Scaling properties of direct photons from gluon saturation in heavy ion collisions

Vladimir Khachatryan
Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794-3800, USA,
Michał Praszalowicz
M. Smoluchowski Institute of Physics, Jagiellonian University, S. Łojasiewicza 11, 30-348 Kraków, Poland.

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A recent analysis from the PHENIX collaboration on direct photon production has shown a universal, within experimental uncertainties, multiplicity scaling, in which photon $p_T$-spectra for transverse momenta up to 2 GeV/c are scaled with charged hadron pseudorapidity density at midrapidity raised to power $\alpha = 1.25$. On the other hand particle production in hadron and nucleus collisions, photons included, exhibits geometrical scaling in the similar $p_T$ range. The geometrical scaling follows from gluon saturation and collision geometry. We show that these two scaling laws are interconnected and discuss physical conditions needed to relate one to another.

INTRODUCTION

Measurements of direct photons provide unique opportunities in probing and studying the properties and evolution of the matter produced in heavy ion collisions (HIC). These photons are defined to be produced from all the sources except for hadronic decays. Since they hardly interact with the “fireball” of quarks and gluons due to a small interaction cross section with the medium, the information they carry from the time of their production is not washed out by final state interactions. Experimentally measured low momentum direct photon $p_T$-spectra by PHENIX [1-7] and ALICE [8] collaborations in HIC are enhanced with respect to $N_{\text{coll}}$ (number of binary nucleon collisions) scaled (measured or calculated) reference yield in p+p collisions. Low momentum direct photon measurements by STAR collaboration show less enhancement above $N_{\text{coll}}$ scaled p+p yield [9]. Direct photons in HIC also show large anisotropy (elliptic flow) [7, 10, 11]. The attempts to explain the large yield and large anisotropy have given rise to a challenging problem, dubbed as direct (thermal) photon flow puzzle [12]. Fig. 1 shows several data sets of direct photon $p_T$-spectra at low- and intermediate-$p_T$ regions.

Two types of scalings have been observed in direct photon $p_T$ distributions in HIC and hadron collisions:

1. Direct photon multiplicity scaling (MS): an experimental observation in HIC, which shows that direct photon invariant yields follow one universal curve within experimental statistical and systematic uncertainties at low-$p_T$ for various colliding species and system sizes at different center-of-mass energies, when scaled by charged hadron multiplicities at midrapidity raised to power $\alpha = 1.25$ [3]:

$$\frac{1}{(dN_{\text{ch}}/d\eta)_{|\eta|>0}^0} \frac{dN_{\gamma}}{d^2p_T dy} = \frac{1}{Q_0^2} G(p_T), \quad (1)$$

where $G$ is a universal energy- and multiplicity-independent function of $p_T$ (see Fig. 2) and $Q_0 \sim 1$ GeV/c. The high-$p_T$ direct photon yields [2] [8] seem to scale only at discrete energies, being independent of the colliding species and system sizes.

2. Geometrical scaling (GS): observed for the first time in deep inelastic scattering (DIS) in e+p collisions [13], where the invariant yields of direct pho-

1 We will henceforth simply use $dN_{\text{ch}}/d\eta$, skipping the midrapidity notation $\eta \approx 0$ in the text, and keeping it only in the figures.
tons (and also of charged hadrons) can be related to a universal energy-independent function of scaling variable $\tau$ [14, 18]:
\[
\frac{1}{S_T} \frac{dN_{\gamma,\text{ch}}} {d^2p_T dy} = F_{\gamma,\text{ch}}(\tau),
\]
where $S_T$ is a parameter characterising the geometrical overlap area of colliding nuclei. The variable $\tau$ is given by
\[
\tau = p_T / Q_s(x).
\]
Here $Q_s(x)$ is the saturation scale, i.e., the transverse momentum at which number of gluons (partons) with longitudinal momentum fraction $x$, called as Bjorken variable, cannot grow any more due to the nonlinearity of QCD evolution. This happens at gluon occupancy numbers being of the order of $1 / \alpha_s$ ($\alpha_s$ is the strong coupling constant) The phenomenon of gluon saturation is generated from such dynamics [19–23] (however, see [24, 25]).

The geometrical scaling follows from the Color Glass Condensate (CGC) effective theory of high energy scattering, which explains experimental observables based on the gluon saturation [26–28].

For the case of photons see Fig. 3.

In this letter we will show how these two scaling laws are interconnected.

**BASICS OF GEOMETRICAL SCALING**

We start from the reminder of the origin of geometrical scaling, which concerns gluon production. It is assumed that bulk properties of charged particles and photons share scaling properties encoded in the initial state wave function of colliding hadrons/nuclei. Detailed models of direct photon production in the initial stages of evolution of the matter produced in HIC can be found in Refs. [24–33].

