Topological p-n junctions in helical edge states

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Quantum spin Hall effect is endowed with topologically protected edge modes with gapless Dirac spectrum. Applying a magnetic field locally along the edge leads to a gapped edge spectrum with opposite parity for winding of spin texture for conduction and valence band. Using Pancharatnam’s prescription for geometric phase it is shown that mismatch of this parity across a p-n junction, which could be engineered into the edge by electrical gate induced doping, leads to a phase dependence in the two-terminal conductance which is purely topological (0 or \( \pi \)). This fact results in a \( \mathbb{Z}_2 \) classification of such junctions with an associated duality. Current asymmetry measurements which are shown to be robust against electron-electron interactions are proposed to infer this topology.

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\textbf{Introduction} : In an classic paper\cite{1}, Berry introduced the cyclic and adiabatic geometric phase that had implications across disciplines\cite{2} ranging from high energy physics to condensed matter physics. It was soon realised that both the conditions of adiabaticity\cite{3, 4} as well as cyclicity on parameters\cite{5} were not at all necessary and the geometric phase was a property of the Hilbert space itself. It turned out that a generalised form of this idea was already anticipated \cite{6, 7} by Pancharatnam in his seminal work on interference of polarised light\cite{8}. Pancharatnam’s connection (or rule) gives a natural way to compare the phases of any two non-orthogonal states, \(|A\rangle\) and \(|B\rangle\). If \( \langle A|B \rangle \) is real and positive, they are said to be ”in phase” or ”parallel”. Geometrically, the norm of the resultant vector \((|A\rangle + |B\rangle)|\) is maximum while physically, the probability is maximum. This rule is both reflexive and symmetric but not transitive. For a two-state quantum system, if there are three arbitrary non-orthogonal states \(|A\rangle, |B\rangle, |C\rangle\) such that pairwise \(|A\rangle, |B\rangle\) and \(|B\rangle, |C\rangle\) are in phase, it turns out \(|C\rangle\) is not in phase with \(|A\rangle\). The phase of the complex number \(\langle A|B \rangle \langle B|C \rangle \langle C|A \rangle\) accumulated in a series of cyclic quantum projections is non-trivial. This phase is given by half the solid angle of the geodesic triangle \(ABC\) subtended at the centre of the Bloch sphere. Applying these ideas to the case of neutrino oscillations, it was found that as long as \(CP\) was conserved, the Pancharatnam phase had topological values \cite{9} (see also Ref.\cite{10}) while switching on \(CP\) violation led to the phase becoming geometric \cite{11}. Here we exploit Pancharatnam’s ideas to establish the \(\mathbb{Z}_2\) classification of the p-n junctions (p and n stand for hole and electron doping respectively).

The central focus of this Letter is to show that transport in helical 1D electron gas (1DEG) with Dirac-like spectrum is dominated by the Pancharatnam phase when gap is opened in the spectrum due to application of magnetic field while transport is induced in the gapped spectrum by pure electrical doping. A typical 2D system that is expected to host such helical 1D states as edge states is the quantum spin Hall (QSH) state\cite{12, 13}. Existence of such state has been experimentally confirmed\cite{14, 15} and it is established that \(z\)-component (component perpendicular to the plane hosting the QSH state) of electron spin on the edge is a conserved quantity\cite{17}. We consider a situation where a gap is opened in the edge spectrum by application of a magnetic field parallel to the plane locally on the edge. Experimentally, such local magnetic field has been already implemented on the edge states in the context of a quantum Hall bar geometry using nano-scale magnets\cite{18} and hence a similar setup in the context of HgTe/CdTe quantum wells which hosts the QSH state is not inconceivable. Now in order to induce electrical transport in the gapped edge spectrum we consider electrostatic doping in the two different patches such that possibility of engineering p-n, n-n or p-p junctions is opened up. Furthermore we will show below that the Pancharatnam phase leads to a topological distinction between such junctions via a \(\mathbb{Z}_2\) index. \textbf{Model} : We consider an infinitely extended helical edge state lying along \(x\)-axis, exposed to a magnetic field

\[ B = \begin{cases} B\hat{x} & \text{for } -\infty < x < 0 \\ (B\cos\phi)\hat{x} + (B\sin\phi)\hat{y} & \text{for } 0 < x \leq \infty \end{cases} \] (1)

