A curvaton with a polynomial potential

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Abstract. In general a weakly self-interacting curvaton field is expected and the curvaton potential takes a polynomial form. The curvaton potential can be dominated by the self-interaction term during the period of inflation if the curvaton field stays at a large vacuum expectation value. We use the $\delta N$ formalism to calculate the primordial curvature perturbation in the various possible scenarios which make the curvaton model much richer.

Keywords: cosmological perturbation theory, inflation, physics of the early universe
1. Introduction

Inflation [1]–[3] is the leading paradigm for solving the puzzles in the hot big bang model. The quasi-exponential expansion during inflation makes our universe almost homogeneous and isotropic. A bonus of inflation is that the quantum fluctuations of the scalar fields during inflation can naturally explain the small temperature fluctuations in the cosmic microwave background radiations and seed the formation of the large-scale structure. At the leading order, these quantum fluctuations are characterized by their amplitudes and tilts. If we only focus on the primordial power spectrum, we cannot distinguish inflation from the curvaton model [4]–[7].

The gravitational dynamics itself introduces important non-linearities, which will contribute to the final non-Gaussianity in the large-scale CMB anisotropies. In fact, the CMB non-Gaussianity [8]–[10] opens a window to probing the physics of the early universe. A well-understood ansatz of non-Gaussianity has a local shape. This kind of non-Gaussianity can be characterized by some non-linearity parameters $f_{NL}$, $g_{NL}$ and so on:

$$\zeta(x) = \zeta_g(x) + \frac{3}{5} f_{NL} \zeta_g^2(x) + \frac{9}{25} g_{NL} \zeta_g^3(x) + \cdots,$$

(1.1)

where $\zeta_g$ is the linear, Gaussian part of curvature perturbation. The current bound from WMAP five-year data [11] is $-9 < f_{NL}^{local} < 111$ at 95% CL. Even though a Gaussian distribution is still consistent with the present experiments, much of the allowed region...
for $f_{\text{NL}}^{\text{local}} < 0$ from WMAP three-year data was cut. In the single-field inflation model $f_{\text{NL}}^{\text{local}} \sim \mathcal{O}(n_s - 1)$ [12], which is constrained by WMAP ($n_s = 0.960^{+0.014}_{-0.013}$) [11] to be much less than unity. However the curvaton model can easily generate a large local-type non-Gaussianity [4]–[7], [13]–[16]. See [17] for a nice review and see [18]–[29] for the recent relevant discussions.

In general we can expect the curvaton field not to have only a mass term in its potential. Instead of the simplest curvaton potential $\frac{1}{2}m^2\sigma^2$, we adopt a form for the potential which allows a range of possibilities:

$$V(\sigma) = \frac{1}{2}m^2\sigma^2 + \sum_{n \geq 4} \lambda_n \frac{\sigma^n}{M^{n-4}}. \quad (1.2)$$

The term with $n > 4$ is non-renormalizable and suppressed by a UV scale $M$. If all of the interaction terms are negligible, $\frac{\delta\rho}{\rho} \sim 2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma}\right)^2$ and thus the second-order or higher order non-Gaussianity parameters ($g_{\text{NL}}, \ldots$) will be 0. In the literature the potential of the curvaton is assumed to be dominated by the mass term. If a subdominant interaction term is taken into account, the non-linear evolution on large scales is possible. The curvaton dynamics after inflation was discussed in [28]–[32]. In this case $f_{\text{NL}}$ can be small even when $f_D \ll 1$, but $g_{\text{NL}}$ should be large [16, 28, 29]. In all of these papers, the authors only focused on the case where the curvaton potential is always dominated by the mass term and the interaction term is taken as a perturbation. However, the self-interaction term can be dominant if the curvaton mass is small enough and the vacuum expectation value of the curvaton during inflation is large enough. If so, the higher order non-Gaussianity parameters are also expected to be larger.

In this paper we will use $\delta N$ formalism [33]–[35] to calculate the primordial curvature perturbation for the curvaton model with a polynomial potential. When the self-interaction term is taken into account, the curvaton model becomes much richer. Our paper is organized as follows. In section 2, we calculate the primordial power spectrum and the non-linearity parameters in various possible scenarios. In section 3, the spectral index of the primordial power spectrum and the enhancement of the second-order non-Gaussianity parameters are discussed. The evolutions of the curvaton before it starts to oscillate, and after it starts to oscillate but before it decays, are investigated in sections 4 and 5 respectively. In section 6, we give some discussion of the curvaton model.

2. Primordial curvature perturbation

In this paper we expand any field or perturbation at each order ($n$) as follows:

$$\zeta(t, x) = \zeta^{(1)}(t, x) + \sum_{n=2}^{\infty} \frac{1}{n!} \zeta^{(n)}(t, x). \quad (2.1)$$

We assume that the first-order term $\zeta^{(1)}$ is Gaussian and higher order terms describe the non-Gaussianity of the full non-linear $\zeta$. Working in the framework of Fourier transformation of $\zeta$, the primordial power spectrum $P_\zeta$ is defined by

$$\langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 P_\zeta(k_1) \delta^3(k_1 + k_2), \quad (2.2)$$
and the primordial bispectrum and trispectrum are defined by
\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta^3(k_1 + k_2 + k_3),
\]
(2.3)
\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\zeta(k_4) \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta^3(k_1 + k_2 + k_3 + k_4).
\]
(2.4)
The bispectrum and trispectrum are respectively related to the power spectrum by
\[
B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{NL} [P_\zeta(k_1)P_\zeta(k_2) + 2 \text{ permutations}],
\]
(2.5)
\[
T_\zeta(k_1, k_2, k_3, k_4) = \tau_{NL} [P_\zeta(k_1)P_\zeta(k_3)P_\zeta(k_4) + 11 \text{ permutations}]
+ \frac{24}{25} g_{NL} [P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + 3 \text{ permutations}].
\]
(2.6)
Here the non-linearity parameter \( \tau_{NL} \) is not an independent non-linearity parameter and it is given by
\[
\tau_{NL} = \frac{26}{25} f_{NL}^2.
\]
(2.7)
But \( g_{NL} \) is an independent parameter which will be calculated in this paper.

