Joint Optimal Operation of Wind Power Plant and Cascade Hydropower Station

Di Jiang¹, Mingkai Wang¹, Wentao Sun¹ and Yang Wu², *

¹State Grid Liaoning Electric Power Co. Ltd, Shenyang, China
²North China Electric Power University, Baoding, China

*Corresponding author: wuyyyang@ln.sgcc.com.cn

Abstract. With the depletion of fossil energy, the whole people advocate energy conservation and emission reduction, making the scale of wind power integration increase. While wind power has fluctuating and intermittent characteristics, this paper develops a short-term combined operation strategy of wind and water using the flexible regulation characteristics of cascade hydropower stations. To ensure full access to wind power, reduce the impact of wind power fluctuations as little as possible. A multi-objective short-term optimal scheduling model with the minimum variation of combined wind and water output and the largest total power generation is established considering the constraints of storage capacity, discharge flow, reservoir water level, output of each power station, and power transmission section. Aiming at the shortcomings of traditional particle swarm optimization algorithm, such as slow convergence rate and fall into optimal local solution easily in the late search, dynamic inertia weight and learning factors are introduced and applied to model solving. The simulation results show that the cascade hydropower station can effectively stabilize the wind power fluctuation, improve the wind power consumption level, increase the power generation of the system, and ensure the safe and stable operation of the power system.

Keywords: Wind power uncertainty, Cascade hydropower station, Particle swarm optimization, Dynamic inertia weight

1. Introduction
Since the Industrial Revolution, human beings have developed and utilized conventional fossil energy on a large scale for hundreds of years, causing the ecological environment to become increasingly worse. Because wind energy is a clean and environmentally friendly renewable energy source, and the power generation technology is mature, the proportion of wind power generation has increased sharply [1]. However, wind power has fluctuating and intermittent characteristics, and wind power access to the grid brings a lot of uncertain factors, which adversely affects the safe operation of the network, mainly focusing on frequency, voltage, system stability, pollution, line loss, and protection devices [2-4]. Difficulties in wind power consumption have always plagued the large-scale development of wind farms at home and abroad.
According to in-depth research by scholars, hydropower has excellent performance to cope with the impact of uncertain new energy on the power grid. By optimizing the operation of wind power and cascade hydropower stations, temporary storage of electricity can be achieved by adjusting the water storage capacity of the reservoir. Besides, the turbine unit is easy to start and stop, and it is adjusted very quickly, which can reduce the adverse impact of short-term and large-scale fluctuations of wind power in unstable weather conditions such as rain and sandstorm [5-8]. In general, it is necessary to combine wind power with hydropower when formulating a dispatching plan, which can achieve artificial control and regulation of wind farms. This is a more feasible strategy that can effectively enhance wind power capacity.

This paper selects a joint optimized operation of a wind farm and a cascade hydropower station to establish a multi-objective model, which minimizes the fluctuation of system output and guarantees power generation. This model avoids the situation that hydropower only works when the wind power is small. The particle swarm algorithm with strong versatility and simple principle is deeply studied. By introducing dynamic inertia weights and learning factors, the shortcomings of the standard particle swarm algorithm in the later convergence process are slow and easy to fall into the local optimum. The improved particle swarm algorithm is used to solve the wind and hydropower complementary joint system to find the most feasible and efficient dispatching scheme to ensure the long-term safe and stable operation of the power system.

2. Joint optimization model

The wind and hydropower combined optimization model established in this paper bundles wind and hydropower into the power grid, to minimize fluctuations in power generation and maximize power generation in a wind-water interconnected system. China mainly focuses on thermal power generation, and research on optimal dispatch of power systems with wind farms has also focused on joint operations with thermal power [9-10]. Utilizing the flexible adjustment of the hydro-turbine units, the impact of the integration of wind power into the power grid on the power system can be reduced to ensure the stable operation of the power system. The remaining load is then supplied by the thermal power plant or other power plants to avoid numerous startup and shutdown and save power generation costs [11-12].

