On an algorithm for generating inhomogeneous porous media

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Abstract. In the article, the results of the development of an algorithm for creating inhomogeneous three-dimensional porous media based on stochastic fractals are presented. The issues of estimating the parameters of porosity of the media are considered. It has been shown that the fractal dimension of porous media is preserved for any slice scales and the size of the fractal grid used to build porous media, as well as the fractal dimension of matrices and porous media depends on the binary filtration parameter that forms the given porosity by changing the ratio of matrices, open and closed pores, to the total sample volume.

1. Introduction

The problem of determining the structure of a porous medium of practically any material can be solved using the methods of radiography, tomography, and microscopy. These methods have significant limitations. The study of the physical properties of porous media is associated with the use of various equipment and is of high cost. Using these methods does not allow the creation of structures of porous media of scales that are larger than the measuring device. As a rule, the sizes of such samples do not exceed several tens of centimeters. Preparation of samples for research as well as the research process itself takes a long time. Process modeling based on the natural structure of the porous medium significantly limits the variability of the modeling process. Variability is limited by modeling parameters, while the modeling environment itself remains unchanged [1,2].

In this paper, there is considered an algorithm that allows to create the structure of a porous medium with any given characteristic of porosity, at any scale (limited by the power of the computing station) and with an unlimited set of variations in the topology of the structure of the porous medium.

Natural media, for example, such as geological porous media are formed over a considerable time period and, as a result of many random and regular processes, consist of heterogeneous objects distributed in an unpredictable order for the researcher.

In this process, many parameters are determined by chance. The use of stochastic fractals allows to reconstruct natural objects with mathematical precision exclusively by mathematical methods, since they are reflected in natural physical processes. To study the physical processes occurring in porous media, the spatial parameters of the model have a significant role. In this regard, the created digital twins - fractal porous media - can easily be reduced to specific geometric and spatial parameters at any scale.
Geometric and spatial parameters allow to determine the area and volume of pore space, the number of open and closed pores.

2. Porous media generation algorithm
Based on the algorithms that generate the base for the porous medium, stochastic fractals were chosen: Perlin Noise and a gas cloud based on the Octahedron and Cube algorithm, which are described in [3-6]. These algorithms allow to form an inhomogeneous (in pore structure) fractal field. A comparative analysis showed that both fractals are capable of forming a matrix of the porous medium with specified porosity parameters with a high degree of adequacy (the porosity parameter was used as a parameter for checking the adequacy) [7]. To determine the complexity of surface topology in the porous medium, the estimation of fractal dimension were used.

To a first approximation, stochastic fractals allow to form any variations in the structures of porous media, as shown in figure 1. However, these structures allow to create homogeneous porous media, as if the rock sample consisted only of pure sandstone that did not contain interspersed elements (inclusions).

![Figure 1. Fractal field created by the gas fractal algorithm.](image1)

Application of the binary filtration algorithm allows to conditionally designate open pore zones and rock structures. As a result of the performance of the proposed algorithm, a surface map of the porous medium can be created (figure 2). The algorithm for generating a homogeneous porous medium is represented by the following steps.

- **Step 0** Choose a fractal field generation algorithm.
- **Step 1** Calculate the fractal field.
- **Step 2** Apply a binary filter to the fractal field.
- **Step 3** Form a polygonal surface structure of the porous medium.

![Figure 2. 3D dimensional polygonal structure of the porous medium.](image2)

To simulate inclusions of more or less dense compounds, additional iterations were introduced into the algorithm.

- **Step 0.** Choose a fractal field generation algorithm.
- **Step 1.** Select an interspersion pattern (by default the field is filled with 0).
- **Step 2.** Determine the number of iterations $n$.
- **Step 3.** Set the parameters for the formation of the porous medium.
Step 4. Calculate the fractal field of the $n$-th iteration.
Step 5. Add the fractal field of the $n$-th iteration to the fractal field obtained at the $n-1$ iteration.
Step 6. Go to Step 3
Step 7. Apply the binary filter to the fractal field of the $n$-th iteration.
Step 8. Form a polygonal surface structure of the porous medium.

As a result of the performance of the proposed algorithm, structures that are inhomogeneous in density can be formed. In figure 3, the homogeneous fractal fields are shown, which were created with a variable parameter of grid dimension $r$ in the range from 1 to 4. It should be noted that the images in figure 3a, 3b and 3c are not mutually connected.