Also we consider the spin-orbit field of the 1D helical electron (1DEG) to be pointing along \(z\)-axis. Introduction of a transverse magnetic field (\(i.e.\) along any direction in \(x\)-\(y\) plane) opens up a gap in the dispersion\cite{19}. By keeping the provision to apply magnetic field in two different directions we have the possibility to manipulate the Pancharatnam phase in a desired way. In addition, we include influence of electrostatic gate voltages \(V_{\phi_1}\) and \(V_{\phi_2}\) acting on the two halves \((x \in [-\infty, 0], x \in [0, \infty])\) respectively which allows doping of electrons or holes depending on the sign of the applied voltage resulting in a finite transport. The Hamiltonian representing the above situation is given by

\[ H = -i\hbar v_F \sigma_z \partial_x + g\mu_B \left( \vec{S}(x) \cdot \vec{B} \right) - eV_{\phi_2}, \] (2)
where \( \hbar = \hbar/2\pi \) is the reduced Planck constant, \( v_F \) is the Fermi velocity, \( \sigma_{x,y,z} \) are the Pauli matrices, \( S(x) = \frac{\hbar}{2} \vec{\sigma} \) is the spin operator, \( g \) is the Landé-g factor for the electron; \( \mu_B \) is the Bohr magneton, \( e \) is the electronic charge, \( V_G \) is the applied gate voltage.

_Landauer conductance_: The energy spectrum for the problem in each patch is given by \( E_k = eV_G \pm v_F \sqrt{k^2 + b^2} \) where \( b = g\mu_B B/2e \) and \( k \) is the momentum of the electron. And the corresponding momentum-dependent eigen-spinor (corresponding to applied \( \vec{B} \) in the \( x-y \) plane with the azimuthal angle \( \phi \)) is

\[
\psi_{\eta,\epsilon} = (\epsilon/N_{\eta,\epsilon}) \left[ (a_{\eta} k_i + \sqrt{k_i^2 + b^2})/(be^{i\phi}) \right] \begin{pmatrix} 1 \\ t \end{pmatrix} .
\]

where \( \eta = R/L \) corresponds to the right- and left- movers respectively, \( a_{\eta} = \pm \) for \( \eta = R/L \) respectively and \( \epsilon = \pm \) for conduction and valence bands respectively. \( N_{\eta,\epsilon} \) is the Normalization constant. Now by demanding continuity of the plane-wave solution of the Schrödinger equation at the junction we obtain the Landauer conductance in the spin response limit which is expressed in terms of transmission probability as

\[
G_{\psi_1,\psi_2} = \frac{e^2}{\hbar} \begin{pmatrix} \rho_i \\ \rho_t \end{pmatrix} S(\alpha, \beta, \gamma) ,
\]

