Modal simulation analysis based on vibration characteristics of Euler Beam

Ya-na Li*,1,a, Li-li Xu1,b
College of Locomotive and Rolling Stock Engineering, Dalian Jiaotong University, Dalian, Liaoning, China
aemail: lyn@djtu.edu.cn, bemail: 1968200974@qq.com
*Corresponding author: aemail: lyn@djtu.edu.cn

Abstract: Taking the Euler Beam as the research object, the finite element models of beam structure under five support conditions, using solid elements, shell elements and beam elements respectively, are established for considering the influence of different boundary conditions. Based on the eigenvalue method, the first three natural frequencies of the beam structure under five kinds of support conditions are calculated. Through the contrast analysis of the theoretical value and simulation value, the most reasonable simulation constraints are given for different element types. Also, the selecting rules of the element numbers and element types for the Euler Beam are offered which provide the reference and basis for the beam structure modal calculation.

1. Introduction
With the development of science and technology, the beam structure is widely used in hot fields such as machinery, architecture, civil engineering, aviation and aerospace [1-2]. It is also the most basic load-bearing structure in housing construction, roads, bridges, railways and other engineering structures. Many researchers have explored and studied the beam structure, and the vibration characteristics of the beam is one of the hot topics. When the external load frequency of the beam is close to its natural frequency, the resonance will occur and produce a certain damage effect. Therefore, the study of the natural frequency of the beam can effectively avoid the damage and loss caused by the resonance phenomenon.

The analysis of beam vibration is main to study the natural frequency. Zhang Yahui and Zhu Dechao [3-4] used eigenvalue method to calculate the theoretical values of several common support conditions; Lu Hongming and Xu Gening et al. [5-6] theoretically analyzed the influence of constraint conditions on structural modes when they studied the dynamic study of short beams. Lu,Yingfa [7] analyzed that the boundary support conditions of continuous girder bridges had great influence on the results of structural analysis. Zhang Liang et al. [8] used three-dimensional deformable solid element, shell element and beam element to establish track finite element model for comparative calculation. Liu Wei, Dai Liling et al. [9-10] showed the calculation method of natural frequency of simply supported beam and compared it with experiment. Wei Xugou et al. [11] compared the results of finite element analysis with theoretical calculation results and experimental results, also obtained a more suitable constraint condition for numerical simulation of simply supported beam. Lu Yingfa et al. [12] studied the boundary condition of cantilever beam in theory. These studies did not consider the comparison of modal theoretical calculation and simulation analysis of beam structures under different
element types and different support conditions, and did not give a detailed definition of finite element simulation constraints.

In this paper, taking Euler Beam as the research object, the finite element model of rectangular section beam is established by solid element, shell element and beam element respectively in the process of simulation. Based on different element types and different support conditions, different constraint conditions should be adopted to calculate the natural frequency. By comparing with the theoretical value, the law and the conclusion are found which can be used for reference in Euler Beam simulation.

2. Theoretical Analysis of Vibration and calculation of Natural Frequency of Euler Beams

It is assumed that the beam structure, a constant section beam, has the characteristic of symmetry. And in the process of bending, the deflection curve, that is, the axis of the beam is always kept in the symmetrical plane. The transverse direction of the beam is x-axis (positive to the right), and the axis perpendicular to the X-axis is the Y-axis (positive upward) in the symmetrical plane.

It can be obtained according to the formula of material mechanics.

\[ \frac{EI}{L} \frac{d^2y}{dx^2} = M \]  \hspace{1cm} (1)

\[ \frac{EI}{L} \frac{d^2y}{dx^2} = Q \]  \hspace{1cm} (2)

Where \( E \) is the elastic modulus, \( I \) is the moment of inertia, \( M \) is the torque on the micro-segment, \( Q \) is the shear force on the micro-segment. And equation (2) is the deflection differential equation of constant section beam. With the application of Darumbo’s principle and applying the inertia force on the beam is \( \frac{\partial Q}{\partial x} = -\frac{\partial^2y}{\partial t^2} \), the differential equation of free bending vibration of beam can be obtained as equation(3) by \( a^2 = EI/\rho \).

