Localization from shattering: higher dimensions and physical realizations

Vedika Khemani,1 Michael Hermele,2 and Rahul M. Nandkishore2

1Department of Physics, Stanford University, Stanford, CA 94305, USA
2Department of Physics and Center for Theory of Quantum Matter, University of Colorado, Boulder, CO 80309

In recent work [arXiv: 1904.04815] two of us explained how the twin constraints of charge and dipole moment conservation ‘shatter’ the Hilbert space of a one dimensional quantum system into emergent disconnected dynamical sectors, giving rise to an exactly localized subspace of dimension exponentially large in volume which the localization does not rely on disorder, and is robust to noise. The dimensions of the emergent dynamical subsectors have a wide distribution, leading to a strong initial state dependence in the dynamics and the coexistence of low and high entanglement eigenstates reminiscent of quantum scarring. Here we explain how this phenomenology may be extended to higher dimensional systems on hypercubic lattices. We also explain how the key phenomena may be readily observed in near term ultracold atom experiments. In experimental realizations, the conservation laws are approximate rather than exact, so the localization only survives up to a prethermal timescale that we estimate. We comment on the implications of these results for recent predictions of Bloch/Stark many-body localization.

A particularly interesting question in many body quantum dynamics is whether a system can robustly fail to come to equilibrium under its own dynamics. One well known class of problems where such robust ergodicity breaking does arise involve the phenomenon of many body localization (MBL) [1–9]. Other phenomena involving ergodicity breaking include integrable systems which possess an extensive number of conserved quantities and, more recently, systems exhibiting so called ‘quantum scars’ in which a vanishing fraction of eigenstates are non-thermal and coexist with thermal states [10–12]. In these latter contexts, however, it is not known to what extent the ergodicity breaking is robust to generic perturbations of the Hamiltonian [13], and explanations for the phenomenology of scarring are still being widely debated [10, 13–21]. The search for alternative mechanisms for robustly and provably breaking ergodicity therefore continues apace.

Recently, it was observed that strictly local quantum dynamics in one dimension, constrained to conserve both charge and the dipole moment of charge, can exhibit ergodicity breaking that (unlike MBL) is robust even to temporal noise [22–24]. The localization in these models lies outside the usual framework of locator expansions, and instead follows from a novel mechanism introduced in Refs [23, 24]: the twin imposed constraints provably cause the Hilbert space to ‘shatter’ or ‘fracture’ into dynamically disconnected emergent subsectors — so that states even with the same global values of charge and dipole moment cannot mix under the local dynamics. The shattering leads to a wide distribution of dimensions for the emergent subsectors, leading to a strong initial state dependence in the dynamics. Most strikingly, the subsectors include exponentially many (in system size) exactly localized subspaces with dimension one.

The phenomenon is robust in that it only relies on spatial locality and two local commuting constraints, and does not depend on details of the Hamiltonian, nor on the presence or absence of spatial or temporal translation symmetry. This is particularly striking since conventional wisdom holds that the presence of a finite (O(1)) number of conservation laws should not generically impede thermalization — which requires instead the presence of extensively many explicit or emergent integrals of motion. However, Refs [23, 24] primarily focused on systems in one spatial dimension, leaving open the question of whether the phenomenon of shattering extends also to higher dimensional systems. Further, while the “toy” models with charge and dipole moment conservation were inspired by work on fracton models [25], it was unclear how constraints of this form may be generated in physical laboratory settings.

In this work we explain how ‘shattering’ of Hilbert space may arise in higher dimensions, and how it may give rise to a subspace exponentially large in system volume in which the dynamics is exactly localized. By ‘localized’ we mean that the system retains a memory of its initial conditions in local observables for infinite times. We also discuss how the relevant models may be realized in near-term ultracold atom experiments. We apply this understanding to the special case of ultra cold atoms in a tilted potential, a problem with a long history [26–31], which has recently been revisited [32–34], especially from the point of view of ‘Bloch’ [32] or ‘Stark’ [33] many body localization.

