Effects of massive gravity on \(p\)-wave holographic superconductor

Cao H. Nam

Phenikaa Institute for Advanced Study and Faculty of Fundamental Sciences,
Phenikaa University, Yen Nghia, Ha Dong, Hanoi 12116, Vietnam

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In this paper, we have analytically investigated \(p\)-wave holographic superconductor in the framework of massive gravity in the probe limit. We obtained the analytical expressions for the critical temperature, the value of the condensate operator, and the difference of the free energy between the superconductor and normal phases. We studied the behavior of these quantities in the presence of graviton mass, which is found to be dependent crucially on the sign of the massive gravity couplings. For the (negative)positive massive gravity couplings, the critical temperature becomes (lower)higher and the condensate value gets (larger)smaller with increasing the coupling amplitude and graviton mass. This fact corresponds to that the superconductor phase is (more)less thermodynamically favored.

I. INTRODUCTION

Massive gravity is a modification of Einstein gravity in the infrared regime with including a mass to graviton. As a result, this modification leads to new physical degrees of freedom and thus it has profound consequences such as the natural resolution for the acceleration of our universe without introducing dark energy. The recent direct detections of the gravitational waves by LIGO on the binary black hole merger have provided an upper bound on the graviton mass as, \(m_g \leq 1.2 \times 10^{-22}\,\text{eV}\) \[1\]. This has made the question regarding whether the graviton has mass become more interesting. The first attempt to construct massive gravity was done by Fierz and Pauli \[2\]. Unfortunately, this massive gravity suffers from a pathology well-known as the van Dam-Veltman-Zakharov (vDVZ) discontinuity \[3, 4\]. As showed by Vainshtein, the vDVZ discontinuity could be resolve in a nonlinear framework by introducing higher order interaction terms \[5\]. However, nonlinear massive gravity encounters the Boulware-Deser (BD) ghost \[6\]. In 2010, de Rham, Gabadadze and Tolley proposed a successful nonlinear massive gravity theory which avoids the vDVZ discontinuity and the BD ghost \[7, 8\].

The AdS/CFT correspondence \[9\] determines a relationship between the weakly coupled grav-
ity theory in \((d + 1)\)-dimensional bulk AdS spacetime and the strongly coupled conformal field theory living on the \(d\)-dimensional boundary. Hence, this correspondence has been regarded as a powerful tool to explore physics of the strongly correlated system by using the weakly interacting gravitational dual in one higher dimension. Based on the ideal of the AdS/CFT correspondence, Hartnoll et al. built a model of \(s\)-wave holographic superconductor at which a complex scalar field is coupled to the \(U(1)\) gauge field in the framework of Einstein gravity \([10, 11]\). It was found that below a critical temperature RN-AdS black brane which is dual to the normal phase in the boundary field is the instability due to the condensate of the scalar field. Black brane with this nontrivial configuration of the scalar field is dual to the superconductor phase in the boundary theory. The model of \(s\)-wave holographic superconductor has been generalized to investigate \(p\)-wave holographic superconductor at which the spin-1 order parameter in the boundary theory is dual to a 2-form field \([12]\), a \(SU(2)\) Yang-Mills gauge field \([13]\), or a complex vector field \([14]\) in the bulk theory.

The properties of holographic superconductors are dependent on the background of the black hole geometry as well as the action form of the \(U(1)\) gauge field. Since the investigation of holographic superconductors has received much attention in the literature, which are considered in alternative theories of gravity as well as in the nonlinear electrodynamics. The \(p\)-wave and \(s\)-wave holographic superconductors were investigated in Einstein-Gauss-Bonnet gravity \([15-23]\) and in the nonlinear electrodynamics \([24-30]\). Also, the \(s\)-wave holographic superconductor was studied in massive gravity \([31, 32]\), in the background of the Lifshitz black hole geometry \([33]\), and in the background of the black hole surrounded by string cloud \([34]\). In addition, the \(p\)-wave holographic superconductor was considered in the background of the scalar hairy black hole \([35]\).

