CMB Anisotropies and Inflation from Non-Standard Spinors

Christian G. Böhmer \textsuperscript{a}, David F. Mota \textsuperscript{b}

\textsuperscript{a}Department of Mathematics, University College London, Gower Street, London, WC1E 6BT, UK
\textsuperscript{b}Institute for Theoretical Physics, University of Heidelberg, 69120 Heidelberg, Germany

Abstract

The apparent alignment of the cosmic microwave background multipoles on large scales challenges the standard cosmological model. Scalar field inflation is isotropic and cannot account for the observed alignment. We explore the imprints, a non-standard spinor driven inflation would leave on the cosmic microwave background anisotropies. We show it is natural to expect an anisotropic inflationary expansion of the Universe which has the effect of suppressing the low multipole amplitude of the primordial power spectrum, while at the same time to provide the usual inflationary features.

Inflation is a successful theory to explain many cosmological puzzles. However, one does not really know what has driven it, since it most probably occurred near the scale of grand unification, hence far beyond the standard model of particle physics.

In this letter we study the possibility of non-standard spinors to drive inflation and investigate the possible imprints of such spinors on the Cosmic Microwave Background (CMB) anisotropies. In fact, we consider the possibility that such an effect has already been detected, in the form of the Axis of Evil: an apparent alignment of the CMB multipoles on very large scales \cite{1,2,3}. While a scalar field driven inflationary epoch is naturally isotropic, an anisotropic expansion might occur within a more complex model. This may lead to the existence of a preferred direction in the primordial power spectrum.

Although the statistical significance of such preferred direction is hard to quantify, a variety of models have been put forward to explain this phenomenon \cite{4,5,6,7,8,9,10,11,12,13}. These are motivated since the large scale anisotropy claimed by \cite{14} in the CMB quadrupole and octupole seems to be present at several cosmological scales and observations. In particular the quadrupole and octupole seem also to align with the dipole \cite{15}. Recently, there are claims that such alignment even extends to higher multipoles \cite{16}. Furthermore, the polarization of radio galaxies and the optical polarizations of quasars also indicate a preferred direction pointing at the same direction \cite{17}. Finally, there are several indications from the SDSS data that deviations from isotropy and homogeneity are also present at cluster and galactic scales \cite{18}. Hence, there is an entire set of observations that disfavor isotropy at high confidence level.

As for the non-standard Wigner class spinors, we consider a spin one half matter field with mass dimension one, named elko spinors \cite{19}. These spinors are based on the eigenspinors of the charge conjugation operator. The resulting field theory has the unusual property $(CPT)^2 = -I$ \cite{36}. This particular model belongs to a wider class of so-called flagpole spinors \cite{20}. The spinors have mass dimension one and therefore the only power counting renormalizable interactions of this field with standard matter take place through the Higgs doublet or with gravity \cite{19}. Consider the left-handed part $\phi_L$ of Dirac spinor $\psi$ in Weyl representation, then an elko spinor is defined by \cite{19}

\begin{equation}
\lambda = \left( \begin{array}{c} \pm \sigma_2 \phi_L^* \\ \phi_L \end{array} \right),
\end{equation}

where $\phi_L^*$ denotes the complex conjugate of $\phi_L$. Since
the helicities of $\phi_L$ and $\sigma^2 \phi_L^*$ are opposite [19], one has to distinguish the two possible helicity configurations, therefore

$$\lambda_{(-,+)} = \left( \frac{\pm \sigma_2 \phi_L^*}{\phi_L^*} \right), \quad \lambda_{(+,-)} = \left( \frac{\pm \sigma_2 \phi_L}{\phi_L} \right).$$

(2)

The first entry of the helicity subscript $\{ - , + \}$ refers to the upper two-spinor while the second to the lower. Let us henceforth denote the helicity subscript by the indices $u,v,\ldots$ and define the elko dual by

$$\bar{\lambda}_u = i \varepsilon^u_{v r} \lambda_r^0,$$

with the anti-symmetric symbol $\varepsilon_{\{ - , + \}} = -1 = -\varepsilon_{\{ + , - \}}$. Note that due to the double helicity structure of the spinors, these have an imaginary bi-orthogonal norm [19] with respect to the standard Dirac adjoint $\psi = \psi^\dagger \gamma^0$. With the dual defined above one finds (by construction)

$$\bar{\lambda}_u(p) \lambda_v(p) = \pm 2m \delta_{uv},$$

(4)

where $p$ denotes the momentum.

