I. INTRODUCTION

The standard model in particle physics has succeeded to describe the physics below the electroweak scale. It is not, however, a complete theory because of the theoretical problems. One is the strong CP problem. Despite the expectation of the existence of the CP-violating $\theta$-term in the Lagrangian, $L = (\theta g_s^2/32\pi^2)G^\mu\nu G_{\mu\nu}$ with $\theta \sim O(1)$, experimentally it is constrained as $\theta \lesssim 10^{-10}$. This is solved by the Peccei-Quinn (PQ) mechanism \[4\, 2\], in which the axion, pseudo-Nambu-Goldstone boson associated with the spontaneous breakdown of the global $U(1)_{\text{PQ}}$ symmetry, dynamically relaxes the $\theta$ to nearly zero. Another problem is the gauge hierarchy problem, which states that the huge difference between the weak scale and the grand unified theory scale requires unnatural fine tuning. In the framework of supersymmetry (SUSY) \[3\], this problem does not arise. Thus it is well motivated that we consider the SUSY axion model.

On the other hand, cosmological observations revealed that the Universe started with inflationary expansion era and is now filled with unknown matter, called dark matter \[1\]. The cosmological inflation and dark matter cannot be accounted for in the framework of the standard model, and hence they strongly indicate the physics beyond the standard model.

In this letter we point out that inflation naturally takes place in the SUSY axion model. It takes the form of hybrid inflation where the waterfall field is identified as a Peccei-Quinn scalar. The Peccei-Quinn scale is predicted to be around $10^{15}\text{GeV}$ for reproducing the large-scale density perturbation of the Universe. After the built-in late-time entropy-production process, the axion becomes a dark matter candidate. Several cosmological implications are discussed.

Therefore this model provides a simultaneous solution to the hierarchy and strong CP problems, inflation and dark matter in a simple and unified framework.

II. MODEL AND COSMOLOGICAL IMPLICATIONS

Our model is described by the following Kähler and superpotential,

$$K = |S|^2 + |\Psi|^2 + |\bar{\Psi}|^2,$$

$$W = \kappa S(\bar{\Psi}\tilde{\Psi} - f_a^2) + \lambda S \Psi X \bar{X} + kSY\bar{Y} + W_0,$$  \(1\)

where $S, \Psi$ and $\bar{\Psi}$ are gauge singlets, $X(\bar{X})$ and $Y(\bar{Y})$ have some gauge charges, and $\kappa, \lambda$ and $k$ are coupling constants, which are taken to be real and positive. Here we keep minimal Kähler potentials only, and effects of non-minimal terms will be discussed later. The constant term $W_0(= m_{3/2}M_P^2)$, where $m_{3/2}$ denotes the gravitino mass and $M_P$ is the reduced Planck scale) ensures that the cosmological constant is nearly zero in the present Universe. This superpotential possesses a global $U(1)_{\text{PQ}}$ symmetry, which is anomalous at the quantum level, and also has the $U(1)_R$ symmetry whose charge assignments are shown in Table. \[1\]. After $\Psi$ and $\bar{\Psi}$ obtain vacuum expectation values (VEV), this PQ symmetry is spontaneously broken and there appears a pseudo-Nambu-Goldstone boson, which dynamically cancels the strong CP phase and solves the strong CP problem.

This is nothing other than the SUSY version of the hadronic (or KSVZ) axion model \[18\], if $X$ and $\bar{X}$ have color charge. In this case we can choose $Y$ and $\bar{Y}$ as minimal SUSY standard model (MSSM) Higgses: $Y = H_u$ and $\bar{Y} = H_d$. For a certain choice of $k$, a sizable $\mu$-term is generated after $S$ gets a VEV \[19\], as we will see later.

It is also possible to choose $X$ and $\bar{X}$ to be MSSM Higgses: $X = H_u, \bar{X} = H_d$. In this case, the present model describes the SUSY version of the DFSZ axion model \[13\]. In this case $Y$ and $\bar{Y}$ are additional chiral supermultiplets. It is also allowed to introduce some additional chiral supermultiplets like heavy quarks in the...
TABLE I: Charge assignments on the field content.

| U(1)$_{PQ}$ | U(1)$_{R}$ |
|-------------|-------------|
| S | +1 | +2 |
| Ψ | −1 | 0 |
| ¯Ψ | −1/2 | 0 |
| X | −1/2 | 0 |
| X | −1/2 | 0 |
| Y | 0 | 0 |
| ¯Y | 0 | 0 |

KSVZ model. In order to maintain gauge coupling unification, these additional multiplets may belong to fundamental representations of SU(5).