Throughout this paper we will restrict to systems on hypercubic lattices with linear extent L in each direction i.e. square lattices in two dimensions, or simple cubic lattices in three dimensions. On each site r, there exists an effective local U(1) ‘charge’. This could be particle number for a bosonic or fermionic model, or Sz (the z component of spin) in a model of qudits with spin S. We will work with spin variables in most of what follows, but our statements are readily translated to the bosonic and fermionic cases. The dynamics will be required to conserve the total U(1) charge \( Q = \sum_r S^z_r \), and certain multipole moments of charge (defined below in the obvious manner). We further assume that the dynamics are generated by strictly local models, such as static Hamiltonians with interactions of maximum spatial range ℓ or,
more generally, models of unitary circuits with local gates that may be chosen randomly in space and time (and no gate acts on two sites separated by more than $\ell$ along any lattice axis). Our arguments are cleanest for a finite $O(1)$ interaction range $\ell$, but we also discuss exponentially local (rather than strictly local) models below. It will not matter whether our models are translationally invariant in space or time.

**Shattering from dipole conservation:** We begin with the case where the dynamics is generated by gates that conserve charge and also conserve dipole moment, defined along the various lattice directions as $P_\alpha = \sum_r r_\alpha S_\alpha^r$, where $r = (r_x, r_y, r_z \cdots)$. In one dimension, it is easy to see that the conservation of $Q$ and $P_z$ provably gives exponentially many strictly localized ‘inert’ states that are left invariant by the dynamics ([23], [24]). Let us denote by $+/−$ the maximum/minimum local charges on a site respectively; these could be the ‘top’ and ‘bottom’ states of a qubit so that $S^z = ±S$, or else the occupied and unoccupied states of a hardcore boson model. Consider product states which are either $+/−$ on every lattice site, with $+$ and $−$ regions separated by domain walls that are at least $\ell$ sites apart (Fig. 1(a)). A range $\ell$ unitary gate acting on such a state locally sees either a region of extremal charge (if it lies within a domain), or a region of extremal dipole moment for a particular total local charge (if it straddles a domain wall), and hence can make no rearrangements. To lower-bound the number of inert states, we can divide the chain into line-segments (blocks) of length $\ell$ randomly pick each block to be ‘all $+$’ or ‘all $−$’. There are $2^{\ell/2}$ such inert states, every one of which has trivial dynamics ([23], [24]). Crucially, even though such states can have the same global $Q, P_z$ quantum numbers as other states, they cannot be connected to these other states under local dynamics.

We now turn to higher dimensions $d > 1$. Of course, the 1d states considered above can be extended in a translationally invariant fashion in directions orthogonal to $\hat{x}$ (Fig. 1(b)) and all such states would still be inert. But there are only exponentially many in $L$ such states. However, we now show that if all components $\{P_\alpha\}$ are conserved, then the number of inert states is $\sim \exp(cL^d)$. For specificity, consider a system in $d = 2$ space dimensions $(\hat{x}, \hat{y})$. Start with a ‘stripe’ state with domain walls parallel to the $y$ axis and at least $\ell$ sites apart in the $x$ direction. Now, note that these domain walls can be allowed to ‘roughen’ slightly while leaving the state inert. For specificity: divide up a domain wall that lives between sites with $x$-coordinate $n\ell$ and $(n\ell+1)$ into blocks of length $\ell$. In each such block, allow the domain wall to uniformly shift in the $+\hat{x}$ direction by either zero or one lattice spacings (Fig. 1(c)). This reduces the spacing between domain walls by at most one in the $\hat{x}$ direction; to wit, all sites with $n\ell+1 < x \leq (n+1)\ell$ are $+$ while all sites with $(n−1)\ell + 1 < x \leq n\ell$ are $−$, and dipole conservation of $P_x$ still prohibits any rearrangement involving these sites. It is only along the line $x = (n\ell+1)$ that we encounter both $+$ and $−$ sites, and can make rearrangements that conserve the $\hat{x}$ component of dipole moment. However, along this line we see alternating $+$ and $−$ regions with domain walls at least $\ell$ apart, and conservation of $P_y$ guarantees that this too must be inert. Thus, in fact any such ‘roughened’ configuration of a domain wall is inert. There are $N(L) \sim 2^{L/\ell}$ inert ‘roughened’ configurations of each domain wall, and $L/\ell$ places where we could choose to place a domain wall (or not), so the total number of inert states is at least $\sim N(L)^{L/\ell} \sim 2^{L^2/\ell^2}$. This argument proceeded by a dimensional reduction to the one-dimensional problem. By the same token, the argument extends to hypercubic lattices in arbitrary dimension, so that conservation of charge and all components of dipole moment is always sufficient to guarantee the existence of an exponential in volume number of inert states.

