Testing Long-Run Neutrality in Japan

Yoji Morita* and Yoshitaka Sawada** and Shigeyoshi Miyagawa*

* Professor emeritus of Kyoto University of Advanced Science
** Faculty of Economics and Business Administration, Kyoto University of Advanced Science

Uzumasa, Kyoto 615-8577, Japan

E-mail: morita.yoji@kuas.ac.jp

Abstract

The simplest “long-run neutrality” proposition specifies that a permanent change of the money stock has no long-run consequences for the level of real output. King and Watson exhibited the basic framework of long-run neutrality. In this paper, we extend King & Watson’s method and succeed direct estimations of unknown parameters. We investigate the neutrality of (real GDP, M3) in Japan during (1980q1,2007q4), where M3 is a wider class of money than M2. We show that this interval rejects the neutrality to the positive direction.

1 Introduction

The simplest “long-run neutrality” proposition specifies that a permanent change of the money stock has no long-run consequences for the level of real output. Most macroeconomists accept the proposition that changes in the money stock has no effect on the real variables, such as the real output, real consumption, real wages, and the real interest rates. King & Watson [1] exhibited the basic framework of long-run neutrality, where two conditions should be satisfied: (i) \( \ln(\text{real GDP}) \) and \( \ln(\text{money}) \) is I(1), that is, the level of each process is nonstationary and the 1st-differenced process is stationary and (ii) there is no cointegration property between two processes. Under these conditions, they showed that the neutrality of money cannot be rejected under a wide range of system parameters in US data during (1949q1,1990q4).

Following King & Watson, Serletis and Koustas [2] investigated international neutrality of money. Ooi, Shiratsuka and Shirota [3] also applied King & Watson’s method to show that the neutrality of (GNP, M2) in Japan cannot be rejected in a very long period (1885, 2003).

Since King & Watson’s method can only give us ranges of system parameters under which the neutrality is rejected and since their method does not estimate those system parameters, we cannot know whether the neutrality of our system is rejected or not. We extend King & Watson’s method and succeed to estimate system parameters in a framework of SVAR (Structural VAR) model. Due to the requirement for variables to be I(1), the interval (1980q1,2007q4) is taken into consideration. We show that this interval rejects the neutrality to the positive direction.

2 Data Properties

Variables and their symbolic notations are given.

\[
y_t = 400 \times \ln(\text{real GDP}(t)),
\]

\[
m_t = 400 \times \ln(M(3)(t)),
\]

where quarterly data during (1980q1,2007q4) is obtained from the data base FRED.

2.1 Unit root test

We carried out two kinds of test for investigating unit root properties; one is ERS test with null hypothesis as a unit root, and the other is KPSS test with the null hypothesis as stationarity. It is shown that each of \( y_t \) and \( m_t \) has a unit root and that each of \( \Delta y_t \) and \( \Delta m_t \) is stationary, where \( \Delta \) is the 1st differenced operator such as \( \Delta y_t = y_t - y_{t-1} \). We can say that \( y_t \) and \( m_t \) processes belong to I(1)-class. See Appendix.

2.2 Cointegration test

Cointegration test between \( y_t \) and \( m_t \) is carried out by Johansen's method [4], and we can say that there is no cointegration property. See also Appendix.

3 Dynamic Simultaneous System Model

3.1 System model

Following King & Watson, we consider the bivariate SVAR (structural vector autoregressive) model;

\[
\Delta y_t = \lambda_{ym} \Delta m_t + \sum_{j=1}^{p} \alpha_{j,yy} \Delta y_{t-j} + \sum_{j=1}^{p} \alpha_{j,ym} \Delta m_{t-j} + \epsilon_{t}^{y},
\]

(3)

\[
\Delta m_t = \lambda_{my} \Delta y_t + \sum_{j=1}^{p} \alpha_{j,my} \Delta y_{t-j} + \sum_{j=1}^{p} \alpha_{j,mm} \Delta m_{t-j} + \epsilon_{t}^{m},
\]

(4)

where \( \epsilon_{t}^{m} \) is a mean-zero serially independent shock of money, \( \epsilon_{t}^{y} \) is vector of shocks, other than money, that affect
output and where \( p \) is the lag order to be decided by AIC.

It should be noted that, in numerical studies, we introduce const. + linear trend on RHSs of Eqs.(3) and (4). With \( \Delta y_t \) and \( \Delta m_t \) replaced by \( \Delta y_t - E(\Delta y_t) \) and \( \Delta m_t - E(\Delta m_t) \), we have once again the same SVAR model in Eqs.(3) and (4).

