Potential of a singlet scalar enhanced Standard Model

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Abstract

We investigate the parameter space of the Standard Model enhanced by a gauge singlet real scalar $S$. Taking into account all the theoretical and experimental constraints, we show the allowed parameter space for two different types of such singlet-enhanced Standard Model. For the first case, the scalar potential has an explicit $Z_2$-symmetry, and may lead to a dark matter candidate under certain conditions. For the second case, the scalar potential does not respect any $Z_2$. This is again divided into two subcategories: one where the Standard Model vacuum is stable, and one where it is unstable and can decay into a deeper minimum. We show how the parameters in the scalar potential control the range of validity of all these models. Finally, we show the effect of one-loop correction on the positions and depths of the minima of the potential.

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1 Introduction

One of the minimalistic extensions of the Standard Model (SM) is that by one or more gauge singlet real (or complex) scalar field(s). Motivations to introduce a singlet scalar to the SM are, amongst others: (i) to provide a viable cold dark matter (CDM) candidate through Higgs portal models [1], (ii) to make the electroweak phase transition a strong first-order one [2, 3], and (iii) to address the naturalness problem of the SM Higgs boson [4]. Phenomenological aspects of such singlets have also been discussed in case of colliders [5, 6, 7, 8, 9], and in the context of electroweak precision constraints [10]. Such a singlet with a mass around 750 GeV may also be responsible for the recently observed excess in the diphoton channel [11, 12] if one adds vectorial fermions to the model.

One often imposes a $Z_2$-symmetry on the scalar potential under which the SM particles are all even and the extra singlet(s) is(are) odd, which can make the lightest $Z_2$-odd particle a CDM candidate. In an analogous way to what happens in R-parity conserving supersymmetry and universal extra dimension models, a $Z_2$-symmetry on the scalar potential under which the singlet $S$ is odd, can in principle lead to the Higgs portal dark matter models where $S$ constitutes the dark matter. A necessary condition for this is zero (or infinitesimally small) vacuum expectation value (VEV) for $S$, so that it cannot mix with the SM doublet scalar $\Phi$. One must remember that this $Z_2$ is rather ad hoc, introduced just for the sake of having a CDM candidate.

The nature of the tree-level scalar potential has also been discussed by several authors [13]. The potential is more complicated than the SM one because of one extra field and the possibility that the CP-even neutral component of the SU(2) doublet $\Phi$, which will be denoted by $\phi$, and the gauge singlet scalar $S$ can both have nonzero VEVs. We denote these VEVs by $v$ and $v_s$ respectively. If there is only one minimum for $v_s$, one often uses the shift symmetry $S \rightarrow (S + \Delta)$ to ensure $v_s = 0$, which in turn ensures a CDM candidate if the model is $Z_2$-symmetric. However, if there are more than one minima for $v_s$, there is no particular advantage in using the shift symmetry, except ensuring that $v_s = 0$ is an extremum.

In this paper, we investigate the nature of the tree-level potential of the SM extended by one real singlet scalar, which we call SM+S, with and without the $Z_2$-symmetry. We find the allowed parameter space for three different SM+S models: (1) the Higgs portal model i.e., the $Z_2$-symmetric model where $S$ can be a CDM candidate, (2) the $Z_2$-symmetric model with no CDM candidate, and (3) the $Z_2$-asymmetric model. Obviously, the allowed parameter space has to be consistent with all theoretical and current experimental constraints. For the third case, $Z_2$ breaking is soft, coming from operators with mass dimension less than 4. Dimension-4 $Z_2$
breaking operators are forbidden from gauge and/or Lorentz symmetry. As we consider only one real singlet scalar, all the couplings are real in these models.

Data from the Large Hadron Collider (LHC) essentially constrains the mixing between $S$ and $\phi$. The mixing angle $\theta$ is constrained to be so small that all SM+S models also satisfy the constraints coming from the oblique parameters. We will discuss it in detail later. The measurement of the $W$ boson mass also puts serious restrictions on the parameter space of SM+S [14].

Apart from these experimental constraints, there are three other important constraints. Firstly, the potential has to be stable at all energy scales till one reaches the range of validity. This range is, in general, way below the Planck scale, apart from some exceptional choices of the parameters. Above this limit, either at least one of the couplings blow up, or the potential becomes unbounded from below along some direction in the field space. The second one is the existence of a minimum, either global or local, where $\langle \phi \rangle = v = 246$ GeV, which will be referred to as the electroweak (EW) vacuum. The third constraint comes from the stability of the EW vacuum; if there is another minimum deeper than the EW vacuum, the tunnelling lifetime should not be less than the age of the universe.

We also study the effect of one-loop corrections to the potential. In general, the one-loop corrections are expected to be small compared to the tree-level potential, unless one looks along a flat direction. Even when both $S$ and $\Phi$ have non-zero mass terms, one can find a direction in the field space, at least for large values of the fields, following the prescriptions of Gildener and Weinberg [15]. One should also choose the regularization scale properly. This choice, in principle, is arbitrary if one uses renormalization group (RG) improved couplings. On the other hand, one can always tune the scale so that the one-loop corrected EW vacuum still has $\langle \phi \rangle = 246$ GeV [16]. However, this makes the choice of the regularization scale dependent on the model parameters.

