Observation of photon-phonon correlations via dissipative filtering

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Cavity-optomechanics enables photon-phonon interaction and correlations by harnessing the radiation-pressure force. Here, we realize a “cavity-in-a-membrane” optomechanical architecture which allows detection of the motion of lithographically-defined, ultrathin membranes via an integrated optical cavity. Using a dissipative filtering method, we are able to eliminate the probe light in situ and observe photon-phonon correlations associated with the low-frequency membrane mechanical mode. The developed method is generally applicable for study of low-frequency light scattering processes where conventional frequency-selective filtering is unfeasible.

I. INTRODUCTION

Optomechanical cavities, involving radiation-pressure-coupled electromagnetic fields and mechanical oscillators, enable measurement and control of acoustic excitations, i.e., phonons, using optical means [1]. Recently, with the phonon-counting technique [2, 3], quantum correlations between photons and phonons in optomechanical cavities have been revealed [4], leading to demonstrations of a mechanical quantum memory [5] and optomechanical teleportation [6]. The phonon-counting technique essentially uses a pump-probe approach for coherent conversion between phonons and photons followed by frequency filtering to selectively address the acoustically-scattered photons. Due to the need of mechanical sideband filtering, this method is most effective for optomechanical systems with high-frequency mechanical modes—typically in the microwave band. On the other hand, a growing number of optomechanical systems involving mechanical resonators with ultralow losses have emerged for applications ranging from sensing to quantum transduction [7–11]. These low-loss mechanical resonators, relying on specific dissipation and stress engineering, have a frequency typically in the megahertz range and below. As a result, the phonon counting technique involving frequency-selective optical sideband filtering becomes difficult for these low-frequency optomechanical systems [12].

Here, we develop a tunable dissipative filtering method, which enables attenuation of the transmission of probe photons while simultaneously optimizes the emission of mechanically-scattered photons from a waveguide-coupled optomechanical cavity with low-frequency mechanical modes. In contrast to the frequency-selective filtering, our method based on cavity dissipation engineering separates external probe photons from intra-cavity scattered photons regardless of their frequency. We apply the dissipative filtering method to an optomechanical system with a waveguide-coupled ring resonator embedded in a free-standing membrane and observe photon-phonon correlations by photon counting. Such “cavity-in-a-membrane” architecture enables detection of membrane vibrational modes with an integrated optical cavity by concentrating optical energy within the wavelength distance of the membrane, in contrast to previous methods utilizing free-space Fabry-Perot cavities [13, 14]. Because the membrane is defined during the lithographic fabrication process, ultrathin membranes with a thickness down to several atomic layers can be made, which might enable new optomechanical sensing and transduction modalities. The dissipative filtering method developed here is generally applicable to other types of optomechanical systems as well as study of low-frequency light scattering processes, such as dynamic light scattering by macromolecules [15], where conventional frequency-selective filtering is unfeasible.

II. CAVITY-IN-A-MEMBRANE OPTOMECHANICAL SYSTEM

The “cavity-in-a-membrane” optomechanical system is illustrated in Fig. 1. The photonic circuit consisting of a waveguide-coupled ring resonator is embedded in a suspended membrane. The waveguide is terminated with a photonic crystal mirror at one end and an adiabatic coupler at the other. In this study, the photonic circuit and membrane are made from thin-film InGaP (150 nm) and silicon dioxide (50 nm) by atomic layer deposition, respectively (see Appendix A for fabrication and an image of the fabricated device). Ultrathin membranes down to a few atomic layers can be fabricated by undercut of the silicon dioxide adjacent to the InGaP circuit. The membrane attached to the ring has an average inner and outer width of 8 µm and 5.5 µm, respectively (for rings with a radius less than 8 µm, the interior region of the ring is thus fully released). We performed finite element simula-

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FIG. 1.  

a. A schematic plot of the “cavity-in-a-membrane” optomechanical system. The photonic circuit consists of a ring resonator side-coupled to a waveguide which is terminated with a photonic crystal mirror and an adiabatic coupler. b and c. Simulated electric field ($E_r$) of the TE$_{00}$ optical mode and total displacement of a few membrane and ring mechanical modes of the 5-µm-radius ring (b) and the 40-µm-radius ring (c). The central membrane of the 5-µm-radius ring is fully released while that of the 40-µm-radius ring is partially released with the anchoring positions labeled. d and e. Simulated optomechanical coupling and mechanical quality factor of the 5-µm-radius ring (d) and the 40-µm-radius ring (e). The breathing ring mode is highlighted by the dashed box.

