In-situ remaining useful life prediction of aircraft auxiliary power unit based on quantitative analysis of on-wing sensing data

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Abstract
The in-situ prognostics and health management of aircraft auxiliary power unit faces difficulty using the sparse on-wing sensing data. As the key technology of prognostics and health management, remaining useful life prediction of in-situ aircraft auxiliary power unit is hard to achieve accurate results. To solve this problem, we propose one kind of quantitative analysis of its on-wing sensing data to implement remaining useful life prediction of auxiliary power unit. Except the most important performance parameter exhaust gas temperature, the other potential parameters are utilized based on mutual information, which can be used as the quantitative metric. In this way, the quantitative threshold of mutual information for enhancing remaining useful life prediction result can be determined. The implemented cross-validation experiments verify the effectiveness of the proposed method. The real on-wing sensing data of auxiliary power unit for experiment are from China Southern Airlines Company Limited Shenyang Maintenance Base, which spends over $6.5 million on auxiliary power unit maintenance and repair each year for the fleet of over 500 aircrafts. Although the relative improvement is not too large, it is helpful to reduce the maintenance and repair cost.

Keywords
Mutual information, quantitative analysis, auxiliary power unit, on-wing sensing data

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Introduction
Auxiliary power unit (APU) is utilized in many kinds of modern aircrafts. One of the most important functions of APU is to help start the main engines. The necessary supplement for starting the main engines are compressed air and electricity. In addition, APU provides fresh air and power into the cabin during the off-working state of the aeroengine. The flight task of the aircraft will be suspended if the on-wing APU is fault or failure. For the electric aircraft, an important function of APU is to provide additional thrust when the aircraft is on the air.\textsuperscript{1} Hence, its on-wing condition is very important for the modern aviation. One promising technology to predict the condition of APU is
prognostics and health management (PHM), which has been applied for monitoring the condition of complex object.\textsuperscript{2,3} In the domain of PHM, the methodologies based on physical model and data-driven model are two kinds of commonly utilized approaches.\textsuperscript{4,5} Although the model-based method can achieve precise and stable prediction result, it is hard to formulate the physical model of complex object.\textsuperscript{6} Hence, it is not applicable in many modern industry systems. On the contrary, data-driven model can be carried out with the available sensing data of the monitored system.\textsuperscript{7} Nowadays, the condition monitoring data can be collected easily, especially with the help of modern sensor technology. Therefore, the available data for formulating the data-driven model are more and more abundant, which make it become the promising method for complex industrial system.

In the area of data-driven model, one of research subjects is to improve the prediction result, especially for implementing remaining useful life (RUL) prediction.\textsuperscript{8,9} For the online application scenario, Yan et al.\textsuperscript{10} propose the online model parameter adaption scheme to enhance the accuracy and precision of diagnosis and prognosis. To carry out the adaptive clustering for bearing and gear fault diagnosis, Wang et al.\textsuperscript{11} aim to solve three real industrial applications by the intelligent method. With the development of deep learning, it has been successfully employed to improve the condition monitoring of bearing and the analogic circuit.\textsuperscript{12,13} In some real scenarios, it is difficult to collect the condition monitoring data of the whole life-cycle. To solve this problem, accelerated degradation testing (ADT) is one kind of efficient method.\textsuperscript{14–16} By applying ADT, RUL prediction can be improved with the abundant ADT data. At the same time, many available RUL prediction methodologies have been proposed to predict the condition of the target system.

Among the existing studies, the rotating machinery (e.g. bearing and gear) has been paid much attention. The application of entropy theories on bearing fault diagnosis has been investigated comprehensively.\textsuperscript{17} The problem of recognizing the early fault in bearing and gear has been considered in Wang et al.\textsuperscript{18} To improve the fault diagnosis of bearing, the novel convolutional neural network can be utilized by enhancing the related features.\textsuperscript{19,20} To identify the different kinds of fault, the method based on smoothness index-based Bayesian inference is adopted.\textsuperscript{21} For realizing the deep fault in the rotating machinery, it needs to denoise and extract the valuable information which can be realized by the integrated model.\textsuperscript{22} To identify the size and position of fault in the bearing, the method based on vertical–horizontal synchronization is proposed.\textsuperscript{23} To mine the fault feature contained in the monitoring data of bearing, the time–frequency manifold sparse reconstruction can be utilized.\textsuperscript{24} To extract the degradation information of bearing and gear, the long short-term memory recurrent neural network can be used.\textsuperscript{25,26} The method based on Bayesian dynamic linear model and Copulas can also be adopted to reach this objective.\textsuperscript{27} The aforementioned researches indeed implement effective methodologies. However, those works mainly focus on how to process the existing data and ignore whether the sensor data are anomalous.

