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ABSTRACT
To build a quantum computing device, which is capable of generating arbitrary input states and performing universal unitary gate operations (UUGOs), is an important goal in the field of quantum information science. However, only a few special quantum computations have been reported by now based on specific input states and well-designed information processors. Here, we demonstrate a flexible scheme for two-qubit quantum computations by employing the polarization and the spatial mode of a single photon. Two-qubit UUGOs both in free-space optics and for arbitrary pure input states consisting of separable states and entangled states are presented. Quantum state tomography and process tomography are used to characterize the fidelity of the output states and the gate operations we considered. Beyond a demonstration, we believe that our work also enriches the techniques of bulk-optics for quantum information study and has a broad application for other fundamental research.

I. INTRODUCTION
A quantum computer has the potential for propelling the information society forward because it could efficiently deal with the problems intractable for a classical computer.1–7 As a fundamental expectation, a quantum computer should be capable of performing universal computation, meaning that it could manipulate arbitrary input states and implement universal unitary gate operations (UUGOs). The implementation of UUGO for multi-qubit is generally thought to be tricky. During the past decades, there were many investigations on UUGO.8–13 In particular, it has been theoretically verified that UUGO can be achieved by employing only rotating one-qubit gates and entangling two-qubit gates. Although several studies on programmable quantum processors have been conducted,14–17 only a small subset of UUGO has been implemented with regard to some particular applications.

Recently, the implementations of two-qubit UUGO for ion-trap systems and silicon photonic circuits18,19 have been reported. In Ref. 18, the UUGO have been demonstrated by decomposing the UUGO into a local equivalence class with some single-qubit operations, and 16 separate pure input states are produced. In Ref. 19, the UUGO have been realized by exploiting high-dimensional entanglement to reduce the number of entangling two-qubit gates, and arbitrary separate pure input states are produced. However, the UUGO scheme with arbitrary pure input states (APISs) consisting of separable and entangled states has not been discussed. In fact, entangled input states appear very often in the field of matrix multiplication with the form of four-dimensional column vectors. It is important to consider such a case in many applications.

In this work, we experimentally demonstrate a flexible scheme for two-qubit quantum computations. Our scheme is based on the polarization and spatial mode of a single photon. UUGOs both in free-space optics and for APISs consisting of separable and entangled states have been realized. Moreover, the function of the two-qubit quantum computation has been certified by quantum process tomography and quantum state tomography. This work could contain more computational situations compared with the previous studies and contribute to the bulk-optics quantum technologies from the experimental aspect, like the realization of two-qubit UUGO based on the polarization and orbital angular momentum (OAM).20 The quantum computation platform we built can also be used for other fundamental studies in the future, such as the diagnosis of an unknown two-qubit state, a two-qubit quantum approximate optimization algorithm, and efficient simulation of Szegedy directed quantum walks in free-space optics.
II. THEORY AND EXPERIMENT

The mathematical form of a quantum computation can be generally written as

$$|\psi\rangle_{\text{out}} = U|\psi\rangle_{\text{in}},$$  \hspace{1cm} (1)

where $U$ represents the UUGO and $|\psi\rangle_{\text{in}}$ ($|\psi\rangle_{\text{out}}$) represents the AIPS (output states). Our experimental scheme is illustrated in Fig. 1. The flexible setup consists of four parts: the preparation of the single-photon source, the preparation of arbitrary input states, UUGO, and the measurement of output states. In the experiment, we have employed the polarized (Po) and spatial (Sp) degrees of freedom (DOFs) of the single photon to encode the two-qubit input states. Horizontal (H) and vertical (V) polarizations are regarded as the basis for the polarized DOF and represented as $(1,0)^T$ and $(0,1)^T$ (the superscript $T$ represents the transpose operation). The same goes for the right (R) and left (L) paths for the spatial DOF. So, the two-qubit pure state can be described as a four-dimensional column vector $(RH, RV, LH, LV)^T$, which is obtained by Sp$\otimes$Po. The symbol $\otimes$ represents the tensor product.

