Solving the Supersymmetric CP Problem with Abelian Horizontal Symmetries

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Models that combine Abelian horizontal symmetries and spontaneous CP violation can
(i) explain the smallness and hierarchy in quark parameters; (ii) satisfactorily suppress
supersymmetric contributions to flavor changing neutral current processes; (iii) solve the
$\mu$-problem; and (iv) suppress supersymmetric contributions to CP violating observables to
an acceptable level. The CKM phase is $O(1)$ and responsible, through Standard Model
box diagrams, to $\epsilon_K$. The supersymmetric CP violating phases are suppressed, $\phi_A \sim \lambda^6$
and $\phi_B \sim \lambda^8$ ($\lambda \sim 0.2$), leading to an electric dipole moment of the neutron that is about
2–3 orders of magnitude below the experimental bound.
1. Introduction

Supersymmetric theories introduce new sources of CP violation. Even with just the minimal supersymmetric extension of the Standard Model, there are two new phases \[ \phi_A = \arg(A^*m_\lambda), \]
\[ \phi_B = \arg(m_\lambda \mu m_{12}^2), \] (1.1)

where \( A \) and \( m_{12}^2 \) are the coefficients of, respectively, the trilinear and bilinear soft SUSY breaking terms and \( m_\lambda \) is the gaugino mass. Unless these phases are \( \lesssim O(10^{-2}) \), or supersymmetric masses are \( \gtrsim O(1 \text{ TeV}) \), the supersymmetric contribution to the electric dipole moment of the neutron \( d_N \) is well above the experimental bound. This is the Supersymmetric CP Problem. Furthermore, the CKM phase \( \delta_{KM} \) contributes to \( K - \bar{K} \) mixing through many new diagrams involving supersymmetric particles. For generic squark masses, these contributions are well above the experimental value of \( \epsilon_K \).

Horizontal symmetries, invoked to explain the smallness and hierarchy in fermion masses and mixing angles, have further interesting implications in the supersymmetric framework. In particular, they constrain the form of the mass-squared matrices of squarks and sleptons and, consequently, are able to solve or, at least, relax the problems of supersymmetric flavor changing neutral currents (FCNC). This idea has been investigated for both Abelian \([3-4]\) and non-Abelian \([5-9]\) symmetries. In both frameworks, even the most stringent FCNC constraints – \( \Delta m_K \) in the quark sector and \( \mu \to e\gamma \) in the lepton sector – can be satisfied: an Abelian symmetry (in combination with holomorphy) can precisely align the fermion mass matrix with the corresponding sfermion mass-squared matrix (leading to highly suppressed gaugino mixing angles), while a non-Abelian symmetry can lead to degeneracy between the first two sfermion generations.

In this work, we investigate whether the Abelian symmetries that lead to satisfactory quark-squark alignment may simultaneously solve the SUSY CP problems.\footnote{1 For a related study, in the framework of non-Abelian symmetries, see [6].} Indeed, it has already been shown in \([3-4]\) that the quark-squark alignment could be precise enough so that the magnitude of the SUSY contribution to \( K - \bar{K} \) mixing is orders of magnitude
below the Standard Model box diagrams. The contribution to $\epsilon_K$ is then very small, even for $O(1)$ phases. However, the contributions to the electric dipole moment of the neutron from phases of the type $\phi_A, \phi_B$ are, in general, not suppressed below those of generic Supersymmetric models. An extra ingredient, beyond the horizontal symmetry, is required. We here show that the required suppression can be achieved when CP breaking is spontaneous. Our basic assumption is that CP is preserved by the sector responsible for supersymmetry breaking, while it is spontaneously broken in the flavor sector. (For studies of spontaneous CP violation in various supersymmetric models, see refs. [10-16].)

Below, we present an explicit model. This is a minimal extension of the quark-squark alignment models of ref. [4] that can accommodate spontaneous CP violation. In our conclusions, we point out which of the ingredients in this model might apply in a more general framework.

