Empirical Study on Characteristics of Angled Spoke-Based Wheels on Granular Media

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Abstract
In this study, the optimal design parameters of angled spoke-based wheels (ASWs) was evaluated to maximize the driving speed of a mobile robot with ASWs on granular media. As a search and rescue mobile robot, it is required to locomote various terrains agilely. When granular media behave as fluids, slippage occurs, and the mobile robot slows down compared with when the media behave similarly to a solid ground. There are no exact criteria to explain the behavior of granular media. Therefore, the Taguchi method and \( L_9(3^4) \) orthogonal array were employed to empirically optimize the shape of the ASW. Base on the geometry feature of precedent design of ASWs design parameters were selected. The design variables evaluated were the motor input velocity, width–height proportion of the foot, radius of curvature, and toe angle of the foot. Grain size of granular media was adopted as the user condition. An orthogonal array and signal-to-noise (S/N) ratio were used to analyze the validity of the design variables. The optimal design variables of the ASW were a motor velocity of 5000 rpm, width–height proportion of 3:7, radius of curvature of 1.0\( h \), and toe angle of 15°. Verification experiments were performed to evaluate the improved driving speed of the ASW. The angular velocity of the ASW increases by 2.79% for \( \varnothing 1 \text{ mm} \) and 10.50% for \( \varnothing 6 \text{ mm} \). Therefore, an optimized ASW design is expected to improve the driving speed of a mobile robot with ASWs.

Keywords Angled spoke-based wheel · Mobile robot · Robust design · Sensitivity analysis · Granular media · Taguchi method

1 Introduction

Angled spoke-based wheel (ASW) and the mobile robot with ASWs called DODO were designed for field of Search and rescue (SAR) and inspection with ability to locomote various terrains [1]. Numerous technologies have been developed for disaster prevention. However, unexpected disasters, such as Deepwater horizon oil spills, have occurred. Approaching method to the cite of maritime disaster is suggested in Fig. 1. When such maritime disaster occurs, ground vehicle should overcome ground, granular media even before it reaches to water. This makes it difficult for a wheeled vehicle to access the site. As shown in Fig. 1a, when a ship is wrecked at sea, robots should pass the granular surface to approach the water. Figure 1b illustrates the maritime rescue carried out on the shore. This requires robots to walk on granular media. Furthermore, SAR should be performed within 72 hours, because it is a critical time to rescue human life [2]. Therefore, this study focused on optimizing design parameters of spokes based on geometry of precedent ASW to increase driving speed on granular media.
Sand is the most representative granular medium. Granular media refers to a collection of particles that behave solid- and liquid-like [3]. Therefore, the locomotion of legged robots can be divided into two types, namely walking and swimming [4]. The motor input velocity and the volume fraction of a granular medium determine the fluidization of the medium [5]. Walking locomotion occurs at a relatively low motor velocity and high volume fraction, whereas swimming locomotion occurs when the motor velocity increases and the volume fraction decreases [6].

Many studies were conducted to explain the characteristics of granular media. Indeed, parameters that affects granular media such as low motor velocity [7], high motor velocity [8], overshoot of force [9], and fluctuation of force [10] has been conducted. Characteristic of granular media significantly affects driving ability. However, mathematical model for behavior of mobile robots in granular media have not yet completed. Therefore, optimization of ASW to maximize driving speed of DODO on granular media has to be done empirically.

