Bregman Gradient Policy Optimization

Feihu Huang \footnote{Feihu and Shangqian contributed equally.}
Department of Electrical and Computer Engineering
University of Pittsburgh, Pittsburgh, USA
HUANGFEIHU2018@GMAIL.COM

Shangqian Gao
Department of Electrical and Computer Engineering
University of Pittsburgh, Pittsburgh, USA
SHG84@PITT.EDU

Heng Huang
Department of Electrical and Computer Engineering
University of Pittsburgh, Pittsburgh, USA
HENGHUANGHH@GMAIL.COM

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Abstract
In this paper, we design a novel Bregman gradient policy optimization framework for reinforcement learning based on Bregman divergences and momentum techniques. Specifically, we propose a Bregman gradient policy optimization (BGPO) algorithm based on the basic momentum technique and mirror descent iteration. At the same time, we present an accelerated Bregman gradient policy optimization (VR-BGPO) algorithm based on a momentum variance-reduced technique. Moreover, we introduce a convergence analysis framework for our Bregman gradient policy optimization under the nonconvex setting. Specifically, we prove that BGPO achieves the sample complexity of $\tilde{O}(\epsilon^{-4})$ for finding $\epsilon$-stationary point only requiring one trajectory at each iteration, and VR-BGPO reaches the best known sample complexity of $\tilde{O}(\epsilon^{-3})$ for finding an $\epsilon$-stationary point, which also only requires one trajectory at each iteration. In particular, by using different Bregman divergences, our methods unify many existing policy optimization algorithms and their new variants such as the existing (variance-reduced) policy gradient algorithms and (variance-reduced) natural policy gradient algorithms. Extensive experimental results on multiple reinforcement learning tasks demonstrate the efficiency of our new algorithms.

Keywords: Bregman Gradient, Mirror Descent, Policy Optimization

1. Introduction
Policy Gradient (PG) methods are a class of popular policy optimization methods for Reinforcement Learning (RL), and have achieved significant successes in many challenging applications (Li, 2017) such as robot manipulation (Deisenroth et al., 2013), the Go game (Silver et al., 2017) and autonomous driving (Shalev-Shwartz et al., 2016). In general, PG methods directly search for the optimal policy by maximizing the expected total reward of Markov Decision Processes (MDPs) involved in RL, where an agent takes action dictated by a policy in an unknown dynamic environment over a sequence of time steps. Since the PGs are generally estimated by Monte-Carlo sampling, such vanilla PG methods usually suffer from very high variances resulted in slow convergence rate and destabilization. Thus,
Table 1: **Sample complexities** of the representative PG algorithms based on mirror descent algorithm for finding an $\epsilon$-stationary point of the nonconcave performance function. Although Liu et al. (2019); Shani et al. (2020) have provided the global convergence of TRPO and PPO under some specific policies based on the convex mirror descent, they still obtain a stationary point of the nonconcave performance function. Note that our convergence analysis does not rely any specific policies.

| Algorithm     | Reference         | Complexity | Batch Size |
|---------------|-------------------|------------|------------|
| TRPO          | Shani et al. (2020) | $O(\epsilon^{-4})$ | $O(\epsilon^{-2})$ |
| Regularized TRPO | Shani et al. (2020) | $O(\epsilon^{-2})$ | $O(\epsilon^{-2})$ |
| TRPO/PPO      | Liu et al. (2019)  | $O(\epsilon^{-8})$ | $O(\epsilon^{-6})$ |
| VRMPO         | Yang et al. (2019) | $O(\epsilon^{-3})$ | $O(\epsilon^{-2})$ |
| MDPO          | Tomar et al. (2020) | Unknown    | Unknown    |
| BGPO          | Ours              | $O(\epsilon^{-4})$ | $O(1)$      |
| VR-BGPO       | Ours              | $O(\epsilon^{-3})$ | $O(1)$      |

Recently, many fast PG methods have been proposed to reduce variances of vanilla stochastic PG. For example, (Sutton et al., 2000) introduced a baseline to reduce variances of the stochastic PG. (Konda and Tsitsiklis, 2000) proposed an efficient actor-critic algorithm by estimating the value function to reduce effects of large variances. (Schulman et al., 2015b) proposed the generalized advantage estimation (GAE) to control both the bias and variance in policy gradient. More recently, some faster variance-reduced PG methods (Papini et al., 2018; Xu et al., 2019a; Shen et al., 2019; Liu et al., 2020) have been developed based on the variance-reduction techniques in stochastic optimization.

Alternatively, some successful PG algorithms (Schulman et al., 2015a, 2017) improve convergence rate and robustness of vanilla PG methods by using some penalties such as Kullback-Leibler (KL) divergence penalty. For example, trust-region policy optimization (TRPO) (Schulman et al., 2015a) ensures that the new selected policy is near to the old one by using KL-divergence constraint, while proximal policy optimization (PPO) (Schulman et al., 2017) clips the weighted likelihood ratio to implicitly reach this goal. Subsequently, Shani et al. (2020) have analyzed the global convergence properties of TRPO in tabular RL based on the convex mirror descent algorithm. Liu et al. (2019) have also studied the global convergence properties of PPO and TRPO equipped with overparametrized neural networks based on mirror descent iterations. At the same time, Yang et al. (2019) tried to propose the PG methods based on the mirror descent algorithm. More recently, mirror descent policy optimization (MDPO) (Tomar et al., 2020) iteratively updates the policy beyond the tabular RL by approximately solving a trust region problem based on convex mirror descent algorithm. In addition, (Agarwal et al., 2019; Cen et al., 2020) have studied the natural PG methods for regularized RL. However, (Agarwal et al., 2019) mainly focuses on tabular policy and log-linear, neural policy classes. (Cen et al., 2020) mainly focuses on softmax policy class.

