Assessment of the anisotropic wood strength on local crushing

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Abstract. Modern structural analysis standards for wooden and glued laminated building structures of civil and industrial buildings, bridges, electric power facilities, communications, etc., on the one hand, have differences, and on the other hand, general design principles (SP). However, the design resistances values are given only for the force vector acting along and across the fibers. The designer has to get the intermediate design resistances values according to the formulas of the 30s, very approximately, and sometimes incorrectly. For example, for cross laminated timber (CLT). Based on previously performed experimental researches and the results of calculations, general patterns of the resistance to local crushing changes were obtained for samples taken from glued laminated timber structures elements (glulam). General approaches have been developed to assess the strength of glulam ($\sigma_{cr}$) and its design resistances to local crushing ($R_{cr}$) with varying static geometrical parameters of the hard stamp ($\psi = d/l$) depending on the external forces vector direction with respect to the wood fibers – lamellas ($\alpha$). All this makes the research present-day and focused on improving reference documents.

1. Introduction
It is known that the physicomechanical properties of wood were first studied in the 19th century by Savart (1830) and Saint-Venant (1856), taking it as orthotropic material in experiments on torsion, bending, and compression. In Russia, D.I. Zhuravsky for the first time standardized the wood strength characteristics for the analysis of the Nikolaev railway wooden bridge trusses.

This question arises again with the advent of new wood-composite materials, since modern standards [1] do not reflect the entire completeness of parameters, impacts and operating conditions. The need to solve this issue is also due to the need to analyze the crushing area in the wooden structures nodes [2]. Most of the researches devoted to contact tasks relate to dowel pin and multi-pin connections, connectors in timber and glulam structures.

The following hypotheses and assumptions are made in the article for a layered anisotropic material that is heterogeneous in structure: glued laminated timber is considered as a homogeneous material due to the infinitely small thickness of the adhesive layers of the glulam array; the difference in elastic characteristics in lamellas (layers), randomly or organizedly oriented in the cross section of the glulam package, are within the statistical error, regardless of the axes $t$, $r$ or $tr$, i.e. we average the physical and mechanical characteristics by the influence of local defects in glued laminated timber (knots, slanting, non-glueing, etc.) that are within the normal range, we neglect it; stress and strain components are
assumed to be connected linearly, i.e. Hooke's law is fair; initial stresses arising during the manufacturing and operation (temperature, humidity, etc.) are taken into account at the stage of constructive analyze according to the standards. Thus, the physical model will correspond to a particular case of anisotropy – the transtropic object [3, 4, 5].

Research goal: to obtain standard and design strength characteristics for glued laminated timber under local stamped load impacts.

2. Research methodology

Testing of timber from factory elements of glulam structures was carried out on prismatic samples with dimensions $b \times l = 60 \times 100$ mm at the base and height $h = 90$ mm (figure 1). Lamella thickness was $20 \pm 0.5$ mm, use FRF-50 glue. The stamp dimensions to the sample width $v$ ratio was 0.25; 0.50; 0.75; 1.00; the inclination angle of the effective load in relation to the layers $\alpha$ ranged from $0^\circ$ to $90^\circ$ ($0^\circ; 15^\circ; 22.5^\circ; 30^\circ; 45^\circ; 60^\circ; 67.5^\circ; 90^\circ$).

![Figure 1. The shape and dimensions of the glulam timber samples.](image)

For the test we used a press MUP-50 with a 25-ton scale. The loading rate was within $15 \pm 2.5$ kN/min. Each point had at least 5 samples. During testing, for each sample, the deformation of the stamp relative to the stationary part of the installation was measured. The deformation of the connection was measured using a dial indicator with graduations of 0.01 mm. The indicator was attached to the fixed bed of the machine, and the pin abutted against the movable element, which fit snugly against the stamp and moved with it. The range of the indicator in 1 cm allowed to leave it on the sample until the latter was destructed.

The start of the samples destruction was recorded on a testing machine with a distinctive stop of the load growth. At $\alpha < 45^\circ$, the start of the destruction moment matched to a further load decrease. At $\alpha > 45^\circ$, a further load increase began after the destruction moment.

3. Results view

According to the tests, the destruction is mainly in the areas adjacent to the stamp. Forms of destructions are very diverse and depend on the angle between the fibers directions and the force action, the complex and very heterogeneous material under the stamp, the heterogeneity of the wood structure, the initial micro destructions and defects that occur during the making of samples.

