Diagnosis for gear tooth surface damage by empirical mode decomposition in cyclic fatigue test

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Abstract
Gear is one of the most important and commonly used components in machine system. Some gear failure may lead to fatal damage of the entire system, or even huge losses in industrial production. Early detection of gear damage is crucial to prevent the machine system from malfunction. This paper provides an intelligent diagnosis method for gear damage based on techniques of empirical mode decomposition and support vector machines. By the data processing of empirical mode decomposition, the original signal are decomposed into a finite set of intrinsic mode functions with frequency bands ranging from high to low. The characteristic energy ratios of intrinsic mode functions are acquired as representative parameters of the signal. Furthermore, statistical parameters of standard deviation, root mean square value, kurtosis and skewness are extracted from the original signal. Characteristic energy ratios and statistical parameters are combined as failure feature vectors to be input to the support vector machines classifiers for gear damage diagnosis. The validity of the presented method is confirmed by the application of monitoring gear conditions during the cyclic fatigue test. The vibration accelerations of gear box are acquired to illustrate the progression of pitting damage. Most of the gear conditions are identified, indicating the effectiveness of the proposed method.

Key words : Gear, Damage diagnosis, Empirical mode decompositions, Support vector machines, Failure feature vectors, Pitting

1. Introduction
Gear is one of the most important and commonly used components in machine system. Minor gear damage may cause fatal failures of the entire equipment, even huge economic losses. Consequently, failure diagnosis especially early detection of the gear damage is crucial to prevent the mechanical system from malfunction. Researchers have developed many techniques to detect gear faults using vibration signal (Koide, et al., 2013, Hood, et al., 2013, Lim and Mba, 2013), acoustic emission (Al-Balushi, and Samanta, 2002), tooth root strain (Board, 2003), laser scattering (Tanaka, et al., 2011) and so on. Especially, analyzing the vibration signal of gear motion is one of the most effective methods applied in the detection of gear failures. For processing the vibration signal, various technologies in time domain, frequency domain and time-frequency domain are developed and widely applied in condition monitoring and diagnosis of gear device, such as cepstrum (Randall, 1982, Dalpiaz, et al., 2000 ), time synchronous average and related techniques (Houjoh, et al., 2007), wavelet transform techniques (Djebala, et al., 2012, Wang, et al., 2010) and advanced statistical methods (Martin, et al., 1990). Local faults in gears always produce transient modifications in vibration signals. Therefore, these signals have to be considered as non-stationary. Empirical mode decomposition (EMD) as a new data processing method was recently introduced by Huang et al. (1996, 1999, 1998 and 2003), especially for analyzing data from nonlinear and non-stationary processes. The main purpose of EMD is to decompose any linear or
Empirical mode decomposition (EMD) is a new adaptive data processing technique developed by Norden E. Huang et al. (1996, 1998, 1999), especially to deal with data from non-stationary and nonlinear processes (Huang and Shen, 2005). In contrast to most of other decomposition methods, it is intuitive and adaptive with a basis function based on and derived from the analytical data. The algorithm is based on a simple assumption that any data consists of different simple intrinsic modes of oscillations which have the same number of extrema and zero-crossings. With the initial processing of EMD, any signal can be decomposed into a finite set of intrinsic mode functions (IMFs) with frequency bands ranging from high to low, each of which must satisfy the following definitions (Huang and Shen, 2005):

(i) in the whole dataset, the number of extrema and the number of zero-crossings must either equal each other or differ at most by one, and

(ii) at any point, the mean value of the envelopes defined by the local maxima and local minima is zero.

An IMF represents a simple oscillatory mode as a counterpart to the simple harmonic function, however, it is much more general as the IMF can have a variable amplitude and frequency as functions of time. With the definition of IMF, a sifting process is employed for identifying IMFs in time-series signal (De Lima, et al., 2006). Supposing a time-series signal is \( S(t) \), it can be decomposed into its constituent IMFs by the following steps:

(i) Identify all the local extrema, then produce the upper and lower envelopes by connecting the local maxima and local minima with a spline curve respectively. Their mean value is computed as follows:

\[ m_1 = \frac{(S(t)_{\text{max}} + S(t)_{\text{min}})}{2}. \]  

