Non-Gaussianity and finite length inflation

Shiro Hirai
Department of Digital Games, Osaka Electro-Communication University
1130-70 Kiyotaki, Shijonawate, Osaka 575-0063, Japan
E-mail: hirai@isc.osakac.ac.jp

Tomoyuki Takami
Department of Digital Games, Osaka Electro-Communication University
1130-70 Kiyotaki, Shijonawate, Osaka 575-0063, Japan
E-mail: takami@isc.osakac.ac.jp

Abstract: In the present paper, certain inflation models are shown to have large non-Gaussianity in special cases. Namely, finite length inflation models with an effective higher derivative interaction, in which slow-roll inflation is adopted as inflation and a scalar-matter-dominated period or power inflation is adopted as pre-inflation, are considered. Using Holman and Tolley’s formula of the nonlinearity parameter $f_{\text{NL}}^{\text{flattened}}$, we calculate the value of $f_{\text{NL}}^{\text{flattened}}$. A large value of $f_{\text{NL}}^{\text{flattened}}(f_{\text{NL}}^{\text{flattened}} > 100)$ can be obtained for all of the models considered herein when the length of inflation is 60-63 e-folds and $f_{\text{NL}}$ has strong dependence on the length of inflation. Interestingly, this length is similar to that for the case in which the suppression of the CMB angular power spectrum of $l = 2$ was derived using the inflation models described in our previous papers.

Keywords: .
Contents

1. Introduction 2
2. Scalar perturbations 3
3. Calculation of the nonlinearity parameter 5
4. Discussion 7
1. Introduction

Non-Gaussianity of primordial perturbations is one of the most interesting problems implied by the WMAP data [1, 2]. The observational limits on the nonlinearity parameter from WMAP seven-year data [2] are $-10 < f_{\text{local}}^{\text{NL}} < 74$ (95% CL), $-214 < f_{\text{equil}}^{\text{NL}} < 266$ (95% CL) and $-410 < f_{\text{orthog}}^{\text{NL}} < 6$ (95% CL). However, the standard simple inflation model predicts approximately Gaussian fluctuation, the deviation from Gaussian of which is very small. Several studies have attempted to achieve such large non-Gaussianity. Holman and Tolley [3] showed that if the effective action for the inflaton contains a higher-derivative interaction, which is derived, for example, from k-inflation [4] or DBI inflation [5], and the initial state of inflation is not the Bunch-Davies vacuum, then enhanced non-Gaussianity is derived in the "flattened" triangle configurations, the contribution of which is also discussed in [6]. In their paper, the initial state of the curvature perturbation in inflation was assumed not to be the Bunch-Davies vacuum, i.e., squeezed states, but they did not report a concrete value or the physical mechanism that generates the initial state in inflation, although the value of the coefficient of the initial state in inflation has a very important effect on the non-Gaussianity.

On the other hand, the effect of the initial condition in inflation on the power spectrum of curvature perturbations has been considered [7-9] and the effect of the length of inflation and pre-inflation physics on the power spectrum and the angular power spectrum of scalar and tensor perturbations has been examined by the present authors. [8-9]. The suppression of the spectrum at $l = 2$ as indicated by Wilkinson Microwave Anisotropy Probe (WMAP) data [1] may be explained to a certain extent by the finite length of inflation for an inflation of 50-60 $e$-folds [9]. Of course, there are many attempts [10] to derive this suppression.

Based on the physical conditions before inflation, we have shown that the initial state of scalar perturbations in inflation is not simply the Bunch-Davies state, but rather a more general state (a squeezed state), where a scalar-matter-dominated period, a radiation-dominated period, or another inflation is considered as pre-inflation, and the general initial states in inflation were calculated analytically. In the present paper, we demonstrate a new property of the proposed inflation model. Using Holman and Talley’s formula for the nonlinearity parameter $f_{\text{flattened}}^{\text{NL}}$, we calculate the value of $f_{\text{flattened}}^{\text{NL}}$ for the case in which the proposed finite inflation models have effective higher-derivative interactions, where slow-roll inflation is adopted as inflation and a scalar-matter-dominated period or power-law inflation period is adopted as pre-inflation. The obtained results are very interesting.
2. Scalar perturbations