The mechanical spectroscopy was performed at room temperature and ambient environment with the setup illustrated in Fig. 2 (see Appendix D for more details). We describe the result of a device with a $R = 5$ µm ring as an example. The loaded quality factor of the TE$_{00}$ 1550 nm-band resonance of the fabricated ring resonator is $Q = 1.2 \times 10^5$ for the waveguide-ring gap of 250 nm. The intrinsic quality factor of ring resonators with radius ranging from 5 µm to 80 µm is similar, which is about $Q_i \approx 2.5 \times 10^5 \ [16]$. Because the frequency of radiation-pressure coupled mechanical modes is smaller than the optical resonance linewidth, a probe beam blue-detuned from the resonance at the half linewidth is used to drive the optomechanical cavity. An adiabatically tapered fiber-optic coupler is used to transmit light into the device with an efficiency over 70% [16]. The reflected light from the device by the photonic crystal mirror is sent to a high-speed photodetector and the noise power spectrum of the photocurrent is measured. Fig. 2b and c show the optically-transduced mechanical spectrum, where both the flexural membrane modes and breathing ring modes are observed. There are multiple modes near the breathing ring mode, which might be due to structural mode splitting or the hybridization of the ring mode and higher-order membrane modes. The linewidth of a membrane mode $b_1$ at 4.5 MHz and a ring mode $b_2$ at 156.0 MHz (highlighted in 2b and c) is found to be 0.15 MHz and 3.8 MHz, respectively. The dissipation of the mechanical modes is primarily due to gas damping [17]. When varying the pump power, the peak noise power of $b_1$ and $b_2$ modes changes linearly with the cavity photon number $n_c$ expectedly (Fig. 2d). The optomechanical coupling of $b_1$ and $b_2$ modes is found to be $g/2\pi = 1.7$ kHz and $g/2\pi = 6.4$ kHz, respectively (Appendix D). The difference from the simulation might be due to mode distortion in the presence of the waveguide and the actual
The rate of the cavity mode thus is approximately given by

\[ \kappa = \frac{2\pi}{2} (1 + \sin 2\varphi) \]

where \( \varphi \) is the waveguide propagation phase between the ring and the photonic crystal mirror. As a result, by changing \( \varphi \), the cavity external loss rate can be continuously tuned to approach the critical coupling condition, i.e., \( \kappa_c = \kappa_i \), when the on-resonance reflection of the external probe from the one-port cavity-waveguide system vanishes. On the other hand, any light generated inside the cavity due to, for example, here mechanical vibration induced Stokes and anti-Stokes scattering is immune to such dissipative cancellation, regardless of the frequency of the scattered light. The output power of the on-resonance scattered light, proportional to \( \kappa_e^2/\kappa^4 \) (for \( \omega_m \ll \kappa \)), is in fact maximized under the critical coupling condition. More rigorous modeling of the dissipative filtering is provided in Appendix C.

In this study, tuning of the waveguide phase is realized using a tapered optical fiber (taper angle \( \approx 2^\circ \)) in contrast with the waveguide top. The introduction of the tuning fiber changes the effective mode index \( (n_{\text{eff}}^i) \) of the waveguide and the incremented propagation phase is given by \( \Delta \varphi = 2\pi n_{\text{eff}} L/\lambda_0 \), where \( L \) is the length of the fiber on the waveguide. Thus, \( \varphi \) can be continuously tuned by changing \( L \), which in our experiment is controlled by a motorized stage. For example, according to numerical simulation, for a fiber with 1 \( \mu \)m radius on top of an InGaP waveguide with dimension of 800 nm \( \times \) 115 nm, an incremented \( 2\pi \)-phase shift can be achieved with \( L \approx 37 \mu \)m and a resolution of \( 8.5 \times 10^{-3} \) radians (for 50-\( \mu \)m stage step size). A better resolution can be achieved with a narrower fiber. This method is applied to the \( R = 5 \mu \)m ring device to tune one of the standing-wave resonance close to the critical coupling condition with a minimum reflection coefficient of \( 1.4 \times 10^{-4} \) (Fig. 3b).

The fitting of the reflection spectrum near the critical coupling shows an internal quality factor of \( Q_i = 2.6 \times 10^5 \) (Fig. 3c).

The minimum reflection achieved via the dissipative filtering reveals a quadratic dependence of the residual...
FIG. 3. a. Schematic plots showing the comparison between the frequency-selective filtering (left) and the dissipative filtering via a one-port cavity-waveguide coupled system (right). Yellow light in the right panel is generated in the cavity. See text. b. Normalized optical reflection spectrum for different tuning phases (inset), showing the process of tuning one resonance (left) to be critically-coupled (red curve). c. Model fitting of the critically coupled resonance spectrum. Inset shows the reflection near the resonance dip measured by the SNSPD. d. The minimum reflection versus $R^{-1}$. The blue curve is a quadratic fitting of the measured data. The red and yellow curves are the $R^{-1}$ and $R^{-2}$ components corresponding to the mechanically-scattered light and the thermorefractive noise, respectively.

IV. PHOTON-PHONON CORRELATIONS

The dissipative attenuation of the probe beam allows the measurement of the correlations of the photons scattered by the low-frequency mechanical modes as well as the phonon statistics. Under a parametric pump, the optomechanical cavity with a low-frequency mechanical mode ($\omega_m \ll \kappa$) is described by a linearized Hamiltonian $\hat{H}_{\text{OM}} = G(a + a^\dagger)(b + b^\dagger)$, where $a(b)$ is the annihilation operator of the optical/mechanical mode, $G = g\sqrt{n_c}$ and $n_c$ is the cavity photon number. The output light from the cavity contains the mechanically-scattered photons, which inherit the statistical property of the phonons $\mathbb{B}$, and the residual pump light, i.e., $a_{\text{out}}(t) = sa_{\text{in}}(t) + ub_{\text{in}}(t) + v\hat{b} \ket_{\text{in}}(t)$, where $s$, $u$ and $v$ are the coefficients of direct reflection, anti-Stokes and Stokes scattering, respectively. As a result, the self-correlation of the output field contains the cross-correlation between photons and phonons under a continuous pump. For a thermal mechanical state, the normalized second-order correlation function of the output light is given by (Appendix E)

