A fatigue approach to wind turbine control

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Abstract. Conventional design of wind turbine controllers is focused on speed and produced electric power. As fatigue loads is an important design consideration, the resulting design is evaluated also with respect to the fatigue loads inflicted on the turbine structure. This is normally done by performing simulations using tools like FLEX, HAWC or FAST, followed by rainflow counting in the resulting time series. This procedure constitutes an iterative design procedure involving realisations of the stress processes in order to obtain the time series needed for fatigue estimates.

The focus of this paper is the elimination of the need for process realisation. To this end, known techniques for approximative fatigue load assessment based on the spectral moments of the inflicted stress histories are applied. Assuming a linearised system model, we present a novel scheme for efficient computation of these spectral moments. The scheme is applied to obtain rapid evaluation of cost functions including fatigue loads, hereby allowing efficient numerical optimisation of the controller. Three different controller design examples are given, all defined directly in terms of component life times.

1. Introduction

As wind turbine controller complexity grows, a demand for numerical optimisation of controller parameters arise. As is the case for any numerical optimisation, efficient cost function evaluation is crucial. Fatigue load assessment is essential to structural wind turbine design. Therefore, an estimate of the fatigue load load inflicted on the turbine using a given controller is a vital part of the cost function used. That is, given an iterant (a candidate controller), we must be able to estimate the resulting fatigue load in an efficient manner.

We will define the following controller design objective: Determine the controller that maximizes the shortest component life time under the constraint that the variabilities of generator speed and produced power do not exceed some predefined limits.

Thus, the controller design problem is formulated in terms of reliability (component life times) and hard constraints on the generator speed and the quality of the produced power. This should be compared to e.g. LQ designs, where the design weights relate to the (cross)variances of the state variables and control signals. As fatigue damage is not well quantified by the stress history variance, a tuning scheme based on variances addresses the fatigue issue in a quite indirect way.

In section 2 we present a medium-complexity turbine model, resulting in a linear, stochastic model, with turbulence being modelled as a linear process. Section 3 describes the applied class of state-feedback controllers. The fatigue model used is presented in section 4. In section 5 we present an efficient scheme for computing spectral moments for linear processes, as the spectral...
moments form the basis for the fatigue damage approximations presented in section 6. Finally, in section 7, these results are used for solving the controller design problem outlined above in an efficient way using numerical optimisation.

2. Turbine model

Turbine dynamics is modelled using a state-space model combining a wind model and a structural model.

2.1. Wind model

We will describe the wind speed \( v \) averaged over the rotor plane as a mean wind speed \( \bar{v} \) perturbed by a turbulent contribution, \( \tilde{v} \):

\[
v = \bar{v} + \tilde{v},
\]

with both contributions being perpendicular to the rotor plane. The turbulence \( \tilde{v} \) is being modelled as a 2nd order, linear process:

\[
\begin{pmatrix}
\dot{\tilde{v}} \\
\ddot{\tilde{v}}
\end{pmatrix} =
\begin{pmatrix}
0 & -\frac{1}{\tau_1} \\
\frac{1}{\tau_1 \tau_2} & -\frac{1}{\tau_2}
\end{pmatrix}
\begin{pmatrix}
\tilde{v} \\
\dot{\tilde{v}}
\end{pmatrix} +
\begin{pmatrix}
0 \\
\varepsilon
\end{pmatrix}.
\]

In (1), \( \varepsilon \) is a white noise process with intensity \( \sigma_\varepsilon^2 = \left( \frac{\alpha}{\tau_1 \tau_2} \right)^2 \). Parameters \( \alpha, \tau_1, \) and \( \tau_2 \) are functions of the mean wind speed \( \bar{v} \).

2.2. Structural model

The structure of the turbine model is depicted in figure 1. The states of the mechanical model are rotor speed \( \omega_r \), generator speed \( \omega_g \), drive train torsional deformation \( \theta \), nacelle displacement \( z \), and nacelle velocity \( \dot{z} \). Further, due to actuator dynamics, pitch angle \( \beta \) and generator torque \( T_g \) become model states.

![Figure 1](attachment:image.png)

**Figure 1.** Interactions in the wind turbine model. Blade pitch angle reference \( \beta_{\text{ref}} \) and power reference \( P_{\text{ref}} \) are controllable inputs. Notice how nacelle velocity \( \dot{z} \) affects the wind speed \( v_r \) seen by the rotor.

