Cosmic birefringence fluctuations and cosmic microwave background B-mode polarization

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ABSTRACT

Recently, BICEP2 measurements of the cosmic microwave background (CMB) B-mode polarization has indicated the presence of primordial gravitational waves at degree angular scales, inferring the tensor-to-scalar ratio of $r = 0.2$ and a running scalar spectral index, provided that dust contamination is low. In this Letter, we show that the existence of the fluctuations of cosmological birefringence can give rise to CMB B-mode polarization that fits BICEP2 data with $r < 0.11$ and no running of the scalar spectral index. When dust contribution is taken into account, we derive an upper limit on the cosmological birefringence, $A^2 < 0.0075$, where $A$ is the amplitude of birefringence fluctuations that couple to electromagnetism with a coupling strength $\beta$.

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The spatial flatness and homogeneity of the present Universe strongly suggest that a period of de Sitter expansion or inflation had occurred in the early Universe [1]. During inflation, quantum fluctuations of the inflaton field may give rise to energy density perturbations (scalar modes) [2], which can serve as the seeds for the formation of large-scale structures of the Universe. In addition, a spectrum of gravitational waves (tensor modes) is produced from the de Sitter vacuum [3].

Gravitational waves are very weakly coupled to matter, so once produced, they remain as a stochastic background until today, and thus provide a potentially important probe of the inflationary epoch. Detection of these primordial waves by using terrestrial wave detectors or the timing of millisecond pulsars [4] would indeed require an experimental sensitivity of several orders of magnitude beyond the current reach. However, horizon-sized tensor perturbation induces large-scale temperature anisotropy of the cosmic microwave background (CMB) via the Sachs–Wolfe effect [5]. In addition, the tensor modes uniquely induce CMB B-mode polarization that is the primary goal of ongoing and future CMB experiments [6].

Recently, WMAP+LPT CMB data has placed an upper limit on the contribution of tensor modes to the CMB anisotropy, in terms of the tensor-to-scalar ratio, which is $r < 0.18$ at 95% confidence level, tightening to $r < 0.11$ when also including measurements of the Hubble constant and baryon acoustic oscillations (BAO) [7]. Planck Collaboration XVI has quoted $r < 0.11$ using a combination of Planck, SPT, and ACT anisotropy data, plus WMAP polarization; however, the constraint relaxes to $r < 0.26$ (95% confidence) when running of the scalar spectral index is allowed with $dn_s/d\ln k = -0.022 \pm 0.010$ (68%) [8]. More recently, BICEP2 CMB experiment has found an excess of B-mode power at degree angular scales, indicating the presence of tensor modes with $r = 0.20^{+0.07}_{-0.05}$ and $dn_s/d\ln k = -0.028 \pm 0.009$ [9]. If this result is confirmed, it would give very strong support to inflation model and open a new window for probing the inflationary dynamics. However, lately a joint analysis of BICEP2/Keck Array and Planck data has indicated that the BICEP2 B-mode signal could be mainly due to dust foreground [10].

In this Letter, we investigate another source for generating CMB B-mode polarization. The generated B-mode power spectrum can explain the BICEP2 excess B-mode power, while complying to the limit $r < 0.11$ and $dn_s/d\ln k = 0$. Here we consider a nearly massless pseudoscalar $\Phi \equiv M\phi$ that couples to the electromagnetic field strength via $-2\beta/\phi F_{\mu\nu}F^{\mu\nu}$, where $\beta$ is a coupling constant and $M$ is the reduced Planck mass. The effect of this coupling to CMB polarization has been previously studied in different contexts such as new high-energy physics [11], a massless...
pseudo-Nambu–Goldstone spectator field [12], and scalar quantum fluctuations of the vacuum-like cosmological constant in an axiverse [13,14]. It is well known that the above \( \phi \)-photon interaction may lead to cosmic birefringence [15] that induces rotation of the polarization plane of the CMB, thus resulting in a conversion of \( E \)-mode into \( B \)-mode polarization without affecting the temperature anisotropies [16,17]. For such a pseudoscalar, we consider the contribution of \( \phi \) perturbation to cosmic birefringence, resulted from dressed photon propagators with \( \phi \) perturbation. Furthermore, \( \phi \)-photon elastic scatterings can lead to CMB lensing; however, this is a second-order effect which is subdominant to the gravitational lensing of large-scale structures in the present consideration.

