Shell-model description for the properties of the forbidden $\beta^-$ decay in the region “north-east” of $^{208}$Pb

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In the present work, we report a comprehensive shell-model study of the log $ft$ values for the forbidden $\beta^-$ decay transitions in the north-east region of $^{208}$Pb. For this we have considered $^{210-215}$Pb $\to$ $^{210-215}$Bi and $^{210-215}$Bi $\to$ $^{210-215}$Po transitions. We have performed shell-model calculation using KHPE interaction in valence shell 82-126 for protons and 126-184 for neutrons without any truncation. We have also calculated half-lives and Q-values for the concerned nuclei. Recently several log $ft$ values are observed corresponding to $\beta^-$ decay from (8-) isomeric state of $^{214}$Bi at CERN-ISOLDE facility [Phys. Rev. C 104, 054301 (2021)], and for the first time we have reported shell-model results for these transitions.

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I. INTRODUCTION

The r-process plays a significant role in the nucleosynthesis of heavier nuclei in astrophysics [1]. It is believed to occur in core-collapse supernovae [2] or neutron star mergers [3], but the actual site of r-process nucleosynthesis is still an open area for investigation [4]. The main conditions required for the r-process nucleosynthesis [5] are high temperature and large neutron density [6, 7]. Further, the abundance pattern of r-process nuclei shows enhanced peaks near neutron shell closures. $^{208}$Pb is the heaviest doubly magic stable nucleus with 82 protons and 126 neutrons. Due to the increased stability of doubly magic nuclei compared to the single magic nuclei and others, the region around these nuclei has always been an area of great interest for investigation. The measurement of half-lives of nuclei near N = 126 is difficult. Thus, it is highly desirable to give theoretical predictions for half-lives around this region. Suzuki et al. [8] have evaluated beta-decay properties of r-process nuclei near N=126 isotones using shell-model calculations. First forbidden beta-decay competes with allowed Gamow-Teller and Fermi beta-decay in the region of N=126 r-process nuclei [9, 10]. It is thus crucial to study these nuclei in this region for a better understanding of the abundance pattern of these r-process nuclei.

Beta-decay can be divided into two categories: allowed and forbidden, based on the value of the angular momentum of the emitted leptons. Allowed transitions correspond to the no change in parity and $l = 0$ state of the emitted leptons relative to the nucleus, whereas $l > 0$ corresponds to the forbidden transitions. Further, based on the value of spin angular momentum, these transitions can be characterized in two categories, Fermi and GT transitions. Fermi transitions correspond to spin non-flip with $S = 0$, whereas GT transitions correspond to spin-flip with $S = 1$. Further, forbidden transitions can be divided into the unique forbidden beta-decay and the non-unique forbidden beta-decay. In unique forbidden beta-decay, $\Delta J = l + 1$, while in non-unique forbidden beta-decay, $\Delta J = l - 1, l$, where $l$ is the degree of forbiddenness. Change in parity for forbidden beta-decay is positive (even) for an even degree of forbiddenness, while negative (odd) for an odd degree of forbiddenness. Beta-decay properties have been evaluated using effective values of the weak coupling constants in Refs. [11, 12]. Recently, using shell-model, our group has calculated first-forbidden beta-decay properties of $^{207}$Hg $\to$ $^{207}$Tl in Ref. [13].

There are several approaches that can be used to calculate beta-decay properties. First is the macroscopic approach, i.e., Gross theory of beta-decay [14] and second is global semi-microscopic approaches such as quasi-particle random phase approximation (QRPA) [15], density functional theory (DFT) [16], Hartree-Fock-Bogoliubov method (HFB) [17], etc. These models underestimate the residual interaction between nucleons, which reduces the Gamow-Teller (GT) strengths towards lower excitation energies [18–20]. The third approach is the microscopic approach, i.e., shell-model. We are using large-scale shell-model in the study of beta-decay properties. Further, in shell-model, quenching factors for the weak axial and vector coupling constants are needed to reproduce reliable data [21, 22]. Several efforts have been done to calculate quenching factors in Pb region: Warburton [23] found that the quenching factors for the axial and vector coupling constants are different and it comes out to be $(g_A/g_A^{free}, g_V/g_V^{free})=(0.47, 0.64)$ in the Pb region. Further, the mesonic enhancement factor has been calculated in Refs. [24, 25] for the first forbidden beta-decay with $\Delta J^\pi = 0^-$ for $A = 205-212$, where the enhancement factor is defined as the proportion of the axial-charge matrix element $\gamma_5$ in first-forbidden beta-decay to its impulse-approximation value. This value comes out

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to be $\epsilon_{\text{MEC}} = 1 + \delta_{\text{MEC}} = 2.01 \pm 0.05$. Later on, Rydström et al. in Ref. [26] found out two sets of quenching factors in the Pb region: first one being $(g_A/g_A^{\text{free}},
g_N/g_N^{\text{free}}) = (0.34, 0.67)$ and the second one was $(g_A/g_A^{\text{free}},
g_N/g_N^{\text{free}}) = (0.51, 0.30)$. Furthermore, Zhi et al. [27] performed large-scale shell-model calculations for $r$-process waiting point nuclei with $N = 50, 82, 126$, including both GT and first forbidden transitions. They found that the shell-model overestimate the transition strengths in the GT and forbidden beta-decays. Thus, they found the quenching factors i.e. $(g_A/g_A^{\text{free}},
g_N/g_N^{\text{free}}) = (0.38, 0.51)$.

In the present work, the log $ft$ values, average shape factor, and half-lives have been calculated for $^{210-215}$Pb $\rightarrow$ $^{210-215}$Bi and $^{210-215}$Bi $\rightarrow$ $^{210-215}$Po transitions and compared with the available experimental data. These beta-decay properties have been computed using two sets of quenching factors: the first one being calculated from our work using the chi-squared fitting method, whereas the second one is taken from Ref. [27]. To the best of our knowledge, theoretical estimates for these nuclei mentioned are carried out for the first time except for log $ft$ values for $^{211}$Pb to $^{211}$Bi transitions [24], which were calculated by using old experimental data with truncated model space using KHEPE interaction [28]. In our work, we have performed large-scale shell-model calculations without using any truncation. Since $\beta$-decay is very sensitive to the Q-value, so, it is crucial to use a precise Q-value. Thus, we have also calculated Q-value using shell-model calculations and used them in the calculation of beta-decay properties. The shell-model results have also been calculated where experimental data is not available. Based on our calculated log $ft$ values, we have confirmed spin and parity of several states where experiments were unable to make unique assignments.

Recently, a new experiment has been performed at the CERN-ISOLDE facility to study $^{214}$Bi isotope using $\gamma$ ray spectroscopy, and a new isomeric state ($8^-$) has been identified. Further, beta-decay of this $^{214}$Bi$^{m}$ isomer to various spin parity states of $^{214}$Po was studied, and its log $ft$ values and beta-decay feeding fractions have been reported. Moreover, the experimental energy spectra of $^{214}$Po and yrast and yrare states of $^{214}$Bi were compared with the KHEPE and H208 interactions, but log $ft$ values using these shell-model interactions were not calculated. Thus, for the first time, these log $ft$ and average shape factor values have been calculated within the framework of shell-model and compared with the experimental values according to the decay mode as presented in Ref. [29].

The content of this paper is organized as follows: Section II depicts theoretical formalism in which shell-model Hamiltonian, beta-decay theory, and quenching factor are briefly discussed. Further, the calculation of the quenching factor with the help of the chi-squared fitting method is given in this section. In Section III, log $ft$ values, average shape factors, and half-lives for the concerned nuclei are evaluated, and computations of Q-values are also carried out. Finally, conclusion is given in Section IV.

II. FORMALISM

A. Shell-model Hamiltonian

The nuclear shell-model Hamiltonian can be expressed as combination of a single-particle energy term and a two-nucleon interaction term [30]. The shell-model Hamiltonian has the form

$$H = T + V = \sum_\alpha \epsilon_\alpha c^\dagger_\alpha c_\alpha + \frac{1}{4} \sum_{\alpha \beta \gamma \delta} v_{\alpha \beta \gamma \delta} c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta,$$

where $\alpha = \{n, l, j, t\}$ stands for single-particle state and the corresponding single particle energy is denoted by $\epsilon_\alpha$. $c^\dagger_\alpha$ and $c_\alpha$ stands for creation and annihilation operators. $v_{\alpha \beta \gamma \delta} = (\alpha | V | \gamma \delta)$ are the antisymmetrized two-body matrix elements.