2.1. Objective function

On a particular day, the inflow of hydropower stations at all levels and time-varying curve of wind speed are known. Without considering the complicated changes of short-term fluctuations in wind speed, the wind power output of the system is fully connected to the grid, and the hydropower station plays a role of peak regulation, so that the system's joint output curve is more stable, and short-term dispatching schemes for hydropower stations at all levels are derived. However, in the process of optimizing the solution, the common problem is that the joint output power is small. Therefore, this paper establishes a multi-objective mathematical model with the minimum output fluctuations and the maximum total power generation in a combined wind and hydropower system, as shown in equation (1). The scheduling period is one day, and the minimum scheduling period is one hour.

\[
\begin{align*}
\min F_1 &= \frac{1}{n} \sum_{t=1}^{T} \left( P_{\Sigma}^t - \bar{P} \right)^2 \\
\min F_2 &= \sum_{t=1}^{T} P_{\Sigma}^t \times \Delta t
\end{align*}
\]

Where \( P_{\Sigma}^t \) is the combined output of the combined power generation system at time \( t \), and \( \bar{P} \) is the average output power.

The expression of wind power output is as follows.
Where $v_{in}$, $v_{out}$ and $v_r$ are the cut-in wind speed, cut-out wind speed and rated wind speed of the wind turbine, respectively, and $P_r$ is the rated output power.

The expression of power output of each level of hydropower station is as follows.

$$P_h = 9.81\eta QH$$

(3)

Where $\eta$ is the power generation efficiency of the hydropower station, $H$ is the net head of the hydropower station, and $Q$ is the discharged flow.

To effectively and simply analyze the joint optimization system model, this chapter uses the approximate linear weighting coefficient method for multi-objective function processing, as shown below:

$$\min F = R_f \left( \frac{F_{1,\min}}{1} + \frac{1}{n} \sum_{i=1}^{T} \frac{P_{i,\min} \times \Delta t}{F_{2,\max}} \right)$$

(4)

Where $R_f$ is the output fluctuation penalty coefficient, and $R_p$ is the power generation income coefficient.

2.2. Constraints

The constraints of the model are as follows:

(1) Reservoir capacity constraints

$$V_{i,\min} \leq V_{i}^{'} \leq V_{i,\max}$$

(5)

Where $V_{i,\max}$ and $V_{i,\min}$ represent the maximum and minimum reservoir capacities of the $i$-th hydropower station, respectively.

(2) Discharged flow constraints

$$Q_{i,\min} \leq Q_{i}^{'} \leq Q_{i,\max}$$

(6)

Where $Q_{i,\max}$ and $Q_{i,\min}$ represent the maximum and minimum discharged flow of the $i$-th hydropower station, respectively.

(3) Reservoir water level constraints

$$Z_{i,\min} \leq Z_{i}^{'} \leq Z_{i,\max}$$

(7)

Where $Z_{i,\max}$ and $Z_{i,\min}$ are the highest and lowest water levels of the $i$-th hydropower station, respectively.

(4) Hydropower station output constraints

$$P_{i,\min} \leq P_{i}^{'} \leq P_{i,\max}$$

(8)
Where $P_{i,max}$ and $P_{i,min}$ respectively indicate the maximum and minimum output of the $i$-th hydropower unit.

(5) Water capacity balance constraints

$$V_i^t = V_i^{t-1} + Q_{io}^t \times \Delta t - Q_{ir}^t \times \Delta t$$

(9)

Where $V_i^{t-1}$ and $V_i^t$ respectively indicate the initial and final reservoir capacity of the $i$-th hydropower station in the $t$-th period; $Q_{io}^t$ and $Q_{ir}^t$ represent the inflow flow and discharged flow.

(6) Wind farm power constraints

$$P_{w,min} \leq P_w^t \leq P_{w,max}$$

(10)

Where $P_{w,max}$ and $P_{w,min}$ represent the maximum and minimum output of the wind farm unit, respectively.

3. Improved Particle Swarm Optimization

3.1. Traditional Particle Swarm Optimization

Traditional Particle Swarm Optimization (PSO) is an evolutionary computation method based on intelligent foraging of bird swarm systems. In PSO, each particle knows its optimal historical position and optimal global position so far, that is, the personal optimal solution ($pbest$) and optimal global solution ($gbest$). In each iteration, the particle continuously changes its running speed in the solution space according to the distance between the current position and the two extreme values, as close as possible to the optimal position in the group.

