Entanglement purification of multi-mode quantum states

J. Clausen, L. Knöll, and D.-G. Welsch
Friedrich-Schiller-Universität Jena,
Theoretisch-Physikalisches Institut,
Max-Wien-Platz 1, D-07743 Jena, Germany
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I. INTRODUCTION

The impossibility to isolate a physical system completely from its environment represents a major difficulty in the experimental realization of devices processing quantum information. Of crucial interest are therefore schemes which tolerate a certain amount of environmental coupling yet protect the actual information, e.g., by accepting some redundancy in the quantum state preparations. For example, a pure entangled state may be recovered from a number of mixed entangled states of spatially extended signals shared by separated parties [1]. A potential application of such entanglement purification schemes is the realization of a secure data transmission despite using a lossy channel and apparatus [2]. Apart from this, the treatment of entanglement purification helps to deepen the understanding of entanglement itself [3, 4]. Related questions which have been addressed are whether it is sufficient to process each signal state copy individually [5] or how to deal with copies which cannot be factorized into separate states [6]. Entanglement swapping [6, 7] has been discussed to propose schemes operating more efficient, despite loss.

While for certain classes of states such a superpositions of coherent states, methods solely based on linear optical elements like beam splitters and photodetections could be found [8], an implementation covering other classes of entangled states remains a challenge. Schemes based on cross-Kerr couplers have been described for Gaussian continuous variable entangled states [9, 10, 11]. Alternatively to the generalization to continuous variable states shared by two stations, multi-user schemes have been considered which involve more than two parties [12].

The aim of this work is to investigate a range of truncated mixtures of entangled multi-mode states, whose periodic nature allows a relatively simple separation of their pure components despite the fact that the states themselves are infinite. Here, focus is on the theoretical possibility of a purification. From a practical point of view, an implementation of the suggested scheme may be realistic only for special signal states. The effect of loss is discussed in a selected example of application.

II. INPUT STATES

Within this work, we assume that the mixture describing the input signal state $\hat{\varrho}$ can be truncated at a sufficiently large number $M$,

$$\hat{\varrho} = \sum_{n=0}^{M} p_n |\Psi_n\rangle\langle\Psi_n|, \quad \langle\Psi_n|\Psi_n\rangle = \delta_{n'n}.$$  \hfill (1)

By entanglement purification we understand the extraction of some pure component $|\Psi_0\rangle\langle\Psi_0|$ of $\hat{\varrho}$ by effectively implementing a transformation $\hat{\varrho} = p^{-1}\hat{\varrho}\hat{\varrho}^+$, $p = \langle\hat{\varrho}\hat{\varrho}\rangle$ with $\hat{\varrho} \sim |\Psi_0\rangle\langle\Psi_0|$. It may therefore be understood as a multi-mode state purification. We assume that in each signal mode $j$, whose entity is denoted by $I_j = 1, \ldots, N$, a repeater is located whose operation involves some auxiliary modes $b_j$. If the repeaters can only use local preparations $|F\rangle_{b_j}$, unitary transformations $\hat{U}_j$, and detections $b_j|G_j\rangle$, they can solely implement local operators $\hat{Y}_j|G_j\rangle = \hat{U}_j|G_j\rangle b_j|G_j\hat{U}_j,b_j\rangle|F\rangle_{b_j}$, which may depend on the results $G_j$ of the local detections in the auxiliary modes. In contrast, the $|\Psi_n\rangle$ may be arbitrary entangled states of the spatially separated signal modes. Therefore, the repeaters must share an entangled state in general. If this state has not been provided previously by a separate resource similar to teleportation setups, then the only alternative is to equip each of the repeaters with a second signal input mode $-j$, whose entity is denoted by $II = -1, \ldots, -N$, and which is fed with a second copy of the signal state $\hat{\varrho}$. If the repeaters periodically receive preparations $\hat{\varrho}$ the purification can be implemented in a loop (using, e.g., a cavity) as a repeated transformation of the signal, each time applying a further copy of the signal state.

