Distribution of ion velocity near an isolated dust particle

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Abstract. In the current paper, a numerical model with self-consistent ion velocity distribution function calculated near an isolated dust particle is presented. This dust particle is under the influence of an external electrostatic field. As a result of calculation, ion velocity distribution function along an axis aligned with the external electrostatic field is obtained. The calculated data allows obtaining the set of parameters for which the approximation of the shifted Maxwell distribution function is applicable.

1. Introduction
Dust is a solid micron-sized substance often revealing itself in etching or deposition plasmas [1]. The appearance of such particles in gas discharge plasma is generally referred to as a negative phenomenon. In plasma, coagulated dust grains are usually negatively charged with their charge being equal to $Z_d = -10^3 - 10^5$ e. Investigations of dust particles have proved that in an external electromagnetic field an oscillating structure behind dust particles is formed. This structure is called “wake” [2]. But the majority of these studies do not take into consideration that ion velocity distribution function is non-Maxwellian [12].

Theoretical studies have shown that the ion velocity distribution function affects the charge of the dust particles in a significant way [13]. Using the PIC simulation, it is shown how the ion velocity distribution function affects the charge of a Langmuir probe [14]. Hence, the calculation of the self-consistent ion velocity distribution functions is of great importance. There are currently numerical models that calculate the ion velocity distribution functions depending on a given plasma drift velocity [15], [16]. However, to this date there is no such study for the 3D-models.

To achieve the solution of abovementioned problem, a new numerical model was created on the basis of methods utilized in [17]–[19]. In the presented model the ion velocity distribution function is determined in a close proximity to an isolated spherical dust particle.
Model

The model given in this paragraph describes a solution region in which an isolated spherical dust particle and the surrounding plasma are present. Based on the task, the geometry of the domain was set as a cube. A sphere $r_0 << \lambda_i$ ($\lambda_i$ is an ion Debye length) was placed in the center of the cube. In this system an external electrostatic field is also present and directed along the $z$-axis.

At the beginning, an ion was generated, coordinates and velocity components of which were set randomly. These ion velocities obeyed the Maxwell distribution for a temperature $T_i = 300$ K. In order to calculate this ion trajectory, Newton’s motion equations were used.

The computational domain was divided into cells ($i, j$) by azimuthal angle and radius with each cell’s volume being equal to $V_{i,j} = 2\pi r_i^2 \Delta r_i \sin \theta_j \Delta \theta_j$.

The time $T_{i,j}$ that the ions have spent in the $i, j$ cell is summed up and normalized to $V_{i,j}$:

$$n_i(i, j) = \frac{T_{i,j}}{V_{i,j}}. \tag{1}$$

The value of $n_i(i, j)$ is proportional to the ion density $n_i(r, \theta)$ in a cell $i, j$. The coefficient of proportionality is determined by the formula:

$$A = \frac{n_i}{\langle n_i(i, j) \rangle_{\text{border}}}, \tag{2}$$

where $\langle n_i(i, j) \rangle_{\text{border}}$ is the time accumulated in the boundary cells of the domain.

The average velocity spatial distribution is calculated similarly:

$$u_i(i, j) = \sum_{\text{time}} \frac{u_i dt}{T_{i,j}}. \tag{3}$$

where $dt$ is the time step chosen for calculating the Newton equations.

The $z$-component of ion velocity is of the most importance, because it is parallel to the external electric field. In this paper, the ion velocity distribution function was measured on an $z$-axis ($x=0, y=0$).

The velocity distribution function is defined as follows: an array is set, each element of which corresponds to a small range $\Delta u_z$. The total time during which the ion had a velocity from $u_z - 0.5\Delta u_z$ to $u_z + 0.5\Delta u_z$ is recorded. Then this time is normalized to $N$.

