Maximizing Social Welfare in Selfish Multi-Modal Routing using Strategic Information Design for Quantal Response Travelers

Sainath Sanga,
Venkata Sriram Siddhardh Nadendla,
Sajal K. Das

Abstract

Traditional selfish routing literature quantifies inefficiency in transportation systems with single-attribute costs using price-of-anarchy (PoA), and provides various technical approaches (e.g. marginal cost pricing) to improve PoA of the overall network. Unfortunately, practical transportation systems have dynamic, multi-attribute costs and the state-of-the-art technical approaches proposed in the literature are infeasible for practical deployment. In this paper, we offer a paradigm shift to selfish routing via characterizing idiosyncratic, multi-attribute costs at boundedly-rational travelers, as well as improving network efficiency using strategic information design. Specifically, we model the interaction between the system and travelers as a Stackelberg game, where travelers adopt multi-attribute logit responses. We model the strategic information design as an optimization problem, and develop a novel approximate algorithm to steer Logit Response travelers towards social welfare using strategic Information design (in short, LoRI). We demonstrate the performance of LoRI on a Wheatstone network with multi-modal route choices at the travelers. In our simulation experiments, we find that LoRI outperforms SSSP in terms of system utility, especially when there is a motive mismatch between the two systems and improves social welfare. For instance, we find that LoRI persuades a traveler towards a socially optimal route for 66.66\% of the time on average, when compared to SSSP, when the system has 0.3 weight on carbon emissions. However, we also present a tradeoff between system performance and runtime in our simulation results.

Introduction

Smart navigation systems (e.g. GPS devices, navigation applications on mobile/smart devices) have transformed the transportation domain in terms of reducing cognitive overload in travelers. However, such technological advancements have had little impact on several fundamental issues such as mitigating congestion [INRIX 2020] and reducing carbon emissions [Literacy 2020], which have only worsened over time. For instance, current state-of-the-art navigation systems employ traditional shortest paths algorithms, such as Dijkstra’s algorithm [Lanning, Harrell, and Wang 2013], Bellman–Ford or Warshall-Floyd, and A*-algorithms [Cormen et al. 2009] to recommend routes and mitigate travelers’ cognitive overload. On the other hand, selfish travelers exhibit multi-attribute preferences, which are typically misaligned from system’s interests. As a result, travelers often reject route recommendations that involve non-personal transport modalities, such as public transportation, ridesharing services and other micro-mobility services [Bureau 2014; McKenzie et al. 2015]. Although unintentional, people have steered away from personal car usage during the ongoing COVID-19 pandemic in 2020 [Wash 2020], which have resulted in significant cost reductions in terms of congestion, carbon emissions as well as collisions.

Our goal in this paper is to steer selfish travelers away from personal car usage (even under non-pandemic conditions), via offering them alternative routing choices in a persuasive manner.

Selfish routing is a strategic framework where travelers employ their best-response routes selfishly according to their respective preferences to form an equilibrium. However, the central authority (e.g., a city transportation department) chooses a social-welfare objective that is not necessarily aligned with all travelers’ interests. This leads to system inefficiency, which can be quantified by price-of-anarchy (PoA) [Roughgarden and Tardos 2002]. Several techniques have been proposed to drive PoA towards unity, which happens when the equilibrium outcome is optimal in terms of the system’s objective. A seminal example is marginal cost pricing, where selfish travelers are imposed taxes based on their marginal contribution to the system’s objective [Ruggles 1949]. Although the idea of marginal cost pricing has been floating around for several decades, the technique remains practically infeasible due to our inability to estimate marginal costs accurately.

