RARE KAON DECAYS

A. Pich

Departament de Física Teòrica, IFIC, Universitat de València – CSIC
E-46100 Burjassot, València, Spain

Abstract

Rare K decays are an important testing ground of the electroweak flavour theory. They can provide new signals of CP–violation phenomena and, perhaps, a window into physics beyond the Standard Model. The interplay of long-distance QCD effects in strangeness-changing transitions can be analyzed with Chiral Perturbation Theory techniques. Some theoretical predictions obtained within this framework for radiative kaon decays are reviewed, together with the present experimental status.

*Invited Talk at the Workshop on K Physics, Orsay, France, May 30 – June 4, 1996
1 Introduction

High-precision experiments on rare kaon decays offer the exciting possibility of unravelling new physics beyond the Standard Model. Searching for forbidden flavour-changing processes at the $10^{-10}$ level $[\text{Br}(K_L \to \mu e) < 3.3 \times 10^{-11}$, $\text{Br}(K_L \to \pi^0 \mu e) < 3.2 \times 10^{-9}$, $\text{Br}(K^+ \to \pi^+ \mu e) < 2.1 \times 10^{-10}, \ldots]$], one is actually exploring energy scales above the 10 TeV region. The study of allowed (but highly suppressed) decay modes provides, at the same time, very interesting tests of the Standard Model itself. Electromagnetic-induced non-leptonic weak transitions and higher-order weak processes are a useful tool to improve our understanding of the interplay among electromagnetic, weak and strong interactions. In addition, new signals of CP violation, which would help to elucidate the source of CP-violating phenomena, can be looked for.

Since the kaon mass is a very low energy scale, the theoretical analysis of non-leptonic kaon decays is highly non-trivial. While the underlying flavour-changing weak transitions among the constituent quarks are associated with the $W$-mass scale, the corresponding hadronic amplitudes are governed by the long-distance behaviour of the strong interactions, i.e. the confinement regime of QCD.

The standard short-distance approach to weak transitions makes use of the asymptotic freedom property of QCD to successively integrate out the fields with heavy masses down to scales $\mu < m_c$. Using the operator product expansion (OPE) and renormalization-group techniques, one gets an effective $\Delta S = 1$ hamiltonian,

$$H_{\Delta S=1}^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i + \text{h.c.},$$

which is a sum of local four-fermion operators $Q_i$, constructed with the light degrees of freedom ($u, d, s; e, \mu, \nu$), modulated by Wilson coefficients $C_i(\mu)$ which are functions of the heavy ($W, t, b, c, \tau$) masses. The overall renormalization scale $\mu$ separates the short- ($M > \mu$) and long- ($m < \mu$) distance contributions, which are contained in $C_i(\mu)$ and $Q_i$, respectively. The physical amplitudes are of course independent of $\mu$; thus, the explicit scale (and scheme) dependence of the Wilson coefficients, should cancel exactly with the corresponding dependence of the $Q_i$ matrix elements between on-shell states.

Our knowledge of the $\Delta S = 1$ effective hamiltonian has improved considerably in recent years, thanks to the completion of the next-to-leading logarithmic order calculation of the Wilson coefficients. All gluonic corrections of $O(\alpha_s^n t^n)$ and $O(\alpha_s^{n+1} t^n)$ are already known, where $t \equiv \log (M/m)$ refers to the logarithm of any ratio of heavy-mass scales ($M, m \geq \mu$). Moreover, the full $m_t/M_W$ dependence (at lowest order in $\alpha_s$) has been taken into account.

Unfortunately, in order to predict the physical amplitudes one is still confronted with the calculation of the hadronic matrix elements of the quark operators. This is a very difficult problem, which so far remains unsolved. We have only been able to obtain rough estimates using different approximations (vacuum saturation, $N_C \to \infty$ limit, QCD low-energy effective action, . . . ) or applying QCD techniques (lattice, QCD sum rules) which suffer from their own technical limitations.

Below the resonance region ($\mu < M_\rho$) the strong interaction dynamics can be better understood with global symmetry considerations. We can take advantage of the fact that the pseudoscalar mesons are the lowest energy modes of the hadronic spectrum: they correspond to the octet of Goldstone bosons associated with the dynamical chiral symmetry breaking of QCD, $SU(3)_L \otimes SU(3)_R \to SU(3)_V$. The low-energy implications of the QCD symmetries can then be worked out through an effective lagrangian containing only the Goldstone modes. The effective chiral perturbation theory (ChPT) formulation of the Standard Model is an ideal framework to describe kaon decays. This is because in K decays the only physical states which
appear are pseudoscalar mesons, photons and leptons, and because the characteristic momenta involved are small compared to the natural scale of chiral symmetry breaking ($\Lambda_\chi \sim 1$ GeV).

Fig. 1 shows a schematic view of the procedure used to evolve down from $M_W$ to the kaon mass scale. At the different energy regimes one uses different effective theories, involving only those fields which are relevant at that scale. The corresponding effective parameters (Wilson coefficients, chiral couplings) encode the information on the heavy degrees of freedom which have been integrated out. These effective theories are convenient realizations of the fundamental Standard Model at a given energy scale (all of them give rise to the same generating functional and therefore to identical predictions for physical quantities). From a technical point of view, we know how to compute the effective hamiltonian at the charm–mass scale. Much more difficult seems the attempt to derive the chiral lagrangian from first principles. The symmetry considerations only fix the allowed chiral structures, at a given order in momenta, but leave their corresponding coefficients completely undetermined. The calculation of the chiral couplings from the effective short–distance hamiltonian, remains the main open problem in kaon physics.