The paper is arranged as follows. In Section 2, we give a brief outline of the singlet-enhanced SM, and discuss the constraints. Section 3 discusses the $Z_2$-symmetric model where the CDM is allowed, and Section 4 is on models with a non-zero singlet-doublet mixing. While these are all tree-level results, we discuss the one-loop corrections to the potential in Section 5. In Section 6, we summarize and conclude.

## 2 The real singlet scalar enhanced SM

Let us consider the most general potential for SM+S, the single real-scalar extended SM:

$$V(\Phi, S) = -\mu^2 \phi^4 \Phi - M^2 S^2 + \lambda (\phi^2 \Phi)^2 + a_1 \phi^4 \Phi S + a_2 \phi^4 \Phi S^2 + b_1 S + b_3 S^3 + b_4 S^4,$$  \hspace{1cm} (1)

where $\Phi$ is the SM doublet and $S$ is a gauge singlet scalar field. We denote the CP-even neutral component of $\Phi$ by $\sqrt{2} \phi$, and the VEVs are given as $\langle \phi \rangle = v$, $\langle S \rangle = v_s$. There are six new parameters in Eq. (1) over and above those in the SM. Of them $a_2$, $M^2$, and $b_4$ respect the $Z_2$ symmetry of $S \to -S$, while $a_1$, $b_1$ and $b_3$ break it softly. Note that $\mu^2, M^2 > 0$ stand for wrong-sign mass terms in the potential. All the couplings are, of course, real, because we have only one real singlet $S$ in this model.

The stability conditions are obtained from the requirement that the potential should not become negative along any direction of the field space, which gives

$$\lambda > 0, \quad b_4 > 0, \quad a_2 + 2\sqrt{\lambda b_4} > 0,$$  \hspace{1cm} (2)

along the directions $S = 0$, $\Phi = 0$, and $\sqrt{\lambda} \phi \Phi = \sqrt{v_s} S^2$ directions respectively.

In case there is mixing between $\phi$ and $S$, the mass eigenstates $(h, s)$ are defined as

$$h = \phi \cos \theta + S \sin \theta, \quad s = -\phi \sin \theta + S \cos \theta,$$  \hspace{1cm} (3)

where $\theta$ is the mixing angle.

In terms of VEVs $v$ and $v_s$, the potential in Eq. (1) becomes

$$V(v, v_s) = -\frac{1}{2} \mu^2 v^2 - M^2 v_s^2 + \frac{1}{4} \lambda v^4 + \frac{1}{2} a_1 v^2 v_s + \frac{1}{2} a_2 v^2 v_s^2 + b_1 v_s + b_3 v_s^3 + b_4 v_s^4,$$  \hspace{1cm} (4)

and the extremization conditions are

$$-\mu^2 v + \lambda v^3 + a_1 vv_s + a_2 vv_s^2 = 0,$$  \hspace{1cm} (5)

$$\frac{1}{2} a_1 v^2 + a_2 v^2 v_s + b_1 - 2M^2 v_s + 3b_3 v_s^2 + 4b_4 v_s^3 = 0.$$  \hspace{1cm} (6)
One can always apply the shift symmetry $S \rightarrow (S + \Delta)$ to Eq. (1), where $\Delta$ is some constant, since this shift in $S$ does not change the physics. We can use this freedom\(^1\) to remove one of the independent terms of the potential. Let us choose

$$b_1 = \frac{1}{2} a_1 v^2.$$  \hspace{1cm} (7)

Use of Eq. (7) in Eq. (4) removes the linear terms in $S$ and simplifies the potential to

$$V(v, v_s) = \frac{1}{2} \mu^2 v^2 - M^2 v_s^2 + \frac{1}{4} \lambda v^4 + \frac{1}{2} a_2 v^2 v_s^2 + b_3 v_s^3 + b_4 v_s^4.$$  \hspace{1cm} (8)

The extremization conditions now guarantee one extremum line along $v = 0$ and another along $v_s = 0$ (because of the shift symmetry):

$$v \left( -\mu^2 + \lambda v^2 + a_2 v_s^2 \right) = 0,$$  \hspace{1cm} (9)

$$v_s \left( a_2 v_s^2 - 2M^2 + 3b_3 v_s + 4b_4 v_s^2 \right) = 0.$$  \hspace{1cm} (10)

From Eqs. (9) and (10) one finds that there are three extrema for $v_s$, and for each $v_s$ there are three extrema for $v$, depending on the existence of real solutions\(^2\). For $v = 0$, Eq. (10) can be simplified to

$$v_s \left( -2M^2 + 3b_3 v_s + 4b_4 v_s^2 \right) = 0.$$  \hspace{1cm} (11)

For the other two nonzero extrema in the $\Phi$-direction, one uses $v^2 = (\mu^2 - a_2 v_s^2)/\lambda$ from Eq. (9) and get

$$v_s \left( \left[ 4b_4 - \frac{a_2^2}{\lambda} \right] v_s^2 + 3b_3 v_s + \left[ \frac{a_2 \mu}{\lambda} - 2M^2 \right] \right) = 0.$$  \hspace{1cm} (12)

Given the parameters of the potential, Eq. (12) gives the condition for real non-zero solutions for $v_s$ along $v \neq 0$. Note that for a $Z_2$-symmetrizable potential ($b_3 = 0$), the stability criteria ensure that $(a_2 \mu^2/\lambda - 2M^2) < 0$.