In [Sharon et al. 2019], the authors studied the effects of underestimating marginal costs on the optimality in terms of system objectives, and showed that taxing underestimating marginal costs produces an outcome that is at least as good as having no taxes. Although attempts have been made to implement such solutions by authoritarian regimes [Yang, Purevjav, and Li 2020], the friction to adopt marginal cost pricing continues to persist due to various political reasons in democratic nations. Another powerful idea to influence traveler behavior is Stackelberg routing, where a fraction of agents are routed centrally, while the remaining agents are allowed to choose their routes selfishly.
behavior, while the travelers (followers) exhibit logit
the stochastic utility maximization framework (Luce 1959).
EUM at each traveler are captured by the randomness within
quantal response equilibrium (QRE), where deviations from
strategic information design framework as stated below.
are far from reality. Therefore, in this paper, we consider a
costs and unimodal transportation networks, all of which
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getic information design in transportation settings make sev-
ors in transportation domain when the network congestion
state is uncertainly available at the travelers. For example, in (Das, Kamenica, and Mirka 2017), the authors computed best-response signals under first-best, full information, public-signal and optimal information structure scenarios in the context of Wheatstone Network; they demonstrated that optimal information structures reveal only partial information revelation to mitigate network congestion. Similar results have been found in (Wu and Amin 2019) in the case of Pigou networks (graphs with parallel routes between a single-source and a single-destination) in the presence of state uncertainty on one of the routes. Optimal information structures have been found using Bayesian persuasion framework to reduce average traffic spillover on a specific route in a Pigou network.

Recently, strategic information design has been studied in the transportation domain where the network congestion state is uncertainly available at the travelers. For example, in (Das, Kamenica, and Mirka 2017), the authors computed best-response signals under first-best, full information, public-signal and optimal information structure scenarios in the context of Wheatstone Network; they demonstrated that optimal information structures reveal only partial information revelation to mitigate network congestion. Similar results have been found in (Wu and Amin 2019) in the case of Pigou networks (graphs with parallel routes between a single-source and a single-destination) in the presence of state uncertainty on one of the routes. Optimal information structures have been found using Bayesian persuasion framework to reduce average traffic spillover on a specific route in a Pigou network.

Despite the above development, existing works in strategic information design in transportation settings make several impractical assumptions. Since this is still a fledging topic, almost all efforts assume that travelers are expected utility maximizers (EUM). However, there has been a strong evidence from real-world observations that travelers deviate from EUM behavior quite frequently. Such an effort was first made in (Nadendla, Langbort, and Basar 2018), which studied strategic information design in a single-sender, single-receiver setting when both are prospect-theoretic agents. Nevertheless, this framework is not applicable to transportation domain where there are multiple receivers. Another impractical assumption is the consideration of single-attribute costs and unimodal transportation networks, all of which are far from reality. Therefore, in this paper, we consider a more realistic transportation framework and develop a novel strategic information design framework as stated below.

First, we assume that the travelers’ responses exhibit quantal response equilibrium (QRE), where deviations from EUM at each traveler are captured by the randomness within the stochastic utility maximization framework (Luce 1954). We model the strategic interaction with the system as a novel Stackelberg-QRE game, where the system (leader) exhibits EUM behavior, while the travelers (followers) exhibit logit

Let a multi-modal transportation network consisting of $\Lambda_t$ travelers at time $t$, be represented as a graph $G = \{V, E\}$, where $V = \{0, 1, \cdots , N\}$ represents the set of physical locations (vertices), and $E$ represents the transport interconnections (edges) between various locations in $V$. Let $G$ support a gamut of transport modalities $M = \{1, \cdots , M\}$. For the sake of convenience, we expand the network $G$ into a multi-layered graph $G_{exp}$, using unimodal subgraphs $\{G_m\}_{m \in M}$, and switch edge sets $E_{s,t}$ which interconnect $i^{th}$ modality to $j^{th}$ modality within each vertex. For example, consider a Wheatstone road network with four vertices and ten edges, as illustrated in Figure 1. Consider $M = 3$ transport modalities on this network, and $M = \{Private \text{ Car (colored black),.Metro Train (colored blue) and Walking (colored green)}\}$. Using unimodal subgraphs and switch edges (depicted using dashed lines), we expand the example network into a multi-layered graph $G_{exp}$, as shown in Figure 2.

We model the network state as $s_t = \{c_{e,t}\}_{e \in E}$, where $c_{e,t}$ is the number of travelers on edge $e \in E$ at time $t$.

