VELOCITY AND DISTRIBUTION OF PRIMORDIAL NEUTRINOS.

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ABSTRACT

The Cosmic Neutrinos Background (CNB) are Primordial Neutrinos decoupled when the Universe was very young. Its detection is complicated, especially if we take into account neutrino mass and a possible breaking of Lorentz Invariance at high energy, but has a fundamental relevance to study the Big-Bang. In this paper, we will see that a Lorentz Violation does not produce important modification, but the mass does. We will show how the neutrinos current velocity, with respect to comobile system to Universe expansion, is of the order of $10^6 \frac{\text{km}}{\text{s}}$, much less than light velocity. Besides, we will see that the neutrinos distribution is complex due to Planetary motion. This prediction differs totally from the usual massless case, where we would get a correction similar to the Dipolar Moment of the CMB.

INTRODUCTION

From the beginning, the photons and all particles were coupled forming a plasma that was evolving under the influence of the Universe expansion. In such a moment, when the photons were dominating the expansion, the neutrinos were decoupling from the plasma and evolved in an independent way. One of the last discoveries about neutrino is its mass. This has relevant effects in the Standard Model and in some of its characteristics, distinguishing it from the photons. One of them, which we will study, is its velocity. Thus, we will analyze the evolution of the neutrinos’s kinetic energy since its decoupling till today.

Other phenomenon that we will study, that is directly related with the first one, is the neutrinos distribution. The detection of the Cosmic Microwave Background of Photons (CMB) is the best proof of the Big-Bang scenario [1] that helped to check or refute models that describe it, and study the composition of the Universe. Because of this, it is important to study the Cosmic Neutrinos Background (CNB), especially the form of their Distribution Function to consider the effect of the peculiar velocity of the planet, named Dipolar Moment in the CMB, and optimize the detection. This is already complicated due to the low interaction that the neutrinos have with ordinary matter. The calculation will be done for photons and neutrinos in parallel.

Finally, we will include a Lorentz Invariance Violation (LIV) represented by an alteration to the Dispersion Relation of energy given by $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$...
\[ E^2 = v_{\text{max}}^2 p^2 + m^2 c^4 \]

Where \( v_{\text{max}} = c(1 - \alpha) \) is the maximum attainable particle velocity with \( \alpha \sim (10^{-22} - 10^{-23}) \). The motivation to use this LIV comes from the possibility that, at the high energies available in the Big Bang there take place some LIV due to Quantum Gravity \([2, 4, 5]\). If such a LIV exists, the first problem is the appearance of a privileged reference system, but fortunately exists a natural candidate, the one where the CMB is isotropic. A LIV without a preferred frame as in Double Special Relativity \([6]\), will not be considered here.

1 ENERGY AND VELOCITY OF PRIMORDIAL NEUTRINOS.

Initially the neutrinos were in thermal equilibrium with the rest of matter. For this, is necessary that \( \Gamma_i \gg H \), where \( \Gamma_i \) is the rate of interactions of the species \( i \), \( H \propto T^2 \) is Hubble’s constant and \( T \) the temperature. While the neutrinos are kept in equilibrium, its distribution will be given by Fermi-Dirac’s statistics:

\[ f_{eq}(E, T) = \frac{1}{e^{E/k_B T} + 1} \]

During cosmic expansion, the temperature will be diminishing down to a point where \( \Gamma_\nu \lesssim H \) and \( \Gamma_{i\neq \nu} \gg H \). This means that the neutrinos lost the equilibrium and are decoupled from the rest of the matter. We will name \( T_{\nu, D} \) the neutrinos decoupling temperature that are obtained when we impose \( \Gamma_\nu \simeq H(T) \). To see what is happening with its distribution we will do the following analysis. For a time \( t_0 \), an observer sees in any direction a quantity \( dN = f d^3r d^3p \) of neutrinos in an volume \( d^3r \) and with momentum between \( \vec{p} \) and \( \vec{p} + d\vec{p} \). After a \( dt \) time, the neutrinos have not interacted, so \( dN \) remains constant, but the volume in which they are, have increased in a factor \( \left( \frac{R(t_0 + dt)}{R(t_0)} \right)^3 \) and the momentum has diminished in \( \frac{R(t_0)}{R(t_0 + dt)} \), because of the expansion of the Universe. This means that \( f(E, T_\nu) \) is constant in time. Therefore, for \( t > t_D \) (or \( T_\nu < T_{\nu, D} \)) with \( t_D \) the moment in which is produced the decoupling, the distribution function is given by \([7]\):

\[ f[E(p(t)), T_\nu(t)] = f_{eq}[E(p_D), T_{\nu, D}] = f_{eq} \left[ E \left( \frac{R(t)}{R_D} \right), T_{\nu, D} \right] \]