Although these specific PG methods based on mirror descent iteration have been recently studied, which are scattered in empirical and theoretical aspects respectively, it lacks a universal framework for these PG methods without relying on some specific RL tasks. In
particular, there still does not exist the convergence analysis of PG methods based on
the mirror descent algorithm under the nonconvex setting. Since mirror descent iteration
adjusts gradient updates to fit problem geometry, and is useful in regularized RL (Geist
et al., 2019), there exists an important problem to be addressed:

Could we design a universal policy optimization framework based on the mirror
descent algorithm, and provide its convergence guarantee under the non-convex
setting?

In the paper, we firmly answer the above challenging question with positive solutions and
propose an efficient Bregman gradient policy optimization framework based on Bregman
divergences and momentum techniques. In particular, we provide a convergence analysis
framework of the PG methods based on mirror descent iteration under the nonconvex
setting. Our main contributions are provided as follows:

a) We propose an effective Bregman gradient policy optimization (BGPO) algorithm
based the basic momentum technique, which achieves the sample complexity of \( O(\epsilon^{-4}) \)
for finding \( \epsilon \)-stationary point only requiring one trajectory at each iteration.

b) We further propose an accelerated Bregman gradient policy optimization (VR-BGPO)
algorithm based on the momentum variance-reduced technique. Moreover, we prove
that the VR-BGPO reaches the best known sample complexity of \( O(\epsilon^{-3}) \) under the
nonconvex setting.

c) We design a unified policy optimization framework based on the mirror descend it-
eration and momentum techniques, and provide its convergence analysis under the
nonconvex setting.

In Table 1 shows the sample complexities of the representative PG algorithms based on
mirror descent algorithm. Shani et al. (2020); Liu et al. (2019) have established global
convergence of a mirror descent variant of PG under some pre-specified setting such as
over-parameterized networks (Liu et al., 2019) by exploiting these specific problems’ hid-
den convex nature. Without these special structures, global convergence of these methods
cannot be achieved. However, our framework does not rely on any specific policy classes,
and our convergence analysis only builds on the general nonconvex setting. Thus, we only
prove that our methods convergence to stationary points.

Geist et al. (2019); Jin and Sidford (2020); Lan (2021); Zhan et al. (2021) studied a
general theory of regularized MDPs based on policy function space that generally is discon-
tinuous. Since both the state and action spaces \( S \) and \( A \) generally are very large in practice,
the policy function space is large. While our methods build on policy parameter space that
is generally continuous Euclidean space and relatively small. Clearly, our methods and
theoretical results are more practical than the results in (Geist et al., 2019; Jin and Sidford,
2020; Lan, 2021; Zhan et al., 2021). (Tomar et al., 2020) also proposes mirror descent PG
framework based on policy parameter space, but it does not provide any theoretical results
and only focuses on Bregman divergence taking form of KL divergence. Our framework can
collaborate with any Bregman divergence forms. At the same time, our framework can also
flexibly use the momentum and variance reduced techniques.
2. Related Works

In this section, we review some related work about mirror descent algorithm in RL and (variance-reduced) PG methods, respectively.

2.1 Mirror Descent Algorithm in RL

Due to easily deal with the regularization terms, mirror descent algorithm has shown significant successes in regularized RL. For example, Neu et al. (2017) have shown both the dynamic policy programming (Azar et al., 2012) and TRPO (Schulman et al., 2015a) algorithms are approximate variants of mirror descent algorithm. Subsequently, Geist et al. (2019) have introduced a general theory of regularized MDPs based on the convex mirror descend algorithm. More recently, Liu et al. (2019) have studied the global convergence properties of PPO and TRPO equipped with overparametrized neural networks based on mirror descent iterations. At the same time, Shani et al. (2020) have analyzed the global convergence properties of TRPO in tabular policy based on the convex mirror descent algorithm. Wang et al. (2019) have proposed divergence augmented policy optimization for off-policy learning based on mirror descent algorithm. MDPO (Tomar et al., 2020) iteratively updates the policy beyond the tabular RL by approximately solving a trust region problem based on convex mirror descent algorithm.

2.2 (Variance-Reduced) PG Methods

PG methods have been widely studied due to their stability and incremental nature in policy optimization. For example, the global convergence properties of vanilla policy gradient method in infinite-horizon MDPs have been recently studied in (Zhang et al., 2019). Subsequently, Zhang et al. (2020) have studied asymptotically global convergence properties of the REINFORCE (Williams, 1992), whose policy gradient is approximated by using a single trajectory or a fixed size mini-batch of trajectories under soft-max parametrization and log-barrier regularization. To accelerate these vanilla PG methods, some faster variance-reduced PG methods have been proposed based on the variance-reduction techniques of SVRG (Johnson and Zhang, 2013), SPIDER (Fang et al., 2018) and STORM (Cutkosky and Orabona, 2019) in stochastic convex and non-convex optimization. For example, fast SVRPG (Papini et al., 2018 Xu et al., 2019a) algorithm have been proposed based on SVRG. Fast HAPG (Shen et al., 2019) and SRVR-PG (Xu et al., 2019a) algorithms have been presented by using SPIDER technique. More recently, the momentum-based PG methods (i.e., IS-MBPG and HA-MBPG) (Huang et al., 2020) have been developed based on variance-reduced technique of STORM.

