A panorama of CDCC calculations for deuteron induced reactions: from elastic cross sections to transfer and inelastic ones.

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Abstract. In this contribution, an overview of the calculations that can be performed within the Continuum Discretized Coupled Channels (CDCC) approach to analyze deuteron induced reactions is given. First, we briefly remind the CDCC formalism. In the second part, we present an extension of the CDCC formalism which accounts for the target excitations allowing us to determine \((d,d')\) cross sections off deformed nuclei. After the derivation of the coupled equations, we compare some calculated inelastic cross sections with experimental data. Then it is shown that the CDCC formalism can also be a useful tool to determine \((d,p)\) or \((d,n)\) cross sections. We illustrate this point for nuclei with \(N \approx 32\).

1. Introduction
In the seventies, the Continuum Discretized Coupled Channels (CDCC) formalism was independently proposed by R C Johnson [1] and G H Rawitscher [2] to describe the deuteron induced reactions by explicitly including the breakup channels. Since then, it has been widely studied and quite successfully used to analyze deuteron elastic cross sections (see e.g. [3, 4, 5, 6] and references therein). With the advent of high intensity radioactive beams, the study of exotic nuclei (see e.g. [7]) has been performed involving some weakly bound projectiles such as \(\text{Li}\); the CDCC approach has thus been extended to analyse this kind of reactions. It has also been extended to include projectile excitations [8] and to deal with 4-body systems [9]. In the paper, we will focus our attention on deuteron induced reactions showing that elastic, breakup and inelastic channels can be treated on the same footing and that the CDCC formalism is a quite efficient fully quantum mechanical tool to investigate these reactions. We will also perform some calculations to predict transfer cross sections and to extract spectroscopic factors.

2. The Continuum Discretized Coupled Channels framework for deuteron induced reactions
2.1. The Continuum Discretized Coupled Channels formalism
The deuteron and the target can be described as a 3-body system whose effective hamiltonian reads:

\[
\hat{H}_{\text{eff}} = \hat{T}_R + U_p(\vec{r}_p) + U_n(\vec{r}_n) + \hat{T}_p + V_{pn}(\vec{\rho}) + V_p^{\text{(Coul)}}(R),
\]

where \(\vec{R} = (\vec{r}_p + \vec{r}_n)/2\) denotes the center of mass coordinate and \(\vec{\rho} = \vec{r}_n - \vec{r}_p\) the relative one where \(\vec{r}_n\) and \(\vec{r}_p\) are the coordinates of the neutron and the proton, respectively. The total angular momentum is defined by \(J = L + I\) where \(L\) is the orbital angular momentum associated...
to \( \vec{R} \) and \( I = 1 \). From the expression (1) of the hamiltonian, one can note that it is assumed that the target remains inert and that the Coulomb interaction is a function of the center mass coordinate which means that the Coulomb breakup will be neglected.

The wave function of this system is written as

\[
\Psi_{M}^{J} = \sum_{L=|J-1|}^{J+1} \left[ \Phi_{0}(\vec{\rho}) \otimes \chi_{0}(L, J ; \vec{R}) \right]_{M}^{J} + \sum_{\tau=\alpha, \beta} \sum_{L=|J-1|}^{J+1} \int_{0}^{\infty} \left[ \Phi_{\tau}(I = 1 ; k, \vec{\rho}) \otimes \chi(I, \tau, L, J ; P_{k}, \vec{R}) \right]_{M}^{J} dk .
\]

In equation (2) the breakup term \( BU_{lLJ} \) depends on independent states \( \Phi_{\alpha} \) and \( \Phi_{\beta} \) as defined in [5] (each one has a component with \( l = 0 \) and another one with \( l = 2 \)) and represents the continuum contribution but it can not be handled since it involves an infinite number of states. In the CDCC approach it is thus assumed that the continuum can be divided into bins and that for each bin, the wave function can be replaced by an averaged wave function. It is also assumed that a finite number of bins can be taken into account.