2. The Model

The model of ref. [4] assumed an Abelian horizontal symmetry

$$\mathcal{H} = U(1)_1 \times U(1)_2.$$  \hfill (2.1)

The symmetry is spontaneously broken by the VEVs of two Standard Model gauge singlets, $S_1$ and $S_2$, with $\mathcal{H}$-charges

$$S_1(-1, 0), \quad S_2(0, -1).$$  \hfill (2.2)

The information about the horizontal symmetry breaking is communicated to the observed quarks at a high energy scale, possibly the Planck scale $m_{\text{Pl}}$, thus providing two small breaking parameters (this is the Froggatt-Nielsen mechanism [14-18]):

$$\epsilon_1 \equiv \frac{\langle S_1 \rangle}{m_{\text{Pl}}} \sim \lambda, \quad \epsilon_2 \equiv \frac{\langle S_2 \rangle}{m_{\text{Pl}}} \sim \lambda^2,$$  \hfill (2.3)

where $\lambda$ is taken to be of the order of the Cabibbo angle, $\lambda \sim 0.2$. With this scalar content, it is impossible to have spontaneous CP violation, because any phase in $\langle S_i \rangle$ can be rotated away by means of a $U(1)_i$ rotation. In order to have spontaneous CP violation, at least
one additional singlet $S_3$ is required, which transforms under either or both of $U(1)_1$ and $U(1)_2$. We choose

$$S_3(-3, -1), \quad \epsilon_3 \equiv \frac{\langle S_3 \rangle}{m_{Pl}} \sim \lambda^5. \quad (2.4)$$

Without loss of generality, we can take $\epsilon_1$ and $\epsilon_2$ real, while $\epsilon_3$ is complex. (Both $|\langle S_3 \rangle/\langle S_1 \rangle^3 \langle S_2 \rangle|$ $\sim 1$ and $\arg[\langle S_3 \rangle/\langle S_1 \rangle^3 \langle S_2 \rangle]$ $\sim 1$ will be shown to arise naturally.)

We assign the following $\mathcal{H}$ charges to the matter supermultiplets ($Q_i$ are quark doublets, $\bar{d}_i$ and $\bar{u}_i$ are quark singlets, $\phi_u$ and $\phi_d$ are the Higgs doublets):

$$
\begin{align*}
&Q_1 \quad Q_2 \quad Q_3 \quad \bar{d}_1 \quad \bar{d}_2 \quad \bar{d}_3 \\
&(3, 0) \quad (0, 1) \quad (0, 0) \quad (-2, 3) \quad (5, -1) \quad (1, 1) \\
&\bar{u}_1 \quad \bar{u}_2 \quad \bar{u}_3 \quad \phi_u \quad \phi_d \\
&(-1, 2) \quad (1, 0) \quad (0, 0) \quad (0, 0) \quad (-1, 0)
\end{align*}
$$

(2.5)

To find the quark mass matrices and the squark mass-squared matrices, we note that the following selection rules hold in the effective theory below the Planck scale:

(i) Terms in the superpotential that carry charge $(n_1, n_2)$ are suppressed by $\lambda^{n_1+2n_2}$ if $n_1 \geq 0$ and $n_2 \geq 0$ and vanish otherwise.

(ii) Terms in the Kahler potential that carry charge $(n_1, n_2)$ are suppressed by $\lambda^{|n_1|+2|n_2|}$.

We now present the form of the various mass matrices for quarks and squarks that follow from the selection rules in our specific model. We emphasize that in each entry in the mass matrices below, we omit an unknown coefficient of order 1. However, assuming that the only source of CP violation is $\arg(\langle S_3 \rangle^* \langle S_1 \rangle^3 \langle S_2 \rangle)$, all these coefficients are real.

For the quarks,

$$M^d \sim \langle \phi_d \rangle \begin{pmatrix}
\epsilon_3^3 & 0 & \epsilon_3^3 \epsilon_2 + \epsilon_3 \\
0 & \epsilon_1^4 & \epsilon_2^2 \\
0 & 0 & \epsilon_2
\end{pmatrix}, \quad (2.6)$$

$$M^u \sim \langle \phi_u \rangle \begin{pmatrix}
\epsilon_1^2 \epsilon_2 & \epsilon_1^4 & \epsilon_1^3 \\
0 & \epsilon_1 \epsilon_2 & \epsilon_2 \\
0 & \epsilon_1 & 1
\end{pmatrix}. \quad (2.7)$$

There are no higher order corrections to these entries because there is no holomorphic combination of breaking parameters that is $\mathcal{H}$-invariant.

