Predictions for $\mu \rightarrow e\gamma$ in SUSY from non trivial Quark-Lepton complementarity and flavor symmetry

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Abstract

We compute the effect of non diagonal neutrino mass in $l_i \rightarrow l_j \gamma$ in SUSY theories with non trivial Quark-Lepton complementarity and a flavor symmetry. The correlation matrix $V_M = U_{CKM} U_{PMNS}$ is such that its $(1,3)$ entry, as preferred by the present experimental data, is zero. We do not assume that $V_M$ is bimaximal. Quark-Lepton complementarity and the flavor symmetry strongly constrain the theory and we obtain a clear prediction for the contribution to $\mu \rightarrow e\gamma$ and the $\tau$ decays $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$. If the Dirac neutrino Yukawa couplings are degenerate but the low energy neutrino masses are not degenerate, then the lepton decays are related among them by the $V_M$ entries. On the other hand, if the Dirac neutrino Yukawa couplings are hierarchical or the low energy neutrino masses are degenerate, then the prediction for the lepton decays comes from the $U_{CKM}$ hierarchy.

keywords: Neutrino mass matrices, SUSY, Quark-Lepton complementarity, flavor symmetry, $\mu \rightarrow e\gamma$

1 Introduction

The present experimental situation is such that we are very close to obtain a theory of flavor that is able to explain in a clear way all the Standard Model masses and mixing. The last but not least experimental ingredient has been the neutrino data and the determination of $\Delta m^2_{12}$, $|\Delta m^2_{23}|$, $\theta_{12}$ and $\theta_{23}$. From all these results we are able to extract strong constraints on the flavor structure of the SM. In particular the neutrino data were determinant to clarify the role of the discrete symmetry in flavor physics.

The disparity that nature indicates between quark and lepton mixing angles has been viewed in terms of a 'Quark-Lepton complementarity' (QLC) [1] [2] which can be expressed in the relations

$$\theta_{12}^{PMNS} + \theta_{12}^{CKM} \simeq 45^\circ; \quad \theta_{23}^{PMNS} + \theta_{23}^{CKM} \simeq 45^\circ. \quad (1)$$

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Despite the naive relations between the PMNS and CKM angles, a detailed analysis shows that the correlation matrix $V_M = U_{CKM}U_{PMNS}$ is phenomenologically compatible with a tribimaximal pattern, and only marginally with a bimaximal pattern. Future experiments on neutrino physics, and in particular in the determination of $\theta_{23}$ and the CP violating parameter $J$, will be able to better clarify if a trivial Quark-Lepton complementarity, i.e. $V_M$ bimaximal, is ruled out in favor of a non trivial Quark-Lepton complementarity, i.e. $V_M$ tribimaximal or even more structured \cite{3}. From present experimental evidences a non trivial Quark-Lepton complementarity arises \cite{4}. Moreover the clear non trivial structure of $V_M$ and the strong indication of gauge coupling unification allow us to obtain in a straightforward way constraints on the high energy spectrum too. Within this framework we get information about flavor physics from the correlation matrix $V_M$ too. It is very impressive that for some discrete flavor symmetries such as $A_4$ dynamically broken into $Z_3$ \cite{5,6,7} or $S_3$ softly broken into $S_2$ \cite{8,9,10} the tribimaximal structure appears in a natural way.

In supergravity theories if the effective Lagrangian is defined at a scale higher than the Grand Unification scale, the matter fields have to respect the underlying gauge and flavor symmetry. Hence, we expect quark-lepton correlations among entries of the sfermion mass matrices. In other words, the quark-lepton unification seeps also into the SUSY breaking soft sector \cite{12}. In general we do not get strongly renormalization effects on flavor violating quantities from the heavy neutrino scale to the electroweak scale because of the absence of flavor violation. In fact the remaining flavor violation related to the low energy neutrino sector gives a negligible contribution with the exception of the case with highly degenerate neutrinos and $\tan \beta > 40$ \cite{13,14}.

In this work we compute the effect of non diagonal neutrino mass in $l_i \rightarrow l_j \gamma$ in SUSY theories with non trivial Quark-Lepton complementarity and flavor symmetry. In comparison with previous works (i.e. \cite{15,16}), where a bimaximal $V_M$ matrix is assumed, in the present work the correlation matrix $V_M = U_{CKM}U_{PMNS}$ is such that its $(1,3)$ entry, as preferred by experimental data, is zero. All the other entries are assumed to vary as allowed by the experimental data \cite{3,4}. Nevertheless We obtain a clear prediction for the contribution to $l_i \rightarrow l_j \gamma$. By using the non trivial Quark-Lepton complementarity, flavor symmetry, and the see-saw mechanism we will compute the explicit spectrum of the heavy neutrinos. This will allow us to investigate the relevance of the form of $V_M$ in $l_i \rightarrow l_j \gamma$. There are three cases. They depend on the spectrum of the Dirac neutrino mass matrix and the low energy neutrinos. We may have: 1) hierarchical Dirac neutrino eigenvalues (in this case we have very hierarchical right-handed neutrino masses); 2) degenerate Dirac neutrino eigenvalues, with non degenerate low energy neutrino masses (in this case the hierarchy of the right-handed neutrino masses is close to the one of the low energy spectrum); 3) degenerate Dirac neutrino eigenvalues and low energy neutrino spectrum (that implies right-handed neutrinos close to degenerate). For each of these cases we have different contributions to $l_i \rightarrow l_j \gamma$. 

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We will show that only when Dirac neutrino eigenvalues are degenerate and low energy neutrino masses are not degenerate, the explicit form of $V_M$ plays an important role.

The plan of the work is the following. In Sec. 2 we explain our notations and clarify the meaning of the correlation matrix $V_M$ in flavor theories. In Sec. 3 we introduce the relation between $l_i \rightarrow l_j \gamma$ and the Dirac neutrino matrix. In Sec. 4 we relate the Dirac neutrino Yukawa coupling to the $CKM$ mixing matrix by using the non trivial Quark-Lepton complementarity and flavor symmetry. Then we compute the heavy neutrino spectrum. In Sec. 5 we compute the value of the contribution to the $l_i \rightarrow l_j \gamma$ processes from a non diagonal Dirac neutrino Yukawa coupling. Finally in Sec. 6 we report our conclusions.

