Berry Curvature Engineering in Magnetic Topological Insulator Heterostructures

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The Berry phase picture provides important insights into the electronic properties of condensed matter systems. The intrinsic anomalous Hall (AH) effect can be understood as a consequence of non-zero Berry curvature in momentum space. The realization of the quantum anomalous Hall effect provided conclusive evidence for the intrinsic mechanism of the AH effect in magnetic topological insulators (TIs). Here we fabricated magnetic TI/TI heterostructures and found both the magnitude and sign of the AH effect in the magnetic TI layer can be altered by tuning the TI thickness and/or the electric gate voltage. The sign change of the AH effect with increasing TI thickness is
attributed to the charge transfer across the TI and magnetic TI layers, consistent with first-principles calculations. By fabricating the magnetic TI/TI/magnetic TI sandwich heterostructures with different dopants, we created an artificial “topological Hall (TH) effect”-like feature in Hall traces. This artificial “TH effect” is induced by the superposition of two AH effects with opposite signs instead of the formation of chiral spin textures in the samples. Our study provides a new route to engineer the Berry curvature in magnetic topological materials that may lead to potential technological applications.

The Berry phase is a quantum geometrical phase which has provided deep insights into the topological electronic properties of quantum materials \(^1,^2,^3,^4\). Since the Berry phase encodes the adiabatic evolution of occupied eigen wave functions in the first Brillouin zone (BZ) of momentum space \(^4,^5\), it is also important for understanding other electronic properties of quantum materials such as the intrinsic anomalous Hall (AH) effect \(^6\). In the 1950s, Karplus and Luttinger first proposed the idea that an AH effect of intrinsic origin can arise from the properties of the electronic band structure of a material \(^7\). This connection was reformulated in the language of the Berry phase in the early 2000s \(^6,^8,^9,^10\). The Berry curvature \(\Omega(\vec{k})\) of the occupied Bloch bands is equivalent to an effective magnetic field in momentum space, this can affect the motion of electrons and gives rise to the AH effect in ferromagnetic (FM) materials \(^6\). Therefore, the Hall conductance \((\sigma_{xy})\) of the intrinsic AH effect in a two-dimensional (2D) FM material can be in essence calculated from the Fermi-sea integration of the Berry curvature \(\Omega\) in its first BZ:
\[\sigma_{xy} = -\frac{e^2}{2\pi\hbar} \int_{\text{BZ}} \Omega(\vec{k})d^2\vec{k}\]  \hspace{1cm} (1)

where \(e\) is the elementary charge, \(\hbar\) is the Planck constant and \(\vec{k}\) is the momentum wavevector. Because the integration of Berry curvature \(\Omega\) for occupied Bloch bands in the first BZ of a 2D FM insulator equals to its Chern number \(C\) multiple of \(2\pi\), i.e. \[\int_{\text{BZ}} \Omega(\vec{k})d^2\vec{k} = 2\pi C,\] \(\sigma_{xy}\) is therefore quantized as the following equation:

\[\sigma_{xy} = -\frac{e^2}{\hbar} C\]  \hspace{1cm} (2)

The experimental realization of the quantum anomalous Hall (QAH) effect in Cr-doped (Bi,Sb):Te film, confirming Eq. (2) with \(C = 1\), also establishes convincingly that the intrinsic mechanism is solely responsible for the AH effect in magnetic TIs at least when the chemical potential of the sample is tuned near or located in the magnetic exchange gap \(^{11,12,13,14,15,16,17,18}\). However, when the chemical potential of magnetic TI is tuned away from the magnetization gap and crosses the bulk bands, it is not clear whether the intrinsic AH effect in magnetic TI is still dominant.

The origin of the AH effect in the metallic regime can be studied by exploring the scaling relationship between the anomalous Hall resistance (\(\rho_{yx}\)) and the longitudinal resistance (\(\rho_{xx}\)). A quadratic dependence (i.e. \(\rho_{yx} \propto \rho_{xx}^2\)) usually indicates that the AH effect is induced by scattering independent mechanisms, i.e. either intrinsic \(^6\) or extrinsic side-jump \(^{19}\), while a linear dependence (i.e. \(\rho_{yx} \propto \rho_{xx}\)) can only be a result of extrinsic skew-scattering \(^{20,21}\). It has been theoretically proposed \(^8\) and experimentally demonstrated \(^{22,23}\) that the AH effect in metallic diluted magnetic semiconductors (DMSs) (e.g. Mn-doped GaAs) is dominated by intrinsic mechanisms invoking the Berry phase \(^6\). Since the magnetically doped
TI is in principle also a class of DMSs, it is expected that the intrinsic AH contribution is still dominant when the bulk channels are introduced in the magnetic TI. Provided the intrinsic AH contribution is still dominant, it is not clear whether the AH effect and the corresponding Berry curvature \( \Omega \) in metallic magnetic TIs can be altered.

In this Article, we deposited TI films of different thicknesses on top of the magnetic TI film to form TI/magnetic TI bilayer heterostructures and systematically carry out Hall measurements under different temperatures \((T)\) and different gate voltage \((V_g)\). We observed a quadratic dependence between \( \rho_{yx} \) and \( \rho_{xx} \) (i.e. \( \rho_{yx} \propto \rho_{xx}^2 \)) in the metallic regime of the magnetic TI samples, demonstrating the scattering independent origin of the AH effect. By tuning the TI thickness and/or the electric gate voltages, the magnitude and the sign of the AH effect for the magnetic TI heterostructures can be changed. We then fabricated the magnetic TI/TI/magnetic TI sandwich heterostructures with different dopants and were at first surprised to find a topological Hall (TH) effect-like behavior in the Hall traces. However, our careful analysis of the data shows that this apparent “TH effect” is not a result of the formation of chiral spin textures in the samples but due to the superposition of two AH effects with opposite signs. The Berry curvature engineering realized in magnetic TI heterostructures opens a new avenue for systematic studies of the intrinsic AH effect of magnetic topological materials.

