V-Tail flutter analysis of wing-in-surface-effect (WISE) aircraft using a structural analysis software

M Kusni*, A Taufiqurrahman, and L Gunawan

Lightweight Structure Research Group, Faculty of Mechanical and Aerospace Engineering, Institut Teknologi Bandung; Jl. Ganesha 10, Bandung 40132, Indonesia

*Corresponding e-mail: kusni.ac.itb.ac.id

Abstract. Flutter is a dynamic aeroelastic instability that may cause structural failure and limits flight envelope of an aircraft. A passenger aircraft is required to be free from flutter and other aeroelastic instability phenomena, as stated in the regulations such as FAR 25.33 / FAR 23.33. This paper presents the flutter analyses of the WISE aircraft using MSC Nastran. The analyses were carried out to the V-tail, one of the component where flutter might occur, by assuming rigid fuselage. Results of each analysis, in the form of velocity-damping and velocity-frequency curves, were evaluated to determine the critical flutter speed and frequency. First the analysis were conducted for sea level operation using KE, PK and PKNL methods which predict respectively the flutter speed of 1036 knot, 1037 knot, and 1037 knot. The three methods also consistently predict that the mode shapes involved in the flutter are the 5th mode and the 3rd mode. Then by using the PK-method, the analysis were repeated for air density variation. It is shown that the lower the air density, the higher the flutter speed is. It is concluded that tail flutter does not occur during the operation of the WISE craft with max operating speed of 80 knot.

Keywords: flutter, aeroelastic, V-Tail, wing in surface effect (WISE)

1. Introduction
Passenger aircraft must be safe from aeroelastic failures, that is, safe against divergence, control reversal, and flutter such as stated in FAR 25.33 / FAR 23.33. Flutter occurs due to the interaction between structure, inertia and aerodynamic forces and is indicated with vibration with increasing amplitude. This phenomenon may cause structure failure and hence must be avoided.

This paper presents the flutter analysis of a WISE aircraft. The analysis was focused on the V-tail flutter case. This analysis was limited to flexible tail and rigid fuselage. The tail structure comprising two vertical tail and one horizontal tail, as shown in Figure 1, is modeled as stick beams with concentrated mass distributed along the length of the beam at certain offsets.
2. Theory

2.1 Aeroelastic Equations of Motion

Aeroelastic equation of motion can be written in the form of the matrix equation:\n\[
[M] \ddot{z} + [B] \dot{z} + [K] z = \{A\}(geometry, \rho, \mu, U, t),
\]
with:
- \([m] = \text{mass matrix}\)
- \([c] = \text{damping matrix}\)
- \([k] = \text{stiffness matrix}\)

The aerodynamic forces comprises external and motion dependent aerodynamic forces. For instability analysis, only the latter is considered and can be written as:
\[
\{A\} = \frac{1}{2} \rho U^2 \{f\}[z]
\]
Hence the aeroelastic equation of motion for flutter analysis is:
\[
[M] \ddot{z} + [B] \dot{z} + [K] z = \frac{1}{2} \rho U^2 \{f\}[z]
\]

2.2 Modal Transformation

The analysis is carried out by analysing the undamped free vibration of the aircraft to obtain the natural frequencies and the corresponding mode shapes. Then, by using the modal matrix, equation 3 is transformed into the equation of motion in modal coordinates by assuming that the displacement \([z]\) is the superposition of displacement in each mode shape:
\[
\{z\} = [\varphi](q)
\]
where \([\varphi]\) is the modal matrix and \([q]\) is the modal coordinates vector.

