Heavy dileptons from nonequilibrium QGP

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Abstract
The rate of emission of heavy dileptons from QGP is found without an assumption of its complete thermal equilibrium. We base on the real-time quantum field kinetic approach [1] and use the expansion up to the second order with respect to strong coupling constant $g$. The final answer is not free from the collinear singularities and we show that this is the actual issue. As a result the main contribution to the rate of the heavy dileptons production at $M/T \sim 10$ comes from the process $qg \rightarrow \gamma^*$. 

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Introduction

Transparency of hadronic plasma for electromagnetic signals inspires hope that they may carry information about the intimate details of interactions in the quark-gluon media. For this reason they are considered as promising probes of the hot hadronic matter\[2\]. Initial theoretical efforts were put basically to the study of an influence of hydrodynamic background on the total yield of the dileptons [3]. The production rate was taken from the first Born’s diagram, $q\bar{q} \rightarrow \gamma^*$. The dynamics of quark-gluon interactions was present only virtually via an assumption that it has led to a thermal distribution of quarks. The further improvement of the microscopic picture also took into account the gluon interactions[4] up to the $\alpha_s$-order.

The most encouraging result was obtained by Baier, Pire an Schiff [5]. They found that the rate of emission of the dileptons in the $\alpha_s$ order is defined as well as the first Born’s term: all the infrared and mass singularities has cancelled out in their approach. It was the direct check of the Lee-Nauenberg theorem for the emission of dileptons from the equilibrium thermal bath. This result was confirmed in somewhat different manner in [6].

The most important consequence of this study was a statement that even apart from the factor $\exp(-M/T)$ the rate of dilepton emission is a decreasing function of $M/T$.

Nevertheless there remain some questions. First, any deviation from the detail balance relationships of the true thermal ensemble breaks a zero balance of mass singularities. The calculation given below is at least an example. Second, the rate of dilepton emission in the process $q\bar{q}g \rightarrow \gamma^*$ increases (apart from $\exp(-M/T)$ ) with the growth of the dilepton mass. The effect is due to specific features of the phase-space volume of the reaction and is not sensitive to the nature of the infrared cut-off. The leading role of this reaction was noticed in [7]. Recently it was confirmed [8] by numerical calculations. In this paper we give an analytic estimate.

We confine ourselves to the same first $\alpha_s$-order also. The main difference of this work from the previous ones is that we consider the quark gluon plasma without thermal and chemical equilibrium. We show that both effects, the absence of collinear singularities and suppression of the process $q\bar{q}g \rightarrow \gamma^*$, are the artifacts of ideal thermal equilibrium. If the equilibrium is broken then at $M/T \sim 10$ the rate of this process overwhelms the Born’s rate while the rates of all the other processes are suppressed at least by the factor $T/M$.

An initial goal of this study was to design a tool which could have been used for calculations of the dilepton emission against the background of partons distributions generated from cascade simulation of the $A-A$ collision. Some scenario show [9] that cascade do not reach thermal equilibrium and that quarks are relatively suppressed with respect to hot glue. So we used some flexible analytic approximation for distribution of the partons in favour of the opportunity to get an analytic answer and
reliable estimate of the leading terms.

While choosing this approximation we kept in mind the following scenario of the heavy ions collision. The initial stage of a collision at RHIC or LHC energies ($\tau \sim 0.5 \text{fm}$) is a region of nucleons fragmentation and development of the initial parton cascade. Dileptons are emitted only due to the hard Drell-Yan process. Our region begins a little bit later when partons are already chaotized and may be described by the one-particle distributions.

The most general density matrix which simulate any given ahead form of the one-particle distribution is of the next form,

$$\rho = \prod_N \prod_p e^{-f_j(N,p)a_j^+(N,p)a_j(N,p)}$$  \hfill (1)

where $N$ label the space cells on the hypersurface of the initial data and $n_j(N,p) = a_j^+(N,p)a_j(N,p)$ is an operator of the number of partons of the sort $j$ and quantum numbers $p$ in the $N$-th sell. Thus we completely neglect all the correlation effects in the phase space of the partons.

The density matrix (1) allows one to apply the field theory to the description of the further evolution and results in a set of habitual elements like cross-sections, self-energies, vertices. If the functions $f_j(N,p)$ which define the shapes of partons distributions are completely arbitrary then we are totally confined to numerical calculations. The only piece of theoretical study will be connected with the necessity to transform some singular expressions to the form when numerical calculations will be unambiguous.

Allover this paper we use the Boltzmann-like distributions damaged by introduction of specific parameters $\zeta_Q$ and $\zeta_G$ which all together will be considered as the measure of nonequilibrium in the quark-gluon system. We adopt for quarks and gluons

$$n_F(p) = \zeta_Q e^{-|p_u|/T}, \quad n_B(p) = \zeta_G e^{-|p_u|/T},$$  \hfill (2)

where $u^\mu$, the 4-velocity of the nonequilibrium partonic media, fugacities $\zeta$ and temperature are very smooth functions of space-time coordinates. Scenario of the hot glue [9] leads to $\zeta_Q < \zeta_G < 1$ and $T$ is rather a formal parameter than a thermodynamic temperature. We hope that distributions (2) are enough flexible to approximate the cascade approaching to the thermal equilibration.

An important advantage of this parametrization is that all the calculations can be performed analytically to the very end and result in no more than one-dimensional convergent integrals.

The paper is organized as follows. In Sec.1 we briefly remind the basic definitions as they were derived in ref.[1] and trace the way of perturbative expansion assuming the weak coupling between the quarks and gluons. In Sec.2 we split the general
expression for the rate of the dilepton emission into the pieces which at some heuristic level may be called as real processes and radiative corrections. In Sec.3 the rate of the real processes is divided into more specified parts, annihilation and Compton and the integration over the distributions of initial partons is carried out. We carefully analyze the kinematic regions where the further radiative corrections will be needed. In Sec.4 we do the same for the virtual corrections. In Sec.5 we perform the remaining integration over the final state of the partons. At this step we still have a set of the IR-divergent one-dimensional integrals. Sec.6 describes the assembling procedure which results in the IR-finite answer for the rate of emission. We show that scale which is responsible for IR-finiteness of the observable rate is $T$ (not $gT'$) and put forward the arguments why the double logarithms should be exponentiated into the K-factor.

In conclusion we analyze the mass singularities and show that they are naturally restricted by the amplitude of the forward scattering of hard quark in the partonic media. By some chance this cut-off coincide with the so-called thermal mass. This mass can not be associated with the rest mass of any quasi-particle.

1. Perturbative expansion for the rate of emission

For sake of completeness we shall start with reminding the definitions and the main framework of calculations[1]. We shall close this section by the formal expansion of the rate of emission defined in the scheme of quantum field kinetics up to the first order in $\alpha_s$.

The inclusive rate of the dilepton emission is given by

$$\frac{dN_{e^+e^-}}{dk_1 dk_2 d^4x} = -ie_0 \frac{L_{\mu\nu}(k_1, k_2)}{4(2\pi)^6} \Delta_{10}^{\mu\nu}(-k),$$

(1.1)

where $k = k_1 + k_2$, $L^{\mu\nu} = k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - g^{\mu\nu}(k_1 k_2 - m_e^2)$, is a trace of lepton spinors,

$$\Delta_{10}^{\mu\nu}(-k) = -i \int d^4(x - y)(T^+(A^\mu(x)S^+)T(A^\nu(y)S))e^{-ik(x-y)},$$

(1.2)

is a kind of photon Wightman function averaged with $\rho^{QCD}$ and $e_0$ is an electric charge of the lepton.

We suppose that an explicit separation of long-range and short-range orders takes place and have passed from the inclusive cross-sections to the emission rates per unit volume. That is we have assumed that even not reaching thermal equilibrium cascade produces at least locally homogeneous distributions of the partons.

The integral equation for $\Delta_{10}$ can be effectively solved in a manner described in [1] and we get the rate of dileptons emission expressed via polarization operator $\Pi_{10}(-k)$,
\begin{align}
\frac{dN_{e^+e^-}}{d^4k}\frac{dN_{e^+e^-}}{d^4k'} = \frac{i\epsilon_0^2 L_{\mu\nu}(k_1, k_2) \Pi_{10}^{\mu\nu}(-k)}{4(2\pi)^6 |k|^2},
\end{align}

where $|k|^2$ stands for a product $D_{ret}(k)D_{adv}(k)$ out of the photon mass shell.