In what follows, we assume that there exists a single-mode basis $\{|\Phi_n\rangle\}$ allowing the representation of the sig-
ormal eigenstates in the form of

$$|\Psi_n\rangle = (M+1)^{-N/2} \sum_{I=0}^{\infty} q(n) \sum_n^{(n)} |\Phi_{n_1,\ldots,n_N}\rangle,$$

(2)

where $l=l_1,\ldots,l_N$, and to ensure normalization we must have $\sum_{n=0}^{\infty} |q(n)|^2 = 1$. The expression $\sum_{n=0}^{(n)}$ denotes the sum over all $(M+1)^{N-1}$ different $N$-tuples $n = n_1,\ldots,n_N$ of numbers $n_j$ that can each take the values 0,\ldots,M but must obey $\sum_{j=1}^{N} n_j = n$. Here, we have used the notation

$$[m] \equiv \text{Mod}(m, M+1).$$

(3)

In order to explain the operation of the purification scheme, it will be convenient to distinguish between the states

$$\hat{R}_I = \sum_{n=0}^{M} p_n |\Psi_n\rangle_I \langle \Psi_n|,$$

(4a)

$$\hat{O}_II = \sum_{n=0}^{M} p_n |\Psi_n\rangle_{II} \langle \Psi_n|,$$

(4b)

where $\hat{R}_I$ is the state of the signal currently processed in the loop and $\hat{O}_{II}$ that of the original signal just entering the second signal input ports of the repeaters.

III. IMPLEMENTATION OF THE QUANTUM REPEATER

To describe the transformation of the input states Eqs. (4), let us take a closer look at the quantum repeater illustrated in Fig. 1(a). $\hat{V}$ is a unitary single-mode oper-

![Figure 1](image_url)

FIG. 1: Implementation of an $N$-mode entanglement purification. The overall setup shown in (b) consists of quantum repeaters located in each of the $N$ signal modes $I = 1,\ldots,N$. Their operation requires a second copy of the signal state to be fed into the inputs $I_I = -1,\ldots,-N$. A detailed view of a single repeater is given in (a).

ator that transforms the basis states $|\Phi_n\rangle$ to Fock states $|n\rangle = \hat{V}|\Phi_n\rangle$, i.e., the single-mode basis $\{ |\Phi_n\rangle \}$ is supposed to be known. A realization of a desired single-mode operator or detection in a truncated space also based on beam splitter arrays, zero and single photon detections as well as cross-Kerr elements is presented in [13]. (Note that the $\hat{V}$ are not required if attention is limited to the photon number basis.) Enclosed between devices implementing $\hat{V}$ and $\hat{V}^\dagger$, an array of $2(M+1)$ cross-Kerr elements is placed, whose operation is described by the operator

$$\hat{K} = e^{i\frac{\pi}{2\sqrt{M+1}} (a_j^\dagger a_j - a_j a_j^\dagger)} \sum_{m=0}^{M} m b_m^\dagger b_m.$$  

(5)

This array couples the signal modes $j$ and $-j$ to $M+1$ auxiliary modes $0,\ldots,M$, themselves coupled by two $(2M+1)$-port beam splitter arrays implementing the operators $\hat{U}$ and $\hat{U}^\dagger$, respectively. The latter are defined by the matrix elements

$$U_{kl} = \langle \varphi_k^* | \hat{U} | \varphi_l \rangle = \frac{1}{\sqrt{M+1}} e^{i kl \frac{\pi}{2\sqrt{M+1}}},$$

(6a)

$$|\varphi_k \rangle = \hat{b}^\dagger_k |0\rangle \cdots |0\rangle_M.$$  

(6b)

Initially, these auxiliary modes are prepared in the state $|\varphi_0 \rangle$, i.e., a single-excited Fock state is fed into input port 0 of the $\hat{U}$-array, while the remaining input ports are left in the vacuum. Assuming a final detection of the auxiliary modes in the state $|\varphi_k \rangle$, i.e., the photodetectors at the output ports $m$ of the $\hat{U}^\dagger$-array detect $\delta_{mk}^j$ photons, the action of the $j$th repeater without the detector placed in the signal output port $-j$ can be formally described by the operator

$$\hat{Y}_{-j}(k) = \hat{V}_j^\dagger \langle \varphi_k | \hat{U}^\dagger \hat{U} | \varphi_0 \rangle \hat{V}_{-j} \hat{V}_j,$$

