Control of a Quadrotor Equipped with a Fixed-wing by Tilting Some of Four Rotors

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Abstract—Unmanned aerial vehicles (UAVs) are being expected to be used for the vegetational observation and the information collection of disaster sites. Especially, rotorcrafts typified by helicopters are attractive, because they are able to hover and achieve vertical take-off and landing (VTOL). However, rotorcrafts have a disadvantage that it cannot have a long-distance flight, because they fly by the thrust of upward direction. Aircrafts with tilt rotors are developed in order to overcome such disadvantages. Such aircrafts can be hovering and take a VTOL and also a long-distance flight by changing the angle of the rotor. In this research, it is aimed at proposing a VTOL-type UAV with a fixed-wing and four tiltable rotors and controlling it.

Index Terms—Tilt rotors, Fixed-wing, Dynamical model, VTOL-type UAV

I. INTRODUCTION

Unmanned aerial vehicles (UAVs) are being expected to be used for the vegetational observation and the information collection of disaster sites [1]. Especially, rotorcrafts typified by helicopters are attractive, because they are able to hover and achieve vertical take-off and landing (VTOL) [2]. This feature facilitates the operation of the UAVs. However, rotorcrafts have a disadvantage that it cannot have a long-distance flight, because they fly by the thrust of upward direction. Aircrafts with tilt rotors are developed in order to overcome such disadvantages. Such aircrafts can be hovering and take a VTOL and also a long-distance flight by changing the angle of the rotor [3] [4] [5]. In other words, aircrafts with tilt rotors have several advantages, compared to both rotorcrafts and fixed-wing aircrafts.

In this research, it aims to propose a VTOL-type UAV with a fixed-wing and four tiltable rotors and to control it. Four tiltable rotors are placed on the front and rear of the airframe and the both sides of a fixed wing. When this UAV takes a hovering and a VTOL, each rotor is upward like a quadrotor. When the UAV shifts to a level flight, the thrust generated from the front and rear rotors maintains the airframe altitude, so that this UAV can move forward by obtaining driving force from the right and left rotor thrust. Since this type of UAV with a fixed wing can obtain the lift force by moving forward, it is possible of achieving an efficient level flight, compared to conventional quadrotor-like UAVs. Thus, the present objective is to perform the position and attitude control by switching a rotorcraft mode, which is able to take a VTOL and hovering, and a fixed-wing aircraft mode, which is able to have a fast and long-distance flight. Thus, this paper proposes a VTOL-type UAV with a fixed-wing and four tiltable rotors. A controller is designed to control a position and attitude by changing tilt angle of rotors placed on the both side of fixed-wing. The effectiveness of the control method is verified by some simulations.

II. OVERVIEW OF THE UAV WITH FOUR TILTABLE ROTORS AND A FIXED-WING

A. Structure of a UAV with Four Tiltable Rotors and a Fixed-wing

The UAV with four tiltable rotors and a fixed-wing proposed in this research is shown in Fig. 1. This airframe has a fixed wing and four tiltable rotors, where each rotor can be tilted back and forward against the airframe. Fig. 2 shows the case that the airframe is viewed from the plus y-axis, where each rotor is tilted backward. The angle of each rotor is referred to from a vertically upward axis. Note here that the rotational direction of the rotor 2 and the rotor 4 is mutually reverse to cancel a counter torque generated by their revolutions during a level flight.
B. Definition of the UAV with a Fixed-wing and Tiltable Rotor

The definition of coordinates is shown in fig. 2, and the robot coordinate C is defined such that the origin is the center of the airframe, positive x-axis is set as the forward direction of the airframe, positive y-axis is set as the right direction of the airframe, and positive z-axis is set to be downward perpendicular to the airframe. Similarly, the world coordinate E is a right-handed coordinate where positive Z-axis is set to vertically downward. The center position of the airframe is represented by \( \mathbf{z} = [x, y, z]^T \) in the world coordinate, and the rotational angles for roll, pitch and yaw in the robot coordinate system are represented as \( \phi, \theta, \psi \) respectively, then the attitude of the airframe is represented by \( \mathbf{\eta} = [\phi, \theta, \psi]^T \). A rotation matrix \( R \) to transform the robot coordinate to the world coordinate is defined by

\[
R = \begin{bmatrix}
A_1 & A_2 & A_3 \\
A_4 & A_5 & A_6 \\
A_7 & A_8 & A_9
\end{bmatrix}
\]

where \( A_1 \) to \( A_9 \) are denoted by

\[
\begin{align*}
A_1 &= \sin \theta \cos \psi \\
A_2 &= \sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi \\
A_3 &= \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\
A_4 &= \cos \theta \sin \psi \\
A_5 &= \sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi \\
A_6 &= \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\
A_7 &= -\sin \theta \\
A_8 &= \sin \phi \cos \theta \\
A_9 &= \cos \phi \cos \theta
\end{align*}
\]