where, \( S(\alpha, \beta, \gamma) = (1 - |\alpha|^2)(|\beta|^2 + |\alpha|^2|\gamma|^2 - 2\alpha\beta\gamma \cos\Omega) \), the parameters \( \alpha, \beta \) and \( \gamma \) are the spinor overlaps given by \( \alpha = \langle \psi_r|\psi_i \rangle, \beta = \langle \psi_t|\psi_i \rangle, \gamma = \langle \psi_t|\psi_r \rangle \) where the indices \( i, r, t \) stand for the incident, reflected and transmitted spinors respectively evaluated at the average Fermi energy across the junction. The exact expressions for \( \alpha, \beta \) and \( \gamma \) would depend upon \( V_{\psi_1}, V_{\psi_2} \) which decides the doping on the two sides of the junction and the relative angle between the applied magnetic fields, \( \phi \) via Eq[3]. \( \rho_i \) and \( \rho_t \) are the density of states of the incident (and also reflected) branch and transmitted branch given by \( \rho_i = \sqrt{k_i^2 + b^2}/v_F k_i \), \( \rho_t = \sqrt{k_t^2 + b^2}/v_F k_t \). And \( \Omega \) is the Pancharatnam phase which is the phase of the complex number \( \chi = \alpha\beta\gamma = \langle \psi_r|\psi_i \rangle \langle \psi_t|\psi_i \rangle \langle \psi_t|\psi_r \rangle \). Note that this product is indeed representing a cyclic loop made from projection closing on to itself hence giving rise to the geometric phase which can be understood in terms of Pancharatnam connection[7]. It is worthwhile to remark that the two-terminal linear conductance across the junction (Eq[4]) depends upon two quantities: (a) the mismatch of density of states across the junction at the Fermi level, and (b) \( S(\alpha, \beta, \gamma) \) which only depends on the spin texture mismatch of the dispersion across the junction at the Fermi level. The non-trivial aspect of the spin-texture mismatch lies in its dependence on \( \Omega \) which is actually the only phase which influences the transmission probability and hence the two terminal conductance. Next, we analyse the influence of \( \Omega \) on the electrical transport across the junction.

_Topological phase in \( \phi = 0 \) case and the \( \mathbb{Z}_2 \) index_: For \( \phi = 0 \), the applied magnetic field points only along the \( x \)-direction and the spin-orbit field points along \( z \)-axis, hence the incident, reflected and the transmitted spinors represent three distinct points on the Bloch sphere all of which lie on a great circle contained in \( x-z \) plane irrespective of details of the doping. As a consequence the spherical triangle formed by connecting these three points along the geodesic path[4] governed by the collapses in \( X \) has two distinct possibilities: either the collapse process in the \( x-z \) plane encloses the centre (call it \( g_1 \)) or it goes back and forth on a finite patch of the great circle (call it \( g_2 \)) (see also 9, 10). The solid angle subtended by the closed curve \( g_1 \) at the centre is \( 2\pi \) and that enclosed by \( g_2 \) is zero respectively. Using Pancharatnam’s idea, we can immediately predict that \( \Omega \), which is the phase of \( \chi \), should be equal to half the solid angle[7] subtended by the geodesic triangles at the centre of Bloch sphere formed by either \( g_1 \) or \( g_2 \). This implies that \( g_1 \) should correspond to \( \Omega = \pi \) and \( g_2 \) should correspond to \( \Omega = 0 \) respectively, i.e. \( \chi(g_1) \) is real and negative and \( \chi(g_2) \) is real and positive. So, \( g_1 \) type of closed path which encloses the origin are topologically distinct from that of \( g_2 \) type of path which do not enclose the origin and this fact manifests itself as sign of \( \chi \) in the transmission amplitude. Hence the conductance corresponding to the case of \( g_1 \) and \( g_2 \) can be distinguished in terms of a \( \mathbb{Z}_2 \) index given by

\[
\nu = \text{sgn}\{\chi\} = \text{sgn}\{\langle \psi_r|\psi_i \rangle \langle \psi_t|\psi_i \rangle \langle \psi_t|\psi_r \rangle\} .
\]

Next we need to understand which situation corresponds to \( g_1 \) type of paths and which ones corresponds to \( g_2 \) type of paths on the Bloch sphere. To quantify this, we define parity of winding of the spin-texture for left and right moving electrons in the conduction and the valence band given by

\[
\lambda_{R,\pm} = \text{sgn}\left[ \int_0^{+\infty} \frac{\partial}{\partial k} \left( \theta_k^{\pm} \right) dk \right] ,
\]

