Indicators that influence prospective mathematics teachers representational and reasoning abilities

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Abstract. Representational and mathematical reasoning ability are very important ability as basic in mathematics learning process. The 2013 curriculum suggests that the use of a scientific approach emphasizes higher order thinking skills. Therefore, a scientific approach is required in mathematics learning to improve ability of representation and mathematical reasoning. The objectives of this research are: (1) to analyze representational and reasoning abilities, (2) to analyze indicators affecting the ability of representation and mathematical reasoning, (3) to analyze scientific approaches that can improve the ability of representation and mathematical reasoning. The subject of this research is the students of mathematics prospective teachers in the first semester at Private Higher Education of Bandung City. The research method of this research was descriptive analysis. The research data were collected using reasoning and representation tests on sixty-one students. Data processing was done by descriptive analysis specified based on the indicators of representation ability and mathematical reasoning that influenced it. The results of this first-year study showed that students still had many weaknesses in reasoning and mathematical representation that were influenced by the ability to understand the indicators of both capabilities. After observing the results of the first-year research, then in the second and third year, the development of teaching materials with a scientific approach in accordance with the needs of prospective students was planned.

1. Introduction

The ability of mathematical representation is one of the standard processes in learning mathematics that increasingly gets special attention. NCTM [11] described this ability as the means in which people can understand and use these ideas. In addition to the ability of mathematical representation, mathematical reasoning is an important component that are necessary to be developed in mathematics learning process.

PISA 2012 results showed that Indonesian students are poor at reasoning ability. From the combined results of math, science and reading tests, Indonesia ranks 64th out of 65 participating countries [1]. According to Pranoto [1] this results may indicate that: 1) students are not accustomed to solving non-routine problems; 2) students are weak in modeling real situations into mathematics and interpreting mathematical situations to real situations; and 3) students comprehension and analysis are still weak. Actually, ability to solve non-routine problems and communicate is more required and more complex at this time.

Japan, China, Singapore and South Korea are countries at the top list of PISA and TIMMS result. It might be due to mathematics learning in these countries put greater emphasis on reasoning and problem
solving. Therefore, they were able to produce high-achiever students in mathematics tests. Learning to solve problem essentially is about learning to think or learning to reason. Thinking and reasoning are used to apply obtained knowledge to solve unfamiliar problems [9].

The Director General of Teachers and Education Personnel (GTK) reported that based on the results of Teacher Competency Test (UKG) administered at the end of 2015, the average scores was 53.02. The competencies tested are professional competence and pedagogic competency. Based on the data, it is still necessary to improve the quality of teachers in Indonesia.

The improvement of the mathematical representation and reasoning ability of the prospective teachers is a necessity as it might influence students' mathematical ability. This study aims at: (1) analyzing prospective mathematics teachers’ representational and reasoning ability; (2) analyzing factors affecting prospective mathematics teachers’ representational and reasoning ability, (3) analyzing alternative of the scientific approach to improve prospective mathematics teachers’ representational and reasoning ability. In accordance with prior studies, students with high level of comprehension, mastery of material, and problem solving ability of students will determine high level of representational and reasoning ability [13].

Reasoning was described by Copi [6] as a special kind of thinking and drawing conclusions based on premises. It means that reasoning is a thinking process to draw conclusions or construct a new statement by referring to true statements. According to Romberg and Chair [2], the indicators of mathematical reasoning ability are: (1) drawing logical conclusions; (2) providing an explanation using models, facts, traits, and relationships; (3) estimating answers and solutions; (4) using patterns and relationships to analyze mathematical situations, drawing analogies and generalizations; (5) constructing and testing conjectures; (6) giving a counterexamples; (7) following the rules of inference; (8) checking the validity of arguments, compiling valid arguments; and (9) establishing direct, indirect and indirect use of mathematical induction.

Darta [7] suggests that representations are classified into: 1) visual representations (e.g. images, charts, or tables); 2) symbolic representations (e.g. mathematical statement, mathematical notation, numerical/algebraic symbol); and 3) verbal representations (e.g. written text or words). The ability of mathematical representation is the ability to express mathematical ideas (e.g. problems, statements, solutions, definitions, etc.) into certain form: 1) drawings, graphs, or tables; 2) mathematical notation, numerical/algebraic symbols; and 3) written text/words, as an interpretation of his thoughts.

In terms of reasoning ability and mathematical representation, researchers conducted interviews with prospective mathematics teachers’ attending algebra and trigonometry courses over four years. The courses included difficult subjects required a high level of reasoning, difficult to understand, it is difficult to communicate material because it contains complex concepts. These problems could be observed through the following table.

| Academic year | 2012/ 2013 | 2013/ 2014 | 2014/ 2015 | 2015/ 2016 |
|---------------|------------|------------|------------|------------|
| Final Scores  | 3.1        | 2.9        | 2.9        | 2.9        |

(Source: Student Data Education Mathematics Study Program of FKIP)

Table 1 shows that the average scores of Algebra and Trigonometry courses achieved by students decreases annually. In general, the value is still under expectations, so it is necessary to make an effort to improve the attained scores thus it might satisfy students, lecturers, and related institutions’ needs.

