Finite-time adaptive sliding mode control for compressor surge with uncertain characteristic in the presence of disturbance

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Abstract
In this paper, a novel control approach is designed for surge instability in the compressor system using the finite-time adaptive sliding mode scheme. The primary novelty of this study lies in the development of a finite-time adaptive control for the surge instability avoidance in a compressor system in the presence of disturbance and uncertainty in the characteristic curve of the compressor and also throttle valve. The Lyapunov method is utilized to verify the finite-time stability of the closed-loop system. The performance of the presented method is compared against other methods in the literature through simulations in Matlab. The results suggest that our designed controller outperforms the existing ones in terms of surge avoidance and robustness against uncertainties and disturbances.

1. Introduction

With the advancement of technology, the compression systems have attracted great interest from the academic and industrial communities due to their promising utility. The compressors act as the beating-heart for the operation of industrial sectors and are used in a broad range of modern applications, including turbojet engines, industrial gas turbines, turbochargers, and transport pipelines in the petrochemical and mining industries for pressurization of gas and fluids. Thus, extensive efforts have been devoted to improving the reliability and performance of these systems (An & Suzuki, 2017; Jang et al., 2007; Jiang et al., 2006; Mojaddam & Pullen, 2019; Zhang et al., 2020).

Two frequently used compressors are the centrifugal and axial flow. According to Boyce (2011), the centrifugal compressor is more efficient in high pressure, low flow rate applications, while the axial one is preferred for higher flow rate and higher speed machinery. The compressor systems commonly face two major instabilities, surge and stall, which limit their operational range. As the mass flow through the compressor drops down to a critical point, the surge occurs reflected by an unstable flow pattern. The ‘surge point’ separates the stable and unstable regions in the compressor’s characteristic curve. Likewise, the ‘surge line’ connects the surge points on the characteristic curves obtained at various compressor speeds. Surge instability can impact the entire compression system, and as a limit cycle, it is characterized by extreme fluctuations in the compressor’s pressure and flow, leading to a substantial loss in the efficiency and performance of the system.

Concerning the destructive effects of the surge phenomenon in the compression systems, significant attempts have been made to control this instability in centrifugal compressors. Two principal categories of surge instability control include surge prevention and surge active control (Epstein et al., 1989; Willems & De Jager, 1999). Due to the drawbacks associated with the application of the surge preventive method that leads to the compressor’s efficiency loss, active surge control has emerged as a more effective tool for compensation of surge instability. The active controller might extend the stable operating envelope of the compressor system through (1) improved aerodynamic design, (2) reducing the variations in operating conditions, (3) including the components influencing the flow, and (4) active suppression of aerodynamic instabilities.

In recent years, several methods for active surge control in constant-speed compressors have been proposed using various operators (Arnulf et al., 2001; Bøhagen & Gravdahl, 2008; Wang et al., 2006). Some studies have utilized the benefits of the feedforward control method for the compressor’s pressure control (Imani, Jahed-Motlagh, et al., 2018; Imani, Jahed-Motlagh, Salahshoor, 2020).
Ramazani, et al., 2017; Imani, Jahed-Motlagh, Salahshoor, Ramezani, et al., 2017; Imani, Malekizade, et al., 2018), with the designed controller being robust to the uncertainty and disturbance (Imani, Jahed-Motlagh, et al., 2018; Imani, Jahed-Motlagh, Salahshoor, Ramezani, et al., 2017). Others have applied the adaptive and back-stepping approaches to overcome the effects of uncertainty and disturbance (Ghanavati, Salahshoor, Jahed-Motlagh, et al., 2018; Ghanavati, Salahshoor, Motlagh, et al., 2018; Ziabari et al., 2017). However, several important issues have been neglected in previous researches. Greitzer’s model, commonly used in these articles (Ghanavati, Salahshoor, Motlagh, et al., 2018; Ghanavati, Salahshoor, Motlagh, et al., 2018; Imani, Jahed-Motlagh, et al., 2018; Imani, Jahed-Motlagh, Salahshoor, Ramazani, et al., 2017; Imani, Jahed-Motlagh, Salahshoor, Ramezani, et al., 2017; Imani, Malekizade, et al., 2018; Ziabari et al., 2017) to describe the compressor system, assumes the compressor’s characteristic curve as accurate and known. This is while the characteristic curve varies under the effects of different compressor parts, such as pipes, upstream and downstream segments, and other parameters, and it might be unknown. Finite time stabilization of the compressor system by noticing the operating limitations of the actuator is another problem that has been considered in this study. Other challenges include the disturbances in the flow and pressure of the compressor and uncertainty on the throttle valve characteristic that all will be addressed in this paper.