Integrating this distribution over the relative momentum of the leptons we get a distribution of the dileptons over their masses and total momenta:

\begin{align}
\frac{dN_{e^+e^-}}{d^4k} = 2k_0 \frac{dN_{e^+e^-}}{d^3k M^2} = \frac{i\pi\epsilon_0^2 \epsilon_\mu \Pi_{10}^{\mu\nu}(-k)}{3(2\pi)^6 M^2}
\end{align}

Polarization matrix in these equations is given by an expression

\begin{align}
\Pi_{AB}(x, y) = -i(-1)^{A+B} e^2 \sum_{R,S=0}^1 (-1)^R S \int d\xi d\eta \gamma^\mu G_{AR}(x, \xi) E_{RSB}(\xi, \eta; y) G_{SA}(\eta, x),
\end{align}

where $e$ stands for the electric charge of a quark and a subsequent summation over the $u$- and $d$-quarks is assumed, $e^2 = (5/18)\epsilon_0^2$. It contains an electromagnetic vertex dressed by strong interaction,

\begin{align}
E_{RS, P}(x, y; z) = (-1)^{R+S+P} \frac{\delta[G^{-1}(x, y)]_{RS}}{e\delta A_\lambda(z_P)}.
\end{align}

The matrix Schwinger-Dyson equation can be solved in the form,

\begin{align}
[G^{-1}]_{AB} = [G_0^{-1}]_{AB} - M_{AB},
\end{align}

where the matrix of quark self-energy looks as

\begin{align}
M_{AB}(x, y) = i(-1)^{A+B} \frac{e^2 \sum_{R,S=0}^1 (-1)^R S}{g_0} \int d\xi d\eta \times
\end{align}

\begin{align}
\times \gamma^\mu G_{AR}(x, \xi) \Gamma_{RB,S}(\xi, \eta; y) D_{SA,\lambda\mu}(\eta, x),
\end{align}

and $\Gamma$ is the vertex of strong interaction. The inverse Greenian $G_0^{-1}$ is nothing but the Dirac operator with the "external" field $A(x)$,

\begin{align}
[G_0^{-1}]_{AB}(x, y) = (-1)^A \delta_{AB} \delta(x - y)[i\hat{\partial}_x + e\xi \hat{\mathcal{A}}(x_A) - m]
\end{align}

Proceeding from the Eq.(1.6) and confining ourselves to the two lowest orders of the vertex expansion we get

\begin{align}
\Pi_{10}^{\mu\nu}(-k) = ie^2 N_c \int \frac{d^4p}{(2\pi)^4} \gamma^\mu G_{10}(p - k) \gamma^\nu G_{01}(p) + e^2 g^2 N_c C_F \sum_{R,S=0}^1 (-1)^R S \times
\end{align}

\begin{align}
\times \int \frac{d^4p d^4q}{(2\pi)^8} D_{SR}(q) \gamma^\mu G_{1R}(p - k) \gamma^\nu G_{R0}(p + q - k) \gamma^\nu G_{0S}(p + q) \gamma^\lambda G_{S1}(p)
\end{align}
The second term of this equation is already restricted to the $\alpha_s$ order. The further expansion of the exact Greenians in the first term is performed by means of equations\[1\],

\[G_{01,10} = G_{01,10} - G_{ret}M_{01,10}G_{adv} + G_{ret}M_{ret}G_{01,10} + G_{01,10}M_{adv}G_{adv}, \quad (1.11)\]

where in the order we are interested in all the propagators are to be taken bare and self-energies should be calculated to the first nonvanishing order in strong coupling constant. The later are as following,

\[M_{AB}(s) = i(-1)^A\bar{B}g_0^2C_F(2\pi)^4\int d^4q\gamma^\mu G_{AB}(s + q)\gamma_\lambda D^{\lambda\mu}_{BA}(q), \quad (1.12)\]

and

\[M_{ret,adv}(s) = ig_0^2C_F2(2\pi)^4\int d^4q\left[\gamma^\mu G_{ret,adv}(s + q)\gamma_\lambda D_1^{\lambda\mu}(q) + \gamma_\mu G_1(s + q)\gamma_\lambda D_1^{\mu\nu}_{adv,ret}(q)\right]. \quad (1.13)\]

2. Separation of real processes and radiative corrections.

Eqs. (1.11) - (1.14) give us the rate of emission in the closed form containing all contributions of all possible processes. To split them we should notice that an indication of the type of the process comes from its statistical weight. Thus we can single out the direct annihilation, $q\bar{q} \rightarrow \gamma^*$, annihilation with the emission or adsorption of a gluon, $q\bar{q} \rightarrow g\gamma^*$ and $q\bar{g} \rightarrow \gamma^*$, and Compton scattering of quarks on a gluon, $qg \rightarrow q\gamma^*$ and $\bar{q}g \rightarrow \bar{q}\gamma^*$.

If in the first line of Eq.(1.11) we substitute instead of exact Greenians $G_{10,01}$ their unperturbed values we get the first Born’s term, related to direct process $q\bar{q} \rightarrow \gamma^*$:

\[\Pi^{\mu\nu}_{Born}(-k) = ie^2N_c\int \frac{d^4p}{(2\pi)^4}\gamma^\mu G_{10}(p - k)\gamma^\nu G_{01}(p). \quad (2.1)\]

The diagram of the Born’s term is given at Fig1.

If we insert terms with the off-diagonal elements of the self-energy we get two equal terms related to the real processes, which contribute to the second Born’s approximation term

\[\Pi^{\mu\nu}_{Born}(-k) = 2e^2g^2N_cC_F\int \frac{d^4qd^4q}{(2\pi)^8}D_{10}(q)Tr\{\gamma^\mu G_{10}(p - k)\gamma^\nu \times \times G_{ret}(p)\gamma_\lambda G_{01}(p + q)\gamma_\lambda G_{adv}(p)\} \quad (2.2)\]

The second contribution to the real processes comes from the two equal items with $R \neq S$ in the sum in Eq.(1.11).
\[
\Pi_{\mu\nu}^b(-k) = -2e^2g^2N_cC_F \int \frac{d^4p d^4q}{(2\pi)^8} D_{10}(q) \times \\
\times Tr\{\gamma^\mu G_{10}(p - k)\gamma^\lambda G_{00}(p + q - k)\gamma^\nu G_{01}(p + q)\gamma_\lambda G_{11}(p)\}\tag{2.3}
\]

In a short while we shall rearrange \(\Pi_{\text{real}} = \Pi_a + \Pi_b\) into a sum \(\Pi_{\text{real}} = \Pi_{\text{em}} + \Pi_{\text{abs}} + \Pi_{\text{com}}\) of the inclusive subprocesses of the \(q\bar{q}\)-annihilation with the emission or absorption of the gluons and the Compton (bremsstrahlung) process. The diagrams of \(\Pi_a\) and \(\Pi_b\) are given at Fig.2. The crossed lines present the on-mass-shell partons with density matrix (1). Their statistical weights originate from the off-diagonal densities \(G_{AB}\) and \(D_{AB}\), \(A \neq B\), which carry a certain sign of the energy.

The rest items from equations (1.11) and (1.12) are the self-energy and vertex corrections.

The two remaining terms of the Eq.(1.12), containing \(M_{\text{ret}}\) and \(M_{\text{adv}}\) give rise to the radiative corrections of the self-energy type. They account both for the vacuum effects and for the amplitude of the forward scattering of quark on the partons produced by the cascade.

\[
\Pi_{\text{mass}}^\mu\nu(-k) = e^2g^2N_cC_F \int \frac{d^4p d^4q}{(2\pi)^8} \left\{\gamma^\mu G_{10}(p - k)\gamma^\nu G_{01}(p) \times \\
\times \{[\gamma^\lambda G_{\text{ret}}(p + q)\gamma_\lambda D_1(q) + \gamma^\lambda G_{1}(p + q)\gamma_\lambda D_{\text{adv}}(q)]G_{\text{ret}}(p) + \\
+ [\gamma^\lambda G_{\text{adv}}(p + q)\gamma_\lambda D_1(q) + \gamma^\lambda G_{1}(p + q)\gamma_\lambda D_{\text{ret}}(q)]G_{\text{adv}}(p)\} \right\} \tag{2.4}
\]

This expression contains several functions with overlapping singularities and it seems better to transform it

\[
\Pi_{\text{mass}}^\mu\nu(-k) = e^2g^2N_cC_F \int \frac{d^4p d^4q}{(2\pi)^8} \left\{\gamma^\mu G_{s}(p + q)\gamma^\nu G_{10}(p - k)\gamma_\lambda G_{01}(p)\gamma_\lambda \right\} \times \\
\times (G_s(p + q)D_1(q) + G_1(p + q)D_s(q)) \tag{2.5}
\]