In addition, research conducted by Darta [8] indicated that final year students’ self-efficacy are still low which is still below 7 (mean = 6.76) with a scale of 1-9. This situation encourages researchers to conduct further research in order to obtain more comprehensive and in-depth answers about representation and reasoning ability as these abilities contribute positively to self-efficacy.
2. Methods
This study used Research and Development considering four-D model proposed by Thiagarajan [4] with multiphase design through four stages: Define, Design, Develop, and Disseminate. This article describes the results of the first year of research that is defining ability of representation and reasoning of prospective mathematics teacher students as the starting point of the development of the next year. From this analysis, we get a description of facts, hopes, and alternatives to solve basic problems, which facilitate the determination or selection of teaching materials that will be developed.

Analysis of the study of the characteristics of prospective teachers is to get a description of the characteristics of the level of ability of representation and mathematical reasoning based on the Student's Initial Ability (SIA) of each capability indicators under study. Conceptual analysis is conducted to identify the basic concepts of Algebra and Trigonometry to be taught, to compile them in the form of hierarchy, and to break down individual concepts into critical and irrelevant terms. The next analysis is to determine the factors that affect the ability of representation and generalization based on the indicators that make up the two capabilities.

3. Results and Discussion
The results of SIA data were analyzed to determine students' reasoning and mathematical representational ability before the study. The descriptive statistics is presented in Table 2

| Tabel 2. Descriptive Statistics | N | Minimum | Maximum | Mean | Std. Deviation |
|---------------------------------|---|---------|---------|------|----------------|
| Reasoning test score            | 61| .57     | 3.66    | 2.3172 | .61475         |
| Representation test score       | 61| .00     | 3.45    | 1.9382 | .72562         |

Based on Table 2, the number of participants of the study is 61 students. The scores of Achievement Index (AI) scale range from 0.00 to 4.00. The minimum score of reasoning and representation tests is 0.57 and 0.00, respectively. It indicates that there are several students whose representational abilities is low. While the maximum scores of reasoning and representation ability tests are 3.66 and 3.45 respectively. It indicates that there are several students whose reasoning and representational ability is high. The average of reasoning and representational ability tests is 2.32 and 1.94 respectively. It indicates that students' reasoning and mathematical representational ability are still low. Furthermore, if criteria of the ability of early mathematics (AEM) is used, the number of students with the category of high, moderate, and low amounted to 4 people, 47 people, and 10 people respectively. Its descriptive statistics of test score across AEM is shown in Table 3.

| Table 3. Descriptive Statistics of Test Score across AEM | Test Score | AEM | Statistic |
|--------------------------------------------------------|------------|-----|-----------|
| Reasoning score                                        | High       | Mean| 3.2575    |
|                                                        | Moderate   | Mean| 2.4264    |
|                                                        | Low        | Mean| 1.428     |
| Representation score                                   | High       | Mean| 2.8625    |
|                                                        | Moderate   | Mean| 2.067     |
|                                                        | Low        | Mean| 0.963     |

Based on Table 3, the mean of reasoning ability tests from high, moderate, and low achiever are 3.26, 2.43, and 1.43, respectively. While the average score of representation skills of high, moderate, and low achiever respectively are 2.86, 2.07, and 0.96. It might indicate that high, moderate, and low achiever had better in prior mathematical reasoning abilities than in prior mathematical representation ability. It might be seen at the percentage of representation abilities of each indicators shown in Figure 1.
Finding of this initial study also revealed the fact that students' mathematical reasoning ability is still relatively low. This can be seen at the graph of the percentage of reasoning ability of each indicator in Figure 2.

Figures 1 and 2 have shown that mathematical representational and reasoning ability are still low. The possible reason is that mathematics learning has not been direct students to develop their mathematical knowledge. Students depend heavily on lecturers. The dependency might be internalized by students through learning process. However, students are expected to develop reasoning, bring up ideas or ideas that are creative, and represent problems faced. Based on Figure 2, it appears that the indicators follow the rules of inference, give the opponent the example, and check the validity of the argument in a row is an indicator that least controlled by students.

This is in accordance with the research of Peressini and Bassett [12] that without communication especially representations in mathematics we will have little information, data, and facts about students' understanding of mathematical processes and applications. This means that representations that are part of the communication in mathematics which help educators understand the ability of learners to interpret and express their understanding of the concepts and processes of mathematics they learn.