We assume a conformally flat metric, \( ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2) \), where \( a(\eta) \) is the cosmic scale factor and \( \eta \) is the conformal time defined by \( dt = a(\eta)d\eta \). The \( \Phi \bar{F} \) term leads to a rotational velocity of the polarization plane of a photon propagating in the direction \( \hat{n} \) [15],

\[
\omega(\eta, \hat{n}) = -\frac{\beta}{2} \left( \frac{\partial \phi}{\partial \eta} + \bar{\Phi} \hat{n} \cdot \hat{n} \right). \tag{1}
\]

Thomson scatterings of anisotropic CMB photons by free electrons give rise to linear polarization, which can be described by the Stokes parameters \( Q(\eta, \hat{x}) = Q(\eta, \hat{x}) \) and \( U(\eta, \hat{x}) \). The time evolution of the linear polarization is governed by the collisional Boltzmann equation, which would be modified due to the rotational velocity of the polarization plane (1) by including a temporal rate of change of the Stokes parameters:

\[
\dot{Q} \pm i\dot{U} = \mp i \omega(Q \pm iU), \tag{2}
\]

where the dot denotes \( d/d\eta \). This gives a convolution of the Fourier modes of the Stokes parameters with the spectral rotation that can be easily incorporated into the Boltzmann code.

Now we consider the time evolution of \( \phi \). Decompose \( \phi \) into the vacuum expectation value and the perturbation: \( \phi(\eta, \hat{x}) = \phi(\eta) + \delta \phi(\eta, \hat{x}) \). For the metric perturbation, we adopt the synchronous gauge: \( ds^2 = a^2(\eta)(d\eta^2 - [\delta_{ij} + h_{ij}(\eta, \hat{x})]dx^idx^j) \). Neglecting the back reaction of the interaction, we obtain the mean field evolution as

\[
\ddot{\phi} + 2H\dot{\phi} + \frac{a^2}{M^2} \frac{\partial V}{\partial \phi} = 0, \tag{3}
\]

where \( H \equiv \dot{a}/a \) and \( V(\phi) \) is the scalar potential. The equation of motion for the Fourier mode \( \delta \phi_k^l \) is given by

\[
\ddot{\delta \phi_k^l} + 2\dot{H}\delta \phi_k^l + \left( k^2 + \frac{a^2}{M^2} \frac{\partial^2 V}{\partial \phi^2} \right) \delta \phi_k^l = -\frac{1}{2} h_{ij} \delta \phi_i^j, \tag{4}
\]

where \( h_{ij} \) is the Fourier transform of the trace of \( h_{ij} \).

If \( \phi \) is nearly massless or its effective mass is less than the present Hubble parameter, the mass term and the term second in Eq. (4) can be neglected. In this case, \( V(\phi) \) can be either null or it behaves just like a cosmological constant with \( \dot{\phi} = 0 \). However, its perturbation is dispersive and can be cast into \( \delta \phi_i^j(\eta) = \delta \phi_i^j(\dot{\phi}(\eta) \hat{n}) \), where \( \delta \phi_i^j \) is the initial perturbation amplitude and \( f(\eta) \) is a dispersion factor. For a super-horizon mode with \( k_0 \ll 1 \), \( f(\eta) = 1 \); the factor then oscillates with a decaying envelope once the mode enter the horizon. Let us define the initial power spectrum \( P_{\delta \phi}(k) \) by