For the diagonal blocks of the squark mass-squared matrices,

$$\tilde{M}^q_{LL} \sim \tilde{m}^2 \begin{pmatrix}
1 & \epsilon_3^3 \epsilon_2 + \epsilon_3^3 \epsilon_2 & \epsilon_3^3 + \epsilon_3^3 \epsilon_2 \\
\epsilon_3^3 \epsilon_2 & 1 & \epsilon_3^3 \\
\tilde{m}_3 & \epsilon_3^3 \epsilon_2 & 1
\end{pmatrix}, \quad (2.8)$$
\[ \tilde{M}_{RR}^{q2} \sim \tilde{m}^2 \begin{pmatrix} 1 & \epsilon_1^4 \epsilon_2^4 & \epsilon_1^3 \epsilon_2^3 + \epsilon_3 \epsilon_2^3 \\ \epsilon_1^3 \epsilon_2^3 + \epsilon_3^3 \epsilon_2^3 & 1 & \epsilon_1^2 \epsilon_2^3 + \epsilon_3^2 \epsilon_2^3 \\ \epsilon_1^3 \epsilon_2^3 + \epsilon_3^3 \epsilon_2^3 & \epsilon_1 \epsilon_2 & 1 \end{pmatrix}, \quad (2.9) \]

\[ \tilde{M}_{RR}^{u2} \sim \tilde{m}^2 \begin{pmatrix} 1 & \epsilon_1^2 \epsilon_2^2 & \epsilon_1 \epsilon_2 \\ \epsilon_1 \epsilon_2 & 1 & \epsilon \end{pmatrix}, \quad (2.10) \]

where \( \tilde{m} \) is the SUSY breaking scale. In each entry, we explicitly wrote the subleading contributions to \( \mathcal{O}(\lambda^4) \).

The mass-squared matrices for squarks that arise from the \( A \) terms are similar in form to the quark mass matrices. Both \( M^q \) and \( \tilde{M}_{LR}^{q2} \) get non-holomorphic contributions when the kinetic terms are rescaled to the canonical form \[4\]. However, there are two additional (in general, non-holomorphic) contributions to the effective \( \tilde{M}_{LR}^{q2} \) matrices: first, terms in the Kahler potential with one power of the SUSY breaking spurion \( \eta \equiv \tilde{m}\theta^2 \) and, second, insertions of the soft masses \( \tilde{M}_{LL,RR}^{q2} \) on virtual squark lines. All these sources can effectively be accounted for by estimating \( [(\tilde{M}_{LR}^{q2})_{ij}]_{\text{eff}} \sim [\tilde{M}_{LL}^{q2} M^q \tilde{M}_{RR}^{q2}]_{ij} \):

\[ \begin{pmatrix} \tilde{M}_{LR}^{d2} \end{pmatrix}_{\text{eff}} \sim \tilde{m} \langle \phi_d \rangle \begin{pmatrix} \epsilon_2^3 & \epsilon_1^3 \epsilon_2^3 + \epsilon_3 \epsilon_2^3 & \epsilon_1 \epsilon_2 \\ \epsilon_1 \epsilon_2 & \epsilon_2 & \epsilon_2 \\ \epsilon_2 & \epsilon_2 & \epsilon_2 \end{pmatrix}, \quad (2.11) \]

\[ \begin{pmatrix} \tilde{M}_{LR}^{u2} \end{pmatrix}_{\text{eff}} \sim \tilde{m} \langle \phi_u \rangle \begin{pmatrix} \epsilon_1^2 \epsilon_2^2 + \epsilon_3 \epsilon_1 \epsilon_2 \epsilon_2 & \epsilon_1^3 \epsilon_2^3 & \epsilon_1 \epsilon_2 \\ \epsilon_1^3 \epsilon_2^3 & \epsilon_1 \epsilon_2 & \epsilon_2 \end{pmatrix}. \quad (2.12) \]

In each term of (2.11) and (2.12) we wrote subleading contributions (to \( \mathcal{O}(\lambda^4) \)) only to the extent that they carry non-trivial phases (namely, are \( \epsilon_3 \)-dependent). Note that the zeros of (2.3), (2.7) are lifted. The non-holomorphic corrections are often ignored in the literature, but they are important for CP violation.

