Some comments on the recent results about the measurement of the Lense-Thirring effect in the gravitational field of the Earth with the LAGEOS and LAGEOS II satellites.

Lorenzo Iorio

Dipartimento Interateneo di Fisica dell’ Università di Bari
Via Amendola 173, 70126
Bari, Italy

e-mail: lorenzo.iorio@libero.it

Abstract

In a recently published paper Ciufolini reports on the so far performed tests aimed at the detection of the general relativistic gravito-magnetic Lense-Thirring effect in the gravitational field of the Earth by means of the analysis of the laser-ranged data of the existing LAGEOS and LAGEOS II geodetic satellites. In this paper we will critically discuss his claims by showing that the total error, mainly due to the systematic bias due to the mismodelling in the static and time-varying parts of the multipolar expansion of the Newtonian terrestrial gravitational potential, is larger than that claimed by Ciufolini. E.g., the systematic error due to the mismodelling in the static part of the geopotential in the tests performed with the EGM96 Earth gravity model and the combination involving the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II realistically amounts to more than 80% (1-σ): the claimed total uncertainty, including also the non-gravitational perturbations which especially affect the perigee of LAGEOS II, is, instead, 20-25%. The claimed accuracy in the more precise tests performed with the 2nd generation CHAMP-only EIGEN2 Earth gravity model and a combination involving the nodes of LAGEOS and LAGEOS II over 10 years is 18%. With numerical simulations we will show that, instead, it is \( \leq 51\% \) (1-σ) if the impact of the secular variations of the even zonal harmonics over a so long observational time span (\(~14\%\)) is accounted for.
Contents

1 Introduction 3

2 The Lense-Thirring effect on the orbit of a test particle 3
  2.1 The gravitational error 4
    2.1.1 The linear combination approach 5

3 The performed Lense-Thirring tests with the LAGEOS satellites 8
  3.1 The node-node-perigee tests 8
    3.1.1 The gravitational error 8
    3.1.2 The non-gravitational error 9
  3.2 The node-node tests 10
    3.2.1 The gravitational error 10
    3.2.2 The impact of the secular variations of the even zonal harmonics: a quantitative estimate

4 Conclusions 16
1 Introduction

Recent years have seen increasing efforts aimed to directly detecting various phenomena connected to the general relativistic gravitomagnetic field \[1, 2, 3, 4\] of the rotating Earth. It should be noted that, according to K. Nordtvedt \[5\], the multidecadal analysis of the Moon’s orbit by means of the Lunar Laser Ranging (LLR) technique yields a comprehensive test of the various parts of order $O(c^{-2})$ of the post-Newtonian equation of motion. The existence of gravitomagnetism as predicted by the Einstein’s General Theory of Relativity would, then, be indirectly inferred from the high accuracy of the lunar orbital reconstruction. In \[6\] the same arguments are applied to the radial motion of the LAGEOS satellite.

The extraordinarily sophisticated and expensive Gravity Probe B (GP-B) mission \[7, 8\] has been launched in April 2004; it is aimed at the detection of the gravitomagnetic precession of the spins \[9\] of four superconducting gyrosopes carried onboard at a claimed accuracy of 1% or better.

The Lense-Thirrring effect on the orbital motion of a test particle \[10\] could be measured by analyzing the orbital data of certain Earth artificial satellites with the Satellite Laser Ranging (SLR) technique \[11\]. Up to now, the only performed tests are due to Ciufolini and coworkers.

In this paper we will analyze the latest results presented in \[12\] from a critical point of view in order to show that the claimed accuracies are optimistic.

2 The Lense-Thirrring effect on the orbit of a test particle and the strategy to measure it

The gravitomagnetic field of a spinning mass of proper angular momentum $J$ induces tiny secular precessions on the longitude of the ascending node $\Omega$ and the argument of pericentre\(^1\) $\omega$ of a test particle \[10, 13, 2, 14\]

\[
\dot{\Omega}_{LT} = \frac{2GJ}{c^2a^3(1-e^2)^{3/2}}, \quad \dot{\omega}_{LT} = -\frac{6GJ \cos i}{c^2a^3(1-e^2)^{3/2}},
\]

where $G$ is the Newtonian constant of gravitation, $c$ is the speed of light in vacuum, $a, e$ and $i$ are the semimajor axis, the eccentricity and the inclination, respectively, of the test particle’s orbit. In the terrestrial space environment the gravitomagnetic precessions are very small: for the geodetic SLR LAGEOS satellites, whose orbital parameters are listed in Table

\(^1\)In their original paper Lense and Thirring use the longitude of pericentre $\varpi = \Omega + \omega$. 

Table 1: Orbital parameters ($a$ semimajor axis, $e$ eccentricity, $i$ inclination) of the existing LAGEOS and LAGEOS II satellites and their Lense-Thirring node precessions $\dot{\Omega}_{LT}$ in mas yr$^{-1}$.

| Satellite  | $a$ (km) | $e$     | $i^\circ$ | $\dot{\Omega}_{LT}$ (mas yr$^{-1}$) |
|------------|----------|---------|-----------|-------------------------------------|
| LAGEOS     | 12270    | 0.0045  | 110       | 31                                  |
| LAGEOS II  | 12163    | 0.0135  | 52.64     | 31.5                                |

they amount to a few tens of milliarcseconds per year (mas yr$^{-1}$ in the following)

The extraction of the Lense-Thirring precessions from the orbit data analysis is very difficult due to a host of competing classical orbital perturbations of gravitational [15, 16, 17, 18] and non-gravitational [19, 20, 21, 22, 23, 24] origin which have various temporal signatures and are often quite larger than the relativistic signal of interest. The most insidious ones are the perturbations which have the same temporal signature of the Lense-Thirring precessions$^2$, i.e. secular trends. Indeed, whatever the length of the adopted observational time span $T_{obs}$ is, they cannot be fitted and removed from the time series without removing the relativistic signal as well. Then, it is of the utmost importance to assess as more accurately and reliably as possible their aliasing impact on the measurement of the Lense-Thirring effect.

