Decays of $X(3940)$ as $c\bar{c}$ States

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Partial decay widths of the $X(3940)$ are evaluated in the $^3P_0$ quark model with all model parameters predetermined. The work reveals that it is difficult to accommodate the $X(3940)$ with any $c\bar{c}$ meson state.

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The $X(3940)$ was first observed by the Belle Collaboration as an enhancement at $(3943 \pm 6 \pm 6)$ MeV in the spectrum of mass recoiling against the $J/\psi$ in the process $e^+e^- \rightarrow J/\psi X$ via $X(3940) \rightarrow D^*\bar{D}$ decay mode. The decay width of the state is determined to be less than 52 MeV at the 90\% C.L. A new measurement of the $X(3940)$ was performed in the processes $e^+e^- \rightarrow J/\psi D(\ast)\bar{D}(\ast)$ later by the same collaboration. It is noted that the $X(3940)$ has been observed in the $D^*\bar{D}$ channel but neither in the $D\bar{D}$ nor the $\omega J/\psi$ decay mode. Since all lower-mass states observed in the process $e^+e^- \rightarrow J/\psi X$ recoiling against $J/\psi$ have $J = 0$ (the $\eta_c(1S)$, $\chi_{c0}$ and $\eta_c'(2S)$ as shown in Fig. 1 of Ref.), it is natural to propose the $X(3940)$ to be $\eta_c'(3S)$. The reaction $e^+e^- \rightarrow J/\psi X(3940)$ was studied in the framework of light cone formalism supposing that the $X(3940)$ is a $3^1S_0$ state or one of the $2^3P$ states. The results suggested that the $X(3940)$ is a $3^1S_0$ charmonium. In this work, we study the decay reactions $X(3940) \rightarrow D(\ast)\bar{D}(\ast)$, assuming a $3S$, $2P$ or $2D$ $c\bar{c}$ state to the $X(3940)$.

We work in the nonperturbative $^3P_0$ quark dynamics in which a $q\bar{q}$ pair is created from or destroyed into vacuum. The model, originally introduced by Micu, has made considerable successes in understanding low-energies hadron physics. At least for meson decay, this approximation has been given a rigorous basis in strong-coupling QCD. The $^3P_0$ model was first applied to evaluate strong decay partial widths of the three $c\bar{c}$ states $\psi(3770)$, $\psi(4040)$, and $\psi(4415)$ in 1970’s. Barnes et al. calculated recently in the $^3P_0$ model all open-charm strong decay widths of 40 $c\bar{c}$ states up to 4.4 GeV, where a universal coupling strength is employed for the $^3P_0$ vertex and all mesons take spherical harmonic oscillator wave functions with a single length parameter.

We intend to evaluate the open-charm partial decay widths of the $X(3940)$ in the $^3P_0$ model with all model parameters well predetermined. The effective vertex of the $^3P_0$ model takes the form as in Refs.,

$$V_{ij} = \lambda \sigma_{ij} \cdot (p_i - p_j) \hat{F}_{ij} \hat{C}_{ij} \delta(p_i + p_j)$$

$$= \lambda \sum \mu \sqrt{4\pi/3} (-1)^{\mu} \sigma_{ij}^{\mu} Y_{1\mu}(p_i - p_j) \hat{F}_{ij} \hat{C}_{ij} \delta(p_i + p_j)$$

where $\sigma_{ij}^{\mu}, \hat{F}_{ij}, \hat{C}_{ij}$ are respectively the spin, flavor and color operators projecting a $q\bar{q}$ pair to the respective vacuum quantum numbers. The wave functions of all mesons are approximated with the Gaussian form,

$$\Psi_{nlm}(\vec{p}) = N_{nl} e^{-\vec{p}^2/2} L_n^{l+1/2}(b \vec{p}) Y_{lm}(\theta, \phi)$$

where $L_n^{l+1/2}(x)$ are the generalized Laguerre polynomial, $\vec{p}$ is the relative momentum between the quark and antiquark in a meson, and $b$ is the length parameter of the Guassian-type wave function.

There are three model parameters, the length parameters of the radial wave functions of the $D(\ast)$ and $X(3940)$ mesons and the effective coupling strength $\lambda$ of the $^3P_0$ vertex, which must be nailed down before the $X(3940)$ is studied.

