A transparent expression of the $A^2$-Condensate’s renormalisation

Ph. Boucaud$^a$, F. De Soto$^b$, A. Le Yaouanc$^a$, J.P. Leroy$^a$, J. Micheli$^a$, H. Moutarde$^c$, O. Pène$^a$, J. Rodríguez–Quintero$^d$

$^a$ Laboratoire de Physique Théorique
Université de Paris XI, Bâtiment 210, 91405 Orsay Cedex, France

$^b$Dpto. de Física Atómica, Molecular y Nuclear
Universidad de Sevilla, Apdo. 1065, 41080 Sevilla, Spain

$^c$Centre de Physique Théorique Ecole Polytechnique, 91128 Palaiseau Cedex, France

$^d$Dpto. de Física Aplicada e Ingeniería eléctrica
E.P.S. La Rábida, Universidad de Huelva, 21819 Palos de la fra., Spain

We give a more transparent understanding of the vacuum expectation value of the renormalised local operator $A^2$ by relating it to the gluon propagator integrated over the momentum. The quadratically divergent perturbative contribution is subtracted and the remainder, dominantly due to the $O(1/p^2)$ correction to the perturbative propagator at large $p^2$ is logarithmically divergent. This provides a transparent derivation of the fact that this $O(1/p^2)$ term is related to the vacuum expectation value of the local $A^2$ operator and confirms a previous claim based on the operator product expansion (OPE) of the gluon propagator. At leading logarithms the agreement is quantitative, with a standard running factor, between the local $A^2$ condensate renormalised as described above and the one renormalised in the OPE context. This result supports the claim that the BRST invariant Landau-gauge $A^2$ condensate might play an important role in describing the QCD vacuum.

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I. INTRODUCTION

In a series of lattice studies [1–4] the gluon propagator in QCD has been computed at large momenta, and it was shown that its behavior was compatible with the perturbative expectation provided a rather large $1/p^2$ correction was considered. In an OPE approach this correction has been shown [2,3] to stem from an $A^2$ gluon condensate which does not vanish since the calculations are performed in the Landau gauge. It was also claimed [4] that this condensate might be related to instantons.

The role of such a condensate in the non-perturbative properties of QCD, in particular its relation to confinement, has been studied by several groups [5,6]. Of course any physics discussion about the $A^2$ condensate necessitates a clear definition of what we speak about, i.e. it needs a well defined renormalisation procedure to define the renormalised local $A^2$ operator, since $A(0)^2$ is a quadratically divergent quantity as can easily be seen in perturbation theory. A renormalisation of $A^2$ was defined in [2,3] within the OPE context which we now briefly summarise. It uses the notion of “normal order product” in a “perturbative vacuum” which is annihilated by the fields $A$. It implies that $< : A(0)^2 : >_{\text{pert}} = 0$ in the perturbative vacuum\(^\dagger\). The contribution to $< : A(0)^2 : >$ in the true QCD vacuum is then of non-perturbative origin. It has only logarithmic divergences and it is multiplicatively renormalised. Of course this notion of a perturbative vacuum in which Fock expansion could be performed has not a very transparent physical meaning especially in a non-perturbative context such as the numerical Euclidean path integral method.

\(^\dagger\)The symbol “; ... ;” represents the normal ordered product in this perturbative vacuum.

\(^\star\)Unité Mixte de Recherche du CNRS - UMR 8627

\(^\dagger\)The symbol “; ... ;” represents the normal ordered product in this perturbative vacuum.
$A^2$ is not a gauge invariant operator but the bare $A^2$ condensate is a very special object since, by definition, it is a minimum of the gauge orbit \[6\]. In other words, some important physics seems to lie beneath the BRST invariance of $A^2$ in Landau gauge. The authors of ref. \[5\] discussed on the generalised composite operator $A_\mu A^\mu + 2i\alpha\bar{c}c$, which is BRST invariant in the manifestly Lorentz covariant gauge, and examined the survival of this invariance after renormalisation. In this note, although in a different context, we also examine the same point: the subtle relationship between the minimum of bare $A^2$ in the gauge orbit and any gauge-independent physical phenomenology associated to the renormalised condensate \[6\], emerging for instance from the OPE analysis \[1–4\]. To this aim, we will derive the renormalised $A^2$ vacuum expectation value without using the normal ordering but using only the OPE expansion of the gluon propagator \(\dagger\). It will provide a more transparent definition, related directly to a quantity which is actually measured.

