Two-Dimensional Solitons at Finite Temperature

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Abstract

The partition function of two-dimensional solitons in a heat bath of mesons is worked out to one-loop. For temperatures large compared to the meson mass, the free energy is dominated by the meson-soliton bound states and the zero modes, a consequence of Levinson’s theorem. Using the Bethe-Uhlenbeck formula we compare the soliton energy-shift to the shift expected in the pole mass using a virial expansion. We construct the partition function associated to a fast moving soliton at finite temperature, and found that the soliton thermal inertial mass is no longer constrained by Poincare’s symmetry. At finite temperature, the concept of quasiparticles is process dependent.
1. The statistical mechanics of mesons in the presence of solitons has been studied extensively in the past using two-dimensional models [1]. These systems arise naturally in physical settings involving polymeric structures [2], and have been used successfully to describe a variety of thermodynamical quantities [3].

At zero temperature, solitons provide an ideal set up for describing extended particles in a way that is fully consistent with the Poincare symmetry. Meson-soliton and soliton-soliton scattering can be systematically analyzed using a $1/h$-expansion. In four dimensions, the semi-classical expansion is motivated by QCD in the large number of colors, and provide an interesting way of organizing the strong coupling expansion. Given the interest in finite temperature problems in QCD, it is natural to ask how meson-soliton description fetch at finite temperature.

In this letter, we will address the finite-temperature issue in two-dimensional models, as a way to develope some insights to the behaviour of classical and extended objects in a heat bath of mesons, using semi-classical techniques. We start by constructing the partition function of a soliton in a heat bath of mesons. The free energy is calculated to one-loop, using the exact meson-soliton scattering amplitude. The large T behaviour of the free energy is generic, and solely determined by the number of meson-soliton bound states and zero modes, a direct consequence of Levinson’s theorem. The relation to the soliton mass-shift at finite temperature is elucidated using the Bethe-Uhlenbeck formula. Finally, we find that at finite temperature the energy-mass relation is no longer supported by the extended nature of the soliton. The relevance of these results to four-dimensional soliton models is discussed in our concluding remarks.

2. Consider a field theory in two-dimensions with a given Lagrangian density

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - U(\Phi)$$

For a class of time-translational potentials [4], the action associated to (1) supports classical soliton or kink configurations with finite energy. They will be referred to as by $\Phi_0(x - X)$, where $X$ indicates that the finite energy configuration is space-translational invariant. The quantum theory associated to (1) supports particle and meson states. The classical soliton configurations are precursors to the quantum particle states. Their characterization and properties will be assumed here [5].
In the one-soliton sector, the meson-soliton dynamics can be organized systematically in $1/\hbar$. Indeed, by elevating $X$ to the rank of a quantum variable, we have

$$\Phi(x,t) = \Phi_0(x - X(t)) + \phi(x - X(t),t)$$

the meson fluctuation $\phi$ is made transverse to the zero mode $\Phi'_0$, to insure decoupling. The meson-nucleon Hamiltonian to order $\hbar^0$ reads

$$H = M_0 + \frac{1}{2} \int \left( \pi_0^2 + \phi^2 + U''(\Phi_0)\phi^2 \right)$$

where $M_0 = \int \Phi'_0$ is the classical soliton mass. The soliton kinetic energy $P^2/2M_0$ is of order $\hbar^{-1}$.

In the one-soliton sector the partition function associated to (3) at temperature $T = 1/\beta$, is given by

$$Z_s = \sqrt{M_0 e^{-\beta M_0}} Z_\phi$$

where,

$$Z_\phi = \left( \prod_{bs \neq zm} \frac{e^{-\beta \omega_n/2}}{1 - e^{-\beta \omega_n}} \right) \left( \prod_{sc} \frac{e^{-\beta \omega_n/2}}{1 - e^{-\beta \omega_n}} \right)$$

The first product runs over the possible meson-soliton bound states which are distinct from the zero modes ($zm$). The second product runs over the meson scattering states ($sc$). The energy modes $\omega_n$ diagonalize the $\hbar^0$ part of the Hamiltonian (3). Since the soliton kinetic energy has been ignored, (5) is valid for temperatures $T << M_0$. To account for vacuum renormalization, we will consider the ratio of $Z_s$ (5) to $Z_0$, the partition function in the absence of the soliton. In the dilute gas approximation, the ratio $\hat{Z}_s = Z_s/Z_0$ provides the rationale for a virial expansion. This ratio will be used below.