The mass matrices given above lead \[4\] to the observed hierarchy in quark masses and mixing angles; highly suppress supersymmetric contributions to \( \Delta m_K \) and \( \epsilon_K \); and induce \( D - \bar{D} \) mixing which is close to the experimental bound. In addition, as \( (M^d)_{13} \) carries a phase of order 1, the CKM matrix is complex with \( \delta_{KM} = \mathcal{O}(1) \).

The gauginos do not transform under the horizontal symmetry. Therefore, their masses are not suppressed by any of the small parameters. Moreover, they are real, since all holomorphic \( \mathcal{H} \) invariants vanish.
The combination $\varphi_u\varphi_d$ carries $\mathcal{H}$-charge $(-1,0)$. Consequently, the $\mu$-term cannot arise from the superpotential (it cannot be holomorphic in both $\epsilon_1$ and $\epsilon_3$). It can still arise from the Kahler potential. The selection rules imply then

$$
\mu \sim \tilde{m}\epsilon_1^*, \quad m_{12}^2 \sim \tilde{m}^2\epsilon_1^*.
$$

(2.13)

The magnitude of the $\mu$ term shows that the Supersymmetric $\mu$ problem is solved. This is a specific realization of the solution suggested in ref. [19]. We note that this scenario implies $\tan\beta \sim 5$, and requires fine tuning of order $\lambda$ to get the correct $m_Z/\tilde{m}$ [20-21].

3. The Electric Dipole Moment of the Neutron

We now consider the various contributions to $d_N$. The same analysis, with similar conclusions, applies to the broader class of nuclear electric dipole moments. (We use the calculations of ref. [22].) First, we examine phases of the type $\varphi_A$. In our framework, where the $A$ terms are not proportional to the Yukawa terms, there are many phases of this type. The ones that are most crucial for $d_N$ are

$$
\phi_A^u = \arg\left(\frac{[V_L^u(M^u)_{\text{eff}}V_R^{u\dagger}]_{11}}{[V_L^u(M^u_{LR})_{\text{eff}}V_R^{u\dagger}]_{11}}\right), \quad (3.1)
$$

and the similarly defined $\phi_A^d$. In (3.1), $V_L^u$ and $V_R^u$ are the diagonalizing matrices for $(M^u)_{\text{eff}}$ ($V_L(M^u)_{\text{eff}}V_R^{u\dagger} = M^u_{\text{diag}}$); $(M^u)_{\text{eff}}$ is the up-quark mass matrix in the basis where the kinetic terms are canonically normalized; and $(M^u_{LR})_{\text{eff}}$ takes into account both the rotation to this basis and the other contributions discussed in the previous section. Indeed, the effect of the diagonalizing matrices $V_M^q$ can be considered a fourth source of non-analytic contributions to the effective $A$-terms. While the various entries of $V_M^q$ can be thought of as carrying $\mathcal{H}$-charges similar to $M^{q2}_{MM}$, one cannot apply quite the same selection rules, since $V^q$ depends also on inverse powers of the $\epsilon_i$’s.

The experimental bound on $d_N$ requires that $\phi_A^u, \phi_A^d \leq \mathcal{O}(10^{-2})$. Examining eqs. (2.6), (2.7) (2.11) and (2.12), we find

$$
\phi_A^u, \phi_A^d = \mathcal{O}(\lambda^6) \sim 6 \times 10^{-5}.
$$

(3.2)
The reason for this strong suppression is that, to a good approximation, the relevant Yukawa coupling and the corresponding $A$ term are dominated by one and the same combination of breaking parameters and, therefore, there is no relative phase between them. (Remember that the $\mathcal{O}(1)$ coefficients in each of them are real.) Therefore, the SUSY contribution to $d_N$ from this source is about two orders of magnitude below the experimental bound.

Heavy quarks may contribute to $d_N$ through two-loop diagrams. The relevant phases are $\phi_A^c$, etc. These have to be smaller than $\sim 10^{-1}$, and we find that in our model they indeed are $\lesssim \mathcal{O}(\lambda^6)$. Their effects are therefore smaller than those of (3.2).

