A scalable mining of frequent quadratic concepts in d-folksonomies

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Abstract—Folksonomy mining is grasping the interest of web 2.0 community since it represents the core data of social resource sharing systems. However, a scrutiny of the related works interested in mining folksonomies unveils that the time stamp dimension has not been considered. For example, the wealthy number of works dedicated to mining tri-concepts from folksonomies did not take into account time dimension. In this paper, we will consider a folksonomy commonly composed of triples <users, tags, resources> and we shall consider the time as a new dimension. We motivate our approach by highlighting the battery of potential applications. Then, we present the foundations for mining quadri-concepts, provide a formal definition of the problem and introduce a new efficient algorithm, called QUADRICONS for its solution to allow for mining folksonomies in time, i.e., d-folksonomies. We also introduce a new closure operator that splits the induced search space into equivalence classes whose smallest elements are the quadri-minimal generators. Carried out experiments on large-scale real-world datasets highlight good performances of our algorithm.

Keywords—Quadratic Context; Formal Concept Analysis; Quadratic Concepts; Folksonomies; Algorithm; Social Networks

I. INTRODUCTION

Folksonomy (from folk and taxonomy) is a neologism for a practice of collaborative categorization using freely chosen keywords [1]. Folksonomies (also called social tagging mechanisms) have been implemented in a number of online knowledge sharing environments since the idea was first adopted by the social bookmarking site del.icio.us in 2004. The idea of a folksonomy is to allow the users to describe a set of shared objects with a set of keywords, i.e., tags, of their own choice. The new data of folksonomies systems provides a rich resource for data analysis, information retrieval, and knowledge discovery applications. The rise of folksonomies, due to the success of the social resource sharing systems (e.g., Flickr, Bibsonomy, YouTube, etc.) also called Web 2.0, has attracted interest of researchers to deal with the Folksonomy mining area. However, due to the huge size of folksonomies, many works focus on the extraction of lossless concise representations of interesting patterns, i.e., triadic concepts [2] [3] [4].

Recently, in [5], the new TriICONS algorithm outperforms its competitors thanks to a clever sweep of the search space. Nevertheless, a scrutiny of these related work unveils that the time stamp dimension has not been considered yet. Time is considered one of the most important factors in detecting emerging subjects. Agrawal and Srikant show in [6] the importance of sequential patterns which may be useful to discover rules integrating the notion of temporality and sequence of events. In our case, such rules shall be of the form : users which shared the movie “Alcatraz” using the tag prison will shared it later with the tag escape.

With this paper, we initiate the confluence of three lines of research, Formal Concept Analysis, Folksonomy mining and Mining Sequential Patterns. Formal Concept Analysis (FCA) [7] has been extended since fifteen years ago to deal with three-dimensional data [8]. However, Triadic Concept Analysis (TCA) has not garnered much attention for researchers until the coming of folksonomies as they represent the core data structure of social networks. Thus, we give a formal definition of the problem of mining all frequent quadri-concepts (the four-dimensional and sequential version of mining all frequents tri-concepts) and introduce our algorithm QUADRICONS for its solution, which is an extension of the TriICONS algorithm to the quadratic case. We also introduce a new closure operator that splits the induced search space into equivalence classes whose smallest elements are the quadri-minimal generators (QGs); QGs are helpful for a clever sweep of the search space [5] [9].

The remainder of the paper is organized as follows. In the next section, we motivate our conceptual and temporal clustering approach for solving the problem of mining all frequent quadri-concepts of a given dataset. We thoroughly study the related work in Section III. In Section IV we provide a formal definition of the problem of mining all frequent quadri-concepts. We introduce a new closure operator for the quadratic context as well as the QUADRICONS algorithm dedicated to the extraction of all frequent quadri-concepts, in Section V. In Section VI, we carried out experiments about performances of our algorithm in terms of execution time, consumed memory and compacity of the quadri-concepts. Finally, we conclude the paper with a summary and we sketch some avenues for future works in Section VII.

II. MOTIVATION : CONCEPTUAL AND TEMPORAL CLUSTERING OF FOLKSONOMIES

The immediate success of social networks, i.e., social resource sharing systems is due to the fact that no specific skills are needed for participating [2]. Each individual user is able to share a web page, a personal photo, an artist he like or a movie he watched without much effort.

1http://del.icio.us
2http://flickr.com
3http://last.fm
4http://movielens.org
The core data structure of such systems is a folksonomy. It consists of three sets \(\mathcal{U}, \mathcal{T}, \mathcal{R}\) of users assigning tags to resources as well as a ternary relation \(Y\) between them. To allow conceptual and temporal clustering from folksonomies, an additional dimension, i.e., \(D\), is needed: time. Indeed, the special feature of folksonomies under study is their unceasing evolution \(\mathbf{10}\). Such systems follow trends and evolve according to the new user’s taggings \(\mathbf{11}\). The increasing use of these systems shows that folksonomy-based works are then able to offer a better solution in the domain of Web Information Retrieval (WIR) \(\mathbf{12}\) by considering time increasing use of these systems shows that folksonomy-based works are then able to offer a better solution in the domain of Web Information Retrieval (WIR) \(\mathbf{12}\) by considering time.

In this section, we focus on the largest concepts of the data. In \(\mathbf{19}\), inspired by work of Wille \(\mathbf{8}\) extending Formal Concept Analysis to three dimensions, the author created a framework for analyzing \(n\)-dimensional formal concepts. He generalized the triadic concept analysis to \(n\) dimensions for arbitrary \(n\), giving rise to Polyadic Concept Analysis.

The \(n\)-adic contexts give rise, in a way analogous to the triadic case, to \(n\)-adic formal concepts. \(\mathbf{19}\) the author gives examples of quadratic concepts and their associated quadri-lattice. Despite robust theoretical study, no algorithm has been proposed by Voutsadakis for an efficient extraction of such \(n\)-adic concepts. Recently, Cerf et al. proposed the DATA-PEELER algorithm \(\mathbf{4}\) in order to extract all closed concepts from \(n\)-ary relations. DATA-PEELER enumerates all the \(n\)-adic formal concepts in a depth first manner using a binary tree enumeration strategy. When setting \(n\) to 4, DATA-PEELER is able to extract quadri-concepts.

In the following, we give a formal definition of the problem of mining all frequent quadri-concepts as well as the main notions used through the paper.

IV. THE PROBLEM OF MINING ALL FREQUENT QUADRI-CONCEPTS

In this section, we formalize the problem of mining all frequent quadri-concepts. We start with an adaptation of the notion of folksonomy \(\mathbf{2}\) to the quadratic context.

**Definition 1:**\(\mathbf{(D-FOLKSONOMY)}\) A \(d\)-folksonomy is a set of tuples \(\mathbb{F}_d = (\mathcal{U}, \mathcal{T}, \mathcal{R}, \mathcal{D}, Y)\) where \(\mathcal{U}, \mathcal{T}, \mathcal{R}\) and \(\mathcal{D}\) are finite sets which elements are called users, tags, resources and dates. \(Y \subseteq \mathcal{U} \times \mathcal{T} \times \mathcal{R} \times \mathcal{D}\) represents a quaternary relation where each \(y \in Y\) can be represented by a quadruple \(y = \{(u, t, r, d) | u \in \mathcal{U}, t \in \mathcal{T}, r \in \mathcal{R}, d \in \mathcal{D}\}\) which means that the user \(u\) has annotated the resource \(r\) using the tag \(t\) at the date \(d\).

**Example 1:** Table \(\mathbf{1}\) depicts an example of a \(d\)-folksonomy \(\mathbb{F}_d\) with \(\mathcal{U} = \{u_1, u_2, u_3, u_4\}\), \(\mathcal{T} = \{t_1, t_2, t_3\}\), \(\mathcal{R} = \{r_1, r_2\}\) and \(\mathcal{D} = \{d_1, d_2\}\). Each cross within the quaternary relation indicates a tagging operation by a user from \(\mathcal{U}\), a tag from \(\mathcal{T}\) and a resource from \(\mathcal{R}\) at a date from \(\mathcal{D}\), i.e., a user has tagged a particular resource with a particular tag at a date \(d\). For example, the user \(u_1\) has tagged the resource \(r_1\) with the tags \(t_1\), \(t_2\) and \(t_3\) at the date \(d_1\).

The following definition introduces a (frequent) quadri-set.

**Definition 2:**\(\mathbf{(A (FREQUENT) QUADRI-SET)}\) Let \(\mathbb{F}_d = (\mathcal{U}, \mathcal{T}, \mathcal{R}, \mathcal{D}, Y)\) be a \(d\)-folksonomy. A quadri-set of \(\mathbb{F}_d\) is a quadruple \((A, B, C, E)\) with \(A \subseteq \mathcal{U}\), \(B \subseteq \mathcal{T}\), \(C \subseteq \mathcal{R}\) and \(E \subseteq \mathcal{D}\) such as \(A \times B \times C \times E \subseteq Y\).
**Definition 3:** (Frequent Quadratic Concept) A quadratic concept (or a quadri-concept for short) of a d-folksonomy \( F_d = (U, T, R, D, Y) \) is a quadruple \((U, T, R, D)\) with \( U \subseteq U, T \subseteq T, R \subseteq R \) and \( D \subseteq D \) with \( U \times T \times R \times D \subseteq Y \) such that the quadruple \((U, T, R, D)\) is maximal, i.e., none of these sets can be extended without shrinking one of the other three dimensions. A quadri-concept is said to be frequent whenever it is a frequent quadri-set.

**Problem 2:** (Mining all frequent quadri-concepts) Let \( F_d = (U, T, R, D, Y) \) be a d-folksonomy and let \( \text{minsupp}_u, \text{minsupp}_t, \text{minsupp}_r, \text{minsupp}_d \) be user-defined minimum thresholds. The task of mining all frequent quadri-concepts consists in determining all frequent quadri-concepts \((U, T, R, D)\) of \( F_d \) with \( |U| \geq \text{minsupp}_u, |T| \geq \text{minsupp}_t, |R| \geq \text{minsupp}_r, |D| \geq \text{minsupp}_d \). The set of all frequent quadri-concepts of \( F_d \) is equal to \( QC = \{qc | qc = (U, T, R, D)\} \) is a frequent quadri-concept).

