{29} This technical feature is of great importance not only in high energy density physics studies, but also in magnetic fusion plasma studies by enabling poloidal magnetic and electric field measurement in 2D profiles.\textsuperscript{30}

An implication of creating a large $V_d$ in a proton and modest-Z ion plasma of $\sim10^2$ keV in temperature would be a first laboratory experiment on the aneutronic CNO fusion chain\textsuperscript{12} reaction rates. By replacing $^\text{11}\text{B}$ with some of the stable isotopes in this reaction chain ($^\text{12}\text{C}$, $^\text{13}\text{C}$, $^\text{14}\text{N}$, or $^\text{15}\text{N}$), a fusion device could be so arranged as to measure the corresponding fusion reaction rates, albeit at minute yet detectable levels. Estimates of these $Q_{p13C}$ values (of the order 0.0001) for the ranges of $V_d$ and $(n_i/\rho_{Ei} T_i)$ values considered in Figure 3 are shown in Figure 4. Although the values estimated could be uncertain at the present time by an order of magnitude, the potential contribution of such a laboratory test to the field of aneutronic fusion physics in the stars would be large, and therefore deserves consideration.

It is of importance to point out that this theory assumes the maintenance of the protons and the $^\text{11}\text{B}$ ions that are not by themselves in thermal equilibrium. It has been calculated\textsuperscript{31} that in this case the externally applied recirculating power density needed to maintain this condition would be higher than the fusion power density so produced. However, a combination of particle, momentum, and energy input into and loss from such a “toroidal magnetically confined high-temperature plasma of multiple ion species” can lead to a physical system in which apparently non-equilibrium components are maintained as part of a more complex macroscopic fluid equilibrium.\textsuperscript{36} A surprising feature of such a plasma equilibrium is a theoretical possibility of substantial spatial separation of a relativistic electron fluid from the thermal electron fluid and the two thermal ion fluids of different charge-to-mass ratios and flow velocities. The co-location requirement assumed in [31] is no longer appropriate, leaving open the question of the required external power to maintain this apparent absence of thermal equilibrium.
It is further necessary to point out that the four-fluid equilibrium model contains a freedom to choose as input the spatial distributions of temperatures, densities, and velocities of these two ion fluids. The stability of near sonic or supersonic velocities and velocity differentials in such an equilibrium becomes an important open question that must be analyzed during the next stage of this theoretical work.

**Conclusion**

This work establishes a theoretical basis for lowering the Lawson criterion of triple product \((n_i \rho_i E_i T_i)\) for \(^{11}\text{B}\) magnetic fusion plasma burn to about one order of magnitude above the ITER goal, by assuming \(x_p = 0.9\), \(T_i/T_e = 4\), and \(V_d = 2C_s\). This result indicates that a net energy gain via \(^{11}\text{B}\) magnetic plasma fusion is in theory possible. A scientific bonus of this finding would be a possibility to measure the CNO fusion chain reaction rates in a suitably enabled toroidal fusion experimental device in a laboratory. This new experimental test could be enhanced by the application of the CLAPA technology. The development of techniques and magnetic configurations that maintain a high, stable and stationary \(V_d/C_s\) value while reducing the recirculating power substantially is therefore of great importance to the goal of realizing a practical energy gain in a \(^{11}\text{B}\) magnetic fusion plasma.

**Declarations**

**Acknowledgement**

The authors of this paper acknowledge multiple opportunities of helpful discussion with Peipei Chen, Minsheng Liu, Yang Li, and Huasheng Xie during this work. Y.-K. M. Peng also thanks Wenjun Liu for assistance in the preparation of the manuscript.

**Author contributions**

Y.-K. M. Peng initiated and guided the research reported in this article; Y. Shi and M. Wang developed the theoretical model, carried out the computations, and produced the figures; B. Liu carried out the broad survey of related literature; X. Yan provided the scientific input related to CLAPA; all contributed to the writing of the manuscript.

**References**

1. Bethe, H. A. Energy Production in Stars. *Physical Review* 55, 434(1939).
2. Bethe, H. A. Energy production in stars. *Science* 161, 541-547 (1968).
3. Weaver, T., Zimmerman, G. & Wood, L. “Prospects for Exotic Thermonuclear Fuel Usage in CTR Systems”, UCRL-74191, UCRL-74352, Lawrence Livermore National Laboratory (1972); see also “Exotic CTR Fuels: Non-Thermal Effects and Laser Fusion Applications,” UCRL-74938, Lawrence Livermore National Laboratory (1973).
4. Shuy, G. W. & Conn, R. W. “Physics Phenomena in the Analysis of Advanced Fusion Fuel Cycles”, PPG-522, University of California, Los Angeles (1980); see also "Charged Particle Cross Section Requirements for Advanced Fusion Fuel Cycles," Proc. Int. Conf Nuclear Cross Sections for Technology, NBS Special Publication 594, p. 254, U.S. National Bureau of Standards (1980).

