Anisotropy of exciton spectrum and spin-orbit interactions in quantum wells in tilted magnetic field

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Abstract. We study theoretically excitonic energy spectrum and optical absorption in narrow-gap semiconductor quantum wells in strong magnetic field. We show that, in the presence of an in-plane field component, the absorption coefficient exhibit a double-peak structure due to hybridization of bright and dark excitons. If both Rashba and Dresselhaus spin-orbit terms are present, the spectrum is anisotropic in in-plane field orientation with respect to [100] axis. In particular, the magnitude of the splitting can be tuned in a wide interval by varying the azimuthal angle of the in-plane field. The absorption spectrum anisotropy would allow simultaneous measurement Dresselhaus and Rashba spin-orbit coefficients.

1. Introduction
The role of spin-orbit (SO) interactions in magnetooptics has been studied since the original work of Rashba[1]. Most of the theoretical work was devoted to the effect of the SO-induced nonparabolicity on cyclotron resonance lineshape[2]. In quantum wells (QW), the anticrossings of Landau levels (LL) due to SO coupling lead to an intricate structure of the absorption spectrum due to the interplay of Coulomb and SO interactions in two-dimensional (2D) electron gas [3]. In a strong magnetic field, such anticrossings can be observed in narrow-gap QWs when an in-plane component of magnetic field is used to bring the Zeeman-split adjacent LLs into resonance. While measurements of SO-induced beats of Shubnikov-de-Haas oscillations have long become a standard method for determining the SO interaction constants[2], direct observations of SO effects in optical spectroscopy were reported only recently [4].

In this paper, we study effect of SO interactions on QW excitons in a strong magnetic field[5, 6]. We find that, in the presence of an in-plane field, the interplay between Zeeman, Coulomb, and SO interactions leads to a drastic change in the magnetoeexciton energy spectrum. When separation between adjacent Landau levels with opposite spins becomes of the order of the magnetoeexciton binding energy, the bright and dark exciton dispersions exhibit anticrossing as a function of center-of-mass momentum. With varying in-plane field, the anticrossing moves to zero momentum leading to a spin-orbit-induced splitting of the excitonic absorption spectrum. In the presence of both Rashba and Dresselhaus spin-orbit terms, the spectrum is anisotropic and depends explicitly on the in-plane orientation of the magnetic field. In particular, by varying the azimuthal angle, the splitting of excitonic absorption peak can be tuned in a wide interval. This would allow to extract both Dresselhaus and Rashba SO coupling coefficients from absorption lineshape at two different angle values.
2. Spin-orbit interactions and exciton states in a tilted field

We consider QW in the presence of a strong tilted magnetic field $\mathbf{B} = \mathbf{B}_\perp + \mathbf{B}_||$ with tilt angle $\theta$ and azimuthal angle $\varphi$ with respect to crystallographic axes of the $[001]$ plane. We assume that QW is sufficiently narrow so that the effect of the in-plane field component on the orbital motion is negligibly small. The total two-band Hamiltonian is $H = H_e + H_h + H_{eh}$, where $H_e$ and $H_h$ are Hamiltonians for an electron and a hole, respectively, and $H_{eh}$ is Coulomb interaction.

The electron Hamiltonian has the form $H_e = H_0 + H_Z + H_{so}$, where $H_0$ is the orbital part while $H_Z = \frac{1}{2} g^* \mu_B \sigma \mathbf{B}$ and $H_{so} = H_R + H_D$ are Zeeman and spin-orbit terms, respectively. Here $m$ and $g^*$ and electron effective mass and $g$-factor, respectively, $\mu_B$ is the Bohr magneton, $\sigma$ is the Pauli matrices vector, and $\mathbf{k} = -i \nabla \pm e \mathbf{A}$ is the in-plane momentum with upper/lower sign corresponding to electron/hole [we use the Landau gauge $\mathbf{A} = (0, x B_{\perp})$]. The two contributions to $H_{so}$ correspond to Rashba and Dresselhaus terms, $H_R = \alpha_R (\sigma_z k_y - \sigma_y k_x)$ and $H_D = \alpha_D (\sigma_x k_z - \sigma_y k_y)$, respectively. We are interested in low-lying exciton states and adopt a simple one-band Hamiltonian for the heavy hole states, $H_h = H_0 + H_Z$, where $H_Z = \frac{1}{2} g^h \mu_B B_{\perp, z}$, with eigenvalues corresponding to total angular momentum $J_z = \pm 3/2$. The in-plane $g$-factor for heavy holes vanishes so that, in the absence of heavy-light hole mixing, $J$ is quantized along the $z$-axis. At the same time, for electron states in the conduction band, the “natural” spin quantization axis is along the total field $\mathbf{B}$.

