Brane-induced supersymmetry breaking

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Abstract: We study spontaneous supersymmetry breaking induced by brane-localized dynamics in five-dimensional supergravity compactified on $S^1/Z_2$. We consider a model with gravity in the bulk and matter localized on tensionless branes at the orbifold fixed points. We assume that the brane dynamics give rise to effective brane superpotentials that trigger the supersymmetry breaking. We analyze in detail the super-Higgs effect. We compute the full spectrum and show that the symmetry breaking is spontaneous but nonlocal in the fifth dimension. We demonstrate that the model can be interpreted as a new, non-trivial implementation of a coordinate-dependent Scherk-Schwarz compactification.

Keywords: Supersymmetry Breaking, Supergravity Models, Field Theories in Higher Dimensions.
1. Introduction

Models with extra dimensions have attracted considerable attention because they provide a geometrical approach to the hierarchy problems that afflict modern particle physics: the gauge hierarchy and the cosmological constant. It is widely believed that (broken) supersymmetry may play a role in generating and stabilizing these hierarchies.

In this paper we study supersymmetry breaking induced by brane-localized dynamics in higher dimensional theories [1, 2]. (Related work can be found in [3, 4]). For simplicity we consider compactifications from five to four dimensions on an $S^1/Z_2$ orbifold, but the mechanism we describe can be extended to higher dimensions. We assume that the bulk contains pure five dimensional supergravity; the discussion can be generalized to include bulk vector multiplets and hypermultiplets as well. We imagine that tensionless branes are placed at the orbifold fixed points. The branes do not generate a warp factor, so the bosonic vacuum is flat.

We start by writing down a five-dimensional action that describes bulk supergravity interacting with the vevs of the brane superpotentials. The action is invariant under the full set of supersymmetries that are consistent with the orbifold construction. Since the fifth dimension is compactified on an orbifold, the five-dimensional supersymmetries split into an infinite number of four dimensional supersymmetries. All but one are nonlinearly realized.

We then study how brane-localized matter can spontaneously break the remaining $N = 1$ supersymmetry. Our construction is independent of the details of the brane physics; we simply assert that each brane has an effective superpotential, the remnant of some localized brane dynamics. We assume that each superpotential receives a constant expectation
value (vev). The superpotential vevs do not affect the brane tensions, so our construction describes a scenario with vanishing \( F \) - and \( D \)-terms for the brane-localized matter. The construction reproduces the main features of gaugino condensation [1] in \( M \)-theory [5], at the level of an effective five-dimensional Lagrangian.

The initial part of our discussion follows, albeit in a simpler context, the treatment of ref. [2]. However, we go further in two important respects. First, we provide a complete analysis of the super-Higgs effect and derive the spectrum for all the Kaluza-Klein modes. Second, we apply the results of [6] and demonstrate that the model can be interpreted as a new type of coordinate-dependent Scherk-Schwarz compactification [7]. We show that our brane action induces a set of generalized boundary conditions on the gravitino fields, with field discontinuities at the orbifold fixed points.

At low energies, our five-dimensional theory reduces to spontaneously broken no-scale supergravity [9], provided the supersymmetry-breaking mass splittings are small relative to the Kaluza-Klein scale. The order parameter for supersymmetry breaking is nonlocal in the fifth dimension [1], and is proportional to the average of the superpotential vevs on the branes. We illustrate how this non-locality ameliorates the instability problems [10] of standard no-scale supergravities [9]. (This was previously stressed in [11] for conventional Scherk-Schwarz compactifications.)

2. The bulk action and its spectrum

Our starting point is pure five-dimensional Poincaré supergravity [12] in its on-shell formulation. The supergravity multiplet contains the fünfbein \( e^A_M \), the gravitino \( \Psi_M \) and the graviphoton \( B_M \). The five-dimensional bulk Lagrangian reads

\[
\kappa L_{\text{bulk}} = -\frac{1}{2\kappa^2} e_5 R_5 - \frac{1}{4} e_5 F_{MN} F^{MN} - \frac{\kappa}{6\sqrt{6}} e^{MNOPQ} F_{MN} F_{OP} B_Q + i \epsilon^{Mnopq} \Psi_M \Sigma_{NO} D_P \Psi_Q - i \sqrt{\frac{3}{2}} \kappa e_5 F_{MN} \Psi^M \Psi^N + i \sqrt{\frac{3}{2}} \kappa e^{MNO} F_{MN} \Psi_O \Gamma_P \Psi_Q + 4\text{-fermion terms}.
\]

Unless otherwise stated, our five-dimensional notation is identical to ref. [13]; our four-dimensional notation is the same as in ref. [14]. In particular, the five-dimensional coordinates are \( x^M = (x^m, x^5) \); \( \hat{5} \) is the fifth tangent-space index; \( M_5 \equiv \kappa^{-1} \) is the (reduced) five-dimensional Planck mass; \( e_5 = \det e^A_M \); \( R_5 \) is the five-dimensional scalar curvature; \( e_4 = \det e^a_A \), where the latter are the components of the fünfbein with four-dimensional indices; and \( e^{MNO} = e_5 e^A_M e^B_N e^C_O e^D_P e^E_Q e^{abcdef} \), \( e^{mnop} = e_4 e^m_a e^n_b e^o_c e^p_d e^{abcd} \), \( \epsilon^{01235} = \epsilon_{0123} = +1 \).

