Robust adaptive model-based compensator for the real-time hybrid simulation benchmark

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Summary
This study presents a robust adaptive model-based compensation framework for real-time hybrid simulation (RTHS), capable of minimizing synchronization errors with uncertain experimental substructures. The initial conditions of the compensator are defined using a nominal model of the transfer system without consideration of specimen–actuator interaction. Then, robust calibration of the compensator is obtained through offline numerical simulations using particle swarm optimization. The proposed methodology is validated in a virtual RTHS benchmark problem but incorporates more complex scenarios such as uncertain and nonlinear experimental substructures for the same compensator design. The results show excellent accuracy and robustness of the proposed methodology, with quick adaptation for different substructuring scenarios. Furthermore, this methodology proves that a robust compensator designed independently from the experimental substructure can be helpful to avoid tedious calibration and early tests of the physical specimen, with unintentional premature damage effects.

KEYWORDS
adaptive compensation, benchmark problem, real-time hybrid simulation, robustness, optimization

1 | INTRODUCTION

Real-time hybrid simulation (RTHS) is a technique for structural testing that combines experimental testing with numerical simulation. A structural component of interest is studied experimentally in the laboratory, while the rest of the structure is represented numerically. This technique has emerged as a cost-effective alternative compared to shake table testing because only a particular structural component is physically tested, reducing the costs of each specimen and the laboratory capacity requirements. Additionally, since the experimental and numerical substructure is tested in real time, including the interaction between substructures, this technique results in a realistic loading history over the experimental substructure. At the same time, it allows studying the effects of the experimental substructure on the global structural response.

In RTHS, the equation of motion (EOM) of the numerical substructure is solved using numerical integration schemes. Then, for the compatibility of displacements, the numerical substructure's response is imposed on the...
The synchronization in the boundary conditions between numerical and experimental substruc-
tures is crucial for RTHS. Since the transfer systems have intrinsic dynamics, its response is delayed from the commanded displacements and could also have amplitude errors. The delay error in the boundary conditions results in delayed experimental forces sent back to the numerical substructure. This error produces an effect equivalent to negative damping, which can lead to an unstable behavior of the hybrid system. Although the delay mainly depends on the transfer system and the experimental substructure, the hybrid system’s sensitivity to the delay depends principally on the partitioning configuration. Methods such as the predictive stability indicator or the dynamical stability analysis serve to estimate the critical delay that causes instability for a particular partitioning, avoiding choices that are highly susceptible to instability. Another robust approach includes uncertainties and delay compensation in the stability analysis.

The delay error affects not only the stability of the RTHS test but also the reliability of the results. Therefore, it is necessary to reduce the synchronization error as much as possible. Several dynamic compensation methods are available in the literature to control the transfer system connected to the experimental substructure. One of the most utilized methods is the polynomial extrapolation, where the delay of the transfer system is assumed as a constant value. However, the delay produced by the transfer system is frequency dependent, and it is affected by the interaction between the transfer system and the experimental substructure. This phenomenon is known as control–structure interaction. Therefore, more sophisticated compensation methods have been developed, such as model-based compensation, where the compensation is based on a model of the transfer system connected to the experimental substructure. A more recent approach by Zhou and Li takes account of the modeling error of the identified control plant and handles it through a two-stage feedforward compensation method. Model-based compensation techniques exhibit excellent performance in RTHS. Still, two major disadvantages are recognized in this framework. First, a previous test is generally performed to obtain a control plant model, including the experimental substructure, which can prematurely alter the physical properties of the specimen. Second, the performance of the compensation method is not guaranteed if there are considerable uncertainties, nonlinear behavior, or time-varying properties in the control plant.

To achieve a better performance in the presence of uncertainties or nonlinearities, several authors have introduced adaptive control to the RTHS problem. Cha et al. and Palacio-Betancur and Gutierrez Soto utilize a feedforward controller where the control parameters are updated in the time domain using least-squares methods. Other approaches adjust the control parameters using a frequency domain analysis such as Tao and Mercan or Xu et al. Some techniques consider discrete models of the control plant. Similarly, Tao and Mercan utilize model-based compensation with adaptation in discrete form using a least-squares approach. Moreover, nonlinear control combined with adaptive control such as sliding mode control, self-tuning regulator, and backstepping have been proposed. In general, adaptive compensation methods have demonstrated excellent performance, but the design and calibration depend on several parameters, initial conditions, and adaptation constraints. Therefore, it generally requires a good a priori knowledge of the control plant, including the interaction with the experimental substructure. Furthermore, there are no robustness guarantees in adaptive control for uncertain plants with noisy experimental data.

This study aims to design a robust controller for a well-known transfer system model with an uncertain experimental substructure. For this purpose, adaptive model-based compensation (AMBC) is implemented with initial conditions based on a nominal model of the transfer system without interaction. Then, during the test, the control parameters are updated using adaptive control to capture the specimen interaction. The adaptive law depends on a set of adaptive gains calibrated to ensure fast adaptation and excellent compensation. Therefore, an optimization process is carried out to find optimal gains for several numerical simulations with stochastic entries.

