A* Tree Search for Portfolio Management

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Abstract

We propose a planning-based method to teach an agent to manage portfolio from scratch. Our approach combines deep reinforcement learning techniques with search techniques like AlphaGo. By uniting the advantages in A* search algorithm with Monte Carlo tree search, we come up with a new algorithm named A* tree search in which best information is returned to guide next search. Also, the expansion mode of Monte Carlo tree is improved for a higher utilization of the neural network. The suggested algorithm can also optimize non-differentiable utility function by combinatorial search. This technique is then used in our trading system. The major component is a neural network that is trained by trading experiences from tree search and outputs prior probability to guide search by pruning away branches in turn. Experimental results on simulated and real financial data verify the robustness of the proposed trading system and the trading system produces better strategies than several approaches based on reinforcement learning.

1 Introduction

In the last decade, artificial intelligence(AI) achieves explosive development and people try to apply this technique almost in every domain of modern society. The advantages of employing AI are significant and AI has obtained abilities in areas such as object detection, image understanding, natural language processing with high level [4, 12, 13]. In these tasks, there always exists some kind of labels to guide the agent to adjust its parameter to minimize a human-defined criterion. Also, huge data of different types play an important role in the training process. In a sense, the achievements in these tasks mainly depends on the better performance in speed or volume of machines. However, there are other areas where supervised learning is hardly applicable because of either inaccuracy or unavailability of labels.

Reinforcement learning(RL) is an approach to learn from interaction with a specific environment and artificial guide for supervised learning is no longer needed [27]. Over the past few years, RL has been developed greatly because of the rise of deep learning(DL). A deep Q-network is developed and attains a human level in Atari 2600 games [17]. Deep RL also robustly solves a variety of domains with continuous action spaces [14, 15]. All these successes in RL are because of using DNN to extract features in observation space and mapping them to generate some kind of policy with good behavior. However, these deep RL networks concentrate mainly on mapping from features to a policy distribution over actions and lack explicit planning.

With Monte Carlo tree search(MCTS) as its planning module, AlphaGo Fan defeated the European champion Fan Hui [25]. Later, AlphaGo Zero, a much stronger version than AlphaGo Fan, was created and might be the strongest Go player so far. However, without planning, AlphaGo Zero with only raw network becomes a little bit weaker than AlphaGo Fan [26]. The reason may be that DNN only attains an imperfect level of generation under computing budget and fusing the estimated

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values of each state by tree search could eliminate a certain degree of errors. Also, the more precise estimation will be obtained since it is one step closer to the goal by looking ahead. Anyway, the achievement in AlphaGo has exhibited the potential of deep RL together with tree search. And the most powerful ability of deep RL is that it doesn’t need a precise model of the environment and has the potential to be promoted to other practical tasks.

In this paper, we propose a new deep RL framework, as well as path search algorithm designed for sequential decision problems, in which the output of the DNN could help reduce the search space and the search feedback could guide the learning direction of the DNN the other way round. Furthermore, we add the insight of exploration from MCTS into the famous A* algorithm by introducing tree structure[8, 25, 26, 28]. For the two-player problem in the game of Go, MCTS uses the average information to lead the construction of the search tree because some states can only be attained by the two players jointly. Since our approach is designed for one-player problem and states are dominant by the only agent, the idea of using of the optimal value to guide the following search is retained. To the best of our knowledge, this is the first attempt to use the tree search in real-time financial trading.

2 Related Works

Traditional financial study about stock investment started about 60 years ago. In the 1950s, Markowitz raised the modern portfolio theory[16] and later Sharpe developed this method into the capital asset pricing model(CAPM)[23]. CAPM assumes that there is one common factor affecting all the stock prices and each individual stock price is also affected by its own specific factor. Several years later, Ross came up with arbitrage pricing theory(APT)[22], which assumes that one common factor is not enough to explain the price change of each stock. Besides modeling the financial market, Markowitz proposed the Standard Markowian Portfolio Optimization to choose optimal portfolio. Based on this, Sharpe developed the famous sharp ratio which is an effective target to be optimized in practice[24].

