Classical information deficit and monotonicty on local operations

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We investigate classical information deficit: a candidate for measure of classical correlations emerging from thermodynamical approach initiated in [1]. It is defined as a difference between amount of information that can be concentrated by use of LOCC and the information contained in subsystems. We show nonintuitive fact, that one way version of this quantity can increase under local operation, hence it does not possess property required for a good measure of classical correlations. Recently it was shown by Igor Devetak [2], that regularised version of this quantity is monotonic under LO. In this context, our result implies that regularization plays a role of “monotoniser”.

INTRODUCTION

Correlations are a fundamental property of compound quantum distributed system. The study of quantum correlations was initiated by Einstein, Podolsky and Rosen and Schrödinger. They were concerned with entanglement - quantum correlation, which are nonexistent in classical physics. Usefulness of entanglement in Quantum Information Theory to such task like teleportation, dense coding has motivated extensive study of it. However, an exciting subject of characterizing other interesting types of correlations has emerged. Namely, quantum correlation has been studied as a notion independent of entanglement. There were trials to divide total correlation into classical and quantum part, defined and analysed in [3] and strange properties of classical correlation of quantum states were discovered in [4]. A measure of classical correlation has also been proposed in [5].

In [1] an operational measure of quantum correlations was proposed. It was based on the idea that by using a system in state $\varrho$ one can draw $(N - S(\varrho))kT\ln 2$ of work from a single heat bath, where $N$ is amount of qubits in state $\varrho$ and $S(\varrho)$ is von Neumann entropy of given state. So that information function given by:

$$I(\varrho) = N - S(\varrho)$$

(1)
can be treated as equivalent to work. This scenario was used in the distributed quantum system, where Alice and Bob are allowed to perform only local operations and communicate classically with each other (These are so called LOCC operations) to concentrate information contained in the state to local subsystems. For nonclassical states the amount of work drawn by LOCC (equivalently amount of information $I_{LOCC}$ we can concentrate by LOCC operation into local subsystem) is usually smaller than work extractable by global operations (equivalently information $I_{GO}$, to which we have access by using global operation). The resulting difference $\Delta = I_{GO} - I_{LOCC}$ called information deficit or work deficit and it accounts for the part of correlation that must be lost during classical communication, thus describe purely quantum correlation. Similar definitions we can apply for one-way (Alice to Bob) information deficit $\Delta_{\rightarrow}$. It differs from $\Delta$ by using in definition one way LOCC operation instead (two way) LOCC. (For one way, from Alice to Bob, LOCC operations only communication from Alice to Bob is allowed.)

In [3] a complementary quantity, that could account for classical correlation was defined - classical information deficit $\Delta_{cl}$.

$$\Delta_{cl} = I_{LOCC} - I_{LO}$$

(2)

where $I_{LO}$ is amount of information accessible by using only local operations performed on $N_A$ qubits of subsystem A and $N_B$ qubits of subsystem B. (i.e. $I_{LO} = N_A - S(\varrho_A) + N_A - S(\varrho_B)$. One can see, that the two measures of correlations add up to quantum mutual information given by:

$$I_M = S(\varrho_A) + S(\varrho_B) - S(\varrho_{AB})$$

(3)

where $S(\varrho)$ is the von Neumann entropy of state $\varrho$ and $\varrho_{AB}(B) = Tr_B(\varrho_{AB})$, i.e. we have:

$$\Delta_{cl} + \Delta = I_M$$

(4)

Analogously, we have the following formula for one-way version of classical information deficit:

$$\Delta_{cl}(\varrho_{AB}) = I_{LOCC} - I_{LO}$$

(5)

In this paper we show a nonintuitive fact, i.e. that one-way classical information deficit $\Delta_{cl}(\varrho_{AB})$ is not a measure of classical correlation, because can increase under local operation. Remarkable, it was recently shown by Devetak [2] that quantity, which is equal to regularised classical information deficit $\Delta_{cl}^\infty(\varrho_{AB})$ is monotonic under LO. Combining those results with ours, we obtain, that regularisation may play a role of monotoniser. An asymptotic version of a function may be monotonic, even though one copy version is not.

