EDM–free supersymmetric CP violation with
non–universal soft terms

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Abstract

Non–universality in the soft breaking terms is a common feature in most superstring inspired SUSY models. This property is required to obtain sizeable CP violation effects from SUSY and, on the other hand, can be used to avoid the Electric Dipole Moment constraints. We take advantage of these qualities and explore a class of SUSY models based on type I string theory where scalar masses, gaugino masses and trilinear couplings are non–universal. In this framework, we show that, in the presence of large SUSY phases, the bounds on the Electric Dipole Moments can be controlled without fine–tuning. At the same time, we find that these phases, free from EDM constraints, lead to large contributions to the observed CP phenomena in Kaon system and, in particular, to direct CP violation in $\varepsilon'/\varepsilon$. 
1 Introduction

CP violation constitutes one of the main open questions in high energy physics at the beginning of the 21st century. The Standard Model (SM) of electroweak interactions is able to accommodate the experimentally observed CP violation through a phase, $\delta_{CKM}$, in the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix. In spite of this, there exist strong hints from other fields (for instance, electroweak baryogenesis) that suggest that this can not be the only source of CP violation present in nature.

In fact, most of the extensions of the SM include new phases that may modify the SM predictions in CP violation phenomena. For example, even in the simplest supersymmetric extension of the SM, the so–called Constrained Minimal Supersymmetric Standard Model (CMSSM), we have new phases in the gaugino masses, $A$–terms and the $\mu$–term [1]. However, it is known since the early 80s [2], that the presence of these phases for SUSY masses around the electroweak scale gives rise to supersymmetric contributions to the Electric Dipole Moment (EDM) of the electron and the neutron roughly two orders of magnitude above the experimental bounds. Hence, given these strong constraints, most of the people working in SUSY phenomenology take these phases as exactly vanishing. Although this might be the most direct choice, it has been recently shown that there exist some other mechanisms which allow large SUSY phases while respecting EDM bounds. For instance, one of these mechanisms consists in a possible destructive interference among different contributions to the EDM that can occur in some regions of the SUSY parameter space [3, 4, 5]. A second option is to have approximately degenerate heavy sfermions for the first two generations [6] and finally, a third possibility (and maybe more natural) is to have non–universal soft supersymmetry breaking terms [7, 8, 9, 10].

In the presence of one of these mechanisms, large CP phases will be present in the SUSY sector and one may expect important effects on CP physics other than the EDM, e.g. in the $K$ and $B$ systems. However, in Ref. [11], it has been shown that, for vanishing $\delta_{CKM}$, a general SUSY model with all possible phases in the soft–breaking terms, but no new flavor structure beyond the usual Yukawa matrices can never generate a sizeable contribution to $\varepsilon_K$, $\varepsilon'/\varepsilon$ or hadronic $B^0$ CP asymmetries. This means that the presence of non–universal soft breaking terms besides large SUSY phases is crucial to enhance these CP violation effects. In agreement with this, it has been explicitly shown that
contributions to $\varepsilon_K$ are small within the dilaton–dominated SUSY breaking of the weakly coupled heterotic string model [12], where $A$–terms as well as gaugino masses are universal. On the other hand, it is well–known that the strict universality in the soft breaking sector is a strong assumption not realized in many supergravity and string inspired models [13]. All these arguments indicate not only that the presence of non–universal soft terms can solve the problem of too large contributions to EDMs but also that it allows for large SUSY contributions in CP violation experiments. Hence, in this work we will follow this avenue and analyze the effects of non–universal soft terms in both EDM and CP violation in the $K$–system.

In particular, non–universality of $A$–terms has been shown to be very effective to generate large CP violation effects [7, 8, 9, 10, 14]. In fact, the presence of non–degenerate $A$–terms is essential for enhancing the gluino contributions to $\varepsilon'/\varepsilon$ through large imaginary parts of the $L–R$ mass insertions, $\text{Im}(\delta_{LR})_{12}$ and $\text{Im}(\delta_{RL})_{12}$, as recently emphasized in Ref. [8]. These SUSY contributions can, indeed, account for a sizeable part of the recently measured experimental value of $\varepsilon'/\varepsilon$ [15, 16]. In the following, we will present an explicit realization of such mechanism in the framework of a type I superstring inspired SUSY model. Within this model, it is possible to obtain non–universal soft breaking terms, i.e. scalar masses, gaugino masses and trilinear couplings. We show that here EDMs can be sufficiently small while the SUSY phases of the off diagonal $A$–terms are large, and enough to generate sizable contribution to $\varepsilon'/\varepsilon$.

