Dynamical Likelihood Method and Top Quark Mass Measurement at CDF

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Abstract. We report the current status of the top quark mass measurement at CDF (Collider Detector at Fermilab). There are several methods of analysis in CDF, which are called by different names: (a) template method, (b) matrix element method, (c) quantized dynamical likelihood method. The purpose of this talk is to focus on methodologies and review these different methods in a coherent way.

1. Introduction
In this talk, we report results of the top quark mass measurements at Tevatron as of today, and overview the common and different features of the analysis methods being used for these measurements. Significance of the top quark mass measurement has been well recognized. The mass value is important by itself as a basic physics constant, but it also provides the mass range of the Standard Model Higgs boson when combined with the mass of the $W$ boson (Fig.1(a)).

The top quark pairs $t\bar{t}$ are produced in the $p\bar{p}$ collisions by the quark-antiquark annihilations or by the gluon fusions, where the former is the dominant process. A top quark ($t$ or $\bar{t}$) decays into a $W$ boson plus a $b$ quark which is observed as a jet. According to the decay modes of $W$ boson, the processes are classified as (a) lepton+4jets ($l+4j$), where one $W$ makes leptonic ($l(e, \mu)\bar{\nu}$ or its charge conjugate) and the other hadronic ($qq'$) decay, (b) dilepton, where two $W$’s make the leptonic decays, and (c) all-hadronic, where two $W$’s make the hadronic decays.

2. Dynamical Likelihood Method (DLM)
In this talk, we use the word 'DLM' in its wide sense. The concept was originally proposed by the author in Refs [1], and a normalization of the cross section was proposed by Dalitz and Goldstein [2]. The applications of the method to real data have been developed by D0 [3] and CDF groups at Tevatron.

Motivation and goal of DLM Since early days of the hadron collider experiments, uncertainties in collider events have been well recognized. These are uncertainties in the momentum-energy measurements of jets, parton-jet identification, the unknown initial state, momenta of missing particles, and the signal vs. background event identification. On the other hand, the general purpose $4\pi$ detectors widely employed in today’s collider experiments have an advantage that they give overall pictures of individual events.
Figure 1. (a) The Standard Model Higgs mass region constrained by experiments. The top quark mass uncertainty from Tevatron in the future is tentatively estimated as $\pm 3 \text{ GeV}/c^2$ in this figure.

(b) Results of the top quark mass measurement as of summer 2005. The data sample classified according to the $W$ decay modes are indicated. The CDF and D0 average value from Run 1 (1994-95) is $M_{\text{top}} = 178.0 \pm 4.3 \text{ GeV}/c^2$ and the preliminary value of the CDF average up to present is $172.7 \pm 4.3 \text{ GeV}/c^2$.

The DLM was proposed to make a probabilistic full reconstruction of each event and derive the dynamical parameters for the process. The parameters include the mass, the decay width, the coupling constants and the signal vs. background probability.

**General feature of DLM** Three different versions of the method are currently used at CDF.

In all these methods, a theoretical input, i.e. the differential cross section formula, is introduced to the event reconstruction in one way or another. This is primarily because the true mass of the top quark that is in a virtual state can not be measured directly but included in the theory as a pole mass, which we call a “dynamical” parameter. The traditional way of measuring the pole mass of a resonant state is to compare the distribution of the invariant mass of decay products from the resonance with that of Monte Carlo (M.C.) events by changing the input pole mass.

In DLM, the form of the differential cross section or its approximate form is explicitly used in the event reconstruction. The quantities to be measured are not invariant masses of top quarks $\sqrt{s_{11}}$ or $\sqrt{s_{22}}$, where suffices 1 and 2 are used to specify the top pair ignoring the charge information, but is the pole mass as a parameter which is obtained by the statistical inference. This statistical inference is in principle equivalent to generation of M.C. events, but the regenerated events in DLM are only those which are consistent with the measured quantities, making M.C. generation more efficient than the traditional method.
A comment in passing is on comparison with the Neural Network Method (N.N.M). In N.N.M., one has to be clever to choose an appropriate set of observables, while the cross section used in DLM consists of the necessary and satisfactory set of observables.

