CHECKERBOARD-MASK CORONAGRAPH CORONAGRAPH CORONAGRAPH FOR HIGH-CONTRAST IMAGING

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ABSTRACT

We present yet another new family of masks for high-contrast imaging, as required for the planned Terrestrial Planet Finder space telescope. We call these masks “checkerboard” masks. They consist of two “bar-code” masks, one rotated 90° with respect to the other. Each bar-code mask provides contrast to the 10⁻⁵ level. The checkerboard mask then achieves a 10⁻¹⁰ level of contrast everywhere except along the two axes of symmetry, where the contrast remains at the 10⁻⁵ level. With these designs, we are able to reduce the inner working angle to 2λ/D for each bar code, which translates to 2√2λ/D along the diagonal of the associated checkerboard mask. We show that by combining a Lyot-plane checkerboard mask with an image-plane occulter we can achieve even tighter inner working angles, although as with occulting designs in general pointing error and stellar size become nontrivial issues. Checkerboard masks can be thought of as the binary-mask analog of Nisenson’s apodized square aperture concept.

Subject headings: instrumentation: miscellaneous — planetary systems

1. INTRODUCTION

Following our previous work on pupil-plane masks for high-contrast imaging in the context of terrestrial planet finding (see Kasdin et al. 2003; Vanderbei et al. 2003b, 2003c), we present in this paper further new pupil masks, as well as some pupil-plane/image-plane combinations of masks. Most of our earlier pupil-plane mask designs have an inner working angle (IWA) of 4λ/D, which implies a main mirror in the 12 m range (as usual, λ is wavelength and D is aperture). One of the main objectives leading to the designs presented in this paper was to significantly decrease this IWA. If the IWA can be reduced to 2λ/D, then the main mirror can be dropped down to the 6 m range and still be capable of studying the same set of stars. This is our goal.

One of the pupil masks presented here achieves a contrast ratio of 10⁻¹⁰ in four rectangular regions of the image plane given by \{(|x|, |y|): 2 < |x| < 20, 2 < |y| < 20\}. The points of high contrast that are closest to the center of the star’s image occur along the diagonals and therefore correspond to an IWA of 2√2 = 2.8λ/D.

The masks considered in this paper consist of rectangular arrays of rectangular openings. We call them “checkerboard” masks. They can be built as two identical striped masks, one placed on top of the other but oriented so that the stripes on one mask are orthogonal to the stripes on the other (see Fig. 1). We call the striped masks “bar-code” masks (see Kasdin et al. 2004). Each bar-code mask need only provide a contrast ratio of 10⁻⁵; it is then guaranteed that the two-mask overlay can achieve a contrast of 10⁻¹⁰. This is important, since it is quite feasible to check in a laboratory that a bar-code mask achieves 10⁻⁵, but is difficult, if not impossible, to check that a mask achieves 10⁻¹⁰ on the ground. Note that this performance guarantee assumes that the Fraunhofer approximation is adequate and that the instrument has a wave front amplitude and phase control system that is able to achieve the required extreme contrast. These are very serious issues that we and others are working on and will report on in other papers.

We show that one can get an even tighter IWA of 1.4√2 = 2.0λ/D by placing a rectangular pupil-plane mask in the Lyot plane of a traditional coronagraph and using an image-plane hard occulting mask consisting of a “plus-sign” shape, \{(|x|, |y|): 0.6 < |x| < 6, 0.6 < |y| < 6\}. We show that such a coronagraph can tolerate pointing errors as large as about 0.05λ/D.

Finally, in the interest of producing an on-axis coronagraph, we present a Lyot-plane rectangular pupil mask that includes a 2% four-vane spider. A similar spider can then be placed in the entrance pupil and provide the opportunity to place a small secondary mirror at the center of the plane. Having such a spider over the entrance pupil provides the possibility of building an all-mirror on-axis system and in this way greatly reducing the negative effects of differential polarization that are a concern with off-axis designs. Of course, a 2% secondary mirror is very small and perhaps not practical.