When the subscript \( D \) refers to the age of decoupling. In addition, we know that the number of neutrinos, the total energy and the energy per neutrino are given by:

\[ N_\nu = \frac{gV}{(2\pi\hbar)^3} \int f(p, T_\nu) d^3p \]

\[ E_\nu = \frac{gV}{(2\pi\hbar)^3} \int E(p) f(p, T_\nu) d^3p \]

\[ \epsilon_\nu = \frac{E_\nu}{N_\nu} \]

\[ E^2(p) = v_{\text{max}}^2 p^2 + m^2 c^4 \]
to allow for a small LIV in the dispersion relation.

Now we can determine the Distribution Function that they will have after being decoupled. It is possible to express the energy of the neutrinos (high energies and small masses) as $E(t) = v_{\text{max, } \nu} p(t)$ during the decoupling (We use an expansion with zero order in the mass because $f$ depends exponentially on $E$), and as $p_D = p(t) \frac{R(t)}{R_D}$ we obtain using (1):

$$f[p, T_{\nu}] = \frac{1}{e^{\frac{v_{\text{max, } \nu} p}{k_B T_{\nu}}} + 1}$$  \hspace{1cm} (5)

With $T_{\nu} = T_{\nu, D} \frac{R_D}{R(t)}$ and $\mu_{\nu} = 0$ because of the low interaction that they have with matter. This means that the distribution of neutrinos after decoupling is Fermi’s with temperature $T_{\nu}$, therefore $RT_{\nu} = \text{cte}$. Replacing it in (2) and (3):

$$N_{\nu} = \frac{3 g V \zeta(3) (k_BT_{\nu})^3}{4 \pi^2 h^3 v_{\text{max, } \nu}^3}$$  \hspace{1cm} (6)

Where $\zeta(3) = 1.2021$ is the Riemann’s Zeta function. We see that, in fact, $N_{\nu}$ keeps constant in time because $V \propto R^3(t)$ and $T_{\nu} \propto R^{-1}(t)$. To determine $E_{\nu}$, we must compute the integral, which is complicated for the general case. Thus, we will analyze the extreme cases where the neutrinos continue being relativistic and when they do not. Due to the spherical symmetry, the velocity is only radial, therefore we just must determine its modulus. The modulus of the velocity of a particle is given by:

$$v = \frac{\partial \varepsilon}{\partial p}$$

Being $\varepsilon$ and $p$ the energy and the momentum of a particle, related by our dispersion relation:

$$\varepsilon^2 = v_{\text{max, } \nu}^2 p^2 + m^2 c^4$$

While the particle continues being relativistic, developing the derivative till the second order in the mass ($\varepsilon \gg mc^2$), we obtain:

$$v_{\nu} \simeq v_{\text{max, } \nu} \left(1 - \frac{1}{2} \left(\frac{mc^2}{\varepsilon}\right)^2\right)$$  \hspace{1cm} (7)

Notice that we must use $E(p) = v_{\text{max, } \nu} p$ to calculate $E_{\nu}$, to the order of approximation in the mass that we are considering.

Now, if the particle becomes Non-Relativistic, we have that the energy and the velocity of a particle to second order in the momentum ($p v_{\text{max, } \nu} \ll mc^2$) will be:

$$\varepsilon_{\nu} \simeq mc^2 + \left(\frac{v_{\text{max, } \nu}}{c}\right)^2 \frac{p^2}{2m}$$

$$v_{\nu} = \frac{\partial \varepsilon}{\partial p} \simeq \left(\frac{v_{\text{max, } \nu}}{c}\right)^2 \frac{p}{m} = v_{\text{max, } \nu} \sqrt{2 \left(\frac{\varepsilon}{mc^2} - 1\right)}$$  \hspace{1cm} (8)
Where we see that, to keep the order in the momentum, the calculation must be up to second order in the expression of \( E(p) \), therefore \( E(p) = mc^2 + \left(\frac{v_{\text{max},\nu}}{c}\right)^2 \frac{p^2}{2m} \).

