Stability Analysis on the Spread of the Ebola Virus Considering Changes in Human Behavior and Climate (Case Study: Democratic Republic of the Congo)

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Abstract. In this paper construction of the SEIR (Susceptible-Exposed-Infected-Recovery) type mathematical model consider account the influence of changes in human behavior and climate in the Democratic Republic of the Congo (DR Congo) in the form of differential equations to describe the spread of the Ebola virus. The relationship between climate and behavior change is represented in the form of parameters (α). The constructed system is then validated with existence and uniqueness. Furthermore, an analysis of the spread of the virus was carried out with stability analysis. Based on the results of stability analysis, the system around the endemic equilibrium point is unstable. In the final stage a simulation was carried out to obtain a visual picture of the spread of the Ebola virus in DR Congo.

1. Introduction
The Ebola virus is a type of zoonotic virus that can be fatal if not treated immediately. The Ebola virus first appeared in 1976. Infection with the Ebola virus is caused by a person's mucous membrane coming into contact with goods or the environment that is contaminated with bodily fluids from an infected person or damaged skin. Based on data from WHO the mortality rate caused by the Ebola virus is quite high, on average 50% and in the past it can reach the range of 25% -90% [1].

Ebola outbreaks have reappeared in the DR Congo in May 2018, this outbreak is the 9th Ebola outbreak in the past four decades [2]. The location of the emergence of the Ebola virus is in remote areas with difficult road access, causing people to have very little knowledge regarding the Ebola virus. The outbreak of the Ebola virus in DR Congo is allegedly due to the influence of changes in behavior of people in DR Congo that are influenced by the climate.

Research on stability analysis on the spread of disease has been examined by Nihaya et al with the SEITR multi-path epidemic model [3]. Then research about transmission of Ebola virus in Guinea using the SEIT type model has been researched by Li et al [4], model of the spread of the Ebola virus using SLIR type models with two control parameters to control the spread of Ebola virus in Nigeria by Ndanusa et al [5], and then developed by constructing a mathematical model of the impact of behavior change on the spread of the Ebola virus by considering the settings of medical care and different types of burial by Conrad et al [6].
In contrast to previous research, in this study using a SEIR type mathematical model that refers to [4] but it was developed by adding new parameters namely $\alpha$ in the susceptible population and exposed population. The new parameter describes the level of change in human behavior that is influenced by climate.

2. Main Result

2.1. Effects of Climate on Behavior Change, in Transmission Spread of the Ebola Virus

DR Congo is a country located in Central Africa and has a tropical climate that tends to have high temperatures, so the outbreak of the Ebola virus can spread easily in the country. Climate change that often occurs in the DR Congo is extreme temperatures which can cause a prolonged dry season and cause drought. Drought can cause difficulties in obtaining food ingredients because of the large number of dead plants. Besides that, climate change can also cause increased pest attacks and facilitate the spread of disease. The DR Congo people who have difficulty obtaining food ingredients for a long time are at risk of starvation. The risk of starvation can encourage people to behave deviant. The deviant behavior of the DR Congo community based on information from WHO is that the DR Congo people still eat fruit that has been eaten by animals (bats), the DR Congo people still consume wild animals with low meat maturity, the DR Congo tradition in traditional burial ceremonies has direct contact between mourners and corpses, and DR Congo people still rely on traditional medicine. The deviant behavior above can cause an increase in transmission of the spread of the Ebola virus if the animal, person or corpse is infected with the Ebola virus.

Based on the explanation above, there is a correlation between climate and behavior change which can be represented by parameter $\alpha$. The value of $\alpha$ in this paper uses proportion numbers so that the value of $\alpha$ is in interval $[0,1]$ or $0 < \alpha < 1$. The value of alpha in this paper uses propositional numbers of changes in human behavior compared to the number of population, so that we obtain the value of alpha is 0.7 from the journal [7].

2.2. Model Construction

In this paper the object observed is the spread of the Ebola virus in the DR Congo. The mathematical model was constructed by referring to the model of the spread of the Ebola virus that had been previously studied [4]. In contrast to the previous study [4], in this study using a SEIR type mathematical model which was developed by adding new parameters namely $\alpha$. The parameter $\alpha$ is placed in the susceptible population and the population is exposed because of the community in that population has a considerable opportunity to change behavior.

