A fully coupled damage model with stress triaxiality and Lode dependence

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Abstract. In the present work, an advanced CDM model considering stress triaxiality and Lode angle effect is proposed. The framework of thermodynamics of irreversible processes with state variables is used to build the constitutive equations accounting for strong and full coupling between all the dissipative phenomena. The proposed model is implemented into Finite Element (FE) code ABAQUS/Explicit via a user material subroutine (VUMAT). A detailed parametric study with various values of the new material parameters is conducted in order to show the predictive capability of the proposed model. Applications to sheet metal forming simulation have been performed to validate the damage prediction capability of the proposed model, and the numerical simulation results are analysed and discussed.

1. Introduction

Modeling and simulation of damage and fracture of metallic materials are important issues in sheet metal forming industry[1]. In complex forming processes, reliable and accurate predictions of ductile damage is still a challenging task, and advanced constitutive models are still required. Up to date, two kinds of methodologies based on uncoupled and coupled models are used by many researchers and engineers. Among these models, the CDM (Continuum Damage Mechanics) framework is increasingly used in damage prediction for sheet metal forming, since it is convenient in coupling with different physical phenomena[2].

However, the classical CDM model shows limitations in damage prediction under shear dominated and low stress triaxiality stress states[3]. As observed by experiments, the equivalent plastic strain at fracture is highly dependent on the Lode angle and stress triaxiality for many sheet metals[4]. Therefore, more advanced fracture models have been proposed to consider the stress state, i.e., Wierzbicki-type models[4], Lou-Huh model[5], modified GTN model[6], enhanced CDM models[7, 8]. Based on these models, the improvement of the damage prediction accuracy were evident in sheet metal forming processes[9].

The objective of the present study is to enhance the fully coupled damage model by accounting for stress state dependence. The formulation of the model is performed in the framework of thermodynamics of irreversible processes with state variables. The proposed fully coupled damage model is then implemented into Abaqus/Explicit finite element code. After a relatively exhaustive
parametric study, the material parameters are identified using experimental results taken from literature for application purpose. A square cup deep drawing simulation is made, which shows good agreement with the experimental results.

2. Fully coupled damage model

The Helmholtz free energy, a convex and closed function of strain-like state variables in the effective strain space, is taken as a state potential. The elastic strain and Cauchy stress tensor are decomposed into hydrostatic part \( \left( \varepsilon^H, \sigma^H \right) \) and deviatoric part \( \left( \varepsilon^D, \Sigma \right) \); \( (r, R) \) denotes isotropic hardening; \( (\varepsilon, X) \) represents kinematic hardening and finally \( (\alpha, Y) \) represents isotropic ductile damage. The damage scalar variable \( d \) records the isotropic damage state on a material point and evolves from initial undamaged state \( (d = 0) \) to fully damaged state \( (d = 1) \) indicating the final fracture of that material point. To the above presented state variables, appropriate effective state variables can be defined based on the total energy equivalent assumption in the following form:

\[
\begin{align*}
\varepsilon^H &= \sqrt{1 - h(\eta)} d^{-\varepsilon^H} \quad \text{and} \quad \sigma^H = \frac{\sigma^H}{\sqrt{1 - h(\eta)d^{-\varepsilon^H}}} \\
\varepsilon^D &= \sqrt{1 - h(\eta)} d^{-\varepsilon^D} \quad \text{and} \quad \Sigma = \frac{\Sigma}{\sqrt{1 - h(\eta)d^{-\varepsilon^D}}} \\
\varepsilon_k &= \sqrt{1 - h(\eta)} d^{-\varepsilon_k} \quad \text{and} \quad X = \frac{X}{\sqrt{1 - h(\eta)d^{-\varepsilon_k}}} \\
\sigma_k &= \sqrt{1 - h(\eta)} d^{-\sigma_k} \quad \text{and} \quad R = \frac{R}{\sqrt{1 - h(\eta)d^{-\sigma_k}}}
\end{align*}
\]

The parameters \( \gamma_k \) and \( \gamma_r \) are used to differentiate between the effect of isotropic damage on elastic hydrostatic stress and isotropic hardening respectively. The microcracks closure parameter \( h(\eta) \) is dependent on stress triaxiality \( \eta = \frac{J_2(\bar{\sigma})}{J_2(\bar{\sigma})} \) as follows:

\[
h(\eta) = \frac{1 + h_i}{2} + \frac{1 - h_i}{2} \tanh(\xi_\eta) \]

where \( h \) is the value of \( h \) at negative stress triaxiality and the parameters \( \xi_\eta \) control the evolution of \( h \) from positive to negative stress triaxiality.

