Neutrino asymmetry around black holes: Neutrinos interact with gravity

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Propagation of a fermion in curved space-time generates a gravitational interaction due to coupling of its spin with the space-time curvature connection. This gravitational interaction, which is an axial-four-vector multiplied by a four gravitational vector potential, appears as a CPT violating term in the Lagrangian which generates an opposite sign and thus asymmetry between the left-handed and the right handed partners under the CPT transformation. In the case of neutrinos, this property can generate a neutrino asymmetry in the Universe. If the background metric is of the rotating black hole, i.e. the Kerr geometry, this interaction for the neutrino is non-zero. Therefore, the dispersion energy relations for the neutrino and its anti-neutrino are different which give rise to the difference in their number densities and the neutrino asymmetry in the Universe in addition to the known relic asymmetry.

KEY WORDS : neutrino asymmetry, rotating black hole, space-time curvature, CPT violation

PACS NO. : 04.62.+v, 04.70.-s, 11.30.Er, 11.30.Fs

I. INTRODUCTION

The generation of neutrino asymmetry, i.e., the excess of neutrinos over anti-neutrinos, in early Universe is a well known fact. This essentially arises due to the lepton number asymmetry, e.g. via the Affleck-Dine mechanism [1], in the early Universe. A large neutrino asymmetry in the early Universe can have interesting effects on various cosmological phenomena like big-bang nucleosynthesis and cosmic microwave background [2]. Massive neutrinos with large asymmetry can also offer to explain existence of cosmic radiation [3,4] with energy greater than GZK cutoff [5]. Apart from such asymmetry arising in the early Universe, one can always ask whether there is a possibility of the neutrino asymmetry arising when the Universe has cooled down, let us say in the present era. In this paper we present one such scenario when neutrinos are propagating around Kerr black holes.

Since long, propagation of test particles with some inherent structure in curved space-times has been of keen interest at both the classical and quantum realms. A spinning test particle when propagates in the gravitational field, its spin couples with the connection of the background space-time and produces an interaction term [6–8]. A similar coupling effect gets transferred to the phase factor at the quantum mechanical level leading to an interesting geometrical phase shifts (see e.g. [9]). This interaction between the spin of the particle and the spin connection of the background field is analogous to that of the electric current with the vector potential in the case of the electromagnetic field. The way electro-magnetic connection serves as a gauge field, in a similar manner, in case of fermions in curved space, the

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gravitational interaction gives rise to some sort of gauge field \cite{10}.

The propagation of fermions in the curved space-time is well studied in past by several authors (e.g. \cite{10–17}). The interaction term does not seem to preserve CPT \cite{21}, and is similar to the effective CPT as well as Lorentz violating terms as described in other contexts in previous works (e.g. \cite{18–20}). Therefore the interaction due to the curvature coupling of the spinor will give rise to opposite sign for a left-handed and right-handed field which for the case of neutrinos can lead to an asymmetry. A preliminary result, based on this scenario, was already reported by us \cite{21,22}. Also a similar asymmetry was noted by Ahluwalia-Khalilova \cite{23}. In connection with this fact, the gravitationally-induced neutrino oscillations were studied \cite{24–27}. Later, the Lorentz and CPT violation scenario was addressed by Kostelecký in the context of Riemann-Cartan space-times \cite{28}.

In this paper, we elaborate our earlier results \cite{21}, describing various aspects, in detail and showing its application around black holes. We show that such a neutrino asymmetry can arise even in the present epoch like in the black hole space-times. In fact, we would show, it is just the form of the background metric which is responsible for such an effect. If the background metric satisfies a particular form which we discuss below and if the temperature of the bath is large enough, then the favorable conditions for neutrino asymmetry exist. In this connection, obviously the Dirac equation and corresponding Lagrangian in curved background comes into the picture. Under curved space-times Dirac spinors can break the Lorentz invariance in the local frame which provide a background where the ordinary rules of quantum field theory, e.g. CPT invariance, can break down. It is seen that coupling between the fermionic spin and curvature of the space-time gives rise to an extra interaction term in the Lagrangian apart from free part, even if no further interaction is there. This interaction term does not preserve CPT and Lorentz symmetry.

