D-wave baryon resonances with charm from coupled-channel dynamics

J. Hofmann\textsuperscript{a} and M.F.M. Lutz\textsuperscript{a}

\textsuperscript{a} Gesellschaft f"ur Schwerionenforschung (GSI)
Planck Str. 1, 64291 Darmstadt, Germany

Identifying the zero-range exchange of vector mesons as the driving force for the s-wave scattering of pseudo-scalar mesons off the baryon ground states, the spectrum of $\frac{3}{2}^-$ molecules is computed. We predict a strongly bound 15-plet of $C = -1$ states. A narrow crypto-exotic octet of charm-zero states is foreseen. In the $C = +1$ sector a sextet of narrow resonances is formed due to the interaction of D mesons with the baryon decuplet. A strongly bound triplet of double-charm states is a consequence of coupled-channel dynamics driven by the D mesons.

1 Introduction

In this work we further explore the implications of the hadrogenesis conjecture \cite{1–3} that baryon resonances not belonging to the large-$N_c$ ground states are generated dynamically by coupled-channel dynamics. In recent works \cite{1,2,4,5} it was shown that chiral symmetry in combination with coupled-channel dynamics \cite{1,2} predicts the existence of a wealth of s-wave and d-wave baryon resonances. The exciting novelty of these studies lies in the systematics that supports the hadrogenesis conjecture: there are many resonance states that are naturally recovered within a coupled-channel computation based on an interaction unambiguously determined by the chiral properties of QCD.

For the first time it was shown only recently in \cite{5} that d-wave resonances are an immediate consequence of QCD’s chiral SU(3) symmetry. In the early days of hadron physics a few selected candidates, like the $\Lambda(1405)$, were suggested to be hadronic molecules, i.e. their masses and partial decay widths were obtained in terms of a coupled-channel theory with effective hadronic degrees of freedom rather than quarks and gluons \cite{6–11}. This is quite analogous to the physics of nuclei, many properties of which are most economically understood in terms of nucleon degrees of freedom. The success of the early works \cite{6–11},
that addressed s-wave states only, was revived by many authors in the last
decade applying a more systematic framework [12–17] but basically applying
an interaction already used decades ago in [6–11]. The close correspondence
of those works is a consequence of the fact that the leading chiral interaction
may be represented by a zero-range t-channel exchange of vector mesons.

A further important step was taken in [18–21] where for the first time the
chiral coupled-channel dynamics was claimed to be applicable in the charm
and beauty sectors as well. The crucial observation was that chiral symmetry
predicts unambiguously the s-wave interaction of the Goldstone bosons with
the $1^{-}_{2}$ and $3^{-}_{2}$ ground states of non-zero charm quantum numbers. Coupled-
channel dynamics based on the chiral interaction predicted a zoo of s- and
d-wave resonances with charm [19,21].

At present the spectrum of charmed baryon resonances is very poorly known.
It is important to perform detailed computations that help the discovery of
new states. The computation of the $1^{-}_{2}$ spectrum [19] was extended recently for
the inclusion of additional channels involving the $\eta'$, $\eta_c$ and D-mesons [22].
The study was based on a force defined by a zero-range t-channel exchange
of vector mesons that is compatible with constraints set by chiral symmetry.
A rich spectrum of $1^{-}_{2}$ molecules was obtained that reproduces known states
but predicts plenty of unknown states, in part with exotic quantum numbers.
It is stressed that those results were not based on assuming a chiral SU(4)
symmetry, rather chiral SU(3) symmetry was realized in terms of universally
coupled light vector mesons. Additional 3-point vertices involving heavy vector
mesons were estimated by a SU(4) assumption. The purpose of the present
study is to work out the analogous generalization of the $3^{-}_{2}$ spectrum. We
consider the coupled-channel interaction of the 16-plet of $0^{-}$ ground state
mesons with the 20-plet of $3^{-}_{2}$ baryon ground states.

We recover the spectrum of chiral excitations formed by the interaction of the
Goldstone bosons with baryon ground states [18,22]. Those states decouple to
high accuracy from the coupled-channel dynamics driven by the D mesons. The
latter imply a strongly bound 15-plet of $C = -1$ states and a narrow crypto-
exotic octet of charm-zero states. In the $C = +1$ sector a sextet of narrow
resonances is formed due to the interaction of D mesons with the baryon
decuplet. A strongly bound triplet of double-charm states is a consequence of
coupled-channel dynamics driven by the D mesons.
2 Coupled-channel interactions

We study the interaction of the ground-state mesons and baryons with \( J^P = 0^- \) and \( \frac{3}{2}^+ \) quantum numbers composed out of u,d,s,c quarks. The pseudoscalar mesons that are considered can be grouped into multiplet fields \( \Phi[9], \Phi[3] \) and \( \Phi[1] \). We introduce the fields

\[
\begin{align*}
\Phi[9] &= \tau \cdot \pi(139) + \alpha^\dagger \cdot K(494) + K^\dagger(494) \cdot \alpha + \eta(547) \cdot \lambda_8 + \sqrt{\frac{2}{3}} \mathbf{1} \cdot \eta'(958), \\
\Phi[3] &= \frac{1}{\sqrt{2}} \alpha^\dagger \cdot D \cdot (1867) - \frac{1}{\sqrt{2}} D^\dagger(1867) \cdot \alpha + i \tau_2 D^{(s)}(1969), \\
\Phi[1] &= \eta_c(2980),
\end{align*}
\]

which are decomposed further into SU(2) multiplets. The approximate masses in units of MeV as used in the numerical simulation of this work are recalled in brackets [23]. The matrices \( \tau \) and \( \alpha \) are given in terms of the Gell-Mann SU(3) generators \( \lambda_1, ..., \lambda_8 \) with

\[
\tau = (\lambda_1, \lambda_2, \lambda_3), \quad \alpha^\dagger = \frac{1}{\sqrt{2}}(\lambda_4 + i\lambda_5, \lambda_6 + i\lambda_7).
\]

The baryon states are collected into SU(3) multiplet fields \( B_{[10]}, B_{[6]}, B_{[3]} \) and \( B_{[1]} \). The baryon decuplet field is completely symmetric and related to the physical states by

\[
\begin{align*}
B_{[10]}^{111} &= \Delta^{++}, & B_{[10]}^{113} &= \Sigma^+ / \sqrt{3}, & B_{[10]}^{133} &= \Xi^0 / \sqrt{3}, & B_{[10]}^{333} &= \Omega^-, \\
B_{[10]}^{122} &= \Delta^+ / \sqrt{3}, & B_{[10]}^{123} &= \Sigma^0 / \sqrt{6}, & B_{[10]}^{223} &= \Xi^- / \sqrt{3}, \\
B_{[10]}^{122} &= \Delta^{0} / \sqrt{3}, & B_{[10]}^{223} &= \Sigma^- / \sqrt{3}, \\
B_{[10]}^{222} &= \Delta^{-}.
\end{align*}
\]

The sextet, triplet and singlet fields are decomposed into their isospin multiplet components

\[
\begin{align*}
\sqrt{2} B_{[6]} &= \frac{1}{\sqrt{2}} \alpha^\dagger \cdot \Xi_c(2646) + \frac{1}{\sqrt{2}} \Xi'_c(2646) \cdot \alpha + \Sigma_c(2518) \cdot (i \tau_2 \lambda_8) \\
&+ \frac{\sqrt{2}}{3} (1 - \sqrt{3} \lambda_8) \Omega_c(2770), \\
\sqrt{2} B_{[3]} &= \frac{1}{\sqrt{2}} \alpha^\dagger \cdot \Xi_{cc}(3519) - \frac{1}{\sqrt{2}} \Xi'_{cc}(3519) \cdot \alpha + i \tau_2 \Omega_{cc}(3620), \\
\sqrt{2} B_{[1]} &= \sqrt{2} \Omega_{ccc}(4600),
\end{align*}
\]

with their masses taken in this work given in units of MeV. The \( \Omega_c(2770), \Xi_{cc}(3519), \Omega_{cc}(3620) \) and \( \Omega_{ccc}(4600) \) states are not established so far. The
mass of the $\Omega_c(2770)$ is estimated by assuming an equal spacing rule for the SU(3) breaking pattern of the sextet. The mass of the $\Xi_{cc}(3159)$ is obtained by the premise that the double-charm state claimed by the SELEX collaboration [24–26] has $J^P = \frac{3}{2}^+$ quantum numbers. The mass of the $\Omega_{cc}(3620)$ state is estimated from model calculations [27,28] which predict a typical splitting to the $\Xi_{cc}$ mass of about 100 MeV. Finally the mass of the $\Omega_{ccc}(4600)$ is guessed from [27,28] taking into account the too large predictions of the $\Omega_{cc}(3620)$ mass.

In a first step we construct the interaction of the mesons and baryon fields introduced in (1, 4) with the nonet-field of light vector mesons

$$V_{\mu}^{[9]} = \tau \cdot \rho_{\mu}(770) + \alpha^\dagger \cdot K_{\mu}(894) + K^\dagger_{\mu}(894) \cdot \alpha + \left(\frac{2}{3} + \frac{1}{\sqrt{3}} \lambda_8\right) \omega_{\mu}(783) + \left(\frac{\sqrt{2}}{3} - \frac{\sqrt{2}}{3} \lambda_8\right) \phi_{\mu}(1020).$$  \(\text{(5)}\)

The multiplet fields are introduced with

$$V_{\mu}^{[3]} = \frac{1}{\sqrt{2}} \alpha^\dagger \cdot D_{\mu}(2008) - \frac{1}{\sqrt{2}} D^\dagger_{\mu}(2008) \cdot \alpha + i \tau_2 D_{\mu}^{(s)}(2112),$$

$$V_{\mu}^{[1]} = (J/\Psi)_{\mu}(3097).$$  \(\text{(6)}\)

We recall the complete list of SU(3) invariant 3-point vertices that involve the minimal number of derivatives:

$$L_{\text{int}}^{\text{SU(3)}} = i h_{33}^9 \tr ((\partial_{\mu} \Phi_{[3]} \Phi_{[3]}^\dagger V_{\mu}^{[9]} - \Phi_{[3]} \Phi_{[3]}^\dagger \partial_{\mu} V_{\mu}^{[9]}))$$

$$+ i h_{33}^1 \tr ((\partial_{\mu} \Phi_{[3]} \Phi_{[3]}^\dagger - \Phi_{[3]} \partial_{\mu} \Phi_{[3]}^\dagger)) \tr (V_{\mu}^{[9]}))$$

$$+ i h_{99}^9 \tr ((\partial_{\mu} \Phi_{[9]} \Phi_{[9]}^\dagger - \Phi_{[9]} \partial_{\mu} \Phi_{[9]}^\dagger) V_{\mu}^{[9]}))$$

$$+ i h_{33}^3 \tr ((\partial_{\mu} \Phi_{[3]} \Phi_{[3]}^\dagger V_{\mu}^{[1]} - \Phi_{[3]} \partial_{\mu} \Phi_{[3]}^\dagger V_{\mu}^{[1]}))$$

$$+ i h_{33}^9 \tr ((\partial_{\mu} \Phi_{[9]} \Phi_{[9]}^\dagger V_{\mu}^{[1]} - \Phi_{[9]} \partial_{\mu} \Phi_{[9]}^\dagger V_{\mu}^{[1]}))$$

$$+ i h_{39}^3 \tr ((\partial_{\mu} \Phi_{[3]} \Phi_{[3]}^\dagger V_{\mu}^{[1]} - \Phi_{[3]} \partial_{\mu} \Phi_{[3]}^\dagger V_{\mu}^{[1]}))$$

$$+ i h_{39}^3 \tr ((\partial_{\mu} \Phi_{[3]} \Phi_{[3]}^\dagger V_{\mu}^{[3]} - \Phi_{[3]} \partial_{\mu} \Phi_{[3]}^\dagger V_{\mu}^{[3]}))$$

$$+ i h_{39}^3 \tr ((\partial_{\mu} \Phi_{[3]} \Phi_{[3]}^\dagger V_{\mu}^{[3]} - \Phi_{[3]} \partial_{\mu} \Phi_{[3]}^\dagger V_{\mu}^{[3]}))$$

$$+ i h_{31}^3 \tr ((\Phi_{[3]}^\dagger V_{\mu}^{[3]} - \Phi_{[3]} V_{\mu}^{[3]})) \tr (\partial_{\mu} \Phi_{[9]}))$$

$$+ i h_{31}^3 \tr ((\partial_{\mu} \Phi_{[3]} V_{\mu}^{[3]} - \Phi_{[3]} V_{\mu}^{[3]})) \tr (\Phi_{[9]}).$$  \(\text{(7)}\)

It was emphasized in [22] that the terms in (7) are not at odds with the constraints set by chiral symmetry provided the light vector mesons are coupled to matter fields via a gauge principle [29,30]. The latter requires a correlation of the coupling constants $h$ in (7).
with the pion decay constant $f \simeq 92$ MeV. Here the universal vector coupling strength is $g \simeq 6.6$ and the mass of the light vector mesons is $m_{[9]}^{(V)}$. A further constraint is implied by the OZI rule [31,22] which claims

$$h_{33}^0 = \frac{1}{2} \frac{g_{[9]}}{f^2}, \quad h_{33}^1 = -g.$$  

The remaining coupling constants are basically unknown at present. It was pointed out in [22] that a SU(4) ansatz is compatible with the chiral and large-$N_c$ constraints (8, 9), but suggests in addition the relations

$$h_{33}^0 = \sqrt{2} g, \quad h_{33}^1 = 2 g, \quad h_{39}^0 = 2 g,$$
$$h_{30}^3 = \sqrt{2} g, \quad h_{30}^3 = \sqrt{2} g, \quad h_{31}^3 = 2 g, \quad h_{13}^3 = g.$$  

A surprising additional consequence of a SU(4) ansatz is the KSFR relation [32,33]

$$\frac{(m_{[9]}^{(V)})^2}{2 f^2 g} = g.$$  

Confronting (10) with the scarce empirical information available suggests a surprisingly small SU(4) breaking pattern at the level of three-point vertices [22]. The SU(4) estimates appear to be valid up to a factor of two. It is reassuring that a similar breaking pattern is seen also in the weak-decay constants of the pseudo-scalar mesons [23].