We consider curvature perturbations in inflation and a scalar-matter-dominated epoch. The background spectrum considered is a spatially flat Friedman-Robertson-Walker (FRW) universe described by metric perturbations. The line element for the background and perturbations is generally expressed as [11]

\[ ds^2 = a^2(\eta) \{ (1 + 2A) d\eta^2 - 2B dx^i dx^j - [(1 - 2\Psi) \delta_{ij} + 2 \partial_i \partial_j E + h_{ij}] dx^i dx^j \} , \]  

(2.1)

where \( \eta \) is the conformal time, the functions \( A, B, \Psi, \) and \( E \) represent the scalar perturbations, and \( h_{ij} \) represents tensor perturbations. The density perturbation in terms of the intrinsic curvature perturbation of comoving hypersurfaces is given by

\[ \rho = -\Psi - \left( \frac{H}{\dot{\phi}} \right) \delta \phi , \]

where \( \phi \) is the inflaton field, \( \delta \phi \) is the fluctuation of the inflaton field, \( H \) is the Hubble expansion parameter, and \( \rho \) is the curvature perturbation. Overdots represent derivatives with respect to time \( t \), and primes represent derivatives with respect to the conformal time \( \eta \). Introducing the gauge-invariant potential \( u \equiv a(\eta)(\delta \phi + \left( \frac{\dot{\phi}}{H} \right) \Psi) \) allows the action for scalar perturbations to be written as [12]

\[ S = \frac{1}{2} \int d\eta d^3x \{ \left( \frac{\partial u}{\partial \eta} \right)^2 - c_s^2 (\nabla u)^2 + \frac{Z''}{Z} u^2 \} , \]

(2.2)

where \( c_s \) is the velocity of sound, and in inflation \( Z = a \dot{\phi}/H \), and \( u = -Z \mathcal{R} \). The field \( u(\eta, x) \) is expressed using annihilation and creation operators as follows:

\[ u(\eta, x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \{ u_k(\eta) a_k + u_k^*(\eta) a_k^\dagger \} e^{-ikx} . \]

(2.3)

The field equation for \( u_k(\eta) \) is derived as

\[ \frac{d^2 u_k}{d\eta^2} + \left( c_s^2 k^2 - \frac{1}{Z} \frac{d^2 Z}{d\eta^2} \right) u_k = 0 , \]

(2.4)

where \( c_s^2 = 1 \) is assumed in inflation. The solution of \( u_k \) satisfies the normalization condition \( u_k(\eta) = \frac{1}{(2\pi)^{3/2}} \int d^3k \{ u_k(\eta) a_k + u_k^*(\eta) a_k^\dagger \} e^{-ikx} \). The field equation for \( u_k(\eta) \) is derived as

\[ \frac{d^2 u_k}{d\eta^2} + \left( c_s^2 k^2 - \frac{1}{Z} \frac{d^2 Z}{d\eta^2} \right) u_k = 0 , \]

(2.4)

where \( c_s^2 = 1 \) is assumed in inflation. The solution of \( u_k \) satisfies the normalization condition \( u_k(\eta) = \frac{1}{(2\pi)^{3/2}} \int d^3k \{ u_k(\eta) a_k + u_k^*(\eta) a_k^\dagger \} e^{-ikx} \). The field equation for \( u_k(\eta) \) is derived as

\[ \frac{d^2 u_k}{d\eta^2} + \left( c_s^2 k^2 - \frac{1}{Z} \frac{d^2 Z}{d\eta^2} \right) u_k = 0 , \]

(2.4)

where \( c_s^2 = 1 \) is assumed in inflation. The solution of \( u_k \) satisfies the normalization condition \( u_k(\eta) = \frac{1}{(2\pi)^{3/2}} \int d^3k \{ u_k(\eta) a_k + u_k^*(\eta) a_k^\dagger \} e^{-ikx} \).