Let there be a central entity (a.k.a. the system), which evaluates the network state in terms of the overall traffic congestion and carbon emissions using a weighted multi-
attribute cost. Assuming that there are $K$ attributes, each edge $e \in E$ has a multi-attribute cost vector $x(c_{e,t}) = \{x_1(c_{e,t}), \ldots, x_K(c_{e,t})\}$. The system evaluates the cost of each edge $e$ at time $t$ as

$$y(c_{e,t}) = \sum_{k=1}^{K} a_k \cdot x_k(c_{e,t}).$$

(1)

Since centralized systems typically have access to sensing infrastructure across the network to measure the network state in real-time, we assume that the system has greater information regarding the current state $s_t$ than the travelers.

In this paper, we assume that the system constructs a multi-dimensional signal $\mu_{\ell,t} = [\mu_{\ell,e,t}]_{e \in E}$ to steer $\ell^{th}$ traveler’s decision, where

$$\mu_{\ell,e,t}(\eta, \lambda) = [\mathbb{P}_e(c_{e,t+1} = \lambda|c_{e,t} = \eta)]^c_{\lambda=0},$$

(2)

is the state transition probability shared by the system to the $\ell^{th}$ traveler. The system constructs this signal with the goal of steering travelers’ decisions towards system’s optimal (a.k.a. social welfare).

Note that the overall system cost after a finite time horizon $T$ depends on decisions taken by all the active travelers and all the signals presented to the active travelers. It comprises of both past and future costs, and is given by

$$U_{0,T}(\mu_T, p_T) = \sum_{t=1}^{T} \sum_{e \in E} \sum_{\lambda=1}^{\infty} \psi_{\ell,e,t}(\lambda) y(\lambda),$$

(3)

where $\mu_T = [\mu_{1,T}, \ldots, \mu_{\ell,T}]$ is the signal profile sent to all the travelers in the network; $p_T = [p_{1,T}, \ldots, p_{\ell,T}]$ is the path profile chosen by the travelers; and $\psi_{\ell,e,t}(\lambda)$ denotes the a priori system’s belief probability regarding the state of edge $e$ being $c_{e,t} = \lambda$ at time $t$. Then, we define the system’s rationality as follows:

**Definition 1.** The system’s motive is to minimize its cost function that depends on all the travelers’ decisions and the signals presented by the system. The motive is given by:

$$\min_{\mu_T} U_{0,T}(\mu_T, p_T).$$

(4)

Although these signals can be revealed by the system at any time, the travelers can take advantage of this information and change their path only when they are present at some node. We label such agents as *active* travelers. In other words, we can define the state of the $\ell^{th}$ traveler at time $t$ as

$$\alpha_{\ell,t} = \begin{cases} 1 & \text{if the } \ell^{th} \text{ traveler is active}, \\ 0 & \text{if the } \ell^{th} \text{ traveler is inactive}. \end{cases}$$

(5)

In other words, an active traveler’s state gets updated to an inactive state as soon as an active traveler chooses the next edge, and remains so until he/she traverses that edge completely and reaches the other vertex as shown in Figure 3. That is, $c_{\ell,e,t}$ is equal to the total number of inactive travelers $\{\alpha_{\ell,t} = 0\}$ on edge $e$ at time $t$.

Furthermore, we assume that the travelers cannot fully observe the true network state $s_t$ at any given time, but can construct a multi-dimensional belief $\phi_{\ell,e,t} = [\phi_{\ell,e,t}]_{e \in E}$ about $s_t$ at time $t$ based on prior experiences, where

$$\phi_{\ell,e,t}(c_{\ell,e,t}) = \left\{ \phi_{\ell,e,t}(c) \right\}^{\infty}_{c=0}$$

(6)

is the traveler’s belief vector regarding the state of the edge $e \in E$ at time $t$, and $\phi_{\ell,e,t}(c) = \mathbb{P}_e(c_{e,t} = c)$. Assuming that the $\ell^{th}$ traveler’s multi-attribute cost $\psi_{\ell,e,t}$ on edge $e$ at time $t$ is a weighted linear combination of all attribute-wise edge costs $x(c_{e,t})$, as given by

$$z_{\ell}(c_{e,t}) = \sum_{k=1}^{K} b_{\ell,k} \cdot x_k(c_{e,t}),$$

(7)

we model the $\ell^{th}$ traveler’s stochastic expected cost for choosing a path $p_{\ell,T}$ as

$$V_{\ell,T}(\mu_{\ell,T}, \pi_{\ell,T}) = \mathbb{E}_{\pi_{\ell,T}}[U_{t,T}(\mu_{\ell,T}, p_{\ell,T})] + \epsilon_{p_{\ell,T}},$$