3. Preliminaries

In the section, we will review some preliminaries of Markov decision process and policy gradients.
3.1 Notations

Let \([n] = \{1, 2, \cdots, n\}\) for all \(n \in \mathbb{N}_+\). For a vector \(x \in \mathbb{R}^d\), let \(\|x\|\) denote the \(\ell_2\) norm of \(x\), and \(\|x\|_p = \left(\sum_{i=1}^d |x_i|^p\right)^{1/p} (p \geq 1)\) denotes the \(p\)-norm of \(x\). For two sequences \(\{a_k\}\) and \(\{b_k\}\), we denote \(a_k = O(b_k)\) if \(a_k \leq C b_k\) for some constant \(C > 0\), and \(\tilde{O}(\cdot)\) hides logarithmic factors. \(\mathbb{E}[X]\) and \(\mathbb{V}[X]\) denote the expectation and variance of random variable \(X\), respectively.

3.2 Markov Decision Process

Reinforcement learning generally involves a discrete time discounted Markov Decision Process (MDP) defined by a tuple \(\{\mathcal{S}, \mathcal{A}, \mathbb{P}, r, \gamma, \rho_0\}\). \(\mathcal{S}\) and \(\mathcal{A}\) denote the state and action spaces of the agent, respectively. \(\mathbb{P}(s'|s,a) : \mathcal{S} \times \mathcal{A} \rightarrow \triangle(\mathcal{S})\) is the Markov kernel that determines the transition probability from the state \(s\) to \(s'\) under taking an action \(a \in \mathcal{A}\). \(r(s,a) : \mathcal{S} \times \mathcal{A} \rightarrow [\mathbb{R}] (\mathbb{R} > 0)\) is the reward function of \(s\) and \(a\), and \(\rho_0 = p(s_0)\) denotes the initial state distribution. \(\gamma \in (0,1)\) is the discount factor. Let \(\pi : \mathcal{S} \rightarrow \triangle(\mathcal{A})\) be a stationary policy, where \(\triangle(\mathcal{A})\) is the set of probability distributions on \(\mathcal{A}\).

Given the current state \(s_t \in \mathcal{S}\), the agent executes an action \(a_t \in \mathcal{A}\) following a conditional probability distribution \(\pi(a_t|s_t)\), and then the agent obtains a reward \(r_t = r(s_t,a_t)\). At each time \(t\), we can define the state-action value function \(Q_\pi(s_t,a_t)\) and state value function \(V_\pi(s_t)\) as follows:

\[
Q_\pi(s_t,a_t) = \mathbb{E}_{s_{t+1}, a_{t+1}, \cdots} \left[ \sum_{l=0}^\infty \gamma^l r_{t+l} \right], \quad V_\pi(s_t) = \mathbb{E}_{a_t, s_{t+1}, \cdots} \left[ \sum_{l=0}^\infty \gamma^l r_{t+l} \right].
\]

We also define the advantage function \(A_\pi(s_t,a_t) = Q_\pi(s_t,a_t) - V_\pi(s_t)\). The goal of the agent is to find the optimal policy by maximizing the expected discounted reward

\[
\max_\pi J(\pi) := \mathbb{E}_{s_0 \sim \rho_0} [V_\pi(s_0)].
\]

Consider a time horizon \(H\), the agent collects a trajectory \(\tau = \{s_t, a_t\}_{t=0}^{H-1}\) under any stationary policy. Then the agent obtains a cumulative discounted reward \(r(\tau) = \sum_{t=0}^{H-1} \gamma^t r(s_t,a_t)\). Since the state and action spaces \(\mathcal{S}\) and \(\mathcal{A}\) are generally very large, directly solving the problem (2) is difficult. Thus, we let the policy \(\pi\) be parametrized as \(\pi_\theta\) for the parameter \(\theta \in \Theta \subseteq \mathbb{R}^d\). Given the initial distribution \(\rho_0 = p(s_0)\), the probability distribution over trajectory \(\tau\) can be obtain

\[
p(\tau|\theta) = p(s_0) \prod_{t=0}^{H-1} \mathbb{P}(s_{t+1}|s_t,a_t)\pi_\theta(a_t|s_t).
\]

Thus, the problem (2) will be equivalent to maximize the expected discounted trajectory reward:

\[
\max_{\theta \in \Theta} J(\theta) := \mathbb{E}_{\tau \sim p(\tau|\theta)} [r(\tau)].
\]
3.3 Policy Gradients

The policy gradient methods (Williams, 1992; Sutton et al., 2000) are a class of effective policy-based methods to solve the above RL problem (4). Specifically, the gradient of $J(\theta)$ with respect to $\theta$ is given as follows:

$$\nabla J(\theta) = \mathbb{E}_{\tau \sim p(\tau|\theta)} \left[ \nabla \log (p(\tau|\theta)) r(\tau) \right].$$

(5)

Given a mini-batch trajectories $B = \{\tau_i\}_{i=1}^n$ sampled from the distribution $p(\tau|\theta)$, the standard stochastic policy gradient ascent update at $(k+1)$-th step, defined as

$$\theta_{k+1} = \theta_k + \eta \nabla J_B(\theta_k),$$

(6)

where $\eta > 0$ is learning rate, and $\nabla J_B(\theta_k) = \frac{1}{n} \sum_{i=1}^n g(\tau_i|\theta_k)$ is stochastic policy gradient. Given $H = O(\frac{1}{\eta^2})$ as in (Zhang et al., 2019; Shani et al., 2020), $g(\tau|\theta)$ is the unbiased stochastic policy gradient of $J(\theta)$, i.e., $\mathbb{E}[g(\tau|\theta)] = \nabla J(\theta)$, where

$$g(\tau|\theta) = \left( \sum_{t=0}^{H-1} \nabla_\theta \log \pi_\theta(a_t, s_t) \right) \left( \sum_{t=0}^{H-1} \gamma^t r(s_t, a_t) \right).$$

(7)

