Geometrical thinking of junior high school students on the topic of lines and angles according to Van Hiele theory

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Abstract. Unconsciously, we use the concept and skill that we learned from the result of doing math questions, because the laws of mathematics govern everything. We can shaping up mathematics with geometry. Furthermore, this study aims to analyze geometrical thinking of junior high school students on the topic of lines and angles according to Van Hiele theory. This research use qualitative method with subject of research were four junior high school students in Bandung District, Indonesia. The data collection technique were used test and interview. The result showed that students with high abilities were at level 1 (analysis), and students with medium and low abilities were at level 0 (visualization).

1. Introduction
Mathematics is all around us. Unconsciously, we use the concept and skill that we learned from the result of doing math questions, because the laws of mathematics govern everything. Including learning in the 21st century, mathematic as a language that offers students to see the world, make a connection with other disciplines, and can help students solve complex problems [1].

From the survey result of Assessment and Teaching of 21st Century Skills (AT21CS) in [2] found that the majority of secondary school graduates were still minimum in mathematics and problem solving skills. Whereas we can shaping up mathematics with geometry [3]. This forming activity is a kind of systematic long-term instruction that help students more than just learn geometry, but can have better mathematical skills. Students become more equipped to deal with all schoolwork when they have a geometrical sense as a tool for thinking [3]. Geometry can also connect each topic in mathematics and mathematical topics to the real life [4] So, mathematics learning should be able to give emphasis to students’ thought processes, which one is geometrical thinking.

One of theory that discusses about the capability of geometrical thinking is Van Hiele’s theory. This theory explains the level of geometry understanding where students cannot achieve a level of thinking without going through the previous level, it mean consecutive from basic to abstract. The students’ level of geometric thinking divides into 5 levels, level 0 (visualization), level 1 (analysis), level 2 (informal deduction), level 3 (formal deduction), dan level 4 (rigor) [5]. Research [5] also explains, even though the generated data is enough to set an indicator of the development of geometric thought, however the research only provides the indicators for level 0 to 3, for the level 4 (rigor) there are no junior high school students who achieve in it. This level intended for higher levels of learning. Even though based on research [6], the students abilities of geometrical thinking have not been reached the rigor level, it is still between formal and informal deduction abilities when solving geometry questions.
The ability of Van Hiele’s geometrical thinking itself has been widely studied, for example research \cite{5} explain that some students are consistent in identifying and naming (level 0) and able to provide informal deductive explanations (level 2) \cite{7}. This is also supported by the result of research showed that the junior high school students’ level of geometric thinking is the highest at level 2 (informal deduction) and most of them are at level 0 (visualization) \cite{5}. Meanwhile if it is reviewed by gender, female students are at level 1 (analysis) and male students are at level 0 (visualization) \cite{8}. According to these results, this research is limited to analyze geometrical thinking of junior high school students’ based on Van Hiele’s theory but only at level 0 to level 2.

Furthermore, NCTM \cite{9} mentioned that one of five mathematical standard content is geometry, where the standard geometry in elementary school students up to 12th grade discusses four main fields i.e a) properties of shapes; b) locations and spatial relationships; c) transformations and symmetry; and d) visualization. As in this study, the geometry problem that students will solve is related to the lines and angles material. This material is taught in mathematic subject grade 7th junior high school. The result showed that there were still many students who made mistakes in answering questions related to the material of lines and angles, i.e error facts, principles, concepts, and operations \cite{10}, also students have difficulties such as an inaccuracy in writing symbols or calculations and students’ lack in understanding the concept even the presented questions \cite{11}. Therefore, this study aims to analyze geometrical thinking abilities of junior high school students on the topic of lines and angles according to Van Hiele theory.

2. Methods

This study used qualitative research method. The subjects of the study were 4 students of grade VII in the academic year of 2019/2020 in one of state Junior High School in Bandung District, Indonesia. We were choosen 4 students of the VII grade junior high school as the research subjects, where students have learned about lines and angles. The data were obtained by the test, followed by interview with the subject. The instruments of this study used the students’ geometrical thinking ability test based on Van Hiele’s level thinking geometry descriptor, and the interview guidelines that made based on an explanation of Van Hiele’s geometrical thinking test indicator. The data analysis methods include reducing, displaying, and getting conclusions.

