Baryogenesis via Density Fluctuations with a Second Order Electroweak Phase Transition

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Abstract

We consider the presence of cosmic string induced density fluctuations in the universe at temperatures below the electroweak phase transition temperature. Resulting temperature fluctuations can restore the electroweak symmetry locally, depending on the amplitude of fluctuations and the background temperature. The symmetry will be spontaneously broken again in a given fluctuation region as the temperature drops there (for fluctuations with length scales smaller than the horizon), resulting in the production of baryon asymmetry. The time scale of the transition will be governed by the wavelength of fluctuation and, hence, can be much smaller than the Hubble time. This leads to strong enhancement in the production of baryon asymmetry for a second order electroweak phase transition as compared to the case when transition happens due to the cooling of the universe via expansion. For a two-Higgs extension of the Standard Model (with appropriate CP violation), we show that one can get the required baryon to entropy ratio if fluctuations propagate without getting significantly damped. If fluctuations are damped rapidly, then a volume factor suppresses the baryon production. Still, the short scale of the fluctuation leads to enhancement of the baryon to entropy ratio by at least 3 - 4 orders of magnitude compared to the conventional case of second order transition where the cooling happens due to expansion of the universe.

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I. INTRODUCTION

Baryogenesis at the electroweak phase transition in the universe is one of the most attractive models of solving the matter-antimatter problem of the universe \[1\]. As compared to other models such as those based on Grand Unified Theories (GUT), primordial black holes, etc., much of the physics of electroweak theory is well understood and is accessible to laboratory experiments. Unfortunately, it is by now generally accepted that it does not seem possible to get sufficient baryon asymmetry entirely within the standard model. The requirements of a strong first order phase transition, large CP violation, and suppression of baryon violating interactions in the broken phase (so that produced asymmetry survives), seem only possible if one considers extensions of the standard model. It is also known that in the case of a second order phase transition, when the temperature decreases due to the expansion of the universe, the resulting baryon asymmetry is 8 to 9 orders of magnitude below the required value \[2\]. This happens essentially because the cooling rate of the universe, and hence the rate of change of the vacuum expectation value of the Higgs field (say, in two Higgs extension of the Standard Model), is many orders of magnitude smaller than the electroweak scale \[4\]. There have been some proposals such as those based on topological defects \[3\] (e.g., utilizing the symmetric core of the defect), and reheating of local regions due to evaporation of primordial black holes \[4,5\], where it has been argued that sufficient baryon asymmetry may arise even with a second order phase transition.

An important aspect of phase transitions in the universe, which has not received much attention, is the fact that the universe is necessarily inhomogeneous. There are density fluctuations present in the universe, which eventually lead to the formation of structure we see today. Density fluctuations, and hence temperature fluctuations, even if they are of small magnitude, can affect the phase transition dynamics in crucial ways. There has been some discussion of the effects of inhomogeneities on the dynamics of a first order phase transition, in the context of quark-hadron transition in the universe \[6,7,8\]. For example, Christiansen and Madsen have discussed \[6\] heterogeneous nucleation of hadronic bubbles due to presence of impurities. It is mentioned in ref. \[6\] that possible sources of such impurities could be primordial black holes, cosmic strings, magnetic monopoles, or relic fluctuations from the electroweak scale. Hadronic bubbles are expected to nucleate at these impurities with enhanced rates. Recently, Ignatius and Schwarz have proposed \[7\] that the presence of density fluctuations (those arising from inflation) at quark-hadron transition will lead to splitting of the region in hot and cold regions with cold regions converting to hadronic phase first. Baryons will then get trapped in the (initially) hotter regions. Estimates of sizes and separations of such density fluctuations were made in ref. \[7\] using COBE measurements \[9\] of the temperature fluctuations in the cosmic microwave background radiation (CMBR). In an earlier work \[8\], we have considered the effect of cosmic string induced density fluctuations on quark-hadron transition and have shown that it can lead to formation of planar regions of baryon inhomogeneity which may have important effects on nucleosynthesis.

There are many possible sources of density fluctuations in the early universe such as inflation and cosmic defects (strings, monopoles, textures, domain walls). There has been extensive study of density fluctuations generated by cosmic strings from the point of view of structure formation \[10\]. Though recent measurements of temperature anisotropies in the microwave background by BOOMERANG, and MAXIMA experiments \[11\] at angular scales
of $\ell \simeq 200$ disfavor models of structure formation based exclusively on cosmic strings \cite{12,13}, still, due to many uncertainties in the scaling models of cosmic string network evolution one cannot rule them out as candidates of sources of required density fluctuations. Further, even with present models, it is not ruled out that cosmic strings may contribute to some part in the structure formation in the universe. Above all, cosmic strings generically arise in many Grand Unified Theory (GUT) models. If the GUT scale is somewhat lower than $10^{16}$ GeV then the resulting cosmic strings will not be relevant for structure formation (for a discussion of these issues, see \cite{13}). However, they may still affect various stages of the evolution of the universe in important ways.

In this paper we will study the implications of density fluctuations produced by cosmic strings on electroweak baryogenesis for the case of a second order electroweak phase transition. We will also briefly comment on the first order transition case, a detailed discussion of that case is postponed for a future work where we will discuss the effects of inflation generated density fluctuations on electroweak baryogenesis.