The $D(\ast)$ length parameter can be determined with the process $D^+ \rightarrow \mu^+\nu_\mu$. The partial decay width of the
reaction $D^+ \rightarrow \mu^+ \nu_\mu$ is given by

$$\Gamma = \frac{p f}{32 M_D \pi^2} \int \left| T_{D^+ \rightarrow \mu^+ \nu_\mu} \right|^2 d\Omega$$

(3)

with

$$T_{D^+ \rightarrow \mu^+ \nu_\mu} = \int \frac{d\vec{p}}{(2\pi)^3/2} \psi(\vec{p}) \frac{\sqrt{2M_D}}{\sqrt{2E_1\sqrt{2E_2}}} T_{cd \rightarrow \mu^+ \nu_\mu}$$

(4)

where $T_{cd \rightarrow \mu^+ \nu_\mu}$ is the transition amplitude of the process $u\bar{d} \rightarrow \mu^+ \nu_\mu$ and $\psi(\vec{p})$ is the $D$ meson wave function in momentum space. Used as inputs the weak coupling constant $G = 1.166 \times 10^{-3}$ GeV$^{-2}$, the CKM element $|V_{cd}| = 0.230$, the $D^+$ meson mass $M_D = 1.870$ GeV, the $c$ quark mass $m_c = 1.27$ GeV, the $d$ quark mass as the constituent mass $m_d = 0.35$ GeV, and the experimental value of $\Gamma_{D^+ \rightarrow \mu^+ \nu_\mu} = 2.42 \times 10^{-7}$ eV, we derive the length parameter of the $D$ meson to be $B_D = 2.28$ GeV$^{-1}$. This value is larger than the one employed in [8], i.e. 2.0 GeV$^{-1}$. Note that it is impossible to estimate an error range for the length parameter as the CKM element $|V_{cd}|$ alone would lead to a sizable error bar for the $D$ meson decay width.

The investigation of the reactions $\psi(2S) \rightarrow e^+e^-$ and $\psi(3770) \rightarrow e^+e^-$ reveals that the $\psi(2S)$ possess a small D-wave component but the $X(3770)$ is mainly a $1D$ state with some S-wave component. These two mesons may couple as

$$\psi(2S) = \cos \theta |2S\rangle - \sin \theta |1D\rangle$$

$$\psi(3770) = \sin \theta |2S\rangle + \cos \theta |1D\rangle$$

(5)

where $\theta$ is the mixing angle between the $2S$ and $1D$ states. By fitting the theoretical decay widths to the experimental values [11] of $\Gamma_{\psi(2S) \rightarrow e^+e^-} = (2.35 \pm 0.04)$ keV and $\Gamma_{\psi(3770) \rightarrow e^+e^-} = 0.262 \pm 0.018$ keV, the length parameter and mixing angle were obtained as $B_{\psi(3770)} = (1.95 \pm 0.01)$ GeV$^{-1}$ and $\theta = (10.69 \pm 0.63)^o$ or $(-27.6 \pm 0.69)^o$ in the present model [12]. The mixing angle derived here is in agreement with results from other works [13, 14].

Using the mixing angle $\theta = 10.7^o$ or $-27.6^o$, the length parameters $B_{\psi(3770)} = 1.95$ GeV$^{-1}$ and $B_D = 2.28$ GeV$^{-1}$ to fit the experimental decay widths of $\Gamma_{\psi(3770) \rightarrow D^+D^-} = (11.15 \pm 1.09)$ MeV and $\Gamma_{\psi(3770) \rightarrow D^0\bar{D}^0} = (14.14 \pm 1.36)$ MeV [11], we get the corresponding effective coupling strength $\lambda = (0.68 \pm 0.04)$ or $(4.15 \pm 0.20)$.

We expect that the reaction $e^+e^- \rightarrow D\bar{D}$ at energy threshold is dominated by the two-step process in accordance with other work using an effective Lagrangian approach [15] and with studies of various reactions at low energies in the $^3P_0$ models [9, 10, 16]. In the two-step process, the $e^+e^-$ pair annihilates into a virtual time-like photon which decays into a $c\bar{c}$ pair; these created $c\bar{c}$ firstly form an intermediate vector meson and then the vector meson finally decays into $D\bar{D}$. With only the $J/\psi$, $\psi(2S)$ and $\psi(3770)$ included as intermediate mesons, we fit the lineshape of

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FIG. 1: Theoretical results for the cross section of the reaction $e^+e^- \rightarrow D\bar{D}$ at energy threshold. The $J/\psi$, $\psi(2S)$ and $\psi(3770)$ are included as the intermediate mesons. The upper panel is resulted from the parameters \{15, 10, 16\} while the lower panel is from the parameters \{15, 10, 16\}. The experimental data are from the Belle [17] and the BaBar [18].
the $\psi(3770)$ meson in the $e^+e^- \rightarrow D\bar{D}$ cross section with two sets of model parameters: \{\theta = 10.69^\circ, \lambda = 0.68, B_D = 2.28 \text{ GeV}^{-1}, B_{\psi(3770)} = 1.95 \text{ GeV}^{-1}\} and \{\theta = -27.6^\circ, \lambda = 4.15, B_D = 2.28 \text{ GeV}^{-1}, B_{\psi(3770)} = 1.95 \text{ GeV}^{-1}\} as shown in Fig. [1]. It is found that the experimental data strongly favor the first set of parameters as the second set of parameters leads to a $\psi(3770)$ peak over 10 times higher than the experimental values \cite{17,13}.