\[
\delta \phi_i^j(\dot{\phi}(\eta) \hat{n}) = \int \frac{dk}{2\pi} P_{\delta \phi}(k) \delta \tilde{\phi}(k - \hat{n} - \hat{\tilde{k}}), \tag{5}
\]

where \( \alpha(\dot{n}) = \sum_n \alpha_n V_n(\dot{n}) \), \( \eta_0 \) is the present time, and \( \eta_0 \) denotes the time when the primordial CMB polarization is generated on the last scattering surface or the recombination surface. The rotation power spectra for the recombinations and the realizations with \( A\beta^2 = 0.0046 \) are shown in Fig. 3. Using this ro-

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**Fig. 1.** Cosmological birefringence induced \( B \)-mode power spectrum through the perturbed nearly massless scalar field with \( A\beta^2 = 0.0046 \) (short-dashed). Also shown are the theoretical power spectra of lensing induced \( B \)-modes (long-dashed) and gravity-wave induced \( B \)-modes (dot-dashed) with \( r = 0.168 \). The thick solid curve is the best-fitting averaged \( B \)-mode band powers that are the sum of these three \( B \)-mode power spectra convolved with the BICEP2 (\( \ell < 400 \)) and the POLARBEAR (\( \ell > 400 \)) window functions. BICEP2 data [9] (diamonds) and POLARBEAR data [19] (triangles and inverted solid triangle representing the absolute value of a negative band) are shown.

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**Fig. 2.** Likelihood plot of the parameters \( r \) and \( A\beta^2 \), showing 1-sigma and 2-sigma contours. Solid contours use WMAP, Planck, BICEP2, and POLARBEAR data, with the maximum likelihood values of \( r = 0.168 \) and \( A\beta^2 = 0.0046 \). Dashed contours use BICEP2 and POLARBEAR data only for comparison.
The rotation-induced $B$-mode polarization can be approximated by

$$C_l^{BB} = \frac{1}{\pi} \sum_{l_1, l_2} (2l_1 + 1)(2l_2 + 1)C_l^{EE}(\eta_0)|C_l^{\alpha}|^2,$$

where $l$ is the multipole order and $\eta_0$ is the value of $\eta$ at the time of recombination. This expression is based on the assumption that the pivot scale of the primordial power spectrum is $l_0 = 100$.

Note that both $C_l^{TB}$ and $C_l^{BB}$ power spectra vanish due to the fact that $(\delta \phi) = 0$. On the other hand, the conversion of $E$ to $B$ would diminish both $C_l^{TE}$ and $C_l^{EE}$. We have found that the effect of the birefringence on CMB power spectra predominately comes from low-$l$ $C_l^\alpha$. As such, the observed power spectra basically follow the shapes of the original power spectra and can be approximated by $C_l^{BB, obs} \approx C_l^{EE} \cos^2(2\alpha)$, and $C_l^{EE, obs} \approx C_l^{EE} \cos(2\alpha)$, where $\alpha$ is a root-mean-square rotation angle. Recent observations have confirmed that $|\alpha| < 3$ [22]. In the present consideration, the birefringence $C_l^{BB} \propto 0.0025 C_l^{EE}$, implying a root-mean-square rotation angle of about 1.4$^\circ$, which is still within the above constraint. Certainly, a simultaneous fitting of these power spectra to all of the currently available polarization data is required to give an accurate answer.

Recently, the gravitational lensing $B$-mode polarization has been detected by cross correlating $B$ modes measured by the SPTpol experiment with lensing $B$ modes inferred from cosmic infrared background fluctuations measured by Herschel and $E$ modes measured by SPTpol [23]. Another CMB experiment called POLARBEAR has also confirmed this cross correlation [24]. However, we note that this detection has no constraint on the rotation-induced $B$-mode polarization because the rotation power spectrum and the lensing power spectrum are uncorrelated.

