Mapping the hydrodynamic response to the initial geometry in heavy-ion collisions

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with

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Motivation

- The Almond Shape and Elliptic Flow
  Smooth & Realistic Initial Conditions

Mapping the hydrodynamic response

- How to map?
- The elliptic flow case;
- Generalization to higher harmonics
- Improving the predictor

Conclusion
The azimuthal distribution of outgoing particles in a hydro event can be written as

\[ \frac{2\pi}{N} \frac{dN}{d\phi_p} = 1 + 2 \sum_n v_n \cos[n(\phi_p - \Psi_n)] \]

or, equivalently:

\[ \{ e^{in\phi_p} \} = v_n e^{-in\Psi_n} \]

\{ \ldots \} = \text{average in one event}

The largest source of uncertainty in hydro models is the initial conditions.

Anisotropic flow \( v_n \) and the event plane \( \Psi_n \) are determined by initial conditions.

We need to understand which properties of the initial state determine \( v_n \) and \( \Psi_n \), so as to constrain models of initial conditions from data.
The almond shape and $\nu_2$

**Average Initial Conditions - the smooth case**

- With smooth initial conditions, the **participant eccentricity** $\varepsilon_2$ is proportional to the elliptic flow $\nu_2$,
- And the **participant plane** $\Phi_2$ is aligned with the event plane $\Psi_2$.

$$\varepsilon_2 e^{i2\Phi_2} = -\frac{\left\{ r^2 e^{i2\phi} \right\}}{\left\{ r^2 \right\}}.$$  

$\{\cdots\} = \text{average over initial density profile}$

- But, in real collisions there are fluctuations, event-by-event.
- In event-by-event hydrodynamics are these relations, $\nu_2 \approx k\varepsilon_2$ and $\Psi_2 \approx \Phi_2$, valid?

figures by R. Andrade
Motivation

**Fluctuations: Event-by-event hydro**

**NeXSPheRIO**
- NeXus: initial condition generator;
- SPheRIO: solves the equations of relativistic ideal hydrodynamics.

Scatter plot of $v_2$ versus $\varepsilon_2$

Distribution of $\Psi_2 - \Phi_2$

$\nu_2 \approx k \varepsilon_2$ and $\Psi_2 \approx \Phi_2$?

Reasonable, but not perfect.

See also, F.G.G. et al 1110.5658, Petersen et al 1008.0625, Qiu & Heinz 1104.0650.
Our goal

- Propose a simple quantitative measure of the correlation between $(v_2, \Psi_2)$ and $(\varepsilon_2, \Phi_2)$;
- Generalization to higher harmonics;
- Find better scaling laws.
Previously, the correlation of the flow with the initial geometry was studied through

- Distribution of $\psi_2 - \phi_2$
- Scatter plot $v_2$ versus $\varepsilon_2$

Our Proposal: A GLOBAL ANALYSIS

$$v_2 e^{i2\psi_2} = k\varepsilon_2 e^{i2\phi_2} + \mathcal{E}$$

- $k$: It is the same for all events (in each centrality class).
- $\mathcal{E}$: event-by-event error.

The best linear fit is achieved minimizing the mean-square error

$$\langle |\mathcal{E}^2| \rangle (\langle \cdots \rangle \equiv \text{average over events})$$

- $k = \langle \varepsilon_2 v_2 \cos[2(\psi_2 - \phi_2)] \rangle / \langle \varepsilon_2^2 \rangle$
- $\langle |\mathcal{E}^2| \rangle = \langle v_2^2 \rangle - k^2 \langle \varepsilon_2^2 \rangle$
Elliptic flow as a response to the almond-shaped overlap area

The quality response is given by:

\[ \text{Quality} = \frac{k \sqrt{\langle \varepsilon_2^2 \rangle}}{\sqrt{\langle v_2^2 \rangle}} \]

The closer Quality to 1, the better the response.

- Central collisions: All anisotropies due to fluctuations
  Quality 81%

- Mid-central collisions: Elliptic flow is driven by the almond shape: Quality 95%

\( \varepsilon_2 \) is a very good predictor of \( v_2 \)!
Results

Generalization to higher harmonics

- Natural estimators are:
  \[ v_n e^{in\Psi_n} = k \varepsilon_n e^{in\Phi_n} + \mathcal{E} \]

- Generalizing \( \varepsilon_n \) (Petersen et al 1008.0625).
  \[ \varepsilon_n e^{in\Phi_n} = -\frac{\left\{ r^n e^{in\phi} \right\}}{\left\{ r^n \right\}} \]

Teaney&Yan (1010.1876) showed \( \varepsilon_n \) come from a cumulant expansion of the initial density energy

- \( n=3 \):
  \( \varepsilon_3 \) is a very good predictor of \( v_3 \).

- \( n=4,5 \):
  Good quality for central collisions

  Then decrease and even become negative: \( \Psi_n \) and \( \Phi_n \) are anticorrelated.

  Qiu&Heinz, 1104.0650
Finding better estimators

The Almond Shape and $\nu_4$

With smooth IC, inspired by NeXus IC in the 30 – 40% centrality bin

There is no $\varepsilon_4$, so where does $\nu_4$ come from?

- $\nu_4$ is generated by $\varepsilon_2$!
- $\Psi_4$ is in the reaction plane, as $\Psi_2$.

Comparing with NeXSPheRIO (30-40%), $\langle \nu_2 \rangle \approx .066$ and $\langle \nu_4 \rangle \approx .01$

_F.G.G. et al 1110.5658._
Finding better estimators

\( \nu_4 \) induced by \( \varepsilon_2 \) in event-by-event

- A natural estimator is:
  \[
  \nu_4 e^{i4\psi} = k (\varepsilon_2 e^{i2\phi_2})^2 + \mathcal{E}
  \]
  (preserves rotational symmetry)

- For mid-central collisions, where \( \varepsilon_2 \) is large, the non-linear term is important!
- This estimator is not as good as previous estimators of \( \nu_2 \) and \( \nu_3 \).

How to improve the estimator?
Finding a better estimator

Combining both effects?

- Defining
  \[ \nu_4 e^{i4\Psi_4} = k\varepsilon_4 e^{i4\Phi_4} + k' (\varepsilon_2 e^{i2\Phi_2})^2 + \mathcal{E} \]

- And minimizing \( \langle |\mathcal{E}|^2 \rangle \), with respect to \( k \) and \( k' \).

- Then, the mean-square error is
  \[ \langle |\mathcal{E}|^2 \rangle = \langle \nu_2^2 \rangle - \langle |k\varepsilon_4 e^{i4\Phi_4} + k' (\varepsilon_2 e^{i2\Phi_2})^2|^2 \rangle \]

  This error is always smaller than with one parameter.

The combined estimator results in an excellent predictor for all centralities!

For \( \nu_5 \), it is also possible to use both, linear and non-linear, terms to obtain the best estimator: \( \varepsilon_5 \) and \( \varepsilon_2\varepsilon_3 \) preserves rotational symmetry.

**Mapping the hydrodynamic response to the initial geometry in heavy-ion collisions**

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We have defined a quantitative measure of the quality of estimators of $v_n$ from initial conditions event-by-event hydro;

$v_2$ can be understood as a response to the almond-shaped overlap area $\varepsilon_2$, even for central collisions;

The triangularity $\varepsilon_3$ is a very good predictor to $v_3$;

Non-linear terms are necessary to predict $v_4$ (and $v_5$) from initial energy density, for all centralities. (See TeaneyYan 1206.1905)

These results provide an improved understanding of the hydro response to the initial state in realistic heavy-ion collisions.