The destruction of the samples occurred as a result of the longitudinal bending of separate fibers, the indentation of late wood into earlier wood, tearing across the fibers and splitting. With an increase in $\alpha$ to $45^\circ$, the destruction occurred as a result of tearing across the fibers and splitting, and for $\alpha$ more than $45^\circ$ as a result of deformation of weaker layers, i.e. by analogy with compression across the fibers. It is specific to "clean" wood. Therefore, the moment of samples destruction from $\alpha$ to 45 can
be considered as the shear strength, and the transition to the area of the non-linear relationships between deformations and load can be considered for samples with $\alpha$ more than $45^\circ$. The test results are given in the work [6].

In assessing the results of statistical processing of the glulam timber strength to local crushing it can be observed that there are high accuracy rates ($p = 1.5 \ldots 6.7\%$) obtained over the entire range of angles $\alpha$ and variation coefficients that do not go beyond $20\%$.

It was possible to identify regularities of change in the design resistance to crushing at an angle to the fibers (lamellas) $R_{cr,\alpha}$ after statistical processing of the results and determination of the average values of tensile strength $\sigma_{cr,\alpha}$ depending on the ratio $\psi$ and angle $\alpha$.

For this purpose, we marked the mechanical characteristics $\sigma_{cr,\alpha}$ for $Y$ and the stamp dimensions to the sample dimensions $\psi$ ratio for $X$. We got the relationship equations between the values of $Y$ and $X$ by using the method of least squares (OLS).

To address this issue we used a set of standard programs to get the unknown coefficients values $A$ and $B$. The approximating expressions, some of which are linear equations, and some are non-linear equations (exponential and logarithmic):

$$
Y = AX + B; \quad Y = AX^\theta; \quad Y = AX^X; \quad Y = A + B/X; \quad Y = 1/(AX + B); \quad Y = X/(AX + B); \quad Y = AlnX + B.
$$

(1)

The experimental pairs $X$ and $Y$ were used to solve each of the equations. 16 relationship equations were considered. The values of $X$, $Y$, $\alpha$, coefficients $A$, $B$ for seven relationship equations are given in this research.

According to the table the following equations $Y = AX + B$ (linear) and $Y = AlnX + B$ (non-linear) are most accurately show the quantitative changes in $\sigma_{cr,\alpha}$ depending on $\psi$ for constant environmental parameters ($t = 20^\circ C$, $W = 60\%$). Based on the smallest error for the equation $Y = AlnX + B$ was obtained at angles of $45^\circ \ldots 90^\circ$, we take this relationship equation as the baseline one for subsequent calculations.

We obtained a function for defining of tensile strength (crushing) of glulam timber for various $\psi$ and $\alpha$ by using the numerical coefficients values of the relationship equations and the double approximations method.

$$
\sigma_{cr,\alpha} = B + (-AlnX),
$$

(2)

here $B = Ne^{-n\alpha} = 47.352e^{-0.0324\alpha}$; $A = Ma^n = 34.173\alpha^{0.462}$.

In formula (2), the second term ($AlnX$) takes into account an increase in the tensile strength of glulam timber with a decrease in the relative crushing area size depending on the angle $\alpha$. For $X = d/l = 1$, the second term is equal to zero, and expression (2) determines the ultimate compressive strength (crushing) over the entire surface. For $d/l < 1$, it has negative values. Of course, all the changing parameters ($\psi$, $\alpha$) indicate a sharp difference between the obtained the strength and anisotropy values of glulam elements.

Table 1. The approximating relationship equations of the tensile strength (crushing) with changing the relative stamp dimensions and fiber (lamellas) angle.

| $\alpha$, degree | X   | Y   | Approximating equations                      | Equation error |
|------------------|-----|-----|---------------------------------------------|----------------|
|                  | 0.0 | 0.25| $Y = -16.000X + 58.000$                     | 0.000          |
|                  | 0.0 | 0.50| $Y = 58.919 -0.715X$                        | 0.1683         |
|                  | 0.0 | 0.75| $Y = 1/(0.007X + 0.0166)$                   | 0.3445         |
|                  | 0.0 | 1.00| $Y = -8.420 lnX + 43.017$                   | 0.8858         |
|                  | 0.50| 50.0| $Y = -8.420 lnX + 43.017$                   | 0.8858         |
|                  | 0.75| 46.0| $Y = 43.090X^{0.175}$                       | 1.0494         |
|                  | 1.00| 42.0| $Y = 40.564 + 3.569/X$                      | 1.6579         |
|                  | 1.00| 42.0| $Y = X/(0.0255X + 0.0023)$                  | 3.7320         |
Based on the results expression (2) can be written in the general equation form (3) that allows to calculate the changing values of \( \sigma_{cr,\alpha} \) in the presence of data at three characteristic points for angles 0°, 45°, 90°, plot the graphs on these points:

\[
\sigma_{cr,\alpha} = \frac{\sigma_{cr,\alpha}}{1 + \beta \alpha m} + (-D \ln \psi)
\]

