Numerical study of diamagnetic regime in open magnetic trap

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Abstract. The article presents a two-dimensional axially symmetric numerical model of a diamagnetic plasma confinement regime in a linear magnetic system as applied to laboratory experiments at the BINP SB RAS, Novosibirsk. The model is based on the kinetic approximation for the ion components of the background plasma and the injected beam. The MHD approximation is used for the electron component, taking into account the mechanisms of energy dissipation due to conductivity and thermal conductivity. The particle-in-cell method is used to solve the Vlasov equation. The dependences of the magnetic field profile and plasma density on the current of the ion beam and its energy are found.

1. Introduction
Despite the long history of theoretical and experimental studies of plasma confinement and heating, there remain a number of issues that need to be addressed to create a prototype of fusion reactor. The main direction in solving the problem of controlled thermonuclear fusion is associated with toroidal traps tokamaks. The well-known significant disadvantages of tokamaks (technical complexity, running electrons, accumulation of high-charge impurity) lead to search the new methods of plasma heating and confinement. One of these methods is based on a recently proposed new method of plasma confinement in the diamagnetic regime of an open magnetic trap [1]. The simple linear structure of the magnetic field and the possibility of creating a compact fusion reactor with $\beta \sim 1$ are among of attractive features of the diamagnetic trap and stimulate experimental and theoretical studies of the regime of diamagnetic plasma confinement. The first experiments to study the diamagnetic regime of a mirror trap with the powerful neutral particles beam injection were carried out at a new CAT facility in the Budker Institute of Nuclear Physics in Novosibirsk [2]. At this stage of research, a significant role is played by numerical modeling of the main processes of a magnetic field cavity (diamagnetic “bubble”) formation, depending on the plasma parameters, the configuration of the magnetic field and the characteristics of the injected beam. The one-dimensional as well as two-dimensional models of equilibrium processes in the diamagnetic trap based on MHD approximation are presented in [1, 3]. These models predict the confinement time of particles and distributions of plasma density in the ”bubble” regime of an open magnetic trap. However, assumption of small Larmor radius of ions in the MHD models is not applicable to the problem, because of several new
effects appearing when kinetic effects are taken into account. One of the effects is collisionless
particle losses due to non-adiabaticity of motion. These collisionless losses are proportional to
the Larmor radius of ions in mirrors and disappear in the MHD limit (when Larmor radius is
zero) [4].

In this paper, the processes of the diamagnetic regime formation and the plasma confinement
were studied on the base of computer simulation. The anomalous coefficients associated with
the electric conductivity, the electron thermal conductivity, and the size of ion Larmor radius
were taken into account. The new hybrid 2D code is based on the kinetic description for the
plasma ion component and the MHD approximation for the electron component [5, 6]. The ion
motion is calculated by the particle-in-cell (PIC) method; explicit finite-difference schemes are
used to calculate the electromagnetic field and particle dynamics.

2. Statement of the problem
Let us consider the following problem of plasma dynamic in an open magnetic trap. At initial
time \( t = 0 \), there is a uniform hydrogen plasma of density \( n_0 = \text{const} \) that occupies a cylindrical
chamber of radius \( R_0 \) and length \( L \). The axisymmetric magnetic field of the system under
consideration \( \mathbf{B} = (B_r, 0, B_z) \) is created by the current of the coils located at the edges \( z = 0 \)
and \( z = L \). The configuration of the magnetic field corresponds to the magnetic field structure
of open traps with a mirror ratio \( \rho = 2 \) (Fig. 1). The current and the energy of the ion
beam entering the center of the chamber depends on the number of injected particles, and the
beam velocity \( \mathbf{V} = (V_r, 0, V_z) \). The injected beam has Maxwellian velocity distribution with
the temperature \( T_b \). The electrons and ions of the background plasma are assumed to be cold:
\( T_e = 0, T_i = 0 \). Due to axial symmetry, we consider the problem in a two-dimensional cylindrical
geometry.

Despite the fact that full kinetic simulation via PIC method is the best tool for studying
phenomena in an open magnetic trap, its application is limited by severe requirements for
memory and performance of computing systems. To solve this problem, a hybrid numerical
model applying the PIC method only for the ion plasma components is used. The possibility of
using this approximation is based on the need to study low-frequency processes, the characteristic
dimensions of which are much larger than the Debye radius and the speed is much less than
the speed of light. Nowadays, various modifications of hybrid models are widely used in solving
astrophysics problems [6, 7].

