Fano-type interference in quantum dots coupled between metallic and superconducting leads

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We analyze the quantum interference effects appearing in the charge current through the double quantum dots coupled in $T$-shape configuration to an isotropic superconductor and metallic lead. Owing to proximity effect the quantum dots inherit a pairing which has the profound influence on nonequilibrium charge transport, especially in the subgap regime $|V| < \Delta/|e|$. We discuss under what conditions the Fano-type lineshapes might appear in such Andreev conductance and consider a possible interplay with the strong correlation effects.

I. INTRODUCTION

Heterostructures with nanoobjects (such as quantum dots, nanowires, molecules, etc) hybridized to one conducting and another superconducting electrode seem to be promising testing fields where the strong electron correlations (responsible e.g. for Coulomb blockade and Kondo physics [1]) can be confronted with the superconducting order [2]. Coulomb repulsion between electrons in the solid state physics is known to suppress the local ($s$-wave) pairing and, through the spin exchange mechanism, eventually promotes the intersite ($d$-wave) superconductivity [3]. Mutual relation between such repulsion and the local pairing is however rather difficult for studying, both on theoretical grounds and experimentally. In nanoscopic heterostructures some of these limitations can be overcome by a suitable adjustment of the hybridization and the gate-voltage positioning of energy levels involved in the charge transfer [4]. They enable a controllable changeover between the Kondo regime and opposite case dominated by the induced on-dot pairing.

Quantum dot (QD) coupled with the strength $\Gamma_N$ to metallic conductor (N) and with $\Gamma_S$ to superconducting electrode (S) can exhibit the features characteristic both for the on-dot pairing and the Kondo effect (including their coexistence) [5]. Their efficiency depends on the ratio $\Gamma_S/\Gamma_N$. In the limit $\Gamma_S \gg \Gamma_N$ the underlying physics is controlled by on-dot pairing and manifests itself e.g. by the particle-hole splitting of the quasiparticle levels. On the other hand for $\Gamma_S \ll \Gamma_N$ the strong correlations take over. Non-trivial aspects related to such interplay between the Coulomb interactions and the proximity induced on-dot pairing has been addressed theoretically using various methods like: the mean field slave boson approach [6], the noncrossing approximation [7], perturbative scheme [8], constrained slave boson technique [9], numerical renormalization group [10–12] and other [13–16]. Also the cotunneling regime of a Coulomb blockaded quantum dot sandwiched between a normal and superconducting lead, where charge fluctuations are strongly suppressed, has been discussed emphasizing the role of in-gap resonances [17].

As far as the experimental situation is concerned it has been less intensively explored. The earliest transport measurements for N-QD-S interface have been obtained using the multi-wall carbon nanotube deposited between Au and Al electrodes [18]. Those investigations concentrated however on the specific regime $k_B T_K \geq \Delta$, when the Coulomb correlations dominated over the proximity effect. Other studies of the same group have been done for similar structures replacing a metallic electrode by a ferromagnet [19]. Several recent efforts focused on the multiterminal structures involving two normal and one superconducting electrodes as useful schemes for realization of: the crossed Andreev reflections tunable via gate voltages [20], the Cooper pair splitters [21], and the QD spin valves [22].

Very useful understanding of a subtle interplay between the correlations and the induced on-dot pairing has been gained from recent measurements by R.S. Deacon et al [23]. The authors have explored the subgap transport originating from the Andreev-type scattering processes for several representative ratios $\Gamma_S/\Gamma_N$ using the self-assembled InAs quantum dots deposited between the golden (N) and aluminium (S) electrodes. Their measurements provided the unambiguous experimental evidence for: a) particle-hole splitting of the subgap conductance of the Andreev states when $\Gamma_S \geq \Gamma_N$, and b) enhancement of the zero-bias Andreev conductance due to formation of the Kondo resonance at the Fermi level of metallic
lead, as has been qualitatively suggested by our studies [16] and also indicated by other groups [23].