**Parton process, observables and transfer function**

The parton level process, or ‘hard scattering’ process, is written as

\[ a/A + b/B \rightarrow \cdots \rightarrow c_1 + \cdots + c_n \equiv C, \]  

where \( a \) and \( b \) are the initial partons, each representing a quark or an anti-quark or a gluon, in beam particles \( A \) and \( B \) respectively, and \( c(c_1, c_2, \ldots, c_n) \) are the final state partons. States of partons in Eq. (1) are after the initial- and before the final-state radiations.

States of the initial partons are given by the parton distribution functions. If a final parton \( c_i \) is a quark or gluon, it generates parton showers, which fragment into hadrons, and are observed as a jet. We denote in general the observed quantities (observables) by \( y(y_1, \ldots, y_m) \) and parton level quantities by \( x(x_1, \ldots, x_{3n})(m \leq 3n) \). For observables, we assume \( x_i \) one-to-one corresponds to \( y_i \).

**Transfer function**

The cross section used in DLM is for process (1). We need to infer the parton level quantities \( x \) from observables \( y \). The probability density function for \( y \) when \( x \) is given is called the transfer function (TF) \( w(y|x; \alpha) \) where \( \alpha \) is a dynamical parameter set.

The TF generally depends on the assumptions in the reconstruction, (a) process; signal or background, and if background, which background, and (b) topology; identification of jets with quarks. The notation \( w(y|x; \alpha) \) needs to be modified to specify these ambiguities, which we abbreviate here for simplicity.

In the event reconstruction, one assumes a process and a topology, and infers \( x \) from \( y \) using a posterior TF \( w(x|y; \alpha) \). Once \( x \) is inferred, the 4-momenta of the initial, intermediate and final partons in process (1) are determined by the energy-momentum conservations in vertexes of the Feynman diagram except for the neutrino momentum ambiguities which we discuss below.

**Neutrino momentum ambiguity**

The only observable associated with the neutrino(s) is the missing transverse energy (MET), a two-dimensional vector,

\[ \vec{E}_T = \vec{E}^{\text{obs}}_T = -(\vec{E}^{\text{cal}}_T + \sum \mu_T). \]  

where \( \vec{E}^{\text{cal}}_T \) is the vector sum of transverse component of the energy flow measured by calorimeters, and \( \mu_T \) is the transverse momentum of a muon. We assume that \( \vec{E}_T \) corresponds to the sum of transverse momenta of neutrinos involved in the process. There are \( 1 \) and \( 2 \) neutrinos in the 1+4j and dilepton processes, respectively. To get neutrino momenta, we infer the invariant masses of a lepton+neutrino system,

\[ s_W \equiv s_{l\nu} = (l + \nu)^2 \]  

according to the propagator factor in the cross section. For 1+4j process, there are 4 constraints; 2 from \( \vec{E}_T \), 1 from Eq. (3) and 1 from the mass relation of a neutrino. For dilepton process, there are 8 constraints; 2 from \( \vec{E}_T \), 2 from \( s_W \)'s and 2 from \( s_l \)'s, and 2 from mass relations for neutrinos. The equations for neutrino momenta are reduced to a quadratic (quartic) equation for the 1+4j (dilepton) process. In general, there are plural solutions for neutrino momenta.

3. Three versions of the likelihood used at CDF

Many people/groups are working on the top mass measurement at CDF. We pick up typical algorithms that already gave preliminary results; “Template Method”, “Matrix Element Method” and “QDLM” in CDF.
Template Method (T.M.) [5] [6] In the first observation of the top quark at CDF [4], a method currently called 'Template method' was used. The $\chi^2$ for the likelihood $L(M)$ is defined by

$$\chi^2(M) \equiv -2 \ln[L(M)] = \chi^2_p + \chi^2_t,$$

where

$$\chi^2_p = \sum_{i=1}^{2} \frac{(\sqrt{s}_{W_i} - M_W)^2}{\Gamma_W^2} + \sum_{i=1}^{2} \frac{(\sqrt{s}_{t_i} - M_t)^2}{\Gamma_t^2},$$

$$\chi^2_t = \sum_{\text{lepton}} \frac{(p_T^l - p_{T}^{l'})^2}{\sigma_{pT}^2} + \sum_{\text{jet}} \frac{(p_T^j - p_{T}^{j'})^2}{\sigma_{pT}^2}.$$

In these equations, $s$'s and $p$'s are parton level, and $p^*$ are observed, quantities. The uncertainties $\sigma$'s are obtained from the energy-scale study of particles. The $\chi^2_p$ term is the non-relativistic, Gaussian approximation of the propagator factors for $W$'s and $t$'s, while the $\chi^2_t$ term is a Gaussian approximation of the TF for MET and jet energies.