![Figure 3](image3.png)

**Figure 3.** 3D homogeneous fractal fields: a) $r = 1$; b) $r = 2$; c) $r = 4$.

In figure 4, the inhomogeneous fractal fields are shown, which were created by the extended algorithm for iterations $n = 3$ and the given grid dimension $r (1..4)$. These fractal fields are based on a randomly generated initial fractal field. Images in figure 4a, 4b and 4c are mutually connected and created sequentially based on the previous iteration.

![Figure 4](image4.png)

**Figure 4.** 3D inhomogeneous fractal fields: a) $n = 1, r = 1$; b) $n = 2, r = 2$; c) $n = 3, r = 4$.

In the analysis of porous media, an averaged description of the ratio of matrix segments and porous space is often used, which does not quite correspond to the structure of the density distribution in natural samples. As a result of the performance of the algorithm (figure 5), it can be observed that porous space is not uniformly distributed throughout the sample. More dense areas are concentrated in the fractal field obtained at the first iteration.

![Figure 5](image5.png)

**Figure 5.** 3D polygonal structure of an inhomogeneous porous medium.
To achieve greater approximation of the samples generated by the algorithm to natural samples of porous media, artificial fields can be taken as the basis for generating a fractal field. Artificial fields are areas of increased or decreased density deposited according to a given pattern.

Figure 6. An example of a 3D field with inclusions of high densities: a) an artificial field of inclusions; b) a 3D inhomogeneous fractal field with n = 2, r = 2; c) 3D polygonal structure of an inhomogeneous porous medium.

For the formation of models of porous media of the required size, which are homogeneous and inhomogeneous in structure and density, a specialized software product “Kernaliz” was developed. By changing the noise generation parameters and the binary filtration parameters, the structural characteristics of the model can be changed and the required porosity and permeability can be achieved. The dimension of the matrix is determined by the size of the generated fractals and the dimension of the grid forming the framework of the internal structure. By setting the parameters of the initial field of high densities, maximum closeness to samples of geological structures of porous media is achieved.

3. Evaluation of the fractal dimension of the porous medium
To assess the complexity of the inner surface of the porous medium and its topology, the Hausdorff dimension was used. This assessment allows to get the dimension of the object, regardless of the method of its construction and structure. The Hausdorff dimension $D$ is defined as follows:

$$D = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log \frac{1}{\varepsilon}}$$

In three-dimensional space, there are many points $N_0$. To cover this set, $N(\varepsilon)$- cubes are necessary with a characteristic size $\varepsilon$, in this case, $N(\varepsilon) \approx \frac{1}{\varepsilon^3}$ when $\varepsilon \to 0$ is determined by the law of similarity. In this paper, the fractal dimensions were calculated for porous media created by the algorithm described above with a given number of iterations $n = 5$. In this case, the fractal generation parameter “grid size” is assumed to be equal to $s_i = 2^{i-1}$ ($i=1..n$). In Table 1, the fractal dimensions with a fixed filtration parameter $f=500$ are shown.

Table 1. Fractal dimensions of porous media.

| Octaves | Number of iterations | 1     | 2       | 3       | 4       | 5     | 6     | 7     |
|--------|----------------------|-------|---------|---------|---------|-------|-------|-------|
|        | 1                    | 2.566 | 2.566   | 2.566   | 2.566   | 2.566 | 2.566 | 2.566 |
|        | 2                    | 2.567 | 2.590   | 2.582   | 2.587   | 2.583 | 2.584 | 2.583 |
|        | 3                    | 2.566 | 2.581   | 2.563   | 2.561   | 2.561 | 2.561 | 2.561 |
|        | 4                    | 2.570 | 2.573   | 2.576   | 2.575   | 2.573 | 2.573 | 2.573 |
|        | 5                    | 2.567 | 2.555   | 2.566   | 2.561   | 2.561 | 2.561 | 2.573 |
Regardless of the selected noise parameter (octave), the maximum value of the fractal dimension is achieved at the second iteration. An increase in the number of iterations leads to the separation of regions into finely dispersed components (figure 4c) and the appearance of new regions of porous space. This process leads to a decrease in the fractal dimension of the porous medium. The fractal dimension depends on the fractal parameters to a lesser extent and is more stochastic, which is confirmed by the data from Table 1. This is due to the fact that the algorithm for constructing a fractal for any input parameters forms a very similar structure, regardless of scale.

4. Conclusion
Modeling of porous media using fractal structures has great potential. The generation of an unlimited number of porous media shall allow to create full-scale “digital twins” of porous media that are closest to natural conditions. As a result, the capabilities of the developed software complex “Kernaliz” for generating stochastic fractals with a homogeneous and inhomogeneous structure of porous media are shown.

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