This paper is organized as follows. In section 2 we show our starting models based on type I string theory. We emphasize that it is quite natural to obtain non–universal $A$–terms in these models. In section 3 we discuss the impact of these new flavor structures on the sfermion mass matrices. We show that non–universality of the $A$–terms, in particular, can generate sizable off–diagonal entries in the down squark mass matrix. Section 4 is devoted to the discussion of the constraints from the EDMs of the electron and neutron. We explain that, in the model we consider with non–universal $A$–terms, the EDMs can be kept sufficiently small while there are still two phases completely unconstrained. The effect of these two phases in other CP violating process are given in section 5, where we study explicitly the effect of these phases in the K–system. Finally, in section 6, we give our conclusions.
2 Type I models

In this section we explain our starting model, which is based on type I string models. The purpose of the paper is to study explicitly several CP aspects in models with non-universal soft breaking terms. Type I models can realize such initial conditions, in particular, it is possible to obtain non-universality in the scalar masses, $A$-terms and gaugino masses. To obtain non-universal gaugino masses, we must assign the gauge groups to different branes. Type I models contain nine-branes and three types of five-branes ($5_a$, $a = 1, 2, 3$). Phenomenologically there is no difference between the 9-brane and the 5-branes. A gauge multiplet is assigned on one set of branes. Only if the SM gauge group is not associated with a single set of branes, the gaugino masses can be non-universal. Here we assume that the gauge group $SU(3)$ on one of the branes and the gauge group $SU(2)$ on another brane. We call these branes the $SU(3)$-brane and the $SU(2)$-brane, respectively.

Now we assign chiral matter fields and the brane corresponding to $U(1)_Y$ such that we obtain non-universal $A$-terms. Chiral matter fields correspond to open strings spanning between branes. Thus, chiral matter fields have non-vanishing quantum numbers only for the gauge groups corresponding to the branes between which the open string spans. For example, the chiral field corresponding to the open string between the $SU(3)$ and $SU(2)$ branes can have non-trivial representations under both $SU(3)$ and $SU(2)$, while the chiral field corresponding to the open string, which starts and ends on the $SU(3)$-brane, should be an $SU(2)$-singlet. Furthermore, it is required that $U(1)_Y$ should correspond to one of the $SU(3)$-brane and $SU(2)$-brane but not another brane such that quark doublets have non-vanishing $U(1)_Y$ charges.

While there is only one type of the open string which spans between different branes, there are three types of open strings which start and end on the same brane, that is, the $C_i$ sectors ($i=1,2,3$), which corresponding to the $i$-th complex compact dimension among the three complex dimensions. If we assign the three families to the different $C_i$ sectors each other, we obtain non-universality. That is the only possible non-universality and it is important for model building. That implies that we can not derive non-universality for

\[1\] Different possibilities can be found in \[17\].
the squark doublets, i.e. the left–handed sector. Non–universality can appear in the right–handed sector only if $U(1)_Y$ corresponds to the $SU(3)$–brane and the families are assigned to different $C_i$ sectors each other. Therefore, the model leading to both non–universal gaugino masses and non–universal $A$–terms is unique, that is, we assign $SU(3) \times U(1)_Y$ and $SU(2)$ to different branes. The quark doublets correspond to the open string between the $SU(3) \times U(1)_Y$–brane and the $SU(2)$–brane. The quark singlets correspond to three different sectors on the $SU(3)$–brane. Hence, non–universality of soft SUSY terms can appear only for the right–handed sector, while soft SUSY breaking terms are universal for the left–handed sector.

Here we assume that the gauge group $SU(3) \times U(1)_Y$ is originated from the 9–brane and the gauge group $SU(2)$ is originated from the 51–brane like Ref. In this case $SU(2)$–doublet fields, e.g. quark doublets and the Higgs fields, should be assigned to the open string, which spans between the 51 and 9–branes and is denoted by the $C^{951}_1$ sector. On the other hand, the $SU(2)$–singlet fields, e.g. quark singlets, correspond to the open string, which starts and ends on the 9–brane. Such open string includes three sectors denoted by $C^9_{i1}$ ($i = 1, 2, 3$).

