Geophysical Modelling Techniques and their Benefit to Ultrasonic Measurement Tools

by

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Abstract

Modelling of medical ultrasonic fields requires prediction of acoustic waves as they propagate through fluid, elastic and visco-elastic media. These are characteristics that are shared with the modelling of seismic waves in geophysics (and particularly geophysical prospecting). This paper introduces a Finite Difference (FD) model based upon formulations developed to study seismic propagation, and describes it relevance to the ultrasonic community. Examples are provided that validate the models ability to predict secondary wave effects such as diffraction, mode conversion and Lamb wave generation.

A small case study is then presented. Within this study the propagation of ultrasonic waves in the immediate vicinity of the tip of a needle hydrophone are considered. The numerical simulations confirm the suggestion of other authors that the low frequency response of needle hydrophones is governed by diffraction phenomena. The propagation of waves around a revised needle tip geometry is then modelled and improvements in frequency response are noted. In conjunction with the numerical simulations, experimental investigation of the frequency response of some needle hydrophones is also conducted. The design changes suggested by the modelling process are implemented on these devices, and the performance enhancements seen experimentally are found to be entirely consistent with the predicted data.

Introduction

The rapid increase in speed and storage modern desktop computers years has facilitated the widespread use numerical modelling techniques without having to resort to the use of supercomputers. The simulation of acoustic fields is one such area that has seen significant activity. However there are certain basic requirements and thus any model of acoustic propagation should:

- Predict the propagation within, and across the interface between, media with significantly different material parameters (e.g. fluid-solid boundaries)
- Cater for a wide range of problem geometries
- Be capable of simulating transient and continuous source functions
- Not introduce numerical errors and artefacts (such as numerical dispersion/instability/anisotropy)

This paper will start by discussing a few modelling options and where limitations of these techniques occur. A modelling method capable of meeting all of the requirements listed above will then be presented and examples of its use will also be discussed.
Overview of Possible Numerical Methods

There exists a wide range of numerical methods that have been used to examine the propagation of ultrasonic and acoustic field including: Finite and Boundary Element methods, Finite Integration techniques, Integral Transform techniques and Pseudo-Spectral methods. This paper will concentrate only on another popular choice – Finite Difference (FD) methods, although the relative merits of the many different options is discussed within Hurrell (2002).

Although this paper concentrates only on FD methods it is important to note that there are number of different formulations. One approach that has proved useful in the simulation of oceanographic research is the parabolic approximation of the acoustic wave equations – the so-called “One Way Wave Equation”. Several examples of this method can be found in Jensen et al (1994). This method has relatively low computational requirements but tends to yield results that are of lower accuracy (particularly at large angles of propagation relative to the computational grid). Solutions of this nature are often a mono-frequency and oceanic simulations tend to involve fluid media only.

An alternative technique that is popular in the NDE/NDT communities was proposed by Bond (1979) and Harker (1984). This method is expressed in terms of the horizontal and vertical components of particle displacement and is based upon the elasto-dynamic equations of motion rather than the acoustic wave equation. However the NDE community is primarily concerned with propagation within elastic, solid media that have quasi-free boundary conditions (metal samples bounded by air), thus there is little need for solutions that easily accommodate fluid-solid boundaries.

Both of the approaches discussed thus far overlay a single grid onto the computational domain to obtain the discrete nodes at which the values of the field variables are calculated. However, single grid models suffer from a noticeable limitation – they tend to exhibit significant numerical dispersion as well as anisotropic propagation speed with faster propagation in directions aligned with the grid. It should also be observed that formulations based on either the acoustic or elasto-dynamic wave equations leads to solution that are inherently homogeneous. Consequently boundary conditions (e.g. continuity of the normal component of velocity) need to be explicitly included if these techniques are used to model geometries involving more than one medium (heterogeneous problems)

In contrast to oceanic and NDT environments, the geophysics community often has to with fluid-solid boundaries. Consider for example oil exploration where a model would need to cater for a large fluid (oil or natural gas) regions contained with elastic or visco-elastic media (rock). Such models much also be able to cater with transient sources since these are encounter either with seismic events or when geophysical prospecting with explosive charges. When the highly variable nature of geological structures is also taken into account it is reasonable to conclude that models developed for the geophysics community also meet all of the requirements for ultrasonic modelling that are listed in the introduction. In fact the only difference between the requirements of the two communities are related to wavelength, with ultrasonic modelling conducted on a much smaller length scale.