It turns out that the perigees of the LAGEOS-like satellites are severely affected by the non-gravitational perturbations, contrary to the nodes. Moreover, since the non-conservative forces depend on the structure, the shape and the rotational status of the satellite their correct modelling is not a trivial task and, as we will see later, introduces large uncertainties in the correct assessment of the error budget in some of the performed gravitomagnetic tests.

2.1 The gravitational error

The even ($\ell = 2, 4, 6...$) zonal ($m = 0$) harmonic coefficients $J_\ell$ of the multipolar expansion of the Earth’s gravitational potential, called geopotential,

$^2$Also the perturbations which grow quadratically in time are, of course, very dangerous. Those induced by the secular variations of the even zonal harmonics of the Earth’s geopotential fall in this category, as we will see in detail in Section 3.2.4. Time-dependent periodic perturbations with periods longer than the observational time span may also be insidious because they would resemble superimposed linear trends [15].
induce secular precessions\(^3\) on the node and the perigee of any near-Earth artificial satellite \([25]\) which, of course, depend only on its orbital configuration and are independent of its physical structure. Such aliasing effects are many orders of magnitude larger than the Lense-Thirring precessions; the precision with which the even zonal harmonics are known in the currently available Earth gravity models \([26, 27, 28, 30, 31, 32, 33, 34]\) would yield errors amounting to a significant fraction of the Lense-Thirring precessions or even larger.

Even more dangerous are the perturbations induced by the secular variations of the low degree even zonal harmonics \(\dot{J}_\ell, \ell = 2, 4, 6\ \[35, 36]\). Indeed, such perturbations grow quadratically in time if the shifts in mas are considered and linearly in time if the rates in mas yr\(^{-1}\) are considered. Their impact on the orbital elements of the LAGEOS satellites have been worked out in \([37]\). It turns out that, by using the results of \([35]\), the errors induced by \(\dot{J}_2\) would amount to 8%, 14% and 5.4% for the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II, respectively, over an observational time span \(T_{\text{obs}}\) of just one year at 1\(−\sigma\) level. This clearly shows that it would be impossible to analyze single orbital elements.

The time-dependent periodic perturbations \([15, 17, 18]\) are less dangerous because if their periods are shorter than the adopted observational time span they can be fitted and removed from the time series. The most insidious tidal perturbation is that induced by the even zonal constituent which has a period of 18.6 years and whose nominal impact on the orbital elements of the LAGEOS satellites amounts to thousands of mas \([15]\). However, it turns out that it does not affect the observables which have been adopted for the performed Lense-Thirring tests because its main component is of degree \(\ell = 2\) and order \(m = 0\).

### 2.1.1 The linear combination approach

The problem of reducing the impact of the mismodeling in the even zonal harmonics of the geopotential with the currently existing satellites can be coped in the following way \([38]\).

Let us suppose we have at our disposal \(N\) (\(N > 1\)) time series of the residuals of those Keplerian orbital elements which are affected by the geopotential with secular precessions, i.e. the node and the perigee: let them be \(\psi^A\), \(A=\text{LAGEOS}, \text{LAGEOS II}, \text{etc}\). Let us write explicitly down the expressions

\(^3\)Also the subtle non-gravitational Yarkovsky-Rubincam force, which is due to the interaction of the Earth’s electromagnetic IR radiation with the physical structure of the LAGEOS satellites, induces secular effects on their nodes and perigees \([21]\).
of the observed residuals of the rates of those elements $\delta \dot{\psi}_A^{\text{obs}}$ in terms of the Lense-Thirring effect $\dot{\psi}_A^{\text{LT}}$, of N-1 mismodeled classical secular precessions $\dot{\psi}_A^{\text{class}}$ induced by those even zonal harmonics whose impact on the measurement of the gravitomagnetic effect is to be reduced and of the remaining mismodeled phenomena $\Delta$ which affect the chosen orbital element

$$\delta \dot{\psi}_A^{\text{obs}} = \dot{\psi}_A^{\text{LT}} \mu_{\text{LT}} + \sum_{N-1 \text{ terms}} \dot{\psi}_A^{\text{class}} \delta J_\ell + \Delta^A, \quad A = \text{LAGEOS, LAGEOS II, ...}$$

The parameter $\mu_{\text{LT}}$ is equal to 1 in the General Theory of Relativity and 0 in Newtonian mechanics. The coefficients $\dot{\psi}_A^{\text{class}}$ are defined as

$$\dot{\psi}_\ell = \frac{\partial \dot{\psi}_\text{class}}{\partial J_\ell}$$

and have been explicitly worked out for the node and the perigee up to degree $\ell = 20$ in [11, 16]: they depend on some physical parameters of the central mass ($GM$ and the mean equatorial radius $R$) and on the satellite’s semimajor axis $a$, the eccentricity $e$ and the inclination $i$. We can think about eq. (2) as an algebraic nonhomogeneous linear system of $N$ equations in $N$ unknowns which are $\mu_{\text{LT}}$ and the $N-1 \delta J_\ell$: solving it with respect to $\mu_{\text{LT}}$ allows to obtain a linear combination of orbital residuals which is independent of the chosen N-1 even zonal harmonics. In general, the orbital elements employed are the nodes and the perigees and the even zonal harmonics cancelled are the first N-1 low-degree ones.

This approach is, in principle, very efficient in reducing the impact of the systematic error of gravitational origin because all the classical precessions induced by the static and time-dependent parts of the chosen N-1 $J_\ell$ do not affect the combination for the Lense-Thirring effect. Moreover, it is flexible because it can be applied to all satellites independently of their orbital configuration, contrary to the butterfly configuration in which the cancellation of the even zonal harmonics can be achieved only for supplementary orbital planes and identical orbital parameters. Apart from the first orbital element which enters the combination with 1, the other elements are weighted by multiplicative coefficients $c_i(a, e, i) \neq 1$ which are built up with $\dot{\psi}_\ell$ and, then, depend on the orbital elements of the considered satellites. Their magnitude is very important with respect to the non-gravitational perturbations, which in general are not cancelled out by the outlined method,

\[4\text{It can be expressed in terms of the PPN \(\gamma\) parameter [39] as } \mu_{\text{LT}} = (1 + \gamma)/2.\]
and to the other time-dependent perturbations of gravitational origin with $\ell \neq 2, 4, 6, ..., m \neq 0$. Values smaller than 1 for the $c_i$ coefficients are, in general, preferable because they reduce the impact of such uncancelled perturbations. It is important to note that the order with which the orbital elements enter the combination is important: indeed, while the systematic error due to the even zonal harmonics of the geopotential remains unchanged if the orbital elements of a combination are exchanged, the coefficients $c_i$ do change and, consequently, also the non-gravitational error. The best results are obtained by choosing the highest altitude satellite as first one and by inserting the other satellites in order of decreasing altitudes.

