Considering Time by Quantities in Economic History: The Performance of the Box-Jenkins ARIMA Approach on the 19th Century Smyrna Harbor Foreign Trade Data

**Abstract**

The time-series analysis usually undertakes various estimation procedures by utilizing the quantifiable representations of the research phenomenon. The economic history research, beside the contemporary others, gradually becomes another well repudiated discipline for the same approach especially in cases where the quantifiable observations reside within their findings. However, contrasting with the research areas for which the availability of data sets succinctly reach to the threshold values in terms of sample sizes, in economic history research rarely sufficient amount of observations would be found available. The limiting quantities is often about the multivariate considerations where still various univariate techniques would yield beneficial results. In the same favor, herein, one of the leading approaches, namely the Box-Jenkins (ARIMA) approach to the time-series analysis is applied to the 19th century foreign trade figures compiled for the Smyrna (Izmir) harbor as a historical data-set and the performance of utilized univariate techniques is questioned.

**1. Introduction**

The time-series analysis undertakes an estimation process on the basis of the univariate or the multivariate data types to conclude with a model for forecasting\(^1\). In contemporary terms, the named type of analysis is a fairly well known one in various areas of economics and the economic history research gradually becomes another realm in which the element of economists, after it was initially phrased by A. Marshall "The element of time . . . is the centre of the chief difficulty of almost every economic problem." Hood, W.C., (1948). In specific to the cyclical analysis, as one

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\(^1\) The earliest contributors of the time-series analysis is often cited as Yule (1927), Wold (1938) and Box-Jenkins (1967, 1970) with strict emphasizes to the statistical and econometric contributions (Box, Jenkins & Reinsel (1994)). However, not only in engineering sciences and statistics but also in economics, the same concern, namely the time based analysis of economic phenomena has rather earlier emphasizes.

\(^\Delta\) Yazar(lar) bu çalışmaların tüm süreçlerinin araştırma ve yayın etğine uyun olduğunu, etik kurallara ve bilimsel atif gösterme ilkelерine uygunu beway etmiştir. Akşı bir durumda Akdeniz İİF Dergisi sorununudur değildir.
time-series analysis would also yield beneficial results especially when the quantifiable observations reside within their findings.

Whereas, the initial attempts to quantify the economic history research, in its well formalized systematic, often refers to the scholar definitions of the early 20th century and likely approaches with strong advocacy of quantitative analysis is not indicating the unique effort that adopts similar tools. In that sense, various other approaches of economic history research would also be found adopting different quantitative exercises while questioning their presented findings. However, regardless to the scholar tradition or the scope of the research, the critical issue is given a rise because of the same practical concern, and evenly speaking, this is not the adopted research methodology which favors the quantitative analysis or not, but this is often about the availability of the data, or the quantifiable representations of the research phenomenon to infer.

In a comparison to clarify, the contemporary research areas of the time-series based analysis in economics (i.e. capital market dynamics, the dynamics of tourism and agriculture, various other research areas utilizing the macroeconomic and the microeconomic forecasting) usually infer under the succinct availability of data series which can be found in a quick proximity of the researchers within the suggested threshold values in terms of sample sizes. Contrarily, when the economic history becomes any research area to appeal to the time-series analysis, rarely the sufficient amount of observation would be found available for a candidate research. This contradicting nature of the data provision usually holds for both types of data in economic history research and imposes scarcity for both of the univariate and the multivariate data types. Especially for any multivariate analysis, the lack of requested sample size would be a frequent concern while the availability of univariate data would have a better chance to be kept in the right numbers. Consequently, the multivariate econometric models in economic history research most likely to be neglected from the time-series analysis, however, even in likely cases that the availability of data could not reach up to the requested limits for multivariate analysis, still various forms of univariate techniques would be found beneficial with respect to their results.

Here, into this article, a similar type of constraint has given a lead to the following analysis of the historical foreign trade data, where one of an earlier and a leading approaches of the time-series analysis is applied to the sample in a comparison with the straightforward trend line estimations. The named models will be estimated by utilizing the 19th century foreign trade figures as mentioned and the data belongs to a Mediterranean harbor, namely the Smyrna(Izmir). On this basis, the herein presented workaround, with its interdisciplinary essentials, would not be identified as an econometric research while it is also at a fair distance to be called as an economic history research in its entirety. By being at their intersection the discussion is representing a certain case enclosing historical findings and namely these are about the data compiled for the 19th century foreign trade of an Ottoman harbor.

The Smyrna(Izmir) is often repudiated as an important marine trade hub of the Ottoman Empire in the same century and mainly identified with its export oriented role in the foreign trade analysis. The compiled set of samples consists of various statistical figures residing within the trade reports of the same era. As it is phrased into the following lines, the subject data keeps a size of 48 years on annual basis and it is found very close to the suggested sample sizes. The whole compilation of data is of various type, and found available either as primary or as in the type of secondary. The overall compilation is enclosing the total nominal volume of trade, in terms of exportation and importation, the detailed breakdown of trade items, as well as the secondary data, i.e. the price index derivations and the concluded terms of trade figures. The secondary series is involved into tiny derivations, as an instance, in the shape of an explicit economic phenomenon residing among the many different discussion points, the following would also be mentioned. The Kuznets(1928) refers to the just initiated concerns of the same era, the very early years of 20th century, that was amplified together with the upcoming contributions on the same topics and those were reached to the late 20th century in an identifiable extend. Since the very beginning of the namely discussions i.e. Kuznets(1928) the well known contributions, in the same favor, is mostly addressed to Konradieff and the cyclical formations in time-series, together with the initial attempts to formalize a statistical toolbox i.e. Bratt, E.C. (1938), Fels, R. (1951), George, G.(1943), Hood, W.C. (1948), Rostow, W.W. (1951), Koopmans, T.C. (1947), Kuznets, S. (1928), Tinther, G. (1940) are enclosing nearly all current issues in time based analysis. The main references for the time based analysis into the cited works is often referring to the initial core of discussions about business cycles which were contributed by Konradieff, the Juglar, the Kitchin and also the Kuznets cycles. The same topic of business cycles that the previous citations refers as a common case for their time-series analysis discussions is known reaching up to the Schumpeterian pattern of development on the same pace Kuznets, S. (1940), Schumpeter, J.A.(1935), Rostow, W.W.(1975).