It is well known that cosmic strings can produce wake like over-densities with magnitude which can be large \cite{10}. (As we will discuss below, this is also true for strings moving through a relativistic ideal fluid). We consider the universe to be at a temperature $T_b$ which is below the electroweak transition temperature $T_c$, and consider density fluctuations via string wakes. Typical wavelengths associated with such wakes will be much smaller than the horizon (for ultrarelativistic strings). The evolution of such density fluctuations is simple for small amplitudes \cite{14}; they propagate as plane waves with speed of sound $c_s = 1/\sqrt{3}$. Even for larger amplitudes, the propagating density disturbance generated by string wakes will typically retain short wavelength \cite{14}. At a given region, through which the fluctuation propagates, the density, and hence the temperature, undergoes an oscillation, with the oscillation time scale being governed by the typical wavelength of the fluctuation. As a propagating density disturbance will be like a pulse of short wavelength, the temperature in a given spatial region in its path will typically undergo one oscillation cycle with short time period.

Depending on the amplitude of temperature oscillation, and the background temperature $T_b$ of the universe, it may happen that the temperature temporarily exceeds the electroweak transition temperature $T_c$, thereby restoring the electroweak symmetry. As the temperature oscillates back to values lower than $T_c$, the symmetry will be broken again spontaneously. During this re-occurrence of the electroweak phase transition, baryon asymmetry will be generated. For a second order phase transition no baryons would have been generated when the whole universe went through the electroweak phase transition, due to extremely slow cooling of the universe \cite{2}. However, the cooling time scale, and hence the time scale of the phase transition, due to the density (temperature) fluctuation pulse will be governed by the wavelength of the fluctuation, and can be much shorter than the Hubble time scale. The baryon asymmetry generated in this case, therefore, can be enhanced by many orders of magnitude. We will show that for a $10^{16}$ GeV GUT scale cosmic string, the resulting density fluctuations can give the required value of the baryon to entropy ratio (in the context of a two Higgs doublet model as in \cite{4}, with the CP violation parameter of order 10) if the fluctuation propagates without getting significantly damped. If fluctuations are dissipated rapidly, then a volume suppression factor reduces the produced baryon asymmetry, though it is still 3-4 orders of magnitude larger than the conventional case where baryons are produced.
during the cooling of the universe via expansion.

The paper is organized in the following manner. In section II we briefly recall the relevant results for the electroweak baryogenesis for the case when the electroweak phase transition is of second order. Here, the baryon to entropy ratio is expressed in terms of \( \dot{\theta} \), the time derivative of the (relative) Higgs phase, and consequently \( \dot{T} \), where \( T \) is the temperature. Since largest baryon asymmetry will arise from the largest value of \( \dot{\theta} \), which translates to the requirement of largest value of \( \dot{T} \), it is clear that the most relevant density fluctuations in our model are those with the shortest wavelength (for a given value of the amplitude of density fluctuation). In section III, we discuss such density fluctuations as expected from cosmic strings moving through a relativistic fluid. Section IV presents our results for the baryon to entropy ratio with the value of \( \dot{T} \) governed by short wavelength density fluctuations. Section V briefly discusses the expected results when the transition is first order. Conclusions are presented in section VI.

II. BARYON TO ENTROPY RATIO WITH A SECOND ORDER ELECTROWEAK TRANSITION

We first briefly outline the results for baryon to entropy ratio \( (B/s) \) when the electroweak phase transition is of second order and the phase transition proceeds by the cooling of the universe due to its expansion. For this, we will mostly follow the discussion in ref. [2]. As discussed in ref. [15], in two-Higgs doublet extensions of the standard model, there are terms in the effective action for the gauge-Higgs sector at high temperature, which are of the form \( \sim \chi F \tilde{F} \). Here \( \chi \) is some combination of the Higgs fields, \( F \) is the SU(2) field strength tensor, and \( \tilde{F} \) is its dual. Integration by parts shows (with the use of the chiral anomaly equation) that this term is equivalent to a chemical potential term \( \dot{\chi} B \) for the baryon number, where \( B \) is the baryon number density [2,15] (see, also, ref. [16]). The value of this chemical potential \( \mu_B \) can be written as [2,15],

\[
\mu_B \equiv \dot{\chi} \simeq 7\zeta(3)\left(\frac{m_t}{\pi T}\right)^2 \frac{v_1^2}{v_1^2 + v_2^2} \dot{\theta},
\]

where \( \zeta(3) \simeq 1.2 \), and \( \theta \) is the relative phase between the two Higgs fields. \( v_1 \) and \( v_2 \) are the magnitudes of the vacuum expectation values of the two Higgs fields, and \( m_t \) is the top quark mass. In the presence of chemical potential given in Eq.(1), one can show [2] that the rate at which the baryon number relaxes to equilibrium, is given by

\[
\dot{B} = -\Gamma'(B - c_n \mu_B T^2),
\]

where \( \Gamma' = \frac{N_F \Gamma}{c_n T^3} \), and \( c_n \) is a slowly varying parameter with value remaining close to about 0.4. \( N_F \) is the number of quark generations, and \( \Gamma \) is the sphaleron rate per unit volume in the broken phase [17]. Using the Green’s function method, the solution to this equation is obtained as

\[
B(t) = \int_{t_0}^{t} \Gamma'(t') c_n \mu_B(t') T^2(t') e^{-\int_{t'}^{t} \Gamma'(t'')dt''} dt'.
\]
If we make the assumption that \( \int_{t_{\text{in}}}^{t} \Gamma' dt \gg 1 \), then the upper bound of this equation is given by the maximum of the function \( c_n \mu_B(t) T^2(t) \) which we call as \( B_0 \). The solution is then simply

\[
B(t) \leq B_0 \int_{t_{\text{in}}}^{t} \Gamma'(t') [e^{-\int_{t'}^{t} \Gamma''(t'') dt''}] dt' = B_0 \equiv |c_n \mu_B(t) T^2(t)|_{\text{max}}. \tag{4}
\]