The long-run elasticities \( \gamma_m \) from money to output and \( \gamma_{my} \) from output to money are respectively given by parameters in the above equations:

\[
\gamma_m = \frac{\lambda_{ym} + \sum_{j=1}^{p} \alpha_{j,ym}}{1 - \sum_{j=1}^{p} \alpha_{j,yy}}, \quad \gamma_{my} = \frac{\lambda_{my} + \sum_{j=1}^{p} \alpha_{j,my}}{1 - \sum_{j=1}^{p} \alpha_{j,mm}}.
\]

Denoting \( X_t = (\Delta y_t, \Delta m_t)' \), \( \varepsilon_t = (\varepsilon_t^0, \varepsilon_t^m)' \) and letting \( A_0 \) be defined by

\[
A_0 = \begin{pmatrix} 1 & -\lambda_{ym} \\ \lambda_{my} & 1 \end{pmatrix},
\]

we rewrite the system model:

\[
A_0 X_t = \sum_{j=1}^{p} \alpha_j X_{t-j} + \varepsilon_t,
\]

where the covariance matrix \( \Sigma_{\varepsilon} \) of \( \varepsilon_t \) is assumed to be:

\[
\Sigma_{\varepsilon} = \begin{pmatrix} s_1^2 & 0 \\ 0 & s_2^2 \end{pmatrix}.
\]

The reduced VAR model becomes

\[
X_t = \sum_{j=1}^{p} \Phi_j X_{t-j} + u_t,
\]

where

\[
\Phi_j = A_0^{-1} \alpha_j, \quad u_t = A_0^{-1} \varepsilon_t.
\]

The residuals \( u_t = (u_{1t}, u_{2t})' \) satisfies the relation

\[
\Sigma_u = A_0^{-1} \Sigma_{\varepsilon} (A_0^{-1})' ,
\]

where \( \Sigma_u \) is a covariance matrix of \( (u_{1t}, u_{2t})' \). It should be noticed that we have 4 unknown parameters: \( \lambda_{my}, \lambda_{ym}, s_1 \) and \( s_2 \), while covariance matrix \( \Sigma_u \) gives us only 3 relationship.

3.2 Estimation of \( \gamma_m \) and 95% confidence intervals

Following King & Watson, Eqs.(3) and (4) are taken into consideration. They use IV method, not Structural VAR method.

3.2.1 \( \gamma_m \) as a function of \( \lambda_{my} \)

In Eq.(4), a value of \( \lambda_{ym} \) is fixed, and \( \Delta m_t - \lambda_{my} \Delta y_t \) is regressed by OLS. The residuals of this estimation are inserted into Eq.(3) as one of variables of IV method and parameters in Eq.(3) are estimated to give the value of \( \gamma_m \) in Eq.(5). Furthermore, the 95% confidence intervals of \( \gamma_m \) are calculated by delta method associated with covariance matrix of parameters obtained. Another value of \( \lambda_{my} \) is fixed and estimation procedures for \( \gamma_m \) are iterated. Thus, we can obtain

\[
\gamma_m \text{ as a function of } \lambda_{my}.
\]

Estimation results are depicted in Fig.1, where a blue line means \( \gamma_m \) with \( \lambda_{my} \) as a horizontal axis, and where red lines are \( \gamma_m \pm 2 \times \text{s.e.} \) (standard error). Since \( \gamma_m - 2 \times \text{s.e.} > 0 \) for a range of \( \lambda_{my} < 0.2 \), we can say that the neutrality (\( \gamma_m = 0 \)) is rejected for \( \lambda_{my} < 0.2 \).

3.2.2 \( \gamma_m \) as a function of \( \lambda_{ym} \)

In Eq.(3), a value of \( \lambda_{ym} \) is fixed, and \( \Delta y_t - \lambda_{ym} \Delta m_t \) is regressed by OLS. Estimated parameters together with the fixed \( \lambda_{ym} \) are inserted into Eq.(5) and \( \gamma_m \) is calculated as well as 95% confidence intervals. Another value of \( \lambda_{ym} \) is fixed and estimation procedures for \( \gamma_m \) and 95% confidence intervals are iterated. Thus, we can obtain \( \gamma_m \) as a function of \( \lambda_{ym} \). Estimation results are depicted in Fig.2. It can be seen that the neutrality (\( \gamma_m = 0 \)) is rejected for \( \lambda_{ym} > -1.07 \).

