Image reconstruction for interrupted-beam x-ray CT on diagnostic clinical scanners

Matthew J Muckley, Baiyu Chen, Thomas Vahle, Thomas O'Donnell, Florian Knoll, Aaron D Sodickson, Daniel K Sodickson and Ricardo Otazo

1 New York University School of Medicine, New York, NY, United States of America
2 Mayo Clinic, Rochester, MN, United States of America
3 Siemens Healthcare GmbH, Erlangen, Germany
4 Siemens Healthineers, Malvern, PA, United States of America
5 Harvard Medical School, Boston, MA, United States of America
6 Memorial Sloan-Kettering Cancer Center Department of Medical Physics, New York, NY, United States of America

E-mail: Matthew.Muckley@nyulangone.org and Ricardo.Otazo@mskcc.org

Keywords: low-dose CT, interrupted-beam CT, compressed sensing

Abstract

Low-dose x-ray CT is a major research area with high clinical impact. Compressed sensing using view-based sparse sampling and sparsity-promoting regularization has shown promise in simulations, but these methods can be difficult to implement on diagnostic clinical CT scanners since the x-ray beam cannot be switched on and off rapidly enough. An alternative to view-based sparse sampling is interrupted-beam sparse sampling. SparseCT is a recently-proposed interrupted-beam scheme that achieves sparse sampling by blocking a portion of the beam using a multislit collimator (MSC). The use of an MSC necessitates a number of modifications to the standard compressed sensing reconstruction pipeline. In particular, we find that SparseCT reconstruction is feasible within a model-based image reconstruction framework that incorporates data fidelity weighting to consider penumbra effects and source jittering to consider the effect of partial source obstruction. Here, we present these modifications and demonstrate their application in simulations and real-world prototype scans. In simulations compared to conventional low-dose acquisitions, SparseCT is able to achieve smaller normalized root-mean square differences and higher structural similarity measures on two reduction factors. In prototype experiments, we successfully apply our reconstruction modifications and maintain image resolution at quarter-dose reduction level. The SparseCT design requires only small hardware modifications to current diagnostic clinical scanners, opening up new possibilities for CT dose reduction.

1. Introduction

X-ray computed tomography (CT) is one of the frontline imaging examinations used in disease diagnosis. However, the drawback of ionizing radiation in x-ray CT has motivated extensive research into low-dose CT for the purpose of mitigating radiation concerns while preserving image quality (Liang et al. 2017). Dose reduction factors as large as 20 have been achieved in the commercial and academic realms (Gordic et al. 2014). Despite extensive advances in low-dose CT techniques over the last decade, widespread implementation on clinical systems is an ongoing process.

There are several approaches for implementing low-dose CT. Tube-current reduction methods are the most common, but result in increased sinogram noise during acquisition. This increased noise is commonly addressed by using advanced algorithms to minimize a cost function that takes into account statistics of the system noise and prior knowledge of the image volume. These algorithms can incorporate noise models in either the transmission (La Riviére et al. 2006, Xu and Tsui 2009) or post-log domain (Thibault et al. 2007, Beister et al. 2012) for improved data consistency. They can also incorporate prior information via regularizers, e.g. Markov random fields (Thibault et al. 2007, Geman and Geman 1984) and edge-preserving penalties such as total variation (Defrise et al. 2011). These methods have been an ongoing area of extensive research in CT, with advances in
algorithm design in recent years improving practicality for the clinic (Kim et al.
2015, McGaffin and Fessler 2015, Nien and Fessler 2016).

Sparse sampling based on compressed sensing (Candes and Wakin 2008) is an alternative to tube-current
reduction. Many sparse sampling methods for CT are based on view undersampling (Snyder et al.
2006, Liu et al. 2012). Aside from enabling dose reduction in combination with compressed sensing, another advantage of these
schemes is that they reduce the amount of data collected, and thus decrease the time needed for reconstruction.
Dose reduction factors as large as 8-fold have been demonstrated for sparse-view sampling schemes (Luo et al.
2016). The performance of sparse-view CT has been evaluated in cardiac CT perfusion imaging (Chen
2008). The previous results to encompass a more com-
prehensive reconstruction scheme and test it both on simulation data with realistic sampling models and a real-
world beam-interrupting collimator prototype. Our simulation experiments include noise-altering penumbra
effects and techniques to maintain uniform resolution in the reconstruction at different radiation dose reduction
factors. With our collimator prototype data, we demonstrate the reconstruction pipeline on diagnostic clinical hardware has not previously been demonstrated.

In this paper we investigate image reconstruction for an interrupted-beam approach tailored for use on di-
agnostic clinical scanners, which we call ‘SparseCT’. The SparseCT interrupted-beam method had initially been
investigated in simulation with physically unrealistic sampling models (Koesters et al. 2017). These have indicated that MSC designs may be able to outperform other sparse sampling
schemes (Abbas et al. 2013, Dong et al. 2013, Koesters et al. 2017). These indicate that MSC designs may be able to outperform other sparse sampling
schemes (Abbas et al. 2013) or even current commercial low-dose imaging methods (Koesters et al. 2017). MSC
-based designs introduce new challenges that must be addressed: penumbra effects from the finite size of the
source (Chen et al. 2017) and corresponding coherence drop-offs related to collimator design parameters (Muck-
ley et al. 2017) have been modeled and simulated. However, while a number of studies have examined MSC effects
in isolation, a reconstruction pipeline on diagnostic clinical hardware has not previously been demonstrated.