AI techniques used in the stock market for automated trading exist for a long time. The most common one is supervised learning, in which the neural network is fed data with labels. In [1, 9], the neural network is trained used past stock price data and optimized decisions derived by maximizing sharp ratio or modified sharp ratio. However, since training on label data is a two-step optimization process, these methods will yield suboptimal performance[19].

RL is very suitable to apply in financial domains because of the dynamic behavior of the market. The typical RL can be generally categorized into two types as value-based and policy-based methods. Since the trading environment is too complicated to be estimated in a discrete space, Q-learning is not a good paradigm for trading[6, 18]. Then policy-based approaches are raised to trade one asset or portfolio in [3, 18, 19]. Recently, DL is used in trading areas for its power of feature extraction. To trade one share of the asset, DL is used to sense the dynamic market condition and RL makes trading decisions in [4]. For portfolio management, deep RL could also be directly applied to solve portfolio management problem[11, 29]. However, since the policy in these methods is optimized based on the gradient, it may get stuck at the local minimum and becomes invalid when utility function is non-differential.

3 Background

Reinforcement learning is about an agent interacting with the environment and learning to behave tactfully according to rewards in essence. It solves the problem of how to map situations to actions so as to maximize a numerical reward[27]. Unlike unsupervised learning, tasks in RL have some kind of learning target, also unlike supervised learning with specific object i.e. labels. Learning target in RL is generally changeable and uncertain, even not totally right. In a sense, this characteristic makes learning feasible in situations that human cannot summarize an effective learning rule.

RL is usually formalized into a Markov Decision Process(MDP) which is a sequential decision making based not only immediate rewards but delayed rewards. At each step \( t \), the agent faces some state \( S_t \in S \) and selects an action \( A_t \in A(s) \) based on \( S_t \). One time step later, the agent receives a numerical reward \( R_{t+1} \in \mathbb{R} \subset \mathbb{R} \) and gets to a new state \( S_{t+1} \). The trajectory of agent is \( S_0, A_0, R_1, S_1, A_1, R_2, ... \). The MDP problem in stock market is generally regarded as infinite, while in this paper it is treated as finite MDP by setting a investment period. In finite MDP, the sets of states, actions and rewards have a finite number of elements. Then, the environment transfers to a
new state $S_{t+1}$ at probability $p(S_{t+1} = s', R_{t+1} = r|S_t = s, A_t = a)$ and the agent want to learn the optimal policy $p(A_t = a|S_t = s)$ which is a distribution towards action set and is denoted as $\pi$. However, direct output of policy from neural network has a certain variance as we can see from [26] that a well-trained raw network doesn’t attain the best performance.

Here we consider an RL problem called episode task which starts at $t = 0$ and ends at $t = T$. In this paper, the discounted factor is set to 1 since divergence is not a problem under this condition. When applying policy $\pi$, the accumulated reward obeys a distribution and the state-value function of each state is defined as $v_\pi(s) = \mathbb{E}_\pi \left[ \sum_{k=0}^{T-t} R_{t+k+1}|S_t = s \right]$.

Then we can further get $v_\pi(s) = \mathbb{E}_\pi \left[ R_{t+1} + v_\pi(S_{t+1}) | S_t = s \right]$, which is the Bellman equation for $v_\pi$. The optimal policy is denoted by $\pi^*$, and the Bellman optimality equation of state-value function $v^*$ is denoted as $v^*(s) = \max_\pi v_\pi(s)$, for all $s \in S$.

When a complete and accurate model of the environment is obtainable, the RL problem degenerates to a dynamic programming problem and is well developed mathematically. If not, we then have two choices. The first one is building an extra model to represent the environment and training the agent to use this information, which is termed model-based RL. The second one is simply not to model environment and doesn’t consider the influence to the environment, which is termed model-free RL [27].

