The CSLDV Technique for Vibration Measurement of a Plate Structure with Arbitrary Rectangular Holes

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Abstract. The Continuous Scanning Laser Doppler Vibrometry (CSLDV) developed on the base of the galvanometer scanner system has made it possible to quickly obtain the full-field vibration responses within an intact structure without any hole. In order to make CSLDV also suitable for testing structures with arbitrary rectangular holes, a novel method of area-partitioned continuous scanning test and mode reconstruction is further proposed in this paper. In the first step, the surface of a flat plate structure with arbitrary rectangular holes is divided into several rectangular sub regions by an appropriate area-partitioned method, and the operational deflection shape (ODS) of each sub region is obtained by a continuous scanning at constant speed. Then, considering that the ODSs of all sub regions are spliced and the relative phases among them are different, the operational deflection shape of the whole structure is reconstructed based on linear function and other process. Finally, a plate structure with two arbitrary rectangular holes was taken as an example for modal test using the proposed method. At the same time, a validated experiment was performed using the SLDV measurement. The results show the mode shapes derived from the extended CSLDV are in agreement with those from SLDV and the Modal Assurance Criterion (MAC) between the two are all greater than 0.95. This method has great potential in practical engineering applications with the advantages of high efficiency and the full-field measurement.

1. Introduction
Scanning Laser Doppler Vibrometry (SLDV) is now widely used in engineering due to its advantages such as non-contact measurement. However, the resolution of the measured mode shape is relatively poor compared with the full field measurement and sometimes difficult to meet the requirement of the further work such as model correction and validation in structural design. With the number of measurement points increasing, the time consuming and the measurement cost of test are also increased. The continuous scanning laser Doppler vibration (CSLDV) measurement technology has been well developed\cite{1,2}. It has been well applied to continuously scanning of the surface of thin-walled structures with intact and regular shapes, such as plates, beams, disks, and blades \cite{3-5} etc. Chiariotti et al. expanded the application of the CSLDV test technology in the vibration laboratory \cite{6} and proposed an automatic data processing method suitable for time-domain signals obtained from the CSLDV test. They also applied this technology to structural damage identification, including structures of multiple materials and structures of multiple types of damage \cite{7}. Chen\cite{8} and Song\cite{9} developed the constant-speed scanning and sinusoidal-speed continuous scanning methods to measure the beam and plate structures.

In practical engineering, with the increasing complexity of mechanical structures, many porous structures, such as aero-engine, pump body and valves, have arbitrary holes and require a suitable
method for vibration test. When CSLDV method is directly used to the whole surface of these structures, there may be in difficulty and cause some problems. If the whole surface is treated as the unbroken and tested by skipping the holes, the follow-up analysis may be greatly affected due to the mutation of scanning distance and it is difficult to ensure the accuracy of the results. If the holes on the structure are skipped during scanning, the scanning path will be interrupted and the laser will jump. Such jump will unpredictably impact on the test data and may result in the decrease of the credibility of the experimental results, or even the failure of obtaining results. Zhang and others\cite{10} have implemented the coordinate transformation method to improve the vibration test of engine external bending pipeline, but it was only suitable for curve scanning, not for plane scanning. Li\cite{11} realized the continues scanning measurement of the three-dimensional ODS of a large torsion blade based on the mapping relationship between the normalized scanning and the actual scanning areas. However, the fitting relationship of bivariate quadratic polynomial becomes more difficult to build if the scanning area looks complex.

In this paper, a novel technique for vibrational ODS measurement of a plate structure with arbitrary rectangular holes using the continuous scanning laser Doppler vibration is proposed. First of all, the structure is divided into several regular rectangular sub regions based on two requirements and an optimization standard. Then continuous scanning test and analysis were carried out on the surfaces of all areas. Finally, according to the difference of phase, amplitude and datum plane of ODS between subregions, a method of reconstruction is proposed. It can be seen as repeatedly splicing an area to an existing part. Firstly, the phase of the ODS to be spliced is ensured to be the same as the original ODS. Then a number of points are selected in the coincident region, and their amplitudes in the two regions are obtained by interpolation or polynomial fitting. Finally, according to the minimum difference of the amplitude of the corresponding point as the selection criterion, the coefficients K and B of the linear function are obtained. They are used to handle the corresponding sub regions, so that the high resolution mode shape of this type of structure can be obtained. A flat plate structure with two interfering rectangular holes is taken as an example to carry out the CSLDV test of this method. Meanwhile, test based on the SLDV is also undertaken to validate the proposed approach.