The samples used in this work are \( m \) quintuple layers (QL) Sb₂Te₃ with \( m = 0, 1, 2, 3, \) and 4 on top of 5 QL V-doped Sb₂Te₃ bilayers and 5 QL Cr-doped Sb₂Te₃/3 QL Sb₂Te₃/5 QL V-doped Sb₂Te₃ sandwiches. All samples were grown on 0.5 mm thick heat-treated SrTiO₃ (111) substrates in molecular beam epitaxy (MBE) chambers. Electrical transport
measurements were performed in a Quantum Design Physical Property Measurement System (PPMS) (2 K/9 T) with the magnetic field applied perpendicular to the film plane. The excitation current used in our measurements is 1μA. Six-terminal Hall bars with a bottom-gate electrode were used for electrical transport studies. The Hall transport results shown here were anti-symmetrized as a function of the magnetic field and the ordinary Hall effect contributions were subtracted.

We first examined the relationship between the intrinsic AH effect and the integration of Berry curvature Ω of all the occupied Bloch bands in FM materials. As shown in Eq. (1), in the metallic regime of the 2D ferromagnets, the non-quantized Hall conductance σ_{xy} can be calculated by integrating the Berry curvature Ω in the first BZ (∫Ω) but with an opposite sign 3. When an external magnetic field B is applied perpendicular to the sample, the spontaneously ordered magnetic moments deflects the motion of electrons and the AH effect emerges (Figs. 1a and 1b). When the internal magnetization M points upward, the mobile electrons move to the high potential side for ∫Ω < 0, so σ_{xy} > 0 (Fig. 1a) and the intrinsic AH hysteresis exhibits a positive sign (Fig.1c). If ∫Ω > 0 in the FM materials, the mobile electrons will move to the lower potential side under positive internal magnetization M, so σ_{xy} < 0 (Fig. 1b) and the sign of the intrinsic AH effect becomes negative (Fig. 1d). Therefore, the intrinsic AH effect can be used as an electrical transport probe to detect the Berry curvature Ω of the FM materials.

In order to demonstrate the dominance of the intrinsic AH effect in the metallic regime of magnetically doped TI films, we studied the dependence between ρ_{yx} and ρ_{xx} in a series of 5QL V-doped Sb_{2}Te_{3} (Sb_{2-x}V_{x}Te_{3}) samples with different V concentration x (x ≥
Figure 1e shows the $\rho_{yx}$ under zero magnetic field (labeled as $\rho_{yx}(0)$) versus $\rho_{xx}$ under zero magnetic field (labeled as $\rho_{xx}(0)$) curve plotted on a log-log scale in the low-temperature regions. This curve is linearly fitted and gives the scaling exponent $\alpha$ in the formula 

$$\rho_{yx}(0) \propto \rho_{xx}^{\alpha}(0).$$

We found $\alpha \sim 1.84 \pm 0.02$, $1.94 \pm 0.01$, and $1.91 \pm 0.02$ for $x = 0.10$, 0.16, and 0.20 samples, respectively. Since the $\alpha$ values are all very close to 2, we conclude the scattering independent (i.e. the intrinsic and/or extrinsic side-jump) mechanisms are primarily responsible for the AH effect in the metallic magnetic TI films. However, since the TI materials have strongly spin-orbit coupled bands, the intrinsic contribution is expected to be dominant, similar to the AH effect in the well-studied magnetic Heusler and half-Heusler metals. We will next focus on altering this intrinsic AH effect by adding additional TI layer(s) or changing electric gating voltages.

Figures 2a to 2e show the AH hysteresis loops of $m$ QL Sb$_2$Te$_3$/5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ bilayer samples with $0 \leq m \leq 4$. For the $m = 0$ and $m = 1$ samples, the sign of the AH hysteresis loops is positive (i.e. $\rho_{yx} > 0$ for $M > 0$). Under zero magnetic field, $\rho_{yx}(0)$ are $\sim 62$ $\Omega$ and $\sim 7.5$ $\Omega$, respectively. Both values are much lower than the quantized values ($\sim 25.8$ k$\Omega$), indicating that the chemical potential is crossing the bulk valence bands and the sample is located in the metallic regime. When we increase the thickness ($m \geq 2$) of the Sb$_2$Te$_3$ layer, the sign of the AH effect reverses and becomes negative (i.e. $\rho_{yx} < 0$ for $M > 0$). The absolute value of $\rho_{yx}$ at zero magnetic field increases from $\sim 1.2$ $\Omega$ for the $m = 2$ sample to $\sim 10$ $\Omega$ for the $m = 4$ sample.

We summarized the temperature dependence of $\rho_{yx}(0)$ for these five samples in Fig. 2f. For the $m = 0$ sample, the temperature dependence of $\rho_{yx}(0)$ is monotonic and $\rho_{yx}(0)$ becomes
0 at $T = 40$ K, suggesting the Curie temperature ($T_C$) of 5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ is $\sim$40 K. This temperature dependence is typical in conventional DMS and magnetically doped TI samples. For the $m = 1$ sample, $\rho_{yx}(0)$ first increases and then decreases with temperature showing a maximum at $T = 15$ K. $\rho_{yx}(0)$ vanishes at $T = 40$ K. For the $m = 2$ sample, as noted above, $\rho_{yx}(0) < 0$ at $T = 2$ K. With further increasing temperature, the magnitude of $\rho_{yx}(0)$ decreases monotonically and becomes zero at $T = 8$ K. $\rho_{yx}(0)$ of the $m = 3$ sample shows an unusual temperature dependence, specifically it changes its sign from being negative to positive near $T = 11$ K. In other words, this sample shows a linear Hall trace at $T = 11$ K. Note that the linear Hall trace observed here does not mean that the ferromagnetic order is absent because the sample still shows a butterfly magnetoresistance curve (see Supplementary Information). Instead, $\rho_{yx}(0) = 0$ at $T = 11$ K indicates $\int \Omega = 0$ of the occupied Bloch bands in the 3 QL Sb$_2$Te$_3$/5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ bilayer sample. For the $m = 4$ sample, $\rho_{yx}(0) < 0$ in the entire temperature range and its absolute value decreases monotonically with increasing temperature $T$. We note that the butterfly structure of $\rho_{xx}$ in all samples disappears at $T = 40$ K, suggesting that $T_C$ of all these samples is $\sim$ 40 K and the V atoms in the bottom V-doped TI layer do not diffuse into the top TI layer. We, therefore, concluded the magnitude and the sign changes in these bilayer heterostructures are results of the change in the integration of the Berry curvature $\int \Omega$. This will be discussed in more details below after we show the effect of tuning the sample by a gate voltage.