The present work extends our former studies by taking into account interference effects arising from the additional degrees of freedom. As the simplest prototype for Fano-type interference [24] we consider the setup (see figure 1) with a side-attached quantum dot contributing an extra pathway for electrons transmitted between the metallic and superconducting leads. Our analysis is complementary to the previous study by Y. Tanaka et al who considered the double quantum dots coupled between N and S leads in a T-shape setup but assuming $U_1 = 0$, $U_2 \neq 0$ [23] and in a series configuration [20].

In section 2 we introduce the microscopic model and briefly outline basic notes on the nonequilibrium subgap transport. In the next section 3 we discuss a unique way in which the Fano-type lineshapes might be observed in Andreev conductance, focusing on the uncorrelated quantum dots. In the last part (section 4) we discuss the influence of correlations at the interfacial quantum dot which seem to have remarkable influence on the low bias transport. We end with the summary and suggestions for the future studies.

II. THE MODEL

For description of the heterojunction illustrated in figure 1 we use the Hamiltonian

$$
\hat{H} = \hat{H}_N + \hat{H}_{N-DQD} + \hat{H}_{DQD} + \hat{H}_{S-DQD} + \hat{H}_S
$$  \hspace{1cm} (1)

where the double quantum dot (DQD) is represented by

$$
\hat{H}_{DQD} = \sum_{\sigma, i} \epsilon_i \hat{d}_{i\sigma}^\dagger \hat{d}_{i\sigma} + U_1 \hat{n}_{1\uparrow} \hat{n}_{1\downarrow} + \left( t \hat{d}_{1\uparrow}^\dagger \hat{a}_{2\downarrow} + \text{h.c.} \right). \hspace{1cm} (2)
$$

The energies of each ($i = 1, 2$) quantum dot electrons are denoted by $\epsilon_i$ and $t$ stands for the usual interdot hopping. We restrict considerations of the correlation effects (section 4) to the Coulomb repulsion $U_1$ between opposite spin electrons $\sigma = \uparrow, \downarrow$ at the interfacial quantum dot.

The external reservoirs N and S of charge carriers are described by $\hat{H}_N = \sum_{k, \sigma} \xi_{kN} \hat{c}_{k\sigma N}^\dagger \hat{c}_{k\sigma N}$ and correspondingly $\hat{H}_S = \sum_{k, \sigma} \xi_{kS} \hat{c}_{k\sigma S}^\dagger \hat{c}_{k\sigma S} - \sum_{k} \Delta \hat{c}_{k\uparrow S}^\dagger \hat{c}_{k\downarrow S} + \Delta^* \hat{c}_{-k\downarrow S}^\dagger \hat{c}_{-k\uparrow S}$ assuming the isotropic energy gap $\Delta$. As usually $\xi_{k\sigma} = \epsilon_{k\sigma} - \mu_\beta$ denote the electron energies measured from the individual chemical potentials $\mu_\beta$ which become detuned $\mu_N - \mu_S = eV$ if a bias $V$ is applied across the junction inducing the nonequilibrium charge flow $I(V)$. Fano-type quantum interference effects originating from the hopping $t$ to side-coupled quantum dot $i = 2$ are discussed here assuming that only the interfacial quantum dot $i = 1$ is directly coupled to external leads

$$
\hat{H}_{\beta-DQD} = \sum_{k, \sigma} \left( V_{k\beta} \hat{d}_{1\sigma}^\dagger \hat{c}_{k\sigma \beta} + \text{h.c.} \right). \hspace{1cm} (3)
$$

In the wide-band limit approximation it is convenient to introduce the structureless coupling constants $\Gamma_\beta = 2\pi \sum |V_{k\beta}|^2 \delta (\omega - \epsilon_k)$ which shall be used here as the energy units.

Interplay between the proximity induced on-dot pairing, the correlations and the quantum interference effects can be in practice detected by measuring the differential conductance $dI(V)/dV$. Particularly valuable for this purpose is the low voltage (subgap) regime $|eV| \ll \Delta$. Under such conditions the charge current is provided by the anomalous Andreev scattering in which electrons from the metallic lead are converted into the Cooper pairs in superconductor with a simultaneous reflection of the electron holes back to the normal lead. On a formal level the resulting Andreev current can be expressed by the Landauer-type formula [5, 13]

$$
I_A(V) = \frac{2e}{h} \int d\omega T_A(\omega) \left[ f(\omega - eV) - f(\omega + eV) \right], \hspace{1cm} (4)
$$

where $f(\omega, T)$ is the Fermi distribution function and the transmittance $T_A(\omega) = \Gamma_N^2 \left| G_{12}(\omega) \right|^2$ depends on the off-diagonal part (in the Nambu notation) of the retarded Green’s function (5) of the interfacial quantum dot.

