Optical response of superfluid state in dilute atomic Fermi-Dirac gases

J. Ruostekoski
Abteilung für Quantenphysik, Universität Ulm, D-89069 Ulm, Germany
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We theoretically study the propagation of light in a Fermi-Dirac gas in the presence of a superfluid state. BCS pairing between atoms in different hyperfine levels may significantly increase the optical linewidth and line shift of a quantum degenerate Fermi-Dirac gas and introduce a local-field correction that, under certain conditions, dramatically dominates over the Lorentz-Lorenz shift. These optical properties could possibly unambiguously sign the presence of the superfluid state and determine the value of the BCS order parameter.

After the first observations of atomic Bose-Einstein condensates there has been an increasing interest in the studies of Fermi-Dirac (FD) gases. One especially fascinating property of FD gases is that with effectively attractive interaction between different particles the ground state of the system may become unstable with respect to formation of bound pairs of quasi-particles or Cooper pairs. This effect is analogous to the BCS transition in superconductors. The particles near the Fermi surface having opposite momenta and different internal quantum number tend to appear in pairs. This leads, e.g., to a finite energy gap in the excitation spectrum of the system and to a nonvanishing anomalous expectation value \( \langle \psi_\uparrow (r) \psi_\downarrow (r) \rangle \). Here the two internal sublevels are referred to as \( \uparrow \) and \( \downarrow \).

In this paper we study the optical response of a superfluid state in a zero-temperature FD gas at low atom densities for low-intensity light. We show that the presence of the coherent pairing between atoms in different hyperfine levels may dramatically enhance optical interactions and the scattering of light in FD gas.

One particularly promising candidate to undergo the BCS transition and to become a superfluid is spin-polarized atomic \(^6\)Li. Atoms in two different internal levels can interact via s-wave scattering and the \(^6\)Li atom has an anomalously large and negative s-wave scattering length \( a_s \approx -2160 \text{a}_0 \), where \( a_0 \) is the Bohr radius. The nuclear spin states \( m_i = 1 \) and \( 0 \) of \(^6\)Li have been predicted to undergo a superfluid transition at \( 10^{-8} \) K with the density of \( 10^{12} \text{cm}^{-3} \) at zero temperature.

We study the propagation of light by introducing the dipole approximation for atoms and the corresponding Hamiltonian in the length gauge obtained in the Power-Zienau-Woolley transformation. FD gas is assumed to occupy two different hyperfine levels \( |g, \uparrow \rangle \) and \( |g, \downarrow \rangle \) of the same atom. For simplicity, we consider here a situation where there are only two electronically excited levels \( |e, \uparrow \rangle \) and \( |e, \downarrow \rangle \). All the dipole matrix elements for optical transitions between the levels are assumed to vanish, except \( d_{g\uparrow e\uparrow} \) and \( d_{g\downarrow e\downarrow} \), where \( d_{g\uparrow e\uparrow} \) denotes the dipole matrix element for the transition \( |e, \uparrow \rangle \rightarrow |g, \uparrow \rangle \). For instance, the electric dipole moment between levels \( \uparrow \) and \( \downarrow \) vanish, if the levels refer to different nuclear spin states that are decoupled from the levels involved in optical transitions by an external magnetic field.

In the absence of the driving light field atoms in the electronic ground state are described in second quantization by the Hamiltonian density \( \mathcal{H}_1 \): \( \mathcal{H}_1 = \sum_\nu \psi_\nu^\dagger (H_{\text{Fermi}} - \mu_{\nu}) \psi_\nu + i \hbar u_{\nu e} \psi_\nu^\dagger \psi_{e\nu} \psi_{e\nu}^\dagger \), \( \psi_\nu (r) \) is the atom field operator for level \( |g, \nu \rangle \) in the Heisenberg picture, \( \mu_{\nu} \) is the corresponding chemical potential, and \( H_{\text{Fermi}} \) denotes the center-of-mass (c.m.) Hamiltonian. We have approximated the finite-range inter-particle potential by a contact interaction with the strength given by \( u_g = 4\pi a_g \hbar /m \), where \( a_g \) is the s-wave scattering length and \( m \) is the mass of the atom. The atoms in different hyperfine levels can interact via s-wave scattering. On the other hand, there only is a very weak p-wave scattering between two atoms in the same hyperfine state which is ignored in Eq. (1).