(7a)

$$\langle \varphi_k | \hat{U}^\dagger \hat{U} | \varphi_0 \rangle = \frac{1}{M+1} \sum_{m=0}^{M} e^{i kl \frac{\pi}{2\sqrt{M+1}}} (a_j^\dagger a_j - a_j a_j^\dagger) m.$$  

(7b)

The detector placed in the signal output port $-j$ is assumed to perform a Pegg-Barnett phase measurement such that each trial gives a value $m_j$ that can take the values $0,\ldots,M$ and is described by the Positive Operator Valued Measure (POVM)

$$\hat{P}_{-j}(m_j) = \sum_{l=0}^{\infty} \langle l, m_j | l, m_j \rangle,$$

(8a)

$$\langle l, m_j | = \frac{1}{\sqrt{M+1}} \sum_{n=0}^{M} e^{i kl \frac{\pi}{2\sqrt{M+1}} n} \times |l(M+1)+n\rangle_{-j}. $$  

(8b)

In turn, the value $m_j$ determines the operator

$$\hat{V}_j^\dagger = \hat{V}_j^\dagger e^{i \frac{\pi}{2\sqrt{M+1}} m_j a_j^\dagger a_j}$$

in Eq. (7a).
IV. INPUT-OUTPUT RELATIONS

We now consider the overall setup including all signal modes as depicted in Fig. 11(b). If states $|\hat{\psi}_k\rangle$ and POVM values $m_j$ are detected in the auxiliary and second signal output modes of the quantum repeaters, the reduced state of the first signal output modes becomes

$$\hat{R}_I(m, k) = \frac{Tr_{II} \left[ \hat{Y}(k)\hat{R}_I \otimes \hat{\sigma}_{II} \hat{Y}(k)\hat{I}(m) \right]}{p(m, k)} = \frac{(M + 1)^{1-2N}}{p(m, k)} \hat{R}_I \hat{W}^{jk} \hat{\sigma}_j \hat{W}^k. \quad (10)$$

In the first line of Eq. (10), $k = k_1, \ldots, k_N$ and

$$\hat{Y}(k) = \prod_{j=1}^{N} \hat{Y}_{j,-j}(k_j) \quad (11)$$
denotes the product of the individual operators Eq. (8b). Similarly, $m = m_1, \ldots, m_N$ and

$$\hat{I}(m) = \prod_{j=1}^{N} \hat{I}_{j,-j}(m_j) \quad (12)$$
is the product of the respective single-mode POVM's Eq. (8b). In the second line of Eq. (10), an operator $\hat{W}$ has been introduced to express a rotation of the eigenstate indices according to $\hat{W}^{jk} |\Psi_n\rangle = |\Psi_{n+1}\rangle$, and $k = [\sum_{j=1}^{N} k_j]$. $\hat{\sigma}_I$ is the original input state $\hat{\sigma}_{II}$ given by Eq. (4b) but transferred from $II$ to $I$, i.e., using a formal mode transfer operator $\hat{I}_{I,II} = \prod_{j=1}^{N} \hat{I}_{j,-j}$, where

$$\hat{I}_{jk} = \sum_{n=0}^{\infty} |n\rangle_j k \langle n| = k \langle 0 | e^{\hat{a}_j^\dagger \hat{a}_k} |0\rangle_j = \hat{I}_{k,j} \quad (13)$$

we may write $\hat{\sigma}_I = \hat{I}_{I,II} \hat{\sigma}_{II} \hat{I}_{I,II}^\dagger$. Since only the signal modes $I$ appear in the second line of Eq. (10), let us leave out this mode index in what follows. From Eq. (10), we see that the state $\hat{R}'(m, k)$ only depends on $k$, so that the signal output state in case of an event $k$ can be written as

$$\hat{R}'(k) = \frac{1}{p(k)} \sum_{m=0}^{M} \sum_{k} \left( k \right) p(m, k) \hat{R}'(m, k) = \frac{1}{p(k)} \hat{R} \hat{W}^{jk} \hat{\sigma} \hat{W}^k. \quad (14)$$