Matrix Element Method (M.E.M.) [7] The formulation of the method was proposed by Dalitz and Goldstein [2], and the method was applied by D0 group to the top mass measurement in the $l^++4j$ channel [3]. The essential feature of the likelihood is expressed as

$$L(M) = \frac{1}{\epsilon \sigma_T(M)} \int \frac{d\sigma}{d\Phi_{n}} w(x|y; M) dx' dx.$$

where $\sigma_T(M)$ is the total cross section for the process, and $\epsilon$ is the detection efficiency, $x$ is a set of the parton level kinematic variables observed as $y$, $x'$ is a set of the parton level kinematic variables which are not observed.

Quantized DLM (QDLM) [5] [8] This is a new formulation of DLM being developed at CDF. For the formulation, one postulates [9],

1. In each event, the final state partons occupy a unit phase space volume:

$$d\Phi_{n}^{(f)} = \prod_{i=1}^{n} \frac{d^3p_i}{(2\pi)^3 2E_i} = 1,$$

hence the differential cross section for a single event is given by $d\sigma/d\Phi_{n}^{(f)}$.

2. The likelihood for a single event is defined by the Poisson probability for one event,

$$L(\mu) = \mu \exp(-\mu), \quad \mu = l_1 d\sigma/d\Phi_{n}^{(f)},$$

where the Poisson probability is used in a posterior way, namely, the likelihood is a p.d.f. for $\mu$ when $n = 1$ is given. In Eq. (6), $l_1$ is the luminosity required for a given event to be produced, and is unknown on the event-by-event basis. We assume $l_1$ is an arbitrary constant that is common to all events. In QDLM analysis to be discussed in this talk, we take $\mu << 1$ hence $L = \mu$.

Comparison of the three algorithms The M.E.M. is formulated according to the traditional form of the cross section, and normalized by the total cross section. The QDLM emphasizes the posterior nature of the probability: a given event is assumed to be independent of nearby events in the phase space, and the probability does not depend on the final state density. The T.M. picks up the essential part of QDLM; the Gaussian approximations for the propagator factor and the TF.

4. Data samples in the three methods

In Table 1 and 2 we show the number of selected candidate events for data and M.C. Background contents are also shown. The event selection criteria in the three methods are those which optimize the methods.
Figure 2. (a) The first observation of the top quark at CDF. The Template Method was used. Yellow: background M.C., Blue: signal plus total background M.C., Red: real data. The inset is the $\chi^2$ of the likelihood as a function of $M_{\text{top}}$. (b) Typical background process to $t\bar{t}$ production.

| Source | Events |
|--------|--------|
| Expected $t\bar{t}$ ($M_{\text{top}} = 178$ GeV/c$^2$, $\sigma = 6.1$ pb) | $15.9 \pm 1.4$ |
| Expected Background | $10.5 \pm 1.9$ |
| Drell-Yan ($Z/\gamma$) | $5.5 \pm 1.3$ |
| Misidentified Lepton | $3.5 \pm 1.4$ |
| Diboson ($WW/WZ$) | $1.6 \pm 0.2$ |
| Total Expected | $26.4 \pm 2.3$ |
| Run II Data | 33 |

Table 1. Expected numbers of signal and background events for dilepton M.E.M. The integrated luminosity of the data sample is $340 \text{ pb}^{-1}$.

| Source | Events $\geq b$ tag QDLM sample |
|--------|---------------------------------|
| Expected Background | $26.6 \pm 3.0$ | $9.2 \pm 1.8$ |
| $W$+jets | $19.6 \pm 2.4$ | $5.3 \pm 1.1$ |
| Multi-jets | $4.7 \pm 0.7$ | $3.1 \pm 1.0$ |
| Other | $2.3 \pm 3.0$ | $0.8 \pm 0.1$ |
| Selected $t\bar{t}$ events in data | 121 | 63 |

Table 2. The background composition and the number of $t\bar{t}$ candidates for events with $\geq 1b$ tag (T.M.) and for subset used in QDLM analysis.

