On making predictions in a multiverse: conundrums, dangers, and coincidences

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Abstract. The notion that there are many “universes” with different properties is one answer to the question of “why is the universe so hospitable to life?” This notion also naturally follows from current ideas in eternal inflation and string/M theory. But how do we test such a “multiverse” theory: which of the many universes do we compare to ours? This paper enumerates would would seem to be essential ingredients for making testable predictions, outlines different strategies one might take within this framework, then discusses some of the difficulties and dangers inherent in these approaches. Finally, I address the issue of whether there may be some general, qualitative predictions that multiverse theories might share.

1. Introduction

The standard model of particle physics and the standard model of cosmology are both rife with numerical parameters that must have values fixed by hand to explain the observed world. The world would be a radically different place if some of these constants took a different value. In particular, it has been argued that if any one of six (or perhaps a few more) numbers did not have rather particular values, then life as we know it would not be possible: atoms would not exist, or no gravitationally bound structures would form in the universe, or some other calamity would occur that would appear to make the (alter)-universe a very dull and lifeless place. How, then, did we get so lucky as to be here?

This question is an interesting one because all of the possible answers to it that I have encountered or devised entail very interesting conclusions. An essentially exhaustive list of such answers is:

(i) We just got very lucky: all of the numbers could have been very different, in which case the universe would have been barren – but they just happened by pure chance to take values in the tiny part of parameter space that would allow life. We owe our existence to one very, very, very lucky roll of the dice.†

† If the parameters were explained – by some deeper theory – in terms of fewer or no parameters, this does not change much: it would explain the origin of the parameters, but their hospitality to life would still be dumb luck.
(ii) We weren’t particularly lucky: almost any set of parameters would have been fine, because life would find a way to arise in nearly any type of universe. This is quite interesting because it implies (at least theoretically) the existence of life forms radically different from our own, existing for example in universes with no atoms or with no bound structure, or overrun with black holes, etc.

(iii) The universe was specifically designed for life. The choice of constants only happened once, but their values were determined in some way by the need for us to arise. This might be divine agency, or some radical form of Wheeler’s “self-creating universe”, or super-advanced beings that travel back in time to set the constants at the beginning of the universe, etc. However the reader feels about this possibility, they must admit that it would be interesting if true.

(iv) We did not have to get lucky, because there are many universes with different sets of constants – i.e., the dice were rolled many, many times. We are necessarily in one of the universes that allows life, just as we necessarily reside on a planet that supports life, even when most others may not. This is interesting because it means that there are other very different universes coexisting with ours in a “multiverse”.

These four answers – luck, *elan vital*, design, and multiverse – will appeal at different levels to different readers. But I think it is hard to argue that the multiverse is necessarily less reasonable than the alternatives. Moreover, as is discussed at length elsewhere in this volume, there are quite independent reasons to believe, on the basis of inflation, quantum cosmology, and string/M theory, that there might quite naturally be many regions larger than our observable universe, governed by different sets of low-energy physics. I am not aware of any independent scientific argument for the other three possible explanations.

Whether they are contemplated as an answer to the “why are we lucky” question, or because they are forced upon us from other considerations, multiverses come at a high price. Even if we have in hand a physical theory and cosmological model that lead to a multiverse, how do we test it? If there are many sets of constants, which ones do we compare to those we observe? In the next section of this chapter I will outline what I think a sound prediction in a multiverse would look like. As will become clear, this requires many ingredients, and there are some quite serious difficulties in generating some of these ingredients, even with a full theory in hand. For this reason, many shortcuts have been devised to try to make predictions more easily. In the third section I will describe a number of these, and show the cost that this convenience entails. Finally, in Section 4 I will focus on the interesting question of whether the anthropic approach to cosmology might lead to any general conclusions about how the study of cosmology will look in coming years.

2. Making predictions in a multiverse

Imagine that we have a candidate physical theory and set of cosmological boundary conditions (hereafter denoted $\mathcal{T}$) that predicts an ensemble of physically realized
systems, each of which is approximately homogeneous in some coordinates and can be characterized by a set of parameters (i.e. the constants appearing in the standard models of particle physics and cosmology; I assume here that the laws of physics themselves retain the same form). Let us denote each such system a “universe” and the ensemble a “multiverse”. Given that we can observe only one of these universes, what conclusions can we draw regarding the correctness of $\mathcal{T}$, and how?

One possibility would be if there were a parameter for which none of the universes in the ensemble had the value we observe. In this case $\mathcal{T}$ would be ruled out. (Note that any $\mathcal{T}$ in which at least one parameter has a range of values that it does not take in any universe is thus rigorously falsifiable, which is a nice thing for a theory to be). Or perhaps some parameter takes only one value in all universes, and this value matches the observed one. This would obviously be a significant accomplishment of the theory. Both possibilities are good as far as they go, and seem completely uncontroversial. But they do not go far enough. What if our observed parameter values appear in some but not all of the universes? Could we still rule out the theory if those values are incredibly rare, or gain confidence if they are extremely common?

I find it hard to see why not. If some theory predicts outcome A of some experiment with $p = 0.99999999$ probability, and outcome B with probability $1 - p$, I think we would be reluctant to accept the theory if a single experiment were performed and showed outcome B, even if we did not get to repeat the experiment. In fact, it seems consistent with all normal scientific methodology to rule out the theory at 99.999999% confidence – the problem is just that without repeating our measurements we will not be able to increase this confidence. This seems to be exactly analogous to the multiverse if we can compute, given our $\mathcal{T}$, the probability that we should observe a given value for some observable.