B. $\beta$ decay theory for allowed and forbidden transitions

The theoretical formalism of beta-decay theory is briefly explained here. Here, we will give brief details of formalism about allowed and forbidden beta-decay. One can find more detailed formalism in Refs. [31, 32]. This formalism is based on impulse approximation [30] i.e., the decaying nucleon does not feel strong interaction with the remaining nucleons and only feels weak interaction at the instance of decay. Here, the remaining nucleons will act as spectator. The total half-life is the inverse of the decay rate, and it can be defined as

$$\frac{1}{T_{1/2}} = \sum_k \frac{1}{t_{1/2}^{(k)}},$$

where $t_{1/2}^{(k)}$ is the partial half-life to the final state $k$. The partial half-life is related to transition probability as

$$t_{1/2} = \int_{W_0}^{W_e} P(W_e) dW_e,$$

where the integrand in the denominator is the transition probability, and $m_e$ is the electron mass. The probability of the emitted beta particle to have energy between $W_e$ and $W_e + dW_e$ has the form

$$P(W_e) dW_e = \frac{G_F^2}{(\hbar c)^6} \frac{1}{2\pi^3} C(W_e) p_e c W_e (W_0 - W_e)^2$$

$$\times F_0(Z, W_e) dW_e,$$

where $G_F$ is the effective coupling constant, i.e., the Fermi coupling constant determines the strength of beta interaction, $p_e$ and $W_e$ are the momentum and energy of
the emitted beta particle, respectively. \( W_0 \) is the endpoint energy, i.e., the maximum energy attained by the emitted beta particle in the beta-decay process. The \( C(W_e) \) is the shape factor that depends on electron energy, and \( F_0(Z, W_e) \) is the Fermi function included in the expression for Coulomb interaction between the beta particle and remaining nucleus. For the simplification of the integration, dimensionless quantities are introduced such as \( w_0 = W_0/m_e c^2 \), \( w_e = W_e/m_e c^2 \), and \( p = p_e c/m_e c^2 = \sqrt{(w_e^2 - 1)} \). Thus, the dimensionless integrated shape function has the form

\[
f = \int_1^{w_0} C(w_e)pw_e(w_0 - w_e)^2 F_0(Z, w_e)dw_e. \tag{5}\]

The shape factor does not depend on electron energy in case of allowed transition i.e. \( C(w_e) = B(GT) \), where \( B(GT) \) is the Gamow-Teller transition probability. Thus,

\[
C(w_e) = \frac{g_A^2}{2J_i + 1} |\mathcal{M}_{GT}|^2, \tag{6}\]

where the \( J_i \) is the initial angular momentum and \( g_A \) is the axial-vector coupling constant, and the \( \mathcal{M}_{GT} \) stands for the Gamow-Teller nuclear matrix element [33].

Thus, the phase-space factor becomes

\[
f_0 = \int_1^{w_0} pw_e(w_0 - w_e)^2 F_0(Z, w_e)dw_e. \tag{7}\]

For forbidden beta-decay, the shape factor is given by

\[
C(w_e) = \sum_{k_e, k_e, K} \lambda_{k_e} \left[ M_K(k_e, k_e)^2 + m_K(k_e, k_e)^2 \right. \n - 2\gamma_{k_e} m_K(k_e, k_e) m_K(k_e, k_e) \left\], \tag{8}\]

where \( K \) is the forbiddenness order and \( k_e, k_e \) are the positive integers emerging from partial wave expansion of the leptonic wave function. The quantities \( M_K(k_e, k_e) \) and \( m_K(k_e, k_e) \) are expressed in terms of the nuclear matrix elements (NMEs) containing nuclear structure information and the leptonic phase space factors. The auxiliary quantities \( \gamma_{k_e} \) and \( \lambda_{k_e} \) can be written as

\[
\gamma_{k_e} = \sqrt{k_e^2 - (\alpha Z)^2} \quad \text{and} \quad \lambda_{k_e} = F_{k_e-1}(Z, w_e)/F_0(Z, w_e), \]

where \( \lambda_{k_e} \) stands for Coulomb function and \( F_{k_e-1}(Z, w_e) \) is the generalized Fermi function [34, 35] which has the form

\[
F_{k_e-1}(Z, w_e) = 4^{k_e-1}(2k_e)(k_e + \gamma_{k_e})[[2k_e - 1]]2^{k_e - y} (2\gamma_{k_e} - y) \sqrt{\pi} \Gamma((\gamma_{k_e} + y)/2) \left( 1 + 2 \gamma_{k_e} \right)^{-k_e}. \tag{9}\]

The auxiliary quantity \( y = (\alpha Z w_e/p_e c) \), where \( \alpha = 1/137 \) is the fine structure constant.

The NMEs [36, 37] can be described as

\[
V/A m_{KLS}^{(N)}(k_e, m, n, \rho) \tag{10}\]

\[
= \sqrt{2\pi} \sum_{\rho} V/A m_{KLS}^{(N)}(k_e, m, n, \rho)(\Psi_f \parallel \{\epsilon^k \} \| \Psi_i),
\]

where \( \tilde{J}_i = \sqrt{2\tilde{J}_i + 1} \) with \( J_i \) being the initial angular momentum and the summation runs over protons and neutrons single particle states. The quantity \( V/A m_{KLS}^{(N)}(k_e, m, n, \rho) \) stands for the single particle matrix elements (SPMEs) which is independent of the choice of nuclear models. \( (\Psi_f \parallel \{\epsilon^k \} \| \Psi_i) \) is the one body transition density (OBTDs) which vary for different nuclear models. Here, \( \Psi_i \) and \( \Psi_f \) are the initial and final nuclear states. The SPMEs are calculated with the help of formalism given in Ref. [38] whereas the OBTDs are calculated from the shell-model using NuShellX [39] and KSHELL [40]. The partial half-life is usually expressed as comparative half-life or the reduced half-life, which is given as

\[
f t_1/2 = \kappa, \tag{11}\]

where \( \kappa \) is constant value which is expressed as [41]

\[
\kappa = \frac{2\pi^3 h^7 \ln(2)}{m_e^3 c^4 (G_F \cos \theta_C)^2} = 6289 \ s, \tag{12}\]

where \( \theta_C \) is the Cabibbo angle which is the mixing angle between two generation of quarks.

Usually, \( ft \) are expressed in terms of ‘log \( ft \) values’ because \( ft \) values are large. Thus,

\[
\log (ft) \equiv \log(f_0 t_1/2).
\]

The phase space factor is sensitive to the Q-value. Therefore, it is essential to evaluate it precisely. Thus, the Q-value [42] can be expressed as

\[
Q(\beta^-) = E^{par}_{g.s.} - E^{dau}_{g.s.} + \delta m, \tag{13}\]

where \( \delta m = (m_n - m_p - m_e) c^2 = 0.78 \) MeV. \( E^{par}_{g.s.} \) and \( E^{dau}_{g.s.} \) stands for ground state binding energy for the parent and daughter nuclei, respectively. The binding energy of the ground state is given by

\[
E = E_{SM} + E_{core} + E_{C}(Z, N). \tag{14}\]

Here \( E_{SM} \) is the shell-model calculated binding energy and \( E_{core} \) is the binding energy of the core considered, and \( E_{C}(Z, N) \) is the Coulomb energy which can be calculated from the formalism given in Refs. [43, 44].
C. Quenching factor

