Multi-criteria optimization of reinforced concrete beams using genetic algorithms

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Abstract. A strategy for finding optimal solutions for beam reinforced concrete structures according to several criteria has been proposed. The analysis and systematization of the optimization objectives for reinforced concrete framed systems is performed. To solve the problem, a modified genetic algorithm is used, which involves the selection of design options in a single iterative process into several databases containing solutions for each target criterion. When forming the current set of solutions, the migration of the best options from any base under consideration is allowed. As a general criterion of optimality, the J. Nash equilibrium principle is considered for penalties under failure to comply with constraints in relation to reinforced concrete structures.

1. Introduction
One of the new relevant objectives in the design and scientific research of the reinforced concrete structures is the search for solutions that are optimal immediately according to several target criteria. Separate optimality criteria are formulated in the recommendations of the Concrete and Reinforced Concrete Research Institute [1]. In the practice of designing the reinforced concrete structures, the following basic criteria can now be formed:

- the total cost of the reinforced concrete structure in the "case", taking into account the technology of its construction;
- the total cost function taking into account the initial costs and risk associated with damage due to financial losses in accidents;
- the survivability of the structural system in case of single or multiple local damages;
- the cost of the life cycle of a structural system;
- the cost of materials, taking into account the assortments of their classes (concrete, reinforcement);
- consumption of concrete, reinforcement
- other criteria (the complexity of the work performing, the time of construction of structures, etc.).

One of the promising optimization methods is genetic algorithms [1-5], which have proven to be efficient in solving a number of problems associated with the design of both reinforced concrete [6-8] and steel structures [9, 10]. At the same time, various types of impacts were considered both under normal operation conditions [11, 12], and during emergency impacts. Most of the works took into account local mechanical damages [13, 14]; structural optimization under high temperature impacts is hardly considered. Multi-criteria methods are also being developed [15-17], in which the V. Pareto
frontier is used as an approximation to the multi-criteria optimum.

2. Formulation of the optimization problem

We will formulate the problem of multi-criteria optimization taking into account the D. Nash equilibrium principle. This principle interpreted for reinforced concrete structures as follows: a solution according to several criteria is optimal if the potential total penalty for possible violation of conditional constraints for each of the optimization criteria is minimal, and the penalties for constraints related to each optimization criterion, approximately equal to each other. By conditional constraints, we deem such systems of constraints, the penalties for non-fulfillment of which will amount to a certain amount with strict satisfaction of the actual constraints. Conditional constraints imply some slight tightening of the actual constraints. We will solve the problem of parametric optimization. To solve this problem using a genetic algorithm, we formulate a system of objective functions:

\[
\begin{align*}
C_1 &= \left( f_1 \left( \{Y_{p1}\} \right) + \chi(r)R_1 \right) \rightarrow \min; \\
C_2 &= \left( f_2 \left( \{Y_{p2}\} \right) + \chi(r)R_2 \right) \rightarrow \min; \\
&\vdots \\
C_N &= \left( f_N \left( \{Y_{pN}\} \right) + \chi(r)R_N \right) \rightarrow \min
\end{align*}
\]

Where \( \{Y_{p1}\} = \{y_{p1}, ..., y_{pn}\} \), \( \{Y_{p2}\} = \{y_{p2}, ..., y_{pn}\} \), and \( \{Y_{pN}\} = \{y_{pN}, ..., y_{pn}\} \) are discrete sets of variable parameters for which the condition \( \{Y_{p1}\} \subseteq \{Y_{p2}\} \subseteq \ldots \subseteq \{Y_{pN}\} \) is satisfied, \( N \) is the number of optimality criteria; \( n \) is the number of variable parameters, \( R_1...R_N \) are risks associated with structural failures as a result of normal operation or beyond design basis impacts, \( \chi(r) \) is the Heaviside function that determines the need to take risks into account depending on the specified value of the control parameter \( r \). We formulate a system of real and conditional constraints of the problem taking into account the D. Nash equilibrium. The system of valid constraints \( H_1 \) and generated penalties \( P_i \) (\( i \) is the number of constraints) for their violation for flexible reinforced concrete bar elements according to the criterion \( C_1 \rightarrow \min \) can be represented as

