Neutrino mass from cosmology: impact of high-accuracy measurement of the Hubble constant

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Abstract. Non-zero neutrino mass would affect the evolution of the Universe in observable ways, and a strong constraint on the mass can be achieved using combinations of cosmological data sets. We focus on the power spectrum of cosmic microwave background (CMB) anisotropies, the Hubble constant $H_0$, and the length scale for baryon acoustic oscillations (BAO) to investigate the constraint on the neutrino mass, $m_\nu$. We analyze data from multiple existing CMB studies (WMAP5, ACBAR, CBI, BOOMERANG, and QUAD), recent measurement of $H_0$ (SHOES), with about two times lower uncertainty (5%) than previous estimates, and recent treatments of BAO from the Sloan Digital Sky Survey (SDSS). We obtained an upper limit of $m_\nu < 0.2$ eV (95% C.L.), for a flat $\Lambda$CDM model. This is a 40% reduction in the limit derived from previous $H_0$ estimates and one-third lower than can be achieved with extant CMB and BAO data. We also analyze the impact of smaller uncertainty on measurements of $H_0$ as may be anticipated in the near term, in combination with CMB data from the Planck mission, and BAO data from the SDSS/BOSS program. We demonstrate the possibility of a $5\sigma$ detection for a fiducial neutrino mass of 0.1 eV or a 95% upper limit of 0.04 eV for a fiducial of $m_\nu = 0$ eV. These constraints are about 50% better than those achieved without external constraint. We further investigate the impact on modeling where the dark-energy equation of state is constant but not necessarily $-1$, or where a non-flat universe is allowed. In these cases, the next-generation accuracies of Planck, BOSS, and 1% measurement of $H_0$ would all be required to obtain the limit $m_\nu < 0.05 - 0.06$ eV (95% C.L.) for the fiducial of $m_\nu = 0$ eV. The independence of systematics argues for pursuit of both BAO and $H_0$ measurements.

Keywords: neutrino masses from cosmology, cosmological parameters from CMBR, cosmological neutrinos, baryon acoustic oscillations

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1 Introduction

Neutrino mass is now one of the most important targets in cosmology. Since neutrino mass affects the evolution of the Universe in some observable ways, a mass constraint can be obtained from the cosmological data such as cosmic microwave background (CMB) [1–5], galaxy clustering [6], Lyman-α forest [7], and weak lensing data [8, 9]. Although atmospheric, solar, reactor and accelerator neutrino oscillation experiments can probe mass differences precisely ($|\Delta m^2_{21}| = 7.65^{+0.23}_{-0.20} \times 10^{-5}\text{ eV}^2$ and $|\Delta m^2_{31}| = 2.40^{+0.12}_{-0.11} \times 10^{-3}\text{ eV}^2$, where $m_i$ is the mass of the $i$-th neutrino mass eigenstate and errors are 1σ [10]), they are not sensitive to absolute values. Absolute mass may be inferred from terrestrial experiments such as tritium beta decay and neutrino-less double beta decay [11–13], but, cosmological measurements from several observables have the potential to deliver higher accuracy.

Analyses of the CMB data alone constrain neutrino masses, but there is a relatively large degeneracy between neutrino masses and the Hubble constant [1]. A combination of different data sets can tighten limits. CMB data and matter power spectrum measurements can be usefully combined, since the suppression of the power spectrum below the damping scale is dependent on the mass of free-streaming neutrinos. A neutrino mass constraint has previously been derived [5] that combines CMB data with measurements of the baryon acoustic oscillation (BAO) scale, supplemented by luminosity distance measurements from type-Ia supernovae. Since these non-CMB observations directly characterize the distance scale, a high accuracy prior on the Hubble constant can have a qualitatively similar effect, as was first demonstrated for WMAP first-year data by [1].

In this paper, we examine the extent to which priors on the Hubble constant improve constraint on neutrino mass, without using matter power spectrum data (see also [14, 15]). Because systematic errors for estimating the Hubble constant and the matter power spectrum are different, these parallel routes to estimate neutrino mass are important. For the approach using matter power spectrum including nonlinear effects, we refer to refs. [16, 17]. We focus on the newly reported measurement of the Hubble constant [18] that has a rather higher central value and about two times smaller error than that of previous work [19]. We also study the effect of tighter priors on the Hubble constant, anticipating that the accuracy of estimates will improve over time, at least through measurement of distance to a large
number of galaxies in the Hubble flow that host water maser sources [20–22]. In this paper, we denote the Hubble constant in unit of km/s/Mpc as $H_0$.

In the next section, we discuss the neutrino mass constraints using the current CMB data set and the Hubble constant measurements. We demonstrate the degeneracy between neutrino masses and the Hubble constant in analysis of CMB data alone, where we consider WMAP5, ACBAR, CBI, BOOMERANG and QUAD. We also demonstrate improvement in the constraint through addition of the Hubble constant determination of ref. [18]. The degeneracy in the CMB mainly originates from that in the angular diameter distance to last scattering surface, so another observation probing such distances will also be helpful to obtain the constraint. Hence we also make an analysis that includes the BAO scale measurement released recently [23]. In section 3, future expected limits for neutrino masses are presented, with particular attention to the role of the prior on $H_0$. For this purpose, we use projected Planck CMB data and uncertainties in $H_0$ that may plausibly be anticipated from estimation of distances to galaxies containing water maser sources [20–22]. Furthermore, we discuss the effect of future BAO data such as the Baryon Oscillation Spectroscopic Survey (BOSS) on the neutrino mass constraint. In section 2 and 3, a flat universe and a cosmological constant for dark energy are assumed. However, in section 4, we investigate the constraint for the case of a constant equation of state for dark energy and a non-flat universe. The final section summarizes the paper.

2 Neutrino mass constraint in $\Lambda$CDM model

2.1 Current CMB and $H_0$ measurements

We first derive limits on neutrino mass using CMB data from WMAP and other small angular scale measurements, including recently released QUAD data [24]. We demonstrate the degeneracy between neutrino masses and the Hubble constant in the CMB data analysis, which originates from geometry. In light of this, we examine how the mass limit is improved by adding a prior on $H_0$ obtained from the HST Key Project (KP) [19] and a newly available one from the Supernovae and $H_0$ for the Equation of State (SHOES) program [18].

