A weighted evolving network model more approach to reality

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In search of many social and economical systems, it is found that node strength distribution as well as degree distribution demonstrate the behavior of power-law with droop-head and heavy-tail. We present a new model for the growth of weighted networks considering the connection of nodes with low strengths. Numerical simulations indicate that this network model yields three power-law distributions of the node degrees, node strengths and connection weights. Particularly, the droop-head and heavy-tail effects can be reflected in the first two ones by this new model.

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I. INTRODUCTION

Complex networks have been studied across many fields of science and society, stimulated by the fact that many systems in nature can be described by complex networks with nodes representing individuals or organizations and edges mimicking the interactions among them[1]. In the past study, there have been a variety of important properties from the real-life networks as well as several artificial networks. Notably, it is found that many real-life networks are scale-free networks, which means that the degree distributions of these networks follow a power law form \( P(k) \sim k^{-\gamma} \) for large \( k \), where \( P(k) \) is the probability that a node in the network is connected to \( k \) other nodes and \( \gamma \) is a positive real number determined by the given network. Since power laws are free of characteristic scale, such networks are called "scale-free network". Examples of such networks are numerous: these include the Internet, the World Wide Web, social networks of acquaintance or other relations between individuals, metabolic networks, integer networks, food webs, etc.[2].

From 1999, because of the ubiquity of scale-free networks, much attention has been focused on how to model scale-free networks. One of the most well-known model is introduced by Barabási and Albert[3], which is very similar to Price’s model[4]. The BA model suggests that two main ingredients of self-organization of a network in a scale-free structure are growth and preferential attachment. These point to the facts that most networks continuously grow by the addition of new vertices, and new vertices are preferentially attached to existing vertices with large number of neighbors. However, not all the edges should be viewed as the same because many real-life networks display different interaction weights between nodes such as various scientific collaboration network and ecosystems[5]. This weighted networks[6] are usually described by a matrix \( w_{ij} \) specifying the weight on the edge connecting the vertexes \( i \) and \( j \), with \( i, j = 1, \ldots, N \), where \( N \) is the size of the network. For simplicity, we only consider undirected network models so that the weight be merely viewed symmetrically \( w_{ij} = w_{ji} \). Then the strength of the node \( i \) can be defined as[7, 8]:

\[
s_i = \sum_{j \in V(i)} w_{ij}
\]

where the sum runs over the set \( V(i) \) of neighbors of \( i \). What should be pointed out is that in most cases the empirical distribution of \( s \) has the behavior of power-law with droop-head and heavy-tail[8], analogous to the power-law decay of the degree distribution in weighted networks, while in unweighted networks the empirical distribution of degree also has the droop-head and heavy-tail[3].

Recently, Barrat et al. proposed a weighted evolving network[6], which recovers an effective preferential attachment. Their model indeed demonstrates the property of scale-free, but it loses the properties of droop-head and heavy-tail. In the present paper, we propose a new model for the growth of weighted networks considering the connection of nodes with low strengths. Numerical simulations indicate that this network model yields three power-law distributions of...
the node degrees, node strengths and connection weights. Particularly, the droop-head and heavy-tail effects can be reflected in the first two ones by this new model.

The outline of this paper is as follows. In Sec. 2, we define the model. In Sec. 3, we report the experimental results. Finally, in Sec. 4, we give our conclusion and a short discussion.

II. THE NEW MODEL

The proposed model is defined as the following scheme:

First, start from an initial seed of $N_0$ nodes connected by links with assigned weight $w_0$. Next, at each time step, a new node $n$ is added with $m$ edges that are randomly attached to a previously existing node $i$ according to the probability distribution

$$\Pi_{n \rightarrow i} = \frac{s_i}{\sum_j s_j}$$

(2)

This rule relaxes the usual degree preferential attachment, focusing on a strength driven attachment in which new nodes connect more likely to nodes handling larger weights and which are more central in terms of the strength of interactions. The weight of each new edge is fixed to a value $w_0$. Moreover, the presence of the new edge $(n, i)$ will introduce variations of the existing weights across the network. In particular, we consider the local rearrangements of weights between $i$ and its neighbors $j \in V(i)$ according to the simple rule

$$w_{ij} \rightarrow w_{ij} + \Delta w_{ij}$$

(3)

where

$$\Delta w_{ij} = \delta \frac{w_{ij}}{s_i}$$

(4)

and

$$s_i \rightarrow s_i + \delta + w_0$$

(5)

which induces a total increase of traffic $\delta$. In this paper, we only focus on the simplest model with $\delta$ a constant (See Fig. 1 a).