$$
g^{(2)}(\tau) = 1 + \frac{(|u|^2 + |v|^2)^2 I_b^2 |g_b^{(1)}(\tau)|^2 + 2|s|^2(|u|^2 + |v|^2)I_a |g_b^{(1)}(\tau)| \cos(\omega_m \tau) + 2|uv|^2 I_b^2 |g_b^{(1)}(\tau)|^2 \cos(2\omega_m \tau)}{(|s|^2 I_a + (|u|^2 + |v|^2)I_b)^2}, \quad (1)$$
FIG. 4. a. \(g^{(2)}(\tau)\) of a \(R = 5\) \(\mu\)m ring. b. Fourier transformation (FT) of \(g^{(2)}(\tau)\) and noise power spectrum of the 5 \(\mu\)m ring. The curves are offset for clarity. c. \(g^{(2)}(\tau)\) of a \(R = 40\) \(\mu\)m ring. d. Fourier transformation of \(g^{(2)}(\tau)\) and noise power spectrum of the 40 \(\mu\)m ring. e and f. Model fitting (red) of the \(g^{(2)}(\tau)\) of the 5-\(\mu\)m ring (e) and the 40-\(\mu\)m ring (f).

where \(|g^{(1)}_m(\tau)| = e^{−\gamma \tau}/\gamma\), \(I_a = P_{in}/\hbar \omega_{p}\) and \(I_b = \gamma \bar{n}_m/4\) (\(\bar{n}_m\) is the average phonon occupation of the mechanical mode and \(\gamma\) is the mechanical dissipation rate) are the input photon and phonon flux, respectively. Because the mechanical modes are in classical states, the correlation function between photons and phonons are factorized. It is seen from Eq. 4.1 that, when \(|s|^2 \approx |u|^2, |v|^2\), the correlations between photons and phonons could be observed.

We measured the correlation of output photons of the \(R = 5\) \(\mu\)m ring device with a cavity photon number \(n_c \approx 2\). The device is tuned to be critically coupled with a dip reflection of \(1.4 \times 10^{-4}\) (Fig. 3a). The probe beam is frequency locked near the dip with a stable reflection coefficient (Appendix E). When the reflection coefficient is sufficiently small, the second-order correlation function of the output light is manifested with \(g^{(2)}(0) > 1\) due to the thermal mechanical state as well as oscillations due to the beating between the mechanically-scattered photons and the residual probe photons. Fig. 4b shows the measured \(g^{(2)}(\tau)\) for reflection coefficient \(|s|^2 = 2.9 \times 10^{-4}\). For this measurement, since the residual probe light is at a level higher than the amount of scattered light, the dominant term in Eq. 4.1 is \(g^{(1)}_m(\tau) \cos(\omega_m \tau)\), whose Fourier transform yields the mechanical spectrum [20]. This is shown in Fig. 4b, where the Fourier transform of \(g^{(2)}(\tau)\) and the noise power spectrum measured by the photodetector are almost identical. The low-frequency rising slope in the noise power spectrum is due to the thermorefractive noise [18, 19]. Fig. 4c and d show the measured photon correlation (resonance dip of \(1.8 \times 10^{-5}\) and locked reflection level of \(6.9 \times 10^{-5}\)) and the corresponding mechanical spectrum of a \(R = 40\) \(\mu\)m ring, respectively. The measured photon correlation, containing multi-frequency oscillations due to multiple optically-coupled mechanical modes, can be fitted using a multimode generalization of Eq. 4.1 including the thermorefractive noise (Appendix E), as shown in Fig. 4e and f for the \(R = 5\) \(\mu\)m and \(R = 40\) \(\mu\)m ring, respectively. From the fitting, the amount of photons scattered by each mechanical mode as well as the thermorefractive noise can be inferred.

V. DISCUSSION

The “cavity-in-a-membrane” optomechanical architecture enables detection of mechanical motion of ultrathin membranes defined via lithographic fabrication, which might lead to new modalities of sensing and signal transduction. Further studies of the acoustic loss of atomically-thin membranes at low temperature and in the vacuum could be relevant to phonon dynamics in low-dimensional materials. Despite the observed photon correlation being classical, same approach can be applied to probe quantum correlations between photons and phonons when the mechanical oscillator is prepared in a quantum state. The dissipative filtering method, facilitating the detection of scattered light by mechanical oscillators with frequencies in the MHz range and beyond, will also be useful for study of other low-frequency light scattering processes where optical frequency-selective filtering is difficult.

