Modeling the formation of non-diffraction parabolic beams

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Abstract. In this article we perform a numerical study of the generalization of parabolic non-diffraction beams of two orders formed from an analytically given distribution of the ring spatial spectrum. The formation and propagation of non-diffraction parabolic beams are simulated using the Fourier transform and the Fresnel transform. The influence of the radius and width of the circular spatial spectrum on the non-diffraction properties of the beams, as well as the nature of the symmetry of the transverse beam patterns depending on the orders of the beam are researched.

1. Introduction

Diffraction-free beams [1-3] attract increased attention of researchers in connection with their ability to resist the influence of diffraction during propagation. This property of diffraction-free beams is effectively applied in various fields, such as optical capture and manipulation of micro- and nano-objects [4-7], metrology and microscopy [8-10], coding and transmission of information [11, 12].

The most famous among diffraction-free beams are the Bessel modes [1-3], Mathieu beams [13], parabolic beams [14] and their generalizations [15-18] are somewhat less known. Beams with properties similar to non-diffraction beams are also known, for example, Airy beams [19], Olver beams [20] and their generalizations [21-25].

A common property of classical non-diffraction beams corresponding to solutions of the Helmholtz equation in separable coordinate systems is the concentration of the spatial spectrum on a narrow ring. This property is often used to generate various diffraction-free beams [1, 26, 27]. However, in this case, when radiation falls on a narrow annular gap, a significant part of the energy is lost. The formation of diffraction-free beams is more energy-efficient, when refractive or diffractive optical elements are used [28-32]. In addition, a simple method can be used for the energy-efficient formation of various non-diffraction laser beams using partial aperture of the annular spatial spectrum [33-35] created by a conventional axicon.

Obviously, in the experimental realization of diffraction-free beams based on the ring distribution, it is possible to form diffraction-free beams only approximately, since the width of the ring is not infinitely narrow with infinite energy, as theoretically assumed. To study the conservation of diffraction-free properties of such beams, numerical simulation can be performed.

In this article, a two-order generalization of parabolic non-diffraction beams [17] is numerically investigated, formed from an analytically given distribution of the annular spatial spectrum. The simulation of the formation and propagation of diffractionless parabolic beams is performed using the Fourier transform and the Fresnel transform. The influence of the radius and width of the annular spatial spectrum on the diffraction-free properties of the beams is investigated.
2. The theoretical basis
Diffraction-free beams are described in the general case as follows [13-18]:

\[ U(x, y, z) = \exp(-ikz) \int_0^{\pi} A(\varphi) \exp[-ik(x \cos \varphi + y \sin \varphi)] d\varphi, \]  

where \( A(\varphi) \) is the angular spectrum.

Classical parabolic non-diffraction beams are completely described by the angular spectrum of the following form [14]:

\[ A(\varphi; a) = \frac{1}{2(\pi |\sin \varphi|)^{1/2}} \exp \left[ i\alpha \ln \left( \frac{tg \frac{\varphi}{2}}{g \frac{\varphi}{2}} \right) \right]. \]  

Function (2) has only one continuous parameter \( a \), which was called the order of the beam.

In [17], a generalization of parabolic beams having two orders was considered — the previously proposed continuous parameter \( a \) and the integer index \( m \), providing new properties of beams. The generalized spatial spectrum is described by the following formula:

\[ A_m(\varphi; a) = \frac{1}{2} (\pi |\sin (m \varphi)|)^{-1/2} \exp \left[ i\alpha \ln \left( \frac{tg \left( \frac{m\varphi}{2} \right)}{g \frac{m\varphi}{2}} \right) \right], \]  

where \( m \) is an integer.

Consider the properties of function (3): it has zeros of the denominator; the integral of this function is convergent; singular points of phase condensation coincide with the zeros of the denominator.

To analyze the propagation of these beams, the Fresnel transform is used the Fourier transform through.

The Fourier transform of the function \( f(x) \) of the real variable is described in general form by follow integral:

\[ \hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ix\omega} dx. \]  

| Input Phase | Fourier Transform |
|-------------|------------------|
| m=1         | ![Input Phase m=1](image1)  |
| m=2         | ![Input Phase m=2](image2)  |
| m=3         | ![Input Phase m=3](image3)  |
Table 2. The result of the program when \( a = 5 \)

| \( m \) | Input Phase | Fourier Transform |
|-------|-------------|-------------------|
| 1     | ![Input Phase](Image1) | ![Fourier Transform](Image2) |
| 2     | ![Input Phase](Image3) | ![Fourier Transform](Image4) |
| 3     | ![Input Phase](Image5) | ![Fourier Transform](Image6) |
| 5     | ![Input Phase](Image7) | ![Fourier Transform](Image8) |

Fourier transform of functions defined on \( \mathbb{R}_n \):

\[
f(\omega) = \frac{1}{(2\pi)^{n/2}} \int_{\mathbb{R}^n} f(x) e^{-ix\omega} \, dx.
\]

(5)

Here, \( \omega \) and \( x \) are the vectors of space, \( \omega x \) is their scalar product.