The process of solving the function optimization problem shown in equation (11) using PSO will be expressed by a mathematical model:

$$F = \text{max} f\{x_1, x_2, \cdots, x_n\}$$

$$\text{s.t. } x_{i,\text{min}} \leq x_i \leq x_{i,\text{max}}, i = 1, 2, \cdots, n$$

(11)

Suppose that $N$ particles are searched in a $D$-dimensional space, the positions and velocities of the particles are randomly selected. The individual optimal solution and the optimal global solution of the particles are obtained according to equations (12) and (13), respectively.

$$pbest_i^{k+1} = \begin{cases} pbest_i^k, & f(x_i^k) \leq f(pbest_i^k) \\ x_i^k, & f(x_i^k) > f(pbest_i^k) \end{cases}$$

(12)

$$gbest = \text{max}\left\{ f(pbest_1^k), f(pbest_2^k), \cdots, f(pbest_N^k) \right\}$$

(13)

The standard particle swarm keeps updating its running speed and position according to the following formula:

$$v_{i,d}^{k+1} = ov_{i,d}^k + c_1 \times rand_1 \times (pbest_i^k - x_{i,d}^k) + c_2 \times rand_2 \times (gbest - x_{i,d}^k)$$

(14)

$$x_{i,d}^{k+1} = x_{i,d}^k + v_{i,d}^{k+1}, i = 1, 2, \cdots, N, d = 1, 2, \cdots, D$$

(15)
Where \( k \) is the number of iterations, \( \omega \) is the inertia weight, \( c_1 \) and \( c_2 \) are learning coefficients, and \( \text{rand}_1 \) and \( \text{rand}_2 \) are random numbers obeying \([0,1]\) uniform distribution.

PSO has attracted the academic community's attention for its advantages, such as easy implementation and high accuracy. It has been widely used in function optimization problems in many fields [13-14]. However, the convergence speed in the later stage of the search is slow, and it is easy to fall into a local optimum. If the updating strategy of fitness and inertia weight is improved, the algorithm's global search ability can be increased. This paper introduces a dynamic inertia weight and learning coefficients to improve PSO's performance, which is applied to the scheduling problem of the joint optimization system of wind power and hydropower.

### 3.2. Improvement of Particle Swarm Optimization

This paper adopts a new and improved Particle Swarm Optimization by dynamically adjusting the inertia weight and learning coefficients. The ability to balance the strength of global particle search and local search is particularly vital for the algorithm's success. Therefore, the inertia weight is introduced in equation (16), which is a weighting coefficient related to the previous speed. Choosing a reasonable inertia weight, which can make the search reach such a balance, is the key to avoid falling into a local optimal and efficient search [15-16]. During the iteration process, the value of the inertia weight is continuously changed. At the early stage of the iteration, the value is larger, indicating that the particle has a robust global search ability at this time, and can locate the optimal solution position faster. As the number of iterations of the algorithm increases, as the particles gradually approach the optimal solution, the value gradually decreases exponentially, and the particles focus on excellent local search to improve the convergence accuracy.

\[
\omega = \omega_{\text{max}} - \frac{(\omega_{\text{max}} - \omega_{\text{min}})k}{k_{\text{max}}}
\]  

(16)

Similarly, choose a dynamically changing learning coefficients [17-18]. In the early stage of the search, the value of \( c_1 \) should be larger, and \( c_2 \) should be smaller, so that the particles learn more from the individual extreme values and less from the extreme global values, increasing the global searchability. The following situation is the opposite, and the local optimization ability of the particles is greater than the global optimization ability. The learning coefficients change linearly in different trends, to ensure that during the process of particle velocity update, the particles can reasonably allocate their own cognitive ability and global cognitive ability. The changing rule of inertia weight and learning coefficients is shown in equation (17).

\[
\begin{align*}
\text{c}_1 &= c_{\text{max}} + \frac{(c_{\text{min}} - c_{\text{max}})k}{k_{\text{max}}} \\
\text{c}_2 &= c_{\text{min}} + \frac{(c_{\text{max}} - c_{\text{min}})k}{k_{\text{max}}}
\end{align*}
\]

(17)

The specific steps of the improved Particle Swarm Optimization in solving the joint optimization system scheduling problem are as follows:

1. Initialize \( N \) particles randomly. Within the variation range of the allowed discharged flow in each period, arbitrarily generate \( N \) sets of discharged flow change sequences at the end of each period: \((Q_{1,1}, Q_{1,2}, \cdots, Q_{1,D}), \cdots, (Q_{N,1}, Q_{N,2}, \cdots, Q_{N,D})\). These are set to the current position coordinates of the particles. Randomly generate \( N \) sets of discharged flow speed sequences at the end of each period: \((v_{1,1}, v_{1,2}, \cdots, v_{1,D}), \cdots, (v_{N,1}, v_{N,2}, \cdots, v_{N,D})\).
(2) Set the fitness value of each particle calculated according to the objective function as the initial individual optimal value, as follows: \( p_{best_i}^1 = [x_{1,i}^1, x_{2,i}^1, \ldots, x_{D,i}^1] \). Find the largest one of the individual optimal values of the particles as the initial global optimal value and record it: \( g_{best} = \max \{ f(p_{best_1}^1), f(p_{best_2}^1), \ldots, f(p_{best_N}^1) \} \).