The structure of the paper is presented as follows. Section 2 explains the AMBC method and its calibration process. Then, Section 3 presents the implementation of the proposed compensation method in the virtual RTHS benchmark problem, which is a recognized problem to evaluate compensation approaches. Also, additional validation scenarios are formulated for the benchmark problem to include different experimental substructures with more uncertainties and nonlinear behavior. The nonlinear models considered in this study include stiffness and strength degradation of the experimental substructure properties. Following, we present a robust adaptive compensator for the benchmark problem in Section 4, where a detailed explanation of the design and calibration process is given. Subsequently, the results of different simulations are presented in Section 5, demonstrating excellent accuracy and robustness. Finally, Section 6 discusses the conclusions of this study.
2 METHODOLOGY

2.1 Adaptive model-based compensation

The control problem in RTHS consists of minimizing the synchronization error between the numerical and experimental substructure displacements. Therefore, the goal is to control the transfer system connected to the experimental substructure to achieve the desired displacement.

For this purpose, adaptive model-based compensation (AMBC) based on Chen et al\(^{30}\) is employed. The general architecture is presented in Figure 1, where the displacement to be imposed is defined as target displacement \(x_t\). A feedforward control that depends on a set of parameters \(A = \{a_i\}\) takes the target displacement to generate the command signal \(x_c\) for the control plant. Consequently, the control plant achieves a measured displacement \(x_m\), and the goal is to find the control parameters \(A\) such that the synchronization error \(\tilde{x} = x_m - x_t \to 0\) in finite time.

Let the control plant to be approximated in the frequency domain by a third-order transfer function without zeros, denoted as \(G_p(s)\). Then, Equation (1) provides the dynamics of the control plant:

\[
x_m(s) = G_p(s)x_c(s) = \frac{1}{a_3s^3 + a_2s^2 + a_1s + a_0}x_c(s),
\]

where \(A = \{a_i\}, i = \{0, 1, 2, 3\}\), are the coefficients of the transfer function \(G_p(s)\); \(s\) is the Laplace variable; \(x_c\) is the command displacement; and \(x_m\) is the measured displacement. To compensate for the control plant dynamics, the inverse of \(G_p(s)\) is implemented to generate the command signal from the target displacement as shown in Equation (2):

\[
x_c(s) = G_p^{-1}(s)x_t(s) = (a_3s^3 + a_2s^2 + a_1s + a_0)x_t(s),
\]

where \(G_p^{-1}(s)\) is the inverse feedforward controller and \(x_t\) is the target displacement from the numerical substructure. Notice that the transfer function presented in Equation (2) is improper; however, the controller can be implemented using a backward difference approximation as shown in Equation (3):

\[
X = \begin{bmatrix} x[i] \\ \dot{x}[i] \\ \ddot{x}[i] \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/\Delta t & -1/\Delta t & 0 & 0 \\ 1/\Delta t^2 & -2/\Delta t^2 & 1/\Delta t^2 & 0 \\ 1/\Delta t^3 & -3/\Delta t^3 & 3/\Delta t^3 & -1/\Delta t^3 \end{bmatrix} \begin{bmatrix} x[i] \\ x[i-1] \\ x[i-2] \\ x[i-3] \end{bmatrix},
\]

where \(X\) is the regressor vector containing the derivatives of signal \(x[i]\) at time step \(i\) and \(\Delta t\) is the discrete-time step of the simulation. Nevertheless, other finite difference rules can be used. Finally, the command signal for the actuator can be obtained from Equation (4):

\[
x_c = AX_t = [a_0 \ a_1 \ a_2 \ a_3][x_t \ \dot{x}_t \ \ddot{x}_t]^T,
\]
where $A$ is the control parameters vector and $X_t$ is the target regressor vector obtained from Equation (3) with target displacement $x_t$ as argument.

Notice that the control signal is generated with an inverse feedforward controller with the estimated control plant parameters. Although the AMBC the controller is formulated in the frequency domain, the implementation in the time domain in Equation (4) is equivalent to the feedforward control presented in Adaptive Time Series (ATS)$^{19}$ and Conditional Adaptive Time Series (CATS)$^{20}$ The difference between the AMBC, the ATS, and CATS is mainly the algorithm chosen to update the control parameters. In the AMBC, the parameters are updated with an adaptive gradient law. In contrast, the ATS updates its parameters with the standard least-squares method considering a moving window of measured and commanded data. In contrast, the CATS parameters are updated with the recursive least-squares method. These approaches can reach similar results, but they also depend on carefully chosen design parameters to provide effective adaptation. Furthermore, both CATS and ATS are presented in the literature for first and second-order compensators, and there is no evidence of its performance for higher order models.

In the proposed methodology, the initial parameters for $a_i$ are obtained from a priori identified model of the transfer system without specimen interaction. Then, these parameters are updated in real time with the adaptive law presented in Chen et al.$^{30}$ In the original AMBC, the parameter $a_0$ (known as DC gain) was taken equal to unity and is not subject to adaptation. But in this study, the $a_0$ term is considered in the adaptation process since the experimental substructure properties can alter the DC gain of the control plant.