Research has been conducted to design the limbs of mobile robots to locomote on granular media. The most representative research is the study on RHex-class robots such as Sandbot, Amphihex-I, Amphihex-II, and a flipper-based robot called Fbot. RHex-class robots have vertically spinning one-degree-of-freedom curved-shaped legs. Studies on Sandbot [5, 11–13] focused on the direction and degree of curvature. In addition, prior studies also examined Amphihex-I [14, 15] focused on the influence of the shape of its elliptical leg. These studies showed that the greater the curvature, the faster do the mobile robots advance. Another approach that shifts materials with different stiffness has been performed with Amphihex-II [4]. A lower stiffness can result in a faster driving speed on granular media. Meanwhile, Fbot [16] has legs with two degrees of freedom that push off the granular media horizontally. It was shown that legged robots can not only move on solid terrain but also locomote on granular media. However, although both types of walking methods are capable of locomotion on a granular surface, their speed is significantly low for SAR. RHex classed legs spin in a circle when granular media fluidized owing to high motor velocity. In addition, Fbot advances only a short distance with a single swing. The ASW mechanism was adopted in this study. ASW has advantages for disaster relief and SAR such as high driving speed, various surface drivability, and the ability to overcome obstacles. By changing the shape of the preceding ASW, the driving speed on granular media was increased. Precedent ASWs experienced a speed drop owing to the slippage on granular media [1]. It had a cylindrical shape for a single point contact with the ground. Although the penetration of spokes into granular media was high, the contact area was limited. Therefore, its driving speed drops owing to the slippage caused by the low drag force. To prevent slippage and increase the driving speed, a new design for ASW is required. A newly introduced design can adjust parameters that affect the driving speed of the ASW.

The remainder of this paper is organized as follows. Section 2 defines the problem of ASWs on granular media. Section 3 explains robust design planning, including the objective function, design variables, and user condition. Section 4 details the experimental results and discussion. Finally, conclusions are presented in Sect. 5.

2 Configuration of ASW

2.1 Problem Definition

Granular media have a unique characteristic that they can exhibit solid- and fluid-like behavior [3]. Granular media become fluidized when slippage occurs between the grains. The speed of ASW on granular media drops when granular media behave fluid-like. Three factors that decrease the driving speed of ASW such as motor velocity [5], volume fraction [6], and grain mass [17] were selected.
When the motor velocity is low, granular media behave similarly to a solid ground, whereas they behave fluid-like for higher motor velocity [5]. When a legged robot locomotes on solidified granular media, limb kinematics determines the speed of the legged robot [17]. However, during swimming locomotion on fluidized granular media, slippage occurs and the speed drops [7]. The volume fraction $\phi$ is another main factor that determines the driving speed of the ASW. The volume fraction is defined by equation (1), which is the volume of granular medium $v_{\text{solid}}$ over the volume occupied $v_{\text{occupied}}$ [18].

$$\phi = \frac{v_{\text{solid}}}{v_{\text{occupied}}} \quad (1)$$

The drag force is influenced by the penetration depth and contact area [17]. Densely packed granular media with a high volume fraction prevent ASW from penetrating deeply. However, loosely packed granular media with a low volume fraction can allow the ASW to penetrate deeply. As the drag force of an ASW is influenced by the penetration depth and contact area [17], the driving speed is dependent on the volume fraction. However, if the penetration is excessively high, the driving speed of the ASW can be decreased [6]. Therefore, an empirical study is necessary to ascertain the optimal conditions for the ASW.

The mass of the grain determines the critical force $F_c$ to fluidize the granular media as shown in equation (2). If the applied force is higher than $F_c$, slippage occurs. The critical force was calculated as follows [17]:

$$F_c = A m g j \quad (2)$$

where $A$ is a constant depending on the characteristics of granular media, $g$ is gravity, $m_g$ is the mass of a grain, and $j$ is the depth of the penetration length of the ASW. The larger grain mass, the higher is the critical force. Therefore, advancing on granular media with heavy grains can result in less slippage.

### 2.2 Foot and Spoke Configuration

To improve the driving speed, the newly introduced spoke design shown in Fig. 2 consists of the following three parts: a wheelbase, spoke, and foot. The design variables of the foot configuration were determined to maximize the driving speed of the ASW. The design variables determine the contact area, the ability to penetrate granular media, and the thrust force of a foot. The design was optimized to maximize the driving velocity of the ASW on granular media.

The ASW has a unique workspace that is tilted at 45°. The newly designed model shown in Fig. 3 follows the same mechanism. This tilted workspace changes the contact area of the ASW projected onto the XZ plane. The mobile robot with ASW advances along the Y axis, and this change affects the driving speed.