A repeated run of the purification can then be described as an iteration such that the $j$th iteration step generates a signal state

$$\hat{R}^{(j)}(k) = \frac{1}{p(k)} \hat{R}^{(j-1)} \hat{W}^{jk} \hat{\sigma} \hat{W}^k \quad (15)$$

from its precursor $\hat{R}^{(j-1)}$, originally descending from a signal state copy, $\hat{R}^{(0)} = \hat{\sigma}$. At each step, $k = k^{(j)}$ takes some value $0, \ldots, M$. An observer unaware of any detection result simply observes the average

$$\hat{R}^{(j)} = \sum_{k=0}^{M} p(k) \hat{R}^{(j)}(k) = \hat{R}^{(j-1)}, \quad (16)$$

so that $\hat{R}^{(j)} = \hat{R}^{(0)} = \hat{\sigma}$, which is just the unchanged original signal state. Thinking of the complete purification procedure as a linear array of repeating stations rather than a loop, the resulting setup therefore constitutes a ‘quantum state guide’ which only may increase the observer’s knowledge rather than changing the signal state. In the special case of an equipartition, $p_n = (M+1)^{-1}$, no additional knowledge can be gained even from the measurements, so that a purification is not possible, $\hat{R}^{(j)}(k) = \hat{R}^{(j-1)}$. By taking into account all detection results $k^{(1)}, \ldots, k^{(j)}$ obtained during the steps 1, $\ldots$, $j$, the explicit expression of the iterated state becomes

$$\hat{R}^{(j)} \left[ k^{(1)}, \ldots, k^{(j)} \right] = \frac{\hat{\sigma} \prod_{l=0}^{j} \hat{W}^{k_l} \hat{\rho} \hat{W}^{k_l}}{p \left[ k^{(1)}, \ldots, k^{(j)} \right]} = \frac{M}{\sum_{n=0}^{M} p_{n}^{j}} \prod_{n=0}^{M} p_{n}^{j} \quad (17)$$

with

$$p_{n}^{j} = \frac{1}{p \left[ k^{(1)}, \ldots, k^{(j)} \right]} \prod_{l=1}^{j} p_{n-k^{(l)}} = \frac{1}{p \left[ k^{(1)}, \ldots, k^{(j)} \right]} \prod_{l=0}^{M} p_{n-l}^{j}, \quad (18)$$

where $s_j$ is the number of events for which $k = l$. From Eq. (18) we see that $\hat{R}^{(j)}$ only depends on $s = s_0, \ldots, s_M$, so that we can refer the iterated state to this event,

$$\hat{R}^{(j)}(s) = \frac{1}{p(s)} \sum_{k^{(1)}, \ldots, k^{(j)}} p \left[ k^{(1)}, \ldots, k^{(j)} \right] \hat{R}^{(j)} \left[ k^{(1)}, \ldots, k^{(j)} \right] = \frac{1}{p(s)} \left( j \right) s \sum_{n=0}^{M} p_{n} \prod_{l=0}^{M} \hat{\rho}_{n-l} \hat{W}^{k} \hat{W}^{k^\dagger} \hat{\rho}^\dagger \hat{W}^{k^\dagger} \hat{\rho} \hat{W}^{k}. \quad (19)$$

Here $\sum^*$ denotes the sum over all $j$-tuples $k^{(1)}, \ldots, k^{(j)}$ containing $s_j$ times the numbers $l = 0, \ldots, M$, whose number is given by the polynomial coefficient $\langle j \rangle$. Since a pure signal input state results in a pure signal output state, the random walk of the state may eventually end in a pure state. If this happens, the probability of obtaining $|\Psi_n\rangle$ is $p_n = \langle \Psi_n | \hat{\rho} | \Psi_n \rangle$ as becomes plausible from Eq. (10).
V. BINARY OPERATION

Let us return to Eq. \( \text{(15)} \). Since we are only interested in obtaining the state \( |\Psi_0\rangle \), we may limit attention to a binary detection by only discriminating between the event \( k=0 \) and its complement \( k \neq 0 \). Eq. \( \text{(15)} \) then gives the transformation

\[
\tilde{R}^{(j)}(0) = \frac{1}{p(0)} \tilde{R}^{(j-1)} \hat{\theta}, \quad (20a)
\]

\[
\tilde{R}^{(j)}(-0) = \frac{1}{1 - p(0)} \tilde{R}^{(j-1)} (I - \hat{\theta}). \quad (20b)
\]