Notice that the cosmology of such spinors will be different from the ones investigated by Saha and collaborators [21]. Firstly, the scalar field like equations of motion are second order equations opposed to the first order equations for standard spinors. Moreover, only intrinsically massless Dirac spinors are power counting renormalizable, which is one of the main motivations to analyze non-standard spinors. The natural potential is power counting renormalizable and their structure is much richer than that given by standard spinors.

The introduction of Elko spinor fields into an arbitrary curved spacetime can be found in [22]. This resulting theory is based on the following matter action

$$S = \frac{1}{2} \int \left( g^{\mu \nu} \nabla_{(\mu} \bar{\lambda} \nabla_{\nu)} \lambda - m^2 \bar{\lambda} \lambda + a(\bar{\lambda} \lambda)^2 \right) \sqrt{-g} \, d^4 x,$$

(5)

where $m$ is the mass of the field and $a$ is a coupling constant.

With the aim to understand possible effects of non-standard spinors in cosmology we investigate a quite general metric given by

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2) - b(t)^2dz^2,$$

(6)

where $a(t)$ and $b(t)$ are two expansion parameters, that define two Hubble parameters by $H_a = \dot{a}/a$ and $H_b = \dot{b}/b$. Note that this reduces to the isotropic FRW metric in the case where $a = b$. The presence of the spin-connection in the matter part leads to additional couplings between the field and the geometry. Hence, such a spinor driven inflationary epoch can naturally result in anisotropic expansion.

We assume that the non-standard spinors only depend on the time coordinate $t$. Following [22], the cosmological spinors are given by

$$\lambda_{(-,+)} = F(t) \xi, \quad \lambda_{(+,-)} = F(t) \zeta,$$

with their respective dual spinors $\tilde{\xi}$ and $\tilde{\zeta}$, where $\xi$ and $\zeta$ are constant non-standard spinors [22] satisfying

$$\tilde{\lambda}_{(-,+)} \lambda_{(-,+)} = \tilde{\lambda}_{(+,-)} \lambda_{(+,-)} = \pm 2F^2.$$

(8)

Henceforth we consider the self-dual spinors with $\tilde{\xi} = \zeta, \xi = \tilde{\zeta} = +2$.

We are interested in finding a solution of the form

$$a(t) = e^{H_at}, \quad b(t) = e^{H_b t}, \quad F(t) = F_0 = \text{const}.$$

(9)

Plugging this ansatz into the Einstein field equations, the equations of motion reduce to a system of algebraic equations

$$H_a^2 + 2H_aH_b = \frac{8\pi}{m_{pl}^2} \left( \frac{2}{4} H^2_a + \frac{1}{4} H^2_b + m^2 + \alpha F_0^2 \right) F_0^2,$$

$$- (H_a H_b + H_a^2 + H_b^2) = \frac{8\pi}{m_{pl}^2} \left( \frac{1}{4} H^2_a - m^2 - \alpha F_0^2 \right) F_0^2,$$

$$- 3H_a^2 = \frac{8\pi}{m_{pl}^2} \left( \frac{2}{4} H^2_a - \frac{1}{4} H^2_b - m^2 - \alpha F_0^2 \right) F_0^2.$$

(10)

These three equations can be simultaneously satisfied with $H_b > H_a > 0, F_0 > 0$ and $m > 0, \alpha > 0$. These rather complicated expressions can be greatly simplified after the following considerations are taken into account.