**Remark 1:** It is important to note that the extracted representation of quadri-concepts is information lossless. Hence, after solving Problem 2 we can easily solve the Problem 1 by enumerating all quadri-concepts \((A, B, C, E)\) such as it exists a frequent quadri-concept \((U, T, R, D)\) such as \( A \subseteq U, B \subseteq T, C \subseteq R, E \subseteq D \) and \( |A| \geq \text{minsupp}_u, |B| \geq \text{minsupp}_t, |C| \geq \text{minsupp}_r, |D| \geq \text{minsupp}_d \).

In the following, we introduce the QUADRICONS algorithm for mining all frequent quadri-Concepts before discussing its performances versus the DATA-PEELER algorithm for the quadratic case in the section after.

V. THE QUADRICONS ALGORITHM FOR MINING ALL FREQUENT QUADRI-CONCEPTS

In this section, we introduce new notions that would be of use throughout the QUADRICONS algorithm. Hence, we introduce a new closure operator for a d-folksonomy which splits the search space into equivalence classes as well as an extension of the notion of minimal generator [5]. Then, we provide an illustrative example of our algorithm.

A. Main notions of QUADRICONS

Before introducing our closure operator for a d-folksonomy/quadratic context, we define a general definition of a closure operator for a n-adic context.

**Definition 4:** (Closure Operator of a n-adic Context) Let \( S = (S_1, S_2, \ldots, S_n) \) be a n-set, with \( S_1 \) being maximal for \( S_1 \times \cdots \times S_n \subseteq Y \), of a n-adic context \( \mathbb{K}^n \) with \( n \) dimensions, i.e., \( \mathbb{K}^n = (D_1, D_2, \ldots, D_n, Y) \). A mapping \( h \) is defined as follows:

\[ h(S) = h(S_1, S_2, \ldots, S_n) = (C_1, C_2, \ldots, C_n) \text{ such as:} \]
\[ C_1 = S_1 \]
\[ \land C_2 = \{C_2^1 \in D_2 | (c_1^1, C_2^1, \ldots, C_n^1) \in Y \forall c_1^1 \in C_1, \forall C_2^2 \in S_2, \ldots, \forall C_n^2 \in S_n\} \]
\[ \vdots \]
\( C_n = \{ C_n' \in D_n \mid (c_1', c_2', \ldots, c_{n-1}', C_n') \in Y \land c_1' \in C_1, \ldots, \land c_{n-1}' \in C_{n-1} \} \)

**Proposition 1:** \( h \) is a closure operator.

**Proof:** To prove that \( h \) is a closure operator, we have to prove that this closure operator fulfills the three properties of extensivity, idempotency and isotony [21].

(1) **Extensivity:** Let \( S = (S_1, S_2, \ldots, S_n) \) be a \( n \)-set of \( \mathbb{K}^n \Rightarrow h(S) = (C_1, C_2, \ldots, C_n) \) such that: \( C_1 = S_1 \), \( C_2 = \{ C_2' \in D_2 \mid (c_1', c_2', c_3', \ldots, c_n') \in Y \land c_1' \in C_1, \forall c_2' \in S_2, \ldots, \forall c_n' \in S_n \} \supseteq S_2 \) since \( C_1 = S_1 \), \( \ldots, C_n = \{ C_n' \in D_n \mid (c_1', c_2', \ldots, c_{n-1}', C_n') \in Y \land c_1' \in C_1, \forall c_2' \in C_2, \ldots, \forall c_{n-1}' \in C_{n-1} \} \supseteq S_n \) since \( C_1 = S_1, C_2 \supseteq S_2, \ldots, C_{n-1} \supseteq S_{n-1} \). Then, \( C_1 = S_1 \) and \( S_i \subseteq C_i \) for \( i = 2, \ldots, n \Rightarrow S \subseteq h(S) \) (cf., Lemma [1]).

(2) **Idempotency:** Let \( S = (S_1, S_2, \ldots, S_n) \) be a \( n \)-set of \( \mathbb{K}^n \Rightarrow h(S) = (C_1, C_2, \ldots, C_n) \Rightarrow h(C_1, C_2, \ldots, C_n) = (C_1', C_2', \ldots, C_n') \) such that: \( C_1' = C_1, C_2' = \{ C_2' \in D_2 \mid (c_1', c_2', c_3', \ldots, c_n') \in Y \land c_1' \in C_1, \forall c_2' \in S_2, \ldots, \forall c_n' \in S_n \} \supseteq C_2 \) since \( C_1 = S_1 \), \( \ldots, C_n = \{ C_n' \in D_n \mid (c_1', c_2', \ldots, c_{n-1}', C_n') \in Y \land c_1' \in C_1, \forall c_2' \in C_2, \ldots, \forall c_{n-1}' \in C_{n-1} \} \supseteq C_n \) since we have \( C_1' = C_1, C_2' = C_2, \ldots, C_{n-1}' = C_{n-1} \). Then, \( C_i = C_1 \) for \( i = 1, \ldots, n \Rightarrow h(S) = h(S) \) (cf., Lemma [1]).

(3) **Isotony:** Let \( S = (S_1, S_2, \ldots, S_n) \) and \( S' = (S_1', S_2', \ldots, S_n') \) be two \( n \)-sets of \( \mathbb{K}^n \) with \( S \subseteq S' \), i.e., \( S_i \subseteq S_i' \) and \( S_i \subseteq S_i' \) for \( i = 2, \ldots, n \) (cf., Lemma [1]). We have \( h(S) = (C_1, C_2, \ldots, C_n) \) and \( h(S') = (C_1', C_2', \ldots, C_n') \) such that:

- \( C_1 = S_1, C_1' = S_1' \) and \( S_i \subseteq S_i' \Rightarrow C_1' \subseteq C_1 \)
- \( C_2 = \{ C_2' \in D_2 \mid (c_1', c_2', c_3', \ldots, c_n') \in Y \land c_1' \in C_1, \forall c_2' \in S_2, \ldots, \forall c_n' \in S_n \} \subseteq C_2 \) for \( i = 2, \ldots, n \)
- \( C_n = \{ C_n' \in D_n \mid (c_1', c_2', \ldots, c_{n-1}', C_n') \in Y \land c_1' \in C_1, \forall c_2' \in C_2, \ldots, \forall c_{n-1}' \in C_{n-1} \} \subseteq C_n \) for \( i = 2, \ldots, n \)

Then, \( C_i' \subseteq C_i \) and \( C_i \subseteq C_i' \) for \( i = 2, \ldots, n \Rightarrow h(S) \subseteq h(S') \).

According to (1), (2) and (3), \( h \) is a closure operator.

For \( n = 4 \), we instantiate the closure operator of a quadratic context, i.e., a d-\textit{folksonomy} as follows:

**Definition 5:** (Closure Operator of a d-Folksonomy)
Let \( S = (A, B, C, E) \) be a quasi-set of \( \mathbb{F}_d \) with \( \mathbb{A} \) being maximal for \( A \times B \times C \times E \subseteq Y \). The closure operator \( h \) of a d-folksonomy \( \mathbb{F}_d \) is defined as follows:

\[
\begin{align*}
\text{h}(S) &= \text{h}(A, B, C, E) = (U, T, R, D) \mid U = A \\
&\land T = \{ (t_i \in T \mid (u_i, t_i, r_i, d_i) \in Y \land u_i \in U, \land r_i \in C, \land d_i \in E) \}
\end{align*}
\]

\( \land R = \{ r_i \in R \mid (u_i, t_i, r_i, d_i) \in Y \land u_i \in U, \land t_i \in T, \land d_i \in E \} \)

\( \land D = \{ d_i \in D \mid (u_i, t_i, r_i, d_i) \in Y \land u_i \in U, \land t_i \in T, \land r_i \in R \} \)

**Remark 2:** Roughly speaking, \( h(S) \) computes the largest quadruple-set in the \( d \)-folksonomy \( \mathbb{F}_d \) which contains maximal sets of tags, resources and dates shared by a group of users. The application of the closure operator \( h \) on a quadruple-set gives rise to a \textit{quadruple-concept} \( qc = (U, T, R, D) \).

Like the dyadic and triadic case, the closure operator splits the search space into equivalence classes, that we introduce in the following:

**Definition 6:** (Equivalence Class) Let \( S_1 = (A_1, B_1, C_1, E_1) \), \( S_2 = (A_2, B_2, C_2, E_2) \) be two quadruple-sets of \( \mathbb{F}_d \), and \( qc \) be a frequent quadruple-concept. \( S_1 \) and \( S_2 \) belong to the same equivalence class represented by the quadruple-concept \( qc \), i.e., \( S_1 \equiv qc \) \( S_2 \) iff \( h(S_1) = h(S_2) = qc \).

![Figure 1. Example of an equivalence class extracted from the d-folksonomy depicted by Table II](image)
2) \((B_1 \subseteq B \land C_1 \subseteq C \land E_1 \subseteq E) \lor (B_1 \subseteq B \land C_1 \subseteq C \land E_1 \subseteq E)\), and

3) \(h(g) = h(g_1) = q_c\).

**Example 3:** Let us consider the d-folksonomy \(F_d\) shown in Table 1. Figure 1 shows an example of an equivalence class. For example, we have \(h(g) = \{\{u_1, u_2, u_3\}, \{t_3, r_1, d_1\}\} = \{\{u_1, u_2, u_3\}, \{t_2, t_3, t_4\}, r_1, \{d_1, d_2\}\} = q_c\) such as \(g_1\) is a quadri-generator. Thus, \(q_c\) is the quadri-concept of this equivalence class which is the largest unsubsumed quadri-set and it has two quadri-generators. However, \(g_2 = \{\{u_1, u_2, u_3\}, \{t_3, t_4\}, r_1, d_1\}\) is not a quadri-generator of \(q_c\) since it exists \(g_1\) such as \(g_1\).extent = \(g_3\).extent, \(g_1\).intent = \(g_3\).intent \& \(g_1\).modus \(\subseteq\) \(g_3\).modus \& \(g_1\).variable = \(g_3\).variable.