The driving light field introduces additional terms for the system Hamiltonian. In the length gauge the basic dynamical degree of freedom for the light field is the electric displacement \( \mathbf{D}(r) \) that interacts with the atomic polarization \( \mathbf{P}(r) \)

\[
\mathcal{H}_2 = -\frac{1}{\epsilon_0} \mathbf{P}(r) \cdot \mathbf{D}(r). \tag{2}
\]

In the present case the positive frequency component of the polarization is given by

\[
\mathbf{P}^+(r) = d_{g\uparrow e\uparrow} \psi_{g\uparrow}^\dagger (r) \psi_{e\uparrow} (r) + d_{g\downarrow e\downarrow} \psi_{g\downarrow}^\dagger (r) \psi_{e\downarrow} (r) \equiv \mathbf{P}_{\uparrow}^+(r) + \mathbf{P}_{\downarrow}^+(r). \tag{3}
\]

The polarization self-energy was shown in Ref. [13] to be inconsequential for dipole atoms. We assume that to leading order all remaining interactions between the atoms and the light field, that cannot be accounted for when the atoms are modeled as point dipoles, are governed by the following interactions:

\[
\mathcal{H}_3 = \sum_\nu \psi_{e\nu}^\dagger (H_{\text{c.m.}} + \hbar \omega_0 - \mu_{e\nu}) \psi_{e\nu} + i \hbar u_{e\nu} \psi_{e\nu}^\dagger \psi_{e\nu} \psi_{e\nu}^\dagger \\
+ \sum_{\nu,\sigma} \hbar u_{g\nu e\sigma} \psi_{g\nu}^\dagger \psi_{e\nu}^\dagger \psi_{e\sigma} \psi_{g\nu}. \tag{4}
\]
Here \( u_{\text{ge}} = 4\pi \hbar a_{\text{ge}} / m \) and \( u_e = 4\pi \hbar a_e / m \) describe the two-body s-wave scattering between the atoms. For simplicity, the frequency of the optical transition \( \omega_0 \) is assumed to be independent of the hyperfine level. For typical values of the optical linewidth the c.m. motion for the excited atoms may be omitted \([2]\).

The positive frequency component of the electric field \( E^+ \) may be obtained by solving the Heisenberg equations of motion \([4]\)

\[
\epsilon_0 E^+(\mathbf{r}) = D^+_\epsilon(\mathbf{r}) + \int d^3 r' G(\mathbf{r} - \mathbf{r}') P^+(\mathbf{r}') , \tag{5a}
\]

\[
G_{ij}(\mathbf{r}) = \left[ \frac{\partial}{\partial r_i} \frac{\partial}{\partial r_j} - \delta_{ij} \nabla^2 \right] \frac{e^{i \mathbf{k} \cdot \mathbf{r}}}{4\pi r} - \delta_{ij} \delta(\mathbf{r}) . \tag{5b}
\]

Here \( D^+_\epsilon \) is the positive frequency component of the driving electric displacement with the frequency \( \Omega, \) and \( k = \Omega / c. \) The monochromatic dipole radiation kernel \( G(\mathbf{r}) \) is precisely the classical expression of the dipolar field, including the delta function at the origin \([4]\).

In the limit of low light intensity we have derived from the Heisenberg equations of motion a hierarchy of equations for correlation functions involving atomic polarization and atom density \([4][3]. \) In the case of the present system we may proceed similarly. As far as the optical response is concerned it is again assumed that we can concentrate on the dynamics of internal degrees of freedom for the atoms and the light. Hence, in the equation of motion for the atomic polarization the kinetic energy of the atoms is neglected.