Can we compute this probability distribution in a multiverse? Perhaps. I will argue that to do so in a sensible way, we would need seven successive ingredients.

(i) First, of course, we require a multiverse: an ensemble of regions, each of which would be considered a universe to observers inside it (i.e. its properties would be uniform for as far as those observers could see), but each of which may have different properties.

(ii) Next we need to isolate the set of parameters characterizing the different universes. This might be the set of 20-odd free parameters in the standard model of particle physics (see, e.g., Ref. [2] and references therein), plus a dozen or so cosmological parameters \[3, 4\]. There might be additional parameters that become important in other universes, or differences (such as different forms of the physical laws) that cannot be characterized by differences in a finite set of parameters. But for simplicity let us assume that some set of $N$ numbers $\alpha_i$ (where $i = 1..N$) fully specify each universe.

(iii) Given our parameters, we need some measure with which to calculate the multi-dimensional probability distribution $P(\alpha_i)$ for the parameters. We might, for
example, “count each universe equally” to obtain the probability $P_U(\alpha_i)$, defined to be the chance that a randomly chosen universe from the ensemble would have the parameter values $\alpha_i$. This can be a bit tricky, however, because it depends on how we delineate the universes: suppose that $\alpha_1 = a$ universes happen to be $10^{10}$ times larger than $\alpha_1 = b$ universes. What would then prevent us from “splitting” each $\alpha_1 = a$ universe into 10, or 100, or $10^{10}$ universes, thus radically changing the relative probability of $\alpha_1 = a$ vs. $\alpha_1 = b$? These considerations might lead us to take a different measure such as volume, e.g. to define $P_V(\alpha_i)$, the chance that a randomly chosen universe would reside in a universe with parameter values $\alpha_i$. But in an expanding universe volume increases, so this would depend on the time at which we choose to evaluate the volume in each universe. We might then consider some “counting” object that endures, say a baryon (which is relatively stable), and define $P_B(\alpha_i)$, the chance that a randomly chosen baryon would reside in a universe with parameter values $\alpha_i$. But now we have excluded from consideration universes with no baryons. Do we want to do that? This will be addressed in step (v). For now, note only that it is not entirely clear, even in principle, which measure we should place over our multiverse. We can call this the “measure problem.”

(iv) Once we choose a measure object $M$, we still need to actually compute $P_M(\alpha_i)$, and this may be far from easy. For example, in computing $P_V$, some universes may have infinite volume. In this case values of $\alpha_i$ leading to universes with finite volume will have zero probability. How, though, do we compare two infinite volumes? The difficulty can be seen by considering how we would count the fraction of red vs. blue marbles in an infinite box. We could pick one red, then one blue, and find a 50-50 split. But we could also repeatedly pick 1 red, then 2 blue, or 5 red, then 1 blue. We could do this forever and so obtain any ratio we like. What we would like to do is just “grab a bunch of marbles at random” and count the ratio. But in the multiverse case it is not so clear how to perform this random ordering of marbles to pick. This difficulty, which might be termed the “ordering problem” [4], has been discussed a number of times in the context of eternal inflation [5, 6, 7, 8] and a number of plausible prescriptions have been proposed. But there does not seem to be any generic solution, or convincing way to prove that one method is correct.

(v) If we have managed to calculate $P_M(\alpha_i)$, do we have a prediction? Sort of. We have an answer to the question: “given that I am (or can associate myself with) a randomly chosen $M$-object, which sort of universe am I in?” But this is not necessarily the same as the more general question: “what sort of universe am I in?” First of all, different $M$-objects will generally give different probabilities, and they cannot all be the answer to the same question. Second, we may not be all that closely associated with our $M$-object (which was chosen mainly to provide some way to compute probabilities) because it does not take into account important

‡ Note that this is really shorthand for $\frac{dP}{d\alpha_1 \ldots d\alpha_N} d\alpha_1 \ldots d\alpha_N$, the probability that $\alpha_i$ are all within the interval $[\alpha_i, \alpha_i + d\alpha_i]$, where $P(\alpha_i)$ is a cumulative probability distribution.
requirements for our existence. For example, if $M$ were volume, I would be asking what I should observe given that I am at a random point in space; but we are not at a random point in space (which would on average have a density of $10^{-29}$ g/cc), but rather at one of the very rare points with density $\sim 1$ g/cc. The reason for this improbable situation is obviously “anthropic” – we just do not worry about it because we can observe many other regions at the proper density (if we could not see such regions, we might be more reluctant to accept a cosmological model with such a low average density.) Finally, it might be argued that the question we have answered through our calculation is not nearly as specific a question as we could ask, because we know a lot more about the universe than that it contains volume, or baryons. We might, instead, ask “given that I am in a universe with the properties we have already observed, what should I observe in the future?”

As discussed at length in [9], these different specific questions can be usefully thought of as arising from different choices of conditionalization. The probabilities $P_M(\alpha_i)$ are conditioned on as little as possible, whereas the anthropic question of “given that I am a randomly chosen observer, which should I measure” specifies probabilities conditioned on the existence of an “observer”, while the approach of “given what I know now, what will I see” specifies probabilities conditioned on being in a universe with all of the properties that we have already observed. These are three genuinely different approaches to making predictions in a multiverse that may be termed, respectively, “bottom-up”, “anthropic”, and “top down”.