Based on the gradient estimator in (7), we can obtain the existing well-known policy gradient estimators such as REINFORCE (Williams, 1992), PGT (Sutton et al., 2000). Specifically, the REINFORCE obtains a policy gradient estimator by adding a baseline $b$, defined as

$$g(\tau|\theta) = \left( \sum_{t=0}^{H-1} \nabla_\theta \log \pi_\theta(a_t, s_t) \right) \left( \sum_{t=0}^{H-1} \gamma^t r(s_t, a_t) - b_t \right).$$

The PGT is a version of the REINFORCE, defined as

$$g(\tau|\theta) = \sum_{t=0}^{H-1} \sum_{j=t}^{H-1} (\gamma^j r(s_j, a_j) - b_j) \nabla_\theta \log \pi_\theta(a_t, s_t).$$

Alternatively, based on the policy gradient Theorem (Sutton et al., 2000; Zhang et al., 2019), we also have the following policy gradient form

$$\nabla J(\theta) = \mathbb{E}_{(s,a) \sim P_\theta(s,a)} \left[ \nabla \log \pi_\theta(s,a) Q^{\pi_\theta}(s,a) \right],$$

(8)

where $P_\theta(s,a) = P_\theta(s) \pi_\theta(a|s)$ and $P_\theta(s) = \sum_{t=0}^{H-1} \gamma^t P(s_t = s|s_0, \pi_\theta)$ denotes a valid probability measure over the state $S$. Here $P(s_t = s|s_0, \pi_\theta)$ is the probability that state $s_t = s$ given initial state $s_0$ and policy parameter $\theta$. Since any function $h(\cdot): S \rightarrow \mathbb{R}$ is independent of action $a$, the policy gradient (8) is equivalent to

$$\nabla J(\theta) = \mathbb{E}_{(s,a) \sim P_\theta(s,a)} \left[ \nabla \log \pi_\theta(s,a) \left( Q^{\pi_\theta}(s,a) - h(s) \right) \right],$$

where $h(s)$ is a baseline function. When choose the state-value function $V^{\pi_\theta}(s)$ as a baseline function $h(s)$, we can obtain the advantage-based policy gradient

$$\nabla J(\theta) = \mathbb{E}_{(s,a) \sim P_\theta(s,a)} \left[ \nabla \log \pi_\theta(s,a) A^{\pi_\theta}(s,a) \right].$$

(9)
4. Bregman Gradient Policy Optimization

In the section, we propose a novel Bregman gradient policy optimization framework based on Bregman divergences and momentum techniques. We first let $f(\theta) = -J(\theta)$, the goal of policy-based RL is to solve the following problem:

$$\max_{\theta \in \Theta} J(\theta) \iff \min_{\theta \in \Theta} f(\theta).$$

(10)

So we have $\nabla f(\theta) = -\nabla J(\theta)$.

**Definition 1** (Zhang and He, 2018) Given a function $h(x)$ defined on a closed convex set $\mathcal{X}$, and the Bregman distance $D_\psi$: $D_\psi(y,x) = \psi(y) - \psi(x) - \langle \nabla \psi(x), y - x \rangle$, $\forall x, y \in \mathcal{X}$, then we define a proximal operator $P_{\lambda,h}^\psi$ of function $h(x)$,

$$P_{\lambda,h}^\psi(x) = \arg \min_{y \in \mathcal{X}} \{ h(y) + \frac{1}{\lambda} D_\psi(y,x) \},$$

(11)

where $\lambda > 0$, $\psi(x)$ is a continuously-differentiable and $\nu$-strongly convex function, i.e., $\langle x - y, \nabla \psi(x) - \nabla \psi(y) \rangle \geq \nu \| x - y \|$, $\nu > 0$.

Based on the proximal operator of $P_{\lambda,h}^\psi$ as in Zhang and He (2018); Ghadimi et al. (2016), we define the Bregman gradient as follows:

$$B_{\lambda,h}^\psi(x) = \frac{1}{\lambda} (x - P_{\lambda,h}^\psi(x)).$$

(12)

If $\psi(x) = \frac{1}{2} \| x \|^2$ and $\mathcal{X} = \mathbb{R}^d$, $x^*$ is a stationary point of $h(x)$ if and only if $B_{\lambda,h}^\psi(x^*) = \nabla h(x^*) = 0$. Thus, this Bregman gradient can be regarded as a generalized gradient.

4.1 BGPO Algorithm

In the subsection, we propose a Bregman gradient policy optimization (BGPO) algorithm based on the basic momentum technique. The pseudo code of BGPO Algorithm is provided in Algorithm 1.

In Algorithm 1, the step 5 uses the stochastic Bregman gradient descent (a.k.a., stochastic mirror descent) to update the parameter $\theta$. Let $h(\theta) = \langle \theta, u_k \rangle$ be the first-order approximation of function $f(\theta)$ at $\theta_k$, where $u_k$ is a stochastic gradient of function $f(\theta)$ at $\theta_k$. By the step 5 of Algorithm 1 and the above equality (12), we have

$$B_{\lambda,h}^\psi(\theta_k) = \frac{1}{\lambda} (\theta_k - \tilde{\theta}_{k+1}),$$

(13)

where $\lambda > 0$. Then by the step 6 of Algorithm 1, we have

$$\theta_{k+1} = \theta_k - \lambda \eta_k B_{\lambda,h}^\psi(\theta_k),$$

(14)

where $0 < \eta_k \leq 1$. Due to the convexity of set $\Theta$ and $\theta_1 \in \Theta$, we choose the parameter $\eta_k \in (0,1]$ to ensure the updated sequence $\{\theta_k\}_{k=1}^K$ in the set $\Theta$.