This method was explicitly adopted for the first time in [38] with the nodes of the LAGEOS satellites and the perigee of LAGEOS II. The obtained combination is

$$\delta \dot{\Omega}_{\text{LAGEOS}}^{\text{obs}} + c_1 \delta \dot{\Omega}_{\text{LAGEOS II}}^{\text{obs}} + c_2 \delta \omega_{\text{LAGEOS II}}^{\text{obs}} \sim \mu_{LT} 60.2,$$

where $c_1 = 0.304$, $c_2 = -0.350$ and 60.2 is the slope, in mas yr$^{-1}$, of the expected gravitomagnetic linear trend. Eq. (4) is insensitive to the first two even zonal harmonics $J_2$ and $J_4$. It has been used in [40] when the level of accuracy of the JGM3 [26] and EGM96 [27] Earth gravity models, available at that time, made it necessary to consider a combination of observables which is independent of errors in both $J_2$ and $J_4$.

In view of the great improvements in the Earth gravity field modelling with the CHAMP [41] and, especially, GRACE [42] missions an extensive search for alternative combinations has been subsequently performed [43, 37, 44, 45, 46, 47]. In [37, 44, 45] the following combination has been proposed

$$\delta \dot{\Omega}_{\text{obs}}^{\text{LAGEOS}} + k_1 \delta \dot{\Omega}_{\text{obs}}^{\text{LAGEOS II}} \sim \mu_{LT} 48.2,$$

where $k_1 = 0.546$ and 48.2 is the slope, in mas yr$^{-1}$, of the expected gravitomagnetic linear trend. It has been adopted for the tests performed in [42] with the 2nd generation CHAMP-only EIGEN2 Earth gravity model [30] and the 1st generation GRACE-only GGM01S [33] Earth gravity model. Eq. (5) allows to cancel out the first even zonal harmonic $J_2$.

The possibility of using only the nodes of the LAGEOS satellites in view of the improvements in the Earth gravity models from GRACE has been proposed for the first time in [42], although without quantitative details. In [42] it seems that Ciufolini refers to it as a proper own result with his reference [6] which includes [40] of the present work and an announced paper. [40] is not concerned with eq. (5) because it deals with eq. (4) and its analysis by means of EGM96. Moreover, Iorio retains the e-mails in which he passed to Ciufolini, with whom he was long in contact, the combination of eq. (5) along with the estimates of the systematic error obtained with EIGEN2 and is disposed to make them publicly available on request.
3 The performed Lense-Thirring tests with the LAGEOS satellites

The only performed tests aimed at the detection of the Lense-Thirring precessions of eq. (1) in the gravitational field of the Earth with the existing LAGEOS satellites have been performed, up to now, by Ciufolini and coworkers. They have used the node-node-perigee combination of eq. (4) [40, 12] and the node-node combination of eq. (5) [12].

In [12] it is claimed that “[...] the Lense-Thirring effect exists and its experimental value, [...] fully agrees with the prediction of general relativity.” in regard to both the tests with the EGM96 and EIGEN-2 Earth gravity models. In this Section we will disprove such statements.

The main objections to the results presented in these works can be summarized as follows:

- Ciufolini has not performed tests by varying the length of the adopted observational time span, running backward and forward the initial epoch of the analysis, varying the secular rates of the even zonal harmonics in order to check their impact over different time spans, using different Earth gravity models in order to obtain a scatter plot of the obtained results.

- The total error budget has been underestimated, especially the systematic error of gravitational origin. E.g., the impact of the secular variations of the even zonal harmonics of the geopotential, which may become a very limiting factor over time spans many years long as those used, has not been addressed. Almost always $1-\sigma$ results have been presented without any explicit indication of this fact.

3.1 The node-node-perigee tests

The combination of eq. (4) has been analyzed by using the EGM96 [27] Earth gravity model over 4 years in [40] and over 7.3 years in [12]. The claimed total error budget amounts to 20-25% over 4 years and to 20% over 7.3 years.

3.1.1 The gravitational error

The impact of the remaining uncancelled even zonal harmonics of the geopotential $J_6, J_8, J_{10}, ...$ on eq. (4) has been estimated by Ciufolini and coworkers with the full covariance matrix of EGM96 in a root-sum-square calcula-
tion. In [10] and, six years later, in [12] it is claimed to be \( \lesssim 13\% \). Apart from the fact that this is a 1–\( \sigma \) level estimate, in [48], as later acknowledged in a number of papers [43, 16, 37, 45, 47], the use of the full covariance matrix of EGM96 has been questioned. Indeed, it has been noted that in the EGM96 solution the recovered even zonal harmonics are strongly reciprocally correlated; it seems, e.g., that the 13\% value for the systematic error due to geopotential is due to a lucky correlation between \( J_6 \) and \( J_8 \) which are not cancelled by eq. (4). The point is that, according to [48], nothing would assure that the covariance matrix of EGM96, which is based on a multi–year average that spans the 1970, 1980 and early 1990 decades, would reflect the true correlations between the even zonal harmonics during the particular time intervals of a few years adopted in the analyses by Ciufolini and coworkers. Then, a more conservative, although pessimistic, approach would be to consider the sum of the absolute values of the errors due to the single even zonal as representative of the systematic error induced by our uncertainty in the terrestrial gravitational field according to EGM96 [45, 37]. In this case we would get a conservative upper bound of 83\% (1-\( \sigma \)). If a root-sum-square calculation is performed by neglecting the correlations between the even zonals a 45\% 1–\( \sigma \) error is obtained [16, 45, 37, 47].