**Figures 3a to 3e** show the magnetic field ($\mu_0 H$) dependence of $\rho_{yx}$ under different $V_g$s at $T = 11$ K for the $m = 3$ sample. As noted above, $\rho_{yx}$ shows a linear behavior at $V_g = 0$ V (**Fig. 3c**). By applying a negative $V_g$ to introduce hole carriers into the sample, the AH hysteresis
with a positive sign appears, suggesting $\int \Omega < 0$ (Figs. 3a and 3b). When applying a positive $V_g$, the electron carriers are introduced, the AH hysteresis with a negative sign appears, suggesting $\int \Omega > 0$ (Figs. 3d and 3e). Therefore, the samples tend to show the AH effect with the positive (negative) sign when injecting the hole (electron) carriers. It is interesting to note that our observation here is opposite to that reported in (Bi,Sb)$_2$Te$_3$/Cr-doped (Bi,Sb)$_2$Te$_3$ bilayer samples. Figure 3f shows the 2D color contour plot of $\rho_{yx}(0)$ on the $T$ and $V_g$ plane, which is divided into two regions by a dashed line that extends to zero temperature. Positive $\rho_{yx}(0)$ is favored at higher $T$ and negative $V_g$ with more hole carriers injected, while negative $\rho_{yx}(0)$ is favored at lower $T$ and positive $V_g$ with more electron carriers injected. Therefore, both the magnitude and the sign of $\rho_{yx}$ can also be changed by tuning the chemical potential, i.e., by varying the gating voltage.

The magnitude and the sign changes of $\rho_{yx}$ induced by varying the thickness of the TI layers and gating voltages can be interpreted based on the variation of carrier density in the V-doped Sb$_2$Te$_3$ layer, modulated by either the electron transfer from the Sb$_2$Te$_3$ layer or the external electric gates. Prior studies have demonstrated that the V dopants in Sb$_2$Te$_3$ can introduce more hole carriers. In other words, the chemical potential of the V-doped Sb$_2$Te$_3$ layer compared to that of the Sb$_2$Te$_3$ layer is more deeply buried in the bulk valence bands (Fig. 4a). When an additional Sb$_2$Te$_3$ layer is deposited on top of the 5QL V-doped Sb$_2$Te$_3$ layer to form the Sb$_2$Te$_3$/V-doped Sb$_2$Te$_3$ bilayer heterostructures, additional hole carriers in the V-doped Sb$_2$Te$_3$ layers can move to the Sb$_2$Te$_3$ layers, leading to a built-in electric field. Correspondingly, the chemical potentials of the Sb$_2$Te$_3$ and V-doped Sb$_2$Te$_3$ layers move downward and upward, respectively (Fig. 4a). The undoped Sb$_2$Te$_3$ layer plays...
the same role as a positive gate voltage on the heterostructure sample. Thicker Sb$_2$Te$_3$ layer can absorb more hole carriers from the V-doped Sb$_2$Te$_3$ layer.

To support this picture, we carried out first-principles calculations on band structures and the corresponding AH conductance $\sigma_{xy}$ of 5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ films (Fig. 4b). As noted above, the additional Sb$_2$Te$_3$ layer with the thickness of $m$ QL plays the role of a positive gate $V_g$ on the V-doped Sb$_2$Te$_3$ layer (Fig. 4a). Therefore, we can simulate the intrinsic AH effect of the Sb$_2$Te$_3$/V-doped Sb$_2$Te$_3$ bilayer heterostructures through a simplified model of 5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ film through tuning the chemical potential. The calculated AH conductance $\sigma_{xy}$ can be either positive or negative with the chemical potential located at different energy levels (Figs. 4b). The sign change of $\sigma_{xy}$ is likely a result of the accidental crossing and/or anti-crossing in the bulk valence bands. The Berry curvature near the $\Gamma$ point is also calculated with the chemical potential located at different energy levels (Figs. 4c to 4g). The sign of the Berry curvature changes when the chemical potential is tuned across the accidental band crossing and/or anti-crossing points shown as red circles in Fig. 4b (see more in Supplementary Information). Our calculations show that as the chemical potential is increased, the sign of the AH effect in 5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ can indeed change from positive to negative (Figs. 2 and 3).