### 3 Experiment Design

#### 3.1 Taguchi Method

The Taguchi method was used to demonstrate the robustness of the ASW [19, 20]. As the characteristics of granular media change with the motor velocity and volume fraction, an empirical study is required for the optimal design of ASWs. The Taguchi method was used to optimize the variables of the ASW with the least number of experiments [21, 22]. Three levels of the four design variables were used. The corresponding $L_9(3^4)$ Taguchi orthogonal array was used.
3.2 Objective and Design Variables

The purpose of this research was optimizing design parameters of ASW to maximize the driving speed of ASW on granular media. The driving speed was measured as the average angular velocity. Design variables that influence the driving speed of the ASW are determined. Figure 4 shows four design variables, that is, motor velocity, width–height proportion, radius of curvature, and toe angle. The levels of each design variable for the first and second experiments are presented in Tables 1 and 2. Based on the results of the sensitivity analysis from the first experiment, the levels of each design variable were adjusted.

The motor velocity determines the characteristics of granular media. The width–height proportion influences the penetration and drag forces. It is the proportion between the width and height $h$ of the projected area of a foot. If the proportion of height is greater, it can penetrate better. If the height is larger than the width, the friction drag will be greater. The radius of curvature influences the contact time. A longer contact time causes a higher $y$ axis reaction force and levitates ASW. Finally, the toe angle was considered because ASW has a $45^\circ$ tilted work plane. The angle between the contact area and driving direction of the ASW was defined as the toe angle. The toe angle $\varphi$ generates a directional force.

3.3 User Condition

The grain size was selected as the user condition. Grain size is an important factor that affects both the volume fraction and critical force. As grain size gets bigger, volume fraction gets larger [23]. In addition, when the mobile robot with ASWs is relieved, the SAR grain size cannot be predetermined. This must advance regardless of the grain size. Therefore, two different sizes of the ocher balls in Table 3 were determined. The size is presented as the diameter of the grain with $\Phi$ (mm). For variable control, a controlled grain volume is required. Therefore, isometric ocher balls of two different sizes were selected.

3.4 S/N Ratio

To maximize the driving speed of the ASW, the mesh characteristic equation (3) was used.

$$S/N\text{ratio} = -10\log \left( \frac{1}{y_1^2} + \frac{1}{y_2^2} + \cdots + \frac{1}{y_n^2} \right)$$  \hspace{1cm} (3)

where $y_i (i = 1, 2, \ldots, n)$ denotes the measured Average velocity of ASW and $n$ represents the repetitions of an experiment under both user conditions.

3.5 Test Bench

A test bench for the experiments is shown in Fig. 5. The test bench consists of three units, namely a stationary unit, measuring unit, and mobile unit. The stationary unit consists of a frame of the test bench and a circular rail. The circular rail is filled with ocher balls, and the mobile unit moves along the rail.
The mobile unit consists of a ECX TORQUE 22L motor with 14:1 gear head, one set of ASWs, and a squeeze. The motor input velocity is controlled by EPOS2 24/5 controller. The mobile robot with ASWs for relief and SAR has a light weight. Its weight is approximately 1.2 kg. The payload on one set of ASW is approximately 300 g. Therefore, the payload of the ASW was set to 300 g. As in the previous ASW, the spokes and feet are connected to a motor at 45° angle. As the motor rotates, the foot in contact with the ochre ball propels, and the mobile unit advances along with the rail. The squeeze was connected to the opposite side of the motor. After the ASW walks on the ochre ball, it is left on the ochre ball. The squeeze flattens the surface of the ochre ball to initialize the surface conditions for the next revolution.

The measuring unit consists of an encoder and a rod, and they are placed so as to have the same concentricity with the rail. The length of the moment arm and the length between the center of the test bench and the motor is fixed at 250 mm. The rod was connected to the HE40B-6-1024-6-L encoder which is shown in Table 4, squeeze, and motor. The rod was fixed with an encoder with a shaft so that the encoder measures the angle of the rod. The squeeze was fixed with the rod. However, the motor is connected to the rod with two ball bushes, which creates degrees of freedom along the Z-axis. The angular velocity can be calculated on the basis of the measured angle.

### 4 Experiment and Result

#### 4.1 Experimental Method

The experiments were conducted with five revolutions. The assigned motor input velocity, which makes the ASW spin, was applied. The ASW generates advancing movements and rotates along with a rod connected to a rotary encoder at the center of the test bench. The angle data of the rod was measured using an encoder, and the angular velocity was calculated based on the angle data. Please refer to the Multimedia extension for the experimental set-up and results. A total of five revolutions were conducted, and data from the second to fourth revolutions were used. The driving speed was calculated with the average angular velocity. In addition, the S/N ratios were calculated using equation (3).

