Convergence of Dümbgen’s Algorithm for Estimation of Tail Inflation

ABSTRACT

Given a density \( f \) on the non-negative real line, Dümbgen’s algorithm is a routine for finding the (unique) log-convex, non-decreasing function \( \hat{\phi} \) such that \( \int \hat{\phi}(x)f(x)dx = 1 \) and such that the likelihood \( \prod_{i=1}^{n} f(x_i)\hat{\phi}(x_i) \) of given data \( x_1, \ldots, x_n \) under density \( x \mapsto \hat{\phi}(x)f(x) \) is maximized. We summarize Dümbgen’s algorithm for finding this MLE \( \hat{\phi} \), and we present a novel guarantee of the algorithm’s termination and convergence. We also discuss the asymptotic behaviour of the algorithm, and discuss rate of convergence.