III. FANO RESONANCES

Fano resonances appear in many physical systems due to the quantum interference of the waves transmitted res-
and \( \hat{\Psi} = (\hat{\Psi}^{\Sigma} \noninteracting \text{ case} \) (the deep subgap regime

of the underlying physics let us start by considering freedom mixed with one and other. To have a clear pic-

are very specific because of the particle and hole degrees of

factors appearing in the anomalous Andreev current, which

level is responsible for forming the Fano resonance on a

linebroadenings [28, 29]. In the latter case the narrower

other simple possibility takes place in the electron tunnel-

quantum dot and directly via a shortcut bridge [27]. An-

two external electrodes are in parallel coupled through a

are present for instance in the electron transport when

states. In nanoscale physics such resonances are feasi-

transmittance contributed from a continuum of other

onantly via some discrete energy level combined with

Fig. 3: (color online) Changeover of the interfacial quantum
dot spectrum from the Fano (resonance and antiresonance)
lineshapes to the effective four-peak structure upon increasing
the interdot hopping \( t \) for the same parameters as in figure 2.

simplifies to (see the appendix)

\[
\Sigma^0(\omega) = \begin{pmatrix}
-\frac{i}{2} & \frac{t^2}{2} \\
-\frac{\Gamma^c}{2} & -\frac{\Gamma^c}{2} + \frac{t^2}{2}
\end{pmatrix},
\]

When the interference effects caused by the hopping \( t \)
to the side-coupled QD are neglected the expression \( 9 \)
becomes static (\( \omega \)-independent) and nontrivial physics
of this, so called atomic superconducting limit, has been
explored in detail by several groups [2, 10, 30] including
ourselves [16].

Taking into account the quantum interference \( t \neq 0 \) we
show in figure 2 the proximity induced on-dot pairing
formally arising from the off-diagonal parts of \( 6 \) illustrating
the energy spectrum \( \rho(\omega) = -\frac{1}{\pi} \text{Im}G_{11}(\omega + i0^+) \)
obtained for strong coupling to the superconducting lead \( \Gamma_S = 5\Gamma_N \). Such coupling \( \Gamma_S \) is responsible for
the particle-hole splitting of the effective quasiparticle
states formed at \( \pm \sqrt{\varepsilon_1^2 + (\Gamma_S/2)^2} \) whereas the coupling \( \Gamma_N \) controls their broadening. In the particular case \( \varepsilon_1 = 0 \) the quasiparticle peaks appearing at \( \pm E_1 \) (where
\( E_1 \equiv \sqrt{\varepsilon_1^2 + \Gamma_S^2/4} \)) are symmetric, but for arbitrary \( \varepsilon_1 \) they are weighted by the corresponding BCS coefficients
\( u^2, v^2 = \frac{1}{2} (1 \pm \varepsilon_1/E_1) \) \[16\]. On top of such behavior we
clearly notice that hopping to the side-coupled quantum
dot induces additional features appearing in the effective
spectrum near \( \pm \varepsilon_2 \) as the Fano resonance and antireso-
nance. For the case of both metallic leads there would
survive just the single Fano structure at \( \varepsilon_2 \) which in very
pedagogical way has been discussed by R. Žítko [28].

Fano-type lineshapes (see the lower panel in figure 2) are
present only in the weak hopping regime \( t \ll \Gamma_N \). For increasing \( t \) the Fano structures gradually evolve into
separate quasiparticle peaks illustrated in figure 3. Physically
this can be assigned to the induced pairing on the
side-attached QD \( \langle d_{2\downarrow}d_{2\uparrow} \rangle \neq 0 \) transmitted there indirectly via the interfacial quantum dot. Such effect again
qualitatively differs from the structures of the DQD coupled
to both metallic leads [28, 29].