#### 4.2 Experimental Result

The experiments were performed twice. The levels of design variables for the second experiment were determined from the results of the first experiment shown in Table 5. The result of second experiment was shown in Table 6. Based on the sensitivity analysis in Fig. 6, the redirect scopes of levels are higher for motor input velocity, width–height proportion, and radius of curvature. The scope of the toe angle is narrowed down the range because it has a maximum S/N ratio at level 2. Since physical configuration, limitation of width–height proportion and radius of curvature are 1:9 and $1.0 \times h$.

From the sensitivity analysis of the first experiment shown in Fig. 6, it was found that the driving speed of the ASW increases as the height proportion increases. Therefore, the second experiment was conducted using design variables with a higher length proportion. The results show that the width and length proportion of the projected area with a 3:7 proportion shows the greatest driving speed. Proportion of width determine penetration of feet into granular media.
### Table 5 \( L_9(3^4) \) Orthogonal array of 1st experiment

| Number of experiment | Design variables | User conditions | S/N ratio (dB) |
|----------------------|-----------------|----------------|--------------|
|                      | A   | B   | C   | D   | Grain size \( \phi 1 \) mm | Grain size \( \phi 6 \) mm | |
|                      |     |     |     |     | Average angular velocity \( y \) (deg/s) | |
| 1                    | 1   | 1   | 1   | 1   | 10.72 | 15.46 | 10.58 | 29.64 | 31.65 | 33.40 | 23.83 |
| 2                    | 1   | 2   | 2   | 2   | 35.64 | 35.18 | 35.86 | 48.20 | 47.40 | 47.88 | 32.12 |
| 3                    | 1   | 3   | 3   | 3   | 69.01 | 68.73 | 69.35 | 72.75 | 73.08 | 73.25 | 37.02 |
| 4                    | 2   | 1   | 2   | 3   | 31.02 | 31.66 | 32.13 | 44.48 | 43.11 | 44.48 | 31.20 |
| 5                    | 2   | 2   | 3   | 1   | 27.67 | 28.25 | 30.83 | 52.35 | 57.72 | 54.81 | 31.15 |
| 6                    | 2   | 3   | 1   | 2   | 63.07 | 62.18 | 60.84 | 93.20 | 88.57 | 97.15 | 37.26 |
| 7                    | 3   | 1   | 3   | 2   | 60.06 | 58.87 | 60.07 | 122.89 | 124.08 | 127.19 | 37.63 |
| 8                    | 3   | 2   | 1   | 3   | 74.58 | 74.58 | 67.74 | 84.34 | 79.36 | 77.31 | 37.59 |
| 9                    | 3   | 3   | 2   | 1   | 49.50 | 47.97 | 51.28 | 71.35 | 72.73 | 76.75 | 35.28 |

### Table 6 \( L_9(3^4) \) Orthogonal array of 2nd experiment

| Number of experiment | Design variables | User conditions | S/N ratio (dB) |
|----------------------|-----------------|----------------|--------------|
|                      | A   | B   | C   | D   | Grain size \( \phi 1 \) mm | Grain size \( \phi 6 \) mm | |
|                      |     |     |     |     | Average angular velocity \( y \) (deg/s) | |
| 1                    | 1   | 1   | 1   | 1   | 162.8 | 161.66 | 176.4 | 206.27 | 202.3 | 200.78 | 45.21 |
| 2                    | 1   | 2   | 2   | 2   | 189.4 | 185.23 | 171.41 | 201.36 | 206.32 | 199.14 | 45.62 |
| 3                    | 1   | 3   | 3   | 3   | 248.7 | 225.78 | 247.14 | 218.03 | 225.64 | 223.19 | 47.25 |
| 4                    | 2   | 1   | 2   | 3   | 308.52 | 293.14 | 289.11 | 301.6 | 301.42 | 299.85 | 49.51 |
| 5                    | 2   | 2   | 3   | 1   | 229.94 | 218.84 | 234.27 | 248.91 | 245.98 | 243.63 | 47.47 |
| 6                    | 2   | 3   | 1   | 2   | 212.49 | 198.18 | 195.66 | 270.51 | 250.4 | 259.21 | 47.05 |
| 7                    | 3   | 1   | 3   | 2   | 251.36 | 270.49 | 295.93 | 317.02 | 296.79 | 290.74 | 49.09 |
| 8                    | 3   | 2   | 1   | 3   | 286.66 | 300 | 305.79 | 324.92 | 293.15 | 270.52 | 49.41 |
| 9                    | 3   | 3   | 2   | 1   | 176.17 | 196.22 | 205.67 | 289.14 | 291.11 | 292.94 | 47.09 |

