Deformation, non-commutativity and the cosmological constant problem

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In this talk we provide arguments on possible relation between the cosmological constant in our space and the non-commutativity parameter of the internal space of compactified string theory. The arguments are valid in the context of D3/D7 brane cosmological model of inflation/acceleration.

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The current set of cosmological data strongly suggests that the universe experienced at least two separate stages of inflation, which is an accelerated expansion of the universe \((\dot{a} > 0)\) in a de Sitter-like state. The first inflationary stage began 13.7 billion years ago, immediately after the Big Bang. It could be very short, lasting only approximately \(10^{-35}\) seconds, but it was long enough to make our universe extremely large and to produce density perturbations responsible for the formation of galaxies. The initial value of the Hubble constant \(H\) in the beginning of inflation could be arbitrarily large, but its value at the last stages of inflation should be several orders of magnitude smaller than the Planck mass, \(H_{\text{init}} \leq 10^{-5} M_P\), since otherwise we would see gravitational waves produced during inflation. At this stage the vacuum energy was given by \(V \sim H^2 M_P^2 < 10^{-10} M_P^4\).

After inflation there was a long stage when the universe kept expanding, \(\dot{a} > 0\), but it was decelerating, \(\dot{a} < 0\). But later, approximately 5 billion years ago, the second stage of accelerated expansion began, with the Hubble parameter \(H_{\text{accel}} \sim 10^{-60} M_P\). At this stage the vacuum energy is given by \(V \sim H^2 M_P^2 \sim 10^{-120} M_P^4\); it constitutes about 70% of the total energy density of the universe at present. Thus, string theory faces a challenge to explain these two stages of accelerated expansion of the universe.

String theory and cosmology is an emergent topic. All observations so far fit 4-dimensional Einstein general relativity. The major problem here is how to get this picture and the explanation of the data from the compactified fundamental 10-dimensional string theory or 11-dimensional M-theory and supergravity. First of all, we should learn how to get any 4-dimensional de Sitter space from some string theory construction. Secondly, we would like to be more ambitious and explain the numbers like \(H_{\text{init}} \leq 10^{-5} M_P\) and \(H_{\text{accel}} \sim 10^{-60} M_P\).

Over the last few years there were several developments in string theory which attempted to address the challenge. In type-IIB string theory Giddings, Kachru and Polchinski have found a way to stabilize the dilaton-axion moduli using the 3-form fluxes. Also the proposal was made about the possibility of volume stabilization mechanism by Kachru, R. K., Linde and Trivedi, the KKLT model of de Sitter space. New interesting developments in this direction with the name of “Landscape of string theory” were proposed by Susskind and “Statistics of flux vacua” by Douglas in the spirit of Boussso-Polchinski proposal to tune the cosmological constant using the large number of flux vacua. Explicit string models with stabilized moduli have been constructed. KKLT model of de Sitter space was developed into a model of inflation by Kachru, R. K., Maldacena, McAllister, Linde and Trivedi, the KLMT model. The picture describing this model (as well as the KLMT symbol) is taken from: The throat geometry has a highly warped region related to the tip of the resolved conifold:

\[ w_1^2 + w_2^2 + w_3^2 + w_4^2 = z, \]

where the warping factor \(e^{2A}\) in \(ds^2 = e^{2A(z)} ds^2_{4D} + ds^2_{y}\) reaches its minimal value:

\[ e^{A_{\text{min}}} \sim z^{1/3} \sim e^{-\frac{2\pi k}{3y}}, \quad e^{2A_{\text{min}}} \ll 1. \]

FIG. 1: Schematic picture of the KLMT geometry: a warped Calabi-Yau manifold with throats, identified under a \(Z_2\) orientifold.
Warped geometry of the compactified space and nonperturbative effects allow to obtain AdS space with unbroken SUSY and stabilized volume. One can uplift AdS space to a metastable dS space by adding anti-D3 brane at the tip of the conifold. The small warping factor plays a significant role in the uplifting AdS vacuum to dS. The smallness of $z$ is due to the resolution of the conifold singularity. The KKLT potential consists of two parts. The $V_F$ part comes from the non-perturbative superpotential and permits an AdS vacuum (the blue plot with the minimum at negative energy). The second contribution proportional to $\sigma^{-3}$ (the red runaway curve) comes from anti-D3 brane, placed at the tip of the throat

$$V = V_F + V(\bar{D}3) \quad V(\bar{D}3) = \frac{C}{\sigma^3}.$$  

Here $C \sim z^{2/3} \sim e^{A_{min}} \ll 1$. In our example $C$ was $10^{-9}$. Small $C$ is necessary for dialing the anti-D3 energy to AdS scale to preserve and uplift the minimum to the position with a small positive value of $V$, as shown in Fig. 2 (in the black curve). The redshift in the throat plays the key role in this construction.

Thus, the advantage of using highly warped geometry is that we have a source of small parameters in string theory, where otherwise we have only the string scale and discrete fluxes and a priori no small parameters.

