Introducing constrained matched filters for improved separation of point sources from galaxy clusters

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Abstract

Matched filters (MF) are elegant and widely used tools to detect and measure signals that resemble a known template in noisy data. However, they can perform poorly in the presence of contaminating sources of similar or smaller spatial scale than the desired signal, especially if signal and contaminants are spatially correlated. We introduce new multi–component MF and matched multifilter (MMF) techniques that allow for optimal reduction of the contamination introduced by sources that can be approximated by templates. The application of these new filters is demonstrated by applying them to microwave and X–ray mock data of galaxy clusters with the aim of reducing contamination by point–like sources, which are well approximated by the instrument beam. Using microwave mock data, we show that our method allows for unbiased photometry of clusters with a central point source but requires sufficient spatial resolution to reach a competitive noise level after filtering. A comparison of various MF and MMF techniques is given by applying them to Planck multi–frequency data of the Perseus galaxy cluster, whose brightest cluster galaxy hosts a powerful radio source known as Perseus A. We also give a brief outline how the constrained MF (CMF) introduced in this work can be used to reduce the number of point sources misidentified as clusters in X–ray surveys like the upcoming eROSITA all–sky survey. A python implementation of the filters is provided at https://github.com/j-erler/pymf.

Key words: galaxies: clusters: general – methods: data analysis – techniques: image processing

1 INTRODUCTION

Matched filtering (MF) is a technique for the extraction of the flux of sources with a well known spatial template at optimal signal-to-noise ratio (SNR). Matched filtering was first proposed for the study of the kinetic Sunyaev–Zeldovich (kSZ) signal from clusters of galaxies by Haehnelt & Tegmark (1996) and subsequently developed and generalized by Herranz et al. (2002) and Melin, Bartlett & Delabrouille (2006) for the extraction of the thermal Sunyaev–Zeldovich (tSZ) signal from multi–frequency data sets like those delivered by the Planck mission, giving rise to what is now known as the matched multifilter (MMF). These filters have since been adopted with great success by the SPT, ACT and Planck Collaborations to extract the tSZ signal of clusters from their respective multi–frequency data sets (Hasselfield 2013; Bleem et al. 2015; Planck Collaboration 2016a).

While matched filters perform admirably in separating diffuse galactic foregrounds and primary cosmic microwave background (CMB) anisotropies from the SZ signal of clusters, contamination by point sources remains an issue (e.g. Bartlett & Melin 2006; Melin, Bartlett & Delabrouille 2006) and can lead to significant biases in the measured cluster parameters (Knox, Holder & Church 2004; Aghanim, Hansen & Lagache 2005; Lin & Mohr 2007; Sehgal et al. 2010). This problem is mitigated to a degree by MMFs due to the prior knowledge of the tSZ spectrum that is used to construct these multifilters, but accurate photometry of clusters that contain a central radio source remains challenging. Point source confusion is also a central concern for the detection of clusters in X–ray observations (e.g. Biffi, Dolag & Merloni 2018; Koulouridis et al. 2018; Tarrio et al. 2016; Tarrio, Melin & Arnaud 2018) recently demonstrated that point source confusion can be reduced by a joint SZ and X–ray MMF analysis, making use of their very different spectral characteristics at microwave frequencies compared to the X–ray regime.
In this work we present a multi-component extension of the matched filter concept that can improve the separation of contaminants that can be approximated by well known templates (e.g. point sources) based purely on their spatial characteristics. This approach is mathematically identical to the generalized multi-component internal linear combination (ILC) algorithms introduced by Remazeilles, Delabrouille & Cardoso (2011a,b) and Hurier, Maclas-Pérez & Hildebrandt (2013), which can be thought of as matched filters in frequency space, and allows for an unbiased photometry of clusters with a central point source. Generalizing our method to multi–frequency data gives rise to a new matched multifi ltering technique that combines spatial and spectral constraints to provide an optimal separation. A similar but less general approach was presented by Herranz et al. (2005), who showed that the tSZ and kSZ signals of clusters can be separated with matched multi fi lters that use the different spectra of the two effects but take the same spatial template for the two components, which restricts the method from being applied to other contaminating sources. In this work we derive our new filters and demonstrate their application using mock microwave and X–ray data of clusters, as well as Planck data of the Perseus galaxy cluster.

This article is structured as follows: Section 2 introduces matched filters and multifilters for galaxy clusters and our proposed constrained filters in detail. Section 3 describes our simulation pipeline for the creation of mock data that are used to test the performance of the constrained matched filters. The results obtained on both simulations and on data from the Planck mission are presented in Section 4. In Section 5 we provide a discussion of our new technique and give an outlook to its application in future experiments. Section 6 provides a summary and concludes our analysis.

Throughout this paper we assume a flat ΛCDM cosmology with \( \Omega_m = 0.7, \Omega_b = 0.05, h = 0.7, \) and \( T_{\text{CMB}} = 2.7255 \text{K}. \) \( E(z) \equiv H(z)/H_0 = (\Omega_m(1+z)^3 + \Omega_b)/(1+z)^2 \) denotes the redshift-dependent Hubble ratio and \( \rho_{\text{crit}}(z) = 3H(z)^2/(8\pi G) \) the critical density of the universe at redshift \( z. \) Unless noted otherwise, the quoted parameter uncertainties refer to the 68 per cent confidence interval. All–sky maps were processed with HEALPIX (v3.31; Górski et al. 2005).

2 MATCHED FILTERING

Setting up a matched filter requires only very limited knowledge about the astrophysical content of a dataset. We assume that an observed map \( l_v \) at frequency \( v \) represents a linear combination of the desired signal, e.g. the SZ signal from galaxy clusters with the spectrum \( f(v) \), plus a noise map \( N_v \) that contains both instrumental noise and astrophysical emission:

\[
l_v = f(v) \cdot A \cdot y + N_v.
\]

(1)

The signal must be well approximated by a known spatial template \( y \) like the projected pressure profile of clusters. We now would like to construct a filter \( \Psi \) that returns the signal (i.e. the amplitude \( A \) of the source template if \( y \) is normalized to unity) at maximum significance. Using the flat sky approximation and changing to Fourier space, a matched filter \( \Psi \) can be constructed by minimizing the variance of the filtered map (e.g. Schäfer et al. 2006)

\[
\sigma^2 = \Psi^T C \Psi,
\]

