Efficient and Robust Equilibrium Strategies of Utilities in Day-Ahead Market With Load Uncertainty

Tianyu Zhao, Hanling Yi, Minghua Chen, Fellow, IEEE, Chenye Wu, Member, IEEE, and Yunjian Xu, Member, IEEE

Abstract—We consider the scenario where $N$ utilities strategically bid for electricity in the day-ahead market and balance the mismatch between the committed supply and actual demand in the real-time market, with uncertainty in demand and local renewable generation in consideration. We model the interactions among utilities as a noncooperative game, in which each utility aims at minimizing its per-unit electricity cost. We investigate utilities’ optimal bidding strategies and show that all utilities bidding according to (net load) prediction is a unique pure strategy Nash equilibrium with two salient properties. First, it incurs no loss of efficiency; hence, competition among utilities does not increase the social cost. Second, it is robust and $(0, N - 1)$ fault immune. That is, fault behaviors of irrational utilities only help to reduce other rational utilities’ costs. The expected market supply–demand mismatch is minimized simultaneously, which improves the planning and supply-and-demand matching efficiency of the electricity supply chain. We prove the results hold under the settings of correlated prediction errors and a general class of real-time spot pricing models, which capture the relationship between the spot price, the day-ahead clearing price, and the market-level mismatch. Simulations based on real-world traces corroborate our theoretical findings. Our article adds new insights to market mechanism design. In particular, we derive a set of fairly general sufficient conditions for the market operator to design real-time pricing schemes so that the interactions among utilities admit the desired equilibrium.

Index Terms—Bidding strategy, electricity market, electricity price, fault immunity, load uncertainty, Nash equilibrium.

I. INTRODUCTION

T HE modern power system has been actively practicing deregulated electricity supply chain since the reform in 1990s [1]. As illustrated in Fig. 1, the deregulated supply chain usually consists of generation companies, utility companies, and sectors in charge of transmission and distribution networks [2]. In particular, utilities obtain power supply from the regional electricity market and local renewable sources to serve households and microgrids. The market operator (known as independent system operator (ISO), e.g., ISO New England (ISO-NE) [3]) provides a trading place and matches the supply offerings and demand bids at two different timescales and prices [4]–[6].

1) Day-ahead market: Generation companies (utilities) submit offers (bids) for selling (buying) electricity one day before the actual dispatch, based on generation (net load) forecasting. They are cleared at a market clearing price.

2) Real-time market: Real-time market is designed to resolve the imbalance between the actual real-time demand and the committed supply purchased from the day-ahead market on an hourly basis. We remark that the real-time electricity price depends on both the day-ahead clearing price as well as the real-time market level imbalance.

New York Independent System Operator reports that while roughly 95% of the electricity load is scheduled in the day-ahead market transaction, 5% of that remains to be settled in the real-time market [7]. The cost of the utility is composed of the payment in the day-ahead market and the expense in the real-time market to resolve the imbalance.1

Although the existing electricity market is not a free market and is regulated to a certain degree, utilities can still play strategically to lower their own costs. In particular, in the day-ahead market, utilities can overbuy (respectively underbuy) electricity given the load forecasting results, expecting they can sell the surplus in the real-time market at a higher price (respectively, buy the shortage at a lower price). We remark that utilities interact with each other in this process. The cost of a utility depends on both its own bidding strategy and those of all other utilities since the latter affect the real-time market level imbalance and consequently the real-time spot price.

A. Related Work

There have been a number of works studying the optimal bidding strategies of generation companies in order to minimize

1Similar to [8]–[13], we do not consider the intraday market as its trading quantity is negligible as compared to the day-ahead and real-time markets.
their operating costs on the supply side of the electricity supply chain [14]–[30]; see, e.g., [19] for a survey. In particular, generation companies derive their optimal decisions by solving the scheduling problems that seek optimal production schedules. For example, Kazempour and Zareipour [15] proposed an equilibrium problem with equilibrium constraints (EPEC) framework to analyze the strategic behaviors of electricity producers owning dispatchable wind power units. Zhang et al. [16] formulate the Cournot competition model to analyze the interactions among generation firms with production uncertainty. The impact of coalition on the game equilibrium is explored. As the intermediary between consumers and the deregulated electricity market, utilities’ optimal bidding strategies and their interactions have also been actively investigated in many studies on the demand side of the electricity market, mainly on two aspects.

The first is on the optimal bidding strategies of utilities. This includes characterizing the maximum profit for individual utility in the market [31], [32] and designing optimal bidding decisions [33]–[35]. We note that the existing works mainly focus on formulating the utilities’ optimization problems to obtain the optimal decisions under various scenarios, e.g., time-series methods, data-driven predictions, and genetic algorithms are discussed in [8] and [36]. However, the analysis on the market equilibrium structure and properties may not be included, especially concerning demand uncertainty in the two-settlement electricity market. The first category is investigating the optimization methods and frameworks considering demand response [9], [11], [37]–[42], e.g., a short-term planning model to determine the bidding strategies for utilities with flexible power demand is proposed in [9]. The second category is applying stochastic programming models to incorporate price and demand uncertainties [43]–[47]. This includes deriving utilities’ optimal strategies and the market clearing mechanism under various probabilistic scenarios, e.g., Habibian et al. [46] present a multistage stochastic demand-side management model to solve a large-scale time-horizon electricity consumption scheduling problem. A stochastic market clearing mechanism that considers deviation costs and demand uncertainty is introduced in [45]. The third category focuses on the market equilibrium characterization [35], [45], [48]–[50], which involves utilities’ interactions, e.g., Hu and Ralph [49] employ the EPEC optimization model to represent market participants interactions and establishes the sufficient conditions for the equilibrium existence. Atzeni et al. [35] formulate the day-ahead grid optimization as a generalized Nash equilibrium problem and obtains the corresponding converging algorithm. Besides focusing on attaining the market equilibrium, Srinivasan et al. [51] propose a coevolutionary approach to investigate the individual and cooperative coalition strategies of utilities.

In addition to the three categories described earlier, there has been a line of research on studying utilities’ behaviors [52], [53] and market efficiency [54] under some specific scenarios. Fang et al. [52] propose a scheduling model for utilities’ energy storage operation with CVaR as a risk metric to cope with price and load uncertainties. Roozbehani et al. [54] characterize a real-time retail pricing model to investigate the electricity market stability and efficiency. The interactions between utilities, consumers, and wholesale electricity market are studied in [53].