**Shattering from multipole conservation:** The localized states considered above were fundamentally one dimensional (i.e. ‘stripe-like’). We now consider a class of intrinsically $d$-dimensional localized states, with number exponential in system volume, which become available if the system conserves the first $d$ multipoles of charge.

Consider a two dimensional system on a square lattice constrained to conserve charge $q$, dipole moment $\{P_\alpha\}$, and also quadrupole moment $\{P_{\alpha\beta}\}$. Note that in two dimensions quadrupole moment is a rank two symmetric traceless tensor with two independent entries $P_{xy}$ and $P_{xx} − P_{yy}$, corresponding to the ‘dipole of a dipole’ in the directions perpendicular and parallel to the dipole vector respectively. The components of $P_{\alpha\beta}$ are defined in $d$ space dimensions as $P_{\alpha\beta} = \int dV \rho(r)[dr \delta_{\alpha\beta} − r^2\delta_{\alpha\beta}]$, where $dV$ indicates a volume integral, $\rho$ is the charge density, $\delta_{ij}$ is the Kronecker delta function, and the definition depends on the choice of origin (with obvious lattice generalizations) Assume that charge, dipole, and $P_{xy}$ are all locally conserved (conservation of $P_{xx} − P_{yy}$ is not necessary). Now consider ‘checkerboard’ states made by dividing up the system into $\ell \times \ell$ squares, and allowing every square to be either all $+$ or all $−$ randomly (Fig. 1(d)). There are $2^{L^2/\ell^2}$ such states. Any gate acting on such a state acts across either zero or one corners. If it acts across zero corners, then it acts on a state which is locally either maximum charge, or maximum dipole given its charge, and charge and dipole conservation suffices to guarantee that the gate must act trivially (i.e. as a pure phase). Meanwhile, if the gate acts across one corner, then it acts on a state that is locally of extremal $P_{xy}$ given its charge and dipole moment, such that charge, dipole and $P_{xy}$ conservation again forces the gate to act trivially. It then follows that every ‘checkerboard’ state of this form is an exact eigenstate of the dynamics, concluding our proof that there is an exactly localized subspace of dimension at least $2^{L^2/\ell^2}$. Likewise, one can generalize to three and higher dimensions, so long all higher multipoles of charge are conserved. For instance, three dimensions would require conservation of charge, dipole moment, all off diagonal components of the quadrupole moment $\{P_{xy}, P_{xz}, P_{yz}\}$, and the fully off diagonal $P_{xyz}$
component of the octupole moment – in which case all $2^{L/\ell^3}$ cubic tilings of space with maximal local charge would be inert.

**Shattering and shielding with multipolar conservation laws:** We now show that in addition to the ‘exactly localized’ subspace discussed above, there also arises a broad distribution of dynamical subsectors of various sizes, akin to the shattering phenomenon discussed in [23, 24]. If we consider the time evolution operator constructed by repeated applications of the gates, then a ‘dynamical subsector’ consists of a set of $S^z$ product states such that the time evolution operator has non-zero matrix elements between them. The ‘inert’ states discussed above lie in subsectors of size one (i.e. they do not mix with any other $S^z$ product state). Subsectors of size larger than one may be simply constructed by embedding ‘active’ regions into the otherwise inert states. At first glance one may worry whether active regions can be stably embedded, or whether they ‘destabilize’ the localization in the entire system, say through a mechanism akin to the ‘avalanche’ discussed in [25]. In the present problem, though, the avalanche problem is straightforward to avoid - one simply surrounds the active region by a ‘shielding’ region of maximal multipole [23]. In one dimension, a suitable shielding region would be ‘all +’ to the right and ‘all −’ to the left, with the size of the shielding region at least as large as the active region. In two dimensions, a suitable shielding region would be all + in the first and third quadrant, and all − in the second and fourth quadrant. In either case, any process by which the active region ‘melts’ the shielding region necessitates increasing the multipole moment of the active region (dipole moment in one dimension, $P_{xy}$ in two dimensions, etc), which makes the active region less active. Moreover, the deeper the rearrangements extend into the shielding region, the bigger the change in the multipole moment in the shielding region, and thus the bigger the back action on the active region. For a finite sized active region and a sufficiently large finite shielding region, the time evolution operator cannot have any matrix elements to product states which differ from the initial condition at the outer boundaries of the shielding region, or beyond. The initial condition can thus only mix with a finite number of other product states. In this manner, one may construct dynamical subspaces of a range of sizes by embedding one or more finite volume active regions into the otherwise localized subspace, hence ‘shattering’ Hilbert space.