In fact, our algorithm unifies many popular policy optimization algorithms. When the mirror mappings $\psi_k(\theta) = \frac{1}{2} \| \theta \|^2$ for $\forall k \geq 1$, then the update (14) is reduced to classic policy
Algorithm 1 BGPO Algorithm

1: **Input**: Total iteration $K$, tuning parameters $\{\lambda, b, m, c\}$ and mirror mappings $\{\psi_k\}^K_{k=1}$ are $\nu$-strongly convex functions;
2: **Initialize**: $\theta_1 \in \Theta$, and sample a trajectory $\tau_1$ from $p(\tau | \theta_1)$, and compute $u_1 = -g(\tau_1 | \theta_1)$;
3: **for** $k = 1, 2, \ldots, K$ **do**
4:     Compute $\eta_k = \frac{b}{(m+k)^{1/2}}$;
5:     Update $\tilde{\theta}_{k+1} = \arg \min_{\theta \in \Theta} \{ \langle u_k, \theta \rangle + \frac{1}{\lambda} D_{\psi_k}(\theta, \theta_k) \}$;
6:     Update $\theta_{k+1} = \theta_k + \eta_k (\tilde{\theta}_{k+1} - \theta_k)$;
7:     Update $\beta_{k+1} = \gamma \beta_k$;
8:     Sample a trajectory $\tau_{k+1}$ from $p(\tau | \theta_{k+1})$, and compute $u_{k+1} = -\beta_{k+1} g(\tau_{k+1} | \theta_{k+1}) + (1 - \beta_{k+1}) u_k$;
9: **end for**
10: **Output**: $\theta_\zeta$ chosen uniformly random from $\{\theta_k\}^K_{k=1}$.

gradient algorithms (Sutton et al., 2000; Zhang et al., 2019, 2020). In the other words, given $u_k = -g(\tau_k | \theta_k)$ and $\psi_k(\theta) = \frac{1}{2} \| \theta \|^2$, we have $B^\psi_k(\theta_k) = -g(\tau_k | \theta_k)$ and

$$\theta_{k+1} = \theta_k + \eta_k g(\tau_k | \theta_k).$$

When the mirror mappings $\psi_k(\theta) = \frac{1}{2} \theta^T F(\theta_k) \theta$ with $F(\theta_k) = \mathbb{E}_{(s,a) \sim P_{\theta_k}(s,a)} [\nabla_\theta \pi_{\theta_k}(s,a) \nabla_\theta \pi_{\theta_k}(s,a)^T]$ for $\forall k \geq 1$, the update (14) is reduced to natural policy gradient algorithms (Kakade, 2001; Liu et al. 2020). Similarly, given $u_k = -g(\tau_k | \theta_k)$ and $\psi_k(\theta) = \frac{1}{2} \theta^T F(\theta_k) \theta$, we have $B^\psi_k(\theta_k) = -F(\theta_k)^+ g(\tau_k | \theta_k)$ and

$$\theta_{k+1} = \theta_k + \lambda \eta_k g(\tau_k | \theta_k),$$

where $F(\theta_k)^+$ denotes the Moore-Penrose pseudoinverse of the Fisher information matrix $F(\theta_k)$.

When mirror mapping $\psi_k$ is Boltzmann-Shannon entropy function Shannon (1948), i.e., $D_{\psi_k}$ is the KL divergence, the update (14) is reduced to relative mirror descent policy optimization (MDPO) (Tomar et al., 2020).

4.2 VR-BGPO Algorithm

In the subsection, we propose a faster variance-reduced Bregman gradient policy optimization (VR-BGPO) algorithm based on a variance-reduced technique. The pseudo code of VR-BGPO algorithm is provided in Algorithm 2.

Consider the problem (4) is non-oblivious that the distribution $p(\tau | \theta)$ depends on the variable $\theta$ varying through the whole optimization procedure, we apply the importance sampling weight (Papini et al., 2018; Xu et al., 2019a) in estimating our policy gradient $u_{k+1}$, defined as

$$w(\tau_{k+1} | \theta_k, \theta_{k+1}) = \frac{p(\tau_{k+1} | \theta_k)}{p(\tau_{k+1} | \theta_{k+1})} = \prod_{t=0}^{H-1} \frac{\pi_{\theta_k}(a_t | s_t)}{\pi_{\theta_{k+1}}(a_t | s_t)}.$$


When algorithms, defined as In this subsection, we given a reasonable metric to measure the convergence of our Algorithm 5.1 Convergence Metric

are $\nu$-strongly convex functions;

In this section, we will analyze the convergence properties of the proposed algorithms. All related proofs are provided in the Appendix.

5. Convergence Analysis

In this section, we will analyze the convergence properties of the proposed algorithms. All related proofs are provided in the Appendix.

5.1 Convergence Metric

In this subsection, we given a reasonable metric to measure the convergence of our Algorithms, defined as

$$G_k = \frac{\nu}{\lambda} \|\hat{\theta}_{k+1} - \theta_k\| + \|\nabla f(\theta_k) - u_k\|, \quad (17)$$

where $\nu > 0$ depends on the $\nu$-strongly convex function $\psi_k$. This convergence metric form is also used in the SUPER-ADAM algorithm (Huang et al., 2021). When $G_k = \frac{\nu}{\lambda} \|\hat{\theta}_{k+1} - \theta_k\| + \|\nabla f(\theta_k) - u_k\| \to 0$, we have $u_k \to \nabla f(\theta_k)$ and $\hat{\theta}_{k+1} = P_{\psi_k}^{\lambda}(\theta, u_k)$.

Thus, we have

$$G_k \to \nu \|B_{\psi_k}^{\lambda}(\theta, \nabla f(\theta_k))\| \to 0. \quad (18)$$

In fact, our metric is a generalized metric to $\|B_{\psi_k}^{\lambda}(\theta, \nabla f(\theta_k))\|$ used in (Zhang and He, 2018; Yang et al. 2019). In particular, when $\Theta = \mathbb{R}^d$ and $\psi_k(\theta) = \frac{1}{2} \|\theta\|^2$, we have $\nu = 1$ and

$$G_k = \|\nabla f(\theta_k) - u_k\| + \|u_k\| \geq \|\nabla f(\theta_k)\|. \quad (19)$$

When $G_k \to 0$, we can obtain $\|\nabla f(\theta_k)\| \to 0$. 