Prabowo [14] revealed several students’ weaknesses in effort to master the concept and cultivate a scientific attitude that affect their representational ability. The weaknesses are: a) the inaccuracy of educators in providing visualization of concepts, and lack of explanation of the physical meaning of each mathematical formulation in learning activities; b) problem in construction that spur on the taxonomy of Bloom's lane without the support of process skills; c) educators are not aware and evaluate students’ learning progress; d) demonstration and problem-solving activities that do not meet the requirements of teachers with concentration on the fulfillment of material targets; e) the learning

| Representation Type                        | Percentage |
|-------------------------------------------|------------|
| Representation of verbal to notation      | 71.31%     |
| Representation of verbal to visual        | 69.26%     |
| Representation of notation to verbal      | 48.77%     |
| Representation of notation to visual      | 34.02%     |
| Representation of visual to verbal        | 2.87%      |
| Representation of visual to notation      | 80.33%     |

**Figure 1. Percentage Ability of Student Mathematical Representation**
environment has not been anticipated by educators in determining teaching and learning strategies. The weaknesses mostly related to the learning process undertaken by educators. Many studies agree that mathematical representational ability is important to communicate ideas for skeptical, critical, creative, and innovative thinking. This is in line with Darta’s research [7] which states that representational ability is a tool for improving students’ mathematical competence. In addition, students’ representational ability can provide information for teachers about how students think about a context or mathematical idea, and students’ strategy in understanding a concept.

In the wake of learning process, we should provide a structured task related to mathematical reasoning. This is done so that the scope of student thinking is not formally constrained, because the more we reduce and formalize the scope of what is considered a mathematical reasoning, the more we limit the potential multidimensionality of learners. In addition, the interaction of teachers and students during the learning process should be minimized so that children can develop their own reasoning. This is in the opinion of [5] stated that we have witnessed in the afterschool program that the more we reduce and formalize the scope of what counts as mathematical reasoning, the more we constrain the multidimensionality of learners’ potential. When communicating with students, teachers tend to refine students’ work by correcting what students think or how they use language, rather than trying to understand what students is trying to do.

Completion of mathematical representation and reasoning found different answers between students. Here is a description of the answers of high and low achiever students in solving problems whose reasoning indicator is to provide explanations by using models, facts, and relationships, and representational abilities indicators is to change algebraic notation into the image.

Given \( f(x) = x^2 - 8x + 16 \). (The test adopted from Larson and Palvo [10])

a. Change it into the general form of \( f(x) = a(x - h)^2 + k, a \neq 0 \)? (2 points)
b. From the general form properties, specify the vertex and the cut point? (2 points)
c. Draw the sketch of the function and explain its properties based on the sketch? (4 points).

Figure 3. Best Student’s Answer

Based on Figure 3, students can solve the problem perfectly and correctly. The student has good reasoning ability. It can be seen at problem 2b (providing an explanation using models, facts, traits, and relationships) in which he applies the formula with discriminant because the student might forgot the peak concept of the standard form \((x - h)^2 + k\) where the peak point \((h, k)\). He told me that he forgot the fastest way, but he reasoned in different ways that produced the same answer. In addition, he has a good representational ability. It could be discerned from the image with the representational abilities indicator namely, providing an explanation by using models, facts, traits, and relationships that he can do. He
knows the image properties of the parabola, in this case when $a > 0$, then the parabola opens upward. Furthermore, the answers of the low students are shown in Figure 4.

In Figure 4, the student can only solve the problem with the reasoning indicator namely, providing the explanation using the model, facts, traits, and relationships. The student has a poor reasoning ability as he was not able to solve it. He faced difficulty so there was no response provided. In addition, he has a poor representational ability as well. It can be seen from the image of the 2c (indicators of representation of the form of notation (algebra) into the image) which he can do by means of substitution and producing incorrect answers. Another difficulty is that he does not know the shape of the drawing of the squared function, making the scale on the wrong x and y-axis, resulting in the graph being straight and not curved, the incomplete point retrieval process, consequently, the graph has not cut the x-axis. In accordance with the research result Bergqvist and Lithner [3], most task solutions are based on available algorithms, often without arguments that justify the reasoning, which may lead to rote learning.

![Figure 4. Low Student's Answer](image)

In relation to relevant research results and preliminary analyses, the researcher think that scientific approach encouraged by the government could increase prospective teachers’ representational and reasoning ability. Because scientific learning can develop all potential learners covering all aspects. This is in accordance with the implementation of the Curriculum 2013 nationally in which a scientific approach is recommended to use. In addition, studies on scientific approaches, representational and reasoning abilities are necessary to be conducted. In order for the policy to get adequate support from research results in various subjects such as students, teachers, and prospective mathematics teachers as well.

4. Conclusion
This study is an initial sequence of research planned for three years. The results of this first-year study showed that students still had many weaknesses in the representation and mathematical reasoning that was influenced by the ability to understand the indicators of both capabilities. After seeing the results of this first-year study, then the researcher planned following studies in the second and third year for the development of teaching materials with the scientific approach in accordance with the needs of prospective students.

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