Fourier transform and Fresnel transform simulation results

Table 1, Table 2, and Table 3 show the results of modeling the phase diagram of the input function and the Fourier transform graph when changing the parameter \( a \neq 0 \).

After analyzing the obtained graphs, we can conclude about the symmetry order of the amplitude of the Fourier graph. If \( m \) is an odd number, then the order will be equal to \( m \). If \( m \) is an even number, then the order is \( 2m \). For example, with \( m = 2 \), the symmetry order will be 4.

After analyzing the obtained Fourier graphs, you can see that with increasing module of parameter \( a \), the branches of the graph begin to move away from each other.

According to the graphs obtained in Table 3, we can enclose that with \( a < 0 \), the branches of the graph of the Fourier transform will look up, and with \( a > 0 \) they look down.

After analyzing all the obtained phase graphs, we can draw a conclusion that for a fixed value of \( m \) and change in the values of \( a \), the phase has the property to change.

For the subsequent analysis, we fix \( m \) and change the parameter \( a \) modulo. The simulation results are shown in table 4.
**Table 3.** The result of the program when \( a = -9 \)

| \( m \) | Input Phase | Fourier Transform |
|--------|-------------|-------------------|
| 1      | ![Image]    | ![Image]          |
| 2      | ![Image]    | ![Image]          |
| 3      | ![Image]    | ![Image]          |
| 5      | ![Image]    | ![Image]          |

**Table 4.** The result of the program for a fixed \( m = 2 \)

| \( a \) | Input Phase | Fourier Transform |
|--------|-------------|-------------------|
| 1      | ![Image]    | ![Image]          |
| 5      | ![Image]    | ![Image]          |
| 10     | ![Image]    | ![Image]          |
According to the results obtained, it can be enclosed that with increasing parameter $a$, the Fourier image will go to the periphery, and with decreasing $a$, on the contrary, it will concentrate towards the center of the image.

The Fresnel transform plays an important role in describing the free propagation of coherent optical fields and in analyzing of diffraction under less restricted conditions than those required for the Fourier transform. The Fresnel transform can be defined as follows:

$$F(u, v, z) = \iint f(x, y) \exp\left[\frac{i k}{2 z} (x - u)^2 + (y - v)^2\right] \, dx \, dy.$$  \hspace{1cm} (6)

where $z$ is the distance from the optical element.

If collecting lens with focal length $f$ and transmission function $\exp\left[-\frac{i k}{2 f} (x^2 + y^2)\right]$, is added into an input field, then after simplification we get

$$F(u, v, z) = \exp\left[-\frac{i k}{2 f} (x^2 + y^2)\right] \iint f(x, y) \exp\left[\frac{i k}{2 z} \left(\frac{1}{z} - \frac{1}{f}\right) (x^2 + y^2)\right] \, dx \, dy \, \exp\left[-\frac{i k}{z} (xu + yv)\right] \, dx \, dy.$$  \hspace{1cm} (7)

In this way, if we take the Fresnel transform with an added lens near the focal plane, there will be noticeable significant graphic similarity with the graph of the Fourier transform.

To visualize the result obtained, Table 5 and Table 6 show the graphs of the Fresnel transform and the Fourier transform of the input function.

After analyzing the obtained graphs, we can conclude that when the distance to the optical element changes by more than half the focal length, the diffraction-free properties will be lost.

**Table 5. The result of the program**

| $a=2$, $m=2$ |
|----------------|
| $z=150$ |  
| $f=200$ |  
| $z=175$ |  
| $f=200$ |  
| $z=225$ |  
| $f=200$ |

### 3. Conclusion

In this article, using the Fourier transform and the Fresnel transform, numerical modeling of the formation and propagation of diffraction-free parabolic beams of two orders is performed. For modeling, Python and Octave programs were written to form an analytically given distribution of the annular spatial spectrum and then apply the Fourier and Fresnel transforms to it.

The calculated transverse patterns of beams have symmetry of the order of $m$ for odd values of the index, while for even values of the index $m$ symmetry of $2m$ is observed. Modeling the propagation of
diffractionless parabolic beams showed an increasing in the distance of conservation of the
diffractionless properties of the beams with a decrease in the width of the ring, and reduction in the
non-diffraction distance with an increase in its radius. Numerical studies have revealed a violation of
diffractionlessness with an even value of the index m.

| Table 6. The result of the program |
|-----------------------------------|
| **Fresnel Transform** | **Fourier Transform** |
| a=5, m=1 |
| $z=250$ | $z=250$ |
| $f=300$ | $f=300$ |
| $z=275$ | $z=275$ |
| $f=300$ | $f=300$ |
| $z=300$ | $z=300$ |
| $f=300$ | $f=300$ |
| $z=350$ | $z=350$ |
| $f=300$ | $f=300$ |
| $z=490$ | $z=490$ |
| $f=300$ | $f=300$ |

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