THE JANUS HEAD OF THE HD 12661 PLANETARY SYSTEM

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ABSTRACT

In this work, we perform a global analysis of the radial velocity curve of the HD 12661 system. Orbital fits that are obtained by the genetic and gradient algorithms of minimization reveal the proximity of the system to the 6 : 1 mean motion resonance. The orbits are locked in the secular resonance with apsidal axes librating about 180°, with a full amplitude of ±90°–180°. Our solution incorporates the mutual interaction between the companions. A stability analysis with the MEGNO (Mean Exponential Growth factor of Nearby Orbits) indicator shows that the system is located in an extended stable zone of quasi-periodic motions. These results are different from those obtained on the basis of the orbital fit published by Fischer et al.

Subject headings: celestial mechanics — methods: n-body simulations — methods: numerical — planetary systems — stars: individual (HD 12661) — stellar dynamics

1. INTRODUCTION

The recently discovered planetary systems HD 12661 and HD 38529 (Fischer et al. 2003) are among about a dozen multiplanetary systems. Their dynamics are the subject of extensive work carried out by many researchers. The knowledge of these systems changes rapidly as more and more observational data are gathered. Remarkably, most of the new multiplanetary systems have a resonant nature. The mean motion resonances (MMRs) and/or the secular apsidal resonances (SARs) are most likely present in υ And, HD 82943, Gliese 876, 47 UMa, 55 Cnc, and HD 12661 (see Table 8 in Fischer et al. 2003).

The orbital fit to the radial velocity (RV) observations of the HD 12661 system has been determined by Fischer et al. (2003). They discovered two giant planets, b and c, with masses $m_b \approx 2.3 M_J$ and $m_c \approx 1.57 M_J$, revolving around the parent star in elongated orbits ($e_b \approx 0.35$, $e_c \approx 0.2$) and with semimajor axes $(a_b \approx 0.83 \text{ AU}$, $a_c \approx 2.56 \text{ AU})$. These data permit us to classify the HD 12661 system as a planetary hierarchical triple system. Recently, using the initial conditions provided by Fischer et al. (2003), Gozdziewski (2003) and Lee & Peale (2003) investigated the dynamics of the HD 12661 system. Lee & Peale (2003) studied coplanar dynamics in the framework of an analytical secular theory. The work of Gozdziewski (2003), mostly numerical, is devoted to a stability analysis of the system, in a wide neighborhood of the initial condition, and can also be used for noncoplanar systems. The results of these papers are in accord. They reveal that the HD 12661 system is close to the 11 : 2 MMR, accompanied by the SAR, with the critical angle $\sigma_0 - \sigma$, librating about 180°. This type of SAR is the first such case found among the known planetary systems (Lee & Peale 2003). The octupole-level secular theory of these authors explains that the planetary system is located in an extended resonance island in the phase space, and it helps to understand why the resonance persists in wide ranges of orbital parameters. This is also true for noncoplanar systems and wide ranges of planetary masses (Gozdziewski 2003).

Our dynamical study of the HD 12661 system (Gozdziewski 2003) is based on the first two-planetary Keplerian fit published by the California and Carnegie Planet Search Team on their Web site.3 Recently, an updated fit to the RV data appeared (see Table 2 in Fischer et al. 2003). A preliminary dynamical analysis of this initial condition does not bring qualitative changes to the system dynamics. However, the results of remarkable papers by Laughlin & Chambers (2001) and Lee & Peale (2003) inspired us to analyze the RV data again, in the framework of the full, nonlinear $N$-body dynamics. Although the RV observations are most commonly modeled by additive Keplerian signals, the mutual interaction between giant planets can introduce substantial changes to such a simple model. A further nuance flows from the fact that the two-planetary Keplerian fits are mostly interpreted as oscillating, astrocentric elements. In fact, the fits should be interpreted as Keplerian elements related to the Jacobi coordinates (Lee & Peale 2003). These refinements seem to change qualitatively the view of the dynamical state of the HD 12661 system, as it can be derived from the current RV data.

2. ORBITAL FITS

We used the updated RV observations of the HD 12661 system (see Table 4 in Fischer et al. 2003), containing 86 data points, collected at the Keck and Lick Observatories. The reported uncertainties are $\approx 3$–6 m s$^{-1}$ for the Keck data and $\approx 7$–17 m s$^{-1}$ for the Lick observations.