One may notice that the first term in the superpotential (2) introduced to stabilize the PQ scalar at large field value coincides with that used for the hybrid inflation [3, 13], after identification of $S$ with the inflaton and $Ψ(Ψ)$ with the waterfall fields. Thus we reach the interesting possibility : the PQ sector for solving the strong CP problem naturally causes inflation. We do not need any additional fields and interactions. According to a recent analysis including the effect of constant term in the superpotential (2) [12], the correct magnitude of the density perturbation is reproduced for $f_α \sim 10^{10}$GeV and $κ \sim 10^{-3}$ if $m_{3/2} \sim 1$ TeV. At first sight this may seem to be disappointing, because such large PQ symmetry breaking scale leads to axion overproduction, as is well known [20, 21]. In this inflationary scenario, the PQ symmetry is restored during inflation and broken after that. Thus the phase of the axion takes random values for different patch of the Universe, and it is not allowed to tune the initial misalignment angle to avoid the axion overproduction.

However, the situation is much better than the first thought. This is because the late-time entropy production mechanism, which dilutes the axion abundance to the acceptable level, is already built in the present model. Therefore, the large PQ scale, $f_α \sim 10^{15}$GeV, is rather an appealing feature considering that the axion can take a role of the dominant component of dark matter after the entropy-production process.

Now we discuss the scalar field dynamics after inflation. The scalar potential is given by

$$V = κ^2 |Ψ|^2 + κ^2 |S|^2 (|Ψ|^2 + |Ψ|^2).$$

Here we have taken $X = X = 0$ since they quickly settle at the origin due to the Hubble mass term during inflation. The global minimum is located at $S = 0$ and $Ψ = f_α$. In other words, there is a flat direction along which the scalar fields do not feel the potential, ensured by the U(1)$_{PQ}$ symmetry extended to a complex U(1) due to the holomorphy. The SUSY breaking effect lifts up the flat direction, saxion, and gives a mass of order $m_{3/2}$,

$$V_{SB} = c_1 m_{3/2}^2 |Ψ|^2 + c_2 m_{3/2}^2 |Ψ|^2,$$

where $c_1$ and $c_2$ are $O(1)$ constants. This stabilizes the flat direction at $|Ψ| \sim |Ψ| \sim f_α$. We denote deviation from this minimum along the flat direction as $σ$, and call it as saxion. The $Ψ$ field also receives a finite-temperature effective potential, $V_T \sim α_s^2 T^4 \log Ψ$, where $α_s$ is the QCD gauge coupling constant, coming from two-loop effects even if heavy quarks are decoupled from thermal bath [22].

After inflation ends, the inflaton $S$ and waterfall fields $Ψ(Ψ)$ oscillates around the minimum, $S = 0, |Ψ| = |Ψ| = f_α$, noting that the flat direction at this stage obtains a mass of $κ|S|$. The scalar degrees perpendicular to this direction, which fully mixes with $S$, decays much earlier than the saxion since they have masses of $m_S \sim κ f_α$. The decay is induced by the third term in (2), and the reheating temperature is around $T_R \sim 10^{11}$GeV for $m_S \sim 10^{12}$GeV and $κ \sim κ$. After that, the thermal logarithmic comes to dominate and drives the saxion to $|Ψ| \sim α_s M_P$ where the effective thermal mass becomes equal to the Hubble parameter, and the saxion stops there until the thermal effect becomes irrelevant. When the Hubble parameter decreases to $m_{3/2}$, the mass term dominates over the thermal correction, and the saxion begins to oscillate around the minimum, $|Ψ| \sim |Ψ| \sim f_α$, with an initial amplitude of $σ_0 \sim α_s M_P$. The abundance of the saxion coherent oscillation, in terms of the energy density to entropy ratio, is then given by

$$ρ_σ = \frac{90}{π^2 g_*} \sqrt{m_σ M_P} \frac{σ_0^2}{8 M_P^2} \approx 1 \times 10^{8} \text{GeV} \left(\frac{m_σ}{1 \text{TeV}}\right)^{1/2} \left(\frac{σ_0}{α_s M_P}\right)^2 ,$$

where $m_σ \sim m_{3/2}$ is the saxion mass. This comes to dominate the Universe well before the QCD phase transition. In the case of KSVZ model, the saxion decays into gluons with the rate