FORMULA FOR $\Delta_{cl}^\infty$ AND COMPARISON WITH HENDERSON-VEDRAL MEASURE

In this section we provide formula for $\Delta_{cl}^\infty$ and compare it with measure of classical correlation introduced
by Henderson and Vedral. To this end we have to determine formula for the maximal amount of information, which can be concentrated to subsystems via a protocol, in which one-way classical communication is allowed. The most general such protocol is the following. Alice makes a measurement on her part of state and tells her results to Bob. The amount of concentrable information is then equal to the information of Alice plus average final information of Bob. The protocol transforms the state in following way:

$$\rho_{AB} \rightarrow \rho'_{AB} = \sum_i p_i \otimes I \rho_{AB} P_i \otimes I$$  \hfill (6)

where \( p_i \) given by

$$p_i = \text{Tr}(P_i \otimes I \rho_{AB} P_i \otimes I)$$  \hfill (7)

is probability that Bob gets the state \( \rho'_i \), which is of the form:

$$\rho'_i = \text{Tr}_A(P_i \otimes I \rho_{AB} P_i \otimes I)/p_i$$  \hfill (8)

and \( \{P_i\} \) are projectors constituting von Neumann measurement. Usually, in LOCC paradigm one would allow for POVM. However POVM requires adding ancillas, which we have to take into account, if we are estimating the amount of information that we can concentrate. Thus, we include from very beginning all needed ancillas and consider von Neumann measurement. In such a way we take into account POVM’s, too. (There is an open question, if it pays to add ancillas at all, we will discuss this later.)

The amount of information \( I(\mathcal{P}) \), which can be concentrated into subsystems in one-way protocol \( \mathcal{P} \) is thus equal to:

$$I(\mathcal{P}) = I^{\text{out}}_A + I^{\text{out}}_B$$  \hfill (9)

$$= N_A - S(\rho'_A) + N_B - \sum_i p_i S(\rho'_B)$$  \hfill (10)

$$= N - S(\rho'_A) - \sum_i p_i S(\rho'_B)$$  \hfill (11)

where \( N_{A(B)} \) is amount of qubits in Alice (Bob) part of state, \( (N_A + N_B = N) \). \( \rho'_A = \text{Tr}_B \rho'_{AB} \). The maximal information that can be concentrated by one-way protocols \( \mathcal{P} \rightarrow \) is denoted by \( I^\rightarrow \):

$$I^\rightarrow = \sup_{\mathcal{P} \rightarrow} I(\mathcal{P} \rightarrow)$$  \hfill (12)

Having formula for \( I^\rightarrow \) we can express \( \Delta_q^\rightarrow \) as:

$$\Delta_q^\rightarrow = N - S(\rho_{AB}) - \sup_{\{P_i\}} \{(N - \sum_i p_i S(\rho'_B) - S(\rho'_A))$$

$$= \inf_{\{P_i\}} \{\sum_i p_i S(\rho'_B) + S(\rho'_A)\} - S(\rho_{AB})$$  \hfill (13)

Since \( \Delta_q^\rightarrow \) is equal to the difference between total information \( N - S(\rho_{AB}) \) and \( I^\rightarrow \), we obtain:

$$\Delta_q^\rightarrow(\rho_{AB}) = \sup_{\{P_i\}} \{(S(\rho_A) - S(\rho'_A))$$

$$+ \{S(\rho_B) - \sum_i p_i S(\rho'_B))\}$$  \hfill (14)

where the supremum is taken over all local dephasings on Alice’s side. Note that protocol is determined by choosing Alice’s measurement. Note also, that the optimal measurement is a complete one, i.e. \( P_i \) can be chosen to be one dimensional projectors. Indeed, given any incomplete measurement, Alice can always refine it, in such a way, that her entropy will not increase, and of course, any refinement will not increase Bob’s average entropy. In eq. (14), we have distinguished two terms. The second term

$$S(\rho_B) - \sum_i p_i S(\rho'_B)$$

shows the decrease of Bob’s entropy after Alice’s measurement. The first one

$$S(\rho_A) - S(\rho'_A)$$

denotes the cost of this process on Alice side, and is non-positive. It vanishes only if Alice measures in the eigenbasis of her local density matrix \( \rho_A \). Thus, the expression for \( \Delta_q^\rightarrow \) is very similar to the measure of classical correlation introduced by Henderson and Vedral:

$$C_{HV}(\rho_{AB}) = \sup_{P_i} (S(\rho_B) - \sum_i p_i S(\rho'_B)).$$  \hfill (15)