We start from the observation that the non-renormalised $\langle A(0)^2 \rangle$ is related to the integral of the gluon propagator over momentum. Hence it is expected that the non-perturbative contribution to $A^2$ has to do with the non-perturbative contribution to the gluon propagator. The latter contains precisely $1/p^2$ contributions due to the $A^2$ condensate at large momenta, and also strong deviations from perturbative QCD at small momenta, see fig. 1 (taken from \[1–4\]). How does this fit together?

### II. BARE, PERTURBATIVE AND NON-PERTURBATIVE $A^2$

It is possible in principle from lattice calculations to define the non-perturbative gluon propagator in the Landau gauge. Lattice calculations provide the bare gluon propagator. From the gluon propagator computed with a series of different values of the lattice spacing one can in principle compute the renormalised gluon propagator from zero momentum up to as large a momentum as one wishes. An example of such a non-perturbative propagator is shown in fig. 1. We can choose for example the MOM renormalisation scheme**\*, such that

$$G^{(2)}_R(p^2 = \mu^2) = \frac{1}{\mu^2}. \tag{1}$$

This implies a renormalisation of the gluon fields

$$A^{\mu}_R = Z_3(\mu) \cdot \frac{1}{2} A^{\mu}_{\text{bare}}, \quad Z_3(\mu) \equiv \mu^2 G^{(2)}_{\text{bare}}(\mu^2) \tag{2}$$

The renormalisation constant $Z_3$ has to be understood as related to any regularisation method and any value of the UV regulator provided that the latter is larger than the momenta carried by the gluons. The coupling constant is also renormalised in the MOM scheme. The Yang-Mills theory is thus fully renormalised and from now on we will consider only renormalised gauge fields and propagators.

The propagator is defined in Euclidean space by

$$\int d^4x e^{ip \cdot x} \langle A^a_{\mu R}(0) A^b_{\nu R}(x) \rangle = \delta_{a,b} \left[ \delta_{\mu,\nu} - \frac{p_\mu p_\nu}{p^2} \right] G_R(p^2) \tag{3}$$

Inverting the Fourier transform,

$$\sum_{a,\mu} \langle A^a_{\mu R}(0) A^a_{\mu R}(0) \rangle = \frac{3(N_c^2 - 1)}{(2\pi)^4} \int d^4p G^{(2)}(p^2) = \frac{3(N_c^2 - 1)}{16\pi^2} \int p^2 dp^2 G^{(2)}(p^2) \tag{4}$$

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\(\dagger\) Landau gauge is recovered in the limit $\alpha \to 0$

\(\ddagger\) Of course the normal ordering has been used in \[2,3\] to compute the anomalous dimension of $A^2$ and the Wilson coefficient $c_2$

** Notice that the chosen renormalisation scheme is not relevant in our argument in this paper, but we clearly need a scheme in which non-perturbative quantities coming from lattice simulations can be accommodated. MOM is one of the simplest. On the contrary the \(\overline{\text{MS}}\) scheme does not fulfill this condition.
FIG. 1. Gluon propagator extracted from lattice calculations renormalised at $\mu = 10$ GeV and plotted between 0 and 9 GeV. The curve corresponds to the fit written in eq. (7). It results that the infrared cut-off $p_{\text{min}}$ can be safely taken around 2.6 - 3.0 GeV.

This integral is quadratically divergent in the ultraviolet. Indeed, if the gauge fields and the coupling constant have been renormalised, the local $A^2$ operator has not yet. Let us introduce an ultraviolet cut-off $\Lambda$ and define

$$< (A_R(\mu))^2 >_{\Lambda} = \frac{3(N_c^2 - 1)}{16\pi^2} \int_0^{\Lambda^2} p^2 dp^2 G_R^{(2)}(p^2).$$

where $(A_R(\mu))^2$ refers to the square of the gauge fields renormalised at the scale $\mu$, but where $A^2$ has not been renormalised as a local product of operators. The symbol $"< ... >"$ represents the vacuum expectation value. $(A_R)^2$ is clearly an UV divergent quantity. The index $\Lambda$ refers to the ultraviolet cut-off and $\mu$ to the renormalisation point for the gauge fields and the coupling constant. The cut-off $\Lambda$ has nothing to do with the lattice cut-off $a^{-1}$. The renormalisation in eqs. (1) and (2) has eliminated any dependence in the different lattice spacings which have been used to produce the renormalised propagator. $\Lambda$ is introduced simply to control the quadratic and logarithmic divergences we encounter here.