\[ \frac{\partial^2y}{\partial x^2} + \frac{1}{a^2} \frac{\partial^2y}{\partial t^2} = 0 \]  \hspace{1cm} (3)

Suppose the solution of equation (3) can be written as

\[ y(x,t) = X(x)Y(t) \]  \hspace{1cm} (4)

By using the method of separating variables, the following can be obtained:

\[ Y(t) = A\sin \omega t + B\cos \omega t \]  \hspace{1cm} (5)
Among them, "\( \omega \) is the natural frequency of the beam", \( \beta \) is the root of the characteristic equation, and \( A, B, D_1, D_2, D_3, D_4 \) are all undetermined constants, which are determined by the support conditions.

Then, the condition of support can be written as

- Fixed end \( X=0 \):
  \[
  \frac{dX}{dx} = X' = 0
  \]

- Simply supported end \( X=0 \):
  \[
  \frac{d^2X}{dx^2} = X'' = 0
  \]

- Free end \( X=0 \):
  \[
  \frac{d^3X}{dx^3} = X''' = 0
  \]

In large-scale equipment, the local vibration is more obvious, while the overall vibration can often reach more than hundreds of hertz or more in the low-order frequency domain, so only the low-order frequency domain characteristics of the beam structure are generally considered when calculating the natural frequency of the beam structure. The first three theoretical natural frequencies of Euler Beam under five different boundary conditions: fixed at both ends, free at one end, free at both ends, simply supported at both ends and fixed at one end, simply supported at another end, as shown in Table 1.

**Table 1  Theoretical results**

| Support conditions                              | Modes | Frequency f/Hz |
|------------------------------------------------|-------|----------------|
| Fixed at both ends                             | 1     | \( f_1 = \left(\frac{4.73}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 2     | \( f_2 = \left(\frac{7.853}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 3     | \( f_3 = \left(\frac{1100}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
| Fixed at one end, free at another end           | 1     | \( f_1 = \left(\frac{1.875}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 2     | \( f_2 = \left(\frac{4.694}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 3     | \( f_3 = \left(\frac{7.85}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
| Free at both ends                               | 1     | \( f_1 = \left(\frac{4.73}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 2     | \( f_2 = \left(\frac{7.853}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 3     | \( f_3 = \left(\frac{1100}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
| Simply supported at both ends                   | 1     | \( f_1 = \left(\frac{\pi}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 2     | \( f_2 = \left(\frac{3\pi}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 3     | \( f_3 = \left(\frac{3\pi}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
| Fixed at one end, simply supported at another end| 1     | \( f_1 = \left(\frac{3.927}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 2     | \( f_2 = \left(\frac{7.069}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
|                                                | 3     | \( f_3 = \left(\frac{10.210}{L}\right) \sqrt{\frac{EI}{\rho^4}} \) /2\( \pi \) |
3. Finite element simulation analysis

Three element types, solid element, shell element, and beam element, are used respectively to establish Euler Beam structure finite element model. Taking a rectangular finite element beam structure to simulate, as shown in figure 2, the basic parameters are: B=20mm (the section width), H=10mm (section height), L=500mm (length), \( \rho = 7.85 \times 10^{-9} \text{kg/mm}^3 \) (density), \( E = 210000 \text{MPa} \) (elastic modulus), \( \mu = 0.3 \) (Poisson's ratio) and element length is 5mm. Considering different support conditions and different element types, the simulation conditions which are closest to theoretical calculation value are obtained. The simulation calculation results are compared to the theoretical results calculated by the characteristic value, as shown in Table 2.