2. CSLDV for structures with arbitrary rectangular holes
Considering the structural surface with arbitrary rectangular holes, the laser scanning path cannot jump over the holes during the test process. To avoid the holes in laser scanning, the scanning path on the structural surface is initially partitioned into several regular rectangular sub-areas with the holes split. The laser continuous scanning test with a constant speed is then performed on all sub-regions and the measured time domain signals are further processed using delay optimization to obtain the ODS of all sub-regions. Finally, the ODS of an intact structure with arbitrary rectangular holes can be obtained by reconstructing the ODS of all sub-regions. The flowchart of the procedure is shown in Figure 1.
The reconstruction can be seen as a repetitive process of splicing the ODS of a sub region onto the exciting part. First of all, their phases are ensured to be the same, that is, their vibrational directions are the same. Then, several points are selected at the overlap between the spliced area and the existing region, and the corresponding amplitudes in the two regions can be obtained by interpolation or polynomial fitting respectively. After these amplitudes are substituted into the linear function, the coefficients of the linear function are determined according to the minimum sum of the difference of the amplitudes of the corresponding points.

2.1. Subarea partition for scanning path
During the process of laser testing, the laser optical path includes two stages. First of all, the laser is emitted through the spectroscope and reaches the surface of the test piece. In this process, the light source is relatively stationary, the observation point moves relatively, and the frequency of the laser after reflection \( (f'_1) \) is as follows:

\[
f'_1 = \frac{c + u}{c} f_0
\]

(1)

Among them, \( u \) represents the speed of vibration, \( c \) represents the speed of light, and \( f_0 \) represents the original frequency.
After the diffuse reflection, reach the spectroscope again. In this process, the light source moves relatively, the observation point is relatively static, and the observed laser frequency \( f_2 \) is as follows:

\[
f_2 = \frac{c}{c - u} f_1
\]

The difference in the frequency of the laser received by the photodetector is:

\[
\Delta f = f_2 - f_0 = \frac{2u \cos \theta}{c} f = \frac{2u}{\lambda} \cos \theta
\]

Therefore, as long as the difference of the frequency measured by the photoelectric sensor is obtained, the vibration velocity of the object surface in the direction parallel to the laser can be obtained. The laser beam will not reflect back when the scanning is passing through the holes, so the scanning path cannot go through the hole. If the scanning path jumps over whenever it encounters a hole, the data near the hole will fail due to problems such as the inertia of the scanning mirror.

According to the characteristics of structures with arbitrary rectangular holes, the partition of sub areas for scanning laser beam path to split the holes should meet the following requirements:

- The scanning path needs to cover the whole surface of a structure to ensure that the ODS of the whole structure can be reconstructed;
- Each of sub regions should have a certain overlap with others, so as to facilitate the subsequent mode reconstruction.

A plate structure with two interfering rectangular holes at any position is taken as an example, i.e., in Figure 3. The interference means that a hole will intersect with another hole after extending along one side. It is made of 304 stainless steel with a length of 200 mm, a width of 80 mm and a thickness of 3 mm. According to the above requirements and the minimum number of sub regions, the structure is divided into seven parts, as shown in Figure 3.

![Figure 3 Subarea division of structural surface](image)

Taking the center position of the structure as the coordinate origin, the rectangular coordinate system is established, taking the transverse direction as the X axis and the longitudinal direction as the Y axis. The location and size of each region are described by its two endpoints, the lower left corner and the upper right corner. The coordinates of the endpoints of each region are shown in Table 1.

| Area number | Coordinates of the lower left point | Coordinates of the upper right endpoint |
|-------------|------------------------------------|----------------------------------------|
| 1           | (-30, -100)                        | (30, 50)                               |
| 2           | (-30, -100)                        | (0, 50)                                |
| 3           | (20, -100)                         | (30, 0)                                |
| 4           | (-30, 0)                           | (30, 50)                               |
| 5           | (-30, 0)                           | (-20, 75)                              |
| 6           | (10, 0)                            | (30, 100)                              |
| 7           | (-30, 75)                          | (30, 100)                              |