Therefore the CDCC trial wave function is given by:

\[
\Psi_{M}^{J} \sim \sum_{L=|J-1|}^{J+1} \left[ \Phi_{0}(\vec{\rho}) \otimes \chi_{0}(L, J ; \vec{R}) \right]_{M}^{J} + \sum_{\tau=\alpha, \beta} \sum_{L=|J-1|}^{J+1} \sum_{i=1}^{N} \left[ \Phi_{1,\tau}(I = 1 ; \vec{\rho}) \otimes \tilde{\chi}_{i}(I, \tau, L, J ; \vec{R}) \right]_{M}^{J} .
\]

(3)

If \( c \) stands for the quantum numbers \((i, \tau, I, L, J)\) and \( u_{c}^{J} \) denote the radial part of \( \tilde{\chi}_{c} \) then a set of coupled channels equations can be derived by left-multiplying the Schrödinger equation and integrating over angular variables and \( \rho \):

\[
\left( -\frac{\hbar^{2}}{2\mu_{R}} \frac{d^{2}}{d\vec{R}^{2}} + \frac{\hbar^{2}L(L+1)}{2\mu_{R}R^{2}} + V_{p}^{(\text{Coul})}(R) - E_{i} \right) u_{c}^{J}(R) = - \sum_{c'} F_{cc'}^{J}(R) u_{c'}^{J}(R)
\]

(4)

where the form factors \( F_{cc'}^{J} \) are derived by folding the nucleon-nucleus optical potentials with the proton-neutron wave functions:

\[
F_{cc'}^{J} = \left\langle \Phi_{1,\tau} \otimes Y_{L}(\hat{R}) \right\rangle_{JM} U_{p} + U_{n} \left\langle \Phi_{\gamma,\tau'} \otimes Y_{L}(\hat{R}) \right\rangle_{JM} \delta_{\gamma,\tau',c'} .
\]

(5)

In equation (5) the brackets \( \langle \ \rangle \) denote the integration over the angular variables and \( \rho \). The solutions must satisfy the following boundary conditions:

\[
u_{c}^{J}(R) \rightarrow \delta_{\gamma,\rho} U_{\gamma}(\hat{R}, R) - \sqrt{R_{p}/R_{0}} S_{\gamma,\rho}(\hat{R}, R) U_{\gamma}(\hat{R}) .
\]

From the solutions, one can then deduce the S-matrix elements \( S_{\gamma,\rho}^{(J)} \) and the cross sections.

On figure 1, the ratio of the elastic differential cross section to the Rutherford one has been plotted. The dashed line, dotted line, the dashed-dotted line and the solid one have been calculated by using 0, 2, 4 and 10 bins to discretize the continuum. One can notice that the cross sections converge with an increasing bin number, that the experimental data plotted as dots on figure 1 can be reproduced by the CDCC cross sections and that a better agreement is obtained when including the breakup channel even if some discrepancies are observed.
2.2. Calculations of inelastic cross sections for rotational nuclei

To include the target degrees of freedom, the wave function of the system $|\Phi_{J_T M_T}(\vec{p}, \vec{R})\rangle$ with total angular momentum $J_T$ and projection $M_T$ along the z-axis is now extended as follows:

$$|\Phi_{J_T M_T}(\vec{p}, \vec{R})\rangle = \sum_{dS}\sum_{l J t} U_c(R) i^L \left( \left[ \left( \varphi_{d,l}(\vec{p}) \otimes \chi_S \right)^I_{p} \otimes Y_L(\vec{R}) \right]^J \otimes \psi_l(\xi_t) \right)_{M_T}$$

where $\vec{p}$, $\vec{R}$ and $\xi_t$ are the proton-neutron relative coordinates, the proton-neutron center of mass ones and the target variables, respectively. In equation (6), $c$ denotes the channel labeled by $(i S I_p L J I_t)$ where $i$ represents the bin number for the proton-neutron wave function, $l$ is the orbital angular momentum associated to $\vec{p}$, $I_p$ is obtained by coupling $l$ and $S = 1$, $L$ is the orbital angular momentum associated to $\vec{R}$, $J$ is the coupling between $L$ and $I_p$, $I_t$ is the spin of the target level and $\varphi_{d,l}$ and $\psi_l$ represent the deuteron wave function and the target state.

