Parametric Resonance and Cosmological Gravitational Waves

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Abstract

We investigate the production of gravitational waves due to quantum fluctuations of the vacuum during the transition from the inflationary to the radiation-dominated eras of the universe, assuming this transition to be dominated by the phenomenon of parametric resonance. The energy spectrum of the gravitational waves is calculated using the method of continuous Bogoliubov coefficients, which avoids the problem of overproduction of gravitons at large frequencies. We found, on the sole basis of the mechanism of quantum fluctuations, that the resonance field leaves no explicit and distinctive imprint on the gravitational-wave energy spectrum, apart an overall upward or downward translation. Therefore, the main features in the spectrum are due to the inflaton field, which leaves a characteristic imprint at frequencies of the order of MHz/GHz.

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I. INTRODUCTION

Although gravitational waves have not yet been directly detected, those of cosmological origin are at present the object of an important research effort, as they will provide us with a unique telescope to the very early stages of the evolution of the universe.

In this work we investigate the production of gravitational waves during the transition from the inflationary period to the radiation-dominated era of the universe. We assume this transition to be dominated by parametric resonance, triggered by the coupling of the modes of an inhomogeneous bosonic field χ to the homogeneous inflaton field φ. We shall concentrate our efforts on those gravitational waves created by quantum fluctuations of the vacuum [1, 2, 3]. This mechanism is independent from the generation of gravitational waves directly sourced by large inhomogeneities in matter distribution occurring during parametric resonance [4]. Although order of magnitude calculations suggested that the last mechanism would dominate the production of gravitational waves, during parametric resonance, we thought it would be of interest to settle the issue with an accurate calculation based on the quantum fluctuations of the vacuum.

Parametric resonance may play a fundamental role in the reheating of the universe after the inflationary period [3]. Based on previous work on the subject [5], we shall investigate a simplified model of parametric resonance which, nevertheless, keeps the most important features of the more complicated model. For instance, this is the case when, as done in this paper, we neglect the backreaction from the metric perturbations and also perturbations of the inflaton field itself. This considerably simplifies the calculations, without missing the most important feature, namely, the sudden increase in the modes of the χ field, due to their coupling to the coherent oscillations of the inflaton φ.

We shall consider two different potentials for the inflaton field, namely, a quadratic potential $V(\phi) = m_\phi^2 \phi^2 / 2$ and a quartic potential $V(\phi) = \lambda \phi^4 / 4$. For the coupling between the inflaton field φ and the bosonic field χ we shall use $g^2 \phi^2 \chi^2 / 2$. The field χ will be taken to have a mass term $V(\chi) = m_\chi^2 \chi^2 / 2$. We incorporate reheating through an elementary decay mechanism of the two scalar fields into a relativistic radiation fluid characterized by a density $\rho_{\text{rad}}$. The decay rates of the fields φ and χ are determined by the decay constants $\Gamma_\phi$ and $\Gamma_\chi$, respectively. Arguments have been given that the χ field is strongly suppressed before the transition period begins [6]. The enormous increase of the scale factor during the inflationary era results in an exceedingly small value for χ at the beginning of the transition. We are then justified in treating the field χ as arising from its own quantum fluctuations, without any primordial classical part.

Gravitational wave production will be calculated by the use of the Bogoliubov coefficients as continuous functions of time, which obey differential equations first introduced by Parker [1] (see also Ref. [8] for a different derivation of these differential equations). Besides being a theoretically correct method of calculation of the graviton production, it has advantages over the frequently used sudden transition approximation. Associated with the sudden transition there is always an overproduction of gravitons of large frequencies, requiring the introduction of an explicit cut-off for frequencies above the rate of expansion of the universe. This problem is automatically

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solved by the use of the continuous Bogoliubov coefficients \[9\]. During the inflationary period, the potential term of the inflaton field dominates over the kinetic term, yielding a situation that closely resembles the behavior of a fluid with an equation of state of the form \(p = -\rho\), also characterizing a cosmological constant. This, plus the fact that gravitational waves produced during the inflationary period will be completely red-shifted, simplifies the calculations, making it reasonable to begin our numerical simulations at the end of the inflationary period, with simple initial conditions for the Bogoliubov coefficients appropriate to a cosmological constant.

