TWO LOOP UNIFICATION OF NON-SUSY SO(10) GUT WITH TEV SCALARS

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Abstract. In this paper we examine gauge coupling unification at the two loop level in the non-SUSY SO(10) grand unified theory proposed by Babu and Mohapatra [1]. This GUT, which breaks down to the standard model in a single step, has the distinguishing feature of containing non-standard model scalars at the TeV scale. This leads to a plethora of interesting effects in the TeV range, most prominently predicting the possibility of discovering new particles at the LHC in run 2. This model also gives rise to measurable proton decay, neutron-antineutron oscillations, provides a mechanism for baryogenesis, and contains potential dark matter candidates. In this paper, we compute the two loop beta function and show that this model unifies to two loop order around $10^{15}$ GeV. We then compute the proton lifetime and argue that threshold effects place it comfortably above the Super Kamiokande limit. In this paper, we demonstrate that this model passes the baseline for physical plausibility and therefore is worth studying due to its interesting low energy phenomenology.

1. Introduction

The standard model is one of the most predictively successful theories in physics, providing a successful description of nearly all high energy observations. However, since its inception in the early 1970's, this model has been shown to have many issues ranging from naturalness to observational discrepancies, indicating its incompleteness. The most prominent of these problems is the observation of neutrino mass, a feature excluded from standard model [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Grand unified theories solve this problem in a simple and elegant manner. They also pose resolutions to many other issues, including the origin of charge quantization and standard model gauge couplings. In addition, they also suggest explanations to cosmological phenomena, providing both mechanisms to explain baryogenesis and potential dark matter candidates.

Among this broad class of theories, the subset of minimal SO(10) theories are arguably the most natural. They describe each generation of the standard model in an irreducible representation of the gauge group and have inherent left-right chiral symmetry. These theories also provide a natural explanation for the origin of neutrino masses and flavor oscillation through a high scale see-saw mechanism [2, 14, 15, 16, 17, 18] coming from symmetry breaking of a $10\oplus126$-plet Higgs sector [19].

A key test of grand unification theories is through proton decay experiments. In the standard model, both baryon and lepton numbers are conserved. This prevents protons, the lightest baryon, from decaying through perturbative interactions. However, one of the salient features of GUTs is that due to having a larger gauge group, they most often predict the decay of protons through the interactions of additional heavy gauge bosons. Fortunately, due to the Higgs mechanism, these gauge bosons acquire a mass on the order of the unification scale, suppressing proton decay to an order $1/M^4_{U}$, in agreement with the observation that protons are effectively stable.

This places a special emphasis on proton decay experiments, making them a powerful tool for determining the viability of a GUT model. In experiment, the most commonly sought decay modes are the $p \rightarrow e^{+}\pi^{0}$ and $p \rightarrow \bar{\nu}K^{+}$, the second of which shows up primarily in SUSY GUTs [20]. By measuring these decay channels, the latest experiments have placed the lifetime of the proton on the order of $10^{34}$ years [21, 22]. This places heavy restrictions on the class of physical theories, providing a minimum requirement for all GUT models.

However, there are many effects which makes the theoretical prediction of the proton decay lifetime hard to estimate. There are two main sources of error in this calculation. The first stems from the fact that proton is a composite particle made up of quarks. Unfortunately, quark confinement is still not well understood and is thought to be a non-perturbative effect. Because of this, numerical lattice QCD must be used to approximate decay amplitudes, which introduces error inherent to numerical methods. The second source of error, is the fact that there are many heavy Higgs after symmetry breaking which are not taken into account. These can lead to uncertainties in the proton lifetime up to 2.1 orders of magnitude [23]. These uncertainties make it hard to determine if a GUT model agrees with experiment if it predicts a proton lifetime within one or two orders of magnitude of the experimental value.
2. Babu-Mohapatra Model

