The interplay between mathematics and physics in prospective primary teacher education

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Abstract. Mathematics and sciences, in particular physics, are structurally related to each other. Understanding the specific nature of these subjects and their reciprocal interplay is important particularly for future teachers in primary school, where they have to be competent in building the literacy starting from a multidisciplinary perspective. One basic aspect of the educational process in primary school is the gradual building of scientific thought. The primary teachers’ competence is based on different disciplinary areas and cultural spheres related to each other. The weak competence in the mathematical and scientific scope has to be transformed into the capacity to offer a grounded scientific education and mathematical instruction, accompanying the identification of the disciplines with concrete examples from everyday life. Therefore primary teachers need insight into the basics of the subjects. This research analyses the spontaneous ideas about the relationship between math and physics of prospective primary teachers attending the course of primary education at the University of Udine. Results of a teaching intervention module focused on the reflection on this aspect are analyzed in the framework of corresponding models in literature in order to study the problem of how to improve teachers’ awareness about the Interplay between Mathematics and Physics (IMP).

1. Introduction

The interplay between mathematics and physics (IMP) is one of the most important cultural and epistemological achievements of humans. Those two disciplines are structurally intimately related to each other. Their relationship is as well epistemologically as also culturally important in being the basis of a modern world description. Despite this importance in many studies it becomes apparent that the lack of success with mathematics in physics is due to the failed transfer of mathematical skills to physics lessons \cite{1} or insufficient understanding of formula \cite{2}. The basis of more success are laid quite early as already in primary school instruction where both subjects, mathematics and science, are introduced. Pupils learn the basic concepts of math in primary school. At the same age, in playing, they build abstract worlds that they transform into games by inventing and applying rules \cite{3}. By drawing they build abstract entities, using symbols and the language of representations \cite{4}. By associating concrete entities and words they build concepts \cite{5}. These playful nevertheless scientific activities can be built upon to lay the basis of the children’s awareness about the scientific description of the world including an insight into formal thinking\cite{6} which is an important educational goal, related to the identification of the nature of physics itself \cite{6}. In many textbooks for primary school formal representations (Venn diagram, table and graphs) are introduced. However, as most given examples refer to simple objects such as e. g. candy or money the opportunity of using them in creating relationships is not exploited. Therefore, in order to

\footnote{1 Formal thinking on primary level includes the steps of abstraction, systematization, stepwise increasing the precision of meaning and representation by well-defined formal means such as tables, diagrams or numbers.}
find more significant examples with respect to science, primary teachers should have insight into the nature of physics including the basics of the role of mathematics for physics. To make this correlation explicit in educational scopes from the earliest stages on is one of the requirements of the Italian science education curriculum. But, how this can be done is an open problem which is related to the ideas on math and science of prospective primary teachers (PPT). These ideas are of course closely related to the story of the personal and curricular experiences, which vary widely because access to university is open for everybody with a diploma from any type of secondary school. Starting from these ideas an environment has to be created in which prospective primary teachers (PPT) find the opportunity to analyze the interplay of both subjects in order to make it explicit, to clarify, to shape and to identify it first to themselves, then to reflect it to take it into account in building scientific education for the children. Teachers should support primary school children in developing formal thinking in physics as an important educational goal [6].

In the last 20 years the interest of physics education research in primary teacher education has grown significantly. Many international surveys show that often primary school teachers show scientific illiteracy and a lack of motivation for science. As teachers’ professional development is one of the most important aspects for improving education [7, 8] we look for ways to impart the IMP to prospective primary teachers. Because in the academic education of PPT, the scientific part is often very weak, this requires a balance between theoretical background of the IMP and practical aspects with respect to teaching in primary school (grade 1 to 4, age 6 to 10 years). The overarching goal is to provide the children with the tools to notice and to use this interplay from early on.

1.1. Theoretical Framework

Physics is considered as a science relying on experiments and on formalization (implying abstraction, idealisation, systematization, increasing precision of meaning, use of representations, using and interpreting formal (mathematical) expressions and deduction of new insights) as well as on their fruitful interplay. Mathematics on its part is characterized by deduction and logical procedures. Accordingly, both disciplines are taught from two different perspectives with different aims, but despite this the link between mathematics and physics is expected to happen automatically during teaching and learning in school and in university. This missed link very often leads to problems with the students [9, 10]. The main problem is the transfer between the concrete physical phenomena and the abstract mathematical world [11]. In this section we will first give examples suitable for PPT. Then, in order to attack this inherent difficulty different models of describing the IMP were developed which describe the interplay from different perspectives. The two considered different models offer the reflection on IMP focusing on: a) how and which kind of math plays a role in the interpretative, descriptive and applicative epistemic goals of physics as a formalized science (functional model) and b) how we can reframe the IMP with respect to the representational, technical and structural roles of mathematics in physics and how these roles are addressed (procedural model).

