Angular Momentum about the Contact Point for Control of Bipedal Locomotion: Validation in a LIP-based Controller

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Abstract—In the control of bipedal locomotion, linear velocity of the center of mass has been widely accepted as a primary variable for summarizing a robot’s state vector. The ubiquitous massless-legged linear inverted pendulum (LIP) model is based on it. In this paper, we argue that angular momentum about the contact point has several properties that make it superior to linear velocity for feedback control. So as not to confuse the benefits of angular momentum with any other control design decisions, we first reformulate the standard LIP controller in terms of angular momentum. We then implement the resulting feedback controller on the 20 degree-of-freedom bipedal robot, Cassie Blue, where each leg accounts for nearly one-third of the robot’s total mass of 35 Kg. Under this controller, the robot achieves fast walking, rapid turning while walking, large disturbance rejection, and locomotion on rough terrain. The reasoning developed in the paper is applicable to other control design philosophies, whether they be Hybrid Zero Dynamics or Reinforcement Learning.

I. INTRODUCTION

Maintaining “balance” is widely viewed as the most critical problem in bipedal locomotion. The notion of “balance” needs to be quantified so that it can be transformed into a feedback control objective. Some represent “balance” with an asymptotically stable periodic orbit [3], [11]. A common approach is to summarize the status of a nonlinear high-dimensional robot model with a few key variables. The most frequently proposed variables as surrogates for “balance” include Center of Mass (COM) velocity [6], [9], [13], [15], [19], COM position [5], Capture Point [7], [23], Zero Moment Point [17], [27], and Angular Momentum [10], [20].

In this paper we choose to regulate angular momentum about the contact point step-to-step as our primary control objective. Some of angular momentum’s desirable properties in bipedal locomotion have been highlighted and exploited for feedback control in [10], [12], [22], [26], [30]. We choose to demonstrate our results using a reformulation of the Linear Inverted Pendulum (LIP) model [18] in terms of angular momentum about the contact point, as in [20], [21]. We emphasize that the LIP model, when reformulated in terms of angular momentum, has higher fidelity when applied to realistic robot models, than when based on linear velocity.

Powell and Ames [20] developed a similarly reformulated LIP model and they chose to regulate the angular momentum at the beginning of the next step through touch down timing and transfer of momentum at impact. Here we take advantage of the higher fidelity of predicted angular momentum about the contact point and choose to regulate it at the end of next step, which can then be more effectively controlled by foot placement [24], [28].

To demonstrate that our results transfer in practice to a realistic bipedal robot, we implement the resulting feedback controller on the 20 degree-of-freedom bipedal robot, Cassie Blue, where each leg accounts for nearly one-third of the robot’s total mass of 32 Kg. In experiments, Cassie Blue is able to execute walking in a straight line up to 2.1 m/s, simultaneously walking forward and diagonally on grass at 1 m/s, make quick, sharp turns, and handle very challenging undulating terrain. For the purpose of completeness, we note that a LIP-inspired controller organized around COM velocity has been implemented on a Cassie-series robot in [31].

The main contributions of the paper are as follows:

• Demonstrate that the one-step-ahead prediction of angular momentum about the contact point provided by a LIP model is superior to a one-step-ahead prediction of linear velocity of the center of mass when applied to realistic robots;
• Formulate a foot placement strategy based on the one-step-ahead prediction of angular momentum.
• Demonstrate the resulting controller can achieve highly dynamic gaits on a 3D bipedal robot with legs that are far from massless; see Fig. 1

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Fig. 1: Cassie Blue, by Agility Robotics, on the iconic University of Michigan Wave Field.
We will use both Rabbit [4] and Cassie Blue to illustrate our developments in the paper. Experiments will be conducted exclusively on Cassie. Rabbit is a 2D biped with five links, four actuated joints, and a mass of 32 Kg; see Fig. 2. Each leg weighs 10 kg, with 6.8 kg on each thigh and 3.2 kg on each shin.

The bipedal robot shown in Fig. 1, named Cassie Blue, is designed and built by Agility Robotics. The robot weighs 32kg. It has 7 deg of freedom on each leg, 5 of which are actuated by motors and 2 are constrained by springs; see Fig. 2. A floating base model of Cassie has 20 degrees of freedom. Each foot of the robot is blade-shaped and provides 5 holonomic constraints when it is on ground. Though each of Cassie’s legs has approximately 10 kg of mass, most of the mass is concentrated on the upper part of the leg. In this regard, the mass distribution of Rabbit is a bit more typical of bipedal robots, which is why we include the Rabbit model in the paper.