(3)

here \( \sigma_{cr,\alpha} \) is the compressive strength (crushing) of glulam timber along the fibers, MPa; \( \alpha \) is the inclination angle of the external force vector with respect to the direction of the fibers (layers); \( \psi \) – relative crushing area sizes, \( \psi = d/l' \); \( \beta = (c - e)\sigma_0/45^\circ \); \( c = 1/\sigma_{45^\circ}; \sigma_0 = 1/\sigma_{90^\circ} \); \( m = \log(a - e)/(c - e) \); \( a = 1/\sigma_{90^\circ} \); \( D \) is a coefficient that takes account of the strength change depending on the angle \( \alpha \), (obtained after processing the tests results); \( D = 100 / (pa + f) = 100 / (0.164a + 9.15) \).

We used expression (3) to plot the dependence which show the relationship between \( \sigma_{cr,\alpha} \) and the relative stamp dimensions \( \psi \) and \( \alpha \) (figure 2). The graphs data clearly show the boundary character of

\[
\begin{array}{|c|c|c|}
\hline
\alpha & \psi & \sigma_{cr,\alpha} \\
\hline
0.25 & 34.0 & Y = 1/(0.0214X + 0.0226) \\
0.50 & 29.4 & Y = -7.781X + 23.891 \\
0.75 & 21.3 & Y = 21.327 + 3.271/X \\
1.00 & 14 & Y = X/(0.0504X + 0.0070) \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\alpha & \psi & \sigma_{cr,\alpha} \\
\hline
0.25 & 26.2 & Y = 1/(0.0395X + 0.0240) \\
0.50 & 20.9 & Y = 16.891X^{0.378} \\
1.00 & 13.3 & Y = X/(0.0857X + 0.0147) \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\alpha & \psi & \sigma_{cr,\alpha} \\
\hline
0.25 & 20.5 & Y = 1/(0.0371X + 0.0411) \\
0.50 & 16.7 & Y = 16.96X^{0.378} \\
1.00 & 13.3 & Y = X/(0.0857X + 0.0147) \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\alpha & \psi & \sigma_{cr,\alpha} \\
\hline
0.25 & 11.0 & Y = 1/(0.2108X + 0.0353) \\
0.75 & 5.3 & Y = 4.187X^{0.717} \\
1.00 & 4.0 & Y = X/(0.3015X + 0.0675) \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\alpha & \psi & \sigma_{cr,\alpha} \\
\hline
0.25 & 8.8 & Y = 1/(0.2850X + 0.0343) \\
0.75 & 4.0 & Y = 3.247X^{0.756} \\
1.00 & 3.1 & Y = X/(0.3950X + 0.0919) \\
\hline
\end{array}
\]
the angle $\alpha = 45^\circ$, because the concavity of the curves changes to convexity when passing through this value. Also this is an additional confirmation of the transition from brittle fracture to plastic fracture.

![Figure 2. The dependence of the glulam timber tensile strength and $\psi$ for various angles $\alpha$: 1 – 22.5°; 2 – 30.0°; 3 – 45.0°; 4 – 60.0°; 5 – 67.5°; 6 – 90.0°.](image)

Due to the fact that we conducted limited tests along the fibers (lamellas), we will use our data and take conditionally $\sigma_0 = 42$ MPa to draw the anisotropy curves using the formulas [1] and the formulas proposed in the present researches. The calculation results in comparison with the tests for $\psi = 1.0$ are given in table 2. Data analysis shows we obtained the closest values in almost the entire range of angles with our formula (3). Results according to the formulas of E.A. Ashkenazi [3] have quite acceptable values, which are widely used in modern scientific and technical literature to assess the strength of wood and wood materials.