2 Chiral Perturbation Theory

In the absence of quark masses, the QCD lagrangian is invariant under independent $SU(N_f)$ transformations of the left– and right–handed quarks in flavour space $[q_L \rightarrow g_L q_L, q_R \rightarrow g_R q_R, g_{L,R} \in SU(N_f)_{L,R}]$. This $SU(N_f)_L \otimes SU(N_f)_R$ chiral symmetry, which should be approximately good in the light quark sector (u,d,s), is however not seen in the hadronic spectrum: although hadrons can be nicely classified in $SU(3)_V$ representations, degenerate multiplets with opposite parity do not exist. To be consistent with this experimental fact, the ground state of the theory (the vacuum) should not be symmetric under the chiral group. The $SU(3)_L \otimes SU(3)_R$ symmetry spontaneously breaks down to $SU(3)_{L+R}$ and, according to Goldstone’s theorem, an octet of pseudoscalar massless bosons appears in the theory. The eight lightest hadronic states
The quark mass matrix \( L \equiv \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^- & K^+ \\ \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 & K^0 \\ -\frac{1}{\sqrt{6}} \eta_8 & -\frac{1}{\sqrt{6}} \eta_8 & \eta_8 \end{pmatrix} \).

\[ L \equiv \begin{pmatrix} \lambda & \pi^0 & \pi^- \\ \pi^+ & K^0 & K^0 \\ K^- & K^- & \eta_8 \end{pmatrix} \] \tag{2}

\[ U^{ij}(\phi) \] parametrizes the Goldstone excitations over the vacuum quark condensate \( \langle \bar{q}_L q_R \rangle \). Under the chiral group, it transforms as \( U \rightarrow g_R U g_L^* \).

To get a low-energy effective lagrangian realization of QCD, for the light–quark sector, we should write the most general lagrangian involving the matrix \( U(\phi) \), which is consistent with chiral symmetry. The lagrangian can be organized in terms of increasing powers of momentum \( \eta_6 \). At lowest order in the number of derivatives:

\[ \mathcal{L}_{\text{eff}}(U) = \sum_n \mathcal{L}_{2n} \] \tag{3}

In the low–energy domain we are interested in, the terms with a minimum number of derivatives will dominate.

The lowest–dimensional effective chiral lagrangian is uniquely given by

\[ \mathcal{L}_2 = \frac{f^2}{4} \left( \langle D_\mu U D^\mu U^\dagger \rangle + 2B_0 \langle \mathcal{M} U^\dagger + U \mathcal{M} \rangle \right), \] \tag{4}

where \( f \approx f_\pi = 92.4 \text{ MeV} \) is the pion decay constant (to lowest order), \( \langle \cdot \rangle \) denotes the trace of the corresponding matrix and the covariant derivative

\[ D_\mu U \equiv \partial_\mu U - i r_\mu U + i U l_\mu, \] \tag{5}

accounts for the coupling to electromagnetism (and the weak gauge bosons),

\[ r_\mu \equiv v_\mu + a_\mu = eQ A_\mu + \cdots \quad l_\mu \equiv v_\mu - a_\mu = eQ A_\mu + \cdots \] \tag{6}

with the charge matrix \( Q = \frac{1}{3} \text{diag}(2, -1, -1) \).

The second term in \( \mathcal{L}_2 \) is an explicit breaking of chiral symmetry due to the presence of the quark mass matrix \( \mathcal{M} = \text{diag}(m_u, m_d, m_s) \) in the QCD lagrangian. The parameter \( B_0 \) (\( \simeq - < \bar{u}u > / f^2 \)) relates the squares of the pseudoscalar meson masses to the quark masses,

\[ B_0 = \frac{M_{2\pi}^2}{m_u + m_d} = \frac{M_{2K}^2}{m_u + m_s} = \frac{M_{2K}^2}{m_d + m_s}. \] \tag{7}

The effect of strangeness-changing non-leptonic weak interactions with \( \Delta S = 1 \) is incorporated as a perturbation to the strong effective lagrangian \( \mathcal{L}_{\text{eff}} \). At lowest order in the number of derivatives, the most general effective bosonic lagrangian, with the same \( SU(3)_L \otimes SU(3)_R \) transformation properties as the short–distance hamiltonian \( \mathcal{H} \), contains two terms:

\[ \mathcal{L}_{\text{2}}^{\Delta S = 1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ g_8 \langle \lambda \bar{L}_\mu L^\mu \rangle + g_{27} \left( L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) \right\}, \] \tag{8}
where the matrix $L_\mu = i f^2 U^\dagger D_\mu U$ represents the octet of $V-A$ currents, and $\lambda \equiv (\lambda_6 - i \lambda_7)/2$ projects onto the $s \to d$ transition $[\lambda_{ij} = \delta_{3j}\delta_{j2}]$. The chiral couplings $g_8$ and $g_{27}$ measure the strength of the two parts of the effective hamiltonian transforming as $(8_L, 1_R)$ and $(27_L, 1_R)$, respectively, under chiral rotations. Their values can be extracted from $K \to 2\pi$ decays.\[^{14}\]

$$|g_8| \simeq 5.1, \quad |g_{27}/g_8| \simeq 1/18.$$ \hspace{1em} (9)

The huge difference between these two couplings\[^{14}\] shows the well-known enhancement of the octet $|\Delta I| = 1/2$ transitions.

Using the lagrangians (5) and (8), the rates for decays like $K \to 3\pi$ or $K \to \pi\pi\gamma$ can be predicted at $O(p^2)$ through a trivial tree-level calculation. However, the data are already accurate enough for the next-order corrections to be sizeable. Moreover, due to a mismatch between the minimum number of powers of momenta required by gauge invariance and the powers of momenta that the lowest-order effective lagrangian can provide\[^{14}\] the amplitude for any non-leptonic radiative K decay with at most one pion in the final state ($K \to \gamma\gamma, K \to \gamma\ell^+\ell^-$, $K \to \pi\gamma\gamma, K \to \pi\ell^+\ell^-$, . . . ) vanishes to $O(p^2)$. These decays are then sensitive to the non-trivial quantum field theory aspects of ChPT.

At the one-loop level, corresponding to $O(p^4)$, we need to add to the effective lagrangian all possible terms with four powers of momenta, satisfying the symmetry constraints. Each term will introduce an additional coupling constant, not fixed by chiral symmetry. These constants can be seen as remnants of the fundamental theory after quarks and gluons have been integrated out; they contain both long- and short-distance information, and some of them (like $g_8$) have in addition a CP-violating imaginary part. Since the one-loop divergences are reabsorbed by the $O(p^4)$ couplings, these constants will depend, in general, on an arbitrary renormalization scale.