3.1.2 The non-gravitational error

Another important class of systematic errors is given by the non–gravitational perturbations which affect especially the perigee of LAGEOS II. The main problem is that it turned out that their interaction with the structure of LAGEOS II changes in time due to unpredictable modifications in the physical properties of the LAGEOS II surface (orbital perturbations of radiative origin, e.g. the solar radiation pressure and the Earth albedo) and in the evolution of the spin dynamics of LAGEOS II (orbital perturbations of thermal origin induced by the interaction of the electromagnetic radiation of solar and terrestrial origin with the physical structure of the satellites, in particular with their corner–cube retroreflectors). Moreover, such tiny but insidious effects were not entirely modelled in the GEODYN II software at the time of the analysis of [10, 12], so that it is not easy to correctly and reliably assess their impact on the total error budget of the measurement performed during that particular time span. According to the evaluations in [21], the systematic error due to the non–gravitational perturbations over a time span of 7 years amounts to almost 28\%. However, according to [48], their impact on the measurement of the Lense–Thirring effect with the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II is, in general,
quite difficult to be reliably assessed.

So, by adding quadratically the gravitational and non–gravitational errors of [21] we obtain for the systematic uncertainty $\delta \mu_{\text{systematic}} \sim 54\%$ if we assume a 45\% error due to geopotential. The sum of the absolute values of the errors due to geopotential added quadratically with the non–gravitational perturbations would yield a total systematic error of $\delta \mu_{\text{systematic}} \sim 88\%$. It must be noted that the latter estimate is rather similar to those released in [48]. Note also that they are 1-$\sigma$ evaluations. Moreover, it should be considered that the perigee of LAGEOS II is also sensitive to the eclipses effect on certain non–gravitational perturbations. Such features are, generally, not accounted for in all such estimates. An attempt can be found in [19] in which the impact of the eclipses on the effect of the direct solar radiation pressure on the LAGEOS–LAGEOS II Lense–Thirring measurement has been evaluated: it should amount to almost 10\% over an observational time span of 4 years.

3.2 The node-node tests

In this Section we will deal with the node-node combination of eq. (5). Such observable only cancels out the gravitational bias of the first even zonal harmonic $J_2$, but has the great advantage of discarding the perigee of LAGEOS II and its insidious non-gravitational perturbations.

3.2.1 The gravitational error

In [12] the node-node combination of eq. (5) has been analyzed with the 2nd generation CHAMP-only EIGEN2 [30] and the 1st generation GRACE-only GGM01S [33] Earth gravity models over a time span of almost 10 years. In [37] the impact of the static part of the geopotential, according to the CHAMP-only EIGEN2 Earth gravity model, is evaluated as 18\% (1-$\sigma$ root-sum-square covariance calculation), 22\% (1-$\sigma$ root-sum-square calculation) and 37\% (1-$\sigma$ upper bound). Ciufolini reports 18\% obtained in a root-sum-square fashion with the full covariance matrix of EIGEN2 for which the same remarks as for EGM96 holds. Moreover, he does not consider the fact that EIGEN2 is only based on six months of data and that the released sigmas of the even zonal harmonics of low degree, which are the most relevant in this kind of analyses with the LAGEOS satellites, are rather optimistic, as explicitly pointed out in [30] and acknowledged in [37]. In regard to the GGM01S model, the covariance matrix was not publicly released. Ciufolini correctly presents a 19\% which is the 1–$\sigma$ upper bound obtained in [37].
However, GGM01S is only based on 111 days of data.

In our opinion the author’s conclusion “We conclude, using the Earth gravity model EIGEN-2S, that the Lense-Thirring effect exists and its experimental value, $\mu = 0.98 \pm 0.18$, fully agrees with the prediction of general relativity” is optimistic. Indeed, he claims that in his 18% total error budget all the error sources are included. Ciufolini neglects the impact of the time-dependent gravitational perturbations on eq. (5). Indeed, they may turn out to be a serious limiting factor mainly due to the secular variations of the even zonal harmonics $\dot{J}_\ell$. Indeed, eq. (5) allows to cancel out $\dot{J}_2$, but is sensitive to $\dot{J}_4$, $\dot{J}_6$, ..., as pointed out in [50]. The uncertainties in the $\dot{J}_\ell$ are still quite large: see Table 1 of [36]. From it the values of Table 2 can be inferred.

Table 2: Weighted means and standard deviations from Table 1 of [36] of the secular rates of the first three even zonal harmonics in units of $10^{-11}$ yr$^{-1}$.

| $\ell$   | $\dot{J}_\ell$ | $\sigma_{\dot{J}_\ell}$ |
|----------|----------------|--------------------------|
| 2        | -2.113         | 0.0810                   |
| 4        | -0.6992        | 0.2029                   |
| 6        | -0.3594        | 0.1765                   |

On the other hand, their impact on the Lense-Thirring measurement grows linearly in time$^7$. Indeed, the mismodelled shift, in mas, of eq. (5) due to the secular variations of the uncanceled even zonal harmonics can be written as

$$\sum_{\ell=4}^{\infty} \left( \dot{\Omega}_{\ell}^{\text{LAGEOS}} + k_1 \dot{\Omega}_{\ell}^{\text{LAGEOS II}} \right) \frac{\sigma_{\dot{J}_\ell}}{2} T_{\text{obs}}^2,$$

where the coefficients $\dot{\Omega}_{\ell}$ are $\partial \dot{\Omega}_{\text{class}}/\partial J_\ell$ and have explicitly been calculated up to degree $\ell = 20$ in [11, 16]. It must be divided by the gravitomagnetic shift, in mas, of eq. (5) over the same observational time span

$$\left( \dot{\Omega}_{\text{LT}}^{\text{LAGEOS}} + k_1 \dot{\Omega}_{\text{LT}}^{\text{LAGEOS II}} \right) T_{\text{obs}} = 48.2 \text{ mas yr}^{-1} T_{\text{obs}}.$$

$^6$The problem of the secular variations of the even zonal harmonics in post-Newtonian tests of gravity with LAGEOS satellites has been quantitatively addressed for the first time in [51]. In regard to the Lense-Thirring measurement with eq. (5), it has been, perhaps, misunderstood in [57].