Rotor aerodynamics are modelled by the following equations governing the aerodynamic power \( P_a \) and the rotor thrust \( F_t \), respectively:

\[
P_a = \frac{1}{2} \rho \pi R^2 v_r^3 C_P(\lambda, \beta), \quad F_t = \frac{1}{2} \rho \pi R^2 v_r^2 C_T(\lambda, \beta).
\]
Here, \( \rho \) is the air density, \( R \) is the rotor radius, and \( C_P \) is the power efficiency coefficient. As indicated, \( C_P \) and \( C_T \) are both functions of the tip speed-ratio \( \lambda \) and the blade pitch angle \( \beta \), with tip speed-ratio being defined as 
\[
\lambda \equiv \frac{\omega_r R}{v_r}.
\]
The effects of the flexible tower is modelled by the nacelle position \( z \) being the displacement in a 2nd order spring-damper system:
\[
m_t \ddot{z} = -K_t z - D_t \dot{z} + F_t.
\]
For the drive train we use the model depicted in figure 2 and governed by the equations:
\[
I_r \ddot{\omega}_r = T_a - K_d \dot{\theta} - B_d \left( \frac{\omega_r - \omega_g}{N_g} \right) - B_r \omega_r,
\]
\[
I_g \ddot{\omega}_g = -T_g + \frac{K_d}{N_g} \dot{\theta} + \frac{B_d}{N_g} \left( \frac{\omega_r - \omega_g}{N_g} \right) - B_g \omega_g,
\]
\[
\dot{\theta} = \omega_r - \frac{\omega_g}{N_g}.
\]
The pitch actuator as well as the generator torque control dynamics will be modelled as first-order systems with time constants \( \tau_\beta \) and \( \tau_g \), respectively:
\[
\dot{\beta} = \frac{1}{\tau_\beta} (\beta_{ref} - \beta),
\]
and
\[
\dot{T}_g = \frac{1}{\tau_g} \left( \frac{P_{ref}}{\omega_g} - T_g \right), \quad P_e = \omega_g T_g.
\]
Combining the wind model and the structural model results in a 9th order model on the form:
\[
\frac{d}{dt} \begin{bmatrix} \omega_r \ T_a \ I_r \ B_r \ B_d \ K_d \ N_g \ \theta \ \dot{\theta} \ \ddot{\theta} \ z \ \dot{z} \ \ddot{z} \ \dddot{z} \end{bmatrix} = f(x,u,w;\bar{v}),
\]
\[
y = g(x).
\]
where
\[
x \equiv \begin{bmatrix} \omega_r & \omega_g & \theta & z & \dot{z} & T_g & \beta & \dot{\beta} & \ddot{\beta} & \dddot{\beta} \end{bmatrix}^T,
\]
\[
u \equiv \begin{bmatrix} \beta_{ref} & P_{ref} \end{bmatrix}^T,
\]
\[
w \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,
\]
\[
y \equiv \begin{bmatrix} \omega_g & P_e \end{bmatrix}^T.
\]
This model can be linearised to obtain:
\[
\frac{d}{dt} \begin{bmatrix} \dot{x} \end{bmatrix} = A \dot{x} + B \dot{u} + w,
\]
\[
y = C \dot{x}.
\]
\[\text{Figure 2. Mechanical equivalent for the drive train.}\]

\[\text{1 Note that, due to the turbulence model being parameterised by the mean wind speed } \bar{v}, \text{ the combined model is also parameterised by the mean wind speed.}\]
\[\text{2 In this work we use a model linearised around a mean wind speed } \bar{v} = 16 \text{ m/s.}\]
3. A state-feedback control scheme with integral action