This paper is organized as follows. In Sect. II we introduce a \(p\)-wave holographic superconductor model in the probe limit in the black brane geometry background with the presence of graviton mass. In Sect. III we use the Sturm-Liouville analytical method to obtain an expression for the critical temperature as a function of the charge density and then investigate the effect of massive gravity on the behavior of the critical temperature. In Sect. IV we determine analytically the condensate value and study its behavior in the presence of graviton mass. In Sect. V we calculate the free energy for the superconductor and normal phases. Finally, we conclude our main results in Sect. VI.
II. HOLOGRAPHIC DUAL MODEL

In this section, we construct a $p$-wave holographic superconductor model which is considered in the context of massive gravity. The corresponding action is described by

$$S = \frac{1}{2} \int d^d x \sqrt{-g} \left[ R + \frac{(d - 1)(d - 2)}{l^2} + m_g^2 \sum_{i=1}^4 c_i \mathcal{U}_i(g, f) + \mathcal{L}_{\text{mat}} \right],$$

where $R$ is the scalar curvature of the spacetime, $l$ is the AdS radius, $m_g$ is the mass of graviton, $c_i$ are the coupling parameters, $f$ is the reference metric which is of course a symmetric tensor, $\mathcal{U}_i$ are symmetric polynomials in terms of the eigenvalues of the $4 \times 4$ matrix $\mathcal{K}_\mu^\nu = \sqrt{g} \delta_{\mu}^\lambda f_{\lambda \nu}$ given as

$$\begin{align*}
\mathcal{U}_1 &= [\mathcal{K}], \\
\mathcal{U}_2 &= [\mathcal{K}]^2 - [\mathcal{K}^2], \\
\mathcal{U}_3 &= [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3], \\
\mathcal{U}_4 &= [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4],
\end{align*}$$

with $[\mathcal{K}] = \mathcal{K}_\mu^\mu$. The matter term is given as

$$\mathcal{L}_{\text{mat}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} \left( D_\mu \rho_\nu - D_\nu \rho_\mu \right) \dagger \left( D^\mu \rho^\nu - D^\nu \rho^\mu \right) - m^2 \rho_\mu \rho^\mu + iq \gamma \rho_\mu \rho^\mu F^{\mu \nu},$$

with $F_{\mu \nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ with $A_\mu$ representing the $U(1)$ gauge field, $\rho_\mu$ is the complex vector field of the mass $m$ and the charge $q$, $D_\mu = \nabla_\mu - iq A_\mu$, and $\gamma$ refers to the magnetic moment of the complex vector field $\rho_\mu$. In this work we consider the case without the external magnetic field and hence the last term in (2) is set to zero.

We find a black brane solution with the metric ansatz

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 h_{ij} dx^i dx^j,$$

where $h_{ij} dx^i dx^j = dx_1^2 + dx_2^2 + ... + dx_{d-2}^2$ is the line element of the $(d-2)$-dimensional planar hypersurface. Following Ref. [39], we consider the reference metric in the form as $f_{\mu \nu} = \text{diag}(0, 0, h_{ij})$.

In addition, we adopt the following ansatz for the gauge field and complex vector field as