Figure 1. The map of magnetic field lines at \( t = 0 \) \((\rho = 2)\).
The hybrid model system of equations is comprised of the Vlasov equation for the ion component of the background plasma and the injected ion beam, magnetohydrodynamics equations for the electrons, and the Maxwell equations:

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \frac{\partial f_i}{\partial \mathbf{r}} + \frac{\mathbf{F}_i}{m_i} \frac{\partial f_i}{\partial \mathbf{v}} = 0$$  \hspace{1cm} (1)

$$\mathbf{F}_i = e \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) + \mathbf{R}_i$$  \hspace{1cm} (2)

$$-e \left( \mathbf{E} + \frac{1}{c} \mathbf{v}_e \times \mathbf{B} \right) - \frac{\nabla p_e}{n_e} + \mathbf{R}_e = 0$$  \hspace{1cm} (3)

$$n_e \left( \frac{\partial T_e}{\partial t} + (\mathbf{v}_e \cdot \nabla) T_e \right) + (\gamma - 1) p_e \nabla \cdot \mathbf{V}_e = (\gamma - 1)(Q_e - \nabla q)$$  \hspace{1cm} (4)

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j}$$  \hspace{1cm} (5)

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$$  \hspace{1cm} (6)

$$\nabla \cdot \mathbf{B} = 0$$  \hspace{1cm} (7)

$$\nabla \cdot \mathbf{E} = 4\pi(n_i - n_e)$$  \hspace{1cm} (8)

Here, $f_i$ is the distribution function of ions, $\mathbf{E}$ and $\mathbf{B}$ are the electric and the magnetic fields, $\mathbf{V}_e$ is the electron velocity, $\mathbf{V}_i = \int f_i(\mathbf{r}, \mathbf{v}, t) \mathbf{v} d\mathbf{v} / \int f_i(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$ is the average velocity of ions, $p_e = T_e n_e$ and $T_e$ are the pressure and the temperature of the electrons respectively, $n_i$ and $n_e$ are the densities of the ion and electron plasma components. The collision force $\mathbf{R}_e = -\mathbf{R}_i$ takes into account the pulses exchange between the electron and the ion components of the plasma and the beam. $\mathbf{R}_e = -m_e(\mathbf{V}_e - \mathbf{V}_i) / \tau_{ei}$, where $\tau_{ei}$ is characteristic ion-electron collision time. From the dissipation mechanisms, the plasma conductivity and the electronic thermal conductivity are taken into account. The heat generated in the electrons is $Q_e = j^2 / \sigma$, $j = ne(\mathbf{V}_i - \mathbf{V}_e)$, where $\sigma = n \epsilon^2 / m_e \nu$ and the electron heat flux is $\mathbf{q} = -k \nabla T_e$. We assume that the coefficients of thermal conductivity $k$ and the frequency of collisions $\nu = 1 / \tau_{ei}$ caused by anomalous processes of scattering on fluctuations of electromagnetic fields do not depend on the parameters of the plasma and magnetic field ($k = \text{const}$, $\nu = \text{const}$). In the calculations the adiabatic index $\gamma = 5/3$ is used. In the hybrid model, the condition of plasma quasi-neutrality is assumed, and displacement currents are neglected due to low-frequency processes considered. The condition of plasma quasineutrality allows one to eliminate equation (8), replacing it with the condition $n_e = n_i$. The electric field is determined from the equation of motion of the electrons under the assumption $m_e = 0$. This approximation ignores the dispersion effects associated with the electron component of plasma.

The author’s modification of PIC is used for the computer implementation of the solution of the Vlasov kinetic equation [8]. The equation for the electron component, the equation for the temperature, and Maxwell’s equations are solved on a uniform rectangular grid by using explicit finite-difference schemes of the first-order accuracy in time and space $O(\tau, h)$ with the specification of the boundary and the initial conditions [9].
3. Results of simulation