The hamiltonian of the system is:

$$\hat{H} = \hat{T}_R + V_{pA}(\vec{r}_p, \xi_t) + V_{nA}(\vec{r}_n, \xi_t) + V_{Coul} + \hat{H}_{pn} + \hat{H}_{A}$$

with

$$\hat{H}_{pn} \varphi_{d,l}(\vec{p}) = \varepsilon_i \varphi_{d,l}(\vec{p}) \text{ and } \hat{H}_{A} \psi_l(\xi_t) = \varepsilon_I \psi_l(\xi_t).$$

As shown by Tamura [10] the optical potentials $V_{aA}(\vec{r}_a, \xi_t)$ for rotational nuclei ($a$ stands for $n$ or $p$) can be written as:

$$V_{aA}(\vec{r}_a, \xi_t) = \sum_{\lambda\mu} \sqrt{\frac{4\pi}{2\lambda + 1}} Y_{aA}^{(\lambda)}(r_a) D_{\mu 0}^{\lambda}(\Theta_k) Y_{\lambda \mu}(\theta_a, \phi_a)$$

where $\Theta_k (k = 1, 2, 3)$ are the Eulerian angles. In equation (7), $V_{aA}^{(\lambda)}$ depends only on the radial variable $r_a$ and on the deformation parameters of the target. Introducing Solid Spherical Harmonics and using an addition theorem, it is shown that the nucleon-target optical potentials can be expanded as:

$$V_{aA}(\vec{r}_a, \xi_t) = \sum_{\lambda p} \frac{V_{aA}^{(\lambda)}(r_a) x_a^p R^p y_a^{\lambda - p} \rho^{\lambda - p}}{r_a^\lambda} D_{\mu 0}^{\lambda}(\Theta_k) C_{\lambda,p} \left[ Y^{p}(\vec{R}) \otimes Y^{\lambda - p}(\vec{\rho}) \right]^\lambda_{\mu}$$

where $C_{\lambda,p} = \sqrt{\frac{4\pi(\lambda + 1)!}{(2\lambda - p + 1)!(2p + 1)!}}$ and $\vec{r}_a = x_a \vec{R} + y_a \vec{\rho}$ and $0 \leq p \leq \lambda$. A set of coupled differential equations can then be derived:

$$\left[ -\frac{\hbar^2}{2\mu R} \left( \frac{d^2}{dR^2} - \frac{L(L + 1)}{R^2} \right) + F_{cc} + V_{Coul} - E_c \right] U_{c'} = -\sum_{c''} F_{c c''}(R) U_{c''}$$

with $E_c = E - \varepsilon_i - \varepsilon_I$ and where the coupling $F_{c c'} = \langle i S I_p L J I_t; J_T M_T | V_{pA} + V_{nA} | i' S' I'_p L' J' I'_t; J_T M_T \rangle$ are deduced from equation (8) and the Wigner-Eckart theorem. From the solutions satisfying the appropriate boundary conditions, one deduces the $S$-matrix elements and the cross sections.

We have firstly checked the convergence of the calculations by increasing the number of bins used to discretize the continuum. An illustration is given on figure 2: the elastic and inelastic on the first 24 cross sections for deuteron incident on $^{24}\text{Mg}$ at 70 MeV have been plotted on the left and right panel, respectively. The bin number to discretize the continuum has been increased from 0 (no continuum state) to 10: the thin full line, the dashed one, the
The following calculations have been performed by using the Reid softcore potential to derive the deuteron ground state and $^3S_1$ and $^3D_1$ proton-neutron continuum wave functions discretized as done by G H Rawitscher et al in [5]. To determine the form factors we have used the Koning-Delaroche global parameterization for the nucleon-nucleus optical potentials. On figures 3-4, the elastic and inelastic differential cross sections for deuteron incident on $^{24}$Mg for incident energy ranging from 60.0 MeV to 90.0 MeV have been plotted. The calculated cross section are represented by solid lines and the experimental data measured by A Kiss et al [11] are denoted by the dots. A quite good agreement has been obtained without any adjustment within this energy range.

2.3. Transfer reactions

For the transfer reactions, we have used a simplified approach by assuming that the coupling effect of the transfer channel on the elastic, breakup and inelastic ones is small and can be neglected. We have thus solved the following equations:

$$\begin{cases} [E_\alpha - K_\alpha - \langle \alpha| V_{\alpha\alpha} |\alpha \rangle] u_\alpha (\vec{r}_\alpha) & \approx 0 \\ [E_\beta - K_\beta - \langle \beta| V_{\beta\beta} |\beta \rangle] u_\beta (\vec{r}_\beta) & = \int d\vec{r}_\alpha d\chi_\alpha d\psi_\alpha^* V_{\beta\alpha} (\vec{r}_{\alpha}, \vec{r}_\beta) \psi_\beta u_\alpha (\vec{r}_\alpha) \end{cases}$$