In the field of sensor data analysis, there are some typical studies which have paid attention to the above theme. Liu et al.\textsuperscript{28} propose one kind of quantitative sensor selection method for RUL prediction based on the improved permutation entropy. The sensing data of the aircraft engine are selected and implemented anomaly detection to reach better prognostics result.\textsuperscript{29} The anomalous sensor data in the space battery have been carried out to ensure the corrective monitoring result.\textsuperscript{30} The outlier condition of sensor data in the unmanned aerial vehicle is studied by the local regulated optimization method.\textsuperscript{31} If the anomalous sensor data are adopted for the condition monitoring of system, they may bring seriously deviated prediction result. One kind of effective method is to recover those data for the input of condition monitoring algorithm.\textsuperscript{32,33} These researches have provided valuable guide to carry out APU PHM with its sensing data. APU belongs to one kind of typical complex system, which is mainly consisted of the load compressor, the power section, the gearbox, and so on. Some works have been implemented to predict its fault or failure. To evaluate the performance deterioration of APU, the generalized renewal process which is based on Weibull distribution can be adopted.\textsuperscript{34} The works of Wang et al.\textsuperscript{35} and Liu et al.\textsuperscript{36} aim to ensure that exhaust gas temperature (EGT) can reflect its condition correctly, which is one of the most important parameters. Liu et al.\textsuperscript{37} propose to carry out RUL prediction of APU by using its on-wing sensing data. However, this work only considers the simple combination of EGT with the other sensing data.

To the best of our knowledge, there is no study which has considered the sparse dimension feature of APU on-wing sensing data, that is, only one or a few available sensing data for RUL prediction. This study aims to mine the other potential parameters in the quantitative manner, which can help to enhance RUL prediction of APU. One kind of data-driven method based on mutual information analysis among the on-wing sensing data is proposed, which is expected to find the quantitative threshold for determining other valuable sensing data for RUL prediction of APU. This method utilizes quantitative analysis of the raw sensing data, which has been neglected by many researchers. As introduced in Gorinevsky et al.,\textsuperscript{38} the most appropriate sensing data when APU helps start the main engines are also adopted in the proposed method. The quantitative of mutual information ratio of EGT with other sensing data is the key element in the proposed
Methodology

In this section, the proposed method for implementing RUL prediction of APU is introduced. The utilized theories for this method are also presented, and metrics for measuring RUL prediction result are illustrated.

The proposed RUL prediction method for APU

Due to the complex structure of APU, its physical model cannot be formulated accurately. When APU is in the on-wing utilization, its condition monitoring data are transmitted to the airline datacenter during each flight task. Among these available on-wing sensing data, some are appropriate for implementing its RUL prediction. Based on the most important performance parameter (EGT), one kind of data-driven method is proposed to predict its RUL, as illustrated in Figure 1.

The condition monitoring data of on-wing APU are consisted of many segments, including CC, C1, CE, and so on. These sensing data are collected during the working process of APU, which can reflect its real working condition. They are also the only available source to evaluate the on-wing condition of APU. The proposed method aims to mine these condition monitoring data to achieve the degradation analysis of APU. The detailed implication of these sensing data will be introduced in the following section.

The most important step in Figure 1 is how to evaluate the useful information of those on-wing sensing data (i.e. appropriate input for prediction method). As given in Figure 1, this step is named as information

Figure 1. The proposed RUL prediction method for APU.
analysis. To realize this target, the parameter EGT which can denote the performance of APU is confirmed. To mine the other parameters which can bring positive influence on RUL prediction result, mutual information is adopted.

The quantitative information Δ of other on-wing sensing data with EGT is first calculated. The proposed method supposes that there is a threshold (i.e. C in Figure 1) of this information to bring positive influence on RUL prediction result of APU. If mutual information between another parameter and EGT is larger than the half mutual information value of EGT itself, it may bring more valuable information of APU degradation. Therefore, the value of threshold C is half mutual information value of EGT itself. The reason is that EGT contains much degradation information. If another parameter can provide additional degradation information, it should be helpful for enhancing RUL prediction of the in-situ APU.

The on-wing sensing data of APU have the sparse feature, which means that only a few dimension data contain the degradation information. As introduced in the “Introduction” section, most studies focus on only using EGT to achieve condition assessment of APU. To mine other potential monitoring data for realizing RUL prediction of APU, GPR and RVM, which are two typical methods in this domain, are used to evaluate the effectiveness of the proposed quantitative analysis among the on-wing sensing data. The details of two prediction methods and evaluation metrics are introduced in the following sub-sections.

