Poincaré gauge gravity: An emergent scenario

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The Poincaré gauge gravity (PGG) with the underlying vector fields of tetrads and spin-connections is perhaps the best theory candidate for gravitation to be unified with the other three elementary forces of nature. There is a clear analogy between local frame in PGG and local internal symmetry space in the Standard Model. As a result, the spin-connection fields, gauging the local frame Lorentz symmetry group $SO(1,3)_{LF}$, appear in PGG much as photons and gluons appear in SM. We propose that such an analogy may follow from their common emergent nature allowing to derive PGG in the same way as conventional gauge theories. In essence, we start with an arbitrary theory of some vector and fermion fields which possesses only global spacetime symmetries, such as Lorentz and translational invariance, in flat Minkowski space. The two vector field multiplets involved are proposed to belong, respectively, to the adjoint ($A_{ij}^{\mu}$) and vector ($e_i^{\mu}$) representations of the starting global Lorentz symmetry. We show that if these prototype vector fields are covariantly constrained, $A_{ij}^{\mu} A_{ij}^{\nu} = \pm M_A^2$ and $e_i^{\mu} e_i^{\nu} = \pm M_e^2$, thus causing a spontaneous violation of the accompanying global symmetries ($M_{A,e}$ are their proposed violation scales), then the only possible theory compatible with these length-preserving constraints is turned out to be the gauge invariant PGG, while the corresponding massless (pseudo)Goldstone modes are naturally collected in the emergent gauge fields of tetrads and spin-connections. In a minimal theory case being linear in a curvature we unavoidably come to the Einstein-Cartan theory. The extended theories with propagating spin-connection and tetrad modes are also considered and their possible unification with the Standard Model is briefly discussed.

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I. INTRODUCTION

A conventional view on the problem of unification of all fundamental (electromagnetic, weak and strong) interactions with gravity is based on the modern superstring theory. However, this approach, as is well known, has serious problems to be adapted to the present particle physics phenomenology. Actually, only a few qualitative prescriptions stemming from superstrings can really be taken into account at lower energies. In this connection, one may think that a better understanding of some aspects of this problem is still related to the quantum field theory framework provided that the gravity, like as the other basic forces, is given by a true gauge theory.

A. PGG: an introductory commentary

The general relativity (GR) is, of course, a gauge theory in a sense that it is invariant under local general linear transformation group $GL(4)$ being considered according to the principle of general covariance as a linear part of some fundamental world symmetry for our spacetime\textsuperscript{1}. The point is, however, that it is not a theory which is similar to other gauge theories incorporated into the Standard Model (SM) where gauge fields are in fact propagating physical fields like photons, gluons and weak bosons being related to generators of internal symmetry groups. In this connection, a natural question arises whether a similar view could be applied to local spacetime symmetries that could lead to a deep conceptual analogy between gravity and the other three elementary forces described by modern gauge theories.

Indeed, GR can be described by a simple extension of the methods used to describe Yang-Mills theory\textsuperscript{1}, as was argued by Utiyama\textsuperscript{2} a long ago. Traditionally, one might start with coordinate transformations being a local generalization of translations, since gravity is generically defined as a force that couples to energy-momentum in much the same way as electromagnetism couples to charge. However, this is not enough to define spinors, since there no representations of $GL(4)$ which behave like spinors under its

\textsuperscript{1} This is certainly the most natural choice for a gauge group in GR if one works in an ordinary metric formulation.
Lorentz subgroup. So, one must introduce some extra local Lorentz symmetry to include spinors into the theory. Remarkably, regardless of spinors, this is also required by the Einstein equivalence principle for the local spacetime structure being identified with Minkowski space. One might then propose \textsuperscript{2} that a true theory of gravity can be seen as a gauge theory based on the local Lorentz group $SO(1,3)$ in the same manner as the Yang-Mills gauge theory appears on the basis of the internal isospin gauge group $SU(2)$. In this formulation, the affine connection in a local Lorentz frame could be obtained as the gravitational counterpart of the Yang-Mills gauge fields. However, this seems to be only half a story. In order to relate this local Lorentz frame to the global world space (being curved in general), one needs some extra tools. They are tetrad fields being built into all spacetime points to define local reference frames. Tetrads may be considered as objects given a priori \textsuperscript{2} or one can try to treat them on the gauge base as well relating them to the local translations in a general frame \textsuperscript{3}. So, the gauge fields in the complete Poincaré gauge gravity (PGG) include the tetrads $e^i_\mu$ as well as the local affine connections $A^i_\mu$ which are usually referred to as the spin-connections in PGG. Remarkably, though PGG can be thought as a field theory in flat Minkowski space, it successfully mimics curved space geometry when is expressed in terms of the general affine connection and metric. Consequently, the local Poincaré symmetry appears to correspond to a world space with torsion as well as curvature: just torsion is a feature with which the base space should be endowed to accommodate spinor fields. In this sense, PGG has the geometric structure of the Riemann–Cartan space $U_4$.

We have, therefore, in PGG the world coordinate transformation group, which includes translations and the orbital part of Lorentz transformations acting in the global base space, and a locally acting Lorentz group, which includes the spin part of Lorentz transformations in the flat Minkowski spacetime. To make a distinction between them we use Greek indices $(\mu, \nu, \rho, ... = 0, 1, 2, 3)$ for the former and Latin indices $(a, b, c, ..., i, j, k, ... = 0, 1, 2, 3)$ for the latter. All field variables taken in the theory, including tetrads and spin-connections, are generally proposed to be functions of the global world 4-coordinate $x_\mu$. Clearly, this coordinate itself and therefore its partial derivatives $\partial_\mu$, are not affected by the spin part of Lorentz transformations, while all spinor and vector fields are acted on. In essence, the vector gauge fields of tetrads $e^i_\mu$ ($e^i_\mu$) satisfying the following orthogonality conditions

$$ e^i_\mu e^j_\nu = \delta^i_\nu , \quad e^i_\mu e^i_\mu = \delta^i_\mu \quad (1) $$

set the local coordinate frame, while those of the spin-connections $A^i_\mu$ are in fact connections with respect to the tetrad frame. Remarkably, this ”global-local” duality is in a natural accordance with the Einstein equivalence principle which is somewhat hidden in the standard GR and should be stated per se.

An important point to stress is that the Poincaré gauge theory of gravitation is basically related to the vector fields of tetrads and spin-connections as the primary notions in theory. This is quite understandable since gauge field is a tool which extends an ordinary derivative to the covariant one. Thus, such a field may be only a vector field, by definition. In this sense, an effective tensor field of metric

$$ g_{\mu\nu} = e^i_\mu e^j_\nu \eta_{ij} \quad (2) $$

(where $\eta_{ij}$ is the metric in the Minkowski spacetime, $\eta_{00} = -\eta_{11} = -\eta_{22} = -\eta_{33} = 1$) being in GR traditionally identified with a graviton, appears as an essentially secondary tool in this scheme. In this connection, PGG is in principle different theory which may have some common root with conventional gauge theories, as can also be clearly seen from its emergent scenario presented below. In any case, it certainly is a valuable alternative to the standard GR and has extra opportunities to describe gravity in a self-consistent way \textsuperscript{4-8}. Some excellent presentations can be found in the reviews \textsuperscript{9-13}.

B. The present paper

All the above allows to think that PGG with the underlying vector fields of tetrads and spin-connections is perhaps the best theory candidate for gravitation to be unified with the other three elementary forces in the quantum field theory framework. Remarkably, there is some clear analogy between a local frame in PGG and a local internal symmetry space in conventional quantum field theories. As a result, the vector fields of the spin-connections $A^i_\mu(x)$ gauging the local frame Lorentz group $SO(1,3)_{LF}$ as its adjoint representation, appear in PGG much as photons and gluons appear in in the Standard Model. We propose that such an analogy may follow from their common emergent nature allowing to derive PGG in the same way as conventional gauge theories. To make this analogy clearer we give in the next Section II a detailed presentation of the emergent gauge theories. We begin with an underlying emergence...
conjecture \cite{14,15} according to which the vector field theory possessing only some input global internal symmetry may convert this symmetry into the local one provided that the vector field (or vector field multiplet) involved is covariantly constrained, $A_\mu A^\mu = \pm M^2$ (where $M$ is some mass scale). Normally, this constraint might mean a spontaneous Lorentz violation with the vector Goldstone bosons produced, while the vector Higgs component appears frozen to its vacuum expectation value (VEV) $M$. This is what usually happens in the non-linear $\sigma$-model for pions \cite{16} with a spontaneously broken chiral symmetry. However, in the vector field theories due to gauge invariance emerged the physical Lorentz invariance is left intact. Remarkably, the emergence conjecture may be applied to any vector field theory with an input global Abelian or non-Abelian internal symmetry and, consequently, the conventional gauge invariant QED or Yang-Mills theory are emerged. This is explicitly demonstrated at some length in the Sections II A and II B, respectively.

In the Section III we turn to the construction of an emergent PGG theory. We start with an arbitrary theory of some vector and fermion fields which possesses only global spacetime symmetries, such as Lorentz and translational invariance, in flat Minkowski space $M_4$. The two vector field multiplets involved are proposed to belong, respectively, to the adjoint ($A_{ij}^\mu$) and vector $(e_\mu^\mu)$ representations of the starting global Lorentz symmetry. We show that if these prototype vector fields are covariantly constrained, $A_{ij}^\mu A_{ij}^\mu = \pm M_A^2$ and $e_\mu^\mu e_\mu^\mu = \pm M_e^2$, thus causing a spontaneous violation of the accompanying global symmetries ($M_A,e$ are their proposed violation scales), then the only possible theory compatible with these constraints is turned out to be the standard PGG (with gauged translations and local frame Lorentz transformations), while the corresponding massless (pseudo)Goldstone modes are naturally collected in the emergent gauge fields of tetrads and spin-connections. In essence, the local Poincaré invariance emerges as a necessary condition for the prototype vector fields $A_{ij}^\mu$ and $e_\mu^\mu$ not to be superfluousy restricted in degrees of freedom, apart from the above length-fixing constraints. Again, due to gauge invariance emerged the physical Lorentz and translation invariance remains in the final theory. In minimal theory case being linear in curvature we unavoidably come to the Einstein-Cartan theory that is thoroughly presented in the Section IV. The extended theories with propagating spin-connection and tetrad modes are also considered and their possible unification with the Standard Model is briefly discussed. And in the final Section V we conclude.

II. EMERGENT GAUGE THEORIES

The dynamical origin of massless particle excitations for spontaneously broken internal symmetries \cite{17} allows to think that possibly only global symmetries are fundamental symmetries of nature, whereas local symmetries and associated massless gauge fields could solely emerge due to spontaneous breaking of underlying spacetime symmetries involved, such as relativistic invariance. It is conceivable that spontaneous Lorentz invariance violation (SLIV) could provide a dynamical approach to quantum electrodynamics, gravity and Yang-Mills theories with the photon, graviton and gluons appearing as massless Nambu-Goldstone bosons. This rather old idea \cite{18–21} has gained a further development \cite{22–26} in recent years.

We will follow here a somewhat different point of view \cite{14,15} according to which SLIV in itself could be only one of the consequences of the gauge symmetry emergence scenario rather than its underlying cause. Presumably, such a scenario is basically related to some covariant constraint(s) which, for one reason or another, is put on a vector field system possessing only some global internal symmetry. As a matter of fact, the simplest holonomic constraint of this type for vector field (or vector field multiplet) $A_\mu$ may be the ”length-fixing” condition

$$C(A) = A_\mu A^\mu - n^2 M^2 = 0 , \quad n^2 = n_\mu n^\mu = \pm 1 \quad (3)$$

where $n_\mu$ is a properly oriented unit Lorentz vector, while $M$ is some high mass scale. We will see that gauge invariance appears unavoidable in the proposed theory, if the equations of motion involved should have enough freedom to allow a constraint like (3) to be fulfilled and preserved over time. Namely, gauge invariance in such theories has to appear in essence as a response of an interacting field system to putting the covariant constraint (3) on its dynamics, provided that we allow parameters in the corresponding Lagrangian density to be adjusted so as to ensure self-consistency without losing too many degrees of freedom. Otherwise, a given field system could get unphysical in a sense that a superfluous reduction in the number of degrees of freedom would make it impossible to set the required initial conditions in an appropriate Cauchy problem. Namely, it would be impossible to specify arbitrarily the initial values of
the vector and other field components involved, as well as the initial values of the \( \theta \) conjugated to them. Furthermore, in quantum theory, to choose self-consistent equal-time commutation relations would also become impossible \(^2\).