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Appendix A: Device fabrication

The devices are fabricated from 115 nm thick (measured by scanning electron microscopy) disordered InGaP films grown on GaAs substrate (2 degree off-cut toward the [110]) by metal-organic chemical vapor deposition (T = 545°C, V/III = 48, precursors: trimethylindium, trimethylgallium and PH3). The device pattern is defined using 150 keV electron beam lithography and 150 nm negative tone resist hydrogensilesquioxane (HSQ). A 20 nm thick layer of silicon dioxide is deposited on InGaP via plasma-enhanced chemical vapor deposition (PECVD) to enhance the adhesion of HSQ. The pattern is transferred to InGaP via inductively coupled plasma reactive-ion etch (ICP-RIE) using Cl2/CH4/Ar gas mixture with a selectivity of InGaP: HSQ: PECVD SiO2 = 240: 90: 80. After a short buffered oxide etch to remove the residual oxide (both HSQ and PECVD oxide), a layer of 50 nm thick silicon dioxide is deposited on the chip via atomic layer deposition. A second electron beam lithography and subsequent ICP-RIE using CHF3 gas are applied to pattern etch-through holes in the silicon dioxide layer for undercut of the InGaP device. The etch-through holes are 200 nm in radius and separated from each other by 2 µm. Finally, the InGaP device is released from the GaAs substrate using citric acid-based selective etching [21], leading to a 11 µm wide membrane along the waveguide oriented along the [011] and [011] direction. Because the wet etching of GaAs is crystal surface selective, the membrane is anisotropic along the ring with a varying width from 11 to 16 µm (see Fig. 5). The suspended InGaP device is mechanically anchored to the silicon dioxide membrane. See Ref. [16] for more details.

Appendix B: Device measurement

A continuous-wave telecom laser (Newport Velocity, TLB-6700) is used to probe the device. The laser beam is filtered by a narrow-bandwidth optical filter (LUNA, 50 GHz FSR, 400 Finesse) to eliminate side-mode emission and broadband amplified spontaneous emission. The filtered light is coupled into the on-chip device via an adiabatic fiber coupler. The reflected light from the device passes through a polarization controller and a polarizer to filter the unpolarized and/or orthogonal polarization component of the laser light or scattered light in the fiber. The light is then directed either to a high-speed photodetector for measuring the noise power spectral density or a Hanbury-Brown and Twiss setup with two superconducting single-photon detectors (Quantum Opus, 85% efficiency, 100 Hz dark count rate) for measuring the second-order correlation. For the latter, the reflection signal is locked near the resonance dip by controlling the laser frequency in response to the drift of the resonance due to thermo-optic effect and water vapor in the atmosphere.

Appendix C: Modeling of dissipative filtering

For an ideal ring resonator, the clockwise (cw) and counterclockwise (ccw) resonances are degenerate with frequency ω0. In fabricated ring resonators, the clockwise and counterclockwise resonances could couple due to the surface roughness. The input-output equation of the cw and ccw resonances (Fig. 6) is given by

\[ a_{\text{in},1} = a_{\text{out},1} \]

\[ a_{\text{in},2} = a_{\text{out},2} e^{i\phi} \]

FIG. 6. Schematic of the input-output relation of the device. A tapered fiber (blue) is used to tune the waveguide phase \( \varphi \).
\[
\frac{d}{dt} \begin{pmatrix} a_{cw} \\ a_{ccw} \end{pmatrix} = -i \begin{pmatrix} \omega_0 - i \frac{\kappa_{ccw}}{2} & V \\ -V & \omega_0 - i \frac{\kappa_{ccw}}{2} \end{pmatrix} \begin{pmatrix} a_{cw} \\ a_{ccw} \end{pmatrix} + \begin{pmatrix} 0 \\ \sqrt{\kappa_{ccw, e}} \end{pmatrix} a_{in,1} + \begin{pmatrix} \sqrt{\kappa_{cw, e}} e^{i\phi} \end{pmatrix} a_n, \quad (C1)
\]

\[
\begin{pmatrix} a_{out,1} \\ a_{out,2} \end{pmatrix} = \begin{pmatrix} a_{in,1} \\ a_{in,2} \end{pmatrix} - \begin{pmatrix} 0 \\ \sqrt{\kappa_{cw, e}} \end{pmatrix} \begin{pmatrix} a_{cw} \\ a_{ccw} \end{pmatrix}, \quad (C2)
\]

where \( \kappa_{cw(ccw)} = \kappa_{cw(ccw), e} + \kappa_{cw(ccw), i} \).

The photonic crystal mirror with non-unity reflection acts like a beamsplitter, which relates \( a_{out,1} \) and \( a_{in,2} \) by

\[
a_{in,2} = e^{i\phi} (ire^{i\phi}a_{out,1} + ta_n), \quad (C3)
\]

where \( \phi \) is the propagation phase between the ring and the photonic crystal mirror, \( r \) and \( t \) are the real reflection and transmission coefficient of the photonic crystal mirror, and \( a_n \) represents the noise operator.

By eliminating \( a_{out,1} \) and \( a_{in,2} \) from Eqs. (C1)-(C3) we obtain

\[
\frac{d}{dt} \begin{pmatrix} a_{cw} \\ a_{ccw} \end{pmatrix} = -i \begin{pmatrix} \omega_0 - i \frac{\kappa_{ccw}}{2} & V + \sqrt{\kappa_{cw, e}\kappa_{ccw, e}} re^{i2\phi} \\ -V & \omega_0 - i \frac{\kappa_{ccw}}{2} \end{pmatrix} \begin{pmatrix} a_{cw} \\ a_{ccw} \end{pmatrix} + \begin{pmatrix} i\sqrt{\kappa_{cw, e}} re^{i2\phi} \\ \sqrt{\kappa_{ccw, e}} \end{pmatrix} a_{in,1} + \begin{pmatrix} \sqrt{\kappa_{cw, e}} e^{i\phi} \end{pmatrix} a_n, \quad (C4)
\]

\[
a_{out,2} = ire^{i2\phi} a_{in,1} - \sqrt{\kappa_{cw, e}} a_{cw} - i\sqrt{\kappa_{cw, e}} re^{i2\phi} a_{ccw} + te^{i\phi} a_n. \quad (C5)
\]