Interrelation between the interference and proximity
effect can be practically investigated by measuring the

FIG. 4: (color online) The differential Andreev conductance
\( G_A(V) \) versus the bias \( V \) revealing the quasiparticle peaks
(near \( \pm \varepsilon_2 \)) and Fano-type lineshapes (near \( \pm \varepsilon_2 \)) for
the set of parameters used in figure 2 and \( t = 0.1\Gamma_N \).
tunneling current. In figure 5 we show bias voltage $V$ dependence of the differential Andreev conductance $G_A(V) = dI_A(V)/dV$ determined at zero temperature from 1 over a broad regime covering both the sub-gap quasiparticle peaks. Figure 5 illustrates the resulting Fano-type lineshapes $G_A = G_0[1 + e^{-(x+\epsilon_2)/\Gamma_N}]$ with $x = [eV + \epsilon_2]/\Gamma_N$ and the asymmetry parameter $q$ gradually decreases upon increasing the hopping integral $t.$ Our results can be thought as extension of the predictions obtained for the normal electron tunneling using the $T$-shape DQD coupled to both metallic leads 28, 32 onto the anomalous Andreev current where the particle hole mixing has the essential importance.

**IV. INTERPLAY WITH CORRELATIONS**

Coulomb repulsion between electrons of opposite spins can have an important influence on the spectral and transport properties of various nanostructures. For the case of quantum dots coupled to both conducting leads such interactions are known to be responsible for: a) the charging effect (if a given energy level $\epsilon_i$ is attempted to be occupied by more than a single electron this costs the system an extra energy $U_i$), b) the Kondo effect when the singlet state is formed between QD and itinerant electrons from the leads 31. In spectroscopic properties they are manifested by appearance of the Coulomb satellite around $\omega = \epsilon_i + U_i$ and the narrow Kondo resonance at the Fermi level. For heterostructures with the superconducting electrodes the situation is more complex due to a competition between the induced on-dot pairing and Coulomb repulsion.

The rich interplay between the quantum interference, correlations and proximity effect for the configuration shown in figure 1 have been so far addressed using the density functional technique 33 (which does not capture the Kondo physics) and by the numerical renormalization group calculations 22. In latter case the authors focused on $U_1 = 0, U_2 \neq 0$ when the side-attached quantum dot can indirectly form the Kondo state with electrons of the metallic lead affecting the Andreev transport.

![FIG. 5: (color online) Differential conductance $G_A(V)$ of the subgap Andreev current versus the source-drain bias $V$ in a vicinity of the Fano structure appearing at $V = \pm \epsilon_2/\epsilon.$](image)

To account for the correlation effects predominantly originating from the interfacial quantum dot we extend here the procedure previously used by us for studying the single quantum dot 10. The main idea is to approximate the correlation self-energy $\Sigma^U(\omega)$ by the diagonal matrix

$$\Sigma^U(\omega) \approx \begin{pmatrix} \Sigma_N(\omega) & 0 \\ 0 & -\Sigma_N(-\omega) \end{pmatrix}. \tag{7}$$

Such assumption (applied also in the NRG studies 10) can be thought as the simplest ansatz for the many-body selfenergy $\Sigma^U(\omega)$ allowing to combine the proximity effect (6) with the correlations, brought separately from the particle and hole channels. In more advanced treatments one should take into account the possible feedback effects between these normal and anomalous channels. We nevertheless hope that by imposing (7) we can get some insight at least on a qualitative level which might stimulate the future studies.

Within qualitative studies of the correlation effects we can describe the Coulomb blockade and Kondo effects using the following equation of motion expression 31

$$\Sigma_N(\omega) = \omega - \epsilon_1 - \left[\tilde{\omega} - \epsilon_1 - U_1 - \Sigma_3(\omega) + U_1 \Sigma_1(\omega)\right] / \left[\omega - \epsilon_1 - \Sigma_3(\omega) + U_1 (1-n_{1,\sigma})\right] \tag{8}$$

