Analysis of Kinematic Motion Deviations of Machining Centers
Based on Geometric Tolerances

Naoki SATONAKA1, Nobuhiro SUGIMURA1, Yoshitaka TANIMIZU1 and Koji IWAMURA1
1 Graduate School of Engineering, Osaka Prefecture University, Japan, sugimura@me.osakafu-u.ac.jp

Abstract:
Machine tools are recognized as key components of manufacturing systems, and product quality and cost mainly depend on performances of the machine tools. Much progress has been made in the machine tool technologies, aimed at improving the performances of the machine tools from various viewpoints, such as accuracy, reliability, productivity, and flexibility. The machining accuracy is one of the most important characteristics of the machine tools. From the viewpoints of the design and the manufacturing of the machine tools and their components, one of the important issues is to clarify the relationships between the kinematic motion deviations of the machine tools and the geometric tolerances of the components, such as the guide ways and the bearings. The objective of the present research is to establish mathematical models representing the kinematic motion deviations of the machine tools, on the basis of the geometric tolerances of the components, and to apply the models to theoretical analysis of the kinematic motion deviations of the machine tools.

Keywords: Machine Tools, Shape Generation Motions, Geometric Tolerances, Kinematic Motion Deviations

1. Introduction

One of the most important revolutions in the later part of the last century is introduction of NC (Numerical Control) machine tools, which are able to carry out various complicated machining processes without human interaction. Various types of NC machine tools are now being designed and applied to machining processes of complicated machine products. The machining accuracy is one of the most important characteristics of the NC machine tools for generating the products with the high accuracy and the complicated geometries.

Some researches have been carried out to analyze the machining accuracy of the machine tools based on the deviations of the shape generation motions between the tools and the workpieces [1]-[4].

However, the motion deviations of the machine tools are deeply influenced by the geometric deviations of the components, such as guide ways and bearings. Therefore, it is now required to clarify the relationships between the kinematic motion deviations of the machine tools and the geometric deviations of the components, from the viewpoints of the design and the manufacturing of the machine tools and their components.

As regards the geometric deviations of the machine components, many researches have been carried out to deal with the dimensional tolerances and the geometric tolerances, aimed at realizing systematic analysis and design methodologies for the three dimensional machine products [5]-[8]. However, the relationships between the kinematic motion deviations and the geometric deviations of the components have not yet been clarified.

The objectives of the present research are to establish mathematical models representing the kinematic motion deviations of the machine tools, on the basis of the geometric tolerances of the components, and to apply the models to theoretical analysis of the kinematic motion deviations of the machine tools.

Main issues discussed in the present paper are summarized as follows.
(1) Modeling of geometric deviations of guide-ways.
(2) Modeling of geometric deviations of linear tables.
(3) Modeling and analysis of kinematic motion deviations of 3-axis machining centers.

2. Geometric Deviations of Components and Kinematic Motion Deviations of Machine Tools

The shape generation processes of the machine tools are carried out by the shape generation motions between the tools and the workpieces, and the shape generation motions are realized by a set of the relative motions between the constituting components. Therefore, the kinematic deviations of the shape generation motions are influenced by the geometric deviations of the components and the motion deviations between these components.

Figure 1 summarizes the relationships between the kinematic deviations of the shape generation motions and
the deviations of the components. In the figure, WO, CT and unit-i represent the workpieces to be machined, the tools, and the constituent components, respectively.

The kinematic deviations of the shape generation motions of the machine tools are defined as the kinematic motion deviations between the tools CT against the workpieces WO, and the deviations are influenced by various types of deviations, such as the position deviations in setting up the tools and the workpieces, the relative position deviations between the units, and the geometric deviations of the features. The features mean here the geometric elements, such as plane faces of guide-ways and cylindrical faces of bearings, which connect and constrain the relative motions between the pairs of the units.

The objective of the present research is to represent and to analyze the kinematic motion deviations of the tools against the workpieces, based on the geometric deviations of the features of the components.

3. Geometric Deviations of Plane Features [9]-[10]

The geometric tolerances of the features specify the allowable areas named “tolerance zones,” which constrain the position and orientation deviations of the associated features against the nominal features, as shown in Fig. 2. The associated features and the nominal features mean the features of the manufactured products and the ideal features defined in the design phase, respectively. The geometric deviations of the associated features from the nominal features are represented by sets of parameters named “deviation parameters [9].” For example, one position parameter w and two rotational parameters α and β are required to represent the geometric deviations of the associated plane features against the nominal plane features, for the case where the tolerance zone is given by the area between a pair of parallel planes.

In the research, the followings are assumed for the ease of the modeling and the analysis of the geometric deviations.