This paper is organized as follows. In Section II we describe the method used to calculate the gravitational-wave energy spectrum, present the equations to determine the continuous Bogoliubov coefficients and derive appropriate initial conditions for these equations. The simplified model of parametric resonance, describing the transition between the inflationary period and the radiation-dominated era, as well as the equations describing the evolution of the universe from the end of the transition period till the present time, are introduced in Section III. Our numerical simulations, for two different potentials of the inflaton field (quadratic and quartic), are described in Section IV. Finally, in Section V we present our conclusions.

II. COSMOLOGICAL GRAVITATIONAL WAVES

We assume a flat universe and write the perturbed metric, in conformal time \(\eta\), as

\[
ds^2 = a^2(\eta) \left\{-d\eta^2 + [\delta_{ij} + h_{ij}(\eta, \mathbf{x})] \, dx^i \, dx^j \right\},
\]

where the tensor perturbations \(h_{ij}\) are expanded in terms of plane waves

\[
h_{ij}(\eta, \mathbf{x}) = \sqrt{8\pi G} \sum_{p=1}^{2} \int \frac{d^3k}{(2\pi)^{3/2} a(\eta)\sqrt{2k}} \times \left[a_p(\eta, \mathbf{k})\varepsilon_{ij}(\mathbf{k}, p)e^{ik\cdot\mathbf{x}}(\eta, k) + H.c.\right].
\]

(2)

In the previous expression, \(G\) is the gravitational constant, \(p\) runs over the two polarizations of the gravitational waves, \(k = |\mathbf{k}| = 2\pi a/\lambda = \omega \lambda\) is the co-moving wave number, \(a_p\) is the annihilation operator, \(\varepsilon_{ij}\) is the polarization tensor, and the mode function \(\xi\) obeys the equation of a parametric oscillator,

\[
\xi'' + \left(k^2 - \frac{\alpha''}{\alpha}\right) \xi = 0,
\]

(3)

where a prime denotes a derivative with respect to conformal time. Under appropriate circumstances, we may have exponentially growing solutions, with the gravitational field pumping energy into the gravitational waves. Quantum mechanically this corresponds to graviton production. We shall calculate the amount of gravitons produced using Parker’s method \[11\].

In a expanding universe the ground-state and the operators change with time,

\[
|0\rangle_\eta \neq |0\rangle_\eta,
\]

(4)

meaning that the annihilation \(a_p(\eta, \mathbf{k})\) and creation \(a_p^\dagger(\eta, \mathbf{k})\) operators also change, such change being codified in terms of time-fixed annihilation \(\bar{A}_p(\mathbf{k})\) and creation \(\bar{A}_p^\dagger(\mathbf{k})\) operators through the Bogoliubov transformation

\[
a_p(\eta, \mathbf{k}) = \alpha(\eta, k)\bar{A}_p(\mathbf{k}) + \beta^*(\eta, k)\bar{A}_p^\dagger(\mathbf{k}).
\]

(5)

In the previous expression, the Bogoliubov coefficients \(\alpha\) and \(\beta\) obey the condition \(|\alpha|^2 - |\beta|^2 = 1\). Comparing any matter field expansion at two different times, using Eq. (5) and the corresponding one for \(a_p^\dagger(\eta, \mathbf{k})\), it can be shown that the coefficient \(\beta\) gives the number of gravitons created, \(|\beta|^2 = \langle N_k(\eta)\rangle\).

To find the power spectrum, \(P(\omega)\), we use the definition of the density of states, \(\omega^2 d\omega/(2\pi^2 c^3)\), and the fact that each graviton contributes with two polarizations, \(2\hbar \omega\); then, from the definition of the energy density in terms of the power spectrum, \(dE = P(\omega)d\omega\), we obtain

\[
P(\omega) = \frac{\hbar\omega^4}{\pi^2 c^3} |\beta|^2.
\]

(6)

Taking into account that the gravitational-wave energy density is given by \(\rho_{GW} = \int P(\omega)d\omega\), we finally get the dimensionless relative logarithmic energy spectrum of the gravitational waves at the present time \(\eta_0\):

\[
\Omega_{GW}(\omega, \eta_0) = \frac{1}{\rho_{crit}(\eta_0)} \frac{d\rho_{GW}}{d\ln \omega}(\eta_0) = \frac{8\hbar G}{3\pi c^3 H^2(\eta_0)} \omega^4 |\beta|^2(\eta_0),
\]

(7)

where \(\rho_{crit}\) is the critical density of the universe.