Based on those new introduced notions, we propose in the following our new QUADRICONS algorithm for a scalable mining of frequent quadri-concepts from a d-folksonomy.

**B. The QUADRICONS Algorithm**

In the following, we introduce a test-and-generate algorithm, called QUADRICONS, for mining frequent quadri-concepts from a d-folksonomy. Since quadri-generators are minimal keys of an equivalence class, their detection is largely eased. QUADRICONS operates in four steps as follows: the FINDMINIMALGENERATORS procedure as a first step for the extraction of quadri-generators. Then, the CLOSURECOMPUTE procedure is invoked for the three next steps in order to compute respectively the modus, intent and variable parts of quadri-concepts. The pseudo code of the QUADRICONS algorithm is sketched by Algorithm 1. QUADRICONS takes as input a d-folksonomy \(F_d = (U, T, R, D, Y)\) as well as four user-defined thresholds (one for each dimension) : \(\text{minsupp}_u, \text{minsupp}_r, \text{minsupp}_d\) and \(\text{minsupp}_d\). The output of the QUADRICONS algorithm is the set of all frequent quadri-concepts that fulfil these thresholds. QUADRICONS works as follows: it starts by invoking the FINDMINIMALGENERATORS procedure (Step 1), which pseudo-code is given by Algorithm 2 in order to extract the quadri-generators stored in the set \(M_G\) (Line 3). For such extraction, FINDMINIMALGENERATORS computes for each triple \((t, r, d)\) the set \(U_d\) representing the maximal set of users sharing both tag \(t\) and resource \(r\) at the date \(d\) (Algorithm 2 Line 3). If \(|U_u|\) is frequent w.r.t \(\text{minsupp}_u\) (Line 4), a quadri-generator is then created (if it does not already exist) with the appropriate fields (Line 5). Algorithm 2 invokes the ADDQUADRI function which adds the quadri-generator \(g\) to the set \(M_G\) (Line 7).

Hereafter, QUADRICONS invokes the CLOSURECOMPUTE procedure (Step 2) for each quadri-generator of \(M_G\) (Lines 5-7), which pseudo-code is given by Algorithm 3: the aim is to compute the modus part of each quadri-concept. At this step, the two first cases of Algorithm 3 (Lines 3 and 6) have to be considered w.r.t the extent of each quadri-generator. The CLOSURECOMPUTE procedure returns the set \(Q_S\) formed by quadri-sets. The indicator flag (equal

**ALGORITHM 1 : QUADRICONS**

**Data :**

1) \(F_d (U, T, R, D, Y)\) : A d-folksonomy.
2) \(\text{minsupp}_u, \text{minsupp}_r, \text{minsupp}_d, \text{minsupp}_d\) : User-defined thresholds.

**Results :**

\(Q_C\) : {Frequent quadri-concepts}.

1 Begin
2 /*Step 1 : The extraction of quadri-generators*/
3 FINDMINIMALGENERATORS\((F_d, M_G, \text{minsupp}_u)\);
4 /*Step 2 : The computation of the modus part*/
5 Foreach quadri-gen \(g \in M_G\) do
6 CLOSURECOMPUTE(M_G, \text{minsupp}_u, \text{minsupp}_r, \text{minsupp}_d, g, QS, 1);
7 End
8 PRUNEINFREQUENTSETS(QS,\text{minsupp}_u);
9 /*Step 3 : The computation of the intent part*/
10 Foreach quadri-set \(s \in QS\) do
11 CLOSURECOMPUTE(QS, \text{minsupp}_u, \text{minsupp}_r, \text{minsupp}_d, s, QS, 2);
12 End
13 PRUNEINFREQUENTSETS(QS,\text{minsupp}_u);
14 /*Step 4 : The computation of the variable part*/
15 Foreach quadri-set \(s \in QS\) do
16 CLOSURECOMPUTE(QS, \text{minsupp}_u, \text{minsupp}_r, \text{minsupp}_d, s, QC, 3);
17 End
18 PRUNEINFREQUENTSETS(QC,\text{minsupp}_d);
19 End
20 return \(Q_C\);

**ALGORITHM 2 : FINDMINIMALGENERATORS**

**Data :**

1) \(M_G\) : The set of frequent quadri-generators.
2) \(F_d (U, T, R, D, Y)\) : A d-folksonomy.
3) \(\text{minsupp}_u\) : User-defined threshold of user’s support.

**Results :**

\(M_G\) : {The set of frequent quadri-generators}.

1 Begin
2 Foreach triple \((t, r, d)\) of \(F_d\) do
3 \(U_d = \{u_i \in U \mid (u_i, t, r, d) \in Y\}\)
4 If \(|U_\text{d}| \geq \text{minsupp}_u\) then \(g_\text{.extent} = U_u; g_\text{.intent} = r; g_\text{.modus} = t; g_\text{.variable} = d\)
5 If \(g \notin M_G\) then
6 ADDQUADRI(M_G, g)
7 End
8 End
9 End
10 End
11 End
12 return \(M_G\);
ALGORITHM 3: CLOSURECOMPUTE

Data:
1) $S_{IN}$: The input set.
2) $\min_u$, $\min_i$, $\min_r$: User-defined thresholds.
3) $q$: A quadri-generator/quadri-set.
4) $S_{OUT}$: The output set.
5) $i$: an indicator.

Results: $S_{OUT}$: The output set.

Begin

Foreach quadri-set $q' \in S_{IN}$ do

If $i=1$ and $q$.intent = $q'$.intent and $q$.extent $\subseteq q'$.extent then

$s$.intent = $q$.intent; $s$.extent = $q$.extent; $s$.variable = $q$.variable; $s$.modus = $q$.modus $\cup$ $q'$.modus;

ADDQUADRI($S_{OUT}$, $s$);

Else if $i=1$ and $q$.intent = $q'$.intent and $q$ and $q'$ incomparable then

$g$.extent = $q$.extent $\cap$ $q'$.extent; $g$.modus = $q$.modus $\cup$ $q'$.modus; $g$.intent = $q$.intent; $g$.variable = $q$.variable;

If $g$ u-frequent then ADDQUADRI($MG$, $g$);

Else if $i=2$ and $q$.extent $\subseteq q'$.extent and $q$.modus $\subseteq q'$.modus and $q$.intent $\neq q'$.intent then

$q_{s}.extent = q$.extent; $q_{s}.modus = q$.modus; $q_{s}.variable = q$.variable; $q_{s}.intent = q$.intent $\cup$ $q'$.intent;

ADDQUADRI($S_{OUT}$, $q_{s}$);

Else if $i=2$ and $q$ and $q'$ incomparable then

$s$.extent = $q$.extent $\cap$ $q'$.extent; $s$.modus = $q$.modus $\cap$ $q'$.modus; $s$.variable = $q$.variable; $s$.intent = $q$.intent $\cap$ $q'$.intent;

If $s$ is u-frequent and t-frequent then ADDQUADRI($S_{OUT}$, $s$);

Else if $i=3$ and $q$.extent $\subseteq q'$.extent and $q$.modus $\subseteq q'$.modus and $q$.intent $\subseteq q'$.intent and $q$.variable $\neq q'$.variable then

$q_{c}.extent = q$.extent; $q_{c}.modus = q$.modus; $q_{c}.intent = q$.intent; $q_{c}.variable = q$.variable $\cup$ $q'$.variable;

ADDQUADRI($S_{OUT}$, $q_{c}$);

Else if $i=3$ and $q$ and $q'$ incomparable then

$s$.extent = $q$.extent $\cap$ $q'$.extent; $s$.modus = $q$.modus $\cap$ $q'$.modus; $s$.intent = $q$.intent $\cap$ $q'$.intent; $s$.variable = $q$.variable $\cap$ $q'$.variable;

If $s$ is u-frequent, t-frequent and r-frequent then ADDQUADRI($S_{OUT}$, $s$);

End

End

return $S_{OUT}$ ;

end

C. Structural properties of QUADRICONS

Proposition 2: The QUADRICONS algorithm is correct and complete. It retrieves accurately all the frequent quadri-concepts.

Proof: The FINDMINIMALGENERATORS procedure allows to extract all quadri-generators from the $d$-folksonomy $F_d$ since all the context’s triples are enumerated in order to group maximal users w.r.t each triple $(t,r,d)$ (Algorithm 2, Lines 2-10). This allows to extract accurately all the quadri-generators. From quadri-generators already extracted, QUADRICONS calls the CLOSURECOMPUTE procedure three times in order to compute, respectively, the modus, intent and variable parts of each quadri-generator.

At each call, i.e., $i = 1, 2, 3$, for each couple of candidates $q$ and $q'$, two cases have to be considered:

1) (Algorithm 3, lines 3, 10, 18) $q$ and $q'$ are comparable. Hence a quadri-set (quadri-concept when $i = 3$) is created from the union of different parts of both candidates.

2) (Algorithm 3, lines 6, 14, 22) $q$ and $q'$ are incomparable. Hence, a new quadri-set (quadri-generator when $i = 1$) is created matching the different parts of $q$ and $q'$.

Thus, all cases of comparison between candidates are enumerated. Finally, the PRUNEINFRQUENTSETS procedure prune infrequent quadri-concepts w.r.t minimum thresholds (Algorithm 1, lines 8, 13 and 18). We conclude that QUADRICONS faithfully extracts all frequent quadri-concepts. So, it is correct.

Proposition 3: The QUADRICONS algorithm terminates.

Proof: The number of quadri-generators generated by QUADRICONS is finite. Indeed, the number of QGs candidate generated from a context $(U, T, R, D)$ is at most $|T| \times |R| \times |D|$. Since the set $MG$ of quadri-generators is
finite, the three loops of Algorithm 1 running this set are thus finite. Moreover, the total number of quadri-concepts generated by QUADRICONS is equal to \(2^{\|T\|+\|R\|+\|D\|}\). Therefore, the algorithm QUADRICONS terminates.