Light has to be present in order to produce population in the electronically excited levels. In the present paper we consider the limit of low intensity of the driving light. This is done by retaining only those products of operators that involve at most one excited state field operator or the driving electric displacement \([4]\). Then, e.g., the term proportional to \( u_e \) in Eq. (6) has no contribution to the equation of motion for \( P^+(\mathbf{r}). \)

To simplify further we assume that \( \mathbf{d}_{g\tau\tau'} = \mathbf{d}_{g\epsilon\epsilon'}. \) It is useful to introduce the following projection operator

\[
P = \mathbf{d}_{g\tau\epsilon} \mathbf{d}_{g\tau\epsilon}^\dagger / |\mathbf{d}_{g\tau\epsilon}|^2 , \tag{6}
\]

that projects the internal degrees of freedom onto the subspace defined by the four hyperfine levels in consideration. For the expectation values we use the notation \( P_{1\nu} \equiv \langle P^+_\nu \rangle, \) for \( \nu \) denoting the hyperfine state. The steady-state solution of \( P_{1\nu} \) is given by

\[
P_{1\nu}(\mathbf{r}_1) = \alpha \rho_\nu P \cdot D^+_\epsilon(\mathbf{r}_1) + \sum_{\sigma} F_{\sigma\nu} P_2(\mathbf{r}_1 \sigma; \mathbf{r}_1 \nu)
+ \alpha \sum_{\sigma} \int d^3 r_2 P \cdot G' (\mathbf{r}_1 - \mathbf{r}_2) P_2(\mathbf{r}_1 \nu; \mathbf{r}_2 \sigma) . \tag{7}
\]

Here \( \alpha = -D^2 / [i \hbar c (\delta + i \gamma)] \) is the polarizability of a single atom, \( D \) is the reduced dipole matrix element, \( \rho_\nu \) denotes the atom density in level \( \nu, \) \( \gamma = D^2 k^3 / (6\pi \hbar c) \) is the spontaneous linewidth, and \( \delta \) the atom-light detuning. We have also defined

\[
P_2(\mathbf{r}_1 \nu; \mathbf{r}_2 \sigma) = \langle \psi_{\nu}^\dagger (\mathbf{r}_1) P_{\sigma}^\dagger (\mathbf{r}_2) \psi_{\sigma} (\mathbf{r}_1) \rangle , \tag{8}
\]

\[
F_{\sigma\nu} = \frac{1}{\delta + i \gamma} \left[ u_{\text{ge}} - (1 - \delta_{\sigma\nu}) u_{\sigma} \right] . \tag{9}
\]

The normally ordered expectation value \( P_2(\mathbf{r}_1 \nu; \mathbf{r}_2 \sigma) \) describes correlations between an atomic dipole at \( \mathbf{r}_2 \) in hyperfine level \( \sigma \) and a ground state atom at \( \mathbf{r}_1 \) in hyperfine level \( \nu. \) The tensor \( F_{\sigma\nu} \) generates the collisionally-induced level shifts.

Due to the hard-core interatomic potential we remove the contact dipole-dipole interactions between different atoms. In Eq. (7) this is done by introducing the propagator \( G'_{ij}(\mathbf{r}) = G_{ij}(\mathbf{r}) + \delta_{ij} \delta(\mathbf{r}) / 3. \) The purpose of this definition is to yield a vanishing integral for \( G'(\mathbf{r}) \) over an infinitesimal volume enclosing the origin \([3]\).

So far, we have obtained a steady-state solution for the atomic polarization \([7] \) that acts as a source for the secondary radiation in Eq. (6). Equation (6) involves unknown correlation function \( P_2. \) Basically, we could continue the derivation and obtain the equations of motion for \( P_2 \) and for the higher order correlation functions. This would eventually result in an infinite hierarchy of equations analogous to the equations in Ref. \([4]\). However, even in the case of a simple level structure and in the absence of the s-wave interactions the solution for the whole system by stochastic simulations is demanding on computer time \([8]\). In the studies of the refractive index of a quantum degenerate Bose-Einstein gas Morice et al. \([14]\) derived a density expansion in terms of the number of atoms repeatedly exchanging a photon by introducing certain approximations in the ground state atom correlations. Although the lowest order density correction for the electric susceptibility of a zero-temperature FD gas may be obtained analytically \([3], \) in the presence of non-trivial statistical position correlations a rigorous density expansion is in most cases a very challenging task. In this paper we consider low atom densities (in terms of \( \rho / k^3 \)) and approximate Eq. (6) by the decoupling that corresponds to the lowest order correction in Ref. \([4]\)

