Fine structure generation in double-diffusive system

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Double-diffusive convection in a horizontally infinite layer of a unit height in a large Rayleigh numbers limit is considered. From linear stability analysis it is shown, that the convection tends to have a form of travelling tall thin rolls with width about 30 times less than height. Amplitude equations of ABC type for vertical variations of amplitude of these rolls and mean values of diffusive components are derived. As a result of its numerical simulation it is shown, that for a wide variety of parameters considered ABC system have solutions, known as diffusive chaos, which can be useful for explanation of fine structure generation in some important oceanographical systems like thermohaline staircases.

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I. INTRODUCTION

Double-diffusive or thermohaline convection plays important role in a heat-mass transfer processes in the ocean. It also essentially influences on different small scale processes, like formation of vertical temperature and salinity fine structure. Such phenomena are not well understood at present day. There are only few works, devoted to analytical models of fine structure generation in the sea. But no one of them essentially considers the role of double diffusion in such processes. As an exception one can mention work, where double-diffusive step-like fine structure is simulated by numerical Monte Carlo methods. The purpose of this article is to develop one more mathematical model of two dimensional double-diffusive convection in a horizontally infinite layer, based on system of amplitude equations, describing formation of vertical fine structure, which in some aspects resembles actual experimental and observational data. The main idea of this work consists in combining the result, that in the limit of large Rayleigh numbers convective cells tend to be narrow and tall, with constructing of ABC-system of amplitude equations with respect to vertical coordinate in the case of such cells. So this article includes two main sections apart from this one. In the section from linear stability problem we determine sizes of the most prominent cells in large Rayleigh numbers limit. In the section in weakly nonlinear approximation by multi-scale decomposition technique we derive the ABC-system. Its numerical simulation gives vertical fine structure via so called diffusive chaos solutions.

The initial equations describe two-dimensional thermohaline convection in a liquid layer of thickness $h$, bounded by two infinite plane horizontal boundaries. The liquid moves in a vertical plane and the motion is described by the stream function $\psi(t, x, z)$. The horizontal $x$ and vertical $z$ space variables are used; the time is denoted by $t$. It is assumed, that there are no distributed sources of heat and salt, and on the upper and lower boundaries of the area these quantities have constant values. Hence, basic distribution of temperature and salinity is linear along the vertical and is not depend on time. The variables $\theta(t, x, z)$ and $\xi(t, x, z)$ describe variations in the temperature and salinity about this main distribution. There are two types of thermohaline convection: the fingering, in which the warmer and more saline liquid is at the upper boundary of the area, and the diffusive type, in which the temperature and salinity are greater at the lower boundary. In this paper we study the later case.

The governing equations in the Boussinesq approximation in dimensionless form are a system of nonlinear equations in first order partial derivatives with respect to time, that depend on four parameters: the Prandtl number $\sigma$ (usual value - 7.0), the Lewis number $\tau$ (0 $< \tau < 1$, usually 0.01 - 0.1), and the temperature $R_T$ and salinity $R_S$ Rayleigh numbers:

$$
(\partial_t - \sigma \Delta) \psi + \sigma (R_S \partial_z \xi - R_T \partial_z \theta) = J(\Delta \psi, \psi), \\
(\partial_t - \tau \Delta) \xi - \partial_z \psi = J(\theta, \psi), \\
(\partial_t - \tau \Delta) \xi - \partial_z \psi = J(\xi, \psi).
$$

(1)