From now on, we assume the cw and ccw resonances have the same dissipation rate and omit the label. To obtain the reflection coefficient of the device, we solve Eqs. (C4) and (C5) in the frequency domain by ignoring the noise operator, which yields

\[
a_{out,2}[\omega] = i \left[ re^{i2\phi} - \kappa_e \begin{pmatrix} 1 & ire^{i2\phi} \\ re^{i2\phi} & 1 \end{pmatrix} \begin{pmatrix} \omega - \omega_0 + i \frac{\kappa_e}{2} \\ -V \end{pmatrix} re^{i2\phi} \begin{pmatrix} \omega - \omega_0 + i \frac{\kappa_e}{2} \\ -V \end{pmatrix} \right]^{-1} \begin{pmatrix} ire^{i2\phi} \\ 1 \end{pmatrix} a_{in,1}[\omega]. \quad (C6)
\]

The reflection is zero (i.e., \( a_{out,2}[\omega] = 0 \)) when

\[
\left( \cos 2\phi + \frac{r \Delta \kappa^2}{2V \kappa_e} \right)^2 = 1 - \frac{r^2 \Delta \kappa^2}{\kappa_e^2} \quad (C7)
\]

and

\[
\omega = \omega_0 \pm \sqrt{V^2 + \Delta \kappa^2 + V(\kappa_e/r) \cos 2\phi} \quad (C8)
\]

\[
= \omega_0 \pm \sqrt{V^2 + (\Delta \kappa/2)^2 \pm V(\kappa_e/r)^2 - \Delta \kappa^2},
\]

where \( \Delta \kappa = \kappa_i - \kappa_e \).

1. Large splitting limit

In the large splitting limit, i.e., \( V \gg \kappa \), Eq. (C4) can be easily diagonalized, leading to

\[
\frac{d}{dt} \begin{pmatrix} a_e \\ a_o \end{pmatrix} = -i \begin{pmatrix} \omega_e - i \frac{\kappa_e}{2} & \omega_o - i \frac{\kappa_o}{2} \\ -\omega_o - i \frac{\kappa_o}{2} & \omega_e - i \frac{\kappa_e}{2} \end{pmatrix} \begin{pmatrix} a_e \\ a_o \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} i\sqrt{\kappa_e} re^{i2\phi} + \sqrt{\kappa_e} \\ i\sqrt{\kappa_e} re^{i2\phi} - \sqrt{\kappa_e} \end{pmatrix} a_{in,1} + \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\kappa_e} e^{i\phi} \\ \sqrt{\kappa_e} e^{i\phi} \end{pmatrix} a_n, \quad (C9)
\]
The zero-reflection condition at the large splitting limit is

\[
\sin 2\varphi = \pm \frac{r\Delta\kappa}{\kappa_e}
\]  
(C13)

and

\[
\omega = \omega_0 \pm (V + \frac{1}{2} r \kappa_e \cos 2\varphi)
\]  
(C14)

which is consistent with Eqs. C7 and C8. From Eq. C13 it is seen that the zero-reflection condition can be achieved only if \( \kappa_i \leq (1 + \frac{1}{r})\kappa_e \). Note Eq. C13 is not the same as \( \kappa_{e(o),e} = \kappa_{e(o),i} \) for \( r < 1 \) because of the Fano interference.

### Appendix D: Optically transduced mechanical spectrum

Consider an optomechanical cavity with the Hamiltonian

\[
\hat{H} = \omega_e a^\dagger a + \omega_m b^\dagger b + ga^\dagger a(b + b^\dagger),
\]  
(D1)

where \( a^\dagger(a) \) and \( b^\dagger(b) \) are the creation(annihilation) operator of the optical and mechanical modes, respectively. The optical mode is subject to a parametric drive with frequency \( \omega_p \), and thus the optomechanical Hamiltonian can be linearized in the rotating frame of the pump given by

\[
\hat{H} = \Delta a^\dagger a + \omega_m b^\dagger b + G(a + a^\dagger)(b + b^\dagger),
\]  
(D2)

where \( \Delta = \omega_e - \omega_p \), \( G = g\sqrt{\kappa_e} \), \( n_e \) is the cavity photon number, and \( a \) corresponds to the quantum fluctuation of the cavity field. The Heisenberg-Langevin equations of motion for the coupled optical and mechanical modes are given by

\[
\dot{a} = (-i\Delta - \frac{\kappa}{2})a - iG(b + b^\dagger) + \sqrt{\kappa_e}a_{in} + \sqrt{\kappa_i}a_{in,i},
\]  
(D3)

\[
\dot{b} = (-i\omega_m - \frac{\gamma}{2})b - iG(a + a^\dagger) + \sqrt{\gamma}b_{in},
\]  
(D4)

with the input-output relation

\[
a_{out} = a_{in} - \sqrt{\kappa_e}a_i,
\]  
(D5)

where \( a_{in,i} \) is the vacuum noise from the intrinsic loss channel.

