Fabry Perot Cavity Length Demodulation Method Based on Continuous Thinning Algorithm of Fast Fourier Transform Spectrum

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Abstract. Optical fiber Fabry Perot sensors are widely used in industrial production and life. It is of great significance to improve the performance of sensor demodulation algorithm. Based on the analysis of the principle and error source of the cavity length demodulated by the Fast Fourier Transform (FFT) algorithm, the FFT-FT algorithm with continuous refinement of FFT spectrum is introduced. By using the feature that FFT-FT algorithm can refine at the peak of discrete spectrum, the length of F-P cavity demodulated by FFT algorithm is optimized to improve the accuracy of the demodulation. Through simulation experiments, the F-P cavity length of FFT demodulation algorithm and FFT-FT demodulation algorithm under the same light source signal are compared and analyzed, and the goodness of fit of the fitting curve of continuous demodulation is analyzed. The simulation results show that the error of FFT demodulation algorithm is more than 100 times that of FFT-FT algorithm when it is refined 100 times near the peak value of FFT discrete spectrum. The goodness of fit of the demodulation fitting curve calculated by FFT-FT algorithm is more than 2 times that of FFT algorithm, which proves that the improved algorithm demodulation F-P cavity length value has better consistency with the real value.

1. INTRODUCTION
Fiber optic Fabry-Perot sensor (F-P) is widely used in temperature, stress, magnetic field strength measurement due to its small size, wide response band, high sensitivity, immunity from electromagnetic interference and suitable for harsh environment and other advantages \cite{1}. The basic principle is that the external temperature, strain, stress, magnetic field intensity and other physical parameters will change the cavity length of the fiber F-P cavity, and then cause the change of the output spectrum of the reflection and transmission. By checking the light intensity, phase and polarization changes of the optical signal, the length of F-P cavity can be demodulated to detect the measured physical quantity.

According to the different optical parameters used in demodulation, the commonly used demodulation methods include light intensity demodulation method, fringe counting method, cavity
length matching method and Fourier transform, FT method \[^2\], etc. The intensity demodulation method is simple to calculate, but the error is large and the hardware such as the light source is high. Fringe counting method demodulation is simple, but the error is larger, and the demodulation error is on the order of micron. Cavity length demodulation method does not need expensive spectral receiver devices, and its cost is low, but its hardware implementation is difficult. The demodulation method based on the Fourier transform method usually uses the fast Fourier transform (FFT) to calculate the spectrum of the sensor. Due to the fence effect of FFT, the calculated frequency has a maximum error of 0.5 times of the spectral line interval \[^3\], at this time, if the cavity length is demodulated directly through the peak frequency obtained by FFT algorithm, a large error will be generated.

At present, the technology of optical fiber F-P sensor is developing towards the direction of long distance, array and high precision. Therefore, it is of great significance to study the high precision F-P cavity length demodulation algorithm suitable for engineering application. In order to improve the accuracy of the demodulation method based on Fourier transform, FFT-FT algorithm is introduced in this paper. The essence of this method is to firstly use FFT to calculate the panoramic spectrum, and then refine the peak value of the FFT discrete spectrum. Firstly, the resolution of the spectrum is determined, and then the range of the spectrum sequence to be refined is determined. Finally, the real part and imaginary part are calculated by Fourier transform (FT) continuous spectrum analysis, and the amplitude spectrum and phase spectrum are synthesized. The refining density can be set according to the actual requirements, so as to obtain the high precision spectrum of local refining. The proposed algorithm overcomes the error of the traditional FFT algorithm, which has the maximum 0.5 times spectral interval, and greatly improves the spectral resolution and the demodulation accuracy of the cavity length.

2. PRINCIPLE OF DEMODULATION

2.1. Principle of Fourier transform method to demodulate the Bree-Perot cavity length

The working principle of Fabry-Perot sensor is based on multi-beam interference \[^4\], and its structure is shown in Fig. 1.