**Theoretical Complexity issues:** As in the triadic case \([2]\), the number of (frequent) quadri-concepts may grow exponentially in the worst case. Hence, the theoretical complexity of our algorithm is around \(O(2^n)\) with \(n = |T| + |R| + |D|\). Nevertheless, and as it will be shown in the section dedicated to experimental results, from a practical point of view, the actual performances are far from being exponential and QUADRICONS flags out the desired scalability feature. Therefore we focus on empirical evaluations on large-scale real-world datasets.

### D. Illustrative example

Consider the d-folksonomy depicted by Table 1 with \(\text{minsupp}_a = 2\), \(\text{minsupp}_a = 2\), \(\text{minsupp}_a = 1\) and \(\text{minsupp}_d = 1\). Figure 2 sketches the execution trace of QUADRICONS above this context. As described above, QUADRICONS operates in four steps:

1. *(Step 1)* The first step of QUADRICONS involves the extraction of quadri-generators (QGs) from the context (Algorithm 1, Line 3). QGs are maximal sets of users following a triple of tag, resource and date. Thus, eleven QGs (among twelve) fulfill the minimum threshold \(\text{minsupp}_a\) (cf., Figure 2 Step 1).

2. *(Step 2)* Next, QUADRICONS invokes the CLOSURE-COMPUTE procedure a first time on the quadri-generators allowing the computation of the computation of the *modus* part (the set of tags) of such candidates (Algorithm 1, Lines 5-8). For example, since the *extent* part (the set of users) of \(\{u_1, u_2, u_4\}, t_1, r_1, d_1\) is included into that of \(\{u_1, u_2, u_3, u_4\}, t_2, r_1, d_1\), the *modus* part of the first QG will be equal to \(\{t_1, t_2\}\). In addition, new QGs can be created from intersection of the first ones (Algorithm 1 Lines 6-9) : it is the case of the two QGs (a) and (b) (cf., Figure 2 Step 2). Finally, candidates that not fulfill the minimum threshold \(\text{minsupp}_d\) are pruned (cf., the three last ones).

3. *(Step 3)* Then, QUADRICONS proceeds to the computation of the *intent* part (the set of resources) of each candidate within a second call to the CLOSURE-COMPUTE procedure (Algorithm 1, Lines 10-13). For example, the candidate \(\{u_1, u_2, u_4\}, \{t_1, t_2\}, r_1, d_1\) has an *extent*, *modus* and *variable* included or equal into those of the candidate \(\{u_1, u_2, u_4\}, \{t_1, t_2\}, r_2, d_1\). Then, its *intent* will be equal to \(\{r_1, r_2\}\). At this step, four candidates fulfill the minimum threshold \(\text{minsupp}_d\) (cf., Figure 2 Step 3). By merging comparable candidates, this step allow reducing at the same time their number.

4. *(Step 4)* Via a last call to the CLOSURE-COMPUTE procedure, QUADRICONS computes the *variable* part (the set of dates) of each candidate while pruning infrequent ones (Algorithm 1, Lines 15-18). For example, since the candidate \(\{u_1, u_2\}, \{t_1, t_2\}, r_1, d_2\) has an *extent*, *modus* and *intent* included into those of \(\{u_1, u_2, u_4\}, \{t_1, t_2\}, \{r_1, r_2\}, d_1\) its *variable* will be equal to \(\{d_1, d_2\}\) (cf., Figure 2 Step 4).

After the Step 4, QUADRICONS terminates. The four frequent quadri-concepts given as output are:

1. \(\{u_1, u_2, u_4\}, \{t_1, t_2\}, \{r_1, r_2\}, d_1\)
2. \(\{u_1, u_3, u_4\}, \{t_2, t_3\}, \{r_1, r_2\}, d_1\)
3. \(\{u_1, u_4\}, \{t_1, t_2\}, \{r_1, r_2\}, d_1\)
4. \(\{u_1, u_2\}, \{t_1, t_2\}, \{r_1, d_1, d_2\}\)

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**VI. Evaluation and Discussion**

In this section, we show through extensive carried out experiments, the assessment of the QUADRICONS performances vs. the state-of-the-art DATA-PEELER algorithm in

\[\text{minsupp}_a = 2\], \(\text{minsupp}_a = 2\), \(\text{minsupp}_a = 1\) and \(\text{minsupp}_d = 1\). Figure 2 sketches the execution trace of QUADRICONS above the d-folksonomy depicted by Table 1.

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6. Concretely, it means that the users \(u_1\) and \(u_2\) who shared the resource \(r_1\) with the tags \(t_1\) and \(t_2\) at the date \(d_2\) also shared it at the date \(d_1\).
terms of execution time. We also put the focus on the differences between the consumed memory of both algorithms. Finally, we compare the number of frequent quadri-concepts versus the number of frequent quadri-sets in order to assess the compacity of the extracted representation. We have applied our experiments on two real-world datasets described in the following. Both datasets are freely downloadable and statistics about these snapshots are summarized into Table II.

- **MOVIELENS** ([http://movielens.org](http://movielens.org)) is a movie recommendation website. Users are asked to annotate movies they like and dislike. Quadruples are sets of users sharing movies using tags at different dates.

- **LAST.FM** ([http://last.fm](http://last.fm)) is a music website, founded in 2002. It has claimed 30 million active users in March 2009. Quadruples are sets of users annotating artists through tags at different dates.

| Dataset | Type | # Quadruples | # Users | # Tags | # Resources | # Dates (timestamps) | Periods |
|---------|------|--------------|--------|--------|-------------|----------------------|---------|
| MOVIELENS | Dense | 95580 | 4010  | 15227  | 11272 (movies) | 81601 | 07/08/2011 - 02/12/2008 |
| LAST.FM  | Sparse | 186479 | 1892  | 9749   | 12523 (artists) | 3549  | 07/02/2006 - 12/01/2008 |

**Table II**  
**CHARACTERISTICS OF THE CONSIDERED SNAPSHTOS.**

| Datasets | Dates | Users | Tags | Resources |
|----------|-------|-------|------|-----------|
| MOVIELENS | 03/12/05 | krycek | kids fantasy darkness magic | Harry Potter The Prisoner of Azkaban |
|          | 16/07/06 | maria  |        | The Order of the Phoenix |
|          | 21/02/08 |        |        |             |
| LAST.FM  | 07/05/10 | csmdavis | pop concert dance | Britney Spears Madonna |
|          | 02/06/11 | franny rossanna | concert dance |             |

**Table III**  
**EXAMPLES OF FREQUENT QUADRI-CONCEPTS OF MOVIELENS AND LAST.FM.**

A. Examples of quadri-concepts

Table III shows two examples of frequent quadri-concepts extracted from the MOVIELENS and LAST.FM datasets. The first one depicts that the users krycek and maria used the tags kids, fantasy, darkness and magic to annotate the movie *Harry Potter* and its sequels successively in 03/12/2005, in 16/07/2006 and then in 21/02/2008. Such concept may be exploited further for recommending tags for that movie or analyze the evolution of tags associated to “Harry Potter”.

The second quadri-concept shows that the users csmdavis, franny and rossanna shared the tags pop, concert and dance to describe the artists Britney Spears and Madonna in 07/05/10 and then in 02/06/11. We can use such quadri-concept to recommend the users franny and rossanna to the first one, i.e., csmdavis as they share the same interest for both artists using the same tags. It will be also useful to study the evolution of the artist’s fans and the vocabulary they used to annotate them through time.

In the following, in order to assess the performances of QUADRICONS vs. DATA-PEELER while extracting quadri-concepts, we ran both algorithms on both datasets and we vary the values of minimum thresholds as depicted by Tables IV and V.

B. Execution Time

Tables IV and V show the different runtimes of the QUADRICONS algorithm vs. those of DATA-PEELER for the different values of quadruples, which grows from 20000 to 95580 for the MOVIELENS dataset and from 40000 to 186479 for the LAST.FM dataset, and for different values of minimum thresholds. We can observe that for both datasets and for all values of the number of quadruples, DATA-PEELER algorithm is far away from QUADRICONS in terms of execution time. QUADRICONS ran until 332 times faster than DATA-PEELER on LAST.FM and until 124 times on MOVIELENS. Indeed, the poor performance flagged out by DATA-PEELER, is explained by the strategy adopted by this later which starts by storing the entire dataset into a binary tree structure, which should facilitate its run and then the extraction of quadri-concepts. However, such structure is absolutely not adequate to support a so highly sized data, which is the case of the real-world large-scale datasets considered in our evaluation. Contrariwise, The main thrust of the QUADRICONS algorithm stands in the localisation of the quadri-generators (QGs), that stand at the “antipodes” of the closures within their respective equivalence classes. Then, in an effort to improve the existing work, our strategy to locate these QGs have the advantage of making the extraction of quadri-concepts faster than its competitor. This is even more significant in the case of our real-world datasets where the number of data reaches thousands.

C. Consumed Memory

Tables IV and V show the memory consumed by both algorithms on both datasets for the different values of quadruples. We observe that QUADRICONS consumes memory far below its competitor: less than 40000 KB and 20000 KB on both datasets versus millions of KB for DATA-PEELER. Such difference is explained by the fact that QUADRICONS, unlike DATA-PEELER, does not store...
the dataset in memory before proceeding the extraction of quadri-concepts. Furthermore, QUADRICONS generates fewer candidates thanks to the clever detection of quadri-generators that reduce the search space significantly. For example, to extract the 167 quadri-concepts from LAST.FM when \( \text{minsupp}_u = 3 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 1 \) and \( \text{minsupp}_d = 1 \), QUADRICONS requires only 1754 KB in memory while detecting the 939 quadri-generators of the dataset. However, despite the few number of extracted quadri-concepts, DATA PEELER requires 788021 KB in memory to store the entire dataset before generating candidates. Hence, detecting quadri-generators before extracting quadri-concepts allows QUADRICONS consuming until 54 and 115 times less memory than DATA PEELER on respectively MOVIELENS and LAST.FM datasets.

| \( Y \) | QUADRI CONS (sec) | Consumed Memory (kilobytes) | DATA PEELER (sec) | Consumed Memory (kilobytes) |
|---|---|---|---|---|
| \( \text{minsupp}_u = 3 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 1 \), \( \text{minsupp}_d = 1 \) | 6.86 | 542 | 43.10 | 209843 |
| \( \text{minsupp}_u = 2 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 1 \), \( \text{minsupp}_d = 1 \) | 0.97 | 431 | 107.71 | 378907 |
| \( \text{minsupp}_u = 2 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 2 \), \( \text{minsupp}_d = 1 \) | 3.79 | 567 | 227.65 | 667006 |
| \( \text{minsupp}_u = 2 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 1 \), \( \text{minsupp}_d = 1 \) | 5.76 | 2491 | 421.44 | 769822 |
| \( \text{minsupp}_u = 2 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 1 \), \( \text{minsupp}_d = 1 \) | 15.92 | 5246 | 1269.70 | 979200 |
| \( \text{minsupp}_u = 2 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 2 \), \( \text{minsupp}_d = 1 \) | 29.22 | 9845 | 2037.73 | 1153401 |
| \( \text{minsupp}_u = 2 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 1 \), \( \text{minsupp}_d = 1 \) | 48.92 | 16556 | 3478.98 | 1446242 |

Table IV
PERFORMANCES OF QUADRICONS vs. DATA-PEELER ABOVE THE MOVIELENS DATASET.

