The Theoretical Value of the Hubble Constant $H_o$ and Unification of the Fundamental Forces of Nature

Paul C. Rivera

ABSTRACT

The Hubble constant $H_o$ represents the speed of expansion of the universe and various cosmological observations and modeling methods were utilized by astronomers for a century to pin down its exact value. Determining $H_o$ from cosmological observations is a long and tedious process requiring highly accurate datasets. To circumvent this need, a simple theoretical approach is introduced in this study which uses the concept of gravitational weakening and seismic-induced recession. As tremors occur among celestial objects, their gravitational fields would also change. This resulted in a fundamental relation of $H_o$ and the computed rate of recession that gives a theoretical value for $H_o=69.921$ Km/s/Mpc. Using the newly discovered seismic-induced gravitational weakening and time dilation, it is possible that various astrophysical methods using different measurement methods would converge to this theoretical $H_o$ value when cosmological distances and time delay measurements are corrected with the simple formulas we derived. The new model assumes that, as quakes occur in celestial objects, luminosity-induced acceleration and high-energy collision of protons and electrons may produce a massive number of neutrinos, quarks and other subatomic particles. Furthermore, the fine structure constant was found to be inversely proportional to $H_o$-squared and that the fine-structure constant obtained in this study gives a new physical interpretation of $\alpha$. New relations for the speed of light, orbital velocity, gravitational force and the Hubble constant were further derived from the new recession constant using approximate relations for the Newtonian and electric force constant. This resulted in a modified gravitational law that is both repulsive and attractive and a theoretical explanation of the phenomenon of light-induced gravitation analogous to the electromagnetic force where photon is the force-carrier. Finally, the fundamental forces of gravitation, electromagnetism and strong nuclear force are now unified.

Keywords: Hubble constant, gravitational weakening, light-induced gravitation, photo-fragmentation, seismic-induced recession, time delay.

I. INTRODUCTION

The rate of expansion and the exact age of the universe are dictated by the Hubble constant $H_o$. It is an important cosmological parameter as it governs the size and fate of the universe and various studies had been conducted to determine its value for almost a century. Various observational methods of estimating the value of this cosmological constant had been introduced in a number of astrophysical studies. Using the early and late universe, many values of $H_o$ had been reported in each literature along with their uncertainties and limitations, and there appears to be an unresolved conflict between the reported values that fuels discussion of a new physics. The recent study conducted by Freedman et al. [1] reported a value of 69.8±0.8 Km/s/Mpc. The LIGO/VIRGO collaboration of Abbott et al. [2] reported a value of 70.0 Km/s/Mpc. Furthermore, Riess et al [3] also reported a value of 73.4 Km/s/Mpc using Cepheid variables while CMB-based methods reported different but generally lower values with a mean of 67.4 Km/s/Mpc. Another CMB-based method using only WMAP data [4] also obtained a value of 70 Km/s/Mpc with a 3% accuracy. Bennett et al. [5] also reported a value of 69.6 Km/s/Mpc using a combination of methods. Recently, Blakeslee et al. [6] reported a value of about 73.3 Km/s/Mpc using infrared surface brightness fluctuation distances of about 63 bright and early-type galaxies.

The exact value of the Hubble constant is generally unknown and different measurement techniques have led to an increasing tension among observational cosmologists. Such studies may have a very high accuracy and precision, but the values obtained therein cannot be compared to any known exact value of $H_o$. There is thus a need to determine the exact theoretical value of the Hubble constant to be able to compare cosmological observations against it. This study aims to determine the exact value of the Hubble constant.
using the theoretical foundation originally proposed in Rivera [7].

II. THEORY AND METHODS

Gravitational jerks occur in episodic manner resulting in a temporal change in the gravitational force. Assuming an initially unperturbed gravitational force \( F_o \), a concomitant gravitational weakening would ensue due to massive tremors leading to recession and orbital expansion of celestial objects [7]. A modified Newtonian gravitational law was previously derived where changes in the orbits of celestial objects would result to small changes in the gravitational force \( F \) given by:

\[
F = G \frac{Mm}{R^2} \left(1 - 2 \ln \frac{R}{R_o}\right)
\]  

(1)

where \( R_o \) is the average circular orbit and \( R \) is the seismic-perturbed orbital radius. The Newtonian gravitational model is now extended by adding a factor in the parenthesis to account for the change in the orbits of celestial bodies. A new gravitational potential can also be derived from the modified gravitational law and is given by:

\[
\varphi = - \frac{GM}{R_o} \left(1 + 2 \ln \frac{R}{R_o}\right)
\]  

(2)

Based on the original Newtonian law, the ratio of the distances \( R \) and \( R_o \) should be given by the square root of the gravitational forces \( F \) and \( F_o \) as in:

\[
\frac{R}{R_o} = \left(\frac{F_0}{F}\right)^{1/2}
\]  

(3)