5. McNally, Jr., J.R. Physics of fusion fuel cycles. Nuclear Technology – Fusion 2, 9-28 (1982).

6. Nevins, W. M. A review of confinement requirements for advanced fuels. Fusion Energy 17, 25-32 (1998).

7. Huba, J. D. NRL Plasma Formulary, NRL/PU/6790-09-523, Naval Research Laboratory (2009).

8. Gould, R. J. Thermal bremsstrahlung from high-temperature plasmas. The Astrophysical Journal 238, 1026-1033 (1980).

9. Putvinski, S.V. Fusion reactivity of the pB11 plasma revisited. Fusion 59, 076018 (2019).

10. Bubukina, I. I. & Koryagin, S. A. Bremsstrahlung from collisions of low-energy electrons with positive ions in a magnetic field. Journal of Experimental and Theoretical Physics 108, 917-927 (2009).

11. Imazu, S., Irisawa, J. & Takano, S. Bremsstrahlung rates in fully ionized gases in a magnetic field. Journal of the Physical Society of Japan 52, 1224-1229 (1983).

12. Shimada, M. et al. Progress in the ITER Physics Basis - Chapter 1: Overview and summary. Fusion 47, S1-S17 (2007).

13. Galambos, J. D. & Peng, Y.-K. M. Ignition and Burn Criteria for D-3He Tokamak and Spherical Torus Reactors. Fusion Technology 19, 31-42 (1991).

14. Bell, M. G., Arunasalam, V., & Barnes, C. W. (1989). An overview of TFTR confinement with intense neutral beam heating. Plasma physics and controlled nuclear fusion research 1988. Vol. 1, IAEA, Vienna 27 (1989).

15. Meade, M.D. et al. Recent TFTR results. Plasma physics and controlled nuclear fusion research 1990. V1, 9-25 (1991).

16. JET Team. Recent JET results and future prospects. Plasma physics and controlled nuclear fusion research 1990. V1, 27-51 (1991).

17. Hay, M. J. & Fisch, N. J. Ignition threshold for non-Maxwellian plasmas. Plasmas 22, 112-116 (2015).

18. Lawson, J. D. Some criteria for a power producing thermonuclear reactor. Proceedings of the physical society. Section B 70, 6 (1957).

19. Sikora, M. H. & Weller, H. R. A New Evaluation of the $^{11}$B (p, α) αα Reaction Rates. Journal of Fusion Energy 35, 538-543 (2016).

20. Munch, M., Kirsebom, O. S., Swartz, J.A. & Fynbo, H. O. U. Resolving the $^{11}$B(p, α0) cross-section discrepancies between 0.5 and 3.5 MeV. Phys. J. A 56, 17 (2020).

21. Moir, R. W. & William L. B. Venetian-blind” direct energy converter for fusion reactors. Nuclear Fusion 13, 35 (1973).

22. Ikeda, K. ITER on the road to fusion energy. Fusion 50, 014002 (2010).
23. Baylor, L. R. et al. Comparison of toroidal rotation velocities of different impurity ions in the DIII-D tokamak. *Physics of Plasmas* **11**, 3100-3105 (2004).
24. Testa, D., Giroud, C., Fasoli, A., Zastrow, K.-D. & EFDA-JET Team. On the measurement of toroidal rotation for the impurity and the main ion species on JET. *Fusion* **9**, 243-250 (2002).
25. Crombé, K. et al. Poloidal rotation dynamics, radial electric field, and neoclassical theory in the jet internal-transport-barrier region. *Physical review letters* **95**, 155003 (2005).
26. Hirshman, S. P. & Sigmar, D. J. Neoclassical Transport of Impurities in Tokamak Plasmas. *Fusion* **21**, 1079 (1981).
27. Zhu, J. G. et al. Experimental demonstration of a laser proton accelerator with accurate beam control through image-relaying transport. *Physical review accelerators and beams* **22**, 061302 (2019).
28. Geng, Y. X. et al. Generating Proton Beams Exceeding 10 MeV Using High Contrast 60TW Laser. *Chinese Physics Letters* **35**, 092901 (2018).
29. Ma, W. J. et al. Laser acceleration of highly energetic carbon ions using a double-layer target composed of slightly underdense plasma and ultrathin foil. *Physical review letters* **122**, 014803 (2019).
30. Yang, X. Y. at al. 2D profile of poloidal magnetic field diagnosed by a laser-driven ion-beam trace probe (LITP) , *Review of Scientific Instruments* **87**, 11D608 (2016).
31. Rider, T. H. Fundamental limitations on plasma fusion systems not in thermodynamic equilibrium. *Plasmas* **4**, 1039-1046 (1997).
32. Zijderhand, F. & Van der Leun, C. Strong M2 transitions. *Nuclear Physics* **A460**, 181-200 (1986).
33. King, J. D. et al. Cross section and astrophysical S-factor for the $^{13}$C (p, γ) $^{14}$N reaction. *Nuclear Physics* **A567**, 354-376 (1994).
34. Genard, G., Descouvemont, P. & Terwagne, G. S-factor measurement of the $^{13}$C (p, γ) $^{14}$N reaction in reverse kinematics. *Journal of Physics: Conference Series* **202**, 012015 (2010).
35. Xu, Y., Takahashi, K., Goriely, S. & Arnould, M. NACRE II: an update of the NACRE compilation of charged-particle-induced thermonuclear reaction rates for nuclei with mass number A< 16. *Nuclear Physics* **A918**, 61-169 (2013).
36. Ishida, A., Peng, Y.-K. M. & Liu, W. Four-Fluid Axisymmetric Plasma Equilibrium Model Including Relativistic Electrons and Computational Method. *Under peer review by Physics of Plasmas.*