### 3.1 Monte Carlo Tree Search

The key point of Monte Carlo Methods to solve RL problem is averaging random sample returns [27]. It is the most trivial way to calculate expectation when the environment model is difficult to obtain. The average state value using Monte Carlo Methods under policy $\pi$ can be calculated as $v(s) = \frac{1}{n} \sum_{i=1}^{n} G_i(s)$, where $G_i(s)$ is the total reward from state $s$ in $n$th episode [1]. According to Law of large numbers, $v(s)$ converges to $v_\pi(s)$ as $n$ goes infinity. Therefore, this approximation method needs enough experiences to get a good estimation.

MCTS is a method for finding optimal decisions in the sequential decision problem. By interacting with the environment randomly and building a tree structure to count the average outcome, the method could give quite good decision under certain scouting times [2]. It was first proposed by Coulom to choose an action in Go-playing program [5] and applied in computer Go widely [7]. MCTS consists of 4 steps in each simulation process: selection, expansion, simulation, backup. AlphaGo Fan [25] uses a meticulous rollout policy in the simulation phase rather than random policy while AlphaGo Zero [26] abandons rollout simulation.

### 3.2 A* Search

A* search algorithm is widely used to solve pathfinding and graph traversal problem by using heuristics to guide the search. Hart et al first described A* and demonstrated the optimality property [8]. A* combines the advantage of best-first search [20] with Dijkstra’s algorithm and its attention always focuses on the state that appears to be most potent. According to A* algorithm, a node is selected in the OPEN list containing all the unexpanded nodes by a policy

$$s^* = \arg \min_{s \in \text{OPEN}} f(s), \quad f(s) = g(s) + h(s),$$

Each state is assumed to appear only once in an episode.
where \( g(s) \) is cost of path from the start node to current node \( s \), and \( h(s) \) is a heuristic estimation of cost from current node \( s \) to the goal. Then \( s^* \) is put into the CLOSED list while its child nodes are put into the OPEN list again. This heuristic estimation function is problem-specific and directly affect the efficiency of the algorithm. For example, in the shortest path problem, the heuristic function could be the Euclidean distance. However, an admissible heuristic value is generally hard to obtain thus the complexity of the algorithm is still the same as Dijkstra’s algorithm.

An improvement named CNNEIM-A was proposed by Xu et al in [28] featured by lookahead scouting. The intuition comes from the idea that estimation of node closer to goal is more precise. The policy in Equation (5) is modified as

\[
s^* = \arg \min_{s \in \text{OPEN}} \bar{f}(s),
\]

(6)

where \( \bar{f}(s) \) is the average value of several \( f \)-values which are obtained by scouting the corresponding sub-path. In AlphaGo, since the \( g \)-value is hard to compute based on current board state DNN directly estimates an \( f \)-value of the current state and the estimated value is refined by scouting technique together with backup updating.

4 Algorithm

In this section, we first demonstrate the relationship and bridge the gap between A* and MCTS algorithm. Then we describe a new algorithm named A* tree search, which combines the insights in A* algorithm [8, 28] together with AlphaGo [25, 26].

In A* algorithm, a node is directly selected from the OPEN list with the optimal \( f \)-value only to expand and evaluate. After completion of evaluating new nodes, they are inserted into the OPEN list according to the rule that maintains the OPEN list ordered. However, the nodes in the OPEN list, as well as the CLOSED list, are not updated anymore.

By contrast in MCTS, the leaf node\(^2\) in search tree could be viewed in the OPEN list assessed only by prior probabilities and nodes with leaves in CLOSED list evaluated by priors and values. Then, a leaf node is chosen by traversing the tree from the root by selecting the most perspective child each time. After evaluation, the newly-gained information is returned along the path to root and the value of each node is the synthesis of all its child nodes by average information.

In a word, A* uses the optimal information to guide decisions while MCTS uses average information. The difference is because of the different reachability of each state in the one-player problem and the two-player problem. And the tree structure in MCTS makes it convenient to execute exploration with the help of prior probability.