2 Notations

In this section we explain the relation between the product $V_M = U_{CKM}U_{PMNS}$ and the diagonalization of the right-handed neutrino mass.

2.1 $V_M$ in theories with see-saw of type I

Let’s fix the notations in the lepton sector. Let $Y_l$ be the Yukawa matrix for charged leptons. It can be diagonalized by

$$Y_l = U_l Y^\Delta_l V_l^T.$$  \(2\)

Let $M_R$ be the Majorana mass matrix for the right-handed neutrino and $M_D$ the Dirac mass matrix. Under the assumption that the low energy neutrino masses are given by the see-saw of Type I we have that the light neutrino mass matrix is given by

$$M_\nu = M_D \frac{1}{M_R} M^T_D.$$  \(3\)

Let us introduce $U_0$ from the diagonalization of the Dirac mass matrix

$$M_D = U_0 M^\Delta_D V_0^T,$$  \(4\)

then we define $V_M$ by the diagonalization of the light neutrino mass

$$M_\nu = U_\nu M_\nu^\Delta U_\nu^T$$
$$= U_0 V_M M_\nu^\Delta (V_M)^T U_0^T,$$  \(5\)

with the constraint that $U_0 V_M$ is an unitary matrix. Finally the lepton mixing matrix is

$$U_{PMNS} = U_l^T U_\nu = U_l^T U_0 V_M.$$  \(6\)
Let us introduce the following symmetric complex matrix $C$

$$C = M^\Delta_V V_0^\dagger \frac{1}{M_R} V_0^* M^\Delta_D ,$$  \hspace{1cm} (7)$$

where $V_0$ is the mixing matrix that diagonalizes on the right the Dirac neutrino mass matrix. From eqs. (4-5) we see that the inverse of $V_M$ diagonalizes the symmetric matrix $C$, in fact we have

$$V_M M^\Delta V^T_M = C. \hspace{1cm} (8)$$

2.2 Flavor symmetry implies $V_M$ as correlation matrix

In the quark sector we introduce $Y_u$ and $Y_d$ to be the Yukawa matrices for up and down sectors. They can be diagonalized by

$$Y_u = U_u Y^\Delta_u V^\dagger_u \quad \text{and} \quad Y_d = U_d Y^\Delta_d V^\dagger_d , \hspace{1cm} (9)$$

where the $Y^\Delta$ are diagonal and the $U$s and $V$s are unitary matrices.

Then the quark mixing matrix is given by

$$U_{CKM} = U_u^\dagger U_d . \hspace{1cm} (10)$$

To relate the $U_{CKM}$ with the $U_{PMNS}$ normally one makes use of GUT models, such as generic $SO(10)$ or $E_6$, where there are some natural Yukawa unifications. In fact these cases give an interesting relation between the $U_{CKM}$ quark mixing matrix, the $U_{PMNS}$ lepton mixing matrix and $V_M$ obtained from eq. (7). The mixing matrix $V_M$ turns out to be the correlation matrix defined in eq. (12). The reason for it is that in $SO(10)$ or $E_6$ one has intriguing relations between the Yukawa couplings of the quark sector and that of the lepton sector. For instance, in minimal renormalizable $SO(10)$ with Higgs in the $10, 126,$ and $120$, we can have $Y_l \approx Y^T_d$.

However this feature is much more general and may depend on the flavor symmetry instead of the gauge grand unification. The presence of a flavor symmetry usually implies the structure of the Yukawa matrices and the equivalent entries of $Y_l$ and $Y_d$ are of the same order of magnitude. We conclude that, as long as the flavor symmetry fully constrains the mixing matrices that diagonalize the Yukawa matrices, we have $U_l \simeq V_d^*$. Notice that if there is a flavor symmetry that constrains the Yukawa couplings in such a way that the diagonalizing unitary matrices are fixed, then the entries of $Y_l$ can still be very different from the entries of $Y_d^T$. However both Yukawa matrices are diagonalized by the same mixing matrices. This is exactly the case in the presence of an $A_4$ discrete flavor symmetry dynamically broken into $Z_3 \ [3, 6, 7]$ and can be partially true in the case of $S_3$ softly broken into $S_2 \ [8, 10]$.

From eq. (6) we get

$$U_{PMNS} \simeq V_d^T U_0 V_M .$$
If we denote by $Y_\nu$ the Yukawa coupling that generates the Dirac neutrino mass matrix $M_D$, we have also the relation

$$Y_\nu \approx Y_u^T \rightarrow U_0 \simeq V_u^*.$$  \hspace{1cm} (11)

This relation, together with the previous one, implies

$$U_{PMNS} \simeq V_d^* V_u^* V_M.$$  

If the Yukawa matrices are diagonalized by a similar matrix on the left and on the right, for example in minimal renormalizable $SO(10)$ with only small contributions from the antisymmetric representations such as $120$ or more important in models where the diagonalization is strongly constrained by the flavor symmetry, the previous relationship translates into a relation between $U_{PMNS}$, $U_{CKM}$ and $V_M$. In fact we have

$$Y_u \simeq Y_u^T \rightarrow V_u^* = U_u \hspace{0.5cm} \text{and} \hspace{0.5cm} Y_d \simeq Y_d^T \rightarrow V_d^* = U_d.$$  

The first relation tells us that

$$U_{PMNS} = V_d^T U_u V_M.$$  

Finally, using the second relation in eq. (12) and the definition of the CKM mixing matrix of eq. (10) we get

$$V_M = U_{CKM} \cdot \Omega \cdot U_{PMNS},$$  \hspace{1cm} (12)

where we introduced the matrix

$$\Omega = \text{diag}(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3})$$  \hspace{1cm} (13)

to allow us to write the CKM and PMNS matrices in their standard form (i.e. three rotation angles and one phase for the CKM and the equivalent for the PMNS) and to take into account the phase mismatching between quarks and leptons. The form of $V_M$ can be obtained under some assumptions about the flavor structure of the theory. Some flavor models give for example a correlation $V_M$ with $(V_M)_{13} = 0$. As a consequence of the from of the non trivial Quark-Lepton complementarity there are some predictions from the model, such as for $\theta_{13}^{PMNS}$ from [4] and the correlations between CP violating phases and the mixing angle $\theta_{12}$ of [3].

