A study of neutron-deuteron scattering in configuration space

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A new computational method for solving the configuration-space Faddeev equations for the breakup scattering problem \cite{1} has been applied to nd scattering both below and above the two-body threshold. To perform numerical calculations for a general nuclear potential and with arbitrary number of partial waves retained, we use the approach proposed in \cite{2}. Calculations of the inelasticities and phase-shifts, as well as cross-sections and analyzing powers for elastic nd scattering at $E_{\text{lab}}=14.1$ MeV were made with the charge independent AV14 potential. The results are compared with those of other authors and experimental data.

1. Formalism

The Faddeev equations projected onto the MGL basis \cite{2} are:

$$
\left[ E + \frac{\hbar^2}{m}(\partial_x^2 + \partial_y^2) - v_{\alpha}(x, y) \right] \Phi_{\lambda,s_0,M_0}(x, y) = \sum_{\beta} v_{\alpha\beta}(x) \left[ \Phi_{\lambda_0,s_0,M_0}(x, y) + \int_{-1}^{1} du \sum_{\gamma} g_{\beta\gamma}(x, y, u) \Phi_{\lambda_0,s_0,M_0}(x', y') \right].
$$

(1)

Here Greek subindexes denote state quantum numbers: $\alpha = \{l, \sigma, J, s, \lambda, t\}$, where $l$, $\sigma$, $J$ and $t$ are the orbital, spin, total angular momenta and isospin of a pair of nucleons, $\lambda$ is the orbital momentum of the third nucleon relative to the c.m. of a pair nucleons, and $s$ is the total ”spin” ($s = 1/2 + J$). $M = \vec{\lambda} + s$ is the three-particle angular momentum. The geometrical function $g_{\beta\gamma}(x, y, u)$ is the representative of the permutation operator $P^+ + P^-$ in MGL basis (details see in \cite{2}). In Eq. (1) $v_{\alpha}^{M}$ is the centrifugal potential, and nucleon-nucleon potentials are $v_{\alpha\alpha'}(x) = \delta_{\lambda\lambda'}\delta_{s_s'}\delta_{\sigma\sigma'}\delta_{J,J'}v_{\alpha\alpha'}^{M}$, where $v_{\alpha\alpha'}^{M}$ are the potential representatives in the two-body basis $Y_{J\lambda}(\hat{x})$.

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The asymptotic conditions for the radial parts are

\[ \Phi_{\alpha}^{0s0M_0}(x, y) = \delta_{\alpha\lambda \delta_{s0}\delta_{s0}} \delta_{\sigma1} \delta_{J_0J_{\lambda}(qy)\psi_l(x)} + ia_{\alpha\lambda \delta_{s0}M_0}^{M_0} h_\lambda^{(+)}(qy)\psi_l(x) + O(y^{-1}) \] (2)

and

\[ \Phi_{\alpha}^{0s0M_0}(x, y) = A_{\alpha\lambda \delta_{s0}M_0}^{M_0} e^{i\sqrt{E_X}} \sqrt{X} + O(X^{-3/2}), \quad X^2 = x^2 + y^2. \] (3)

In these formulae \( \hat{j}_\lambda(qy) \) is the regularized spherical Bessel function, \( \psi_l(x) \) is the deuteron wave function \((l=0,2)\), \( h_\lambda^{(+)}(qy) \) is the regularized Hankel function, \( a_{\alpha\lambda \delta_{s0}M_0}^{M_0} \) and \( A_{\alpha\lambda \delta_{s0}M_0}^{M_0} \) are the elastic and breakup amplitudes, respectively, \( q \) an \( E \) are the c.m momentum and final energy.

Table 1

| \( a_{\lambda \delta_{s0}M_0}^{M_0} \) | Ref. \[3\] | present | Ref. \[3\] | present |
|---------------------------------|----------|--------|----------|--------|
| \( a_{1/2}^{1/2} \), \( a_{1/2}^{1/2} \) | -0.291 + i 0.093 | -0.2903 + i 0.093 | -0.4691 + i 0.3272 | -0.4679 + i 0.324 |
| \( a_{1/2}^{3/2} \), \( a_{1/2}^{3/2} \) | -0.0175 + i 0.0003 | -0.0173 + i 0.0003 | -0.0682 + i 0.0048 | -0.0616 + i 0.0044 |
| \( a_{1/2}^{1/2} \), \( a_{1/2}^{1/2} \) | 0.2085 + i 0.0459 | 0.208 + i 0.0457 | 0.3749 + i 0.176 | 0.3739 + i 0.1743 |
| \( a_{3/2}^{1/2} \), \( a_{3/2}^{1/2} \) | -0.4985 + i 0.5383 | -0.4985 + i 0.5382 | -0.3145 + i 0.888 | -0.3153 + i 0.8874 |
| \( a_{3/2}^{3/2} \), \( a_{3/2}^{3/2} \) | 0.0101 + i 0.0001 | 0.0108 + i 0.0001 | 0.0420 + i 0.0018 | 0.0437 - i 0.0019 |
| \( a_{3/2}^{1/2} \), \( a_{3/2}^{1/2} \) | -0.0187 + i 0.0004 | -0.0181 + i 0.0004 | -0.0735 + i 0.006 | -0.0721 + i 0.0057 |
| \( a_{3/2}^{3/2} \), \( a_{3/2}^{3/2} \) | -0.0718 + i 0.0052 | -0.0716 + i 0.0052 | -0.1227 + i 0.016 | -0.1212 + i 0.0157 |
| \( a_{3/2}^{1/2} \), \( a_{3/2}^{1/2} \) | 0.2392 + i 0.061 | 0.2401 - i 0.0615 | 0.396 + i 0.1959 | 0.3974 - i 0.1978 |
| \( a_{3/2}^{3/2} \), \( a_{3/2}^{3/2} \) | 0.0021 + i 7.1E-6 | 0.0023 + i 7.6E-6 | 0.0158 + i 0.0003 | 0.0174 + i 0.0004 |

