The generation of signal and background sky maps for ultra-high energy gamma ray source with a Monte Carlo method

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Abstract. Studying the ultra-high energy gamma ray source is an important way to reveal the origin of the cosmic ray, which is also a main topic of many ground gamma ray experiments. To expect the observation ability during the design of the experiment, as well as to test the analyse methods in the observation, a Monte Carlo procedure for the signal and background generation is necessary. In this paper, such a procedure is introduced. And the source significance for a Blazar source Mrk421 is expected based on this procedure. Finally, an unfolding method for the source spectrum measurement is also established bashed on this Monte Carlo data, which gives fitting errors around ±10.0% both for the flux and the spectrum index.

1. Introduction
Blazar, which is one category of Active Galaxy Nuclear (AGN) and has strong emits at all wavelengths, is the brightest observed extra-galactic objects. The observation by both the satellite/balloon and ground experiments indicate that the Spectral Energy Distribution (SED) of these objects exhibits double-hump structure [1]. The first hump, around the soft to medium X-ray range, is thought to be synchrotron radiation from high energy electrons [2]. The origin of the second hump, around GeV/TeV range, is unclear. In addition, the composition of these jets is also not known. It is not clear whether they are made of electron-positron plasma (lepton origin) or electron-proton plasma (hadron origin).

Mrk421 and Mrk501 are the nearest and brightest BL Lac type of Blazars in the X-ray and UV sky. They are also the first and second detected extra-galactic sources at TeV energies [3, 4]. Many space and ground experiments have observed these two objects at all wavebands. The associated analysis of these experiments at multiple wavebands identifies its common features. They are the double-hump structure, the strong correlation between KeV/MeV X ray and GeV/TeV gamma ray wavebands, as well as the spectrum becomes harder at higher energies for all wavebands [5]. All these features are flavoured with the synchrotron self-Compton (SSC) model. However, several flare observations indicate that the flux becomes strong only at gamma ray band and the experiments at X ray band do not observe the flare (orphan flares) [6], which cannot be explained by the SSC model.

A long-term, unbiased without seasonal and temperature, and durative stable monitoring can help to reveal the features of these ultra-high energy sources. Some sensitive experiments, like LHAASO-WCDA [7] and HAWC [8] are designed to achieve this goal. To understand the physical mechanicals of the observation, as well as to test the analysis method, the uncertainties initiated by the detected effects should be examined clearly. Therefore, a fast Monte Carlo (MC) procedure to generate the
signal and background maps for these objects, in which the detector responding is parameterized, is necessary.

In this paper, the detection principle of these experiments is described briefly at first. Then the method of the generation of the signal and the background maps are presented, followed by an example of the sky map generation of Mrk421 observed by a Water Cherenkov experiment. An energy spectrum fitting procedure based on MINUIT package [9], which is a tool to search the minimum value of a multi-parameter function, is constructed with the maps generated by this method. The summary for this MC procedure is given at last.

2. Monte Carlo method

2.1. Event detection principle
While an ultra-high energy gamma ray travels through the atmosphere, an extensive air shower (EAS) will be initiated. Due to the neutrality of the gamma rays, they will not be deflected by the interstellar magnetic field. While the travelling speed of the particles in the EAS exceed the light speed, Cherenkov photons will be emitted. Through detecting such Cherenkov photons, one can observe the EAS. By detecting such an EAS, the ground experiment can obtain the primary information of the gamma, so as to get the information of the objects in the universe where these gamma rays are originated and accelerated. While the EAS propagates in pure water, the generation of Cherenkov photons are more than in the air. Such a water Cherenkov (WC) technique is used by some ground experiments, locating the photo multiplier tube (PMT) in the pure water to detect Cherenkov lights. In the universe, the intensity of cosmic ray (CR) particles (mostly hadron) are about more than four orders stronger than gamma. And they also can produce EAS in the atmosphere, so as to be captured by the detector. That is, CRs are the strong background detected in the WC experiment. Therefore, in the MC simulation, both the gamma and hadron maps are generated.