We begin by showing a neutrino mass constraint from current CMB data sets alone. In the analysis of this section, we assume a flat universe and a cosmological constant for dark energy. For neutrinos, we assume that there are three generations of active neutrinos with equal mass $m_\nu$, so that $\sum m_\nu = 3m_\nu$. We also assume there is no lepton asymmetry in the Universe. We used CMB data from WMAP5 [4, 28], ACBAR [29], CBI [30], BOOMERANG [31–33] and QUAD [24].

We performed a Monte Carlo (MC) analysis with some modifications to MultiNest [35], integrated in CosmoMC [36]. Convergence of MC samples is diagnosed by applying the Gelman-Rubin test on two independent chains. Typically, $R - 1 < 0.01$ is achieved in our analysis. We explored 7+1 dimensional parameter space $(\omega_b, \omega_c, \theta_s, \tau, \omega_\nu, n_s, A_s, A_{SZ})$. Their prior ranges are shown in table 1. Here, $\omega_b = \Omega_b h^2$,

1There are several independent methods to determine $H_0$. In particular, [25] recently offers a constraint $H_0 = 70.6 \pm 3.1$ using the time-delay distance from the observation of strong gravitational lens systems. Since the errors on $H_0$ is similar to those of SHOES, we expect a competitive constraint on neutrino mass would be obtained from the time-delay distance.

2The constraint on neutrino masses including lepton asymmetry has been investigated in [26, 27].

3Since some of these data sets probe the same regions of the sky on the same angular scale, in a strict sense, correlation among them should be taken into account as is done for example in ref. [34]. We consider this procedure would not change our result since small angular scale data scarcely change the WMAP alone limit as is shown below.
prior ranges

Table 1. Adopted prior ranges for cosmological parameters described in the main text. In addition to top-hat priors on the primary parameters, we also impose an additional prior \( H_0 \in [40, 100] \), which is hardcoded in CosmoMC by default. Also note that the prior ranges for \( \Omega_k \) and \( w_X \), shown in the parentheses, are adopted only when they are varied in section 4.

| parameters | prior ranges |
|------------|--------------|
| \( \omega_b \) | \( 0.020 \rightarrow 0.025 \) |
| \( \omega_c \) | \( 0.08 \rightarrow 0.14 \) |
| \( \theta_s \) | \( 1.03 \rightarrow 1.05 \) |
| \( \tau \) | \( 0.01 \rightarrow 0.20 \) |
| \( \Omega_k \) | \( (-0.1 \rightarrow 0.1) \) |
| \( \omega_\nu \) | \( 0 \rightarrow 0.02 \) |
| \( w_\nu \) | \( (-3 \rightarrow 0) \) |
| \( n_s \) | \( 0.8 \rightarrow 1.1 \) |
| \( \ln(10^{10} A_s) \) | \( 2.8 \rightarrow 3.5 \) |
| \( A_{SZ} \) | \( 0 \rightarrow 4 \) |
| \( H_0 \) | \( 40 \rightarrow 100 \) |

Table 2. Constraints on the \( \Lambda \)CDM+\( m_\nu \) model from current CMB data (WMAP5+ACBAR+CBI+BOOMERANG+QUAD) with and without priors on \( H_0 \). We adopt two different priors on \( H_0 \) from direct measurements, the KP prior of \( H_0 = 72 \pm 8 \) [19] and the SHOES prior of \( H_0 = 74.2 \pm 3.6 \) [18]. In the table, errors are at 68 % C.L., except for upper limits at 95 % C.L. on \( \omega_\nu \) and \( m_\nu \), which are not bounded from below.
| parameters          | CMB+BAO       | CMB+BAO+KP    | CMB+BAO+SHOES |
|---------------------|---------------|---------------|---------------|
| $\omega_b \times 10^2$ | $2.264^{+0.047}_{-0.057}$ | $2.266^{+0.053}_{-0.050}$ | $2.279^{+0.047}_{-0.053}$ |
| $\omega_c$          | $0.1093^{+0.0038}_{-0.0041}$ | $0.1092^{+0.0038}_{-0.0041}$ | $0.1090^{+0.0038}_{-0.0037}$ |
| $\theta_s \times 10^2$ | $104.13^{+0.22}_{-0.21}$ | $104.13^{+0.22}_{-0.21}$ | $104.16^{+0.24}_{-0.17}$ |
| $\tau$              | $0.087^{+0.016}_{-0.015}$ | $0.088^{+0.016}_{-0.015}$ | $0.088^{+0.015}_{-0.016}$ |
| $\omega_\nu$        | $< 0.0090$ (95%) | $< 0.0086$ (95%) | $< 0.0071$ (95%) |
| $n_s$               | $0.955^{+0.011}_{-0.012}$ | $0.955^{+0.011}_{-0.012}$ | $0.958^{+0.011}_{-0.011}$ |
| $\ln[10^{10}A_s]$   | $3.050^{+0.035}_{-0.037}$ | $3.051^{+0.035}_{-0.038}$ | $3.054^{+0.040}_{-0.033}$ |
| age [Gyr]           | $13.88^{+0.14}_{-0.16}$ | $13.87^{+0.14}_{-0.16}$ | $13.80^{+0.10}_{-0.15}$ |
| $\Omega_m$          | $0.291^{+0.018}_{-0.025}$ | $0.289^{+0.019}_{-0.023}$ | $0.277^{+0.014}_{-0.020}$ |
| $z_{re}$            | $10.4^{+1.2}_{-1.4}$ | $10.4^{+1.2}_{-1.4}$ | $10.5^{+1.2}_{-1.4}$ |
| $H_0$               | $68.4^{+2.3}_{-1.6}$ | $68.6^{+2.1}_{-1.7}$ | $69.5^{+1.7}_{-1.6}$ |
| $m_\nu$ [eV]        | $< 0.28$ (95%) | $< 0.27$ (95%) | $< 0.22$ (95%) |

