New Enhancement Mechanism of the Transitions in the Earth of the Solar and Atmospheric Neutrinos Crossing the Earth Core

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It is shown that the $\nu_2 \to \nu_e$ and $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_\mu(\tau)$) transitions respectively of the solar and atmospheric neutrinos in the Earth in the case of $\nu_e - \nu_\mu(\tau)$ mixing in vacuum, are strongly enhanced by a new type of resonance when the neutrinos cross the Earth core. The resonance is operative at small mixing angles but differs from the MSW one. It is in many respects similar to the electron paramagnetic resonance taking place in a specific configuration of two magnetic fields. The conditions for existence of the new resonance include, in particular, specific constraints on the neutrino oscillation lengths in the Earth mantle and in the Earth core, thus the resonance is a “neutrino oscillation length resonance”. It leads also to enhancement of the $\nu_2 \to \nu_e$ and $\nu_e \to \nu_\mu$ transitions in the case of $\nu_e - \nu_\mu$ mixing and of the $\bar{\nu}_\mu \to \bar{\nu}_e$ (or $\nu_\mu \to \nu_e$) transitions at small mixing angles. The presence of the neutrino oscillation length resonance in the transitions of solar and atmospheric neutrinos traversing the Earth core has important implications for current and future solar and atmospheric neutrino experiments, and more specifically, for the interpretation of the results of the Super-Kamiokande experiment.

1. Introduction

When the solar and atmospheric neutrinos traverse the Earth, the $\nu_2 \to \nu_e$ and $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_\mu(\tau)$) transitions/oscillations they undergo due to small $\nu_\mu - \nu_e$ mixing in vacuum\textsuperscript{1} can be dramatically enhanced by a new type of resonance which differs from the MSW one and takes place when the neutrinos cross the Earth core\textsuperscript{2}. The resonance is present in the $\nu_2 \to \nu_e$ and $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_\mu(\tau)$) transition probabilities, $P_2$ and $P(\nu_\mu(\tau) \to \nu_e(\mu;\tau))$, if the neutrino oscillation length (and mixing angles) in the Earth mantle and in the Earth core obey specific conditions\textsuperscript{3}. When satisfied, these conditions ensure that the relevant oscillating factors in the probabilities $P_2$ and $P(\nu_\mu(\tau) \to \nu_e(\mu;\tau))$ are maximal\textsuperscript{4} and that this produces a resonance maximum in $P_2$ and $P(\nu_\mu(\tau) \to \nu_e(\mu;\tau))$. Accordingly, the term “neutrino oscillation length resonance” or simply “oscillation length resonance” was used in \textsuperscript{5} to denote the resonance of interest. There exists a beautiful analogy between the neutrino oscillation length resonance and the electron spin-flip resonance realized in a specific configuration of magnetic fields \textsuperscript{6} (see \textsuperscript{7} for further details).

At small mixing angles ($\sin^2 2\theta \lesssim 0.05$) the maxima due to the neutrino oscillation length resonance in $P_2$ and $P(\nu_\mu(\tau) \to \nu_e(\mu;\tau))$ are absolute maxima and dominate in $P_2$ and $P(\nu_\mu(\tau) \to \nu_e(\mu;\tau))$: the values of the probabilities at these maxima in the simplest case of two-neutrino mixing are considerably larger - by a factor of $\sim (2.5 - 4.0)$ ($\sim (3.0 - 7.0)$), than the values of $P_2$ and $P(\nu_\mu \to \nu_e) = P(\nu_e \to \nu_\mu(\tau))$ at the local maxima associated with the MSW effect taking place in the Earth core (mantle). The magnitude of the enhancement due to the oscillation length resonance depends on the neutrino trajectory through the Earth core: the enhancement is maximal for the center-crossing neutrinos\textsuperscript{8}.

\textsuperscript{1}As is well-known, the $\nu_2 \to \nu_e$ transition probability accounts for the Earth effect in the solar neutrino survival probability in the case of the MSW two-neutrino $\nu_e \to \nu_\mu(\tau)$ and $\nu_e \to \nu_e$ transition solutions of the solar neutrino problem, $\nu_e$ being a sterile neutrino.

\textsuperscript{2}Note that, in contrast, the MSW effect is a resonance amplifying the neutrino mixing.