An alternative to view-based undersampling are interrupted-beam sampling approaches in which the source
is partially blocked using a multislit collimator (MSC). Previous studies have investigated beam blocking for
applications such as scatter correction (Zhu et al. 2009) and dose reduction (Abbas et al. 2013, Dong et al. 2013,
Koesters et al. 2017). These have indicated that MSC designs may be able to outperform other sparse sampling
schemes (Abbas et al. 2013) or even current commercial low-dose imaging methods (Koesters et al. 2017). MSC
-based designs introduce new challenges that must be addressed: penumbra effects from the finite size of the
source (Chen et al. 2017) and corresponding coherence drop-offs related to collimator design parameters (Muck-
ley et al. 2017) have been modeled and simulated. However, while a number of studies have examined MSC effects
in isolation, a reconstruction pipeline on diagnostic clinical hardware has not previously been demonstrated.

In this paper we investigate image reconstruction for an interrupted-beam approach tailored for use on di-
agnostic clinical scanners, which we call ‘SparseCT’. The SparseCT interrupted-beam method had initially been
investigated in simulation with physically unrealistic sampling models (Koesters et al. 2017, Muckley et al. 2017)
or in simulation without modeling for uniform resolution (Chen et al. 2017). A first demonstration was recently
achieved for a prototype experiment (Chen et al. 2019). We extend the previous results to encompass a more com-
prehensive reconstruction scheme and test it both on simulation data with realistic sampling models and a real-
world beam-interrupting collimator prototype. Our simulation experiments include noise-altering penumbra
effects and techniques to maintain uniform resolution in the reconstruction at different radiation dose reduction
factors. With our collimator prototype data, we demonstrate the reconstruction pipeline for the first time on a
real-world SparseCT system. Our paper begins by giving an overview of the MSC scanner design and the relevant
system model in section 2. Then, we outline which steps in the reconstruction process must be modified in the
presence of the MSC: this includes the topic of partial source obstruction in section 2.2 and the preprocessing
pipeline in section 3.2. Finally, we demonstrate the results of these modifications in section 4 and discuss future
directions in section 5.

2. Theory

The SparseCT MSC design replaces the standard adaptive collimator with a new collimator plate that contains
many slits as shown in figure 1. Specifications of these slits are denoted in terms of their footprint on the detector
array such that a one-row opening on the MSC would illuminate one row of the detector with an ideal point
source. One example design is to open four rows out of every 16 rows—we indicate this notationally with W4S16,
for a width of four rows and a spacing period of 16 rows. The W4S16 pattern would be repeated from one end of
the collimator to the other, until all rows of the detector are accounted for. Figure 2 shows an illustration of the
MSC and the source, including the location of the source, the MSC, and the undersampled x-ray beam. As shown
in the figures, the rays begin at the source, are partially blocked by the MSC, pass through the subject, and then
illuminate the detector.

As the gantry rotates around the subject, two motions occur: the first is flying focal spot motion accomplished
by steering of the x-ray beam. This motion allows the scanner to effectively acquire two or more projections for
a single projection angle, in order to increase z-axis resolution. When the MSC is included, this source motion
leads to a shift of the MSC shadow on the detector, and thus continues to produce extra views for one projection
angle, but with a predictable detector irradiation pattern determined by the source motion and the MSC design.
The second motion that occurs throughout the scan is movement of the MSC itself under an actuator. Motion
occurs along the z-direction. This paper discusses a linear motion pattern, with study of other motion patterns
left to future work.
2.1. System model

Our system model relies on that of previous work (Thibault et al 2007). The $p$th data point, $b_p$, can be modeled as

$$b_p = f_p + \epsilon_p,$$

$$f_p \sim \text{Poisson}(I_pe^{-\bar{y}_p}),$$

$$\epsilon_p \sim \mathcal{N}(0, \sigma^2),$$

where $\bar{y}_p = a_p^T x$, and $x \in \mathbb{R}^N$ is the object being imaged with $N$ voxels and $I_p$ is the source intensity (Thibault et al 2007). The line integral projections, $\bar{y}_p$, are attenuated by the object, $x$, as described by the ray path in $a_p$. The Poisson variable $f_p$ with mean and variance $\bar{y}_p$ is called the \textit{quantum noise}, while the Gaussian variable $\epsilon_p$ with variance $\sigma^2$ is called the \textit{electronic noise}.

For simplification, we work with the postlog data, $y_p$, instead of working with (1) directly. We use the following linear model (Thibault et al 2007):

$$y_p = a_p^T x,
$$

where $x \in \mathbb{R}^N$ is the object being imaged with $N$ voxels and $I_p$ is the source intensity (Thibault et al 2007). The line integral projections, $\bar{y}_p$, are attenuated by the object, $x$, as described by the ray path in $a_p$. The Poisson variable $f_p$ with mean and variance $\bar{y}_p$ is called the \textit{quantum noise}, while the Gaussian variable $\epsilon_p$ with variance $\sigma^2$ is called the \textit{electronic noise}.