2.2. CSLDV test for all partitioned sub-areas

When the CSLDV test is carried out, the path control module generates the corresponding voltage signal according to the designed path, which is provided to the scanning system, as shown in Figure 4. At the same time, the time domain data of path, vibration and excitation are collected for subsequent processing.
When the measured structure is excited by sinusoidal excitation with frequency of \( \omega \), the velocity response of the structure parallel to the laser beam direction (i.e. direction z in Figure 4) at any measuring point is as follows:

\[
V_z(x, y, t) = V_R(x, y, t) \cos(\omega t) + V_I(x, y, t) \sin(\omega t)
\]  

(4)

Among them, \( V_R \) is the real part vibration component and \( V_I \) is the imaginary part vibration component, both of which are related to the excitation level.

For the regular rectangular sub area, the method of constant speed continuous scanning test is adopted. The speed of laser point moving in X and Y directions is constant, which is recorded as \( v_x \) and \( v_y \) respectively. Therefore:

\[
\begin{cases}
    x = v_x t \\
    y = v_y t
\end{cases}
\]  

(5)

After the constant speed linear motion in two directions is synthesized, it is still a constant speed motion. The location of the laser spot is only related to time. The output of the laser vibrometer is a modulated signal. The envelope of the signal is the mode along the scanning path. In order to process the velocity signal to get the ODS, the velocity signal is multiplied by the simple harmonic signal in the same direction and orthogonal direction to the excitation frequency:

\[
\begin{cases}
    v_x \cos(\omega t) = V_R(t) \cos^2(\omega t) + V_I(t) \sin(\omega t) \cos(\omega t) = \frac{V_R(t)}{2} \cos(2\omega t) + \frac{V_I(t)}{2} \sin(2\omega t) \\
    v_x \sin(\omega t) = V_R(t) \cos(\omega t) \sin(\omega t) + V_I(t) \sin^2(\omega t) = \frac{V_R(t)}{2} \sin(2\omega t) + \frac{V_I(t)}{2} \cos(2\omega t)
\end{cases}
\]  

(6)

The vibration component is low frequency component, and the term containing \( \omega \) is a high frequency component. If the threshold value of low-pass filter is set reasonably, the high-frequency term can be filtered out, so that only real part vibration component \( V_R \) or imaginary part vibration component \( V_I \) is included in the result.

2.3. ODS reconstruction of the whole structure from the measurement of subareas

When the ODS of the whole structure is to be reconstructed from the ODSs of all subregions, three parameters such as phases, amplitudes and datum are required to be carefully considered. A method is proposed and the specific process is as follows:

(1) First of all, the nodal lines of the desired vibration modes are obtained by simulation calculation or low-resolution SLDV test. If the common area has nodal lines, take a point on each side of it, as shown in Figure 5. The red line in the picture is the nodal line. An and An and B 'are two sets of corresponding points, and they will coincide after splicing.
Figure 5 Schematic diagram of point selection for judging whether the nodal line vibrates reversely or not

The amplitude of each pitch line is taken as reference. If the amplitude at A is bigger than that at the nodal line, it is necessary to make the amplitude at A’ to be bigger than it. B and B’ are treated in the same way. If the pitch line is close to the boundary of the coincident area, only a group of points far away from the pitch line are needed.

(2) According to the size of the overlapping area and the accuracy of the results, a certain number of points are selected. The coincident area is rectangular, so generally 5 points are selected, and their position relationship is shown in Figure 6. If high precision is required, more points can be selected.