1.1.1. Elaboration on IMP with examples. The intention of the examples is that the PPT gain a view point that allows them to meaningfully introduce primary pupils to scientific and formal thinking, not just knowing examples from primary level. These examples are oriented by the Ministry guidelines for primary scientific education and cover several aspects.

(1) It is basic to learn what a physical quantity and its measurement means: what is a measurement and how to organize an ordered relationship on the base of a property, e. g. comparing mass or thermal sensation or temperature. (2) Students should be able to relate the different properties and the behavior of an object (system) to a class of phenomena, e. g. to identify it as a fluid or a force. This includes reading a table of physics properties or a graph. Suitable physics examples are often found in textbooks for math. (3) An additional aspect is the attribution of physical meaning to mathematical operators such as “+” or “−” in the sense of “symbolic forms” [12] or the definition of new quantities such as e. g. velocity \( v = \frac{\Delta s}{\Delta t} \) by the ratio between displacement \( \Delta s \) and time interval \( \Delta t \). Also the conceptual meaning of distribution of the operation of the division can be used to understand that pressure is a distribution
of force on a surface or the density is the distribution of the mass in a volume. (4) An important aspect is how a physics model is built: e. g. it is discussed how in physics we decide to replace a car that climbs a mountain with a material point on an inclined plane, using geometry; how we chose the point and how we represent the forces acting on it on an inclined plane [13].

A central aspect of the IMP is the different meaning of those algebraic expressions which appear as equations. These meanings can be: a) relationships expressing conditions producing an effect: examples are: an acting force producing a deformation, or an heating producing an increase in temperature; b) characterization of a quantity by the related variables: e. g. the increase in temperature in a heating process depends on the heating power, on the quantity of the substance and on the kind of substance, or the current in a circuit (the brilliance of a bulb) depends on the voltage (number of equal batteries) applied and on the numbers of bulbs in series; c) expressions defining a new entity: in the cases of density or pressure mentioned above, d) description of a spatial or time evolution of a quantity: as e. g. change of temperature in time or in the different points of a room.

As physicists we are aware that in the case of a) we have an algebraic expression, an equation describing the second law of dynamics or the fundamental law of calorimetry. The conceptual role of those equations can be identified and discussed even if the differential equations which lie behind are not in the pre-knowledge of students. For example it might be discussed how to emphasize the general role of the second law of dynamics and in particular how it – as a theoretical referent – indicates the cause of an acceleration on a material point mass with the specific force law acting. Examples of the case b) are the laws of the specific forces: elastic, central, friction, viscose, surface force etc. The case c) is strongly related to aspect (3) above where the meaning of the math operator offers the conceptual meaning of the physics quantities. Additional examples of the case d) are found in generalized kinematics, such as time evolution of the position or of the velocity or of the acceleration or the spatial description of a quantity (a scalar or vector field). The discussion of such a higher stand point without getting lost in technicalities is important for PPT as they shape the development of formal thinking.

1.1.2. Functional model. This model focusses on epistemological aspects of both mathematics and physics. Therefore an important aspect in this model is that physics is characterized as an experimental and at the same time formalized science. A central aspect is the search for a relationship between the role of math in building the scientific activities such as description, interpretation and prediction, and the physics view namely how to simplify a system to study a phenomenon. From this basic characterization mathematics is attributed a descriptive, an interpretative and a predictive function. In the descriptive function mathematics is used to communicate physics. In the interpretative function it contributes to the interpretation of physical phenomena and processes. In the predictive function it serves for predicting results for physical processes. In addition – but not exclusively – mathematics serves as a tool for physics. Here, mathematics includes not only algebraic expressions but also geometrical tools. Especially geometry can help in the simplification of a physical system in order to create a more accessible model [13] and can also identify the nature and the properties of physics quantities such as e. g. vectors for forces. Mathematics provides help in the formalization of the relationship between quantities for the description of processes and gives operational definitions of quantities e. g. as heat or force [14].