Thus, the most significant contribution of this paper lies in the implementation of a finite-time approach for the compressor’s surge control. Given the significance of the convergence rate of the system states, many applied programmes require a rapid convergence using a finite-time control strategy (Huang et al., 2021; Liu et al., 2009; Wang et al., 2020). Concerning the detrimental impacts of surge instability on performance, adopting a finite-time control approach for fast control of surge instability in the compressor system offers a range of beneficial properties, such as good disturbance rejection and robustness against uncertainties. Other major advantages include faster transient response, higher precision tracking performance, and faster convergence rate compared to the asymptotic method (Fu & Wang, 2016; Li et al., 2016).

Given the importance of robust control approaches in various applications (Chen et al., 2020; Shi et al., 2017; Wang et al., 2021; Xiong et al., 2016; Yu et al., 2020; Zhang et al., 2018; Zhu et al., 2020), this paper presents a novel finite-time adaptive sliding mode control scheme for surge instability in the compressor system, in which the adaptive approach is used to estimate the uncertainties associated with the compressor characteristic curve and throttle valve. Our proposed method takes into account the upper bound of uncertainty, as well as the upper bounds of pressure- and flow-related disturbances. Besides, the Lyapunov method is applied to guarantee the finite-time stability of the closed-loop system. Our findings suggest that the finite-time adaptive sliding mode controller enables the capability to fast control of surge instability despite the uncertainty in the compressor’s characteristic curve, the unknown opening percentage of the throttle valve, and the presence of disturbance with unknown upper bound.

Overall, the organization of this paper is as follows: in section II, the Greitzer’s model for the compressor system is presented. In section III, the designed controller is presented. Finally, the simulation results and conclusions are provided in sections IV and V of the paper.

2. Model of the compressor system

The modelling of the compressor systems for surge instability control has long been studied with the incentive to extend the stable operating envelope for centrifugal and axial compressors. A comprehensive study of modelling techniques for compressor systems can be found in (Gravdahl & Egeland, 2012) and (Longley, 1994). In (Badmus et al., 1991), the existing mathematical models for compressor systems were divided into either one-dimensional models for surge prediction or two-dimensional (2D) models for surge and rotating stall prediction. The rotating stall is more related to axial compressors, and the 2D models are particularly developed for such turbomachines.

Despite the introduction of various advanced models for compression systems, the original Greitzer’s model is still the prior choice for active surge control in centrifugal compressors due to its low order equations and simple structure. Numerous studies in the literature have applied Greitzer’s model for surge control using linear (Amuifi et al., 2006; Blanchini et al., 2002; Boïnov et al., 2006) and nonlinear (Ananthkrishnan et al., 2003; Badmus et al., 1991; Chaturvedi & Bhat, 2006; Fontaine et al., 2004; Krstic et al., 1998) control theory. Greitzer’s model is utilized in this paper as a starting point for control of the compression system. This model can provide a decent qualitative description of the relevant phenomena, while its simplicity facilitates the physical interpretation of the model parameters and their impact on the overall dynamics. Besides, a set of ordinary differential equations (ODEs) is unlikely to cause computational complications in real-time executables. The reader is referred to the reviews provided in (Abed et al., 1993; Weigl et al., 1997) for comprehensive information on compressor system models and their application in surge control design.
The centrifugal compressor in this paper is modelled using the following equations:

\[
\begin{align*}
\frac{d\psi}{d\xi} &= \frac{1}{4B^2lc}(\phi - \phi_T(\psi)) \\
\frac{d\phi}{d\xi} &= \frac{1}{lc} \left( \psi_c(\phi) - \psi - \frac{3H}{4} \left( \frac{\phi}{W} - 1 \right) J \right) \\
\frac{dJ}{d\xi} &= J \left( 1 - \left( \frac{\phi}{W} - 1 \right)^2 - \frac{J}{4} \right) \delta
\end{align*}
\] (1)

where \( \psi \) and \( \phi \) are the coefficients of the compressor’s pressure rise and mass flow, respectively. \( J \) stands for the square of the rotational stall range, and \( \xi \) is the dimensionless time. Also, \( \phi_T(\psi) \) and \( \psi_c(\psi) \) represent the characteristics of the throttle valve and the compressor, respectively. \( lc \) specifies the length of ducts and compressors, and \( \delta \) is a constant coefficient. \( B \) is the Greitzer’s parameter obtained from the following relation:

\[
B = \frac{U}{2\alpha_t \sqrt{V_p/A_{clc}}}
\] (2)