1.1 Relativistic Neutrinos

As we said, to determine \( E_\nu \) we must use \( E(p) = v_{\text{max}}p \). With this, we obtain the expression:

\[
E_\nu = \frac{7\pi^2 gV(k_B T_\nu)^4}{240 \hbar^3 v_{\text{max},\nu}^3}
\]  

(9)

Using (6) and (9) in (4) and (7), we obtain:

\[
\varepsilon_\nu = \frac{7\pi^4}{180 \zeta(3)} k_B T_\nu
\]

(10)

\[
v_\nu = v_{\text{max},\nu} \left(1 - \frac{1}{2} \left(\frac{180 \zeta(3) m_\nu c^2}{7\pi^4 k_B T_\nu}\right)^2\right)
\]

(11)

If we define the relative velocity between the neutrinos and the photons as \( \Delta v = c - v_\nu \), result:

\[
\Delta v = \Delta v_{\text{max}} + \frac{v_{\text{max},\nu}^2}{2} \left(\frac{180 \zeta(3) m_\nu c^2}{7\pi^4 k_B T_\nu}\right)^2
\]

Where \( \Delta v_{\text{max}} = c - v_{\text{max},\nu} = c\alpha_\nu \). We can see that this factor vanishes if the violation does not exist. Evaluating numerically:

\[
\frac{\Delta v}{c} = \alpha_\nu \left(1 - 5.04 \times 10^{-2} \left(\frac{M_\nu}{k_B T_\nu}\right)^2\right) + 5.04 \times 10^{-2} \left(\frac{M_\nu}{k_B T_\nu}\right)^2
\]

(12)

Where we have separated the LIV dependent part from the rest.

1.2 Non-Relativistic Neutrinos

In this case we have that \( E(p) = m_\nu c^2 + \left(\frac{v_{\text{max},\nu}}{c}\right)^2 \frac{p^2}{2m_\nu} \), therefore, when we evaluate in \( E_\nu \) using (6), we obtain:

\[
E_\nu = N_\nu m_\nu c^2 \left(1 + \frac{1}{2} \left(\frac{k_B T_\nu}{m_\nu c^2}\right)^2 \frac{I_4}{I_2}\right)
\]

(13)

With \( I_n = \int_0^\infty \frac{x^n}{e^x + 1} dx = (1 - \frac{1}{2^n}) n! \zeta(n + 1) \). Then, evaluating in (4) and (8), we have:

\[
\varepsilon_\nu = m_\nu c^2 \left(1 + 15 \frac{\zeta(5)}{2 \zeta(3)} \left(\frac{k_B T_\nu}{m_\nu c^2}\right)^2\right)
\]

(14)

\[
v_\nu = v_{\text{max},\nu} \sqrt{15 \frac{\zeta(5) k_B T_\nu}{\zeta(3)^2 M_\nu}}
\]

(15)

Giving a relative velocity.
\[
\frac{\Delta v}{c} = \alpha \nu \sqrt{15 \frac{\zeta(5)}{\zeta(3)}} k_B T_\nu \left( 1 - \sqrt{15 \frac{\zeta(5)}{\zeta(3)}} M_\nu \right) \tag{16}
\]

Where we have separated the LIV part from the rest and \( \zeta(5) = 1.0369 \). Since the neutrino velocity cannot be higher than its maximum velocity, we must see to what temperatures this approximation is valid. We have that \( v_\nu > v_{\text{max},\nu} \) if \( k_B T_\nu > \sqrt{\frac{\zeta(3)}{15 \zeta(5)}} M_\nu \). It means that the approximation is valid if \( k_B T_\nu \ll \sqrt{\frac{\zeta(3)}{15 \zeta(5)}} M_\nu \sim 0.28 M_\nu \).

1.3 Numerical Results and Analysis

In the age of decoupling of the neutrinos, we know that \( k_B T_{\nu,D} \simeq (2 - 4) \text{ [MeV]} \) and, currently, \( k_B T_{\nu,0} = 1.68 \times 10^{-4} \text{ [eV]} \). In addition to this, for cosmological parameters, we know [8]:

\[
\sum_i m_{\nu_i} \leq 0.17 \text{ [eV]}
\]

That clearly indicates that they are relativistic in the moment of the decoupling. There exist many estimations of the masses of the neutrinos, but none of them are very precise. Thus, we will use \( m_{\nu} \simeq 0.17 \text{ [eV]} \). This way, we are sure of being inside the correct limits and we will find the maximum effect that the mass could have in the velocity of neutrinos. This way, none of these estimations is below \( k_B T_{\nu,0} \), therefore they are Non-Relativistic nowadays.