First, slow-roll inflation is considered. The slow-roll parameters are defined as [13,14]:

\[ \epsilon = 2 \frac{\dot{\phi}^2}{H^2} (\frac{\dot{\phi}}{2} + V)^{-1} = 2M_P^2 \left( \frac{H'(\phi)}{H(\phi)} \right)^2 , \]

(2.5)

\[ \delta = 2M_P^2 \frac{H''(\phi)}{H(\phi)} , \]

(2.6)

\[ \xi = 4M_P^2 - \frac{H'(\phi) H''(\phi)}{(H(\phi))^2} . \]

(2.7)

The quantity \( V(\phi) \) is the inflation potential, and \( M_P \) is the reduced Plank mass. Other slow-roll parameters \( (\epsilon_v, \eta_v, \xi_v) \) can be written in terms of the slow-roll parameters \( \epsilon, \delta, \) and \( \xi \) for first-order slow roll, i.e., \( \epsilon = \epsilon_v, \delta = \eta_v - \epsilon_v, \) and \( \xi = \xi_v - 3\epsilon_v \eta_v + 3\epsilon_v^2, \) where
\[ \epsilon_v = \frac{M_P^2}{2(V'/V)^2}, \eta_v = \frac{M_P^2(V''/V^2)}, \text{and } \xi_v = \frac{M_P^2(V'''/V^2)}. \]

Using the slow-roll parameters, \( \frac{d^2Z}{d\eta^2}/Z \) is written exactly as

\[
\frac{1}{Z} \frac{d^2Z}{d\eta^2} = 2a^2H^2(1 + \epsilon - \frac{3}{2} \delta + \epsilon^2 - 2\epsilon\delta + \frac{\delta^2}{2} + \frac{\xi}{2}),
\]

and the scale factor is written as \( a(\eta) = -((1 - \epsilon)\eta H)^{-1}. \) Here, the slow-roll parameters are assumed to satisfy \( \epsilon < 1, \delta < 1, \text{and } \xi < 1. \) As only the leading-order terms of \( \epsilon \) and \( \delta \) are adopted, \( \epsilon \) and \( \delta \) may be considered to be constant, allowing the scale factor to be written as \( a(\eta) \approx (\eta)^{-1-\epsilon[14]} \). Equation (2.4) can be rewritten as

\[
\frac{d^2u_k}{d\eta^2} + (k^2 - \frac{2 + 6\epsilon - 3\delta}{\eta^2})u_k = 0.
\]

The solution to Eq. (2.9) is written as [13]

\[
f_k^I(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\nu+1/2)\pi/2} (-\eta)^{1/2} H_{\nu}^{(1)}(-k\eta),
\]

where \( \nu = \frac{3}{2} + 2\epsilon - \delta, \text{and } H_{\nu}^{(1)} \) is the Hankel function of the first kind of order \( \nu. \) The mode functions \( u_k(\eta) \) of a general initial state in inflation are written as

\[
u_k(\eta) = c_1 f_k^I(\eta) + c_2 f_k^{*I}(\eta),
\]

where the coefficients \( c_1 \) and \( c_2 \) obey the relation \( |c_1|^2 - |c_2|^2 = 1. \) The important point here is that the coefficients \( c_1 \) and \( c_2 \) do not change during inflation. In ordinary cases, the field \( u_k(\eta) \) is considered to be in the Bunch-Davies state, i.e., \( c_1 = 1 \text{ and } c_2 = 0, \) because as \( \eta \to -\infty, \) the field \( u_k(\eta) \) must approach plane waves \( e^{-ik\eta}/\sqrt{2k}. \) Second, in the case of power-law inflation, where \( a(t) \propto t^q, \) a similar method can be used, and the solution is obtained as Eq.(2.10) with \( \nu = \frac{3}{2} + 1/(q - 1). \) Third, the curvature perturbations in the scalar matter are calculated using a method similar to that used for inflation [7, 12, 15]. The field equation \( u_k \) can be written in a form similar to Eq. (2.4) with a value of \( c_s^2 = 1 \) and with \( Z \propto a_p(\eta)[\mathcal{H}^2 - \mathcal{H}']^{1/2}/\mathcal{H}, \) (where \( \mathcal{H} = a'_p/a_p \)). The solution to Eq. (2.4) is then written as \( f_k^S = (1 - i/(k\eta)\exp[-ik\eta])\sqrt{2k}. \)
3. Calculation of the nonlinearity parameter