The dominant contribution to this integral is the perturbative one. To separate the perturbative contribution from the non perturbative we will now use the results of [3]

$$p^2 G_R^{(2)}(p^2, \mu^2) = \frac{p^2 G^{(2)}(p^2)}{\mu^2 G^{(3)}(\mu^2)} = c_0 \left( \frac{p^2}{\mu^2}, \alpha(\mu) \right) + c_2 \left( \frac{p^2}{\mu^2}, \alpha(\mu) \right) \frac{< (A^2)_{R}(\mu) >_{\Lambda}}{4(N_c^2 - 1)},$$

where $G^{(2)}(p^2)$ is the bare propagator. This expansion does not exactly separate the perturbative form the non-perturbative contribution because of the denominator $\mu^2 G^{(2)}(\mu^2)$ which contains a non perturbative contribution. It is therefore convenient to introduce a slightly different renormalisation $R'$:

$$p^2 G_R^{(2)}(p^2, \mu^2) = \frac{p^2 G^{(2)}(p^2)}{\mu^2 G^{(2)}_{\text{pert}}(\mu^2)} = \frac{c_0 \left( \frac{p^2}{\mu^2}, \alpha(\mu) \right)}{c_0(1, \alpha(\mu))} + \frac{c_2 \left( \frac{p^2}{\mu^2}, \alpha(\mu) \right)}{c_0(1, \alpha(\mu))} \frac{< (A^2)_{R}(\mu) >_{\Lambda}}{4(N_c^2 - 1)}.$$

where $(A^2)_{R}(\mu)$ represents the $A^2$ operator renormalised as a local operator at the scale $\mu$. Here the denominator is only the perturbative contribution to the Green function whence the first term in eq. (7) is purely perturbative: it
runs perturbatively with a perturbative MOM renormalisation condition at $p^2 = \mu^2$. Let us define for simplicity the constant

$$z_0 \equiv \frac{1}{c_0(1, \alpha(\mu))} = \frac{G^{(2)}(\mu^2)}{G^{(2)}_{\text{pert}}(\mu^2)} = 1 + O\left(\frac{1}{\mu^2}\right).$$  

(8)

From [3] the first term in eq. (7), $z_0 c_0 \left(p^2/\mu^2, \alpha(\mu)\right)$, represents the three loop perturbative contribution and the second, $z_0 c_2 \left(p^2/\mu^2, \alpha(\mu)\right)$, the leading logarithm Wilson coefficient of the $O(1/p^2)$ nonperturbative correction attributed to the vacuum expectation value of the renormalised local operator $< (A^2)_R(\mu) >$.

Let us now introduce some notations:

$$< (A^2)_R(\mu) > = Z^{-1}_A(\mu) : A^2_{\text{bare}} : = Z^{-1}(\mu) : (A_R(\mu))^2 : ,$$  

(9)

where $Z(\mu) \equiv Z^{-1}(\mu) Z_{A^2}(\mu)$, and

$$\gamma_{A^2}(\alpha(\mu)) \equiv \frac{d}{d \ln \mu^2} \ln Z_{A^2}(\mu) = -\frac{35 N_C}{12} \frac{\alpha(\mu)}{4 \pi} + ... ,$$

$$\hat{\gamma}(\alpha(\mu)) = \frac{d}{d \ln \mu^2} \ln \hat{Z}(\mu) = -\gamma_0 \frac{\alpha(\mu)}{4 \pi} + ... = -\frac{35 N_C}{12} \frac{\alpha(\mu)}{4 \pi} + ...$$  

(10)

where the symbol : ... : represents the normal ordered product in the perturbative vacuum$^{\dagger\dagger}$. Our main goal in this paper is to understand better the connection between $< (A^2)_R(\mu) >$ defined in [3] and the $(A_R(\mu))^2$ object considered here.

The expansion in (7) is only valid above some momentum $p \geq p_{\text{min}}$. Typically we have taken $p_{\text{min}} = 2.6$ GeV for our fits reported in [1] - [4].