By solving the Heisenberg-Langevin equations in the frequency domain, we obtain

\[
a_{out}(\omega) = s[\omega]a_{in}(\omega) + n_{opt}[\omega]a_{in,i}(\omega) + u[\omega]b_{in}(\omega) + v[\omega]b_{in,i}(\omega),
\]  
(D6)

where \( a_{in,i}^\dagger(\omega) = (a_{in}(\omega))^\dagger \),

\[
s[\omega] = \frac{(\kappa_i - \kappa_e)/2 - i(\omega - \Delta)}{\kappa/2 - i(\omega - \Delta)},
\]

\[
n_{opt}[\omega] = \frac{-\sqrt{\kappa_i\kappa_e}}{\kappa/2 - i(\omega - \Delta)},
\]

\[
u[\omega] = \frac{-\sqrt{\kappa_e}}{\kappa/2 - i(\omega - \Delta)\gamma/2 - i(\omega - \omega_m)},
\]

and we have omitted higher order terms \( O(G^2/\kappa^2) \) given the weak coupling condition. Note \( |s[\omega]|^2 + |n_{opt}[\omega]|^2 = 1 \). For the sideband-unresolved case \( \omega_m \ll \kappa \), the cavity is driven with a blue-detuned pump at the half linewidth, i.e., \( \Delta = -\kappa/2 \), and Eq. D7 is simplified to be

\[
s[\omega] \approx \frac{(\kappa_i - \kappa_e)/2 - i\kappa/2}{\kappa/2 - i\kappa/2},
\]

\[
n_{opt}[\omega] \approx \frac{-\sqrt{\kappa_i\kappa_e}}{\kappa/2 - i\kappa/2},
\]

\[
u[\omega] \approx \frac{-\sqrt{\kappa_e}}{\kappa/2 - i\kappa/2\gamma/2 - i(\omega - \omega_m)},
\]

The spectral density of the normalized photocurrent due to the beating between the carrier and mechanical sidebands, \( I(\omega) = a_{out}(\omega) + a_{out}^\dagger(\omega) \), is given by

\[
a_{out,2} = iRe^{i2\varphi}a_{in,1} - \frac{1}{\sqrt{2}}(i\sqrt{\kappa_e}e^{i2\varphi} + \sqrt{\kappa_e})a_e - \frac{1}{\sqrt{2}}(-i\sqrt{\kappa_e}e^{i2\varphi} + \sqrt{\kappa_e})a_o + te^{i\varphi}a_n,
\]  
(C10)
The EDFA gain is thus given by 

\[ G_{\text{EDFA}} = \zeta P_{\text{out}}/P_{\text{in}}. \]

The optomechanical coupling \( g \) is inferred from the mechanical resonance noise peak above the noise floor, given by 

\[ 32\frac{G_{\text{EDFA}}^2 S_N^2}{\eta} \frac{\gamma}{\kappa} \cdot \frac{\bar{n}_m}{\kappa^2} \]  

according to Eq. (D10).

Appendix E: Photon-phonon correlations

According to Eqs. (D6) and (D7), the temporal input-output operator relation for a monochromatic input is given by

\[ a_{\text{out}}(t) = sa_{\text{in}}(t) + ub_{\text{in}}(t) + vb_{\text{in}}^\dagger(t), \]

where

\[ s = \frac{(\kappa_i - \kappa_e)/2 + i\Delta}{\kappa/2 + i\Delta}, \]

\[ u = iG \frac{\sqrt{\kappa_e}}{\kappa/2 + i\Delta} \frac{2}{\sqrt{\gamma}}, \]

\[ v = iG \frac{\sqrt{\kappa_e}}{\kappa/2 + i\Delta} \frac{2}{\sqrt{\gamma}}. \]

Note the \( a \) operator here corresponds to the total optical field and thus is dominated by the coherent pump. We have omitted the vacuum noise of the intrinsic loss channel because the correlation function is normally ordered. Using this, the second-order correlation function of the output photons can be computed as

\[ S_{11}(\omega) = \int d\omega' \langle I(\omega)I(\omega') \rangle \]

\[ = \int d\omega' \langle s^*[-\omega]s[\omega]a_{\text{in}}(\omega')a_{\text{in}}^\dagger(\omega) + s^*[-\omega]s[\omega]a_{\text{in}}^\dagger(\omega')a_{\text{in}}(\omega) \rangle \]

\[ + n_{\text{opt}}^*[-\omega]n_{\text{opt}}[\omega]a_{\text{in}}^\dagger(\omega)a_{\text{in}}(\omega') + n_{\text{opt}}^*[-\omega]n_{\text{opt}}[\omega]a_{\text{in}}(\omega)a_{\text{in}}^\dagger(\omega') \]

\[ + (u^*[-\omega] + v[\omega])(u[\omega'] + v^*[\omega'])b_{\text{in}}^\dagger(\omega)b_{\text{in}}(\omega') + (u^*[-\omega] + v[\omega'])(u[\omega] + v^*[\omega])b_{\text{in}}(\omega)b_{\text{in}}^\dagger(\omega') \]

\[ = |s|^2 + |n_{\text{opt}}[\omega]|^2 + \frac{4\kappa_e|G|^2}{\kappa^2} \frac{\gamma\bar{n}_m}{(\gamma/2)^2 + (\omega + \omega_m)^2} + \frac{\gamma(\bar{n}_m + 1)}{(\gamma/2)^2 + (\omega - \omega_m)^2} \]

\[ = 1 + \frac{8\kappa_e|G|^2}{\kappa^2} \bar{s}_m, \]

where \( \bar{n}_m = \frac{\hbar T}{\hbar \omega_m} \) is the average phonon occupation number and we have used \([a_{\text{in}}(\omega), a_{\text{in}}^\dagger(\omega')] = [a_{\text{in}}^\dagger(\omega), a_{\text{in}}(\omega')] = \delta(\omega + \omega'), \]