Figure 6 Schematic diagram of point selection in coincidence area

(3) The resolution of the result of continuous scanning is very high, and a large number of points can be obtained in the whole region, so the corresponding amplitude of the selected points can be obtained by interpolation in both sub-regions. The coordinates (x,y) of one point correspond to an amplitude S(x, y), so we can get the following results:

\[ S(x, y) = Z_1(1-u)(1-v) + Z_2u(1-v) + Z_3uv + Z_4(1-u)v \]  

(7)

Among them, \( Z_1=S(x_1, y_1) \), \( Z_2=S(x_2, y_2) \), \( Z_3=S(x_3, y_3) \), \( Z_4=S(x_4, y_4) \) are the amplitudes of the four adjacent points with the point (x, y) as the midpoint, and u and v are the basis functions:

\[ u = \frac{x - x_2}{x_2 - x_1} \quad \text{and} \quad v = \frac{y - y_1}{y_2 - y_1}. \]

Mark the coordinates of the selected points as \( (x_i, y_i) \) in turn \( (i=1,2,3...n) \), and n is the total number of selected points. By substituting them into equation (7), the amplitudes of all selected points can be obtained as \( S_i \), and the amplitudes of the selected points in another area can be marked as \( S_i' \).

The coefficients K and B in the linear equation \( y = kx + b \) can be determined by using the amplitudes of corresponding points. The error of each group is:

\[ \Delta S_i = S_i - (k \times S_i' + b) \]  

(8)

Based on the minimum sum of errors of all points \( \sum_{i=1}^{n} \Delta S_i \), the best coefficient is obtained.

(4) The coefficient obtained in the previous step is used for correction. The ODS of the whole structure can be reconstructed by drawing the modified sub-regions together.

3. Experimental verification of the plate structure with multiple arbitrary rectangular holes

3.1. Comparative test with SLDV
The plate structure used in this experiment is fixed on the bracket by four pairs of bolts and nuts and a splint to simulate the fixed state of one end. The experimental device is shown in Figure 7.

![Experimental setup for a flat plate with two rectangular holes](image)

**Figure 7** Experimental setup for a flat plate with two rectangular holes

In SLDV test, acoustic excitation is used, $7 \times 21 - 4 - 3 = 140$ test points are arranged on the flat plate, and the analysis frequency is 1500Hz. The natural frequencies and modes of the first five modes of the plate are measured by using the single input multiple output modal analysis method, as shown in Table 2 and

| Order | Natural frequencies /Hz |
|-------|--------------------------|
| 1     | 52.73                    |
| 2     | 313.28                   |
| 3     | 355.47                   |
| 4     | 832.81                   |
| 5     | 1122.66                  |

**Table 2** Natural frequencies of plates with multiple rectangular holes in 1500Hz

![Mode results of SLDV test](image)

**Figure 8** Mode results of SLDV test

3.2. *The approached CSLDV measurement for the plate*

Keeping the test device unchanged and the test environment unchanged, the natural frequencies of each order measured by SLDV are used as the excitation frequencies respectively. The single frequency excitation of the flat plate is also realized through the horn. According to the length and width of the regular rectangular area, the scanning frequency of the short side direction is 8Hz, and that of the long side direction is 0.125hz. The scan path for area 2 is taken and shown in Figure 10. Under the excitation frequency at 1122.66Hz, the collected time domain velocity signal of area 3 is shown in Figure 10.
After collecting the velocity signal in time domain, the vibration modes of all sub regions of each order can be obtained, and the ODS at the third frequency is shown in Figure 11.

Figure 11 The ODSs of all sub region at third natural frequency

In the splicing reconstruction, the third natural frequency is the first torsional vibration mode. Take the process of splicing area 3 to the existing area as an example to explain how to select the points and determine the coefficient. After dealing with the phase angle, the part of the existing area is shown in Figure 12 (a), and if it is drawn together directly, it is shown in Figure 12 (b).

(a) The existing area       (b) Unprocessed splicing       (c) Processed splicing

Figure 12 Schematic diagram of the splicing of area 3

The coincident region is from 20 to 30 in the x direction and from -100 to -50 in the y direction. Referring to Figure 6, select five points and calculate their amplitudes in the two regions respectively. The results are shown in Table 3. The best coefficients are $k=0.6929$, $b=-0.0572$. The coefficients are used to process area 3 and the result is painted in the original picture, as shown in Figure 12 (c).
### Table 3 Coordinates and amplitude of the selected points

| Serial number | Coordinate | Amplitude in the original area | Amplitude in area 3 |
|---------------|------------|-------------------------------|--------------------|
| 1             | (21, -51)  | -3.7372                       | -6.0875            |
| 2             | (29, -51)  | -4.1869                       | -5.0303            |
| 3             | (25, -75)  | -1.8469                       | -2.5349            |
| 4             | (21, -99)  | -0.2323                       | -0.3776            |
| 5             | (29, -99)  | -0.2184                       | -0.3089            |

Whether the phase angle of each region needs to be changed, and the coefficients $k$ and $b$ are shown in Table 4. The vibration mode results after splicing are shown in Figure 13. (The above part of the view angle in the figure is cantilever end and the lower part is fixed support end.) The process of mode reconstruction is basically the same in other natural frequencies. Their results are shown in Figure 14.