**Table 3.** Constraints on the ΛCDM+$m_\nu$ model from current CMB and the BAO data, with and without priors on $H_0$.

**Figure 1.** Regions of 68% and 95% C.L. from current CMB data with no prior on $H_0$ (black solid line), with the KP prior $H_0 = 72 \pm 8$ [19] (red dashed line) and with the SHOES prior $H_0 = 74.2 \pm 3.6$ [18] (green and yellow shaded regions). Vertical lines show priors on $H_0$ at 1σ and 2σ level from the KP (blue dot-dashed line) and the SHOES (magenta dotted line) measurements. Inner (outer) contours and shaded regions show regions of 68% (95%) C.L.
however, such large masses would affect the distance to the last scattering surface, which is normalized to the sound horizon $d_{\text{ls}}$, which is given by the ratio of sound horizon $d_{\text{ls}}$ to the last scattering surface, $d_{\text{ls}}$. When $m_\nu$ increases for given $\omega_c$, there is a larger non-relativistic component in the present universe (which makes expansion faster), and hence $d_{\text{ls}}$ is smaller. Meanwhile, $d_{\text{ls}}$ is inversely proportional to $H_0$, and thus larger $H_0$ gives smaller $d_{\text{ls}}$. Therefore, the effect of increasing $m_\nu$ is cancelled by decreasing $H_0$. Note that the sound horizon could be affected by $m_\nu$ if it is so large that neutrinos become non-relativistic before the epoch of recombination. However, such large masses would modify the heights of the CMB acoustic peaks in a characteristic manner, which is excluded by current CMB data. (This fact gives the upper bound on $m_\nu$ described in table 2.)

This argument can be quantified by calculating how the first peak position of CMB power spectrum $l_1$ responds as $m_\nu$ or $H_0$ vary. (For a detailed description of how this can be done, see [1, 38, 39].) Note that $l_1$ becomes smaller (moves toward larger scales) when $d_{\text{ls}}$ decreases. We find $\Delta l_1/\Delta m_\nu = -19.7$ and $\Delta l_1/\Delta H_0 = -0.643$. Therefore, the direction with no change in $l_1$ is $\Delta m_\nu/\Delta H_0 = 0.033$, which roughly describes the slope of the correlation in figure 1.

A tighter upper limit on $m_\nu$ flows directly from a lower bound on $H_0$, independent of CMB data [1]. The results for two priors, from the KP $H_0 = 72 \pm 8$ [19] and SHOES $H_0 = 74.2 \pm 3.6$ [18] are presented in table 2, where the upper bound on neutrino mass is as low as $m_\nu < 0.20$ eV.\footnote{The sound horizon $r_s$ at the recombination epoch $a_{\text{rec}}$ is given by}

$$r_s(a_{\text{rec}}) = \int_0^{a_{\text{rec}}} \frac{c_s}{a^2 H} da$$

where $c_s$ is the sound speed of the photon-baryon fluid.

\footnote{The epoch when neutrinos become non-relativistic can be roughly evaluated by taking the time of $\langle p_\nu \rangle \sim m_\nu$ with $\langle p_\nu \rangle$ being the average momentum of neutrinos which is roughly $\sim 3T_\nu$ where $T_\nu$ is the temperature of neutrinos. This gives $1 + z_{\text{eNR}} \approx 2 \times 10^3 \left( \frac{m_\nu}{1 \text{eV}} \right)$.}

\footnote{The latter case is also investigated in ref. [15] and our result is consistent with theirs.}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameter & Value & Notes \\
\hline
$\Omega_b h^2$ & 0.0226 & \\
$\Omega_c h^2$ & 0.1195 & \\
$\Lambda$ & 0.6504 & \\
$\theta_s$ & 1.0328 & \\
$n_s$ & 0.9689 & \\
$s$ & 0.721 & \\
$s_{\text{low}}$ & 0.701 & \\
$\Delta l_{\text{ls}}$ & 19.7 & \\
$\Delta H_0$ & 0.643 & \\
$m_\nu$ & 0.20 eV & (upper bound) \footnote{The sound horizon $r_s$ at the recombination epoch $a_{\text{rec}}$ is given by}
\end{tabular}
\end{table}

\section{Current CMB and BAO measurements}

Measurement of the BAO scale also fixes geometry and may be used to constraint $m_\nu$. We follow up with an analysis of $m_\nu$ in the context of current CMB data, augmented by
Table 4. Fiducial cosmological parameters and prior ranges used for predictive analysis. The fiducial values here correspond to the mean values of the flat ΛCDM model from analysis of the WMAP5 data alone. Regarding the fiducial neutrino mass, we consider two cases, $m_\nu = 0$ and $0.1$ eV, assuming degenerate mass hierarchy (three neutrino species have the same mass). We fix the energy density of dark matter and the Hubble constant to be $\omega_{dm} = 0.1099$ and $H_0 = 71.9$, and hence $\omega_c$ and $\theta_s$ vary with $m_\nu$. Values of $\omega_\nu$, $\omega_c$ and $\theta_s$ are shown for $m_\nu = 0 \ [0.1] \ eV$. Also note that the prior ranges for $w_\chi$ and $\Omega_k$, shown with parentheses, are adopted only when they are varied in section 4.

| parameters | fiducial values | prior ranges |
|------------|----------------|--------------|
| $\omega_b$ | 0.02273        | 0.20 $\rightarrow$ 0.25 |
| $\omega_c$ | 0.1099[0.1067] | 0.08 $\rightarrow$ 0.14 |
| $\theta_s$ | 1.04062[1.04684] | 1.03 $\rightarrow$ 1.05 |
| $\tau$    | 0.87           | 0.01 $\rightarrow$ 0.2 |
| $\Omega_k$ | 0              | $(-0.1 \rightarrow 0.1)$ |
| $\omega_\nu$ | 0[0.0032]     | 0 $\rightarrow$ 0.02 |
| $w_\chi$  | -1             | $(-2 \rightarrow 0)$ |
| $n_s$     | 0.963          | 0.8 $\rightarrow$ 1.1 |
| $\ln(10^{10}A_s)$ | 3.063 | 2.8 $\rightarrow$ 3.5 |
| $H_0$     | 71.9           | —             |
| $m_\nu$ [eV] | 0[0.1]       | —             |

the recent BAO data from [23]. In table 3, constraints on the cosmological parameters are shown for the analyses of CMB+BAO, CMB+BAO+KP and CMB+BAO+SHOES. As anticipated, the constraints on $m_\nu$ from the CMB+BAO analyses are improved as well. However, some cautionary words are in order. The results indicate a tighter upper limit for CMB+SHOES analysis than CMB+BAO analysis. This comes from the fact that BAO data prefers a relatively low value of $H_0$, which is outside the SHOES confidence interval. The joint CMB+BAO+SHOES analysis provides a less tighter limit on $m_\nu$ compared to that of CMB+SHOES, and it does not resolve friction between the contributing data and should be interpreted with care. In light of their different sources of systematic error, it may be more advisable, for the time being, to interpret the SHOES and BAO data separately.