To be able to count the number of meson states, we consider the system in a spatial box of dimension $L$, with periodic boundary conditions. As a result, the meson-soliton phase-shifts $\delta_n(k)$ for a meson of energy $\omega_n = \sqrt{m^2 + k_n^2}$, satisfy

$$Lk_n + \delta(k_n) = 2n\pi$$

As a result, we the pressure is just given by

$$- \log \hat{Z}_s = +\beta \left( M_0 - \frac{\Lambda\delta(\Lambda)}{2\pi} + \int \frac{dk}{2\pi} \delta'(k) \frac{\omega}{2} \right)$$
\[ + \sum_{bs \neq zm} \left( \frac{1}{2} \beta \omega_n + \log(1 - e^{-\beta \omega_n}) \right) \]
\[ + \int_{\Lambda_-}^{\Lambda_+} \frac{dk}{2\pi} \delta'(k) \log(1 - e^{-\beta \omega_k}) \]  
(7)

where \( \Lambda_{\pm} = \pm(\Lambda - \delta(\Lambda)/L) \). The ultraviolet cut-off for the soliton in free space is chosen to be \( \Lambda = 2N\pi/L \), where \( N \) is some fixed but large number. The ensuing logarithmic divergence in the mode sum, results in a finite renormalization of the soliton mass [6]. If we denote it by \( c \) (\( c \) is zero for the sine-Gordon model) we have

\[ - \log \hat{Z}_s = +\beta \left( M_0 - \frac{\Lambda \delta(\Lambda)}{2\pi} + c \right) \]
\[ + \sum_{bs \neq zm} \left( \frac{1}{2} \beta \omega_n + \log(1 - e^{-\beta \omega_n}) \right) \]
\[ + \int_{\Lambda_-}^{\Lambda_+} \frac{dk}{2\pi} \delta'(k) \log(1 - e^{-\beta \omega_k}) \]  
(8)

If we denote by \(-\log Z_T\), the contribution of the meson fluctuations to the pressure in the one-soliton sector, then as

\[ - \log Z_T = + \sum_{bs \neq zm} \log(1 - e^{-\beta \omega_n}) + \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \delta'(k) \log(1 - e^{-\beta \omega_k}) \]  
(9)

The hight temperature limit of (9), \( m \ll T < M_0 \), is quoted in Table 1, using the explicit form of the phase shifts for the Sine-Gordon model [4] and the \( \Phi^4 \) model. These results can be understood independently of the detailed knowledge of the phase-shifts as we now show.

3. To unravel the behaviour of (9) in the high temperature regime \( T \gg m_\pi \), we can expand (9) in \( \beta = 1/T \). A Taylor expansion of (9) gives

\[ \log Z_T = - \sum_{bs \neq zm} \log \beta \omega_n + \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} k \delta(k) \frac{1}{\omega^2} + n \log \beta m_\pi + O(\beta m_\pi) \]  
(10)

The integral over the phase shift in (10) can be unwound using Jost functions [10]. The result for the integral part is

\[ - \log \left( -im_\pi \hat{f}(-im_\pi) \prod_{bs \neq zm} \frac{m_\pi}{m_\pi - k_n} \right) \]  
(11)

where the derivative of the Jost function \( f(k) = S(k) = e^{2i\delta(k)} \) is related to the behaviour of the scattering amplitude in the lower half of the complex \( k \)-plane, at \( k = -im_\pi \) and
\( k = -i k_n \) corresponding to the locations of the bound states (bs) with the zero modes (zm) included. For instance,

\[
\dot{f}(-im_\pi) = 2(-im_\pi)c_+c_- M_0
\]

(12)

where \( c_\pm \) are two constants to be determined (see below).