Analogous to Eq. \( \text{(19)} \), the explicit expression of the iterated state

\[
\tilde{R}^{(j)}(q) = \frac{1}{p(q)} \left( \frac{j}{q} \right) q^q (I - \hat{\theta})^{j-q} \quad (21)
\]

can be expressed in terms of the number \( q \) of events \( k = 0 \). To give an example, in a situation where our desired state \( |\Psi_0\rangle \) is still dominant over the impurities, \( p_0 > p_n \forall n \neq 0 \), the state \( |\Psi_0\rangle \) is obtained in situations when \( j^{-1}q \) is sufficiently large. This case is illustrated in Fig. 2 monitoring on the basis of the expectation value of the von Neumann entropy \( \hat{S} = -\ln \tilde{R}^{(j)} \) a successful example of a simulated random purification process calculated according to Eqs. \( \text{(20)} \).

![FIG. 2: Typical evolution of the von Neumann entropy \( S = -\text{Tr}[\tilde{R}^{(j)} \ln \tilde{R}^{(j)}] \) of the signal state \( \tilde{R}^{(j)} \) as seen by an observer aware of the measurement results occurring during the first \( j \) cycles of a left alone (random) purification process according to Eqs. \( \text{(20)} \). The initial probabilities are assumed to be \( p_n \sim e^{-n} \), where \( n = 0, \ldots, 100 \).](image)

VI. PURIFICATION IN A SINGLE INSTANT

If the signal state is a mixture of the \((M+1)^N\) states

\[
|\Psi_n\rangle = \sum_{l=0}^{\infty} c_l(n)|\Phi_{l_1(M+1)+n_1}\rangle \cdots |\Phi_{l_N(M+1)+n_N}\rangle_N
\]

instead of Eq. \( \text{(2)} \), each copy can be purified individually without need of further inputs. All we have to do is feeding the second signal input ports of the repeaters with vacuum states \( |0\rangle_{-j} \) and removing the devices implementing \( \hat{V}_{-j} \) thus making the \( m_j \)-measurements redundant. Together with \( \hat{\theta}_l = \sum_{n=0}^M p_n|\Psi_n\rangle\langle \Psi_n| \), Eq. \( \text{(10)} \) is then replaced with

\[
\hat{\theta}_l(k) = \frac{\text{Tr}_{II} \left[ \hat{Y}(k) \hat{\theta}_l \otimes |0\rangle_{II} \langle 0| \hat{Y}^\dagger(k) \right]}{p(k)} = |\Psi_k\rangle\langle \Psi_k|. \quad (23)
\]

The probability of obtaining \( |\Psi_k\rangle \) is just \( p_k \).

VII. INFLUENCE OF LOSS IN AN EXAMPLE

Let us return to mixtures of states Eq. \( \text{(2)} \). From a practical point of view, the loss occurring in the purification loop must be taken into account. The problem is that its effect depends on the respective circulating signal state \( \hat{\rho} \) itself. Let us therefore choose the particular example of a superposition

\[
\hat{\theta}(r) = \frac{(|\alpha\rangle\langle \alpha| + |\alpha\rangle\langle \alpha|)}{2(1 + re^{-2|\alpha|^2})}
\]

\[
= \sum_{n=0}^1 p_n|\Psi_n\rangle\langle \Psi_n|
\]

of two \( N \)-mode coherent states \( |\alpha\rangle = |\alpha_1\rangle \cdots |\alpha_N\rangle \), characterized by their complex amplitude \( \alpha_1, \ldots, \alpha_N \) according to \( |\alpha_j\rangle = e^{\alpha_j a_j^\dagger - \alpha_j^* a_j}|0\rangle_j \). The eigenvalues and eigenstates are given by

\[
p_n = \frac{1 + (1)^n e^{-2|\alpha|^2}}{2(1 + r e^{-2|\alpha|^2})}[1 + (1)^n r], \quad (25a)
\]

