Projection Effects of Large-scale Structures on Weak-lensing Peak Abundances

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Abstract

High peaks in weak lensing (WL) maps originate dominantly from the lensing effects of single massive halos. Their abundance is therefore closely related to the halo mass function and thus a powerful cosmological probe. However, besides individual massive halos, large-scale structures (LSS) along lines of sight also contribute to the peak signals. In this paper, with ray-tracing simulations, we investigate the LSS projection effects. We show that for current surveys with a large shape noise, the stochastic LSS effects are subdominant. For future WL surveys with source galaxies having a median redshift \( z_{\text{med}} \sim 1 \) or higher, however, they are significant. For the cosmological constraints derived from observed WL high-peak counts, severe biases can occur if the LSS effects are not taken into account properly. We extend the model of Fan et al. by incorporating the LSS projection effects into the theoretical considerations. By comparing with simulation results, we demonstrate the good performance of the improved model and its applicability in cosmological studies.

Key words: gravitational lensing: weak – large-scale structure of universe

1. Introduction

Being an important cosmological probe, the weak lensing (WL) effect is one of the key science drivers for a number of ongoing and future large surveys (e.g., Albrecht et al. 2006; LSST Dark Energy Science Collaboration 2012; Amendola et al. 2013; Weinberg et al. 2013; Fu & Fan 2014; Kilbinger 2015; Dark Energy Survey Collaboration et al. 2016). Unlike strong lensing effects, where individual lens systems can be investigated, WL analyses are statistical in nature. Therefore it is important to explore different statistics to enrich the cosmological gains from WL data.

The cosmic shear two-point (2pt) correlation/power spectrum analyses are the most widely studied ones in WL cosmology (e.g., Kilbinger et al. 2013; Becker et al. 2016; Hildebrandt et al. 2017). However, WL signals have reached the nonlinear scales, and thus the 2pt statistics cannot uncover the full cosmological information therein. Three-point correlation measurements are done for a number of surveys with the realization of many more complications, both observational and theoretical, than in 2pt correlations (Pen et al. 2003; Semboloni et al. 2011; Fu et al. 2014). Concentrating on high-signal regions, WL peak statistics has emerged as another promising means to probe nonlinear structures and cosmology, complementary to cosmic shear correlation analyses (Shan et al. 2012, 2014; J. Liu 2015, X. K Liu 2015; Kacprzak et al. 2016; Martinet et al. 2018; Shan et al. 2018). WL peaks, particularly high peaks, arise primarily from the lensing effects of massive halos along their lines of sight (White et al. 2002; Hamana et al. 2004; Dietrich & Hartlap 2010; Fan et al. 2010; Yang et al. 2011; Lin & Kilbinger 2015; Reischke et al. 2016; Shirasaki et al. 2015). The high-peak abundance is thus a reflection of the halo mass function and a sensitive cosmological probe considering further the cosmology-dependent lensing kernel. It is less affected by baryonic physics than normal cluster abundances, where certain baryonic observable-mass relations are needed. However, apart from massive halos, other effects can also affect the peak signals, notably the projection effect of large-scale structures (LSS) and the shape noise resulting from the intrinsic ellipticities of source galaxies (Tang & Fan 2005; Fan et al. 2010; Yang et al. 2011; Hamana et al. 2012; J. Liu & Haiman 2016). To predict accurately the WL peak abundance for cosmological studies, we should carefully take them into account.

In principle, numerical simulations can include various effects, and we can build empirical templates of peak counts for different cosmological models incorporating different observational effects with respect to specific surveys (Dietrich & Hartlap 2010). By comparing with observational peak counts, we therefore are able to derive cosmological constraints (J. Liu et al. 2015; Kacprzak et al. 2016).

Such an approach is numerically intensive given the high dimensions of the cosmological parameter space and different astrophysical and observational effects. Thus, theoretical models are highly desirable for performing cosmological studies efficiently. Furthermore, the physical picture related to WL peaks can be seen more clearly in theoretical models, which need to specify different effects explicitly (Marian et al. 2009; Maturi et al. 2010; Hamana et al. 2012; Lin & Kilbinger 2015; Shirasaki et al. 2015).

In Fan et al. (2010; hereafter F10), the WL high-peak abundance is modeled by assuming that the true WL peaks are from the lensing effects of individual massive halos. In addition, the shape noise effect is carefully included, which not only generates false peaks but also influences the peak signals from halos. The comparison with simulations shows that the model works very well in the case of surveys with source galaxies having a shallow redshift \( z \sim 0.7 \), a number density \( n_g \sim 10 \text{arcmin}^{-2} \), and a survey area \( \sim 150 \text{deg}^{2} \). For such surveys, the projection effect of LSS is minor compared to
the shape noise. This model has been applied in cosmological studies by analyzing WL peak counts using data from the KiDS survey (Shan et al. 2018), the CFHTLenS survey (X. K. Liu et al. 2016), and the CFHT Stripe 82 survey (Shan et al. 2014; X. K. Liu et al. 2015).

For the ongoing and upcoming surveys, the survey depth can be improved considerably to detect more faraway galaxies for WL analyses. This will result in suppression of the shape noise as well as growth of the LSS projection effects. In such cases, the LSS effects must be carefully included in the theoretical modeling. In addition, the sky coverage will be enlarged by orders of magnitude, and the statistical errors of WL peak counts are expected to decrease. Therefore, even when the LSS projection effect is minor, it is still necessary to consider this effect for accurate modeling.

Recently, comparisons between WL peak counts from a large set of simulations and from the halo-based Monte Carlo model named CAMELUS (Lin & Kilbinger 2015) are shown in Zorrilla Matilla et al. (2016). It is found that for high peaks, CAMELUS works well for the source galaxies at \( z_s = 1 \) and with the cosmological parameters \( \Omega_m \) and \( \sigma_8 \) close to the current best values, where \( \Omega_m \) and \( \sigma_8 \) are the dimensionless matter density of the universe at present and the linear extrapolated density perturbations smoothed over a top-hat scale of \( 8.0 h^{-1} \text{Mpc} \), respectively. For higher values, for example at \( \Omega_m \sim 0.5 \) and \( \sigma_8 \sim 0.9 \), the deviations between the results from simulations and those from CAMELUS show up. We note that for high peaks, CAMELUS is essentially the same as that of F10, and it does not include the LSS contributions beyond halos. For high \( \Omega_m \) and \( \sigma_8 \), we expect stronger LSS projection effects than for low \( \Omega_m \) and \( \sigma_8 \). This should at least partly explain the differences of the WL high-peak counts seen in Zorrilla Matilla et al. (2016).

In this paper, we investigate in detail the LSS projection effect on WL high-peak counts, and we improve the model of F10 by taking the projection effect into the theoretical considerations. We perform extensive tests using numerical simulations, demonstrating the applicability of the improved model for future WL studies.

The rest of the paper is organized as follows. Section 2 presents the WL peak analyses and the improved model for high-peak abundances including the LSS projection effects. In Section 3, we show the simulation tests in detail and validate the model performance for different survey settings. A summary and discussions are given in Section 4.

2. Modeling Weak-lensing Peak Abundance Including the LSS Projection Effect

2.1. Weak Gravitational Lensing Effect

Photons are subject to the gravity of cosmic structures and are deflected when they propagate toward us. As a result, the observed images differ from their original ones. This phenomenon is referred to as the gravitational lensing effect. In the WL regime, the effect leads to small changes in size and shape of the images.