D. Compacity of Quadri-Concepts

Figure 4 shows the number of frequent quadri-concepts versus the number of frequent quadri-sets on both MOVIELENS and LAST.FM datasets for the different values of quadruples. We observe that for both datasets, the number of frequent quadri-sets increase massively when the number of quadruples grows. Indeed, frequent quadri-concepts become more large, i.e., containing more users, tags, resources and dates. Thus, such concepts cause the steep increase of frequent quadri-sets. For both datasets, the frequent quadri-concepts represent until 3.68 % and 28.99 % of the number of frequent quadri-sets. Hence, computing frequent quadri-sets is a harder task than computing frequent quadri-concepts while providing the same information.

| \( Y \) | QUADRI CONS (sec) | Consumed Memory (kilobytes) | DATA PEELER (sec) | Consumed Memory (kilobytes) |
|---|---|---|---|---|
| \( \text{minsupp}_u = 3 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 1 \), \( \text{minsupp}_d = 1 \) | 0.65 | 114 | 7.13 | 309453 |
| \( \text{minsupp}_u = 2 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 1 \), \( \text{minsupp}_d = 1 \) | 0.99 | 309453 | 445431 |
| \( \text{minsupp}_u = 2 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 2 \), \( \text{minsupp}_d = 1 \) | 15.00 | 320540 | 1561992 |
| \( \text{minsupp}_u = 2 \), \( \text{minsupp}_t = 2 \), \( \text{minsupp}_r = 1 \), \( \text{minsupp}_d = 1 \) | 186479 | 1754 | 255.71 | 788021 |

Table V
PERFORMANCES OF QUADRICONS vs. DATA-PEELER ABOVE THE LAST.FM DATASET.

VII. CONCLUSION AND PERSPECTIVES

In this paper, we considered the quadric context formally described by a \textit{d-folksonomy} with the introduction of a new dimension : time stamp. Indeed, we extend the notion of closure operator and tri-generator to the four-dimensional case and we thoroughly studied their theoretical properties. Then, we proposed the QUADRICONS algorithm in order to extract frequent quadri-concepts from \textit{d-folksonomies}. Several experiments show that QUADRICONS provides an efficient method for mining quadri-concepts in large scale conceptual structures. It is important to highlight that mining quadri-concepts stands at the crossroads of the avenues for future work : (i) analyse evolution of users, tags and
resources through time, (ii) define the quadratic form of association rules according to quadri-concepts.

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A scalable mining of frequent quadratic concepts in d-folksonomies

Abstract—Folksonomy mining is grasping the interest of web 2.0 community of as far user as freely tag resources. However, a scrutiny of the related work unveils that the time stamp dimension has not been considered. For example, the wealthy number of works dedicated to mining tri-concepts from folksonomies did not take into account time dimension. In this paper, we will consider a folksonomy commonly composed of triples \(<\text{users}, \text{tags}, \text{resources}>\) and we shall consider the time as a new dimension. We motivate our approach by highlighting the battery of potential applications and we introduce a new algorithm, called QUADRICONS, as an extension of TRICONS dedicated to the triadic contexts. QUADRICONS aims at getting out quadratic concepts, i.e., quadri-concepts from quadratic contexts. We also introduce a new closure operator that splits the induced search space into equivalence classes whose smallest elements are the quadri-minimal generators. Carried out experiments on large-scale real-world snapshots of social networks highlight very interesting results about analyzing trend detection in folksonomies starting from quadri-concepts.

Keywords—Quadratic Context; Formal Concept Analysis; Quadratic Concepts; Folksonomies; algorithm

I. INTRODUCTION

-FCA extended depuis 95 au Triadic Concept Analysis : not much attention coming of folksonomies
-Folksonomy (definition informelle, rise of folk) / web 2.0
[1] Folksonomy (from folk and taxonomy) is a neologism for a practice of collaborative categorization using freely chosen keywords. Folksonomies (also called social tagging mechanisms) have been implemented in a number of online knowledge sharing environments since the idea was first adopted by social bookmarking site del.icio.us in 2004. The idea of a folksonomy is to allow the users to describe a set of shared objects with a set of keywords of their own choice.

-folksonomy mining triconcepts : repre.condense des folk tricons les surpase
-timestamp forgot:importance du temps en 1 frase

Time is considered one of the most important factors in detecting emerging subjects. In this paper, confluence of both lines of research : FCA (cas 4-aire:QCA) + mining sequential patterns, donner exemple de harry potter

The remainder of the paper is organized as follows. Section 2 recalls the key notions used throughout this paper. We thoroughly study the related work in Section 3. In Section 4, we introduce a new closure operator for the quadratic context as well as the QUADRICONS algorithm dedicated to the extraction of frequent quadri-concepts. In Section 5, carried out experiments about performances of our algorithm and analyzing trend detections. Finally, we conclude the paper with a summary and we sketch some avenues for future works in Section 6.

II. MOTIVATION : CONCEPTUAL AND TEMPORAL CLUSTERING OF FOLKSONOMIES

The immediate success of social networks, i.e., social resource sharing systems is due to the fact that no specific skills are needed for participating. Each individual user is able to share a web page, a personal photo, an artist he like or a movie he watched without much effort.

The core data structure of such systems is a folksonomy. It consists of three sets U, T, R of users assigning tags to resources as well as a ternary relation Y between them. To allow conceptual and temporal clustering from folksonomies, an additional dimension is needed: time. Within this new dimension, our goal is to detect hidden sequential conceptualizations in folksonomies. An exemple of such a concept is that users which tagged "Harry Potter" will tag "The Prisoner of Azkaban" and then tag "The Order of the Phoenix", probably with the same tags.

Our algorithm solves the problem of frequent closed patterns mining for this kind of data. It will return a set of (frequent) quadruples, where each quadruple \((U, T, R, D)\) consists of a set \(U\) of users, a set \(T\) of tags, a set \(R\) of resources and a set \(D\) of dates. These quadruples, called (frequent) quadri-concepts, have the property that each user in \(U\) has tagged each resource in \(R\) with all tags from \(T\) at the different dates from \(D\), and that none of these sets can be extended without shrinking one of the other three dimensions. Hence, they represent the four-dimensional extension of tri-concepts. Moreover, we can add minimum support constraints on each of the four dimensions in order to focus on the largest concepts of the folksonomy, i.e., by setting higher values of minimum supports.

III. THE PROBLEM OF MINING ALL FREQUENT QUADI-CONCEPTS

In this section, we formalize the problem of mining all frequent quadri-concepts. We start with an adaptation of the notion of folksonomy to the quadratic context.

Definition 1: (D-FOLKSONOMY) A d-folksonomy is a set of tuples \(\mathbb{F}_d = (\mathcal{U}, \mathcal{T}, \mathcal{R}, \mathcal{D}, Y)\) where \(\mathcal{U}, \mathcal{T}, \mathcal{R}\) and \(\mathcal{D}\) are

1http://del.icio.us
2http://flickr.com
3http://last.fm
4http://movielens.org
finite sets which elements are called users, tags, resources and dates. \( Y \subseteq U \times T \times R \times D \) represents a quaternary relation which each \( y \subseteq Y \) can be represented by a quadruple: \( y = \{ (u, t, r, d) \mid u \in U, t \in T, r \in R, d \in D \} \) which means that the user \( u \) has annotated the resource \( r \) using the tag \( t \) at the date \( d \).

**Example 1:** Table I depicts an example of a d-folksonomy \( F_d \) with \( U = \{ u_1, u_2, u_3, u_4 \} \), \( T = \{ t_1, t_2, t_3 \} \), \( R = \{ r_1, r_2 \} \) and \( D = \{ d_1, d_2 \} \). Each cross within the quaternary relation indicates a tagging operation by a user from \( U \), a tag from \( T \) and a resource from \( R \) at a date from \( D \), i.e., a user has tagged a particular resource with a particular tag at a date \( d \). For example, the user \( u_1 \) has tagged the resource \( r_1 \) with the tags \( t_1 \), \( t_2 \) and \( t_3 \) at the date \( d_1 \).

| \( F_d \) | \( Ud/T \) | \( r_1 \) | \( r_2 \) |
|-------|----------|--------|--------|
| \( d_1 \) | \( u_1 \) | \( x \) | \( x \) | \( x \) |
|       | \( u_2 \) | \( x \) | \( x \) | \( x \) |
|       | \( u_3 \) | \( x \) | \( x \) | \( x \) |
|       | \( u_4 \) | \( x \) | \( x \) | \( x \) |
| \( d_2 \) | \( u_1 \) | \( x \) | \( x \) |        |
|       | \( u_2 \) | \( x \) | \( x \) |        |
|       | \( u_3 \) | \( x \) |             |
|       | \( u_4 \) | \( x \) |             |

Table I: A d-folksonomy.

The following definition introduces a (frequent) quadri-set.

