SELECTIONS FOR PARACONVEX-VALUED MAPPINGS ON NON-PARACOMPACT DOMAINS

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Abstract. We prove that Michael’s paraconvex-valued selection theorem for paracompact spaces remains true for $C'(E)$-valued mappings defined on collectionwise normal spaces. Some possible generalisations are also given.

1. Introduction

For a topological space $E$, let $2^E$ be the family of all nonempty subsets of $E$, and $\mathcal{F}(E)$ be the subfamily of $2^E$ consisting of all closed members of $2^E$. A set-valued mapping $\varphi : X \to 2^E$ is lower semi-continuous, or l.s.c., if the set

$$\varphi^{-1}(U) = \{x \in X : \varphi(x) \cap U \neq \emptyset\}$$

is open in $X$ for every open $U \subset E$. A set-valued mapping $\psi : X \to 2^E$ is upper-semi continuous, or u.s.c., if the set

$$\psi^#(U) = \{x \in X : \psi(x) \subset U\}$$

is open in $X$ for every open $U \subset E$. Equivalently, $\psi$ is u.s.c. if $\psi^{-1}(F)$ is closed in $X$ for every closed subset $F \subset E$. A single-valued mapping $f : X \to E$ is a selection for $\varphi : X \to 2^E$ if $f(x) \in \varphi(x)$ for every $x \in X$.

Let $E$ be a normed space. Throughout this paper, we will use $d$ to denote the metric on $E$ generated by the norm of $E$. Following [11], a subset $P$ of $E$ is called $\alpha$-paraconvex, where $0 \leq \alpha \leq 1$, if whenever $r > 0$ and $d(p, P) < r$ for some $p \in E$, then

$$d(q, P) \leq \alpha r \text{ for all } q \in \text{conv}(B_r(p) \cap P).$$

Here, $B_r(x) = \{y \in E : d(x, y) < r\}$, and conv$(A)$ is the convex hull of $A$. The set $P$ is called paraconvex if it is $\alpha$-paraconvex for some $\alpha < 1$. A closed set is 0-paraconvex if and only if it is convex. In the sequel, we will use $\mathcal{F}_\alpha(E)$ to denote all $\alpha$-paraconvex members of $\mathcal{F}(E)$ (i.e., all nonempty closed $\alpha$-paraconvex subsets of $E$).

Recall that a space $X$ is paracompact if it is Hausdorff and every open cover of $X$ has a locally finite open refinement. In [8], E. Michael proved that if $X$ is
paracompact and $E$ is a Banach space, then every l.s.c. convex-valued mapping $\varphi : X \to \mathcal{F}(E)$ has a continuous selection (see, [8, Theorem 3.2]). In [11], E. Michael generalised this result by replacing “convexity” with “$\alpha$-paraconvexity” for a fixed $\alpha < 1$, and proved the following theorem.

**Theorem 1.1** ([11, Theorem 2.1]). Let $X$ be a paracompact space, $E$ be a Banach space, and let $\varphi : X \to \mathcal{F}_\alpha(E)$ be an l.s.c. mapping, where $\alpha < 1$. Then, the following hold:

(a) $\varphi$ has a continuous selection.

(b) If $r > 0$ and $g : X \to E$ is continuous such that $d(g(x), \varphi(x)) < r$ for all $x \in X$, then there exists $\delta > 0$ and a continuous selection $f$ for $\varphi$ such that $d(g(x), f(x)) < \delta r$, $x \in X$.

It should be remarked that Theorem 1.1 is not true for $\alpha = 1$. Indeed, V. Klee [7] proved that every subset of an inner-product space $H$ (in particular, of a Hilbert space $H$) is 1-paraconvex, while not every l.s.c. mapping $\varphi : X \to \mathcal{F}_1(H)$, from a paracompact space $X$ has a continuous selection, because in this case, $\mathcal{F}_1(H) = \mathcal{F}(H)$.

