Low momentum nucleon-nucleon potentials
with half-on-shell T-matrix equivalence

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Abstract

We study a method by which realistic nucleon-nucleon potentials $V_{NN}$ can be reduced, in a physically equivalent way, to an effective low-momentum potential $V^{low-k}$ confined within a cut-off momentum $k_{cut}$. Our effective potential is obtained using the folded-diagram method of Kuo, Lee and Ratcliff, and it is shown to preserve the half-on-shell T-matrix. Both the Andreozzi-Lee-Suzuki and the Andreozzi-Krenciglowa-Kuo iteration methods have been employed in carrying out the reduction. Calculations have been performed for the Bonn-A and Paris NN potentials, using various choices for $k_{cut}$ such as 2 fm$^{-1}$. The deuteron binding energy, low-energy NN phase shifts, and the low-momentum half-on-shell T-matrix given by $V_{NN}$ are all accurately reproduced by $V^{low-k}$. Possible applications of $V^{low-k}$ directly to nuclear matter and nuclear structure calculations are discussed.

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INTRODUCTION

Recently there has been much interest in studying nuclear physics problems, particularly the two-nucleon problem, using effective field theory (EFT). The basic idea of the EFT approach is to shrink the full-space theory to a small-space one which contains only the low-momentum modes. This is accomplished by integrating out the high momentum modes, thus generating effective couplings which implicitly contain the effects of the high-momentum modes. In this way, one can derive a low-momentum nucleon-nucleon (NN) potential, which is specifically designed for low-energy nuclear physics. The low-momentum NN potentials so constructed have indeed been quite successful in describing the two-nucleon system at low energy.

In the present work, we would like to derive also a low-momentum NN potential, although along a different direction. There are a number of realistic nucleon-nucleon potentials $V_{NN}$, such as the Bonn and Paris potentials, which all describe the observed deuteron and NN scattering data very well. They have both low- and high-momentum components. In fact because of the strong short range repulsion contained in them, their momentum space matrix elements $V(k, k')$ are still significant at large momentum. We would like to reduce such realistic potentials, in a physically equivalent way, to certain effective low-momentum NN potentials, $V_{\text{low}-k}$, which have only slow momentum components, below a chosen cut-off momentum $k_{\text{cut}}$.

TRANSFORMATION METHOD

The Schroedinger equation for the full-space two-nucleon problem is written as

$$H | \Psi \rangle = E | \Psi \rangle; \quad H = H_0 + V_{NN},$$

(1)

where $H_0$ is the kinetic-energy operator for the two-nucleon system. A model space $P$ is defined as a momentum subspace with $k \leq k_{\text{cut}}$, $k$ being the two-nucleon relative momentum. We want to transform the above equation to a model-space one

$$PH_{\text{eff}} P | \Psi \rangle = EP | \Psi \rangle; \quad H_{\text{eff}} = H_0 + V_{\text{low}-k},$$

(2)

where the low-momentum effective interaction is denoted as $V_{\text{low}-k}$. As far as the low-energy physics is concerned, we would like to have $H_{\text{eff}}$ to be physically equivalent to $H$. 
Specifically, this means the requirement that the deuteron binding energy, low-energy NN phase shifts and the low-momentum half-on-shell T-matrix of H are all reproduced by $H_{eff}$. The full-space half-on-shell T-matrix for $V_{NN}$ is defined as

$$\langle p' | T(\omega) | p \rangle = \langle p' | V_{NN} | p \rangle + \int_0^\infty k^2 dk \langle p' | V_{NN} | k \rangle \frac{1}{\omega - H_0(k)} \langle k | T(\omega) | p \rangle; \quad \omega = \varepsilon_p,$$

where $\varepsilon_p$ is the unperturbed energy for state $| p \rangle$. The corresponding model-space T-matrix given by $V_{low-k}$ is

$$\langle p' | T_{eff}(\omega) | p \rangle = \langle p' | V_{low-k} | p \rangle + \int_0^{k_{cut}} k^2 dk \langle p' | V_{low-k} | k \rangle \frac{1}{\omega - H_0(k)} \langle k | T_{eff}(\omega) | p \rangle; \quad \omega = \varepsilon_p.$$

Note for $T_{eff}$ the intermediate states are integrated up to $k_{cut}$. The boundary conditions associated with the free Green’s function are not written out, for simplicity. For p and p' both belonging to P (i.e. both $\leq k_{cut}$), we require

$$\langle p' | T(\omega = \varepsilon_p) | p \rangle = \langle p' | T_{eff}(\omega = \varepsilon_p) | p \rangle.$$

Base on the Kuo-Lee-Ratcliff (KLR) folded-diagram method \[5, 6\], Bogner, Kuo and Coraggio \[7\] have recently shown that the above requirements can be satisfied when the low-momentum effective interaction is given by the folded-diagram series

$$V_{low-k} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} + \cdots,$$

where $\hat{Q}$, often referred to as the $\hat{Q}$-box, is an irreducible vertex function in the sense that its intermediate states must be outside the model space P. The integral sign appearing above represents a generalized folding operation \[5, 6\]. $\hat{Q}'$ is obtained from $\hat{Q}$ by removing terms of first order in the interaction $V_{NN}$.