When a beam of flat light with illumination intensity of \(I_0\) is incident into F-P cavity through optical fiber, multiple reflections and refraction will occur at the two end faces of the cavity to form multi-beam interference. The relationship between the output reflected light intensity and the cavity length and wavelength is as follows:

\[
I_\lambda = \frac{r_1 + r_2 - 2\sqrt{r_1r_2}\cos(4\pi nl/\lambda)}{1 + r_1r_2 - 2\sqrt{r_1r_2}\cos(4\pi nl/\lambda)} I_0
\]

(1)

\(r_1, r_2\) are respectively the reflectance of optical fiber end faces at both ends, \(l\) is the cavity length of F-P sensor, and \(n\) is the refractive index of medium in the cavity. When the reflectivity of the end faces of two optical fibers is the same and both of them are low, \(r_1 = r_2 = r\). When the medium in the cavity is air, \(n \approx 1\). Under these conditions, Equation (1) can be simplified as two-beam interference:

\[
I_\lambda = \frac{2r - 2r\cos(4\pi l/\lambda)}{2r - 2r\cos(4\pi l/\lambda)} I_0
\]

(2)
From Equation (2), it can be seen that the reciprocal of the output reflected light intensity and wavelength is a cosine-like relationship, and because the relationship between the wavelength $\lambda$ and frequency of light $v$ is:

$$\lambda = \frac{c}{v}$$  \hspace{1cm} (3)

The relation between light frequency and reflected light intensity is as follows:

$$I_r = [2r - 2r \cos(4\pi v t/c)]I_0$$  \hspace{1cm} (4)

Fourier transform of formula (4) is applied to obtain the spectrum of reflected light intensity:

$$F(\Omega) = \int_{-\infty}^{\infty} I_r e^{-j\Omega t} dt = \int_{-\infty}^{\infty} 2r[1 - \cos(4\pi v t/c)]I_0 e^{-j\Omega t}$$

$$d\nu = 2\pi rI_0[2\delta(\Omega) - \delta(\Omega + 4\pi v t/c) - \delta(\Omega - 4\pi v t/c)]$$  \hspace{1cm} (5)

According to Equation (5), the relationship between F-P cavity length and angular frequency is as follows:

$$l = \Omega c/(4\pi)$$  \hspace{1cm} (6)

Therefore, Fourier transform is carried out on the reflected spectral signal of the sensor to find the spectrum corresponding to the maximum value in the spectrum (filter out the DC component in the signal), and the cavity length of the sensor can be obtained by Formula (6).

In actual demodulation, since the computer can only calculate the discrete data, the above continuous Fourier transform should be changed into the discrete Fourier transform:

$$X(k) = \sum_{n=0}^{N-1} x(n)\exp(-j2\pi nk/N)$$  \hspace{1cm} (7)

$x(n)$ is the frequency-light intensity sampled signal after high-pass filtering. Find the number subscript $k_i$ corresponding to the peak value of the $X(k)$ amplitude, and the relationship between the number angular frequency $\omega_i$ and the angular frequency $\Omega$:

$$\omega_i = \Omega_i \delta\nu$$  \hspace{1cm} (8)

The cavity length of F-P can be obtained as:

$$l = c\omega_i/(2N\delta\nu)$$  \hspace{1cm} (9)

According to Equation (9), when sampling interval $\delta\nu$ and sampling points $N$ are constant, the precision of F-P cavity length depends on the resolution of $\omega_i$. Through Equation (9), it is easy to obtain the maximum error of F-P cavity length demodulated by the fast Fourier transform algorithm as follows:

$$\Delta l = c\delta k/(4N\delta\nu)$$  \hspace{1cm} (10)

2.2. Principle of FFT spectrum refinement

According to Equation (10), in order to improve the demodulation accuracy, sampling points and sampling intervals can be increased, and spectral resolution can be reduced. As a result, the increase of sampling points and sampling intervals is limited due to the limitation of the light source. The FFT-FT algorithm is introduced to improve the frequency resolution without increasing the sampling points and the calculation amount.