At the string level, only the $C^9_1$ sector is allowed in the 3–point $C^{951}_1C^{951}_1C^9_1$ coupling. However, we assume that the Yukawa couplings for the other sectors $C^9_i$ ($i = 1, 2$) are allowed through higher dimensional operators after symmetry breaking within the framework of effective field theory. Such effective Yukawa couplings originated from higher dimensional operators naturally lead to suppressed values of couplings, while the $C^{951}_1C^{951}_1C^9_1$ coupling would correspond to the large Yukawa coupling of the top quark as well as the bottom quark. Within such framework, the hierarchical structure of fermion mass matrices could be realized. Then we allow all of the $C^9_i$ ($i = 1, 2, 3$) as candidates of quark singlets. In particular, we assign the $C^9_1$ sector to the third family. Also we assign the first and second families of quark singlets to $C^9_3$ and $C^9_2$, respectively, in order to derive non–universal $A$–terms.

\footnote{It is possible to assign $U(1)_Y$ as a linear combination of $U(1)$ symmetry on the $SU(3)$–brane and $U(1)$ symmetries on other branes including the $SU(2)$–brane. However, in this case, phenomenological results are same.}

\footnote{Different assignment of branes lead to phenomenologically similar results as emphasized above.}
Under the above assignment of the gauge multiplets and the matter fields, soft SUSY
breaking terms are obtained, following the formulae in Ref. [17]. The gaugino masses are obtained

\[ M_3 = M_1 = \sqrt{3} m_{3/2} \sin \theta \ e^{-i \alpha_S}, \]

\[ M_2 = \sqrt{3} m_{3/2} \cos \theta \ \Theta_1 e^{-i \alpha_1}. \]

While the A-terms are obtained as

\[ A_{C_1^0} = -\sqrt{3} m_{3/2} \sin \theta \ e^{-i \alpha_S} = -M_3, \]

for the coupling including \( C_1^0 \), i.e. the third family,

\[ A_{C_2^0} = -\sqrt{3} m_{3/2} (\sin \theta \ e^{-i \alpha_S} + \cos \theta \ (\Theta_1 e^{-i \alpha_1} - \Theta_2 e^{-i \alpha_2})), \]

for the coupling including \( C_2^0 \), i.e. the second family and

\[ A_{C_3^0} = -\sqrt{3} m_{3/2} (\sin \theta \ e^{-i \alpha_S} + \cos \theta \ (\Theta_1 e^{-i \alpha_1} - \Theta_3 e^{-i \alpha_3})), \]

for the coupling including \( C_3^0 \), i.e. the first family. Here \( m_{3/2} \) is the gravitino mass, \( \alpha_S \)
and \( \alpha_i \) are the CP phases of the F-terms of the dilaton field \( S \) and the three moduli fields
\( T_i \), and \( \theta \) and \( \Theta_i \) are goldstino angles, and we have the constraint, \( \sum \Theta_i^2 = 1. \)

Thus, if quark fields correspond to different \( C_i^0 \) sectors, we have non–universal A–
terms. Then we obtain the following A–matrix for both of the up and down sectors,

\[ A = \begin{pmatrix} A_{C_1^0} & A_{C_2^0} & A_{C_3^0} \\ A_{C_1^0} & A_{C_2^0} & A_{C_3^0} \\ A_{C_1^0} & A_{C_2^0} & A_{C_3^0} \end{pmatrix}. \]

Note that the non–universality appears only for the right–handed sector. The trilinear
SUSY breaking matrix, \( (Y^A)_{ij} = (Y)_{ij} (A)_{ij} \), itself is obtained

\[ Y^A = \begin{pmatrix} Y_{ij} \end{pmatrix} \begin{pmatrix} A_{C_1^0} & 0 & 0 \\ 0 & A_{C_2^0} & 0 \\ 0 & 0 & A_{C_3^0} \end{pmatrix}, \]

in matrix notation.
In addition, soft scalar masses for quark doublets and the Higgs fields are obtained,

\[ m_{C_{951}}^2 = m_{3/2}^2 \left( 1 - \frac{3}{2} \cos^2 \theta \left( 1 - \Theta_1^2 \right) \right). \] (8)

The soft scalar masses for quark singlets are obtained as

\[ m_{\zeta_i}^2 = m_{3/2}^2 \left( 1 - 3 \cos^2 \theta \Theta_i^2 \right), \] (9)

if it corresponds to the \( C_i^9 \) sector.

Finally, we fix the magnitudes of the \( \mu \)-term and \( B \)-term by using the minimization conditions of the Higgs potential. This completes the whole set of initial conditions in our type I string inspired model.

