Quantum optical phenomena in semiconductor quantum dots

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Abstract

Quantum optical phenomena are explored in artificial atoms well known as semiconductor quantum dots, in the presence of excitons and biexcitons. The analytical results are obtained using the conventional time-dependent perturbation technique. Numerical estimations are made for a realistic sample of CdS quantum dots in a high-Q cavity. Quantum optical phenomena such as quantum Rabi oscillations, photon statistics and collapse and revival of population inversion in exciton and biexciton states are observed. In the presence of biexcitons the collapse and revival phenomenon becomes faster due to the strong coupling of biexciton with cavity field.

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I. INTRODUCTION

Recent progress in crystal growth techniques for the fabrication of nanostructure materials has enabled us to realize the possibility of high-quality epitaxial layers enough to demonstrate excitonic optical properties of quantum dots (QDs) in the ultraviolet [1] to infrared [2] range including room-temperature excitonic lasing, high-temperature excitonic stimulated emission and high characteristic temperature for optical threshold power [3, 4]. Such nonlinear optical effects would be much more enhanced if biexcitons were involved in the optical processes because of the giant oscillator strength effect [5, 6]. In fact, low-threshold lasing based on optical processes associated with biexcitons has been theoretically predicted [7, 8] and observed [4].

Since its proposal by Purcell [9] and early work involving atoms in cavities [10], cavity quantum electrodynamics has been actively pursued for its potential insight into fundamental problems in light-matter interaction. Here, the energies and modes of photon states, to which electronic transitions of atoms can couple, are changed by the cavity confinement. Recently, efforts have been made to extend this work to the optical regime, which requires cavities with sizes of the order of few micrometers. This work has been made possible by the epitaxial fabrication of high quality layered semiconductors from which optical cavities have been made that have strong photonic confinement in one direction [11–13]. However, the preparation of optical microcavities which confine the propagation of light in all three dimensions is presently a challenge for modern microtechnologies. New techniques are being developed to overcome the difficulties in developing such microcavities using quantum dot structures. Quasi atomic light emitters in these resonators can be mimicked by semiconductor quantum dots that exhibit discrete density of electronic states. Arakawa et al, [4] fabricated QD lasers with vertically stacked self-assembled dots with a columnar shape which achieved a room-temperature threshold current of 5.6 mA, comparable to or even less than that of quantum-well lasers. The threshold current density has also been lowered to 90 A/cm$^2$ [12]. Pulsed and continuous-wave operation of 1.3 $\mu$m quantum dot lasers at room temperature has been realized, which is a milestone towards the application of QDs to ultrafast fiber optic data transmission and optical interconnects [2]. Artemyev et al, [14] have developed high-Q microcavities using tiny hollow microspheres made of the transparent polymer polymethylmethacrylate (PMMA) with smooth, highly reflecting surfaces that
exhibit all properties of an optimum optical cavity. Such improved laser performance has now enabled physicists to study various unique properties of interaction between QDs and light.

The study of interaction of quantized radiation with matter is important in QDs, because the typical sizes of QDs are much smaller than the wavelength of light being used (for example the typical sizes of QDs are CdS - 1.7nm [15], and CdSe \( \approx 4\)nm [16]). The smaller cross sectional areas of these QDs allow only few photons to interact with each of them. Hence, the application of quantum optical tool to QD may yield interesting results. Accordingly, with the recent developments in the fabrication of high-Q microcavities having 3-dimensional photonic confinement on one hand and the enhanced excitonic nonlinear optical characteristics of the QDs in 3-dimensional electronic confinement on the other, we have made an attempt to explore the quantum optical phenomena in these structures in the presence of both excitons and biexcitons.

II. THEORETICAL CALCULATIONS

Time-dependent perturbation technique under the interaction picture has been employed to study the interaction of a single mode of cavity radiation with a small QD embedded in a high-Q microcavity. The atomic-like energy-level structure existing in these QD nanostructures allows the photo-induced electronic transitions to take place between the ground (\( |0\rangle \)) and exciton (\( |e\rangle \)) states. For large excitation intensities, this simplistic two-level picture is modified by the creation of biexcitons. Accordingly, in the present paper, while studying the nonclassical phenomena in QDs, we have incorporated the exciton and biexciton (\( |b\rangle \)) states via a three-level system (3-LS) as schematically represented in Fig. 1. The allowed electronic transitions (solid arrows) take place between the states \( |0\rangle \equiv |e\rangle \) and \( |e\rangle \equiv |b\rangle \). The parity violating two-photon transitions (dashed arrow) between \( |0\rangle \equiv |b\rangle \) are ignored in the present calculations.