It is not hard to check that the bulk supergravity Lagrangian is invariant, up to a total derivative, under the following supersymmetry transformations,

\[
\delta e^A_M = i\kappa (\bar{\eta} \Gamma^A \Psi_M - \bar{\Psi}_M \Gamma^A \eta),
\]

\( ^3 \)The physical dimensions of the fields depend, of course, on the powers of \( M_5 \) that are inserted in the different terms of \( L_{\text{bulk}} \). Our conventions are chosen so that all fields have canonical dimension after compactification from five to four dimensions.
\[ \delta B_M = -i \sqrt{\frac{3}{2}} (\eta \Psi M - \overline{\Psi} M \eta), \]

\[ \delta \Psi_M = \frac{2}{\kappa} D_M \eta - \sqrt{\frac{2}{3}} \left( \Gamma^N F_{NM} - \frac{1}{4} e_5 \epsilon_{MNOP} F^{NO} \Sigma^{PQ} \right) \eta \]

+ 3-fermion terms. \hspace{1cm} (2.2)

The transformation parameter \( \eta(x^M) \) is a five-dimensional Dirac spinor. Here and hereafter, we neglect all three- and four-fermion terms.

In what follows, we take the fifth dimension to be compactified on the orbifold \( S^1/\mathbb{Z}_2 \), obtained by the identification \( x^5 \leftrightarrow -x^5 \). For convenience, we choose to work on the orbifold covering space, so we let \( x^5 \) vary in the interval \([-\pi \kappa, \pi \kappa]\). We define the bulk action to be \(^2\)

\[ S_{\text{bulk}} = \frac{1}{2} \int d^4 x \int_{-\pi \kappa}^{+\pi \kappa} dx^5 L_{\text{bulk}}, \] \hspace{1cm} (2.3)

where the factor of \((1/2)\) avoids double-counting equivalent points.

We take our fields to be fluctuations off the following background,

\[ \langle g_{MN} \rangle = \begin{pmatrix} \eta_{mn} & 0 \\ 0 & r^2 \end{pmatrix}, \] \hspace{1cm} (2.4)

where \( r \neq 0 \) is an undetermined real constant and all other background fields are assumed to vanish. (In principle, \( \langle B_5 \rangle \) is a second undetermined real constant, but we set it to zero as well.) This background is a solution to the five-dimensional equations of motion. From (2.4) we deduce the relation between the four-dimensional Planck mass \( M_4 \) and \( M_5 \equiv \kappa^{-1}, \)

\[ M_4^2 = \pi R \cdot M_5^3, \] \hspace{1cm} (2.5)

where \( R = r/M_5 \) is the physical compactification radius.

We define the action of the orbifold symmetry in such a way that the action, the transformation laws and the background are all invariant. Decomposing the five-dimensional spinor \( \Psi \), and its conjugate \( \overline{\Psi} \), into four-dimensional form, and following the convention that \( \Psi \equiv (\psi^1_m, \psi^2_5) \) and \( \overline{\Psi} \equiv (\bar{\psi}^a_1, \bar{\psi}^a_5) \), we assign even \( \mathbb{Z}_2 \)-parity to

\[ e_m^a, \ e_5^a, \ B_5, \ \psi^1_m, \ \psi^2_5, \ \eta^1, \] \hspace{1cm} (2.6)

and odd \( \mathbb{Z}_2 \)-parity to

\[ e_m^a, \ e_5^a, \ B_5, \ \psi^2_m, \ \psi^1_5, \ \eta^2. \] \hspace{1cm} (2.7)

From a four-dimensional point of view, the physical spectrum contains one massless \( N = 1 \) gravitational multiplet, with spins \((2, 3/2)\), built from the zero modes of \( e_m^a \) and \( \psi^1_m \); one massless \( N = 1 \) chiral multiplet, with spins \((1/2, 0)\), composed of the zero modes

\(^2\)Note that the limits of the integration over the fifth coordinate are set by convention: If we let \( x^5 \) vary between \(-\pi/m \) and \( \pi/m \), the physical compactification radius is \( R = r/m \). We choose to set \( m = M_5 \).
of $\psi^2_5$, $e_{55}$ and $B_5$; and an infinite series of (short) massive multiplets of $N = 2$ supergravity, with spins $(2, 3/2, 3/2, 1)$ and squared masses

$$M_n^2 = \frac{n^2}{\kappa^2}, \quad (n = 1, 2, \ldots).$$

The Kaluza-Klein tower gains mass through an infinite series of Higgs and super-Higgs effects, each occurring at its own mass level [13]. The Kaluza-Klein gravitons and graviphotons gain mass by eating the Fourier modes of the fields $g_{m5}$, $g_{55}$ and $B_5$, while the massive gravitinos eat the Fourier modes of the field $\Psi_5$. This is consistent with the fact that the parameter of five-dimensional supersymmetry, $\eta(x^M)$, has an infinite number of Fourier modes. Each of the modes generates a supersymmetry; in the absence of matter, all but one are spontaneously broken. The broken supersymmetries implement an infinite series of super-Higgs effects for the massive gravitinos. The remaining supersymmetry is the $N = 1$ supersymmetry of the four-dimensional low-energy effective action.

3. Brane action and modified supersymmetry transformations

Having described the bulk action, we now construct the brane action. We are not interested in the details of the brane physics, so we imagine that the brane fields are integrated out, leaving a constant superpotential vev on each brane. We assume the physics is such that the tension vanishes on each brane. We shall see that the superpotential vevs can spontaneously break the remaining $N = 1$ supersymmetry.