**Definition 2:** (A (Frequent) Quadri-set) Let \( F_d = (U, T, R, D, Y) \) be a d-folksonomy. A quadri-set of \( F_d \) is a quadruple \( (A, B, C, E) \) with \( A \subseteq U, B \subseteq T, C \subseteq R \) and \( E \subseteq D \) such as \( A \times B \times C \times E \subseteq Y \).

D-Folksonomies have four dimensions which are completely symmetric. Thus, we can define minimum support thresholds on each dimension. Hence, the problem of mining frequent quadri-sets is then the following:

**Problem 1:** (Mining all frequent quadri-sets) Let \( F_d = (U, T, R, D, Y) \) be a d-folksonomy and let \( \text{minsupp}_u, \text{minsupp}_t, \text{minsupp}_r, \text{minsupp}_d \) be user-defined minimum thresholds. The task of mining all frequent quadri-sets consists in determining all quadri-sets \( (A, B, C, E) \) of \( F_d \) with \( | A | \geq \text{minsupp}_u, | B | \geq \text{minsupp}_t, | C | \geq \text{minsupp}_r, \) and \( | E | \geq \text{minsupp}_d \).

Our thresholds are antimonotonic constraints: If \( (A_1, B_1, C_1, E_1) \) with \( A_1 \) being maximal for \( A_1 \times B_1 \times C_1 \times E_1 \subseteq Y \) is not u-frequent\(^5\) then all \( (A_2, B_2, C_2, E_2) \) with \( B_1 \subseteq B_2 \) and \( C_1 \subseteq C_2 \) are not u-frequent either. The same holds symmetrically for the other two dimensions. In [2], the authors demonstrate that above the two-dimensional case, the direct symmetry between monotonicity and antimonotonicity breaks. Thus, they introduced a lemma which results from

the triadic Galois connection [2] induced by a triadic context.

In the following, we adapt that lemma to our quadratic case.

**Lemma 1:** Let \( (A_1, B_1, C_1, E_1) \) and \( (A_2, B_2, C_2, E_2) \) be quadri-sets with \( A_1 \) being maximal for \( A_1 \times B_1 \times C_1 \times E_1 \subseteq Y \), for \( i = 1, 2 \). If \( B_1 \subseteq B_2, C_1 \subseteq C_2 \) and \( E_1 \subseteq E_2 \) then \( A_2 \subseteq A_1 \). The same holds symmetrically for the other three directions. In the sequel, the inclusion \( (A_1, B_1, C_1, E_1) \subseteq (A_2, B_2, C_2, E_2) \) holds if and only if \( B_1 \subseteq B_2, C_1 \subseteq C_2, E_1 \subseteq E_2 \) and \( A_2 \subseteq A_1 \).

**Example 2:** Let \( F_d \) be the d-folksonomy of Table I and let \( S_1 = \{ \{ u_3, u_4 \}, t_3, \{ r_1, r_2 \}, \{ d_1, d_2 \} \} \) and \( S_2 = \{ \{ u_1, u_3, u_4 \}, \{ t_2, t_3 \}, \{ r_1, r_2 \}, d_1 \} \) be two quadri-sets of \( F_d \). We have \( S_1 \subseteq S_2 \) since \( \{ u_3, u_4 \} \subseteq \{ u_1, u_3, u_4 \} \), \( t_3 \subseteq \{ t_2, t_3 \} \), \( \{ r_1, r_2 \} \subseteq \{ r_1, r_2 \} \) and \( d_1 \subseteq \{ d_1, d_2 \} \).

As the set of all frequent quadri-sets is highly redundant, we consider a specific condensed representation, i.e., a subset which contains the same information: the set of all frequent quadri-concepts. The latter’s definition is given as follows:

**Definition 3:** (Frequent Quadratic Concept) A quadratic concept (or a quadri-concept for short) of a d-folksonomy \( F_d = (U, T, R, D, Y) \) is a quadruple \( (U, T, R, D) \) with \( U \subseteq U, T \subseteq T, R \subseteq R \) and \( D \subseteq D \) with \( U \times T \times R \times D \subseteq Y \) such that the quadruple \( (U, T, R, D) \) is maximal, i.e., none of these sets can be extended without shrinking one of the other three dimensions. A quadri-concept is said to be frequent whenever it is a frequent quadri-set.

**Problem 2:** (Mining all frequent quadri-concepts) Let \( F_d = (U, T, R, D, Y) \) be a d-folksonomy and let \( \text{minsupp}_u, \text{minsupp}_t, \text{minsupp}_r, \text{minsupp}_d \) be user-defined minimum thresholds. The task of mining all frequent quadri-concepts consists in determining all quadri-concepts \( (U, T, R, D) \) of \( F_d \) with \( | U | \geq \text{minsupp}_u, | T | \geq \text{minsupp}_t, | R | \geq \text{minsupp}_r, \) and \( | D | \geq \text{minsupp}_d \). The set of all frequent quadri-concepts of \( F_d \) is equal to \( QC = \{ QC \mid QC = (U, T, R, D) \) is a frequent quadri-concept\}. In the remainder, we will scrutinize the state-of-the-art propositions aiming to mine quadratic concepts from d-folksonomies.

**IV. RELATED WORK**

- voutsadakis
- datapeeler

**sans critiques!!! (laisser pr comparaison)**

In [3], Voutsadakis generalized the constructs and results of Wille [2] to the n-adic contexts. The author gives a
definition of an n-adic concept as well as that of a complete n-lattice of a n-adic context. Moreover, it was shown that the n-adic concepts of an n-adic context $K$ form a complete n-lattice with respect to component-wise defined quasi-orders. To illustrate those new definitions, Voutsadakis gives an example of quadratic concepts and their associated complete Boolean 4-lattice. Despite robust theoretical study, no algorithm has been proposed by Voutsadakis for an efficient extraction of quadratic concepts. In addition, despite that of an n-adic concept, no basic notion of data mining (minimal generator, equivalence class, etc.) was adapted to the n-adic context. Finally, no potential applications were proposed in order to illustrate the usefulness of such concepts. Recently, Cerf et al. proposed the DATA-PEELER algorithm [4] which is able to extract all closed concepts from n-ary relations. It enumerates all the n-dimensional closed patterns in a depth first manner using a binary tree enumeration strategy. When $n = 4$, the DATA-PEELER algorithm is able to extract quadratic concepts. However, DATA-PEELER is hampered by the large number of elements that may contain any of the dimensions and its strategy becomes ineffective and leads to a complex computation of n-adic concepts. In the following, we review some approaches dealing with trend detection in folksonomies which illustrates the usefulness of the quadratic concepts and the consideration of the time dimension in folksonomies.

V. THE QUADRICONS ALGORITHM FOR MINING ALL FREQUENT QUADRI-CONCEPTS

A. Main Notions of QUADRICONS

Before introducing our closure operator for a d-folksonomy, we define a closure operator of a n-adic context. In [5], Voutsadakis define n-closure operators for a n-adic context. Each i-closure operator aims to compute the closed part related to the dimension i for a given n-set (1 ≤ i ≤ n). In what follows, we introduce a new closure operator $h$ which is able to compute the closure of a given n-set. Contrariwise to [5], we use a single closure operator that computes a single time all closed parts of the resulting n-adic concept.

**Definition 4:** (Closure Operator of a n-Adic Context) Let $S = (S_1, S_2, \ldots, S_n)$ be a n-set, with $S_1$ being maximal for $S_1 \times \ldots \times S_n \subseteq Y$, of a n-adic context $\mathbb{K}^n$ with n dimensions, i.e., $\mathbb{K}^n = (D_1, D_2, \ldots, D_n, Y)$. A mapping $h$ is defined as follows:

$h(S) = h(S_1, S_2, \ldots, S_n) = (C_1, C_2, \ldots, C_n)$ such that:

$C_1 = S_1$

$\wedge C_2 = \{c_2^i \in D_2 \mid (c_1^i, c_2^i, c_3^i, \ldots, c_n^i) \in Y \forall c_1^i \in C_1, \forall c_3^i \in S_3, \ldots, \forall c_n^i \in S_n\}$

$\vdots$

$\wedge C_n = \{c_n^i \in D_n \mid (c_1^i, c_2^i, \ldots, c_{n-1}^i, c_n^i) \in Y \forall c_1^i \in C_1, \ldots, \forall c_{n-1}^i \in C_{n-1}\}$

**Proposition 1:** $h$ is a closure operator.

**Proof:** To prove that $h$ is a closure operator, we have to prove that this closure operator fulfills the three properties of extensivity, idempotency and isotony [6].

1. **Extensivity:** Let $S = (S_1, S_2, \ldots, S_n)$ be a n-set of $\mathbb{K}^n \Rightarrow h(S) = (C_1, C_2, \ldots, C_n)$ such that: $C_1 = S_1, C_2 = \{c_2^i \in D_2 \mid (c_1^i, c_2^i, c_3^i, \ldots, c_n^i) \in Y \forall c_1^i \in C_1, \forall c_3^i \in S_3, \ldots, \forall c_n^i \in S_n\} \supseteq S_2$ since $C_1 \subseteq S_1$ since $C_1 = S_1$ . . . , $C_n = \{c_n^i \in D_n \mid (c_1^i, c_2^i, \ldots, c_{n-1}^i, c_n^i) \in Y \forall c_1^i \in C_1, \forall c_2^i \in C_2, \ldots, \forall c_{n-1}^i \in C_{n-1}\} \supseteq S_n$ since $C_1 = S_1, C_2 \supseteq C_2, S_3 \ldots, S_n \supseteq S_n$. Then, $C_1 = S_1$ and $S_i \subseteq C_i$ for $i = 2, \ldots, n \Rightarrow S \subseteq h(S)$ (cf., Lemma 1).