(8)

1If some attribute $k$ is not applicable to a given edge $e \in E$, then we let $x_k(c_{e}) = 0$. For example, the attribute ‘CO emissions’ is not applicable to all the edges of mode ”walking”, for these edges, we let $x_{CO}(c_{e}) = 0.$
where $\pi_{\ell, T}$ is a probability distribution over the set of all paths $\mathcal{P}_T$, $U_{\ell,T}(\mu_{\ell,T}, \pi_{\ell,T})$ denotes the nominal (known) expected cost of the traveler, and $\epsilon_{\ell, T}$ is the noise (random parameter) term that captures any uncertainty regarding $\ell^{th}$ traveler’s rationality. The decision policy adopted by the $\ell^{th}$ traveler at time $t$ is denoted as the path $p_{\ell, t} \in \mathcal{P}_T$, where $\mathcal{P}_T$ represents the set of all paths available for the $\ell^{th}$ traveler.

Let $p_{\ell, 1:T}$ denote the sequence of edges that the $\ell^{th}$ traveler has already taken (committed) until time $T$. Then, the $\ell^{th}$ traveler’s expected cost $U_{\ell,T}(\mu_{\ell,T}, p_{\ell,T})$ comprises of two terms: the incurred (deterministic) cost from traversed, and the future (unknown) cost from the remaining path to be traversed. In other words, we have

$$U_{\ell,T}(\mu_{\ell,T}, p_{\ell,T}) = \sum_{e \in p_{\ell, 1:T}} z\ell(e, \ell, e) + \sum_{e \in p_{\ell, 1:T} - p_{\ell, 1:T}} \left( \sum_{\lambda=1}^{\infty} \phi\ell, e, t(\lambda) \cdot z\ell(\lambda) \right), \quad (9)$$

where $t\ell, e$ is the time at which the traveler is at the head of edge $e$, and $p_{\ell, 1:T} - p_{\ell, 1:T}$ represents the sequence of edges that the traveler will travel in the future, if he/she continues to stay on the same decision policy $p_{\ell, T}$. Then, the traveler’s rationality is defined as follows:

**Definition 2.** The traveler’s motive is to minimize the random cost function that depends on the signals presented by the system and the path chosen by the traveler, which is given as:

$$\min_{\pi \in \Delta(\mathcal{P}_T)} V_{\ell,T}(\mu_{\ell,T}, \pi_{\ell,T}), \quad (10)$$

Given that both the system and travelers have non-identical utilities (i.e., mismatched motives), it is natural to model their interaction as a one-shot Stackelberg-Quantal-Response (SQR) game, where the system commits to its signaling strategy as defined in Definition [1] before travelers choose their stochastic policies as per Definition [2] (Fudenberg and Tirole [1991]).

**Definition 3.** The equilibrium of an SQR game between the system and travelers is defined as the pair $(\mu_{\ell, t}^*, \pi_{\ell, t}^*)$, where

$$\mu_{\ell, t}^* = \arg\min_{\mu_{\ell,t}} U_{0,T}(\mu_{\ell,t}, \mu_{\ell, t-1}, p_{\ell, t}, p_{\ell-1, t}) \quad (11)$$

$$\pi_{\ell, t}^* = \arg\min_{\pi_{\ell,t}} V_{\ell, t}(\mu_{\ell, t}^*, \pi_{\ell, t})$$

Similar to solving traditional Stackelberg-Nash games, we propose a novel solution approach named LoRI based on backward induction, which evaluates travelers’ quantal response equilibrium as a function of system’s signal $\mu_{T}$, and then evaluate the best response signal at the system. We present the technical details of our approach in the following section, and later analyze its performance in simulation experiments.