The remainder of the paper is organized as follows. Section II and III introduce angular momentum and the LIP model. In Section IV, we show how to predict the evolution of angular momentum with a LIP model and how to use the prediction to decide foot placement. This provides a feedback controller that will stabilize a 3D LIP. In Section V, we provide our path to implementing the controller on Cassie Blue. Additional reference trajectories are required beyond a path for the swing foot, and we provide “an intuitive” method for their design. Section VI addresses several practical challenges associated with the passage from simulation to experimental implementation on Cassie. Section VII shows the experiment results. Conclusion are given in Sect. VIII.

II. ANGULAR MOMENTUM ABOUT CONTACT POINT

Some of the properties of angular momentum in bipedal locomotion have been discussed in [12], [22], [30], and in [10], non-holonomic virtual constraints are created with angular momentum. The view we take here is considerably different for these references. In the following, we will address two questions: why we can replace linear momentum with angular momentum for the design of feedback controllers and what are the benefits of doing so.

Initially, we address the single support phase of walking, meaning only one leg is in contact with the ground. Moreover, we are considering a point contact.

Let $L$ denote the angular momentum about the contact point of the stance leg. The relationship between angular momentum and linear momentum for a 3D biped robot is

$$L = L_{CoM} + p \times m_{tot}v_{CoM},$$  \hspace{1cm} (1)

where $L_{CoM}$ is the angular momentum about the center of mass, $v_{CoM}$ is the linear velocity of the center of mass, $m_{tot}$ is the total mass of the robot, and $p$ is the vector emanating from the contact point to the center of mass.

For a biped robot that is walking instead of doing somersaults, the angular momentum about the center of mass must oscillate about zero. Hence, (1) implies that the difference between $L$ and $p \times m_{tot}v_{CoM}$ also oscillates around zero, which we will write as

$$L - p \times m_{tot}v_{CoM} = L_{CoM}$$ oscillates about 0. \hspace{1cm} (2)

From (2), we see that we approximately obtain a desired linear velocity by regulating $L$.

The discussion so far has focused on a single support phase of a walking gait. Bipedal walking is characterized by the transition between left and right legs as they alternately take on the role of stance leg (aka support leg) and swing leg (aka non-stance leg). In double support, the transfer of angular momentum between the two contact points satisfies

$$L_2 = L_1 + p_{2 \rightarrow 1} \times m_{tot}v_{CoM}$$ \hspace{1cm} (3)

where $L_i$ is the angular momentum about contact point $i$ and $p_{1 \rightarrow 2}$ is the vector from contact point 1 to contact point 2.

Hence, one can replace the control of linear velocity with control of angular momentum about the contact point. But what are the advantages?

(a) The first advantage of controlling $L$ is that it provides a more comprehensive representation of current walking status by including both $L_{CoM}$ and $p \times m_{tot}v_{CoM}$, between which momentum moves forth and back during a step.

(b) Secondly, $L$ has a relative degree three with respect to motor torques, if ankle torque is zero. Indeed, in this case,

$$\dot{L} = p \times m_{tot}g.$$ \hspace{1cm} (4)

where $g$ is the gravitational constant. Consequently, $L$ is very weakly affected by peaks in motor torque that often occur in off nominal conditions. Moreover, if a limb, such as the swing leg, is moving quickly in response to a disturbance, it will strongly affect the angular momentum about the center of mass and the robot’s linear velocity, while leaving the angular momentum about the contact point only weakly affected.
(c) Thirdly, $\dot{L}$ is ONLY a function of the center of mass position, making it easy to predict its trajectory over a step.

(d) Angular momentum about a given contact point is invariant under impacts at that point, and the change of angular momentum between two contact points depends only on the vector defined by the two contact points and the CoM velocity. Hence, we can easily determine the angular momentum about the new contact point by \((3)\) when impact happens without approximating assumptions about the impact model. Moreover, if the vertical component of the $v_{\text{CoM}}$ is zero and the ground is level, then $p_{2-1} \times m_{\text{tot}} v_{\text{CoM}} = 0$ and hence $L_2 = L_1$.