Let us now state the main purpose of this paper. Namely, in Section 2, we prove a collectionwise normal version of Theorem 1.1 (see, Theorem 2.1), thus generalising [8, Theorem 3.2] (see also [1]; for alternative proofs, see [5, 13]) in terms of paraconvex sets. In Section 3, we show how our arguments can be used to generalise further some of these results.

2. **Collectionwise Normality, Paraconvexity and Selections**

Recall that a $T_1$-space $X$ is $\tau$-collectionwise normal, where $\tau$ is an infinite cardinal number, if for every discrete collection $\mathcal{D}$ of closed subsets of $X$, with $|\mathcal{D}| \leq \tau$, there exists a discrete collection $\{U_D : D \in \mathcal{D}\}$ of open subsets of $X$ such that $D \subset U_D$ for every $D \in \mathcal{D}$. A space $X$ is collectionwise normal if it is $\tau$-collectionwise normal for every $\tau$. It is well known that $X$ is normal if and only if it is $\omega$-collectionwise normal. Clearly, collectionwise normality lies between paracompactness and normality.

In what follows, for a space $E$, let $\mathcal{C}(E) = \{S \in \mathcal{F}(E) : S$ is compact$\}$, and $\mathcal{C}'(E) = \mathcal{C}(E) \cup \{E\}$. Also, for a normed space $E$, we will use the subscript $\alpha$ to denote all $\alpha$-paraconvex members of $\mathcal{C}(E)$ or $\mathcal{C}'(E)$. Finally, $w(E)$ denotes the topological weight of $E$.

**Theorem 2.1.** Let $X$ be a $\tau$-collectionwise normal space, $E$ be a Banach space with $w(E) \leq \tau$, and $\varphi : X \to \mathcal{C}'_\alpha(E)$ be an l.s.c. mapping, for some $\alpha < 1$. Then, the following hold:

(a) $\varphi$ has a continuous selection.

(b) If $r > 0$ and $g : X \to E$ is continuous such that $d(g(x), \varphi(x)) < r$ for all $x \in X$, then there exists $\delta > 0$ and a continuous selection $f$ for $\varphi$ such that $d(g(x), f(x)) < \delta r$, $x \in X$. 
To prepare for the proof of Theorem 2.1, we need the following proposition. In the proof of this proposition and what follows, a set-valued mapping \( \psi : X \to 2^E \) is a multi-selection (or, a set-valued selection) for another set-valued mapping \( \varphi : X \to 2^E \) if \( \psi(x) \subset \varphi(x) \), for every \( x \in X \).

**Proposition 2.2.** Let \( X \) be a \( \tau \)-collectionwise normal space, \( E \) be a completely metrizable space with \( w(E) \leq \tau \), \( \{V_n : n \in \mathbb{N}\} \) an increasing open cover of \( E \), and \( \varphi : X \to \mathcal{C}(E) \) an l.s.c. mapping. Then, there exists an increasing closed cover \( \{A_n : n \in \mathbb{N}\} \) of \( X \) such that \( A_n \subset \varphi^{-1}(V_n) \), for every \( n \in \mathbb{N} \).

**Proof.** Since \( \{V_n : n \in \mathbb{N}\} \) is an increasing open cover of \( E \) and \( E \) is normal and countably paracompact (being metrizable), there exists an increasing closed cover \( \{F_n : n \in \mathbb{N}\} \) of \( E \) such that \( F_n \subset V_n \), for every \( n \in \mathbb{N} \). We then have

\[
(2.1) \quad \varphi^{-1}(F_n) \subset \varphi^{-1}(V_n), \text{ for every } n \in \mathbb{N}.
\]

By a result of Choban and Valov [1] (see also Nedev [13]), there exists a u.s.c. multi-selection \( \psi : X \to \mathcal{C}(E) \) for \( \varphi \). Since \( \psi \) is u.s.c., each \( \psi^{-1}(F_n), n \in \mathbb{N} \), is closed in \( X \). Since \( \psi(x) \subset \varphi(x), x \in X \), we have

\[
\psi^{-1}(F_n) = \{x \in X : (\psi(x) \cap F_n) \neq \emptyset\} \\
\subset \{x \in X : \varphi(x) \cap F_n \neq \emptyset\} = \varphi^{-1}(F_n).
\]