We will use this upper limit \( B_0 \) as an estimate of the value of baryon asymmetry created. We write \( B_0 \) as follows,

\[
B_0 = |c_n \dot{\chi} T^2|_{\text{max}} = |c_n \frac{d\chi}{dT} \dot{T} T^2|_{\text{max}}. \tag{5}
\]

The value of \( (\frac{d\chi}{dT})_{\text{max}} \) will crucially depend on the details of the phase transition dynamics. Uncertainties in these estimates have been discussed in the literature \[2\]. For simple estimates, one can use,

\[
T \frac{d\chi}{dT} \approx \frac{d\phi}{dT} \epsilon \approx \epsilon, \tag{6}
\]

where \( \epsilon \) characterizes CP violation in the model, \( \phi \) being the Higgs field. Assuming order one CP violation, one can take \( T \frac{d\chi}{dT} \approx 1 \). It has been argued \[2\] that depending on the details of the specific models, this value may be larger by about two orders of magnitude. One can thus use \( \epsilon \) as a parameter, with its value ranging from 1 to about 200 (ref. \[2\]).

If the departure from equilibrium is due to the expansion of the universe, then the Friedmann equation for the radiation era can be used to relate the time derivative to the temperature derivative. In the radiation era,

\[
H(T) = \left( \frac{8\pi^3 G g_* T^4}{90} \right)^{1/2} T = \frac{1}{2t}. \tag{7}
\]

Here \( G \) is the Newton’s constant, \( H^{-1} \) is the Hubble time, and we have used the plasma energy density \( \rho = \frac{\pi^2 g_* T^4}{30} \) with \( g_* \) being the number of degrees of freedom. We get,

\[
\dot{T} = -H(T) T, \quad H(T_c) = H(T) \frac{T_c^2}{T^2}. \tag{8}
\]

Using these equations, we can write \( B_0 \) as,

\[
B_0 = c_n \epsilon H(T) T^2. \tag{9}
\]

The equation for the entropy density is given by

\[
s = \frac{2\pi^2}{45} g_* T_c^3. \tag{10}
\]

Resulting baryon to entropy ratio can now be written as,

\[
\frac{B}{s} \approx \frac{45 c_n \epsilon H(T_c)}{2\pi^2 g_* T_c}. \tag{11}
\]
For the electroweak case the value of $g_*$ is about 100. With $T_c = 110 GeV$ and $H/T_c \sim 10^{-16}$, we get,

$$\frac{B}{s} \simeq 10^{-18}\epsilon.$$  \hfill (12)

Thus, as discussed in the literature [2], even with optimistic estimates of $\epsilon \sim 200$, required baryon asymmetry cannot be generated for the case of second order transition when the transition proceeds by the cooling of the universe due to expansion.

In the next sections we will discuss the density fluctuations present in the universe. We will see that in the presence of these fluctuations, the time scale of phase transition can be smaller by several orders of magnitude, such that required baryon asymmetry may be generated with a second order transition.

### III. DENSITY FLUCTUATIONS IN THE UNIVERSE

Results for $B/s$ obtained in the previous section correspond to the situation when the universe is perfectly homogeneous, and undergoes second order phase transition everywhere as the temperature falls below $T_c$ due to the expansion of the universe. As mentioned in the Introduction, presence of density fluctuations in the universe is unavoidable, as then only structures observed today can arise. In this section we will discuss the nature of density fluctuations produced via cosmic strings moving through the cosmic fluid. For the second order transition case, density fluctuations produced via inflation will not be helpful in generating a baryon asymmetry in our model, as we will explain at the end of Sec.IV. For simplicity, we discuss the case of straight cosmic strings which give rise to wake like density fluctuations. It is important to note that for cosmic strings, another important contribution to density fluctuations comes from small loops. However, the exact nature of density fluctuations in this case is more complicated due to the fact that loops rapidly oscillate, and the issue of time dependence is of crucial importance for our results. In a more detailed investigation their contribution must be taken into account. (Segments of very large string loops, which have sizes not much smaller than the horizon, will effectively behave as straight strings).