King & Watson claims that the neutrality (\( \gamma_m = 0 \)) cannot be rejected for reasonable ranges of \( \lambda_{my} \) and \( \lambda_{ym} \) from eco-

![Fig. 1: \( \gamma_m \) as a function of \( \lambda_{my} \) (blue line) and 95% confidence intervals (red lines)](image1)

![Fig. 2: \( \gamma_m \) as a function of \( \lambda_{ym} \) (blue line) and 95% confidence intervals (red lines)](image2)
nomics viewpoints. However, it is difficult to judge whether each range of $\lambda_{my} \geq 0.2$ and $\lambda_{ym} \leq -1.07$ is reasonable or not. Therefore, in the next section, we try to estimate true values of system parameters $\lambda_{my}$, $\lambda_{ym}$, $s_1$ and $s_2$ in SVAR model.

4 Estimation of $\lambda_{my}$, $\lambda_{ym}$, $s_1$ and $s_2$ in SVAR

Structural Var model in Eqs.(3) and (4) has a restriction of 4 unknown parameters $\lambda_{my}$, $\lambda_{ym}$, $s_1$ and $s_2$ with 3 equations given by Eq.(11). Therefore, setting a value of one parameter can determine the other three parameters. So, we investigate mutual relationship among those parameters.

4.1 Mutual relationship among $\lambda_{my}$, $\lambda_{ym}$, $s_1$ and $s_2$

4.1.1 Relation between $\lambda_{my}$ and $\lambda_{ym}$

Setting values of $\lambda_{ym}$ and/or $\lambda_{my}$ properly, it can be shown that $(\lambda_{ym}, \lambda_{my})$ satisfies a hyperbolic curve with asymptotes $\lambda_{ym} = -53$ and $\lambda_{my} = -11$ in Fig.3. It should be noted that setting $\lambda_{ym}$ (or $\lambda_{my}$) gives us a unique solution $(\lambda_{my}, s_1, s_2)$ (or $(\lambda_{ym}, s_1, s_2)$). Since our concern is whether $\gamma_{ym} = 0$ or not, we only have to consider the curve neighboring around $(\lambda_{ym}, \lambda_{my}) = (0, 0)$. Setting several values of $\lambda_{ym}$ properly around $-2.5 < \lambda_{ym} < 2.5$, $\lambda_{my}$ is calculated and is depicted in Fig.4. Notice that, roughly speaking, around $(\lambda_{my}, \lambda_{ym}) = (0, 0)$, $\lambda_{ym} > 0$ implies $\lambda_{my} < 0$ and vice versa.

4.1.2 Relation between $(\lambda_{ym}, \lambda_{my})$ and $s_1$

Letting $s_1$ be set as 2.9 to 3.5 with interval 0.1, each set of the estimated $(\lambda_{my}, \lambda_{ym})$ has two kinds of values, and is depicted in Fig.5, where for simplicity values of $s_2$ are omitted. In the case of $(\lambda_{my}, \lambda_{ym})$ corresponding to $s_1 = 2.85$ is additionally shown as a neighborhood of the lower limit of $s_1$. At $s_1 = 2.85$, two points of $(\lambda_{my}, \lambda_{ym})$ almost coincide each other. One value of $s_1$ gives two different sets of $(\lambda_{my}, \lambda_{ym})$ with almost symmetry at the center $\lambda_{ym} = -0.09$.

4.1.3 Relation between $(\lambda_{ym}, \lambda_{my})$ and $s_2$

Letting $s_2$ be set as 1.3 to 1.5 with interval 0.02, each set of the estimated $(\lambda_{my}, \lambda_{ym})$ has two kinds of values, and is depicted in Fig.6. Here and where $(\lambda_{my}, \lambda_{ym})$ corresponding to $s_2 = 1.29$ is additionally shown as a neighborhood of the lower limit of $s_2$. At $s_2 = 1.29$, two points of $(\lambda_{my}, \lambda_{ym})$ almost coincide each other. One value of $s_2$ gives two different sets of $(\lambda_{my}, \lambda_{ym})$ with almost symmetry on $\lambda_{my} = -0.02$.

Letting $s_1 = 65.7$ and $s_2 = 1100$, each parameter gives respectively two sets of $(\lambda_{ym}, \lambda_{my})$ which are drawn in Fig.7.