The condition that any extremum of $V(v, v_s)$ is a minimum, and not a maximum or saddle point, is

$$\frac{\partial^2 V}{\partial v^2} > 0,$$  \hspace{1cm} (13)

To get the mixing between the CP-even component of the SM Higgs doublet $\phi$ and the real scalar $S$, we expand the potential in Eq. (1) about the VEVs of the fields. The terms quadratic in fields are given by

$$V(\phi, S) \supset \phi^2 \left[ \frac{\mu^2}{2} + \frac{3}{2} \lambda v^2 + \frac{a_1}{2} v_s + \frac{a_2}{2} v_s^2 \right] + S^2 \left[ -M^2 + \frac{a_2}{2} v^2 + 3b_3 v_s + 6b_4 v_s^2 \right] + 2\phi S \left[ \frac{a_1}{2} v + a_2 v v_s \right]$$

$$\equiv \left( \begin{array}{c} \phi \\ S \end{array} \right) \mathcal{M} \left( \begin{array}{c} \phi \\ S \end{array} \right),$$  \hspace{1cm} (14)

where

$$\mathcal{M} = \begin{pmatrix}
\frac{1}{2} \left( -\mu^2 + 3\lambda v^2 + a_1 v_s + a_2 v_s^2 \right) & \frac{a_1}{2} v + a_2 v v_s \\
\frac{a_2}{2} v + a_2 v v_s & -M^2 + \frac{a_2}{2} v^2 + 3b_3 v_s + 6b_4 v_s^2
\end{pmatrix},$$  \hspace{1cm} (15)

in its most general form. It should be noted that the mixing between $\phi$ and $S$ also depends on the parameter $a_1$, which does not appear in the minimization conditions in Eq. (9) and (10), because of the choice in Eq. (7). If $M_1, M_2$ are the eigenvalues of $\mathcal{M}$, the masses of the physical states are given by

$$m_h = \sqrt{2M_1}, \quad m_s = \sqrt{2M_2},$$  \hspace{1cm} (16)

and the mixing angle $\theta$ is the angle which parametrize the $2 \times 2$ rotation matrix that diagonalizes $\mathcal{M}$.

\(^1\)One can always remove the tadpole term if the potential is $Z_2$-symmetric. However, for a $Z_2$-asymmetric potential, there can be two minima with different values of $v$, and removal of the tadpole at one minimum does not ensure its removal at the other.

\(^2\)For any given value of $v_s$, there is one extremum at $v = 0$. If Eq. (10) has three real solutions, and Eq. (9) has three real solutions for each of the solutions for $v_s$, there can be nine such extrema. Out of these nine extrema, three have $v = 0$ and thus cannot be the EW vacuum. The two solutions for nonzero $v$ for a given solution of $v_s$ are symmetrically placed about $v = 0$.  

3
2.1 Zero VEV for at least one field

One instructive case is when one of the minima has either \( v = 0 \) or \( v_s = 0 \) or both.

2.1.1 Minimum at \( v = v_s = 0 \)

The point \( v = v_s = 0 \) is an extremum where

\[
\frac{\partial^2 V}{\partial v^2} = -\mu^2, \quad \frac{\partial^2 V}{\partial v_s^2} = -2M^2, \quad \frac{\partial^2 V}{\partial v \partial v_s} = 0, \tag{17}
\]

and will be a minimum if \( \mu^2, M^2 < 0 \) i.e. the mass terms are right-sign.

From Eq. (9) one can see for this case that \( v \) will also have two non-zero real roots if (\( a_2 v_s^2 - \mu^2 \)) < 0, which requires a large negative \( a_2 \), resulting in a large singlet-doublet mixing.

2.1.2 Minimum at \( v = 0, v_s \neq 0 \)

If there is a minimum for \( v = 0, \) \( \partial^2 V/\partial v \partial v_s = 2a_2 v v_s = 0 \), and the condition for minimum translates to

\[
\frac{\partial^2 V}{\partial v^2} > 0 \Rightarrow a_2 v_s^2 - \mu^2 > 0, \quad \frac{\partial^2 V}{\partial v_s^2} > 0 \Rightarrow 8b_4 v_s^2 + 3b_3 v_s > 0 . \tag{18}
\]

The first condition shows, from Eq. (9), that there is only one real solution for \( v = 0 \). As \( b_4 > 0 \) from stability criterion, the second condition yields interesting bounds. If \( b_3 < 0 \) and \( v_s < 0 \) or \( b_3 > 0 \) and \( v_s > 0 \), \( \partial^2 V/\partial v_s^2 \) is definitely positive. On the other hand, if \( v_s > 0 \) but \( b_3 < 0 \), there is a lower bound on \( v_s \),

\[
v_s > \frac{3|b_3|}{8b_4}. \tag{19}
\]

Similarly, if \( b_3 > 0 \) and \( v_s < 0 \), there is an upper bound on \( v_s \):

\[
v_s < -\frac{3b_3}{8b_4}. \tag{20}
\]

2.1.3 Minimum at \( v \neq 0, v_s = 0 \)

For \( v \neq 0 \) and \( v_s = 0 \), the minima along \( v \) are symmetric, at \( v = \pm \sqrt{\mu^2/\lambda} \). This means \( \mu^2 > 0 \) for \( v \) to be real and hence \( \partial^2 V/\partial v^2 = 2\mu^2 > 0 \). So the condition for minimum at \( v \neq 0, v_s = 0 \) reduces to (\( a_2 v^2 - 2M^2 \)) > 0.