(1) The deviation parameters δ representing the position and orientation deviations of the associated features follow the normal distribution N(μ, σ), and μ = 0.

Where, μ and σ are the mean values and the standard deviations, respectively.

(2) The manufacturing processes of the components are well controlled, and the proportion of the non-conforming components is as small as a value Pd called “percent defective.” Here, the non-conforming components mean the components, in which the tolerated features exceed the tolerance zones.

(3) Equation (1) represents the relationships between the standard deviations σi of the deviation parameters of

\[ \sigma_i = \frac{\delta_{\text{max}}}{Cpd} \]

where, \( \delta_{\text{max}} \): maximum values of the deviation parameters δ, if the other deviation parameters \( \delta_j = 0 \), \( i \neq j \).

\( Cpd \): a constant representing the ratio of the maximum values \( \delta_{\text{max}} \) and the standard deviations \( \sigma_i \).

Let us consider a case shown in Fig. 2, as an example. The maximum values \( \delta_{\text{max}} \) are given as follows.

\[ \delta_{1} = w, \delta_{2} = \alpha, \delta_{3} = \beta \]

\[ \delta_{\text{max}} = t/2; \delta_{\text{max}} = t/L_1; \delta_{\text{max}} = t/L_2 \]

where, \( L_1, L_2 \): Length and width of the plane feature.

\( t \): Tolerance values, e.g. the distance between two planes representing the tolerance zones.
The following equation gives the conditions that the plane features are included within the tolerance zone between a pair of planes.

\[-t/2 < \delta_1 + L_1 \delta_2 / 2 + L_2 \delta_3 / 2 < t/2\]  

\((3)\)

The probability that the toleranced features are included within the tolerance zones is given by the following equation.

\[1 - P_d = \left( \frac{2}{\sqrt{2\pi}} \right)^3 \int_0^\infty \int_0^\infty \int_0^\infty \left( \prod_{i=1}^3 \exp \left( -\frac{x_i^2}{2} \right) \right) dx_1 dx_2 dx_3\]  

\((4)\)

where,

\[x_1 = 2C_{P_2} \delta_1 / t, \quad x_2 = L_1 C_{P_2} \delta_2 / t, \quad x_3 = L_2 C_{P_2} \delta_3 / t\]

If the percent defective \(P_d\) is less than 0.27%, the constant \(C_{pd}\) can be estimated as \(\sqrt{C_{pd}} = 5.83\). This value is applied in the following analysis.

4. Geometric Deviation of Linear Tables

Figure 3(a) shows a typical example of the linear tables utilized for the machine tools. A base supports and guides a table by four plane faces called guide-ways, which are indicated by \(a, b, c,\) and \(d\). In the case, the relative position and orientation of the table against the base are given by the following equation, if the guide-ways coincide with the nominal features.

\[x_i = A_a A_a'^{-1} x_{i+1} = A_b A_b'^{-1} x_{i+1} = \ldots = A_d A_d'^{-1} x_{i+1}\]  

\((5)\)

where,

\(x_i: \) Position vector of a point \(P\) in the base coordinate system \(C_i,\)

\(x_{i+1}: \) Position vector of \(P\) in the table coordinate system \(C_{i+1},\)

\(A_j (j = a, \ldots, d): 4 \times 4\) homogeneous transformation matrices representing the positions and orientations of the guide-ways \(j\) in the base coordinate system \(C_i,\)

\(A_j' (j = a, \ldots, d): 4 \times 4\) homogeneous transformation matrices representing the positions and orientations of the guide-ways \(j\) in the table coordinate system \(C_{i+1},\)

However, the relative positions of the table against the base are influenced by the position and orientation deviations of the guide-ways, if the guide-ways do not coincide with the nominal features. The positions of the table against the base are represented by the following equation corresponding to the individual guide-ways.

\[x_i = A_j D_j D_j'^{-1} A_j'^{-1} x_{i+1} \quad (j = a, \ldots, d)\]  

\((6)\)

\(D_j: 4 \times 4\) homogeneous transformation matrices representing the position and orientation deviations of the guide-ways \(j.\)

Equation (6) means that various values of the relative position of the table against the base are obtained for the individual guide-ways, as shown in Fig. 3(b). A systematic method is therefore required to evaluate one value of the relative position for the cases where the guide-ways have the geometric deviations. It is because that both the base and the table are rigid bodies and the relative position between the rigid bodies should be represented by one value. The following two methods are proposed here to solve the problem.