Let us outline their proof. A general term of the above T-matrix can be written as

$$\langle p' | (V + V_{1e(p)} V + V_{1e(p)} V_{1e(p)} V + \cdots) | p \rangle$$

where $e(p) \equiv (\varepsilon_p - H_0)$. Note that the intermediate states cover the entire space (1=P+Q where P denotes the model space and Q its complement). Writing it out in terms of P and Q, a typical term of T is $V_{e}^Q V_{e}^{Q} V_{e}^{P} V_{e}^{P} V_{e}^{P} V$. Note it has three segments partitioned by two $\frac{P}{e}$ propagators. Let us define a $\hat{Q}$-box as $\hat{Q} = V + V_{e}^{Q} V + V_{e}^{Q} V_{e}^{P} V + \cdots$, where all intermediate states belong to Q. One readily sees that the previous term is just a part of the three-$\hat{Q}$-box term,
and in general we have $T = \hat{Q} + \hat{Q}^P e \hat{Q} + \hat{Q}^P e \hat{Q}^P e \hat{Q} + \cdots$. By performing a folded-diagram factorization of each term with more than one $\hat{Q}$-box, one can rewrite the T-matrix as $T = V^{\text{low} - k} + V^{\text{low} - k} [ e V^{\text{low} - k} + V^{\text{low} - k} [ e V^{\text{low} - k} + \cdots. The above result then follows.

**RESULTS AND DISCUSSION**

The above $V^{\text{low} - k}$ may be calculated using iteration methods. We have done so using both the Andreozzi-Lee-Suzuki (ALS) and Andreozzi-Krenciglowa-Kuo (AKK) iteration methods\cite{8}, for both the Bonn-A and Paris potentials. We note that $V^{\text{low} - k}$ is energy independent, and it contains less information than the full-space $V_{NN}$. The ALS method converges to the lowest (in energy) d states of $H$, d being the dimension of the model space. (Here we have discretized the momentum space, writing $H$ as a finite matrix.) In contrast, the AKK method converges to the d states of $H$ with maximum P-space overlaps. We have found that the $V^{\text{low} - k}$ given by both methods are very close to each other, an indication that the intruder-state problem \cite{9} does not seem to be present in our present calculation.

The deuteron binding energy given by $V_{NN}$ is very accurately reproduced by $V^{\text{low} - k}$, for a wide range of $k_{\text{cut}}$. In Fig.1, we compare the phase shifts given by $V_{NN}$ and those by $V^{\text{low} - k}$. They agree quite well. Empirical phase shifts are determined up to $E_{\text{lab}} \approx 300\text{MeV}$, and they are given by the fully-on-shell T-matrix. Hence we need to use $k_{\text{cut}} \sim 2 \text{fm}^{-1}$, if we want $V^{\text{low} - k}$ to reproduce the phase shifts up to this energy. In Fig. 2, we compare the half-on-shell T-matrices (calculated with the principal-value boundary condition) given by $V_{NN}$ and $V^{\text{low} - k}$, they also agree quite well. Note that plotted are the $(k',k)$ matrix elements with $k^2 = E_{\text{lab}} M / 2 \hbar^2$, $M$ being the nucleon mass. There are a number of similarities between the model-space reduction method used here and the renormalization group method employed in effective field theory, and it would be useful to elucidate the connection between them. Since our method exactly preserves the half-on-shell T-matrix, it may provide a convenient way to study the flow equation which describes the change of the low-momentum effective interaction with respect to the momentum cutoff.
To summarize, we have reduced realistic NN potentials $V$ (Bonn-A and Paris) to corresponding effective low-momentum potentials $V_{\text{low}-k}$ for a model space of $k \leq k_{\text{cut}}$. The deuteron binding energy, low-energy phase shifts and the low-momentum half-on-shell T-matrix given by $V_{NN}$ are all reproduced by $V_{\text{low}-k}$. Because of the strong short-range repulsion contained in $V_{NN}$, it is well known that we can not use it directly in shell-model calculations of nuclei and/or in Hartree-Fock calculations of nuclear matter; we need first to convert $V_{NN}$ into a G-matrix, to take care of the short-range correlations. The G-matrix so obtained is energy dependent. We have found that our $V_{\text{low}-k}$ is a generally smooth potential (without strong short range repulsion), and it is energy independent. It may be suitable to use $V_{\text{low}-k}$ directly in the above calculations, without the need of first calculating the usual Brueckner G-matrix. This would be of interest and desirable. We have done some shell model calculations [7] in this direction, and obtained rather encouraging results. Nuclear matter calculations using $V_{\text{low}-k}$ are in progress.

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$k_{\text{cut}} = 1.7 \text{ fm}^{-1}$

Phase Shifts (Degrees)

Lab Energy (MeV)
$E_{\text{lab}}=25$ MeV, $k_{\text{cut}}=1.7$ fm$^{-1}$

$T(k',k)$ (fm)

Relative momentum $k'$ (fm$^{-1}$)

- from $V$–Paris
- from $V$–lowk