### 3 The observables

As explained in the introduction, in this work we are interested in extracting informations from non trivial quark-lepton complementarity and flavor symmetry about the $l_i \rightarrow l_j \gamma$ decays. We report here the usual formula obtained in the literature on these processes. It is obtained in the weak eigenstate neutrino base, where charged lepton and Majorana right-handed neutrino mass matrices and weak interactions are diagonal. These processes depends on $M_D$, the Dirac neutrino mass in the weak base.
3.1 \( l_i \to l_j \gamma \)

The contribution at first order approximation to the process \( l_i \to l_j \gamma \) in SUSY models is given by

\[
BR(l_i \to l_j \gamma) \propto \frac{\Gamma(l_i \to e\nu\nu)}{\Gamma(l_i)} \frac{\alpha^2}{G_f m_0^2 v_u^4} \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2 \left| (\tilde{M}_D L \tilde{M}_D^T)_{ij} \right|^2
\]

where \( m_0 \) is the universal scalar mass, \( A_0 \) is the universal trilinear coupling parameter, \( \tan \beta \) is the ratio of the vacuum expectation values of the up and down Higgs doublets, and \( m_s \) is a typical mass of superparticles with \( m_s^8 \approx 0.5m_0^2M_{1/2}^2(m_0^2 + 0.6M_{1/2}^2)^2 \), where \( M_{1/2} \) is the gaugino mass. The matrix \( L_{ij} = \delta_{ij} \log M_3/M_1 \) takes into account the RGE effects on the Majorana right-handed neutrino masses. In fact the eq. (14) is computed in the base where the Yukawa of the charged lepton and the Majorana neutrino mass are diagonal. Eq. (14) is valid in the base where right-handed Majorana neutrino mass matrix, charged lepton mass matrix and weak gauge interactions are diagonal. The experimental limit for the branching ratio of \( \mu \to e\gamma \) is \( 1.2 \times 10^{-11} \) at 90\% of confidence level [18] and it could go down to \( 10^{-14} \) as proposed by MEG collaboration.

4 \( \tilde{M}_D \) from non trivial Quark-Lepton complementarity and flavor symmetry

Let us investigate the value of Dirac neutrino mass matrix \( \tilde{M}_D \) in the base where right-handed Majorana neutrino mass matrix, charged lepton mass matrix and weak gauge interactions are diagonal. The part of the Standard Model Lagrangian containing the leptons is

\[
\mathcal{L} = \bar{\nu}_L Y_D \nu_R H + \nu_R^T C M_R \nu_R + \bar{\ell}_L Y_\ell \ell_R H + \bar{\nu}_L W \ell_L .
\] (15)

We want to redefine the fields in such a way that the only source of flavor violation is in the Dirac neutrino Yukawa coupling. We introduce the following definitions

\[
l'_R = V_i^\dagger l_R, \quad \nu'_R = V_R^T \nu_R, \quad l'_L = U_i^\dagger \ell_L, \quad \nu'_L = U_i^\dagger \nu_L,
\]

where the unitary matrices \( V_i, U_i \) are defined in eqs. (2). The unitary matrix \( V_R \) is defined by the diagonalization of \( M_R \)

\[
V_R M_R^{\Delta R} V_R^T = M_R .
\]

Consequently we have

\[
l_R = V_i l'_R, \quad \nu_R = V_R^* \nu'_R, \quad \ell_L = U_i l'_L \quad \text{and} \quad \nu'_R = (\nu'R)^T V_R^\dagger, \quad \ell_L = U_i^\dagger U_i^\dagger, \quad \bar{\nu}_L = \bar{\nu}'_L U_i^\dagger .
\]
In this primed base we get
\[ \mathcal{L} = \bar{\nu}_L U_l^\dagger M_D V_R^* \nu'_R + (\nu'_R)^T C M_R^\Delta \nu'_R + \bar{l}_L M^\Delta l_R + \bar{\nu}_L W \nu'_L \] (18)
and we define
\[ \tilde{M}_D = U_l^\dagger M_D V_R^* \] (19)

We want now to relate this \( \tilde{M}_D \) matrix to the CKM mixing matrix by using the non trivial Quark-Lepton complementarity and flavor symmetry. First of all we rewrite this matrix as
\[ \tilde{M}_D = U_l^\dagger M_D V_R^* = U_l^\dagger U_0 M_D^\Delta V^{*T}_0 V_R^* \] (20)

Then we notice that the matrix \( V^{*T}_0 V_R^* \) is related via the \( C \) matrix to the diagonal low energy neutrino mass matrix \( m^\Delta_{\text{low}} \) and to \( V_M \). In fact we have
\[ V_M m^\Delta_{\text{low}} V_M^T = C = M_D V^{*T}_0 V_R^* \] (21)

where we used the inverse of eq. (17)
\[ V_R^* \frac{1}{M^\Delta_R} V_l^\dagger = \frac{1}{M_R} \] (22)

We multiply on the left and on the right both sides of eq. (21) by \( 1/M_D^\Delta \) and we get
\[ V^{*T}_0 V_R^* \frac{1}{M^\Delta_R} V_R^T V^{*T}_0 = \frac{1}{M_D^\Delta} V_M m^\Delta_{\text{low}} V_M^T \frac{1}{M_D^\Delta} \] (23)

If one uses the method of [19] one can extract the matrix \( V^{*T}_0 V_R^* \) by making the square root of the matrices in eq. (23). One has
\[ V^{*T}_0 V_R^* \frac{1}{M^\Delta_R} = \frac{1}{M_D^\Delta} V_M \sqrt{m^\Delta_{\text{low}}} R^T \] (24)

where \( R \) is a complex orthogonal matrix such that \( R^T R = 1 \), and one obtains
\[ V^{*T}_0 V_R^* = \frac{1}{M_D^\Delta} V_M \sqrt{m^\Delta_{\text{low}}} R^T \] (25)
Finally one concludes that

$$\tilde{M}_D = U_D^\dagger U_0 M_D^\Delta \frac{1}{M_D} V_M \sqrt{m_{\text{low}}^\Delta R^T \sqrt{M_R^\Delta}}$$  \hspace{1cm} (26)

$$= U_{PMNS} \sqrt{m_{\text{low}}^\Delta R^T \sqrt{M_R^\Delta}}.$$  \hspace{1cm} (27)

Notice that in eq. (27) does not appear the matrix $V_M$, and any information from $V_M$ is hidden into the $R$ matrix.