The basic requirement to generate the neutrino asymmetry by this mechanism is that the background metric should deviate from spherical symmetry, like that of a Kerr black hole. If the black hole is chosen to be non-rotating (e.g. Schwarzschild type), then the CPT violating interaction term disappears and the neutrino asymmetry is ruled out. In next section, we give the mathematical formalism, which clearly shows the neutrino asymmetry is possible to generate in present era. In §3, we give an example where this asymmetry can arise in the black hole space-time. At last, in §4, we make conclusions.

II. FORMALISM TO PRODUCE NEUTRINO ASYMMETRY

The general Dirac Lagrangian density, which shows the covariant coupling of fermion of spin-1/2 to gravity, can be given as

$$\mathcal{L} = \sqrt{-g} \left( i \bar{\psi} \gamma^a D_a \psi - m \bar{\psi} \psi \right), \quad (1)$$

where the covariant derivative and spin connection are defined as

$$D_a = \left( \partial_a - \frac{i}{4} \omega_{bca} \sigma^{bc} \right), \quad (2)$$

$$\omega_{bca} = \epsilon_{bkl} \left( \partial_a e^\lambda_c + \Gamma^\lambda_{\gamma \mu} e^\gamma_c e^\mu_a \right). \quad (3)$$
We would work in units of $c = \hbar = k_B = 1$. We have assumed a torsion-less space-time and the Lagrangian is invariant under the local Lorentz transformation of the vierbien, $e^a_\mu$, and the spinor field, $\psi(x)$, as $e^a_\mu(x) \rightarrow \Lambda^a_b(x)e^b_\mu(x)$ and $\psi(x) \rightarrow \exp(ie_{ab}(x)\sigma^{ab})\psi(x)$, where $\sigma^{bc} = \frac{i}{2}[\gamma^c, \gamma^b]$ is the generator of tangent space Lorentz transformation, the Latin and Greek alphabets indicate the flat and curved space coordinate respectively. Also

$$e^a_\mu e^\mu_a = g^{\mu\nu}, \quad e^a_\nu e^\nu_a = \eta^{ab}, \quad \{\gamma^a, \gamma^b\} = 2\eta^{ab},$$

(4)

where $\eta^{ab}$ represents the inertial frame of the Minkowski metric and $g^{\mu\nu}$ is the curved space-time metric, here the Kerr geometry.

Now from (1) and (2), it is clear that the product of three Dirac matrices appears in the Lagrangian and which is

$$\gamma^a\gamma^b\gamma^c = \eta^{ab}\gamma^c + \eta^{bc}\gamma^a - \eta^{ac}\gamma^b - ie^{abcd}\gamma^5\gamma_d.$$

(5)

Thus the spin connection terms are reduced into the combination of an anti-hermitian, $\bar{\psi}\Lambda_\alpha\gamma^\alpha\psi$, and a hermitian, $\bar{\psi}B^d\gamma^5\gamma_d\psi$, interaction terms. The anti-hermitian interaction term disappears when its conjugate part of Lagrangian is added to (1). The only interaction survives in $\mathcal{L}$ is the hermitian part and (1) reduces to

$$\mathcal{L} = \text{det}(e)\bar{\psi} \left[(i\gamma^a\partial_a - m) + \gamma^a\gamma^5B_a\right]\psi,$$

(6)

where

$$B^d = e^{abcd}e^\beta_\lambda \left(\partial_\beta e^\lambda_c + \Gamma^\lambda_{\alpha\beta}e^\alpha_c e^\mu_\lambda\right)$$

(7)

and in terms of tetrads, Christoffel connection is reduced as

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}\eta^{ij}e^a_i e^b_j \left[\left(e^d_{\beta\nu}e^\mu_\nu + e^d_{\beta\mu}e^\nu_\mu\right)\eta_{dp} + \left(e^d_{\beta\mu}e^\mu_p + e^d_{\beta\nu}e^\nu_p\right)\eta_{d\mu} - \left(e^p_{\mu\beta}e^a_{\nu\lambda} + e^p_{\nu\beta}e^a_{\mu\lambda}\right)\eta_{pq}\right].$$

(8)