From eq. (5), (7) we decompose

$$< (A_R(\mu))^2 >_A = < (A_R(\mu))^2 >_A^{\text{pert}} + < (A_R(\mu))^2 >_A^{\text{OPE}} + < (A_R(\mu))^2 >_A^{\text{IR}}$$  

(11)

where

$$< (A_R(\mu))^2 >_A^{\text{pert}} = \frac{3(N_c^2 - 1)z_0}{16\pi^2} \int_{p_{\text{min}}^2}^{A^2} dp^2 c_0 \left(\frac{p^2}{\mu^2}, \alpha(\mu)\right);$$  

(12)

$$< (A_R(\mu))^2 >_A^{\text{OPE}} = \frac{3(N_c^2 - 1)z_0}{16\pi^2} \int_{p_{\text{min}}^2}^{A^2} \frac{dp^2}{p^2} c_2 \left(\frac{p^2}{\mu^2}, \alpha(\mu)\right) \frac{< (A^2)_R(\mu) >}{4(N_c^2 - 1)};$$

(13)

$$< (A_R(\mu))^2 >_A^{\text{IR}} = \frac{3(N_c^2 - 1)}{16\pi^2} \int_0^{p_{\text{min}}^2} p^2 dp^2 G^{(2)}_R(p^2).$$

(14)

A few comments are in order here. $< (A_R(\mu))^2 >_A^{\text{pert}}$ corresponds indeed to the perturbative computation of the vacuum expectation value of $A^2$, i.e. to the connected diagrams with no external legs and with one $A^2$ inserted. However, the coupling constant and the gluon fields in the diagrams have been consistently renormalised at the scale $\mu$. To leading order eq. (12) leads to

$$< (A_R(\mu))^2 >_A^{\text{pert}} \xrightarrow{\Lambda \to \infty} \frac{3(N_c^2 - 1)}{16\pi^2} \Lambda^2 \int_{\ln \left(\frac{\mu}{\Lambda_{\text{QCD}}}/\Lambda_{\text{QCD}}\right)}^{\ln \left(\frac{A}{\Lambda_{\text{QCD}}}\right)} \left(1 + O\left(\frac{1}{\ln \left(\frac{\Lambda}{\Lambda_{\text{QCD}}}\right)}\right)\right) \Lambda^2$$

(15)

which diverges more than quadratically. Note that the dependence in $p_{\text{min}}^2$ is subdominant.

$^{\dagger\dagger}$The : ... : symbols have been erroneously omitted in [3].
In equation (13) the left hand side has been defined from the decomposition of the integral (5) according to (7). The right hand side contains $<(A^2)_R(\mu)>$ already discussed. The latter is just a number which factorises out of the integral in (13). We thus see that $<(A_R(\mu))^2>_{\Lambda}^{\text{OPE}}$ and $<(A^2)_R(\mu)>$ are proportional.

Our next task is to compute the proportionality coefficient and to compare $<(A_R(\mu))^2>_{\Lambda}^{\text{OPE}}$ with the other subleading term, $<(A_R(\mu))^2>_{\Lambda}^{\text{IR}}$. From [3] and eq. (8) we know that $z_0 = 1 + O(1/\mu^2)$. Our calculation of the integral in eq. (13) being performed to leading logarithm we will take $z_0c_2 = c_2$ in the following. From (23) in [3],

$$c_2 \left( \frac{p^2}{\mu^2}, \alpha(\mu) \right) = 12\pi\alpha(p) \left( \frac{\alpha(p)}{\alpha(\mu)} \right)^{-\gamma_0/\beta_0}. \tag{16}$$

Let us also recall

$$Z_3(\mu) \propto (\alpha(\mu))^{\gamma_0/\beta_0}, \quad <(A^2)_R(\mu)> \propto (\alpha(\mu))^{-\gamma_0/\beta_0} \tag{17}$$

with

$$\beta_0 = 11, \quad \gamma_0 = 13/2, \quad \gamma_{A^2} = 35/4, \quad \gamma_{A^2} - \gamma_0 = \frac{9}{4} \tag{18}$$

From eq. (13) and the leading logarithm relation

$$dp^2/p^2 = d\log(p^2) \approx \frac{4\pi}{\beta_0} \frac{d\alpha}{\alpha^2} \tag{19}$$

we find

$$< (A_R(\mu))^2 >_{\Lambda}^{\text{OPE}} = \frac{3(N^2_c - 1)}{16\pi^2} \frac{(12\pi)}{(\alpha(\mu))^{\gamma_0/\beta_0}} \frac{<(A^2)_R(\mu)>}{4(N^2_c - 1)} \frac{4\pi}{\beta_0} \int_{\alpha(\Lambda)}^{\alpha(p_{\text{min}})} d\alpha \alpha^{-1} \left( \frac{\alpha(\Lambda)}{\alpha(\mu)} \right)^{-\gamma_0/\beta_0} \tag{20}$$

It is interesting to notice that the coefficient $\beta_0/\gamma_0$ stemming from the integration over $\alpha$ is exactly compensated by the prefactors outside the integral, the origin of which does not appear at first sight to be related to the anomalous dimension of $A^2$. Had we taken any other anomalous dimension instead of $\gamma_0$, say some $\gamma'$, we would have ended with a constant $9/(4\gamma')$ in front of the r.h.s of eq. (20).