Figure 3 shows simulation plots of $L$, $L_{\text{CoM}}$, and $v_{\text{CoM}}$ for the planar biped robot, Rabbit, and the 3D biped robot, Cassie Blue. It is seen that the angular momentum about the contact point has the advantages discussed above.

III. LINEAR INVERTED PENDULUM MODEL

This section provides the ubiquitous Linear Inverted Pendulum (LIP) model of Kajita et al. \[18\]. The LIP model assumes the center of mass moves in a plane, the angular momentum about the center of mass is constant, and the legs are massless. Here, we will express the model in terms of its original coordinates, namely position and angular momentum about the contact point. The dynamics of the inverted pendulum are exactly linear. Moreover, the 3D dynamics in the $x$ and $y$ directions are decoupled, and hence we only need to consider a 2D pendulum.

Let $H$ denote the height of the center of mass. For a 2D model, the dynamics in the $x$ direction is

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ g/H & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix},$$

where $x$ is the position of CoM in the frame of contact point. If we assume there is no ankle torque. The solution of this linear system is

$$\begin{bmatrix} x(T) \\ \dot{x}(T) \end{bmatrix} = \begin{bmatrix} \cosh(\ell(T - t)) & 1/\ell \sinh(\ell(T - t)) \\ \ell \sinh(\ell(T - t)) & \cosh(\ell(T - t)) \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix},$$

where $\ell = \sqrt{g/H}$, $t$ is the current time and $T$ is the (predicted) time of the end of the step.

We assume the body is a point mass and is moving on a horizontal plane, that is the height of the center of mass is constant. Because we are assuming a point mass, the angular momentum about the center of mass is zero. We now replace the states $\{x, \dot{x}\}$ with $\{x, L^y\}$, where $L^y$ is the $y$-component of angular momentum about the contact point. The corresponding dynamic model is

$$\begin{bmatrix} \dot{x} \\ \dot{L}^y \end{bmatrix} = \begin{bmatrix} 0 & 1/mH \\ mg & 0 \end{bmatrix} \begin{bmatrix} x \\ L^y \end{bmatrix},$$

and its corresponding solution is

$$\begin{bmatrix} x(T) \\ L^y(T) \end{bmatrix} = \begin{bmatrix} \cosh(\ell(T - t)) & 1/mH \ell \sinh(\ell(T - t)) \\ mH \ell \sinh(\ell(T - t)) & \cosh(\ell(T - t)) \end{bmatrix} \begin{bmatrix} x(t) \\ L^y(t) \end{bmatrix},$$

where $t$ is the current time and $T$ is the (predicted) time of the end of the step.

For a point-mass inverted pendulum, where the mass moves on a horizontal plane, representations \((5)\) and \((7)\) are exactly the same. So what have we gained? Importantly, for a real robot, where the two representations are only approximate, the second one is better for making predictions on the robot’s state, as we discussed in Sec. II. Figure 5 compares the predictions of linear velocity and angular velocity about the contact point for a seven degree of freedom 2D model of Rabbit and a 20 degree of freedom 3D model of Cassie. In the figure, the simulated instantaneous values of $v_{\text{CoM}}^x(t)$ and $L^y(t)$ are shown in blue. The red line shows the evolution of the predicted values at the end of a step for $v_{\text{CoM}}^x(t)$ and $L^y(t)$ from \((6)\) and \((8)\) plotted against the corresponding instantaneous values. In a perfect predictor, the predicted values would be straight lines. It is clear that, when extrapolated to a realistic model of a robot, the prediction of angular momentum about the contact point is significantly more reliable than the estimate of linear velocity.

IV. HIGH-LEVEL CONTROL STRATEGY IN TERMS OF ANGULAR MOMENTUM

Our overall control objective will be to regulate walking speed. Because of the four advantages of angular momentum versus linear velocity that we listed in Sect. III, we will use angular momentum about the contact point as a primary control variable. In this section, we explain our method for deciding where to end one step by initiating contact between the ground and the swing foot, thereby beginning the next step. In robot locomotion control, this is typically called “foot placement control”.

A. Notation

We need to distinguish among the following time instances when specifying the control variables.

- $T$ is the step time.
- $T_k$ is the time of the $k$th impact.
- $T_{k-}$ is the end time of step $k$, so that $T_{k-} = T_k$ is the beginning time of step $k + 1$ and $T_{k+1-}$ is the end time of step $k + 1$.
- $(T_{k-} - t)$ is the time until the end of step $k$.