Theoretically, the WL effect can be described by the lensing potential \( \phi \). Its gradient gives rise to the deflection angle, and the second derivatives are related directly to the observational consequence of the lensing effect. Specifically, the convergence \( \kappa \) and the shear \( \gamma \), characterizing the size and the shape changes, respectively, are given by Bartelmann & Schneider (2001):

\[
\kappa = \frac{1}{2} \nabla^2 \phi, \tag{1}
\]

\[
\gamma_1 = \frac{1}{2} \left( \frac{\partial^2 \phi}{\partial x_1^2} - \frac{\partial^2 \phi}{\partial x_2^2} \right), \quad \gamma_2 = \frac{\partial^2 \phi}{\partial x_1 \partial x_2}, \tag{2}
\]

where \( x = (x_1, x_2) \) is the two-dimensional angular vector. Under the Born approximation, the convergence \( \kappa \) is the projected density fluctuation weighted by the lensing kernel and given by

\[
\kappa(x) = \frac{3H_0^2\Omega_m}{2} \int_0^{\chi_m} d\chi \int_0^{\chi_m} d\chi' \int_0^{\chi_m} d\chi'' \delta[f_k(\chi'; \chi, \chi')], \tag{3}
\]

where \( \chi \) is the comoving radial distance, \( \chi_m = \chi(z = \infty) \), \( a \) is the cosmic scale factor, \( f_k \) is the comoving angular diameter distance, \( \delta \) is the 3D density fluctuation, and \( p_s \) is the source distribution function. The cosmological parameter \( H_0 \) is the Hubble constant.

We define the lensing window function as

\[
w(x') = \int_{x'}^{\chi_m} d\chi p_s(\chi)f_k(\chi - x')/f_k(\chi); \tag{4}
\]

the corresponding power spectrum of \( \kappa \) is then

\[
C_l = \frac{9H_0^4\Omega_m^2}{4} \int_0^{\chi_m} d\chi' w^2(\chi') P_s(\ell, \chi'), \tag{5}
\]

where \( P_s \) is the power spectrum of 3D matter density perturbations.

Observationally, the brightness quadrupole moment tensor of a source galaxy can be measured, and from that, the source ellipticity can be extracted. The WL effect on observed images can then be described by the Jacobian matrix of the lensing equation, which reads

\[
A = \begin{pmatrix}
1 - \kappa & -\gamma_1 & -\gamma_2 \\
-\gamma_1 & 1 - \kappa & \gamma_1 \\
-\gamma_2 & \gamma_1 & 1 + \kappa
end{pmatrix}
\]

\[
= (1 - \kappa) \begin{pmatrix}
1 - g_1 & -g_2 \\
-g_2 & 1 + g_1
end{pmatrix}
\]

where \( g_1 = \gamma_1/(1 - \kappa) \) is the reduced shear. Considering the intrinsic ellipticity of a source galaxy, the observed ellipticity written in the complex form (Seitz & Schneider 1997) is

\[
\epsilon = \begin{pmatrix}
\epsilon_s + g \epsilon_i \\
1 + g \epsilon_s \end{pmatrix}; \quad \text{for } |g| \leq 1
\]

\[
\begin{pmatrix}
\epsilon_s + g \epsilon_i \\
1 + g \epsilon_s \end{pmatrix}; \quad \text{for } |g| > 1
\]

where \( \epsilon \) and \( \epsilon_i \) are the observed and intrinsic ellipticities of a source, respectively. The symbol * represents the complex conjugate operation. It is seen that the observed ellipticity is closely related to the WL shear. For \( \kappa \ll 1, \epsilon \approx \epsilon_i + \gamma \). Without considering the intrinsic alignments, the correlation analyses of \( \epsilon \) can thus give rise directly to an estimate of the WL shear correlation (e.g., Fu et al. 2008; Kilbinger et al. 2013;
In Figure 1, we zoom in to two high peaks from our ray-tracing simulations. The upper panels show the contribution of each lens plane to the peak in the noise-free convergence map, i.e., the ratio $K_{\text{LSS}}/K$, where $K = K_{\text{LSS}} + K_{\text{noise}}$. The contribution of the dominant halo is labeled as $K_{\text{dom}}/K$, as a red bar, and the LSS projection is labeled as $K_{\text{LSS}}/K$ as a black bar. The bottom panels are for the cases with shape noise added ($n_s = 10$ arcmin$^{-2}$ for left and $n_s = 20$ arcmin$^{-2}$ for right; the smoothing scales are both $\theta_G = 2.0$ arcmin) where $K_{\text{noise}} = K_{\text{dom}} + K_{\text{LSS}} + \chi$. The target peak is in the center of the corresponding stamp, labeled as a red $x$, and the halos ($M \gtrsim 10^{14} h^{-1} M_\odot$) are shown as black circles with the sizes of their projected angular virial radii. The contribution of the shape noise $K_{\text{noise}}/K_{\text{noise}}$ is shown as the blue bar.

In this paper, we concentrate on WL peaks identified in convergence fields, and in particular we study the LSS projection effects on high-peak abundances.

2.2. WL High-peak Abundance with Stochastic LSS

Physically, the WL convergence field reflects the projected density distribution weighted by the lensing kernel. Peaks there should correspond to the projected mass concentrations. Studies show that for a high peak, its signal is primarily contributed by a single massive halo located in the line of sight (e.g., Kaiser 1993; Bartelmann 1995; Seitz & Schneider 1995; Squires & Kaiser 1996; Jullo et al. 2014). We note that $\kappa$ is the weighted projection of the density fluctuations, and thus the structures can be better seen in the $\kappa$ field than in the shear field. Compared to previous observations targeting individual clusters, the current survey cameras have a large field of view, typically $\sim 1^\circ \times 1^\circ$, and thus the boundary effects on the convergence reconstruction can be in good control. Cosmological studies using the reconstructed convergence fields have been carried out for different WL surveys (e.g., Shan et al. 2012; Van Waerbeke et al. 2013; Shan et al. 2014; J. Liu et al. 2015; X. K. Liu et al. 2015, 2016).

In this paper, we concentrate on WL peaks identified in convergence fields, and in particular we study the LSS projection effects on high-peak abundances.

We take $\theta_G = 2.0$ arcmin in this paper. For this peak, $K_{\text{peak}} \approx 0.0835$ for the noiseless case (upper), and $K_{\text{peak}} \approx 0.114$ for the noisy case (lower). It is seen clearly that the lens plane at $z \approx 0.18$ contributes dominantly to the peak signal. Further examination finds that a massive halo is located there. In the noiseless case, the LSS effect from other lens planes only accounts for less than 10% (negative) of the peak signal, as indicated by the black bar. In the noisy case, the dominant halo contributes $\sim 80\%$ of the peak signal. The shape noise as indicated by the blue bar contributes $\sim 25\%$, and LSS from other planes contributes $\sim 5\%$.

These two examples show that for high peaks, the halo approach to modeling the WL peak abundances is a physically viable approach. The shape noise and the LSS projection effect can be regarded as perturbations to the signal from the dominant halo. For relatively shallow surveys, the shape noise is much larger than the LSS effect, and we expect the good performance of the F10 model, which takes into account the shape noise but without including the LSS effect. For deep surveys, however, the two perturbations are comparable, and the stochastic LSS effect cannot be neglected.