The single-photon source shown in the source box of Fig. 1 is enabled by exploiting the signal photon of the photon pairs generated through the spontaneous parametric down conversion process of a 3 mm long beta-barium-borate (BBO) crystal cutting under type-I phase matching conditions. The crystal is pumped by the second harmonic of a Ti:sapphire picosecond laser (Tsunami Spectra-Physics Inc.) with a center wavelength of 390 nm and repetition frequency of 80 MHz. The pump laser with H polarization passes through the crystal and two correlated photons with V polarization are generated. Then, the signal photon with H polarization is obtained by the half-wave plate (HWP) rotated 22.5°. The signal photon passes through the quarter-wave plate (QWP), beam-splitter (BS), and polarizing beam-splitter (PBS) in the setup, and finally, it is detected by the single-photon detector. The other photon of the pair (the idler photon) is employed as a trigger.

Based on the above single-photon source, we can prepare arbitrary pure two-qubit input states using the setup as shown in the input state box in Fig. 1. The input states include separable states and entangled states. First, the superposition state with H and V polarizations is obtained by rotating the HWP with angle $\theta$, which can be expressed as $(\cos 2\theta, \sin 2\theta)^T$. Then, after the PBS, the photon is in the superposition state of the R and L paths with V and H polarizations, respectively. The superposition state is expressed as $(0, \cos 2\theta, \sin 2\theta, 0)^T$ in the representation of Sp$\otimes$Po. In the R and L paths, we use one HWP in the middle of two QWPs (it is called a Q-H-Q in Ref. 21) to modulate the polarizations. The function of the Q-H-Q is described in detail in Ref. 21. Briefly, any polarization states can be produced by the Q-H-Q, and arbitrary single-qubit states are expressed as $(\cos \varphi, e^{i\alpha} \sin \varphi)^T$. So the superposition state is equivalent to $(\cos 2\theta \cos \varphi, \cos 2\theta e^{i\alpha} \sin \varphi, \sin 2\theta \cos \psi, \sin 2\theta e^{i\beta} \sin \psi)^T$.

Note that a relative phase is generated by the Q-H-Q, but it is offset by the following piezoelectric ceramic (PZT) operation. With the help of PZT (PZT-1 shown in Fig. 1) operation, any relative phase $e^{i\delta}$ between the R and L paths can be realized. So, the superposition state is expressed as $(\cos 2\theta \cos \varphi, \cos 2\theta e^{i\alpha} \sin \varphi, \sin 2\theta \cos \psi, e^{i\delta} \sin 2\theta e^{i\beta} \sin \psi)^T$. This is a general expression for pure two-qubit states because the probability amplitudes in the R and L paths can be arbitrarily changed by adjusting $\theta$ and $\varphi$, and the polarizations can be arbitrarily adjusted in two paths. The arbitrary two-qubit states can be described by six independent parameters, which satisfy the above expression. For example, the separable state $|L\rangle_S |V\rangle_F$ can be produced when we set $\theta = 45^\circ$, $\varphi = 0^\circ$, $\psi = 90^\circ$, and $\alpha = \beta = \delta = 0^\circ$. The entangled state $\frac{1}{\sqrt{2}}(|L\rangle_S |H\rangle_F + |L\rangle_S |V\rangle_F)$ can be realized when $\theta = 22.5^\circ$, $\varphi = 0^\circ$, $\psi = 90^\circ$, and $\alpha = \beta = \delta = 0^\circ$. This means that arbitrary pure two-qubit input states can be produced using our scheme.