Let us consider the results of computer simulation of “bubble” formation in open magnetic trap due to the injection of an ion beam. Hereinafter, the spatial dimensions are expressed in units of the ion dispersion size \( c/\omega_{0i} \), \( \omega_{0i} = \sqrt{4\pi n_0 e^2/m_i} \), the time is measured in the units of reciprocal ion cyclotron frequency \( \omega_{-1} \). The velocity is normalized to the Alfvén velocity \( V_A = B_0/\sqrt{(4\pi n_0 m_i)} \), where \( n_0 \) is the density of the background plasma at the initial time \( t = 0 \), \( B_0 \) is the magnitude of magnetic field in the center of simulation box \( r = 0, z = L/2 \) (Fig. 1). The special features of the studied processes include various scales of spatially and temporal characteristics that must be considered in the numerical simulation. For the typical experimental parameters, there is a hierarchy of time scales: \( 2\pi \omega_{-1} \ll T_0 \ll \tau_{ei} \). Here \( T_0 \) is the period of bounce-oscillations of ions that is of the order of the ratio of distance between mirrors to typical ion velocity. This hierarchy leads to difficulties in computer simulation of formations of “bubble”-like magnetic field structure. The propagation of the continuously injecting beam is accompanied by the displacement of the magnetic field and the magnetic cavity formation. The dynamics of the formation of the magnetic cavity and the distribution of ions of the injected beam at successive times are shown in Fig. 2.

![Figure 2](image-url)

**Figure 2.** The maps of magnetic field lines and position of the beam ions (black dots) at different moments of time \( t = 20, 60, 120 \) (\( |V| = 0.1, J = 1 \)).

The magnetic field pressure inside the cavity can reach less than 1% of the pressure of initial field \( B_0 \). The jump in the magnetic field at the cavity boundary has a size of the order of several Larmor radii of injected ions. The amplitude of the magnetic jump reaches value \( B^* = 1.1 - 3 \) in the central section \( z = 0 \). The observed process is characterized by the displacement of the background plasma from the region of the magnetic cavity and the formation of a layer of increased plasma density at the boundary of the magnetic cavity.

The trajectories of several particles of the injected beam for \( 0 < t < 100 \) is presented in Fig. 3. The motion of the observed beam particles is determined by rotation in a magnetic field and reflection from the boundary of the magnetic cavity, where a region of increased pressure of the magnetic field is formed.

The spatial size of the magnetic cavity \( R_c \) is determined by the current \( J \) of the injected beam. Fig. 4 shows the non-linear dependence of the transverse size of the magnetic cavity on the value of the ion beam current \( J \) for various values of the beam velocity \( V \). An example of a non-linear dependence of the size of the magnetic cavity \( R_c \) on the beam velocity \( |V| \) is shown in Fig. 5 at time \( t = 40 \), the beam current \( J = 1 \).

The radial distribution of the main functions, which is typical for the diamagnetic regime of an open magnetic trap, is shown in Fig. 6. The presented distributions correspond to the moment of time \( t = 40 \), when the process of formation of the magnetic field cavity approaches the quasi-stationary phase. In this case, the number of injected ions becomes comparable to the number of particles that leave the trap along the field lines.

The presented results of numerical simulation were obtained for the parameters corresponding to the conditions of laboratory experiments on the CAT setup: \( B_0 = 0.2T, n_0 = 10^{12} cm^{-3} \),
Figure 3. The trajectories of the individual ions of the beam ($|V| = 0.1, J = 1$). The color scale defines the time intervals $0 < t < 100$.

Figure 4. The dependence of the magnetic cavity size on the current of the injected beam for $|V| = 0.1$ – solid line, $|V| = 0.2$ – dotted line ($z = L/2, t = 40$).

Figure 5. The dependence of the magnetic cavity size on the beam velocity $|V|$ ($z = L/2, J = 1, t = 40$).

$V_A = 4.3 \cdot 10^8 cm/s, c/\omega_0 = 22 cm, T_0 = 10 eV, \rho = 2$, the velocity of the injected beam was varied in the range from $0.05V_A$ to $0.5V_A$.

4. Conclusion
The article proposes a description of the plasma dynamics in a diamagnetic trap based on the kinetic approximation for ions and the magnetohydrodynamics approximation for magnetized electrons. The simulation performed demonstrates the accumulation of plasma, the displacement of the magnetic field from the region occupied by the plasma, and the formation of a quasi-
stationary field configuration in the regime with continuous injection of ions into the trap. The performed series of computational experiments made it possible to obtain the dependences of the spatial size of the formed magnetic cavity on the velocity and current of the injected ion beam.

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