Modeling of thermomechanical processes in woven composite material at blow by the striking element

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The paper studies mechanical and thermomechanical interaction processes between striking element and multilayered woven barrier made of polymeric fabric. A simplified mathematical model is proposed. This model describes the inhibition of the damaging element due to energy dissipation, that is spent on irreversible strains of fabric threads stretching, friction during the mutual slippage of threads and barrier material heating. The defining equations parameters are estimated by identification according by the experimental data. The computational experiment results using the mathematical model can be used to diagnose armor protection according to the measurement of dynamic temperature fields.

1. Introduction
One of perspective methods of armored protection quality assessment is the analysis of the dynamic temperature field on a surface at interaction with the striking element [1, 2]. However, full-scale experimental research is an expensive action, and therefore the development of a theoretical model is relevant. The model for determining the energy absorption capacity of armored barriers and the dynamic temperature field on the surface allows for a comparative evaluation of materials and their optimal placement in armor structures without expensive experiments.

2. Thermomechanical model for interaction processes between armored protection and striking element
The squared layered woven sample is considered (Figure 1). Each layer is formed by two mutually perpendicular families of filaments: the weft and the base. The sample is on a malleable base. At the moment of impact, the central part of the sample contacts the striker, the mass M and its initial velocity V0 are known.

Figure 1. Layered woven pattern with a striking element: 1 - impactor, 2 - layer of tissue, 3 - base.
The speed of the impactor is slowed down in the interacting process; kinetic energy turns into frictional energy threads, work of structural changes and destruction of threads material, irreversible stress energy in the basis and heating of a sample, the impactor and threads. At the same time variable conditions of model change in time: movements $U$, speed $\dot{U}$, internal stress $\Sigma$ and temperature $T$. It is required to construct mathematical model which displays variables impacts $\Xi = \{M, V_0\}$ on variable states $H = \{U, \dot{U}, \Sigma, T\}$:

$$H = \Psi(\Xi).$$ (1)

We will enter Lagrangian coordinates for each fabric layer (Figure 2) where axes $\alpha$ and $\beta$ are sent to along threads, and the axis $n$ is perpendicular the layer planes. These coordinates are time-independent in the process of layer movement, and Euler coordinates $(x, y, z)$ change over time. We will demand that axes $n$ of all Lagrangian systems remained parallel to an axis $z$ of Euler system.

![Figure 2. Euler coordinates (x, y, z) and Lagrangian coordinates (α, β, n)](image)

The provision of a layer of fabric is defined by the current provision of a coordinate surface $(\alpha, \beta)$ – a median surface of a layer. Each point of the surface moves along an axis $z$. Full points movement of threads consists of the middle surface's motion and relative movement of thread within a layer. Thus, the figurative movement is defined by shifts along an axis $z$, and relative – along Lagrangian coordinate lines. Simplifying model, we will accept that the relative shift of threads points occurs along the direction of the same thread. The movement components along the reference vectors that are tangent to lines $\alpha$ and $\beta$ refer to different between (base and weft) of the same layer.

Taking into account the made simplification, knowing figurative movements, relative lengthening along threads of a basis and a weft will be equal

$$\lambda_\alpha = \sqrt{1 + \left( \frac{\partial u_z}{\partial \alpha} \right)^2}, \quad \lambda_\beta = \sqrt{1 + \left( \frac{\partial u_z}{\partial \beta} \right)^2},$$ (2)

and relative rapprochement of layers

$$\lambda_n = \frac{\partial u_z}{\partial z} \sqrt{1 + \left( \frac{\partial u_z}{\partial \alpha} \right)^2 + \left( \frac{\partial u_z}{\partial \beta} \right)^2}.$$ (3)

The accounting dissipative forces is the most complex challenge of the model creation. Friction forces are the most essential, they arise at the mutual shift of layers and when slipping threads in woven layers. We will allocate two components of dissipative force – "viscous" "rigid". The "rigid"
component of dissipative force limits growth of mutual shifts at a small tension, and the "viscous" component limits the speed of threads shift basis relatively a weft.

The movement of the modelled object is unambiguously described by functions of two coordinates: figurative movements of layers $u_z$, relative movements of basis threads of each fabric layer $u_{\alpha}$ and relative movements of threads weft $u_{\beta}$. We will apply the variation principle of Lagrange according to which the variation of work external forces is equal to work internal forces and inertial forces on variations the generalized movements:

$$\int_{V} \rho V_i \delta u_i dV + \int_{V} \sigma_{k} \delta \varepsilon_{k} dV + \int_{V} \tau_{nk} \delta \gamma_{nk} dV + \int_{S} f_k \delta u_k dS = P_i \delta u_i. \tag{4}$$

Here the index $i$ belongs to Euler basis, an index $k$ – to Lagrangian. Integration on the volume $V$ is carried out at calculation of an inertia force variation works, tension in threads and the conditional tension of interlayered shift. The works variation of dissipative forces at relative slipping of threads is calculated as integration on layers surfaces $S$.