For simplification, we work with the postlog data, $y_p$, instead of working with (1) directly. We use the following linear model (Thibault et al 2007):

\begin{equation}
\text{Figure 1. Illustration of a MSC and source, showing periodic slit openings in the z-direction. The rays from the x-ray source on top pass through open rows of the MSC, which replaces the normal adaptive collimator on current diagnostic scanners. A subsampled set of rays then pass through the subject and are recorded at the detector.}
\end{equation}

\begin{equation}
\text{Figure 2. Schematic of partial source obstruction. In the absence of the MSC, the assumed point source location for detector row q would correspond to the true source centroid. However, in the presence of the MSC, the observed centroid is shifted. This shift was applied to each of the ray casting steps in calculating A. This figure previously appeared in Muckley et al (2018).}
\end{equation}
where \( \mathbf{y} = [y_1, ..., y_p]^T \) is the preprocessed data and \( \mathbf{A} = [\mathbf{a}_1, ..., \mathbf{a}_p]^T \) is the system matrix of line integrals. Section 2.2 describes the construction of \( \mathbf{A} \) that takes into account partial obstruction of the x-ray source. \( \mathbf{n} \sim \mathcal{N}(0, \mathbf{W}^{-1}) \) is Gaussian noise designed to approximate the combined effects of both noise sources and the preprocessing steps. For clarity, we note that \( \bar{y}_p \) is the ideal, noiseless version of \( y_p \), whereas \( y_p \) is the estimate of \( \bar{y}_p \) that arises from log transform preprocessing. The covariance matrix, \( \mathbf{W}^{-1} \), is assumed to be diagonal, with its inverse being

\[
\mathbf{W} = \begin{bmatrix}
w_1 & & \\
& \ddots & \\
& & w_p
\end{bmatrix},
\]

where \( w_p \) is the variance of the \( p \)th data point. Based on previous results (Thibault et al. 2007), we use \( w_p = b_p \), which leads to high-intensity pre-log data being more important in the reconstruction. Calculating \( y_p \) from \( b_p \) requires a logarithm and some spectral corrections that we describe in section 3.2.2.

Given the signal model in (2), one could obtain the penalized weighted least squares (PWLS) estimate of \( \mathbf{x} \) by solving the following optimization problem:

\[
\hat{\mathbf{x}} = \arg\min_{\mathbf{x} \geq 0} \left\{ \psi(\mathbf{x}) := \frac{1}{2} ||\mathbf{y} - \mathbf{A}\mathbf{x}||^2_W + R(\mathbf{x}) \right\},
\]

where the above estimate also enforces a non-negativity constraint on the attenuation coefficients in \( \mathbf{x} \). To compensate for aliasing due to MSC-based undersampling, we incorporate \( R(\mathbf{x}) \), a regularization function that promotes sparsity. We define \( R(\mathbf{x}) \) to have the following general form:

\[
R(\mathbf{x}) = \sum_{j=1}^{J} \psi_j (|\mathbf{C}\mathbf{x}|),
\]

where \( \mathbf{C} \in \mathbb{R}^{J \times N} \) is a finite difference operator and \( |\mathbf{C}\mathbf{x}| \) is the \( j \)th output of \( \mathbf{C}\mathbf{x} \). The function \( \psi_j(t) \) is a potential function. The classic compressed sensing choice for \( \psi_j(t) \) would be the absolute value function, thus yielding the \( \ell_1 \) regularizer. However, in our experiments we found that this could lead to salt-and-pepper noise artifacts as demonstrated in previous results (Thibault et al. 2007). Instead, we use the hyperbola function (Long et al. 2010),

\[
\psi_j(t) = \beta_j \delta^2 \left[ \sqrt{1 + (t/\delta)^2} - 1 \right].
\]

With small values of \( \delta \), (6) approximates the classic Total Variation function used for compressed sensing. \( \beta_j \) is a space-varying regularization weight. We allow the regularization parameter to vary since the data fidelity weighting can induce resolution variation on the reconstructed image (Fessler and Rogers 1996). In particular, low weighting of projections through the center of the image volume can lead to over-regularization in this area. We compensate for this in our selection of the \( \beta_j \) weights. First, we calculate

\[
\kappa_{n} = \sqrt{\left( \sum_{p=1}^{p} a_{pn} w_p \right) / \left( \sum_{p=1}^{p} a_{pn} \right)},
\]

where \( a_{pn} \) is the \( p \)th, \( n \)th value of \( \mathbf{A} \). We then apply

\[
\beta_j = \beta \prod_{\forall n, \bar{c}_{n} \neq 0} \max (\kappa_{n}, 0.01 \kappa_{\text{max}}),
\]

where \( \bar{c}_{n} \) is the \( k \)th, \( n \)th value of \( \mathbf{C} \) and \( \beta \) is a baseline value that governs the regularization level in the entire image volume. This choice of \( \beta_j \) values promotes uniform resolution in the reconstructed image (Fessler and Rogers 1996, Kim et al. 2015).