The complete list of $O(p^4)$ terms describing strong and electromagnetic interactions can be found in ref.\[^{14}\], where the numerical values of the corresponding couplings have been determined using experimental information. Two of those terms are relevant for our purposes:

$$\mathcal{L}_{4}^{\text{em}} = -ieL_{10}F^{\mu\nu}\langle QD_{\mu}U D_{\nu}U^\dagger + QD_{\mu}^\dagger U^\dagger D_{\nu}U \rangle + e^2L_{10}F^{\mu\nu}F_{\mu\nu}\langle UQU^\dagger Q \rangle + \cdots \hspace{1em} (10)$$

When combined with the lowest-order $\Delta S = 1$ lagrangian, the couplings (10) give rise to physical contributions to the various Kaon decays we are going to consider here.

Another source of $O(p^4)$ contributions comes from direct $\Delta S = 1$ terms. Although the complete list of possible chiral structures\[^{14}\] is rather long, only a few terms are relevant\[^{14}\] for the kind of processes we are going to discuss (radiative K decays with at most one pion in the final state):

$$\mathcal{L}_{4}^{\Delta S=1,\text{em}} = -\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^* g_8 \left\{ -\frac{ie}{f^2}F^{\mu\nu}\left[ w_1 \langle Q\lambda L_{\mu}L_{\nu} \rangle + w_2 \langle QL_{\mu}\lambda L_{\nu} \rangle \right] 
\quad + e^2f^2w_4 F^{\mu\nu}F_{\mu\nu}\langle \lambda QU^\dagger QU \rangle + \text{h.c.} \right\}. \hspace{1em} (11)$$

### 3 $K \to \pi\nu\bar{\nu}$

The decay $K^+ \to \pi^+\nu\bar{\nu}$ is a well-known example of an allowed process where long-distance effects play a negligible role.\[^{14}\] Thus, this mode provides a good test of the radiative structure of the Standard Model. The decay process is dominated by short-distance loops ($Z$ penguin,
involving the heavy top quark, but receives also sizeable contributions from internal charm–quark exchanges. The resulting decay amplitude,

\[ T(K \to \pi \bar{\nu}) \sim \sum_{i=e,t} F(V_{id}V_{is}^{*} x_i \bar{\nu}_L \gamma_{\mu} \nu_L) \langle \pi | \bar{s}_L \gamma_{\mu} d_L | K \rangle, \quad x_i \equiv m_i^2/M_W^2, \tag{12} \]

involves the hadronic matrix element of the \( \Delta S = 1 \) vector current, which (assuming isospin symmetry) can be obtained from \( K_{l3} \) decays. In the ChPT framework, the needed hadronic matrix element is known at \( \mathcal{O}(p^4) \); this allows to make a reliable estimate of the relevant isospin–violating corrections.\[3,4]\]

Summing over the three neutrino flavours and expressing the quark–mixing factors through the Wolfenstein parameters, one can write the approximate formula:\[5,6\]

\[ \text{Br}(K^+ \to \pi^+ \nu \bar{\nu}) \approx 1.93 \times 10^{-11} A^4 x_1^{1.15} [\eta^2 + (\rho_0 - \rho)^2]; \quad \rho_0 \approx 1.4. \tag{13} \]

The departure of \( \rho_0 \) from unity measures the impact of the charm contribution.

With the presently favoured values for the quark–mixing parameters, the branching ratio is predicted to be in the range:\[6\]

\[ \text{Br}(K^+ \to \pi^+ \nu \bar{\nu}) = (9.1 \pm 3.2) \times 10^{-11}, \tag{14} \]

to be compared with the present experimental upper bound\[28\]

\[ \text{Br}(K^+ \to \pi^+ \nu \bar{\nu}) < 2.4 \times 10^{-9} \quad (90\% \text{ CL}). \]

What is actually measured is the transition \( K^+ \to \pi^+ \pi^0 \) nothing; therefore, the experimental search for this process can also be used to set limits on possible exotic decay modes like \( K^+ \to \pi^+ X^0 \), where \( X^0 \) denotes an undetected light Higgs or Goldstone boson (axion, familon, majoron, . . . ).

The CP–violating decay \( K_L \to \pi^0 \nu \bar{\nu} \) has been suggested\[23\] as a good candidate to look for pure direct CP–violating transitions. The contribution coming from indirect CP violation via \( K^0 \to \bar{K}^0 \) mixing is very small\[6\]

\[ \text{Br}_i \approx 5 \times 10^{-15}. \]

The decay proceeds almost entirely through direct CP violation, and is completely dominated by short–distance loop diagrams with top quark exchanges:\[6\]

\[ \text{Br}(K_L \to \pi^0 \nu \bar{\nu}) \approx 8.07 \times 10^{-11} A^4 \eta^2 x_1^{1.15}. \tag{15} \]

The present experimental upper bound\[4\]

\[ \text{Br}(K_L \to \pi^0 \nu \bar{\nu}) < 5.8 \times 10^{-5} \quad (90\% \text{ CL}), \]

is still far away from the expected range.

\[ \text{Br}(K_L \to \pi^0 \nu \bar{\nu}) = (2.8 \pm 1.7) \times 10^{-11}. \tag{16} \]

Nevertheless, the experimental prospects to reach the required sensitivity in the near future look rather promising.\[6\] The clean observation of just a single unambiguous event would indicate the existence of CP–violating \( \Delta S = 1 \) transitions.

## 4 \( K_S \to \gamma \gamma \)

The symmetry constraints do not allow any direct tree–level \( K_1^0 \gamma \gamma \) coupling at \( \mathcal{O}(p^4) \) (\( K_{1,2}^0 \) refer to the CP–even and CP–odd eigenstates, respectively). This decay proceeds then through a loop of charged pions as shown in Fig. 2 (there are similar diagrams with charged kaons in the loop, but their sum is proportional to \( M_{K^0}^2 - M_{K^+}^2 \) and therefore can be neglected). Since there are no possible counter–terms to renormalize divergences, the one–loop amplitude is necessarily finite. Although each of the four diagrams in Fig. 2 is quadratically divergent, these divergences cancel in the sum. The resulting prediction\[4,12\]

\[ \text{Br}(K_S \to \gamma \gamma) = 2.0 \times 10^{-6}, \]

is in very good agreement with the experimental measurement:\[13,14\]

\[ \text{Br}(K_S \to \gamma \gamma) = (2.4 \pm 0.9) \times 10^{-6}. \tag{17} \]
5 \( K_{L,S} \rightarrow \mu^+\mu^- \)

There are well–known short–distance contributions\( ^3 \) (electroweak penguins and box diagrams) to the decay \( K_L \rightarrow \mu^+\mu^- \). However, this transition is dominated by long–distance physics. The main contribution proceeds through a two–photon intermediate state: \( K_0^0 \rightarrow \gamma^*\gamma^* \rightarrow \mu^+\mu^- \). Contrary to \( K_1^0 \rightarrow \gamma\gamma \), the prediction for the \( K_2^0 \rightarrow \gamma\gamma \) decay is very uncertain, because the first non-zero contribution occurs\( ^\dagger \) at \( O(p^6) \). That makes very difficult any attempt to predict the \( K_L \rightarrow \mu^+\mu^- \) amplitude.