Thus from (6), the free part of the Lagrangian is, $\mathcal{L}_f = \text{det}(e)\bar{\psi}(i\gamma^a\partial_a - m)\psi$, which is exactly same as the Dirac Lagrangian in the flat space, and the interaction part due to the curvature of space-time is, $\mathcal{L}_I = \text{det}(e)\bar{\psi}\gamma^a\gamma^5\psi B_a$. It is known that Lagrangian for any fermionic field is invariant only under local Lorentz transformation [15]. However, if the gravitational four vector field $B_a$, is chosen as constant background in the local frame, then $\mathcal{L}_I$ violates CPT as well as particle Lorentz symmetry in the local frame. For example, if $B_a$ is constant and space-like (what we will show later according to Kerr geometry), then the corresponding fermion will have different interactions if its direction of motion or spin orientation changes, and thus the breaking of Lorentz symmetry in the local frame is natural. It should be noted that similar interaction terms are considered in CPT violating theories and string theory (e.g. [18], [29]). Here the terms come into the picture automatically, due to the interaction with background curvature, and therefore the physical origin is very clear. Following [15], [18], we call the interaction, $\mathcal{L}_I$, is observer Lorentz invariant but there the particle Lorentz symmetry is broken. Here, both the kinds of Lorentz symmetry are different obviously as neutrinos are considered moving under gravitational field and thus they are no longer free. Now $\mathcal{L}_I$ is CPT violating if it changes sign under CPT transformation. Actually under CPT transformation, $\bar{\psi}\gamma^a\gamma^5\psi$, which is an axial-vector (pseudo-vector), changes sign. If $B_a$ does not change sign under CPT, then $\mathcal{L}_I$ is CPT violating (CPT odd) interaction as well otherwise the interaction is CPT even. It is the nature of background metric which determines whether the
The functional form of $B_a(x, y, z, t)$ is odd (changes sign) under CPT or not. Overall we can say, $L_I$ is CPT as well as particle Lorentz violating interaction (it can be noted that CPT violation necessarily implies the Lorentz violation in local field theory [30]). However, if $B_a$ does not break the symmetry of particle Lorentz transformations in the local frame, the CPT also cannot be broken. As we would see, for the propagation of neutrinos in the Kerr black hole space-times the interaction term is CPT violating. Thus, the vector $B_a$ causes breakdown of Lorentz invariance and CPT violation.

We would here like to mention that our analysis is different from earlier studies of interactions violating Lorentz invariance but which were mainly CPT even [31]. These studies were based on interactions in the flat space-time and thus excluded interactions of fermions with background gravitational field. The purpose of these studies was to have high energy high precision tests of special relativity. One can then obtain bound on terms in Lagrangian violating Lorentz invariance, through various experiments like cosmic ray observations (e.g. [31,32]), neutrino oscillations (e.g. [18,33]) etc. We in this paper, focus on the general relativistic effects on propagation of fermions and we establish that the background gravitational field plays an interesting role in disguise of vector $B_a$ to cause CPT violation and hence the neutrino–anti-neutrino asymmetry. Further, our analysis, unlike that of [31] is based on considering interaction terms which violate CPT. As applied to phenomenology our motivation would be to seek possible generation of neutrino–anti-neutrino asymmetry in the Universe by putting bounds on parameters of the background black hole space-times. It would be interesting to extend this analysis to study the phenomenological applications e.g. neutrino oscillation as studied earlier [31].

The important factor for our application is that the interaction term ($L_I$) in (6) has different signs for left and right chiral fields. The coupling term for particles $\psi$ and anti-particles $\psi^c$ may be expressed as

$$\bar{\psi} \gamma^a \gamma^5 \psi = \bar{\psi}_L \gamma^a \psi_L - \bar{\psi}_R \gamma^a \psi_R,$$

(9)

$$\bar{\psi}^c \gamma^a \gamma^5 \psi^c = (\bar{\psi}^c)_L \gamma^a (\psi^c)_L - (\bar{\psi}^c)_R \gamma^a (\psi^c)_R.$$

(10)

Now, if we consider the spinor field as neutrino and since according to the standard model, particles (neutrinos) have left chirality and anti-particles (anti-neutrinos) have only right chirality, the second term in (9) and the first term in (10) will not be present. Thus the spin-connection interaction will have opposite sign for the (left-handed) neutrino and the (right-handed) anti-neutrino. Therefore the dispersion relation of the left and right chirality fields including the Lorentz and CPT violating term can be written as

$$(p_a \pm B_a)^2 = m^2,$$

(11)

where the ‘+’ and ‘−’ signs correspond to the left handed and right handed partners.