As we have just seen, in order to obtain the gravitational-wave spectrum we need to compute the Bogoliubov coefficient \(\beta\). This is done by solving the set of differential equations \[11\ 12\]

\[
\begin{align*}
\alpha' &= \frac{i}{2k} \left(\alpha + \beta e^{2ik(\eta-\eta_0)}\right) \frac{a''}{a},
\beta' &= -\frac{i}{2k} \left(\beta + \alpha e^{-2ik(\eta-\eta_0)}\right) \frac{a''}{a},
\end{align*}
\]

(8)

(9)

which, upon the redefinition

\[
\begin{align*}
\alpha &= \frac{1}{2}(X + Y)e^{i\kappa(\eta-\eta_0)},
\beta &= \frac{1}{2}(X - Y)e^{-i\kappa(\eta-\eta_0)},
\end{align*}
\]

(10)

(11)

become

\[
\begin{align*}
X' &= -ikY',
Y' &= -\frac{i}{k} \left(k^2 - \frac{a''}{a}\right) X.
\end{align*}
\]

(12)

(13)
Because we are interested in graviton production during the transition between the inflationary period and the radiation-dominated era, the above set of differential equations should be integrated from the end of the inflationary period right up to the present time. This requires us to specify a model of evolution of the universe for the entire period under consideration in order to determine $a''/a$ (see next section) and also to specify, at the end of the inflationary period, the initial conditions for $X(\eta)$ and $Y(\eta)$.

Clearly, the set of differential equations (12) and (13) should be solved numerically. However, we can take advantage of the fact that it admits an exact analytical solution for the case of a de Sitter universe, in order to specify appropriate initial conditions for $X(\eta)$ and $Y(\eta)$. Indeed, during the inflationary period, the potential term of the inflaton field dominates over the kinetic term, yielding a situation that closely resembles the behavior of a cosmological constant. Since, in this case, the scale factor is given by $a(\eta) = [H(\eta - \eta_1)]^{-1}$, where $\eta < \eta_1$ ($\eta_1$ is an arbitrary constant and $H$ is the Hubble parameter), Eqs. (12) and (13) yield the solution

$$X(\eta) = c_1 \left[ 1 + \frac{i}{k(\eta_1 - \eta)} \right] e^{ik(\eta_1 - \eta)} + c_2 \left[ 1 - \frac{i}{k(\eta_1 - \eta)} \right] e^{-ik(\eta_1 - \eta)},$$

$$Y(\eta) = c_1 \left[ 1 + \frac{i}{k(\eta_1 - \eta)} - \frac{1}{k^2(\eta_1 - \eta)^2} \right] e^{ik(\eta_1 - \eta)} + c_2 \left[ -1 + \frac{i}{k(\eta_1 - \eta)} + \frac{1}{k^2(\eta_1 - \eta)^2} \right] e^{-ik(\eta_1 - \eta)}.$$  

The condition $|\alpha|^2 - |\beta|^2 = 1$ imposes a constraint on the integration constants $c_1$ and $c_2$, namely, $c_1^2 - c_2^2 = 1$. Let us choose $c_1 = 1$ and $c_2 = 0$. Then, at the end of the inflationary period,

$$X(\eta_1) = \left( 1 + \frac{ia(\eta_1)H}{k} \right) e^{ik[\eta(\eta_1)H]},$$

$$Y(\eta_1) = \left( 1 + \frac{ia(\eta_1)H}{k} - \frac{a^2(\eta_1)H^2}{k^2} \right) e^{ik[\eta(\eta_1)H]}.$$  

These expressions will now be used as initial conditions.

Note that a slow-rolling inflaton field, as the one in our model (see next section), does not mimic exactly a cosmological constant and, therefore, the Hubble parameter is not constant during inflation, as assumed above. However, during the slow-roll phase, the Hubble parameter decreases so slowly, that Eqs. (16) and (17), with $H = H(\eta_1)$, constitute a very good approximation.