We will restrict the domain of admissible control schemes to the class of full-order state-feedback controllers, governed by the control law
\[ u = -L \dot{x}, \]
where \( L \) is the state feedback gain. Here, we have introduced the augmented state vector \( \dot{x}_a = (\dot{x} \ x_i)^T \), where \( x_i \) denotes the integral of the output deviation variable \( \dot{y} \), i.e:
\[ \dot{x}_i = \dot{y} = C \dot{x}. \]
As the augmented system is now described by
\[ \dot{\dot{x}}_a = (A_0 \ 0 \ 0 \ C_0 \ C ) \begin{pmatrix} \dot{x} \\ \dot{x}_i \\ \end{pmatrix} = (A_0 \ 0 \ 0 \ C_0 \ C ) \begin{pmatrix} \dot{x} \\ \dot{x}_i \\ \end{pmatrix} + \begin{pmatrix} B_0 \\ 0 \\ \end{pmatrix} \dot{u} + \begin{pmatrix} \dot{w}_0 \\ 0 \\ \end{pmatrix}, \]
the augmented system is now described by
\[ \dot{\dot{x}}_a = (A_0 \ 0 \ 0 \ C_0 \ C ) \begin{pmatrix} \dot{x} \\ \dot{x}_i \\ \end{pmatrix} + (B_0 \ 0 \ 0 ) \dot{u} + (w_0 \ 0 \ 0 ), \]
application of the control law leaves us with the following closed-loop description:
\[ \dot{\dot{x}}_a = A_{cl} \begin{pmatrix} \dot{x} \\ \dot{x}_i \\ \end{pmatrix} + w_a, \quad A_{cl} = (A_0 \ 0 \ 0 \ C_0 \ C ) - (B_0 \ 0 \ 0 \ 0 \ 0 ) L, \quad w_a = ( \dot{w}_0 \ 0 \ 0 \ 0 \ 0 ). \]
That is, the augmented state vector \( x_a \) is modelled as a linear, stochastic process. Even though the driving process \( w_a \) is formally a vectorial process, it contains only one non-zero element (the turbulence driving process \( \varepsilon \)), effectively leaving the closed-loop model as a SIMO system. This, in turn, allows for the individual state elements being described as white noise processes being filtered through scalar SISO transfer functions \( H(s) \) extracted from the matrix \((sI - A_{cl})^{-1}\).

The state-feedback control scheme assumes complete knowledge of the system state. In a real-world application, system state knowledge will have to be obtained by use of a state estimator like a Kalman filter, for instance. This issue will not be pursued further in this article.

4. Fatigue model

A material’s ability to withstand cyclic stress-histories is often described through the SN curve, defined by the relationship
\[ s^k N = K, \]
where \( s \) is the stress range (twice the amplitude in a sinusoidal stress history) and \( N \) is the lifetime in cycles. The quantities \( K \) and \( k \) are material properties, with \( k \) being denoted the Wöhler-coefficient. Further, we assume a linear damage accumulation rule known as Palmgren-Miner’s damage rule, stating that the total damage \( D \) imposed by a given stress history is found as the sum of the damages imposed by the individual \( M \) cycles in the total stress history:
\[ D = \sum_{i=1}^{M} \frac{1}{K} s_i^k, \]
Here, it should be noted that we neglect the effects of the stress history mean value. Assuming that the stress history is an ergodic process we introduce the expected damage rate \( d \), given in terms of the number of cycles pr. time unit, \( \nu_c \), and the probability density function \( p_s(s) \) for the stress ranges:
\[ d = \frac{\nu_c}{K} \int_0^\infty s^k p_s(s) ds. \]
Finally, as fatigue failure occurs when the total damage exceeds unity, we note that the expected lifetime of a component is the inverse of the damage rate.

Equation (3) gives the damage as the combined damage from a number of stress cycles. The interpretation of a cycle is apparent when it comes to constant-amplitude, cyclic stress histories like a sine wave, for instance. In real-life applications, though, stress histories exhibit complex waveforms with the meaning of a cycle not being obvious.

A solution to this problem is to convert a given, arbitrary waveform into a number of equivalent stress ranges. That is, a number of stress ranges that will cause the same amount of fatigue damage as the original waveform. The so-called rainflow-counting method is considered as being the superior method for this conversion [Ryc93]. In the present work, the rainflow counting procedure is used as the state-of-the reference for the approximations introduced later.
5. Spectral moments of a linear process

We will define the $m$th spectral moment $\lambda^x_m$ for the process $x(t)$ as follows:

$$\lambda^x_m \equiv \frac{1}{\pi} \int_0^\infty \omega^m S_x(\omega) d\omega,$$

where $S_x(\omega)$ denotes the power spectral density of the process. Two important properties follow from this definition:

$$\lambda^x_0 = \sigma^2_x \quad \text{and} \quad \lambda^x_2 = \sigma^2_x \dot{x}.$$

That is, the variance is given by the 0th spectral moment, and the variance of the first derivative of $x(t)$ is given by the second spectral moment. [New84].