We first discuss the evolution of such small wavelength density fluctuations [14]. It is well known that the density fluctuations in a relativistic ideal fluid which have wavelength of order, or larger, than the horizon, grow with the scale factor. However, when wavelength of the fluctuation is much smaller than the horizon, which is the relevant case for us, then the density fluctuation simply propagates as a plane wave with the speed of sound $c_s = 1/\sqrt{3}$, at least when the amplitude of density fluctuation is small [14]. Thus, such a density fluctuation will evolve as,

$$\frac{\delta \rho}{\rho} \sim A e^{i(kr-\omega t)},$$ \hfill (13)

where $A$ is the amplitude of the fluctuation, $k$ is the wave vector and $\omega (= 2\pi c_s/\lambda)$ is the angular frequency of the fluctuation with wavelength $\lambda$. Our interest will be in the evolution of density disturbances for times shorter than the Hubble time, so the effects of universe
expansion can be neglected for these fluctuations for an order of magnitude estimate. Also, the wake like density fluctuation arising from cosmic string moving through a relativistic ideal fluid (with zero chemical potential) may not lead to sustained oscillations. What one expects is that the resulting density fluctuation (hence temperature fluctuation) will propagate as a single pulse.

It is important to note here that the main factor responsible for baryon production enhancement in our model will be (as we will see below) the short time scale associated with these small scale density fluctuations compared to the Hubble time. This remains true irrespective of the fact whether these fluctuations propagate for significant distances, or they decay rapidly. If the fluctuation decays without propagating significantly, still the time scale for the temperature fluctuation will be the same as for the propagating case, as again it will be typically governed by the speed of sound $c_s$ and the length scale of the fluctuation. As we will see below, for the first case, the propagating disturbances can lead to required baryon production as they sweep the entire horizon volume. If fluctuations decay rapidly, then there will be a volume suppression factor which will lead to much smaller baryon production, though it will still be several orders of magnitude larger than the conventional case when baryons are produced only due to the overall cooling of the universe.

Once the nature of density fluctuations is known (depending on the source of density fluctuations), we can use Eq.(13) to determine the time variation of temperature resulting from such propagating density fluctuations. An important assumption we make in this regard is that temperature at any time and at any point of space, is determined by the local density $\rho$, even in the presence of short wavelength density fluctuations. This assumption of local thermal equilibrium is justified here because the wavelengths we will be considering will be about $10^{-6}$ cm, with the relevant time scale of density variation being of order $10^{-16}$ sec. This time scale, though much shorter than the Hubble time, is still many orders of magnitude larger than the time scale of relevant interactions, which keep the particles in equilibrium.

With the use of local thermal equilibrium assumption, the density $\rho$ is related to the temperature $T$ as, $\rho = \frac{\pi^2 g_*}{30} T^4$, where $g_* \simeq 100$ is the number of degrees of freedom at the electroweak scale. From this we can relate the temperature fluctuation to the density fluctuation,

$$\frac{\delta \rho}{\rho} \equiv \frac{\rho - \rho_b}{\rho_b} = \frac{T^4 - T_b^4}{T_b^4},$$

where $T_b$ is the background temperature of the plasma. Then using Eq.(13) we obtain the time - temperature relation for propagating density fluctuations.

$$\frac{T^4 - T_b^4}{T_b^4} = A e^{i(k \cdot r - \omega t)}. \quad (15)$$

Given this, we take the time dependence of the temperature at a given point in space to be given by,

$$T(t) = T_b[1 + A \sin \omega t]^{1/4}. \quad (16)$$

Thus, at $t = 0$, $T = T_b$, the background temperature.
As discussed above, although we are using the above expression for \( T(t) \) with a periodic time dependence, the propagating density fluctuation actually constitutes only a single pulse. Thus the above expression for \( T(t) \) is to be used only for a single cycle. Of course that is enough for our purpose as the baryon asymmetry is to be generated during the cooling part of the cycle, irrespective of the number of cycles.

Propagation of density fluctuation given in Eq.(13), and resulting oscillation of temperature (Eq.(16)) is valid for the case when the amplitude \( A \) is small, so the disturbance propagates as acoustic wave. For our case, density fluctuations produced in string wakes can have large amplitudes, with \( A \sim 1 \). Evolution of such density fluctuations is not as simple as given above. In fact, for a general fluctuation (i.e. with large amplitude), the propagating disturbance can lead to transfer of energy to shorter wavelengths, leading to steepening of the disturbance [14]. In our case the initial profile of the density fluctuation will be given by the wake like shock which forms behind a cosmic string moving with supersonic velocity. Also, the baryon asymmetry produced in our model will be larger for shorter wavelengths due to shorter time scale of temperature variation. Thus, a propagating disturbance even with large amplitude will lead to baryon production at least as large as given by the initial wavelength of the disturbance. This is assuming that the disturbance does not decay away very rapidly, e.g. by subsequent shock developments etc. Though, as we have discussed above, even if fluctuation decays rapidly, the typical time scale of temperature evolution will still be given by the wavelength of the fluctuations. With this in mind, we will use the temperature variation given by Eq.(16) (again, for one cycle only as appropriate for a pulse of over-density) even for the case when the amplitude \( A \) of density fluctuation is not too small, as in the case of a relativistic cosmic string wake.

