A novel fault-tolerant control strategy based on inverse bicausal bond graph model in linear fractional transformation

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Abstract
This article presents the development of a novel fault-tolerant control strategy. For this task, a bicausal bond graph model-based scheme is designed to generate online information to the inverse controller about the faults estimation. Secondly, a new approach is proposed for the fault-tolerant control based on the inverse bicausal bond graph in linear fractional transformation form. However, because of the time delay for fault estimation, the PI controller is used to reduce the error before the fault is estimated. Hence, the required input that compensates the fault is the sum of the control signal delivered by the PI controller and the control signal resulting from the inverse bicausal bond graph for fast fault compensation and for maintaining the control objectives. The novelties of the proposed approach are: (1) to exploit the power concept of the bond graph by feeding the power generated by the fault in the inverse model (2) to suitably combining the inverse bicausal bond graph with the PI feedback controller so that the proposed strategy can compensate for the fault with a very short time delay and stabilize the desired output. Finally, the experimental results illustrate the efficiency of the proposed strategy.

Keywords
Fault-tolerant control, inverse bond graph, linear fractional transformation, fault estimation, bicausality

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Introduction
Today’s modern engineering systems are increasingly equipped with intelligent actuators, sensors, and embedded software. However, their increasing complexity makes them more prone to malfunctions (fauxts) that may lead to critical situations if not correctly dealt with in a timely manner. Therefore, it is very interesting to design new fault-tolerant control (FTC) and fault diagnosis (FD) techniques to ensure these complex systems’ safety and reliability. A wide variety of approaches have been proposed, and considerable researches have been devoted to FTC and FD methods in the last years.1–6 A recent detailed review on FD and FTC with hybrid/active-based and knowledge-based techniques is given in Gao et al.7 and that for signal-based and model-based techniques is presented in Gao et al.8

The FTC techniques are generally classified into active (AFTC) and passive FTC (PFTC).9 The passive.
technique uses a fixed robust structure with predefined parameters in both faulty and nominal situations (off-line mode) that enable the system to deal with a particular class of anticipated faults.\textsuperscript{10-12} For example, the problem of fault-tolerant perfect tracking in the event of additive faults has been formulated in Stefanovski,\textsuperscript{10} where a new PFTC consisting of the three-block controller was employed. Nasiri et al.\textsuperscript{11} developed a new PFTC strategy based on a sliding mode scheme for an affine class of multi-input multi-output (MIMO) non-linear systems. In a recent study by Jadidi et al.,\textsuperscript{12} two new PFTC schemes based on model predictive control and fuzzy logic have been proposed for an advanced hybrid microgrid benchmark under realistic fault scenarios in the presence of microgrid uncertainties and disturbances. Moreover, the cited approaches are fast enough to act instantly to any abnormal condition. However, they can accommodate a limited set of faults defined at the design stage.

In comparison with PFTC, AFTC requires an FD unit. The FD consists of Fault detection and isolation (FDI) and Fault estimation. The FDI provides information to the controller about the fault and its location, and then, the AFTC reacts by reconfiguring the controller based on FDI information.\textsuperscript{13-17} For instance, in Ji et al.,\textsuperscript{13} an adaptive AFTC strategy is developed for a longitudinal dynamical system of hypersonic flight vehicles subject to actuator faults. This scheme consists of nonlinear observer model-based fault detection for the system model and combined sliding mode control with back-stepping for a finite-time convergence FT controller. A new AFTC algorithm for a nonlinear three-tank system with partial faults in the actuator has been proposed in He et al.,\textsuperscript{14} where the AFTC includes the FDI unit and a new controller reconfiguration strategy. Qin et al.\textsuperscript{15} presented an advanced AFTC scheme that combines the external-loop PD controller with the fault estimation algorithm for a quadrotor in the presence of sensor faults. An AFTC scheme is developed in Wang et al.\textsuperscript{16} for uncertain systems subject to sensor faults. This scheme consists of an adaptive interval observer to detect and identify the faults and a compensation controller with a fault estimation module to improve performance with reduced conservatism. Moreover, these approaches can deal with diverse types of faults and necessarily lead to optimal performance. However, for detecting and isolating the fault, the FDI step requires excessive computations, thereby making the time response of the AFTC system slow compared to the PFTC system.

Many techniques for integrating FTC and fault estimation have been proposed in the last years. These techniques exclude FD in favor of fault estimation. The reason is the fact that all fault estimation inaccuracies are taken into account while designing FTC systems. This straightforward fact has led to a rising interest in the application of such integrated procedures.\textsuperscript{18-21} For example, Li et al.\textsuperscript{18} proposed an integrated scheme of fuzzy adaptive observer-based fault estimation and non-fragile FTC for Takagi–Sugeno fuzzy systems in the presence of both sensor and actuator faults. For a class of Markovian jump systems with uncertainties subjected to input faults, output faults, and disturbances, Li et al.\textsuperscript{19} presented a novel approach integrating uncertain fault estimation and the FTC method. However, in most of the previous works, the faults in the parameter components are rarely considered. Due to the causal and structural properties, the BG approach can be an alternative solution for dealing with actuator and parameter faults, which have rarely been studied.

The BG approach is a multidisciplinary and unified tool based on the graphical representation of the energetic analogy between the physical domains and power exchanges within the components of the system.\textsuperscript{22} It has been extensively exploited for dynamical modeling\textsuperscript{23,24} for Robust FDI,\textsuperscript{25-28} and fault estimation.\textsuperscript{29,30} Due to its properties, BG has been used in the last decade for the FTC and failure prognosis tasks.\textsuperscript{31} Recently, several BG model-based FTC approaches have been developed.\textsuperscript{32-35} For instance, in Djeziri et al.,\textsuperscript{32} a novel BG model-based FTC method was implemented on an intelligent autonomous vehicle. Loureiro et al.\textsuperscript{33} proposed a new BG model-based structural fault-tolerance analysis to verify faults diagnosability and faults recoverability conditions. This method has been then extended by integrating an FD and FTC for monitoring an electric vehicle.\textsuperscript{34} Most of these studies make a decision to change the control law based on FDI.

Compared with PFTC methods,\textsuperscript{10-12} which have limited fault acceptability only for the minor amplitude fault, the proposed FTC developed in this article provides more accurate fault estimation and fault compensation, such that the closed-loop control performance is guaranteed even in the presence of large-amplitude faults. Compared with active FTC methods,\textsuperscript{13-21,32-35} the proposed FTC scheme developed in this article does not need a reconfigurable mechanism. Thus, it will not lead to the time-delay problem of fault compensation. The developed FTC strategy proposes the direct use of fault estimation in the control system to compensate for the effects of the faults without the need for reconfiguration mechanism. To the best of our knowledge, the direct exploitation of fault estimation results is very limited, which remains challenging and motivates us to do this study.

Inspired by the above background, this paper develops a novel FTC strategy based on the inverse bicausal bond graph model in linear fractional transformation. The main contributions of this article are stated as follows:
• Adoption of the same graphical tool (bicausal BG approach) for fault modeling, estimation, and FTC. This brings significant convenience and allows the handling of the activities above systematically and straightforwardly.
• Integration of fault estimation method and inverse bicausal BG by exploiting bicausality notion properties to feed the power provided by the fault directly in the exact place so that the inverse bicausal BG can use it. Moreover, the direct use of fault estimation without a controller reconfiguration process may significantly decrease the computational complexity.
• We propose a novel FTC scheme that combines the inverse bicausal BG model in linear fractional transformation with a PI controller. The PI controller is designed to be robust against modeling uncertainties, while the inverse bicausal BG is employed for fault compensation. Because of the time delay for fault estimation, the PI controller also reacts quickly to reduce somewhat the error before the fault magnitude is estimated. Hence, the proposed scheme can ensure both fast fault compensation and optimal performance.

Moreover, the property of the proposed strategy is attractive because as long as the power generated by the faulty junction is known, component fault isolation is not a necessary condition.