In beta-decay, the Gamow-Teller and forbidden strengths get overestimated in the shell-model calculations. The weak coupling constants $g_V$ and $g_A$ are included in these Gamow-Teller and forbidden strengths, where $g_V$ is the vector coupling constant decided by CVC (Conserved Vector Current) theory and $g_A$ is the axial-vector coupling constant determined by PCAC (Partial Axial Vector Current) theory. The free nucleon values of these weak coupling constants are $g_V=1.0$ and $g_A=1.27$. These values get affected by many nucleon correlations like model space truncation in shell-model calculations and other nuclear medium effects. Therefore, the values of weak coupling constants get heavily quenched in the heavier mass region. Thus, we use effective values of these weak coupling constants. According to Behrens and Bühring [31], the average shape factor is given by

$$\overline{C(w_c)} = f/f_0. \quad (15)$$

Here $f$ is the phase space factor and $f_0$ is the phase space factor for allowed transitions. For allowed GT transitions, this shape factor is independent of electron energy. However, for $n^{th}$ forbidden transition, the average shape factor [24] comes out to be

$$\overline{C(w_c)}(m_2^n) = \frac{6289\lambda_{Ce}^2}{ft}. \quad (16)$$

Thus, for the first forbidden transition, the average shape factor [45] has the form

$$\overline{C(w_c)}(m^2) = \frac{6289\lambda_{Ce}^2}{ft} = \frac{9378 \times 10^5}{ft}, \quad (17)$$

where $\lambda_{Ce}$ is the reduced Compton wavelength of electron. Now, for allowed GT transitions, the operator is just $\sigma \tau$, whereas there are six Nuclear Matrix Elements (NMEs) for the first forbidden transition [30]. Out of which, four non-relativistic NMEs are extracted from wave-function expansion of p-wave leptons, and rest two relativistic NMEs come out from the small components of Dirac spinors. These are

$$O(0^-) : O_{RA} = g_A(\sigma \cdot p_e), \quad O_{SA} = g_A(\sigma \cdot r)$$
$$O(1^-) : O_{RV} = g_V p_e, \quad O_{VA} = g_A(\sigma \times r), \quad O_{VV} = g_V r$$
$$O(2^-) : O_{TA} = g_A[\sigma r | 2], \quad (18)$$

where $O(0^-)$ is rank zero operator with $\Delta J=0$. The terms $O_{RA}$ and $O_{SA}$ are recoil-axial matrix element and scalar-axial matrix element, respectively. $O(1^-)$ is rank one operator with $\Delta J=1$. The terms $O_{RV}$, $O_{VA}$ and $O_{VV}$ are recoil-vector matrix element, vector-axial matrix
element and vector-vector matrix element, respectively. \(O(2^-)\) is rank two operator with \(\Delta J=2\). The term \(O_{TA}\) is tensor-axial matrix element. These six operators changes parity during transition i.e. \(\pi_i \pi_f = -1\). Further, the recoil-axial matrix element \(\gamma_5\) gets enhanced over the impulse approximation with the aid of meson enhancement factor [46] which is depicted by \(\epsilon_{MEC}\). We have used the value \(\epsilon_{MEC} = 2.01\) of meson enhancement factor in these calculations for the rank zero nuclear matrix element \(\gamma_5\) which corresponds to \(\epsilon_{MEC} = 2.01\pm0.05\) given in [24].

We have obtained the quenching factor for \(^{210-215}\)Pb \(\rightarrow^{210-215}\)Bi and \(^{210-215}\)Bi \(\rightarrow^{210-215}\)Po transitions using the chi-square fitting method. We have compared theoretical and experimental average shape factor values for these transitions to get the quenching factor. In our calculation for the average shape factor, we have included the next-to-leading order terms [47]. First we have performed calculations without including the quenching factor by taking the bare values of the weak coupling constant, i.e., \(g_A=1.27\) and \(g_V=1.00\). Corresponding results are shown in Fig. 1(a); we can conclude that the theoretical and experimental values of the average shape factor are very far from each other. Hence, we have used chi-square fitting method to obtain the quenching factor, it comes out to be 0.38 such that \(g_A^{eff}=gg_A^{free}=0.4826\) and \(g_V^{eff}=1.00\). Using these values of weak coupling constants, we have plotted Fig. 1(b) for the same transitions. In this figure, the data points come close to the central line, which means that theoretical values are approaching to the experimental ones. We have also calculated values of the average shape factor using another set of weak coupling constants taken from Ref. [27]. Corresponding results are shown in Fig. 1(c) for \((g_A/g_A^{free}, g_V/g_V^{free})=(0.38, 0.51)\). This figure shows the best fit for the average shape factor. In our further calculations, we have used these two sets of weak coupling constants i.e. set I: \((g_A/g_A^{free}, g_V/g_V^{free})=(0.38, 1.00)\) and set II: \((g_A/g_A^{free}, g_V/g_V^{free})=(0.38, 0.51)\).

### III. RESULTS AND DISCUSSION

In the present work, large-scale shell-model calculations for the allowed and forbidden beta-decay transitions have been carried out in the north-east region of doubly-magic nucleus \(^{208}\)Pb and compared these results with the recently available experimental data [48]. In this work, for the first time, we present theoretical \(log ft\) values, average shape factors, and half-lives for \(^{210-215}\)Pb \(\rightarrow^{210-215}\)Bi and \(^{210-215}\)Bi \(\rightarrow^{210-215}\)Po transitions. We have also calculated the quenching factor (as discussed in the above section) and theoretical Q-value for these nuclei, and we have compared the shell-model results with the experimental data. In the present work we have performed shell-model calculations by taking \(^{208}\)Pb as a core using KHPE effective interaction [28] given by Kuo Herling for \(^{208}\)Pb (\(Z=82, N=126\)). Model space for this interaction is \(0h_{9/2}, 1f_{7/2}, 1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 0h_{13/2}\) for protons \((82 < Z < 126)\) and \(0h_{11/2}, 1g_{9/2}, 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 0j_{15/2}\) for neutrons \((126 < N < 184)\). In Table I, \(log ft\) values and average shape factor have been calculated from the ground states of \(^{210-215}\)Pb to the ground, and several excited states of \(^{210-215}\)Bi using both sets of the weak coupling constants. At some places, double bracket represents those states which are not confirmed experimentally. These results have been computed using experimental Q-values taken from [48]. On comparing experimental and theoretical results in Table I, we notice that both sets of weak coupling constants give promising results for the \(log ft\) values. As we can observe that \(log ft\) values for the transition from \(^{210}\)Pb((0\(^+\))) to \(^{210}\)Bi((0\(^-\))) at energy 46.539(1) keV is 5.469 for both of the sets which agrees quite well the experimental value i.e., 5.4(1). In the case of \(^{211}\)Pb((9/2\(^+\))) to \(^{211}\)Bi((9/2\(^-\))) transition at energy 831.960(12) keV, the theoretical value of \(log ft\) is 5.779 for the set I and 5.771 for the set II, which is near to the experimental value, i.e., 5.7330(18). Shell-model results for \(^{211}\)Bi to \(^{211}\)Bi transitions have also been calculated by Warburton [24] with KHPE interaction using old data. Further, for the transition from \(^{213}\)Pb((9/2\(^+\))) to \(^{213}\)Bi((9/2\(^-\))) theoretical \(log ft\) value for the set I is 6.447 and for the set II is 6.459, while the experimental value is 6.5. In the case of \(^{215}\)Pb((9/2\(^+\))) to \(^{215}\)Bi((9/2\(^-\))) transition, experimentally, the \(log ft\) value should be greater than or equal to 6.1, which is confirmed by shell-model results as \(log ft\) value for the set I is 6.713 and for the set II is 6.741. Also, in the \(^{215}\)Pb((9/2\(^+\))) to \(^{215}\)Bi((7/2\(^-\))) transition at energy 183.5(3) keV, this \(log ft\) value should be greater than 6.6, which can be verified via theoretical results, i.e., for the set I, it is 7.224, and for the set II it is 8.486. There are several such transitions for which theoretical \(log ft\) values match very well with the experimental values, which shows the authenticity of shell-model calculations. We have also calculated \(log ft\) results corresponding to the transitions for which experimental data is unavailable. For instance, \(log ft\) value for \(^{212}\)Pb((0\(^+\))) to \(^{212}\)Bi((2\(^-\))) transition at energy 115.183(5) keV is 10.143 for the set I and 10.140 for the set II. In case of \(^{214}\)Pb((0\(^+\))) to \(^{214}\)Bi((2\(^-\))) transition at energy 53.2260(15) keV, this \(log ft\) value comes out to be 9.446 for the set I and 9.445 for the set II while for \(^{214}\)Pb((0\(^+\))) to \(^{214}\)Bi((2\(^-\))) transition at energy 377.03(4) keV, this value comes out to be 10.211 for the set I and 10.197 for the set II. Furthermore, in case of \(^{215}\)Pb((9/2\(^+\))) to \(^{215}\)Bi((7/2\(^-\)),(5/2\(^-\))) at energy 592.72(8) keV, experiments do not confirm one suitable spin state thus predict two possible spin states. Therefore, we have performed shell-model calculations for both possible spin states 7/2\(^-\) and 5/2\(^-\). It is inferred from these results that the spin and parity of the state at 592.72(8) keV is 7/2\(^-\) as its \(log ft\) value is 8.184 for the set I and 8.018 for the set II, which is close to the experimental value, i.e., 7.5. However, for the \(^{214}\)Pb((0\(^+\))) to \(^{214}\)Bi((0\(^+\)),1\(^-\))) transition at energy 351.9323(21) keV, our calculated shell-model results for the \(log ft\) values give
good results for both possible spins of the state. Thus it is difficult to distinguish the spin-parity of these two states. There are some transitions in this table for which shell-model results overestimates or underestimates the experimental log $ft$ values. For instance, for $^{210}$Pb($0^+$) to $^{210}$Bi($1^-$) transition, the experimental log $ft$ value is 7.9(1), while for the set I, it is 6.719, and for the set II it is 8.517. Similarly, for $^{212}$Pb($0^+$) to $^{212}$Bi($1^-$), the experimental value is 6.73(4), while it is 7.486 for the set I and it is 7.288 for the set II.