\[
\begin{align*}
H_1(P_1, ..., P_3) &= \begin{cases} 
\forall (\hat{k} = \left( M_{ult} \left( \{Y_{p1}\} \right) - M \right) / M) \exists P_1 = \begin{cases} 
0; & \hat{k} > 1 \\
\hat{k} & \hat{k} \leq 1
\end{cases}; \\
\forall (\hat{k} = \left( Q_{ult} \left( \{Y_{p1}\} \right) - Q_{ult} \right) / Q) \exists P_2 = \begin{cases} 
0; & \hat{k} > 1 \\
\hat{k} & \hat{k} \leq 1
\end{cases}; \\
\forall (\hat{k} = \left( \{N_{ult}\} \left( \{Y_{p1}\} \right) - N \right) / N) \exists P_2 = \begin{cases} 
0; & \hat{k} > 1 \\
\hat{k} & \hat{k} \leq 1
\end{cases}; \\
\forall (\hat{k} = \left( [\delta] - \delta \right) / \delta) \exists P_3 = \begin{cases} 
0; & \hat{k} > 1 \\
\hat{k} & \hat{k} \leq 1
\end{cases}; \\
\forall (\hat{k}' = \left( a_{crc} - a_{crc} \right) / a_{crc}) \exists P_4 = \begin{cases} 
0; & \hat{k}' > 1 \\
\hat{k}' & \hat{k}' \leq 1
\end{cases}.
\end{cases}
\end{align*}
\]

Here \( \hat{k} \), \( k \), \( \hat{k} \), \( k' \) are the degrees of violation of constraints in fractions of one; \( \Omega_1 - \Omega_3 \) are coefficients that determine the amount of the penalty depending on the value of the objective function; \( M_{ult} \), \( Q_{ult} \), \( N_{ult} \) are the limiting values of bending moments, transverse and longitudinal forces that
can be perceived by the cross section of the reinforced concrete element; \([\delta], [a_{crc}]\) are the values of deflections and widths of crack opening allowed during normal operation; \(M,Q,N\) are the calculated values of the internal forces in the element due to external loads and influences.

Then the general system of valid constraints will take the form:

\[ H = \{H_1, \ldots, H_N\} \cdot \quad (3) \]

It should be noted that the definition of constraint systems of type (2) for other optimality criteria must contain its constraints. We write the general system of conditional constraints in the form:

\[ \{\bar{P}\}_j \approx \{\bar{P}\}_1 \approx \ldots \approx \{\bar{P}\}_N \]  

where the system \(\bar{P}\) can be obtained from expression (2) by substituting:

\[ \sqrt{\delta} = \Delta_4 [\delta]; [a_{crc}] = \Delta_5 [a_{crc}] \cdot \quad (5) \]

In this case, the condition for achieving the optimality of the solution as much as possible in accordance with the equilibrium condition according to D. Nash can be written as:

\[ \frac{\sum_{i=1}^{N} \sum_{j=1}^{m} \{\bar{P}\}_i}{\{\bar{P}\}_1} \rightarrow \min; \quad \{\bar{P}\}_1 \approx \{\bar{P}\}_2 \approx \ldots \approx \{\bar{P}\}_N \]  

\[ \{\bar{P}\}_1 \sim \{\bar{P}\}_2 \sim \ldots \sim \{\bar{P}\}_N \]  

\[ \text{Condition (2) is written for the case of normal operation of beam reinforced concrete structures, while the system survivability criterion, which is used as the optimality criterion, is of interest. Moreover, the system of constraints for the supporting beams of floors and coatings means the fulfilling the conditions for the absence of destruction of concrete and reinforcement, which is simplified by the level of ultimate relative deformations for these materials, and the condition of unacceptably large changes in geometry. Changes in the system geometry that exceed the conditions for the safe evacuation of people and equipment are unacceptable. These changes can be described by deflections \([f_y]\), the values of which depend on the height of the floor } \quad (6) \]