In this data set (1980q1,2007q4), we have two kinds of
MLE. Standard errors of estimated parameters depend on

4.2 Estimation of parameters

From Figs. 5 and 6:

lower bound: $s_1 \geq 2.85$ and $s_2 \geq 1.29$.

4.2.1 Standard errors in $\lambda_{ym}, \lambda_{my}, s_1, s_2$ through standard error analysis

When a value of $s_1$ (or $s_2$) is set, then the other three parameters ($\lambda_{ym}, \lambda_{my}, s_2$) (or ($\lambda_{ym}, \lambda_{my}, s_1$)) are calculated by MLE. Standard errors of estimated parameters depend on $s_1$ (or $s_2$), and therefore, it is expected that the true value of $s_1$ (or $s_2$) will minimize standard errors of the other parameters. Noting that a given value of $s_1$ (or $s_2$) produces two sets of the other parameters ($\lambda_{ym}, \lambda_{my}, s_2$) (or ($\lambda_{ym}, \lambda_{my}, s_1$)) as a multivalued function of $s_1$ (or $s_2$), we classify four cases of estimated parameters from Figs. 5 and 6:

- (case-i) $s_1 \rightarrow \lambda_{my} < 0, \lambda_{ym} > 0$ (case-iv) $s_2 \rightarrow \lambda_{ym} > 0$
- (case-ii) $s_1 \rightarrow \lambda_{my} > 0, \lambda_{ym} < 0$ (case-iv) $s_2 \rightarrow \lambda_{ym} < 0$

Fig. 7: $(\lambda_{ym}, \lambda_{my})$ as a function of $s_1 = 65.7$ and $s_2 = 1100$

Fig. 8: $\lambda_{ym}, \lambda_{my}, s_2$ as a function of $s_1$, cases-i and ii

Let us set $s_1$ properly and estimate $(\lambda_{ym}, \lambda_{my}, s_2)$ as a function of $s_1$. Two kinds of $(\lambda_{ym}, \lambda_{my}, s_2)$ are estimated by multivalued function of $s_1$, corresponding to (case-i) and (case-ii). See Fig. 8. Behaviors of $(\lambda_{ym}, \lambda_{my}, s_1)$ driven by $s_2$ in cases-i and case-ii) are shown in Fig. 9.

4.2.1 Standard errors in $\lambda_{ym} > 0$ and $\lambda_{my} < 0$ (case-i) and (case-iii)

(cases-i) Let $se_{ym}, se_{my}$ and $se_{s2}$ be standard errors of $\lambda_{ym}, \lambda_{my}$ and $s_2$, respectively, where these standard errors are functions of $s_1 = 3.28, 3.32, 3.36, \ldots, 3.52$. In the region of $\lambda_{ym} > 0$ and $\lambda_{my} < 0$, these errors are depicted in Fig. 10.

Fig. 9: $\lambda_{ym}, \lambda_{my}, s_1$ as a function of $s_2$, case-iii and iv

Fig. 10: $se_{ym}, se_{my}, se_{s2}$ as a function of $s_1$, (case-i) in a region of $(\lambda_{ym} > 0, \lambda_{my} < 0)$

It can be seen that

- $se_{my}$ is minimized at $s_1 = 3.44$,
- $se_{ym}$ is minimized at $s_1 = 4.10$,
- $se_{s2}$ is minimized at $s_1 = 2.875$.

In this case, there are three different results of minimizations. However, minimization of $se_{ym}$ with respect to $s_1$ can give us estimation of true parameters, while minimization of $se_{ym}$ or $s_2$ with respect to $s_1$ cannot estimate true
values of parameters. For the reason of this judgement, see simulation studies in section 5, where true parameters of the system are known in data-generation and estimation procedures are carried out. We can judge which minimization in three candidates is true.

**(case-i)** In the above (case-i), each \( s_1 \) set as 3.28, 3.32, \( \cdots \), 3.52 estimates \( s_2 \) to be 1.445598, 1.461548, \( \cdots \), 1.541546 respectively. So, here in (case-iii), we set \( s_2 \) as 1.445598, 1.461548, \( \cdots \), 1.541546 and estimate \( (\lambda_{ym}, \lambda_{my}, s_1) \) and their standard errors, where estimated values of \( (\lambda_{ym}, \lambda_{my}, s_1) \) also take same ones as in setting \( s_1 \), but their standard errors are newly calculated as a function of \( s_2 \). Without loss of generality, we use the same notation: \( se_{ym}, se_{my} \) and \( se_{s1} \) as standard errors of \( \lambda_{ym}, \lambda_{my} \) and \( s_1 \) respectively, where those standard errors are functions of \( s_2 \) in Fig.11.