![Fig. 6](image.png) Result of sensitivity analysis of 1st and 2nd experiments
As width decreases pressure at the tip of feet increases, and penetration gets larger. Deeper penetration generates higher thrust force of DODO. However, if the penetration gets too deep, it slows speed of DODO. Based on the sensitivity analysis of second experiment 2:8 proportion showed lower S/N ration than 3:7. Therefore, the fittest width and height proportion was found as 3:7 proportion.

From the first and second experiments, the curvature showed an increasing sensitivity, while the radius $R$ increased. Owing to the physical limit, $R$ should be equal to or greater than the height $h$ as shown in Fig. 4. Larger radius of curvature generate larger projected area when it contacts to granular media. Larger projected area prevent feet from sinking into granular media. Therefore, the optimal value of $R$ to make the curvature is when $R$ is equal to $h$.

The optimal toe angle was 0° in the first experiment. Therefore, the range of the toe angle was narrowed down, and the second experiment was conducted near 0°. The second experiment showed that 15° had the highest sensitivity. This is because ASW has 45° tilted area, and it changes the projected contact area through the path of the foot.

The speed drops caused by fluidization did not occur under both user conditions. User conditions were selected for robust optimal result under two extreme conditions. The experimental results showed a tendency that driving speed under $\varnothing 6$ mm was faster than $\varnothing 1$ mm. This implies that less slippage occurred in larger sized Ocher balls. Since the mass of $\varnothing 6$ mm grain is greater than that of $\varnothing 1$ mm grain, and volume fraction of $\varnothing 6$ mm is greater, $\varnothing 6$ mm grain behaves like solid ground.

### 4.3 Verification Experiment

Verification experiments were conducted. The optimal foot features of the design variable combination of both experiments are shown in Table 7. The validation experiments were conducted under the same conditions. Average angular velocities were used to compare the optimal and orthogonal array experiments. The results are presented in Fig. 7 The optimal design showed a speed improvement of 2.79 % and 10.50% on ochre balls of $\varnothing 1$ mm and $\varnothing 6$ mm, respectively, compared to the fastest result of the orthogonal array.

### 5 Conclusion

In this study, the shape of the ASW was optimized. The design variables used in this study were the motor velocity, width and height proportion of the projected area of the foot, radius of curvature, and toe angle. The Taguchi method was used and based on the S/N ratio from $L_9(3^4)$, orthogonal array sensitivity analysis was conducted. Through sensitivity analysis, it was confirmed that the design variables are sensitive to the driving speed of ASW on granular media.

| Design variables       | Optimal value |
|------------------------|---------------|
| A Motor input velocity (rpm) | 5000          |
| B Width–height proportion    | 3:7           |
| C Radius of curvature       | 1.0           |
| D Toe angle               | 15            |

![Fig. 7 Result of validity experiments: a result of 1st validity experiment; b result of 2nd validity experiment](image-url)
The optimal conditions for a higher driving speed of ASW on granular media were achieved through two experiments, as follows: the motor input velocity was 5000 rpm, width–height proportion was 3:7, radius of curvature was $1.0\times\phi$, and toe angle was 15°. Optimized ASW showed angular velocity of 305.78°/s on $\phi 1$ mm ocher ball, and 333.19°/s on $\phi 6$ mm ocher ball. The optimal ASW showed a speed improvement of 2.79% and 10.50% on ocher balls of $\phi 1$ mm and $\phi 6$ mm, respectively, compared to the fastest combination of orthogonal arrays.

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