$$A_\mu = \phi(r) \delta_\mu^t, \quad \rho_\mu = \rho_x(r) \delta_\mu^x.$$

By varying the action (1) with the ansatz for the fields given above, we obtain the equations of
motion in the probe limit as
\[
\frac{d - 2}{2r^4} [r^2(d - 3)f + r^3f'] - \frac{(d - 1)(d - 2)}{2r^2} - \frac{m_q^2}{2} \left[ \frac{(d - 2)c_1}{r} + \frac{(d - 2)(d - 3)c_2}{r^2} \right] = 0,
\]
\[
\frac{(d - 2)(d - 3)(d - 4)c_3}{r^3} + \frac{(d - 2)(d - 3)(d - 4)(d - 5)c_4}{r^4} = 0,
\]
where \( r_+ \) is the event horizon radius. The temperature of the boundary field theory is identified as the Hawking temperature of the black brane and given by
\[
T_H = \frac{(d - 1)r_+}{4\pi l^2} + \frac{m_q^2}{4\pi} \left[ c_1 + \frac{(d - 3)c_2}{r_+} + \frac{(d - 3)(d - 4)c_3}{r_+^2} + \frac{(d - 3)(d - 4)(d - 5)c_4}{r_+^3} \right].
\]
For the fields well-behaving at the event horizon, we require the regularity condition as, \( \phi(r_+) = 0 \) and \( \rho_x(r_+) = m^2\rho_x(r_+)/f'(r_+) \). The asymptotic behavior of the fields near the AdS boundary (\( r \to \infty \)) is given by
\[
\phi(r) = \mu - \frac{\rho}{r^{d-3}},
\]
\[
\rho_x(r) = \frac{\langle O_{x-} \rangle}{r^{\Delta_-}} + \frac{\langle O_{x+} \rangle}{r^{\Delta_+}},
\]
where \( \Delta_\pm = \left[ d - 3 + \sqrt{(d - 3)^2 + 4m^2l^2} \right] / 2 \), \( \mu \) and \( \rho \) are the chemical potential and charge density, respectively. It should be noted that first the mass of the complex vector field satisfies the Breitenlohner-Freedman bound \([37]\) as, \( m^2 \geq -\frac{(d-3)^2}{d^2} \). Second, according to AdS/CFT correspondence, either \( \langle O_{x-} \rangle \) or \( \langle O_{x+} \rangle \) is interpreted as the source and the other is interpreted as the vacuum expectation value of the \( x \)-component of the vector operator in the dual field theory at the AdS boundary. From requiring that the condensate appears spontaneously, we impose the vanishing source \( \langle O_{x-} \rangle = 0 \).

By introducing the new coordinate \( z = \frac{r_+}{r} \), one can rewrite Eqs. (6) and (7) as
\[
\phi''(z) + \frac{4 - d}{z} \phi'(z) - \frac{2q^2\rho_x^2(z)}{z^2f(z)} \phi(z) = 0,
\]
\[
\rho_x''(z) + \left[ \frac{6 - d}{z} + \frac{f'(z)}{f(z)} \right] \rho_x'(z) + \frac{r_+^2}{z^4} \left[ \frac{\phi^2(z)}{f^2(z)} - \frac{m^2}{f(z)} \right] \rho_x(z) = 0.
\]
In the following sections, we will solve these equations to investigate the properties of $p$-wave holographic superconductor in the black brane geometry background in the presence of graviton mass. In what follows, we fix $q = 1$ and $l = 1$ which can be chosen by using the scaling symmetries.

III. CRITICAL TEMPERATURE VERSUS CHARGE DENSITY

In this section, we determine the relation between the critical temperature $T_c$, below which the condensate value $\langle O_{x^+} \rangle$ is nonzero, and the charge density $\rho$. In order to obtain this relation, we employ the analytical approach based on the Sturm-Liouville eigenvalue problem.

At the critical temperature $T_c$ the condensate value $\langle O_{x^+} \rangle$ vanishes and hence Eq. (12) becomes

$$\phi''(z) + \frac{4 - d}{z} \phi'(z) = 0. \quad (14)$$

The solution of this is obtained as

$$\phi(z) = r_{+c} \lambda (1 - z^{d-3}) \equiv r_{+c} \lambda \xi(z), \quad (15)$$

where $r_{+c}$ is the horizon radius of the black brane with the temperature $T_c$, $\lambda \equiv \frac{\rho}{r_{+c}^d}$, and we have used the boundary condition $\phi(1) = 0$ and the asymptotic behavior given at Eq. (10).

In the limit that the temperature is near the critical temperature $T_c$, we can express $\rho_{x}(z)$ in the following form

$$\rho_{x}(z) = \langle O_{x^+} \rangle \frac{z^\Delta_{+}}{r_{+}^\Delta_{+}} F(z), \quad (16)$$

where $F(z)$ is the trivial function which satisfies the conditions $F(0) = 1$ and $F'(0) = 0$ consistent to the asymptotic behavior given at Eq. (11). By substituting this form of $\rho_{x}(z)$ and the solution of $\phi(z)$ given at Eq. (15) into Eq. (13), we obtain the equation in the form of Sturm-Liouville equation as