**Physical Realizations:** We now discuss how conservation laws on multipole moments may be generated in physically realistic settings. Dipole moment couples directly to electric field, so if the system is placed in a sufficiently large static electric field (or equivalently, a tilted potential) then the Hilbert space will (approximately) split into symmetry sectors labelled by dipole moment (equivalently, center of mass position), with states at different dipole moment having sharply different energies. A minimal model which ‘conserves charge and dipole moment’ in one dimension would thus be given by a model of hardcore bosons (or spinless fermions) in a linearly varying scalar potential (Fig. 2(a)) so that $H = H_0 + V$:

$$H_0 = J \sum_{x} b_x^\dagger b_{x+1} + U \sum_i n_x n_{x+1}$$  \hspace{1cm} (1)

$$V = F \sum_x x n_x = FP_x$$  \hspace{1cm} (2)

where $n_x = 1$ ($n_x = 0$) corresponds to the + (−) state i.e. the presence or absence of a boson on site $x$. This model can be mapped to a spin 1/2 system with spin up and down states mapping to $n_x = 1, 0$ respectively. Likewise, we could choose $H_0$ to be a standard Fermi-Hubbard Hamiltonian with spinful fermions, in which case the $n_x \in \{0, 1, 2\}$ states can be mapped to the $S^z$ states of a spin-1 qudit, with the extremal $\pm$ states corresponding to empty ($n_x = 0$) and doubly occupied ($n_x = 2$) respectively.

Now if one prepares an initial state at high temperature with respect to $H_0$, say a charge density wave (CDW) of small amplitude $A$ and wavenumber $k$ with $\langle n(x, t = 0) \rangle = \pi + A \cos(kx + \phi)$, we expect the system to evolve towards thermal equilibrium by
A one-dimensional system in a ‘tilted potential’ conserves dipole moment up to an exponentially long prethermal timescale. (b) When a ‘tilted potential’ is applied at an angle in two dimensions, dipole moment in the direction of the tilt is conserved but one has to worry about hopping in an ‘equipotential shell’ perpendicular to the tilt direction (shown). The color scale denotes the potential energy due to a tilt at 60 degrees relative to the $\hat{x}$ axis. Sites within the equipotential shell see a quasiperiodic potential set by the distance from the equipotential line. (c) Quadrupole conservation in two dimensions may be obtained by placing the system in a harmonic trap, with inequivalent trap frequencies along two orthogonal directions, and with the trap rotated with respect to lattice axes.

exchanging energy between the tilt and ‘non-tilt’ parts of $H$ [23]. However, we note that the spectrum of $F$ is super-extensive ($\sim L^2$) while that of $H_0$ is merely extensive. Thus, if we prepare an initial state with $\langle P_x \rangle_0 \sim L^2$, the tilt energy cannot be dissipated through $H_0$ and the final equilibrium state maintains the same value of $\langle P_x \rangle$, regardless of $F$ (modulo $O(L)$ corrections). In contrast, for initial states with merely extensive $\langle P_x \rangle_0 \sim L$, the tilt energy can potentially be dissipated by heating the system towards infinite temperature with respect to $H_0$ which would take the system towards a uniform density profile at late times (in a possibly subdiffusive manner [34]). For such states, we expect the behavior to be qualitatively different for large and small $F$, and the relaxation towards infinite temperature is only appropriate for small $F$.