### 2.2. System operator and partial source obstruction

We construct the data operator, \( \mathbf{A} \), based on the classic Siddon line integral method (Siddon 1985). We choose to do this since the Siddon method introduces minimal parameterization to the system geometry, allowing adaptation to correct for the effects of partial source obstruction, a distortion the MSC introduces to the standard system geometry. Partial source obstruction results in each row of the detector only seeing a portion of the source. This shifts the effective location of the ideal point source, resulting in row-variant cone beam magnification effects. To compensate for partial source obstruction effects, we shift the location of the source for each row of the detector. We determine the size of the shift from offline simulation experiments on the scanner geometry for each...
MSC position and MSC design. Then, in our Siddon ray-tracing procedure (Siddon 1985), we apply this shift for each row of the detector and MSC position.

Figure 2 shows a schematic of the partial source obstruction effect. To estimate what portion of the source was observable, we approximated the source as a summation of Gaussian basis functions based on experimental data. Then, we calculated the centroid of the observable source and used this information to perturb the point source location for each row of the detector in the projector.

2.3. Optimization algorithm

There are several approaches for minimizing the cost function in (4): our chosen method was to use the ordered subsets/momentum algorithm (OS-momentum) (Kim et al 2015) due to its speed and ease of implementation. The OS-momentum method develops an optimization transfer algorithm for (4) and accelerates the optimization transfer algorithm using ordered subsets and Nesterov momentum (Nesterov 1983). We review this procedure for (4).

The first step is the creation of the surrogate, which can be done by upper-bounding the Hessian of (4), $H$ with a diagonal matrix, $D$. The Hessian can be decomposed into two parts given by the data-fitting term and the regularizer, i.e. $H = H_f + H_R$. Then, the following is a diagonal upper bound:

$$D_f + D_R = D \succeq H = H_f + H_R,$$

provided that $D_f \succeq H_f$ and $D_R \succeq H_R$ where $D \succeq H$ implies that $D - H$ is positive semidefinite. Thus, we can consider the data-fitting and regularizer terms separately in two stages. A diagonal upper bound for the data-fitting part of the Hessian, $A^TWA$, can be found using the method of De Pierro (1995):

$$D_f = \text{diag}\{ |A^T| W |A| 1 \} \succeq A^TWA,$$

where $| \cdot |$ is the element-wise absolute value operation. This bound is commonly used for quadratic surrogates (SQS) methods (Erdoğan and Fessler 1999). The method of De Pierro also gives a bound for the regularizer (De Pierro 1995),

$$D_R = \text{diag}\{ |C^T| \text{diag}\{ \tilde{v}_j(0) \} |C| 1 \}.$$ Combining these into $D = D_f + D_R$ gives the following separable quadratic surrogate (Erdoğan and Fessler 1999) at iteration $k$ for the cost function in (4):

$$\phi_k(x) = \frac{1}{2} \| x - \left( k^{(k)} - D^{-1} \nabla \Psi \left( x^{(k)} \right) \right) \|^2_D,$$

where $\nabla \Psi \left( x^{(k)} \right)$ is the gradient computed at the point, $x^{(k)}$. Iteratively applying the update in $k$ will descend the cost function in (4).

Such an algorithm is typically slow, so we add ordered subsets and momentum accelerations (Kim et al 2015). The ordered subsets acceleration assumes that the overall cost function can be decomposed into sums of functions that have the same shape. If this is satisfied, then we apply a similar update to that in (12), but instead using the ordered subset cost function:

$$\Psi_m(x) := \frac{1}{2} \| y_m - A_m x \|^2_{W_m} + \frac{1}{M} R(x),$$

for $m = 1, \ldots, M$. Further, we assume that $\nabla \Psi \left( x^{(k)} \right) \approx M \nabla \Psi_1 \left( x^{(k)} \right) \approx \ldots \approx M \nabla \Psi_M \left( x^{(k)} \right)$ and replace $\nabla \Psi \left( x^{(k)} \right)$ in (12) with $\nabla \Psi_m \left( x^{(k)} \right)$. This can also be combined with Nesterov momentum by introducing auxiliary variables $v$ and $z$ (Kim et al 2015), giving the overall OS-Momentum Algorithm. All subsets are updated in the FFT bit-reversal order to ensure incoherence between consecutive updates.

3. Methods

We performed two sets of experiments to investigate the efficacy of the interrupted-beam design: simulation experiments and real-world prototype experiments. Section 3.1 describes the simulation experiments, while section 3.2 describes the real-world prototype experiments. All experiments used the Siddon line integral method (Siddon 1985) implemented in the EMrecon toolbox (Kösters et al 2011) (available at emrecon.uni-muenster.de).

3.1. Simulation experiments

Our simulation experiments used data from the Low Dose CT Grand Challenge (McCollough et al 2017). The data consisted of a central $736 \times 64 \times 4006$ (detector channels $\times$ detector rows $\times$ projections) section of the original helical sinogram of a liver examination from patient 67. This section of data was cropped from the
original data set from $z = 125\text{mm}$ to $z = 165\text{mm}$, yielding 4,006 projections. From this, we reconstructed a $768 \times 768 \times 32$ image volume with $0.66\text{mm} \times 0.66\text{mm} \times 3\text{mm}$ resolution. The images displayed in subsequent figures are from slice 15 of the image volume.