The effect of background gravitational field on the propagation of neutrinos is to modify the dispersion relation. The vector $B_a$ violates CPT, breaks Lorentz invariance and causes the above modification.

Thus, expanding out (11) for particles ($E_\nu$) and anti-particles ($E_{\bar{\nu}}$) the dispersion energy becomes

$$E_\nu = \sqrt{(p - \vec{B})^2 + m^2 + B_0},$$

$$E_{\bar{\nu}} = \sqrt{(p + \vec{B})^2 + m^2 - B_0},$$

(12)
where we only consider the positive energy solutions. Clearly the energy splitting between neutrino and anti-neutrino disappears when the gravitational coupling, $B_a \rightarrow 0$ (in the flat space).

An important point to be noted here that while propagating under a strong gravity neutrinos and anti-neutrinos acquire an effective mass due to the coupling with the space-time (they no longer are free now). It is this effective mass (which is helicity dependent) appears in an opposite sign in the Lagrangian with different helicity and then dispersion relations [see above equation (12)]. Neutrinos and anti-neutrinos propagating in gravitational fields would thus have different energies. This energy difference between particles and anti-particles is the direct result of the presence of $B_a$ which violates CPT. We can further evaluate the difference in number density of neutrinos and anti-neutrinos propagating in a gravitational background as

$$\Delta n = \frac{g}{(2\pi)^3} \int_{R_i}^{R_f} dV \int d^3|\vec{p}| \left[ \frac{1}{1 + \exp(E_\nu/T)} - \frac{1}{1 + \exp(E_{\bar{\nu}}/T)} \right],$$  \hspace{1cm} (13)

where $R_i$ and $R_f$ refer to two extreme points of the interval over which the asymmetry is measured and $dV$ is the small volume element in that interval.

Here the above modifications to dispersion relations and then the asymmetry is something similar to the effect on baryogenesis of spontaneous CPT violation in string based scenario [34]. In that case fermions are considered with extra interactions responsible for different chemical potentials and the asymmetry between quarks and anti-quarks. Here a similar interaction originates inherently to bring the neutrino-antineutrino asymmetry.

In the case, when $B_0$ is vanishing, the integrand is an odd function and hence $\Delta n = 0$. Any non zero value of $B_0$ would yield a $\Delta n \neq 0$ and hence neutrino asymmetry. Thus to create any neutrino asymmetry, a non-zero $B_0$ is required, and it does not matter whether $B_i$'s ($i = 1, 2, 3$) are vanishing or not. This is the reason, why the metric should have a non-zero off-diagonal spatial components for the neutrino asymmetry to occur.

### III. NEUTRINO ASYMMETRY AROUND BLACK HOLES

An example of origin of the neutrino asymmetry in a black hole space-time can be given for the Kerr geometry. For simplicity of analysis we would write the Kerr metric in Cartesian-like form, i.e., our variables are $t(=x_0), x(=x_1), y(=x_2), z(=x_3)$. We would however here stress that the conclusions are independent of the choice of coordinate system to describe the background space-time, as we comment in §4. In the Cartesian form, the Kerr metric with signature $[+---]$ can be written as [35]

$$ds^2 = \eta_{ij} \, dx^i \, dx^j - \left[ \frac{2\alpha}{\rho} s_i v_j + \alpha^2 v_i v_j \right] dx^i \, dx^j$$  \hspace{1cm} (14)

where

$$\alpha = \frac{\sqrt{2Mr}}{\rho}, \quad \rho^2 = r^2 + \frac{a^2 z^2}{r^2},$$  \hspace{1cm} (15)

$$s_i = \left( 0, \frac{rx}{\sqrt{r^2 + a^2}}, \frac{ry}{\sqrt{r^2 + a^2}}, \frac{r}{r} \frac{z\sqrt{r^2 + a^2}}{r} \right),$$  \hspace{1cm} (16)
\[ v_i = \left( 1, \frac{ay}{a^2 + r^2}, \frac{-ax}{a^2 + r^2}, 0 \right). \]  