Let us dwell upon this point in more detail. Generally, while a conventional variation principle requires the equations of motion to be satisfied, it is possible to eliminate one spacetime component of a general 4-vector field \( A_\mu \) in order to describe a pure spin-1 particle by imposing a supplementary condition. In the massive vector field case there are three physical spin-1 states to be described by the \( A_\mu \) field. Similarly in the massless vector field case, although there are only two physical (transverse) photon spin states, one cannot construct a massless 4-vector field \( A_\mu \) as a linear combination of creation and annihilation operators for helicity \( \pm 1 \) states in the Lorentz covariant way, unless one fictitious state is added \(^3\). So, in both the massive and massless vector field cases, only one component of the \( A_\mu \) field may be eliminated and still preserve physical Lorentz invariance. Now, once the constraint (3) is imposed, it is therefore not possible to satisfy another supplementary condition since this would superfluously restrict the number of degrees of freedom for the vector field. To avoid this, its equation of motion should not lead by itself to any new constraint that is only possible if it is automatically divergenceless, as normally appears in the gauge invariant theory. Thus, due the constraint (3), being the only possible covariant and holonomic vector field constraint, the theory has to acquire on its own a gauge-type invariance, which gauges the starting global symmetry of the interacting vector and matter fields involved.

### A. A QED primer

To see how technically a global internal symmetry may be converted into a local one in an Abelian \( U(1) \) symmetry case, let us consider in detail the question of consistency of the constraint for vector field \(^3\) with its equations of motion in an arbitrary relativistically invariant Lagrangian \( L(A, \psi) \) which also contains some charged fermion field \( \psi \). In the presence of the constraint (3), it follows that the equations of motion can no longer be independent. The important point is that, in general, the time development would not preserve the constraint. So, the parameters in the Lagrangian have to be chosen in such a way that effectively we have one less equation of motion for the vector field. This means that there should be some relationship between all the vector and matter field Eulerians (\( E_A, E_\psi \)) involved\(^2\). Such a relationship can quite generally be formulated as a functional - but by locality just a function - of the Eulerians, \( F(E_A, E_\psi) \), being put equal to zero at each spacetime point with the configuration space restricted by the constraint \( C(A) = 0 \),

\[
F(C = 0; \ E_A, E_\psi) = 0. \tag{4}
\]

for the one matter fermion case proposed. This relationship, which we shall call the emergence equation for what follows, must satisfy the same symmetry requirements of Lorentz and translational invariance, as well as all the global internal symmetry requirements, as the general starting Lagrangian does. This Lagrangian is supposed to also include the standard Lagrange multiplier term with the field \( \lambda(x) \)

\[
L^{tot}(A, \psi, \lambda) = L(A, \psi) - \frac{\lambda}{2} (A_\mu A^\mu - n^2 M^2) \tag{5}
\]

the variation under which results in the above constraint \( C(A) = 0 \). In fact, the relationship (4) is used as the basis for an emergence of gauge symmetries in the constrained vector field theories \(^4\) \(^5\). We propose that initial values for all fields (and their momenta) involved are chosen so as to restrict the phase space to values with a vanishing multiplier function \( \lambda(x) \), \( \lambda = 0 \). Actually, due to an automatic conservation of the Noether matter current in the theory an initial value \( \lambda = 0 \) will then remain for all time so that the Lagrange multiplier field \( \lambda \) never enters in the physical equations of motions for what follows\(^3\).

\(^2\) Hereafter, the notation \( E_A \) stands for the vector field Eulerian \( (E_A)^\mu \equiv \partial L/\partial A_\mu - \partial_\nu [\partial L/\partial (\partial_\nu A_\mu)] \). We use similar notations for other field Eulerians as well.

\(^3\) A more direct way to have this solution with the vanished Lagrange multiplier field \( \lambda(x) \) would be to include in the Lagrangian \(^5\) an additional Lagrange multiplier term of the type \( \xi \lambda^2 \), where \( \xi(x) \) is a new multiplier field. One can then easily confirm that a variation of the modified Lagrangia \( L' + \xi \lambda^2 \) with respect to the \( \xi \) field leads to the condition \( \lambda = 0 \), whereas a variation with respect to the basic multiplier field \( \lambda \) preserves the vector field constraint (3). Such an approach with additional Lagrange multiplier terms can be then properly generalized to the emergent Yang-Mills theory and PGG theory cases considered later.
Let us consider a “Taylor expansion” of the function $F$ expressed through various combinations of the fields involved, their combinations with the Eulerians, as well as the derivatives acting on them. The constant term in this expansion is of course zero since the relation (4) must be trivially satisfied when all the Eulerians vanish, i.e. when the equations of motion are satisfied. We basically consider the terms with the lowest mass dimension 4, corresponding to the Lorentz invariant expressions

$$\partial_\mu (E_A)^\mu, \ A_\mu (E_A)^\mu, \ E_\psi \psi, \overline{\psi} E_{\overline{\psi}}.$$  

(6)

to eventually have an emergent gauge theory at a renormalizable level. All the other terms in the expansion contain field combinations and derivatives with higher mass dimension and must therefore have coefficients with an inverse mass dimension. We expect the mass scale associated with these coefficients should correspond to a large fundamental mass (e.g. the Planck mass $M_P$). Hence we conclude that such higher dimensional terms must be properly suppressed and can be neglected.

Now, under the assumption that the constraint (3) is preserved under the time development given by the equations of motion, we show how gauge invariance of the starting Lagrangian $L(A, \psi)$ in (4) is established. A conventional variation principle applied to the total Lagrangian $L_{\text{tot}}(A, \psi, \lambda)$ requires the following equations of motion for the vector field $A_\mu$ and the auxiliary field $\lambda$ to be satisfied

$$(E_A)^\mu = 0, \quad C(A) = A_\mu A^\mu - n^2 M^2 = 0.$$  

(7)

However, in accordance with general arguments given above, the existence of five equations for the 4-component vector field $A^\mu$ (one of which is the constraint) means that not all of the vector field Eulerian components can be independent. Therefore, there must be a relationship of the form given in the emergence equation (4). When being expressed as a linear combination of the Lorentz invariant terms (9), this equation leads to the identity between the vector and matter field Eulerians of the following type

$$\partial_\mu (E_A)^\mu = it E_\psi \psi - it \overline{\psi} E_{\overline{\psi}}.$$  

(8)

(where $t$ is some constant) which is in fact identically vanished when the equations of motion are satisfied\(^\text{4}\). This identity immediately signals about invariance of the basic Lagrangian $L(A, \psi)$ in (4) under vector and fermion field local $U(1)$ transformations whose infinitesimal form is given by

$$\delta A_\mu = \partial_\mu \omega, \quad \delta \psi = it \omega \psi.$$  

(9)

Here $\omega(x)$ is an arbitrary function, only being restricted by the requirement to conform with the nonlinear constraint (3)

$$(A_\mu + \partial_\mu \omega)(A^\mu + \partial^\mu \omega) = n^2 M^2.$$  

(10)

Conversely, the identity (8) follows from the invariance of the physical Lagrangian $L(A, \psi)$ under the transformations (9). Indeed, both direct and converse assertions are particular cases of Noether’s second theorem (29).

So, we have shown how the constraint (3) enforces the choice of the parameters in the starting Lagrangian $L_{\text{tot}}(A, \psi, \lambda)$, so as to convert its global $U(1)$ charge symmetry into a local one, thus demonstrating an emergence of gauge symmetry (9) that allows the emerged Lagrangian to be completely determined. For a theory with renormalizable couplings, it is in fact the conventional QED Lagrangian

$$L_{\text{QED}}(A, \psi) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi} (i\gamma \partial + m) \psi - e A_\mu \overline{\psi} \gamma^\mu \psi$$  

(11)

extended by the Lagrange multiplier term, which provides the constraint (3) imposed on the vector field $A_\mu$. Interestingly, this type of the QED theory with the constrained vector potential was considered by Nambu (30) quite a long ago.

\(^{4}\) Note the term proportional to the vector field itself, $A_\mu (E_A)^\mu$, which would correspond to the selfinteraction of vector field, is absent in the identity (8). In presence of this term the transformations of the vector field given below in (9) would be changed to $\delta A_\mu = \partial_\mu \omega + c \omega A_\mu$. The point is, however, that these transformations cannot in general form a group unless the constant $c$ vanishes, as can be readily confirmed by constructing the corresponding Lie bracket operation for two successive vector field variations. We shall see later that non-zero $c$-type coefficients necessarily appear in the non-Abelian internal symmetry case, resulting eventually in a emergent gauge invariant Yang-Mills theory.
Let us make it clearer what does the constraint \( a \) mean in the gauge invariant QED framework. This constraint is in fact very similar to the constraint appearing in the nonlinear \( \sigma \)-model for pions \( \sigma \). It means, in essence, that the vector field \( A_\mu \) develops some constant background value, \( (A_\mu) = n_\mu M \), and the Lorentz symmetry \( SO(1, 3) \) formally breaks down to \( SO(3) \) or \( SO(1, 2) \) depending on the time-like \( (n^2 = 1) \) or space-like \( (n^2 = -1) \) nature of SLIV with the corresponding vector Goldstone mode associated with a photon. Nonetheless, despite an evident similarity with the nonlinear \( \sigma \)-model for pions, which really breaks the corresponding chiral \( SU(2) \times SU(2) \) symmetry in hadron physics, the QED theory \( \sigma \) with the supplementary vector field constraint \( a \) involved leaves the physical Lorentz invariance intact.

Some heuristic argument may look as follows. Let us turn to the convenient SLIV parametrization

\[
A_\mu = a_\mu + n_\mu H, \quad n_\mu a^\mu = 0
\]

\[
H = (M^2 - n^2 a^2)^{1/2} = M - \frac{a^2}{2M} + \cdots \quad (a^2 \equiv a_\mu a^\mu)
\]

which could be referred to as the "symmetry broken phase" being determined by the constraint \( a \) itself. Indeed, the new \( a^\mu \) field appears here as the vector Goldstone mode which is orthogonal to the vacuum direction given by the unit vector \( n_\mu \), while \( H \) stands for the effective Higgs mode. Substituting the parametrization \( A_\mu \) into the emergent Lagrangian \( \sigma \) and properly redefining the fermion field with the phase linear in coordinate

\[
\psi \rightarrow e^{iM(x-n)} \psi
\]

to exclude the fictitious noncovariant mass term \( eM\bar{\psi}n_\mu \gamma^n \psi \) one comes to the theory expressed solely in the vector Goldstone modes \( a_\mu \) emerging in the "axial gauge" \( \sigma \)

\[
L(a, \psi) = -\frac{1}{4} [f_{\mu\nu} + (\partial_\mu n_\nu - \partial_\nu n_\mu)H][f^{\mu\nu} + (n^\mu \partial^\nu - n^\nu \partial^\mu)H] - \frac{1}{2} \delta(n_\mu a^\mu)^2
\]

\[
+ \bar{\psi}(i\gamma\partial + m)\psi - e\bar{\psi}\gamma^\mu \gamma^n \psi + (H - M)\bar{\psi}n_\mu \gamma^n \psi.
\]

where we have denoted the Goldstone field strength tensor by \( f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \) and also retained the notation for the redefined fermion field. Now, one can see that though the Lagrangian \( \sigma \) contains a plethora of Lorentz violating couplings (stemming from the effective Higgs field expansion \( \sigma \) in it) they all disappear in the limit \( M \rightarrow \infty \) that leads to the theory which is completely equivalent to a conventional QED taken in an axial gauge. Thus, a possible Lorentz violation, even if it were the case, should only be extremely small being significantly suppressed by inverse powers of the large mass scale \( M \). However, as was shown in the tree \( \sigma \) and one-loop \( \sigma \) approximations, there is no physical Lorentz violation in the emergent QED for any mass value \( M \) in the vector field constraint \( a \). Later this result was also confirmed for many other gauge theories with the supplementary vector field constraints, such as the spontaneously broken massive QED \( \sigma \), non-Abelian theory \( \sigma \), tensor field gravity \( \sigma \), and also their supersymmetric extensions \( \sigma \).

So, in contrast to a spontaneous violation of internal symmetries, SLIV caused by the length-preserving constraint does not necessarily imply a physical breakdown of Lorentz invariance. Rather, we are concerned here only with a "spontaneous breakdown" of an input covariance of the gauge condition \( a \) for the starting vector field \( A_\mu \) to the noncovariant axial gauge \( \sigma \) for the appearing vector Goldstone boson \( a_\mu \). In this connection, though it may seem counterintuitive, these two aspects could be allowed to coexist: an appearance of vector Goldstone bosons, from the one side, and a non-observability of spontaneous Lorentz violation caused by a covariant constraint, from the other. Actually, the constrained gauge fields are turned out to be, at the same time, the vector Goldstone bosons manifesting themselves in the "symmetry broken phase" \( \sigma \). However, gauge invariance in QED and other gauge theories always leads to a total conversion of SLIV into gauge degrees of freedom of massless vector Goldstone bosons. We will hereafter refer to this case of SLIV as an "inactive" SLIV, as opposed to an "active" SLIV leading to physical Lorentz violation which appears if gauge invariance in the theory is explicitly broken\(^5\).