Here the Jacobian $J(f, g) = \partial_x f \partial_z g - \partial_x g \partial_z f$ is introduced. First equation in describes liquid particle pulse evolution in terms of stream function, second and third ones describe temperature and salt diffusion respectively. The boundary conditions for the dependent variables are chosen to be zero, which implies that the temperature and salinity at the boundaries of the area are constants, the vorticity vanishes at the boundaries, and the boundaries are impermeable:

$$
\psi = \partial_z^2 \psi = \theta = \xi = 0 \text{ on } z = 0, 1.
$$

These boundary conditions are usually called free-slip conditions because the horizontal velocity component at the boundary does not vanish.
As a space scale the thickness of the liquid layer $h$ is used. As a time scale value $t_0 = h^2/\chi$ is used, where $\chi$ is the thermal diffusivity of the liquid. Velocity field components are determined as $v_x = (\chi/h)\partial_x \psi$ and $v_z = - (\chi/h)\partial_z \psi$. For temperature $T$ and salinity $S$ we have relations:

$$T(t, x, z) = T_- + \delta T \left[ 1 - z + \theta(t, x, z) \right],$$

$$S(t, x, z) = S_- + \delta S \left[ 1 - z + \xi(t, x, z) \right].$$

Here $\delta T = T_+ - T_-$, $\delta S = S_+ - S_-$, where $T_+$, $T_-$ and $S_+$, $S_-$ are the temperatures and salinities on the lower and upper boundaries of the region, respectively. The temperature and salinity Rayleigh numbers can be expressed as follows:

$$R_T = \frac{g\alpha h^3}{\chi \nu} \delta T, \quad R_S = \frac{g\gamma h^3}{\chi \nu} \delta S,$$

where $g$ is the acceleration of gravity, $\nu$ is the viscosity of the liquid, $\alpha'$ and $\gamma'$ are the temperature and salinity coefficients of volume expansions.

II. FORM OF CONVECTIVE CELLS AT LARGE RAYLEIGH NUMBERS

Consider theromhaline convection in a limit of large $R_S$, which is true for the most of oceanographically important applications ($R_S \approx 10^9 - 10^{12}$). After rescaling of the time $t = (\sigma R_S)^{-1/2} \tau$, and the stream function $\psi = (\sigma R_S)^{1/2} \psi'$, we can rewrite basic system (1) in a singularly disturbed form (primes are omitted):

$$(\partial_t - \sigma \varepsilon^2 \Delta) \Delta \psi + (\partial_x \xi - (1 - N^2) \partial_z \theta) = J(\Delta \psi, \psi),$$

$$(\partial_t - \varepsilon^2 \Delta) \theta - \partial_z \psi = J(\theta, \psi),$$

$$(\partial_t - \tau \varepsilon^2 \Delta) \xi - \partial_x \psi = J(\xi, \psi),$$

Here a small parameter $\varepsilon^4 = 1/\sigma R_S$ and a buoyancy frequency $N^2 = 1 - R_T/R_S$ are introduced. In this system singular perturbations are present as $\varepsilon^2$ before Laplacians. If we let $\varepsilon = 0$, then our system (3) turns into common equations, describing two-dimensional internal waves with the constant buoyancy frequency $N$ in the Boussinesq approximation.

For investigating of a linear stability problem for the system (3) with boundary conditions (2) we omit nonlinear terms in the right part of the system and choose a solution in a form of normal mode:

$$\psi(x, z, t) = Ae^{\lambda t - ikx} \sin n \pi z,$$

$$\theta(x, z, t) = a_T e^{\lambda t - ikx} \sin n \pi z,$$

$$\xi(x, z, t) = a_\xi e^{\lambda t - ikx} \sin n \pi z,$$

where $\lambda$ is an eigen value, describing growth rate of the mode, $k$ is a horizontal wave number, $n$ is a number of the mode and $A$ is an amplitude of the mode. After substitution of the expressions (4) into the system (3) we get a system of algebraic equations with solvability condition, having a form of a third order polynomial with respect to $\lambda$:

$$(\lambda + \sigma \varepsilon^2 \Delta)(\lambda + \varepsilon^2 \Delta)(\lambda + \tau \varepsilon^2 \Delta) + \frac{k^2 N^2}{\varepsilon^2} (\lambda + \gamma \varepsilon^2 \Delta) = 0. \quad (5)$$

Here $\Delta = k^2 + n^2 \pi^2$ is a full wave number and $\gamma$ is a constant: $\gamma = \tau + (1 - \tau)/N^2$. Equation (5) is known as dispersive relation and has three roots, two of which can be complex conjugates for a sufficiently small value of $\varepsilon$.