In contrast, for large $F$, the tilt is the dominant term in $H$ and the spectrum splits into sectors labelled by dipole moment which now becomes a conserved quantity. Of course, this is only strictly true at infinite $F$ (or if $F \sim L$). However, even when $F$ is finite so that sectors with different values of $P_x$ overlap, it is possible to obtain long-lived approximate conservation of $P_x$ provided $F \gg J$. In this case, rearrangements of the system that change $P_x$ can only occur at a high order $\sim (F/J)$ and one can appeal to the theory of prethermalization [36–38] which predicts that $P_x$ will be conserved up to an exponentially long timescale $t_* \sim \exp(F/J)$.

On timescales short compared to $t_*$, the system is described by an ‘effective Hamiltonian’ that is constructed as a power-series in $1/F$, and which displays an emergent dipole conservation for an operator $\tilde{P}_x$ which agrees with $P_x$ to leading order. The terms in the effective Hamiltonian which are off diagonal in the local density basis must conserve center of mass. The lowest order off diagonal term is of the form $b_i^\dagger b_{i+1}b_{i+2}b_{i+3}^\dagger + h.c.$ (for an explicit derivation, see [39]). If we stop here then the effective Hamiltonian exhibits ‘strong fracture’ [23, 24] in that a finite size motif (e.g. a string of five consecutive occupied sites) can ‘cut’ the chain in two, with no charge transport being possible across this motif. In this case a typical state will exhibit localized charge dynamics, with charge being exponentially unlikely to wander far from its initial position. Moreover, a density wave that alternates between + and − with domain walls at least four sites apart will be an exact inert eigenstate of the dynamics. This is however an approximation to the dynamics obtained by truncating the expansion at the lowest non-trivial order in $1/F$.

At higher orders, the effective Hamiltonian acquires terms of all possible spatial ranges, with terms of spatial range $R$ (assumed large) being generated at order $aR$ in perturbation theory, where $a$ is an $O(1)$ number. Such terms will then have amplitude $(J/F)^aR$, and will become important on a timescale $t(R) \sim \exp(aR\log(F/J))$. Once the longer range terms are incorporated into the Hamiltonian, the fracture will be only ‘weak’ [23, 24], and so a typical state will only exhibit localized dynamics up to a timescale set by the least weak off diagonal term of range larger than some critical value $R_c \geq 4$. This timescale will be $t(R_c)$. However, an initial condition that alternates between $n_i = 1$ and $n_i = 0$ with domain walls at least $\ell$ sites apart will remain an exact eigenstate of the dynamics up to the timescale $t_c \sim \min(t(\ell)), t_*$, beyond which longer range terms or dipole non-conserving effects will become important. On timescales longer than $t_*$ we expect the system to thermalize to the energy density set by the initial condition. Finally, if $t(\ell) < t_*$ then on the intermediate timescales the thermalization will be to an energy shell and restricted to a certain dipole sector, and this may have interesting features that are beyond the scope of the present work.

To summarize, the sharpest signature of fracturing at
large tilts is a strong initial state dependence in the dynamics. States that would be strictly inert with exact dipole conservation (such as a CDW with \textit{maximal} amplitude for density fluctuations with wavelengths greater than $\ell$) will still look inert, albeit only up to a long timescale $t_c$. On the other hand, a state with small overlap on the inert states (say CDW states with small amplitudes for density fluctuations) will relax towards a uniform density profile, perhaps subdiffusively \cite{54}. We note that $t_c$ may look infinite in a finite-sized system for which a large enough tilt could lead to actual — rather than prethermal — conservation of $P_x$.

This initial state dependence also emphasizes that the origin of the observed Bloch/Stark MBL at large tilts is entirely distinct from the usual MBL phenomenology in disordered systems, which relies on the existence of exponentially many emergent integrals of motion and predicts localization for any typical initial state. Instead, the numerical observations of Stark MBL \cite{32,33} follow from Hilbert space shattering. To wit, the main diagnostics presented in \cite{32,33} were (i) a lack of level repulsion in the energy spectrum which is explained by the presence of exponentially many emergent dynamical sectors with $P_x$ conservation so that the eigenvalues in different sectors do not feel each other and (ii) persistence of local memory starting from certain staggered CDW initial states, which happen to be inert for the effective Hamiltonian with $P_x$ conservation to leading order. This analysis also predicts that the observed ‘transition’ must become a crossover at large sizes once dipole is not strictly conserved — although various ‘additional’ ingredients in the models in Refs. \cite{32,33} such as onsite disorder and non-linearities in the tilt might preclude the eventual thermalization. As an example, while the bare disorder strength in the model in \cite{32} looks weak compared to the bare hopping, it may be sizeable compared to the \textit{effective} dipole-conserving hopping and hence lead to MBL via more conventional routes.