The formulated model is closed, but the equations can't be solved analytically. For integration of the equation of the movement (4) it is necessary to use a numerical method.

3. The numerical scheme for the allowing system of the equations

The multilayered sample, rectangular in the plan is a set of final elements – parallelepipeds. Knots of elements settle down on a median surface of a layer. The pliable basis is allocated as a separate layer which moves together with the lower layer of fabric. Thus, the knots layer of a grid is connected with each fabric layer.

Initial knots coordinates is Lagrangian coordinates which don't change in the course of the movement. The Lagrangian basis is knotted with everyone. In this basis the axis $\alpha$ is directed along a fabric basis, $\beta$ axis - lengthways a weft, and the axis $z$ is directed down and doesn't change the situation at deformation.

Movements of all knots unambiguously define the current configuration of model, and they are the generalized movements. We will designate the generalized movement corresponding to degree of freedom $\Delta$, through $u_{\Delta}$ (numbers of degrees of freedom are designated by capital Greek letters). The number of freedom degrees of discrete model is final, therefore movements variations $\delta u$ can't be any coordinates functions in (5), the only variation of the movements field answers each freedom degree.

$$\delta u^{(\Delta)}(\alpha, \beta) = \psi_{\Delta}(\alpha, \beta) \cdot \delta u_{\Delta}. \tag{5}$$

Here $\delta u_{\Delta}$ - variation of the generalized movement $u_{\Delta}$, $\psi_{\Delta}(\alpha, \beta)$ - basic function of coordinates, $\delta u^{(\Delta)}(\alpha, \beta)$ - variation of the movements field at a variation of the generalized movement $u_{\Delta}$.

For receiving a discrete analog of the inertial member (first composed in equality (4)) it is necessary to consider that the generalized movements are referred to Lagrangian basis, and the vector of acceleration has to be expressed in Euler coordinates. It is necessary to cover separately the movement of threads basics and threads a weft as their movements and speeds are various.

Speed of each of families of threads consists of the figurative speed directed along an Euler axis $z$, and the relative speed directed on a tangent to the deformed layer surface. Resulting relative speed in Euler basis, we will receive:

$$\dot{u}_x = \dot{u}_\alpha \cos(x, \alpha) + \dot{u}_\beta \cos(x, \beta),$$

$$\dot{u}_y = \dot{u}_\alpha \cos(y, \alpha) + \dot{u}_\beta \cos(y, \beta). \tag{6}$$
\[
\dot{u}_z = \dot{u}_c + \dot{u}_\alpha \cos(z, \alpha) + \dot{u}_\beta \cos(z, \beta),
\]

where \( \dot{u}_c \) – transport velocity.

Further, we interpolate figurative and relative speeds, using basis (6), and we will find kinetic energy by integration on volume in Lagrangian coordinates. Lowering intermediate calculations, we will receive that kinetic energy \( T \) has an appearance of a square form of rather figurative and relative speeds of final element model knots \( \dot{u} \), and square form matrix depends on the current movements:

\[
T = \frac{1}{2} \dot{u}_\Delta M_{\Delta \Gamma} \dot{u}_\Gamma.
\tag{7}
\]

We will receive inertial composed discrete analog from Lagrange's equation of the second sort:

\[
\int_V \rho \dot{u}_i \delta t_i dV = \delta t_\Delta \left[ M_{\Delta \Gamma} \dot{u}_\Gamma + M_{\Delta \Gamma} \dot{u}_\Gamma \right].
\tag{8}
\]

Further we will consider third and fourth composed in the equation (8) which, contain two components – "rigid", linearly depending from the generalized movements, and "viscous", proportional to the generalized speeds of discrete model. The variation of work of these forces can be presented in shape:

\[
\int_V \tau_k \delta u_k dV = \delta t_\Delta \left( R_f + K_f \dot{u}_\Gamma + C_f \dot{u}_\Gamma \right),
\tag{9}
\]

where \( R_f \) - constant composed,

\( K_f \) - slipping rigidity matrix,

\( C_f \) - slipping viscosity matrix.

Finally, the discrete analog of the equation (4) has an appearance system of the linear differential equations with float factors

\[
M_{\Delta \Gamma} \dot{u}_\Gamma + C_{\Delta \Gamma} \dot{u}_\Gamma + K_{\Delta \Gamma} u_\Gamma = R_\Delta,
\tag{10}
\]

where \( M \) – masses matrix,

\( C \) – total viscosity matrix with \( \dot{M}_{\Delta \Gamma} \),

\( K \) – total rigidity matrix,

\( R \) – total vector of constants (on a small time step) the generalized forces.