Solving for \( R \) in (1) and (3) and equating the result yields a simple non-linear transcendental equation of the form:

\[
\frac{F}{F_o} = \exp\left(\frac{F}{F_o} - 1\right)
\]  

(4)

Equation (4) can be solved using trial and error method with a hand-held calculator but a short computational program was described in Rivera [7] using the secant method which yielded the unknown variable \( \chi = F/F_o = 0.99999 \), or \( F = 0.99999 F_o \). Clearly, there is an associated reduction of the gravitational force \( F \) as the object recedes from \( R_o \) to \( R \). This is in accordance with the original Newtonian law. An increase in \( F \) also follows if \( R \) decreases to less than \( R_o \) as (1) dictates. The gravitational force exerted by the mysterious lunar wobble (to-and-fro motion) can now be determined using this extended Newtonian gravitational law that allows repulsion. Similarly, for distant celestial objects undergoing tremors, the changes of the gravitational force due to changes in the orbital distances could now be determined. Apparently, even a small change in the gravitational force would yield a large change in the orbit \( R \) of objects in distant locations outside our galaxy. The proper distance \( R \) can now be obtained from the unperturbed orbital distance \( R_o \) using (3):

\[
R = 1.000005R_o
\]  

(5)

This shows that celestial objects undergoing gravitational weakening due to tremors (i.e., pulsations) should constantly recede to a distance proportional to their original distances. The farther the celestial object is, the farther it should recede. This is similar to the original law derived and observed by Lemaître and Hubble almost a century ago. The new gravitational model shows that the orbital distance \( R \) increases and expands for each seismic action within the celestial object itself. This should also result in a corresponding change in the momentum of the orbiting object as shown by the earth-moon and earth-sun system [7]. This resultant motion could be the physical mechanism that governs the rate of expansion of planets, stars and galaxies in the cosmos as described by the Hubble’s law.

To determine the recession rate (or current expansion rate) of celestial objects defined by the Hubble constant, a simple equation can now be derived from the previous equations as:

\[
\frac{dR}{dt} = \frac{\Delta R}{\Delta t} = \frac{5 \times 10^{-6} R_o}{\Delta t}
\]  

(6)

in which \( \Delta R (=R-R_o) \) and the recession constant equals \( 5.0 \times 10^{-6} \). Dividing (6) by \( R_o \) simply reduces to a constant:

\[
\frac{dR}{R_o dt} = H_o = \frac{k}{\Delta t}
\]  

(7)

where \( k = 5.0 \times 10^{-6} \). This represents the expansion and recession rate of celestial objects and this is defined as the Hubble constant at the present epoch. Therefore, the rate of expansion could be due mainly to gravitational weakening and recession of celestial objects. The product of the recession constant with the cosmological distance scale \( 1 \) Megaparsec and the speed of light \( c \) (noting that 1 Megaparsec is about 3,261,633 light-years) yields the constant of interest. The exact theoretical value of the Hubble constant \( H_o \) can then be determined from (\( \Delta t \) of 1 year cancels out):

\[
H_o = (\beta c / k)^{1/2}
\]  

(8)

where \( \beta \) is the equivalent distance of 1 megaparsec in light-years (i.e., \( \beta = 3,261,633.44 \)). With the value of the speed of light \( c = 2.99792458 \times 10^8 \) m/s, the term inside the parenthesis of (8) is about 4.8890653 \times 10^8 and the square root of this number gives a theoretically exact value for the Hubble constant \( H_o \approx 69.92185 \) m/s/Mpc or 69.92185 Km/s/Mpc in cosmological units.

III. RESULTS AND DISCUSSION

From the inverse of the Hubble constant, the age of the universe can be computed and is equal to 13.984 billion years. The exact size of the universe can also be estimated now (e.g., radius of 13.98Gly). This is less than 1/3 of the accepted radius of 46.5 Gly but it is much closer to the observed
farthest galaxy of about 13.4 Gly.

With the simple model presented here, all the estimated \( H_0 \) values since Hubble began analyzing astronomical observations in the 1920’s up to the present can now be compared with a theoretical value. Table I gives a comparison of the most recent astrophysical observations (\( H_{obs} \)) and the computed theoretical value of the Hubble constant. The difference of the computed and measured \( H_o \) (using \( H_o = 69.92 \, \text{Km/s/Mpc} \)) values are shown in the last column (\( \Delta H_o \)).

It can be seen that the observed \( H_o \) using the gravitational wave (GW) siren is the most accurate so far with a very small error of -0.08 Km/s/Mpc. More measurements using GW will reduce the uncertainty involved in the estimated \( H_o \). The measured \( H_o \) value using the tip of the Red Giant Branch (TRGB) of Freedman et al. [1] is also very accurate with a very small error of about 0.12 Km/s/Mpc.

The BAO-method used by Cheng et al. [8] has a relatively small error of about 1.82. New measurements by Riess et al. [9] also showed a value of 73.2 Km/s/MPC almost similar to Riess et al. [9] with a reported higher accuracy but farther from the theoretical value of \( H_o \).