Besides the introduction section, “Robust FDI using uncertain BG model” presents the robust FDI of uncertain systems using the BG approach. Section “Bicausality for fault estimation” discusses the fault modeling and fault estimation procedures using the bicausal BG-LFT model. After that, a novel method is developed in the section “The proposed FTC strategy using a combined approach,” which includes bicausal BG model inversion technique and the proposed FTC strategy. Section “Application” presents the applicability of the proposed FTC strategy by resorting to an omnidirectional mobile robot named Robotino. In addition, a comparative study is carried out used experimental data to assess the betterment of our FTC strategy. Finally, the section “Conclusions and further works” concludes this article with suggestions and remarks for further works.

Robust FDI using uncertain BG model

Uncertain ARRs generation for robust fault detection

In the BG framework, robust fault detection is mainly based on generating uncertain analytical redundancy relations (ARRs) with separable nominal and uncertain parts systematically from the BG model in preferred derivative causality.\(^{36}\) Preferred derivative causality (exploited for diagnosis purposes) is assigned to the nominal BG, by dualizing sensors when possible. Then, the uncertainties are modeled on the BG model to obtain uncertain BG. The candidate ARRs represent the constraint relations generated from an over-constrained subsystem/system (from “0” or “1” junction) using the covering causal paths methodology.\(^{37}\)

- In the case of “0” junction

\[
ARR_a : \sum f_a - \sum |w_a| \quad (1)
\]

- In the case of “1” junction

\[
ARR_b : \sum e_b - \sum |w_b| \quad (2)
\]

Where \(w_a\) represents the flow (equation (1)) or the effort (equation (2)) generated by the uncertainty, \(f_a\) is the flow by the nominal value from the “0” junction, and \(e_b\) is the effort by the nominal value from the “1” junction.

The junctions “0” or (respectively “1”) in the BG modeling are used to connect different elements of the system (BG elements) having common efforts (respectively common flows). The structural equations deduced from these junctions represent a set of conservation laws and/or equilibrium equations. The difference between them is explained as follows:

- “0” junction (common effort junction): It associates elements with the same effort, and the algebraic sum of flows is equal to zero. For example, series circuit in mechanics, and parallel circuit in electricity and hydraulics.
- “1” junction (common flow junction): It associates elements with the same flow, and the algebraic sum of efforts is equal to zero. For example, parallel circuit in mechanics, and series circuit in electricity and hydraulics.

Note that the threshold \(a\) is generated by the sum of the maximum absolute values of the ARRs different parts containing the parameter and measurement errors.\(^{38}\) The residual \(r\) describes the nominal part of ARR.