Iggers, G.G. (1997) presents a detailed discussion on the evolving pathway of historical analysis, where the changing agenda and the research programs of different approaches to historical research is also presented with references to their academic pathways. Into this context, the named text encloses the role and the place of the quantitative analysis as well. In the same part of the discussion, the initial introduction of the quantitative analysis to the historical research and the academic discussions are also mentioned within their relevancy (2003:42-48 the herein given references is for the Turkish Translation). In addition to the mentioned discussion, one of the leading researchers from the same genre of economic history, North, D.C. (1977) also presents the brief distinctions and the demarcating instances for the known approaches to economic history with clear emphasis on the quantitative research.

Without being a rule to hold, and merely as being a rule of thumb, the well suited sample size for time-series analysis is usually denoted as 40-50 samples i.e. Box, Jenkins and Reinsel (1994:32) emphasizes 50 samples. However, with regards to the confidence intervals, often the admissible sample sizes would exhibit a dependency to the test statements, to the statistical inferences and these are changing from one case to another with respect to this essentials. The common calculations for the reliable sample sizes would be found available in various number of statistical analysis text.

Various definitions of the multivariate and univariate data types would be found available in different texts of statistics and econometrics, however the following would also be cited within their relevancy and as example, where in Caldwell(1971:11,16) clarification regarding to the multivariate and univariate variable types is given in the context of Box-Jenkins methodology, as well as, the Box, Jenkins and Reinsel's own explanations are also available in their text of (1994) where a specific part is dedicated to multivariate analysis beside the discussions given for the essential topics of time-series analysis. The provided discussion with this regard is almost entirely about the principles of univariate analysis and questioning the different cases on the basis of one variable.

In addition to many others, with their references on the Mediterranean trade and on the role of the Smyrna(Izmir) harbor in marine trade for the 18th and 19th century, Burke III, E. (2013), Frangakis-Syrett, E. (1988a), (1988b), (2001), Issawi, C. (1966) (1980) , Jackson, M. (2012), Küçükkalay, A.M. (2008), Küçükkalay, A.M. & N. Elibol(2006), Kurmu, O.(2007), Pumuk, Ş. (2010), Yürekli, S.(2019) would be mentioned.
of undertaken adjustments or completions needed during the workarounds and in favor of establishing the appealed price indexes, namely the Paasche, Laspeyres and Fischer.

In parallel to the priority requirements of the utilized estimation techniques, to the potential inquiries i.e. stationarity, in order to provide necessary clarifications, the main concerns of the data preparation and the manipulation will be mentioned into the first section. On the basis of the compiled samples, the performance of few univariate techniques will be questioned afterwards.

In this favor, into the second section, the ordinary trend line estimations and the Box-Jenkins contribution to the time-series analysis, namely the autoregressive integrated moving averages (ARIMA) and the enclosed sub components or sub classes of the integration, the autoregressive (AR) and the moving averages (MA) estimations will be questioned as the univariate structures. The autocorrelation, as being the main concern of the time-series analysis, will hold the initials and with the same understanding, the Durbin-Watson test will be applied to the samples. Following these steps, the results of the estimations will also be evaluated with regards to their usability. During the ARIMA workaround, the main strategy will be kept in parallel to the Box-Jenkins’ own suggestions and their priority emphasizes i.e. the parsimony principle, the stationarity, therefore the lag orders will be chosen with those in mind.

Following the above emphasized sections, lastly, a comparison that leads to the identified strengths and weaknesses together with the conclusive remarks will be presented on the basis of estimation results.

2. Data

Herein, the named models of the linear trend-equations and the AR(I)MA class of models is exercised on the basis of the 19th century foreign trade figures enclosing almost a 50 years of registered statistical information. With a sample size of the subject data-set becomes a proper candidate for a time-series analysis to be undertaken as mentioned. However, few additional remarks would be found necessary to release any potential concern that might have phrased in parallel to the appealed estimation techniques. The clarifications to these are provided in a relevancy, as follows.

As mentioned previously, the data belongs to a Mediterranean harbor, namely the Smyrna(Izmir) which is often repudiated as an important marine trade hub of the Ottoman Empire in the 19th century. In common, the same harbor is known identified with its export oriented role in the foreign trade analysis. The main statistics for the 19th century marine trade of Smyrna(Izmir) is compiled by using the trade registers residing within the commercial reports of the same years and the appealed report entries is consisting of a various information which is provided in a detailed form for the trade items loaded from or landed to the Smyrna(Izmir) harbor for the respective years (1864-1912). While the enclosed information has peculiarities in its own, some common concerns of historical sampling was also experienced within the compilation process. To phrase the few as an example to the mentioned concerns, the following would be emphasized. The enclosed items, as an instance, could not be found presenting the same breakdown of commodities for each respective year of reporting. In addition to those, the different data items is found available while some of the items could not be. In likely cases, if the references to the various other report sections was found reliable enough to ensure the consistency, these were appealed in order to structure a continuous representation of the data series. While trying to built up some secondary series, the manipulation, in the form of simple calculations (i.e. appealing to simple or weighted averages, subtractions and additions) is always a known process in data analysis, (whereas each of them would also be called as an estimation process in their very limited realm of inputs and outputs) and it was the same for the utilized samples of the herein presented time-series analysis as well.

\[
P_P^{ti} = \frac{\sum_{i=1}^{n} p_i^{t}q_i^{0}}{\sum_{i=1}^{n} q_i^{0}} \quad (1)
\]

\[
P_L^{ti} = \frac{\sum_{i=1}^{n} p_i^{t}q_i^{t}}{\sum_{i=1}^{n} q_i^{t}} \quad (2)
\]

\[
P_F^{ti} = \left( P_P^{ti} P_L^{ti} \right)^{1/2} \quad (3)
\]

where:

- \( p_p^{ti} \) = Paasche index for the year \((t)\)
- \( p_L^{ti} \) = Laspeyres index for the year \((t)\)
- \( p_F^{t} \) = Fischer index for the year \((t)\)
- \( p_i^{0} \) = price of the \(i^{th}\) commodity for the base year \((0)\)
- \( q_i^{0} \) = quantity of the \(i^{th}\) commodity for the base year \((0)\)
- \( p_i^{t} \) = price of the \(i^{th}\) commodity for the year \((t)\)
- \( q_i^{t} \) = quantity of the \(i^{th}\) commodity for the year \((t)\)

In specific terms, as an example, the nominal trade volume data, both for the exportation and the importation is found accessible within the enclosed years. However, in contrary academic service providers that this collection would be found available within their archives).