The specific Helmholtz free energy \( \Psi(\varepsilon, \varepsilon_k, \varepsilon_r, T, d) = \Psi(\varepsilon, \varepsilon_k, \varepsilon_r, T) \), defined in the fictive undamaged configuration, is taken as a state potential. In which, \( \Psi \) is the elastic part and \( \Psi \) plastic part:

\[
\begin{align*}
\rho'\varepsilon &= \rho'\varepsilon^e(\varepsilon^e, \varepsilon^D) + \rho'\Psi(\varepsilon_k, \varepsilon_r) + \rho'\Psi(\varepsilon^e, \varepsilon^D, d) + \rho'\Psi(\varepsilon, \varepsilon_r, d) \\
&= \frac{1}{2} \left(1 - h(\eta)d^{-\varepsilon^e}\right) \kappa_e(\varepsilon^e, \varepsilon^D) + (1 - h(\eta)d) \mu_k(\varepsilon^D, \varepsilon^D) + \frac{1}{3} \left(1 - h(\eta)d\right) C \varepsilon_k + \frac{1}{2} \left(1 - h(\eta)d^{-\varepsilon^e}\right) Q r^2
\end{align*}
\]

Based on the state potential defined above, the stress-like variables are derived and summarized in Table 1. In these equations \( \kappa_e \) is the compressibility modulus defined by the following equation \( \kappa_e = (3\lambda_0 + 2\mu) / 3 = E / (3(1-2\nu)) \), \( \mu_k \) and \( \lambda_0 \) are the Lame’s constants \( (\lambda_0 = \nu E / ((1+\nu)(1-2\nu)) \) and \( \mu_k = E / (2(1+\nu)) \), \( \varepsilon \) and \( \nu \) are the Young’s modulus and the Poisson’s ratio, \( C \) and \( Q \) are the kinematic and isotropic hardening moduli, \( \rho \) is the material density.

Since this paper focuses on developing damage model with stress state dependence, the classical isotropic von Mises quadratic criterion is used. Remarkably, more advanced quadratic and non-quadratic anisotropic yield function can be easily adopted. Assuming the non-associative plasticity
theory, the quadratic von Mises yield criterion \( f(\sigma, X, R, d) \) and plastic dissipation potential \( F(\sigma, X, R, Y, d) \) are defined as follows:

\[
f(\sigma, X, R, d) = \frac{\|\sigma - X\|_2}{\sqrt{1 - h(\eta)d}} - \frac{R}{\sqrt{1 - h(\eta)d}} - \sigma_r \leq 0
\]

\[
F(\sigma, X, R, Y, d) = f + \frac{3a(X : X)}{4C(1 - h(\eta)d)} + \frac{bR^2}{2Q(1 - h(\eta)d)} + \frac{S(\bar{\theta})}{(s + 1)(1 - h(\eta)d)\theta} \left( Y - Y_s \right)^{(s+1)}
\]

where \( \sigma_r \) is the yield stress, and \( a \) and \( b \) are material coefficients defining the nonlinear evolution of isotropic and kinematic hardening, and \( S, s, Y, \beta \) are damage parameters. From Eq. (8), all the fluxes governing the evolution of the dissipative phenomena can be derived, as given in Table 1. To enhance the shear fracture prediction of CDM model, the damage parameter \( S \) is defined as appropriate function of normalized Lode angle \( \bar{\theta} \) to differentiate between the damage evolution under tension and shear loading paths:

\[
S(\bar{\theta}) = S_{sa} + (S_{so} - S_{sa}) \tanh\left(\frac{\bar{\theta}}{\bar{\theta}_0}\right)
\]

where the normalized Lode angle \( \bar{\theta} \) \((-1 \leq \bar{\theta} \leq 1)\) is defined as follows:

\[
\bar{\theta} = 1 - \frac{6\theta}{\pi} = 1 - \frac{2}{\pi} \arccos\left(\frac{3\sqrt{3}J_2}{2J_2}\right)
\]

The parameters \( S_{sa} \) and \( S_{so} \) represent ductility of material in shear (i.e. \( S(\bar{\theta} = 0) = S_{sa} \)) and in tension (i.e. \( S(\bar{\theta} = \pm 1) = S_{so} \)) respectively, while \( \xi \) adjust the effect of \( \bar{\theta} \).