In this relation, \( \alpha \) indicates the positive constant of the reversed dead time, and \( U \) is the constant of compressor’s tangent speed. \( \alpha_t \) is the sound speed, \( V_p \) the plenum volume, and \( A_{clc} \) the cross-section of the compressor.

The basic equations of a compressor with constant speed, flow rate \( m \), pressure difference \( \Delta P \), and real-time \( t \) can be normalized according to the following terms:

\[
\phi = \frac{m}{\rho A_{clc} U}, \quad \psi = \frac{\Delta P}{\rho U^2}, \quad \xi = \frac{U}{R} t
\] (3)

Normalization maps a class of compressor curves into a single characteristic curve for each speed. The characteristic curve describes the nonlinear relationship between the compressor pressure \( \psi \) and flow \( \phi \). Following Moor and Greitzer (Ziabari et al., 2017), the well-established cubic characteristic of the compressor is defined as:

\[
\psi_c(\phi) = \psi_{c0} + H \left( 1 + \frac{3}{2} \left( \frac{\phi}{W} - 1 \right) - \frac{1}{2} \left( \frac{\phi}{W} - 1 \right)^3 \right)
\] (4)

where \( \psi_{c0} \) shows the value of the characteristic curve in zero dB. \( H \) is the half-height and \( W \) the half-width of the characteristic curve.

Based on (Ziabari et al., 2017), the throttle valve characteristic is given by:

\[
\phi_T(\psi) = \gamma_T \sqrt{\psi}
\] (5)

where \( \gamma_T \) indicates the valve’s yield.

Taking \( J = 0 \) in equations (1), the surge expressions are obtained as:

\[
\begin{align*}
\dot{\psi} &= \frac{1}{4B^2lc}(\phi - \phi_T(\psi)) \\
\dot{\phi} &= \frac{1}{lc} (\psi_c(\phi) - \psi)
\end{align*}
\] (6)

Like all other physical systems, the presence of disturbance in the compressor system is inevitable and may impact the stability and active surge controller performance. In this model, the disturbances in the flow and pressure of the compressor are formulated as:

\[
\begin{align*}
\dot{\psi} &= \frac{1}{4B^2lc}(\phi - \phi_T(\psi) + d\phi(\xi)) \\
\dot{\phi} &= \frac{1}{lc} (\psi_c(\phi) - \psi + d\phi(\xi))
\end{align*}
\] (7)

Figure 1 shows the schematic of the compression system with a closed coupled valve (CCV). The CCV means that there is no mass storage of gas between the compressor outlet and the valve and the pressure rise in the compressor and the pressure drop through the valve can be joined into an equivalent compressor. Considering CCV as a control actuator, the dynamical equations of the compressor system are given by:

\[
\begin{align*}
\dot{\psi} &= \frac{1}{4B^2lc}(\phi - \phi_T(\psi) + d\phi(\xi)) \\
\dot{\phi} &= \frac{1}{lc} (\psi_c(\phi) - \psi - \psi_V(\phi) + d\phi(\xi))
\end{align*}
\] (8)

\( \psi_V(\phi) \) acts as the input for the control system. Taking \( x_1 = \psi \) and \( x_2 = \phi \), the equations of compressor’s state space will be:

\[
\begin{align*}
\dot{x}_1 &= \frac{1}{4B^2lc}(x_2 - \phi_T(x_1) + d\phi(\xi)) \\
\dot{x}_2 &= \frac{1}{lc} (\psi_c(x_2) - x_1 - u + d\phi(\xi))
\end{align*}
\] (9)

The governing equations of the system exhibit the unmatched characteristic for the flow disturbance, whereas the disturbance in its second state is of type matched.
3. Robust finite-time adaptive sliding mode control for surge instability