In the later case a Hopf bifurcation take place when at some values of $N$ and $\varepsilon$ real part of the complex conjugates roots turns to zero. It is true when

$$\varepsilon^4 < \frac{k^2 \Delta^3}{\tau^2 N^6} \left( \frac{1 - \tau}{1 + \sigma} \right),$$

$$N_*^2 = \frac{1 - \tau}{1 + \sigma} - \varepsilon^4 \Delta^3 \left( \sigma + \tau(1 + \tau + \sigma) \right),$$

$$\omega^2 = \frac{k^2 \Delta}{\varepsilon^2} \left( \frac{1 - \tau}{1 + \sigma} \right) - \varepsilon^4 \Delta^3.$$

where $\omega = \text{Im}(\lambda)$ is a Hopf frequency.

Because dispersive relation (5) explicitly contains small parameter $\varepsilon^4$, we can choose one of the complex conjugates roots and express $\lambda$ in the form of an asymptotic expansion by the powers of $\varepsilon$:

$$\lambda = \lambda_0 + \varepsilon^2 \lambda_1 + \varepsilon^4 \lambda_2 + \varepsilon^6 \lambda_3 + \cdots.$$

After substitution of this expression in (5) we have for $\lambda_i$:

$$\lambda_0^2 = -(k^2/\Delta^2) N^2, \quad \lambda_1 = \Delta F_1,$$

$$\lambda_2 = -(\Delta^2/\lambda_0) F_2, \quad \lambda_3 = -\Delta^4/(k^2 N^2) F_3,$$

$$F_1 = (\gamma - C_1)/2 > 0,$$

$$F_2 = (3F_1^2 + 2C_1 F_1 + C_2)/2 > 0,$$

$$F_3 = 4F_1^3 + 4C_1 F_1^2 + (C_1^2 + C_2) F_1 + (\tau + \sigma)(C_1 + \tau C_2)/2 > 0,$$

where constants are: $C_1 = 1 + \tau + \sigma$ and $C_2 = \tau + \sigma + \tau \sigma$. It is interesting to note, that when $N_* > N > 0$ functions $F_1, F_2, F_3$ are positive. The growth rate, caused by thermohaline convective instability can be written as follows (11):

$$\text{Re}(\lambda) = \varepsilon^2 \Delta F_1 - \varepsilon^6 \frac{\Delta^3}{k^2 N^2} F_3 + \cdots. \quad (6)$$

One can see (FIG. 1), that for a given mode with number $n$, the growth rate is maximal for some $k$, which determines horizontal size of the most prominent convective cells. Also the first convective mode has maximal growth rate, so that convective cells tend to be tall and thin. Really about 30 first modes for $N = 0.3$ have positive growth rate (FIG. 2), but the most prominent cells any way will be tall and thin, as having relatively more large
rate of growth. For simplicity further we consider only the first convective mode.

Although above developed perturbation approach gives qualitatively true estimates, for more accurate results one should immediately solve algebraic equation (5). Rewrite it in another form, with new introduced variables $P^2 = \varepsilon^2 \kappa^2 \approx \varepsilon^2 k^2$, $X = \lambda/P^2$ and $Y = N^2/P^4$.

$$(X + \sigma)(X + 1)(X + \tau) + Y(X + \gamma) = 0.$$ Roots of this equation depend on parameter $Y$, so that finally $\lambda$ depends on horizontal wave number $P$.