In the large $\Lambda$ limit, $\alpha(p_{\text{min}}) \gg \alpha(\Lambda)$ whence, since $\gamma_0$ is positive, the main result of this note comes from:

$$< (A_R(\mu))^2 >_{\Lambda}^{\text{OPE}} \sim < (A^2)_R(\mu) > \left( \frac{\alpha(\Lambda)}{\alpha(\mu)} \right)^{-\gamma_0/\beta_0}. \tag{21}$$

To leading logarithms and keeping $\mu$ fixed,

$$<(A_R(\mu))^2>_{\Lambda}^{\text{OPE}} \alpha(\Lambda)^{-\gamma_0/\beta_0} \Lambda \rightarrow \infty. \tag{22}$$

On the other hand, from eq. (14)

$$<(A_R(\mu))^2>_{\Lambda}^{\text{IR}} = \text{constant}. \tag{23}$$

since it does not depend on $\Lambda$. It results that $<(A_R(\mu))^2>_{\Lambda}^{\text{OPE}}$ is dominant over $<(A_R(\mu))^2>_{\Lambda}^{\text{IR}}$ in the decomposition (11). This dominance will lead to the coming interpretation (next section) of the decomposition (11), which is indeed the main result of this note. Of course, we perform our analysis in the Landau gauge because of its conceptual [6] and numerical (Lattice Green functions [3]) particular interest. However the survival after renormalisation of BRST invariance in covariant gauges for the generalised composite operator showed in ref. [5] seems to point out that an analogous analysis, with similar results, for these gauges could be performed.

As an interesting special case, if $\mu = \Lambda$

$$<(A_R(\Lambda))^2>_{\Lambda}^{\text{OPE}} \Lambda \rightarrow \infty < (A^2)_R(\Lambda) > \propto \alpha(\Lambda)^{-\gamma_0/\beta_0}. \tag{24}$$
Our conclusion is summarised in

\[ < (A^2)_R (\mu) > \simeq < (A_R(\mu))^2 >_{\Lambda}^{\text{OPE}} \left( \frac{\alpha(A)}{\alpha(\mu)} \right)^{\frac{\mu}{\Lambda}} \]
\[ \simeq \left[ < (A_R(\mu))^2 >_{\Lambda} - < (A_R(\mu))^2 >_{\Lambda}^{\text{pert}} \right] \left( \frac{\alpha(A)}{\alpha(\mu)} \right)^{\frac{\mu}{\Lambda}}. \]  

(25)

The notations being not conventional let us recall that the \(< \ldots >_{\Lambda}\)'s in the r.h.s represent the gluon propagator integrated over momentum up to an UV cut-off, \(\Lambda\), see eqs. (5) and (12). The gluon fields and coupling constants are renormalised in all the terms appearing in these equation. Thus we learn that the further renormalisation of the local operator \(A^2\) proceeds by subtracting to the plain vacuum expectation value of \(A^2\) the same object computed in perturbation. This logarithmically divergent difference is then renormalised by the powers of \(\alpha\) in the r.h.s of eq. (25).

Not unexpectedly, we retrieve in essence the initial expression of the renormalisation of the \(A^2\) operator through normal ordering (i.e. subtraction of the perturbative v.e.v), followed by the multiplicative, logarithmic renormalisation \(Z_{A^2}\). But apart from a non trivial consistency check, involving in particular the detailed expression of the Wilson coefficient, we obtain an expression which is more transparent, since it only involves a measurable quantity, the integral over the renormalised propagator.

Eq. (25) presents a separation between perturbative and non-perturbative contributions to the integrated propagator i.e. to \(A^2\). Of course, such a separation depends on the renormalisation scheme, and on the order in perturbation theory in which the Green functions are computed. It is also well known that summing to infinity the perturbative series may generate renormalons which behave like non-perturbative condensates. To avoid any such problem we stick to a finite order in the perturbative series. Furthermore, if the quantitative separation between perturbative and non-perturbative contributions depends on these prescription, the results summarised in eq (25) \textit{do not depend on them} provided that we use \textit{the same} scheme and order when computing both sides of eq. (25). Of course the anomalous dimensions to leading logarithms do not either depend on them.