where $\Sigma_{i=1,3}(\omega) = \sum_k |V_{kN}|^2 \left[2e^{i\omega T} \right] \frac{\tilde{\omega}}{\tilde{\omega} - \epsilon_{kN}} [\tilde{\omega} - \xi_{kN}]^{-1} +$.
We have studied a unique nature in which the Fano-type quantum interference manifest itself in the energy spectrum and differential conductance of the heterojunction where a metallic lead is coupled via double quantum dot to superconducting electrode. In the regime of subgap source-drain voltage $|eV| < |\Delta|$ nonequilibrium charge transport is contributed only through the anomalous Andreev channel when electron from the metallic electrode is converted into the Cooper pair (propagating further in superconductor) with a simultaneous reflection of hole back to the metallic lead. Transmittance of such Andreev scattering is a sensitive probe of the proximity induced on-dot pairing as well as the quantum interference and correlation effects.

Since on-dot pairing mixes the particle with hole states the interference effects are doubled in a comparison to similar junctions without the superconducting electrode. In particular, for $T$-shape configuration schematically shown in figure 4 we notice that effective spectrum of the interfacial quantum dot develops the resonance and antiresonance, correspondingly at $\pm \varepsilon_2$ (figure 2). These Fano-type structures are present whenever the hopping integral $t$ to the side-attached quantum dot ($i = 2$) is much smaller than the linebroadening $\Gamma_N$ (whereas $\Gamma_S$ merely controls the induced quasiparticle splitting). Upon increasing $t$ the Fano-type features disappear, evolve into the new quasiparticle peaks (figure 3) being a consequence of the proximity effect indirectly spread onto the side-attached quantum dot.

Correlation effects play an important role with regard to the following aspects: a) the charging effect which causes appearance of the Coulomb satellite near $\varepsilon_1 + U_1$, b) the Kondo singlet state (when the interfacial quantum dot spin is effectively screened by electrons of the metallic electrode leading to formation of a narrow resonance at $\mu_N$), and c) eventual suppression the on-dot pairing. We have previously shown [16] that the Kondo effect enhances the zero-bias Andreev conductance as indeed reported experimentally [5]. In the present work we indicate that in the double quantum dots the quantum interference can (destructively) affect such feature if the Fano-type structures appear nearby the Kondo peak.

A more detailed analysis of the Fano-Kondo interplay could be a challenging task in the future studies. For this purpose one should resort either to nonperturbative techniques, like the numerical renormalization group, or to some sophisticated perturbative methods capable to interpolate between the limits $t \to 0$, $\Gamma_\beta \to 0$ and $U \to 0$.

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Appendix: Self-energy of the noncorrelated DQD

Using the standard Nambu notation we can express the retarded Green’s functions of the metallic lead

$$g_N^r(k, \omega) = \begin{pmatrix} 1 & 0 \\ \frac{1}{\omega - i \xi_{kn}} & 0 \end{pmatrix}$$  \hspace{1cm} (A.1)$$

the (unperturbed) side-attached quantum dot

$$g_s^r(\omega) = \begin{pmatrix} \frac{1}{\omega - \varepsilon_2} & 0 \\ 0 & \frac{1}{\omega + \varepsilon_2} \end{pmatrix}$$  \hspace{1cm} (A.2)$$

and the isotropic superconductor

$$g_s^r(k, \omega) = \begin{pmatrix} \frac{u_k^2}{\omega - E_k} + \frac{v_k^2}{\omega + E_k} & -\frac{u_k\delta_{kS}}{E_k} \\ \frac{-u_k\delta_{kS}}{E_k} & \frac{u_k^2}{\omega - E_k} + \frac{v_k^2}{\omega + E_k} \end{pmatrix}$$  \hspace{1cm} (A.3)$$

In the equation (A.3) we applied the BCS coefficients

$$u_k^2,v_k^2 = \frac{1}{2} \left[ 1 \pm \frac{\xi_{ks}}{E_k} \right]$$

$$u_k^2v_k = \frac{\Delta}{2E_k}$$

where $E_k = \sqrt{\xi_{ks}^2 + \Delta^2}$.