3 Flavor physics and soft breaking terms

In the previous section, we have defined our string inspired model. Below the string scale, this model is simply a MSSM (understood as with the minimal particle content from the SM) with non-trivial soft-breaking terms from the point of view of flavor. Scalar mass matrices and trilinear terms have completely new flavor structures, as opposed to the supergravity inspired CMSSM or the SM, where the only connection between different generations is provided by the Yukawa matrices.

This model includes in the quark sector 7 different structures of flavor, \( M_{Q}^2, M_{U}^2, M_{D}^2, Y_d, Y_u, Y_d^A \) and \( Y_u^A \). From these matrices, \( M_{Q}^2 \), the squark doublet mass matrix, is proportional to the identity matrix, and hence trivial, then we are left with 6 non-trivial flavor matrices. Notice that we have always the freedom to diagonalize the hermitian squark mass matrices and then Yukawa and trilinear matrices are completely fixed. This implies that, in this case, these four matrices are observable, as opposed to the SM or CMSSM case, where only quark masses and the CKM matrix are observable.

At this point, to specify completely the model, we need not only the soft-breaking terms but also the complete Yukawa textures. The only available experimental information is the CKM mixing matrix and the quark masses. In this work, as an estimate of possible effects we choose our Yukawa texture following two simple assumptions: i) the CKM mixing matrix originates from the down Yukawa couplings and ii) Yukawa matrices
are hermitian \([19]\). With these two assumptions we fix completely the Yukawa matrices at the string scale, \(M_X\),

\[
Y_u = \frac{1}{v_2} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad Y_d = \frac{1}{v_1} K^\dagger \cdot \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \cdot K
\] (10)

with \(v = v_1/(\cos \beta) = v_2/(\sin \beta) = \sqrt{2}M_W/g\). Through all the paper we fix \(\tan \beta = v_2/v_1 = 2\) and \(K\) is the CKM matrix. In principle, generic Yukawa matrices in this basis of diagonal squark masses could be different \([19]\), but other matrices lead to physically similar results for the following analyses. Hence, the texture in Eq.(10) is enough for our purposes.

In this basis we can analyze the down trilinear matrix that with Eqs.(7) and (10) is,

\[
Y_A^d(M_X) = \frac{1}{v_1} K^\dagger \cdot M_d \cdot K \cdot \begin{pmatrix} A_{C_2^g} & 0 & 0 \\ 0 & A_{C_2^q} & 0 \\ 0 & 0 & A_{C_1^q} \end{pmatrix}
\] (11)

with \(M_d = \text{diag.}(m_d, m_s, m_b)\).

Hence, together with the up trilinear matrix we have our model completely defined. The next step is simply to use the MSSM Renormalization Group Equations (RGE) \([20]\) to obtain the whole spectrum and couplings at the low scale, \(M_W\). The dominant effect in RGEs of the trilinear terms is due to the gluino mass which produces the well-known alignment among A–terms and gaugino phases. However this RG effect is always proportional to the Yukawa matrices and not to the trilinear terms themselves, that is, roughly the RGEs are \(dY_d^A/dt \sim F(\alpha_s) m_{\tilde{g}} Y_d + G(\alpha_s, \alpha_W, Y_d, Y_u, \ldots)Y_d^A\) \([20]\). This implies that, in the SCKM basis the gluino effects are diagonalized in excellent approximation, while due to the different flavor structure of the trilinear terms large off–diagonal elements remain with phases \(O(1)\) \([8]\). To see this more explicitly, we can roughly approximate the RGE effects as,

\[
Y_d^A(M_W) = c_{\tilde{g}} m_{\tilde{g}} Y_d + c_A Y_d \cdot \begin{pmatrix} A_{C_2^g} & 0 & 0 \\ 0 & A_{C_2^q} & 0 \\ 0 & 0 & A_{C_1^q} \end{pmatrix}
\] (12)
with $m_{\tilde{g}}$ the physical gluino mass and $c_{\tilde{g}}, c_A$ coefficients order 1 (typically $c_{\tilde{g}} \simeq 5, c_A \simeq 1$).