The universal operation of the unitary gate for the above single-photon two-qubit states is realized as shown in the gate box in Fig. 1. The detailed theory has been given in Ref. 11. Briefly, first, we can discretely create a theoretical unitary operation $U$. Secondly, based on the chosen $U$, we can obtain four matrices to describe the transformations of the Q-H-Q. Finally, according to the four matrices, the parameters of QWPs and HWPs are obtained. In the experiment, we construct the setup in the gate box and modify the wave plates to realize the UUGO. For example, we create a unitary gate operation $U$ in theory by the random number generator of the MATLAB toolbox, which is expressed as

$$U = \begin{pmatrix}
0.2756 & -0.4269i & 0.09792 & -0.1938i \\
-0.5253 & -0.4895i & -0.1734 & 0.5173i \\
-0.1133 & 0.0531i & -0.2755 & -0.0525i \\
-0.2063 & 0.4099i & 0.6532 & -0.3872i \\
\end{pmatrix}.$$  \hspace{1cm} (2)
According to the above $U$ and theory, we obtain four unitary polarization matrices,

$$
V_1 = \begin{pmatrix}
0.9412 - 0.1485i & 0.2173 - 0.212i \\
0.2173 + 0.212i & -0.9412 - 0.1485i
\end{pmatrix},
$$

$$
V_2 = \begin{pmatrix}
-0.1496 + 0.562i & 0.7036 + 0.4038i \\
0.7036 - 0.4038i & 0.1496 + 0.562i
\end{pmatrix},
$$

$$
V_R = \begin{pmatrix}
-0.5204 - 0.5542i & 0.1986 - 0.6185i \\
-0.5926 - 0.2663i & 0.0636 + 0.7576i
\end{pmatrix},
$$

$$
V_L = \begin{pmatrix}
0.4471 - 0.8806i & 0.1104 - 0.1111i \\
0.1317 - 0.0849i & -0.7684 - 0.6205i
\end{pmatrix}.
$$

The transformations of Q-H-Qs in the gate box can be described by the above four matrices except for that in the sign L path. From these matrices, detailed parameters of Q-H-Qs for HWPs and QWPs can be obtained. However, although the theoretical design is provided in Ref. 11, some problems need to be solved when the corresponding experiment is performed.

In the experiment, the dielectric film is often used as a beamsplitter (BS) and a relative phase shift between H and V polarizations is generated when the photon is reflected. In this work, the relative phase shift is not small, which would cause lots of errors. The diagram of the phase shift is shown in Fig. 2(a). The displayed matrices are related to the space of polarization. A positive and a negative phase shift $y = \pm \pi/9$ are produced for the V polarization when the photon is incident from different paths, no phase shift when the photon is transmitted. In order to correct the phase shift, we add four special phase plates as shown in Fig. 2(b). The special phase plates provide a phase shift only for the V polarization and the function can be written as

$$
\begin{pmatrix}
1 & 0 \\
0 & e^{iy/2}
\end{pmatrix}.
$$

The green (yellow) plate is $e^{iy/2} (e^{-iy/2})$ phase retarder.

The Q-H-Q in the sign L path is used to realize the function of one green plate. All the phase plates mentioned above are not used in the experiment, because the functions of these plates are merged into $V_1$, $V_2$, $V_R$, and $V_L$ by the matrix multiplication. In fact, we obtain the detailed parameters of HWPs, QWPs, and PZTs by using new matrices of $V_1$, $V_2$, $V_R$, and $V_L$. So, our UUGO setup can be performed very well.

