Avoiding the uncertainty from correlation between $|\Delta m^2_{31}|$ and CP phase $\delta$
in $\nu_\mu \rightarrow \nu_\mu$ long baseline experiments

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Abstract

We introduce a new index $I_{\Delta m^2_{31}}$ to find where is the better setup of the baseline length
and energy to avoid as well as possible the uncertainty from the correlation between $\Delta m^2_{31}$
and $\cos \delta$ in $\nu_\mu \rightarrow \nu_\mu$ long baseline experiments.

Detection of the CP effect in lepton sector (MNS matrix\cite{1}) is one of the remaining most
important subjects in not only elementary particle physics but also particle cosmology. To
confirm the existence of the CP phase, many long baseline experiments\cite{2,3,4,5} by using
$\nu_\mu \rightarrow \nu_e$ oscillation mode are proposing. After finding the CP effects in $\nu_\mu \rightarrow \nu_e$, as the next
step, it will be an important subject to check whether they are consistent with the standard
model(SM). To do so, we need to measure the CP effects(phase) independently by using the
other oscillation mode. Measuring CP phase by $\nu_\mu \rightarrow \nu_\mu$ mode is going to be more important to
confirm the SM and to investigate the existing possibility of new physics. We have to confirm the
consistency and the unitarity in lepton sector\cite{6} by comparing the observables extracted from the
different oscillation modes. We are investigating the CP effect in $\nu_\mu \rightarrow \nu_\mu$ mode in our work\cite{7}.

The dependence of the probability on the CP phase $\delta$ with the maximal 2-3 mixing $\theta_{23} = 45^\circ$ is
written as follows\cite{7,8,9}:

$$P_{\mu\mu} = A_{\mu\mu} \cos \delta + C_{\mu\mu} + D_{\mu\mu} \cos 2\delta \approx A_{\mu\mu} \cos \delta + C_{\mu\mu} + O(\sin \theta_{13} \Delta m^2_{21}),$$ \hspace{1cm} (1)

where $D_{\mu\mu}$ as a coefficient of $\cos 2\delta$ is negligible because the magnitude should be proportional
to the quite small parameter $\sin \theta_{13} \Delta m^2_{21}$. $A_{\mu\mu}$ and $C_{\mu\mu}$ are the quantities determined by
the parameters except for the CP phase $\delta$. The effect from $\cos \delta$ depends on the magnitude of $A_{\mu\mu}$
so that it is an index to know the CP dependence. We discussed where is better set of the
baseline length $L$ and the neutrino energy $E$ to extract the CP effect from $\nu_\mu \rightarrow \nu_\mu$ experiment
and pointed out it favors $E < 2\text{GeV}$, $L > 2000\text{km}$. At once, we showed it seems to be difficult to determine the CP phase because there is a correlation between $\Delta m^2_{31}$ and $\cos \delta$ in small $L/E$.

In this letter, we introduce a new index $I_{\Delta m^2_{31}}$ to look for the better region in $(E, L)$ plane and to avoid the uncertainty from $\Delta m^2_{31}$-$\cos \delta$ correlation. Here we are using the following input parameters: $\Delta m^2_{21} = 8.1 \times 10^{-5} \text{eV}^2$, $\sin^2 \theta_{12} = 0.31$, $\sin^2 2\theta_{23} = 1$, and for an unknown parameter $\theta_{13}$, the upper bound $|\sin^2 2\theta_{13}| = 0.16$ is used. In the estimation of the probability $P_{\mu \mu}$, we are using the exact solution for the neutrino oscillation in matter.

The $\Delta m^2_{31}$-$\cos \delta$ correlation is plotted in Fig.1, where we assume $(\cos \delta, \Delta m^2_{31}^{\text{true}}) = (0, 2.5 \times 10^{-3} \text{eV}^2)$ as the true values and the plotted points show where the probability $P_{\mu \mu}$ at the fake values $(\cos \delta', \Delta m^2_{31})$ are almost same with $P_{\mu \mu}^{\text{true}}$ at true value. The figure shows the linear relation between the fake parameters $\Delta m^2_{31}$ and $\cos \delta'$. As we discussed in our previous work, there is a relation between the true value and fake one which are producing same probability approximately within the $L/E < 1000$ as follows:

\[
\left( |\Delta m^2_{31}| - |\Delta m^2_{31}^{\text{true}}| \right) = -4J_e \Delta m^2_{21} (\cos \delta' - \cos \delta) \quad (2)
\]