To implement A* algorithm via tree search, we first come up with an updating style using inside information flow to choose the maximum node to be expanded. We claim a maximum tree structure that has the following property:

Lemma 1. If all parent nodes contain the maximum action value of its child nodes, the node with global maximum action value can be chosen greedily from the root node.

Proof. Suppose we finally choose \( C_1 \) greedily from root node in Figure 1, it has global maximum value trivially. If not, we assume there exists \( C_n \) somewhere else in the tree such that \( v_f(C_n) > v_f(C_1) \). Since \( P_1 \) and \( P_n \) are ancestors of \( C_1 \) and \( C_n \) respectively and contain the maximum action value of its child nodes, we have

\[
\begin{align*}
Q_f(R, a_n) &= \max\{Q_f(P_n, a_1), \ldots, Q_f(P_n, a_i)\} \\
&= v_f(C_n) > v_f(C_1) \\
&= \max\{Q_f(P_1, a_1), \ldots, Q_f(P_1, a_j)\} \\
&= Q_f(R, a_1),
\end{align*}
\]

(7)

which contradicts with greedy strategy.

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\(^2\)A leaf node is the edge that is not expanded yet and has no child.
With the tree structure in Lemma 1, the action value of each node is equivalent to

$$Q_f(s,a) = \max_{s'|s,a \rightarrow s'} v_f(s'),$$

where $s, a \rightarrow s'$ indicates that a simulation eventually reached a leaf node with state $s'$ after taking an action $a$ from state $s$. Then in Fig(2-a), the most promising state $s_L$ can be chosen by selecting policies with prior probability [21, 26]. In Fig(2-b), we adopt a DNN $f_\theta$ with parameters $\theta$ to output the action priors and h-values of each child states $\{s_i\}$ of $s_L$ in batch style by

$$\langle P, v \rangle = f_\theta(\{s_i\}).$$

After evaluation, each node for $\{s_i\}$ stores prior probabilities $p_i$ of its child nodes and estimation $v_i$ for itself.

If newly expanded nodes could be added into the tree meanwhile keeping the characteristic of the tree in Lemma 1, then A* algorithm is realized by removing the OPEN and the CLOSED lists. In Fig(2-c), when a leaf state $s_L$ is expanded and $f$-values of its children are calculated by

$$v_f(s_i) = f_Q(v_{gi}, v_i),$$

where $v_{gi}$ is calculated by current information and $f_Q$ is an evaluation function. Note that this function can be more delicate although it is simple summation in A*. Then its ancestor nodes are updated along with each step $t < L$ in a backward pass by

$$Q_f(s_t, a_t) = \max_{s_{t+1}|s_t, a_t \rightarrow s_{t+1}} Q_f(s_{t+1}, a_{t+1}).$$

And visiting times of each affected node is also updated. Since new nodes are added to the tree and the affected nodes are all updated to reflect its maximum action value from leaf to root, the tree still preserves the feature in Lemma 1 after backup operation. We refer our algorithm as A* tree search (ATS, Algorithm 1).

ATS algorithm improves A* algorithm by adding exploration through upper confidence bound $U^2$. Also, combining prior probability with value to leads to better node selection and pruning of inferior branches, which owes to the hierarchical structure of the tree. Furthermore, ATS ensures that each node is judged by prior and value while only priors are guaranteed for the latest nodes in AlphaGo [25, 26].

5 Method

Portfolio management (PM) is a kind of sequential decision problem with numerical rewards, although PM is pretty much like play the game of Go since at each state there are a number $b$ of actions to be chosen and the task could be implemented in a predefined number of $d$ steps. The complexity of these kinds of problems lies in the scale of order $b^d$. When $b$ and $d$ are quite big, it’s impossible to use brute force method to solve these problems even though current computing capacity is huge. AlphaGo tackles the problem very well by using DNN as the heuristic function and MCTS as scouting method. It is amazing the generalization ability of DNN after feeding very few training data relative to the huge state space of Go, i.e. $3^{361}$. Although PM is also path searching problem, there are several differences between PM and Go indeed:
Figure 2: A* Tree Search. a, Each simulation traverses the tree by selecting the edge with maximum action value $Q$, plus an upper confidence bound $U$ that gives more visit opportunities to those nodes with less visit count $N$. b, The leaf node $s_L$ expands all its child nodes and the associated child states $\{s_i\}$ are all evaluated by the neural network $(p(s_i, \cdot), v(s_i)) = f_\theta(\{s_i\})$; the pre-stored prior $p_i$ in $s_L$ for each child node and the vector of $p_i$ are stored in the outgoing edges from $s_L$. c, Action value $Q_f$ is evaluated by $I_f$ and updated to track the optimum of all evaluations $v_f$ in the subtree below that action. Also, the visit count $N$ for each node in this path is updated.