The mathematical model in [4] that has been developed by adding the $\alpha$ parameter is as follows:

$$\begin{align*}
\frac{dS}{dt} &= \Lambda - \beta_{10}IS - \beta_{20}ES - \mu S + \gamma_0 R + \alpha S \\
\frac{dE}{dt} &= \beta_{10}IS + \beta_{20}ES - \varepsilon_0 E - (\mu + \mu_{10})E + \alpha E \\
\frac{dI}{dt} &= \varepsilon_0 E - \nu_0 I - (\mu + \mu_{20})I \\
\frac{dR}{dt} &= \nu_0 I - \gamma_0 R - (\mu + \mu_{30})R
\end{align*}$$

with initial conditions,

$S(0) = S_0, E(0) = E_0, I(0) = I_0, R = R_0$

The total population at time $t$ is represented by $N(t) = S(t) + E(t) + I(t) + R(t)$. With $S(t)$ is the susceptible population at time $t$, $E(t)$ is the exposed population at time $t$, $I(t)$ is the infected population
at time $t$, $R(t)$ is the recovery population / free from the Ebola virus at time $t$, $\beta_{10}$ rate of Ebola virus transmission in population $I$ and $S$, $\beta_{20}$ rate of Ebola virus transmission in population $E$ and $S$, $\mu$ rate natural mortality, $\alpha$ rate of change in human behavior affected by climate, $\varepsilon_0$ transition rate from population $E$ to population $I$, $v_0$ rate transition from population $I$ to population $R$, $\gamma_0$ rate transition from population $R$ back to population $S$, and $\mu_{10}, \mu_{20}, \mu_{30}$ is the rate of death due to disease or virus.

2.3. Existence and Uniques Solution (Validation Model)

Analysis of existence and uniqueness solutions can be seen by looking for the value of the \textit{lipschitz constant} $k(t)$ that satisfies:

$$\|f(x^1, t) - f(x^2, t)\| \leq k(t)\|x^1 - x^2\|$$

Assuming that

$$\|f(x^1, t) - f(x^2, t)\| = \begin{vmatrix} f(S^1, t) - f(S^2, t) \\ f(E^1, t) - f(E^2, t) \\ f(I^1, t) - f(I^2, t) \\ f(R^1, t) - f(R^2, t) \end{vmatrix} = \begin{vmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{vmatrix}$$

$$\|f(x^1, t) - f(x^2, t)\| \leq k(t)\|a_i\|, \text{with } \|a_i\| = \text{maks}\|\Sigma_{i=1}^4 a_i\|$$

Based on the results of simplifying the system with model reduction, the results are:

$$\leq \max \left\{ \left| (-\lambda_1 - \lambda_2 - \mu + \lambda_3 + \alpha)(S^1 - S^2) \right|, \left| (\lambda_4 + \lambda_5 - \varepsilon_0 - (\mu + \mu_{10}) + \alpha)(E^1 - E^2) \right|, \left| (\lambda_6 - v_0 - (\mu + \mu_{20}))(I^1 - I^2) \right|, \left| (\lambda_7 - \gamma_0 - (\mu + \mu_{30}))(R^1 - R^2) \right| \right\}$$

$$\leq k(t)\begin{vmatrix} (S^1 - S^2) \\ (E^1 - E^2) \\ (I^1 - I^2) \\ (R^1 - R^2) \end{vmatrix}$$

so it can be obtained

$$k(t) = \left\{ \left| (\lambda_3 + \alpha)_{\text{max}} - (\lambda_1 + \lambda_2 + \mu)_{\text{min}} \right|, \left| (\lambda_4 + \lambda_5 + \alpha)_{\text{max}} - (\varepsilon_0 + (\mu + \mu_{10}))_{\text{min}} \right|, \left| (\lambda_6)_{\text{max}} - (v_0 + (\mu + \mu_{20}))_{\text{min}} \right|, \left| (\lambda_7)_{\text{max}} - (\gamma_0 + (\mu + \mu_{30}))_{\text{min}} \right| \right\}$$

It is assumed that the infected population can spread the Ebola virus widely in DR Congo. Therefore, observations are only made to the infected population. The mathematical model of the spread of the Ebola virus in this study has a single and complete solution at the time $k(t) =$
Because the model in this study has a solution and uniqueness, the constructed model is valid.

2.4 Equilibrium Point

The equilibrium point is obtained from \( \frac{dS}{dt} = 0 \), \( \frac{dE}{dt} = 0 \), \( \frac{dI}{dt} = 0 \), \( \frac{dR}{dt} = 0 \).

