Reliability Analysis of Wheel fatigue strength based on active learning Kriging Model

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Abstract. The reliability analysis of fatigue strength of rail vehicle wheels is studied in this paper. On the basis of deterministic analysis, a basic model for reliability analysis of wheel fatigue strength is established by comprehensively considering many uncertain factors. A reliability analysis method based on active learning Kriging (ALK) model is introduced to solve the fatigue strength failure probability of wheels. The results of reliability analysis are verified, and the results show that the reliability analysis based on ALK model can significantly improve the efficiency on condition the accuracy is guaranteed.

1. Introduction

High-speed and heavy-haul have become the main symbols of modern railway trains, which puts forward higher requirements for the safety and reliability of railway vehicles. As the moving of the rail vehicle, the wheel bears the complex stress in all directions, and the working environment is bad. Once the fatigue fracture occurs during the running process, the vehicle derailment will happen, and the result will be catastrophic. Therefore, the scientific and effective fatigue reliability analysis of wheels is of great significance to ensure the safety of train operation.

The key point of reliability analysis is to estimate the probability that the response of a system fails to meet the design requirement, considering the randomness of input variables [1]. For evaluating the failure probability, it requires to frequently obtain the response of performance function. In many engineering applications, the performance function generally needs to be calculated by the finite element simulation, which could bring serious computation burden to the computer. In order to reduce performance function computational demands, various efficient reliability analysis methods have been proposed, such as the first-order reliability method (FORM), the second-order reliability method (SORM), the importance sampling method and response surface method. These methods have their own characteristics and limitations. In recent years, a demand-oriented approximate model method (ALK model) based on active learning strategy has been proposed, and it has become a hot spot in the field of reliability research. The most representative active learning methods are EGRA method proposed by Bichon et al. [2] and the AK-MCS method proposed by Echard et al [3]. On this basis, many scholars have conducted in-depth and extensive research on ALK method. Zheng et al. [4] proposed a new active learning method based on a widely used learning function U, which can improve the speed of convergence of the AK-MCS method for problems with a connected domain of failure. Wen et al. [5] used an adaptive sampling region and parallelizability strategy to improve the ALK method, which further
improved the efficiency of the ALK method. In order to make ALK models applicable to estimation of small failure probabilities, Yang et al. [6] proposed the ALK-MM-IS method by combining ALK model with multimodal important sampling. And similarly, Huang et al. [7] proposed the AK-SS method by combining the ALK model with the subset simulation method. From the reports of high-level articles such as Refs. [2][3][8], for low and medium dimensional problems (below 30-dimensional), whether it is simple, strongly nonlinear or a multi-failure-region performance functions, the reliability analysis methods based on the ALK model can obtain high-precision (less than 5% error) results with only a few performance function evaluations. Therefore, this paper uses ALK model to analyze the wheel fatigue reliability.

At present, the fatigue strength evaluation standard of railway vehicle wheels has not been established in our country, and the fatigue strength analysis of wheels is mainly based on the UIC510-5 standard of the International Railway Union or the EN13979-1 standard of the European Union. The regulations of these two standards are roughly the same. According to these two standards, domestic scholars have put forward different analysis methods. Mi and Li [9] calculated the dynamic stress in the direction of the maximum principal stress under all working conditions, and then combined the Haigh-Goodman curve to evaluate the fatigue strength of the wheel. Liu et al. [10] proposed to simultaneously calculate the dynamic stress in three directions: the maximum first principal stress, the maximum second principal stress and the minimum third principal stress. Finally, the worst case was selected, and the equivalent fatigue stress was used to evaluate the fatigue strength of the wheel. Wang et al. [11] calculated the maximum and minimum dynamic stress by selecting the principal stress and the corresponding direction of each working condition, and finally took the safety factor as the standard of fatigue assessment. According to the theoretical analysis and practical calculation, Liu et al. [12] concluded that the effect of multiple cross section loading on axisymmetric wheels was equivalent to that of single cross section loading, and the rotation of wheel did not affect the maximum dynamic stress. Xiao et al. [13] calculated the fatigue strength of the electric multiple units (EMU) force measuring wheel according to the UIC510-5 standard. In Ref. [14], Liu et al. found that the uniaxial fatigue criterion was more suitable for the evaluation of the fatigue strength of the web plate, while the multiaxial fatigue criterion was safer for the evaluation of the web plate hole. Xu et al. pointed out in Ref. [15] that when the loading cross section changes, the maximum von-Mises stress in the web plate hole changed greatly, while the maximum von-Mises stress in other areas of the web plate rarely changed. In addition, they also compared four commonly used multiaxial fatigue criteria to evaluate the safety of the web plate hole. On the basis of Archard wear model and rolling contact fatigue model, Wu et al. [17] established a wheel wear fatigue prediction model based on dynamics and studied and predicted the wheel wear and rolling contact fatigue of different profiles.

