Analysis of a proportional pressure regulator

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Abstract. The paper deals with the theoretical and experimental analysis of a proportional pneumatic device. Such a piece of equipment can be integrated in the structure of an intelligent system that works with compressed gases. Different constructive variants of this type of equipment are analyzed. Authors focus their attention on a device that makes use of an electro-mechanic actuator operating with a proportional electromagnet. For the mathematical model some simplifying assumptions are identified. The mathematical model is numerically integrated. The behavior of the proportional pneumatic device at the variation of the inlet pressure and outlet mass flow rate consumption is pointed out.

1. Introduction
Modern pneumatic drive systems have in their structure besides “conventional” equipment of regulation and control also proportional equipment [1, 2]. What makes different the two families of equipment are first of all the command they use. For the first family the two parameters – pressure and flow rate can be modified only by manual adjustment, and the electric drive, when is used is of the kind” all or nothing”. For the second family the parameter adjustment is continuously achieved, electronically following a pre-established program.

In the present paper authors intend to make a careful analysis of the existing solutions of pieces of equipment for the proportional control of pressure and to find out some new, original ones.

Any attempt in augmenting the heat transfer performance of a heat exchanger is accompanied by an increase in the pressure drop.

2. Constructive variants for proportional regulators
Nowadays there are several solutions to build up pneumatic proportional regulators.

A first possibility [3, 4] consists in using a piece of equipment with a cylindrical drawer with a translation movement (Figure 1).

This one can be used, with small changes, both for the flow and pressure regulation. When is used as a pressure regulator, from the left room Cs is missing the compression spring that exist when the structure is used as proportional distributor and through a circuit, the pressure is supplied from the consumer hole (2). In this case against the drawer acts to forces: the force developed by the proportional electromagnet \( F_{EMP} \) and the pressure force \( F_P \).
Figure 1. A three-hole equipment: a - an image of the equipment, b - the principal schematic, c – the static characteristic of the equipment.

This one can be used, with small changes, both for the flow and pressure regulation. When is used as a pressure regulator, from the left room $C_s$ is missing the compression spring that exist when the structure is used as proportional distributor and through a circuit, the pressure is supplied from the consumer hole (2). In this case against the drawer acts to forces: the force developed by the proportional electromagnet $F_{EMP}$ and the pressure force $F_P$.

If $F_{EMP} < F_P$ the consumer hole (2) is connected to the atmosphere, and pressure decreases until balance is achieved and the hole (2) is blocked. If $F_{EMP} > F_P$ the hole (2) is connected at the pressure source and pressure increases until the equilibrium is achieved. In this way pressure at the hole level (2) is kept constant at value proportional to the intensity of the driving current even when perturbations are present; pressure is calculated such that:

$$p_2 = k_{EP} \cdot i$$  \hspace{1cm} (1)

where $k_{EP}$ represents the constant of the proportional piece of equipment.

$$k_{EP} = \frac{4k_{EMP}}{\pi D_s^2}$$  \hspace{1cm} (2)

Based on the same idea five-hole equipment can be provided. These can control both the movement sense of the motor and the pressure value in one of its active rooms. The SMC Company produces pressure regulators that operate on the present principle.

In Figure 1 a three-hole equipment is presented (an image of it - Figure 1.a, the principle schematic – Figure 1.b and the pressure characteristic – intensity for three variants of proportional regulators, with different regulation fields – Figure 1.c).

Such a piece of equipment, with three consumers is presented in Figure 1 (Figure 1.a - an image of the equipment, Figure 1.b - the principal schematic and Figure 1.c – the static characteristic of the equipment).

A second solution comes from the well-established constructive solution for a manual pneumatic pressure regulation [2, 3]. The only difference comes from the fact that in this case the discharge to the atmosphere valve is differently achieved. Such equipment (Figures 2 and 3) has in its structure two mains subassemblies:

- the main stage for amplification,
- the pilot subassembly.

Then main stage for amplification contains the following elements:

- The mobile assemblies composed of a plain valve 1, the rod 2, membrane 3 and the two discs 4 that make up the rigid of the membrane;
- The plane valve 5 which can slide along the rod 2;
- The body of the equipment 8 that contains the holes (1), (2), (3).

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**Figure 2.** Pneumatic controlled pilot "all or nothing".

**Figure 3.** Proportional electromagnet pilot.

Regarding the pilot subassembly two variants may exist:
- with pneumatic controlled pilot ("all or nothing" Figure 2)
- with proportional electromagnet (Figure 3).