2. PUPIL APODIZATIONS AND MASKS

In this paper, we assume that telescope optics follows the Fraunhofer approximation. Hence, given a pupil-plane apodization function 0 ≤ A(x, y) ≤ 1, the image-plane electric field corresponding to an on-axis point source is given by the two-dimensional Fourier transform of the apodization function:

\[ E(\xi, \zeta) = \hat{A}(\xi, \zeta) \equiv \int \int e^{2\pi i (\xi x + \zeta y)} A(x, y) dy dx. \]

If the apodization function takes only the values zero and one, then the function represents a “mask.” The intensity in the image plane is the square of the magnitude of the electric field.
For an on-axis point source, this intensity function is called the “point spread function” (PSF).

Certain performance metrics guide our choice of the best pupil apodization (or mask). For example, the inner working angle $\rho_{\text{IWA}}$ and outer working angle $\rho_{\text{OWA}}$ specify the desired high-contrast region. For circularly symmetric designs, high contrast is specified by the requirement that

$$\left| E(\xi, \zeta)/E(0, 0) \right|^2 \leq 10^{-10}, \quad \rho_{\text{IWA}} \leq \sqrt{\xi^2 + \zeta^2} \leq \rho_{\text{OWA}}. \quad (2)$$

Another important consideration is the amount of light that gets into the main lobe of the PSF. We express this as a fraction of the total light available to an open unapodized aperture and call it the “Airy throughput,” denoted $T_{\text{Airy}}$. Finally, the full width at half-maximum (FWHM) of the main lobe of the PSF is an important measure of the sharpness of the PSF. For comparison purposes, we review these metrics for some simple apodizations.

2.1. Clear Aperture

For an open circular aperture, $\text{FWHM} = 1.02, T_{\text{Airy}} = 84.2\%$, and, if we define the inner working angle as the angle of the first null, which defines the border of the Airy disk, then $\rho_{\text{IWA}} = 1.24$. Of course, with this definition of $\rho_{\text{IWA}}$, the...
contrast constraint (2) is not satisfied. This is bad. If we choose $\rho_{\text{OWA}} = \infty$ and then pick the smallest $\rho_{\text{IWA}}$ for which the contrast constraint (2) is satisfied, we get $\rho_{\text{IWA}} = 743$, far too large for a practical planet-finding system.

2.2. Optimal Circularly Symmetric Apodization

In Vanderbei et al. (2003b), we presented a circularly symmetric apodization that we found by maximizing $E(0, 0)$, which is a simple surrogate for Airy throughput, subject to contrast constraints as given by equation (2) and some mild smoothness constraints on the apodization function. In this way, we found an apodization function for which $\text{FWHM} = 2$, $\rho_{\text{IWA}} = 4$, $\rho_{\text{OWA}} = \infty$, and $T_{\text{Airy}} = 9\%$. Except for the imposition of smoothness constraints, this apodization represents the best that one can expect to achieve with any circularly symmetric apodized or shaped pupil design. In particular, the Airy throughput of 9% should be thought of as an upper bound on the throughput one can obtain if one wishes there to be no light brighter than $10^{-10}$ outside of the inner working angle of $4\lambda/D$. To get higher throughput, one must accept either a larger inner working angle, a smaller outer working angle, or a design in which the dark zone is not an entire annulus. If we assume that a throughput of 9% is adequate, then the only downside to this design is that it is not possible with current technology to apodize a pupil to the level of precision required.