The algorithm for robust fault detection is given in Table 1. The residual \(r_n\) is the numerical evaluation of the ARR. The threshold \(a_n\) is obtained using the measurement \(\delta_{SSe,SSf}\) and modeling \(\delta_m\) errors, the parameters \(\theta_n\). In the BG modeling: (Se) and (Sf) are the sources of effort and flow, respectively. Sensors are modeled by dualized (inverted) effort (SSe) and flow (SSf) detectors. (MSe) and (MSf) represent modulated (controlled) effort and flow inputs, respectively. The nominal part of ARR \((r_n)\) is characterized by point

```
Table 1. Robust fault detection algorithm with uncertain ARRs.

| Input: \{ \Psi'_k(SSe(k), Se, MSE, SSf(k), Sf, MSf, \theta_n) \} | Output: \{ \Psi'_2(\xi_{SSf, SSE}, \sigma_n, \theta_n, SSe(k), SSf(k)) \} |
|--------------------------|--------------------------|
| Variable: i | variable detection |
| Residuals generation | a_k=n(\Psi'_1(SSe(k), Se, MSE, SSf(k), Sf, MSf, \theta_n)) |
| Adaptive thresholds generation | a_k=n(\xi_{SSf, SSE}, \sigma_n, \theta_n, SSe(k), SSf(k)) |
| Online Fault detection | \begin{cases} \text{false} & \text{if } r(k) \geq -a(k) \text{ and } r(k)<a(k) \\ \text{true} & \text{otherwise} \end{cases} |
| End If |

valued function \( \Psi'_k \) for any known variables \( K \). It is perfectly separated from the uncertain part of ARR \( a_n \). The latter is also identified as the point valued part \( \Psi'_2 \). Thus, by using the robust fault detection algorithm (Table 1), the change in flow/effort caused by the fault value is detected successfully by the residuals. In this case, the adaptive thresholds \( a_n \) are exceeded by their sensitive residuals \( r_n \). One fault can be detected by one or more residuals.

Adaptive thresholds are used to achieve robustness in the FD step by accounting for the parameter and measurement uncertainties so that the FD module can minimize false alarms and non-detection. An uncertain parameter \( \theta \), can be represented in multiplicative form as,

\[
\theta = \theta_n(1 + \delta_n) \\
\theta = \theta_n + \theta_n\delta_n
\]

Where \( \theta \) corresponds to parameters associated with the model, \( \delta_n \) is the relative deviation of the nominal parameter value \( \theta_n \). The multiplicative form of representation of uncertainty is useful for accounting relative uncertainty. Adaptive thresholds using the BG approach can be used, in which uncertainties in parameters are detached from their nominal parameters model. These uncertainties are modeled as feedback loops of internal variables.

As for the measurement uncertainties, they are modeled in an uncertain BG model by an additive source of flow \( (\xi_{SSf}) \) (in the case of a flow sensor) and of effort \( (\xi_{SSE}) \) in the case of an effort sensor. The idea is based on the fact that the measurement signal is composed of a real part \( (SSf_r, SSE_r) \) and a measurement uncertainty part \( (\xi_{SSf, SSE}) \) as \( SSf = SSf_r + \xi_{SSf} \) and \( SSE = SSE_r + \xi_{SSE} \). The maximum of the derivative measurement uncertainty is estimated as follows

\[
\max \left( \frac{d\xi_{SSf, SSE}}{dt} \right) = \frac{2(\Delta_{SSf, SSE})}{t_i - t_{i-1}} = \frac{2(\Delta_{SSf, SSE})}{\Delta t}
\]

Where \( \Delta t \) represents the sampling time.

To this end, for all ARRs, the adaptive thresholds are systematically generated from the uncertain BG model by adding the maximal absolute values of the different parts of the ARRs containing the parameter and measurement uncertainties. The faulty junctions must be deduced once all candidate ARRs are derived from the uncertain BG model using the robust fault detection algorithm. Thus, the concept of isolable junctions will be introduced.

Fault isolation through the isolable junctions

Another key element in the FDI algorithm is the Boolean fault signature matrix (BFSM), representing the faults' influence on the residuals. Hence, the BFSM allows knowing the faulty elements. Note that the FTC strategy requires information about the location of the fault (Fault isolation) and its magnitude (Fault estimation).

The fault isolation procedure is achieved through the BFSM. In this BFSM (Table 2), each entry holds boolean values \( (S_{i,j} = \{0, 1\}) \), and each component signature \( C_i \) is represented by the row vector: \( V_G = [S_{1,1}, S_{1,2}, \ldots, S_{1,m}] \), \( i = 1, \ldots, n \). The value \( S_{i,j} \) is the relation between the residuals and the system components and is assigned as follows

\[
S_{i,j} = \begin{cases} 1 & \text{if } r_j \text{ influential by fault on component } C_i \\ 0 & \text{otherwise} \end{cases}
\]

For a residual generation, \( r_j = eval(ARR_j), (j = 1, \ldots, m) \), where \( eval(ARR_j) \) defined as the

Table 2. Boolean fault signature matrix (BFSM).

| C_n | r_1 = eval(ARR_1) | r_2 = eval(ARR_2) | \ldots | r_m = eval(ARR_m) | Dt | It |
|-----|------------------|------------------|-------|------------------|----|----|
| C_1 | S_{1,1}          | S_{1,2}          | \ldots | S_{1,m}          | dt_1 | it_1 |
| C_2 | S_{2,1}          | S_{2,2}          | \ldots | S_{2,m}          | dt_2 | it_2 |
| \vdots | \vdots          | \vdots          | \vdots | \vdots          | \vdots | \vdots |
| C_n | S_{n,1}          | S_{n,2}          | \ldots | S_{n,m}          | dt_n | it_n |
yields residuals $r_j$. It expresses the consistency between the real system and the reference behavioral model described by the $ARR_j$. If the mathematical expression of $r_j$ contains the parameter of the component $C_i$; then $S_{i,j} = 1$. This means that $r_j$ is sensitive to the fault affecting $C_i$. For the ARRs obtained from the BG model, the number of the obtained ARRs is defined and denoted by $\text{Card}(ARRs) = m$.

In addition to the signature vectors, the BFSM contains two other columns that representing fault detectability $Dt = [d_1, d_2, \ldots, d_m]^\top$ and fault isolability $It = [i_1, i_2, \ldots, i_n]^\top$. $d_i$ and $i_t$ are equal to “1” if the component fault is detectable and isolable, respectively. A fault in $C_i$ detectable $(Dt = (dt_i))$ if at least one boolean value $S_{i,j}$ of its signature $V_{C_i}$ is not “0” $(\exists j (j = 1, \ldots, m) : S_{i,j} \in V_{C_i} \neq 0)$. The fault in a component is isolable $(It(i_t))$ if all signatures are different from its signature vector $V_{C_i}$.

$$i_t = \begin{cases} 1 & \text{if } \forall f (f = 1, \ldots, n) : V_{C_i} \neq V_{C_i}(i \neq f) \\ 0 & \text{otherwise} \end{cases}$$

As can be concluded, Table 2 can obtain the detectable and isolated faults directly by analyzing the ARRs. Let us consider in this part the isolable junction(s) Table (Table 3). The latter can be structurally obtained from the BFSM illustrated in Table 2.

The idea is to group the components ($\{C_1, \ldots, C_m\}$) that have the same signature $V_{C_i} = [S_{i,1}, S_{i,2}, \ldots, S_{i,n}]$ in the same row. Thereby, when the signature of a row vector is triggered, we can know the faulty junction. In this case, we can say that the fault affects one of the components belonging to the triggered signature. Finally, by using the isolable junctions table, we can know the junction ($J_n$) affected by the fault, together with possibly faulty components.

### Bicausality for fault estimation

As explained in the previous section via the FDI algorithms, a robust alarm can be generated allows detecting and isolating the fault as quickly as possible. However, the fault magnitude is obtained through another technique named fault estimation. Once the fault magnitude estimation is obtained, the FTC system must instantly react to the estimated fault using an appropriate controller. In this context, the BG approach becomes a suitable technique for estimating faults, especially in dynamic systems belonging to multi-energetic domains.

#### Fault modeling by BG-LFT

This subsection describes the procedure of fault modeling. This procedure is based on the BG in linear fractional transformation form (BG-LFT) to represent the faults that can occur in the system. For fault modeling, the BG elements are replaced by their corresponding BG-LFT elements. Thus, the fault estimation equation can be obtained directly and systematically from the obtained BG in the LFT form.

For example, when a parameter fault occurs, the latter generates an effort in the case of 1-junction or flow in the case of 0-junction. These faults can occur in: dissipative R-elements (hydraulic valve, electrical resistor, damper); I-elements (electrical inductance, mechanical inertia); C-elements (spring, tank, electrical capacitor).

For more illustration, an R-element (imposed flow) is illustrated (Figure 1). BG-LFT can represent it by decoupling the nominal part ($R_n$) from the faulty part ($F_R$). The additional effort generated by the fault is brought in at junction 1. It is represented on BG-LFT by a virtual effort $De^*: Z_{F_R}$ and effort fictitious source $Mse : W_{F_R}$. In fact, in the BG framework, fictitious sources represent the power generated by the fault.