We go to the SCKM basis after diagonalizing all the Yukawa matrices (i.e. $K \cdot Y_d \cdot K^\dagger = M_d/v_1$). In this basis we obtain the trilinear couplings as,

$$v_1 Y_d^A(M_W) = (c_{\tilde{g}} m_{\tilde{g}} M_d + c_A M_d \cdot K \cdot \begin{pmatrix} A_{C_3^0} & 0 & 0 \\ 0 & A_{C_2^0} & 0 \\ 0 & 0 & A_{C_1^0} \end{pmatrix} \cdot K^\dagger).$$

(13)

From this equation we can get the $L$–$R$ down squark mass matrix

$$M_{LR}^{(d)} = v_1 Y_d^{A^*} - \mu e^{i\phi_\mu} \tan \beta M_d.$$  

(14)

Finally using unitarity of $K$ we obtain for the $L$–$R$ mass insertions,

$$(\delta_{LR})_{ij} = \frac{1}{m_{\tilde{q}}} m_i \left( \delta_{ij} (c_A A_{C_3}^{*0} + c_{\tilde{g}} m_{\tilde{g}} - \mu e^{i\phi_\mu} \tan \beta) + K_{i2} K_{j2}^{*} c_A (A_{C_2}^{*0} - A_{C_3}^{*0}) + K_{i3} K_{j3}^{*} c_A (A_{C_1}^{*0} - A_{C_3}^{*0}) \right)$$

(15)

where $m_{\tilde{q}}$ is an average squark mass and $m_i$ the quark mass.

This expression shows the main effects of the non–universal $A$–terms. In the first place, we can see that the diagonal elements are still very similar to the universal $A$–terms situation. Apart of the usual scaling with the quark mass, these flavor–diagonal mass insertions receive dominant contributions from the corresponding $A_{C_i^0}$ terms (due to the fact that the CKM mixing matrix is close to the identity) plus an approximately equal contribution from gluino to all three generations and an identical $\mu$ term contribution. Hence, given that the gluino RG effects are dominant in Eq.(12), also the phases of these terms tend to align with the gluino phase, as in the CMSSM. Therefore, EDM bounds constrain mainly the relative phase between $\mu$ and gluino (or chargino) and give a relatively weaker constraint to the relative phase between $A_{C_3^0}$ (the first generation $A$–term) and the relevant gaugino [21] as we will show in the next section. Effects of different $A_{C_i^0}$ in these elements are suppressed by squared CKM mixing angles. However, flavor–off–diagonal elements are completely new in our model. They do not receive significant contributions from gluino nor from $\mu$ and so their phases are still determined by the $A_{C_i^0}$ phases and, in principle, they do not directly contribute to EDMs . It is also important to notice that in these off–diagonal elements the relevant quark mass is the one of the
left–handed quark, see Eqs.(13,15). In section 5, we will analyze the effects of these mass insertions in the kaon system.

In the same way, we must also apply the same rotations to the $L–L$ and $R–R$ squark mass matrices,

$$M_{LL}^{(d)}(M_W) = K \cdot M_Q^2(M_W) \cdot K^\dagger,$$

$$M_{RR}^{(d)}(M_W) = K \cdot M_D^2(M_W) \cdot K^\dagger.$$  \hspace{1cm} (16)

From Eq.(8) we have the universal mass for the squark doublets. This matrix remains approximately universal at $M_W$ and hence the off–diagonal elements after the rotation to the SCKM basis are sufficiently small. However the case of $M_{RR}^{(d)}$ is different. The masses of the squark singlets, Eq.(9), are not universal and hence sizeable off–diagonal elements are generated after the rotation to the SCKM basis. These entries could cause problems with the bounds from mass insertions [22]. However, this non–universality is diluted by the universal and dominant contribution from gluino to the squark mass matrices in the RGE. In the next sections we will analyze some CP violation observables in this framework.

4 EDM in models with non degenerate $A$–terms

In this section we show that, in the class of models with non degenerate $A$–terms, the EDM of the electron and neutron can be naturally smaller than the experimental limits,

$$d_n < 6.3 \times 10^{-26} \text{ e} \cdot \text{cm},$$

$$d_e < 4.3 \times 10^{-27} \text{ e} \cdot \text{cm},$$  \hspace{1cm} (17)

even in the presence of new supersymmetric phases $O(1)$. As mentioned in the introduction, in the universal $A$–term scenarios we have severe constraints on the SUSY phases from EDMs apart from a few points in the parameter space where cancellations occur. In the absence of cancellations among different contributions $\varphi_\mu$, the phase of the $\mu$ term, is constrained to be $O(10^{-2})$, while $\varphi_A$ is not strongly constrained [21]. The cancellation mechanism allows for somewhat larger phases at special regions in the parameter space. However in this points where we could have large phases, we can not generate any sizable
SUSY contribution to CP violation in the absence of new flavor structure [11, 12]. Moreover, this mechanism, when not justified by a symmetry argument, involves necessarily a certain degree of fine tuning [12]. In this paper, we will show that in the non-universal situation there is no need to restrict our parameter space to the cancellation regions to have large supersymmetric phases and, more important, large contributions to CP violation observables exist.