The last inclusion and (2.1) imply that the family \( \{A_n : n \in \mathbb{N}\} \), with \( A_n = \psi^{-1}(F_n) \), is an increasing closed cover of \( X \) such that \( A_n \subset \varphi^{-1}(V_n) \), for every \( n \in \mathbb{N} \). \( \square \)

Recall that if \( (E, d) \) is a metric space, a mapping \( \psi : X \to 2^E \) is \( d \)-l.s.c. (resp. \( d \)-u.s.c.) if, given \( \varepsilon > 0 \), every \( x \in X \) admits a neighborhood \( V \) such that \( \psi(x) \subset B_{\varepsilon}(\psi(z)) \) (resp. \( \psi(z) \subset B_{\varepsilon}(\psi(x)) \)), for every \( z \in V \). A mapping \( \psi \) is said to be \( d \)-continuous if it is both \( d \)-l.s.c. and \( d \)-u.s.c.; \( \psi \) is continuous if it is both l.s.c. and u.s.c.; and \( \psi \) is \( d \)-proximal continuous if it is both l.s.c. and d-u.s.c. (see, [4]). Every \( d \)-continuous or continuous mapping is \( d \)-proximal continuous, but the converse is not true (see, for instance [1, Proposition 2.5]). The following Lemma also plays an essential role in the proof of Theorem 2.1.

**Lemma 2.3 ([6, Lemma 4.2]).** Let \( X \) be a \( \tau \)-collectionwise normal space, \( E \) be a Banach space with \( w(E) \leq \tau \), \( \psi : X \to \mathcal{F}(E) \) be a \( d \)-proximal continuous convex-valued mapping, and \( \varphi : X \to \mathcal{F}(E) \) be an l.s.c. convex-valued multi-selection for \( \psi \) such that \( \varphi(x) \) is compact whenever \( \varphi(x) \neq \psi(x), x \in X \). Then, \( \varphi \) has a continuous selection.

**Proof of Theorem 2.1.** Let \( X, E, \alpha, \) and \( \varphi \) be as in that theorem. We first prove (b), and then (a).

(b). Since \( \alpha < 1 \), there exists \( \gamma \in \mathbb{R} \) such that \( \alpha < \gamma < 1 \). Then, \( \sum_{i=0}^{\infty} \gamma^i < \infty \) (i.e. the series \( \sum_{i=0}^{\infty} \gamma^i \) converges). So, take \( \delta \) such that \( \sum_{i=0}^{\infty} \gamma^i < \delta \). To show
that this $\delta$ works, by induction, we shall define a sequence of continuous maps $f_n : X \to E$, $n < \omega$, such that for all $n$ and all $x \in X$,

\[(2.2) \quad d(f_n(x), \varphi(x)) < \gamma^n r,\]

\[(2.3) \quad d(f_n(x), f_{n+1}(x)) \leq \gamma^n r.\]

This will be sufficient because by (2.3), $\{f_n : n < \omega\}$ is a Cauchy sequence in $E$ which is complete, so it must converge to some continuous map $f : X \to E$. By (2.2), $f(x) \in \varphi(x)$, for every $x \in X$, and by (2.3)

\[
d(g(x), f_{n+1}(x)) = d(f_0(x), f_{n+1}(x)) \\
\leq d(f_0(x), f_1(x)) + d(f_1(x), f_2(x)) + \cdots + d(f_n(x), f_{n+1}(x)) \\
\leq r + \gamma r + \gamma^2 r + \gamma^3 r + \cdots + \gamma^n r \\
= r \sum_{i=0}^{n} \gamma^i.
\]

Therefore, $d(f(x), g(x)) \leq r \sum_{i=0}^{\infty} \gamma^i < \delta r$.