Another possible source of large contribution to $d_N$ is a relative phase between the $\mu$ parameter of the superpotential and the SUSY breaking $m^2_{12}$ term in the scalar potential. Eq. (2.13) reveals that there is no relative phase between $\mu$ and $m^2_{12}$ and, therefore, no contribution to $d_N$ from this source. More precisely, both $\mu$ and $m^2_{12}$ get additional contributions of order $\tilde{m}\epsilon^2_1\epsilon_2\epsilon^*_3$ and $\tilde{m}^2\epsilon^2_1\epsilon_2\epsilon^*_3$, respectively. Consequently,

$$\phi_B = \mathcal{O}(\epsilon_1\epsilon_2\epsilon_3) \sim 2 \times 10^{-6}$$

(3.3)

which is safely below the $d_N$ bound.

We conclude: in our model, combining an Abelian horizontal symmetry and spontaneous CP violation, all the supersymmetric contributions to FCNC processes and to CP violating quantities are suppressed below the experimental bounds. $D - \bar{D}$ mixing is very close to the bound, while $d_N$ is about two orders of magnitude below the bound.

4. The Higgs Potential

We would like to show that spontaneous CP violation could naturally arise in our framework. For that purpose, we have to introduce yet another Standard Model gauge singlet, $S_4$, to which we assign $\mathcal{H}$-charge $(6, 2)$. The most general superpotential involving the $S_i$ fields is

$$W(S_i) \sim a S_4 S_3^2 + \frac{b}{m^3_{Pl}} S_4 S_3 S_1 S_2 + \frac{c}{m^6_{Pl}} S_4 S_1^6 S_2^2 + \cdots,$$

(4.1)
where the ellipses stand for terms with higher powers of $S_4/m_{Pl}$; $a, b, c$ are $O(1)$ coefficients.

Requiring $F_{S_1} = F_{S_2} = F_{S_3} = 0$ can be solved by $\langle S_4 \rangle = 0$. This prevents any change in our analysis of quark and squark mass matrices of the previous section due to the introduction of $S_4$. On the other hand, $F_{S_4} = 0$ leads to

$$a \epsilon_3^2 + b \epsilon_1^3 \epsilon_2 \epsilon_3 + c \epsilon_1^6 \epsilon_2^2 = 0 \implies \frac{\epsilon_3}{\epsilon_1 \epsilon_2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \quad (4.2)$$

We learn that

(i) The ratio of VEVs that we used, $\epsilon_3 \sim \epsilon_3^3 \epsilon_2$, arises naturally as a consequence of the $\mathcal{H}$-charge assignments;

(ii) A relative complex phase between $\langle S_3 \rangle$ and $\langle S_1 \rangle^3 \langle S_2 \rangle$ arises for $b^2 - 4ac < 0$. This is similar to the mechanism used in [16].

The overall scale of the VEVs $\langle S_i \rangle$ is not determined by (4.2). However, it is attractive to assume that the horizontal $U(1)$’s are gauged and have anomalies [23] which are cancelled by the Green-Schwarz mechanism [24]. This indeed requires that the scales $\langle S_i \rangle$ are not far below $m_{Pl}$ [25-27].

5. Conclusions

The combination of Abelian horizontal symmetries and spontaneous CP violation could give viable models where, without any fine tuning, the following features arise:

(a) Quark masses and mixing angles exhibit the observed smallness and hierarchy;

(b) Supersymmetric contributions to FCNC are suppressed. The suppression of the contribution to $K - \bar{K}$ mixing is satisfactory only in a special class of models, where the Cabibbo angle is generated in the up sector and not in the down sector;

(c) The $\mu$ problem is solved: the $\mu$ term arises only from the Kahler potential and is somewhat below the supersymmetry breaking scale;

(d) Supersymmetric contributions to CP violation and, in particular, to the electric dipole moment of the neutron, are suppressed below the experimental bounds.

Our solution is particularly relevant for squark masses of a few hundred GeVs. If squarks are very light, i.e. $\sim 100$ GeV, then the quark-squark alignment solution of the
\(\Delta m_K\) problem runs into problems with the \(\Delta m_D\) bound. (Notice however that the gluino dominance of the RG evolution at low energy can easily induce a mild \(\mathcal{O}(10\%)\) degeneracy among the squarks of the first two families. This allows to naturally satisfy the bounds on \(D - \bar{D}\) mixing for squark masses as low as \(\sim 200\) GeV.) If, on the other hand, squark masses are \(\sim 1\) TeV, then the suppression of SUSY contributions to \(d_N\) relaxes the requirement that CP violating phases are small.