2.2 Minimum at \( v \neq 0, v_s \neq 0 \)

The constraints for \( v \neq 0, v_s \neq 0 \) are obtained using the concavity condition in Eq. (13), along with the expression for the second derivatives, to be

\[
\frac{\partial^2 V}{\partial v^2} = 2 (\mu^2 - a_2 v^2_s) = 2\lambda v^2, \quad \frac{\partial^2 V}{\partial v_s^2} = 3b_3 v_s + 8b_4 v^2_s, \quad \frac{\partial^2 V}{\partial v \partial v_s} = 2a_2 v v_s . \tag{21}
\]

For the \( Z_2 \)-symmetric potential, \( b_3 = 0 \), and hence the condition for a minimum at \( v \neq 0, v_s \neq 0 \) simplifies to \( 4\lambda b_4 > a_2^2 \), which is nothing but one of the stability criteria for the potential as shown in Eq. (2).

Further simplifications occur if we have only one minimum of the potential. In this case, it has to be at \( |v| = 246 \text{ GeV} \), and hence solutions for \( v_s \neq 0 \) can be obtained in a straightforward way from Eq. (10):

\[
v_s = -\frac{3b_3 \pm \sqrt{9b_3^2 - 16b_4(a_2 v^2 - 2M^2)}}{8b_4}. \tag{22}
\]

Thus, for the \( Z_2 \)-symmetric case, the condition for a minimum at \( v_s \neq 0 \) is \( a_2 v^2 - 2M^2 < 0 \), as \( b_4 > 0 \) from stability criterion. For nonzero \( b_3 \), the condition is

\[
a_2 v^2 - 2M^2 < 9b_3^2/16b_4 . \tag{23}
\]

One can easily have more than one minima with \( v \neq 0 \) and \( v_s \neq 0 \). However, if one minimum is at the origin, the second minimum at nonzero \( v \) and \( v_s \) requires \( a_2 \) to be large and negative, as discussed in Section 2.1.1. Such large values of \( a_2 \) are under severe kosh from the LHC data. A detailed discussion on the nature of the scalar potential related to the electroweak phase transition can be found in Ref. [2].
2.3 LHC constraints

If there is no mixing between $\phi$ and $S$, there are no constraints on $S$ coming from the LHC data, except that it cannot be so light ($< m_h/2$) that $h \to SS$ is allowed and the branching ratio is more than 34% at 95% CL [17]. Similarly, there is no constraint from electroweak precision observables as $S$ does not have any gauge coupling.

If there is a mixing between $\phi$ and $S$ parametrized by an angle $\theta$, the number of events, which is just the production cross-section times the branching ratio, goes down by $\cos^2 \theta$. Denoting the production cross-section times the decay width scaled to that in the SM by $\mu$, the ATLAS and CMS combined result shows

$$\mu = 1.09^{+0.11}_{-0.10},$$

from which we can put a limit of $\theta \leq 0.1$ at 1$\sigma$, which we will use for our subsequent discussion. This helps us to avoid both LHC data and precision constraints at one stroke.

2.4 Oblique parameters

Only the $T$ parameter may be significant in the small mixing case. In this model, the $T$ parameter is given by [5]

$$T_{\text{SM}+S} = - \left( \frac{3}{16\pi^2} \right) \left\{ \cos^2 \theta \left[ \frac{1}{\epsilon_W} \left( \frac{m_1^2}{m_1^2 - m_Z^2} \right) \ln \frac{m_1^2}{m_1^2 - m_W^2} - \left( \frac{m_1^2}{m_1^2 - m_W^2} \right) \ln \frac{m_1^2}{m_W^2} \right] + \sin^2 \theta \left[ \frac{1}{\epsilon_W} \left( \frac{m_2^2}{m_2^2 - m_Z^2} \right) \ln \frac{m_2^2}{m_2^2 - m_W^2} - \left( \frac{m_2^2}{m_2^2 - m_W^2} \right) \ln \frac{m_2^2}{m_W^2} \right] \right\},$$

where $m_1 (\approx 125 \text{ GeV})$ and $m_2$ are the two mass eigenstates, and $\theta$ is the mixing angle. The SM expression for $T$ can be found by putting $\theta = 0$ and $m_1 = m_h$. The quantity constrained by the electroweak fit, $\Delta T$, is given by [18]

$$\Delta T = T_{\text{SM}+S} - T_{\text{SM}} = 0.01 \pm 0.12,$$

$T$ is related with the $\rho$-parameter by $\rho - 1 = \alpha T$. For small mixing ($\theta \leq 0.1$), the constraints coming from the oblique parameters are not significant.