| Initial ($J^+_i$) | Final ($J^+_f$) | Decay mode | Energy (keV) | log $ft$ | $\log([C(w_e)]^{1/2}$ |
|------------------|----------------|-------------|-------------|----------|----------------|
|                  |                |             |             | Expt. | Set I | Set II | Expt. | Set I | Set II |
| $^{210}$Pb($0^+$) | $^{210}$Bi($1^-$) | 1st FNU | 0.0 | 7.9(1) | 6.719 | 8.517 | 3.436 | 13.380 | 1.689 |
| $^{210}$Bi($0^-$) | 1st FNU | 46.539(1) | 5.4(1) | 5.469 | 5.469 | 61.102 | 56.422 | 56.422 |
| $^{211}$Bi($9/2^+$) | $^{211}$Bi($7/2^-$) | 1st FNU | 404.866(9) | 7.19(3) | 6.239 | 7.183 | 7.781 | 23.262 | 7.847 |
| $^{211}$Bi($9/2^-$) | 1st FNU | 831.960(12) | 5.7330(18) | 5.779 | 5.771 | 41.644 | 39.480 | 39.881 |
| $^{212}$Pb($0^+$) | $^{212}$Bi($1^-$) | 1st FNU | 1109.485(23) | 5.58(4) | 5.507 | 5.513 | 49.665 | 54.049 | 53.646 |
| $^{213}$Pb($9/2^+$) | $^{213}$Bi($9/2^-$) | 1st FNU | 0.0 | 6.73(4) | 7.486 | 7.288 | 13.215 | 5.534 | 6.952 |
| $^{213}$Bi($7/2^-$) | 1st FNU | 238.632(2) | 5.179(10) | 5.208 | 5.208 | 78.805 | 76.179 | 76.179 |
| $^{213}$Bi($9/2^-$) | 1st FNU | 415.272(11) | 5.342(17) | 4.551 | 5.156 | 65.321 | 162.418 | 80.915 |
| $^{214}$Pb($0^+$) | $^{214}$Bi($1^-$) | 1st FNU | 0.0 | 6.5 | 6.447 | 6.459 | 17.221 | 18.297 | 18.057 |
| $^{214}$Bi($2^-$) | 1st FU | 115.183(5) | - | 10.143 | 10.140 | - | 0.260 | 0.261 |
| $^{214}$Bi($3^-$) | 1st FU | 53.2260(15) | - | 9.446 | 9.445 | - | 0.580 | 0.580 |
| $^{214}$Bi($4^-$) | 1st FU | 258.869(24) | 8.04(12) | 8.665 | 8.688 | 2.924 | 1.424 | 1.387 |
| $^{214}$Bi($5^-$) | 1st FU | 295.2236(19) | 5.250(24) | 5.080 | 5.080 | 72.620 | 173.437 | 95.335 |
| $^{214}$Bi($6^-$) | 1st FU | 351.9323(21) | 5.073(18) | 5.079 | 5.079 | 89.342 | 88.348 | 88.348 |
| $^{214}$Bi($7^-$) | 1st FU | 377.03(4) | - | 10.211 | 10.197 | - | 0.240 | 0.244 |
| $^{214}$Bi($8^-$) | 1st FU | 533.672(14) | 6.23(4) | 7.374 | 7.881 | 23.499 | 6.304 | 3.511 |
| $^{214}$Bi($9^-$) | 1st FU | 838.994(22) | 4.43(9) | 4.238 | 4.238 | 0.483 | 0.597 | 0.597 |
| $^{215}$Pb($9/2^+$) | $^{215}$Bi($9/2^-$) | 1st FNU | 0.0 | ≥6.1 | 6.713 | 6.741 | ≤27.293 | 13.472 | 13.045 |
| $^{215}$Bi($7/2^-$) | 1st FNU | 183.5(3) | >6.6 | 7.224 | 8.486 | <15.348 | 7.480 | 1.751 |
TABLE II. The same as in Table I for the Bi→Po transitions.

