Students’ Representation in Mathematical Word Problem-Solving: Exploring Students’ Self-efficacy

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Abstract. This descriptive qualitative research aims at investigating student represented in mathematical word problem solving based on self-efficacy. The research subjects are two eighth graders at a school in Surabaya with equal mathematical ability consisting of two female students with high and low self-efficacy. The subjects were chosen based on the results of test of mathematical ability, documentation of the result of middle test in even semester of 2016/2017 academic year, and results of questionnaire of mathematics word problem in terms of self-efficacy scale. The selected students were asked to do mathematical word problem solving and be interviewed. The result of this study shows that students with high self-efficacy tend to use multiple representations of sketches and mathematical models, whereas students with low self-efficacy tend to use single representation of sketches or mathematical models only in mathematical word problem-solving. This study emphasizes that teachers should pay attention of student’s representation as a consideration of designing innovative learning in order to increase the self-efficacy of each student to achieve maximum mathematical achievement although it still requires adjustment to the school situation and condition.

1. Introduction

Representation is a description an object or process i.e. words, diagrams, graphics, computer simulations, mathematical equations and others. It can also be used to describe cognitive processes to the understanding of an idea in mathematics [1]. Representation of mathematical requires students to discover and develop a tool or way of thinking in communicating mathematical ideas from the abstract to the concrete, making it easier to understand. It takes place in two stages: internal and external representation. Internal representation is knowledge and structure in memory that is part of the cognitive process i.e. personal notation systems, nature language, visual images and problem-solving strategies while external representations are knowledge and structures in environments such as physical symbols, objects, or dimensions [2]. The process of internal representation is certainly not observable visually and cannot be assessed directly because it is a person's mental activity in mind. In other words, a person who performs an internal representation process in learning mathematics will think about the ideas, thought, or mathematical concepts he is studying in order to clearly understand the problem, connect the problem with the student’s knowledge, and make a plan.

External representation is used to describe what the student, teacher, and mathematician. It is grouped into three forms, namely: visual form, symbol, and verbal. The visual form includes diagrams, tables, or graphs, geometry drawings and sketch drawings. The symbols include mathematical models and symbols or mathematical expressions. The verbal form includes both written and spoken words or
sentences [3]. When faced with new and complex ideas, it is possible for students to try different representations as a manifestation of their strategies in mathematics problem-solving. The use of various representations in solving a problem is called multiple representation. To be able to identify and represent the same concept in different representational models, conceptual understanding in mathematics it is important [4,5]. Generally, math problems can be a word problem. The word problem is presented in the form of a meaningful narration that can be understood and answered mathematically based on prior learning experience, as well as related to the situation experienced by students in everyday life [6,7]. In solving word problem, the students must understand the text to identify the missing information, obtain a plan to solve for it, and perform calculations to search for it. Solving the word problem in mathematics is an important aspect in mathematic learning and mathematical understanding. The steps to mathematical word problem solving according to Polya includes: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, (4) looking back [8].

Mathematical word problem give students challenges in applying mathematical thinking in various situations. The solving requires integration of some cognitive processes in which the students need to understand the language and factual information in the problem, translate the problem by using relevant information to create the appropriate mental representation, compile and monitor problem-solving plans, and carrying out appropriate procedural calculations [9]. In addition, knowledge in mathematical word problem can give students confidence to accomplish the task. The lack of strategy in mathematical word problem can decrease the confidence and belief of being success. The success of students in mathematical word problem is supported by psychological factors, not only about the method of learning but also about the beliefs about the ability possessed by students. The beliefs that the students have in performing a particular task or action is called self-efficacy [10].

Math self-efficacy affects mathematical achievement of the students [11]. High self-efficacy created tranquility ahead of the task and confidence in facing the difficult activities. Conversely, a person who doubts about his ability, thinks things in more difficult way than the real truth. [12]. Trust in one's academic ability is an essential component of school success. When individuals are familiar with task demands or activities, they tend to bring out self-efficacy that has been developed as a result of previous experience with similar tasks. Students’ self-confidence in the success of problem-solving or mathematical word problems is called mathematical word problem self-efficacy [13]. There is a relationship between self-efficacy and academic achievement. The higher the self-efficacy, the higher the academic ability is. Self-efficacy is also a determinant that influences individual choices in an escape effort, persistence in the face of adversity and the mindset and emotional reactions they experience [14,15]. Thus, representation and self-efficacy are very important parts in the mathematical word problem-solving process, so the solution of mathematical word problem is more structured and in accordance with the goals which are going to be achieved. Therefore, this study aims at examining students’ representation in mathematical word problem based on their self-efficacy.

2. Method
This research is descriptive qualitative research, researcher use Polya’s problem-solving step. The research subjects are two students of class VIII MTs Negeri 2 Surabaya with equal mathematic ability consisting of two female students, one of them with high self-efficacy and the other with low self-efficacy. The selection of study subjects was according to the results of mathematical ability test, documentation of the result of middle test even semester of 2016/2017 academic year, and results of questionnaire of mathematical word problem self-efficacy scale. The instrument of mathematical word problem self-efficacy scale is an adaptation of the original instrument with an adjustment to the psychological and cultural students of MTs Negeri 2 Surabaya. The data obtained were the provision of mathematical word problem-solving task and interview which were done twice. Giving the second mathematical word problem-solving task and the second interview led to a triangulation of data.
3. Results and Discussion