So far, we have investigated the cosmological constraints on $m_\nu$ using currently available measurements of CMB, BAO and $H_0$. However, measurements of these are expected to be more precise with new observations in the near future such as Planck, BOSS and the $H_0$ measurement mentioned in the introduction. Thus, in the next section, we consider future constraints on the neutrino mass based on CMB, BAO and $H_0$.

3 Impact of improved cosmological data on ΛCDM estimation of $m_\nu$

Next-generation cosmological data will tighten the limits that can be placed on $m_\nu$. We investigate the impact of data from the Planck CMB mission, a high-accuracy prior on $H_0$, as may
Table 5. Sensitivities of the BOSS survey [43] adopted in the analysis (http://www.sdss3.org/collaboration/description.pdf). Expected errors (%) for angular diameter distance $d_A(z)$ and the Hubble expansion rate $H(z)$ are presented. Correlations among errors are assumed to be absent.

| redshifts | $\sigma_{d_A(z)}$ (%) | $\sigma_{H(z)}$ (%) |
|-----------|-----------------------|---------------------|
| 0.35      | 1.0                   | 1.8                 |
| 0.6       | 1.1                   | 1.7                 |
| 2.5       | 1.5                   | 1.5                 |

Figure 2. Regions of 68% and 95% C.L. on the $m_\nu-H_0$ plane from projected data of Planck with no other data (black dotted line), with $H_0$ at 1% accuracy (red dashed line), with BAO scales from BOSS (blue solid line) and both $H_0$ at 1% and BAO scales from BOSS combined (green and yellow shaded regions). Inner (outer) contours and shaded regions show 68% (95%) C.L. limits. The fiducial values for $m_\nu$ are taken to be $m_\nu = 0$ eV (left panel) and 0.1 eV (right panel).

be achieved from study of water masers [20–22],\(^7\) and analysis of the SDSS BAO survey [43].

We again perform a Monte Carlo analysis to obtain a constraint on $m_\nu$. Regarding the expected data from Planck, we analyze it for multipoles $l < 2500$, obviating the need to consider the SZ effect. Thus, we do not include the parameter $A_{SZ}$ in the analysis here. We make use of the lensing of CMB in the likelihood calculation, which is effective at constraining masses of neutrino in the future observations of CMB [44]. We note that our Planck forecast assumes an ideal lensing extraction, which could be achieved with very detailed understanding of noise and foregrounds. For the analyses in this section, we assume fiducial cosmological parameters summarized in table 4. For the projected Planck data, we adopt the instrumental specifications found in ref. [44]. Regarding a future BAO data set, we adopt predicted performance of the BOSS survey, which will measure the angular diameter distance $d_A(z)$ and the Hubble expansion rate $H(z)$ of the Universe over a broad range of redshifts (table 5). Roughly

\(^7\) $H_0$ may also be constrained tightly by the gravitational lens time delays [40–42]. For recent results, we refer to e.g. [25].
Table 6. Constraints on the ΛCDM+$m_\nu$ model from analysis of Planck, Planck+$H_0$, Planck+BOSS and Planck+BOSS+$H_0$ data. The fiducial value for the neutrino mass is taken to be $m_\nu = 0$ eV. We assume a prior on $H_0$ with 1% uncertainty (1σ) provided by a future direct measurement.

| parameters | Planck | Planck+$H_0$ | Planck+BOSS | Planck+BOSS+$H_0$ |
|------------|--------|--------------|-------------|------------------|
| $\omega_b \times 10^2$ | $2.267^{+0.013}_{-0.013}$ | $2.273^{+0.011}_{-0.011}$ | $2.274^{+0.012}_{-0.010}$ | $2.275^{+0.012}_{-0.010}$ |
| $\omega_c$ | $0.1101^{+0.0009}_{-0.0012}$ | $0.10949^{+0.00080}_{-0.00085}$ | $0.10946^{+0.00078}_{-0.00073}$ | $0.10936^{+0.00078}_{-0.00072}$ |
| $\theta_s \times 10^2$ | $104.055^{+0.026}_{-0.027}$ | $104.067^{+0.023}_{-0.024}$ | $104.067^{+0.023}_{-0.023}$ | $104.069^{+0.023}_{-0.023}$ |
| $\tau$ | $0.0878^{+0.0040}_{-0.0047}$ | $0.0883^{+0.0038}_{-0.0037}$ | $0.0882^{+0.0039}_{-0.0040}$ | $0.0883^{+0.0038}_{-0.0040}$ |
| $\omega_\nu$ | $< 0.0028$ (95%) | $< 0.0014$ (95%) | $< 0.0012$ (95%) | $< 0.0011$ (95%) |
| $n_s$ | $0.9617^{+0.0036}_{-0.0028}$ | $0.9632^{+0.0024}_{-0.0026}$ | $0.9632^{+0.0029}_{-0.0031}$ | $0.9635^{+0.0026}_{-0.0027}$ |
| $\ln[10^{10}A_s]$ | $3.0647^{+0.0070}_{-0.0096}$ | $3.0643^{+0.0072}_{-0.0090}$ | $3.0640^{+0.0076}_{-0.0093}$ | $3.0640^{+0.0075}_{-0.0093}$ |
| age [Gyr] | $13.757^{+0.035}_{-0.068}$ | $13.713^{+0.024}_{-0.026}$ | $13.710^{+0.019}_{-0.025}$ | $13.706^{+0.018}_{-0.020}$ |
| $\Omega_m$ | $0.2688^{+0.0073}_{-0.0142}$ | $0.2597^{+0.0056}_{-0.0047}$ | $0.2591^{+0.0048}_{-0.0047}$ | $0.2586^{+0.0047}_{-0.0042}$ |
| $z_{re}$ | $10.53^{+0.31}_{-0.42}$ | $10.52^{+0.33}_{-0.37}$ | $10.51^{+0.32}_{-0.39}$ | $10.51^{+0.33}_{-0.38}$ |
| $H_0$ | $70.64^{+1.24}_{-0.85}$ | $71.51^{+0.54}_{-0.46}$ | $71.58^{+0.48}_{-0.44}$ | $71.68^{+0.44}_{-0.41}$ |
| $m_\nu$ [eV] | $< 0.088$ (95%) | $< 0.042$ (95%) | $< 0.038$ (95%) | $< 0.034$ (95%) |