Due to the uncertain demand and the fast penetration of intermittent renewables, there is a series of studies further study the impact of load uncertainty, i.e., how renewables and demand uncertainty would affect utilities’ net load predictions [55] and costs [56], besides containing such modeled uncertainty [31] in the optimization programs discussed above. It is observed that renewable penetration is likely to deteriorate load prediction errors [56], and consequently, net load uncertainty will affect utilities’ costs [55]–[60]. For instance, in our previous work [55], Yi et al. show that larger renewable penetration degrades prediction accuracy, leading to higher load uncertainty. The corresponding local impact and global impact of such uncertainty are investigated. It is demonstrated that load uncertainty can cause an increase in utilities’ costs.

It should be noted that electricity price forecasting is also fundamental for energy companies’ decision-making mechanisms [61], [62]. Utilities involved in the electricity market are faced with the uncertainty challenge of volatile power market prices in their daily operations, and prior knowledge of such price fluctuations can help them set up corresponding bidding strategies so as to maximize their profit [63].

Hourly energy price is a complex signal due to its nonlinearity, nonstationarity, and time-variant behavior [64], [65]. Multiple probabilistic price forecasting models and frameworks are proposed to approximate the parameters of the probability density function for the hourly price variables based on past prices, power loads, chronological information, etc., [66]–[68].

Our article differs from the existing literature in that, we consider the demand side bidding under the game-theoretical setting with load uncertainty in the two-settlement electricity market and we investigate the uniqueness and efficiency of the market equilibrium according to its structure. Equilibrium robustness and the aggregate impact of all utilities’ bidding strategies on the individual utility’s cost are also studied. The explicit form of utilities’ optimal bidding strategies and the market equilibrium can be derived. Furthermore, besides designing the optimal decisions of utilities, our study adds new insights to market mechanism design and market operation. In particular, we show that if the market operator can design the real-time pricing scheme to meet a set of fairly general sufficient conditions described in Section IV, then the utility bidding game will admit a unique pure strategy Nash equilibrium without loss of efficiency, and it is robust to irrational utilities’ fault behaviors. Meanwhile, the expected total market supply–demand mismatch can be minimized simultaneously, which improves the planning and supply-and-demand matching efficiency of the electricity supply chain.

### B. Contributions

We formulate the interactions among utilities as a noncooperative game. Utilities aim at minimizing individual costs by optimizing their own bidding strategies, taking into account
uncertainty in load and local renewable generation. We seek answers to the following three critical questions.

1) What is the outcome of such a utility bidding game? In particular, does there exist a pure strategy Nash Equilibrium\(^2\)? If so, is it unique?

2) Does competition introduce efficiency loss compared with the social optimal under the coordinated setting? That is, what is the loss of efficiency\(^3\) at the equilibrium?

3) How robust is the equilibrium against irrational fault behaviors\(^4\)? In particular, will rational utilities suffer in a fault-ridden setting with irrational utilities?

Answers to these questions provide new understandings of the effectiveness of the electricity market design, as well as the impact of load uncertainty. We conduct a comprehensive study and make the following contributions.

To better bring out the insights and intuitions, we first focus on a baseline setting where the load prediction errors across utilities are mutually independent and the spot pricing model, describing the relationship among the spot price, day-ahead clearing price, and real-time market-level mismatch, is a class of piecewise linear functions similar to the ones in [71] and [72].

1) We show that all utilities bidding according to (net load) prediction is a unique pure strategy Nash equilibrium.

2) We show that the Nash equilibrium incurs no loss of efficiency, i.e., the social cost of the equilibrium is the same as the optimal one under the coordinated setting. Furthermore, the equilibrium is robust and \((0, N - 1)\) fault immune. That is, irrational fault behaviors of any subset of the utilities only help reduce the costs of the remaining rational utilities [70].

We then generalize the results to the setting with correlated prediction errors and general pricing models.

1) We present a set of sufficient conditions on the spot pricing model for observing the unique, efficient, and \((0, N - 1)\) fault immune robust pure strategy Nash equilibrium. In particular, we show that our above results hold for correlated prediction errors and a general class of real-time spot pricing models which can be nonlinear.

In Section V, we conduct extensive simulations based on price and load data from the ISO-NE electricity market. The results corroborate our theoretical findings and highlight that it is possible to design effective real-time pricing schemes satisfying the sufficient conditions derived in our article, such that the interactions among utilities admit a unique, efficient, and robust pure strategy Nash equilibrium.

The structure of this article is as follows. We first introduce the utility bidding game formulation in Section II. Based on the proposed game-theoretical model, Section III investigates the desired properties of the equilibrium. Section IV provides the generalized conditions to still admit the preferred equilibrium. Experimental results are included in Section V. Section VI delivers the concluding remarks. Due to the space limitation, all proofs, the summary of theoretical results, and more discussions are presented in our technical report [73].

\(^2\) A pure strategy corresponds to the common practice that individual utility places one bidding curve (quantity) for each trading hour in day-ahead market.

\(^3\) The ratio between the social cost at the equilibrium and the social optimal quantifies the loss of efficiency due to competition [69].

\(^4\) An equilibrium is \((\epsilon, K)\) fault immune if nonfault utilities’ expected costs increase by at most \(\epsilon\) when up to \(K\) other utilities deviate arbitrarily [70].

II. Problem Formulation

We consider the scenario where \(N\) utilities strategically bid for electricity in the day-ahead market and balance the mismatch between the committed supply and actual demand in the real-time market, with uncertainty in demand and local renewable generation in consideration. Since the transactions are settled on an hourly basis, we focus on a particular hour without loss of generality. We use \(p_d\) and \(p_s\) (unit: $/MWh) to denote the corresponding day-ahead price and the spot price, respectively. Similar to [12], [13], [8], [10], [43], [11], we consider the setting where a single utility does not have market power to manipulate the day-ahead clearing prices, i.e., utilities are considered as price-takers in the day-ahead market. This setting is consistent with the situation in many liberalized electricity markets where the trading volume of most utilities is too small to influence the price as compared to the total market turnover [9], [10], [74]. We also consider that utilities’ bidding strategies can affect the real-time market spot price, i.e., utilities are viewed as price-makers in the real-time market. This is reasonable as utilities’ strategic bidding behaviors can have significant impact in real-time balancing markets where only a small amount of energy is traded [8]. In practice, at the time of day-ahead bidding, utilities usually do not know precisely its load and local renewable generation. Instead, they have distributional information of the net load by load forecasting. For ease of analysis, we first focus on the setting that utilities only place one quantity bid for each trading hour in the day-ahead market. Our results can be generalized to the case in which utilities submit demand curves as bidding pairs (price, quantity); see Section IV-C for a discussion.