The situation is completely different for the \( K_S \) decay. A straightforward chiral analysis\( ^35 \) shows that, at lowest order in momenta, the only allowed tree–level \( K_0^0 \mu^+\mu^- \) coupling corresponds to the CP–odd state \( K_0^0 \). Therefore, the \( K_0^0 \rightarrow \mu^+\mu^- \) transition can only be generated by a finite non-local two–loop contribution. The explicit calculation\( ^35 \) gives:

\[
\frac{\Gamma(K_S \rightarrow \mu^+\mu^-)}{\Gamma(K_S \rightarrow \gamma\gamma)} = 2 \times 10^{-6}, \quad \frac{\Gamma(K_S \rightarrow e^+e^-)}{\Gamma(K_S \rightarrow \gamma\gamma)} = 8 \times 10^{-9},
\]

well below the present (90% CL) experimental upper limits: \( ^36, 37 \) \( \text{Br}(K_S \rightarrow \mu^+\mu^-) < 3.2 \times 10^{-7} \), \( \text{Br}(K_S \rightarrow e^+e^-) < 2.8 \times 10^{-6} \). Although, in view of the smallness of the predicted ratios, this calculation seems quite academic, it has important implications for CP–violation studies.

The longitudinal muon polarization \( \mathcal{P}_L \) in the decay \( K_L \rightarrow \mu^+\mu^- \) is an interesting measure of CP violation. As for every CP–violating observable in the neutral kaon system, there are in general two different kinds of contributions to \( \mathcal{P}_L \): indirect CP violation through the small \( K_1^0 \) admixture of the \( K_L \) (\( \varepsilon \) effect), and direct CP violation in the \( K_2^0 \rightarrow \mu^+\mu^- \) decay amplitude.

In the Standard Model, the direct CP–violating amplitude is induced by Higgs exchange with an effective one–loop flavour–changing \( \bar{s}dH \) coupling.\( ^38 \) The present lower bound on the Higgs mass, \( M_H > 66 \text{ GeV} \) (95% CL), implies a conservative upper limit \(|\mathcal{P}_{L,Direct}| < 10^{-4} \). Much larger values, \( \mathcal{P}_L \sim O(10^{-2}) \), appear quite naturally in various extensions of the Standard Model.\( ^39, 40 \) It is worth emphasizing that \( \mathcal{P}_L \) is especially sensitive to the presence of light scalars with CP–violating Yukawa couplings. Thus, \( \mathcal{P}_L \) seems to be a good signature to look for new physics beyond the Standard Model; for this to be the case, however, it is very important to have a good quantitative understanding of the Standard Model prediction to allow us to infer, from a measurement of \( \mathcal{P}_L \), the existence of a new CP–violation mechanism.

\( ^\dagger \) At \( O(p^4) \), this decay proceeds through a tree–level \( K_2^0 \rightarrow \pi^0, \eta \) transition, followed by \( \pi^0, \eta \rightarrow \gamma\gamma \) vertices. Because of the Gell–Mann–Okubo relation, the sum of the \( \pi^0 \) and \( \eta \) contributions cancels exactly to lowest order. The decay amplitude is then very sensitive to \( SU(3) \) breaking.
The chiral calculation of the $K^0 \to \mu^+\mu^-$ amplitude allows us to make a reliable estimate of the contribution to $P_L$ due to $K^0-K^0$ mixing:

\[ 1.9 < |P_{L,\varepsilon}| \times 10^2 \left( \frac{2 \times 10^{-6}}{\text{Br}(K_S \to \gamma\gamma)} \right)^{1/2} < 2.5. \] (19)

Taking into account the present experimental errors in $\text{Br}(K_S \to \gamma\gamma)$ and the inherent theoretical uncertainties due to uncalculated higher–order corrections, one can conclude that experimental indications for $|P_L| > 5 \times 10^{-3}$ would constitute clear evidence for additional mechanisms of CP violation beyond the Standard Model.

6 $K \to \pi\gamma\gamma$

The most general form of the $K \to \pi\gamma\gamma$ amplitude depends on four independent invariant amplitudes $A$, $B$, $C$ and $D$:

\[ A[K(p_K) \to \pi(p_1)\gamma(q_1)\gamma(q_2)] = \epsilon_{\mu}(q_1) \epsilon_{\nu}(q_2) \left\{ \frac{A(y, z)}{M_K^2} (q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu}) \right. \]
\[ + \frac{2B(y, z)}{M_K^2} (p_K \cdot q_1 q_2^\mu p_K^\nu + p_K \cdot q_2 q_1^\mu p_K^\nu - q_1 \cdot q_2 p_K^\mu p_K^\nu - p_K \cdot q_1 p_K \cdot q_2 g^{\mu\nu}) \]
\[ + \frac{C(y, z)}{M_K^2} \epsilon^{\mu\nu\rho\sigma} q_1 \rho q_2 \sigma \]
\[ + \frac{D(y, z)}{M_K^2} \left[ \epsilon^{\mu
u\rho\sigma} (p_K \cdot q_2 q_1 \rho + p_K \cdot q_1 q_2 \rho) p_K \sigma + \left( p_K^\mu \epsilon^{\nu\rho\gamma} + p_K^\nu \epsilon^{\mu\rho\gamma} \right) p_K \sigma q_1 q_2 \gamma \right] \}, \] (20)

where $y \equiv |p_K \cdot (q_1 - q_2)|/M_K^2$ and $z = (q_1 + q_2)^2/M_K^2$. In the limit where CP is conserved, the amplitudes $A$ and $B$ contribute to $K_2 \to \pi^0\gamma\gamma$ whereas $K_1 \to \pi^0\gamma\gamma$ involves the other two amplitudes $C$ and $D$. All four amplitudes contribute to $K^+ \to \pi^+\gamma\gamma$. Only $A(y, z)$ and $C(y, z)$ are non-vanishing to lowest non-trivial order, $O(p^4)$, in ChPT.