**Mutual information**

As one effective information measurement, mutual information has been frequently adopted to mine the relationship among different data. Compared with the correlation coefficient, mutual information can effectively weigh the linear and nonlinear relationship. Its definition is based on the basic information entropy. Therefore, the physical connotation of entropy is given first. If the variable \( x \) is discrete, its entropy can be realized by Shannon’s work\(^{39} \)

\[
H = -\sum_{i=1}^{N} p_i(x) \log p_i(x) 
\]

where \( p_i(x) \) refers to the probability of each variable over all kinds of variables. The number of all kinds of variables is denoted by \( N \).

If the variable \( x \) is continuous, it can be expressed by the distribution function \( f(x) \). The corresponding information entropy is defined by Shannon’s work\(^{39} \)

\[
H = -\int_{S} f(x) \log f(x) dx 
\]

where \( S \) denotes the aggregation of all kinds of random variable.

In the above two equations, the base of log can select different values. The corresponding unit of entropy is determined by the value of base. According to the different requirement of specific application, we can set the log base to be any value. In this study, the base of the logarithm is set to be 2 for all mutual information calculation, which will not affect the result measurement.

In order to calculate the mutual information of different variables, the first step is to calculate the conditional entropy of one variable with the other variables. For the conditional entropy \( H(X|Y) \), its definition is illustrated by

\[
H(X|Y) = \sum_{j=1}^{m} p(y_j)H(X|Y = y_j) 
\]

\[
= -\sum_{j=1}^{m} \sum_{i=1}^{n} p(y_j,x_i) \log p(x_i|y_j) 
\]

where \( X \) refers to the data series \( \{x_i, x_2, \ldots, x_n\} \) and \( Y \) indicates the data series \( \{y_1, y_2, \ldots, y_m\} \).

For the conditional entropy \( H(Y|X) \), its definition is illustrated by

\[
H(Y|X) = \sum_{i=1}^{n} p(x_i)H(Y|X = x_i) 
\]

\[
= -\sum_{i=1}^{n} \sum_{j=1}^{m} p(x_i,y_j) \log p(y_j|x_i) 
\]

where \( X \) and \( Y \) have the same definition as those in equation (3).

By utilizing the above definitions, the mutual information \( I(X;Y) \) of two variables is calculated by

\[
I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) 
\]

\[
= \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} 
\]

In equation (5), the mutual information denotes the subtraction of the information entropy to the conditional entropy. By analyzing the above three kinds of definitions, the relationship is given in Figure 2.

In this study, we utilize mutual information to mine other potential data with EGT to predict RUL of APU. The other determined sensing data are expected to bring positive influence on the final prediction result.

**GPR**

Gaussian process has been adopted in different scenarios to solve the problem of stochastic process.\(^{41} \) For \( D = \{x_n\}_{n=1}^{N}, x \in \mathbb{R}^d, f(x_1), \ldots, f(x_N) \) include a group of
The conditional distribution represents the output by using test data, and with the Gauss joint distribution. The expression of \( f \) random variables, which comply with the joint Gaussian distribution. In this way, the group of \( f(x_1), \ldots, f(x_N) \) can be expressed by

\[
\begin{align*}
    f(x) &\sim GP(m(x), k(x_i, x_j)) \\
    m(x) &\equiv E[f(x)] \\
    k(x_i, x_j) &\equiv E[(f(x_i) - m(x_i))(f(x_j) - m(x_j))] 
\end{align*}
\]

where \( k(x_i, x_j) \) refers to the covariance function and \( m(x) \) indicates the mean function.

For the typical application, \( f(x) \) contains the noise and can be written by

\[
y = f(x) + \varepsilon
\]

where \( \varepsilon \in N(0, \sigma^2) \) represents the white noise. For the adopted \( f(x) \), the observation \( y \) also complies with the Gaussian process, which can be given by

\[
y \sim GP(m(x), k(x_i, x_j + \sigma^2\delta_{ij}))
\]

where \( \delta_{ij} \) denotes the Dirac function.