Algorithm 1 ATS algorithm

1: Initialize search tree by current state $s_0$
2: for $i = 1, n_{scout}$ do
3: Initialize starting state by $s_0$
4: Choose a leaf state $s_t$ by tree policy
5: if $s_t$ is not end state then
6: expand $s_t$ using pre-stored priori probabilities
7: if $s_t+2$ exists then
8: $(P, v) \leftarrow f_\theta$(all child states of $s_t$)
9: All child nodes store priori $p_i$ and $v_i$
10: end if
11: State value $v_f$ is obtained by Eqn[10]
12: end if
13: Update value of each affected node by $\max v_f$ and visiting times
14: end for
15: $\pi(a|s_0) \leftarrow N(s_0, a)/\sum_b N(s_0, b)$ for each action $a$

- The path to be searched in PM is not all the same subject to the pattern changes of the market, while the rule in Go is time-invariant.
- A numerical reward is received after each action in PM, while a binary reward in Go can only be obtained at the end of the game.
- The agent in PM can withdraw at any time step to prevent further loss while the ending point is decided by two players together in Go.
- The target of the market in PM is hard to describe while the target is identical for two players in their own viewpoints respectively.

In this paper, we consider the practical example of choosing a daily sequence of portfolio board lots in order to maximize the portfolio value while taking into account transaction costs.

The agent makes decision at each time step $t$ by yielding a portfolio lots $l_t = (l_{t,1}, l_{t,2}, \ldots, l_{t,j})$ for $j$ risky assets and each lot configuration is mapping to an action. After each time interval, the newly

\[^3\text{It is the transaction unit of an asset, which can only be an integer.}\]
net gain $g_t$ consists of the loss $L_{t-1}$ at interval beginning and the return $R_t$ and is given by

$$g_t = R_t - L_{t-1}. \quad (12)$$

For simplicity, we assume that a transaction with value change $\delta v$ will cost $c_r |\delta v|$ where $c_r$ is fee rate. Then loss $L_t$ due to transaction is

$$L_t = \sum_{i=1}^j c_r p_{t,i} |l_{t,i} - l_{t-1,i}|, \quad (13)$$

where $p_{t,i}$ represents the price of asset $i$ at time $t$. The return $R_t$ due to price changes from time point $t-1$ to time $t$ is

$$R_t = \sum_{i=1}^j (p_{t,i} - p_{t-1,i}). \quad (14)$$

Note that for ease of analysis, we have ignored returns due to the risk-free rate on capital. At time step $t_s$, the agent comes to a state $s$ and the return rate accumulated in past trading trajectory is given by

$$r_s = \frac{1}{W} \sum_{t=1}^{t_s} g_t, \quad (15)$$

where $W$ is initial wealth.

The Sharp ratio is commonly used to evaluate the performance of invest\[19\]. The Sharp ratio is defined to be

$$SR = \frac{\text{Average}(g_t)}{\text{Std}(g_t)}, \quad (16)$$

where the average and standard deviation are estimated for periods $t = \{1, ..., t_s\}$. In this paper, we define a path utility function based on Sharp ratio because Sharp ratio encourages fluctuation when $r_s$ is slightly negative and discourages return when its denominator is close to zero. At time step $t_s$, the trajectory utility is calculated by

$$U_p(s) = \begin{cases} a \cdot \frac{\text{Average}(g_t)}{\text{Std}(g_t) + b} & \text{Average}(g_t) < 0 \\ \text{Average}(g_t) & \text{Average}(g_t) \geq 0 \end{cases}, \quad (17)$$

where $a$ and $b$ are constants to balance magnitude with exploration term in Eqn(18) and incite reward respectively.