A. Utility Bidding and Load Mismatch Modeling

We define \(D_i\) (unit: MWh) as utility \(i\)'s actual net load at a particular hour, and it is only revealed to the utility in the real-time manner. When utility \(i\) participates in day-ahead market, it has a prediction of \(D_i\), denoted as \(\hat{D}_i\), modeled as follows:

\[
\hat{D}_i = D_i + \epsilon_i \tag{1}
\]

where \(\epsilon_i\) is a random variable representing the load prediction error. It is affected by the uncertainties of demand and local renewable generation owned by the utilities and microgrids.

Given the load prediction \(\hat{D}_i\), utility \(i\) can strategically participate in the day-ahead market by bidding a quantity \(Q_i \triangleq \hat{D}_i - \mu_i\), where 1) \(\mu_i = 0\): Utility bids precisely according to prediction; 2) \(\mu_i > 0\): Utility strategically underbids; 3) \(\mu_i < 0\): Utility strategically overbids. We use \(\mu_i\) to represent the bidding strategy of utility \(i\).

The bidding strategy of utility \(i\) will affect its mismatch between the real-time actual net load and the day-ahead purchased supply in the real-time market, which is denoted as \(\Delta_i\) (unit: MWh). By definition, we have

\[
\Delta_i \triangleq D_i - Q_i = \mu_i - \epsilon_i. \tag{2}
\]

Whenever there is an imbalance, i.e., \(\Delta_i \neq 0\), the utility has to settle this imbalance in the real-time market at the spot price \(p_s\). It either sells the residual electricity back to the market when \(\Delta_i < 0\), or buys the deficient electricity from the market when \(\Delta_i > 0\).
\( \Delta_i > 0 \). For ease of presentation, we define

\[
\Delta = \sum_{i=1}^{N} \Delta_i, \quad \Delta_{-i} = \sum_{j \neq i}^{N} \Delta_j
\]

(3)
as the market-level mismatch and the aggregate mismatch of all other utilities except utility \( i \), respectively.

**B. Real-Time Market Spot Pricing Model**

The real-time market price generally depends on the market-level supply and demand imbalance, i.e., the difference between the day-ahead scheduled supply and the real-time actual demand. Deficient supply in the market leads to a higher spot price (up-regulation), whereas excessive supply results in a lower spot price (down-regulation). To capture their relationship, we consider the following piecewise linear pricing model\(^5\) [72]; see Fig. 2 for illustration

\[
p_s = \begin{cases} 
    p_d, & \Delta = 0 \\
    (a_1 \Delta + b_1)p_d, & \Delta > 0 \\
    (a_2 \Delta + b_2)p_d, & \Delta < 0
\end{cases}
\]

(4)

where, \( a_1, a_2, b_1, b_2 \in \mathbb{R}_+ \) are parameters of the pricing model.

Remark:

i) The model was proposed in [72] by curve-fitting the historical data. In general, \( a_1 \neq a_2, b_1 \neq b_2 \). Specifically, Neupane et al. [72] suggest \( a_1 = 0.0034, a_2 = 0.0005, b_1 = 1.2378 \), and \( b_2 = 0.6638 \).

ii) The spot price function is discontinuous at \( \Delta = 0 \), i.e., \( b_1 > 1 > b_2 \). This discontinuity models the *premium of readiness* that utilities need to pay for the generation companies, since they have to generate urgent regulating power [71].

iii) In this article, we first focus on the piecewise linear symmetric pricing model, i.e., \( a_1 = a_2 > 0, (b_1 p_d + b_2 p_d)/2 = p_d \), and \( b_1 > b_2 \).

Then, in Section IV, we generalize our results to a larger class of pricing models, which can be nonlinear or continuous at the origin.

\(^5\)Generation imbalance from generation companies’ side also proposes an effect on the real-time market electricity price, e.g., in case of generator failure or the uncertainty from large-scale renewable generation. Such imbalance may increase the market mismatch variability. This paper mainly focuses on the demand side of the electricity market, and the effect of generation uncertainty on the market imbalance is not considered. We leave the incorporation of generation uncertainty into market equilibrium analysis for future study.

**C. Cost Function of Utilities and Strategic Bidding Game**

For utility \( i \), its electricity procurement is settled in the following two timescales: (i) an amount of \( \tilde{D}_i - \Delta_i \) is settled in the day-ahead market at price \( p_d \), and (ii) the remaining amount \( \Delta_i \) is settled in the real-time market at the spot price \( p_s \). Hence, the total electricity cost of utility \( i \) is given as follows (unit: $)

\[
\tilde{C}_i = p_d (\tilde{D}_i - \mu_i) + p_s \Delta_i = p_d (\tilde{D}_i - \Delta_i) + p_s \Delta_i.
\]

(5)
The average buying cost per unit electricity (ABC) for utility \( i \) is simply

\[
ABC_i = \frac{\tilde{C}_i}{\tilde{D}_i} = \frac{1}{\tilde{D}_i} [p_d (\tilde{D}_i - \Delta_i) + p_s \Delta_i].
\]

(6)

Consider the net load uncertainty, the cost function for utility \( i \) is defined as the expected \( ABC_i \), i.e.,

\[
C_i(\mu_i, \mu_{-i}) \triangleq \mathbb{E}[ABC_i].
\]

(7)

Note that the cost of utility \( i \) not only depends on its own strategy \( \mu_i \) but also depends on \( \mu_{-i} = \sum_{j \neq i}^{N} \mu_j \), the aggregate strategies of all other utilities. The underlying reason is that the real-time spot price is determined by the market-level mismatch, and other utilities’ strategic behavior can affect the cost of utility \( i \) through the spot price \( p_s \).

Given the model of strategic behaviors and utilities’ cost functions, we model their interactions as a noncooperative game with \( N \) utilities where each utility aims to minimize its own cost \( C_i(\mu_i, \mu_{-i}) \) by choosing a strategy represented by \( \mu_i \). Formally, a strategy profile \( \mu^* = (\mu_1^*, \mu_2^*, \ldots, \mu_N^*) \) constitutes a Nash equilibrium if for each \( i = 1, 2, \ldots, N \)

\[
C_i(\mu_i, \mu_{-i}^*) \leq C_i(\mu_i^*, \mu_{-i}^*) \quad \forall \mu_i \in \mathbb{R}
\]

(8)

where \( \mathbb{R} \) is the set of real numbers.

**III. MAIN RESULTS: MARKET EQUILIBRIUM ANALYSIS**

Under the two-settlement market mechanism, we study the existence, uniqueness, efficiency, and robustness of the equilibrium of the game among utilities. In this section, we consider the scenario that the net load prediction errors \( \epsilon_1, \ldots, \epsilon_N \) are mutually independent and the real-time spot pricing model is piecewise linear symmetric as defined in (4) with \( a_1 = a_2, (b_1 p_d + b_2 p_d)/2 = p_d \), and \( b_1 > b_2 \).