(1) Priority among connections

Let us consider the case shown in Fig. 4 (a) as an example. The unit-\(i\) and the unit-\(i+1\) is connected by two sets of the faces indicated by \(A\) and \(B,\) which are perpendicular to each other. The deviations of the relative positions of the unit-\(i+1\) to the unit-\(i,\) which are specified by \(D_j D_j'^{-1}\) in Eq. (6), are given by the following equations for the individual sets of the connecting faces.

\[D_j D_j'^{-1} = \begin{pmatrix} 1 & 0 & \delta_{iA} & 0 \\ 0 & 1 & -\delta_{iA} & 0 \\ -\delta_{iA} & \delta_{iA} & 1 & \delta_{ii} \\ 0 & 0 & 0 & 1 \end{pmatrix} \]  

\((7)\)

\(D_j D_j'^{-1} = \begin{pmatrix} 1 & 0 & \delta_{iB} & 0 \\ 0 & 1 & -\delta_{iB} & 0 \\ -\delta_{iB} & \delta_{iB} & 1 & \delta_{ii} \\ 0 & 0 & 0 & 1 \end{pmatrix} \)
In this case, two values are obtained for defining the relative positions of the unit-i and the unit-i+1. Priority may be set to the connecting faces, based on the areas of the faces and the direction of the gravity. Higher priority given to the face set A in the case, if the direction of the gravity is –Z. Therefore, the deviations of the relative positions of the unit-i+1 to the unit-i are given as follows.

\[
D_{AB}D_{AB}^{-1} = \begin{pmatrix}
1 & -\delta_{AB} & \delta_{B\|} & \delta_{B\perp} \\
\delta_{AB} & 1 & -\delta_{A\|} & 0 \\
-\delta_{B\|} & \delta_{A\|} & 1 & \delta_{A\perp} \\
0 & 0 & 0 & 1
\end{pmatrix}
\] (8)

(if \(A > B\))

where, “\(A > B\)” specifies that the face set A has higher priority to the face set B.

Non-priority among connections
For the case shown in Fig. 4 (b), there are two sets of the connecting faces between the unit-i and the unit-i+1. However, there is not any priority between the face sets A and B. In this case, the deviations for the both connecting face sets are averaged, and the deviations of the relative position of the unit-i+1 to the unit-i are given as follows.

\[
D_{AB}D_{AB}^{-1} = \begin{pmatrix}
1 & 0 & (\delta_{B\|}+\delta_{B\perp})/2 & 0 \\
0 & 1 & -\delta_{A\|}/2 & 0 \\
-(\delta_{B\|}+\delta_{B\perp})/2 & (\delta_{A\|}+\delta_{B\perp})/2 & 1 & (\delta_{B\|}+\delta_{B\perp})/2 \\
0 & 0 & 0 & 1
\end{pmatrix}
\] (9)

where, “\(A = B\)” means that the face sets A and B do not have any priority.

5. Analysis of Kinematic Motion Deviations of Linear Tables and Machining Centers
5.1 Modeling of Kinematic Motion Deviations
The geometric deviations of the linear tables are estimated based on the geometric deviations of the guide-ways of a, b, c, and d in Fig. 3 (a). Equation (10) is obtained to describe the motion deviations of the linear tables, under the assumption that the priority among the guide-ways is “\((a = c) > (b = d)\)” In Eq. (10), \(y\) is the relative linear motions of the tables against the bases.

A machining center illustrated in Fig. 5 (a) is considered in the research, and the kinematic motion deviations of the tools against the workpieces are analyzed by applying the models proposed in the previous sections. The analysis process is summarized in the followings.

STEP 1: Modeling of linear tables
The machining center shown in Fig. 5 (a) has three linear tables to carry out the liner motions in X, Y and Z-directions, and all the linear tables have same structure shown in Fig. 3 (a). The kinematic motions including the deviations are given by Eq. (10).

\[
A_2(y) = \begin{pmatrix}
1 & -\delta_{y\|} + \delta_{y\perp} & \delta_{y\|} + \delta_{y\perp} & \frac{1}{2} \left(\delta_{xy} - \frac{1}{2} L y \left(\delta_{x\|} + \delta_{x\perp} + \delta_{a\|}' + \delta_{a\perp}'\right)\right) \\
\frac{1}{2} \left(\delta_{x\|} + \delta_{x\perp} + \delta_{a\|}' + \delta_{a\perp}'\right) & 2 & -\frac{1}{2} \left(\delta_{y\|} + \delta_{y\perp} + \delta_{y\perp} + \delta_{y\perp}'\right) & \frac{1}{2} \left(\delta_{xy} + \frac{1}{2} L y \left(\delta_{x\|} + \delta_{x\perp} + \delta_{a\|}' + \delta_{a\perp}'\right)\right) \\
-\delta_{y\|} + \delta_{y\perp} & 1 & \frac{1}{2} \left(\delta_{y\|} + \delta_{y\perp} + \delta_{y\perp} + \delta_{y\perp}'\right) & \frac{1}{2} \left(\delta_{xy} - \frac{1}{2} L y \left(\delta_{x\|} + \delta_{x\perp} + \delta_{a\|}' + \delta_{a\perp}'\right)\right) \\
\frac{1}{2} \left(\delta_{y\|} + \delta_{y\perp} + \delta_{y\perp} + \delta_{y\perp}'\right) & 0 & 0 & 1
\end{pmatrix}
\] (10)
STEP 2: Modeling of whole machining centers
The model of the machining centers is constructed as follows, by combining the linear table models.