(3) The fitness value of the new particles after the iteration is calculated according to the objective function. If it is higher than the particle's current individual optimal value, set it as the particle's position, and the individual optimal value and the optimal global value are updated.

(4) Dynamic inertia weight and learning coefficients are introduced according to equations (16) and (17).

(5) The positions and velocities of the particles are updated according to equations (14) and (15).

(6) Determine whether the value of each dimension of the particle meets the constraints. If it exceeds the limit, the value is taken according to the boundary.

(7) Check whether the termination condition of the iteration is satisfied. If the current number of iterations reaches the preset maximum number, the iteration is terminated, and the extreme global points are recorded as the optimal scheduling of the reservoir; otherwise, it goes to step 3 and continues the iteration.

The algorithm flow chart is shown in Figure 1.

![Flow chart of improved Particle Swarm Optimization](image-url)
4. Case analysis
In order to verify the practicability of the proposed joint optimization model of wind power and hydropower, the selected power generation system includes a wind farm with a capacity of 300 MW and a cascade hydropower station. Hydropower stations at all levels are named Class I, Class II, and Class III hydropower stations in order from high to low altitude [19]. The structure of the cascade hydropower station is shown in Figure 2. The necessary information about the three hydropower stations is shown in the table below.

![Figure 2. Schematic diagram of serial cascade hydropower station](image)

**Table 1.** Necessary hydraulic data of the cascade hydropower station

| Hydropower Station | Class I | Class II | Class III |
|--------------------|---------|----------|-----------|
| Installed capacity (MW) | 454     | 306      | 66        |
| Maximum discharged flow (m³/s) | 950     | 1500     | 1000      |
| Minimum discharged flow (m³/s) | 200     | 230      | 150       |
| Maximum reservoir capacity (10⁸×m³) | 44.36   | 33.15    | 4.40      |
| Maximum reservoir capacity (10⁸×m³) | 18.58   | 15.99    | 3.50      |
| Maximum output (MW) | 454     | 306      | 66        |
| Guaranteed output (MW) | 76.7    | 60.8     | 19        |

This paper selects the wind farm with an installed capacity of 300 MW to analyze the model. The hourly wind speed change curve during the day is shown in the figure below.

![Figure 3. Hourly wind speed curve of the wind farm](image)
Table 2. Improved Particle Swarm Optimization parameters

| Parameter                        | Value       |
|----------------------------------|-------------|
| Population size $N$              | 380         |
| The maximum number of iterations $k_{max}$ | 500         |
| Inertia weight $w$               | $1.2-k_{max} \times (1.2-0.8)/k_{max}$ |
| Learning coefficient $c_1$       | $2.1+k_{max} \times (0.8-2.1)/k_{max}$ |
| Learning coefficient $c_2$       | $2.1+k_{max} \times (2.1-0.8)/k_{max}$ |
| Output fluctuation penalty coefficient $R_f$ | 0.65         |
| Power generation income coefficient $R_p$ | 0.35         |

For the multi-objective function established in this paper, the linear weighted method is used to convert the multi-objective into a single target; the improved Particle Swarm Optimization is used to simulate the case. Before and after optimization, the output of wind farms and tertiary hydropower stations is shown in Figure 4-7.

Figure 4. Wind power output during the scheduling period

Figure 4 shows the fluctuation of the wind farm output in one day, with an average output power of 187.7 MW. The wind speed is higher in the early morning, before sunset, and late at night. The specific description is: between 4:00-6:00, 7:00-11:00, 15:00-19:00, and 23:00-1:00, wind power output is higher than the average daily output power, the highest is 300 MW. At noon and in the evening, the wind speed is lower, and the output power is lower between 1:00-4:00, 11:00-15:00, and 19:00-23:00, and the lowest is only 88.99 MW. The power output fluctuates frequently, and the amplitude is large, which has certain hidden dangers to the safe and stable operation of the power system.

