Probing anharmonic properties of nuclear surface vibration by heavy-ion fusion reactions

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Abstract

Describing fusion reactions between $^{16}$O and $^{154}$Dy and, between $^{16}$O and $^{144}$Sm by the $sd$− and $sdf$− interacting boson model, we show that heavy-ion fusion reactions are strongly affected by anharmonic properties of nuclear surface vibrations and nuclear shape, and thus provide a powerful method to study details of nuclear structure and dynamics.

1. Introduction

Extensive experimental as well as theoretical studies of heavy-ion fusion reactions at energies near and below the Coulomb barrier in the past decade have offered many basic ideas on the effects of nuclear collective excitations such as surface vibrations and rotation, or particle transfer on the fusion cross section\cite{1}. Recent very accurate data at ANU and at Legnaro are providing us with the possibility to step up to a more advanced stage of the research. For example, one can extract detailed informations on nuclear structure and dynamics from the data of heavy-ion fusion reactions. For instance, the sign of the hexadecupole deformation can be determined by studying the fusion barrier distribution, as was exemplified for the $^{16}$O + $^{186}$W, $^{154}$Sm fusion reactions \cite{2, 3}. Also, Stefanini et al. \cite{4} demonstrated the important role played by the double phonon excitations in the $^{58}$Ni + $^{60}$Ni fusion reactions. In this contribution we address to the effects of anharmonic properties of nuclear surface vibrations. Our study was motivated by the discussions with Jack, Nand and David. When we visited Canberra last spring, they introduced us their puzzle about the role of octupole vibrations in the fusion reactions between $^{16}$O and $^{208}$Pb and between $^{16}$O and $^{144}$Sm. The problem is the following: in the former, a better agreement between data and theory concerning the fusion barrier distribution is obtained if one includes the double phonon coupling\cite{5}. On the other hand, in the latter, a good agreement obtained by a single phonon coupling is destroyed if one adds the double
phonon coupling. $^{144}$Sm has low lying quartet states which seem to correspond to the double octupole phonon excitations. Therefore, it is quite puzzling that the inclusion of the double phonon coupling spoils the agreement between the data and theory. However, there exists a noticeable anharmonicity in the energy spectrum. We therefore expected that the puzzle raised by Jack et al. can be attributed to the anharmonic properties of nuclear surface vibrations. This is what we are going to report in this contribution. Actually important anharmonic properties are concerned not with the energy spectrum, but rather with the transition matrices, especially reorientation effects.

2. Coupled-channels formalism in the IBM – a case study for $^{16}$O + $^{154}$Dy fusion reactions

In this section we make a case study for the $^{16}$O + $^{154}$Dy fusion reactions by using the U(5) limit of the sd–IBM. It is known that the U(5) limit of the IBM corresponds to the anharmonic vibrator in the geometrical model of Bohr and Mottelson. We choose the IBM to reduce the number of free parameters.

We assume the following Hamiltonian for the total system

$$H = -\frac{\hbar^2}{2\mu} \nabla^2 + H_{IBM} + V_{coup}(r, \xi),$$

where $r$ is the coordinate of the relative motion between the projectile and target, $\mu$ is the reduced mass, and $\xi$ represents the internal degrees of freedom of the target nucleus. $H_{IBM}$ is the IBM Hamiltonian for the quadrupole vibration in the target nucleus, for which we assume the harmonic limit

$$H_{IBM} = \epsilon_d \hat{n}_d$$

Here $\hat{n}_d$ is the number operator of the $d$ bosons and $\epsilon_d$ is the excitation energy of the quadrupole vibration. In the numerical analyses we also introduce a deviation from the harmonic spectrum and study its effects.