6 The common reference to the herein mentioned time-series analysis is often credited on behalf of Box-Jenkins, however, the references to the forthcoming parts of the text is mainly based on the Box, Jenkins and Reinsel (1994), therefore the given references to the models mainly refers to the presented discussions residing into this text.

7 ibid (5).

8 The named reports is referring to one of the main sources of data for various towns and countries. Currently these are available in online academic databases under a special collection named as the UK Parliamentary Papers (http://www.proquest.com is one of the available

9 The Dridi & Zieschang (2004) presents a comprehensive discussion on the index formulations, as well as the potential problems that might be experienced by the researchers who is involved in some subject requires the analysis of trade statistics. Their emphasizes are all about the current state of statistical compilations, however, many of the given highlights would also be read with the same priority even for the archival sources as being in similarity with those were utilized herein.
to the cumulated trade series, as mentioned, because the breakdown of the trade items is identified exhibiting certain discrepancies between the same years, the strategy to close these gaps was become another concern. While any individual item was registered in terms of the quantity with its metric or imperial references, and the subject amount of the trade is given in terms of monetary values, the price becomes an average for the given unit of commodity. The subject series is calculated alike and on the basis of different commodity categories these were all structured during the survey of archival sources.

The priority purpose of this classification was practical in a large extend, in that sense, by adopting such a classification, the available variety of the data has gained an advantage to reflect the price changes, even in the absence of different items for various time points. In addition to this, by keeping the composition of production factors in mind, and trying to classify the available commodities with this factor in mind, in its applicability, such a classification also yield another basis to discuss the price fluctuations. The resultant weightings of the categories were also utilized to reach an index value of the exports or the imports. Finally, the structured data sets became the basis to calculate the price indexes of Paasche, Laspeyres and Fisher’s geometric mean for the exportation and importation as shown in (Eq. 1,2 and 3) respectively 10.

Even after the mentioned compilation process, the availability of foreign trade figures was found still in a halt for few number of years, and for the similar gaps, several other trade indicators were chosen as the representative. In such an absence of data, the adopted strategy was as follows. Wherever the marine trade statistics was found available, in terms of the entered and cleared vessels, the tonnages of the sailing vessels were appealed to construct gap-filling index values in order to keep the continuum in acceptable margins. The initial cross-check was undertaken by using the years for which both of the data series were found available, namely for the maritime traffic and for the trading information with specific breakdowns. While it was an already identified fact that the vast majority of trading transactions was being undertaken by the maritime traffic, such an indicator was accepted as a reliable candidate to conclude about the trade dynamics under the below clarified presumptions.

The theoretical basis of this strategic choice adopted during the data analysis would be found into the earlier discussions of trade theories, again which refers almost to the same years for which the data was collected. Literally emphasizing, the total share of the cargo load into the overall tonnage values provided for the entering and clearing vessels, is simply another statement of previously defined “representative bales” concept that is often cited to A. Marshall 11. Almost being in the same favor, within the considered years, the traded bales, or explicitly denoting, the cargo load of vessels was accepted has almost the same composition of commodities in terms of their factor endowments and without exhibiting drastic changes. The underlying presumptions is accepted with respect to the main variables which would have a potential to degrade and/or to dequalify the concluded series. With the same understanding, in any similar exercise undertaken to establish a likely data-series, the priority would be given to the following factors of, the composition of the inputs or the capital-labor ratios for the commodities being subject to the trade, either as an export or as an import item, as well as, the shares of the trading countries as an exporter or as an importer. Being in the same favor, the same factors were considered for the herein evaluated data-set. Since the mentioned factors were found not exhibiting drastic changes within the years that the mentioned manipulation is undertaken for the subject case, the resultant series was accepted qualified as an acceptable representation of the subject variable, may be not with full grading, however with sufficiency at worst 12.

In their compilation process because the nature of the data is kept aside from heavy manipulation and because these were kept in a proximity to its origin, the randomness would be deemed remain as it is in parallel to the references given into the Box-Jenkins methodology. Therefore, while the data provided for a very specific harbor, and provided from the same data sources, these limitations is accepted would not cause an extraneous shift for the subject years, and the concluded indicator values are also deemed to be stable enough to profile the stationarity of the time-series. The trend is resident, however, because the subject process is not a mechanical one. The process is also about the economic development and growth and not solely refers to the foreign trade. Depending to this fact the differing is considered in various cases during the herein presented evaluation and the appealed model structures will be reviewed as follows.

3. Model Descriptions

The linear or the non-linear trend estimations are mostly undertaken by the ordinary least squares (OLS) procedure. The named methodology essentially aims to minimize the error terms in a stochastic definition. Together with the requested measures and the test statistics, the same procedure yields an optimized fit for subject regression. Residing into the same class of regression models, the trend-lines consist of the most favorable model types which is often appealed in time-series analysis estimations, especially in history research, and the same procedure is also utilized to end up with an estimator in order to forecast the future values of the subject time-based variable (Schroeder, L.D. et al. 1986).

10 The initial calculations was undertaken by using the same formulae as mentioned in Yürekli (1992:Ek 2), however, an extensive discussion about the same indexes would also be found available in Dridi and Zieschang(2004).

11 The bales, as the units of transportation for the traded commodities, is an essential concept since the earlier discussions undertaken in international trade theories. Several earlier works would be mentioned as follows and with regards to the treatment of representative bales in international trade analysis, Benham, F. (1940), Bhagwati, J. & H.G. Johnson (1960), Currie,J.M. et.al(1971), Das Gupta, A. K. (1954), Enke, S. (1961), Graham,F.D. (1932), Haberler, G. (1955), Jaffé, W. et.al (1946), Stevens, R.W. (1951), Young, A.A. (1924). In addition to the previous reference also Yilmaz, Ş. (2010:71:105) would be mentioned for an overview of the discussion involving into the terms of trade topic, where the representative bales subject resides within.