\(^5\) Indeed, the only way for an inactive SLIV to to be activated (thus causing a physical Lorentz violation) would appear only if gauge invariance in the theory were really broken rather than merely constrained by some gauge condition. Such a violation of gauge invariance could provide some extension of the considered model with high-dimension operators induced presumably by quantum gravity at very small distances \( \sigma \).
B. Yang-Mills theories

We still have considered only a single vector field case with an underlying global \( U(1) \) symmetry. An extension to a theory possessing from the outset some global non-Abelian symmetry \( G \) follows the same logic, though includes some peculiarities \[14, 15\]. Suppose that this theory contains an adjoint vector field multiplet \( A^\mu_\alpha \) and some fermion matter field multiplet \( \psi \) belonging to one of irreducible representations of \( G \) given by matrices \( t^\mu \)

\[
[t^\mu, t^\nu] = if^{\mu\nu\rho}t^\rho, \quad Tr(t^\mu t^\nu) = \delta^{\mu\nu} \quad (p, q, r = 0, 1, ..., N - 1)
\]

where \( f^{\mu\nu\rho} \) stand structure constants, while \( N \) is a dimension of the \( G \) group. The corresponding Lagrangian \( L^{\text{tot}} \) is supposed to also include the standard Lagrange multiplier term with the field function \( \lambda(x) \)

\[
L^{\text{tot}}(A_\mu, \psi, \lambda) = L(A_\mu, \psi) - \frac{\lambda}{2}(A^\mu_\mu A_{\mu\mu} - n^2 M^2), \quad n^2 \equiv n_\mu n^{\mu} = \pm 1
\]

where \( n_\mu \) stands now for some properly-oriented ‘unit’ rectangular matrix both in Minkowski spacetime and internal space. As in the above Abelian case, the Lagrange multiplier term does not contribute to the vector field equations of motion by the proper choice of initial values for all fields (and their momenta) involved so as to restrict the phase space to values with a vanishing multiplier function \( \lambda(x) \) (see also the footnote\(^3\)).

The variation of the Lagrangian \( L^{\text{tot}}(A_\mu, \psi, \lambda) \) leads to following equations of motion for the vector field multiplet \( A^\mu_\mu \)

\[
(E^\mu_A)_\mu = 0, \quad p = 0, 1, ..., N - 1
\]

and the auxiliary field \( \lambda \)

\[
C(A) = A^\mu_\mu A_{\mu\mu} - n^2 M^2 = 0
\]

which is the covariant length-preserving constraint for vector fields in the non-Abelian symmetry case. Thus, there are a total of \( N + 1 \) equations for \( N \) 4-vector fields \( A^\mu_\mu \), one of which is the constraint \( C(A) = 0 \) being preserved in time. This means, in accordance with general arguments given above, that the equations of motion for the vector fields \( A^\mu_\mu \) cannot be all independent. As a result, the appropriate emergence equations, analogous to the equation \( 1 \) in an Abelian theory, inevitably occur

\[
F^\mu(C = 0; \ E_A, E_\psi) = 0, \quad p = 0, 1, ..., N - 1.
\]

When being expressed as a linear combination of the basic mass dimension-4 terms, this equation leads to the identities between all field Eulerians involved

\[
\partial_\mu (E^\mu_A)_\mu = f^{\rho\sigma\mu} A^\rho_\mu (E^\sigma_A)_\mu + E_\psi (it^\rho) \psi + \overline{\psi} (-it^\rho) E_\psi.
\]

An identification of the coefficients of the Eulerians on the right-hand side of the identities \( 20 \) with the structure constants \( f^{\rho\sigma\mu} \) and generators \( t^\mu \) \( 15 \) of the group \( G \) is quite transparent. This readily comes from the fact that the right-hand side of this identity must transform in the same way as the left-hand side, which transforms as the adjoint representation of the \( G \) group. The terms and their coefficients in the identity \( 23 \) are typically chosen so as to satisfy the Lee bracket operation to close the symmetry algebra once the corresponding field transformations are identified\(^6\). Note that, in contrast to the Abelian case, the term proportional to the vector field multiplet \( A^\mu_\mu \) itself which corresponds to a self-interaction of non-Abelian vector fields, also appears in the above identity. Again, Noether’s second theorem \( 23 \) can be applied directly to this identity in order to derive the gauge invariance of the Lagrangian \( L(A, \psi) \)

\(^6\) In order to avoid generating too many symmetry transformations, which would only be consistent with a trivial Lagrangian (i.e. \( L = \text{const} \)), we typically require that the general symmetry transformations following from the emergence identities like \( 20 \) have to constitute a group. This means that they have to satisfy the Lie bracket operations to close the symmetry algebra. This requirement puts strong restrictions on the form of the emergence identities in all the emergent symmetry cases considered \[14, 15\].
For a theory with renormalizable coupling constants, this emergent gauge symmetry leads to the conventional Yang-Mills type Lagrangian

$$L_{\text{YM}}(A, \psi, \lambda) - \frac{\lambda}{2} A_{\mu}^p A^{\mu p} - n^2 M^2$$ (22)

where we also include the corresponding Lagrange multiplier term. This term, as was mentioned above, does not contribute to the vector field equation of motion in the identity (20).

Now let us turn to the spontaneous symmetry violation which may be caused by the nonlinear vector field constraint (18) determined by the Lagrange multiplier term in (22). Although the Lagrangian $L_{\text{YM}}(A, \psi, \lambda)$ only has an $SO(1,3) \times G$ invariance, the constraint term in it possesses the much higher accidental symmetry $SO(N,3N)$ according to the dimension $N$ of the adjoint representation of $G$ to which the vector fields $A_{\mu}^p$ belong. This symmetry is indeed spontaneously broken at a scale $M$

$$\langle A_{\mu}^p \rangle = n_{\mu}^p M$$ (23)

with the vacuum direction determined now by the 'unit' rectangular matrix $n_{\mu}^p$ which describes simultaneously both of the possible symmetry breaking cases, time-like

$$SO(N,3N) \to SO(N-1,3N)$$ (24)

or space-like

$$SO(N,3N) \to SO(N,3N-1)$$ (25)

depending on the sign of $n^2 \equiv n_{\mu}^p n^{\mu p} = \pm 1$. In both cases the matrix $n_{\mu}^p$ has only one non-zero element, subject to the appropriate $SO(1,3)$ and (independently) $G$ rotations. They are, specifically, $n_0^3$ or $n_3^0$ provided that the VEV (23) is developed along the $p = 0$ direction in the internal space and along the $\mu = 0$ or $\mu = 3$ direction, respectively, in the ordinary four-dimensional spacetime.

As was argued in [33, 34], side by side with one true vector Goldstone boson corresponding to the spontaneous violation of the actual $SO(1,3) \otimes G$ symmetry of the Lagrangian $L_{\text{YM}}(A, \psi, \lambda)$, the $N - 1$ pseudo-Goldstone vector bosons (PGB) related to the breakings (24, 25) of the accidental symmetry $SO(N,3N)$ of the constraint (18) per se are also produced\footnote{Note that in total there appear $4N - 1$ pseudo-Goldstone modes, complying with the number of broken generators of $SO(N,3N)$. From these $4N - 1$ pseudo-Goldstone modes, $3N$ modes correspond to the $N$ three-component vector states as will be shown below, while the remaining $N - 1$ modes are scalar states which will be excluded from the theory.}. Remarkably, in contrast to the familiar scalar PGB case [16], the vector PGBs remain strictly massless being protected by the simultaneously generated non-Abelian gauge invariance. Together with the above true vector Goldstone boson, they also come into play properly completing the whole gauge multiplet of the internal symmetry group $G$ taken.

Now we come again, as in the QED case, to the symmetry broken phase which in accordance with the VEV equation (23) is defined as

$$A_{\mu}^p = a_{\mu}^p + n_{\mu}^p H , \quad n_{\mu}^p a^{\mu p} = 0 \quad (a^2 \equiv a_{\mu}^p a^{\mu p})$$ (26)

in which the new vector fields $a_{\mu}^p$ appear as the vector Goldstone modes. Indeed, this multiplet is orthogonal to the vacuum direction given by the 'unit' rectangular matrix $n_{\mu}^p$, as is determined by the constraint (18) itself together with the effective Higgs mode

$$H = (M^2 - n^2 a^2)^{1/2} = M - \frac{n^2 a^2}{2M} + \cdots$$ (27)

Note that, apart from the pure vector fields, general zero modes $a_{\mu}^{p'}$ contain $N - 1$ scalar modes, $a_0^{p'}$ or $a_3^{p'}$ ($p' = 1, ..., N - 1$), for the time-like ($n_{\mu}^p = n_0^0 g_{00} g^{\mu 0}$) or space-like ($n_{\mu}^p = n_3^0 g_{33} g^{\mu 3}$) SLIV, respectively.
They can be eliminated from the theory, if one imposes appropriate supplementary conditions on the $N - 1$ fields $a^p_\mu$ which are still free of constraints. Using their overall orthogonality \([20]\) to the physical vacuum direction $n_\mu$, one can formulate these supplementary conditions in terms of a general axial gauge for the entire $a^p_\mu$ multiplet

$$ n \cdot a^p = n_\mu a^{p\mu} = 0, \quad p = 0, 1, ..., N - 1. \quad (28) $$

Here $n_\mu$ is the unit Lorentz vector being analogues to the vector introduced in the Abelian case, which is now oriented in Minkowski spacetime so as to be "parallel" to the vacuum unit $n_\mu$ matrix. This matrix can be taken hereafter in the "two-vector" form

$$ n^p_\mu = n_\mu \epsilon^p, \quad \epsilon^p \epsilon^p = 1 \quad (29) $$

where $\epsilon^p$ is unit $G$ group vector belonging to its adjoint representation. As a result, in addition to an elementary Higgs mode excluded earlier by the above orthogonality condition \([20]\), all the other scalar fields are also eliminated. Consequently only the pure vector fields, $a^p_\mu (i = 1, 2, 3)$ or $a^{p\mu}_\mu (\mu' = 0, 1, 2)$, for the time-like or space-like SLIV respectively, are left in the theory. Clearly, the components $a^{p\mu=0}_i$ and $a^{p\mu=0}_i$ correspond to the true Goldstone bosons in these cases, while all the other (for $p = 1, ..., N - 1$) are vector PGBs.

Substituting the parameterization \([20, 27]\) into the Lagrangian \([22]\), one is led to the non-Abelian gauge theory in terms of the pure emergent modes $a^p_\mu$. However, as in the above Abelian case, one should first use the local invariance of the Yang-Mills Lagrangian $L_{YM}(A, \psi)$ in the emergent theory \([22]\) to gauge away the apparently large but fictitious Lorentz violating terms which appear in the symmetry broken phase \([20]\). As one can readily see, they stem from the effective Higgs field $H$ expansion \([27]\) when it is applied to some vector-vector and vector-fermion field interaction couplings in the Lagrangian $L_{YM}$ in \([22]\). To exclude them we can make the appropriate transformations (similar to the transformation \([18]\) taken above in the Abelian case) of the fermion ($\psi$) and new vector ($a_\mu \equiv a^p_\mu$) field multiplets:

$$ \psi \rightarrow U(\omega)\psi, \quad a_\mu \rightarrow U(\omega)a_\mu U(\omega)^\dagger, \quad U(\omega) = e^{igM(x)\cdot n}. \quad (30) $$

Since the phase of the transformation phase $\omega$ is linear in the spacetime coordinate the following equalities are evidently satisfied:

$$ \partial_\mu U(\omega) = igMn_\mu U(\omega) = igMU(\omega)n_\mu, \quad n_\mu \equiv n^p_\mu t^p. \quad (31) $$

One can readily confirm now that the above-mentioned fictitious Lorentz violating terms in Lagrangian \([22]\) are thereby cancelled with the analogous terms stemming from kinetic terms of vector and fermion fields. So, the final Lagrangian for the emergent Yang-Mills theory in the symmetry broken phase takes the form

$$ L_{em}(A, \psi) = -\frac{1}{4}[f^p_\mu + \bar{f}^p_\mu (H - M)][f^{p\mu\nu} + \bar{f}^{p\mu\nu}(H - M)] - \frac{1}{2}i(n^p_\mu a^p_\mu)^2 + \bar{\psi}(i\gamma\partial - m)\psi + g[a^p_\mu + (H - M)n^p_\mu]\bar{\psi}(\gamma^p t^p)\psi. \quad (32) $$

Here the stress-tensor $f^p_\mu$ is, as usual,

$$ f^p_\mu = \partial_\mu a^p_\nu - \partial_\nu a^p_\mu + g f^{pqr} a^q_\nu a^r_\mu, \quad (33) $$

while $\bar{f}^p_\mu$ stands for the new SLIV oriented tensor

$$ \bar{f}^p_\mu \equiv n^p_\mu \partial_\mu - n^p_\mu \partial_\nu + g f^{pqr} a^q_\mu n^r_\nu - a^q_\mu n^r_\nu. \quad (34) $$

acting on the effective Higgs field expansion terms in \([32]\). We also included the overall gauge fixing term for the entire $a^p_\mu$ multiplet to remove all scalar modes from the theory and retained the original notations for the fermion and vector fields after the transformations \([30]\) were carried out. One can easily see that these transformations actually amounts to the replacement of the effective Higgs field $H$ by the combination $H - M$ in the emergent Lagrangian \([32]\). As a result, all the large Lorentz breaking
terms proportional to the scale M appear cancelled\(^8\). Expanding the effective Higgs field \( H \) in powers of \( a^2/M^2 \) in the Lagrangian \( \mathcal{L}_3 \), one comes to highly nonlinear theory in terms of the zero vector modes \( a_\mu \), which contains many the properly suppressed Lorentz violating couplings. However, as in the Abelian symmetry case, they do not lead to physical Lorentz violation effects which turn out to be strictly cancelled among themselves \(^{33,34}\).