Consider actual oceanographical system such as an inversion of thermohaline staircase. Let it has thickness $h = 250$ cm and temperature difference $\delta T = 0.1^\circ C$, also $\sigma = 7$ and $\tau = 1/81$. In this case $\varepsilon = 1.53 \times 10^{-3}$ and non-dimensional critical buoyancy frequency $N_c = 35136$. For $N = 0.2764$ the most unstable mode has $P_c = 0.1599579$ and width of convective cell $l_c = \pi \varepsilon h/P = 7.7$ cm. For comparison formula (6) gives $P_c = 0.126$, i.e. somewhat less than the exact value. From (6) one can extract dependence of $P_c$ from $N$, having form $P_c = [N^2 F_1/(3F_3)]^{1/4}$. From picture (FIG. 3) one can easily see, that when value of $N$ becomes slightly small than its critical value $N_c \approx 35136$, value of $P_c$ abruptly (as $(N_c - N)^{1/4}$) increases and becomes maximal at $N \approx 0.3$. When $N$ becomes even more less $P_c$ decreases to $P_c \approx 0.13688$ for $N = 0$. It should be emphasized that $P_c$ is nearly independent of $N$ in considered case.

This result for thermohaline convection at large $R_s$ is sufficiently different from that for small $R_s$, when critical wave number is $k_c = \pi/\sqrt{2}$ [12]. For our case typical wave numbers are of the order $0.1/\varepsilon$. This estimate is more accurate, than mentioned in [1]. Thus convective cells for large $R_s$ have tall and thin geometry from linear stability analysis. Physically one can understand this effect from consideration that when Rayleigh numbers are large, the buoyancy forces acting vertically are also large in comparison with forces of inertia of liquid particles determining width of cells.

### III. FINE STRUCTURE GENERATION AND DIFFUSIVE CHAOS

Now we will study nonlinear vertical modulations of amplitude of our tall thin convective cells.

At first, introduce new small parameter, extracted from geometry of the convective cells: $e = l_c/h = \pi \varepsilon h/P_c \approx 20\varepsilon$. Rescale variables $\psi = e^2 \psi'$, $\theta = e\theta'$, $\xi = e\xi'$ (prime will be omitted), and introduce one more small parameter $E = \varepsilon/e \approx 1/20$. After changing of the space scale from $h$ to $l_c$ basic system appears in the form:

\begin{align*}
(\partial_t - \sigma E^2 \Delta) \Delta \psi + (\partial_t \xi - (1 - N^2) \partial_x \theta) = J(\Delta \psi, \psi), \\
(\partial_t - E^2 \Delta) \theta - \partial_x \psi = J(\theta, \psi), \\
(\partial_t - \tau E^2 \Delta) \xi - \partial_x \psi = J(\xi, \psi).
\end{align*}

FIG. 2: Grows rate $\text{Re}(\lambda(k, n))$ for convective modes with different mode number $n$. All parameters are the same as in the FIG. [a]
At second, introduce slow vertical variable $Z = ez$ and slow time $T = e^2t$. In accordance with multi-scale decomposition technique we get prolonged derivatives 

\[
\begin{align*}
\partial_z & \to e\partial_Z, \\
\partial_t & \to \partial_t + e^2\partial_T, \\
\Delta & \to \partial_z^2 + e^2\partial_Z^2, \\
\Delta^2 & \to \partial_z^2 + 2e^2\partial_z^2 + e^4\partial_z^4, \\
\partial_t\Delta & \to \partial_t\partial_z^2 + e^2\partial_T\partial_z^2 + e^2\partial_z^2\partial_Z^2.
\end{align*}
\]