DFT of discrete signal $\{x_n\}, n = 0, 1, 2, \ldots, N-1$ can be expressed by real and imaginary parts as follows:

$$X(k) = \sum_{n=0}^{N-1} x_n \cos(2\pi kn/N), k = 0, 1, 2, \ldots, N/2-1$$  \hspace{1cm} (11)

$$X(k) = \sum_{n=0}^{N-1} x_n \sin(2\pi kn/N), k = 0, 1, 2, \ldots, N/2-1$$  \hspace{1cm} (12)

In essence, the discrete Fourier transform is obtained by the continuous Fourier transform through the following four steps: integrating operation into summation operation, time domain discretization, frequency domain discretization and time domain windowing truncation to finite length. If not in the four steps in the frequency domain discretization, and only through the continuous Fourier transform through the integral operation into a sum operation, time domain discretization, and the time domain truncation for finite length these three steps, then continuous Fourier transform into discrete time, continuous spectrum of special Fourier transform [5]:
$$X(f) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nf/N}, n = 0, 1, 2, ..., N-1$$ (13)

Where $x(n) = A\cos(2\pi f_0 n/(N+\theta))$ is a single harmonic signal sequence, $A$ is the amplitude, $f_0$ is the frequency, $\theta$ is the initial phase, $f_0^\prime = f_0/\Delta f$ is the normalized frequency normalized according to the frequency resolution $\Delta f = f_s/N$, $N$ is the number of FFT points, $f_s$ is the sampling frequency. By writing the real and imaginary parts of Equation (13), can be obtained:

$$x_R(f) = \sum_{n=0}^{N-1} x_n \cos(2\pi nf/f_s), 0 \leq f \leq f_s/2$$ (14)
$$x_I(f) = \sum_{n=0}^{N-1} x_n \sin(2\pi nf/f_s), 0 \leq f \leq f_s/2$$ (15)

Frequency resolution $f$ is a continuous frequency variation that is no longer limited by the number of sampling points. However, only discrete spectrum can be obtained by computer calculation in practice. But as long as the specified frequency range $[f_1, f_2]$ is suitable and narrow, the frequency resolution can be greatly improved and the accuracy of the analysis can be greatly improved.

Then, refine the specified frequency interval $[f_1, f_2]$, and set the refining multiple $D$ according to the accuracy of the frequency to be obtained in advance. Then, the refined frequency resolution is:

$$\Delta f' = \Delta f / D$$ (16)

Then the refined calculated spectral sequence is:

$$\{f_1, f_1 + \Delta f', f_1 + 2\Delta f', ..., f_2\}$$ (17)

The amplitude spectrum and phase spectrum of the spectrum sequence needed to be calculated are obtained by Equation (13). The frequency value corresponding to the maximum amplitude value is searched in each thinning spectrum line, which is the estimated frequency of the harmonic signal.

When using this method to demodulate the length of F-P cavity, the speed of directly using Formula (14) and (15) to calculate the panoramic spectrum of the output spectrum is slow, so FFT can be used to calculate the panoramic spectrum of the output spectrum first, and then using Formula (14) and (15) to perform local refining of the spectrum near the peak value. This way can speed up the calculation. However, it should be noted that this refinement does not increase the number of sampling points.

3. SIMULATION AND ANALYSIS

FFT-FT algorithm to obtain the F-P cavity length value. First, the original reflected spectral signal is processed by high-pass filtering to filter the DC component in the signal. Then the FFT algorithm is used to obtain the spectrum value, and then the FFT-FT algorithm is used to refine the frequency at the peak, so as to obtain the peak frequency with higher precision. Finally, the cavity length is calculated according to the relationship between frequency and cavity length. The specific demodulation steps are shown in Fig. 2:

Let the reflectivity of the optical fiber end face be $r = 0.04$, cavity length $l = 400\mu m$, the light source is a flat light source with wavelength $\lambda$ in the range of 1510-1590nm, light intensity is 1. Take 1024 points at equal wavelength intervals, Fig. 3 is a wavelength spectrogram.