Remarkably, this paper is the first to present a finite-time adaptive scheme for surge control in the presence of uncertainty in the compressor’s characteristic curve and also the opening percentage of the throttle valve, while both compressor system’s states are subjected to disturbances (9). The design steps for robust finite-time adaptive control are as follows:

First, the error variables are derived as:
\[ e_1 = x_1 - x_{1d} \]
\[ e_2 = x_2 - \alpha \] (10)

By differentiating the relation (10) and from (9), the systems error equations are obtained as follows:
\[ \dot{e}_1 = \frac{1}{4BC_2} (e_2 + \alpha - \phi_T(x_1) + d_\phi(\xi)) - \dot{x}_{1d} \]
\[ \dot{e}_2 = \frac{1}{L_c} (e_c(x_2) - e_1 - x_{1d} - u + d_\psi(\xi)) - \dot{\alpha} \] (11)

The nonlinear functions \( f_1 \) and \( f_2 \) include the uncertainties and disturbances of the system and are defined as:
\[ f_1(\cdot) = -\phi_T(x_1) + d_\phi(\xi) - 4B_2^2L_c x_{1d} \]
\[ f_2(\cdot) = \psi_c(x_2) - x_{1d} + d_\psi(\xi) - L_c \dot{\alpha} \] (12)

Thus, the system’s error equations reduce to:
\[ \dot{e}_1 = \frac{1}{4B_2^2L_c} (e_2 + \alpha + f_1(\cdot)) \]
\[ \dot{e}_2 = \frac{1}{L_c} (-e_1 - u + f_2(\cdot)) \] (13)

The Lyapunov candidate, \( V \), is then defined as:
\[ V = 2B_2^2L_c e_1^2 + \frac{L_c e_2^2}{2} + \frac{1}{2\lambda_1} \dot{f}_1^2 + \frac{1}{2\lambda_2} \dot{f}_2^2 \] (14)

where:
\[ \dot{f}_1 = f_1 - \dot{\hat{f}}_1 \]
\[ \dot{f}_2 = f_2 - \dot{\hat{f}}_2 \] (15)

In these relations, \( \dot{\hat{f}}_1 \) and \( \dot{\hat{f}}_2 \) estimate the bounds of \( f_1 \) and \( f_2 \), respectively. The function \( V \) is differentiated to obtain the following expression:
\[ \dot{V} = e_1 (e_2 + \alpha + f_1(\cdot)) + e_2 (-e_1 - u + f_2(\cdot)) - \frac{1}{\lambda_1} \dot{f}_1 \dot{\hat{f}}_1 \]
\[ - \frac{1}{\lambda_2} \dot{f}_2 \dot{\hat{f}}_2 \] (16)

The virtual control input \( \alpha \), and the control input \( u \) in this relation are defined as:
\[ \alpha = -k_1 e_1 - k_2 e_1^* - \dot{\hat{f}}_1 sgn(e_1) \]
\[ u = k_3 e_2 + k_4 e_2^* + \dot{\hat{f}}_2 sgn(e_2) \] (17)

where \( \nu = (p/q) \) and \( p, q > 0 \), \( k_1, k_2, k_3, \) and \( k_4 \) are the system gains, and \( sgn(.) \) is the sign function.

From relations (16) and (17), we have:
\[ \dot{V} = -k_1 e_1^2 - k_2 e_1 e_1^* + e_1 (f_1 - \dot{\hat{f}}_1 sgn(e_1)) \]
\[ - k_3 e_2^2 - k_4 e_2 e_2^* + e_2 (f_2 - \dot{\hat{f}}_2 sgn(e_2)) \]
\[ - \frac{1}{\lambda_1} \dot{\hat{f}}_1 \]
\[ - \frac{1}{\lambda_2} \dot{\hat{f}}_2 \] (18)

The recent relation is simplified to yield:
\[ \dot{V} \leq -k_1 e_1^2 - k_2 e_1 e_1^* - k_3 e_2^2 - k_4 e_2 e_2^* + |e_1| f_1 - |e_1| \dot{\hat{f}}_1 - k_3 e_2^2 \]
\[ - k_4 e_2 e_2^* + |e_2| f_2 - |e_2| \dot{\hat{f}}_2 - \frac{1}{\lambda_1} \dot{\hat{f}}_1 \]
\[ - \frac{1}{\lambda_2} \dot{\hat{f}}_2 \] (19)