These two settings allow us to better highlight the impacts of utilities’ strategies and the market equilibrium characteristics. We later extend the results to the case with correlated prediction errors and general pricing models in Section IV.

**A. Load Imbalance Distribution and Analysis of Utilities’ Strategic Behaviors**

We model the net load prediction error \( \epsilon_i \) as a general symmetric unimodal random variable with zero mean and variance \( \sigma_i^2 \).

**Definition 1:** A probability density function \( f_X(\cdot) \) is symmetric unimodal if it is

i) Symmetric w.r.t. its mean \( \xi \)

\[
f_X(\xi + x) = f_X(\xi - x) \quad \forall x \in \mathbb{R}.
\]

(9)

(ii) Central dominant

\[
f_X(x) \leq f_X(y), \text{ if } |x - \xi| \geq |y - \xi| \quad \forall x, y \in \mathbb{R}.
\]

(10)

Here, \( \xi = \int_{-\infty}^{+\infty} x \cdot f_X(x) dx \) is the expected value of \( X \).
Many prediction error distributions are symmetric unimodal, including Gaussian distribution and Laplace distribution. We say that a random variable is symmetric unimodal if it follows a symmetric unimodal distribution. The specific meaning of central dominant comes from that utilities tend to make larger prediction errors with smaller probability compared with the case of smaller errors with larger probability. Symmetric distribution implies that utilities have equal chances to encounter positive or negative prediction errors; see Section V-B for details.

Given a utility’s bidding strategy \( \mu_i \), the real-time imbalance \( \Delta_i \) follows a symmetric unimodal distribution with mean \( \mu_i \) and variance \( \sigma_i^2 \)

\[
\Delta_i \sim \mathcal{P}(\mu_i, \sigma_i^2), \quad i = 1, 2, \ldots, N.
\]  

**Lemma 1:** The sum of independent symmetric unimodal random variables is still symmetric unimodal.

Lemma 1 states the property of the convolutions of symmetric unimodal random variables [75]. We have a stronger observation on such distribution convolutions. We refer interested readers to Section III-A in the technical report [73] for a discussion. Applying Lemma 1, we have

\[
\Delta \sim \mathcal{P}(\mu, \sigma^2), \quad \text{and} \quad \Delta_{-i} \sim \mathcal{P}_{-i}(\mu_{-i}, \sigma_{-i}^2)
\]

where \( \mu \triangleq \sum_{i=1}^{N} \mu_i, \mu_{-i} \triangleq \sum_{j \neq i} \mu_j, \sigma^2 \triangleq \sum_{i=1}^{N} \sigma_i^2, \text{and} \quad \sigma_{-i}^2 \triangleq \sum_{j \neq i} \sigma_j^2. \)

Based on the above-mentioned observations, Theorem 1 characterizes the cost function \( C_i(\mu, \mu_{-i}) = \mathbb{E}[\text{ABC}_i] \) of utility \( i \).

**Theorem 1:** Under independent prediction errors and piecewise linear symmetric spot pricing model, the expectation of \( \text{ABC}_i \) is given as

\[
\mathbb{E}[\text{ABC}_i] = p_d + \frac{p_d}{D_i} \left[ \frac{a_1 + a_2}{2} (\mu_i - \mu_{-i} + \sigma_i^2 + \mu_i^2) \right] + (b_1 - b_2) \mathbb{E} \left[ \Delta_i \cdot \tilde{F}(\Delta_i) \right]
\]

where \( \tilde{F}(\Delta_i) \triangleq \int_{\Delta_i} f_{\Delta_i}(\delta_{-i})d\delta_{-i}, f_{\Delta_i}(\cdot) \) is the PDF of \( \Delta_i \) with mean \( \mu_{-i} \), and coefficients \( a_1, a_2, b_1, b_2 \) are parameters of the spot pricing model defined in (4).

**Remarks:** Utility estimates its expected cost via (13) considering load uncertainty. The expectation of \( \text{ABC}_i \) in (13) depends on three terms. The first term is simply the day-ahead market clearing price. The second and the third terms represent the real-time market operation cost to balance the mismatch. It is clear that utility’s strategic behavior and the aggregation of all other utilities’ strategies affect both the second and the third terms. In addition, the second term reveals the influence of the pricing model slope, and the third term presents the discontinuous part of the pricing model. Meanwhile, given \( \mu_{-i} = 0 \), both term two and term three are positive and \( \mathbb{E}[\text{ABC}_i] \) will increase under the following conditions. 1) The day-ahead market clearing price \( p_d \) increases. 2) The slope of the real-time market pricing model increases, i.e., \( a_1, a_2 \) increases. 3) The discontinuity gap of the pricing model \( b_1 - b_2 \) increases.

In Section V-D, our simulation results verify these observations. The results show that utilities suffer higher costs under higher day-ahead clearing price and larger real-time market sensitivity. Note that in day-ahead market operations, the actual net load \( D_i \) is not realized precisely to utilities. Our following results show that utilities’ optimal bidding strategies are independent of the specific value of \( D_i \). Therefore, utilities can still derive their optimal operations even in absence of the full knowledge of \( D_i \); accordingly, the market equilibrium can be described.

### B. Existence and Uniqueness of the Nash Equilibrium

A Nash equilibrium in the utility bidding game is a strategic profile in which all utilities choose the optimal strategy that minimizes its own cost given others’ behaviors. Recall that utilities only place one quantity for one bid in the market; it is sufficient to focus on the pure strategy Nash equilibrium.

**Theorem 2:** Under independent prediction errors and piecewise linear symmetric spot pricing model, the strategy profile \( \mu^* = 0 \) is the unique pure strategy Nash equilibrium.

**Proof idea:** We start by understanding the characteristics of utilities’ cost functions (13). Specifically,

1) we find that the third term in (13) will increase if the utility deviates from bidding according to prediction, given that the aggregation of all other utilities’ strategies \( \mu_{-i} \) is zero. This implies the strategy profile \( \mu^* = 0 \) is a pure strategy Nash equilibrium; and

2) we present a necessary condition for all pure strategy Nash equilibria. We then prove that such a condition only admits one solution \( \mu^* = 0 \).

Theorem 2 shows utilities’ optimal bidding strategies at the unique equilibrium. We reveal the insights behind the result from the perspective of cost minimization and best response. Note that when the utility’s real-time mismatch has the same sign as the market-level mismatch, the utility will suffer a loss; otherwise, it will gain. For example, when the market-level mismatch is positive, the real-time price is higher than the day-ahead price according to the pricing model. If the utility’s mismatch is negative, it means that the utility buys excessive energy in the day-ahead market and it can sell it back to the market at a higher price. Thus, the utility will gain.