We continue with the construction of the three-point vertices involving baryon fields. A complete list of SU(3) invariant terms reads:

$$\mathcal{L}_{int}^{SU(3)} = -\frac{1}{2} g_{[9]}^0 \text{tr} \left( (\bar{B}^{[10]}_\nu \gamma_\mu B_{[10]}^\nu) V^{[9]}_{[9]} \right) - \frac{1}{2} g_{[9]}^1 \text{tr} \left( \bar{B}^{[10]}_\nu \gamma_\mu B_{[10]}^\nu \right) \text{tr} \left( V^{[9]}_{[9]} \right)$$
$$- \frac{1}{2} g_{33}^0 \text{tr} \left( \bar{B}^{[3]}_\nu \gamma_\mu B_{[3]}^\nu V^{[9]}_{[9]} \right) - \frac{1}{2} g_{33}^1 \text{tr} \left( \bar{B}^{[3]}_\nu \gamma_\mu B_{[3]}^\nu \right) \text{tr} \left( V^{[9]}_{[9]} \right)$$
$$- \frac{1}{2} g_{66}^0 \text{tr} \left( \bar{B}^{[6]}_\nu \gamma_\mu B_{[6]}^\nu V^{[6]}_{[6]} \right) - \frac{1}{2} g_{66}^1 \text{tr} \left( \bar{B}^{[6]}_\nu \gamma_\mu B_{[6]}^\nu \right) \text{tr} \left( V^{[6]}_{[6]} \right)$$
$$- \frac{1}{2} g_{11}^1 \text{tr} \left( \bar{B}^{[1]}_{[1,\nu]} \gamma_\mu B_{[1]}^\nu \right) \text{tr} \left( V^{[1]}_{[1]} \right) - \frac{1}{2} g_{11}^1 \text{tr} \left( \bar{B}^{[1]}_{[1,\nu]} \gamma_\mu B_{[1]}^\nu V^{[1]}_{[1]} \right)$$
$$- \frac{1}{2} g_{90}^0 \text{tr} \left( (\bar{B}^{[10]}_\nu \gamma_\mu \cdot B_{[10]}^{\nu}) V_{[9]}^{\mu} \right) - \frac{1}{2} g_{90}^1 \text{tr} \left( \bar{B}^{[10]}_\nu \gamma_\mu B_{[10]}^{\nu} \right) \text{tr} \left( V_{[9]}^{\mu} \right)$$
$$- \frac{1}{2} g_{66}^0 \text{tr} \left( \bar{B}^{[6]}_\nu \gamma_\mu B_{[6]}^\nu V_{[6]}^{\mu} \right) - \frac{1}{2} g_{66}^1 \text{tr} \left( \bar{B}^{[6]}_\nu \gamma_\mu B_{[6]}^\nu \right) \text{tr} \left( V_{[6]}^{\mu} \right)$$
$$- \frac{1}{2} g_{33}^3 \text{tr} \left( \bar{B}^{[3]}_\nu \gamma_\mu \text{tr}(B_{[3]}^{\nu} V_{[3]}^{\mu}) - 2 \bar{B}^{[3]}_\nu \gamma_\mu B_{[3]}^{\nu} V_{[3]}^{\mu} \right)$$
$$+ \bar{B}^{[3]}_\nu \gamma_\mu V_{[3]}^{\mu} \text{tr} \left( B_{[3]}^{\nu} \right) - 2 \bar{B}^{[3]}_\nu \gamma_\mu B_{[3]}^{\nu} V_{[3]}^{\mu} \right).$$
\[-\frac{1}{2} g_{31}^3 \text{tr} \left( \bar{B}_{[1],\nu} \gamma_\mu B_\nu^{[3]} V_{[3]}^\mu + \bar{B}_{[3],\nu} \gamma_\mu B_\nu^{[1]} V_{[3]}^\mu \right) \]
\[-\frac{1}{2} g_{66}^3 \text{tr} \left( (\bar{B}_{[6]}^{[1]} \gamma_\mu \cdot B_\nu^{[6]} V_{[6]}^\mu - (\bar{B}_{[6]}^{[6]} \gamma_\mu \cdot B_\nu^{[1]} V_{[6]}^\mu) \right), \tag{12} \]

where we apply the notation

\[
[\bar{B}_{[10]} \cdot B_{[6]}]_{ab} = \epsilon^{abk} \bar{B}_{[10],ijk} \cdot B_{ij}^{[6]}, \quad [\bar{B}_{[6]} \cdot B_{[10]}]_{ab} = \epsilon_{abk} \bar{B}_{[6],ij} \cdot B_{ik}^{[10]}, \tag{13} \]

Within the hidden local symmetry model \cite{30} chiral symmetry is recovered with

\[
g_{00}^9 = 3 g, \quad g_{66}^9 = -g_{33}^9 = 2 g. \tag{14} \]

It is acknowledged that chiral symmetry does not constrain the coupling constants in (12) involving the SU(3) singlet part of the fields. The latter can, however, be constrained by a large-\(N_c\) operator analysis \cite{34}. At leading order in the \(1/N_c\) expansion the OZI rule \cite{31} is predicted. As a consequence the estimates

\[
g_{00}^1 = g_{11}^1 = g_{66}^1 = 0, \quad g_{33}^1 = g. \tag{15} \]

follow.

It is instructive to construct also the SU(4) symmetric generalization of the interaction (12). The baryons form a 20-plet in SU(4). Its field is represented by a tensor \(B_{ijk}^{[20]}\), which is completely symmetric. The indices \(i, j, k\) run from one to four, where one can read off the quark content of a baryon state by the identifications \(1 \leftrightarrow u, 2 \leftrightarrow d, 3 \leftrightarrow s, 4 \leftrightarrow c\). We write:

\[
\mathcal{L}_{\text{int}}^{\text{SU}(4)} = -\frac{3}{4} g \sum_{i,j,k,l=1}^4 \bar{B}_{ijk}^{[20]} \gamma_\mu V_{[16]}^{[i\mu,j,k]} B_{ijkl}^{[20]}, \tag{16} \]

\[
B_{[20]}^{11} = \Delta^{++}, \quad B_{[20]}^{12} = \frac{1}{\sqrt{3}} \Delta^+, \quad B_{[20]}^{12} = \frac{1}{\sqrt{3}} \Delta^0, \quad B_{[20]}^{22} = \Delta^-, \quad B_{[20]}^{13} = \frac{1}{\sqrt{3}} \Sigma^+, \quad B_{[20]}^{13} = \frac{1}{\sqrt{3}} \Sigma^0, \quad B_{[20]}^{23} = \frac{1}{\sqrt{3}} \Sigma^-, \quad B_{[20]}^{14} = \frac{1}{\sqrt{3}} \Xi^+, \quad B_{[20]}^{14} = \frac{1}{\sqrt{3}} \Xi^0, \quad B_{[20]}^{24} = \frac{1}{\sqrt{3}} \Xi^0, \quad B_{[20]}^{33} = \frac{1}{\sqrt{3}} \Omega^- \tag{17} \]

where we use the vector field \(V_{[16]}^{[i\mu]}\) as introduced in \cite{22}. It is pointed out that the relations (14, 15) follow from the form of the interaction (16). In addition the SU(4) symmetric vertex (16) implies
\[
g_{06}^3 = \sqrt{\frac{3}{2}} g, \quad g_{03}^3 = 2 g, \quad g_{31}^3 = -\sqrt{3} g, \\
g_{06}^0 = 0, \quad g_{06}^0 = \sqrt{2} g, \quad g_{33}^0 = 2 \sqrt{2} g, \quad g_{11}^0 = 3 \sqrt{2} g.
\] (18)

Unfortunately there appears to be no way at present to check on the usefulness of the result (18). Eventually simulations of QCD on a lattice may shed some light on this issue. The precise values of the coupling constants (10, 18) will not affect the major results of this work. This holds as long as those coupling constants range in the region suggested by (10, 18) within a factor three.
3 Coupled-channel scattering

We consider the s-wave scattering of the pseudo-scalar mesons fields (1) off the baryon fields (3, 4). The scattering process is described by the amplitudes that follow as solutions of the Bethe-Salpeter equation,

\[ T_{\mu\nu}(\vec{k}, k; w) = K_{\mu\nu}(\vec{k}, k; w) + \int \frac{d^4 l}{(2\pi)^4} K_{\alpha\beta}(l, w) G_{\alpha\beta}(l, w) T_{\beta\nu}(l, k; w), \]

where we suppress the coupled-channel structure for simplicity. The meson and decuplet propagators, \( D(q) \) and \( S_{\mu\nu}(p) \), are used in the notation of [3]. We introduced convenient kinematics:

\[ w = p + q = \bar{p} + \bar{q}, \quad k = \frac{1}{2} (p - q), \quad \bar{k} = \frac{1}{2} (\bar{p} - \bar{q}), \]

where \( q, p, \bar{q}, \bar{p} \) are the initial and final meson and baryon 4-momenta.

The scattering kernel is approximated by the t-channel vector meson exchange force defined by (7, 12), where we apply the formalism developed in [2,35]. The scattering kernel has the form

\[ K^{(I,S,C)}_{\mu\nu}(\vec{k}, k; w) = \frac{1}{4} \sum_{V \in [16]} C^{(I,S,C)}_V \left( \frac{\hat{q} + \hat{q}}{2} - \left( \frac{q^2}{t - m_V^2} \right) \frac{\hat{q}}{2 m_V^2} \right) g_{\mu\nu}, \]

with \( t = (\bar{q} - q)^2 \).

In (21) the scattering is projected onto sectors with conserved isospin (I), strangeness (S) and charm (C) quantum numbers. The latter are introduced with respect to the states collected in Tabs. 1-4, where we use the notation of [2,3]. The Pauli matrices \( \sigma_i \) act on the isospin doublet fields, like \( \Xi \) or \( K \). The 2 × 4 transition matrices \( T \) are normalized according to

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\( (1, 0, -1) \) & \( (2, 0, -1) \) & \( (\frac{1}{2}, -1, -1) \) & \( (\frac{1}{2}, -1, -1) \) \\
\hline
\( (\sqrt{2} D^i \sigma_2 T^i \Delta) \) & \( (\sqrt{2} D^i \sigma_2 (\sigma_1 T^i_1 + \sigma_2 T^i_2) \Delta) \) & \( (\frac{1}{2} \epsilon \cdot a \bar{D}) \) & \( (\frac{1}{2} \epsilon \cdot a \bar{D}) \) \\
\hline
\( (0, -2, -1) \) & \( (1, -2, -1) \) & \( (\frac{1}{2}, -3, -1) \) & \( (0, -4, -1) \) \\
\hline
\( (\sqrt{2} D^i \sigma_2 \Xi) \) & \( (\frac{1}{2} \epsilon \sigma_1 \Xi) \) & \( (\frac{1}{2} \epsilon \sigma_1 \Xi) \) & \( (\frac{1}{2} \epsilon \sigma_1 \Xi) \) \\
\hline
\end{tabular}
\caption{Coupled-channel states with isospin (I), strangeness (S) and charm (C = -1).}
\end{table}
Table 2
Coupled-channel states with isospin (I), strangeness (S) and charm (C = 0).