1.1.3. Procedural model. This model is inspired by the modeling cycle from mathematics education and makes it possible to describe the translation process from physics to mathematics (mathematization) and back again (interpretation) [9, 10]. In order to analyze the detailed processes of the mathematization of a physical process different aspects of mathematics in the construction of the physical world description were highlighted: a technical, a structural and a representational one [15]. Students and teachers should be aware of the specifics of each of these aspects in the context of the physical method. The structural aspect contributes to the meaning making of mathematical elements or structures in the context of physics, e. g. building mathematical models and equations, or using formal mathematical analogies in treating physical processes from different areas of physics. Hence it refers to the semantic
aspect of mathematization. The technical aspect comprises the processes of calculation and plays in the
realm of mathematics. The model includes the identification of the elements of mathematics and of
physics and of the interplay between these elements.

1.2. Specific background of prospective primary teachers
Teacher education and professional development is one of the most important aspects for improving
education [6, 16, 17]. This is especially valid on the level of primary school [7, 18]. The main focus has
to be on the teachers’ needs to integrate subject matter and pedagogical aspects to gain professional
competence in conducting active learning by pupils. These competences require specific dedicated time
and own activities by the primary teachers [1, 19, 20] .

Prospective primary school teachers (PPT) are often educated under a programme that strongly
focuses on pedagogical education. Content-related parts are included only to a relatively small extent as
e. g. in Italy. The corresponding study programs form the context of our research. The university Master
curriculum for PPT education lasts 5 years starting with the freshmen first year (not Bologna process
but unique cycle) and ends with the habilitation for teaching in kindergarten and primary school. It
includes overall 69 credits of pedagogical subject courses. The courses in mathematics and the sciences
have the following amount: Didactics of Math: 11 credits, Didactics of Physics: 9 credits and Didactics
of Ecology and Biology: 13 credits. These together comprise less than half the credits of the pedagogical
courses. Nevertheless in the field of scientific education, the PPT have to deal with topics that concern
mathematics and physics, as recommended in literature [21]. The small number of credits and the lack
of real integration between pedagogical education and disciplinary didactic education present a problem
for the acquisition of the transversal knowledge in this area. A general lack of basic competences in
physics (missing CK) is another obstacle. Fertile strategies for overcoming conceptual knots and/ or to
activate interpretative models building scientific thinking are needed. A second problematic issue is the
lack of resources (research resources or documentation and suggestions) or experience in school practice
as referent. Therefore, in our research, a formative model focusing on the construction of a flexible
Pedagogical Content knowledge (PCK) has been studied [22]. This approach, implemented since 2001
in Udine University [23] is based on an integration of metacultural, experiential and situated teacher
education models [16] in a specific PCK model. In this the most relevant parts are the personal
involvement of PPT in analysing research based educational proposals for lesson planning and
implementing in primary school, and then analysing the learning outcomes of the involved children. In
the following we discuss the case of PPT competence gain during such a course specifically on the
relation of math and physics.

2. Research goal and method
The goal of the (exploratory) study is to analyse the preliminary ideas of prospective primary teachers
concerning the interplay of mathematics and physics and to evaluate the effect of a teaching intervention
to introduce PPT to the relevance of the interplay mathematics and physics and to identify possible ways
to promote the ownership about this interplay. PPT are given examples for clarification and
understanding of the functional and procedural models described above.

2.1. Research questions
The research questions are:

1. Which aspects of the role of mathematics in physics do PPT describe on their own after the
academic courses of Math Education and Physics Education?
2. How does a specific intervention module focused on active reflection by means of two
perspectives contribute to the gain of the awareness of the role of mathematics in physics?
Which aspects do PPT describe after the short intervention?

2 Habilitation is the licence of teaching: in Italy it is necessary to pass the habilitation exam to be able to teach in
primary school and in particular to participate in the competitions for a teaching position in primary school.
2.2. Method
As the specific topic of the study is not intensely researched until now we undertook an exploratory study. The study involved 100 students attending the course of didactics of physics in the third year of the primary education master degrees at the University of Udine (Italy).