\textsuperscript{3}This analogy was brought to the attention of the author by L. Wolfenstein.
over a distance $X$ oscillation length $L$ matter effect on the oscillation length $L$ and the core, having different constant density profile of the Earth in the two-layer model. It also exhibits strong energy dependence.

The presence of the oscillation length resonance in the transitions of solar and atmospheric neutrinos traversing the Earth has important implications for the interpretation of the results, e.g., of the Super-Kamiokande experiment.

The Earth enhancement of the two-neutrino transitions of interest has been discussed rather extensively, see, e.g., refs. [5,8]. Some of the articles contain plots of the probabilities $P_{22}$ and/or $P(\nu_\mu \to \nu_e)$ or $P(\nu_e \to \nu_\mu(c))$ on which one can recognize now the dominating maximum due to the neutrino oscillation length resonance (see, e.g., [5]). However, this maximum was invariably interpreted to be due to the MSW effect in the Earth core before the appearance of [1].

### 2. The Neutrino Oscillation Length Resonance (NOLR)

All the interesting features of the solar and atmospheric neutrino transitions in the Earth, including those related to the neutrino oscillation length resonance, can be understood quantitatively in the framework of the two-layer model of the Earth density distribution [12,10]. The density profile of the Earth in the two-layer model is assumed to consist of two structures - the mantle and the core, having different constant densities, $\bar{\rho}_{\text{man}}$ and $\bar{\rho}_c$, and different constant electron fraction numbers, $Y_{e\text{man}}$ and $Y_e$ [1].

The transitions of interest of the neutrinos traversing the Earth are essentially caused by two-neutrino oscillations taking place i) first in the mantle over a distance $X'$ with a mixing angle $\theta'_m$ and oscillation length $L_{\text{man}}$, ii) then in the core over a distance $X''$ with different mixing angle $\theta''_m$ and oscillation length $L_{\text{man},c} = L_{\text{vac},c}$ and iii) again in the mantle over a distance $X'$ with $\theta'_m$ and $L_{\text{man}}$. Due to the matter effect $\theta'_m, \theta''_m \neq 0$ and $L_{\text{man},c} \neq L_{\text{vac},c}$ being the oscillation length in vacuum (see, e.g., [13]). For fixed $X'$ and $X''$ the neutrino oscillation length resonance occurs [1] if i) the relative phases acquired by the energy eigenstate neutrinos in the mantle and in the core, $\Delta E'X' = 2\pi X'/L_{\text{man}}$ and $\Delta E''X'' = 2\pi X''/L_c$, are correlated, being odd multiples of $\pi$, so that

$$\frac{X'}{L_{\text{man}}} = k + \frac{1}{2}, \quad \frac{X''}{L_c} = k' + \frac{1}{2},$$

where $k, k' = 0, 1, 2, ..., \text{ and if ii) the inequality}$

$$\cos(2\theta'_m - 4\theta''_m + \theta) \left(\cos(2\theta''_m - 4\theta'_m)\right) < 0$$

is fulfilled. Condition (2) is valid for the probability $P_{22} (P(\nu_\mu \to \nu_e))$. When equalities (1) hold, (2) ensures that $P_{22} (P(\nu_\mu \to \nu_e))$ has a maximum. In the region of the NOLR maximum where, e.g., $\Delta E'X' = \Delta E''X'' = \pi (2k + 1)$, $P_{22}$ is given in the case of $\nu_e - \nu_\mu$ mixing by [1]:

$$P_{22} \cong \sin^2 \theta + \frac{1}{4} \left[1 - \cos \Delta E'X''\right] \times \left[\sin^2 (2\theta''_m - 4\theta'_m + \theta) - \sin^2 \theta\right].$$

At the NOLR maximum $P_{22}$ takes the form [1]

$$P_{22}^{\text{max}} = \sin^2 (2\theta''_m - 4\theta'_m + \theta).$$

The analogs of eqs. (3) - (4) for the probability $P(\nu_{e(c)} \to \nu_{e(\mu,\tau)})$ can be obtained by formally setting $\theta = 0$ while keeping $\theta''_m \neq 0$ and $\theta'_m \neq 0$ in (3) - (4). Note that one of the two NOLR requirements $\Delta E'X' = \Delta E''X'' = \pi$ is equivalent at $\sin^2 2\theta \approx 0.02$ to the physical condition [1]

$$\pi \left(\frac{1}{X'} + \frac{1}{X''}\right) \cong \sqrt{2} G_F (Y_e \bar{\rho}_e - Y_{e\text{man}} \bar{\rho}_{\text{man}}).$$