In the following step, let \( D_1 = \{x_i, y_i\}_{i=1}^N \) and \( D_2 = \{x'_i, y_i\}_{i=1}^N \) denote the training data and test data, respectively. The \( d \) indicates the dimension of the input data. The \( m \) and \( m^* \) are the mean vectors. The \( f(x_i) \) represents the output by using test data, and \( y \) refers to training data. In the above equations, \( f \) and \( y \) comply with the Gauss joint distribution. The expression of these two variables is given in the following equation

\[
\begin{bmatrix}
y \\ f(x) \\
\end{bmatrix} \sim \begin{bmatrix}
m \\
m^* \\
\end{bmatrix}, \begin{bmatrix}
C(X, X) & K(X, X_i) \\
K(X, X_i) & K(X_i, X_i) \\
\end{bmatrix}
\]

where \( C(X, X) \) refers to the covariance of training data, as shown by \( C(X, X) = K(X, X) + \delta_{ij}I \). The \( I \in \mathbb{R}^{N \times N} \) is unit matrix and the \( K(X_i, X_i) \) is covariance of test data. Then, the conditional distribution \( f_x \) is able to be realized by

\[
f_x | X, y, X_s \sim N(\mu, \text{cov}(f_x))
\]

\[
\mu = E[f_i | X, y, X_s] = m + K(X_s, X)C(X, X)^{-1}(y - m)
\]

\[
\text{cov}(f_x) = K(X_i, X_s) - C(X, X)^{-1}K(X, X_s)
\]

The first step of utilizing GPR is to determine the mean and variance functions. Meanwhile, the parameters in these two functions need to be initialized. Then, according to their prior distribution, the training data are utilized as the input. The iterative algorithm is adopted to achieve the posterior distribution. Finally, RUL prediction is realized by using the posterior distribution and test data.

When GPR is used to analyze the on-wing sensing data of APU, the distribution of long-time monitoring data can be recognized. The degradation trend is the most valuable information for predicting RUL, which can be mined by GPR. This procedure is mainly realized during the training period of GPR. The related parameters of GPR model are automatically optimized by the adopted training data. Then, the trained GPR model is utilized to achieve the following RUL prediction of the in-situ APU.

**RVM**

For \( \{x_n, t_n\}_{n=1}^N \), \( x_n \in \mathbb{R}^d \), and \( t_n \in \mathbb{R} \), the definition of RVM is given by Tipping’s work

\[
y(x_n, \omega) = \sum_{i=1}^N \omega_i K(x_n, x_i) + \omega_0
\]

where \( \omega_i \) is the weight, \( N \) denotes the total number of input data, and \( K(x_n, x_i) \) refers to the utilized kernel function which has different selection.

In the real application, the data are with noise which are represented by

\[
t_n = y(x_n, \omega) + \varepsilon_n
\]

where \( \varepsilon_n \) is the noise process, and it is often supposed to be the normal distribution \( \varepsilon_n \sim N(0, \sigma^2) \).

If \( t_n \) is independent, the likelihood \( p(t_n | x) = N(t_n | y(x_n), \sigma^2) \) belongs to be Gaussian distribution over \( t_n \). The mean and the variance are \( y(x_n) \) and \( \sigma^2 \), respectively. The likelihood is rewritten as

\[
p(t_n | \omega, \sigma^2) = (2\pi\sigma^2)^{-N/2} \exp \left\{ \frac{-||t - \Phi \omega||^2}{(2\sigma^2)} \right\}
\]

where \( t = (t_1, \ldots, t_n)^T \), \( \omega = (\omega_0, \ldots, \omega_N)^T \), and \( \Phi \) are the matrices with \( \Phi = [\phi(x_1), \phi(x_2), \ldots, \phi(x_n)]^T \) and \( \phi(x_n) = [1, K(x_n, x_1), \ldots, K(x_n, x_N)] \).
The maximum-likelihood of $\omega$ and $\sigma^2$ in equation (17) may result in the over-fitting. To avoid this problem, Bayesian perspective is adopted, as given by

$$p(\omega/\alpha) = \prod_{i=0}^{N} N(\omega_i|0, \alpha_i^{-1}) = \prod_{i=0}^{N} \frac{\alpha_i}{\sqrt{2\pi}} \exp\left(\frac{-\omega_i^2 \alpha_i}{2}\right)$$  

(18)

where $\alpha$ is the hyper-parameter vector. It independently associates with each weight.

In this way, the posterior distribution is given by

$$p(\omega/t, \alpha, \sigma^2) \sim N(\mu, \sum) = (2\pi)^{-\frac{N+1}{2}} |\sum|^{-1/2} \exp\left\{-\frac{(\omega - \mu)^T \sum^{-1}(\omega - \mu)}{2}\right\}$$  

(19)

In equation (19), the covariance and the mean can be achieved by

$$\sum = (\sigma^2 \Phi^T \Phi + A)^{-1}$$  

(20)

$$\mu = \sigma^2 \sum \Phi^T t$$  

(21)

with $A = \text{diag}(\alpha_0, \alpha_1, \ldots, \alpha_N)$.