The supersymmetric contributions to the EDM include gluino, chargino and neutralino loops. In first place, we consider the gluino contribution which gives usually the major contribution. The gluino contribution for the EDM of the quark \( u \) and \( d \) in SCKM basis are given by

Figure 1: Allowed values for \( \alpha_2 - \alpha_S \) (open blue circles) and \( \alpha_3 - \alpha_S \) (red stars)
\[
\begin{align*}
    d_{d}^{g}/e & = -\frac{2}{9} \frac{\alpha_{S}}{\pi} \frac{1}{m_{\tilde{g}}^{2}} M_{1}(x) \text{Im}\{m_{\tilde{g}}(\delta_{LR}^{(d)})_{11}\} \\
    d_{u}^{g}/e & = \frac{4}{9} \frac{\alpha_{S}}{\pi} \frac{1}{M_{q}^{2}} M_{1}(x) \text{Im}\{m_{\tilde{g}}(\delta_{LR}^{(u)})_{11}\}
\end{align*}
\]  

(18)

where \(m_{\tilde{g}}\) is complex in this model and \(x = m_{\tilde{g}}^{2}/m_{\tilde{q}}^{2}\). The function \(M_{1}(x)\) is given by

\[
M_{1}(x) = \frac{1 + 4x - 5x^{2} + 4x \ln(x) + 2x^{2} \ln x}{2(1-x)^{4}}
\]

and by using the non–relativistic quark model approximation of the EDM of neutron we can calculate it in terms of the mass insertions \((\delta_{LR}^{(d)})_{11}\) and \((\delta_{LR}^{(u)})_{11}\).

It is important to notice in Eq.(18), that the relevant phase for the gluino contribution is the relative phase between the gluino mass and the \(L-R\) mass insertion \([14, 18]\). Thus, the physical phases entering in gluino contributions are \(\alpha'_{1} = \alpha_{1} - \alpha_{S}\), \(\alpha'_{2} = \alpha_{2} - \alpha_{S}\), \(\alpha'_{3} = \alpha_{3} - \alpha_{S}\) and \(\varphi'_{\mu} = \varphi_{\mu} - \alpha_{S}\). A very similar situation happens in the chargino contributions. These contributions are given by the squark and chargino mass matrices. Hence, in the same way as with the gluino contributions, the relevant phases are the relative phases between chargino masses, \(\alpha_{1}\), \(\varphi_{\mu}\), and \(L-R\) mass insertions.

As explained above, we can see from Eq.(14) that these flavor–diagonal mass insertions tend to align with the gluino phase (this is also true for the up squark mass matrices). Hence, to have a small EDM it is enough to have the phases of the gauginos and the \(\mu\) term approximately equal, \(\alpha_{S} = \alpha_{1} = \varphi_{\mu}\). On the other hand, the phases of the \(A\)–terms are not strongly constrained by the EDM bounds and so \(\alpha_{2}\) and \(\alpha_{3}\) can still be \(\mathcal{O}(1)\). This situation was already present even in the CMSSM \([21]\). Furthermore, in this case, we have the additional freedom of independent phases for different elements of the trilinear matrix, Eqs.(3,4,5).

In figure we show the allowed values for \(\alpha_{S}\), \(\alpha_{2}\) and \(\alpha_{3}\) assuming \(\alpha_{1} = \varphi_{\mu} = 0\). All other parameters in the model are scanned in the range: \(60\text{GeV} < m_{3/2} < 300\text{GeV}\), \(0.6 < \theta < 0.9\) and \(0 < \Theta_{i} < 1\) with the constraint \(\sum\Theta_{i}^{2} = 1\). Moreover, we have imposed all the usual constraints:

- Squark masses above 100 GeV with the only possible exception of the lightest stop and sbottom above 80 GeV.
- Charginos heavier than 80 GeV
• Branching ratio of the $b \to s\gamma$ decay, including supersymmetric contributions from chargino and gluino, from $2 \times 10^{-4}$ to $4.5 \times 10^{-4}$.

• Gluino and chargino contributions to $d_n$ and $d_e$ independently smaller than the phenomenological bounds.

• Gluino and chargino contributions to $\varepsilon_K$ smaller than $2.25 \times 10^{-3}$.