By using a substitution method, an equilibrium point for the spread of Ebola virus in the DR Congo is obtained as follows:

\[
S^* = \frac{\Lambda(-B - B\varepsilon_0A - B\mu A - B\mu_{10}A + B\alpha A + \gamma_0 \nu_0 \varepsilon_0)}{B(\alpha)}
\]

\[
E^* = \frac{\varepsilon_0 A(-\Lambda + \mu S^* - \alpha S^*)}{B(\nu_0 + (\mu + \mu_{20}))}
\]

\[
I^* = \frac{\varepsilon_0 (\mu + \mu_{20})}{B(\nu_0 + (\mu + \mu_{10}))}
\]

\[
R^* = \frac{\nu_0 \varepsilon_0 (\mu + \mu_{30})}{B(\nu_0 + (\mu + \mu_{20}))}
\]

with

\[
A = (\gamma_0 + (\mu + \mu_{20}))(\nu_0 + (\mu + \mu_{20})),
\]

\[
B = A(-\varepsilon_0 - (\mu + \mu_{10}) + \alpha) + \gamma_0 \nu_0 \varepsilon_0.
\]

2.5 Stability Analysis

The mathematical model in equation (1) is linearized. The results of linearization are then written into the form of the Jacobian matrix.

So that the Jacobian matrix is obtained from the system of differential equations as follows

\[
J = \begin{bmatrix}
-\beta_{10}l - \beta_{20}E - \mu + \alpha & -\beta_{20}S & -\beta_{10}S & \gamma_0 \\
\beta_{10}l + \beta_{20}E & \beta_{20}S - \varepsilon_0 - (\mu + \mu_{10}) & \beta_{10}S & 0 \\
0 & \varepsilon_0 & -\nu_0 - (\mu + \mu_{20}) & 0 \\
0 & 0 & \nu_0 & -\gamma_0 - (\mu + \mu_{30})
\end{bmatrix}_{l^*, E^*, I^*, R^*}
\]

Assuming that

\[
a_{11} = -\beta_{10}l^* - \beta_{20}E^* - \mu + \alpha
\]

\[
a_{12} = -\beta_{20}S^*
\]

\[
a_{13} = -\beta_{10}S^*
\]

\[
a_{14} = \gamma_0
\]

\[
a_{21} = \beta_{10}l^* + \beta_{20}E^*
\]

\[
a_{22} = \beta_{20}S^* - \varepsilon_0 - (\mu + \mu_{10})
\]

\[
a_{23} = \beta_{10}S^*
\]

\[
a_{24} = \varepsilon_0
\]

\[
a_{32} = -\nu_0 - (\mu + \mu_{20})
\]

\[
a_{33} = \nu_0
\]

\[
a_{44} = -\gamma_0 - (\mu + \mu_{30})
\]
So obtained

\[
|J - \lambda I| = \begin{vmatrix}
\lambda - a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & \lambda - a_{22} & a_{23} & 0 \\
0 & a_{32} & \lambda - a_{33} & 0 \\
0 & 0 & a_{43} & \lambda - a_{44}
\end{vmatrix}
\]

So we can get the characteristic equation

\[
|J - \lambda I| = a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0
\]

with assumption

\[
a_4 = -1
\]
\[
a_3 = a_{11} + a_{22} + a_{33} + a_{44}
\]
\[
a_2 = -a_{11}a_{22} - a_{22}a_{33} - a_{11}a_{33} - a_{22}a_{44} - a_{11}a_{33} - a_{33}a_{44} + a_{23}a_{32} + a_{12}a_{21}
\]
\[
a_1 = a_{11}a_{22}a_{33} + a_{11}a_{22}a_{44} + a_{22}a_{33}a_{44} + a_{11}a_{33}a_{44} - a_{23}a_{32}a_{44} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{44}
\]
\[
- a_{12}a_{21}a_{33} - a_{13}a_{23}a_{32}
\]
\[
a_0 = -a_{11}a_{22}a_{33}a_{44} - a_{11}a_{23}a_{32}a_{44} + a_{12}a_{21}a_{33}a_{44} + a_{13}a_{21}a_{32}a_{44} + a_{14}a_{21}a_{32}a_{43}
\]

To find out the sign of the coefficient \(a_i\) use the following \textit{routh-hurwitz} criteria table:

| Table 1. Table Routh-Hurwitz | Coefficients |
|-------------------------------|--------------|
| \(\lambda^4\)                | \(a_0\)      |
| \(\lambda^3\)                | \(a_1\)      |
| \(\lambda^2\)                | \(b_1\)      |
| \(\lambda^1\)                | \(c_1\)      |
| \(\lambda^0\)                | \(d_1\)      |

The system around the endemic equilibrium point is said to be stable if the signs from the first column (\(a_0, a_1, b_1, c_1, d_1\)) in the \textit{routh-hurwitz} table have the same sign that is positive.