\[
\begin{array}{cccccc}
(1, 1, 0) & (2, 1, 0) & (\frac{1}{2}, 0, 0) & (\frac{1}{2}, 0, 0) \\
(\sqrt{2}K^i \sigma_2 \pi^i \Delta) & (\sqrt{2}K^i \sigma_2 (\sigma_i T^i_j + \sigma_j T^i_j) \Delta) & \left( \begin{array}{c}
\frac{1}{2} \pi \cdot T^i \Delta \\
\frac{1}{2} \Sigma \cdot \sigma K \\
\frac{1}{2} \Sigma_\omega \cdot \sigma \bar{D}
\end{array} \right) & \left( \begin{array}{c}
\frac{1}{2} \pi \cdot T^i \Delta \\
\frac{1}{2} \Sigma \cdot \sigma K \\
\frac{1}{2} \Sigma_\omega \cdot \sigma \bar{D}
\end{array} \right)
\end{array}
\]

\[
\begin{array}{cccccc}
(\frac{1}{2}, 0, 0) & (0, -1, 0) & (1, -1, 0) & (2, -1, 0) & (\frac{1}{2}, -2, 0) & (\frac{1}{2}, -2, 0) \\
\left( \begin{array}{c}
\frac{1}{2} \pi \cdot \Sigma \\
\frac{1}{2} \Sigma \cdot \sigma_2 \Xi
\end{array} \right) & \left( \begin{array}{c}
\frac{1}{2} \pi \cdot \Sigma \\
\frac{1}{2} \Sigma \cdot \sigma_2 \Xi
\end{array} \right) & \left( \begin{array}{c}
\frac{1}{2} \pi \cdot \Xi \\
\frac{1}{2} \Sigma \· \bar{D}_s \Xi
\end{array} \right) & \left( \begin{array}{c}
\frac{1}{2} \pi \cdot \Xi \\
\frac{1}{2} \Sigma \· \bar{D}_s \Xi
\end{array} \right) & \left( \begin{array}{c}
\frac{1}{2} \pi \cdot \Xi \\
\frac{1}{2} \Sigma \· \bar{D}_s \Xi
\end{array} \right)
\end{array}
\]

\[
\begin{array}{cccccc}
(0, -3, 0) & (1, -3, 0) & (\frac{1}{2}, -4, 0) \\
\left( \begin{array}{c}
\frac{1}{2} \Sigma \· \Xi
\end{array} \right) & \left( \begin{array}{c}
\frac{1}{2} \Sigma \· \Xi
\end{array} \right) & \left( \begin{array}{c}
\frac{1}{2} \Sigma \· \Xi
\end{array} \right)
\end{array}
\]

Table 2

\[
\vec{T} \cdot \vec{T}^\dagger = 1, \quad T^i_j T^j_i = \delta_{ij} - \frac{1}{3} \sigma_i \sigma_j.
\]

The coupled-channel structure of the matrices \(C^{(I,S,C)}_{V,ab}\) is detailed in the Appendix. Only non-vanishing elements are displayed. Owing to the 'chiral' identifications (8, 14) and the KSFR relation (11) we reproduce the coupled-channel structure of the Weinberg-Tomozawa interaction identically. Summing
Table 3
Coupled-channel states with isospin \((I)\), strangeness \((S)\) and charm \((C = 1)\) over the light vector meson states

\[
\sum_{V \in [0]} C^{(I,S,C)}_V = 4g^2 C^{(I,S,C)}_{WT},
\]

we reproduce the matrices \(C_{WT}\) as given previously in [5]. The first term of the interaction kernel matches corresponding expressions predicted by the leading order chiral Lagrangian if we put \(t = 0\) in (21) and use the common value for the vector-meson masses suggested by the KSFR relation (11). The second term in (21) is formally of chiral order \(Q^3\) for channels involving Goldstone bosons. Numerically it is a minor correction but nevertheless it is kept in the computation.

Given (10, 18) one may decompose the interaction into SU(4) invariant tensors:
Table 4
Coupled-channel states with isospin \((I)\), strangeness \((S)\) and charm \((C = 2, 3, 4)\).

\[
\frac{1}{4} g^2 \sum_{V \in [16]} C^\langle I, S, C \rangle_V = 7 C_{[20]} + 4 C_{[20_s]} - 3 C_{[120]} + C_{[140]},
\]

15 \otimes 20 = 20 + 20_s \oplus 120 \oplus 140. \tag{24}

The normalization of the matrices \(C_{[\ldots]}\) is such that their weight factors in (24) give the eigenvalues of \(\sum V C^\langle I, S, C \rangle_V / (4 g^2)\). Strongest attraction is foreseen in the 20-plets, repulsion in the 120-plet. It is interesting to observe that (24) predicts weak attraction in the 140-plet. This is an exotic multiplet. To digest this abstract group theory we further decompose the SU(4) multiplets into the more familiar SU(3) multiplets

\[
[20]^{SU(4)} = [8]^{SU(3)}_{C=0} \oplus [6]^{SU(3)}_{C=1} \oplus [3]^{SU(3)}_{C=1} \oplus [3]^{SU(3)}_{C=2},
\]

\[
[20]^{SU(4)} = [6]^{SU(3)}_{C=0} \oplus [3]^{SU(3)}_{C=1} \oplus [3]^{SU(3)}_{C=2}.
\]
\[ [20]^{SU(4)}_s = [10]^{SU(3)}_{C=0} \oplus [6]^{SU(3)}_{C=1} \oplus [3]^{SU(3)}_{C=2} \oplus [1]^{SU(3)}_{C=3} , \]

\[ [120]^{SU(4)}_* = [15]^{SU(3)}_{C=-1} \oplus [10]^{SU(3)}_{C=0} \oplus [35]^{SU(3)}_{C=0} \oplus [60]^{SU(3)}_{C=1} \oplus [24]^{SU(3)}_{C=1} \oplus [3]^{SU(3)}_{C=2} \oplus [15]^{SU(3)}_{C=3} \oplus [8]^{SU(3)}_{C=3} \oplus [3]^{SU(3)}_{C=4} , \]

\[ [140]^{SU(4)}_* = [15]^{SU(3)}_{C=-1} \oplus [8]^{SU(3)}_{C=0} \oplus [10]^{SU(3)}_{C=0} \oplus [27]^{SU(3)}_{C=0} \oplus [3]^{SU(3)}_{C=1} \oplus [6]^{SU(3)}_{C=1} \oplus [24]^{SU(3)}_{C=1} \oplus [3]^{SU(3)}_{C=2} \oplus [6]^{SU(3)}_{C=2} \oplus [15]^{SU(3)}_{C=3} . \]

(25)

It should be stressed that the decomposition (24) is useful only to perform some consistency checks of the computation. A SU(4) decomposition ignores the important physics whether possible attraction is provided by the t-channel exchange of the light or heavy vector mesons.

Following [2,3,5] an effective interaction kernel is introduced that can be decomposed into a set of covariant projectors with well defined total angular momentum, \( J \), and parity, \( P \),

\[
V_{\mu\nu}(\vec{k}, k; w) = \sum_{J,P} V^{(J,P)}(\sqrt{s}) \mathcal{Y}^{(J,P)}(\vec{q}, q, w) ,
\]

\[
\mathcal{Y}^{(3/2,-)}(\vec{q}, q; w) = \frac{1}{2} \left( g_{\mu\nu} - \frac{w_{\mu} w_{\nu}}{w^2} \right) \left( 1 + \frac{\psi}{\sqrt{w^2}} \right) - \frac{1}{6} \left( \gamma_{\mu} - \frac{w_{\mu}}{w^2} \psi \right) \left( 1 - \frac{\psi}{\sqrt{w^2}} \right) \left( \gamma_{\nu} - \frac{w_{\nu}}{w^2} \psi \right) .
\]

(26)

where we recall the projector relevant for s-wave scattering. At leading order the effective scattering kernel \( V_{\mu\nu}(\vec{k}, k; w) \) may be identified with the on-shell projected Bethe-Salpeter kernel \( K_{\mu\nu}(\vec{k}, k; w) \). The merit of the projectors is that they decouple the Bethe-Salpeter equation (19) into orthogonal sectors labelled by the total angular momentum \( J \). Here we suppress an additional matrix structure that follows since for given parity and total angular momentum, \( J \geq 3/2 \), two distinct angular momentum states couple. In general, for given \( J \), the projector form a \( 2 \times 2 \) matrix, for which we displayed in (26) only its leading 11-component. The effect of the remaining components is phase-space suppressed and not considered here.

In this work we neglect the t-dependence of the interaction kernel insisting on \( t = 0 \) in (21). Following [3,5] the s-wave projected effective scattering kernel, \( V^{(I,S,C)}(\sqrt{s}) \), is readily constructed:

\[
V^{(I,S,C)}(\sqrt{s}) = \sum_{V \in [16]} \frac{C^{(I,S,C)}_V}{8 m_V^2} \left( 2 \sqrt{s} - M - \bar{M} + (\bar{M} - M) \frac{m^2 - \bar{m}^2}{m_V^2} \right) .
\]

(27)

where \( M, \bar{M} \) and \( m, \bar{m} \) are the masses of initial and final baryon and meson...
states. In (27) and below we suppress the reference to the angular momentum and parity \( J^P = \frac{3}{2}^- \). The partial-wave scattering amplitudes, \( M^{(I,S,C)}(\sqrt{s}) \), take the simple form

\[
M^{(I,S,C)}(\sqrt{s}) = \left[ 1 - V^{(I,S,C)}(\sqrt{s}) J^{(I,S,C)}(\sqrt{s}) \right]^{-1} V^{(I,S,C)}(\sqrt{s}).
\]  

(28)

The unitarity loop function, \( J^{(I,S,C)}(\sqrt{s}) \), is a diagonal matrix. Each element depends on the masses of intermediate meson and baryon, \( m \) and \( M \), respectively:

\[
J(\sqrt{s}) = (E + M) \left( \frac{5}{9} + \frac{2E}{9M} + \frac{2E^2}{9M^2} \right) (I(\sqrt{s}) - I(\mu)),
\]

\[
I(\sqrt{s}) = \frac{1}{16\pi^2} \left( \frac{p_{cm}}{\sqrt{s}} \ln \left( 1 - \frac{s - 2p_{cm}\sqrt{s}}{m^2 + M^2} \right) - \ln \left( 1 - \frac{s + 2p_{cm}\sqrt{s}}{m^2 + M^2} \right) \right) + \left( \frac{1}{2} \frac{m^2 + M^2}{m^2 - M^2} - \frac{m^2 - M^2}{2s} \right) \ln \left( \frac{m^2}{M^2} + 1 \right) + I(0),
\]  

(29)

where \( \sqrt{s} = \sqrt{m^2 + p_{cm}^2} + \sqrt{m^2 + p_{cm}^2} \) and \( E = \sqrt{M^2 + p_{cm}^2} \) [3]. The finite widths of the \( \Delta(1232) \) and \( \Sigma(1385) \) states is taken into account by folding the loop functions \( J(\sqrt{s}) \) of (29) with normalized spectral functions (see [3]).

A crucial ingredient of the approach developed in [2,35] is its approximate crossing symmetry guaranteed by a matching scheme. The matching scale \( \mu \) in (29) depends on the quantum number \( (I,S,C) \) but should be chosen uniformly within a given sector. It also must be independent on the \( J \) and \( P \) [2,35,4,5,22]. We insist on

\[
\mu = \sqrt{m_{th}^2 + M_{th}^2},
\]  

(30)

where \( m_{th} \) and \( M_{th} \) is the mass of the lightest hadronic channel. Note that almost all matching scales were encountered already in [22] when studying s-wave resonances. As a consequence of (30) the s-channel and u-channel unitarized amplitudes involving the lightest channels can be matched smoothly at the matching point \( \mu \) [2,35,4,5]. The construction (30) implies in addition that the effect of heavy channels on the light channels is suppressed naturally [22].
4 Numerical results

In order to explore the formation of baryon resonances we study generalized
speed functions [36,35,22] of the simple form

\[
\text{Speed}_{ab}(\sqrt{s}) = \left| \frac{d}{d\sqrt{s}} [M_{ab}(\sqrt{s})] \right|. \tag{31}
\]

If a partial-wave scattering amplitude develops a resonance or bound state,
close to that structure it may be approximated by a pole and a background
term. We write

\[
M_{ab}(\sqrt{s}) \simeq -\frac{g_a^* g_b}{\sqrt{s} - M_R + i \Gamma_R/2} + b_{ab}, \tag{32}
\]

with the resonance mass \(M_R\) and width \(\Gamma_R\). The dimension less coupling con-
stants \(g_b\) and \(g_a\) parameterize the coupling strength of the resonance to the
initial and final channels. The background term \(b_{ab}\) is in general a complex
number. If the scattering amplitude has the form (32) its speed takes a maxi-
mum at the resonance mass \(M_R\). The ratio of coupling constants to total decay
width \(\Gamma_R\) is then determined by the value the speed takes at its maximum

\[
\text{Speed}_{aa}(M_R) = \left| \frac{2 g_a}{\Gamma_R} \right|^2. \tag{33}
\]

We determine the resonance position and coupling constants by adjusting the
parameters \(M_R\) and \(g_a\) to the Speed of its associated amplitudes. This is an
approximate procedure, fully sufficient in view of the schematic nature of the
computation.

It should be stressed that the properties of the resonance states presented in
the following ( see Tabs. 5-9 ) are subject to changes expected when incor-
porating additional channels involving the \(1^-\) mesons and \(1/2^+\) baryons. The
latter channels are required to arrive at results that are consistent with the
heavy-quark symmetry.

4.1 \(J^P = \frac{3}{2}^-\) resonances with charm minus one

We begin with a presentation of results obtained for d-wave resonances with
negative charm. The possible existence of such states was discussed first by
Gignoux, Silvestre-Brac and Richard twenty years ago [37,38]. Such states
Table 5
Spectrum of $J^P = \frac{3}{2}^-$ baryons with charm minus one.