The coupling between the relative motion and the intrinsic motion of the target nucleus is described by $V_{coup}$ in Eq. (1). We use the linear coupling model and the no-Coriolis approximation. We then have

$$V_{coup}(r, \xi) = U_N(r) + \frac{Z_P Z_T e^2}{r} + \frac{\beta}{\sqrt{4\pi N}} f(r) \hat{Q}_{20}$$

where the coupling form factor $f(r)$ consists of the nuclear and Coulomb parts and read

$$f(r) = -R_T \frac{dU_N}{dr} + \frac{3}{5} Z_P Z_T e^2 R_T^2 \frac{1}{r^3}$$

$N$ is the boson number, the subscripts $P$ and $T$ refer to the projectile and target nuclei, respectively. The scaling of the coupling strength with $\sqrt{N}$ is introduced to ensure the equivalence of the IBM and the geometric model results in the large $N$ limit. $\beta$ is the
quadropole deformation parameter. We represent the quadropole moment operator \( \hat{Q}_{2\mu} \) by

\[
\hat{Q}_{2\mu} = s^\dagger d_\mu + s d_\mu^\dagger + \chi (d^\dagger d)^{(2\mu)}
\]  

where tilde is defined as \( \tilde{b}_\mu = (-)^{l+\mu} b_{l-\mu} \). When the \( \chi \) parameter is zero, the quadropole moments of all states vanish, and one obtains the harmonic limit in the large \( N \) limit. Non-zero values of the \( \chi \) generate quadropole moments and are responsible for the anharmonicities. This can be clearly seen from the structure of the matrix elements appearing in the coupled-channels calculations, which read, for example,

\[
H_{IBM} + V_{coup} = U_N(r) + \frac{Z_P Z_T \epsilon^2}{r} + \begin{pmatrix}
F(r) & 0 & 2\epsilon_d + \delta - \frac{4}{\sqrt{14N}} \chi F(r) \\
\epsilon_d - \frac{2}{\sqrt{14N}} \chi F(r) & 0 & \sqrt{2(1 - 1/N)} F(r) \\
0 & \sqrt{2(1 - 1/N)} F(r) & 2\epsilon_d + \delta - \frac{4}{\sqrt{14N}} \chi F(r)
\end{pmatrix}
\]  

if we truncate by the two phonon states. \( F(r) \) is \( \frac{\beta}{4\pi} f(r) \). We introduced a parameter \( \delta \) in order to represent the deviation of the energy spectrum from the harmonic limit.

We now apply this formalism to the fusion reactions between \(^{16}\text{O}\) and \(^{154}\text{Dy}\) treating \(^{16}\text{O}\) as inert[11]. The boson number for \(^{154}\text{Dy}\) is estimated to be 4 by considering 64 and 82 as the proton and the neutron magic numbers, respectively. The data of the E2 transition and the intrinsic quadropole moment lead to \( \beta = 0.24 \) and \( \chi = -2.36 \). The excitation energy of the \( d \) boson, \( \epsilon_d \), has been identified with the excitation energy of the first excited state of \(^{154}\text{Dy}\), 0.335 MeV.

Fig.1 shows the role of anharmonicity in this reaction by comparing the fusion excitation function and the fusion barrier distribution calculated in four different ways. The dotted line is the result of the one phonon coupling in the harmonic oscillator limit. The dott-dashed line is the result of the one phonon coupling where the reorientation term is included. The dashed line is the result when one takes double phonon states into account in the harmonic oscillator limit. The solid line is the corresponding calculations where the anharmonicities are taken into account. The effects of anharmonicity can be hardly seen in the fusion excitation function. However, the fusion barrier distribution clearly shows a significant change from the harmonic oscillator limit by the anharmonicity.

Fig.2 shows the dependence of the fusion barrier distribution on the deviation of the energy spectrum from the harmonic oscillator limit (the upper panel) and on the boson number (the lower panel). The upper panel compares the fusion barrier distribution calculated for four different choices of the parameter \( \delta \) in Eq. (6). It shows that the fusion barrier distribution is insensitive to the anharmonicity concerning the energy spectrum. The lower panel compares the fusion barrier distribution calculated for various boson numbers \( N \). For comparison, it also contains the fusion barrier distribution calculated in the harmonic oscillator limit in the case, where there exist one and two phonon states. One clearly observes a strong dependence of the fusion barrier distribution on the boson number, and also the effects of anharmonicity.