\fig{1}{The left panel shows then dispersion for \( n \) doped \( x < 0 \) and \( p \) doped \( x > 0 \) sides of a n-p junction where the arrows show the spin texture. The right panel shows the position of the spinors corresponding to the incident (i, spin direction in orange ), reflected (r, magenta) and transmitted (t, red) electron wave functions at the Fermi level represented as three dots placed on the great circle contained in x-z plane on the Bloch sphere.}
\[ \lambda_{L, \pm} = \text{sgn} \left[ \int_{0}^{\infty} \frac{\partial}{\partial k} (\theta_k^\pm) \, dk \right], \]  

(7)

where \( \theta_k^\pm \) is expressed as \( \tan^{-1}(S_{k,z}^\pm/S_{k,x}^\pm) \) and \( S_{k,z}^\pm \) is the expectation value of the \( z/x \) components of the spin operator at a momentum \( k \) on the conduction and the valence band respectively. \( R \) and \( L \) stands for left and right movers respectively. This is an interesting quantity as it tells us which way is the spin twisting as we move from \( k = \pm \infty \) to \( k = 0 \) on the conduction and the valence band. Note that, as \( k \to \infty \), the spin-orbit field dominates over external B-field and dictates the orientation of spin associated with a momentum mode while at \( k = 0 \) the spin-orbit field vanishes and spin direction is decided by the externally applied magnetic field. Essentially these two facts decide the parity \( \lambda_{R/L, \pm} \). Also, note that the unphysical looking infinite momentum limits of the integrals can be replaced by a appropriate band cut-off imposed by the bulk gap of the QSH effect while we assume that the locally applied magnetic field on the edge induces a gap much smaller than the bulk gap. The relevance of this opposite winding can be seen as we evaluate the quantity \( \chi \) for the cases of n-p and n-n junctions. It is straightforward to check that all junctions which have an opposite parity of \( \lambda_{R/L} \) for electron bands participating in transport at the Fermi level on the two sides of the junction (i.e. p-n and n-p junctions) always correspond to the \( g_1 \) kind (see Fig.1) and hence corresponds to \( \nu = -1 \). On the other hand, if parity of \( \lambda_{R/L} \) is same on the two sides of junction (i.e. n-n and p-p junctions) then it always correspond to \( g_2 \) kind and hence corresponds to \( \nu = 1 \). This observation completes the topological classification of such junctions with negative value for the \( \mathbb{Z}_2 \) index for p-n, n-p junctions and positive value for the \( \mathbb{Z}_2 \) index for n-n, p-p junctions. This is one of the central results of this Letter. Next we show that this topological phase (\( \Omega \)) becomes geometric as we turn on a finite \( \phi \).

Non-topological phase for \( \phi \neq 0 \) and junction dualities:

Now we consider a situation where the two sides of the junction are exposed to magnetic field pointing along two different directions in the \( x-y \) plane (as given in Eq[1]) which implies that all momentum eigenstates belonging to the patch \( -\infty \leq x < 0 \) can be represented by points on Bloch sphere contained on a great circle lying in the \( x-z \) plane. While all states belonging to patch \( 0 < x \leq \infty \) will be contained on great circle contained in the Bloch sphere lying in the \( n-z \) plane where \( n = (B \cos \phi) \hat{x} + (B \sin \phi) \hat{y} \). From this fact it is clear that the incident, reflected and the transmitted spinor which goes into the construction of \( \chi \) can no longer be represented by three distinct points on the Bloch sphere which could be spotted on a single great circle. This in turn implies that half the solid angle subtended by the geodesic triangles formed by cyclic projection of these three states can never be equal to \( 2\pi \) (i.e. \( \Omega \neq \pi \)) and its value in general will depend on details of the position of these three states on the Bloch sphere hence rendering it non-topological or geometric (see Fig.2 for variation of \( \Omega \) (see [11]). Actually the angle \( \phi \) provides an efficient handle on this geometric phase which could be tuned to be topological in two limits, i.e. for \( \phi = 0 \) (studies above) and \( \phi = \pi \) when all the states once again lie on a single great circle on the Bloch sphere. The \( \phi = \pi \) is an interesting case as it flips the signs of both \( \lambda_{R, \pm} \) and \( \lambda_{L, \pm} \) hence resulting in switching between \( g_1 \) type of paths with \( g_2 \) type of paths and vice-versa. This amounts to saying that the a n-p junction for \( \phi = 0 \) case is topologically equivalent to n-n junction for the \( \phi = \pi \) case and similarly p-n junction for \( \phi = 0 \) case is topologically equivalent to the p-p junction for the \( \phi = \pi \). Hence \( \phi = 0 \to \phi = \pi \) defines a duality transformation between the n-n and n-p junctions and similarly between n-p and p-p junctions. Actually \( \phi = 0 \to \phi = \pi \) results in switching of two-terminal conductance between \( \{p-n, n-n\} \) junctions with \( \{p-p, n-p\} \) junctions respectively which is depicted in the plot in Fig.2.