The properties of the $C = -1$ states as generated by the coupled-channel equations (27, 28, 29) are collected in Tab. 5. The spectrum is computed insisting on the chiral relations (8, 14) together with the leading order large-$N_c$ relations (9, 15). Relying on the KSFR relation (11) the binding energies are determined by the universal vector coupling constant for which we take the value $g = 6.6$ from [22]. It is emphasized that none of the coupling constants (10, 18) that are estimated by an SU(4) ansatz are relevant for the spectrum. Various molecules are formed. Their SU(3) multiplet structure is readily worked out:

$$3 \otimes 10 = 15_1 \oplus 15_2,$$

$$[15_1] \ni \begin{pmatrix} (1, +0) \\ (\frac{1}{2}, -1), (\frac{3}{2}, -1) \\ (0, -2), (1, -2) \\ (\frac{1}{2}, -3) \end{pmatrix}, \quad [15_2] \ni \begin{pmatrix} (2, +0) \\ (\frac{3}{2}, -1) \\ (1, -2) \\ (\frac{1}{2}, -3) \\ (0, -4) \end{pmatrix}. \quad (34)$$

Attraction is predicted in the first 15-plet and repulsion in the second 15-plet. Summing the coefficient matrix $C_V$ over the nine light vector mesons we
obtain:

\[
\frac{1}{4g^2} \sum_{V \in [9]} C_V^{(C=-1)} = C_{[15_1]} - 3C_{[15_2]}.
\] (35)

The strengths in the two multiplets reflect the coefficient in front of the 140- and 120-plet of the SU(4) decomposition in (24). This follows since charm-exchange does not contribute here. The algebra (35) is directly reflected in Tab. 5, which collects the masses and coupling constants of the bound 15-plet. The SU(3) breaking pattern in the multiplet is significant. The lightest state with \((I, S) = (1, 0)\) comes at 2867 MeV, the heaviest state with \((I, S) = (1/2, -3)\) a mass of 3211 MeV. So far the states predicted in Tab. 5 have not been observed. Since the \((1, 0)\) state has a mass below the \(D N\) mass we do not see any indication that the H1 signal could possibly be a result of a d-wave state.

\[4.2\] \[J^P = \frac{3}{2}^- \] resonances with zero charm

We turn to the resonances with \(C = 0\). The spectrum falls into two types of states. Resonances with masses above 3 GeV couple strongly to mesons with non-zero charm content. In the SU(3) limit those states form an octet. All other states have masses below 2.5 GeV. In the SU(3) limit they group into an octet and decuplet. The presence of the heavy channels does not affect that part of the spectrum at all. We reproduce the previous coupled-channel computation [5]. The effect of using a spectral distribution for the \(\Delta(1232)\) and \(\Sigma(1385)\) does not alter the spectrum qualitatively [48]. There is nothing we want to add to those results at this stage.

Most spectacular are the resonances with hidden charm above 3 GeV. The multiplet structure of such states is readily understood. The mesons with \(C = -1\) form a triplet which is scattered off the charmed baryons forming a sextet. We decompose the product into irreducible tensors

\[3 \otimes 6 = 8 \oplus 10.\] (36)

The interaction is attractive in the crypto-exotic octet and repulsive in the decuplet. For the formation of the states the charm-exchange processes are irrelevant. This holds as long as the SU(4) estimates (10, 18) give the coupling constants within a factor of three. Thus the crypto-exotic sector may be characterized by the decomposition

\[
\frac{1}{4g^2} \sum_{V \in [9]} C_V^{(C=0)} = C_{[8]}^{\text{crypto}} - 2C_{[10]}^{\text{crypto}},
\] (37)

16
where we assume the chiral and large-$N_c$ relations (8, 14) and (9, 15). The binding energies of the crypto-exotic states are large. This is in part due to the large masses of the coupled-channel states: the kinetic energy the attractive t-channel force has to overcome is reduced. A second kinematical effect, which further increases the binding energy, is implied by the specific form of the t-channel exchange (27). It provides the factor $2\sqrt{s - M - \bar{M}}$. If evaluated at threshold it scales with the meson mass. We emphasize that the results are quite stable with respect to small variations of the matching scale $\mu$ introduced in (30): if we lower or increase $\mu$ by 20% the binding energies of the crypto exotic states change by less than 20 MeV.

In Tab. 6 the spectrum of crypto-exotic $J^P = \frac{3}{2}^-$ baryons with charm zero.

Table 6
Spectrum of crypto-exotic $J^P = \frac{3}{2}^-$ baryons with charm zero.

| $C = 0$: $(J, S)$ | state | $M_R$ [MeV] | $\Gamma_R$ [MeV] | $|g_R|$ |
|-------------------|-------|-------------|-----------------|--------|
| $(\frac{1}{2}, 0)$ | $\pi \Delta$ | 3430 | 0.50 | 0.05 |
|                   | $K \Sigma$ | 0.50 | 0.04 |
|                   | $\bar{D} \Sigma_c$ | 5.6 | 0.04 |
| $(0, -1)$         | $\pi \Sigma$ | 3538 | 0.63 | 0.05 |
|                   | $K \Xi$ | 0.63 | 0.05 |
|                   | $\bar{D} \Xi_c$ | 5.5 | |
| $(1, -1)$         | $\pi \Xi$ | 3720 | 0.83 | 0.01 |
|                   | $K \Delta$ | 0.83 | 0.01 |
|                   | $\eta \Sigma$ | 0.03 | 0.04 |
|                   | $K \Xi$ | 0.03 | 0.04 |
|                   | $\eta' \Sigma$ | 0.01 | 0.04 |
|                   | $\eta \Xi$ | 0.20 | 0.04 |
|                   | $D_{1/2} \Sigma_c$ | 4.5 | 0.04 |
|                   | $\bar{D} \Xi_c$ | 2.8 | 0.04 |
| $(\frac{1}{2}, -2)$ | $\pi \Xi$ | 3742 | 1.1 | 0.06 |
|                   | $K \Sigma$ | 1.1 | 0.06 |
|                   | $\eta \Xi$ | 0.06 | 0.06 |
|                   | $K \Omega$ | 0.06 | 0.06 |
|                   | $\eta' \Xi$ | 0.06 | 0.06 |
|                   | $\eta \Xi$ | 0.16 | 0.06 |
|                   | $D_{1/2} \Xi_c$ | 3.2 | 0.06 |
|                   | $D \Omega_c$ | 4.2 | 0.06 |

where we assume the chiral and large-$N_c$ relations (8, 14) and (9, 15). The binding energies of the crypto-exotic states are large. This is in part due to the large masses of the coupled-channel states: the kinetic energy the attractive t-channel force has to overcome is reduced. A second kinematical effect, which further increases the binding energy, is implied by the specific form of the t-channel exchange (27). It provides the factor $2\sqrt{s - M - \bar{M}}$. If evaluated at threshold it scales with the meson mass. We emphasize that the results are quite stable with respect to small variations of the matching scale $\mu$ introduced in (30): if we lower or increase $\mu$ by 20% the binding energies of the crypto exotic states change by less than 20 MeV.

In Tab. 6 the spectrum of crypto-exotic baryons is shown. The charm-exchange contributions are estimated by the SU(4) ansatz (10, 18). For the universal vector coupling constant we use $g = 6.6$ as before [22]. The octet of states is narrow as a result of the OZI rule. The mechanism is analogous to the one explaining the long life time of the $J/\Psi$-meson. The precise values of the width parameters are sensitive to the SU(4) estimates. In contrast to our previous study of crypto-exotic s-wave resonances [22], channels that involve the $\eta'$ meson do not to play a special role in the d-wave spectrum. This is in part a consequence that the $\eta'$ channels decouple from the crypto-exotic sector in the SU(3) limit. It is interesting to observe that a narrow nucleon resonance is predicted at 3.42 GeV. One may speculate that the crypto-exotic resonance claimed at 3.52 GeV in [49] may be a d-wave state. Given the uncertainties of the claim [49] we refrain from fine tuning the model parameters as to push up the $(\frac{1}{2}, 0)$ state.
The results collected in Tab. 6 are subject to large uncertainties. The evaluation of the total width as well as a more reliable estimate of the binding energies of the crypto-exotic states requires the consideration of further partial-wave contributions. The large binding energy obtained suggest a more detailed study that is based on a more realistic interaction taking into account in particular the finite masses of the t-channel exchange processes.
4.3 $J^P = \frac{3}{2}^-$ resonances with charm one

At present we know very little about open-charm d-wave resonances. Only two states $\Lambda_c(2625)$ and $\Xi_c(2815)$ are quoted by the Particle Data Group [23]. So far no quantum numbers are determined experimentally. The assignments are based on a quark-model bias. An additional state $\Sigma_c(2800)$ was discovered recently by the BELLE collaboration [50]. We consider it as a candidate for a d-wave state.

In a previous coupled-channel computation the effect of the Goldstone bosons as they scatter off the sextet of charmed baryons with $J^P = \frac{3}{2}^+$ was studied [21]. We confirm the striking prediction which suggest the existence of bound $3, 6, 15$ systems where attraction is foreseen in the anti-triplet, sextet and 15-plet with decreasing strength. This result is reproduced quantitatively if the charm-exchange reactions are neglected. In this case the decomposition

$$\frac{1}{4g^2} \sum_{V \in [9]} C^{(C=1)}_V = 5C^{\text{chiral}}_3 + 3C^{\text{chiral}}_6 + C^{\text{chiral}}_{15} - 2C^{\text{chiral}}_{24}, \quad (38)$$

holds, where the suffix 'chiral' indicates that the multiplets are realized with states that involve Goldstone bosons. The chiral and large-$N_c$ relations (8, 14) and (9, 15) are assumed in (38).

Further multiplets are generated by the scattering of the anti-triplet mesons of the decuplet baryons. We derive

$$\frac{1}{4g^2} \sum_{V \in [9]} C^{(C=1)}_V = 5C^{\text{heavy}}_6, \quad (39)$$

where we use the suffix 'heavy' to indicate that the multiplets are formed by states involving the D mesons. In Tab. 7 the spectrum of resonances is displayed where the charm-exchange contributions are considered based on the SU(4) estimates (10, 18). It should again be emphasized that the amount of binding predicted is insensitive to the SU(4) estimates. The latter is a consequence of the universally coupled light vector mesons.

The multiplet structure anticipated in (38, 39) is clearly reflected in the spectrum. For the readers’ convenience we recall the $(I, S)$ content of the various SU(3) multiplets:
\[ \Gamma [\text{MeV}] \]

| \( C = 1 \) : \( I, S \) | state \(| \bar{\Omega} \rangle \) | \(| \bar{\gamma} \rangle \) | \(| \bar{\Omega} \rangle \) |
|----------------|----------------|----------------|----------------|
| \((\frac{1}{2}, -1)\) | \( K \Sigma_c \) | 2988 | 1.6 | -- |
| \((0, 0)\) | \( K \Xi_c \) | 2660 | 2.3 | 3100 | 0.50 |
| \((0, 0)\) | \( K \Xi_c \) | 53 | 0.5 | 17 | 2.3 |
| \((0, 0)\) | \( D \Xi_{cc} \) | 1.4 | 0.1 | 0.08 |
| \((1, 0)\) | \( \pi \Sigma_c \) | 4268 | 0.08 | -- |
| \((1, 0)\) | \( \pi \Sigma_c \) | 1.4 | 0.06 | -- |
| \((1, 0)\) | \( \eta \Sigma_c \) | 2613 | 0.01 | 2716 | 0.9 |
| \((1, 0)\) | \( \eta \Sigma_c \) | 128 | 3.1 | -- |
| \((1, 0)\) | \( \eta \Sigma_c \) | 0.74 | 0.09 | 0.0 |
| \((1, 0)\) | \( \eta \Sigma_c \) | 3064 | 0.01 | 0.3 |
| \((1, 0)\) | \( \eta \Sigma_c \) | 0.18 | 0.12 | -- |
| \((\frac{1}{2}, -1)\) | \( \pi \Xi_c \) | 2762 | 0.01 | 2838 | 0.43 |
| \((\frac{1}{2}, -1)\) | \( \pi \Xi_c \) | 16 | 4.4 | 0.2 |
| \((\frac{1}{2}, -1)\) | \( \eta \Xi_c \) | 0.09 | 0.01 | 0.0 |
| \((\frac{1}{2}, -1)\) | \( \eta \Xi_c \) | 0.37 | 0.15 | 0.05 |
| \((\frac{1}{2}, -1)\) | \( \eta \Xi_c \) | 0.06 | 0.06 | 0.06 |
| \((\frac{1}{2}, -1)\) | \( \eta \Xi_c \) | 0.06 | 0.06 | -- |
| \((0, 2)\) | \( K \Xi_c \) | 2843 | 0.12 | 2.5 |
| \((0, 2)\) | \( K \Xi_c \) | 5.5 | 6.2 | 1.4 |
| \((0, 2)\) | \( D \Xi_c \) | 0.09 | 0.01 | 0.01 |
| \((0, 2)\) | \( D \Xi_c \) | 0.01 | 0.09 | 0.01 |
| \((0, 2)\) | \( \eta \Xi_c \) | 0.76 | 0.03 | 0.12 |

Table 7
Spectrum of \( J^P = \frac{3}{2}^- \) resonances with charm one.

\[
[3] \ni \begin{pmatrix} \begin{pmatrix} (0, 0) \\ (\frac{1}{2}, -1) \end{pmatrix} \\ (1, 0) \end{pmatrix}, \quad [6] \ni \begin{pmatrix} \begin{pmatrix} (1, 0) \\ (\frac{1}{2}, -1) \end{pmatrix} \\ (0, -2) \end{pmatrix},
\]

20
Strongest binding is predicted for the sextet states with $\Sigma_c(2613)$, $\Xi_c(2762)$ and $\Omega_c(2843)$. Such states are difficult to detect empirically since they couple only very weakly to open channels involving Goldstone bosons. As a consequence the widths of those states is typically below 1 MeV.