The adaptation process is presented as follows. The commanded signal $x_c$ can be approximately estimated by inverting Equation (1), resulting in the following expression:

$$\hat{x}_c = AX_m = [a_0 \ a_1 \ a_2 \ a_3] [x_m \ \dot{x}_m \ \ddot{x}_m \ \dddot{x}_m]^T,$$

where $\hat{x}_c$ is the estimated command signal and $X_m$ is the measured regressor vector obtained from Equation (3) with measured displacement $x_m$ as argument. Also, the derivatives of $x_m$ are obtained in two steps. First, $x_m$ is filtered with a Butterworth filter with to remove high-frequency noise. Then, the measured regressor vector $X_m$ is calculated with Equation (3). In the original AMBC, a low-pass filter $\Lambda(s) = 1/(1+s)^3$ is employed to make proper transfer functions and obtain the derivatives of $x_m$. This low-pass filter affects the amplitude of the measured signal deteriorating the adaptation process. For this reason, the filter is changed to a Butterworth filter, just like in other methods such as ATS$^{19}$ and CATS.$^{20}$

With Equation (5) and the command signal $x_c$, an indirect estimation error of the parameters $a_i$ can be obtained as shown in Equation (6):

$$e = \frac{z - \hat{x}_c}{m^2_e},$$

where $e$ is the parameter estimation error, $z$ is the command signal $x_c$ filtered with the same Butterworth filter to synchronize with the filtered measured signal $x_m$, $\hat{x}_c$ is the approximated command signal obtained with Equation (5) with the estimated parameters $a_i$, and $m^2_e$ is a normalizing signal used to bound $e$ and taken as $m^2_e = 1 + (X_m^T X_m)$. Then, to update the parameters $a_i$, a cost function is formulated in Equation (7):

$$C = \frac{e^2 m^2_e}{2},$$

where $C$ is defined as the cost function. Finally, Equation (8) is the adaptive law based on the gradient descent method, which minimizes the cost function $C$ and is implemented to update the parameters $a_i$:

$$\dot{A} = \Gamma e X_m,$$

where $\dot{A} = [a_0 \ \dot{a}_1 \ \dot{a}_2 \ \dot{a}_3]$ contains the rate of change of the adaptive parameters $a_i$ and $\Gamma$ is a diagonal adaptive gain matrix associated with the adaptation rate of parameters $a_i$. The implementation of the AMBC is summarized in the flowchart of Figure 2.
Additionally, it is worth mentioning that the adaptive controller presented in this paper can be augmented with feedback control, such as PI control or LQG control. However, this study did not consider feedback control for two reasons. First, to demonstrate the efficiency and robustness of the AMBC by itself. It is worth mentioning that a feedforward controller has more authority to compensate for delay errors than feedback controllers. The second reason is that modern feedback control requires an accurate control plant model, including the interaction with the experimental substructure. Notice that adequate feedback control can improve the compensation, but a bad design can result in instability of the close-loop of the controller–plant system. The robustness of a feedback controller depends on the level of accuracy of the control plant model and whether model uncertainty, exogenous disturbances, and noise are considered in the controller design. More often than not, modern feedback control design is an ad hoc process, where robustness can be a competing objective to other controller specifications (e.g., reference tracking).

### 2.2 Robust calibration of AMBC

The compensator must be designed to provide excellent synchronization and quick adaptation, so special attention is given to selecting initial conditions $A_{\text{init}}$ and adaptive gains $\Gamma$. Initial conditions are determined according to the transfer system dynamics, but choosing the adaptive gains could be challenging. Slow adaptation occurs with small adaptive gains, and as a consequence, it can result in uncompensated delay during a part of the test. On the other hand, high adaptive gains can result in instability of the adaptive law, especially in the presence of noise or disturbances. Therefore, it is necessary to choose the adaptive gains carefully to achieve both robustness and performance.

This study proposes a robust calibration of adaptive gains in AMBC (rAMBC). The design and calibration process is schematized in Figure 3. The first step is to obtain a model of the transfer system without specimen interaction in the form of Equation (1). Then, a feedforward controller is designed, taking the inverse of the initial model. Next, the adaptive gains $\Gamma$ are robustly calibrated through an offline optimization procedure. This robust calibration requires numerical simulations with the following components:

1. A set of target displacements $x_t$ with sufficient information on the frequency range of interest (i.e., persistently exciting signals). These data can be obtained from calibration structures subjected to ground motions of interest. Notice that the ground excitation is extremely uncertain in earthquake engineering, but it is chosen a priori for structural performance assessment in RTHS.
2. A controller based on an initial model, with initial parameters $A_{\text{init}}$.
3. A low-pass filter with a cut-off frequency according to the frequencies of interest according to the test specifications.

It is necessary to consider the frequency content of ground motions and the natural frequencies of the reference structure. Additionally, the operation range of the transfer system is crucial to define the target frequency range of interest.
4. Virtual control plants with random parameters. These plants are required to be different from the identified model of the transfer system without a physical specimen. Consequently, the controller requires adaptation to reach an acceptable tracking of the virtual plant response $x_m$.