In equation (17), the likelihood distribution is able to be calculated by

$$p(t|\alpha, \sigma^2) = \int p(t|\omega, \sigma^2) \cdot p(\omega|\alpha) d\omega$$  

(22)

Therefore, the detail of formation of the function is shown by

$$p(t|\alpha, \sigma^2) = N(0, C) = (2\pi)^{-N/2} |\sigma^2 I + \Phi A^{-1} \Phi^T|^{-1/2} \exp\left\{-\frac{1}{2} t^T (\sigma^2 I + \Phi A^{-1} \Phi^T)^{-1} t\right\}$$  

(23)

with $C = \sigma^2 I + \Phi A^{-1} \Phi^T$.

The values of $\alpha$ and $\sigma^2$ cannot be solved in the analytical form. The parameter of $\alpha$ can be achieved by equating the differentiation of equation (23), and the new $\alpha$ is updated by the approach of MacKay

$$\alpha_i^\text{new} = \frac{\gamma_i}{\mu_i}$$  

(24)

where $\mu_i$ is the $i$th posterior mean weight and the $\gamma_i$ refers to the $i$th posterior weight covariance.

The new $\sigma^2$ can be calculated by

$$\sigma_i^2 = \frac{\|y - \Phi \mu_i\|}{N - \sum_i \gamma_i}$$  

(25)

where $N$ is the number of input data.

If there is a new point $x_i$, the related prediction can be reached as $t_i$. Therefore, $p(t_i|t)\sim N(\mu, \sigma^2)$ complies with Gaussian distribution. Its mean and variance are $\mu$ and $\sigma^2$, as given by

$$\mu = \mu^T \Phi(x_i)$$  

(26)

$$\sigma^2 = \sigma^2_{\text{MP}} + \Phi(x_i)^T \sum \Phi(x_i)$$  

(27)

where $\mu$ is the mean of $x_i$ and $\sigma^2$ is the variance of two components.

When RVM is adopted to realize RUL prediction, the key step is to train RVM model using likelihood function and priori knowledge. The likelihood is to obtain the potential feature of training data, and the priori knowledge is to describe the complexity of the model. Then, the iterative algorithm is utilized to train the model. In this way, the stability equalization can be achieved.

RVM has been verified that it can achieve relatively accurate RUL prediction results under a few dimension sensing data. For example, RVM has been utilized to realize RUL prediction of the lithium battery based on only the current and voltage data. For the on-wing APU, RVM is expected to use EGT and other data for improving prediction results. The potential sensing data, which contain the degradation information of APU, may provide positive influence on the final prediction results.

**Metrics for RUL prediction result**

To evaluate the performance of RUL prediction, *mean absolute error (MAE)* is utilized to measure the accuracy and *root mean square error (RMSE)* is adopted to measure the stability of RUL prediction result. The definitions of these two metrics are given in the following two equations

$$\text{MAE} = \frac{1}{N} \sum_{k=1}^{N} |P_k - R_k|$$  

(28)

$$\text{RMSE} = \sqrt{\frac{\sum_{k=1}^{N} (P_k - R_k)^2}{N}}$$  

(29)

In equations (28) and (29), $P_k$ denotes the predicted RUL of APU. $R_k$ refers to the actual RUL of APU. The number of the implemented prediction is denoted by $N$. For measuring prediction result, smaller MAE refers to better accuracy, and smaller RMSE indicates better stability.

**Evaluation experiments and discussion**

In this section, the utilized on-wing sensing data of APU are introduced. The evaluation experiments are
carried out and results are analyzed, discussed, and compared in detail.

**APU data description**

The on-wing condition of APU is collected by the previous deployed sensors. Then, these sensing data are transmitted to the datacenter of the airline by Aircraft Communications Addressing and Reporting System (ACARS). The A13 message in these data indicates the condition of APU and other essential information of the aircraft, as illustrated in Table 1.

**Mutual information between sensing data**

Based on the expert knowledge of APU, there are some important parameters which are directly relative to its on-wing condition. As introduced in the “APU data description” section, the original data in the segments of N1 and S1 are utilized. Among these two segments, there are 13-dimension data. Besides the control word and the stable parameters, four important sensing data are selected for the following experiments and analysis. Among these available condition monitoring data, EGT is the most important parameter that can reflect the condition of the on-wing APU. Mutual information is used to measure the quantitative relationship between EGT and other three important sensing data. In this way, the potential parameters for enhancing RUL prediction can be determined. By using the definition of mutual information, the numerical values among these data are summarized in Table 2.

