Spontaneous breaking of Lorentz invariance may take place in string theories, possibly endowing the photon with a mass. This leads to the breaking of the conformal symmetry of the electromagnetic action allowing for the generation within inflationary scenarios of magnetic fields over Mpc scales. We show that the generated fields are consistent with amplification by the galactic dynamo processes and can be as large as to explain the observed galactic magnetic fields through the collapse of protogalactic clouds.

1 Introduction

Magnetic field of nearby galaxies coherent over Mpc scales are estimated to be of order $B \sim 10^{-6}$ G. The most plausible explanation for these fields involves some sort of dynamo effect. Indeed, if one assumes that a galactic dynamo has operated during about 10 Gyr then a seed magnetic field could be amplified by a factor of $e^{30}$ and the observed galactic magnetic fields at present may have had its origin in a seed magnetic field of about $B \sim 10^{-19}$ G. On the other hand, the galactic magnetic fields can emerge directly from the compression of a primordial magnetic field, in the collapse of protogalactic clouds. In this case, it is required a seed magnetic field of $B \sim 10^{-9}$ G over a scale $\lambda \sim$ Mpc, the comoving size of a region which condenses to form a galaxy. Since the Universe through most of its history has behaved as a good conductor it implies that the evolution of any primeval cosmic magnetic field will conserve magnetic flux. Thus, the ratio denoted by $r$, of the energy density of a magnetic field $\rho_B = \frac{B^2}{8\pi}$ relative to the energy density of the cosmic microwave background radiation $\rho_{\gamma} = \frac{\pi^2}{15}T^4$ remains essentially constant and provides a invariant measure of magnetic field strength. It then follows that pregalactic magnetic fields of about $r \approx 10^{-34}$ are required if one invokes dynamo amplification processes, and $r \approx 10^{-8}$ if one assumes only the collapse of protogalactic clouds.

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In what follows we shall describe a mechanism to generate primordial magnetic fields that is based on a putative violation of the Lorentz invariance in string field theory solutions and that relies on inflation for the amplification of quantum fluctuations of the electromagnetic field. Invoking a period of inflation to explain the creation of seed magnetic fields is a quite attractive suggestion as inflation provides the means of generating large-scale phenomena from microphysics that operates on subhorizon scales. Indeed, inflation, through de Sitter-space-produced quantum fluctuations, generates excitations of the electromagnetic field allowing for an increase of the magnetic flux before the Universe is filled with a highly conducting plasma. Furthermore, in this amplification process, long-wavelength modes for which $\lambda \geq H^{-1}$, are enhanced.

However, it is not possible to produce the required seed magnetic fields from a conformally invariant theory as is the usual U(1) gauge theory. The reason being that, in a conformally invariant theory, the magnetic field decreases as $a^{-2}$, where $a$ is the scale factor, and during inflation, the total energy density in the Universe is constant, so the magnetic field energy density is strongly suppressed, yielding $r = 10^{-10} \lambda_{\text{Mpc}}^{-4}$.

In the context of string field theory, there exists solutions where conformal invariance may be broken, actually due to the possibility of spontaneous breaking of the Lorentz invariance. This possibility arises explicitly from solutions of string field theory of the open bosonic string, as interactions are cubic in the string field and these give origin in the static field theory potential to cubic interaction terms of the type $SSS$, $STT$ and $TTT$, where $S$ and $T$ denote scalar and tensor fields. Lorentz invariance may then be broken as can be seen, for instance, from static potential involving the tachyon, $\langle \varphi \rangle$, and a generic vector field. The vacuum of this model is unstable and gives rise to a mass-squared term for the vector field that is proportional to $\langle \varphi \rangle$. If $\langle \varphi \rangle$ is negative, then the Lorentz symmetry itself is spontaneously broken as the vector field can acquire a non-vanishing vacuum expectation values. This mechanism can give rise to vacuum expectation values to tensor fields inducing for the fields that do not acquire vacuum expectation values, such as the photon, mass-squared terms proportional to $\langle T \rangle$. Hence, one should expect from this mechanism terms for the photon of the form $\langle T \rangle A_\mu A^\mu$, $\langle T_{\mu\nu} \rangle A^\mu A^\nu$ and so on. Naturally, these terms break explicitly the conformal invariance of the electromagnetic action.