Let us denote by $O$ the conditionalization object used to specify these conditional probabilities. In bottom-up reasoning, it would be the same as the $M$-object; in the anthropic approach it would be an “observer”, and in the top-down approach it could be a universe with the currently-known properties of our universe. It can be seen that they inhabit a spectrum, from the weakest conditionalization (bottom-up) to the most stringent (top-down). Like our initial $M$-object, choosing a conditionalization is unavoidable and important, and there is no obviously correct choice to make. (See Refs. [10, 11] for similar conditionalization schemas.)

(vi) Having decided on a conditionalization object $O$, the next step is to compute the number of $N_{O,M}(\alpha_i)$ of $O$-objects per $M$-object, for each set of values of the parameters $\alpha_i$. For example, if we have chosen to condition on observers, but have used baryons to define our probabilities, then we need to calculate the number of observers per baryon as a function of cosmological parameters. We can then calculate $P_O(\alpha_i) \equiv P_M(\alpha_i)N_{O,M}(\alpha_i)$, i.e. the probability that a randomly chosen $O$-object (observer) resides in a universe with parameters $\alpha_i$. There are a few possible pitfalls in doing this. First, if $N_{O,M}$ is infinite, then the procedure clearly breaks because $P_O$ then becomes undefined. This is why the $M$-object should be chosen to requires as little as possible for its existence (and hence be associated with the minimal-conditionalization bottom-up approach). This difficulty will generically occur if the existence of an $O$-object does not necessarily entail the existence of an $M$-object. For example, if the $M$-object were a baryon but the $O$-object were a
bit of volume, then $N$ would be infinite for $\alpha_i$ corresponding to universes with no baryons. The problem arises because baryons require volume to be in, but volume does not require a baryon to be in it. This seems straightforward, but gets much murkier when we consider the second difficulty when calculating $N$, which is that we may not be able to precisely define what an $O$-object is, or what it takes to make one. If we say that the $O$-object is an observer, what exactly does that mean? A human? A carbon-based life form? Can observers exist without water? Without heavy elements? Without baryons? Without volume? It seems quite hard to say. We are forced, then, to choose some proxy for an observer, e.g. a galaxy, or a star with possible planets, etc. But our probabilities will perforce depend on the chosen proxy and this must be kept in mind.

It is worth noting a small bit of good news here. If we do manage to consistently compute $N_{O,M_1}$ for some measure object $M_1$, then insofar as we want to condition our probabilities on $O$-objects, we have solved the measure problem: if we could consistently calculate $N_{O,M_2}$ for a different measure object $M_2$, then we should obtain the same result for $N_{O}$, i.e. $N_{O,M_1}P_{M_1} = N_{O,M_2}P_{M_2}$. Thus our choice of $M_1$ (rather than $M_2$) becomes unimportant.

(vii) The final step in making predictions is to make the assumption that the probability that we will measure some set of $\alpha_i$ is given by the probability that a randomly chosen $O$-object will. This assumption really entails two others: first, that we are some how directly associated with $O$-objects, and second that we have not, simply by bad luck, observed highly improbable values of the parameters. The assumption that we are typical observers has been termed the “principle of mediocrity” [12]. One may argue about this assumption, but some assumption is necessary if we are to connect our computed probabilities to observations, and it is difficult to see what alternative assumption would be more reasonable.

The result of all this work would be the probability $P_O(\alpha_i)$ that a randomly selected $O$-object (out of all of the $O$-objects that exist in multiverse) would reside in a universe governed by parameters $\alpha_i$, along with a reason to believe that this same probability distribution should govern what we will observe. We can then make the observations (or consider some already-made ones). If the observations are highly improbable according to our predictions, we can rule out the candidate $T$ at some confidence that depends on how improbable our observations were. Apart from the manifest and grave difficulties involved in actually completing the seven listed steps in a convincing way, I think the only real criticism that can be leveled at this approach is that unless $P = 0$ for our observed parameters, there will always be the chance that the $T$ was correct and we measured an unlikely result. Usually, we can rid ourselves of this problem by repeating our experiments to make $P$ as small as we like (at least in principle), while here we do not have that option – once we have “used up” the measurement of all of the parameters required to describe our universe (which appears to be rather surprisingly few, at least according to current theories), we are done.
3. Making predictions in a multiverse more easily: a bestiary of shortcuts

Although the idea of a multiverse has been around for quite a while, no one has ever really come close to making the sort of calculation outlined in the previous section. Instead, those wishing to make predictions in a multiverse context have made strong assumptions about which parameters $\alpha_i$ actually vary across the ensemble, about the choice of $O$-object, and about the quantities $P_M(\alpha_i)$ and $N_{O,M}(\alpha_i)$ that go into predicting their probabilities for measurement. Some of these shortcuts aim simply to make a calculation tractable; others are efforts to avoid anthropic considerations, or alternatively to use anthropic considerations to avoid other difficulties.

I would not have listed any ingredients that I thought could be omitted from a really sound calculation, thus all of these shortcuts are necessarily incomplete (some, in my opinion, disastrously so). But by listing and discussing them, I hope to give the reader both a flavor for what sort of anthropic (or anthropic-esque) arguments have been made in the literature, and where they may potentially go astray.