It can be seen that
- \( se_{ym} \) is minimized at \( s_2 = 1.509516, (s_1 = 3.44 = f^{-1}(1.509516)) \),
- \( se_{my} \) is minimized at \( s_2 = 1.84 \),
- \( se_{s1} \) is minimized at \( s_2 = 1.32 \).

**[Results from (case-i) and (case-iii)]** Minimizing \( se_{my} \) and \( se_{ym} \) with respect to \( s_1 \) and \( s_2 \) respectively gives us the same result of estimated values

\[
(\lambda_{ym}, \lambda_{my}, s_1, s_2) = (1.368308, -0.28889, 3.44, 1.509516).
\]

\[(12)\]

![Graph](image)

Fig. 11: \( se_{ym}, se_{my}, se_{s1} \) as a function of \( s_2 \), (case-iii) in a region of \( (\lambda_{ym} > 0, \lambda_{my} < 0) \)

We may have a conjecture such that \( s_1 \) given deterministically with zero standard error has much bias in estimating the standard error \( se_{ym} \) of \( \lambda_{ym} \) in the same equation, while \( s_1 \) has less bias in estimating \( se_{my} \) of \( \lambda_{my} \) located for another equation. At the same reason, \( s_2 \) given deterministically with zero standard error has much bias in estimating the standard error \( se_{my} \) of \( \lambda_{my} \) in the same equation, while \( s_2 \) has less bias for estimation of \( se_{ym} \) of \( \lambda_{ym} \) located for another equation.

Therefore, \( \lambda_{my} \) is estimated by minimizing \( se_{my} \) at \( s_1 = 3.44 \) and \( \lambda_{ym} \) is also estimated by minimizing \( se_{ym} \) at \( s_2 = 1.509516 \), that is, the corresponding \( s_1 \) becomes \( s_1 = 3.44 \) by a transformation \( s_1 = f^{-1}(s_2) \) with \( s_2 = 1.509516 \). The same value of \( s_1 = 3.44 \) determines both of \( \lambda_{my} \) and \( \lambda_{ym} \).

**4.2.2 Standard errors in \( \lambda_{ym} < 0 \) and \( \lambda_{my} > 0 \) (case-ii) and (case-iv))**

**(case-ii)** Let \( se_{ym}, se_{my} \) and \( se_{s2} \) be standard errors of \( \lambda_{ym}, \lambda_{my} \) and \( s_2 \) respectively, where these standard errors are functions of \( s_1 = 3.28, 3.32, 3.36, \cdots, 3.52 \). In the region of \( \lambda_{ym} < 0 \) and \( \lambda_{my} > 0 \), these errors are depicted in Fig.12. Since minimization of \( se_{ym} \) with respect to \( s_1 \) is essential for estimating true parameters, we only show the region minimizing \( se_{ym} \) with respect to \( s_1 \), neglecting minimization of \( se_{ym} \) and \( se_{s2} \) in Fig.12.

![Graph](image)

Fig. 12: \( se_{ym} \) (left axis), \( se_{my} \) (right axis), \( se_{s2} \) (left axis) as a function of \( s_1 \), (case-ii) in a region of \( (\lambda_{ym} < 0, \lambda_{my} > 0) \)

It can be seen that
- \( se_{my} \) is minimized at \( s_1 = 3.36 \),

**(case-iv)** In (case-ii) stated above, each of \( s_1 = 3.28, 3.32, 3.36, \cdots, 3.52 \) estimates another set of \( s_2 = 1.51322, 1.533518, \cdots, 1.634853 \) respectively. Here, therefore in (case-iv), we set \( s_2 \) as 1.51322, 1.533518, \( \cdots \), 1.634853 and estimate \( (\lambda_{ym}, \lambda_{my}, s_1) \) and their standard errors, where estimated values of \( (\lambda_{ym}, \lambda_{my}, s_1) \) also take same ones as in setting \( s_1 \), but their standard errors are newly calculated as a function of \( s_2 \). Without loss of generality, we use the same notation: \( se_{ym}, se_{my} \) and \( se_{s1} \) as standard errors of \( \lambda_{ym}, \lambda_{my} \) and \( s_1 \) respectively, where those standard errors are functions of \( s_2 \) in Fig.13, where only the region of minimizing \( se_{ym} \) with respect to \( s_2 \) is depicted.