**Equilibrium Analysis**

In order to carry out equilibrium analysis, it is necessary to evaluate the path costs at the $\ell^{th}$ traveler, which depend on the network state. However, given that the network state evolves over time with all the active travelers’ path choice updates, we first compute the network state based on travelers’ strategy profiles using Algorithm 1. Given the network state, we evaluate the cost of traversing a path $p_{\ell, t} \in \mathcal{P}_T$ at the $\ell^{th}$ traveler using Algorithm 2. Note that the term $z\ell(s_i[e])$ in Algorithm 2 represents the cost of traveling on edge $e$ at time $t$ at the $\ell^{th}$ traveler, when its state is given by $s_i[e] = c_{e, t}$. Given the cost matrix, we now proceed to evaluating the equilibrium of the proposed SQR game using backward induction, i.e. evaluate travelers’ QRE as a function of system’s signal, and then compute the best-response signal at the system.
Traveler’s Quantal Response Analysis

Given the system’s signal \( \mu_{t,t} \), the traveler updates his prior belief defined in Equation (5) using Bayes rule to obtain the following posterior belief regarding the network state:

\[
\phi_{\ell,e,t+1}(\lambda) = \frac{\phi_{\ell,e,t}(\eta) \cdot \mu_{\ell,e,t}(\eta, \lambda)}{\sum_{\lambda=0}^{\infty} \phi_{\ell,e,t}(\eta) \cdot \mu_{\ell,e,t}(\eta, \lambda)}. \tag{12}
\]

We assume that the denominator in Equation (12) always converges to some value in the region \([0, 1]\) and every traveler’s belief regarding the future state of the network remains stationary until the system presents a signal.

The cost that the traveler attains by choosing a path \( p_{\ell,t} \in \mathcal{P}_{\ell,t} \) is decomposed into (i) known (nominal) cost at the traveler, and (ii) an unknown random cost \( \epsilon_{\ell,T} \), as shown in Equation (8). In this paper, we assume that the noise term \( \epsilon_{\ell,T} \) in the traveler’s expected cost is independently identically distributed extreme value, also known as Gumbel distribution.

**Theorem 1** (Luce 1959). The \( \ell \)-th traveler’s logit choice probability for the path \( p_{\ell,t} \) at time \( t \) is given by:

\[
\pi_{\ell,T}(p_{\ell,T}) = \frac{\exp(\alpha \cdot U_{\ell,T}(\mu_{\ell,T}, p_{\ell,T}))}{\sum_{p_{\ell,T} \in \mathcal{P}_{\ell,T}} \exp(\alpha \cdot U_{\ell,T}(\mu_{\ell,T}, p_{\ell,T}'))}, \tag{13}
\]

where \( \alpha \geq 0 \) is the parameter of the quantal response model.

To compute the Quantal Response Equilibrium for the travelers, we use Gambit (McKelvey, McLennan, and Turocy 2006). Gambit is a library of game theory software and tools for the construction and analysis of finite extensive and strategic games. We build a strategic game (Normal-Form game) between all the travellers and use Gambit’s tool gambit – logit to solve for QRE. Gambit computes the principle branch of the (logit) quantal response correspondence using the predictor-corrector method based on the procedure described in (Turocy 2005). The predictor-corrector method first generates a prediction using differential equations describing the branch of the correspondence, followed by a corrector step which refines the prediction using Newton’s method for finding a zero of a function.

Approximate-Response Signaling

At any time \( t \), let there be a total of \( \Lambda_t \) travelers on the network. Assuming that the \( \ell \)-th traveler is on edge \( e_{\ell,t} \) at time \( t \) due to the decision \( p_{\ell,t} \), we can compute the total number of travelers on edge \( e \in \mathcal{E} \) at the time \( t \) as

\[
c_{e,t} = \sum_{\ell=1}^{\Lambda_t} \mathbb{I}(e_{\ell,t} = e), \tag{14}
\]

where \( \mathbb{I}(\cdot) \) represents the indicator function which takes the value 1 whenever the argument holds true. Given \( c_{e,t} \) at time

\[
t \text{ on every edge } e \in \mathcal{E}, \text{ we can now compute the state transition probability } \psi_{e,t+1} \text{ as follows:}
\]

\[
\psi_{e,t+1}(\eta, \lambda | \Lambda_t) = \mathbb{P}(c_{e,t+1} = \lambda | c_{e,t} = \eta)
\]

\[
= \mathbb{P} \left( \sum_{\ell=1}^{\Lambda_t} \mathbb{I}(e_{\ell,t+1} = e) = \lambda | \sum_{\ell=1}^{\Lambda_t} \mathbb{I}(e_{\ell,t} = e) = \eta \right).
\tag{15}
\]