We can see that, similarly to the CMSSM situation, $\varphi_\mu$ is constrained to be very close to the gluino and chargino phases (in the plot $\alpha_S \simeq 0, \pi$), but $\alpha_2$ and $\alpha_3$ are completely unconstrained. It is also important to notice that we do not consider the possible cancellation regions. In any case, these special regions would only enlarge our allowed parameter space, mainly with larger relative phases between $\mu$ and gauginos. However, we will see in the next section that, without these additional regions, large effects in CP violation observables are already present.

## 5 CP violation in the Kaon system

We have shown in the previous section, that the presence of non–universal $A$–terms allows the existence of large phases in the supersymmetry soft–breaking sector while keeping EDMs sufficiently small. However, the important question from the phenomenological point of view is whether these phases are observable in other CP violation experiments [11]. It has been recently pointed out that, in general string inspired SUSY models with non–universal $A$–terms, it is possible to have large effects in CP violation observables, and in particular in $\varepsilon'/\varepsilon$ [8]. In the following, we will show that this mechanism is realized in our model and large effects are indeed present, while, at the same time, coping with EDM constraints.

We will mainly concentrate on the effects in the kaon system and, in the line of Refs. [1, 8, 9], we will consider the effects of $L-R$ mass insertions. In our model, as defined in section 4, flavor–off–diagonality is mainly present in the down squark mass matrix. Hence, it is clear that gluino contributions are dominant and we can directly apply the mass insertion bounds obtained in Ref. [22].
Supersymmetric contributions to $K^0-\bar{K}^0$ mixing are mainly given by $(\delta^{(d)}_{LR})_{12}$ and $(\delta^{(d)}_{LR})_{21} = (\delta^{(d)\ast}_{LR})_{12}$,

$$
\langle K^0|H_{eff}|\bar{K}^0\rangle_G = \frac{\alpha_s^2}{216m_{\bar{q}}^2}m_Kf_K^2 \left\{ \left( (\delta^{(d)}_{LR})_{12} + (\delta^{(d)}_{RL})_{12} \right) \left[ 44 \left( \frac{m_K}{m_s + m_d} \right)^2 x f_6(x) \right] + (\delta^{(d)}_{LR})_{12} \left[ 48 \left( \frac{m_K}{m_s + m_d} \right)^2 - 28 \right] \tilde{f}_6(x) \right\}
$$

(19)

where $m_K$ and $f_K$ denote the mass and decay constant of the Kaon, $x = m_{\bar{q}}^2/m_q^2$ and the functions $f_6(x)$ and $\tilde{f}_6(x)$,

$$
f_6(x) = \frac{6(1 + 3x) \ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5}
$$

$$\tilde{f}_6(x) = \frac{6x(1 + x) \ln x - x^3 + 9x^2 + 9x + 1}{3(x - 1)^5}.
$$

(20)

From this matrix element, the contributions to $\Delta m_K$ and $\varepsilon_K$ are,

$$
\Delta m_K = 2Re\langle K^0|H_{eff}|\bar{K}^0 \rangle
$$

$$\varepsilon_K = \frac{e^{i\phi}}{\sqrt{2}} \frac{Im\langle K^0|H_{eff}|\bar{K}^0 \rangle}{\Delta m_K}.
$$

(21)

Similarly, these $L-R$ mass insertions contribute to the direct CP violation observable $\varepsilon'/\varepsilon$. Here, the $L-R$ mass insertions enter mainly in the chromomagnetic penguin operators

$$
Re \left( \frac{\varepsilon'}{\varepsilon} \right)_G = \frac{11\sqrt{3}}{64\pi} \frac{\omega}{|\varepsilon|} Re(A_0) \frac{m_{\bar{q}}^2 m_K^2}{f_\pi (m_s + m_d)} \frac{\alpha_s(m_{\bar{q}})}{m_{\bar{q}}} Im\{ (\delta^{(d)\ast}_{LR})_{12} + (\delta^{(d)}_{LR})_{21} \} G_0(x)
$$

(22)

with $A_i = \langle (\pi\pi)_I = 1 | H_{eff} | K^0 \rangle$, $\omega = ReA_2/ReA_0$. In this convention we have $ReA_0 = 3.326 \times 10^{-4}$ and $f_\pi = 131$ MeV. The loop function $G_0(x)$ is,

$$
G_0(x) = \frac{x(22 - 20x - 2x^2 + 16x \ln(x) - x^2 \ln(x) + 9 \ln(x))}{3(1 - x)^4}.
$$