In general, the physics is different on the two branes, so the superpotential vevs need not be the same. The brane action we adopt, in analogy with [1, 2], is

$$S_{brane} = \frac{\kappa^2}{2} \int d^4x \int_{-\pi\kappa}^{+\pi\kappa} dx^5 \, e_4 \left[ \delta(x^5)P_0 + \delta(x^5 - \pi\kappa)P_\pi \right] \psi^a_1 \sigma^{ab} \psi^b_1 + \text{h.c.},$$

where $P_0$ and $P_\pi$ are complex constants with the dimension of $(mass)^3$ which parametrize the vevs of the superpotentials.

It is not hard to compute the variation of the brane action using the transformations inherited from the bulk. We find

$$\delta S_{brane} = \frac{\kappa^2}{2} \int d^4x \int_{-\pi\kappa}^{+\pi\kappa} dx^5 \, e_4 \left[ \delta(x^5)P_0 + \delta(x^5 - \pi\kappa)P_\pi \right] \cdot \left( 4 \psi^a_1 \sigma^{mn} D_n \eta^1 + i \sqrt{\frac{3}{2}} F^{5n} \psi^a_1 \eta^1 \right) + \text{h.c.},$$

where $D_n \eta^1$ contains the spin connection $\omega_{nab}$. In writing eq. (3.3), we exploit the fact that $\omega_{na5}$ vanishes on the branes, consistently with the bosonic jump conditions.

The variation (3.2) can be cancelled by modifying the transformation laws of $\psi^2_5$,

$$\delta \psi^2_5 = \delta \psi^2_5 \big|_{old} + 2\kappa^2 \left[ \delta(x^5)P_0 + \delta(x^5 - \pi\kappa)P_\pi \right] \eta^1,$$
where $\delta \psi^2_{i\text{odd}}$ is as in eq. (2.2). With this new transformation, the bulk action is not invariant,

$$
\delta S_{\text{bulk}} = \frac{\kappa^2}{2} \int d^4x \int_{-\pi\kappa}^{+\pi\kappa} dx^5 e_4 \left[ \delta(x^5) \overline{P}_0 + \delta(x^5 - \pi\kappa) \overline{P}_\pi \right] \cdot \left( \frac{4}{\kappa} \psi^1_m \sigma^{mn} D_n \eta^1 + i \sqrt{\frac{3}{2}} \hat{F}^5_n \psi^1_n \eta^1 \right) + \text{h.c.},
$$

(3.4)

where again we exploit the fact that $\omega_{nab}$ vanishes at the fixed points. If we define the total action to be $S = S_{\text{bulk}} + S_{\text{brane}}$, the variation $\delta S_{\text{bulk}}$ precisely cancels $\delta S_{\text{brane}}$, for any values of $P_0$ and $P_\pi$.

4. The super-Higgs effect

In the previous section we constructed a five-dimensional bulk-plus-brane action that is supersymmetry invariant. The brane action is purely fermionic, so the superpotential vevs do not change the bosonic equations of motion. In particular, the branes remain tensionless, so the bosonic background does not warp.

In this section we study the supersymmetry breaking induced by the superpotential vevs. We give a complete description of the super-Higgs effect and the resulting spectrum for all the gravitino modes. The initial part of our discussion is close to the treatment of [1], but avoids many of the complications that arise from Horava-Witten theory, such as the additional moduli of the Calabi-Yau manifold, the warp factors, etc.

We start by searching for solutions to the Killing spinor equations, which are determined by the right-hand sides of the supersymmetry transformations, evaluated in the bosonic background. The only nontrivial equations arise from the variations of $\psi^1_5$ and $\psi^2_5$.

We find

$$
\begin{align*}
2 \kappa \partial_5 \eta^1 &= 0, \\
2 \kappa \partial_5 \eta^2 &= -2\kappa^2 \left[ \delta(x^5) \overline{P}_0 + \delta(x^5 - \pi\kappa) \overline{P}_\pi \right] \eta^1.
\end{align*}
$$

(4.1)

These equations have no solution on the circle except when $\overline{P}_0 = -\overline{P}_\pi$. In this case supersymmetry is preserved, and

$$
\eta^1 = \zeta,
\eta^2 = -\kappa^3 \left( \frac{\overline{P}_0 - \overline{P}_\pi}{4} \right) \epsilon(x^5) \zeta,
$$

(4.2)

is the Killing spinor for constant $\zeta$; $\epsilon(x^5)$ is the ‘sign’ function defined on $S^1$. When $\overline{P}_0 \neq -\overline{P}_\pi$, there is no Killing spinor, and supersymmetry is broken spontaneously. The amount of supersymmetry breaking is fixed by the order parameter $F = \kappa(\overline{P}_0 + \overline{P}_\pi)$.

This analysis indicates that the supersymmetry breaking is controlled by the vevs of the superpotentials on the two branes. These vevs are determined independently, by the physics on each brane, separated by a finite distance in the $x^5$ direction. In this sense the order parameter for supersymmetry breaking is nonlocal, as in [1].
The non-locality of the order parameter makes it worthwhile to study in detail how the supersymmetry breaking is realized. In particular, one would like to identify the Goldstone fermion and investigate the super-Higgs effect. To see how this works, we focus on the fermion bilinears and set all the bosonic fields to their background values. We find

\[ S_{2f}^{(5)} = \frac{1}{2} \int d^4 x \int_{-\pi \kappa}^{+\pi \kappa} dx^5 \left\{ \frac{\kappa}{2} e^{\epsilon mnpq} (\psi^{\dagger}_m \sigma_n \partial_p \psi^1_q - \psi^2_m \sigma_n \partial_p \psi^{\dagger}_q) \right\} \]

\[ + \frac{2}{\kappa} e_4 \left( -\psi^2_5 \sigma^{mn} \partial_5 \psi^1_n + \psi^1_5 \sigma^{mn} \partial_5 \psi^1_n + \psi^2_5 \sigma^{mn} \partial_5 \psi^1_n \right) \]

\[ - \psi^1_5 \sigma^{mn} \partial_n \psi^1_5 + \psi^2_5 \sigma^{mn} \partial_n \psi^1_5 - \psi^1_5 \sigma^{mn} \partial_n \psi^1_5 \right) \]

\[ + (e_4 \kappa^2 \left[ \delta(x^5) \mathcal{P}_0 + \delta(x^5 - \pi \kappa) \mathcal{P}_{\pi} \right] \psi^1_5 \sigma^{mn} \psi^1_n + \text{h.c.}) \right\} . \tag{4.3} \]