All the above allows one to conclude that the Yang-Mills theory can naturally be interpreted as emergent gauge theory caused by the generic length-preserving constraint put on the vector field multiplet in some prototype theory possessing only global non-Abelian internal symmetry. Such a constraint could in principle lead to the spontaneous Lorentz violation which, however, appears unobservable due to the gauge invariance \(^{21}\) emerged, thus giving one more example of an inactive SLIV. In this connection, the emergence conjecture itself could be reformulated as a principle \(^{22}\) of a generic non-observability of the spontaneous Lorentz violation being caused by some constant background value of the vector field (or vector field multiplet). Presumably, this principle may provide the origin of all gauge internal symmetries observed whether they are exact as in QED and quantum chromodynamics or spontaneously broken as in the electroweak theory and grand unified models.

### III. TOWARDS AN EMERGENT POINCARÉ GRAVITY

We have mentioned above that PGG looks in essence as a gauge field theory in flat Minkowski space which successfully mimics curved space geometry when making the transition to the base world space with general affine connections and metric expressed, respectively, in spin-connections and tetrads. Conventionally, one has in PGG the world space (WS) symmetry \( ISO(1,3)_{WS} \), which includes translations and the orbital part of Lorentz transformations, and a local frame (LF) Lorentz symmetry \( SO(1,3)_{LF} \), which only includes the spin part of Lorentz transformations acting on representation indices. Remarkably, this duality is in an automatic accordance with the Einstein equivalence principle which, therefore, need not to be specially postulated in PGG as is in the standard GR. On the other hand, this suggests some clear analogy between the local frame symmetry space in PGG and internal charge space in conventional quantum field theories. Such a unique property of PGG allows us to proceed in precisely the same way as before in the Yang-Mills theory case that eventually leads us to the emergent PGG theory. In this connection, we begin with the entirely global spacetime symmetries, both \( ISO(1,3)_{WS} \) and \( SO(1,3)_{LF} \), and our starting objects are the two vector field multiplets which are 4-vectors of \( ISO(1,3)_{WS} \) and belongs, respectively, to the adjoint \( (A^\mu_{ij}) \) and vector \( (\epsilon^\mu_i) \) representations of the Lorentz group \( SO(1,3)_{LF} \) (antisymmetry in the indices \( ij \) in \( A^\mu_{ij} \) is imposed). In what follows, we will refer to these prototype fields as the spin connections and tetrads. As we will see, they are really turned out to be those when an emergence procedure given above in Section II B is applied to them. As a result, the local frame Lorentz symmetry \( SO(1,3)_{LF} \) and translation subgroup in \( ISO(1,3)_{WS} \) appear gauged, while the orbital Lorentz transformations are actually absorbed by the latter. So, eventually one has the local translations and local \( SO(1,3)_{LF} \) transformations being gauged by the emergent tetrad and spin-connection fields, respectively. The gauge tetrads field \( e^\mu_i \) set the local coordinate frame in the emergent PGG, while the gauge spin-connection fields \( A^\mu_{ij} \) are in fact connections with respect to the tetrad frame (we retain the starting notations for them). Again, due to gauge invariance emerged the physical Lorentz (and translation invariance) remains in the final theory. This means that the covariant length-preserving constraints only lead to an inactive SLIV case even if they are applied to vector fields related to spacetime symmetries.

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\(^{8}\) Let us note that the constant background value of the gauge vector field can always be cancelled out depending no whether the Higgs field is effective or elementary. Indeed, the vacuum shift of a vector field in an Abelian case is a gauge transformation in itself with a gauge function linear in coordinate, as can be readily seen in \(^{23}\). In non-Abelian case, the vacuum shift of a vector field multiplet \( A_\mu \) given in \(^{26,27}\) being accompanied by a subsequent "rotation" \(^{30,31}\) of an emergent multiplet \( a_\nu \) can be written entirely as

\[
A_\mu = U a_\mu U^{-1} - \frac{i}{g} \left( \partial_\mu U \right) U^{-1} + n_\mu (H - M).
\]

with the transformation phase linear in the coordinate. Now one can clearly see that the first two terms present the pure gauge transformation of vector field multiplet \( a_\mu \) and, thereby, only the third term can not be gauged away. That is why the emergent Lagrangian \(^{32}\) contains just the combination \( H - M \). Due to this fact the Lagrangian \( \mathcal{L}^{em} \) goes to a conventional Yang-Mills theory in the limit \( M \rightarrow \infty \). Otherwise, it would become infinite in this limit.
A. Constrained tetrad and spin-connection fields

First of all, as we could learn above from the emergent gauge theories, the tetrad and spin-connection fields have to be properly constrained to induce an appropriate emergence process. The essential point is, however, that the tetrad field is generically constrained by definition \( \delta \). To see clearer what does this constraint mean, let first notice that whereas the spin-connection field \( A_{\mu}^{ij} \) has a canonical vector field mass dimension, the tetrad field \( e_{\mu}^{i} \) appears to have zero mass dimension. Treating it as all other boson fields having a canonical dimension of mass we introduce some fundamental mass scale in the above definition of tetrad fields \( e_{\mu}^{i} \) changing their orthogonality equations to

\[
e_{\mu}^{i}e_{\nu}^{j} = \delta_{\mu}^{\nu}M_{e}^{2}, \quad e_{\mu}^{i}e_{\nu}^{i} = \delta_{\mu}^{\nu}M_{e}^{2}, \quad e_{\mu}^{i}e_{\nu}^{i} = n^{2}M_{e}^{2},
\]

where the first two conditions could be considered as those which define the inverse tetrads \( e_{i}^{\mu} \), whereas the third one is their length-fixing constraint. Here \( n^{2} \) stands for

\[
n^{2} \equiv \delta_{\mu}^{\nu}\delta_{\nu}^{\mu} = \delta_{i}^{i}\delta_{i}^{i} = \delta_{j}^{j}\delta_{j}^{j} = 4.\tag{36}
\]

Since a general metric tensor \( g_{\mu\nu} \) is generally assumed to be dimensionless one also has

\[
g_{\mu\nu} = \frac{1}{M_{e}^{2}}\eta_{ij}e_{i}^{\mu}e_{j}^{\nu}.\tag{37}
\]

rather than \( \eta \). We can readily see that the last constraint in \( e_{\mu}^{i} \) is indeed similar to the constraints we have above for conventional vector fields \( \delta_{\mu}^{\nu} \). This constraint actually means that PGG is a spontaneously broken theory that manifests itself at some input mass scale \( M_{e} \) which could be in principle associated with the Plank mass \( M_{P} \). Generally, this violation may concern both the world spacetime symmetry and the local frame Lorentz symmetry being developed along some particular directions, just as it took place in the non-Abelian vector field case considered above. However, one can choose this violation in a way that the vacuum of the PGG theory is flat Minkowski space rather than breaks Lorentz invariance. This lead, as we will see shortly, to spontaneous violation of some accidental global symmetry and generation of the Goldstone vector bosons properly gauging translation in the world space.

The similar length-fixing constraint is proposed to be put on the spin-connection fields \( A_{\mu}^{ij} \)

\[
A_{\mu}^{ij}A_{\mu}^{ij} = n^{2}M_{A}^{2}, \quad n^{2} \equiv n_{ij}^{ij}n_{ij}^{ij} = \pm 1\tag{38}
\]

being analogous to the constraints \( \delta_{\mu}^{\nu} \) for ordinary vector fields (here \( n_{ij}^{ij} \) stands now for some properly-oriented ‘unit’ rectangular matrix, and also antisymmetry in the \( ij \) indices is imposed). The constraint \( \eta_{ij} \) actually means that we also have a spontaneous Lorentz violation in PGG that appears at some high mass scale \( M_{A} \) which could be in principle close to the Plank mass \( M_{P} \) as well. This will cause, as we confirm later, the generation of Goldstone vector bosons gauging Lorentz symmetry in the local frame, while the physical Lorentz invariance is left intact. Remarkably, we have here the very special case of an inactive SLIV which induces a local Lorentz invariance.

B. From global to local symmetries

We have already emphasized above that there is a generic analogy between the local frame symmetry space in PGG and internal charge space in conventional quantum field theories. As a result, the vector fields of spin-connections \( A_{\mu}^{ij}(x) \) gauging the local Lorentz group \( SO(1,3)_{\lambda,F} \) look like as gauge bosons appearing in the Standard Model. As to tetrads \( e_{\mu}^{i}(x) \), though they are gauge fields of the coordinate-dependent translations in the world spacetime, they transform like as ordinary matter fields w.r.t. the local Lorentz frame. As such, they belong to the vector multiplet of \( SO(1,3)_{\lambda,F} \) rather than its adjoint representation as the spin-connection fields \( A_{\mu}^{ij}(x) \).

All that looks very similar to the situation we had above in the Yang-Mills theory case, and actually represents some precondition for PGG to be also emerged from pure global symmetries once the vector field constraints \( \delta_{\mu}^{\nu} \) and \( \eta_{ij} \) come into play. Accordingly, we start with some prototype theory possessing only global symmetries \( ISO(1,3)_{\mu,S} \) and \( SO(1,3)_{\lambda,F} \) operating in the two flat Minkowski spaces with constant metrics \( \eta_{\mu\nu} \) and \( \eta_{ij} \), respectively. This yet arbitrary theory contains some prototype vector
fields having form of spin-connections $A^I_{ij}(x)$ and tetrads $e^i_{\mu}(x)$ and may also contain some matter fields (say, fermions $\psi$). The theory have in general all possible interactions between all vector and matter fields involved. The corresponding Lagrangian $\mathcal{L}^{\text{tot}}$ is supposed to also include the standard Lagrange multiplier terms with the field functions $\lambda_A(x)$ and $\lambda_e(x)$

$$\mathcal{L}^{\text{tot}}(e, A, \psi; \lambda_e, \lambda_A) = \mathcal{L}(e, A, \psi) - \frac{\lambda_A}{2}(A^I_{ij} A^\mu_{ij} - n^2 M_A^2) - \frac{\lambda_e}{2}(e^\mu_i e_i^\mu - n^2 M_e^2). \quad (39)$$

As in the above QED and Yang-Mills theory cases (and for the same reason, see also footnote$^3$), these terms do not contribute to the vector field equations of motion. The variations under $\lambda_A(x)$ and $\lambda_e(x)$ result, accordingly, in the covariant length-preserving constraints for the spin-connection and tetrad fields

$$C_A = A^I_{ij} A^\mu_{ij} - n^2 M_A^2 = 0, \quad C_e = e^\mu_i e_i^\mu - n^2 M_e^2 = 0 \quad (40)$$

in the PGG theory. Therefore, we again face the question of consistency of these extra constraint equations with the equations of motion for the vector fields of tetrads $e^\mu_i$ and spin-connections $A^I_{ij}$

$$(\mathcal{E}^i_{\mu})_\mu = 0, \quad (\mathcal{E}^j_{\mu})_\mu = 0 \quad (i, j = 0, 1, 2, 3; \mu = 0, 1, 2, 3). \quad (41)$$

For an arbitrary Lagrangian $\mathcal{L}(e, A, \psi)$, the time development of the fields would not preserve in general the constraints$^{10}$. So, the parameters in the Lagrangian $\mathcal{L}$ must be chosen so as to give a relationship between the Eulerians for all the fields involved. The need to preserve the constraints $C_A = 0$ and $C_e = 0$ with time implies, as in the emergent Yang-Mills theory case, that the equations of motion for the vector fields of spin-connections $A^I_{ij}$ and tetrads $e^\mu_i$, respectively, cannot be all independent. As a result, the special emergence equations for spin-connection fields

$$\mathcal{F}^{ij}(C_A = 0; E_A, E_e, \psi, ...) = 0 \quad (i, j = 0, 1, 2, 3) \quad (42)$$

and tetrad fields

$$\mathcal{F}_\mu(C_e = 0; E_e, E_A, \psi, ...) = 0 \quad (\mu = 0, 1, 2, 3), \quad (43)$$

necessarily appear. Remind one more time that an antisymmetry in the Lorentz indices $(i, j, k, ...)$ is everywhere imposed.