For small quantum dots of size \( R \) smaller than the exciton Bohr radius \( (a_B) \), the energy gap between the vacuum and exciton states is defined as [17]

\[
\hbar \omega_{oe} = \hbar \omega_g + \frac{\hbar^2}{2m_r} \left( \frac{\kappa_{ml}}{R} \right)^2,
\]

where \( \hbar \omega_g \) is the band gap energy and \( \kappa_{ml} \) is \( m \)th root of the \( l \)th order Bessel function with
m and l corresponding to the 1s, 1p, 1d, ... 2s, 2p, 2d, ... levels of the electrons and holes. 

$m_r$ is the reduced mass of an electron-hole pair. Due to the finite Coulombic attraction between two excitons, the energy of the biexciton is reduced by an amount equal to the binding energy ($\Delta E$) and defined by [18]

$$\hbar \omega_{eb} = \hbar \omega_{oe} - \Delta E.$$  (2)

We have assumed that at time $t = 0$, all the electrons are in the ground state. For time $t > 0$, the interaction of cavity photons with the QD creates one and two electron-hole pairs (i.e., excitons and biexcitons). Accordingly, the total wavefunction of QD-radiation can be defined under dressed state representation as

$$|\Psi(t)\rangle = c_{0,n+1}(t)|0,n+1\rangle + \sum_{ml} c_{e,n}(t)|e,n\rangle + \sum_{ml} c_{b,n-1}(t)|b,n-1\rangle.$$  (3)

c_{0,n+1}(t), c_{e,n}(t) and $c_{e,n-1}(t)$ are the probability amplitudes of the vacuum, exciton and biexciton states, respectively. While writing the total wavefunction, we assume for the field mode that the initial state is coherent, such that

$$|c_n(0)|^2 = \frac{\bar{n}^n e^{-\bar{n}}}{n!},$$  (4)

where, $\bar{n}$ is the initial photon number.

The interaction Hamiltonian of the exciton and biexciton states is represented by

$$\mathcal{H}_I = \mathcal{H}_e + \mathcal{H}_b,$$  (5a)

with

$$\mathcal{H}_e = -g_{oe}[a \pi_{eo} e^{-i \omega_{oe} t} + a^\dagger \pi_{oe} e^{i \Delta_{oe} t}]$$  (5b)

and

$$\mathcal{H}_b = -g_{eb}[a \pi_{be} e^{-i \Delta_{eb} t} + a^\dagger \pi_{eb} e^{i \Delta_{eb} t}],$$  (5c)

where $a^\dagger$ and $a$ are the creation and annihilation operators of the electromagnetic field, $\pi_{ij} = |i\rangle\langle j|$. The detuning parameter is defined as $\Delta_{oe(eb)} = \omega - \omega_{oe(eb)}$. $g_{oe} = |\mu_e E/\hbar|\sqrt{n+1}$ and $g_{eb} = |\mu_b E/\hbar|\sqrt{n}$ are the coupling parameters of the exciton and biexciton, respectively
with the cavity field. Also $E$ is the field per photon and

$$\mu_e = \langle e|\hat{\mu}|0\rangle = \mu_{oe} \int \phi(r, r) dr$$

$$\mu_b = \langle b|\hat{\mu}|e\rangle = -\sqrt{2}\mu_{oe} \times \int \int \int \phi(r, r, r, r, r) \phi(r, r) dr dr dr.$$  \hspace{1cm} (6a) \hspace{1cm} (6b) \hspace{1cm} (6c)

for the same spin states. \cite{19} Here, $\mu_{oe} = [e|p_{oe}|/(m_o \omega_{oe})]$ is the transition dipole moment of the vacuum to exciton state transition and $m_o$ is the free electron mass. $\phi(r, r)$ and $\phi(r, r, r, r)$ are the exciton and biexciton wave functions, respectively. They are related to the single particle wave function $\phi(r)$ as \cite{18, 19} $\phi(r, r) = \phi(r) \phi(r)$ and $\phi(r, r, r, r) = \phi(r) \phi(r) \phi(r) \phi(r)$, with

$$\phi(r) = \sqrt{\frac{2a_3^2 j_l(\kappa nl r/R)}{R^3 j_{l+1}(\kappa nl)}}.$$  \hspace{1cm} (7)