To finish this section, let us point out that measurements of the cosmic microwave background radiation impose an upper limit on the energy spectrum for angular frequencies corresponding to the present size of the horizon, $h_0^2 \Omega_{GW}(\omega_{\text{max}}, \eta_0) < 2.0 \times 10^{-8}$ for $\omega_{\text{rad}} = 2.5 \times 10^{-8}$ rad/s [11] and $h_0^2 \Omega_{GW}(\omega_{\text{Cas}}, \eta_0) < 0.014$ for $\omega_{\text{Cas}} = 7.5 \times 10^{-6}$ rad/s [12]. For higher angular frequencies, of the order of a few hundred rad/s, an upper limit is available from the Laser Interferometer Gravitational-Wave Observatory (LIGO), namely, $h_0^2 \Omega_{GW}(\omega, \eta_0) < 3.4 \times 10^{-5}$ [13]. Finally, an integral bound can be derived from standard Big Bang nucleosynthesis, $h_0^2 \Omega_{GW}(\omega, \eta_0) \approx 10^{-9}$ rad/s [3, 10].

### III. Parametric Resonance

Assuming that the transition between the inflationary and the radiation eras is dominated by the phenomenon of parametric resonance, the Einstein equations for the transition period are written in the form

$$\left( \frac{a''}{a} \right)^2 = \frac{8\pi G}{3} a^2 \rho_{\text{tot}},$$

$$\frac{a''}{a} = \frac{4\pi G}{3} a^2 (\rho_{\text{tot}} - 3p_{\text{tot}}),$$  

where the total energy density and pressure are given by

$$\rho_{\text{tot}} = \frac{1}{a^2} \left[ \frac{1}{2} \phi'^2 + a^2 V(\phi) + \frac{1}{2} \langle \chi^2 \rangle + \frac{1}{2} a^2 m_x^2 \langle \chi^2 \rangle \right] + \rho_{\text{rad}},$$

$$p_{\text{tot}} = \frac{1}{a^2} \left[ \frac{1}{2} \phi'^2 - a^2 V(\phi) + \frac{1}{2} \langle \chi^2 \rangle - \frac{1}{2} a^2 m_x^2 \langle \chi^2 \rangle \right] - \frac{\Omega_{GW}}{10^6} \chi^2 \langle \chi^2 \rangle + \frac{1}{3} \rho_{\text{rad}},$$  

where $V(\phi)$ represents the quadratic and quartic potentials, and $g^2(\chi^2)\phi^2/2$ the coupling between the two fields $\phi$ and $\chi$.

The equations for the inflaton and the relativistic radiation fluid are, respectively,

$$\phi'' + 2\frac{a'}{a} \phi' + a^2 V(\phi) + a^2 g^2(\chi^2) \phi = -a \Gamma \phi'$$  

and

$$\rho'_{\text{rad}} + \frac{4\pi G}{3} \rho_{\text{rad}} = \frac{1}{a^2} \Gamma_{\phi} \phi'^2 + \frac{1}{a^2} \Gamma_{\chi} \langle \chi^2 \rangle.$$  

Note that when terms involving $\chi^2$ appear in homogeneous equations, like the ones above, the ensemble average

$$\langle \chi^2 \rangle = \frac{1}{(2\pi)^3} \int d^3k \chi_k^* \chi_k$$  

should be used [14]. The same applies to terms involving $\phi^2$. 

Doppler tracking of the Cassini spacecraft also provide bounds on the gravitational-wave energy spectrum, respectively, $h_0^2 \Omega_{GW}(\omega_{\text{max}}, \eta_0) < 2.0 \times 10^{-8}$ for $\omega_{\text{rad}} = 2.5 \times 10^{-8}$ rad/s [11] and $h_0^2 \Omega_{GW}(\omega_{\text{Cas}}, \eta_0) < 0.014$ for $\omega_{\text{Cas}} = 7.5 \times 10^{-6}$ rad/s [12]. For higher angular frequencies, of the order of a few hundred rad/s, an upper limit is available from the Laser Interferometer Gravitational-Wave Observatory (LIGO), namely, $h_0^2 \Omega_{GW}(\omega, \eta_0) < 3.4 \times 10^{-5}$ [13]. Finally, an integral bound can be derived from standard Big Bang nucleosynthesis, $h_0^2 \Omega_{GW}(\omega, \eta_0) \approx 10^{-9}$ rad/s [3, 10].
The non-homogeneous equation for the field \( \chi \) is written in the \( \kappa \)-component form, for the mode functions \( \chi_\kappa \)

\[
\chi'' + \frac{2\alpha'}{a} \chi' + \left( \kappa^2 + a^2 m_\chi^2 + a^2 g^2 \phi^2 \right) \chi_\kappa = -a \Gamma \chi_\kappa \).
\]