The items with \(R = S\) in Eq.(1.11) produce the vertex-type radiative corrections,

\[
\Pi_{\text{vert}}^\mu\nu(-k) = -e^2g^2N_cC_F \int \frac{d^4p d^4q}{(2\pi)^8} \times \\
\times \left\{\gamma^\mu G_{10}(p - k)\gamma^\lambda G_{00}(p + q - k)\gamma^\nu G_{00}(p + q)\gamma_\lambda G_{01}(p)D_{00}(q) + \\
+ \gamma^\mu G_{10}(p - k)\gamma^\lambda G_{11}(p + q - k)\gamma^\nu G_{11}(p + q)\gamma_\lambda G_{01}(p)D_{11}(q)\right\} \tag{2.6}
\]

The vacuum part of the \(\pi_{\text{vert}}\) is reasonable to calculate just in the form given above. It is due to the \(T\)-independent parts of \(G_{00}\) and \(G_{11}\). For the thermal part it is better
to transform,

\[ \Pi^\mu_{\text{vert}}(-k) = -\epsilon^2 g^2 N_c C_F \int \frac{d^4p d^4q}{4(2\pi)^8} Tr\{\gamma^\lambda G_{01}(p)\gamma^\mu G_{10}(p-k)\gamma^\lambda \times \}
\]

\[ \times (G_s(p + q - k)\gamma_\sigma G_s(p + q)D_1(q) + 2G_1(p + q - k)\gamma^\nu G_s(p + q)D_s(q)) \]  \hspace{1cm} (2.7)

These transformations present "real" and "virtual" processes in the same form: now all thermal diagrams have three cuts! The difference is that in radiative corrections the extra cut correspond to "elastic scattering on the distribution of the partons". The typical graphs of this corrections are plotted at Fig.3. Statistical weights of the additional cuts always originate from the local density of states \( G_1 \) (or \( D_1 \)). They are contributed both by induced emission and absorption.

3. Real processes. Integration over the initial states.

In this section we begin calculation of the real processes. We do not assume that quarks and gluons are in thermal equilibrium and use the partons distributions (2) for the analytic calculations.

At the end of this section the reader will also find physical discussion, what part of the phase-space is important for the heavy dileptons production.

In what follows we shall calculate only the distribution of the emitted dileptons over their total 4-momenta, so we calculate only \( \pi(k) = g_{\mu\nu}\Pi^\mu_{\text{vert}}(-k) \).

For the Born’s term the first integration is the last one,

\[ \pi_{\text{Born}}(k) = -\frac{3ie^2}{4\pi}(1 + \frac{2m^2}{M^2})\sqrt{1 - \frac{4m^2}{M^2}} M^2 e^{-ku/T}. \]  \hspace{1cm} (3.1)

In the next perturbative order we have,

\[ \pi_{\text{real}} = -\frac{ie^2 g^2 N_c C_F}{2\pi^5} \int d^4p d^4q \delta(q^2)\delta((p-k)^2 - m^2)\delta((p+q)^2) \times \]

\[ \times (SW_{\text{em}} + SW_{\text{abs}} + SW_{\text{com}}) \{ 1 + \frac{A}{(p^2 - m^2)^2} + \frac{B}{p^2 - m^2} + \frac{C}{p^2 - m^2 + 2kq} \} \]  \hspace{1cm} (3.2)

where we denoted:

\[ A = 2m^2(k^2 + 2m^2), \quad B = 3m^2 + k^2 + 2(kq) - \frac{4m^4 - (k^2)^2}{2(kq)} \]

\[ C = (k^2 + m^2) + \frac{4m^4 - (k^2)^2}{2(kq)} \]  \hspace{1cm} (3.3)

The expression in the curly brackets is nothing but a sum of the squared moduli of the matrix elements of the annihilation processes with emission, \( q\bar{q} \rightarrow g\gamma^* \), and absorption, \( q\bar{q}g \rightarrow \gamma^* \), of a real gluon or the Compton process, \( qg \rightarrow q\gamma^* \) and \( \bar{q}g \rightarrow \bar{q}\gamma^* \).
Just for the purpose of the following calculations it is written not it terms of habitual Mandelstam variables \((s, t, u)\). Specification of the process is due to statistical weights.

The statistical weight of the annihilation process with emission of a gluon looks as

\[
SW_{em} = \theta(k_0 - p_0)\theta(q_0 + p_0)\theta(q_0) n_F(k_0 - p_0)n_F(q_0 + p_0)\theta(q_0)\psi_k \approx \theta(k_0 - p_0)\theta(q_0 + p_0)\theta(q_0)\psi_k e^{-(ku)/T}(e^{-(qu)/T} + \zeta_G e^{-2(qu)/T}) \quad (3. 4)
\]

The statistical weight of the annihilation process with absorption of a gluon is

\[
SW_{abs} = \theta(k_0 - p_0)\theta(q_0 + p_0)\theta(-q_0)n_F(k_0 - p_0)n_F(q_0 + p_0)\theta(-q_0)\psi_k \approx \theta(k_0 - p_0)\theta(q_0 + p_0)\theta(-q_0)\psi_k e^{-(ku)/T}. \quad (3. 5)
\]

For the Compton rate of the dilepton emission the statistical weight equals to

\[
SW_{com} = -\theta(p_0 + q_0)\theta(p_0 - k_0)\theta(-q_0)n_F(p_0 + q_0)[1 - n_F(p_0 - k_0)]n_B(-q_0) - \theta(-p_0 - q_0)\theta(k_0 - p_0)\theta(-q_0)n_F(k_0 - p_0)[1 - n_F(-p_0 - q_0)]n_B(-q_0) - \approx -\zeta_G\psi_k \{\theta(p_0 + q_0)\theta(p_0 - k_0)\theta(-q_0)e^{-pu/T}[1 - \zeta_G e^{-(pu-ku)/T}] + \}
\]

\[
+\theta(-p_0 - q_0)\theta(k_0 - p_0)\theta(-q_0)e^{-ku/T}e^{(qu+pu)/T}[1 - \zeta_G e^{(qu+pu)/T}]\} \quad (3. 6)
\]

The 4-vector \(k^\mu + q^\mu\) is time-like. So we can perform integration over \(p\) using the Breit reference system where \(k + q = 0\). The exact analytic result has the next covariant form:

\[
\pi_{em} = \frac{ie^2 q^2 N_c C_F}{4\pi^4} \zeta_Q^2 e^{-(ku)/T} \int d^4q\delta(q^2)\theta(q_0)\theta[(k + q)^2 - 4m^2]\mathcal{F}_a(kq)\{e^{qu/T} + \zeta_G e^{-2qu/T}\},
\]

\[
\pi_{abs} = \frac{ie^2 q^2 N_c C_F}{4\pi^4} \zeta_Q \zeta_G e^{-(ku)/T} \int d^4q\delta(q^2)\theta(q_0)\theta[(k - q)^2 - 4m^2]\mathcal{F}_a(-kq) \quad (3. 7)
\]

where

\[
\mathcal{F}_a(x) = -(1 + \frac{M^2 + 2m^2}{x} + \frac{M^4 - 4m^4}{2x^2})\ln \frac{1 + \sqrt{1 - 4m^2/(M^2 + 2x)}}{1 - \sqrt{1 - 4m^2/(M^2 + 2x)}} + 
\]

\[
+(1 + \frac{M^2 + 2m^2}{x} + \frac{M^2(M^2 + 2m^2)}{2x^2})\sqrt{1 - \frac{4m^2}{M^2 + 2x}} \quad (3. 9)
\]

We emphasize that the "nonequilibrium" parameters \(\zeta_Q\) and \(\zeta_G\) carry a significant additional information. They allow to separate contributions of spontaneous and
induced processes. This information would have been important even in the case of true thermal equilibrium but it is completely hidden if we use the detail balance relations at the early stage of calculations. For instance, examination of the expression for $\pi_{\text{em}}$ immediately shows that the first of the "gluon exponents" originates from quark and anti-quark distributions and momentum conservation. It describes the spontaneous emission of a gluon and it is not sensitive to a shape of the initial gluons distribution.

Practically, introduction of $\zeta$'s prevents the eventual cancellation of mass singularities between real processes and vacuum or thermal mass and vertex radiative corrections. The level of vacuum fluctuations and the rate of spontaneous processes are fixed de fault by $\zeta_{\text{vac}} = 1$.