(ii) at any point, the mean value of the envelopes defined by the local maxima and local minima is zero.
(ii) The first component $h_1$ is obtained as the following equation:

$$h_1 = S(t) - m_1$$

(2)

(iii) If $h_1$ is an IMF, the procedure moves to step (iv). Otherwise, if $h_1$ does not fulfill the criteria defining an IMF, it is treated as data $S(t)$ and the step (i)-(ii) are repeated up to $k$ times until $h_k$ can be considered as an IMF, that is:

$$h_k = h_{k-1} - m_{k-1}$$

(3)

(iv) The first IMF $c_1$ and the residual data $r_1$ are designated as:

$$c_1 = h_k$$

(4)

$$r_1 = S(t) - c_1$$

(5)

(v) The residue $r_1$ is treated as the new data and iterated as the same sifting process (i)-(iv).

The sifting process is completed until the resulting signal is monotonous or lower than the pre-specified value (Cheng, et al., 2013). Thus the analytical signal consists with $n$-empirical modes and a residue $r_n$ is achieved as:

$$S(t) = \sum_{i=1}^{n} c_i + r_n$$

(6)

Note that the sifting process is adaptively and based solely on the analytical data, which is able to find appropriate time-scales that may reveal intrinsic characteristics of the original signal. Furthermore, the IMFs include different frequency bands ranging from high to low and vary with the change of the original signal.

2.2 Support vector machines

Support vector machines (SVMs) are a machine learning method developed from the theory of limited samples Statistical Learning Theory by Vapnik in early 1990s, which are originally designed for binary classification. In case of linear classification, supposing a data set is $\{(x_i,y_i)\}$, $i = 1, 2, \ldots, n$ $n$ is the total number of samples, $y_i = \{1, -1\}$ is the class label of samples. As shown in Fig. 1, squares stand for class A and circles stand for class B, $x_1$ and $x_2$ are feature parameters value of input vector $x_i$. SVMs try to search for an optimal linear boundary to identify the two classes precisely and to ensure the margin between two classes is maximum to improve the classification accuracy (Samanta, 2004). The nearest data points used to determine the margin are called support vectors. When the feature space of input vector $x_i$ is high-dimensional ($>2$), the boundary is called separating hyper-plane which can be defined as:

$$f(x) = \langle w \cdot x \rangle + b = 0$$

(7)

Where, $w$ is a weight vector with the same dimensions of $x$, $\langle \cdot \rangle$ denotes a scalar product of vectors, $b$ is a scalar threshold, $w$ and $b$ are used to determine the position of the hyper-plane. A separating hyper-plane must satisfy the following constraint (Vapnik, 1998):

![Fig. 1 Linear SVMs for two classes](image-url)
In order to search an optimal separating hyper-plane, one has to find a maximum margin $2/\|\mathbf{w}\|^2$ subject to the linear constraint Eq. (8). Then the optimal hyper-plane problem transforms into the quadratic optimization problem. If the samples are not perfectly separable, outliers would affect the determination of hyper-plane even reduce the margin. To address this problem, slack variable $\varepsilon_i$ is introduced to reduce the effect of outliers on the boundary. The optimal hyper-plane can be obtained as a solution to the following quadratic optimization problem:

$$$egin{align*}
\text{Minimize } J(\mathbf{w}) = & \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^{n} \varepsilon_i \\
\text{Subject to, } & y_i \left(\langle \mathbf{w} \cdot \mathbf{x}_i \rangle + b \right) \geq 1 - \varepsilon_i, \quad \varepsilon_i \geq 0, \quad i = 1, \ldots, n
\end{align*}$$$

Where $\varepsilon_i$ is the slack variable measuring the degree of permitted deviation of outliers, $C$ is the penalty constant controlling the weight between objectives of searching the optimal hyper-plane and guaranteeing the minimum deviation of outliers in the function. It is found that Eq. (9) is a classical convex quadratic programming problem. Hence, the calculation can be converted into the equivalent Lagrange dual problem illustrated as follows:

$$$egin{align*}
L(\mathbf{w}, b, \alpha) &= \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^{n} \varepsilon_i - \sum_{i=1}^{n} \alpha_i \left( y_i \left(\langle \mathbf{w} \cdot \mathbf{x}_i \rangle + b \right) - 1 + \varepsilon_i \right) - \sum_{i=1}^{n} \varepsilon_i \alpha_i
\end{align*}$$$