Finally, the above set of equations is complemented with the equations describing the evolution of the universe from the end of the transition period till the present time, namely,

\[
\frac{a''}{a} = \frac{4\pi G}{3} a^2 \left[ \rho_{\text{mat}}(\eta_0) \left( \frac{a(\eta_0)}{a} \right)^3 + (3w + 1) \rho_{\text{de}}(\eta_0) \left( \frac{a(\eta_0)}{a} \right)^{3(1-w)} \right],
\]

where the dark energy we take \( w = 0.78 \) and the density of radiation, matter and dark energy at the present time are, respectively, \( \rho_{\text{rad}}(\eta_0) = 4.6 \times 10^{-31} \text{ kg/m}^3 \), \( \rho_{\text{mat}}(\eta_0) = 2.6 \times 10^{-27} \text{ kg/m}^3 \) and \( \rho_{\text{de}}(\eta_0) = 6.9 \times 10^{-27} \text{ kg/m}^3 \).

The above set of equations (13)–(26), together with Eqs. (12) and (13) for the Bogoliubov coefficients, are solved numerically in order to obtain the energy spectrum of the gravitational waves.

### IV. NUMERICAL SIMULATIONS

Our numerical simulations begin at the present redshift and continue up to the present epoch. We use a Runge-Kutta method with variable step to solve the system of differential equations (14)–(26). Equations (15) and (27) are used to check the accuracy of the numerical solution. The integral in Eq. (24) is computed using the Simpson rule, with a cut-off at \( \kappa = 2\pi a H \) and a interval of integration divided in 20 segments of equal length \( \delta = \pi/10 a'/a \). Having determined the time evolution of \( a'/a \), we then solve numerically Eqs. (12) and (13) (again with a Runge-Kutta method with variable step) for different values of \( \omega = k/a \), compute \( \beta \) with Eq. (11) and, finally, obtain \( \Omega_{\text{GW}}(\omega, \eta_0) \) from Eq. (7).

The energy spectrum of the gravitational waves, \( \Omega_{\text{GW}}(\omega, \eta_0) \), is computed for values of \( \omega \) in the interval \( \omega_{\text{min}} \leq \omega \leq \omega_{\text{max}} \). The lower limit corresponds to a gravitational wave with a wavelength equal, today, to the Hubble distance: \( \omega_{\text{min}} = 2\pi/d_{\text{Hubble}}(\eta_0) \approx 2\pi H(\eta_0) = 1.4 \times 10^{-17} \text{ rad/s} \), where the Hubble parameter at the present time was taken to be \( H(\eta_0) = 71 \text{ km s}^{-1}\text{Mpc}^{-1} \). The upper limit corresponds to a gravitational wave which, at the beginning of the transition period between the inflationary and radiation-dominated eras, had a wavelength equal to the Hubble distance at that time; because of red-shifting, the angular frequency of this gravitational wave is, today, \( \omega_{\text{max}} \approx 2\pi H(\eta_0) \phi(\eta_0)/a(\eta_0) \), where \( \eta_0 \) is the conformal time at the beginning of the transition period. Within the model we are considering, the maximum angular frequency is about \( (10^9 - 10^{10}) \text{ rad/s} \).

There are two more characteristic values of \( \omega \) in the gravitational-wave energy spectrum, corresponding to gravitational waves which had a wavelength equal to the Hubble distance at the moments \( \eta_{\text{infl}} \rightarrow \text{rad} \) and \( \eta_{\text{rad-mat}} \), these being, respectively, the time when the energy density of the inflaton becomes equal to the energy density of radiation (marking the end of the transition period between the inflationary and radiation-dominated eras) and the time when the energy density of radiation becomes equal to the energy density of matter (corresponding to the transition from a radiation-dominated to a matter-dominated universe). These angular frequencies are, today, \( \omega_{\text{infl}} \rightarrow \text{rad} \approx 10^7 \text{ rad/s} \) and \( \omega_{\text{rad-mat}} \approx 10^{-15} \text{ rad/s} \).