It turns out to be convenient to refer to a fictitious isotropic metric (with expansion parameter $\dot{a}(t)$), defined via an averaged Hubble parameter

$$\bar{H} = \frac{2H_a + H_b}{3},$$

(11)

which can be used to parameterize deviations from isotropy by

$$\epsilon_H = \frac{2H_b - H_a}{\bar{H}}.$$

(12)

The parameter $\epsilon_H$ turns out to be expressed solely in terms of the spinorial part $F_0$. The function $\epsilon_H(F_0)$ is increasing and vanishes at the origin. Since we are interested in a geometry with only small deviations from isotropy, we assume $F_0^2 \ll 1$. This guarantees the usual post-inflation isotropic expansion, and that non-standard spinors never dominate a cosmological epoch.

Expanding the anisotropy parameter and the mean Hubble parameter for small $F_0$ yields

$$\bar{H} = mF_0 \sqrt{\frac{8\pi}{3} \left( 1 + \frac{\alpha}{m^2} F_0^2 \right) + O(F_0^4)},$$

$$\epsilon_H = \frac{8\pi}{3} F_0^2 - 2 \left( \frac{8\pi}{3} F_0^2 \right)^2 + O(F_0^6).$$

(13)

(14)

In order to treat the anisotropy as a perturbation around the background, we furthermore assume that
Next we verify that the usual inflationary parameters (number of e-folds, near scale invariant spectral index, small non-gaussianities) are also in agreement with the present model. In order to calculate them we express the field equations in terms of the averaged Hubble parameter (11) and the deviation from isotropy (12).

It turns out that the terms linear in $\epsilon_H$ vanish identically. Therefore, by neglecting term of the order $O(\epsilon_H^2)$ and higher, we find that the average Hubble parameter and the equation of motion for the spinor field are given by

$$\dot{H}^2 = \frac{8\pi}{3m^2_{pl}} \left( \frac{1}{2} \partial_\lambda \partial_\lambda + 3 \dot{H} \right) + V(\hat{\lambda} \lambda),$$  \hspace{1cm} (15)

$$\partial_\lambda \lambda + 3 \dot{H} \partial_\lambda - \frac{3}{4} H^2 \partial_\lambda + \partial_\lambda \partial_\lambda \lambda = 0,$$ \hspace{1cm} (16)

respectively, where the latter equation is indeed exact. Requiring a power counting renormalizable theory uniquely determines the potential to have the form $V(\lambda) = m^2 \lambda^2 + \alpha |\lambda|^2$. Note that in contrast to the scalar field case, the matter part (right-hand side) now also contains the Hubble parameter. One can then (cosmologically) re-interpret the non-standard spinors as a scalar field with a time dependent mass. However, since both Hubble parameters are assumed to be constant throughout inflation this merely leads to a shift of the mass parameter. Therefore, although this model naturally allows for anisotropic inflation it is effectively equivalent to standard single field inflation. This greatly simplifies the interpretation of all equations. Since the expressions for the usual inflationary quantities will be similar in this theory. However, as one will see below, there are some cosmological imprints which are very particular to an anisotropic inflationary epoch driven by a non-standard spinor, which are not present in the usual scalar field models.

Eq. (15) can be solved for $\dot{H}$ and yields

$$\dot{H}^2 = \frac{8\pi}{3m^2_{pl}} \left( \frac{1}{2} \partial_\lambda \partial_\lambda + V(\hat{\lambda} \lambda) \right),$$ \hspace{1cm} (17)

where we neglected terms of the order $\lambda^2/m_{pl}^2$. Where $m_{pl}$ is the Planck mass.

For the slow-roll conditions: $\lambda^2/2 \ll V(\hat{\lambda} \lambda)$ and $|\dot{\lambda}| \ll 3H|\lambda|$ we therefore obtain

$$\dot{H}^2 \simeq \frac{8\pi}{3m^2_{pl}} V(\hat{\lambda} \lambda), \hspace{1cm} 3 \dot{H} \partial_\lambda \lambda \simeq - V(\hat{\lambda} \lambda) + \frac{3}{4} \dot{H}^2 \lambda.$$ \hspace{1cm} (18)

The last term containing $\dot{H}^2$ can again be replaced by the actual expression for $\dot{H}^2$ and results in a rather complicated expression. However, the factors of $m_{pl}$ as before make all additional contributions small.

