SPM BULLETIN

ISSUE NUMBER 36: November 2013 CE

1. Proceedings of the Fourth Workshop on Coverings, Selections and Games in Topology

Volume 160, Issue 18, of Topology and its Applications (1 December 2013, Pages 2233–2566), is dedicated to the proceedings of the Fourth Workshop on Coverings, Selections and Games in Topology, on the occasion of Ljubiša D.R. Kočinac’s 65th birthday. This issue is now available online at

http://www.sciencedirect.com/science/journal/01668641/160/18

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2. LONG ANNOUNCEMENTS

2.1. Completeness and related properties of the graph topology on function spaces. The graph topology \( \tau_T \) is the topology on the space \( C(X) \) of all continuous functions defined on a Tychonoff space \( X \) inherited from the Vietoris topology on \( X \times \mathbb{R} \) after identifying continuous functions with their graphs. It is shown that all completeness properties between complete metrizability and hereditary Baireness coincide for the graph topology if and only if \( X \) is countably compact; however, the graph topology is \( \alpha \)-favorable in the strong Choquet game, regardless of \( X \). Analogous results are obtained for the fine topology on \( C(X) \). Pseudocompleteness, along with properties related to 1st and 2nd countability of \( (C(X), \tau_T) \) are also investigated.

http://arxiv.org/abs/1304.6628

Lubica Holá, László Zsilinszky

2.2. Topological games and Alster spaces. In this paper we study connections between topological games such as Rothberger, Menger and compact-open, and relate these games to properties involving covers by \( G_\delta \) subsets. The results include: (1) If Two has a winning strategy in the Menger game on a regular space \( X \), then \( X \) is an Alster space. (2) If Two has a winning strategy in the Rothberger game on a topological space \( X \), then the \( G_\delta \)-topology on \( X \) is Lindelof. (3) The Menger game and the compact-open game are (consistently) not dual.

http://arxiv.org/abs/1306.5463
2.3. Strongly Summable Ultrafilters, Union Ultrafilters, and the Trivial Sums Property. We answer two questions of Hindman, Steprāns and Strauss, namely we prove that every strongly summable ultrafilter on an abelian group is sparse and has the trivial sums property. Moreover we show that in most cases the sparseness of the given ultrafilter is a consequence of its being isomorphic to a union ultrafilter. However, this does not happen in all cases: we also construct (assuming \( \text{cov}(\mathcal{M}) = \mathfrak{c} \)), on the Boolean group, a strongly summable ultrafilter that is not additively isomorphic to any union ultrafilter.

http://arxiv.org/abs/1306.5421
David J. Fernández Bretón

2.4. Topological games and productively countably tight spaces. The two main results of this work are the following: if a space \( X \) is such that player II has a winning strategy in the game \( G_1(\Omega_x, \Omega_x) \) for every \( x \in X \), then \( X \) is productively countably tight. On the other hand, if a space is productively countably tight, then \( S_1(\Omega_x, \Omega_x) \) holds for every \( x \in X \). With these results, several other results follow, using some characterizations made by Uspenskii and Scheepers.

http://arxiv.org/abs/1307.7928
Leandro F. Aurichi and Angelo Bella

2.5. On a game theoretic cardinality bound. The main purpose of the paper is the proof of a cardinal inequality for a space with points \( G_\delta \), obtained with the help of a long version of the Menger game. This result improves a similar one of Scheepers and Tall.

http://arxiv.org/abs/1310.0409
Leandro F. Aurichi and Angelo Bella

2.6. Pytkeev \( \aleph_0 \)-spaces. A regular topological space \( X \) is defined to be a Pytkeev \( \aleph_0 \)-space if it has countable Pytkeev network. A family \( \mathcal{P} \) of subsets of a topological space \( X \) is called a Pytkeev network in \( X \) if for a subset \( A \subset X \), a point \( x \in A \) and a neighborhood \( O_x \subset X \) there is a set \( P \in \mathcal{P} \) such that \( x \in P \subset O_x \) and moreover \( P \cap A \) is infinite if \( x \) is an accumulation point of \( A \). The class of Pytkeev \( \aleph_0 \)-spaces contains all metrizable separable spaces and is (properly) contained in the class of \( \aleph_0 \)-spaces. This class is closed under many operations over topological spaces: taking a subspace, countable Tychonoff product, small countable box-product, countable direct limit, the hyperspace. For an \( \aleph_0 \)-space \( X \) and a Pytkeev \( \aleph_0 \)-space \( Y \) the function space \( C_k(X, Y) \) endowed with the compact-open topology is a Pytkeev \( \aleph_0 \)-space. A topological space is second countable if and only if it is Pytkeev \( \aleph_0 \)-space with countable fan tightness.

http://arxiv.org/abs/1311.1468
Taras Banakh

2.7. Selective covering properties of product spaces, II: \( \gamma \) spaces. We study productive properties of \( \gamma \) spaces, and their relation to other, classic and modern, selective covering properties. Among other things, we prove the following results:

(1) Solving a problem of F. Jordan, we show that for every unbounded tower set \( X \subset \mathbb{R} \) of cardinality \( \aleph_1 \), the space \( C_p(X) \) is productively Fréchet–Urysohn. In particular, the set \( X \) is productively \( \gamma \).
(2) Solving problems of Scheepers and Weiss, and proving a conjecture of Babinkostova–Scheepers, we prove that, assuming the Continuum Hypothesis, there are $\gamma$ spaces whose product is not even Menger.

