Analysis and multiple control measures for a typhoid fever disease model

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Abstract. Typhoid fever is one of the diseases that is affecting humanity globally especially in the developing countries where access to clean environment and food are limited. We formulate a dynamical system model to study the effects of multiple control measures (i.e, vaccination, treatment and environmental sanitation) in mitigating typhoid fever disease. The qualitative analysis of our model was carried out accordingly. The impact of introducing the control measures were estimated using appropriate mathematical analysis and numerical simulations.

1. Introduction

Typhoid fever is a dangerous disease caused by Salmonella Typhi [1]. The disease spread mainly by eating contaminated food [1]. The symptoms of the disease include prolonged fever, headache, loss of appetite etc [1]. To decrease typhoid fever infections, environmental sanitation and cleanliness of food and water should be considered as one of the control measures. Other control measures that have been successfully used in reducing the spread of the disease includes treatment of infected individuals and vaccination. Typhoid fever can be treated by using antibiotics that kill the bacteria. There are some available vaccines for controlling typhoid fever disease in the endemic and epidemic regions of the world. About three of the available vaccines are recommended by the World Health Organization (WHO) [1]. Since each of these three control measures are independent, therefore three of them can be considered simultaneously (multiple control measures) in fighting typhoid fever.

Dynamical system models have been successfully used in studying the dynamics and control for infectious disease [2, 3, 4, 5, 6, 7, 8]. In this work, utilize mathematical dynamical system model to study the effects of multiple control measures in mitigating the typhoid fever infections.

2. Model development

We consider a community with total population $N$ where there is typhoid fever disease outbreak. We assumed that the community is fighting the disease with multiple control measures (i.e, vaccination, treatment and cleanliness of food and water with environmental sanitation). Assumed that the total population is partitioned into susceptible individuals denoted by $S(t)$, vaccinated individuals represented by $V(t)$, infected individuals denoted by $I(t)$, treated individuals represented by $T(t)$ and recovered individuals denoted by $R(t)$. Pathogens concentration in the environment or food and water is denoted by $P(t)$. Typhoid fever can
be transmitted either indirectly through exposure to the pathogen causing the disease in the environment, food, water etc or directly through closed contact with an infected person. The dynamics of in the system is as follows. Individuals are recruited into \( S(t) \) at a rate \( \Lambda \). \( S(t) \) get infected either by contact with \( I(t) \) at a rate \( \beta \) or by contact with \( P(t) \) at a rate \( \alpha \). \( S(t) \) get vaccinated at a rate \( \phi \) with a vaccine whose efficacy is \( \varepsilon \). \( I(t) \) get treatment at a rate \( \sigma \) and those that have high immune system can recover naturally at a rate \( \gamma \). \( T(t) \) recover at a rate \( \rho \). The recovered people after sometime can loss immunity and becomes susceptible to the disease again at a rate \( \omega \). Natural mortality occur in the entire human population at a rate of \( \mu \). \( I(t) \) shed pathogens into \( P(t) \) at a rate \( \nu \). \( P(t) \) decay or die naturally at rate \( \xi \) while cleanliness of food and environment lead to the death of \( P(t) \) at a rate \( \theta \). From the above assumptions, we derived the model for typhoid fever given by

\[
\begin{align*}
\dot{S}(t) &= \Lambda - \beta I(t)S(t) - \alpha P(t)S(t) - (\mu + \phi)S(t) + \omega R(t), \\
\dot{V}(t) &= \phi S(t) - (1 - \varepsilon)I(t)\beta V(t) + (1 - \varepsilon)\alpha V(t)P(t) - \mu V(t), \\
\dot{I}(t) &= \beta S(t)I + \alpha S(t)P(t) + (1 - \varepsilon)\beta I(t)V(t) + (1 - \varepsilon)\alpha P(t)V(t) - (\sigma + \mu + \gamma)I(t), \\
\dot{T}(t) &= \sigma I(t) - (\mu + \rho)T(t), \\
\dot{R}(t) &= \gamma I(t) + \rho T(t) - (\mu + \omega)R(t), \\
\dot{P}(t) &= \nu I(t) - (\xi + \theta)P(t).
\end{align*}
\]  

Parameters of the model and their meaning are presented in Table 1 respectively.

