RHO MESON PROPERTIES IN NUCLEAR MATTER FROM QCD SUM RULES

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We study the properties of rho mesons in nuclear matter by means of QCD sum rules at finite density. For increased sensitivity, we subtract out the vacuum contributions. With the spectral function as estimated in the literature, these subtracted sum rules are found to be not well satisfied. We suppose that Landau singularities from higher resonance states in the nearby region in this channel are the cause for this failure.

1 Introduction

It is believed that the mass and width of particles in a medium differ from their values in the vacuum. Such an effect might be observable in various physical systems such as the transition from a quark-gluon plasma to the hadronic phase in the early universe or in the interior of neutron stars. Furthermore, such conditions are created in heavy ion collisions at present and future colliders.

In order to analyze these systems, it is necessary to obtain a good theoretical understanding of the properties of the particles in a medium, for instance of the $\rho$-meson. There are various approaches and models in the literature which predict the behavior of the mass and width of the $\rho$-meson as function of the temperature and the density. We will use here the method of QCD sum rules which has been quite successfully applied in the vacuum case. This approach was later extended to finite temperature and density.

Since the medium breaks Lorentz invariance, more operators and unknown condensates enter on the QCD side of the sum rules (operator product expansion) and also the hadronic side (spectral function) is only poorly known in the medium. Nevertheless, one can perhaps gain insight into the chiral phase transition from experimental data on $\rho$-mesons in the medium, since these sum rules relate the mass and width of the $\rho$-meson to condensates like $\langle \bar{q}q \rangle$, which serves as an order parameter of chiral symmetry breaking.

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Most previous studies found at zero temperature a drop of the $\rho$-meson mass of about 10-20% at nuclear saturation density $\bar{n}_s$, although in Ref. [4] no significant shift was observed. In view of these conflicting results we want to scrutinize here and in Ref. [5] on some aspects of QCD sum rules in the medium.

2 Operator product expansion and spectral representation

We consider the ensemble average of the $T$-product of two vector currents

$$T_{\mu\nu}^{ab}(q) = i \int d^4xe^{iq\cdot x} \langle T\{V_{\mu}^{a}(x)V_{\nu}^{b}(0)\}\rangle,$$  \hspace{1cm} (1)

with $V_{\mu}^{a}(x) = \bar{q}(x)\gamma_{\mu}(\tau^a/2)q(x)$. The ensemble average is defined by $\langle O \rangle = \text{Tr}[\exp(-\beta(H - \mu_B N))]/\text{Tr}[\exp(-\beta(H - \mu_B N))]$ where $\beta = 1/T$. The nucleon chemical potential has been denoted by $\mu_B$ and the QCD Hamiltonian and the nucleon number operator by $H$ and $N$, respectively. The invariant decomposition of $T_{\mu\nu}^{ab}$ is given by

$$T_{\mu\nu}^{ab}(q) = \delta^{ab}(Q_{\mu\nu}T_l + P_{\mu\nu}T_l),$$  \hspace{1cm} (2)

with $P_{\mu\nu} = -g_{\mu\nu} + (q_{\mu}q_{\nu}/q^2) - (q^2/q^2)\bar{u}_\mu\bar{u}_\nu$ and $Q_{\mu\nu} = (q^\mu/q^2)\bar{u}_\mu\bar{u}_\nu$, where $\bar{u}_\mu = u_\mu - \omega q_\mu/q^2$ and $\bar{q} = \sqrt{\omega^2 - q^2}$. The amplitudes $T_{l,t}$ are functions of $q^2$ and $\omega = u \cdot q$. We have introduced the four-velocity of the medium $u_\mu$, in order to formally restore Lorentz invariance which is broken by the medium.

The sum rules are obtained by equating the short-distance expansion of the product of the two currents in Eq. (1), which results in a series of condensates of QCD operators, with the spectral representation for the invariant amplitudes $T_{l,t}$, where various intermediate hadronic states will contribute.