The primordial density perturbation can be described in terms of the non-linear curvature perturbation on uniform-density hypersurfaces [36]:
\[
\zeta(t, x) = \delta N(t, x) + \frac{1}{3} \int \frac{\rho(t, x)}{\bar{\rho} + \bar{p}},
\]
(2.8)
where \( \mathcal{N} = \int H \, dt \) is the integrated local expansion, \( \bar{\rho} \) is the homogeneous density in the background model, \( \bar{\rho} \) is the local density and \( \bar{p} \) is the local pressure.

For simplicity, the potential of the curvaton field is assumed to contain a mass term and a self-interaction term as follows:
\[
V(\sigma) = \frac{1}{2} m^2 \sigma^2 + \frac{1}{n(n - 1)} \lambda \sigma^n.
\]
(2.9)
The coupling constant \( \lambda \) takes dimensions of \( E^{4-n} \). The effective mass of the curvaton is given by
\[
\tilde{m} = \sqrt{m^2 + \lambda \sigma^{n-2}}.
\]
(2.10)
The potential is dominated by the interaction term if
\[
\sigma > \sigma_c = \left( \frac{n(n - 1)m^2}{2\lambda} \right)^{1/(n-2)}.
\]
(2.11)
For a weakly coupled field, the quantum fluctuations can be well described by a Gaussian random field [37]. Since we are also interested in the case where the self-interaction term dominates the potential, we want to estimate the effects of the non-linear quantum fluctuations in a curvaton field at Hubble exit during inflation. The fluctuation of the curvaton is expanded to the third order as follows:
\[
\sigma_* = \bar{\sigma}_* + \delta\sigma_*^{(1)} + \frac{1}{2} \delta\sigma_*^{(2)} + \frac{1}{6} \delta\sigma_*^{(3)},
\]
(2.12)
where \( \ast \) denotes that the quantities are evaluated at the Hubble exit during inflation. The perturbations of a self-interacting scalar field during inflation are discussed in the appendix. The second-order and the third-order perturbations are respectively related to
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\( \delta \sigma^{(1)} \) by equations (A.7) and (A.8). Here the curvaton potential deviates from the exactly quadratic form and then the non-linear evolution of the curvaton field on large scales is expected. Generally the initial amplitude of curvaton oscillations is some function of the field value at the Hubble exit:

\[
\sigma_o = \sigma_o(\sigma_s). \tag{2.13}
\]

Thus we can expand \( \sigma_o \) around \( \bar{\sigma}_o = \sigma_o(\bar{\sigma}_s) \) as follows:

\[
\sigma_o = \bar{\sigma}_o [1 + X + \frac{1}{2}(h_2 + \kappa_2)X^2 + \frac{1}{6}(h_3 + 3h_2\kappa_2 + \kappa_3)X^3], \tag{2.14}
\]

where

\[
X = \frac{\delta \sigma^{(1)}_o}{\sigma_o}, \tag{2.15}
\]

\[
h_2 = \frac{\bar{\sigma}_o \sigma''_o}{\sigma_o^2}, \quad \kappa_2 = -(n - 2)\frac{N_k \lambda \bar{\sigma}^{n-2}_o}{3\bar{H}^2} \tag{2.16}
\]

\[
h_3 = \frac{\bar{\sigma}_o^2 \sigma'''_o}{\sigma_o^3}, \quad \kappa_3 = -(n - 2)(n - 3)\frac{N_k \lambda \bar{\sigma}^{n-2}_o}{3\bar{H}^2} \tag{2.17}
\]

Here the prime denotes the derivative with respective to \( \sigma_s \). Usually \( \sigma'_o \sim \mathcal{O}(1) \), and then \( -\kappa_2 \sim -\kappa_3 \sim N_k \lambda \bar{\sigma}^{n-2}\bar{H}^2 \lesssim N_k \bar{\eta}^2/H^2 \ll 1 \). So it is also reasonable to consider that the quantum fluctuations of the curvaton at Hubble exit during inflation can be well described by a Gaussian random field, namely \( \sigma_s = \bar{\sigma}_s + \delta \sigma_s \). The higher order terms can be neglected even when the curvaton self-interaction term dominates its potential. In the following discussions, we will ignore all of the terms with \( \kappa_2 \) and \( \kappa_3 \). These terms can be recovered by taking \( h_2 \rightarrow h_2 + \kappa_2 \) and \( h_3 \rightarrow h_3 + 3h_2\kappa_2 + \kappa_3 \) if we want.

In order to make our calculations clearer, we calculate the curvature perturbations in the various possible scenarios separately.

### 2.1. The curvaton potential is dominated by the mass term during inflation

In this case, the value of the curvaton during inflation satisfies \( \sigma_s \ll \sigma_c \) and the interaction term can be taken as a perturbation. The curvature perturbation in this case has been discussed very thoroughly. To make our paper complete, we directly quote the results from [16].

The amplitude of the primordial power spectrum and the non-linearity parameters are respectively given by

\[
P_\zeta = \frac{1}{9\pi^2} f_D^2 g^2 \frac{H_0^2}{\sigma_*^2}, \tag{2.18}
\]

\[
f_{NL} = \frac{5}{4f_D} (1 + h_2) - \frac{5}{3} - \frac{5f_D}{6}, \tag{2.19}
\]

\[
g_{NL} = \frac{25}{54} \left[ \frac{9}{4f_D^2}(h_3 + 3h_2) - \frac{9}{f_D}(1 + h_2) + \frac{1}{2}(1 - 9h_2) + 10f_D + 3f_D^2 \right], \tag{2.20}
\]
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where

\[ q = \frac{\sigma_s \sigma'_0}{\sigma_0}, \quad f_D = \frac{3 \Omega_{\sigma,D}}{4 - \Omega_{\sigma,D}}, \tag{2.21} \]

where $\Omega_{\sigma,D}$ is the fraction of the curvaton energy density in the energy budget at the time of curvaton decay. If the curvaton potential is purely quadratic, $h_2 = h_3 = 0$ and then

\[ g_{NL} + \frac{10}{3} f_{NL} \simeq 0. \tag{2.22} \]

Any deviation from the above relation implies that the curvaton potential does not take the purely quadratic form.