- **Nominal case** (Figure 1(a)): From a mathematical point of view, the characteristic law with parameter in the nominal situation (without any faults) can be done as follows

$$e_R = f_R \cdot R$$

- **Faulty case** (Figure 1(b)): With a multiplicative parameter fault $F_R$, the following constraints can be obtained

$$e_F = f_R \cdot R_n(1 + f_R)$$

$$e_F = f_R \cdot R_n - W_{F_R}$$

Where $W_{F_R} = -f_R \cdot R_n \cdot F_R$. The effort $Z_{F_R} = f_R \cdot R_n$ is brought to $Mse : W_{F_R}$ by the virtual detector $De^*: Z_{F_R}$. Figure 1(c) shows the block diagram of the BG-LFT in Figure 1(b).

For actuator fault modeling, the procedure replaces the flow source $Sf$ (or effort $Se$) by $Sf : F_s$ (or $Se : F_s$),

| Junctions ($J_n$) | Signaturevectors | Components |
|-------------------|------------------|------------|
| $J_1$             | $[S_{1,1}, \ldots, S_{1,m}]$ | $\{C_{1,1}, \ldots, C_{1,m}\}$ |
| $J_2$             | $[S_{2,1}, \ldots, S_{2,m}]$ | $\{C_{2,1}, \ldots, C_{2,m}\}$ |
| $\vdots$          | $\vdots$         | $\vdots$   |
| $J_n$             | $[S_{n,1}, \ldots, S_{n,m}]$ | $\{C_{n,1}, \ldots, C_{n,m}\}$ |
respectively, as shown in Figure 2. From the latter, the equations can be obtained as follows

\[
\begin{align*}
    f_{FSf} &= S_{fn} / C_0 \quad (10) \\
    e_{FSe} &= S_{en} / C_0 \quad (11)
\end{align*}
\]

When a fault occurs in a sensor, it can be modeled by a flow source (Figure 3(a)) or effort (Figure 3(b)) depending on the used sensor.

From the BG-LFT model of Figure 3, the following equations can be obtained

\[
\begin{align*}
    a &\rightarrow \begin{cases} 
    f_1 = SSf : f_1 \\
    e_1 = SSE : e_1 
    \end{cases} \quad (12) \\
    b &\rightarrow \begin{cases} 
    f_2 = SSf : f_F - F_{SSF} \\
    e_2 = SSE : e_F - F_{SSE} 
    \end{cases} \quad (13)
\end{align*}
\]

Where \( f_r \) and \( e_r \) are respectively the flow and effort, \( F_{SSF} \) and \( F_{SSE} \) are the faults on the flow and effort sensor.

**Bicausal BG-LFT for fault estimation equations generation**

Due to the properties and computation capabilities of the bicausal BG, the fault estimation equations can be obtained directly and systematically from the bicausal BG-LFT model. Moreover, the bicausality concept enables the two power variables (flow and effort) in the same direction. From the bicausal BG-LFT model, the fault estimation equations are generated systematically by the following algorithm.
Step 1. Model the fault by replacing the obtained BG elements with the BG-LFT ones, where the faults are modeled and represented by a modulated source $MS_f$ (or $MSe$).

Step 2. Then, the bicausality is applied to the virtual detector and the fictitious source that represents the fault. Thus, the bicausal path propagated from the sensor of the residual to the modulated source.

Step 3. Finally, the fault estimation equation can be easily obtained from the bicausal BG-LFT in a systematic way.

A fault is considered in the actuator to explain the above algorithm (Figure 4). Figure 4(a) displays a BG model in preferred derivative causality. Then, the actuator fault is represented in the bicausal BG-LFT by $MSe : -F_{MSe}$ (Figure 4(b)).

Finally, to generate the expression of fault estimation, the bicausality notion was applied (Figure 4(c)). The bicausality is propagated from the detector ($SS_f : f$) to the modulated source ($MSe : -F_{MSe}$), representing the introduced fault. Finally, the following constraints can be obtained

\[
\begin{align*}
f_F &= f = f_2 = f_3 \\
e &= e_F - e_2 - e_3 \\
e &= e_F - e_2 - e_3 \\
e &= 0 \\
e_F &= e_2 + e_3 \\
f_F &= f_{ac} = f_{MSe} \\
e_F &= e_{ac} + e_{MSe} \\
e_{MSe} &= MSe : -F_{MSe}
\end{align*}
\]

Where $e_{ac}$ is the nominal effort, $e_{MSe}$ is the effort generated by the actuator fault. The inactive variable of the flow sensor is equal to zero.

Finally, the fault estimation equation (14) can be generated as follows

\[
F_{MSe} = - e_{ac} + e_2 + e_3 \quad (14)
\]

The next section describes the proposed strategy. Moreover, to help understand our strategy, some illustrative examples are given.

The proposed FTC strategy using a combined approach

The FTC step is an important technique that allows online reconstructing the control law to accommodate the fault. The system produces an actual output close to the desired output behavior even in the presence of the fault. In other terms, the FTC system can maintain the overall system stability and optimal performance in the event of faults. Once the fault is detected, the nominal feedback controller aims at compensating the fault. However, large faults may exceed the controller robustness. They can degrade the nominal behavior that weakens reliability or safety, so the AFTC system is required for fast fault compensation and maintaining the desired control objectives.

The AFTC community usually considers the robust FDI algorithms as information that furnishes the fault estimations and locations. However, the FDI and reconfiguration steps can be based on large computations and thus can lead to a time delay, thereby making the response of the system quite slow (slow response). Moreover, robust FDI is a challenging problem for systems with a closed-loop controller as its objectives are opposite to the controller objective. In other words,
there is a trade-off between good FDI and good closed-loop performance.

This article has proposed a novel FTC strategy that combines the inverse bicausal bond graph with a PI feedback controller to overcome the mentioned problems effectively. As an alternative to using the robust traditional FDI algorithms in FTC strategies, our approach proposes a direct use of the fault estimation signals consideration once the fault has been detected. These estimated fault signals are generated from the bicausal BG-LFT as described in section “Bicausality for fault estimation.” The latter are used directly in the bicausal inverse BG model in LFT form to compensate for the faults. In this case, the more complex step (robust FDI) is prevented. However, because of the time delay for fault estimation, the PI feedback controller is used to reduce somewhat the error before that the magnitude of the fault is estimated. The PI controller is also intended to compensate for modeling uncertainties and disturbances. In this case, the PI feedback controller can respond rapidly to faults, while the bicausal inverse BG optimizes the system’s performance. Thus, the proposed strategy can compensate for the faults in the systems with a minimal time delay and stabilize the system on the desired output behavior.

Bicausal BG model inversion

A direct also called forward model of a dynamic system is used to predict the system outputs to given inputs under the supposition that there are no faults in the system. Hence, if accurate modeling is achieved, the vector of the physical system output is approximately the same as the vector of the predicted model output.

Contrary to the direct model, the use of inversion of the direct model is to know the required inputs that give the desired outputs. Due to the bicausality properties, the bicausal BG modeling tool has proven to be powerful for modeling the direct model. For this reason, inverse bicausal BG modeling is exploited in this article to get the system inversion. The idea of using the inverse bicausal BG model for system inversion is motivated by the computation capability of the bicausality concept. Moreover, the bicausality concept enables the possibility to impose the two power variables (effort and flow) independently in the same direction. The two power variables indicated by causal half strokes are fixed or known. Figure 5 displays a general representation of the direct BG model and inverse bicausal BG model.

**Proposition 1.** A linear system modeled by the BG methodology is invertible if there is at least one bicausal path connecting the control input variable to the system output variable.39

**Proof.** Let \( M \) be the bicausal BG model of a linear system with \( I \) control inputs and \( O \) system outputs, and

\[
\begin{align*}
G_i, (i = 1, \ldots, I) & \text{ defined as a bicausal path where } I \text{ is the number of bicausal paths. } G_i & \text{ links the control input which is represented in bicausal BG by } DeDf/\text{DF}De \text{ element to the output SFSe/SeSF. If there is at least the covering of one } G_i & \text{ in } M, \text{ so } M \text{ is invertible.}
\end{align*}
\]

Figure 5(a) displays the BG model for the direct system. The system receives \( e_0 = u \) as a controlled input, while the output is \( f_1 = H \). Hence, the effort \( (e_1) \) conjugated with the flow \( (f_1) \) is deduced from the system. On the other hand, if we want to obtain the inverse bicausal BG (Figure 5(b)), the power variable of flow \( f_1 = H \) and its conjugated effort \( e_1 \) are the inputs of the system. However, not provide power to the system, the input effort must be set to zero \( (e_1 = 0) \). After the bicausality propagation from output to controlled input, we reach the input variable \( (e_0 = u) \) while leaving \( f_0 \) free. Thus, the proposed approach enables propagating the bicausality to construct in a simple and systematic way the inverse bicausal BG model of a system with \( n \) inputs and \( n \) outputs.