Next, we turn to higher dimensions, and see how one can achieve conservation of all components of dipole moment. In two or higher dimensions, a tilted potential will result in (prethermal) conservation of only one component of dipole moment if the field/tilt is aligned with a lattice axis ($\hat{x}$). Meanwhile relaxation in directions orthogonal to $\hat{x}$, corresponding to motion along an equipotential surface, will not be inhibited by the applied field.

Instead, if the field is applied at an angle $\theta$ with respect to the $\hat{x}$ axis then it has projections along all the different lattice axes and could potentially engineer long-lived conservation of all components of dipole moment if $F \gg J$. Specializing to 2d, consider $V = F \sum_{\mathbf{x}} \cos(\theta) r_{x}\hat{\mathbf{x}} + \sin(\theta) r_{y} \hat{\mathbf{y}}$. Now, an important point is that if $F_y/F_x = \tan(\theta)$ is rational, then ‘flat’ equipotential lines orthogonal to the tilt direction pass directly through lattice sites and the system can once again relax along these directions. If $\tan(\theta + \pi/2) = p/q$ then sites along the equipotentials are connected to each other at $O(p+q)$ in the bare nearest-neighbor hopping, giving an effective hopping along the equipotential line $J_{\text{eff}} = (J/F)^{p+q}$, which sets the time-scale for relaxation (the factor of $F$ in the denominator comes from the component of the bare hopping that is against the strong field).

On the other hand, if $\tan(\theta)$ is irrational, there will still be ‘equipotential surfaces,’ although these will not contain more than one lattice site (Fig. 2), so that in this case one can get (prethermal) $P_x, P_y$ conservation along both lattice directions. If we pick a strip of some $O(1)$ width $\epsilon$ about the flat equipotential line, then lattice sites within this strip see a \textit{quasiperiodic} potential set by $F\delta x$, where $\delta x$ is the displacement of the target site from the true equipotential surface. If the system were non-interacting, the particles would be Aubry-Andre localized along the flat direction for strong enough $F$. In the interacting problem, two sites in this strip will be ‘resonant’ if the effective hopping/interaction matrix between them exceeds the potential energy difference. This sets a new relaxation timescale $t'_c$ for hopping between distinct lattice sites in an equipotential shell, beyond which it will become apparent that only one component of dipole moment is conserved. If this scale exceeds the prethermal scale $t_*$, then it is irrelevant for the dynamics in which case we expect to find exp(volume) product states that will be eigenstates of the dynamics, up to the prethermal timescale $t_*$. The incommensurate potential may also lead to quasiperiodic MBL \cite{30,31} along the flat direction for strong enough tilt, in which case only $t_*$ will be relevant and both $P_x, P_y$ look conserved for this time.

Finally, let us now discuss how one may generate conservation of quadrupole moment in $d = 2$. Quadrupole moment couples to the gradient of the electric field. Thus, the addition of a scalar potential of the form $V(x, y) = F(Ax^2 + By^2 + Cxy)$ will, for $O(1)$ coefficients $A, B, C$ and sufficiently large $F$, cause the spectrum to split into symmetry sectors labelled by quadrupole moment, again modulo the same considerations as before on prethermalization. However, a scalar potential of this form may be rewritten (at least for $AB > C^2$) simply as $\tilde{A}(x')^2 + \tilde{B}(y')^2$, where the $x'$ and $y'$ axes are rotated with respect to the lattice. This may simply be recognized as the potential for a \textit{harmonic trap}, with inequivalent trap frequencies along the $x'$ and $y'$ directions which is easily realized in experiments (Fig. 2). However, it is not presently clear how to establish conservation of both components of dipole moment \textit{and} quadrupole conservation, in an infinite system. Naively we would think to do this via the addition of a linear component to the potential at an irrational direction with respect to the lattice vectors, which simply shift the trap center along the irrational direction. However, a small region far from the trap center will locally only ‘see’ an approximately uniform potential tilt, which along certain ‘far field’ directions will be aligned with lattice axes, leading to conservation of only one component of dipole moment but not both. In a finite size experimental system it may be possible by judicious choice of parameters to evade this issue.
Discussion and conclusions: We have discussed how the phenomenon of localization from shattering introduced in \cite{23,24} may be extended to higher dimensions, and in particular how one may obtain an exactly localized subspace that is exponentially large in system volume when certain multipole moments of charge are conserved.