Fig.3 shows how the sign of nuclear deformation, i.e. the sign of the parameter \( \chi \) in the quadropole moment (see Eq. (5)), affects the excitation function of the fusion cross section and the fusion barrier distribution. Though the former is almost insensitive to the
sign of the deformation, the latter drastically changes with the sign of the deformation. This enables us to determine the shape of nuclei with anharmonic vibrational modes of excitation by analysing the fusion barrier distribution.

3. Anharmonic effects of $^{144}$Sm on the $^{16}$O + $^{144}$Sm fusion reactions

We now come to the question raised by Jack et al. (see Ref. [12] for details). Our main concern is the anharmonic effects of the octupole excitation of $^{144}$Sm. We therefore extend the model in the previous section by including the $f$ bosons, and assume that the IBM Hamiltonian is given by

$$H_{IBM} = \epsilon_d \hat{n}_d + \epsilon_f \hat{n}_f. \quad (7)$$

where $\hat{n}_f$ is the number operator for $f$ bosons and $\epsilon_f$ is the excitation energy of the octupole vibration. Since the study in the previous section has shown that the anharmonicity concerning the energy spectrum does not so much influence both the excitation function of the fusion cross section and the fusion barrier distribution, we assume here a harmonic spectrum. The coupling Hamiltonian now includes also the coupling to the octupole vibration. Further, we modify the linear coupling model in the previous section by treating the nuclear coupling to full order, while we keep the Coulomb coupling in the linear coupling approximation [10, 13]. The Coulomb and the nuclear parts of the coupling Hamiltonian are now given by

$$V_C(r, \xi) = \frac{Z_P Z_T e^2}{r} \left( 1 + \frac{3 R_T^2}{5} \beta_2 \hat{Q}_{20} + \frac{3 R_T^2}{7} \beta_3 \hat{Q}_{30} \right),$$

$$V_N(r, \xi) = -V_0 \left[ 1 + \exp \left( \frac{1}{a} \left( r - R_0 - R_T (\beta_2 \hat{Q}_{20} + \beta_3 \hat{Q}_{30})/\sqrt{4\pi N} \right) \right) \right]^{-1}. \quad (8)$$

$\beta_2$ and $\beta_3$ in Eq. (8) are the quadrupole and octupole deformation parameters, which are usually estimated from the electric transition probabilities using the expression $\beta_\lambda = 4\pi (B(E\lambda) \uparrow)^{1/2} / Z_T e R_T^2$. However, this formula does not hold for anharmonic vibrators. Therefore, we treat $\beta_2$ and $\beta_3$ as free parameters and look for their optimal values to reproduce the experimental data. $\hat{Q}_2$ and $\hat{Q}_3$ in Eq. (8) are the quadrupole and the octupole operators in the IBM, which we take as

$$\hat{Q}_2 = s^\dagger d + s d^\dagger + \chi_2 (d^\dagger d)^{(2)} + \chi_2 (f^\dagger f)^{(2)},$$

$$\hat{Q}_3 = sf^\dagger + \chi_3 (d f^\dagger)^{(3)} + h.c., \quad (9)$$

The model parameters are determined as follows. The standard prescription for boson number (i.e. counting pairs of nucleons above or below the nearest shell closure) would give $N = 6$. However, it is well known that the effective boson numbers are much smaller due to the $Z = 64$ subshell closure [8]. The suggested effective numbers in the literature vary between $N = 1$ and 3. We adopted $N = 2$ in our calculations, since there are experimental signatures for the two-phonon states, but no evidence for three-phonon states in $^{144}$Sm. The parameters of the IBM Hamiltonian Eq. (7) are simply determined from the excitation energies of the first $2^+$ and $3^-$ states in $^{144}$Sm as $\epsilon_d = 1.66$ MeV and
\( \epsilon_f = 1.81 \text{ MeV} \). The nuclear potential parameters are taken from the exhaustive study of this reaction in Ref. [6] as \( V_0 = 105.1 \text{ MeV} \), \( R_0 = 8.54 \text{ fm} \) and \( a = 0.75 \text{ fm} \). Finally, the target radius is taken to be \( R_T = 5.56 \text{ fm} \).