$$Γ_σ → gg = \frac{α_s^2}{32π^4} \frac{m_σ^3}{f_α^2}.$$

Then the Universe is reheated again by the saxion decay. The temperature after the saxion decay is estimated as

$$T_σ \sim 3 \text{MeV} \left(\frac{m_σ}{10 \text{TeV}}\right)^{3/2} \left(\frac{10^{15} \text{GeV}}{f_α}\right),$$

and hence is compatible with the lower bound on the reheating temperature [20] for $m_σ \gtrsim 10 \text{TeV}$. Notice that the saxion also decays into a SUSY particle pair and then produces the lightest SUSY particles (LSP) nonthermally, which easily exceed the dark matter abundance.

1 Here we have assumed that the $S$ decays before the saxion begins to oscillate. Otherwise, the presence of the $κ|S|$ mass term for the saxion makes the saxion oscillation amplitude exponentially suppressed [23].
2 Decay into two axions, $σ → 2a$, must be suppressed for successful reheating in the KSVZ model. This requires $c_1 \approx c_2$ in Eq. (4) [24, 25].
Thus we need to introduce small R-parity violation in order for the LSP to decay well before BBN begins, or to assume SUSY particles are heavy enough not to be produced by the saxion decay. In the case of DFSZ model, the saxion decays into Higgs pair or fermion pairs. For example, the decay width into the lightest Higgs boson pair is

$$\Gamma_{\sigma \rightarrow hh} = \frac{1}{8\pi} \frac{m_3^3}{f_\sigma} \left( \frac{\mu}{m_\sigma} \right)^4,$$

where $\mu = \lambda(\Psi)$ gives the higgsino mass, and hence we obtain

$$T_\sigma \sim 5 \text{ MeV} \left( \frac{m_\sigma}{1\text{TeV}} \right)^{3/2} \left( \frac{10^{15}\text{GeV}}{f_\sigma} \right) \left( \frac{\mu}{m_\sigma} \right)^2.$$  \hfill (9)

Thus in this case we need $m_\sigma \sim 1$ TeV and decay into a SUSY particle pair can naturally be forbidden.

Now let us discuss the abundance of the axion, gravitino and axino after the dilution by the saxion decay. The axion abundance, in terms of the density parameter, after the dilution is estimated as $\Omega_a h^2 \sim 5 \times 10^{-2} \left( \frac{T_\sigma}{1\text{MeV}} \right) \left( \frac{f_\sigma}{10^{15}\text{GeV}} \right)^2$, \hfill (10)

hence it is consistent with the WMAP observation of the dark matter abundance. This is appealing, since the PQ scale $f_\sigma \sim 10^{15}$GeV is required for generating the density perturbation of the Universe, while this large PQ scale leads to the efficient late-time entropy production, making the axion plausible candidate of dark matter. Note that the axion does not have an isocurvature perturbation in this model, since the PQ symmetry is restored during inflation.

As for the gravitino, they are produced both thermally and nonthermally from the inflaton decay, but diluted sufficiently. The thermally produced gravitino abundance, in terms of the number to entropy ratio, is estimated as

$$Y_{3/2} \simeq 1 \times 10^{-22} \left( \frac{1 \text{TeV}}{m_\sigma} \right)^{1/2} \times \left( \frac{T_R}{10^{11}\text{GeV}} \right) \left( \frac{T_\sigma}{1\text{MeV}} \right) \left( \frac{\alpha_s M_P}{\sigma_i} \right)^2.$$ \hfill (11)

This satisfies the bound on the unstable gravitino abundance from BBN $m_{1/2} Y_{3/2} \lesssim 10^{-13} \times 10^{-9}$GeV for $m_{3/2} \sim 1-10$TeV \cite{27,30,31} for an unstable gravitino.