We have also explained how the requisite conservation laws may be naturally introduced in near term ultracold atom experiments. A key signature of the resulting physics lies in the exquisite sensitivity of the dynamics to the initial conditions. Initial conditions lying in the “localized” subspaces should be exact eigenstates of the dynamics, up to a prethermal timescale. Importantly, the prethermal timescale is always finite in the thermodynamic limit, so localization from shattering will manifest experimentally as a prethermal crossover rather than a true transition, although the two may be difficult to distinguish in finite size systems.

Our work explains the origin of the recent numerical observations of Stark/Bloch MBL \cite{32,33} in tilted finite-size systems, sharpening how the observed non-ergodicity follows from Hilbert space shattering in a large tilt.

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[1] P. W. Anderson, Phys. Rev. 109, 1492 (1958), URL http://link.aps.org/doi/10.1103/PhysRev.109.1492.
[2] D. Basko, I. Aleiner, and B. Altshuler, Annals of Physics 321, 1126 (2006), ISSN 0003-4916, URL http://www.sciencedirect.com/science/article/pii/S0003491605002830.
[3] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. Lett. 95, 206603 (2005), URL https://link.aps.org/doi/10.1103/PhysRevLett.95.206603.
[4] A. Pal and D. A. Huse, Phys. Rev. B 82, 174411 (2010), URL http://link.aps.org/doi/10.1103/PhysRevB.82.174411.
[5] V. Oganesyan and D. A. Huse, Phys. Rev. B 75, 155111 (2007), URL http://link.aps.org/doi/10.1103/PhysRevB.75.155111.
[6] M. Znidarič, T. c. v. Prosen, and P. Prelovšek, Phys. Rev. B 77, 064426 (2008), URL http://link.aps.org/doi/10.1103/PhysRevB.77.064426.
[7] J. Z. Imbrie, Journal of Statistical Physics 163, 998 (2016), ISSN 1572-9613, URL http://dx.doi.org/10.1007/s10955-016-1508-x.
[8] R. Nandkishore and D. A. Huse, Annual Review of Condensed Matter Physics 6, 15 (2015), https://doi.org/10.1146/annurev-conmatphys-031214-014726, URL https://doi.org/10.1146/annurev-conmatphys-031214-014726.
[9] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Rev. Mod. Phys. 91, 021001 (2019), URL https://link.aps.org/doi/10.1103/RevModPhys.91.021001.
[10] N. Shiraishi and T. Mori, Phys. Rev. Lett. 119, 030601 (2017), URL https://link.aps.org/doi/10.1103/PhysRevLett.119.030601.
[11] S. Mondal, S. Rachel, B. A. Bernevig, and N. Regnault, Phys. Rev. B 98, 235155 (2018), URL https://link.aps.org/doi/10.1103/PhysRevB.98.235155.
[12] C. Turner, A. Michailidis, D. Abanin, M. Serbyn, and Z. Papić, Nature Physics (2018).
[13] V. Khemani, C. R. Laumann, and A. Chandran, Phys. Rev. B 99, 161101 (2019), URL https://link.aps.org/doi/10.1103/PhysRevB.99.161101.
[14] S. Choi, C. J. Turner, H. Pichler, W. W. Ho, A. A. Michailidis, Z. Papić, M. Serbyn, M. D. Lukin, and D. A. Abanin, Phys. Rev. Lett. 122, 220603 (2019), URL https://link.aps.org/doi/10.1103/PhysRevLett.122.220603.
[15] W. W. Ho, S. Choi, H. Pichler, and M. D. Lukin, Phys. Rev. Lett. 122, 040603 (2019), URL https://link.aps.org/doi/10.1103/PhysRevLett.122.040603.