![Figure 1. Simplest presentation of the computational region division into cells.](image)
In the classical case the ion velocity distribution function will coincide with the shifted Maxwell distribution function:

\[ f_{\text{M}}(u_z) = \frac{m_i}{\sqrt{2\pi kT_i}} \exp\left[\frac{-m_i(u_z - u_d)^2}{2kT_i}\right] \]

where \( u_d \) is the drift velocity.

To calculate self-consistent potential spatial distributions the dimensionless variables were utilized. The dimensionless dust particle charge and the dimensionless external electric field strength were calculated by the ratios:

\[ Q = \frac{e^2 Z_i}{\lambda_i kT_i}; \quad E = \frac{eE \lambda_i}{kT_i}. \]

The initial potential in the computational domain was set as a superposition of the Debye-Huckel and external electric field potentials:

\[ U(r, \theta) = -\frac{Q}{r} e^{-r} - \tilde{E} r \cos \theta. \]

After statistics were accumulated (3), the space charge was expanded into the Legendre polynomials:

\[ n(r, \theta) = \frac{n_i(r, \theta) - n_e(r, \theta)}{n_e} = \sum_{k=0}^{\infty} n_k(r) P_k(\cos \theta), \]

\[ n_k(r) = \frac{2k+1}{2} \int_0^r n(r, \theta) P_k(\cos \theta) \sin \theta d\theta. \]

Here, the electron density spatial distribution \( n_e(r, \theta) = n_e \exp(-eU(r, \theta)/kT_i) \).

The selfconsistent potential of the computational domain was calculated next:

\[ U(r, \theta) = -\frac{Q}{r} \int \sum_{k=0}^{\infty} U_k(r) P_k(\cos \theta) - \tilde{E} r \cos \theta = -\frac{Q}{r} + \sum_{k=0}^{\infty} \frac{P_k(\cos \theta)}{2k+1} \times \]

\[ \left[ \frac{1}{r^{k+1}} \int_0^r n_k(r) x^{k+2} dx + r^k \int_0^\infty n_k(r) x^{1-k} dx \right] - \tilde{E} r \cos \theta. \]

The iterative scheme for the self-consistent potential determination is as follows:

1) The ion trajectories are calculated by the Newton’s motion equations, which are calculated for the potential given in a form (7).

2) From the trajectories it is calculated how long ions were present in each segment of space, what their average velocities were and what the spread of named velocities was.

3) Distribution \( n(r, \theta) \) is calculated and \( n_i(r, \theta) \) expanded into polynomials (8).

4) A potential \( U(r, \theta) \) is calculated (9).

5) The calculation process repeats itself, recurring to step one.
3. Results

The data presented in the current paragraph obtained for the following set of parameters: ion temperature $T_i = 273 \text{ K}$, temperature ratio $T_e/T_i = 100$, dust particles radii $r_0 = 1 \mu \text{m}$.

Figures 2 and 3 show the dependence of ion velocity distribution function on the distance to the dust particle for the value of the external electrostatic field $\vec{E} = 0.3$. In the Figure 2, ion velocity distribution function downstream of the dust particle is depicted and in the Figure 3 - upstream of the dust particle. In these figures are shown a shifted Maxwellian distributions of the ion drift velocities corresponding to the value of the external electrostatic field $\vec{E} = 0.3$. From these dependences it can be seen how more distorted ion velocity distribution functions become with the approach to the dust particle. The ion velocity distribution functions calculated for $z > 1 \lambda_i$ coincide with the shifted Maxwell distribution functions. At distances $0.1 \lambda_i$ ion velocity distribution function is almost symmetrical, which corresponds to the ion orbiting around dust particles. In both cases - upstream and downstream of the dust particle - ion velocity distribution function distortions at distances $z > 1 \lambda_i$ become insignificant.