Remarkably enough, for the $\nu_\mu \to \nu_e$ and $\nu_\tau \to \nu_\mu(c)$ transitions in the Earth, the NOLR conditions (1) with $k = k' = 0$ are approximately fulfilled at small mixing angles ($\sin^2 2\theta \lesssim 0.05$) in the regions where (2) holds [1]. The associated NOLR maxima in $P_{22}$ and $P(\nu_\mu \to \nu_e)$ are absolute maxima (Figs. 1 - 2).

Let us note that the study performed in [1] and discussed briefly above differs substantially from

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4The densities $\bar{\rho}_{\text{man},c}$ should be considered as mean densities along the neutrino trajectories. In the Earth model [11] one has: $\bar{\rho}_{\text{man}} \cong (4 - 5) \text{ g/cm}^3$ and $\bar{\rho}_c \cong (12) \text{ g/cm}^3$. For $Y_e$ one can use the standard values [11,12] (see also [5]) $Y_{e\text{man}} = 0.49$ and $Y_e = 0.467$.

5For analysis of the NOLR effects in the $\nu_\mu \to \nu_e$ and $\nu_\tau \to \nu_\mu(c)$ transitions ($\nu_e - \nu_\mu$ mixing) and in the $\nu_\mu \to \bar{\rho}_e$ (or $\nu_\mu \to \nu_e$) transitions at small mixing angles see [13].
the studies \[14\]. The authors of \[14\] considered the possibility of resonance enhancement of the \(\nu_e \rightarrow \nu_\mu(\tau)\) transitions of neutrinos propagating in matter with density, varying periodically along the neutrino path (parametric resonance). It was found, in particular, that at small mixing angles strong enhancement is possible only if the neutrinos traverse at least 2 - 3 periods (in length) of the density oscillations. The density distribution in the Earth is not periodic \[1\] and in order for the oscillation length resonance \[1\] to occur periodic variation of the density is not required.

In \[16\] the \(\nu_\mu \rightarrow \nu_\tau\) transitions in the Earth were considered for \(\sin^2 2\theta \cong 1\). It was noticed that in the region where \(\sqrt{2} G_F N_n^{\text{man},c} \gg \Delta m^2/E, N_n^{\text{man},c}\) being the neutron number density, a new maximum in \(P(\nu_\mu \rightarrow \nu_\tau)\) appears when \(\sqrt{2} G_F N_n^{\text{man}(c)} X(\nu) \cong 2\pi\), which was found to hold at \(h \sim 28^0\). The height of the maximum is comparable to the heights of the other “ordinary” maxima present in \(P(\nu_\mu \rightarrow \nu_\tau)\) for \(\sin^2 2\theta = 1\). It is stated in \[4\] that the effect does not take place in the \(\nu_\mu(c) \rightarrow \nu_\tau(c)\) transitions, which is incorrect both for \(\sin^2 2\theta \ll 1\) and \(\sin^2 2\theta \cong 1\) \[12\].