It is obvious to see that by using a transformation of \( s_1 = f^{-1}(s_2) \)
5 Simulation Studies

Equations (7) and (9) are rewritten with const.+trend term:

\[ A_0 X_t = c_0 + c_1 * trend + \sum_{j=1}^{p} \alpha_j X_{t-j} + \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} \begin{pmatrix} \epsilon^{\eta}_t \\ \epsilon^{m}_t \end{pmatrix} \quad (14) \]

\[ X_t = c_0 + \tilde{c}_1 * trend + \sum_{j=1}^{p} \tilde{\alpha}_j X_{t-j} + A_0^{-1} \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} \begin{pmatrix} \epsilon^{\eta}_t \\ \epsilon^{m}_t \end{pmatrix} \quad (15) \]

Letting \( \bar{X}_t = X_t - E\{X_t\} \), \( \bar{X}_t \) is defined as the deviation from \( E\{X_t\} \) and satisfies Eqs.(7) and (9) without const. and trend term. Simulation procedures are given below:

- Assume true values of unknown parameters as \( (\lambda_{ym}, \lambda_{my}, s_1, s_2) = (1.4, -0.3, 3.5, 1.5) \).
- Set \( \alpha_j \) as the same ones in the reduced VARs calculated from the real data.
- Generate \( \epsilon^{\eta}_t \) and \( \epsilon^{m}_t \) as mutually i.i.d. sequences with \( N(0,1) \).
- Generate \( X_t \) with sample size 500.
- Estimate \( (\lambda_{ym}, \lambda_{my}, s_1, s_2) \) by minimizing \( se_{ym} \) and \( se_{my} \) w.r.t. \( s_2 \) and \( s_1 \) respectively.

Covariance matrix between \( \epsilon^{\eta}_t \) and \( \epsilon^{m}_t \) are given as

\[
\text{cov}(\epsilon^{\eta}_t, \epsilon^{m}_t) = \begin{pmatrix} 1.00000045 & 0.0000365 \\ 0.0000365 & 1.00000045 \end{pmatrix}
\]

5.1 Estimation of \( (\lambda_{my}, s_1, s_2) \) when \( \lambda_{ym} \) is deterministically given

Estimated values of \( (\lambda_{my}, s_1, s_2) \) are depicted as a function of \( \lambda_{ym} \) given deterministically in Fig.14.

![Fig. 14: Estimation of \( (\lambda_{my}, s_1, s_2) \) as a function of \( \lambda_{ym} \) given deterministically](image)

It can be seen that \( s_1 \) becomes minimum when \( \lambda_{ym} = -0.15169 \) \( (\lambda_{my} = 0) \), and that \( s_2 \) becomes minimum when \( \lambda_{ym} = 0 \).

Standard errors of \( (\lambda_{my}, s_1, s_2) \) are shown in Fig.15. It can be seen that \( se_{s1} \) becomes minimum when \( \lambda_{ym} = -0.15169 \) \( (\lambda_{my} = 0) \), and that each of \( se_{s2} \) and \( se_{my} \) becomes minimum when \( \lambda_{ym} = 0 \). We can say that when \( \lambda_{ym} \) is deterministically given, minimum values of standard errors for the other three variables cannot show any information concerning true values.
5.2 Estimation of \((\lambda_{ym}, s_1, s_2)\) when \(\lambda_{my}\) is deterministically given

Estimated values of \((\lambda_{ym}, s_1, s_2)\) are depicted as a function of \(\lambda_{my}\) given deterministically in Fig. 16.

It can be seen that \(s_1\) becomes minimum when \(\lambda_{my} = 0\) \((\lambda_{ym} = -0.15169)\), and that \(s_2\) becomes minimum when \(\lambda_{my} = -0.03027\) \((\lambda_{ym} = 0)\).

Standard errors of \((\lambda_{ym}, s_1, s_2)\) are shown in Fig. 17. It can be seen that \(se_{s2}\) becomes minimum when \(\lambda_{my} = -0.03027\) \((\lambda_{ym} = 0)\), and that each of \(se_s1\) and \(se_{ym}\) becomes minimum when \(\lambda_{ym} = -0.15169\). We can say that when \(\lambda_{my}\) is deterministically given, minimum values of standard errors for the other three variables cannot show any information concerning true values.