As shown in Table 2, the largest mutual information is 4.61 bits, which is the EGT itself. As the most important parameter of APU, there are three values of mutual information. Both bleed air pressure and gearbox oil temperature have stronger relationship with EGT. The mutual information between these two parameters and EGT are 2.90 bits and 2.40 bits, respectively. The mutual information of bleed flow itself and with other parameters are much smaller. The original on-wing sensing data of these four parameters are illustrated in Figure 3.

For following simple analysis, let BF denote the bleed flow, BAP denote the bleed air pressure, and GOT denote the gearbox oil temperature. EGT is the exhaust gas temperature.
points in BAP, its monotonous trend may represent the degradation. The GOT has obviously monotonous trend in the latter working process. Compared with other three parameters, the BF has a few different values. As presented in the “The proposed RUL prediction method for APU” section, the aim of this study is to find the threshold \( C \) for determining the RUL prediction of APU. This insight will be verified and discussed in the following evaluation experiments.

**Experimental results**

In this subsection, RUL prediction of APU is carried out by GPR and RVM. To select the training set and the test set, the general basis is to provide enough training data for optimizing the utilized data-driven methods. In this study, more than half of total data are used to train GPR and RVM. The other part of data set is used to test the performance of the trained models. Experimental results are discussed in detail to evaluate the performance of the proposed method.

RUL prediction of APU using GPR. Two comparison experiments for RUL prediction of APU using GPR are mainly carried out to evaluate the performance of the proposed method. The details of these two experiments are introduced as follows.

The first experiment is that only EGT is utilized to realize RUL prediction of APU. In Shannon’s study,\(^ {39} \) EGT is considered as the important parameter of APU and it can be adopted to predict the degradation of APU. The second experiment is that EGT and the other sensing data are utilized to predict RUL of APU. These utilized sensing data have the maximal numerical value of mutual information with EGT. As the insight discovered in Gorinevsky et al.,\(^ {38} \) the RUL prediction of APU can be improved by these two input sensing data.

Besides, the cross-validation experiments which are based on the combination of different sensing data with EGT are implemented. To be specific, two-dimension sensing data which include EGT and another one-dimension data are utilized as the input. Three-dimension sensing data which include EGT and another two-dimension data are utilized as the input. All four-dimension sensing data are utilized as the input.

By comparing and analyzing the above experimental results, the performance of the proposed RUL prediction method will be evaluated. It is expected that the proposed method can achieve better prediction result. For the first kind of implemented experiment, which utilizes only EGT as the input of GPR, experimental results are given in Figure 4.

![Figure 3. The original condition monitoring data of APU.](image1)

The dotted line and the star line represent the actual RUL and the predicted RUL, respectively. In general, the curve of the predicted RUL has the feature of drastic change. Most of the predicted RUL values are far from the actual RUL values. Especially at the 13th current cycle, the prediction value is –5 cycle, which means that APU has reached failure condition. The reason is that the data-driven GPR model only relies on the input data. If these data cannot denote the degradation of APU well enough at that moment, it may result in wrong prediction result. The aforementioned two metrics for evaluating the prediction result are calculated as follows

\[
\begin{align*}
MAE & = 5.80 \text{ cycle} \\
RMSE & = 6.89 \text{ cycle}
\end{align*}
\]

For the second kind of implemented experiment, which utilizes two-dimension sensing data as the input of GPR, three experiments are carried out. The utilized input data for RUL prediction include three kinds of combination, including EGT and BF, EGT and BAP, and EGT and GOT. As the same in Gorinevsky et al.,\(^ {38} \) the experimental results of the first combination are illustrated in Figure 5.

![Figure 4. RUL prediction result with only EGT parameter using GPR.](image2)
Compared with Figure 4, the predicted RUL of APU in Figure 5 has relatively small change. But, at the beginning and the end of prediction, the predicted RUL values are far away from the actual RUL values.

The two metrics of RUL prediction result in Figure 5 are given as follows

\[ MAE = 4.80 \text{ cycle} \]
\[ RMSE = 5.52 \text{ cycle} \]

For the third kind of implemented experiment, which utilizes three-dimension sensing data as the input of GPR, three experiments are carried out. The corresponding input sensing data include (1) EGT, BF, BAP; (2) EGT, BF, GOT; and (3) EGT, BAP, GOT. Here, only the experimental results of the third data combination are given in Figure 6.

Compared with Figures 4 and 5, the predicted RUL in Figure 6 is closer to the actual RUL, which means that the experimental results are more precise and stabler. The two metrics of RUL prediction result in Figure 6 are given as follows

\[ MAE = 2.90 \text{ cycle} \]
\[ RMSE = 3.59 \text{ cycle} \]

For the fourth kind of implemented experiment, all four-dimension sensing data are utilized by GPR. Experimental results are shown in Figure 7.