\[
P_2(\mathbf{r}_1 \nu; \mathbf{r}_2 \sigma) \approx \rho_2(\mathbf{r}_1 \nu, \mathbf{r}_2 \sigma) P_{1\sigma}(\mathbf{r}_2) / \rho_\sigma , \tag{10}
\]

where the ground state pair correlation function \( \rho_2 \) is defined by

\[
\rho_2(\mathbf{r}_1 \nu, \mathbf{r}_2 \sigma) = \langle \psi_{\nu}^\dagger (\mathbf{r}_1) \psi_{\sigma}^\dagger (\mathbf{r}_2) \psi_{\sigma} (\mathbf{r}_2) \psi_{\nu} (\mathbf{r}_1) \rangle . \tag{11}
\]

It is important to point out that the predictions of the expansion by Morice et al. \([14]\) were tested for a zero-temperature FD gas in one dimension \([8]. \) The agreement with the exact solution obtained by the numerical simulations was found to be semi-quantitative and in the low-density limit excellent.

Before the light is switched on, the system is described by the Hamiltonian density \( \mathcal{H} = \mathcal{H}_1 \) [Eq. (4)]. The assumption that the driving light only weakly disturbs the
system allows us to evaluate the pair correlation functions for the ground state atoms [Eq. (1)] from $H_1$ even in the presence of the driving light. We assume a homogeneous sample and introduce a plane wave basis for the field operators: $\psi_{\nu\sigma}(\mathbf{r}) = V^{-1/2} \sum_k b_{\nu \sigma} \exp(ik \cdot \mathbf{r})$. The Hamiltonian [1] is diagonalized by the standard canonical transformation to the Bogoliubov quasi-particles [10,11]

$$\begin{align*}
\alpha_k &= u_k b_k + v_k b_k^{-1}, \quad (12a) \\
\beta_{-k} &= u_k b_{-k} + v_k b_{-k}^{-1}, \quad (12b)
\end{align*}$$

where $u_k$ and $v_k$ are real, depend only on $|k|$, and satisfy $u_k^2 + v_k^2 = 1$. The requirement that $H_1$ in Eq. [1] is diagonal in the quasi-particle representation sets an additional constraint and we obtain

$$u_k^2 = \frac{1}{2} \left( 1 + \frac{\xi_k}{E_k} \right), \quad v_k^2 = \frac{1}{2} \left( 1 - \frac{\xi_k}{E_k} \right), \quad (13)$$

where $E_k = \sqrt{\Delta^2 + \xi_k^2}$, $\xi_k = \epsilon_k - \mu + nu_g(\rho_\uparrow + \rho_\downarrow)/2$, and the energy gap $\Delta = -nu_g V^{-1} \sum_k u_k v_k (1 - n_{\alpha k} - n_{\beta \bar{k} k})$. In equilibrium, the quasi-particle occupation numbers $\bar{n}_{\alpha k} \equiv \langle \alpha \rangle$ and $\bar{n}_{\beta \bar{k} k} \equiv \langle \beta \rangle$ satisfy FD statistics with $\bar{n}_{\alpha k} = \bar{n}_{\beta \bar{k} k} = (e^{E_k/k_B T} + 1)^{-1}$. The dispersion relation for free particles is given by $\epsilon_k = \hbar^2 k^2/(2m)$ and the average of the chemical potentials is $\bar{\mu} = (\mu_\uparrow + \mu_\downarrow)/2$. For simplicity, from now on we assume $\mu_\uparrow = \mu_\downarrow$.