2.3. Concentric Ring and Spiderweb Masks

Also in Vanderbei et al. (2003b) we show that, if one relaxes the smoothness constraint, then the optimal solution turns out to be zero/one valued, i.e., a mask. The metrics for this design are $\text{FWHM} = 1.9$, $\rho_{\text{IWA}} = 4$, $\rho_{\text{OWA}} = 60$, and $T_{\text{Airy}} = 9\%$. The only downside to this design is that the mask consists of concentric rings that must be supported somehow. If they are laid on glass, then there is concern that imperfections in the glass will introduce too much scattered light. Another possibility is to use a large number of spiders to support the rings. If the number of spiders is large enough, say more than 150, then a reasonably large dark zone is preserved. However, again, concerns about manufacturability resurface. In addition, throughput is reduced somewhat by the presence of the spider vanes.

3. BAR-CODE AND CHECKERBOARD PUPIL MASKS

In the previous section, we reviewed our prior work on circularly symmetric pupil mask design. With those designs, the best we were able to do was to get an inner working angle of 4 and an Airy throughput of 9%. In addition, manufacturability was an issue in all of those designs. In this section, we introduce new pupil designs that break the $\rho_{\text{IWA}} = 4$ barrier and are more amenable to manufacture.

We should note that one can also break the 9% throughput barrier by limiting the discovery zone to less than 360°. The original Spergel pupil (Spergel 2000) is an example of this, as are some of the designs we have developed earlier (see, e.g., Fig. 8 in Vanderbei et al. 2003a).

If the apodization function depends only on one of the coordinates (either $x$ or $y$), then we say that it is a one-dimensional apodization or a one-dimensional mask. We also refer to one-dimensional masks as “bar-code” masks. This family of masks was studied in Kasdin et al. (2004). In this paper, we are interested in masks that correspond to the tensor product of a pair of one-dimensional masks:

$$ A = A_x \otimes A_y \iff A(x, y) = A_x(x)A_y(y). \quad (3) $$

The electric field corresponding to a tensor product is itself a tensor product:

$$ E = \hat{A} = \hat{A}_x \otimes \hat{A}_y = \hat{A}_x \otimes \hat{A}_y. \quad (4) $$

In other words,

$$ E(\xi, \zeta) = \hat{A}(\xi)\hat{A}(\zeta). \quad (5) $$

Tensor products of smooth apodizations were first proposed for terrestrial planet finding in Nisenson & Papaliolios (2001).

Since intensity is the square of the magnitude of the electric field, it follows that for a contrast of $10^{-10}$ we seek apodization functions that provide the following contrast inequalities:

$$ |E(\xi, \zeta)| \leq 10^{-5}E(0, 0), \quad (\xi, \zeta) \in \mathcal{O}, \quad (6) $$

where $\mathcal{O}$ denotes the points in the image plane at which high contrast is to be achieved. Suppose as before that the apodization function is a tensor product. If the set $\mathcal{O}$ is a “generalized rectangle”

$$ \mathcal{O} = \{ (\xi, \zeta) : \xi \in \mathcal{O}_\xi, \zeta \in \mathcal{O}_\zeta \} \quad (7) $$

(i.e., $\mathcal{O} = \mathcal{O}_\xi \times \mathcal{O}_\zeta$), then the contrast inequalities can be achieved by giving two one-dimensional apodizations that each achieve a contrast ratio of $10^{-5}$:

$$ |\hat{A}_x(\xi)| \leq 10^{-2.5} \hat{A}_x(0), \quad (\xi, \zeta) \in \mathcal{O}_\zeta, \quad (8) $$

$$ |\hat{A}_y(\zeta)| \leq 10^{-2.5} \hat{A}_y(0), \quad (\xi, \zeta) \in \mathcal{O}_\xi. \quad (9) $$

Figure 1 shows a checkerboard mask corresponding to $\mathcal{O}_\xi = \mathcal{O}_\zeta = \{ \xi : 2 \leq |\xi| \leq 25 \}$. The mask has a 28.1% open area. Its Airy throughput, defined as

$$ T_{\text{Airy}} = \int_{-\xi_0}^{\xi_0} \int_{-\zeta_0}^{\zeta_0} |E(\xi, \zeta)|^2 d\zeta d\xi, \quad (10) $$

where $\xi_0$ denotes the inner working angle for the bar-code mask, is 15.1%. The sharpness measured along the diagonal is $\text{FWHM} = 0.94\sqrt{2}$, and its inner working angle is $\rho_{\text{IWA}} = 2\sqrt{2}$. This design breaks both the $4\lambda/D$ inner working angle limit and the 9% Airy throughput limit. These gains come at the expense of introducing a plus sign–shaped “diffraction spike” at the $10^{-5}$ level that extends beyond the inner working angle as well as a smallish outer working angle.