We define the adaptive laws as:
\[ \dot{\hat{f}}_1 = \lambda_1 |e_1| \]
\[ \dot{\hat{f}}_2 = \lambda_2 |e_2| \] (20)

which gives the following expression:
\[ \dot{V} \leq -k_1 e_1^2 - k_2 e_1 e_1^* - k_3 e_2^2 - k_4 e_2 e_2^* \] (21)

From (21), we can infer that our designed control laws guarantee the stability of the compressor system under the effect of the uncertainties and disturbances described earlier.

**Lemma 1:** (Zou et al., 2013)

Suppose there exists a continuous positive definite function \( V(t) \) satisfying the following differential inequality:
\[ \dot{V} \leq -\rho_1 V - \rho_2 V^\alpha, \forall t > t_0 \] (22)

where \( \rho_1 > 0, \rho_2 > 0, \) and \( 0 < \alpha < 1. \) Then \( V(t) \) converges to the equilibrium point in the finite-time \( t_f \) defined by:
\[
\tau \leq t_0 + \frac{1}{\rho_1(1 - \rho_2)} \ln \left( \frac{\rho_1 V^{1-\rho} (t_0) + \rho_2}{\rho_2} \right)
\]

(23)

**Lemma 2:** (Zou et al., 2013)

Suppose that \(\tau_1, \tau_2, \ldots, \tau_n\) and \(0 < \rho < 1\) are positive constants, then the following inequality holds:

\[
(\tau_1^2 + \tau_2^2 + \cdots + \tau_n^2)^{2h} \leq (\tau_1^{2h} + \tau_2^{2h} + \cdots + \tau_n^{2h})^2
\]

(24)

Let the Lyapunov function candidate be:

\[
V = 2B^2lce_1^2 + \frac{l^2}{2}e_2^2
\]

(25)

Taking the time derivative of the \(V\) and from the equation (17), the following inequality is obtained:

\[
\dot{V} \leq -k_1e_1^2 - k_2e_1e_1' - k_3e_2^2 - k_4e_2e_2' + |e_1|^2 + |e_2|^2
\]

(26)

When considering the different segments of the above expression, the following inequalities hold:

\[
|e_1|^2 \leq c_1e_1^2 + \frac{l^2}{4c_1}
\]

(27)

\[
|e_2|^2 \leq c_2e_2^2 + \frac{l^2}{4c_2}
\]

(28)

\[
-k_1e_1^2 - k_3e_2^2 + |e_1|^2 + |e_2|^2
\]

\[= -\left( \frac{k_1 - c_1}{2B^2l_c} \right) \left( 2B^2l_ce_1^2 \right) - \left( \frac{2(k_3 - c_2)}{l_c} \right) \left( \frac{l^2}{2}e_2^2 \right)
\]

\[+ \frac{l^2}{4c_1} + \frac{l^2}{4c_2}
\]

\[\leq -\gamma_1 \left( 2B^2lce_1^2 + \frac{l^2}{2}e_2^2 \right) + c_3
\]

(29)

where:

\[
\gamma_1 = \min \left( \frac{k_1 - c_1}{2B^2l_c}, \frac{2(k_3 - c_2)}{l_c} \right)
\]

\[
c_3 = + \frac{l^2}{4c_1} + \frac{l^2}{4c_2}
\]

(30)

Again, from relation (26) and Lemma 2, the following inequality is satisfied:

\[
k_2e_1^{\nu+1} + k_4e_2^{\nu+1} \leq \min(k_2, k_4)(e_1^{\nu+1} + e_2^{\nu+1})
\]

\[\leq \min(k_2, k_4)(e_1^2 + e_2^2) \frac{\gamma_1}{\nu}
\]

\[\leq \min(k_2, k_4) \min \left( \frac{1}{2B^2l_c}, \frac{2}{l_c} \right)
\]

\[\times \left( 2B^2lce_1^2 + \frac{l^2}{2}e_2^2 \right)^{\frac{\nu+1}{2}}
\]

\[\leq \gamma_2 V^{\frac{\nu+1}{2}}
\]