Using Schrodinger’s equation under the interaction picture, the equations of motion of probability amplitudes are found to be

$$\dot{c}_{0,n+1}(t) = i \sum c_{e,n}(t) \exp(i \Delta_{oe} t),$$

$$\dot{c}_{e,n}(t) = i g_{oe} c_{0,n+1}(t) \exp(-i \Delta_{oe} t)$$

$$+ i g_{eb} c_{b,n-1}(t) \exp(i \Delta_{eb} t),$$

$$\dot{c}_{b,n-1}(t) = i g_{eb} c_{e,n}(t) \exp(-i \Delta_{eb} t).$$  \hspace{1cm} (7a) \hspace{1cm} (7b) \hspace{1cm} (7c)

The above set of coupled equations can be solved by assuming

$$c_{0,n+1}(t) = A \exp(i \Omega t),$$

$$c_{e,n}(t) = B \exp[i(\Omega - \Delta_{oe})t],$$

$$c_{b,n-1}(t) = C \exp[i(\Omega - \Delta_{eb})t],$$  \hspace{1cm} (8a) \hspace{1cm} (8b) \hspace{1cm} (8c)

where $A$, $B$ and $C$ are the time-independent constants, $\Omega$ is an unknown parameter having the dimension of frequency and $\Delta_{ob} = \Delta_{oe} + \Delta_{eb}$. Use of (7) and (8) yields a cubic equation in $\Omega$ as

$$\Omega^3 + a_2 \Omega^2 + a_1 \Omega + a_0^3 = 0$$  \hspace{1cm} (9)

with $a_0^3 = -|g_{oe}|^2 \Delta_{ob}$; $a_1^2 = \Delta_{oe} \Delta_{eb} - |g_{oe}|^2 - |g_{eb}|^2$ and $a_2 = -\Delta_{oe} - \Delta_{eb}$. The solution of
Ω are found to be

\[
\Omega_1 = -\frac{a_2}{3} + \frac{X^2 - 12Y^2}{6X},
\]

(10a)

\[
\Omega_3 = -\frac{a_2}{3} - \frac{X^2 e^{\pm i\frac{\pi}{3}} - 12Y^2 e^{\pm i\frac{\pi}{3}}}{6X}.
\]

(10b)

with

\[
X^3 = 36a_1^2a_2 - 108a_0^3 - 8a_2^3 + \sqrt{3} \sqrt{4a_1^6 - a_1^4a_2^2 - 18a_1^3a_2a_0^2 + 27a_0^6 + 4a_0^3a_2^3}
\] and \( Y^2 = a_1^2 - \frac{a_2^2}{3} \).