Again, the symmetry constraints do not allow any tree–level contribution to $K_2 \to \pi^0\gamma\gamma$ from $O(p^4)$ terms in the lagrangian. The $A(y, z)$ amplitude is therefore determined by a finite loop calculation. The relevant Feynman diagrams are analogous to the ones in Fig. 2, but with an additional $\pi^0$ line emerging from the weak vertex; charged kaon loops also give a small contribution in this case. Due to the large absorptive $\pi^+\pi^-$ contribution, the spectrum in the invariant mass of the two photons is predicted to have a very characteristic behaviour (dotted line in Fig. 4), peaked at high values of $m_{\gamma\gamma}$. The agreement with the measured two–photon distribution is remarkably good. However, the $O(p^4)$ prediction for the rate $\text{Br}(K_L \to \pi^0\gamma\gamma) = 0.67 \times 10^{-6}$, is smaller than the experimental value $\text{Br}(K_L \to \pi^0\gamma\gamma) = (1.70 \pm 0.28) \times 10^{-6}$. (21)

Since the effect of the amplitude $B(y, z)$ first appears at $O(p^6)$, one should worry about the size of the next–order corrections. A naive vector–meson–dominance (VMD) estimate through the decay chain $K_L \to \pi^0, \eta, \eta' \to V\gamma \to \pi^0\gamma\gamma$ results in a sizeable contribution to $B(y, z)$,

\[ A(y, z)\big|_{\text{VMD}} = \tilde{a}_V \left( 3 - z + \frac{M_S^2}{M_K^2} \right), \]
\[ B(y, z)\big|_{\text{VMD}} = -2\tilde{a}_V, \]
\[ \tilde{a}_V \equiv \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* g_8 \frac{M_K^2 \alpha}{\pi} \alpha_V, \] (22)
Figure 4: 2γ-invariant-mass distribution for $K_L \rightarrow \pi^0\gamma\gamma$: $O(p^4)$ (dotted curve), $O(p^6)$ with $a_V = 0$ (dashed curve), $O(p^6)$ with $a_V = -0.9$ (full curve). The spectrum is normalized to the 50 unambiguous events of NA31 (without acceptance corrections).

with $a_V \approx 0.32$. However, this type of calculation predicts a photon spectrum peaked at low values of $m_{\gamma\gamma}$, in strong disagreement with experiment. As first emphasized in ref. 48, there are also so-called direct weak contributions associated with $V$ exchange, which cannot be written as a strong VMD amplitude with an external weak transition. Model-dependent estimates of this direct contribution suggest a strong cancellation with the naive vector–meson–exchange effect; but the final result is unfortunately quite uncertain.

A detailed calculation of the most important $O(p^6)$ corrections has been performed in ref. 49. In addition to the VMD contribution, the unitarity corrections associated with the two–pion intermediate state (i.e. $K_L \rightarrow \pi^0\pi^+\pi^- \rightarrow \pi^0\gamma\gamma$) have been included. Fig. 4 shows the resulting photon spectrum for $a_V = 0$ (dashed curve) and $a_V = -0.9$ (full curve). The corresponding branching ratio is:

$$\text{Br}(K_L \rightarrow \pi^0\gamma\gamma) = \begin{cases} 0.67 \times 10^{-6}, & O(p^4), \\ 0.83 \times 10^{-6}, & O(p^6), a_V = 0, \\ 1.60 \times 10^{-6}, & O(p^6), a_V = -0.9. \end{cases}$$  \quad (23)$$

The unitarity corrections by themselves raise the rate only moderately. Moreover, they produce an even more pronounced peaking of the spectrum at large $m_{\gamma\gamma}$, which tends to ruin the success of the $O(p^4)$ prediction. The addition of the $V$ exchange contribution restores again the agreement. Both the experimental rate and the spectrum can be simultaneously reproduced with $a_V = -0.9$. A more complete unitarization of the $\pi^-\pi$ intermediate states including the experimental $\gamma\gamma \rightarrow \pi^0\pi^0$ amplitude, increases the $K_L \rightarrow \pi^0\gamma\gamma$ decay width some 10%, leading to a slightly smaller value of $|a_V|$.

For the charged decay $K^+ \rightarrow \pi^+\gamma\gamma$, the sum of all 1–loop diagrams gives also a finite $O(p^4)$ amplitude $A(y, z)$. However, chiral symmetry allows in addition for a direct tree–level

Figure 5: Measured 2γ-invariant-mass distribution for $K_L \rightarrow \pi^0\gamma\gamma$ (solid line). The dashed line shows the estimated background. The experimental acceptance is given by the crosses. The dotted line simulates the $O(p^4)$ ChPT prediction.
contribution proportional to the renormalization–scale–invariant constant
\[ \hat{c} = 32\pi^2 \left[ 4(L_9 + L_{10}) - \frac{1}{3}(w_1 + 2w_2 + 2w_3) \right]. \tag{24} \]

There is also a contribution to \( C(y, z) \), generated by the chiral anomaly. Since \( \hat{c} \) is unknown, ChPT alone cannot predict \( \Gamma(K^+ \to \pi^+\gamma\gamma) \); nevertheless, it gives, up to a twofold ambiguity, a precise correlation between the rate and the spectrum. Moreover, one can derive the lower bound \( \text{Br}(K^+ \to \pi^+\gamma\gamma) \geq 4 \times 10^{-7} \).

From naïve power–counting arguments one expects \( \hat{c} \sim O(1) \), although \( \hat{c} = 0 \) has been obtained in some models. The shape of the \( z \) distribution is very sensitive to \( \hat{c} \) and, for reasonable values of this parameter, is predicted again to peak at large \( z \) due to the rising absorptive part of the \( \pi\pi \) intermediate state. The preliminary results of the BNL-E787 experiment show indeed a clear enhancement of events at large \( z \), in nice agreement with the theoretical expectations.