$^7$For a possible alternative combination which would cancel out the first three even zonal harmonics along with their temporal variations see [46, 47].
By assuming, e.g., $\sigma_{j_4} = 0.6 \times 10^{-11} \text{ yr}^{-1}$ and $\sigma_{j_6} = 0.5 \times 10^{-11} \text{ yr}^{-1}$, it turns out that the percent error on the combination eq. (5) grows linearly with $T_{\text{obs}}$ and would amount to 1% over one year at 1-σ level. This means that, over 10 years, their impact is $\sim 10$% (1-σ). In Section 3.2.2 we will quantitatively support this evaluation.

3.2.2 The impact of the secular variations of the even zonal harmonics: a quantitative estimate

Here we describe a numerical experiment aimed at a quantitative evaluation of the impact of $\dot{J}_\ell$.

The first step consists in simulating the time series of $\delta \Omega_{\text{LAGEOS}} + k_1 \delta \Omega_{\text{LAGEOS II}}$ in order to obtain the qualitative and quantitative features of Figure 4 of [12]. It refers to EIGEN2 and shows the raw residual time series with a straight line which fits it. The post-fit residuals amounts to 12 mas. In our model, called Input Model (IM), we include

- $\text{LT} \equiv S_{\text{LT}} t$ with $S_{\text{LT}} = 48.2 \text{ mas yr}^{-1}$. Lense-Thirring trend as predicted by the General Theory of Relativity in order to simulate the fact that the residuals of LAGEOS and LAGEOS II have been built up by dealing with the gravitomagnetic force as a totally unmodelled feature.

- $\text{ZONDOT} \equiv \sum_{\ell=4}^{6} \{r\} \left( \dot{\Omega}_{\ell}^{\text{LAGEOS}} + c_1 \dot{\Omega}_{\ell}^{\text{LAGEOS II}} \right) \left( \frac{\dot{J}_\ell}{2} \right) t^2$. Quadratic term due to the $\dot{J}_\ell$ according to Table 2. The numbers $\{r\}$ are randomly generated from a normal distribution with mean zero, variance one and standard deviation one. Note that EIGEN2 does not solve for $\dot{J}_\ell$. In [12] there are no details about the values included in the dynamical force models of the orbital processor; thus we treat the secular rates of the even zonal harmonics as unmodelled features fully absorbed by the residuals.

- $\text{ZONALS} \equiv p \left( \frac{x}{100} \right) S_{\text{LT}} t$. Linear trend with a slope of $x\%$ of the Lense-Thirring signal. For EIGEN2 $x = 37$ (sum of the absolute values of the individual errors) is assumed. The number $p$ is randomly generated as for the $\{r\}$.

- $\text{TIDE} \equiv \sum \{a_c\} \sigma_{A_c} \cos \left( \left( \frac{2\pi}{P} \right) t + \{f_c\} \right) + \sum \{a_s\} \sigma_{A_s} \sin \left( \left( \frac{2\pi}{P} \right) t + \{f_s\} \right)$. Set of various tidal perturbations of known periods $P$. For the impact of such kind of perturbations on the orbits of the LAGEOS satellites see [14]. The sets of numbers $\{a_c\}, \{a_s\}, \{f_c\}, \{f_s\}$ are randomly generated as $p$ and the $\{r\}$.  

12
• NOISE. White gaussian noise with variable amplitude which simulates the observational errors of the laser-ranged measurement

The full IM used in our analysis is thus

\[ IM = LT + ZONDOT + ZONALS + TIDE + NOISE. \] (8)

We include in our model the possibility of varying the length of the time series \( T_{\text{obs}} \), the temporal step \( \Delta t \) which simulates the orbital arc length, the amplitude of the noise and of the mismodelling in the perturbations and the initial phases of the sinusoidal terms in order to simulate different initial conditions and uncertainties in the dynamical force models of the orbital processors. More precisely, the magnitude of the mismodelling in the various effects is randomly varied within the currently accepted ranges (1-\( \sigma \)) by using random numbers generated from a normal distribution with mean zero, variance one and standard deviations one. The same also holds for ZONALS because it is impossible to know, a priori, the sign of the slope of the residual trend due to the even zonal harmonics. The so built IM represents the basis of our subsequent analyses.

With the so built IM we perform a set of 5000 runs by randomly varying the initial phases, the noise and the mismodelling amplitudes within the accepted intervals in order to simulate a wide range of initial conditions and measurement errors which can occur in the real world. The length of the time series is keep fixed at \( T_{\text{obs}} = 9.5 \) years. In every run we fit the IM with a straight line only (LF) and record the obtained slope \( \mu \) with the related error \( \delta \mu \). Then, we calculate the averages of \( \mu \) and \( \delta \mu \) and the standard deviation of the mean \( \Sigma \). For EIGEN2, i.e. \( x = 37 \), we obtain

\[ \left\langle \frac{\mu - \mu_{LT}}{\mu_{LT}} \right\rangle \sim 30\%. \] (9)

Also the post-fit residuals are calculated. Figure 1 shows the complete IM, its straight line fit compared with the nominal Lense-Thirring trend and the post-fit residuals for a given set of randomly chosen initial conditions. The RMS post-fit amounts to 12.7 mas. The averaged RMS post-fit is 11 mas. This shows that our procedure represents a realistic starting point for our analyses.

A first interesting result is that the departure of the measured slope \( \mu \) from the nominal gravitomagnetic slope \( \mu_{LT} \), which is included in IM, amounts to \( \sim 30\% \) on average, while for Ciufolini is 2% only (\( \mu = 0.98 \)).