In our discussion however eq. (23) unequivocally fixes $V_0^\dagger V_R^*$ and the $R$ matrix, once we know the eigenvalues of the Dirac neutrino mass matrix and the low energy neutrino spectrum. In fact the $V_M$ matrix is assumed to be known because of the non trivial Quark-Lepton complementarity. Once we computed the $V_0^\dagger V_R^*$ matrix form eq. (23), by using eq. (20), we get

$$\tilde{M}_D = U_D^\dagger U_0 M_D^\Delta V_0^\dagger V_R^*$$

$$= U_{PMNS} V_M^\dagger M_D^\Delta V_0^\dagger V_R^*$$

$$= \Omega U_{CKM}^\dagger M_D^\Delta V_0^\dagger V_R^*,$$  \hspace{1cm} (28)

where in the last line we used the relations in eq. (6) and (12).

### 4.1 Full determination of $V_0^\dagger V_R^*$ and $M_R^\Delta$

Eq. (28) is the equivalent of the general eq. (27) in presence of non trivial Quark-Lepton complementarity and flavor symmetry. We observe that the main modification is the presence of $U_{CKM}^\dagger$ instead of $U_{PMNS}$ thanks to the fact the these matrices are related to each other through $V_M$ as shown in eq. (12). Moreover the $R$ is absent and is substantially substituted by the known $V_0^\dagger V_R^*$ matrix, computed with eq. (23). Let us now compute the $V_0^\dagger V_R^*$ matrix in a general scenario.

In the following we use the experimental constraint from [4] that says that $(V_M)_{13}$ is zero. With this single constraint on $V_M$ we write

$$V_M = \begin{pmatrix}
\cos \theta_{12} & \sin \theta_{12} & 0 \\
-\sin \theta_{12} \cos \theta_{23} & \cos \theta_{12} \cos \theta_{23} & \sin \theta_{23} \\
\sin \theta_{12} \sin \theta_{23} & -\cos \theta_{12} \sin \theta_{23} & \cos \theta_{23}
\end{pmatrix}$$  \hspace{1cm} (29)

and the allowed ranges for $\theta_{12}^{V_M}$ and $\theta_{23}^{V_M}$ are [4]

$$\tan^2 \theta_{12}^{V_M} \in [0.3, 1.0] \quad \text{and} \quad \tan^2 \theta_{23}^{V_M} \in [0.5, 1.4].$$  \hspace{1cm} (30)

Let us denote by $m_i$ the complex low energy neutrino masses obtained after the see-saw $(m_{\text{low}}^\Delta = \{m_1, m_2, m_3\})$, and $M_1$ the eigenvalues of the Dirac neutrino mass matrix $M_D$ ($M_D^\Delta = \{M_1, M_2, M_3\}$). We have $V_M m_{\text{low}}^\Delta V_M^\dagger$ equal to

$$\begin{pmatrix}
(m_1 c_{12}^2 + m_2 s_{12}^2) & -(m_1 - m_2) c_{12} c_{23} s_{12} & (m_1 - m_2) c_{12} s_{12} s_{23} \\
-(m_1 - m_2) c_{12} c_{23} s_{12} & (m_1 s_{12}^2 c_{23}^2 + m_2 c_{12}^2 c_{23}^2 + m_3 s_{23}^2) & s_{23} c_{23} (m_3 - m_2 c_{12}^2 - m_1 s_{12}^2) \\
(m_1 - m_2) c_{12} s_{12} s_{23} & s_{23} c_{23} (m_3 - m_2 c_{12}^2 - m_1 s_{12}^2) & s_{23} (m_1 s_{12}^2 + m_2 c_{12}^2) + m_3 c_{23}^2
\end{pmatrix}$$  \hspace{1cm} (31)
and from eq. (25) we get

\[
\begin{pmatrix}
\frac{(m_1^2 + m_2^2 s_{12}^2)}{M_1^2} & -\frac{(m_1 - m_2) c_{12} s_{12}}{M_1 M_2} & \frac{(m_1 - m_2) c_{12} s_{12} s_{23}}{M_1 M_3} \\
\frac{-(m_1 - m_2) c_{12} c_{23} s_{12}}{M_1 M_2} & \frac{(m_1 s_{12}^2 + m_2 c_{12}^2 s_{12} + m_3 s_{23})}{M_2^2} & \frac{s_{23} c_{23} (m_3 - m_2 c_{12} - m_1 s_{12})}{M_2 M_3} \\
\frac{(m_1 - m_2) c_{12} s_{12} s_{23}}{M_1 M_3} & s_{23} c_{23} (m_3 - m_2 c_{12}^2 - m_1 s_{12}^2) & \frac{s_{23}^2 (m_1 s_{12}^2 + m_2 c_{12}^2) + m_3 c_{23}^2}{M_3^2}
\end{pmatrix}.
\]

Eq. (32) is general and must be specified depending on the explicit form of \( V_M \). For example for \( V_M \) tribimaximal we get

\[
V_0^\dagger V_R^* \frac{1}{M_R^\Delta} V_R^\dagger V_0^* = \begin{pmatrix}
\frac{2m_1 + m_2}{3M_1^2} & \frac{m_1 - m_2}{3M_1 M_2} & \frac{m_1 - m_2}{3M_1 M_3} \\
\frac{m_1 - m_2}{3M_1 M_2} & \frac{m_1 + 2m_2 + 3m_3}{6M_2^2} & \frac{m_1 + 2m_2 + 3m_3}{6M_2 M_3} \\
\frac{m_1 - m_2}{3M_1 M_3} & \frac{m_1 + 2m_2 + 3m_3}{6M_2 M_3} & \frac{m_1 + 2m_2 + 3m_3}{6M_3^2}
\end{pmatrix}
\]

(33)

where we remind the reader that \( m_i \) are complex numbers, and their sign is not defined.