3.1. The “Maybe anthropic considerations only allow one set of parameters” hope

This assumption underlies a sort of anthropic reasoning that has earned the anthropic principle a lot of ill will. It goes something like: “Let’s assume that lots of universes governed by lots of different parameter values exist. Then since only universes with parameter values almost exactly the same as ours allow life, we must be in one of those, and we should not find it strange if our parameter values seem special.” In the conventions I have described, this is essentially equivalent setting $O$-objects to be observers, then hoping that the “hospitality factor” $N_{\text{obs},M}(\alpha_i)$ is very narrowly peaked around one particular set of parameters. In this case, the a priori probabilities $P_M$ are pretty much irrelevant because the shape of $N_{\text{obs},M}$ will pick out just one set of parameters. Because our observed values $\alpha_{\text{obs}}^i$ definitely allow observers, the allowed set must then be very near $\alpha_{\text{obs}}^i$.

Three problems with this type of reasoning are as follows. First, it is rather circular: it entails picking the $O$-object to be an observer, but then quickly substituting a “universe just like ours” for the $O$-object, with the reasoning that such universes will definitely support life. Thus we have arrived at: the universe we observe should be pretty much like the observed universe. The way to avoid this silliness is to allow at least the possibility that there are life-supporting universes with $\alpha_i \neq \alpha_{\text{obs}}^i$, i.e. to discard the unproven assumption that $N_{\text{obs},M}$ has a single, dominant, narrow peak.

The second problem is that if $N_{\text{obs},M}$ were really so narrowly peaked as to render $P_M$ irrelevant, then we would be in serious trouble as theorists, because we would lose

The most ambitious attempt is probably the recent one by Tegmark [4].

One can also argue that if there were other, more common, universes that supported life, we ought to be in them; since we are, not, we should assume that almost all life-supporting universes are like ours. But this is also circular in assuming that the whole anthropic argument works, in formulating the argument.
any ability to distinguish between candidates for our fundamental theory: unless our observed universe is impossible in the theory, then the anthropic factor would force the predictions of the theory to match our observations. As discussed in the next section, this is not good.

The third problem with \( N_{\text{obs},M} \) being an extremely peaked function is that it does not appear to be true! As discussed in below, it appears that for any reasonable surrogate for observers (e.g. galaxies like ours, or stars with heavy elements, etc.), calculations done using our current understanding of galaxy and structure formation indicate that the region of parameter space in which there can be many of those objects may be small compared to the full parameter space, but it is much larger than the region compatible with our observations.

3.2. The “Just look for zero probability regions \((P = 0)\) in parameter space” approach

As mentioned in Section 2, there is a (relatively!) easy thing to do with a multiverse theory \( \mathcal{T} \): work out which parameters combinations cannot occur in any universe. If the combination we actually observe is one of these, then the theory is ruled out. This is unobjectionable, but a rather weak way to test a theory because given two theories that are not ruled out, we have no way whatsoever of judging one to be better, even if the parameter values we observe are in some sense generic in one and absurdly rare in the other.¶

This is not how science usually works. For example, suppose our theory is that a certain coin-tossing process is unbiased. If our only way to test this theory was to look for experimental outcomes that are impossible, then the theory would unfalsifiable: we would have to accept it theory for any coin we are confronted with, because no sequence of tosses would be impossible in it! Even if 10,000 tosses in a row all came up heads, we would have no grounds for doubting our theory because while getting heads 10,000 times in a row on a fair coin is absurdly improbable, it is not impossible. Nor would we have reason to prefer the (seemingly much better) “nearly every toss comes out heads” theory. Clearly this is a situation we would like to improve on, as much in universes as in coin tosses.

3.3. The “Let’s look for overwhelmingly more probable values” suggestion

One possible improvement would be to assume that we will observe a “typical” set of parameters in the ensemble, i.e. that we will employ “bottom-up” reasoning as described in the first section, by using the \( a \text{ priori} \) (or “prior”) probabilities \( P_M \) for some choice of measure-object such as universes, and just ignore the conditionalization factor \( N_{O,M} \). There are two possible justifications for this. First, we might simply want to avoid any

¶ Amusingly, in terms of testing \( \mathcal{T} \), this approach which makes no assumptions about \( N_{O,M} \) is equivalent to the approach just described of making the very strong assumption that \( N_{O,M} \) allows only one specific set of parameter values, because in either case a theory can only be ruled out if our observed values are impossible in that theory.
sort of anthropic issues on principle. Second, we might hope that some parameter values are much, much more common than others, to the extent that the \( N_{O,M} \)-factor becomes irrelevant – in other words that \( P_M \) (rather than \( N_{O,M} \)) is a very strongly peaked around some particular parameters.

The problem with this approach is the “measure problem” discussed above: there is an implicit choice of basing probabilities on universes (say) rather than on (say) volume elements or baryons. Each of these measures has problems – for example, it seems that probabilities based on “universes” depends on how the universes are delineated, which can be ambiguous. Moreover there seems to be no reason to believe that predictions made using any two measures should agree particularly well. For example, as discussed elsewhere in this volume, in the string theory “landscape” there are many possible parameter sets, depending on which metastable minimum one chooses in a potential that depends in turn on a number of fluxes that can take a large range of discrete values. Imagine that exponentially many more minima lead to \( \alpha_1 = a \) than lead to \( \alpha_1 = b \). Should we expect to observe \( \alpha_1 = a \)? Not necessarily, because the relative number of \( a \)-universes vs. \( b \)-universes that \emph{actually come into existence} may easily differ exponentially from the relative number of \( a \)-minima vs. \( b \)-minima. (This seems likely to me in an eternal-inflation context, where the relative number universes could depend on exponentially-suppressed tunnelings between vacua.) Worse yet, these may in turn differ exponentially (or even by an infinite factor) from the relative numbers of baryons, or relative volumes.