Next, we evaluate the partial decay widths of the $X(3940)$ meson in the $3^3P_0$ quark model with all model parameters predetermined. We assume in this work that the length parameter of $X(3940)$ is given by $B_{X(3940)} = B_{\psi(3770)}=1.95$ GeV$^{-1}$ and we use $B_{D^{(*)}}=2.28$ GeV$^{-1}$ and the effective coupling strength of the $3^3P_0$ vertex $\lambda = 0.68 \pm 0.04$. We consider the $X(3940)$ as either a $3S, 2P$ or $2D$ $c\bar{c}$ meson since potential models predict a mass around 4 GeV for a $c\bar{c}$ meson in the $3S, 2P$ and $2D$ states. Listed in Table I are the theoretical results compared with the Belle data \cite{24} in the last row. The values in parentheses are decay widths derived with the parameters in Ref. \cite{8}, where the $3^3P_0$ strength $\lambda = 0.4$ and a size parameter 2.0 GeV$^{-1}$ is applied to all mesons. It is noted that our results are fairly consistent with the ones derived with the model parameters from Ref. \cite{8}.

Since the $X(3940)$ has been observed in the $D^*\bar{D}$ channel but not in the $D\bar{D}$ decay mode, the states $3^3S_1, 2^3P_0, 2^3P_2, 2^3D_1$ and $2^3D_3$ are clearly ruled out and possible states are then $3^1S_0, 2^1P_1, 2^3P_1, 2^1D_2$ and $2^3D_2$. However, when considering that the reaction $e^+e^- \rightarrow J/\psi X$ is dominated by the one photon exchange process, the charge conjugation invariance demands that the $X(3940)$ must be a $C = 1$ state. Therefore, the only possible candidates for the $X(3940)$ are the states $3^1S_0, 2^3P_1$ and $2^1D_2$ which have positive charge conjugation.

It is difficult to accommodate the $X(3940)$ with the $2^1D_2$ state as the mass of the $2^1D_2$ $c\bar{c}$ meson obtained in the framework of potential approaches is 4150 – 4210 MeV \cite{19}. The large decay width of the $2^3P_1$ derived in this work makes this state unlikely to accommodate the $X(3940)$. The prediction for the decay width of the $3^1S_0$ state is more or less consistent with the experimental data, and hence one might consider interpreting the $X(3940)$ as a $3^1S_0$ $c\bar{c}$ state. However, one problem with this interpretation is that the mass of the $3^1S_0$ $c\bar{c}$ meson is derived as 4040 – 4060 MeV in potential approaches \cite{16}. Given the success of the potential quark model (see Ref. \cite{19,20} for a recent review), to assign the $3^1S_0$ $c\bar{c}$ meson a mass of 3940 MeV will cause great concern about the non-relativistic $c\bar{c}$ phenomenology.

One may argue that the prediction of the $3^3P_0$ quark model may not be so reliable. We have to admit that if the same model parameters are applied to both the light and heavy meson sectors, one may expect only qualitative results and sometime even misleading results. For instance, the theoretical $\rho$ decay width in Ref. \cite{21} is only half its experimental value, and the partial decay width of the process $\psi(4040) \rightarrow D\bar{D}$ in Ref. \cite{8} is almost zero which is strongly inconsistent with experimen-

| States     | $\Gamma_{D\bar{D}}$ (MeV) | $\Gamma_{D^*\bar{D}}$ (MeV) |
|------------|---------------------------|-------------------------------|
| $3^1S_0$   | -                         | 99.8 ± 12.0 (74.1)            |
| $3^3S_1$   | 80.3 ± 9.6 (56.2)         | 66.5 ± 8.0 (49.4)             |
| $2^1P_1$   | -                         | 109.3 ± 13.1 (140.4)          |
| $2^3P_0$   | 90.5 ± 10.9 (130.8)       | -                             |
| $2^3P_1$   | -                         | 204.5 ± 24.5 (271.4)          |
| $2^3P_2$   | 65.7 ± 7.9 (66.2)         | 8.5 ± 1.0 (8.7)               |
| $2^1D_2$   | -                         | 15.3 ± 1.8 (17.2)             |
| $2^3D_1$   | 54.6 ± 6.6 (64.3)         | 12.5 ± 1.5 (16.1)             |
| $2^3D_2$   | -                         | 22.6 ± 2.7 (25.6)             |
| $2^3D_3$   | 7.6 ± 0.9 (6.4)           | 0.27 ± 0.03 (0.23)            |

$X(3940)$: $37^{+26}_{-15} \pm 8$
tal data \cite{17, 18}. In this work, however, the model parameters have been fitted with very relevant reactions and hence the theoretical results listed in Table I reflect the physics to a large extent. We strongly argue that the $^3P_0$ quark model should be applied with the model parameters carefully fixed with very relevant reactions.

In summary we may conclude that it is difficult to accommodate the $X(3940)$, observed in the process $e^+e^- \rightarrow J/\psi X$ via the $X(3940) \rightarrow D^*\bar{D}$ decay mode \cite{1}, with any $c\bar{c}$ meson state.

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