A. Rabi Oscillations

For analytical confirmation, we have reduced the generalized equations obtained for a three-level system (3-LS) to a conventional two-level atomic system (2-LS). For a 2-LS, in the absence of biexcitons the coefficients are corrected by substituting \( g_{eb} = 0 \) and \( \Delta_{ob} = 0 \) and it is found that \( a_0 = 0, a_1^2 = -|g_{oe}|^2 \) and \( a_2 = -\Delta_{oe} \), which reduces the cubic equation to a quadratic equation in \( \Omega \). We find that \( \Omega_2 \) vanishes in the absence of biexcitons and the other two solutions remain finite. In a 2-LS, the magnitudes of these eigenvalues decide the frequencies of oscillation of the probability of the upper and lower levels (in the present case between vacuum and exciton states) [20]. This frequency of oscillation is widely termed as Rabi frequency. The three solutions of \( \Omega \) obtained in the present case of a 3-LS also exhibit a similar feature: \( \Omega_j \) obtainable from (10) may correspond to the Rabi oscillations occurring between the vacuum-exciton and exciton-biexciton states. However, in the presence of biexcitons, analytical results of \( \Omega \) could not be obtained due to the complex nature of the three solutions obtained in (10). Hence, we applied our analysis to a realistic sample of quantum dot of CdS embedded in an undamped microcavity. The material parameters of CdS taken from the experimental paper of Butty et al [15] are: \( \hbar \omega_g = 2.56eV, \Delta E = 28meV, R = 1.7nm, a_B = 2.9nm \) and \( m_h/m_e = 4.2 \). Also, to make the present solutions compatible with the conventional eigenvalues [20], we define \( \Omega'_j = \Omega_j - a_2/3 \). The nature of dependence of \( \Omega'_j \) on exciton detuning parameter \( \Delta_{oe} \) are exhibited in Figs. 2 and 3. In obtaining Fig. 2, we have incorporated the contribution of excitons only, while in Fig. 3, the contribution of both exciton and biexcitons are incorporated. The solid lines are obtained in the presence of finite cavity field and the dashed lines represent the same
obtained for \( n = 0 \). To increase the clarity of the curves, a thicker line has been drawn for \( \Omega'_2 \) while \( \Omega'_1 \) and \( \Omega'_3 \) share same line-thickness. Parts of the dashed lines of \( \Omega'_1 \) (below resonance) and \( \Omega'_3 \) (above resonance) are not visible since \( \Omega'_2 \) for \( n = 0 \) as well as \( n \neq 0 \) overlaps with them. The curves obtained in the absence of biexcitons agree with the conventional results obtained for a 2-LS system.[20] For \( n = 0 \), all the curves vanish at \( \Delta_{oe} = 0 \). However, for \( n \neq 0 \), the eigenvalues show repulsion at \( \Delta_{oe} = 0 \) by an amount equal to \( \pm g_{oe} \). The repulsive behavior of the two eigenvalues is attributed to the finite value of QD-cavity field coupling. The phenomenon of repulsion of the eigenvalues are also called as AC Stark shift or dynamic stark shift since the energy levels show shift due to the dynamic nature of the amplitude of the radiation. As expected from the analytical results for a 2-LS, one of the solution \( (\Omega_2 = \Omega'_2 - a_2/3) \) remains unchanged at zero.

For \( n = 0 \), all the three curves obtained in the presence of both exciton and biexciton shown in Fig. 3, display biexcitonic signatures between \( \Delta_{oe} = 0 \) and \( \Delta_{eb} = 0 \). A close observation of the curves obtained for \( n = 0 \) yields the following: \( \Omega_{1,2,3} \) vanish neither at \( \Delta_{oe} = 0 \) nor at \( \Delta_{eb} = 0 \). However, (i) \( \Omega_1 \) shows its minimum value at \( \Delta_{oe} + \Delta E/2 \) and is repelled from the zero line by an amount \( \Delta E/6 \); (ii) for \( \Delta_{oe} \geq 0 \leq \Delta_{eb} \), the second solution remains constant at \( -\Delta E/3 \); for \( \Delta_{oe} \leq 0 \), it steadily increases to a value of \( \Delta E/6 \) and again starts decreasing until \( \Delta_{eb} \) becomes equal to zero, and (iii) for decreasing detuning parameter, the final solution steadily increases to \( -\Delta E/3 \) till \( \Delta_{oe} = 0 \); further, for \( \Delta_{oe} \leq 0 \geq \Delta_{eb} \) it remains constant at \( -\Delta E/3 \). For \( \Delta_{eb} \leq 0 \), it starts decreasing steadily.