We then write this action in terms of four dimensional fields, which we define through a Fourier expansion:

\[ \psi^+(x^5) = \frac{1}{\sqrt{\pi} r} \left[ \psi^0 + \sqrt{2} \sum_{\rho = 1}^{\infty} \psi^0_\rho \cos \rho M_5 x^5 \right] , \]

\[ \psi^-(x^5) = \frac{1}{\sqrt{\pi} r} \left[ \sqrt{2} \sum_{\rho = 1}^{\infty} \psi^-_\rho \sin \rho M_5 x^5 \right] . \tag{4.4} \]

In this expression, \( \psi^+ \) stands for \( (\psi^1_5, \psi^3_5) \) and \( \psi^- \) for \( (\psi^2_5, \psi^4_5) \). The expansion is consistent with the boundary conditions and the \( Z_2 \)-parity assignments for the fields. We substitute these expressions into (4.3) and integrate over \( x^5 \) to obtain

\[ \mathcal{L}^{(4)}_{2f} = \frac{1}{2} e^{\epsilon mnpq} \left( \psi_{\rho,0} \sigma_q \partial_m \psi^1_{n,0} + \sum_{\rho = 1}^{\infty} \psi_{\rho,0} \sigma_q \partial_m \psi^1_{n,\rho} + \sum_{\rho = 1}^{\infty} \psi_{\rho,0} \sigma_q \partial_m \psi^2_{n,\rho} \right) \]

\[ + \frac{2}{\kappa} e_4 \left( \psi^2_{5,0} \sigma^{mn} \partial_m \psi^1_{n,0} + \sum_{\rho = 1}^{\infty} \psi^2_{5,0} \sigma^{mn} \partial_m \psi^1_{n,\rho} - \sum_{\rho = 1}^{\infty} \psi^1_{5,0} \sigma^{mn} \partial_m \psi^2_{n,\rho} \right) \]

\[ + \frac{2}{\kappa} e_4 \sum_{\rho = 1}^{\infty} (\rho M_5) \psi^2_{m,\rho} \sigma^{mn} \psi^1_{n,\rho} \]

\[ + \frac{\kappa^2}{2 \pi} e_4 \mathcal{P}_0 \left[ \psi^1_{m,0} + \sqrt{2} \sum_{\rho = 1}^{\infty} \psi^1_{m,\rho} \right] \sigma^{mn} \left[ \psi^1_{n,0} + \sqrt{2} \sum_{\sigma = 1}^{\infty} \psi^1_{n,\sigma} \right] \]

\[ + \frac{\kappa^2}{2 \pi} e_4 \mathcal{P}_{\pi} \left[ \psi^1_{m,0} + \sqrt{2} \sum_{\rho = 1}^{\infty} (-1)^\rho \psi^1_{m,\rho} \right] \sigma^{mn} \left[ \psi^1_{n,0} + \sqrt{2} \sum_{\sigma = 1}^{\infty} (-1)^\sigma \psi^1_{n,\sigma} \right] \]

\[ + \text{h.c.} \] \tag{4.5}

This expression shows that the brane superpotentials induce mixings between the different Fourier modes. These mixings considerably complicate the discussion of the super-Higgs effect, as in [2]. For generic values of \( P_0 \) and \( P_\pi \), with \( P_0 \neq -P_\pi \), the fields \( \psi^1_{5,\rho} \),
ψ^2_{5,0} and ψ^2_{5,ρ} (ρ > 0) are all goldstinos. They are absorbed by the gravitinos through the following field transformations:

ψ^1_{m,0} \rightarrow ψ^1_{m,0} + \frac{2π}{κ^2 (P_0 + P_π)} \partial_m ψ^2_{5,0} + \sum_{ρ=1}^{∞} \frac{\sqrt{2}}{ρ M_5} \left[ \frac{P_0 + (-1)^{ρ} P_π}{P_0 + P_π} \right] \partial_m ψ^1_{5,ρ}, \quad (4.6)

ψ^1_{m,ρ} \rightarrow ψ^1_{m,ρ} - \frac{1}{ρ M_5} \partial_m ψ^1_{5,ρ}, \quad (4.7)

ψ^2_{m,ρ} \rightarrow ψ^2_{m,ρ} - \frac{\sqrt{2}}{ρ M_5} \left[ \frac{P_0 + (-1)^{ρ} P_π}{P_0 + P_π} \right] \partial_m ψ^2_{5,0} + \sum_{σ=1}^{∞} \frac{1}{π ρσ (P_0 + P_π)} [1 - (-1)^{ρ}] [1 - (-1)^{σ}] \partial_m ψ^1_{5,σ} + \frac{1}{ρ M_5} \partial_m ψ^2_{5,ρ}. \quad (4.8)

These transformations define the “unitary gauge”: they eliminate all the terms containing ψ^1_{5,ρ}, ψ^2_{5,0} and ψ^2_{5,ρ} (ρ > 0), from the Lagrangian (4.5). Moreover, they permit us to read the infinite-dimensional gravitino mass matrix directly from eq. (4.5).