Let $f_0 = g$, which satisfies (2.2). Suppose that $f_n$ has been constructed for some $n \geq 0$, and let us construct $f_{n+1}$. Define a mapping $\psi_{n+1} : X \to \mathcal{F}(E)$ by $\psi_{n+1}(x) = B_{\gamma^n r}(f_n(x))$, $x \in X$. Then, $\psi_{n+1}$ is $d$-proximal continuous (being $d$-continuous) and convex-valued. Define another mapping $\varphi_{n+1} : X \to \mathcal{F}(E)$ by

\[\varphi_{n+1}(x) = \text{conv}(\varphi(x) \cap B_{\gamma^n r}(f_n(x))), \quad x \in X.\]

By the inductive assumption, $\varphi_{n+1}(x)$ is never empty for every $x \in X$, because $f_n$ satisfies (2.2) above. Furthermore, by [8, Propositions 2.3, 2.5, and 2.6], $\varphi_{n+1}$ is l.s.c. and it is clearly convex-valued. Finally, $\varphi_{n+1}$ is a multi-selection of $\psi_{n+1}$ and $\varphi_{n+1}(x) \neq \psi_{n+1}(x)$ implies that $\varphi_{n+1}(x)$ is compact. Then, by Lemma 2.3, $\varphi_{n+1}$ has a continuous selection $f_{n+1}(x) : X \to E$ such that

\[d(f_n(x), f_{n+1}(x)) \leq \gamma^n r,\]

which is (2.3). Since $\varphi(x)$ is $\alpha$-paraconvex for every $x \in X$, and $\alpha < \gamma$, we have

\[d(f_{n+1}(x), \varphi(x)) \leq \alpha \cdot \gamma^n r < \gamma^{n+1} r, \quad \text{for all} \ x \in X,\]

that is, $f_{n+1}$ satisfies also (2.2).

(a) Take $\beta > 1$ such that $\varphi(x) \cap B_{\beta}(0) \neq \emptyset$ for some $x \in X$, where 0 is the origin of $E$. Let

\[V_n = B_{\beta^n}(0), \quad \text{for each} \ n \in \mathbb{N}.\]

Then, each $V_n$ is open in $E$, and the family $\{V_n : n \in \mathbb{N}\}$ is an increasing open cover of $E$. By Proposition 2.2, there exists an increasing closed cover $\{B_n : n \in \mathbb{N}\}$ of $X$ such that $B_n \subset \varphi^{-1}(V_n)$, for every $n \in \mathbb{N}$. Since $X$ is normal, there are open sets $U_n \subset X$ such that $B_n \subset U_n \subset \overline{U_n} \subset \varphi^{-1}(V_n)$ and $U_n \subset U_{n+1},$
for each $n \in \mathbb{N}$. Letting $A_n = \overline{U_n}$ and following the construction in the proof of (a) in [11, Theorem 2.1], we get (using (b) above) a sequence of continuous selections $f_n : A_n \to E$ for $\varphi | A_n$ such that $f_{n+1} | A_n = f_n$, $n \in \mathbb{N}$. Then, the mapping $f : X \to E$ defined by $f | A_n = f_n$, $n \in \mathbb{N}$, is a selection for $\varphi$ which is continuous because each $f | U_n = f_n | U_n$, $n \in \mathbb{N}$, is continuous and $\{U_n : n \in \mathbb{N}\}$ is an open cover of $X$. The proof is completed. □

Remark 2.4. As it was already mentioned, the proof of (a) in Theorem 2.1 follows the proof in Theorem 1.1 (see, [11, Theorem 2.1]). However, the proof in [11] contains a minor gap where the sets $A_n$, $n \in \mathbb{N}$, were only assumed to be closed rather than $A_n = \overline{U_n}$, for some increasing open cover $\{U_n : n \in \mathbb{N}\}$ of $X$. If the condition $A_n = \overline{U_n}$, $n \in \mathbb{N}$, is not explicitly required, then the resulting selection $f : X \to E$ defined by $f | A_n = f_n$, $n \in \mathbb{N}$, may fail to be continuous. An example of such a situation is given by the function $f(x) = \sin(1/x)$, $0 < x \leq 1$, $f(0) = 0$ and $A_n = \{0\} \cup [n^{-1}, 1]$.