At each iteration of the optimization process, a specific set of adaptive gains $\Gamma_k$ are defined, where index $k$ corresponds to the $k$th iteration. A finite number of simulations $N$ are carried out with random target displacements, and a performance indicator $J_{2n}$ is computed, corresponding to the normalized root mean square error (NRMSE) between target and measured displacements:

$$J_{2n}(\Gamma_k) = \frac{\mathbb{E}[\left( (x_{tn} - x_{mn})^2 \right)]}{\mathbb{E}[x_{tn}^2]} ,$$

where $\mathbb{E}[\cdot]$ is the expectation operator and index $n$ is associated with the $n$th simulation. Subsequently, the average for the $N$ simulations corresponds to the objective function $R_2$ as a function of the specific set of adaptive gains $\Gamma_k$.

$$R_2(\Gamma_k) = \frac{1}{N} \sum_{n=1}^{N} J_{2n}(\Gamma_k).$$

Then, new adaptive gains are defined through the optimization algorithm to minimize the objective function $R_2$. This process is carried out until the optimization algorithm converges to the optimal gains $\Gamma^*$. Notice that the goal of the calibration process is to find adaptive gains $\Gamma^*$ that ensure the adaptation capacity maintaining excellent tracking performance and robustness guarantees and not to find specific adaptive parameters $a_i$. This procedure is repeated with the initial nominal model for different perturbed virtual plants. Finally, once the adaptive gains converge to a solution, the calibration of the rAMBC is over, and the controller is ready for implementation for RTHS.

It is worth mentioning that the number of simulations $N$ depends on the number of selected ground motions and the number of samples required to quantify the uncertainty in the calibration process. On the other hand, the number
of iterations depends on the optimization method of choice. Both quantities must be selected according to each specific application. A detailed design example for the robust AMBC is presented in Section 4.

3 | NUMERICAL APPLICATION: THE BENCHMARK PROBLEM

The virtual RTHS benchmark problem from Silva et al.\textsuperscript{31} is selected as a platform to evaluate the performance of the proposed compensation method. The reference structure consists of a three-story moment frame with three lateral degrees of freedom, as shown on the left side of Figure 4. The reference structure is divided into a numerical substructure and a linear single-degree-of-freedom (SDOF) experimental substructure, as shown on the right side of Figure 4. The equation of motion (EOM) of the reference structure in state-space form is expressed in Equations (11) and (12):

\[
\begin{align*}
\{ \ddot{x}_r \} = & \begin{bmatrix} 0 & I \\ -M_r^{-1}K_r & -M_r^{-1}C_r \end{bmatrix} \{ x_r \} + \begin{bmatrix} 0 \\ -Y \end{bmatrix} \{ \ddot{u}_g \}, \\
\{ x_r \} = & \begin{bmatrix} I & 0 \end{bmatrix} \{ x_r \} + \begin{bmatrix} 0 \end{bmatrix} \{ \ddot{u}_g \},
\end{align*}
\]

(11)

(12)

where \( M_r, K_r, \) and \( C_r \) are the reference mass, stiffness, and damping matrices, respectively. \( x_r(t), \dot{x}_r(t), \) and \( \ddot{x}_r(t) \) are the reference displacement, velocity, and accelerations vectors, respectively, all measured relative to the ground motion. \( \ddot{u}_g(t) \) is the ground acceleration, and \( Y = [1 \ 1 \ 1]^T \) is the seismic influence vector.

Once the reference structure is partitioned, the EOM of the numerical substructure is reordered in Equations (13) and (14) with ground acceleration and experimental force as inputs and the lateral displacements as outputs:

\[
\begin{align*}
\{ \ddot{x}_n \} = & \begin{bmatrix} 0 & I \\ -M_n^{-1}K_n & -M_n^{-1}C_n \end{bmatrix} \{ x_n \} + \begin{bmatrix} 0 \\ -M_r^{-1}M_rY -M_n^{-1}P \end{bmatrix} \{ \ddot{u}_g \}, \\
\{ x_n \} = & \begin{bmatrix} I & 0 \end{bmatrix} \{ x_n \} + \begin{bmatrix} 0 \\ \ddot{f}_e \end{bmatrix},
\end{align*}
\]

(13)

(14)

where \( M_n, K_n, \) and \( C_n \) are the numerical mass, stiffness, and damping matrices, respectively. \( x_n(t), \dot{x}_n(t), \) and \( \ddot{x}_n(t) \) correspond to the displacement, velocity, and acceleration of the numerical substructure, all relative to the ground.
Meanwhile, $f_e$ corresponds to the feedback force from the experimental substructure acting with an influence vector $p = [1 \ 0 \ 0]^T$. $f_e$ for a linear experimental substructure is defined in Equation (15):

$$f_e = k_e x_m + c_e \dot{x}_m + m_e \ddot{x}_m,$$

(15)

where $m_e$, $k_e$, and $c_e$ are the mass, stiffness, and damping of the experimental substructure. $x_m, \dot{x}_m, \ddot{x}_m$ correspond to the measured (experimental) displacement, velocity, and acceleration imposed on the experimental substructure. Then, the response of the first degree of freedom of the numerical substructure $x_{n1}$ is defined as the target displacement $x_t$ to be imposed on the experimental substructure to enforce compatibility in the interface between substructures. It means, in the ideal case, the following condition must be satisfied: $x_m = x_t = x_{n1}$.

Henceforth, a transfer system (i.e., actuator) is connected to the experimental substructure to impose the target displacement. The interaction between the transfer system and experimental substructure is defined as a control plant. A linear control plant model is illustrated in Figure 5 where the parameters are listed in Table 1. The parameters are represented as uncorrelated random variables with normal distribution with mean (nominal) and a standard deviation provided in Table 1. Additionally, to get a more realistic representation of the virtual control plant, the measured force $f_e$ and displacement $x_m$ signals are contaminated with random noise modeled as band-limited white noise to represent physical sensors. The numeric implementation is realized in Simulink using a fourth-order Runge-Kutta solver (ode4) with a fixed time step of $\Delta t = 1/4096$ s.

