Physics beyond Causality

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Abstract

The notion of the space-time in physics is analyzed. The existence of space-time continua different from the observer’s one is investigated. The physical reality represented with help of causality seems incomplete and the reconsideration of the paradigm of cognoscibility is needed. The theory of sets predicts an infinite number of levels of cognition, where the world around seems more and more disordered and chaotic. The possibilities of different levels of cognition are estimated from this point of view. It is shown, that relativistic and quantum theories operate on different levels of cognition, so their unification seems doubtful.

Keywords: Space-time, General relativity, Quantum theories, Theory of cognition.

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Introduction

Whether space-times, different from the observer’s one, can exist? How can we describe them, and what physical situation it may concern? How could we observe the physical objects from different space-times, and do they have correspondences with known ones? All of these questions together with the arising epistemological problems are being considered and discussed.

1 Space-Time

An observer perceives matter through the frame of references \( S : x \equiv (x^0, \ldots, x^{n-1}) \), which is a set of marks, i.e. coordinates or events in space and time. In the Newtonian-Galilean scheme (NG) and in Special Relativity (SR) the space-time \( G \) is equivalently described from a class of inertial frames \( \{S|T\} \) related with each other by a group of transformations \( T \). Transformations \( T \) defines the Galileo group in NG, and they are generalized by enlarging to the Lorentz one in SR (see, for example, L. Ryder Ref.[11], 2009).

The ‘origin’ of inertiality is unknown and usually postulated. Without this postulate, the space-time is not determined. For example, two continuums \( G : \{(x^0, \ldots, x^{n-1})\} \) and \( \tilde{G} : \{(\tilde{x}^0, \ldots, \tilde{x}^{n-1})\} \) seem equivalent for the space-time description. However, if we declare the transformation \( x^i = \arctan \tilde{x}^i \) (or \( \tilde{x}^i = \arctan x^i \)) between their coordinates, so one continuum is mapped ‘point-to-point’ into the \( n \)-dimensional cube of another one (see Figure[1A]). Which continuum will be appropriate for the space-time description now?

In practice, to distinguish the inertial frame from the arbitrary, non-inertial one, the observer needs to use a test particle. However, any particle is a part of matter investigated by the observer, so all particles belong to this matter by definition and ‘off-site’ particles different from the ‘ordinary’ matter cannot exist in SR (and NG) representations. The observer
describes the physical objects to which he belongs himself, so there is a ‘hidden’ axiomatic interconnection between space-time $G_{SR}$ with some initial inertial frame $S$ of the observer and matter $M$:

$$G_{SR} : \{S|T\} \leftrightarrow M.$$  \hspace{1cm} (1)

Thus, space-time representations in SR are logically close for the observer: he tests his own space-time by particles of matter, to which he belongs himself. Off-site matter, off-site space-times, such parallel worlds, even if exist, are undistinguished for the observer in SR from nonexistent.

The ‘hidden’ axiomatic interconnections between space-time and matter become explicit in General relativity (GR), where the gravitating matter can influence to the space-time topology. According to the equivalence principle between accelerating frames and gravitational fields, one may compensate gravitational fields in a small enough local region $U_x$ of any point $x$, so the local frame of references in GR is an SR one $G_{GR}^U \approx G_{SR} : \{S(x)|T\}$. Local frames $S(x)$ are interconnected with each other and with ‘infinitely distant’ from matter the initial inertial frame $S$ by continuous coordinate transformations, which depend on distribution of the gravitating matter $M$. Thus, SR representations of a space-time are generalized to the GR space-time manifold:

$$G_{GR} : \{S, \{S(M)|T\}\} = M.$$  \hspace{1cm} (2)

As in SR, the observer in GR perceives his own space-time, his ‘ordinary’ matter to which he belongs himself, so this system of representations seems also logically closed for him. In other words, the space-time in GR correlates with the ‘ordinary’ matter $M$ and, hence, with the observer, who is a part of this matter.

However, if one closed system exists, we do not have reasons to decline the existence of other such systems – i.e. off-site space-times $\tilde{G}$ correlated with their own matters $\tilde{M}$ and ‘initial frames’ $\tilde{S}$ of other ‘off-site observers’. There is a possibility in GR to make off-site continuums ‘distinguished from nonexistent’. As far as in GR the ‘ordinary’ matter can explicitly influence to the space-time, we may suppose that off-site matters $\tilde{M}$ with corresponding space-times $\tilde{G}$ can be involved in gravitational interactions with other continuums, thus unifying in Global system (GS):

$$\tilde{G} : \{\tilde{S}, \{\tilde{S}(M, \tilde{M}(1), ...) | \tilde{T}(1)\}\} = \tilde{M}(1).$$  \hspace{1cm} (3)

$$\tilde{G}(1) : \{\tilde{S}(1), \{\tilde{S}(1)(M, \tilde{M}(1), ...) | \tilde{T}(1)\}\} = \tilde{M}(1).$$

$$\vdots$$

Figure 1: Simplified relations between continuums (A) and the Global system of interconnected continuums (B).
Off-site space-times do not need to have similar topologies, i.e. the same dimensionalities, invariant groups of transformations, connectivities, arrangements, etc. It is underlined by using separate notations $\tilde{T}_{(i)}$ for groups of equivalent frames in off-site space-times $\tilde{G}_{(i)}$.

In the general case, not all matter, but some parts of it from common regions $D \subset G$ and $\tilde{D} \subset \tilde{G}$ may be involved in gravitational interaction. There are shown three possible cases of interconnections between continuums on Figure 1B: the general case when $G \supset D \rightleftharpoons \tilde{D} \subset \tilde{G}$ ($\tilde{G}_{(j)}$) and two particular cases when $D \equiv G$ ($\tilde{G}_{(j)}$) and $\tilde{D} \equiv \tilde{G}$ ($\tilde{G}_{(k)}$).