As a comparison, we show in Figure 2 a low peak with $z_s = 2.05$ and $n_s = 20$ arcmin$^{-2}$. Here the dominant halo is at $z \approx 0.7$. In this case, the LSS effect increases to $\sim 15\%$ because of the increase in $z_s$. The shape noise contribution is $\sim 10\%$. The two examples show that for high peaks, the halo approach to modeling the WL peak abundances is a physically viable approach. The shape noise and the LSS projection effect can be regarded as perturbations to the signal from the dominant halo. For relatively shallow surveys, the shape noise is much larger than the LSS effect, and we expect the good performance of the F10 model, which takes into account the shape noise but without including the LSS effect. For deep surveys, however, the two perturbations are comparable, and the stochastic LSS effect cannot be neglected.

As a comparison, we show in Figure 2 a low peak with $z_s = 2.05$ and $n_s = 20$ arcmin$^{-2}$. The peak signal is $K_{\text{peak}} \approx 0.0422$. It is seen that the signals are from the cumulative effect of the line-of-sight mass distribution, and no dominant halo contribution can be found. For these peaks, a modeling methodology other than the halo approach is needed.
In this paper, we focus on high peaks, and we present our model for high-peak abundances including LSS projection effects. Similar to F10, we assume that the signal of a true high peak is mainly from a single massive halo. The shape noise contributes a random component to the reconstructed convergence field. For stochastic LSS, from Figure 1, we see that they add onto the final peak signal in a zigzag way, leading to a statistically random perturbation. Thus it is appropriate to also model the LSS projection effect as a random field. Therefore, in our model, the convergence field in a halo region can be written as

$$\mathcal{K} = \mathcal{K}_H + \mathcal{K}_{\text{LSS}} + \mathcal{N},$$

where $\mathcal{K}_H$ is the contribution from the halo convolved with a window function corresponding to the smoothing operation made in the convergence reconstruction. It is regarded as a known quantity given the density profile of the halo. The smoothed shape noise field $\mathcal{N}$ is assumed to be Gaussian due to the central limit theorem (e.g., van Waerbeke 2000). We note that, in general, the overall convergence field from simulations shows non-Gaussianity due to the nonlinearity of structure formation. In our consideration here, however, $\mathcal{K}_{\text{LSS}}$ is the stochastic LSS contribution excluding the massive halo part, which is already explicitly split out as $\mathcal{K}_H$. It can be more Gaussian than the overall convergence field. Also, $\mathcal{K}_{\text{LSS}}$ is from small additive contributions from different lens planes in the high-peak case, and $|\mathcal{K}_{\text{LSS}}| \ll |\mathcal{K}_H|$. Therefore, as an approximation, we assume that $\mathcal{K}_{\text{LSS}}$ is also a Gaussian random field. Its validity will be extensively tested in Section 3 by comparing the model predictions for high-peak counts with the results from simulations.

A similar consideration was mentioned in Shirasaki et al. (2015) but without really calculating the LSS contribution. Also, they only concentrated on the influence of the random field on the central peak signals of halos. In our modeling here, we take into account specifically the stochastic LSS and calculate the total peak counts, including both the central ones from massive halos and the peaks from the random field $\mathcal{K}_{\text{LSS}} + \mathcal{N}$ inside halo regions as well as outside halo regions. In other words, to apply our model for cosmological studies, we can simply use all of the high peaks identified from convergence maps without the need to go through additional analyses to locate true halo-associated peaks.

With the Gaussian assumptions for the two random fields, the total field $\mathcal{K}$ in Equation (9) is also a Gaussian random field. More specifically, it is the Gaussian random field $\mathcal{K}_{\text{LSS}} + \mathcal{N}$ modulated by the known halo contribution $\mathcal{K}_H$. Following the same procedures shown in F10, we can then calculate the number of peaks in a halo region. Two features need to be addressed. First, the original peak signal from the halo is affected by the existence of the two random fields, which not only generates scatters but also leads to a positive shift for the signal (F10; Shirasaki et al. 2015). Second, the height distribution of peaks generated purely by the stochastic part $\mathcal{K}_{\text{LSS}} + \mathcal{N}$ is modulated by the halo convergence profile $\mathcal{K}_H$.

In formulae, for high-peak abundances, we have (F10)

$$n_{\text{peak}}(\nu) d\nu = n_{\text{peak}}^c(\nu) d\nu + n_{\text{peak}}^n(\nu) d\nu,$$

where $\nu = \mathcal{K}/\sigma_0$ with $\sigma_0^2 = \sigma_{\text{LSS},0}^2 + \sigma_{\text{N},0}^2$ being the total variance of the field $\mathcal{K}_{\text{LSS}} + \mathcal{N}$, and $n_{\text{peak}}^c(\nu)$ and $n_{\text{peak}}^n(\nu)$ are, respectively, the number density of peaks per unit $\nu$ centered at $\nu$ inside and outside halo regions.

We emphasize that our model is applicable for high peaks in which the signals of true peaks are dominated by single massive halos (see Figure 1). Thus we consider halos with mass $M \geq M_0$ as major contributors to the halo regions. Simulation analyses show that $M_0 \sim 10^{14} h^{-1} M_\odot$ is a proper choice (Wei et al. 2018). The rest from smaller halos and the correlations between halos are included in the stochastic LSS part. Then, for $n_{\text{peak}}^c(\nu)$, we have

$$n_{\text{peak}}^c(\nu) = \int dz \frac{dV(z)}{d\Omega z} \int_{M_0}^{\infty} dM n(M, z) \times \int_0^{\theta_{\text{vir}}} d\theta (2\pi\theta) \tilde{n}_{\text{peak}}^c(\nu, M, z, \theta),$$

where $n(M, z)$ is the comoving halo mass function, $\tilde{n}_{\text{peak}}^c(\nu, M, z, \theta)$ is the number density of peaks at $\theta$, and $\theta_{\text{vir}} = R_{\text{vir}}(M, z)/D_A(z)$ is the angular virial radius of a halo with mass $M$ at redshift $z$. Here $R_{\text{vir}}$ and $D_A$ are the virial radius of the halo and the angular diameter distance to the halo, respectively.

Considering the Gaussian random field $\mathcal{K}_{\text{LSS}} + \mathcal{N}$ modulated by the halo term $\mathcal{K}_H$, following the calculations in F10, we have

$$\tilde{n}_{\text{peak}}^c(\nu, M, z, \theta) = \exp \left[ -\frac{(K_H^2 + K_{\text{LSS}}^2)}{\sigma_1^2} \right] \times \frac{1}{2\pi\theta_{\text{vir}}^2} \exp \left[ -\frac{1}{2} \left( \nu - \frac{\mathcal{K}_H}{\sigma_0} \right)^2 \right] \times \frac{1}{2\pi(1-\gamma^2)} \exp \left[ -\frac{x + (K_H^2 + K_{\text{LSS}}^2) / \sigma_2 - \gamma(\nu - \mathcal{K}_H/\sigma_0)^2}{2(1-\gamma^2)} \right] \times F(x)$$

(12)
and

\[
F(x) = \exp\left[\frac{-(k_{1H} - k_{2H})^2}{\sigma^2}\right] \\
\times \int_0^{\ell/2} d\psi (x' e)^2(1 - 4e^2) \exp(-4x^2 e^2) \\
\times \int_0^\pi \frac{d\psi}{\pi} \exp\left[-4\epsilon e \cos(2\psi)\left(\frac{k_{1H} - k_{2H}}{\sigma}\right)^2\right].
\]

(13)

Here, $\theta^2 = 2\sigma_1^2/\lambda^2$, $\gamma = 2\sigma_1^2/\sigma_{02}\sigma_2$, $K_{1H} = \partial/\partial r_{\text{th}}$, and $K_{2H} = \partial/\partial r_{\text{th}}$. Different from that in F10, here the quantities $\sigma_i^2$ ($i = 0, 1, 2$) are, respectively, the moments of the total random field $\kappa_{\text{LSS}} + \mathcal{N}$ and its first and second derivatives. Specifically, $\sigma_i^2 = \sigma_{\text{LSS},i}^2 + \sigma_{\text{th},i}^2$.