Originally, in definition of \( C_{HV} \) the supremum was taken over POVMs, but as mentioned, we take the state acting already on a suitably larger Hilbert space, unless stated otherwise explicitly. The difference between the Henderson-Vedral classical correlation measure and the one given in eq. (14) is that the former does not include Alice’s entropic cost of performing dephasing. Hence in general

$$\Delta_q^\rightarrow \leq C_{HV}$$

**WHEN \( \Delta_q^\rightarrow \) CAN BE EQUAL TO \( C_{HV} \)**

In this section we prove the following lemma:

**Lemma 1** Let \( \rho_{AB} \) be any bipartite state. Then \( C_{HV}(\rho_{AB}) = \Delta_q^\rightarrow(\rho_{AB}) \) if and only if there exist projectors \( \{P_i\} \) such that they commute with \( \rho_A(= \text{tr}_B \rho_{AB}) \) and they are optimal for both \( C_{HV} \) and \( \Delta_q^\rightarrow \) for the state \( \rho_{AB} \).
Remark 1 Note, that eigenbasis of $\varrho_A$ may not be unique.

Proof. For specific measurement, let us use the following notation:

\[ c_{HV} = S(\varrho_B) - \sum_i p_i S(\varrho_{i}^B) \]  
\[ \delta_{cl}^{-} = S(\varrho_A) - S(\varrho_A') + S(\varrho_B) - \sum_i p_i S(\varrho_{i}^B) \]

The quantity $c_{HV}$ and $\delta_{cl}^{-}$ are functions of state and a measurement. We have

\[ C_{HV} = \sup_{\{P_i\}} c_{HV} \]  
\[ \Delta_{cl}^{-} = \sup_{\{P_i\}} \delta_{cl}^{-}. \]

"⇒" Let us proof the "only if" part: Suppose that

\[ C_{HV} = \Delta_{cl}^{-} \]

Consider measurement (i) which achieves $C_{HV}$ and measurement (ii), which achieves $\Delta_{cl}^{-}$. Let $c_{HV}^{(i)}$ be the values $c_{HV}$ for measurement $i$ and $S(\varrho_{i}^{A})$ is Alice’s part entropy after measurement (ii). Then

\[ \Delta_{cl}^{-} = S(\varrho_A) - S(\varrho_A') + c_{HV}^{(i)} = c_{HV}^{(i)} = C_{HV} \]

We know that for arbitrary measurement $S(\varrho_A) - S(\varrho_A') \leq 0$ and $c_{HV}^{(i)} \leq c_{HV}^{(i)}$. If we want the equality to hold, then it must be that

\[ S(\varrho_A) - S(\varrho_A') = 0 \]  
\[ c_{HV}^{(i)} = c_{HV}^{(i)} \]

It follows that measurement (ii) is also optimal for $C_{HV}$. Moreover, notice, that this measurement is made in eigenbasis, otherwise it would increase entropy $S(\varrho_A)$ violating eq. 20.

"⇐" The "if" proof is obvious. Since we assume that the measurement achieving $C_{HV}$ and $\Delta_{cl}^{-}$ is the same and is made in eigenbasis of $\varrho_A$, so then $S(\varrho_A) - S(\varrho_A') = 0$, so that $\Delta_{cl}^{-}$ and $C_{HV}$ must be equal. This ends the proof of lemma.

WHEN $\Delta_{cl}^{-}$ CAN INCREASE UNDER LOCAL OPERATION.

Lemma 2 If $\Delta_{cl}^{-} \neq C_{HV}$ then the quantity $\Delta_{cl}^{-}$ can be increased by local operations.