In order to evaluate the impact of the secular rates of the even zonal harmonics in every run we also fit the IM with a quadratic polynomial (QF)
Figure 1: Simulated time series, straight line fit and post-fit residuals for $T_{\text{obs}} = 9.5$ years and $\Delta t = 15$ days. The RMS of the post-fit residuals is 12.7 mas. The slope of the trend simulating the impact of the mismodelled even zonal harmonics has been fixed to 37% of the Lense-Thirring effect, according to EIGEN2.
and compare the so obtained slope $\mu_{QF}$ with the slope obtained in LF $\mu_{LF}$. Note that procedure is analogous to that adopted in [18] for the periodic perturbations. On average, the difference between the two slopes, i.e. the systematic error due to $\dot{J}_4, \dot{J}_6$, amounts to

$$\left\langle \left| \frac{\mu_{LF} - \mu_{QF}}{\mu_{LT}} \right| \right\rangle \sim 14\%$$

of the Lense-Thirring effect. This result holds for $1 - \sigma$. It is important to note that Figure 1 has been obtained by assuming that the combined residuals of the LAGEOS satellites absorb the quadratic signature of $\dot{J}_\ell$ according to Table 2; i.e. IM also includes ZONDOT. Nonetheless, it is difficult to discern the parabolic signal which, instead, is present and does affect the recovery of the slope. This means that a simple visual inspection of the plots of the combination of eq. (5) cannot be considered conclusive about the effect of $\dot{J}_4, \dot{J}_6$.

Another important point is that the combination of eq. (5) cannot be used to reliably constrain the zonals’ rates by measuring a $\dot{J}_{4\text{eff}}$. Indeed, it turns out that the errors $\delta Q$ in the quadratic parameters of QF are always larger than the estimated values themselves $Q$ and their mean $<\delta Q/Q>$ over a given set of 5000 runs amounts to $\sim 260\%$ with a standard deviation of the mean of 68%. Moreover, these figures change for different sets of 5000 runs.

If we repeat the same numerical experiments for $T_{\text{obs}} = 9.5$ years with $x = 18$ (1-$\sigma$ root-mean-square full covariance calculation) and $x = 22$ (1-$\sigma$ root-mean-square variance calculation) the situation does not substantially change

$$\left\langle \left| \frac{\mu_{LF} - \mu_{LT}}{\mu_{LT}} \right| \right\rangle \sim 20\%, \quad \left\langle \left| \frac{\mu_{LF} - \mu_{QF}}{\mu_{LT}} \right| \right\rangle \sim 14\%.$$  

Then, a more conservative 1-$\sigma$ estimate of the total systematic error of the measurement of the Lense-Thirring effect with the combination of eq. (5) and the EIGEN2 Earth gravity model is

$$\delta \mu_{LT}^\text{total error} \leq 51\%.$$  

While the forthcoming solutions from CHAMP and, especially, GRACE will be able to improve the static part of the terrestrial gravitational potential, i.e. the $J_\ell$, it is not so for their secular rates $\dot{J}_\ell$. This fact sets for the systematic error of gravitational origin a sort of threshold below which it will not be possible to go unless much more accurate determinations of $\dot{J}_4, \dot{J}_6$ will be available.
4 Conclusions

In this paper we have performed a detailed critical analysis of the reliability and robustness of the so far performed tests aimed to the detection of the Lense-Thirring effect in the gravitational field of the Earth with the existing or proposed LAGEOS satellites.

We can summarize our conclusions as follows

- In regard to the node-node-perigee LAGEOS-LAGEOS II combination, the claimed $20 - 25\%$ total accuracy obtained with the EGM96 Earth gravity model, still presented in [12], is not realistic because of the impact of the non-gravitational perturbations on the perigee of LAGEOS II and the mismodelling in the even zonal harmonics of the geopotential whose $1 - \sigma$ upper bound is 83%.

- In regard to the node-node LAGEOS-LAGEOS II combination of eq. [13], extensive numerical tests have been performed in order to quantitatively assess the impact of the uncancelled secular variations of the even zonal harmonics on the proposed measurement of the Lense-Thirring effect. A simulated time series curve has been fitted with a straight line and a quadratic polynomial and the so obtained slopes have been compared. This procedure has been repeated over 5000 runs performed by randomly varying the initial phases, the noise and the mismodelling level within the currently accepted ranges of the simulated signal. It turns out that the bias due to $\dot{J}_4$ and $\dot{J}_6$ over 9.5 years amounts to $\sim 14\%$ on average. This yields an upper bound of the total systematic error of the performed tests with EIGEN2 of $\sim 51\%$.

- Alternative combinations involving the use of existing laser-ranged targets other than the LAGEOS satellites should be analyzed. The most promising combination is, in principle, the one that involves the nodes of LAGEOS, LAGEOS II, Ajisai and Jason-1 [46, 47]. It cancels out the first three even zonal harmonics $J_2, J_4, J_6$ along with their temporal variations at the price of introducing the relatively huge non-gravitational perturbations on Jason-1 which, however, should have a time-dependent periodic signature with short periodicities. According to the recently released combined CHAMP+GRACE+terrestrial gravimetry/altimetry EIGEN-CG01C Earth gravity model [31], the systematic error due to the remaining even zonal harmonics would amount to $0.7\%$ (root-sum-square calculation) and $8\%$ (upper bound)

---

\(^8\)The $1-\sigma$ errors for the node-node LAGEOS-LAGEOS II combination of eq. [13] are 5%
The forthcoming more accurate and robust Earth gravity model solutions from GRACE should especially improve the higher degree even zonal harmonics, so that it might happen that the difference between the node-only LAGEOS-LAGEOS II and the node-only LAGEOS-LAGEOS II-Ajisai-Jason-1 combination will further enforce to the detriment of the former one, at least in regard to the gravitational error. The possibility of getting long time series of the Jason’s node should seriously be investigated with real data tests. Moreover, Jason-1 is also affected by the orbital maneuvers but they are mainly in-plane.

References

[1] Mashhoon, B., Gronwald, F., and Lichtenegger, H. (2001). Gravitomagnetism and the Clock Effect. In: Gyros, Clocks, Interferometers...: Testing Relativistic Gravity in Space, C. Lämmerzahl, C. W. F. Everitt, and F. W. Hehl (Eds.), Springer, Berlin, pp. 83–108.

[2] Ciufolini, I., and Wheeler, A. (1995). Gravitation and Inertia, Princeton University Press, Princeton.