Before discussing the results, we will analyze the effect of the LIV. Thus, we will compare our relativistic expressions with our Non-Relativistic ones. If we observe these expressions, we can see that both are proportional to \( v_{\text{max},\nu} \), which is the only thing that depends on \( \alpha \nu \). It means that the difference in percentage between the case with and without LIV is always:

\[
\frac{v_\nu(0) - v_\nu(\alpha \nu)}{v_\nu(0)} \times 100\% = \alpha \nu \times 100\% = 1 \times 10^{-20}\%
\]

Therefore, it is not possible that this LIV has an important effect in the neutrinos, then we will continue our calculations using \( \alpha = 0 \).

Previously we mentioned that our Non-Relativistic approximation is valid if \( \frac{k_B T_\nu}{M_\nu} \leq 0.28 \). At present we have that \( \frac{k_B T_\nu}{M_\nu} \simeq 10^{-3} \) fulfilling the Non-Relativistic bound, but with a mass 100 times minor \( (\sim 2 \times 10^{-3} \text{ [eV]}) \) the bound is not respected. However it does not correspond to the relativistic case either. To be kept relativistic, we need a mass 10000 times smaller or less \( (\sim 2 \times 10^{-5} \text{ [eV]}) \).

In Figure 3 the evolution of the velocity of the neutrinos due to the expansion of the Universe is represented graphically. We define the adimensional quantities \( z = \frac{M_\nu}{k_B T_\nu} \), \( y = \frac{v_\nu}{c} \). It is indicated in the graph that the time grows towards bigger values in \( z \). Clearly, we see that the neutrinos suffer a rapid deceleration from the time of decoupling. Then, this deceleration begins to diminish slowly, approaching a zero velocity.

All the estimations of \( M_\nu \) indicate that we are in a zone dominated by the Non-Relativistic approximation. The estimation for the smallest masses ranges between \( 10^{-4} \) and \( 10^{-3} \text{ [eV]} \). Remembering that our top limit is 0.17 [eV], we see that we are currently in the region \( 0.6 < z < 1012 \), which is a very wide range. Evaluating numerically in (15), we obtain \( v_\nu = 3.55 \times 10^{-3}c = 1065 \left( \frac{\text{km \ s}}{\text{c}} \right) \), with a mass of 0.17 [eV]. This velocity will be bigger if we use smaller neutrino masses.
Up to now, we have assumed that the neutrinos are not affected by the galactic potential, they are free particles and are not relics from the Milky Way [9]-[10]. To check this point, we consider the relation between kinetic and potential energy of the neutrino in the Milky Way. That is:

\[ \frac{m_\nu v^2}{2} = \frac{GMm_\nu}{R} \]

Where \( v \) would be the limit velocity where the potential energy is comparable with the kinetic energy.

Evaluating in \( M \simeq 2 \times 10^{42} \text{[kg]} \) and \( R \simeq 4.7 \times 10^{20} \text{[m]} \), mass and radius of the Milky Way respectively, we obtain \( v \simeq 754 \text{[km/s]} \). Since \( v_\nu \gtrsim 1000 \text{[km/s]} \), our supposition is correct.

2 THE CNB DISTRIBUTION.

To determine the effective neutrinos distribution (distribution from Earth), we need to use equation (5) in the comobile system of the Universe. In addition to this, we will use the photons distribution when they are decoupled, that is:

\[ f(p,T) = \frac{1}{e^{\frac{pc}{kT}} - 1} \]

That corresponds to an ultra-relativistic Bose-Einstein’s distribution with \( RT = cte \). Currently, it has a temperature of 2.73 [K]. In addition, we saw in the previous chapter that a LIV of the form:

\[ E^2 = v_{\text{max}}^2 p^2 + m^2 c^4 \]

Is not markedly different in its energy and velocity in comparison to the usual Dispersion Relation \((v_{\text{max}} = c)\). Because of this the usual special relativity rules are valid. For instance, Lorentz’s Transformations for the coordinates of space-time and of energy - momentum. Then, we can compare the neutrinos distribution in the comobile system and Earth. It is possible to demonstrate (See Appendix):

\[ f'(p',T') = f(p,T) \]  \hspace{1cm} (17)

\[ E = \gamma(E' - v_t p' \cos(\theta')) \]  \hspace{1cm} (18)

Where the primed elements refer to the reference system of Earth and the non primed to the comobile. \( \theta' \) is the angle that is formed between the vision line and the direction of Earth motion and \( v_t \) is the Earth’s velocity [11]. We can see that the distribution function is invariant under Lorentz’s Transformation and the energy changes with the angle of vision.