To see how (12) comes about, a small digression into the scattering problem is necessary. For the normal modes, the scattering equation associated to (3) reads

\[
\left[-\frac{\partial^2}{\partial x^2} + U''(\Phi_0) - m^2_\pi\right] y_\pm(k, x) = k^2 y_\pm(k, x)
\]

(13)

Here \( y_+(k, x) \) and \( y_-(k, x) \) are the two independent solutions of the scattering equation with boundary conditions \( y_+(k, x \to +\infty) = e^{-ikx} \) and \( y_-(k, x \to -\infty) = ie^{ikx}/2k \). As in the three dimensional scattering radial equation the Jost function \( f(k) \) relates these solutions through

\[
y_-(k, x) = \frac{i}{2k} y_+(k, x)f(k)
\]

(14)

Proceeding with analytic continuation in the lower half of the complex plane of \( k \) the discrete solutions (bs) of (13) occur for the zeros of the Jost function. In particular \( \Phi'_0(x) \) being the zero mode solution at \( k = -im_\pi \), we have

\[
y_\pm(-im_\pi, x) = c_\pm \Phi'_0(x)
\]

(15)

In terms of (11-120 and (15), the meson induced pressure in the one-soliton sector at high temperature (10) reads

\[
- \log Z_T = + \log \left(-i\dot{f}(-im_\pi)T \prod_{bs \neq zm} \frac{\omega_n}{m_\pi - k_n} \right) + O\left(\frac{m_\pi}{T}\right)
\]

(16)

This equation is remarkable. It shows that to leading order in the temperature, the soliton pressure to one-loop is totally driven by the lowest lying states (zero modes and bound states). We expect this result to extend to four dimensions [7]. The value of the Jost function \( f(-im_\pi) \) can be obtained by using (14) along with the boundary conditions on \( y_\pm \) to obtain \( c_\pm \). With \( c_\pm \) we then make use of (11) to get \( f(-im_\pi) \). The only explicit form is contained in the zero mode \( \Phi'_0(x) \). To gain more insight we work out the cases for both the Sine-Gordon and \( \Phi^4 \) models. The results are displayed in Table 1. Both formula
(9) and (15) are independently used to check the validity of our manipulation. We point out that $-\log Z_T$ through (9) was worked out in [3].

| Model | sine-Gordon | $\Phi^4$ |
|-------|-------------|-------|
| $U(\Phi)$ | $-\frac{m^2}{2\pi} (\cos g \Phi - 1)$ | $\frac{1}{2}g^2(\Phi^2 - \frac{m^2}{4\pi^2})^2$ |
| $\Phi_0(x)$ | $\frac{4}{9} \arctan e^{mx}$ | $\frac{m}{g} \tanh(mx)$ |
| $M_0$ | $8m/g^2$ | $4m^3/3g^2$ |
| b.s. | $-im_x = im$ | $-2im; -ik_1 = im$ |
| $c_+$ | $g/4m$ | $g/4m^2$ |
| $c_-$ | $-g/8m^2$ | $-g/16m^3$ |
| $\dot{f}(-im_\pi)$ | $i/2m$ | $i/12m$ |
| $-\log Z_T(15)$ | $-\log 2\beta m$ | $-\log 4\sqrt{3}\beta m$ |
| $\delta(k)$ | $2 \arctan \frac{m}{k}$ | $2(\arctan \frac{2m}{k} + \arctan \frac{m}{k})$ |
| $-\log Z_T(9)$ | $-\log 2\beta m$ | $-\log 4\sqrt{3}\beta m$ |

**Table 1.**

4. Given the pressure $P_s = -\ln Z_T$, the energy of the one-soliton in the heat bath of mesons follows through $E_s = -T\partial P_s/\partial T$. Using (15), we have to one-loop

$$E_s = \left( M_0 - \frac{\Lambda \delta(\Lambda)}{2\pi} + c \right) + \sum_{b \neq z} \omega_n \left( \frac{1}{2} + \frac{1}{e^{\omega_n} - 1} \right) + \int_{-\Lambda}^{+\Lambda} \frac{dk}{2\pi} \delta'(k) \frac{\omega_k}{e^{\omega_k} - 1}$$