### Table 1. Stress-like variables and evolution equations.

| Stress-like variables | Evolution equations |
|-----------------------|---------------------|
| Cauchy stress \( \sigma'' \) | \( D^p = \lambda \frac{\partial F}{\partial \sigma} = \lambda \hat{\eta} = \frac{\lambda}{\sqrt{1 - h(\eta)d}} \hat{n} \) |
| \( S = \rho \frac{\partial \Psi}{\partial \epsilon^c} = 2(1 - h(\eta)d) \mu_c \epsilon^c \) | \( \hat{n} = H(\sigma - X) \left\| \sigma - X \right\|_2 \) |
| Kinematic hardening \( X = \rho \frac{\partial \Psi}{\partial \epsilon} = \frac{2}{3} (1 - h(\eta)d)C \epsilon \) | \( \hat{\epsilon} = \lambda \frac{\partial F}{\partial \epsilon} = \lambda \left( \frac{\hat{n}}{\sqrt{1 - h(\eta)d}} - a \alpha \right) \) |
| Isotropic hardening \( R = \rho \frac{\partial \Psi}{\partial \tau} = (1 - h(\eta)d)\gamma \) | \( \hat{\tau} = \lambda \frac{\partial F}{\partial \tau} = \lambda \left( \frac{1}{\sqrt{1 - h(\eta)d}} - b \right) \) |
| Damage \( Y = -\rho \frac{\partial \Psi}{\partial \alpha} = \frac{1}{2} h(\eta)\gamma \cdot d^{-1} \kappa_c (\epsilon^m : \epsilon^m) + \frac{1}{2} h(\eta)\gamma \cdot \alpha \) | \( \hat{\alpha} = \lambda \frac{\partial F}{\partial \alpha} = \lambda \left( \frac{1}{\sqrt{1 - h(\eta)d}} \left( \frac{Y - Y_s}{S(\bar{\theta})} \right)^\gamma \right) \) |

### 3. Parametric study of the proposed model

In this work, the stress triaxiality effect is embedded in the microcracks closure effect with the function \( h \) depending on stress triaxiality as shown in Eq. (6). From this dependent function, the desired values of \( h \) in tension, shear and compression can be achieved by adjusting the two parameters \( h_c \) and \( \xi_h \). Figure 1a displays the effect of parameter \( h_c \) on the evolution of accumulated plastic strain (computed by time integration of \( \dot{\rho} = \sqrt{(2/3)D^p : D^p} \)) at fracture according to stress...
triaxiality. Note that the case $h_c=1.0$ corresponds to $h$ independent from stress triaxiality ($h=1.0$). While with the decrease of $h_c$, the fracture strain at low triaxiality is increased. As $h_c$ decreases from $h_c=1.0$ to $h_c=0.1$, the final fracture plastic strain (i.e. ductility) increases significantly for lower stress triaxiality. Since the negative value and positive value of triaxiality represent the compressive and tensile loading respectively, the parameter $h_c$ controls the different damage evolutions from tension to compression.

![Figure 1](image1.png)

**Figure 1.** Effect of parameter $h_c$ on the curves accumulated plastic strain vs. stress triaxiality.

The Lode angle effect is taken into account in ductility parameter $S$ which becomes function of the normalized Lode angle as given in Eq. (9). The parameter $S_{sh}$, $S_{st}$, and $\xi$ are used to adjust the value of the damage parameters $S$ according to different stress states. When the case $S_{st}=S_{sh}$ is considered the Lode angle will not have any effect on the damage evolution. When the Lode angle effect is taken into account (with $S_{st}=0.5$ and $S_{sh}=0.1$), the equivalent plastic strain at fracture is no longer a monotonic evolution with stress triaxiality, as shown in Figure 1b. It increases with the increase of stress triaxiality within the range $(0 \leq \eta \leq 0.33)$ and $(0.566 \leq \eta \leq 0.66)$, meanwhile, it decreases with the increase of stress triaxiality within the remaining ranges $(-0.33 \leq \eta \leq 0)$ and $(0.33 \leq \eta \leq 0.566)$.