(31)

where

\[
\gamma_2 = \min(k_2, k_4) \min \left( \frac{1}{2B^2l_c}, \frac{2}{l_c} \right)
\]

is a positive constant. By using equations (29-31), the relation (26) is rewritten as:

\[
\dot{V} \leq -\gamma_1 V - \gamma_2 V^{\frac{\nu+1}{2}} + c_3
\]

(32)

Then, the relation (32) can be presented in the following forms:

\[
\dot{V} \leq -\left( \frac{c_3}{V} \right) V - \gamma_2 V^{(\nu+1)/2}
\]

(33)

\[
\dot{V} \leq -\gamma_1 V - \left( \frac{c_3}{V^{(\nu+1)/2}} \right) V^{(\nu+1)/2}
\]

(34)

From the relation (33), if

\[
(\gamma_1 - \frac{c_3}{V}) > 0,
\]

then the finite-time stability would still be guaranteed using Lemma 1, which implies that \(V\) converges to the region \(V \leq c_3/\gamma_1\) in the finite-time. Hence, in the finite-time the sliding manifold \(s\) will converge to the region:

\[
||s|| \leq \frac{c_3}{\min(k_2, k_4) \min \left( \frac{1}{2B^2l_c}, \frac{2}{l_c} \right)}
\]

(35)

From (34), if

\[
\left( \gamma_2 - \frac{c_3}{V^{(\nu+1)/2}} \right) > 0,
\]

then the finite-time stability would still be guaranteed by using Lemma 2, which implies that \(V\) converges to the region \(V \leq (c_3/\gamma_2)^{2/(\nu+1)}\) in the finite-time. Therefore, the sliding manifold \(s\) will converge to the region:

\[
||s|| \leq \frac{c_3}{\min(k_2, k_4) \min \left( \frac{1}{2B^2l_c}, \frac{2}{l_c} \right)}^{2/(\nu+1)}
\]

(36)

Ultimately, the finite-time convergence region \(s \leq \Delta\) for the sliding manifold would be:

\[
||s|| \leq \Delta = \min \left\{ \frac{c_3}{\min(k_2, k_4) \ min \left( \frac{1}{2B^2l_c}, \frac{2}{l_c} \right)} ,
\right\}

(37)
4. Simulation results

In this section, the robustness and effectiveness of the proposed approach are verified by its simulation in Matlab (R2014, Mathworks, Natick, USA) software, and the obtained results are compared against the feedback control and robust adaptive fuzzy back-stepping methods presented in (Shafieian et al., 2019; Sheng et al., 2014). The simulations are conducted through three different scenarios. Until $t = 30$ s, the opening percentage of the throttle valve equals to $\gamma_T = 0.65$. This means that the working point of the compressor lies at the right of the surge line. After $t = 30$s, the opening percentage of the throttle valve reduces to $\gamma_T = 0.6$, causing the compressor’s working point to displace to the left of the surge line. As a result, the system enters the limit cycle (surge). The simulation parameters for the compressor are extracted from (Greitzer, 1976).

$$B = 1.8, I_c = 3, H = 0.18, W = 0.25, \psi_{c0} = 0.3 \quad \text{(38)}$$

The initial points in the process were taken as $(x_1(0), x_2(0)) = (0.15, 0.4)$. The controller parameters are chosen as:

$$k_1 = 10, k_2 = 1, k_3 = 10, k_4 = 1, \lambda_1 = 0.01,$$
$$\lambda_2 = 0.01, p = 3, q = 5 \quad \text{(39)}$$

For the first scenario, it is assumed that no external disturbance is applied to the compressor system:

$$d_\phi(\xi) = 0$$
$$d_\psi(\xi) = 0 \quad \text{(40)}$$

Figures 2 and 3 demonstrate the system states after the employment of the proposed method to the compressor’s pressure and flow, respectively. Based on Figure 2, the robust adaptive fuzzy back-stepping method has a smaller transient time and faster convergence to its final value. However, the pressure rise related to the presented method is higher, and according to Figure 3, the stabilization occurs in the lower flow.

Figure 4 presents the control signals for the first scenario. Here, CCV has been regarded as a control actuator. Therefore, its output must be positive, which serves as a physical constraint for the model. While our proposed scheme successfully generated a non-negative signal output throughout the simulation, this limitation has not been met by the controllers of (Shafieian et al., 2019; Sheng et al., 2014).