In short, while we are free to use bottom-up reasoning with any choice of measure object we like, we are not free to assert that other choices would give similar predictions, or that conditionalization can be rendered irrelevant. So we had better have a pretty good reason for the choice we make.

3.4. The “Let’s fix some parameters to the observed values and predict others” shortcut

Another way in which one might hope to circumvent anthropic issues is to condition the probabilities on some or all observations that have already been made. In this “top-down” (or perhaps “pragmatic”) approach we ask: given everything that has been observed so far, what will we observe in some future measurement? It has a certain appeal, as this is often what is done in experimental science: we do not try to \emph{predict} what our laboratory will look like, just what will happen given that the lab is in a particular state at a given time. In the conventions of Section 2, the approach could consist of choosing the \( O \)-object to be universes with parameters agreeing with the measured values.

While appealing, this approach suffers some deficiencies:

- It still does not completely avoid the measure problem, because even once we have limited our consideration to universes that match our current observations, we must still choose a measure with which to calculate the probabilities for the remaining ones.
• Through our conditioning, we may accept theories for which our parameter values are wildly improbable, without supplying any justification as to why we observe such improbable values. This is rather strange. Imagine that I have a theory in which the cosmological constant $\Lambda$ is (with very high probability) much higher than we observe, and the dark matter particle mass $m_{\text{DM}}$ is almost certainly $> 1000$ GeV. I condition on our observed $\Lambda$, simply accepting that I am in an unusual universe. Now say I measure $m_{\text{DM}} = 1$ GeV. I would like to say my theory is ruled out. Fine, but here is where it gets odd: according to top-down reasoning, I should also have already ruled it out if I had done my calculation in 1997, before $\Lambda$ was measured. And someone who invented the very same theory next week—but had not been told that I have already ruled it out—would not rule it out, but instead just take the low value of $m_{\text{DM}}$ (along with the observed $\Lambda$) as part of the conditionalization!

• If we condition on everything we have observed, we obviously give up the possibility of explaining anything we have observed (which at this point is quite a lot in cosmology) through our theory.

The last two issues motivate variations on the top-down approach in which only some current observations are conditioned on. Two of which I am aware are:

(i) We might start by conditioning on all observations, then progressively condition on less and less and try to “predict” the things we have decided not to condition on (as well, of course, as any new observations)\textsuperscript{13, 14}. The more we can predict, the better our theory is. The problem is that either (a) we will get to the point where we are conditioning on as little as possible (the bottom-up approach), and hence the whole conditionalization process will have been a waste of time, or (b) we will still have to condition on some things, and admit either that these have an anthropic explanation, or that we just choose to condition on them (leading to the funny issues discussed above).

(ii) We might choose at the outset to condition on things that we think may be fixed anthropically (without trying to actually generate this explanation), then try to predict the others \textsuperscript{15}. This is nice in being relatively easy, and in providing a justification for the conditionalization. It suffers from the problems of (a) guessing which parameters are anthropically important and which are not, (b) even if a parameter is anthropically unimportant, it may be strongly correlated in $P_M$ with one that is, and (c) we still have to face the measure problem, which we cannot avoid by counting conditioning on observers, because we are avoiding anthropic considerations.

3.5. The “Let’s assume just one parameter varies” simplification

Most of the “shortcuts” discussed so far have been attempts to avoid anthropic considerations. But we may, instead, consider how we might try to formulate an anthropic prediction (or explanation) for some observable, without going through the
full calculation outlined in Section 2. The way of doing this that has been employed in
the literature (largely in the efforts of Vilenkin and collaborators) is as follows.

First, one fixes all but one (or perhaps two) of the parameters to the observed
values. This is done for tractability and/or because one hopes that they will have non-
anthropic explanations. Let us call the parameter that is allowed to vary across the
ensemble \( \alpha \).

Next, an \( O \)-object is chosen such that given that only \( \alpha \) varies, it is hoped that
(a) the number \( N_{O,M} \) of these objects (per baryon, or per comoving volume element)
in a given universe is calculable, and (b) this number is arguably proportional to the
number of observers. For example, if only \( \Lambda \) varies across the ensemble, galaxies might
make reasonable \( O \)-objects because a moderately different \( \Lambda \) will probably not change
the number of observers per galaxy, but will change the number of galaxies in a way that
can be computed using fairly well-understood theories of galaxy and structure formation
(for examples see \[16, 17, 18, 19, 20, 21, 1\]).

Third, it is assumed that \( P_M(\alpha) \) is either flat or a simple power-law, without any
complicated structure. This can be done just for simplicity, but it is often argued to be
natural \[22, 23, 24\]. The flavor of this argument is as follows. If \( P_M \) is to have interesting
structure over the relatively small range in which observers are abundant, there must
be a parameter of order the observed \( \alpha \) in the expression for \( P_M \). But precisely this
absence is what motivated the anthropic approach. For example, if the expression for
\( P_M(\Lambda) \) contained the energy scale \( \sim 0.01 \) eV corresponding to the observed \( \Lambda \), the origin
of that energy scale would probably be more interesting than our anthropic argument,
as it would provide the basis for a (non-anthropic) solution to the cosmological constant
problem!