The spectral index is given in terms of the slow roll parameters $n = 1 - 6\epsilon + 2\eta$. Both the parameters can be calculated straightforwardly from the above equations. The parameter $\epsilon$ is given by

$$\epsilon \simeq \frac{m^2_{pl} V(\hat{\lambda} \lambda)}{16\pi} V^2 \simeq \frac{m^2_{pl} V(\hat{\lambda} \lambda)}{16\pi} \frac{\lambda \lambda}{V^2}.$$ \hspace{1cm} (19)

as it is usual in scalar field inflation. On the other hand, the parameter $\eta$ acquires one non-trivial extra term. This term is obtained by differentiating Eq. (18) with respect to $t$ and dividing the resulting equation by $9H^2\partial_\lambda \lambda$, which yields an additional term of 1/12 to $\eta$, then we find

$$\eta \simeq \frac{m^2_{pl} V(\hat{\lambda} \lambda)}{8\pi} - \frac{1}{12},$$ \hspace{1cm} (20)

plus some lower order terms that can be neglected. Hence, we find that $\eta$ should be smaller for non-standard spinor inflation. Similarly, for the number of e-folds we get

$$N_e = \log \frac{a_f}{a} = \int_{t_i}^{t_f} \frac{8\pi}{m^2_{pl}} \int \frac{V(\hat{\lambda} \lambda)}{\dot{\lambda}} d\lambda.$$ \hspace{1cm} (21)

Similarly to the single field inflation scenario, the non-gaussianity parameters within this scenario are given by

$$f_{NL} = \frac{5}{6}(\eta - 2\epsilon), \hspace{0.5cm} \tau_{NL} = (\eta - 2\epsilon)^2 = \frac{36}{25} f_{NL}^2,$$ \hspace{1cm} (21)

where we neglect the parameter $g_{NL}$ which contains the third derivatives of the potential because of its smallness. For these non-standard spinors the additional contribution of 1/12 in $\eta$ will therefore yield a slightly smaller $f_{NL}$ parameter with respect to the usual slow roll inflationary scalar field models

$$f_{NL} = \frac{5}{6}(\eta - 2\epsilon) - \frac{5}{6} \frac{1}{12}.$$ \hspace{1cm} (22)

From WMAP3, $-54 \leq f_{NL} \leq 114$, and the PLANCK satellite’s design aim is, among others, to constrain the parameter $|f_{NL}| \leq 5$. Hence, we can conclude that non-standard spinor inflation cannot be ruled out by this new data alone.

In standard inflation the primordial power spectrum $P(k)$ only depends on the magnitude of the vector $k$ which follows from the rotational invariance. An inflationary epoch driven by non-standard spinors results in anisotropic expansion where rotational invariance is broken by a small unit vector $n$. The imprint of such an anisotropy on the density perturbation power spectrum has the following most general form

$$P(k) = P(k) \left( 1 + A(k)(\hat{k} \cdot n)^2 \right),$$ \hspace{1cm} (23)}
power spectrum is characterized by a single function \( A(k) \), which is given by
\[
A(k) = \frac{9}{2} \epsilon_H \log \left( \frac{k}{a(t_*) H} \right).
\] (24)

Since we assume around 60 e-folds before the end of inflation, we find that \( \log(k/a(t_*) H) \) is of the order \(-60 \) [35] for a wide range of scales which are cosmologically relevant today. CMB measurements probe \( k/a(t_*) \) up to \( 10^3 \). Hence, one can roughly assume that \( A(k) \) is \( k \)-independent at the astrophysical scales of interest. However, the additional effect on \( k \) decrease with \( k/H \).