In our exploration, the V-doped Sb$_2$Te$_3$ with doping concentration $x > 0.08$ always shows the positive AH sign, indicating that the 5QL Sb$_{1.9}$V$_{0.1}$Te$_3$ samples should be located within the positive $\sigma_{xy}$ region of our theoretical calculations (Figs. 4c and 4d). When an additional Sb$_2$Te$_3$ layer is deposited on 5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ layer, the electron transfer will yield a built-in electric field and lift the chemical potential of 5QL Sb$_{1.9}$V$_{0.1}$Te$_3$ higher and the AH
conductance $\sigma_{xy}$ value changes from being positive (Figs. 4c and 4d) to negative (Figs. 4e to 4g). We note that a threshold of $\sigma_{xy}$ exists when the thickness of the Sb$_2$Te$_3$ layer increases. When the thickness of the Sb$_2$Te$_3$ layer exceeds the effective penetration length of the built-in electric field, the thicker Sb$_2$Te$_3$ layer will not contribute to the negative AH conductance, while the magnitude of the averaged AH conductance $\sigma_{xy}$ starts to decrease because of the zero AH conductance of newly-added Sb$_2$Te$_3$ layers. Finally, we note that the calculations of the intrinsic $\sigma_{xy}$ are entirely from the band structures of an ideal crystal structure with the absence of scattering effects. In reality, the carriers in the samples are inevitably scattered by the impurities and defects and thus have a finite relaxation time $^6$, which would reduce the AH velocity and significantly suppress the intrinsic AH conductance $\sigma_{xy}$ in the dirty limit $^{35}$. Therefore, the calculated values of $\sigma_{xy}$ here are generally larger than the experimental values shown in Fig. 2.

The temperature dependence of the AH resistance $\rho_{yx}$ (Fig. 2f) can also be qualitatively interpreted by the above picture. For the $m = 1, 2$, and 3 samples, a large number of holes can be thermally excited with increasing temperature in both V-doped and undoped Sb$_2$Te$_3$ layers, thus weakening the electron transfer effect between these two layers, so $\rho_{yx}$ of the Sb$_2$Te$_3$/V-doped Sb$_2$Te$_3$ heterostructure will become more similar to the $m = 0$ sample. The weakening of charge transfer effect explains the sign change of the AH resistance $\rho_{yx}$ for the $m = 3$ sample with increasing temperatures. When the temperature approaches towards $T_c$, the magnitude of $\rho_{yx}$ always decreases towards zero due to the disappearance of magnetization.

We next studied the AH effect of the 5QL Cr-doped Sb$_2$Te$_3$/3QL Sb$_2$Te$_3$/5QL V-doped
Sb$_2$Te$_3$ sandwich heterostructures (Fig. 5), in which the top and bottom magnetic TI layers have opposite AH signs and different coercive fields ($H_c$). These two AH effects can be added together to form unusual AH hysteresis loops because the middle 3QL Sb$_2$Te$_3$ layer weakens the interlayer coupling effects between the top and bottom magnetic TI layers. When the absolute value of $\rho_{yx}$ in Cr-doped TI is larger than that of V-doped TI, the AH effect of the Cr-doped TI layer is dominant, and the total AH hysteresis loop shows a positive sign (Fig. 5a), we named this as “Type 1” AH hysteresis loop. When the absolute value of $\rho_{yx}$ in Cr-doped TI is less than that of V-doped TI, the AH effect of the V-doped TI layer is dominant, and the total AH hysteresis loop shows a negative sign (Fig. 5b), we named it as “Type 2” AH loop. In both cases, two anti-symmetric humps appear when the external magnetic field $\mu_0 H$ is between $\mu_0 H_1$ and $\mu_0 H_2$. Here $\mu_0 H_1$ and $\mu_0 H_2$ are the coercive fields of the top Cr-doped TI and the bottom V-doped TI layers, respectively. A “hump” feature in the AH hysteresis loop can result from the formation of chiral spin textures in samples and is usually known as the TH effect 36, 37, 38, 39. The hump feature observed in our sandwich samples, on the other hand, is the result of the superposition of two AH effect with opposite signs. We performed Hall measurements on the 3QL Sb$_2$Te$_3$/5QL Sb$_{1.92}$V$_{0.08}$Te$_3$ and 5QL Sb$_{1.84}$Cr$_{0.16}$Te$_3$/3QL Sb$_2$Te$_3$ bilayer samples. As discussed above, the former sample does show the AH effect with a negative sign (Figs. 5d and 5g), while the latter shows the AH effect with the positive sign (Figs. 5e and 5h), consistent with prior studies 29, 30 and our theoretical calculations (see Section VIII of Supplementary Information). Because the temperature dependences of $\rho_{yx}$ in these two bilayer samples are different, both types of AH loops can be realized in the 5QL Sb$_{1.84}$Cr$_{0.16}$Te$_3$/3QL Sb$_2$Te$_3$/5QL Sb$_{1.92}$V$_{0.08}$Te$_3$ samples in
different temperature regions (see Supplementary Information). Since the absolute value of
$\rho_{yx}$ in $5QL\ Sb_{1.84}Cr_{0.16}Te_3/3QL\ Sb_2Te_3$ is larger than that of $3QL\ Sb_2Te_3/5QL\ Sb_{1.92}V_{0.08}Te_3$ at
$T=2\ K$, “Type 1” AH loop is observed (Fig. 5f). At $T=12\ K$, the absolute value of $\rho_{yx}$ in $5QL\ Sb_{1.84}Cr_{0.16}Te_3/3QL\ Sb_2Te_3$ is comparable to or less than that of $3QL\ Sb_2Te_3/5QL\ Sb_{1.92}V_{0.08}Te_3$, therefore “Type 2” AH loop appears (Fig. 5i). All observations here are
consistent with our analysis in Figs. 5a and 5b, confirming that the TH effect-like hump
feature observed in our Cr-doped TI/TI/V-doped TI sandwiches are indeed from the
superposition of two AH effects with opposite signs. Our findings here, together with two
prior studies $^{40,41}$ suggest humps/dips features along with the AH effect observed in a number
of heterostructure samples might not be induced by the formation of the chiral spin textures
in samples $^{31,42,43,44,45}$.

