Game-Theoretic Models for Usage-based Maintenance Contract

H Husniah¹, R Wangsaputra², A Cakravastia² and B P Iskandar²

¹ Industrial Engineering, Langlangbuana University
Karatipan 116, Bandung 40261, Indonesia
² Department of Industrial Engineering, Bandung Institute of Technology
Ganesha 10, Bandung, 40132, Indonesia

Email: hennie.husniah@gmail.com, bermawi@mail.ti.itb.ac.id

Abstract. A usage-based maintenance contracts with coordination and non coordination between two parties is studied in this paper. The contract is applied to a dump truck operated in a mining industry. The situation under study is that an agent offers service contract to the owner of the truck after warranty ends. This contract has only a time limit but no usage limit. If the total usage per period exceeds the maximum usage allowed in the contract, then the owner will be charged an additional cost. In general, the agent (Original Equipment Manufacturer/OEM) provides a full coverage of maintenance, which includes PM and CM under the lease contract. The decision problem for the owner is to select the best option offered that fits to its requirement, and the decision problem for the agent is to find the optimal maintenance efforts for a given price of the service option offered. We first find the optimal decisions using coordination scheme and then with non coordination scheme for both parties.

1. Introduction

In an open pit mining, heavy equipments are used for loading and hauling mining material and these equipments play an important role to support business processes in a mining company. As the equipments deteriorate with age and usage, an effective maintenance is needed to keep the equipments in a good condition which in turn gives a high availability of the equipments. In general, maintenance is not considered as the core competencies of a mining company, and hence it is outsourced to an external agent. Furthermore, the technology of such equipments tends to be complex and having maintenance facilities and maintenance specialists for conducting maintenance activities of such complex equipments in house is far from economical. In many mining companies, maintenance activities (including preventive maintenance (PM) and corrective maintenance (CM)) are outsourced to an OEM (original equipment manufacturer) or an external agent. OEM or an agent can do a partial or full coverage of the maintenance actions (Preventive Maintenance (PM) or/and Corrective maintenance (CM)). Preventive maintenance (PM) actions are done to prevent the excessive degradation (whether it is an age based or condition based maintenance), while corrective maintenance (CM) is performed to restore the failed equipment to the operational state.

From the mining company (as the owner of the equipment) perspective, the decision problem facing is to choose the best option that results in high availability of the equipments. As a result, it needs to control closely the equipment availability as it influences very much the production and
revenue of the company. On the other hand, the OEM’s (agent’s) decision problem is to determine the price of each option offered that maximises its profit.

In this paper, we propose a new maintenance service contract (MSC) characterized by $L$ representing a time period of contract and $U_{\text{max}}$, representing the maximum usage allowed (See Fig. 1). Here, the contract will not terminate when the usage at time $t$ ($< L$) is greater than $U_{\text{max}}$, but the owner will be charged some additional cost. The contract will terminate only due to the time limit. This contract can be viewed as the extension of the two dimensional service contract by [1] and [2] where the usage limit is no longer acting as the limit of contract but it is only a maximum usage allowed.

![Figure 1. Key elements of maintenance service contract considered](image)

In addition, we study two MSC models based on cooperative and non-cooperative games, where the cooperative case is similar to that studied by [3] but we consider a different maintenance policy (i.e. imperfect PM policy) which fits with the case of dump trucks used in a mining site.

For the cooperative game, it requires two major criteria. First, cooperation should lead to a win-win situation for the owner and OEM, such that the profits of both players become higher compared to independent individual profit [3]. Second, the players should have no incentive to deviate from the non-cooperative solution, i.e., they should modify their profits such that the total maximum solution becomes identical to a Nash equilibrium (for the simultaneous move case) [4]. We have allocated the maximum total profit between the players to satisfy the first criterion. In this game, decision makers negotiate over different contract terms.

Next, for the non-cooperative game, we considered: non-cooperative simultaneous move game where the owner and OEM choose their strategies simultaneously. We derive the Nash equilibrium. It is of interest to compare the performance of the non-cooperative contracts and the cooperative alternative. One measure of performance is the difference between the total profit of a non-cooperative contract and that of a cooperative one (which has the maximum total profit).