12 A detailed clarification about the mentioned subjects would also be found into the unpublished copy of the M.A. dissertation work completed by Yürekli (1992:EK 2).
In common terms, the OLS methodology, beside the other options, would be defined as an admissible estimation technique if any regression between two or more variables are deemed to have a causal relation as described and without being effected by another variable in such a way how it is initially defined (Box, G.E.P. 1966). Whereas, the trend equations, as a specific case of linear regressions, again refers to two or more variables, namely the dependent and the independent, within a causal relation and consists of another possible variation into the same class of models. However, the causal statement in likely estimations mainly refers to the subject variable and the time statement or the representation of time, as such that by presuming the time points carrying out a memory effecting the subject variable's representations for furthering observations (Anderson, O.D. 1977:4). In this form a trend equation would be accepted as a representation of variable's time-based realization, in the shape of its progressing, developing, changing pattern and proposes a candidate model for the history research where the time-series is not exceptional findings as potential outcomes. The structural definition of the mentioned causality on time-dependent basis is almost the same area of concern from where the more grained methodologies of time-series analysis was given a rise, because, a likely presumption also poses a critical concern of econometric analysis, namely the autocorrelation.

An autocorrelation basically refers to a correlation between the same variable's observation values and the initial emphasizes to identify the potential cases having imprints of any suspected autocorrelation is often cited together with Durbin-Watson test. By appealing to the Durbin-Watson test together with their presumptions as a common statistical pinpoint of robustness in relevancy, the same type of models exhibits a potential to present the time based structure of the subject variable or the trend pattern within reliable margins.

As a well known pathway for any case study that might have chosen in favor of a time based econometric estimation, not only including the economic history research, but for all probable application areas, the trend-line based regression model will be briefed as follows and following these lines dedicated to the trend-line estimation, the Box-Jenkins time-series analysis will also be given as another option to appeal for similar time based cases.

### 3.1 Trend Equations and The Durbin-Watson Test

The common form of any regression between two variables of \(X\) and \(Y\) would be defined as its shown in (Eq. 4).

\[
Y_t = \alpha + \beta X + e
\]

As its availed within (Eq. 4) the general form of the regression defines a linear relation between the variables together with the stochastic term of error \((e)\) indicating the amount of effect that would not be explained by the independent variable \((X)\) and the coefficients \((\alpha, \beta)\) together.

While the common representation of a regression would be given in this form, the trend equation for any variable \((Y)\) would also be defined on the same basis. Any trend equation of \((Y)\) presents a similar form by considering the time index \((T)\) as an independent variable. In this shape, the previous definition becomes a regression between these two variables, namely a regression for a trend-line. The common form is given by (Eq. 5).

\[
DW= \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}
\]

where \(e_t = y_t - \hat{y}_t\) for the \(th\) observation

Once the presumed relation between the different time points \((T)\) and the subject variable values \((Y)\) is regressed in a non-linear form, this equation can also lead to several varieties. Likely considerations mostly requires a logarithmic transformation and the OLS procedure would hold even in these cases if some other procedure is not chosen for the estimation. Any estimation utilizing the OLS is presumed to yield the error optimized estimators with respect to the constants of slope and interception \((\alpha, \beta)\). In this shape the overall result becomes an optimized fit for \((Y)\) values. Again, on the basis of the same linear form, variations between the occurrences of \((Y)\) and the estimated values of the variable \((\hat{Y})\) provides the variations from the sampled values. In an accordance with the residuals, namely the variations from the sampled values, the correlation and the determination coefficient are also identified within the same workaround.

However, the time based nature of the series involved into the trend estimation is known being subject to several other concerns than those would be identified with respect to the cases that the cross-section data is utilized. These concerns is usually underlined with regards to the potential correlations between the lagged observation values which decreases the strength of estimators. Consequently, several additional test procedures becomes a necessity for likely analysis in order to reach an unbiased estimator. In this genre of test statistics, one of the earlier contributions belongs to J. Durbin and G.S. Watson\(^\text{13}\). The Durbin-Watson (DW) statistics simply refers to the achieved results and the residuals reflected to the error term \((e)\) of the estimator. In parallel to its inputs the DW statistics would be explicitly defined as shown in (Eq. 6) (Hines & Montgomery 1990:529-532).

With this understanding, the DW procedure causally bridges the lagged observation values by the help of error terms and produces a test statistic. The DW statistic is utilized over the lagged pairs of the same variable in order to reach a conclusion about any potential autocorrelation. As it is shown by its underlying rationale it indicates a check between the error terms or the residuals \((e_i)\) of the trend estimation and the first order lags \((e_{i-1})\) for the variable under consideration (Eq. 6). On the basis of this statistic a suggestion would be made with regards to any potential

\(^{13}\) A comprehensive discussion about time-series regression would be found in Hibbs, D.A. Jr. (1973 – 1974), as well as in Durbin, J. & G.S. Watson (1950, 1951).
autocorrelation between the lagged terms in first order. The relation between the lagged values of time based variables refers to a phenomenon defined alike and it deems to reduce the predictive strength of the result no matter if the estimator is in the form of linear or non-linear. Therefore, the same statistic becomes a decision criterion to identify the similar problems, and avoiding the autocorrelation by leaving out or, by eliminating the effecting factors.

In contrary, the Box-Jenkins proposal is simply to build up a model to enclose any potential effect originated by the lagged observations. Rather than eliminating the autocorrelation, their proposal is detailed in order to reach a more gained estimation together with the imprints of identified autocorrelation. With this understanding, they formalize the potential memory of the time by using its strength to refine forecasting efforts (Anderson, O.D. 1977:4). The adoptable choice to utilize the conclusion would exhibit a certain variety. Once after the estimation process is undertaken, the estimator would be utilized to filter the quantified effects in order to enclose the subject variable within a more comprehensive model structure or simply to forecast the variable values solemnly by using the concluded structure.