5.1 Training Phase

In the training phase, this decision problem is formulated as an optimal path finding case. The agent conducts self-trade given the past stock price data and accumulates invest experience by trial and error.

We adopt a DNN $f_\theta$ with parameters $\theta$ which uses the state $s$ consisting of previous market information as input and outputs the action probabilities and an approximation of the value. The current position of the agent is not regarded as network input because fee rate could be neglected compared to market fluctuation for appropriate frequency trading. Prior probabilities given to each action that would be impossible to attain due to account balance are set to zero and the remaining renormalized.

At each trading day $t_s$ with state $s$, an ATS is executed to return a posterior probability $\pi(s)$. The constructed search tree only contains actions of agent and the behaviors of the market are not included since the distribution of market price is assumed independent of the agent. In the selection phase, a child node is selected and traversed by PUCT policy\[26\]

$$a^* = \arg \max_a \left[ Q_f(s, a) + c_{\text{puct}} p(s, a) \sqrt{N(s) + \frac{N(s, a)}{1 + N(s, a)}} \right], \quad (18)$$

where $c_{\text{puct}}$ is a constant determining the level of exploration together with prior probability $p(s, a)$.

Until a leaf node $s_L$ is reached, all of its feasible child nodes $\{s'\}$ are expanded with prior probability
Figure 3: Learning Pipeline. The action prior probability is used to prune away bad choices previously considered. And the influence of prior is weakened as simulation time increases and a more reliable posterior $\pi$ is generated to implement reinforcement learning.

stored in $s_L$ in advance and evaluated by DNN in a batch style which attains a high efficiency of utilizing computing resource.

Since the g-value of each child node are already available, $v_f(s'_i)$ is directly estimated by $U_p(s'_i)$ in Eqn(17) and $\max v_f(s'_i)$ is used to backup to its ancestor nodes to update their action values in this path following Eqn(11). Note that values from DNN are not used and Eqn(10) is degenerated since the utility function is not additive. When a certain number of ATS is implemented, posterior $\pi(s)$ is obtained by visiting frequency of each action. Then, the action with maximum posterior probability is conducted to carry forward the simulation episode.

Until an invest episode is ended at $s_T$, the actual f-value $v_f^*$ in this path is obtained as $U_p(s_T)$ and the data for each state $s$ in this path is stored as $(s, \pi, v_f^*)$. Then the neural network $f_\theta$ is trained by self-trade experiences of recent episodes to minimize the following loss function

$$l = -\pi^T \log p + \alpha(v_f - v_f^*)^2 + c||\theta||^2,$$

where $\alpha$ is used to balance the magnitude of value loss and policy loss, and $c$ is a constant controlling the L2 weight regularization. We set $v_f^*$ as one of learning objectives to alleviate overfitting. Also, this loss function reflects a constraint condition named path consistency implicitly since $v_f^*$ in the same path is identical. The complete learning pipeline is shown in Fig(3).

5.2 Testing Phase

In the testing phase, no future market prices are available until that day and prior probability from DNN is directly used without conducting ATS. The action with the biggest prior is chosen to implement. When new market prices are obtained, the ATS is conducted to get a latest policy and the DNN is retrained in an adaptive style.

5.3 Network Structure

The neural network $f_\theta$ takes as input a representation of the current market state which is a $1 \times 4j$ vector comprising of accumulated price growth rates of previous 1, 3, 5, 10 days of $j$ risky assets. In Fig(4), the input feature vector is connected to a common block with two hidden layers and the node
numbers are 64 and 128. Then the output from the common block is sent to a policy block and a value block both with 64 hidden nodes. All hidden nodes are followed by batch normalization\[10\]. By assuming a maximum lot of each risky asset is \(l_m\), the policy block outputs a vector of size \((l_m + 1)^j\) corresponding to logit probabilities for all possible actions and value block outputs a scalar.