For the number density of peaks contributed by those outside halo regions, $n_{\text{peak}}(\nu)$, we have

\[
n_{\text{peak}}(\nu) = \frac{1}{d\Omega} \times \left\{n_{\text{sub}}(\nu) \left[ d\Omega - \int dz \frac{dV(z)}{dz} \int_M \Delta N(M, z)(\pi \theta_{\text{vir}}^2) \right]\right\},
\]

(14)

where $n_{\text{sub}}(\nu)$ is the number density of peaks from the random field $\kappa_{\text{LSS}} + \mathcal{N}$ without halo modulations, and it can be calculated from Equation (12) by setting the halo-related quantities to zero.

From the above, we see that the stochastic LSS effects occur specifically in quantities of $\sigma_i^2 = \sigma_{\text{LSS},i}^2 + \sigma_{\text{th},i}^2$. For the shape noise part, we have (e.g., van Waerbeke 2000)

\[
\sigma_{\text{N,i}}^2 = \int_0^\infty \frac{ddl}{2\pi^2} L(l)^2 C_i^N,
\]

(15)

where $C_i^N$ is a power spectrum of the smoothed noise field $\mathcal{N}$. For the Gaussian smoothing, we have

\[
\sigma_{\text{N,i}}^2 = \frac{\sigma^2_{\epsilon}}{4\pi\eta_i^2\theta_{\text{G}}^2},
\]

(16)

where $\sigma_{\epsilon}$ is the rms amplitude of the intrinsic ellipticities. We further have $\sigma_{\text{N,0}} = \sigma_{\text{N,1}} = 1 : 2\sqrt{2}/\theta_{\text{G}} : 2\sqrt{2}/\theta_{\text{G}}$.

For $\sigma_{\text{LSS}}, \rho$ they are physical quantities and need to be computed in a cosmology-dependent way. In other words, $\sigma_{\text{LSS}}, \rho$ also contributes to the cosmological information embedded in WL peak counts. Given a power spectrum for the LSS convergence field, $C_i^{\text{LSS}}$, we have

\[
\sigma_{\text{LSS},i}^2 = \int_0^\infty \frac{ddl}{2\pi^2} L(l)^2 C_i^{\text{LSS}}.
\]

(17)

To calculate $C_i^{\text{LSS}}$, we adopt the following approach. From Equation (5), the WL power spectrum $C_i$ can be obtained from the integration of the weighted 3D nonlinear power spectrum $P_i$. For $P_i$, it can be computed using the simulation-calibrated halo model (Takahashi et al. 2012) and has been included in different numerical packages, such as CAMB (Lewis et al. 2000). In the language of the halo model, the overall $P_i$ consists of contributions from the one-halo term of all halos and the two-halo term considering the correlations between halos. In our model here, halos with $M > M_* \Delta_0$ have been separated out as $H_{\text{th}}$. Thus, to compute the leftover stochastic LSS effects, we subtract the one-halo term from halos with $M > M_*$ from the overall power spectrum:

\[
P_{i}^{\text{LSS}}[k, \chi(z)] = P_i[k, \chi(z)] - P_i^{\text{HI}}|_{M > M_*}[k, \chi(z)].
\]

(18)

It is seen that $P_i^{\text{LSS}}$ contains the one-halo term from halos with $M < M_*$ and the two-halo term between all the halos including the ones with $M > M_*$. For the one-halo term $P_i^{\text{HI}}|_{M > M_*}$, we have (e.g., Cooray & Sheth 2002)

\[
P_i^{\text{HI}}|_{M > M_*}[k, \chi(z)] = \frac{4\pi}{\bar{\rho}^2} \int dM M^2 W^2(k, M)n(M, z),
\]

(19)

where $\bar{\rho}$ is the mean matter density of the universe, and $W(k, M)$ is the Hankel transformation of the spherically symmetric halo density profile $\rho(r, M)$, given by

\[
W(k, M) = \frac{1}{M} \int_0^{\infty} dr \frac{\sin(kr)}{kr} 4\pi r^2 \rho(r, M).
\]

(20)

From $P_i^{\text{LSS}}$, we can obtain $C_i^{\text{LSS}}$ by

\[
C_i^{\text{LSS}} = \frac{9H_0^2\Omega_m^2}{4} \int_0^\infty d\chi \frac{\mathcal{K}_s}{\chi} \frac{\mathcal{P}_{\text{LSS}}}{\ell_k} \frac{\ell}{\ell_k} \chi',
\]

(21)

and subsequently $\sigma_{\text{LSS},i}^2$ by Equation (17).

For the calculations of the one-halo term in Equation (19), we take the Navarro–Frenk–White (NFW) halo density profile given by Navarro et al. (1996, 1997):

\[
\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2},
\]

(22)

where $\rho_s$ and $r_s$ are the characteristic density and scale of a halo, respectively. The scale $r_s$ reflects the compactness of a halo and is often given through the concentration parameter $c_{bh} = R_b/r_s$, with $R_b$ being the radius inside which the average density of a halo is $\Delta_0$ times the cosmic density. Here we
For the halo mass function \( n(M, z) \), we use the one given in Watson et al. (2013), an empirical fitting formula derived from \( N \)-body simulations.

In Figure 3, we show \( C_{\ell}^{\text{LSS}} \) together with the overall \( C_{\ell} \) and the massive one-halo term \( C_{\ell}^{\text{H}} \) under different source galaxy distributions \( p(z) \), for which we adopt the following form:

\[
p(z) \propto z^2 \exp \left[ -\left( \frac{1.414}{z_{\text{med}}} \right)^{1.5} \right],
\]

where \( z_{\text{med}} \) is the median redshift. In the plots, we also show the power spectra of aperture mass under the massive one-halo term and use the concentration–mass relation from Duffy et al. (2008) with

\[
c_{\text{vir}}(M, z) = 5.72 \left( \frac{M}{10^{14} h^{-1} M_\odot} \right)^{-0.081} (1 + z)^{-0.71}.
\]

For the halo mass function \( n(M, z) \), we use the one given in Watson et al. (2013), an empirical fitting formula derived from \( N \)-body simulations.

In Figure 4, we show the results from this model (solid lines) and the ones from F10 without the LSS (red lines). For the model with LSS effects, we also show the contributions of peaks inside \( n_{\text{peak}}^p \) (dashed-dotted line) and outside \( n_{\text{peak}}^a \) (dashed) halo regions. It is seen that in the considered cases, peaks with \( \nu_0 \gtrsim 4 \) are dominantly from halo regions. For higher \( z_{\text{med}} \), such domination shifts a little more toward higher \( \nu_0 \).