Although the tungsten MSC has a very high attenuation factor and completely blocks the beam, the size of the focal spot leads to a non-binary sampling pattern corrupted by penumbra effects (Chen et al. 2017, 2019). The signal in the penumbra region may experience different physical effects than the main signal region—this can be caused by a spectral shift due to the source location on the anode or a partially attenuated path through a corner of the MSC. For these reasons, we decided to exclude elements from the penumbra region that had more than an 80% attenuation due to the MSC. We corrupted the data using a noise model that incorporated penumbra attenuation effects (Chen et al. 2017) based on the W4S16 and W2S16 sampling patterns (4-fold and 8-fold reduction factors). Then, we reconstructed by minimizing (4) using the OS-Momentum algorithm (Kim et al. 2015). We reconstructed images across a large array of values for the regularization parameter. We used a $\delta = 5\text{HU}$ based on a previous study (Long et al. 2009). These results were compared to quarter and eighth dose tube-current reduction simulations based on the Siemens ReconCT software (Yu et al. 2012), also reconstructed by minimizing (4) with the OS-Momentum algorithm. A reference standard was reconstructed with uncorrupted data. Lastly, our experiments used a sinogram oversampling factor of 3 to reduce ringing from the Siddon ray-tracing procedure.

We did not simulate the effects of partial source obstruction in our simulation experiments. Our main task was to isolate the sampling and noise properties of the MSC to ascertain feasibility of the approach on in vivo data. However, these effects are fully-realized in the real-world experiments.

### 3.2. Prototype experiments

#### 3.2.1. Data Collection

Our real-world prototype consisted of a Siemens SOMATOM Force scanner equipped with a W4S16 collimator in place of the standard adaptive collimator. Figure 3 shows the W4S16 collimator secured in the adaptive collimator jaws of the Siemens scanner.

In our prototype experiments, we used the American College of Radiology CT (ACR) phantom. The phantom has inserts at known locations that can be used to determine the resolution of the system. Four data sets were acquired with the prototype scanner: a no-MSC, 1/4 tube current air scan, a no-MSC, 1/4 tube current ACR scan, an MSC air scan, and an MSC ACR scan. The air scans were acquired for log-domain preprocessing (described in section 3.2.2). From these data, we retrospectively simulated a scan in which the MSC has a piecewise linear motion from the first to the sixteenth position over one rotation. This generated a $920 \times 96 \times 2100$ data set. Of this data set, elements that had greater than 80% attenuation due to the MSC were excluded. From this, we reconstructed a $768 \times 768 \times 32$ image volume at $0.66\text{mm} \times 0.66\text{mm} \times 3\text{mm}$ resolution. The reconstruction process used the custom preprocessing pipeline described in section 3.2.2 and the partial source obstruction model shown in section 2.2.

#### 3.2.2. Prototype preprocessing

The prototype experiments used a custom preprocessing pipeline to convert the prelog data points $b_p$ to postlog data points $y_p$. Preprocessing pipelines consist of several corrections for non-ideal system factors. These factors include alterations to the spectral distribution of the x-ray source due to beam hardening, negative values that can arise in $b_p$ due to the electronic noise, geometric distortions, and anatomy-specific factors. Corrections for these factors can vary between vendors, and the details are often proprietary. Our goal is to build upon past literature on preprocessing while making modifications necessary for SparseCT. These include air calibration, log transformation, and beam-hardening correction. We apply these modifications to the data in our prototype experiments.

We first correct the negative values due to the electronic noise in $b_p$. Current systems have signal-dependent filters (SDFs) for handling small values that occur behind highly-attenuating structures such as bone. However, the MSC itself is an abnormal high-attenuating structure, so we do not expect these filters to work in our setting. Instead, we apply clipping:

$$s_p = \max(b_p, b_{p, \text{min}}),$$  

(14)

where $b_{p, \text{min}} = 0.000\text{01I}_p$. This limits the minimum value to be five orders of magnitude below the incident value. Clipping can create some bias in low-signal regions, but we design the reconstruction weights in (4) such that low-signal regions have relatively small effects on the final reconstruction.

We next apply the logarithm. This requires knowledge of the fluence, $I_p$, which is measured with an air scan. We collected air scans for each MSC position with our prototype. With these scans, we apply the logarithm as

$$t_p = -\log\left(\frac{s_p}{I_p}\right).$$  

(15)
The final step is beam-hardening correction. For this, we relied on a proprietary 2-compartment model from Siemens (related, Raupach et al (2001)). Since we apply the method in the post-log domain after the air scan calibration, we assume that the beam-hardening method is performing in normal operating conditions and that no further modifications are necessary due to the MSC. This gives the final postlog data for reconstruction:

$$y_p = BHC(u_p),$$

where $BHC(\cdot)$ is the beam-hardening correction function. We did not perform scatter correction due to the anti-scatter grid on the detector.