Thus, Special relativity generalizes the Newton-Galilean scheme by introducing the mutual dependence of space and time. SR is generalized in General relativity by including the dependence of space-time on matter. GR is being generalized in Global system by introducing the dependence of the observer’s space-time also on off-site matters.

2 Paradigm of Cognoscibility

In spite of the principal possibility, the existence of off-site matter still meets some objections. The first one: “if we have no particular answer to a question: ‘what physical situation does this describe?’, then it remains an interesting case, but perhaps only in the mathematical sense” (L. Ryder, 2010). Another objection comes from the theory of cognition, but it occurs in close connection with the first one.

The paradigm of cogniscibility in science declares that the world around is cognizable. One usually interprets it as follows: the world around is accessible for human cognition and even if the unknown regions exist, they are only undiscovered yet. Such interpretation has logical justification. If something can influence us, so we are able to perceive it and to cognize. If not, in practice, it is undistinguished for us from nonexistent. That is why science deals with cognizable real world, called physical reality, and considers ‘off-site matters’ and ‘outside worlds’ only as some speculations.

However, what does ‘cognizable’ mean? The fundamental properties of objects in quantum physics are declared as ‘outside the human imagination’, ‘outside the everyday experience’, etc. The quantum mechanics is considered as “an anti-intuitive discipline ... full of mysteries and paradoxes, which we do not understand well enough, but are able to use” (M. Gell-Mann Ref.[5], 1981). The observations in cosmology lead to the ‘absurd model’ of the universe, where “most of the universe is made of something fundamentally different from the ordinary matter we are made of. ... The contribution of ordinary matter to the overall mass-energy budget has been shown to be small, with more than 95% of the universe existing in new and unidentified forms of matter and energy” (W. Freedman, M. Turner Ref.[4], 2003).

All of this casts doubts on the above interpretation of the paradigm of cogniscibility and, hence, on standard GR representations. Indeed, the observer creates the system of representations about the world around by interpreting events and interconnecting them by cause-effect chains. According to the existing paradigm of cogniscibility, we believe, that it is possible to interconnect by cause-effect chains all events at any infinitesimal interval in the space-time and to arrange them. It predetermines the use of real numbers for the space-time description, predetermines the continuality and connectivity of the space-time for us. In fact, representing (and even ‘constructing’) the space-time as a continuum, we limit the kind of interconnections between events of the world around by continuous functions, basing only on the convenience for the observer to arrange events by cause-effect chains. However, it is known from the mathematical theory of
sets (see, for example, Refs. [2, 3]) that continuous interconnections are not the only ones. The power or the cardinal number of continuous functions is \(\aleph_1\), but, for example, real functions of any kind from real argument have cardinality \(\aleph_2\). There exist sets with cardinalities \(\aleph_3, \aleph_4\ldots\) up to infinity. Thus, generally, cause-effect chains cannot represent all possible interconnections, and possibilities of deterministic methods based on causality seem quite limited.

For example, two sets of integer numbers \(\{n\}\) and \(\{m\}\), where \(n = m\), are equivalent. However, if we ‘shift’ them related each other, putting \(n = m + \frac{1}{2}\) (or \(m = n - \frac{1}{2}\)), sets would not even intersect with each other. In this sense, they are ‘parallel’ to each other. What set \(\{n\}\) or \(\{m\}\) is now a set of integer numbers? – it depends on the ‘observer’, i.e. is a matter of choice of ‘initial’ frame. One can continue. Two sets of rational numbers \(\{x\}\) and \(\{y\}\) are equivalent, but when \(x = y + \pi\) (or \(y = x - \pi\)), they do not intersect with each other. That is why, when we understand ‘continuum’ as a set of elements or events continuously interconnected with each other, any discontinuous transformation of its elements (or a part of elements) will create ‘off-site’ continuum nonintersecting with the space-time of the observer’s continuum and unreachable from it. Continuums will seem like ‘parallel worlds’ differed from each other by the ‘point of view’, i.e. by the ‘initial frame of references’ or by the ‘internal observer’.

It looks like ‘mysteries and paradoxes’ mentioned above and ‘something fundamentally different from the ordinary matter we are made of’ lie exactly beyond the continuum of the observer, beyond his cause-effect chains, i.e. beyond causality. It was shown in previous Section, that the GR representations about space-time are a closed system for the observer. It is a crucial point, but not the end of cognition, only a reason to use other methods. For example, the representations of Global system include interconnections between off-site space-times, so exceed the possibilities of description with help of causality inside each space-time continuum.

We may specify the new interpretation of Paradigm of Cognoscibility as:

- The world around is cognizable, but the possibility of its cognition with help of causality is limited.

Thus, we have a background to consider off-site matters and off-site space-times in representations of Global system Eq. (3) with a hope to find correspondences with physical objects like ones of quantum physics and objects beyond ‘the ordinary matter we are made of’ in cosmology.