We note that in our model calculation, we directly obtain peak counts at different \( \nu = K/\sigma_0 \). On the other hand, observationally, we can only estimate the shape noise part \( \sigma_{N,0} \) by randomly rotating the observed galaxies. Thus to be consistent with observational analyses, we first make a binning in terms of \( \nu_0 = K/\sigma_0 \) and then convert it to the binning in \( \nu \) using the ratio of \( \sigma_{N,0}/\sigma_0 \) for model calculations. The results shown are the peak counts versus \( \nu_0 \). The corresponding \( K \) values are also listed in the upper horizontal axis. It is seen clearly that with the increase in the median redshift of source galaxies, the LSS projection effects become increasingly important.

In the next section, we will compare our theoretical results with those from ray-tracing simulations to validate the model performance.

### 3. Simulation Tests

In this section, we test our model performance using ray-tracing simulations. We describe the simulations and the mock WL data generation with respect to different source galaxy distributions in Section 3.1, and we present the comparison results in detail in Section 3.2.

#### 3.1. WL Simulations

We carry out ray-tracing simulations up to \( z = 3.0 \) based on large sets of \( N \)-body simulations. The simulation setting is the same as that in X. K. Liu et al. (2015), but with the number of simulations doubled. The fiducial cosmology is the flat ΛCDM model with the parameters of \( \Omega_m \), dark energy density \( \Omega_\Lambda \), baryonic matter density \( \Omega_b \), Hubble constant \( h \), the power index
of initial matter density perturbation power spectrum \( n_s \) and \( \sigma_8 \) set to be \((\Omega_m, \Omega_{\Lambda}, h, n_s, \sigma_8) = (0.28, 0.72, 0.046, 0.7, 0.96, 0.82)\).

For each set of ray-tracing calculations, we use 12 independent \( N \)-body simulation boxes to fill up to the region of a comoving distance \( \sim 4.5 h^{-1} \) Gpc to \( z = 3.0 \), as illustrated in Figure 5. Among them, eight small boxes each with the size of \( 320 h^{-1} \) Mpc are padded between \( z = 0.0 \) and \( z = 1.0 \). In the redshift range of \( 1.0 < z < 3.0 \), we pad four boxes of size \( 600 h^{-1} \) Mpc. The number of particles of \( N \)-body simulations for both small and large boxes is \( 640^3 \), and the corresponding mass resolution is \( \sim 9.7 \times 10^9 h^{-1} M_{\odot} \) and \( \sim 6.4 \times 10^{10} h^{-1} M_{\odot} \), respectively. For each of the boxes, we start at \( z = 50 \) and generate the initial conditions using 2LPTic\(^6\) based on the initial power spectrum from CAMB\(^7\) (Lewis et al. 2000). The simulations are run by GADGET-2\(^8\) (Springel 2005) with the force softening length being \( \sim 20 h^{-1} \) kpc.

In the multiplane ray-tracing calculations, up to \( z = 3 \), we use 59 lens planes with the corresponding redshifts \( z_i \) being listed in Table 1. We run the ray-tracing WL simulations using the same code described in X. K. Liu et al. (2014) in which we deal with the crossing-boundary problem of halos following the procedures used in Hilbert et al. (2009). We then generate \( 4 \times (3.5^2 \times 3.5^2) \) convergence and shear maps at each lens plane, denoted as \( \kappa(z_{\text{lens}}) \) and \( \gamma(z_{\text{lens}}) \), respectively, from a set of 12 \( N \)-body simulations. Each of the \( 3.5^2 \times 3.5^2 \) maps is pixelized into \( 1024 \times 1024 \) grids with a pixel size of \( \sim 0.205 \) arcmin. We perform in total 24 sets of \( N \)-body simulations, which give rise to \( 24 \times 4 \times (3.5^2 \times 3.5^2) = 1176 \) deg\(^2\) of \( \kappa(z_{\text{lens}}) \) and \( \gamma(z_{\text{lens}}) \). From them, we construct the final \( \kappa \) (or \( \gamma \)) maps corresponding to different source galaxy distributions as follows:

\[
\kappa_{\text{mock}}(\theta) = \sum_{\text{lens}} p(z_{\text{lens}}) \kappa(\theta; z_{\text{lens}})(z_{\text{lens}}+1 - z_{\text{lens}}),
\]

(25)

where \( p(z) \) is the normalized source galaxy redshift distribution given in Equation (24). For a given \( p(z) \), we obtain \( 24 \times 4 = 96 \) maps, each with the size of \( 3.5^2 \times 3.5^2 \).

Because we aim to test our WL high-peak model, we concentrate on convergence maps directly here. We include the shape noise by adding a Gaussian noise field to the pixels of each of \( 24 \times 4 = 96 \) convergence maps \( \kappa_{\text{mock}}(\theta) \) with the variance given by

\[
\sigma^2_{\text{pix}} = \frac{\sigma_e^2}{2n_g \theta_{\text{pix}}^2},
\]

(26)

where we take \( \sigma_e = 0.4 \) and the pixel size of maps \( \theta_{\text{pix}} = 0.205 \) arcmin. We then apply a Gaussian smoothing given by Equation (8) with \( \theta_G = 2.0 \) arcmin to obtain the final smoothed noisy convergence maps for peak analyses.

\[^6\] http://cosmo.nyu.edu/roman/2LPT/
\[^7\] http://camb.info/
\[^8\] http://wwwmpa.mpa-garching.mpg.de/gadget/
3.2. Model Test

To analyze the LSS effects and test our model performance, we consider different survey parameters, including the median redshift $z_{\text{med}}$, the number density $n_g$ of the source galaxies, and the survey area $S$. These are listed in Table 2.

From our simulations, for each set of $(z_{\text{med}}, n_g)$, we generate 96 noiseless convergence maps each with the size of $3.5^\circ \times 3.5^\circ$. For each map, we then add a Gaussian-shape noise field and apply smoothing as described above. To suppress the fluctuations caused by a particular realization of the noise field, we perform noise-adding 20 times for each map with different seeds. Therefore, we have $20 \times 96$ maps with the shape noise included for each set of $(z_{\text{med}}, n_g)$.

We first compare peak counts between model predictions and the simulation results. We identify a peak in a pixelized convergence map from simulations if its convergence value is higher than those of its eight neighboring pixels. Because Fourier transformations are involved in ray-tracing calculations and in the smoothing operations, there can be boundary effects in each of the $3.5^\circ \times 3.5^\circ$ convergence maps. To avoid such a problem, in our peak analyses, we exclude the outermost 20 pixels along each side of the map. The leftover area is $\sim 11.31 \text{deg}^2$ for each map, and the total is $\sim 96 \times 11.31 \approx 1086 \text{deg}^2$ for each set of noise field realizations.