(2)

where \( C \) is the azimuthally-averaged noise power spectrum of the unfiltered map expressed as a diagonal matrix \( C = \text{diag}(N(k)) \). Here \( k \) denotes the two-dimensional spatial frequency that corresponds to the two dimensional sky position \( x \) in Fourier space. At the same time, we demand the filtered field to be an unbiased estimator of the deconvolved amplitude of the signal template at the position of sources. This condition can be written as

\[
\Psi^T \tau = 1,
\]

(3)

where \( \tau \) is the Fourier transform of the source template \( y \) convolved with the instrument beam. A solution to this optimization problem is found by introducing a Lagrange multiplier \( \lambda \), which leads to a system of linear equations

\[
\begin{bmatrix}
2 \cdot C & -C \cdot \tau^T \\
\tau \cdot C^T & 0
\end{bmatrix}
\begin{bmatrix}
\Psi \\
\lambda
\end{bmatrix} =
\begin{bmatrix}
0 \\
1
\end{bmatrix}.
\]

(4)

the solution to which is:

\[
\Psi = [\tau^T C^{-1} \tau]^{-1} \tau C^{-1}.
\]

(5)

The matched filter derived here is optimal in the least square sense and was first proposed for the study of galaxy clusters by Haehnelt & Tegmark (1996). Although it is most commonly applied to data sets with Gaussian noise, Gaussianity is not a strict requirement. Non–Gaussian noise will not cause a bias but the solution might no longer be optimal (Melin, Bartlett & Delabrouille 2006). However, optimal matched filters were recently derived for the low–number count Poisson noise regime that is relevant for X–ray and γ–ray observations (Ofek & Zackay 2018; Vio & Andreani 2018).

2.1 Constrained matched filters (CMF)

We now show that the matched filter concept can be generalized to multiple sources with known spatial templates. For this we assume that the observed sky is a linear combination of \( n \) sources with known templates \( y_i \) plus noise:

\[
l_v = f_i(v) \cdot A_1 y_1 + \cdots + f_n(v) \cdot A_n y_n + N_v.
\]

(6)

Our goal is to construct a filter that minimizes the variance of the filtered map as defined in equation (2) and at the same time has an unbiased response to the chosen source template. We now place additional constraints by e.g. demanding the filter to have zero response to contaminating sources with well known spatial templates:

\[
\Psi^T \tau_1 = 1
\]

\[
\Psi^T \tau_2 = 0
\]

\[
\vdots
\]

\[
\Psi^T \tau_n = 0.
\]

(7)

In the following it is convenient to construct a matrix \( T \) of dimensions \( n \times n \) from the \( n \) space templates \( \tau_i \):

\[
T = \begin{bmatrix}
\tau_1[1] & \tau_2[1] & \ldots & \tau_n[1] \\
\vdots & \vdots & \ddots & \vdots \\
\tau_1[n] & \tau_2[n] & \ldots & \tau_n[n]
\end{bmatrix}.
\]

(8)
We can derive the form of the new filter by solving a system of linear equations analogous to equation (4)
\[
\begin{pmatrix}
2 \cdot C & -T \\
T^T & 0
\end{pmatrix}
\begin{pmatrix}
\Psi \\
\lambda
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix},
\]
where \(e = (1, 0, \ldots)^T\) is a vector that contains the response of the filter to the \(n\) constraints defined in equation (7) and \(\lambda\) are the \(n\) Lagrange multipliers. The solution for the constrained matched filter is
\[
\Psi = e^T \left( T^T C^{-1} T \right)^{-1} T C^{-1},
\]
which is similar to the one of the traditional matched filter in equation (5). A possible application of this new filter is the reduction of point source contamination in observations of galaxy clusters, which will be explored in Section 4. However, any other contaminating source with a well known template or even multiple sources could be set to zero using this approach. This benefit will come at the cost of a reduced SNR, which will be discussed in Section 4. A comparison of the two filters using simulated microwave data of galaxy clusters and point sources is shown in Fig. 1.

## 2.2 Constrained matched multifilters (CMMF)

Both the matched filter and constrained matched filter presented previously were built to be applied to a single-frequency map. However, the matched filter concept can be generalized to multi-frequency datasets like the ones delivered by Planck. These generalized techniques are known as matched multifilters (MMF; Herranz et al. 2002; Melin, Bartlett & Delabrouille 2006; Lanz et al. 2010; Melin et al. 2012; Tarrio et al. 2016; Tarrio, Melin & Arnaud 2018) and are designed to use prior spatial and spectral information about a source to return an optimally filtered map of A in the least-square sense. We will show here that the matched multifilter concept can be generalized to separate multiple components with known spatial and spectral templates in an analogous way to the single-frequency filter. We start again by constructing a simple model of the observed sky. As before, we can represent observations of the sky as a linear mixture of astrophysical emission and noise:
\[
I(x) = f_{0} \cdot A y(x) + N(x).
\]
Different from equation (1) we now describe the observed maps as vectors in frequency space with \(n_v\) components at each sky position \(x\) in order to simplify the notation. Using this formalism and changing to Fourier space, the multi-frequency source template will be given at each \(k\) as a vector \(F\) in frequency space
\[
F(k) = f_{0} y(k) B_{0}(k),
\]
where \(B_{0}(k)\) denotes the Fourier transform of the beam, which in general will be frequency dependent. We now aim to find a filter \(\Psi(k)\) that, as before, has unit response to the multi-frequency source template:
\[
\int d^2 k \, \Psi^T(k) F(k) = 1.
\]
We therefore construct a series of \(n_v\) filters \(\Psi(k)\) that are the components of \(\Psi(k)\). The final result will be a single filtered map that is the linear combination of the observed maps, each convolved with their respective frequency-dependent filter. The matched multifilter is derived analogously to the single-frequency case by demanding minimum variance of the filtered map at each spatial scale
\[
\sigma^2 = \Psi^T(k) P(k) \Psi(k),
\]
where \(P\) is the noise power spectrum, a matrix in frequency space with \(n_v \times n_v\) components for each \(k\) that are defined as \(P_{jk}(k) = \langle \delta(k-k') \rangle = \langle N_{v}^{*}(k) N_{v}(k) \rangle\). The asterisk denotes the complex conjugate. The matched multifilter is then given by
\[
\Psi(k) = \sigma^2_{MMF} P^{-1}(k) F(k),
\]
with the variance of the filtered map:
\[
\sigma^2_{MMF} = \int d^2 k \, \Psi^T(k) P^{-1}(k) \Psi(k). \tag{14}
\]
The matched multifilter derived here was employed with great success for the detection and photometry of galaxy clusters by the ACT, SPT, and Planck collaborations (Hasselfield et al. 2013; Bleem et al. 2015; Planck Collaboration 2016a).

