Inflationary universe model with $\Omega > 1$ and tachyon field

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Abstract.

In this work we study closed inflationary universe models by using a tachyonic field theory. We determine and characterize the existence of an inflationary universe with $\Omega > 1$. We found that these models are less restrictive compared to the standard ones with a scalar field. We use recent astronomical observations to constraint the parameters appearing in the model.

1. Introduction

Due to recent results [1, 2], it may be interesting to consider other inflationary universe models where the spatial curvature is taken into account [3]. In fact, it is interesting to check if the flatness in the curvature, as well as in the spectrum, are indeed reliable and robust predictions of inflation [4]. In the context of an open scenario, several authors have proposed models in which open universes may be realized [5]. The possibility to inflationary universe models with $\Omega > 1$ has been analyzed in [4, 6]. In this article we would like to describe this kind of models.

On the other hand, an inflationary phase described by the potential of a tachyon field has been considered in a quite diverse topics [7, 8]. An open inflationary universe dominated by tachyon matter is studied in Ref. [9]. In this work we adopt the point of view considered by Linde [4] but where inflation is driven by a tachyon field.

2. Cosmological Equations in the Tachyon Models

The action for our model is given by [10]

$$S = \int \sqrt{-g} \, d^4x \left[ \frac{R}{2\kappa} - V(\phi) \sqrt{1 - \partial^{\mu} \phi \partial_{\mu} \phi} \right],$$

(1)

where $\kappa = 8\pi G = 8\pi / M_p^2$ (here $M_p$ represent the Planck mass) and $V(\phi)$ is the scalar tachyon potential.

The metric is described by $ds^2 = dt^2 - a(t)^2 \, d\Omega_k^2$, where $a(t)$ is the scale factor, $t$ represents the cosmic time and $d\Omega_k^2$ is the spatial line element corresponding to the hypersurfaces of homogeneity. We will restrict ourselves to the case $k = 1$ only. Using the metric in the action
(1), we obtain the following field equations:

\[ \frac{\ddot{a}}{a} = \frac{\kappa}{3} \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \left( 1 - \frac{3}{2} \dot{\phi}^2 \right), \]

\[ \frac{\ddot{\phi}}{1 - \dot{\phi}^2} = -3 \frac{\dot{a}}{a} - \frac{1}{V(\phi)} \frac{dV(\phi)}{d\phi}, \]

where the dot denotes derivative with respect to \( t \).

3. Constant Potential

First let us take a simple model with the following step-like effective potential: \( V(\phi) = V = \text{const.} \) for \( \phi > \phi_0 \), and \( V(\phi) \) is extremely steep for \( \phi < \phi_0 \). We consider that the birth of the inflating closed universe can be created "from nothing", in a state \( \phi = \phi_0 \) at the point with \( \dot{a} = 0, \dot{\phi} = 0 \) and potential energy density \( V(\phi_0) \geq V \). If the effective potential for \( \phi < \phi_0 \) grows very sharply, then the tachyon field instantly falls down to the value \( \phi_0 \), with potential energy \( V(\phi_0) = V \), and its initial potential energy \( V(\phi_0) \) becomes converted to the kinetic energy. Since this process happens instantly we can consider \( \dot{a} = 0 \), so that tachyon field arrives to the the plateau with a velocity given by:

\[ \dot{\phi}_0 = +\sqrt{1 - \left( \frac{V}{V(\phi_0)} \right)^2}. \]

Thus, in order to study the inflation in this scenario, we have to solve Eqs. (2) and (3) in the interval \( \phi \geq \phi_0 \), with initial conditions \( \dot{\phi} = \dot{\phi}_0, a = a_0 \) and \( \dot{a} = 0 \). In particular, if we insert Eq. (4) into Eq.(2), we obtain:

\[ \frac{\ddot{a}}{a} = \frac{\kappa}{6} V(\phi_0) \left[ 3 \left( \frac{V}{V(\phi_0)} \right)^2 - 1 \right]. \]

Then, there are three different scenarios, depending on the value of \( V(\phi_0) \). First, when universe remain static. In the second case universe rapidly collapses. In the third case \( \ddot{a} > 0 \), the universe enters into an inflationary stage and corresponds to \( V < V(\phi_0) < \sqrt{3} V \). In this work, we are going to make a simple analysis of the cosmological equations of motion when the last case is satisfied.