With this in mind, let us look at the case when utility \( i \) chooses the bidding strategy \( \mu_i > 0 \), given \( \mu_{-i} = 0 \). Recall that the utility’s prediction error follows a symmetric unimodal distribution with mean zero, which indicates it has the same possibility to encounter particular positive or negative errors. Consequently, if the utility strategically underbids when participating in the day-ahead market, i.e., \( \mu_i > 0 \), its real-time imbalance will tend to be negative. Since the market-level mismatch follows a symmetric unimodal distribution \( \mathcal{P}(\mu_i, \sigma^2) \), when \( \mu_i > 0 \), the market-level mismatch and the utility’s mismatch tend to be positive simultaneously, thus the utility \( i \) tends to suffer a loss. Similarly, when \( \mu_i < 0 \), given \( \mu_{-i} = 0 \), the utility \( i \) will also suffer a loss. Due to the space limitation, we refer interested readers to Sec. III-B in the technical report [73] for a discussion and proofs.

Theorem 2 implies that for any day-ahead clearing price \( p_d \), if all utilities except utility \( i \) bid according to prediction, then bidding according to prediction is the best response of utility \( i \) (see Fig. 9 based on real-world data for illustration). Consequently, all utilities bid exactly according to (net load) prediction is the unique pure strategy Nash equilibrium. In Section IV, we generalize the results to the case of correlated prediction errors (across utilities), general pricing models, and the setting in which utilities submit bidding curves considering the uncertainty of the day-ahead clearing prices.

### C. Efficiency and Robustness of the Nash Equilibrium

We have shown that \( \mu^* = (u^*_1, u^*_2, \ldots, u^*_N) = 0 \) is the unique pure strategy Nash equilibrium. The next natural question is: 

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*The day-ahead clearing price is not known precisely to utilities beforehand. See Section I-A for discussions on electricity price forecasting and uncertainty.*
what is the corresponding loss of efficiency? Recall that loss of efficiency is characterized as the gap between the social costs under the game-theoretical strategic setting, i.e., the social cost at the equilibrium, and the coordinated setting.

The optimal social cost under the coordinated setting is obtained by solving the following cost minimization problem:

$$\min_{\mu_i \in \mathbb{R}} \left\{ \min_{\forall i \in \{1,2,\ldots,N\}} \ E[ABC_{total}] \right\}$$

(14)

where $ABC_{total}$ is defined as

$$ABC_{total} \triangleq \frac{C_{total}}{D_{total}} = \frac{\sum_{i} D_i \cdot ABC_i}{\sum_{i} D_i} = \frac{1}{D_{total}} \left[ p_d (D_{total} - \Delta) + p_s \Delta \right].$$

(15)

Here, $\Delta$ follows a symmetric unimodal distribution with mean $\mu = \sum_{i=1}^{N} \mu_i$, and $ABC_{total}$ (unit: $$/MWh) can be interpreted as the unit cost of the market to settle $D_{total} \triangleq \sum_{i=1}^{N} D_i$ amount of electricity.

**Theorem 3:** Under independent prediction errors and piecewise linear symmetric spot pricing model, the unique pure strategy Nash equilibrium $\mu^* = 0$ incurs no loss of efficiency.

**Remarks:** Theorem 3 shows the market operator that the social cost at the equilibrium is the same as the optimal one under the coordinated setting in (14). The intuition behind Theorem 3 lies in that we can treat the whole market as an entity. From the market-level perspective, an amount of $(D_{total} - \Delta)$ power is committed at the price $p_d$ and the imbalance $\Delta$ is settled at the spot market price $p_s$. When the market has a particular positive (respectively negative) real-time mismatch, this mismatch is settled at a price $p_s > p_d$ (respectively $p_s < p_d$); hence the market will suffer a loss.

Since the market mismatch $\Delta$ follows a symmetric unimodal distribution $P(\mu, \sigma^2)$, when $\mu > 0$, the market-level mismatch tends to be positive, thus, the market tends to suffer a loss facing a higher spot price. Similar analysis can be applied to the case when $\mu < 0$. We conclude that the unique pure strategy Nash equilibrium is efficient; see Fig. 9 for illustration.

The guarantee provided by a Nash equilibrium is that each player’s strategy is optimal, assuming all others play their designated strategies. However, there could exist fault behaviors, wherein some utilities are irrational, and their actions can be arbitrary or even adversarial. In the following theorem, we illustrate a strong observation on the robustness of the equilibrium. That is, the suboptimal response faults behaviors of irrational utilities not only increase their own costs but also benefit other rational utilities.

**Theorem 4:** Under independent prediction errors and piecewise linear symmetric spot pricing model, the equilibrium $\mu^* = 0$ is $(0, N - 1)$ fault immune. That is, consider all utilities except a group of utilities $S, S \subseteq \{1,2,\ldots,N\}$ and $1 \leq |S| \leq N - 1$. Given $\mu_j = 0$, $\forall j \in \{1,2,\ldots,N\} \setminus \{S\}$, $E[ABC_j]$ is nonincreasing w.r.t. $|\mu_j|$ where $\mu_j = \sum_{i \in S} \mu_i$.

Theorem 4 shows the robustness of the unique pure strategy Nash equilibrium $\mu^* = 0$. This is a desirable property as discussed in [70]. It provides the utility an understanding of its cost change characteristic w.r.t. irrational market participants’ actions. The result indicates that fault behaviors of irrational utilities will not increase the costs of other rational utilities; see Fig. 4 for illustration. We have a stronger observation on the equilibrium structure. We refer interested readers to Section III-C in the technical report [73] for a discussion.

Fig. 3. Load prediction error histogram from case study based on real-world data.

Fig. 4. Illustration of the $(0, N - 1)$ fault immune robustness of the pure strategy Nash equilibrium under piecewise linear symmetric pricing model and Gaussian distributed prediction errors.

IV. MAIN RESULTS: EQUILIBRIUM GENERALIZATION

Previous analysis focuses on the scenario where the real-time market pricing model is piecewise linear symmetric and prediction errors of utilities are mutually independent. In this section, we extend our results to the setting with correlated prediction errors, general pricing models, and utilities submitting bidding curves considering the uncertain day-ahead prices.

A. Beyond Piecewise Linear Symmetric Spot Pricing Model

The uniqueness, efficiency, and robustness of the pure strategy Nash equilibrium hold for a large class of pricing models, which can be nonlinear or continuous at the origin.