The gravitational-wave energy spectrum is then naturally divided in three regions. The first, from \( \omega_{\text{min}} \rightarrow \omega_{\text{rad-mat}} \), marked by a sudden decrease of \( \Omega_{\text{GW}} \), the second, from \( \omega_{\text{rad-mat}} \rightarrow \omega_{\text{infl}} \rightarrow \text{rad} \), where \( \Omega_{\text{GW}} \) is constant (corresponding to a radiation-dominated universe, in which there is no production of gravitational waves), and the third, from \( \omega_{\text{infl}} \rightarrow \text{rad} \rightarrow \omega_{\text{max}} \) where \( \Omega_{\text{GW}} \) has some complex structure (peaks and valleys), which depends on the details of the transition from the inflationary period to the radiation-dominated era.

Our investigation is carried out for two different potential of the inflaton field \( \phi \), namely, \( V(\phi) = m_\phi^2 \phi^2/2 \) and \( V(\phi) = \lambda \phi^4/4 \). The values of the parameters \( m_\phi \) and \( \lambda \) are chosen such that the bounds from the temperature anisotropy of the cosmic microwave background radiation (CMBR) and from large-scale structure (LSS) are satisfied.

\[
m_\phi = \frac{\sqrt{3\pi P_x(k_c)}}{2N(k_c) + 1} m_{\text{pl}} \tag{28}
\]

and

\[
\lambda = \frac{3\pi^2 P_x(k_c)}{2N(k_c) + 1}^3 \tag{29}
\]

In the previous expressions, \( P_x(k) \) is the power spectrum for density perturbations and \( N(k) \) is the number of e-folds of expansion between the time when a co-moving distance scale labelled by \( k \) exited the horizon during inflation and the end of inflation, both \( P_x(k) \) and \( N(k) \) being evaluated at the CMBR/LSS scale \( k_c = 0.05 \text{ Mpc}^{-1} \). For \( 47 \leq N(k_c) \leq 62 \) and \( P_x(k_c) = (2.45 \pm 0.23) \times 10^{-9} \), we obtain the constraints \( 1.2 \times 10^{-6} \leq m_\phi/m_{\text{pl}} \leq 1.7 \times 10^{-6} \) and \( \lambda \leq 3.6 \times 10^{-13} \).

In the slow-roll approximation, inflation ends when the parameter \( \epsilon \equiv \frac{m_{\text{pl}}^2}{(16\pi)} |(\partial\phi/V)^2| \approx 1 \), implying that \( \phi(\eta_1) \approx m_{\text{pl}}/(2\sqrt{\pi}) \) for the case \( V(\phi) = m_\phi^2 \phi^2/2 \) and
\( \phi(\eta_i) \approx m_{\text{pl}}/\sqrt{\rho_i} \) for the case \( V(\phi) = \lambda \phi^4/4 \). The initial value of \( \phi' \) is chosen such that \( \phi^2(\eta_i)/[2a^2(\eta_i)] \lesssim V[\phi(\eta_i)] \). We also choose \( \chi(\eta_i) \) and \( \chi'(\eta_i) \), as well as \( m_\chi \) and \( g \), such that \( \rho_{\text{tot}} \) [see Eq. (20)] is dominated by the potential and kinetic terms of the inflaton field. Since any pre-existing radiation fluid is dilute during inflation, we choose \( \rho_{\text{rad}}(\eta_i) = 0 \). Finally, the decay constants \( \Gamma_\phi \) and \( \Gamma_\chi \) are chosen to be small enough to allow a significant growth of the resonant field \( \chi \) before the evolution of the universe becomes dominated by radiation.

A. Case \( V(\phi) = \frac{1}{2}m_{\phi}^2\phi^2 \)

The masses of the fields \( \phi \) and \( \chi \), the coupling constant \( g \) and the decay constants \( \Gamma_\phi \) and \( \Gamma_\chi \) are chosen to take the following values: \( m_\phi = 1.5 \times 10^{-6} m_{\text{pl}} \), \( m_\chi = 10^{-3} m_{\text{pl}} \), \( 0 \leq g \leq 10^{-2} \), \( \Gamma_\phi = \Gamma_\chi = 5 \times 10^{-10} m_{\text{pl}} \).