However, the above studies treat the important parameters that affect the working state of the wheel as determinate values, such as stress, strength, safety factor, load, material properties, etc. [17]. The uncertainty in the service process of the wheel is ignored. Through investigation, it is found that there are few researches on the reliability of wheel fatigue life. On the basis of traditional wheel fatigue analysis, various factors uncertainties are considered and the fatigue reliability analysis of rail wheel is researched in this paper. By combining with the latest development of reliability analysis in recent years, this paper proposes a wheel fatigue reliability analysis method based on active learning Kriging (ALK) model, which provides an effective solution for the safety check of rail vehicle wheels.

2. Reliability assessment of wheel fatigue strength

As a component directly contacting with the track, the wheel will be subjected to complex alternating stress in the process of running. On the one hand, due to the irregularity of the track, the hunting motion will occur when the vehicle is running, which leads to the contact parts between the wheels and rails to change constantly. At the same time, the magnitude and direction of the stress at the contact parts also change. On the other hand, due to the periodic rotation of the wheel, the stress
at each point on the wheel will also change. In addition, the vehicle will experience different load conditions during the driving process. All of the above factors will lead to alternating stress, which makes the wheel fatigue analysis quite complicated. In order to facilitate the analysis of engineering application, the International Union of Railways formulated a convenient standard for checking the fatigue strength of wheels, which is offered in UIC510-5 [18]. In this standard, only the alternating stress caused by the load condition and the wheel periodic rotation is considered.

Then, according to the phenomenon that the direction of the fatigue crack is perpendicular to the maximum principal stress direction, the three-dimensional stress state is simplified to the unidirectional stress state. Finally, by calculating the stress amplitude and stress mean value of the stress cycle, the Goodman curve can be used to evaluate the fatigue strength of the wheel.

The fatigue strength of wheels is mainly calculated by finite element method. Suppose the stress matrix $S^{(j)}$ of a node on the wheel under the $j$-th working condition is obtained by the finite element method:

$$
S^{(j)} = \begin{bmatrix}
\sigma_x & \tau_{xy} & \tau_{xz} \\
\tau_{xy} & \sigma_y & \tau_{yz} \\
\tau_{xz} & \tau_{yz} & \sigma_z
\end{bmatrix} \quad j = 1, 2, ..., n
$$

(1)

Where $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{yz}$ and $\tau_{xz}$ are the stress components of the node, and $n$ denotes the number of working conditions.

The principal stress matrix $\sigma^{(i)}$ of the node and the direction vector $v^{(i)}$ of each principal stress can be solved by using the Jacobian method mentioned in Ref. [19].

$$
\sigma^{(i)} = \begin{bmatrix}
\sigma_i & 0 & 0 \\
0 & \sigma_i & 0 \\
0 & 0 & \sigma_i
\end{bmatrix}
$$

(2)

$$
v^{(i)} = (l_i, m_i, n_i)^T \quad i = 1, 2, 3
$$

(3)

Where $\sigma_i$, $\sigma_i$, and $\sigma_i$ are the first, second, and third principal stresses respectively; $i$ is the $i$-th principal stress; $l_i$, $m_i$ and $n_i$ denote the direction cosines of the $i$-th principal stress.