Let buoyancy frequency somewhat less than its critical value $N^2 = N_e^2 - e^2 R$. Parameter $R$ here is a forcing of the system. Equations (7) now get a form:

\[
\begin{align*}
\left(\partial_t - \sigma E^2\partial_z^2\right)\psi_1 & + (\partial_z\xi - (1 - N^2_e)\partial_z\theta) = \\
& - eJZ(\psi, \partial_z^2\psi) - e^2(\partial_T\partial_z^2 + \partial_z\partial_Z^2) \\
& - 2e^2\sigma E^2\partial_z^2\psi - R\partial_z\theta,
\end{align*}
\]

\[
\begin{align*}
\left(\partial_t - E^2\partial_z^2\right)\theta & - \partial_z\psi = \\
& - eJZ(\psi, \theta) - e^2(\partial_T - E^2\partial_Z^2)\theta,
\end{align*}
\]

\[
\begin{align*}
\left(\partial_t - \tau E^2\partial_z^2\right)\xi & - \partial_z\psi = \\
& - eJZ(\psi, \xi) - e^2(\partial_T - \tau E^2\partial_Z^2)\xi.
\end{align*}
\]

Solutions of these equations we will find as the asymptotic sets by powers of the small parameter $e$:

\[
\begin{align*}
\psi & = e\psi_1 + e^2\psi_2 + e^3\psi_3 + \cdots \\
\theta & = e\theta_1 + e^2\theta_2 + e^3\theta_3 + \cdots \\
\xi & = e\xi_1 + e^2\xi_2 + e^3\xi_3 + \cdots.
\end{align*}
\]

After substitution of these expressions into equations (7) collect terms at the same powers of $e$. As a result we have systems of equations for determining of the terms of the sets (8). Thus, at $e^1$ we have following system:

\[
\begin{align*}
\left(\partial_t - \sigma E^2\partial_z^2\right)\partial_z\psi_1 & + (\partial_z\xi_1 - (1 - N_e^2)\partial_z\theta_1) = 0,
\end{align*}
\]

\[
\begin{align*}
\left(\partial_t - E^2\partial_z^2\right)\partial_z\psi_1 & + (\partial_z\xi_1 - (1 - N_e^2)\partial_z\theta_1) = 0,
\end{align*}
\]

\[
\begin{align*}
\left(\partial_t - \tau E^2\partial_z^2\right)\xi_1 & - \partial_z\psi_1 = 0.
\end{align*}
\]

Choose for this system solution in the form of normal convective mode travelling to the right, with constants of integration $B(T, Z)$ and $C(T, Z)$, depending on slow variables.

\[
\begin{align*}
\psi_1 & = A(T, Z)e^{i\omega t - iKx} + c.c., \\
\theta_1 & = \sigma A(T, Z)e^{i\omega t - iKx} + B(T, Z) + c.c., \\
\xi_1 & = \sigma B(T, Z)e^{i\omega t - iKx} + C(T, Z) + c.c.
\end{align*}
\]

Here wave number $K = K_e(N)$ is a horizontal wave number, corresponding to the most unstable waves of convection, and maximal value of $K = \pi$ from choice of the space scale, related with convective cells. It is attained when $N \approx 0.3$. Parameters of the normal mode (9) are related as follows:

\[
\begin{align*}
\sigma a_T = -\frac{iK}{i\omega + E^2K^2}, \\
\sigma a_S = -\frac{iK}{i\omega + \tau E^2K^2},
\end{align*}
\]

\[
\begin{align*}
(\omega + \sigma E^2K^2)(i\omega + E^2K^2)(i\omega + \tau E^2K^2) & + N^2(i\omega + \gamma E^2K^2) = 0.
\end{align*}
\]

System of equations at $e^2$ is the same as (10) and does not lead to any new results. System at $e^3$ is:

\[
\begin{align*}
\left(\partial_t - \sigma E^2\partial_z^2\right)\partial_z^2\psi & + (\partial_z\xi - (1 - N_e^2)\partial_z\theta) = \\
& - eJZ(\psi, \partial_z^2\psi) - e^2(\partial_T\partial_z^2 + \partial_z\partial_Z^2) \\
& - 2e^2\sigma E^2\partial_z^2\psi - R\partial_z\theta,
\end{align*}
\]

\[
\begin{align*}
\left(\partial_t - E^2\partial_z^2\right)\theta & - \partial_z\psi = \\
& - eJZ(\psi, \theta) - e^2(\partial_T - E^2\partial_Z^2)\theta,
\end{align*}
\]

\[
\begin{align*}
\left(\partial_t - \tau E^2\partial_z^2\right)\xi & - \partial_z\psi = \\
& - eJZ(\psi, \xi) - e^2(\partial_T - \tau E^2\partial_Z^2)\xi.
\end{align*}
\]