III. DERIVATION OF DYNAMICAL MODEL

A dynamical model of the UAV with four tiltable rotors and a fixed-wing is derived by the Lagrangian method. Here, the mass of the airframe is \( m \) and the moment of inertia around each axis is represented by \( I_x, I_y \) and \( I_z \) respectively, the lift force and the drag force are \( L \) and \( D \) respectively, and the moment of inertia of the rotor is \( J_r \). Letting \( u_x \) be the input of translational motion of \( x \) direction, \( u_z \) be the input of translational motion of \( z \) direction, \( u_3 \) be the input of roll motion, \( u_3 \) be the input of pitch motion, and \( u_4 \) be the input of yaw motion, the dynamical model of the UAV with a fixed-wing and four tiltable rotors is derived as follows:

\[
\begin{align*}
\dot{x} &= A_1 \frac{u_x}{m} - A_4 \frac{u_z}{m} - A_3 \frac{L}{m} - A_1 \frac{D}{m} \\
\dot{y} &= A_4 \frac{u_x}{m} - A_6 \frac{u_z}{m} - A_5 \frac{L}{m} - A_4 \frac{D}{m} \\
\dot{z} &= g + A_7 \frac{u_x}{m} - A_9 \frac{u_z}{m} - A_8 \frac{L}{m} - A_7 \frac{D}{m} \\
\phi &= (\dot{\psi} I_y - I_z) - J_y \dot{\Theta}_z + u_2) / I_x \\
\dot{\theta} &= (\dot{\psi} I_y - I_z) + J_y (\dot{\Theta}_z - \psi \dot{\Theta}_x + u_3) / I_x \\
\dot{\psi} &= (\dot{\phi} I_x - I_y) + J_x \dot{\Theta}_z + u_4) / I_x
\end{align*}
\]

where \( \Omega_z \) is the sum of x-directional components of rotation of rotors, \( \Omega_z \) is the sum of z-directional components of rotation of rotors, which are calculated by

\[
\begin{align*}
\Omega_x &= \omega_3 \sin \theta_3 + \omega_4 \sin \theta_4 - \omega_3 \sin \theta_2 \\
\Omega_z &= \omega_3 \cos \theta_3 + \omega_4 \cos \theta_4 - \omega_3 \cos \theta_1 - \omega_2 \cos \theta_2
\end{align*}
\]

IV. CONTROLLER FOR THE UAV

A. Controller for the Attitude Angle (\( \phi, \theta \))

In this paper, a PD controller is used for controlling the UAV with four tiltable rotors and a fixed-wing. Letting \( k_1, k_3 \) be P gains, \( k_2, k_4 \) be D gains, and \( \phi, \theta \) be the target values of the roll and pitch angle, a PD controller for the roll and pitch angle is represented by

\[
\begin{align*}
u_x &= -k_1 (\phi - \phi_3) - k_2 \dot{\phi} \\
u_z &= -k_3 (\theta - \theta_3) - k_4 \dot{\theta}
\end{align*}
\]

B. Controller for the Attitude (\( \psi \)) by Tiltable Rotors

A rotational direction of rotors of the UAV with a fixed wing and four tiltable rotors is different from a conventional quadrotor. In other words, it cannot use a conventional control method for \( \psi \) by applying the difference of torques between rotors. Therefore, the present airframe controls \( \psi \) by changing
the tilt angle of rotors placed on the edge of a fixed-wing. However, when the second rotor is tilting backward against the airframe and the fourth rotor is tilting forward, the airframe cannot rotate in a direction of positive \( \psi \). Therefore, the input \( u_{\psi_d} \) required for controlling the attitude of \( \psi \) is calculated by a PD controller. The inputs \( u_{\theta_1} \) and \( u_{\theta_2} \) for controlling the tilt angles and the attitude of \( \psi \) are decided by comparing the direction of \( u_{\psi_d} \) with the current tilt angle \( \theta_i \) \( (i = 2, 4) \). The input \( u_{\psi_d} \) required for \( \psi \) is denoted by

\[
 u_{\psi_d} = -k_3(\psi - \psi_d) - k_6 \dot{\psi}
\]