Compared with the above three figures, most of the predicted RUL values are smaller than the actual RUL values. There are also some minus RUL prediction results, as the situation in Figure 4. The two metrics of RUL prediction result in Figure 7 are given as follows

\[ MAE = 8.10 \text{ cycle} \]
\[ RMSE = 9.82 \text{ cycle} \]

Finally, all the experimental results implemented with GPR for RUL prediction of APU are summarized in Table 3. As shown in Table 3, the minimum values of \( MAE \) and \( RMSE \) of RUL prediction result are 2.90 cycle and 3.59 cycle, respectively. The maximal values of \( MAE \) and \( RMSE \) of RUL prediction result are 8.10 cycle and 9.82 cycle, respectively, which is achieved by using four-dimension sensing data. Experimental results show that GPR can reach relatively better prediction results by the quantitative analysis between EGT and other two sensor data, which have larger mutual information with EGT.

RUL prediction of APU using RVM. Similar to the four kinds of implemented experiments in RUL prediction of APU using GPR, the input data for RUL prediction using RVM are carried out in this subsection. The dimension data for RVM are also from one to four, and the combination of input is same with the situation of GPR. Experimental results are compared and discussed in detail.
The sensing data adopted in Gorinevsky et al.\textsuperscript{38} and Shannon\textsuperscript{39} are also selected as the important comparison experiment. If the experimental results implemented by RVM can achieve relatively better prediction result, the adaptation and the scalability of the proposed method are verified at a large degree.

In the first experiment using RVM, only EGT is used to predict RUL of APU and experimental results are shown in Figure 8.

For the experimental results illustration in this subsection, the curves have the same meaning with those in the “RUL prediction of APU using GPR” subsection. The predicted RUL in Figure 8 is a little bit far away from the actual RUL, especially at the beginning of prediction. The reason may be that RVM cannot reach effective performance when it only using EGT. Besides, the curves of predicted RUL in Figures 4 and 8 have similar change, which both utilizes only EGT. This verifies that data-driven method only mines the information contained in the sensing data. The metric values for measuring RUL prediction result which adopts only EGT using RVM are as follows

\[
\begin{align*}
\text{MAE} & = 4.96 \text{ cycle} \\
\text{RMSE} & = 5.88 \text{ cycle}
\end{align*}
\]

In the second experiment, two-dimensional data (i.e. EGT and BAP) are adopted as the input of RVM. The setting of this experiment is also same with the situation in Gorinevsky et al.\textsuperscript{38} The only difference is that the prediction algorithm. Experimental results are shown in Figure 9.

The variation of predicted RUL in Figure 9 is smaller than that in Figure 8. \textit{MAE} and \textit{RMSE} of RUL prediction result are as follows

\[
\begin{align*}
\text{MAE} & = 4.81 \text{ cycle} \\
\text{RMSE} & = 5.74 \text{ cycle}
\end{align*}
\]

In the third experiment, three-dimensional data (i.e. EGT, BAP, and GOT) are utilized as the input of RVM. Experimental results are provided in Figure 10.

Compared with the above two experiments using RVM, the predicted RUL is obviously superior. \textit{MAE} and \textit{RMSE} of this experiment result are given as follows

| Sensing data dimension | Input sensing data | \textit{MAE} (cycle) | \textit{RMSE} (cycle) |
|------------------------|-------------------|---------------------|---------------------|
| 1                      | EGT               | 5.80                | 6.89                |
| 2                      | EGT, BF           | 5.95                | 7.16                |
|                        | EGT, BAP         | 4.80                | 5.52                |
|                        | EGT, GOT         | 5.10                | 6.46                |
| 3                      | EGT, BF, BAP     | 5.30                | 6.28                |
|                        | EGT, BF, GOT     | 5.55                | 6.52                |
|                        | EGT, BAP, GOT    | 2.90                | 3.59                |
| 4                      | EGT, BF, BAP, GOT| 8.10                | 9.82                |

\textbf{Table 3.} RUL prediction result of APU using GPR.

\textbf{RUL:} remaining useful life; \textbf{APU:} auxiliary power unit; \textbf{GPR:} Gaussian process regression; \textbf{MAE:} mean absolute error; \textbf{RMSE:} root mean square error; \textbf{EGT:} exhaust gas temperature; \textbf{BF:} bleed flow; \textbf{BAP:} bleed air pressure; \textbf{GOT:} gearbox oil temperature.
In the fourth experiment, all four-dimension sensing data are adopted as the input of RVM. Experimental results are given in Figure 11.