The equipment has a pressure transducer $T_p$ that transforms pressure $p_2$ into a signal. At the level of the electronic block BEC permanently the seated pressure $p_{2,r}$ is compared to the effective one $p_2$ and depending on the comparison result, digital command signals are generated, or not, for the distributors $D_P_a$ and $D_P_e$ according to the logic presented in the Table 1.

| Condition         | $x_1$  | $x_2$  |
|-------------------|--------|--------|
| $p_2 < p_{2,r}$   | 24 V   | 0      |
| $p_2 = p_{2,r}$   | 0      | 0      |
| $p_2 > p_{2,r}$   | 0      | 24 V   |

When the command signal $x_1$ is null, the mobile assemble of the main stage of amplification is in the position presented in figure due to the forces developed by the tow helical springs 6, 7.

The increase in the command signal generates signal $x_1$ that switches distributor $D_P_a$; in this way the inlet pressure $p_1$ reaches the upper room of membrane $C_s$ and develops on its surface the pressure force $F_s$ that will determine the moving down of the mobile assemble and the establishing of the connection (1) to (2) and in this way the increase in pressure $p_2$. 
Through the reaction pipe c, pressure $p_2$ reaches the lower room of membrane $C_i$ where it develops, on the lower surface, pressure $F_i$; this force is lower than force $F_s$ due to the smaller surfaces against which pressure acts. In this moment, against the membrane acts the pressure force $(F_s - F_i)$, under whose affect the principal valve opens. When pressure $p_2$ reaches the set value $p_{2,r}$, signal $x_1$ vanishes and the distributor DP$_a$ moves to the preferential position.

If the inlet electrical signal decreases, signal $x_2$ is generated and distributor DP$_e$ switches and makes contact to the atmosphere of the upper room of the membrane $C_s$. The mobile assemble is displaced up under the action of the elastic force developed by spring 7 and it will train in motion valve 5, what determines to establish connection (2) to (3), fact that leads to the decrease in pressure $p_2$. Company SMC produces this type of equipment [4]. Figure 4 shows such equipment with its characteristics.

![Figure 4. Company SMC equipment.](image)

3. Theoretical analysis of the proportional regulator

The theoretical analysis has been achieved on a model of a proportional regulator for which the electromagnetic actuator is a proportional electromagnet.

The principle schematic of this regulator is presented in Figure 5 and the mathematical model is composed of the following attached equations:

a. differential equation of the pressure in the room of volume $V$ of the regulator

$$\frac{dP_r}{dt} = \frac{X\cdot R\cdot T}{V} \cdot (\dot{m}_t - \dot{m}_e) - \frac{X^P}{V} \cdot \frac{dV}{dt}$$

(4)

where:

$$\dot{m}_t = \frac{k_{EMP} \cdot S_c}{\sqrt{P_a}} \cdot N \left( \frac{P_r}{P_a} \right)$$

(5)

$$N \left( \frac{P_r}{P_a} \right) = \begin{cases} 
1 & \text{if } 0 \leq \frac{P_r}{P_a} \leq 0.528 \\
\alpha \cdot \left( \left( \frac{P_r}{P_a} \right)^{\frac{1}{X}} - \left( \frac{P_r}{P_a} \right)^{\frac{1}{X+1}} \right)^{\frac{1}{2}} & \text{if } \frac{P_r}{P_a} > 0.528
\end{cases}$$

(6)

$$K = \sqrt{\frac{X}{R} \left( \frac{2}{X+1} \right)^{\frac{X+1}{X-1}}}$$

(7)

$$a = \sqrt{\frac{2}{X-1} \cdot \left( \frac{X+1}{2} \right)^{\frac{X+1}{X-1}}}$$

(8)
Coefficients in Equations (4)-(8) have the following values [5]:

$$\chi = 1.4 \quad [-]$$

$$R = 287.04 \begin{bmatrix} m^2 \\ s^2 \cdot K \end{bmatrix}$$

for which it follows:

$$K = 0.04042 \begin{bmatrix} \sqrt{K \cdot s} \\ m \end{bmatrix}$$

$$a = 3.8639 \quad [-]$$

The flow section has the following relationship:

$$S_c = \pi \cdot d \cdot x = k_1 \cdot x$$ (9)

b. the movement equation of the mobile subassembly:

$$m_r \cdot \frac{d^2 x}{dt^2} + B \cdot \frac{dx}{dt} = k_{EMP} \cdot i - A_m \cdot (P_r - P_0) - P_a \cdot S_t - k_{arc} \cdot x - F_{em}$$ (10)

where [3]:

$$A_m = \frac{\pi}{12} \cdot (D^2 + D \cdot d + d^2) - \frac{F_{em}}{P_r - P_0}$$ (11)

$$F_{em} = \frac{2 \cdot \pi \cdot E \cdot h \cdot x}{\sqrt{1 + \left(\frac{2x}{D-d}\right)^2}} \cdot \ln \left(\frac{1 + \left(\frac{2x}{D-d}\right)^2}{\ln \left(\frac{D}{d}\right)}\right)$$ (12)