This simple result has several interesting consequences. First, it has been advocated [4] that the \(A^2\) condensate could be dominantly due to the contribution to the path integral of semi-classical gauge field configurations such as instantons liquids. It is useful to consider this hypothesis through a background field picture, i.e. factorising the path integral into an integral over semi-classical gauge field configurations, and for each value of these an integral over quantum fluctuations around this background configuration. It means that the hermitian matrix \(A_\mu\) is decomposed into:

\[ A_\mu = B_\mu + Q_\mu(B) \]
\[ A^2 = B^2 + \{B,Q\}_+ + Q^2(B) \]  

(26)

\(B_\mu\) being the background, assumed to be non-perturbative, and \(Q_\mu\) the quantum fluctuations assumed to be perturbative. \(\{B,Q\}_+ \equiv B.Q + Q.B\). In principle, \(Q_\mu\) depends on \(B_\mu\) and differs from the quantum fluctuations around the trivial vacuum \(B_\mu = 0\) which is what perturbative QCD computes. The hypothesis that \(< (A^2)_R >\) is due \(\dagger\dagger\) to these semi-classical gauge configurations is translated into: \(< (A^2)_R > \simeq B^2\). From eqs. (25), (26)

\[ < (A^2)_R > \simeq < B^2 > \simeq < A^2 > - < Q^2(B = 0) > \]  

(27)

i.e. that \((Q^2(B) - Q^2(B = 0))\) is subleading\(\S\S\). In other words the dependence of \(Q_\mu\) on \(B_\mu\) is subleading. The hard quantum fluctuations are not sensitive to the soft background field.

A most interesting consequence of our result is related to some discussions in [6]. These authors extend to QCD some remarks stemming from compact \(U(1)\). They attribute a special role to the \(A^2\) condensate, even if a gauge dependent quantity, by arguing that \(A^2\) in the Landau gauge is the minimum of \(A^2\) on the gauge orbit. One difficulty

\(\dagger\dagger\)This discussion is qualitative and we do not know how to define rigorously the corresponding scale \(\mu\). We therefore prefer to omit writing \(\mu\) here.

\(\S\S\)If \(B\) is a classical solution of the field equations, the term linear in \(Q\) will vanish. \(B\) should be close to such a solution and we therefore neglect \(\{B,Q\}_+\).
in this argument is the following: Fixing the Landau gauge amounts to minimize the $\langle A_{\text{bare}}^2 \rangle$ while the condensate refers to some renormalised quantity free of the quadratic and logarithmic divergences. In compact $U(1)$ life is simpler:

$$\langle A_{\text{bare}}^2 \rangle = \langle A_{\text{pert}}^2 \rangle + \langle A_{\text{nonpert}}^2 \rangle,$$

the perturbative theory is trivial and the nonperturbative contribution is due grossly speaking to the topology. A phase transition when the coupling constant varies allows to measure directly the non-perturbative contribution. We refer to [6] for more details. Our result eq. (25) exhibits in QCD, up to subleading contributions, a linear decomposition similar to eq. (28), although such a similarity is not at all obvious at first sight. The next question could be whether in some sense the $\langle A^2 \rangle_{\text{OPE}}$ computed in the Landau gauge is the minimum of some quantity on the gauge orbit.

Last but not least, let us simply say that the result in eq. (25) provides a fairly simple understanding of what the $A^2$ condensate is. It confirms that indeed the $O(1/p^2)$ correction to perturbative QCD at large momenta has to do with the $A^2$ condensate. Indeed, if one start with some doubt about the relation of the r.h.s of eq. (13) with an $A^2$ condensate, just considering it as an unidentified $1/p^2$ contribution, we end-up with the conclusion that it yields a non-perturbative contribution to the $A^2$ v.e.v. The fact that in our derivation this term has precisely the anomalous dimension of an $A^2$ condensate comes form the fact that $c_2$ in the r.h.s of eq. (13) has been computed under the assumption that it is due to an $A^2$ condensate, an assumption which has been shown to fit fairly well the lattice data. Had we used another scale dependence for $c_2$ we would have ended with a wrong scale dependence for the resulting non-perturbative contribution to the $A^2$ v.e.v. We would have also ended with a constant different from 1 in front of the r.h.s of eq. (25), see the discussion following eq. (20). Thus the picture is fully consistent.

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