Table 7. Same table as in table 6, but for the fiducial neutrino mass $m_\nu = 0.1$ eV.

speaking, the BOSS survey adopted here measures geometry of the Universe both in transverse and the line of sight direction with uncertainties of about 1% level at redshifts $z = 0.35$, 0.6 and 2.5. For the mass of neutrino, we assume two fiducial values of $m_\nu = 0$ eV and 0.1 eV. The fiducial value of 0.1 eV is motivated from the KATRINT neutral mass direct search exper-
iment, which will achieve a sensitivity of $m_\nu < 0.2$ eV (95% C.L.) within the next decade [45].

In tables 6 and 7, we summarize the projected constraints on cosmological parameters for the cases with the fiducial values of $m_\nu = 0$ and 0.1 eV with and without 1% prior on $H_0$. The cases where we include the BAO data or a combination of Planck, BAO and $H_0$ are also shown. In figure 2, contours of 68% and 95% C.L. constraints are depicted for two different fiducial values for the neutrino mass $m_\nu = 0$ and 0.1 eV. In both cases, the degeneracy between $m_\nu$ and $H_0$ is significant for analyses of CMB data alone. However, imposing a Gaussian prior for the Hubble constant with the error of 1% improves the constraint significantly. For the case of the fiducial value of $m_\nu = 0$ eV, 95% C.L. limit for Planck data is $m_\nu < 0.088$ eV, which improves to $m_\nu < 0.042$ eV for a prior on $H_0$ with 1% error. When we assume $m_\nu = 0.1$ eV as a fiducial value, the neutrino mass is constrained to be $m_\nu < 0.197$ eV if no prior on $H_0$ is adopted. Very interestingly, when we impose the prior on the Hubble constant with 1% error, the constraint becomes $m_\nu = 0.107^{+0.021}_{-0.020}$ eV (68% C.L.), which indicates that we can detect neutrino masses at about 5$\sigma$ level if $m_\nu = 0.1$ eV. Here it should be noted that even if we vary the fiducial value of $H_0$ by 10%, the limit is almost unchanged. As seen from figure 2, when we assume no prior on $H_0$, the massless neutrino is still allowed at 2$\sigma$ level for this case. This analysis shows that an accurate determination of $H_0$ may lead us towards the detection of the absolute mass of neutrino from cosmology and therefore this direction research should be pursued along with other methods.

The degree of constraint on $m_\nu$ is in fact a continuous function of measurement uncertainty on $H_0$, and thus far 1% has been illustrative. For a fiducial mass $m_\nu = 0$ eV and an expected error $\Delta H_0/H_0$ (1$\sigma$), the constraints on $m_\nu$ can be fitted as, by marginalizing over other cosmological parameters

$$m_\nu < 0.045 \left( \frac{\Delta H_0}{H_0} [\%] \right)^{0.36} \text{(95\% C.L.)}. \quad (3.1)$$

On the other hand, for a fiducial $m_\nu = 0.1$ eV, the constraints can be fitted as

$$m_\nu = 0.1 \pm 0.022 \left( \frac{\Delta H_0}{H_0} [\%] \right)^{0.41} \text{(68\% C.L.)}. \quad (3.2)$$

Eqs. (3.1) and (3.2) are valid for the range 1% $\leq \Delta H_0/H_0 \leq 5\%$ in the $\Lambda$CDM+$m_\nu$ model.

When the BAO data from BOSS are combined with Planck data, similar limits can be obtained as seen from tables 6 and 7. These analyses indicate that observations of the geometric distances can very usefully constrain the neutrino mass. As demonstrated, in the near term, accurate observations of $H_0$ and the BAO scale would give almost comparable limits when these are combined with CMB data. Needless to say, when both an $H_0$ measurement and BAO data are combined with CMB, we can obtain a tighter constraint on $m_\nu$ than that with CMB data alone.

As a closing remark of this section, we note that 5$\sigma$ detection of the neutrino mass of $m_\nu = 0.1$ eV requires a 1% accuracy of $H_0$ measurement. When the accuracy is 3%, it becomes 3$\sigma$. Thus for the detection of the neutrino mass with high significance, a very accurate measurement such as that with 1% error would be necessary.

4 Effects of dark energy equation of state and the curvature of the Universe

So far, we have investigated constraints on neutrino masses, assuming a flat $\Lambda$CDM cosmological model. We have discussed that there exists a degeneracy between $m_\nu$ and $H_0$ in CMB
power spectrum, but by adopting the prior on $H_0$ and/or combining the BAO data, we can break the degeneracy to some extent and obtain a tighter constraint on $m_\nu$. However, we note that the extent of the degeneracy depends on distance scale, and this is affected by the equation of state for dark energy and the curvature of the Universe, which we have previously fixed to be $w_X = -1$ and $\Omega_k = 0$. Therefore here we relax these assumptions and investigate how the variations of $w_X$ and $\Omega_k$ affect the determination of $m_\nu$, paying particular attention to the combination of present-day CMB data and constraints on $H_0$ and the BAO scale. Furthermore, as we did in the previous section, we also investigate an attainable bound on $m_\nu$ in the near future using the projected characteristics of Planck data and anticipated constraints on $H_0$ and the BAO scale.