(17)

Here \( a \) and \( M \) are respectively the specific angular momentum and mass of the Kerr black hole and \( r \) is positive definite satisfying the following equation,

\[ r^4 - r^2 \left( x^2 + y^2 + z^2 - a^2 \right) - a^2 z^2 = 0. \]  

(18)

The corresponding non-vanishing component of tetrads (vierbiens) are [35]

\[ e_i^0 = 1, \quad e_i^1 = -\frac{\alpha}{\rho} s_1, \quad e_i^2 = -\frac{\alpha}{\rho} s_2, \quad e_i^3 = -\frac{\alpha}{\rho} s_3, \]

\[ e_i^1 = 1 - \frac{\alpha}{\rho} s_1 v_1, \quad e_i^2 = -\frac{\alpha}{\rho} s_2 v_1, \quad e_i^3 = -\frac{\alpha}{\rho} s_3 v_1, \]

\[ e_i^y = -\frac{\alpha}{\rho} s_1 v_2, \quad e_i^y = 1 - \frac{\alpha}{\rho} s_2 v_2, \quad e_i^3 = -\frac{\alpha}{\rho} s_3 v_2, \quad e_i^3 = 1 - \frac{\alpha}{\rho} s_3 v_3. \]

(19)

Using (7), (8), (14) and (19), the gravitational scalar potential can be evaluated as

\[ B^0 = e_{1\lambda} \left( \partial_3 e_2^\lambda - \partial_2 e_3^\lambda \right) + e_{2\lambda} \left( \partial_1 e_3^\lambda - \partial_3 e_1^\lambda \right) + e_{3\lambda} \left( \partial_2 e_1^\lambda - \partial_1 e_2^\lambda \right). \]

(20)

Similarly, the gravitational vector potentials \( B^1, B^2, B^3 \) can be calculated. From (20), it is clear that \( B_0 \) will become zero, if all the off-diagonal spatial components of the metric are zero (i.e. \( g_{ij} = 0 \), where, \( i \neq j \rightarrow 1, 2, 3 \)). In other words we can say, there should be a minimum space-space curvature coupling effect to give rise to a nonzero scalar potential, \( B^0 \).

One can easily check from (20) along with (19) that under CPT transformation, form of \( B_0 \) would not behave as odd function, more precisely, \( B_0 \) neither flips its sign \( [B_0(-x, -y, -z, -a, M) \neq -B_0(x, y, z, a, M)] \) nor be invariant \( [B_0(-x, -y, -z, -a, M) \neq B_0(x, y, z, a, M)] \). The same would hold for \( B_1, B_2, B_3 \). Therefore, \( B_a \) leads to CPT violation in the Lagrangian. As mentioned earlier, this nature of \( B_a \) under CPT totally depends on the choice of background metric, the space-time, where the neutrino is propagating. A case of the space-time was studied earlier [17] where \( B_0 \) flips its sign (odd function) under CPT and thus overall \( \mathcal{L}_t \) is CPT invariant. However, the present case where the space-time is chosen around a rotating black hole gives rise to an actual CPT and Lorentz violating situation.

Now we will show, how does the above mentioned property of neutrino, along with the effect of curvature, generates its asymmetry. For simplicity, let us consider a special case of a black hole space-time with \( \vec{B} \cdot \vec{p} \ll B_0 p^0 \) and the black hole curvature effect is such that \( B_a B^a \) term can be neglected, and thus only the first order curvature effect is important. Then in the ultra-relativistic regime, we get from (13),

\[ \Delta n = \frac{g}{(2\pi)^2 T^3} \int_{R_i}^{R_f} \int_0^\infty \int_0^\pi \left[ \frac{1}{1 + e^{u e B_0/T}} - \frac{1}{1 + e^{-u e B_0/T}} \right] u^2 d\theta du dV \]

(21)

where \( u = |\vec{p}|/T \). If \( B_0 \ll T \), then

\[ \Delta n \sim g T^3 \left( \frac{B_0}{T} \right), \]

(22)