### TABLE I: DIFFERENCE BETWEEN COMPUTED AND RECENTLY OBSERVED \( H_o \) USING VARIOUS METHODS

| Observation          | \( H_{obs} \) (Km/s/Mpc) | \( \Delta H_o \) (Km/s/Mpc) |
|----------------------|--------------------------|-----------------------------|
| Blakeslee et al. [6] | 73.30                    | -3.38                       |
| Riess et al. [8]     | 73.20                    | -3.28                       |
| Freedman et al. [1]  | 69.80                    | 0.12                        |
| Riess et al. [3]     | 73.48                    | -3.56                       |
| Plank [10]           | 67.40                    | 2.52                        |
| GW (Abbott et al. [2]) | 70.00                  | -0.08                       |
| Riess et al. [11]    | 73.24                    | -3.32                       |
| Cheng et al. [8]     | 68.11                    | 1.82                        |

As far as celestial objects are used in the cosmological observations and estimation of \( H_o \) values, red giant appears to be more accurate since no additional correction to distance is needed. In the case of the Cepheid variables and Type Ia supernovae, additional correction to the observed distances must be imposed since every pulsation or tremor implies an increase in the proper distance. In general, the proper distance \( R \) of any celestial object undergoing seismic action increases as in [7]:

\[
R = R_o (1 + kQT) \tag{9}
\]

where \( R_o \) is the initial time, \( k (=5 \times 10^{-6}) \) is the constant of orbital expansion or recession, and \( Q \) is the number of tremors occurring for each time \( T \). \( Q \) is simply an integer number \( (1...n) \) which is the number of seismic pulsations occurring in celestial objects. As tremors occur, the object continually recedes, and \( R \) increases as \( F \) decreases. For instance, a 1000-pulsation per year in 10 years will result to a 1.05\( R_o \). When this correction is used in the estimated distance using Cepheid variables, a value of \( H_o=73.4 \, \text{Km/s/Mpc} \) is reduced to 69.9 Km/s/Mpc which is equal to the theoretical value computed above.

When the radial expansion equation (28) in Rivera [7] is used to determine the evolution of the universe, it can be extrapolated that the universe did not possibly start from a singularity, particularly when seismic-induced forces are assumed to occur. If the force of the ‘big bang’ is mimicked with about 28,000 quakes, the universe may have started from a radial distance \( R_o=0.5 \, \text{m} \) at \( t=t_o \). Using the predicted age of the universe of about 13.984 \times 10^9 \) years, this would result to a radius \( R=4.413 \times 10^{20} \) m (46.6 Gly). With a slightly lower number of massive tremors of about 27460, this would give a radius of the universe of \( R=13.984 \) Gly.

The Tully-Fisher distance needs correction as already described in Rivera [12]. When the cosmological distance of pulsating objects is corrected using (9), the Hubble constant estimated using Cepheid variables and supernovae may converge to the theoretical \( H_o \) value of 69.921 Km/s/Mpc.

This could also be true in time-delay distance estimations and the CMB-modeled \( H_o \) value. When the proper time is used in the early universe model (and in determining time-delay distances), the extrapolation may also yield the correct Hubble constant. It is thus recommended to use the seismic-induced gravitational time dilation equation given by [7]:

\[
t = t_o (1 + kQ t_o) \tag{10}
\]

in which \( r \) is the dilated time and \( t_o \) is the proper time. This is a quadratic but non-relativistic gravitational time dilation equation that is not dependent on the speed of light. Owing to seismic-induced recession of objects, there is a corresponding time delay equivalent to:

\[
\Delta t = kQ t_o^2 \tag{11}
\]

This is comparable to the Shapiro time delay but with a constant value of \( k (=5 \times 10^{-6}) \) that is almost equal to \( GM/c^2 \) where \( M \) is the solar mass. The \( k \) constant is almost similar but slightly different from the Gaussian gravitational constant \( (4.77 \times 10^{-6} \text{ rad/sec}) \) whose square has been reported in units of \( G \). An example of the computed time delay using the present gravitational time dilation model is shown in Fig. 1 below.

![Fig. 1. Computed time delay from the new seismic-induced gravitational time dilation model.](image-url)

For a proper time of 1 sec, a time delay of 5 microsecond is computed. After 10 seconds of proper time, the time delay increases to 500 microseconds. The time delay increases quadratically to 64.8 seconds for a proper time of 1 hour, giving a dilated time of 3664.8 seconds.

When the proper distances of galactic objects and time delays are determined correctly by imposing the corrections given in (9), (10), \( H_o \) values from different cosmological measurements using CMB and various celestial markers would yield an exact theoretical value of the Hubble constant.
Furthermore, it should be noted that the MOND acceleration constant $a_0 (\propto \frac{H_0}{2\pi})$ is also governed by seismic-induced recession and gravitational weakening since $H_0$ is dictated by the recession constant (8). The external field effect noted by MOND researchers of galactic motion may be due largely to seismic-induced increase in the orbital distances of stars and planets of distant galaxies.