$$[T(z)F'(z)]' - Q(z)F(z) + \lambda^2 P(z)F(z) = 0, \quad (17)$$
where

\[ T(z) = \exp \left\{ \int dz \left[ \frac{6 - d + 2\Delta_+}{z} + \frac{g'(z)}{g(z)} \right] \right\}, \]

\[ = z^{4-d+2\Delta_+} \left\{ 1 - z^{d-1} + m^2_g \left[ \frac{\bar{c}_1 z}{d-2} (1 - z^{d-2}) + \bar{c}_2 z^2 (1 - z^{d-3}) \right. \right. \]

\[ + (d-3)\bar{c}_3 z^3 (1 - z^{d-4}) + (d-3)(d-4)\bar{c}_4 z^4 (1 - z^{d-5}) \}

\[ Q(z) = -T(z) \left[ \frac{\Delta_+}{z} \left( \frac{5 - d + \Delta_+}{z} + \frac{g'(z)}{g(z)} \right) - \frac{m^2}{z^4 g(z)} \right], \]

\[ P(z) = \frac{T(z)\xi^2(z)}{z^4 g^2(z)}; \]

\[ g(z) = \frac{1 - z^{d-1}}{z^2} + m^2_g \left[ \frac{\bar{c}_1}{(d-2)z} (1 - z^{d-2}) + \bar{c}_2 (1 - z^{d-3}) \right. \]

\[ + (d-3)\bar{c}_3 z (1 - z^{d-4}) + (d-3)(d-4)\bar{c}_4 z^2 (1 - z^{d-5}) \].

(18)

with \( \bar{c}_n \equiv c_n/r_n \) for \( n = 1, 2, 3, 4 \). From the Sturm-Liouville eigenvalue problem, the eigenvalue \( \lambda^2 \) in Eq. (17) is obtained by minimizing the following expression

\[ \lambda^2 = \frac{\int_0^1 T(z)F^2(z)dz + \int_0^1 Q(z)F^2(z)dz}{\int_0^1 P(z)F^2(z)dz}, \]

(19)

where the trial function \( F(z) \) is chosen as \( F(z) = 1 - \beta z^2 \). As a result, we can express the critical temperature \( T_c \) in terms of the charge density \( \rho \) as

\[ T_c = \frac{1}{4\pi} \left\{ d - 1 + m^2_g \left[ \bar{c}_1 + (d-3)\bar{c}_2 + (d-3)(d-4)\bar{c}_3 + (d-3)(d-4)(d-5)\bar{c}_4 \right] \right\} \]

\[ \times \left( \frac{\rho}{\lambda_{\text{min}}} \right)^{\frac{1}{d-2}}, \]

(20)

where \( \lambda_{\text{min}} \) is the minimum value obtained from minimizing Eq. (19). The effect of massive gravity on the critical temperature \( T_c \) is showed through which it enters into both the expression of \( T_c \) and \( \lambda_{\text{min}} \). It should be noted that only the couplings \( c_1, c_2 \) affect the critical temperature in the case of four dimensions. Whereas, the couplings \( c_3 \) and \( c_4 \) modify the critical temperature if the spacetime dimension is larger than four and five, respectively.

In figure 1, we show the behavior of the critical temperature as a function of the charge density for various values of the massive gravity couplings. We observe that the critical temperature in the presence of graviton mass is lower/higher than that in the case of massless graviton if the massive gravity couplings are negative/positive. It is also indicated that increasing the amplitude of the massive gravity couplings makes the critical temperature increasing or decreasing dependently on the sign of the massive gravity couplings. In figure 2, we plot the critical temperature in terms of the charge density for different values of graviton mass. It is found that, if the massive gravity
FIG. 1: The critical temperature as a function of the charge density at \( m_g = 1 \). In the left panel: the blue, brown, purple, and red curves correspond to the following coupling values \((\tilde{c}_1, \tilde{c}_2) = (-1, -1), (-0.5, -0.5), (0.5, 0.5), \) and \((1, 1)\), respectively. In the right panel: the blue, brown, purple, and red curves correspond to the following coupling values \((\tilde{c}_1, \tilde{c}_2, \tilde{c}_3) = (-0.5, -0.5, -0.8), (-0.25, -0.25, -0.5), (0.25, 0.25, 0.5), \) and \((0.5, 0.5, 0.8)\), respectively. The dashed black curves refer to the case of \( m_g = 0 \).