The first term in (17) is the zero temperature mass-shift to order $\hbar^0$. In the Sine-Gordon model $c = 0$, and the $-\Lambda \delta(\Lambda)/2\pi = -m/\pi$ [3]. The temperature effects in (17) are just the ones expected from meson thermal weights in phase space, which is a good check on
From (17) the soliton mass shift (or energy shift) at finite temperature is just given by

$$\delta M_T = \sum_{b \neq m} \frac{\omega_n}{e^{\beta\omega_n} - 1} + \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \frac{\omega \delta'(k)}{e^{\beta\omega} - 1}$$  \hspace{1cm} (18)$$

For temperatures $T \gg m_\pi$ knowledge of the meson-soliton phase-shift is not needed. Indeed,

$$\delta M_T = T(n-1) + T\frac{\delta(\infty) - \delta(0^+)}{\pi} + O(m \log \beta m)$$  \hspace{1cm} (19)$$

Using Levinson’s theorem [10], we obtain

$$\delta M_T = -T + O(m \log \beta m)$$  \hspace{1cm} (20)$$

This result is generic to soliton models in two-dimensions. In the temperature range $m_\pi \ll T \ll M_0$, the soliton mass shift as given by the energy definition, is negative and linear in $T$. The slope is controlled by (minus) the total number of zero modes. In (20) the slope is just $-1$, since there is only one translational zero-mode in two-dimensions, in models without isospin. The result (20) can be directly checked from (18) using the explicit forms of the phase shifts. The results are summarized in Table 2. Is the concept of a thermal soliton mass shift discussed in this section unique?

5. For an extended particle the mass definition is unique both at the classical and quantum level. The definition is process independent. It is the same whether we compute the energy of a soliton, its recoil mass or measure its time-like correlation functions. The uniqueness is insured by the fact that the Poincare symmetry is properly enforced to all orders in $\hbar$. At finite temperature, this is no longer the case as we now show.

The soliton pressure and hence energy has provided us with one possible definition of the thermal soliton mass shift. This definition gives $\Delta M_T$ that is real to all orders in $1/\hbar$. Alternatively and in a dilute meson gas, a propagating soliton may receive a correction to its mass through local re-scattering processes. The latter cause the soliton pole mass to shift. The shift can be organized in powers of the pion density (virial expansion). To first order in the pion density,

$$\delta M_T^* = -\int \frac{dk}{2\pi} \frac{t^{lab}}{2\omega} \frac{1}{e^{\beta\omega} - 1}$$  \hspace{1cm} (21)$$
where \( t_{lab} \) is the forward scattering meson-soliton amplitude. To get a simple estimate on (21) at high temperature, consider a simple meson-fermion Yukawa-coupling \( \mathcal{L} = \bar{f} \psi i \gamma_5 \gamma_\mu \psi \partial^\mu \pi \), where \( \psi \) refers to a fermion of mass \( M \). The result is \( \delta M^*_T = f^2 m_\pi T/8M \). The soliton shift at high temperature is subleading in \( 1/\hbar \) and linear in \( T \). Since \( M \sim /\hbar \), the mass shift is actually zero to order \( \hbar^0 \).

To understand the discrepancies between the mass-shift \( \delta M_T \) (18) provided by the energy-definition, and the mass-shift \( \delta M^*_T \) (21) provided by the pole-definition, we call upon the Bethe-Uhlenbeck formula

\[
\delta'(k) = \frac{d}{dk} \left( \frac{Re \ t(k)}{2k} \right) + \frac{i}{8k^2} (tt^* - t't^*)
\]

with the convention

\[
S = e^{i\delta(k)} = 1 - \frac{t(k)}{2k}
\]

In the absence of low-lying resonances, the quadratic term in the scattering amplitude can be ignored [11]. In the present discussion, this is not possible since in the soliton sector there are low-lying zero modes, besides the zero modes (recoiling fermion in the intermediate state). Inserting (22) into (17) yields

\[
\delta M_T = \sum_{bs \neq zm} \frac{\omega_n}{e^{\beta \omega_n} - 1} - \int \frac{dk \ Re \ t}{2\pi} \frac{1}{2\omega} \frac{e^{\beta \omega}}{e^{\beta \omega} - 1} + \int \frac{dk \ Re \ t}{2\pi} \frac{\beta e^{\beta \omega}}{2} \left( \frac{e^{\beta \omega} - 1}{e^{\beta \omega}} \right)^2
\]

\[
+ \int \frac{dk}{2\pi} \frac{i}{8k^2} (tt^* - t't^*) \frac{\omega}{e^{\beta \omega} - 1}
\]