When \(P_0 = -P_π\), the transformations (4.6-4.8) are singular and it is not possible to remove all the modes of ψ^1_5 and ψ^2_5 from the Lagrangian. There is one linear combination that remains massless,

ψ_5^{(0)} = \alpha \left( ψ^1_{n,0} - \sqrt{2} \frac{κ^3}{π} \frac{P_0}{P_π} \sum_{ρ=0}^{∞} \frac{1}{2ρ + 1} ψ^1_{n,2ρ+1} \right)

= \alpha M_5 \sqrt{\frac{r}{4π}} \int_{-π/M_5}^{+π/M_5} dx^5 \left[ ψ^1_5 + \left( \frac{κ^3 P_0}{2} \right) \epsilon(x^5) ψ^2_5 \right], \quad (4.9)

where \(α = (1 - κ^6 |P_0|^2/4)^{-1/2}\) is a normalization. This form is consistent with the solution to the Killing spinor equations, which fixes the gravitino zero mode to be

ψ^1_n = ψ^{(0)}_n, \quad ψ^2_n = -\frac{κ^3 P_0}{2} \epsilon(x^5) ψ^{(0)}_n. \quad (4.10)

The massless gravitino indicates that four-dimensional \(N = 1\) supersymmetry is left unbroken. There is also one two-component massless fermion that is not absorbed by the super-Higgs mechanism. It is described by the combination

ψ_5^{(0)} = \alpha \left( ψ^2_{5,0} + \sqrt{2} \frac{κ^3}{π} \frac{P_0}{P_π} \sum_{ρ=0}^{∞} \frac{1}{2ρ + 1} ψ^1_{5,2ρ+1} \right)

= \alpha M_5 \sqrt{\frac{r}{4π}} \int_{-π/M_5}^{+π/M_5} dx^5 \left[ ψ^2_5 - \left( \frac{κ^3 P_0}{2} \right) \epsilon(x^5) ψ^1_5 \right]. \quad (4.11)

An additional linear transformation in the space \((ψ^{(0)}_n, ψ^{(0)}_5)\) is needed to diagonalize the kinetic terms for the massless fermions.
In the appendix we derive the gravitino mass spectrum. Taking for simplicity $P_0$ and $P_\pi$ to be real, we find:

\[ M_{3/2}^{(\rho)} = \frac{\rho}{R} + \frac{\delta_0 + \delta_\pi}{2\pi R}, \quad (\rho = 0, \pm 1, \pm 2, \ldots), \tag{4.12} \]

where

\[ \delta_0(\pi) = 2 \arctan \left( \frac{\kappa P_0(\pi)}{2} \right). \tag{4.13} \]

Note that when $(P_0 + P_\pi) \neq 0$, the gravitino masses are shifted with respect to their supersymmetric partners. Moreover, the lightest gravitino has a non-vanishing mass. These facts show that supersymmetry is indeed spontaneously broken.

5. Relation to coordinate-dependent compactifications

Our bulk-plus-brane construction gives a mass spectrum that is reminiscent of the conventional Scherk-Schwarz mechanism \[7\], in which all fields are smooth, and the boundary conditions are twisted by a global symmetry of the five-dimensional theory.\(^3\) The twist shifts each of the eigenvalues of the gravitino mass matrix by the same amount, exactly as in (4.12).

The analogy between the two cases is not limited to the fermionic spectrum. In each case, the bosonic fields have the same masses as when supersymmetry is unbroken; the bulk and brane contributions to the vacuum energy vanish classically; and the compactification radius is a classical flat direction (together with its super-partner, the axionic phase associated with the fifth component of the graviphoton). There is, however, an important difference. In the conventional Scherk-Schwarz mechanism, the universal mass shift arises from a bulk mass term. In our construction, the shift arises from mass terms localized at the orbifold fixed points. The localized masses induce mixings between all levels of the Kaluza-Klein decomposition.

As we have shown in a companion paper \[8\], a suitable generalization of the Scherk-Schwarz mechanism can give rise to the localized mass terms. The generalization makes use of twisted boundary conditions, as in the usual Scherk-Schwarz mechanism, but allows the fields to have cusps and discontinuities (or ‘jumps’) at the orbifold fixed points. In this section we will see that the five-dimensional supergravity action, with smooth gravitinos and twisted boundary conditions, is equivalent to a bulk-plus-brane action, with periodic gravitinos and jumps at the orbifold fixed points.

We start by recalling the essential features of the conventional Scherk-Schwarz mechanism, for the case of five-dimensional Poincaré supergravity compactified on $S^1/Z_2$. The Lagrangian has a global $SU(2)_R$ invariance, under which $\Phi_M \equiv (\psi^1_M, \psi^2_M)^T$ transforms as a doublet. $[\Phi_M$ should not be confused with $\Psi_M \equiv (\psi^1_M, \psi^2_M)^T.]$ The gravitino boundary conditions are twisted by a $U(1)_R \subset SU(2)_R$ transformation,

\[ \Phi^c_M(x^5 + 2\pi \kappa) = e^{-i\beta \sigma^2} \Phi^c_M(x^5), \tag{5.1} \]

\(^3\)Some similarities between gaugino condensation in Horava-Witten theory and the Scherk-Schwarz mechanism were noticed in \[4\].
where $\sigma^2$ is a Pauli matrix acting on the space of $(\psi^1_M, \psi^2_M)^T$. This twist is consistent with the orbifold projection defined in eqs. (2.6) and (2.7). The label ‘c’ indicates that the fields are continuous across the orbifold fixed points,

$$\Phi^c_M(\pm \xi) = \Phi^c_M(-\xi), \quad \Phi^c_M(\pi \kappa + \xi) = \Phi^c_M(\pi \kappa - \xi), \quad (0 < \xi \ll 1). \quad (5.2)$$