A recent analysis of the main \( O(p^6) \) corrections, analogous to the one previously performed for the \( K_L \) decay mode, suggests that the unitarity corrections generate again a sizeable (\( \sim 30\text{–}40\% \)) increase of the decay width.

7 \( K \to \pi l^+l^- \)

In contrast to the previous processes, the \( O(p^4) \) calculation of \( K^+ \to \pi^+l^+l^- \) and \( K_S \to \pi^0l^+l^- \) involves a divergent loop, which is renormalized by the \( O(p^4) \) lagrangian. The decay amplitudes can then be written as the sum of a calculable loop contribution plus an unknown combination of chiral couplings,

\[
\begin{align*}
    w_+ &= -\frac{1}{3}(4\pi)^2 [w_1^r + 2w_2^r - 12L_9^r] - \frac{1}{3}\log \left( M_K M_\pi / \mu^2 \right), \\
    w_S &= -\frac{1}{3}(4\pi)^2 [w_1^r - w_2^r] - \frac{1}{3}\log \left( M_K^2 / \mu^2 \right),
\end{align*}
\tag{25}
\]

where \( w_+ \), \( w_S \) refer to the decay of the \( K^+ \) and \( K_S \) respectively. These constants are expected to be of \( O(1) \) by naïve power–counting arguments. The logarithms have been included to compensate the renormalization–scale dependence of the chiral couplings, so that \( w_+ \), \( w_S \) are observable quantities. If the final amplitudes are required to transform as octets, then \( w_2 = 4L_9 \), implying \( w_S = w_+ + \frac{1}{3}\log \left( M_\pi / M_K \right) \). It should be emphasized that this relation goes beyond the usual requirement of chiral invariance.

The measured \( K^+ \to \pi^+e^+e^- \) decay rate determines two possible solutions for \( w_+ \). The two–fold ambiguity can be solved, looking to the shape of the invariant–mass distribution of the final lepton pair, which is regulated by the same parameter \( w_+ \). A fit to the BNL–E777 data gives

\[ w_+ = 0.89^{+0.24}_{-0.14}, \tag{26} \]

in agreement with model–dependent theoretical estimates. Once \( w_+ \) has been fixed, one can predict the rates and Dalitz–plot distributions of the related modes \( K^+ \to \pi^+\mu^+\mu^- \), \( K_S \to \pi^0e^+e^- \) and \( K_S \to \pi^0\mu^+\mu^- \). The preliminary value \( \text{Br}(K^+ \to \pi^+\mu^+\mu^-) = (5.0\pm0.4\pm0.6) \times 10^{-8} \), reported at this workshop by the BNL-787 experiment, is in excellent agreement with the theoretical prediction \( \text{Br}(K^+ \to \pi^+\mu^+\mu^-) = (6.2^{+0.8}_{-0.6}) \times 10^{-8} \).
The rare decay $K_L \to \pi^0 e^+ e^-$ is an interesting process in looking for new CP–violating signatures. If CP were an exact symmetry, only the CP–even state $K^0_L$ could decay via one–photon emission, while the decay of the CP–odd state $K^0_L$ would proceed through a two–photon intermediate state and, therefore, its decay amplitude would be suppressed by an additional power of $\alpha$. When CP violation is taken into account, however, an $O(\alpha)$ $K_L \to \pi^0 e^+ e^-$ decay amplitude is induced, both through the small $K^0_L$ component of the $K_L$ ($\varepsilon$ effect) and through direct CP violation in the $K^0_L \to \pi^0 e^+ e^-$ transition. The electromagnetic suppression of the CP–conserving amplitude then makes it plausible that this decay is dominated by the CP–violating contributions.

The short–distance analysis of the product of weak and electromagnetic currents allows a reliable calculation of the direct CP–violating $K^0_L \to \pi^0 e^+ e^−$ amplitude. The corresponding branching ratio has been estimated to be: \[ \text{Br}(K_L \to \pi^0 e^+ e^-)_{\text{Direct}} = (4.5 \pm 2.6) \times 10^{-12}. \] (27)

The indirect CP–violating amplitude induced by the $K^0_1$ component of the $K_L$ is given by the $K_S \to \pi^0 e^+ e^-$ amplitude times the CP–mixing parameter $\varepsilon$. Using the octet relation between $w_+$ and $w_4$, the determination of the parameter $\omega_+$ in (26) implies \[ \text{Br}(K_L \to \pi^0 e^+ e^-)_{\text{Indirect}} \leq 1.5 \times 10^{-12}. \] (28)

Comparing this value with (27), we see that the direct CP–violating contribution is expected to be bigger than the indirect one. This is very different from the situation in $K \to \pi \pi$, where the contribution due to mixing completely dominates.

Using the computed $K_L \to \pi^0 \gamma \gamma$ amplitude, one can estimate the CP–conserving two–photon exchange contribution to $K_L \to \pi^0 e^+ e^-$, by taking the absorptive part due to the two–photon discontinuity as an educated guess of the actual size of the complete amplitude. At $O(p^4)$, the $K_L \to \pi^0 e^+ e^-$ decay amplitude is strongly suppressed (it is proportional to $m_e$), owing to the helicity structure of the $A(y, z)$ term.\[ \text{Br}(K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 e^+ e^-)_{O(p^4)} \sim 5 \times 10^{-15}. \] (29)

This helicity suppression is, however, no longer true at the next order in the chiral expansion. The $O(p^6)$ estimate of the amplitude $B(y, z)$ gives rise to \[ \text{Br}(K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 e^+ e^-)_{O(p^6)} \sim \begin{cases} 0.3 \times 10^{-12}, & a_V = 0, \\ 1.8 \times 10^{-12}, & a_V = -0.9. \end{cases} \] (30)

Thus, the decay width seems to be dominated by the CP–violating amplitude, but the CP–conserving contribution could also be important. Notice that if both amplitudes were comparable there would be a sizeable CP–violating energy asymmetry between the $e^−$ and the $e^+$ distributions.