Observational constraints on the breaking of the Lorentz invariance arising from the measurements of the quadrupole splitting time dependence of nuclear Zeeman levels along Earth’s orbit, have been performed over the years, the most recent one indicating that $\delta < 3 \times 10^{-21}$. Bounds on the violation of momentum conservation and existence of a preferred reference frame can be
also extracted from limits on the parametrized post-Newtonian parameter $\alpha_3$ obtained from the period of millisecond pulsars, namely $|\alpha_3| < 2.2 \times 10^{-20}$ implying the Lorentz symmetry is unbroken up to this level. These limits indicate that if the Lorentz invariance is broken then its violation is suppressed by powers of energy over the string scale. Similar conclusions can be drawn for possible violations of the CPT symmetry.

In order to relate the theoretical possibility of spontaneous breaking of Lorentz invariance to the observational limits discussed above we parametrize the vacuum expectation values of the Lorentz tensors in the following way:

$$< T > = m_L^2 \left( \frac{E}{M_S} \right)^{2l},$$

where $m_L$ is a light mass scale when compared to string typical energy scale, $M_S$, where we assume that $M_S \approx M_P$, $M_P$ being the Planck mass; $E$ is the temperature of the Universe in a given period and $2l$ is a positive integer. We shall further replace the temperature of the Universe by the inverse of the scale factor, given that expansion of the Universe is adiabatic. Parametrization (1) is similar to the one used in previous work.

2 Generation of Seed Magnetic Fields

We consider spatially flat Friedmann-Robertson-Walker cosmologies with the metric given in the conformal time, $\eta$, the corresponding scale factor being, $a(\eta)$, and the stress tensor of a perfect fluid. The Hubble constant is written as $H_0 = 100 \, h_0 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$ and the present Hubble radius is $R_0 = 10^{26} \, h_0^{-1} \, \text{m}$, where $0.4 \leq h_0 \leq 1$. We shall assume the Universe has gone through a period of exponential inflation at a scale $M_{\text{GUT}}$ and whose associated energy density is given by $\rho_I \equiv M_{\text{GUT}}^4$. Hence, from the Friedmann equation, $H_0 = (\frac{8\pi}{3})^{1/2} \frac{M_{\text{GUT}}^2}{M_S^2}$. From our discussion on the breaking of Lorentz invariance we consider for simplicity only the term, $(T)A_\mu A^\mu$, from which follows the Lagrangian density for the photon:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + M_L^2 a^{-2l} A_\mu A^\mu,$$

where $M_L^2 = \frac{m_L^2}{M_P^2}$. One can readily obtain the wave equation for the magnetic field:

$$\frac{1}{a^2} \frac{\partial^2}{\partial \eta^2} a^2 \vec{B} - \nabla^2 \vec{B} + \frac{n}{\eta^2} \vec{B} = 0.$$
The corresponding equation for the Fourier components of $\vec{B}$ is given by:

$$\ddot{\vec{F}}_k + k^2 \vec{F}_k - \frac{n}{\eta^2} \vec{F}_k = 0,$$

(4)

where the dots denote derivatives according to the conformal time and $\vec{F}_k(\eta) \equiv a^2 \int d^3x e^{i\vec{k}.\vec{x}} \vec{B}(\vec{x}, \eta)$, $\vec{F}_k$ being a measure of the magnetic flux associated with the comoving scale $\lambda \sim k^{-1}$. The energy density of the magnetic field is given by $\rho_B(k) \propto \frac{|\vec{F}_k|^2}{a^4}$.

For modes well outside the horizon, $a\lambda >> H^{-1}$ or $|k\eta| << 1$, solutions of Eq. (4) are given in terms of the conformal time $\eta$:

$$|\vec{F}_k| \propto \eta^{m_{\pm}}$$

(5)

where $m_{\pm} = \frac{1}{2} \left[ 1 \pm \sqrt{1 - 4n} \right]$.

By requiring that $n$ is not a growing function of conformal time, it follows that $n$ has to be either a constant or that $2l$ is negative, which is excluded by our assumption (3). Hence for different phases of evolution of the Universe:

(I) Inflationary de Sitter (dS) phase, where $a(\eta) \propto -\frac{1}{\eta H_{dS}}$, it follows that $l = 0$ and

$$n = -\frac{M_{dS}^2}{H_{dS}^2},$$

(6)

where we refer $M_L$ by its index in the relevant phase of evolution of the Universe.