In the superfluid phase transition the atoms in the different hyperfine levels $\uparrow$ and $\downarrow$ form a quasi-particle condensate that results in a nonvanishing anomalous correlation $\langle \psi_\uparrow(\mathbf{r}_1) \psi_\downarrow(\mathbf{r}_2) \rangle$. The effect of this macroscopic two-particle coherence on the pair correlation function [1] can be clearly seen by considering the ground state of $H_1$ [Eq. [1]] that is the vacuum of the Bogoliubov quasi-particles [Eq. (12a)]. Then (for $\nu \neq \sigma$)

$$\begin{align*}
\rho_2(\mathbf{r}_1 \nu, \mathbf{r}_2 \sigma) &= \rho_\uparrow \rho_\downarrow + |\langle \psi_{\nu \uparrow} \mathbf{r}_1 | \psi_{\sigma \downarrow} \mathbf{r}_2 \rangle|^2, \quad (14a) \\
\rho_2(\mathbf{r}_1 \nu, \mathbf{r}_2 \nu) &= \rho_\uparrow^2 - |\langle \psi_{\nu \uparrow} \mathbf{r}_1 | \psi_{\nu \downarrow} \mathbf{r}_2 \rangle|^2. \quad (14b)
\end{align*}$$

The optical response may now be evaluated by eliminating $D_0^\uparrow$ and $P_2$ from Eqs. (5a), (7), and (10). Because we are dealing with a linear theory, the electric field and the polarization are related by the susceptibility as $P^\uparrow = \varepsilon_0 \chi E^+$. We consider a situation where FD gas fills the half-infinite space $z > 0$. We assume that the constant atom densities for the hyperfine states are equal $\rho_\uparrow = \rho_\downarrow \equiv \rho$. For simplicity, it is also assumed that the s-wave scattering length $a_{\nu \sigma \nu \sigma}$ is independent of hyperfine states $\nu$ and $\sigma$ resulting in $F^+ = F^+ \uparrow \downarrow$ and $F^+ = F^+ \uparrow \downarrow$. The incoming free field is written $D_\nu^\uparrow(\mathbf{r}) = D_\nu^\uparrow \mathbf{e} \exp(ikz)$, and we assume that the polarization $\mathbf{e}$ is parallel to the electric dipole moments $d_{\nu \gamma \uparrow \downarrow}$ and $d_{\nu \gamma \uparrow \downarrow}$. With the ansatz $P_{\nu \gamma}^\uparrow(\mathbf{r}) = P_{\nu \gamma}^\uparrow(\mathbf{r}) = \mathbf{P} \mathbf{e} \exp(ikz)$ for $\text{Im}(k') > 0$, by using Eq. (4), and by ignoring the effects of the surface of the atomic gas [3], we obtain a spatially constant susceptibility for the sample as

$$\chi = \frac{k'^2}{k^2} - 1 = \frac{2\alpha \rho}{1 - 2\alpha \rho/3 + \Sigma_1 + \Sigma_2}, \quad (15)$$

with

$$\begin{align*}
\Sigma_1 &= -\frac{\alpha}{\rho} \int d^3 r e^{-ikz} G'(\mathbf{r}) \left[ |\langle \psi_{\nu \uparrow} \mathbf{r} | \psi_{\sigma \downarrow} \mathbf{r} \rangle|^2 \right. \quad (16) \\
\Sigma_2 &= -\frac{1}{\rho} \sum_{\nu, \sigma} F_{\nu \sigma} \rho_2(\mathbf{r}_\nu, \mathbf{r}_\sigma) \quad (17)
\end{align*}$$

Here we have used the obvious relations $\rho_2(\mathbf{r}_\uparrow \uparrow, \mathbf{r}_\downarrow \downarrow) = \rho_2(\mathbf{r}_\downarrow \uparrow, \mathbf{r}_\uparrow \downarrow)$ and $\rho_2(\mathbf{r}_\uparrow \downarrow, \mathbf{r}_\downarrow \uparrow) = \rho_2(\mathbf{r}_\downarrow \uparrow, \mathbf{r}_\uparrow \downarrow)$.