Now we will analyze some cases. First, the photons of the CMB to guide us because they are already very well known, and secondly, the neutrinos. Using expression (18) we will determine \( p' \) as a function of \( p \).
2.1 Photons

In this case, we have that \( E = cp \), therefore the expression (18) is reduced to:

\[
p = \frac{1 - \frac{v}{c} \cos(\theta)}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} p'
\]

Replacing in (17), we obtain:

\[
f'(p', T'_\gamma) = f\left(\frac{1 - \frac{v}{c} \cos(\theta')}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} p', T_\gamma\right)
\]

As the photons, after being decoupled, continue with a distribution of the form:

\[
f_\gamma = \frac{1}{e^{\frac{p}{k_B T_\gamma}} - 1}
\]

We can leave our expression as:

\[
f'(p', T'_\gamma) = f\left(p', T_\gamma \sqrt{1 - \left(\frac{v}{c}\right)^2} \frac{1}{1 - \frac{v}{c} \cos(\theta')}\right)
\]

Therefore, the photons distribution detected from Earth, \( f' \), in a specific direction, will be of the same form that the one detected in the comobile system to the Universe, but with a different temperature given by:

\[
T'_\gamma = T_\gamma \sqrt{1 - \left(\frac{v}{c}\right)^2} \frac{1}{1 - \frac{v}{c} \cos(\theta')}
\]

If we consider that \( v_t \ll c \), we have:

\[
T'_\gamma \simeq T_\gamma \left(1 + \frac{v_t}{c} \cos(\theta')\right)
\]

\[
\frac{\Delta T_\gamma}{T_\gamma} \simeq \frac{v_t}{c} \cos(\theta')
\]

that is known as the Dipolar Moment, and is of the order of \( 10^{-4} \).

2.2 Neutrinos

Now, we have particles with mass. Currently, the neutrinos are Non-Relativistic, therefore \( E = m_\nu c^2 + \frac{p^2}{2m_\nu} \) for both the comobile and Earth systems. Evaluating in (18) and using the approximation \( v_t \ll c \) up to second order in \( p' \) and \( v_t \), we obtain:

\[
p^2 = p'^2 - 2m_\nu v_t p' \cos(\theta') + m_\nu^2 v_t^2
\]

Evaluating in (17), we have:

\[
f'(p', T'_\nu) = f\left(\sqrt{p'^2 - 2m_\nu v_t p' \cos(\theta')} + m_\nu^2 v_t^2, T_\nu\right)
\]
| $a(\theta')$ | Direction of Observation                                      |
|-----------|--------------------------------------------------------------|
| 1         | In favour of the Terrestrial Movement                        |
| 0.5       | 60° deflected to the Terrestrial Movement                    |
| 0         | Perpendicular to the Terrestrial Movement                    |
| −0.5      | 120° deflected to the Terrestrial Movement                   |
| −1        | Against the Terrestrial Movement                             |

Table 1: Directions of Observation.

Values of $a(\theta')$ used in Figure 4 with the corresponding direction of observation.

In this case is impossible to find a relation between $T'_\nu$ and $T_\nu$, but we know that the distribution is given by (5). Seemingly, we can only notice the effects graphically. To facilitate our analysis, it will be helpful to define the number of neutrinos per solid angle $d\Omega'$ of momentum as:

$$
\frac{dN}{d\Omega'} = \frac{gV}{(2\pi\hbar)^3} f'(p', T'_\nu) p'^2 dp
$$

With this, we can obtain the distribution function of the number of particles:

$$
F'(p', T'_\nu) = \frac{gV}{(2\pi\hbar)^3} f'(p', T'_\nu) p'^2
$$

(21)

In our case, the distribution function $F'$ will be:

$$
F'(p', T'_\nu) \propto \frac{p'^2}{e^{\frac{p'^2}{k_B T'_\nu} - \frac{2m_\nu v t}{k_B T'_\nu} \cos(\theta') + \frac{m_\nu^2 v^2}{2 k_B T'_\nu} + 1}}
$$