### 3 Off-site Action

E. Schrödinger Ref. [13] had come to possibility of the existence of off-site continuums as long ago as 1950: “The general theory of relativity generates conservation laws inside itself and not in the form of consequences of the field equations, but in the form of identities. There is the following. If you are considering the integral \(I = \int R d^4 x\), in which \(R\) is, of course, an invariant density, so the four identical ratios between Hamiltonian derivatives of \(R\) just follow from the only one fact of the general invariance of this integral and these ratios are of type of conservation laws. ... These identities are not the only ones. Any scalar density generates some system of identities. ... If one has two different densities, it may seem that their conservation laws and field equations do not have any correspondences, as though stay off-site from each other”. Nevertheless, there is no possibility in standard GR representations to interpret different scalar densities.
In GS, different scalar densities $\tilde{R}$ are quite appropriate for the description of off-site continuums $\tilde{G}$. Analogously to continuum in GR, one may consider the off-site integral:

$$\tilde{I} = \int_{\tilde{G}} \tilde{R} d\tilde{x} = |\tilde{M}|.$$  \hfill (4)

Such integrals are taken over the whole space-time, so their values may depend only on quantity characteristic of matter, for example, on amounts of matter $|M|$ or $|\tilde{M}|$ in corresponding continuums, or with corresponding amounts of matter $|M|^D$ or $|\tilde{M}|^D$ in some regions $D \subset G$ or $\tilde{D} \subset \tilde{G}$. If integrals $I$ and $\tilde{I}$ are infinite (they usually are), the ratio $I/\tilde{I} = |M|/|\tilde{M}|$ may have sense in passage to the limit.

According to GS representations Eq.(3), we define the off-site action between continuums as:

- **Continuums are in off-site action with each other, if corresponding parts of matter from their common regions are involved in gravitational interaction in both continuums.**

Thus, if continuums $G$ and $\tilde{G}$ are interconnected on common regions $G \subset D \rightleftarrows \tilde{D} \subset \tilde{G}$, their general integrals $I = \int_G R d\tilde{x}$ and $\tilde{I} = \int_{\tilde{G}} \tilde{R} d\tilde{x}$ include the influence of corresponding parts of matter $|M|^D = |M'|^D + |\tilde{M}'|^D = |\tilde{M}|^D$ of previously independent continuums (marked with apostrophes) and the integrals over common regions are:

$$\int_D R d\tilde{x} = \int_{D'} R' d\tilde{x}' + \int_{D'} \tilde{R}' d\tilde{x}' = \int_D \tilde{R} d\tilde{x}.$$ \hfill (5)

Off-site action has an integral character and does not install the point-to-point correspondence between common regions. In this sense (as it was shown in previous Section), continuums do not intersect with each other, even if common regions coincide with their space-time continuums $(G \equiv D \rightleftarrows \tilde{D} \equiv \tilde{G})$. Thus, the interacting continuum is still a closed system for the internal observer, so he perceives the off-site continuums and off-site matter as ‘parallel worlds’. Such ‘nonintersecting’ continuums and the alteration of their space-time curvature during the off-site action is schematically shown on Figure 2.

The off-site matter acts inside common region $D$, so Eq.(5) may be represented on the observer’s continuum as an action integral over this region of some function $\Lambda$ including the influence of off-site matter:

$$I^D = \int_D \sqrt{-g} d\tilde{x}, \Lambda = \left\{ \Lambda^c + \Lambda_s, \Lambda^c \cdot \Lambda_s \right\},$$ \hfill (6)

where $R = \Lambda \sqrt{-g}$ and $g$ is a determinant of the metric tensor $g_{ik}$ of the observer’s continuum. If the off-site action is stable in time, the action integral needs to be invariant and its integrand $\Lambda \sqrt{-g}$ needs to be a scalar density, so $\Lambda$ needs to be a scalar field. Thus, the accidental off-site source $\Lambda_s$ needs to be accompanied by complemented ‘ordinary field’ $\Lambda^c$ to fill $\Lambda$ to the scalar field. The off-site source and complemented field of stable in time off-site object have deep interconnection.
The continuum includes all cause-effect chain available for the internal observer, so interconnections between continuums cannot be expressed by continuous functions, otherwise continuums will coincide (see also section 2). Anyway, the cardinality of functions of any kind is greater than continuous ones, so the possibility to observe the continuous interconnections is equal to zero, and one may consider that interconnections between continuums are always discontinuous. Thus, in the general case, the observer’s and off-site space-time continuums do not intersect with each other, even if common regions coincide with corresponding continuums $G \equiv D \rightleftharpoons \bar{D} \equiv \bar{G}$. The function $\Lambda$ represented in any analytical form will always be only some approximation to off-site action.

For example, if the off-site source $\Lambda_{S}$ is not a scalar field $\mu(x)$, but a covariant vector $B_{i}(x)$, the complemented field needs to be a contravariant one $\Lambda^{C} = A^{i}(x)$, if $\Lambda_{S} = D_{ik}$ is a covariant tensor, $\Lambda^{C}$ needs to be contravariant. It defines representations of tensor algebra used in relativistic theories:

$$\Lambda = \mu + A^{i}B_{i} + C^{ik}D_{ik} + E^{ijk}G_{ijk} + \cdots \tag{7}$$

It is possible to represent the off-site source as a scalar complex field $\Lambda_{S} = \psi(x)$, thence the complex conjugated field will be a complemented one to it $\Lambda^{C} = \bar{\psi}(x)$, so $\Lambda = \bar{\psi}(x)\psi(x) = \Lambda^{C}\Lambda_{S}$ (or, more generally, $\Lambda = \bar{\phi}(x)\psi(x)$). One may continue these representations for spinor or matrix complex fields as:

$$\Lambda = \mu + \bar{\psi}\psi + \bar{\psi}_{i}\psi_{i} + \bar{\psi}^{ij}\psi_{ij} + \cdots \tag{8}$$

where the summation over repeating indices is meant and up indices mean transposition, so, in this definition, $\psi^{i}$ is a component of a row-vector, $\psi_{i}$ of a column-one, $\psi_{ik} = \psi^{ki}$.