(3) Solving a problem of Scheepers–Tall, we show that the properties $\gamma$ and Gerlits–Nagy (*) are preserved by Cohen forcing. Moreover, every Hurewicz space that Remains Hurewicz in a Cohen extension must be Rothberger (and thus (*)).

We apply our results to solve a large number of additional problems, and use Arhangel’-skii duality to obtain results concerning local properties of function spaces and countable topological groups.

http://arxiv.org/abs/1310.8622

Arnold W. Miller, Boaz Tsaban, Lyubomyr Zdomskyy

2.8. Productively countably tight spaces of the form $C_k(X)$. Some results in $C_k$-theory are obtained with the use of bornologies. We investigate under which conditions the space of the continuous real functions with the compact-open topology is a productively countably tight space, which yields some applications on Alster spaces.

http://arxiv.org/abs/1311.2011

Leandro Fiorini Aurichi, Renan Maneli Mezabarba

3. Short announcements

3.1. Partitioning bases of topological spaces.

http://arxiv.org/abs/1304.0472

Daniel T. Soukup, Lajos Soukup

3.2. Compactness of $\omega^\lambda$.

http://arxiv.org/abs/1304.0486

Paolo Lipparini

3.3. Non-meager free sets for meager relations on Polish spaces.

http://arxiv.org/abs/1304.2042

Taras Banakh and Lyubomyr Zdomskyy

3.4. On strong $P$-points.

http://www.ams.org/journal-getitem?pii=S0002-9939-2013-11518-2

Andreas Blass; Michael Hrusak; Jonathan Verner

3.5. Products and countable dense homogeneity.

http://arxiv.org/abs/1307.0184

Andrea Medini

3.6. Strong colorings yield kappa-bounded spaces with discretely untouchable points.

http://arxiv.org/abs/1307.1989

Istvan Juhasz and Saharon Shelah

3.7. Selective and Ramsey ultrafilters on $G$-spaces.

http://arxiv.org/abs/1310.1827

O.V. Petrenko, I.V. Protasov
3.8. Seven characterizations of non-meager P-filters.  

http://arxiv.org/abs/1311.1677

Kenneth Kunen, Andrea Medini, Lyubomyr Zdomsky

4. UNSOLVED PROBLEMS FROM EARLIER ISSUES

Issue 1. Is \( \left( \frac{\Omega}{\Gamma} \right) = \left( \frac{\Omega}{\Gamma} \right) ? \)

Issue 2. Is \( U_{\text{fin}}(O, \Omega) = S_{\text{fin}}(\Gamma, \Omega)? \) And if not, does \( U_{\text{fin}}(O, \Gamma) \) imply \( S_{\text{fin}}(\Gamma, \Omega)? \)

Issue 4. Does \( S_1(\Omega, T) \) imply \( U_{\text{fin}}(\Gamma, \Gamma)? \)

Issue 5. Is \( p = p^* \)? (See the definition of \( p^* \) in that issue.)

Issue 6. Does there exist (in ZFC) an uncountable set satisfying \( S_{\text{fin}}(B, B)? \)

Issue 8. Does \( X \notin \text{NON}(M) \) and \( Y \notin D \) imply that \( X \cup Y \notin \text{COF}(M)? \)

Issue 9 (CH). Is \( \text{Split}(\Lambda, \Lambda) \) preserved under finite unions?

Issue 10. Is \( \text{cov}(M) = \text{od}? \) (See the definition of \( \text{od} \) in that issue.)

Issue 12. Could there be a Baire metric space \( M \) of weight \( \aleph_1 \) and a partition \( \mathcal{U} \) of \( M \) into \( \aleph_1 \) meager sets where for each \( \mathcal{U} \subseteq \mathcal{U}, \bigcup \mathcal{U}' \) has the Baire property in \( M? \)

Issue 14. Does there exist (in ZFC) a set of reals \( X \) of cardinality \( \mathfrak{d} \) such that all finite powers of \( X \) have Menger’s property \( S_{\text{fin}}(O, O)? \)

Issue 15. Can a Borel non-\( \sigma \)-compact group be generated by a Hurewicz subspace?

Issue 16 (MA). Is there \( X \subseteq \mathbb{R} \) of cardinality continuum, satisfying \( S_1(B_\Omega, B_\Gamma)? \)

Issue 17 (CH). Is there a totally imperfect \( X \) satisfying \( U_{\text{fin}}(O, \Gamma) \) that can be mapped continuously onto \( \{0, 1\}^\mathbb{N}? \)

Issue 18 (CH). Is there a Hurewicz \( X \) such that \( X^2 \) is Menger but not Hurewicz?

Issue 19. Does the Pytkeev property of \( C_p(X) \) imply that \( X \) has Menger’s property?

Issue 20. Does every hereditarily Hurewicz space satisfy \( S_1(B_\Gamma, B_\Gamma)? \)

Issue 21 (CH). Is there a Rothberger-bounded \( G \leq \mathbb{Z}^\mathbb{N} \) such that \( G^2 \) is not Menger-bounded?

Issue 22. Let \( \mathcal{W} \) be the van der Waerden ideal. Are \( \mathcal{W} \)-ultrafilters closed under products?

Issue 23. Is the \( \delta \)-property equivalent to the \( \gamma \)-property \( \left( \frac{\Omega}{\Gamma} \right)? \)