2. Results

In the present run we used the charge independent AV14 potential and included all angular momenta of subsystems, \( l \) and \( \lambda \leq 4 \), total angular momentum of a pair nucleons \( J \leq 3 \), and total three-body angular momentum \( M_0 \) up to 9/2. The cutoff radius was taken 80 fm. In Table 1, our results on the \( nd \) elastic amplitudes are given. For comparison, accurate results of Bochum and Pisa group are also presented.

In Figure 1 the elastic differential cross-section is shown together with that of Bochum group \[3\] and the experimental data of \[4,5\]. Agreement is good except for angles in the vicinity of 180 degrees. In Figures 2 and 3 vector analyzing powers are shown along with the experimental data.
Figure 1. \( nd \) elastic differential cross section at \( E_{\text{lab}} = 14.1 \) MeV. The solid line is our results. The dashed one corresponds to the Bochum group prediction \cite{3}. The data are from \cite{4} (open circles) and \cite{5} (open diamonds).

Figure 2. The vector analyzing power \( A_y \) at \( E_{\text{lab}} = 14.1 \) MeV. The solid line is our results. The dashed one is the Bonn B prediction with \( J \leq 3 \) of \cite{6}. The data are from \cite{7}. The \( pd \) data at \( E_{\text{lab}} = 15.0 \) MeV are from \cite{8}.

As one can see agreement with experimental data is enough good in both cases. However one to take into account our limitations on the maximal values for \( M, l \) and \( \lambda \). Convergence with these values is presently under study.

To check accuracy of our calculations we applied the optical theorem. Presenting it in the form \( \text{Im} \ a = \text{R.H.S} \), where the expression for \( \text{R.H.S} \) may be found in \cite{2}, we get the results shown in Table 2.
Table 2
The optical theorem results for $E_{\text{lab}} = 14.1$ MeV and $M_0^x = 1/2^+, 3/2^+$

| $s_0, \lambda_0$ | Im$M_{s_0,\lambda_0 0}^{1/2+}$ | R.H.S. | $s_0, \lambda_0$ | Im$M_{s_0,\lambda_0 0}^{3/2+}$ | R.H.S. |
|------------------|-------------------------------|--------|------------------|-------------------------------|--------|
| 1/2, 0           | 0.7172                        | 0.7182 | 3/2, 0           | 0.8942                        | 0.8971 |
| 3/2, 2           | 0.0258                        | 0.0263 | 1/2, 2           | 0.0516                        | 0.0522 |
| -                | -                             | -      | 3/2, 2           | 0.0359                        | 0.0363 |

Presenting the total cross-section $\sigma_{\text{tot}}$ as a sum of its elastic $\sigma_{\text{el}}$ and inelastic $\sigma_{\text{in}}$ parts we obtained values for them shown in Table 3.

Table 3
Cross-sections of $nd$ breakup scattering for $E_{\text{lab}} = 14.1$ MeV in [mb]

| $M_0$ | $\sigma_{\text{tot}}$ | $\sigma_{\text{el}}$ | $\sigma_{\text{in}}$ |
|-------|-----------------------|-----------------------|-----------------------|
| 1/2   | 183.25                | 132.47                | 50.78                 |
| 3/2   | 584.82                | 471.14                | 113.68                |
| 5/2   | 786.61                | 637.14                | 149.47                |
| 7/2   | 804.25                | 648.20                | 156.05                |
|       | 824. ± 10             | 694. ± 42             | 130. ± 43 [5]         |
|       | 807. [9]              | 172. ± 12 [10]        |

3. Conclusion

1. The proposed method of the solution of the Faddeev equations in configuration space by the direct integration of the set of partial differential equations is fully feasible and reliable, and give results of the same precision as obtained by different methods.
2. This shows that the method can be generalized to include the Coulomb interaction in the $pd$ scattering below and above the deuteron threshold, where others methods experience considerable difficulties especially for the breakup channel.

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