Differential gluon production cross-section in hadron-hadron scattering can be written as [34]:
\[
\frac{d\sigma}{dyd^2p_T} = \frac{C}{p_T^2} \int d^2\vec{k}_T \alpha_s(k_T^2) \varphi^{(1)}(x_1, \vec{k}_T^2) \varphi^{(2)}(x_2, (\vec{k} - \vec{p}_T)^2),
\]
where $C$ contains color factors and numerical constants. The Bjorken $x$’s of colliding partons read
\[
x_{1,2} = \frac{p_T}{\sqrt{s}} e^{\pm y}.
\]

We are interested in the midrapidity region, therefore $y \approx 0$.

There exist many models of unintegrated gluon distributions $\varphi(x, \vec{k}_T^2)$ (see e.g. [35–37]) that enter [14]. Most of them share two important features: geometrical scaling and dependence on the transverse area parameter $A_T$:
\[
\varphi(x, \vec{k}_T^2) = A_T \phi(k_T^2 / Q_s^2(x)),
\]
where $\phi$ is a dimensionless function of the scaling variable $r^2 = k_T^2 / Q_s^2(x)$. The precise meaning of $A_T$ is best understood in a picture where also the impact parameter is taken into account [38, 39].

Ignoring momentum dependence of the strong coupling constant, on dimensional analysis grounds, we arrive at
\[
\frac{d\sigma}{dyd^2p_T} = A_T^{(1)} A_T^{(2)} F(\tau),
\]
where $F$ is a universal, energy-independent function of the scaling variable $\tau$ in (3).

If one assumes that $A_T$ is an energy-independent constant, which is true in the case of the GBW model for DIS, then it is the differential cross-section that should exhibit GS. Indeed, it has been shown in Ref. [40] for charged particles the differential cross-section in p+p scattering scales better (over larger $p_T$ interval) than multiplicity, and the exponent $\lambda \approx 0.3$ is compatible with the DIS scaling [41]. In contrast, for multiplicity scaling $\lambda \approx 0.2$.

For multiplicity distribution we have to divide (7) by an appropriate inelastic cross-section

$$\frac{dN}{dyd^2p_T} = \frac{A_T^{(1)} A_T^{(2)}}{\sigma_{\text{inel}}} F(\tau). \quad (8)$$

This distribution would scale if

$$S_T = \frac{A_T^{(1)} A_T^{(2)}}{\sigma_{\text{inel}}} \quad (9)$$

were energy-independent. This assumption leads to [2]. It is the energy dependence of $S_T$ that leads to different scaling properties of the cross-section and multiplicity in p+p collisions mentioned above [41].

In the case of heavy ion collisions for a fixed centrality, $S_T$ has geometrical interpretation as an overlap area of two colliding nuclei [36, 42], and depending on the system size it scales like $N^{2/3}_{\text{part}}$ for large systems or as $O(1)$ in small systems. Throughout this paper we shall assume $Q_s \sim S_T \sim N^{\delta}_{\text{part}}$ for A+A and $Q_s \sim l \sim N^{3\delta/2}_{\text{part}}$ for p+A or d+A or He$^3$+A collisions (where $l$ denotes thickness of A nucleus), and – as will be explained later – we will also need weak energy dependence on $S_T$ that leads to the violation of geometrical scaling.

The functional form of the saturation scale $Q_s(x)$ follows from the nonlinear QCD evolution [23] and, in the case of heavy ions, from the geometry of the collision [36, 42], resulting in the following form of the scaling variable $\tau$:

$$\tau = \frac{1}{N^{3/4}_{\text{part}}} \left( \frac{p_T}{Q_0} \right)^{\lambda/2} \quad \text{or} \quad \tau = \frac{1}{N^{3\delta/8}_{\text{part}}} \left( \frac{p_T}{Q_0} \right)^{\lambda/2} \quad (10)$$

for large and small system collisions, respectively. $W = \sqrt{3N_N} \times 10^{-3}$ is the center-of-mass energy. Should there be any energy dependence of $S_T$ (which enters the definition of $Q_s$), it is already included in $\lambda$, as we fix $\lambda$ from the fits to data that do not distinguish different sources (QCD nonlinear evolution, $S_T$ energy dependence) of the energy increase of $Q_s(W)$. These fits indicate [18] that for direct photons $\lambda \approx 0.2$, rather than 0.3. **RELATING SCALING LAWS**