The superscripts $+$ and $-$ on $T_k$ are due to the impact map; see \[30\].

- $p_{\text{st} \rightarrow \text{CoM}}$. Vector emanating from stance foot to CoM. Here, the stance foot can be thought of as the current contact point.
- $p_{\text{sw} \rightarrow \text{CoM}}$. Vector emanating from swing foot to CoM. Here, the swing foot is defining the contact for the next impact and hence will be a control variable.
Fig. 3: Comparison of $L$, $L_{\text{CoM}}$, $v_{\text{CoM}}^x$ in simulation for the bipedal robots Rabbit and Cassie, while $v_{\text{CoM}}^z$ is carefully regulated to zero. The angular momentum about the contact point, $L$, has a convex trajectory, both of which are similar to the trajectory of a LIP model, while the trajectory of the longitudinal velocity of the center of mass, $v_{\text{CoM}}^x$, has no particular shape. In this figure, $L$ is continuous at impact, which is based on two conditions: $v_{\text{CoM}}^z = 0$ at impact and the ground is level. Even when these two conditions are not met, the jump in $L$ at impact can be easily calculated with (3).

B. Foot placement in longitudinal direction

There are multiple means to stabilize gaits [30]. Because we seek to focus on the benefits of using angular momentum instead of linear velocity, the controller we give here simply uses “foot placement” as is commonly associated with the LIP model. The principal idea is to regulate the angular momentum at the end of the next step by choosing the foot placement position for the end of the current step.

The Angular Momentum at the end of the next step is related to the Angular Momentum and the position of the center of mass at the beginning of next step by

$$L^y(T_{k+1}^-) = mH\ell \sinh(\ell T) p_{st\rightarrow CoM}(T^+_{k}) + \cosh(\ell T)L^y(T^+_{k}).$$

However, the CoM position with respect to the stance foot at the beginning of the next step is the same as the CoM position with respect to the swing foot at the end of the current step,

$$p_{st\rightarrow CoM}(T^+_{k}) = p_{sw\rightarrow CoM}(T^-_{k}).$$

Because $p_{sw\rightarrow CoM}(T^-_{k})$ is what we can control in the current step, we are motivated to re-write (9) as

$$L^y(T_{k+1}^-) = mH\ell \sinh(\ell T) p_{st\rightarrow CoM}(T^+_{k}) + \cosh(\ell T)L^y(T^+_{k}).$$

Solving (11) for the desired foot position would not yield a causal formula. However, because the CoM height is assumed constant and the ground is flat, the angular momentum about the next contact point is equal to the angular momentum about the current stance leg,

$$L^y(T^+_{k}) = L^y(T^-_{k});$$

see (3). This yields

$$L^y(T_{k+1}^-) = mH\ell \sinh(\ell T) p_{sw\rightarrow CoM}(T^-_{k}) + \cosh(\ell T)L^y(T^-_{k}).$$

Now, we are almost there. From the second row of (8), an estimate for the angular momentum about the contact point at
Fig. 5: Comparison of the ability to predict velocity vs angular momentum at the end of a step. The most crucial decision in the control of a bipedal robot is where to place the next foot fall. In the standard LIP controller, the decision is based on predicting the longitudinal velocity of the center of mass. In Sect. III we use angular momentum about the contact point. We do this because on realistic bipeds, the LIP model provides a more accurate and reliable prediction of $L_y$ than $v_{CoM}$. The comparison is more significant on Rabbit, whose leg center of mass is further away from the overall CoM.

Fig. 6: Definition of $T_k$

Fig. 7: Definition of $p_{st→CoM}$ and $p_{sw→CoM}$

With a perfect estimator for the angular momentum about that stance leg, such as is possible with an ideal LIP model, the desired foot position would be constant. Here, it varies because in a real robot, our estimate for angular momentum evolves with time.
C. Lateral Control

From (4), the time evolution of the angular momentum about the contact point is decoupled about the $x$- and $y$-axes. Therefore, once a desired angular momentum at the end of next step is given, Lateral Control is essentially identical to Longitudinal Control and (15) can be applied equally well in the lateral direction. The question becomes how to decide on $L^x \text{des}(T_{k+1})$.