### 4.2 Hierarchical \( M_D \)

First of all let us investigate the case where the \( M_D \) eigenvalues have a hierarchical structure as well as any other Dirac mass matrix \( M_u, M_d, M_l \). As it is well known in this case the heavy neutrino masses are very hierarchical and the lighter one is very light compared to the unification scale. For example if we take the eigenvalues of the Dirac mass matrix \( M_D \) to be \( M_3 \{ \lambda^{2n}, \lambda^n, 1 \} \) with \( n \) of order 1, we get\(^2\)

\[
\frac{1}{M_R^\Delta} = \begin{pmatrix}
m_\alpha / (\lambda^{4n} M_3^2) & 0 & 0 \\
0 & m_\beta / (\lambda^{2n} M_3^2) & 0 \\
0 & 0 & m_\gamma / M_3^2
\end{pmatrix} (1 + O(\lambda))
\]

\[
V_0^\dagger V_R^* = \begin{pmatrix}
1 - \alpha^2 \lambda^{2n} / 2 & \alpha \lambda^n & \beta \lambda^{2n} \\
-\alpha \lambda^n & 1 - (\alpha^2 + \gamma^2) \lambda^{2n} & \gamma \lambda^n \\
(-\beta + \alpha \gamma) \lambda^{2n} & -\gamma \lambda^n & 1 - \gamma^2 \lambda^{2n} / 2
\end{pmatrix} + O(\lambda^{3n})
\]

(34)

where

\[
m_\alpha = m_1 \cos^2 \theta_{12} + m_2 \sin^2 \theta_{12} + O(\lambda^{2n})
\]

\[
m_\beta = \frac{m_1 m_2}{m_\alpha} \cos^2 \theta_{23} + m_3 \sin^2 \theta_{23} + O(\lambda^{2n})
\]

\[
m_\gamma = \frac{m_1 m_2 m_3}{m_\alpha m_\beta}
\]

(35)

\(^2\)We neglect here the cases \( m_1 \simeq m_2 \tan^2 \theta_{12} \) and \( m_3 \tan^2 \theta_{23} \simeq m_1 m_2 / (m_1 \cos \theta_{12} + m_2 \sin \theta_{12}) \).
\[
\alpha = - \frac{(m_1 - m_2)}{2m_\alpha} \sin(2\theta_{12}) \cos \theta_{23} + O\left(\lambda^{2n}\right)
\]
\[
\gamma = \frac{m_1m_2 - m_3m_\alpha}{2m_\alpha m_\beta} \sin(2\theta_{23}) + O\left(\lambda^{2n}\right)
\]
\[
\beta = \frac{(m_1 - m_2)}{2m_\alpha} \sin(2\theta_{12}) \sin \theta_{23} + O\left(\lambda^{2n}\right), \quad (36)
\]

The numbers \(\alpha, \beta, \gamma\) are of order 1 but the corresponding angles must be computed up to order \(\lambda^{6n}\) to obtain the right heavy neutrino masses. The parameters \(m_\alpha, m_\beta, m_\gamma\) are of order of the low energy neutrino masses. Notice that the rotation angles \((1,2)\) and \((2,3)\) in \(V^\dagger_0V_R^*\) are of order \(\lambda^n\) while the \((1,3)\) angle is of order \(\lambda^{2n}\).

We observe that in this scenario, with hierarchical Dirac neutrino eigenvalues, the result depends on the explicit value of the angle \(\theta_{12}^{V_M}\) and \(\theta_{23}^{V_M}\) only at higher order in \(\lambda\) and via the value of \(m_\alpha, m_\beta, m_\gamma\). For example, if the \((2,3)\) angle of \(V_M\) is \(\pi/4\), i.e. for \(V_M\) maximal, we obtain

\[
m_\alpha = m_1 \cos^2 \theta_{12} + m_2 \sin^2 \theta_{12} + O\left(\lambda^{2n}\right) \quad (37)
\]
\[
m_\beta = \frac{m_1m_2 + m_3m_\alpha}{2m_\alpha} + O\left(\lambda^{2n}\right)
\]
\[
m_\gamma = \frac{m_1m_2m_3}{m_\alpha m_\beta}
\]
\[
\alpha = - \frac{\sqrt{2}(m_1 - m_2) \sin(2\theta_{12})}{4m_\alpha} + O\left(\lambda^{2n}\right)
\]
\[
\gamma = 1 - \frac{m_\gamma}{m_3} + O\left(\lambda^{2n}\right)
\]
\[
\beta = -\alpha + O\left(\lambda^{2n}\right) \quad (38)
\]

and for \(V_M\) tribimaximal we get

\[
m_\alpha = \frac{2m_1 + m_2}{3} + O\left(\lambda^{2n}\right) \quad (39)
\]
\[
m_\beta = \frac{3m_1m_2 + 2m_1m_3 + m_2m_3}{2(2m_1 + m_2)} + O\left(\lambda^{2n}\right) \quad (40)
\]
\[
m_\gamma = \frac{6m_1m_2m_3}{3m_1m_2 + 2m_1m_3 + m_2m_3}. \quad (41)
\]

For any \(V_M\), the heavy neutrino spectrum is hierarchical with ratios given mainly by

\[
M_1^R : M_2^R : M_3^R \simeq (M_1)^2 : (M_2)^2 : (M_3)^2. \quad (42)
\]

In fact on one hand we have that, for normal low energy neutrino hierarchy \(m_\alpha\) is of order \(m_2\), \(m_\beta\) is of order \(m_3\), and \(m_\gamma\) is of order \(m_1\). Then we obtain

\[
|m_\alpha|/\lambda^{4n} \gg |m_\beta|/\lambda^{2n} \gg |m_\gamma|. \quad (43)
\]
On the other hand, for inverted low energy neutrino hierarchy $m_\alpha$ is of order $m_2$, $m_\beta$ is of order $m_1$ ($\approx m_2$), and $m_\gamma$ is of order $m_3$ ($< m_1, m_2$) and then

$$|m_\alpha|/\lambda^{4n} >> |m_\beta|/\lambda^{2n} >> |m_\gamma|.$$  

Moreover the mixing matrix $V_0^\dagger V_R^*$ is close to the identity. Notice that the lightest right-handed neutrino has a mass smaller than $M_{\text{Planck}}(M_1/M_3)^2$ if we want the mass of the heaviest right-handed neutrino to be smaller than $M_{\text{Planck}}$.