| Parameter | Meaning |
|-----------|---------|
| \( \beta \) | Contact rate between \( S(t) \) and \( I(t) \) |
| \( \alpha \) | Contact rate between \( S(t) \) and \( P(t) \) |
| \( \phi \) | Rate of vaccination of \( S(t) \) |
| \( \varepsilon \) | The efficacy of vaccination |
| \( \mu \) | Natural mortality rate of individuals |
| \( \sigma \) | Treatment rate of \( I(t) \) |
| \( \gamma \) | Natural recovery rate of \( I(t) \) |
| \( \rho \) | Recovery rate due to treatment rate of \( I(t) \) |
| \( \nu \) | Shedding rate of pathogen into the environment by \( I(t) \) |
| \( \xi \) | Natural decay or death rate of pathogens |
| \( \theta \) | Decay rate of \( P(t) \) due to sanitation of environment and cleanliness of food and water |

Initial conditions of the model 1 variables are given as follows:

\[
S(0) > 0, \ V(0) \geq 0, \ I(0) \geq 0, \ T(0) \geq 0, \ R(0) \geq 0, \ P(0) > 0. \tag{2}
\]

### 3. Model analysis

The dynamical system analysis of model 1 is presented in this section. This analysis will enhance our knowledge on the dynamics and control measures for typhoid fever infections. A unique disease-free equilibrium (DFE) exist in model 1 and is given by

\[
(S^0, \ V^0, \ I^0, \ T^0, \ R^0, \ P^0) = \left( \frac{\Lambda}{\phi + \mu}, \frac{\phi S^0}{\mu}, 0, 0, 0, 0 \right).
\]

The basic reproduction number can be defined as the expected number of new infections of typhoid fever produced when an infected individual is brought into contact with a given
population that is susceptible to typhoid fever. Using the next generation matrix approach of [4], the basic reproduction number of model 1 can be determined as

$$R_0 = \frac{\beta S^0 + (1 - \varepsilon)\beta V^0}{\mu + \gamma + \sigma} + \frac{\nu(\alpha S^0 + (1 - \varepsilon)\alpha V^0)}{(\theta + \xi)(\mu + \gamma + \sigma)}. \quad (4)$$

Epidemiologically, when $R_0 < 1$, the disease can easily be eradicated from the population using the control measures. This can be shown by proving that DFE is stable when $R_0 < 1$ [4, 7, 6]. The control measures ensure that $R_0$ is not above unity so that the disease will not persist in the community. However, if the control measures are not effective in reducing the $R_0$ below unity, an outbreak of typhoid fever might occur in the population. To understand the typhoid fever disease dynamics when the control measures are not effective, we perform the analysis of the model 1 when no control measure is introduced into the model.

When no control measure is introduced (i.e., $\phi = 0, \sigma = 0, \theta = 0$), the control model 1 reduces to the no-control model

$$\begin{align*}
\dot{S}(t) &= \Lambda - \beta I(t)S(t) - \alpha P(t)S(t) - \mu S(t) + \omega R(t), \\
\dot{I}(t) &= \beta I(t)S(t) + \alpha P(t)S(t) - (\gamma + \mu)I(t), \\
\dot{R}(t) &= \gamma I(t) - (\omega + \mu)R(t), \\
\dot{P}(t) &= \nu I(t) - \xi P(t).
\end{align*} \quad (5)$$

The DFE of the no-control model 5 is

$$(S^o, I^o, R^o, P^o) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right). \quad (6)$$

Using the next generation matrix approach, the basic reproduction number ($R_0$) of the no-control model 5 becomes

$$R_0 = \frac{\beta S^o}{\gamma + \mu} + \frac{\nu\alpha S^o}{\xi(\gamma + \mu)}. \quad (7)$$

Epidemiologically, if the rate of generation of secondary infections is not high such that $R_0 < 1$, the disease can be eradicated without any formal control measures. However, if the rate of generation of secondary infections is high such that $R_0 > 1$ the disease can become endemic in the population. This is the situation with many developing countries today where typhoid fever is endemic. Mathematically, when $R_0 > 1$ an unique endemic equilibrium (EE) occurs in the no-control model 5 and is

$$(S^*, I^*, R^*, P^*) = \left(\frac{\xi(\mu + \gamma)}{\beta\xi + \alpha\nu}, \frac{\Lambda - \mu S^*}{\mu + \gamma}, \frac{\nu I^*}{\xi}, \frac{\gamma I^*}{\mu}\right). \quad (8)$$

The epidemiological implication of stability of EE 8 is that the disease will remain endemic in the population when $R_0 > 1$ [2, 4, 7, 6].

