Epileptic seizures regularities, revealed from encephalograms time series by nonlinear mechanics methods

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Abstract. Recurrence quantification analysis and multifractal method were used for investigation of the encephalograms time series of the children with epileptic seizures. It has been shown that epileptic seizures are accompanied by an increase in determinism of the brain electric process due to neuron activity synchronization. This behavior is typical for nonlinear systems catastrophes of different nature.

1. Introduction
Nonlinear mechanic methods are widely used in different fields of science, such as applied mathematics [1-3], physics and mechanics of fracture [4-6], earth sciences [7-10], biomedicine [11,12], economics [13,14] etc. In this work we use recurrence quantification analysis (RQA) [15] and multifractal formalism in order to find out if there exist some peculiar features in encephalograms (EEG) related with epileptic seizures which are common to other types of catastrophic behavior in nonlinear systems.

2. Experimental data
Experimental data was obtained at the Children’s Hospital in Boston. It consists of EEG recordings from pediatric subjects with intractable seizures and published in Physionet database [12,16]. The children were monitored for up to several days following withdrawal of anti-seizure medication in order to characterize their seizures and assess their candidacy for surgical intervention. All EEG signals were sampled at 256 Hz with 16-bit resolution. The International 10-20 system of EEG electrode positions was used for these recordings. In addition four other electrodes (T7, T8, P7, P8) standardized by the American Electroencephalographic Society were also used. Most files included 23 EEG signals. We studied only those files that contained one or more epileptic seizures. The beginning and end of each seizure was annotated in the database. We emphasize that the aim of the work was not forecasting or early detection of seizures. But instead we tried to reveal the regularities which appear to be general to other nonlinear systems during catastrophes.

3. Principal components transform
We started with transforming original EEG recordings to principal components. Figure 1 represents an example of the percent variability described by principle components (PC). One can notice that the first 10 PC describe about 95% of variability, so the number of variables may be considerably reduced.
Next we divide PC time series into not intersecting intervals with length of 1024 points each. Taking into account the discreteness of 256 samples per second such an interval is equal to 4 s. Each of these time intervals was analyzed by means of RQA and multifractal method.

4. Recurrence plot quantification analysis
The main quantity of recurrence plot – the recurrence matrix is defined as

\[ R_{ij}(\epsilon_i, m) = \theta(\epsilon_i - \|X_i(m) - X_j(m)\|), \ i, j = 1..N \]  

where \( X_i(m) \) is a phase space trajectory in an \( m \) dimensional phase space (we chose \( m=1 \)), \( \theta \) is a Heaviside function, \( \| \) is a norm, \( N=1024 \) in our case, and \( \epsilon_i \) is a predefined threshold distance. Being plotted as 2d graph the matrix (1) represents a set of black and white dots on the plane. Black dots refer to the case of \( R_{ij} = 1 \) (close or recurrent points), while the white dots mean that \( R_{ij} = 0 \) (non recurrent points). Vicinity of points is controlled by \( \epsilon_i \) parameter. The choice of \( \epsilon_i \) plays a crucial role: if one takes \( \epsilon_i \) too large, most points will be recurrent (black) and the whole plot would be almost black. On the contrary, if \( \epsilon_i \) is chosen too small the recurrent plot would be too sparse. According to recommendations of paper [15] we chose \( \epsilon_i \) for each point \( i \) in such a way that the recurrence point density stayed approximately equal to 0.1 (so called FAN criterion). Free software package CRP Toolbox [15] has been used. Recurrence matrix is a basic quantity for all other RQA parameters.

Some RQA results are displayed in figure 2. Time dependencies of the maximum diagonal line \( L_{max} \) and determinism \( DET \) [15] are shown for PC3 principle component and four different patients. Beginnings of epileptic seizures are noted by arrows. One can notice that in most cases (a-c) epileptic seizures are accompanied by an increase in RQA diagonal measures. This means that the brain electric process becomes more deterministic (simple) while the epileptic seizure occurs. An exception is shown in figure 2(d). The level of determinism in this case is high almost everywhere (\( DET \approx 0.9 \) ) and a decrease of this measure precedes the epileptic seizure. Nevertheless the high value of \( DET \) restores during the seizure, so again it is accompanied with the increase of \( DET \).

5. Hurst coefficients behavior
Maxima of multifractal singularity spectra (Hurst coefficients \( H \)) are shown versus time in figure 3. Wavelet leaders [17, 18] were used in order to calculate singularity spectra. Wavelet leader \( L(j,k) \) is the largest wavelet coefficient \( d(j,k) \) of the discrete wavelet transform computed at all finer scales.
$2^j \leq 2^k$ within a narrow time neighborhood $(k-1) \cdot 2^j \leq k \cdot 2^j \leq (k+1) \cdot 2^j$ [17]. Wavelet leaders reproduce Hölder exponents $h$ which can be computed by a standard multifractal formalism: constructing the generalized partition function

$$Z(j,q) \propto \sum_k L(j,k)^q \propto 2^{j \tau(q)};$$

extracting from (2) scaling exponents $\tau(q)$; and finally obtaining singularity spectrum $D(h)$ via Legendre transform. One can see that epileptic seizures are accompanied by an increase in Hurst coefficients $H$. This increase means that the fractal dimension of the signal $D = 2 - H$ decreases – the signal becomes more regular.