For the system (2), let $H(s) = \Phi(s)/\Theta(s)$ denote the rational transfer function from the driving noise process $\varepsilon$ to the state variable $x$. The power spectral density $S_x(\omega)$ for the process $x(t)$ is given by:

$$S_x(\omega) = \sigma^2_\varepsilon H(j\omega) H(-j\omega) = \sigma^2_\varepsilon \frac{\Phi(j\omega) \Phi(-j\omega)}{\Theta(j\omega) \Theta(-j\omega)} = \frac{P(\omega)}{Q(\omega)},$$

giving, for the spectral moments

$$\lambda^x_m = \frac{1}{\pi} \int_0^\infty \omega^m \frac{P(\omega)}{Q(\omega)} d\omega = \frac{1}{\pi} \int_0^\infty \frac{\bar{P}(\omega)}{Q(\omega)} d\omega.$$

That is, the spectral moments are given by integrals of rational functions. It is readily shown that the set of roots $\bar{P}$ of $Q(\omega)$ is related to the set of poles $P$ in $H(s)$ as

$$\bar{P} = jP \cup jP^*.$$

This mapping is depicted in figure 3 and has two important properties:

(i) A real pole in $H(s)$ maps to a pair of purely imaginary, complex conjugated poles in $S_x(\omega)$.

(ii) A complex conjugated pole pair in $H(s)$ maps to a quadruple of poles in $S_x(\omega)$ distributed symmetrically around both the real axis and the imaginary axis.

Further, notice that $S_x(\omega)$ will have no real poles. In [Ham06] it is shown that, due to this symmetry, the spectral moments can be computed as

$$\lambda^x_m = \frac{1}{\pi} \int_0^\infty \frac{\bar{P}(\omega)}{Q(\omega)} d\omega = -\sum_{p \in jP} \Im \left( \frac{\bar{P}(p)}{Q'(p)} \right) + \frac{1}{\pi} \sum_{p \in jP} \left\{ \Re \left( \frac{\bar{P}(p)}{Q'(p)} \right) \log |p|^2 + 2 \Im \left( \frac{\bar{P}(p)}{Q'(p)} \right) \arctan \left( \frac{\Re(p)}{\Im(p)} \right) \right\},$$

where general results found in [SM92] regarding integrals of rational functions have been applied.

Thus, numerical integration of the spectral density is avoided and the spectral moments can be computed by means of polynomial evaluation and differentiation along with evaluation of a logarithm and an inverse tangens.

6. Spectrally based fatigue damage approximation

Now we will present a fatigue estimate based on the spectral properties of the stress history. The results presented here follow from [Ryc93, BT05].
6.1. Fatigue for narrow-banded stress histories

Consider a narrow-banded, Gaussian process. The peak amplitudes of such a process are Rayleigh distributed, thus having a probability density function \( p_a(a) \) given by:

\[
p_a(a) = \frac{a}{\sigma^2} e^{-\frac{a^2}{2\sigma^2}} = \frac{a}{\lambda_0} e^{-\frac{a^2}{2\lambda_0}}, \quad a \geq 0.
\]

When performing rainflow counting in a narrow-band process, all peaks will pair to a trough with similar amplitude, producing equivalent ranges with magnitudes twice the amplitudes of the narrow-band process. As a result, the rainflow-counting procedure will result in a range density \( p_s(s) \) related to \( p_a(a) \) as:

\[
p_s(s) = \frac{1}{2} p_a \left( \frac{s}{2} \right) = \frac{1}{2} \frac{s/2}{\lambda_0} e^{-\frac{(s/2)^2}{2\lambda_0}} = \frac{s}{4\lambda_0} e^{-\frac{s^2}{8\lambda_0}}, \quad s \geq 0.
\]  

For a Gaussian process, the number of peaks per unit time, denoted \( \nu_p \), is given by:

\[
\nu_p = \frac{1}{2\pi} \frac{\sqrt{\lambda_4}}{\lambda_2}.
\]  