For our theoretical model calculations, the quantity $M_*$ corresponds to the lower mass limit of halos above which the halos dominate the WL high-peak signals. We have performed $\chi^2$ tests with respect to the simulated peak counts to find suitable $M_*$. We note that for $z_{\text{med}} = 0.7$ and $n_g = 10 \text{arcmin}^{-2}$, the shape noise is much larger than that of the LSS effects, and the model of F10 works equally well. In that case, using F10, $M_* = 10^{13.9} h^{-1} M_\odot$ gives results that are in good agreement with those from simulations. In our current model with the LSS effects, for all of the cases including the one with $z_{\text{med}} = 0.7$ and $n_g = 10 \text{arcmin}^{-2}$, $M_* = 10^{14.0} h^{-1} M_\odot$ is a proper choice. We comment that, physically, the suitable choice of $M_*$ depends on the halo mass function used in the model calculations. The specific value of $M_*$ may also have a mild cosmology dependence, which may need to be

![Figure 7](image-url)
The red contours are the constraining results of 1σ, 2σ, and 3σ using the F10 model, and the blue ones are from the model in this work. The input cosmological parameters for the simulation are indicated by black crossing lines.

In this paper, we do not include this subtle effect.

The peak count comparison results are shown in Figures 6 and 7 corresponding to the survey conditions listed in Table 2. The green symbols are the results averaged over the corresponding $20 \times 96$ maps and then scaled to the considered survey area. The error bars are the corresponding Poisson errors. The blue lines are the results from our model including the LSS effect, and the red lines are from F10 without the LSS effect. Again, the results shown are the peak counts versus $\nu_0$ defined by the shape noise $\sigma_{N,0}$. The corresponding $\nu$ values are indicated in the upper horizontal axes. In the bottom part of each panel, we show the fractional differences of the two models with respect to the simulation results.

Figure 6 shows the results for S10small with $z_{\text{med}} = 0.7$, $n_g = 10 \text{arcmin}^{-2}$, and $S = 150 \text{deg}^2$, similar to the current accomplished WL surveys. In this case, $\sigma_{\text{LSS,0}} = 0.0057$ and $\sigma_{N,0} = 0.0178$ for $\theta_G = 2.0 \text{arcmin}$. Thus the contribution from LSS is much smaller than that from the shape noise, and its effect on WL peak counts is rather weak considering the relatively large error bars. This is evident in the lower part of the panel. Both the blue and red lines agree with the simulation results very well for high peaks with $\nu_0 \gtrsim 4$ with the fractional differences less than 10%.

In the upper left panel of Figure 7, we show the results of S10. In this case, the number density and the redshift distribution of source galaxies are the same as those in Figure 6, but with a larger survey area with $S = 1086 \text{deg}^2$. Thus the statistical errors of WL peak counts are smaller by $\sim 2.7$ times than that of S10small. We see again that both models work well, and the model including the LSS effect (blue) gives better results for the left two bins.

In the upper right panel of Figure 7, the results of S20 with $z_{\text{med}} = 0.7$, $n_g = 20 \text{arcmin}^{-2}$, and $S = 1086 \text{deg}^2$ are presented. In this case, the LSS effect is the same as that of S10, but the shape noise is lower with $\sigma_{N,0} = 0.0126$. Thus the relative contribution of the LSS effect should be stronger than for S10. We see that for $\nu_0 > 5$, although both the blue and red line agree with the simulation results within 10%, the red line is systematically lower, showing that the LSS effect starts to be important. For $\nu_0 < 5$, the red line deviates significantly from the simulation results, but our current model including the LSS effect can give excellent predictions out to $\nu_0 \sim 4$.

The results for M20 with $z_{\text{med}} = 1.0$, $n_g = 20 \text{arcmin}^{-2}$, and $S = 1086 \text{deg}^2$ are shown in the lower left panel of Figure 7. Here the LSS contribution increases to $\sigma_{\text{LSS,0}} = 0.0082$. Comparing to the upper right panel, we see that the model prediction without the LSS effect (red line) significantly underestimates the peak counts over the whole considered range. Taking into account the LSS effect, our improved model works very well to $K \gtrsim 0.06$, corresponding to $\nu_0 \gtrsim 4.5$.

With even higher $z_{\text{med}}$, the LSS projection effect gets larger. The lower right panel of Figure 7 shows the results of D20 with $z_{\text{med}} = 1.4$, $n_g = 20 \text{arcmin}^{-2}$, and $S = 1086 \text{deg}^2$. In this case, $\sigma_{\text{LSS,0}} = 0.0108$ is comparable to that from the shape noise with $\sigma_{N,0} = 0.0126$. Without including the LSS projection effect, the underestimation is at the level of 30%, much larger than the statistical errors. While including the LSS effect, the model predictions (blue lines) are in excellent agreement with the simulation results in $K \gtrsim 0.063$, or $\nu_0 \gtrsim 5$.

Note that for the three cases with $n_g = 20 \text{arcmin}^{-2}$, the WL lensing signal from a halo increases with the increase of $z_{\text{med}}$. Thus we see a somewhat increased lower limit of $\nu_0$, above which our high-peak halo model applies from $z_{\text{med}} = 0.7$ to $z_{\text{med}} = 1.4$.

3.3. Cosmological Constraints

To demonstrate explicitly the LSS effect on the cosmological constraints derived from WL high-peak counts and how our improved model performs, here we run Markov chain Monte Carlo (MCMC) fitting using WL mock data.

For S10, S20, M20, and D20 in Table 2, we generate the WL peak count mock data by averaging over the $20 \times 96$ maps. For S10small, we scale the peak counts obtained for S10 to $S = 150 \text{deg}^2$. The central data points in different bins $\{N_i\}$ for different cases are the same as those shown in Figures 6 and 7. Note that for the upper end of the peaks, we only use bins with $N_i \gtrsim 10$.

We employ the $\chi^2$ fitting to constrain cosmological parameters from WL mock data. The $\chi^2$ is defined as

$$\chi^2 = \Delta^T \Sigma^{-1} \Delta,$$  \hspace{1cm} (27)

where $\Delta$ is $\Delta = \hat{N} - \bar{N}$, with $\hat{N}$ being the mock data vector consisting of WL peak counts of different bins and $\bar{N}$ being the model predictions for these bins. The quantity $\Sigma$ is the covariance matrix for peak counts between different bins. We calculate it using the simulation at the fiducial cosmology. Specifically, for each case in Table 2, we first obtain the covariance for an area $S_0$ corresponding to an individual simulated convergence map by calculating the variance of peaks $[C^0]_{ij}$ between the $i$th and $j$th bin from $20 \times 96$ maps. We then scale $C^0$ to the mock survey area $S$ considered in different cases by

$$C = \frac{S}{S_0} C^0.$$  \hspace{1cm} (28)

Studies have shown that this scaling can lead to a slight underestimate of the covariance for large $S$ (Kratochvil
et al. 2010). This, however, should not affect our conclusions regarding the bias resulting from neglecting the LSS effect and the validity of our new model. It is also noted that the cosmology dependence of the covariance is not considered here.

From $C$, we can calculate its inverse $C^{-1}$ and further the $\widetilde{C}^{-1}$:

$$
\widetilde{C}^{-1} = \frac{R - N_{\text{bin}} - 2}{R - 1} C^{-1},
$$

(29)

Table 3

| Mocks    | S10small\(^a\) | S10small | S10 | S20 | M20 | D20 |
|----------|----------------|----------|-----|-----|-----|-----|
| $\alpha$ | 0.434          | 0.452    | 0.456 | 0.493 | 0.465 | 0.417 |
| $\Sigma_8$ | 0.833 $\pm$ 0.045 | 0.791 $\pm$ 0.050 | 0.814 $\pm$ 0.016 | 0.836 $\pm$ 0.011 | 0.838 $\pm$ 0.009 | 0.834 $\pm$ 0.008 |

Note. The degeneracy is defined by $\Sigma_8 = \sigma_8 (\Omega_0 / 0.27)^{\alpha}$.