\[ H_{2,b}(P_1, P_2, P_3) = \begin{cases} \forall (k = (e_{b,ult} \left(f_y \right) - e_{b,ult}^{\max})/e_{b,ult}^{\max}) \exists P_1 = \begin{cases} 0, \cdots k > 1 \\
\tilde{k} \Omega_1, \cdots \tilde{k} \leq 1 \end{cases} \\
\forall (k = (f_y - f_y^{\max})/f_y^{\max}) \exists P_2 = \begin{cases} 0, \cdots k > 1 \\
\tilde{k} \Omega_2, \cdots \tilde{k} \leq 1 \end{cases} \\
\begin{bmatrix} f_y \end{bmatrix} = \begin{cases} 0.2 H_f \quad (2.5 \leq H_f \leq 3.2) \\
0.3 H_f \quad (3.2 < H_f \leq 4.6) \end{cases} \end{cases} \]  

Here \(e_{b,ult}^{\max}, e_{s,ult}^{\max}\) are the ultimate relative deformations of concrete and reinforcement corresponding to fracture conditions \(e_{b,ult}, e_{s,ult}\) are the ultimate relative deformations of concrete and reinforcement obtained by calculation under the conditions of an accident and maintaining the structure resistance to progressive destruction; \(f_y\) is the actual estimated vertical deflection of the damaged structure.
For columns, the survivability condition will be interpreted as preventing loss of stability during dynamic loading, as well as a constraint on horizontal movements $\delta_x$, leading to a change in the design scheme and a possible loss of concrete strength due to compression with bending. In this case, we write the system of constraints for reinforced concrete columns:

$$
H_{2,\text{col}}(P_1, P_2, P_3) = \begin{cases}
\forall(k = (\varepsilon_{b,\text{ult}}(Y_{p2}) - \varepsilon_{b,\text{ult}})/\varepsilon_{b,\text{ult}}) \exists P_1 = \left\{ \begin{array}{ll}
0; & \kappa > 1 \\
\kappa \Omega_1; & \kappa \leq 1
\end{array} \right.; \\
\forall(k = ([\delta_x] - \delta_x)/\delta_x) \exists P_2 = \left\{ \begin{array}{ll}
0; & \kappa > 1 \\
\kappa \Omega_2; & \kappa \leq 1
\end{array} \right.; \\
\forall(k = ([N_{\text{cr}}] - N)/N) \exists P_3 = \left\{ \begin{array}{ll}
0; & \kappa > 1 \\
\kappa \Omega_2; & \kappa \leq 1
\end{array} \right.;
\end{cases}
$$

where $[N_{\text{cr}}]$ is the ultimate longitudinal force causing loss of stability of the element during beyond design basis exposure.

3. Optimization method
Parametric synthesis of load-bearing structures based on several optimization criteria can be formulated in the form of a block diagram shown in Fig. 1. Let us explain the main stages of this optimization algorithm. After the formation of the target criteria, all the necessary data about the object of study are set taking into account its calculation model. Here, permissible sets of variable parameters are formed. One of the guarantees for obtaining a solution is the defining such sets that would synthesize objects that satisfy actual constraints. In the blocks of the formation of constraint systems in relation to reinforced concrete structures, the conditions of strength, stiffness, stability, survivability, etc. are recorded. These constraints are valid active. That is, in case of violation of any of them, a solution cannot be obtained. In addition, a number of constraints are formed that are similar to them, but more stringent. Their tightening can be achieved using the dependence (5) in the task $\Delta_i = 0.75 \div 0.95$. We consider these constraints conditional. They are necessary to verify that the solution matches the optimum according to several criteria. The blocks for choosing a calculation analysis technique may contain the following types of calculations:
- stress-strain state analysis of reinforced concrete structures based on the methods described in the code specification;
- stress-strain state analysis of reinforced concrete structures based on the finite element method during normal operation;
- calculation of the stress-strain state of reinforced concrete structures based on the finite element method taking into account physical, geometric and structural nonlinearity in case of local damage;
- calculation of the initial design reliability and risks based on the full or conditional probability of failure caused by natural or man-made causes for one or more structural elements;
- calculation of the stress-strain state of reinforced concrete structures during impacts, high temperature exposure, corrosion damage, etc.
- other special calculation methods (climatic effects, solar radiation, etc.).