2.5 Renormalization Group equations

We would also like to see how the couplings evolve with energy. The one-loop $\beta$-functions are [19]

$$16\pi^2 \beta_\lambda = 12\lambda^2 + 6g_t^2\lambda + a_2^2 - \frac{3}{2}\lambda(g_1^2 + 3g_2^2) - 3g_t^4 + \frac{3}{16}(g_1^4 + 2g_1^2g_2^2 + 3g_2^4),$$

$$16\pi^2 \beta_{b_4} = 36b_4^2 + a_2^2,$n

$$16\pi^2 \beta_{a_2} = \left[ 6\lambda + 12b_4 + 4a_2 + 6g_t^2 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2 \right] a_2,$n

$$16\pi^2 \beta_{g_t} = \left[ -\frac{9}{4}g_1^2 - \frac{17}{24}g_1^2 - \frac{9}{8}g_2^2 - 4g_2^2 \right] g_t$$

(27)

where $\beta_h \equiv dh/dt$, and $t \equiv \ln(Q^2/\mu^2)$. The $\beta$-functions for all gauge couplings are identical to that of the SM. For simplicity, we have put all the SM Yukawa couplings equal to zero except for that of the top quark. This hardly changes our conclusions.

For the $Z_2$-asymmetric case, the trilinear couplings $a_1$ and $b_3$ also evolve:

$$16\pi^2 \beta_{a_1} = a_1 (9\lambda + 4a_2) + 6a_2b_3,$n

$$16\pi^2 \beta_{b_3} = 2a_1a_2 + 36b_3b_4.$n

(28)

Note that $\beta_{b_4}$ is always positive and hence can only increase, starting from a positive value. $\beta_\lambda$ also gets a positive contribution on top of the SM ones. These make the couplings blow up at a much lower scale than the Planck scale ($\sim 10^{19} \text{ GeV}$), unless one starts with very small values of $b_4$. Similarly, $\beta_{a_2}$ is proportional to $a_2$ itself and can lead to a blow-up for large $a_2$.

For our analysis, we have taken the initial values of the couplings at the electroweak scale in such a way that the Higgs boson mass is correctly reproduced as $m_h \in [124 - 126] \text{ GeV}$. The threshold effects are taken at the singlet mass scale, however it has been seen that the final results are not very sensitive on the exact choice of this scale, and moreover, uncertainties coming from possible higher-loop contributions are larger compared to the uncertainties coming from the threshold corrections. The constraints on the parameter space are all obtained with the one-loop improved values of the couplings.
Figure 1: (a) Left: Schematic diagram of an asymmetric double-well potential with a single field. Parameters $\epsilon$ and $\delta$ control the tunnelling lifetime. If there are more than one fields, one gets a multi-dimensional contour. (b) Right: Example of a two-dimensional contour for SM+S. The line shows the shortest path joining the two minima along which the tunnelling probability should be calculated. Notice that the path does not pass through the local maximum. Contour values denote the potential at that point in the unit $10^{10}$ GeV$^4$.

2.6 The unstable vacuum case

If there are two minima of the potential and the EW vacuum is shallower, the universe can tunnel down to the deeper vacuum. In such cases, the parameters must be chosen such that the lifetime of the shallower vacuum should be at least as large as the lifetime of the universe, which is about 13.7 billion years.

To calculate the lifetime of the metastable state [20], let us assume that the potential at the EW vacuum is zero (this can always be achieved by a constant shift), the true vacuum has a depth $-\epsilon$ ($\epsilon > 0$), and the height of the barrier with respect to the SM vacuum is $\delta$, as shown in Fig. 1. The decay width density of the universe, $\Gamma/V$, is given by

$$\Gamma/V = A\exp(-B),$$

where $A$ is a small pre-factor, and $B$ is estimated in the thin-wall approximation as [21]

$$B = \frac{211\pi^2}{3\lambda} \left(\frac{\delta}{\epsilon}\right)^3.\quad (30)$$

This expression is true for a single field whose self-quartic coupling is given by $\lambda$. With more than one fields, there are several dimensionless couplings and the tunnelling path may not even go through under the hill. However, for SM+S, what we have done numerically is to find the two minima and then calculate the tunnelling along the straight line joining them, as shown in Fig. 1. This is effectively a single-field approximation, where the field is a combination of $h$ and $S$. The height $\delta$ is taken to be the maximum height above zero-level along this path and not the local maximum of the field space.

The ratio $\delta/\epsilon$ should be greater than 0.1 to make the shallower vacuum stable with respect to the lifetime of the universe. Keeping in mind of the necessary simplifications, we have chosen only those models for which this ratio is more than unity, and therefore the stability is assured.

2.7 One-loop corrections to the potential

The one-loop effective potential in the SM is given by

$$V_1(\phi_c) = \frac{1}{64\pi^2} \sum_i n_i m_i(\phi_c)^4 \left(\ln \frac{m_i(\phi_c)^2}{Q^2} - C_i\right),$$

where $i$ runs over $h, G, W, Z$ and $t$, and

$$n_i = 1, 3, 6, 3, -12, \quad C_i = \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{3}{2}.\quad (32)$$
for \( i = h, G, W, Z \) and \( t \) respectively. The masses are field-dependent and depend on the classical minimum \( \phi_c \). \( Q \) is the arbitrary regularization scale.