Let us first calculate the charged particle multiplicity for large systems from the scaling formula [2]. To this end we will need a Jacobian to change integration variables from $p_T$ to $\tau$:

$$p_T dp_T = \frac{2Q_0^{2\lambda/\delta}}{2 + \lambda} N_{\text{part}} \left( \frac{W}{Q_0} \right)^{2\lambda/\delta} \tau^{2\lambda/\delta} d\tau. \quad (11)$$

which yields

$$\frac{dN_{\text{ch}}}{dy} = \int d^2p_T \frac{dN_{\text{ch}}}{dp_T} dy$$

$$= S_T Q_0^{\delta/2} \left( \frac{W}{Q_0} \right)^{2\lambda/\delta} \int F_{\text{ch}}(\tau) \tau^{2\lambda/\delta} d\tau$$

$$= N_{\text{part}}^{3\delta/8} \left( \frac{W}{Q_0} \right)^{2\lambda/\delta} \kappa, \quad (12)$$

where constant $\kappa$ includes the integral of $F_{\text{ch}}$ and other irrelevant constants. Note that energy dependence $W^{2\lambda/(2+\delta)}$ is compatible with multiplicity growth in p+p as measured at the LHC [43], and $N^{3\delta/8}_{\text{part}} \approx N_{\text{part}}$ for small $\lambda$ and $\delta = 2/3$, as expected [42].

Let us rewrite the scaling laws (2) and (1) in the following way

$$S_T F_\gamma(\tau(p_T)) = \frac{dN_{\text{ch}}}{d^2p_T dy} = \left( \frac{dN_{\text{ch}}}{dy} \right) \frac{1}{Q_0^\lambda} G(p_T). \quad (13)$$

It follows from Eq. (13) that both left and right hand sides should have the same $p_T$, $W$ and $N_{\text{part}}$ dependence.

It is clear that one cannot proceed further without explicit knowledge of functions $F_\gamma$ and $G$. To this end we shall assume a power-law dependence

$$F_\gamma(\tau) \sim \frac{1}{\tau^n} \quad \text{and} \quad G(p_T) \sim \left( \frac{Q_0}{p_T} \right)^m. \quad (14)$$

Indeed, as shown in Figs. 2 and 3, where we plot $F_\gamma$ and $G$ for Au+Au [13], Cu+Cu [4] and Pb+Pb [8] direct photon data at different energies and centralities for $\lambda = 0.2$ and $\delta = 2/3$, the power-law fall-off [16] with $n, m \sim 4, 4.5$ works pretty well at small and intermediate $\tau$. Note that $m = 4$ is a generic prediction for radiation from the CGC [44] at intermediate $p_T$, while – as we have checked by looking at prompt photons up to transverse momenta $\sim 2 \text{ GeV}/c$ – fits to the data prefer $m = 5$.

Hence

$$\frac{dN_{\text{ch}}}{d^2p_T dy} \sim N^{\delta/4}_{\text{part}} \left( \frac{N^{\delta/4}_{\text{part}}}{Q_0} \left( \frac{W}{Q_0} \right)^{2\lambda/\delta} \right)^n$$

$$\sim \left( N^{\delta/4}_{\text{part}} \right)^{3\lambda/2} \left( \frac{W}{Q_0} \right)^{2\lambda/\delta} \left( \frac{Q_0}{p_T} \right)^m, \quad (15)$$

where we have used $S_T \sim N^{\delta/4}_{\text{part}}$. Functions in (15) are proportional if

$$m = \frac{2 + \lambda}{2} n, \quad \frac{4 + n}{4} = \frac{3 + \lambda}{2} \delta, \quad \frac{\lambda}{2} n = \frac{2}{2 + \lambda} \alpha. \quad (16)$$
From the first equation we see that the power-like fall-off should be faster for MS than for GS, and this prediction works quite well, as can be seen from Figs. 2 and 3.

We get from \( \frac{4 + n}{4} \frac{2 + \lambda}{3 + \lambda} \bigg|_{n=4, \lambda=0.2} = 1.375 \),
\[
\alpha = \frac{2 + \lambda}{4} \bigg|_{n=4, \lambda=0.2} = 2.2. \tag{17}
\]

The first estimate gives power \( \alpha \) compatible with the experimental value of 1.25 (given our crude assumptions and approximations), while the second one fails. Note that the first estimate follows from the assumptions about geometry of A+A collisions (although it does not depend on the actual value of \( \delta \)), whereas the second one follows from the energy dependence.

The reason why the second estimate in Eq. (17) overshoots both the experimental value and the first estimate can be related to the violation of GS. First, it has been observed in Ref. [15] that the energy scaling of \( N_A \), in HIC is of much worse quality than in p+p, and the same tendency is observed for direct photons [18]. Secondly, even in p+p collisions, as mentioned already in the previous section, the cross-section scales better than the multiplicity [40]. This is related to the implicit assumption in [4] that \( S_T \) is energy-independent. We do not have any experimental handle on this dependence, however recent Glauber model calculations [46] show slight increase of \( S_T \) with energy, Fig. 3, which is slower than the rise of \( NN \) inelastic cross-section and can be effectively parametrised as a power-law:

\[
S_T = \left( \frac{W}{Q_0} \right)^{\lambda'} \frac{N_{\text{part}}^5}{Q_0^5} \tag{18}
\]

with \( \lambda' \approx 0.1 \). This energy dependence introduces small violation of GS and leads to the modification of Eq. (15). Its theoretical origin can be most probably related to the weak energy dependence of the strong coupling constant that we have systematically neglected throughout this paper, and deserves an independent study.