\(^a\) Derived from the F10 model.
where $R = 20 \times 96$ and $N_{\text{bin}}$ is the number of bins of WL peak counts used in deriving cosmological constraints.

In our analyses, we concentrate on the constraints on $\Omega_m$ and $\sigma_8$, and we set all of the other cosmological parameters fixed to be the input values of the simulations. We implement the MCMC technique to explore the posterior probabilities of ($\Omega_m$, $\sigma_8$) (X. K. Liu et al. 2015, 2016).

The constraining results for S10small are shown in Figure 8, where the blue and red contours are the results using the model presented in this paper including the LSS effect and the model of F10 without the LSS effect, respectively. The black cross indicates the input values of the two parameters for WL simulations. Consistent with that shown in Figure 6, the two constraints overlap significantly and the two models perform equally well. In this case, the LSS effect is negligible, and the application of the F10 model is well justified without introducing notable biases in the parameter constraints.

The results for S10, S20, M20, and D20 are presented in Figure 9. Because the survey area is larger than that of S10small, the statistical errors are reduced considerably, resulting in smaller contours. For S10 (upper left), the blue and red contours still have a large overlap. The WL simulation input values are at the edge of the 1σ red region. The blue constraints from our improved model including the LSS effect, on the other hand, give better results.

For S20 (upper right), $\sigma_{8,0} = 0.0126$, $\sigma_{\text{LSS},0} = 0.0057$, and the total $\sigma_0 = 0.0138$. The fractional contribution from LSS is $\sigma_{\text{LSS},0}/\sigma_0 \approx 0.41$. Thus the LSS effect is already apparent. The constraints obtained by using the model of F10 are biased by more than 2σ. For M20 (lower left) and D20 (lower right), the shape noise is the same as that of S20. But the LSS effect is stronger with $\sigma_{\text{LSS},0} = 0.0082$ and 0.0109, and the corresponding fractional contribution to $\sigma_0$ is $-0.55$ and $-0.65$ for M20 and D20, respectively. Without the LSS effect, the derived constraints are severely biased by more than 3σ for M20 and even larger for D20. On the other hand, in all of the cases, our new model incorporating the LSS effect works excellently with the input values being aligned with the degeneracy direction and well within the 1σ region, as shown in blue.

It is known that WL effects depend on $\Omega_m$ and $\sigma_8$ in a degenerate way, and the derived constraints of the two parameters are highly correlated, as seen from Figures 8 and 9. Such a correlation is often described by a relation $\Sigma_k = \sigma_8(\Omega_m/0.27)\alpha$. In Table 3, we list the values of $\alpha$ and $\Sigma_k$ for different cases. These values are derived from the principal component analysis of the MCMC samples (for details, please see Section 4.1 of Tereno et al. 2005 or the PCA method in getdist\footnote{http://cosmologist.info/cosmomc}). Because the F10 model works well for S10small, for this case we also list the values obtained from the constraints using F10. For the other cases, we only show the results derived from the blue regions in Figure 9. We see that for S10small, we have $\alpha \approx 0.434$ from F10 and $\alpha \approx 0.452$ from our improved model. The two results are very similar and consistent with the one we obtained from WL peak analyses using CS82 (X. K. Liu et al. 2015). For S20, M20, and D20, the $\alpha$ value decreases somewhat with the increase of $z_{\text{med}}$. We show their 1σ contours together with the derived degeneracy directions in Figure 10. This indicates the potential of tomographic WL peak analyses, which we will explore in detail in our future studies. We also note that the $\alpha$ values derived from WL high-peak abundances are systematically smaller than those from cosmic shear correlations (Kitzbinger 2015), showing the complementary nature of the two types of statistical analyses.

### 3.4. Further Tests

The previous analyses show the results with a set of fiducial parameters. In this subsection, we test the validity of the model for different cases. In Figure 11, we show the results with Gaussian smoothing of different smoothing scales with $\theta_e = 1.5$ arcmin (upper) and 3 arcmin (lower) for S20 on the left and M20 on the right. We see the model performs equally well as that of $\theta_G = 2$ arcmin.

In our model, we consider massive halos with $M \geq M_*$ as the dominant sources of high peaks. Simulations show that $M_* \sim 10^{14} h^{-1} M_\odot$ is an appropriate choice. The very precise value can have dependencies on, for example, the halo mass function and cosmological models. In our fiducial analyses, we take $M_* = 10^{14} h^{-1} M_\odot$. To test the $M_*$ sensitivity of our model predictions, in Figure 12, we show the differences of the model predictions with $M_* = 10^{13.9} h^{-1} M_\odot$ and $10^{14.1} h^{-1} M_\odot$ with respect to that of the fiducial results. The data points are the differences between the simulation results and the fiducial model predictions with $M_* = 10^{14} h^{-1} M_\odot$. The large and small error bars are for the survey area of $\sim 150$ deg$^2$ and $\sim 1080$ deg$^2$, respectively. It is seen, as expected, that a different choice of $M_*$ has no effect on the predicted abundance of very high peaks. For peaks around $\nu_0 \approx 4$ in the considered cases, they show some effects. For surveys of $\sim 150$ deg$^2$ and $z_{\text{med}} = 0.7$, the differences arising from a 0.1 dex variation of $M_*$ are within the statistical errors for $\nu_0 \geq 4$. For surveys of $\sim 1000$ deg$^2$, or higher $z_{\text{med}}$, the

\begin{figure}[h]
  \centering
  \includegraphics[width=0.5\textwidth]{figure10}
  \caption{Degeneracy of ($\Omega_m$, $\sigma_8$). The contours are the 1σ regions from the peak abundance model including LSS projection for S20, M20, and D20. The dashed lines are the corresponding degeneracy curves defined by $\Sigma_k = \sigma_8(\Omega_m/0.27)\alpha$. The input cosmological parameters for the mock data are shown as the black cross.}
\end{figure}
dependence on $M_\text{ap}$ becomes significant. We will investigate these issues in more detail in our future studies.

In Figure 12, we extend the horizontal axis to $\nu_0 = 3$. We see that at $\nu_0 \sim 3$, there are some deviations between the model predictions and the simulation results: the higher the $z_{\text{med}}$, the larger the deviations. From Figure 4, we see that in the considered cases, for peaks of $\nu_0 \sim 3$, a significant fraction of them are from the field regions resulting from the combined effects of LSS and shape noise. Thus they are more sensitive to the LSS properties than high peaks that are mainly from halo regions. The Gaussian approximation of the LSS effects needs to be improved to better account for these relatively low peaks, particularly for higher $z_{\text{med}}$, where the LSS effects are comparable to or even larger than the shape noise effects. This is another important effort in our future studies.