2.3 ANALYSIS

To do our analysis, it is useful to introduce the adimensional variables $x = \frac{p' c}{k_B T'_\nu}$, $a(\theta') = \frac{m_\nu v t}{k_B T'_\nu} \cos(\theta')$ and $b = a(0)$. With these parameters, our distribution is:

$$
F' \propto \frac{x^2}{e^{\frac{x^2}{2ax + b^2} + 1}}
$$

(22)

Considering a terrestrial velocity $v_t \simeq 300 \left( \frac{km}{s} \right)$, we can see that $b \simeq 1$ for $M_\nu = 0.17$. It means that $-1 \leq a(\theta') \leq 1$. This range will be smaller if we use a smaller mass, but then the Non-Relativistic approximation is less precise.

In Figure 4 it is shown $F'$; here we have used a value of $b \sim 1$ and some representative values of $a(\theta')$ (See Table 1).

Let’s remind that the distribution $F'$ represents the particles number that come from certain direction and momentum. We see in Figure 4 that the distribution suffers a loss of homogeneity, which is translated in more neutrinos observed in favour of the Earth’s movement, but simultaneously the form of the distribution function is altered much more with regard to the distribution of the comobile system. If we move away from this direction, the neutrino number detected diminishes considerably and the small momentum are favored.

The distribution maximum must fulfill the equation:
\[
(2\sqrt{x_{\text{max}}^2 - 2ax_{\text{max}} + b^2} - x_{\text{max}} + ax_{\text{max}}) e^{\sqrt{x_{\text{max}}^2 - 2ax_{\text{max}} + b^2}} + 2\sqrt{x_{\text{max}}^2 - 2ax_{\text{max}} + b^2} = 0 \quad (23)
\]

It is complicated to find a general expression for \(x_{\text{Max}}\), although a numerical treatment is readily available. As an example, we can study the extreme cases \(a(\theta) = b\) and \(a(\theta) = -b\) where \(b \simeq 1\) for \(M_\nu = 0.17\) [eV]. Evaluating in (23), we obtain:

\[
x_{\text{max}}(a = b) = 2.463
\]

\[
x_{\text{max}}(a = -b) = 2.091
\]

This means that the momentum of the majority of detected neutrinos will be:

\[
2.091 \leq \frac{p'c}{k_B T_\nu} \leq 2.463
\]

This will be a useful information to plan the detectors.

Now, if we use a smaller mass, the differences between different directions in the distributions diminish. In Figures [5] and [9] we can see the distributions with a mass 10 and 100 times smaller, where the Non-Relativistic approximation can be still valid. To compare, the photons distribution appears in Figure [7] for different observation angles. Comparing this with Figure [9] we see that the effect produced in the neutrinos distribution is bigger always than the produced in the photons.

This happens because the photons always go to a bigger velocity than the terrestrial (\(c \gg v_t\)), therefore Earth would be almost still with regard to the comobile system, doing that the isotropy almost does not change. On the other hand, if the neutrinos acquire mass, they will be found submitted to a deceleration as the Universe expands (See Figure [3]) so that that currently the neutrinos are Non-Relativistic, with a velocity not much higher than \(v_t\). This means that the effect of the terrestrial movement begins being important in the velocities addition and will be increased with the time due to the constant cooling of the neutrinos. This will reach the point in which the neutrino velocity will be much smaller than \(v_t\) and, practically, the planetary movement will predominate. This will be reflected in an increase of the distribution in the direction of the terrestrial movement.

CONCLUSION.

The mass of the neutrinos brought important modifications to its velocity. Without mass, the neutrinos would have supported a constant velocity and equal to the light velocity, \(c\). On the other hand, with non zero masses, its velocity is affected by a strong deceleration (See Figures [3]), therefore they are Non-Relativistic nowadays. As we have developed an expression for the velocity with regard to the comobile system to the expansion of the Universe, it is necessary to use the addition of velocities to determine the mean neutrino velocity relative to Earth. Thus, we use Lorentz’s Transformations since the LIV did not bring any important effect. The difference that is produced in its velocity with and without LIV is of \(\sim 10^{-20}\) %, which is totally negligible. Then, we can use the invariance of the distribution function to relate the comobile system to the terrestrial.