3. SOME POSSIBLE GENERALISATIONS

By [11, Corollary 2.2], if $X$ is paracompact, $A \subset X$ is closed and $Y$ is a closed paraconvex subset of a Banach space $E$, then every continuous $g : A \to Y$ can be extended to a continuous $f : X \to Y$. According to Dowker’s extension theorem [2], this implies that the same remains valid for $X$ being only collectionwise normal. As a rule, the theorems for the existence of continuous selections for l.s.c. mappings originated as a natural generalisation of extension theorems, see Michael [8, 9]. In view of the above, this brings the question for a more natural setting of Theorem 2.1. Namely, given $0 \leq \alpha < 1$ and a closed $\alpha$-paraconvex set $Y$ of a Banach space $E$, let $\mathcal{C}_\alpha(Y) = \{S \in \mathcal{C}_\alpha(E) : S \subset Y\}$ and $\mathcal{C}'_\alpha(Y) = \mathcal{C}_\alpha(Y) \cup \{Y\}$. Since $Y$ is $\alpha$-paraconvex, each member of $\mathcal{C}'_\alpha(Y)$ is also $\alpha$-paraconvex, so it is in good accordance with the families $\mathcal{F}_\alpha(E)$ and $\mathcal{C}_\alpha(E)$. The following question was posed to the author by V. Gutev.

Question 3.1. Let $X$ be a $\tau$-collectionwise normal space, $E$ be a Banach space, $0 \leq \alpha < 1$, and $Y$ be a nonempty $\alpha$-paraconvex closed subset of $E$, with $w(Y) \leq \tau$. Then, is it true that every l.s.c. $\varphi : X \to \mathcal{C}'_\alpha(Y)$ has a continuous selection?

To resolve Question 3.1, one can try to follow the proof of (b) of Theorem 2.1. A particular difficulty to do this is that even to take $f_0 : X \to Y$ and construct $f_1$ in a similar way, some values of $f_1$ may already go out of the set $Y$.

For an infinite cardinal number $\tau$, a $T_1$-space $X$ is called $\tau$-paracompact if every open cover $\mathcal{U}$ of $X$, with $|\mathcal{U}| \leq \tau$, has a locally finite open refinement. In the special case of $\tau = \omega$, an $\omega$-paracompact space is called countably paracompact. In contrast to paracompactness, there are $\tau$-paracompact spaces which are not normal. Of course, a space is paracompact if and only if it is $\tau$-paracompact for every $\tau$.

It is well known that if $X$ is $\tau$-paracompact and normal, $E$ is a Banach space, with $w(E) \leq \tau$, then every l.s.c. convex-valued mapping $\varphi : X \to \mathcal{F}(E)$ has a
continuous selection (see, [12]). Using exactly the same proof as for the case of paracompact spaces and the above fact, one gets the following theorem.

**Theorem 3.2.** Let $X$ be a $\tau$-paracompact and normal space, $E$ be a Banach space, with $w(E) \leq \tau$, and let $\varphi : X \to \mathcal{F}_\alpha(E)$ be an l.s.c. mapping, where $\alpha < 1$. Then, the following hold:

(a) $\varphi$ has a continuous selection.
(b) If $r > 0$ and $g : X \to E$ is continuous such that $d(g(x), \varphi(x)) < r$ for all $x \in X$, then there exists $\delta > 0$ and a continuous selection $f$ for $\varphi$ such that $d(g(x), f(x)) < \delta r$, $x \in X$.

In the special case of $\tau = \omega$, the above theorem implies the following consequence.

**Corollary 3.3.** Let $X$ be a countably paracompact and normal space, $E$ be a separable Banach space, and let $\varphi : X \to \mathcal{F}_\alpha(E)$ be an l.s.c. mapping, where $\alpha < 1$. Then, the following hold:

(a) $\varphi$ has a continuous selection.
(b) If $r > 0$ and $g : X \to E$ is continuous such that $d(g(x), \varphi(x)) < r$ for all $x \in X$, then there exists $\delta > 0$ and a continuous selection $f$ for $\varphi$ such that $d(g(x), f(x)) < \delta r$, $x \in X$.