| Initial \((J^+_I)\) | Final \((J^+_F)\) | Decay mode | Energy (keV) | log \(ft\) Expt. Set I Set II | C(\(w_c\)) \(^{1/2}\) Expt. Set I Set II |
|------------------|------------------|------------|-------------|-------------------------------|-------------------------------|
| 210Bi\((1^-)\)  | 210Po\((0^+)_1\) | 1st FNU    | 0.0         | 8.0                           6.690 8.114                   | 3.062 13.842 2.686           |
| 211Bi\((9/2^-)\) | 211Po\((9/2^+)\) | 1st FNU    | 0.0         | 5.99(2)                      6.143 6.140                   | 30.978 25.982 26.053         |
| 212Bi\((1^-)\)  | 212Po\((0^+)_1\) | 1st FNU    | 0.0         | 7.266(4)                     7.330 8.951                   | 7.126 6.621 1.024            |
|                  | 212Po\((2^+)_2\) | 1st FNU    | 727.330(9)  | 7.720(11)                   7.017 7.609                   | 4.227 9.498 4.803            |
|                  | 212Po\((2^+)_2\) | 1st FNU    | 1512.70(8)  | 7.093(13)                   6.554 6.645                   | 8.701 16.179 14.581          |
|                  | 212Po\((1^+)_1\) | 1st FNU    | 1620.738(10)| 6.748(11)                   6.079 6.384                   | 12.944 27.973 19.679         |
|                  | 212Po\((2^+)_2\) | 1st FNU    | 1679.450(14)| 7.51(6)                     5.850 6.447                   | 5.383 36.411 18.295          |
|                  | 212Po\((0^+)_1\) | 1st FNU    | 1800.9(2)   | 8.05(9)                      8.103 7.750                   | 2.891 2.721 3.945            |
|                  | 212Po\((2^+)_2\) | 1st FNU    | 1805.96(10)| 6.695(21)                   6.546 6.926                   | 13.758 16.327 10.543         |
| 213Bi\((9/2^-)\) | 213Po\((9/2^+)\) | 1st FNU    | 0.0         | 6.31(1)                     6.367 6.358                   | 21.235 20.068 20.287         |
|                  | 213Po\((11/2^+)\) | 1st FNU | 292.805(8) | 8.45(10)                     5.386 5.869                   | 1.910 62.110 35.590          |
|                  | 213Po\((7/2^+)\) | 1st FNU    | 440.446(9)  | 6.08(1)                     9.381 9.302                   | 28.252 0.624 0.684           |
|                  | 213Po\((5/2^+)\) | 1st FU     | 600.87(17)  | 10.03(9)                    13.513 13.462                  | 0.448 0.005 0.006            |
|                  | 213Po\((13/2^+)\) | 1st FU | 867.98(3)  | 8.64(5)                     10.457 10.458                  | 1.589 0.181 0.181            |
|                  | 213Po\((9/2^+)\) | 1st FNU    | 1003.605(22)| 7.49(3)                     6.096 6.091                   | 5.452 27.435 27.586          |
|                  | 213Po\((9/2^+)\) | 1st FNU    | 1045.65(9)  | 7.85(7)                     7.196 7.211                   | 3.640 7.727 7.592            |
|                  | 213Po\((11/2^+)\) | 1st FNU | 1045.65(9) | 7.85(7)                     8.293 8.591                   | 3.640 2.184 1.551            |
| 214Bi\((1^-)\)  | 214Po\((0^+)_1\) | 1st FNU    | 0.0         | 7.872(11)                   7.314 8.311                   | 3.586 6.746 2.142            |
|                  | 214Po\((2^+)_1\) | 1st FNU    | 609.318(5)  | 9.06(7)                     7.649 8.318                   | 0.904 4.590 2.124            |
|                  | 214Po\((3^-)_1\) | 2nd FNU    | 1274.765(9)| 9.5(3)                      11.161 11.542                  | 210.292 31.069 20.037        |
|                  | 214Po\((2^+)_1\) | 1st FNU    | 1377.681(7)| 7.374(11)                   6.695 6.978                   | 6.296 13.754 9.934           |
|                  | 214Po\((2^-)_1\) | 1st FNU    | 1415.408(8)| 8.25(3)                     7.380 7.644                   | 2.296 6.250 4.616            |
|                  | 214Po\((2^+)_1\) | 1st FNU    | 1661.282(14)| 8.21(4)                     6.302 6.929                   | 2.405 21.624 10.510          |
|                  | 214Po\((2^+)_1\) | 1st FNU    | 1729.613(7)| 6.654(12)                   6.713 6.240                   | 14.423 13.470 23.241         |
|                  | 214Po\((2^+)_1\) | 1st FNU    | 1847.446(9)| 6.859(13)                   6.509 7.170                   | 11.391 17.034 7.961          |
|                  | 214Po\((2^+)_1\) | 1st FNU    | 2010.831(13)| 7.422(15)                   6.381 6.730                   | 5.957 19.749 13.212          |
| 215Bi\((9/2^-)\) | 215Po\((9/2^+)\) | 1st FNU    | 0.0         | >6.9                        6.653 6.642                   | <10.866 14.442 14.619        |
|                  | 215Po\((7/2^+)\) | 1st FNU    | 271.11(10)  | >8.2                        10.020 8.737                   | <2.432 0.299 1.312           |
|                  | 215Po\((11/2^+)\) | 1st FNU | 293.53(10) | 6.0(1)                      5.349 5.813                   | 30.623 64.770 38.001         |
|                  | 215Po\((5/2^+)\) | 1st FU     | 401.6(10)   | 7.7                         13.796 13.714                  | 4.326 0.004 0.004            |
|                  | 215Po\((7/2^+)\) | 1st FNU    | 517.53(17)  | 7.8                         9.041 8.965                   | 3.855 0.924 1.008            |
|                  | 215Po\((9/2^+)\) | 1st FNU    | 517.53(17)  | 7.8                         6.619 6.608                   | 3.855 15.024 15.20           |
|                  | 215Po\((11/2^+)\) | 1st FNU | 609.0(5)   | 7.4                         9.058 10.090                  | 6.110 0.905 0.276            |
|                  | 215Po\((13/2^+)\) | 1st FU | 609.0(5)   | 7.4                         9.931 9.933                   | 6.110 0.332 0.331            |
Table II shows the log $ft$ and average shape factor values for the transitions from the ground states of $^{210-215}$Bi to the ground and different excited states of $^{210-215}$Po, calculated using both sets of the weak coupling constants. Decay mode and experimental data are also included in this table. The theoretical results are in quite good agreement with the experimental data. For instance, the log $ft$ value for $^{210}$Bi($^{1-}$) to $^{210}$Po($^{0+}$) transition is 8.114 for the set II, which is close to the experimental value i.e. 8.0. In the case of $^{211}$Bi($9/2-$) to $^{211}$Po($9/2^+$) transition, the experimental log $ft$ value is 5.99(2), which is close to the shell-model results; i.e., for the set I, it is 6.143, and for the set II it is 6.140. Also for $^{212}$Bi($1^-$) to $^{212}$Po($2^+_1$) transition at energy 1805.96(10) keV, the theoretical log $ft$ value for the set I is 6.546 and for the set II it is 6.926, which matches with the experimental value i.e. 6.695(21). For $^{213}$Bi($9/2-$) to $^{213}$Po($9/2^+$) transition, this value is 6.367 for the set I and 6.358 for the set II which is very near to the experimental value, i.e., 6.31(1). On moving toward the transition from $^{214}$Bi($1^-$) to $^{214}$Po($0^+$), the experimental log $ft$ value is 7.872(11), which matches with shell-model results, i.e., for the set I, it is 7.314, and for the set II it is 8.311. Further, in $^{215}$Bi($9/2^-$) to $^{215}$Po($ll/2^+$) transition at energy 293.53(10) keV, the experimental and shell-model results for the set II are in good agreement with each other, i.e., experimentally log $ft$ value is 6.0(1) while for the set I, it is 5.349 and for the set II it is 5.813. There are several transitions for which our computations overestimate or underestimate the log $ft$ value. For instance, there are small discrepancies in the results of $^{213}$Bi decay. For the transition from $^{213}$Bi($9/2^-$) to $^{213}$Po($5/2^+$) at energy 600.87(17) keV, the experimental log $ft$ value is 10.03(9) whereas the theoretical value for the set I is 13.513 and for the set II is 13.462. Also, for $^{215}$Bi($9/2^-$) to $^{215}$Po($5/2^+$) transition at energy 401.6(10) keV, the experimental log $ft$ value is 7.7 while the theoretical value for the set I is 13.796, and for the set II is 13.714, which are approximately twice the experimental value. It can be because of several reasons. Firstly, the $9/2^-$ state of $^{215}$Bi is not yet confirmed, and secondly, there are several approximations (such as impulse approximation) assumed while calculating log $ft$ and half-lives in the shell-model calculations. Further, there are several transitions where unique assignments of spin-parity are not possible. We have calculated shell-model results for all possible spins and parities of the states. However, it is difficult to make unique assignments for these states because their results are in close proximity to each other. For instance, in the case of $^{213}$Bi($9/2^-$) to $^{213}$Po($9/2^+_3$, $11/2^+_3$) at energy 1045.65(9) keV, the experimental log $ft$ value is 7.85(7), and the shell-model results for both $9/2^+_3$ and $11/2^+_3$ in $^{213}$Po give values close to the experimental value. Thus, we cannot make a unique assignment of spin-parity for the state of $^{213}$Po at 1045.65(9) keV.

As Q-values play very important roles in the calculation of beta-decay properties, we have also calculated theoretical Q-values for the concerned nuclei which are listed in Table III. In this paper, we have used experimental Q-values for the calculation of shell-model results. Further, for better comparison, we have also used theoretical Q-values to calculate log $ft$ and half-lives of the transitions included in this paper. Column I and II show initial and final ground states of the transitions of the concerned nuclei, and column III and IV shows shell-model binding energies of these nuclei. Column V shows experimental Q-values and column VI show calculated shell-model Q-values. It can be concluded that these shell-model Q-values are in close proximity with the experimental ones. For instance, the theoretical Q-value for $^{215}$Po to $^{215}$Bi transition is 2.765 MeV while the experimental value is 2.770(10) MeV. These two values are very close to each other.

In Table IV, a comparison between theoretical and experimental half-lives has been given for Pb to Bi transitions using both the sets of the coupling constants. We have calculated half-lives using experimental Q-values. As we know, beta-decay half-lives are very sensitive to the Q-values, we have calculated half-lives using theoretical Q-values also. These half-lives have been calculated using the property of transition probability that it is additive in nature. It can be concluded from this table that our calculations of half-lives match well with the experimental half-lives in most of the cases. For instance, experimental half-life value for $^{211}$Po to $^{211}$Bi transition is 36.1(2) min while theoretical value for the set I is 36.052 min with experimental Q-value. In case of the transition from $^{210}$Po($0^+$) to $^{210}$Bi, experimental half-life does not match well with the result of the set I but matches well with the result of the set II. Minor discrepancies in the results can be explained as there are several transitions.
TABLE III. Comparison between the shell-model Q-values and experimental [48] Q-values.

| Transition | E (SM) (MeV) | Q-value (MeV) | Expt. | Theoretical |
|------------|--------------|---------------|-------|-------------|
| Initial    | Final        | Initial | Final | |
| $^{210}$Pb$(0^+)$ | $^{210}$Bi$(1^-)$ | -9.091 | -8.403 | 0.0635(5) | 0.092 |
| $^{211}$Pb$(9/2^+)$ | $^{211}$Bi$(9/2^-)$ | -12.936 | -13.512 | 1.367(6) | 1.356 |
| $^{212}$Pb$(0^+)$ | $^{212}$Bi$(1^-)$ | -18.034 | -17.882 | 0.5691(18) | 0.628 |
| $^{213}$Pb$(9/2^+)$ | $^{213}$Bi$(9/2^-)$ | -21.762 | -23.044 | 2.030(8) | 2.062 |
| $^{214}$Pb$(0^+)$ | $^{214}$Bi$(1^-)$ | -26.888 | -27.149 | 0.5691(18) | 0.628 |
| $^{215}$Pb$(9/2^+)$ | $^{215}$Bi$(9/2^-)$ | -30.370 | -32.355 | 2.770(10) | 2.765 |
| $^{210}$Bi$(1^-)$ | $^{210}$Po$(0^+)$ | -8.403 | -8.762 | 1.1622(8) | 1.139 |
| $^{211}$Bi$(9/2^-)$ | $^{211}$Po$(9/2^+)$ | -13.512 | -13.309 | 0.574(5) | 0.577 |
| $^{212}$Bi$(1^-)$ | $^{212}$Po$(0^+)$ | -17.882 | -19.212 | 2.2515(17) | 2.110 |
| $^{213}$Bi$(9/2^-)$ | $^{213}$Po$(9/2^+)$ | -23.044 | -23.598 | 1.422(5) | 1.334 |
| $^{214}$Bi$(1^-)$ | $^{214}$Po$(0^+)$ | -27.149 | -29.440 | 3.269(11) | 3.071 |
| $^{215}$Bi$(9/2^-)$ | $^{215}$Po$(9/2^+)$ | -32.355 | -33.661 | 2.189(15) | 2.086 |