Figure 5. Comparison of the output before and after optimization of Class I hydropower station
It can be seen from Figures 5 to 7 that from 1:00-4:00, the power generation of the three hydropower stations has increased slightly compared with that before optimization. The power generation from 4:00-11:00 is approximately equal to that before optimization. During the periods of 11:00-15:00 and 19:00-23:00, the amount of power generation increased, but the outputs change faster than the period of 1:00-4:00, and the amplitudes are also larger. However, during the periods of 15:00-19:00 and 23:00-1:00, the output of the three hydropower stations decreased significantly. In summary, the cascade hydropower station output after optimization has a changing trend that is opposite to that of wind power, and the frequency and amplitude of the change are consistent with wind power fluctuations. Compared with Class I and Class II hydropower stations, Class III hydropower stations have the best optimization effect. This is because the Class III hydropower station is an anti-regulation hydropower station, which mainly plays a regulatory role, rather than increasing the efficiency of power generation.
Figure 8. Comparison of the output before and after optimization of the joint optimization system

Table 3. Comparison of power generation of each power source before and after optimization

| Output power ($10^4\times$kW·h) | Class I hydropower station | Class II hydropower station | Class III hydropower station | Total hydropower station | Joint Optimization System |
|---------------------------------|---------------------------|----------------------------|-----------------------------|--------------------------|--------------------------|
| Before                          | 482.15                    | 429.07                     | 98.32                       | 1009.54                  | 1460.10                  |
| By PSO                          | 517.84                    | 399.87                     | 96.57                       | 1014.28                  | 1464.84                  |
| By improved PSO                 | 532.72                    | 418.44                     | 101.59                      | 1052.75                  | 1503.31                  |

From the data in Figure 8 and Table 3, it can be concluded that the optimization effect of the improved PSO algorithm is significantly better than the traditional PSO algorithm. After optimization, the output of the hydropower station changes rapidly with the change of wind power. The maximum fluctuation value of the total output curve obtained by the traditional PSO algorithm is reduced from 605.39 MW to 176.75 MW. The total output deviation var is decreased from 31470.00 to 2815.10. The result of using the improved PSO algorithm is that the maximum fluctuation is 205.57 MW, and the variance value is 2611.77. Except for it is slightly larger during the particular period, the total output change in each period is controlled within 80 MW, and the output curve is more stable than before. It can be concluded that the hydropower station can effectively compensate for the fluctuation of wind power and meet the requirements of the power grid. In addition, the total output of the cascade hydropower station optimized by the traditional PSO algorithm, and the total daily power generation has increased slightly by 47,400 kW·h. Surprisingly, after the optimized PSO algorithm is used for optimization, the total daily power generation is raised by 432,100 kW·h. The optimization result of the improved PSO algorithm is better than the traditional PSO algorithm, which verifies the effectiveness of the improved method. To sum up, hydropower can smooth and restrain the random fluctuations of wind power, and increase the total power generation. The combined operation of wind power and hydropower plays a positive role in the long-term safe and stable operation of the power system.

5. Conclusions
This paper focuses on the short-term joint optimal dispatching of the wind farm and cascade hydropower stations for theoretical studies and feasibility research, as follows: Ensure that the wind power is fully connected to the grid, and quickly adjust the output of the turbine to stabilize the fluctuation of wind power. A multi-objective optimal scheduling model is established from the aspects of joint output fluctuations and total power generation, and a linear weighted method is used to convert the multi-objective into a single target for processing. In order to solve the problem of low efficiency of the traditional Particle Swarm Optimization in the later search, the dynamic inertia
weight and dynamic learning factor are introduced to improve the algorithm, and it is used to solve the model. By analyzing the simulation results, it can be known that the combined power generation system of wind power and hydropower makes use of the flexibility and controllability of cascade hydropower stations to reduce the impact of wind power grid connection on the power grid and achieve the goal of more power generation and stable output. Therefore, the coordinated operation of wind and hydropower brings new opportunities for the consumption of clean energy.

Although this paper has obtained some conclusions on the combined optimization of wind power and hydropower, there is still much work that needs to be studied in the future: (1) Because large-scale wind power access will have a time and space impact on the power grid, it is necessary to design a multi-time scale (from second to year) active power dispatching system for wind power and hydropower and study in detail the scheduling methods at different time scales. (2) The paper simplifies the output of wind farms. In the next step, factors such as load forecast errors and wind power forecast errors will be introduced to more accurately describe the optimal dispatching process of wind and hydropower.

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