The remainder of this paper is organised as follows. In section 2 we give model formulation for the service contract studied. The cooperative and the non-cooperative methodologies are described in Section 3. Section 4 analyzes the game solutions, the Nash equilibrium is determined and compares it results with the cooperative solution. Section 5 presents numerical examples to illustrate the MSC models and to investigate their performance. Finally, Section 6 concludes the paper.

2. Model Formulation
The following notations will be used in model formulation.

- $W, U$: Warranty time and usage limits
- $X_i$: Downtime caused by the $i$-th failure and waiting time
- $\zeta$: Total repair time allowed
We consider that the performance of the equipment degrades with age and usage and often the usage contributes more impact to the deterioration of the equipment, then the owner will be charged an additional cost when the usage rate goes beyond the nominal value. For a given usage rate (y) of a dump truck, the warranty ceases at $W_y=W$ for $y \leq U/W$, or $W_y = U/y$, for $y > U/W$ (See Fig.2).

2.1. Warranty Policy
In this paper we assume that each dump truck purchased is covered by a two-dimensional warranty. We also assume that to provide more protection to the buyer, the warranty covers PM. Further, every CM is covered by the warranty and rectified at no cost to the buyer. This means, after the warranty ends, the responsibility of the CM and PM actions shifts to the buyer (the owner). To express the warranty coverage we use a rectangle region $\Omega_y = [0,W] \times [0,U]$ where $W$ and $U$ are the time, and the usage limits. For a given usage rate (y) of a dump truck, the warranty ceases at $W_y = W$ for $y \leq U/W$, or $W_y = U/y$, for $y > U/W$ (See Fig.2).

2.2. Maintenance Service Contract (MSC)
The OEM proposes a two-dimensional MSC for a period of $L$ (e.g. $L=2$ year) with a fixed price $P_G$. Here, the MSC has no usage limit or in other words, the contract coverage forms a region $\Omega_S = \left[ W_y, L \right] \times \left[ yW_y, \infty \right]$ (See Fig. 2). The contract starts at the end of warranty, $W_y$. However, since the performance of the equipment degrades with age and usage and often the usage contributes more impact to the deterioration of the equipment, then the owner will be charged an additional cost when the usage rate goes beyond the nominal value. This is considered as a compensation to the OEM due to the increase in repair cost.

In general, the OEM provides a full coverage of maintenance, which includes PM and CM, for each MSC dump truck. In other words, all failures under MSC contract are fixed at no cost to the owner and all PM actions are performed without any charge to the owner. It is also stated that the OEM as a service contract provider assures a minimum down time (repair time and waiting time) for each failure and penalty cost incurred when the down time exceeds the predetermined target.

The MSC is defined as follows.

For a price of $P_G$, the OEM proposes a two-dimensional MSC for a period of $L$. When the total usage $(U_y)$ at $L$ is greater than $U_{max}$, an additional cost is charged to the owner. The amount of the additional cost is proportional to $\Delta = U_y - U_{max}$ given by $C_{ac}(U_y, -U_{max})$. The additional cost is viewed as a compensation for the OEM as the total usage $> U_{max}$. And a penalty cost is born to the OEM when the down time exceeds the down time target. Under this option, the OEM agrees to carry out PM and CM in $(W_y, W_y + L)$ or the maintenance service is full coverage.

We study the MSC proposed from the view points of the OEM and the owner. The objective of the OEM is to minimize the expected maintenance cost according to various usage pattern and the mining operational condition whilst the objective of owner is to maximize the expected profit.

2.2.1. Equipment Failures and Repairs. Let $Y$ be the constant usage rate for a given truck and varies across the trucks. For $Y = y$, the conditional hazard function for the time to first failure is given by $r(t)$ which is a non-decreasing function of t (the age of the truck) and y. Here we consider that the degradation of the truck is strongly affected by usage rate of the truck and a land contour of a mining EV.
area where the truck is operated. An AFT (accelerated failure time) model is proposed to model the effect of usage rate and the operating condition on degradation of the truck.