The spectrum of chiral excitations is much richer and presumably easier to confirm experimentally. A triplet and sextet of states is formed. The isospin singlet of the triplet at mass 2659 MeV should be identified with the $\Lambda_c(2625)$. We underestimate the binding somewhat. The isospin doublet of the triplet comes at 2838 MeV. Its small width of about 16 MeV only reflects its small coupling strength to the $\pi\Xi_c$ channel. In [21] the latter state was identified with the $\Xi_c(2815)$ of the Particle Data Group [23]. The attraction predicted in the chiral sextet channels leads to additional resonances $\Sigma_c(2716)$, $\Xi_c(3180)$ and $\Omega_c(3008)$. In [21] the isospin triplet state was suggested as a candidate for Belle’s state at 2800 MeV [50]. However, the identification is troublesome due to a too large width.

Crypto-exotic states with $cc\bar{c}$ content are formed by the scattering of the 3-plet mesons with $C = -1$ off the triplet baryons with $C = 2$:

\[
\frac{1}{4} g^2 \sum_{V \in [9]} C_V^{(C=1)} = C_{[3]}^\text{crypto} - C_{[6]}^\text{crypto}, \tag{41}
\]

where we predict strong attraction in the anti-triplet sector only. The associated narrow states have masses 4277 MeV and 4491 MeV. The binding energies and widths of these states are expected to be quite model dependent and more detailed studies are required.

4.4 $J^P = \frac{3}{2}^-$ resonances with charm two

Double-charm baryon systems are very poorly understood at present. There is a single published isospin doublet state claimed by the SELEX collaboration at 3519 MeV [24] that carries zero strangeness. There are hints that this state can not be the ground state with $J^P = \frac{1}{2}^+$ quantum numbers [25]. We assigned it $J^P = \frac{3}{2}^+$ quantum numbers.
There are two types of molecules formed in the coupled-channel computations. The chiral excitations of the $\frac{3}{2}^+$ baryons with $C = 2$ form a strongly bound triplet and a less bound sextet of resonance. This part of the spectrum is analogous to the one of the chiral excitations of open-charm mesons: attraction is predicted in the triplet and anti-sextet sectors [18,20]:

$$8 \otimes 3 = 3 \oplus \bar{6} \oplus 15, \quad 3 \otimes 6 = 3 \oplus 15. \quad (42)$$

Further molecules are formed by the systems composed of open-charm mesons and open-charm baryons. The multiplet decomposition for the sextet baryons is given also in (42). Strong attraction is predicted again in the triplet only. In the SU(3) limit there is no interaction in the crypto-exotic $\bar{D} \Omega_{ccc}$ sector if the charm-exchange reactions are neglected. All together we have

$$\frac{1}{4 \, g^2} \sum_{V \in [9]} C_V^{(C=2)} = 3 C_{[3]}^{\text{chiral}} + C_{[6]}^{\text{chiral}} - C_{[15]}^{\text{chiral}} + 4 C_{[3]}^{\text{heavy}}. \quad (43)$$

For the readers’ convenience we recall the isospin strangeness content of the various multiplets:
\[
[3] \ni \left( \begin{array}{c}
\left( \frac{1}{2}, 0 \right) \\
(0, -1)
\end{array} \right), \\
[6] \ni \left( \begin{array}{c}
(0, +1) \\
\left( \frac{1}{2}, 0 \right) \\
(1, -1) \n\end{array} \right), \\
[15] \ni \left( \begin{array}{c}
(1, +1) \\
\left( \frac{1}{2}, 0 \right), \left( \frac{3}{2}, 0 \right) \\
(0, -1), (1, -1) \\
\left( \frac{1}{2}, -2 \right) \n\end{array} \right). \tag{44}
\]

In Tab. 8 the masses and coupling constants of the double-charm molecules are collected. Strongest binding is foreseen for the \( \Xi_{cc}(3671) \) and \( \Omega_{cc}(3761) \) states which couple dominantly to the D mesons. Those resonances are quite narrow since their decay into final states with Goldstone bosons is suppressed. The relevant coupling constants reflect the SU(4) estimates (10, 18). A further triplet of double-charm molecules with \( \Xi_{cc}(3723) \) and \( \Omega_{cc}(3863) \) couple strongly to the final states with Goldstone bosons. The isospin-doublet state is broad due to its large coupling constant to the open \( \pi \Xi_{cc} \) channel. The weakly bound sextet of chiral excitations is identified most clearly in the \((0, 1)\) and \(\left( \frac{1}{2}, 0 \right)\) sectors: states of mass 3983 MeV and 4046 MeV are predicted. Their \((1, +1)\) partner is difficult to discriminate from the \( \bar{K} \Xi_{cc} \) threshold and therefore not included in Tab. 8. Due to their shallow binding correction terms in the interaction may or may not lead to their disappearance.

We expect the results for those resonances which couple strongly to the final states with Goldstone bosons to be more reliable than those for resonances which couple dominantly to the D mesons. For instance in the \(\left( \frac{1}{2}, 0 \right)\) sector, one might wonder about the effect of the s-wave \( D^* \Sigma_{cc}(2455) \) state, which is just about 80 MeV higher in energy than the \( D \Sigma_{cc}(2520) \) state. Heavy quark symmetry predicts great similarities, not only between the charmed \( 0^- \) and \( 1^- \) mesons, but also between the charmed baryons with \( \frac{1}{2}^+ \) and \( \frac{3}{2}^+ \). Thus a more reliable computation asks for the inclusion of additional channels as to arrive at results compatible with the constraints set by heavy-quark symmetry.

4.5 \( J^P = \frac{3}{2}^- \) resonances with charm three

We close the result section with a discussion of triple-charm baryon states. They are formed by scattering the triplet baryons with \( C = 2 \) of the anti-

| \( C = 3 \) : \((I, S)\) | state | \( M_R \) [MeV] | \( \Gamma_R \) [MeV] | \( |g_R| \) |
|---|---|---|---|---|
| \((0, 0)\) | \( \eta \Omega_{ccc} \) | 4369 | 0 | 0.02 |
| | \( D \Xi_{cc} \) | | 5.5 |
| | \( \eta' \Omega_{ccc} \) | | 0.03 |
| | \( D_1 \Omega_{ccc} \) | | 2.8 |
| | \( \eta_c \Omega_{ccc} \) | | 1.3 |

Table 9
Spectrum of \( J^P = \frac{3}{2}^- \) baryons with charm three.
triplet mesons with $C = 1$. It holds

$$\frac{1}{4} g^2 \sum_{V \in [9]} \delta^{(C=3)} C_{V} = 3 C_{[1]}^{\text{heavy}},$$

where the chiral and large-$N_c$ relations (8, 14) and (9, 15) are assumed. The decomposition (45) reflects the fact that Goldstone bosons do not interact with the SU(3) singlet $\Omega_{ccc}$ at leading order in the chiral expansion. In Tab. 9 we predict the properties of the bound singlet state at mass 4369 MeV. This analogous to the s-wave spectrum observed in [22].

5 Summary

We have performed a coupled-channel study of d-wave baryon resonances with charm $-1, 0, 1, 2, 3, 4$. A rich spectrum is predicted in terms of a t-channel force defined by the exchange of light vector mesons. All relevant coupling constants are obtained from chiral and large-$N_c$ properties of QCD. The results of this work should be taken cautiously since it remains to study the effect of additional terms in the interaction kernel. Moreover, in order to restore the heavy-quark symmetry additional channels involving $1^- \text{ and } \frac{1}{2}^+$ states have to be incorporated.

Two distinct types of states are dynamically generated. A spectrum of chiral excitations is formed due the interaction of the Goldstone bosons with the baryon ground states [18,21]. Additional states are due to the coupled-channel dynamics of the D mesons. Those states decouple from the spectrum of chiral excitations to a large extent. The mixing of the two spectra requires charm-exchange interactions that are suppressed naturally. We estimated the poorly known strength of the charm-exchange by making a SU(4) ansatz for three-point vertices.

Coupled-channel dynamics driven by the D mesons lead to a strongly bound 15-plet of $C = -1$ states and a narrow crypto-exotic octet of charm-zero states. In the $C = +1$ and $C = +2$ sector a sextet and triplet of narrow resonances is formed due to the interaction of D mesons with the baryon decuplet and sextet respectively. A singlet triple-charm state is foreseen below 4.5 GeV. We do not have a good candidate for the possible $C = -1$ state of the H1 collaboration [40]. Though we predict attraction in the isospin triplet and strangeness zero sector, our state below 2.9 GeV is too strongly bound. Most spectacular is the prediction of narrow crypto-exotic baryons with charm zero forming below 4 GeV. Such states contain a $c\bar{c}$ pair. Their widths parameters are small due to the OZI rule, like it is the case for the $J/\Psi$ meson. The narrow nucleon
resonance close to 3.5 GeV may be a natural candidate for the exotic signal claimed in [49].

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6 Appendix A
| $(J, S)$ | $(a, b)$ | $V$ | $C_{V,ab}^{(l, m, \pi)}$ | charm minus one |
|--------|--------|-----|------------------|----------------|
| $(1, +0)$ | $(1, 1)$ | $\rho$ | $\pm g_{\omega 0} h_{33}^0$ | $8 g_{\omega 0} h_{33}^1 + 4 g_{\omega 0} h_{33}^2 + \pm g_{\omega 0} h_{33}^0 + g_{\omega 0} h_{33}^3$ |
| | | $\omega$ | $4 g_{\omega 0} h_{33}^1 + 4 g_{\omega 0} h_{33}^2$ | $g_{\omega 0} h_{33}^3 + 2 g_{\omega 0} h_{33}^3$ |
| | | $\phi$ | | $2 g_{\omega 0} h_{33}^3$ |
| | | $J/\Psi$ | | |
| $(2, +0)$ | $(1, 1)$ | $\rho$ | $-g_{\omega 0} h_{33}^0$ | $8 g_{\omega 0} h_{33}^1 + 4 g_{\omega 0} h_{33}^2 + g_{\omega 0} h_{33}^0 + g_{\omega 0} h_{33}^3$ |
| | | $\omega$ | $4 g_{\omega 0} h_{33}^1 + 2 g_{\omega 0} h_{33}^3$ | $g_{\omega 0} h_{33}^0 + 2 g_{\omega 0} h_{33}^0$ |
| | | $\phi$ | | $2 g_{\omega 0} h_{33}^3$ |
| | | $J/\Psi$ | | |
| $(3/2, -1)$ | $(1, 1)$ | $\omega$ | $\pm g_{\omega 0} h_{33}^0$ | $8 g_{\omega 0} h_{33}^1 + 4 g_{\omega 0} h_{33}^2 + 4 g_{\omega 0} h_{33}^2 + 2 h_{\omega 0} h_{33}^3$ |
| | | $\phi$ | $4 g_{\omega 0} h_{33}^1 + 2 g_{\omega 0} h_{33}^3$ | $-\sqrt{2} g_{\omega 0} h_{33}^3$ |
| | | $J/\Psi$ | | |
| | | $K$ | | $-\sqrt{2} g_{\omega 0} h_{33}^3$ |
| $(2, -2)$ | $(1, 2)$ | $\rho$ | $g_{\omega 0} h_{33}^0$ | $8 g_{\omega 0} h_{33}^1 + \pm g_{\omega 0} h_{33}^1 + 2 g_{\omega 0} h_{33}^3 + 2 h_{\omega 0} h_{33}^3$ |
| | | $\omega$ | $4 g_{\omega 0} h_{33}^1 + 2 g_{\omega 0} h_{33}^3$ | $\pm g_{\omega 0} h_{33}^1 + 2 h_{\omega 0} h_{33}^3$ |
| | | $\phi$ | | $2 g_{\omega 0} h_{33}^3$ |
| | | $J/\Psi$ | | |
| $(0, -2)$ | $(1, 1)$ | $\rho$ | $g_{\omega 0} h_{33}^0$ | $8 g_{\omega 0} h_{33}^1 + \pm g_{\omega 0} h_{33}^1 + 2 g_{\omega 0} h_{33}^3 + 4 h_{\omega 0} h_{33}^3$ |
| | | $\omega$ | $4 g_{\omega 0} h_{33}^1 + 2 g_{\omega 0} h_{33}^3$ | $\pm g_{\omega 0} h_{33}^1 + 4 h_{\omega 0} h_{33}^3$ |
| | | $\phi$ | | $2 g_{\omega 0} h_{33}^3$ |
| | | $J/\Psi$ | | |
| $(1, -2)$ | $(1, 2)$ | $\omega$ | $\pm g_{\omega 0} h_{33}^1 + 4 g_{\omega 0} h_{33}^2 + 2 g_{\omega 0} h_{33}^3 + 4 h_{\omega 0} h_{33}^3$ |
| | | | $4 g_{\omega 0} h_{33}^1 + 2 g_{\omega 0} h_{33}^3$ |
| | | $J/\Psi$ | | |
| | | $K$ | | $\pm g_{\omega 0} h_{33}^1$ |
| | | | $2 g_{\omega 0} h_{33}^3$ |
| $(2, -3)$ | $(1, 1)$ | $\omega$ | $\pm g_{\omega 0} h_{33}^1 + 4 g_{\omega 0} h_{33}^2 + 2 g_{\omega 0} h_{33}^3 + 4 h_{\omega 0} h_{33}^3$ |
| | | | $4 g_{\omega 0} h_{33}^1 + 2 g_{\omega 0} h_{33}^3$ |
| | | $J/\Psi$ | | |
| | | $K$ | | $\pm g_{\omega 0} h_{33}^1$ |
| | | | $2 g_{\omega 0} h_{33}^3$ |
| | | | $-\sqrt{2} g_{\omega 0} h_{33}^3$ |
| $(0, -4)$ | $(1, 1)$ | $\omega$ | $\pm g_{\omega 0} h_{33}^1 + 4 g_{\omega 0} h_{33}^2 + 2 g_{\omega 0} h_{33}^3 + 4 h_{\omega 0} h_{33}^3$ |
| | | | $4 g_{\omega 0} h_{33}^1 + 2 g_{\omega 0} h_{33}^3$ |
| | | $J/\Psi$ | | |