\( B_0 \) indicates the integrated value of \( B_0 \) over the space.
It should be noted that the sign of above asymmetry would depend on the overall sign of $B_0$, which depends on details of mass and angular momentum of the black hole. A large asymmetry can be achieved in practical situations as in accretion disks and case of Hawking radiation bath. In the first case, the virial temperature of thermal bath for the neutrinos can be as high as $10^{12}$ K $\sim$ 100 MeV. Therefore, to have a neutrino asymmetry around a Kerr black hole, the space-time curvature effect has to be at least one order of magnitude weaker, say, $B_0 \leq 10$ MeV, than the energy of bath. Moreover, the phenomena of a Hawking bath looks very promising, where small primordial black holes are produced in copious amounts. We know, all the primordial black holes of mass less than $10^{15}$ gm have been evaporated already. Only black holes of mass, $M > 10^{15}$ gm, still exist today. The temperature of Hawking bath can be given as

$$T = \frac{\hbar}{8\pi k_B M} \sim 10^{-7} K \left(\frac{M_\odot}{M}\right).$$

Thus the primordial black hole of masses of the order $10^{15}$ gm can generate Hawking temperature of the order $T \sim 10^{11}$ K $\sim$ 10 MeV. Hence, to generate a neutrino asymmetry, the restriction on curvature effect should be, $B_0 \leq 1$ MeV. If we consider, temperature of bath, $T \sim 10^{11}$ K $\sim 1.6 \times 10^{-5}$ erg, $B_0 \sim 1.6 \times 10^{-6}$ erg, then $\Delta n \sim 10^{-16}$. If there are typically $10^6$ number of black holes with same sign of $B_0$, $\Delta n \sim 10^{-10}$, which agrees with the observed neutrino asymmetry in the Universe.

IV. CONCLUSION

We have proposed a new mechanism to generate the neutrino asymmetry in the present epoch of the Universe. Such a mechanism can provide the neutrino asymmetry in addition to the relic neutrino asymmetry arising due to the leptogenesis in the early Universe. We have explicitly demonstrated this through an example where neutrinos are propagating around Kerr black holes. Here, for convenience, we have chosen the Kerr metric in Cartesian-like coordinates ($x, y, z, t$). It is seen that, in presence of any off-diagonal spatial component of the metric ($g_{ij}, i \neq j \rightarrow 1, 2, 3$) the scalar potential part ($B_0$) of the space-time interaction is non-zero. According to the present mechanism, this scalar potential is actually responsible for the neutrino asymmetry in the Universe. If all the $g_{ij}$s are zero, $B_0$ vanishes and hence $\Delta n = 0$. Although, this restriction on $g_{ij}$ as well as $B_0$, to have a non-zero neutrino asymmetry, is made here on the basis of a fixed coordinate system, in principle we can choose any other kind of coordinate system to describe the background geometry and to generate neutrino asymmetry. One can easily check that, in the Boyer-Lindquist coordinate system [36], $B_0$ is zero. But in that case, at least one non-zero $B_i$ ($i \rightarrow 1, 2, 3 \equiv \rho, \theta, \phi$) is required i.e., for example, presence of $g_{03}$ is enough, to give rise to neutrino asymmetry. Thus the restriction to generate the neutrino asymmetry around the black hole is that the black hole must be rotating and hence the system is symmetric axially.

The asymmetry can be produced in accretion disks or/and Hawking radiation baths, which provide high enough temperature for such an effect to occur. Assume that, there are $N_i$ number of $i$-type black holes in Universe, each producing a net curvature effect $B_{0i}$ in a typical temperature of the system $T_i$, then the neutrino asymmetry due to the presence of a black hole of kind-$i$ can be given as

$$\Delta n = \frac{N_i}{M_i} B_{0i} \sim 10^{-10}.$$
\[ \Delta n_i = 10^{-10} \left( \frac{N_i}{10^6} \right) \left( \frac{B_{0i}}{10^{-6} \text{erg}} \right) \left( \frac{T_i}{10^{-5} \text{erg}} \right)^2. \]  

(24)

If all the black holes in Universe are of \( i \)-kind and there are \( 10^6 \) such black holes, the curvature effect and temperature of the system are \( 10^{-6} \text{ erg} \) and \( 10^{-5} \text{ erg} \) respectively, then the neutrino asymmetry in Universe is \( 10^{-10} \). Any change of \( N_i, B_{0i} \) and \( T_i \) will affect \( \Delta n \). In general the net neutrino asymmetry in the Universe can be written as

\[ \Delta n = \sum_i \Delta n_i, \]

