Riemann-Hilbert Problems

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Abstract: Historically, the development of modern Harmonic Analysis has been inexorably linked with the theory of Complex Variables in the plane. One of the central topics in Complex Analysis deals with the issue of identifying natural conditions that uniquely determine a holomorphic function. A prominent role in this regard is played by the One-Sided Riemann-Hilbert problem formulated as the boundary value problem

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\begin{align*}
\bar{\partial} u &= 0 \quad \text{in } \Omega, \\
\Re(\Phi u) &= f \quad \text{on } \partial \Omega,
\end{align*}
\]

where \(\Phi\) is a fixed function and \(f\) is the boundary datum. This problem was first proposed by B. Riemann in the 1851 and the first significant progress has been registered by D. Hilbert who suggested an approach based on Singular Integral Operators. In this lecture I will use the One-Sided Riemann-Hilbert problem to show the fruitful interplay between Complex Analysis, Potential Theory, Geometric Measure Theory, and Index Theory, and time permitting I will discuss recent sharp results (joint work with M. Mitrea and M. Taylor).