FIG. 2: The critical temperature as a function of the charge density at \( d = 4 \) and \( m^2 = 1/2 \). The dashed black, blue, pink, brown, purple, and red curves correspond to \( m_g = 0, 0.2, 0.4, 0.6, 0.8, \) and 1, respectively.

couplings are positive, the critical temperature increases with the growth of graviton mass. On the contrary, if the massive gravity couplings are negative, the critical temperature decreases with increasing of graviton mass. Therefore, the high temperature superconductors can be achieved in the framework of massive gravity with the proper parameters.

IV. CONDENSATE VALUE

In this section, we obtain an analytical expression for the condensate value \( \langle O_{x+} \rangle \) and investigate the effect of massive gravity on the behavior of \( \langle O_{x+} \rangle \) in terms of the temperature. In order to
do this, we need to study the behavior of the gauge field by solving Eq. (12). Near the critical temperature $T_c$, the condensate value $\langle O_{x+} \rangle$ is small. Hence one can expand the function $\phi(z)$ near $T_c$ in terms of $\langle O_{x+} \rangle$ as

$$\phi(z) = r_+ \lambda \xi(z) + r_+ \frac{\langle O_{x+} \rangle^2}{r_+^{2\Delta_+}} \chi(z), \tag{21}$$

where the function $\chi(z)$ satisfies the boundary condition $\chi(1) = \chi'(1) = 0$. By substituting this expansion of $\phi(z)$ and the expression of $\rho_x(z)$ given at Eq. (16) into Eq. (12), we obtain

$$\chi''(z) + \frac{4 - d}{z} \chi'(z) - \frac{2\lambda (1 - z^{d-3})}{f(z)} z^{2(\Delta_+ - 1)} F^2(z) = 0,$$ \tag{22}

which can be rewritten in the following form

$$\left[ \frac{\chi'(z)}{z^{d-4}} \right]' = \frac{2\lambda (1 - z^{d-3})}{f(z)} z^{2(1 + \Delta_+ - d)} F^2(z). \tag{23}$$

By integrating this equation with the boundary condition $\phi'(1) = 0$, we find

$$\chi'(z) = z^{d-4} \int_1^z \frac{2\lambda (1 - z^{d-3})}{f(z)} z^{2(1 + \Delta_+ - d)} F^2(\bar{z}) d\bar{z}. \tag{24}$$

Expanding of $\phi(z)$ near the AdS boundary, corresponding to Eq. (21), is given by

$$\phi(z) = \lambda r_+ \left(1 - z^{d-3}\right) + \frac{\langle O_{x+} \rangle^2}{r_+^{2\Delta_+ - 1}} \left[ \chi(0) + \chi'(0)z + \frac{\chi''(0)}{2} z^2 + \cdots + \frac{\chi^{(d-3)}(0)}{(d-3)!} z^{d-3} + \cdots \right]. \tag{25}$$

By comparing the coefficients of $z^{d-3}$ in the right-hand sides of Eqs. (10) and (25), we find

$$\rho_{r_+^{d-2}} = \lambda - \frac{\langle O_{x+} \rangle^2}{r_+^{2\Delta_+}} \frac{\chi^{(d-3)}(0)}{(d-3)!} \tag{26}.$$ 

Finally, we obtain the expression for the condensate value $\langle O_{x+} \rangle$ as a function of the temperature as

$$\langle O_{x+} \rangle = \left(\frac{4\pi}{d-1+a}\right)^{1+\Delta_+} \sqrt{\frac{d-3}{A} T^{1+\Delta_+}} \sqrt{\frac{T_c}{T}}^{d-2} - 1,$$

$$\approx \left(\frac{4\pi}{d-1+a}\right)^{1+\Delta_+} \sqrt{\frac{(d-3)(d-2)}{A} T_c^{1+\Delta_+} \sqrt{1 - \frac{T}{T_c}}}, \tag{27}$$