As the rainflow counting procedure will create one equivalent cycle for each peak, the number of cycles per unit time equals the number of peaks per unit time. That is, \( \nu_c = \nu_p \). Now, inserting (5) and (6) into (4) yields the expected damage rate \( d_\lambda \) for a narrowband process:

\[
d_\lambda = \frac{1}{2\pi} \sqrt{\frac{\lambda_4}{\lambda_2} \Gamma \left( \frac{1}{2} \right)} \int_0^\infty s^{k} e^{-\frac{s^2}{8\lambda_0}} ds = \frac{1}{2\pi} \sqrt{\frac{\lambda_4}{\lambda_2} \Gamma \left( \frac{1}{2} \right) \left( 2\sqrt{2\lambda_0} \right)^k} \Gamma \left( 1 + \frac{k}{2} \right),
\]

where the last equal sign results from the integral being recognised as the \( k' \)th moment of a Rayleigh density function with parameter \( \sqrt{4\lambda_0} \). \( \Gamma(\cdot) \) denotes the Gamma function.

6.2. Benasciutti’s approximation

In [BT05], Benasciutti proposes an estimate of the expected fatigue damage rate given as the narrow-band result modified by a correction factor to account for the process not necessarily being narrow-band:

\[
d \approx \frac{1}{2\pi} \sqrt{\frac{\lambda_4}{\lambda_2} \Gamma \left( \frac{1}{2} \right) \left( 2\sqrt{2\lambda_0} \right)^k} \Gamma \left( 1 + \frac{k}{2} \right) \left( b + (1-b)\alpha_k^{k+1} \right)
\]  

\[ \text{Correction} \]
where
\[ b = \frac{(\alpha_1 - \alpha_2) \left[ 1.112 \left( 1 + \alpha_1 \alpha_2 - (\alpha_1 + \alpha_2) \right) e^{2.11\alpha_2 + (\alpha_1 - \alpha_2)} \right]}{(\alpha_2 - 1)^2} \] (7b)
and
\[ \alpha_1 = \frac{\lambda_1}{\sqrt{\lambda_0 \lambda_2}}, \quad \alpha_2 = \frac{\lambda_2}{\sqrt{\lambda_0 \lambda_4}}. \] (7c)

Formulating (7) as the functional relationship
\[ d \approx \Lambda(\lambda_0, \lambda_1, \lambda_2, \lambda_4; k, K) \]
emphasizes the fact that the fatigue damage is now expressed purely in terms of the spectral properties of the stress history, and parameterized by the material properties \( k \) and \( K \). That is, a fatigue estimate that does not require any realisation of the process.

7. Application

The results in the preceding sections provide a means for efficient estimation of damage rates for load histories being linear in the state variables in a linear system. This, in turn, allows for efficient evaluation of a performance cost function involving fatigue damage. In this section we will demonstrate how this allows for numerical optimisation of a state-feedback controller for the wind turbine.

Assuming a state-feedback controller defined by the gain matrix \( L \) as introduced in section 3, we will formulate the controller design problem presented in the introduction as a minimax optimisation problem as follows:
\[ L^* = \arg \min_L \left\{ \max_i d_i(L) \right\} \] (8a)
subject to
\[ c_1 : \quad c_{v,\omega_y}(L) - \Gamma_{\omega_y} < 0 \] (8b)
\[ c_2 : \quad c_{v,P_e}(L) - \Gamma_{P_e} < 0, \] (8c)

where \( d_i \) denotes the damage rate for the \( i \)th component. In the present case, three different damage rates are computed, namely the low-speed shaft damage rate \( d_\theta \), the tower damage rate \( d_z \), and the blade bearing damage rate \( d_\beta \). Thus, the damage rate vector \( d \) is defined as \( d \equiv (d_\theta \quad d_z \quad d_\beta)^T \). The variability coefficients are defined as the ratio between standard deviation and mean value: \( c_{v,x} \equiv \frac{\sigma_x}{\mu_x} \).

7.1. Damage in the wind turbine model

For the tower and for the low-speed shaft we will assume proportionality between deformations and stresses as follows:\(^3\)
\[ s_z = C_z z \quad , \quad s_\theta = C_\theta \theta, \] (9)
where the constants \( C_z \) and \( C_\theta \) are functions of the specific geometries involved. These assumptions allow for the fatigue to be described in terms of the spectral moments for the system state variables \( z \) and \( \theta \). Further, we will assume that the damage rate for the blade bearings is proportional to the standard deviation of the pitch rate:\(^4\)
\[ d_\beta = C_\beta \sqrt{\lambda_2}. \] (10)

\(^3\) Note that only single-axis stress conditions are considered.
\(^4\) This is based on a crude assumption of constant blade root moment for a given mean wind.
7.2. Normalised damage rates
The proportionality factors $C_z$, $C_\theta$, and $C_\beta$ are normalised to yield a component lifetime of 100 years when the state feedback matrix $L$ is found using an off-the-shelf LQI-controller with diagonal weight matrices. This gain matrix that inherently provides a stable closed-loop system, is used as the starting point for the numerical optimisations that follow.