[3] Ruggiero, M. L., and Tartaglia, A. (2002). Gravitomagnetic Effects, Nuovo Cim. B 117, 743.

[4] Schäfer, G. (2004). Gravitomagnetic Effects, Gen. Rel. Grav. 36, 2223.

[5] Nordtvedt, K. (2003). Some considerations on the varieties of frame dragging. In: Nonlinear Gravitodynamics, R. Ruffini, and C. Sigismondi (Eds.), World Scientific, Singapore, pp. 35–45.

[6] Nordtvedt, K. (1988). Gravitomagnetic interaction and laser ranging to Earth satellites, Phys. Rev. Lett. 61, 2647.

[7] Everitt, F. (1974). The Gyroscope Experiment I. General Description and Analysis of Gyroscope Performance Proc. Int. School Phys. "Enrico Fermi" Course LVI, B. Bertotti (Ed.), Academic Press,

(root-sum-square calculation with the variance matrix) and 6% (upper bound), according to EIGEN-CG01C, while they are 3% and 4%, according to the 2nd generation GRACE-only EIGEN-GRACE02S model.
New York, pp. 331–360. Reprinted in: (2003). *Nonlinear Gravitodynamics*, R. Ruffini, and C. Sigismondi (Eds.), World Scientific, Singapore, pp. 439–468.

[8] Everitt, F., et al. (2001). Gravity Probe B: Countdown to Launch. In: *Gyros, Clocks, Interferometers...: Testing Relativistic Gravity in Space*, C. Lämmerzahl, C. W. F. Everitt, and F. W. Hehl (Eds.), Springer, Berlin, pp. 52–82.

[9] Schiff, L. (1960). Motion of a Gyroscope According to Einstein’s Theory of Gravitation, *Proc. Nat. Acad. Sci. Am.* **46**, 871. Reprinted in: (2003). *Nonlinear Gravitodynamics*, R. Ruffini and C. Sigismondi (Eds.), World Scientific, Singapore, pp. 427-438.

[10] Lense, J., and Thirring, H. (1918) Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie, *Phys Z* **19**, 156 see also English translation by Mashhoon, B., Hehl, F. W., and Theiss, D. S. (1984). *Gen Rel Grav* **16**, 711 reprinted in: (2003). *Nonlinear Gravitodynamics*, R. Ruffini, and C. Sigismondi (Eds.), World Scientific, Singapore, pp. 349–388.

[11] Iorio, L. (2002). Recent developments in testing general relativity with satellite laser ranging, *Riv. Nuovo Cim.* **25**, (5).

[12] Ciufolini, I. (2004). Frame Dragging and Lense-Thirring Effect, *Gen. Rel. Grav.* **36**, 2257.

[13] Allison, T., and Ashby, N. (1993). Canonical planetary equations for velocity-dependent forces, and the Lense-Thirring precession, *Celest. Mech. Dyn. Astron.* **57**, 537.

[14] Iorio, L. (2001a). An alternative derivation of the Lense-Thirring drag on the orbit of a test body, *Nuovo Cim. B* **116**, 777.

[15] Iorio, L. (2001b). Earth Tides and Lense-Thirring Effect, *Celest. Mech. Dyn. Astron.* **79**, 201.

[16] Iorio, L. (2003). The Impact of the Static Part of the Earth’s Gravity Field on Some Tests of General Relativity with Satellite Laser Ranging, *Celest. Mech. Dyn. Astron.* **86**, 277.

[17] Iorio, L., and Pavlis, E. (2001). Tidal Satellite Perturbations and the Lense-Thirring Effect, *J. Geod. Soc. Jpn.* **47**, 169.
[18] Pavlis, E., and Iorio, L. (2002). The impact of tidal errors on the determination of the Lense-Thirring effect from satellite laser ranging, *Int. J. Mod. Phys. D* 11, 599.

[19] Vespe, F. (1999). The perturbations of Earth penumbra on LAGEOS II perigee and the measurement of Lense-Thirring gravitomagnetic effect, *Adv. Sp. Res.* 23, 699.

[20] Lucchesi, D. (2001). Reassessment of the error modelling of non–gravitational perturbations on LAGEOS II and their impact in the Lense–Thirring determination. Part I, *Plan. Space Sci.* 49, 447.

[21] Lucchesi, D. (2002). Reassessment of the error modelling of non–gravitational perturbations on LAGEOS II and their impact in the Lense–Thirring determination. Part II, *Plan. Space Sci.* 50, 1067.

[22] Lucchesi, D. (2003). The asymmetric reflectivity effect on the LAGEOS satellites and the germanium retroreflectors, *Geophys. Res. Lett.* 30, 1957.

[23] Lucchesi, D. (2004). LAGEOS Satellites Germanium Cube-Corner-Retroreflectors and the Asymmetric Reflectivity Effect, *Celest. Mech. Dyn. Astron.* 88, 269.

[24] Lucchesi, D. et. al. (2004). LAGEOS II perigee rate and eccentricity vector excitations residuals and the Yarkovsky–Schach effect, *Plan. Space Sci.* 52, 699.

[25] Kaula, W. M. (1966). *Theory of Satellite Geodesy*, Blaisdell, Waltham.

[26] Tapley, B. D., et al. (1996). The Joint Gravity Model 3, *J Geophys Res* 101, 28029.

[27] Lemoine, F. G., et al. (1998). The Development of the Joint NASA GSFC and the National Imagery Mapping Agency (NIMA) geopotential Model EGM96 NASA/TP-1998-206861.

[28] Gruber, Th., et al. (2000). GRIM5-C1: Combination solution of the global gravity field to degree and order 120, *Geophys. Res. Lett.* 27, 4005.

[29] Reigber, Ch., et al. (2002). A high quality global gravity field model from CHAMP GPS tracking data and Accelerometry (EIGEN-1S), *Geophys. Res. Lett.* 29, 10.1029/2002GL015064.
[30] Reigber, Ch., et al. (2003). The CHAMP-only Earth Gravity Field Model EIGEN-2, *Adv. Sp. Res.* **31**, 1883.