Denote the $i$-th principal stress in Eq. (2) under the $j$-th working condition as

$$
\sigma_i^{(j)}
$$

(4)

If the node obtains the maximum principal stress value at the $i$-th principal stress of the $k$-th working condition, that is

$$
\sigma_{\text{max}}^{(k)} = \sigma_i^{(k)}
$$

(5)

and the direction vector corresponding to the maximum principal stress is $v_i^{(k)}$, then the projection of all working conditions in the direction of the maximum principal stress is:

$$
\sigma_{\text{project}}^{(i)} = (v_i^{(j)})^T S^{(j)} v_i^{(j)} \quad j = 1, 2, ..., n
$$

(6)

The minimum principal stress corresponding to the minimum value of the projections:

$$
\sigma_{\text{min}} = \min \{\sigma_{\text{project}}^{(1)}, \sigma_{\text{project}}^{(2)}, ..., \sigma_{\text{project}}^{(n)}\}
$$

(7)

With the maximum and minimum principal stresses, the average stress $\sigma_m$ and stress amplitude $\sigma_a$ of the node are obtained by:

$$
\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}
$$

(8)

$$
\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}
$$

(9)
Finally, the fatigue strength of the wheel is checked by drawing the Haigh-Goodman curve, as shown in Fig. 1 [20].

![Haigh-Goodman curve](image)

**Fig. 1. Haigh-Goodman curve.**

In the figure, $\sigma_s$, $\sigma_b$, and $\sigma_{-1}$ are the yield limit, strength limit and fatigue limit of the wheel material respectively. The coordinates of points A and B are

$$
\left( \frac{\sigma_b (\sigma_{-1} - \sigma_s)}{\sigma_b + \sigma_{-1}}, \sigma_{-1} \right)
$$

and

$$
\left( \frac{\sigma_b (\sigma_{-1} - \sigma_s)}{\sigma_b - \sigma_{-1}}, \sigma_{-1} \right)
$$

respectively.

If the mean stress value $\sigma_m$ and the stress amplitude value $\sigma_a$ fall in the colored area in the figure, the fatigue failure will not occur on this node; otherwise, the structure will lose efficacy. Accordingly, the performance function $G(x)$ can be expressed as

$$
G(x) = \begin{cases} 
(\sigma_s + \sigma_a(x)) - \sigma_s(x), & \sigma_m(x) \in \left[ -\sigma_s, \frac{\sigma_b (\sigma_{-1} - \sigma_s)}{\sigma_b + \sigma_{-1}} \right] \\
\sigma_s (1 - \frac{\sigma_a(x)}{\sigma_b}) - \sigma_s(x), & \sigma_m(x) \in \left[ \frac{\sigma_b (\sigma_{-1} - \sigma_s)}{\sigma_b + \sigma_{-1}}, \frac{\sigma_b (\sigma_s - \sigma_{-1})}{\sigma_b - \sigma_{-1}} \right] \\
(\sigma_s - \sigma_a(x)) - \sigma_s(x), & \sigma_m(x) \in \left[ \frac{\sigma_b (\sigma_s - \sigma_{-1})}{\sigma_b - \sigma_{-1}}, \sigma_s \right]
\end{cases}
$$

(10)

Where $x = (x_1, x_2, \ldots, x_n)$ denotes random variable; $\sigma_m(x)$ and $\sigma_a(x)$ are implicit functions of mean stress and stress amplitude about $x$. When $G(x) < 0$, fatigue failure will occur, which corresponds to the area beyond the fatigue envelope in the Goodman curve. Define the failure domain as $F = \{x | G(x) < 0\}$, and then the wheel failure probability $P_f$ can be expressed as:

$$
P_f = P\{x \in F\} = \iiint \cdots \int I(x \in F) f(x) dx
$$

(11)

Where $P\{\cdot\}$ denotes the probability of an event; $f(\cdot)$ is the joint probability density function (PDF); $I(\cdot)$ is an indicator function of an event, whose value is 1 if the event is true and 0 otherwise.

It is difficult to directly solve $P_f$ by Eq. (11), which is generally obtained by Monte Carlo simulation (MCS) method.

$$
\hat{P}_f = \frac{1}{N_{MCS}} \sum_{i=1}^{N_{MCS}} I(G(x_i) < 0)
$$

(12)

the coefficient of variation ($Cov$) is given as:
\[ \text{Cov}(\hat{P}_j) = \frac{1 - \hat{P}_j}{N_{\text{MCS}} \hat{P}_j} \]  

where \( x_i \) is the \( i \)-th random sample and \( N_{\text{MCS}} \) is the number of samples.