The Staggered Grid Model

The formulation that is discussed here is the “staggered grid” formulation originally proposed by Madariaga (1976) and then enhanced by Virieux (1986) and Levander (1988). A brief description of the scheme follows, but the reader is encouraged to consult the latter two of these references for a more rigorous discussion. The staggered grid model consists of four grids upon which 5 field variables (3 stresses and 2 particle velocities) are calculated. Each of these grid is spatially offset from one another by half a grid spacing such that all grids are interleaved as shown in Figure 1.

The staggered grid model makes use of an equation set based upon Newton's 2nd Law and an the constitutive equations for an elastic material. Virieux (1986) manipulated these equations into a form where the calculation of stress at any given timestep relies only upon values of the velocities at the same timestep, and vice versa for the calculation of velocity. This can be seen in Figure 1 by considering a velocity node in the middle of the grid. The only nodes connecting to it (either vertically or horizontally) are stress nodes. Similarly the nearest horizontal and vertical neighbours of any stress node are velocities. Since stresses and velocities are now completely decoupled they can be evaluated at consecutive timesteps – this can be seen in Figure 2.
As discussed by Levander (1988), the staggering of grids both spatially and temporally produces a model that is much less prone to numerical dispersion and anisotropy and numerically stable for a wide range of different media. Additionally the 5 variable formulation is now a heterogeneous solution with the same equations applying everywhere within the computational domain. Boundaries between one media and another are simply catered for by changing the value of the material parameters at nodes on either side of the boundary.

Modelling a Circular Plane Piston Source

Although the staggered grid model has thus far only been discussed in terms of a two dimensional Cartesian co-ordinate system, the addition of an extra variable allow rapid extension to an axi-symmetric cylindrical co-ordinate system. Although the staggered grid model has been used extensively to simulate ultrasonic propagation 2D Cartesian co-ordinates (Hurrell (2002)), this paper will only discuss results obtained with 3D axi-symmetric cylindrical co-ordinates.

The circular plane piston source mounted in a rigid baffle has been extensively studied. For the simple case of constant frequency motion radiating into a fluid medium Archer-Hall and Gee (1980) derived a compact and computationally simple expression for the pressure at any point in the fluid

$$ p(x, z) \propto \left\{ \begin{array}{ll} 1 & : x < a \\ \frac{1}{2} & : x = a \\ 0 & : x > a \end{array} \right\} e^{-jkz} + \frac{1}{\pi} \int_0^\infty e^{-jk\psi} \frac{ax \cos \psi - a^2}{a^2 + x^2 - 2ax \cos \psi} d\psi. \quad \text{Equation 1} $$

where $k$ is the wave number, $a$ is the radius of the circular piston, $\psi$ is the variable of integration and $s$ is given by $s^2 = a^2 + x^2 - 2ax \cos \psi$. Furthermore the pressure along the acoustic axis of the transducer has no angular and radial dependences and Equation 1 can be greatly simplified to

$$ p(z,t) \propto \sin \left( \frac{kz}{2} \left[ \sqrt{1 + \left( \frac{a}{z} \right)^2} - 1 \right] \right). \quad \text{Equation 2} $$

A comparison study between the Finite Element codes running at the National Physical Laboratory (London) and the staggered grid model presented here has been conducted and the full results of this are due
to be published soon. However, a subset of the data will now be presented to demonstrate the effectiveness of the staggered grid model in its ability to simulate a circular plane piston source. The radius of the plane piston was 5mm, and it was driven by a continuous 500 kHz sine wave, radiating into water. The staggered grid FD simulation used 60,000 nodes and was able to compute 1000 timesteps in a 98 seconds on a 1533 MHz Athlon PC, whilst the FE simulation used 7124 nodes and ran in 120 seconds on a Sun Ultra 2. Figure 3 shows the axial time averaged acoustic pressure from FE and FD models compared against the analytical solution provided by Equation 2. Similarly Figure 3 compares the transverse acoustic pressure profile measured parallel to the face of the piston at a separation of 4.38 mm from the two numerical models, with that obtained from Equation 1.