In tables 8–10, we summarized the constraints on cosmological parameters for the cases where (i) the equation of state for dark energy $w_X \neq -1$ is allowed (but constant in time) and a flat universe is assumed ($w\text{CDM}$ model), (ii) $w_X = -1$ but a non-flat universe is allowed ($\Omega\text{CDM}$ model) and (iii) $w_X \neq -1$ and a non-flat universe are allowed ($\Omega w\text{CDM}$ model), respectively. Since the geometric degeneracy between $\Omega_k$ and $H_0$ is particularly very large in the CMB analysis, we do not obtain meaningful cosmological constraints on them through analysis of CMB data alone for non-flat cases, such as $\Omega\text{CDM}$ and $\Omega w\text{CDM}$ models. (Hence these CMB analyses are not presented in tables 9–10.) Although this problem is mitigated somewhat through combinations of an $H_0$ prior and BAO data with CMB data, a degeneracy remains among $H_0$, $w_X$, and $\Omega_k$ because each affects the distance scales. Thus even if we include the geometry measurements such as $H_0$ and the BAO scale, the constraint on $m_\nu$ is not so improved when $w_X$ and/or $\Omega_k$ are varied since the change of the geometric distance made by varying $m_\nu$ by the amount $\mathcal{O}(0.1)$ eV can be easily compensated by a relatively small adjustment of $w_X$ and $H_0$, and then the neutrino mass effectively decouples from the degeneracy in the geometric distance (tables 8–10). This is in contrast to analysis of the flat $\Lambda\text{CDM}$ model, where the degeneracy involves just two quantities rather than four. Nonetheless, the combination of distance scale measurements substantively improve limits on $w_X$ and $\Omega_k$. Furthermore, we note that for $w\text{CDM}$ model, the limit on $m_\nu$ from CMB+BAO data is looser than that for CMB alone. This is because the BAO data [23] favor a smaller value for $w_X$, and $w_X$ and $m_\nu$ are negatively correlated.

Finally we investigate future constraints on neutrino masses in $w\text{CDM}$, $\Omega\text{CDM}$ and $\Omega w\text{CDM}$ model. We repeat the analysis using the expected data from Planck, direct $H_0$ measurement and BAO scale as we did in section 3 for these models. We summarized the marginalized constraints on $m_\nu$ in tables 11 and 12 for fiducial values of $m_\nu = 0$ and 0.1 eV, respectively. Hereafter we mainly discuss the constraints for the case of the fiducial value of the neutrino mass of $m_\nu = 0$ eV, because there is no qualitative difference between fiducial models of $m_\nu = 0$ and 0.1 eV.

Let us first discuss the case with the $w\text{CDM}$ model. For the Planck only analysis, $w_X$ degenerates with $H_0$ most significantly, which can be seen in figure 4 where we present 2d contours of marginalized posterior distributions among $m_\nu$, $w_X$ and $H_0$. As mentioned for the constraints from current data, the neutrino mass is effectively irrelevant to the degeneracy in the geometric distance, particularly in the CMB analysis in this model. Due to this strong degeneracy between $w_X$ and $H_0$, even if we add the data from the $H_0$ measurement, the constraint on $m_\nu$ does not improve much unlike the case of the $\Lambda\text{CDM}$ model since the effect of $m_\nu$ can be easily cancelled by changing $w_X$. On the other hand, when we add the BAO data to Planck, the constraint on $m_\nu$ becomes slightly stronger. Since the BAO scale measurement can probe the geometric distance up to $z = 2.5$, it is sensitive to both $w_X$ and...
the BAO data and the direct measurement of $\omega$ parameters CMB CMB+SHOES CMB+BAO CMB+BAO+SHOES

| parameters | CMB+SHOES | CMB+BAO | CMB+BAO+SHOES |
|------------|-----------|---------|---------------|
| $\omega_b \times 10^2$ | $2.248^{+0.054}_{-0.066}$ | $2.249^{+0.058}_{-0.060}$ | $2.231^{+0.049}_{-0.064}$ | $2.230^{+0.049}_{-0.060}$ |
| $\omega_c$ | $0.1103^{+0.0048}_{-0.0051}$ | $0.1103^{+0.0052}_{-0.0051}$ | $0.1121^{+0.0051}_{-0.0043}$ | $0.1123^{+0.0050}_{-0.0043}$ |
| $\theta_s \times 10^2$ | $104.12^{+0.23}_{-0.20}$ | $104.13^{+0.25}_{-0.19}$ | $104.10^{+0.21}_{-0.22}$ | $104.10^{+0.21}_{-0.22}$ |
| $\tau$ | $0.086^{+0.015}_{-0.017}$ | $0.086^{+0.013}_{-0.018}$ | $0.084^{+0.015}_{-0.016}$ | $0.084^{+0.015}_{-0.015}$ |
| $\omega_\nu$ | $< 0.014$ (95%) | $< 0.013$ (95%) | $< 0.015$ (95%) | $< 0.015$ (95%) |
| $w_X$ | $-1.29^{+0.55}_{-0.47}$ | $-1.27^{+0.27}_{-0.15}$ | $-1.41^{+0.52}_{-0.24}$ | $-1.38^{+0.32}_{-0.19}$ |
| $n_s$ | $0.949^{+0.016}_{-0.016}$ | $0.948^{+0.018}_{-0.014}$ | $0.943^{+0.015}_{-0.017}$ | $0.943^{+0.017}_{-0.013}$ |
| $\ln[10^{10} A_s]$ | $3.049^{+0.037}_{-0.037}$ | $3.050^{+0.040}_{-0.035}$ | $3.049^{+0.034}_{-0.039}$ | $3.050^{+0.032}_{-0.040}$ |

Table 8. Constraints on the $wCDM+m_\nu$ model from current CMB data, combined with and without the BAO data and the direct measurement of $H_0$.