Figure 2. Time dependencies of RQA measures $L_{\text{max}}$ and DET for patients: (a) chb01_16, (b) chb15_54, (c) chb17_03 and (d) chb14_04 in Physionet notation. Arrows denote beginnings of epileptic seizures

6. Discussion
The increase in determinism and regularity are often observed in different nonlinear systems during catastrophic situations. Patients with severe heart failure show much narrower range of $h$ and higher $H$ values, indicating loss of multifractal complexity with a life-threatening disease [e.g. 19]. Singularity spectra of lateral surface profiles in mechanically loaded metals and metal glasses measured by means of probe microscopy become significantly narrower when materials approach mechanical fracture. The decrease in fractal dimension (increase of $H$) and narrowing of singularity spectra of seismic time series are well known phenomena accompanying earthquakes [21-22]. Same peculiarities were observed in acoustic emission time series during main crack formation in mechanically loaded laboratory rock samples by means of RQA and multifractal analysis [5, 24]. In fracture process these phenomena are caused by structure defects collectivization at various space and time scales leading to fractal self-organization of defect structure. It is known [e.g. 16] that epileptic seizures are
accompanied by neuron activity synchronization, so collectivization of structure elements takes place in this case either.

Figure 3. Time dependencies of Hurst coefficients $H$ for patients: (a) chb08_11, (b) chb15_54, (c) chb02_16+ and (d) chb14_04. Arrows denote beginnings of epileptic seizures

7. Conclusion
Obtained results suggest that epileptic seizures are accompanied by the raise of the process determinism and regularity demonstrated by the recurrence plots diagonal lines measures as well as the increase in the Hurst coefficient (increase of persistence). This means that during the seizure process changes in time from more complicated to a simpler one.

References
[1] Katok A and Hasselblatt B 1996 Introduction to the Modern Theory of Dynamical Systems (Cambridge: Cambridge University Press) p 829
[2] Olson C C, Nichols J M and Virgin L N 2012 Nonlinear Dynamics 70 381–91
[3] Hegger R, Kantz H and Schreiber T 1999 Chaos 9 413–34
[4] Nichols J M, Trickey S T and Seaver M 2006 Mech. Syst. Signal Process. 20 421–37
[5] Hilarov V L 2015 Solid State Phys. 57 2271–34
[6] Kacimi S and Laurens S 2009 Journal of Applied Physics 106 024909
[7] Ponyavin D I, Barliaeva T V and Zolotova N V 2006, Memorie della societa astronomica italiana – series 76 1026
[8] Lyubushin A A 2015 Journal of Seismology 19 329–40
[9] Zakharov V S 2011 Moscow University Geology Bulletin 66 385-92
[10] Smirnov V B, Ponomarev A V, Jiadong Q and Cherepantsev A S 2005 Izvestiya. Physics of the Solid Earth 41 428–48
[11] Mekler A A 2008 International journal of psychophysiology 69 258–59
[12] Goldberger A L, Amaral L A N, Glass L, Hausdorff J M, Ivanov P Ch, Mark R G, Mietus J E,
Moody G B, Peng C K and Stanley H E 2000 *Circulation* 101 E215–20

[13] Soofi A S and Liangyue C 2002 *Modelling and Forecasting Financial Data* (Norwell: Kluwer Academic Publishers) p 488

[14] Dieci R, He X Z and Hommes C 2014 *Nonlinear Economic Dynamics and Financial Modelling* (Cham: Springer International Publishing) p 389

[15] Marwan N, Romano M C, Thiel M and Kurths J 2007 *Phys. Rep.* 438 237-329

[16] Shoeb A 2009 *Application of Machine Learning to Epileptic Seizure Onset Detection and Treatment. PhD Thesis* (Massachusetts Institute of Technology)

[17] Wendt H, Abry P and Jaffard S 2007 *IEEE Signal Process.* 24 38–48

[18] Wendt H, Roux S G, Jaffard S and Abry P 2009 *Signal Process.* 89 1100–14

[19] Ivanov P Ch, Amaral N L A, Goldberger A L, Havlin S, Rosenblum M G, Struzikk Z R and Stanley H E 1999 *Nature* 399 461–65

[20] Korsukov V E, Knyazev S A, Butenko P N, Gilyarov V L, Korsukova M M, Nyapshaev I A and Obidov B A 2017 *Phys. Solid State* 59 316-320

[21] Gibowicz S J and Lasocki S 2001 *Advances in Geophysics*, ed R Dmowska and B. Saltzman vol 44 (San Diego: Elsevier Academic Press) p 39–181

[22] Kasimova V A, Kopylova G N and Lyubushin A A 2018 *Izvestiya. Physics of the Solid Earth* 53 269–83

[23] Damaskinskaya E E, Hilarov V L, Panteleev I A, Gafurova D R and FrolovD I 2018 *Phys. Solid State* 60 1821–26