The exact value of Newton’s gravitational constant $G$ may also be derived from the recession constant i.e. $G=Gc/(3600+24)=5.787037037\times10^{-11}$. This could be the original value of $G$. The changes in $G$ as observed on earth (i.e., CODATA value of $G=6.67430\times10^{-11}$Nm²/kg²) possibly indicate the general recession of earth from the sun to maintain a constant force of gravitation on earth. A theoretical study on the Faint Young Sun paradox showed that the astronomical unit in the distant past was 0.866 times the present value [7]. This is the same as the ratio of the initial $G$ derived from this study and the newly accepted CODATA value of $G$. The anomalous recession of planets and moons could be governed mainly by seismic-induced forces. More studies are needed to verify this new idea including the elusive dark matter which could be due to celestial jerks or tremors as earlier [7, 12].

A. Physical Mechanism Generated by Massive Quakes

Gravitational weakening by quake action may govern the accelerating expansion rate of the universe and the repulsive dark energy is due mainly to seismic-induced gravitational weakening and not due to a cosmological constant. Thus, the new model presented here may help in the modification of the accepted $\Lambda$CDM cosmological model. What physical mechanism present in quakes that make this possible? In particular, as quakes occur in celestial objects, the associated high energy acceleration of protons and their collisions may produce a massive number of neutrinos, quarks and other particles that facilitate gravitational repulsion and cosmic expansion. It is highly feasible that an intense light of unlimited brightness or luminosity is present during quakes that triggers every conceivable particle collision and interaction. A physical mechanism similar to flash photolysis accompanied by gamma ray emission may occur during massive quakes. The ultra-intense light pulse of very short duration (femto-millisecond) which may be several orders of magnitude of solar luminosity, may result in the fragmentation of atoms and protons, mobilization or loss of electrons and momentary production of positron and many other elementary particles. Photon-photon collision possibly occurs during seismic action inducing the generation of electron-positron pair with a collision energy that is higher than their rest-mass energy and annihilate each other in a short period of time. Such intense light should induce an irradiant force $F_i$ whose magnitude per unit mass is proportional to the temperature gradient or the luminosity $L$ as [12]:

$$F_i = \frac{16}{3c} \sigma T^4 \frac{dT}{dr} = \frac{3k}{4\pi r^2} L$$  \hspace{1cm} (12)

where $k$ is the opacity [12].

The Friedmann equations governing the Hubble parameter $H$ may need to be revised. Using the Friedmann equation ($\rho_0=3H^2/8\pi G$), the critical mass density $\rho$ of the universe is now estimated to be $9.18\times10^{-27}$kg/m³ using the new $G$ from the recent CODATA value. This is over 13% smaller than ($\rho_0=1.059\times10^{-26}$kg/m³) when the value of $G$ was slightly higher at $5.787\times10^{-11}$Nm²/kg². As gravity weakens and $G$ increases, the density appears to decrease due to cosmic expansion. It is possible that the matter density was previously higher (about 6.34 hydrogen atoms/m³) and decreased to 5.5 atoms/m³ as the universe expands. The observed high matter density in space could be due to photo-induced fragmentation of hydrogen atoms, protons and other particles triggered by intense light during the occurrence of celestial tremors.

It should be noted that the Planck’s constant $h = 8\pi G/\alpha$ [14] may also be influenced by seismic-induced recession because of the dependence of $H$ on $k$, and this implies that the new recession constant may hold from atomic to galactic scales.

The seismic-induced photo-fragmentation of atoms and protons that is proposed here may need experimental verification. It may be related to the mechanism governing flash photolysis (and also the photoelectric effect) applied to hydrogen atoms and its compounds. Initial calculations of its possible effects also yield the mysterious gravitational time reversal problem. If confirmed, this may lead to a successful explanation of some lingering physics problems like the matter-antimatter anomaly, the violation of symmetries in particle physics, and unknown sources of stellar explosion, gamma ray bursts (GRB) and fast radio bursts (FRB).

B. Linking Hubble Constant and Gravitation to Particle Physics and Quantum Mechanics

It is possible that slight changes in the fine structure constant $\alpha$ is also governed by the same process dictating cosmic expansion. This can be seen by using the new Hubble (8) and the Hubble-dependent Planck’s constant of Nikitin (2018) in the original expression for $a = (2\pi Ke^2/\hbar c)$ in which the $K$ is the Coulomb’s constant or electric force constant. Using the recession constant to bridge the gap between gravitation and electromagnetism, the constant $K$ can be approximated by:

$$K = G k c^3$$  \hspace{1cm} (13)

where $k$ is the new recession constant. The physical dimension for $K$ can be obtained by multiplying this by a dimensional constant. This would give a revised permittivity of free space and link it to the magnitude of electro-magnetic fluxes. This would also give a new formula for the Newtonian gravitation constant as:

$$G = \frac{K}{k c^3}$$  \hspace{1cm} (14)