\([a_{\text{in}}(\omega), a_{\text{in}}(\omega)] = [a_{\text{in}}^\dagger(\omega), a_{\text{in}}^\dagger(\omega)] = \delta(\omega + \omega'), \]

\([a_{\text{in}}^\dagger(\omega) a_{\text{in}}(\omega)] = (a_{\text{in}}^\dagger(\omega') a_{\text{in}}(\omega)) = 0, \]

\([b_{\text{in}}(\omega') b_{\text{in}}(\omega)] = \bar{n}_m \delta(\omega + \omega'). \]

Thus, the measured noise power spectral density is

\[ S(\omega) = S_e + \frac{G_{\text{EDFA}}^2}{R_I} \left[ S_{\text{EDFA}} + G_{\text{EDFA}}^2 S_N^2 \left( 1 + \frac{8\kappa_e|G|^2}{\kappa^2} \bar{s}_m \right) \right], \]  

where \( S_e \) is the electronic noise of the detector, \( S_{\text{EDFA}} \) is the noise induced by EDFA, \( S_N = \sqrt{2P_{\text{shot}}\hbar \omega_o} \) is the shot noise of the light into the detector, \( G_{\text{EDFA}} = 900 \text{V/W} \) is the conversion gain of the detector, \( R_I = 50 \Omega \) is the impedance of the spectrum analyzer, and \( G_{\text{EDFA}} \) is the EDFA gain. \( \eta = \eta_s \eta_f \eta_d = 0.41 \) is the total efficiency of the setup, including \( \eta_s = 0.70, \eta_f = 0.86, \) and \( \eta_d = 0.68 \) for the fiber-optic coupler efficiency, the fiber transmission efficiency from the chip to the detector, and the detector efficiency, respectively. To characterize the EDFA gain, we measured the input \( (P_{\text{in}}) \) and output \( (P_{\text{out}}) \) power of the EDFA and the ratio \( (\zeta) \) of coherent laser power to amplified spontaneous emission power using an optical spectrum analyzer. The EDFA gain is thus given by

\[ G_{\text{EDFA}} = \zeta P_{\text{out}}/P_{\text{in}}. \]
\[
\langle a_{\text{out}}^\dagger(0)a_{\text{out}}^\dagger(\tau)a_{\text{out}}(\tau)a_{\text{out}}(0) \rangle
\]
\[= |s|^4 \langle a_{\text{in}}^\dagger(0)a_{\text{in}}^\dagger(\tau)a_{\text{in}}(\tau)a_{\text{in}}(0) \rangle + |u|^2 + |v|^2 \langle b_{\text{in}}^\dagger(0)b_{\text{in}}^\dagger(\tau)b_{\text{in}}(\tau)b_{\text{in}}(0) \rangle
\]
\[+ |su|^2 \left( \langle a_{\text{in}}^\dagger(0)a_{\text{in}}(\tau)b_{\text{in}}^\dagger(\tau)b_{\text{in}}(0) \rangle + \langle a_{\text{in}}(\tau)a_{\text{in}}(0)b_{\text{in}}^\dagger(0)b_{\text{in}}(\tau) \rangle \right)
\]
\[+ |sv|^2 \left( \langle a_{\text{in}}^\dagger(0)a_{\text{in}}(\tau)b_{\text{in}}(0)b_{\text{in}}(\tau) \rangle + \langle a_{\text{in}}^\dagger(0)a_{\text{in}}(\tau)b_{\text{in}}(0)b_{\text{in}}^\dagger(\tau) \rangle \right)
\]
\[+ |uw|^2 \left( \langle b_{\text{in}}^\dagger(0)b_{\text{in}}^\dagger(\tau)b_{\text{in}}(\tau)b_{\text{in}}(0) \rangle + \langle b_{\text{in}}^\dagger(\tau)b_{\text{in}}^\dagger(0)b_{\text{in}}(0)b_{\text{in}}(\tau) \rangle \right)
\]
\[+ |sw|^2 \left( \langle a_{\text{in}}^\dagger(0)a_{\text{in}}(\tau)b_{\text{in}}(0)b_{\text{in}}(\tau) \rangle + \langle a_{\text{in}}(\tau)a_{\text{in}}(0)b_{\text{in}}^\dagger(0)b_{\text{in}}(\tau) \rangle \right)
\]
\[= |s|^4 G_a^{(2)}(\tau) + |u|^2 + |v|^2 (G_b^{(1)}(0)^2 + |G_b^{(1)}(\tau)|^2) + 2|s|^2 (|u|^2 + |v|^2) |G_a^{(1)}(\tau)G_b^{(1)}(\tau)| \cos(\omega_m \tau)
\]
\[+ 2 |uv|^2 |G_b^{(1)}(\tau)|^2 \cos(2\omega_m \tau) + 2|s|^2 (|u|^2 + |v|^2) G_a^{(1)}(0) G_b^{(1)}(0),
\]
where we have used Wick’s theorem for calculation of the second-order correlation function of the thermal mechanical state. For thermal mechanical state, \(G_b^{(1)}(\tau) = \gamma \bar{n}_m \tau/2\.