### 2.2. The curvaton potential is dominated by the interaction term during inflation

In this subsection, we focus on the cases in which the self-interaction term dominates the curvaton potential during inflation. The value of the curvaton is roughly the same as that when it starts to oscillate. So we also assume that the curvaton energy density is dominated by the self-interaction term when it starts to oscillate. According to equation \((2.14)\), the curvaton density fluctuation during the curvaton oscillation can be expanded as

\[ \rho_{\sigma_o} = \bar{\rho}_{\sigma_o} \left( 1 + nX + \frac{n}{2}(n-1+h_2)X^2 + \frac{n}{6}(n-1)(n-2) + 3(n-1)h_2 + h_3 \right). \tag{2.23} \]

As demonstrated in [38], the energy density of an oscillating scalar field in an expanding universe with potential $V \sim \sigma^n$ scales as

\[ \rho_{\sigma} \sim a^{-6n/(n+2)}. \tag{2.24} \]

Or equivalently, the pressure of the curvaton when it is oscillating is related to its energy density by

\[ p = \frac{n-2}{n+2}\rho. \tag{2.25} \]

When the curvaton starts to oscillate, but before it decays, the non-linear curvature perturbation on uniform-curvaton density hypersurfaces is given by

\[ \zeta_{\sigma_o}(t, x) = \bar{\delta}\mathcal{N}(t, x) + \frac{n + 2}{6n} \int_{\bar{\rho}_{\sigma_o}}^{\rho_{\sigma_o}(t, x)} \frac{d\bar{\rho}_{\sigma_o}}{\bar{\rho}_{\sigma_o}}. \tag{2.26} \]

Therefore the curvaton density on spatially flat hypersurfaces is

\[ \rho_{\sigma_o}|_{\delta\mathcal{N}=0} = \exp \left[ \frac{6n}{n+2} \zeta_{\sigma_o} \right] \bar{\rho}_{\sigma_o}. \tag{2.27} \]

Considering equation \((2.23)\), order by order, we obtain

\[ \zeta_{\sigma_o}^{(1)} = \frac{n+2}{6}X, \tag{2.28} \]

\[ \zeta_{\sigma_o}^{(2)} = -\frac{6}{n+2}(1 - h_2) \left( \zeta_{\sigma_o}^{(1)} \right)^2, \tag{2.29} \]

\[ \zeta_{\sigma_o}^{(3)} = \left( \frac{6}{n+2} \right)^2 (2 - 3h_2 + h_3) \left( \zeta_{\sigma_o}^{(1)} \right)^3. \tag{2.30} \]
The energy density of the oscillating curvaton decreases as $\rho_\sigma \sim a^{-6n/(n+2)}$. If $n < 4$, the energy density of the curvaton increases with respect to radiation, but it decreases with respect to radiation if $n > 4$. On the other hand, the amplitude of the curvaton oscillations also decreases, and it is possible that the self-interaction term becomes subdominant before it decays. But it is also possible that this transition does not happen before the curvaton decays. We will investigate these two possibilities in section 5 in detail. Here we calculate the primordial curvature perturbation for these two cases separately.

2.2.1. The curvaton potential is dominated by the interaction term before it decays. The curvaton decay hypersurface is a uniform-density hypersurface and thus from equation (2.8) the perturbed expansion on this hypersurface is $\delta N = \zeta$, where $\zeta$ is the total curvature perturbation at the curvaton decay hypersurface. Before the curvaton decays, there have been radiations which are the products of inflaton decay. Since the equation of state of the radiations is $p_r = \rho_r/3$, the curvature perturbation related to radiations is

$$\zeta_r = \zeta + \frac{1}{4} \ln \frac{\rho_r}{\bar{\rho}_r}.$$  \hspace{1cm} (2.31)

The pressure of the oscillating curvaton is $p = ((n - 2)/(n + 2))\rho$ and thus

$$\zeta_\sigma_o = \zeta + \frac{n + 2}{6n} \ln \frac{\rho_\sigma_o}{\bar{\rho}_\sigma_o}.$$  \hspace{1cm} (2.32)

In the absence of interactions between radiations and the curvaton, the curvature perturbations $\zeta_r$ and $\zeta_\sigma_o$ are conserved individually and the above two equations can be written as

$$\rho_r = \bar{\rho}_r \exp [4(\zeta_r - \zeta)],$$  \hspace{1cm} (2.33)

$$\rho_\sigma_o = \bar{\rho}_\sigma_o \exp \left[\frac{6n}{n + 2}(\zeta_\sigma_o - \zeta)\right].$$  \hspace{1cm} (2.34)

At the time of curvaton decay, the total energy density $\rho_{tot}$ is conserved, i.e.

$$\rho_r(t_D, \mathbf{x}) + \rho_\sigma_o(t_D, \mathbf{x}) = \bar{\rho}_{tot}(t_D).$$  \hspace{1cm} (2.35)

Requiring that the total energy density is uniform on the decay surface, we have

$$(1 - \Omega_{\sigma,D}) e^{4(\zeta_r - \zeta)} + \Omega_{\sigma,D} e^{(6n/(n+2))(\zeta_\sigma_o - \zeta)} = 1,$$  \hspace{1cm} (2.36)