Therefore let us assume that $\Delta_{cl}^{-} < C_{HV}$ for the state $\varrho_{AB}$. (Recall that, in general, $\Delta_{cl}^{-} \leq C_{HV}$.) Let us consider an optimal measurement $\{P_i^{HV}\}$ achieving $C_{HV}$. After the measurement, the state is of the form

\[ \varrho_{AB}' = \sum_i p_i P_i^{HV} \otimes \varrho_i^B. \]

We know that $C_{HV}$ cannot increase after local operations. Then

\[ C_{HV}(\varrho_{AB}) \leq C_{HV}(\varrho_{AB}') = S(\varrho_B) - \sum_i p_i S(\varrho_{i}^B) \]

so that \{ $P_i^{HV}$ \} is an optimal measurement for the state $\varrho_{AB}'$ also. Now if we repeat the measurement \{ $P_i^{HV}$ \} on $\varrho_{AB}'$, we get the same value of $C_{HV}(\varrho_{AB}')$ as before since $\varrho_{AB}'$ and the created Bob ensemble do not change under that particular measurement. Thus

\[ C_{HV}(\varrho_{AB}) = C_{HV}(\varrho_{AB}') . \]

Note that \{ $P_i^{HV}$ \} corresponds to the eigenbasis of $\varrho_A'$, where $\varrho_A'$ is the reduced matrix of $\varrho_{AB}$. Then

\[ \Delta_{cl}^{-}(\varrho_{AB}') = C_{HV}(\varrho_{AB}') \]

so that

\[ \Delta_{cl}^{-}(\varrho_{AB}') \geq \Delta_{cl}^{-}(\varrho_{AB}) \]

i.e. $\Delta_{cl}^{-}$ increase after local operation of dephasing by $P_i$.

Having Lemma 2 the question is whether there exist states for which $\Delta_{cl}^{-} \neq C_{HV}$. We know that in such case, for such states there should not exist any measurement optimizing both $\delta_{cl}^{-}$ and $c_{HV}$, which is made in eigenbasis. Equivalently, there should not exist a measurement that optimizes $C_{HV}$, which is made in eigenbasis of $\varrho_A$. To show this, the following results of Schumacher and Westmoreland and King, Nathanson and Ruskai connected with classical capacity of a quantum channel are helpful.

Suppose a source produces states $\varrho_k$ with probabilities $p_k$. For this ensemble, the authors in Ref. 11, 12 considered a quantity called Holevo quantity, defined as

\[ \chi = S(\varrho) - \sum_k p_k S(\varrho_k) \]

where \[ \varrho = \sum_k p_k \varrho_k. \]

They were interested in maximizing $\chi$ for the output ensemble $\{ p_k, \Lambda (|\psi_k\rangle \langle \psi_k|) \}$, where $\Lambda$ is fixed completely positive map (channel).

It turns out that for some channels, to maximize $\chi$, one needs a non-orthogonal input ensemble. This was first shown by Fuchs. 13 An example of such a channel is given by the following map 11:

\[ A_1(\varrho) = A_1 \varrho A_1^\dagger + A_2 \varrho A_2^\dagger \]  
\[ \text{where } A_1 = \sqrt{\frac{1}{2}} |1\rangle \langle 1| + |0\rangle \langle 0| \]
\[ A_2 = \sqrt{\frac{1}{2}} |0\rangle \langle 1| \]
where \(|0\rangle,|1\rangle\) is the standard basis in \(\mathbb{C}^2\). For this channel, maximum \(\chi\) is obtained for non-orthogonal input states.

On the other hand, it has been recently shown \[12\] that sometimes the number of states in the optimal ensemble must be greater than dimension of the system. An example is the map given by

\[
\Lambda_2(\varrho) = \frac{1}{2} \left( I + [0.6w_1, 0.6w_2, 0.5 + 0.5w_3, \vec{\sigma}] \right)
\]

where

\[
\varrho = \frac{1}{2}(I + \vec{w}\vec{\sigma})
\]

and \(\vec{w} = (w_1, w_2, w_3)\) with \(\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)\) and \(\sigma_i\) being the Pauli matrices. In this case \(\chi\) is maximized by a three component ensemble.

The above examples can lead us to a bipartite state \(\varrho_{AB}\), for which \(C_{HV}\) is not achieved by the measurement in the eigenbasis of \(\varrho_A\). (Note that these examples act only as indications. The results of channel capacities are not used directly, although such a direct connection is not ruled out.)

More precisely, given any channel and ensemble, we will construct some bipartite state and a measurement on one of its subsystems. We will expect that the measurement will give high value of \(c_{HV}\) on that state. In particular, if the ensemble is two component but non-orthogonal we obtain a better value of \(C_{HV}\) than the eigenbasis measurement then the optimal measurement for \(C_{HV}\) on Alice’s part of the state will not be in the eigenbasis of \(\varrho_A\). Moreover, if the ensemble is three component and the measurement will give a better value than any von Neumann measurement, the optimal measurement for attaining \(C_{HV}\) will not be a von Neumann measurement, but POVM. We show that it is indeed the case in both situations.