If the variation in volume V is neglected ($\frac{dV}{dt} = 0$), with the substitution

$$\frac{dx}{dt} = v$$

the mathematical model becomes:

$$\begin{cases} 
\frac{dP_r}{dt} = \frac{\chi R T}{\nu} \cdot \left[ \frac{k p_a}{\sqrt{T_a}} \cdot N \left( \frac{P_r}{P_a} \right) - m_e \right] \\
\frac{dv}{dt} = \frac{1}{m_r} \cdot \left[ k_{EMP} \cdot i - A_m \cdot (P_r - P_0) - P_a \cdot S_t - k_{arc} \cdot x - F_{em} - B \cdot v \right] \\
\frac{dx}{dt} = v
\end{cases}$$ (13)

The mathematical system of differential equations is solved numerically.
If one wants to solve it analytically, simplifying assumptions should be made [6, 7, 8]. The technique to simplify a mathematical model contains the following procedures:
- replacement of complex functions with more simple approximated relationships
- the use of mathematical development in series
- the use of empirical or statistical relationships instead of analytical functions
- linearization of non-linear functions around the average operation situations
- the modifying or omission of some factors
- consideration that certain variable factors are constant factors (as friction coefficients, flow coefficients, flow velocities etc.).

In the following the mass \( m \) of the mobile ensemble and the elastic force of the membrane \( F_{em} \) are neglected [9,10]. For these conditions the mathematical model is simplified so fare:

\[
\begin{align*}
B \frac{dx}{dt} &= k_{EMP} \cdot i - A_m \cdot (P_r - P_0) - P_a \cdot S_t - k_{arc} \cdot x \\
\dot{m}_t &= \dot{m}_e + \frac{V}{\chi R T} \cdot \frac{dP_r}{dt}
\end{align*}
\]

This system can be solved based on the Laplace transform:

\[
\begin{align*}
B \cdot s \cdot x &= k_{EMP} \cdot i - A_m \cdot (P_r - P_0) - P_a \cdot S_t - k_{arc} \cdot x \\
\dot{m}_t &= \frac{m_e}{\chi R T} \cdot s \cdot P_r
\end{align*}
\]

Based on that the following relationships could be determined:

\[
\begin{align*}
\chi &= \frac{k_{EMP} \cdot 1 - A_m \cdot (P_r - P_0)}{B \cdot s + k_{arc}} \\
\dot{m}_e &= \dot{m}_t - \frac{V}{\chi R T} \cdot s \cdot P_r
\end{align*}
\]

On the other side the mass flow rate becomes:

\[
\dot{m}_t = k_1 \cdot x \cdot \left\{ \frac{p}{R T^\alpha} \cdot \frac{p}{P_a} \cdot \frac{2}{(k-1) \cdot R} \cdot \left[ \left( \frac{p}{P_a} \right)^{2/k} - \left( \frac{p}{P_r} \right)^{(k+1)/k} \right] \right\}
\]

The term in parenthesis on the right-hand term of Eq. (17) can be written such that:

\[
T = \left( \frac{p}{P_a} \right)^{2/k} - \left( \frac{p}{P_r} \right)^{(k+1)/k} = \left( 1 - \frac{\Delta P}{P_a} \right)^{2/k} - \left( 1 - \frac{\Delta P}{P_r} \right)^{(k+1)/k}
\]

and then, by developing each term in a binomial series and neglecting the terms that have greater power than three one gets:

\[
T = \frac{k-1}{k} \cdot \frac{\Delta P}{p_a} \cdot \left[ 1 - \frac{3}{2k} \cdot \frac{\Delta P}{P_a} - \frac{5k-7}{6k^2} \cdot \left( \frac{\Delta P}{P_a} \right)^2 \right]
\]

Considering that the process is adiabatic, we can write:

\[
\frac{P_a}{P_r} = \frac{\rho_r}{\rho_a} \quad \frac{k}{P_a}
\]

and accounting for the state equation for a perfect gas

\[
\rho_a = \frac{P_a}{R \cdot T_a}
\]