The results of the coupled-channels calculations are compared with the experimental data in Fig. 4. The upper and the lower panels in Fig. 4 show the excitation function of the fusion cross section and the fusion barrier distributions, respectively. The experimental data are taken from Ref. [3]. The dotted line is the result in the harmonic limit, where couplings to the quadrupole and octupole vibrations in \(^{144}\text{Sm}\) are truncated at the single-phonon levels. The deformation parameters are estimated to be \( \beta_2 = 0.11 \) and \( \beta_3 = 0.21 \) from the electric transition probabilities. The dotted line reproduces the experimental data of both the fusion cross section and the fusion barrier distribution reasonably well, though the peak position of the fusion barrier distribution around \( E_{cm} = 65 \text{ MeV} \) is slightly shifted. As was shown in Ref. [6], the shape of the fusion barrier distribution becomes inconsistent with the experimental data when the double-phonon channels are included in the harmonic limit (the dashed line). The good agreement is recovered when one takes the effects of anharmonicity of the vibrational motion into account. These results are shown in Fig. 4 by the solid line. This calculation has been performed using the parameters, \( \beta_2 = 0.13 \), \( \beta_3 = 0.23 \), \( \chi_2 = -3.30 \), \( \chi_{2f} = -2.48 \), and \( \chi_3 = 2.87 \), which are obtained from a \( \chi^2 \) fit to the fusion cross sections. The \( \chi^2 \) fit gave a unique result, regardless of the starting values. The non-zero \( \chi \) values indicate the anharmonic effects in the transition operators. The slight change in the values of the deformation parameters from those in the harmonic limit results from the renormalization effects due to the extra terms in the operators given in Eq. (9). Note that the solid line agrees with the experimental data much better than the dotted line.

One of the pronounced features of an anharmonic vibrator is that the excited states have non-zero quadrupole moments. Using the \( \chi \) parameters in the E2 operator, \( T(E2) = e_B \hat{Q}_2 \), extracted from the analysis of fusion data, we can estimate the static quadrupole moments of various states in \(^{144}\text{Sm}\). Here, \( e_B \) is the effective charge, which is determined from the experimental \( B(E2; 0 \rightarrow 2^+) \) value as \( e_B = 0.16 \text{ eb} \). For the quadrupole moment of the first \( 2^+ \) and \( 3^- \) states, we obtain \(-0.28 \text{ b} \) and \(-0.70 \text{ b} \), respectively. The negative sign of the quadrupole moment of the octupole-phonon state is consistent with that suggested from the neutron pick-up reactions on \(^{145}\text{Sm} \) [14].

In the case of rotational coupling, fusion barrier distributions strongly depend on the sign of the quadrupole deformation parameter through the reorientation term. Also, as mentioned in the introduction it has been reported that fusion barrier distributions are very sensitive to the sign of the hexadecapole deformation parameter [3]. Similarly, it is likely that the shape of fusion barrier distributions changes significantly when one inverts the sign of the quadrupole moment in a spherical target if there exists a strong anharmonicity. Fig. 5 shows the influence of the sign of the quadrupole moment of the excited states on the fusion cross section and the fusion barrier distribution. The solid line is the same as in Fig. 4 and corresponds to the optimal choice for the signs of the quadrupole moments of the first \( 2^+ \) and \( 3^- \) states. The dotted and dashed lines are obtained by changing the sign of the \( \chi_2 \) and \( \chi_{2f} \) parameters in Eq. (9), respectively, while the dot-dashed line is the result where the sign of both \( \chi_2 \) and \( \chi_{2f} \) parameters are
inverted. The change of sign of $\chi_2$ and $\chi_{2f}$ is equivalent to taking the opposite sign for the quadrupole moment of the excited states. Fig. 5 demonstrates that subbarrier fusion reactions are indeed sensitive to the sign of the quadrupole moment of excited states. The experimental data are reproduced only when the correct sign of the quadrupole moment are used in the coupled-channels calculations. Notice that the fusion excitation function is completely insensitive to the sign of the quadrupole moment of the first $2^+$ state, but strongly depends on that of the first $3^-$ state. In contrast, the fusion barrier distribution can probe the signs of the quadrupole moments of both the first $2^+$ and $3^-$ states. This study shows that the sign of quadrupole moments in spherical nuclei can be determined from subbarrier fusion reactions, especially through the barrier distribution.