Where, $\alpha_i$ and $\varepsilon_i$ denote Lagrange multipliers. $\mathbf{w}$ is acquired as: $\mathbf{w} = \sum_{i=1}^{n} \alpha_i y_i \mathbf{x}_i$, after solving the dual variable $\alpha_i$ the optimal separating hyper-plane can be determined. Therefore the decision function $\text{sgn}(f(\mathbf{x}))$ for linearly classifying the input test data as either belonging to class A or class B is acquired as follows:

$$$egin{align*}
h(\mathbf{x}) = \text{sgn}(f(\mathbf{x})) = \text{sgn} \left( \sum_{i=1}^{n} \alpha_i y_i \langle \mathbf{x}_i \cdot \mathbf{x} \rangle + b \right)
\end{align*}$$$

In case of nonlinear classification, SVMs can also be effective by using kernel function $k(\mathbf{x}_i, \mathbf{x})$ to map the input variables into a high-dimensional feature space, where the linear classification is possible (Widodo, et al., 2009). Then, the decision function for nonlinear classification is acquired as, (Vapnik, 1998):

$$$egin{align*}
h(\mathbf{x}) = \text{sgn} \left( \sum_{i=1}^{n} \alpha_i y_i k(\mathbf{x}_i, \mathbf{x}) + b \right)
\end{align*}$$$

Where, $k(\mathbf{x}_i, \mathbf{x})$ is the kernel function. Various existent kernel functions like Linear function, Gaussian radial basis function and Polynomial function can be adopted in SVMs (Chen, et al., 2013). However, there are not any standards to select the appropriate kernel function in present. In this paper, the kernel function of Gaussian radial basis function $k(\mathbf{x}_1, \mathbf{x}_2) = \exp(-|\mathbf{x}_1 - \mathbf{x}_2|^2/2\mu^2)$ is adopted for its high flexibility and broad application, $\mu$ is the width of Gaussian function which can be determined by an iterative program (Xue, et al., 2013).

3. An intelligent diagnosis method for gear damage

In this study, a diagnosis method for gear damage is developed using multiple classifiers of support vector machines with extracting failure feature vectors from the measured data, which can be adopted to monitor the conditions of gears. The procedure of this method mainly includes extracting failure feature vectors and diagnosis with multi-classifiers of SVMs.

3.1 Extracting failure feature vectors

Because the diagnostic accuracy considerably depends on the failure feature vectors, it is crucial to extract suitable parameters from the analytical signal. Statistical parameters such as standard deviation, kurtosis, root mean square value, crest factor and so on, are sensitive to the alteration of the signal, which are usually adopted to illustrate characteristics of the time-domain waveform. In this paper, statistical parameters of standard deviation ($\sigma$), kurtosis($\beta_1$), skewness ($\beta_2$) and root mean square value ($\gamma$) are calculated from the vibration data of gear box.

The method of EMD decomposes the original signal into a number of IMFs, each of which corresponds to various
Fig. 2 Diagram of gear damage diagnosis using Multi-classifiers of SVMs

frequency bands from high to low and represents the local characteristics of the original signal. Since the IMFs are adaptively derived from the measured data, the amplitude of each IMF will change along with the variation of gear conditions. Thus, the energy of IMFs from first to \(n\)-th can be served as representative parameters to demonstrate characteristics of various gear conditions. The energy of IMFs is computed as:

\[
E_i = \sum_{j=1}^{M} |c_i(j)|^2
\]  

Where, \(E_i\) is the energy of \(i\)-th IMF, \(M\) is the discrete data length of \(i\)-th IMF and \(c_i(j)\) is the sample amplitude of \(i\)-th IMF. In order to eliminate the impact of dimension and make convenience for the following data processing, the energy of IMFs are normalized by using the total energy \(E = \sum_{i=1}^{n} E_i\). Thus, the characteristic energy ratios of IMFs \(E_R\) are computed as representative parameters of gear conditions as follows:

\[
E_R = (e_{r1}, e_{r2}, \ldots, e_{rn}) = (E_1/E, E_2/E, \ldots, E_i/E)
\]  

The statistical parameters in time domain and characteristic energy ratios of IMFs are combined as failure feature vector \( \mathbf{V} = (\sigma, \beta_1, \beta_2, \gamma, e_{r1}, \ldots, e_{rn}) \) for the diagnosis of gear conditions. Since the high-dimensionality of feature vector complicates the calculation and extends the processing time, the method of principal component analysis is adopted to reduce the dimensions of feature vectors by synthesizing characteristics of each feature parameter into a composite index. Then, the processed parameters by principal component analysis are denoted as input vectors for SVMs classifiers.