\[ X_w = A_3(-z_0)A_3(x)A_3(-z_1)A_2(y)A_3(z_2)A_3(z_3)x_c \]  

(11)

where,
- \( A_3(-z_0), A_3(-z_1), A_3(z_2), A_3(z_3) \): Initial positions of linear tables shown in Fig. 5(b).
- \( A_2(y) \): Linear motions by Y-axis linear tables represented by Eq. (10), which include the kinematic motion deviations.
- \( A_3(x), A_3(z) \): Linear motions by X-axis linear tables and Z-axis linear tables. The transformation matrices are obtained by modifying the matrix given in Eq. (10).

STEP 3: Estimation of kinematic motion deviations
The kinematic motion deviations of the machining centers are estimated by applying the following procedures.

1. The standard deviations of the deviation parameters of the individual guide-ways are estimated from the sizes of the tolerance zones of the guide-ways.
2. The standard deviations of the kinematic motions of the individual linear tables are estimated based on the standard deviations of the guide-ways, by applying Eq. (10).
3. The standard deviations of the positions and orientations of the tools against the workpieces are estimated, by applying Eq. (11).

5.2 Analysis Results
The kinematic motion deviations of the cutting tools against the workpieces were analyzed for the machining center shown in Fig. 5. In particular, the standard deviations of the kinematic motions were estimated under the following conditions.

1. Table Size
   - X-axis table: \( L_{x_1} = 470[\text{mm}] \), \( L_{x_2} = 50[\text{mm}] \), \( L_{x_3} = 30[\text{mm}] \)
   - Y-axis table: \( L_{y_1} = 1,080[\text{mm}] \), \( L_{y_2} = 120[\text{mm}] \), \( L_{y_3} = 60[\text{mm}] \)
   - Z-axis table: \( L_{z_1} = 470[\text{mm}] \), \( L_{z_2} = 50[\text{mm}] \), \( L_{z_3} = 30[\text{mm}] \)
2. Initial Table Positions
   - \( z_0 = 500[\text{mm}] \), \( z_1 = 200[\text{mm}] \), \( z_2 = 1,700[\text{mm}] \), \( z_3 = 500[\text{mm}] \)
3. Travels of Tables
   - \(-510[\text{mm}] \leq x \leq 510[\text{mm}] \), \(-285[\text{mm}] \leq y \leq 285[\text{mm}] \)
   - \(-285[\text{mm}] \leq z \leq 285[\text{mm}] \)
4. Sizes of Tolerance Zones
   - \( t = 1 \times 10^{-2} [\text{mm}] \) for all the guide-ways

Table 1 shows the standard deviations of the positions of the tools against the workpieces estimated by applying the model developed here. The left column gives the table positions, and the remaining columns are the estimated standard deviations in X, Y and Z-directions, respectively. It is found that the standard deviations in Y-direction are relatively large against ones in the other directions. Therefore, sensitivity analysis was carried out to find the sensitive deviation parameters which affect the deviations in Y-direction.

It was found that some of the deviation parameters of the guide-ways affect the tool positions, in particular, that the rotational deviations increase the position deviations in Y-directions. Therefore, the size of the tolerance zones of these sensitive deviation parameters were set to be 1/10 of the others, and a modified model was generated. The analysis results of the modified machining center model
are shown in Table 2. The standard deviations of the tool positions have almost the same values in all directions. This means that the effects of the sizes the tolerance zones of the toleranced features are investigated with use of the proposed model, and that the proposed method provides a method to select the suitable sizes of the tolerance zones, based on the analysis of the kinematic motion deviations.

5. Conclusions
Following remarks are concluded.

(1) A systematic method was proposed here to establish mathematical models representing the kinematic motion deviations of the machine tools, based on the geometric tolerances of the components.

(2) The geometric deviations of the guide-ways were discussed, and a method was proposed to estimate the standard deviations of the geometric deviations of the guide-ways. The geometric deviations of the linear tables were formulated, based on the priorities among the guide-ways of the linear tables.

(3) A mathematical model was developed to estimate the standard deviations of the positions and the orientations of the tools against the workpieces. The proposed method provides us with a systematic method to analyze and to estimate the kinematic motion deviations of the machine tools in the tolerance design phase.

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