3.2 Box-Jenkins AR, MA and ARIMA Models

Time-series in economics is often being subject to the analysis of different determinants effecting the subject variables on temporal basis and the initial discussions indicates a very early start15. The cyclical fluctuations, as one of the frequent discussion topic in economic analysis, is well cited under various terms as an example. The most common reference is given to the business cycles, or by some other wording, refers to the seasonal regularities indicating fluctuating structure of consumption and/or production16. In addition to the initial statements' holistic understanding of the economic growth and economic development realms, some more specific sub-sections would also be emphasized as previously mentioned17. Whereas, from econometrics’ point of view, the efforts formalized under the time-series analysis is also initially established in the earlier stages of similar models and in its extended end, the contemporary research on the time-series modeling involves into different realms of economics with its more refined procedures as well18.

\[ \text{AR}(p) \quad \text{Autoregressive Model } p^{\text{th}} \text{ order } \phi(B)z_t = a, \]
\[ \tau_t = \phi_1 \tau_{t-1} + \phi_2 \tau_{t-2} + \ldots + \phi_p \tau_{t-p} + a, \quad (7.1.1) \]
\[ X_t = \delta + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + A_t \quad (7.1.2) \]

\[ A, a = \text{ the white noise} \]
\[ \mu = \text{ the mean of the process (sample set)} \]
\[ p = \text{ the order of the AR model} \]
\[ \delta = \left(1 - \sum_{i=1}^{p} \phi_i\right) \mu \]
\[ \tau_t = z_t - \mu \]

The main idea that stands behind the time-series analysis is the identification of the effecting lagged observations. As an earlier contribution to the similar analysis, the AR, MA or mixed ARIMA techniques hold the same understanding, and this is often emphasized as to capture the shares of the effecting lag terms for the same subject variable in univariate analysis, however, the analysis would also be undertaken with the multivariate time-series as well as for various research realms (Box, G.E.P. et.al(1994), De Gooijer, J.G. and R. J. Hyndman (2006) and Paul, M.T (2017)).

The herein mentioned class of models is questioned with respect to the lagged samples of the same variable, and since the enclosed number of variables is not more than one, these simply refer to the univariate analysis. The most common definitions of the named univariate models are formulated as given by herein provided forms. The autoregressive structure is as shown in (Eq. 7.1.1 and Eq. 7.1.2).

In this form, the autoregressive (AR) component of estimation simply denotes a relation between the lagged terms of the considered variable. The estimation yields the autoregressive term for each sample occurrences in lag and reflects their share into the overall change while leaving the random shocks, or the white noise, well separated away from the rest of all effecting elements.

\[ \text{MA}(q) \quad \text{Moving Average Model } q^{\text{th}} \text{ order } \theta(B)\zeta_t = a, \]
\[ \tau_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q}, \quad (7.2.1) \]
\[ X_t = \mu + A_t - \theta_1 A_{t-1} - \theta_2 A_{t-2} - \ldots - \theta_q A_{t-q}, \quad (7.2.2) \]

\[ A, a = \text{ the white noise} \]
\[ \mu = \text{ the mean of the process (sample set)} \]
\[ q = \text{ the order of the MA model} \]

14 Within this part and into the rest of the text, the relevant references are all given to the G.E.P. Box, et.al. (1994) and for the model formulations the NIS(2003) would also be cited together with G.E.P. Box et.al. (1994).
15 ibid. (1)
16 ibid. (1)
17 In addition to the previously cited subjects, another well known topic in this area of research would be mentioned as the Cobweb discussion (Ezekiel(1938)). The named analysis essentially refers to the adoptive production patterns into the agriculture, however it again relies on the time based analysis undertaken with references to the different terms of production and market equilibrium.
18 To provide an extend that which the time-series analyses is already reached, De Gooijer, J.G. and R. J. Hyndman (2006) and Paul, M.T (2017) would be mentioned with references to the enclosed surveys.
The initial (Eq. 7.1.1) form, the naive definition of the Box-Jenkins, leaves the variations into the term of \( \bar{Z}_t = z_t - \mu \), where, the later (Eq. 7.1.2) form of autoregression estimator encloses the variations into the term of \( (\delta) \). However, those two essentially exhibits the same definition and filters the effects of the lagged terms from the white noise, up to the \( (p)^{th} \) order backwards. While \((B)\) denotes a backward operator, an (AR) component can also be denoted as \((\varphi(B)\bar{Z}_t = \alpha_t)\) in common terms\(^{19}\).

The second class of models consists of the moving averages (MA), and this component is based on the moving averages of the subject variable for different lag orders \((q)\). Again a very common form of the MA component would be defined as shown in (Eq. 7.2.1 and Eq. 7.2.2).

Herein, the potential effects of variations from the mean is enclosed for sequential lagged orders by presuming that the removal of all effects would also be achieved by smoothing the subject variables' own variations. Identified by the (MA) terms, this component would yield a white noise that demonstrates the random external shocks depending on different reasons other than the examined variable itself. In such a case, the remainder indicates the variable's own variation in a relation with the external shocks which can be shown by each form of (Eq. 7.2.1 and Eq. 7.2.2).

These models would be reduced to the overall definition of which identifies the filtered effects and denoting them by the backward operator \((B)\) and its share by \((\theta)\), the (MA) component would also be defined as \(z_t - \mu = \bar{Z}_t = \theta(B)\alpha_t\) in its general form.

In some cases, as proposed by Box-Jenkins, both effects would be considered together in favor of a better fit, and this type of considerations indicates a mixed structure, namely an ARMA\((p,q)\) model, as shown in (Eq. 7.3).

\[
ARMA(p,q) \\
X_t = \delta + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \ldots + \varphi_p X_{t-p} + A_t - \theta_1 A_{t-1} - \theta_2 A_{t-2} - \ldots - \theta_q A_{t-q} \tag{7.3}
\]

In essential terms, all the given classes of models is based on the stationarity assumption which simply indicates that the subject variable is drawn from a population having a constant mean and variance, or by other terms, those don’t exhibit a systematic change (Box et.al. 1994). The main strategy to identify any potential concern regarding to the stationarity is an advised check on the basis of autocorrelation functions for various lag orders (G.E.P. Box et.al. 1994). In suspicious cases that the examination of samples ends up with contradicting results against stationarity, Box-Jenkins-Reinsel propose several strategies. One of them is differencing the data-series, yet a different proposal is known as de-trending (Matridakis, S. and M. Hibon 1997:155-158). Where its needed, the rest of the estimation procedure would be completed by the differenced data for several lag orders. As being in the same favor, the same results would also be achieved by the removal of a trend pattern, which is often denoted as de-trending or de-seasoning the samples if any seasonal fluctuations identified\(^{20}\). In cases that an analysis of data yields such a result, the Box-Jenkins proposal ends up with an ARIMA\((p,d,q)\) after differencing the data up to the \((d)^{th} \) order of lag backwards. However, in the absence of any differentiation, by keeping the \((d=0)\) factor in mind, the model becomes a naive ARMA type, and can be read as a reduced form of an ARIMA i.e. ARMA\((p,q)\)=ARIMA\((p,0,q)\).

\[
ARIMA(1,0,0) = AR(1) \tag{7.4}
\]

\[
ARIMA(0,0,1) = MA(1)
\]

\[
ARIMA(1,0,1) = ARIMA(1,1)
\]

\[
ARIMA(1,1,1) = ARIMA(1,1,1)
\]

With this understanding, the common definition of ARIMA would be utilized to identify various combinations chosen from the available class of mentioned models. In this shape, the mentioned models would be summarized from another way around. This is to say any ARIMA\((p,d,q)\) definition would be accepted referring to any one of the sub structures while reducing the estimated regression to another component, as shown by (Eq. 7.4).