Here, an inflation model is considered. Since we consider slow-roll inflation to have a finite length, we assume a pre-inflation period to be a scalar-matter-dominated period in which the scalar field is the inflaton field, or is power-law inflation, i.e., double inflation. A simple cosmological model is assumed, as defined by

Pre-inflation: \( a_p(\eta) = b_1(-\eta - \eta_j)^r \), \( r = 2 \)

Slow-roll inflation: \( a_1(\eta) = b_2(-\eta)^{-1-\epsilon} \)

where

\[ \eta_j = -(\frac{r}{1+\epsilon} + 1)\eta_1, b_1 = (\frac{-1-\epsilon}{r})^r(-\eta_1)^{-1-\epsilon-r}b_2. \]

The scale factor \( a_1(\eta) \) represents slow-roll inflation. Here, de-Sitter inflation (\( \epsilon = 0 \)) is not considered. Slow-roll inflation is assumed to begin at \( \eta = \eta_1 \). In pre-inflation, for the case of \( r = 2 \), the scale factor \( a_p(\eta) \) indicates that pre-inflation is a scalar-matter-dominated period, and, for the case of \( r = -q/(q-1) \), the pre-inflation is power-law inflation, where the scale factor \( a_p(t) \propto t^q \).

Using above the pre-inflation model, the initial state of inflation given by Eq. (2.11) will be fixed as follows. The coefficients \( c_1 \) and \( c_2 \) are fixed using the matching condition in which the mode function and first \( \eta \)-derivative of the mode function are continuous at the transition time \( \eta = \eta_1 \). (\( \eta_1 \) is the time at which slow-roll inflation begins.) For simplicity pre-inflation states are assumed to be the Bunch-Davies vacuum. The coefficients \( c_1 \) and \( c_2 \) can be calculated analytically in the case of the scalar-matter-dominated period:

\[
c_1 = -\frac{i}{8z^{3/2}} \sqrt{\frac{\pi}{2}} e^{i((-1+\delta-2\epsilon)\pi/2 - 2z/(1+\epsilon))} \left\{ 2z(-1 - 2iz - \epsilon)H_{\nu_1}^{(2)}(z) + (4z^2 + (3 - 2\delta + 3\epsilon)(1 + \epsilon + 2iz))H_{\nu_2}^{(2)}(z) \right\}, \tag{3.4}
\]

\[
c_2 = -\frac{i}{8z^{3/2}} \sqrt{\frac{\pi}{2}} e^{-i((-1+\delta-2\epsilon)\pi/2 + 2z/(1+\epsilon))} \left\{ 2z(-1 - 2iz - \epsilon)H_{\nu_1}^{(1)}(z) + (4z^2 + (3 - 2\delta + 3\epsilon)(1 + \epsilon + 2iz))H_{\nu_2}^{(1)}(z) \right\}, \tag{3.5}
\]

and in the case of the double inflation model:

\[
c_1 = -\frac{\pi}{4\sqrt{q(q-1)(1+\epsilon)}} e^{i\pi(6+2/(q-1)-2\delta+4\epsilon)/4} \left\{ -qzH_{5/2+1/(q-1)}^{(1)}(zz)H_{\nu_1}^{(2)}(z) - H_{3/2+1/(q-1)}^{(1)}(zz)(-qzH_{\nu_2}^{(2)}(z) + (1 + q(-2 + \delta - 4\epsilon) + \epsilon)H_{\nu_1}^{(2)}(z) \right\}, \tag{3.6}
\]