4. Results

4.1. Simulation experiments

In our simulation experiments we compared to the full-dose reference scan shown in figure 4(a). We reconstructed this image with mild regularization to reduce the influence of the noise present in the original data. After calculating the reference image, we simulated SparseCT acquisitions with the W4S16 and the W2S16 collimator design. For comparison, we also simulated 1/4th dose and 1/8th dose tube-current acquisitions and reconstructed them with the same algorithm as the full-dose scan. Then, we calculated the normalized root-mean-square difference (NRMSD) and structural similarity measure (SSIM) across six slices between the low-dose image volumes and the reference image volume. The SSIM calculations used 400 HU for the dynamic range since this was the viewing window size. All displayed images in figures 4 and 5 were reconstructed with the NRMSD-optimal $\beta$ for that method. The results of these reconstructions are shown in figures 4 and 5. Both tube-current reduction and SparseCT acquisitions show image degradation as the dose is reduced in the simulated images in figures 4 and 5. At high dose reduction levels, the SparseCT acquisitions showed less image degradation, as illustrated in figure 5. To more fully characterize the methods, we also plotted SSIM and NRMSD versus the ground truth image as a function of regularization parameter. These are shown in figure 6. The SparseCT methods perform better for the metrics at lower regularization levels. Among our experiments the minimum W4S16 NRMSD was 2.56% at $\beta = 1270$ with an SSIM of 0.8752. The minimum 1/4 tube current NRMSD was 2.69% at $\beta = 3002$ with an SSIM of 0.8810. The minimum W2S16 NRMSD was 3.19% at $\beta = 1270$ with an SSIM of 0.8336. The minimum 1/8 tube current MSE was 4.19% at $\beta = 3999$ with an SSIM was 0.8015.
4.2. Prototype experiments

4.2.1. Partial source obstruction

Figure 7 shows images illustrating the effects of partial source obstruction on one view of the sinogram. The original, open-collimator scan is shown in figure 7(a). Figure 7(b) shows how the operating table becomes slanted in the presence of the MSC—this is due to variations in the cone beam magnification effect caused by changes in the effective source location. Figure 7(c) shows one view from \( x_{\text{no msc}} \), where \( x_{\text{no msc}} \) is the reconstructed image volume from no-collimator sinogram (i.e. that shown in figure 7(a)). Figure 7(c) shows the effect of the simulated partial source obstruction model. The model reproduces the slant of the scanner table. The model does not smoothly interpolate into regions where the MSC incurs full obstruction, but the effects of this region are negligible due to sinogram-domain masking and the weights matrix, \( W \).

4.2.2. Resolution

Figure 8 compares results with a quarter-dose open collimator versus the MSC with linear motion by zooming in on the 7 line pairs per cm (lp cm\(^{-1}\)) insert of the ACR phantom. For these reconstructions, a \( \beta \) of 128 was used for all methods. The open collimator results were obtained with the same dose reduction achieved for the SparseCT method and with the same reconstruction algorithm. Both methods resolve the 7 lp cm\(^{-1}\) insert. Although both methods were able to resolve the 7 lp cm\(^{-1}\) insert in figure 8, we found inserts finer than 7 lp cm\(^{-1}\) were more difficult.

5. Discussion

5.1. Comparisons between SparseCT and tube current reduction

Figures 4 and 5 suggest that SparseCT and tube current have different error characteristics. Although SSIM and NRMSD are similar for both methods, the errors for the tube current reduction method are heavily concentrated on the interior of the abdomen, whereas the errors for SparseCT are more diffuse. This effect becomes greater at higher dose reduction factors. Model-based reconstruction methods for low-dose CT often rely on detailed system models and priors to compensate for this increased noise. Figure 6 suggests that with these models, the
extent of regularization necessary to maintain high image quality is reduced with an interrupted-beam design. Nonetheless, the trends suggested by figures 4–6 should be considered a proof-of-concept of the potential of an interrupted-beam design. Further validation is necessary in future work.

5.2. **New considerations of interrupted-beam image reconstruction**

Our reconstruction pipeline shares a number of aspects with other low-dose reconstruction methods. Although data fidelity weighting and regularization for uniform resolution were originally developed for tube-current reduction schemes (Fessler and Rogers 1996, Thibault et al 2007), we found they readily extend to the SparseCT setting to consider penumbra effects. We also found that much of the standard preprocessing pipeline can be preserved after air calibration.
Still, SparseCT introduces some new considerations that are not present in other sparse sampling schemes such as view-based undersampling. In particular, we discovered that partial source obstruction can affect the data. Our correction for this required simulations of the source distribution and an alteration to the projection operator. Many implementations of modern projection operators (such as separable footprints (Long et al 2010)) are highly parameterized, but the jittering necessary for partial source obstruction precludes use of detailed parameterizations. This was the primary motivation for our use of the very general Siddon method (Siddon 1985). In the future, we will examine whether more parameterized projection operators can be adapted for SparseCT.

5.3. Extensions to a fully-functional prototype

Here, we simulated a linear motion of the MSC by retrospectively combining scans at different MSC positions. These results readily extend to a prospective linear motion with a few modifications under development. First, the air calibration scan must be registered to each location of the MSC in the sinogram. This requires a process for detecting the location of the MSC, then interpolating between the two closest corresponding air scan locations in order to perform the appropriate air calibration. Another modification will register the partial source obstruction effects to the detected location of the MSC by first fitting a polynomial for partial source obstruction corrections based on simulations, then applying this polynomial to the appropriate position determined from the MSC registration procedure. After these modifications are applied, the rest of the pipeline described in this paper can be used for reconstruction.