The present experimental upper bound on \[ \text{Br}(K_L \to \pi^0 e^+ e^-)_{\text{Exp}} < 4.3 \times 10^{-9} \quad (90\%\text{CL}), \] (31) is still far away from the expected Standard Model signal, but the prospects for getting the needed sensitivity of around $10^{-12}$ in the next few years are rather encouraging.
To be able to interpret a future experimental measurement of the decay rate as a (direct) CP-violating signature, it is first necessary, however, to pin down more precisely the actual size of the three different components of the decay amplitude. Some possible improvements are:

- The size of the indirect CP-violating amplitude in eq. (28) uses the octet relation between \( w_+ \) and \( w_S \). Although consistent with this assumption, the explicit calculations of those chiral couplings\(^{56}\) do not exclude sizeable deviations which could imply a larger contribution to the decay amplitude. A more reliable estimate is then required.\(^{60}\)

- A measurement of \( \text{Br}(K_S \to \pi^0 e^+ e^-) \) would directly determine the size of the indirect CP-violating amplitude. To bound this contribution below \(10^{-12}\), one needs an experimental upper bound on the \( K_S \) branching ration below \(3 \times 10^{-10}\), to be compared with the present value\(^{61}\) \( \text{Br}(K_S \to \pi^0 e^+ e^-) < 3.9 \times 10^{-7} \) (90% CL).

- A careful fit to the \( K_L \to \pi^0 \gamma \gamma \) data, taking the experimental acceptance into account, would allow to extract the actual value of \( a_V \), and fix the absorptive contribution to the CP-conserving amplitude. A better understanding of the dispersive piece\(^{44, 47, 58, 62}\) is also needed.

9 The Chiral Anomaly in Non-Leptonic \( K \) Decays

The chiral anomaly also appears in the non-leptonic weak interactions. A systematic study of all non-leptonic \( K \) decays where the anomaly contributes at leading order, \( O(p^4) \), has been performed in refs. \(^{63, 64}\). Only radiative \( K \) decays are sensitive to the anomaly in the non-leptonic sector.

The manifestations of the anomaly can be grouped in two different classes of anomalous amplitudes: reducible and direct contributions. The reducible amplitudes arise from the contraction of meson lines between a weak non-leptonic \( \Delta S = 1 \) vertex and the Wess-Zumino-Witten functional.\(^{65, 66}\) In the octet limit, all reducible anomalous amplitudes of \( O(p^4) \) can be predicted in terms of the coupling \( g_8 \). The direct anomalous contributions are generated through the contraction of the \( W \) boson field between a strong Green function on one side and the Wess-Zumino-Witten functional on the other. Their computation is not straightforward, because of the presence of strongly interacting fields on both sides of the \( W \). Nevertheless, due to the non-renormalization theorem of the chiral anomaly,\(^{67}\) the bosonized form of the direct anomalous amplitudes can be fully predicted.\(^{68}\) In spite of its anomalous origin, this contribution is chiral invariant. The anomaly turns out to contribute to all possible octet terms of \( \mathcal{L}_{\Delta S=1} \) proportional to the \( \varepsilon_{\mu \nu \alpha \beta} \) tensor. Unfortunately, the coefficients of these terms get also non-factorizable contributions of non-anomalous origin, which cannot be computed in a model-independent way. Therefore, the final predictions can only be parametrized in terms of four dimensionless chiral couplings, which are expected to be positive and of order one.

The most frequent anomalous decays \( K^+ \to \pi^+ \pi^0 \gamma \) and \( K_L \to \pi^+ \pi^- \gamma \) share the remarkable feature that the normally dominant bremsstrahlung amplitude is strongly suppressed, making the experimental verification of the anomalous amplitude substantially easier. This suppression has different origins: \( K^+ \to \pi^+ \pi^0 \) proceeds through the small 27-plet part of the non-leptonic weak interactions, whereas \( K_L \to \pi^+ \pi^- \) is CP violating. The remaining non-leptonic \( K \) decays with direct anomalous contributions are either suppressed by phase space \([K^+ \to \pi^+ \pi^0 \pi^0(\gamma), K^+ \to \pi^+ \pi^+ \pi^- \gamma(\gamma), K_L \to \pi^+ \pi^- \pi^0 \gamma, K_S \to \pi^+ \pi^- \pi^0 \gamma(\gamma)]\) or by the presence of an extra photon in the final state \([K^+ \to \pi^+ \pi^0 \gamma \gamma, K_L \to \pi^+ \pi^- \gamma \gamma]\).


10 Summary

Rare K decays are an important testing ground of the electroweak flavour theory. With the improved experimental sensitivity expected in the near future, they can provide new signals of CP–violation phenomena and, perhaps, a window into physics beyond the Standard Model.

The theoretical analysis of these decays is far from trivial due to the very low mass of the hadrons involved. The delicate interplay between the flavour–changing dynamics and the confining QCD interaction makes very difficult to perform precise dynamical predictions. Fortunately, the Goldstone nature of the pseudoscalar mesons implies strong constraints on their low–energy interactions, which can be analyzed with effective lagrangian methods. The ChPT framework incorporates all the constraints implied by the chiral symmetry of the underlying lagrangian at the quark level, allowing for a clear distinction between genuine aspects of the Standard Model and additional assumptions of variable credibility usually related to the problem of long–distance dynamics. The low–energy amplitudes are calculable in ChPT, except for some coupling constants which are not restricted by chiral symmetry. These constants reflect our lack of understanding of the QCD confinement mechanism and must be determined experimentally for the time being. Further progress in QCD can only improve our knowledge of these chiral constants, but it cannot modify the low–energy structure of the amplitudes.

It is important to emphasize that the experimental verification of the chiral predictions does not provide a test of the detailed dynamics of the Standard Model; only the implications of the underlying symmetries are being proved. The dynamical information is encoded in the chiral couplings. Thus, one needs to derive those chiral constants from the Standard Model itself, to actually test the non-trivial low–energy dynamics. Although this is a very difficult problem, the recent attempts done in this direction look quite promising.