A. Density Fluctuations arising due to Straight Cosmic Strings

We now give a brief review of the structure of density fluctuations produced by a cosmic string moving through a relativistic fluid. The space-time around a straight cosmic string (along the \( z \) axis) is given by the following metric [18],

\[
ds^2 = dt^2 - dz^2 - dr^2 - (1 - 4G\mu)^2 r^2 d\psi^2,
\]

where \( \mu \) is the string tension. This metric describes a conical space, with a deficit angle of \( 8\pi G\mu \). This metric can be put in the form of the Minkowski metric by defining angle \( \psi' = (1 - 4G\mu)\psi \). However, now \( \psi' \) varies between 0 and \( (1 - 4G\mu)2\pi \), that is, a wedge of opening angle \( 8\pi G\mu \) is removed from the Minkowski space, with the two boundaries of the wedge being identified. It is well known that in this space-time, two geodesics going along the opposite sides of the string, bend towards each other [19]. This results in binary images of distant objects, and can lead to planar density fluctuations. These wakes arise as the string moves through the background medium, giving rise to velocity impulse for the particles in the direction of the surface swept by the moving string. For collisionless cold dark matter particles the resulting velocity impulse is [10,20],

\[ v_{\text{impls}} \approx 4\pi G\mu v_{\text{st}}\gamma_{\text{st}} \]

(where \( v_{\text{st}} \) is the transverse velocity of the string). This leads to a wedge like region of overdensity, with the wedge angle being of order of the deficit angle, i.e. \( 8\pi G\mu (\sim 10^{-5} \text{ for } 10^{16} \text{ GUT strings}) \). The density fluctuation in the wake is of order one. Subsequent growth of this
wake by gravitational instability in the matter dominated era has been analyzed in great
detail in literature \cite{20}.

The structure of this wake is easy to see for collisionless particles (whether non-
relativistic, or relativistic). Each particle trajectory passing by the string bends by an
angle of order $4\pi G\mu$ towards the string. In the string rest frame, take the string to be at the
origin, aligned along the $z$ axis, such that the particles are moving along the $-x$ axis. Then
it is easy to see that particles coming from positive $x$ axis in the upper/lower half plane will
all be above/below the line making an angle $\mp 4\pi G\mu$ from the negative $x$ axis. This implies
that the particles will overlap in the wedge of angle $8\pi G\mu$ behind the string leading to a
wake with density twice of the background density. One thus expects a wake with half angle
$\theta_w$ and an overdensity $\delta \rho/\rho$ where \cite{10,20},

$$\theta_w \sim 4\pi G\mu, \quad \frac{\delta \rho}{\rho} \sim 1. \quad (18)$$

However, the case of relevance for us is cosmic strings moving through a relativistic
plasma of elementary particles at temperatures of order 100 GeV. At that stage, it is not
proper to take the matter as consisting of collisionless particles. A suitable description of
matter at that stage is in terms of a relativistic fluid which we will take to be an ideal fluid
consisting of elementary particles. Generation of density fluctuations due to a cosmic string
moving through a relativistic fluid has been analyzed in the literature \cite{21,22,23}. The study
in ref. \cite{21} focused on the properties of shock formed due to supersonic motion of the string
through the fluid. In the weak shock approximation, one finds a wake of overdensity behind
the string. In this treatment one can not get very strong shocks with large overdensities. In
refs. \cite{22,23}, a general relativistic treatment of the shock was given which is also applicable
for ultra-relativistic string velocities. The treatment in ref. \cite{23} is more complete in the
sense that the equations of motion of a relativistic fluid are solved in the string space-
time (Eq. (17)), and both subsonic and supersonic flows are analyzed. One finds that for
supersonic flow, a shock develops behind the string, just as in the study of ref. \cite{21,22}. In
the treatment of ref. \cite{23} one recovers the usual wake structure of overdensity (with the
wake angle being of order $G\mu$) as the string approaches ultra-relativistic velocities. Also the
overdensity becomes of order one in this regime.

We mention here that it is not expected that the string will move with ultra-relativistic
velocities in the early universe. Various simulations have shown \cite{24} that rms velocity of
string segments is about 0.6 for which the shock will be weak. For this case, the opening angle
of the wedge $\theta_w$ will also be large. This will then imply a large wavelength for the density
fluctuation, and consecutively, small value of $\dot{T}$ and small baryon to entropy ratio. It is still
possible that various segments of string may move with ultra-relativistic velocities. (Note
that the friction dominated regime for GUT strings ends long before these temperatures are
reached \cite{25}.) We will take this to be the case, and will consider the situation when string
produces strong shock, with a small wedge angle.

For ultra-relativistic case, we use expressions from ref. \cite{23}. Resulting density fluctuation
in the wake of the moving string is expressed in terms of fluid and sound four velocities,

$$\frac{\delta \rho}{\rho} \simeq \frac{16\pi G\mu u_f^2 (1 + u_s^2)}{3u_s \sqrt{u_f^2 - u_s^2}}, \quad sin \theta_w \simeq \frac{u_s}{u_f}, \quad (19)$$

$$\text{where } u_s = \text{the sound velocity, and } u_f = \text{the fluid four velocity.}$$
where $u_f = v_f / \sqrt{1 - v_f^2}$ and $u_s = c_s / \sqrt{1 - c_s^2}$. In this case, when string velocity $v_f$ is ultra-relativistic, then one can get strong overdensities (of order 1) and the angle of the wake approaches the deficit angle $\simeq 8\pi G\mu$. This is the structure of wake which is also expected for a collision-less matter as discussed above. We will assume that this is the case, and hence, take the properties of the shock as given by Eq.(18). The density fluctuations resulting from (straight) cosmic strings, as characterized by Eq.(18), have the amplitude $A$ of order 1, and average wavelength $\lambda$ of order $(1/2)(8\pi G\mu d_H)$ where $d_H$ is the horizon size. For a GUT scale cosmic string with $8\pi G\mu \sim 10^{-5}$, $\lambda$ is of order $10^{-6}$ cm. Note that this value of $\lambda$ is much larger than the diffusion length scales $l_{diff}$ of various particles at these temperatures which is of order $10^{-14}$ cm [26]. Hence these fluctuations will not be dissipated by diffusion processes.