### 4.3 Degenerate $M_D$

We remind the reader that the fact that the non trivial quark-lepton complementarity can come from a flavor symmetry implies that the Dirac neutrino may have a different hierarchical structure than the up sector, as clarified in sec. 2.2. For example the same argument applies to the charged lepton and down sectors, where we know that the hierarchical structure differs from each other. The idea beyond this fact, as explained in Sec. 2 is that the quark-lepton complementarity comes both from an unified gauge theory and from a flavor theory. It is supposed that, as the recent progresses show us [3, 4, 5, 6, 7, 8, 9, 10, 11], the nature of the mixing angles and that of the mass come from different type of flavor symmetries. For this reason, the non trivial quark-lepton complementarity can survive even if there is no Yukawa matrices unification. The important point is that the mixing in the Yukawa are related among them. In Sec. 2 we assumed these relations, but from recent literature about flavor physics we know that this is the case.

#### 4.3.1 Non degenerate $m_{\text{low}}$

If the Dirac neutrino mass eigenvalues are degenerate then, from eq. (23), we obtain

$$V_0^\dagger V_R^* \frac{1}{M_R^2} V_R^T V_0^* \approx V_M \frac{1}{M_D^2} m_{\text{low}} \frac{1}{M_D^2} V_M^T.$$  

(43)

In this case, if the low energy neutrino masses are not degenerate, $V_0^\dagger V_R^*$ is close to $V_M$ and $M_R^2 \approx m_{\text{low}}^2/(M_D^2)^2$. Let us define $\delta M_i = M_3 - M_i$. By performing the full computation up to orders $(\delta M_i/M_3)^2$, we get

$$\frac{1}{M_R^2} \approx \begin{pmatrix} m_\alpha/M_3^2 & 0 & 0 \\ 0 & m_\beta/M_3^2 & 0 \\ 0 & 0 & m_\gamma/M_3^2 \end{pmatrix}$$

$$V_0^\dagger V_R^* \approx V_M \begin{pmatrix} 1 - \alpha^2/2 & \alpha & \beta \\ -\alpha & 1 - (\alpha^2 + \gamma^2) & \gamma \\ -\beta + \alpha \gamma & -\gamma & 1 - \gamma^2/2 \end{pmatrix} \equiv V_M V_\epsilon$$  

(44)
where

\[
\begin{align*}
    m_α &\simeq m_1 \left(1 - \frac{δM_1}{M_3} \left(1 + \frac{\cos(2θ_{12})}{2}\right) + \frac{δM_2}{M_3} \left(-\frac{1 - \cos(2θ_{12})}{2} - \cos(2θ_{23}) \sin^2 θ_{12}\right)\right) \\
    m_β &\simeq m_2 \left(1 - \frac{δM_1}{M_3} \left(1 - \frac{\cos(2θ_{12})}{2}\right) + \frac{δM_2}{M_3} \left(-\frac{1 - \cos(2θ_{12})}{2} - \cos(2θ_{23}) \cos^2 θ_{12}\right)\right) \\
    m_γ &\simeq m_3 \left(1 - \frac{δM_2}{M_3} \left(1 + \cos(2θ_{23})\right)\right)
\end{align*}
\]

\[
α \simeq -\frac{m_1 + m_2}{4(m_1 - m_2)} \frac{2δM_1 - δM_2 - δM_2 \cos(2θ_{23})}{M_3} \sin(2θ_{12})
\]

\[
γ \simeq \frac{m_2 + m_3}{2(m_2 - m_3)} \frac{δM_2}{M_3} \sin(2θ_{23}) \cos θ_{12}
\]

\[
β \simeq \frac{m_1 + m_3}{2(m_1 - m_3)} \frac{δM_2}{M_3} \sin(2θ_{23}) \sin θ_{12} \, .
\]

(45)

The parameters \(m_α, m_β, m_γ\) are of order of the low energy neutrino masses. The angles \(α, β, γ\) are of order \(δM_i/M_3\) with the exception of degenerate low energy neutrino masses. In this case \(α\) is enhanced by a factor \(m^2/δm^2_{12}\), while the other two angles \(β\) and \(γ\) have a factor \(m^2/δm^2_{13}\), and our approach here is not valid any more because the three angles can be small only if the degeneracy of the Dirac neutrino eigenvalues is such that \(δM_1/M < 10^{-5}\). We notice that there is not any substantial difference for normal (\(m_1 < m_2 < m_3\)) or inverted hierarchy (\(m_3 < m_1 < m_2\)) of the low energy neutrino masses, and the only effect is to change the sign of \(β\) and \(γ\) angles.

From eq. (28) we get

\[
\tilde{M}_D = Ω^1 U_{CKM}^T \Delta_D V_M V_ε
\]

(46)

and \(\tilde{M}_D\) can be computed using the expressions in eq. (15) and \(U_{CKM}\). Notice that in this case the resulting \(\tilde{M}_D\) strongly depends on the \(V_M\) matrix.

For any \(V_M\), the heavy neutrino spectrum is degenerate. However the mixing matrix \(V_0^T V_R^*\) is close to the \(V_M\) matrix.

### 4.3.2 Degenerate \(m_{\text{low}}\)

If the low energy neutrino masses \(m_i\) and the Dirac neutrino eigenvalues are degenerate then we get

\[
V_0^T V_R^* \frac{1}{M_R^2} V_R^i V_0^* \simeq \frac{1}{M_D^2} \frac{m_{\text{low}}}{M_D^2} \, .
\]

(47)

In this case the value of \(V_M\) plays a marginal role. The mixing matrix \(V_0^T V_R^*\) is close to a small rotation in the (1, 3) plane and the heavy neutrino spectrum is
degenerate too:

\[
M_1^R = \frac{m}{M^2} \left( 1 - \frac{\delta M_1}{M} \left( 1 + \sqrt{1 - \frac{1}{3} \frac{\delta M_1}{M} + \left( \frac{\delta M_1}{M} \right)^2} \right) \right)
\]

\[
M_2^R = \frac{m}{M^2} \left( 1 - 2 \frac{\delta M_2}{M} + \sqrt{\frac{\delta m^2_{\text{atm}}}{m}} \right)
\]

\[
M_3^R = \frac{m}{M^2} \left( 1 - \frac{\delta M_1}{M} \left( 1 - \sqrt{1 - \frac{1}{3} \frac{\delta M_1}{M} + \left( \frac{\delta M_1}{M} \right)^2} \right) \right).
\]

For any \(V_M\) compatible with the experiments, the heavy neutrino spectrum is almost degenerate. Moreover the mixing matrix \(V_0^*V^R\) is close to the identity matrix.