The tachyon field satisfies the Eq.(3) with \( V = \text{constant} \), which implies

\[ \phi^2(t) = \frac{1}{1 + Ca^0(t)}, \]

where \( C \) is a positive integration constant.

The Eq.(6) implies that the evolution of the universe rapidly falls into an exponential regimen (inflationary stage) where the scale factor becomes \( a \sim e^{Ht} \) with \( H = \sqrt{\frac{2V}{3}} \). The tachyon field moves by an amount \( \Delta \phi_{in} \) and then stops. From Eq. (6) we get:

\[ \Delta \phi_{in} = \frac{1}{3H} \sinh^{-1} \left( \frac{1}{\sqrt{C}} \right). \]

Before inflation take place, we can write conveniently the equation for the scale factor (2) introducing:

\[ \beta(t) = \frac{1}{2} \frac{1}{\sqrt{1 - \dot{\phi}^2}} \left( 1 - \frac{3}{2} \dot{\phi}^2 \right). \]
Now we proceed to make a simple analysis for $\beta(0) = \beta_0 < 1$. At the beginning, we have $a(t) \approx a_0$ and $\beta(t) \approx \beta_0$, then for small $t$ the solution of equation (2) is

$$a(t) = a_0 \left(1 + \frac{\kappa \beta_0 V}{3} t^2 \right). \quad (9)$$

From Eqs.(6) and (9) we can find the time interval $\Delta t_1$ where $\beta(t)$ becomes twice as large as $\beta_0$. Consequently we obtain that the tachyonic field increases by

$$\Delta \phi_1 \sim \dot{\phi}_0 \Delta t_1 \sim \frac{1}{\sqrt{\kappa V}}. \quad (10)$$

After the time $\Delta t_2 \approx \Delta t_1$, the tachyonic field increases by the amount $\Delta \phi_2 \approx \Delta \phi_1$, and the rate of growth of $a(t)$ also increases. This process finishes when $\beta(t) \rightarrow 1/2$. Since at each interval $\Delta t_i$ the value of $\beta$ doubles, the number of intervals $n_{int}$ after which $\beta(t) \rightarrow 1/2$ is $n_{int} = -1 - \frac{\ln \beta_0}{\ln 2}$. Thus, the value of the tachyon field at which the inflation begins is:

$$\phi_{inf} \sim \phi_0 - \left(1 + \frac{\ln \beta_0}{\ln 2} \right) \frac{1}{\sqrt{\kappa V}}. \quad (11)$$

This result is sensitive to the choice of value of the $V$ and the initial velocity of the tachyon field immediately after it rolls down to the plateau of the potential energy. If the tachyon moves with a large initial velocity the inflation is delayed, but once the inflation begins, it never stops.

Now we estimate the conditional probability that the universe is created with an energy density equal to $\sqrt{3} V - \beta_0 V$. Assuming that this energy is smaller than $V(\phi_{inf}) = \sqrt{3} V$, for the probability we get [11]

$$P \sim \exp \left(\frac{-M_p^4 \beta_0}{8 V} \right), \quad (12)$$

where we have used that $\dot{\phi}^2 \ll 1$. This implies that the process of quantum creation of an inflationary universe is not exponentially suppressed by $\beta_0 < 8V/M_p^4$.

### 4. Exponential potential

Now we proceed with a more realistic case, where $V(\phi) \approx V_0 e^{-\lambda \phi}$ for $\phi > \phi_0$ and becomes extremely steep at $\phi < \phi_0$. Here $\lambda > 0$ and $V_0$ are free parameters, besides $\lambda$ is related with the tachyon mass $[8]$.