[31] Reigber, Ch., et al. (2005). Earth Gravity Field and Seasonal Variability from CHAMP. In: *Earth Observation with CHAMP - Results from Three Years in Orbit*, Ch. Reigber, H. Lühr, P. Schwintzer, and J. Wickert (Eds.), Springer, Berlin, pp. 25–30.

[32] Reigber, Ch., et al. (2005). An Earth gravity field model complete to degree and order 150 from GRACE: EIGEN-GRACE02S, *J. Geodyn.* **39**, 1.

[33] Tapley, B. D., et al. (2004). The Gravity Recovery and Climate Experiment: Mission Overview and Early Results, *Geophys. Res. Lett.* **31**, L09607.

[34] Reigber, Ch., et al. (2004). A High Resolution Global Gravity Field Model Combining CHAMP and GRACE Satellite Mission and Surface Gravity Data: EIGEN-CG01C. Submitted to *J. of Geodesy*

[35] Eanes, R. J, and Bettadpur, S. V. (1996). Temporal variability of Earth’s gravitational field from satellite laser ranging. In: *Global Gravity Field and its Temporal Variations (IAG Symp. Ser. 116)*, R. H. Rapp, A. Cazenave, and R. S. Nerem (Eds.), Springer, New York, pp 30–41.

[36] Cox, C., et al. (2003). Time-Variable Gravity: Using Satellite Laser Ranging as a Tool for Observing Long-Term Changes in the Earth System. In: *Proc. 13th Int. Laser Ranging Workshop NASA CP 2003-212248*, R. Noomen, S. Klosko, C. Noll, and M. Pearlman (Eds.), (NASA Goddard) (Preprint [http://cddisa.gsfc.nasa.gov/~lw/lw Proceedings.html#science](http://cddisa.gsfc.nasa.gov/~lw/lw Proceedings.html#science)).

[37] Iorio, L., and Morea, A. (2004). The impact of the new Earth gravity models on the measurement of the Lense-Thirring effect, *Gen. Rel. Grav.* **36**, 1321. (Preprint [http://www.arxiv.org/abs/gr-qc/0304011](http://www.arxiv.org/abs/gr-qc/0304011)).

[38] Ciufolini, I. (1996). On a new method to measure the gravitomagnetic field using two orbiting satellites, *Nuovo Cim. A* **109**, 1709.

[39] Will, C. M. (1993). *Theory and Experiment in Gravitational Physics* 2nd edition, Cambridge University Press, Cambridge.
[40] Ciufolini, I., et al. (1998). Test of General Relativity and Measurement of the Lense-Thirring Effect with Two Earth Satellites, *Science* **279**, 2100.

[41] Pavlis, E. (2000). Geodetic Contributions to Gravitational Experiments in Space. In: *Recent Developments in General Relativity*, R. Cianci, R. Collina, M. Francaviglia, and P. Fré (Eds.), Springer, Milan, pp 217–233.

[42] Ries, J.C., Eanes, R.J., Tapley, B.D., and Peterson, G.E. (2003). Prospects for an Improved Lense-Thirring Test with SLR and the GRACE Gravity Mission. In: *Proc. 13th Int. Laser Ranging Workshop NASA CP 2003-212248*, R. Noomen, S. Klosko, C. Noll, and M. Pearlman (Eds.), (NASA Goddard) (Preprint [http://cddisa.gsfc.nasa.gov/lw13/lw_proceedings.html#science](http://cddisa.gsfc.nasa.gov/lw13/lw_proceedings.html#science)).

[43] Iorio, L. (2002). Is it possible to improve the present LAGEOS-LAGEOS II Lense-Thirring experiment?, *Class. Quantum Grav.* **19**, 5473.

[44] Iorio, L. (2003a). The new Earth gravity models and the measurement of the Lense-Thirring effect. Paper presented at Tenth Marcel Grossmann Meeting on General Relativity Rio de Janeiro, July 20-26. (Preprint [http://www.arxiv.org/abs/gr-qc/0308022](http://www.arxiv.org/abs/gr-qc/0308022)).

[45] Iorio, L. (2005). The impact of the new CHAMP and GRACE Earth gravity models on the measurement of the general relativistic Lense–Thirring effect with the LAGEOS and LAGEOS II satellites. In: *Earth Observation with CHAMP - Results from Three Years in Orbit*, Ch. Reigber, H. Lühr, P. Schwintzer, and J. Wickert (Eds.), Springer, Berlin, pp. 187–192. (Preprint [http://www.arxiv.org/abs/gr-qc/0309092](http://www.arxiv.org/abs/gr-qc/0309092)).

[46] Iorio, L., and Doornbos, E. (2005). How to reach a few percent level in determining the Lense-Thirring effect?, *Gen. Rel. Grav.*, in press. (Preprint [http://www.arxiv.org/abs/gr-qc/0404062](http://www.arxiv.org/abs/gr-qc/0404062)).

[47] Vespe, F., and Rutigliano, P., (2004). The improvement of the Earth gravity field estimation and its benefits in the atmosphere and fundamental physics *35th COSPAR Scientific Assembly Paris, France, 18 - 25 July 2004 COSPAR04-A-03614* submitted to *Adv. Sp. Res.*
[48] Ries, J. C., Eanes, R. J., and Tapley, B. D. (2003). Lense-Thirring Precession Determination from Laser Ranging to Artificial Satellites. In: *Nonlinear Gravitodynamics*, R. Ruffini, and C. Sigismondi (Eds.), Singapore, World Scientific, pp. 201–211.

[49] Iorio, L. (2003b). A reassessment of the systematic gravitational error in the LARES mission, *Gen. Rel. Grav.* **35**, 1263.

[50] Iorio, L. (2004). Towards a few-percent measurement of the Lense-Thirring effect with the LAGEOS and LAGEOS II satellites?, (Preprint [http://www.arxiv.org/abs/gr-qc/0408031](http://www.arxiv.org/abs/gr-qc/0408031)).

[51] Lucchesi, D. (2003). LAGEOS II perigee shift and Schwarzschild gravitoelectric shift, *Phys. Lett. A* **318**, 234.