where $\Omega_{\sigma,D} = \bar{\rho}_{\sigma,D}/\bar{\rho}_{tot}$ is the fraction of the curvaton energy density in the energy budget at the time of curvaton decay. Here we assume that the curvaton suddenly decays into radiation. In the curvaton model, usually we also assume that the curvature perturbation generated by the inflaton is very small and can be ignored, e.g. $\zeta_r = 0$. Order by order
from equation (2.36), we have
\[
\zeta^{(1)} = f_D \zeta^{(1)}_o, \quad (2.37)
\]
\[
\zeta^{(2)} = \left[ \frac{6(n-1+h_2)}{(n+2)f_D} - \frac{8(n-1)}{n+2} - \frac{2(4-n)}{n+2} f_D \right] (\zeta^{(1)})^2, \quad (2.38)
\]
\[
\zeta^{(3)} = \left[ \frac{36}{(n+2)^2 f_D^2} [(n-1)(n-2) + h_3 + 3(n-1)h_2] - \frac{144}{(n+2)^2 f_D} (n-1)(n-1+h_2) 
+ \frac{4}{(n+2)^2} [44n^2 - 121n + 68 - 9(4-n)h_2] 
- \frac{80}{(n+2)^2} (n-1)(n-4)f_D + \frac{12}{(n+2)^2} (4-n)^2 f_D^2 \right] (\zeta^{(1)})^3, \quad (2.39)
\]
where
\[
f_D = \frac{3n\Omega_{\sigma,D}}{2(n+2) - (4-n)\Omega_{\sigma,D}}. \quad (2.40)
\]
Therefore the amplitude of the primordial power spectrum is
\[
P_\zeta = \left( \frac{n+2}{12\pi} \right)^2 f_D^2 q^2 H^2 / \sigma^2, \quad (2.41)
\]
Making the identification \(\zeta^{(1)} = \zeta_o\) and recalling \(\zeta = \zeta^{(1)} + \frac{1}{2} \zeta^{(2)} + \frac{1}{6} \zeta^{(3)}\), from equation (1.1) the non-linearity parameters are given by
\[
f_{NL} = \frac{5}{6} \left[ \frac{6(n-1+h_2)}{(n+2)f_D} - \frac{8(n-1)}{n+2} - \frac{2(4-n)}{n+2} f_D \right], \quad (2.42)
\]
\[
g_{NL} = \frac{25}{54} \left[ \frac{36}{(n+2)^2 f_D^2} [(n-1)(n-2) + h_3 + 3(n-1)h_2] 
- \frac{144}{(n+2)^2 f_D} (n-1)(n-1+h_2) + \frac{4}{(n+2)^2} [44n^2 - 121n 
+ 68 - 9(4-n)h_2] - \frac{80}{(n+2)^2} (n-1)(n-4)f_D 
+ \frac{12}{(n+2)^2} (4-n)^2 f_D^2 \right]. \quad (2.43)
\]
For \(n = 2\), these results are just the same as those in section 2.1. For \(n \neq 2\), if \(f_D \ll 1\), \(g_{NL} \approx 50(n-1)(n-2)/(3(n+2)^2 f_D^2)\) which is large, and
\[
g_{NL} \approx 2(n-2)/(3(n-1)f_D^2). \quad (2.44)
\]
For \(n > 2\), \(g_{NL}\) is positive.

These results can be easily understood. The energy density of the curvaton is \(\rho_\sigma \sim \sigma^n\).

Considering \(\sigma \rightarrow \sigma + \delta \sigma\), we have
\[
\frac{\delta \rho_\sigma}{\rho_\sigma} \sim n \frac{\delta \sigma}{\sigma} + \frac{1}{2} n(n-1) \left( \frac{\delta \sigma}{\sigma} \right)^2 + \frac{1}{6} n(n-1)(n-2) \left( \frac{\delta \sigma}{\sigma} \right)^3. \quad (2.45)
\]
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Since $\zeta_g \simeq ((n+2)/6)f_D(\delta \sigma /\sigma)$, the curvature perturbation reads

$$\zeta \simeq \zeta_g + \frac{3(n-1)}{(n+2)f_D}\zeta_g^2 + \frac{6(n-1)(n-2)}{(n+2)^2f_D^2}\zeta_g^3.$$  \hfill (2.46)

Using equation (1.1), we find $f_{NL} \simeq \frac{5(n-1)}{(n+2)f_D}$ and $g_{NL} \simeq \frac{50(n-1)(n-2)}{3(n+2)^2f_D^2}$.

2.2.2. The mass term becomes dominant before the curvaton decays. The equation of state of the oscillating curvaton is $p = ((n-2)/(n+2))\rho$ when $\lambda \sigma^n$ is dominant, and $p = 0$ when the mass term is dominant. In this case, there is a transition from $p = ((n-2)/(n+2))\rho$ to $p = 0$ when the amplitude of the curvaton oscillations is roughly $\sigma_c$. Since the pressure of an oscillating curvaton field is a unique function of its energy density, the energy conservation implies that the curvature perturbation $\zeta_c$ is conserved [39] even when the equation of state of the oscillating curvaton changes.

In this case, the pressure of the oscillating curvaton is $p = 0$ before it decays and thus

$$\zeta_c = \zeta + \frac{1}{3} \ln \frac{\rho_c}{\rho_c}.$$  \hfill (2.47)

Similarly, on the curvaton decay hypersurface, we have

$$(1 - \Omega_{\sigma,D})e^{4(\zeta - \zeta)} + \Omega_{\sigma,D}e^{3(\zeta - \zeta)} = 1.$$  \hfill (2.48)

Order by order, the curvature perturbation reads

$$\zeta^{(1)} = f_D \zeta_c^{(1)},$$  \hfill (2.49)

$$\zeta^{(2)} = \left[ \frac{3(n+2h_2)}{(n+2)f_D} - 2 - f_D \right] (\zeta^{(1)})^2,$$  \hfill (2.50)

$$\zeta^{(3)} = \left[ \frac{9}{(n+2)^2f_D^2}(n(n-2) + 4h_3 + 6nh_2) - \frac{18}{(n+2)f_D}(n + 2h_2) 
 + \frac{2}{n+2}(5 - 2n - 9h_2) + 10f_D + 3f_D^2 \right] (\zeta^{(1)})^3,$$  \hfill (2.51)

where

$$f_D = \frac{3\Omega_{\sigma,D}}{4 - \Omega_{\sigma,D}}.$$  \hfill (2.52)

The amplitude of the primordial power spectrum and the non-linearity parameters are

$$P_\zeta = \left( \frac{n+2}{12\pi} \right)^2 f_D^2 \frac{\sigma_c^2}{H_*^2},$$  \hfill (2.53)

$$f_{NL} = \frac{5}{6} \left[ \frac{3(n+2h_2)}{(n+2)f_D} - 2 - f_D \right],$$  \hfill (2.54)

$$g_{NL} = \frac{25}{54} \left[ \frac{9}{(n+2)^2f_D^2}(n(n-2) + 4h_3 + 6nh_2) 
 - \frac{18}{(n+2)f_D}(n + 2h_2) + \frac{2}{n+2}(5 - 2n - 9h_2) + 10f_D + 3f_D^2 \right].$$  \hfill (2.55)

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For \( n = 2 \), these results are also the same as those in section 2.1. For \( n \neq 2 \), if \( f_D \ll 1 \),
\[ g_{NL} \simeq \frac{2(n-2)}{3n} f_{NL}^2, \]
which is different from that in section 2.2.1.