The twisted boundary conditions break the four-dimensional supersymmetry. To see how this works, we change to gravitino fields $\tilde{\Phi}_M(x^5)$ that are periodic on the circle. The twisted fields are related to the untwisted fields by

$$\Phi^c_M(x^5) = V(y) \tilde{\Phi}_M(x^5), \quad (5.3)$$

where

$$V(y) = \exp\left(-\frac{i\beta \sigma^2 x^5}{2\pi \kappa}\right). \quad (5.4)$$

We then substitute (5.3) into the Lagrangian (2.1). The only new terms are those in which the $x^5$ derivatives act on the fermionic fields. We find

$$V^\dagger \partial_5 \Phi^c_M = \left[\partial_5 + V^\dagger \partial_5 V\right] \tilde{\Phi}_M \equiv \tilde{D}_5 \tilde{\Phi}_M, \quad (5.5)$$

which implies that $\tilde{D}_5 \tilde{\Phi}_M$ is a covariant derivative, with constant connection $A_5 \equiv V^\dagger \partial_5 V = -\frac{i\beta \sigma^2}{2\pi \kappa}. \quad (5.6)$

The connection gives rise to a supersymmetry-breaking gravitino mass term, one that shifts the gravitino spectrum at each mass level:

$$\mathcal{M}^{(\rho)}_{3/2} = \frac{\rho}{R} - \frac{\beta}{2\pi R}, \quad (\rho = 0, \pm 1, \pm 2, \ldots). \quad (5.7)$$

We are now ready to show that the bulk-plus-brane action has an alternative interpretation in terms of a generalized Scherk-Schwarz mechanism [6]. Because we have compactified on the orbifold $S^1/Z_2$, the gravitino boundary conditions are characterized by an overall twist and by discontinuities at the orbifold fixed points. We start with the conventional Scherk-Schwarz fields $\Phi^c_M$, with twist parameter $\beta$. We then perform the following field redefinition:

$$\Phi^c_M(x^5) = e^{i\alpha(x^5)\sigma^2} \Phi_M(x^5), \quad (5.8)$$

where

$$\alpha(x^5) = \frac{\delta_0 - \delta_{\pi}}{4} \epsilon(x^5) + \frac{\delta_0 + \delta_{\pi}}{4} \eta(x^5). \quad (5.9)$$

In this expression, $\epsilon(x^5)$ is the ‘sign’ function, and

$$\eta(x^5) = 2n + 1, \quad n\pi \kappa < x^5 < (n + 1)\pi \kappa, \quad (n \in Z), \quad (5.10)$$

is the ‘staircase’ function that steps by two units every $\pi \kappa$ along $x^5$. 
From these expressions, it is not hard to check that the fields $\Phi_M(x^5)$ obey the following jump conditions at the orbifold fixed points:
\[
\Phi_M(+\xi) = e^{i\delta_0} \sigma^2 \Phi_M(-\xi), \quad \Phi_M(\pi \kappa + \xi) = e^{i\delta_\pi} \sigma^2 \Phi_M(\pi \kappa - \xi). \tag{5.11}
\]
The fields $\Phi_M(x^5)$ also have twist $\beta + \delta_0 + \delta_\pi$. Indeed, if we choose
\[
\beta = - (\delta_0 + \delta_\pi), \tag{5.12}
\]
the fields $\Phi_M(x^5)$ are periodic.

The bulk action is not invariant under this field redefinition. As before, the $x^5$ derivatives give rise to a connection $A_5$. Now, however, the connection is singular; it generates a brane action that is localized at the orbifold fixed points,
\[
S_{brane} = \frac{1}{2\kappa} \int d^4x \int_{-\pi \kappa}^{+\pi \kappa} dx^5 e_4 \left[ \delta(x^5) \delta_0 + \delta(x^5 - \pi \kappa) \delta_\pi \right] \left( \psi^{1a}_a \sigma^{ab} \psi^{1}_b + \psi^{2a}_a \sigma^{ab} \psi^{2}_b \right) + \text{h.c.} \tag{5.13}
\]

Supersymmetry invariance of the total action $S = S_{bulk} + S_{brane}$ is guaranteed by the fact that we have redefined the fields of an invariant bulk action. The supersymmetry transformations for the fields $\psi^{1,2}_5$ are easily derived,
\[
\delta \psi^{1}_5 = \left[ \delta \psi^{1}_5 \right]_{old} - 2\kappa^2 \left[ \delta(x^5) P_0^0 + \delta(x^5 - \pi \kappa) P_\pi^\pi \right] \eta^2, \\
\delta \psi^{2}_5 = \left[ \delta \psi^{2}_5 \right]_{old} + 2\kappa^2 \left[ \delta(x^5) P_0^0 + \delta(x^5 - \pi \kappa) P_\pi^\pi \right] \eta^1, \tag{5.14}
\]
where $\left[ \delta \psi^{1,2}_5 \right]_{old}$ is as in (2.2). For the special parameter choice $P_0^0 = -P_\pi^\pi$, supersymmetry is not broken, and the Killing spinor is given by
\[
\eta^1 = \cos \left( \frac{\epsilon(x^5) \kappa^2 P_0^0}{2} \right) \zeta, \quad \eta^2 = -\sin \left( \frac{\epsilon(x^5) \kappa^2 P_0^0}{2} \right) \zeta, \tag{5.15}
\]
in analogy with (4.2).