Four partitioning cases (cases I to IV) are proposed in the benchmark problem with different reference structure properties but identical experimental substructures. Meanwhile, this study includes two additional cases to consider a broad spectrum of scenarios. The reference structures are listed in Table 2. We propose cases V and VI to consider different linear experimental substructures with random properties uniformly distributed between the limits detailed in Table 3.

![Figure 5](image_url)  
**Figure 5** Block diagram of the control plant. Adapted from Silva et al. 31

| Parameter | Mean       | Standard deviation | Units     |
|-----------|------------|--------------------|-----------|
| $\alpha_1\beta_0$ | $2.13 \cdot 10^{13}$ | --                  | m $\cdot$ Pa/s |
| $\alpha_2$ | $4.23 \cdot 10^6$ | --                  | m $\cdot$ Pa    |
| $\alpha_3$ | $3.3$ | $1.3$              | 1/s       |
| $\beta_1$ | $425$ | $3.3$              | -          |
| $\beta_2$ | $10^5$ | $3.31 \cdot 10^3$ | 1/s       |
| $m_e$    | $29.1$ | --            | kg        |
| $c_e$    | $114.6$ | --                | N s/m     |
| $k_e$    | $1.19 \cdot 10^6$ | $5 \cdot 10^4$       | N/m      |

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TABLE 1 Control plant parameters 31
### TABLE 2  Reference structure properties for each case

| Case | Mass/floor (kg) | Natural frequencies (Hz) | Modal damping ratio  
|------|-----------------|--------------------------|-----------------|
| I    | 1000            | (3.61; 16.00; 38.09)     | 5%              |
| II   | 1100            | (3.44; 15.25; 36.32)     | 4%              |
| III  | 1300            | (3.17; 14.03; 33.40)     | 3%              |
| IV   | 1000            | (3.61; 16.00; 38.09)     | 3%              |
| V    | 1400            | (3.05; 13.52; 32.30)     | 5%              |
| VI   | 900             | (3.81; 16.86; 40.15)     | 4%              |

*Each case considers the same damping ratio for the three modes.

### TABLE 3  Bounds of random experimental substructure properties for cases V and VI

| Parameter | Minimum | Maximum | Units |
|-----------|---------|---------|-------|
| $m_e$     | 10      | 40      | kg    |
| $c_e$     | 50      | 1000    | N s/m |
| $k_e$     | $10^4$  | $2 \cdot 10^6$ | N/m   |

*Note: Random parameters with uniform distribution.*

### FIGURE 6  Control plant and Butterworth filter Bode diagrams

### 4  ROBUST AMBC FOR THE BENCHMARK PROBLEM

This section provides details of the necessary steps to design a robust adaptive model-based compensator (rAMBC) for the virtual RTHS benchmark. The source code developed to solve this problem is openly available in a Zenodo repository.\(^{35}\)

The rAMBC requires a model of the transfer system without any specimen interaction, namely, as the initial model. In laboratory, this model can be obtained from system identification; but in this numerical example, the initial model is assumed from the control plant model in Figure 5 evaluated with a dummy experimental substructure with small properties ($m_e = 0, c_e = 0$, and $k_e = 1$ N/m). The continuous initial model is presented in Equation (16):
\[ x_m(s) = G_p^0(s)x_c(s) = \left( \frac{1}{1.986 \cdot 10^{-7}s^3 + 8.440 \cdot 10^{-5}s^2 + 1.99 \cdot 10^{-2}s + 1} \right)x_c, \]  

(16)

where \( G_p^0(s) \) is the initial model and its Bode diagram is presented in Figure 6. Notice that the initial model presents a considerable time delay, especially for low frequencies (between 15 and 20 ms in the 0–20 Hz frequency range). Furthermore, including a realistic experimental substructure drastically increases the time delay and affects the magnitude error. It is expected that a controller designed for the initial model results in insufficient compensation for the different control plants; therefore, the controller must adapt its parameters to reach acceptable performance.

The initial feedforward controller is formulated taking the inverse of the initial model \( G_p^0(s) \):

\[ x_c = A_{\text{init}}X_t = \begin{bmatrix} 1 & 1.99 \cdot 10^{-2} & 8.440 \cdot 10^{-5} & 1.986 \cdot 10^{-7} \end{bmatrix} [x_t \ x_t \ x_t \ x_t]^T, \]  

(17)

where \( A_{\text{init}} \) is the initial condition for the controller and \( X_t \) is the target regressor vector.

For the noise filter design, three principal factors should be considered: (i) the frequency content of the earthquake; (ii) the frequency content of the structural response, which is mainly affected by the natural frequencies of the reference structure; and (iii) the frequency range of operation of the control plant. The frequency range of interest is considered between 0 to 20 Hz for this study. Although the third natural frequency of reference structure for all cases is higher than 20 Hz, the structural response will be primarily controlled by the first and second modes due to the frequency content of the seismic excitation. Therefore, the selected noise filter is a fourth-order Butterworth with a cut-off frequency of 20 Hz. The Bode diagram of this filter is illustrated in Figure 6. This filter keeps an approximately unity gain for low frequencies but delays the filtered signal. However, this time delay only affects the adaptation process and does not affect the compensation directly.