We now show that a constrained matched multifilter can be constructed in similar fashion as before. The aim is to find a filter that allows us to constrain multiple well known multi-frequency source templates to reduce the impact of well characterized contaminants on the filtered map. We begin by assuming that the observed sky is a linear mixture of known sources plus noise:
\[
I(x) = f_{v1} \cdot A_1 y_1(x) + \cdots + f_{vn} \cdot A_n y_n(x) + N(x). \tag{17}
\]
Next, we define the desired response of the filter to our known source templates:
\[
\begin{align*}
\int d^2 k \, \Psi^T(k) F_{1}(k) &= 1 \quad \text{.....} \\
\int d^2 k \, \Psi^T(k) F_{2}(k) &= 0 \\
&\vdots \\
\int d^2 k \, \Psi^T(k) F_{n}(k) &= 0.
\end{align*}
\]
For each \(k\) the constraints can be written as a matrix \(U\) with dimensions \(n_v \times n\):
\[
U(k) = \begin{pmatrix}
F_{1}[1](k) & F_{2}[1](k) & \cdots & F_{n}[1](k) \\
\vdots & \vdots & \ddots & \vdots \\
F_{1}[n_v](k) & F_{2}[n_v](k) & \cdots & F_{n}[n_v](k)
\end{pmatrix},
\]
By minimizing the variance of the filtered map, we find that the constrained matched multifilter is
\[
\Psi(k) = e^T S^{-1} P^{-1}(k) U(k),
\]
with the \(n \times n\) matrix \(S\) defined as:
\[
S = \int d^2 k \, U^T(k) P^{-1}(k) U(k). \tag{20}
\]
The variance of the filtered map can be computed as:
\[
\sigma_{\text{CMMF}}^2 = \int \, d^2k \, \Psi^*(k) \mathcal{P}(k) \Psi(k).
\]

The constrained matched multifilter this way can be used to separate sources in a similar fashion as the single frequency constrained matched filter, but for multi-frequency datasets. This will require both a spatial and a spectral template for each constrained source, e.g. if we want to extract galaxy clusters from multi-frequency microwave data while minimizing point source contamination we need to know the beam as well as the spectral energy distribution (SED) of the point sources. This makes the method very efficient in cleaning the data, but it will be limited to a specific type of source. Reducing the contamination of radio and far-infrared point sources at the same time can be achieved by placing two additional constraints using the same spatial template (i.e. the beam) but two different SEDs. We will compare the performance of the different filters presented here by applying them to Planck High Frequency Instrument (HFI) data of the Perseus galaxy cluster in Section 4.

3 SIMULATIONS

3.1 The SZ effect of galaxy clusters

In order to test the performance of the constrained matched filter and compare it to the traditional matched filter we prepared a pipeline for the creation of mock images of the microwave sky. We use the tSZ effect signal (Sunyaev & Zeldovich 1970, 1972; Birkinshaw 1999; Carlstrom, Holder & Reese 2002) of galaxy clusters as our sources of interest.

The tSZ effect is a secondary anisotropy of the CMB spectrum with a temperature decrement at low (\(\lesssim 217\) GHz) and a temperature increment at high (\(\gtrsim 217\) GHz) frequencies. Peculiar motion of clusters will cause a red/blue-shift of the CMB in their rest frame, which gives rise to the kSZ effect. The spectra of the SZ signals are commonly expressed as a temperature shift relative to the CMB monopole, which can be written as

\[
\frac{\Delta T_{\text{SZ}}}{T_{\text{CMB}}} = f(x, T_\text{c}) y - \tau_e \frac{\nu_{\text{spec}}}{c},
\]

where \(T_{\text{CMB}}\) is the CMB temperature, \(c\) is the speed of light, \(\nu_{\text{spec}}\) is the peculiar velocity along the line of sight, \(f(x, T_\text{e})\) is the relativistic tSZ (rSZ) spectrum (e.g., Wright 1979; Ikhsan & Nozawa 1998; Chluba et al. 2012), \(y = h\nu/(k_B T_{\text{CMB}})\) is the dimensionless frequency, \(\tau_e = \sigma_T \int n_e(r) dl\) is the optical depth of the plasma and \(y\) is the Comptonization parameter:

\[
y(r) = \frac{\sigma_T}{n_e r c^2} \int_{\text{L.o.s.}} dl \, n_e(r) k_B T_e(r).
\]

Here, \(k_B\) is the Boltzmann constant, \(\sigma_T\) is the Thomson cross-section, \(m_e\) is the electron rest mass and \(n_e\) and \(T_e\) are the number density and temperature of the electrons in the ICM.

The Comptonization parameter is a measure of the gas pressure integrated along the line of sight (L.o.s.) and is computed by projection of the Generalized Navarro-Frenk-White (GNFW) pressure profile (Nagai, Kravtsov & Vikhlinin 2007) using the parametrization presented by Arnaud et al. (2010)

\[
\frac{P_e(r)}{\text{keV cm}^{-3}} = 1.65 \times 10^{-3} E_{\text{c}}^{8/3} \left( \frac{M_{500}}{3 \times 10^{14} \text{M}_\odot} \right)^{0.79} P \left( \frac{r}{r_{500}} \right).
\]
The simulated clusters are added to an artificial CMB map computed using CAMB (Lewis, Challinor & Lasenby 2000). We account for emission from the cosmic infrared background (CIB) by adding maps of the resolved and the clustered CIB provided by the WebSky Extragalactic CMB Mocks team\footnote{The mocks are provided at https://mocks.cita.utoronto.ca} to our simulation pipeline. We make use of the Python sky model (PSM; Thorne et al. 2017) to obtain maps of galactic synchrotron, free–free, spinning dust and thermal dust components. The PSM uses the most recent foreground maps published by the Planck Collaboration (2016b) for the latter three components and adds small scale fluctuations to all maps following an approach similar to the one presented by Miville-Deschênes et al. (2007).

Compact radio sources are modeled by including all sources from the NVSS point source catalog (Condon et al. 1998). The measured flux densities at 1.4 GHz are extrapolated to microwave frequencies assuming a power law SED, \(I(\nu) \propto \nu^{-\alpha}\) with a spectral index \(\alpha\) randomly drawn for each source from a Gaussian distribution with a mean of 0.5 and a standard deviation of 0.1. Galactic and extragalactic near-infrared point sources are included by adding the sources listed in the IRAS point source catalog (Beichman et al. 1988) by following the approach presented by Delabrouille et al. (2013) to extrapolate the reported flux densities to lower frequencies.

