Properties of relativistically rotating quark stars

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Abstract. In this work, quasi-equilibrium models of rapidly rotating triaxially deformed quark stars are computed in general relativistic gravity, assuming a conformally flat spatial geometry (Isenberg-Wilson-Mathews formulation) and a polynomial equation of state. Especially, since we are using a full 3-D numerical relativity initial data code, we are able to consider the triaxially deformed rotating quark stars at very high spins. Such triaxially deformed stars are possible gravitational radiation sources detectable by ground based gravitational wave observatories. Additionally, the bifurcation from axisymmetric rotating sequence to triaxially rotating sequence hints a more realistic spin up limit for rotating compact stars compared with the mass-shedding limit. With future observations such as sub-millisecond pulsars, we could possibly distinguish between equation of states of compact stars, thus better understanding strong interaction in the low energy regime.

1. Introduction
The direct detection of gravitational waves (GWs) [1, 2] has opened a brand new GW astronomy window for us to explore the Universe. In addition to binary black hole and binary compact star mergers, rapidly rotating compact stars have also been considered as important candidate of GW sources that could be detected by ground based GW observatories [3, 4, 5, 6, 7]. Therefore, it will be interesting and essential to study the configuration of rapidly rotating compact stars.

The equilibrium models of self-gravitating, uniformly rotating incompressible fluid stars has been systematically studied in a Newtonian scheme long ago [8]. Depending on the rotational kinetic energy, the configuration could be axisymmetric Maclaurin ellipsoids as well as nonaxisymmetric ellipsoids, such as Jacobian (triaxial) ellipsoid. For the GW astronomy of rotating compact stars, however, general relativity is required to replace Newtonian gravity. The configuration of relativistic rotating stars is also studied in [9, 10].

By assuming more realistic equation of states (EoS), such as piecewise polytropic EoS, scientists have conducted more realistic calculations for rotating neutron stars. It has been shown that a rotating neutron star will spontaneously break its axial symmetry if the rotational kinetic energy to gravitational binding energy ratio \( \frac{T}{|W|} \) exceeds a critical value [11]. This high \( \frac{T}{|W|} \) ratio is possible to be reached for a newly born rotating compact star during a core collapse supernova or for a neutron star which is spun up by accretion. Quasi-equilibrium configurations of triaxially rotating neutron stars have also been created and studied in a full general relativity scheme [12].

Nevertheless, it’s worth noting that the EoS of compact stars is not fully clear yet since it’s related to the non-perturbative quantum chromodynamics. Other EoS model such as strange quark star has been favored after Bodmer and Witten have suggested that strange quark matter
composed of de-confined u,d and s quarks could be absolutely stable [13, 14]. There are also some observational hints of the existence of quark stars (for example, see [15]). Unlike neutron stars which are bound by gravity, quark stars are self-bound by strong interaction. Consequently, rotating quark stars can reach a much larger $T/|W|$ ratio compared with neutron stars and the triaxial instability could play a more important role for quark stars [16, 17, 18]. In particular, the triaxial bar mode (Jacobi-like) instability for MIT bag models has also been investigated in a relativistic scheme [19]. In this work, we compute initial data for both axisymmetric and triaxial rotating quark star, as well as sequences with various quark star EoSs (MIT bag model and LaiXu09 model [20]) and different compactnesses.

2. Configuration of triaxially rotating quark stars

As an illustration of the calculated triaxial solution, we have shown the surface plot for the $C=0.2$ case of MIT bag model, see Fig.1.

Figure 1. Illustration of the solution with the largest triaxial deformation in our simulation. The colors in this plot illustrates the z-axis value of the surface of the star while the white circles are the equal-phi/theta contours. The fact that the equal-phi circles are not parallel with the color patterns highlights the triaxial deformation, since in an axisymmetric case equal-phi contours should be identical with equal-z contours.

In Fig.2 and 3, the relation between the $T/W$ ratio, $M\Omega$ and the ellipticity of the star has been plotted for 3 different compactness ($C = 0.1, 0.15$ and $0.2$) for both the LaiXu09 model and the MIT bag model.

Unlike in the Newtonian case of an incompressible star, for which the bifurcation to triaxial deformation happens at $T/W \approx 0.1375$ for any compactness, it is found that in a full relativistic
case, for quark stars the $T/W$ ratio at bifurcation point strongly depends on the compactness. Similar results have been obtained for triaxially rotating neutron stars ([12]). The largest $T/W$ ratio for the onset of secular instability is found to be roughly 0.17 for rotating quark stars in the configurations that we considered, and it will be even larger for a higher compactness.

It’s worth noting that compared with the rotating neutron stars calculated by [12], rotating quark stars have much longer triaxial sequences. Especially, at the $C = 0.2$, the triaxial sequence terminates for neutron stars with $n = 0.5$ EoS, while for quark stars, the bifurcation point happens much earlier than the mass shedding limit is reached and there is still reasonably long triaxial sequence which are stable. This justifies the discussion mentioned above, that triaxial instability plays a more important role for quark stars since these stars can reach a much larger $T/W$ ratio.

![Figure 2](image)

**Figure 2.** Plots for $T/|W|$ (left panel) and $\Omega M$ (right panel) versus eccentricity $e := \sqrt{1 - (\bar{R}_z/\bar{R}_x)^2}$ (in proper length) for MIT bag model sequences. Solid curves are axisymmetric solution sequences, and dashed curves are triaxial solution sequences. In each panel, from the top to the bottom the curves correspond to $M/R = 0.2$ (green), 0.15 (red) and 0.1 (blue) respectively.

3. Discussion

Triaxially rotating compact stars are important sources for ground based gravitational wave observatories. Our calculation has shown that for rotating quark stars with different EoSs, the bifurcation point to triaxial sequence happens at roughly 1 millisecond of the spin period. Therefore, the corresponding frequency of the gravitational waves from such sources will be at 2 kHz roughly, which is still detectable by GW observatories such as AdvLIGO or VIRGO. It has been shown that with the largest triaxial deformation solution in our calculation (which is also the one that can be seen in Fig.1), the GW amplitude can be as high as of the order of $10^{-23}$ at a distance of 30 Mpc, which is sufficient for detections.

Another astrophysical implication of the existence of the triaxial sequences is on the spin-up limit for rotating compact stars. A rough estimation is that pulsars can spin up to the mass shedding limit, which is obtained by requiring axisymmetry. This usually gives a spin period of 0.7 ms for a compact star with mass of 1.4 $M_\odot$ and radius of 10 km. However, as shown by both the calculation in this work and in [12], when triaxial deformations are taken into account, the spin period of a compact star actually decreases as it gains angular momentum (by accretion, for example) in the triaxial sequence. In another word, the ‘spin-up’ mechanism is actually
spinning down the pulsar when the bifurcation point is reached. Therefore, it’s reasonable to believe that no pulsar could spin up faster than the period at the bifurcation point in a realistic astrophysical scheme. Therefore, in the future, as we detect more and more fast spinning pulsars when projects such as SKA and FAST are completed, we can also try to rule out EoSs with the observation of sub-millisecond pulsars, as the period of the bifurcation point also depends on the EoSs.

Figure 3. Same as Fig. 2 but for LaiXu09 EoS sequences. Dashed curves and solid curves from the top to the bottom in each panel correspond to $M/R = 0.2$ (green), 0.15 (red) and 0.1 (blue) respectively.

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