\[
c_2 = -\frac{\pi}{4\sqrt{q(q-1)(1+\epsilon)}} e^{i\pi(-q(1+\delta-2\epsilon)+2q-1)/2(q-1)} \left\{ -qzH_{5/2+1/(q-1)}^{(1)}(zz)H_{\nu_1}^{(1)}(z) - H_{3/2+1/(q-1)}^{(1)}(zz)((1 + q(\delta - 2 - 4\epsilon) + \epsilon)H_{\nu_1}^{(1)}(z) - qzH_{\nu_2}^{(1)}(z) \right\}. \tag{3.7}
\]
where \( \nu_1 = 3/2 + 2\epsilon - \delta, \nu_2 = 5/2 + 2\epsilon - \delta, z = -k\eta_1 \) and \( zz = qz/((q - 1)(1 + \epsilon)) \).

The initial states of inflation can be written in terms of the slow-roll parameters, the start time of slow-roll inflation \( \eta_1 \), and the double inflation parameter \( q \). Here, three slow-roll inflation models are adopted: the new inflation model with the potential term given by \( V(\phi) = \lambda^2 \nu^4(1 - 2(\phi/\nu)^p) \), where \( p = 3, 4 \) and \( \nu \approx M_p \), the chaotic inflation model with the potential term given by \( V(\phi) = \frac{M^4}{2(\phi^2/m)} \), where \( a = 2, 4, 6 \) and \( m \approx M_p \) and the hybrid model \( V(\phi) = \alpha((\nu^2 - \sigma^2)^2 + m^2/2\phi^2 + g^2 \phi^4) \simeq \alpha(\nu^4 + m^2/2\phi^2) \), where \( \nu \approx 10^{-2}M_p \) and \( m \approx 2 \times 10^{-5}M_p \) [16]. Using the normalization value from the WMAP five-year data, we obtain the values of the spectral index and the slow-roll parameters, such as

New inflation: \( n_s = 0.935, \epsilon = 1.027 \times 10^{-9}, \delta = -0.03228 \)

Hybrid inflation: \( n_s = 0.9816, \epsilon = 0.00504, \delta = 0.000878 \)

Chaotic inflation model:
- \( \phi^2 \) model: \( n_s = 0.967, \epsilon = 0.00828, \delta = 0.000022 \)
- \( \phi^4 \) model: \( n_s = 0.950, \epsilon = 0.01655, \delta = 0.008928 \)
- \( \phi^6 \) model: \( n_s = 0.9334, \epsilon = 0.0248, \delta = 0.01657 \).

Now, we calculate the values of the nonlinearity parameter \( f_{\text{NL}}^{\text{flattened}} \). Holman and Tolley [3] showed that if the effective action for the inflaton contains the higher-derivative interaction [17] \( \mathcal{L} = \sqrt{-g} \frac{1}{8\Lambda^2} ((\nabla\phi)^2)^2 \), which is derived, for example, from k-inflation or DBI inflation, and the initial state of inflaton is not the Bunch-Davies vacuum, then the enhanced non-Gaussianity is derived as follows:

\[
\begin{align*}
\hat{f}_{\text{NL}}^{\text{flattened}} &\approx \frac{\hat{\phi}^2}{M^4|c_2|}(\frac{k}{a(\eta_1)H}) = \frac{2\epsilon M_p^2}{H^2 z^3 |c_2|},
\end{align*}
\]