Figure 5 depicts the compressor trajectory in the performance curve for the first scenario. As shown, the designed controller exhibits good performance to prevent the compressor from entering the surge area.

In the case of the second scenario, some transient disturbances are assumed to be exerted on the system. The
Transient disturbances are modeled as:

\[
\begin{align*}
    d_{\phi}(\xi) &= 0.05e^{-0.015\xi} \cos(0.2\xi) \\
    d_{\psi}(\xi) &= 0.01e^{-0.005\xi} \sin(0.3\xi)
\end{align*}
\]  

Figures 6 and 7 illustrate the compressor’s pressure and flow, respectively. Strikingly, the presented controller outperforms the controllers in (Shafieian et al., 2019; Sheng et al., 2014) in terms of the surge avoidance.

Figure 8 displays the control signal for compressor surge avoidance in the second scenario. As shown, our proposed method has effectively met the physical limitation of the control actuator. Conversely, the control signals obtained from feedback control and robust adaptive fuzzy back-stepping methods are negative at the initial time of the simulation.

The compressor trajectory is illustrated in Figure 9, which presents the ability of the controller in active surge control while achieving transient disturbance rejection.
For the third scenario, it is assumed that some stable disturbances are imposed on the system. The stable disturbances are modelled as:

\[
\begin{align*}
    d_\phi(\xi) &= 0.02 \sin(0.1\xi) + 0.02 \cos(0.4\xi) \\
    d_\psi(\xi) &= 0.02 \sin(0.1\xi) + 0.02 \cos(0.4\xi)
\end{align*}
\]  

Based on Figures 10 and 11, the presented method shows superiority in stabilizing the compressor pressure and flow compared to the controllers reported in (Shafieian et al., 2019; Sheng et al., 2014).

The CCV output for the third scenario is presented in Figure 12. According to the figure, in contrast to the feedback control and robust adaptive fuzzy back-stepping practices, our developed controller guarantees the non-negative control signal for the compressor.

Figure 13 shows the performance curve of the compressor. Notably, despite the existence of disturbance, the controller demonstrates excellent capability in surge prevention.

While the compressor performance curve and disturbance bound in our proposed method were unknown, similar to the controllers introduced in (Shafieian et al., 2019; Sheng et al., 2014), unlike them, we did not have access to the throttle valve characteristic. However, our results suggest that in comparison to the feedback control and robust adaptive fuzzy back-stepping methods, the presented controller demonstrates superior performance in surge avoidance and is more robust to uncertainties imposed on the system. Furthermore, our controller generates a positive control signal throughout the simulations, which is consistent with the physical constraint of the system. Although the feedback control and adaptive fuzzy back-stepping methods were able to ensure a high increase in pressure and operation in lower flow, respectively, they failed to satisfy the restriction that...
the signals obtained from the control actuator must be non-negative. As a result, the control signal output from these controllers is an imaginary signal that would be impractical to real-world applications. In general, it can be said that using the proposed method, the finite time robust stability is guaranteed in the presence of uncertainty and disturbance, the physical limitations of the actuator are met, the disturbance effects on the flow and pressure of the compressor as well as uncertainty on the compressor characteristic curve and the throttle valve are covered, while the upper bounds of these uncertainties are unknown.

5. Conclusion

The present paper covers several aspects of the surge control problem in the compressor system, including (1) uncertainty of the compressor’s characteristic curve, (2) unknown opening percentage of the throttle valve, (3) disturbance imposed on flow and pressure, (4) unspecified upper bound of disturbance, and (5) finite-time assumption of the control approach. These effects were incorporated into the developed model via the employment of a finite-time sliding mode control scheme along with the adaptive method. The finite-time surge control is achieved through the terminal slide mode technique, while the adaptive control allows for the estimation of upper bounds of disturbance and uncertainty. The simulation results in Matlab verify that the designed controller can guarantee the stabilization and active control of the compressor system, along with higher pressure rise and working at lower flow in finite-time. Also, the proposed approach can efficiently mitigate the effects of the disturbance and uncertainties present in the system. Future studies on compressor systems can be designed to include the pipe effects, controller optimization, neural network-based intelligence to detect the movement of the compressor trajectory toward the surge, centralized and decentralized control of parallel and series compressors, etc.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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