To summarize, we carried out Berry curvature engineering in magnetic TI samples by
depositing an undoped TI layer on top to form bilayer heterostructures. Our observation is
well interpreted by our first-principles calculations based on the model of charge transfer
through the interface between TI and magnetic TI layers. We fabricated the magnetic
TI/TI/magnetic TI sandwich sample with different dopants on two surfaces and observed two
atypical humps in its AH loops. We demonstrated that the unusual AH hysteresis loops are
induced by the superposition of two AH effects with opposite signs rather than the formation
of the chiral spin textures in samples. Our work provides a new route for the “engineering” of
the AH effect in magnetic topological materials and insights into the underlying mechanism
for the TH effect-like humps observed in nonhomogeneous materials.

Methods
MBE growth of magnetic TI heterostructures.

The magnetic TI films and heterostructures were grown on heat-treated 0.5 mm thick insulating SrTiO$_3$(111) substrates using two MBE chambers (Omicron Lab 10 and Vecco Applied EPI 620) with a base pressure of $2.0 \times 10^{-10}$ mbar. Prior to sample growth, the SrTiO$_3$ substrates are first degassed at $\sim 600$°C for 1h. High-purity Sb(6N), Te(6N), V(5N), and Cr(5N) are evaporated from Knudsen cells. During the growth of the magnetic TI films, the substrate is maintained at $\sim 240$°C. The flux ratio of Te/(Sb+V/Cr) is set to be $>$10 to avoid possible Te deficiency in samples. The growth rate of magnetic TI or TI films is around $\sim 0.2$ QL/min. Finally, the samples are capped with a 10 nm Te layer to prevent its degradation during the ex-situ electrical transport measurements.

Electrical transport measurements.

All samples for transport measurements were scratched into Hall bar geometries ($\sim 1$ mm $\times$ 0.5 mm) $^{46}$. The bottom gate electrode is made by pressing an indium foil on the backside of the SrTiO$_3$ (111) substrate. Note that for all Hall data shown in this paper, unless pointed out, the ordinary Hall term has been subtracted by fitting the linear slope of the Hall data in a high magnetic field regime.

First-principles calculations.

The V atom is doped in $2 \times 2 \times 5$ supercell of Sb$_2$Te$_3$ (5 QL thickness) without breaking the spatial reversal symmetry. The concentration of vanadium is about 5% (i.e. Sb$_{1.9}$V$_{0.1}$Te$_3$). The lattice parameters and the positions of atoms were fixed as the bulk ones, and we relaxed the dopants and atoms in the nearest neighbor of the dopants with an energy tolerance of $10^{-5}$ eV. A 20 Å vacuum layer is placed above the sample. The density functional theory (DFT)
calculations are performed via the Vienna Ab-initio Simulation Package (VASP) with exchange-correlation energy described by the generalized gradient approximation (GGA) in the Perdew-Burke-Ernzerhof (PBE) parametrization. We set the energy cutoff to $\sim 300$ eV for the plane-wave expansion. The van der Waals (vdW) correction is also included with the optB86b-vdW type in the relaxation process. An $8 \times 8 \times 1$ $\Gamma$-centered $k$-mesh is used in structural relaxation with the energy tolerance converged to $10^{-6}$ eV. To consider strongly correlated $3d$ electrons of the V atoms, an LDA+U correction is implemented in the whole calculation with the parameter $U = 2.7$ eV and $J = 0.7$ eV. The spin-orbit coupling effect is included in band structure calculations. The AH conductance $\sigma_{xy}$ is calculated in Wannier representation via the Wannier90 code. In the projection from Bloch representation to Wannier representation, considering the computation capability, the $\Gamma$-centered $k$-mesh is set to $5 \times 5 \times 1$. To include the disorder effect in the magnetic TI samples, a $0.07$eV energy range is averaged in the calculated Hall conductance $\sigma_{xy}$. The maximal energy range is estimated by $E_{\text{range}} = \frac{\hbar}{2\tau} = 0.09$eV, where $\tau$ is the relaxation time estimated by the carrier mobility data from experiments and the effective mass of valence band edge from the $m = 0$ DFT calculations.

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**Author contributions**

C. Z. C. conceived and designed the experiment. Y. F. Z. and D. X. grew the magnetic TI films and heterostructure samples with the help of N. S. and C. Z. C.. F. W., Y. F. Z. and L. Z. carried out the PPMS transport measurement with the help of M. H. W. C. and C. Z. C.. X. W, C. L., and H. Z. provided the theoretical support and did the first-principles calculations. F. W., X. W., C. L., H. Z., and C. Z. C. analyzed the data and wrote the manuscript with contributions from all authors.

**Additional information**

Supplementary information is available in the [online version of the paper](#). Reprints and permissions information is available online at [www.nature.com/reprints](http://www.nature.com/reprints).

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**Competing financial interests**

The authors declare no competing financial interests.

**Data availability**

The data that support the findings of this study are available from H. Zhang or C. -Z. Chang upon reasonable request.
Figures and figure captions:

**Figure 1 | Intrinsic anomalous Hall (AH) effect and Berry curvature $\Omega$.** (a,b) The intrinsic AH effect formed in the ferromagnetic materials with positive (a) and negative (b) Berry curvature $\Omega$ integration of the occupied Bloch bands when the internal magnetization points upward. (c, d) The corresponding AH hysteresis in the ferromagnetic materials with positive (c) and negative (d) Berry curvature $\Omega$ integration of the occupied bands. The sign and value of the AH effect are determined by the sign and magnitude of Berry curvature $\Omega$ integration of occupied bands. (e) A plot of the AH resistance at zero magnetic field $\rho_{yx}(0)$ as a function of the longitudinal resistance at zero magnetic field $\rho_{xx}(0)$ on a log-log scale for three 5 QL Sb$_{2-x}$V$_x$Te$_3$ samples ($x=0.10$, $0.16$, and $0.20$). The dashed lines are the fits using $\rho_{yx} \propto \rho_{xx}^\alpha$, $\alpha = 1.84 \pm 0.02$, $1.94 \pm 0.01$, and $1.91 \pm 0.02$ for $x = 0.10$, $0.16$, and $0.20$ samples, respectively.
Figure 2 | Berry curvature engineering in TI/magnetic TI heterostructures. (a-e) Magnetic field ($\mu_0H$) dependence of the Hall resistance $\rho_{yx}$ in $m$ QL Sb$_2$Te$_3$/5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ bilayer heterostructures. $m = 0$ (a); $m = 1$ (b); $m = 2$ (c); $m = 3$ (d); $m = 4$ (e). The blue (red) curve represents the process for decreasing (increasing) $\mu_0H$. All measurements were taken at $T = 2$ K. The linear contribution from the ordinary Hall effect was subtracted. (f) Temperature dependence of the Hall resistance at zero magnetic field $\rho_{yx}(0)$ in $m$ QL Sb$_2$Te$_3$/5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ bilayer heterostructures.
Figure 3 | Electric field control of the Berry curvature in TI/magnetic TI bilayer heterostructures. (a-e) $\mu_0H$ dependence of $\rho_{yx}$ in 3 QL Sb$_2$Te$_3$/5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ bilayer heterostructure at different gates $V_g$. $V_g = -200V$ (a); $V_g = -100V$ (b); $V_g = 0V$ (c); $V_g = +100V$ (d); $V_g = +200V$ (e). Blue (red) curve represents the process for decreasing (increasing) $\mu_0H$. All measurements were taken at $T = 11$ K. The linear contribution from the ordinary Hall effect was subtracted. (f) 2D color contour plot of $\rho_{yx}(0)$ as a function of both $T$ and $V_g$ in 3 QL Sb$_2$Te$_3$/5 QL Sb$_{1.9}$V$_{0.1}$Te$_3$ bilayer heterostructure.
Figure 4| Schematic of the charge transfer between the TI and magnetic TI layers and band structures and Berry curvature of the magnetic TI films. (a) Schematic of the charge transfer through the interface between the Sb$_2$Te$_3$ and V-doped Sb$_2$Te$_3$ layers. The V-doped Sb$_2$Te$_3$ layer has more hole carries than the Sb$_2$Te$_3$ layer. A built-in electric field is generated at the Sb$_2$Te$_3$/V-doped Sb$_2$Te$_3$ interface. (b) The calculated band structures (left) and corresponding Hall conductance $\sigma_{xy}$ (right) of 5 QL Sb$_{1.84}$V$_{0.16}$Te$_3$ based on the intrinsic AH effect mechanism. A few band crossings or anti-crossings in the valence bands are labeled using red circles. (c, d, e, f, g) The calculated Berry curvature distribution of 5 QL Sb$_{1.84}$V$_{0.16}$Te$_3$ for chemical potential located at +145 meV(c), +120 meV(d), +90 meV(e), +25 meV(f), and -40 meV(g). The corresponding chemical potential positions are labeled in (a) by dashed lines.
Figure 5 | The artificial “TH effect” in magnetic TI/TI/magnetic TI sandwich heterostructures. (a, b) Two types of artificial “TH effect” formed by the parallel connection of two AH effects with opposite signs. Type 1 is constructed by the AH effect with the larger coercive field ($\mu_0 H_c$) showing smaller $\rho_{yx}$, while Type 2 is constructed by the AH effect with the larger $\mu_0 H_c$ showing larger $\rho_{yx}$. (c) Schematic for the 5 QL Cr-doped Sb$_2$Te$_3$/3 QL Sb$_2$Te$_3$/5 QL V-doped Sb$_2$Te$_3$ sandwich sample. (d, g) $\mu_0 H$ dependence of $\rho_{yx}$ of the 3 QL Sb$_2$Te$_3$/5QL Sb$_{1.92}$V$_{0.08}$Te$_3$ bilayer heterostructure measured at $T = 2$ K (d) and $T = 12$ K (g). (e, h) $\mu_0 H$ dependence of $\rho_{yx}$ of the 5QL Sb$_{1.84}$Cr$_{0.16}$Te$_3$/3 QL Sb$_2$Te$_3$ bilayer heterostructure measured at $T = 2$ K (e) and $T = 12$ K (h). (f, i) $\mu_0 H$ dependence of $\rho_{yx}$ of the 5QL Sb$_{1.84}$Cr$_{0.16}$Te$_3$/3 QL Sb$_2$Te$_3$/ 5QL Sb$_{1.92}$V$_{0.08}$Te$_3$ sandwich heterostructure measured at $T = 2$ K (f) and $T = 12$ K (i).
References

1. Hasan M. Z., Kane C. L. Colloquium: Topological Insulators. *Rev. Mod. Phys.* **82**, 3045-3067 (2010).

2. Qi X. L., Zhang S. C. Topological Insulators and Superconductors. *Rev. Mod. Phys.* **83**, 1057-1110 (2011).

3. Weng H. M., Yu R., Hu X., Dai X., Fang Z. Quantum Anomalous Hall Effect and Related Topological Electronic States. *Adv. Phys.* **64**, 227-282 (2015).

4. Xiao D., Chang M. C., Niu Q. Berry phase effects on electronic properties. *Rev. Mod. Phys.* **82**, 1959-2007 (2010).

5. Berry M. V. Quantal Phase-Factors Accompanying Adiabatic Changes. *Proc R Soc Lon Ser-A* **392**, 45-57 (1984).

6. Nagaosa N., Sinova J., Onoda S., MacDonald A. H., Ong N. P. Anomalous Hall effect. *Rev. Mod. Phys.* **82**, 1539-1592 (2010).

7. Karplus R., Luttinger J. M. Hall Effect in Ferromagnetics. *Phy. Rev.* **95**, 1154-1160 (1954).

8. Jungwirth T., Niu Q., MacDonald A. H. Anomalous Hall Effect in Ferromagnetic Semiconductors. *Phys. Rev. Lett.* **88**, 207208 (2002).