Let \( \rho_{\ell,t}(e, e') \) denote the probability that the \( \ell \)-th traveler is on edge \( e \) at time \( t \) given that he is on edge \( e' \) at time \( t-1 \). Then, the state transition probability \( \psi_{e,t+1} \) can be evaluated using the following recursive relation:

\[
\psi_{e,t+1}(\eta, \lambda | \Lambda_t) = \rho_{\ell,t+1}(e, e') \cdot \psi_{e,t+1}(\eta, \lambda - 1 | \Lambda_t - 1)
\]

\[
+ (1 - \rho_{\ell,t+1}(e, e')) \cdot \psi_{e,t+1}(\eta, \lambda | \Lambda_t - 1)
\]

\tag{16}
\]

where

\[
\rho_{\ell,t+1}(e, e', \alpha_{e,t}) = \sum_{p_{\ell,t} \in \mathcal{P}_{\ell,t}} \pi_{\ell,t}(p_{\ell,t}|\mu_{\ell,t}, e \in p_{\ell,t}, e' \in p_{\ell,t-1}, \alpha_{e,t} = 1)
\]

\tag{17}
\]

The leader’s optimal strategy is to minimize its cost \( U_{0,T} \) which can be computed as:

\[
\min_{\mu_{0,T}} U_{0,T}(\mu_{0,T} - \pi_{0,T}(p_{0,T}|\mu_{0,T}, \mu_{-T}), p_{0,T} - \pi_{0,T}) \tag{P1}
\]

Using Equation (16), we write the term \( \psi_{e,t}(\lambda) \) and expand \( U_{0,T} \) as shown in Equation (18).

We further expand this using Equation (17). For better representation, we write \( \rho_{\ell,t+1}(e, e', \alpha_{e,t}) = \)
\[ U_{0,T} = \sum_{t=1}^{T} \sum_{e \in E} y(c_{e,t}) + \sum_{t=T+1}^{\infty} \sum_{e \in E} \sum_{\lambda=1}^{\infty} \left[ \mu_{e,t}(e', e') \cdot \psi_{e,t}(\eta, \lambda - 1|\Lambda_{t-1} - 1) ight. \\
+ \left. \left(1 - \mu_{e,t}(e', e')\right) \cdot \psi_{e,t}(\eta, \lambda|\Lambda_{t-1} - 1)y(\lambda) \right] \\
= \sum_{t=1}^{T} \sum_{e \in E} y(c_{e,t}) + \sum_{t=T+1}^{\infty} \sum_{e \in E} \sum_{\lambda=1}^{\infty} \left[ \left(\sum_{p_{\ell,t} \in P_{\ell,t}} \pi_{e,t}(p_{\ell,t}|\mu_{\ell,t})\right) \cdot \psi_{e,t}(\eta, \lambda - 1|\Lambda_{t-1} - 1) ight. \\
+ \left. \left(1 - \sum_{p_{\ell,t} \in P_{\ell,t}} \pi_{e,t}(p_{\ell,t}|\mu_{\ell,t})\right) \cdot \psi_{e,t}(\eta, \lambda|\Lambda_{t-1} - 1)y(\lambda) \right] \] (18)

\[
\sum_{p_{\ell,t} \in P_{\ell,t}} \pi_{\ell,t}(p_{\ell,t}|\mu_{\ell,t}) \text{. By Definition 2, } \mu_{\ell,t} = [\mu_{e,\ell,t}]_{e \in E}
\]
is a vector of \( \mu_{\ell,t} \) for every \( e \in E \) in the network.