| Table 8. Constraints on the $wCDM+m_\nu$ model from current CMB data, combined with and without the BAO data and the direct measurement of $H_0$. |

| parameters | CMB+SHOES | CMB+BAO | CMB+BAO+SHOES |
|------------|-----------|---------|---------------|
| $\omega_b \times 10^2$ | $2.271^{+0.050}_{-0.057}$ | $2.262^{+0.045}_{-0.056}$ | $2.271^{+0.059}_{-0.064}$ |
| $\omega_c$ | $0.1076^{+0.0047}_{-0.0047}$ | $0.1085^{+0.0044}_{-0.0049}$ | $0.1089^{+0.0053}_{-0.0039}$ |
| $\theta_s \times 10^2$ | $104.17^{+0.23}_{-0.21}$ | $104.15^{+0.23}_{-0.21}$ | $104.17^{+0.22}_{-0.21}$ |
| $\tau$ | $0.088^{+0.016}_{-0.017}$ | $0.088^{+0.015}_{-0.017}$ | $0.088^{+0.013}_{-0.019}$ |
| $\Omega_\nu$ | $0.006^{+0.006}_{-0.010}$ | $0.000^{+0.005}_{-0.010}$ | $-0.0018^{+0.005}_{-0.0084}$ |
| $\omega_\nu$ | $< 0.011$ (95%) | $< 0.012$ (95%) | $< 0.011$ (95%) |
| $n_s$ | $0.957^{+0.014}_{-0.012}$ | $0.954^{+0.014}_{-0.012}$ | $0.9560^{+0.0065}_{-0.0084}$ |
| $\ln[10^{10} A_s]$ | $3.046^{+0.036}_{-0.038}$ | $3.047^{+0.036}_{-0.038}$ | $3.0510^{+0.037}_{-0.037}$ |

Table 9. Constraints on $\Omega\Lambda CDM+m_\nu$ model from current observations.

$H_0$. Thus the effect of varying $m_\nu$ on the distance measure can only be partly compensated by the change of $w_X$ and $H_0$ in this case. Hence the situation is a bit different from the
Table 10. Constraints on $\Omega wCDM+\nu$ model from current observations. Note that constraints on $w_X$ from CMB+SHOES and CMB+BAO are not presented, since the posterior probabilities of $w_X$ are not bounded in the prior range, $w_X \in [-3,0]$.

| parameters | CMB+SHOES | CMB+BAO | CMB+BAO+SHOES |
|------------|-----------|---------|---------------|
| $\omega_b \times 10^2$ | $2.246^{+0.052}_{-0.060}$ | $2.241^{+0.052}_{-0.058}$ | $2.241^{+0.045}_{-0.063}$ |
| $\omega_c$ | $0.1097^{+0.0049}_{-0.0061}$ | $0.1099^{+0.0045}_{-0.0064}$ | $0.1100^{+0.0043}_{-0.0050}$ |
| $\theta_s \times 10^2$ | $104.13^{+0.22}_{-0.22}$ | $104.12^{+0.21}_{-0.22}$ | $104.11^{+0.24}_{-0.19}$ |
| $\tau$ | $0.086^{+0.016}_{-0.016}$ | $0.086^{+0.015}_{-0.017}$ | $0.086^{+0.014}_{-0.017}$ |
| $\Omega_k$ | $0.009^{+0.014}_{-0.016}$ | $-0.006^{+0.006}_{-0.010}$ | $-0.007^{+0.006}_{-0.010}$ |
| $\omega_\nu$ | $< 0.013$ (95%) | $< 0.013$ (95%) | $< 0.013$ (95%) |
| $w_X$ | — | — | $-1.47^{+0.29}_{-0.27}$ |
| $n_s$ | $0.948^{+0.016}_{-0.013}$ | $0.947^{+0.016}_{-0.013}$ | $0.947^{+0.015}_{-0.015}$ |
| $\ln [10^{10} A_s]$ | $3.046^{+0.037}_{-0.037}$ | $3.046^{+0.034}_{-0.034}$ | $3.047^{+0.032}_{-0.039}$ |
| age[Gyr] | $14.4^{+0.8}_{-1.0}$ | $14.32^{+0.42}_{-0.46}$ | $14.29^{+0.41}_{-0.40}$ |
| $\Omega_m$ | $0.256^{+0.024}_{-0.023}$ | $0.250^{+0.039}_{-0.046}$ | $0.254^{+0.023}_{-0.024}$ |
| $z_{re}$ | $10.4^{+1.4}_{-1.3}$ | $10.4^{+1.3}_{-1.3}$ | $10.4^{+1.5}_{-1.1}$ |
| $H_0$ | $73.5^{+3.6}_{-3.5}$ | $75.0^{+5.1}_{-7.9}$ | $73.9^{+2.7}_{-3.5}$ |
| $m_\nu$ [eV] | $< 0.40$ (95%) | $< 0.42$ (95%) | $< 0.41$ (95%) |

Figure 3. Confidence regions of 68% and 95% C.L. from current CMB data on the $m_\nu-H_0$ plane for the $wCDM$ model (left), $\Omega\Lambda CDM$ model (middle) and $\Omega wCDM$ model (right). Presented are the results from the present CMB data alone (black dotted line), CMB with SHOES prior $H_0 = 74.2 \pm 3.6$ (red dashed line), CMB with BAO data (blue solid line), and CMB combined with BAO and SHOES (green and yellow shaded regions). Inner (outer) contours and shaded regions show regions of 68%(95%) C.L. Note that $H_0$ is not explored at $H_0 > 100$ nor $H_0 < 40$ in MC analyses. Note that analyses with CMB data alone are not performed for the $\Omega\Lambda CDM$ and $\Omega wCDM$ models (See text for details).

The case of Planck+$H_0$ where $w_X$ can be relatively freely adjusted to cancel the effect of varying $m_\nu$. In any case, to obtain a severe constraint on $m_\nu$, we need to determine the values of both $H_0$ and $w_X$ accurately, which motivates us to include both measurements of $H_0$ and the...
Figure 4. Regions of 68% and 95% C.L. for the $w$CDM model using expected data of future observations. We assume $m_\nu = 0$ eV as a fiducial value. Shown are constraints from Planck alone (black dotted line), Planck+$H_0$ (red dashed line), Planck+BOSS (blue solid line) and Planck+$H_0$+BOSS (green and yellow shaded regions). Inner (outer) contours and shaded regions show regions of 68% (95%) C.L.

BAO scale. As seen from table 11, the constraint on $m_\nu$ from Planck+BOSS+$H_0$ is much severer than that from Planck alone. Thus the direct measurement of $H_0$ should also have a strong power in constraining $m_\nu$ in combination with some other distance measurement such as BAO in this case as well.