In the absence of SO interactions, the free electron eigenstates are given by products of Landau wave-function and two-component spinors, $\psi_{k,n}^\pm (\mathbf{r}) = \psi_{k,n} (\mathbf{r}) \chi_0^\pm$ with $\chi_0^+ = (1, 0)$, and $\chi_0^- = (0, 1)$ corresponding to eigenenergies $E_{0s} = \omega_c (n + 1/2) - s \omega_Z / 2$, where $\omega_c = e B_{\perp} / m_e$, and $\omega_Z = -g^* \mu_B B$ are cyclotron and Zeeman frequencies, respectively (and similarly for holes). In this basis, SO matrix elements that couple adjacent LLs with different spin polarizations, $t_{mn+1}^{-\pm} = \langle n - |H_{so}| n + 1^\pm |$, are given by [7]

$$t_{mn+1}^{-\pm} = -\frac{\sqrt{2} m_l}{l} \left( \alpha_R \sin^2 \theta e^{i \varphi} + i \alpha_D \cos^2 \theta e^{-i \varphi} \right),$$

where $l = (e B_{\perp})^{-1/2}$ is the magnetic length and we assumed that, for LLs with low $n$, SO constants are $n$-independent. Note that, the coupling magnitude depends on orientation (with respect to $[100]$ axis) of the in-plane field component. This $\varphi$-dependence results from the interference between the two types of spin-orbit interaction and is analogous to spatial anisotropy of electron spectrum at zero field[8, 9].

We now turn to the effect of SO interactions on the excitonic energy spectrum. We assume that the cyclotron energy is much larger than the characteristic Coulomb interaction energy, $\omega_c \gg e^2 / \kappa l$ ($\kappa$ is the dielectric constant), so the Coulomb-induced inter-LL transitions are suppressed. In the absence of SO interaction, the excitonic wave-function is given by $\Psi_{ss'}^{nn'} (\mathbf{r}_e, \mathbf{r}_h) = \Psi_{p,nn} (\mathbf{r}_e, \mathbf{r}_h) S^{ss'}$; the orbital part is the Kallin-Halperin wave-function ($\mathbf{p}$ is the center-of-momentum of e-h pair). The spin part is a diadic product of electron and hole spins. If electron and hole spins are quantized along $\mathbf{B}$ and $\mathbf{B}_{\perp}$, respectively, then $S^{ss'} = \chi_0^s \otimes \chi_0^{s'}$. We use the convention that first and second indices stand for electron and hole degrees of freedom, respectively (LL and spin). The optically-active (bright) exciton states, $\Psi_{z, +}^{nn}$ and $\Psi_{z, -}^{nn}$, excited by left and right circularly-polarized light, respectively, correspond to $n$th LL electron-hole pairs with spin contents $(\frac{1}{2}, -\frac{1}{2})$ and $(-\frac{1}{2}, +\frac{1}{2})$. All other exciton states are dark. In a strong field, $\omega_c \gg e^2 / \kappa l$, the bright and dark states are the eigenstates of the system, and so are decoupled from each other.

In the presence of SO interactions, the coupling of adjacent spin-split LLs changes both the orbital and spin content of excitonic eigenstates. The upper bright state, $\Psi_{00}^{++}$, couples to the dark state $\Psi_{10}^{++}$ in the region where $|E_{00}^{++} - E_{00}^{+}| \sim \Delta$, with $E_{00}^{++} (p) = \omega_Z / 2 + U_{00} (p)$ and $E_{10}^{+} (p) = \omega_c - \omega_Z / 2 + U_{10} (p)$. Here $U_{mn} (p) = -\int d\mathbf{r}_e d\mathbf{r}_h |\Psi_{mn} (\mathbf{p}; \mathbf{r}_e, \mathbf{r}_h)|^2 V (\mathbf{r}_e - \mathbf{r}_h)$ are the Coulomb matrix elements. The region of bright-dark exciton hybridization is given by $\delta_p \sim \Delta$, where $\delta_p = -\int d\mathbf{r}_e d\mathbf{r}_h |\Psi_{mn} (\mathbf{p}; \mathbf{r}_e, \mathbf{r}_h)|^2 V (\mathbf{r}_e - \mathbf{r}_h)$ are the Coulomb matrix elements.
polarized light incident normal to the plane can excite only bright excitons with spin projection $\sigma = \pm 1$.

For $|\delta_p|/\omega_c \ll 1$, it is sufficient to include only the coupling between adjacent LLs so that the lowest energy eigenstates can be presented as superpositions of bright and dark excitons, $\Psi = a \Psi_{00}^{\pm 0} + b \Psi_{10}^{\pm 0}$, where the coefficients $a$ and $b$ are determined by diagonalizing the full Hamiltonian $H = H_e + H_{lh} + H_{eh}$. We then obtain the new eigenenergies as

$$E_p^{(\pm)} = \frac{1}{2} \left[ \omega_c + U_{00}(p) + U_{10}(p) \pm \sqrt{\delta_p^2 + \Delta^2} \right],$$

where $\Delta = 2|\epsilon_{00}|$ is given by

$$\Delta^2 = 8 \left[ \frac{\alpha_R^2}{l^2} \sin^2 \frac{\theta}{2} + \frac{\alpha_D^2}{l^2} \cos^2 \frac{\theta}{2} + \frac{\alpha_R \alpha_D}{2l^2} \sin^2 \theta \sin 2\varphi \right].$$

The anticrossing gap magnitude depends on the orientation of in-plane field component. While $\theta$ is fixed by the resonance condition, we can vary $\Delta$ by changing the azimuthal angle $\varphi$. Remarkably, for $\varphi = -\pi/4$, and $\alpha_D/\alpha_R = \tan^2(\theta/2)$, the gap vanishes in the first order in SO coupling. The latter condition can be achieved, e.g., by tuning the Rashba coupling $\alpha_R$ with gate voltage.