After substitution into the right parts of these equations expressions (11) we get a system with resonating right parts, breaking regularity of the asymptotic expansions (12). The condition of the absence of secular terms in this case takes form of so called ABC system (13) (intermediate calculations are omitted):

\[
\begin{align*}
\partial_t A & = E^2\beta_1\partial_z^2 A + R\beta_2 A - \beta_3 A\partial_z^2 B + \beta_4 A\partial_z^2 C, \\
\partial_t B & = E^2\partial_z^2 B - E^2\beta_5\partial_z|A|^2, \\
\partial_t C & = \tau E^2\partial_z^2 C - \tau E^2\beta_6\partial_z|A|^2.
\end{align*}
\]

Here coefficients are:

\[
\beta_0 = 1 + \frac{1}{i\omega + E^2K^2} \left((i\omega + \gamma E^2K^2)\right)
\]
In this case mean profiles of temperature and salinity were in the range from 0 till 50. Number of grid points is 2048. Dimensional variations in temperature and salinity are proportional to $\delta T$ and can be estimated as $6 \cdot 10^{-5}$·$B$·$\delta T$·$C$ and $6.0 \cdot 10^{-5}$·$C$·$\delta S$%, respectively. Amplitude of stream function is proportional to $|A|$ and can be estimated as $6.9 \cdot 10^{-4}$·$|A|$·$cm^2/sec$. Maximal variation in temperature, for instance, is about 20% of $\delta T$.

Thus in this article we have derived ABC system of amplitude equations for travelling waves of double-diffusive convection in a limit of high Hopf frequency (large $R S$) in the infinite horizontal layer.

Equations (12) have nontrivial solutions, describing such phenomena as diffusive chaos (16) and can be used for simulation of formation of patterns, like vertical fine structure of temperature and salinity in some areas of the ocean, for instance, in inversions of thermohaline staircases.

Transform system (12) to more convenient form by introducing new time variable $T' = E^2 T$, and applying following substitutions:

$$A' = A E^{-1} \sqrt{\beta_5/\beta_3} \exp(-i R \beta_2 R T' E^{-2})$$
$$B' = |\beta_3| E^{-1} B \quad C' = |\beta_4| E^{-2} C.$$

System (12) now gets form (primes are omitted):

$$\partial_T A = \beta_1 \partial_Z^2 A + \alpha_2 R A - \alpha_3 A \partial_Z B + \alpha_4 A \partial_Z C,$$
$$\partial_T B = \partial_Z^2 B - \partial_Z |A|^2,$$
$$\partial_T C = \tau \partial_Z^2 C - \tau \alpha_6 \partial_Z |A|^2.$$

Here are coefficients: $\alpha_2 = \beta_2 R E^{-2}$, $\alpha_3 = \beta_3 / |\beta_3|$, $\alpha_4 = \beta_4 / |\beta_4|$ and $\alpha_6 = \beta_6 / |\beta_6| / (|\beta_4| / |\beta_3|)$.

We developed numerical models for parallel calculations of system (13) based on explicit and Dufort-Frankel schemes. For numerical experiments it were chosen parameters $\sigma = 7$, $K = \pi$, $E = 0.05$, and two values of Lewis number: $\tau = 1/10$ and $\tau = 1/81$. Governing parameter $R$ was in the range from 0.1 till 50. Number of vertical dots $n$ was from 256 till 2048 to resolve microstructure.