where \( M \) is the cutoff scale, which is the limit of effective theory, and we assume \( M \approx k/a(\eta_1) \) where \( \eta_1 \) is the beginning time of slow-roll inflation, and \( z = -k\eta_1 \). The present treatment considers the effect of the length of inflation, where \( z = 1 \) indicates that inflation starts at the time when the present-day size perturbation \( k = 0.002(1/\text{Mpc}) \) exceeds the Hubble radius in inflation (i.e., inflation of close to 60 \text{e-folds}). Using the values of the above parameters we can calculate the values of \( |c_1|, |c_2| \), and \( f_{\text{NL}}^{\text{flattened}} \) in terms of \( z(= -k\eta_1) \). The values of \( |c_2| \) change only slightly among the models, but vary with the value of \( z \), as 0.0063 for \( z = 8 \), 0.004 for \( z = 10 \), and 0.001 for \( z = 20 \), and \( |c_1| \approx 1 \). From all of the models except for the \( \phi^2 \) model, similar values of \( f_{\text{NL}}^{\text{flattened}} \) are calculated, i.e., \( f_{\text{NL}}^{\text{flattened}} \approx 120 \) at \( z = 8 \), and \( f_{\text{NL}}^{\text{flattened}} \approx 40 \) at \( z = 10 \). Details are shown in Table 1. With respect to the other values of \( z \), larger values of \( f_{\text{NL}}^{\text{flattened}} \) can be derived at smaller \( z(< 8) \), and small values of \( f_{\text{NL}}^{\text{flattened}} \) can be derived at larger \( z(> 20) \). Based on the above results, the value of \( f_{\text{NL}}^{\text{flattened}} \) appears to depend strongly on the value of \( z \), which represents the length of inflation, and the difference of the value of \( f_{\text{NL}}^{\text{flattened}} \) among our three slow-roll inflation models is not large. Since the \( z \)-dependence of \( f_{\text{NL}}^{\text{flattened}} \) is very steep, any value of \( f_{\text{NL}}^{\text{flattened}} \) can be derived at some point of \( z \). We next consider the case of double inflation, the value of \( f_{\text{NL}}^{\text{flattened}} \) is 100 at \( 3 < z < 4 \) in the chaotic inflation, at \( 4 < z < 5 \) in the case of new inflation, and at \( z \approx 3 \) in the case of hybrid inflation. With respect to the \( q \)-dependence (\( a(t) \propto t^q \)), the values of \( f_{\text{NL}}^{\text{flattened}} \) are similar at very large \( q \) but change at \( q \approx 100 \). The details are shown in Tables 2-4.
4. Discussion

We have derived a new property of the proposed finite inflation model. The possibility of large non-Gaussianity is demonstrated. The proposed inflation model is a finite length inflation model with an effective higher derivative interaction, where slow-roll inflation is adopted as inflation and a scalar-matter-dominated period or power inflation is adopted as pre-inflation. Owing to the existence of pre-inflation, the initial state in inflation is not the Bunch-Davies state, but is instead a more general state. The coefficients $c_1$ and $c_2$ can be analytically calculated. Using Holman and Tolleys formula of the nonlinearity parameter $f_{\text{flattened}}$, we calculated the value of $f_{\text{flattened}}$. For the case in which the scalar-matter-dominated period is considered to be pre-inflation, large values of $f_{\text{flattened}}(f_{\text{flattened}} \approx 100)$ are obtained at $8 < z < 10$ in all the models considered herein, and similar results are derived for the case of double inflation at $3 < z < 4$. These ranges can be written as 60-63 $e$-folds. This length is similar to that obtained when the suppression of CMB angular power spectrum of $l = 2$ was derived using the inflation models described in previous papers [9], although such spectral suppression is not inconsistent when considering cosmic variance.

On the experimental value of $f_{\text{flattened}}$, the orthogonal shape ($f_{\text{orthog}}$) is peaked both on equilateral-triangle configurations ($f_{\text{equil}}$) and on flattened-triangle configurations ($f_{\text{flattened}}$) [18], but we think we need further considerations to derive the constraint of $f_{\text{flattened}}$ from the constraints of $f_{\text{orthog}}$ and $f_{\text{equil}}$. Therefore, we do not show it here. We assume such a high-derivative interaction in order to obtain non-linearity and effective interactions for slow-roll interaction. This high-derivative interaction appears to influence the parameters of slow-roll inflation. In order to clarify this problem, we must investigate a concrete inflation model such as k-inflation or DBI inflation. In the future, we would like to apply the proposed method to other inflation models and investigate the dependence of the length of inflation on $f_{\text{flattened}}$.