The disadvantage of relying totally on high warping shows up in the basic version of the inflationary KKLMIT model, highly warped region of Klebanov-Strassler geometry [9] corresponds to conformal coupling of the inflaton field (position of D3-brane in the throat region) which leads to

$$M^2_{\text{infl}} \sim H^2.$$  

However, the observed flatness of the spectrum of inflationary perturbations requires that

$$M^2_{\text{infl}} \sim 10^{-2} H^2.$$  

Few possibilities to improve the basic KKLMIT model are already known. Here, however, we will proceed with an alternative scenario, initiated by the recent work of Burgess, R. K. and Quevedo [10], where the role of uplifting of AdS to dS was given not to an anti-D3 brane, but to a D7 brane with 2-form fluxes on its world-volume, which also lead to the term proportional to $\sigma^{-3}$. Here again, to get a dS stabilization of the volume we had to place D7 in the highly redshifted region of the moduli space.

The main proposal of this talk is to switch to a new source of uplifting which is based on a non-commutative nature of the space, orthogonal to D3 brane in the D3/D7 brane system.

The D3/D7 cosmological model proposed by Dasgupta, Herdeiro, Hirano and R. K. a while ago [11], before the issue of volume stabilization was developed, has been recently reconsidered. This model gives a stringy realization of Linde’s hybrid inflation model [12], in particular, of its version related to the D-term inflation model of Binetruy-Dvali and Halyo [13]. This follows the general setting of brane inflation concept by Dvali and Tye [14].
inflaton shift symmetry: in certain situations the motion of branes does not destabilize the volume, which was also discussed by Firouzjahi and Tye [11]. The proposed inflaton shift symmetry of this model was confirmed more recently by Hsu and R. K. [16] using an important input from the work of Angelantonj, D’Auria, Ferrara and Trigiante [12] as well as Koyama, Tachikawa and Watari [19] describing this model in the context of special geometry. In familiar case of near extremal black holes duality symmetry protects exact entropy formula from large quantum corrections. In the cosmological context duality (inflaton shift symmetry) protects the flatness of the potential in D3/D7 inflation model from large quantum corrections.

\[ V = S^2 \Phi^\dagger \Phi + \frac{g^2}{2} D^2, \quad \tilde{D} = \Phi^\dagger \tilde{\sigma} \Phi - \tilde{\xi}. \]  (1)

Here \( \Phi \) is the charged hypermultiplet and \( \tilde{\xi} \) is an FI triplet, providing the resolution of the small instanton singularity, \( S \) is the distance between branes.

The mass of D3-D7 strings (hypers) is split due to the presence of the deformed flux on D7. This leads to the one-loop effect correction to the potential:

\[ V_{1-\text{loop}} = \frac{g^2 \xi^2}{2} [1 + \frac{g^2}{8 \pi^2} \ln \left( \frac{S^2}{S_{\text{cr}}^2} \right)]. \]  (2)

Here \( S_{\text{cr}} \) is the critical value of the inflaton field where dS minimum turns into dS maximum. De Sitter stage of the model (the long uplifted valley in Fig. 5):

\[ \Phi = 0, \quad S \gg S_{\text{cr}}, \quad \tilde{D} = \tilde{\xi}, \quad V \approx \frac{g^2 \xi^2}{2} \]

can explain inflation or current acceleration, depending on the parameters of the model. The D3 brane is attracted to the D7 brane due to the presence of the anti-self-dual deformed flux on D7 in accordance with spontaneously broken supersymmetry in de Sitter valley. When the critical distance between branes has been reached, the waterfall stage brings the system into the supersymmetric ground state:

\[ \Phi^\dagger \tilde{\sigma} \Phi = \tilde{\xi}, \quad S = 0, \quad \tilde{D} = 0, \quad V = 0. \]

D3/D7 bound state corresponds to the Higgs branch and non-commutative instantons. It is described by Nekrasov-Schwarz non-commutative instantons [20]. D3 can move away from D7 when the deformation parameter \( \tilde{\xi} \) vanishes, the moduli space is singular: there is no de Sitter space as shown in Fig. 7.

Resolution of singularity of the moduli space of instantons in D3/D7 Higgs branch requires \( \tilde{\xi} \neq 0 \) so that the Coulomb branch \( \tilde{D} = \Phi^\dagger \tilde{\sigma} \Phi - \xi \) with \( \Phi = 0 \) has a non-vanishing D-term potential \( V_D = \frac{2g^2 \xi^2}{2} \). Thus we see the relation between the deformation, non-commutativity and resolution of singularity which leads to de Sitter space in effective 4-dimensional space. The relation between Nekrasov-Schwarz non-commutative instantons...
and non-linear deformed instantons in Dirac-Born-Infeld theory was explained by Seiberg and Witten \textsuperscript{21}.