With modern-day values of $K$ and $c$, this would give $G=6.671281907\times10^{-11}$m³/kg·s². Since $k$ is also approximately equal to $GM/c^3$, this gives new approximate formulas for $K$ and $G$ as:

$$K = G^2 M, \quad G = \sqrt{\frac{K}{M}}$$  \hspace{1cm} (15)
This shows that $G$ apparently increases as the solar mass $M$ decreases. This also gives a new model for the central mass $M=KGM^2$ which shows that mass decreases as $G$ increases. Additionally, this further gives for the gravitational mass parameter of the sun, $GM = (KM)^{1/3}$. Coupled with these, (13)-(15) gives a fundamental link of electromagnetic force and gravitational force as one electro-gravity force! This can give us a quantitative description of the speed of gravity or gravitational waves. The speed of light, and apparently the speed of gravity, can now be derived from the new recession constant $k$, the electric force constant $K$ and the Newtonian constant $G$ as:

$$c = \left( \frac{GM}{k} \right)^{1/2} = \left( \frac{K}{kG} \right)^{1/2} = \left( \frac{KM}{k^2} \right)^{1/2}$$

(16)

It is of course possible to relate the speed of gravity to the Hubble constant by manipulating (8) and this is given by $c = k[H_o/k^2]$ in which $\beta$ is the equivalent distance of 1 megaparsec in light-years (i.e. $\beta=3.261,633.44$) and $t$ is time or number of seconds in 1 year. Alternatively, the speed of gravity can be derived easily from (8) as $c = H_o^2/\beta k$ when $H_o$ is given in cosmological units of m/s/MPC.

Using (13) in the original equation for $a$, a new expression for the fine-structure constant that is governed by the Hubble and gravitation constants, can now be obtained as:

$$\alpha = \frac{\pi^2 G e^4 M}{4H_o^2}$$

(17)

where $e$ is the electron charge and $M$ is the solar mass. This leads to a clear fundamental relation between the cosmological Hubble constant, gravitation and quantum particle physics. This gives a value of about 1/137 which is equal to the original value of the fine-structure constant introduced by Sommerfeld a century ago. This further leads to a new fine-structure model showing the interaction between gravitational and electromagnetic forces, and the light-induced electron displacement given by:

$$\alpha = \left[ \frac{32(\pi G e c^5)}{m_\beta} \right]^{1/7} \left( \frac{1}{G} \right)$$

(18)

where, $\beta = 3.261,633.44$ is the cosmic distance scale in light-year/MPC and $m_\beta$ is the electron mass in kg. This is also dimensionless, they give rise to a new theoretical explanation of the origin of $a$ and its relation to the Hubble constant. The 1/17th power law for $a$ would also yield a new energy relation for the photo-induced fragmentation of atoms and collision of photons by virtue of the intense luminosity during celestial tremors as:

$$E = \zeta mc^7$$

(19)

with a dimensional constant $\zeta$ to yield $E$ in joules. This is far more powerful than the conventional energy-mass relation $E=mc^2$. This new energy relation may be behind the mysterious high burst of energy in stellar explosions and neutron stars/magnetars captured by powerful telescopes and those observed by neutrino detectors in the south pole and elsewhere. After a very long-distance propagation from the source of cosmic radiation, it may be possible that the corresponding recorded energy is still within the Peta-electron volt range. A cubic power relation may also be possible corresponding to a lower energy magnitude similar to $E = \zeta mc^7$.

Assuming that only $G$ varies, Eq. (18), the relative change in $a$ is now given by:

$$\frac{d\alpha}{\alpha} = 2 \frac{dG}{G} \left( \frac{1}{137} \right)$$

(20)

This is in the order of $-10^{-5}$-10$^{-6}$ which is the same magnitude that recent measurements indicate [15], [16]. This implies that there is a mean decrease of $a$ that can be explained in the quantum level as a possible change in the internal structure of hydrogen atoms [7] via photo-fragmentation driven by seismic-induced irradiant force. In addition, the photo-mobilization of protons (and electrons) should indicate a slight change in the momentum of the protons and justifies the possible increase in their mass. This would result to a higher proton to electron mass ratio ($\mu$) corresponding to a decrease in $a$ as indicated simply by $a=13.6h^2/\mu$ as introduced earlier in [7]. By virtue of (13)-(15), the gravitational and electromagnetic forces and their response to intense light pulses and photo-fragmentation induced by photons are now interrelated. This unifies gravitation, electromagnetism and quantum mechanics using only simple equations of physics.