Into the above mentioned classifications only the naive ARIMA structure is given and other variations are not enclosed (i.e. the seasonal SARIMA). The different variations of the named class of Box-Jenkins models would also be considered in specific cases\(^{21}\).

### 3.3 ARIMA Estimation Procedure

#### Illustration 1: The Algorithm of Box-Jenkins Approach to Time-Series Analysis

The estimation process of previously mentioned models are all undertaken by using the R\(\text{\textregistered}\) system and spreadsheet softwares. The common strategy is followed straightforward for the linear trend equations, however, the procedure involved into the Box-Jenkins class of models follows the proposed steps of their own approach, as it is often emphasized. This approach, rather than being straightforward, usually requires various checks with respect to the different states of the analysis. The basic structure of any algorithm that reflects the essential steps of

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\(^{19}\) The term “naive” often refers to the basic ARIMA (Autoregressive Integrated Moving Averages) or IARMA (Integrated Autoregressive Moving Averages) as how the Box-Jenkins initially presented their contribution with the sub components of AR and MA together.

\(^{20}\) Box-Jenkins approach, as its presented in Box, G.E.P. et.al. (1994) gives priority to differencing in similar cases, and another integration emphasized as an estimator when an identifiable cyclical formation resides into the variable data, this is called as Seasonal ARIMA (SARIMA) as they denote in Box, G.E.P. et.al (1994).

\(^{21}\) In Box, G.E.P. et.al. (1994) the potential sub structures are also introduced with references to their essentials, the justifying concerns and the necessary clarifications.
the Box-Jenkins approach would be outlined as given in (Illustration 1)\(^2\).

In this favor, during the Box-Jenkins estimations, the autocorrelation (acf) and the partial autocorrelation functions (pacf) are separately evaluated for the subject data-series in order to examine the stationarity with priority. In cases that the stationary would not be met as requested, previously mentioned manipulations are all advised to ensure the stationarity (i.e. differencing or de-trending). Once after the stationarity is ensured, the applicable model structure is chosen on the basis of the lag orders provided in parallel to the achieved results\(^3\).

The Box-Jenkins parsimony principle is kept in mind to guide in necessary cases that the different number of lags poses mild variations. In likely cases the same principle simply suggests favoring the less number of lag in order to strengthen the forecasting performance. This principle is defined by their own wordings referring to Tukey:

“It is important, in practice, that we employ the smallest possible number of parameters for adequate representations. The central role played by this principle of parsimony (Tukey [1961]) in the use of parameters will become clearer as we proceed.” (Box et.al. 1994:16).

As it is mentioned within the emphasize, the parsimony, as a principle to follow, simply promotes the cost-benefit assessment of the overall estimation process and positions the econometricians on a cost efficient workaround. In this sense the named principle is also to optimize the named workload. With the same understanding the time-series analysis would be justified against the multivariate and more sophisticated models. Reading together with the above emphasize on the “memory” of previous instances or “time”, as its mentioned in (Anderson, O.D. 1977:4), the above emphasis on the “memory” of previous instances or “time”, as its mentioned in (Anderson, O.D. 1977:4), the past realizations of any variable would be accepted carrying the sufficient amount of reflexions or realizations originated from other variables in its measurable observations, which again together with the parsimony principle in mind, would yield a choice that favors the time series analysis rather than building up a sophisticated, multivariate models. Herein, lead by the absence of necessary data in their relevancy with the analyzed foreign trade information, these two factors, the parsimony and the presumed memory of past instances get the priority and the exercise is undertaken with these in mind. During the estimation process which is undertaken in parallel to the provided algorithm (Illustration 1), the same order is followed except than the last step of application. However, because of the main motivation which is simply to compare the different estimation techniques on the basis of their performance, the application stage of the mentioned algorithm is not undertaken and evaluation is completed by the help of residual analysis.

4. Estimation Results

On the basis of concluded estimations, the usability of the respective models would be compared with regards to their estimation strength, which refers to the below given essentials.

4.1 Trend-line estimations

| Trend Eq. | Exportation | Importation | Export Prices | Import Prices |
|-----------|-------------|-------------|---------------|---------------|
| X = a + b t | I = a + b t | P = a + b t | Pm = a + b t |
| (a) | 4997.4 | 3490.17 | 186.67 | 140.14 |
| (b) | 4.64 | 9.51 | -3.9 | -2.45 |
| Det. Coef. | 0.02 | 0.05 | 0.58 | 0.66 |
| F Stat. | 0.99 | 2.25 | 63.42 | 89.47 |
| Sig. Result | b ≠ 0 \(^2\) | b ≠ 0 | b ≠ 0 | b ≠ 0 |
| DW Stat. | 1.1394 | 0.7662 | 0.4357 | 0.1965 |

The estimation process of the trend-line is undertaken on the basis of linear forms with respect to four different data-series. These are the nominal trade volumes (exportation and importation) and the price indexes for the exportation and the importation. The results is given into the (Table 1).

The concluded results suggests that, the linear trend estimation does not exhibit a requested strength as a potential forecasting structure, as it's reflected to the determination coefficients, where these are found weak enough. Though they are also found with low significance on the basis of their parameter tests. Comparing the estimations, the price indexes yield a stronger fit than the nominal trade data however, again their weakness in terms of their forecasting strength is reflected to the basic outcomes. In addition to the on the basis of the undertaken tests, the DW statistics for each of them is indicating that all series exhibits an autocorrelation between the lagged values. The results suggested by the statistics also carry the imprints of a requested alternative consideration as an option to structure the furthering appeals. This is also a concluded indication that an ARIMA examination would get a Justifiable basis to filter the autoregressive components, and in its availability, to structure a better model to forecast.

Lastly, in parallel to the herein presented estimations, the following can also be emphasized. The examination is kept limited with the linear forms and the possible other model structures is not included into this article, however, the initial observations yields higher determination coefficients in favor of polynomial forms of estimators, where exponential options would also be considered. On the basis of same tentative checks the linear trend estimation of the real trade data is also found exhibiting better estimations, but, in order to keep the pace in its initially phrased structure, only the mentioned linear forms are considered.