\( \mu_{\ell,t} = \left[ \mathbb{P}(l(c_{e,t+1} = \lambda|c_{e,t} = \eta)) \right]_{\lambda=0}^{\infty} \) is a vector of probabilities \( \mathbb{P}_e(l(c_{e,t+1} = \lambda|c_{e,t} = \eta)) \) for all possible values of \( \lambda \). We assume that the upper bound of \( \lambda \) is the capacity \( c_e \) of the edge \( e \). Since the system has a cost minimization rationality, it will send a signal \( \mu_{\ell,t} \) at time \( t \) to the \( \ell^{th} \) traveler such that the path chosen by the traveler minimizes the system’s overall cost. Since we have a leader-follower game, we use backward induction to solve for the optimal leader strategy, i.e., the optimal signal at the system. System can- not send signal to every traveller at the same time as every traveller’s decision depend on all the other travellers as well. Therefore, for every time step, the system sends to signals travellers in a round-robin fashion. The system sends a signal \( \mu_{\ell,t} \) to the \( \ell^{th} \) traveler while all the other travellers are fixed on their respective paths.

The search space in this optimization problem comprises of all right stochastic matrices which can be shown as a convex set. However, it is analytically hard to verify whether or not, the objective function \( U_{0,T} \) stated in Equation 13 is convex in \( \mu \). Note that the term \( \pi_{\ell,t} \) represents logit probabilities which are known to be non-convex. Equation 13 comprises of convex combination of sum of logit probabilities whose convexity properties are hard to verify. Therefore, we employ interior point algorithms to compute the approximate signal. In our simulation experiments, we use CVXPY (Diamond and Boyd 2016) package to implement interior point search in Algorithm 5.

### Results and Discusssions

In this section, we discuss our simulation experiments along with our findings in terms of the performance of LoRI, in comparison to single-source shortest path (SSSP) algorithms used by traditional navigation systems in the context of a Wheatstone network shown in Figure 2. We assume that SSSP algorithms are constructed based on a single attribute, namely Travel Time, whereas our proposed algorithm (LoRI) relies on two attributes, namely Travel Time, and CO Emissions. Depending on the transport mode, we employed well-known cost models found in the literature, to carry out our simulation experiments. For example, travel time \( TT_e \) on edge \( e \) can be calculated for transport modes serviced on a road network (e.g. car, taxi, bus) using Bureau of Public Roads (BPR) formula (Manual 1964):

\[
TT_e(c_{e,t}) = f_e \left[ 1 + \alpha \left( \frac{n_{e,t+1}}{c_e} \right)^\beta \right],
\]

(19)

where \( n_{e,t} \) is the number of vehicles at time \( t \), \( c_e \) is the capacity, of the edge on edge \( e \), \( f_e \) denotes the free-flow travel time of edge \( e \), \( \alpha \) and \( \beta \) are constants in the BPR function (usually \( \alpha \) is 0.15 and \( \beta \) is 4). Similarly, the rate of carbon emissions per vehicle can be calculated using a non-linear, static emission model for network links proposed by Wallace et al. (Wallace et al. 1998), as shown below:

\[
CO_e(TT_e(n_{e,t})) = 0.2038TT_e(n_{e,t}) \exp \left( \frac{0.7962l_e}{TT_e(n_{e,t})} \right),
\]

(20)

where \( l_e \) is the link length (in kilometers), \( TT_e(n_{e,t}) \) is the travel time (in minutes) for link \( e \), and \( CO_e \) is measured in grams per vehicle per hour.

The travel time for edges that support subway mode can be extracted from their arrival and departures time. In our example network in Figure 2 we assume travel times as \{a \rightarrow b : 3, b \rightarrow d : 4, d \rightarrow c : 2, c \rightarrow a : 4\}. For simplicity, we assume CO emissions per traveler on a subway to be \( 0.5 \times \text{travel time} \). For the edge corresponding to walking modality, we assume the travel time \{b \rightarrow c : 4\} and CO emissions to be simply 0. In this example, we consider the total cost of traveling on a switch edge to be 1. We implement our simulation experiments in two different scenarios using the following Python packages: python-igraph v0.9.6, gambit v1.6.0.1, cvxpy v1.1.14, numpy v1.21.1, matplotlib v3.4.3 and all their dependencies.