The masses of pions and electrons can also be estimated directly from the newly found theoretical value of the Hubble constant and the fine-structure constant. In particular, Perkovic [17] reported that the mass of the pion from Weinberg is given by $m_\pi = (\pi^3H_o/4\pi^2 Gc)^{1/3}$. On the other hand, the mass of the electron can be computed from $m_e = (\pi^2H_o/3\pi^2 Gc)^{1/3}$ in which $G$ is the original value of the Newtonian constant of gravitation when the earth was still at a closer radial distance from the sun (i.e., $G = 5.787037037 \times 10^{-11}$).

C. Light-induced Gravitation

The conventional idea that the gravitational field of a mass can attract other masses has been studied for centuries. We modified this by introducing the concept of seismic-induced forces on celestial masses that could change the gravitational field leading to a model of gravitation that accounts for both repulsive and attractive motion of celestial objects [7]. While it is true that masses can attract other masses, it also is possible that light attracts light and a new mechanism in which light induces gravitational attraction is hereby proposed. The idea of a gravitational field produced by light has been proposed in Tolman et al. [18] but a simpler approach to determine the gravitational force produced by light is presented here. It should be noted that the Newtonian gravitational model can also be modified upon using (13)-(15). This would lead to a modification of the conventional circular acceleration as:
where $V$ is the radial or circular velocity of an orbiting body around a central mass $M$, $K$ is the Coulomb's electric force constant, and $R$ is the orbital radius. For our solar system, $M$ is simply the solar mass. The last two equalities, when multiplied with a dimensional constant similar to $G$, yields a gravitational acceleration based on the action of electromagnetic radiation or light. This further leads to a new model for the orbital velocity that is now related to the strength of the electromagnetic force in a quartic equation given by (first and last equalities):

$$V^4 = \frac{KM}{R^2}$$  \hspace{1cm} (22)

This is similar to the conventional Tully-Fisher relation in which mass is proportional to the $4^{th}$ power of the orbital velocity $V$. Using (13) and (22), the Hubble constant can now be written as:

$$H_0 = \frac{1}{c} \sqrt{\frac{\beta K}{G}} = \frac{V^2 R}{c^2} \sqrt{\frac{\beta}{GM}} = \frac{V^2 R}{c^2} \sqrt{\frac{\beta}{kc}}$$  \hspace{1cm} (23)

in which $V$ is the earth's orbital velocity with $R$ being its radial distance from the sun (i.e., the astronomical unit). With the cosmic-distance scale $\beta = 3.261,564$, (23) yields $H_0$ in units of m/s/Mpc. When the speed of gravity is derived from the first equality, a fundamental inverse relation between the speed of a gravitational wave and the Hubble constant is given by $\phi = \frac{c}{H_0} (\beta K/G)^{1/2}$. Again, the (23) can be multiplied with a dimensional constant so that the dimension of $H_0$ is consistent with cosmological units. The resulting Hubble constant can be converted to SI units of inverse time by diving the results by $3.08567 \times 10^{22}$ m/MPC. Equation (23) now gives a clear fundamental relation between the Hubble constant, the electric force constant, the speed of light, and the earth's dynamic motion around the sun. The last 2 forms of the equation show that the Hubble constant largely depends on the orbital velocity and radial distance of the planet earth from the sun. It was shown earlier in [7] how the anomalous increase of the astronomical unit is governed by gravitational weakening caused by massive quakes. Hence, the accelerating tendency of the expansion rate of the universe as observed on earth may just be due to the accelerating changes in the orbital distance of the planet earth from the sun, its orbital velocity and the gravitational constant which are all inter-related. It is worthwhile to note that the second equality containing the term $V^2 R/c$ shows how the expansion rate is governed by a dispersion term in units of m$^2$/s. This is analogous to the rate of dispersion of particles in a fluid medium by the diffusive action of the fluid flow itself. Equation (23) is also inversely proportional to the speed of light and indicates that the Hubble rate of expansion is a gravity-related problem and that the speed of gravity is governed by the speed of light. This is not surprising since observations like the gravitational wave event GW 170817 already showed that a gravitational wave travels at the speed of light. Hence, the speed of cosmic expansion must also be governed by light propagation, travelling as an electromagnetic radiation.

From (21), the Newtonian gravitational law may now be written as:

$$F = \frac{G M m}{R^2} = \frac{m k c^3}{R^2} = \frac{m \sqrt{K M}}{R^2}$$  \hspace{1cm} (24)

where $m$ is the mass of the orbiting body and $k$ is our newly-found recession or expansion constant. For the solar system, the gravitational force exerted by the sun on earth is about $3.560735 \times 10^{22}$ N when using the second equality. This is almost the same magnitude of force computed from the conventional Newtonian model. This shows that photons, and not the hypothetical gravitons, can be the mediator for gravitational force similar to electromagnetic force and this confirms the fundamental linkage of the two forces. The observed speed of gravitational waves travelling at the speed of light also confirms that photons are the carrier of gravitational force. Multiplying the last two terms of (24) by a dimensional constant also yields $F$ in Newton as the standard unit of force.