TABLE IV. Comparison between theoretical and experimental [48] half-lives values for Pb → Bi transitions for experimental and theoretical Q-values. The calculations are carried out through two sets of quenching factors in the weak coupling constants $g_V$ and $g_A$. The values of quenching factor for set I: ($g_A$/$g_A^{\text{free}}$, $g_V$/$g_V^{\text{free}}$)=(0.38, 1.00) and set II: ($g_A$/$g_A^{\text{free}}$, $g_V$/$g_V^{\text{free}}$)=(0.38, 0.51). The quenching factors of set I have been calculated from this work and set II have been taken from [27].

| Transition | Half-life | Expt. | Set I (Exp. Q) | Set I (Theo. Q) | Set II (Exp. Q) | Set II (Theo. Q) |
|------------|-----------|-------|----------------|-----------------|-----------------|-----------------|
| Initial    | Final     |       |                |                 |                 |                 |
| $^{210}$Pb$(0^+)$ | $^{210}$Bi | 22.20(22) y | 5.974 y | 0.886 y | 23.087 y | 1.350 y |
| $^{211}$Pb$(9/2^+)$ | $^{211}$Bi | 36.1(2) min | 36.052 min | 40.534 min | 41.743 min | 43.157 min |
| $^{212}$Pb$(0^+)$ | $^{212}$Bi | 10.622(7) h | 8.804 h | 4.069 h | 11.004 h | 4.476 h |
| $^{213}$Pb$(9/2^+)$ | $^{213}$Bi | 10.2(3) min | 15.452 min | 16.246 min | 19.542 min | 18.661 min |
| $^{214}$Pb$(0^+)$ | $^{214}$Bi | 27.06(7) min | 10.7329 min | 10.558 min | 33.306 min | 33.792 min |
| $^{215}$Pb$(9/2^+)$ | $^{215}$Bi | 147 s | 617.223 s | 675.112 s | 3486.387 s | 813.294 s |

TABLE V. The same as in Table IV for the Bi→Po transitions.

| Transition | Half-life | Expt. | Set I (Exp. Q) | Set I (Theo. Q) | Set II (Exp. Q) | Set II (Theo. Q) |
|------------|-----------|-------|----------------|-----------------|-----------------|-----------------|
| Initial    | Final     |       |                |                 |                 |                 |
| $^{210}$Bi$(1^-)$ | $^{210}$Po | 5.012(5) d | 0.212 d | 0.227 d | 5.631 d | 5.894 d |
| $^{211}$Bi$(9/2^-)$ | $^{211}$Po | 2.14 (2) min | 2.769 min | 2.750 min | 2.754 min | 2.736 min |
| $^{212}$Bi$(1^-)$ | $^{212}$Po | 60.55(6) min | 56.694 min | 79.610 min | 297.021 min | 466.002 min |
| $^{213}$Bi$(9/2^-)$ | $^{213}$Po | 45.59(6) min | 13.461 min | 27.040 min | 28.880 min | 49.984 min |
| $^{214}$Bi$(1^-)$ | $^{214}$Po | 19.71(2) min | 5.598 min | 8.885 min | 17.775 min | 30.015 min |
| $^{215}$Bi$(9/2^-)$ | $^{215}$Po | 7.6(2) min | 2.103 min | 3.409 min | 5.275 min | 8.152 min |
Half-lives and those obtained for the set I with both the experimental and theoretical Q-values approach the experimental value significantly better than others. There is a slight deviation at N=133. However, the set I using both the experimental and theoretical Q-values is more favorable than others at N=129, while there is a good agreement between the experimental values and set II values for both theoretical and experimental Q-values at N=130 to 132.

For a better and more detailed comparison of the shell-model and experimental half-lives, we have calculated the error in the theoretical half-lives compared to the experimental ones. Table VI shows the deviation in calculated half-lives compared to the experimental half-lives on the log scale because the magnitude of half-lives listed in Tables IV and V vary in a wide range. Therefore in this table, we have tried to show the mean deviation and fluctuation [49–51] from the experimental data on the log scale for both sets of the weak coupling constants, including both theoretical and experimental Q-values. Here, \( r \) is the measure of deviation which is defined as,

\[
\bar{r} = \frac{1}{n} \sum_{i=1}^{n} r_i, \quad \sigma = \left[ \frac{1}{n} \sum_{i=1}^{n} (r_i - \bar{r})^2 \right]^{1/2},
\]

where \( n \) is the number of transitions taken to calculate the half-lives and index \( i \) goes from 1 to \( n \). In these calculations, the value of \( n \) is 12. The mean error value and standard deviation should be equal to zero for minimal deviations because we have calculated it on the log scale. Our results are approaching zero for both experimental and theoretical Q-values, so we can conclude that our calculations show good agreement with the experimental data. Use of experimental Q-values is generally better than the case of theoretical Q-values. Use of the set II improves the agreement with the experimental data compared with the set I. We have also shown the mean and standard deviation in powers of 10 to eliminate the log scale.

Recently, a new beta-decaying isomeric state \((8^-)\) for \(^{214}\text{Bi}^m\) is predicted at the CERN-ISOLDE facility, which decays to various excited states of \(^{214}\text{Po}\) listed in table VII with a half-life of 9.39(10) min. In Ref. [29], experimental energy spectra are compared with the shell-model, but shell-model results for beta-decay properties such as log \( ft \) are not reported. We here give theoretical estimates for these results. Table VII shows the log \( ft \) and average shape factor for \(^{214}\text{Bi}(8^-)\) isomer. Computations have been carried out for various transitions from

\[
T_{\text{calc}} \rightarrow T_{\text{theo}} = \frac{T_{\text{calc}}}{T_{\text{exp}}},
\]

However, for which unique spin-parity assignments are not possible. Therefore, we have to exclude these transitions in our calculations, leading to some deviation in the results.

Table V compares theoretical and experimental half-lives for Bi to Po transitions using both sets of the weak coupling constants. Calculations for half-lives using both theoretical and experimental Q-values have been done. The theoretical results for the half-lives agree pretty well with the experimental half-lives in most of the cases. For instance, experimental half-life value for \(^{210}\text{Bi} \rightarrow ^{210}\text{Po}\) transition is 5.012(5) d while theoretical value for the set II is 5.631 d with the experimental Q-value and it comes out to be 5.894 d with the theoretical Q-value. For better comparison of half-lives, we have plotted calculated and experimental half-lives with the use of both the experimental and shell-model Q-values in Fig. 2 on the log scale for Pb to Bi transitions. This plot shows half-lives for both the sets of the weak coupling constants using both experimental and theoretical Q-values. It can be concluded from Fig. 2 here that the set II with the experimental Q values matches extremely well with the experimental half-lives up to N=130. Further, at N=131, there is a good agreement between the experimental half-lives and those obtained for the set I with both the experimental and theoretical Q-values. There is a slight deviation at N=133. However, the set I using both the experimental and theoretical Q values approach the experimental value significantly better than others.