As explained previously, the inverse bicausal BG is used for inverse system modeling in nominal conditions (no-fault). Nevertheless, once the fault \( \hat{F} \) has occurred, it modifies the power exchanged between the system components regarding the nominal condition. Since the bicausal BG displays these exchanges, it is easy to capture this modification of power, which is relatively by a variation of the power variables (flow/effort). Therefore, the presence of a fault in one of the passive elements \( \{C, I, R\} \) or sources \( \{SF, SE, MSF, MSe\} \) associated with \( \{TF, GY, 1, 0\} \) junction, modifies this junction’s power. Finally, to obtain an accurate inverse system modeling under the faulty condition, the effect of the fault is injected into the system using BG-LFT. This new model to construct the inverse system under faulty conditions will be referred to as the inverse bicausal BG model in LFT form. Note that the bicausal inverse BG in LFT form is obtained by propagating the bicausal path from the outputs SFSe and SeSF to the inputs DFDe and DeDF, as depicted in Figure 6.

**Remark 1.** The introduction of the LFT form for modeling the faults does not affect the invertibility of the system.
To clarify bicausal BG model inversion, let us consider an illustrated example (Figure 7). Figure 7(a) represents an example of an acausal BG model with two junctions $J_1$ and $J_2$, where $MS_e : u$ is a controlled effort input and $DF : H$ is a flow detector.

**Definition 1.** An acausal (non-causal) BG is a forward BG without a causality assignment. That is, reference directions for the energy flows have been defined, though not computational causalities.

To obtain the inverse bicausal BG model as shown in Figure 7(b), the detector is replaced by a double source $SF_e : H_{des}$, where $H_{des}$ is the desired output. Moreover, the input source is replaced by $DeD_f : u_{inv}$, where $u_{inv}$ is the required input obtained by the inverse bicausal BG model. Thus, by covering the bicausal path (red arrow), one can derive simply and systematically the required input (equation (15)) that attains the desired output as follows:

$$H_{des} = f_6 = f_5 = f_4 = f_3$$
$$e_3 = e_5 + e_4 - e_6, \ e_6 = 0$$
$$f_5 = f_2 = f_1$$
$$e_1 = e_3 - e_2, \ e_1 = u_{req}$$
$$u_{req} = e_3 - e_2$$  \(15\)

In the conclusion of this subsection, we can say that the concept of bicausal BG model inversion proposed in this article gives the basis to calculate appropriate control actions for fault compensation. In this case, the fault estimation information can be considered by feeding the power generated by the fault in the inverse bicausal BG model so that the controller can consider them.

**FTC using a combination of inverse bicausal BG with a closed-loop PI controller**

Figure 8 displays the principal of our proposed FTC strategy in this article. There are three colored dashed boxes indicate.
Fault estimation: In this part, we propose a novel way to consider fault information using the bicausality notion. This step allows determining the magnitude of the fault $\hat{F}$. By exploiting the computational capabilities of the bicausality, the estimated fault is then injected into the inverse bicausal BG model to compensate for it. Note that the calculated fault estimation can contain a derivative of sensor signals $\{SfSe, SeSf\}$ which amplify the measurement noises. To solve this problem and enhance the fault estimation step, we proposed using a specific form of filter. However, the latter leads to delays in fault estimation.

FDI module: This part concerns the design of an uncertain BG model in the presence of uncertainties to give information about the presence of the fault and faulty junction. The latter is not used for fault compensation, and just the fault estimation is injected into the combined control system.

Combined control: The third part resulted from combining the inverse bicausal BG model in LFT form and closed-loop PI controller. First, the developed bicausal inverse BG in LFT form is designed offline based on the physical understanding of the differential-algebraic equation system. Then, based on fault estimation $\hat{F}$ and system desired output $\hat{y}_{des}$, the inverse bicausal BG model in LFT form is therefore updated in real-time. Second, the closed-loop PI controller was independently designed from the FTC and FDI problem. This means that the control law may have been intended so that the PI controller is robust to modeling uncertainties and to compensate a particular class of faults. Nevertheless, a large fault in the actuator or the physical system may exceed the robustness of the PI, so the FTC system is very required. Consequently, the required input delivered to the system is the sum of the control signal predicted by the inverse bicausal BG $u_{inv}$ and the control signal resulting from the PI controller $u_{PI}$. Therefore, the control signal $u_{req}$ applied to the system is

$$u_{req} = u_{inv} + u_{PI}$$ (16)

Let $G$ denote the function of the PI controller. The output of the PI controller can be exhibited as:

$$u_{PI} = G(\hat{y}_{des} - \hat{y}_m)$$ (17)

In fact, $u_{inv}$ in the faulty situation is equal to the command variable plus the power generated by the fault. So, the control law is expressed by

$$u_{req} = u_{inv} + G(\hat{y}_{des} - \hat{y}_m)$$ (18)

Remark 2. It was excluded to insert the inverse bicausal BG model in LFT form between the closed-loop PI controller and the system because of the high sensitivity of the inverse bicausal BG model in LFT form to measurement noises that would have caused large errors in the required control signal $u_{req}$ if the output of the closed-loop PI controller were used as input to the inverse bicausal BG.

Once fault $\hat{F}$ has occurred, it takes some time to estimate it. At the same time, the system measured output $\hat{y}_m$ deviates from the system desired output $\hat{y}_{des}$, and the closed-loop PI controller aims at compensating the
fault. Once the fault has been estimated, the fault magnitude is feed into the inverse bicausal BG model in LFT form. Using the fault magnitude $F$ and system desired output $\hat{y}_{des}$ as input, the inverse bicausal BG model provides an output signal $u_{inv}$. This signal is added to the PI signal to compensate for the effects of the fault. At this time, the fault effects have already been somewhat reduced by the PI controller. The resulting signal ($u_{inv} + u_{PI}$) delivered to the system drives the error ($\hat{y}_{des} - \hat{y}_{m}$) to zero despite the fault. If small disturbances on the system occur after fault compensation, it is compensated by the PI feedback controller.

**Application**

This section displays the application of our strategy to an omnidirectional mobile robot named Robotino. In addition, experimental results are carried out to gain practical insights into the proposed approach’s applicability and effectiveness.

**System description**

The mobile robot used in this study is composed of three electromechanical traction systems (Figure 9). Each system has a DC motor equipped with two sensors, a gear and a wheel. Whose wheels are placed symmetrically at 120° from one to another, allowing it to move in all directions. For actuation, every omnidirectional wheel is entrained by a DC motor. The Robotino technical specifications are:

- **Dimensions:**
  - Height: 0.21 m.
  - Diameter: 0.37 m.
- **Weight:** 11 kg.
- **Maximum speed:** 10 km/h.

**System BG modeling**

The overall BG model of the Robotino is displayed in Figure 10. As shown in Figure 10, the dynamics of Robotino is composed of

- Traction systems (first, second, and third electromechanical system).
- Longitudinal, lateral, and yaw dynamics of Robotino $\{\dot{v}_x, \dot{v}_y, \dot{\psi}_x\}$.

Each $i$th traction system with $i \in [1, 2, 3]$ is composed of three parts. The electrical part of the DC motor is characterized by an electrical inductance.
\( I : L_{ai} \), an electrical resistance \( R : R_{ai} \), and a controlled voltage source \( MSe : U_{reqi} \). The mechanical part is represented by inertia \( I : J_{ei} \) and a motor friction coefficient \( R : f_{ei} \). The Gyrator (with a constant \( GY : K_{ei} \)) describes the phenomenon of energy transformation from the electrical part to the mechanical part. The gear in the link between the mechanical part and the wheel part. It is characterized by a \( TF \) element \((TF : N_i)\). The wheel in interaction with the ground is modeled by an inertia \( I : J_{si} \) and friction coefficient \( R : f_{si} \). The torque generated by the contact effort \((F_{li}, i = 1, 2, 3)\), which is considered constant (equal to 2 N), representing a punctual coulomb effort. Under the assumption that the robot moves at low velocities and the road is identical, the contact effort is estimated experimentally by identifying the minimum voltage source applied to start the wheels on a homogeneous surface. Finally, each traction system is equipped with two flow sensors \((Df : i_{mi} \) and \( Df : \omega_{mi} \)) for the current and the angular velocity measurements.