IV. BARYON TO ENTROPY RATIO DUE TO DENSITY FLUCTUATIONS

In the previous section, we have determined the time variation of the temperature resulting from density fluctuations. From Eqs.(5),(6), and (16), we can write,

$$B_0 = (c_n\epsilon T \dot{T})_{max} = \frac{c_n\epsilon A \omega T_b^4}{4T^2}. \quad (20)$$

This estimate of maximum baryon asymmetry has been obtained with the optimistic assumption that sphaleron processes turn off when the source of baryon number has optimum value [2]. As the sphaleron rate changes by many orders of magnitude when $T$ decreases below $T_c$, relevant value of $T$ in the above equation will be close to $T_c$. For rough estimates, we will take $T \sim T_b$ (note that $T_b$ is of same order as $T_c$ in our model as we want density fluctuations to be able to restore the electroweak symmetry locally). With this, Eq.(20) is obtained with maximum value of $\cos \omega t = 1$. The resulting baryon to entropy ratio can now be expressed in terms of the amplitude $A$, and the wavelength $\lambda$ of the density fluctuation as,

$$\frac{B}{s} = \frac{45c_n c_s A}{\pi g_\ast} \frac{A}{4\lambda T_b \epsilon} \simeq \frac{0.01 \epsilon A}{\lambda T_b}. \quad (21)$$

Here we have used $g_\ast = 100$.

For density fluctuations generated due to cosmic string wakes, the amplitude $A \sim 1$, and the shortest wavelength is given by the average width of the generated wake $\sim \frac{1}{2} 8\pi G\mu d_H$ (since the wake has the shape of a wedge) where $d_H$ is the horizon size at that time. Since $d_H$ is of the order of 0.1 cm at the time of the electroweak scale, and $8\pi G\mu \sim 10^{-5}$ for a GUT string, we get $\lambda_{smc} \simeq 10^{-6} cm$. The resulting value of baryon to entropy ratio from Eq.(21) is $B/s \simeq 10^{-11} \epsilon$. We see that for the case of cosmic string, with $\epsilon$ of order 10, one is able to get the required baryon asymmetry. (We mention here that with the amplitude of density fluctuations of order one, $T_b$ can be much lower than $T_c$ in the cosmic string case. Thus, there may be possibility of having a weaker bound on the Higgs mass which is needed to avoid sphaleron wash out of the created baryon asymmetry, see ref. [5].)
It is important to realize that, if fluctuations propagate without getting significantly damped, then essentially all regions will participate in this generation of baryon asymmetry, so there is no volume suppression factor here. For example, the structure of density fluctuations here is in form of planar sheets (wakes). These planar density fluctuations will propagate in the direction normal to the plane, and sweep the entire horizon volume. At any given point within the horizon volume, the temperature will undergo an oscillation as given by Eq.(16). There may be some horizon volumes which will not have any strings present to generate significant density fluctuations at the relevant stage. However, the probability of occurrence of such regions will be small, and hence the order of magnitude estimate of the baryon to entropy ratio as given above will not be affected. There will be some change in the wavelengths of these fluctuations during the time they propagate across the entire horizon, but again, one will expect it not to affect the final numbers within an order of magnitude. Some level of dissipation of density fluctuations, while they propagate, may be compensated by the fact that there will be multiple sources of density fluctuations in a given horizon volume. For example, string simulations have shown [27] that the number of long strings in a given horizon volume is expected to be about 15. Each of these strings will lead to wakes of density fluctuations, which will sweep the horizon volume. This may also compensate for the effects of shorter wakes of a given strings (possibly getting truncated behind the string due to other strings/fluctuations). It is also possible that the baryon asymmetry generated due to one string may first get wiped out when temperature there rises above $T_c$ due to another string wake. In that region, the surviving baryon asymmetry will correspond to the density fluctuations which are last to pass by. Eventually, the temperature in that region will be sufficiently low so that any subsequent cosmic string wakes are not able to re-heat the region enough to destroy the already created baryon asymmetry. This will be the stage when baryon asymmetry will get frozen.