In fact, the representation Eq.(8) forms on $D$ the infinite dimensional complex vector space. It exactly coincides with the mathematical apparatus of quantum mechanics. Indeed, in the infinite dimensional complex vector space some functions $f(x), h(x), \ldots$ defined on $D$ are understood as vectors, for which there are usually defined the operations of summation $f + h$; multiplication on scalar $\alpha f$; scalar product $(f, h) = \int_{D} fh\gamma dx$, where $\gamma \geq 0$ is a weight function (in many applications $\gamma \equiv 1$ or $\gamma dx$ is a volume element). Metrics is usually introduced as $\| f + h \| = \left[ \int_{D} |f - h|^{2}\gamma dx \right]^{1/2}$, which in case $h \equiv 0$ defines the norm: $\| f \| = \left[ \int_{D} |f|^{2}\gamma dx \right]^{1/2}$. The series of functions $\{u_{0}, u_{1}, \ldots\}$ are orthogonal and normalized, if $(u_{i}, u_{k}) \equiv \int_{D} u_{i} u_{k}\gamma dx = \delta_{ik}$, where $\delta_{ik} = 1$ when $i = k$ and $\delta_{ik} = 0$ otherwise.

Thus, any element of introduced vector space may satisfy to off-site or complemented source of Eq.(8). The behavior of such objects in space-time is described by differential equations, which are operators $L$ acting on some function $\psi(x)$ put in correspondence with the physical object. The Hermit operators $(L : (f, Lh) - (Lh, f) = \int_{D}|\Pi_{i}u_{k}\gamma dx = \delta_{ik}$, where $\delta_{ik} = 1$ when $i = k$ and $\delta_{ik} = 0$ otherwise.

Putting the eigenvalues $q_{k}$ of the Hermit operator $L_{k} = -i\partial_{k}$ in a correspondence to the energy-momentum 4-vector $p^{k} = (E/c, p)$ of a particle of a rest mass $m_{0}$ and applying $hq_{k} = p_{k}$ in $p^{k}p_{k} = m^{2}c^{2}$, one can get the Klein-Gordon equation $-h^{2}\partial^{k}\partial_{k}\psi = m^{2}c^{2}\psi$. The Schrödinger equation $-ihc\partial_{t}\psi = (\hbar^{2}/2m)\partial^{2}\partial_{t}\psi + U(x^{a})\psi$ comes from $E' = p^{2}/2m + U(x^{a})$, where $E'$ and $U$ are the kinetic energy and the potential. The Dirac equation for the spinor $\Psi(x) = (\psi_{0}, \ldots, \psi_{3})$ is a result of matrix factorization of the Klein-Gordon equation.

One may see that each next term in Eqs.(7,8)
are the generalization of the previous one. Indeed, a vector field seems as a generalization of a scalar one, tensor – of vector, and so on. It would be interesting to introduce and to consider a tensor algebra of Eq.(7) on the functional space of Eq.(8). Note that in such ‘unification’, vectors and tensors may have both off-site and complemented elements. It looks like the analysis of different groups $T_{(i)}$ from GS representations Eq.(3). We leave the regularized analysis of these cases for future investigations with the remark that one may find elements of such analysis in gauge theories (there are some examples in the next Section).

Thus, “General relativity is, conceptually, a completely different sort of theory from the other field theories, because of its explicitly geometric nature” (L. Ryder Ref.[11]), but, nevertheless, Global system includes quite logically both relativistic and quantum theories.

4 Physical Objects

As a first approximation, the integral Eq.(6) observed on time period $\tau \in [1,2]$ coincides with the Action $S$ of the physical system:

$$I_{1-2} = \int_{1}^{2} d\tau \int_{V} \Lambda \sqrt{-g} dV = -cS,$$

where $D : \tau \times V$, $d\tau = dx^{0}$ and $dV = dx^{1} \cdots dx^{n-1}$. The integral $\int_{V} \Lambda \sqrt{-g} dV$ has sense of a Lagrangian function, its integrand $\Lambda \sqrt{-g}$ the Lagrangian density. Thus, one may represent the influence of off-site matter by some energy-momentum tensor $T^{ik}$. This way, the Lagrangian density has sense of the distribution of the gravitating matter and corresponds to the first term approximation of $\Lambda = \mu$ in Eqs.(7,8) as a scalar field.

Other terms in approximations of $\Lambda$ in Eqs.(7,8) may be also interpreted as physical objects. For example, one can associate off-site sources $B_{i}$ from Eq.(7) with the currents of electric charges $B_{i} = j_{i}$, which are complemented by a 4-vector potential $A^{i}$ of electromagnetic fields and 4-tensors of the electromagnetic field $C^{ik} = F^{ik}$ and $D_{ik} = F_{ik}$ in the continuum of the observer.

It is also possible to get the electromagnetic field representations using the complex field approximation of $\Lambda$ by Eq.(8). In field theories, a classical scalar complex field is a continuation of the Lagrangian formulation of particle mechanics and has a Lagrange density $\Lambda = m^{2}\psi\bar{\psi} + (\partial\psi\bar{\psi}) (\partial_{i}\psi)$ (compare with second and third terms of Eq.(8)). By virtue of the rotation symmetry in the complex plane of this Lagrangian, there is a conserved current $j^{i} = i(\bar{\psi}\partial^{i}\psi - \psi\partial^{i}\bar{\psi})$ corresponding by the Noether’s theorem to a conserved (time independent) quantity $Q = \int_{V} j^{0} dV$ identified with an electric charge. On making the symmetry a local character, the invariance of the Lagrangian $\Lambda$ may be restored by introducing a field $A^{i}$ and associated 4-dimensional curl $F_{ik} = \partial_{i}A_{k} - \partial_{k}A_{i}$. This is the electromagnetic field, whose source is electric charge (see L. Ryder Refs.[10,11]).