![Flowchart of FFT-FT demodulation F-P cavity length algorithm](image)

Fig.2 Flowchart of FFT-FT demodulation F-P cavity length algorithm
The frequency spectrum of 1024 points generated by interpolation was transformed by FFT and the modulus value was taken to find out the frequency at its peak, shown in Fig. 4.

As can be seen from Figure 4, the peak position of the 30th point is located. Because the fence effect of FFT algorithm and the truncation interval of the signal are not equal to the integer multiple of the signal period, the 30th point in the amplitude spectrum is not the true peak value, but it is a point closest to the true peak value. The true peak value should be between the 31st point and its adjacent points. Because the window function added by signal truncation is a rectangular window, and the rectangular window belongs to the symmetric convex non-negative window function. According to the properties of symmetric window functions\[6\]:

1. The amplitude spectrum of the symmetric convex non-negative window function reaches its maximum value at \( \omega = 0 \).
2. There are at least two FFT lines in the main lobe of the amplitude spectrum of convex window function, and the two lines closest to \( \omega = 0 \) are the largest and the second largest, respectively. So the real peak is somewhere between the second highest and the highest, somewhere between 30 and 31. FT spectrum refinement of 100 points was done between these two points, and the refinement spectrum line is shown in Fig. 5.
Fig. 5 100 points FT spectrum refinement spectrum map

By finding the abscissa corresponding to the peak value of the amplitude after the refinement of the FT spectrum, the cavity length of the F-P cavity can be calculated as 399.9744μm, and the absolute error is not more than 26nm, and the relative error is not more than 0.0064%. The proposed method has high accuracy in the estimation of the frequency spectrum of the F-P cavity length, which can meet the accuracy requirements in actual demodulation. If the spectrum refinement is not carried out, the cavity length calculated by using the spectrum value obtained from FFT is 397.3256μm, and the absolute error and relative error are more than 100 times that after FT refinement.

Based on the above simulation experiments, the cavity length was changed from 400μm to 410μm with an interval of 1μm. The simulation results are shown in Fig. 6. The goodness of fit of the demodulation fitting curve of FFT-FT algorithm to the true cavity length value $R^2$ is 0.9996, while the goodness of fit of the demodulation fitting curve of FFT algorithm to the true cavity length value $R^2$ is only 0.4731. It can be seen that the demodulation value of F-P cavity length obtained by FFT-FT algorithm has high demodulation accuracy.

4. CONCLUSIONS

The improvement of demodulation performance of optical fiber Fabry Perot sensor is of great significance to its practical application. In order to improve the accuracy of Fabry-Perot sensor demodulated by fast Fourier transform method, FFT-FT algorithm is proposed to demodulate the Fabry-Perot sensor. By theoretical analysis, under the restriction of optical wavelength bandwidth of actual light source, and under the condition of constant sampling number N and sampling interval, the frequency near the peak value is refined, and more accurate demodulation results can be obtained. The simulation results show that the proposed algorithm can obtain the F-P cavity length with higher demodulation accuracy. The error of the cavity length demodulated by FFT algorithm is more than one hundred times that of FFT-FT algorithm when the frequency near the peak is refined by 100 times. The goodness of fit of the cavity length curve calculated by FFT-FT algorithm is more than 2 times that of FFT algorithm, which proves that the improved algorithm has better demodulation accuracy and consistency. Thinning factor affects the speed of demodulation algorithm. In order to maintain the speed of demodulation algorithm as much as possible and improve the demodulation accuracy, the next step is to study the relationship between the demodulation accuracy and demodulation time of FFT-FT demodulation algorithm and thinning factor and find out the best demodulation multiple.
(a) Comparison diagram of real value and demodulation value

(b) Local enlargement of true value and demodulation value

Fig.6 Comparison of real value and demodulation value

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