In our first attempt to find the initial condition of the RV data, we applied the genetic algorithm (GA). To the best of our knowledge, this method of minimization was used for analyzing the RV of υ And (Butler et al. 1999; Stepinski, Malhotra, & Black 2000), the Gliese 876 system (Laughlin & Chambers 2001), and 55 Cnc (Marcy et al. 2002). Basically, the genetic scheme makes it possible to find the global minimum of the $\chi^2$ function. Our experiments confirm the remarks of Stepinski et al. (2000) that the method is inefficient for finding very accurate best-fit solutions but provides good starting points for fast and precise gradient methods, like the Levenberg-Marquardt (LM) scheme (Press et al. 1992). These two methods of minimization complement each other. We used the publicly available code PIKAIA,4 version 1.2 by Charbonneau (1995), a great advocate of the GAs.

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3 See http://exoplanets.org.

4 See http://www.hao.ucar.edu/public/research/si/pikaia/pikaia.html.
In the simplest case, the minimized function is given by the following formula:

\[
\chi^2 = \sum_{i=1}^{N} \frac{(V_k(t_i, p) + \bar{V}_0 - V(t_i))^2}{\sigma(t_i)},
\]

where \(N\) is the number of observations of the RV, \(V(t_i)\), at moments of time \(t_i\), with uncertainties \(\sigma(t_i)\), modeled by a sum of Keplerian signals \(V_k(t_i, p) = V_0(t_i, p) + V(t_i, p)\) and the velocity offset \(\bar{V}_0\). The fit parameters, \(p = (p_b, p_b^*)\), where \(p_b = (K_p, n_p, e_p, \omega_p, T_p)\), for every planet \(p\), are the amplitude \(K_p\), the mean motion \(n_p\), the eccentricity \(e_p\), the argument of periastron \(\omega_p\), and the time of periastron passage \(T_p\). The orbital elements emerging from these parameters should be related to the Jacobi coordinates (Lee & Peale 2003). Recently, we drew a similar conclusion independently (K. Goździewski, M. Konacki, & A. Maciejewski 2003, in preparation).

In our fit model, the planetary system’s reference frame is chosen in such a way that the \(z\)-axis is directed from the observer to the system’s barycenter and \(V(t)\) is the \(z\)-coordinate of star velocity. The relation between \(K_p\) and the mass of a planet \(m_p\), the mass of the host star \(m_\ast\), and the geometric elements, related to the Jacobi coordinates, is given by \(K_p = \sigma_b a_p n_p \sin i\), where, for the three-body system, \(\sigma_b = Gm_\ast/(m_\ast + m_b + m_\ast)\) for the first planet and \(\sigma_b = Gm_\ast/(m_b + m_b + m_\ast)\) for the second companion. For efficiency reasons, the code for the dynamical analysis, incorporating the MEGNO (Mean Exponential Growth factor of Nearby Orbits) indicator, works internally in astrocentric coordinates; thus, when it is required, the Jacobi coordinates are transformed to these coordinates. In this sense, the astrocentric elements are considered only as a formal representation of the initial condition. Together with \(\chi^2\), we use \((\chi^2)^{1/2} = \left(\chi^2/\nu\right)^{1/2}\), where \(\nu = N - N_p - 1\) is the number of degrees of freedom and \(N_p\) is the total number of parameters.

The PIKAIA code is controlled by a set of 11 parameters. We set the most relevant as follows: the population number to 256, the number of generations to a rather high value of 8000–12,000, and the variable mutation rate in between 0.005 and 0.07 with a crossover probability equal to 0.95. The PIKAIA runs, restarted many times, resulted in a number of qualitatively similar fits, with \((\chi^2)^{1/2} = 1.44\) and an rms of \(=8.2\) m s\(^{-1}\). The RV data have been modeled by a two-planetary Keplerian signal and one RV offset. Surprisingly, we found that not only do all these solutions have a slightly smaller \((\chi^2)^{1/2}\) and rms than reported by Fischer et al. (2003) for the two-planetary Keplerian model \([\chi^2^{\text{LM2}} = 1.46\) and an rms of \(=8.8\) m s\(^{-1}\)], but they also differ substantially from the former fit. The period ratio is very close to 6 : 1, and not to 11 : 2! Also, the initial \(e_c = 0.1\) is smaller than previous estimates.