Let us consider first the emergence equations$^{12}$. Again, when being expressed as a linear combination of the basic mass dimension-4 terms, this equation leads to the identities between all field Eulerians involved

$$\partial^\mu (E_A)^{ij}_\mu = c^{ij}_{[kl][mn]} A^k_{\mu} (E_A)^{\mu, mn} + e^j_{[\mu} (E_e)^{j]\mu} + E_\psi S^{ij} \psi + \overline{\psi} S^{ij} \overline{\psi} \quad (44)$$

which are precisely analogous to those we had above in$^{19}$ for the Yang-Mills theory. An appropriate identification of the Eulerian terms on the right-hand side of the identity$^{11}$ with the structure constants $c^{ij}_{[kl][mn]}$ and the fermion representation matrices $S^{ij}$ of the Lorentz symmetry group $SO(1, 3)_{\mu\nu}$ is indeed quite clear. The point is that the right-hand side of this identity must transform in the same way as its left-hand side, which transforms as the adjoint representation of $SO(1, 3)_{\mu\nu}$. As to their coefficients and other possible terms in the identity$^{11}$, there remained only terms which satisfy the Lee bracket operation to close the symmetry algebra once the corresponding field transformations are identified (in this connection, see our comment to the Yang-Mills theory case in the footnote$^6$).

As to the basic identities following from the emergence equations for tetrad fields$^{13}$, the non-trivial lowest mass dimension terms constructed from the Eulerians for this case will necessarily include the translation operator expression $T^\mu = -\partial^\mu_x$ for all the fields involved. Consequently they take the following form

$$e^\mu_i \partial_\mu (E_e)_i^\nu + A^I_{ij} \partial_\mu (E_A)^{ij}_\mu + (\partial_\mu \psi) \psi + \overline{\psi} (\partial_\mu \overline{\psi}) = 0 \quad (45)$$

which consist of all the terms having mass dimension 5.

Now again, Noether’s second theorem$^{20}$ can be applied directly to the above identities$^{13}$ and$^{15}$ in order to derive the gauge invariance of the Lagrangian $\mathcal{L}(e, A, \psi)$ in$^{39}$. Indeed, with the constraint
implied, this Lagrangian tends to be invariant under local transformations of the spin-connection, tetrad and matter fields of the type

\[ \delta A^i_{\mu j} = \varepsilon^i_k A^k_{\mu j} + \varepsilon^i_k A^k_{\mu i} - \partial_\mu \xi^i A^i_{\mu j} - \partial_\mu \varepsilon^i j, \]
\[ \delta \varepsilon^i_\mu = \varepsilon^i_i \partial_\mu \xi^i - \varepsilon^i_k \varepsilon^k_\mu, \delta \psi = \frac{1}{2} \varepsilon^i j S^i j \psi. \] (46)

Note that the first two terms in \( \delta A^i_{\mu j} \) correspond to local Lorentz rotations of the spin-connection fields \( A^i_{\mu j} \) with parameters \( \varepsilon^{ij}(x) \), while the third term is due to the local translations conditioned by the parameters \( \xi^i(x) \). The last terms in \( \delta A^i_{\mu j} \) means that the spin-connection fields \( A^i_{\mu j} \) gauge just the local Lorentz rotations. The tetrad field in \( \delta \varepsilon^i_\mu \) is Lorentz-rotated (in the local Lorentz frame) and, simultaneously, subject to the coordinate-dependent translations (in the world spacetime). And finally, the transformation of the fermion field \( \psi \) in (46) is, as usual, determined by the fermion representation matrices \( S^{ij} \) which are simply given by the commutators of the \( \gamma \)-matrices

\[ S_{ij} = \frac{1}{4} [\gamma_i, \gamma_j] \equiv \gamma_{ij}/2. \] (47)

The local transformations (46) shows that the somewhat arbitrarily introduced prototype vector fields \( A^i_{\mu j} \) and \( \varepsilon^i_\mu \) are really turned out to be the PGG spin-connection and tetrad fields once they satisfy the length-preserving constraints (40). Moreover, the induced gauge symmetry (40) unavoidably leads to the emergent PGG Lagrangian

\[ \mathcal{L}^{\text{em}}(e, A, \psi; \lambda_e, \lambda_A) = \mathcal{L}^{\text{em}}(e, A, \psi)_{\text{PGG}} - \frac{\lambda_A}{2} (A^i_{\mu j} A^i_{\mu j} - n^2 M^2_A) - \frac{\lambda_e}{2} (\varepsilon^i_\mu \varepsilon^i_\mu - n^2 M_e^2) \] (48)

where \( \mathcal{L}^{\text{em}}(e, A, \psi)_{\text{PGG}} \) is solely constructed from the covariant curvature and torsion tensors

\[ R^i_{\mu \nu} = \partial_\nu A^i_{\mu \nu} + \eta_{kl} A^i_{\nu \nu} A^i_{\mu \nu}, \quad T^i_{\mu \nu} = \partial_\nu \varepsilon^i_\mu + \eta_{kl} A^i_{\nu \nu} \varepsilon^i_\mu \] (49)

and a covariant derivative for the fermion field

\[ \bar{\psi} \gamma^i \gamma^j \psi = \bar{\psi} \gamma^i (\partial_\mu \psi) - (\partial_\mu \bar{\psi}) \gamma^i \psi + \frac{1}{4} A^a_{\mu} \bar{\psi} \{ \gamma^i, \gamma_a \} \psi \] (50)

We also included the corresponding Lagrange multiplier terms which, as was mentioned above, do not contribute to the physical field equations of motion. Now, for a theory with the lowest dimension coupling constants, containing at most the quadratic terms in the curvature and torsion one has

\[ \mathcal{L}^{\text{em}}(e, A, \psi)_{\text{PGG}} = \mathcal{L}^{(1)}(e, A, \psi) + \mathcal{L}^{(2)}(e, A, \psi) \] (51)

where the first term correspond to the minimal Einstein-Cartan theory being linear in the curvature

\[ \mathcal{L}^{(1)}(e, A, \psi) = \frac{e}{2\kappa} \frac{\varepsilon^i_\mu e^i_{\nu}}{M_e} R^i_{\mu \nu} + e \frac{\varepsilon^i_\mu}{2M_e} \bar{\psi} \gamma^i \gamma^j \psi \] (52)

(where \( \kappa \) stands for the modified Newtonian constant \( 8\pi G \)), while in the second term \( \mathcal{L}^{(2)} \) all eight possible quadratic terms (36, 57) are generally collected.

C. Broken symmetry phase: zero spin-connection modes

We have found above that the presence of the spin-connection and tetrad field constraints (40) in the theory unambiguously converts the global symmetry \( ISO(1,3)_{WS} \times SO(1,3)_{LF} \) we started into the local Poincaré symmetry \( T(1,3)_{WS} \times SO(1,3)_{LF} \) that leads to the conventional PGG theory\(^9\). The point is, however, that these constraints mean at the same time that this global symmetry is spontaneously broken

\(^9\) The orbital part of Lorentz symmetry transformations in the starting \( ISO(1,3)_{WS} \) group is in fact absorbed by local translations, as mentioned above.
thus inducing the Goldstone spin-connection and tetrad field modes in which the PGG theory has to be eventually expressed.

To see it in more detail, let us consider first the spin-connection fields. Note above all, whereas the emergent PGG Lagrangian \( L_{|\text{PGG}} \) in (43) possesses the local Poincaré symmetry \( T(1,3)_{WS} \times SO(1,3)_{LF} \), the accidental global symmetry of the length-fixing spin-connection constraint (38) appears much higher, \( ISO(6,18)_{WS} \). This symmetry is indeed spontaneously broken at a scale \( M_A \)

\[ \langle A_{ij}^\mu \rangle = n_{ij}^\mu M_A \]  

(53)

with the vacuum direction determined now by the ‘unit’ rectangular matrix \( n_{ij}^\mu \) which describes simultaneously both of the SLIV cases, time-like

\[ ISO(6,18) \rightarrow ISO(5,18) \]  

(54)

or space-like

\[ ISO(6,18) \rightarrow ISO(6,17) \]  

(55)

depending on the sign of \( n^2 = \pm 1 \). In both cases the matrix \( n_{ij}^\mu \) has only one non-zero element, subject to the appropriate \( ISO(1,3)_{WS} \) and (independently) \( SO(1,3)_{LF} \) transformations. They are, specifically, \( n_0^{(ij)} \) or \( n_3^{(ij)} \) provided that the VEV (53) is developed along the \( \langle ij \rangle \) direction in the local Lorentz frame and along the \( \mu = 0 \) or \( \mu = 3 \) direction, respectively, in the world spacetime.

As was argued in the above non-Abelian vector field case, side by side with one true vector Goldstone boson corresponding to spontaneous violation of an actual \( ISO(1,3)_{WS} \times SO(1,3)_{LF} \) symmetry of the PGG Lagrangian, the five pseudo-Goldstone vector bosons related to the breakings (54, 55) of the accidental symmetry \( ISO(6,18) \) of the constraint (38) per se are also produced. Remarkably, the vector PGBs remain strictly massless being protected by the simultaneously generated Lorentz gauge invariance. Together with the above true vector Goldstone boson, they also come into play thus properly completing the entire adjoint gauge multiplet of spin-connection fields of the local Lorentz symmetry group \( SO(1,3)_{LF} \).

Due to the constraint (38), which virtually appears as a single condition put on the spin-connection field multiplet \( A_{ij}^\mu \), one can identify the pure Goldstone field modes \( A_{ij}^\mu \) using the parametrization

\[ A_{ij}^\mu = A_{ij}^\mu + n_{ij}^\mu \sqrt{M_A^2 - n^2 A^2}, \quad n_{ij}^\mu A_{ij}^\mu = 0 \quad (A^2 \equiv A_{ij}^\mu A_{ij}^\mu). \]  

(56)

and an effective “Higgs” mode

\[ H = \sqrt{M_A^2 - n^2 A^2} = M_A - \frac{n^2 A^2}{2M_A} + \cdots \]  

(57)

determined by the constraint itself. Note that, apart from the pure vector fields, the general zero modes \( A_{ij}^\mu \) contain the five scalar modes, \( A_0^{ij} \) or \( A_3^{ij} \), for the time-like \( (n_{ij}^{(ij)} = n_0^{(ij)} g_{\mu 0} \delta^{(ij)(ij)}) \) or space-like \( (n_{ij}^{(ij)} = n_3^{(ij)} g_{\mu 3} \delta^{(ij)(ij)}) \) SLIV, respectively. They can be eliminated from the theory, if one imposes appropriate supplementary conditions on the five fields \( A_{ij}^\mu \) which are still free of constraints. Using their overall orthogonality (56) to the physical vacuum direction \( n_{ij}^\mu \), one can formulate these supplementary conditions in terms of a general axial gauge for the entire \( A_{ij}^\mu \) multiplet

\[ n_{ij}^\mu A_{ij}^\mu = 0. \]  

(58)

---

10 This symmetry being treated as the world space symmetry is determined by a proper number of the spacetime directions related to the (local frame) Lorentz group representations of the vector fields involved (just like as in the Yang-Mills theory case considered above such a total symmetry was determined by a number of the internal space directions related to an adjoint representation of the vector field multiplet invoked). In this way, the length-fixing constraint for spin-connection fields (38) possesses the global symmetry \( ISO(6,18)_{WS} \), whereas a similar constraint for tetrad fields (39) the lower global symmetry \( ISO(4,12)_{WS} \), as is claimed below.