By substituting equation 4 into equation 3, and pre multiplying the result with the transposed modal matrix, the following results is obtained:
\[
M_{hh}\ddot{q} + B_{hh}\dot{q} + K_{hh}q = \frac{1}{2} \rho U^2 Q_{hh}q
\]
where:
- \(M_{hh} = [\varphi]^T [M] [\varphi]\) : modal mass matrix
- \(B_{hh} = [\varphi]^T [B] [\varphi]\) : modal damping matrix
- \(K_{hh} = [\varphi]^T [K] [\varphi]\) : modal stiffness matrix
- \(Q_{hh} = [\varphi]^T [f] [\varphi]\) : modal aerodynamic matrix

The modal mass and stiffness matrices are diagonal with dimensions \([n \times n]\), where \(n\) is the number of mode shapes used in the modal matrix.
2.3 Flutter Analysis Method

Several parameters are needed to find the solution of flutter equation. Several methods have been developed and implemented in NASTRAN, such as KE-method, the PK-method, and the PKNL-method.

2.3.1 KE-method

KE-method is special case of K-method where viscous damping is neglected. This basic principle of this method is based on the assumption that the solution of the flutter equation is harmonic motion, \( \{q\} = \{\hat{q}\}e^{i\omega t} \), with artificial structural damping being added to the system. The equation for flutter analysis is [6]:

\[
- \left[ M_{hh} + \frac{\rho}{2} \left( \frac{c}{2k} \right)^2 Q_{hh}(M_\infty, k) \right] \frac{\omega^2}{1 + iq} + K_{hh} \{\hat{q}\} = 0
\]  

(6)

This method compute flutter roots \((\omega^2/(1 + g))\) for specified values of density \((\rho)\), Mach number \((M_\infty)\), reduced frequency \((k = \omega c/(2U))\), where \(c\) is reference length. From the roots and the inputs, the value of \(U\), \(g\), and \(\omega\) are determined. Flutter speed is obtained from values of \(\rho\), \(M_\infty\), and \(k\) for which \(g = 0\).

2.3.2 PK-method

In PK-method, the motion is harmonic with exponentially varying amplitude, \(\{q\} = \{\hat{q}\}e^{pt}; p = \omega(\gamma \pm i)\), and the eigenvalues of flutter equations is complex. The unsteady aerodynamic forces depends only on the reduced frequency \(k = \omega c/(2U)\). The basic equation for flutter analysis is [6]:

\[
\left[ M_{hh} p^2 + \left( B_{hh} - \frac{1}{4} \rho c U Q_{hh}^l/k \right) p + \left( K_{hh} - \frac{1}{2} \rho U^2 Q_{hh}^R \right) \right] \{\hat{q}\} = 0
\]  

(7)

where:

- \(Q_{hh}^l\) = modal aerodynamic damping matrix, a function of \(M_\infty\) and \(k\).
- \(Q_{hh}^R\) = modal aerodynamic stiffness matrix, a function of \(M_\infty\) and \(k\).
- \(p\) = eigenvalue = \(\omega(\gamma \pm i)\)
- \(\gamma\) = transient decay rate coefficient.

The PK-method extracts these roots for user-specified values of \(\rho\), \(M_\infty\), and \(U\). From these roots, the frequency and damping can be determined.

2.3.3 PKNL-method

PKNL or PK No Looping method is a variation of the PK-method that does not loop on all combinations of users input \(\rho\), \(M_\infty\), and \(U\) [7], but only for combination of \(\rho\) and matching pairs of \(M_\infty\), and \(U\). Hence this method is more efficient than the PK-method.

3. Aeroelastic Analysis using MSC Nastran

Aeroelastic analysis using MSC Nastran is divided into two steps, the free vibration analysis and flutter analysis. Free vibration analysis requires stiffness and mass model of the structure, and flutter analysis requires the aerodynamic model and relation between structure and aerodynamic motion.

3.1 Free Vibration Analysis

Free vibration analysis is performed using SOL103 to obtain the dynamic characteristics of the V-tail structure, i.e. the natural frequencies and the corresponding mode shapes. This analysis needs structure data to create the \([K]\) and \([M]\) in equation 3, and the results are used as the basis of \([\varphi]\) in equation 4.