where \( k_3 \) is a P gain, \( k_6 \) is a D gain and \( \psi_d \) is the target value of \( \psi \). In Fig. 3(a) and (b), since the second and fourth rotors can generate a thrust with the component of \( u_{\psi_d} \) - direction by the current tilt angles \( \theta_i \) \( (i = 2, 4) \), the input of \( \psi \) is \( u_{\psi} = u_{\psi_d} \). However, in Fig. 3(c) and (d), since the second and fourth rotors cannot generate a thrust with the component of \( u_{\psi_d} \) - direction by the current tilt angle, the input of \( \psi \) is \( u_{\psi} = 0 \) and the input for controlling the angle of rotors \( \theta_i \) \( (i = 2, 4) \) is \( u_{\theta_i} = -u_{\psi_d} \). According to the relationship among these variables, a logical switching rule is represented by

Rule 1:
If \( u_{\psi_d} > 0 \) and \( \theta_2 \leq 0 \) and \( \theta_4 \geq 0 \)
then \( u_{\theta_1} = -u_{\psi_d} \), \( u_{\psi} = u_{\psi_d} \)

Rule 2:
If \( u_{\psi_d} \leq 0 \) and \( \theta_2 > 0 \) and \( \theta_4 < 0 \)
then \( u_{\theta_1} = -u_{\psi_d} \), \( u_{\psi} = u_{\psi_d} \)

Rule 3:
If \( u_{\psi_d} \geq 0 \) and \( \theta_2 \geq 0 \) and \( \theta_4 \leq 0 \)
then \( u_{\theta_1} = -u_{\psi_d} \), \( u_{\psi} = 0 \)

Rule 4:
If \( u_{\psi_d} < 0 \) and \( \theta_2 < 0 \) and \( \theta_4 > 0 \)
then \( u_{\theta_1} = -u_{\psi_d} \), \( u_{\psi} = 0 \)

When controlling \( \psi \), the rotors 1 and 3 are fixed such as \( \theta_1 = 0 \) and \( \theta_3 = 0 \).

C. Controller for the Position \((y, z)\)

Position control for the UAV with four tiltable rotors and a fixed-wing is performed by changing or without changing the attitude of the airframe. The UAV can move to x-direction without tilting the airframe to \( \theta \) - direction. However, the moment of y-direction should be performed by tilting the airframe to \( \phi \) - direction. Therefore, the position control of y-direction is performed by changing the target value \( \phi_d \) of a controller for the attitude. The target value of the attitude angle is made from the error of the target position and the current position of the airframe. Letting \( k_7 \) be P gain, \( k_8 \) be D gain of a controller, and \( y_d \) be the target value of y-direction, a PD controller of y-direction is designed by

\[
 \phi_d = -k_7(y - y_d) - k_8 \dot{y}
\]

However, since \( \psi \) is not used for the position control, it results in \( u_{\psi} = 0 \). In this paper, the position control of x-direction is performed by tilting some rotors, keeping \( \theta_2 = 0 \).

Control of the position in z-direction is affected by the gravity, the input of x-direction, the lift force and the drag force. Therefore, the position controller for z-direction is represented by

\[
 u_z = \frac{mg + A_x}{A_y} - \frac{DA_x}{A_y} - m(-k_9(z - z_d) - k_{110}z) / A_y
\]

where \( k_9 \) is a P gain and \( k_{110} \) is a D gain.

D. Controller for the x-directional Position by Tiltable Rotors

![Fig. 4. Relationship among inputs and tilt angles for controlling x-directional position](image)

In this study, controlling position in x-direction is performed by controlling the magnitude of the thrust and the tilt angle of rotors. However, when the second rotor and fourth rotor are tilting backward against the airframe, the airframe cannot move to the direction of positive x. Therefore, the input \( u_{\psi} \) required for controlling the position in x-direction is calculated by a PD controller. The input \( u_{\theta_1} \) and \( u_{\psi} \) for controlling the tilt angle and the position in x-direction are decided by comparing the direction of \( u_{\psi} \) with the current tilt angles \( \theta_i \) \( (i = 2, 4) \). A PD controller \( u_{\psi} \) for x-direction is given by

\[
 u_{\psi} = -k_{1}(x - x_d) - k_{12} \dot{x}
\]

where \( k_{1} \) is a P gain, \( k_{12} \) is a D gain and \( x_d \) is the target value of x-direction. In Fig. 4(a) and (b), since the second and fourth rotors can generate a thrust with the component of \( u_{\psi} \) - direction by the current tilt angle \( \theta_i \) \( (i = 2, 4) \), the input of x-direction is \( u_{\psi} = u_{\psi_d} \). However, in Fig. 4(c) and (d), since the second and fourth rotors cannot generate a thrust with the component of \( u_{\psi} \) - direction by the current tilt angle, the input of x-direction is \( u_{\psi} = 0 \) and the input for controlling the angle of rotors \( \theta_i \) \( (i = 2, 4) \) is \( u_{\theta_i} = -u_{\psi_d} \). According to the relationship among these variables, a logical switching rule is represented as follows:

Rule 1:
If \( u_{ad} > 0 \) and \( \dot{\theta}_i < 0 \) then \( u_{\dot{\theta}_i} = -u_{ad}, \ u_\phi = u_{ad} \)

Rule 2: 
If \( u_{ad} < 0 \) and \( \dot{\theta}_i > 0 \) then \( u_{\dot{\theta}_i} = -u_{ad}, \ u_\phi = u_{ad} \)

Rule 3: 
If \( u_{ad} > 0 \) and \( \dot{\theta}_i \geq 0 \) then \( u_{\dot{\theta}_i} = -u_{ad}, \ u_\phi = 0 \)

Rule 4: 
If \( u_{ad} < 0 \) and \( \dot{\theta}_i \leq 0 \) then \( u_{\dot{\theta}_i} = u_{ad}, \ u_\phi = 0 \)

Here, the tilt angles of the rotor 1 and 3 are fixed such as \( \dot{\theta}_1 = 0 \) and \( \dot{\theta}_3 = 0 \).

V. SIMULATIONS

A. Controlling for x-directional Position by Tiltable Rotors

| TABLE I. PARAMETERS OF THE UAV WITH A FIXED-WING AND FOUR TILTABLE ROTORS |
| Parameter | Description | Value | Unit |
|-----------|-------------|-------|------|
| \( m \)   | Mass        | 0.8   | kg   |
| \( l \)   | Distance    | 0.4   | m    |
| \( I_x \) | Roll inertia| 0.3   | kg⋅m^2|
| \( I_y \) | Pitch inertia| 0.2  | kg⋅m^2|
| \( I_z \) | Yaw inertia  | 0.4   | kg⋅m^2|

This simulation is intended to verify that the x-directional position of the airframe is controlled by controlling the magnitude of the thrust and tilt angle, keeping a constant height. The physical parameters used for the simulation are shown in Table 1. The initial state of the UAV with four tiltable rotors and a fixed-wing is \( q_0^T = [x(0) \ y(0) \ z(0) \ \phi(0) \ \theta(0) \ \psi(0)] = [0 \ 0 \ 0 \ 0 \ 0 \ 0] \) and the goal state is \( q^T_r = [x_r \ y_r \ z_r \ \phi_r \ \theta_r \ \psi_r] = [10 \ 0 \ -1 \ 0 \ 0 \ 0] \). The feedback gains are decided such as \( k_1 = 4.0, k_2 = 1.0, k_3 = 4.0, k_4 = 1.0, k_5 = 1.2, k_6 = 0.5, k_7 = 0.04, k_8 = 0.15, k_9 = 0.6, k_{10} = 1.0, k_{11} = 0.05 \) and \( k_{12} = 0.5 \), through trials and errors.

It is found from Fig. 5 that the position, i.e., the states \( x, y \) and \( z \) converge from the initial position to the goal position. Then, the attitudes \( \phi, \theta \) and \( \psi \) are not changed as shown in Fig. 6. Fig. 7 shows that the tilt angles for the second and fourth rotors changed so as to approach the desired position of x-axis, keeping a desired height (i.e. 1 m).

B. Controlling of Yaw Angle by Tiltable Rotors

Fig. 5. Positional result when controlling x-direction

Fig. 6. Attitude result when controlling x-direction

Fig. 7. Tilt angle result when controlling x-direction

Fig. 8. Positional result when controlling the yaw angle
It is found from Fig. 8 that the position $z$ converges from the initial position to the goal position, keeping $x$ and $y$ at the initial state. Then, the attitude $\psi$ converges from the initial attitude to the goal attitude, while $\phi$ and $\theta$ are not changed as shown in Fig. 9. Fig. 10 shows that the tilt angles for the second and fourth rotors change so as to approach the desired attitude of $\psi$.

VI. CONCLUSION

In this paper, a VTOL-type UAV that has a fixed-wing and four tiltable rotors has been introduced. A PD controller was designed to control the position and attitude by controlling tiltable rotors. It was found from some simulations that the airframe position in X-direction and attitude of $\psi$ was controlled. For future work, it needs to design a controller for a level flight and verify the effectiveness of such a controller, together with the present controller, through experiments for an actual machine.

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