**Theorem 5:** Suppose the load net prediction errors are mutually independent. Denote the pricing model as

$$p_s = \begin{cases} p_d, & \Delta = 0 \\ p(\Delta) + b_1 p_d, & \Delta > 0 \\ p(\Delta) + b_2 p_d, & \Delta < 0 \end{cases}$$

(16)

Here, $b_1 + b_2 = 2$, $b_1 \geq b_2$, $p(\Delta)$ is a nondecreasing odd function, i.e., $p(\Delta) = -p(-\Delta)$ and $p(x) \geq p(y), \forall x \geq y$, and $p(\Delta)$ is continuous at $\Delta = 0$. Then, $\mu^* = (\mu_1^*, \mu_2^*, \ldots, \mu_N^*) = 0$ is a pure strategy Nash equilibrium. In addition, the following statements are true.

(1) If $p(\cdot)$ is differentiable for all $x \in \mathbb{R}$, and

$$p'(x_1) \geq p'(x_2) > 0 \quad \forall x_1 > x_2 \geq 0$$

$$p'(x_1) \geq p'(x_2) > 0 \quad \forall x_1 < x_2 \leq 0$$

(17)

then $\mu^* = 0$ is the unique pure strategy Nash equilibrium.

(2) The pure strategy Nash equilibrium $\mu^* = 0$ incurs no loss of efficiency.
Suppose net load prediction errors are mutually correlated; see, e.g., [76]. In this section, we generalize our results and consider the case that utilities’ prediction errors $\epsilon_i$, $i = 1, 2, \ldots, N$, are zero mean Gaussian random variables with nonnegative correlations, then the strategy profile $\mu^* = 0$ is the unique, efficient, and $(0, N - 1)$ fault immune robust pure strategy Nash equilibrium.

Remarks:

i) The general pricing models defined in (16) are described by the discontinuous gap $(b_1 - b_2) \cdot p_d$ and the imbalance related term $p(\Delta)$. The results in Section III are for the special case of Theorem 5 of the piecewise linear symmetric pricing model.

ii) Theorem 5 indicates that the results of the uniqueness, efficiency, and $(0, N - 1)$ fault immune robustness of the equilibrium obtained in Section III can be extended to the case with general pricing models, which can be nonlinear or continuous at the origin.

iii) Pricing models satisfying (17) are convex when $\Delta > 0$ and concave when $\Delta < 0$.

The pricing models satisfying (18) or (19) are concave when $\Delta > 0$ and convex when $\Delta < 0$; see Fig. 5 for illustration. Therefore, the piecewise linear symmetric pricing model with discontinuous gap at the origin is the only one that satisfies both (17) and (18) or (19) and thus admits the unique, efficient, and $(0, N - 1)$ fault immune robust pure strategy Nash equilibrium.

C. Beyond Submitting Quantity Bid

In practice, market participants are always allowed to react to the uncertain market clearing prices since they may encounter short-term price fluctuation [9]–[11], [53]. Specifically, utilities (generation companies) submit their buying (selling) bidding curves, then the ISO compiles these bidding curves and calculates the clearing price for each trading period. A certain volume of electricity is agreed for a utility according to the clearing price and its bidding curve. In this part, we extend our results to the setting in which utilities submit bidding pairs (price, quantity) that form their bidding curves for each hour considering the uncertain market clearing prices.

We use $Q_i(p_d)$ to denote utility $i$’s bidding curve, which represents its willingness to procure different amounts of power from the market under different day-ahead uncertain clearing prices $P_d$. Hence, utility $i$’s bidding strategy can be represented by $\mu_i(p_d) = D_i - Q_i(p_d)$. We have the following results.

Theorem 7: Suppose net load prediction errors are mutually independent. Denote the pricing model as (16). Then, $\mu^*(p_d) = (\mu_1^*(p_d), \mu_2^*(p_d), \ldots, \mu_N^*(p_d)) = 0$ is a pure strategy Nash Equilibrium. In addition, the following statements are true.

1) If (17) holds, then $\mu^*(p_d) = 0$ is the unique pure strategy Nash equilibrium.

2) The pure strategy Nash equilibrium $\mu^*(p_d) = 0$ incurs no loss of efficiency.

3) If (18) or (19) holds, then the pure strategy Nash equilibrium $\mu^*(p_d) = 0$ is $(0, N - 1)$ fault immune.

Theorem 7 states that the strategy profile that all utilities submit vertical bidding curves exactly at the predicted demand is a unique and $(0, N - 1)$ fault immune robust pure strategy Nash equilibrium, which incurs no loss of efficiency; see Fig. 6(a) for illustration.

We further extend the results to setting of nonnegatively correlated Gaussian errors as the case of Theorem 6.

3We focus on the setting that the net load $D_i$ is inelastic and does not change with the day-ahead clearing price $p_d$. In reality, utilities may have price-related flexible demands due to dynamic pricing [9]. Therefore, the net loads $D_i(p_d)$ may change with price $p_d$. Our results still hold under such setting in which utilities submit bidding curves exactly at $Q_i(p_d) = D_i(p_d)$ that react to the market prices with flexible demands and can be generalized to the scenario with correlated prediction errors; see Fig. 6(b) for illustration.
Table I: Parameters of Pricing Models

| Pricing model and Parameters       | $a_1$ | $a_2$ | $b_1$ | $b_2$ |
|-----------------------------------|-------|-------|-------|-------|
| Symmetric pricing model           | 0.0034| 0.0034| 1.2378| 0.7622|
| Asymmetric pricing model          | 0.0034| 0.0005| 1.2378| 0.6638|

Theorem 8: Suppose the real-time spot pricing model is piecewise linear symmetric, and $\epsilon_i$, $i = 1, 2, \ldots, N$, are zero mean Gaussian random variables with nonnegative correlations, then the strategy profile $\mu^*(p_d) = 0$ is the unique, efficient, and $(0, N - 1)$ fault immune robust pure strategy Nash equilibrium.

Theorem 8 highlights that the equilibrium with desired properties is still admitted when utilities submit bidding curves with nonnegatively correlated Gaussian prediction errors.

Due to the space limitation, we refer interested readers to Section IV-C in the technical report [73] for detailed construction of the game-theoretical formulation and the corresponding Nash equilibrium descriptions.

D. Impact of Load Uncertainty on Equilibrium Strategy and Equilibrium Social Cost

Note that load uncertainty is captured by the variance of the load prediction error. In this part, we discuss the impact of load uncertainty on market equilibrium strategy and social cost.

Corollary 1: The utilities’ market equilibrium strategies $\mu_i^*$ are independent of load uncertainty variances $\sigma_i^2$, and equilibrium social cost $E(ABC_{\text{total}})$ is increasing w.r.t. each $\sigma_i^2$.