5.3 Estimation of \((\lambda_{ym}, \lambda_{my}, s_2)\) when \(s_1\) is deterministically given

Although \(s_1\) is a multivalued function, our interest is concerned with case-i \((\lambda_{ym} \leq -0.15169, \lambda_{my} \leq 0)\), and hence we only show standard errors corresponding to case-i in the following Fig. 18.

In Fig. 18, we can see three minimum points of standard errors.

- \(se_{s2}\) has a minimum value at \(s_1 = 2.897\), which gives us \((\lambda_{ym}, \lambda_{my}, s_1, s_2) = (0, -0.0303, 2.897, 1.291)\), while in Fig. 15 with \(\lambda_{ym}\) given deterministically \(se_{s2}\) has the minimum value at \(\lambda_{ym} = 0\). Therefore, we can explain that the minimum value of \(se_{s2}\) does not bring us any information of true values of unknown parameters.

- \(se_{ym}\) has a minimum value at \(s_1 = 3.474\), which gives us \((\lambda_{ym}, \lambda_{my}, s_1, s_2) = (1.336, -0.285, 3.474, 1.487)\). We can say that true values \((1.4, -0.3, 3.5, 1.5)\) are estimated by minimizing \(se_{ym}\) with respect to \(s_1\).

- \(se_{ym}\) has a minimum value at \(s_1 = 4.06\) which gives us estimates \((\lambda_{ym}, \lambda_{my}, s_1, s_2) = (2.066, -0.4229, 4.06, 1.7199)\). Therefore, minimizing \(se_{ym}\) with respect to \(s_1\) is not useful for estimation of true values.
5.4 Estimation of \((\lambda_{ym}, \lambda_{my}, s_1)\) when \(s_2\) is deterministically given

Although \(s_2\) is a multivalued function, our interest is concerned with case-iii \((\lambda_{ym} \geq -0.15169, \lambda_{my} \leq 0)\), and hence we only show standard errors corresponding to case-iii in the following Fig.19.

Both minimizing results exhibit good estimations for true values \((\lambda_{ym}, \lambda_{my}, s_1, s_2) = (1.4, -0.3, 3.5, 1.5)\). Therefore, we can conclude that results in section 4 are efficient.

6 Conclusions

Neutrality of money in Japan is investigated in a bivariate system of (realGDP,M3). King & Watson’s method is extended, and we succeed direct estimations of system parameters in SVAR model analysis. During (1980q1,2007q4) in Japan, the neutrality of money was rejected toward a positive direction. Our method was applied to USA data associated with (realGDP,M3) during (1960q1,2019q4), and the neutrality of money in USA was rejected. Neutrality problems concerning money, Fisher effect and Phillips curve in USA will be reported in near future.

Appendix

For unit root tests, ERS test has a null hypothesis of “unit root”, while KPSS test has a null hypothesis of “stationarity”.

For cointegration tests, quadratic deterministic trend is assumed, because each of \(\Delta y\) and \(\Delta m\) is shown to have \(c + trend\) in unit root tests.

| vrbls | ERS(t-stat.) | KPSS(LM-stat.) | c+trend |
|-------|-------------|----------------|---------|
| \(y\) | 0.775851 | 1.123203 | *** c |
| \(\Delta y\) | -2.947952* | 0.111096 | c+trend |
| \(m\) | -0.037738 | 1.121261*** | c |
| \(\Delta m\) | -2.9497* | 0.074206 | c+trend |

*, ** and *** means “rejection of null hypothesis” with 10%, 5% and 1% significance level respectively.

Table 1: Unit root test

| Trace test |
|------------|
| Hypo. No. of CE(s) | Eigen-value | Trace Eigen stat. | 0.05 crit. value | prob.* |
| None | 0.0515 | 8.6177 | 18.3977 | 0.619 |
| At most 1 | 0.0265 | 2.9058 | 3.8415 | 0.088 |

| Maximum Eigenvalue test |
|--------------------------|
| Hypo. No. of CE(s) | Eigen- value | Max- Eigen stat. | 0.05 crit. value | prob.* |
| None | 0.0515 | 5.7112 | 17.1477 | 0.8436 |
| At most 1 | 0.0265 | 2.9058 | 3.8415 | 0.088 |

* MacKinnon-Haug-Michelis[5] p-value
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