4. Summary

We discussed the effects of anharmonicity in nuclear surface vibrations on heavy-ion fusion reactions by using the interacting boson model. Our analyses clearly show that the fusion barrier distribution is very sensitive to the anharmonicity of the nuclear surface vibrations, and suggest that the puzzle raised by Jack et al. concerning the $^{16}\text{O} + ^{144}\text{Sm}$ fusion reactions can be solved by considering the anharmonicities of the octupole surface vibrations of $^{144}\text{Sm}$. We have also shown that one can determine nuclear shape by using this high sensitivity of the fusion barrier distribution to the anharmonicity through reorientation effects.

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**Figure Captions**

**Fig. 1:** Effects of anharmonicity in the $^{16}\text{O} + ^{154}\text{Dy}$ fusion reactions.

**Fig. 2:** Dependence of the fusion barrier distribution on the energy spectrum and the boson number in the $^{16}\text{O} + ^{154}\text{Dy}$ fusion reactions.

**Fig. 3:** Dependence of the excitation function of the fusion cross section for the $^{16}\text{O} + ^{154}\text{Dy}$ reactions and of the fusion barrier distribution on the boson number.

**Fig. 4:** Comparison of the experimental fusion cross section (the upper panel) and fusion barrier distribution (the lower panel) with the coupled-channels calculations for $^{16}\text{O} + ^{144}\text{Sm}$ reaction. The experimental data are taken from Ref. [3]. The solid line shows the results of the present IBM model including the double-phonon states and anharmonic effects. The dotted and the dashed lines are the results of the single- and the double-phonon couplings in the harmonic limit, respectively.

**Fig. 5:** Dependence of the fusion cross section and barrier distribution on the sign of the quadrupole moment of the excited states in $^{144}\text{Sm}$. The meaning of each line is indicated in the inset.
\[ \sigma_{\text{fus}} \] (mb)

\[ \frac{d^2(E\sigma)}{dE^2} \] (mb / MeV)

\(^{16}\text{O} + ^{154}\text{Dy}\)

- 2 phonon (AHV)
- 2 phonon (HO)
- 1 phonon (AHV)
- 1 phonon (HO)

=E \text{ cm (MeV)}
\[ \frac{d^2(E\sigma)}{dE^2} \text{ (mb / MeV)} \]

- \( E_2 = 1.5 \ E_1 \)
- \( E_2 = 2 \ E_1 \)
- \( E_2 = 2.5 \ E_1 \)
- \( E_2 = 3 \ E_1 \)

- 2 phonon (HO)
- 1 phonon (AHV)
- N = 2
- N = 4
- N = 8
\[
\frac{d^2(E\sigma)}{dE^2} \text{ (mb / MeV)}
\]

\[
\sigma_{\text{fus}} \text{ (mb)}
\]

\(\chi = -2.36\)

\(\chi = 0\)

\(\chi = +2.36\)
$d^2(E\sigma) / dE^2$ (mb / MeV) vs. $E_{c.m.}$ (MeV)

- **1ph (HO)**
- **2ph (HO)**
- **2ph (AHV)**
- **Expt.**

The graph shows the fusion cross section $\sigma_{fus}$ (mb) as a function of energy $E$ (MeV) in the center-of-mass frame. The data points represent experimental results (Expt.). The various curves represent different theoretical models: 1-ph (HO), 2-ph (HO), and 2-ph (AHV).
\[ \frac{d^2(E\sigma)}{dE^2} \text{ (mb/MeV)} \]

\[ \sigma_{\text{fus}} \text{ (mb)} \]

\[ E_{\text{c.m.}} \text{ (MeV)} \]

- \( Q(2^+) < 0, Q(3^-) < 0 \)
- \( Q(2^+) > 0, Q(3^-) < 0 \)
- \( Q(2^+) < 0, Q(3^-) > 0 \)
- \( Q(2^+) > 0, Q(3^-) > 0 \)

Expt.

\[ Q(2^+), Q(3^-) \]

\[ Q(2^+), Q(3^-) \]

\[ Q(2^+), Q(3^-) \]

\[ Q(2^+), Q(3^-) \]