In order to prove the flexibility of our setup with respect to two-qubit quantum computations, the measurements related to the quantum process and state tomography have to be done. In this work, the measurement bases for the spatial DOF are taken as $|R\rangle_S$, $|L\rangle_S$, $|+\rangle_S = 1/\sqrt{2}(|R\rangle_S + |L\rangle_S)$ and $|\rangle_S = 1/\sqrt{2}(i|L\rangle_S + |R\rangle_S)$ for the polarization DOF, the measurement bases are chosen as $|H\rangle_{po}$, $|V\rangle_{po}$, $|+\rangle_{po} = 1/\sqrt{2}(|H\rangle_{po} + |V\rangle_{po})$, and $|\rangle_{po} = 1/\sqrt{2}(i|V\rangle_{po} + |H\rangle_{po})$. So, a set of complete information measurements is confirmed, which consists of 16 different measurement states formed by the tensor products of the above bases. The eight measurements related to $|R\rangle_S$ and $|L\rangle_S$ can be realized directly by the measurement setup shown in the measurement box in Fig. 1. However, another cascaded interferometer is necessary for the eight measurements related to $|+\rangle_S$ and $|\rangle_S$, which will impose more burden on the experimental setup considering that two cascaded interferometers have already been employed. Here, we use a transformation method to realize the measurements related to the bases $|+\rangle_S$ and $|\rangle_S$. For the measurements related to the basis state $|+\rangle_S$, we use the transformation $U_1=\text{Hadamard} \otimes I_{po}$. For the measurements related to the basis state $|\rangle_S$, we use the transformation $U_2=\text{Hadamard} \otimes I_{po}U$, where $U_{\text{Hadamard}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & i \\
1 & -i
\end{pmatrix}$. Thus, the results of $|\rangle_S$ for $U_2$ are obtained by measuring the results of $|R\rangle_S$ for $U_1$. This is because that the operation $H_{po}$ can transform $|+\rangle_S$ into $|R\rangle_S$. Similarly, for the measurements related to the basis state $|\rangle_S$, we use the transformation $U_3=\text{Hadamard} \otimes I_{po}U$, where $U_{\text{Hadamard}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & i \\
1 & -i
\end{pmatrix}$. Thus, the results of $|\rangle_S$ for $U_3$ are obtained by measuring the results of $|R\rangle_S$ for $U_2$. The operation $U_{\text{Hadamard}}$ connects $|\rangle_S$ and $|R\rangle_S$. In the following, we provide the experimental results on quantum process tomography and quantum state tomography.

Quantum process tomography is a method that determines the completely positive map $\epsilon$ generated by the chosen gate operation. The map acted on the arbitrary input state $\rho$ can be represented as $\epsilon(\rho) = \sum_{mn} \chi_{mn} E_m \rho E_n^\dagger$, where the operators $\{E_m\}$ form a basis for the set of operators acting on $\rho$ and the elements of the quantum process matrix $\chi_{mn}$ can be used to describe the map $\epsilon$ completely and uniquely once $\{E_m\}$ have been fixed. In our experiment, the input states are taken to be the same as the above measurement states and $\{E_m\}$ are constructed by the tensor product of identity matrix $I$ and Pauli matrices $X$, $Y$, and $Z$. Figures 3(a) and 3(b) show the real part and imaginary part of experimental process matrix $\chi_{XX}$, in which the maximum-likelihood estimation technique is used to guarantee physical results. For comparison, the corresponding theoretical results are shown in Figs. 3(c) and 3(d). We find that the experimental results agree with the theoretical results very well. Based on these data, we calculate the process fidelity ($F_F$) as $F_F = \text{Tr}(\chi_{XX}^\dagger \chi_{XX}) = 90.49\% \pm 0.17\%$, which shows a well performance of the constructed UUGO.

Now, we perform quantum state tomography to reconstruct the density matrices of output states $\rho_{out} = \rho_{out}(\psi)$ and test the function of two-qubit quantum computation. The output density matrices $\rho_{out}$ are generated when the input states are processed by the gate operations. In the experiment, the input states are taken as the 16 separable states used in the above quantum process tomography and four entangled states (Bell states). Here, Bell
states are chosen just because of their notability; in fact, arbitrary entangled states can be chosen in the setup. As an example, in Figs. 4(a) and 4(b), we present the experimental and theoretical results of output states for one separable input state $|l⟩_S|V⟩_P$. The corresponding results for one entangled input state $\frac{1}{\sqrt{2}}(|R⟩_S|H⟩_P + |L⟩_S|V⟩_P)$ are shown in Figs. 4(c) and 4(d). The bar and color heights show the absolute values of output density matrices. The solids correspond to the experimental data, and the semi-transparent color corresponds to the theoretical results.