In this work, the vertical and horizontal tail structure are modeled using stick beams along the elastic axis of each component. The mass of the structure is lumped at the c.g. of the beam by a rigid element.
Figure 2. The division of the (a) VT structure and (b) HT structure

3.1.1. Vertical Tail Model
The vertical structure of the tail is divided into four sections based on the number of ribs, as shown in figure 2.a. Data of vertical tail dimensions: root chord = 1700 mm, tip chord = 1040 mm, mean chord = 1400 mm. For each section, the area and stiffness are shown in table 1 and the inertial properties are shown in table 2.

Table 1. Area and stiffness of vertical tail

| Section | Area (mm²) | $I_{11}$ (mm⁴) | $I_{22}$ (mm⁴) | $I_{33}$ (mm⁴) | E (KN/m²) | G (KN/m²) |
|---------|------------|----------------|----------------|----------------|------------|------------|
| 1       | 5980.17    | 2.59E+7        | 7.63E+8        | 2.74E+8        |            |            |
| 2       | 5516.68    | 2.04E+7        | 5.91E+8        | 2.1E+8         |            |            |
| 3       | 4682.61    | 1.33E+7        | 3.48E+8        | 1.27E+8        | 1.630E+7   | 6.52E+6    |
| 4       | 4256.95    | 9.2E+6         | 2.62E+8        | 8.97E+7        |            |            |

3.1.2. Horizontal Tail Model
The horizontal structure of the tail is divided into 10 sections based on ribs, divided as shown in figure 2.b. For each section, the area and stiffness are shown in table 3 and the inertial properties are shown in table 4. Each section is modeled with one beam element on the elastic axis of the structure. Overall, the horizontal tail structure is modeled with ten beam elements.

Table 2. Inertias of vertical tail

| Section | Mass (kg) | Xcg (mm) | Ycg (mm) | Zcg (mm) | $I_{11}$ (kg.mm²) | $I_{22}$ (kg.mm²) | $I_{33}$ (kg.mm²) |
|---------|-----------|----------|----------|----------|-------------------|-------------------|-------------------|
| 1       | 17.95     | 13640    | -1182.2  | 1754.6   | 2.096E+7         | 1.053E+7         | 2.485E+7         |
| 2       | 11.59     | 13908    | -1435.2  | 2457.6   | 2.452E+7         | 8.789E+6         | 2.989E+7         |
| 3       | 10.14     | 14848    | -2142.9  | 3113.6   | 4.098E+7         | 6.785E+6         | 4.528E+7         |
| 4       | 2.69      | 15348    | -2465.8  | 3800.4   | 1.355E+7         | 8.58+E5          | 1.423E+7         |

Each section is modeled with 3 beam elements on the elastic axis of the structure. Hence, model of each vertical tail (right and left) consists of 12 beam elements.

Table 3. Area and stiffness of horizontal tail

| Section | Area (mm²) | $I_{11}$ (mm⁴) | $I_{22}$ (mm⁴) | $I_{33}$ (mm⁴) | E (KN/m²) | G (KN/m²) |
|---------|------------|----------------|----------------|----------------|------------|------------|
| All     | 10088.23   | 1.15E+09       | 2.00E+09       | 1.12E+09       | 1.41E+7   | 5.24E+6   |

Table 4. Inertia properties of half HT (symmetric to xz plane)

| Section | Mass (kg) | Xcg (mm) | Ycg (mm) | Zcg (mm) | $I_{11}$ (kg.mm²) | $I_{22}$ (kg.mm²) | $I_{33}$ (kg.mm²) |
|---------|-----------|----------|----------|----------|-------------------|-------------------|-------------------|
| 1       | 4.87      | 15850    | 4900     | 3780     | 1.2983E+7         | 1.148E+6         | 1.355E+7         |
| 2       | 9.30      | 15914    | 3640     | 3756     | 1.2983E+7         | 1.148E+6         | 1.355E+7         |
| 3       | 8.14      | 15999    | 2373     | 3788     | 2.6086E+7         | 2.255E+6         | 2.711E+7         |
| 4       | 7.18      | 16042    | 1510     | 3840     | 2.6086E+7         | 2.255E+6         | 2.711E+7         |
| 5       | 7.74      | 16043    | 510      | 3840     | 2.2742E+7         | 2.914E+6         | 2.330E+7         |
3.1.3. V-Tail Model
Figure 3a shows the structural model of the V-tail where rigid elements are used to connect the vertical to the horizontal tail. The vertical tails are rigidly clamped at the bottom ends. Thin plate elements with zero properties are added to the beam elements using rigid elements to visualize the mode shapes of the tail, as shown in figure 3.b.