Note that in Theorems 1.1 and 2.1, $\alpha$ is a fixed constant. Regarding this, the following question is naturally raised: do both theorems remain true if to each $x \in X$, there corresponds an $\alpha(x) < 1$ (possibly different for different $x$) for which $\varphi(x)$ is $\alpha(x)$-paraconvex? A first attempt in answering the above question was proposed by P. Semenov [14], who generalized [11, Theorem 2.1] by replacing the constant $\alpha$ by a function $h : (0, +\infty) \to [0, 1)$ satisfying a certain property (PS).

**Theorem 3.4 ([14]).** Suppose that a function $h : (0, +\infty) \to [0, 1)$ has property (PS), $X$ is a paracompact space, and $E$ is a Banach space. Then, every l.s.c. mapping $\varphi : X \to \mathcal{F}(E)$ whose values are $h$-paraconvex has a continuous selection.

Here, for an arbitrary function $H : (0, \infty) \to [0, 1)$, a functional sequence $\{H_n : n < \omega\}$ is defined such that

$$H_0(t) = 1; \text{ and } H_{n+1}(t) = H(H_n(t)t) \cdot H_n(t), \ n < \omega.$$ 

A function $h : (0, +\infty) \to [0, 1)$ has property (PS) if there is a function $H : (0, +\infty) \to [0, 1)$ strictly dominating $h$ and such that the series $\sum_{n=0}^{\infty} H_n(t)$ converges for all $t > 0$. For a function $h : (0, +\infty) \to [0, 1)$, a closed nonempty subset $P$ of a Banach space $(E, d)$ is called $h$-paraconvex if for any open ball $B$ of radius $r$ that intersects the set $P$ and for any point $q \in \text{conv}(P \cap B)$, then $d(q, P) \leq h(r)r$. A closed nonempty subset of a Banach space is said to be functionally paraconvex if it is $h$-paraconvex for some function $h : (0, +\infty) \to [0, 1)$ (see, [14]). Using the technique in the proof of Theorem 2.1, the following result is easily proved.
Theorem 3.5. Suppose that a function \( h : (0, \infty) \to [0, 1) \) has property (PS), \( X \) is a \( \tau \)-collectionwise normal space, and \( E \) is a Banach space with \( w(E) \leq \tau \). Then, every l.s.c. mapping \( \varphi : X \to \mathcal{C}^0(E) \) whose values are \( h \)-paraconvex has a continuous selection.

Note that if the function \( h \) is equal to a constant \( \alpha < 1 \), then \( h \)-paraconvexity is equivalent to \( \alpha \)-paraconvexity; and Theorem 3.4 obviously implies Theorem 1.1, while Theorem 2.1 is a consequence of Theorem 3.5.

Theorem 1.1 remains true for arbitrary domain \( X \), provided that the continuity of \( \varphi : X \to \mathcal{F}_\alpha(E) \) is strengthened to \( d \)-continuity; that is, the following holds.

Theorem 3.6. Let \( X \) be a topological space, \( E \) be a Banach space, and \( \varphi : X \to \mathcal{F}_\alpha(E) \) be a \( d \)-continuous mapping, for some \( 0 \leq \alpha < 1 \). Then, \( \varphi \) has a continuous selection.

It is unclear whether the above theorem holds when one further relaxes the continuity of the mapping \( \varphi \) to \( d \)-proximal continuity. The following question was posed to the author by V. Gutev.

Question 3.7. Let \( X \) be a topological space, \( E \) be a Banach space, and \( \varphi : X \to \mathcal{F}_\alpha(E) \) be a \( d \)-proximal continuous mapping, for some \( 0 \leq \alpha < 1 \). Then, is it true that \( \varphi \) has a continuous selection?

Question 3.7 is open even for the special case of continuous set-valued mappings.

The author would like to express his deep gratitude to Professor V. Gutev for introducing him to this topic and guiding him in the preparation of this paper. The author would also like to thank the referee for his valuable comments and suggestions.

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