![Figure 2. Warranty region $\Omega_w$ and service contract region $\Omega_s$ for (a) $y \leq \gamma$ and (b) $y > \gamma$](image)

In AFT model the distribution function for $T_y$ is given by $F(t, \alpha_y)$, with a scale parameter given by $\alpha_y = \left(\frac{y_0}{y}\right)\gamma \alpha_0$ with $\gamma \geq 1$ where $\gamma$ is a parameter for the operating condition of a truck. The hazard and the cumulative hazard functions associated with $F(t, \alpha_y)$ are given by $r(t) = f(t, \alpha_y)/(1 - F(t, \alpha_y))$ and $R(t) = \int_0^t r(s)ds$ respectively where $f(t, \alpha_y)$ is the associated density function.

2.2.2. Preventive Maintenance Policy. The PM policy is defined as follows. For a truck with usage rate $y$, the PM policy is characterised by single parameter $r_y$ during $\Omega_w$ [$\Omega_s$]. Conditional on $Y = y$, the equipment is periodically maintained at $k\gamma$, $k = 1, 2, \ldots$. This involves $k$ disjoint intervals $[0, \gamma], \ldots, [\ell\gamma, \ell\gamma + \gamma)$ in which all failures within PM period are minimally repaired, where $\ell$ is an integer value. Note $(\ell + 1)\gamma = W$ where $k[\ell]$ is an integer value. The effect of PM actions are modelled through the reduction in the intensity function after PM at $t_j$, $j \geq 1$ is $\delta$. Since any failure occurring between PM is minimally repaired and $\delta^* = \delta$, then the expected total number of minimal repairs in $(t_{j-1}, t_j), 1 \leq j \leq k + 1$ is given by $N = \sum_{j=1}^{k+1} \int_{t_{j-1}}^{t_j} r_j(t)dt$.

2.2.3. Modelling Cost. The OEM’s expected total cost consists of PM cost, repair cost, and penalty cost (incurred when the down time exceeds the predetermined target). If $J^I(k, \delta_j), J^M(k, \delta_j)$ and $J^F(k, \delta_j)$ are the expected total PM cost, the expected total repair costs and the expected penalty cost over the MSC period $(W_s, W_s + L)$ for a given usage rate $y$, respectively, then the expected total cost for the OEM, $\Pi^*[k, \delta_j]$ is given by

$$\Pi^*[k, \delta_j] = J^I(k, \delta_j) + J^M(k, \delta_j) + J^F(k, \delta_j)$$

Let $C_{pm}(\delta_j)$ and $C_j$ be the cost of the $j$-th PM and the cost of each minimal repair. If $C_{pm}(\delta_j) = C_0 + C_j\delta_j$ as in [5] and [6] then the expected total PM cost over the MSC period $(W_s, W_s + L)$ is given by
\[ J^1(k, \delta_j) = \sum_{j=1}^{k} C_{pm}(\delta_j) = kC_0 + C_j \delta_j \]  

And the expected total minimal repair cost is given by
\[
J^2(k, \tau_r, \delta_j) = C_j \left( R_j(L) - \sum_{j=1}^{k} (L - j\tau_r) \delta_j \right) 
\]

Let \( D \) and \( \bar{D} \) be the down time (consisting repair time and waiting time) for each failure and the down time allowed, respectively. The OEM incurs a penalty cost when \( D > \bar{D} \), and the penalty cost is assumed to be proportional to the excess of down time, \( (D - \bar{D}) \). Then, the expected penalty cost is given by
\[
\hat{G}(\bar{D}) - \left\{ \int_{\bar{D}}^\infty (z - \bar{D}) dF(z) \right\} 
\]

where \( \hat{G}(\bar{D}) \) is the expected value of penalty, and \( Z_i \) (downtime caused by the \( i \)-th failure) are assumed i.i.d with distribution function \( F(z) \). After simplification we have the expected total cost of the OEM given by
\[
\Pi^*\left[ k, \tau_r, \delta_j \right] = \hat{C}R_j(L) - \left\{ \hat{C} \sum_{j=1}^{k} \left[ (L - C_j \delta_j) - j\tau_r \right] \delta_j - kC_0 \right\} 
\]

where \( \hat{C} = (C_j + C_p \hat{G}(\bar{D})) \).

2.2.4. Modelling Profit. We assume that OEM and the owner have the same attitudes to risk, with the utility function \( \phi \), where \( \phi \) is the owner’s profit function in order to make the solution reach equilibrium.