5 Contribution to \(l_i \rightarrow l_j \gamma\)

Using the result in eq. (28) and the general eq. (14), we get

\[
BR(l_i \rightarrow l_j \gamma) \propto \left| \left( \Omega^\dagger U_{\text{CKM}}^\dagger M_D^\Delta V L V^\dagger M_D^\Delta U_{\text{CKM}} \Omega \right)_{ij} \right|^2
\]

where \(\Omega\) is the mixing matrix computed with eq. (23). Notice that the \(\Omega\) phase differences \(\exp^{i(\phi_i - \phi_j)}\) cancel because we take the absolute value. We want to stress here that the result in eq. (49) depends on the Quark-Lepton complementarity (and the underlying flavor symmetry) assumption only, and not on the explicit form of the correlation matrix \(V_M\).

At zero approximation we neglect the different normalizations for different right-handed neutrinos. We assume \(L = \hat{L} = 1 \log M_X/M_R\) where \(M_R\) is the common heavy neutrino mass. The \(BR(\mu \rightarrow e \gamma)\) can be rewritten as

\[
BR(\mu \rightarrow e \gamma) \propto \frac{\Gamma(\mu \rightarrow e \nu \nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m^2_v \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2} \hat{L}^2
\]

\[
\left| \left( U_{\text{CKM}}^\dagger (M_D^\Delta)^2 U_{\text{CKM}} \right)_{21} \right|^2
\]

\[
= \frac{\Gamma(\mu \rightarrow e \nu \nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m^2_v \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2} \hat{L}^2
\]

\[
\left| (M^2_2 - M^2_1) \lambda (1 + O(\lambda^2)) + M^2_2 A^2 (\rho - i\eta) \lambda^5 (1 + O(\lambda^6)) \right|^2
\]

where \(\lambda\) is the sine of the Cabibbo angle, and \(A, \rho\) and \(\eta\) are the other parameters of the unitary CKM matrix. For each Dirac neutrino mass we introduced, its first
contribution. Similarly to the process $\mu \to e\gamma$ we can compute the contribution to the $\tau$ decays. For $\tau \to e\gamma$ we get

$$BR(\tau \to e\gamma) \propto \frac{\Gamma(\tau \to e\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_\nu^4 v_u^2} \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2 \hat{L}^2 \left| (1 - (\rho - i\eta))M_1^2 - M_2^2 + M_3^2(\rho - i\eta))A\lambda^3(1 + O(\lambda^2)) \right|^2.$$  

The other $\tau$ decay process that violates the individual lepton number is such that

$$BR(\tau \to \mu\gamma) \propto \frac{\Gamma(\tau \to \mu\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_\nu^4 v_u^2} \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2 \hat{L}^2 \left| (-M_1^2\lambda^2 - M_2^2 + M_3^2)A\lambda^2(1 + O(\lambda^2)) \right|^2.$$  

To understand the main contribution we must make some assumptions about the hierarchy of the Dirac neutrino masses $M_i$. Moreover to include the effect of non-degeneration for heavy neutrino masses we must include $V$, whose form depends also on the hierarchy of the low energy neutrino masses.

### 5.1 Hierarchical $M_D$

For hierarchical $M_D$ the factor $L$ in eq. (49) cannot be neglected. If we introduce the full form of $L$ then the form of $V$ is relevant. Under the assumption of hierarchical $M_D$, $V$ is close to the identity and we get

$$BR(\mu \to e\gamma) \propto \frac{\Gamma(\mu \to e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_\nu^4 v_u^2} \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2 \left| \left( M_2^2 \log \frac{M_X}{M_2^R} - M_1^2 \log \frac{M_X}{M_1^R} \right) \lambda + M_3^2 \log \frac{M_X}{M_1^R} A^2(\rho - i\eta) \lambda^5 \right|^2,$$

where we introduced the structure of $L$ to take into account the hierarchical structure of heavy neutrino masses too. For example if we assume that

$$M_1 : M_2 : M_3 \propto m_u : m_b : m_t$$

at the unification scale, then we obtained in Sec. 4.3 that

$$M_1^R : M_2^R : M_3^R \propto m_u^2 : m_c^2 : m_t^2.$$  

For the BR we have

$$BR(\mu \to e\gamma) \propto \frac{\Gamma(\mu \to e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_\nu^4 v_u^2} \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2 \log^2 \frac{M_X}{M_3} \left( \frac{M_3}{m_t} \right)^4 \left| m_c^2 \lambda \log \frac{m_t^2}{m_c^2} + m_t^2 \log \frac{m_t^2}{m_u^2} A^2(\rho - i\eta) \lambda^5 \right|^2.$$  

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Similarly to the process \( \mu \to e\gamma \) we can compute the contribution to \( \tau \to e\gamma \) and \( \tau \to \mu\gamma \). We get

\[
BR(\tau \to e\gamma) \propto \frac{\Gamma(\tau \to e\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s v_u^4} \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2 \hat{L}^2 \left( \frac{M_3}{m_t} \right)^4 \left| m_t^2 \log \frac{m_t^2}{m_u^2} A(\rho - i\eta) \lambda \right|^2,
\]

where in the last line we used a hierarchical structure for the Dirac neutrino masses and introduced the structure of \( L \). We observe that \( BR(\mu \to e\gamma) \) is suppressed by a factor \( \lambda^4 \) with respect to \( BR(\tau \to e\gamma) \).

The other \( \tau \) decay is the least suppressed process that violates the individual lepton number. In fact we have

\[
BR(\tau \to \mu\gamma) \propto \frac{\Gamma(\tau \to \mu\nu\nu)}{\Gamma(\tau)} \frac{\alpha^3}{G_f m_s v_u^4} \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2 \hat{L}^2 \left( \frac{M_3}{m_t} \right)^4 \left| m_t^2 \log \frac{m_t^2}{m_u^2} \lambda^2 \right|^2.
\]

We observe that \( BR(\mu \to e\gamma) \) is in general suppressed by a factor \( \lambda^6 \) with respect to \( BR(\tau \to \mu\gamma) \), and \( BR(\tau \to \mu\gamma) \) by a factor \( \lambda^2 \). Our conclusions are equivalent to the one in [15, 16], and also in our analysis it can be a further suppression of the branching ratios if the leading term in eq. (53) cancels. We can conclude that in this case, for general values of the SUSY parameters, the expected branching ratios are compatible with the actual experimental data, and will be observable only for high value of the low energy neutrino masses and for particular point in the SUSY parameter space. However our discussion is more general since in fact we showed that these results do not depend on the form of the correlation matrix \( V_M \).