| Table 2. Parameter Value Used | Parameter | Value | Source |
|-------------------------------|-----------|-------|--------|
| \(\Lambda\)                  | 1,159     | [4]   |
| \(\beta_{10}\)               | 0.055     | [5]   |
| \(\beta_{20}\)               | 0.394     | [5]   |
| \(\mu\)                      | 0.0097    | [4]   |
| \(\mu_{10}\)                 | 0.5       | [4]   |
| \(\mu_{20}\)                 | 0.6647    | [4]   |
| \(\mu_{30}\)                 | 0.8       | [4]   |
| \(\varepsilon_0\)            | 0.0596    | [4]   |
Parameter | Value | Source
--- | --- | ---
\(\nu_0\) | 0.8613 | [4]
\(\gamma_0\) | 0.2995 | [4]
\(\alpha\) | 0.70519 | [7]

By entering the parameter values found in table 2, the values obtained:

\[ a_0 = 0.67182 \]
\[ a_1 = -1.07500 \]
\[ b_1 = 0.56263 \]
\[ c_1 = 0.36345 \]
\[ d_1 = -1 \]

Because \(a_1\) and \(d_1\) are negative, the system around the endemic equilibrium point is unstable. Thus, the spread of the Ebola virus is categorized as high at time \(t\) and the population transition rate \(E\) to \(I\) is quite large.

2.6 Numerical Simulation

In this section a simulation is done using matlab software with the aim to see the effect of \(\alpha\) on the spread of the Ebola virus in the DR Congo.

The data used in this paper are data that spread the Ebola virus that occurred in DR Congo in 1976, 1977, 1995, 2007, 2008-2009, 2012, 2014, 2017 and may – July 2018. Related information from WHO and journals ([1],[2]) obtained data on the total population of DR Congo is 77,267,000; \(S = 77,265,875\); \(E = 5,625\); \(I = 1,125\); \(R = 285\).

![Simulation Without Alpha Parameter](image)

**Figure 1.** Graph simulation the rate of spread the ebola virus without \(\alpha\) parameter
Figure 2. Graph simulation the rate of spread the ebola virus with $\alpha$ parameter

Figure 1 shows the spread of the Ebola virus without the influence of changes in human behavior or the same as the initial model in [4]. While Figure 2 shows the spread of the Ebola virus due to the influence of changes in human behavior ($\alpha$).

The population value $S$ without the parameter $\alpha$ is 1.078 when $t = 3$ months. After entering the parameter $\alpha$, the population value $S$ has increased by 87.17% so that it becomes 8.4 when $t = 3$ months. This shows that changes in human behavior influenced by climate can increase the population of people who are susceptible to infection with the Ebola virus.

The value of population $E$ without the $\alpha$ parameter is $1.76 \times 10^{-5}$ when $t = 3$ months. After entering the parameter $\alpha$, the population value $S$ has increased by 99.36% so that it becomes $2.76 \times 10^{-3}$ when $t = 3$ months. This shows that changes in human behavior influenced by climate can increase the population of people who are susceptible to infection with the Ebola virus. This shows that changes in human behavior can also increase the number of people showing signs of being infected with the Ebola virus.

3. Conclusion
From the results of the analysis carried out in the epidemic model of the spread of the SEIR type Ebola virus, the following conclusions were drawn:

1) The results of the analysis of the mathematical models that have been constructed show that the system of equations has a single and a solution. With the Lipschitz Constant value $k(t) = \left(\lambda_0 + (\nu_0 + (\mu + \mu_{20}))\right)_{\min}$ which shows that the maximum condition of the spread of the Ebola virus occurs when more population $E$ moves to population $I$. While the minimum conditions for the spread of the Ebola virus in the DR Congo occur when the mortality rate increases in population $I$ and more people recover from Ebola virus infection.

2) From the stability analysis, it was concluded that the system in this study was unstable around the endemic equilibrium point $(S^*, E^*, I^*, R^*)$. This shows that the Ebola virus still infects the DR Congo community with a transition from population $E$ to a fairly high population $I$ with a parameter indicator value $\varepsilon_0 = 0.4$.

3) Simulation results with parameter values $\Lambda = 0.03602$, $\beta_{10} = 0.055$, $\beta_{20} = 0.394$, $\mu = 0.0097$, $\mu_{10} = 0.5$, $\mu_{20} = 0.5061$, $\mu_{30} = 0.8$, $\varepsilon_0 = 0.4$, $\nu_0 = 0.5140$, $\gamma_0 = 0.2$, and $\alpha =$
0.70519 which shows that deviant behavior can increase populations $S$ and $E$ by an average of 93.26%.

References

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