Under these (fairly strong) assumptions we can then actually calculate \( P_O(\alpha) \) and
see whether or not the observed value is reasonably probable given this predicted
distribution. For example when \( \Lambda \) alone is varied, a randomly chosen galaxy is predicted
to lie in a universe with \( \Lambda \) comparable to (but somewhat larger than) the value we
see \[17\].\textsuperscript{+}

I actually think this sort of reasoning is pretty respectable, given the assumptions
made. In particular, the anthropic argument in which only \( \Lambda \) varies is a relatively clean
one. But there are a number of pitfalls when it is applied to parameters other than \( \Lambda \),
or when one allows multiple parameters to vary simultaneously.

- Assuming that the abundance of observers is strictly proportional to that of galaxies
  only makes sense if the number of galaxies – and not their properties – changes
  as \( \alpha \) varies. However, changing nearly any cosmological parameter will change
  the properties of typical galaxies. For example, increasing \( \Lambda \) will decrease galaxy
  numbers, but also make galaxies smaller on average, because a high \( \Lambda \) squelches
  structure formation at late times when massive galaxies form. Increasing the

\textsuperscript{+} This is for a “flat” probability distribution \( dP_M/d\lambda \propto \lambda^\alpha \) with \( \alpha = 0 \). For \( \alpha > 0 \), higher values
would be predicted, and with \( \lambda < 0 \) lower values would be favored.
amplitude of primordial perturbations would similarly lead to smaller, denser – but more numerous – galaxies, as would increasing ratio of dark matter to baryons. In these cases, we must specify in more detail what properties an observer-supporting galaxy should have, and this is very difficult to do without falling into the circular-argument trap of assuming that only galaxies like ours support life. Finally, this sort of strategy seems unlikely to work if we try to change non-cosmological parameters, as this could lead to radically different physics and the necessity of thinking very hard about what sort of observers there might be.

- The predicted probability distribution clearly depends on $P_M$, and the assumption that $P_M$ is flat, or a simple power law, can break down. This can happen even for $\Lambda$ [17] but perhaps more naturally for other parameters such as the dark matter density for which particle physics models can already yield sensible values. Moreover, this breakdown is much more probable if (as discussed below and contrary to the assumption made above) the hospitality factor $N_{O,M}(\alpha)$ is significant over many orders of magnitude in $\alpha$.

- Calculations of the hospitality factor $N_{O,M}(\alpha)$ can go awry if $\alpha$ is changed more than a little. For example, a neutrino mass slightly larger than we observe would suppress galaxy formation by erasing small-scale structure. But neutrinos with a large ($\geq 100$ eV) mass would act as dark matter and lead to strong halo formation. Whether these galaxies would be hospitable is questionable (they would be very baryon-poor), but the point is that the physics becomes qualitatively different. As another example, a lower photon/baryon ratio $n_\gamma/n_b$ would lead to earlier-forming, denser galaxies. But a much smaller value would lead to qualitatively different structure formation, as well as the primordial generation of heavy elements [3]. As discussed at length in ref. [3], these changes are very dangerous because over orders of magnitude in $\alpha$, $P_M(\alpha)$ will tend to change by many orders of magnitude. Thus even if these alter-universes only have a few observers in them, they may dominate $P_O$ and hence qualitatively change the predictions.

- Along the same lines, but perhaps even more pernicious, when multiple parameters are varied simultaneously, the effects of some variations can offset the effect of others so that universes quite different from ours can support many of our chosen $O$-objects. For example, increasing $\Lambda$ cuts off galaxy formation at a given cosmic density, but raising the perturbation amplitude $Q$ causes galaxies to form earlier (thus nullifying the effect of $\Lambda$). This can be seen in the calculations of [25, 18], and is discussed explicitly in [3, 21, 4]. Many such deneneracies exist, because raising $\Lambda$, $n_\gamma/n_b$, or the neutrino mass all decrease the efficiency of structure formation, while raising $\Omega_{DM}/\Omega_b$ or $Q$ increase the efficiency. As an extreme case, it was shown in [3] that if $Q$ and $n_\gamma/n_b$ are allowed to vary with $\Lambda$, then universes with $\Lambda$ of $10^{17}$ times our observed value could arguably support observers! Including more cosmological parameters, or non-cosmological parameters, can only make this problem worse.

These problems indicate that while anthropic arguments concerning $\Lambda$ in the...
literature are relatively “clean”, it is unclear whether other parameters (taken individually) will work as nicely. More importantly, a number of issues arise when several parameters are allowed to vary at once, and there does not seem to be any reason to believe that success in explaining one parameter anthropically will persist when additional parameters are allowed to vary. In some cases, it may: for example, allowing neutrino masses to vary in addition to $\Lambda$ does not appear to spoil the anthropic explanation of a small but nonzero cosmological constant [20]. On the other hand, allowing $Q$ to vary does, unless $P_M(Q)$ is strongly peaked at small values of $Q$ [21]. I suspect that allowing $\Omega_{DM}/\Omega_b$ or $n_{\gamma}/n_b$ to vary along with $\Lambda$ would have a similar effect.