2. **Idempotency:** Let $S = (S_1, S_2, \ldots, S_n)$ be a n-set of $\mathbb{K}^n \Rightarrow h(S) = (C_1, C_2, \ldots, C_n)$ such that: $C_1 = C_1, C_2 = C_2 \in D_2$ ($c_1^i, c_2^i, c_3^i, \ldots, c_n^i) \in Y \forall c_1^i \in C_1, \forall c_3^i \in S_3, \ldots, \forall c_n^i \in S_n$) $C_2$ since $C_1 = S_1$ , . . . , $C_n = C_n \in D_n \mid (c_1^i, c_2^i, \ldots, c_{n-1}^i, c_n^i) \in Y \forall c_1^i \in C_1, \forall c_2^i \in C_2, \ldots, \forall c_{n-1}^i \in C_{n-1}\}$ $C_n$ since we have $C_1 = C_1, C_2 = C_2, \ldots, C_n = C_n$. Then, $C_1$ = $C_i$ for $i = 1, \ldots, n \Rightarrow h(h(S)) = h(S)$.

3. **Isotony:** Let $S = (S_1, S_2, \ldots, S_n)$ and $S' = (S_1', S_2', \ldots, S_n')$ be two n-sets of $\mathbb{K}^n$ with $\subseteq S'$, i.e., $S_i \subseteq S'_i$ and $S_i \subseteq S_i'$ for $i = 2, \ldots, n$ (cf., Lemma 1). We have $h(S) = (C_1, C_2, \ldots, C_n)$ and $h(S') = (C_1', C_2', \ldots, C_n')$ such that:

$C_1 = S_1, C_1' = S_1'$ and $S_i \subseteq S_i' \Rightarrow C_i \subseteq C_i'$

$C_2 = \{c_2^i \in D_2 \mid (c_1^i, c_2^i, c_3^i, \ldots, c_n^i) \in Y \forall c_1^i \in C_1, \forall c_3^i \in S_3, \ldots, \forall c_n^i \in S_n\}$ and $C_2' = \{c_2^i \in D_2 \mid (c_1^i, c_2^i, c_3^i, \ldots, c_n^i) \in Y \forall c_1^i \in C_1, \forall c_3^i \in S_3, \ldots, \forall c_n^i \in S_n\} \Rightarrow C_2 \subseteq C_2'$ since $S_i \subseteq S_i'$ for $i = 3, \ldots, n$ and $C_1 \subseteq C_1$. (cf., Lemma 1).

$C_n = \{c_n^i \in D_n \mid (c_1^i, c_2^i, \ldots, c_{n-1}^i, c_n^i) \in Y \forall c_1^i \in C_1, \forall c_2^i \in C_2, \ldots, \forall c_{n-1}^i \in C_{n-1}\}$ and $C_n' = \{c_n^i \in D_n \mid (c_1^i, c_2^i, \ldots, c_{n-1}^i, c_n^i) \in Y \forall c_1^i \in C_1', \forall c_2^i \in C_2', \ldots, \forall c_{n-1}^i \in C_{n-1}'\} \Rightarrow C_n \subseteq C_n'$ since $C_n \subseteq C_1$, $C_2 \subseteq C_2'$, . . . , $C_{n-1} \subseteq C_{n-1}'$. (cf., Lemma 1).

Then, $C_i \subseteq C_i$ and $C_i \subseteq C_i'$ for $i = 2, \ldots, n \Rightarrow h(S) \subseteq h(S')$.

According to (1), (2) and (3), $h$ is a closure operator. □

For $n=4$, we instantiate the closure operator of a quadratic context, i.e., a d-folksonomy as follows:

**Definition 5:** (Closure Operator of a D-Folksonomy) Let $S = (A, B, C, E)$ be a quadri-set of $\mathbb{K}_d$ with $A$ being maximal for $A \times B \times C \times E \subseteq Y$. The closure operator $h$ of a d-folksonomy $\mathbb{K}_d$ is defined as follows:

$h(S) = h(A, B, C, E) = (U, T, R, D)$ $U = A$

$\wedge T = \{t_i \in T \mid (u_i, t_i, r_i, d_i) \in Y \forall u_i \in U, \forall r_i \in C, \forall d_i \in E\}$

$\wedge R = \{r_i \in R \mid (u_i, t_i, r_i, d_i) \in Y \forall u_i \in U, \forall r_i \in T, \forall d_i \in E\}$
An Example of an Equivalence Class from the d-folksonomy depicted by Table I

Minimal Generators (MGs) have been shown to play an important role in many theoretical and practical problem settings involving closure systems. Such minimal generators can offer a complementary and simpler way to understand the concept, because they may contain far fewer attributes than closed concepts. Indeed, MGs represent the smallest elements within an equivalence class. Complementary to closures, minimal generators provide a way to characterize formal concepts [7]. In the following, we introduce an extension of the definition of a MG to the d-folksonomy.

Definition 7: (Quadri-Minimal Generator) Let \( q = (A, B, C, E) \) be a quadri-set of \( \mathbb{F}_d \) such as \( A \subseteq U, B \subseteq T, C \subseteq R \) and \( E \subseteq D \) and \( QC \in QC \). The quadraple \( g \) is a quadri-minimal generator (quadri-generator for short) of \( QC \) iff \( h(g) = QC \) and \( h(g_1) = (A_1, B_1, C_1, E_1) \) such as:

1) \( A = A_1 \),
2) \( B_1 \subseteq B \land C_1 \subseteq C \land E_1 \subseteq E \) or \( (B_1 \subseteq B \land C_1 \subseteq C \land E_1 \subseteq E) \), and
3) \( h(g) = h(g_1) = QC \).

Example 3: Let us consider the d-folksonomy \( \mathbb{F}_d \) shown in Table I. Figure 1 shows an example of an equivalence class. For example, we have \( h(g_1)=\{u_1, u_2, u_3, t_3, r_1, d_1\} = QC \) such as \( g_1 \) is a quadri-generator. The largest unsubsumed quadri-set \( QC \) has two quadri-generators \( g_1 \) and \( g_2 \). However, \( g_3 = \{u_1, u_2, u_3\}, \{t_3, t_4, r_1, d_1\} \) is not a quadri-generator of \( QC \) since it exists \( g_1 \) such as \( g_1,ext\equiv g_2.\)

B. The QuadriCons Algorithm

In the following, we introduce a test-and-generate algorithm, called QuadriCons, for mining frequent quadri-concepts from a d-folksonomy. Since quadri-generators are minimal keys of an equivalence class, their detection is largely eased. QuadriCons operates in four steps as follows: the FindMinimalGenerators procedure as a first step for the extraction of quadri-generators. Then, the ClosureCompute procedure is invoked for the three next steps in order to compute respectively the modus, intent and variable parts of quadri-concepts. The pseudo code of the QuadriCons algorithm is sketched by Algorithm 1. QuadriCons takes as input a d-folksonomy \( \mathbb{F}_d = (U, T, R, D, Y) \) as well as four user-defined thresholds (one for each dimension): \( \minsupp_u, \minsupp_t, \minsupp_r, \minsupp_d \). The output of the QuadriCons algorithm is the set of all frequent quadri-concepts that fulfill these thresholds. QuadriCons works as follows: it starts by invoking the FindMinimalGenerators procedure (Step 1), which pseudo-code is given by Algorithm 2, in order to extract the quadri-generators stored in the set \( \mathcal{MG} \) (Line 3). For such extraction, FindMinimalGenerators computes for each triple \((t, r, d)\) the set \( U_t \) representing the maximal set of users sharing both tag \( t \) and resource \( r \) at the date \( d \) (Algorithm 2, Line 3). If \( |U_t| \) is frequent w.r.t \( \minsupp_u \) (Line 4), a quadri-generator is then created (if it does not already exist) with the appropriate fields (Line 5). Algorithm 2 invokes the AddQuadri function which adds the quadri-generator \( g \) to the set \( \mathcal{MG} \) (Line 7).

Hereafter, QuadriCons invokes the ClosureCompute procedure (Step 2) for each quadri-generator of \( \mathcal{MG} \) (Lines 5-7), which pseudo-code is given by Algorithm 3: the aim is to compute the modus part of each quadri-concept. At this point, the two first cases of Algorithm 3 (Lines 3 and 6) have to be considered w.r.t the extent of each quadri-generator. The ClosureCompute procedure returns the set \( \mathcal{QS} \) formed by quadri-sets. The indicator flag (equal
ALGORITHM 3: ClosureCompute

Data:
1) $S_{IN} :$ The set of frequent quadri-generators/quadri-sets.
2) $min_u, min_t, min_r :$ User-defined thresholds of extent, modus and intent support.
3) $q :$ A quadri-generator/quadri-set. $S_{OUT} :$ {The set of frequent quadri-sets/quadri-concepts}.
4) $i :$ an indicator.

Results: $S_{OUT} :$ {The set of frequent quadri-sets/quadri-concepts}.

Begin
Foreach quadri-set $q' \in S_{IN}$ do
  If $i=1$ and $q$.intent $= q'$.intent and $q$.extent $\subseteq q'$.extent then
    $s$.intent $= q$.intent; $s$.extent $= q$.extent; $s$.variable $= q$.variable; $s$.modus $= q$.modus $\cup q'$.modus;
    ADDQUADRI($S_{OUT}, s$);
  End
  Else if $i=1$ and $q$.intent $= q'$.intent and $q$ and $q'$ incomparable then
    $g$.extent $= q$.extent $\cap q'$.extent; $g$.modus $= q$.modus $\cup q'$.modus; $g$.intent $= q$.intent; $g$.variable $= q$.variable;
    If $|g$.extent$| \geq min_u$ then ADDQUADRI($M_{G}, g$);
  End
  Else if $i=2$ and $q$.extent $\subseteq q'$.extent and $q$.modus $\subseteq q'$.modus and $q$.intent $\neq q'$.intent then
    $QC$.extent $= q$.extent; $QC$.modus $= q$.modus; $QC$.variable $= q$.variable; $QC$.intent $= q$.intent $\cup q'$.intent;
    ADDQUADRI($S_{OUT}, QC$);
  End
  Else if $i=2$ and $q$ and $q'$ incomparable then
    $s$.extent $= q$.extent $\cap q'$.extent; $s$.modus $= q$.modus $\cap q'$.modus; $s$.variable $= q$.variable; $s$.intent $= q$.intent $\cup q'$.intent;
    If $|s$.extent$| \geq min_u$ and $|s$.modus$| \geq min_t$ then ADDQUADRI($S_{OUT}, s$);
  End
  Else if $i=3$ and $q$.extent $\subseteq q'$.extent and $q$.modus $\subseteq q'$.modus and $q$.intent $\subseteq q'$.intent and $q$.variable $\neq q'$.variable then
    $QC$.extent $= q$.extent; $QC$.modus $= q$.modus; $QC$.intent $= q$.intent; $QC$.variable $= q$.variable $\cup q'$.variable;
    ADDQUADRI($S_{OUT}, QC$);
  End
Else if $i=3$ and $q$ and $q'$ incomparable then
  $s$.extent $= q$.extent $\cap q'$.extent; $s$.modus $= q$.modus $\cap q'$.modus; $s$.intent $= q$.intent $\cap q'$.intent;
  $s$.variable $= q$.variable $\cup q'$.variable;
  If $|s$.extent$| \geq min_u$ and $|s$.modus$| \geq min_t$ and $|s$.intent$| \geq min_r$ then ADDQUADRI($S_{OUT}, s$);
End
End

return $S_{OUT} ;$
**ALGORITHM 1 : QUADRICONS**

Data :
1) $\mathcal{F}_d (U, T, R, D, Y) : A$ d-folksonomy.
2) $\minsupp_u, \minsupp_t, \minsupp_r, \minsupp_d : \text{User-defined thresholds.}$