We restrict our analysis to the extragalactic sky that is relevant for studies of galaxy clusters and cosmological studies by applying a 40 per cent Galactic dust mask to our mock maps. We furthermore exclude the region of the sky that has not been observed by the NVSS in order to keep the properties of our sky model homogeneous.

All maps are processed at HEALPix \(n_{\text{side}} = 8192\), which allows to generate mock data with a minimum FWHM of 1 arcmin. Maps that come at a lower native resolution are oversampled and smoothed with a narrow Gaussian beam to avoid pixelization artifacts. The microwave sky is simulated at 150 GHz and 350 GHz with different spatial resolutions ranging from 1 arcmin to 20 arcmin, assuming circular Gaussian beams and white instrumental noise with \(\sigma^{\text{noise}}_{150\text{GHz}} = 6.4 \mu\text{K}_{\text{CMB}}\text{-arcmin}\) and \(\sigma^{\text{noise}}_{350\text{GHz}} = 25 \mu\text{K}_{\text{CMB}}\text{-arcmin}\). The wide range of simulated spatial resolutions allows us to test our filtering techniques for instruments ranging from Planck to current and future ground–based experiments. The resulting maps are shown in Fig. 2.

### 3.3 Simulating X–ray data

In addition to tests using the simulated microwave data that was previously introduced, we apply the single–frequency filters presented in Section 2 to simulated X-ray data. We chose to create mock images of the upcoming extended RÖntgen Survey with an Imaging Telescope Array (eROSITA, Merloni et al. 2012; Predehl 2017). Following Clerc et al. (2018), the images are simulated in the 0.5–2 keV energy band. Each image has a size of 3.6° × 3.6°, with a pixel size of 4 arcsec and a simulated exposure time of 1.6 ks. We include the X-ray and instrumental background model presented in Table 1 of Born et al. (2014). A randomly distributed population of point sources, which is described by the Moretti et al. (2013) log \(N\) – log \(S\) relation, is added. For simplicity, the images only contain a single type of isothermal cluster, simulated using a projected \(\beta\)–model (Cavaliere & Fusco-Femiano 1976)

\[
S_X(\theta) \propto \int_{0,3} dl \, n_c(\nu)^2 \propto \left[ 1 + \frac{\theta}{\theta_c} \right]^{-2-3\beta},
\]

with a fixed flux of \(5 \times 10^{-13} \text{erg s}^{-1} \text{cm}^{-2}\), core radius \(\theta_c\) of 20 arcsec, and \(\beta\) of 2/3. All sources are convolved with a Gaussian PSF with a FWHM of 28 arcsec, which is the expected value in survey mode for eROSITA (Merloni et al. 2012). We derived the count–rates of the sources from the physical fluxes for a given spectral emission model and using the instrumental response file erosita_1v_7telfov_ff.rsp\footnote{The eROSITA response file is available at http://www2011.mpe.mpg.de/erosita/response/}. In this work, we assume an APEC thermal plasma model (Smith et al. 2001) having a metal abundance of 0.3 \(Z_\odot\) along with a Galactic hydrogen column density corresponding to \(1.7 \times 10^{20} \text{cm}^{-2}\) (Kalberla et al. 2005; Born et al. 2014). The simulated X-ray images will be used in Section 4.3 to demonstrate how constrained matched filters can aid the separation of galaxy clusters and point sources in X–ray surveys.

### 4 RESULTS

#### 4.1 Photometry of clusters with a central point source

Using the simulation pipeline introduced in the previous section, we first investigate how the new filtering technique presented in this work can improve the photometry of clusters that harbour a bright central point source. We do so by creating 150 GHz mock observations of clusters with masses ranging from \(10^{14} M_\odot\) to \(10^{15} M_\odot\) at a constant redshift of 0.2. Each simulated cluster features a central radio source with a fixed flux density of 10 mJy at 150 GHz. The beam
Figure 2. Orthographic view of the simulated microwave sky at 150 and 350 GHz that was used to test the different filtering techniques. The projection is centered on the Galactic north and south pole. The maps are shown in histogram equalized scale to enhance the dynamic range. The composition of the maps is described in Section 3. We remove the brightest parts of the galactic disk by applying a 40 per cent Galactic mask and exclude the part of the sky that has not been observed by the NVSS.

is assumed to have a FHWM of 1 arcmin. The values of the central Comptonization parameter computed from the measured cluster flux after filtering are shown in the left–hand panel of Fig. 3. By construction, the constrained matched filter always returns an unbiased result, while the values obtained through matched filtering are biased low with a linear dependence on the brightness of the point source. For the given flux density this bias has a significance of $\sim 4\sigma$ and increases to $\sim 5\sigma$ for lower cluster masses due to the decreasing cluster size. The biased fluxes can therefore lead to a non-detection of low mass clusters and biased inferred cluster properties for high mass systems.

We also consider a potential offset of a bright central point source relative to the cluster center. If a central point source is not aligned with the cluster, both methods will find a bias due to ringing artifacts around the filtered point source. We find that for small angular separations up to $\sim 0.75\text{FWHM}$ the constrained matched filter returns a value with a bias that is smaller than the one observed in the values returned by the matched filter, while both methods find similar values for larger offsets.
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Figure 3. Left–hand panel: Impact of (radio) point source contamination on the measured Comptonization parameter. We simulate the tSZ decrement at 150 GHz for a range of clusters with different masses at a constant redshift of $z = 0.2$ using the GFW profile and add a central point source with a fixed flux density of 10 mJy to each of them. The blue data points show the estimates of the central Comptonization parameter $y_0$ obtained using matched filters, while the data shown in red were obtained using constrained matched filters. The solid black line indicates the expected relation. The constrained matched filter allows an unbiased measurement of a cluster’s flux in the presence of a central point source, while the matched filter will return a biased value. Right–hand panel: Impact of an offset in the point source location on the previous results. The x–axis gives the positional offset relative to the cluster center. For this test we assume the same resolution, frequency, point source flux, and source redshift, but only consider a single cluster of mass $M_{500} = 10^{15} M_\odot$. The shaded regions indicate the uncertainty of the flux estimates. We find that the constrained matched filter performs better than the matched filter up to an offset of $\sim 0.75$ FWHM. For higher values, both methods provide a similar bias.