3.2 Diagnosis with multiple classifiers of support vector machines

Figure 2 illustrates the diagram of diagnosis using classifiers of SVMs. In the step of diagnosis, multi-classifiers based on the binary tree are designed to classify the condition of test gear into three types, namely normal, slight failure and severe failure. SVM1 and SVM2 are the classifiers built with the training data. Variable \(x\) represents the output result: \(x=1\) for normal, \(x=2\) for slight failure, while \(x=3\) for severe failure. Firstly, the test data are input to classifier SVM1, if \(x=1\) the sample is determined as normal gear and the procedure completes; otherwise the sample will be automatically input to classifier SVM2, it is diagnosed as slight failure gear when \(x=2\), or else is severe failure gear.

4. Experiment
4.1 Experimental apparatus and procedure

In order to investigate the generation and progression of gear failures and confirm the validity of the proposed method, cyclic fatigue test was implemented on the power circulating type gear testing machine.

Figure 3 shows a scheme of experimental apparatus, which consists of a driving motor, a test gear box, a slave unit, bearing boxes, loading device and a photo sensor. An accelerometer was set at the center of the upper part of test gear box, by which the vibration accelerations were measured and recorded by computer through the amplifier and AD.
Fan, Ikejo, Nagamura, Kawada and Hashimoto, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.8, No.3 (2014)

Table 1 Conditions of the cyclic fatigue test

| Torque $T$ [N-m] | 70 |
|------------------|----|
| Rotation speed $n$ [rpm] | 1800 |
| Cycles $N$ | $0, 5 \times 10^4, 1 \times 10^5, 2 \times 10^5, 5 \times 10^5, 1 \times 10^6, 2 \times 10^6, 3 \times 10^6, 5 \times 10^6, 7 \times 10^6, 1 \times 10^7$ |

Table 2 Dimensions of test gears

| Module $m$ [mm] | 4 |
|-----------------|---|
| Number of teeth $z_1/z_2$ | 29/29 |
| Pressure angle $\alpha_0$ [deg] | 20 |
| Addendum $h_a$ [mm] | 4 |
| Dedendum $h_d$ [mm] | 5 |
| Pitch circle diameter $d$ [mm] | 116 |
| Tip circle diameter $d_a$ [mm] | 124 |
| Face width $b$ [mm] | 10 |
| Contact ratio $\varepsilon$ | 1.65 |
| Material | JIS S45C Thermal refining steel |
| Surface finishing | Hobbing |

Table 3 Classification of test gears

| Gear condition | Cycles | Pitting area ratio of driving gear ($A_P$) |
|----------------|--------|--------------------------------------|
| Normal | $N=0 \sim 1 \times 10^6$ | 0 |
| Slight failure | $N=2 \times 10^6 \sim 3 \times 10^6$ | $<2\%$ |
| Severe failure | $N=5 \times 10^6 \sim 1 \times 10^7$ | $\geq 2\%$ |

The lubrication method was oil bath, and turbine oil ISO VG32 was selected as the lubrication oil. Table 1 presents the conditions of cyclic fatigue test, test gears were driven until $1 \times 10^7$ cycles with the rotation speed $n=1800$ rpm and load torque 70 N-m. During the fatigue test, the vibration accelerations of gear box were measured at cycles $N=0, \cdots, 1 \times 10^7$ to investigate the progression of pitting failure. Table 2 shows the dimensions of test gears. The test gears are the involute spur gears whose module is 4 mm, number of teeth is 29, pressure angle is 20° and material is thermal refining steel JIS S45C. For the purpose of classification, we separate the gear condition into normal, slight failure and severe failure according to the pitting area ratio ($A_P$) of driving gear, which is shown in table 3. The pitting area ratio is defined as a ratio of the whole pitting area to the entire contact area of a gear.