### Scenario 1

We simulate three different travelers with unique origin-destination pairs: \{(a, d), (d, b), (c, b)\}, each of whom interacts with SSSP for a route recommendation. For simplicity, we assume the LoRI’s weight for travel time to be 0.7, and traveler’s weights for travel time to be \{0, 0.25, 0.5, 0.75, 1\}. We compute the empirical average costs across different traveler motives at both traveler and system, and plot them in Figure 4. Although the travelers’ average cost remains the same for both SSSP or LoRI, the system cost reduces by about 14% when the travelers interact with the LoRI in lieu of SSSP. Specifically, LoRI reduces the congestion rate by about 10%
and CO emissions rate by 4% across the entire multi-modal network.

In our second experiment, we assume that the three traveler’s weights for travel time are 0.8, 0.6, 0.5, and varied LoRI’s weights across \( \{0,0.1,0.2,\ldots,1.0\} \). We evaluated average system costs across different origin-destination pairs and plot them as shown in Figure 5. It is quite evident that LoRI’s costs are at least as good as that of SSSP. Specifically, system obtains a tremendous gain by adopting LoRI when there is a motive mismatch between SSSP and the system. For example, when the system’s weight for travel time is 0, the adoption of LoRI reduces the overall network congestion by 45%.

In our third experiment, we assume every travelers’ weight for travel time to be 0.5 for all possible origin-destination pairs. In this experiment, we observe that LoRI successfully persuades the traveler with 66.66% probability, i.e. LoRI’s strategically designed information was able to successfully steer travelers’ routes towards socially optimal choices over 66.66% of all origin-destination pairs.

**Scenario 2**

In this scenario, we distribute 30 travelers across the entire multi-modal transportation network. We perform this experiment with \( x \) number of travelers who interact with LoRI, while all the other travelers \((30-x)\) interact with SSSP. We assume the system’s weight for travel time to be 0.7. We calculated average system costs across different traveler paths and present them in Table 1. Note that the system’s social welfare consistently improves as the number of travelers interacting with LoRI increases. However, there is a significant tradeoff in terms of run time. Table 2 shows how the runtime of LoRI and SSSP varies with \( x \). Although SSSP’s runtime remains almost unchanged with increasing number of travelers in our experiment, LoRI’s runtime increases exponentially with increasing number of interacting travelers. This exponential increase in runtime happens because of significant increase in the number of possible signaling strategies at the system, which in turn depends on all possible combinations of all the paths available at every active travelers \((\alpha_{t,t} = 1)\) interacting with LoRI.

**Conclusion**

In summary, we proposed a novel Stackelberg signaling framework to improve the inefficiency of selfish routing in the presence of behavioral agents. We modeled the interaction between the system and quantal response travelers as a Stackelberg game, and developed a novel approximate algorithm \( LoRI \) that constructs strategic, personalized information regarding the state of the network. The system presents this information as a private signal to each traveler to steer their route decisions towards socially optimal outcomes. We demonstrate the performance of LoRI and compare with that of a SSSP algorithm on a Wheatstone network with multi-modal routes. We presented the tradeoff between system’s costs and runtime within strategic information design framework. In the future, we will design computationally efficient, approximate algorithms at the system with better run-time performance. We will also consider strategic information design for diverse agent rationalities.

| Number of Travelers | LoRI          | SSSP          |
|---------------------|---------------|---------------|
| 1                   | 0.24670171737670898 | 0.001127958297729322 |
| 2                   | 0.41092681884765625 | 0.002007007598876953 |
| 3                   | 2.2496159076690674  | 0.002805948257446289 |
| 4                   | 909.9852938652039   | 0.0027680397033691406 |

Table 1: System costs with \( x \) travelers interacting with LoRI under Scenario 2

| Number of Travelers | LoRI          | SSSP          |
|---------------------|---------------|---------------|
| 1                   | 20.211234567901233 | 23.624074074074073 |
| 2                   | 20.135671283950616 | 23.595292592592592 |
| 3                   | 19.85057613168724  | 23.010582010582010 |

Table 2: Run Time under Scenario 2
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