Table 10
The coupled-channel structure of the t-channel exchange in (21).
| $(I, S)$ | $(a, b)$ | $V$ | $C^{(1, S, 0)}_{V,a,b}$ |
|-----------|-----------|------|----------------------|
| $(1, +1)$ | $(1, 1)$  | $\rho$ | $\frac{10}{3}$ $g_{90}^0 h_{99}^0$ |
|           |           | $\omega$ | $-4 g_{90}^0 h_{99}^0 - 2 g_{90}^0 h_{99}^0$ |
|           |           | $\phi$  | $4 g_{90}^0 h_{99}^0$ |
| $(2, +1)$ | $(1, 1)$  | $\rho$ | $-2 g_{90}^0 h_{99}^0$ |
|           |           | $\omega$ | $-4 g_{90}^0 h_{99}^0 - 2 g_{90}^0 h_{99}^0$ |
|           |           | $\phi$  | $4 g_{90}^0 h_{99}^0$ |
| $(\frac{1}{2}, +0)$ | $(1, 1)$ | $\rho$ | $\frac{10}{3}$ $g_{90}^0 h_{99}^0$ |
|           |           | $\omega$ | $-4 g_{90}^0 h_{99}^0 - 2 g_{90}^0 h_{99}^0$ |
|           |           | $\phi$  | $4 g_{90}^0 h_{99}^0$ |
|           | $(1, 2)$  | $K$    | $\sqrt{\frac{3}{2}} g_{90}^0 h_{99}^0$ |
| $(2, 2)$  | $\omega$ | $-4 g_{90}^0 h_{99}^0 + \frac{3}{2} g_{90}^0 h_{99}^0$ |
|           | $\phi$   | $\sqrt{\frac{3}{2}} g_{96}^0 h_{39}^0 + \sqrt{\frac{3}{2}} g_{96}^0 h_{39}^0$ |
| $(1, 3)$  | $D^*$    | $2 g_{96}^0 h_{39}^3$ |
|           | $(2, 3)$  | $D^*_1$ | $8 g_{96}^0 h_{33}^1 + 4 g_{96}^0 h_{33}^3 + 2 g_{96}^0 h_{33}^3 + y_{96}^0 h_{33}^0$ |
|           | $(3, 3)$  | $\rho$ | $4 g_{96}^0 h_{33}^3 + 2 g_{96}^0 h_{33}^3 + y_{96}^0 h_{33}^0$ |
|           |           | $\omega$ | $2 g_{96}^0 h_{33}^3$ |
|           |           | $\phi$  | $2 g_{96}^0 h_{33}^3$ |
| $(\frac{1}{2}, +0)$ | $(1, 1)$ | $\rho$ | $\frac{10}{3}$ $g_{90}^0 h_{99}^0$ |
|           |           | $\omega$ | $-4 g_{90}^0 h_{99}^0 - 2 g_{90}^0 h_{99}^0$ |
|           |           | $\phi$  | $4 g_{90}^0 h_{99}^0$ |
|           | $(1, 3)$  | $K$    | $\sqrt{\frac{3}{2}} g_{90}^0 h_{99}^0$ |
|           | $(2, 3)$  | $K$    | $\sqrt{\frac{3}{2}} g_{96}^0 h_{39}^0$ |
| $(3, 3)$  | $\rho$ | $-\frac{3}{2} g_{90}^0 h_{99}^0$ |
|           | $\omega$ | $-4 g_{90}^0 h_{99}^0 - 2 g_{90}^0 h_{99}^0$ |
|           | $\phi$   | $4 g_{90}^0 h_{99}^0 + \frac{3}{2} g_{90}^0 h_{99}^0$ |
| $(1, 6)$  | $D^*$    | $\sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3 + \sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3$ |
|           | $(2, 6)$  | $D^*$    | $\sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3 + \sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3$ |
|           | $(3, 6)$  | $D^*_1$ | $\sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3 + \sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3$ |
|           | $(4, 6)$  | $D^*$    | $\sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3 + \sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3$ |
|           | $(5, 6)$  | $D^*$    | $-2 g_{96}^0 h_{39}^3 - 2 g_{96}^0 h_{39}^3$ |
|           | $(6, 6)$  | $\rho$ | $-g_{96}^0 h_{33}^3$ |
|           |           | $\omega$ | $8 g_{96}^0 h_{33}^1 + 4 g_{96}^0 h_{33}^3 + 2 g_{96}^0 h_{33}^3 + y_{96}^0 h_{33}^0$ |
|           |           | $\phi$  | $4 g_{96}^0 h_{33}^3 + 2 g_{96}^0 h_{33}^3 + y_{96}^0 h_{33}^0$ |
|           |           | $J/\Psi$ | $2 g_{96}^0 h_{33}^3$ |
| $(\frac{1}{2}, +0)$ | $(1, 1)$ | $\rho$ | $-4 g_{90}^0 h_{99}^0$ |
| $(0, -1)$ | $(1, 1)$  | $\rho$ | $\frac{10}{3}$ $g_{90}^0 h_{99}^0$ |
|           |           | $\omega$ | $-4 g_{90}^0 h_{99}^0 - 2 g_{90}^0 h_{99}^0$ |
|           |           | $\phi$  | $4 g_{90}^0 h_{99}^0$ |
|           | $(1, 2)$  | $K$    | $\sqrt{\frac{3}{2}} g_{90}^0 h_{99}^0$ |
| $(2, 2)$  | $\omega$ | $-4 g_{90}^0 h_{99}^0 - 2 g_{90}^0 h_{99}^0$ |
|           | $\phi$   | $4 g_{90}^0 h_{99}^0 + \frac{3}{2} g_{90}^0 h_{99}^0$ |
| $(1, 3)$  | $D^*$    | $\sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3 + \sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3$ |
|           | $(2, 3)$  | $D^*_1$ | $\sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3 + \sqrt{\frac{3}{2}} g_{96}^0 h_{39}^3$ |
|           | $(3, 3)$  | $\rho$ | $8 g_{96}^0 h_{33}^1 + 2 g_{96}^0 h_{33}^3 + y_{96}^0 h_{33}^0$ |
|           |           | $\omega$ | $4 g_{96}^0 h_{33}^3 + 2 g_{96}^0 h_{33}^3 + y_{96}^0 h_{33}^0$ |
|           |           | $\phi$  | $2 g_{96}^0 h_{33}^3$ |

Table 11
The coupled-channel structure of the t-channel exchange in (21).
Table 12
The coupled-channel structure of the t-channel exchange in (21).
| \((J, S)\) | \((a, b)\) | \(V\) | \(C^{(J, S, 0)}\) | charm zero |
|---|---|---|---|---|
| (2, 7) | \(D^*\) | \(\sqrt{\frac{1}{2}} g_{100} h_{33} + \sqrt{\frac{1}{2}} g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (3, 7) | \(D^*\) | \(- \sqrt{\frac{1}{2}} g_{100} h_{33} - \sqrt{\frac{1}{2}} g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (5, 7) | \(D^*\) | \(4 g_{100} h_{33} + 4 g_{100} h_{33} - \frac{1}{2} g_{100} h_{33} - \frac{1}{2} g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (6, 7) | \(D^*\) | \(- \sqrt{\frac{1}{2}} g_{100} h_{33} - \sqrt{\frac{1}{2}} g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (7, 7) | \(\omega\) \(\phi\) | \(J/\Psi\) | \(8 g_{100} h_{33} + 2 g_{100} h_{33} + 4 g_{100} h_{33} + g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (1, 8) | \(D^*\) | \(g_{100} h_{33} + g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (3, 8) | \(D^*\) | \(g_{100} h_{33} + g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (4, 8) | \(D^*\) | \(\sqrt{\frac{1}{2}} g_{100} h_{33} + \sqrt{\frac{1}{2}} g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (5, 8) | \(D^*\) | \(\sqrt{\frac{1}{2}} g_{100} h_{33} - \sqrt{\frac{1}{2}} g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (6, 8) | \(K\) | \(\sqrt{\frac{1}{2}} g_{100} h_{33} - \sqrt{\frac{1}{2}} g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |
| (8, 8) | \(\omega\) \(\phi\) | \(J/\Psi\) | \(4 g_{100} h_{33} + g_{100} h_{33} + g_{100} h_{33} + g_{100} h_{33}\) | \(2 g_{100} h_{33}\) |

\[\begin{align*}
(3, -2) & \quad (1, 1) \quad \rho \quad - \frac{1}{2} g_{100} h_{99} \\
(1, 2) & \quad K \quad \frac{1}{2} g_{100} h_{99} \\
(2, 2) & \quad \rho \quad - \frac{1}{2} g_{100} h_{99} \\
(2, 1) & \quad \omega \quad 4 g_{100} h_{99} + \frac{1}{2} g_{100} h_{99} \\
\phi & \quad - 4 g_{100} h_{99} - \frac{1}{2} g_{100} h_{99} \\
(0, -3) & \quad (1, 1) \quad \rho \quad 2 g_{100} h_{99} \\
(1, 2) & \quad \omega \quad 4 g_{100} h_{99} + \frac{1}{2} g_{100} h_{99} \\
\phi & \quad - 4 g_{100} h_{99} - \frac{1}{2} g_{100} h_{99} \\
(1, 5) & \quad \rho \quad - 4 g_{100} h_{99} \\
(2, 5) & \quad D^* \quad \sqrt{\frac{1}{2}} g_{100} h_{33} + \sqrt{\frac{1}{2}} g_{100} h_{33} \\
(3, 5) & \quad \rho \quad - \sqrt{\frac{1}{2}} g_{100} h_{33} - \sqrt{\frac{1}{2}} g_{100} h_{33} \\
(4, 5) & \quad \omega \quad 8 g_{100} h_{33} + 4 g_{100} h_{33} \\
\phi & \quad J/\Psi \quad 4 g_{100} h_{33} + 4 g_{100} h_{33} \\
(1, -3) & \quad (1, 2) \quad K \quad - \sqrt{\frac{1}{2}} g_{100} h_{99} \\
(2, 2) & \quad \rho \quad - \frac{1}{2} g_{100} h_{99} \\
(2, 1) & \quad \omega \quad 4 g_{100} h_{99} + \frac{1}{2} g_{100} h_{99} \\
\phi & \quad - 4 g_{100} h_{99} - \frac{1}{2} g_{100} h_{99} \\
(1, 2) & \quad \rho \quad - 4 g_{100} h_{99} \\
(2, 1) & \quad \omega \quad 4 g_{100} h_{99} + \frac{1}{2} g_{100} h_{99} \\
\phi & \quad - 4 g_{100} h_{99} - \frac{1}{2} g_{100} h_{99} \\
(1, 1) & \quad \omega \quad 4 g_{100} h_{99} \\
\phi & \quad - 4 g_{100} h_{99} - 4 g_{100} h_{99} \\
\end{align*}\]