9. Fang Z., Nagaosa N., Takahashi K. S., Asamitsu A., Mathieu R., Ogasawara T., Yamada H., Kawasaki M., Tokura Y., Terakura K. The anomalous Hall effect and magnetic monopoles in momentum space. *Science* **302**, 92-95 (2003).

10. Yao Y. G., Kleinman L., MacDonald A. H., Sinova J., Jungwirth T., Wang D. S., Wang E. G., Niu Q. First principles calculation of anomalous Hall conductivity in ferromagnetic bcc Fe. *Phys. Rev. Lett.* **92**, 037204 (2004).

11. Chang C. Z., Zhang J. S., Feng X., Shen J., Zhang Z. C., Guo M. H., Li K., Ou Y. B., Wei P., Wang L. L., Ji Z. Q., Feng Y., Ji S. H., Chen X., Jia J. F., Dai X., Fang Z., Zhang S. C., He K., Wang Y. Y., Lu L., Ma X. C., Xue Q. K. Experimental Observation of the Quantum Anomalous Hall Effect in a Magnetic Topological Insulator. *Science* **340**, 167-170 (2013).

12. Chang C. Z., Zhao W. W., Kim D. Y., Zhang H. J., Assaf B. A., Heiman D., Zhang S. C., Liu C. X., Chan M. H. W., Moodera J. S. High-Precision Realization of Robust Quantum Anomalous Hall State in a Hard Ferromagnetic Topological Insulator. *Nat. Mater.* **14**, 473-477 (2015).
13. Checkelsky J. G., Yoshimi R., Tsukazaki A., Takahashi K. S., Kozuka Y., Falson J., Kawasaki M., Tokura Y. Trajectory of the Anomalous Hall Effect towards the Quantized State in a Ferromagnetic Topological Insulator. *Nat. Phys.* **10**, 731-736 (2014).

14. Kou X. F., Guo S. T., Fan Y. B., Pan L., Lang M. R., Jiang Y., Shao Q. M., Nie T. X., Murata K., Tang J. S., Wang Y., He L., Lee T. K., Lee W. L., Wang K. L. Scale-Invariant Quantum Anomalous Hall Effect in Magnetic Topological Insulators beyond the Two-Dimensional Limit. *Phys. Rev. Lett.* **113**, 137201 (2014).

15. Chang C. Z., Li M. D. Quantum Anomalous Hall Effect in Time-Reversal-Symmetry Breaking Topological Insulators. *J. Phys. Condens. Matter* **28**, 123002 (2016).

16. Liu C. X., Zhang S. C., Qi X. L. The Quantum Anomalous Hall Effect: Theory and Experiment. *Annu. Rev. Condens. Matter Phys.* **7**, 301-321 (2016).

17. Grauer S., Schreyeck S., Winnerlein M., Brunner K., Gould C., Molenkamp L. W. Coincidence of Superparamagnetism and Perfect Quantization in the Quantum Anomalous Hall State. *Phys. Rev. B* **92**, 201304 (2015).

18. Kandala A., Richardella A., Kempinger S., Liu C. X., Samarth N. Giant anisotropic magnetoresistance in a quantum anomalous Hall insulator. *Nat. Commun.* **6**, 7434 (2015).

19. Berger L. Side-jump mechanism for the Hall effect of ferromagnets. *Phys. Rev. B* **2**, 4559-4566 (1970).

20. Smit J. The Spontaneous Hall Effect in Ferromagnetics-I. *Physica* **21**, 877-887 (1955).

21. Smit J. The Spontaneous Hall Effect in Ferromagnetics-II. *Physica* **24**, 39-51 (1958).

22. Edmonds K. W., Campion R. P., Wang K. Y., Neumann A. C., Gallagher B. L., Foxon C. T., Main P. C. Magnetoresistance and Hall effect in the ferromagnetic semiconductor Ga$_{1-x}$Mn$_x$As. *J. Appl. Phys.* **93**, 6787-6789 (2003).

23. Chun S. H., Kim Y. S., Choi H. K., Jeong I. T., Lee W. O., Suh K. S., Oh Y. S., Kim K. H., Khim Z. G., Woo J. C., Park Y. D. Interplay between carrier and impurity concentrations in annealed Ga$_{1-x}$Mn$_x$As: Intrinsic anomalous Hall effect. *Phys. Rev. Lett.* **98**, 026601 (2007).

24. Shekhar C., Kumar N., Grinenko V., Singh S., Sarkar R., Luetkens H., Wu S. C., Zhang Y., Komarek A. C., Kampert E., Skourski Y., Wosnitza J., Schnelle W., McCollam A., Zeitler U., Kuebler J., Yan B. H., Klauss H. H., Parkin S. S. P., Felser C. Anomalous Hall Effect in Weyl Semimetal Half-Heusler Compounds RPtBi (R = Gd and Nd). *Proc. Natl. Acad. Sci.* **115**, 9140-9144 (2018).
25. Suzuki T., Chisnell R., Devarakonda A., Liu Y. T., Feng W., Xiao D., Lynn J. W., Checkelsky J. G. Large anomalous Hall effect in a half-Heusler antiferromagnet. *Nat. Phys.* **12**, 1119-1123 (2016).

26. Manna K., Muechler L., Kao T. H., Stinshoff R., Zhang Y., Gooth J., Kumar N., Kreiner G., Koepernik K., Car R., Kubler J., Fecher G. H., Shekhar C., Sun Y., Felser C. From Colossal to Zero: Controlling the Anomalous Hall Effect in Magnetic Heusler Compounds via Berry Curvature Design. *Phys. Rev. X* **8**, 041045 (2018).