5. Single event likelihood

From a given observable set $y$ of an event, one infers the parton state (kinematics) of the event. To define the parton state, one first has to assume the process, i.e. signal or a background, and the topology, i.e. quark-jet identifications. Then one infers the top quark mass $M_{\text{top}}$ and the parton
kinematic variable \( x \). The definition of the single event likelihood with respect to the process and topology assignments depends on the methods. Details of these options are found in Refs. [5], [6], [7], [8] and [9].

**Inferences of \( M_{\text{top}} \) and \( x \)** The top mass \( M_{\text{top}} \) is inferred uniformly, and for a single value of \( M_{\text{top}} \), variables \( x \) is inferred a large number of times, typically of the order of \( 10^5 \). In case of M.E.M. this is made as a numerical integration over \( x \). We need a large number of inferences, because there are many components of \( x \) (including \( s \)) to be inferred independently. The inferences of the neutrino momenta are made by solving the neutrino equations. By scanning of \((x, M_{\text{top}})\), one gets the likelihood \( L(M_{\text{top}}) \) as a function of \( M_{\text{top}} \), the maximum likelihood mass \( \hat{M}_{\text{top}} \) and the maximum likelihood parton variables \( \hat{x} \).

In Fig.3, we show event-by-event \( L(M_{\text{top}}) \) by QDLM and \( \hat{M}_{\text{top}} \) from M.T. for \( l^+4j \bar{t}t \) candidate events. We see the global consistency, but we also recognize the differences of the two methods, i.e. the peak positions of \( L(M_{\text{top}}) \) from QDLM do not necessarily agree with the \( \hat{M}_{\text{top}} \)’s from T.M.

**Transfer function and its comparison with real data** By the full event reconstruction in DLM, the maximum likelihood energies of individual jets are determined. The shift of the quark energy from the corresponding jet energy, \( \hat{E}_{\text{quark}} - E_{\text{jet}} \), is supposed to distribute according to the TF. Figure 4 shows an example of QDLM TF’s: the distribution of \( \xi \equiv (\hat{E}_{\text{quark}} - E_{\text{jet}})/\hat{E}_{\text{quark}} \) (a) for \( W \)-jets (jets by \( q \) and \( q' \) from \( W \) ) and (b) for \( b \)-jet. The M.C. set includes backgrounds in Table 2. The distribution of the shifts in the real events are consistent with the TF determined by the M.C. events.

**Likelihood functions for signal and background events** The \( M_{\text{top}} \) dependences of \( \chi^2 \) of the \( t\bar{t} \) likelihood function \( L_{\text{eq}}(M_{\text{top}}) \) by QDLM are shown in Fig.5: (a) for the parton level and (b) for the fully simulated events. Samples are signal and background sets. We observe in (a) that the \( \chi^2 \) dip for the signal events are centered around the input mass 175 GeV/c\(^2\), while the \( \chi^2 \) minimum for the background is located at or near the edge of the search region. The difference of \( L_{\text{eq}}(M_{\text{top}}) \) between the signal and background for the simulated events is less clear, but the result shows the effectiveness of the use of DLM for the signal vs. background identification.

**The process likelihood of each event as a signal or a background** Each candidate event of the data sample is given after the selection by certain event selection criteria. But on the event-by-event basis one does not know whether the given event is a signal or a background. Since the precise value of \( M_{\text{top}} \) is not available at this stage, we evaluate the process likelihood of the event by integrating the likelihood function in the mass range by

\[
< L >_{\text{sig/bkgd}} \equiv \int_{\text{min}}^{\text{max}} L_{\text{sig/bkgd}}(M) dM.
\]

The definition of the process by \( < L >_{\text{sig/bkgd}} \) may not be quite efficient, since the differences on \( M_{\text{top}} \) dependence as discussed in the preceding paragraph are masked by the integration. This point is to be improved in the future.

The distributions of the process likelihood are shown in Fig.6: (a) \( t\bar{t} \) likelihood by QDLM, (b) \( t\bar{t} \) and (c) \( Z+4j \) likelihood by M.E.M.

**Event-by-event maximum likelihood mass distribution** Figure 7 shows the distribution of \( \hat{M}_{\text{top}} \) for individual events in \( l^+4j \) channel: (a) by T.M. and (b) by QDLM. The contributions of the signal and the background processes are shown. The distribution in the real data including background (arrows in (b)) is consistent with that of the total M.C. processes.