11 Note that in total there appear the 23 pseudo-Goldstone modes, complying with the number of broken generators of \( SO(6,18) \). From these 23 pseudo-Goldstone modes, 18 modes correspond to the six three-component vector states, as will be shown below, while the remaining 5 modes are scalar states which will be excluded from the theory.
where \( e^{ij} \) is the unit Lorentz group tensor belonging to its adjoint representation. As a result, in addition to the “Higgs” mode excluded earlier by the orthogonality condition \( (50) \), all the other scalar fields are eliminated. Consequently only the pure vector fields, \( A^{ij}_{\mu} (\mu = 1, 2, 3) \) or \( A^{ij}_{\mu} (\mu'' = 0, 1, 2) \), for the time-like or space-like SLIV respectively, are left in the theory. Clearly, the components \( A^{(ij)=(ij)}_{\mu} \) and \( A^{(ij)=(ij)}_{\mu''} \) correspond to the true Goldstone vector boson, for each type of SLIV, respectively, while all the other five ones (with \((ij) \neq (ij)\)) are vector PGBs. Consequently these six modes altogether represent the fundamental spin-connection field multiplet in the PGG theory in the final symmetry broken phase.

**D. Broken symmetry phase: zero tetrad modes**

Let us now turn to the tetrad fields. Again, as one can readily confirm, the tetrad length-fixing constraint in \( (53) \) possesses the high total global symmetry \( ISO(4,12)_{WS} \) rather than \( ISO(1,3)_{WS} \times SO(1,3)_{LF} \) as other terms in the emergent PGG Lagrangian \( (48) \). This symmetry then spontaneously breaks to some its ”diagonal” subgroup \( ISO(1,3) \) that results in an appearance of the corresponding Goldstone and Higgs modes. Note that this violation actually appears as the 16-dimensional Poincaré symmetry violation down to the ordinary 4-dimensional one. As it is well known for spontaneously broken spacetime symmetries \( (38) \), such a violation can solely lead to the Goldstone modes corresponding to the broken translational generators. There are no additional modes corresponding to the broken Lorentz generators. So, we eventually have only twelve Goldstone modes (according to the number of the broken translation generators) which may be given by the non-diagonal \( e^i_\mu \) components \( (e^0_1, e^1_2, e^2_3, e^3_2, e^2_1, e^1_3) \), whereas the Higgs mode by some combination of their diagonal ones \( (e^0_0, e^1_1, e^2_2, e^3_3) \). Indeed, the above Goldstone modes are in fact pseudo-Goldstone modes since, as was mentioned above, the symmetry of the PGG Lagrangian \( \mathcal{L}_{PGG}^{em} \) is much lower than the symmetry of the tetrad field constraint \( (53) \).

All that can be readily seen by using the familiar parametrization

\[
e^i_\mu = e^i_\mu + n^i_\mu \sqrt{M^2 - \epsilon^2} (\epsilon^2 \equiv e^i_\mu e^\mu_i / n^2)
\]

with \( e^i_\mu \) appearing as the vector Goldstone fields which correspond to the spontaneous violation of the high-dimensional translation invariance. For the unit vacuum direction tensors chosen accordingly as \( n^i_\mu = \delta^i_\mu \) and \( n^i_\mu = \delta^i_\mu \) one therefore has

\[
\delta^i_\mu e^i_\mu = 0 , \delta^i_\mu e^i_\mu = 0 (\delta^i_\mu \delta^i_\mu = \delta^i_\mu \delta^i_\mu = \delta^i_\mu \delta^i_\mu = 4).
\]

At the same time, the vector Goldstone fields \( e^i_\mu \) and \( e^i_\mu \) fields are turn out to be the gauge fields of local translations, as directly follows from the tetrad transformation law in \( (48) \). Meanwhile the second (diagonal) term in the parametrization \( (60) \) represents the effective Higgs mode

\[
\mathfrak{h} = \sqrt{M^2 - \epsilon^2} = M_e - \frac{\epsilon^2}{2M_e} + \cdots
\]

Note that with this ”mixed” Kronecker symbols \( \delta^i_\mu \) and \( \delta^i_\mu \) one also has some new orthogonality equation

\[
e^i_\mu e^i_\mu = \delta^i_\mu e^i_\mu = \delta^i_\mu e^\mu_i = M_e^2 \delta^i_\mu
\]

provided that the standard orthogonality conditions \( (33) \) work\(^{12}\). Substituting the parametrizations \( (60) \)

\(^{12}\) Actually, there are more orthogonality equations for all possible tetrad components

\[
e^i_\mu e^i_\mu = \delta^i_\mu e^i_\mu = \delta^i_\mu e^i_\mu = \delta^i_\mu e^\mu_i = M_e^2 \delta^i_\mu, \quad e^i_\mu e^i_\mu = \delta^i_\mu e^i_\mu = \delta^i_\mu e^\mu_i = M_e^2 \delta^i_\mu
\]

in addition to the standard equations \( (33) \).
into all them one can readily receive the constraints put on the Goldstone fields $e^i_\mu$ and $e^\mu_i$

$$
\begin{align*}
\xi^\mu_i e^i_\nu - \delta^\mu_i \xi^2 + (\delta^\mu_i e^i_\nu + \delta^\nu_i e^\mu_i) h &= 0, \\
\xi^\mu_i e^\mu_j - \delta^\mu_i \xi^2 + (\delta^\mu_i e^\mu_j + \delta^\nu_j e^\nu_i) h &= 0, \\
\xi^\mu_i e^\mu_j - \delta^\mu_i \xi^2 + 2 \xi^\mu_i h &= 0.
\end{align*}
$$

(64)

For a general metric tensor $g_{\mu\nu}(x)$ which corresponds to the tetrad $e^i_\mu$ one consequently has from a conventional metric definition (37) and equations (60)

$$
g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_2^2} [h(\delta^\mu_i e^\mu_j + \delta^\nu_j e^\nu_i) + e^\mu_i e_{\mu\nu} - \eta_{\mu\nu} \xi^2]
$$

(65)

where $\eta_{\mu\nu}$ stands for a flat metric $\eta_{ij} = \delta^i_j$ in the world space and, therefore, the second term in (65) represents a deviation from the flat metric. As is readily seen from (65), the vacuum in the PGG theory is a largely flat Minkowski spacetime that allows to treat gravity as a generically spontaneously broken theory. Though this point was discussed in many different contexts (see, for example, the review [4]), it looks the most transparent just in the emergent PGG framework. Indeed, one can readily see that the above-mentioned deviation from a flat metric is naturally small once the symmetry breaking scale $M_2$ related to the tetrad field $e^i_\mu$ is associated with the Planck mass scale $M_P$. Respectively, an inverse metric tensor $g^{\mu\nu}(x)$ corresponding to the tetrad $e^\mu_i$ has a similar form with an extremely small deviation from a flat metric $\eta^{ij} = \delta^i_j$ given as in (65) by an appropriate Goldstone tetrad field combinations. Indeed, due the constraints (64), a conventional relationship between general metrics, $g_{\mu\nu} g^{\nu\rho} = \delta^\rho_\mu$, is automatically satisfied.

IV. EMERGENT EINSTEIN-CARTAN THEORY AND BEYOND

A. Non-propagating tetrads and spin-connections

We start with the minimal theory part $L^{(1)}$ in the basic emergent Lagrangian [23]. Without kinetic terms, the tetrad and spin-connection Goldstone modes in this minimal Lagrangian are not propagating physical fields, though their variations may lead to some non-trivial constraint equations. We will see below that, varying this Lagrangian under Goldstone tetrad modes $e^i_\mu$, one comes to the Einstein-Cartan equation, while variation under Goldstone spin-connection modes $A^i_\mu$ may reveal some spin-spin gravitational interaction trace in this equation.

Let us note first that for a variation of tetrad fields and their determinant we have now taking into account that tetrads are dimensionful fields,

$$
\delta e^\mu_i = -e^\nu_j e^\mu_i \delta e^j_\nu / M_e^2, \ \delta e = e e^\mu_i \delta e^\mu_i / M_e^2.
$$

(66)

Multiplying the both sides of the first equation by $\delta^\mu_i$ and using the tetrad orthogonality condition in its right side one has

$$
\delta(\delta^\mu_i e^\mu_i) = -\delta(\delta^\mu_i e^\mu_i)
$$

(67)

that due to the Goldstone condition (51) for the $e^\mu_i$ and $e^i_\mu$ modes, respectively, eventually gives

$$
\delta h = 0, \ \delta e^\mu_i = \delta e^i_\mu.
$$

(68)

Thus, the effective Higgs field $h$ does not vary and a total variation of the starting tetrad fields $e^i_\mu (e^\mu_i)$ amounts to the variation of the pure Goldstone modes $e^i_\mu (e^\mu_i)$. In terms of these modes the variation equations (66) acquire the simple forms

$$
\delta e^\mu_i = \delta e^i_\mu = -e^\nu_j e^\mu_i \delta e^j_\nu / M_e^2, \ \delta e = e e^\mu_i \delta e^\mu_i / M_e^2.
$$

(69)

This in turn means that the variation of the minimal Lagrangian $L^{(1)}$ under the Goldstone tetrad fields $e^i_\mu$ will lead to the same equations of motion as the variation under the total tetrad fields $e^\mu_i$. 
In contrast to tetrads, there is no the similar orthogonality conditions for spin-connection fields $A^i_{\mu j}$. As a result, not only its Goldstone mode $A^i_{\mu j}$ but also its effective Higgs mode $H$ will vary

$$
\delta A^i_{\mu j} = \delta A^i_{\mu j} + n^i_{\mu j} \delta H
$$
$$
= \delta A^i_{\mu j} - n^i_{\mu j} n^2 \delta A^j_{\mu i} - \frac{\delta A^2_{\mu i}}{2M_A}
$$

(70)

that, therefore, might lead to the corrections of the order $O(A^2/M_A^2)$ to the spin-connection constraint equation along the vacuum direction given by the unit tensor $n^i_{\mu j}$. However, as we show below, all these corrections are unavoidably cancelled in the final Einstein-Cartan equation.

With these preliminary comments, let us now rewrite the minimal PGG theory $L^{(1)}$ in the symmetry broken phase. Indeed, substituting the spin-connection field parameterization, one is led to the Einstein-Cartan theory expressed in terms of the pure emergent modes $A^i_{\mu j}$. At the same time, one can still keep the total tetrad field $e^i_{\mu j}$ in the theory (being properly dimensioned by mass scale $M_e$) rather than its Goldstone modes $\epsilon^i_{\mu j}$ since, as was mentioned above, they both lead to the same equations of motion in the minimal theory. However, as in the Yang-Mills theory case considered in Section II B, one should first use the local invariance of the emergent Lagrangian $L^{em}$ to gauge away the apparently large but fictitious Lorentz violating terms (being proportional to the scale $M_A$) which appear in the symmetry broken phase. As one can readily see, they stem from the effective Higgs field $H$ expansion when it is applied to some spin-connection field couplings following from the corresponding covariant derivatives in the Lagrangian $L^{em}$. To exclude them we can make some appropriate Lorentz rotations of all the fields involved, namely, spin-connection fields

$$
A^i_{\mu j} \rightarrow A^i_{\mu j} + \epsilon^i_{k \mu} A^k_{\mu j} + \epsilon^j_{k \mu} A^i_{\mu k}
$$

(71)
tetrads

$$
e^i_{\mu j} \rightarrow e^i_{\mu j} - \epsilon^i_{k \mu} e^k_{\mu j}
$$

(72)

and matter fermions

$$
\psi \rightarrow \left(1 + \frac{1}{4} \epsilon^{ij} \gamma_{ij}\right) \psi
$$

(73)

with a phase $\epsilon^{ij}(x)$ being linear function in the 4-coordinate, $\epsilon^{ij} = -(n^i_{\mu j} x^\mu) M_A$. These transformations lead to an exact cancellation of the large constant term in the effective Higgs field $H$ expansion so that the transformed Lagrangian appears to contain everywhere just the combination $H - M_A$ as an effective Higgs field. Thus, the emergent Einstein-Cartan theory following from the minimal Lagrangian in the symmetry broken phase takes the form (we retain the same notations for fields)

$$
e^{-1} L^{em}_{ECC} = 1 \frac{e^\mu e^\nu}{2M_e^2} \left[R^{ij}_{\mu \nu} + R^{ij}_{\mu \nu} (H - M_A) \right] + \frac{1}{2} \delta(n^\mu A^j_{\mu i})^2
$$
$$
+ \frac{e^i_{\mu j}}{2M_e^2} \left\{ \psi \gamma^i (\overleftrightarrow{\partial_{\mu}} \psi) + \frac{1}{4} n^a_{\mu j} (H - M_A) \bar{\psi} \gamma^i [\gamma^a, \gamma_{ab}] \psi \right\}
$$

(74)

where $R^{ij}_{\mu \nu}$ is the stress tensor of emergent spin-connection modes $A^i_{\mu j}$

$$
R^{ij}_{\mu \nu} = \partial_{\nu} A^i_{\mu j} - \partial_{\mu} A^j_{\nu i} + n_{k i} (A^i_{\nu k} A^j_{\mu i} - A^i_{\nu k} A^j_{\mu k})
$$