In a recent paper [1], Babu and Mohapatra pointed out that a non-SUSY SO(10) GUT model can be constructed so that the seesaw scale is close to the GUT scale. But the inherent quark-lepton unification in GUTs implies that $\Delta L = 2$ breaking required for seesaw give rise to observable $\Delta B = 2$ processes leading to neutron-antineutron oscillation. Key to this observation is the existence of a TeV mass color sextet scalar ($\Delta_{u,c,d}$) transforming like $(6,1,1/3)$ which is necessary for one loop unification. Strictly speaking, unification additionally requires the TeV scale color sextet $\Delta_{u,c,d}$ to be accompanied by 2 complex weak triplets, which could arise from 45-Higgs field required for SO(10) symmetry breaking. These scalars together lead to one loop unification of the gauge couplings near $10^{15}$ GeV.

This leads to several experimentally interesting features: (i) the TeV scalars have a low enough mass to potentially be discovered at the LHC in run 2, (ii) due to unification scale around $10^{15}$ GeV, it predicts observable proton decay, (iii) by virtue of having color sextets at low mass there is a mechanism for neutron-antineutron oscillation which could be measured in the next generation of experiments [24], (iv) this color sextet provides a mechanism for GUT scale baryogenesis which is unaffected by electroweak sphaleron processes, and (v) the $45 \oplus 45$ component of the Higgs sector has no Yukawa coupling, so the weak triplets have incredibly low coupling to the standard model, providing an excellent candidate for dark matter.

Since neutron-anti-neutron oscillation goes in this model like $M_{\Delta_{u,c,d}}^{-4}$, it is important to know the precise value of the color sextet scalar mass. Similarly to have a reliable prediction for proton decay, one needs a precise value of the unification scale. To accomplish these goals, it is necessary to carry out a two loop analysis of the gauge coupling evolution which can determine the color sextet mass as well as the unification scales more precisely than the one loop analysis of [1]. It is the goal of this paper to carry out this program.

3. Coupling Unification

The standard method for constructing non-SUSY SO(10) models is by implementing intermediate symmetry groups, each of which are broken at a different energy level, creating a multi-step chain from SO(10) to the Standard Model. A detailed two loop analysis of many SO(10) breaking chains were carried out in [25]. However the breaking pattern of this model was not included there.

$$SO(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$$ (1)

We will now take a bottom-up perspective and assume that in addition to the standard model, there exists a color sextet $\Delta_{u,c,d}$ which transform as $(6,1,1/3)$, and two weak triplets $\omega_i$ which transform as $(1,3,0)$ [1].

The formula for the two loop beta function is given in [26, 27] as:

$$\beta(g) = \frac{g^3}{(4\pi)^2} \left\{ \frac{2}{3} C_2(F) + \frac{1}{3} C_2(S) - \frac{11}{3} C_2(G) \right\}$$

$$+ \frac{g^5}{(4\pi)^4} \left\{ \left[ 2C_2(F) + \frac{10}{3} C_2(G) \right] S_2(F) \right\}$$

$$- \frac{34}{3} [C_2(G)]^2 + \left[ 4C_2(S) + \frac{2}{3} C_2(G) \right] S_2(S) \right\}$$

where the different field components are implicitly summed over. Here, $C_2(R)$ and $S_2(R)$ are the Casimir element and Dynkin index of a representation $R$ respectively, and $F$, $S$, $G$ correspond to the fermionic, scalar, and adjoint representation of a gauge group $G$.

When $G$ is the direct product of semi-simple terms ($G = G_1 \times G_2 \times ... G_n$), the two loop term allows for mixing of the gauge subgroups corresponding to contributions to the gauge boson propagator from field multiplets transforming non trivially under multiple subgroups. These fields change the formula such that in addition to the diagonally embedded representation terms, there are also terms that go as:

$$... + \sum_j \frac{g_i^3 g_j^2}{(4\pi)^4} [2S_2(F_i)C_2(F_j) + 4S_2(S_i)C_2(S_j)] \right\}$$ (3)

where $F_i$ is the sub-representation of $F$ transforming under the gauge group $G_i$. Now, writing the beta function as:

$$\beta(g_i) = \frac{g_i^3}{(4\pi)^2} b_i + \sum_k \frac{g_i^3 g_k^2}{(4\pi)^2} b_i k$$ (4)

the numerical coefficients are given by:

$$b_i = \begin{pmatrix}
\frac{127}{30} \\
-\frac{11}{6} \\
-\frac{37}{6}
\end{pmatrix} \quad b_{ij} = \begin{pmatrix}
\frac{613}{150} & \frac{37}{10} & \frac{172}{15} \\
\frac{9}{10} & \frac{43}{10} & 12 \\
\frac{37}{6} & \frac{9}{2} & \frac{37}{3}
\end{pmatrix}$$ (5)

These match with the calculation from [1]. Following Babu and Mohapatra, we will also consider a second Standard Model Higgs doublet $H(1,2,1/2)$ which produces the beta function:

1Here we use the normalization for the hypercharge so that $C_2(F) = \left( \frac{Y}{3} \right)^2$
the $\beta_i$ can be numerically integrated using Mathematica.

We found that with a single standard model Higgs doublet, the couplings unify at $1.42 \times 10^{15}$ GeV with $M_\omega = 6.1587$ TeV and $M_\Delta = 1$ TeV, and $\alpha_U(M_U)^{-1} = 38.47$. We also found that with two standard model Higgs doublets, the couplings unify at $1.06 \times 10^{15}$ with $M_\omega = 76.75$ TeV, $M_\Delta = 1$ TeV, and $\alpha_U(M_U)^{-1} = 38.18$. 

### 3.1. Sensitivity

In general, the unification of this model is not very sensitive to the mass of the scalar multiplets. If use the previous unification parameters for a single standard model Higgs doublet and change the mass of the sextet from 1 TeV to 2 TeV we find a very small unification triangle forming (see Figure 2). The size of this triangle is on the order of $110$ TeV with a single standard model Higgs doublet and change the mass of the sextet from 1 TeV to 2 TeV (dashed line) while keeping the mass of the weak triplets constant.

![Figure 2](image2.png)

**Figure 2.** A small unification triangle forms when changing the mass of the color sextet from 1 TeV (solid line) to 2 TeV (dashed line) while keeping the mass of the weak triplets constant.

### 4. Proton Decay

From our calculation for the unification scale we can determine the proton lifetime in this model due to the decay mode: $p \rightarrow e^+\pi^0$. In general there are 5 types of dimension 6 operators which lead to nucleon decay. In the notation of Weinberg, they are denoted $[28, 29, 30, 31]$

\[
O^{(1)}_{abcd} = (d_{a\alpha R}u_{b\beta R})(u_{c\gamma L}L_{j=1}d_{L})\epsilon_{\alpha\beta\gamma\epsilon_{ijkl}} \\
O^{(2)}_{abcd} = (q_{a\alpha L}q_{b\beta L})(u_{c\gamma R}L_{j=1}d_{R})\epsilon_{\alpha\beta\gamma\epsilon_{ijkl}} \\
O^{(3)}_{abcd} = (q_{a\alpha L}q_{b\beta L})(q_{k\gamma C}L_{j=1}d_{L})\epsilon_{\alpha\beta\gamma\epsilon_{ijkl}} \\
O^{(4)}_{abcd} = (q_{a\alpha L}q_{b\beta L})(q_{k\gamma C}L_{j=1}d_{R})\epsilon_{\alpha\beta\gamma\epsilon_{ijkl}} \\
O^{(5)}_{abcd} = (d_{a\alpha R}u_{b\beta R})(u_{c\gamma R}L_{j=1}d_{R})\epsilon_{\alpha\beta\gamma}
\]

where $\alpha, \beta, \gamma$ denote SU(3) color indices, $i,j,k,l$ are SU(2) isospin indices, $a,b,c,d$ are generation indices, and $L$ and $R$ refer to the chirality. Only 4 of these $(1,2,4,5)$ are relevant for proton decay. Of these only:

\[
Q^{(1)} = O^{(1)}_{1111} = (d_{a\alpha R}u_{b\beta R})(u_{c\gamma L}L_{j=1}d_{L})\epsilon_{\alpha\beta\gamma} \\
Q^{(2)} = -\frac{1}{2}O^{(2)}_{1111} = (d_{a\alpha L}u_{b\beta L})(u_{c\gamma R}L_{j=1}d_{R})\epsilon_{\alpha\beta\gamma} \\
Q^{(3)} = O^{(3)}_{1111} = (d_{a\alpha L}u_{b\beta L})(u_{c\gamma L}L_{j=1}d_{R})\epsilon_{\alpha\beta\gamma} \\
Q^{(4)} = O^{(4)}_{1111} = (d_{a\alpha R}u_{b\beta R})(u_{c\gamma R}L_{j=1}d_{R})\epsilon_{\alpha\beta\gamma} \\
Q^{(5)} = O^{(5)}_{1111} = (d_{a\alpha R}u_{b\beta R})(u_{c\gamma R}L_{j=1}d_{R})\epsilon_{\alpha\beta\gamma}
\]

lead to proton decay in the mode: $p \rightarrow e^+\pi^0$. We will denote the first part of these operators (the parts with a $e^+$ leptonic term) $O_{1\Gamma \Gamma'}$ where $\Gamma, \Gamma' = L, R$. For example: $O_{LR} = Q^{(2)}$. In order to calculate the proton lifetime we need to calculate the amplitude:

It is important to note that the mass of the scalars are not fixed. Once the mass is fixed for one scalar, the other is fixed by the condition of gauge unification. The given scalar masses were selected to give the maximum proton decay lifetime so that the scalar masses are $\geq 1$ TeV.
\[
\langle \pi^0, e^+ | O_{\Gamma V} | p \rangle = -\langle \pi^0, e^+ | O_{\Gamma V} | p \rangle \tag{10}
\]

where \( \tilde{R} = L \) and \( \tilde{L} = R \). From this, the formula for the decay width is given by [31]:

\[
\Gamma(p \rightarrow \pi^0 + e^+) = \frac{m_p}{32\pi} \left(1 - \left(\frac{m_\pi}{m_p}\right)^2\right)^2 \times \left| \sum_i C_i^W \langle p \rightarrow \pi^0 + e^+ \rangle \right|^2 \tag{11}
\]

where the \( W_i^j \) are the form factors associated to the operators \( Q^{(i)} \) (i.e. \( W_0^0 = \langle \pi^0, e^+ | Q^{(i)} | p \rangle \)) and the \( C_i^W \) are the Wilson coefficients coming from the renormalization of gauge couplings. These coefficients are given by running the coupling constants from mass scale from \( \mu_1 \) down to \( \mu_2 \) [29, 30]:

\[
\begin{align*}
C_1(\mu) &= \left[ \frac{\alpha_3(\mu_2)}{\alpha_3(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_1}} \left[ \frac{\alpha_2(\mu_2)}{\alpha_2(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_2}} \left[ \frac{\alpha_1(\mu_2)}{\alpha_1(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_3}} C_{1(i)}^{\mu_1}
C_2(\mu) &= \left[ \frac{\alpha_3(\mu_2)}{\alpha_3(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_1}} \left[ \frac{\alpha_2(\mu_2)}{\alpha_2(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_2}} \left[ \frac{\alpha_1(\mu_2)}{\alpha_1(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_3}} C_{2(i)}^{\mu_1}
C_3(\mu) &= \left[ \frac{\alpha_3(\mu_2)}{\alpha_3(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_1}} \left[ \frac{\alpha_2(\mu_2)}{\alpha_2(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_2}} \left[ \frac{\alpha_1(\mu_2)}{\alpha_1(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_3}} C_{3(i)}^{\mu_1}
C_4(\mu) &= \left[ \frac{\alpha_3(\mu_2)}{\alpha_3(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_1}} \left[ \frac{\alpha_2(\mu_2)}{\alpha_2(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_2}} \left[ \frac{\alpha_1(\mu_2)}{\alpha_1(\mu_1)} \right]^{\frac{\Delta\mu}{\mu_3}} C_{4(i)}^{\mu_1}
\end{align*}
\]