**Owner Expected Profit**

The revenues for the owner consist of the revenue generated from the operation of the equipment plus the penalty cost paid by the OEM. Hence, the expected profit is given by
\[
E[\phi_r] = K \left\{ L - E[D^*_r(L)] \right\} + J^3(k, \delta_j) - P_o - C_o^* 
\]

where \( E[D^*_r(L)] = \mu N_r \), \( \mu \) (expected downtime), \( N_r \) (expected number of failures), \( K \) is the revenue ($/hour) received by the owner as a result of transporting mining materials from a mining area to a processing unit, and \( P_o \) is the MSC price.

**OEM Expected Profit**

The revenues for the OEM consist of the revenue received as a payment of the MSC contract plus the additional charges paid by the owner. The costs consist of PM cost, repair cost, penalty cost. Hence, the expected profit is given by
\[
E[\pi_r] = P_o + \Phi(y) - \Pi^*\left[ k, \tau_r, \delta_j \right] 
\]

The optimal PM interval for the OEM is obtained by maximizing \( E[\pi_r] \) with the respect to \( \tau_r \).

3. Model Analysis

3.1 Cooperative Solution [case 1]

We first consider that the owner and OEM follow in a cooperative fashion. It is clear that the owner’s objective is to maximize the process’ expected uptime, but the OEM’s objective is to minimize the total expected maintenance cost. Consequently, an agreement may not be achievable between the two
parties. Here, owner and the OEM want to get a win-win solution in order to have PM interval that maximizes the expected profit both OEM and the owner.

As a result, the terms and condition of the service contract are negotiated and decided jointly (or coordinated) between the two parties involved. In coordination scheme, the strategy set of the OEM and the owner is \( Q = \{(\delta, \tau, k, b) \mid 0 \leq \delta \leq 1, \tau \geq 0, k, b \geq 0\text{int}\} \). Both players choose the set of strategies \( q^* \in Q \) that solves \( \max_{q} E[\Pi_j] = \max_{q} \{E[\phi_j] + E[\pi_j]\} \) where \( E[\phi_j] \) and \( E[\pi_j] \) are the total expected profit for the owner and OEM. The coordination solution is obtained by solving the following problem:

\[
\max_{\delta, \tau, k, b} E[\Pi_j] = KL - C_0 - (\mu K + C_0) R(L) - \left[kC_0 - (\mu K + C_r - C_0) \sum_{j=1}^{k} (L - j\tau) \delta^j\right] \\
\text{s.t.} \quad \delta > 0, \tau > 0, k, b > 0 \text{ and integer} 
\]

(7)

3.2. Non-Cooperative (case 2)

Next, we consider that the owner and OEM follow in a non-cooperative fashion, and the governing optimization equilibrium concept underlying non-cooperative behaviour is that of Nash. Based on Nash Bargaining solution, to find Nash equilibrium, the owner and OEM negotiate over the share and moreover, they equally split the joint profit gain, \( E[\Pi_j] \). In this scheme, the strategy set of the OEM and the owner is \( Q_{\text{nash}} = \{(\delta, \tau, k, b) \mid 0 \leq \delta \leq 1, \tau \geq 0, k, b \geq 0\text{int}\} \). Both players choose the set of strategies \( q^* \in Q_{\text{nash}} \) that solves \( \max_{q} E[\phi_j] = \max_{q} E[\pi_j] \) where \( E[\phi_j] \) and \( E[\pi_j] \) are the total expected profit for the owner and OEM. Next, we determine proper values of the set of strategies \( q^* \in Q_{\text{nash}} \) for the owner and OEM by solving the following problem:

\[
\max_{\delta, \tau, k, b} E[\Pi_{\text{nash}}] = \frac{1}{2} \left(KL - C_0 - (\mu K + C_r) R(L) - \left[kC_0 - (\mu K + C_r - C_0) \sum_{j=1}^{k} (L - j\tau) \delta^j\right]\right) \\
\text{s.t.} \quad \delta > 0, \tau > 0, k, b > 0 \text{ and integer} 
\]

(8)

\( q^* \in Q \) can be obtained by simultaneously solving the two first order conditions for the owner and OEM.