(75)

\textit{As in the non-Abelian theory case discussed in the footnote, the vacuum shift of a spin-connection field multiplet $A^i_{\mu j}$ given in being accompanied by a subsequent "rotation" of an emergent multiplet $A^{ij}_{\mu}$ can be written entirely as}

$$
A^{ij}_{\mu} = \epsilon^i_{k} A^k_{\mu i} + \epsilon^j_{k} A^k_{\mu j} - \partial_{\mu} \epsilon^{ij} + n_{ij}^i (H - M_A).
$$

with the transformation phase linear in the 4-coordinate. As can be readily seen, the first two terms here present a pure gauge transformation of the spin-connection field multiplet $A^i_{\mu j}$. Therefore, only the third term can not be gauged away. Consequently, the transformed Lagrangian contains just the combination $H - M_A$, as its effective Higgs mode.
while \( \mathcal{R}^{ij}_{\mu
u} \) stands for the new SLIV oriented tensor of the type
\[
\mathcal{R}^{ij}_{\mu
u} = n^{ij}_{\nu} \partial^\nu - n^{ij}_{\nu} \partial^\mu + n_{kl} \left[ (n^{ik}_{\nu} A^{ij}_{\nu} + n^{lj}_{\nu} A^{ik}_{\nu}) - (n^{ik}_{\mu} A^{lj}_{\nu} + n^{lj}_{\mu} A^{ik}_{\nu}) \right]
\] (76)
acting on the effective Higgs field expansion terms in (74). The "standard" Lorentz covariant derivative \( \overrightarrow{D}_\mu \) for fermion \( \psi \), though written in terms of the emergent \( A \) fields, is defined exactly as in (50). We have also introduced a general axial gauge fixing term for the entire \( A_{ij}^{\mu} \) multiplet to remove all scalar modes from the theory\(^{14}\). After variation of the Lagrangian (74) under tetrad field one comes to some extended equation of motion that can be written in the form
\[
R^{\rho\sigma} - g^{\rho\sigma} R / 2 + \kappa \partial^{\rho\sigma}
\]
(77)
when going from local to general frame. Here the left side presents the standard Einstein-Cartan equation terms including the energy-momentum tensor
\[
\partial^{\rho\sigma} = \frac{1}{2M_e} (g^{\rho\sigma} e_i^\mu - g^{\mu\sigma} e_i^\rho) \bar{\psi} \gamma^i \overrightarrow{D}_\mu \psi
\] (78)
expressed, however, in terms of the emergent \( A_{ij}^{\mu} \) modes, whereas the right side corresponds to the Lorentz breaking background terms newly appeared. The \( R \) and \( \mathcal{R} \) tensors are defined as usual
\[
(\mathcal{R}, \mathcal{R})^{\rho\sigma} = (R, \mathcal{R})^{ij}_{\mu\nu} e_i^\sigma e_j^\rho / M_e^2, \quad (R, \mathcal{R}) = (R, \mathcal{R})^{ij}_{\mu\nu} e_i^\mu e_j^\nu / M_e^2 .
\] (79)

The theory is not yet fully determined until the constraint equations for the spin connection modes \( A_{ij}^{\mu} \) are found. Varying the Lagrangian (74) w.r.t. these modes one has such equations in the symmetry broken phase
\[
D_\mu \left( e_{a}^{[\mu} e_{b}^{\nu]} \right) + A_{ab} D_\nu \left( n^{ij}_{\mu} e_i^{[\mu} e_j^{\nu]} \right) + (\mathcal{H} - M_A) \left[ n^{ij}_{\mu a} e_i^{[\mu} e_j^{\nu]} + n^{ij}_{\nu b} e_i^{[\mu} e_j^{\nu]} \right]
\]
(80)
where the first covariant derivative term in (80)
\[
(D_\mu - e^{-1} \partial_\mu) \left( e_{a}^{[\mu} e_{b}^{\nu]} \right) = A_{ab} e_{a}^{[\mu} e_{b}^{\nu]} + A_{ba} e_{b}^{[\mu} e_{a}^{\nu]}
\] (81)
is indeed the standard one, while the second one
\[
(D_\nu - e^{-1} \partial_\nu) \left( n^{ij}_{\mu} e_i^{[\mu} e_j^{\nu]} \right) = \frac{n^{ij}_{\nu}}{2H} \eta_{kl} (n^{ik}_{\mu} A_{\nu}^{lj} + n^{lj}_{\mu} A_{\nu}^{ik}) e_i^{[\mu} e_j^{\nu]}
\] (82)
is related to spontaneous Lorentz violation and disappears when its scale \( M_A \) goes to infinity. Expanding the effective Higgs field \( \mathcal{H} \) (57) in (74), one comes to the highly nonlinear theory in terms of the zero spin-connection modes \( A_{ij}^{\mu} \) which contains some properly suppressed Lorentz violating couplings. The point is, however, that all these terms are precisely cancelled in the basic equation of motion (77) once the constraint equations (80) are used. Thus, one eventually is led to the standard Einstein-Cartan equation terms given solely by the left side of the equation (77).

Let us show first how this cancellation works for the largest extra terms in the equation (77). These terms correspond to the case when the tetrad fields take the constant background value, \( e_i^\mu = \delta_i^\mu M_e \) (\( e = 1 \)). In this approach, which allows to omit all the tetrad derivative terms in the constraint equations

---

\(^{14}\) Note that we have omitted the higher order term \( n_{kl}(n^{ij}_{\nu} n^{ij}_{\mu} - n^{ik}_{\nu} n^{lj}_{\mu})(\mathcal{H} - M_A)^2 \) in the Lagrangian (74) since it automatically vanishes. Indeed, the background unit tensor \( n^{ij}_{\mu} \) may always be chosen to be nonzero for only one world space component \( \mu \).
and also leaving in them only terms linear in spin-connection modes $A^i_{\mu}$, one comes to the "zero-order" constraint equations

$$A^i_{\mu b}\delta^i_{\mu a}\phi_a + A^i_{\mu a}\delta^i_{\mu b} = -\frac{\kappa}{4} \left( \delta_{\mu b} \bar{\psi} \{ \gamma_{\mu}^k, \gamma_{\mu k} \} \psi - n^2 \frac{A_{c b}}{2M_A} \delta_{\mu b} \bar{\psi} \{ \gamma_{\mu}^k, \gamma_{\mu i j} \} \psi \right). \tag{83}$$

They consequently give the following solution for spin-connection fields expressed in pure local frame Lorentzian components

$$A_{abc} = -\frac{\kappa}{4} \bar{\psi} \gamma_{abc} \psi \left( 1 + \frac{\kappa}{8M_A} n^2 \gamma_{ijk} \bar{\psi} \gamma_{ijk} \psi \right) \tag{84}$$

where the combination of the $\gamma$-matrices $\gamma_{ijk}$ and the matrix $\gamma_{ijk}$ are defined according to the following (anti)symmetrization of indices

$$\gamma_{ijk} \equiv \frac{1}{2} \left( \gamma_{i[jk]} - \gamma_{k[ij]} + \gamma_{j[ki]} + \eta_{ik} \eta_{j}^{ab} \gamma_{a[k]} + \eta_{jk} \eta_{i}^{ab} \gamma_{b[i]} \right), \quad \gamma_{k[ij]} \equiv \{ \gamma_{k}, \gamma_{ij} \}. \tag{85}$$

Note that the "zero-order" solution (84) holds in fact for the contortion tensor $K_{\mu ab}$ part in the total spin-connection field $A_{abc} = A^0_{\mu abc} + K_{\mu abc}$ since an ordinary part $A^0_{\mu abc}$ vanishes in the absence of the fermion source. Putting this solution into the equation of motion (77) taken in the same approximation (the background value for tetrads, no tetrad derivative terms, no terms higher than linear in $A^i_{\mu}$) one receives for the right side the vanishing sum of contributions

$$\pm \frac{e \kappa^2}{128M_A} \hat{n}^{ijk} \left( \bar{\psi} \gamma_{ijk} \psi \right) \left( \bar{\psi} \gamma_{abc} \psi \right) \left( \bar{\psi} \gamma_{abc} \psi \right). \tag{86}$$

stemming from its first and second terms, respectively. Thereby, one unavoidably comes to the standard Einstein-Cartan equation in (77). Though these six-fermion contributions are much smaller even regarding to the standard four-fermion interaction in the Einstein-Cartan theory, their cancellation is strictly provided by the Poincaré gauge invariance emerged. Applying these arguments order by order in spin-connection modes $A^i_{\mu}$ one may come to the same conclusion in a general case as well.

**B. Theories with physical tetrad and spin-connection fields**

After the minimal Einstein-Cartan theory, let us now turn to a general PGG Lagrangian (51) containing in its second part $L^{(2)}$ all possible quadratic combinations of the Poincaré torsion and curvature, $T^{i}_{\mu \nu}$ and $R^{ij}_{\mu \nu}$ (10), respectively. Substituting the field parametrizations (50) and (60) into the quadratic Lagrangian part in (51) and expanding the square roots there in powers of $A^2/M_A^2$ and $c^2/M_2^2$ we come, as in the above minimal model, to a highly nonlinear theory in terms of the propagating tetrad and spin-connection emergent Goldstone modes, $A^i_{\mu}$ and $e^i_{\mu}$. Apart from the standard vector field couplings, this theory contains many Lorentz and translation violating couplings stemming from their field strengths $T^{i}_{\mu \nu}$ and $R^{ij}_{\mu \nu}$ in the symmetry broken phase.

For some particular Lagrangian with propagating tetrad and spin-connection fields due to the pure "Yang-Mills type" extension

$$L^{cm} (e, A, \psi) = L^{cm}_{EC} - e \frac{1}{4K_c} T^{i}_{\mu \nu} T^{i}_{\mu \nu} - e \frac{1}{4K_A} R^{ij}_{\mu \nu} R^{ij}_{\mu \nu} \tag{87}$$

one has applying the expressions (56) (57) (60) and properly redefining fields according to the transformations (71) (72) (73)

$$T^{i}_{\mu \nu} = T^{i}_{\mu \nu} + \mathcal{T}^{i}_{\mu \nu}, \quad R^{ij}_{\mu \nu} = R^{ij}_{\mu \nu} + \mathcal{R}^{ij}_{\mu \nu} (\mathcal{H} - M_A). \tag{88}$$

Here $\mathcal{T}^{i}_{\mu \nu}$ is an ordinary tetrad field stress tensor expressed, however, in terms of emergent modes $A^{i}_{\mu}$ and $e^{i}_{\mu}$

$$\mathcal{T}^{i}_{\mu \nu} = \partial_{\nu} e_{\mu}^{i} - \partial_{\mu} e_{\nu}^{i} + \eta_{ki} (A_{\nu}^{i} e_{\mu}^{k} - A_{\mu}^{i} e_{\nu}^{k}), \tag{89}$$
while $T_{\mu\nu}^i$ stands for a new tetrad strength appearing in the symmetry broken phase

$$
T_{\mu\nu}^i = [\delta_{\mu}^i \partial_\nu - \delta_{\nu}^i \partial_\mu + \eta_{kl}(A_{\mu}^{ik} \delta_{\nu}^l - A_{\nu}^{ik} \delta_{\mu}^l)]b + \eta_{kl}[n_{\nu}^{ik} e_{\mu}^l - n_{\mu}^{ik} e_{\nu}^l + (n_{\nu}^{ik} \delta_{\mu}^l - n_{\mu}^{ik} \delta_{\nu}^l)](H - MA).
$$

(90)

The spin-connection stress tensors $R_{\mu\nu}^{ij}$ and $T_{\mu\nu}^i$, have already been given in (75) and (76), respectively.

We see that the starting tetrad fields $e_{\mu}^i$ in the Lagrangian (87) plays in fact the role of the Higgs multiplet for the spin-connection fields $A_{\mu}^{ij}$, due to which a part of them gets mass terms of the type

$$
- \frac{M_e^2}{2k_e}(A_{\mu}^{ij} A_{\nu}^{il} - A_{\mu}^{ij} A_{\nu}^{lk} \delta_{\nu l}^i).
$$

(91)

One can readily confirm that, similar to the conventional vector field theories [33, 34], the couplings related to the spin-connection fields will not lead to physical Lorentz or translation violation effects. They turn out again to be strictly cancelled among themselves in all processes involved, like as the Compton scattering of $A$ boson off the matter fermion, the $A - A$ scattering and others. Actually, their tree level amplitudes are essentially determined by an interrelation between the longitudinal $A$ boson exchange diagrams and the corresponding contact $A$ boson interaction diagrams following from the higher terms in $A^2/M_A^2$ in the Lagrangian (87). These two types of diagrams are always exactly cancelled for any processes taken at least in the tree approximation.