Acknowledgments

The authors would like to thank the staff of Osaka Electro-Communication University for their valuable discussions.
References

[1] Spergel D N et al. 2007 *Astrophys.J.Suppl.* 170 377 (astro-ph/0603449); Hinshaw G et al. 2007 *Astrophys.J.Suppl.* 170 288 (astro-ph/0603451); Page L et al. 2007 *Astrophys.J.Suppl.* 170 335 (astro-ph/0603450); Komatsu E et al 2009 *Astrophys.J.Suppl.* 180 330 (arXiv:0804.4142)

[2] Komatsu E et al 2010 (arXiv:1001.4538)

[3] Holman R and Tolley A J 2008 *JCAP* 0805 001

[4] Armendariz-Picon C, Damour T, and Mukhanov V F, 1999 *Phys.Lett.B* 458 209; Garriga J and Mukhanov V F, 1999 *Phys.Lett.B* 458 219

[5] Silverstein E and Tong D, 2004 *Phys.Rev.* D 70 103505; Alishahiha M, Silverstein E, and Tong D 2004 *Phys.Rev.* D 70 123505

[6] Chen X, Huang M x, Kachru S, and Shiu G 2007 *JCAP* 0701 002 (arXiv:hep-th/0605045)

[7] Hirai S 2003 *Class.and Quant.Grav.* 20 1673 (hep-th/0212040)

[8] Hirai S 2003 (hep-th/0307237); Hirai S 2005 *Class.and Quant.Grav.* 22 1239 (astro-ph/0404519)

[9] Hirai S and Takami T 2006 *Class.and Quant.Grav.* 23 2541 (astro-ph/0506473); Hirai S and Takami T 2007 (arXiv:0710.2385)

[10] Bastero-Gil M, Freese K, Mersini-Houghton L 2003 *Phys.Rev.* D 68 123514; Niarchou A, Jaffe A H and Pogosian L 2004 *Phys.Rev.* D 69 063515; Cline J M, Crotty P and Lespourgues J 2003 *JCAP* 0309 3010 (astro-ph/0304558); Contraldi C R, Peloso M, Kofman L and Linde A 2003 *JCAP* 0307 002 (astro-ph/0304558); Martin J and Ringevel C 2004 *Phys.Rev.* D 69 083515; Bridle S L, Lewis A M, Weller J and Efstathiou G 2003 *Mon.Not.R.Aston.Soc.* 342 L72 (astro-ph/0302306); Piao Y S, Feng B and Zhang X 2004 *Phys.Rev.* D 69 103520; Shankaranayanan S and Sriramkumar L 2004 *Phys.Rev.* D 70 123520;

[11] Bardeen J M 1980 *Phys.Rev.* D 22 1882; Kodama H and Sasaki M 1984 *Prog.Theor.Phys.Suppl.* 78 1

[12] Mukhanov V F, Feldman H A and Brandenberger R H 1992 *Phys.Rep.* 215 203

[13] Lidsey J E, Liddle A R, Kolb E W, Copeland E J, Barreiro T and Abney M 1997 *Rev.Mod.Phys.* 69 373

[14] Mattin J and Brandenberger R 2003 *Phys.Rev.* D 68 063513

[15] Albrecht A, Ferreira P, Joyce M and Prokopec T 1994 *Phys.Rev.* D 50 4807

[16] Ichikawa K, Suyama T, Takahashi T, Yamaguchi M 2008 *Phys.Rev.* D 78 023513

[17] Creminelli P 2003 *JCAP* 0310 003

[18] Senatore L, Smith K M and Zaldarriaga M 2010 *JCAP* 1001 28
Table 1: Values of $f_{\text{flattened}}^{\text{NL}}$ for the case of the matter-dominated period as pre-inflation

|       | New inflation $p = 3$ | Hybrid $p = 4$ | Chaotic inflation $\phi^2$ | $\phi^4$ | $\phi^6$ |
|-------|-----------------------|----------------|-----------------------------|-----------|-----------|
| $z = 8$ | 123.8                | 123.7          | 122.7                       | 123.8     | 187.7     | 126.4     |
| $z = 10$ | 40.5                | 40.4           | 40.1                        | 40.4      | 61.3      | 41.3      |
| $z = 20$ | 1.26                | 1.26           | 1.25                        | 1.26      | 1.91      | 1.28      |