6 Experimental Results

In this section, we illustrate the effectiveness of our approach by short-term investment problem. Our system is compared with other RL systems for online trading. The first competitor is a deep RL system using deep deterministic policy gradient (DDPG) algorithm\[29\]. The second one is the direct reinforcement learning (DRL) system for portfolio\[19\]. All these systems are all testified on simulated data and real market data. The latter two trading systems directly output the portfolio weights for the four assets and are irrespective of the asset prices since the growth rate is enough to calculate final return rate. Please refer to \[18\] for details.

6.1 Implementation Details

We consider the task of managing a portfolio which consists of four securities, i.e. three risky assets and a risk-free asset. We ignore the fixed deposit interest rate of the risk-free asset in the following simulation. The whole trading period is cut into episode pieces days and the state in the last episode is directly used as the initial state in the next. In each episode, the depth of search tree i.e. trading length \(T\) is set to 10 and each ATS uses 40 simulations with \(c_{puct}\) as 0.5 for action optimization in following experiments. The parameters \(a\) and \(b\) in utility function \(U_p\) is set to 500 and 0.005. During testing, the trading systems are re-trained whenever 5 new data are available. In each case, we fix the trading cost to 0.1% of the transaction value and short sell is not allowed.

The predictor type in \[29\] is set to CNN with window length as 3. Since market performance imposes a leading impact on the final income, we also employ a naive strategy that capital is equally apportioned among three risky assets at the beginning and held until the end to reflect the market trend.

The evaluation functions are final return rate (FRR), Sharp ratio (SR) and downside deviation ratio (DDR). DDR is defined as\[18\]

\[
DDR = \frac{Average(g_t)\left(\frac{1}{n} \sum_{t=1}^{n} \min\{g_t, 0\}^2\right)^{\frac{1}{2}}}{(\frac{1}{n} \sum_{t=1}^{n} \min\{g_t, 0\}^2)^{\frac{1}{2}}}, \tag{20}
\]
where \( n \) is the total length of the trading period. Only downside return is considered as a risk in DDR while upside volatility is also regarded as a risk in SR.

### 6.2 Evaluation on the Simulated Data

We first demonstrate the performance of our approach using simulated data of three risky assets. The data are generated from log price series as random walks with autoregressive trend processes\( ^{[19]} \). The top panel in Fig(5) shows the three artificial price series with 400 data used in the simulation. The first 300 data are used to train each system while the rest for testing.

![Figure 5: A view of 100 time periods from a simulation of the different portfolio management systems.](image)

The bottom panel shows the cumulative gain rate of each system over this time period.

Since our method is based on combinatorial operations and the other two depend on continuous portfolio weight, we need to distribute our capital into different assets. The amount in one lot for each asset is set to be proportional to their prices and relatively close to each other to spread risk. The maximum lot \( l_m \) of each risky asset is set to 2 and the initial wealth of the agent is 2000.

Table 1: Performance in simulated data

| Method    | FRR   | SR    | DDR   |
|-----------|-------|-------|-------|
| ATS(ours)| 0.247 | 0.288 | 0.556 |
| DDPG     | 0.199 | 0.168 | 0.288 |
| DRL      | 0.226 | 0.342 | 0.777 |
| Hold     | 0.185 | 0.239 | 0.433 |

From the result in the bottom panel of Fig(5), it is observed that the market is on an upward trend and volatility is gentle in the latter half of the testing data. When compared with the magenta curve (DDPG) and the green curve (DRL), the red curve (ATS) makes more profits. In Table[1], DRL attains the best performance for risk-adjusted return. Our method performs a little worse because the action space is discrete and leads to some fluctuations. However, the continuous weights are actually needed to be converted to discrete operations and are hardly to achieve arbitrary weights in reality.
6.3 Verifications on the Global Markets

The performances of each system are also testified on the American market to trade portfolio consisting of S&P 500, DJIA, and NASDAQ index and a risk-free asset. Daily closing prices of the stock indices covering the period from January 2017 to August 2018 are downloaded from Yahoo Finance. The first 300 days are used to train each system and last 100 days are used to test. The amount in one lot for each asset is set to be proportional to their index values and $l_m$ is 2, while the initial wealth of the agent is 1000 in following experiments.