References

1. L. Littenberg, these proceedings.
2. B. Winstein, these proceedings.
3. G. Buchalla, A.J. Buras and M.E. Lautenbacher, Weak Decays Beyond Leading Logarithms, Rev. Mod. Phys. (1996) [hep-ph/9512380]; A.J. Buras, these proceedings.
4. S. Weinberg, Physica 96A (1979) 327.
5. J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465, 517, 539.
6. G. Ecker, Prog. Part. Nucl. Phys. 35 (1995) 1.
7. A. Pich, Rep. Prog. Phys. 58 (1995) 563.
8. E. de Rafael, Chiral Lagrangians and Kaon CP–Violation, in CP Violation and the Limits of the Standard Model, Proc. TASI’94, ed. J.F. Donoghue (World Scientific, Singapore, 1995).
9. G. Ecker, A. Pich and E. de Rafael, Rare Kaon Decays in Chiral Perturbation Theory, to appear.
10. A. Pich, B. Guberina and E. de Rafael, Nucl. Phys. B277 (1986) 197.
11. J. Kambor, J. Missimer and D. Wyler, *Phys. Lett.* B261 (1991) 496.
12. J. Kambor *et al.,* *Phys. Rev. Lett.* 68 (1992) 1818.
13. G. Ecker, A. Pich and E. de Rafael, *Nucl. Phys.* B291 (1987) 692.
14. G. Ecker, A. Pich and E. de Rafael, *Phys. Lett.* B189 (1987) 363.
15. G. Ecker, A. Pich and E. de Rafael, *Nucl. Phys.* B303 (1988) 665.
16. J. Kambor, J. Missimer and D. Wyler, *Nucl. Phys.* B346 (1990) 17.
17. G. Ecker, *Geometrical aspects of the non-leptonic weak interactions of mesons,* in Proc. IX Int. Conf. on the Problems of Quantum Field Theory, ed. M.K. Volkov (JINR, Dubna, 1990).
18. G. Esposito–Farèse, *Z. Phys.* C50 (1991) 255.
19. G. Ecker, J. Kambor, and D. Wyler, *Nucl. Phys.* B394 (1993) 101.
20. D. Rein and L.M. Sehgal, *Phys. Rev.* D39 (1989) 3325.
21. J.S. Hagelin and L.S. Littenberg, *Prog. Part. Nucl. Phys.* 23 (1989) 1.
22. M. Lu and M.B. Wise, *Phys. Lett.* B324 (1994) 461.
23. S. Fajfer, *Long distance contribution to $K^+ \to \pi^+\nu\bar{\nu}$ decay and $O(p^4)$ terms in CHPT,* hep-ph/9602322.
24. C.Q. Geng, I.J. Hsu and Y.C. Lin, *Phys. Rev.* D54 (1996) 877.
25. H. Leutwyler and M. Roos, *Z. Phys.* C25 (1984) 91.
26. W.J. Marciano and Z. Parsa, *Phys. Rev.* D53 (1996) R1.
27. L. Wolfenstein, *Phys. Rev. Lett.* 51 (1983) 1945.
28. S. Adler *et al.,* *Phys. Rev. Lett.* 76 (1996) 1421.
29. L.S. Littenberg, *Phys. Rev.* D39 (1989) 3322.
30. M. Weaver *et al.,* *Phys. Rev. Lett.* 72 (1994) 3758.
31. G. D’Ambrosio and D. Espriu, *Phys. Lett.* B175 (1986) 237.
32. J.L. Goity, *Z. Phys.* C34 (1987) 341.
33. G.D. Barr *et al.,* *Phys. Lett.* B351 (1995) 579.
34. H. Burkhardt *et al.,* *Phys. Lett.* B199 (1987) 139.
35. G. Ecker and A. Pich, *Nucl. Phys.* B366 (1991) 189.
36. S. Gjesdal *et al.,* *Phys. Lett.* 44B (1973) 217.
37. A.M. Blick *et al.,* *Phys. Lett.* B334 (1994) 234.
38. F.J. Botella and C.S. Lim, *Phys. Rev. Lett.* 56 (1986) 1651.
39. C.Q. Geng and J.N. Ng, Phys. Rev. D42 (1990) 1509.
40. R.N. Mohapatra, Prog. Part. Nucl. Phys. 31 (1993) 39.
41. L. Cappiello and G. D’Ambrosio, Nuovo Cimento 99A (1988) 155.
42. G.D. Barr et al, Phys. Lett. B284 (1992) 440; B242 (1990) 523.
43. V. Papadimitriou et al, Phys. Rev. D44 (1991) 573.
44. L.M. Sehgal, Phys. Rev. D38 (1988) 808; D41 (1990) 161.
45. T. Morozumi and H. Iwasaki, Progr. Theor. Phys. 82 (1989) 371.
46. J. Flynn and L. Randall, Phys. Lett. B216 (1989) 221.
47. P. Heiliger and L.M. Sehgal, Phys. Rev. D47 (1993) 4920.
48. G. Ecker, A. Pich and E. de Rafael, Phys. Lett. B237 (1990) 481.
49. A.G. Cohen, G. Ecker and A. Pich, Phys. Lett. B304 (1993) 347.
50. L. Cappiello, G. D’Ambrosio and M. Miragliuolo, Phys. Lett. B298 (1993) 423.
51. J. Kambor and B.R. Holstein, Phys. Rev. D49 (1994) 2346.
52. T. Shinkawa, these proceedings.
53. T. Nakano, First Observation of the Decay $K^+ \rightarrow \pi^+\gamma\gamma$, Proc. Moriond 1996.
54. G. D’Ambrosio and J. Portolés, Unitarity and vector meson contributions to $K^+ \rightarrow \pi^+\gamma\gamma$, hep-ph/9606213.
55. C. Alliegro et al, Phys. Rev. Lett. 68 (1992) 278.
56. C. Bruno and J. Prades, Z. Phys. C57 (1993) 585.
57. J.F. Donoghue, B.R. Holstein and G. Valencia, Phys. Rev. D35 (1987) 2769.
58. J.F. Donoghue and F. Gabbiani, Phys. Rev. D51 (1995) 2187.
59. D.A. Harris et al, Phys. Rev. Lett. 71 (1993) 3918.
60. A. Pich and E. de Rafael, to appear.
61. G. Thomson, these proceedings.
62. T.P. Cheng, Phys. Rev. 162 (1967) 1734.
63. G. Ecker, H. Neufeld and A. Pich, Phys. Lett. B278 (1992) 337.
64. G. Ecker, H. Neufeld and A. Pich, Nucl. Phys. B413 (1994) 321.
65. J. Wess and B. Zumino, Phys. Lett. 37B (1971) 95.
66. E. Witten, Nucl. Phys. B223 (1983) 422.
67. S.L. Adler and W.A. Bardeen, Phys. Rev. 182 (1969) 1517.
68. J. Bijnens, G. Ecker and A. Pich, Phys. Lett. B286 (1992) 341.