[16] F. M. Surace, P. P. Mazza, G. Giudici, A. Lerose, A. Gambassi, and M. Dalmonte, arXiv preprint arXiv:1902.09551 (2019).
[17] J. Park, Y. Kuno, and I. Ichinose, Physical Review A 100 (2019), ISSN 2469-9934, URL http://dx.doi.org/10.1103/PhysRevA.100.013629.
[18] A. J. James, R. M. Konik, and N. J. Robinson, Physical Review Letters 122 (2019), ISSN 1079-7114, URL http://dx.doi.org/10.1103/PhysRevLett.122.130603.
[19] T. Iadecola, M. Schecter, and S. Xu, arXiv e-prints arXiv:1903.10517 (2019), 1903.10517.
[20] T. Iadecola and M. Znidaric, arXiv e-prints arXiv:1811.07903 (2018), 1811.07903.
[21] S. Pai and M. Pretko, Fractons from confinement in one dimension (2019), 1909.12306.
[22] S. Pai and M. Pretko, and R. M. Nandkishore, Physical Review X 9 (2019), ISSN 2160-3308, URL http://dx.doi.org/10.1103/PhysRevX.9.021003.
[23] V. Khemani and R. Nandkishore, Local constraints can globally shutter hilbert space: a new route to quantum information protection (2019), 1904.04815.
[24] P. Sala, T. Rakovszky, R. Verresen, M. Knap, and F. Pollmann, arXiv e-prints arXiv:1904.04266 (2019),
[25] M. Pretko, Phys. Rev. B 95, 115139 (2017), URL https://link.aps.org/doi/10.1103/PhysRevB.95.115139.
[26] G. H. Wannier, Phys. Rev. 117, 432 (1960), URL https://link.aps.org/doi/10.1103/PhysRev.117.432.
[27] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, nature 415, 39 (2002).
[28] S. Sachdev, K. Sengupta, and S. M. Girvin, Phys. Rev. B 66, 075128 (2002), URL https://link.aps.org/doi/10.1103/PhysRevB.66.075128.
[29] D. O. Krimer, R. Khomeriki, and S. Flach, Phys. Rev. E 80, 036201 (2009), URL https://link.aps.org/doi/10.1103/PhysRevE.80.036201.
[30] S. Mandt, A. Rapp, and A. Rosch, Phys. Rev. Lett. 106, 250602 (2011), URL https://link.aps.org/doi/10.1103/PhysRevLett.106.250602.
[31] M. Eckstein and P. Werner, Phys. Rev. Lett. 107, 186406 (2011), URL https://link.aps.org/doi/10.1103/PhysRevLett.107.186406.
[32] E. van Nieuwenburg, Y. Baum, and G. Refael, Proceedings of the National Academy of Sciences 116, 9269 (2019).
[33] M. Schulz, C. Hooley, R. Moessner, and F. Pollmann, Physical review letters 122, 040606 (2019).
[34] E. Guardado-Sanchez, A. Morningstar, B. M. Spar, P. T. Brown, D. A. Huse, and W. S. Bakr, arXiv e-prints arXiv:1909.05848 (2019), 1909.05848.
[35] W. De Roeck and F. m. c. Huveneers, Phys. Rev. B 95, 155129 (2017), URL https://link.aps.org/doi/10.1103/PhysRevB.95.155129.
[36] A. Rosch, D. Rasch, B. Binz, and M. Vojta, Phys. Rev. Lett. 101, 265301 (2008), URL https://link.aps.org/doi/10.1103/PhysRevLett.101.265301.
[37] D. A. Abanin, W. De Roeck, and F. Huveneers, Physical review letters 115, 256803 (2015).
[38] T. Mori, T. Kuwahara, and K. Saito, Phys. Rev. Lett. 116, 120401 (2016), URL https://link.aps.org/doi/10.1103/PhysRevLett.116.120401.
[39] S. Moudgalya, A. Prem, R. Nandkishore, N. Regnault, and B. Bernevig, ArXiv e-prints (2019).
[40] S. Iyer, V. Oganesyan, G. Refael, and D. A. Huse, Phys. Rev. B 87, 134202 (2013), URL https://link.aps.org/doi/10.1103/PhysRevB.87.134202.
[41] V. Khemani, D. N. Sheng, and D. A. Huse, Phys. Rev. Lett. 119, 075702 (2017), URL https://link.aps.org/doi/10.1103/PhysRevLett.119.075702.