Let us now present our construction of the state and measurement from a given channel \(\Lambda\) and ensemble \(\{p_i, \psi_i\}\). We will first exhibit two ways of obtaining ensemble \(\{p_i, \psi_i\}\) from a pure bipartite state \(\psi_{AB}\), where \(\psi_i = \Lambda(\psi_i)\). Let \(\psi_{AB}\) be a state for which

\[
\text{Tr}_A|\psi_{AB}\rangle\langle\psi_{AB}| = \sum_i p_i |\psi_i\rangle\langle\psi_i|.
\]

One can write it in the form \(|\psi_{AB}\rangle = \sum_i \sqrt{p_i} |i\rangle |\psi_i\rangle\), where \(|i\rangle\) are orthogonal. Note that when we make a measurement in the basis \(|i\rangle\) at Alice’s side, the ensemble \(\{p_i, \psi_i\}\) is created at Bob’s side. Then one obtains ensemble \(\{p_i, \psi_i\}\) by letting \(\psi_i\) to evolve through the channel \(\Lambda\). But one can achieve \(\{p_i, \psi_i\}\) in a different way. First, the state \(\psi_{AB}\) is prepared and the operation \(I_A \otimes \Lambda_B\) is performed, producing state \(\varrho_{AB}\):

\[
\varrho_{AB} = (I_A \otimes \Lambda_B) (\psi_{AB})
\]

Then Alice makes the measurement in the basis \(|i\rangle\) and this produces the ensemble \(\{p_i, \varrho_i\}\) at Bob’s site. The connection between the scenarios is illustrated by the commuting diagram below. Starting from \(\psi_{AB}\), we can achieve the ensemble \(\{p_i, \varrho_i\}\) in two ways.

Here \(M_A\) denotes the measurement by Alice and \(\{*, *\}_B\) denotes the corresponding ensemble at Bob’s site. If we want to find the needed state \(\varrho_{AB}\) for which \(\Delta^2_\varrho \neq C_{HV}\), we should construct pure state \(\psi_{AB}\) and then perform operation \(I_A \otimes \Lambda_B\). First we use the channel (given by eq. \ref{eq:channel}) and ensemble from Ref. \[11\] to obtain \(\varrho_{AB}\) for which \(C_{HV}\) for some measurement is greater than for measurement in eigenbasis.

An example of a non-orthogonal ensemble, for the channel \(\Lambda_1\) given by eq. \ref{eq:channel}, which gives greater \(\chi\), than any orthogonal one is \(\{(\frac{1}{2}, \varphi_0), (\frac{1}{2}, \varphi_1)\}\), where

\[
|\varphi_0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\]

\[
|\varphi_1\rangle = \frac{4}{5}|0\rangle + \frac{3}{5}|1\rangle.
\]

Then we have

\[
\varrho_{AB} = (I_A \otimes \Lambda_1^B) |\psi_{AB}\rangle\langle\psi_{AB}|
\]

where

\[
|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A |\varphi_0\rangle_B + |1\rangle_A |\varphi_1\rangle_B)
\]

Now, we can check directly that for Alice’s measurement in basis \(|0\rangle, |1\rangle\) (which prepares the non-orthogonal ensemble \(\{(\frac{1}{2}, \varphi_0), (\frac{1}{2}, \varphi_1)\}\) on Bob’s side), the Henderson-Vedral quantity \(c_{HV}\) (see eq. \ref{eq:HV}) attains the value \(c_{HV}^{(1)} = 0.45667\). But for Alice’s measurement in the eigenbasis of \(\varrho_A\), \(c_{HV}\) attains the value \(c_{HV}^{(2)} = 0.3356\). Therefore \(c_{HV}^{(1)} > c_{HV}^{(2)}\), i.e. there exists Alice’s measurement which gives better value for \(c_{HV}\) than the measurement in the eigenbasis of \(\varrho_A\). The optimal measurement
is therefore clearly not in eigenbasis. This fact, as follows from Lemma 1, implies that for state given by formula (25), one has $C_{HV} \neq \Delta_{cl}^\rightarrow$. The three component ensemble for which $\chi$ for the channel $\Lambda_2$ given by eq. (24) is greater than that for any two component ensemble is $\{p_i, |\phi_i\rangle\}$ where $p_0 = 0.4023$, $p_1 = p_2 = 0.29885$ and