4. Numerical Example

Let us recall that for a given usage rate \( \gamma \), \( F(t; \gamma) \) is the time to the first failure which follows the Weibull distribution with \( F(t; \alpha, \gamma) = 1 - \exp(-t / \alpha, \gamma) \). Its failure rate function is \( \lambda(t) = \beta(t^{\beta-1} / \alpha, \gamma) \) where \( \alpha_\gamma \) as in (1). The other parameter values are: \( \alpha_0 = 0.4, \beta = 2.5, W = 12 \text{ (months)}, L= 12 \text{ (months)}, U=24 \text{ (1x10^4Km)}, K = 24 \text{ (1x10^4Km)} \) \( (\gamma = U/W = 1) \), \( y_0 = 1, \rho = 1.5 \) and \( C_r = 0.5C_m, \zeta = 80 \text{ (hours)} \) or 4 (days).

Table 1 shows optimal solutions for cases 1 and 2. In Case 1, after warranty ends the optimization is carried out jointly for both players. As the usage rate increases, profit obtained for the agent (OEM) increases whilst the profit for the owner decreases.


**Table 1. Results for Case 1 and Case 2**

| Warranty Region (W=12 months) | MSC Region (L=12 months) | Non-Coordination |
|-------------------------------|--------------------------|------------------|
| Coordination                  |                          |                  |
| \( \bar{y} \)                | \( k_i^*; r_i^*; E[\text{cost}] \) | \( k_i^*; r_i^*; \delta_i^* \) |
| \( y \)                      | \( k_i^*; E[\pi_i^*]; E[\phi_i^*] \) | \( E[\Pi_i^*] \) |
| \( [10^{5}] \)               | \( [10^{6}] \)           | \( E[\pi_i^*] = E[\phi_i^*] \times 10^5 \) |
| \( E[G] \)                   | \( [10^{5}] \)           |                  |
| 1.2                           | 6; 1.36; 866.07          | 5.91; 5.85       |
| 1.09                          | 2.0; 2.19; 2.20; 9.05    | 5.87; 5.77       |
| 866.07                        | 3.0; 2.30; 1.13; 1.09    | 5.77; 5.78       |
| 4.0                           | 2.38; 8.88              |                  |

This is as expected since the agent performs a more effective PM (with \( \delta^* (= 1.09) \)) and the PM in turn will decrease the number of failures and hence the total maintenance cost and downtime. Compare with the profit resulting from the Nash bargaining solution (Case 2), the profit for the agent (OEM) and for the owner always equal. This is due to the bargaining strategy that maximize both profits. Since the number of failures increase as the usage increases then it in turn decreases the profit for both players. The similar result exists for the MSC prices, it decreases as the usage rate increases.

**5. Conclusion**

In this paper we study a usage based MSC for dump trucks after the expired of a two-dimensional warranty, where the MSC is characterized by two parameters – age and usage limits which form a region. An imperfect PM (which reduces the age of the equipment) is performed during the MSC and the optimal imperfect PM is obtained by maximizing the expected total profit for the both players. The paper models the contract using the cooperative and non-cooperative game approach with one dimensional approach. One can models using a bivariate approach with multi players, and considers other shapes of a contract region. This is one topic for future research.

**Acknowledgements**

This work is funded by the Ministry of Research, Technology, and Higher Education of the Republic of Indonesia through the scheme of “PUPT 2017” with contract number SP DIPA-042.06.1.401516/2017.

**References**

[1] Husniah H, Pasaribu U S and Iskandar B P 2014 Proc. of ICMIT Performance-based maintenance contract for equipment used in mining industry Singapore.

[2] Husniah H, Pasaribu U S and Iskandar B P 2016 Proc. of IEEM Two dimensional maintenance contract with coordination between owner and agent Bali.

[3] Taracki H, Tang K, Moskowitch H and Plante R N 2006 IIE Transactions Incentive Maintenance Outsourcing Contracts for Channel Coordination dan improvement 366 71–684.

[4] Iskandar B P, Husniah H and Pasaribu U S 2014 Quality Technology and Quantitative Management Maintenance service contract for equipment sold with two dimensional warranties 11 3.

[5] Sheu S H, Chang C C, Chen Y L and Zhang Z G 2015 Rel. Eng.&Sys.Safety Optimal preventive maintenance and repair policies for multi-state systems 140 78-87.

[6] Wang Y, Liu Z L and Liu Y 2015 Rel. Eng.&Sys Safety Optimal preventive maintenance strategy for repairable items under two-dimensional warranty 142 326–333.