The first term \( \delta M_{bs} \) is the contribution of the bound states viewed as sharp resonances, the second term \( \delta M_{sol} \) is the (real part) analog of (21), the third term \( \delta M_{entro.} \) is the entropy-counterpart of the second term, and the fourth term \( \delta M_{quadra.} \) contains the contributions from possible resonances.

In the high temperature limit \( \delta M_{sol} = -T/2 \) for both the Sine-Gordon and the \( \Phi^4 \) models. The contributions \( \delta M_{entro.} + \delta M_{quadra.} \) are respectively \(-T/2\) and \( -3T/2 \) for the sine-Gordon and the \( \Phi^4 \) models. In Fig. 1 we plot these various contributions to the soliton mass shift in the sine-Gordon model for the low and high temperature regimes. Clearly the quadratic term following from the Bethe-Uhlenbeck formula cannot be ignored when assessing the mass shift using the energy definition.
The discrepancy between $\delta M_{\text{sol.}} = -T/2$ at high temperature following from (24) and $\delta M^* \sim T$ following from (21) using a Yukawa-coupling can be traced back to the schematic nature of the scattering amplitude. Indeed, in weak coupling limit, the meson-soliton $t$-matrix is known exactly. It contains a Born term with intermediate bound and continuum states plus a seagull term $K$, following from an equal-time commutator [12]. The contribution of the bound states (including zero modes) to the mass shift (24)

\[
\delta M^0_{\text{sol.}} = - \int \frac{dk}{2\pi} \frac{t_0^{lab}}{2\omega} \frac{1}{e^{\beta \omega} - 1}
\]

where

\[
t_0^{lab} = -\frac{k^4}{M^2 \omega^2} f_0^2(k^2)
\]

Here $f_0(k^2)$ is the meson-soliton Yukawa form factor [12]. We obtain, for both the Sine-Gordon and $\Phi^4$, $\delta M^0_{\text{sol.}} = +T/2$. Obviously some important contributions still need to be evaluated. For instance $\delta M_K$ and $\delta M_{\text{cont.}}$ respectively associated to the $K$ (seagull) and $t_{\text{cont.}}$ (continuum+excited states) scattering amplitudes [12]. The results are displayed in Table 2. In Fig. 2, we show the respective contributions for finite $T$ in the Sine-Gordon model. While the soliton-meson Yukawa interaction drives the mass contribution (25) up, the seagull part drives it down. We note that in both the Sine-Gordon model and the $\Phi^4$ model,

\[
\delta M_{\text{sol.}} = \delta M^0_{\text{sol.}} + \delta M_K + \delta M_{\text{cont.}} = -T + O \left( \frac{T}{m_\pi} \right)
\]

in agreement with (24), implying that the bound-state, entropy, and quadratic contributions as illustrated in (24) cancel out at high temperature. In general, however, the two-definitions of the soliton mass shift (17) and (21) (or equivalently (25)) are not necessarily the same.
6. The above discrepancy may imply that in a heat bath, the dispersion relation of
a soliton no longer follows the general lore of the vacuum, where Poincare’s invariance
implies
\[ E_s(v) = M_s + \frac{1}{2} M_s v^2 + \mathcal{O}(v^2) \]  
with \( M_s = M_0 + \Lambda \delta(\Lambda)/2\pi + c \), is the one-loop corrected soliton mass \([13,14]\). To see how
the soliton disperses at finite temperature, consider a fast moving soliton in a periodic
box of size \( L \). A rerun of the above arguments, show that the analog of (6) is
\[ L\gamma(k_n + \omega_n v) + \delta_k = 2n\pi \]  
where the energy \( \omega_n \) and the momentum \( k_n \) of a meson are in the soliton rest frame.
Here \( \gamma = 1/\sqrt{1-v^2} \) is the Lorentz contraction factor. The partition function for a
fast moving soliton follows from a proper quantization of the meson-soliton system with
boosted configurations \([14]\). To one-loop, the result is (the \( bs \neq zm \) contribution is
dropped for simplicity)
\[ -\log Z_s(v) = +\beta \gamma M_0 + \sum_n \left( \frac{1}{2} \beta \gamma \omega_n + \log(1 - e^{-\beta \gamma \omega_n}) \right) \]
The partition function of a boosted soliton follows from the one at rest \((8)\) by changing \(M_0 \rightarrow \gamma M_0\) and \(\omega_n \rightarrow \gamma \omega_n\). The scattering mesons around a fast moving extended soliton are Doppler-shifted. Subtracting the vacuum part for the moving system and after mass renormalization, we obtain the reduced partition function