When the observer is outside the common region, he looks to off-site objects externally. To describe the internal structure of off-site objects, the observer in GR tries to extend his ‘external representations’ approximating the integrand of $\Lambda \sqrt{-g}$ of Eq.(6) by continuous functions in accordance with the paradigm of cognoscibility in science. From GS point of view, such efforts of the GR observer to ‘include’ off-site objects into GR space-time continuum seem quite unreasonable and lead to explicit limitations and paradoxes. For example, the capturing of matter inside nuclei will always be unexplainable for the observer in GR. He will be obliged to introduce and postulate the ‘fundamental’ strong interactions to ‘explain’ a confinement.

It does not need in GS. For example, the simplified interconnections $r = \arctan \tilde{r}$ between continuums from Figure 1A map ‘point-to-point’ the real axis $\tilde{r} \in (-\infty, \infty)$ of off-site
continuum $\hat{G}$ onto the interval $r \in (-\pi/2, \pi/2)$ of the observer’s continuum $G$, so any off-site objects will seem captured inside this interval.

Indeed, let coordinates of continuums $G : (\tau, r)$ and $\hat{G} : (\hat{r}, \hat{r})$ are interconnected by double continuously differentiable functions $\hat{\tau} = \tau$, $\hat{r} = \hat{r}(r)$ (we put here $\hat{\tau} = \tau$ to preserve the stability of the object) and the test particle in $\hat{G} : (\hat{r}, \hat{r})$ moves along the trajectory $\hat{r} = \hat{r}(\hat{\tau})$ with the velocity $\dot{\hat{v}} = d\hat{r}/d\hat{\tau}$ and with the acceleration $\ddot{\hat{v}} = d^2\hat{r}/d\hat{\tau}^2$. Denoting $\hat{\tau} = dr/d\tau$ and $\hat{r}' = d\hat{r}/dr$ one can get: $\dot{\hat{v}} = \hat{r}' \hat{v}' = \hat{r}' v$ and $\ddot{\hat{v}} = \hat{r}'^2 \dot{\hat{v}}' + \hat{r}^2 \ddot{\hat{v}}' = w^2 + v^2 \hat{r}''$. If the test particle is moving along $\hat{r}$ with the constant speed $\hat{v}$ (hence, its acceleration $\ddot{\hat{v}} = 0$), its velocity and acceleration in $G$ will be:

$$v = \hat{v}, \quad w = -v^2 \frac{\hat{r}''}{(\hat{r}')^3}. \quad (9)$$

Thus, for the simplified interconnections $r = \arctan \hat{r}$ the off-site test particle – material point from $\hat{G}$ moving rectilinearly along $\hat{r}$ with constant velocity $\hat{v}$ will be observed in $G$ as moving with the velocity $v = \hat{v} \cos^2 r$ and the acceleration $w = \hat{v}^2 \sin 2r \cos^2 r$. The test particle seems as ‘captured’ inside $(-\pi/2, \pi/2)$, because for any $\hat{v}$: $v \to 0$ with $r \to \pm\pi/2$, and having the centrifugal acceleration (see Figure 3A). Thus, in GS the confinement is visible for the observer, the behavior of off-site objects is caused by the internal topology of off-site continuums, which, of course, is not coincident with the topology of the observer’s one. The ‘fundamental’ strong interactions responsible for a confinement are the complemented fields in GS from Eq.(6) (see also remarks on a ‘group representation’ in GS in Section 3).

In the internal observation, when the observer is inside the common region, i.e. in case reversed to just considered, $\hat{r} = \arctan r$, $r \in (-\infty, \infty) \Rightarrow \hat{r} \in (-\pi/2, \pi/2)$, the test particle will be seen as moving with the velocity $v = \hat{v}(1 + r^2) \to \infty$ with $r \to \pm\infty$ and with the centrifugal acceleration $w = 2\hat{v}^2 r(1 + r^2)$ (see Eq.(9) and Figure 3B). Thus, in internal observation, the off-site objects seem as ‘flying away’ from each other with the radial acceleration. Such processes also seem unreasonable in GR, where the observer’s continuum is considered as the unique one. The observer in GR again, as in the external observation, tries to interpret the off-site matter in frames of his continuum, getting new forms of matter with paradoxical unphysical characteristics, “something fundamentally different from the ordinary matter we are made of”, for example, such objects as dark energy and dark matter.

During the internal observation, when $G \equiv \mathbb{D} \equiv \mathbb{D} \subset \hat{G}$, the observer in GR does not identify the gravitational influence of off-site matter, considering it as a space-time property. It leads to introduction of the cosmological constant $\Lambda_0$ responsible for the general curvature of the space-time. In our notations the term $2\Lambda_0$ is added to the function $\Lambda$ from Eq.(6), which leads to the Einstein field equations with cosmological $\Lambda_0$ term. “Einstein had abandoned the term, calling its introduction the ‘greatest blunder’ of his scientific life. The fact that the expansion of the Universe is now seen to be accelerating indicates that there is, after all, a place for $\Lambda$, so Einstein’s blunder might turn out to be not such a bad idea after all!” (L. Ryder Ref.[11], 2009).

Cosmological objects such as galaxies or galaxy clusters are usually observed externally. This way, in case of two or more spatial coordinates, supposing, according to off-site action Eq.(5), that transformations need to conserve the total energy of isolated test particle, one may conclude that decreasing the radial component of the velocity of the particle will lead to the corresponding increasing of the tangential component. It looks appropriate to explain, for example, the rotational curves of spiral galaxies or peculiar velocities in galaxy clusters. However, generally, one needs to take into account that the situation with the obser-
vation of off-site objects is more complicated than in considered simplified examples.