3.6. So what should we do?

For those serious about making predictions in a multiverse, I would propose that rather than working to generate additional incomplete anthropic arguments by taking shortcuts, a much better job must be done in each of the individual ingredients. For one example, our understanding of galaxy formation is sufficiently strong that the multidimensional hospitality factor $N_{O,M}(\alpha_i)$ could probably be computed for $\alpha_i$ within a few orders of magnitude of the observed values, for $O$—objects of galaxies with properties within some range. Second, despite some nice previous work, I think the problem of how to compute $P_M$ in eternal inflation is a pretty open one. Finally, the string/M theory landscape (which is generating a lot of interest in the present topic right now) cannot hope to say much of anything about $P_M$ until its place in cosmology is understood—in particular, we need both a better understanding of the statistical distribution of field values that result from evolution in a given potential, and also an understanding of how transitions between vacua with different flux values occur, and exactly what is transitioning.

4. Cosmic coincidences and living dangerously: are there general predictions of anthropic reasoning?

The preceding sections should have suggested to the reader that it will be a huge project to compute a sound prediction of cosmological and physical parameters from a multiverse theory in which they vary. It may be so hard that it will be a very long time before any such calculation is at all believable. It is worth asking then: is there any way nature might give us an indication as to whether the anthropic approach is a sensible one, i.e. does the anthropic approach make any sort of general predictions even without the full calculation of $P_O$? Interestingly, I think the answer might be yes: I am aware of two such general (though somewhat vague) predictions of the anthropic approach.

To understand the first, assume that only one parameter, $\alpha$, varies, and consider $p(\log \alpha) = \alpha P_M(\alpha)$, the probability distribution in $\log \alpha$, given by some theory $\mathcal{T}$. For $\log \alpha$ near the observed value $\log \alpha_{\text{obs}}$, $p$ can basically only be doing one of three things: it can rise with $\log \alpha$, fall with $\log \alpha$, or be approximately constant. In the first two
cases, the theory $T$ would predict that we should see a value of $\alpha$ that is, respectively, higher or lower than we actually do if no anthropic conditionalization $N_{\text{obs, M}}$ is applied. Now suppose we somehow compute $N_{\text{obs, M}}(\alpha)$ and find that it falls off quickly for values of $\alpha$ much smaller or larger than we observe, i.e. that only a range $\alpha_{\text{min}} < \alpha < \alpha_{\text{max}}$ is “anthropically acceptable”. Then we have an anthropic argument explaining $\alpha_{\text{obs}}$, because this falloff means that $P_{\text{obs}}$ will only be significant near $\alpha_{\text{obs}}$. But now note that within the anthropically acceptable range, $P_{\text{obs}}$ will be peaked near $\alpha_{\text{max}}$ if $p$ is increasing with $\alpha$, or near $\alpha_{\text{min}}$ if $p$ is decreasing with $\alpha$. That is, we should expect $\alpha_{\text{obs}}$ at one edge of the anthropically acceptable range. This idea has been called the “principle of living dangerously” [26]. It asserts that for a parameter that is anthropically determined, we should expect that a calculation of $N_{\text{obs, M}}$ would reveal that observers would be strongly suppressed either for $\alpha$ slightly larger or slightly smaller than $\alpha_{\text{obs}}$, depending on whether $p$ is rising or falling.

Now, this is not a very specific prediction: exactly where we would expect $\alpha_{\text{obs}}$ to lie depends both on how steep $p(\log \alpha)$ is, and how sharp the cutoff in $N_{\text{obs, M}}$ is for $\alpha$ outside of the anthropically acceptable range. And it would not apply to anthropically-determined parameters in all possible cases. (For example, if $p$ were flat near $\alpha_{\text{obs}}$, but also very high at $\alpha \gg \alpha_{\text{obs}}$, anthropic effects would be required to explain why we do not observe the very high value; but any region within the anthropically acceptable range would be equally probable, so we would not expect to be, so to speak, living on the edge.) Despite these caveats, this is a prediction of sorts, because the naive expectation would probably be for our observation to place us somewhere in the interior of the region of parameter space that is hospitable to life, rather than at the edge.

A second sort of general prediction of anthropic reasoning is connected to what might be called “cosmic coincidences.” For example, many cosmologists have asked themselves (and each other) why the current density in vacuum energy, dark matter, baryons, and neutrinos are all within a couple of orders of magnitude of each other – making the universe a much more complicated place than it might be. Conventionally, it has been assumed that these coincidences are just that, and follow directly from fundamental physics that we do not understand. But if the anthropic approach to cosmology is really correct (that is, if it is the real answer to the question of why these densities take the particular values they do), then the explanation is quite different: the densities are bound together by the necessity of observers’ existence, because only certain combinations will do.

More explicitly, suppose several cosmological parameters are governed by completely unrelated physics, so that their individual prior probabilities $P_M$ simply multiply to yield the multidimensional probability distribution. For example, we might have $P_M(\Lambda, \Omega_{DM}/\Omega_b, Q) = P_M(\Lambda)P_M(\Omega_{DM}/\Omega_b)P_M(Q)$. But even if $P$ factors, the hospitality factor $N_{\text{obs, M}}$ will almost certainly not: if galaxies are $O$-objects, the number of galaxies formed at a given $\Lambda$ will depend on both other parameters, and only certain combinations will give a significant number of observers. Thus $P_O = N_{\text{obs, M}}P_M$ will likewise have correlations between the different parameters that lead to only particular
combinations (for example those with $\Omega_{\text{DM}}/\Omega_b \sim 1 - 10$ for a given $Q$ and $\Lambda$) having high probabilities. The cosmic coincidences would be explained in this way.