3. The spectral index of the primordial power spectrum and a mixed scenario

The spectral index is an important quantity for characterizing the primordial power spectrum. In the curvaton model, the scale dependence of the primordial power spectrum is the same as that of \( \delta \sigma^{(1)} \). So the spectral index of the primordial power spectrum in the curvaton model takes the form
\[ n_{cv} = 1 + \frac{2\tilde{m}_{*}^2}{3H_*^2} - 2\epsilon, \quad (3.1) \]

where \( \tilde{m}_* \) is the effective mass of the curvaton at \( \sigma = \sigma_* \) and \( \epsilon = -\dot{H}_*/H_*^2 \) is a slow-roll parameter. This result is valid for all of the previous scenarios. Since we have \( \tilde{m}_* \ll H_* \) in the curvaton model, a small value of \( \epsilon \) and a closely scale-invariant power spectrum are expected. However WMAP five-year data favor a red-tilted power spectrum.

In \cite{26}, we suggested a mixed scenario in which the curvature perturbation generated by the inflaton also makes a significant contribution to the primordial power spectrum, in order to naturally obtain a red-tilted power spectrum in the curvaton model. Denote the curvature perturbation generated by the curvaton as \( P_{\zeta}^{cv} \).
\[ P_{\zeta}^{cv} = \beta P_{\zeta}^{tot}, \]
the spectral index of the total primordial power spectrum becomes
\[ n_s = \beta n_{cv} + (1 - \beta)n_{inf}^{s}, \quad (3.2) \]

where \( n_{inf}^{s} = 1 - 6\epsilon + 2\eta \) is the spectral index of the power spectrum generated by the inflaton. Now the bispectrum and trispectrum are respectively related to the power spectrum by
\[ B_{\zeta}(k_1, k_2, k_3) \simeq \frac{6}{5} f_{NL}^{cv}[P_{\zeta}^{cv}(k_1)P_{\zeta}^{cv}(k_2) + 2 \text{ permutations}], \quad (3.3) \]
\[ T_{\zeta}(k_1, k_2, k_3, k_4) \simeq \tau_{NL}^{cv}P_{\zeta}^{cv}(k_1)P_{\zeta}^{cv}(k_2) + 11 \text{ permutations} \]
\[ + \frac{54}{25}g_{NL}^{cv}[P_{\zeta}^{cv}(k_2)P_{\zeta}^{cv}(k_3)P_{\zeta}^{cv}(k_4) + 3 \text{ permutations}], \quad (3.4) \]

where we ignore the contribution to the non-linearity parameters from the fluctuation of the inflaton. Since \( P_{\zeta}^{cv} = \beta P_{\zeta}^{tot} \), the observed non-Gaussianity parameters become
\[ f_{NL} \simeq \beta^2 f_{NL}^{cv}, \quad \tau_{NL} \simeq \beta^3 \tau_{NL}^{cv}, \quad g_{NL} \simeq \beta^3 g_{NL}^{cv}. \quad (3.5) \]

Considering equation (2.7), we have
\[ \tau_{NL} = \frac{36}{25\beta}(f_{NL})^2. \quad (3.6) \]

In section 2, we conclude that the second-order non-Gaussianity parameter \( g_{NL}^{cv} \) is proportional to \( (f_{NL}^{cv})^2 \), i.e. \( g_{NL}^{cv} = c(f_{NL}^{cv})^2 \) where the coefficient \( c \) is different in different
cases. Similarly we have
\[ g_{NL} = \frac{c}{\beta}(f_{NL})^2. \] (3.7)
If \( \epsilon \simeq 0, n_s \simeq 1 + 2(1 - \beta)\eta \) and a red-tilted primordial power spectrum is obtained if \( \beta < 1 \). Now the second-order non-Gaussianity parameters are enhanced by a factor \( 1/\beta \) for a fixed \( f_{NL} \). Or equivalently, the bound on \( g_{NL} \) from experiments will give a bound on \( \beta \) for a given \( f_{NL} \).

4. Curvaton dynamics and the non-linearity parameters

After inflation, our universe is dominated by radiation and the Hubble parameter goes like \( H = 1/(2t) \). Usually we assume that the curvaton field does not evolve until the Hubble parameter drops below the effective mass of the curvaton. Once \( H \sim \mathcal{O}(\tilde{m}) \), the curvaton starts to oscillate around \( \sigma = 0 \). However the curvaton field evolves slowly even when \( H > \tilde{m} \) and the non-linear evolution is also expected if the interaction term is taken into account. The evolution of the curvaton after inflation, but before it oscillates, has been discussed in [28]–[32] where the interaction term is regarded as a perturbation.

Here we pay attention to the case where the interaction term is dominant before the curvaton starts to oscillate. The curvaton equation of motion after inflation is
\[ \ddot{\sigma} + \frac{3}{2t}\dot{\sigma} = -\frac{\lambda}{n - 1}\sigma^{n-1}. \] (4.1)
It is difficult to find an analytic solution for this non-linear differential equation. Before the curvaton starts to oscillate, the effective curvaton mass is smaller than the Hubble parameter. So the curvaton slowly rolls down its potential. Taking the slow-roll approximation, the curvaton equation of motion is simplified to
\[ \frac{3}{2t}\dot{\sigma} \simeq -\frac{\lambda}{n - 1}\sigma^{n-1}, \] (4.2)
whose solution with the initial condition \( \sigma_{ini} = \sigma_* \) at \( t = 0 \) is given by
\[ \sigma(t) \simeq \sigma_* - \frac{\lambda t^2}{3(n - 1)}\sigma_*^{n-1}. \] (4.3)
The curvaton begins to oscillate roughly at the time of \( t_o = 1/(2\tilde{m}_o) \) which corresponds to \( H = \tilde{m}_o \simeq \sqrt{\lambda}\sigma_*^{n-2} \). Now the parameters \( q \), \( h_2 \) and \( h_3 \) are respectively given by
\[ q \simeq 1 - \frac{x_o^2}{3}, \] (4.4)
\[ h_2 \simeq -(n - 2)\frac{x_o^2}{3}, \] (4.5)
\[ h_3 \simeq -(n - 2)(n - 3)\frac{x_o^2}{3}, \] (4.6)
where \( x_o = \tilde{m}t_o \simeq 1/2 \). For \( n = 4 \), \( q \simeq 0.92, h_2 \simeq h_3 \simeq -0.17 \). The corrections to the non-linearity parameters from the curvaton dynamics after inflation, but before the curvaton begins to oscillate, are at the sub-leading order for the case where the interaction term is dominant before it starts to oscillate.
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5. Evolution of the curvaton after the curvaton starts to oscillate