In some degree of approximation, the interconnections with off-site continuum $\tilde{G}$ may be considered as the coordinate transformations to curvilinear frame of references $\tilde{S}$. It is declared in GR that even in this approximation, not all of possible reference frames may be ‘physically realized’ (L. Landau, E. Lifschitz Ref.[6]), only when a time $d\tilde{\tau} = \sqrt{\tilde{g}_{00}}dx^0$ and a distance $d\tilde{l}^2 = (\tilde{g}_{\alpha\beta} + \tilde{g}_{0\alpha}\tilde{g}_{0\beta}/\tilde{g}_{00})dx^\alpha dx^\beta$, $\alpha,\beta=1\ldots3$ are introduced in curvilinear frame and they have the same physical sense as in GR space-time (so a metric tensor $\tilde{g}_{ik}$ is determined). Thus, in GR, the frame $\tilde{S}$ with the metric tensor $\tilde{g}_{ik}$ may be realized by ‘real bodies’ (L. Landau, E. Lifschitz Ref.[6]) only if:

$$\begin{vmatrix} \tilde{g}_{00} > 0, & \tilde{g} < 0, & \tilde{g}_{00} \tilde{g}_{01} \tilde{g}_{02} \\ \tilde{g}_{00} & \tilde{g}_{01} & 0 \\ \tilde{g}_{10} & \tilde{g}_{11} & \tilde{g}_{12} \end{vmatrix} > 0. \quad (10)$$

There are no limitations for ‘real bodies’ in GS, because off-site continuums consist exactly from off-site matter. When Eq.(10) are not satisfied, our test particle - material point in off-site continuum cannot be even identified in space and time as a ‘real body’ in the observer’s continuum. Coming from the analysis of curvilinear frames of references in GR, A.V. Novikov-Borodin Ref.[7] (2008) had proposed to separate the common regions of off-site action on fields depending on whether $\tilde{g}_{00}$ and $\tilde{g} = \det \tilde{g}_{ik}$ are positive or negative. According to these representations, when $\tilde{g}_{00} > 0$ and $\tilde{g} < 0$ the observer will see the ‘ordinary matter’, but with ‘strange motion’ as it was shown in considered examples on Figure 3. When $\tilde{g}_{00} < 0$ and $\tilde{g} < 0$: “the non-fulfillment of the condition $g_{00} > 0$ ($\tilde{g}_{00} > 0$) would mean only, that the corresponding frame of references cannot be realized by real bodies; thus if the condition on principal values is carried out, it is possible to achieve $g_{00}$ ($\tilde{g}_{00}$) to become positive by appropriate transformation of coordinates” Ref.[6]. Thus, the space and time properties of off-site objects cannot be identified by the observer, but objects still possess the physical characteristics and may take part in gravitational interactions, so are identified as ‘dark matter’. When $\tilde{g}_{00} > 0$: “The tensor $\tilde{g}_{ik}$ ($\tilde{g}_{ik}$) cannot correspond to any real gravitational field at all, i.e. the metrics of the real
space-time” Ref. [9], so physical objects may possess quite unusual, even unphysical characteristics, usually interpreted as ‘dark energy’.

Thus, we may conclude that our hopes to find correspondences of off-site matter with objects of quantum physics and objects beyond ‘the ordinary matter we are made of’ in cosmology are coming true. It looks like such objects are only different perceptions or approximate representations of off-site matter.

5 Quantization

It was mentioned in Section 4 that currents $j^i$ of the electric charges from Eq. (7) may be associated in GS with off-site sources complemented by electromagnetic fields $A^i (F^{ik})$ in the continuum of the observer. Thence, for the stable in time off-site object the currents $j^i$ captured inside the common region $D$ (as before $D : \tau \times V$), would permanently excite (or absorb) the electromagnetic fields according to the wave equation. Thus, to preserve the energy conservation, the electromagnetic fields $A^i$ excited by off-site currents $j^i$ cannot exist outside the region $D$, except for the stationary fields, which do not transfer the energy, so one may come to the wave equation with edge conditions [9]:

$$\partial^k \partial_k A^i = j^i, \quad A^i, j^i : \begin{cases} \neq 0, & x \in D \\ = 0, & x \notin D \end{cases}. \quad (11)$$

When the off-site excitation is periodic with the frequency $\omega$ and amplitude $J(r)$ ($j(\tau, r) = J(r)e^{i\omega \tau}$, where $x = (\tau, r)$, the excited fields $u(\tau, r) = U(r)e^{i\omega \tau}$ have sense of coefficients of the Fourier transform for $\tau$ and is described for each component by stationary Helmholtz equation (see, for example, V.S. Vladimirov Ref. [15]):

$$(\nabla^2 + |k|^2) U(r) = J(r), \quad |k|^2 = \omega^2. \quad (12)$$

Off-site currents $j^i$ excite the electromagnetic fields in the continuum of the observer, so these fields do not need to be ‘captured’ as off-site objects in common regions. It seems that the only possibility to satisfy to the edge conditions for electromagnetic fields is mechanism of compensation. It is when different off-site sources are synchronized and compensate the electromagnetic fields excited by them outside the common region. Thus, sources need to be self-matched with electromagnetic fields excited by them.