To be more specific, in the brane construction we consider Dirac-Born-Infeld $\kappa$-symmetric action and non-linear deformed instantons of Marino, Minassian, Moore and Strominger \textsuperscript{22} which were originally constructed for D0/D4 system. The construction of a D3/D7 bound state with unbroken supersymmetry is based on definition of unbroken supersymmetry of a set of branes via the equation

$$(1 - \Gamma)\epsilon = 0,$$

where $\Gamma$ is the $\kappa$-symmetry operator on D7 brane in the background of the D3 brane with some constant two-form $B$. According to Bergshoeff, R. K., Ortín and Papadopoulos \textsuperscript{23}, the complete dependence of $\Gamma$ of the deformed 2-form flux on the world-volume, $\mathcal{F} = dA - B$, is given by a “rotating factor” $a$

$$\Gamma = e^{-a/2} \Gamma_0 e^{a/2},$$

where $a$ depends on a 2-form $Y$

$$a = \frac{1}{2} Y_{ik} \Gamma^{ik} \sigma_3,$$

which in turn is a complicated (but known) non-linear function of $\mathcal{F}$

$$\mathcal{F} = \text{“} \tan \text{“} Y.$$ This $\kappa$-symmetric construction is a generalization and formalization of the work of Berkooz, Douglas and Leigh on unbroken supersymmetry of a combination of branes intersecting at angles \textsuperscript{24}. Non-linear deformed Abelian instanton equation

$$\frac{\mathcal{F}^-}{1 + Pf \mathcal{F}} = - \frac{B^-}{1 + Pf B}$$

follows from $(1 - \Gamma)\epsilon = 0$, it has finite energy due to non-vanishing deformation (non-commutativity) parameter $B^-$. Here $\mathcal{F}^-$ is an anti-self-dual part of the two-form $\mathcal{F}$ and $B^-$ is an anti-self-dual part of the two-form $B$.

Now we can go back to the D-term volume stabilization issue: instead of anti-D3 brane we add D7 brane. The D-term potential depends on the anti-self-dual deformed flux $\mathcal{F}^-$ and volume modulus

$$V_D \sim \frac{(\theta_1 - \theta_2)^2}{\sigma^3},$$

where $\theta_1 = \arctan \mathcal{F}_{67}$ and $\theta_2 = \arctan \mathcal{F}_{89}$. When $\theta_1$ and $\theta_2$ are small we have

$$V_D \sim \frac{(\theta_1 - \theta_2)^2}{\sigma^3} \approx \left( \frac{\mathcal{F}^-}{\sigma} \right)^2.$$

There are 2 possibilities to make this mechanism working for the uplifting of AdS minimum to dS minimum:

A) Place D7 in highly warped region of space as proposed in Burgess, R. K., Quevedo \textsuperscript{10}. The required small fine-tuning parameter $C$ is due to the warping associated with the deformation of the conifold $z$.

B) Use non-commutativity of the internal space: irrational $B$ cannot be gauged away into quantized $F = dA$. Quantization of the flux $\int F = 2\pi n$ may have prevented the D-term contribution from D7 of the form $\frac{\mathcal{F}^-}{\sigma}$ to provide a small $C$ when D7 is not at the tip of the conifold but somewhere in the part of Calabi-Yau space. Here the deformation parameter $B^-$ in $\mathcal{F}^-$ comes to rescue, it is not quantized, it can be small! We may choose therefore the vanishing quantized part of the 2-form so that $\mathcal{F}^-$ may be tuned via deformation parameter to uplift any AdS minimum into a de Sitter minimum since $C \sim (B^-)^2$.

In the context of non-commutative Nekrasov-Schwarz instantons and of Seiberg-Witten non-linear instantons in DBI action, Fayet-Iliopoulos terms are necessary to make the Abelian instantons non-singular. It is tempting to speculate that in D3/D7 cosmological model with volume stabilization mechanism there is an explanation of the non-vanishing effective cosmological constant: the non-commutativity parameter (FI term in effective theory) is needed to remove the instanton moduli space singularity in the description of the supersymmetric D3/D7 bound state when D3 is dissolved into D7. The same cosmological model must have a non-supersymmetric de Sitter stage when D3 is separated from D7.

Both methods A) and B) can be used for uplifting the AdS vacuum to dS vacuum, which is necessary for describing the present state of acceleration of the universe with $H_{\text{accel}} \sim 10^{-60} M_P$. However, the method B) based on the use of FI terms and non-commutativity seems to have an important advantage when we want to make the second uplifting of the vacuum energy in order to describe the stage of inflation: This method allows us to obtain in a natural way an inflationary flat direction with $m^2 \ll \dot{H}^2$, as shown in Figs. 4 and 5.

It is quite interesting that the amplitude of inflationary perturbations of metric and the resulting CMB anisotropy in the theory with the potential (1), (2) coincides with the FI term $\xi$, up to a numerical coefficient $O(1)$ \textsuperscript{25}. This suggests an intriguing possibility that one can measure the non-commutativity parameters of the internal space by looking at the sky.

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