### Table 4 ODS splicing parameters at the third natural frequency

| Serial number to be spliced | whether need to change the phase angle | $k$  | $b$  |
|-----------------------------|---------------------------------------|------|------|
| 1                           | No                                    | 1    | 0    |
| 2                           | No                                    | 1.185| 0.2644|
| 3                           | Yes                                   | 0.6929| 0.572|
| 4                           | Yes                                   | 1.1378| 0.9280|
| 5                           | Yes                                   | 0.9922| 0.9280|
| 6                           | Yes                                   | 0.9388| 0.1486|
| 7                           | Yes                                   | 0.7348| 6.9528|

Figure 13 Intermediate results of ODS splicing at the third natural frequency

Figure 14 CSLDV test results of multiple rectangular hole structures

3.3. **Comparison of results**

The results of CSLDV are compared with that of SLDV. The continuous scan test method proposed in this paper has the following differences:

1. The SLDV test time increases linearly with the number of test points. There are 140 test points, and each point needs to be measured three times to take the average value. Each measurement takes 6.4s, and it takes a total of 45 minutes. The continuous scanning method needs at most 10 cycles for each region, 8 seconds for each cycle, and 9 minutes for seven sub regions, which significantly shortens the test time.

2. Due to the limited number of test points, the SLDV test results can not fit the actual boundary closely by using a small number of points around the hole. This will make it difficult to obtain the vibration near the hole. If the test points are concentrated near the missing hole, the test time will increase linearly. The continuous scanning method has high resolution, so the results can be very smooth in the similar boundary, which is more practical.

In order to compare the mode shape results of the two sets of experiments, obtain the correlation between them, and determine the modal pair between the modes, the correlation analysis was carried out. And use Modal Assurance Criterion (MAC) as the criterion:

$$MAC(i, j) = \frac{\left| \{\psi_X\}^T \{\psi_A\} \right|^2}{\left| \{\psi_X\}^T \{\psi_X\} \right| \left| \{\psi_A\}^T \{\psi_A\} \right|_j}$$  \hspace{1cm} (9)
Among them, $\psi_X$ represents the modal data of the test model, $\psi_A$ represents the modal data of the finite element calculation, and $T$ represents the transposition.

The value of MAC is between 0 and 1. When the MAC value of the two models is closer to 1, it means that the modal correlation between the two models is very high, and the two models have consistent modal correlation, which may be a pair of modals. The correlation analysis of the two groups of experimental results is shown in Table 5 and Figure 15, and the MAC value is basically above 0.95, which proves that the continuous scan test in this paper has high accuracy.

Table 5 The table of MAC between SLDV and CSVDV

| Order | 1  | 2  | 3  | 4  | 5  |
|-------|----|----|----|----|----|
| MAC   | 0.99| 0.96| 0.97| 0.93| 0.96|

Figure 15 The picture of MAC between SLDV and CSVDV

4. Conclusion
In this paper, a novel method of area-partitioned continuous scanning test and mode reconstruction suitable for testing objects with arbitrary rectangular holes on the surface is put forward to obtain full-field vibration measurement. This method is helpful to extend the CSVDV to the structure with other types of holes. First of all, according to certain rules, the surface of the structure is divided into several rectangular regions, and each of them is scanned continuously to get their ODS at a uniform speed. Then, based on the difference of phase, amplitude and datum between the ODSs, a method of mode reconstruction is applied to get the ODS of the whole structure. An example of a plate with two rectangular holes was taken for modal tests using the proposed method. At the same time, a validated experiment was performed in SLDV. Results show the mode shapes derived from the proposed method are highly matched with that from SLDV and the MAC values between the two are almost close to unit. This approach shows great potential to industrial engineering applications.

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