where the first error is statistical and the second is systematic. We now have the final formula for the proton decay width:

\[
\Gamma(p \rightarrow \pi^0 + e^+) = \frac{m_p}{32\pi} \left(1 - \left(\frac{m_\pi}{m_p}\right)^2\right)^2 \times \left(1 - \left(\frac{m_\pi}{m_p}\right)^2\right)^2 \tag{14}
\]

Using the values given above for the QCD factor and the renormalized Wilson coefficients, we find:

\[
\tau(p \rightarrow \pi^0 + e^+) \simeq (1.09 \pm 0.90) \times 10^{33} \text{ yrs.} \tag{15}
\]

where the error comes from the QCD factor. In addition to this, we expect this value to have a total error of 0.9-2.1 orders of magnitude due to threshold effects [23]. This puts the lifetime of the proton comfortably outside the lower limit set by the Super Kamiokande experiment (1.4 \times 10^{34} years [21, 22]), but within reach of future experiments.

### 4.1. Extension of the Babu-Mohapatra Model.

It is interesting to note that the proton lifetime of this model could be lengthened by extending the Babu-Mohapatra model. This can easily be done by adding more scalar sextets. It seems that in general full two loop unification can be achieved if there is 1 more triplet than sextet. This causes the point of intersection between the U(1)\(_Y\) and SU(3)\(_C\) to move so that the unification scale increases and the coupling at unification decreases.
Table 1. Comparison of extensions to the Babu-Mohapatra model by adding additional color sextets (Δ) and weak triplets (ω). In these models $m_\Delta = 1$ TeV in order to achieve the maximum proton lifetime.

| $(N_\Delta, N_\omega)$ | $m_\omega$ (TeV) | $\tau(p \rightarrow \pi^0 + e^+)$ yrs. |
|------------------------|-----------------|-------------------------------------|
| (1,2)                  | 6.1587          | $(1.09 \pm 0.90) \times 10^{34}$    |
| (2,3)                  | 20.6187         | $(3.78 \pm 3.00) \times 10^{37}$    |
| (3,4)                  | 47.749          | $(9.80 \pm 7.81) \times 10^{34}$    |
| (4,5)                  | 100.714         | $(5.43 \pm 4.32) \times 10^{34}$    |
| (5,6)                  | 216.166         | $(3.83 \pm 3.04) \times 10^{34}$    |

Some values are plotted in Figure 3 with corresponding unification values in Table 1. These types of models can also be unified by having the same number of scalar sextets and triplets but these can only be unified by increasing $m_\Delta \geq 1000$ TeV. These extensions can be used to further manipulate the mass of the light scalars to conform to measurements such as those from LUX and the LHC while maintaining a sufficiently long proton lifetime.

5. Conclusion

We have demonstrated in this paper, that the non-SUSY SO(10) GUT model put forth by Babu and Mohapatra unifies to two loop order at $M_U = 1.42 \times 10^{15}$ GeV with scalars at the TeV scale [1]. We have presented the calculations to show that this model predicts the proton lifetime to be $(1.09 \pm 0.90) \times 10^{34}$ yrs, which when taking threshold effect into account places the proton lifetime above of the Super-Kamiokande limit. Since this model has been shown to meet the minimum requirement for proton lifetime, this model is physically realizable and should be further investigated for its phenomenology. This model will potentially produce observable effects due to the relatively low mass of the scalar sextet and triplets. It will be especially interesting due to the prediction of scalars with low enough mass to observed at the LHC in run 2. This model is also of great interest due to its potentially testable predictions of neutron-antineutron oscillation and new mechanism for scalar mediated baryogenesis [1].

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