where

$$a \equiv m_g^2 \left[ \tilde{c}_1 + (d-3)\tilde{c}_2 + (d-3)(d-4)\tilde{c}_3 + (d-3)(d-4)(d-5)\tilde{c}_4 \right],$$

$$A \equiv 2 \int_0^1 z^{4-d+2\Delta_+} (1 - z^{d-3})(1 - \beta z^2)^2 \left\{ 1 - z^{d-1} + m_g^2 \left[ \frac{\tilde{c}_1}{d-2} (1 - z^{d-2}) + \tilde{c}_2 z^{2}(1 - z^{d-2}) + (d-3)\tilde{c}_3 z^3(1 - z^{d-4}) + (d-3)(d-4)\tilde{c}_4 z^4(1 - z^{d-5}) \right] \right\}^{-1} dz. \tag{28}$$
FIG. 3: The dimensionless condensate value $\langle O_{x^+} \rangle / T^{1+\Delta_+}_c$ as a function of $T/T_c$ at $m_g = 1$. In the left panel: the blue, brown, purple, and red curves correspond to the following coupling values $(\bar{c}_1, \bar{c}_2) = (-1, -1), (-0.5, -0.5), (0.5, 0.5),$ and $(1, 1)$, respectively. In the right panel: the blue, brown, purple, and red curves correspond to the following coupling values $(\bar{c}_1, \bar{c}_2, \bar{c}_3) = (-0.5, -0.5, -0.8), (-0.25, -0.25, -0.5), (0.25, 0.25, 0.5),$ and $(0.5, 0.5, 0.8)$, respectively. The dashed black curves refer to the case of $m_g = 0$.

In figures 3 and 4 we plot the dimensionless condensate value $\langle O_{x^+} \rangle / T^{1+\Delta_+}_c$ as a function of $T/T_c$ for various values of the massive gravity parameters. From these figures, we see that for the positive massive gravity couplings as their amplitude or the graviton mass increases, the value of the condensate operator becomes smaller. This suggests that the condensate gets easier to form, which is consistent with the behavior of the critical temperature obtained in the previous section. On the contrary, for the negative massive gravity couplings, the value of the condensate operator becomes larger when increasing the amplitude of the massive gravity couplings and the graviton mass. On the other hand, in this situation, the condensate gets harder to form.

V. FREE ENERGY FOR THE SUPERCONDUCTOR AND NORMAL PHASES

In this section, we study the behavior of the free energy near the critical temperature in the grand canonical ensemble (where the chemical potential is kept fixed) to see the superconductor/normal phase transition. The free energy is given by

$$\Omega = TS_E,$$  \hspace{1cm} (29)
The dimensionless condensate value $\langle O^+ \rangle / T_{c}^{1+\Delta c}$ as a function of $T/T_c$ at $d = 4$ and $m^2 = 1/2$.

The dashed black, blue, pink, brown, purple, and red curves correspond to $m_g = 0, 0.2, 0.4, 0.6, 0.8$, and $1$, respectively.

where $S_E$ is the Euclidean on-shell action computed as

$$-S_E = \frac{i}{2} \int d^dx \sqrt{-g} \left[ -\frac{1}{2} \nabla_\mu (A_\nu F^{\mu \nu}) - \nabla_\mu \left( \rho_\nu^\dagger (D^\mu \rho^\nu - D^\nu \rho^\mu) \right) + \frac{1}{2} A_\nu \nabla_\mu F^{\mu \nu} \right],$$

$$= -\frac{V_{d-2}}{2T} \left[ -\frac{1}{2} \sqrt{-h} n_\mu A_\nu F^{\mu \nu} \right]_{r \to \infty} - \sqrt{-h} n_\mu \rho_\nu^\dagger (D^\mu \rho^\nu - D^\nu \rho^\mu) \bigg|_{r \to \infty} + \frac{1}{2} \int_{r+}^\infty dr \sqrt{-g} A_\nu \nabla_\mu F^{\mu \nu} \right],$$

$$= -\frac{V_{d-2}}{2T} \left[ \frac{(d-3)}{2} \mu \rho - \int_{r+}^\infty \frac{\rho^2 (\phi^2)}{r^{d-1} f(r)} dr \right], \quad (30)$$

where $\int \tau dx_1 ... dx_{d-2} = V_{d-2} / T$ with $\tau$ to be the Euclidean time, $h_{ij} = \text{diag} \left[ -f(r), r^2, ..., r^2 \right]$, and $n_\mu = (n_r, n_i) = \left( 1 / \sqrt{f(r)}, 0 \right)$. Then, the scaled free energy for the superconductor phase is given by

$$\frac{2\Omega_S}{V_{d-2}} = \frac{d}{2} \mu \rho - \int_{r+}^\infty \frac{\rho^2 (\phi^2)}{r^{d-1} f(r)} dr. \quad (31)$$