Figure 4 shows the pitting area ratio of test gears. As shown in this figure, the pitting failure begin to occur on tooth surface of driving gear since $N=2 \times 10^6$, and begin to generate on tooth surface of driven gear since $N=5 \times 10^6$. The
The pitting area of the driving gear is larger than that of the driven gear. Additionally, the pitting area becomes larger with the increase of cycles $N$. Especially after $N = 3 \times 10^6$, the pitting area increases significantly and almost every tooth has pitting failure after $N = 7 \times 10^6$. Figure 5 represents photographs of the tooth surface of the test driving gear. Photographs

![Fig. 4 Pitting area ratio of test gears](image1)

(a) Normal ($N = 0$)  (b) Slight failure ($N = 2 \times 10^6$)  (c) Severe failure ($N = 1 \times 10^7$)

Fig. 5 Photographs of tooth surface of test driving gear

![Fig. 6 Change in tooth profile error of test gears](image2)

(a) Normal ($N = 0$)  (b) Slight failure ($N = 2 \times 10^6$)  (c) Severe failure ($N = 1 \times 10^7$)

Fig. 6 Change in tooth profile error of test gears
respectively correspond to cycles $N=0$, $2 \times 10^6$ and $1 \times 10^7$. For normal condition, there are only tool marks on the gear tooth surface. The dashed line represents the tracing line for measuring tooth profile error of test gears. Figure 6 shows the change in tooth profile error of test driving gear. The difference of tooth profile error between normal and slight failure is not obvious. However, the tooth profile error of severe failure is larger than that of slight failure. From Figs. 4-6, it can be seen that the tooth profile error becomes larger with the increase of driving time.

4.2 Experimental results and discussions

The vibration accelerations on test gear box in one rotation shown in Fig. 7 was measured under conditions of applied torque $T=70N\cdot m$ and rotation speed $n=1800rpm$. As shown in Fig. 7, the waveform changes slightly from cycles $N=0$ to $N=3 \times 10^6$. After $N=5 \times 10^6$, the vibration accelerations and its fluctuations increase rapidly as the increase of cycles $N$ and pitting area. Comparing with the normal teeth which have no failure on tooth surface, the vibration accelerations of the failure teeth are a little larger and manifest an increasing tendency with the growth of pitting area. The reason is that the tooth profile error becomes larger with the increase of pitting area, which will cause stronger vibration when meshing with the failure teeth.

Since the amplitudes of waveform are large and fluctuate strongly when the pitting area is large, the gear condition for severe failure can be diagnosed roughly according to the measured waveform, such as graphs of $N=7 \times 10^6$ and $N=1 \times 10^7$ shown in Fig. 7. However, it is difficult to discriminate the gear condition for slight failure from that for normal when the pitting area is small at $N=2 \times 10^6$ and $N=3 \times 10^6$. Therefore, it is necessary to extract some representative features from the acquired data to diagnose the early gear damage.

Fig. 7 Vibration accelerations on gear box
5. Application of the proposed method on the monitoring of gear conditions

5.1 Extracting failure features from vibration accelerations

For practical application, we analyzed the vibration accelerations on test gear box by the proposed method. The failure feature vectors of statistical parameters and characteristic energy ratios are extracted from the acquired vibration accelerations for representing characteristic features of gear conditions. Then, the method of principal component analysis is adopted to transform the failure feature vectors into a smaller set of input vectors for SVM.

Figure 8 shows the statistical parameters of standard deviation ($\sigma$), kurtosis ($\beta_1$), skewness ($\beta_2$) and root mean square value ($\gamma$) which are calculated from the vibration accelerations of test gear box at various cycles. When the gear condition is normal from $N=0$ to $1\times10^6$, the values of statistical parameters are nearly stable and relatively smaller. After the generation of pitting failure on tooth surface, the values of statistical parameters become larger with the increase of cycles. Particularly after $N=3\times10^6$, the statistical parameters increase obviously. This is because the vibration of gear is considerably influenced by the tooth profile error which becomes larger with the increase of pitting area. As shown in Fig. 4, the failure area on tooth surface becomes larger as the increase of cycles after the generation of pitting damage, especially after $N=3\times10^6$. Therefore, after the failure occurs on tooth surface, the measured vibration acceleration and its statistical parameters increase with the progression of pitting area. The statistical parameters can be employed to represent characteristic features of various gear conditions.