For example, in 1D case, denoting $x = (\tau, r)$, Eq. (11) and Eq. (12) are: $(\partial_\tau \partial_\tau - \partial_r \partial_r) u = j; \quad (\partial_\tau \partial_r + k^2) U = J, \quad k = \omega$. If $J = \delta(x - p) + \delta(x - q)$, where $\delta$ is a delta-function, the excited fields are $U = \frac{1}{2\pi} (e^{ik|x-p|} + e^{ik|x-q|}), \quad k = \omega$ Ref. [15]. When the quantization conditions $q - p = \lambda(n + \frac{1}{2})$, $\lambda = 2\pi/k, \quad n = 0, 1, \ldots$ are satisfied, the solutions $U = \frac{1}{k} \sin k(x-p) \neq 0$ on $x \in [p, q]$ and $U = 0$ on ‘external’ regions $x \in (-\infty, p) \cup (q, +\infty)$. The quantization conditions define the allowed discrete spatial ‘sizes’ of objects in the observer’s continuum. The observer will interpret the standing waves $U = \frac{1}{k} \sin k(x-p)$ on $x \in [p, q]$ as if two sources in points $x = p$ and $x = q$ are in permanent exchange by field quanta with each other, exactly as it seems in field theories. Thus, only states $S_n$ with discrete distances $q-p = \lambda(n + \frac{1}{2})$ between sources are allowed and the lowest state $S_0$ corresponds to $q - p = \lambda/2$. Standing waves have an energy, so the passing between states $S_n$ and $S_m$ needs to be accompanied by emission in opposite directions of two field quanta with ‘length’ $\lambda(n - m)$ if $n > m$ (or absorption if $n < m$), which is illustrated on Figure 4A.

$^1$ In curved space-time the wave equations are: $F^{ik} = \gamma^{-1} \partial_k (\gamma F^{ik}) = -j^i, \quad \gamma = \sqrt{-g}$ Ref. [9]. Here we neglect the gravitational influence of off-site matter to the observer’s continuum, because it is known that on the scale of elementary particles, which we are going to consider, the influence of gravitational interactions is negligibly small in comparison with other ones. Thus, $\sqrt{-g} = 1$ is also considered for simplicity.
One may analogously consider two ‘antiphase’ sources \( J = \delta(x - q) - \delta(x - p) \), which produce the same standing waves \( U = \frac{1}{k} \sin k(x - p) \) on \( x \in [p, q] \), but with the quantization conditions \( q - p = \lambda n \). In this case, if there are no other conserved quantities, the state with \( q = p \) is allowed and off-site sources may simply annihilate with emitting the field quanta. Thus, we have even and odd branches of solutions depending on synchronization of the sources. It looks for the observer that the off-site objects consist exactly from excited fields and their intrinsic energy coincides with the energy of excited standing waves.

The analysis of the spherically symmetric 3D case is analogous to 1D one. The Helmholtz equation in spherical polar coordinates \( r = (r, \vartheta, \varphi) \), is \( \nabla^2 U + |k|^2 U = 0 \) homogeneous everywhere in space except for the surface of the region \( D \), which is of Lebesque measure null. In many of such cases it is possible to get solutions from the class of continuous functions by joining them on this surface, but derivatives will be discontinuous here.

\[ H - R = n\lambda/2, \lambda = 2\pi/k. \] Also only one source \( J = \delta(r - R) \) on sphere of radius \( R \) can create the stable state, if \( 2R = \lambda(m + \frac{1}{2}) \), but excited fields \( U = \frac{1}{2\pi} \cos kr \) will be unlimited in \( r = 0 \) (see Figure 4B).

The solutions limited in \( r = 0 \) exist when the sources on sphere of radius \( R \) depend on \( \vartheta, \varphi \): \( J = \delta(r - R) Y^m_l(\vartheta, \varphi) \), where \( Y^m_l \) are spherical Bessel functions. In this case steady states of Eq. 12 with edge conditions \( U_{r=R} = 0 \) are Ref. 15: \( u^{(l)}_{jm} = e^{-\omega^{(l)}_{jm} r} \frac{C}{\sqrt{r}} J_{l+\frac{1}{2}} (k_j^{(l)} r/R) Y^m_l(\vartheta, \varphi) \), where \( C \) is some constant, \( J_{l+\frac{1}{2}} \) Bessel functions, \( k = \omega \).

The quantization equations are \( J_{l+\frac{1}{2}} (k_j^{(l)} R) = 0 \). Thus, the self-matched compact quantized electromagnetic fields can complement off-site sources in the continuum of the observer.

In above examples, instead of Eq. 12 we have considered the Helmholtz equation \( (\nabla^2 + |k|^2) U(r) = 0 \) homogeneous everywhere in space except for the surface of the region \( D \), which is of Lebesque measure null. In many of such cases it is possible to get solutions from the class of continuous functions by joining them on this surface, but derivatives will be discontinuous here.

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**Figure 4: Excited fields: 1D (A) and 3D (B) cases.**
It is possible to observe the similar self-matched compact electromagnetic fields in macro-objects, so-called, optical solitons (see, for example, N. Rosanov, Ref.[9], 2007) existing in non-linear optical media. The non-linearities of optical media play a role of off-site sources. Certainly, optical solitons differ from the off-site objects by nature, but the way of formation of compact electromagnetic fields is similar. Solitons also possess quite explicit quantum characteristics. There are fundamental and vortical solitons, which can interact with each other, can create dynamically stable systems, merge with each other or divide to different parts. The fireballs also seem as compact electromagnetic fields described by Eqs. (11, 12) created by self-matched sources in excited media (for example, in the ionized air).

There are no limitations on the spatial ‘sizes’ of off-site objects in Eqs. (11, 12). However, small-size objects can destroy the compact electromagnetic fields of ‘larger’ objects, so the last ones seem unstable and will decay due to external influences. Thus, the stable off-site objects will seem with quantized spatial ‘sizes’ and their ‘scale’ is determined by the energy level of the ‘surroundings’.