7.3. Solving the optimisation problem
The MATLAB optimisation toolbox provides the function $\text{fminimax}$ for solving nonlinear, constrained minimax problems. We will use this routine for solving the problem (8), even though it actually only finds local minima in the cost function ((8) is a global optimisation problem). The dimension of the optimisation problem is determined by the number of entries in the gain matrix. In the present case, $L \in \mathbb{R}^{2 \times 11}$, which implies a decision vector of dimension 22.

7.4. Design examples
We will demonstrate the controller optimisation by three examples:

**Design example #1** Find the controller that minimizes the largest damage rate subject to the variabilities of the generator speed and the produced power not exceeding 0.01.

**Design example #2** Assume that the requirement for constant power is relaxed such that the variability of the power is allowed to reach 0.05, and find the optimal controller.

**Design example #3** Assume that the tower base diameter $D$ is increased by 10%. As the stress $s_z$ at the tower base is related to the tower deflection $z$ approximately as $s_z \propto \frac{z}{D^4}$, the increase in diameter would decrease $C_z$ by a factor of $1.1^4 = 1.46$. Find the optimal controller using the new $C_z$ value.

The results of these design examples are summarised in table 1. The estimated values in the second column are the life times in years resulting from evaluating the cost function at the solution, and the expected variabilities resulting from evaluation of the constraint functions at the solution. That is, the life times equal the Benasciutti estimates and the variabilities are analytic results.

The simulation values in the third column result from simulating 10,000 seconds of operation of the closed-loop system using the gain resulting from the minimax optimisation. The lifetimes for the tower and for the drive train are computed from the simulation output using rainflow counting and Palmgren-Miner’s damage rule. The stress histories $s_z$ and $s_\theta$ are obtained by multiplying the $z$ and $\theta$ trajectories with $C_z$ and $C_\theta$, respectively, cf. (9). The simulated damage rate for the blade bearings is found by computing the sample standard deviation of $\dot{\beta}$ and multiply by $C_\beta$, cf. (10). The variabilities $c_v, \omega_g$ and $c_v, P_e$ are computed directly from the sample standard deviations and mean values of the $\omega_g$ and $P_e$ trajectories.

Finally, the last column states the percentage by which the life times predicted by (7) differ from the simulation result obtained by conventional rainflow-counting. For illustrative purposes, the first 50 seconds of each simulation are depicted in figure 4.

A few comments should be added to each of the design results.

**Design example #1** The resulting variabilities for the generator speed and the produced power for the baseline controller indicate that there is room for improvement as these variabilities are only approximately one fourth of their upper limits 0.01. As the optimisation result shows, this overhead is turned into doubled lifetimes for all three components, compared to the baseline controller. The price paid is, as expected, larger fluctuations in generator speed and produced power. Finally, we notice that the Benasciutti
### Table 1. Design results. Rightmost column illustrates accuracy of the approximation (7).

|                  | Estimate | Simulated | Estimate off by |
|------------------|----------|-----------|-----------------|
| **Baseline controller** |          |           |                 |
| Shaft lifetime   | 100.00   | 100.33    | 0 %             |
| Tower lifetime   | 100.00   | 106.28    | 6 %             |
| Blade bearing lifetime | 100.00   | 99.91     | 0 %             |
| Gen. speed variability | 0.0025   | 0.0024    |                 |
| Elec. power variability | 0.0023   | 0.0023    |                 |
| **Design example # 1 – variability constraints equal 0.01** |          |           |                 |
| Shaft lifetime   | 211.26   | 260.47    | 19 %            |
| Tower lifetime   | 211.30   | 219.99    | 4 %             |
| Blade bearing lifetime | 211.30   | 210.96    | 0 %             |
| Gen. speed variability | 0.0100   | 0.0098    |                 |
| Elec. power variability | 0.0097   | 0.0097    |                 |
| **Design example # 2 – electrical power variability relaxed to 0.05** |          |           |                 |
| Shaft lifetime   | 226.78   | 248.40    | 9 %             |
| Tower lifetime   | 226.82   | 242.81    | 7 %             |
| Blade bearing lifetime | 226.82   | 226.47    | 0 %             |
| Gen. speed variability | 0.0100   | 0.0098    |                 |
| Elec. power variability | 0.0136   | 0.0133    |                 |
| **Design example # 3 – tower base diameter increased by 10%** |          |           |                 |
| Shaft lifetime   | 264.84   | 232.98    | -14 %           |
| Tower lifetime   | 264.86   | 280.26    | 5 %             |
| Blade bearing lifetime | 264.86   | 263.37    | -1 %            |
| Gen. speed variability | 0.0100   | 0.0097    |                 |
| Elec. power variability | 0.0100   | 0.0097    |                 |