For walking in place or walking with zero lateral velocity, it is sufficient to obtain $L^x \text{des}$ from a periodically oscillating LIP model,

$$
L^x \text{des}(T_{k+1}) = \frac{1}{2} m H W \frac{\ell \sinh(\ell T)}{1 + \cosh(\ell T)},
$$

where $W$ is the desired step width. The sign is positive if next stance is left stance and negative if next stance is right stance.

D. Turning

In each step, we suppose there is an angle $D_k$ which defines the forward direction of that step. When walking in a straight line, $D_k$ is a constant. When the robot makes turns, $D_k$ will change step by step, $\Delta D_k = D_{k+1} - D_k \neq 0$. The $L^x \text{des}$ and $L^y \text{des}$ will then be first decided in their corresponding frame defined by $D_k$, then transformed back in the world frame and used to decide foot placement in the world frame.

V. IMPLEMENTING THE LIP-BASED ANGULAR MOMENTUM CONTROLLER ON A REAL ROBOT

In this section we introduce the control variables for Cassie Blue and generate their reference trajectory. As in [9], we leave the stance toe passive. Consequently, there are nine (9) control variables, listed below from the top of the robot to the end of the swing leg,

$$
h_0 = \begin{bmatrix}
\text{torso pitch} \\
\text{torso roll} \\
\text{stance hip yaw} \\
\text{swing hip yaw} \\
p^s_{\text{st}}\rightarrow\text{CoM} \\
p^s_{\text{sw}}\rightarrow\text{CoM} \\
p^y_{\text{sw}}\rightarrow\text{CoM} \\
p^y_{\text{sw}}\rightarrow\text{CoM} \\
\text{swing toe absolute pitch}
\end{bmatrix}.
$$

For later use, we denote the value of $h_0$ at the beginning of the current step by $h_0(T_{k-1}^+)$. When referring to individual components, we’ll use $h_{0i}(T_{k-1}^+)$, for example.

We first discuss variables that are constant. The reference values for torso pitch, torso roll, and swing toe absolute pitch are constant and zero, while the reference for $p^x_{\text{st}}\rightarrow\text{CoM}$, which sets the height of the CoM with respect to the ground, is constant and equal to $H$.

We next introduce a phase variable

$$
s := \frac{t - T_{k-1}^+}{T}
$$

that will be used to define quantities that vary throughout the step to create “leg pumping” and “leg swinging”. The reference trajectories of $p^x_{\text{sw}}\rightarrow\text{CoM}$ and $p^y_{\text{sw}}\rightarrow\text{CoM}$ are defined such that:

- at the beginning of a step, their reference value is their actual position;
- the reference value at the end of the step implements the foot placement strategy in (15); and
- in between a half-period cosine curve is used to connect them, which is similar to the trajectory of an ordinary (non-inverted) pendulum.

The reference trajectory of $p^y_{\text{sw}}\rightarrow\text{CoM}$ assumes the ground is flat and the control is perfect:

- at mid stance, the height of the foot above the ground is given by $z_{CL}$, for the desired vertical clearance.

The reference trajectories for the stance hip and swing hip yaw angles are simple straight lines connecting their initial actual position and their desired final positions. For walking in a straight line, the desired final position is zero. To include turning, the final value has to be adjusted. Suppose that a turn angle of $\Delta D_k^{\text{des}}$ radians is desired. One half of this value is given to each yaw joint:

- $+\frac{1}{2} \Delta D_k^{\text{des}} \rightarrow$ swing hip yaw; and
- $-\frac{1}{2} \Delta D_k^{\text{des}} \rightarrow$ stance hip yaw

The signs may vary with the convention used on other robots.

The final result for Cassie Blue is

$$
h_0(s) := \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & (1 - s) h_{03}(T_{k-1}^+) + s (-\frac{1}{2}(\Delta D_k)) & (1 - s) h_{04}(T_{k-1}^+) + s (\frac{1}{2}(\Delta D_k)) \\
1 & (1 + \cos(\pi s)) h_{06}(T_{k-1}^+) + (1 - \cos(\pi s)) h_{07}(T_{k-1}^+) + (1 - \cos(\pi s)) h_{08}(T_{k-1}^+) & 4 z_{CL} (s - 0.5)^2 + (H - z_{CL}) \\
0 & 0 & 0 & 0
\end{bmatrix}.
$$

When implemented with an Input-Output Linearizing Controller so that $h_0$ tracks $h_4$, the above control policy allows Cassie to move in 3D in simulation. Fig. 8 shows Cassie starts from a walking in place gait and accelerates to a speed of 2.8 m/s.