Figure 4. Ratio of the noise level in the CMF and MF filtered maps defined in equation (2) as a function of cluster size. Different colors correspond to different beam FWHMs. The solid lines correspond to results obtained from the 150 GHz mock data while the results shown with dashed lines were obtained using the 350 GHz mock data.

The comparison above highlights a clear advantage of the constrained matched filter, which however is bought with an increase in the noise level in the filtered map that limits the usefulness of the method in some cases. This noise increase results from placing additional constraints that inevitably lower the degrees of freedom available for the optimization of the variance of the filtered map. Figure 4 shows the ratio of the noise in the filtered maps as a function of apparent cluster size and instrument beam. This ratio scales linearly with the cluster-size-to-beam ratio if the map noise is Gaussian. The differences between the results obtained at 150 GHz and 350 GHz are due to the different foreground properties. We find that the constrained matched filter provides maps with a marginally increased noise level for most modern ground–based mm telescopes that offer a typical resolution of $\sim 1$ arcmin. The use case for low-resolution instruments like Planck is however restricted to large, mostly nearby clusters with radii of several tens of arcminutes.

4.2 Application to Planck data

In addition to tests on simulated microwave images we apply the matched filters and multifilters presented in Section 2 to Planck HFI data of the Perseus galaxy cluster at $z = 0.0179$. The brightest cluster galaxy (BCG) of the Perseus cluster (NGC 1275) is a powerful radio source known as Perseus A that is unresolved in all Planck bands. While the matched multifilter and constrained matched multifilter are applied directly to the HFI data without any pre–processing other than converting the 545 and 857 GHz maps to units of $K_{\text{CMB}}$, the HFI maps are combined into a single map before applying the single–frequency filers. This is achieved by smoothing the maps to a common resolution of 9.68 arcmin after which they are combined into a $\gamma$–map with ILC or constrained ILC (CILC) algorithms (Remazeilles, Delabrouille & Cardoso 2011a; see Appendix C of Erler et al. 2018 for details).
The radio galaxy Perseus A appears as a bright source with negative amplitude in the ILC $\gamma$-map due to its diminishing brightness with increasing frequency, which is also seen in the MILCA and NILC $\gamma$-maps published by the Planck Collaboration (2016c). In contrast, the CILC algorithm allows to constrain the Perseus A SED and thus remove its contamination to the $\gamma$-map. The Perseus A SED used for the CILC and constrained matched filter algorithms is extracted directly from the Planck HFI data using constrained matched filters that remove the tSZ contamination by the ICM of the cluster and found to be well approximated by a power law with spectral index $\alpha = 0.78 \pm 0.05$ (see Table 1). We model the tSZ signal of the Perseus cluster with a GNFW pressure profile with $\theta_{500} = 59.7\,\text{arcmin}$ (Urban et al. 2014) and use a non-relativistic approximation of the tSZ spectrum. All maps are $10^3 \times 10^3$ fields centered on (R.A., Dec.) = (03h19m47.2s, +41°30’47”).

We summarize our results by providing the extracted values for the central Comptonization parameter $y_0$ and the derived integrated value $Y_{500}$ in Table 2. The latter is integrated in a cylindrical aperture with the radius $\theta_{500}$

$$Y_{500} = y_0 \frac{2\pi}{(100\,\text{pc})^2} \int_0^{\theta_{500}} \, dr \, y(r),$$

where $y(r)$ is the cluster template that has been normalized to unit amplitude and $D_A$ is the angular diameter distance of the cluster. The processed maps are shown in Fig. 5.

If neither a spatial or a spectral constraint for Perseus A is used, as in the matched multfilter and ILC + matched filter scenarios, we extract a strongly biased negative value for $y_0$ and thus $Y_{500}$. This provides a plausible explanation for the necessity of point source masks that are the reason why the Perseus cluster is not listed in the Planck SZ cluster catalog (PSZ and PSZ2, Planck Collaboration 2014a, 2016a), which were built using two matched multfilter pipelines (MMF1 and MMF3) and the Bayesian PowellSnakes (PwS) algorithm.

The bias introduced by Perseus A is removed by applying a constrained matched filter to the same $\gamma$-map, which yields $y_0 = (9.4 \pm 0.7) \times 10^{-5}$. For the application of a constrained matched filter to an ILC $\gamma$-map it is critical to smooth all maps to a common resolution before combining them. Combining the maps in Fourier space at their native resolution will distort the beam in the $\gamma$-map, which increases the complexity of constraining the beam for point source removal.

Using the Perseus A SED to construct a CILC $\gamma$-map before filtering is an alternative way to remove the bias introduced by the radio source. In that case, both the traditional and the constrained matched filter yield similar values for $y_0$, both of which are consistent with the previous result. Placing a spectral constraint in the CILC step however results in a noisier $\gamma$-map and thus a slightly lower SNR in both cases.

Finally, applying a constrained matched multfilter that uses both the SED of Perseus A and our knowledge of the Planck beams yields $y_0 = (10.0 \pm 0.42) \times 10^{-5}$, which is in agreement with the previous values and with a SNR of 24 offers the strongest signal of all methods compared here. This SNR is comparable to the the SNR of 22 we obtain by applying a matched multfilter to Planck HFI maps of the Coma cluster, a system of similar mass at $z=0.0231$.

Using the $M_{500} - Y_{500}$ scaling relation from the Planck Collaboration (2014b, 2016d) and converting to $Y_{500}^{\text{phys}}$ we find a mass of $(6.97 \pm 0.24) \times 10^{14} M_{\odot}$ for the Perseus cluster, which is consistent with the value obtained by Urban et al. (2014). For Coma, all six methods yield similar values for $y_0$ due to the lack of a bright central radio or FIR source. We find however that the two multfilters deliver an almost identical SNR as the ILC plus matched filter techniques, while the CILC approach gives a slightly lower SNR of 17. This indicates that the additional constraints are “cheaper” for multfilters but come at the drawback that multiple constraints have to be placed for sources with identical spatial template but different SEDs. Combining an ILC map and constrained matched filtering will remove sources just based on their spatial signature with no need to have constraints on their SED.