For \( n \neq 0 \), dramatic changes occur. Both \( \Omega'_1 \) as well as \( \Omega'_3 \) get repelled from \( \Omega'_2 \). In the presence of biexciton, \( \Omega'_2 \) plays a dominant role by repelling both \( \Omega'_1 \) and \( \Omega'_3 \) for \( n = 0 \) and \( n \neq 0 \). This leads to the conclusion that \( \Omega'_2 \) couples the biexciton states to the exciton states and hence the ground state. In addition to the shift along Y-axis, a small shift along X-axis can also be noted for all three curves in the presence of biexcitons. For \( n \neq 0 \) the number of biexciton becomes finite, such that \( a_1 \) as defined earlier modifies the resonance frequency. This leads to small red and blue shifts of the peak values of the eigenvalues. These shifts indicate strong evidence for biexciton signatures in the eigenvalues \( \Omega_1 \) and \( \Omega_2 \). And \( \Omega_3 \) does not show significant change in the presence and absence of biexcitons at \( \Delta_{oe} = 0 \) and \( \Delta_{eb} = 0 \).

The three solutions of \( \Omega \) obtained in (10) suggest the following modifications in the
and assumed solution (8)

\[ c_{0,n+1}(t) = A_1 e^{i \Omega_1 t} + A_2 e^{i \Omega_2 t} + A_3 e^{i \Omega_3 t}, \quad (11a) \]
\[ c_{e,n}(t) = (B_1 e^{i \Omega_1 t} + B_2 e^{i \Omega_2 t} + B_3 e^{i \Omega_3 t}) e^{-i \Delta_{oc} t}, \quad (11b) \]
\[ c_{b,n-1}(t) = (C_1 e^{i \Omega_1 t} + C_2 e^{i \Omega_2 t} + C_3 e^{i \Omega_3 t}) e^{-i \Delta_{ob} t}. \quad (11c) \]

In order to calculate the nine time-independent unknown coefficients \( A_i, B_i \) and \( C_i \) \((i = 1, 2, 3)\), we assume that, at \( t = 0 \), the QD is in the ground state \( |0\rangle \) and the cavity field is in a coherent state. The corresponding boundary conditions at \( t = 0 \), are \( c_{0,n+1}(t) = c_n(0) \) and \( c_{e,n}(t) = c_{b,n-1}(t) = 0 \). Use of these conditions in (7), (8) and (11) yields the solution of the unknown coefficients as

\[
\begin{pmatrix}
A_1 & B_1 & C_1 \\
A_2 & B_2 & C_2 \\
A_3 & B_3 & C_3
\end{pmatrix} = \frac{-c_n(0)}{|M|} \begin{pmatrix} 
\beta_{23} (\alpha_{23} + |g_{oc}|^2) & g_{oc} \beta_{23} (\Delta_{oc} - \gamma_{23}) & g_{oc} g_{eb} \beta_{23} \\
\beta_{31} (\alpha_{31} + |g_{oc}|^2) & g_{oc} \beta_{31} (\Delta_{oc} - \gamma_{31}) & g_{oc} g_{eb} \beta_{31} \\
\beta_{12} (\alpha_{12} + |g_{oc}|^2) & g_{oc} \beta_{12} (\Delta_{oc} - \gamma_{12}) & g_{oc} g_{eb} \beta_{12}
\end{pmatrix}, \quad (12)
\]

with \( M = \begin{pmatrix} 1 & 1 & 1 \\ \Omega_1 & \Omega_2 & \Omega_3 \\ \Omega_1^2 & \Omega_2^2 & \Omega_3^2 \end{pmatrix} \), \( \alpha_{ij} = \Omega_i \Omega_j \), \( \beta_{ij} = \Omega_i - \Omega_j \) and \( \gamma_{ij} = \Omega_i + \Omega_j \). For a 2-LS the above set of equations reduces to

\[
\begin{pmatrix}
A_1 & B_1 & C_1 \\
A_2 & B_2 & C_2 \\
A_3 & B_3 & C_3
\end{pmatrix} = \frac{-c_n(0)}{\Omega_1 - \Omega_2} \begin{pmatrix}
\Omega_2 & g_{oc} & 0 \\
\Omega_1 & -g_{oc} & 0 \\
0 & 0 & 0
\end{pmatrix}. \quad (13)
\]

In obtaining the above equations, we have assumed the biexciton-cavity coupling term \( g_{eb} \) to be zero. Comparison of (13) with the standard solutions [20] for a 2-LS shows good agreement and confirms the validity of the model to a two-level system.