For the Compton rate we start with a chain of changes of variables aimed to reduce the statistical weights to the uniform shape. These are $p \rightarrow -p + k - q$, in the first term and $q \rightarrow -q - p$ in both terms. Then we may perform an exact integration over $p$ using the same Breit reference system. It gives

$$\pi_{\text{compt}} = \frac{-ie^2 g^2 N_c C_F}{4\pi^4} \zeta Q \zeta G e^{-ku/T} \int d^4q \delta(q^2 - m^2) \theta(q_0) F_c(kq) [e^{-qu/T} - \zeta Q e^{-2qu/T}]$$

(3.10)

where $q$ is the momentum of the (anti)quark in the final state and

$$F_c(x) = -\frac{4m^2 + M^2 - 2x - 2(M^4 - 4m^4)/(M^2 + 2x)}{2\sqrt{x^2 - M^2m^2}} \ln \frac{m^2 + x + \sqrt{x^2 - M^2m^2}}{m^2 + x - \sqrt{x^2 - M^2m^2}} + \frac{4(M^2 + 2m^2)}{M^2 + 2x} \frac{(M^2 + 2x)(m^2 + x)}{(M^2 + m^2 + 2x)^2}$$

(3.11)

Besides an opportunity of analytic integration another advantage of the Breit system is that it reveals some additional information about the virtual state of the quark field. Namely, in annihilation process the energy $p_0$ of virtual state in the Breit system equals to ratio of squared dilepton mass to twice energy of initial quarks (or dilepton plus gluon). For the most profitable configuration of heavy dilepton and soft gluon this is about half of dilepton mass; the 4-vector $p_\mu$ is space-like and lies very close to the light cone. For the same reasons the energies of the initial quarks are almost the same but their 4-momenta are slightly time-like. For the Compton process we get the same estimate. In the most probable case of heavy dilepton with low momentum $k$ the Breit system almost coincide with the rest frame of the media. So when we will need further mass corrections to the quark propagators they should be calculated just at these momenta, $p_0 \sim M/2$ and $p^2 \sim 0$.

Another important information is that at any finite variable $q$ the very integrands in Eqs. (3.7,8,10) are divergent at $m \rightarrow 0$. This mean that the collinear divergen-
cies have explicitly different origin than the IR-divergencies and should be treated separately.

4. Radiative corrections. Integration over the initial states.

This section is technical. It could have been safely berried under the common "As it can be shown..." if there were no difficulties associated with the extremely singular form of the integrand. Being worked out naively they can essentially change the final answer.

Following Eqs.(2.5) and (2.7) we calculate contributions of quarks and gluons from the initial partons distributions to the vertex and self-energy corrections separately.

The vertex radiative correction is divided into the two parts contributed by the Fermi and Bose parts of the partons distribution, $\Pi_{\text{vert}} = \Pi^B_{\text{vert}} + \Pi^F_{\text{vert}}$:

$$
\pi^B_{\text{vert}} = \frac{i e^2 g^2 N_c C_F}{4\pi^5} \int d^4p d^4q \delta(q^2) \delta([p - k]^2 - m^2) [\delta([p^2 - m^2]) SW^B_{\text{vert}} \times
\times \frac{4m^4 - k^4 - 2kq(m^2 + k^2) - 4(kp)^2 + 4pq(k^2 + m^2 + kq)}{(p + q - k)^2 - m^2}(p + q)^2 - m^2)] SW^B_{\text{vert}}.
$$

(4. 1)

$$
\pi^F_{\text{vert}} = \frac{i e^2 g^2 N_c C_F}{2\pi^5} \int d^4p d^4q \delta(q^2 - m^2) \delta([p - k]^2 - m^2) \delta([p^2 - m^2]) SW^F_{\text{vert}} \times
\times \frac{-m^2k^2 - 2kqm^2 - 4(kp)^2 + 2pq(k^2 + m^2 + 2kq)}{(q + k)^2 - m^2}(p - q - k)^2, \quad (4. 2)
$$

In the same way we single out the two parts of the mass corrections, $\Pi_{\text{mass}} = \Pi^B_{\text{mass}} + \Pi^F_{\text{mass}}$:

$$
\pi^B_{\text{mass}} = \frac{i e^2 g^2 N_c C_F}{2\pi^5} \int d^4p d^4q \delta(q^2 - m^2) \delta([p - k]^2 - m^2) \delta([p^2 - m^2]) SW^B_{\text{mass}} \times
\times \text{Tr} \mathcal{P} \frac{p}{(p^2 - m^2)(p + q)^2 - m^2}.
$$

(4. 3)

$$
\pi^F_{\text{mass}} = \frac{i e^2 g^2 N_c C_F}{2\pi^5} \int d^4p d^4q \delta(q^2 - m^2) \delta([p - k]^2 - m^2) \delta([p^2 - m^2]) SW^F_{\text{mass}} \times
\times \text{Tr} \mathcal{P} \frac{p}{(p^2 - m^2)(p - q)^2}.
$$

(4. 4)

where $\mathcal{P}$ means the principal value of the integrals and $\text{Tr} \mathcal{P}$ stands for the trace of spinors from the corresponding Greenians,

$$
\mathcal{P} = 2[(p^2 - m^2)^2 + (kq + pq - kp - m^2)(p^2 - m^2) - 2m^2(pq - kp) - 2(kp)(pq) + 2m^4]
$$

(4. 5)
The corresponding statistical weights for these graphs are

$$SW_{\text{vert,mass}}^B = \theta(k_0 - p_0)\theta(p_0)n_F(k_0 - p_0)n_F(p_0)[1 + 2n_B(|q_0|)] \approx \theta(k_0 - p_0)\theta(p_0)\zeta_Q^2e^{-(kq)/T}[1 + 2\zeta_Qe^{-|q|/T}] \quad (4.6)$$

$$SW_{\text{vert,mass}}^F = \theta(k_0 - p_0)\theta(p_0)n_F(k_0 - p_0)n_F(p_0)[1 - 2n_F(|q_0|)] \approx \theta(k_0 - p_0)\theta(p_0)\zeta_Q^2e^{-(kq)/T}[1 - 2\zeta_Qe^{-|q|/T}] \quad (4.7)$$

One may notice that two of three statistical factors are modified by the theta-functions. These are due to the real partons in the initial or final states. The third factor works at both signs of energy which means a simultaneous account for both absorption of quanta from some state in the partonic bath and backward emission to the same state.

The first integration $d^4p$ over the initial partons distributions is naturally performed in the reference frame of the dilepton. For the vertex corrections they result in

$$\pi_{\text{vert}}^B = \frac{ie^2g^2N_cF}{4\pi^4}\zeta_Q^2\theta(k^2 - 4m^2)e^{-(kq)/T}\int d^4q\delta(q^2)\left\{\frac{1}{2} + \zeta_Qe^{-(kq)/T}\right\} \times \left[\frac{k^4 - 4m^4}{2(kq)^2}\ln 1 + v - \frac{1 + v}{1 - v}\right] \quad (4.8)$$

$$\pi_{\text{vert}}^F = \frac{ie^2g^2N_cF}{4\pi^4}\zeta_Q^2\theta(k^2 - 4m^2)e^{-(kq)/T} \times \int d^4q\delta(q^2 - m^2)\left\{\frac{1}{2} - \zeta_Qe^{-(kq)/T}\right\}\left[\frac{2m^2}{k^2 + 2kq} + \frac{kq - k^2 - 2m^2}{k^2 + 2kq}v\right] \quad (4.9)$$

$$+ \left[\frac{(k_2 + 2m^2)(kq + m^2) - m^2(k^2 + 2kq)}{2}\right] \ln \left[\frac{2m^2 + kq + v\sqrt{(kq)^2 + k^2m^2}}{2m^2 + kq - v\sqrt{(kq)^2 + k^2m^2}}\right].$$

When calculating the self-energy corrections we meet several ill defined products of the singular functions like $x\delta(x)\mathcal{P}(1/x)$. They need much care and the way to handle them is described in the Appendix A. This is hardly the most cumbersome part of calculations. It results in

$$\pi_{\text{mass}}^B = \frac{ie^2g^2N_cF}{4\pi^4}\zeta_Q^2\theta(k^2 - 4m^2)e^{-(kq)/T}\int d^4q\delta(q^2)\left\{\frac{1}{2} + \zeta_Qe^{-(kq)/T}\right\} \times \left\{\left[1 + \frac{m^2(k^2 - 2m^2)}{k^2(kq)}\right]\ln 1 + v + \left[\frac{2m^2}{k^2} - \frac{k^2}{kq}\right]v\right\} \quad (4.10)$$
\[\pi^F_{\text{mass}} = \frac{i e^2 g^2 N_c C_F}{4\pi^4} \zeta^2_Q \theta(k^2 - 4m^2)e^{-ku/T} \int d^4q \delta(q^2 - m^2) \left\{ \frac{1}{2} - \zeta_Q e^{-qu/T} \right\} \times\]
\[\times \left[ -\frac{k^2 + m^2 + kq}{\sqrt{(kq)^2 + k^2m^2}} \ln \frac{2m^2 + kq + v\sqrt{(kq)^2 + k^2m^2}}{2m^2 + kq - v\sqrt{(kq)^2 + k^2m^2}} - \frac{2m^2}{k^2v} \right], \quad (4.11)\]

where we denoted, \(v = \sqrt{1 - 4m^2/k^2} \).