\[
|\Psi_n\rangle = \frac{2^{-N/2}}{\sqrt{2[1 + (1)^n e^{-2|\alpha|^2}]}} e^{\alpha_j a_j^\dagger - \alpha^*_j a_j}|0\rangle_j, \quad (25b)
\]

where we have defined \( |\alpha|^2 = \sum_{j=1}^N |\alpha_j|^2 \). In the mesoscopic case, \( e^{-2|\alpha|^2} \ll 1 \), the \( |\Psi_n\rangle \) can be written in the form of Eq. \( \text{(2)} \),

\[
|\Psi_n\rangle \approx \frac{2^{-N/2}}{\sqrt{2[1 + (1)^n e^{-2|\alpha|^2}]}} e^{\alpha_j a_j^\dagger - \alpha^*_j a_j}|0\rangle_j, \quad (26a)
\]

\[
|\Phi_{n_j}\rangle = \frac{2^{-N/2}}{\sqrt{2[1 + (1)^n e^{-2|\alpha|^2}]}} e^{\alpha_j a_j^\dagger - \alpha^*_j a_j}|0\rangle_j. \quad (26b)
\]

The pure states \( |\Psi_n\rangle \), to which Eq. \( \text{(24)} \) reduces for \( r = (1)^n \), just represent the superpositions \( |0, \ldots, 0| \pm |1, \ldots, 1| \) considered in \( \text{(12)} \). Modeling the decoherence of the state Eq. \( \text{(24)} \) by the master equation

\[
L \frac{d\hat{\rho}}{dx} = \sum_{j=1}^N \eta_j (2\hat{a}_j \hat{a}_j^\dagger - \hat{a}_j^\dagger \hat{a}_j - \hat{\eta}_j \hat{\eta}_j), \quad (27)
\]
where $x$ may be, e.g., the propagation distance, we see that the purity parameter $r$ and the individual complex amplitudes $\alpha_j$ decrease according to

$$r(x) = r(0)e^{2(|\alpha|^2 - |\alpha_0|^2)} x \lesssim L r(0)e^{-4|\alpha_0|^2 \frac{x}{\eta}}.$$  

$$\alpha_j(x) = \alpha_j(0)e^{-\eta_j x} x \lesssim \alpha_j(0),$$

where $\eta = \frac{1}{N} \sum_{j=1}^{N} \eta_j$ is determined by the $\eta_j \geq 0$ defining the (e.g., scattering) losses and $\alpha_0 = \alpha(0)$. For distances small compared to the classical transparency length, $x \ll L$, the damping of the complex amplitudes can be neglected. For given complex amplitudes, the state Eq. (24) is then determined by its purity parameter $r$.

The aim of our purification process is therefore to reduce the signal state to an $\hat{\rho}$, which satisfies $\hat{\rho}(\infty) = \hat{\rho}(0)$, and is able to walk. To allow an increase of $|R^{(j)}|$ above its initial value $|r|$, the condition $|R^{(\infty)}| \geq |r|$ must hold, which gives

$$\eta_F \geq 1 + \frac{r^2}{2}.$$  

It follows that the required feedback efficiency increases with the initial purity. For $\eta_F \leq 1/2$, the scheme is of no use at all. On the other hand, the ideal value $R^{(\infty)} = 1$ can only be approached in practice. In order to keep the bound sufficiently close to one, $|R^{(\infty)}| \geq 1 - \varepsilon$ with some given $\varepsilon \ll 1$, the requirement

$$\eta_F \geq 1 - \frac{2|\varepsilon|}{1 + |r|}.$$  

must hold. For sufficiently small $\varepsilon$, a behavior similar to the perfect case is observed. Note that the case considered here is analogous to the example of a related two-mode state discussed in [13]. For a more detailed analysis including numerical simulations, we therefore refer to that work.

VIII. CONCLUSION AND OUTLOOK

We have described a procedure allowing a purification of a class of $N$-mode quantum states as an iterative random process. While in general, a number of identical signal state preparations is applied, only a single preparation is required in certain cases of mixed states. The physical implementation suggested involves beam splitter arrays, zero and single photon detections as well as cross-Kerr elements. The role of imperfections of the procedure is modeled in the example of a superposition state discussed in [14]. For a more detailed analysis including numerical simulations, we therefore refer to that work.

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