![Figure 3. Structural model of V-tail (a) beams (b) beams with plates elements](image)

3.2 Flutter Analysis
Flutter analysis is carried out using Nastran SOL 145. In this analysis, aerodynamic model is prepared to create the aerodynamic matrix $Q_{hh}$ in equation 5 using the Doublet Lattice method. Four lift surfaces were defined, namely, the right and the left vertical tail, and the left half and the right half of the horizontal tail. Each lift surface was divided into aerodynamic panels using CAERO1 card.

3.2.1. Aerodynamic Model of Vertical Tail
Each vertical tail was defined as one lift surface which was divided into 96 aerodynamic panels (24 spanwise and 4 chordwise). With this division, each panel has an aspect ratio of 0.35. Figure 4.a shows the aerodynamic panels of the vertical tail.

3.2.2. Aerodynamic Model of Horizontal Tail
Two lift surfaces were defined for the horizontal tail, left and right half of the horizontal tail. Each lift surface was divided into 32 aerodynamic panels (8 spanwise and 4 chordwise). The aerodynamic panel has an aspect ratio of 0.683. Figure 4.b shows the 64 aerodynamic panels of the horizontal tail.

![Figure 4. Aerodynamic panels of (a) the vertical tail and (b) the horizontal tail](image)

3.2.3. Interpolation of Structural Models and Aerodynamic Models
Interpolation is performed so that the aerodynamic model moves according to the structural motion. SPLINE2 card was used for this purpose. Since four lift surfaces were defined for the V-Tail, four cards were used to interpolate motion of the panels to that of the structure.

3.2.4. Flutter Analysis Parameters
The parameters for flutter analysis are defined using FLUTTER card such as the methods, air density, Mach number, reduced frequency or velocity, number of eigenvalues. From the results displayed as $v$-g and $v$-f graphs, flutter characteristics are determined.
4. Results and Discussions

First, the dynamic characteristics of the V-Tail was determined. Then flutter characteristics at sea level, which practically has the same air density as that in operating altitude of 3 m, was analyzed using the KE, PK, and PKNL methods. Finally, sensitivity of the flutter characteristics of V-Tail to (altitude/air density) was evaluated using the PK method.

4.1. Structural Dynamic Characteristics

Undamped free vibration analysis was carried out to determine the natural frequencies and the corresponding mode shapes of the V-Tail structure. Table 3 shows the lowest 14 frequencies and mode shapes of the tail, and figure 14 depicts the first 8 mode shapes. It can be seen that mode shapes at lower frequencies are dominated by bending of vertical tail in symmetric or anti-symmetric configuration. At higher frequencies the mode shapes involve the torsion of horizontal tail. At even higher frequencies, the mode shapes involve bending and torsion of both vertical and horizontal tail.

4.2. Flutter Characteristics

Flutter analysis at sea level was carried out by using the 14 mode shapes for $M = 0.2$ and a flight altitude of 0 m as the air density at 3 m. Figure 6.a shows the $U-g$ curves, where damping curve that first intersects with intersects the damping axis ($g = 0\%$) first is that of the $5^{th}$ mode shape at 1037 knot. The $U-f$ curves in figure 6.b shows that the frequencies of the $5^{th}$ mode at 1037 knot is 11.37 Hz. Figure 6.b also shows that the frequencies of the $5^{th}$ and the $3^{rd}$ modes are getting closer, as the air speed increases. This shows that the flutter occurs from the coupling between the $5^{th}$ and the $3^{rd}$ modes. From this information, it can be concluded that flutter at sea level occurs first for $5^{th}$ mode at velocity of 1037 knot and frequency of 13.37 Hz.