Terms corresponding to 1/2 in curly brackets originate from vacuum fluctuations. They are both IR- and UV-divergent. Removal of this divergence is a subject for renormalization. We adopt a standard procedure of the on-mass-shell renormalization in the asymptotic states both for quark self-energy and electromagnetic vertex [10]. Thus we insist that vacuum part of the quark self-energy \(M(p)\) has a zero of the second order at \(p^2 = m^2\). In the same way we adopt that when all three momenta of the vertex function,
\[\Gamma^\mu(k^2) = \gamma^\mu F_1(k^2) + \frac{\sigma^{\mu\nu}k_\nu}{2m} F_2(k^2), \quad (4.12)\]
are on the asymptotic mass shells of the free particles then the form-factor \(F_1(0) = 0\).

This kind of renormalization immediately wash out vacuum part of the quark self-energy and leaves the following contribution of quark electromagnetic form-factors to the dilepton emission as the residue [10]:
\[\pi^v_{\text{vert}} = \frac{i e^2 g^2 N_c C_F}{4\pi^3} \zeta^2_Q \theta(k^2 - 4m^2)e^{-ku/T} M^2 \times\]
\[\times \{-h(v) \int_0^m \frac{dq}{q} + (1 + \frac{2m^2}{M^2})v - [\frac{3}{4} + \frac{m^2}{M^2}v] \ln \frac{1+v}{1-v} + 2(1 - \frac{4m^2}{M^2}) \int_0^{\text{atanh} v} \frac{xdx}{\text{tanh} x} \}, \quad (4.13)\]

The IR-divergent integral \(dq/q\) originates from the soft space-like gluon exchange between the fermion legs of the electromagnetic vertex and the function \(h(v)\) is given by
\[h(v) = -(1 - \frac{4m^4}{M^4}) \ln \frac{1 + v}{1 - v} + (1 + \frac{2m^2}{M^2})v \quad (4.14)\]

An asymptotic of the last integral in (4.14) at \(m/M << 1\) looks as
\[2 \int_0^{\text{atanh} v} \frac{xdx}{\text{tanh} x} \approx \frac{\pi^2}{6} + \frac{1}{4} \ln^2 \frac{1 + v}{1 - v} \approx \frac{1}{4} \ln^2 \frac{M^2}{m^2} \quad (4.15)\]
and produce large negative contribution to the total rate of emission. This double logarithm appeared from the collinear configuration of the vertex and comes to be infinite for massless quarks. It may be quite safe if it is properly balanced with the terms of the opposite sign produced by the other processes which are coherent with this one. As it was shown in [5,6] such a tiny balance takes place in the ideal thermal ensemble. For our distributions it does not happen. But after examination
of the scales of different processes we shall see that this double logarithm should be exponentiated into the K-factor.

5. Final integration.

There are two time-like 4-vectors in the theory, the 4-momentum of the dilepton, $k^\mu$, and the 4-velocity of the media, $u^\mu$. So we may continue calculations either in the rest frame of the media or in the rest frame of the dilepton. We will confine ourselves to the rest frame of the media, $u = 0$, basically because it is more natural to work with the isotropic distribution.

Let us consider the term responsible for the $q\bar{q}$-annihilation accompanied by spontaneous emission of the gluon. Its ”weight” is $\zeta_2^2 \zeta_{vac} = \zeta_2^2$. After using of the delta-function and one trivial asimutal integration we come to the two-dimensional integral,

$$-\pi T \int_0^\infty d[e^{-q/T}] \int_{-1}^1 dz q F_4(a_0 q - |k| q z). \quad (5.1)$$

Here and further on $k_0$ stands for $ku$ and $|k|^2 = (ku)^2 - M^2$. The trick which allows one to reduce this double integral to a simple quadrature is as follows [11,12]:

i) change the angular variable $z = \cos \theta$ for the new one, $q(k_0 - |k| z) = y$;

ii) integration with respect to $q$ by parts: in a miraculous way the integral $dy$ disappears;

iii) changes of variables: $q = k_+ y$ (or $q = k_- y$) where $k_\pm = k_0 \pm |k|$.

The result reads as

$$\pi T \int_{|k|}^{\infty} (1 - e^{-y/k_- T}) F_4(y) dy + \int_{|k|}^{\infty} (e^{-y/k_- T} - e^{-y/k_+ T}) F_4(y) dy \quad \quad \quad \quad \quad (5.2)$$

Though both lower and upper limits of the first integral tend to zero when $\lambda \to 0$ the very integral is finite. Indeed, if $f(0) \neq 0$ then

$$\lim_{\lambda \to 0} \int_{|k|}^{\infty} dy \frac{dy}{y} (f(0) + y f'(0) + ...) = f(0) \ln \frac{k_+}{k_-} + O(\lambda), \quad \quad \quad \quad (5.3)$$

and

$$\lim_{\lambda \to 0} \lambda \int_{|k|}^{\infty} \frac{dy}{y^2} (f(0) + y f'(0) + ...) = \frac{2|k|}{M^2} f(0) + O(\lambda). \quad \quad \quad \quad (5.4)$$

Foreseeing the future cancellation of soft divergencies we can safely put the lower limit equal to zero and write the final result for the annihilation with the gluon emission in the following form,

$$\pi_{em} = \frac{ie^2 g^2 N_c g_F}{4 \pi^3} \zeta_2^2 e^{-ku/T} M^2 [(1 + \zeta_G) \frac{k_+}{2|k|} h(v) \ln \frac{k_+}{k_-} +$$

$$+ 2 \int_0^\infty S(y) F_4(M^2 y) y dy + 2 \zeta_G \int_0^\infty S(2y) F_4(M^2 y) y dy]. \quad \quad \quad (5.5)$$
The terms with the extra factor $\zeta_G$ relate to the induced emission of a gluon and we denoted,

$$S(y) = e^{-ky/T} \frac{\sinh(\frac{|k|y}{T})}{(\frac{|k|y}{T})}$$  \hspace{1cm} (5.6)$$

In a quite similar way we obtain the expression for annihilation with the absorption of a gluon,

$$\pi_{abs} = \frac{ie^2 g^2 N_c C_F}{4\pi^3} \zeta_Q \zeta_G e^{-ku/T} M^2 \left[ (-1 + \frac{k_+}{|k|} \ln \frac{k_+}{k_-}) h(v) + 2 \int_0^{(1-4m^2/M^4)/2} \mathcal{F}_a(-M^2y) ydy \right]$$  \hspace{1cm} (5.7)$$

where it is helpful to denote $v(y) = \sqrt{1 - 4m^2/(M^2 + 2y)}$, (now $v(0) = v$) and rewrite,

$$\mathcal{F}_a(M^2y) = -(1 + (1 + \frac{2m^2}{M^2}) \frac{1}{y} + (1 - \frac{4m^4}{M^4}) \frac{1}{2y^2}) \ln \frac{1 + v(y)}{1 - v(y)} + (1 + (1 + \frac{2m^2}{M^2}) \frac{1}{y} + (1 + \frac{2m^2}{M^2}) \frac{1}{2y^2}) v(y)$$  \hspace{1cm} (5.8)$$

Assembling all gluon contributions to self-energy and vertex radiative corrections together we get

$$\pi_{rad}^B = \frac{ie^2 g^2 N_c C_F}{4\pi^3} \zeta_Q \zeta_G e^{-ku/T} M^2 \left[ - \frac{k_+}{|k|} h(v) \ln \frac{k_+}{k_-} + 2 \int_0^\infty (\mathcal{R}(y) + \mathcal{R}(-y)) S(y) ydy \right]$$  \hspace{1cm} (5.9)$$

where function $\mathcal{R}(y)$ in the integrand has a form

$$\mathcal{R}(y) = \ln \frac{1 + v}{1 - v} - (1 - \frac{2m^2}{M^2} v - \frac{h(v)}{2y^2})$$  \hspace{1cm} (5.10)$$

Equations (5.5), (5.7), (2.9) and the vacuum contribution to the vertex (4.13) form a closed system. Though each of them contains IR-divergent integrals over gluon momentum, when taken together, they produce finite result.