Although load uncertainty affects utilities’ costs [55], their optimal bidding strategies at equilibrium only depend on the mean of the load prediction error. In particular, for zero-mean prediction error, bidding according to prediction is exactly the equilibrium strategy and does not rely on the variance. Toward nonzero mean uncertainties, the equilibrium is admitted when all utilities bid at their specific error mean, i.e., $\mu_i^* = E(\epsilon_i)$, $i = 1, 2, \ldots, N$. This implies that each utility seeks to make its individual real-time mismatch $\Delta$ have zero mean bias, which is invariant with load uncertainty variances $\sigma_i^2$.

We further investigate the market social cost $E(ABC_{\text{total}})$. Note that for independent or nonnegatively correlated Gaussian load prediction errors, the increase of $\sigma_i^2$ will lead to higher $\sigma_i^2$, the variance of the market level total mismatch $\Delta$. By applying the approach in [55] to (15), it can be observed that the increase load uncertainty will increase $E(ABC_{\text{total}})$.

V. Simulation Results

A. Simulation Setting

We conduct simulations of the ISO-NE electricity market with 8 virtual utilities, each in charge of a state in New England region. The piecewise linear pricing model is obtained from [72]. We study the market equilibrium under symmetric pricing models and the impact of the asymmetricity of the pricing model whose parameters are presented in Table I. The hourly electricity net loads of the 8 utilities from January 1, 2011 to December 31, 2018 are obtained from ISO-NE market [3]. The day-ahead price is obtained from the mean of the hourly day-ahead prices from ISO-NE market, which is 358/MWh. Utilities’ costs at each time slot are calculated according to (5). Here, we investigate the market equilibrium at a specific day-ahead clearing price.

B. Prediction Error Distribution

Our theoretical analysis focuses on the scenario that the prediction errors of utilities follow symmetric unimodal distributions with zero mean, i.e., symmetric w.r.t. zero and central dominant. In this part, we verify these two conditions.

In practice, utilities predict their demands based on historical loads, weather information, holiday/weekend information, etc. [76]. Multiple types of load prediction methods are applied in short-term load forecasting, e.g., similar day method, artificial neural network (ANN), and time-series regression [3]. In our study, we use ANN to forecast load. Prediction errors are computed as the difference between the predicted values and the corresponding actual demands. We plot the load prediction error histogram of the utility in Maine state in Fig. 3. Similar error histograms can be observed for other utilities.

In our simulation, we observe that the sample mean of the load prediction error is $-0.068$ (MWh), and the sample standard deviation is 38.7 (MWh). Note that we model the load prediction error $\epsilon_i$ as a symmetric unimodal random variable with zero mean. For testing the symmetry around a specific center, we apply Two-sample Kolmogorov–Smirnov test on the prediction error samples [77], [78]. It is used to test whether two underlying one-dimensional probability distributions differ. Let $\{X_1, \ldots, X_N\}$ be the observed values of $\epsilon_i$. The result shows that the null hypothesis (i.e., “The sample data $\{X_1, \ldots, X_N\}$ and $\{-X_1, \ldots, -X_N\}$ are from the same continuous distribution”) is not rejected at the 5% significance level. This observation verifies the symmetry setting on the load prediction error, i.e., $\epsilon_i$ follows a symmetric distribution with zero mean. The central dominant condition can also be justified. As seen from the error histogram in Fig. 3, when the amplitude of the prediction error becomes larger, it has a smaller frequency of occurrence accordingly. We have similar observations for other utilities. Note that here we do not assume the independence of utilities’ load prediction errors $\epsilon_i$. It is observed that there are positive correlation coefficients in $\epsilon_i$ among utilities, as large as 0.66. In the following section, we find that our previous results still hold under this scenario, which shows the robustness of our analysis.

C. Market Equilibrium and Efficiency Performance

We now investigate the impact of the strategic behaviors of utilities on the ABC and the market equilibrium. We obtained the empirical ABC, with respect to the bidding strategy $\mu_i$ of the utility in Maine state by calculating the two-settlement average hourly cost across the consecutive 8 years, for different values of $\mu_i$.

As seen in Fig. 9, the utility’s cost takes the minimum when the utility bids according to prediction, given all others are bidding at prediction. Thus, the strategy profile that all utilities bid according to prediction is a pure strategy Nash equilibrium. Furthermore, we investigate the market-level ABC only
relates to \( \mu = \sum_{i=1}^{N} \mu_i \). For a particular \( \tilde{\mu} \), we randomly decompose it such that \( \tilde{\mu} = \sum_{i=1}^{N} \tilde{\mu}_i \). The corresponding market-level \( \text{ABC}_{\text{total}} \) is computed as the average of \( \text{ABCs} \) of different strategy profiles. We observe that the social cost under the game-theoretical strategic setting is the same as the optimal one under the coordinated setting, i.e., the equilibrium strategy profile \( \mu^* = 0 \) is the optimal solution that minimizes the social cost. Fig. 9 shows that this equilibrium incurs no loss of efficiency with respect to the market-level \( \text{ABC}_{\text{total}} \). We remark that when a utility deviates from the equilibrium, its cost will increase while all other utilities’ costs will decrease; see Figs. 10 and 4 for illustration. Furthermore, the market-level \( \text{ABC}_{\text{total}} \), average of all utilities’ costs, will also increase. This corresponds to our theoretical results in Theorem 3 and implies that deviating from equilibrium leads to efficiency loss.

D. Market Size Analysis and Sensitivity of Real-Time Market

The strategic potential of utilities arises from the dynamically changed real-time market electricity price. The sensitivity of the real-time market and the market size may propose impacts on utilities’ costs. These two aspects of the real-time market can be regarded as the price-changing characteristics with respect to the total imbalance, i.e., the slope and the discontinuous gap of the pricing model, and the number of participants \( N \), respectively. We study the impact of these three parameters. Toward this end, we equally separate each one of the 8 utilities into 2 to 5 subutilities as expanding the market size and calculate each new utility’s \( \text{ABC} \). Meanwhile, we vary the slope of the symmetric pricing model to be 0.005, 0.034, and 0.068, and we change \((b_1, b_2)\) to be \((1, 1)\), \((1.2378, 0.7622)\) and \((1.8, 0.2)\), which are sufficient to illustrate the impacts of the real-time market sensitivity to imbalance. The corresponding utility’s cost is studied. The trend of cost change can be observed when the market size expands and the market sensitivity increases. We use a utility split from the original utility in Maine state as an example to show the cost change trend. Previously, we have shown that when a utility deviates from the equilibrium, both its cost and the market-level cost will increase. In order to study the impacts of market size and market sensitivity on the aggregate cost of all utilities, we further investigate the market-level cost \( \text{ABC}_{\text{total}} \) change under different market settings both at the equilibrium and the strategy deviation conditions.