Results : $\mathcal{QC} : \{\text{Frequent quadri-concepts}\}.$

Begin
1/*Step 1 : The extraction of quadri-generators*/
2 FINDMINIMALGENERATORS$(\mathcal{F}_d, \mathcal{MG}, \minsupp_u);$ 
3 /*Step 2 : The computation of the modulus part*/ 
4 Foreach quadri-gen $g \in \mathcal{MG}$ 
5 ClosureCompute$(\mathcal{MG}, \minsupp_u, \minsupp_t, \minsupp_r, r, QS, 1);$ 
6 end
7 /*Step 3 : The computation of the intent part*/
8 Foreach quadri-set $s \in QS$ do 
9 ClosureCompute$(QS, \minsupp_u, \minsupp_t, \minsupp_r, s, QC, 2);$ 
10 end
11 /*Step 4 : The computation of the variable part*/
12 Foreach quadri-set $s \in QC$ do 
13 ClosureCompute$(QC, \minsupp_u, \minsupp_t, \minsupp_r, s, QC, 3);$ 
14 end
15 PRUNEINFREQUENTSETS$(TC, \minsupp_d);$ 
16 End
17 return $\mathcal{QC} ;$

**ALGORITHM 2 : FINDMINIMALGENERATORS**

Data :
1) $\mathcal{MG} : \text{The set of frequent quadri-generators.}$
2) $\mathcal{F}_d (U, T, R, D, Y) : A$ d-folksonomy.
3) $\minsupp_u : \text{User-defined threshold of user’s support.}$

Results : $\mathcal{MG} : \{\text{The set of frequent quadri-generators}\}.$

Begin
1 Foreach triple $(t, r, d)$ of $\mathcal{F}_d$ do 
2 $U_s = \{ u_i \in U \mid (u_i, t, r, d) \in Y \};$
3 If $| U_s | \geq \minsupp_u$ then 
4 $g.\text{extent} = U_s; \text{g.intent} = r; \text{g.modus} = t; \text{g.variable} = d;$
5 If $g \notin \mathcal{MG}$ then 
6 ADDQUADRI$(\mathcal{MG}, g)$  
7 End 
8 End 
9 End 
10 return $\mathcal{MG} ;$ 

The fourth and final step of QUADRICONS invokes a last time the ClosureCompute procedure with an indicator equal to 3. This will allow to focus on quadri-sets having different variable parts before generating quadri-concepts. QUADRICONS comes to an end after this step and returns the set of the frequent quadri-concepts which fulfills the four thresholds $\minsupp_u, \minsupp_t, \minsupp_r$ and $\minsupp_d$. The QUADRICONS algorithm invokes the PRUNEINFREQUENTSETS function (Lines 8, 13 and 18) in order to prune infrequent quadri-sets/concepts, i.e., whose the modulus/intent/variable cardinality does not fulfill the aforementioned thresholds.

C. Structural properties of QUADRICONS

Proposition 2: The QUADRICONS algorithm is correct and complete. It retrieves accurately all the frequent quadri-concepts.

Proof: 

Proposition 3: The QUADRICONS algorithm terminates.

Proof: 

Theoretical Complexity issues:

D. Illustrative example

Consider the d-folksonomy depicted by Table I, with $\minsupp_u = 2, \minsupp_t = 2, \minsupp_r = 1$ and $\minsupp_d = 1$. Figure 2 sketches the execution trace of QUADRICONS above this context. As described above, QUADRICONS operates in four steps:

Step 1: The first step of QUADRICONS involves the extraction of quadri-generators ($QGs$) from the context. $QGs$ are maximal sets of users following a triple of tag, resource and date. Thus, the eleven $QGs$ that fulfill the minimum threshold $\minsupp_u$ are described by Figure 2 (Step 1).

Step 2: Next, QUADRICONS invokes the ClosureCompute procedure a first time on the quadri-generators allowing the computation of the modulus part of such candidates. For example, since the extent part of $\{\{u_1, u_2, u_4\}, t_1, r_1, d_1\}$ is included into that of $\{\{u_1, u_2, u_3, u_4\}, t_2, r_1, d_1\}$, the modulus part of the first $QG$ will be equal to $\{t_1, t_2\}$. Moreover, new $QGs$ can be created from the intersection of the first ones (see Algorithm 3, Line xx): it is the case of the two $QGs$ (a) and (b) (cf.,
Figure 2, Step 3. Finally, candidates that not fulfill the minimum threshold \( \text{minsupp} \) are pruned (cf., the three last ones).

Step 3. Then, QUADRICONS proceeds at the computation of the intent part of each candidate within a second call to the CLOSURECOMPUTE procedure. For example, the candidate \( \{u_1, u_2, u_4, \{t_1, t_2\}, r_1, d_1\} \) has an extent, modus and variable included or equal to those of the candidate \( \{u_1, u_2, u_4, \{t_1, t_2\}, r_2, d_1\} \). Then, its intent will be equal to \( \{r_1, r_2\} \). At this step, four candidates fulfill the minimum thresholds over the intent part (Figure 2, Step 3). By merging comparable candidates, this step allow reducing at the same time their number.

Step 4. Via a last call to the CLOSURECOMPUTE procedure, QUADRICONS computes the variable part of each candidate while pruning infrequent ones. Since the candidate \( \{u_1, u_2, \{t_1, t_2\}, r_1, d_2\} \) has an extent, modus and intent included into those of \( \{u_1, u_2, u_4, \{t_1, t_2\}, r_1, r_2, d_1\} \), its variable will be equal to \( \{d_1, d_2\} \).

After the Step 4, QUADRICONS terminates. The four frequent quadri-concepts given as output are:

1. \( \{u_1, u_2, u_4, \{t_1, t_2\}, r_1, r_2, d_1\} \)
2. \( \{u_1, u_2, u_4, \{t_2, t_3\}, r_1, r_2, d_1\} \)
3. \( \{u_1, u_2, \{t_1, t_2\}, r_1, r_2, d_1\} \)
4. \( \{u_1, u_2, \{t_1, t_2\}, r_1, d_1, d_2\} \)

VI. EVALUATION AND DISCUSSION

In this section, we show through extensive carried out experiments, the assessment of the QUADRICONS performances vs. DATA-PEELER. We also put the focus on the differences between the consumed memory of both algorithms. Moreover, we compare the number of frequent quadri-concepts versus the number of frequent quadri-sets in order to assess the compacity of the extracted representation. We have applied our experiments on two real-world datasets described in the following. Statistics about these snapshots are summarized into Table II.

**MOVIELENS** (http://movielens.org) is a movie recommendation website. Users are asked to note movies they like and dislike. The MOVIELENS dataset used for our experiments is freely downloadable [8].

**LAST.FM** (http://last.fm) is a music website, founded in 2002. It has claimed 30 million active users in March 2009. The LAST.FM dataset used for our experiments is freely downloadable [8].

Table III shows two examples of frequent quadri-concepts extracted from the MOVIELENS and LAST.FM datasets. The first one depicts that the users krycek and maria used the tags kids, fantasy, darkness and magic to annotate the movie Harry Potter and its sequels successively in 03/12/2005, in 16/07/2006 and then in 21/02/2008. Such concept may be exploited further for recommending tags for that movie or analyze the evolution of tags associated to "Harry Potter".

The second quadri-concept shows that the users csmdavis, franny and rossanna shared the tags pop, concert and dance to describe the artists Britney Spears and Madonna at two different dates. We can use such quadri-concept to recommend the users franny and rossanna to the first one, i.e., csmdavis as they share the same interest for both artists using the same tags.

| Datasets    | Dates         | Users       | Tags            | Resources                |
|-------------|---------------|-------------|-----------------|--------------------------|
| MOVIELENS   | 03/12/2005    | krycek     | kids            | Harry Potter             |
|             | 16/07/2006    | maria      | fantasy         | The Prisoner of Azkaban  |
|             | 21/02/2008    |            | darkness        | The Order of the Phoenix |
| LAST.FM     | 07/05/2010    | csmdavis   | pop             | Britney Spears           |
|             | 02/06/2011    | franny     | concert         | Madonna                  |
|             |               | rossanna   | dance           |                          |

### A. Execution Time

### B. Consumed Memory

### C. Compacity of Quadri-Concepts

VII. CONCLUSION AND PERSPECTIVES

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Figure 2. Example of an equivalence class extracted from the d-folksonomy depicted by Table I

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| # Quandruples | Minimum Thresholds | QUADRICONS  | DATA PEELER | Minimum Thresholds | QUADRICONS  | DATA PEELER | Minimum Thresholds | QUADRICONS  | DATA PEELER |
|---------------|--------------------|-------------|-------------|--------------------|-------------|-------------|--------------------|-------------|-------------|

Table IV

Performances of QUADRICONS vs. DATA-PEELER above the MovieLens and Last.fm datasets.