VI. MODIFICATIONS FOR PRACTICAL IMPLEMENTATION

This section discusses several issues that prevent the basic controller from being implemented on Cassie Blue. Similar issues may or may not arise on your robot.

A. IMU and EKF

In a real robot, an IMU and an EKF are needed to estimate the linear position and rotation matrix at a fixed point on the robot, along with their derivatives. Cassie uses a VectorNav IMU. We used the Contact-aided Invariant EKF developed in [14], [16] to estimate the torso velocity. With these signals in hand, we could estimate angular momentum about the contact point.
translating the method to a floating base model of the robot; and
• using the phase variable (18).
This provided improved tracking performance over the straight-up PD implementation in [9].

E. Springs
On the swing leg, the spring deflection is small and thus we are able to assume the leg to be rigid. On the stance leg, the spring deflection is not negligible. An offset is added on the knee motor to compensate this spring deflection. While there are encoders to measure the spring deflection, direct use of this leads to oscillations. The deflection of the spring is instead calculated through a simplified model.

F. COM Velocity in the Vertical Direction
When Cassie is walking over one meter per second, the assumption that $v_{\text{CoM}} = 0$ breaks down and (12) is no longer valid. Hence, we use

$$L^y(T_k^{-}) = L^y(T_k^{+}) + v_{\text{CoM}}(T_k^{+}) (p_{\text{sw} \rightarrow \text{CoM}}(T_k^{-}) - p_{\text{st} \rightarrow \text{CoM}}(T_k^{+})).$$

(22)

From this, the foot placement is updated to

$$p_{\text{sw} \rightarrow \text{CoM}}(T_k^{-}) = \frac{L^y(T_k^{+}) - \frac{L^y(T_k^{-})}{m(H\ell \sinh(T) - v_{\text{CoM}}^{z}) \cosh(T)}}{m(H\ell \sinh(T) - v_{\text{CoM}}^{z}) \cosh(T)}$$

(23)

VII. EXPERIMENTAL RESULTS
The overall controller summarized in Fig. 9 was implemented on Cassie Blue. The closed-loop system consisting of robot and controller was evaluated in a number of situations that are itemized below.

• Walking in a straight line on flat ground. Cassie could walk in place and walk stably for speeds ranging from zero to 2.1 m/s.
• Diagonal Walking. Cassie is able to walk simultaneously forward and sideways on grass, at roughly 1 m/s in each direction.
• Sharp turn. While walking at roughly 1 m/s, Cassie Blue effected a 90° turn, without slowing down.
• Rejecting the classical kick to the base of the hips. Cassie was able to remain upright under “moderate” kicks in the longitudinal direction. The disturbance rejection in the lateral direction is not as robust as the longitudinal, which is mainly caused by Cassie’s physical design: small hip roll motor position limits.
• Finally we address walking on rough ground. Cassie Blue was tested on the iconic Wave Field of the University of Michigan North Campus. The foot clearance was increased from 10 cm to 20 cm to handle the highly undulating terrain. Cassie is able to walk through the “valley” between the large humps with ease at a walking pace of roughly 0.75 m/s, without falling in all tests. The row of ridges running east to west in the Wave Field are roughly 60 cm high, with a sinusoidal structure.
We argued that angular momentum about the contact point is a superior variable for planning step placement in a LIP-based controller. We believe the same will hold on many other control strategies. This paper limited itself to a LIP-style controller so that other locomotion groups could clearly assess the benefits of angular momentum about the contact point.

Using our new controller, Cassie was able to accomplish a wide range of tasks with nothing more than common sense task-based tuning: a higher step frequency to walk at 2.1 m/s and extra foot clearance to walk over slopes exceeding 15 degrees. Moreover, in the current implementation, there is no optimization of trajectories used in the implementation on Cassie. The robot’s performance is currently limited by the hand-designed trajectories leading to joint-limit violations and foot slippage. These limitations will be alleviated by incorporating optimization.

The current controller tries its best to maintain a zero center of mass velocity in the $z$-direction. This simplifies the transition formula for the angular momentum at impact. Our next publication will explain how to exploit changes in the vertical component of the center of mass velocity in order to better achieve a desired angular momentum: foot placement plus $v^{z_{\text{CoM}}}$!

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