Mathematically, we can see that the basic reproduction number of the control model 1 is always less than the basic reproduction number of the no-control model 5,

$$R_0^c < R_0. \quad (9)$$

This inequality implies that the multiple control measures have significant effects in decreasing the number of secondary infections in the population. Further analysis on the effects of control measures in reducing typhoid fever infections will be conducted using numerical simulations.

4. Numerical results

Numerical simulations is presented here to complement our analytical results. The parameter values used can be found in Table 2.
Table 2. Parameter values used for the numerical simulations

| Symbol of the parameters | Parameter values | Source |
|--------------------------|------------------|--------|
| \( \mu \)               | 0.0200           | [8, 2] |
| \( \beta \)              | 0.0002           | Estimated |
| \( \alpha \)             | 0.00045          | Estimated |
| \( \omega \)             | 0.001            | Estimated |
| \( \gamma \)             | 0.0310           | [9]    |
| \( \rho \)               | 10\( \gamma \)   | Estimated |
| \( \epsilon \)           | 0.78             | [10]   |
| \( \phi \)               | 0.500            | Estimated |
| \( \sigma \)             | 0.450            | Estimated |
| \( \xi \)                | 0.0333           | [6, 2] |
| \( \nu \)                | 5\( \xi \)       | Estimated |
| \( \theta \)             | 3\( \xi \)       | Estimated |

Figure 1. Figure illustrating the effect of considering multiple control measures over no-control measure.

Figure 1 illustrates that considering multiple control measures such as treatment, vaccination and sanitation has significant impact in reducing typhoid fever infected individuals. From the figure, we discover that about 5000 individuals will be freed from the disease when the multiple control measures are introduced to fight typhoid fever. Based on this findings, multiple control measures is strongly recommended for reducing the spread typhoid fever whenever the outbreak occurs.
Figure 2. Figure illustrating the effects of vaccination in reducing typhoid fever infections.

Figure 2 demonstrates the effects of vaccination in mitigating typhoid fever infections. From the figure, we discover that an increase in vaccination rate leads to a decrease in typhoid fever susceptible and infected individuals in the population. In particular, we discover from the figure that a 20% increase in vaccination rate can save about 1000 individuals from contacting the disease. Therefore, effective vaccination only can be recommended for fighting typhoid fever if the multiple control measures are not available.

Figure 3. Figure illustrating the effects of treatment in reducing typhoid fever disease.

Figure 3 illustrates the effects of treatment in mitigating typhoid fever infections. From the figure, we discover that an increase in treatment rate decreases typhoid fever infected individuals. Also, we notice from the figure that a 20% increase in treatment rate can save approximately 1000 individuals from contacting the disease. Thus, effective treatment can be recommended for fighting typhoid fever if the multiple control measures are not available.
Figure 4 illustrates the impact of environmental sanitation and cleanliness of food and water as a control measure for reducing the spread of typhoid fever. From the figure, we discover that increasing the environmental sanitation and cleanliness of food decreases the number of individuals infected with typhoid fever. Thus, effective environmental sanitation and cleanliness should be implemented for possible eradication typhoid fever disease and other food and water borne diseases.

5. Discussion
Typhoid fever is a very dangerous disease affecting individuals globally especially in countries where individuals have limited access to clean environment and consumables. This work used a dynamical system model to study the effects of multiple control measures like treatment, vaccination and sanitation in mitigating typhoid fever infections in endemic communities. By formulating and analysing an appropriate dynamical system model for typhoid fever, we explored the dynamics of the disease. We discovered that typhoid fever can be eradicated using multiple control measures provided that the associated basic reproduction number is kept below unity. However, if no control measure is considered, we discovered that the disease can persist and remain endemic in the population.

The impact of the multiple control measures over no-control measure were investigated analytically and numerically. The results of our analyses revealed that considering multiple control measures has significant effect in decreasing typhoid fever infections over the no-control measure and single control measure. However, if all the multiple control measures are not available, our analysis revealed that any of the single control measures (i.e, vaccination, treatment or environmental sanitation) have some effects in decreasing typhoid fever disease infections.

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