It follows that:

\[
\frac{P_a}{R \cdot T_a} = \frac{\rho_r}{\left( 1 - \frac{\Delta P}{P_r} \right)^\frac{1-k}{k}}
\]

By developing the numerator in a binomial series Eq. 16 becomes:

\[
\frac{P_a}{R \cdot T_a} = \frac{\rho_r}{1 - \frac{1}{k} \frac{\Delta P}{P_a} + \frac{1-k}{2k^2} \left( \frac{\Delta P}{P_a} \right)^2}
\]
Replacing Equations (15), (19) and (21) into Equation (17) one gets:
\[
\dot{m}_t = k^* \cdot k_1 \cdot \frac{k_{EMP} \cdot A_m \cdot (P_r - P_0)}{B \cdot S + k_{arc}} \cdot \sqrt{2 \cdot \rho_r \cdot (P_a - P_r)}
\]  
(22)

where:
\[
k^* = \frac{\frac{3}{2} \cdot \frac{\Delta P}{P_a} \cdot \frac{5-k-7 \cdot (\Delta P)^2}{6 \cdot \rho_r^2}}{1 - \frac{3}{2} \cdot \frac{\Delta P}{P_a} \cdot \frac{5-k-7 \cdot (\Delta P)^2}{6 \cdot \rho_r^2}}
\]
(23)

The outlet mass flow rate becomes:
\[
\dot{m}_e = k^* \cdot k_1 \cdot \frac{k_{EMP} \cdot A_m \cdot (P_r - P_0)}{B \cdot S + k_{arc}} \cdot \sqrt{2 \cdot \rho_r \cdot (P_a - P_r)} = \frac{V}{\chi \cdot R \cdot T} \cdot S \cdot P_r
\]
(23.1)
or:
\[
\dot{m}_e = k^* \cdot K_1 \cdot K_2 \cdot \frac{(P_r - P_0)}{\tau \cdot s + 1} \cdot \sqrt{(P_a - P_r)} = \frac{V}{\chi \cdot R \cdot T} \cdot S \cdot P_r
\]
(23.2)

where:
\[
K_1 = \frac{k_{EMP} \cdot A_m}{k_{arc}}
\]
(24)
\[
K_2 = \frac{k_1 \cdot A_m \cdot \sqrt{2 \cdot \rho_r}}{k_{arc}}
\]
(25)
\[
\tau = \frac{B}{k_{arc}}
\]
(26)

where \(\tau\) is the time constant of the regulator.

The mathematical model (13) has been numerically integrated based on Matlab Simulink and AMESIM. In the same time experimental tests to verify the reaction in time of the equipment were performed. By closing the outlet hole and applying an inlet stage signal, one can observe the variation of the regulated pressure, like shown in Figure 6. This dynamic reaction is considered of the second order and was verified experimentally.

![Figure 6. Set pressure variation at an applied signal step.](image)

The results of the numerical simulation with AMESIM are presented as follows:

- the reaction of the system for a stage variation of the input pressure, when the outlet flow passage of the regulator is 50% of the nominal flow one (Figure 7);
- the reaction of the system for a sinusoidal variation of the input pressure when the outlet flow passage of the regulator is 50% of the nominal flow one (Figure 8);
Figure 7. The reaction of the system for a stage variation of the input pressure

Figure 8. Reaction of the system for a sinusoidal variation of the input pressure

- the system response considering a stage variation of the outlet flow section of the regulator, when the supply pressure is constant (Figure 9).

Figure 9. System response considering a stage variation of the outlet flow section of the regulator.

4. Conclusions
The current stage of knowledge and future prospects for development point out the importance of pneumo-automatics among top technologies.
Some of the negative aspects connected to the physical properties of the working medium as low viscosity and high compressibility should be solved.

The computerization of pneumatic systems represents an important qualitative gain for this field.

The pneutronics concept developed in a natural way as the result of the connection between three fields: pneumatics – electronics – informatics.

As a result of this connection systems with pneutronic drive have been developed. The base of a pneutronic system is represented by the proportional pneumatic equipment.

Besides them, in the system structure one can find: sensors, transducers, electronic circuits for signal processing, convertors A/D and D/A, controllers or microprocessors.

With such a configuration, a pneutronic system becomes more stable, much more accurate and rapid with a certain level of intelligence.

The paper presents a mathematical model for proportional equipment for the pressure regulation. With such equipment, in all the presented situations, pressure is regulated and stabilized and kept constant with a certain deviation around the regulated value.

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