As initial conditions, at the beginning of the transition period, we choose \( a(\eta_i) = 1 \), \( \phi(\eta_i) = 0.3 m_{\text{pl}} \), \( \phi'(\eta_i) = -4.5 \times 10^{-7} m_{\text{pl}}^2 \), \( \rho(\eta_i) = 0 \), \( \chi(\eta_i) = 10^{-25} \) and \( \chi'(\eta_i) = 0 \); the constraint equation (18) gives then \( a'(\eta_i) \approx 1.30 \times 10^{-6} m_{\text{pl}} \).

The relevant parameter to consider is the resonance parameter, defined as \( q \equiv g^2\phi^2/(4m_{\phi,\text{eff}}^2) \), with the effective mass of the inflaton given by \( m_{\phi,\text{eff}}^2 = m_{\phi}^2 + g^2(\chi^2) \). In order to achieve a significant resonance we need values of \( q > 1 \), the so-called broad resonance regime.

Our numerical simulations show that the energy density of the field \( \chi \), \( \rho_\chi = (\chi^2)/(2a^2) + m_\chi^2(\chi^2)/2 \), increases by many orders of magnitude in a short time interval, reaching values comparable to the values of the energy density of the inflaton field, \( \rho_\phi = \phi^2/(2a^2) + m_\phi^2\phi^2/2 \). In Fig. 1 we can see that jump in the value of \( \rho_\chi \). However, when switching off the resonant field \( \chi \), the most relevant change occurring in the spectrum \( \Omega_{\text{GW}} \) is an overall translation upwards or downwards (see Fig. 2).

FIG. 1: (Color online) The jump in the value of \( \rho_\chi \). After a short time interval, the energy density of the field \( \chi \) (lower/blue curve) becomes comparable to the energy density of the inflaton (upper/red curve). Both curves refer to case with \( g = 6.5 \times 10^{-3} \).

FIG. 2: (Color online) Gravitational-wave spectra for the model with \( V(\phi) = m_\phi^2\phi^2/2 \). The effect of switching on the resonant field \( \chi \) is an overall translation upwards or downwards. Upper/red curve refers to the case with \( g = 6.5 \times 10^{-3} \), middle/blue curve to \( g = 0 \) (field \( \chi \) switched off) and lower/green curve to \( g = 10^{-3} \).

FIG. 3: The change in time of the effective mass of the inflaton. Its value increases sharply, when the energy density of the resonant field \( \chi \) becomes comparable to the energy density of the field \( \phi \).

No explicit and distinctive signal left by the resonance can be read from \( \Omega_{\text{GW}} \). At first we assumed that this was due to the fact that \( \rho_\chi \) did not dominate over \( \rho_\phi \) and \( \rho_{\text{rad}} \) for a sufficiently long period of time, and that, by increasing \( g \) (and, therefore, the resonant parameter \( q \)), this could be solved. This does not happen and, from Figs. 3 and 4, we may understand why: when \( \langle \chi^2 \rangle \) begins to increase, the same happens to the effective mass of the inflaton, due to the presence of the term \( g^2(\chi^2) \); as a consequence, the value of \( q \) decreases and the resonance is killed.

As expected, the transition between the inflationary period and the radiation-dominated era leaves its imprint in the gravitational-wave spectrum \( \Omega_{\text{GW}} \) at frequencies of the order of MHz/GHz. However, this imprint is due to the form of the potential of the inflaton field and to the values of the parameters \( m_\phi \) and \( \Gamma_\phi \); the presence...
of the resonant field $\chi$ only induces an overall upward or downward translation of the gravitational-wave spectrum. This upward or downward translation depends on the values of the mass of the resonant field $m_\chi$, the coupling constant $q$ and the decay constant $\Gamma_\chi$. In Fig. 5 we compare the energy spectrum in the presence of the resonant field $\chi$ (for $0 < q \leq 10^{-2}$) with the energy spectrum in the absence of this field. The comparison is made at the flat part of the spectrum, namely, at $\omega = 10^2$ rad/s. As clearly seen in Fig. 5, the spectrum tends to move downwards for small values of $g$, while for large values of $g$ it tends to move upwards.