Proposed experimental protocol: We have established a topological difference between the n-n and n-p junctions. To quantify this difference in terms of experimentally measurable quantity like conductance, we consider a situation where left gate (\( V_{G_1} = \delta_1 \)) is tuned to a n-type doping (i.e. \( \delta_1 < 0 \)) while the right gate \( V_{G_2} \) is kept flexible so that it can be tuned to either a n-type making the system a n-n junction or to a p-type making it a n-p junction by tuning the sign of \( V_{G_2} = \mp \delta_2 \) respectively where \( \delta_2 \to 0 \). We also assume that the applied voltage bias is such that it is driving an electronic current from left to right. Owing to the particle-hole symmetry of the Dirac spectrum, the value of \( \rho_{t}/\rho_{i} \) which appears in the expression of conductance (see Eq[2]) will be identical for a n-n and n-p junction provided we keep \( V_{G_2} = \delta_2 \) fixed and switch between the two types of junction only by changing the sign of \( V_{G_2} \) while keeping its magnitude fixed. Hence the difference in conductance (denoted by \( \Delta G_{\delta_1, \delta_2}^{\phi} \) of a n-n and n-p junction for a given value of \( V_{G_1} \), common to both while \( V_{G_2} \) being different for the two junction only in sign but with same magnitude leads to a quantity which depends linearly on the difference of \( S(\alpha, \beta, \gamma) \) (see Eq[3]) call it \( \Delta S \) as the dependence on the ratio of density of states (\( \rho_{t}/\rho_{i} \)) can be pulled out of the expression as overall scale factor. Now, note that \( S(\alpha, \beta, \gamma) \) represents a part of the conductance which depends only on the spin-texture mismatch at the given junction and is solely responsible for the topological classification of that junction. Hence measurement of \( \Delta G_{\delta_1, \delta_2}^{\phi} = (G_{\delta_1, \delta_2} - G_{\delta_1, -\delta_2}) \propto \Delta S \) could be viewed as a direct quantification of the topological difference between the n-n and n-p junction. Further more, due to the duality defined earlier between n-n and n-p junction, one expects the sign of \( \Delta G_{\delta_1, \delta_2} \) to flip keeping its magnitude.
The problem of


As the problem exhibits the well-known renormalisation group (RG) flow for conductance typical of single impurity problem in Luttinger liquids[20][22]. The details of the RG flow depends on Luttinger parameter of the first and the second semi-infinite Luttinger liquid. Once interactions are switched on, if the RG flow for the n-n and n-p junctions are different then this could lead to change in sign of \( \Delta G_{\delta_1,\delta_2} \) under RG flow rendering \( \Delta G_{\delta_1,\delta_2} \) useless for experimental protocol described above. Hence to understand the influence of interactions on \( \Delta G_{\delta_1,\delta_2} \) we need to calculate the dependence of Luttinger parameter \( K \) on n-type doping and p-type doping. To proceed further we first linearize the spectrum around the Fermi level so that the electron creation operator could be written as