Table 2: Values of $f_{\text{flattened}}^{\text{NL}}$ in the hybrid inflation for double inflation

|       | $q = 10^5$ | $q = 10^4$ | $q = 10^3$ | $q = 10^2$ | $q = 10$ |
|-------|------------|------------|------------|------------|----------|
| $z = 3$ | 108.5      | 109.1      | 115.3      | 190.6      | 1096.5   |
| $z = 4$ | 23.4       | 23.6       | 25.3       | 45.1       | 266.8    |
| $z = 5$ | 7.24       | 7.30       | 7.93       | 14.8       | 88.6     |

Table 3: Values of for $f_{\text{flattened}}^{\text{NL}}$ the new inflation case of $n = 3$ and for the new inflation case of $n = 4$ for double inflation

$n = 3$

|       | $q = 10^5$ | $q = 10^4$ | $q = 10^3$ | $q = 10^2$ | $q = 10$ |
|-------|------------|------------|------------|------------|----------|
| $z = 4$ | 254.0      | 254.1      | 256.0      | 275.1      | 478.5    |
| $z = 5$ | 81.8       | 81.9       | 82.5       | 89.1       | 157.7    |
| $z = 6$ | 32.5       | 32.6       | 32.8       | 35.6       | 63.6     |

$n = 4$

|       | $q = 10^5$ | $q = 10^4$ | $q = 10^3$ | $q = 10^2$ | $q = 10$ |
|-------|------------|------------|------------|------------|----------|
| $z = 4$ | 194.9      | 195.1      | 197.0      | 216.5      | 424.2    |
| $z = 5$ | 62.8       | 62.9       | 63.5       | 70.2       | 140.1    |
| $z = 6$ | 25.0       | 25.0       | 25.3       | 28.0       | 56.5     |
Table 4: Values of $f_{NL}^{\text{flattened}}$ for the Chaotic inflation in the cases $\phi^2$, $\phi^4$, and $\phi^6$ for double inflation

$\phi^2$ model

|       | $q = 10^5$ | $q = 10^4$ | $q = 10^3$ | $q = 10^2$ | $q = 10$ |
|-------|------------|------------|------------|------------|---------|
| $z = 3$ |             |            |            |            |         |
| $z = 3.5$ | 227.6      | 228.2      | 234.7      | 306.9      | 1196.5  |
| $z = 4$  | 100.6      | 100.9      | 104.2      | 140.0      | 561.2   |

$\phi^4$ model

|       | $q = 10^5$ | $q = 10^4$ | $q = 10^3$ | $q = 10^2$ | $q = 10$ |
|-------|------------|------------|------------|------------|---------|
| $z = 3$ |             |            |            |            |         |
| $z = 3.5$ | 181.1      | 181.5      | 185.6      | 242.9      | 1130.3  |
| $z = 4$  | 76.1       | 76.4       | 78.6       | 108.1      | 530.0   |

$\phi^6$ model

|       | $q = 10^5$ | $q = 10^4$ | $q = 10^3$ | $q = 10^2$ | $q = 10$ |
|-------|------------|------------|------------|------------|---------|
| $z = 3$ |             |            |            |            |         |
| $z = 3.5$ | 165.5      | 165.5      | 165.4      | 191.2      | 1061.5  |
| $z = 4$  | 67.1       | 67.1       | 67.1       | 81.4       | 497.6   |

|       | $q = 10^5$ | $q = 10^4$ | $q = 10^3$ | $q = 10^2$ | $q = 10$ |
|-------|------------|------------|------------|------------|---------|
| $z = 3$ |             |            |            |            |         |
| $z = 3.5$ | 30.6       | 30.6       | 30.6       | 39.0       | 257.6   |