3. ALK model

3.1. Kriging model

Kriging model is actually an interpolation model, which uses Gaussian process to approximate the performance function. The performance function \( G(x) \) of the structure is given as:

\[ G(x) = f(x) \hat{\beta} + Z(x) \]  

(14)

Where \( f(x) = [f_1(x), f_2(x), \ldots, f_p(x)]^T \) is the basis functions; \( \beta = [\beta_1, \beta_2, \ldots, \beta_p]^T \) is the vector of regression coefficients; \( p \) denotes the number of the basis function in regression model. \( Z(x) \) is a stationary Gaussian process with zero mean (and covariance defined next). In this paper, we choose the regression model as constant, i.e.

\[ G(x) = \beta + Z(x) \]  

(15)

The covariance of the Gaussian process \( Z(a) \) is defined as:

\[ \text{Cov}[z(a), z(b)] = \sigma_z^2 \mathcal{R}(\theta, a, b) \]  

(16)

In which \( a \) and \( b \) denote two arbitrary points, \( \sigma_z^2 \) is the process variance, \( \mathcal{R}(\cdot) \) is a correlation function with parameter \( \theta \).

Several models are available to define the correlation function. The Gaussian correlation function is selected, which is defined as

\[ \mathcal{R}(\theta, a, b) = \exp \left[ -\sum_{i=1}^{n} \theta_i (a_i - b_i)^2 \right] \]  

(17)

Where \( a_i, b_i \) and \( \theta_i \) are the \( i \)-th coordinates of \( a, b \) and \( \theta \), respectively.

When building a kriging model, a DoE is required: \( x = [x_1, x_2, \ldots, x_n]^T \) and the response \( G = [G_1(x_1), G_2(x_2), \ldots, G_n(x_n)]^T \). Then the mean and variance of the unknown point \( u \) predicted by the Kriging model can be obtained by:

\[ \mu_u(u) = \hat{\beta} + r(u)^T R^{-1} \left( G - 1 \hat{\beta} \right) \]  

(18)

\[ \sigma_u^2(u) = \hat{\sigma}_z^2 \left[ 1 + \frac{r(u)^T R^{-1} r(u) - 1}{r(u)^T R^{-1} R^{-1} r(u)} \right] \]  

(19)

Where \( r(u) = \left[ \mathcal{R}(\theta, u, u_1), \mathcal{R}(\theta, u, u_2), \ldots, \mathcal{R}(\theta, u, u_n) \right]^T \) is a vector defined by Eq. (16) and \( R \) is an \( n \times n \) matrix. \( R \), parameters \( \hat{\beta} \) and \( \sigma_z^2 \) can be expressed by the following equations respectively.

\[
\begin{bmatrix}
\mathcal{R}(\theta, x_1, x_1) & \cdots & \mathcal{R}(\theta, x_1, x_n) \\
\vdots & \ddots & \vdots \\
\mathcal{R}(\theta, x_n, x_1) & \cdots & \mathcal{R}(\theta, x_n, x_n)
\end{bmatrix}
\]

\( R = \left[ \begin{bmatrix}
\mathcal{R}(\theta, x_1, x_1) & \cdots & \mathcal{R}(\theta, x_1, x_n) \\
\vdots & \ddots & \vdots \\
\mathcal{R}(\theta, x_n, x_1) & \cdots & \mathcal{R}(\theta, x_n, x_n)
\end{bmatrix}
\right] \]

\( \hat{\beta} = (1^T R^{-1} 1)^{-1} 1^T R^{-1} G \)  

(21)

\( \hat{\sigma}_z^2 = \frac{1}{n} (G - \hat{\beta} 1)^T R^{-1} (G - \hat{\beta} 1) \)  

(22)
The parameter $\theta$ can be determined with maximum likelihood estimation, that is, by solving the following optimization problem.

$$\theta^* = \arg\min_{\theta} \left| R_\theta \right|$$

The research shows that the value of the parameter $\theta$ will greatly affect the accuracy of the Kriging model [21]. As a global optimization algorithm, DIRECT algorithm is not affected by the initial point and can obtain the global optimal solution compared with local optimization algorithm or gradient-based optimization algorithm [22]. Therefore, we choose the global optimization algorithm to directly solve the equation (23).

3.2. Learning Function.

Eq. (12) shows that the failure probability $P_f$ is only affected by the sign of the objective function $G(x)$. According to the prediction information of Kriging model, quite a lot of learning functions can be proposed to find the point at which the sign of $G(x)$ has the largest probability to be wrongly predicted. For example, Bichon et al. proposed the famous expected feasibility function (EFF) [2]; Echard et al. proposed the widely used U learning function [3]; Yang et al. proposed the expected risk function (ERF) [23]. In this paper, we use EFF to build ALK model.