### 3. Implications of the Neutrino Oscillation Length Resonance

The implications of the oscillation length resonance enhancement of the probability \(P_{e2}\) for the Earth core crossing solar neutrinos, for the tests of the MSW \(\nu_e \rightarrow \nu_\mu(\tau)\) and \(\nu_e \rightarrow \nu_\tau\) solutions of the solar neutrino problem are discussed in refs. \[1\] and \[4\]. It is remarkable that for values of \(\Delta m^2 \cong (4.0 - 8.0) \times 10^{-6} \text{ eV}^2\) from the small mixing angle (SMA) MSW solution region and the geographical latitudes at which the Super-Kamiokande, SNO and ICARUS detectors are located, the enhancement takes place in the \(\nu_e \rightarrow \nu_\mu(\tau)\) case for values of the \(^{13}\)B neutrino energy lying in the interval \(\sim (5 - 12) \text{ MeV}\) to which these detectors are sensitive. The resonance maximum in \(P_{e2}\) at \(\sin^2 2\theta = 0.01\) for the trajectory with \(h = 23^0\), for instance, is located at \(E \cong 5.3 \text{ (10.5) MeV if } \Delta m^2 = 4.0 \text{ (8.0) } \times 10^{-6} \text{ eV}^2\). Accordingly, at small mixing angles the NOLR is predicted \[3\] to produce a much bigger - by a factor of \(\sim 6\), day-night (D-N) asymmetry in the Super-Kamiokande sample of solar neutrino events, whose night fraction is due to the core-crossing neutrinos, in comparison with the asymmetry determined by using the whole night event sample. On the basis of these results it was concluded in \[3\] that it can be possible to test a substantial part of the MSW \(\nu_e \rightarrow \nu_\mu(\tau)\) SMA solution region in the \(\Delta m^2 - \sin^2 2\theta\) plane by performing core D-N asymmetry measurements. The Super-Kamiokande collaboration has already successfully applied this approach to the analysis of their solar neutrino data \[3\]: the limit the
collaboration has obtained on the D-N asymmetry utilizing only the core event sample permitted to exclude a part of the MSW SMA solution region located in the area sin²2θ ≈ (0.007 − 0.01), Δm² ≈ (0.5 − 1.0) × 10⁻⁵ eV², which is allowed by the mean event rate data from all solar neutrino experiments (Homestake, GALLEX, SAGE, Kamiokande and Super-Kamiokande). In contrast, the current Super-Kamiokande upper limit on the whole night D-N asymmetry does not permit to probe the SMA solution region: the predicted asymmetry is too small.

The strong NOLR enhancement of the νµ → νe and νe → νµ;ν(τ) transitions of atmospheric neutrinos crossing the Earth core can take place at small mixing angles practically for all neutrino trajectories through the core, e.g., for the trajectories with h = (0⁰ − 23⁰) (Fig. 2). This is particularly relevant for the interpretation of the results of the atmospheric neutrino experiments and for the future studies of the oscillations/transitions of atmospheric neutrinos crossing the Earth. The Super-Kamiokande collaboration has reported at this Conference strong evidences for oscillations of the atmospheric νµ (ν̄µ) → νe (ν̄e) that is best described in terms of νµ (ν̄µ) ↔ νe (ν̄e) transitions amplified by the oscillation length resonance.

Figure 2. The probability P(νe(µ) → νµ;ν(τ)) as a function of h and E/Δm² for sin²2θ = 0.01. The absolute maximum due to the NOLR for h ≈ (0⁰ − 28⁰) is clearly seen at E/Δm² ≈ (1.3 − 1.6) × 10⁶ MeV/eV². The local maximum at E/Δm² ≈ (2.5 − 3.0) × 10⁶ MeV/eV² is due to the MSW effect in the Earth mantle. The oscillation length resonance Δm² ≈ (0.5 − 6.0) × 10⁻³ eV² and sin²2θ ≈ (0.8 − 1.0). The possibility of two-neutrino νµ (ν̄µ) ↔ νe (ν̄e) large mixing oscillations is disfavored by the data at Δm² ≥ 2 × 10⁻³ eV² it is ruled out.

It is a remarkable coincidence that for Δm² ≈ (0.5 − 6.0) × 10⁻³ eV² and small mixing, sin²2θ ≲ 0.10, the oscillation length resonance in P(νµ → νe) = P(νe → νµ;ν(τ)) occurs for values of the energy E of the atmospheric νe and νµ, which contribute either to the sub-GeV or to the multi-GeV e−-like and µ−-like Super-Kamiokande event samples. For sin²2θ = 0.01, Δm² = 5 × 10⁻⁴; 10⁻⁵; 5 × 10⁻³ eV², and h = 0⁰ (Earth center crossing), for instance, the absolute maximum in P(νµ;ν(τ) → νe) due to the NOLR takes place at E ≃ 0.75; 1.50; 7.5 GeV. Thus, for values of Δm² from the region of the νµ ↔ νe oscillation solution of the atmospheric neutrino problem, the NOLR strongly enhances the νµ → νe (and νe → νµ;ν(τ)) transitions of the atmospheric neutrinos crossing the Earth core, making the transitions detectable even at small mixing angles. It was suggested in that the excess of e−-like events in the region −1 ≤ cos θz ≤ −0.6, θz being the Zenith angle, either in the sub-GeV or in the multi-GeV sample, observed (in both samples) in the Super-Kamiokande experiment, is due to νµ → νe small mixing angle transitions, sin²2θµ ≃ (0.01 − 0.10), with Δm² ≈ (0.5 − 1.0) × 10⁻³ eV² or respectively Δm² ≈ (2 − 6) × 10⁻³ eV², strongly enhanced by the NOLR. The same resonantly enhanced