In an uncorrelated atomic sample theatomic position function is statistically independent and the pair correlation function [11] satisfies $\rho_2(\mathbf{r}_\nu, \mathbf{r}_\sigma) = \rho_\nu \rho_\sigma$. In this case, and in the absence of the s-wave scattering, we would obtain Eq. (17) with $\Sigma_1 = \Sigma_2 = 0$. This is the standard column density result stating that susceptibility equals polarizability of an atom times atom density. Equation (15) also involves the Lorentz-Lorenz local-field correction in the denominator.

The quantum statistical corrections to the column density result are introduced by $\Sigma_1$. It describes the modifications of the optical interactions between neighboring atoms due to the position correlations. The second term in Eq. (16) represents the quantum statistical contributions to the scattering process in which a photon emitted by an atom in hyperfine level $\nu$ at position $\mathbf{r}$ is reabsorbed by another atom in hyperfine level $\nu$ and located at the origin. According to FD statistics two fermions with the same quantum numbers repel each other and other FD statistics forces a regular spacing between the atoms. The optical interactions are dominantly generated at small interatomic distances and the corrections to the susceptibility due to the second term in Eq. (16) correspond to inhibited light scattering. In the absence of superfluid state FD gas exhibits a dramatic line narrowing [3].

The first term in Eq. (17) represents the quantum statistical corrections to the reabsorption process between atoms in different hyperfine levels due to the two-particle coherence. This term is nonvanishing only in the presence of a superfluid state. Because the atom pairs in Eq. (17) interact in the triplet s-state and the total spin of the pair is an integer, the pairs behave as bosons [10]. According to the Bose-Einstein statistics two bosons attract each other and the presence of the BCS pairing favors small interatomic spacing, hence, enhancing the optical interactions and the light scattering.

The line shift induced by $\Sigma_2$ [Eq. (17)] is generated by the s-wave interactions. As far as they can be considered local on the scale of the optical wavelength in Eq. (16) the collisions induce a local-field shift analogous to the Lorentz-Lorenz shift.

The optical line shift for the atomic sample is obtained from Eq. (15)
where we have dropped the equal position coordinates in \( \rho_2 \), used \( \rho_2(\mathbf{r}, \mathbf{r'}) = 0 \), and defined the dimensionless variables \( \bar{\rho} = \rho/k^3 \), \( \bar{\rho}_2 = \rho_2/k^3 \), \( \delta = \delta/\gamma \), \( \bar{\alpha} = -6\pi/(\delta + i) \), and \( \bar{u}_c = 6u k^3/\gamma \). The first two terms form the local-field shift. For \(^6\text{Li}\) the local-field shift due to the s-wave scattering in Eq. (18) is larger than the Lorentz-Lorenz shift, if \( \gamma \lesssim 140[1+(\Delta/\hbar u_0 \rho)^2]^{-1} \), \( \Delta = \rho \), \( \rho_0 = \rho_0 \), and \( \rho_0 = \rho_0 \), where \( \lambda \) is the optical wavelength. Because \( \Delta/\hbar u_0 \rho \approx \) expected to be of the order of one \( \rho \), the local-field shift could strikingly depend on the BCS order parameter \( \Delta \).

If the the effective range \( r_u \) of the triplet s-wave potential in Eq. (4) is very short \( r_u \ll 1/k \), the resonant dipole-dipole interactions may suppress the effect of the s-wave scattering on the line shift just as they cancel the effect of the polarization self-energy [13]. However, for a metastable state \( \gamma^{-1} \) may be large on the time-scale of the atomic interactions. In that case the collisional shift could be observable even for very small \( r_u \).