Further calculation of Compton process and radiative corrections from fermion part of the initial state background meet no infrared problems at low momenta of gluons. So we can safely choose the simplest way of calculations using the rest frame of a dilepton.

$$\pi_{compt} = \frac{ie^2 g^2 N_c C_F}{4\pi^3} \zeta_Q \zeta_G e^{-ku/T} 2m^2 \int_0^\infty \frac{x^2 dx}{\sqrt{1 + x^2}} \mathcal{F}_e(x)[C(x,T) - \zeta_Q C(x, \frac{T}{2})]$$  \hspace{1cm} (5.11)$$
where
\[ C(x, T) = e^{-k_0 m x_0 \sinh(|k|m T x)} / (|k|m T x) \] (5.12)

and
\[ \mathcal{F}_c(x) = \frac{4(M^2 + 2m^2)}{M^2 + 2Mmx_0} + \frac{(M^2 + 2Mmx_0)(m^2 + Mmx_0)}{(M^2 + m^2 + 2Mmx_0)^2} - \frac{1}{2Mmx} \{ 4m^2 + M^2 - 2Mmx_0 - \frac{2(M^4 - 4m^4)}{M^2 + 2Mmx_0} \} \ln \frac{m^2 + Mm(x_0 + x)}{m^2 + Mm(x_0 - x)} \] (5.13)

where \( x_0 = \sqrt{1 + x^2} \). The contribution of the Fermi part of initial partons to radiative corrections is of the next form:
\[ \pi_{\text{rad}}^F = -\frac{i e^2 g^2 N_c C_F}{4\pi^3} \zeta_Q^3 e^{-k_0/T} 2mM \int_0^\infty \frac{x^2 dx}{\sqrt{1 + x^2}} (\mathcal{F}_f(x, x_0) - \mathcal{F}_f(x, -x_0)) C(x, T) \] (5.14)

where
\[ \mathcal{F}_f(x, x_0) = -v(\frac{2m^2 + M^2 - 2Mmx_0}{M^2 + 2Mmx_0} - 2) \] (5.15)

\[ + \frac{1}{x} \left( \frac{(M^2 + 2m^2)(m^2 + Mmx_0)}{(M^2 + m^2 + 2Mmx_0)} - M^2 - \frac{3}{2} m^2 - mMx_0 \right) \ln \frac{2m^2 + Mm(x_0 + x)}{2m^2 + Mm(x_0 - x)} \]

6. Assembling and cancellation of the IR-singularities.

The final answer for the rate of emission is given now as a sum of many terms,
\[ \frac{dN_{e^+e^-}}{d^4kd^4x} = \frac{i\pi e_0^2}{3(2\pi)^6 M^2} \pi(k), \] (6.1)

\[ \pi(k) = \pi_{\text{Born}} + \pi_{\text{em}} + \pi_{\text{abs}} + \pi_{\text{com}} + \pi_{\text{rad}}^B + \pi_{\text{rad}}^F + \pi_{\text{vac}} \]

which are given by Eqs. (3.2), (5.7), (5.10), (5.15) and (4.13). Most of them are the IR-divergent one-dimensional integrals. Nevertheless the whole sum is IR-finite. We show now how it happens and find out the physical scale which is responsible for this phenomenon.

The cancellation of the infrared divergences due to the soft gluons should be looked at separately for terms with the different powers of fugacities \( \zeta_G \) and \( \zeta_Q \). The greatest contribution to the rate of the emission of heavy dileptons comes from the
annihilation with the absorption of a gluon. The sum of the leading IR-singular terms from \( \pi_{\text{abs}} \) and \( \pi_{\text{rad}} \) results in the finite integral,

\[
\pi_{\text{abs}} \approx \frac{i e^2 g^2 N_c C_F}{2 \pi^3} q^2 \zeta G e^{-\mathcal{I}u/T} M^2 \int_0^{q_{\text{max}}} dq \frac{d}{q} \left\{ -\left( 1 - \frac{4m^2}{M^2} \right) (L(-q) - e^{-q/T} L(0)) + \left( 1 + \frac{2m^2}{M^2} \right) (v(-q) - e^{-q/T} v(0)) \right\}
\]

(6.2)

where \( q_{\text{max}} = (M^2 - 4m^2)/2M \sim M/2 \), and

\[
v(q) = \sqrt{1 - \frac{4m^2}{M^2 + 2Mq}}, \quad L(q) = \ln \frac{1 + v(q)}{1 - v(q)}.
\]

(6.3)

The integrand of (6.1) is finite at \( q = 0 \) and takes value \( (2/T) \ln(M/m) \) at this point. It is almost linear function and turns to zero at the upper limit \( q_{\text{max}} \). This means that radiative corrections \( \pi_{\text{rad}} \) effectively screen the process at gluon momenta \( q < T \) (not \( gT \)!). The very integral can be roughly estimated at \( M >> m \) as

\[
\sim 2 \ln \frac{M}{m} \ln \frac{M}{2T}
\]

(6.4)

The result of numerical calculation is drawn at Fig.4 by the bold solid line in comparison with the thin solid line of the Born’s term. In the numerical calculations of Ref.[8] no radiative corrections was introduced and the gluon momentum was cut off at \( q \sim gT \). So the order of magnitude of the effect derived in this way may be different in the parametric scale (obviously there is no difference at \( g \sim 1 \)). The finite part of the \( \pi_{\text{abs}} \) is at least \( M/T \) times less than the leading term and the rest part of the radiative correction is suppressed by the extra factor \( \exp(-M/T) \). They are plotted by the dashed lines.

The induced emission along with the corresponding radiative correction has a statistical weight \( \zeta G \zeta Q^2 \). The IR-finite combination of the leading terms from \( \pi_{\text{em}} \) and \( \pi_{\text{rad}} \) reads as

\[
\pi_{\text{em}}^{\text{ind}} \approx \frac{i e^2 g^2 N_c C_F}{2 \pi^3} q^2 \zeta G e^{-\mathcal{I}u/T} M^2 \int_0^{\infty} dq \frac{d}{q} e^{-q/T} \left\{ -\left( 1 - \frac{4m^2}{M^2} \right) (L(q) e^{-q/T} - L(0)) + \left( 1 + \frac{2m^2}{M^2} \right) (v(q) e^{-q/T} - v(0)) \right\}
\]

(6.5)

The integrand of (6.5) takes value \( (1/T) \ln(M/m) \) at \( q = 0 \) and the thermal exponent effectively cut off the upper limit at \( q \sim T \). So the order of this integral is about \( -\ln(M/m) \) which is also \( M/T \) times less than the leading term and produces negative contribution to the total rate of emission. One more term related to this process is positive and suppressed by the factor \( \exp(-M/T) \). This curves are plotted at the Fig.5 by the thin solid lines. This contribution to the rate of emission is small.
The Compton process and the radiative corrections due to $\pi^F_{rad}$ do not give much also. They are plotted at Fig.5 by the dashed and dotted lines respectively, apart from the factors $\exp(-M/T)$ and $\zeta_{Q,G}$.

The spontaneous emission of a gluon by its statistical weight $\zeta_Q^2$ is going along with the vacuum vertex radiative correction $\pi^\text{vert}_{vac}$. The IR-divergent part of the real process and that of vacuum vertex together give

$$\pi^\text{spont}_{em} \approx \frac{ie^2 g^2 N_c C_F}{2\pi^3} \zeta_Q^2 e^{-ku/T} M^2 \int_0^m \frac{dq}{q} \{-(1 - \frac{4m^2}{M^2})(L(q)e^{-q/T} - L(0)) +

+(1 + \frac{2m^2}{M^2}(v(q)e^{-q/T} - v(0)))\} \quad (6.6)$$

It is easily estimated as $-(2m/T) \ln(M/m)$. Sign minus means the negative contribution of this term to the rate of emission. The factor before $\log$ has replaced a standard $\ln(m/\lambda)$ in course of IR-cancellation. This change is due to the shape of quark distribution and we have no reasons to introduce an artificial mass of a photon. But the collinear singularity, the big negative $-\log^2(M/m)$, originating from the nonsingular integral (4.15) in the Eq. (4.13) had survived. All these terms are plotted at Fig.6. The bold solid line presents (6.6). Thin dotted lines are due do the IR-finite terms of this process.

The bold dashed line relates to the vacuum vertex. We notice that in Refs.[5,6] just this term has killed main contribution (6.4) from the process $3 \rightarrow 1$. This is a remarkable consequence of the detail balance in the equilibrium heat bath. In our case this can not happen because these terms have different powers of fugacities.