We have not addressed the strong CP problem in this work. We would like to mention, however, that horizontal symmetries can naturally solve this problem by setting the bare \(m_u\) to zero \([28]\). Alternatively, there does not seem to be any theoretical obstacle in this scenario to having an axion solution.

Let us now comment on the generality of our mechanism for solving the Supersymmetric CP problem. The suppression of the Supersymmetric contribution to \(\epsilon_K\) is satisfactory in all models of quark-squark alignment \([4]\). However, this is not the case for \(d_N\). Here, additional conditions, all realized in our specific model, should hold:

(i) Some entries in the Yukawa matrices should get comparable contributions from two combinations of breaking parameters which differ in their phases.

(ii) The Yukawa coupling \(Y^d_{11}\) and \(Y^u_{11}\) should each be dominated by a single combination of small parameters.

(iii) The contribution to the Yukawa couplings of the up and down quark mass eigenstates should be dominated (to high order in the small breaking parameter) by the respective \(Y_{11}\) couplings; alternatively, other contributions (e.g. \(Y_{12}Y_{21}/Y_{22}\)) should carry the same phase as \(Y_{11}\).

(iv) The \(\mu\) term should be dominated by a single combination of small parameters.

(v) The non-holomorphic contributions to the effective Yukawa- and \(A\)-terms should either carry no non-trivial phases or be very small.

Condition (i) is necessary in order to get a non-zero CKM phase. In our framework of quark-squark alignment, the Standard Model box diagram is the only possible source of \(\epsilon_K\) and, therefore, \(\delta_{KM} = \mathcal{O}(1)\) is necessary. Condition (ii) is necessary in order that \(\phi^u_A\) is not of order 1. To implement (i) in our framework, the charges of \(S_3\) have to be such that it contributes at leading order to at least one of the entries in \(M^d\) or \(M^u\). On the other
hand, (ii) requires that it does not contribute at leading order to $Y_{11}^q$. This leaves only a limited set of possible charges for $\mathcal{H}(S_3)$. With our assignments of $\mathcal{H}$ charges for quarks, these are $(-3, 0)$, $(-4, 0)$ and $(-3, -1)$. Condition (iii) is achieved when holomorphy requires $Y_{21} = Y_{31} = 0$ in both sectors (which is easy to implement) but also puts further restrictions on $\mathcal{H}(S_3)$, because $Y_{12}^u$ should be real to a good approximation. If $S_3$ does contribute to $Y_{12}^u$ (as would be the case with $S_3(-3, 0)$ or $S_3(-4, 0)$), the phase $\phi_A^u$ is still suppressed, but only by $O(\lambda^2)$. We employ $S_3(-3, -1)$, which gives the strongest possible suppression of $\phi_A^u$ (while keeping the CKM matrix complex). Condition (iv) is necessary to assure that $\phi_B$ is not $O(1)$. Condition (v) imposes various requirements, in particular that the leading $\mathcal{H}$-invariant combination of breaking parameters (in our case, $\epsilon_1^3 \epsilon_2^\dagger \epsilon_3^*$) is extremely small. However (v) requires in general more than just that. For instance, when $M_{13}^d$ carries a non-trivial phase, it constrains $(\tilde{M}_{RR}^d_{31})$ to be relatively small, limiting the choices of $\mathcal{H}$ quantum numbers for the quarks. Both (iv) and (v) are usually satisfied, at least at $O(\lambda^2)$, once the other conditions are fulfilled.

To summarize:

1. Models of quark squark alignment have a satisfactory suppression of supersymmetric contributions to $\epsilon_K$.

2. When combined with spontaneous CP violation, these models give, in general, a satisfactory suppression of $\phi_B$.

3. In a class of these models, with specific choices of the horizontal charges of the scalar whose VEV breaks CP, the $\phi_A$ phases are suppressed but $\delta_{KM}$ is not.

4. With an almost unique choice of these charges, the suppression of $\phi_A$ is well below the experimental bound. Otherwise, $\phi_A$ is close to the bound (but may be acceptable, taking into account the large theoretical uncertainties in the calculation of $d_N$).

More generally, we believe that the basic idea, namely that CP violating phases arise only in connection with small breaking parameters of a horizontal symmetry, might be useful in solving or relaxing the SUSY CP problem.

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