The terms \( m V \dot{V}_x \) and \( m V \dot{V}_y \) are the effects of the yaw dynamics in the lateral and longitudinal dynamics (Figure 10). \( Z \) and \( N \) are the distance from the center of gravity of Robotino to the rear and the front axles. \( L \) is the distance between the first electromechanical traction system and the third electromechanical traction system.

The numerical parameters of Robotino used in this study are identified experimentally. These parameter values are shown in Table 4.

![Schematic model of the electromechanical traction system and its BG in preferred integral causality.](image)

**Figure 11.** Schematic model of the electromechanical traction system and its BG in preferred integral causality.

The properties of the BG model are exploited to calculate the structural and dynamic behavioral equations. Where the flow \( f_n \) and effort \( e_n \) represent the power variables. From the BG model in preferred integral causality (Figure 11), the constructive relations are given as follows:

- Structural equations (constitutive relations of the junctions)

\[
\begin{align*}
1_{1} - \text{Junction:} & \quad \begin{cases} 
 f_1 = f_2 = f_3 = f_4 \\
 e_2 = e_1 - e_3 - e_4 
\end{cases} 
\end{align*}
\]
The Gyrorator $GY : Ke_i$ produces the following relations

$$GY : Ke_i : \begin{cases} e_4 = Ke_i f_3 \\ e_5 = Ke_i f_4 \end{cases}$$

- Behavioral equations

$$f_2 = \frac{1}{L_i} e_2; \quad e_3 = Ra_i f_3$$

$$f_6 = \frac{1}{Je_i} e_6; \quad e_7 = fe_i; \quad e_{11} = Rsi f_{11}; \quad e_{10} = Js_i \frac{d}{dt} f_{10}$$

The transformers $TF : N_i$ produces the following relations

$$TF : N_i : \begin{cases} e_8 = \frac{1}{N_i} f_9 \\ e_9 = \frac{1}{N_i} f_8 \end{cases}$$

The known variables (sensors and actuators) are presented as follows

Sensors: \begin{cases} Df : i_{mi} \\ Df : \omega_{mi} \end{cases} \quad \text{Actuators:} \quad \begin{cases} MSe : Ureq_i \\ MSe : -R.Fl_i \end{cases}$

- Dynamic equations

$$P_3 = Ureq_i - \frac{Rai}{La_i} P_3 - \frac{Ke_i}{Je_i} + \frac{1}{N_i} Js_i P_7$$

$$P_7 = \frac{Ke_i}{La_i} P_3 - \frac{fe_i}{Je_i} + \frac{1}{N_i} Js_i P_7 - R.Fl_i$$

ARRs generation for fault detection and isolable junctions

The $i$th electromechanical traction system contains two dualized detectors connecting to three junctions $(1, 2, 3)$. Thus, according to the uncertain BG model-based robust FDI (see section “Robust FDI using uncertain BG model”), two ARRs equations (26) and (27) can be derived from the uncertain BG model of the traction system (Figure 12).

$$ARR_{11} : Ureq_i - Ra_i i_{mi} - La_i \frac{di_{mi}}{dt} - Ke_i \omega_{mi} + a_{11} = 0$$

$$ARR_{2i} : Ke_i i_{mi} - fe_i \omega_{mi} - Je_i \frac{d\omega_{mi}}{dt} - \left( \frac{Js_i}{N_i^2} \frac{d\omega_{mi}}{dt} + \frac{f_{si}}{N_i^2} \omega_{mi} + F_i \right) + a_{2i} = 0$$

The experimentally collected data (parameter’s nominal values, parameter’s uncertainty values, and sensor’s measurements data) were fed into the uncertain BG model of the traction system in preferred derivative causality for evaluation of residuals and adaptive thresholds. In this article, both modeling and measurement uncertainties are taken into account to generate adaptive residual thresholds. $a_{11}$ and $a_{2i}$ are respectively the uncertain part of $ARR_{11}$ and $ARR_{2i}$.

$$a_{11i} = \max(a_{11}) = W_{La_i} + W_{Ra_i} + W_{Ra_i i_{mi}} + W_{La_i i_{mi}} + Ke_i \xi_{\omega_{mi}}$$

$$a_{2ii} = \max(a_{2i}) = Ke_i \xi_{\omega_{mi}} + W_{Je_i} + W_{fe_i} + W_{Je_i i_{mi}} + W_{fe_i i_{mi}} + W_{Js_i} + W_{f_{si}} + W_{Js_i i_{mi}} + \xi_i$$
Table 5. The BFSM of the ith electromechanical traction system.

| $C_{i1}$ | $r_{i1} = \text{eval}(\text{ARR}_{i1})$ | $r_{i2} = \text{eval}(\text{ARR}_{i2})$ | $D_{i}$ | $I_{i}$ |
|--------|-------------------------------------|-------------------------------------|--------|--------|
| $U_{req_i}$ | 1 | 0 | 1 | 0 |
| $L_{ai}$ | 1 | 0 | 1 | 0 |
| $R_{ai}$ | 1 | 0 | 1 | 0 |
| $K_{ei}$ | 1 | 0 | 1 | 0 |
| $J_{ei}$ | 0 | 1 | 0 | 0 |
| $f_{ei}$ | 0 | 1 | 0 | 0 |
| $J_{si}$ | 0 | 1 | 0 | 0 |
| $f_{si}$ | 0 | 1 | 0 | 0 |
| $N_{i}$ | 0 | 1 | 0 | 0 |
| $F_{li}$ | 0 | 1 | 0 | 0 |
| $R$ | 0 | 1 | 0 | 0 |

Where $W_{Ra_i}$, $W_{La_i}$, $W_{fe_i}$, $W_{fe_i}$, $W_{fe_i}$, $W_{fe_i}$, and $W_{fe_i}$ represent the effort generated by the measurement uncertainties. That is,

$$W_{Ra_i} = Ra_i \Delta_{\omega_i}, W_{La_i} = La_i \Delta_{\omega_i}, W_{fe_i} = fe_i \Delta_{\omega_i}$$

$$W_{fe_i} = J_{ei} \Delta_{\omega_i}, W_{fe_i} = fe_i \Delta_{\omega_i}, W_{fe_i} = fe_i \Delta_{\omega_i}$$

Moreover, $W_{Ra_i}$, $W_{La_i}$, $W_{fe_i}$, $W_{fe_i}$, $W_{fe_i}$, and $W_{fe_i}$ represent the fictitious inputs of modeling uncertainties and are defined as follows

$$W_{Ra_i} = \delta_{Ra_i}Ra_i \omega_{mi}, W_{La_i} = \delta_{La_i}La_i \omega_{mi}, W_{fe_i} = \delta_{fe_i}fe_i \omega_{mi}$$

$$W_{fe_i} = \delta_{fe_i}fe_i \frac{d\omega_{mi}}{dt}, W_{fe_i} = \delta_{fe_i}fe_i \omega_{mi}, W_{fe_i} = \delta_{fe_i}fe_i \frac{d\omega_{mi}}{dt}$$

Where $\delta_{Ra_i}$, $\delta_{La_i}$, $\delta_{fe_i}$, $\delta_{fe_i}$, $\delta_{fe_i}$, and $\delta_{fe_i}$ represent the system parameter uncertainties. $\xi_{\omega}$ and $\xi_{\omega}$ represent the errors in the current and motor angular velocity sensors, respectively. $\xi_{\Phi}(F_i)$ represents the errors in the input effort $\Phi(F_i)$.

The measurement uncertainties were taken into account as bounded random variables. Therefore, we can write max $|\xi_{\omega}| = 0.144$, max $|\Phi(F_i)| = 0.001 Nm$. The absolute maximum of the errors in the angular velocity sensors is considered equal to $\frac{\pi}{180} rad$. The numerical evaluation of the obtained ARRs gives two residuals $r_{i1}$ and $r_{i2}$. These residuals are bounded by $(-a_{i1}, a_{i1})$ and $(-a_{i2}, a_{i2})$, respectively. From the structure of the obtained ARRs equations (26) and (27), the BFSM presented in Table 5 is deduced. We remark that all faults are detectable ($D_{i}$) $[11111111111]^{T}$, but none of them is isolable ($I_{i}$) $[0000000000000]^{T}$.

**Hypothesis 1**: In this study, we only consider parameter and actuator faults. Sensors are supposed to be fault-free.

Table 6. Isolable junctions in the ith electromechanical traction system.

| Junctions($J_{i}$) | Signaturevectors | Components |
|-------------------|-----------------|------------|
| $1_1$ | $[1, 0]$ | $\{U_{req_i}, L_{ai}, R_{ai}\}$ |
| $1_1/1_2$ | $[1, 1]$ | $\{K_{ei}\}$ |
| $1_2/1_3$ | $[0, 1]$ | $\{J_{ei}, f_{ei}, J_{si}, f_{si}, N_{i}, F_{li}, R\}$ |

Now, let us consider the electromechanical traction system’s isolable junction(s) (Table 6). This Table can be easily obtained from the BFSM (Table 5), as explained in subsection “Fault isolation through the isolable junctions.” In this case, when the fault signature is triggered [...], we can directly know the faulty junction(s), together with the element(s) affected by the fault. The junctions $1_1/1_2$ are observable by the dualized detector $SSf : \omega_{mi}$.

**Application of the proposed FTC strategy to Robotino**

The block diagram in Figure 13 shows the scheme of the proposed FTC strategy applied to Robotino based on the proposed inverse bicausal BG in LFT form and a PI feedback controller.

The PI controller coefficients setting is performed by the Control System design Toolbox in MATLAB/Simulink. The PI controller structure (transfer function) used in this study is as follows

$$G(s) = K_p \left(1 + \frac{K_i}{s}\right)$$  \hspace{1cm} (30)

Where, $K_p$ and $K_i$ are proportional and integral coefficients. In our study, $K_p = 0.4066$ and $K_i = 1$.

The Robotino input $MSe : U_{req}$, and measured outputs $SSf : i_{mi}$ and $SSf : \omega_{mi}$ are fed into the Bicausal BG-LFT model, generating an online fault estimate to the inverse bicausal BG model in LFT form. By considering an actuator fault $F_{U_{req}}$ on the controlled voltage input and a parameter fault $F_{fe}$ in the viscous friction coefficient, the inverse bicausal BG model in LFT form is updated in the function of the fault estimation information. By using the proposed fault estimation procedure, the fault estimation equations are obtained from the bicausal BG-LFT model (Figure 13).

- In case of actuator fault $F_{U_{req}}$, in the controlled voltage input

To generate the actuator fault estimation equation $F_{U_{req}}$, the bicausal path (red dashed arrow) $\circ \rightarrow \bullet \rightarrow \circ$ is propagated from the current sensor $SSf : i_{mi}$ to the source with bicausality $MSe : -F_{U_{req}}$. Then, by