estimate for the drive train lifetime is off by 19% compared to the simulation result. Inspection of the simulation output in figure 4 shows that the higher lifetimes result from reduced pitch activity (smaller pitch rate), tower damping, and a lowering of the drive train deformations.

**Design example #2** This example is identical to the previous example except that we now allow the variability of the produced power to reach 0.05 instead of 0.01. The optimisation results shows, though, that even though lifetimes are increased a bit compared to the previous example, the extra overhead is not completely exchanged into increased lifetime. The variability of the produced power is 0.0136. That is, the power variability constraint is not active at the solution. This indicates that there is an upper limit for the damage rate reduction that can be achieved by relaxing the power quality demand.

**Design example #3** The increased tower base diameter would, if no changes in the controller were made, increase the tower lifetime by a factor of approximately $1.46^k = 4.6$. By applying the controller optimisation, however, this lifetime prolongation is shared among the components to reach an overall lifetime prolongation of approximately 25% compared to design example #1. The trajectories in figure 4 illustrates that the nacelle position has larger fluctuations, freeing the pitch system and the torque control from their tower damping duties.
An interesting feature of the resulting drive train deformation is, that the amplitude of the deformation is not significantly lowered. The fatigue damage reduction is achieved through a reduction of the high-frequency contents in the signal. That is, the number of damage inflicting stress cycles per time unit is lowered, hereby reducing the damage rate.

![Graph showing wind speed, generator speed, shaft deformation, nacelle position, pitch angle, pitch velocity, and electrical power over time.](image)

**Figure 4.** Optimisation of state feedback-controller. Black: Baseline controller. Blue: Design example #1. Green: Design example #2. Red: Design example #3

8. Conclusions
The advantage of the Benasciutti approximation is that it gives a fatigue damage estimate based upon the spectral properties of the stress history. That is, expected statistical quantities. This, in
turn, provides a fatigue estimate without any realisation of the process, i.e. no time (or frequency) domain simulation of the system is required. At first glance, the definition of spectral moments indicates that computation of spectral moments would have to include a numerical integration of the spectral density function. Investigation of the symmetry governing the rational spectra of linear systems revealed, though, that general results concerning the integration of rational functions could be applied in a very convenient manner for the definite integral defining the spectral moments. In turn, an algorithm was developed that efficiently computes the spectral moments of linear, stochastic processes.

From the three controller design problems presented, we note the following results:

- The optimisation problem was, with the starting point being a stable textbook controller design, a well-posed problem.
- The optimisation algorithm succeeded in finding the proper trade-offs such that the damage rates of the three components in play were levelled to yield to highest possible minimum lifetime.
- Increased strength in one component was traded for longer lifetimes in the other components to provide equal lifetimes for all components.
- The Benasciutti approximation (7) was off by 19% in one case. This suggests that the validity of this approximation should be studied further.

Rapid evaluation of damage rates in a linear system has been demonstrated through numerical optimisation of a wind turbine controller. It should be stressed, though, that the method presented is valid for any linear model driven by a Gaussian process. In effect, the use of the framework outlined in this work is not restricted to controller design. Performing optimisation of pure mechanical structures described by linear models is a possible application of the methods as well.

We should emphasize that in a real-world application, a more detailed turbine model including blade dynamics and three-dimensional drive train loads including tilt/yaw moments would be required. Having obtained a linearised model, though, the framework described in this article should be readily applicable, effectively by-passing the need for time-domain simulations and subsequent rainflow-counting when evaluating cost functions including fatigue estimates.

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