Additionally, the adaptive parameters \( A = \{a_i, i = 0, 1, 2, 3\} \) can be constrained defining upper and lower bounds for each parameter. In this example, the parameters are lower bound constrained only to strictly positive values \( (a_i > 0) \) because the model in Figure 5 takes only nonnegative coefficients. Also, the adaptive parameters \( a_i \) can be upper bounded according to the expected limits for a particular transfer system and the experimental substructures properties of interest. However, this study does not consider upper bounds to observe free adaptation.

For the robust calibration of adaptive gains, an optimization process is carried out using offline numerical simulations with the Simulink block diagram presented in Figure 7. For each adaptive gain matrix \( \Gamma_k \), a predefined total number of simulations \( N \) are realized with different target displacements and virtual control plants. The target displacement is generated in each simulation using an SDOF calibration structure subjected to a ground motion. Only one ground motion (El Centro 1940 earthquake) is selected for this study’s calibration process, although the proposed methodology can incorporate additional ground motions in the calibration. The peak ground acceleration (PGA) scales and calibration structure properties are modeled as uniform random variables with the specified bounds in Table 4.

On the other hand, the calibration plants do not need accurate models of the specimens tested in RTHS. It only needs to be an approximate model of the transfer system with different levels of specimen interaction. However, the calibration plant must be different from the initial plant to ensure adaptation and assess its robustness. The control plant model can be evaluated with different experimental substructure properties to generate perturbed control plants in the
TABLE 4 Bounds of random parameters considered in calibration structure

| Parameter       | Minimum | Maximum | Units |
|-----------------|---------|---------|-------|
| PGA scale       | 30      | 60      | %     |
| Natural frequency | 2.8    | 4       | Hz    |
| Damping         | 3       | 5       | %     |

Note: Random parameters with uniform distribution.

benchmark problem. In this calibration, the control plants are formulated with the transfer system model in Figure 5 with the nominal parameters of Table 1 and experimental substructures with random uniform properties: $m_c \in [10, 40]$ kg, $c_e \in [50, 1000]$ N s/m and $k_e \in [10^4, 2 \cdot 10^6]$ N/m (i.e., same random properties as in Table 3). The possible range of Bode diagrams for the calibration plants is presented in Figure 6.

For each $n$ th simulation, the NRMSE between $x_m$ and $x_l$ is computed as an error indicator (based on Equation 9):

$$J_{2n}(x_l, x_m) = \sqrt{\frac{\sum_{l=1}^{L} (x_m[l] - x_l[l])^2}{\sum_{l=1}^{L} (x_l[l])^2}} \cdot 100\%,$$

where $J_{2n}$ represents the synchronization error of the $n$ th simulation, $l$ is the discrete-time index, and $L$ is the total length of data in each simulation.

For each set of adaptive gains $\Gamma_k = \text{diag}([\Gamma_0 \ \Gamma_1 \ \Gamma_2 \ \Gamma_3])$, a total of $N = 100$ simulations is considered enough to evaluate the compensator for the random earthquake scales, SDOF properties, and calibration plants. Then, the objective function $R_2$ is computed from Equation (10) for the specific adaptive gains $\Gamma_k$.

An optimization algorithm is utilized to find the optimal $\Gamma^*$ which minimizes the $R_2$. Since $R_2$ depends on four parameters $\Gamma_i, i = \{0,1,2,3\}$, and the $N$ simulations with random inputs, there is no analytical function to compute $R_2(\Gamma)$, so it is very difficult to analyze the search space for $\Gamma$ (i.e., calculate gradients of $R_2$). Consequently, a suitable optimization method should be employed. In this study, we utilize particle swarm optimization in the Global Optimization Toolbox from Matlab. The particle swarm function is utilized with a swarm size of 15 and maximum iterations of 10 (i.e., 150 evaluations with different gains), where each evaluation runs $N = 100$ simulations. This number of evaluations is considerably small for this kind of optimizer. However, it is enough to find excellent results between the selected bounds.

A Matlab function is defined with a vector $\gamma = [\gamma_0, \gamma_1, \gamma_2, \gamma_3]$ as input. This vector $\gamma$ is related to the adaptive gains $\Gamma$ such that

$$\gamma_i = \log_{10}(\Gamma_i).$$

The Matlab function runs the $N$ simulations with the defined adaptive gains and outputs the objective function $R_2$. Additionally, the search space for the adaptive gains is investigated by trial and error to define a constrained but sufficiently wide search space for the optimization. The constraints are selected as $\gamma_0 \in [2,10]$; $\gamma_1 \in [0,8]$; $\gamma_2 \in [-2,6]$ and $\gamma_3 \in [-4,4]$.

After the optimization process, the best result is

$$\Gamma^* = \text{diag}([10^{0.4} \ 10^{6.2} \ 10^{2.1} \ 10^{0.8}]),$$

where the resulting objective function for the robust calibration is $R^*_2 = 0.85\%$.