Figure 2 shows another checkerboard mask. For this mask, a 2% central obstruction was imposed on the design. With such a central obstruction (and implied spiders) one can hang and hide a small secondary mirror and therefore build a telescope with an on-axis optical path. In this design, we have used $\mathcal{O}_\xi = \mathcal{O}_\zeta = \{ \xi : 2 \leq |\xi| \leq 11 \}$. The mask has a 17.2% open area. Its Airy throughput is 5.3%, and its inner working angle (measured along the diagonal) is $\rho_{\text{IWA}} = 2\sqrt{2}$. It remains to be determined whether the reduced throughput and the smallness of the central obstruction outweigh the advantages of having an on-axis design.

We end this section by pointing out that PSFs associated with these masks (and in fact all pupil-plane masks) depend on wavelength only in that the inner working angle is measured in units of $\lambda/D$, where $\lambda$ is wavelength and $D$ is aperture.
Hence, measured in radians, at longer wavelengths the inner working angle is correspondingly larger.

4. A LYOT CORONAGRAPH

Consider an imaging system consisting of an entrance pupil, a first image plane, a reimaged (Lyot) pupil plane, and a final image plane (containing an imaging device). We assume that each of the first three planes can be apodized/masked. Following Kuchner & Traub (2002), we let \( A \) denote the mask function for the entrance pupil, \( \hat{M} \) the mask function for the first image plane, and \( L \) the mask function for the Lyot pupil.

The electric field at point \((\xi, \zeta)\) in the final image plane corresponding to an off-axis point source is a composition of mask multiplication and Fourier transformation:

\[
\mathcal{F}(L \cdot \mathcal{F}(\hat{M} \cdot \mathcal{F}(A \cdot F_{-\xi_0,-\zeta_0}))) (\xi, \zeta) = \\
\hat{\mathcal{L}} \ast (\hat{\mathcal{M}} \ast \delta_{\xi_0,\zeta_0})(\xi, \zeta),
\]

where \( F_{-\xi_0,-\zeta_0} \) denotes the electric field at the entrance pupil corresponding to a point source from direction \((\xi_0, \zeta_0)\), \( \delta_{\xi_0,\zeta_0} \) denotes a unit mass delta function at \((\xi_0, \zeta_0)\), hats denote

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**Fig. 2.—**Centrally obstructed checkerboard mask. This design includes a 2% central stripe to hang/Hide a secondary mirror. The open area of the checkerboard mask is 17.2%, and the Airy throughput is 5.3%. **Top left:** Bar-code mask designed to provide \(10^{-5}\) contrast from \(2\lambda/D\) to \(11\lambda/D\). **Top right:** PSF associated with the bar-code mask. **Bottom left:** Corresponding checkerboard mask. **Bottom right:** PSF corresponding to the checkerboard mask. The gray scale represents a logarithmic stretch with black corresponding to \(10^{-10}\) and white corresponding to 1.
Fourier transforms, dots indicate pointwise multiplication, and stars denote convolutions.

It is easy to see that if each of $L$, $M$, and $A$ are tensor products ($L = L_x \otimes L_y$, $M = M_x \otimes M_y$, $A = A_x \otimes A_y$), then the electric field again factors into a product of electric fields:

$$\hat{L} \star (\hat{M} \star (\hat{A} \star \delta_{\hat{h}, \hat{r}}))(\xi, \zeta) = L_x \star (M_x \star (A_x \star \delta_{h, r}))(\xi) \hat{L}_y \star (\hat{M}_y \star (\hat{A}_y \star \delta_{h, r}))(\zeta).$$

(12)

We use this factorization to greatly simplify the coronagraph optimization problems described in the next section.