As seen, Figs. 7(a) and 10(a) demonstrate that expanding the market size, i.e., increasing the number of utilities \( N \), contributes to decreasing both the utility’s cost and the market-level cost under the equilibrium. This characteristic implies that competition improves efficiency. Based on these observations, market designers have an economic incentive to allow competition and expand market access in order to lower both utilities’ costs and the market-level cost.

Meanwhile, Fig. 8 depicts that when the slope of the piecewise linear symmetric spot pricing model becomes larger, i.e., larger \( a_1 \) and \( a_2 \), given \( \mu_{-i} = 0 \), the utility suffers a larger cost given the same deviation quantity \( \mu_i \). The market-level \( \text{ABC}_{\text{total}} \) presents a similar cost-strategy relationship with respect to \( \mu \) as shown in Figs. 11(b). In addition, Figs. 7(b) and 10(b) show that when the premium of readiness increases, i.e., larger \( b_1 - b_2 \),
both the utility and the market observe an increase in their costs under the same strategy deviation quantity $\mu_i$ and $\mu$, respectively.

Furthermore, it is worth noticing that compared with shrinking $b_1 - b_2$, decreasing $a_1$ and $a_2$ presents a more significant reduction in the deterioration rate (defined as the ratio between the cost increase and the cost under the equilibrium) for both the utility and the market under the same deviation quantity $\mu_i$ and $\mu$. This observation implies that when the real-time market becomes more robust to the total imbalance (corresponding to smaller $a_1$ and $a_2$), the market equilibrium tends to be less sensitive to the fault behaviors of utilities.

We also study the impacts of day-ahead clearing price $p_d$. Figs. 8(b) and 11(b) show that when the day-ahead clearing price $p_d$ increases, both the utility and the market have larger ABCs. Since the costs of utilities are proportional to $p_d$, we observe that there exist linear relationships between ABCs and $p_d$, and between ABCs and $\mu$.

These observations correspond to our theoretical results in Theorems 1 and 3. The abovementioned study suggests that improving the level of competition and the resiliency of the spot pricing against the market-level mismatch cannot only benefit utilities but also reduce the social cost.

E. Performance Under Asymmetric Load Uncertainty

Recall that the probability density functions of net load uncertainties are modeled as symmetric and unimodal. We further study the impact of asymmetric load prediction errors on utilities’ optimal bidding strategies and the market equilibrium. Multipeak asymmetric probability densities are observed when conducting short-term wind power predictions [60], [79], which can be fitted by piecewise exponential distribution, Beta distribution, or Gaussian mixture model (GMM). In this article, we adopt the GMM to describe such prediction errors, which is the combination of several Gaussian distribution components. We then fit load prediction errors according to the GMMs in [60] and scale them to zero mean; see Fig. 12 for illustration.

The GMM parameters are listed in our technical report [73]. Utility’s cost to strategy curve and the market-level cost curve are presented in Fig. 13(a).

It is observed that with such multipeak asymmetric load errors, the utility’s cost still takes the minimum when it bids at prediction, given all others are bidding according to prediction. Therefore, all utility bidding according to prediction is a pure strategy Nash equilibrium. We further investigate the market-level ABCs. The market-level cost curve indicates that the social cost is minimized at the equilibrium, and hence, the equilibrium incurs no loss of efficiency. The above-mentioned results show that the market could still admit an efficient equilibrium even with the multipeak and asymmetric load uncertainty.

Toward the robustness of the equilibrium, we demonstrate the result in Fig. 13(b). We observe that different from the case of symmetric unimodal prediction errors in Fig. 4, the utility’s cost curve is distorted may not exactly decrease w.r.t. other utilities’ irrational behaviors, though the trends perform similarly. This observation provides utilities an incentive to enhance the prediction models toward symmetric unimodal forecasting errors so that the equilibrium can be robust to any fault actions. Meanwhile, we find that utility behaviors cause at most 0.005% cost increase compared with the cost at equilibrium among 8 utilities, indicating the adaptability of equilibrium robustness. We leave the theoretical analysis for multipeak and asymmetric load uncertainty for further work.

F. Performance Under Asymmetric Pricing Model

We now study the impact of the asymmetric pricing model on the market equilibrium and the utility’s cost. The pricing model parameters are listed in Table I. We present the cost to strategy curve of the utility in Maine State. We observe similar cost to strategy relationships for other utilities.

As seen, Fig. 14 shows that when all other utilities bid according to prediction, the utility has an incentive to deviate from bidding according to prediction, which implies that the strategy profile that all utilities bid according to prediction is no longer a Nash equilibrium. Under the asymmetric pricing model, a utility can reduce its cost by bidding higher than the predicted net load. This can be explained intuitively as follows: when the real-time market performs less sensitive to the negative imbalance, i.e., $a_1 > a_2$, utilities can overbuy in the day-ahead market to sell the surplus at a higher price compared with the symmetric pricing model case that we choose.

From the cost to strategy curve in Fig. 14, the optimal nonzero bidding strategy for the utility is to choose $\mu^* = -68.5$ (MWh). Under this case, the utility only witnesses a 0.84% cost reduction compared with choosing $\mu_i = 0$, which indicates that under the realistic asymmetric pricing model suggested in [72], the utility does not have much incentive to deviate from bidding according to prediction given all other utilities bid according to prediction.
VI. CONCLUSION

We study the strategic bidding behaviors of utilities under a game-theoretical setting in the deregulated electricity market, with the uncertainty in demand and local renewable generation in consideration. We show that all utilities bidding according to (net load) prediction is the unique pure strategy Nash equilibrium. Furthermore, it incurs no loss of efficiency and is \((0, N - 1)\) fault immune to irrational fault behaviors. We extend the results to the cases with correlated prediction errors and a general class of real-time pricing schemes. Our simulation results suggest that market designers may improve the level of competition and the resilience of the spot pricing against the market-level mismatch to reduce utilities’ costs and the social cost. Moreover, our study highlights that the market operator can design real-time pricing schemes according to certain conditions, such that the interactions among utilities admit a unique, efficient, and robust pure strategy Nash equilibrium.

This article is based on the setting that utilities are price-takers in the day-ahead market. Under such a scenario, we show that the market admits a unique equilibrium with salient properties. Different from this setting, as it has often been pointed out, in some existing electricity markets, the day-ahead market clearing price is dominated by several large corporations [80]. In that case, our results still hold if we focus on the interactions between small-scale utilities. The competition between price-maker utilities, the market clearing price, and the corresponding market equilibrium remain to be studied. In addition, this article focuses on the demand side game-theoretical economic analysis. We note that bringing the power network constraints, the time-coupling energy storage operations, and the generation imbalance uncertainty into market equilibrium analysis requires further investigation.

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