In addition to the PK method analysis, flutter analysis at sea level was also conducted using the KE and PKNL methods. Figure 7 shows the $U-g$ curves obtained from the three methods which shows that the three methods predict the flutter speed almost close to each other. The three methods consistently predict that the mode shapes involved in the flutter are the $3^{rd}$ and the $5^{th}$ mode shapes.

4.3. Effect of Air Density on the Flutter Speed

To observe the effect of air density on the flutter velocity, additional flutter analyses were conducted with air densities of 1.112 kg/m$^3$, 1.007 kg/m$^3$, and 0.9093 kg/m$^3$ using PK-method.

Figure 8.a shows results of analysis for air density of 1.007 kg/m$^3$, where the damping curve that intersects the damping axis = 0% is that of the $5^{th}$ mode at 1087 knot. The $U-f$ curves in figure 8.b shows that the frequencies of the $5^{th}$ mode at 1087 knot is 11.37 Hz. Figure 8.b also shows that the frequencies of the $5^{th}$ and the $3^{rd}$ modes are getting closer, as the air speed increases. This shows that the flutter occurs from the coupling between the $5^{th}$ and the $3^{rd}$ modes. From this information, it can be concluded that flutter at air density of 1.112 kg/m$^3$ occurs first for $5^{th}$ mode at velocity of 1037 knot and frequency of 13.37 Hz.

Figure 9.a shows results of analysis for air density of 1.007 kg/m$^3$ where the damping curve of $12^{th}$ mode intersects the damping axis = 0% first at 1131 knot. The $U-f$ curves in figure 9.b shows that the frequencies of the $12^{th}$ mode at 1131 knot is 36.63 Hz. Figure 8.b also shows that the frequencies of the $12^{th}$ and the $13^{th}$ modes are getting closer, as the air speed increases. This shows that the flutter occurs from the coupling between the $12^{th}$ and the $13^{th}$ modes. In the flutter analysis at air density of 1.007 kg/m$^3$, the flutter occurs at different mode than at lower densities.

Figure 10.a and 10.b show results of analysis for air density of 0.9093 kg/m$^3$. In this case, flutter occurs similar to that at air density of 1.007 kg/m$^3$. Flutter mode is the $12^{th}$ mode at a frequency of 36.54 Hz at speed of 1173 knot. Table 4 compiles the flutter characteristics of the V-tail at several air densities. The values of flutter speed is increased with lower air density. This result is in agreement with statement of Bisplinghoff [2].
1st mode shape, 4.76 Hz

2nd mode shape, 9.47 Hz

3rd mode shape, 11.39 Hz

4th mode shape, 15.78 Hz

5th mode shape, 16.17 Hz

6th mode shape, 22.14 Hz

7th mode shape, 23.10 Hz

8th mode shape, 29.16 Hz

9th mode shape, 31.49 Hz

10th mode shape, 32.24 Hz

11th mode shape, 33.35 Hz

12th mode shape, 36.58 Hz

13th mode shape, 42.60 Hz

14th mode shape, 43.50 Hz

Figure 5. Mode shapes and natural frequencies of the V-Tail structure
Table 5. Mode shapes and natural frequencies of the V-tail structure