![Figure 2](image1.png)  
*Figure 2. Dependence of the ion velocity distribution function $f(u_z)$ measured behind the dust particle on the distance to the dust particle for $\vec{E} = 0.3$*

![Figure 3](image2.png)  
*Figure 3. Dependence of the ion velocity distribution function $f(u_z)$ measured in front of the dust particle on the distance to the dust particle for $\vec{E} = 0.3$*

Figure 4 shows a comparison of the ion velocity distribution functions calculated for the values of external electric field $\vec{E} = 0.3$, and $\vec{E} = 2.0$ and measured at a distances $0.1 \lambda_i$ and $20 \lambda_i$. The data presented in Figure 4 demonstrate the warping of the ion velocity distribution function, which occurs under the action of an external electric field. From this figure it is clearly visible that the ion velocity distribution function ceases to be almost symmetrical at $z = 0.1 \lambda_i$ and ceases to correspond to the shifted Maxwell distributions at $z = 20 \lambda_i$. Thus, using the data brained in this model, it is possible to establish the boundaries at which the Maxwell approximation for the ion velocity distribution function works.

![Figure 4](image3.png)  
*Figure 4. Comparison of the ion velocity distribution functions calculated for the values of external electric field $\vec{E} = 0.3$, and $\vec{E} = 2.0$ and measured at a distances $0.1 \lambda_i$ and $20 \lambda_i$.*

Figure 5 shows a comparison of the ion velocity distribution functions calculated for the values of mean free path of the ion-neutral charge exchange process $l_i = 5 \lambda_i$, and $l_i = 10 \lambda_i$ and measured at a distances $0.1 \lambda_i$ and $20 \lambda_i$. Figure 5 shows that with the increase of the mean free path, the distribution function measured at $z = 0.1 \lambda_i$ is distorted relative to the zero, while maintaining a semi-symmetrical appearance. More significant differences in the results obtained for $z = 20 \lambda_i$. It can be seen that for $l_i = 10 \lambda_i$, the ion velocity distribution function ceases to correspond to the shifted Maxwell distribution, becomes wider, and its maximum becomes smaller than the shifted Maxwell distribution maximum.

![Figure 5](image4.png)  
*Figure 5. Comparison of the ion velocity distribution functions calculated for the values of mean free path of the ion-neutral charge exchange process $l_i = 5 \lambda_i$, and $l_i = 10 \lambda_i$ and measured at a distances $0.1 \lambda_i$ and $20 \lambda_i$.*
Figure 4. Dependence of the ion velocity distribution function \( f(u) \) measured behind the dust particle for the values of external electric field \( \vec{E} = 0.3 \), and \( \vec{E} = 2.0 \) and measured at a distances \( 0.1 \lambda_i \) and \( 20 \lambda_i \).

Figure 5. Dependence of the ion velocity distribution function \( f(u) \) measured behind the dust particle for the values of mean free path of the ion-neutral charge exchange process \( l_i = 5 \lambda_i \), and \( l_i = 10 \lambda_i \) and measured at distances \( 0.1 \lambda_i \) and \( 20 \lambda_i \).

4. Conclusion

In this paper, a new model was demonstrated. This model allows calculating the ion velocity distribution functions for various parameters of the dusty plasma: the intensity of the external electrostatic field and the ions mean free path of the ion-neutral resonant charge exchange process.

The dependence of the ion velocity distribution function on the distance to the dust particle was demonstrated. It is shown that in the proximity to the dust particle, the ion velocity distribution function is semi-symmetrical, however, at a distances \( r > \lambda_i \) the ion velocity distribution function is close to the shifted Maxwell distribution.

It is shown how, with an increase of the external electric field strength the ion velocity distribution function is distorted. Similar distortions are demonstrated for the case of the increase of the value of ions mean free path of the ion-neutral charge exchange process. From the obtained data it can be concluded that the ion velocity distribution function coincides with the shifted Maxwell distribution, in the case of the external electric field \( \vec{E} < 0.6 \), the value of the mean free path is close to \( l_i \approx 5 \lambda_i \), and when measurements of the ion velocity distribution function are made at distances \( z > \lambda_i \).

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