Next we look at the case of the $\Omega$CDM model. In this case, the situation is somewhat different (See figure 5). One of noticeable differences is that $m_\nu$, $H_0$ and $\Omega_k$ are notably correlated with one another in CMB. This degeneracy shows that the change in the angular diameter distance to last scattering surface made by varying one of the above parameters cannot be efficiently compensated by just adjusting another. Thus other observations of the geometrical distances would be necessary. When the BAO scale measurement is included, the constraint on $m_\nu$ is improved because of a better determination of $\Omega_k$ and $H_0$. For the analysis of Planck+$H_0$, the constraint on $m_\nu$ also becomes stronger. As seen from table 11, Planck+BOSS and Planck+$H_0$ give almost the same limit on $m_\nu$. However, even if both the BAO scale and a direct $H_0$ measurements are included in the analysis, the constraint is not so improved compared to Planck+BOSS and Planck+$H_0$. This is because the BAO scale measurement effectively gives the same information as the direct $H_0$ measurement in this model, which is also the reason why the limit on $m_\nu$ from Planck+BOSS and Planck+$H_0$ are almost the same level.

Now we move on to the case of the $\Omega w$CDM model where both $\Omega_k$ and $w_X$ are varied. In figure 6, 1$\sigma$ and 2$\sigma$ contours are depicted, which shows the degeneracies among the key parameters, $m_\nu$, $\Omega_k$, $w$ and $H_0$. Since BOSS measures the geometry at several, wide-range
redshifts, the inclusion of BOSS to Planck can remove degeneracies among these parameters more compared to the case of Planck+$H_0$ in this model. However, when we combine the BOSS survey with a $H_0$ measurement at 1% accuracy, constraints on some key parameters are much more improved from those without data of the direct $H_0$ measurement. Most significant improvement can be found in constraints on $w_X$. Even though the BAO scale measurement can measure both of $w_X$ and $H_0$ simultaneously to some extent, there remains some degeneracy between these parameters. Thus adding a direct measurement of $H_0$ would be very helpful to break the degeneracy to obtain a better determination of $w_X$. The improvement of constraints on $w_X$ makes the constraint on $m_\nu$ stronger because of the residual degeneracy between $w_X$ and $m_\nu$.

So far we have seen how the measurements of $H_0$ and the BAO scale improve the constraints on the parameters $m_\nu$, $\Omega_k$ and $w_X$, that are severely degenerate with one another in geometrical distances. The BOSS survey is now running and expected to be a powerful tool for constraining these parameters. However, as we have stressed, an $H_0$ measurement has its own cosmological information independent of other observations and if a measurement at 1% accuracy is achieved, it will bring us constraints on these parameters comparable with those from the BOSS survey, or by combining all these together, there would be much more improvements for the constraint on $m_\nu$.

5 Summary

In this paper, we have investigated constraint of neutrino mass from cosmological data with particular attention to direct measurements of the Hubble constant and BAO scale as supplements to CMB data. Since a strong degeneracy exists between $m_\nu$ and $H_0$ in the CMB analysis, which originates from the distance scale, neutrino mass is not effectively constrained.
Figure 6. Same as in figure 4 but for $\Omega_w$CDM model.

| models     | Planck | Planck+$H_0$ | Planck+BOSS | Planck+BOSS+$H_0$ |
|------------|--------|--------------|--------------|------------------|
| $\Lambda$CDM | $< 0.088$ (95%) | $< 0.042$ (95%) | $< 0.038$ (95%) | $< 0.034$ (95%) |
| $w$CDM     | $< 0.10$ (95%) | $< 0.097$ (95%) | $< 0.073$ (95%) | $< 0.064$ (95%) |
| $\Omega$CDM | $< 0.13$ (95%) | $< 0.055$ (95%) | $< 0.055$ (95%) | $< 0.053$ (95%) |
| $\Omega w$CDM | $< 0.14$ (95%) | $< 0.12$ (95%) | $< 0.079$ (95%) | $< 0.070$ (95%) |

Table 11. Summary of constraints on $m_\nu$ from future observations. Presented are the upper limits on $m_\nu$ at 95% C.L. for a fiducial $\Lambda$CDM model with $m_\nu = 0$ eV.

from the current CMB observations alone. However, available measurement of $H_0$ breaks this degeneracy and provides a mass constraint of $m_\nu < 0.20$ eV (95% C.L.). We have also examined the constraint using the BAO data recently released. When the measurement of BAO scales is combined with the present CMB data, we obtain a similar limit for $m_\nu$; current Hubble constant and BAO measurements are competitive in the context of $\Lambda$CDM models,
where we emphasize the EOS of dark energy is constrained to be \(-1\) and curvature is zero.

We have also studied attainable limits on \(m_\nu\) that may be anticipated from next-generation data sets. For this purpose, we have used projected Planck data, assuming an ideal lensing extraction, in combination with a future precise measurement of \(H_0\) and of the BAO scale as may be provided by BOSS before 2015. If the Hubble constant is directly measured with precision \(\Delta H_0/H_0 = 0.01\), then neutrino mass can be detected at the 5\(\sigma\) level for a 0.1 eV mass. Similar accuracy can be achieved through combination of BAO and CMB data. The uncertainty of the Hubble constant degrades the constraint on the neutrino mass approximately as \((\Delta H_0/H_0)^{0.4}\). A more modest accuracy of e.g., 3\% does not provide a useful mass constraint and favors BAO techniques.

In models where \(w_X\) or \(\Omega_k\) are allowed to vary (including more parameters), the geometric degeneracy between \(m_\nu\) and \(H_0\) is broadened, and combinations of data and priors (i.e., CMB+\(H_0\) or CMB+BAO) do not strongly constrain \(m_\nu\). However when both \(H_0\) measurement and BAO are combined with CMB, the constraint becomes tighter. In this respect, a direct measurement of \(H_0\) should be very beneficial along with other cosmological observations.

We have investigated the impact of precision measurement of \(H_0\) on the neutrino mass. The analysis demonstrates that the application of cosmological data has a unique contribution to make in the study of fundamental physics. Analyses similar to ours would plausibly demonstrate strong constraint of other parameters, e.g., \(w_X\) [22, 46], though degeneracies will vary case to case. In general cases (e.g., nonzero curvature and \(w_X \neq -1\)), the constraints imposed by future BAO and \(H_0\) measurements, if of comparable accuracy, are likely contribute at about the same level, but the larger number of model parameters argues pursuit of both BAO and \(H_0\) measurements, so as to leverage the independence of systematic errors in each.

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