5. LYOT CORONAGRAPHS: OPTIMIZATION PROBLEM AND NUMERICAL RESULTS

As in our previous papers (Kasdin et al. 2003; Vanderbei et al. 2003b, 2003c), we use numerical optimization to maximize some measure $\theta$ of throughput subject to imposed contrast constraints. For the designs presented in this section, we solve the following one-dimensional bar-code optimization problem: maximize $\theta$ subject to

$$\theta \leq \hat{L} \star (\hat{M} \star (\hat{A} \star \delta_{\hat{h}, \hat{r}}))(\xi, \zeta), \quad (\xi, \zeta) \in \mathcal{O};$$

$$|\hat{L} \star (\hat{M} \star (\hat{A} \star \delta_{h, r}))(\xi, \zeta)| \leq \frac{\hat{L} \star (\hat{M} \star (\hat{A} \star \delta_{h, r}))(\xi, \zeta)}{10^{2.5}},$$

(13)

$$\quad (\xi, \zeta) \in \mathcal{O}.$$
The optimal bar-code mask is shown in Figure 3 together with the corresponding checkerboard mask. This coronagraph has an off-axis Airy throughput of 6.2%, and its inner working angle is
\[ \text{FWHM} = \frac{1.4}{\mu} \left( \frac{2}{\pi} \right)^{1/2} \]

The second example is the same as the first except that we impose a central obstruction in the pupil-plane masks:
\[
A(x) = \begin{cases} 
1, & 0.01 \leq |x| \leq \frac{1}{2}, \\
0, & \text{otherwise}, 
\end{cases} \tag{18}
\]

\[
|L(x)| \leq 1, \quad 0.01 \leq |x| \leq \frac{1}{2}, \tag{19}
\]

The optimal bar-code mask is shown in Figure 4 together with the corresponding checkerboard mask. This coronagraph has an off-axis Airy throughput of 3.5% and, again, the inner working angle is
\[ \rho_{\text{FWA}} = 1.4/\mu \]

Since the designs presented in this section involve an image-plane mask, the associated PSFs depend on wavelength in a more complicated way than the designs given in §3. To test this wavelength dependency, we did a crude test. We computed the image of our usual three-planet system using four different wavelengths, two shorter (85% and 90%) and two longer (105% and 110%) than the designed wavelength. As Figure 5 shows, the loss of contrast occurs at wavelengths shorter than about 90% and longer than 105% of the design point.
6. CONCLUSIONS

In this paper we have presented a new mask concept and four specific new mask designs for high-contrast imaging. Table 1 summarizes our results in tabular form. We leave it to others to decide which of these four designs most closely matches the needs of the Terrestrial Planet Finder.

We end the paper by reiterating that the basic idea is similar to the apodized square aperture concept of Nisenson & Papaliolios (2001), except that we are working with pupil-plane masks rather than apodizations. In addition, by optimizing the design we get high contrast everywhere outside a narrow plus sign–shaped diffraction spike, whereas the Nisenson & Papaliolios designs provide high contrast only in a neighborhood of the two diagonals.

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| Description                                      | $\rho_{\text{IWA}}$ | $\rho_{\text{OWA}}$ | $T_{\text{Air}}$ |
|--------------------------------------------------|-----------------|-----------------|-----------------|
| Simple Checkerboard                               | $2\sqrt{2}$    | 25              | 15.1            |
| Checkerboard with 2% Central Obstruction          | $2\sqrt{2}$    | 11              | 5.3             |
| Lyot Checkerboard                                | $1.4\sqrt{2}$  | 21              | 6.2             |
| Lyot Checkerboard with 2% Central Obstruction    | $1.4\sqrt{2}$  | 21              | 3.5             |