\[
- \log \hat{Z}_s(v) = + \beta \gamma (M_0 - \frac{\Lambda \delta(\Lambda)}{2\pi} + c) + \int_{\Lambda_l}^{\Lambda_u} \frac{dk}{2\pi} \delta'(k) \log(1 - e^{-\beta \gamma \omega_k}) \tag{31}
\]

The cutoffs \(\Lambda_u, l\) are the same as in the zero-temperature case \([14]\). The result \((31)\) differ from the one discussed by Maki and Takayama \([8]\) by a factor of \(-\beta \gamma k v\) absent in the exponential of \((31)\). The discrepancy lies in the enforcement of the boundary conditions on the scattering mesons in the presence of a fast moving soliton. Following \([14]\), our boundary conditions \((29)\) reflect on the fact that the soliton moves at a speed \(v\) in a periodic box of length \(L\). From \((31)\), it follows that the moving soliton obeys the following dispersion relation

\[
E_s(v) = \gamma \left( M_0 - \frac{\Lambda \delta(\Lambda)}{2\pi} + c \right) - \int_{\Lambda_l}^{\Lambda_u} \frac{dk}{2\pi} \delta'(k) \frac{\gamma \omega_k}{e^{-\beta \gamma \omega_k} - 1} \tag{32}
\]

For small velocities

\[
E_s(v) = E_s(T) + \frac{1}{2} E_K(T)v^2 + \mathcal{O}(v^2) \tag{33}
\]

The thermal inertial mass \(E_K(T)\) differs from \(E_s(T)\) and provides for another definition of the soliton mass-shift at finite temperature. Its high temperature limit is different from the ones discussed above.

7. We have analyzed the partition function of two-dimensional solitons to one-loop. The soliton mass-shift following from the energy-definition was found to asymptote \(-T\) independently of the model used. This result is generic, and only conditioned by the number of zero modes. By construction, the energy-definition yields a mass-shift that is always real. We have also analyzed the soliton mass-shift following from the pole-mass definition. The result is generally complex, and reflects on the possibility of particle absorption in the heat bath. Use of the Bethe-Uhlenbeck formula revealed that the energy-mass definition contains the real part of the pole mass-definition along with other contributions. In the high temperature limit, these contributions cancel out. Our considerations have shown the importance of the use of form factors, seagull as well as continuum in the scattering
amplitude. A simple Born approximation yields erroneous results. Finally, using boosted soliton configurations, we have shown that the soliton dispersion relation at finite $T$ leads yet to a third mass-shift definition using the soliton kinetic energy in a heat bath.

Mass-shifts of extended particles at finite temperature provides for an important way to parameterize their properties at low temperature. While the concepts are process dependent, they reflect on important aspects of the interactions of the quasiparticles in the thermal state. The energy-mass definition captures the essentials of the bulk state relevant for transport properties. The pole-mass definition through its real and imaginary part provides for the essentials of the quasiparticle absorption and shifts. Finally, the kinetic-mass definition provides for parts of the quasiparticle dispersion. In particular in the use of dispersion theories, it is important whether pole-masses or inertial-masses are used. The mass-shifts are found to be sensitive to be very sensitive to the various parts of the scattering amplitude (Born, seagull, continuum). Some of these findings may be important when analyzing thermal shifts of hadrons in four dimensional theories, whether using effective Lagrangians or QCD inspired models.

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Fig. 1
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