To be precise, the radiative corrections, being suppressed by the loop factor, are significant only along a flat or near-flat direction in the field space. If all the dimensionful parameters are zero, the theory becomes scale invariant. The minimization conditions in this case, namely, \( v \left( \lambda v^2 + a_2 v_s^2 \right) = 0 \) and \( v_s \left( a_2 v^2 + 4b_4 v_s^2 \right) = 0 \), yield \( 4b_4 = a_2^2 \) as the consistency condition if neither \( v \) nor \( v_s \) vanishes. The last condition makes the determinant of the mass matrix equal to zero, ensuring a massless mode and hence a flat direction in the field space. If dimensionful couplings are present, there is in general no flat direction in the \( \phi-S \) plane, and one expects the radiative corrections to the potential to have a small effect. Note that \( 4b_4 = a_2^2 \) is the limiting case of the stability condition. However, if there is a strong hierarchy between the two VEVs, the direction along the smaller-VEV field is almost flat. The choice of the regularization scale may change the points with \( S > 0 \) need a very large and negative \( \lambda \), this also shows that putting \( a_1 = -2a_2v_s \) in Eq. (15) does not lead to a CDM candidate, because such a fine-tuned relationship is not stable under radiative corrections.

Thus, \( V \) includes all the SM fields and the singlet.

The regularization scale \( Q \) is arbitrary, but one can choose it in such a way that the one-loop corrected minimum for \( \Phi \) remains unchanged, in other words, \( \phi_c \) falls at \( v \). This keeps all the SM fermion and gauge boson masses invariant, and also keeps the Goldstone bosons massless; thus, we do not need to consider the Goldstone boson contributions for the effective potential. A detailed discussion of the procedure is given in Ref. [16] for the two-Higgs doublet model potential.

### 3 Z\(_2\)-symmetric potential with \( v_s = 0 \)

Let us first go through the well-studied case of the \( Z_2 \)-symmetric potential \( (a_1 = b_1 = b_3 = 0 \text{ in Eq. (4)}) \), for the sake of completeness of this study. One can always get a minimum at \( v_s = 0 \) by applying the shift symmetry \( S \to S + \Delta \). This minimum should better be the only one, or at least the global one (a local one with lifetime more than the age of the universe will also do) if we want to have a CDM candidate in \( S \), with mass of \( \sqrt{a_2 v^2 - 2M^2} \). The points with \( M^2 > 0 \) need a very large and negative \( a_2 \) and are ruled out by the CDM spin-independent scattering cross-section limits. Thus, all such Higgs portal dark matter models must have a right-sign mass term \( (M^2 < 0) \) for \( S \). This is true even for the narrow region of Higgs resonance, at about \( m_S \approx m_h/2 \).

The only way for \( S \) to interact with the SM sector is through the term \( a_2 S^2 \Phi^1 \Phi \) in Eq. (1), since \( a_1 = b_1 = b_3 = 0 \) in this case. The spin-independent CDM-nucleon scattering cross-section is given, in this scenario, by \[ \sigma = \frac{a_2^2 f^2 m_N^2}{\pi m_S^2 m_h^2}, \] where \( m_N \), \( m_S \) and \( m_h \) are the masses of the nucleon, \( S \), and Higgs respectively. The matrix element for scattering is given by \( f \), whose value is approximately 0.3. If \( a_2 \) is very small, the CDM detection cross-section becomes small, but the annihilation rate goes down too, leaving more dark matter in the universe than is allowed, and thus leading to overclosure. If \( a_2 \) is large, the scattering cross-section of CDM with nucleons is also large and hence will be severely constrained by direct detection experiments, in particular LUX, which gives the best limits now [24, 25]. Thus, apart from the narrow Higgs resonance region, only a small wedge for the dark matter mass \( M_{\text{DM}} > 200 \text{ GeV} \) is still allowed, and we focus only on those models that provide \( M_{\text{DM}} \) in this range.

In Fig. 2, we show the allowed regions as a function of \( a_2 \), every dot corresponds to a particular choice of parameters. For all these models \( M^2 < 0 \), which, by our definition of the potential, means a right-sign mass term for the singlet and no symmetry breaking in the \( S \)-direction. Technically, \( M^2 > 0 \) can also lead to a local
Figure 2: Allowed values of the parameter $a_2$ and the spin-independent cross-section of the DM in the $Z_2$-symmetric case with the minimum at $v_s = 0$. (a) Left: Allowed region for $a_2$. Regions outside the wedge are ruled out from direct detection experiments and overclosure ($\Omega > \Omega_{CDM}$). (b) Right: The spin-independent cross-section of the DM as a function of the DM mass. $a_2$ has been varied over the allowed range.

Figure 3: The range of validity as a function of $a_2$ (left) and $b_4$ (right), in the $Z_2$-symmetric case with the minimum at $v_s = 0$. minimum at $\langle S \rangle = 0$, but if we take $a_2$ to be in the perturbative region, such models lead to low $M_{DM}$ and are hence ruled out by the direct detection data.

Depending on the values of $a_2$ and $b_4$, one can also check how far in the energy scale the singlet DM model remains valid. For this, we use the one-loop renormalization group (RG) equations$^3$ and see where the couplings become non-perturbative and ultimately hit the Landau pole, or the potential becomes unstable. Our results are shown in Fig. 3 for the two relevant parameters $a_2$ and $b_4$. Note that there is an upper limit on $b_4 \sim 0.4$ above which the model ceases to be valid even before 50 TeV. While there is no such limit for $a_2$, low-$a_2$ models have a smaller range of validity compared to medium-$a_2$ ($\sim 0.3$) models, where the validity can be as high as $10^{15}$ GeV.