Similar with the four-dimension input data using GPR, this experiment cannot achieve better result. The two metrics of RUL prediction result in Figure 7 are as follows

$$\text{MAE} = 8.10 \text{ cycle}$$
$$\text{RMSE} = 9.82 \text{ cycle}$$

After implementing the above four experiments, the other four kind of sensing data combination for RUL prediction using RVM are also carried out. The details of experiments include two-dimension sensing data and three-dimension sensing data. The first two-dimension sensing data are EGT and BF. The second two-dimension sensing data are EGT and GOT. The first three-dimension sensing data are EGT, BF, and BAP. The second three-dimension sensing data are EGT, BF, and GOT. With the above four experimental results, all eight experimental results are given in Table 4.

The minimum values of $\text{MAE}$ and $\text{RMSE}$ of RUL prediction among these eight experiments are 3.19 cycle and 3.79 cycle, respectively. The largest values of $\text{MAE}$ and $\text{RMSE}$ are 5.85 cycle and 7.12 cycle, respectively, when the sensing data input are EGT and BF. Compared with these two values, the minimum $\text{MAE}$ and $\text{RMSE}$ are reduced by 45% and 47%, respectively. The minimum values $\text{MAE}$ and $\text{RMSE}$ are also achieved by utilizing the same three-dimension input data, which are EGT, BAP, and GOT. Experimental results show that RVM can reach relatively better prediction results by the quantitative analysis between EGT and other two sensor data, which have larger mutual information with EGT.

**Experimental results discussion.** As shown in Table 2, among the four-dimensional sensing data of APU, their relationship of EGT with BAP and GOT is stronger than it with BF. Five insights can be found by analyzing the numerical values in Tables 2–4.

1. For the two-dimensional sensing data experiments using GPR and RVM, the relatively optimal results of RUL prediction are both achieved by using EGT and BAP.
2. For the three-dimensional sensing data experiments using GPR and RVM, the relatively optimal results of RUL prediction are both achieved by using EGT, BAP, and GOT.
3. To summarize the first and the second insight, the on-wing sensing data which has the largest mutual information with EGT help improve RUL prediction of APU. Based on these two-dimensional sensing data, another on-wing sensing data which have the second largest mutual information can help improve RUL prediction of APU in further. Therefore, the claimed $C$ in section “The proposed RUL prediction method for APU” can be determined by the ratio of the second largest mutual information over EGT itself.
4. Compared with the existing research which only utilizes EGT for APU degradation analysis, the performance of RUL prediction can be improved with the help of other on-wing sensing data. This has been realized by the proposed method in this study.
5. For RUL prediction of APU, the proposed data-driven methods based on GPR and RVM are applicable. This insight not only helps to realize RUL prediction of APU, but also provides valuable reference for implementing the similar object.
Conclusion and future work

In this article, we study the on-wing RUL prediction of the aircraft APU. The utilization of mutual information provides the quantitative analysis of the on-wing sensing data, which can help improve the prediction result at a large degree. We also prove that GPR and RVM are available for RUL prediction of APU, which has the sparse dimension feature. By using the cross-validation, the relatively optimal results of RUL prediction are both achieved by the same combination of the on-wing sensing data. There is indeed the quantitative threshold of correlation between EGT and other sensing data to help improve the RUL prediction result. By the implemented experiments, the BAP and GOT can bring positive impact and can be regarded as the input for RUL prediction of APU. The future work will focus on how to automatically determine the dimension sensing data for achieving the optimal RUL prediction results.

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Table 4. RUL prediction result of APU using RVM.

| Sensing data dimension | Input sensing data | MAE (cycle) | RMSE (cycle) |
|------------------------|--------------------|-------------|--------------|
| 1                      | EGT                | 4.96        | 5.88         |
| 2                      | EGT, BF            | 5.85        | 7.12         |
|                        | EGT, BAP           | 4.81        | 5.74         |
|                        | EGT, GOT           | 5.34        | 6.42         |
| 3                      | EGT, BF, BAP       | 4.63        | 5.54         |
|                        | EGT, BF, GOT       | 5.16        | 6.43         |
|                        | EGT, BAP, GOT      | 3.19        | 3.78         |
| 4                      | EGT, BF, BAP, GOT  | 5.08        | 6.24         |

RUL: remaining useful life; APU: auxiliary power unit; RVM: relevance vector machine; MAE: mean absolute error; RMSE: root mean square error; EGT: exhaust gas temperature; BF: bleed flow; BAP: bleed air pressure; GOT: gearbox oil temperature.
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