The process is divided in three parts. The first part corresponds to the creation of the (closed) universe “from nothing” in a state where the tachyon field takes the value $\phi_{inf} \leq \phi_0$ at the point with $\dot{a} = 0$, $\dot{\phi} = 0$, and where the potential energy is $V(\phi_{inf})$. If the effective potential for $\phi < \phi_0$ grows very sharply, then the tachyon field instantly falls down to the value $\phi_0$ given by (4).

The next parts are described by Eqs. (2) and (3) in the interval $\phi \geq \phi_0$ with initial conditions $\phi = \phi_0$, $a = a_0$ and $\dot{a} = 0$. In particular, the second part corresponds to the motion of the tachyon field before the beginning of the inflation stage, and it is well described by the approximation of the Eq.(3) with $V = constant$ and the tachyon field satisfies Eq. (6). Following the section 3 we solve the equation for $a(t)$ by considering $\beta_0 \ll 1$. Then, according to our previous result inflation begin when the tachyon field get the value:

$$\phi_{inf} \sim \phi_0 - \left(1 + \frac{\ln \beta_0}{\ln 2} \right) \frac{1}{\sqrt{\kappa V (\phi_0)}}. \quad (13)$$

The third part corresponds to the stage of inflation where $\phi$ is small enough and $a(t)$ grows up exponentially. We will find analytical solution to the equation of the tachyon field in the
inflationary era, then we considerer the approximation of flat space for the Friedmann equations. Thus, this part is well described by the approximation of the equations of motion of Ref.[12]. Here the scale factor has the following behavior:

$$\frac{a(t)}{a_i} = e^{\gamma t(C-(\lambda^2/12\gamma)t)},$$

(14)

where $a_i$ is the value of the scale factor at the beginning of inflation and $C = e^{-\lambda\ln t}$. The values of the tachyon potential at the beginning and at the end of inflation are related by the number of the e-folds $N$ (see Ref.[12]): $(2N + 1)V_{end} = V(\phi_{inf})$. Then, by using the last relation and Eq.(13), we obtain

$$\beta_0 = \frac{1}{2} \left[ \frac{(2N + 1) \lambda^2 \sqrt{V(\phi_0)}}{2\kappa} \right] \ln(2).$$

(15)

From the discussion of section 3 we know that the probability of the universe with $\beta_0 \neq 0$ will be exponentially suppressed, unless the universe is created very close to the threshold value $V(\phi_{in}) = \sqrt{3}V(\phi_0)$, with

$$\beta_0 < \frac{\kappa^2 V(\phi_0)}{8\pi^2}.$$  

(16)

If we take $\lambda = 10^{-5}\kappa^{-1/2}$ and $V_0 = 10^{-7}\kappa^{-2}$ in the potential, using [12]. And we assume that $\phi_0 \sim 10^5\kappa^{-1/2}$, then we have that $\beta_0 < 2.2 \cdot 10^{-10}$. Following Ref.[4] we can argue that the probability for start with the value $\beta_0 < 2.2 \cdot 10^{-10}$ is suppressed, due to the small phase space corresponding to these values of $\beta_0$. Thus, it is most probable to have $\beta_0 \sim 2.2 \cdot 10^{-10}$, and in that case if we set $\phi_0 = 1.1 \cdot 10^5\kappa^{-1/2}$ which satisfies the condition Eq.(16) and we obtain $N = 60$, this leads to $\Omega = 1.1$ (closed Universe). On the other hand, if we take $\phi_0 = 0.5 \cdot 10^5\kappa^{-1/2}$, we get $N = 171$ and the universe becomes flat.