8A more detailed investigation performed within the indicated three-neutrino mixing scheme reveals, in particular, that the excess of e−-like events in the Super-Kamiokande sub-GeV data at −1 ≤ cos θz ≤ −0.6 seems unlikely to be due to small mixing angle νµ → νe transitions amplified by the oscillation length resonance.
transitions with $\Delta m^2 \sim (2 - 6) \times 10^{-3}$ eV$^2$ ($\Delta m^2 \sim (0.5 - 1.0) \times 10^{-3}$ eV$^2$) should produce at least part of the strong zenith angle dependence, exhibited by the $\mu$-like multi-GeV (sub-GeV) Super-Kamiokande data [2].

The transitions of interest arise in a three-neutrino mixing scheme, in which the small mixing angle MSW $\nu_e \rightarrow \nu_\mu$ transitions with $\Delta m^2_{21} \sim (4 - 8) \times 10^{-6}$ eV$^2$, or large mixing angle $\nu_e \leftrightarrow \nu_\mu$ oscillations with $\Delta m^2_{31} \sim 10^{-10}$ eV$^2$, provide the solution of the solar neutrino problem and the atmospheric neutrino anomaly is due to $\nu_\mu \leftrightarrow \nu_\tau$ oscillations with $\Delta m^2_{23}$ [18].

For $\Delta m^2_{31} \gg \Delta m^2_{21}$ the three-neutrino $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu(\tau)$ transition probabilities reduce to the two-neutrino transition probability $P(\nu_e \rightarrow \nu_\mu)$ (Fig. 2) with $\Delta m^2_{31}$ and $\sin^2 2\theta_{e\mu} = 4U_{e3}^2(1 - |U_{e3}|^2)$ playing the role of the two-neutrino oscillation parameters, where $U_{e3}$ is the $e - \nu_3$ element of the lepton mixing matrix, $\nu_3$ being the heaviest massive neutrino. The data [2] implies: $\sin^2 2\theta_{e3} \lesssim 0.25$. Thus, searching for the $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu(\tau)$ transitions of atmospheric neutrinos, amplified by the oscillation length resonance, can provide also unique information about the magnitude of $U_{e3}$ [19].

4. Conclusions

The neutrino oscillation length resonance should be present in the $\nu_\mu \rightarrow \nu_e$ transitions taking place when the solar neutrinos cross the Earth core on the way to the detector, if the solar neutrino problem is due to small mixing angle MSW $\nu_e \rightarrow \nu_\mu$ transitions in the Sun. The same resonance should be operative also in the $\nu_\mu \rightarrow \nu_\mu$ ($\nu_e \rightarrow \nu_\mu(\tau)$) small mixing angle transitions of the atmospheric neutrinos crossing the Earth core if the atmospheric $\nu_\mu$ and $\bar{\nu}_\mu$ indeed take part in large mixing vacuum $\nu_\mu(\bar{\nu}_\mu) \leftrightarrow \nu_\tau(\bar{\nu}_\tau)$, oscillations with $\Delta m^2 \sim (5 \times 10^{-4} - 6 \times 10^{-3})$ eV$^2$, as is strongly suggested by the Super-Kamiokande data [2] and if all three flavour neutrinos are mixed in vacuum. The existence of three-flavour-neutrino mixing in vacuum is a very natural possibility in view of the present experimental evidences for oscillations/transitions of the flavour neutrinos. In both cases the oscillation length resonance produces a strong enhancement of the corresponding transitions probabilities, making the effects of the transitions observable even at rather small mixing angles. Actually, the resonance may have already manifested itself in the excess of $e$-like events at $-1 \leq \cos \theta_2 \leq -0.6$ observed in the Super-Kamiokande multi-GeV atmospheric neutrino data [14]. And it can be responsible for at least part of the strong zenith angle dependence present in the Super-Kamiokande multi-GeV and sub-GeV $\mu$-like data [2].

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