This example illustrates that there are multiple ways of dealing with point source contamination in clusters. The advantage of the constrained matched filter over using spe-

### Table 1. SED of Perseus A extracted from Planck HFI data using constrained matched filters that remove the tSZ signal of the cluster. It is well approximated by a power law with a spectral index of $0.78 \pm 0.05$. In turn this SED is used to clean tSZ maps of the Perseus cluster in various ILC and MMF approaches.

| $\nu$ (GHz) | FWHM (arcmin) | $S_\nu$ (Jy) |
|-------------|---------------|--------------|
| 100         | 9.68          | 10.36 ± 0.15 |
| 143         | 7.30          | 7.80 ± 0.13  |
| 217         | 5.02          | 5.74 ± 0.25  |
| 353         | 4.94          | 4.12 ± 0.87  |
| 545         | 4.83          | 2.82 ± 2.81  |
| 857         | 4.64          | 2.12 ± 7.90  |

### Table 2. Comparison of the extracted tSZ signal of the Perseus galaxy cluster extracted from Planck HFI data with various ILC and MMF techniques. The corresponding maps are shown in Fig. 5. The ILC–based techniques first combine the 6 HFI maps in an optimal linear combination, after which we apply either a traditional or constrained matched filter. The matched multfilter techniques are directly applied to the HFI maps. The CILC and CMMF techniques use the Perseus A SED given in Table 1. Radio sources like Perseus A will appear as sources with negative $y$ in ILC $\gamma$–maps and MMF maps, leading to biased photometry if not accounted for.

| Technique          | $y_0$ | $Y_{500}$ | SNR |
|--------------------|-------|----------|-----|
| ILC + MF           | -0.74 ± 0.66 | -50.50 ± 0.44 | -1.1 |
| ILC + CMF          | 9.35 ± 0.70  | 6.31 ± 0.47  | 13.4 |
| CILC + MF          | 9.44 ± 0.77  | 6.37 ± 0.52  | 12.3 |
| CILC + CMF         | 9.77 ± 0.82  | 6.59 ± 0.55  | 12.0 |
| MMF                | -2.64 ± 0.39 | -1.77 ± 0.27 | -6.8 |
| CMMF               | 10.0 ± 0.42  | 6.76 ± 0.28  | 24.0 |

The error on the mass includes the uncertainties of the scaling relation parameters given by Planck Collaboration (2016d), which we assume to be uncorrelated.
Introducing constrained matched filters

Figure 5. Filtered maps of the Perseus galaxy cluster processed with the filtering techniques presented in Section 2. The BCG of Perseus hosts a bright radio source called Perseus A that is known to contaminate tSZ observations of the cluster, leading to biased fluxes. We applied all algorithms to $10^\circ \times 10^\circ$ Planck HFI maps centered on (R.A., Dec.) = (03h19m47.2s, +41°30′47″). The maps above show the inner $4^\circ \times 4^\circ$ of the field. In order to apply the single–frequency filters, the six HFI maps were combined into a y–map using ILC and CILC algorithms, the latter of which allows to remove the contamination caused by Perseus A by constraining its SED. Our comparison shows that there are various ways of removing point sources from clusters using either spectral or spatial constraints or a combination of both, all of which find consistent values for the central Comptonization parameter $y_0$ and thus $Y_{500}$. The best SNR is delivered by the constrained matched multifilter, which yields a value of 24. This is comparable to the SNR of clusters with similar mass and redshift to Perseus that do not suffer from point source contamination, like the Coma cluster.
tral constraints is that it is often easier to characterize the instrument source beam than measuring the SED of a source. Radio sources like Perseus A can show variability and extrapolating their fluxes to microwave frequencies based on radio measurements often relies on the assumption of a perfect power-law SED, which can be prone to mistakes since many sources are known to have SEDs that deviate from a power law (Herbig & Readhead 1992). Furthermore, using spectral information will require individual measurements for each source, while a spatial technique can be applied blindly to a large number of objects.

### 4.3 Blind cluster detection and X-ray application

We also investigate the potential application of the constrained matched filter to reduce point source contamination for blind cluster detection. In tSZ surveys below 217 GHz point sources will not be misclassified as galaxy clusters due to the tSZ effect’s characteristic decrement. They can however lower the decrement or even overpower it, which can lead to a biased flux or a non-detection as has been illustrated previously in Section 4.1. At higher frequencies, in the tSZ increment, point sources can bias the flux and might be misclassified as clusters. For instruments like Planck the situation has been mitigated by multi-frequency coverage (e.g. Bartlett & Melin 2006), but prominent examples like the Perseus cluster remain.

Point source contamination is an even greater issue in X-ray surveys due to the stochastic nature of the observed signal. The upcoming eROSITA survey is expected to detect about 100,000 galaxy clusters (Pillepich, Porciani & Reiprich 2012; Clerc et al. 2018) as well as millions of active galactic nuclei (AGN). Separating both source populations presents a major challenge for cluster detection algorithms. The constrained matched filter introduced here presents an additional tool for this task that has the benefit of using reasonable assumptions, like well known cluster profiles and the PSF of the instrument, to deliver an optimal result. In the remaining part of this section we will provide a brief outline how the traditional and constrained matched filters can be combined to detect clusters in X-ray surveys and reduce the number of misclassified point sources.

We perform our tests on the eROSITA mock data that was introduced in Section 3.3. Each field is filtered with both a matched filter and a constrained matched filter. We then apply a simple source finder to the former map to identify bright sources above a fixed threshold, e.g. $5\sigma_{\text{CMF}}$ as defined by equation (2), and determine their centroids. This typically leaves us with around 200 source candidates, the majority of which are point sources. We then determine the values of the map processed with the constrained matched filter at the position of the previously measured centroids and only classify objects for which both values lie above the former threshold as cluster candidates. This procedure is illustrated for a single field in Figs. 6 and 7.

We find that using both filters in conjunction will strongly reduce the number of misclassified point sources. As demonstrated clearly in Fig. 7, the constrained matched filter yields a better segregation of point sources in terms of their SNR but also raises the scatter of the filtered cluster photon counts due to the increased map noise. However this is a very qualitative analysis since we do not account for the Poissonian statistics that govern X-ray observations and do not tune the detection threshold to maximize the number of detected clusters while staying below a fixed rate of spurious detections. We leave a more quantitative analysis of the X-ray application to future work.