While the allowed region for each model depends on the exact values of the parameters chosen, some intuitive insights can be put forward. As there is no mixing, $\lambda$ must start from its SM value $\sim 0.13$. The only modification to its $\beta$-function comes from the $a_2^2$ term, so the range of validity increases with increasing $a_2$, provided $b_4$ is sufficiently small to start with and does not hit its Landau pole earlier ($b_4$ starts from a positive value, and always increases). So, for the low-$a_2$ regions, it is the vacuum stability that mostly controls the allowed range. After $a_2$ passes a certain value, and/or $b_4$ becomes large, the range is controlled by the blowing up of one or more of the couplings. As we have just shown, $a_2 < 0$ is already ruled out for this model.

$^3$This gives a pretty good estimate, although two-loop results are available.
A similar study on the parameter space of Higgs portal dark matter models was performed recently in Ref. [26]. There is always a chance that with improved measurements, the wedge region may go away. In the Higgs resonance region, the allowed values of \( a_2 \) are much smaller, and from Fig. 3, we see that such models cease to be valid at about \( 10^6 \) GeV.

This model, with addition of vectorlike fermions, may explain the recently observed resonance at 750 GeV. However, introduction of such fermions spoils the possibility of a Higgs portal dark matter, as the scalar decays through the fermion loops. On the other hand, there is a new constraint on the singlet mass, which narrows down the parameter space even further. The renormalization group equations also change, with new Yukawa couplings introduced, and may affect the stability of the potential. We will not discuss this extension any further here.

## 4 Singlet-doublet mixing with \( v_s \neq 0 \)

### 4.1 \( Z_2 \)-symmetric case

If the singlet field \( S \) develops a nonzero VEV, \( i.e. \ v_s \neq 0 \), the physical fields \( h \) and \( s \) become orthogonal combinations of \( \phi \) and \( S \), with the mixing angle constrained by the LHC data to be \( \theta < 0.1 \). Hence there is no
For $M^2 < 0$, only negative values of $a_2$ are allowed. This follows from the condition $(a_2 + 2M^2) < 0$. Both positive and negative values of $a_2$ are allowed for $M^2 > 0$.

- There is a correlation between $a_2$ and $\lambda$ for $M^2 < 0$. This follows from the $a_2^2/\lambda$ dependence of $v_s^2$ in Eq. (12).

- Only small positive values of $\mu^2$ are allowed for $M^2 < 0$. This can be understood from the constraint $(a_2 + 2\mu^2 - 2M^2) < 0$ coming from the stability criteria, as mentioned in Sec. 2. It can be shown easily that for $M^2 < 0$, $a_2$ and $\mu^2$ have to be of opposite signs to have real $v_s$, and the only possibility to have a real $v$ as well is to have $\mu^2 > 0$ and $a_2 < 0$. The magnitudes of $a_2$, $\mu^2$ and $M^2$ are, however, restricted by the Higgs mass and the mixing angle $\theta$. For $M^2 < 0$, there is no such constraint and $|\mu^2|$ can be large.

The range of validity for the models is shown in Fig. 5. One can see that large values of $a_2$ or $\lambda$ necessarily mean a smaller range of validity, which follows from the nature of the RG equations. The coupling $b_4$ always increases, so one has to start with a sufficiently small value of $b_4$ not to hit the Landau pole. The other two couplings, $\lambda$ and $a_2$, can be controlled by the negative contributions coming from Yukawa or gauge couplings if they are sufficiently small to start with. Too small a value means instability setting in at a low scale, and too large a value means a quick blowing up of the couplings. Thus, an intermediate range, $a_2 \sim \lambda \sim 0.2$ is the region for maximum validity. We have explicitly checked that the mixing angle is always well within the LHC limit.

One-loop corrections do not change the nature of the potential qualitatively, except changing the depth of the potential. However, the composition of the CP-even neutral scalars change with the one-loop corrections, because the ratio of $\phi_c$ and $\eta_c$, and hence the mixing angle $\theta$ of the mass matrix changes from its tree-level value.

### 4.2 $Z_2$-asymmetric case

In this section, we focus on only those models that allow two non-zero minima for the potential, as discussed in Section 2.2. From Eq. (12) we can see that for $b_3 \neq 0$ the two minima have unequal depths. We demand one of them to be the EW vacuum with $|v| = 246$ GeV. If this is the deeper minimum, the universe is stable; if this is the shallower one, the universe can decay to the true vacuum and in that case the lifetime must at least be equal to the age of the universe.

First, let us focus on the tree-level potential. In Fig. 6, we show the range of validity of these models for various choices of $a_2$ and $\lambda$. The trend is similar to what we have seen before in Section 4.1: large values of $\lambda$, $a_2$, or even $b_4$ make the couplings blow up at a relatively low scale. Note that $a_2 < 0$ ($a_2 > 0$) models tend to have a local (global) minimum at $v = 246$ GeV, but there are exceptions. The range of validity of these models

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4Such a model suffers from the usual domain wall problem. However, existence of multiple vacuum states in the universe is not yet ruled out, and in fact is a distinct possibility in several string theory motivated scenarios.
Figure 7: The dependence of the lifetime of the metastable vacuum on $b_3$ (left) and $\lambda$ (right).