2.2.1. Design of study. The sequence of the study is shown in figure 1. In order to answer the first research question the students were given one open question (Q1) about the interplay between mathematics and physics. This they should answer on the basis of courses on didactics of physics and didactics of mathematics, where no specific attention to the IMP was paid. This open question aimed at exploring in which way the students received the content taught so far and how they incorporated it into their previous knowledge and views. The students had had a short introduction (4 hours = 1 session) into the nature of physics and its relationship with math on the basis of the functional model 7 months before the intervention described below. The focus was on the functional model explained above. Shortly before the intervention the students should answer the open question at home and upload the texts to an online platform.

Figure 1. Sequence of study: On top the location of the intervention in the course of study is shown, at the bottom the sequence of questionnaires (Q1, Q2) and the intervention is indicated.

Then a specifically designed short intervention took place consisting of a module of 4 hours (1 session) with a presentation on the procedural model. The focus was on explaining and exemplifying the technical, structural and communicative role of math in physics. A reflection on the different roles of math for the physical world description was included. Hence the PPT were taught an additional perspective on the role of mathematics in physics hence complementing their previous study of the functional perspective. During the intervention the procedural model was discussed with concrete examples suitable for PPT in the spirit of teacher education described in section 1. These included e. g. numbers with units in everyday situations and interpretation of graphs. Afterwards a reflection was stimulated on how mathematics would contribute to physics in defining the nature of quantities, identifying laws for description and interpretation of phenomena and building modelling. Also the physical interpretation of mathematical operations (e. g. meaning of addition) were discussed. In order to embed this theoretical discussion into practice the students were asked to analyse primary school textbooks with respect to the described roles of mathematics and physics.

After the intervention the students answered to a second, more detailed, questionnaire (Q2) composed of six questions as part of the examination of the course. The questions were discussed together with the students in order to be sure that everyone understood them:
1. Math has a representative role. Explain;
2. Math has a technical role. Explain;
3. Math has a structural role. Explain;
4. Diagrams: what role do they have?;
5. How useful is it for me to classify the roles of mathematics for physics?;
6. How can physics support mathematics?.
2.2.2. *Analysis of data.* The students’ answers were analysed qualitatively by inductively generating categories. These categories were not mutually exclusive so that the same answer could be coded in several categories. The inductively found categories were ordered in main categories deductively derived from the theoretical background of the functional or procedural model. The categories and codings were discussed among the researchers involved. This process was reiterated several times in order to reach agreement and to get a consistent coding scheme. This scheme was described and exemplified with anchor items. The reproducibility was made plausible by the fact that the analysis was repeated after several months and independently done by two researchers with very similar results. The distribution of answers among the main categories obtained from the two questionnaires were compared in order to analyse the impact given by the intervention and students’ becoming conscious of the problem. As software the R-package RQDA was used.

2.3. *Results*

Here we present exemplary results of the questionnaires.

2.3.1. *First Questionnaire Q1.* For the analysis of the data nine different categories were derived from theory. These included theoretical aspects such as epistemology of mathematics or physics, correlations or differences between mathematics and physics or representational aspects and practical aspects such as relevance for teaching at primary school: A. Aspects relevant for teaching at primary school; B. Correlation between math and phys; C. Differences between math and physics; D. Mediators used by math to formalize physics; E. Language; F. Role of geometry; G. Formalization; H. Epistemology of mathematics; I. Epistemology of physics. The distribution of answers coded in each of the categories is shown in figure 2.

![Figure 2](image_url)

*Figure 2.* Percentage of students who gave answers to questionnaire 1 that could be attributed to the corresponding main categories

We describe an exemplary result choosing the main category B. Correlation: 69% of the students gave answers that were categorized as describing the correlation of math and physics and characterizing it. The following subcategories of main category B were derived inductively from the data. These express: 1. Adjectives characterizing the relation; 2. Common aspects of language; 3. Important didactic aspects of the interplay; 4. Common characteristics; 5. Differences; 6. Mutual promotion of development; 7. Role of mutual context; 8. Common tools (see figure 3).

2.3.2. *Second questionnaire Q2.* As the questions of the second questionnaire were more detailed accordingly the answers of the students showed a corresponding depth. In fact both the technical and
the structural role were widely described by the students (318 statements regarding the technical role and 319 regarding the structural role). In characterizing the structural role besides other categories mostly the importance of the role of mathematics in structuring physical thought was particularly stressed (62 out of 319). In comparison the aspect of math as a tool in the service of physics was considered in 37 statements.