Using (11)-(13), we have obtained the temporal variations of probabilities and exhibited these in Fig. 4. The solid, dashed and dotted lines represent \( |c_{a,n-1}|^2 \), \( |c_{e,n}|^2 \) and \( |c_{b,n+1}|^2 \), respectively. The inset shown in the figure demonstrates the same parameters in the absence of biexciton contribution. In the absence of biexciton, the ground and exciton state populations execute oscillation at a frequency equal to \( 2g_e \). Hence, the ground state is bleached for \( g_e t = (2N + 1) \pi / 2 \) with \( N = 0, 1, 2, \ldots \). As expected from earlier analytical simplifications, \( |c_{b,n+1}|^2 \) remains constant at zero. In the presence of biexciton contribution, the boundary
conditions are exactly followed. For \( t > 0 \), the ground state population starts decreasing until \( g_e t = \pi/2 \). Simultaneously, the exciton population reaches its maximum when \( g_e t = \pi/4 \), and later decreases to a minimum value at \( g_e t = \pi/2 \). However, the biexciton population increases very slowly but attains its maximum value when \( g_e t = \pi/2 \). This region is marked as 1 in the figure. This is an interesting region, since the population between \(|b\rangle\) and \(|0\rangle\) as well as between \(|b\rangle\) and \(|e\rangle\) is inverted. This population inverted region occurs again when \( g_e t = 5\pi/2 \). The significance of the region 1 is that for \( g_e t = (5N + 1)\pi/2 \) and \((N + 1)5\pi/2\), low threshold for exciton and biexciton lasing can be achieved. Another region of similar interest is marked as 2, where population inversion occurs between \(|b\rangle\) and \(|e\rangle\). All the three curves rephase for \( g_b t = 4N\pi \).

B. Photon Statistics

The expressions (11) represent the probability amplitudes of the electron to occupy the states \(|0\rangle\), \(|e\rangle\) and \(|b\rangle\) and the probability of \( n \) photons to occupy the states \(|n\rangle\) at any arbitrary time \( t \). The trace over the electronic states yield the probability \( p(n, t) \) of the cavity field as

\[
p(n, t) = \sum_{m} \left[ |c_{0,n}(t)|^2 + |c_{e,n}(t)|^2 + |c_{b,n}(t)|^2 \right]. \tag{14}
\]

Comparison of the above results with the semiclassical [21] results show disagreement, since the semiclassical techniques assume the photon number distribution at \( t = 0 \) as \( c_{0,n+1} = 1 \). However, for an initially coherent state, the Poisson distribution allows the field to propagate in a coherent manner. In Fig. 5, variation of \( p(n, t) \) with time as well as number of photons is demonstrated for \( \bar{n} = 10 \). Fig. 5a and 5b display contour plots in presence of excitons only and in presence of exciton as well as biexciton, respectively. The peak value is found to be at \( \bar{n} \). Both the curves strictly follow Poisson statistics with respect to \( n \), but the envelope oscillates with time. A few peaks are obtained for smaller values of \( n \) while the number of peaks starts increasing with \( n \). The reason for these oscillations are attributed to the definition of \( g_{0e} \) which is proportional to \( \sqrt{n+1} \). In the presence of excitonic contribution only, the photon statistics oscillates at a frequency \( g_{0e} \). In the presence of biexcitons a clear case of chaos is observed as evident from Figs. 5c and 5d, which are obtained for \( n = 10 \).
C. Collapse and revival phenomena

The phenomenon of collapse and revival of Rabi oscillations is studied in atomic systems using atomic inversion operator. The reason for the collapse and revival phenomenon is well understood now. The contributions corresponding to different $n$’s interfere in such a manner that they initially go out of phase. After that they acquire a common phase, and this process is continuously repeated to obtain a series of collapses and revivals. In the present generalized model, we have assumed a three-level structure of excitons and biexcitons. The corresponding population inversion in these states can be found from