2.2. Sky map generation
Given an energy spectrum model \( J(E) \) (1) of a source, the total number of signals detected by a WC experiment can be described as (2).

\[
J(E) = J_0 E^{-\alpha} \exp \left(-\frac{E}{E_{cut}}\right) \left(\text{TeV} \cdot \text{s}^{-1} \cdot \text{cm}^{-2}\right)
\]

where \( J_0, \alpha \) and \( E_{cut} \) are the parameters from the observation. \( E \) is the primary energy of the gamma.

\[
N_{tot} = \int_{E_1}^{E_2} J(E) \cdot A_{eff} \cdot \Delta T \cdot dE
\]

where \( A_{eff} \) is the effective area of the experiment, which includes the effects of the detector area, the trigger efficiency and the event selection efficiency. \( \Delta T \) is the observation duration. \( N_{tot} \) gamma events are sampled by the following steps. Firstly, the energy of each shower is sampled based on the energy spectrum model described in formula (1). Considering the angular resolution of the detector, the events are sampled in the equatorial coordinate system (ECS) obeying a two-dimensional Gauss distribution. The mean is the source location in the ECS. \( \sigma \) is the angular resolution of the detector, which is a function of the energy. For the large hadron background of \( \gamma \) detection, a \( \gamma/P \) discrimination procedure should be developed by each experiment. Some parameters will be constructed based on the different features of the \( \gamma \) and the hadron events. The cut values of the parameter will be chosen to ensure a high \( \gamma \) passing rate while guaranteeing a less hadron contamination. In view of such \( \gamma/P \) discrimination, each event is assigned a weighted factor depending on its \( \gamma \) selected efficiency as a function of energy. Considering all above factors, the signal distribution map of the source is obtained.

The distribution of CR backgrounds is isotropic in the universe. Its primary energy spectrum exhibits a power law as (3). The total background can be calculated by formula (4) during an observation duration \( \Delta T \).

\[
I(E) = I_0 E^{-\alpha} \left(\text{TeV} \cdot \text{s}^{-1} \cdot \text{cm}^{-2} \cdot \text{sr}\right)
\]

\[
N_{bkg} = \int_{E_1}^{E_2} I(E) \cdot A_{eff} \cdot \Delta T \cdot \Delta \Omega \cdot dE
\]
The backgrounds are calculated in an area of interest (AOI) around the source (i.e. ±5°) and the AOI is divided into 100 × 100 bins. The predicted events $N_{i,b0}$ in the $i^{th}$ pixel can be calculated according to the formula (4). The observed events $N_{i,b}$ in this pixel can be obtained by sampling a Poisson distribution $P(N_{i,b}; N_{i,b0})$. It is noted that all the bins should contain an equal solid angle $\Delta \Omega$. Then considering the detected effects and performing the sampling procedures same as the signal distribution on all the $N_{i,b}$ events, a sky map for the background can be obtained.

3. Procedure Applications

In astronomy area, the significance of a source, which is represented by a ratio of the exceed of the detected signals upon the background, is a fundamental parameter. While the significance exceeds a certain value (usually 5). One can declare the discovery of the source and perform other physical analysis to the source. As an example, the detection of the source Mrk421 by a WC Detector Array (LHAASO-WCDA) [7], which is a gamma ray experiment, being constructed in Daocheng, Sichuan Province, has been simulated. Its significance has been calculated in one year’s observation. In addition, to test the MC procedure, its energy spectrum has been fit based on an unfolding procedure.

3.1. Significance map

The primary spectrum of $\gamma$ and proton for Mrk421 are from [8] and [10], respectively. The energy samplings of the events are shown in Figure 1. The orbit of the source in the horizontal coordinate system (local coordinate system) is presented in the left panel of Figure 2, while the zenith distribution as a function of mjd is displayed in the right panel. To ensure a good resolution, the events with the zenith great than 45° are cut. After this cut, the observation time of the source in a sidereal day is about 7.12 hours. Considering the effective area (Figure 3), angular resolution (left panel of Figure 4) and $\gamma$/P selection efficiency (right panel of Figure 4) [11], the signal and background maps of Mrk421 observed in one year are obtained as shown in Figure 5.