Actually, because the Doppler measurements have been performed in two different observatories, we should account for the two independent velocity offsets \(V_b\) and \(V_i\), which are different for the Lick and Keck observations. Data that are inhomogeneous in this sense were analyzed by Stepinski et al. (2000) and Rivera & Lissauer (2001). Indeed, the GA incorporating the RV model with two velocity offsets makes it possible to obtain a significantly better solution \([\chi^2^{\text{LM1}} = 1.37\) and an rms of \(=7.9\) m s\(^{-1}\)].

We verified this fit by determining the quasi-global minimum of \(\chi^2\) as a function of \((P_p, e_p)\). We searched for the best-fit solutions with these two parameters fixed. Because \(P_b\) is determined very well, its value, as well as the value of \(e_c\), approximated by the best GA solution for the two-planetary Keplerian model, was selected for the starting point in the LM method. Next, varying the initial phases (the arguments of periastron and the mean anomalies) of the two companions, with the step \(=30^\circ\), we calculated the best-fit solution with the LM algorithm. The results of this experiment are given in Figure 1. This scan reveals that the best-fit solution is localized in a flat minimum. Its parameters agree very well with the GA solution. This test confirmed that the GA fit is really global. Finally, driven by the full model of the dynamics, we refined this fit by the LM algorithm, incorporating the mutual gravitational interaction between the planets. The best GA fit was used as a starting point for the gradient search. The result is shown in Table 1 and is named the LM1 fit. The synthetic RV curve is shown in Figure 2.

To estimate the errors of this solution, at the first attempt; we used a method that relies on a determination of the \(\chi^2\)
confidence intervals (Press et al. 1992). We were warned by the referees that this method has drawbacks, the most relevant of which is that by using it we do not account for the stellar “jitter.” Even for a chromospherically inactive HD 12661 star, the uncertainty, \( \sigma_{\text{jitter}} \), contributed by the jitter to the RV measurements can be relatively high. The referee, G. Laughlin, suggested that we use the estimate \( \sigma_{\text{obs}} = (\sigma_{\text{jitter}}^2 + \sigma_{\text{obs}}^2)^{1/2} \), where \( \sigma_{\text{obs}} \) are “pure” instrumental errors. Such joint uncertainty, accounted for by calculating \( \chi^2 \), leads to a value of \( \chi^2 = 1 \), thus giving a statistical indication of an adequate model of the RV data. To estimate the parameter’s errors, we synthesized about 250 sets of “observations.” To every original RV measurement, we added a Gaussian noise with the mean dispersion of \( \sigma_{\text{Keck}} = 3.4 \) m s\(^{-1}\) and of \( \sigma_{\text{Lick}} = 7.6 \) m s\(^{-1}\) for observations gathered by the Keck and Lick spectrometers, respectively, and the Gaussian noise of the stellar jitter, with the dispersion of \( \sigma_{\text{jitter}} = 5 \) m s\(^{-1}\). Next, the Newtonian model of dynamics was fitted to every such synthetic data set with the LM algorithm that started from the LM1 solution. The mean values of the fit parameters are given in Table 1 and are called the LM2 fit. Finally, the dispersions of these orbital parameters are adopted as the mean uncertainties of both the LM1 and LM2 fits. In fact, both solutions are the same with respect to the error bounds.

### 3. STABILITY ANALYSIS

To investigate the dynamical stability of the LM1 and LM2 fits, we used the fast indicator called MEGNO. This technique was advocated by us in a series of recent papers (see, e.g., Goździewski et al. 2001; Goździewski 2003). The MEGNO indicator is closely related to the maximal Lyapunov exponent, but it permits us to determine rapidly whether an initial condition leads to a regular, quasi-periodic, or chaotic irregular solution. It is a very efficient tool that helps us to detect orbital resonances, their structure, and unstable regions in the phase space. Looking at a neighborhood of the analyzed initial conditions is a profitable way of resolving the question of whether or not the system dynamics are robust to the fit errors.

The MEGNO tests reveal that both the LM1 and LM2 fits are related to quasi-periodic motions of the HD 12661 system. The MEGNO signature for the LM1 fit is shown in Figure 4c below, and a perfect convergence of MEGNO to 2 indicates a quasi-periodic evolution of the planetary system. The MEGNO scan in the neighborhood of this fit, in the \((a_e, e)\)-plane, is shown in the left panel of Figure 3. Actually, both fits are located in a relatively extended stable zone, in a proximity of the 6 : 1 MMR. The 6 : 1 MMR is separated about 0.2 AU from the 11 : 2 MMR, which has been considered, up to now, as the closest MMR neighboring the system in the phase space. The evolution of orbital elements in the LM1 fit is shown in Figures 4a and 4b.