\[
\psi_\epsilon(x) = \psi_{R,\epsilon}e^{i\epsilon k_F x}a_{R,\epsilon}(x) + \psi_{L,\epsilon}e^{-i\epsilon k_F x}a_{L,\epsilon}(x)
\]

where \( \psi_{R/L} \) is evaluated at the Fermi energy and \( \psi_{n/R}(x) \) represents slow chiral degrees of freedom which could be bosonized using standard bosonization technique[23]. Following Ref[24] it is straightforward to obtain the Luttinger parameter for the conduction(\( \epsilon = 1 \)) and valence(\( \epsilon = -1 \)) band as

\[
K_\epsilon = \frac{1}{\sqrt{\frac{e^2}{\hbar^2} \frac{\pi v_F}{K_F}}} \left( 1 + \frac{V(0)}{\pi v_F} - \frac{V(2K_F)\langle \psi_{L,\epsilon}^\dagger \psi_{R,\epsilon} \rangle^2}{\pi v_F} \right)^{-1/2}.
\]

where \( V(0) \) and \( V(2K_F) \) are the Fourier transforms of screened Coulomb potential at zero and 2\( K_F \) momenta. From Eq[3] it can be checked that the quantity \( \langle \psi_{L,\epsilon}^\dagger \psi_{R,\epsilon} \rangle \) is identical for the conduction and the valence band provided we compare the situations with identical doping with respect to the zero doping situation (i.e. with respect to Fermi level lying in the middle of the gap). This fact is a direct consequence of the particle-hole symmetry of Dirac spectrum. This leads to a symmetry in Luttinger parameter of the conduction and the valence band which immediately implies that \( G_{\delta_1,-\delta_2} \) and \( G_{\delta_1,\delta_2} \) renormalize identically under the influence of interaction. This implies that the RG flow preserves the sign of \( \Delta G_{\delta_1,\delta_2} \) and hence it continues to be reasonable quantity to be measured for confirming the proposed topological classification even in the presence of interactions.

**Discussion on experimental feasibility**: As the proposed experiment requires application of magnetic field on the edge which breaks TRS in general, a study of hierarchy of energy scales, which will protect the topological gap in the bulk but will allow for minimal breaking of TRS on the edge leading to a gapped edge spectrum is desirable. Experiments by König et.al[25] reported that the QSH state is destroyed by a magnetic field which is of the order of 0.7 T for an in-plane field and 28 mT if out of plane. Hence an in-plane field of the order of 50 mT can be considered safe in the sense that that it will not destroy or distort the bulk QSH state but will surely open up a gap in the edge spectrum. A 50 mT field amounts to gap of 145 \( \mu eV \) in the edge spectrum if we assume a Landé-g factor of 50 which is the g-factor for the bulk HgTe. Now, this gap of 145 \( \mu eV \) amount to an equivalent temperature of 1.7K while the temperatures at which the experiments were done are of the order of 30 mK[25]. Hence the present day experiment on QSH samples can resolve these gaps very well which makes our proposal quite feasible. The in-plane field of 50 mT considered above for opening up gap in edge spectrum is motivated from the experiment of Karmakar et. al[18] where nano-magnets were planted on the edge for quantum Hall sample for applying localized magnetic field on the edge states alone. Hence such a nano-magnetic array could be used for implementing the magnetic field to be applied on the QSH edge. Also, we have neglected the possibility of electrical gating induced spin-orbit coupling as the estimated strength is of the order of \( 5 \times 10^3 m/s \) while edge Fermi velocity is of the order of \( 5.5 \times 10^5 m/s \). Hence the percentage change in Fermi velocity induced due to the spin-orbit interaction will be only 0.4 percent which is negligible. Lastly, we point out that the experimentally observed elastic mean free path in these edge states are of the order of 1 \( \mu m \)[25][26]. As our calculations are performed for a ballistic limit, a device of length of a few micrometers could be optimal for implementing our proposal.

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