### 5 DISCUSSION

The new constrained matched filtering and multifiltering techniques presented in this work are straightforward extensions of the matched filtering concept that enable optimal extraction of sources with known templates while at the same time allowing for an optimal reduction of known contaminating sources. The results presented in Section 4 focused on the reduction of point source contamination to SZ and X-ray observations of galaxy clusters, but it is important to stress that the methods presented here are applicable to any contaminating source that can be approximated through a known template. It is also possible to place more than one constraint, yet care has to be taken since every additional constraint will result in a noisier map. As with any matched filter, the values found in the filtered map will be biased if the source template does not match the true shape of a resolved source. We note however that the constrained filters can provide a slightly larger bias than the traditional matched filter if the desired source is more compact than its template and other compact sources are supposed to be removed.

A technique similar to the constrained matched mult-filter presented in this work was explored by (Herranz et al. 2005), who derived an unbiased matched multfilter to minimize the contamination of the tSZ to kSZ maps and vice versa. These authors derived a two-component version of the filter presented here and then use the same spatial but different spectral templates for the two different SZ components to separate them. However, a potential drawback of this method is that the spatial templates of the tSZ and kSZ signals should in general be different, especially for merging systems.

An important detail of the new methods is their dependence on the spatial resolution of the instrument, which has a crucial impact on the noise level of the filtered map. Compact clusters will thus remain spatially indistinguishable from point sources if the instrument beam is large. This also restricts the application of the constrained filters on Planck data to nearby clusters with large apparent radii. The situation improves when the instrument beam has a FWHM of ~ 1 arcmin or less, at which point the noise will only increase by a few percent compared the a matched filtered map for most cluster sizes. Such resolution is quite common for ground-based cluster surveys like the ones performed by the SPT and ACT. However, additional filtering will be applied for ground-based instruments to reduce atmospheric contamination. The impact of these filtering steps on the astrophysical signal has to be understood and characterized before matched filters are applied (e.g. Bleem et al. 2015).

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4 We use the `find_peaks()` function of the python Photutils package.
The new filtering techniques are especially interesting for studies of the kSZ and relativistic tSZ with upcoming instruments like the Simons Observatory (Simons Observatory Collaboration 2018) and CCAT–prime. CCAT–prime will be a 6 m diameter sub-millimeter survey telescope that is going to operate at 5600 m altitude on the summit of the Cerro Chajnantor in the Chilean Atacama Desert (Parshley et al. 2018a,b). The high and dry site offers superb conditions for observations at frequencies raging from 270 GHz up to 860 GHz (Vavagiakis et al. 2018) at up to one order of magnitude better sensitivity than Planck (Erler et al. 2018; Mittal, de Bernardis & Niemack 2018). Combined with mm-data of the advanced ACT–pol survey, CCAT–prime will offer full coverage of the SZ spectrum and allow significant improvements over Planck in measuring cluster parameters (Erler et al. 2018; Stacey et al. 2018). In order to constrain key properties of clusters via the SZ effects, accurate mm and sub-mm photometry will be required and matched filtering techniques including the ones introduced here are an excellent tool for this (Soergel et al. 2016; Erler et al. 2018).

We also briefly outlined how the constrained matched filter can aid the separation of AGN and galaxy clusters in upcoming X-ray surveys. The need for new techniques for better point source separation was recently highlighted by Biffi, Dolag & Merloni (2018), who used X-ray mocks derived from the hydrodynamical Magneticum Pathfinder simulation to investigate the contribution of AGN inside clusters to the X-ray luminosity of ICM. The methods presented in this work are especially tailored to this application, since they use few assumptions and offer an optimal result. An important benefit of the filters presented here is that they are able to separate clusters and point sources even when they are aligned. This however can lead to biased photometry of clusters with compact cool cores if the template does not account for it. The constrained matched filter should not be considered as a replacement for well proven and tested methods but rather presents an additional tool that will work best in conjunction with other methods such as the traditional matched filter or the well-known sliding cell (Harnden et al. 1984) and WAVEDETECT (Freeman et al. 2002).

Figure 6. Zoom-in on a simulated X–ray photon image (left) and the same image convolved with a matched filter (center) and constrained matched filter (right). The colourbar to the right has been cropped at eight counts to highlight faint structures in the filtered maps. The mock data features realistic X–ray and instrumental backgrounds as well as a realistic point source population, but for simplicity only contains multiple realizations of a single simulated cluster (highlighted with yellow circles). A detailed description of the data can be found in Section 3.3. The matched filter yields a SNR amplification of the clusters but its response to point sources is similar to that of clusters, which makes the separation of the two source populations challenging in some cases. In contrast, the constrained matched filter nullifies point sources and leaves behind “doughnuts” at their positions in the map. Since both filters have an identical response to clusters, combining their results allows for a quick and simple separation of the two source populations, which is shown in Fig. 7.

Figure 7. A simplified demonstration of the application of the constrained matched filter to eROSITA mock data. Simulated clusters are shown as stars while point sources are shown as crosses. Applying a matched filter to a map with 1.6 ks exposure time will typically result in 200 sources with centroid values above 5σCMF. When measuring the corresponding values in a map processed with a constrained matched filter, the vast majority of previously detected point sources (blue crosses) will drop below the threshold (red crosses), leaving us with a cleaner sample of cluster candidates.
algorithms, since significant discrepancies between their extracted signals hint at potential point source contamination. Other recent attempts on improving the separation of point sources and galaxy clusters in X-ray cluster searches include the combination with optical data (Green et al. 2017) and a new matched multifilter technique introduced by Tarrio et al. (2016); Tarrio, Melin & Arnaud (2018) who used ROSAT data as an additional Planck channel to make use of the very different source populations in the two data sets.

6 CONCLUSIONS

This work introduced a new way to generalize matched filters and multifilters to separate desired and undesired sources based on just their spatial (CMF) or their spatial and spectral (CMMF) characteristics. Adding additional constraints will reduce the SNR of the sources, but if both source and contaminant are well approximated by given templates the methods introduced here will allow for unbiased photometry and reduced confusion. When applied to Gaussian data, matched filters are optimal in the least-square sense, making them ideal tools for the extraction of the SZ signal of galaxy clusters from microwave data. However, traditional matched filtering techniques can perform poorly if microwave data of galaxy clusters is contaminated by point sources.