5. Analysis

For modes with a wavelength much larger than the horizon ($k \ll aH$), the spectral index $n_s$ is an exact power law with the power spectrum of density perturbations, expressed by

$$P_R(k) \propto k^{n_s},$$

where $k$ is the comoving wave number. In tachyon inflationary models $n_s = 1 - 2\epsilon_1 - \epsilon_2$, and the tensor spectral index is $n_T = -2\epsilon_1$, where the slow-roll parameters are given by [13]. Then we obtain that the running of the scalar spectral index for our model becomes

$$\alpha_s = \frac{dn_s}{d\ln k} \approx -2\lambda^2 V_0^2 e^{2\lambda \phi} = -2\lambda^2 V(\phi)^2.$$  

(17)

Using the WMAP three-year data [2] and the SDSS large scale structure surveys [15], an upper bound $\alpha_s(k_0)$ has been found, where $k_0$=0.002 Mpc$^{-1}$ corresponds to $L = \tau_0k_0 \simeq 30$, with the distance to the decoupling surface $\tau_0=14,400$ Mpc. SDSS measures galaxy distributions at red-shifts $a \sim 0.1$ and probes $k$ in the range 0.016 h Mpc$^{-1}< k < 0.011$ h Mpc$^{-1}$. The recent WMAP three-year data results give the values for the scalar curvature spectrum $P_R(k_0) \equiv 2582^2 H(k_0)/4 \simeq 2.3 \times 10^{-9}$ and the spectral index $n_s \simeq 0.95$. These values allow us to find the constraints on the parameters of our model. Furthermore, from the numerical solution we can obtain their values. In particular, for $k_0=0.002$ Mpc$^{-1}$, we have $n_s \simeq 0.96$ and $n_T \approx -0.03$. Notice that those indices are very close to the Harrison-Zel’dovich spectrum.

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References
[1] Peiris H V et al. 2003 Astrophys. J. Suppl. 148 213. Spergel D N et al. [WMAP Collaboration] 2003 Astrophys. J. Suppl. 148 175. Bennett C L et al. 2003 Astrophys. J. Suppl. 148 1
[2] L. Page et al Preprint astro-ph/0603449
[3] Uzan J P, Kirchner U and Ellis G F R 2003 Mon. Not. Roy. Astron. Soc. 344 L65
[4] Linde A 2003 JCAP 0305 002
[5] Bucher M et al 1995 Phys. Rev. D 52 3314. Bucher M and Turok N 1995 Phys. Rev. D 52 5538. Linde A 1995 Phys. Lett. B 351 99. Linde A and Mezhlumian A 1995 Phys. Rev. D 52 6789. Linde A 1999 Phys. Rev. D 59 023503. Linde A, Sasaki M and Tanaka T 1999 Phys. Rev. D 59 123522. del Campo S and Herrera R 2003 Phys. Rev. D 67 063507. Bouchmadi M and González-Días P F 2002 Phys. Rev. D 65 063510. del Campo S, Herrera R and Saavedra J 2004 Phys. Rev. D 70 023507.
[6] Ellis G, Stoerger W, McEwan P and Dunsby P 2002 Gen. Rel. Grav. 34 1445. Ellis G, Stoerger W, McEwan P and Dunsby P 2002 Gen. Rel. Grav. 34 1461. del Campo S, Herrera R and Saavedra J 2005 Int. J. Mod. Phys. D 14 1. del Campo S and Herrera R 2005 Class. Quant. Grav. 22 2687
[7] Gibbons G W 2002 Phys. Lett. B 537 1.
[8] Fairbairn M and Tytgat M H G 2002 Phys. Lett. B 546 1
[9] Balart L, del Campo S, Herrera R, Labrana P and Saavedra J (Preprint GACG 08/2006 Preprint)
[10] Sen A 2002 JHEP 0204 048 (2002), JHEP 0207 065 (2002)
[11] Koyama K and Soda J 2000 Phys. Lett. B 483 432
[12] Sami M, Chingangbam P and Qureshi T 2002 Phys. Rev. D 66 043530
[13] Steer D A and Vernizzi F 2004 Phys. Rev. D 70 043527
[14] Ballesteros G, Casas J A and Espinosa J R 2006 JCAP 0603 001
[15] Tegmark M et al 2004 Phys. Rev. D 69 103501