It is noted that our analyses are done using the convergence fields from simulations directly. On the other hand, observations measure the shape ellipticities of galaxies, which directly give rise to an estimate of the reduced shear $g_t = \gamma_t/(1-\kappa)$. To perform peak analyses in the convergence fields, in general, we need to first reconstruct them from the shear estimates using the relation between $\gamma$ and $\kappa$. To avoid the reconstructions that may introduce systematic errors, the aperture mass $M_{\text{ap}}$ statistics has been proposed with (e.g., Schneider 1996; van Waerbeke 1998; Jarvis et al. 2004)

$$M_{\text{ap}}(\theta) = \int d^2\theta' Q(|\theta - \theta'|) g_t(\theta'),$$

where $g_t$ is the tangential component of $g$ with respect to $\theta - \theta'$. In the regime of $\kappa \ll 1$ and $g \approx \gamma$, $M_{\text{ap}}$ is equal to applying a $U$ filter to the $\kappa$ field with

$$Q(\theta) = -U(\theta) + \frac{2}{\theta^2} \int \theta' d\theta' U(\theta').$$

It is required that the $U$ filter is compensated with $\int d^2\theta U(\theta) = 0$. Here we present the peak analysis results for $M_{\text{ap}}$ obtained by applying a $U$ filter to the simulated convergence fields to show the applicability of our model. We choose a particular filter set with (van Waerbeke 1998;
This filter has smoothed behaviors both in real and Fourier spaces and can be handled better computationally than sharply truncated filters (van Waerbeke 1998).

\[
U(\theta, \theta_U) = \frac{1}{\pi \theta_U^2} \left( 1 - \frac{\theta^2}{\theta_U^2} \right) \exp \left( -\frac{\theta^2}{\theta_U^2} \right). \tag{32}
\]

This filter has smoothed behaviors both in real and Fourier spaces and can be handled better computationally than sharply truncated filters (van Waerbeke 1998).
distribution of $M_{\text{ap}}$ under the $U$ filter from simulations and also from our model prediction. The results demonstrate that our model works well too in this case, starting from $v_0 \approx 2$.

We should note that the equivalence of obtaining $M_{\text{ap}}$ from $U$ filtering of the convergence fields with its original definition is approximate under the assumption of $g \sim \gamma$. For high peaks, however, such an approximation is not accurate enough. Thus to model high aperture-mass peaks better corresponding to real observational analyses, we need to work on the true $M_{\text{ap}}$ fields derived directly by applying a $Q$ filter to the reduced shear field $g_r$, which is much more computationally complicated and intensive. Great effort has been devoted to building such a model, and we will present it in our forthcoming paper by C. Z. Pan et al. (2018, in preparation).

4. Discussion

In this paper, we analyze the projection effect from stochastic LSS on WL high-peak abundances. Similar to F10, we assume that high peaks are dominantly from individually massive halos with $M \geq M_\text{c}$. To improve F10, we include the LSS effect as a Gaussian random field, and its power spectrum $C_{\text{LSS}}^\ell$ is calculated by subtracting the one-halo contribution from halos with $M \geq M_\text{c}$ from the overall nonlinear power spectrum. In other words, in our modeling, we treat the heavily non-Gaussian contributions from massive halos to WL peaks separately using their halo mass function and the density profiles. The rest of the line-of-sight projection effect is regarded as the LSS effect modeled as a Gaussian random field. We comment that, in line with the halo model, $C_{\text{LSS}}^\ell$ contains contributions from one-halo terms of smaller halos with $M < M_\text{c}$ and the two-halo terms between all of the halos. It is also noted the exclusion of the one-halo terms from halos with $M \geq M_\text{c}$ is important in calculating $C_{\text{LSS}}^\ell$ correctly. Otherwise, the LSS projection effect would be overestimated.

To examine our model performance, we carry out extensive simulation studies by generating WL maps with respect to different survey conditions. Our analyses show that for CFHTLenS-like surveys (S10small), the LSS effect on WL high-peak counts and subsequently the derived cosmological constraints is negligible. This is due to its relatively small contribution to $\sigma_\text{0}$ in comparison to that of the shape noise and the large statistical errors resulting from a small sky coverage. With the same $n_g = 10 \text{ arcmin}^{-2}$ and $z_{\text{med}} = 0.7$ but increasing the survey area to $\sim 1086 \text{ deg}^2$ (S10), the F10 model gives rise to constraints that are at the edge of the 1σ contour. Keeping the same survey depth with $z_{\text{med}} = 0.7$ but increasing $n_g$ to $20 \text{ arcmin}^{-2}$, the LSS projection effect becomes notable. Further increasing the survey depth represented by increasing $z_{\text{med}}$, the LSS projection effect becomes more and more important. With the F10 model without the LSS projection effect, the cosmological constraints derived from WL high-peak counts are biased by more than 2σ and 3σ for S20 and M20, and even larger for D20. This shows clearly that for future large WL surveys, the LSS projection effect on WL high peaks must be taken into account. The model we present in this paper performs very well in capturing the effect. We address that in our improved model, where $C_{\text{LSS}}^\ell$ contributes additional cosmological information.

We also show the good performance of the model for a different smoothing scale $\theta_\text{0}$ and for the aperture-mass peaks with a compensated $U$ filter. For the latter, we should keep in mind that the true aperture-mass fields are calculated from the reduced shears $g$ rather than from the shears $\gamma$.

We note that the mass function and the density profile of halos are important ingredients in our model calculations. Their uncertainties can potentially affect the model predictions. In the analyses here, we take the halo mass function from Watson et al. (2013) and the NFW halo density profile with the mass–concentration relation of Equation (23). They work well in our comparisons with simulated high-peak counts. For very high precision studies in the future, we need to consider these uncertainties more carefully. Considering the complicated mass distributions in real halos, there should be a negative bias ($\approx 10\%$) in the 2D weak-lensing-derived $M$–$c$ relation with respect to that of 3D (Du et al. 2015). As a test, we reduce the $A$ value in the mass–concentration relation by 10%, and the theoretical predictions for high-peak counts decrease at the level of $\sim 10\%$ for the highest bins in Figure 7, and smaller for lower bins. This is still within the statistical uncertainties of the peak counts in our considered cases with $\sim 1100 \text{ deg}^2$. With $S$ being $\sim 15,000 \text{ deg}^2$ for future surveys, such as LSST (LSST Dark Energy Science Collaboration 2012) and Euclid (Amendola et al. 2013), highly accurate knowledge about these ingredients is needed for precision studies. On the other hand, self-calibrated approaches can possibly constrain, for example, the mass–concentration...
relation simultaneously with cosmological parameters from WL peak counts (X. K. Liu et al. 2015). We will investigate these issues in detail in our future studies.

It is also noted that our model applies to high peaks for which the signals are mainly from single massive halos. On the other hand, simulations have shown that low peaks also contain important cosmological information. It is highly desirable to build theoretical models for them. For low peaks, such as that shown in Figure 2, however, we cannot find a single halo that contributes dominantly to the peak signal. Thus, as one of our important future tasks, we need to explore different approaches to modeling the low and medium peaks.

WL peak analysis has shown its power in cosmological studies. Ongoing and future WL surveys will increase the quantity of data by orders of magnitude compared to what we have now. This will lead to a tremendous increase of the statistical power of WL studies. Meanwhile, however, much tighter systematic error controls are needed. Besides the LSS projection effect on WL peaks studied in this paper, there are other systematics that we need to take care to understand, such as the intrinsic alignments of source galaxies, photometric redshift errors, and baryonic effects. Fully exploring the complementarity of WL peak analyses and cosmic shear correlations, not only on cosmological constraints but also on different responses to systematics, is also an important and exciting direction to work on.

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