(25)

where \( B_{0i} \) and then \( \Delta n_i \) can be positive as well as negative depending on the kind of black holes (parameters of background space-times). However, it is difficult to predict about total number of black holes along with the variation of all their parameters. Therefore to estimate the total neutrino asymmetry due to black holes may not be possible physically at this stage. Moreover depending on the sign of \( B_{0i} \)s, we can not say whether the net effects of asymmetry will cancel out completely or not.

It should be reminded that, this kind of the neutrino asymmetry can be achieved in some other space-time geometry where \( B_0 \) is non-vanishing. The Kerr geometry is chosen as an example only in the present paper. However, as the number of black hole may be very high and the physics behind it is very well established, it is advantageous to consider black hole space-times to built up a real feeling about the physics behind this new mechanism. Also the advantage to deal with Cartesian-like coordinate system is that the structure of the Dirac gamma matrices \((\gamma^0, \gamma^i)\) are very well known there.

In our earth, the curvature effect is measured as \( 10^{-34} \text{ MeV} \sim 10^{-40} \text{ erg} \) [17] and the temperature is about \( 10^{-14} \text{ erg} \sim 10^{-2} \text{ eV} \). Thus, according to (24), the neutrino asymmetry comes out as \( 10^{-68} \) which is too small effect to observe. However, in earth’s laboratory, neutrinos can be examined in a high temperature bath. As the asymmetry is proportional to the square of temperature, it can be enhanced by increasing temperature in the laboratory. Moreover, if there are large number of earth like systems exist in the Universe, overall \( \Delta n \) may also increase according to (25).

Similar modifications to the dispersion relations may arise for virtual black holes also. The emergence of a birefringence effects associated with quantum gravity corrections, have already been seen. The modification of dispersion relation due to the quantum gravitational medium effect was shown, first for the photon as a helicity independent manner [37] and then in a helicity dependent way [38]. Those modifications also appear as CPT and Lorentz violation. Those works are involved with the space-time geometry and Maxwell’s field mainly. If one carefully compares those with our present one, it comes that the fundamental origin of the effects are same! In our case, it is the space-time curvature coefficients, which couples with fermions to produce the effect, i.e. the modification to the dispersion relations. In addition, here we bring the neutrino asymmetry based on this gravity effect. That asymmetry only will survive if the space-time deviates from spherical symmetry. Otherwise, say for the expanding FRW cosmology [37], though the dispersion relations get modified but the asymmetry does appear (as explained the reasons in various places in the above text). Point is, whatever be the origin of space-time curvature (due to presence of black holes or expanding era of early Universe with primordial fluctuations etc.) propagating neutrinos in curved space-time always produce this effect.

Our mechanism essentially works in the presence of a pseudo-vector term \((\bar{\psi} \gamma^a \gamma^5 \psi)\) multiplied by a background curvature coupling \((B_a)\). This is the CPT and Lorentz violating term, which picks up an opposite sign between
a neutrino and an anti-neutrino. Thus the CPT violating nature of the gravitational interaction with spinor is an essential condition in the success of the mechanism. Thus we propose, to generate the neutrino asymmetry in presence of gravity, following criteria have to be satisfied as: (i) The space-time should be axially symmetric, (ii) the interaction Dirac Lagrangian must have a CPT violating term which may be an axial-four vector (or pseudo-four vector) multiplied by a curvature coupling four vector potential. (iii) the temperature scale of the system should be large with respect to the energy scale of the space-time curvature. If all these conditions are satisfied simultaneously, our mechanism will give rise to the neutrino asymmetry in Universe. It would be interesting to explore the further theoretical and phenomenological consequences of the role of background gravitational curvature for neutrinos, which might offer new insights in the interplay of gravity and standard model interactions and specially of neutrino physics.

Acknowledgment

Author thanks D. V. Ahluwalia-Khalilova for discussions from two years ago in IUCAA, India. Thanks are also directed to V. A. Kostelecký for his comments and suggestions. This work was supported in part by NSF grant AST 0307433.
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