The short distance expansion of $T_{\mu\nu}^{ab}$ including all operators up to dimension four can be found in Ref. [3]. The availability of the four-vector $u_\mu$ allows one to construct new operators as compared to the vacuum case. In addition to the usual operators 1, $\bar{q}q = \bar{u}d + \bar{d}u$, $G^2 \equiv \frac{g_s}{2}G_{\mu\nu}^aG^{\mu\nu,a}$, there are two new operators $\Theta^q \equiv u^\mu\Theta^q_{\mu\nu}u^\nu$ and $\Theta^q \equiv u^\mu\Theta^q_{\mu\nu}u^\nu$. Here $G_{\mu\nu}^a$, $a = 1, ..., 8$ are the gluon field strengths and $\alpha_s = g^2/4\pi$, $g$ being the QCD coupling constant. $\Theta^q_{\mu\nu}$ are the quark and the gluon parts of the traceless stress tensor $\Theta^q_{\mu\nu} = \bar{q}\gamma_{\mu}D_{\nu}q - \frac{g_s}{2}g_{\mu\nu}\bar{q}q$ and $\Theta^q_{\mu\nu} = -G^{\alpha\beta}_{\mu\lambda}G_{\nu}^{\lambda\epsilon} + \frac{1}{3}g_{\mu\nu}G_{\alpha\beta}^aG^{a\alpha\beta}c$, where $m$ is the quark mass in the limit of SU(2) symmetry. We will take into account the mixing of the operators $\Theta^q$ and $\Theta^\theta$ under the renormalization group.

At finite temperature and chemical potential, we can use the Landau representation of the amplitudes $T_{l,t}$, which is a spectral decomposition in $q_0$ at fixed $|\bar{q}|$

$$T_{l,t}(q_0, |\bar{q}|) = \frac{1}{\pi} \int_{-\infty}^{+\infty} dq_0' \text{Im}T_{l,t}(q_0', |\bar{q}|)/q_0'^2 - q_0 - i\epsilon \text{tanh}(\beta|q_0'|/2),$$  \hspace{1cm} (3)
up to subtraction terms. There will be contributions in the integral from various intermediate hadronic states, first of all from the \( \rho \)-meson. The effects of the medium can be parametrized by employing the operator relation

\[
V^\mu(x) = m^*_\rho F^*_\rho \rho^\mu(x),
\]

where \( m^*_\rho \) and \( F^*_\rho \) denote the in-medium mass and width of the \( \rho \)-meson. This will generate the usual \( \delta \)-function contribution in \( \text{Im} T_{l,t} \).

Moreover, in the nuclear medium there will be a contribution from \( N\bar{N} \) intermediate states. In the vacuum this contribution is small, coming from the cut beginning at threshold, \( q_0^2 = 4m^2_N + |q|^2 \). However, in the nuclear medium the currents can also interact with real nucleons to give rise to a short cut around the origin, \(-|q| < q_0 < +|q|\). The evaluation of this contribution can be found in Ref.\( ^5 \).

Following Refs.\( ^3,^4 \) we ignore the states \( N\bar{N}^* \) with resonances \( N^* \). Furthermore, below we will restrict ourselves to sum rules at finite nuclear density but at zero temperature, therefore the \( 2\pi \) contribution will be eliminated after subtracting out the corresponding vacuum sum rules.

### 3 Sum rules

We obtain the sum rules by equating the spectral representation and the operator product expansion for the two invariant amplitudes \( T_{l,t} \). As usually done in the literature, we take the Borel transform of both sides in order to enhance the contribution from the lowest lying resonance, here the \( \rho \)-meson, and to suppress the contributions from higher dimensional operators. In general, this can only be achieved within a certain region for the so called Borel parameter \( M \). In contrast to earlier work,\( ^3,^4 \) we subtract the vacuum sum rules, assuming that the contribution from the QCD continuum will practically drop out in this way. For \( T \rightarrow 0 \) and \( \mu_B > 0 \) the \( 2\pi \) contribution will also cancel and we obtain the following sum rules in the limit \( |q| \rightarrow 0 \) where the expressions simplify considerably:

\[
F^*_\rho \rho^*_\rho e^{-\frac{m^2_N}{2m^2_N}} + \frac{1}{24\pi^2} \int_{4m^2_N}^{4\nu^2_N} ds e^{-\frac{s}{2m^2_N}} \sqrt{1 - \frac{4m^2_N}{s}(1 + \frac{2m^2_N}{s})} = \frac{\langle O \rangle}{M^2},
\]

\[
m^*_\rho^2 F^*_\rho e^{-\frac{m^2_N}{2m^2_N}} - \frac{1}{24\pi^2} \int_{4m^2_N}^{4\nu^2_N} ds s e^{-\frac{s}{2m^2_N}} \sqrt{1 - \frac{4m^2_N}{s}(1 + \frac{2m^2_N}{s})} = -\langle O \rangle,
\]