Afterward, the neighborhood of the optimal solution is explored by evaluating different adaptive gains. The process consists of fixing two terms of the adaptive gain matrix and generating a grid of adaptive gains for the remaining two terms. The results of $R_2$ for different grids of adaptive gains are presented in Figure 8. For example, the first subgraph (top left) corresponds to different values for $\Gamma_0$ and $\Gamma_1$ with fixed optimum values for $\Gamma_2 = \Gamma_2^*$ and $\Gamma_3 = \Gamma_3^*$. These results show that the optimum is located near a small $R_2$ region, demonstrating a wide zone of feasible results. On the other hand, lower and higher gains result in higher values for $R_2$. The worst scenarios are obtained with larger gains due to overshooting in the adaptation process.
An alternative design is to select smaller gains than optimal values to increase the distance with the unfeasible (yellow) zone while maintaining reasonable suboptimal $R^2$ values. The distance from the unfeasible zone to the selected gains is associated with the degree of robustness. Further studies could take this property into account in the calibration process. However, we utilize the optimal gains $\Gamma^*$ for the controller in all the different vRTHS simulations in Section 5.

5 | SIMULATIONS RESULTS

Different vRTHS simulations are carried out with the same compensator designed in Section 4 to assess its performance and robustness. The results are evaluated through the $J_2$ synchronization indicator defined in Equation (9), which measures the synchronization error between numerical and experimental displacements. Although the benchmark problem considers many other performance indicators, only the $J_2$ is considered in this study because a dynamic compensator aims to minimize this error. Furthermore, if the synchronization error in the numerical-physical boundary is minimized, the response of the hybrid system should be closest to the reference structural response.

5.1 | Compensation and adaptation analysis

The vRTHS simulation results for case I with nominal values are presented and analyzed. The structure is subjected to the El Centro 1940 earthquake scaled to 60% of the PGA. The synchronization results are presented in Figure 9, showing excellent displacement tracking. The synchronization error is mainly due to noise in the measured displacement, and the $J_2$ results are under 2%. In any case, a postprocessing stage can filter the results to remove noise if needed.

The adaptation of control parameters $a_i$ is presented in Figure 10, where after 5 s of simulation (i.e., at the beginning of the strong motion), the control parameters present quick adaptation. After 7 s, the adaptation contains a high-frequency response without a smooth convergence. It is possible to consider lower adaptive gains to obtain a smoother adaptation. However, if the compensation results are excellent in terms of performance and robustness, the smoothness of the adaptation is not an issue.

Additionally, since the initial conditions of the compensator are based on the initial model, the adaptation process can be interpreted as an online identification problem. Although only the parameter $a_1$ shows an evident convergence, an identified value is computed for each $a_i$, taking the mean value between 10 and 35 s of simulation (marked in green
in Figure 10). With these identified values, an identified transfer function is calculated and compared with the initial model and the actual control plant in Figure 11.

The identified control plant match very well with the actual control plant for frequencies below the cut-off frequency of the Butterworth filter of the adaptation process (i.e., $f_c = 20$ Hz), which explains the excellent compensation. However, the identified transfer functions match poorly in the high-frequency range for several reasons. First, the control plant is a fifth-order transfer function; thus, the third-order models cannot capture all the dynamics of the control plant. Further, the data considered in the adaptation process is mainly composed of low-frequency signals due to the filtering effects of the structural system over the ground motion.

5.2 vrTHS simulations with uncertainties and different experimental substructures

This subsection presents the results of several simulations with uncertainties in the control plant. Three different earthquakes are selected: (i) El Centro, 1940 (PGA = 0.298g); (ii) Kobe, Japan, 1995 (PGA = 0.578g); and (iii) Maule, Chile, 2010 (PGA = 0.401g). The three unscaled acceleration records are presented in Figure 12. The same six cases previously
defined are subjected to each earthquake with 20 simulations per case to evaluate the compensator’s robustness. For the simulations, each earthquake was scaled to obtain responses according to the capacity of the benchmark’s transfer system.

The Bode diagrams for the simulated control plants are presented in Figure 13. The original benchmark control plant with uncertainties in experimental stiffness and transfer system parameters is presented on the left side of the figure (cases I–IV). In contrast, the right side presents the different control plants simulated for cases V and VI proposed in this study. Notice that these cases cover a broader spectrum of random scenarios, especially in terms of time delay at low frequencies. The synchronization results are presented in Figure 14 for each simulation. The first observation is that all cases present a low $J_2$ error indicator, below 3%. Additionally, the variability for each case is relatively low, demonstrating that the compensator is robust under the uncertainties in the transfer system and experimental substructure. Also, $J_2$ mean values are computed for each case (dashed lines). These values are different between cases and can be explained because all cases have different structural responses. Due to the excellent compensation, the synchronization error is mainly associated to noise.

**FIGURE 11** Adaptation in frequency domain

**FIGURE 12** Acceleration records of considered earthquakes
In this subsection, the same robustly calibrated controller calibrated for linear experimental substructures in Section 4 is evaluated for vRTHS with nonlinear experimental substructures. Different nonlinear constitutive laws are simulated.

**5.3 | Nonlinear experimental substructures**

In this subsection, the same robustly calibrated controller calibrated for linear experimental substructures in Section 4 is evaluated for vRTHS with nonlinear experimental substructures. Different nonlinear constitutive laws are simulated.
The nonlinear models are based on the Bouc–Wen model with some modifications to include strength and stiffness variations.