In this section, we assume that our universe is dominated by radiation before the curvaton decays in order to have a large non-Gaussianity. The case in which the curvaton potential is always dominated by the mass term has been discussed very well. Here we focus only on the evolution of the oscillating curvaton whose potential is dominated by the interaction term when it starts to oscillate.

Assume $a = 1$ at the time of the curvaton starting to oscillate. At this time the effective mass of the curvaton is

$$m_o = \sqrt{m^2 + \lambda \sigma_*^{n-2}} \approx \sqrt{\lambda \sigma_*^{n-2}},$$

(5.1)

and the energy density of the curvaton is

$$\rho_{\sigma_o} \sim \frac{\lambda}{n(n-1)} \sigma_*^n.$$  

(5.2)

Here we assume $\sigma_o = \sigma_* > \sigma_c$. The curvaton energy density drops as $\rho_{\sigma} = \rho_{\sigma_o} a^{-6n/(n+2)}$. When the universe evolves to

$$a = a_c = \left(\frac{\sigma_*}{\sigma_c}\right)^{(n+2)/6},$$

(5.3)

the mass term begins to be dominant. The curvaton starts to decay at the time when the Hubble parameter drops below the curvaton decay rate $\Gamma_{\sigma}$ and the scale factor is

$$a_{\Gamma} = \sqrt{\frac{m_o}{\Gamma_{\sigma}}}.$$  

(5.4)

If $a_c < a_{\Gamma}$, the curvaton potential is dominated by the mass term before it decays, and then

$$\Omega_{\sigma,D} \approx \frac{\sigma_*^2}{6M_p^2} \frac{m^2}{\tilde{m}_o^{3/2}} \frac{\sigma_*}{\Gamma_{\sigma}}^{n/2-1}.$$  

(5.5)

If $a_c > a_{\Gamma}$, the curvaton potential is always dominated by the interaction term before it decays and we have

$$\Omega_{\sigma,D} \approx \frac{\sigma_*^2}{3n(n-1)M_p^2} \frac{\tilde{m}_o}{\Gamma_{\sigma}}^{(4-n)/(2+n)}.$$  

(5.6)

The curvaton energy density does not decrease until it starts to oscillate. So the curvaton energy density increases with respect to radiation before it starts to oscillate for an arbitrary value of $n$. At the time when the curvaton starts to oscillate ($H = \tilde{m}_o$), we have

$$\Omega_{\sigma,o} = \frac{\lambda \sigma_*^n / (n(n-1))}{3M_p^2 H^2} = \frac{\sigma_*^2}{3n(n-1)M_p^2},$$

(5.7)

which can be $O(1)$ if $\sigma_* \sim M_p$, even though the energy density of the curvaton is negligible during inflation. The energy density of the oscillating curvaton with $n = 4$ goes like $a^{-4}$ which is the same as for the radiation, and then $\Omega_{\sigma,D} = \Omega_{\sigma,o}$. For $n > 4$, the energy density of the oscillating curvaton decreases with respect to radiation and thus $\Omega_{\sigma,D}$ is
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suppressed by a factor \((\Gamma_\sigma/\tilde{m}_o)^{(n-4)/(2+n)}\). One point that we want to stress is that \(\Omega_{\sigma,D}\) can be much larger than the fraction of the curvaton energy density in the energy budget during inflation.

Here we also want to estimate the typical value of the curvaton during inflation. The behavior of a light scalar field in de Sitter space has been studied in [40]–[43]. The quantum fluctuation can be taken as a random walk:

\[
\langle \sigma^2 \rangle = \frac{H^3}{4\pi^2} t. \tag{5.8}
\]

On the other hand, the long wavelength modes of the light scalar field are in the slow-roll regime and obey the slow-roll equation of motion, i.e.

\[
3H_\ast \frac{d\sigma}{dt} = -\frac{dV(\sigma)}{d\sigma} = -m^2 \sigma - \frac{\lambda}{n-1} \sigma^{n-1}. \tag{5.9}
\]

Combining these two considerations, we have

\[
\frac{d\langle \sigma^2 \rangle}{dt} = \frac{H^3}{4\pi^2} - \frac{2m^2}{3H_\ast} \langle \sigma^2 \rangle - \frac{2\lambda}{3(n-1)H_\ast} \langle \sigma^2 \rangle^{n/2}. \tag{5.10}
\]

For \(n = 4\), our result is the same as that in [43]. In the case with a dominant interaction term, the solution of the above differential equation approaches a constant equilibrium value:

\[
\sigma_\ast \simeq \left( \frac{3(n-1)H_\ast^4}{8\pi^2 \lambda} \right)^{1/n}, \tag{5.11}
\]

which can be estimated as the typical value of the curvaton during inflation. Now the requirement of \(\sigma_\ast > \sigma_c\) yields

\[
H_\ast^{2n-4} > \frac{3(n-1)}{8\pi^2} \left( \frac{4\pi^2}{3} \right)^{n/2} \frac{m^n}{\lambda}. \tag{5.12}
\]

As a concrete example, we investigate the case of \(n = 4\) in the following subsection.