| Parameter | Model 1 | Model 2 |
|-----------|---------|---------|
| $a_1$ (GeV) | −20.8 | 90.1 |
| $a_2$ | 0.40 | 0.22 |
| $b_3$ (GeV) | −58.6 | 79.3 |
| $b_4$ | 0.26 | 0.33 |
| $\lambda$ | 0.46 | 0.60 |
| $\mu^2$ (GeV$^2$) | $4.58 \times 10^5$ | $1.16 \times 10^5$ |
| $M^2$ (GeV$^2$) | $4.85 \times 10^5$ | $1.72 \times 10^5$ |
| Global min.: $(v, v_s)$ (GeV) | (247, 1035) | (245, −598) |
| Local min.: $(v, v_s)$ (GeV) | (673, −786) | (362, 406) |

Table 1: Parameter values for the models for which the 1-loop corrections are shown in the Fig. 8.

is almost the same as that of the $Z_2$-symmetric case, because the RG evolutions of the couplings are controlled by the dimensionless couplings. The allowed region for $a_2$, however, is bunched more towards $a_2 \sim 0$.

For models with a shallower minimum at the EW vacuum, one may also estimate the lifetime of the universe. This is bound to be a rough estimate as the path between the two minima need not pass through a local maximum or even a saddle point. Approximately, $B \geq 1$ (see Eq. (30)) leads to a metastable vacuum [21] while $B \leq 1$ tends to make the universe unstable. Assuming the maximum of the quartic couplings to be of the order of unity, this results in an approximate bound of $\delta/\epsilon > 0.05$.

In Fig. 7 we show how the tunnelling lifetime depends on the parameter $b_3$ as a function of the ratio $\delta/\epsilon$ and $\lambda$ as a function of $B$. The dependence of $b_4$ is similar to that of $\lambda$. Fig. 7 shows that the controlling parameter is $b_3$ because that creates the depth difference between the two minima. The smaller $b_3$ is, the larger is the lifetime of the metastable minimum. As a conservative estimate, we have taken the stability limit to be $\delta/\epsilon \geq 1$, which excludes the low $\delta/\epsilon$ and hence low $B$ values. Such excluded models are also shown in the right hand side plot of Fig. 7. The distribution of models does not depend much on the quartic couplings $b_4$ or $\lambda$.

5 One loop corrections to the SM+S potential

The effect of one-loop corrections on the tree-level potential is not very drastic. This is expected because both VEVs are nonzero and neither $\Phi$ nor $S$ direction is a flat one. The one-loop corrections, being perturbative in nature, are suppressed by the standard loop factor of $1/64\pi^2$ and are expected to be significant only if we look at a flat direction.

We show the one-loop corrections for two models, the parameters at the electroweak scale are given in Table 1. We choose the regularization scale $Q$ in such a way that even after the one-loop corrections, the EW vacuum stays at $v \approx 246$ GeV (so that all SM particles have their masses unaffected). This choice of regularization scale
was motivated in Ref. [16]. We show the effect of one-loop corrections in Fig. 8 for these two models. While they both have the global minimum at $v \approx 246$ GeV, the required regularization scales differ by more than one order of magnitude. The nature of change is similar for all models, and thus we do not expect an unstable vacuum model to become metastable (or vice versa) because of the one-loop corrections. However, one may note how much the global minimum has been lowered by the one-loop corrections for the second model. In fact, for all the models scanned, we have never found a switch from global to local minimum induced by the radiative corrections.

6 Summary

The potential of SM+S, a real singlet enhanced SM, shows several interesting features. In this paper, we have investigated the parameter space for several types of SM+S: the Higgs portal dark matter models, the potential with an explicit $Z_2$-symmetry and having singlet-doublet mixing, the $Z_2$-asymmetric potential with a stable EW minimum, or the same with an unstable or metastable EW minimum. The general features can be summarized as follows.

- Adding one more real singlet makes the potential less stable in general at a high energy. This happens because the renormalization group equations for the couplings tend to hit the Landau pole much below the Planck scale, more so if the starting values at the EW scale is large. All the quartic couplings, namely, $\lambda$, $b_4$, and $a_2$, have to be small at the EW scale to keep the model valid up to a high scale, as the $\beta$-functions are coupled. (However, such new scalar couplings are helpful to avoid the vacuum stability bound, coming from the negative pull caused by the large top Yukawa coupling. Again, such conclusions are not valid if
there are more degrees of freedom, like vectorlike fermions.) Our numerical results are shown at one-loop but inclusion of higher-order corrections do not change the result qualitatively. In particular, if we start with a large value of either $\lambda$ or $b_4$, the model hits the Landau pole at a relatively low scale. This can be taken as a possible indication of some new physics taking over and one should consider the effect of higher-dimensional operators on the low-scale physics.

- While there are some minor variations for the allowed range of parameters among different class of models (e.g. $a_2$), the overlap is significant, and so one has to determine all the couplings experimentally to know what class of SM+S it really is. This is, of course, an extremely challenging task, if not outright impossible; the determination of $b_4$ is apparently beyond the reach of any present or upcoming colliders unless there is a significant mixing between the singlet and the doublet and the Higgs self-couplings are determined with sufficient accuracy. For the prospect of determination of the singlet-doublet mixing angle in future colliders, we refer the reader to Ref. [27].

- The tree-level results are robust enough as far as the metastability issue is concerned. This is expected as we are not looking along any flat direction. However, the one-loop corrections can change the position of the minima.

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