$$W(t)_{0e} = \sum_{e,n} (|c_{0,n+1}(t)|^2 - |c_{e,n}(t)|^2) \quad (15a)$$

and

$$W(t)_{eb} = \sum_{e,n} (|c_{e,n}(t)|^2 - |c_{b,n-1}(t)|^2). \quad (15b)$$

In Fig. 6, the temporal variation of $W_{0e}$ is demonstrated in the presence (curve $a$) and absence (curve $b$) of biexciton contribution. Both the curves exhibit collapse and revival of population inversion between the ground and exciton states. A very slow revival is noted in the absence of biexcitons. In the presence of biexcitons, the strong coupling of the biexciton with cavity field reduces the revival time; hence a very small collapse period is observed. However, in the absence of biexcitons the QD behaves like a 2-LS; hence an atomic like collapse and revival phenomenon is noted. In the absence of the biexciton contribution, $W_{0e}$ oscillates around zero. With the biexcitonic contribution, it shows a positive DC shift. The transient behavior of excitonic and biexcitonic inversions, as exhibited in Fig. 7, does not show any remarkable change. In the presence of the biexciton $W_{0e}$, as well as $W_{eb}$ display the same collapse and revival time. However, $W_{0e}$ oscillates around 0.3, while $W_{eb}$ oscillates around -0.2.

III. CONCLUSIONS

A simple quantum optical model has been developed to study nonclassical phenomena in semiconductor quantum dots in an undamped cavity. On reduction of the present calculations to a two-level system, these agree well with the conventional results. The results are applied to a realistic semiconductor QD of CdS of size 1.7nm. The transient nature of
photon statistics exhibits oscillations. The collapse and revival phenomenon in the presence of biexcitons becomes faster due to the strong biexciton-photon coupling.

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[1] Y. F. Chen, D. M. Bagnall, H. J. Ko, K. T. Park, K. Hiraga, Z. Zhu and T. Yao, J. Appl. Phys., 84, 3912 (1998).
[2] M. Sugawara, K. Mukai, Y. Nakata, H. Ishikawa and A. Sakamoto, Phys. Rev. B., 61, 7595 (2000).
[3] D. M. Bagnall, Y. F. Chen, Z. Zhu, T. Yao, S. Koyama, M. Y. Shen, and T. Goto, Appl. Phys. Lett., 70, 2230 (1997); D. M. Bagnall, Y. F. Chen, Z. Zhu, T. Yao, M. Y. Shen, and T. Goto Appl. Phys. Lett., 73, 1038 (1998); D. M. Bagnall, Y. F. Chen, Z. Zhu, T. Yao, M. Y. Shen, and, T. Goto, Nonlinear Opt., 18, 243 (1997).
[4] Y. Arakawa and H. Sakaki, Appl. Phys. Lett., 40, 939 (1982).
[5] E. Hanamura, Phys. Rev. B, 37, 1273 (1988).
[6] U. Hohenester and E. Molinari, Phys. Stat. Sol. (a) 178, 277 (2000).
[7] M. Sugawara, Jpn. J. Appl. Phys., Part 1, 35, 124 (1996).
[8] J. T. Andrews and P. Sen, J. Appl. Phys, 91, 2827 (2002).
[9] E. M. Purcell, Phys. Rev., 69, 681 (1946).
[10] See, for example, Cavity Quantum Electrodynamics, ed. P. Berman (Academic Press, San Diego, 1994).
[11] C. Weisbuch et al., Phys. Rev. Lett., 69, 3314 (1992).
[12] G. Park, D. L. Huffaker, Z. Zou, O. B. Shchekin, and D. G. Deppe, IEEE Photonics Technol. Lett., 11, 301 (1999).
[13] M. Bayer, T. L. Reinecke, F. Weidner, A. Larionov, A. McDonald, and A. Forchel1, Phys. Rev. Lett., 86, 3168 (2001).
[14] M. V. Artemyev, U. Woggon and R. Wannemacher, *Appl. Phys. Lett.*, **78**, 1032 (2001).

[15] J. Butty, Y. Z. Hu, N. Peyghambarian, Y. H. Kao and J. D. Mackenzie, *Appl. Phys. Lett.*, **67**, 2672 (1995); J. Butty, N. Peyghambarian, Y. H. Kao, and J. D. Mackenzie, *Appl. Phys. Lett.*, **69**, 3224 (1996) .

[16] B. C. Hess, I. G. Okhrimenko, R. C. Davis, B. C. Stevens, Q. A. Schulzke, K. C. Wright, C. D. Bass, C. D. Evans, and S. L. Summers, *Phys. Rev. Lett.*, **86**, 3132 (2001).