In the second questionnaire students described the correlation between math and physics (category B from questionnaire 1) mainly in answering the questions 3 (structural role) and 6 (physics is supporting math). We find statements belonging to the subcategories “Common characteristics”, “Differences”, “Mutual promotion” and “Role of mutual context”. “Common aspects of language” is mainly addressed in answers to question 1 (representational role). The subcategory “Common tools” is found in answers to question 2 (“technical role”). The subcategory “Adjectives” is replaced by other categories showing a more detailed view such as e. g. “Creating meaning”. These more detailed answers give a hint that the students developed deeper awareness of the different aspects and have richer and more differentiated views.

![Figure 3. Frequency of answers belonging to the subcategories of main category B. Correlation between math and physics.](image)

2.3.3. Comparison of both questionnaires. The students showed a change in the answers, which is clearly visible in the percentage of answers per category with respect to all coded statements (see table 1). First of all the students write more as seen in the number of overall codings. Clearly the contribution of the procedural perspective in specifying the technical, the structural and the representative role of math in the IMP emerges. This is of course influenced by the specific questions asked in Q2. Even if the number of codings of the three categories concerning the role of math in helping physics (rows 6-8 in table 1) increases the percentage remains similar, indicating that the functional model still is present. Nevertheless the view on math only as a tool is reduced. The relevance of IMP for shaping the lessons at school with its curricular aspects, students’ difficulties and methodological suggestions is elaborated on only in the first questionnaire (category: “didactic general comments”)

Besides the categories presented in the table there are some more observations: mathematical language is considered as having an important function in the IMP. From both questionnaires aspects emerge describing language as related to description, prediction, communication, translation and tool. It is also used to create models for understanding the physical world: in both questionnaires this aspect is mentioned, but only a few students (2 in the pre-test and 9 in the post-test).
Table 1: Key roles (not mutual exclusive) used for IMP by PPT in Q1 and Q2. In the first two columns the number of codings is given, in the last two columns the percentage compared to the number of all codings in each questionnaire.

| Comparison                        | Q1   | Q2   | Q1 in % | Q2 in % |
|-----------------------------------|------|------|---------|---------|
| technical role                    | 11   | 318  | 4%      | 29%     |
| structural role                   | 33   | 319  | 13%     | 29%     |
| representative role               | 81   | 283  | 31%     | 26%     |
| math as tool for physics          | 38   | 37   | 15%     | 3%      |
| descriptive function of math      | 23   | 1    | 9%      | 0%      |
| Math makes the physical thought clear | 9  | 38   | 3%      | 3%      |
| Math helps the predictive function of physics | 5  | 47   | 2%      | 4%      |
| physics makes math concrete       | 17   | 57   | 7%      | 5%      |
| didactical general comments       | 41   | 1    | 16%     | 0%      |

The models are then mentioned in the pre-test among the mediators used by mathematics (15 students). With regard to the support that physics can give to mathematics, the expression "handmaiden" is used sometimes both in the pre-test (2) and in the post-test (10). Physics is also seen as an application context that allows math to acquire concreteness. In the pre-test these aspects were not considered in a significant way: two students mentioned the concreteness of math obtained thanks to physics while 11 did so in the post-test. The role of an application context is treated in the pre-test by 8 students whereas by 57 in the post-test. The aspects related to the formalization of physics through math, proposed by 40 participants, are dealt with less in the second questionnaire.

![Graphical presentation of selected changes between pre- and post-questionnaire as expressed by the students. The boxes represent selected categories. The height of the boxes is proportional to the number of codings (at least ten codings). The arrows indicate which changes took place. The thickness of the arrows is proportional to the number of students indicating the change (occurring at least three times).](image)

Figure 4. Graphical presentation of selected changes between pre- and post-questionnaire as expressed by the students. The boxes represent selected categories. The height of the boxes is proportional to the number of codings (at least ten codings). The arrows indicate which changes took place. The thickness of the arrows is proportional to the number of students indicating the change (occurring at least three times).

The change in the answers is deeper analyzed by contrasting selected categories from pre- and post questionnaires (see fig. 4) containing statements about the interrelation of physics and math. In the post-test the students far more often considered an interplay of mathematics and physics. Especially those
students (13 participants) who did not answer the pretest, wrote about a “close relation” (8/13) or that mathematics and physics “interact and support each other” (5/13) (multiple codings of an answer are possible). The details are found in figure 4 where the most often occurring changes are depicted.