\[
= -0.0146 \times 10^{-3} (\cos \delta' - \cos \delta), \quad (3)
\]

where $J_e = \frac{1}{\text{eVs}^2} \sin 2\theta_{12} \sin 2\theta_{23} \sin \theta_{13} \cos \theta_{13} \simeq 0.045$. If the relation are satisfied, it means one can not determine the magnitude of CP phase without uncertainty. Namely, for the error of $|\Delta m^2_{31}|$, all range of $360^\circ$ is satisfied as the solution. Indeed, even if the error is 1% level, $|\Delta m^2_{31}| = (2.50 \pm 0.02) \times 10^{-3} \text{eV}^2$, we have to consider the uncertainty. From the left of Fig.1, one can find the linear relation between $|\Delta m^2_{31}|$ and $\cos \delta'$ for several baseline lengths. The right figure shows the dependence of fake region of $\Delta m^2_{31}$ on the baseline length $L$ at the case of $\cos \delta' = 1(\delta' = 0^\circ)$ which leads to same probability $P_{\mu \mu}$ with true (input) value $\cos \delta = 0(\delta = 90^\circ)$. From this, we find that the dependence may not be so trivial. One can find that the fake region breaks at several
LS in the right of Fig.1. Hence we investigate around \( L = 505, 1000, 2000, 3000, 4000, 5000\) (km) where the fake region disappears.

Figure 2: The region satisfying \( (P_{\mu\mu}(\delta', |\Delta m^2_{31}|) - P_{\mu\mu}^{true}) / P_{\mu\mu}^{true} < 0.002 \) on \( \cos \delta', |\Delta m^2_{31}| \) is shown in the left figure. The region shows almost same probability with it at \( \cos \delta^{true} = 0 \) and \( \Delta m^2_{31}^{true} = 2.5 \times 10^{-3} \) eV\(^2\) for several \( L \)s. The probability as a function of \( L \) at \( \delta = 0^\circ, 90^\circ, 180^\circ \) in the right.

At \( L = 505, 1000, 2000, 3000, 4000, 5000\) (km), the correlation of \( |\Delta m^2_{31}| \) and \( \cos \delta' \) are plotted in Fig.2(left). The dependence seems to be different with the case in Fig.1 and the almost plotted points are around the true value \( \cos \delta = 0 \) at the \( L \) where the fake regions disappear in Fig.1. It may show that at the several suitable \( L \) one can investigate the CP phase without depending on the error of \( |\Delta m^2_{31}| \) so strong. Where is the region on \( (E, L) \) ? Comparing the right one of Fig.1 with the Fig.2(right) which shows the dependence of the probability on the baseline length \( L \), where the probability shows maximal or minimal. The fake regions also break around \( L = 500, 1500, 2500, 3500, 4500\) (km) but the probability is almost 0 so that we can not extract the CP effect around the \( L \). On the other hand, at \( L = 1010, 2000, \cdots \), \( P_{\mu\mu} \) shows maximal and the CP effects will also be maximal so that it may be possible to determine the CP phase without depending on \( \Delta m^2_{31} \) so hard. They correspond to the region with large \( A_{\mu\mu} \). From the left of Fig.2, one can find the extracted solutions of \( \cos \delta' \) is around the true value for the error of \( \Delta m^2_{31} \) at the special \( L \).

There is the uncertainty in determination of CP phase in \( \nu_\mu \rightarrow \nu_\mu \) oscillation experiments because of \( \Delta m^2_{31} \)-cos\( \delta \) correlation. So we introduce a new index \( I_{\Delta m^2_{31}} \) to search for where is more suitable energy \( E \) and distance \( L \) to avoid the uncertainty. It is defined by the difference of maximum and minimum probabilities (\( P^{max}_{\mu\mu} \) and \( P^{min}_{\mu\mu} \)) within the error of \( \Delta m^2_{31} \) (\( |\Delta m^2_{31}| = 2.50 \pm 0.02 \times 10^{-3} \) eV\(^2\))\(^1\).

\[
I_{\Delta m^2_{31}} = \frac{P^{max}_{\mu\mu}(|\Delta m^2_{31}|) - P^{min}_{\mu\mu}(|\Delta m^2_{31}|)}{P^{max}_{\mu\mu}(|\Delta m^2_{31}|) + P^{min}_{\mu\mu}(|\Delta m^2_{31}|)},
\]

This is the index to indicate how affecting the probability from the error of \( |\Delta m^2_{31}| \). The regions which the new index is as small as possible are favored to avoid the effects from \( |\Delta m^2_{31}| \).