(II) Phase of Reheating (RH) and Matter Domination (MD), where $a(\eta) \propto \frac{1}{4} H_0^2 R_0^3 \eta^2$, yields from the condition $n$ is a constant that $2l = 3$ and

$$n = -\frac{4M_{dS}^2}{H_0^2 R_0^3}.$$  

(7)

(III) Phase of Radiation Domination (RD), where $a(\eta) \propto H_0 R_0^2 \eta$, from which follows that $l = 2$ and

$$n = -\frac{M_{dS}^2}{H_0^2 R_0^3}.$$  

(8)

It is clear that in this case last case $n \ll 1$.

Assuming the Universe has gone through a period of inflation at scale $M_{GUT}$ and that fluctuations of the electromagnetic field have come out from the horizon when the Universe had gone through about 55 $e$-foldings of inflation, yields in terms of $r$:

$$r \approx (7 \times 10^{25})^{-2(p+2)} \times (\frac{M_{GUT}}{M_P})^{4(q-p)/3} \times (\frac{T_{RH}}{M_P})^{2(2q-p)/3} \times (\frac{T_{MD}}{M_P})^{-8q/3} \times \lambda^{-2(p+2)},$$

(9)
where $T_*$ is the temperature at which plasma effects become dominant and can be estimated from the reheating process:

$$T_* = \min\{ (T_{RH} M_{GUT})^{\frac{1}{2}} ; (T_{RH}^2 M_p)^{\frac{1}{3}} \}.$$  

For the reheating temperature we assume either a poor or a quite efficient reheating, $T_{RH} = \{ 10^9 \text{ GeV}; M_{GUT} \}$. Finally, $p \equiv m_{-dS} = \frac{1}{2} \left[ 1 - \sqrt{1 + \left( \frac{2 m_{dS}}{M_{dS}} \right)^2} \right]$ and $q \equiv m_{+RH} = \frac{1}{2} \left[ 1 + \sqrt{1 + 16 \frac{M^2}{M_0^2}} \right]$ are the fastest growing solutions for $\vec{F}_k$ in the de Sitter and reheating phases, respectively.

In order to obtain numerical estimates for $r$ we have to compute $M_L$. At the de Sitter phase we have that $M_L = m_{dS}$. As we have seen $m_L$ is a light energy scale when compared to $M_P$ and $M_{GUT}$, and hence we introduce a parameter, $\chi$, so that $m_{dS} = \chi M_{GUT}$ and $\chi \ll 1$.

At the matter domination phase, we have to impose that the mass term $M_{MD} = m_{MD}(\frac{T}{M_P})^4$, $T_\gamma$ being the temperature of the cosmic background radiation at about the recombination time, is consistent with the present-day limits of the photon mass, $m_\gamma \leq 3 \times 10^{-36} \text{ GeV}$\textsuperscript{11}. Thus, at the matter domination phase, we have to satisfy the condition, $M_{MD} \leq m_\gamma$, which implies that $m_{MD}(\frac{T}{M_P})^{3/2} \leq 3 \times 10^{-36} \text{ GeV}$, following that $m_{MD} \leq 7.8 \times 10^4 \text{ GeV}$. A more stringent bound on $m_{MD}$ could be obtained from the limit $m_\gamma \leq 1.7 \times 10^{-42} h_0 \text{ GeV}$ arising from the absence of rotation in the polarization of light of distant galaxies due to Faraday effect\textsuperscript{12}.

We present in the following table our estimates for the ratio $r$ for $M_{GUT} = 10^{16} \text{ GeV}$. One can see that we obtain values that are in the range $10^{-35} < r < 10^{-9}$, where a poor reheating and the lower values for $\chi$ tend to render $r$ too low even for an amplification via dynamo processes. Estimates for different values of $M_{GUT}$ can be found in Ref. [1].

Table 1: Values of $r = \frac{\rho_B}{\rho_\gamma}$ at 1 Mpc for $M_{GUT} = 10^{16} \text{ GeV}$
3 Summary

We have shown that the strength of the magnetic field produced by considering the spontaneous breaking of the Lorentz symmetry in the context of string theory together with inflation is sensitive to the values of the light mass, $m_L$, (cf. Eq. (1)), $M_{GUT}$, and the reheating temperature, $T_{RH}$. Our results indicate that for rather diverse set of values of these parameters we can obtain values for $r$ that are consistent with amplification via galactic dynamo or collapse of protogalactic clouds.

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