Taking into account only the astrophysical scales of interest for us today \( (A(k) \) becomes roughly \( k \)-independent) we have \( A(k) = A_* \), then we find
\[
\frac{\varphi(lm;lm)}{(a_{lm} a_{lm}^{iso})} = \frac{A_*}{2} \left[ \sin^2 \theta_* + (3 \cos^2 \theta_* - 1) \left( \frac{2l^2 + 2l - 2m^2 - 1}{(2l-1)(2l+3)} \right) \right].
\] (30)

It is interesting to notice that within this scenario one gets a low quadrupole. The multipole spectrum is described by
\[
Q_l = \sqrt{\frac{1}{2\pi(l+1)} \sum_{m=-l}^{l} (a_{lm} a_{lm}^{iso}) |1 + \frac{\varphi(lm;lm)}{(a_{lm} a_{lm}^{iso})}|}.
\] (31)

The observed value of this is \( Q_{2}^{obs} \approx 5.72 \times 10^{-3} \), while the standard concordance model predicts \( Q_{2}^{\Lambda CDM} \approx 13 \times 10^{-3} \) [30]. It has been suggested in previous works that this discrepancy could also be explained by an ellipsoidality of the universe [32], by inhomogeneous cosmological magnetic fields [33], or a dark energy component with an anisotropic equation of state [34]. This would require that the anisotropy of the background is suitably oriented with respect to the intrinsic quadrupole and cancels its power to a sufficient amount. For any orientation then, we should have \( Q_2 \lesssim 19.7 \times 10^{-3} \) to be consistent with observations taking into account the cosmic variance. Inserting the values predicted by the concordance model \( a_{2m} = \sqrt{\pi/3} \cdot 1 \times 10^{-6} \) into Eq. (31) one obtains the following quadrupole moment
\[
Q_{2}^{model} = 13\sqrt{1 + A_*/3} \times 10^{-3},
\] (32)
which must be compared with the observed values for \( a_{2m} \) from the cleaned SILC400 (a), WILC3YR (b) and TCM3YR (c) maps, see [29,30,31], which lead to the following observed quadrupoles
\[
\frac{Q_{2}^{(a)}}{10^{-3}} = 6.08, \quad \frac{Q_{2}^{(b)}}{10^{-3}} = 5.77, \quad \frac{Q_{2}^{(c)}}{10^{-3}} = 5.30.
\]

In order to have agreement between the value predicted by an anisotropic model and the actually observed value, it now becomes clear that models in which the anisotropy is treated as a small quantity can indeed explain the low quadrupole moment we observe. From Eq. (32) we find that \( A_* \) should be around \( A_* \approx -2.41 \). This in turn fixes the spinorial part of the model, namely \( F_0 \) should be of the order of \( F_0 \approx 0.033 \) which in turn leads to \( F_0^2 \approx 1.1 \times 10^{-3} \) which clearly is in agreement with our above assumption \( F_0^2 \ll 1 \).

Taking into account the rather reasonable assumption that for the isotropic background we can assume the modulus of \( a_{lm} \) to be equal for all modes In that case, we can give an explicit expression of the corrected power spectrum
\[
Q_l = \sqrt{l(l+1)c_l/2\pi(1+A_*/3)}.
\] (33)
Hence, within this model multipole moments are suppressed by the factor \((1 + A_k/3)\) where we neglected variations of \(A_k\). Such feature might result in a better agreement between the observational data and the theoretical models, since for the low multipoles there are mild discrepancies between the prediction of the power spectrum from the \(\Lambda\)CDM model and the actually observed values.

In resume, non-standard spinors are a candidate to drive anisotropic inflation. The presence of the spin-connection in the matter part leads to additional couplings between the field and the geometry. Hence, inflation naturally becomes an anisotropic expansion, yielding a preferred direction which might have been detected as the axis of evil. Our derivation of the anisotropy corrected power spectrum is valid for all models in which the anisotropy can be treated as a perturbation around an isotropic background. Remarkably, while the usual inflationary features are obtained (low non-gaussianities, 60 e-folds, etc), one finds that non-standard spinor driven inflation naturally results into a suppression of the lower multipoles of the CMB. This, in particular, cures the quadrupole anomaly that puzzles today’s cosmological observations.

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[35] The minus sign is present due to \(\bar{\alpha}\), in the denominator
[36] The original elko field theory is non-local. However, since \(CPT\) is an anti-unitary operator, a local field theory in principle must exist. We thank Raymond Streater for elucidating this point.