5.1. The case of \(n = 4\) with \(\sigma_\ast > \sigma_c\)

In this subsection, we estimate the value of the curvaton during inflation as the typical value, namely

\[
\sigma_\ast \simeq 0.58\lambda^{-1/4}H_\ast, \tag{5.13}
\]

and the effective mass of the curvaton when it starts to oscillate is

\[
\tilde{m}_o \simeq 0.58\lambda^{1/4}H_\ast. \tag{5.14}
\]

In this case, \(\sigma_c = \sqrt{6\lambda^{-1/2}}m\) and \(\sigma_\ast > \sigma_c\) gives

\[
m < 0.24\lambda^{1/4}H_\ast. \tag{5.15}
\]

We also have

\[
a_c = 0.24\lambda^{1/4}H_\ast \frac{H_\ast}{m}, \quad a_\Gamma = 0.76\lambda^{1/8} \sqrt{\frac{H_\ast}{\Gamma_\sigma}}. \tag{5.16}
\]
Requiring $a_c < a_\Gamma$ yields $\Gamma_\sigma \lesssim \Gamma_c = 10\lambda^{-1/4}m^2/H_*$. If the curvaton decay rate is roughly the same as the gravitational strength decay rate $\Gamma_\sigma = m^3/M_p^2$, $a_\Gamma \sim a_c \sqrt{M_p^2/(mH_*\lambda^{1/4})} > a_c$ and the mass term becomes dominant before the curvaton decays. On the other hand, the gravitational wave perturbation only depends on the inflation scale and $H_*$ is related to the tensor–scalar ratio $r$ by $H_* = 10^{-1}\sqrt{r}M_p$.

- $a_c > a_\Gamma$. The interaction term always dominates the curvaton potential before the curvaton decays. The amplitude of the primordial power spectrum and the non-linearity parameter generated by the curvaton field are respectively given by $P_*^{\text{cur}} = 0.075\sqrt{2}f_D^2$ and $f_{NL}^{\text{cur}} \simeq 5/(2f_D)$. The WMAP normalization [11] is $P_{\zeta,\text{wmap}} = 2.457 \times 10^{-9}$. If the total amplitude of the primordial power spectrum is contributed by the curvaton fluctuation, we have $P_*^{\text{cur}} = P_{\zeta,\text{wmap}}$ which implies $\sqrt{\lambda} \geq 3.3 \times 10^{-8}$ because $f_D = \Omega_{\sigma,D} \leq 1$. On the other hand, we have $f_D = \Omega_{\sigma,D} \simeq 9.3 \times 10^{-3}\lambda^{-1/2}H_*^2/M_p^2$ and then

$$f_{NL} = f_{NL}^{\text{cur}} = 2.7 \times 10^2\frac{\sqrt{\lambda}M_p^2}{H_*^2} \gtrsim 891 \frac{10}{r}.$$  

(5.17)

The limit of $r$ from WMAP5 is $r < 0.2$. The non-Gaussianity parameter $f_{NL}$ is much larger than the upper bound from WMAP five-year data and the above scenario has been ruled out. On the other hand, in order to make this point clearer, let us start with the bound on $f_{NL}^{\text{cur}}$. Requiring $f_{NL}^{\text{cur}} = 5/(2\Omega_{\sigma,D}) = 90M_p^2/\sigma_\star^2 \lesssim 111$ yields $\sigma_\star^2 \gtrsim 0.81M_p^2$. Since $f_D \leq 1$, $P_*^{\text{cur}} = (1/4\pi^2)f_D^2(H_*^2/\sigma_\star^2) \lesssim 3 \times 10^{-10}r < P_{\zeta,\text{wmap}}$. In order to satisfy the bound on the non-Gaussianity from WMAP five-year data, it is natural to assume that the fluctuation of the inflaton makes a significant contribution to the primordial power spectrum, namely $P_*^{\text{cur}} = \beta P_{\zeta,\text{wmap}}$. Now the above constraints are released to give $\sqrt{\lambda} \lesssim 3.3 \times 10^{-8}\beta$ and

$$f_{NL} \geq \frac{891\beta^3}{r}.$$  

(5.18)

Considering $f_{NL} < 111$ and $r < 0.2$ yields $\beta \lesssim 0.3$ and thus $g_{NL} \gtrsim 3c(f_{NL})^2$. If $f_{NL} = 30$ and $r = 10^{-3}$, $\beta \lesssim 0.03$ and $g_{NL} \gtrsim 3 \times 10^4c$ which can be detected by Planck.

- $a_c < a_\Gamma$. We also consider $P_*^{\text{cur}} = \beta P_*^{\text{tot}}$. In this case, $P_*^{\text{cur}} = 0.075\sqrt{2}f_D^2$, $f_{NL}^{\text{cur}} \simeq 5/(3f_D)$, and then $P_*^{\text{tot}} = 0.21\beta^3\sqrt{\lambda}/(f_{NL})^2$. The WMAP normalization [11] reads

$$\lambda \simeq 1.4 \times 10^{-16}(f_{NL})^{1/\beta^6}.$$  

(5.19)

On the other hand, $f_D \simeq \frac{2}{4\Omega_{\sigma,D}} = 2.3 \times 10^{-2}mH_*^{3/2}/(\lambda^{5/8}M_p^2\Gamma_\sigma^{1/2})$. Considering $f_{NL} = \beta^2f_{NL}^{\text{cur}}$, we have

$$f_{NL} \simeq 23\beta^{7/6}\sqrt{r}\left(\frac{m^2}{M_p\Gamma_\sigma}\right)^{1/3},$$  

(5.20)

which is compatible with WMAP five-year data if the curvaton mass is not too large even when the primordial power spectrum is generated by the curvaton field ($\beta = 1$). In this scenario, we have $a_c < a_\Gamma$ and $\sigma_\star > \sigma_c$, namely

$$2 \times 10^{-6}r^{3/4}\sqrt{\Gamma_\sigma M_p/\beta^{1/4}} \leq m \leq 2 \times 10^{-22} M_p^2 \beta/\Gamma_\sigma.$$  

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From equation (5.20), the lower bound on the curvaton mass is automatically satisfied for $f_{\text{NL}} \gtrsim O(1)$. The upper bound on the curvaton mass leads to a bound on $f_{\text{NL}}$ from above as follows:

$$
f_{\text{NL}} \leq 8 \times 10^{-14} \frac{\beta^{1/2} r^{5/2}}{\Gamma_\sigma} M_p. \tag{5.22}
$$