![Fig.2 The finite element model schematic diagram](image)

### Table 2: Theoretical and simulation results

| Element type | Support condition                  | Simulation constraint                                                                 | Modes | Theoretical results /Hz | Simulation results /Hz | Error range |
|--------------|-----------------------------------|---------------------------------------------------------------------------------------|-------|-------------------------|------------------------|-------------|
| Solid element | Fixed at both ends                | Constrain the axial, vertical and lateral displacements of all nodes at both ends.     | 1     | 212.7                   | 213.1                  | 0.19\%      |
|              |                                   |                                                                                       | 2     | 586.2                   | 584.4                  | -0.32\%     |
|              |                                   |                                                                                       | 3     | 1149.0                  | 1143.7                 | -0.46\%     |
|              | Fixed at one end, free at another end | Constrain the axial, vertical and lateral displacements of all nodes at fixed end, and non-constrain at the free end. | 1     | 33.4                    | 33.5                   | 0.30\%      |
|              |                                   |                                                                                       | 2     | 209.4                   | 209.4                  | 0.00\%      |
|              |                                   |                                                                                       | 3     | 584.9                   | 584.7                  | -0.03\%     |
|              | Free at both ends                 | Non-constrain                                                                          | 1     | 212.7                   | 212.3                  | -0.19\%     |
|              |                                   |                                                                                       | 2     | 586.2                   | 583.7                  | -0.43\%     |
|              |                                   |                                                                                       | 3     | 1149.0                  | 1140.0                 | -0.78\%     |
|              | Simply supported at both ends     | Constrain the axial, vertical and transverse displacement of the rigid connection at simply supported one end, and constrain the vertical, lateral displacement of the rigid connection at another end. | 1     | 93.8                    | 93.8                   | 0.00%       |
|              |                                   |                                                                                       | 2     | 375.1                   | 374.1                  | -0.27%      |
|              |                                   |                                                                                       | 3     | 843.9                   | 838.4                  | -0.65%      |
|              | Fixed at one end, simply supported at another end | Constrain the axial, vertical and transverse displacement of the rigid connection at the fixed end, and constrain the axial, vertical displacement of the rigid connection at simply supported one end. | 1     | 146.7                   | 146.7                  | 0.00%       |
|              |                                   |                                                                                       | 2     | 475.2                   | 474.0                  | -0.25%      |
|              |                                   |                                                                                       | 3     | 991.4                   | 985.3                  | -0.62%      |
| Shell element | Fixed at both ends                | Constrain the axial, vertical, transverse displacement and co-rotation of all nodes at both ends. | 1     | 212.7                   | 212.4                  | -0.16%      |
|              |                                   |                                                                                       | 2     | 586.2                   | 583.6                  | -0.45%      |
|              |                                   |                                                                                       | 3     | 1149.0                  | 1139.2                 | -0.86%      |
|              | Fixed at one                      | Constrain the axial, vertical, transverse displacement and co-rotation of all nodes at both ends. | 1     | 33.4                    | 33.4                   | 0.06%       |
4. Calculation results and analysis

As can be seen from Table 2, using solid element, shell element and beam element to simulate Euler Beam under various support conditions, the calculated natural frequencies from the first order to the third order are relatively close to the theoretical values, and the error range is less than 2%, indicating the correctness of the simulation model.

Analyzed from element type, for example, Euler Beam on the condition of fixed at one end and simply supported at another end, the first-order natural frequency simulated by solid element is 146.5Hz and the error range is -0.17%, however by beam element is 146.4Hz and the error range is -0.24%. The error between the three frequency values and the theoretical value is very small. In general, the solid
element can better simulate the natural frequency of Euler Beam, and the simulation accuracy of beam element is slightly lower.

Analyzed from the natural frequency order, with the increase of the modes of the beam structure, the error between the simulation solution and the theoretical solution gradually increases. For example, when the shell element simulation calculation is used in the condition of fixed at both ends, the difference between the first-order frequency simulation calculation and the theoretical value is-0.16%, the second-order frequency is-0.45%, and the third-order frequency is-0.86%. Therefore, it is necessary to increase the number of finite elements in order to ensure the correctness of high-order frequencies.