An appearance of the $\log^2$ is a known phenomena and need the further summation of double logarithms. To understand what should be the result of this summation we remind that spontaneous emission of the gluons and the leading ladder diagrams of the vertex work together and result in the exponential $K$-factor. Related only to spontaneous processes, these diagrams have the common weight $\zeta_Q^2$. We have already got a cancellation of the infrared singularities between soft real gluons and that in the vertex. So we may estimate the total yield of these two processes as Born’s term times $K$-factor,

$$K \sim \exp(-\frac{\alpha_s C_F}{2\pi} \ln^2 \frac{M^2}{m^2}), \quad (6.7)$$

that is as strongly suppressed and even vanishing in the limit of $m = 0$. It is given by the thin dashed line at Fig.6 (with $m \sim gT$). If we had left the logarithmic term (4.15) as it is then we could get the negative total rate of emission in case of $\zeta_G \leq \zeta_Q$.

A spontaneous emission of soft gluons which naturally accompany the Coulomb interaction of the particles with the hard momenta are considered as the common
process with the very interaction of the particles. All parameters of the partons distribution are "external" with respect to this block and do not influence its internal dynamics in the gluon sector. It even does not seems logical to replace the mass $m$ in the $K$-factor by the thermal-one. Though it came from the quark propagator with big momentum $p^0 \sim |p| \sim M/2$ (see Appendix B) the internal length scale of the processes convoluted to the $K$-factor is $1/T$ which is less than the length scale $1/gT$ given by the forward scattering (see below).

An exponentiating of the double-logs is in line with the accepted above scheme of renormalization of the vacuum parts of the self-energy and the vertex. Indeed, an assumption that the UV-divergence is removed by subtractions on the asymptotic mass shell means that we consider the interaction with the vacuum fluctuations as the dominant one and do not allow the interaction with the partons to destroy the mass shell. The existence of such a dominant part of interaction is a footing of any normal perturbation theory for the many-body systems. In fact, this is the main statement which allows to start with the initial distribution of the free partons.

7. Conclusion and Discussion.

This paper presents an analytic calculations of the rate of emission of heavy dileptons from nonequilibrium quark-gluon plasma adjusting the final result to the needs of computer simulation of the parton cascade. Calculations are based on the real-time kinetic approach developed earlier [1,7,11].

An explicit analytic calculation demonstrate that mass singularities remain in the final answer for the rate of the heavy dileptons production from the quark gluon plasma. We show that the previously obtained opposit result [5,6] is an artifact of the true thermal equilibrium in the partons background. One should not expect that in heavy ions collision this kind of equilibrium will be reached. This is in agreement with the known fact that at high momentum transfer the double logarithms in the physical answer is rather a rule than an exception [12].

The main practical consequence of this calculation is a conclusion that the main process beyond the first Born’s one is annihilation of quarks with the absorption of a gluon. The kinematic of this process allows dilepton to store an additional amount of the internal energy. At $M/T \sim 10$ this process gives as much dileptons as the direct process. Ratio of its rate to the rate from the Born’s term grows up almost linearly with the increasing of the dilepton mass,

$$\frac{dN_{e^+e^-}}{d^4kd^4x} = \frac{i\pi e^2}{3(2\pi)^6M^2}(\pi_{\text{Born}}(k) + \pi_{\text{abs}}(k)), \quad (7.1)$$

$$\pi_{\text{Born}}(k) \approx -\frac{3ie^2}{4\pi}\zeta_Q M^2 e^{-ku/T},$$
\[ \pi_{\text{abs}} \approx -\frac{ie^2 g^2 N_c C_F}{2\pi^3} \zeta_2 \zeta_G e^{-k_\mu/T} M^2 \ln \frac{M^2}{m^2} \ln \frac{M}{2T}. \]

The processes of annihilation with the spontaneous and induced emission of a gluon and the Compton process give a vanishing contribution in compare with this process.

The \textit{a priori} unexpected result of the above calculations is the cancellation of all those singularities in the thermal graphs which originate from soft gluons. This seems to be very significant as it reveals some hierarchy of scales and is worth of physical discussion.

We got a new scale: the effective cancellation takes place up to gluon momenta \( \sim T \) and this natural cut-off is greater than an obviously supposed \( gT \). The later is not so surprising mathematically as we compare contributions of the graphs of the same perturbation order. Remember that the gluonic exponent in the finite combinations like \((L(q)e^{-q/T} - L(0))/q\) have appeared in the Eqs.(6.1-5) only from the specific form of the quark distributions and from the conservation laws and without any connection with the distribution of the gluons in the partons cascade. Notice that if statistical weight in this combination was equal to 1 (which is the case of process in the free space) it would not contain the mass singularity also. This would have been in complete agreement with the KNL theorem which implies an absence of any statistical weights for the intermediate states. Probably the true thermal equilibrium is a unique case of populated phase space when a complete cancellation of the mass singularities takes place.

The observed cancellation at \( q < T \) means that the two processes, spontaneous emission of a real gluon and an exchange by the space-like gluons, are mutually coherent at all gluon momenta \( q < T \). As only a sum of these processes is physical it means that gluons with \( q < T \) do not participate this process at all: any attempt of the quark to emit the long-wave gluon is immediately killed by the rearrangement of the proper fields due to static interaction. This is in agreement with the typical size \( 1/T^3 \) of the volume per parton which is free from the ”third bodies”.

The collinear singularities did survive and they do need the additional cut-off. The later can be estimated from the physics of the parallel geometry of the front collision. In this geometry the gluon and the massless quark interact infinitely long until \textit{at least one} interaction with the third parton will interrupt this process and destroy the mutual coherence of the real and virtual diagrams.

The external cut-off should come from the smallest of the three lengths. The first one is the Compton wave-length \( l_c \sim 1/m \). The second one is the mean free path defined via the scattering cross-section, \( l_{mfp} \sim 1/g^2 T \). The third one is connected with the amplitude of the forward scattering, \( l_{fs} \sim 1/gT \sim 1/m_{therm} \). As long as we keep quarks massless and consider coupling as the small parameter we must chose the length \( l_{fs} \) as the smallest one. So whenever we meet the collinear singularity the momenta of both real and virtual quarks should be cut off from the below at \( \sim gT \).
More formal arguments in favor of the scale $gT$ come from calculation of the quark self-energy given in Appendix B. We could not use the previous results by Klimov and Weldon [13] as we deal with essentially different statistical background. It sufficiently changes (simplifies) an analytic structure of the quark self-energy. As it is clearly seen from equations (B.6), (B.7) and (B.11) at high quark momentum the main deviation from the dispersion law of a free quark is formally simulated by the ”thermal mass” of a quark, $m_{\text{therm}} \sim gT$. This does not mean that quark really acquire an additional rest mass. Indeed, the $m_{\text{therm}}^2$ dominates in the dispersion equation (B.6) only when $|\mathbf{p}| >> T$ when its influence onto balance of the energy and momentum is negligible.

We conclude that in the processes with hard quarks a thermal mass establishes the physical boundary of the possible coherence length in different processes. This is the main reason why we prefer to use a cumbersome ”length associated with the amplitude of the forward scattering of the quark in partonic media” instead of a short and simple ”thermal mass”.

An actual need of such a boundary appears only in collinear configuration of the hard processes when its geometric origin is quite evident.

In the kinematic region of the heavy dileptons production (as well as for hard photons and low-mass dileptons of high energy) an approximation of the quark dispersion law by formal introduction of thermal mass occurs to be remarkably exact. The next terms are suppressed by a small factor $\sim T/|\mathbf{p}| \sim T/E$. But this is not true for the case of the small time-like momentum of a quark when these next terms are very large and even divergent.

A significant piece of work was done by R. Pisarski [14] in attempt to overcome specific problems of the soft region, $|\mathbf{p}| < gT$, by means of selective resummation of the perturbation series. This extremely difficult study is confined to the ideal thermal equilibrium. Probably a final solution will demand a nonperturbative approach.

For the heavy dileptons production when the dilepton mass exceeds any of other scales the particular choice of the cut-off scale is not so significant within the reasonable accuracy. The cut-off mass appears only under the logarithm and its variation is not so valuable.

A choice of the cut-off may cause the qualitative changes in calculations of the emission of the low-mass dileptons when we are close to the threshold of the existence of the very Born’s term along with the cascade part of the radiative corrections. They all appear only at $M^2 > 4m^2$ ($\sim 4g^2T^2$ ??) and are absent at the lower masses of the dilepton.