3. Absorption spectrum

Let us turn to the role of SO interactions in the excitonic absorption spectrum. A circularly-polarized light incident normal to the plane can excite only bright excitons with spin projection $S_z = \pm 1$ with right/left ($\sigma_{\pm}$) polarized photon, respectively. The lowest energy absorption peak corresponds to the excitation of $S_z = -1$ electron-hole pair on $n = 0$ LL which, as discussed above, is only weakly affected by SO interactions. The next in energy $n = 0$ LL electron-hole pair with $S_z = 1$ is not an eigenstate of the final-state Hamiltonian but a superposition of states $\Psi_{00}^{00}$ with energies given by Eq. (3). In the anticrossing region, the contribution of higher-lying states is negligible and the absorption coefficient has a simple form

$$\alpha(\omega) \propto \sum_{(\pm)} |f^{(\pm)}|^2 \delta(\omega - E_0^{(\pm)}),$$

where we absorbed the bandgap into photon energy $\omega$. Here $f^{(\pm)}$ is the transition matrix element given by the overlap between optically excited bright exciton and the eigenstates $\Psi^{\pm}$. It can be easily calculated as

$$|f^{(\pm)}|^2 = \mu^2 \cos^2 \frac{\theta}{2} |a_0^{(\pm)}|^2 = \frac{\mu^2 \cos^2 \frac{\theta}{2}}{1 + e^{\pm 2i\beta_0}},$$

with $\sinh \beta_0 = \delta_0/\Delta = (\omega_Z - \omega_c - \epsilon_0/2)/\Delta$, where $\epsilon_0 = \sqrt{\frac{\tau^2}{\kappa^2} m}$ is the Coulomb binding energy of $n = 0$ LL exciton. For InSb QW at $B_\perp = 10$ T, the resonance condition $\delta_0 = 0$ yields $B_\|/B_\perp \approx 2.9$ T, corresponding to the tilt angle $\theta \approx 71^\circ$.

Below we present the results of numerical calculations for $d = 10$ nm wide InSb QW. Since for InSb $\gamma = 160$ eVÅ$^3$ [10], we have $\alpha_D \approx 158$ meV Å. Other parameters for InSb used in our calculations are: $m = 0.014m_0$ ($m_0$ is free electron mass), effective g-factor $g^* = -51$, and dielectric constant $\kappa = 16.5$. For all calculations, the perpendicular component of magnetic field was $B_\perp = 10$ T. In order to assess the accuracy of resonant level model, we included SO coupling between all four lowest spin-split LLs ($n = 0, 1$), but detected virtually no difference in the anticrossing region for the set of parameters used.

In Fig. 1 (upper panel) we show the evolution of the absorption spectrum for different tilt angles $\theta$ and for azimuthal angle $\varphi = 0$. In our calculations, we included the homogeneous...
broadening of the exciton peak $\Gamma = 2.0$ meV which is a typical value for InSb QW [11]. For in-plane field values outside the anticrossing region, the absorption spectrum represents a single peak corresponding to bright $\Psi_{00}^+$ exciton state. When the tilt angle $\theta$ lies within a narrow interval $\sim 5^\circ$ around the resonance value, the spectrum has double-peak structure with maxima at hybrid states with energies $E_0^{(\pm)}$. The splitting is most pronounced at $\theta = 70.7^\circ$ ($B_\parallel/B_\perp = 2.86$) at which bright and dark excitons, $\Psi_{00}^-$ and $\Psi_{10}^+$, contribute equally to the final state.

The SO-induced spatial anisotropy manifests itself in the dependence of splitting on azimuthal angle $\varphi$, as illustrated in Fig. 1 (lower pannel). The maximal and minimal peak-to-peak separation, $\Delta_\pm$, given by

$$\Delta_\pm = \frac{4}{l} \left| \alpha_D \sqrt{1 + B_\perp/B} \pm \alpha_R \sqrt{1 - B_\perp/B} \right|,$$

are achieved at $\varphi = \pm \pi/4$ (or $\varphi = \mp \pi/4$ for opposite sign of $\alpha_D$ and $\alpha_R$). The values of SO coefficients are deduced from splitting as $\alpha_D, R = \frac{8}{l} \frac{\Delta_\pm}{\sqrt{1 \pm B_\perp/B}}$. Thus, an observation of the absorption spectrum anisotropy would allow a direct simultaneous measurement of the magnitudes and relative sign of Dresselhaus and Rashba SO interaction constants.

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