Table 13

The coupled-channel structure of the t-channel exchange in (21).
| (1, S) | (a, b) | V | \( \mathcal{C}^{(1.S.1)}_{V,ab} \) | charm one |
| --- | --- | --- | --- | --- |
| \( \frac{1}{2}^-, +1 \) | (1, 1) | \( \rho \) | \(-4 g_{66}^0 h_{99}^0 \) |
| | | \( \omega \) | \(-4 g_{66}^0 h_{99}^0 - 2 g_{66}^0 h_{99}^0 \) |
| | | \( \phi \) | \(4 g_{66}^0 h_{99}^0 \) |
| | (1, 2) | \( D^* \) | \(-2 g_{66}^0 h_{99}^0 \) |
| | | \( \omega \) | \(-4 g_{66}^0 h_{99}^0 - 2 g_{66}^0 h_{99}^0 \) |
| | | \( \phi \) | \(4 g_{66}^0 h_{99}^0 \) |
| | (2, 2) | \( \omega \) | \(-8 g_{60}^0 h_{13}^1 + 4 g_{60}^0 h_{13}^3 - 4 g_{60}^0 h_{33}^0 - 2 g_{60}^0 h_{33}^2 \) |
| | | \( \phi \) | \(-4 g_{60}^0 h_{13}^1 \) |
| | | | \(-2 g_{60}^0 h_{13}^1 \) |
| \( (0, +0) \) | (1, 1) | \( \rho \) | \(8 g_{66}^0 h_{99}^0 \) |
| | | \( K \) | \(3 g_{66}^0 h_{99}^0 \) |
| | (2, 2) | \( \rho \) | \(-4 g_{60}^0 h_{13}^1 + 2 g_{60}^0 h_{33}^2 + \frac{1}{2} g_{60}^0 h_{33}^2 \) |
| | | \( \omega \) | \(4 g_{60}^0 h_{13}^1 + 2 g_{60}^0 h_{33}^2 + \frac{1}{2} g_{60}^0 h_{33}^2 + g_{60}^0 h_{33}^2 \) |
| | | \( \phi \) | \(2 g_{60}^0 h_{13}^1 \) |
| \( (1, +0) \) | (1, 1) | \( \rho \) | \(4 g_{66}^0 h_{99}^0 \) |
| | | \( D^* \) | \(\sqrt{2} g_{63}^0 h_{33}^3 + \sqrt{2} g_{63}^0 h_{33}^3 \) |
| | | \( \omega \) | \(-8 g_{60}^0 h_{13}^1 + 4 g_{60}^0 h_{13}^3 - 2 g_{60}^0 h_{33}^0 - g_{60}^0 h_{33}^2 \) |
| | | \( \phi \) | \(-4 g_{60}^0 h_{13}^1 \) |
| | \( \omega \) | \(-2 g_{60}^0 h_{13}^1 \) |
| | | \( J/\Psi \) | \(8 g_{63}^0 h_{33}^3 + 2 g_{63}^0 h_{33}^3 + \frac{1}{2} g_{63}^0 h_{33}^3 \) |
| \( (0, -0) \) | (1, 1) | \( \rho \) | \(8 g_{66}^0 h_{99}^0 \) |
| | | \( K \) | \(3 g_{66}^0 h_{99}^0 \) |
| | (2, 2) | \( \rho \) | \(-4 g_{60}^0 h_{13}^1 + 2 g_{60}^0 h_{33}^2 + \frac{1}{2} g_{60}^0 h_{33}^2 \) |
| | | \( \omega \) | \(4 g_{60}^0 h_{13}^1 + 2 g_{60}^0 h_{33}^2 + \frac{1}{2} g_{60}^0 h_{33}^2 + g_{60}^0 h_{33}^2 \) |
| | | \( \phi \) | \(2 g_{60}^0 h_{13}^1 \) |

Table 14
The coupled-channel structure of the t-channel exchange in (21).
| (I, J) | (a, b) | V | \(C^{(I,J)}_{V,ab}\) | charm one |
|---|---|---|---|---|
| (7, 8) | \(\rho\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{0}\) |
| | \(\omega\) | | | \(-4 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(\phi\) | | | \(-\sqrt{2} g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(J/\Psi\) | | | \(-2 g_{\rho}^{0} h_{\rho}^{0}\) |
| (2, 1+) | | \(D^*\) | | \(-4 g_{\rho}^{0} h_{\rho}^{0}\) |
| | | \(D^*\) | | \(-\sqrt{2} g_{\rho}^{0} h_{\rho}^{0}\) |
| | | \(D^*\) | | \(-\sqrt{2} g_{\rho}^{0} h_{\rho}^{0}\) |
| (7, 8) | | | | \(-4 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(\rho\) | | | \(-2 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(\omega\) | | | \(-2 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(\phi\) | | | \(-2 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(J/\Psi\) | | | \(-2 g_{\rho}^{0} h_{\rho}^{0}\) |
| (1, 1) | \(K\) | | | \(4 g_{\rho}^{1} h_{\rho}^{0}\) |
| | \(K\) | | | \(-\sqrt{2} g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(K\) | | | \(-\sqrt{2} g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(K\) | | | \(-\sqrt{2} g_{\rho}^{0} h_{\rho}^{0}\) |
| (2, 3) | \(K\) | | | \(-\sqrt{2} g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(K\) | | | \(-\sqrt{2} g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(K\) | | | \(-\sqrt{2} g_{\rho}^{0} h_{\rho}^{0}\) |
| (2, 4) | \(D^*\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(D^*\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(D^*\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| (4, 4) | \(\rho\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(\omega\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(\phi\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(J/\Psi\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| (1, 5) | \(K\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| (3, 5) | \(K\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| (5, 5) | \(\omega\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| | \(\phi\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| (3, 6) | \(D^*\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| (4, 6) | \(K\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| (5, 6) | \(D^*\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| (6, 6) | \(\omega\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| | \(\phi\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| | \(J/\Psi\) | | | \(\frac{1}{2} g_{\rho} h_{\rho}^{1}\) |
| (4, 7) | \(D^*\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| (6, 7) | \(D^*\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| (1, 8) | \(D^*\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| (3, 8) | \(D^*\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| (5, 8) | \(D^*\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| (7, 8) | \(D^*\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| (8, 8) | \(\omega\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(\phi\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |
| | \(J/\Psi\) | | | \(-8 g_{\rho}^{0} h_{\rho}^{0}\) |

Table 15
The coupled-channel structure of the t-channel exchange in (21).

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### Table 16

The coupled-channel structure of the t-channel exchange in (21).

| $(l, s)$ | $(a, b)$ | $V$ | $C^{(l,s)}_{V_{ab}}$ | charm one |
|---|---|---|---|---|
| (3, 9) | $D^*_s$ | $-\sqrt{\frac{3}{4}} g_{943}^3 h_{39}^3 - \sqrt{\frac{3}{4}} g_{943}^3 h_{39}^3$ | $ho^3_{93} h_{39}^3$ + $\rho^3_{93} h_{39}^3$ + $g_9^3 h_{39}^3$ + $g_9^3 h_{39}^3$ |
| (3, 9) | $D^*_s$ | $-\sqrt{\frac{3}{4}} g_{943}^3 h_{39}^3 + \sqrt{\frac{3}{4}} g_{943}^3 h_{39}^3$ | $\rho^3_{93} h_{39}^3$ + $\rho^3_{93} h_{39}^3$ + $g_9^3 h_{39}^3$ + $g_9^3 h_{39}^3$ |
| (8, 9) | $K$ | $8 g_{93}^3 h_{39}^3 + 4 g_{94}^3 h_{39}^3 + g_{94}^3 h_{39}^3$ | $4 g_{93}^3 h_{39}^3 + 2 g_{93}^3 h_{39}^3$ |
| (9, 9) | $\omega$ | $2 g_{96}^3 h_{39}^3 + 2 g_{96}^3 h_{39}^3$ | $2 g_{96}^3 h_{39}^3 + 2 g_{96}^3 h_{39}^3$ |
| (9, 9) | $\phi$ | $\sqrt{\frac{3}{2}} g_{906}^3 h_{39}^3 + \sqrt{\frac{3}{2}} g_{906}^3 h_{39}^3$ | $-\frac{3}{2} g_{906}^3 h_{39}^3 - g_{906}^3 h_{39}^3$ |
| (4, 10) | $D^*_s$ | $-\sqrt{\frac{3}{2}} g_{906}^3 h_{39}^3 + \sqrt{\frac{3}{2}} g_{906}^3 h_{39}^3$ | $-\frac{3}{2} g_{906}^3 h_{39}^3 - g_{906}^3 h_{39}^3$ |
| (6, 10) | $D^*_s$ | $-\sqrt{\frac{3}{2}} g_{906}^3 h_{39}^3 + \sqrt{\frac{3}{2}} g_{906}^3 h_{39}^3$ | $-\frac{3}{2} g_{906}^3 h_{39}^3 - g_{906}^3 h_{39}^3$ |
| (6, 10) | $D^*_s$ | $-\sqrt{\frac{3}{2}} g_{906}^3 h_{39}^3 + \sqrt{\frac{3}{2}} g_{906}^3 h_{39}^3$ | $-\frac{3}{2} g_{906}^3 h_{39}^3 - g_{906}^3 h_{39}^3$ |

| $(\bar{t}^{-}, l^{-} = 1)$ | $(1, 1)$ | $\rho$ | $-2 g_{906}^0 h_{99}^0$ | $\sqrt{3} g_{906}^0 h_{99}^0$ |
| $(1, 2)$ | $K$ | $-2 g_{906}^0 h_{99}^0$ | $\sqrt{3} g_{906}^0 h_{99}^0$ |
| (2, 2) | $\omega$ | $4 g_{906}^0 h_{99}^0 + 2 g_{906}^0 h_{99}^0$ | $-4 g_{906}^0 h_{99}^0$ |
| (1, 3) | $D^*_s$ | $\sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3 + \sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3$ | $-\frac{3}{2} g_{906}^3 h_{99}^3 - g_{906}^3 h_{99}^3$ |
| (2, 3) | $D^*_s$ | $-\sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3 + \sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3$ | $-\frac{3}{2} g_{906}^3 h_{99}^3 - g_{906}^3 h_{99}^3$ |
| (3, 3) | $\omega$ | $-8 g_{906}^0 h_{93}^0 - \frac{3}{2} g_{906}^0 h_{93}^0 - 2 g_{906}^0 h_{93}^0 + \frac{3}{2} g_{906}^0 h_{93}^0$ | $-4 g_{906}^0 h_{93}^0 - \frac{3}{2} g_{906}^0 h_{93}^0 - 2 g_{906}^0 h_{93}^0 + \frac{3}{2} g_{906}^0 h_{93}^0$ |
| (0, −2) | $(1, 1)$ | $\rho$ | $3 g_{906}^0 h_{99}^0$ | $4 g_{906}^1 h_{99}^0 + h_{99}^0$ |
| $(1, 2)$ | $K$ | $-4 g_{906}^1 h_{99}^0 - 2 g_{906}^1 h_{99}^0$ | $-\sqrt{3} h_{99}^0$ |
| (1, 3) | $D^*_s$ | $-\sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3 + \sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3$ | $-\frac{3}{2} g_{906}^3 h_{99}^3 - g_{906}^3 h_{99}^3$ |
| (2, 3) | $D^*_s$ | $-\sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3 + \sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3$ | $-\frac{3}{2} g_{906}^3 h_{99}^3 - g_{906}^3 h_{99}^3$ |
| (3, 3) | $\omega$ | $-8 g_{906}^0 h_{93}^0 - \frac{3}{2} g_{906}^0 h_{93}^0 - 2 g_{906}^0 h_{93}^0 - \frac{3}{2} g_{906}^0 h_{93}^0$ | $-4 g_{906}^0 h_{93}^0 - \frac{3}{2} g_{906}^0 h_{93}^0 - 2 g_{906}^0 h_{93}^0 - \frac{3}{2} g_{906}^0 h_{93}^0$ |
| (2, 4) | $D^*_s$ | $\sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3 + \sqrt{\frac{3}{2}} g_{906}^3 h_{99}^3$ | $3 g_{906}^0 h_{99}^0$ |
| (3, 4) | $K$ | $8 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0 + 4 g_{906}^0 h_{13}^0$ | $-2 g_{906}^0 h_{93}^0$ |
| (4, 4) | $\omega$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ | $-2 g_{906}^0 h_{93}^0$ |
| (3, 5) | $D^*_s$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ |
| (4, 5) | $D^*_s$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ |
| (1, 6) | $D^*_s$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ |
| (2, 6) | $D^*_s$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ |
| (5, 6) | $D^*_s$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ | $-4 g_{906}^0 h_{93}^0 - 4 g_{906}^0 h_{93}^0$ |
| (6, 6) | $\omega$ | $8 g_{93}^3 h_{39}^3 + 4 g_{93}^3 h_{39}^3 + 4 g_{93}^3 h_{39}^3 + 2 g_{93}^3 h_{39}^3$ | $4 g_{93}^3 h_{39}^3$ |
| | | | | |
| (1, S) | (a, b) | V | $\mathcal{C}^{(1, S, 1)}_{V_{ab}}$ | charm one |
|-------|-------|---|----------------|---------|
| (0, −2) | (3, 7) | $D^*$ | $\sqrt{2} g_{66} h_{13}^3 + \sqrt{2} g_{66} h_{33}^{30}$ | |
| | (4, 7) | $D_7^*$ | $2 g_{66} h_{13}^3 + 2 g_{66} h_{33}^{30}$ | |
| | (6, 7) | $D_7^*$ | $-\sqrt{2} g_{66} h_{13}^3 - \sqrt{2} g_{66} h_{33}^{30}$ | |
| (1, −2) | (1, 2) | $K$ | $-\sqrt{2} g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | (2, 2) | $\rho$ | $\psi$ | $-2 g_{66} h_{13}^3 + 2 g_{66} h_{33}^{30}$ | |
| | (1, 3) | $D^*$ | $-\sqrt{2} g_{66} h_{13}^3 - \sqrt{2} g_{66} h_{33}^{30}$ | |
| | (2, 3) | $D_7^*$ | $-\sqrt{2} g_{66} h_{13}^3 - \sqrt{2} g_{66} h_{33}^{30}$ | |
| | (3, 3) | $\rho$ | $\psi$ | $-2 g_{66} h_{13}^3$ | |
| | (1, −3) | $\omega$ | $-4 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | (2, 2) | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, $\frac{1}{2}$) | (1, 1) | $\omega$ | $-4 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| (1, 1) | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (0, +1) | (1, 1) | $\rho$ | $-3 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, +1) | (1, 1) | $\rho$ | $-3 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, $\frac{1}{2}$) | (1, 1) | $\omega$ | $-4 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| (1, 1) | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, S) | (a, b) | V | $\mathcal{C}^{(1, S, 2)}_{V_{ab}}$ | charm two |
|-------|-------|---|----------------|---------|
| (0, +1) | (1, 1) | $\rho$ | $-3 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, +1) | (1, 1) | $\rho$ | $-3 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, $\frac{1}{2}$) | (1, 1) | $\omega$ | $-4 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| (1, 1) | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (0, +1) | (1, 1) | $\rho$ | $-3 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, +1) | (1, 1) | $\rho$ | $-3 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, $\frac{1}{2}$) | (1, 1) | $\omega$ | $-4 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| (1, 1) | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, S) | (a, b) | V | $\mathcal{C}^{(1, S, 2)}_{V_{ab}}$ | charm two |
|-------|-------|---|----------------|---------|
| (0, +1) | (1, 1) | $\rho$ | $-3 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, +1) | (1, 1) | $\rho$ | $-3 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| | $\omega$ | $-2 g_{66} h_{13}^3$ | |
| (1, $\frac{1}{2}$) | (1, 1) | $\omega$ | $-4 g_{66} h_{13}^3 - 2 g_{66} h_{33}^{30}$ | |
| (1, 1) | $\omega$ | $-2 g_{66} h_{13}^3$ | |

Table 17
The coupled-channel structure of the t-channel exchange in (21).