The results presented above indicate that, as far as the creation of gravitons due to quantum fluctuations of the vacuum is concerned, the main features in the spectrum of the gravitational waves ΩGW are due to the inflaton field. This is different from what happens when we consider gravitational waves directly produced by the motion of matter. In that case, the time-dependent inhomogeneities in the distribution of matter, produced by the rapid growth of the resonant field $\chi$, leave a much stronger imprint in the energy spectrum of the gravitational waves $\Omega_{GW}$ at frequencies of the order of MHz/GHz.

B. Case $V(\phi) = \frac{1}{4}\lambda\phi^4$

The parameters are chosen to take the following values: $\lambda = 3.5 \times 10^{-13}$, $m_\chi = 10^{-9}$ $m_{pl}$, $0 \leq g \leq 2.8 \times 10^{-5}$, $\Gamma_\phi = \Gamma_\chi = 3 \times 10^{-12}$. As initial conditions, at the beginning of the transition period, we choose $a(\eta_i) = 1$, $\phi(\eta_i) = 0.6 m_{pl}$, $\dot{\phi}(\eta_i) = -1.5 \times 10^{-7} m_{pl}^2$, $\rho(\eta_i) = 0$, $\lambda_\chi(\eta_i) = 10^{-25}$ and $\chi^\prime_i(\eta_i) = 0$; the constraint equation gives then $a'(\eta_i) \approx 4.35 \times 10^{-7} m_{pl}$.

In Fig. 6 we show two spectra, one with and the other without the field $\chi$. The same conclusions can be drawn as presented in the previous subsection: no obvious imprint of the resonance of the field $\chi$ in the energy spectrum of the gravitational waves, apart from the overall displacement.

In Fig. 7 we show the bands of resonance which, in this case, are much better marked due to the fact that $q$ is now the ratio between constant parameters, $q \equiv g^2/(4\lambda)$.

If we compare Figs. 2 and 6, something interesting can be observed. While in Fig. 6 we see the oscillations in the MHz/GHz region to be around the value defined by the flat part of the spectrum, in Fig. 2 there is first a pronounced decrease, followed then by the usual oscillations. This can be qualitatively understood as follows. Given an homogeneous scalar field with a potential $V(\phi) \propto \phi^n$, oscillating rapidly relatively to the expansion rate of the universe, for $n = 2$ the energy density of the scalar-field oscillations behaves like non-relativistic
the coupling constant $g$ and the decay constant $\Gamma_\chi$, is quite modest. Consequently, as far as the production of gravitons due to quantum fluctuations of the vacuum is concerned, the main features in the spectrum are due to the inflaton field, which leaves a characteristic imprint at frequencies of the order of MHz/GHz. For the models of chaotic inflation we have considered (quadratic and quartic potentials), the relative energy density of the gravitational waves in this frequency range is quite small, $\Omega_{GW} \lesssim 10^{-17}$. Therefore, our calculations show that, by far, the main contribution to the production of gravitons in the MHz/GHz frequency range, for this type of models, does come from the direct coupling of the anisotropic stress tensor, describing the motion of particles during preheating, as investigated in Ref. [4].

The MHz/GHz region of the spectrum lies far beyond the range of frequencies accessible to ground- or space-based interferometer detectors, as well as to resonant bars and spheres (spanning roughly from $10^{-4}$ to $10^4$ Hz). In recent years, considerable effort has been made in order to develop high-frequency gravitational wave detectors [18]. In a near future, these detectors may reach sensitivities high enough to explore this region of the gravitational-wave spectrum.

Below frequencies of the order of MHz, the influence of the transition regime between the inflationary period and the radiation-dominated era should not be felt anymore and, unless strong anisotropies develop later on, the main source of gravitational waves will then be the quantum fluctuations of the vacuum. However, the level of the flat part of the spectrum is mainly fixed by the parameters defining the kinetic and potential terms of the inflaton field, which are known to obey severe constraints from the observations. What our calculation indicate is that the two popular models of inflation we have used will not be able to generate enough gravitational waves to be seen by the interferometer detectors LIGO, Virgo and LISA or in the cosmic microwave background radiation.

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