[17] J. Thomas Andrews, P. Sen and R. R. Puri, *J. Phys. B, Cond. Matter*, **11**, 6287 (1999).

[18] L. Banyai and S. W. Koch, *Semiconductor Quantum Dots* (World-Scientific, Singapore 1993) pp. 38-49.

[19] Y. Z. Hu, M. Lindberg and S. W. Koch, *Phys. Rev. B*, **42**, 1713 (1990).

[20] C. C.-Tannoudji, B. Diu and F. Laloe, *Quantum Mechanics, Vol-I*, (John-Wiley, New York 1977) pp. 405-415.

[21] R. G. Brewer, *Nonlinear Optics*, eds. P.G. Harper and B.S. Wherrett (Academic, London 1977) pp. 307-363.
FIGURE CAPTIONS

FIG. 1 Schematic diagram of three-level structure of ground, exciton and biexciton states in a quantum dot. Solid lines represent allowed electronic transitions, while the two-photon transition shown by the dashed line is not allowed.

FIG. 2 The nature of dependence of eigenvalues $\Omega'_j$ on exciton detuning parameter $\Delta_{0e}$ in CdS quantum dot in the absence of biexciton contribution.

FIG. 3 Variation of $\Omega'_j$ with detuning parameter in the presence of biexciton contribution in a small quantum dot of CdS.

FIG. 4 Temporal nature of probabilities of ground (dotted), exciton (dashed) and biexciton (solid) states in the presence of exciton and biexciton. The inset shows the same in the presence of exciton only. In the regions marked as 1, population inversion occurs between $|b\rangle$ and $|e\rangle$ as well as between $|b\rangle$ and $|e\rangle$. In the regions 2, inversion occurs between $|b\rangle$ and $|e\rangle$ states.

FIG. 5 Photon statistics $p(n, t)$ in a small quantum dot of CdS embedded in a microcavity for $\bar{n} = 10$. The contour plot a is obtained in the absence of biexcitons while b is obtained in the presence of biexcitons. Plots (c) and (d) are obtained for $n = 10$.

FIG. 6 The temporal variation of excitonic inversion ($W_{oe}$) in a QD of CdS. Curve a is obtained in the presence of biexcitons while curve b is obtained in the absence of biexcitons.

FIG. 7 The nature of dependence of exciton and biexciton population inversion ($W_{0e}$ and $W_{eb}$) on time in a small quantum dot of CdS.
FIG. 1: Schematic diagram of three-level structure of ground, exciton and biexciton states in a quantum dot. Solid lines represent allowed electronic transitions, while the two-photon transition shown by the dashed line is not allowed.
FIG. 2: The nature of dependence of eigenvalues $\Omega'_j$ on exciton detuning parameter $\Delta_{0e}$ in CdS quantum dot in the absence of biexciton contribution.
FIG. 3: Variation of $\Omega'_j$ with detuning parameter in the presence of biexciton contribution in a small quantum dot of CdS.
FIG. 4: Temporal nature of probabilities of ground (dotted), exciton (dashed) and biexciton (solid) states in the presence of exciton and biexciton. The inset shows the same in the presence of exciton only. In the regions marked as 1, population inversion occurs between $|b\rangle$ and $|e\rangle$ as well as between $|b\rangle$ and $|e\rangle$. In the regions 2, inversion occurs between $|b\rangle$ and $|e\rangle$ states.
FIG. 5: Photon statistics $p(n, t)$ in a small quantum dot of CdS embedded in a microcavity for $\bar{n} = 10$. The contour plot $a$ is obtained in the absence of biexcitons while $b$ is obtained in the presence of biexcitons. Plots (c) and (d) are obtained for $n = 10$. 
FIG. 6: The temporal variation of excitonic inversion \( (W_{oe}) \) in a QD of CdS. Curve \( a \) is obtained in the presence of biexcitons while curve \( b \) is obtained in the absence of biexcitons.
FIG. 7: The nature of dependence of exciton and biexciton population inversion ($W_{0e}$ and $W_{eb}$) on time in a small quantum dot of CdS.
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