The axino, which is the fermionic superpartner of the axion, might also have significant effects on cosmology \cite{24,32}. The axino abundance from thermal production \cite{33}, after the dilution, is given by

$$Y_\tilde{a} \simeq 1 \times 10^{-19} \left( \frac{1 \text{TeV}}{m_\sigma} \right)^{1/2} \left( \frac{T_R}{10^{11}\text{GeV}} \right) \left( \frac{T_\sigma}{1\text{MeV}} \right) \left( \frac{10^{15}\text{GeV}}{f_\sigma} \right)^2 \left( \frac{\alpha_s M_P}{\sigma_i} \right)^2.$$ \hfill (12)

In the present model, the axino mass is generated once the $A$-term potential is included: $V_A = A \kappa S f_\sigma^2 + \text{h.c.}$ with $A \sim m_{3/2}$. Then $S$ has a VEV of $\sim A/\kappa$, and it gives an axino mass of $m_\tilde{a} = \kappa(S) \sim A$. Thus the axino mass is comparable to the gravitino. If the axino is not the LSP, it has a similar lifetime to the saxion in the KSVZ model, and it decays before BBN. The constraint is given as $Y_\tilde{a} \lesssim 10^{-12}$ so as not to produce too much LSPs. If the axino is the LSP, the bound reads $m_\tilde{a} Y_\tilde{a} \lesssim 4 \times 10^{-19}$GeV.

In both cases, the constraint is satisfied as is seen in Eq. (12).

### III. DISCUSSION

Here we briefly discuss several remaining issues.

**Coleman-Weinberg correction:** Since the waterfall fields couple to $X(\bar{X})$, there arises a Coleman-Weinberg (CW) effective potential for the $\Psi$ direction \cite{27}. We have explicitly checked that this does not modify the (post-)inflationary scalar dynamics at all. Also the presence of $Y(\bar{Y})$ affects the potential of the inflaton ($S$) through the CW correction. As long as $k$ is not much larger than $\kappa$, the inflaton dynamics is not much affected. Since we need $k > \kappa$ for relaxing to desired vacuum \cite{35}. In this letter we have assumed $k \sim \kappa$.

**Stability of heavy quarks:** In the hadronic axion model, the heavy quarks are stable unless some additional operators are introduced. Thus the existence of too much heavy quarks may be problematic. In our model, however, they are not produced in the early Universe efficiently, since the reheating temperature is lower than the heavy quark mass scale. In the DFSZ model, $Y(\bar{Y})$ have weak scale masses and may also be stable, and are once thermalized after the reheating. But its relic abundance is significantly reduced by the saxion decay and no significant cosmological effects arise.

**Non minimal K"{a}hler potentials:** In the hybrid inflation model in supergravity, the scalar spectral index $n_s$ takes a value from 0.98-1.00 for the range of $f_\sigma \sim 10^{15}$GeV-10^{16}$GeV in the minimal K"{a}hler potential \cite{12,13}. It is possible to make the spectral index more red tilted and fall into the best fit range ($n_s = 0.965 \pm 0.012$ with 68\% C.L. \cite{4}), by introducing a non-minimal K"{a}hler potential $K = k_S |S|^4/M_P^2$ and choosing its coefficient as $k_S \sim 0.01-0.02$. For example, $n_s = 0.96$ (0.95) is obtained for $f_\sigma = 10^{15}$GeV ($4 \times 10^{15}$GeV) for $k_S = 0.01$ \cite{12,13}. We can also introduce non-minimal K"{a}hler potentials, as $K = (k_1 |S|^2 |\bar{\Psi}|^2 + k_2 |S|^2 |\bar{\Psi}|^2)/M_P^2$ with $k_1 \sim k_2 \sim O(1)$. These terms give an additional Hubble mass to the saxion. But this does not modify the saxion dynamics at all, since the saxion mass is dominated by the $\kappa |S|$ term during the $S$ oscillation, and after $S$ decays these terms become irrelevant.

**Domain wall problem:** After inflation $U(1)_{\text{PQ}}$ is broken and cosmic strings are formed. After the QCD phase transition, domain walls appear which is bounded by the strings. Due to the tension of domain walls, strings (and
walls) shrink and finally disappear if the color anomaly number is one, as in the KSZV model. This is not the case for the DFSZ model, hence we need to introduce several heavy quarks in order to make the color anomaly number one.

**Axions emitted from strings**: The axionic strings continuously emit axions with momentum of order of the Hubble scale. After the QCD phase transition it obtains a mass, and the energy density stored in the axions emitted by the strings is comparable to that from the coherent oscillation. Thus these axions may contribute to some non-negligible fraction of the dark matter.

**Baryon asymmetry**: Because of the inevitable late-time entropy production process, any preexisting baryon asymmetry is diluted. One possibility to generate the baryonic isocurvature fluctuation generated by the axion coherent oscillation can be the dark matter. This provides a solution to the strong CP and gauge hierarchy problems and simultaneously explains a cosmic inflation and the presence of dark matter.

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