Equations (21)-(24) are new and show a fundamental connection between gravitation, light and electromagnetism, and the Hubble constant, as the electric force constant $K$ and speed of light $c$ can be used in place of the Newtonian gravitational constant $G$. The gravito-electromagnetic interaction can also be derived upon using (13) where the permeability constant of free space is now given by:

$$\mu_0 = 4 \pi G k c$$  \hspace{1cm} (25)

On the other hand, the permittivity constant of a vacuum is now given by:

$$\varepsilon_0 = \frac{1}{4 \pi G k c^3}$$  \hspace{1cm} (26)

These now give a fundamental link between the electromagnetic and gravitational forces which were not known previously. They now give a modified permittivity and permeability constants of free space and link gravitation to the magnitude of electro-magnetic fluxes. The resulting electric and magnetic forces can now be estimated using these new gravito-electromagnetic constants. In addition, (26) would lead to a fundamental linkage of the Maxwell’s equations of electromagnetism to gravitation. The first equation of Maxwell on the electric field flux $E$ now becomes $\nabla \cdot E = 4 \pi G k e c^2 = k c \nabla^2 \varphi = G M \nabla^2 \varphi = 4 \pi k \rho$, where $\varphi$ is the gravitational potential and $\rho$ is the charge density. Upon using (15), we obtain:

$$G \nabla \times E = k \nabla^2 \varphi$$  \hspace{1cm} (27)

Therefore, Gauss law now includes the effect of gravitation. Using this new form of Gauss Law, the magnetic flux density and its time variation (i.e. Faraday’s Law $\nabla \times E = -\frac{\partial B}{\partial t}$) would then produce the same rotational electric field. Furthermore, the Maxwell-Ampere Law is partly modified in
\( \mu_0 (=4\pi G \kappa c) \) to include gravitational interaction. Hence, the differential equations governing the magnetic field and electric field fluxes in the Maxwell’s equations are tied up to gravitation using (25), (26). Yet, they should essentially produce the same results.

In addition, the magnetic force \( F_m \) and electric force \( F_e \) for two separate charges \( q_1 \) and \( q_2 \) separated by a distance \( r \), are now given by:

\[
F_m = G k_c \frac{q_1 q_2}{r^2} \quad \text{and} \quad F_e = G k_c^3 \frac{q_1 q_2}{r^4}
\]  

Again, the correct dimensions can be obtained by multiplying them with a dimensional unitary constant. The gravitational-magnetic-electric field interaction described in Zhu [19] further yields a modified acceleration formula when (25), (26) are used in the newly found variations of the gravitational acceleration \( \Delta g \). With the correct magnetic flux density \( B \) (and electric field intensity \( E \)), this should lead to a similar variation in the gravitational acceleration

\[
\Delta g = B f (4\pi k_c)^{1/2}
\]

where \( f \) is the gravitational redshift parameter. The term in the parenthesis is constant on the earth’s surface and small changes in \( B \) produces a discernible gravitational acceleration. This may lead to a possible manipulation of gravitational force in the near future.

Equations (24)-(28) could be the theoretical explanation behind the attractive mechanism that was found in a series of experimental measurements conducted in Rancourt [20]. As laser light is applied between two small masses, the attractive force between the two masses appeared to increase slightly. It is also possible that the Casimir effect is just a manifestation of the phenomenon of light-induced gravitational attraction. Following (12), light attracts objects around it and the force of attraction should be proportional to luminosity \( L \) (or brightness \( \mu \) since \( \mu = L/4\pi r^2 \)) and inversely proportional to the speed of light \( c \) as:

\[
F = m \frac{3k_c L}{4\pi c r^2} = \beta mL \frac{\mu mL}{4\pi r^2}
\]

(29)

where the constant \( \beta = \frac{3k_c}{c} \), \( m \) is mass, and \( r \) is radial distance. This and (12) is a new yet simple theoretical explanation of the light-induced gravitational attraction that may explain the results of the physical experiments described above. It can also explain how a large planet can orbit a smaller but brighter star with intense luminosity. The exact value of the proportionality constant may be determined through further experimentation. Applying this to galactic or planetary system should also be feasible with a different value of the constant of proportionality.

Using the new light-induced gravitational law, calculations with (22) and (24) by utilizing the known orbital distances and masses of planets in the solar system also showed similar results with the Newtonian gravitational model both in terms of the orbital velocity \( V \) and the gravitational force \( F \) exerted on the planets by the sun. It also worked well for the larger moons around the gaseous planet Jupiter (by adjusting their dimensional constant). The case of the single moon orbiting the earth may also be described by this simple law of gravitation since the earth is about 50 times brighter than a full moon when viewed from outer space. Hence, the moon should be attracted to the earth by virtue of the luminosity of the earth. Of course, this is just equal to the conventional Newtonian law of gravitation where masses are mutually attracted. However, it may disprove the conventional idea that gravity bends light since it now appears that light induces gravity.