According to the prediction information provided by the Kriging model: $\hat{G}(x) \sim N(\mu_G(x), \sigma_G^2(x))$, the EFF learning function is defined as:

$$EFF(x) = \mu_G \left[ 2\Phi\left(\frac{-\mu_G}{\sigma_G}\right) - \Phi\left(\frac{e - \mu_G}{\sigma_G}\right) - \Phi\left(\frac{e - \mu_G}{\sigma_G}\right) \right] - \sigma_G \left[ 2\phi\left(\frac{-\mu_G}{\sigma_G}\right) - \phi\left(\frac{e - \mu_G}{\sigma_G}\right) - \phi\left(\frac{e - \mu_G}{\sigma_G}\right) \right] + e \left[ \Phi\left(\frac{e - \mu_G}{\sigma_G}\right) - \Phi\left(\frac{e - \mu_G}{\sigma_G}\right) \right]$$

Where $\Phi$ is the cumulative distribution function of standard normal distribution, $\phi$ is the probability density function, and $e = 2\sigma_G^2$. The EFF learning function reflects that how well the true value of the performance function at a point $x$ satisfies the equality constraint $G(x) = 0$ over a region defined by $0 \pm e$. The point with the larger EFF value is more likely to be located near the limit state. Therefore, the next best training point among the candidate points is defined as:

$$x^* = \max_{x \in \Omega_c}(EFF(x))$$

(25)

Where $\Omega_c$ is the set of candidate points. Add the optimal training point to DoE, update the Kriging model, and then iteratively select a new training point until the termination conditions are satisfied. The termination condition of the EFF learning function is defined as:

$$\max(EFF(x)) < 0.0001$$

(26)

4. Summary of ALK

(1) Use Latin hypercube sampling (LHS) to generate a small number of initial training points and the number of initial training point is $n_{DoE} = 6$. Call finite element codes to compute the performance function.

(2) Use MCS method to generate $n_{mc}$ candidate points according to the probability density of $x$. $n_{mc}$ is assigned $10^6$ in this paper.

(3) Build a Kriging model with the current DoE by the improved DACE toolbox.

(4) Identify the optimal training point $x^*$ by Eq.(25).
(5) If the termination condition is not satisfied, calculate the true response value of $G(x)$ at $x^*$, and then add point $(x^*,G(x^*))$ to DoE. Return to step (3) to update the Kriging model. Otherwise, go to step (6).

(6) Based on the Kriging model, the MCS method is used to calculate the failure probability $P_f$ and the coefficient of variation $Cov$.

5. Computational Experiment: Reliability Analysis of Fatigue Strength of HDSA Wheel

HDSA wheel is a kind of wheel widely used in railway wagon in our country, and its maximum running speed is 120km/h. The wheel is made of CL60, with a S-shaped web plate and nominal diameter 840mm. The wheel has an axisymmetric structure and its finite element model is shown in Fig. 2.

![Finite element model of wheel](image1)

**Fig. 2.** Finite element model of wheel.

According to the conclusion of Ref. [12], a loading section is selected and the three working conditions are loaded in the form of Fig. 3. The magnitude of the load is shown in Table 1. $P$ is the wheel weight, that is, half of the axle weight (21t).

![Schematic diagram of load position](image2)

**Fig. 3.** Schematic diagram of load position.
Table 1. Load values under different working conditions.

| Working condition | Vertical load $F_z$ /KN | Transverse load $F_y$ /KN |
|-------------------|------------------------|--------------------------|
| straight line     | $F_{z1} = 1.25P$       | $F_{y1} = 0$             |
| curve             | $F_{z2} = 1.25P$       | $F_{y2} = 0.7P$          |
| fork              | $F_{z3} = 1.25P$       | $F_{y3} = 0.42P$         |

It should be noted that interference fit between wheels and shafts is not considered in this paper. The main reason is that ANSYS takes a long time to solve nonlinear problems, which is not suitable for reliability analysis, so it is simplified accordingly. In addition, although the stress at the interference fit part of the wheel and axle is large, the stress amplitude is not large, and it is not the position where the fatigue damage occurs [10][13]. In practice, the fatigue damage mainly occurs in the web plate. For axisymmetric wheels, all nodes on the inner and outer surfaces of a section of the web plate are selected as fatigue strength assessment points, as marked in Fig. 4.