The parameter $h_c$ does not affect the value of fracture strain when $\eta \geq 0.33$, a smaller value of $h_c$ leads to a high equivalent plastic strain at fracture for the lower stress triaxiality range. The parameters $S_{st}$ and $S_{sh}$ in the CDM damage model make it possible to describe the Lode angle dependence of fracture strains.

![Figure 2](image2.png)

**Figure 2.** Effect of parameter $S_{sh}$ on the curves of accumulated plastic strain at fracture vs. stress triaxiality considering $S_{st}=0.5$ and $h_c=0.1$. 
Figure 2 shows the effect of the parameter $S_{sh}$ (under the condition $S_{sh} < S_{ten}$) on the evolution of accumulated plastic strain at fracture versus stress triaxiality. This figure shows a local maximum for positive triaxiality corresponding to both simple tension ($\eta = 1/3$) and equi-biaxial tension ($\eta = 2/3$) loading paths. The difference between the fracture plastic strain increases when the difference between $S_{ten}$ and $S_{sh}$ is significant. When $S_{ten} = S_{sh}$, Lode angle effect disappears from the accumulated plastic strain at fracture. While with the decrease of the $S_{sh}$, the Lode angle effect is more obvious.

Figure 3. Effect of Lode angle on the damage surfaces displayed in the deviatoric strain plane.$^{[8]}$

The fracture (or damage) loci including the Lode angle effect are plotted in Figure 3. When $S_{ten} = S_{sh}$, there is no Lode angle effect, the fracture surface is a circular surface (dashed line). When $S_{sh} < S_{ten}$, the shape of fracture locus at constant triaxiality varies from a right hexagon to a six-point star. When $S_{sh} > S_{ten}$, the fracture locus at constant stress triaxiality is transferred to a shape of flower with six petals. Note that for this case of initially isotropic von Mises plasticity, the iso-damage surfaces has a $\pi / 3$ symmetry so that fracture locus at six vertices with $\theta = \pm 1$ are the same. It is observed that the ductile fracture prone to happen at the region between the two vertices (shear or plane strain mode), which was observed experimentally.$^{[5]}$

Through the parametric study, the combined effect of stress triaxiality and Lode angle are discussed above. The Lode angle and triaxiality dependence of the damage surfaces is included in our proposed fully coupled CDM model, which could make accurate predictions of damage in sheet metal forming simulations.

4. Application

The proposed isotropic and isothermal model is applied to a square cup drawing process for Al6014-T4. The material parameters are identified using the available experimental results including various tensile tests from literature$^{[10]}$, as given in Table 2.

| Table 2. Calibrated material parameters. |
|------------------------------------------|
| $E$ (GPa)       | $\nu$ | $\sigma_y$ (MPa) | $Q$ (MPa) | $b$ | $C$ (MPa) | $a$ | $S_{sh}$ | $S_{ten}$ |
| 69.0            | 0.3   | 122.37           | 263.5     | 1.2 | 1725.4    | 16.2| 7.2       | 8.9       |
| $h_0$           | $s$   | $\gamma_0$       | $\gamma_r$| $\beta$ | $Y_0$ (MPa) | $\xi$ | $\psi$   |
| 0.2             | 1.0   | 1.0              | 4.0       | 2.0 | 0.0       | 1.0 | 8.0       |

As shown in Figure 4, the force-displacement curve from simulations has good accordance with the experimentally observed one. The distribution of equivalent plastic strain (SDV15) at final rapture is also presented, strain localizations were observed at the four corners of the punch. When the displacement reached at 15 mm, the predicted crack appears at the punch corner of the sheet and the predicted crack path matches well with the experimental observations. From all the comparison of
simulation results and experiments, the proposed fully coupled damage model with stress state dependence could correctly predict the fracture of deep drawing of Al6014-T4 sheets.

Figure 4. Comparison of numerically predicted and experimentally observed results[10].

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