The stable zone of the 6 : 1 MMR is relatively very narrow in the range of small \( e_c \). To illustrate its effect on the motion of the HD 12661 system, we changed \( a_e \) in the LM2 fit from the nominal value of \( a_e \approx 2.78 \) AU to 2.745 AU, still keeping it in the error bound. The MEGNO scan, in the \((a_e, e_c)\)-plane, for such a modified initial condition is shown in the middle panel of Figure 3, and the Jacobi elements are shown in Figures 4d–4f. Note that in both cases, the SAR with the apsidal lines antialigned in the exact resonance is present, so the system still remains in the large libration island found by Lee & Peale (2003).

![Fig. 3.—MEGNO maps for the initial elements found in this Letter. The left panel is the LM1 fit; the middle panel is for the modified LM2 fit (as explained in the text)—these two maps, with the resolution 120 x 100 data points, are for the \((a_e, e)\)-plane. The right panel is for the nominal LM1 fit, and it shows regions of stability in the plane of the system inclination \( i \) vs. the relative inclination of orbits (expressed through the difference of the longitudes of nodes, \( \Delta \Omega = \Omega_1 - \Omega_2 \)). The resolution of the scan is 85 x 90 data points. Zones centered about \( \approx 2 \) correspond to regular, quasi-periodic motions of the HD 12661 system. The intersection of the two lines marks the fitted parameters; the MMRs relevant to our discussion are labeled.](image-url)
Fig. 4.—Evolution of Keplerian orbital elements related to Jacobi coordinates for the initial conditions found in this Letter. The upper panels are for the LM1 fit; the bottom panels are for the modified LM2 fit. Note the presence of the SAR—it’s critical argument $\omega_2 - \omega_3$ librates about 180° (panels b and e). The modified LM2 fit leads to the 6 : 1 MMR (panel f), while for the LM1 fit, the same critical argument circulates. Note the stabilizing effect of the 6 : 1 MMR on the eccentricities of the planets (panels a and d). Panel c is for the MEGNO signature of the LM1 fit, and it indicates a quasi-periodic, stable motion of the planetary system. Time is given in units of $10^5$ yr.

The scan in the $(i, \Delta \Omega)$-plane, where $\Delta \Omega = \Omega - \Omega_p$, is shown in the right panel of Figure 3, and it makes it possible for us to estimate the bounds on masses in the systems with the same $i$, in terms of regular and chaotic motions. For coplanar systems, the stable zone extends up to $i = 25^\circ$, but also two islands about $i = 10^\circ$ and $18^\circ$ exist. Whether the system can survive in the chaotic regions during evolutionary timescales can be verified, in general, only by long-term integrations.

4. CONCLUSIONS

The study of the RV observations of the HD 12661 star revealed fits that lead to significantly different dynamics when compared with the original solution published by Fischer et al. (2003). Combined genetic and gradient methods of optimization helped us to find a global minimum of $\chi^2$ for the RV model that incorporates the mutual interactions between planets. A dynamical analysis shows that the new fits are related to a system close to the 6 : 1 MMR and locked into a deep secular resonance with apsidal lines antialigned. The fits preclude the proximity of the 11 : 2 MMR, which has been suggested by the previously determined initial conditions. In the phase space, the system lies in a relatively extended zone of stable, quasi-periodic motions. The closeness of the HD 12661 system to the low-order 6 : 1 MMR adds a new inquiry to the 3 : 1 MMR in the 55 Cnc system, the 7 : 3 MMR in the 47 UMa system, and the 5 : 2 MMR of Jupiter and Saturn. It is difficult to claim that the HD 12661 system is locked exactly in the 6 : 1 MMR, but our results suggest that very likely it is close to this resonance. Hopefully, the observational window of the HD 12661 will soon cover two orbital periods of the outer companion, and the updated set of measurements will help us to refine the fits again.

The analysis of the RV data benefits from a global approach similar to the one used for the stability studies. For planetary systems with large and not very distant planets, the effects of mutual interactions and a proper dynamical interpretation of the orbital fits are vital for understanding the dynamics and for resolving all the information hidden in the observations. Their analysis requires much care, because omitting even subtle factors can lead to a quite different interpretation of the same data set.

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