![Figure 1. The sampling energy distribution of the gamma (left) from Mrk421 and the proton (right). The red curves are the fitting results by formulas (1) and (3), respectively.](image-url)
Figure 2. Left: The orbit of the source in the horizontal coordinate system. The zenith distribution of the source as a function of mjd. The red line presents the zenith cut in the analysis.

Figure 3. The distribution of $A_{\text{eff}}$ as a function of energy for gamma events (left panel) and proton events (right panel) for different zenith ranges.

Figure 4. The gamma shower angular resolution and the $\gamma$/P selection efficiency of the experiment.
Figure 5. The signal (left) and background (right) distributions of Mrk421 in one year’s observation.

Once the signal and background maps are obtained, the significance of the source can be calculated according to the Li-Ma formula [12]. Considering the angular resolution of the detector, one needs to do the smoothing while calculating the significance. To get an optimal smoothing radius, an optimization procedure is done. The results are shown in the left panel of Figure 6, which indicates that the optimal smoothing radius is 0.57°. Under the optimal smoothing radius, the significance of Mrk421 in one year’s observation are calculated and the significance map is presented in the right panel of Figure 6. The most significant point is 89.3 σ.

3.2. Energy spectrum fitting

3.2.1. Unfolding method. Taking an unfolding method to obtain the primary energy spectrum of the source is a traditional way [13]. In this study, an unfolding procedure by MINUIT package [9] based on a minimum χ² fitting is adopted. The χ² function is constructed as:

\[
\chi^2 = \sum_{i=1}^{N_{\text{bin}}} \frac{(N_{\text{exp}}(i) - N_{\text{fit}}(i))^2}{\sigma^2}
\]

where \(N_{\text{exp}}(i)\) is the observed signal in the \(i^{th}\) pixel during a fixed observed duration. \(N_{\text{fit}}(i)\) is the expected signal in the same pixel under a spectrum model assumption. For a given model (1) with a group of parameters \(f_0\), \(\alpha\) and \(E_{\text{cut}}\), the predicted events \(N(i)\) is calculated following (2). Then the expected signal distribution (i.e. the distribution of \(N_{\text{fit}}(i)\)) is obtained considering the angular resolution and the \(\gamma/P\) discrimination.

Figure 6. Left: The significance distribution as a function of smoothing radius for Mrk421 in one year’s observation. The Red star is the optimized radius. Right: The significance of Mrk421 in one year’s observation under the optimal smoothing radius.
3.2.2. The results. To test the measured accuracy of the energy spectrum, the fitting under one week, one month, three months, one year and two years, are carried out, respectively. For the data of less than three months, the fitting parameter is only $f_0$ because the statistics are not enough to do the spectrum index fitting. The fitting errors are within $\pm 10.0\%$, $\pm 4.6\%$, $\pm 4.4\%$ for one week, one month and three months, respectively. The results indicate that the fitting accuracy is increased with the increase of data statistics. For the data of one year and two years, both the flux $f_0$ and the index $\alpha$ are the fitting parameters. Both the fitting error for the two parameters are around $\pm 10.0\%$.

4. Summary
A fast MC procedure for the study of the ultra-high energy gamma ray source has been established. It includes gamma ray and hadron background generation. The procedure is flexible and can generate the signal and background maps for all kinds of ground astronomy experiments by inputting the detector responding curves as a function of the energy, like the angular resolution, the effective area and $\gamma/P$ selection efficiency et al. To test the MC procedure, two applications based on a water Cherenkov experiment towards the source Mrk421 have been performed. The first is the significance calculation of the source, which is the fundamental work to perform other physical analysis. To obtain a high significance, an optimization procedure for the smoothing radius has been done. Based on the optimal smoothing radius of 0.57°, 89.3 $\sigma$ within one year’s observation, which is agreed well with the real observation, was obtained. The second is the energy spectrum measurement for the gamma source based on an unfolding procedure. The measurement errors both for flux and spectrum index are around $\pm 10.0\%$ while the observation time is greater than three months, which are accurate fitting results towards such a ground WC experiment.

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