covering the bicausal path, the following constraints can be obtained

\[ e_1 = e_2 + e_3 + e_4, \quad e_{13} = 0 \]

\[ e_1 = L_d \frac{di_{mi}}{dt} + Ra_{i_{mi}} + Ke_{i_{mi}} \quad (31) \]

\[ e_{16} = e_1 - e_{17} \]

\[ e_{16} = e_{16} + e_{17} \]

\[ e_1 = U_{reqi} + F_{U_{reqi}} \]

\[ U_{reqi} + F_{U_{reqi}} = L_d \frac{di_{mi}}{dt} + Ra_{i_{mi}} + Ke_{i_{mi}} \quad (32) \]

Finally, the expression of the actuator fault estimation equation \( F_{U_{reqi}} \) can be obtained as follows

\[ F_{U_{reqi}} = -U_{reqi} + Ra_{i_{mi}} + L_d \frac{di_{mi}}{dt} + Ke_{i_{mi}} \quad (33) \]

- In case of parameter fault \( F_{fei} \) in the viscous friction coefficient:

Following the same procedure, while the bicausal path (red dashed arrow) starts from the associated detector which is the velocity sensor \( SS_f : \omega_{mi} \) to the bicausal source of the faulty element \( MS_e : \omega_{des} \). Hence, the constraints can be obtained as follows

\[ e_9 = e_{11} + e_{10} - e_{12}, \quad f_9 = f_{i_{11}} = f_{i_{10}} = f_{i_{12}} \]

\[ e_9 = J_{i_{mi}} \frac{d\omega_{mi}}{dt} + \frac{f_{\omega_{mi}}}{N_i} + FL_i \]

\[ e_8 = \frac{1}{N_i} e_9 \]

\[ e_8 = \left( \frac{J_{i_{mi}} d\omega_{mi}}{N_i^2} + \frac{f_{\omega_{mi}}}{N_i} + FL_i \right) \quad (34) \]

\[ e_7 = e_5 - e_8 - e_6 - e_{14}, \quad e_{14} = 0 \]

\[ e_7 = e_5 - e_6 \]

\[ e_7 = (F_{fei} + 1)\omega_{mi} = F_{fei}\omega_{mi} + \omega_{mi} \quad (35) \]

\[ F_{fei}\omega_{mi} + \omega_{mi} = Ke_{i_{mi}} - J_{e_{i}} d\omega_{mi} \quad \]

\[ -\left( \frac{J_{i_{mi}} d\omega_{mi}}{N_i^2} + \frac{f_{\omega_{mi}}}{N_i} + FL_i \right) \]

Finally, the equation of the parameter fault \( F_{fei} \) in the viscous friction coefficient is then obtained simply and systematically by covering the bicausal path as follows

\[ F_{fei} = \frac{Ke_{i_{mi}} - f_{e_{i_{mi}} - e_{i_{mi}}} - J_{e_{i}} d\omega_{mi}}{f_{e_{i_{mi}}}} \quad (36) \]

The calculated fault estimation values \( F_{U_{reqi}} \) and \( F_{fei} \) and the desired velocity \( \omega_{des} \) are then used as inputs to the inverse BG in LFT form (Figure 13). Using the information about the fault and desired velocity, the inverse BG model in LFT form provides the required input that furnishes the desired velocity. The required input \( U_{inv} \) is derived from the inverse bicausal model in LFT form by covering the bicausal path (green dashed arrow) from the desired velocity \( SS_f : \omega_{des} \)
the error (sum of both inverse bicausal BG model in LFT form): 

\[
\omega_{des} = f_{14} + f_8 = f_7 = f_5 \\
e_5 = e_6 + e_7 + e_8 - e_{14}, \; e_{14} = 0 \\
e_5 = e_6 + e_7 + e_8
\]

In order to obtain \( i_{mi} \), \( e_7 \) is replaced by equations (33) and (34)

\[
e_5 = F_{fei,fei,\omega_{des}} + Fei,\omega_{des} + Jei,\frac{d\omega_{des}}{dt} + \left( \frac{J_{sl}}{N_i^2} \frac{d\omega_{des}}{dt} + \frac{f_{sl}}{N_i^2} \omega_{des} + \frac{F_{li}}{N_i} \right) \]

Thus,

\[
i_{mi} = \frac{F_{fei,fei,\omega_{des}} + Fei,\omega_{des} + Jei,\frac{d\omega_{des}}{dt} + \left( \frac{J_{sl}}{N_i^2} \frac{d\omega_{des}}{dt} + \frac{f_{sl}}{N_i^2} \omega_{des} + \frac{F_{li}}{N_i} \right)}{Kei}
\]

From the 1 junction, we generate

\[
e_1 = e_4 + e_2 + e_3 - e_{13}, \; e_{13} = 0 \\
e_1 = e_4 + e_2 + e_3 \\
e_1 = Ra_i i_{mi} + La_i \frac{di_{mi}}{dt} + Kei,\omega_{des} \\
e_1 = e_{16} + e_{17} \\
e_1 = U_{inv} + F_{Ureqi}
\]

Finally, the required input is obtained from the inverse bicausal BG model in LFT form

\[
U_{inv} = -F_{Ureqi} + Ra_i \\
F_{fei,fei,\omega_{des}} + Fei,\omega_{des} + Jei,\frac{d\omega_{des}}{dt} + \left( \frac{J_{sl}}{N_i^2} \frac{d\omega_{des}}{dt} + \frac{f_{sl}}{N_i^2} \omega_{des} + \frac{F_{li}}{N_i} \right) + \frac{F_{fei,fei,\omega_{des}} + Fei,\omega_{des} + Jei,\frac{d\omega_{des}}{dt} + \left( \frac{J_{sl}}{N_i^2} \frac{d\omega_{des}}{dt} + \frac{f_{sl}}{N_i^2} \omega_{des} + \frac{F_{li}}{N_i} \right)}{Kei} \times \frac{d}{dt} \left[ \frac{e_{16} + e_{17}}{Kei} \right] + Kei,\omega_{mi}
\]

This signal \( U_{inv} \) is added to the PI signal \( U_{pt} \). The sum of both \( U_{inv} \) and \( U_{pt} \) into the traction system drives the error \( (\omega_{des} - \omega_{mi}) \) to zero and forces the traction system to behave as the nominal system even in the presence of the faults. So, the control law \( U_{reqi} \) is

\[
U_{reqi} = U_{inv} + U_{pt}
\]

The inverse bicausal BG model in LFT form derived from that direct BG of the system can be considered as a differential-algebraic equations system of the inverse bicausal BG. A need for using an inverse bicausal BG model in a control system is that its differential-algebraic equations system has a unique solution and that the inverse bicausal BG is stable. Nevertheless, even if the differential-algebraic equations system for inverse bicausal BG can be solved numerically, it is not guaranteed to be stable. In addition, if an inverse bicausal BG is unstable or cannot be inverted, the direct model may be modified before inversion.

**Experimental results**

This subsection illustrates and compares the obtained results of the proposed approach with the classical approach. In the classical approach, the BG is used for FD-FTC integration in the presence of faults. By exploiting the causal and structural properties of the BG model, the FD system provides information to the controller about the fault and its location. Then, the FTC algorithm reacts by reconfiguring the controller based on FD information. The fault scenarios considered in this work are only contained in the first traction system (\( i = 1 \)) because the three traction systems of the Robotino \( (i^{th}, i \in [1, 3]) \) have the same structural model.

- **Scenario 1.** Free-fault case

In this scenario, we carry out the tests without the presence of faults to characterize the nominal behavior of the system. This situation is identified correctly by the two generated residuals \( r_1(V) \) and \( r_2(Nm) \) (Figure 14). These residuals are evaluated by considering adaptive thresholds. We remark that the residual values, in the beginning, are greater than the adaptive thresholds \( (T < 0.2s) \). Modeling uncertainties of the contact effort \( (F_{li}) \) can justify this latter. Although,
after 0.2 s, the two residuals $r_1(V)$ and $r_2(Nm)$ are within the set adaptive thresholds limits.

As long as there is no fault and the real angular velocity $\omega_{m1}(rad/s)$ equals the desired angular velocity ($\omega_{d1} = \omega_{m1}$), which serves as a reference. The input required $U_{req1}(V)$ (Figure 15(c)) to meet the control objectives is determined by means of the control signal predicted by the inverse bicausal BG $U_{inv}(V)$ (Figure 15(a)) that uses in this case only the desired angular velocity as input (no fault is presented in the system) and the control signal resulting from the PI controller $U_{PI}(V)$ (Figure 15(b)).

Figure 16 illustrates the system outputs, which are the current of the electrical motor $i_{m1}(A)$ (Figure 16(a)) and the angular velocity denoted by $\omega_{m1}(rad/s)$ (Figure 16(b)). The reference is depicted in black color. The proposed controller makes sure that the motor angular velocity (blue color) rapidly reaches its steady-state value. As shown in Figure 16(b), the proposed controller is able to follow the desired angular velocity.

- **Scenario 2.** Actuator fault

To validate the proposed FTC strategy experimentally, let us consider different faulty scenarios. In this scenario, an actuator fault is introduced by adding a faulty signal to the exiting control signal $U_{req1}(V)$ with a magnitude equal to $-2V$ at time $T = 5s$. This type of fault modifies the power exchanged in the system. In other words, this fault induces changes in the system dynamical behavior of the Robotino.

The introduced fault is detected since the adaptive thresholds are exceeded by the residual $r_1$, as shown in Figure 17. The residual $r_2$ is not sensitive to this fault. The latter is one of the elements belonging to the first junction 11 with the following fault signature $[1, 0]$. Thus, it is possible to conclude that the introduced fault is identified precisely.

The time evolution of the angular velocity is shown in Figure 18. In addition, it displays the comparison between our proposed approach and the classical approach. At time instant $T = 5s$, the fault value abruptly doubles in this scenario. As a result, the real angular velocity $\omega_{m1}(rad/s)$ deviates increasingly from