| Modes Shape | Natural Frequency (Hz) | Mode Shape |
|-------------|-----------------------|------------|
| 1-st        | 4.76                  | Bending of vertical tail |
| 2-nd        | 9.47                  | Bending of vertical tail |
| 3-rd        | 11.39                 | Bending of vertical tail and torsion of horizontal tail |
| 4-th        | 15.78                 | Bending of vertical tail |
| 5-th        | 16.17                 | Bending and torsion of vertical tail |
| 6-th        | 22.14                 | Bending and torsion of vertical tail, bending of horizontal tail |
| 7-th        | 23.10                 | Bending and torsion of vertical tail, torsion of horizontal tail |
| 8-th        | 29.16                 | Bending of vertical tail, bending of horizontal tail |
| 9-th        | 31.49                 | Bending and torsion of vertical tail |
| 10-th       | 32.24                 | Bending and torsion of vertical tail, bending of horizontal tail |
| 11-th       | 33.35                 | Bending and torsion of vertical tail and horizontal tail |
| 12-th       | 36.58                 | Bending and torsion of vertical tail and horizontal tail |
| 13-th       | 42.60                 | Bending of vertical tail, bending of horizontal tail |
| 14-th       | 43.50                 | Bending and torsion of vertical tail |

Figure 6. (a) U-g and (b) U-f curves at sea level

| Methods   | $V_E$ (knot) | $\omega_E$ (Hz) |
|-----------|--------------|-----------------|
| KE        | 1036         | 13.38           |
| PK        | 1037         | 13.37           |
| PKNL      | 1037         | 13.37           |

Figure 7. U-g curves of V-tail at sea level calculated using PK, K, and PKNL methods
Figure 8. The U-g and U-f curves at air density of 1.112 kg/m³

Figure 9. The U-g and U-f curve at air density of 1.007 kg/m³

Figure 10. The U-g and U-f curves at air density of 0.9093 kg/m³

Table 6. Flutter characteristics of V-Tail at several air density

| Air density (kg/m³) | V_f (knot) | ω_f (Hz) | Flutter mode |
|--------------------|------------|---------|--------------|
| 1.225              | 1037       | 13.37   | 5th mode, bending and torsion of vertical tail |
| 1.112              | 1087       | 13.37   | 5th mode, bending and torsion of vertical tail |
| 1.007              | 1131       | 36.63   | 12th mode, bending and torsion of vertical tail and horizontal tail |
| 0.9093             | 1173       | 36.54   | 17th mode, bending and torsion of vertical tail and horizontal tail |

5. Conclusions

From the results of the flutter analysis that have been performed, it can be concluded that the V-tail structure of the WISE aircraft is safe from flutter for its max operating speed at 80 knots and an altitude of 3 m above sea level. However, further study is needed by using the full structure configuration of the WISE craft to obtain more realistic prediction.
References

[1] Jatiningrum D. Jenie Y I, Muhammad H, Pasaribu HM 2010, Verification of Sail-Flight Testing Procedures of Wing-in-Surface Effect Craft on Engineering Flight Simulator, ICAS 27th Conference, 19-24 Sept 2010, Nice, France.
[2] Bisplinghoff R L, H. Ashley, and R L Halfman 1955 Aeroelasticity, Addison-Wesley Publishing Company, Inc., Massachusetts.
[3] Dowell E H, Curtiss Jr. H C, Scanlan R H, and Sisto F 1980, A Modern Course in Aeroelasticity, Sijthof and Noordhoff, Netherland.
[4] Fung Y C 1996 An Introduction to the Theory of Aeroelasticity, Dover Publications, Inc., New York.
[5] Scanlan R H and Rosenbaum R 1968 Introduction to the Study of Aircraft Vibration and Flutter, Dover Publications, Inc., New York.
[6] Aeroelastic Analysis User’s Guide 2004, MSC.Nastran Version 68, MSC Software Corporation, USA.
[7] Johnson E H 1997, MSC Developments in Aeroelasticity, MSC Aerospace Users’ Conference, The MacNeal-Schwendler Corporation, California, USA.
[8] Dasril H 2007, Pemodelan dan Analisis Dinamika Struktur Pesawat WISE dengan Metode Elemen Hingga, Undergraduate Thesis (unpublished)), Dept. of Aerospace Engineering ITB, Indonesia.
[9] Taufiqurrahman A 2007, Pemodelan dan Analisis Dinamika Struktur Pesawat WISE dengan Metode Elemen Hingga, Undergraduate Thesis (unpublished), Dept. of Aerospace Engineering ITB, Indonesia.