with

\[
\langle O \rangle = \frac{1}{2} \hat{m} \langle \bar{q}q \rangle + \frac{G^2}{24} + \frac{2}{11} \left( \langle \Theta \rangle + \lambda(M^2) \langle \frac{8}{3} \Theta^q - \Theta^g \rangle \right),
\]
where $\langle O \rangle = \langle O \rangle_0 - \langle 0 | O | 0 \rangle$ and $\Theta = \Theta^q + \Theta^g$. The mixing under the renormalization group is taken into account by $\lambda(M^2) = (\alpha_s(\mu^2)/\alpha_s(M^2))^{-d/2b}$ with $d = 4(1/n + n_f)$ and $b = 11 - \frac{2}{3}n_f$. We will take $\mu = 1\text{ GeV}$ below.

In the linear density approximation, we expand the mass and width and the condensates up to first order in the nucleon number density $\bar{n}$,

$$m^*_\rho = m_\rho(1 + a \frac{\bar{n}}{\bar{n}_s}), \quad F^*_\rho = F_\rho(1 + b \frac{\bar{n}}{\bar{n}_s}), \quad \langle O \rangle = C\bar{n}, \quad (8)$$

where $\bar{n}_s = (110 \text{ MeV})^3$ denotes the nuclear saturation density. The coefficient $C$ in Eq. (8) is given by

$$C = \frac{\sigma_0}{2} - \frac{1}{27}(m_N - \sigma) + \frac{3}{22}m_N \left( A^g + A^q + \lambda(M^2)(\frac{8}{3}A^q - A^g) \right), \quad (9)$$

where $\sigma = \langle p|\bar{n}(\bar{u}u + \bar{d}d)|p\rangle/(2m_N) \simeq 45 \text{ MeV}$ denotes the sigma term. The constants $A^{q,g}$ are defined by $\langle p|\Theta^{q,g}|p\rangle = 2A^{q,g}(p_\mu p_\nu - \frac{1}{4}g_{\mu\nu}p^2)$. From parton distribution functions, one obtains the values $A^q = 0.62$ and $A^g = 0.35$.

From the sum rules (3) and (5), we finally obtain the expressions

$$a = \frac{-\bar{n}_s}{2F^2_\rho} \frac{m^2_N}{e\frac{m^2_N}{M^2}} \left[ C \left( \frac{1}{m^2_\rho} + \frac{1}{M^2} \right) - \frac{1}{4m_N} - \frac{m_N^2}{m^2_\rho} - \frac{1}{4m_N}e^{-\frac{4m^2_\rho}{M^2}} \right], \quad (10)$$

$$b = \frac{-\bar{n}_s}{2F^2_\rho} \frac{m^2_N}{e\frac{m^2_N}{M^2}} \left[ C \left( \frac{m^2_N}{M^2} + \frac{1}{4m_N} \left( 1 - \frac{m^2_N}{M^2} \right) - \left( \frac{m_N^2}{M^2} + \frac{1}{4m_N} \left( 1 - \frac{m^2_N}{M^2} \right) \right) e^{-\frac{4m^2_\rho}{M^2}} \right]. \quad (11)$$

### 4 Discussion and Conclusions

In Fig. 1 we have plotted the coefficients $a$ and $b$ as function of the Borel parameter $M$. We note that there is no sign of constancy of $a$ and $b$ in any region of $M$. We thus conclude that the sum rules in Eqs. (7) and (9) cannot give any reliable information about the density dependence of $m_\rho$ and $F_\rho$.

We have included only operators up to dimension four in the sum rules above, but taken into account their mixing which is numerically relevant for $M^2 \neq \mu^2$. Contributions from higher dimensional operators should, however, be relatively small for $M^2 > 1\text{ GeV}^2$. A more detailed analysis is in progress.

Therefore, the failure of our subtracted sum rules can presumably be traced to the hadronic spectral side which is not adequately saturated. In fact, higher resonance states $N\bar{N}^*$ will also contribute a cut for $q^2 \leq (m^*_N - m_N)^2$. The problem with the inclusion of such contributions is, however, that more unknown couplings and masses will enter in the medium.

The sum rules at finite density are given by the vacuum sum rules perturbed by small terms proportional to the density. Since the vacuum sum rules
are stable, this guarantees the stability of the sum rules at finite density as observed in previous work. In contrast, the subtracted sum rules above are much more sensitive to errors in or omissions of any terms.

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