The normalized second-order correlation function thus is
\[
g^{(2)}(\tau) = \frac{\langle a_{\text{out}}^\dagger(0)a_{\text{out}}^\dagger(\tau)a_{\text{out}}(\tau)a_{\text{out}}(0) \rangle}{\langle a_{\text{out}}^\dagger(0)a_{\text{out}}(0) \rangle^2}
\]
\[= 1 + \left[ |u|^2 + |v|^2 \right] 2 \bar{I}_a \bar{I}_b |g_b^{(1)}(\tau)|^2 + 2|s|^2 (|u|^2 + |v|^2) \bar{I}_a \bar{I}_b |g_b^{(1)}(\tau)| \cos(\omega_m \tau) + 2 |uv|^2 \bar{I}_a \bar{I}_b |g_b^{(1)}(\tau)|^2 \cos(2\omega_m \tau),
\]
where \(\bar{I}_a = P_{\text{in}}/\hbar \omega_p\) and \(\bar{I}_b = \gamma \bar{n}_m/4\) are the input photon and phonon flux, respectively. Note \(g^{(2)}(\tau)\) is independent of input optical power, because \(u^2, v^2 \propto G^2 \propto \bar{I}_a\).

\[
g^{(2)}(\tau) = 1 + \left( \sum_i (|u_i|^2 + |v_i|^2) \bar{I}_a \bar{I}_b |g_b^{(1)}(\tau)|^2 + 2|s|^2 (|u_i|^2 + |v_i|^2) \bar{I}_a \bar{I}_b |g_b^{(1)}(\tau)| \cos(\omega_m \tau) + 2 |u_i v_i|^2 \bar{I}_a \bar{I}_b |g_b^{(1)}(\tau)|^2 \cos(2\omega_m \tau)
\]
\[+ \sum_{ij} 2 |u_i v_j^* + u_j v_i^2| \bar{I}_a \bar{I}_b |g_b^{(1)}(\tau) g_b^{(1)}(\tau)^*| \cos(\omega_m - \omega_n) \tau + \cos(\omega_m + \omega_n) \tau \right) \]
\[/ \left( |s|^2 \bar{I}_a + \sum_i (|u_i|^2 + |v_i|^2) \bar{I}_b \right)^2.
\]

\textbf{Appendix F: Thermorefractive noise}

Thermorefractive effect is the fluctuation of refractive index due to the fluctuation of temperature in a finite volume. It will cause jitter of the optical cavity frequency and generation of random photons when the cavity is illuminated. The power spectral density of thermorefractive noise was computed and measured for microsphere and micro-toroid cavities before \cite{18, 19}. It was shown that the thermorefractive cavity frequency jitter, \(\delta \omega_c\),
of the whispering gallery mode is proportional to $1/R$, where $R$ is the cavity radius. The ring optical mode is in good approximation with those whispering gallery modes and we assume $\delta \omega_\nu$ have the same scaling. Under an external pump, the cavity field amplitude is given by $a_\nu = n \sqrt{\frac{\kappa}{\kappa_\nu + \kappa/2}} a_{in}$. The thermorefractive noise amplitude under resonant driving caused by resonance frequency jitter $\delta \omega$ thus is $\delta a_{\nu}[\omega] = i \frac{4 e^{-\kappa R}}{\kappa_\nu} \delta \omega a_{in}$ and the output noise power is $|\delta a_{out}|^2 = \kappa c |\delta a |^2 \propto \frac{1}{R^2}$, given similar $\kappa$ for cavities with different radii.

The second-order correlation function after incorporating the thermorefractive noise is given by

$$g^{(2)}(\tau) = 1 + \frac{\text{photon-phonon} + 2|s|^2 |p|^2 r_s^2 |g_{th}(1)(\tau)|^2 + |p|^4 r_s^2 |g_{th}(1)(\tau)|^2}{\left(\text{photon-phonon} + |p|^2 I_0\right)^2},$$

where $p$ is the effective scattering coefficient of the thermorefractive noise, $|g_{th}(1)(\tau)| = e^{-\tau/\tau_{th}}$ with $n = 1/3$, and photon-phonon terms are given in Eq. (E4). Both $|p|$ and $\tau_{th}$ are fitting parameters for $g^{(2)}(\tau)$. The form of $g_{th}(1)(\tau)$ is inferred from silicon-on-insulator (SOI) ring resonators operated at the similar condition (i.e., critically coupled using dissipative filtering). Because of the structural rigidity, SOI ring resonators are free of low-frequency vibrational modes and the observed $g^{(2)}(\tau) > 1$ at low reflection is purely due to the correlation of the thermorefractive noise. Given the same type of ring resonators, the correlation function of thermorefractive noise in InGaP devices will be the same as the SOI device but with a different $\tau_{th}$ which depends on material parameters and device dimensions.

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