There have been physical constraints on $A$ and $\beta$. Let us assume that inflation generates the initial condition for dark energy perturbation. Then, $n \sim 1$ and $A \sim (H/2\pi)^2/M^2$, where $H$ is the Hubble scale of inflation. The recent CMB anisotropy measured by the Planck mission has put an upper limit on $A < 3.4 \times 10^{-11}$ [8]. This implies that the present spectral energy density of dark energy perturbation relative to the critical energy density, $\Omega_{\phi} < 10^{-15}$, which is negligible compared to that of radiation. The most stringent limit on $\beta$ comes from the absence of a $\gamma$-ray burst in coincidence with Supernova 1987A neutrinos, which would have been converted in the galactic magnetic field from a burst of axion-like particles due to the Primakoff production in the supernova core: $\beta < 2.4 \times 10^7$ for $m_{\phi} < 10^{-9}$ eV [25]. Hence the combined limit is $A\beta^2 < 2 \times 10^4$, which is much bigger than the value that we have used here.

The removal of dust contamination may reduce the tensor contribution to $r = 0.16$ [9]. As such, the contours in Fig. 2 are simply shifted vertically down by about a value of 0.04. With nonzero $A\beta^2 = 0.005 - 0.009$, the solid 1-sigma contour shows $r = 0.09 - 0.1$, which are within the Planck upper limit of $r < 0.11$. Thus, it might be too hasty to conclude that many inflation models with small $r$ are ruled out based on BICEP2 result. With the proper mechanisms like birefringence to induce the large-scale $B$-mode polarization, many inflation models can be still compatible with BICEP2 result, at least at 1-sigma level.

Recently, a joint analysis of BICEP2/Keck Array and Planck data that includes dust contribution have yielded an upper limit $r < 0.12$ [10]. Here we have used a dust $B$-mode power spectrum, $l(l+1)C_l^{BB}/(2\pi) = D(l/100)^{-0.3} \mu K^2$ with a free parameter $D$, which was adopted in Ref. [26], combined with the birefringence $B$ modes and the tensor mode to fit the BICEP2 and POLARBEAR data. The best-fit power spectrum is drawn with $D = 0.01$, $A\beta^2 = 0.0023$, and $r = 0$, as shown in Fig. 4. This power spectrum (with $\chi^2 = 4.83$) is a better $\chi^2$ fitting than that of the pure tensor mode with $r = 0.2$ ($\chi^2 = 7.69$). Fig. 5 is the likelihood plot of the dust power and birefringence and Fig. 6 is that of the tensor and birefringence. The detection level of the dust power and the upper limit on $r < 0.09$ are close to those found in Ref. [26], while we can set an upper limit on $A\beta^2 < 0.0075$.

Cosmological birefringence perturbation can generate a rotation-induced $B$-mode power spectrum. The BICEP2 and POLARBEAR experiments may have barely 1-sigma detection of cosmological birefringence $B$ modes at degree and sub-degree angular scales, if dust emission is ignored. When dust foreground is included, we have set an upper limit on the cosmic birefringence. It would be very important to make precise direct measurements of $B$-mode polarization at sub-degree scales where birefringence $B$ modes peak at and can be mixed with lensing $B$ modes. It thus poses a big challenge to do the separation of different $B$-mode signals. It is apparent that the rotation-induced $B$-mode has acoustic oscillations but to detect or rule out them will require next-generation
or lensing contours. Fig. 6. Likelihood of the parameters $D$ and $A\beta^2$, showing 1-sigma and 2-sigma contours. The maximum likelihood values are $D = 0.01$ and $A\beta^2 = 0.0023$.

Fig. 5. Likelihood plot of the parameters $r$ and $A\beta^2$, showing 1-sigma and 2-sigma contours.

experiments. In principle, one may use de-lensing methods [27] or lensing contributions to CMB bi-spectra [28] to single out the lensing $B$ modes. Furthermore, de-rotation techniques can be used to remove the rotation-induced $B$ modes [29]. More investigations along this line should be done before we can confirm the detection of the genuine $B$ modes.

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