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Algorithm 2 VR-BGPO Algorithm

1: Input: Total iteration $K$, tuning parameters $\{\lambda, b, m, c\}$ and mirror mappings $\{\psi_k\}_{k=1}^K$ are $\nu$-strongly convex functions;
2: Initialize: $\theta_1 \in \Theta$, and sample a trajectory $\tau_1$ from $p(\tau|\theta_1)$, and compute $u_1 = -g(\tau_1|\theta_1)$;
3: for $k = 1, 2, \ldots, K$ do
4: Compute $\eta_k = b (m+k)^{1/4}$;
5: Update $\hat{\theta}_{k+1} = \arg \min_{\theta \in \Theta} \{\langle u_k, \theta \rangle + \frac{1}{\lambda} D_{\psi_k}(\theta, \theta_k)\}$;
6: Update $\theta_{k+1} = \theta_k + \eta_k (\hat{\theta}_{k+1} - \theta_k)$;
7: Update $\beta_{k+1} = c \eta_k^2$;
8: Sample a trajectory $\tau_{k+1}$ from $p(\tau|\theta_{k+1})$, and compute $u_{k+1} = -\beta_{k+1} g(\tau_{k+1}|\theta_{k+1}) + (1 - \beta_{k+1}) (u_k - g(\tau_{k|1}|\theta_{k+1}) + w(\tau_{k+1}|\theta_k, \theta_{k+1}) g(\tau_{k+1}|\theta_k))$;
9: end for
10: Output: $\theta_\zeta$ chosen uniformly random from $\{\theta_k\}_{k=1}^K$. 

Except for different stochastic policy gradients $\{u_k\}$ and tuning parameters $\{\eta_k, \beta_k\}$ using in Algorithms 1 and 2, the steps 5 and 6 in these algorithms for updating parameter $\theta$ are the same. Interestingly, when choosing mirror mapping $\psi_k(\theta) = \frac{1}{2} \|\theta\|^2$, our VR-BGPO algorithm will reduce to a non-adaptive version of IS-MBPG algorithm (Huang et al., 2020).
5.2Some Mild Assumptions

In the subsection, we first give some standard assumptions.

**Assumption 1** For function \( \log \pi_\theta(a|s) \), its gradient and Hessian matrix are bounded, i.e., there exist constants \( C_g, C_h > 0 \) such that

\[
\| \nabla_\theta \log \pi_\theta(a|s) \| \leq C_g, \quad \| \nabla^2_\theta \log \pi_\theta(a|s) \| \leq C_h. \tag{20}
\]

**Assumption 2** Variance of stochastic gradient \( g(\tau|\theta) \) is bounded, i.e., there exists a constant \( \sigma > 0 \), for all \( \pi_\theta \) such that \( \mathbb{V}(g(\tau|\theta)) = \mathbb{E} \| g(\tau|\theta) - \nabla J(\theta) \|^2 \leq \sigma^2 \).

**Assumption 3** For importance sampling weight \( w(\tau|\theta_1, \theta_2) = \frac{p(\tau|\theta_1)}{p(\tau|\theta_2)} \), its variance is bounded, i.e., there exists a constant \( W > 0 \), it follows \( \mathbb{V}(w(\tau|\theta_1, \theta_2)) \leq W \) for any \( \theta_1, \theta_2 \in \mathbb{R}^d \) and \( \tau \sim p(\tau|\theta_2) \).

**Assumption 4** The function \( J(\theta) \) has an upper bound in \( \Theta \), i.e., \( J^* = \sup_{\theta \in \Theta} J(\theta) \).

Assumptions 1 and 2 are standard in the PG algorithms (Papini et al., 2018; Xu et al., 2019a,b). Assumption 3 is widely used in the study of variance reduced PG algorithms (Papini et al., 2018. Xu et al., 2019a). In fact, the bounded importance sampling weight might be violated in some cases such as using neural networks as the policy. Thus, we can clip this importance sampling weights to guarantee the effectiveness of our algorithms as in (Papini et al., 2018). Assumption 4 guarantees the feasibility of the problem (4). Next, we provide some useful lemmas based on the above assumptions.

**Lemma 2** (Proposition 4.2 in Xu et al. (2019b)) Suppose \( g(\tau|\theta) \) is the PGT estimator. Under Assumption 1, we have

1) \( g(\tau|\theta) \) is \( L \)-Lipschitz differential, i.e., \( \| g(\tau|\theta_1) - g(\tau|\theta_2) \| \leq L \| \theta_1 - \theta_2 \| \) for all \( \theta_1, \theta_2 \in \Theta \), where \( L = C_h R/(1 - \gamma)^2 \);

2) \( J(\theta) \) is \( L \)-smooth, i.e., \( \| \nabla^2 J(\theta) \| \leq L \);

3) \( g(\tau|\theta) \) is bounded, i.e., \( \| g(\tau|\theta) \| \leq G \) for all \( \theta \in \Theta \) with \( G = C_g R/(1 - \gamma)^2 \).

**Lemma 3** (Lemma 6.1 in Xu et al. (2019a)) Under Assumptions 1 and 3, let \( w(\tau|\theta_{k-1}, \theta_k) = g(\tau|\theta_{k-1})/g(\tau|\theta_k) \), we have

\[
\mathbb{V}[w(\tau|\theta_{k-1}, \theta_k)] \leq C_w^2 \| \theta_k - \theta_{k-1} \|^2, \tag{21}
\]

where \( C_w = \sqrt{H(2HC_g^2 + C_h)(W + 1)} \).