4.2 ARIMA estimations

The previously mentioned steps of the Box-Jenkins approach were separately undertaken for each sample series. These are briefed as below in parallel to the approach to time-series analysis. In parallel to their essential share into the exercises, those were questioned and clarified from various aspects and by various number of academics. Few of these discussions would be found available in Box, G.E.P., and D.A. Pierce (1970) , Box, G.E.P., G.M. Jenkins and G.C. Reinsel (1994), Newbold, P.(1975) and Ramsey, F.L. (1974), Anderson, O.D.(1977).

\(^2\) The common adopted strategy is quite a well known procedure and would be found in various references beside the Box, G.E.P., G.M. Jenkins and G.C. Reinsel (1994), i.e. Caldwell (1971:5), Matrialdakis & Hibon(1997), NIST(2003) and Newbold, P.(1975) can be revised for the same purpose. The algorithm is mainly structured from Box, Jenkins & Reinsel (1994:17-18) and Matrialdakis & Hibon(1997:149-151).

\(^3\) The autocorrelation, the partial autocorrelation functions and their estimations for specific data-series is given a priority role in Box-Jenkins approach to time-series analysis. In parallel to their essential share into the exercises, those were questioned and clarified from various aspects and by various number of academics. Few of these discussions would be found available in Box, G.E.P., and D.A. Pierce (1970) , Box, G.E.P., G.M. Jenkins and G.C. Reinsel (1994), Newbold, P.(1975) and Ramsey, F.L. (1974), Anderson, O.D.(1977).
calculations of the autocorrelation functions (acf) and the partial autocorrelation functions (pacf). As how its mentioned before, whenever the stationarity is found suspicious, two main strategies are recommended in order to ensure this characteristic. These are the differencing in various lag orders as on the basis of identified lags that the examination suggests, or de-trending. Herein, where the stationarity condition is found requiring, the differencing is adopted for the same purpose.

The initial check was undertaken by using the autocorrelation functions in order to identify the stationarity of the data-series. In parallel to the Box-Jenkins-Reinsel proposals, the autocorrelation functions of the nominal exports (given into the first row of graphics) are found illustrating a stationary structure (Figure B.1). Considering together with the partial autocorrelation function (pacf) structure (which is given into the second row of the graphical illustrations), the results suggest an ARIMA(1, 0, 1) on behalf of the Nominal Exports data. The estimation results is given into the (Table 2).

The second data-series, namely the nominal importation, is also found exhibiting a stationary structure with a mild imprints of effecting trend (Figure B.2). Therefore, without considering the differencing, the AR(1) term, as depicted into the pacf (k=0) together with a MA(1) or MA(2), as depicted into the acf(k=0), would yield the entertained options of ARIMA(1, 0, 1) and ARIMA(1, 0, 2) to examine (Table 3).

In contrary to the previous variables, the third sample set of the exports price index is indicating a weak stationarity as reflected to the acf(k=0) in (Figure B.3). It has a gradual decrease rather than an immediate drop in autocorrelation terms, while, it is not indicating a persistent autocorrelation which suggest a differencing to ensure the stationarity, however the pacf(k=0) is providing a clear indication of an AR(1). The examination of the differentiated values is also suggesting an MA(1) by the acf(k=1) therefore, in parallel to the evaluation, the ARIMA(1, 1, 1) would be identified as the option to entertain (Table 4).

The last data-set is enclosing the import price index. This data also indicates a weak stationarity with its gradual decrease in acf(k=0) terms (Figure B.4). The differentiated series exhibit a stationary structure at the first order acf(k=1) where the results, together with the pacf values, are suggesting either an ARIMA(1,1,0) or ARIMA(1,2,1) on behalf of the imports price index (Table 5).

5. Concluding Remarks

Reminding the initial references given for the time-based analysis and under their shed referring to the residing considerations, in economic history research, several quantitative techniques would be chosen as proper candidates to model the compiled historical data series. Here, as a specific case, the trade figures of the 19th century exportation harbor is chosen for a similar exercise. Two different strategies are examined in the same favor. The initial one is about the linear trend examination and the latter was about the Box-Jenkins approach.

### Table 2
Nominal Exports – Estimation Results

|                         | ARIMA(1, 0, 1)                        |
|-------------------------|--------------------------------------|
| Coefficients            | (ar1) (ma1) (intercept)              |
| Estimations             | (0.1866) (0.2735) (4208.306)         |
| Standard Errors         | (0.3025) (0.2943) (89.627)           |
| (σ²), Log Likelihood, AIC, BIC | (160053) (-355.81) (719.63) (727.11) |

### Table 3
Nominal Imports – Estimation Results

|                         | ARIMA(1, 0, 1)                        | ARIMA(1, 0, 2)                        |
|-------------------------|--------------------------------------|--------------------------------------|
| Coefficients            | (ar1) (ma1) (intercept)              | (ar1) (ma1) (ma2) (intercept)        |
| Estimations             | (0.6370) (-0.0093) (3242.27)         | (-0.0095) (3232.79) (0.645) (0.733)  |
| Standard Errors         | (0.1694) (0.2173) (181.98)           | (0.2260) (0.1612) (0.2261)           |
| (σ²), Log Likelihood, AIC, BIC | (228151) (-364.47) (736.94) (744.42) | (209695) (-362.72) (735.43) (744.79) |

### Table 4
Exports Price Index – Estimation Results

|                         | ARIMA(1, 1, 1)                        |
|-------------------------|--------------------------------------|
| Coefficients            | (ar1) (ma1)                           |
| Estimations             | (0.9485) (-0.1872)                   |
| Standard Errors         | (0.0803) (0.2755)                    |
| (σ²), Log Likelihood, AIC, BIC | (223.4) (-194.78) (395.55) (401.17) |

### Table 5
Imports Price Index – Estimation Results

|                         | ARIMA(1, 1, 0)                        | ARIMA(1, 2, 1)                        |
|-------------------------|--------------------------------------|--------------------------------------|
| Coefficients            | (ar1) (ma1)                           | (ar1) (ma1)                           |
| Estimations             | (0.1463) (0.1342) (-0.9996)          |                                     |
| Standard Errors         | (0.1489) (0.1555) (0.3359)           |                                     |
| (σ²), Log Likelihood, AIC, BIC | (267.3) (-198.03) (400.05) (403.79) | (266.7) (-195.54) (397.08) (402.69) |

24 Detrending and the examination of real figures on the same basis is left to a furthering questioning.
The weakness of the initial choice would be identified as the autocorrelation which clearly reflects itself to the results of the DW test. However, the Box-Jenkins ARIMA approach, with its strength in the identification of time-series based econometric concerns as mentioned, is found more efficient in terms of its estimation results, as well as, in terms of the applicable choices between sub-structures, namely between the AR and MA components. Even while the stationarity would be weaker than expected, the provided support proposes an advantage to structure the quantifiable historical findings, especially in the availability of required sample-sizes, and because of their very nature, the historical data-sets becomes a proper data-structure for the mentioned Box-Jenkins ARIMA time-series analysis if those would be met (Table 7)\(^\text{25}\).