3. Result and Discussion

According to the collecting samples that used to students of the VII grade junior high school, 4 students were choosen as the research subject. These four students have heterogeneous mathematical abilities, that consist of high, medium, and low ability students. These four students have done geometrical thinking test and interviewed. The names of the subjects research are shown in Table 1.

| Table 1. Subject Research |
|---------------------------|
| Initial                  |
| S1                       |
| S2                       |
| S3                       |
| S4                       |

3.1 Analysis of Students’ Geometry Skills at Level 0 (Visualization)

Figure 1 is a question about level 0 (visualization). Visualization or introduction level is the first level in the Van Hiele’s geometrical thinking. At this level, students were are able to recognize geometric shapes based on visual characteristic \cite{11}. From the result of the students’ answer test and interview, overall showed that these four students were able to identify pairs of parallel lines and pairs of intersect lines based on the presented figure.
Specifically, S1 and S3 has been able to write all pairs of parallel lines and perpendicular lines completely. They were able to identify sequentially each lines to the other lines, so there is no missed pairs and it doesn’t write twice. Meanwhile S2 and S4 were still incomplete in writing the pair of lines. Both of them are thorough less in looking the picture where there were lines that were actually a pair of perpendicular lines if the lines are extended.

Furthermore, even though its correct, the four students showed the different ways in writing the answers. S1 answers using the geometry symbol for example, \( \overline{p} \parallel \overline{q} \) to show the parallel lines, meanwhile S2 and S4 answers by redrawing the pair of lines. The differentiation way in answering is based on their differences way of thinking and expressing their thought. This statement then agree with Van de Walle’s [12] that not everyone thinks about geometric ideas in the same way.

3.2 Analysis of Students’ Geometry Skills at Level 1 (Analysis)

Analysis level is the second level in the Van Hiele’s geometrical thinking level. At this level an analysis of concepts and properties has been shown and the used the relationship between these properties to solve the problem. Here students can able to determine the properties of geometrical shape by observing, measuring, experimenting, and drawing. The question at level 1 (analysis) are presented in Figure 2 as follow:

According to the result of test and interview, the subjects who have high mathematical abilities are able to analyze the relationship of the angles on two parallel lines, meanwhile the subjects who have medium and low mathematical abilities have not been able to analyze it. S3 and S4 are not quite right
in answering the angle which is equal to angle 1 along with the reason. This is due to the lack of understanding of their concepts in this sub material. Moreover, many relationships between angles that generated from these two parallel lines also make the subject vulnerable to analyze it. Only S2 that can write down the result of analysis about the angular relationship completely and correctly, namely opposite angle and alternate exterior angle.

3.3 Analysis of Students’ Geometry Skill at Level 2 (Informal Deduction)
The third level in the Van Hiele’s geometrical thinking is informal deduction. This level is also known as the abstract level, the theoretical level, and the linkage level. At this level the subject compares the geometry components and doing problem solving that involving properties itself. However, based on the test and interview result, all subject have not been able to achieve this level of informal deduction. All subjects have difficulty in formulating and using definition, and giving deductive arguments.

The main difficulty factor for the subject is the difficulty in understanding the purpose of the given question. The subject is not accustomed to read carefully to try understand the problem. Also the subject difficult to start from where and bring up and idea for a deduction. Then, this is agree with Clements [13] that students tend to memorize material rather than learning the basic concepts of geometry, and students are also not prepared well during the leaning process.

4. Conclusion
Students’ geometrical thinking based on Van Hiele’s theory has different characteristics. The result showed that students with high abilities were at level 1 (analysis), and students with medium and low abilities were at level 0 (visualization). This level achievement still in the low category, considering there are five levels of Van Hiele’s geometrical thinking which is a hierarchy. So, the students can not achieve the up level if the previous level were not achieve. The main factor of student difficulty is a difficulty in understanding the purpose of the questions that were given and students are weak in the basic concept of angles so its hard to develop the formed angular relationships.

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