We get therefore
\[
\frac{dN}{d^2p_Tdy} \sim \left( \frac{W}{Q_0} \right)^{\lambda'} \left( \frac{N_{\text{part}}^5}{Q_0^5} \right) \left( \frac{W}{Q_0} \right)^{\frac{2\lambda}{2+\lambda}} \left( \frac{Q_0}{p_T} \right)^{2\lambda'} \left( \frac{Q_0}{p_T} \right)^{4n} \sim \left( \frac{N_{\text{part}}^5}{Q_0^5} \right)^{\lambda'} \left( \frac{W}{Q_0} \right)^{2\lambda'} \left( \frac{Q_0}{p_T} \right)^{4n}. \tag{19}
\]

Comparing powers of \( W \) we now get
\[
\frac{\lambda n}{2} + \lambda' = \left( \frac{2\lambda}{2+\lambda} + \lambda' \right) \alpha, \tag{20}
\]

that results in a new condition for \( \alpha \)
\[
\alpha = \frac{\lambda n + 2\lambda'}{2} \left( \frac{2\lambda}{2+\lambda} + \lambda' \right) \bigg|_{n=4, \lambda=0.2} = 1.77. \tag{21}
\]

We see that the small violation of GS introduced above reduces second estimate of (17) by 24%.

**DISCUSSION AND CONCLUSIONS**

Although the violation of geometrical scaling changes the second estimate of Eq. (17) in the right direction, Eq. (21), the discrepancy between both estimates is 24%. One could fine tune this result by decreasing the power \( n \). For example, for \( n = 3.3 \) one obtains \( \alpha = 1.26 \) and 1.53 from (21), reducing the discrepancy to 20%. For higher \( n = 4.5 \) that fits the data over larger \( \tau \) range, we get higher values of \( \alpha = 1.46 \) and 1.95. Nevertheless we find this accuracy quite satisfactory, given a simplicity of the present analysis.

There are many sources that lead to the violation of GS in HIC that we have ignored. Here we list the most obvious ones: factorisation of unintegrated gluon densities in (6) into \( A_T \) and \( \phi \), skipping momentum dependence of the strong coupling constant in (4), simplifying assumptions about \( N_{\text{part}} \) dependence and energy increase of \( S_T \). It is obvious from Fig. 4 that these dependences do not factorise, moreover much better scaling is obtained with power \( \delta = 3/4 \). The latter, however, is of no importance for our analysis, as \( \delta \) drops out from the second equation (16). Finally, simple power-law dependence of the photon spectra is also a simplification that affects accuracy.
of our analysis. A choice of more accurate data-driven functions could give an improved value of $\alpha$ much closer to that extracted from experiment. This would require, however, numerical analysis that we want to avoid for the clarity of argumentation. One should not forget that both scaling laws observed experimentally are only approximate. This concerns also charged particle spectra used here do calculate $dN_{ch}/d\eta$ in Eq. [12].

With all these reservations in mind, one may try to extend our analysis to p+p and p+A/d+A/He+A collisions. Since for p+p collisions we have practically no overlap factor in [2] and no $N_{part}$ dependence of the scaling variable $\tau$, the estimate of $\alpha$ is confined to the second equation [17] with no GS violation encoded in $\lambda'$. For small system collisions $Q_s$ scales like $N_{part}$ (or $N_{part}^{3/2}$) and $S_T=N_{part}$ independent. This results in skipping factors $N_{part}^3$ in Eq. [15] and then replacing $\delta \to \delta/2$ in the remainder. The latter has no effect on the consistency condition, where $\delta$ dependence cancels out. The resulting equation for $\alpha$ is identical to the second Eq. [17]. Therefore for small systems we expect faster increase of the direct proton yields with $dN_{ch}/d\eta$ than in the case of A+A systems.

To conclude: we have linked multiplicity scaling and geometrical scaling of direct photon $p_T$-spectra, estimating the scaling power $\alpha$ from simple assumptions about functional forms of the scaling functions $G$ and $F_\gamma$ and on the energy dependence and $N_{part}$ dependence of the saturation scale. In order to obtain consistent estimates for the power $\alpha$, violation of GS had to be included. We have predicted larger $\alpha$ for small systems.

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