![Figure 3: Axial Profile of Circular Plane Piston](image1)

![Figure 4: Transverse Profile of Circular Plane Piston at 4.38 mm from Surface](image2)

The two figures show excellent agreement between the results of the two numerical models and the analytical solution. The 500 kHz circular piston used thus far has a small $ka$ (10.6), and thus has a relatively simple near field as well as a last axial maximum that is close to the transducer face. In demonstrate the models capability to simulate a more complex field another simulation based upon a piston with $ka$ of 25 was produced, a snapshot of which can be seen below. The axial and transverse profiles of this field show the same quantitative accuracy as the initial case. Also snapshots of the acoustic pressure field at various times (an example shown in Figure 5) provides additional qualitative information. For convenience some of the familiar features are marked.

![Figure 5: Pressure Field generated by a circular plane piston, $ka = 25$](image3)

**Modelling the Frequency Response of a Needle Hydrophone**

Having established the models ability to model relatively simple acoustic fields, a more demanding situation is now examined. Needle hydrophones often exhibit non-flat frequency response, particularly at
lower frequencies. Fay et al (1994) have attributed this response to the interference of the edge waves and head waves caused as an incident wave interacts with the tip of the hydrophone. The first task was to see whether modelled results supported this hypothesis, with a secondary goal of determining whether the frequency response could be improved with the assistance of the staggered grid model. A needle hydrophones with 0.2mm diameter active element was taken and calibrated over the frequency range 1-20 MHz using a multiple frequency technique as described by Smith and Bacon (1990). The exact geometry of the needle hydrophone was then replicated within the staggered grid model and the interaction of the tip of the needle hydrophone with an incident pulsed plane wave was then simulated. The spatial average of acoustic pressure over the area of the active element of the hydrophone was calculated and this can be found in Figure 6, along with an experimentally determined frequency response for the same device. Figure 7 also has a snapshot of the acoustic pressure field from the same simulation.

Figure 6: Modelled and Experimental Frequency Response of an unmodified Needle Hydrophone

Figure 7: Modelled Interaction of a Plane Wave with a Needle Hydrophone Tip

Figure 7 clearly shows circularly spreading waves originating from two diffraction sites (marked). This clearly supports the hypothesis of Fay et al (1994). Given that diffraction from the needle tip is present, and is likely to be the cause of the non-flat frequency response, the model was then used to investigate whether alterations to the geometry of the needle hydrophone tip could achieve a flatter frequency response. Various configurations where examined and a simple taper applied to the tip of the needle hydrophone was found to yield significant improvements. Figure 8 shows the performance of the same hydrophone (Device A) as used in Figure 6 before and after the application of a simple taper. Figure 9 is an example of the modelled pressure field from this simulation showing a much reduced amplitude diffracted component particularly immediately in front of the hydrophones active element. Further modelling has yielded further flattening of needle hydrophone frequency response, and the experimental implementation of these proprietary techniques can be seen in the response of Device B in Figure 8.

Also visible in both Figures 7 and 9 is a diagonally propagating wave to the right of the needle hydrophone tip, but in front of the main wave pulse. The seemingly non-causal effect can be easily explained when Lamb waves are accounted for. In fact this additional pulse arises from a Lamb wave that is travelling faster in the metallic outer case of the hydrophone, and then leaking energy back into the fluid surrounding the hydrophone. Animated sequences of images produced by the model show this effect more clearly than static images contained within this paper, and with longer propagation times both symmetric and anti-symmetric Lamb modes are visible.
Conclusions

A numerical model based on the staggered grid FD formulation and originating in geophysical research community has been successfully applied at ultrasonic frequencies. Its accuracy, both quantitative and qualitative, has been demonstrated both for a well-studied circular plane piston source and for a needle hydrophone. Quantitative results derived from the model support previous authors' assessment of the phenomena affecting the frequency response of needle hydrophones. Visualisation of the wave field, although qualitative, permits better understanding of the physics of the problem that in turn suggests possible remedial action. The model was then also used to predict improved device performance. When implemented, the revised geometry suggested by the modelling process was found to provide much enhanced experimental performance.

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