(23)

Using Eqs.(21) and (22) we can immediately calculate the dominant supersymmetric contributions to these observables in the presence of non–universal $A$ terms for a given set of initial conditions. From [22], with $x = m_{\bar{q}}^2/m_q^2 \approx 1$ the bounds on the $L-R$ mass insertions from $\Delta m_K$, $\varepsilon_K$ and $\varepsilon'/\varepsilon$ are respectively,

$$
\sqrt{|Re(\delta^{(d)}_{LR})_{12}|} < 4.4 \times 10^{-3} \cdot \frac{m_{\bar{q}}(GeV)}{500}
$$

$$
\sqrt{|Im(\delta^{(d)}_{LR})_{12}|} < 3.5 \times 10^{-4} \cdot \frac{m_{\bar{q}}(GeV)}{500}
$$

$$
\sqrt{|Im(\delta^{(d)}_{LR})_{12}|} < 2.0 \times 10^{-5} \cdot \left( \frac{m_{\bar{q}}(GeV)}{500} \right)^2
$$

(24)
Due to the fact that gluino amplitudes are left–right symmetric, these bounds apply exactly the same to \((\delta_{RL}^{(d)})_{12}\) \cite{22}. This means that, for large phases, the most sensitive observable to non–universal \(A\)–terms is always \(\varepsilon'/\varepsilon\); even \(\left|\text{Im}(\delta_{LR}^{(d)})_{21}^2\right| \sim 10^{-5}\) gives a significant contribution to \(\varepsilon'/\varepsilon\) while keeping the contributions from this mass insertion to \(\Delta m_K\) and \(\varepsilon_K\) well bellow the phenomenological bounds. In figure 2 we show a scatter plot of values of \(Im(\delta_{LR}^{(d)})_{21}\) versus the gluino mass in the same regions of parameter space and with the same constraints as in figure 1. Average scalar masses, \(m_{\tilde{q}}\), are close to the gluino mass, i.e. roughly \(x \approx 1, \ldots, 2\). We can see a large percentage of points are above or close to \(1 \times 10^{-5}\), hence, sizeable supersymmetric contribution to \(\varepsilon'/\varepsilon\) can be expected in the presence of non-universal \(A\)–terms. However, if we compare second and third row
in Eq.\((24)\), that is, respectively the bounds from \(\varepsilon'/\varepsilon\) and \(\varepsilon_K\), it is clear that these \(L-R\) mass insertions can never saturate the observed value of \(\varepsilon_K\). Hence, the presence of phases in the CKM matrix is still required. Unfortunately, due to the large uncertainties in the theoretical estimate of \(\varepsilon'/\varepsilon\), the recent experimental measurement in KTeV and NA31 \([15, 16]\) cannot be used to constrain this kind of model at present. In any case, the relative disagreement between the SM predictions and the observed experimental value can be take as a clue of new physics contributions of the kind presented in this paper.

6 Conclusions

Non-universal Supersymmetry soft breaking terms are a natural consequence in many supergravity or string inspired SUSY models. Moreover, non-universality is required besides large SUSY phases to produce observable effects in the low-energy CP violation experiments and, at the same time, provides an efficient mechanism to allow for \(O(1)\) SUSY phases while avoiding EDM bounds. These features have motivated us to make a complete phenomenological analysis of a class of SUSY models based on type I string theory with non-universal scalar masses, gaugino masses and \(A\)-terms.

Within this model, we have studied the supersymmetric contributions to the Electric Dipole Moments of the neutron and the electron. We find that, similarly to the CMSSM situation, the phase of the \(\mu\) term is constrained to be very close to the gluino and chargino phases. However there are still two supersymmetric phases completely unconstrained. This fact is completely independent of the possible existence of cancellations between different SUSY contributions. In any case, these special cancellation regions would only enlarge our allowed parameter space, mainly with larger relative phases between \(\mu\) and gauginos.

In the presence of these large SUSY phases, we have shown that sizeable supersymmetric contribution to CP observables appear. In particular, we have investigated the effects of these phases on the direct CP violation observable \(\varepsilon'/\varepsilon\). It has been recently suggested that, in the presence of non-degenerate \(A\)-terms, large susy contributions to this observable are possible. Here we have demonstrated that, in this completely defined model, this possibility is realized and a very sizeable fraction of the experimentally measured value
can be accounted with these supersymmetric contributions.

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