In the normal phase, we have $\rho_x = 0$, and hence the corresponding scaled free energy is given by $2\Omega_N/V_{d-2} = (d-3) \mu \rho / 2$. By using the solution for $\phi(z)$ and $\rho(x)$ obtained in the previous sections, one can find the difference of the scaled free energy between the superconductor and normal phases as

$$\Delta \Omega = \frac{2(\Omega_S - \Omega_N)}{V_{d-2}} \simeq -\left( \frac{4\pi}{d-1+a} \right)^{d-1} \frac{(d-3)(d-2)}{A} \left( \int_0^1 \frac{\lambda^2 z^{4-d+2\Delta_c} \xi^2(z) F^2(z)}{g(z)} dz \right)$$

$$\times T_c^{d-1} \left( 1 - \frac{T}{T_c} \right). \quad (32)$$

We show the difference of the scaled free energy between the superconductor and normal phases in terms of $T/T_c$ in the figures 5 and 6. These figures indicate that the superconductor phase
FIG. 5: The difference of the scaled free energy between the superconducting and normal phases as a function of $T/T_c$ at $m_g = 1$. In the left panel: the blue, brown, purple, and red curves correspond to the following coupling values $(\tilde{c}_1, \tilde{c}_2) = (-1, -1)$, $(-0.5, -0.5)$, $(0.5, 0.5)$, and $(1, 1)$, respectively. In the right panel: the blue, brown, purple, and red curves correspond to the following coupling values $(\tilde{c}_1, \tilde{c}_2, \tilde{c}_3) = (-0.5, -0.5, -0.8)$, $(-0.25, -0.25, -0.5)$, $(0.25, 0.25, 0.5)$, and $(0.5, 0.5, 0.8)$, respectively. The dashed black curves refer to the case of $m_g = 0$.

FIG. 6: The difference of the scaled free energy between the superconducting and normal phases as a function of $T/T_c$ at $d = 4$ and $m^2 = 1/2$. The dashed black, blue, pink, brown, purple, and red curves correspond to $m_g = 0, 0.2, 0.4, 0.6, 0.8$, and $1$, respectively.

has the free energy smaller than the normal phase and hence it contributes dominantly to the thermodynamics. In this way, below the critical temperature the superconducting phase rather than normal phase is thermodynamically favored. In addition, for the positive massive gravity couplings, the superconducting phase with the stronger coupling amplitude or the more massive graviton is more stable. This is because as indicated in the previous section the value of the condensate operator in this case gets larger with the growth of the coupling amplitude and graviton mass, and thus much more energy is needed to break the condensate.
VI. CONCLUSION

In this paper, we have analytically study the effects of massive gravity on $p$-wave holographic superconductor in the probe limit which the backreaction of the matter fields on the spacetime geometry is ignored. We found that the massive gravity parameters affect significantly on the critical temperature, the value of the order parameter, and the free energy of the superconductor phase, which depends crucially on the sign of the massive gravity couplings. If the massive gravity couplings are positive, it leads to:

- The critical temperature in massive gravity is higher than that in Einstein gravity, whereas the value of the order parameter is smaller compared to the case of massless graviton.

- When the amplitude of the massive gravity couplings and the graviton mass increase, the critical temperature increase, whereas the value of the order parameter gets lower.

These facts suggest that the high temperature superconductors can be achieved in the presence of graviton mass with the proper parameters. In addition, we indicated that for the negative massive gravity couplings, the superconductor phase with the stronger coupling amplitude or the more massive graviton is more thermodynamically favored.