The good consistency between experimental results and theoretical results is observed again. In order to quantitatively characterize the results, we calculated the fidelity of the output state by $f = \left| \text{Tr} \left( \sqrt{\rho_{th} \rho_{ex} \sqrt{\rho_{th}}} \right) \right|^2$; here, $\rho_{th}$ and $\rho_{ex}$ represent theoretical and experimental output density matrices, respectively. Our calculated results show that the fidelity for these 20 input states is $f = 93.83\% \pm 3.95\%$. This means that the computations are performed with high quality. In the experiment, errors mainly come from the stability of cascaded interferometers, imbalanced transmissivity,
and reflectance for the 50:50 BS, and slight precision error is about 3° caused by PZT for the controlling phase. Similar to Ref. 18, we can use the fidelities of the output states and UUGO to prove the computational independence that the performance of two-qubit UUGO is independent of input states. According to the relation $f = \frac{\text{det}(d)}{d}$, we can calculate the mean-state fidelity $f = 92.39\% \pm 0.14\%$, which is identical to our experimental results, and $d$ is the dimension of the system (here, $d = 4$).

The above discussion is only focused on the flexible scheme with pure two-qubit input states and UUGO based on the polarization and spatial DOFs of a single photon. In fact, the case for the arbitrary non-pure state can also be realized by proper design in our scheme. Inspired by the design in Ref. 25, the way of adding ancillary paths and properly destroying coherence among these paths can be used to generate a mixed state. Using HWP and PBS, we can control the intensities of every path. Taking advantage of the combination of Q-H-Qs, parameters of mixed states can be adjusted. Thus, arbitrary mixed states can be successfully prepared and universal two-qubit quantum computation can also be realized by combining two-qubit UUGO shown in the above discussion.

In principle, our scheme can also be extended to multi-qubit cases of quantum computation using multiple DOFs of a single photon, for example, OAM.26–29 Recent experiments have shown that the OAM with more than 100 modes for the single photon can be well modulated.26 In addition, some novel multi-qubit gates have been realized using high-dimensional modes of OAM.28 Meanwhile, this work benefits for the realization of the UUGO based on polarization and OAM28 because they both are established on the same theory.

III. CONCLUSIONS

In conclusion, we have demonstrated a flexible scheme for two-qubit quantum computations with a single photon based on the polarization and spatial DOFs. In our scheme, UUGO both in free-space optics and for APISs consisting of separable and entangled states have been realized. Experimental results have shown good fidelities of the computational output states and the UUGO process compared with ideal cases. Our work contains more computational situations compared with previous studies and will contribute to the bulk-optics quantum technologies from the experimental aspect, like the realization of two-qubit gates based on polarization and OAM. The experimental setup can also be used as a general platform to perform more fundamental studies including diagnosis of unknown two-qubit states, realization of the optimization algorithm, and so on.