Figure 9 shows the intrinsic mode functions (IMFs) of vibration acceleration in one rotation. An IMF represents a simple oscillatory mode as a counterpart to the simple harmonic function, but it is much more general as the IMF can have a variable amplitude and frequency as functions of time. The raw signal can be constituted of several IMFs, each of which contains different frequency component of the raw signal from high to low. Therefore, an IMF is also a modulation signal of the raw signal and varies with the change of original signal. The characteristics of local fault on tooth surface can be extracted from the IMFs. In this study, the raw signal is decomposed into 6 IMFs. In Fig. 9, figures are arranged in order of frequency of IMFs from top to bottom. IMFs 1-3 contain the high frequency bands components of the raw signal, while IMFs 4-6 mainly contain signals of low frequency bands. Moreover, amplitudes of IMFs become weaker and more stable from IMF1 to IMF6. This is because the high-order harmonics and interference signals mainly concentrate in high frequency bands and attenuate rapidly with the decrease of frequency. The interference signals are usually caused by the vibration of other components on the testing machine, the environmental disturbances or the error of measurement system. The signals in low frequency bands are usually periodic and stable, such as signals with meshing frequency in IMF5. In IMF1, the amplitudes are relatively flat throughout the whole rotation under normal condition, while the fluctuation can be observed under slight failure and becomes stronger under severe failure. Additionally, comparing with the raw signal, IMF1 of severe failure magnifies the variation of the signal, which can more clearly manifests the characteristic of pitting fault. The IMF2 of severe failure is stronger than that of normal and slight failure, while IMF3 of normal is a little larger than that of other conditions. In IMFs4-6, the signal periodicity of normal and slight failure is more stable and clearer than that of severe failure. However, the magnitude of vibration acceleration is nearly the same among three conditions. Generally, the whole IMFs are relatively stable in normal case,
Energy of each IMF is computed to constitute the characteristic energy ratios by Eq. (14). Figure 10 shows the characteristic energy ratios of IMFs for various gear conditions. The condition of test driving gear at \( N=0 \) is normal, at \( N=2 \times 10^6 \) is slight failure, and is severe failure at \( N=1 \times 10^7 \). In this figure, the energy ratios of IMFs for normal, slight failure and severe failure are different from each other. In addition, the energy ratio of the first IMF becomes larger in order of normal, slight failure and severe failure. This is because most part of the first IMF is comprised of high frequency bands of vibration accelerations which mainly include high order harmonics of the signal. Especially, the vibration acceleration and its high order harmonics will become stronger with the increase of damaged area.

The statistical parameters and characteristic energy ratios of IMFs are combined as failure feature vector \( \mathbf{V} = (\sigma, \beta_1, \beta_2, \gamma, \epsilon_1, \ldots, \epsilon_n) \) for gear conditions. Since the high-dimensionality of feature vector complicates the calculation, the method of principal component analysis is adopted to reduce the dimensions of feature vectors. Principal component analysis is a statistical technique using an orthogonal transformation to convert an original set of variables into a substantially smaller set of uncorrelated variables called principal components (Widodo, et al., 2007). The original feature vector \( \mathbf{V} \) can be transformed into a new one \( \mathbf{P} \) by:

\[
P = \mathbf{V} \mathbf{W}
\]
\[ P = AV = \begin{bmatrix} a_{1(1)} & \cdots & a_{1(10)} \\ \vdots & \ddots & \vdots \\ a_{10(1)} & \cdots & a_{10(10)} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_{10} \end{bmatrix} = \begin{bmatrix} p_1 \\ \vdots \\ p_{10} \end{bmatrix} \]  

(15)

Where \( A \) is the \( m \times m \) orthogonal matrix whose \( i \)-th row \( a_i \) is the eigenvector of the sample covariance matrix, the new components \( (p_1, \ldots, p_{10}) \) are called principal components.