Note added.– When this paper was being finalized, a work recently appeared [39] which studied the $p$-wave holographic superfluid model in massive gravity, using the numerical method.

[1] B. P. Abbott et al., Phys. Rev. Lett. 116, 221101 (2016).
[2] M. Fierz and W. Pauli, Proc. R. Soc. A 173, 211 (1939).
[3] H. van Dam and M. J. G. Veltman, Nucl. Phys. B 22, 397 (1970).
[4] V. I. Zakharov, Pisma Zh. Eksp. Teor. Fiz. 12, 447 (1970) [JETP Lett. 12, 312 (1970)].
[5] A. I. Vainshtein, Phys. Lett. 39B, 393 (1972).
[6] D. G. Boulware and S. Deser, Phys. Rev. D 6, 3368 (1972).
[7] C. de Rham and G. Gabadadze, Phys. Rev. D 82, 044020 (2010).
[8] C. de Rham, G. Gabadadze, and A.J. Tolley, Phys. Rev. Lett. 106, 231101 (2011).
[9] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998).
[10] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Phys. Rev. Lett. 101, 031601 (2008).
[11] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, JHEP 0812, 015 (2008).
[12] A. Donos and J. P. Gauntlett, JHEP 12, 091 (2011).
[13] S. S. Gubser and S. S. Pufu, JHEP 11, 033 (2008).
[14] R.-G. Cai, L. Li, and L.-F. Li, JHEP 01, 32 (2014).
[15] R. Gregory, S. Kanno, and J. Soda, JHEP 0910, 010 (2009)
[16] R.-G. Cai, Z.-Y. Nie, and H.-Q. Zhang, Phys. Rev. D 82, 066007 (2010)
[17] L. Barclay, R. Gregory, S. Kanno, and P. Sutcliffe, JHEP 1012, 029 (2010).
[18] Q. Pan and B. Wang, Phys. Lett. B 693, 159 (2010).
[19] J. Jing, L. Wang, Q. Pan, and S. Chen, Phys. Rev. D 83, 066010 (2011).
[20] J. Jing, Q. Pan, and S. Chen, Phys. Lett. B 716, 385 (2012).
[21] S.-L. Cui and Z. Xue, Phys. Rev. D 88, 107501 (2013).
[22] D. Parai, S. Gangopadhyay, and D. Ghorai, Ann. Phys. 403, 59 (2019).
[23] S. Liu, Q. Pan, and J. Jing, Phys. Lett. B 765, 91 (2017).
[24] J. Jing, Q. Pan, and S. Chen, JHEP 11, 045 (2011).
[25] R. Banerjee, S. Gangopadhyay, D. Roychowdhury, and A. Lala, Phys. Rev. D 87, 104001 (2013).
[26] D. Ghorai and S. Gangopadhyay, Eur. Phys. J. C 76, 146 (2016).
[27] A. Sheykhi, D. H. Asl, and A. Dehyadegari, Phys. Lett. B 781, 139 (2018).
[28] M. Mohammad and A. Sheykhi, Phys. Rev. D 100, 086012 (2019).
[29] C. H. Nam, Gen. Rel. Grav. 51, 104 (2019).
[30] A. Srivastav, D. Ghorai, and S. Gangopadhyay, Eur. Phys. J. C 80, 219 (2020).
[31] H.-B. Zeng and J.-P. Wu, Phys. Rev. D 90, 046001 (2014).
[32] R. Li and Y. Zhao, Phys. Rev. D 100, 046018 (2019).
[33] Z. Zhao, Q. Pan, and J. Jing, Phys. Lett. B 735, 438 (2014).
[34] C. H. Nam, Phys. Lett. B 797, 134865 (2019).
[35] D. Wen, H. Yu, Q. Pan, K. Lin, and W.-L. Qian, Phys. Lett. B 792, 219 (2019).
[36] D. Vegh, arXiv:1301.0537.
[37] P. Breitenlohner and D. Z. Freedman, Phys. Lett. B 115, 197 (1982).
[38] G. Siopsis and J. Therrien, JHEP 05, 013 (2010).
[39] Z.-Y. Nie, Y.-P. Hu, and H. Zeng, arXiv: 2003.12989.