At microwave frequencies, there are two distinct populations of point-like sources that are spatially correlated with galaxy clusters. The first consists of radio-bright AGN which are found at the centres of many BCGs, and the second being composed of dusty star-forming galaxies. Using realistic microwave mock data we showed that the constrained matched filter introduced in this work allows for unbiased photometry of clusters that harbour a central point source. If applied at multiple frequencies it enables studies of the SZ spectrum of clusters with no need to account for the SED of the point source. We showed that our method requires sufficient spatial resolution to be competitive and otherwise will yield an unbiased but noisy result. Applying constrained and unconstrained matched filters and multifilters to Planck HFI data of the Perseus cluster, which features a bright central radio source, demonstrated that there are multiple ways to remove a central source from actual data, requiring only spatial or spectral constraints, or the combination of both. In the latter case we showed that Perseus can be detected with a SNR typical for a cluster of its mass and redshift. However using only spatial constraints will reduce contamination by point sources regardless of their SED.

The application of the methods presented here is especially interesting to the upcoming CCAT-prime and eROSITA cluster surveys. While CCAT-prime will benefit from unbiased photometry of clusters with central point sources for detailed measurements of the rSZ and kSZ effects, point source confusion during cluster detection is a major concern for X-ray surveys. We illustrated how the constrained matched filter can provide an optimal way to distinguish between clusters and point sources and showed that the new method has the potential to be developed into a competitive cluster finding algorithm.

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REFERENCES

Aghanim N., Hansen S. H., Lagache G., 2005, A&A, 439, 901
Arnaud M., Pratt G. W., Piffaretti R., Böhringer H., Croston J. H., Pointecouteau E., 2010, A&A, 517, A92
Bartlett J. G., Melin J. B., 2006, A&A, 447, 405
Beichman C. A., Neugebauer G., Habing H. J., Clegg P. E., Chester T. J., 1988, IRAS Catalogs and Atlases, Explanatory Supplement, eds. C. Beichman, et al., NASA RP-1190, 1
Biffi V., Dolag K., Merloni A., 2018, preprint (arXiv:1804.01096)
Birkshaw M., 1999, Phys. Rep., 310, 97
Bleem L. E. et al., 2015, ApJS, 216, 27
Born K., Reinpriech T. H., Mohammed I., Lovisari L., 2014, A&A, 567, A65
Carlstrom J. E., Holder G. P., Reese E. D., 2002, ARA&A, 40, 643
Cavaliere A., Fusco-Femiano R., 1976, A&A, 49, 137
Chluba J., Nagai D., Sazonov S., Nelson K., 2012, MNRAS, 426, 510
Clerc N. et al., 2018, preprint (arXiv:1806.08652)
Condon J. J., Cotton W. D., Greisen E. W., Yin Q. F., Perley R. A., Taylor G. B., Broderick J. J., 1998, AJ, 115, 1693
Delabrouille J., 2013, A&A, 553, A96
Freeman P. E., Kashyap V., Rosner R., Lamb D. Q., 2002, ApJS, 138, 185
Erler J., Basu K., Chluba J., Bertoldi F., 2018, MNRAS, 476, 3360
Górski K. M., Hivon E., Banday A. J., Wandelt B. D., Hansen F. K., Reinecke M., Bartelmann M., 2005, ApJ, 622, 759
Green et al., 2017, MNRAS, 465, 4872
Hatchett M. G., Tegmark M., 1996, MNRAS, 279, 545
Harnen Jr. F. R., Fabricant D. G., Harris D. E., Schwarz J., 1984, SAO Special Report, 303
Hasselfield M. et al., 2013, J. Cosmology Astropart. Phys., 7, 008
Herbig T., Readhead A. C. S., 1992, ApJS, 81, 83
Herranz D., Sanz J. L., Holston M. P., Barreiro R. B., Diego

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APPENDIX A: ALL–SKY FORMALISM

The matched filter formalism presented in Section 2 used the flat sky approximation but can be adopted to the full sphere with little effort. Implementing matched filters on the full sphere can have advantages in certain situations, because we can avoid using an approximate projection to a flat-sky geometry. Schäfer et al. (2006) provides an excellent overview on the details. This section is intended to give a summary of the most important points.

Assuming radial symmetry of the sources that we are interested in (i.e. $m = 0$) and using the convolution theorem on the sphere we can relate the spherical harmonic coefficients of the unfiltered map $\hat{a}^{\text{unfilt}}_{\ell m}$ to the ones of the filtered map $\hat{a}^{\text{filt}}_{\ell m}$ by:

$$a^{\text{filt}}_{\ell m} = \sqrt{\frac{4\pi}{2\ell + 1}} \Psi_{\ell 0} a^{\text{unfilt}}_{\ell m} = F_{\ell} a^{\text{unfilt}}_{\ell m}. \quad (A1)$$

The new all–sky matched filter $F$ will thus be

$$F = (\tau T^* C^{-1} \tau)^{-1} T C^{-1}, \quad (A2)$$

where $C$ is the power spectrum of the all–sky map recast as a diagonal matrix as was done in Section 2 and the elements of $\tau$ and $T$ are defined as:

$$\tau_{\ell} = \sqrt{\frac{2\ell + 1}{4\pi}} \tilde{\tau}_{\ell 0} = \sqrt{\frac{2\ell + 1}{4\pi}} \cdot y_{\ell 0} \cdot B_{\ell} \cdot w_{\ell}. \quad (A3)$$

Here, $y_{\ell 0}$ denotes the spherical harmonic transform of the source template profile, while $B_{\ell}$ and $w_{\ell}$ are the beam and pixel window functions. When computing the $C_{\ell}$ it is often useful to mask the brightest regions of the Galaxy to reduce contamination from bright ringing artifacts and ensure that the data is Gaussian.

The constrained matched filter can be applied to the full sphere analogously. Using equation (A1) the all–sky filter can be written as:

$$F = \tau^T (T^T C^{-1} T)^{-1} T C^{-1}.$$  \quad (A4)

As defined in Section 2, $T$ and $\tau$ are matrices build from the $n$ spatial constraints

$$T = \begin{pmatrix} \tau_1[n] & \tau_2[n] & \cdots & \tau_n[n] \\ \vdots & \vdots & \ddots & \vdots \\ \tau_1[n_\ell] & \tau_2[n_\ell] & \cdots & \tau_n[n_\ell] \end{pmatrix}, \quad (A5)$$

$$\tilde{T} = \begin{pmatrix} \tilde{\tau}_1[1] & \tilde{\tau}_2[1] & \cdots & \tilde{\tau}_n[1] \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tau}_1[n_\ell] & \tilde{\tau}_2[n_\ell] & \cdots & \tilde{\tau}_n[n_\ell] \end{pmatrix}, \quad (A6)$$

where the components $\tau_i$ and $\tilde{\tau}_i$ are defined for each template $i$ as done in equation (A3).

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