This anthropic explanation of coincidences, however, should not only apply to things that we have already observed. If it is correct, then it should apply also to future observations; that is, we should expect to uncover yet more bizarre coincidences between quantities that seem to follow from quite unrelated physics.

How might this actually happen? Consider dark matter. We know fairly precisely how much dark matter there is in the universe, and what its basic properties are. But we have no real idea what it actually is, and there are many, many possible candidates that have been proposed in the literature. In fact, we have no observational reason to believe that dark matter is one substance at all: in principle it could be equal parts axions, supersymmetric particles, and primordial black holes. The reason most cosmologists do not expect this is that it would be a strange coincidence if three substances involving quite independent physics all wound up with essentially the same density in our universe. But of course this would be just like the surprising-but-true coincidences that hold in already-observed cosmology.

In the anthropic approach, these comparable densities could be quite natural. To see why, imagine that there are two completely independent types of dark matter permeating the ensemble: in each universe, they have some particular densities $\rho_1$ and $\rho_2$ out of a wide range of possibilities, so that the densities in a randomly chosen universe (or around a randomly chosen baryon, etc.) will be given probabilistically by $P_M(\rho_1)P_M(\rho_2)$. Under these assumptions there is no reason to expect that we should observe $\rho_1 \sim \rho_2$ based just on these a priori probabilities. Now suppose, though, that $N_{O,M}$ picks out a particular narrow range of total dark matter density as anthropically acceptable. That is, $N_{O,M}(\rho_1 + \rho_2)$ is narrowly peaked about some $\rho_{\text{anth}}$. In this case, the peak of the probability distribution $P_O(\rho_1, \rho_2)$, which indicates what values a randomly chosen observer should see, will occur where $P_M(\rho_1)P_M(\rho_2)$ is maximized subject to the condition that $\rho_1 + \rho_2 \simeq \rho_{\text{anth}}$. For simplicity let both prior probabilities be power laws: $P_M(\rho_1) \propto \rho_1^\alpha$ and $P_M(\rho_2) \propto \rho_2^\beta$. Now the coincidence: it is not hard to show that if $\alpha \geq 0$ and $\beta \geq 0$, then the maximum will occur when $\rho_1/\rho_2 = \alpha/\beta$. That is, the two components are likely to have similar densities unless the power law indices of their probability distributions differ by orders of magnitude.* Of course, there are many ways in which this coincidence could fail to occur (e.g. negative power-law indices, or correlated probabilities), but the point is that there is a quite natural set of circumstances in which the components are coincident, even though the fundamental physics is completely unrelated.

* Extremely high power law indices are uncomfortable in the anthropic approach because they would lead to $P_O$ being peaked where $N_{O,M}$ is declining, i.e. we should be living outside the anthropically comfortable range, not just dangerously but downright recklessly.
5. Conclusions

The preceding sections should have convinced the reader that there are good reasons for scientists to be very worried if we live in a multiverse: in order to test a multiverse theory in a sound manner, we must perform a fiendishly difficult calculation of $P_O(\alpha_i)$, the probability that an $O$-object will reside in a universe characterized by parameters $\alpha_i$. And because of the shortcomings of the shortcuts one may (and presently must) take in doing this, almost any particular multiverse prediction is going to be easy to criticize; only a quite good calculation is going to be at all convincing. Much worse, we face an unavoidable and important choice in what $O$ should be: a possible universe, an existing universe, or a universe matching current observations, or a bit of volume, or a baryon, or a galaxy, or an “observer”, etc.

I find it disturbingly plausible that “observers” really are the correct conditionalization object, that their use as such is the correct answer to the measure problem, and that anthropic effects are the real explanation for the values of some parameters (just as for the local density that we observe). Many cosmologists appear to believe that taking the necessity of observers into account is shoddy thinking, and is employed only because it is the easy way out of solving problems the “right” way. But the arguments of this paper suggest that the truth may well be exactly the opposite: the anthropic approach may be the right thing to do in principle, but nearly impossible in practice.

Nonetheless, we cannot do away with multiverses just by wishing them away: we may in fact live in one, whatever the inconvenience to cosmologists. The productive strategy then seems to be one of accepting multiverses as a possibility, and working toward understanding how to calculate the various ingredients necessary to make predictions in one. Whether really performing such a calculation will turn out to be possible, but it is certainly impossible if not attempted.

Even if we cannot calculate $P_O$ in the foreseeable future, however, cosmology in a multiverse may not be completely devoid of predictive power. For example, if anthropic effects are at work, they should leave certain clues. First, if we could determine the region of parameter space hospitable to observers, we should find that we are living in the outskirts of the livable region, rather than somewere in its midst. Second, if the anthropic effects are the explanation of the parameter values – and coincidences between them – that we see, then it ought to predict that new coincidences will be observed in future observations.

If in the next several decades dark matter is resolved into several equally important components, dark energy is found to be three independent substances, and several other “cosmic coincidences” are observed, even someone the most die-hard skeptics might accede that the anthropic approach may have validity – why else would be universe be so very baroque? On the other hand, if we are essentially finished in defining the basic cosmological constituents, and the defining parameters are in the midst of a relatively large region of parameter space that might arguably support observers, then I think the
anthropic approach would lose almost all appeal it has; we would be forced to ask: why isn’t the universe much weirder?

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