This discussion exactly parallels the one we gave for the conventional Scherk-Schwarz mechanism. However, as explained in [6], the brane action (5.13) is inconvenient for deriving the equations of motion. The fields $\psi^{1,2}_m$ are too singular to apply the naive variational principle without regularization. Indeed, as explained in [6], the even fields are not piecewise smooth. (For example, $\psi^{1}_m(0) \neq \lim_{\xi \to 0} [\psi^{1}_m(+\xi) + \psi^{1}_m(-\xi)]/2$, so one cannot apply the standard Fourier decomposition.)

Therefore we follow ref. [6] and consider the equivalent brane action
\[
S_{brane} = \frac{1}{\kappa} \int d^4x \int_{-\pi \kappa}^{+\pi \kappa} dx^5 e_4 \left[ \delta(x^5) \tan \frac{\delta_0}{2} + \delta(x^5 - \pi \kappa) \tan \frac{\delta_\pi}{2} \right] \psi^{1a}_a \sigma^{ab} \psi^{1}_b + \text{h.c.} \tag{5.16}
\]

With this action, the even fields $\psi^{1}_m$ are continuous, so we can apply the naive variational principle and derive the equations of motion. It is immediate to show, using (4.13), that the brane action (5.16) coincides with our original brane action (3.3). By integrating the
equations of motion associated with (5.16), one can derive the discontinuities of the odd fields,
\[
\psi_a^2(\xi) - \psi_a^2(-\xi) = -2\tan\frac{\delta_0}{2} \psi_a^1(0), \quad \psi_a^2(\pi \kappa + \xi) - \psi_a^2(\pi \kappa - \xi) = -2\tan\frac{\delta_\pi}{2} \psi_a^1(\pi \kappa),
\]
and check that they precisely reproduce the jumps of eq. (5.11).

This analysis shows that supersymmetry breaking by brane superpotentials has an alternative description in terms of generalized boundary conditions on the gravitino fields. The supersymmetry breaking is spontaneous because every gravitino becomes massive via a super-Higgs effect. The order parameter, \( F \equiv (\delta_0 + \delta_\pi)/\kappa^2 = -\beta/\kappa^2 \), is manifestly non-local: in one description, it is related to the Scherk-Schwarz twist; in the other, it contains contributions from each of the two fixed points.

As explained in \[6\], one can further generalize this description by allowing for a Scherk-Schwarz twist and for jumps at the orbifold fixed points. This would give two types of gravitino mass terms: one localized at \( x^5 = (0, \pi \kappa) \), and the other constant in the bulk.

6. Conclusions and outlook

In this paper we presented a bulk-plus-brane action that describes spontaneously broken supersymmetry in 4+1 dimensions. Supersymmetry breaking is induced by the expectation values of superpotentials on tensionless branes. The order parameter for the supersymmetry breaking is nonlocal; it is determined by the mismatch of the superpotential vevs on the two branes. The gravitino fields are periodic, but the equations of motion force the odd fields to be discontinuous at the locations of the branes. The construction reproduces the main features of gaugino condensation in M-theory at the level of an effective five-dimensional Lagrangian.

We also showed that our construction is equivalent to a coordinate-dependent compactification on the orbifold \( S^1/Z_2 \), where the gravitino fields and their derivatives are continuous across the orbifold fixed points but obey twisted boundary conditions. The resulting spectrum is identical to that of a conventional Scherk-Schwarz compactification, for an appropriate choice of the twist parameter.

At low energies and in the limit of small supersymmetry breaking, the massive Kaluza-Klein modes can be integrated out to give a four-dimensional effective action for the light graviton and radion supermultiplets. The calculation is relatively easy because the brane action only affects the fermionic fields. The bosonic action is a consistent truncation of the one for the zero modes of five-dimensional supergravity, compactified on a circle \( S^1 \). It corresponds to a four-dimensional no-scale supergravity model, with one chiral supermultiplet, whose interactions are determined by the usual \( SU(1,1)/U(1) \) Kähler potential \[3\]. The fermionic terms are fixed by the Kähler potential, together with a constant superpotential whose value must be adjusted to match the mass of the lightest gravitino.

As is typical in models with extra dimensions and supersymmetry breaking, the flat directions are lifted by quantum corrections. Indeed, it is not hard to compute the one-loop effective potential for the radion \( R \), including contributions from all the Kaluza-Klein
modes. We find

$$V_{eff} = \frac{1}{2} \sum_J (-1)^{(2J+1)} \text{tr} \int \frac{d^4k}{(2\pi)^4} \log(k^2 + M_J^2) \log(k^2 + M_J^2)$$

$$= \frac{1}{8\pi^2} \sum_n \int_0^{+\infty} \frac{dt}{t^3} \left[ e^{-n^2 t/R^2} - e^{-(n+a)^2 t/R^2} \right],$$

(6.1)

where $M_J^2$ denotes the squared mass matrix for the particles of spin $J$. For the case at hand, the graviton and gravitino masses are separated by the constant $a/R$, where

$$a = \frac{\delta_0 + \delta_\pi}{2\pi}.$$  

(6.2)

After performing a Poisson resummation we find

$$V_{eff} = \frac{3}{32\pi^6 R^4} \left[ \zeta(5) - Li_5(e^{2i\pi a}) + \text{h.c.} \right].$$  

(6.3)

The result is finite, despite the divergence occurring at each level in the sum of eq. (6.2). Note that the potential has a minimum at vanishing $R$. In a more realistic model, a nontrivial minimum at a finite non-zero value of $R$ can be obtained by adding matter with appropriate gauge and Yukawa couplings [16].