To calculate the nonlinear linewidth and line shift from integral (16) we need to evaluate the spatial correlation functions by using Eqs. (12) and (13). For instance, the anomalous expectation value reads

\[
\langle \psi_1(\mathbf{r})\psi_0(0) \rangle = \frac{1}{\mathcal{V}} \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{\Delta}{2E_\mathbf{k}} (1 - \bar{n}_\mathbf{k} - \bar{n}_\mathbf{k^*}).
\]

The chemical potential is solved from \( \rho_\nu = \rho_\nu(\bar{\mu}) \). Here \( \langle \psi_1(0)\psi_0(0) \rangle = -\Delta/(\hbar u_0 \rho) \) is ultraviolet-divergent resulting from the assumption of the contact two-body interaction in Eq. (3). This interaction is momentum independent and is not valid at high energies. To estimate the pairing we remove the high-energy divergence by replacing \( \hbar u_0 \rho \) by the two-body T-matrix obtained from the Lippmann-Schwinger equation [4]. This is done by subtracting from Eq. (16) \( \sum \exp(i\mathbf{k}\cdot\mathbf{r})\Delta/(2\mathbf{E}_\mathbf{V}) \). As argued in Ref. [4] the use of the T-matrix may seriously underestimate the overlap in the case of large scattering length \( |a_g| \gg r_u \). Hence, we calculate Eq. (19) also by introducing a high-momentum cut-off \( k_c \).

We have plotted the line shift from Eq. (18) and the linewidth \( \gamma = 1 - 6\pi\text{Im}(\Sigma_{1}/\bar{\alpha}) \) by assuming, for simplicity, \( \bar{u}_g = \bar{u}_{ge} \), \( \lambda = 900 \text{ nm} \), and a moderate value \( \gamma = -12000 \). For the gap parameter at \( T = 0 \) we use the weak coupling approximation \( \Delta \approx 1.76k_BT_c [14] \), where

\[
k_BT_c \approx \frac{8\pi}{\pi} e^{-\gamma/2} \exp \left[ -\frac{\pi}{2kF|a|} \right],
\]

with \( \gamma \approx 0.5772 \) and \( k_F = (6\pi^2 \rho)^{1/3} \).

In Fig. 1 (a) the solid line represents the linewidth in the absence of the superfluid state \( \Delta = 0 \). The line narrows as a function of the density [4]. The presence of the superfluid state broadens the line. The linewidth is finite even without the regularization in the anomalous correlation (the dashed line). This is because the dipole radiation already involves a high-frequency cut-off [14] that regularizes small \( r \) behaviour. We have also plotted the case with the cut-off \( k_c = 1/r_u \) (the dotted line) with the realistic value \( r_u = 100a_0 \) of the triplet s-wave potential [4]. We found that the linewidth is almost independent of the cut-off from \( r_u = 500a_0 \) to \( r_u = 0 \). The line shift from the unregularized anomalous correlation in integral [13] diverges logarithmically for small \( r \). Although the radiation kernel [13] involves a cut-off, the Lamb shift is not treated rigorously. However, for the present purposes we may at least obtain an estimate for the shift by using the T-matrix or the cut-off \( k_c = 1/r_u \) as in the case of the linewidth. For the BCS state, even with \( \bar{u}_g = \bar{u}_{ge} \) and moderately small \( |a_g| \), also the line shift is increased.

FIG. 1. The optical (a) linewidth and (b) line shift as a function of the atom density per cubic optical wave number of the driving light. The dashed-dotted line corresponds to the regularization by the two-body T-matrix, the dotted by the cut-off \( k_c = 0.01a_0^{-1} \), and the dashed line is the unregularized case. The solid line has \( \Delta = 0 \).

We studied the interaction of light with a two-species atomic superfluid gas. The analysis of the quasi-particles followed the standard BCS theory [10]. We assumed a translationally invariant system. Atoms in a harmonic trap may be considered as locally homogeneous [4] provided that the trap length scale \( l = (\hbar/m\omega)^{1/2} \) is much larger than the correlation length, which for intraspecies is \( \xi_{\nu\nu} \sim 1/k_F \) and for the interspecies \( \xi_{\nu\sigma} \sim \epsilon_F/(\Delta k_F) [4] \). Other notable assumptions were zero temperature and low atom density. In the future the present work could be extended to larger values of \( |a_g| \) and \( \bar{\rho} \) by going
beyond the BCS weak coupling limit and by including the cooperative optical linewidths and line shifts \[14\].

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