27. Chang C. Z., Zhao W. W., Kim D. Y., Wei P., Jain J. K., Liu C. X., Chan M. H. W., Moodera J. S. Zero-Field Dissipationless Chiral Edge Transport and the Nature of Dissipation in the Quantum Anomalous Hall State. *Phys. Rev. Lett.* **115**, 057206 (2015).

28. Ohno H., Chiba D., Matsukura F., Omiya T., Abe E., Dietl T., Ohno Y., Ohtani K. Electric-Field Control of Ferromagnetism. *Nature* **408**, 944-946 (2000).

29. Chang C. Z., Zhang J. S., Liu M. H., Zhang Z. C., Feng X., Li K., Wang L. L., Chen X., Dai X., Fang Z., Qi X. L., Zhang S. C., Wang Y. Y., He K., Ma X. C., Xue Q. K. Thin Films of Magnetically Doped Topological Insulator with Carrier-Independent Long-Range Ferromagnetic Order. *Adv. Mater.* **25**, 1065-1070 (2013).

30. Chang C. Z., Liu M. H., Zhang Z. C., Wang Y. Y., He K., Xue Q. K. Field-effect modulation of anomalous Hall effect in diluted ferromagnetic topological insulator epitaxial films. *Sci. China Phys. Mech.* **59**, 637501 (2016).

31. Yasuda K., Wakatsuki R., Morimoto T., Yoshimi R., Tsukazaki A., Takahashi K. S., Ezawa M., Kawasaki M., Nagaosa N., Tokura Y. Geometric Hall Effects in Topological Insulator Heterostructures. *Nat. Phys.* **12**, 555-559 (2016).

32. Zhou Z. H., Chien Y. J., Uher C. Thin-film ferromagnetic semiconductors based on Sb$_{2-x}$V$_x$Te$_3$ with T$_C$ of 177 K. *Appl. Phys. Lett.* **87**, 112503 (2005).

33. Dyck J. S., Hajek P., Losit’ak P., Uher C. Diluted magnetic semiconductors based on Sb$_{2-x}$V$_x$Te$_3$ (0.01 $\leq$ x $\leq$ 0.03). *Phys. Rev. B* **65**, 115212 (2002).

34. Zhao Y.-F., Chang C.-Z. unpublished. (2020).

35. Shiomi Y., Onose Y., Tokura Y. Effect of scattering on intrinsic anomalous Hall effect investigated by Lorenz ratio. *Phys. Rev. B* **81**, 054414 (2010).

36. Neubauer A., Pfeiferer C., Binz B., Rosch A., Ritz R., Niklowitz P. G., Boni P. Topological Hall Effect in the A Phase of MnSi. *Phys. Rev. Lett.* **102**, 186602 (2009).
37. Lee M., Kang W., Onose Y., Tokura Y., Ong N. P. Unusual Hall Effect Anomaly in MnSi under Pressure. *Phys. Rev. Lett.* **102**, 186601 (2009).

38. Kanazawa N., Onose Y., Arima T., Okuyama D., Ohoyama K., Wakimoto S., Kakurai K., Ishiwata S., Tokura Y. Large Topological Hall Effect in a Short-Period Helimagnet MnGe. *Phys. Rev. Lett.* **106**, 156603 (2011).

39. Huang S. X., Chien C. L. Extended Skyrmion Phase in Epitaxial FeGe(111) Thin Films. *Phys. Rev. Lett.* **108**, 267201 (2012).

40. Fijalkowski K. M., Hartl M., Winnerlein M., Mandal P., Schreyeck S., Brunner K., Gould C., Molenkamp L. W. Coexistence of Surface and Bulk Ferromagnetism Mimics Skyrmion Hall Effect in a Topological Insulator. *Phys. Rev. X* **10**, 011012 (2020).

41. Groenendijk D. J., Autieri C., Thiel T. C. v., Brzezicki W., Gauquelin N., Barone P., van den Bos K. H. W., van Aerdt S., Verbeeck J., Filippetti A., Picozzi S., Cuoco M., Caviglia A. D. Berry Phase Engineering at Oxide Interfaces. *arXiv:1810.05619* (2018).

42. Matsuno J., Ogawa N., Yasuda K., Kagawa F., Koshiba W., Nagaosa N., Tokura Y., Kawasaki M. Interface-Driven Topological Hall Effect in SrRuO$_3$-SrIrO$_3$ Bilayer. *Sci. Adv.* **2**, e1600304 (2016).

43. Ohuchi Y., Matsuno J., Ogawa N., Kozuka Y., Uchida M., Tokura Y., Kawasaki M. Electric-Field Control of Anomalous and Topological Hall Effects in Oxide Bilayer Thin Films. *Nat. Commun.* **9**, 213 (2018).

44. Wang L. F., Feng Q. Y., Kim Y., Kim K. H., Lee K. H., Pollard S. D., Shin Y. J., Zhou H. B., Peng W., Lee D., Meng W. J., Yang H., Han J. H., Kim M., Lu Q. Y., Noh T. W. Ferroelectrically tunable magnetic skyrmions in ultrathin oxide heterostructures. *Nat. Mater.* **17**, 1087-1094 (2018).

45. He Q. L., Yin G., Grutter A. J., Pan L., Che X. Y., Yu G. Q., Gilbert D. A., Disseler S. M., Liu Y. Z., Shafer P., Zhang B., Wu Y. Y., Kirby B. J., Arenholz E., Lake R. K., Han X. D., Wang K. L. Exchange-biasing topological charges by antiferromagnetism. *Nat. Commun.* **9**, 2767 (2018).

46. Xiao D., Jiang J., Shin J. H., Wang W. B., Wang F., Zhao Y. F., Liu C. X., Wu W. D., Chan M. H. W., Samarth N., Chang C. Z. Realization of the Axion Insulator State in Quantum Anomalous Hall Sandwich Heterostructures. *Phys. Rev. Lett.* **120**, 056801 (2018).