5.3Convergence Analysis of BGPO Algorithm

In the subsection, we provide convergence properties of the BGPO algorithm. The detailed proof is provided in Appendix A1.
Theorem 4 Assume the sequence \( \{ \theta_k \}_{k=1}^{K} \) be generated from Algorithm 1. Let \( \eta_k = \frac{b}{(m+k)^{1/2}} \) and \( \beta_{k+1} = c\eta_k \) for all \( k \geq 1 \), \( 0 < \lambda \leq \frac{\nu m^{1/2}}{9Lb} \), \( b > 0 \), \( \frac{8\lambda \nu}{b} \leq c \leq \frac{m^{1/2}}{b} \), and \( m \geq \max\{b^2,(cb)^2\} \), we have

\[
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E}[G_k] \leq \frac{2\sqrt{2M}m^{1/4}}{K^{1/2}} + \frac{2\sqrt{2M}}{K^{1/4}},
\]

where \( M = \frac{\nu (J^* - J(\theta_1))}{\lambda^2} + \frac{\nu m^2}{2L^2} + \frac{\nu \sigma^2}{L^2} \ln(m+K) \).

Remark 5 Without loss of generality, let \( b = O(1) \), \( m = O(1) \) and \( \lambda = O(1) \), we have \( M = O(\ln(m+K)) = \tilde{O}(1) \). Theorem 4 shows that the BGPO algorithm has a convergence rate of \( \tilde{O}\left(\frac{1}{K^{1/4}}\right) \). Let \( K^{-\frac{1}{4}} \leq \epsilon \), we have \( K = \tilde{O}(\epsilon^{-4}) \). Since the BGPO algorithm only needs one trajectory to estimate the stochastic policy gradient at each iteration and runs \( K \) iterations, it has the sample complexity of \( 1 \cdot K = \tilde{O}(\epsilon^{-4}) \) for finding an \( \epsilon \)-stationary point.

5.4 Convergence Analysis of VR-BGPO Algorithm

In the subsection, we give convergence properties of the VR-BGPO algorithm. The detailed proof is provided in Appendix A2.

Theorem 6 Suppose the sequence \( \{ \theta_k \}_{k=1}^{K} \) be generated from Algorithm 2. Let \( \eta_k = \frac{b}{(m+k)^{1/3}} \) and \( \beta_{k+1} = c\eta_k \) for all \( k \geq 1 \), \( 0 < \lambda \leq \frac{\nu m^{1/3}}{5Lb} \), \( b > 0 \), \( c \in \left[ \frac{2}{3\lambda^2} + \frac{20L^2\lambda^2}{\nu^2}, \frac{m^{2/3}}{b^2} \right] \) and \( m \geq \max\{2,b^3,(cb)^3,(\frac{5}{6b})^{2/3}\} \), we have

\[
\frac{1}{K} \sum_{k=1}^{K} \mathbb{E}[G_k] \leq \frac{2\sqrt{2M^\prime}m^{1/6}}{K^{1/2}} + \frac{2\sqrt{2M^\prime}}{K^{1/3}},
\]

where \( M^\prime = \frac{\nu (J^* - J(\theta_1))}{\lambda^2} + \frac{m^{1/3} \nu \sigma^2}{16b^2 L^2 \lambda^2} + \frac{\nu^2 \sigma^2}{8L^2 \lambda^2} \) and \( \tilde{L}^2 = L^2 + 2G^2 C_w^2 \).

Remark 7 Without loss of generality, let \( b = O(1) \), \( m = O(1) \) and \( \lambda = O(1) \), we have \( M = O(\ln(m+K)) = \tilde{O}(1) \). Theorem 6 shows that the VR-BGPO algorithm has a convergence rate of \( \tilde{O}\left(\frac{1}{K^{1/3}}\right) \). Let \( K^{-\frac{1}{3}} \leq \epsilon \), we have \( K = \tilde{O}(\epsilon^{-3}) \). Since the VR-BGPO algorithm only needs one trajectory to estimate the stochastic policy gradient at each iteration and runs \( K \) iterations, it reaches a lower sample complexity of \( 1 \cdot K = \tilde{O}(\epsilon^{-3}) \) for finding an \( \epsilon \)-stationary point.

6. Experiments

In this section, we conduct some RL tasks to verify the effectiveness of our methods. We first study the effect of different choices of Bregman divergences with our algorithms (BGPO and VR-BGPO), and then we compare our VR-BGPO algorithm with other state-of-the-art methods such as TRPO (Schulman et al. 2015a), PPO (Schulman et al. et al., 2017), VRMPO (Yang et al., 2019), and MDPO (Tomar et al., 2020).
6.1 Effects of Bregman Divergences

In the subsection, we examine how different Bregman divergences affect the performance of our algorithms. In the first setting, we let mirror mapping $\psi_k(x) = \|x\|_p$ with different $p$ to test the performance of our algorithms. Let $\psi^*_k(y) = (\sum_{i=1}^d |y_i|^q)^{\frac{1}{q}}$ be the conjugate mapping of $\psi_k(x)$, where $p^{-1} + q^{-1} = 1$, $p, q > 0$. According to (Beck and Teboulle, 2003), when $\Theta = \mathbb{R}^d$, the update of $\tilde{\theta}_{k+1}$ in our algorithms can be calculated by $\tilde{\theta}_{k+1} = \nabla \psi^*_k(\nabla \psi_k(\theta_k) + \lambda u_k)$, where $\nabla \psi_k(x_j)$ and $\nabla \psi^*_k(y_j)$ are $p$-norm link functions, and $\nabla \psi_k(x_j) = \frac{\text{sign}(x_j) |x_j|^{p-1}}{\|x\|_p^{p-2}}$, $\nabla \psi^*_k(y_j) = \frac{\text{sign}(y_j) |y_j|^{q-1}}{\|y\|_q^{q-2}}$, and $j$ is the coordinate index of $x$ and $y$.