From above discussion we see that the most important factor responsible for the enhancement of baryon to entropy ratio in our model is the short time scale of these density fluctuations. In addition, propagation of these over-density pulses leads to baryon production in almost entire region (say in a given horizon volume). That is, there is no volume suppression factor here. We now discuss the possibility if these fluctuations do not propagate for large distances, and decay very rapidly. In such a situation, to make a conservative estimate, we can take the over-dense wake of each string as a fixed region in space where the overdensity will dissipate typically with speed of sound $c_s$. This means that the time scale of temperature variation in this wake will again be the same as discussed above. From this we conclude that the resulting baryon to entropy ratio in this case will be suppressed by a volume factor $f_s$ compared to the value given in Eq.(21). $f_s$ will be given by the ratio of the total volume inside the wakes of all strings in a given horizon volume to the horizon volume. Taking account of about 10 long strings in a horizon volume, with typical wake thickness of order $10^{-5}d_H$, we get,

$$f_s = \frac{10 \times 10^{-5}d_H \times d_H^2}{d_H^3} \simeq 10^{-4}$$

With various factors taken as earlier, we get net $B/s \simeq 10^{-15}\epsilon$ for this case. With optimistic value of $\epsilon \sim 100$, resulting value of $B/s$ is $10^{-13}$, about three orders of magnitude smaller than the required one. There may be a possibility of further enhancement in this
value by changing various parameters of cosmic string network e.g. string scale and string velocity (which determine the parameters of string wakes), as well as by properly taking into account the effect of density fluctuations produced by all strings at that stage, i.e. including contributions of all string loops as well, integrated over a suitable duration of time. The main point we would like to make is that even in this case where density fluctuations do not propagate, our model leads to the enhancement of baryon to entropy ratio by at least 3 - 4 orders of magnitude compared to the case when baryogenesis happens only due to the overall cooling of the universe.

An important assumption in deriving equations such as Eq.(5),(6) (using which Eq.(20) has been derived) is that at any stage during the variation of the temperature, the value of the Higgs field is essentially given by the vacuum expectation value (vev) at that temperature. This assumption requires that the time scale of temperature variation should be much larger than the typical time scale of the Higgs field evolution. More importantly, as discussed above, as the temperature rises above $T_c$ (Eq.(16)), the Higgs field should settle down to value zero so that the electroweak symmetry is restored. This is very important as during the rise of temperature, one will get production of antibaryons via above equations (as sign of $\dot{\chi}$ will be reversed during this stage). However, if the electroweak symmetry is restored after that, then all these net antibaryon asymmetry will be erased due to unsuppressed sphaleron processes in the symmetric phase. Thus, only the net baryon asymmetry will remain which will be generated during the stage when the temperature is decreasing.

For checking this requirement on the time scale of temperature variation, we take a simple effective potential to model the second order phase transition (as in ref [28]),

$$V(\phi, T) = D(T^2 - T_c^2)\phi^2 + \frac{\lambda}{4}\phi^4.$$ (23)

We take the values of the parameters as suitable for the Standard model [28], $T_c = 110$ GeV, $D \simeq 0.18$ and $\lambda \simeq 0.1$. The evolution equation of $\phi$ (neglecting spatial dependence) is,

$$\ddot{\phi} + \frac{3}{2} \frac{\dot{\phi}}{t} + V'(\phi, T(t)) = 0,$$ (24)

where the time dependence of $T$ in $V$ is given by Eq.(16). Using this equation, we have checked the evolution of $\phi$ for one oscillation period, starting from the initial time $t = 0$ in Eq.(16), which corresponds to $T = T_b$ (the background temperature) initially. Top figure in Fig.1 shows the time variation of the temperature resulting from cosmic string wakes. Here we have taken $T_b = 100$ GeV, $A = 0.8$ in Eq.(16), and $\omega$ corresponds to $\lambda = 4\pi G_\mu d H \sim 10^{-6}$ cm. The values of $T_b$ and $A$ are taken as sample values such that the maximum value of $T$ (Eq.(16)) reaches slightly above $T_c = 110$GeV. Bottom figure in Fig.1 shows plot of the evolution of the Higgs field $\phi$ (solid line), and the vev of $\phi$ (shown by solid dots at few values of $t$). Note that the plot of $\Delta T/T$ deviates from purely sinusoidal behavior due to large value of $A$.

From Fig.(1) we see that $\phi$ traces the evolution of the vev faithfully. For time duration when $T > T_c$, $\phi$ settles down to the value zero, thereby restoring the electroweak symmetry. This restoration of symmetry happens for the values of $t$ between $0.9 \times 10^7$ to $3.5 \times 10^7$ GeV$^{-1}$. 

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As we mentioned above, this temporary symmetry restoration is necessary to wipe out the antibaryons which would have been generated during the rising phase of the temperature. It is also important to note from these figures that the time duration for which the symmetric phase lasts (i.e. when $T > T_c$) is of the order of $10^7 GeV^{-1}$, which is much larger than the typical time scale of a sphaleron transition in the symmetric phase [29], which is roughly of order $\sim (\alpha_w^2 T)^{-1} \approx 10 GeV^{-1}$. (Spatial extent of regions with density fluctuation is also many orders of magnitude larger than the sphaleron size $\sim (\alpha_w T)^{-1}$.) Thus, equilibrium calculations used in this paper make sense, sphaleron transitions during the time when $T > T_c$, should be able to erase any antibaryon asymmetry generated earlier during the rise of temperature.

We now comment on the density fluctuations generated via inflation. Generic prediction of most inflationary models [30] is that the resulting density fluctuations are scale invariant, with a magnitude of about $10^{-4}$ (when the fluctuation re-enters the horizon), as constrained by COBE observations of temperature fluctuations $\delta T / T \sim 10^{-5}$. In the context of our model, inflationary density fluctuations may contribute to baryon production only if such small temperature increase can lead to significant change in the baryon violation rates via sphaleron transitions. This requires that the background temperature of the universe $T_b$ be very close to the electroweak transition temperature $T_c$. However, in such a situations the produced baryon asymmetry will be erased as the sphaleron transitions at $T = T_b$ will not be much suppressed. Basically, for the scales involved, a fractional temperature difference of order $10^{-5}$ is unlikely to lead to large change in sphaleron rates. However, the situation is entirely different or the case of a first order transition, where even very small temperature differences can lead to significant changes in the bubble nucleation probability. We briefly discuss it in the next section.