In Fig. 3, half-lives have been plotted for Bi to Pb transitions for both sets of the weak coupling constants using both Q-values. For N=127 and 128, results of the set II obtained with experimental and theoretical Q-values agree very well with the experimental results. However,
TABLE VI. Error in calculated half-life values in comparison to experimental [48] half-lives. Comparison is given for both sets of the weak coupling constants using both experimental and shell-model Q-values.

|                | Using Q (Exp.) |          | Using Q (Theo.) |          |
|----------------|----------------|----------|-----------------|----------|
|                | Set I          | Set II   | Set I           | Set II   |
| \( \bar{r} \)  | -0.264         | 0.191    | -0.279          | 0.075    |
| \( \sigma \)    | 0.48           | 0.423    | 0.576           | 0.487    |
| \( 10^g \)      | 0.277          | 0.694    | 0.449           | 1.073    |
| \( 10^\sigma \) | 3.022          | 2.646    | 3.772           | 3.067    |

TABLE VII. Comparison between theoretical and experimental \([29]\) \( \log ft \) values for \( ^{214}\)Bi((8\(^-\))) to the different excited states in \(^{214}\)Po transitions. The calculations are carried out through two sets of quenching factor in the weak coupling constants \( g_V \) and \( g_A \). The values of quenching factor for set I: \( (g_A/g_{A}^{\text{free}}, g_V/g_{V}^{\text{free}})=(0.38, 1.00) \) and set II: \( (g_A/g_{A}^{\text{free}}, g_V/g_{V}^{\text{free}})=(0.38, 0.51) \). The quenching factors of set I have been calculated from this work and set II have been taken from \([27]\).

| Final state | Decay mode | Energy (keV) | \( \log ft \) | \( [C(w_e)]^{1/2} \) |
|-------------|------------|--------------|----------------|----------------------|
| \( J^g \) (Expt.) | \( J^g \) (SM) | Expt. | Set I | Set II | Expt. | Set I | Set II |
| \( (8^+) \) | \( 8_1^+ \) 1st FNU | 1584.4(4) | 6.25(12) | 6.264 | 6.255 | 22.964 | 22.587 | 22.834 |
| \( (8^+, 9^+) \) | \( 8_2^+ \) 1st FNU | 1633.5(4) | 6.62(14) | 6.466 | 6.518 | 14.999 | 17.917 | 16.867 |
| \( (8^+, 9^+) \) | \( 9_1^+ \) 1st FNU | 1633.5(4) | 6.62(14) | 5.468 | 5.941 | 14.999 | 56.523 | 32.786 |
| \( (8^+) \) | \( 8_3^+ \) 1st FNU | 1824.5(4) | 6.25(12) | 5.945 | 6.018 | 22.964 | 32.607 | 30.066 |
| \( (6^+, 7^+) \) | \( 6_1^+ \) 1st FNU | 1843.0(4) | 8.69(17) | 11.130 | 11.147 | 1.384 | 0.083 | 0.082 |
| \( (6^+, 7^+) \) | \( 7_1^+ \) 1st FNU | 1843.0(4) | 7.86(13) | 9.643 | 8.726 | 3.598 | 0.462 | 1.328 |
| \( (8^+, 9^+) \) | \( 8_4^+ \) 1st FNU | 1969.1(4) | 6.93(13) | 6.371 | 6.755 | 10.497 | 12.610 | 5.096 |
| \( (8^+, 9^+) \) | \( 9_2^+ \) 1st FNU | 1969.1(4) | 6.93(13) | 7.005 | 7.517 | 10.497 | 9.631 | 5.341 |
| \( (8^+, 9^+) \) | \( 8_5^+ \) 1st FNU | 2059.5(4) | 6.04(13) | 6.771 | 7.558 | 29.245 | 12.610 | 5.096 |
| \( (8^+, 9^+) \) | \( 9_3^+ \) 1st FNU | 2059.5(4) | 6.04(13) | 10.183 | 10.148 | 3.855 | 4.118 | 3.454 |
| \( (9) \) | \( 9_4^+ \) 1st FNU | 2159.0(4) | 7.80(16) | 7.743 | 7.895 | 3.855 | 4.118 | 3.454 |
| \( (8^+, 9^+) \) | \( 8_6^+ \) 1st FNU | 2197.6(4) | 6.40(14) | 7.297 | 7.297 | 19.322 | 6.887 | 6.883 |
| \( (8^+, 9^+) \) | \( 9_5^+ \) 1st FNU | 2197.6(4) | 6.40(14) | 10.183 | 10.148 | 3.855 | 4.118 | 3.454 |

\(^{214}\)Bi((8\(^-\))) isomer to different excited states of \(^{214}\)Po by using theoretical Q-values. All the transitions in this table are first forbidden non-unique transitions except one transition that is first forbidden unique transition. Thus, we have compared our shell-model results according to the decay mode with the experimental data available in the reference mentioned above. Good results are obtained for the shell-model calculations. For instance, at excitation energy 1584.4(4) keV for a transition from \(^{214}\)Bi((8\(^-\))) to \(^{214}\)Po((8\(^+\))), experimental \( \log ft \) value is 6.25(12) and the theoretical value is 6.26 for the set I and 6.255 for the set II. We have computed \( \log ft \) values for all the transitions where more than one spin-parity assignment is possible. Based on shell-model results it is possible to predict one suitable spin-parity for the state at that energy level. It can be inferred that the spin-parity for the state at energy 1633.5(4) keV can be \( 8_2^+ \) because its theoretical \( \log ft \) value is 6.466 for the set I and 6.518 for the set II, which is close to the experimental value, i.e., 6.62(14). Also, at energy level 1843.0(4) keV, experimental \( \log ft \) will be decided based on whether the decay is unique first forbidden or non-unique first forbidden beta-decay. As a result, the state is predicted to be \( 7_1^+ \) out of two possible spin parity states, \( 6_1^+ \) and \( 7_1^+ \), because the calculated \( \log ft \) value of \( 7_1^+ \) state is close to the experimental value as compared to that for \( 6_1^+ \). At en-
energy level 169.1(4) keV, it is difficult to assign a unique spin-parity for the state out of two $8^+_1$ and $9^+_2$ states, because both spin-parity assignments give good results. At energy 2059.5(4) keV, $8^+_2$ can be suitable as its log $ft$ value for the set I is 6.771 and for the set II is 7.558, whereas its experimental value is 6.04(13). Also, at energy 2197.6(4) keV, $8^+_0$ state can be predicted because its theoretical log $ft$ value is 7.297 and experimental value is 6.40(14).

IV. CONCLUSION

In the present work, we have calculated various beta-decay properties such as log $ft$ values, shape factors, and half-lives of nuclei in the north-east region of $^{208}$Pb nucleus. These computations have been performed for $^{210-215}$Pb $\rightarrow$ $^{210-215}$Bi and $^{210-215}$Bi $\rightarrow$ $^{210-215}$Po transitions using KHPE effective interaction without any truncation. First, we have computed the quenching factor using the chi-square fitting method. Then, using this value of the quenching factor and another set taken from Ref. [27], we have calculated log $ft$, shape factor, and half-lives of these transitions. We have also calculated shell-model Q-values of the concerned nuclei, and used both experimental and theoretical Q-values to study these beta-decay properties for better comparison. Our results for the log $ft$ values and half-lives obtained by the shell-model calculations show good agreement with the available experimental data. Use of experimental Q-values and inclusion of the quenching in both the axial and vector couplings is generally better compared with the case of theoretical Q-values and the quenching in the axial coupling only. We have also calculated log $ft$ values for $^{214}$Bi\textsuperscript{$m$}((8\textsuperscript{-})) isomer recently predicted at the CERN-ISOLDE facility. We have used shell-model Q-values for these calculations. We have also confirmed spins and parities of various states with the help of shell-model calculations, where unique assignments of spin-parity are not possible experimentally. Also, we have calculated log $ft$ values of the transitions whose experimental data is not available. These shell-model results will add more information to the present data and will be very helpful for future experiments. Further, shell-model results can be improved with the inclusion of core polarization effects [25] by allowing nucleonic excitation across $^{208}$Pb core and tuning effective interactions.