### 6. Likelihood from a set of events

**Procedure to determine the maximum likelihood mass and its uncertainty** The likelihood functions for individual events, \( L_i(M)(i = 1, \cdots, N_{\text{ev}}) \), are mutually independent. Hence, we take the joint
Figure 3. Event-by-event mass likelihood functions $L(M_{top})$ by QDLM (curves) and the maximum likelihood masses $\hat{M}_{top}$ by T.M. (black line), for events selected by QDLM. Events with red curves are 2 $b$-tagged.

**Figure 4.** The test of the transfer functions in $l+4j$ channel by QDLM. The energy shift for a jet from the corresponding quark is shown: (a) $W$-jet, (b) $b$-jet.

The likelihood of events of a given data set,

$$L(M_{top}) = \prod_{i=1}^{N_{\text{ev}}} L^{(i)}(M_{top}).$$

The $\chi^2$ of Eq. (8) is fitted with a parabolic function of $M_{top}$. The maximum likelihood mass value, which we denote by $\hat{M}_{top}$, is obtained by the mass at the parabolic bottom. The uncertainty $\sigma_M$ of $\hat{M}_{top}$ is evaluated from the mass values which give $\Delta\chi^2 = +1$ from the parabolic bottom.
Figure 5. The likelihood function $L(M_{\text{top}})$ as a signal event. (a) for events at the parton level, (b) for fully simulated events.

Figure 6. Distribution of the process likelihood as a signal or a background for individual events: (a) Signal ($t\bar{t}$) likelihood in the $l+4j$ channel by the QDLM. (b) A background ($Z+4\text{jets}$) likelihood in the dilepton channel by the M.E.M. (c) Signal ($t\bar{t}$) likelihood in the dilepton channel by the M.E.M.

6.1. Examinations of the reconstructed $\hat{M}_{\text{top}}$

Examinations of the methods can be made on various aspects using M.C. and real events.

Use of parton level events

Reconstruction of $\hat{M}_{\text{top}}$ using the parton level signal events and its
Figure 7. Distribution of the event-by-event maximum likelihood top quark mass $\hat{M}_{\text{top}}$ from the $l+4j$ samples: (a) T.M. and (b) QDLM.

comparison with the input mass $M_{\text{top}}$ of the M.C. events are useful. Typical checks are as follows.

1. To test the basic algorithm of a method, one applies the method to M.C. events at parton level where complete kinematics of process (1) is known.

2. To test the effect of the neutrino solution ambiguity, one sets $E_T = \sum p_{\nu T}$.

3. To test the transfer function for a certain variable, one uses the parton level quantities for the other variables.

Joint likelihood function or its $\chi^2$ The full reconstruction procedure, including the inference of the parton variables $\mathbf{x}$ from observables $\mathbf{y}$ with the TF, and $\hat{s}$ with the propagator factors, gives the likelihood or its $\chi^2$ as a function of $M_{\text{top}}$. Figures 8 show the results of analyses by M.E.M and QDLM on real data; (a) dilepton channel by M.E.M., where the background corrections have been made, (b) $l+4j$ channel by QDLM, where the parabolic bottom gives $\hat{M}_{\text{top}}$ before corrections.

Elimination of the background effect to $\hat{M}_{\text{top}}$ In events selected as $t\bar{t}$ candidates are included the background events. Thus, one should eliminate the background effects to $\hat{M}_{\text{top}}$.

Figures 9(a) and (b) are examples of the background study by QDLM; 9(a) shows the relation between the M.C. input mass $M_{\text{top}}$ and the reconstructed value $\hat{M}_{\text{top}}$. The M.C. samples are prepared such that they contain the background fractions according to the Poisson statistics. 9(b) shows background corrected $\hat{M}_{\text{top}}$ as a function of the background fraction. The background fraction in the QDLM sample is estimated to be 14.5% from the study of the event selection criteria applied to the sample. The signal M.C. sample is for $M_{\text{top}} = 172.5\,\text{GeV}/c^2$. From this study, the background corrected mass value is estimated as $\hat{M}_{\text{top}} = 173.2\,\text{GeV}/c^2$.

In Figs.10 are shown the final calibration for the reconstructed mass by using the signal+background M.C. sample and making the background correction: (a) dilepton events by M.E.M. and (b) $l+4j$ events by QDLM. One sees the consistencies of the methods.