![Graphical comparison of profit gain among the four approaches using test data is shown in Fig(6).](image)

Graphical comparison of profit gain among the four approaches using test data is shown in Fig(6). It is observed that the market fluctuates more severely and more peaks and valleys appear than in simulated data. And the red curve (ATS) is almost above other curves and attains a profit rate of 4.6%. Since DDPG and DRL both perceive the market behavior by continuous gradient, they may be more likely to get stuck in local optimum. On the contrary, our approach could jump between local optimum due to the discretized action space.

It is also interesting to note that our agent stays away from the market by neutral position to prevent loss when the market trend is highly unpredictable. Because of this strategy, ATS gets the best performance for all the three indicators in Table(2). And the magenta curve (DDPG) and the green curve (DRL) are easier to be affected by the market because of slow reaction.

![Table 2: Performance in American market](image)

| Method   | FRR | SR  | DDR  |
|----------|-----|-----|------|
| ATS(ours)| 0.046 | 0.071 | 0.109 |
| DDPG     | 0.034 | 0.038 | 0.052 |
| DRL      | 0.012 | 0.048 | 0.048 |
| Hold     | 0.028 | 0.031 | 0.043 |

Data from the Hong Kong market are also used to justify our approach. The three major indices are Hang Seng Index (HSI), Hang Seng China-Affiliated Corporations Index (HSCCI) and Hang Seng China Enterprises Index (HSCEI) and are used as three risky assets in our experiment. Daily closing prices covering the period from January 1998 to December 1999 are used in order to compare with Scenario II in [3] directly. The number of trading days through this period is 522 and the data of the first 400 samples are used for training while the rest for testing. In [3], return-based method directly
uses index return rates to control portfolio weights while APT-based method uses several hidden factors which are recovered by extra 86 equities from the Hong Kong market.

![Figure 7: The gain curves of different trading systems in the Hong Kong market](image)

We can see that red curve(ATS) still demonstrates a better ability to preserve gained profits in Fig[7]. APT-based and return-based method both rely on learning for continuous weights and are more vulnerable to market fluctuations. We also check the performances of DDPG and DRL in this dataset. From the results in Table[3], the market is downward in total and our method achieves the best outcome for all the three indicators again.

| Method         | FRR | SR  | DDR  |
|----------------|-----|-----|------|
| ATS(ours)      | 0.165 | 0.169 | 0.361 |
| DDPG           | 0.139 | 0.078 | 0.122 |
| DRL            | 0.042 | 0.045 | 0.063 |
| Hold           | -0.037 | -0.02 | -0.028 |
| APT-based      | 0.157 | 0.164 | –    |
| Return-based   | 0.05  | 0.095 | –    |

### 7 Conclusion

In this paper, we generalize the framework of AlphaGo for binary rewards to common areas with numerical reward in a time-variant problem. For the one-player problem, we come up with a new algorithm named A* tree search which pays more attention to the optimal node since the accessibility of each state is only decided by the only player. And the algorithm exhibits amazing ability in portfolio management problem and finds a strategy totally different from other methods. Our deep RL agent learns policy in historical data and reacts quickly to the recent change of environment with randomness. Furthermore, the action from our trading system is directly reachable while the continuous weights from other trading systems are needed to be discretized in the real world. Finally, our approach also shows power in optimizing discrete objective function like that in AlphaGo.

While the capacity of our deep RL framework has been verified in this paper, there are some matters needing attention. First, the regularity in the stock market might vary largely and more delicate discretization of action space could alleviate the dimension explosion problem. Then, in order to
pursue better performance, more information on the market should be included to mitigate the noise in price data.

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