\(^1\)We expect that the experimental error of \( \Delta m^2_{31} \) will be reduced up to 1% level in the future experiments.
other hands, to determine the CP phase, the regions the dependence on \( \cos \delta \) becomes larger are favored. Using \( A_{\mu\mu} \) one can find the regions. \( A_{\mu\mu} \) is a coefficient of \( \cos \delta \) in eq.(1) and it can be also defined as the difference between the maximum and minimum of \( P_{\mu\mu} \) within all range of \( \delta \).

\[
A_{\mu\mu} \simeq \frac{(P_{\mu\mu}|_{\delta=0^\circ} - P_{\mu\mu}|_{\delta=180^\circ})}{2}.
\]

(5)

This corresponds to the numerator of \( I_{CP} \). The region with large \( A_{\mu\mu} \) will be useful to extract the CP phase. The dependence of \( I_{\Delta m^2_{31}} \) and \( A_{\mu\mu} \) on \( L \) at \( E = 1\,\text{GeV} \) are plotted in Fig.3.

From Fig.3, around 1000, 2000, 3000, 4000, 5000, ... km, \( I_{\Delta m^2_{31}} \) become minimum and then \( A_{\mu\mu} \) are showing nonzero values and not so small value. It means that around them, it may be possible to detect the CP phase without depending on \( \Delta m^2_{31} \) so strong.

The same discussion on the \((E,L)\) plane leads to the better setup to extract CP angle. In Fig. 4, the region are shown as yellow(red) area shows \( A_{\mu\mu} > 0.01 \) (0.1) and gray is \( I_{\Delta m^2_{31}} < 0.05 \). From Fig.4, one can roughly estimate the better experimental setup to detect CP phase without depending on the error of \( \Delta m^2_{31} \) so strong. Around 1000km which means T2KK\(^3\), around 0.5GeV or 1GeV is better energy region. Indeed, longer \( L \) is favored for \( A_{\mu\mu} \) but the minimal values of \( I_{\Delta m^2_{31}} \) will depart from zero so that we must more carefully choose the best place\(^3\). In addition, we define \( R_I \) as the ratio between \( I_{\Delta m^2_{31}} \) and \( A_{\mu\mu} \) as following,

\[
R_I \equiv \frac{I_{\Delta m^2_{31}}}{|A_{\mu\mu}|}.
\]

(6)

Around the \((E,L)\) where the ratio is smaller than 1 the dependence of \( P_{\mu\mu} \) on \( \Delta m^2_{31} \) should be smaller than the effect by CP phase. In Fig.5, the region of small \( R_I \) are plotted. It may show that we can constrain \( \delta \) by using the setup of \((E,L)\).

If the experiments are fixed, taking the small and suitable energy bin size, we can avoid the uncertainty from \(|\Delta m^2_{31}|-\cos \delta\) correlation. To estimate where is the better setup of \((E,L)\), the

\(^3\)We are investigating the T2KK case by using numerical analysis, which will be reported in the other paper\(^{14}\).
Figure 4: The better region to extract CP effect without depending on the experimental error of $\Delta m_{31}^2$ are shown as the overlapping area. The yellow region show $A_{\mu\mu} > 0.01$, the red region is $A_{\mu\mu} > 0.1$ and the gray one is $I_{\Delta m_{31}^2} < 0.05$.

Figure 5: The better region to extract CP effect without depending on the experimental error of $\Delta m_{31}^2$ are shown as the area with $R_l < 0.2$(dashed lines), and 1(solid line).
new index may be a powerful tool. As you find from Fig.1, even if the error of $\Delta m^2_{31}$ is reduced, the uncertainty of $\delta$ will remain in almost cases which are not chosen as so good ($E, L$). We expect that the new index is going to be such powerful tool to improve the determination of CP phase in $\nu_\mu \rightarrow \nu_\mu$ oscillation and it will be possible to confirm the consistency with the CP effects in $\nu_\mu \rightarrow \nu_e$.

**Acknowledgment**

The work of T.Y. was supported by 21st Century COE Program of Nagoya University.

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