A large non-Gaussianity is achieved only when the curvaton decay rate is very small compared to $M_p$. In [30] the author pointed out that the curvaton should decay before neutrino decoupling, namely $\Gamma_\sigma > \Gamma_0 = 1.8 \times 10^{-43} M_p$. Otherwise the curvature perturbations may be accompanied by a significant isocurvature neutrino perturbation. This requirement leads to

$$
f_{\text{NL}} \lesssim 4 \times 10^{29} \beta^{1/2} r^{5/2} \left( \frac{M_{\text{CDM}}}{20} \right)^2 / M_p \tag{5.23}
$$

In [30] another constraint on $\Gamma_\sigma$ is $\Gamma_\sigma \gtrsim \Gamma_g = m^3 / M_p^2$ which yields

$$
f_{\text{NL}} \lesssim 23 \beta^{7/6} \sqrt{r} (M_p / m)^{1/3} \tag{5.24}
$$

directly from equation (5.20). According to equation (5.22), we have

$$
f_{\text{NL}} \lesssim 8 \times 10^{-14} \frac{\beta^{1/2} r^{5/2}}{M_p^3 / m^3}, \tag{5.25}
$$

which is more restricted than equation (5.24) if $m / M_p > 3.8 \times 10^{-6} \beta^{-1/4} r^{3/4}$.

To summarize, if the self-interaction term $\lambda \sigma^4$ is always dominant, $\Omega_{\sigma,D}$ will be too small and the non-Gaussianity too large for fitting the WMAP five-year data unless the fluctuation of the inflaton makes the main contribution to the primordial power spectrum. The constraint on the model where the curvaton potential is always dominated by the interaction term with $n > 4$ before the curvaton decays will be more stringent. However, because the energy density of the oscillating curvaton whose potential is dominated by the mass term grows with respect to the radiation, the non-Gaussianity can be compatible with WMAP five-year data even when the primordial power spectrum is mainly generated by the curvaton.

6. Discussion

In this paper we use the $\delta N$ formalism to calculate the primordial curvature perturbation on large scales in the curvaton model with a polynomial potential. The main contribution to the non-Gaussianity in the curvaton model comes from the non-linear gravitational perturbations, rather than the curvaton self-interaction. Our calculations are also straightforward to apply to cases with more complicated curvaton potentials. When the self-interaction term dominates the curvaton potential during inflation, the order of magnitude of the second-order non-linearity parameters $\tau_{\text{NL}}$ and $g_{\text{NL}}$ is roughly $O(f_{\text{NL}}^2)$ if $f_{\text{NL}} \gg 1$. 

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A red-tilted primordial power spectrum can be naturally achieved in the curvaton model if the fluctuation of the inflaton also makes a significant contribution to it [26]. In this mixed scenario, it is also possible to detect the non-Gaussianity generated during inflation in the generalized inflation models [44]–[52]. As another interesting observation for this mixed scenario, the second-order non-Gaussianity parameters $\tau_{NL}$ and $g_{NL}$ are enhanced for fixed $f_{NL}$. In addition, multiple curvatons are generically expected in the fundamental theories, such as string theory. It would be worth investigating the curvature perturbation in the $N$-vaton [26] with a polynomial potential.

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Appendix. Perturbations of a light self-interacting scalar field in the inflationary universe

In this section we consider the perturbations of a light self-interacting scalar field $\sigma$ whose energy density is subdominant in the inflationary universe. The potential of $\sigma$ is given in equation (2.9).

We expand the curvaton field up to third order in the perturbations around the homogeneous background as

$$\sigma(t, x) = \sigma(t) + \delta\sigma^{(1)}(t, x) + \frac{1}{2}\delta\sigma^{(2)}(t, x) + \frac{1}{6}\delta\sigma^{(3)}(t, x).$$

During inflation the equations of motion for the homogeneous part and the perturbations on large scales in cosmic time are given by

$$\ddot{\sigma} + 3H\dot{\sigma} = -m^2\sigma - \frac{\lambda}{n - 1}\sigma^{n-1},$$

$$\ddot{\delta\sigma}^{(1)} + 3H\dot{\delta\sigma}^{(1)} = -\tilde{m}^2\delta\sigma^{(1)},$$

$$\ddot{\delta\sigma}^{(2)} + 3H\dot{\delta\sigma}^{(2)} = -\tilde{m}^2\delta\sigma^{(2)} - (n - 2)\lambda\sigma^{n-3}(\delta\sigma^{(1)})^2,$$

$$\ddot{\delta\sigma}^{(3)} + 3H\dot{\delta\sigma}^{(3)} = -\tilde{m}^2\delta\sigma^{(3)} - 3(n - 2)\lambda\sigma^{n-3}\delta\sigma^{(1)}\delta\sigma^{(2)} - (n - 2)(n - 3)\lambda\sigma^{n-4}(\delta\sigma^{(1)})^3.$$  

In the slow-roll approximation,

$$3H\dot{\delta\sigma}^{(2)} \simeq -(n - 2)\lambda\sigma^{n-3}(\delta\sigma^{(1)})^2,$$

whose solution is roughly given by [17, 53]

$$\delta\sigma^{(2)} \sim -\frac{N_k}{3H^2}(n - 2)\lambda\sigma^{n-3}(\delta\sigma^{(1)})^2,$$

where $N_k = \int_{t_k}^{t_{\text{end}}} H \, dt$ is the number of e-folds between the end of inflation and the time $t_k$ when the scale of wavenumber $k$ leaves the horizon during inflation. Typically $N_k = 60$. Similarly, the solution of $\delta\sigma^{(3)}$ reads

$$\delta\sigma^{(3)} \sim -\frac{N_k}{3H^2}(n - 2)(n - 3)\lambda\sigma^{n-4}(\delta\sigma^{(1)})^3.$$
Both $\delta \sigma^{(2)}$ and $\delta \sigma^{(3)}$ are proportional to the coupling constant. In the limit of $\lambda \to 0$, we only need to expand the curvaton field to $\delta \sigma^{(1)}$ without higher order, non-Gaussian terms.

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