The system of the equations (10) decides under an entry condition:

\[
u_\Delta(0) = 0, \quad \dot{u}_\Delta(0) = v_\Delta(0), \quad \tag{11}
\]

where \( v_\Delta(0) \) - initial velocity.

For numerical integration of the differential equation with matrix coefficients (10) under entry conditions (12) we will enter a small step on time \( \tau \) and we approximate derivatives the differential relations (the top index designates number of timepoint):

\[
Y^{t+1} = \frac{X^{t+1} - X^t}{\tau}, \quad Y^{t+1} = \frac{Y^{t+1} - Y^t}{\tau},
\tag{12}
\]

where \( X \) – generalized movements column matrix,

\( Y = \dot{X} \) - generalized speeds column matrix.

Substitution (12) in (10) gives the following differential equation:
The spectral analysis of the scheme (13) shows her absolute stability [3]. At the same time, this scheme has positive viscosity (isn’t conservative) that demands adjustment of a step depending on the current speed and a posteriori assessment of an error of integration.

4. Computing program implementation technique

The algorithm used for formation system (13), its numerical integration on time and armored barrier material condition parameters calculations on each temporary step consists of a large number of the interconnected. This has led to complexity of the computing program and debugging difficulties. Essential simplification of computer program development is reached by use for a functional and object paradigm [4], which the algorithm is built on functional objects – program structural units. The main component of program realization is the functional and object scheme which reflects functional objects dependence. Algorithm elementary steps are calculation of value, which a functional object is responsible. Functional and object schemes and specialized classes of objects in language C ++ have been developed for solution realization.

Calculation results (movement, speeds, tension and temperature) are unloaded to file; the dynamic graphics creation tool program is developed for their viewing and schedules creation.

5. Power analysis for interaction processes between striking element and armored protection

Kinetic energy of the striking element in the course of interaction with a textile armored package is spent on: work of deformation of threads at their stretching, work of friction forces when slipping (prodyorgivaniye) of threads, the message of kinetic energy to threads of fabric which dissipates further in the form of heat.

Change of kinetic energy of the armored package consisting of 12 layers of fabric of a twill weave with a density of material of 1450 kg/m$^3$ was investigated. The armored package had area density of 2 kg/m$^2$ and settled down on the plasticine basis. The steel ball weighing 1,05 g acted as the striking element.

We will show kinetic energy integrated size distribution in the course of impact of the striking element with textile armor at an initial speed 200 m/s and 400 m/s, kinetic energy of the striker is equal 20 J and 80 J respectively.

In the Figure 3 the curve of distribution of kinetic energy is presented, on a horizontal axis time of contact of the striking element with fabric, and on vertical – kinetic energy is postponed.

![Figure 3](image)

**Figure 3.** A curve of kinetic energy distribution at a striker initial speed: a – 200 m/s, b – 400 m/s.

In the course of impact of the striker with armored fabric growth of kinetic energy of a woven package before achievement of value 3,5 J in a case with initial speed in 200 m/s and 5,6 J is observed at a speed 40 m/s. Further kinetic energy begins to decrease, turning into irreversible work of deformations of threads (Figure 4) and work of internal friction of threads (Figure 5).
At an initial speed of the striker in 400 m/s the saved-up irreversible work of destruction makes 14% of initial kinetic energy of the striking element whereas at speed of 200 m/s only less than 2%. Such distinction is explained by existence of destructions of threads at speed of 400 m/s.

**Figure 4.** A curve of irreversible deformations work of distribution at a striker initial speed: a – 200 m/s, b – 400 m/s

![Figure 4](image)

**Figure 5.** A curve of internal friction work distribution at a striker initial speed: a – 200 m/s, b – 400 m/s

![Figure 5](image)

In the Figure 5, opposite result are observed. At speed of 200 m/s about 50% fall to the share of work of internal friction because of slipping of threads, and at an initial speed of 400 m/s only $10^{-7}$%.

6. Conclusions

The mathematical model is developed. This model adequately describes the physical processes happening at interaction between the striking element and multilayered armored composite material: the joint movement of armored material and the striking element, irreversible deformation of threads, slipping of threads, losses of kinetic energy on friction and irreversible deformation, and also heating. The quality control possibility of an armored composite material an is shown on the basis of the dynamic temperature fields analysis. Results of theoretical researches qualitatively correspond to the known experimental data. Individual control of model by identification of physicomechanical parameters is necessary for receiving quantitative coordination of calculation and an experiment. Calculations allow to assess the striker kinetic energy, which is spent for friction forces and irreversible work of the stretching tension.

References

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