Subjects with high self-efficacy and low self-efficacy exercise all steps of Polya’s problem-solving concept. The subject with high self-efficacy, at a stage, in understanding the problem, tend to use symbols in presenting information and presenting what is asked on the question, but not yet able to explain the meaning of symbols used correctly. In addition, they tend to choose representations in the form of visuals and symbols. The next one is manipulating the visual form and symbols according to the plan, then the results are interpreted in the form of a written text. In devising a plan and carrying out the plan, high self-efficacy subject dominantly employed more than one form of representation or multiple representation. Meanwhile, the subject with high self-efficacy looked back problem-solving strategies and recalculated to prove that the answers obtained were true through sketches and formulas that were made and able to solve mathematical word problems appropriately and consistently along with positive perceptions in mathematics. In terms of behavior, the subject with high self-efficacy in mathematical word problem-solving, dominantly used multiple representation to ensure that the answers obtained is correct. In solving the linear equations problem, the subject with high self-efficacy analyzed the probability in answering without using formal operations taught like substitution and elimination methods. In this case, mathematical word problem-solving is considered more effective and efficient. The subject with high self-efficacy assumed that mathematics have an important and useful role in everyday life [16]. This assumption was made as mathematics is one of the favorite subjects for her.

The subject with low self-efficacy tend to use written words in presenting information and presenting what is asked on the matter, choosing representations in visual form or symbols as a solving strategy. The subject with low self-efficacy have not been able to explain the meaning of the symbols used correctly. Furthermore, at the stage of carrying out the plan, the subject manipulated the visual form or symbol according to the plan, then the results were interpreted into the form of written words. At the stage of devising a plan and carrying out the plan, the subject dominantly planned using visual representation. At the looking back stage, the subject rechecked the problem-solving strategy and recalculated to prove that the answer obtained is correct through drawing the sketch or the formula being made. At each stage, the subject tends to use only one form of representation only. Subjects with low self-efficacy have not showed consistency in mathematical word problem-solving [13]. In terms of behavior, the subject with low self-efficacy in mathematical word problem-solving dominantly used only one form of representation based on learning experience. In solving the linear equations problem, the subject with low self-efficacy used formal operations namely the method of elimination and substitution according to the learning experience gained.

The results show that there are several similarities of representational in mathematical word problem-solving between subjects with high and low self-efficacy as shown in Table 1.

| Problem Solving Step | Similarities between high and low self-efficacy subjects |
|----------------------|----------------------------------------------------------|
| Devising a plan       | Using sketch image based on the subject's understanding of the problem. |
|                      | - Manipulating sketch drawings that are created to get answers from the given problem. |
|                      | - Interpreting the answers obtained into the form of written words. |
| Looking back          | Examining the sketch drawing. |
The results show that there are some differences of representational in mathematics word problem-solving between subjects with high and low self-efficacy as shown in Table 2.

**Table 2.** The results showing the differences of representational in mathematics word problem-solving between subjects with high and low self-efficacy

| Problem-Solving Step | Subject with High Self-efficacy | Subject with Low Self-efficacy |
|----------------------|---------------------------------|-------------------------------|
| Understanding the problem | Presenting data or information that is known to the problem in the form of symbols. | Presenting data or information that is known to the problem in the form of written words matching to the information of the given question. |
| Devising a plan | Presenting what is asked on symbol form issues | Presenting what is asked on verbal form issues. |
| | Choosing a problem-solving strategy from a given problem which tends to be more than one form of representation or multiple representation that is the visual form and symbol based on the learning experience. In addition, it is to assure the results obtained on the visual representation. | Choosing a resolution strategy from a given problem which tends to be one of the representations or single representation. |
| | - Manipulating the drawing of sketches and formulas that have been selected to determine the answers. | - Manipulating the drawing of the selected sketches or formulas to get the expected answers. |
| | - In the linear equations problem, the subject tends to analyze the possibility of the values of x and y in order to obtain the answer. | - In the linear equations problem, the subject tends to do the elimination and substitution method in order to determine the value of the variable which is then used to determine the final answer. |
| Carrying out the plan | The subject tends to use multiple representations of visuals and symbols when looking back the problem-solving strategies and recalculating to prove that the answers obtained are correct. | The subject tends to use only one form of representation of the visuals or symbols when rechecking the problem-solving strategy and recalculating to prove that the answers obtained are correct. |
| Looking back | | |

4. **Conclusion**

Based on the results of data analysis and discussion it can be concluded that students with high self-efficacy in mathematical word problem-solving, presents data or information that is known and asked in relation to the issue into the form of symbols. The strategy of completion of the problem given tend to involve requirement of multiple representation in the visual form of sketches and symbols of formulas or mathematical models, and manipulation of sketch drawings and formulas that have been made. Particularly, for mathematical models on linear equations learning materials, the students analyzed the possible answers that meet the mathematical model that has been made based on the students' experience of working on problems related to linear equations. The results obtained are interpreted in the form of written words. The next one is looking back the problem-solving strategy through drawing sketches and recalculating the conformity of the results on the mathematical models that have been made and also assuring that the steps taken are the best ones. Meanwhile, the student with low self-efficacy presented information that is known and what is asked on the issue in the form of written words. The results are
adapted to the problem by choosing a strategy of completion of the given problem which tends to lead on the use of one representation formula in the form of drawing sketches or symbols which are considered as formulas mathematical models only. In addition, the manipulation of the drawing sketch has been made after that. While in the mathematical model of linear equations learning material, the students used the elimination and substitution methods to find a solution. The results obtained were then interpreted into the verbal form of written text. Next, in looking back the problem-solving strategy through sketch drawing or recalculating the conformity of the results on the mathematical model has been made to ensure that the steps taken are the best ones.

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