In the same way, the mass of the neutrinos brought important changes to the distribution. Unlike the photons, it was not possible to introduce a similar term to the Dipolar Moment because the temperature would depend on \(p'\). Greater the mass greater the effect. In addition, the distribution is widely favored in the Earth’s direction, but if we move away from this direction, the neutrinos number diminishes. In
spite of that the variation depends greatly on the mass; as time goes by, the neutrinos will be cooling diminishing little by little its velocity. This means that in some moment the velocity of the neutrinos will be less than the terrestrial speed. In the future, the neutrinos will be almost still in comparison to the Earth’s velocity. In this moment, we will only detect the neutrino that "crash" with Earth when it advances.

To sum up, we see that the existence of the neutrino mass produces a relatively important effect in its evolution, which is reflected in the perception that we have of them especially in the loss of homogeneity in the distribution function. Thus, for its detection is advisable to use detectors of neutrinos directed in favour to the terrestrial movement or to use a satellite located in someone of Lagrange’s points of the Solar System to keep the isotropic distribution of the comobile system, as the satellite Planck Surveyor that will observe CMB [12].

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APPENDIX.

The special relativity rules say us that two reference systems can be related by:

\[
\mathbf{x} = \mathbf{x}' + \gamma(\mathbf{x}' \cdot \mathbf{v}_t) \\
t = \gamma \left( t' + \frac{\mathbf{v}_t \cdot \mathbf{x}'}{c^2} \right)
\]

and

\[
\mathbf{p} = \mathbf{p}' + \gamma \left( \mathbf{p}' \cdot \mathbf{v}_t + \frac{\mathbf{v}_t}{c^2} E' \right) \\
E = \gamma(E' + \mathbf{v}_t \cdot \mathbf{p}')
\]

Where the primed reference system is moving away from the non primed to a velocity \( \mathbf{v} \). The coefficients with the subscripts \( \parallel \) and \( \perp \) represent the parallel and perpendicular components of the velocity \( \mathbf{v}_t \) respectively. Since we are considering a particle in the universe, our primed and non primed reference systems will be, respectively, Earth and comobile system to the Universe expansion, therefore \( \mathbf{v} \) is the planet velocity. Thus, from now, we will call its \( \mathbf{v}_t \).

If particles go to Earth along the vision line (See Figure 1), the Lorentz’s transformation can be written as:

\[
x_{\perp,i} = x'_{\perp,i} \\
x_{\parallel} = \gamma(x'_{\parallel} + v_t t') \\
t = \gamma \left( t' + \frac{v_t x'_{\parallel}}{c^2} \right)
\]

(24)

\[
p_{\perp,i} = p'_{\perp,i} \\
p_{\parallel} = \gamma \left( p'_{\parallel} - \frac{v_t}{c^2} E' \right) \\
E = \gamma(E' - v_t p'_{\parallel})
\]

(25)

Where \( i = 1, 2 \) label both perpendicular coordinates to \( \mathbf{v}_t \). Its differential form considering an instantaneous measurement from Earth, is \( t' = cte \) or \( dt' = 0 \), giving:

\[
dx_{\perp,i} = dx'_{\perp,i} \\
dx_{\parallel} = \gamma dx'_{\parallel} \\
dt = \gamma \frac{v_t dx'_{\parallel}}{c^2}
\]

(26)

\[
dp_{\perp,i} = dp'_{\perp,i} \\
dp_{\parallel} = \gamma \left( dp'_{\parallel} - \frac{v_t}{c^2} dE' \right) \\
dE = \gamma(dE' - v_t dp'_{\parallel})
\]

(27)
Figure 1: Description of both reference systems. $S$: Comobile reference system to the Universe expansion. Earth has a velocity $v_t$ and the neutrino has momentum $p$. Between both we have the vision angle $\theta$. $S'$: Earth reference system. Earth is still and the neutrino has momentum $p'$. We have the vision angle $\theta'$ measured from Earth. The coordinates system of $S$ and $S'$ are related by the Lorentz’s Transformation.