2.3.4. Individual descriptions. In order to get a better impression of how students might react to the offered theoretical models and how they perhaps might modify their views we selected students who explicitly referred to their personal opinions and remarked extensively on changes or learning effects in their knowledge and views. Out of the 100 students 13 gave an individual description expressed by writing: “I have understood” or “I realized” or “By reflecting” or similar explicit expressions. Out of these we present here some exemplary cases:

- **Student 2:** This student reflects on the relevance for teaching in primary school, by starting from daily life with the goal of leading the children to physical thinking in using mathematical models and mathematical descriptions: “In my opinion it is very important to start from daily life and leading the children to physical thinking in using mathematical models and mathematical descriptions.”

- **Student 13:** This student indicates the role of the intervention to make explicit to the students which different roles the mathematics could play in physics: “I implicitly used the three roles before, but now I can categorize them.”

- **Student 21:** She gives a judgement about the intervention: “What came to me is that the intervention is very useful for the teaching process.”

- **Student 25:** His or her statement shows that it is important to give instances of reflecting on own school experiences and so to gain a more refined and adequate view on the interplay: “From my school career I saw the two disciplines distant because the math was exercised by heart while the physics was definitions by heart... I was fascinated by the part in which the different conceptual meanings of the math operations were analysed, which can profoundly change the logical meaning of a math-physical situation.”

- **Student 45:** The intervention can contribute to a better understanding of the ways in which mathematics and physics interact: “After the intervention I understood better how to use mathematics in describing and understanding physical phenomena.”

- **Student 83:** She sees more intensely the bidirectional relation between mathematics and physics and that this interplay can have many different aspects, especially in explaining physics: “I can see now more intensely the bidirectional relation between mathematics and physics and that this interplay can have many different aspects, especially in explaining physics.”

- **Student 99:** She now sees the connecting point of mathematics and physics and that they have a reciprocal relationship: “I have learned that there is a connecting point of mathematics and physics and that they have a reciprocal relationship.”

On the whole these individual descriptions show that the students often were not aware before of the strong connection between mathematics and physics. The students pronounce the opinion that this experience could help them in shaping the teaching in math and in physics (or science) at primary school in a more informed way. Mostly they see more didactic potential by the possibility of explicitly expressing the interplay and consciously using different roles. This might even be useful for relating the content to daily life.

3. Conclusion

After the courses on math education and physics education prospective primary teachers often have vague ideas on the interplay between math and physics. They mostly name typical characteristics and differences of the disciplines and a certain relation between math and physics still stemming from own school experiences. Their answers to the first questionnaire gave indications of how to shape an intervention introducing them to the importance of the interplay between mathematics and physics. The comparison of the answers given for the two questionnaires demonstrates an improvement of the awareness of the different roles mathematics can play in its interaction with physics and of the didactic
aspects. This study demonstrates the necessity of giving the future teachers a solid scientific education so that they will be able to work in a conscious and careful way.

The results in terms of the learning goals demonstrate an improvement of students’ awareness about the IMP and the typical characteristics of the disciplines and their differences. Especially they no longer only saw math as a maid for physics or physics as maid for math. However, concerning math some stressed the opinion that physics provides a context of application for mathematics. Also the epistemic role of math in physics is appreciated. On the whole they developed a more refined view with math as language of the formalization and a broader role of math in translating the physical thought into formal expressions. The emphasis lied on a mutual support and a reciprocal relation. Therefore we could say that the teaching intervention clarifies the idea of the structural role of math, the representational role and the technical one. The deepening of the reflection was induced by contrasting two different perspectives on the interplay – the procedural and the functional one.

Furthermore PPT get conscious of the possible didactic aspects of the interplay between math and physics and especially the effects in primary education. The PPT evidenced to have gained a reference frame to look at and identify many different roles of math in physics. This gives them more variations in teaching the basics of mathematics and science and allows them to analyse more deeply and critically the existing textbooks. The functional perspective contributes by identifying pathways how to integrate the two disciplines in the instruction, gaining some methodological aspects for their professional work for example in the role of geometry for modeling and abstract referent building. In addition it helped in underlining the role of physics quantities in formula. However, the PCK showing itself in choosing strategies for using the interdisciplinary perspective between the two subjects in teaching has to be improved. Our study indicates that a specific intervention module on Math&Phys interplay is necessary for PPT education to produce at least a reflection on the interplay IMP itself.

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