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Appendix A. Singular integrals

In course of calculations of terms associated with thermal parts of the quark self-energy we had met extremely singular integrals like

\[ I = \int d^4p \delta(p^2 - m^2) \delta((p-k)^2 - m^2) \theta(p_0) \theta(k_0 - p_0) \frac{p}{p^2 - m^2} F(p) \]  
(A. 1)

where \( F(p) \) is a regular function. We choose reference frame \(|k| = 0\) and get

\[ I = \int_{0}^{k_0} dp_0 \int d^3p \delta(p_0^2 - \omega^2) \delta((p_0 - k_0)^2 - \omega^2) \frac{p}{p_0^2 - \omega^2} F(p_0, p) \]  
(A. 2)

where \( \omega^2 = p^2 + m^2 \). Using the known formulae,

\[ 2\pi i \delta(x) = \frac{1}{x - i\epsilon} + \frac{1}{x + i\epsilon} \]

\[ 4\pi i \delta(x) \frac{p}{x} = \frac{1}{(x - i\epsilon)^2} + \frac{1}{(x + i\epsilon)^2} \]  
(A. 3)

and absorbing all the regular part of the integrand into the function

\[ G(p_0, \omega) = \frac{F(p_0, p)}{(p_0 + \omega)^2(p_0 - k_0 - \omega)} \]  
(A. 4)

we get

\[ I = -\int \frac{d^3p}{8\pi^2} \int_{(C)} \frac{G(p_0, p) dp_0}{(p_0 - \omega)^2(p_0 - k_0 - \omega)^2} \]  
(A. 5)

contour \((C)\) is given by \((\omega +; (k_0 - \omega) -; \omega -; (k_0 - \omega) +)\). Now it is easy to see that the internal contour integral differs from zero only at a single point \( \omega - k_0 = k_0 \):

\[ I = -\frac{1}{4} \int d^3p G'(\omega, p) \delta(\omega - \frac{k_0}{2}) \]  
(A. 6)

This equation allows to perform the remaining integration over \(d^3p\) and being applied to Eqs.(4.3) and (4.4) results in Eqs.(4.10) and (4.11) for \( \pi_{mass} \).

Appendix B. Quark self-energy for a nonequilibrium partons distribution.
The invariant decomposition of the “thermal” part of the self-energy of the massive quark against the background of the initial partons distribution can be easily found,

\[ M_{ret}^\beta = -\frac{\alpha^s C_F}{4\pi|\mathbf{p}|^2} \{ 4m\lambda + \hat{p}[\eta - (p_0^2 + |\mathbf{p}|^2)\lambda - 2p_0\xi] - \hat{u}[p_0\eta - p_0p^2\lambda - 2p^2\xi] \}, \quad \text{(B. 1)} \]

Where \( p_0 = pu \) and \( |\mathbf{p}|^2 = p_0^2 - (pu)^2 \). The extra \(|\mathbf{p}|^{-2}\) in quark self-energy originates from the existence of the preferred reference frame and provides additional parameter like \( T/|\mathbf{p}| \) which is small in the region actual for heavy dileptons. In the rest frame of the media we will write it in the form

\[ M_{ret}^\beta = W + \gamma^0 U + \gamma \mathbf{p} V \]

\[ W = \frac{\alpha^s C_F}{4\pi|\mathbf{p}|^2} 4m(\lambda_B + \lambda_F), \quad U = -2\frac{\alpha^s C_F}{4\pi|\mathbf{p}|^2} (\xi_B + \xi_F + p_0\lambda_B), \quad \text{(B. 3)} \]

\[ V = \frac{\alpha s C_F}{4\pi} \left( -\frac{\eta_B + \eta_F}{|\mathbf{p}|^2} + \frac{p_0^2 + |\mathbf{p}|^2 - m^2}{|\mathbf{p}|^2} \lambda_B - \frac{p_0^2 - |\mathbf{p}|^2 + m^2}{|\mathbf{p}|^2} \lambda_F + \frac{2p_0}{|\mathbf{p}|^2} (\xi_B + \xi_F) \right) \]

with the following notations,

\[ \eta_B = \frac{2}{\pi} \int d^4q \delta(q^2) n_B(q_0), \quad \eta_F = \frac{2}{\pi} \int d^4q \delta(q^2 - m^2) n_F(q_0), \quad \text{(B. 4)} \]

\[ \lambda_B = \frac{2}{\pi} \int d^4q \frac{\delta(q^2)}{p^2 - m^2 + 2qp} n_B(q_0), \quad \lambda_F = -\frac{2}{\pi} \int d^4q \frac{\delta(q^2 - m^2)}{p^2 + m^2 - 2qp} n_F(q_0), \]

\[ \xi_B = \frac{2}{\pi} \int d^4q \frac{\delta(q^2)}{p^2 - m^2 + 2qp} q_0 n_B(q_0), \quad \xi_F = -\frac{2}{\pi} \int d^4q \frac{\delta(q^2 - m^2)}{p^2 + m^2 - 2qp} q_0 n_F(q_0) \]

The dispersion equation, \( \det[\hat{p} - m - M_{ret}] = 0 \), can be rewritten in the next form,

\[ p_0^2 - |\mathbf{p}|^2 - m^2 + 2(p_0U + |\mathbf{p}|^2V - mW) + (U^2 - |\mathbf{p}|^2V^2 - W^2) = 0. \quad \text{(B. 5)} \]

The last term of this equation is of the order \( \alpha_s^2 \). So if we are looking for the roots near their unperturbed positions, relaying upon the small value of \( \alpha_s \), we have every reason to omit it. Up to the \( \alpha_s \)-order this equation is as

\[ p_0^2 - |\mathbf{p}|^2 - m^2 - \frac{2\alpha_s}{3\pi} [\eta_B + \eta_F - (p^2 - 3m^2)(\lambda_B + \lambda_F)] = 0. \quad \text{(B. 6)} \]

In the case when the Lagrangian mass obey the inequality, \( m << |p_-| << T << M \), the explicit values of the invariants are as follows (\( E_1 \) is the integral exponent),
\[ \eta_B = 8 \zeta_G T^2, \quad \eta_F = 8 \zeta_Q T^2 \quad (B.7) \]
\[ \lambda_B = \zeta_G L(p), \quad \lambda_F = \zeta_Q L(p), \quad (B.8) \]

\[ L(p) = \frac{T}{|p|} \left[ e^{b^-} E_1(b_-) - e^{-b^-} E_1(-b_-) - e^{b^+} E_1(b_+) + e^{-b^+} E_1(-b_+) \right] \]
\[ \xi_B = \zeta_G K(p), \quad \xi_F = \zeta_Q K(p), \quad (B.9) \]

\[ K(p) = \frac{T^2}{|p|} \left[ e^{b^-} E_1(b_-)(1 - b_-) + e^{-b^-} E_1(-b_-)(1 + b_-) - e^{b^+} E_1(b_+)(1 - b_+) - e^{-b^+} E_1(-b_+)(1 + b_+) \right] \]

where \( b_\pm = (p_0 \pm |p|)/2T \) and \( E_1(-x) = E_1(e^{-i \pi} x) \) We are basically interested in the values of quark self-energy at \( M >> T, \ p_+ \sim M \) and \( p_- \sim 0 \). In this region we can use an approximation,

\[ b^+ e^{b^+} E_1(b_+) \approx 1 \quad \text{and} \quad E_1(b_-) \approx -C_E - \ln(b_-). \quad (B.10) \]

It gives simple expressions,

\[ L(p) \approx -\frac{4T^2}{|p|^2} - i\pi \frac{2T}{|p|}, \quad K(p) \approx \frac{T^2}{|p|} \left[ 2 - 2C_E + i\pi - 2 \ln \frac{p_-}{2T} \right] \quad (B.11) \]

So, as long as we are interested in corrections to the propagator specific for the heavy dileptons emission, when \( |p| \sim M/2 \), the term \( \sim \alpha_s^2 \) in the Eq.(B.5) is suppressed by the additional powers of \( T/M \). Up to the next orders in a scale of \( T/M \) only the invariants \( \eta \) remain significant and we can replace Lagrangian mass by the thermal one,

\[ m_{\text{therm}}^2 = \frac{4g^2}{3\pi^2} (\zeta_Q + \zeta_G) T^2 \quad (B.12) \]

Contrary to a simple calculation of the Green function for the field in QGP the problem of the field quantization in terms of quasiparticles may have no solution. Indeed, to be successive on this way we should expand the propagator using the complete set of the eigenfunctions of the wave equation modified by the self-energy and obeying a certain dispersion law. Even in classical plasma such a set does not exist. The Green function of the Landau kinetic equation allows to solve any problem with the given initial data but a complete set of the solutions of kinetic equation for longitudinal waves in a plasma form the so-called Van-Kampen modes which do not obey any dispersion law [15].

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