Now, when we count the particles number from Earth in a specific direction instantaneously ($dt' = 0$), inside a volume $d^3r'$ we have $dN$ particles with momentum between $\vec{p}'$ and $\vec{p}' + d\vec{p}'$. In addition, we know that $dN$ is given by:

$$dN = f'(p', T')d^3p'd^3r'$$

Where $f'$ is the distribution function on Earth. In the comobile system, the particles are in a volume $d^3r$ and with values of momentum between $\vec{p}$ and $\vec{p} + d\vec{p}$, but in a $dt$ time, given by (26) (different from zero because $dt' = 0$) some particles enter or exit of $d^3r$. Thus, the particle number, in this system, is given by:

$$dN = f(p, T)d^3p + f(p, T)d^3pE \cdot \vec{u}dt$$

Where $\vec{u} = c^2 \frac{\vec{p}}{E}$ is the particle’s velocity and $d\vec{S}$ is the differential area, with normal direction. Both expressions for $dN$ are, simply, a variation of the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

With $\rho = f(p, T)d^3p$. Since $dN$ must be the same in both systems, we must equal (28) and (29). Then:

$$f'(p', T')d^3p'd^3r' = f(p, T)d^3p \left( d^3r + c^2 dt \frac{\vec{E} \cdot \vec{S}}{E} \right)$$

With (See Figure 2):

$$d^3r = dx_\parallel \wedge dx_{\perp,1} \wedge dx_{\perp,2}$$

$$d\vec{S} = -(dx_\parallel \wedge dx_{\perp,1}\hat{x}_{\perp,2} + dx_\parallel \wedge dx_{\perp,2}\hat{x}_{\perp,1} + dx_{\perp,1} \wedge dx_{\perp,2}\hat{x}_\parallel)$$

$$\vec{p} = -(p_{\parallel}\hat{x}_\parallel + p_{\perp,1}\hat{x}_{\perp,1} + p_{\perp,2}\hat{x}_{\perp,2})$$

Where $\wedge$ represents the anti-commutative product between the differentials. Evaluating in (30), using (25) and (26), we have:
Figure 2: Representation of the volume element $d^3r$, where we see the surface elements. We can see, clearly, the vectorial direction of $\vec{p}$ and $d\vec{S}$.

\[
\frac{f'(p', T')d^3p' d^3r'}{E'} = f(p, T)d^3p d^3r E
\] (31)

Replacing (27) in $d^3p = dp_{\bot,1} \wedge dp_{\bot,2} \wedge dp_{\parallel}$, we obtain:

\[
d^3p = dp_{\bot,1}' \wedge dp_{\bot,2}' \wedge \gamma \left( dp_{\parallel}' - \frac{v_{\parallel}}{c^2} dE' \right)
\]

But we know that $E'^2 = c^2(p'^2_{\bot,1} + p'^2_{\bot,2} + p'^2_{\parallel}) + m^2c^4$. Deriving, we obtain the relation $E'dE' = c^2(p'_{\bot,1}dp'_{\bot,1} + p'_{\bot,2}dp'_{\bot,2} + p'_{\parallel}dp'_{\parallel})$. Evaluating:

\[
d^3p = dp_{\bot,1}' \wedge dp_{\bot,2}' \wedge dp_{\parallel}' \gamma \left( 1 - \frac{p_{\parallel}'}{E'} v_{\parallel} \right)
\]

Where we have used that $dp'_{\bot,i} \wedge dp'_{\bot,i} = 0$ for anti-commutativity. Using (25), $d^3p$ stays:

\[
d^3p = d^3p' \frac{E}{E'}
\]

Then, (31) is reduced to:

\[
f'(p', T') = f(p, T)
\] (32)

This means that the distribution function is Lorentz invariant. In reference [13], this has been discussed differently. They used:

\[
dN = f(p, T)d^3pd^3r
\]
Naturally, they obtained that $f$ is not Lorentz invariant.

Figure 3: Neutrino Velocity Representation (Eqs. [11] y [15]). It is had being dominated by the relativistic expression (Blue) and then for the Non-Relativistic (Red). The general expression would be a composition of both.
Figure 4: Primordial Neutrinos Distribution in the current age (Ec 22) for different values of $a$ and $b \simeq 1$ that corresponds to $M_\nu = 0.17$ [eV]. The black curve represents the distribution in the comobile system.

Figure 5: Primordial Neutrinos Distribution in the current age (Ec 22) for different values of $a$ and $b \simeq 0.1$ that corresponds to $M_\nu = 0.017$ [eV]
Figure 6: Primordial Neutrinos Distribution in the current age (Ec 22) for different values of \(a\) and \(b \approx 0.01\) that corresponds to \(M_{\nu} = 0.0017\) [eV]

Figure 7: Primordial Photons Distribution in the current age (Ec 22) for different values of the observation angle \(\theta'\).