In the second setting, we apply diagonal term on the mirror mapping $\psi_k(x) = \frac{1}{2} x^T M_k x$, where $M_k$ is a diagonal matrix with positive values. In the experiments, we generate $M_k = \text{diag}(\sqrt{v_k} + \alpha)$, $v_k = \beta v_{k-1} + (1 - \beta) u_k^2$, and $\alpha > 0, \beta \in (0, 1)$, as in the SUPER-ADAM algorithm (Huang et al., 2021). Then we have $D_{\psi_k}(y, x) = \frac{1}{2} (y - x)^T M_k (y - x)$. Under this setting, the update of $\theta_{k+1}$ can also be analytically solved $\theta_{k+1} = \theta_k - \lambda M_k^{-1} u_k$.

To test the effectiveness of two different Bregman divergences, we evaluate them on three classic control environments from gym Brockman et al. (2016): CartPole-v1, Acrobat-v1, and MountainCarContinuous-v0. In the experiment, categorical policy is used for CartPole and Acrobat environments, and Gaussian policy is used for MountainCar. Gaussian value functions are used in all settings. All policies and value functions are parameterized by multilayer perceptrons (MLPs). For a fair comparison, all settings use the same initialization.
for policies. We run each setting five times and plot the mean and variance of average returns.

For $l_p$-norm mapping, we test three different values of $p = (1.50, 2.0, 3.0)$. For diagonal mapping, we set $\beta = 0.999$ and $\alpha = 10^{-8}$. We set hyperparameters $\{b, m, c\}$ to be the same. $\lambda$ still needs to be tuned for different $p$ to achieve relatively good performance. For simplicity, we use BGPO-Diag to represent BGPO with diagonal mapping, and we use BGPO-$l_p$ to represent BGPO with $l_p$-norm mapping. Details about the setup of environments and hyperparameters are provided in the Appendix B.

From Fig. 1, we can find that BGPO-Diag largely outperforms BGPO-$l_p$ with different choices of $p$. The parameter tuning of BGPO-$l_p$ is much more difficult than BGPO-Diag because each $p$ requires an individual $\lambda$ to achieve the desired performance. To reach the best possible performance $\{b, m, c\}$ may also need to be tuned. On the contrary, the parameter tuning of BGPO-Diag is much simpler, and the performance is also more stable.

### 6.2 Comparison between BGPO and VR-BGPO

To understand the effectiveness of variance reduced technique used in our VR-BGPO algorithm, we compare BGPO and VR-BGPO using the same settings introduced in section 6.1. Both algorithms use the diagonal mapping for $\psi$, since it performs much better than $l_p$-norm.

From Fig. 2, we can see that VR-BGPO can outperform BGPO in all three environments. In CartPole, both algorithms converge very fast and have similar performance, and VR-BGPO is more stable than BGPO. The advantage of VR-BGPO becomes large in Acrobat
and MountainCar environments, probably because the task is more difficult compared to CartPole.

6.3 Compare to other Methods

In this subsection, we apply our VR-BGPO algorithm to compare with the other methods since it has better performances shown in section 6.2. Specifically, we compare VR-BGPO with two popular policy optimization algorithms: PPO (Schulman et al., 2017) and TRPO (Schulman et al., 2015a). We also compare our algorithm with recent proposed mirror-descent based policy optimization algorithms: VRMPO (Yang et al., 2019) and MDPO (Tomar et al., 2020). For VR-BGPO, we continue to use diagonal mapping for $\psi$. For VRMPO, we follow their implementation and use $l_p$-norm for $\psi$. For MDPO, $\psi$ is the negative Shannon entropy, and the Bregman divergence becomes KL-divergence.

To evaluate the performance of these algorithms, we test them on six gym (Brockman et al., 2016) environments with continuous control tasks: Inverted-Pendulum-v2, InvertedDoublePendulum-v2, Walker2d-v2, Reacher-v2, Swimmer-v2, and HalfCheetah-v2. We use Gaussian policies and Gaussian value functions for all environments, and both of them are parameterized by MLPs. To ensure a fair comparison, all policies use the same initialization. For TRPO and PPO, we use the implementations provided by garage (garage contributors, 2019). We carefully implement MDPO and VRMPO following the description provided by the original papers. All methods include our method, are implemented with garage (garage contributors, 2019) and pytorch (Paszke et al., 2019). We run all algorithms ten times on each environment and report the mean and variance of average returns. Details about the setup of environments and hyperparameters are also provided in the Appendix B.

From Fig. 3, we can find that VR-BGPO consistently outperforms all the other methods on six environments, demonstrating the effectiveness of our algorithm. Specifically, our method achieves the best mean average return in InvertedDoublePendulum, Swimmer, and HalfCheetah. Our method converges faster than other methods in all environments, especially in InvertedPendulum, InvertedDoublePendulum, and Reacher. MDPO can achieve good results in some environments, but it can not outperform PPO or TRPO in Swimmer and InvertedDoublePendulum. VRMPO only outperforms PPO and TRPO in Reacher and InvertedDoublePendulum. The undesirable performance of VRMPO is probably because it uses $l_p$ norm for $\psi$, which requires careful tuning of the learning rate.

7. Conclusion

In the paper, we proposed a novel Bregman gradient policy optimization framework for reinforcement learning based on mirror descend iteration and momentum techniques. Moreover, we studied convergence properties of the proposed policy optimization methods under the nonconvex setting.

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