System (13) was integrated numerically on multiprocessor computer MVS-1000/16 with zero boundary conditions and sinusoidal initial conditions for dependent variables. In the most cases the initial state was destroyed after some time via a multiple Eckhaus instability (birth of convective cells, 17) and was followed by diffusive chaos state, with strong space-time irregularity. In this case mean profiles of temperature and salinity become perturbed so that all layer of inversion becomes divided on 10 – 30 small layers (see FIG. 3 for $\tau = 1/81$ and time $t = 9.28$ hours). Buoyancy frequency (FIG. 5)
becomes very irregular, and all this fine structure slowly changes with the time.

IV. CONCLUSION

In this article we developed mathematical model, describing formation of vertical convective patterns in two-dimensional double-diffusive convection in a limit of high Hopf frequency for an infinite horizontal layer. A physical system, corresponding to such model is inversion of thermohaline staircase. Some typical parameters of inversions are presented in the table I. It is known [18] that parameters of stratification in the inversions are often near the onset of convection. Also vertical microstructure (usually step-like) are often observed in the inversions along with small scale turbulence [19]. Results of this work are in qualitative agreement with these observations. For more comparison see also [5] and references therein.

Although we aimed to construct mathematical model with fine structure generation in double-diffusive system without its detailed relation with experimental data, we should note a few points of such relation:

- Developed model predicts that fine structure should exist in given system for a wide range of parameters with typical time of pattern formation of about a few hours.

- Fine structure has very irregular shape, slowly changing with the time in accordance with solution of ABC-system of diffusive chaos type. Mean profiles of temperature and salinity become perturbed so that all layer of inversion becomes divided on $10^{-30}$ small layers.

- Buoyancy frequency structures have peak emissions with amplitude and width with reasonable agreement with observational data [19] in the cases when temperature and salinity differences per layer are relatively small, as in the table I.

Of course, developed model, based on ABC-system of amplitude equations has its limitation of weakly nonlinear approximation, which hardly allows to get "full-fledged" step-like vertical structure of density, as noted in [6]. Partially this defect can be overcome by regarding amplitude equations arising at higher orders of small parameter in multi-scale decomposition method. But this is a matter of further works along with more detailed comparison of predicted fine structure parameters with experimental and observational data.

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**TABLE I: Parameters estimations for inversion of thermohaline staircase.** For all cases it is true $T = 15^\circ C$, $S = 36\%$, $\sigma = 7$, $\tau = 1/81$, $t_0$ - diffusive time scale, $t_0'$ - main time scale, $N_0 = (1 - \tau)/(1 + \sigma)$ - limit of critical buoyancy frequency at $\varepsilon = 0$, $400 t_0'$ - time of establishing of diffusive chaos in the inversion.

| Parameter | 1         | 2         | 3         |
|-----------|-----------|-----------|-----------|
| $h$ [cm]  | 400.0     | 250.0     | 100.0     |
| $\delta T$ [°C] | 1.0       | 0.1       | 0.1       |
| $\delta S$ [%] | 0.33      | 0.033     | 0.033     |
| $R_T$ $\times 10^{11}$ | $9.5 \times 10^{11}$ | $2.3 \times 10^{10}$ | $1.48 \times 10^9$ |
| $R_S$ $\times 10^{12}$ | $1.08 \times 10^{12}$ | $2.6 \times 10^{10}$ | $1.69 \times 10^9$ |
| $\varepsilon$ | 0.0006    | 0.0015    | 0.003     |
| $e$       | 0.012     | 0.03      | 0.06      |
| $l_c$ [cm] | 4.7       | 7.7       | 6.0       |
| $t_0$ [sec] | $1.12 \times 10^8$ | $4.38 \times 10^7$ | $7.0 \times 10^6$ |
| $t_0'$ [min] | 0.68      | 1.7       | 1.07      |
| $N_0$ [cyc/hr] | 4.95      | 1.97      | 3.13      |
| $400 t_0'$ [hr] | 4.5       | 11.4      | 7.14      |
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