### 5.2 Degenerate \( M_D \)

If we assume that the eigenvalues of the Dirac Yukawa matrix are degenerate, as computed in sec. 4.3, we have two cases depending on the degeneration of \( m_{low} \).

#### 5.2.1 Non degenerate \( m_{low} \)

For non degenerate \( m_{low} \) we have the right-handed neutrinos with the same hierarchy of the low energy neutrinos, and \( V_0^\dagger V_R^* \) close to \( V_M \). In this case we get

\[
BR(\mu \to e\gamma) \propto \frac{\Gamma(\mu \to e\nu\nu)}{\Gamma(\mu)} \frac{\alpha^3}{G_f m_s v_u^4} \tan^2 \beta \left( \frac{3m_0 + A_0}{8\pi^2} \right)^2 \left( \frac{M_3}{m_t} \right)^4 \left| m_t^2 \log \frac{m_t^2}{m_u^2} A(\rho - i\eta) \lambda \right|^2.
\]
We obtain that, despite the fact that this case is the most promising to extract the value of the branching ratio of $\mu \rightarrow e\gamma$. If the spectrum of the low energy neutrino is degenerate, then the mixing matrix becomes close to the identity. In this case the branching ratios depend on the common $M_D$ mass and the Cabibbo parameter. By assuming $M_2^2 - M_1^2 > \lambda^4 M_3^2$ we get

$$BR(\mu \rightarrow e\gamma) \propto \left| (M_2^2 - M_1^2) \right|^2 \lambda^2,$$

$$BR(\tau \rightarrow e\gamma) \propto \left| (1 - (\rho - i\eta))M_1^2 - M_2^2 + M_3^2(\rho - i\eta) \right|^2 (A\lambda^3)^2,$$

$$BR(\tau \rightarrow \mu\gamma) \propto \left| (M_3^2 - M_2^2) \right|^2 (A\lambda^2)^2,$$

and the ratios among them are

$$BR(\mu \rightarrow e\gamma) : BR(\tau \rightarrow e\gamma) : BR(\tau \rightarrow \mu\gamma) = 1 : \lambda^4 : \lambda^2.$$
To compare this case with the case of hierarchical $M_D$ of sec 4.2, we observe that here $BR(\mu \rightarrow e \gamma)$ is the largest one, while in the other case it is the smallest one. Moreover the value of the branching ratios here depends on the differences $M_i^2 - M_j^2$ and they are in general smaller then in the other case. For example, if $M_2^2 - M_1^2 \approx \lambda^4 M_3^2$ and $M_i$ are of order $m_t$, we obtain

$$BR(\mu \rightarrow e \gamma) \propto \left(\frac{M_3}{m_t}\right)^4 |m_i^4 \lambda|^2,$$

$$BR(\tau \rightarrow e \gamma) \propto \left(\frac{M_3}{m_t}\right)^4 |m_i^4 \lambda|^2,$$

$$BR(\tau \rightarrow \mu \gamma) \propto \left(\frac{M_3}{m_t}\right)^4 |m_i^4 \lambda|^2.$$

In this case, not only we cannot extract information on the $V_M$ structure, but also we have no hope to observe these branching ratios because they are too small even with respect to the future experimental sensitivities.

### 6 Conclusions

We analized the consequences of a non trivial Quark-Lepton complementarity and a flavor symmetry on $BR(l_i \rightarrow l_j \gamma)$. The non trivial Quark-Lepton complementarity, together with the flavor symmetry, states that the correlation matrix $V_M$, product of the $CKM$ and the $PMNS$ mixing matrix, is related to the diagonalization of the Majorana right-handed and Dirac neutrino mass matrices. In this framework we obtained that $BR(l_i \rightarrow l_j \gamma)$ is related to the $CKM$ mixing matrix and the Dirac neutrino masses.

We have three cases:

1. Hierarchical Dirac neutrino eigenvalues (very hierarchical right-handed neutrino masses, $V_0^t V_R^* \simeq I$) where we get the usual ratios

   $$BR(\mu \rightarrow e \gamma) : BR(\tau \rightarrow e \gamma) : BR(\tau \rightarrow \mu \gamma) = \lambda^6 : \lambda^4 : 1 \propto M_3^4 \lambda^4 \hat{L}.$$ 

   This case is the most promising one for a future observation of the branching ratios. However it will not give us any information about the structure of the $V_M$ matrix.

2. Degenerate Dirac neutrino eigenvalues, with non degenerate low energy neutrino masses (the hierarchy of the right-handed neutrino masses is close to the one of the low energy spectrum, $V_0^t V_R^* \simeq V_M$) where we get

   $$BR(\mu \rightarrow e \gamma) = \tan^2 \theta_{23} V_M \cdot BR(\tau \rightarrow e \gamma) = f(\theta_{12}^{V_M} , \theta_{23}^{V_M}) BR(\tau \rightarrow \mu \gamma) \propto M_3^4 \hat{L}$$

   with $f(\theta_{12}^{V_M} , \theta_{23}^{V_M})$ of order one. This case is the only one where the structure of $V_M$ plays a fundamental role in the determination of the branching ratios.
However it is already excluded for a large part of the SUSY parameters space by the experimental limits.

3. Degenerate Dirac neutrino eigenvalues and low energy neutrino spectrum (right-handed neutrinos close to degenerate, $V_0^†V_R^* \simeq I$) where we have

$$BR(\mu \to e\gamma) : BR(\tau \to e\gamma) : BR(\tau \to \mu\gamma) = 1 : \lambda^4 : \lambda^2 \propto M_3^4\lambda^{10}\hat{L}.$$  

In this case the branching ratios are too small even with respect to the future experimental sensitivities.

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