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REFERENCES

1. R. P. Feynman, Int. J. Theor. Phys. 21, 467 (1982).
2. D. Deutsch, Proc. R. Soc. London, Ser. A 400, 97 (1985).
3. M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, 2000).
4. T. D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J. L. O’Brien, Nature 464, 45 (2010).
5. B. P. Lanyon, T. J. Weinhold, N. K. Langford, M. Barbieri, D. F. V. James, A. Gilchrist, and A. G. White, Phys. Rev. Lett. 99, 250505 (2007).
6. A. Jones, M. Mosca, and R. H. Hansen, Nature 393, 344 (1998).
7. X.-x. Ma, B. Dakic, W. Naylor, A. Zeilinger, and P. Walther, Nat. Phys. 7, 399 (2011).
8. A. Barenco, C. H. Bennett, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. A. Smolin, and H. Weinfurter, Phys. Rev. A 52, 3455 (1995).
9. C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. 75, 4714 (1995).
10. M. A. Nielsen and I. L. Chuang, Phys. Rev. Lett. 79, 321 (1997).
11. B.-G. Englert, C. Kurtsiefer, and H. Weinfurter, Phys. Rev. A 63, 032303 (2001).
12. J. L. O’Brien, G. J. Pryde, A. G. White, T. C. Ralph, and D. Branning, Nature 426, 264 (2003).
13. A. M. Childs, Phys. Rev. Lett. 102, 180501 (2009).
14. L. M. K. Vandersypen, M. Steffen, G. Breyta, C. S. Yannoni, M. H. Sherwood, and I. L. Chuang, Nature 414, 883 (2001).
15. J. Carolan, C. Harrold, C. Sparrow, E. Martin-Lopez, N. J. Russell, J. W. Silverstone, P. J. Shadbolt, N. Matsuda, M. Oguma, M. Itoh, G. D. Marshall, M. G. Thompson, J. C. F. Matthews, T. Hashimoto, J. L. O’Brien, and A. Laing, Science 349, 711 (2015).
16. S. Debnath, N. M. Linke, C. Figgatt, K. A. Landsman, K. Wright, and C. Monroe, Nature 536, 63 (2016).
17. C. Song, X. Xu, W.-X. Liu, C.-P. Yang, S.-B. Zheng, H. Deng, Q.-W. Xie, K.-Q. Huang, Q.-J. Guo, L.-B. Zhang, P.-F. Zhang, D. Xu, D.-N. Zheng, X.-B. Zhu, H. Wang, Y.-A. Chen, C.-Y. Lu, S.-Y. Han, and J.-W. Pan, Phys. Rev. Lett. 119, 180511 (2017).
18. D. Hanneke, J. P. Home, J. D. Jost, J. M. Amini, D. Leibfried, and D. J. Wineland, Nat. Phys. 6, 13 (2010).
19. X. Qiang, X. Zhou, J. Wang, C. M. Wilkes, T. Loke, S. O’Gara, L. Kling, G. D. Marshall, R. Santagati, T. C. Ralph, J. B. Wang, J. L. O’Brien, M. G. Thompson, and J. C. F. Matthews, Nat. Photonics 12, 534 (2018).
20. S. Slussarenko, E. Karimi, B. Piccirillo, L. Marrucci, and E. Santamato, Phys. Rev. A 80, 022326 (2009).
21. V. Bagini, R. Borghi, F. Gori, M. Santarsiero, F. Frezza, G. Schettini, and G. S. Spagnolo, Eur. J. Phys. 17, 279 (1996).
22. Y. S. Teo, B. Englert, J. Reháček, and Z. Hradil, Phys. Rev. A 84, 062125 (2011).
23. A. Gilchrist, N. K. Langford, and M. A. Nielsen, Phys. Rev. A 71, 062310 (2005).
24. M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. A 60, 1888 (1999).
25. Y. Xiao, X.-J. Ye, K. Sun, J.-S. Xu, C.-F. Li, and G.-C. Guo, Phys. Rev. Lett. 118, 140404 (2017).
26. L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, Phys. Rev. A 45, 8192 (1992).
27. O. A. M. Yao and M. J. Padgett, Adv. Opt. Photonics 3, 161 (2011).
28. Y.-B. Song, S.-Y. Fu, X. Zhang, Z.-W. Yang, Q. Zeng, C. Gao, and X. D. Zhang, Sci. Rep. 7, 3601 (2017).
29. V. D’Ambrosio, N. Spagnolo, L. Del Re, S. Slussarenko, Y. Li, L. C. Kwek, L. Marrucci, S. P. Walborn, L. Aolita, and F. Sciarrino, Nat. Commun. 4, 2432 (2013).
30. Q. Zeng, T. Li, X. Song, and X. Zhang, Opt. Express 24, 8186 (2016).