6.2. Pseudo-experiment

To check the uncertainty $\sigma_M$ evaluated for a given data set, we examine the statistical fluctuation of the data as a set. Namely, we prepare M.C. samples each consisting of the same number of events as the real data set. With these samples we get the distribution of the ‘pull’, i.e. the shift of the reconstructed mass from the input, $\Delta \hat{M}_{\text{top}} = \hat{M}_{\text{top}} - M_{\text{top}}$. The evaluated $\sigma_M$ for each set should be rescaled based on the pull distribution. We call such a procedure the pseudo-experiment.
Figure 8. The joint likelihood and its $\chi^2$ of the real data sets: (a) for the dilepton channel obtained by M.E.M. and (b) for the $l+4j$ channel obtained by QDLM. Without any correction, one gets $M_{\text{top}} = 171.8\text{GeV}/c^2$.

Figure 9. The background effects to the reconstructed top quark mass. By using M.C. samples with different background fractions, we get: (a) the relation between the input and output masses with background fraction as a parameter, (b) the reconstructed mass plotted as a function of the background fraction.

Figure 11 shows results of the pseudo-experiments in two methods: (a) the M.E.M. for dilepton channel and (b) the T.M. for $l+4j$ channel. Rescaled values of $\sigma_M$ for the real data set are also shown.

7. Systematic uncertainty of the top quark mass

We show in Table 3 the systematic uncertainties $\Delta M_{\text{top}}$ for $l+4j$ data estimated by QDLM and Template method.

In situ calibration of jet energy scale  In CDF, the jet energy scale is calibrated on the basis of the calorimeter responses in the test beam and jet events by collision accompanied by the photon or $Z(\to l^+l^-)$. The reconstruction of $t\bar{t}$ events makes an in situ calibration of the jet energy scale possible. Figure 12 shows a contour map of the likelihood against the $W$-jet energy scale and $M_{\text{top}}$ in 1-b tagged $l+4j$ events. This calibration is made possible since the mass of $W$ has been measured to a good accuracy.
Figure 10. Reconstructed $\hat{M}_{\text{top}}$ vs. input $M_{\text{top}}$ for M.C. samples: (a) Dilepton events by M.E.M. and (b) $l+4j$ events by QDLM. Background events are included in the sample, but their effects to $\hat{M}_{\text{top}}$ is corrected.

Figure 11. The ‘pull’ distributions in the pseudo-experiments: (a) dilepton data sets by M.E.M., (b) $l+4j$ data set by T.M. In (b), $+\sigma_M$ and $-\sigma_M$ are independently determined. A red line in (a) and arrows in (b) indicate rescaled $\sigma_M$’s for the real data sets in the respective channels.

8. Future prospects

We have reviewed currently available results of the top quark mass measurement at CDF. The applications of the methods discussed in this talk for other channels are in progress, for example, M.E.M. to the $l+4j$ channel, the mass measurement from the all-hadronic events. The work to combine the CDF and D0 results is just going to start. With higher statistics expected to be obtained until 2009, the uncertainty of the top quark mass will be squeezed to below 2 GeV/$c^2$. The quantitative arguments of all these plans are yet to come and beyond the scope of this talk. We believe, however, that we are on the right track for the precision measurement of the top quark mass.

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[2] R.H. Dalitz and G.R. Goldstein : Phys.Rev. D45 (1992) 1531.
| Source          | QDLM $\Delta M_{top}$ (GeV/c$^2$) | Template $\Delta M_{top}$ (GeV/c$^2$) |
|-----------------|------------------------------------|----------------------------------------|
| Jet Energy Scale| 3.0                                | [2.5]                                  |
| ISR/FSR         | 0.6                                | 0.7                                    |
| PDF             | 0.5                                | 0.3                                    |
| Modeling        | 0.8                                | 0.9                                    |
| Method          | 0.5                                | 0.6                                    |
| **Total**       | 3.2                                | 1.3                                    |

**Table 3.** The systematic uncertainties by various sources in the l+4j channel. The results from DLM and Template method are shown.

**Figure 12.** The reconstructed mass as a function of input mass with the background fraction from 0% to 50%.

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[8] Later published as CDF Collaboration: Phys. Rev. D73 (2006) 092002. In this reference, a general term DLM is used to specify the method, while we use acronym QDLM to differentiate the method from others.
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