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| (I, S) | (a, b) | \( V \) | \( \phi \langle 1, S/2 \rangle \) | charm two |
|-------|-------|-------|----------------|---------|
| (5, 6) | \( D_0^* \) | \(-\sqrt{\mathcal{T}_{93}^{23} h_{13}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{31}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{90}^3} + \sqrt{\mathcal{T}_{93}^{23} h_{93}^3} \) | \(-8 \bar{g}_{66} h_{33}^3 - 2 \bar{g}_{66} h_{93}^1 - 4 \bar{g}_{66} h_{93}^0 - \bar{g}_{66} h_{33}^0 \) | \(-2 \bar{g}_{66} h_{33}^0 \) |
| (6, 6) | \( \omega \) | \( \omega \) | \( J/\Psi \) | \(-\sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \(-8 \bar{g}_{66} h_{13}^3 - 2 \bar{g}_{66} h_{13}^3 - 4 \bar{g}_{66} h_{13}^3 - \bar{g}_{66} h_{13}^3 \) | \(-2 \bar{g}_{66} h_{13}^3 \) |
| (1, 7) | \( D_0^* \) | \( J/\Psi \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \(-8 \bar{g}_{66} h_{33}^3 - 2 \bar{g}_{66} h_{33}^3 - 4 \bar{g}_{66} h_{33}^3 - \bar{g}_{66} h_{33}^3 \) | \(-2 \bar{g}_{66} h_{33}^3 \) |
| (2, 7) | \( D_0^* \) | \( \omega \) | \( \omega \) | \( J/\Psi \) | \( -8 \bar{g}_{66} h_{33}^3 - 4 \bar{g}_{66} h_{33}^3 - 2 \bar{g}_{66} h_{33}^3 - \bar{g}_{66} h_{33}^3 \) | \(-4 \bar{g}_{66} h_{33}^3 - 2 \bar{g}_{66} h_{33}^3 - \bar{g}_{66} h_{33}^3 \) | \(-2 \bar{g}_{66} h_{33}^3 \) |
| (3, 7) | \( D_0^* \) | \( J/\Psi \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} + \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} + \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} + \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} + \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) |
| (5, 7) | \( D_0^* \) | \( \omega \) | \( \omega \) | \( J/\Psi \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \(-8 \bar{g}_{66} h_{33}^3 - 2 \bar{g}_{66} h_{33}^3 - 4 \bar{g}_{66} h_{33}^3 - \bar{g}_{66} h_{33}^3 \) | \(-2 \bar{g}_{66} h_{33}^3 \) |
| (7, 7) | \( D_0^* \) | \( \omega \) | \( \omega \) | \( J/\Psi \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \(-8 \bar{g}_{66} h_{33}^3 - 2 \bar{g}_{66} h_{33}^3 - 4 \bar{g}_{66} h_{33}^3 - \bar{g}_{66} h_{33}^3 \) | \(-2 \bar{g}_{66} h_{33}^3 \) |
| (4, 8) | \( D_0^* \) | \( \epsilon \) | \( \epsilon \) | \( J/\Psi \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} + \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} + \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} + \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) |
| (6, 8) | \( D_0^* \) | \( \omega \) | \( \omega \) | \( J/\Psi \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \(-8 \bar{g}_{66} h_{33}^3 - 2 \bar{g}_{66} h_{33}^3 - 4 \bar{g}_{66} h_{33}^3 - \bar{g}_{66} h_{33}^3 \) | \(-2 \bar{g}_{66} h_{33}^3 \) |
| (7, 8) | \( D_0^* \) | \( \omega \) | \( \omega \) | \( J/\Psi \) | \( \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} - \sqrt{\mathcal{T}_{93}^{23} h_{33}^3} \) | \(-8 \bar{g}_{66} h_{33}^3 - 2 \bar{g}_{66} h_{33}^3 - 4 \bar{g}_{66} h_{33}^3 - \bar{g}_{66} h_{33}^3 \) | \(-2 \bar{g}_{66} h_{33}^3 \) |

Table 18
The coupled-channel structure of the t-channel exchange in (21).

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| $(I, S)$ | $(a, b)$ | $V$ | $C(V, ab)$ | charm two |
|---------|---------|-----|------------|-----------|
| $(1, -1)$ | $(1, 2)$ | $K$ | $\frac{2}{\sqrt{2}} g_{13}^3 h_{99}^0$ | |
| $(2, 2)$ | $\rho$ | $\omega$ | $\phi$ | $\frac{1}{\sqrt{2}} g_{13}^3 h_{99}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{99}^0$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{99}^0 - 2 g_{13}^3 h_{99}^0$ |
| $(1, 3)$ | $D^*$ | $\sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0 - \sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0$ | $\sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0 - \sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + 2 g_{13}^3 h_{33}^0 - \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-2 g_{13}^3 h_{33}^0$ |
| $(2, 3)$ | $D^*_{1}$ | $\omega$ | $\phi$ | $-g_{13}^3 h_{33}^0 - g_{13}^3 h_{33}^0$ | $-g_{13}^3 h_{33}^0 - g_{13}^3 h_{33}^0$ |
| $(3, 3)$ | $\rho$ | $\omega$ | $\phi$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + 2 g_{13}^3 h_{33}^0 - \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-2 g_{13}^3 h_{33}^0$ |
| $(1, 1)$ | $\omega$ | $\phi$ | $-4 g_{11}^3 h_{99}^0$ | $4 g_{11}^3 h_{99}^0$ |
| $(1, 2)$ | $D^*$ | $\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ |
| $(2, 2)$ | $\omega$ | $\phi$ | $-4 g_{11}^3 h_{33}^0 + 2 g_{13}^3 h_{33}^0 - 4 g_{11}^3 h_{33}^0 + g_{13}^3 h_{33}^0$ | $-2 g_{11}^3 h_{33}^0$ |
| $(1, 2)$ | $\omega$ | $\phi$ | $-4 g_{13}^3 h_{33}^0 + 2 g_{13}^3 h_{33}^0 - 4 g_{13}^3 h_{33}^0 + g_{13}^3 h_{33}^0$ | $-2 g_{11}^3 h_{33}^0$ |
| $(1, 3)$ | $D^*$ | $\sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0 - \sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-2 g_{13}^3 h_{33}^0 + g_{13}^3 h_{33}^0$ |
| $(2, 3)$ | $D^*_{1}$ | $\omega$ | $\phi$ | $-g_{13}^3 h_{33}^0 - g_{13}^3 h_{33}^0$ | $-g_{13}^3 h_{33}^0 - g_{13}^3 h_{33}^0$ |
| $(3, 3)$ | $\rho$ | $\omega$ | $\phi$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + 2 g_{13}^3 h_{33}^0 - \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-2 g_{13}^3 h_{33}^0$ |
| $(1, 1)$ | $\omega$ | $\phi$ | $-4 g_{13}^3 h_{99}^0$ | $g_{13}^3 h_{39}^3 + g_{13}^3 h_{93}^3$ |
| $(1, 2)$ | $D^*$ | $\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ |
| $(2, 2)$ | $\omega$ | $\phi$ | $-4 g_{13}^3 h_{33}^0 + 2 g_{13}^3 h_{33}^0 - 4 g_{13}^3 h_{33}^0 + g_{13}^3 h_{33}^0$ | $-2 g_{13}^3 h_{33}^0$ |
| $(1, 3)$ | $D^*$ | $\sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0 - \sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-2 g_{13}^3 h_{33}^0 + g_{13}^3 h_{33}^0$ |
| $(2, 3)$ | $D^*_{1}$ | $\omega$ | $\phi$ | $-g_{13}^3 h_{33}^0 - g_{13}^3 h_{33}^0$ | $-g_{13}^3 h_{33}^0 - g_{13}^3 h_{33}^0$ |
| $(3, 3)$ | $\rho$ | $\omega$ | $\phi$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + 2 g_{13}^3 h_{33}^0 - \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-2 g_{13}^3 h_{33}^0$ |
| $(1, 1)$ | $\omega$ | $\phi$ | $-4 g_{13}^3 h_{99}^0$ | $g_{13}^3 h_{39}^3 + g_{13}^3 h_{93}^3$ |
| $(1, 2)$ | $D^*$ | $\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ |
| $(2, 2)$ | $\omega$ | $\phi$ | $-4 g_{13}^3 h_{33}^0 + 2 g_{13}^3 h_{33}^0 - 4 g_{13}^3 h_{33}^0 + g_{13}^3 h_{33}^0$ | $-2 g_{13}^3 h_{33}^0$ |
| $(1, 3)$ | $D^*$ | $\sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0 - \sqrt{\frac{2}{2}} g_{13}^3 h_{33}^0$ | $-\frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0 + \frac{1}{\sqrt{2}} g_{13}^3 h_{33}^0$ | $-2 g_{13}^3 h_{33}^0 + g_{13}^3 h_{33}^0$ |
| $(2, 3)$ | $D^*_{1}$ | $\omega$ | $\phi$ | $-g_{13}^3 h_{33}^0 - g_{13}^3 h_{33}^0$ | $-g_{13}^3 h_{33}^0 - g_{13}^3 h_{33}^0$ |

Table 19

The coupled-channel structure of the t-channel exchange in (21).
| $(I_1, S_1)$ | $(a, b)$ | $V$ | $C^{(I_1, S_1)}_{V_{ab}}$ | \text{charm three} |
|-----------|-----------|----------|-----------------|---|
| $(\frac{1}{2}, -1)$ | (1, 1) | $\omega$ | 4 $g_{11}^1 h_{33}^0$ |  |
| | | $\phi$ | $-4 g_{11}^1 h_{33}^0$ |  |
| | (1, 2) | $D_s^*$ | $9_{31}^3 h_{39}^3 + 9_{31}^3 h_{93}^3$ |  |
| | (2, 2) | $\omega$ | $-8 g_{33}^1 h_{33}^0 - 4 g_{33}^0 h_{33}^1 - 2 g_{33}^1 h_{33}^0 - 2 g_{33}^1 h_{33}^0$ |  |
| | | $\phi$ | $-4 g_{33}^1 h_{33}^0 - 2 g_{33}^1 h_{33}^0$ |  |
| | | $J/\Psi$ | $-2 g_{33}^0 h_{33}^0$ |  |

| $(I_2, S_2)$ | $(a, b)$ | $V$ | $C^{(I_2, S_2)}_{V_{ab}}$ | \text{charm four} |
|-----------|-----------|----------|-----------------|---|
| $(0, +1)$ | (1, 1) | $\omega$ | $-8 g_{11}^1 h_{33}^0 - 4 g_{11}^1 h_{33}^0$ |  |
| | | $\phi$ | $-4 g_{11}^1 h_{33}^1$ |  |
| | | $J/\Psi$ | $-2 g_{11}^1 h_{33}^0$ |  |
| $(\frac{1}{2}, +0)$ | (1, 1) | $\omega$ | $-8 g_{11}^1 h_{33}^0 - 2 g_{11}^1 h_{33}^0$ |  |
| | | $\phi$ | $-4 g_{11}^1 h_{33}^1 - 2 g_{11}^1 h_{33}^0$ |  |
| | | $J/\Psi$ | $-2 g_{11}^1 h_{33}^0$ |  |

Table 20

The coupled-channel structure of the t-channel exchange in (21).
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