Another possibility for extension is that of dual-energy CT. The MSC naturally segments the sinogram into high and low-energy areas. In our case we entirely blocked some segments of the sinogram in order to achieve dose reduction, but an alternative approach would be to only partially attenuate these areas to achieve...
dual-energy CT imaging, an application that has been explored for CBCT systems (Lee et al 2017). The partial source obstruction models developed in this paper would continue to be relevant to reconstruction in this setting. Joint reconstruction techniques (Knoll et al 2014, Rigie and La Rivière 2015, Lee et al 2017) could be used to mitigate the image degradation from the low-energy filter by combining information from both segments of the sinogram.

6. Conclusion

We developed a new image reconstruction pipeline tailored to SparseCT acquisitions. Our method was based on an MBIR method, but included new modifications to consider aspects of the interrupted-beam design. We altered preprocessing methods and updated projectors for the effects of partial source obstruction. We confirmed that data fidelity weighting appropriately considers the signal variation due to penumbra effects. In retrospective simulation experiments, we observed similar NRMSD and SSIM values to tube-current reduction at 4-fold dose reductions and improved NRMSD and SSIM values at 8-fold dose reductions. Errors for tube current reduction were dominated by the interior of the abdomen, whereas in SparseCT this distribution of errors was flattened. Finally, we observed that SparseCT was able to achieve similar resolution to tube-current reduction in prototype experiments. In the future, we will perform experiments with prospective motion, as well as further developing corresponding reconstruction methods to incorporate the latest advances, such as deep learning.

Acknowledgments

We would like to thank NIH Grant U01 EB018760 for providing funding for this project. We would also like to thank the Siemens Engineering team (K Stierstorfer, T Allmendinger, M Berner, B Schmidt and T Flohr) for building the prototype and providing technical expertise for the experiments in this paper. We thank C

Figure 8. Reconstructions of the ACR phantom with an open collimator versus linear motion for a 4-fold dose reduction factor (900 HU–1100 HU window). (a) Reconstructed ACR slice with standard quarter-dose acquisition and reconstruction, with (b) showing a zoomed-in version of the 7 lp cm$^{-1}$ insert. (c) Reconstructed 7 lp cm$^{-1}$ insert with 4-fold dose reduction (W4S16 pattern) and simulated linear motion, with (d) showing a zoomed-in version of the third insert. Both methods show similar resolution.
McCollough, who provided the simulation data in this paper, which was collected under the grant U01 EB017185. We also thank J Fessler, D Faul and D Rigie for many useful discussions. Lastly, we would like to thank D Rigie for proofreading the manuscript.

**ORCID iDs**

Matthew J Muckley [https://orcid.org/0000-0002-6525-8817](https://orcid.org/0000-0002-6525-8817)

Thomas Vahle [https://orcid.org/0000-0002-3043-3101](https://orcid.org/0000-0002-3043-3101)

Florian Knoll [https://orcid.org/0000-0001-5537-8656](https://orcid.org/0000-0001-5537-8656)

Aaron D Sodickson [https://orcid.org/0000-0003-1275-681X](https://orcid.org/0000-0003-1275-681X)

Daniel K Sodickson [https://orcid.org/0000-0002-2436-4664](https://orcid.org/0000-0002-2436-4664)

**References**

 Erdogan H and Fessler JA 1999 Ordered subsets algorithms for transmission tomography Phys. Med. Biol. 44 2835–51

La Riviére P J, Bian J and Vargas P A 2006 Penalized-likelihood sinogram restoration for computed tomography IEEE Trans. Med. Imaging 25 1022–36

Abbas S, Lee T, Chung H, Baek J and Cho S 2013 Effects of sparse sampling schemes on image quality in low-dose CT Proc. of the Int. Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine pp 288–91

Beister M, Kolditz D and Kalender W A 2012 Iterative reconstruction methods in x-ray CT Phys. Med. 28 94–108

Bian J, Siewersden J H, Han X, Sidky E Y, Prince J L, Pelizzari C A and Pan X 2010 Evaluation of sparse-view reconstruction from flat-panel-detector cone-beam CT Phys. Med. Biol. 55 6575

Candes E J and Wakin M B 2008 An introduction to compressive sampling IEEE Sig. Proc. Mag. 25 21–30

Chen B, Kobler E, Muckley A D, O’Donnell T, Flohr T, Schmidt B, Sodickson D K and Otazo R 2019 Sparse CT: system concept and design of multi-slit collimators Med. Phys. accepted

Chen B et al 2017 Realistic undersampling model for compressed sensing using a multi-slit collimator Proc. of the Int. Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine

Chen G H, Tang J and Leng S 2008 Prior image constrained compressed sensing (PICCS): a method to accurately reconstruct dynamic CT images from highly undersampled projection data sets Med. Phys. 35 660–3

De Pierro AR 1995 A modified expectation maximization algorithm for penalized likelihood estimation in emission tomography IEEE Trans. Med. Imaging 14 132–7

Defrise M, Vanhove C and Liu X 2011 An algorithm for total variation regularization in high-dimensional linear problems Inverse Problems 27 065002

Dong X, Petrongolo M, Niu T and Zhu L 2013 Low-dose and scatter-free cone-beam CT imaging using a stationary beam blocker in a single scan: phantom studies Comput. Math. Methods Med. 2013 637614

Fessler JA and Rogers WL 1996 Spatial resolution properties of penalized-likelihood image reconstruction methods: space-invariant tomographs IEEE Trans. Image Process. 5 1346–58