Lastly, the performance of the examined estimation techniques (the ARIMA and the TREND) is evaluated on the basis of residuals. The comparison is undertaken for each class of models with respect to their performance over occurrences. The respective results are separately given in (Figures C.1, C.2, C.3, C.4) for the examined variables. With the same understanding this would be mentioned that, the overall assessment reflects higher percentages of lower residuals for each variable when the ARIMA estimator is considered (Figure 1 and Table 6).

| Variable          | Dickey Fuller statistic | Box-Pierce Test (portmanteau) |
|-------------------|-------------------------|-------------------------------|
| Nominal Exports   | -4.2108                 | 8.2305                        |
| Nominal Imports   | -3.7438                 | 17.3155                       |
| Exports Price Index (k=0) | -10.4275             | 26.8826                       |
| Exports Price Index (k=1) | -6.9318               |                                 |
| Exports Price Index (k=2) | -5.8781               |                                 |
| Imports Price Index | -2.8766                | 33.552                        |

(*) As proposed by Phillips-Peron Unit Root Test.

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\(^{25}\) The historical data sets from various research areas of economics would also be found in Box, Jenkins & Reinsel(1994) example cases.
Appendix A
The Trade Statistics of Smyrna (İzmir) Harbor (1865-1912)

| Nominal Values (000 £) | Index (1880=100) |
|------------------------|------------------|
| **Years** | **Exports** | **Imports** | **Exports** | **Imports** |
| 1865 | 4046 | 2271 | 105.0 | 57.1 |
| 1866 | 3606 | 3763 | 93.6 | 84.5 |
| 1867 | 4008 | 3404 | 114.4 | 55.5 |
| 1868 | 4632 | 3354 | 120.2 | 84.5 |
| 1869 | 4540 | 3587 | 117.9 | 90.1 |
| 1870 | 4620 | 3907 | 104.0 | 75.6 |
| 1871 | 4043 | 3760 | 105.0 | 94.5 |
| 1872 | 4867 | 3461 | 126.3 | 113.5 |
| 1873 | 4999 | 4518 | 116.8 | 134.4 |
| 1874 | 3940 | 4490 | 102.3 | 112.8 |
| 1875 | 3896 | 3433 | 101.1 | 113.5 |
| 1876 | 4630 | 3860 | 120.2 | 71.9 |
| 1877 | 4687 | 3082 | 121.7 | 77.4 |
| 1878 | 3543 | 3140 | 92.0 | 104.0 |
| 1879 | 4407 | 4756 | 114.4 | 71.1 |
| 1880 | 3852 | 2980 | 100.0 | 100.0 |
| 1881 | 3804 | 4656 | 98.7 | 117.0 |
| 1882 | 3542 | 3216 | 99.7 | 86.8 |
| 1883 | 4711 | 3238 | 122.3 | 81.3 |
| 1884 | 4820 | 2929 | 125.1 | 73.6 |
| 1885 | 4315 | 2693 | 112.0 | 67.7 |
| 1886 | 3332 | 2707 | 112.4 | 88.0 |
| 1887 | 3099 | 2709 | 106.4 | 88.0 |
| 1888 | 3867 | 2710 | 100.4 | 88.1 |
| 1889 | 4536 | 3236 | 117.7 | 81.3 |
| 1890 | 3708 | 3031 | 96.3 | 76.1 |
| 1891 | 3927 | 2986 | 101.9 | 75.0 |
| 1892 | 3648 | 3010 | 94.7 | 75.6 |
| 1893 | 3866 | 3084 | 103.5 | 77.5 |
| 1894 | 3234 | 1158 | 122.2 | 67.5 |
| 1895 | 3134 | 2881 | 112.5 | 72.4 |
| 1896 | 3717 | 2566 | 96.5 | 64.5 |
| 1897 | 3100 | 2522 | 80.5 | 66.2 |
| 1898 | 3295 | 2678 | 85.5 | 63.7 |
| 1899 | 3783 | 2563 | 98.2 | 64.4 |
| 1900 | 4137 | 2369 | 107.9 | 59.5 |
| 1901 | 4413 | 2849 | 114.6 | 71.6 |
| 1902 | 4275 | 2805 | 111.0 | 70.5 |
| 1903 | 4834 | 2802 | 125.5 | 70.4 |
| 1904 | 4755 | 3061 | 123.4 | 76.9 |
| 1905 | 4504 | 3215 | 116.9 | 80.8 |
| 1906 | 4973 | 3547 | 129.1 | 89.1 |
| 1907 | 4690 | 1183 | 121.7 | 80.0 |
| 1908 | 4453 | 2938 | 115.6 | 73.8 |
| 1909 | 4814 | 3411 | 125.0 | 85.8 |
| 1910 | 4500 | 4061 | 116.8 | 102.0 |
| 1911 | 4400 | 4138 | 114.2 | 104.0 |
| 1912 | 4000 | 3788 | 103.8 | 95.2 |

Source: Yürekli (2019, [1992])

(*): Nominal trade values that were given for the years 1887, 1893, 1896, represents the averages of the previous and the following years.
Appendix B
Autocorrelation and Partial Autocorrelation Functions
(k=20, n=48, confidence interval (c.i. - confidence interval)=0.95)

Figure B.1: Nominal Exports
Appendix C
Residual Comparisons (residual per observation, n=48)

Figure C.1: Nominal Exports
Figure C.2: Nominal Imports

Figure C.3: Exports Price Index

Figure C.4: Imports Price Index