For the case of uncorrelated quantum dots ($U_i = 0$) we can determine the selfenergy $\Sigma^0(\omega)$ of the interfacial quantum dot from the following equation

$$\Sigma^0(\omega) = \sum_{k,\beta=N,S} V_{k,\beta} g_N^r(k, \omega) V_{k,\beta}^* + t \, g_s^r(\omega) t^*.$$  \hspace{1cm} (A.4)$$

Assuming the wide-band limit we introduce the constant weighted density of states

$$2\pi \sum |V_{k,\beta}|^2 \delta(\omega - \xi_{k,\beta}) = \begin{cases} \Gamma_\beta \text{ for } |\xi_{k,\beta}| < D/2 \\ 0 \text{ elsewhere,} \end{cases}$$  \hspace{1cm} (A.5)$$

where $D$ is the conduction bandwidth. We then easily find that

$$\sum_{k} V_{k,N} g_N^r(k, \omega) V_{k,N}^* = \left( \begin{array}{cc} 0 & -\frac{i\Gamma_N}{2} \\ -\frac{-i\Gamma_N}{2} & 0 \end{array} \right)$$  \hspace{1cm} (A.6)$$

because, according to the Kramers-Krönig relation, the real part disappears. In the same way we obtain from a straightforward algebra that

$$\sum_{k} V_{k,S} g_s^r(k, \omega) V_{k,S}^* = \frac{\gamma(\omega)\Gamma_S}{2i} \left( \begin{array}{cc} 1 & \frac{\Delta}{\omega} \\ -\frac{\Delta}{\omega} & -1 \end{array} \right)$$  \hspace{1cm} (A.7)$$

where

$$\gamma(\omega) = \frac{|\omega| \Theta(|\omega| - |\Delta|)}{\sqrt{\omega^2 - \Delta^2}} - \frac{i\omega \Theta(|\Delta| - |\omega|)}{\sqrt{\Delta^2 - \omega^2}}.$$  \hspace{1cm} (A.8)$$

In the extreme subgap limit $|\omega| \ll |\Delta|$ the function $\gamma(\omega)$ approaches $\gamma(\omega) \rightarrow -i\omega/\Delta$ and in consequence

$$\lim_{|\omega| \ll |\Delta|} \Sigma^0(\omega) = \left( \begin{array}{cc} \frac{-i\Gamma_N}{2} & \frac{|t|^2}{\omega - \varepsilon_2} \\ -\frac{|t|^2}{\omega - \varepsilon_2} & \frac{-i\Gamma_N}{2} \end{array} \right)$$  \hspace{1cm} (A.9)$$

which proves the equation (6).
[20] L.G. Herrmann, F. Portier, P. Roche, A. Levy Yeyati, T. Kontos, and C. Strunk, Phys. Rev. Lett. 104, 026801 (2010).
[21] J. Eldridge, M.G. Pala, M. Governale, J. König, Phys. Rev. B 82, 184507 (2010).
[22] B. Sothmann, D. Futterer, M. Governale, J. König, Phys. Rev. B 82, 094514 (2010).
[23] Y. Yamada, Y. Tanaka, and N. Kawakami, J. Phys. Soc. Jpn. 79, 043705 (2010).
[24] A.E. Miroshnichenko, S. Flach, and Y.S. Kivshar, Rev. Mod. Phys. 82, 2257 (2010).
[25] Y. Tanaka, N. Kawakami, and A. Oguri, Phys. Rev. B 78, 035444 (2008); J. Phys.: Conf. Series 150, 022086 (2009).
[26] Y. Tanaka, N. Kawakami, and A. Oguri, Phys. Rev. B 81, 075404 (2010).
[27] B. Bulka, P. Stefański, Phys. Rev. Lett. 86, 5128 (2001).
[28] R. Zitko, Phys. Rev. B 81, 115316 (2010).
[29] P. Trocha and J. Barnaš, Phys. Rev. B 76, 165432 (2007).
[30] T. Meng, S. Florens, and P. Simon, Phys. Rev. B 79, 224521 (2009).
[31] H. Haug and A.-P. Jauho, Quantum Kinetics in Transport and Optics of Semiconductors, Springer Verlag, Berlin (1996).
[32] S. Sasaki, H. Tamura, T. Akazaki, and T. Fujisawa, Phys. Rev. Lett. 103, 266806 (2009).
[33] A. Karmanyos, I. Grace, and C.J. Lambert, Phys. Rev. B 79, 075119 (2009).
