Randomized Tensor Ring Decomposition and Its Application to Large-scale Data Reconstruction

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Background

- Dimensionality reduction is an essential technique for multi-way large-scale data, i.e., tensor.
- Tensor ring decomposition (TRD) has become popular due to its high representation ability and flexibility.
- The existing TRD algorithms suffer from high computational cost when facing large-scale data.
- Random projection (TP) of matrix has been widely applied to solve large-scale problems.
- Tensor random projection (TRP) is a promising tool to solve large-scale tensor problems.

Tensor ring decomposition

- Decompose tensor \(\mathbf{X} \in \mathbb{R}^{I_1 \times \cdots \times I_N}\) in TR-format:
  \(\mathbf{X}(i_1, i_2, \ldots, i_N) = \text{Trace}\left(\prod_{n=1}^{N} G_n(i_n)\right)\)
- TR core tensor: \(G_n \in \mathbb{R}^{I_1 \times \cdots \times I_N}\)
- TR-rank: \((R_1, R_2, \ldots, R_N)\), where \(R_n = R_{n+1}\)
- \(G_n^{(k)}\) is the kth slice matrix of the nth core tensor.
- Beyond CP and Tucker: the curse of dimensionality free, super compressibility
- Beyond TT format: enhanced representation ability, permutation flexibility of the latent factors, structure information interpretation
- Existing problems: in demand of efficient TRD algorithms

Randomized TRD

Given a large-scale tensor \(\mathbf{X} \in \mathbb{R}^{I_1 \times \cdots \times I_N}\), the randomized tensor ring decomposition is processed as follows:

1. Find the orthogonal projection matrices by
   \[Q_n = QR(X_n,M)\]
   where \(M \in \mathbb{R}^{N}\) is the Gaussian distribution.

2. Process tensor random projection to obtain the projected tensor
   \(P \in \mathbb{R}^{I_1 \times \cdots \times I_N}\)

3. Tensor ring decomposition by TRSVD or TRALS [1] to obtain core tensor \(z_{1}, z_{2}, \ldots, z_{N}\) of \(P\).

4. Back projection to obtain the TRD of the large-scale tensor:
   \(\mathbf{G}_n = Z_n \times_2 Q_n\)

Remarks:
- The random projection is very effective when the large-scale tensor is low-rank in some modes.
- We can take a balance of the accuracy and computational cost by choosing proper size of the projection.
- The orthogonal projection matrices can be generated by more efficient methods, to improve the final performance. See more discussions in [2, 3].

Experiments

The experiments show that the proposed algorithms are much faster than traditional algorithms without loss of accuracy, and our algorithms show superior performance in image denoising and deep learning dataset compression compared to the other randomized algorithms.

Conclusion

- Based on tensor random projection method, we proposed rTRALS and rTRSVD algorithms, by which, without losing accuracy, the large-scale tensor TRD is much faster and outperforms the compared randomized algorithms in image reconstruction and deep learning dataset compression experiments.
- Randomized method is a promising aspect for large-scale data processing. In our future work, we will focus on further improving the performance of decomposition and applying randomized algorithms to very sparse and incomplete tensors.

Reference

[1] G. Zhao, G. Zhou, X. Xie, L. Zhang, and A. Cichocki, “Tensor ring decomposition,” arXiv preprint arXiv:1606.05036, 2016.
[2] P. Li, T. T. Hastie, and K.W. Church, “Very sparse random projections,” Proceedings of the 12th ACM SIGKDD, 2006.
[3] E. Bingham and H. Mannila, “Random projection in dimensionality reduction: applications to image and text data,” Proceedings of the seventh ACM SIGKDD, 2001.
[4] N.B. Erichson, K. Manohat, S. L. Brunton, and J. N. Kutz, “Randomized CP tensor decomposition,” arXiv preprint arXiv:1703.09074, 2017.
[5] G. Zhou, A. Cichocki, and S. Xie, “Decomposition of big tensors with low multilinear rank,” arXiv preprint arXiv:1412.1885, 2014.