$$
|\phi_0\rangle = |0\rangle \\
|\phi_1\rangle = a|0\rangle + b|1\rangle \\
|\phi_2\rangle = a|0\rangle - b|1\rangle
$$

with $a = 0.0701579$, $b = 0.821535$. Then by our prescription, the state for which POVM is better than any von Neumann measurement, as far as $C_{HV}$ is concerned, is

$$
\varrho_{AB} = (I^A \otimes \Lambda_2^B)|\phi_{AB}\rangle\langle\phi_{AB}|
$$

where

$$
|\phi_{AB}\rangle = \sum_{i=0}^{2} \sqrt{p_i} |i\rangle|\phi_i\rangle.
$$

Again, as from Lemma 1 and Lemma 2, for this state $\Delta_{cl}^\rightarrow \neq C_{HV}$, hence $\Delta_{cl}^\rightarrow$ can be increased by Alice’s dephasing in basis $\{|0\rangle, |1\rangle, |2\rangle\}$, which can be treated as a POVM, since Alice’s subsystem has rank two, so it is efficiently qubit. For the measurement in the basis $|i\rangle$ (when the ensemble $\{p_i, |\phi_i\rangle\}$ is prepared on Bob’s side), $c_{HV}$ attains the value $\delta_{cl}^0 = 0.32499$. For von Neumann measurements, $c_{HV} \leq 0.321915$. Equality is obtained for measurement in eigenbasis.

Finally, let us show that POVMs that are good for $C_{HV}$ can be very bad for $\Delta_{cl}$. One can check that the same POVM which gives high value for $c_{HV}$, gives $\delta_{cl}^\rightarrow < 0$. Therefore a POVM which is good for $c_{HV}$ can be very bad for $\delta_{cl}^\rightarrow$.

We have checked that for $\delta_{cl}^\rightarrow$, the best von Neumann measurement is in eigenbasis. Then $\delta_{cl}^\rightarrow$ attains the value $\delta_{cl}^{\rightarrow(vN)} \approx 0.321915$.

This example indicates that $\Delta_{cl}^{\rightarrow}$ might be such a quantity for which POVM does not help. We conjecture that it can be true.

**DISCUSSION**

In this paper we have considered classical information deficit $\Delta_{cl}^{\rightarrow}$ defined as difference between amount of information that can be concentrated by LOCC and information concentrable by LO. It is equal to difference between measure of total correlation and measure of quantum correlation present in state. It was reasonable to expect that it should be a measure of classical correlation. We have shown, that it is not true, because $\Delta_{cl}^\rightarrow$ can increase under local operation. We have proved it through comparison it with measure of classical correlation by Henderson-Vedral. We based on lemma, which tells us, when these quantities can be equal. We showed that, if they are different, then $\Delta_{cl}^\rightarrow$ can increase under local actions. The last thing we did is finding examples of states for which $\Delta_{cl}^\rightarrow \neq C_{HV}$. We also exhibited example, where POVM is very good for $C_{HV}$, but completely bad for $\Delta_{cl}^\rightarrow$. This suggest that POVM’s may be not helpful in one-way protocol of localizing information. This would be compatible with result for two-way protocols, were borrowing ancillas does not help in concentrating information [11].

The above results would mean that $\Delta_{cl}^\rightarrow$ is useless as far as classical correlation of quantum states are concerned. Fortunately, it is not the case.

Recently it was shown in [2] that regularized version of $\Delta_{cl}^\rightarrow$ is a measure of classical correlation, because it is equal to distillable common randomness [6], which in fact is equal to regularised $C_{HV}$. Since the latter is monotonic under local operation, then $\Delta_{cl}^\rightarrow$ if regularised is monotonic, too. It is very puzzling fact, that we have quantity, which defined for one copy of state can increase after local operations, but its regularized version not. Thus, according to our results, the regularization plays a role of “monotoniser” in this case.

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