Geman S and Geman D 1984 Stochastic relaxation, Gibbs distributions, and Bayesian restoration of images IEEE Trans. Pattern Anal. Mach. Intell. 6 721–41

Gordic S et al 2014 Ultralow-dose chest computed tomography for pulmonary nodule detection: first performance evaluation of single energy scanning with spectral shaping Investigative Radiol. 49 465–73

Kim D, Ramani S and Fessler JA 2015 Combining ordered subsets and momentum for accelerated x-ray CT image reconstruction IEEE Trans. Med. Imaging 34 167–78

Knoll F, Koesters T, Otazo R, Block T, Feng L, Vuncio K, Faul D, Nuyts J, Boada F and Sodickson D K 2014 Joint reconstruction of simultaneously acquired MR-PET data with multi sensor compressed sensing based on a joint sparsity constraint EJNMMI Physics vol 1 (Berlin: Springer) p A26

Koesters T, Knoll F, Sodickson A, Sodickson D and Otazo R 2017 SparseCT: interrupted-beam acquisition and sparse reconstruction for radiation dose reduction SPIE Med. Imaging 10132 101320Q

Koesters T, Schäfers K P and Wübbeling F 2011 EMRECON: an expectation maximization based image reconstruction framework for emission tomography data IEEE Nuclear Science Symp. and Medical Imaging Conf. (IEEE) pp 4365–8

Lee D, Lee J, Kim H, Lee T, Soh J, Park M, Kim C, Lee Y J and Cho S 2017 A feasibility study of low-dose single-scan dual-energy cone-beam CT in many-view under-sampling framework IEEE Trans. Med. Imaging 36 2578–87

Lee H, Xing L, Davidi R, Li R, Qian J and Lee R 2012 Improved compressed sensing-based cone-beam CT reconstruction using adaptive prior image constraints Phys. Med. Biol. 57 2287

Liang J Z, Riviére P J L, Fakhri G E, Glick S J and Siewersden J 2017 Guest editorial low-dose CT: what has been done, and what challenges remain? IEEE Trans. Med. Imaging 36 2499–16

Liu Y, Ma J, Fan Y and Liang J 2012 Adaptive-weighted total variation minimization for sparse data toward low-dose x-ray computed tomography image reconstruction Phys. Med. Biol. 57 9293–56

Long Y, Fessler JA and Balter J M 2009 A 3D forward and back-projection method for x-ray CT using separable footprint Proc. of the Int. Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine vol 29, pp 1839–50 (Winner of poster award)

Long Y, Fessler JA and Balter J M 2010 3D forward and back-projection for x-ray CT using separable footprints IEEE Trans. Med. Imaging 29 1839–50

Luo J, Eri H, Can A, Ramani S, Fu L and De Man B 2016 2.5D dictionary learning based computed tomography reconstruction Proc. SPIE 9847 98470L

McCullough C H et al 2017 Low-dose CT for the detection and classification of metastatic liver lesions: results of the 2016 low dose CT grand challenge Med. Phys. 44 e339–52

McGaffin M and Fessler JA 2015 Alternating dual updates algorithm for x-ray CT reconstruction on the GPU IEEE Trans. Comput. Imaging 1 186–99
Muckley M J et al 2018 Reconstruction of reduced-dose SparseCT data acquired with an interrupted-beam prototype on a clinical scanner Proc. 5th Int. Meeting on Image Formation in X-Ray CT

Muckley M J, Chen B, Vahle T, Knoll F, Sodickson A, Sodickson D K and Otazo R 2017 Regularizer performance for SparseCT image reconstruction with practical subsampling Proc. Int. Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine

Nesterov Y 1983 A method of solving a convex programming problem with convergence rate $O(1/k^2)$ Sov. Math. Dokl. 27 372–76

Nien H and Fessler J A 2016 Relaxed linearized algorithms for faster x-ray CT image reconstruction IEEE Trans. Med. Imaging 35 1090–8

Raupach R, Stierstorfer K, Flohr T, Fischbach R, Freund M and Tomandl B 2001 An image-based beam hardening correction technique for CT images Proc. Radiological Society of North America vol 221, p 544

Rigie D S and La Rivière P J 2015 Joint reconstruction of multi-channel, spectral CT data via constrained total nuclear variation minimization Phys. Med. Biol. 60 1741

Siddon R L 1985 Fast calculation of the exact radiological path for a three-dimensional CT array Med. Phys. 12 252–5

Sidky E Y, Kao K M and Pan X 2006 Accurate image reconstruction from few-views and limited-angle data in divergent-beam CT J. X-Ray Sci. Technol. 14 119–39

Thibault J B, Sauer K, Bouman C and Hsieh J 2007 A three-dimensional statistical approach to improved image quality for multi-slice helical CT Med. Phys. 34 4526–44

Xu J and Tsui B M W 2009 Electronic noise modeling in statistical iterative reconstruction IEEE Trans. Image Process. 18 1228–38

Yu L, Shuang M, Jondal D and McCollough C H 2012 Development and validation of a practical lower-dose-simulation tool for optimizing computed tomography scan protocols J. Comput. Assist. Tomogr. 36 477–87

Zhu L, Xie Y, Wang J and Xing L 2009 Scatter correction for cone-beam CT in radiation therapy Med. Phys. 36 2238–68