V. FIRST ORDER TRANSITION CASE

Our conclusions about the second order transition should be applicable in the case of a very weak first order transition (or a cross-over). For example, after a rapid production of bubbles of the broken phase, the bubbles will collide and the transition will be completed. Presence of propagating density fluctuations will again reheat these regions temporarily, restoring the electroweak symmetry. Subsequent cooling will then produce a net baryon asymmetry as described above, with the rate of change of temperature governed by the oscillation frequency of the density fluctuation.

If the first order transition is not very weak, then the effect of density fluctuations should manifest in entirely different manner. Now, the presence of density (and hence) temperature inhomogeneities will split the region into cold regions and hot regions (at a given time). Geometry of such regions will depend on the source of density fluctuations. For example, for inflationary density fluctuations, such regions can be taken as localized overdense, or underdense regions, as discussed in ref. [7]. Phase transition (say, via bubble nucleation) will first happen in the colder regions, forming large interfaces which then extend into the hotter regions. Thus, even for the case of inflationary density fluctuations, the process of electroweak baryogenesis may get affected significantly. For example, it is possible that even for the case of a weak first order transition, different temperature zones may lead
to propagating interfaces emanating from the colder regions and extending into the hotter regions. This in effect may lead to a picture similar to that of a strong first order transition which is one of the requirements for a successful baryogenesis model.

Also, the propagation of the interface will reheat the surrounding region. Due to temperature variations, density of baryons produced in this case may then vary from cold regions to the hot regions. Similarly, for the case of cosmic strings, wedge shaped regions will form where the temperature will be higher than the surrounding (as discussed in ref. [8]). Bubble nucleation and coalescence may happen in the colder regions first, with the interface then extending into the planar hotter regions. This may lead to variations in the baryon density of planar nature. It will be very interesting to find whether any such baryon inhomogeneities can survive until nucleosynthesis to affect elemental abundances.

VI. CONCLUSION

We have studied the implications of small wavelength density fluctuations produced by cosmic strings on electroweak baryogenesis. We have shown that due to rapid changes in the temperature arising from short wavelength density fluctuations, electroweak symmetry may get restored locally, depending on the amplitude of temperature oscillation and the background temperature $T_b$ of the universe. As the temperature oscillates back to a value below the transition temperature, electroweak symmetry will be broken again spontaneously. During this re-occurrence of the electroweak phase transition, baryon asymmetry will be generated. The cooling time scale, and hence the time scale of the phase transition, due to such density (temperature) fluctuation waves will be governed by the wavelength of the fluctuation, and can be much shorter than the Hubble time scale. The baryon asymmetry generated in this case is therefore enhanced by many orders of magnitude. If these short wavelength density fluctuations propagate for significant distances without decaying, then baryogenesis happens in every region of space, as different regions are swept by the propagating over-density (hence temperature) pulse. We have shown that for a $10^{16}$ GeV GUT scale cosmic string, the resulting density fluctuations can give the required value of the baryon to entropy ratio (in the context of a two Higgs doublet model as in [2], with the CP violation parameter of about 10). On the other hand, if density fluctuations decay away rapidly, then there is a volume suppression factor, and the resulting baryon asymmetry is smaller by a factor of order $10^{-4}$ (though it is still larger by at least 3 - 4 orders of magnitude compared to the conventional case where baryons are produced when the universe cools via expansion).

Many aspects of our model need to be worked out in more detail. Exact nature of propagation and decay of these density perturbations needs to be worked out. We have used order of magnitude estimates for the chemical potential for baryons (Eq.(6)), more careful estimates need to be worked out. Further, the baryon asymmetry should be calculated by taking account of several propagating density fluctuation waves. Net asymmetry should be calculated by detailed accounting for the generation of antibaryon asymmetry during the period when $T$ rises, its diffusion outside the region of density fluctuation, and the decay of this antibaryon asymmetry when $T \geq T_c$ so that the sphaleron transitions are unsuppressed. Finally, the first order transition case may show many interesting possibilities, especially for
the case of inflationary density fluctuations.

Similar calculations, as reported in this paper, can be done for other sources of density fluctuations, such as those generated by textures etc. We expect similar results for the baryon asymmetry in those cases. For example, for the case of textures, collapsing textures (at any stage) will give rise to small scale density fluctuations, which will then propagate as density waves. The amplitude of fluctuations may be high for the case of textures also since the relevant fluctuations will be generated during the last stages of texture collapse, though the volume factor will lead to an overall suppression of baryon production.

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Top figure shows time variation of the temperature $T$ (in GeV) for one oscillation period, resulting from density fluctuations with wavelength $\lambda \sim 4\pi G\mu d H$. Time $t$ is in $GeV^{-1}$. Bottom figure shows the evolution of the Higgs field $\phi$ (solid line), and its vev (dots), both in GeV.

FIG. 1.