Generally, the first several principal components sorted in descending order of the eigenvalues perform considerable effect in feature vector \( P \) and can be selected to represent almost all the features of the vector. In this paper, the first 7 principal components contribute 95% effect in vector \( P \). Thus, components \( (p_1, \ldots, p_7) \) are selected as reduced input vector for SVM.

### 5.2 Diagnosis of gear conditions

The vibration accelerations on gear box measured at cycles \( N \) in cyclic fatigue test are utilized as test dataset. To acquire the training data set, several times of experiments were performed on a normal gear, slight failure gear and severe failure gear respectively. All the experiments are carried out under the same conditions of cyclic fatigue test. Finally, 36 experimental results are obtained as training dataset. Then, SVMs classifiers of SVM1 and SVM2 are built with the training data set. Test gear conditions are diagnosed by inputting principal components of test dataset to the training model.

In our previous study (Fan, et al., 2014), a diagnostic method for gear damage using SVMs with extracting amplitude ratios of frequency bands from vibration accelerations as failure feature vectors was provided. To validate the utility of the proposed method, we also diagnosed the conditions of test driving gear using the previous method. According to the method of previous study, amplitude ratios of frequency spectrum \( R = (r_1, r_2, \ldots, r_8) \) and statistical parameters were computed to acquire the failure feature vector \( V = (\sigma, \beta_1, \beta_2, \gamma, r_1, \ldots, r_8) \). The principal component analysis method was also adopted to convert the failure features into a smaller set of input vectors for SVMs.

Figure 11 presents the distribution of test data based on the first three principal components acquired with EMD method and the previous method respectively. PCA1,2,3 represent the first three principal components \( p_1, p_2 \) and \( p_3 \) respectively. Because the features extraction methods are different, the principal components and the distribution of test data with EMD method and without EMD method are also different. Comparing with the data distribution with EMD method shown in Fig. 11 (a), the data distribution without EMD method is a little more scattered. The painted markers are misdiagnosed samples. In Fig. 11 (a), the distribution of normal samples and slight failure samples is a little closer, which indicates the differences of failure feature vectors between the normal condition and slight failure condition are not obvious. Samples of severe failure are distinguished clearly from the samples of normal and slight failure. However, samples at \( N = 5 \times 10^4 \) and \( 5 \times 10^6 \) nearly distribute in the range of slight failure, which are probably misdiagnosed into slight failure. Figure 11 (b) presents the data distribution acquired without EMD method in previous study. Samples at \( N = 5 \times 10^4, 1 \times 10^6 \) and \( 5 \times 10^6 \) distribute a little more closely with samples of slight failure condition. These samples may be misdiagnosed into slight failure.

![Fig. 11 The distribution of test data based on the first 3 principal components](image-url)
Table 4 Diagnostic results of gear conditions

(a) With EMD method

| Cycles  | Actual condition | SVM1 | SVM2 | Diagnostic result |
|---------|------------------|------|------|-------------------|
| $N=0$   | Normal           | 1    |      | Normal            |
| $5 \times 10^4$ | Normal         | -1   | 2    | Slight failure *  |
| $1 \times 10^5$ | Normal         | 1    |      | Normal            |
| $2 \times 10^5$ | Normal         | 1    |      | Normal            |
| $5 \times 10^5$ | Normal         | 1    |      | Normal            |
| $1 \times 10^6$ | Normal         | 1    |      | Normal            |
| $2 \times 10^6$ | Slight failure | -1   | 2    | Slight failure    |
| $3 \times 10^6$ | Slight failure | -1   | 2    | Slight failure    |
| $5 \times 10^6$ | Severe failure | -1   | 2    | Slight failure*   |
| $7 \times 10^6$ | Severe failure | -1   | 3    | Severe failure    |
| $1 \times 10^7$ | Severe failure | -1   | 3    | Severe failure    |

Accuracy 82%

* represents misclassified data.