the desired angular velocity. We remark that the real angular velocity (blue color) decreases less than in the classical approach (red color). Once the actuator fault $F_{\text{Ureq}}$ has occurred, it takes some time to estimate it. Meanwhile, the PI controller aims at compensating the fault (Figure 19(b)). At this time, the PI feedback controller can somewhat reduce the fault before the fault magnitude is estimated. Once the fault magnitude is estimated (Figure 20), it is fed directly into the inverse bicausal BG in LFT form. Thus, with the information about the actuator fault estimation $F_{\text{Ureq}}$ and the desired angular velocity $\omega_{\text{des}}$, the inverse bicausal BG in LFT form provides a voltage input signal $U_{\text{inv}}(V)$, as shown in Figure 19(a). $U_{\text{inv}}(V)$ is then added to the signal resulting from the PI feedback controller $U_{\text{PI}}(V)$. We remark that the signal provided by the PI controller return to its initial value after that the fault has been estimated, and this is because the input voltage provided by the inverse bicausal BG forces the electromechanical traction system to behave as the healthy electromechanical traction system even in the presence of this actuator fault. As directed in Figure 19(c), the sum of both signals $U_{\text{req}}(V)$ provided to the Robotino drives the error to zero ($\omega_{\text{des}} - \omega_{\text{inv}} \approx 0$) and allows compensating actuator fault with a very short time delay.

Finally, it is easily seen from Figure 18 that the proposed FTC approach compensates for the actuator fault effects on the electromechanical traction system by reacting faster and returning the system quicker to its desired angular velocity when compared with the classical approach.

The proposed combination takes into account the modeling uncertainties without complex calculations. Note that, in case there are disturbances on the electromechanical traction system after the fault compensation, they are compensated by the feedback PI controller. The measured current in the presence of the actuator fault is illustrated in shown Figure 21.

- **Scenario 3.** Parameter fault in the viscous friction coefficient

In this scenario, we consider a fault affecting the viscous friction coefficient $f_v$. This fault is introduced at time $T = 5s$ with a magnitude equal to $1N.m.Sec.rad^{-1}$. In fact, many fault scenarios cannot be applied to the Robotino. This is why we used the data acquired from this robot in nominal conditions and added a block of perturbation to validate the proposed FTC strategy in the different scenarios. As structural results conclude (Tables 5 and 6), the residual $r_1$ does not detect this fault (not sensitive to it), while $r_2$ detects this fault and exceeds its adaptive thresholds (Figure 22). The signature in this case $0, 1$. Consequently, this introduced fault is identified correctly.

The parameter fault in the viscous friction coefficient leads to a high level of friction. In this case, the electromechanical traction system deviates from its desired angular velocity, as shown in Figure 23. Once the Parameter fault has occurred in the viscous friction coefficient, the voltage input signal delivered by the PI controller increase, trying to compensate for the introduced fault effects (Figure 24(b)) because the fault magnitude has not been estimated yet.

Since the parameter fault magnitude in the viscous friction coefficient is estimated $F_v$, (Figure 25), it is directly furnished to the inverse bicausal BG model. In this case, the control signal $U_{\text{inv}}$ (Figure 24(a)) increase quickly. The latter is added to the PI controller signal. The resulting signal $U_{\text{req}}$ enables rapid parameter fault compensation and allows compensating the convergence delay (Figure 24(c)). To this end, we can say that
### Discussion

It is clear from the experimental results that the classical FTC Approach does not completely offset the effects of actuator fault (red color) (Figures 18 and 23). It has poor control performance when compared with the proposed FTC. In this study, by combining the inverse bicausal BG and PI controller, the developed FTC approach can compensate for the actuator fault effects and stabilize the electromechanical traction system on the desired angular velocity. It is easily seen that when compared with the classical approach, the proposed FTC compensates for the fault effect by reacting faster and returning the system quicker to its desired angular velocity.

Thanks to the inverse bicausal BG in LFT form, we achieve a fast and effective fault compensation which is not usually attained using some existing FTC approaches. In addition, the inverse bicausal BG model can be adjusted online through the fault estimation information procedure (Figures 20 and 25). As shown in Figures 19 and 24, the proposed controller becomes active in the presence of the faults and provides the required input voltage to the faulty system. Consequently, the faulty electromechanical traction system can be treated as a nominally operating system.

Note that the classical FTC approaches consist of Fault Detection and Isolation module and reconfiguration mechanism, which increases the computational demand significantly and thus may take lots of time, making the system’s response relatively slow. Contrary to the classical FTC method, the developed FTC strategy proposes the direct use of fault estimation in the control system to compensate for the effects of the faults without the need for a reconfiguration mechanism. Due to the elimination of the reconfiguration mechanism, the proposed FTC scheme’s time response becomes very fast compared to classical ones.

By simulation comparisons, it can be concluded that the proposed FTC strategy based on the inverse bicausal BG model in linear fractional transformation has provided a better performance of fault estimation and compensation.

### Conclusions and further works

- It has been successfully demonstrated through an experimental study that the proposed

**Figure 20.** The actuator fault estimation $F_{\text{req}}(V)$.

**Figure 21.** The measured current in the presence of actuator fault.

**Figure 22.** $r_1(V)$ and $r_2(Nm)$ in case of viscous friction coefficient fault.

**Figure 23.** The real angular velocity case of parameter fault in the viscous friction coefficient.
The novel proposed approach combines the inverse bicausal BG model in LFT form and the PI controller. For robust FDI, first, the uncertain BG model has been exploited for uncertain ARRs generation with respect to uncertainties. Second, the isolable junctions table has been proposed to know the junctions affected by the fault, together with possibly faulty components. In addition, this article has proposed a novel way to consider fault estimation information using the bicausality notion. The PI controller has been designed to be robust against modeling uncertainties, while the inverse bicausal BG in LFT form has been employed for fault compensation. Because of the time delay for fault estimation, the PI controller reacted quickly to reduce the error before the fault magnitude was estimated. Then, using the concept of inverse bicausal BG model, the fault effects are compensated. Hence, the proposed combination scheme can ensure both fast fault compensation and optimal performance. For validation purposes, experimental studies were performed. In addition, the comparison has demonstrated that our proposed FTC strategy leads to better performance when compared with the conventional ones.

- For future work, the proposed inverse bicausal BG model can be applied over any uncertain hybrid system. However, it is necessary to consider the controlled junctions containing discrete mode changes during the inversion of the direct BG.
- It is noted that the FTC strategy developed in this work does not consider the sensor faults problem. Therefore, future work will include extending the proposed FTC strategy to compensate for the effects of sensor faults.
- The experimental results show that the developed method has improved the FTC task. However, further developments must be considered to improve the strategy in the presence of uncertainties in fault estimation.
- For non-linear systems, the direct BG and inverse bicausal BG models are to be solved numerically. To be used in a controller, the inverse Bicausal BG must be stable. If the latter is not stable or does not exist, modifications before the direct BG inversion may be a remedy.

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Appendix

Abbreviations

AFTC active fault-tolerant control
PFTC passive fault-tolerant control
FD fault diagnosis
FDI fault detection and isolation
MIMO multi-input multi-output systems
BG bond Graph
BG-LFT bond graph in linear fractional transformation
Acausal bond graph without causality
BGS bond graph with half causal strokes
BFSM boolean fault signature matrix
ARR analytical redundancy relation

Notations

$SSe$ ($SSf$) dualized detector of effort (flow)
$Se$ ($Sf$) sources of effort (flow)
$MSe$ ($MSf$) modulated effort (flow) inputs
$\theta_n$ nominal value of system parameter $\theta$
$\delta_\theta$ multiplicative parameter uncertainty on $\theta$
$\xi_{SSe,SSF}$ measurement errors
$r_n$ numerical evaluation of the nominal part of ARR
$an$ uncertain part of ARR
$k$ known variables
$TF$ transformer element
$GY$ Gyrator element
$\{C, I, R\}$ capacitance, inertial, and dissipation elements
$SfSe$ ($SeSf$) inverse bicausal desired flow (effort) output
$DfDe$ ($DeDf$) inverse bicausal required flow (effort) input