The experimental force is calculated with the following expression:

\[
f_e = r_e + c_v \dot{x}_m + m_e \ddot{x}_m,
\]

where \( r_e \) is a nonlinear restoring force obtained by solving the following ordinary differential equation (ODE):

\[
\dot{r}_e = R_k k_e \left[ 1 - \frac{r_e}{f_y} \left( \eta_1 \text{sign}(r_e \dot{x}_m) + \eta_2 \right) \right] \dot{x}_m,
\]

where \( R_k \) is a stiffness degradation factor, \( k_e \) is the initial elastic stiffness, and \( f_y \) is the yield force. \( M, \eta_1, \) and \( \eta_2 \) are parameters that control the hysteresis shape. The stiffness degradation factor and the yield force degradation are calculated as

\[
R_k = e^{-\alpha H},
\]

\[
f_y = \frac{f_{yo}}{1 + \beta H},
\]

where \( \alpha \) is a parameter that controls the stiffness degradation, \( \beta \) controls the strength degradation, and \( f_{yo} \) is the initial yield force. Finally, \( H \) is the dissipated hysteretic energy that can be calculated in incremental form as

| Model | \( f_{yo} \) (N) | \( M \) | \( \eta_1 \) | \( \eta_2 \) | \( \alpha \) | \( \beta \) | \( k_h \) (N/m³) |
|-------|----------------|-------|-------|-------|-------|-------|-------------|
| A     | 2380           | 25    | 0.5   | 0.5   | 0     | 0     | 0           |
| B     | 1904           | 1     | 0.6   | 0.4   | 0     | 0     | 6 \times 10^9 |
| C     | 2975           | 5     | 0.5   | 0.5   | 0.01  | 0     | 0           |
| D     | 2678           | 2.5   | 0.5   | 0.5   | 0.02  | 0.005 | 0           |

**FIGURE 15**  Force versus displacement for different nonlinear models
\[ \Delta H = \left( \frac{r_e + (r_e + \Delta r_e)}{2} \right) \left( \Delta x_m - \frac{\Delta r_e}{R_k k_e} \right) \]  

Additionally, to consider a hardening effect, a cubic spring is added with a constant cubic stiffness \( k_h \):

\[ f_e = k_h x_m^3 + r_e + c_x x_m + m_c \dot{x}_m. \]  

The reference structure and the initial experimental substructure properties correspond to case I from the original benchmark problem. Four experimental substructures are defined with the parameters presented in Table 5.

Each case is subjected to the El Centro 1940 earthquake scaled to 60% PGA. The relationship between measured force and displacement for each nonlinear case is presented in Figure 15. The measured forces include inertial, viscous, and restoring components and are contaminated with noise. The nonlinear cases are explained as follows: case A

**Figure 16**  Synchronization results for nonlinear simulations A and B

**Figure 17**  Synchronization results for nonlinear simulations C and D
corresponds to an almost bilinear case, case B is a smooth hysteretic model with hardening, case C considers strength degradation, and case D considers both strength and stiffness degradation.

The target and measured displacements of each simulation are presented in Figures 16 and 17. Additionally, the synchronization subspace plots (SSP) are presented in Figure 18, where the almost 1:1 straight lines show the excellent delay compensation without considerable amplitude errors. In Figures 16 and 17, small errors can be observed, and the $J_2$ indicator for every case is under 4% which demonstrates good compensation. Observing the synchronization error of each case, a part of the error can be attributed to the measurement noise, but in some instances, the error grows considerably. The largest error of each case matches the instant of maximum structural response, where the experimental substructure exhibits a highly nonlinear response and large load reversals. On the other hand, this adaptive controller presents excellent behavior for the time-varying properties of nonlinear case D, which presents notorious stiffness degradation in Figure 15.

**6 | CONCLUSIONS**

This study proposes an AMBC to design a robust controller without prior knowledge of specimen interaction with the transfer system. The initial conditions are taken from a nominal model of the transfer system, avoiding system identification that includes the physical specimen. Additionally, the controller is calibrated to reach a quick adaptation and excellent tracking performance under a set of uncertain parameters. Thus, the same robust controller can compensate the transfer system dynamics with different experimental substructures or time-varying properties.

The proposed methodology is implemented and validated in a virtual RTHS benchmark problem. For this study, we included additional cases with different experimental substructures and nonlinear models with stiffness and strength degradation, thus adding more uncertainty to the problem. The designed controller demonstrates an excellent tracking performance and robustness in all different scenarios.

Future work will include the experimental validation of the proposed methodology and study the incorporation of multiple actuators in the transfer system. Finally, we envision an adaptive compensator with robust calibration designed for a specific transfer system in the laboratory capable of compensating for different and complex experimental substructures, expanding the laboratory’s testing capabilities.
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AUTHOR CONTRIBUTION

Cristobal Galmez: Conceptualization (supporting); methodology (lead); software; formal analysis; validation; visualization; writing – original draft (lead); writing – review and editing (equal). Gastón Fermandois: Conceptualization (lead); funding acquisition; project administration; methodology (supporting); supervision; writing – original draft (supporting); writing – review and editing (equal).

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in a Zenodo repository at https://doi.org/10.5281/zenodo.6016125.35

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