When massive quakes govern the motion of celestial bodies in the solar system for instance, this may further lead to a modified gravitational law similar to (1):

\[
F = \frac{mk_c^3}{R_o^2} \left(1 - \ln \frac{R^2}{R_o^2} \right), \quad F = m\sqrt{KM} \left(1 - \ln \frac{R^2}{R_o^2} \right)
\]

(30)

where \( R_o \) is the mean orbital radius and \( R \) is the seismic-perturbed orbital radius. The advantage in using this modified gravitational law is that it allows the gravitational force to be both attractive and repulsive which is analogous to the attraction and repulsion experienced by charged particles under electromagnetism. The modified gravitational potential from the seismic-perturbed motions of celestial bodies may also be derived from (24) as:

\[
\phi = -\frac{k_c^3}{R_o} \left(1 + \ln \frac{R^2}{R_o^2} \right) = -\sqrt{KM} \left(1 + \ln \frac{R^2}{R_o^2} \right)
\]

(31)

where \( \phi \) is the modified gravitational potential. Again, these relations can be adjusted by multiplying them by a unitary dimensional constant to yield the correct physical dimension. This may need further experimental verification.

Aside from a fundamental link with the electromagnetic force, the gravitational force and the strong nuclear force can now be coupled together. Using Nikitin’s relation for the Planck’s constant and the new relation above for the fine structure constant (17), we obtain a fundamental relation between the strong force and the gravitational force. Using further the new relation \( K = G^2 \) (15), we obtain the magnitude of the strong force \( F_s \) as:

\[
F_s = \frac{3\alpha h c}{4r_i^2} = \frac{3\pi G^2 \epsilon^2 M}{2r_i^2} = \frac{3\pi K e^2}{2r_i^2}
\]

(32)

where \( r_i \) is the small distance between the proton and neutron (and other elementary particles) inside the nucleus of an atom. Again, this can be multiplied with a dimensional constant to get the desired unit of force.

**IV. CONCLUSIONS**

The theoretical value of the Hubble constant \( H_0 \) has been determined using the gravitational weakening and seismic-driven recession model. It appears that the rate of cosmic expansion is governed by seismic-induced recession of celestial objects which is parameterized by a new constant of recession due to gravitational weakening. This may represent an external field effect but truly originates from orbital expansion due to seismic-induced gravitational weakening. As shown, the dynamic motion of the earth governs the Hubble constant and it is highly probable that the observed
accelerating rate manifested by the Hubble constant is related to the increasing rate of rotation of the planet earth as its orbit expands. It should be noted that the Hubble constant can also be derived from \( H = (k/r)^{1/2} \) in which \( r \) is time in number of seconds in a year and \( k \) is the new recession constant. When the total number of hours in a day is reduced to 23.89 and not 24 hours, the Hubble constant is about 69.921 Km/s/MPC. It is thus probable that as the earth’s orbit expands due to gravitational weakening induced by massive quakes, the planet rotates faster to conserve angular momentum, leading to a shorter length of day.

Assuming that the solar system is at the center of the cosmos (where the earth is similar to an electron orbiting the sun like a nucleus), the seismic-propelled gravitational weakening and recession of our planet earth may appear to cause shifting motion of distant stars and galaxies as well. This is shown by the observed red-shift of distant galaxies. This was also shown earlier [7] when the apparent shifting positions of celestial objects like distant stars is largely dictated by the anomalous refraction caused by the motion of the planet earth itself. Such a shifting motion of recession is exactly quantified by the new Hubble constant derived in this study. More massive tremors may therefore mean that the speed of cosmic expansion appears accelerating. The accelerating expansion of the universe could be due largely to non-linear increase of seismic activities occurring on earth and in celestial objects scattered throughout the universe. Parallax observations from the Gaia space telescope of the European Space Agency and the upcoming ultra-precise James Webb Space Telescope may help confirm the effect and presence of cosmic tremors on the altered cosmic distances and the newly determined value of the Hubble constant. Uncertainty in the estimated cosmic distances and proper time appears to be the major reason of the Hubble tension.

New relations for the speed of light, speed of gravity, orbital velocity, gravitational force, and the Hubble constant were further derived from the new recession constant and approximate relations for the Newtonian constant \( G \) and electric force constant \( K \). This finally resulted in a modified Newtonian gravitational law and a theoretical explanation of the phenomenon of light-induced gravitational attraction. The new gravitational model allows for both repulsive and attractive forces analogous to the electromagnetic force experienced by charged particles. The results presented here can now be used to unify the fundamental forces of nature – gravitational, electromagnetic, and strong nuclear forces. Even without resorting to a quantum expression of the space-time of General Relativity as required by Quantum Gravity, the gravitational force is now quantized as new expression for light-induced gravitation becomes apparent as this study shows. Photons appear to be the force-carrier for gravitation similar to electromagnetic force, and that light can induce gravitation as confirmed by other investigators.

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