(b) Without EMD method

| Cycles  | Actual condition | SVM1 | SVM2 | Diagnostic result |
|---------|------------------|------|------|-------------------|
| $N=0$   | Normal           | 1    |      | Normal            |
| $5 \times 10^4$ | Normal         | -1   | 2    | Slight failure *  |
| $1 \times 10^5$ | Normal         | 1    |      | Normal            |
| $2 \times 10^5$ | Normal         | 1    |      | Normal            |
| $5 \times 10^5$ | Normal         | 1    |      | Normal            |
| $1 \times 10^6$ | Normal         | -1   | 2    | Slight failure*   |
| $2 \times 10^6$ | Slight failure | -1   | 2    | Slight failure    |
| $3 \times 10^6$ | Slight failure | -1   | 2    | Slight failure*   |
| $5 \times 10^6$ | Severe failure | -1   | 2    | Slight failure*   |
| $7 \times 10^6$ | Severe failure | -1   | 3    | Severe failure    |
| $1 \times 10^7$ | Severe failure | -1   | 3    | Severe failure    |

Accuracy 73%

* represents misclassified data.

Table 4 shows the diagnostic results of test gear conditions with EMD method and without EMD method. The diagnostic accuracy are 82% and 73% respectively. The diagnostic accuracy is defined as a ratio of the number of correctly judged samples to the total number of test data. In both Table 4 (a) and (b), samples for $N=5 \times 10^4$ and $N=5 \times 10^6$ are misclassified into slight failure. The vibration acceleration and its statistical parameters acquired at $N=5 \times 10^4$ are a little larger and close to those of slight failure condition, which can be seen from Fig. 8. The vibration acceleration at $N=5 \times 10^5$ may be strongly affected by the operation conditions or environment disturbances during measurement. The number of failure teeth of test gear at $N=3 \times 10^6$, $5 \times 10^6$ and $7 \times 10^6$ are 9, 12 and 21 respectively. The number of failure teeth at $N=5 \times 10^5$ is much fewer than that of severe failure at $N=7 \times 10^6$. Although the condition of test gear at $N=5 \times 10^6$ is regarded as severe failure according to the average pitting area ratio, the pitting area of most damaged teeth is much smaller than that of severe failure teeth. The vibration accelerations and failure feature vectors acquired at $N=5 \times 10^6$ are more similar with those of slight failure. Thus, the sample at $N=5 \times 10^6$ is more likely to misdiagnosed as slight failure. In Table 4 (b), the sample of normal condition at $N=1 \times 10^6$ is misdiagnose into slight failure. The damage began to generate on tooth surface between cycles $1 \times 10^6$ and $2 \times 10^6$. When the test gear working until $1 \times 10^6$ cycles, the condition of tooth surface begins to deteriorate and the tooth profile error becomes larger, which will intensify the vibration of gears. Therefore, the vibration acceleration and failure features acquired at $N=1 \times 10^6$ is a
little larger, which would be misdiagnosed into slight failure. The diagnostic results show that the proposed method can detect the early gear damage with satisfactory accuracy. In addition, the diagnostic accuracy acquired with the EMD method is better than that acquired with the previous method, which indicates that the EMD method is effective for gear damage diagnosis and classification, even can improve the accuracy of diagnosis.

6. Conclusions

This paper proposes an intelligent diagnostic method for gear damage using techniques of empirical mode decompositions and support vector machines, which mainly includes procedures of extracting failure feature vectors and diagnosis with multiple classifiers of support vector machines. By the technique of empirical mode decomposition, the original signal is decomposed into several intrinsic mode functions. Then, the characteristic energy ratios of intrinsic mode functions and statistical parameters are extracted from the vibration accelerations as failure feature vectors to be input to the support vector machine classifiers for gear damage diagnosis. The validity of the proposed approach is demonstrated by the results of cyclic fatigue test. The conclusions can be summarized as follows:

(1) The vibration accelerations of the waveform gradually become larger and fluctuate more and more strongly as the increase of pitting area. The gear condition of severe failure can be diagnosed according to the original signal. However, it is difficult to discriminate the gear condition of slight failure from normal only based on the waveform.

(2) Statistical parameters of standard deviation, root mean square value, kurtosis and skewness and characteristic energy ratios of intrinsic mode functions are sensitive to the progression of pitting area. They can be served as representative features of gear conditions for diagnosis.

(3) The diagnostic accuracy of gear conditions with empirical mode decomposition method and the previous method are 82% and 73% respectively, which demonstrates that the empirical mode decomposition method is effective for gear damage diagnosis and classification.

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