Let’s consider the block forming current solutions. If during mono-criteria optimization the recommended amount of this pool was in the range from 20 to 30 options of the object, then for multi-criteria search this amount increases in proportion to the criteria considered. That is, for the two criteria under consideration, this amount should be at least 40. At the first iteration, this set is formed on the basis of alternative methods for forming the initial solution pools described in [18], otherwise there is a possibility of not going to the region of feasible solutions. At subsequent iterations, the formation of the current pool is performed on the basis of the procedures indicated in Fig. 1 by arrows 1–4, which we will describe later.
Figure 1. Flow chart of the optimization process
For the generated options of constructive solutions, \( N \) of implementations of calculation methods are performed, as a result of which the calculated values of the VAT components, risks, failure probabilities, etc. are determined. In this case, constraints of type (2), (7), (8) are checked and, if they are not fulfilled the actual penalties are calculated. Further, all analyzed objects are placed in the database in accordance with the values of the objective functions. At the same time, one object can fall into several different bases. For each of the bases, a strategy for keeping the best solution is implemented only for its optimality criterion.

At the next stage, conditional penalties for non-fulfillment of conditional constraints are calculated for all design options from all bases. If a design option has a penalty for failure to comply with valid constraints, then it is added to the corresponding conditional penalty. As a result, each design option has its own total penalty. Design options with a minimum penalty are placed in the database of the best options corresponding to a multi-criteria optimum. In this database, options are sorted according to the condition of equal penalties for each of the systems of constraints of the optimality criterion. Constructions that meet the equality of penalties with their total minimum occupy higher ranks in the database of the best projects. With the same total penalty, priority is given to the construction with more equal penalties.

The main feature of the algorithm is the selection of options for the formation of the current set of solutions. This set is divided into \( N \) parts. For each part, the first options are selected from a multi-criteria database of the best solutions. The selection involved \( 2N \) sorted the best projects selected at random, as shown in the block diagram by link 1 (Fig. 1). The same selection mechanism is also implemented for project databases with the best values of the objective function according to the criteria \( C_1 - C_N \), which is shown by links 2-4. All other options are formed randomly and based on the application of genetic operators to the options already available in the current set.

The criterion for stopping the iterations of the genetic algorithm is traditional here and consists in the absence of changes in the database of the best solutions for several optimality criteria after an integer number of \( N_I \) iterations has passed, this number can be defined as

\[
N_I = N^{m/n}m!,
\]

where \( N \) is the number of optimality criteria; \( m, n \) is the average number of values of the varied parameters and the number of parameters, respectively.

4. Results
Let us consider an example of two-criterion optimization of a double-span reinforced concrete beam, the design scheme and the initial sets of variable parameters are given in [19]. We consider two optimization criteria:

- minimum cost of materials;
- ensuring maximum survivability while removing the middle support.

We set \( [f_y] = 0.2 \text{ m}, q = 50kH/m \). The results of the synthesis of this construction is presented (Fig. 2). The following cost structures are received in modern market prices. For the option in fig. 2,\( e - C_I \approx 3,760 \text{ RUB} \), for the construction of fig. 2,\( f C_{II} \approx 6,350 \text{ RUB} \), for the construction of Fig. 2,\( f - C_{I \setminus II} \approx 5,120 \text{ RUB} \). It is obvious that if a designer set the task of ensuring safety, then the most preferable option is that obtained using two optimization criteria together. A significantly greater effect from the use of these design solutions can be seen when taking into account the risks of possible financial losses in the event of an emergency.

5. Discussion
This multi-criteria optimization algorithm allows performing an efficient search for solutions with a number of criteria not exceeding four. This is due, first of all, to the fact that the number of search options with a large number of variable parameters and their values, and optimization criteria
increases by several orders of magnitude compared to a conventional single-criterion search [19-23]. Therefore, the objective of choosing metaheuristic search strategies for multi-criteria optimization comes to the fore.

Figure 2. The results of the synthesis of a double-span beam: a-d are initial data and optimality criteria; e-g are optimization results.
6. Conclusion
An approach to multi-criteria optimization of reinforced concrete beams is proposed, in which a genetic algorithm is used to search for individual target criteria with the possibility of inter-base migration of design solutions, and a principle similar to the Nash equilibrium condition in game theory is used as an optimum condition for several criteria.

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