5. Conclusion
Based on the theoretical analysis of Euler Beam mode, taking the rectangular section beam structure as the research object, the natural frequency values of Euler Beam under five kinds of support conditions are theoretically calculated with the eigenvalue method. The finite element model of the beam structure is established by using three types of elements, considering different support conditions for simulation. Compared the theory and simulation results, the law is found which provide technical support for the simulation of beam results in the future. The main conclusions are as follows:

(1) In the simulation calculation, due to the different types of elements and different conditions of support, the constraints are also different. Through theoretical calculation and simulation analysis, the most reasonable constraint conditions of finite element simulation of three types of elements under five supporting conditions are obtained, which can be used as a reference for the simulation calculation of beam structures in the future.

(2) After comparison, among the three types of elements, the simulation accuracy of the solid element is the highest, but the beam element is the lowest. And the most of the natural frequencies calculated by the three elements are less than the theoretical solution. The reason is that the model is simplified to a certain extent and the stiffness can not be completely consistent, so it is best to use solid element for simulation analysis when high precision modeling is required.

(3) The error between the natural frequency and the theoretical solution increases with the increase of modes, so fewer elements can be selected when calculating low-order modes, but more elements should be selected when calculating high-order natural frequency. And to ensure that the error is controlled within a reasonable range, reducing the size of the element is needed.

Acknowledgments
This work was supported by Liaoning natural science foundation -- Research on bolt strength checking method of high-speed EMU connecting parts (2019-ZD-0100).

References
[1] Xu Zhilun. Elasticity [M]. Beijing: Higher Education Press, 2016: 26-27.
[2] Wu Jialong. Elasticity[M]. Beijing: Higher Education Press, 2001: 10-11.
[3] Zhang Yahui, Lin Jia Hao. Fundamentals of Structural Dynamics [M]. Dalian University of Technology Press, 2007: 263.
[4] Zhu Dechao. Xing Yufeng. Engineering Vibration Foundation[M]. Beijing: Beijing Aeronautics and Astronautics Press, 2004: 89-96.
[5] Lv Hongming. Study on the influence of boundary conditions on the finite element analysis of short beam[J]. Chinese Journal of Engineering Design, 2013, 20(04): 321-325.
[6] Xu Gening, Feng Xiaolei, Tao Yuanfang, Yang Ruigang. Influence of Boundary Conditions on Finite Element Analysis Results of Mechanical Structures [J]. Hoisting and Conveying Machinery, 2010(02): 60-64.
[7] Lu Yingfa. Deformation and failure mechanism of slope in three dimensions[J]. Journal of Rock Mechanics and Geotechnical Engineering, 2015: 109-119.
[8] Zhang Liang, Hu Zhiren. Finite Element Simulation Analysis of Track Vibration Characteristics Based on ABAQUS [J]. Digital Ocean & Underwater Warfare, 2018, 1(02): 86-90.

[9] Liu Wei, Chen Junjie. An Analysis and Test of the Modal Parameter of Free Beam [J]. Journal of Beijing Union University (Natural Sciences), 2011: 40-43.

[10] DAI Liling. Experimental Testing Methods and Theoretical Analysis of Simple Beam with Natural Frequency and Inherent Vibration [J]. Journal of Kunming University, 2008: 92-95.

[11] Wei Xuhao, Ye Hongling, Zheng Xiaolong. Modal Analysis of Simply Supported Beam Based on ANSYS and MSC Patran & Nastran [A]. Beijing Society of Mechanics. Collection of abstracts of the 15th academic annual meeting of Beijing Society of Mechanics [C]. Beijing Society of Mechanics: Beijing Society of Mechanics, 2009: 2.

[12] Lu Yingfa, Xi Changzhi, Liu Minwei, Liang Chen. Investigation into Generalized Theoretical Solution of Cantilever Beam [J]. J of China Three Gorges Univ. (Natural Sciences), 202, 42(01): 57-62. (in Chinese)