| №  | $\sigma_{cr,\alpha}$, MPa | $\alpha$, degree | Note             |
|----|--------------------------|------------------|------------------|
|    |                          | 0    | 15   | 30    | 45    | 60    | 75    | 90    |           |
| 1  | Test data                | 42.0 | 27.9 | 14.6  | 12.3  | 4.6   | 4.0   | 3.1   | Table 1 [6] |
| 2  | $\sigma_{cr,\alpha} = \frac{\sigma_{cr,0}}{1 + \beta m_{\alpha} + (\mathrm{Dln}\psi)}$ | 42.0 | 28.1 | 15.3  | 9.0   | 5.8   | 4.1   | 3.0   | Formula 3  |
| 3  | $\sigma_\alpha = \frac{\sigma_0}{\cos^4 \alpha + 2 \sin^2 \alpha + \sin^4 \alpha}$ | 42.0 | 36.1 | 19.8  | 9.0   | 4.9   | 3.4   | 3.0   | [3]        |
| 4  | $\sigma_\alpha = \frac{\sigma_0}{1 + \left(\frac{\sigma_0}{\sigma_90} - 1\right) \sin^3 \alpha}$ | 42.0 | 34.3 | 16.0  | 7.5   | 4.4   | 3.3   | 3.0   | SP 64.13330.2017 |
Based on the test data [6], and formula (3) we plotted the dependence of the anisotropy $\sigma_{cr,\alpha}$ on $\psi$ and $\alpha$ (figure 3). The upper and lower boundaries of the confidence interval ($\sigma_{cr,\alpha}^{up}$ and $\sigma_{cr,\alpha}^{down}$) were defined with a confidence probability of 0.95.

$$
\sigma_{cr,\alpha}^{up} = \overline{\sigma_{cr,\alpha}} + tS; \quad \sigma_{cr,\alpha}^{down} = \overline{\sigma_{cr,\alpha}} - tS.
$$

Here $\overline{\sigma_{cr,\alpha}}$ is the arithmetic mean; $S$ is the standard deviation; $t$ is the student’s t-test.

Based on the graphs (figure 3) all the curves fall within the confidence interval boundaries and the actual scattering area of the test results with a probability $p = 0.95$. Only with relative stamp dimensions equal to 0.75 and 0.5, the curves approach the lower confidence border, which is quite acceptable, because quantitative results of $\sigma_a$ belong to the material margin of safety.

Short-term values of tensile strengths can serve as the basis for obtaining the design resistance to crushing of glulam timber on the considered static effects.

**Figure 3.** Dependence of the tensile strength $\sigma_{cr,\alpha}$ anisotropy of glulam timber upon crushing (compression) on the inclination angle of the layers (fibers) $\alpha$ and stamp dimensions $\psi$: a – 1.00; b – 0.75; c – 0.50; d – 0.25; 1 – average values of $\sigma_{cr,\alpha}$; 2 – curves drown by the formula (3).
In light of the above, and also assuming that the design resistances change will not qualitatively differ from the short-term resistances change of glulam timber, expression (3) can be transformed by introducing a reliability coefficient in the right-hand side, but use the standard design resistances values in the left-hand side.

Then the formula for determining the design resistance, taking into account the magnitude of the load applied through the stamp, depending on its relative dimensions and the inclination angle of the force vector with respect to the fibers (lamellas), will take the form:

$$R_{cr,\alpha} = R_0 / (1 + \beta \alpha^m) + (-1/ K_\alpha) D \ln \psi,$$

(4)

here $R_0$ is the design crushing resistance along the fibers, MPa; $\beta = (C - B) \times R_0 / 45^m$; $m = \lg (A - B) / (C - B)$; $A = 1/ R_{90}$; $B = 1/ R_0$; $C = 1/ R_{45}$; $D = 100 / (0.164 \alpha + 9.15)$; $\psi = d/l$ при $(0.1 \leq \psi \leq 1)$.

We omit the methodology for calculating the design resistances [6] and give below, figure 4 shows the dependences $R_{cr,\alpha} = f(\alpha, \psi)$, plotted for five relative dimensions of the crushing area variants with standard design resistances of glulam timber $R_0 = 14.0$ MPa, $R_{90} = 1.8$ MPa, $R_{45} = 4.2$ MPa, taking into account the results smoothing in the range of angles $\alpha$ from 30° to 60°.

![Figure 4. Change in the design crushing resistance of glulam timber at an angle $\alpha$ to the layers (fibers) depending on the relative dimensions of the crushing area $\psi$: 1 – 0.10; 2 – 0.25; 3 – 0.50; 4 – 0.75; 5 – 1.00.](image)

4. Conclusions

1) Based on the above methodology and formulas, it is possible to obtain design characteristics for glued laminated timber and other wood-composite anisotropic materials, taking into account the size of the local impact on the crushing (compression) areas at various angles to the anisotropy axes or layers.

2) It was established experimentally that the nature of fracture during local crushing in the angle range 0 ... 45° is brittle, 45 ... 90° is plastic, which depends on the properties of the wood work under load as an anisotropic (transotropic) material.

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