Fig. 4. Wheel fatigue strength assessment location.

Considering the dispersion of wheel materials, errors in the design and manufacturing process and load changes during vehicle operation, the wheel weight, elastic modulus, strength limit, yield limit and fatigue limit are taken as random variables. The distribution type, mean value and standard deviation are shown in Table 2.

Table 2. Distribution types and parameters of random variables.

| Random Variables | Symbol | Distribution types | mean value | coefficient of variation | standard deviation |
|------------------|--------|--------------------|------------|--------------------------|-------------------|
| Wheel weight P/N | $x_1$  | Normal             | 105000     | 0.05                     | 5250              |
| Elastic modulus Ex/MPa | $x_2$  | Normal             | $2.1 \times 10^5$ | 0.05 | 10500 |
| Strength limit $\sigma_b$ /MPa | $x_3$  | Normal             | 715        | 0.05                     | 35.75             |
| Yield limit $\sigma_s$ /MPa | $x_4$  | Normal             | 350        | 0.05                     | 17.5              |
| Fatigue limit $\sigma_l$ /MPa | $x_5$  | Normal             | 253        | 0.05                     | 12.65             |

Finally, the fatigue failure probability of the wheel is calculated by ALK method. In order to verify the accuracy of the method, 1500 samples were generated by Latin hypercube sampling within the range of $[-5\sigma, 5\sigma]$ of random variables. 1500 times of finite element analysis are performed at those samples and an accurate Kriging model is established. The failure probability
\( \hat{P}_f \) calculated by the Kriging model is used as the reference solution. The calculation results of different methods are shown in Table 3. In the table, the data of ALK method is the average value of 10 times calculation results, because the ALK algorithm has certain randomness. In addition, the proposed method is compared with the SORM method. It can be seen from Table 3 that the “true” failure probability is \( 6.670 \times 10^{-3} \). This means that the wheel has a probability of 99.333% to be free from fatigue damage, which is relatively safe. The SORM method is inferior to the ALK method in accuracy and efficiency. The relative error between the ALK method and the test solution is only 0.432%, which is very accurate. However, the reference solution takes much more time than ALK method. Therefore, ALK method is an algorithm with high accuracy and efficiency.

**Table 3. Calculation results of different methods.**

| Method       | \( N_{\text{calls}} \) | \( \hat{P}_f \)   | Cov | Time consuming (min) |
|--------------|-------------------------|-------------------|-----|----------------------|
| Reference solution | 1500                  | \( 6.670 \times 10^{-3} \) | 2.73% | 230                  |
| SORM         | 48                     | \( 6.636 \times 10^{-3} \) | --  | 7.36                 |
| ALK          | 6+23=29                | \( 6.641 \times 10^{-3} \) | 1.22% | 4.45                 |

Take the results of the 10-th calculation as example, we draw the response at the training points of ALK model in Fig. 5. As show in the figure, most of the training points are concentrated around \( G(x)=0 \). This shows that the ALK model successfully identifies the points near the limit state in the learning process. Using these points as training points, the efficiency can be greatly improved on condition the accuracy is guaranteed. Fig. 6 shows the variation of the maximum value of EFF during the learning process of the ALK method. It can be clearly seen that the convergence speed of the ALK method is very fast. After 4 iterations, the maximum value of EFF almost no longer has a large oscillation, and gradually tends to be stable. Finally, it is close to the convergence value of 0.0001.

![G(x)=0](image)

**Fig. 5.** ALK model training points.
Fig. 6. The variation curve of the maximum value of EFF.

6. Conclusion
(1) According to the UIC 510-5, the basic model of wheel fatigue strength reliability analysis is established by comprehensively considering various uncertain factors. It provides a reference for wheel fatigue strength reliability analysis and has certain engineering significance.

(2) The fatigue strength failure probability of HDSA wagon wheel is calculated by using ALK reliability analysis method. The results show that the failure probability of the wheel is very small and the security is high.

(3) By comparing different reliability analysis methods, we can see that the ALK reliability analysis method has obvious advantages. It can focus much attention on the region of interest instead of blindly searching for training points, and significantly improves the efficiency of reliability analysis on condition the accuracy is assured.

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