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[1] J. J. Cowan, F. K. Thielemann, J. W. Truran, “The $r$-process and nucleochronology”, Phys. Rep. 208, 267 (1991).
[2] S. Wanajo, Y. Ishimaru, “R-process calculations and Galactic chemical evolution”, Nucl. Phys. A 777, 676-699 (2006).
[3] P. -K. Thielemann, M. Eichler, I. V. Panov, and B. Wehmeier, “Neutron Star Mergers and Nucleosynthesis of Heavy Elements”, Annu. Rev. Nucl. Part. Sci. 67 253-274 (2017).
[4] M. Arnould, S. Goriely, K. Takahashi, “The r-process of stellar nucleosynthesis: Astrophysics and nuclear physics achievements and mysteries”, Phys. Rep. 450, 97 – 213 (2007).
[5] E. M. Burbidge, G. R. Burbidge, W. A. Fowler, and F. Hoyle, “Synthesis of the elements in stars”, Rev. Mod. Phys. 29, 547 (1957).
[6] A. G. W. Cameron, “Nuclear reactions in stars and nucleogenesis”, Pub. Astron. Soc. Pacific 69, 201 (1957).
[7] S. E. Woosley et al., “The r-process and neutrino-heated supernova ejecta”, Astrophys. J. 433, 229 (1994).
[8] T. Suzuki et al., “β decays of isotones with neutron magic number of $N = 126$ and r-process nucleosynthesis”, Phys. Rev. C 85, 015802 (2012).
[9] R. J. Carroll et al., “Competition between allowed and first-forbidden β decay: The case of $^{208}$Hg $\rightarrow 208$ Tl”, Phys. Rev. Lett. 125, 192501 (2020).
[10] M. Brunet et al., “Competition between allowed and first-forbidden β decays of $^{208}$At and expansion of the $^{208}$Po level scheme”, Phys. Rev. C 103, 054327 (2021).
[11] J. Kostensalo, M. Haaranen, and Jouni Suohon, “Electron spectra in forbidden β decays and the quenching of the weak axial-vector coupling constant $g_A$”, Phys. Rev. C 95, 044313 (2017).
[12] J. Kostensalo and J. Suohon, “$g_A$-driven shapes of electron spectra of forbidden β decays in the nuclear shell-model”, Phys. Rev. C 96, 024317 (2017).
[13] A. Kumar and P. C. Srivastava, “Shell-model description for the first-forbidden β$^-$ decay of $^{207}$Hg into the one-proton-hole nucleus $^{207}$Tl”, Nucl. Phys. A 1014, 122255 (2021).
[14] K. Takahashi, M. Yamada, T. Kondoh, “Beta-decay half-lives calculated on the gross theory”, At. Data Nucl. Data Tables 12, 101-142 (1973).
[15] I. N. Borzov, S. Goriely, “Weak interaction rates of neutron-rich nuclei and the r-process nucleosynthesis”, Phys. Rev. C 62, 03550, (2000).
[16] K. Nomura, R. Rodríguez-Guzmán, and L. M. Robledo, “β decay of even-A nuclei within the interacting boson model with input based on nuclear density functional theory”, Phys. Rev. C 101, 044318 (2020).
[17] J. Engel et al., “β decay rates of r-process waiting-point nuclei in a self-consistent approach”, Phys. Rev. C 60, 044302 (1999).
[18] J. J. Cuenca-Garcia, G. Martínez-Pinedo, K. Langanke et al., “Shell model half-lives for r-process N = 82 nuclei”, Eur. Phys. J. A 34, 99–105 (2007).
[19] G. Martínez-Pinedo and K. Langanke, “Shell-Model half-lives for N = 82 nuclei and their implications for the r process”, Phys. Rev. Lett. 83, 4502 (1999).
[20] K. Langanke and G. Martínez-Pinedo, “Nuclear weak-interaction processes in stars”, Rev. Mod. Phys. 75, 818 (2003).
[21] J. Suhonen, “Effective value of $g_\alpha$ in $\beta$ and $\beta$ decays”, J. Phys.: Conf. Ser. 1056, 012056 (2018).
[22] A. Kumar, P. C. Srivastava and T. Suzuki, “Shell model results for nuclear $\beta^-$-decay properties of sd-shell nuclei”, Prog. Theor. Exp. Phys. 3, 033D01 (2020).
[23] E. K. Warburton, “Core polarization effects on spin-dipole and first-forbidden $\beta$-decay operators in the lead region”, Phys. Rev. C 42, 2479 (1990).
[24] E. K. Warburton, “First-forbidden decay in the lead region and mesonic enhancement weak axial current”, Phys. Rev. C 44, 233 (1991).
[25] E. K. Warburton, “Mesonic enhancement of the weak axial-vector current evaluated from $\beta$ decay in the lead region”, Phys. Rev. Lett. 66, 1823 (1991).
[26] L. Rydström et al., “Structure of proton-deficient nuclei near $^{208}$Pb”, Nucl. Phys. A 512, 217-240 (1990).
[27] Q. Zhi et al., “Shell-model half-lives including first-forbidden contributions for r-process waiting-point nuclei”, Phys. Rev. C 87, 025803 (2013).
[28] E. K. Warburton, B. Alex Brown, “Appraisal of the Kuo-Herling shell-model interaction and application to A=210-212 nuclei”, Phys. Rev. C 43, 602-617 (1991).
[29] B. Andel et al., “New $\beta$-decaying state in $^{214}$Bi”, Phys. Rev. C 104, 054301 (2021).
[30] J. Suhonen, From Nucleons to Nucleus: Concept of Microscopic Nuclear Theory. (Springer, Berlin 2007).
[31] H. Behrens and W. Bühring, Electron Radial Wave Functions and Nuclear Beta-Decay (Clarendon, Oxford, 1982).
[32] H. F. Schopper, “Weak interaction and Nuclear Beta Decay”. (North-Holland, Amsterdam, 1996).
[33] B. A. Brown and B. H. Wildenthal, “Experimental and theoretical Gamow-Teller beta-decay observables for the sd-shell nuclei”, At. Data Nucl. Data Tables 33, 347 (1985).
[34] M. Haaranen, J. Kotila, and J. Suhonen, “Spectrum-shape method and the next-to-leading-order terms of the $\beta$-decay shape factor”, Phys. Rev. C 95 024327 (2017).
[35] M. T. Mustonen, M. Aumola and J. Suhonen, “Theoretical description of the fourth-forbidden non-unique $\beta$ decays of $^{113}$Cd and $^{115}$In”, Phys. Rev. C 73, 054301 (2006).
[36] A. Kumar, P. C. Srivastava, J. Kostensalo and J. Suhonen, “Second-forbidden nonunique $\beta^-$ decays of $^{24}$Na and $^{36}$Cl assessed by the nuclear shell-model”, Phys. Rev. C 101, 064304 (2020).
[37] A. Kumar, P. C. Srivastava and J. Suhonen, “Second-forbidden nonunique $\beta^-$ decays of $^{59,60}$Fe: possible candidates for $g_\alpha$ sensitive electron spectral-shape measurements”, Eur. Phys. J. A 57, 225 (2021).
[38] H. Behrens and W. Bühring, “Nuclear beta-decay”, Nucl. Phys. A 162, 111 (1971).
[39] B. A. Brown and W. D. M. Rae, “The shell-model code NuShellX@MSU”, Nucl. Data Sheets 120, 115(2014).
[40] N. Shimizu, T. Mizusaki, Y. Utsuno, Y. Tsunoda, “Thick-restart block Lanczos method for large-scale shell-model calculations”, Computer Physics Communications 244, 372-384 (2019).
[41] C. Patrignani (Particle Data Group), “Review of particle physics”, Chin. Phys. C 40, 100001 (2016).
[42] V. Kumar, P.C. Srivastava, and H. Li, “Nuclear $\beta^-$ decay half-lives for $fp$ and $fpg$ shell nuclei” Jour. Phys. G: Nucl. and Part. Phys. 43, 105104 (2016).
[43] J. Duflo and A.P. Zuker, “Microscopic mass formulas”, Phys. Rev. C 52, R23(R) (1995).
[44] E. Caurier et al., “Full $0\hbar\omega$ shell-model calculation of the binding energies of the $1f_{7/2}$ nuclei”, Phys. Rev. C 59, 2033 (1999).
[45] P. Choudhary, A. Kumar, P. C. Srivastava, T. Suzuki, “Structure of $^{46,47}$Ca from the $\beta^-$ decay of $^{46,47}$K in the framework of the nuclear shell-model”, Phys. Rev. C. 103, 064325 (2021).
[46] I. S. Towner, “Enhancement in axial-charge matrix elements from meson-exchange currents”, Nucl. Phys. A 542, 631 (1992).
[47] M. Haaranen, P. C. Srivastava, and J. Suhonen, “Forbidden nonunique $\beta$ decays and effective values of weak coupling constants”, Phys. Rev. C 93 034308 (2016).
[48] “Data extracted using the NNDC World Wide Web site from the ENSDF”,
[49] S. Yoshida, Y. Utsuno, N. Shimizu, and T. Otsuka, “Systematic shell-model study of $\beta$-decay properties and Gamow-Teller strength distributions in A ≈ 40 neutron-rich nuclei”, Phys. Rev. C 97, 054321 (2018).
[50] P. Möller, B. Pfeiffer, K. L. Kratz, “New calculations of gross $\beta$-decay properties for astrophysical applications: Speeding-up the classical r process”, Phys. Rev. C 67, 055802 (2003).
[51] T. Marketin, L. Huther, and G. Martínez-Pinedo, “Large-scale evaluation of $\beta$-decay rates of r-process nuclei with the inclusion of first-forbidden transitions”, Phys. Rev. C 93, 025805 (2016).