Note that the four-dimensional effective theory fails to explain the special ultraviolet properties of the model, connected with the non-local character of supersymmetry breaking. In the four-dimensional theory, one finds a quadratically divergent contribution to the one-loop vacuum energy. The Kaluza-Klein modes of the five-dimensional theory provide the appropriate cutoff for the four-dimensional calculation.

The work presented here is a first step towards a more complete understanding of bulk-plus-brane supersymmetry breaking. In this paper, the Goldstone fermions are all bulk fields. In a more general scenario, the Goldstone fermions can involve brane fields as well. The presence of $F$- and $D$-terms on the branes might well induce non-vanishing brane tensions, which would then require that the bulk background be warped. Such scenarios are presently under investigation.

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A. Appendix

The gravitino mass eigenvalues can be found by solving the five-dimensional equations of motion, subject to the boundary conditions specified in section 5, or by diagonalizing the gravitino mass matrix. In this appendix we take the second approach.

We start with the gravitino mass matrix, which we extract from (4.5). We then make the following redefinitions:

\[ \psi_{m,\rho}^\pm = \frac{\psi_{m,\rho}^1 \pm \psi_{m,\rho}^2}{\sqrt{2}}, \quad (\rho > 0), \quad P_\pm = \frac{\kappa^3}{2\pi} (P_0 \pm P_\pi). \] 

(A.1)

This gives

\[ \mathcal{M}_{3/2} = \frac{1}{R} \begin{pmatrix} P_+ & P_- & P_- & P_+ & P_+ & \ldots \\ P_- & P_+ + 1 & P_+ & P_- & P_- & \ldots \\ P_- & P_+ & P_+ - 1 & P_- & P_- & \ldots \\ P_+ & P_- & P_- & P_+ + 2 & P_+ & \ldots \\ P_+ & P_- & P_- & P_+ & P_+ - 2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}, \]

(A.2)

in the basis \( (\psi_0^1, \psi_1^+, \psi_1^-, \psi_2^+, \psi_2^-, \ldots) \).

We find the mass eigenvalues by extending the techniques of ref. [17]. For simplicity, we take \( P_0 \) and \( P_\pi \) to be real. Defining \( (E \equiv \text{even}, O \equiv \text{odd}) \)

\[ S_E \equiv \sum_{k \in E} (a_k^+ + a_k^-), \quad S_O \equiv \sum_{k \in O} (a_k^+ + a_k^-), \]

(A.3)

and considering for the moment the dimensionless matrix \( \hat{\mathcal{M}} \equiv \mathcal{M}_{3/2} \cdot R \), we rewrite the eigenvalue equations as

\[ P_+ a_0 + P_+ S_E + P_- S_O = \lambda a_0, \quad (n = 0), \]
\[ P_+ a_0 + P_+ S_E + P_- S_O = (\lambda + n) a_n^-, \quad (n \in E), \]
\[ P_+ a_0 + P_+ S_E + P_- S_O = (\lambda - n) a_n^+, \quad (n \in E), \]
\[ P_- a_0 + P_- S_E + P_+ S_O = (\lambda + n) a_n^-, \quad (n \in O), \]
\[ P_- a_0 + P_- S_E + P_+ S_O = (\lambda - n) a_n^+, \quad (n \in O). \]

(A.4)

After a series of manipulations we find:

\[ P_+ a_0 + P_+ S_E + P_- S_O = \lambda a_0, \]

(A.5)
\[ S_E = 2\lambda (P_+ a_0 + P_+ S_E + P_- S_O) \Sigma_E, \]

(A.6)
\[ S_O = 2\lambda (P_- a_0 + P_- S_E + P_+ S_O) \Sigma_O, \]

(A.7)

where:

\[ \Sigma_E \equiv \sum_{n \in E} \frac{1}{\lambda^2 - n^2} = -\frac{1}{2\lambda^2} + \frac{\pi}{4\lambda} \left[ \frac{1 + \cos(\lambda\pi)}{\sin(\lambda\pi)} \right], \]

(A.8)
\[ \Sigma_O \equiv \sum_{n \in O} \frac{1}{\lambda^2 - n^2} = -\frac{\pi}{4\lambda} \left[ \frac{1 - \cos(\lambda\pi)}{\sin(\lambda\pi)} \right] = -\frac{\pi}{4\lambda} \tan\left(\frac{\lambda\pi}{2}\right). \]

(A.9)
Solving (A.6) and (A.7) for $S_E$ and $S_O$, and substituting into (A.5), we find:

$$4\pi P_+ \cos (\lambda \pi) = \left[ \pi^2 (P_-^2 - P_+^2) + 4 \right] \sin (\lambda \pi),$$  \hspace{1cm} (A.10)

or

$$\lambda^{(\rho)} = \frac{1}{\pi} \arctan \left[ \frac{4\pi P_+}{\pi^2 (P_-^2 - P_+^2) + 4} \right] + \rho, \hspace{1cm} (\rho = 0, \pm 1, \pm 2, \ldots),$$  \hspace{1cm} (A.11)

Reinstating the overall factor $1/R$, we derive the mass eigenvalues at each Kaluza-Klein level,

$$M^{(\rho)}_{3/2} = \frac{1}{R} \left\{ \frac{1}{\pi} \arctan \left[ \frac{4\pi P_+}{\pi^2 (P_-^2 - P_+^2) + 4} \right] + \rho \right\}, \hspace{1cm} (\rho = 0, \pm 1, \pm 2, \ldots),$$  \hspace{1cm} (A.12)

or, equivalently, eqs. (4.12) and (4.13).
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