6 Levels of Cognition

The off-site matter was introduced in Section 1 as different from the ‘ordinary’ matter, but taking part in gravitational interactions also in the continuum of the observer. After investigations of off-site matter, one may come ‘back’ to conclusion that off-site matter is only another representation of the ‘ordinary’ matter. However, on the contrary, one can conclude that all gravitating matter is off-site one. Which point of view is right? Surprisingly, it seems both, but to show it, we again need to review in main features the process of human cognition.

We represent the world around by the system of notions interconnected by cause-effect chains, which are needed for prediction of events to provide our survival. Indeed, according to the neurophysiology “All kinds of individual reactions are now related with the spatial organization of a brain, the character of associations of neurons in micro- and macro-ensembles, their arrangement, the relations with each other and with other ensembles” (see Refs.[1, 12]). The science is a human effort to make this logical system consistent: “Science is an indefatigable centuries-old work of thought to bring together by means of a system all cognizable phenomena of our world” (A. Einstein).

The consistency of scientific representations means that the causality principle needs to be valid everywhere inside the logical system, so we believe that any event needs to be interconnected by cause-effect chains with existing logical system to be a part of it. We are trying to use continuous functions for description of physical laws and models of physical objects in such representations. The classical and relativistic descriptions of physical reality are exactly the efforts to create such logically consistent systems.

The general relativity seems the most general representations of physical reality created with help of causality. Indeed, it has “background independent nature” Ref.[14], so seems logically ‘closed’, because its basic notions are

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3 The more detailed analysis of the process of cognition by the human brain shows that the ‘image’ of the world around is ‘constructed’ as the system of notions with help of mechanism: “thesis-antithesis-synthesis”. At first, neurons in brain continuously excited from sensory organs are interconnected (generalized) by interneuronic synapses, axons. It is a ‘thesis’. Then, this ‘model’ from ‘short-time’ memory is compared again with signals from sensory organs with help of the hippocamp, tonsils and hypothalamus of the brain (‘antithesis’) and is corrected until the coincidence (‘synthesis’). Only after that the brain saves the ‘final model’ in long-time memory (see Refs.[1, 12]).
included in the system and there are no something ‘more general’. Experiments in quantum physics and observations in modern cosmology give us explicit evidences of existence of objects and processes in world around principally different from the GR model, so the possibilities of description with help of causality is really limited.

The world around is that we perceive, but he is not obliged to be like this.

Indeed, the world around may not know which way of cognition is more convenient for us to represent him and which is not.

From point of view of the theory of sets Refs.\[2, 3\], the neurophysiological possibilities of our brain to reflect the world around may be estimated as having a cardinality $\aleph_0$ of a discrete countable sets. We may define it as a zero level of cognition – the reflection. Using the causality principle, which is defined by the neurophysiology of our brain, one can extend the possibilities of cognition and reach the possibilities of continuums and continuous functions with the cardinality $\aleph_1$. Certainly, such extension leads to corresponding losses of contents in ‘models’ (notions) and their interconnections during ‘generalization’. This is the first level of cognition, the level of logical systems, classical and relativistic theories. Due to its ‘background independent nature’ General relativity seems the most general ‘closed’ theory on this level of cognition.

Global system has higher level of representations with cardinality at least $\aleph_2$, because it exceeds the possibilities of continuums by including the interconnections of any kind between them. We tried to show in previous sections that the physical objects of quantum physics and dark substances of modern cosmology are from this level of cognition, so, for example, their ‘unification’ with relativistic theories in frames of logical system seems doubtful. It looks like the ‘price’ for the extension of our cognition on this level, where we have additional losses of contents of models and interconnections. It is ‘paradoxes’ and uncertainties in description and prediction.

The off-site action described by Eq.\[5\] is not deterministic, because it does not fix the kind of interconnections between continuums and even do not need them depending on considered level of cognition. We did not suppose the concrete structure of off-site continuums, so one can try to check any symmetry or group of off-site objects by using representations Eq.\[7\] or Eq.\[8\] in approximation of Eq.\[6\]. Thus, one can get more and more exact models of off-site objects, but, probably, never the accurate one, because it is impossible to put in exact correspondence sets with different cardinalities. Such approach seems more general than existing in physics. For example, it is possible to consider the models of string theories just as particular cases of representations of internal structures of off-site continuums (the set theory tells that any ‘string’ as a segment of real axis has the same cardinality as a multi-dimensional set of real numbers).

The world around seems ‘opened’ and one cannot describe it accurately by any ‘closed’ mathematical theory. The off-site action Eq.\[5\] looks like a gate from the observer’s continuum to higher levels of cognition of the world around depending on the degree of chaos or disordering (see Figure 5). These representations surprisingly correspond to epistemic views of ancient philosopher Plato: ‘the man has the only possibility to see the distorted shadows of the bright and multicolor Reality on the wall of the cave, where he is confined and chained up back to the entrance’. We are similarly ‘captured’ by cause-effect chains inside our logic and can only try to reconstruct the models of the ‘multicolor Reality’ basing on the ‘distorted shadows’ given by our senses. This is a way of human cognition, but, it seems, not a limit.
The world around seems more chaotic and disordered than we think before. The theory of sets tells us that there may be an infinite number of levels of chaos. Probably, we have the infinite number of possibilities to go deeper and wider in our representations about the world around with corresponding limitations on this way.

Conclusion

The physical reality is that we perceive with help of causality, but the world around seems much more complicated. The space-time continuums differed from the observer’s one need to exist. The paradigm of cognoscibility has got corrected interpretation: the world is cognizable, but the cognition of it with help of causality is limited and there are many levels of cognition depending on the considered degree of chaos. The relativistic and quantum theories operate on different levels of cognition, so their ‘unification’ seems doubtful. We have no reasons to interrupt the cognition of the world around on some fixed level, so a number of levels of cognition may be infinite.

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