where $\alpha$ and $\beta$ denote two partitions of the system: $\alpha = A + a$, $\beta = B + b$ with $a = p + n$ and $B = A + n$. The calculation of the source term takes into account the non-orthogonality of the two partitions. $u_\alpha$ is replaced by $u_\alpha$ derived by solving the CDCC equations (4), $\psi_\alpha$ is the deuteron ground state and $\psi_\beta$ is the neutron wave function which is a single particle.
**Figure 3.** Elastic and inelastic differential cross sections for deuteron incident on $^{24}$Mg target. The solid line represent the calculated cross sections while the dots represent the measured ones [11]. The incident energy in MeV given near the curves is ranging from 60 MeV to 70.0 MeV.

**Figure 4.** Elastic and inelastic differential cross sections for deuteron incident on $^{24}$Mg target. The solid line represent the calculated cross sections while the dots represent the measured ones [11]. The incident energy in MeV given near the curves is ranging from 72.0 MeV to 90.0 MeV.

**Figure 5.** Differential cross sections and vector analyzing powers for (d,p) reaction for deuteron incident on $^{54}$Cr at $E_{\text{inc}}$=8.0 and 10.0 MeV. The solid lines and the symbols represent the calculations and the data, respectively. $E_x$ is the excitation energy of the occupied level.

**Figure 6.** Differential cross sections and vector analyzing powers for $^{54}$Cr(d,p)$^{55}$Cr reaction for $E_{\text{inc}}$=8.0 and 10.0 MeV and for $E_x$=0.879 MeV. The solid lines and the symbols denote the calculations and the data, respectively. On the left panel the calculations have been performed assuming that the spin of this level is $5/2$ while on the right part it is assumed to be $7/2$. 
wave function calculated with a Woods-Saxon potential whose depth is adjusted to reproduce the experimental binding energy. Many experimental data are available for Cr isotopes (see Table 1).

![Table 1. Mean spectroscopic factors of $^{55}$Cr levels extracted for (d,p) reactions.](image)

| $E_x$ (MeV) | $l_n$ | $j_n^\pi$ | $\langle SF_C \rangle$ | $\langle SF_L \rangle$ [16] | $E_x$ (MeV) | $l_n$ | $j_n^\pi$ | $\langle SF_C \rangle$ |
|------------|------|--------|-----------------|-----------------|------------|------|--------|-----------------|
| 0.000      | 1    | 3/2$^-$| 0.423           | 0.63±0.13       | 0.517      | 3    | 5/2$^-$| 0.194           |
| 0.241      | 1    | 1/2$^-$| 0.123           |                 | 0.564      | 1    | 3/2$^-$| 0.085           |

...e.g. [12, 13, 14, 15] for $^{54}$Cr). We have therefore performed calculations for 100 levels of these isotopes. An illustration of these results are plotted on figures 5-6. On figures 5, we have plotted the differential cross sections and the vector analyzing powers for three levels at 0.564, 1.470 and 2.286 MeV and for two incident energies (8.0 MeV [12] and 10.0 MeV [15, 13, 14]). The solid lines denote the calculations. The measurements performed by D M Rosalky et al. [12] (■), R Bock et al. [15] (▼), A E Macgregor et al. [13] (▲) and T Taylor et al. [14] (⋆) are represented by the symbols. A quite good agreement is obtained. On this figure, some discrepancies are observed between the different sets of data at 10.0 MeV inducing a spreading of the extracted spectroscopic factors. On figure 6 the left panel and the right one show the results for the level at 0.879 MeV assuming that it is a 5/2$^-$ state or a 7/2$^-$ one, respectively. As expected since the differential cross sections mainly depend on the transferred orbital angular momentum (in this case $l=3$), no conclusion can be drawn from the comparison between the calculations and the measured $d\sigma/d\Omega$ but a better agreement between the calculated vector analyzing power and the experimental one has been obtained for $j_\pi = 5/2^-$. Therefore from this experiment, it seems that a spin 5/2$^-$ should be assigned to this level. We have also extracted the spectroscopic factors from these experimental data: some examples are given in table 1.

3. Conclusion

We have shown that the CDCC approach can be used to predict elastic and inelastic cross sections for deuteron induced reactions. The CDCC wave functions can also be used to determine the source term for transfer reactions and to obtain (d,p) cross sections which are a usefull and reliable tool to investigate the spectral properties of nuclei.

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