After given time steps of adding new nodes, the strengths of different nodes diverse greatly to some extend, which can be viewed as the diversities of competitive power of the nodes. It is natural to introduce the rule that the nodes with low strengths prefer to cooperate with one another in order to increase their competitive power. Therefore, in the following evolving process, our model not only add new nodes into the network, but also permit old nodes to evolve with the growth of the network. Define a threshold of strength $s_c$ to tell the nodes with low strengths -consisting of the set $G$ of ”struggling nodes”- apart from the network. This means that when $s_i < s_c$ viz. $i \in G$, the node $i$ struggles to connect with another node $j \in G$ with a given probability $p$ unless there is an existing edge between $i$ and $j$. (See Fig. 1 b) After the connections among old nodes have been done, the growth process is iterated by introducing a new node with the corresponding judgment of and connection among old nodes.

It is agreeable with the reality that the new model contains the mechanics of connection among the nodes with low strength. In the scientific collaboration networks and actor collaboration networks etc., there exists a common phenomena that the people, whose competitive abilities are limited, prefer to collaborate with one another or consist of a team so that they can survive in the environment full of competitions. Take the combination of corporations for another example in reality. Small corporations sometimes have to corporate with one another to increase their competitions too. Therefore, the mechanism of connection among old nodes has its root in reality.

III. EXPERIMENTAL RESULTS

We performed numerical simulations. For the sake of simplicity, we set $w_0 = 1$.

In Fig. 2, we show that the node strength distribution $P(s)$ obeys a power-law distribution with obvious droop-head and heavy-tail which coincides with the statistical results of many real-life networks[7].

In Fig. 3, we show that the degree distribution $P(k)$ also obeys a power-law distribution with droop-head or heavy-tail. This is again consistent with the statistical results of real-life networks.

In Fig. 4, we show the edge weight distribution $P(w)$ obeys power-law without obvious droop-head and heavy tail that is similar to many real-life data[9].

In Fig. 5, we show the time evolution of the strengths of three initial nodes and the weights of two initial edges. As can be seen, both the $s_i(t)$ and the $w_{ij}(t)$ increase linearly with $t$, when $t$ is large enough.
FIG. 1: (a) Illustration of the mechanism of introducing a new node. This introduction obeys the strength preferential attachment, and the strengths and the weights should be rearranged as above. (b) Illustration of the mechanism of the connection between old nodes. The nodes with low strengths prefer to connect with each other by an edge of weight $w_0$.

FIG. 2: The node strength distribution with different parameters $\delta$ and $p$: (a) $\delta = 2.5$ and $p = 0.25$ (b) $\delta = 2$ and $p = 0.25$ (c) $\delta = 1.5$ and $p = 0.29$ (d) $\delta = 1$ and $p = 0.29$. The data are accumulated over 20 networks of size $N = 6000$. All the illustrations display the power law property with the obvious droop-head and heavy-tail.

IV. CONCLUSION AND DISCUSSION

After the weighted evolving model of Barrat, we introduce a new one more approach to reality. We investigated the cause of presence of droop-head and heavy-tail, which gives a clearly physical picture of how a network evolves. A more meaningful thought of the model is that it reveals the mechanism of competition in some social networks. This may shed some new light on the development of modelling the real social networks.

There exist a series of modifications that are allowed. First, $\delta$ being constant is not reasonable. Since not every time step the new node introduces the same amount of $\gamma$ into the network. Second, the threshold of strength $s_c$ should be replaced by a function of $s$. Third, it is worthy to study the property of the spectral density of this model.
FIG. 3: The degree distribution with different parameters $\delta$ and $p$: (a) $\delta = 3$ and $p = 0.17$ (b) $\delta = 1.5$ and $p = 0.17$ (c) $\delta = 0.5$ and $p = 0.13$ (d) $\delta = 0$ and $p = 0.1$. The data are accumulated over 20 networks of size $N = 6000$. All the illustrations display the power law property with the obvious droop-head and heavy-tail.

FIG. 4: The edge weight distribution with different parameters $\delta$ and $p$: (a) $\delta = 0.5$ and $p = 0.2$ (b) $\delta = 1$ and $p = 0.2$ (c) $\delta = 2$ and $p = 0.2$ (d) $\delta = 5$ and $p = 0.2$. The data are accumulated over 1 networks of size $N = 6000$. Compared with the strength and degree distributions, weight distribution obeys relatively more strict power law.
FIG. 5: The left of Fig. 5 illustrates the time evolution of the weights of two edges with the parameters $\delta = 2$ and $p = 0.2$. The right illustrates the evolution of three strengths with the same parameters. As it can be seen, when $t$ is large enough, both the $s_i(t)$ and the $w_j(t)$ increase linearly with $t$.

Forth, the degree-degree and strength-strength auto-correlations and degree-strength cross-correlation are also worth studying.

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