The same can be said about the processes related to tetrad fields. In contrast to the spin-connection couplings which are essentially similar to those in the Yang-Mills theory [33], tetrad couplings are somewhat "hidden". Nevertheless, one can find from the Lagrangians (87, 74) that the lowest dimension ones are given by the operators

$$
- \delta_{\mu}^i (\partial_\mu \bar{e}^j)(\partial_\mu e^2)/2Me, \\
\frac{\xi_{\mu}}{2Me} \bar{\psi} \gamma^j \partial_\mu \psi - \delta_{\mu}^i \frac{e^2}{4M_e^2} \bar{\psi} \gamma^j \partial_\mu \psi,
$$

(92)

where the first one presents the three-linear tetrad field coupling, while the other two are related to the tetrad-fermion interactions. Again, considering the scattering of tetrad $e$ field off the fermion $\psi$, one readily finds that their tree level amplitudes vanish being provided by a mutual cancellation between the longitudinal $e$-boson exchange diagram which is determined by the first $(e^3)$ and the second $(e\psi\bar{\psi})$ vertices in (92), and the corresponding contact $e$-boson interaction diagram $(e^2 \psi \bar{\psi})$. Likewise, it can be shown that all other symmetry breaking processes appear unobservable.

Most likely, the same conclusion can be also derived for the Lorentz (or translation) violation loop contributions of the spin-connection and tetrad fields. However, the considered quadratic Lagrangian (87) contains ghosts and, therefore, should necessarily be complemented by some other terms to exclude them from the theory. Fortunately, there exist the several examples of the unitary PGG theories being free from ghosts and tachyons [36, 37] where the above-mentioned loop calculations may appear sensible. They include the theories with both torsion and curvature ($R + R^2 + T^2$), as well as the theories with only torsion-squared ($R + T^2$) or curvature-squared ($R + R^2$) terms.

Some of these unitary PGG theories could be also used for an unification with the Standard model. There is a point which can help to choose the right PGG candidate. Actually, one may propose that the spin-connection fields $A_{\mu}^{ij}$ could be unified with ordinary SM gauge fields in a framework of some non-compact local symmetry group thus leading to a hyperunification of all gauge forces presented in the local Lorentz frame. As to tetrads $e_{\mu}^i$, however, they transform like as ordinary matter fields belonging to the fundamental vector multiplet of $SO(1,3)_{LF}$ rather than to its adjoint representation. Note that the ordinary gauge theories do not contain the objects like the tetrads, namely, the fundamental vector field multiplets. In this sense, one may only expect a partial unification of PGG with SM unifying only the spin-connection fields with the SM gauge bosons.

Remarkably, there is such an example of the unitary theory containing only the curvature-squared terms [36] that can be written in our notations as

$$
\mathcal{L}^{cm}(A, \psi) = \mathcal{L}^{cm}_{EC} - \frac{e}{4K_A} R_{ijkl}(R^{ijkl} + R^{klij} - 4R^{ikjl}),
$$

(93)

where the curvature tensors (73) in the second term are properly contracted with tetrads, $R^{ijkl} = R^{ijkl}_{\mu\nu} e_{\mu}^i e_{\nu}^j$, and (anti)symmetrized. In this theory the tetrads will only give the constraint equations
causing some extra terms to the Einstein-Cartan equation (77), whereas the spin-connection fields $A_{\mu}^{ij}$ become to propagate like the gauge bosons in the Standard Model. Their entire unification seems to be most obvious inside the pseudo-orthogonal $SO(1, N)$ groups in which a direct embedding of the Lorentz group $SO(1, 3)$ can be readily carried out. Requiring then that such unified group has to contain a non-trivial internal symmetry group giving some grand unification theory (GUT) for three other forces, we come to the condition $N - 4 = 4k + 2$ ($k = 1, 2, ...$) selecting the $SO(N - 4)$ GUTs which have complex representations. So, the minimal possible symmetry group for a hyperunification of all forces appears to be the $SO(1, 13)$ which then spontaneously breaks at some Planck mass order scale into $SO(1, 3) \times SO(10)$ so as to naturally lead to PGG, on the one hand, and $SO(10)$ GUT [39] for quarks and leptons, on the other[15]. However, apart from the $SO(1, N)$ series, some other hyperunification groups are also possible. Particularly, if one keeps an eye on the $SU(N)$ type GUT, then the hyperunification groups may be looked for in the special linear $SL(2N, C)$ groups containing as subgroups the $SL(2, C)$ covering the Lorentz group and some grand unified $SU(N)$ symmetry. Thus, apart from a conventional $SU(5)$ GUT [11] which would stem from the hyperunified $SL(10, C)$, the higher GUTs like as the $SU(8)$ [42, 43] or $SU(11)$ [44] containing all three quark-lepton families could emerge in turn from the hyperunified $SL(16, C)$ or $SL(22, C)$ theories, respectively.

V. CONCLUSION AND OVERLOOK

We have argued that the Poincaré gauge gravity could dynamically appear in a general relativistic framework due to the covariant length-preserving constraints put on some prototype vector fields of spin-connections $A^{ij}_{\mu}(x)$ and tetrads $e^{i}_{\mu}(x)$. Because of these constraints the underlying global Poincaré symmetry in the theory converts into the local one, thus leading to the PGG theory being gauged by these prototype fields. The point is, however, that these gauge fields are turned out to be, at the same time, the vector Goldstone and pseudo-Goldstone modes, $A^{ij}_{\mu}$ and $e^{i}_{\mu}$, manifesting themseves in the symmetry broken phase of the accidental global symmetries accompanying the covariant constraints [35, 38]. As in the other emergent gauge theories, the emergent Poincaré gauge invariance in PGG makes any symmetry violation effects in the theory unobservable. In this connection, the above constraints appear as the covariant gauge conditions being then reduced to the noncovariant gauge choice in the accidental symmetry broken phase. Thus, this global symmetry violation is only manifested as the loss of gauge degrees of freedom of massless vector Goldstone bosons.

Another important point concerns the vector field mediation in PGG at high energies which may appear through the fundamental spin-connection and tetrad fields, $A^{ij}_{\mu}$ and $e^{i}_{\mu}$ (together with the low energy mediation through the effective tensor metric field $g_{\mu\nu}$). This makes the gauge gravity to be closer with other interactions that could lead in principle to its unification with other basic gauge forces provided by the Standard Model. Such hyperunification may presumably be related to a generic analogy between local frame in PGG and internal charge space in conventional quantum field theories, as was mentioned above. Actually, some extended non-compact internal space may contain a local frame spacetime $M_{4}$ as its subspace. This looks quite reverse to the well-known approaches [15, 47] when a part of some extended spacetime appears as an internal space. In contrast, now just geometry may follow from charges rather than charges from geometry[16]. We presented above some possible hyperunifying GUT candidates for such an unification of all four fundamental forces.

One might expect that there would be a potential danger for any hyperunified theory due to the Coleman-Mandula ”no-go” theorem [49] on the impossibility of combining spacetime and internal symmetries. Nonetheless, regarding to the hyperunified theories we consider here, this ”no-go” theorem seems not to be an unavoidable obstacle, as may be seen from the following heuristic arguments. Indeed, the first is that the theorem only works if there is a mass gap in the theory that means difference in energy between the vacuum and the next lowest energy state which is in fact the mass of the lightest particle. However, there is no mass gap in the unified theory in the hyperunification symmetry limit where all fields, both gauge bosons and matter fields, are massless. Apart from the extended gauge invariance in the hyperunified theories, the generic masslessness of all the gauge fields involved (like as photon, gluons

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[15] As we recently learned, such a type of the ”GraviGUT” unification was first discussed, while in the somewhat different context, quite a long ago [10].

[16] An interesting attempt to treat the global Poincaré symmetry as the pure internal one has been made in [45].
and graviton) could also be provided by their Nambu-Goldstone nature being related to the spontaneous breakdown of global spacetime symmetries [22–24]. The second and rather important point seems to be related to the nature of PGG as a theory where a gauge group does not need to be specially linked to the base space manifold. Actually, one may take the space to be either curved [2] or flat [3] being no conditioned by PGG on its own. As usually appears [3, 4], one would have to identify the theory with some space manifold at a later stage. As a result, the local frame Lorentz gauge symmetry rather looks like an internal symmetry in PGG, and as such may then have an unobstructed unification with SM or GUT.

We will return to these interesting issues elsewhere [50].

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[1] C. N. Yang and R.L. Mills, Phys. Rev. 96 (1954) 191.
[2] R. Utiyama, Phys. Rev. 101 (1956) 1597.
[3] T.W. Kibble, J. Math. Phys. 2 (1961) 212.
[4] F. Mansouri and L.N. Chang, Phys. Rev. D 13 (1976) 3192.
[5] K. Hayashi and T. Shirafuji, Prog. Theor. Phys. 64 (1980) 866, 883, 1435, 2222.
[6] E. A. Ivanov and J. Niederle, Phys. Rev. D 25 (1982) 976.
[7] G. Grignani and G. Nardelli, Phys. Rev. D 45 (1992) 2719.
[8] F.W. Hehl, J.D. McCrea, E.W. Mielke and Y. Ne’eman, Phys. Rep. 258 (1995) 1.
[9] D. Ivanenko and G. Sardanashvily, Phys. Rep. 94 (1983) 1.
[10] R.T. Hammond, Rep. Prog. Phys. 65 (2002) 599.
[11] M. Blagojevic, Gravitation and gauge symmetries, IoP Publishing, Bristol, 2002.
[12] S. A. Ali, C. Cafaro, S. Capozziello, C. Corda, Int. J. Theor. Phys. 48 (2009) 3426.
[13] M. Blagojevic and F. W. Hehl, Gauge Theories of Gravitation – a reader with commentaries, Imperial College Press, London, 2013.
[14] J.L. Chkareuli, C.D. Froggatt, H.B. Nielsen, Nucl. Phys. B 848 (2011) 498; arXiv:1102.5440 [hep-th].
[15] J.L. Chkareuli, Phys. Rev. D 90 (2014) 065015, arXiv: 1305.6898 [hep-ph].
[16] S. Weinberg, The Quantum Theory of Fields, v.2, Cambridge University Press, 2000.
[17] Y. Nambu and G. Jona-Lasinio, Phys. Rev. D 25 (1982) 976.
[18] T. Eguchi, Phys.Rev. D 14 (1976) 2755.
[19] M. Suzuki, Phys. Rev. D 37 (1988) 210.
[20] J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, Phys. Rev. Lett. 87 (2001) 091601, hep-ph/0106030.
[21] J.L. Chkareuli and I. Darius, Phys. Rev. Lett. 101 (2008) 261801, arXiv: 0802.1950 [hep-th].
[22] J.L. Chkareuli and H.B. Nielsen, Phys. Rev. D 90 (2014) 065015, arXiv: 1305.6898 [hep-ph].
[36] D.E. Neville, Phys. Rev. D 18 (1978) 3535.
[37] E. Sezgin and P. van Nieuwenhuizen, Phys. Rev. D 21 (1980) 3269.
[38] I. Low and A.V. Manohar, Phys. Rev. Lett. 88 (2002) 101602.
[39] H. Fritzsch and P. Minkowski, Annals Phys. 93 (1975) 193.
[40] R. Percacci, Phys. Lett. B 144 37 (1984), Nucl. Phys. B 353 271 (1991).
[41] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
[42] J.L. Chkareuli, JETP Lett. 32 (1980) 671; Pisma Zh. Eksp. Teor. Fiz. 32 (1980) 684.
[43] C.W. Kim and C. Roiesnel, Phys. Lett. B 93 (1980) 343.
[44] H. Georgi, Nucl. Phys. B 156 (1979) 126.
[45] C. J. Isham, Abdus Salam and J. Strathdee, Phys. Rev. D 8 (1973) 2600; J.C. Huang and P.W. Dennis, Phys. Rev. D 24 (1981) 3125; A. H. Chamseddine, Phys. Rev. D 70 (2004) 084006.
[46] Y. M. Cho, J. Math. Phys. 16 (1975) 2029, Phys. Rev. D 14 (1976) 3335; Y. M. Cho and P. G. O. Freund, Phys. Rev. D 12 (1975) 1711.
[47] L. Smolin, Phys. Rev. D 80 (2009) 124017; A. G. Lisi, L. Smolin and S. Speziale, J. Phys. A 43 (2010) 445401.
[48] C. Wiesendanger, Class. Quant. Grav. 13 (1996) 681.
[49] S. Coleman and J. Mandula, Phys. Rev. 159 (1967) 1251.
[50] J.L. Chkareuli, Poincaré gauge gravity: towards a unified view on all elementary forces (in preparation).